CHARACTERIZING GALAXY CLUSTERS WITH GRAVITATIONAL POTENTIAL

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Received 2010 September 10; accepted 2011 May 17; published 2011 July 19

ABSTRACT

We propose a simple estimator for the gravitational potential of cluster-size halos using the temperature and density profiles of the intracluster gas based on the assumptions of hydrostatic equilibrium and spherical symmetry. Using high-resolution cosmological simulations of galaxy clusters, we show that the scaling relation between this estimator and the gravitational potential has a small intrinsic scatter of $\sim 8\%–15\%$, and it is insensitive to baryon physics outside the cluster core. The slope and the normalization of the scaling relation vary weakly with redshift, and they are relatively independent of the choice of radial range used and the dynamical states of the clusters. The results presented here provide a possible way for using the cluster potential function as an alternative to the cluster mass function in constraining cosmology using galaxy clusters.

Key words: cosmology: theory – galaxies: clusters: general – methods: numerical – X-rays: galaxies: clusters

Online-only material: color figures

1. INTRODUCTION

Clusters of galaxies are useful probes of cosmology. The evolution of their abundance probes the growth of structure and therefore provides constraints on cosmological parameters (e.g., see Voit 2005; Allen et al. 2011 for reviews). Clusters are readily observable in multiple wavelengths. In addition to the optical, the regime where galaxy clusters are first observed, hot gas in the deep gravitational wells of clusters shines in X-ray. In the submillimeter regime, galaxy clusters appear as distortions in the cosmic microwave background (CMB) as the hot electrons in the intracluster medium (ICM) scatter off CMB photons through inverse Compton scattering known as the Sunyaev–Zeldovich (SZ) effect (Sunyaev & Zeldovich 1970; Carlstrom et al. 2002). In recent years, cosmological constraints have been yielded observably in characterizing clusters with their gravitational potential minima using a Gaussian random field formalism (Bardeen 1986) and was used to predict cluster temperature function using simple spherical-collapse modeling of the nonlinear evolution of the cluster potential. On the observational side, Churazov et al. (2008, 2010) measured profiles of the gravitational potential of elliptical galaxies using both X-ray observations and stellar kinematics to constrain non-thermal pressure in those galaxies. It should then be straightforward, at least in principle, to extend these measurements to group and cluster-size systems.

In this paper, we show that the cluster potential can be measured reliably via observables and that the cluster potential can serve as the defining quantity of cluster. We use simulations to estimate the gravitational potential of galaxy clusters by
constructing a simple potential estimator based on intracluster gas profiles, motivated by the observational results of Churazov et al. (2008, 2010). In Section 2, we describe the simulations used in the paper. In Section 3, we describe our estimator of the cluster potential. In Section 4, we show the results of the scaling relations of this potential estimator and the gravitational potential. In Section 5, we give our summary and discussion.

2. SIMULATIONS

The simulation data we used are identical to those in Nagai et al. (2007a, 2007b), where 16 cluster-sized systems are simulated using the Adaptive Refinement Tree N-body + gas-dynamics code (Kravtsov 1999; Kravtsov et al. 2002), which is an Eulerian code that uses adaptive refinement in space and time, and non-adaptive refinement in mass (Klypin et al. 2001) to achieve the dynamic ranges to resolve the cores of halos formed in self-consistent cosmological simulations. The simulations assume a flat ΛCDM model: Ω_m = 1 − Ω_b = 0.3, Ω_b = 0.04286, h = 0.7, and σ_b = 0.9,1 where the Hubble constant is defined as 100h km s^{-1} Mpc^{-1} and σ_b is the mass variance within spheres of radius 8 h^{-1} Mpc. The simulations were run using a uniform 128^3 grid with eight levels of mesh refinement. The box size for CL101–CL107 is 120 h^{-1} Mpc comoving on a side and is 80 h^{-1} Mpc comoving for CL3–CL24. This corresponds to peak spatial resolution of ≈ 7 h^{-1} kpc and 5 h^{-1} kpc for the two box sizes, respectively. Only the inner regions r ≤ 10 h^{-1} Mpc surrounding the cluster center were adaptively refined. The dark matter (DM) particle mass in the region around each cluster was m_p ≃ 9.1 × 10^8 h^{-1} M⊙ for CL101–107 and m_p ≃ 2.7 × 10^8 h^{-1} M⊙ for CL3–24, while other regions were simulated with a bottom mass resolution.

