A Hydrodynamical Analysis of the Burning of a Neutron Star

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Abstract

The burning of a neutron star by strange matter is analyzed using relativistic combustion theory with a planar geometry. It is shown that such burning is probably neither slow combustion nor simple detonation. Fast combustion without detonation is possible under certain circumstances, but would involve very efficient heat transfer mechanisms. It is found, however, that the burning is most likely absolutely unstable with no well defined burn front.

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§1. **Introduction.**

There have recently been a great deal of interest [1] – [7] in the conversion of neutron stars into “strange” stars. As Witten [8] first proposed, nuclear matter may not be the most stable form of matter. Rather, “strange” matter, matter consisting of equal numbers of up, down and strange quarks, is. The average energy per nucleon was shown by Fahri and Jaffe [9] to be lower for this “strange” matter than for nuclear matter. It is believed that if a lump of strange matter comes in contact with a neutron star by whatever means possible, the neutron star will quickly be converted into a strange star. The question is how fast and under precisely what conditions this conversion will take place.

There are currently two methods being used to analyze this conversion process. The first is due to Olinto [2] who uses a non-relativistic diffusion model. As such, theirs is a slow combustion model, with the burn front propagating at a speed of approximately 10 m/sec. This is determined primarily by the rate at which one of the down quarks inside the neutron is converted through a weak decay to a strange quark: $d + u \rightarrow s + u$. The second method was introduced by Horvath and Benvenuto [6] who models the conversion as a detonation. Their conversion rate is several orders of magnitude faster than that predicted by Olinto. We, on the other hand, shall not *a priori* assume that the conversion process is due to either a slow combustion or a detonation. In fact, one of the purposes of our analysis is to establish under what conditions one will have a slow burning of the star or a detonation. Rather, we shall use the standard theory of relativistic combustion to analyze the conversion of the star which depends solely on the equations of state in the two media and the conservation of energy-momentum and baryon number across the combustion front. It is a kinematic analysis and has the advantage of not only being independent
of the details of how the neutrons are absorbed into the strange matter, which at this point is not well understood, but also independent of any assumptions as to the rate at which the conversion will take place. Since the only equation of state for the strange matter is the MIT bag model, any analysis of the conversion of the neutron star into a strange star will be dependent, to a certain extent, on the value of the bag constant, which is not known to any great degree of accuracy. We therefore calculated the velocity of the conversion front for a wide range of values of the bag constant and the density of the neutron star at various temperatures.

§2. Methodology.

We shall effectively be working in one spatial dimension, with the strange matter and the nuclear matter being seperated by a well defined planar combustion wavefront \[10\]. It has been shown that this wavefront is quite abrupt \[6\], \[3\], meaning that the thickness $\delta$ of the wavefront is much smaller than the size of the neutron star. We shall work in the frame which is at rest with the combustion wavefront. The energy-momentum tensor for the whole system is

$$T_{\mu\nu} = (\epsilon + p)u_\mu u_\nu - pg_{\mu\nu}$$

where $u_\mu$ is the covariant fluid velocity, $\epsilon$ is the energy density and $p$ is the pressure. The baryonic number density is $N_\mu = nu_\mu$. We will be using the ideal gas approximation for the quarks in the strange matter with all the quark masses $m_q = 0$ and the strong coupling constant $\alpha_s = 0$. In this zeroth order approximation the fraction of electrons in the strange quark matter $Y_e \equiv n_e/n_q = 0$ and the energy loss by neutrino emission from the quark matter vanishes \[11\]. Even though the process of absorbing neutrons is very exothermic, with each absorbed neutron releasing $\sim10$ MeV of energy \[9\], we shall consider that the reaction, as a whole, conserves energy with the excess energy released in the absorption process going into heating up
the star as a whole. In more realistic processes this approximation need not hold, and we shall comment on these situations in §3. Then $T_{\mu\nu}$ is conserved across the combustion front, and it can be shown that [10],

$$v_q^2 = \frac{(p_q - p_n)(\epsilon_n + p_q)}{(\epsilon_q - \epsilon_n)(\epsilon_q + p_n)} \quad , \quad v_n^2 = \frac{(p_n - p_q)(\epsilon_q + p_n)}{(\epsilon_n - \epsilon_q)(\epsilon_n + p_q)}$$

