Mass hierarchy and flavour mixing from discrete symmetries

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Summary. — We consider a class of discrete flavour symmetries for leptons based on the group $S_3$ and $A_4$ with an hybrid breaking pattern. The aim is to construct models in which the same flavon fields producing the mixing pattern are also responsible for the mass hierarchy.

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1. – Introduction

Nowadays continuous improvement on the knowledge of neutrino oscillation parameters makes desirable a neutrino model building going beyond the mere fitting procedure. The present data [1], at $1\sigma$:

\begin{equation}
\theta_{12} = (34.5 \pm 1.4)^\circ, \quad \theta_{23} = (42.3^{+5.1}_{-3.3})^\circ, \quad \theta_{13} = (0.0^{+7.9}_{-0.0})^\circ,
\end{equation}

are fully compatible with the so-called Tri-Bimaximal (TB) mixing pattern which corresponds to a maximal $\theta_{23}$, an zero $\theta_{13}$ and a “magic” value of solar angle: $\sin^2 \theta_{12} = 1/3$.

It is well known that the lepton mixing angles can be understood by a mechanism of vacuum misalignment in flavour space occurring in theories with non-abelian flavour symmetries [2]. Also the charged lepton mass hierarchy can be achieved via spontaneous breaking of the flavour symmetry. However, in most cases, a separate component of the flavour group is exploited to this purpose. Quite frequently the flavour group is of the type $D \times U(1)_{FN}$ where $D$ is a discrete component that controls the mixing angles and $U(1)_{FN}$ is an abelian continuous symmetry that describes the mass hierarchy.

In the present talk, we will consider a class of flavour symmetries in which the same flavon fields producing the mixing pattern are also responsible for the mass hierarchy employing an hybrid breaking pattern [4]. The idea is to selectively couple charged leptons and neutrinos to two different sets of flavons, $\Phi_e$ and $\Phi_\nu$, respectively. The VEV of $\Phi_\nu$ breaks $G$ down to a residual symmetry in the neutrino sector preferably containing the $\nu_\mu - \nu_\tau$ exchange symmetry as indicated by the oscillation data. While, the VEV of $\Phi_e$ would break $G$ down to a different subgroup, maximally breaking the previous $\nu_\mu - \nu_\tau$ symmetry, guaranteeing a hierarchical and quasi diagonal matrix $m_l$. 
2. Neutrino $\nu_\mu - \nu_\tau$ symmetry, charged lepton hierarchy and $S_3$

In this section we will focus on the approximately vanishing values of $\theta_{13}$ and $\theta_{23} - \pi/4$. Given an arbitrary choice of charged lepton mass matrix $m_l$ and effective neutrino mass matrix $m_\nu$, any change of basis in the generation space modifies the form of $m_l$ and $m_\nu$, but does not change the physics. We can exploit this freedom to render diagonal the charged lepton mass matrix $m_l$:

$$m'_l = \text{diag}(m_e, m_\mu, m_\tau), \quad m'_\nu = U_{PMNS}^\dagger \text{diag}(m_1, m_2, m_3) U_{PMNS}^T.$$  

In this basis, called flavour basis, the effective neutrino mass matrix is completely determined by the measurable quantities $m_i$ and $U_{PMNS}$. In the limit where both $\theta_{13}$ and $\theta_{23} - \pi/4$ vanish, it is easy to verify that $m'_\nu$ exhibits an exact $\nu_\mu - \nu_\tau$ exchange symmetry. However, since $\nu_\mu$ and $\nu_\tau$ are members of the electroweak doublets, a naive extension of such a symmetry to include the charged leptons $\mu$, $\tau$ might be in contrast with the large mass hierarchy $m_\mu \ll m_\tau$. This problem is completely solved in the first paper of [4] by using an hybrid symmetry pattern based on the flavour symmetry $S_3$.

$S_3$ is group of permutations of three distinct objects and is the smallest non-abelian symmetry group. The six elements of the $S_3$ group can be generated by $S$ and $T$ with the following unitary representations (Rep):

$$S = \begin{pmatrix} 1 & S = 1 & T = 1 \\ 1' & S = -1 & T = 1 \end{pmatrix}, \quad 2 \quad S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}.$$ 

$S$ in the two-dimensional Rep corresponds to an interchange symmetry of the two components of a $S_3$ doublet. The tensor products involving pseudo-singlets are given by $1' \times 1' = 1$ and $1' \times 2 = 2$. While the product of two doublets is given by $2 \times 2 = 2 + 1 + 1'$. Given two doublets $\psi = (\psi_1, \psi_2)$ and $\varphi = (\varphi_1, \varphi_2)$, it is easy to see that

$$\psi_1 \varphi_2 + \psi_2 \varphi_1 \in 1, \quad \psi_1 \varphi_2 - \psi_2 \varphi_1 \in 1', \quad \begin{pmatrix} \psi_2 \varphi_2 \\ \psi_1 \varphi_1 \end{pmatrix} \in 2.$$ 