In Table 1 we report r_{500c}, (the radius of the cluster within which its average density is 500 times the critical density), M_{500c}, (the mass within r_{500c}), and our classification of the dynamical state of the cluster based on the morphology of their mock X-ray images. Details of the classification can be found in Nagai et al. (2007b). The cluster center is defined as the location of the potential minimum.

We assess the effects of dissipative gas physics on the relation between the cluster potential and gas properties by comparing two sets of clusters simulated with the same initial conditions but with different prescription of gas physics. In the first set, the gas follows simple physics without any radiative cooling or star formation. We refer to this set of clusters as non-radiative (NR) clusters. In the second set, metallicity-dependent radiative cooling, star formation, supernova feedback, and a uniform UV background were added. We refer to this set of clusters as cooling+star formation (CSF) clusters. For a detailed description of the gas physics implemented in the CSF clusters, please see Nagai et al. (2007a).

3. ESTIMATOR OF THE GRAVITATIONAL POTENTIAL

Following Churazov et al. (2008), we use a simple estimator of the cluster potential based on gas observables. Under the assumptions of hydrostatic equilibrium, the gradient of the gravitational potential is related to the pressure gradient and density of gas by

$$\nabla \phi = -\nabla P \rho_g^{-1},$$

where $\phi$ is the gravitational potential, $\rho_g$ is the density of the intracluster gas, and $P$ is the gas pressure. The gravitational potential can be obtained as a function of gas temperature $T$ and gas density when we integrate Equation (1) by parts:

$$\phi = -\int \frac{d P}{\rho_g} + \text{constant} = -\int \frac{k_B T}{\mu m_H} d \ln \rho_g + \text{constant},$$

(2)

where $k_B$ is the Boltzmann constant, $\mu = 0.59$ is the mean molecular weight for the fully ionized ICM, and $m_H$ is the mass of the hydrogen atom.

Physical properties relate not to the absolute value of the potential but the difference in the potential. The difference of the spherically averaged gravitational potential between two arbitrary radii $r_1$ and $r_2$ can be directly calculated from the temperature and density profiles. We define our potential difference estimator, which is based on the temperature and density profiles of the intracluster gas, as $\Delta \psi$,

$$\Delta \psi \equiv \psi(r_2) - \psi(r_1) = \int r_2 \frac{T(r)}{\mu m_H} d \ln \rho_g dr - \int r_1 \frac{T(r)}{\mu m_H} d \ln \rho_g dr.$$

(3)

One can note that $\psi$ is actually the enthalpy of the intracluster gas. If $\Delta \psi$ is a perfect estimator of the true potential difference $\Delta \phi$, then obviously we have $\Delta \psi = \Delta \phi$. Since in real clusters, the gravitational potential is not strictly spherical and the intracluster gas does not obey perfect hydrostatic equilibrium, we expect $\Delta \psi$ to deviate from $\Delta \phi$. To assess how well $\Delta \psi$ measures $\Delta \phi$ statistically, we fit a scaling relation of the two quantities using ordinary least squares:

$$\log \Delta \phi = \alpha \log(\Delta \psi/\Delta \phi_0) + \log \Delta \phi_0,$$

(4)

where we set $\mu m_H \Delta \phi_0 = 20$ keV. Assuming that the residuals follow a log-normal distribution, we calculate the intrinsic scatter as the root mean square of the residuals $\delta_{\ln,\Delta \phi}$ divided by $\sqrt{N - 2}$, where $N$ is the total number of clusters:

$$\sigma_{\ln,\Delta \psi}^2 = \frac{1}{N - 2} \sum_i (\ln \Delta \phi_i - \alpha (\ln \Delta \psi/\Delta \phi_0) - \ln \Delta \phi_0)^2.$$

(5)
Note that we follow the convention of using natural log for the log-normal scatter.