(1)

where $v_q$ and $v_n$ are the velocities relative to the burn front of the quarks and the neutrons in the strange matter and in the neutron star respectively. $v_n$ is also the velocity of the front relative to the star. (Variables with subscript $q$ ($n$) will denote quantities measured in the strange (nuclear) matter.) Next, baryon number must be conserved across the wavefront, from which we obtain

$$n_q^2 = n_n^2 \frac{(\epsilon_q + p_q)(\epsilon_q + p_n)}{(\epsilon_n + p_n)(\epsilon_n + p_q)}.$$  

For the strange matter, we shall assume a zero strange quark mass and take the MIT bag model equation of state:

$$e_q = \frac{19}{12} \pi^2 T^4 + \frac{9}{2} T^2 \mu^2 + \frac{9}{4 \pi^2} \mu^4 + B$$

$$p_q = \frac{19}{36} \pi^2 T^4 + \frac{3}{2} T^2 \mu^2 + \frac{3}{4 \pi^2} \mu^4 - B$$

$$n_q = T^2 \mu + \frac{1}{\pi^2} \mu^3$$

(2)

where $T$, $\mu$, and $B$ are the temperature, chemical potential of each quark, and bag constant respectively. For the nuclear matter there are various equations of state that we can choose. The two that we shall work with are the zero-temperature Bethe-Johnson (BJ) equation of state [12]

$$\epsilon_n = (236n_n^{1.54} + m_n)n_n \text{ MeV fm}^{-3},$$

$$p_n = 364n_n^{2.54} \text{ MeV fm}^{-3}$$

(3)

where $m_n$ is the mass of the neutron in MeV, and the zero-temperature ideal Fermi-Dirac (FD) neutron gas [13]. The velocities $v_q$ and $v_n$ may
now be determined for any given values of $B$, $n_n$ and $T$. Here we require $0.3 \text{ fm}^{-4} < B$ so that nuclei with high atomic numbers would be stable against decay into non-strange quark matter [9]. In addition, requiring the star to be hydrodynamically stable against small perturbations limits $n_n < 1.5 \text{ fm}^{-3}$ for the hard BJ equation of state, and $n_n < 5 \text{ fm}^{-3}$ for the soft FD equation of state [12].

At this point we need to introduce some terminology [10]. Let $v_q^s$ ($v_n^s$) be the speed of sound in the strange (nuclear) matter. $v_q^s = \frac{1}{\sqrt{3}}$ while

$$v_n^s = \left[ \frac{n_n^{1.54}}{1.01 + 0.648n_n^{1.54}} \right]^{\frac{1}{2}}$$

is the speed of sound for the BJ equation of state. When $v_q < v_q^s$ and $v_n < v_n^s$, the burning is called a deflagration or a slow combustion. When $v_q \leq v_q^s$ and $v_n > v_n^s$, it is called a detonation. When $v_q > v_q^s$ and $v_n < v_n^s$, the burning is absolutely unstable, meaning that in the presence of any small perturbation the wavefront will no longer remain as a well-defined plane and the model itself fails. When $v_q > v_q^s$ and $v_n > v_n^s$, the burning is a fast combustion without detonation which may involve either very efficient heat transfer in the unburnt gas, or reactions which are initially exothermic but are endothermic in their final stages.

§3. Results.

