Environmental Collapse Models

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(Dated: June 6, 2022)

We propose dynamical collapse models in which the stochastic collapse terms affect only photons and/or gravitons. In principle, isolated systems comprising only massive particles could evolve unitarily indefinitely in such models. In practice, since photons and gravitons are ubiquitous and scatter from massive particles, dynamical collapses of the former will effectively induce collapses of the latter.

In non-relativistic models in which particle number is conserved and interactions are modelled by classical potentials, massive systems can be modelled as collections of elementary massive particles bound by potentials, interacting with an environment of photons and gravitons. In this picture, although the photon and/or graviton collapse dynamics effectively localize massive systems, these collapses take the effective form of approximate measurements on the environment whose effect on the massive systems is indirect. We argue that these environmental collapse models, like standard mass-dependent spontaneous localisation models, may be consistent with quantum experiments on microscopic systems while predicting very rapid effective collapse of macroscopic massive systems, and hence a potential solution to the quantum measurement problem. However, the models considered here have different experimental signatures from standard mass-dependent spontaneous localisation models. For example, they predict no deviations from standard quantum interferometry for mesoscopic systems of massive particles isolated from a decohering environment, nor do they predict anomalous spontaneous emission of radiation from isolated matter of the type prediction by standard mass-dependent spontaneous localization models. New experiments and analyses are required to obtain empirical bounds on the decoherence rate in our models.

INTRODUCTION

In previous papers [1–3], we described ideas aimed towards defining realist and Lorentz covariant versions of relativistic quantum theory or quantum field theory. A key idea in these papers is to postulate a fundamental split \( H = H_A \otimes H_B \) in the Hilbert space of nature, together with a suitable asymptotic late time behaviour of the degrees of freedom in one factor. Together with initial conditions on the full state, these are used to define a realist ontology for quantum theory. For example, Refs. [2, 3] propose an ontology for quantum matter defined by the initial state together with the outcomes of fictitious measurements on the electromagnetic or gravitational field at asymptotically late times. In simple models this ontology aligns with common intuitions and with the language commonly used to describe experiments. For example [3], the ontology suggests that, if an unstable excited atom can radiate a photon to infinity without scattering or absorption, it reaches its ground state at an almost precisely defined time that is Poisson distributed, with mean given by the expected photon emission time.

Here we use related intuitions to propose a different type of model. Roughly speaking, we take \( H_A \) to be the degrees of freedom for all massive particles and \( H_B \) for massless particles. Alternatively, we could take \( H_B \) to correspond to force carrier particles. However, we here consider simple non-relativistic models and are primarily interested in the particles associated with long-range forces. We treat hadrons as elementary and ignore gluons; we also ignore W, Z and Higgs bosons. We thus take \( H_A \) to be everything but photons and/or gravitons and \( H_B \) to be electromagnetic and/or gravitational degrees of freedom. We consider dynamical collapse models in which the stochastic “collapse terms” act on \( H_B \) but not \( H_A \). Because matter and radiation are generally entangled, these collapses nonetheless affect matter indirectly.

For example, a \( 10^{-5} \) m diameter dust grain interacts with a flux of \( \sim 10^{13} \) thermal photons per second at 300K. If the dust grain is in a superposition of two macroscopically distinct position states, for which a collapse model predicts that collapses swiftly occur in a way that distinguishes the corresponding states of scattered radiation, then the grain will be swiftly localized to one position state. As this illustrates, the effective spontaneous collapse rate for matter depends both on its environment and on the parameters of a photon (or graviton) collapse model.
NON-RELATIVISTIC PHOTON COLLAPSE