As $\Delta \psi$ and $\Delta \phi$ are dimensionally the same, we expect the slope of the scaling relation between them to be close to unity. We therefore also compute the scatter of the relation assuming unity slope, i.e., $\alpha = 1$. In this case, we compute the mass-weighted temperature. To approximate the temperature inside the radial bin.

We calculate the radial profiles for each cluster using logarithmically spaced bins, with a total of 100 bins spanning a comoving radial range from $1 \, h^{-1} \, kpc$ to $5 \, h^{-1} \, Mpc$. Note that our results are not affected by the exact details of the binning scheme. The potential difference $\Delta \phi$ for each cluster is calculated from the potential profile directly measured from the simulation. The potential at each radial bin is calculated as the volume-average of the potential over all hydrodynamic cells inside the bin.

To calculate $\Delta \psi$, we measure the spherically averaged gas density profile and temperature profile for each simulated cluster. The temperature profile is measured as mass-weighted temperature $T_{mw}$, which is the correct representation of the average specific gas thermal energy for each radial shell. $T_{mw}$ is calculated as

$$ T_{mw} = \frac{\sum_i \rho_i T_i \Delta V_i}{\sum_i \rho_i \Delta V_i}, \quad (6) $$

where $\rho_i$, $T_i$, and $\Delta V_i$ are the gas density, gas temperature, and volume of the cell $i$ inside the radial shell and summed over all cells inside the radial bin.

Although mass-weighted temperature can in principle be measured by combining X-ray and SZ observations (e.g., Ameglio et al. 2007; Mroczkowski et al. 2007; Puchwein & Bartelmann 2007), X-ray observations alone do not measure the mass-weighted temperature. To approximate the temperature measured from X-ray spectra, we calculate $\Delta \psi$ with the spectroscopic-like temperature $T_{sl}$ (Mazzotta et al. 2004) which describes the temperature measured by Chandra and XMM-Newton well:

$$ T_{sl} = \frac{\sum_i \rho_i^2 T_i^{-0.75} \Delta V_i}{\sum_i \rho_i^2 \Delta V_i} \quad (7) $$

This weighting scheme preferentially weighs more toward regions with high gas density and low temperature. In our calculation of the spectroscopic-like temperature, we exclude cells that have temperature less than $10^6 \, K$, which is well below the instrumental response of current X-ray instruments. We denote different $\Delta \psi$ and the fitting parameters that use the mass-weighted temperature and the spectroscopic-like temperature by subscripts "mw" and "sl," respectively.

### 4. TESTING THE ESTIMATOR

We use high-resolution cosmological cluster simulations to compare the potential difference cosmological estimator $\Delta \psi$ with the real potential difference $\Delta \phi$ directly measured from the simulation. We have made the following assumptions for our estimator:

1. Input gas physics has little or no effect on the robustness of the estimator.
2. The assumption of hydrostatic equilibrium is justified.
3. The use of spectroscopic-like temperature is adequate to estimate the potential with little increase in the scatter of the $\Delta \phi$–$\Delta \psi$ relation.

### 4.1. Effects of Dissipation

Assuming hydrostatic equilibrium, we measure $\Delta \phi$ and $\Delta \psi_{mw}$ in the simulated clusters for both CSF and NR clusters at $z = 0$ for different radial distance separations: $[r_1, r_2]$, where we set $r_1/r_{500c} = 0, 0.2$, and $0.4$, and fix $r_2 = r_{500c}$, as it is the typical outermost radius current X-ray surveys are capable of measuring. This is also the radius within which the cluster is relaxed (Evrard et al. 1996) and where the non-thermal pressure is small, $\lesssim 10\%$ (e.g., Evrard 1990; Rasia et al. 2006; Nagai et al. 2007b). However, as we show later, the estimator is robust to the exact choice of radial separation.