There are a few constraints that must be imposed. First, the velocities $v_q$ and $v_n$ must be real and less than the speed of light. Next, for the strange matter to consume the neutron star, the energy per “baryon” in the strange matter must be less than the energy per baryon in the neutron star. The demarcation lines are the $\epsilon_q/n_q = \epsilon_n/n_n$ lines and along these lines the bag constant $B$ may then be solved in terms of $n_n$ for any given $T$. Plots of the lines of equal energy per baryon in the $B$-$n_n$ parameter space are shown with the solid curves in Fig. 1-4 for various choices of $T$. In each case, the
thick solid curve also corresponds to the equal energy line: $\epsilon_q = \epsilon_n$. Along this line the velocities are infinite. Consequently, the regime of physically acceptable velocities for which there is a conversion of the neutron star into a strange star is confined to be below the lowest $\epsilon_q/n_q = \epsilon_n/n_n$, to the left of the vertical line corresponding to the maximum allowed value of $n_n$, and above the horizontal line corresponding to the minimum allowed value of $B$. Notice also that from Eq. (1) the size of $v_q$ and $v_n$ are determined to a large extent by the pressure difference between the nuclear matter and the strange matter. It is only when the $p_q = p_n$ line lies below the $\epsilon_q = \epsilon_n$ line that there will be choices of $B, n_n, and T$ for which $v_q$ or $v_n$ will be substantially less than the speed of light.

The parameter spaces obtained using the FD equation of state are shown in Figs. 1-2. Unlike the BJ equation of state the region of the parameter space for which burning will occur is bounded to the right by the equal energy line. When $T = 0$ this region is contained within a very small triangular region on the right of the graph. As the temperature increases, the $\epsilon_q/n_q = \epsilon_n/n_n$ lines collapse towards the $n_n$ axis and we find that at $T = 60$ MeV this region disappears altogether. We thus conclude that the ideal fermi neutron gas is in general too “soft” to allow the neutron star to burn.

Figs. 3-4 show the graphs of the parameter space using the BJ equation of state. Unlike the FD equation of state, there does exist choice of $B$ and $n_n$ such that burning may occur and we have included in these graphs contour lines of constant $v_q$ (dashed lines) and constant $v_n$ (dotted lines). When $T = 0$ we found that there was no choices of $B$ and $n_n$ for which $v_q < 0.8$ so that $v_q > v^s_q$ throughout the parameter space. Consequently, the starred line, representing the $v_n = v^s_n$ contour, splits the physically allowed parameter space only into two regions. Below this line $v_q > v^s_q$. 


while $v_n < v_n^*$. Consequently, in this region the burning is unstable. Above this line $v_q > v_q^*$ and $v_n > v_n^*$. This is the exotic region in which the burning can only occur due to some exotic heat transfer mechanisms.

As the temperature increases, the $\epsilon_q/n_q = \epsilon_n/n_n$ lines once again collapse towards the $n_n$ axis, but because the physical region is not bounded to the right by the equal energy line, there will still be a region in which the burning will take place. When $T = 60$ MeV we find that once again we have an exotic region and an unstable region in the parameter space, but not we also have a small region lying to the left of the $v_q = 0.6$ contour in which the star will undergo a “slow” combustion. (As the temperature is increased further, this region eventually also disappears.) Even in this slow combustion region $v_n$ is still typically fractions of the speed of light, however. More importantly, we find that in this region $v_n < v_q$ and consequently the burning is unstable [10]. Moreover, by incorporating gravitational effects inside the neutron star and also a surface tension term ascribed to the burn front, Horvath and Benvenuto [6] has shown that the slow combustion is unstable and is likely to become a detonation. In both Figs. 3 and 4 we have included the large dotted line corresponding to $\epsilon_n/n_n - \epsilon_q/n_q \approx 10$ MeV. The most probable velocities for the burning of the neutron star should occur along this line.

We find that the conversion of a neutron star into a strange star is never due to a detonation, although, except for some judicious choices of parameters, it will occur extremely rapidly. If the fast combustion is stable, then some presently unknown exotic mechanism must be present which either allows for a conversion process which must be first exothermic and then endothermic or else the heat conversion must allow for very efficient heat transfers. Although, it has been suggested that the conversion of neutron matter into strange matter is either direct, through two-flavor quark matter
[3], or through a quantum tunnelling process [7]. However, it is unlikely that the conversion has involved reactions which are initially exothermic but endothermic in their final stages. Next, it has been theorized that the core of a neutron state may contain a superfluid state, which permits very efficient heat transfers. It is not clear whether or not the superfluid state may exist near the burn front. Because $v_n$ is typically close to the speed of light, it may be faster than the critical velocity of the superfluid.