Ref. [3] is not a dynamical collapse model, but does consider asymptotic late time fictitious measurements of photons that could be interpreted as the electromagnetic field undergoing spontaneous collapses at an asymptotically late time. Dynamical photon collapses were subsequently considered by Pearle [4] within the framework of non-relativistic mass-dependent continuous spontaneous localization (CSL) models. Mass-dependent non-relativistic CSL models are motivated by the observation that mass density is a natural observable, since it avoids the need to distinguish elementary from composite particles and allows parameter choices that appear consistent with experimental data to date. Pearle’s motivation for considering photon collapse in this context was that, if consistent relativistic versions of these models exist, it seems natural that they would relate collapse to stress-energy and plausible that the relevant quantity is the total stress-energy, to which particles of zero rest mass also contribute. Pearle thus suggests that a non-relativistic limit might relate collapse to energy density (for both photons and massive particles) rather than mass density (which of course is relevant only for massive particles). To implement this, he proposes a CSL dynamical equation in which the operators $\xi^\dagger(x)\xi(x)$ are replaced by the energy density operators $(K^{1/2}\xi^\dagger(3))(K^{1/2}\xi(3))$, where $K^2 = -\nabla^2 + M^2$ and $\xi(x)$ is the annihilation operator of a particle of mass $M$ at location $x$.\[18\] (Pearle’s notation suppresses spin degrees of freedom.) This gives [4] the density matrix evolution equation

$$\frac{\partial}{\partial t} \rho(t) = -i[H, \rho(t)] - \frac{\lambda}{2M^2_N} \int dx \int dx' \exp\left(-\frac{(x-x')^2}{4a^2}\right)[K^{1/2}\xi^\dagger(x)K^{1/2}\xi(x), [K^{1/2}\xi^\dagger(x')K^{1/2}\xi(x'), \rho(t)]] , \quad (1)$$

where $M_N$ is the neutron mass, $\lambda$ the collapse rate and $a$ the collapse range. The vectors $x, x'$ and integrals $\int dx, \int dx'$ are three-dimensional; similarly for momenta below.

As we are considering photon-only collapse here, we need to reconsider the motivations for various possible collapse types.

Collapse based on approximate energy density

We propose photon-only collapse models based on approximate photon energy density, i.e. by a version of Eqn. [2] in which – differently from Ref. [4] – the collapse terms involving quantities related to $K$ and $\xi$ are defined only for photons. Explicitly,

$$\frac{\partial}{\partial t} \rho_{AB}(t) = -i[H, \rho_{AB}(t)] - \frac{\lambda}{2M^2_N} \int dx \int dx' \exp\left(-\frac{(x-x')^2}{4a^2}\right)[K^{1/2}\xi^\dagger(x)K^{1/2}\xi(x), [K^{1/2}\xi^\dagger(x')K^{1/2}\xi(x'), \rho_{AB}(t)]] , \quad (2)$$

where we write $\rho_{AB}(t)$ to emphasize that the density operator represents the complete physical system, defined on $H_A \otimes H_B$. Here $H_A, H_B$ represent matter and electromagnetic degrees of freedom respectively, $K^{1/2}$ is as above with $M = 0$ and $\xi(x)$ is the photon detection or annihilation operator which, with explicit spin summation, is given by

$$\xi(x) = \sum_s \int_k a_{k,s} \epsilon_{k,s} \exp(i k \cdot x) dk$$

Collapse based on approximate photon number

We argue that photon-only collapse models based on approximate photon number density are also well motivated. Photons are not localizable, and there is no density operator whose integral precisely counts the number of photons in a volume. Nonetheless, there are useful approximations to the concept [3].

First, note that in models that use non-relativistic photon-only collapse in idealised environments where photon number is conserved, we can treat photons as free particles that scatter from matter without absorption. Treating massive particles as qualitatively different from photons then avoids the need to discuss further the distinction between elementary and composite particles, which was the main initial motivation for focussing on mass-density collapse models for massive particles. Of course, this leaves questions about how virtual photons would be treated in relativistic models. However, similar questions apply to relativistic versions of any dynamical collapse model. Our main aim here is to propose alternatives to the familiar non-relativistic dynamical collapse models: we leave discussion of relativistic versions for future work.
Second, although experimental data tend to favour conventional mass-density collapse CSL models over particle number collapse CSL models, the latter are not completely excluded [6], and in any case the same conclusion does not necessarily apply for photon-only collapse models. Both types of photon-only collapse model have different properties from their familiar CSL counterparts, and their empirical implications need to be examined independently and afresh.