### Table 2

| $z = 0$ | $[r_1/r_{500c}, r_2/r_{500c}]$ |
|--------|-----------------------------|
|        | [0.0, 1.0]                  |
| $\alpha_{mw}$ | CSF, $0.896 \pm 0.098$    |
|         | NR, $0.965 \pm 0.030$       |
| $\alpha_{sl}$ | CSF, $0.984 \pm 0.059$     |
|         | NR, $0.951 \pm 0.041$       |

### 5. The intrinsic scatter of the $\Delta \phi$–$\Delta \psi$ relation follows log-normal distribution.

In the following, we test the validity of the assumptions (1)–(4) and quantify the amount of scatter in the $\Delta \phi$–$\Delta \psi$ relation. For (5), we need a large sample of clusters to quantify the deviation from log-normal behavior for the scatter which we leave it for future work.

We report the values of the fitted slopes $\alpha$, normalization $\log(\mu_{\psi_{sl}})$, and intrinsic scatter $\sigma_{\Delta \phi, \psi_{sl}}$ of the $\Delta \phi$–$\Delta \psi$ scaling relations in upper halves of Tables 2–4. In the lower halves of these tables, we report the normalization and intrinsic scatter where we fit the $\Delta \phi$–$\Delta \psi$ relation with the slope fixed to unity, $\alpha = 1$. We report values of the fitted parameters estimated using both mass-weighted and spectroscopic-like temperatures wherever possible. Since we are measuring and estimating the potential difference between the inner radius $r_1$ and the outer radius $r_2$ for convenience we denote the radial interval for each scaling relation as $[r_1, r_2]$. 

## Table 2

### Scaling Relation Parameters at $z = 0$ for $r_1 = (0.0, 0.2, 0.4) r_{500c}$ and $r_2 = r_{500c}$

| $z = 0$ | $[r_1/r_{500c}, r_2/r_{500c}]$ |
|--------|-----------------------------|
|        | [0.0, 1.0]                  |
| $\alpha_{mw}$ | CSF, $0.896 \pm 0.098$    |
|         | NR, $0.965 \pm 0.030$       |
| $\alpha_{sl}$ | CSF, $0.984 \pm 0.059$     |
|         | NR, $0.951 \pm 0.041$       |
In Figure 1, we plot the $\Delta \phi$--$\Delta \psi_{mw}$ relations for $[r_1, r_2]/r_{500c} = [0.1, 0.2, 1], [0.4, 1]$ for both CSF and NR clusters. All slopes of the relations are within the expected value of unity within $1\sigma$. The normalization $\log(\mu_{mH} \Delta \psi_{0})$ is also near the expected value of $\log 50 \approx 1.3$. The intrinsic scatter varies from $\sim 6\%$ to $15\%$, depending on the radial separation and input cluster physics. In general the resulted scatters of the fixed slope relations increase slightly compared with those where the slope is a free parameter. The normalizations of the fixed slope relation remain unchanged to within $1\sigma$. The values of the fitted parameters are reported in Table 2.

For $[r_1, r_2]/r_{500c} = [0.2, 1], [0.4, 1]$, the slope, normalization, and scatter are similar for both CSF and NR clusters. Dissipative gas physics has little effect on the potential estimator outside the cluster core $r_1 \geq r_{2500c}$. For $[r_1, r_2]/r_{500c} = [0, 1]$, while the slope is similar in both CSF and NR clusters, the normalization of the CSF relation is offset to a higher value compared with the NR relation. As we discuss in Section 4.2, the higher normalization is due to strong gas rotation in the CSF clusters, where $\Delta \psi$ underestimates $\Delta \phi$, shifting the data points to the left of the $\Delta \phi$--$\Delta \psi_{mw}$ plane. This underestimation is also shown in the upper left panel of Figure 3, where $\Delta \psi_{mw}/\mu_{mH} \Delta \phi$ is plotted as a function of the inner radius $r_1$. The CSF relations have a relatively large scatter of about $18\%$ due to strong dissipation in the cluster core regions.