There is one other mechanism which may allow for very efficient heat transfers which was first suggested by Horvath and Benvenuto [6]. Because the Reynolds numbers for both fluids are extremely large, it may be that the heat transfer mechanism is not due to thermal conduction but rather by turbulent convection of the two materials. This will allow for much more efficient heat transfers. Unlike Horvath and Benevenuto who argued that the burning will quickly reach detonation, we find that because $v_q$ is always greater than the speed of sound in the strange matter, detonation will never occur.

Most of the parameter space which would allow for the burning of the star is taken up by the absolutely unstable burning region. In this region of the parameter space where (condition for absolute unstable burning) the burn front is absolutely unstable, meaning that any perturbation of the planar burn front will very quickly grow and there will be a turbulent mixing of the nuclear and strange matter. Modeling the burn front as an infinite flat plane is no longer realistic and other methods need to be used to analyze the burning.

In conclusion, we find that the conversion of the neutron star into a strange star will never be due to slow combustion or detonation and in fact may not even be describable using a simple infinite planar model of the burn front. If it happens at all, then the conversion could be a fast com-
bustion without detonation. The basic culprit is \( v_q \) which we have found to be greater than the speed of sound in the strange matter for most of the parameter space. As such, detonation will never take place; but then the conditions for stable burning will also be difficult to satisfy. We note, however, that all models of the burning, including our own, used the infinite plane description of the burn front. The neutron star is a sphere, however, and it may be more realistic to use a spherical burn front to model the burning. As the boundary conditions for a spherical geometry is quite different from the infinite plane geometry, namely that the strange matter must be at rest after the burning, a parameter space which will allow for stable burning may be found. If not, however, then the conversion of the neutron star will most probably be absolutely unstable and some way would need to be found to model the turbulent mixing of the strange and nuclear matter. We are in the process of analyzing the burning using a spherical geometry.

In more realistic situations where \( m_s, \alpha_s \) and \( Y_e \) are not zero, thermal energy may be lost by emission of neutrinos produced in URCA processes and the assumption of energy conservation may no longer hold. We note, however, that the neutrino emissivity \( e_\nu \) under URCA processes [11]: 
\[
e_\nu \cong 4.6 \times 10^{32} \left( \frac{T}{\text{MeV}} \right)^6 \text{erg cm}^{-3}\text{sec}^{-1}
\]
where we have taken \( \alpha_s = 0.5, Y_e = 0.01, n_q = 3.4 \times 10^{38} \text{ cm}^{-3} \). The heat production rate per unit volume \( e_s \equiv (e_q - e_q(T = 0))/t \) for the burning can be calculated from Eq. (2). For \( T \leq 10\text{MeV} \) and \( \mu \cong (\pi^2 n_q)^{\frac{3}{2}} \cong 295\text{MeV} \), we find that \( e_s/e_q \cong 0.18(\text{Mev}/T)^4(\text{sec}/t) \).

From the above, we expect the burn front to travel near the speed of light for \( T \leq 10\text{MeV} \). Taking the star radius as 10km and \( v_n \cong 0.1 \), we find that \( t \approx R/v_n \approx 3.3 \times 10^{-4} \text{sec} \). Our analysis may be used as long as \( e_s/e_\nu \gg 1 \), so that \( T < 4.8\text{MeV} \). As we know \( T \) may reach 10MeV or higher. Note, however, that for \( T = 10\text{MeV} \), the neutrino mean free path [11], \( l_\nu \approx 3m<< R \). Consequently, our method of analyzing the burning
process will be applicable even in the more realistic model because neutrino emission is relatively small at low temperatures and at high temperatures, neutrino trapping becomes efficient.

We have also analysed a case in which the equation of state of the neutron gas is of BJ type and $T = 10$ MeV. The result is quite similar to Fig. 3. Moreover, we could not find any choices of $B$ and $n_n$ for $T = 10$ MeV and $m_s = 0$ which would reproduce Olinto’s result.

Preliminary results for the case of finite strange quark mass have shown no qualitative changes to the parameter space. Non-zero strong coupling corrections should have similar results.

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