Third, although it seems reasonable to motivate conventional mass-density collapse CSL models as potentially more natural candidates for some unified theory involving gravity, other ideas in this direction are also worth exploring, particularly given that even consistent (special) relativistic CSL models have not so far been constructed. For example, gravity could plausibly be coupled to mass density in CSL models in which the collapse operators are defined by other quantities. Another interesting idea, which we discuss below, is that quantum theory and gravity are linked by graviton-only collapse models. As in the photon case, these could involve either particle number or energy density collapse.

In summary, photon-only number density collapse models appear well motivated. We thus want to consider the evolution equation

$$\frac{\partial}{\partial t} \rho_{AB}(t) = -i[H, \rho_{AB}(t)] - \lambda \int dx \int dx' \exp\left(-\frac{(x - x')^2}{4a^2}\right)[\xi^\dagger(x)\xi(x), [\xi^\dagger(x')\xi(x'), \rho(t)]] .$$

(4)

Here $\lambda$ is the collapse rate, $a$ the collapse range, $\rho_{AB}(t)$ is a density matrix for the complete physical system (matter and photons), but the collapse terms involve only the photon annihilation and creation operators, $\xi$ and $\xi^\dagger$.

**EFFECTS OF PHOTON COLLAPSE**

Effects on matter

Most lab experiments and most direct observations of matter take place in rich and complex photonic environments. This is also true of the retina and brain, which give rise to our perceptions of definite outcomes of quantum experiments and of matter apparently behaving classically. A detailed analysis of the effects of photon collapse models needs, inter alia, a detailed description of the initial environment state, the effects on that state of scattering from the matter system in question, the effects of subsequent re-scatterings from it and other matter systems, and a careful discussion of how photon number or energy density collapses on the possible scattered states distinguish between different matter states and so induce their effective collapse.

The simplest non-relativistic photon collapse models conserve photon number and so do not include emission or absorption, but more realistic models clearly should. A simple option would be to draw some (ultimately imprecise) distinction between photons that are generally regarded as “free” in between their emission and absorption (such as a laser pulse fired at an absorbing wall) and virtual photons. One could then model the dynamics to allow varying numbers of photons and massive systems with variable energy, and consider the effects of photon collapse in these models.

Our aim here is to get tentative initial insight, using very rough estimates, into whether photon-only collapse models may be consistent with present empirical evidence and candidate solutions to the measurement problem. To do this, we consider discrete collapse models related to the continuous models (4) and (2). Very roughly speaking, these have the effect of approximate measurements of a smeared version of the number of photons of wavelengths $\lambda \ll a$, or the total energy of such photons, within a sphere of diameter $a$.

**Photon number collapse**

Define

$$N(a, x) = \int \exp\left(-\frac{(x' - x)^2}{a^2}\right)\xi^\dagger(x')\xi(x') d^3x' .$$

(5)

Loosely speaking, this approximately represents a Gaussian weighted integral of the number of photons of wavelengths $\ll a$ in a volume centred at $x$. We can define a discrete collapse model in which at each $x$ discrete collapse processes takes place at random times, with the effect that

$$\psi \rightarrow \psi_n^x = \frac{\phi_n^x}{\|\phi_n^x\|} .$$

(6)
where
\[
\phi_n^x = (b/\pi)^{1/4} \exp(-(b/2)(N_n(x) - n)^2)\psi.
\]

These collapses take place with frequency density \( \mu \), so that the probability of a collapse taking place for some \( x \) in a small volume \( V \) during a small time interval \( t \) is \( \mu V t \).

We take the total volume of space to be finite so that the collapses take place at discrete times. Between collapses the state \( \psi \) evolves unitarily. A finite space volume could be imposed by regularizing to a cube of edge length \( L \) with periodic boundary conditions. Here, we simply ignore collapses outside a laboratory or given region of interest. Since such collapses can only increase the effective collapse rate of systems inside the region, lower bounds on collapse rates derived by ignoring them remain valid. In principle, they could affect the discussion of upper bounds on collapse rates. However, in practice, any radiation that has left the region of interest and remains entangled with the system of interest has in any case decohered the system, so that collapses affecting such radiation do not have any additional observable effect within the region. This decoherence is in practice generally irreversible, and special cases where the radiation re-enters the region and recombines with the system can be treated by enlarging the considered region.