The potential estimator $\Delta \psi$ is robust to dissipational gas physics outside the cluster core. As we show in Section 4.2,
once we correct for non-thermal pressure support due the gas motions, the estimator becomes robust to gas physics even when the cluster core is included.

4.2. Non-thermal Pressure Support

We test the validity of the assumption of hydrostatic equilibrium of our potential estimator by explicitly including the contribution of non-thermal pressure support by gas motions, which are mainly responsible for the departure of hydrostatic equilibrium in our simulated clusters (Lau et al. 2009). The effects of gas motions can be corrected by including the following term $\Delta \psi_{\text{gas}}$, in Equation (3), derived from the spherically symmetric radial Jean’s equation (e.g., Binney & Tremaine 2008):

$$\Delta \psi_{\text{gas}} = -\left(\overline{v_r^2}(r_2) - \overline{v_r^2}(r_1)\right) - \int_{r_1}^{r_2} \frac{d \ln \rho}{d r} \overline{v_t}^2 d r,$$

$$- \int_{r_1}^{r_2} \frac{2 \overline{v_t} \overline{v_r} - \overline{v_t^2}}{r} d r,$$

where $\overline{v_r^2}$ and $\overline{v_t^2}$ are the radial and tangential components of the mean-square gas velocities averaged over the spherical radial shell, with respect to the cluster peculiar velocity defined as the average mass-weighted velocity of DM inside $r_{500c}$.

Figure 2 shows the scaling relation with and without the inclusion of the non-thermal pressure term $\Delta \psi_{\text{gas}}$. Including gas motions, i.e., replacing $\Delta \psi_{\text{mw}}$ with $\Delta \psi_{\text{mw}} + \Delta \psi_{\text{gas}}$, results in better estimates of the potential difference by decreasing the scatter of the scaling relation to $\sim 5\%$ for $[0,1]r_{500c}$ for both NR and CSF clusters. This is illustrated more clearly in Figure 3 where we plot the deviations of the estimated potential difference $\Delta \psi_{\text{mw}}$ from the true potential difference $\Delta \phi$ as a function of $r_1$ with $r_2$ fixed at $r_{500c}$, with solid lines representing the dynamically relaxed clusters and dashed lines representing the unrelaxed clusters. Without the inclusion of gas motions, $\Delta \psi_{\text{mw}}$ underestimates $\Delta \phi$ by about $5\%$–$20\%$ for $r_1 \gtrsim 0.1 r_{500c}$ in both NR and CSF clusters, a value that is consistent with gas motions biasing the thermal pressure low by about the same amount (Lau et al. 2009). Most of the scatter is driven by the dynamically disturbed systems in the sample. For $r_1$ approaching $r_2 = r_{500c}$, the scatter in both CSF and NR clusters increases as the outer regions of the cluster are less relaxed. In the inner region $r_1 \lesssim 0.1 r_{500c}$, the deviations from hydrostatic equilibrium are approximately constant for the NR clusters, but for CSF clusters, $\Delta \psi_{\text{mw}}$ increasingly underestimates $\Delta \phi$ as $r_1$ decreases due to strong gas rotation near the core where the gas is rotationally supported induced by gas cooling (Lau et al. 2011). Including gas motions takes into the account the effective potential due to the rotation, leading to a better recovery of the true potential difference and agreement between the scaling relations of the CSF and the NR clusters. This is shown in the upper right panel of Figure 2.