The previous notion on the $S_3$ group is sufficient to construct a simple model for leptons. The left-handed doublets transform as $(1 + 2)$ of $S_3$ and we will call $l_e = (\nu_\mu, e)$ the invariant singlet and $D_l = (l_\mu, l_\tau)$ the $S_3$ doublet. The right-handed charged leptons $e^c, \mu^c, \tau^c$ are all in the non-trivial singlet representation $1'$. The $S_3$ flavour symmetry is spontaneously broken at a scale $\sim$ VEV $\ll \Lambda$, $\Lambda$ being the cutoff scale, by two doublets $\varphi_e, \varphi_\nu$ and a singlet $\xi$ which are all gauge singlets. The flavon fields will develop VEVs of the type

$$\langle \varphi_e \rangle \propto (1, 0), \quad \langle \varphi_\nu \rangle = (1, 1), \quad \langle \xi \rangle \neq 0.$$ 

It is possible to impose an extra abelian symmetry $Z_3$ in such a way that $\varphi_\nu$ and $\xi$ couple only to the neutrino sector and $\varphi_e$ to the charged lepton sector. Since $\langle \varphi_\nu \rangle$ is preserved by $S$, we immediately conclude that in the neutrino sector $S_3$ is broken down to a $Z_2$ subgroup. Being $D_l$ doublet of $S_3$, this residual $Z_2$ parity will exactly lead to the $\nu_\mu - \nu_\tau$ exchange symmetry. Concerning the charged lepton sector, the VEV of $\varphi_e$ breaks the parity symmetry generated by $S$ in a maximal way, since

$$\langle \varphi_e \rangle^T S \langle \varphi_e \rangle = 0.$$
The singlets $e^c$, $\mu^c$ and $\tau^c$ should couple to $(D_l \phi_e)^\dagger$, $(D_l \phi_\mu)^\dagger$ and $l_i (\phi_e \phi_e \phi_e)$ at the leading orders. From the $S_3$ multiplication rules given in [4] we see that the last combinations select respectively $e$, $\mu$ and $\tau$ after the electroweak and flavour symmetry breaking. As a consequence $m_\tau$, $m_\mu$, $m_e$ get their first non-vanishing contribution at the order $\langle \phi_e \rangle / \Lambda$, $(\langle \phi_e \rangle / \Lambda)^2$, $(\langle \phi_e \rangle / \Lambda)^3$, respectively. Then we obtain the correct charged lepton hierarchy assuming $\langle \phi_T \rangle / \Lambda \sim \lambda_e^2$, being $\lambda_e$ the Cabibbo angle.

3. – TB mixing and charged lepton hierarchy from $A_4$

In this section we extend the previous construction to describe the TB mixing pattern by an hybrid symmetry breaking of $A_4$ [3], improving some aspects of the original proposal of [3]. The generators of $A_4$, $S$ and $T$, have the following Reps:

$$\begin{align*}
1 & \quad S = 1 \quad T = 1 \\
1' & \quad S = 1 \quad T = \omega^2 \\
1'' & \quad S = 1 \quad T = \omega
\end{align*}$$

From [2], one can generally show that the most general mass matrix in the flavour basis leading to TB mixing obeys the “magic” symmetry $G_S \simeq Z_2$ generated by $S$ in addition to the $\nu_\mu - \nu_\tau$ exchange symmetry analyzed in the previous section.

We assign the lepton doublets $l_i$ ($i = e, \mu, \tau$) to the triplet $A_4$ representation and the lepton singlets $e^c, \mu^c, \tau^c \sim 1$. The symmetry breaking sector consists of the scalar fields $(\phi_T, \phi_S, \xi)$, transforming as $(3, 3, 1)$ of $A_4$. Under certain conditions, the minimization of the scalar potential naturally leads to the following VEV alignment:

$$\langle \phi_T \rangle \propto (0, 1, 0), \quad \langle \phi_S \rangle \propto (1, 1, 1), \quad \langle \xi \rangle \neq 0.$$ 

In the neutrino sector $A_4$ is broken by $(\phi_S, \xi)$ down to $G_S$. The absence of the scalar singlets $1'$ and $1''$ in the neutrino sector implies that, in addition to $G_S$, the resultant neutrino mass matrix is also automatically symmetric under the exchange of the second and third generations. Then the residual symmetry in the neutrino sector is enhanced and imposes $U_{\text{PMNS}}$ to be of the TB form independently from the mass eigenvalues. In the charged lepton sector, the VEV of $\phi_T$ breaks the $\nu_\mu - \nu_\tau$ exchange symmetry in a maximal way similar to the case of $S_3$. The masses $m_e, m_\mu, m_\tau$ arise at the order $\langle \phi_T \rangle / \Lambda$, $(\langle \phi_T \rangle / \Lambda)^2$ and $(\langle \phi_T \rangle / \Lambda)^3$ respectively leading to a hierarchical mass spectrum.

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