Roughly, then, we can take \( N_n(x) \) to be the number of photons of wavelength \( \ll a \) in a sphere of radius \( a \) centred at \( x \). A collapse constrains its value to within \( \sim b^{-1/2} \), and these collapses take place within a volume \( V \) every \( (\mu V)^{-1} \) on average.

Consider a dust grain of size \( 10^{-4}m \), roughly the smallest size visible by the human eye, in a superposition of two separated centre-of-mass position states, exposed to sunlight, with the line between the two positions roughly perpendicular to the direction of incident sunlight. Although it is not precisely rigorous to speak of the number of photons in a given volume, photodetectors do detect individual photons of given wavelengths, and the intuition that photons can be roughly counted is a helpful guide. In this sense, the flux of sunlight produces \( \sim 3 \times 10^6 \) photons per cm\(^3\), or \( \sim 3 \) photons in a cube of side \( 10^{-4}m \).

We crudely model the effect of sunlight scattering from the dust grain by supposing that, inter alia, a shadow region of cross-section \( 10^{-4}m \times 10^{-4}m \) and length 1m is created behind the dust grain, and that in the “shadow” the number of photons is diminished by \( \frac{1}{4} \), so that it contains \( \sim 2 \) photons in each cube of side \( 10^{-4}m \). In the given configuration, the “shadows” of the two position states of the dust grain do not overlap.

So, if the collapses apply on the scale of cubes of size \( 10^{-4}m \), make the number of optical spectrum photons definite to within \( \sim 1 \), and occur every \( \sim 1s \), they will distinguish the two shadows, and hence the position states, of the dust grain within \( \sim 10^{-4}s \), which is shorter than human perception times of \( \sim 10^{-2}s \). We take the near-definite position state to be reflected as a definite outcome in the ontology. Thus we assume here some resolution of the tails problem (for more extensive discussion of which see e.g. [3]). One possible resolution is that other small contributions to mass density in the unselected region, which will be present in any realistic ontology, swiftly dominate the contributions from the unselected dust grain position state [3]. Another possible stance is that one should not worry overly about espilinics in a model that is at best an approximation to a more compelling underlying theory, in which the tails problem does not arise. Some such assumptions are standardly made to justify the claim that standard dynamical collapse models resolve the measurement problem. Our first goal here is to argue that environmental collapse models should be on the table alongside standard dynamical collapse models as potential solutions, and we see no argument that the tails problem is more serious for environmental collapse models, so we leave more detailed discussion for future work.

In summary, modulo the tails problem, the swift transition of the dust grain into a near-definite position state is consistent with our perceiving within \( \sim 100ms \) that it has an essentially definite position. In other words, perception and ontology are aligned.

**Effects in photon experiments**

A simple Mach-Zehnder interference experiment with a single photon split into two beams and recombined, with path lengths 1m, has flight time \( \sim 3 \times 10^{-9}s \). If we take the paths to run through \( 10^{-4}m \) cubes, a collapse in a cube distinguishing between photon number 0 and 1 corresponds to a detection of a definite path, destroying the interference. The collapse probability is the same as if the photon stayed in a single cube for its flight time, i.e. \( P(\text{collapse}) \sim 3 \times 10^{-9} \). Interference statistics would need to be demonstrated to greater than this precision to observe the predicted anomaly.

In a recent boson sampling experiment [3], 25 photons travel for \( \sim 3m \), with a flight time \( \sim 10^{-8}s \). An anomalous outcome would thus be observed in a run with probability \( \sim 4 \times 10^{-6} \). Given the nature of the experiment, which
produces results that are argued to be essentially impossible to reproduce on classical computers, even if anomalies arose it would be difficult to identify them.

Neither type of experiment thus seems likely to refute a photon-only collapse model, with the given parameters, in the near future.