4.3. Spectroscopic-like Temperature

The spectroscopic-like temperature $T_\text{sl}$ (Equation (7)) is generally less than the mass-weighted temperature $T_{\text{mw}}$ (Equation (6)), as it weights toward colder and denser gas. As shown in Table 2, for different $[r_1, r_{500c}]$ the scatter in the $\Delta \phi - \Delta \psi_{\text{sl}}$ relation is also larger than $\Delta \phi - \Delta \psi_{\text{mw}}$ relation, as $\Delta \psi_{\text{sl}}$ is biased toward the colder and denser gas, whose fractions and locations vary across different clusters. Nevertheless, the scatter is increased only by a few percent, with the exception for $[r_1, r_2] = [0.4, 1]r_{500c}$ where the large scatter is driven by a single cluster which has abnormally low $T_\text{sl}$ due to a dense gas clump.
Comparing the $\Delta \phi - \Delta \psi_{sl}$ relations between the CSF and NR clusters, we find that the normalization for the CSF relations are slightly higher than their NR counterparts at large $r_1$, because of the relatively larger number of cold dense clumps in the CSF clusters than in the NR clusters. Nevertheless their respective normalizations agree to within 1$\sigma$.

To account for the limited angular resolution of X-ray observations, when we measure $T_{sl}$, we average it over a radial window of $\Delta r = 100$ kpc centered on the radius of interest. Varying the size of this window from 50 kpc to 200 kpc has little effect in changing the $\Delta \phi - \Delta \psi_{sl}$ relations.

### 4.4. Evolution with Redshift

Next, we investigate the evolution of the $\Delta \phi - \Delta \psi$ relations by fixing $[r_1, r_2] = [0.2, 1]r_{500c}$ and fit the relations at $z = 0.0, 0.6$, and 1.0. Figure 4 shows the resulting plots. All the relations at higher redshift are consistent with the $z = 0$ results to within 2$\sigma$. Scatter decreases slightly with increasing redshift and there is a weak trend of decreasing slope with redshift. The values of the parameters are shown in Table 3.

The apparent weak evolution in the $\Delta \phi - \Delta \psi$ relation can perhaps be explained by the lack of change in the gravitational potential over time once the DM halo has formed. For example, Li et al. (2007) demonstrated semi-analytically that the circular velocity at the virial radius of the halo remains relatively unchanged throughout much of its mass accretion history. Given that our sample size is quite small (16 clusters), the apparent redshift evolution could be driven by a few clusters. A complete understanding of the redshift evolution of the potential well and a full characterization of the redshift evolution of the $\Delta \phi - \Delta \psi$ relation will require tests using simulations with more clusters and redshift outputs from the current study.

### 4.5. Choice of Radial Separations

In principle the $r_1$ and $r_2$ that define $\Delta \phi \equiv \phi(r_2) - \phi(r_1)$ and $\Delta \psi$ can be chosen arbitrary. In practice, the best choice of $[r_1, r_2]$ would be the one that gives the lowest scatter. Once corrected for pressure from gas motions, the larger $|r_1 - r_2|$ will result in less scatter, as the integral in $\Delta \psi$ (Equation (3)) encompasses a larger radial range thus making the estimator more robust to small-scale variations in gas density and temperature. For example, setting $[r_1, r_2] = [0.1, 2]r_{500c}$ gives a scatter of about 6%, compared to 8% for $[r_1, r_2] = [0.1, 1]r_{500c}$. Decreasing $r_1$ also gives smaller scatter as long as one stays away from the cluster core for the CSF clusters. For example, the scatter decreases from 14% for $[r_1, r_2] = [0.4, 1]r_{500c}$ to 9% for $[r_1, r_2] = [0.2, 1]r_{500c}$. For $r_2 > r_{500c}$, the cluster gas deviates from hydrostatic equilibrium significantly with radius and $\Delta \psi$ underestimates the true potential difference $\Delta \phi$. However, this underestimation is small. As shown in Figure 5, the slope and normalization of the scaling relation are essentially the same as the radial interval changes from $[r_1, r_2] = [0.5, 1]r_{500c}$ to $[r_1, r_2] = [0.1, 2]r_{500c}$.