**Photon energy density collapse**

Although photon energy density collapse models and photon number collapse models have different phenomenologies in general, we can apply similar reasoning regarding their implications in simple discrete collapse models. A dust grain “shadow” that contains \( \sim 1 \) fewer visible photon per \( 10^{-4} \)m cube thereby contains \( \approx h \nu \) J less energy on average, where \( \nu \) is the frequency of a typical sunlight photon. These frequencies mostly range over \( 4 \times 8 \times 10^{14} \) Hz. A discrete collapse model that makes the photon energy within a cube definite to within \( \sim h \times 8 \times 10^{14} \) J thus has only slightly stronger (by a factor of \( \lesssim 2 \)) effects than the photon number model in the situations discussed above. The dust grain shadow model again predicts an effective collapse of the dust grain within \( 10^{-4} \)s, while again predicting undetectably small effects in Mach-Zehnder interference and boson sampling experiments.

**Laser experiments**

Pearle [4] gives some detailed analyses of anomalous effects of photon energy density collapse in possible laser experiments. While Pearle’s discussion considers models that involve both massive particle and photon energy density collapse, the effects on laser beams derive solely from the latter. Pearle concludes that, under heroic experimental assumptions, Eqn. (1) could produce detectable losses of photons from X-ray laser pulses if \( \lambda > 10^{-6} - 10^{-4} \) s\(^{-1} \) and could produce detectable photon “spraying” from high energy CW lasers, monitored for a year, if \( \lambda > 10^{-8} \) s\(^{-1} \).

It would be very interesting to extend these analyses to the various models proposed above, but we postpone this for future work. As far as we are aware, these experiments have not so far been carried out, and it is not clear that – if and when feasible – they would provide the strongest attainable bounds.

**Constraints on collapse parameters from human perception times**

One can indirectly argue, as above, for lower bounds on collapse rates by assuming that a collapse model must imply that a physical system (such as a delocalized dust grain) should collapse within the time it takes for us to observe it in an essentially definite and quasi-classical state. Indeed, authoritative reviews of collapse models (e.g. [6]) often give lower bounds derived in precisely this way. While this appears comfortingly objective, the fact is that our perceptions are the result of physiological processes in the retina and brain, and that a collapse model might possibly predict that the relevant collapses generally occur as a result of these processes. Indeed, in beam-splitter experiments with weak light pulses, or other experiments where a human eye plays the role of photo-detector measuring a superposition of photon states [10], the relevant collapse can only be caused by events after the pulse enters (or does not enter) the retina, according to standard dynamical collapse models.

This is essentially true also of the photon-only collapse models discussed here. Although in principle it is possible for a collapse to effectively determine the path a photon follows from beamsplitter to (or away from) eye, the probability must be negligibly small in order for the model to be consistent with simple photon interferometry experiments. If a collapse takes place within \( \sim 100 \) ms of the photon leaving the source, it must be a collapse associated with differential electromagnetic fields generated by the nervous system, brain or other parts of the anatomy and/or with differential scattering patterns of environmental photons from these systems.

Attempts have been made (e.g. [11–13]) to analyse the mesoscopic mass distribution changes associated with visual perception in order to obtain lower bounds on collapse rates for standard mass-dependent dynamical collapse models. The problem is complex. One problem is that cellular responses to stimuli involve proteins and molecules of various densities diffusing in cellular media, displacing other proteins and molecules, which makes it hard to estimate the net effect on mass density distribution. Another is that it is not clear how far one should pursue the effects along the processing chain (which itself is not known with certainty) within the brain. Beyond this, there is also a case that it might be justifiable to consider physiological responses – slight muscle movements, alterations in heart rate, and so on. These might be unconscious and involuntary, but nonetheless predictable, and they may involve much greater differential displacements of mass density than retinal or neural processing does.
Similar and perhaps even more complex issues arise with photon-only collapse models. The intracellular release of ions from gates certainly generates small electromagnetic fields and has some effect on differential scattering. Brain processing as a whole generates measurable electromagnetic fields. Differential physiological responses would certainly generate differential scattering patterns. Any responses involving more than microscopic motion create mass shifts at the body boundary significantly greater than those of a displaced $10^{-4}$m dust grain. Although our shadow model does not directly apply, similarly modelling differentially reflected radiation suggests that differential scattering would induce effective collapse on timescales short compared to $\sim 100$ms. Of course, detailed analyses of all these effects are needed. That said, at least if it is considered legitimate to allow for possible physiological responses, the relatively macroscopic scale of these responses suggests that our model parameters are adequate to resolve the measurement problem.

**GROVITON COLLAPSE**

The plausibility of photon-based collapse models suggests considering analogous models based on graviton collapse. Intuitively, the idea that collapse is associated with gravity is attractive, offering the hope that the quantum measurement problem and the conceptual problems of quantum gravity could be solved together. Versions of this intuition have been set out by Diosi [14] and Penrose [15]. Diosi’s and Penrose’s proposals appear to have been empirically refuted by recent experimental data [16], suggesting that new ideas in this direction are needed [16].

Collapse models in which the collapse terms only directly apply to gravitons seem good candidates. Rather than proposing, as Penrose and Diosi do, that matter undergoes collapses to prevent macroscopic superpositions, we propose that collapses take place in the quantum gravitational field itself. For an appropriate collapse rate, this offers an alternative mechanism by which superpositions of significantly distinct space-times could be suppressed. Intuitively, one can imagine physics taking place in an approximately classical background space-time that defines a local causal structure but also admits small quantum fluctuations. Because the postulated collapses do not directly apply to matter, the analyses of Ref. [16], and specifically their Eqn. (3), do not apply. The anomalous photon emission from matter implied by these analyses, and excluded by the Gran Sasso experiment [16], need not arise.

Penrose’s proposal that superpositions of space-times differing by one graviton should quickly collapse, although difficult to make precise, is an interesting and intuitively attractive criterion for defining collapse rates. However, it is also interesting to consider the full range of possible collapse rates in graviton-only collapse models. Experimental upper bounds on graviton collapse rates analogous to those (actually or potentially) attainable for photons are not available, as we have no graviton interference experiments, nor graviton lasers. The viability of graviton-only collapse is nonetheless partly testable, given specific quantum gravity models, from constraints based on human perception. The effective (collapse-associated) decoherence rate that graviton-based collapse implies for humans in different perception states cannot be greater than the total (standard) decoherence implied by tracing out gravitational degrees of freedom. Since the differences in gravitational fields associated with different brain perception states are very small, this is already quite a strong constraint, if we restrict attention to brain processing events. Physiological responses produce more strongly differentiated gravitational fields, but even so, whether their scale and speed is sufficient to induce gravitational decoherence within $\sim 100$ms is likely model-dependent. We hope to present detailed analyses elsewhere [17].

One might even further speculate that a slight anomalous energy production in the gravitational field created by graviton collapse could somehow be connected to dark energy.

**CONCLUSIONS**

We have raised the question of whether photon- or graviton-only collapse models might solve the quantum measurement problem while predicting sufficiently small deviations from standard quantum theory that they are consistent with known experiment. Our preliminary and very rough discussions have not found immediately obvious flaws with this idea. Since it has some attractive features, we put it on the table and plan to explore it further.

We have discussed only non-relativistic models. This does not dis favour our models compared to standard dynamical collapse models, which are not presently known to have fully consistent relativistic extensions. Indeed, relativistic versions of photon-only collapse models might perhaps be easier to construct.

Lower bounds on collapse rates in any collapse model are difficult to justify precisely, since they ultimately depend on human perceptions of definiteness. These are produced by very complex physical systems interacting strongly with quite complex environments, which make precise calculations difficult. Potentially questionable assumptions also
need to be made. For example, one needs to decide whether our impression that we perceive definite outcomes within \( \sim 50 - 100 \text{ms} \) is reliable, or if not, whether there is some reliable timescale; whether the collapse ontology should support this perception for typical humans in typical environments or all humans in all conceivable environments; whether possible physiological responses should be considered relevant or whether one should restrict analysis to brain processing events. Still, there are reasonable justifications for particular sets of assumptions (for example, \( \sim 50 - 100 \text{ms} \) is reliable, a collapse ontology should be aligned with this at least in typical cases, physiological responses may (or may not) be relevant), and the questions are fundamentally interesting enough to motivate tackling the complexities.

ACKNOWLEDGEMENTS

I gratefully acknowledge the support of a Foundational Questions Institute (FQXi) grant. This work was partially supported by Perimeter Institute for Theoretical Physics. Research at Perimeter Institute is supported by the Government of Canada through Industry Canada and by the Province of Ontario through the Ministry of Research and Innovation.

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