Following similar logic, the choice of $r_1$ and $r_2$ should not be limited to functions of $r_{500c}$. Defining $r_1$ and $r_2$ through $r_{500c}$ can be undesirable as $r_{500c}$ depends on cosmology through the Hubble parameter $H(z)$. One choice of the radial interval, independent of the halo radius, is simply the physical distance separations. Figure 6 shows the scaling relations for $[r_1, r_2] = [0.2, 1]r_{500c}$ at $z = 0.0, 0.6$ and 1.0. For this radial separation the $\Delta \phi - \Delta \psi$ relation has a relatively small scatter of 8%–9% for the given redshifts. The values of slope and the normalization of the relations are similar to those with $r_1$ and $r_2$ being functions of $r_{500c}$. The relation is the same for the different input gas physics. Redshift evolution of the relation is weak but requires more investigation, as discussed in Section 4.4.

### 4.6. Calibration for N-body Simulations

Although becoming increasingly feasible, full cosmological hydrodynamical simulations of cluster formation with realistic physics like radiative cooling, star formation, and active galactic nucleus feedback are expensive to run. A cheaper way would be to use these simulations to calibrate less expensive dissipation-less N-body simulations. Since both pure N-body simulations
and our NR clusters have gravity as the only driving physics, the results of our NR clusters should be similar to the that of the N-body results. In this subsection, we therefore investigate how well our more realistic CSF clusters measure the gravitational potential wells of the NR clusters, and therefore the potential wells of clusters in N-body simulations. The results will be useful for calibrating a cluster potential function from pure N-body simulations.

We estimate the NR potential difference \( \Delta \phi^{\text{NR}} \) using the potential estimator of the CSF clusters \( \Delta \psi^{\text{CSF}} \). In Figure 7, we show the comparison between the \( \Delta \phi^{\text{NR}} - \Delta \psi^{\text{CSF}} \) relation for \( [r_1, r_2] = [0.2, 1]r_{500c} \) at \( z = 0.0, 0.6, \) and \( 1.0 \) (blue solid lines) and the \( \Delta \phi^{\text{CSF}} - \Delta \psi^{\text{CSF}} \) relation (black dashed lines). The slope of the \( \Delta \phi^{\text{NR}} - \Delta \psi^{\text{CSF}} \) relation is steeper than the \( \Delta \phi^{\text{CSF}} - \Delta \psi^{\text{CSF}} \) relation. This is because dissipation results in deeper potential wells in the CSF clusters compared with their NR counterparts, and this effect is stronger in small size halos which have lower virial temperature. Therefore \( \Delta \psi^{\text{CSF}} \) overestimates the potential difference \( \Delta \phi^{\text{NR}} \) more for the smaller size systems, leading to a steeper slope in the \( \Delta \phi^{\text{NR}} - \Delta \psi^{\text{CSF}} \) relation. The scatter in the \( \Delta \phi^{\text{NR}} - \Delta \psi^{\text{CSF}} \) relation also increases because the NR clusters are not in exact identical dynamical states as their CSF counterparts, as the dynamical evolution of each cluster halo is affected by the dissipative gas physics which changes the structure and hence dynamics of the halos. Table 4 summarizes the fitted parameters.

5. SUMMARY AND DISCUSSION

5.1. Summary of Key Results

In this paper, we propose a simple estimator for the gravitational potential difference in clusters of galaxies. This estimator is based on the density and temperature profiles of the
Gravitational potential has another advantage over mass in that it is more spherical than that of the matter distribution. The assumption of spherical symmetry works better for gravitational potential than matter density, and this is perhaps partly why the potential scaling relation has relatively small scatter compared to mass-observable relations.

An essential characteristic of the gravitational potential scaling relation is that we are free to choose the radius where we measure the potential, not limiting to the halo radius defined in terms of overdensity with respect to the mean density of critical density of the universe, both of them functions of cosmology. This takes away the rather arbitrary nature of cluster mass, where different cluster mass definitions give different cluster mass functions (e.g., White 2002; Tinker et al. 2008).

Despite the advantages given above, there are also several caveats and uncertainties in using the proposed potential estimator for cluster cosmology. We see a weakly evolving redshift evolution in the potential scaling relation that needs to be understood well before using gravitational potential for cluster cosmology. Although the potentials measured at different radial separations result in essentially the same potential scaling relation, it is uncertain whether the different radial separation would result in different cluster potential functions. Furthermore, we have assumed spherical symmetry for the gravitational potential and the gas distribution, but have not tested for the effect of deviation from spherical symmetry, although we expect that the effect to be small compared with the effect on cluster mass. Since our gravitational potential estimator involves an integral over radius of gas temperature and density, which is the dominant term in the estimator (Equation (3)), the gas density and temperature profiles may need to be measured with high spatial resolution. Accurate measurements of gas temperature also require high-resolution X-ray spectrometers. This can be technically challenging, especially if we want to measure the potential of high-z clusters. However, we note that combined X-ray/SZ

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2 We give a comparison of the shape of the dark matter distribution and the shape of the gravitational potential well of the same set of clusters used in this paper in Lau et al. (2011).
analysis is able to recover the temperature profile without the need of X-ray spectra (e.g., Nord et al. 2009).

5.2. Toward the Cluster Potential Function

Given the results presented in the paper, it will be interesting to see whether the cluster potential function is indeed a better alternative for constraining cosmology than the commonly adopted cluster mass function. In this subsection, we discuss ways to improve the potential scaling relations and to construct the cluster potential function.

We suggest that future modeling efforts toward the goal of using the cluster potential function for cluster cosmology should focus on two main areas: (1) further investigation on the intrinsic nature of the potential scaling relation and the properties of the cluster potential function and (2) implementation of the potential scaling relation in actual observations.

The first aspect aims to answer the question whether the cluster potential function can replace the cluster mass function for cosmology. To see whether the cluster potential function has more constraining power than the cluster mass function, comparison between the cluster potential function and the cluster mass function calibrated from the same large-scale simulation is required. The comparison will have to focus on whether the cluster function recovers the fiducial cosmological parameters better in terms of degeneracies between parameters, statistical errors, and systematic uncertainties. Simulations with statistically large sample of clusters at multiple redshifts will help to constrain the weak redshift evolution of the potential scaling relation shown in the current work. They will also help further characterize the scatter and systematics of the potential scaling relation, for example, deviations from log-normal distribution in the scatter.

The second aspect focuses on developing new ways of characterizing gravitational potentials based on cluster observables and apply them to synthetic and real observations. For example, the potential estimator proposed in the current work will benefit from mock X-ray analyses in addressing systematics like gas clumping (Mathiesen et al. 1999; Simionescu et al. 2011; Nagai & Lau 2011) and some of its observational challenges discussed in the previous subsection. It is reasonable to expect that other types of cluster potential estimators exist and may even perform better than the one proposed in this paper. For example, gravitational lensing signals like shear and convergence depend on the projected gravitational potential and may provide a new probe to the gravitational potential. Velocity dispersion of cluster galaxies directly probes the cluster potential, although it can be affected by velocity anisotropy and projection effects. SZ signals measure the integrated gas pressure, and it is similar to the gas enthalpy-based estimator proposed in the current work. Multiple probes of the gravitational field will be helpful in determining systematics, and the covariance of different potential estimator may also improve the precision of the measured potential, as it is for the case of cluster mass proxies (Stanek et al. 2010).

Clearly, much more work is needed to address these issues and we hope that this paper provides a starting point for the investigation of using the cluster potential function for cosmology.

The author thanks the anonymous referee whose suggestions greatly improve the content and presentation of this paper. The author also thanks his PhD advisor, Andrey Kravtsov, for his continuous support, guidance, and patience, and Daisuke Nagai for kindly providing the simulation data and offering invaluable advice. He also thanks Mike Gladders, Nick Gnedin, and Rick Kron for helpful comments. The author is supported by the NSF grant AST-0708154 and by NASA grant NAG5-13274. The cosmological simulations used in this study were performed on the IBM RS/6000 SP4 system (copper) at the National Center for Supercomputing Applications (NCSA). This work made extensive use of the NASA Astrophysics Data System and arXiv.org preprint server.

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