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Critical states in a superconducting plate: Effects of heat generation from flux motion

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Abstract. We have studied vortex penetration into a superconducting plate under a perpendicular magnetic field. Solving the heat transport equation and the Maxwell equations with the current-voltage relation for the superconductor simultaneously, current density and temperature distribution due to vortices motion are obtained. Taking the effect of heat generation into account, there is a size dependence of temperature rise.

1. Introduction

When a perpendicular external magnetic field, which is larger than the lower critical field, is applied to a type II superconducting plate, the superconductor comes to be in a vortex state. Vortices penetrate to the superconducting plate from its edge. There are two kinds of vortex penetrations: a critical state and a vortex avalanche [1]. In the critical state, the Meissner shielding currents flow along an edge of the superconducting plate, and the Lorentz force due to the current drives vortices into the inside of the superconducting plate. When microscopic defects are present in the superconductor, vortices may be pinned by these defects. The pinning force from defects acts against the driving Lorentz force. The local balance between two forces creates a metastable equilibrium state, which is the critical state. In the critical state, magnitude of the shielding current density is adjusted to the critical current density.

On the other hand, the vortex avalanche is caused by thermal fluctuation at finite temperature. The vortex, which is pinned by the pinning center, jumps to another pinning center because of the driving force and the thermal fluctuation. This movement generates heat, and the temperature locally rises. Then superconductivity is weakened. Therefore, vortices further penetrate into the superconductor. Repeating this process, there appears the vortex avalanche.

In this study, we focus on the critical state. The critical states have been studied widely [1, 2, 3]. Although the influence of heat transport has not been taken into consideration, we investigate influence of heat transport on the vortex motion as in the vortex avalanche study [4]. We solved the heat transport equation and the Maxwell equation with a current-voltage relation for the superconductor with three-dimensional finite element method. We obtain distribution of temperature and current density when vortices penetrate into the superconducting plate.
2. Method
In this study, we solve following three equations. The first is the heat transport equation,
\[ \mathcal{C} \frac{\partial T}{\partial t} = \frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} - \alpha (T - T_0) + \mathbf{J} \cdot \mathbf{E}. \] (1)

Here, \( \mathcal{C} \) is a specific heat, \( T(x, y, z, t) \) is temperature of the superconductor, and \( X(T), Y(T), \) and \( Z(T) \) are the heat flow in \( x, y, \) and \( z \) directions, respectively. The forth term on the right hand side is the heat transfer from the superconductor to a substrate, where \( \alpha \) is a heat transfer rate, and \( T_0 \) is temperature of the substrate. The fifth term on the right hand side is the heat generation due to the creep motion of vortices, where \( \mathbf{J} \) is a current density, and \( \mathbf{E} \) is an electric field. These \( X(T), Y(T), \) and \( Z(T) \) are given as follows,
\[
\begin{align*}
X(T) &= \kappa_{11} \frac{\partial T}{\partial x} + \kappa_{12} \frac{\partial T}{\partial y} + \kappa_{13} \frac{\partial T}{\partial z} \\
Y(T) &= \kappa_{21} \frac{\partial T}{\partial x} + \kappa_{22} \frac{\partial T}{\partial y} + \kappa_{23} \frac{\partial T}{\partial z} \\
Z(T) &= \kappa_{31} \frac{\partial T}{\partial x} + \kappa_{32} \frac{\partial T}{\partial y} + \kappa_{33} \frac{\partial T}{\partial z}
\end{align*}
\] (2)

where \( \kappa \) is a heat conductivity tensor.

The second is the Maxwell equations,
\[
\begin{align*}
\text{rot}\{\text{rot} \mathbf{A} - \mathbf{H}_a\} &= \frac{4\pi}{c} \mathbf{J} \\
\mathbf{E} &= \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}
\end{align*}
\] (3)

where \( \mathbf{A} \) is a magnetic vector potential, \( \mathbf{H}_a \) is an external magnetic field, and \( c \) is the speed of light.

The third is the current-voltage relation for the superconductor [4],
\[ \mathbf{E} = \rho(\mathbf{J}) \mathbf{J}. \] (4)

Here \( \rho(\mathbf{J}) \) is a resistivity given by,
\[
\rho(\mathbf{J}) = \begin{cases} 
\rho_0 \left( \frac{\mathbf{J}}{\mathbf{J}_c} \right)^{n-1} & \mathbf{J} \leq \mathbf{J}_c, T \leq T_c \\
\rho_0 & \mathbf{J} > \mathbf{J}_c, T \leq T_c \\
\rho_n & \mathbf{J} > \mathbf{J}_c, T > T_c
\end{cases}
\] (5)

where \( n \) is a creep exponent, \( \rho_0 \) is a resistivity constant of superconducting state, \( \rho_n \) is a resistivity of normal state, \( \mathbf{J}_c \) is a critical current density, and \( T_c \) is the critical temperature. The flux creep exponent is given by \( n - 1 = n_0 \left( 1 - T/T_c \right) \), where \( n_0 \) is a constant. This current-voltage relation represents the fact that due to the current flow, vortices creeps and the resistance rises. The current and temperature dependences of the resistivity in the superconductor are shown in figure 1.
3. Results and Discussion
In this study, we consider two sizes of superconducting plates. The sizes of superconductors were 8×16×0.2 µm³ and 80×160×2 µm³, for smaller and larger superconducting plates, respectively. The superconductors were surrounded by the substrate shown in figure 2. System sizes were 10×18×0.4 µm³ and 100×180×4 µm³ for smaller and larger superconducting plates, respectively.

We use material parameters corresponding to MgB₂, $\mathcal{C} = 35 \text{ kJ/Km}^3 \times (T/T_c)^3$, $\alpha = 220 \text{ kW/Km}^2 \times (T/T_c)^3$, $\kappa = 0.17 \text{ kW/Km} \times (T/T_c)^3$, $\rho_n = \rho_0 = 7 \ \mu\Omega\text{cm}$, $T_c = 39 \text{ K}$, $J_{c0} = 10 \text{ mA/m}^2$, $n_a = 19$. The external applied magnetic field is ramped up from $H_a = 0 \text{ kG}$ at a constant rate, $\dot{H}_a = 5 \times 10^{15} \text{ kG/s}$. The initial temperature of the total system is 10 K. Although the temperature of the superconductor rises, the temperature of the substrate is kept constant 10 K.

Figure 3 shows a three-dimensional distribution of the current density in the smaller superconducting plate, when the external magnetic field is $H_a = 5 \times 10^7 \text{ kG}$. The current density is large around corners of the superconducting plate. Figures 4(a) and (b), and figures 4(c) and (d) show the development of distributions of $x$ component of the current density $J_x$ for smaller and larger superconducting plates, respectively. The external magnetic fields are raised from $H_a = 5 \times 10^7 \text{ kG}$, $H_a = 15 \times 10^7 \text{ kG}$, $H_a = 20 \times 10^7 \text{ kG}$ and $H_a = 25 \times 10^7 \text{ kG}$ in figures 4(a) and (c), and from $H_a = 5 \times 10^7 \text{ kG}$. The current density is large around corners of the superconducting plate.
$23 \times 10^7 \text{kG}, \ H_a = 24 \times 10^7 \text{kG} \text{ to } H_a = 25 \times 10^7 \text{kG} \text{ in figures 4(b) and (d). Comparing figures 4(a) and (c), we find that the current density in the smaller superconducting plate is larger than that in the larger superconducting plate. In figure 4(b), superconductivity is destroyed for } H_a = 25 \times 10^7 \text{kG \text{ where the current density is zero.}}$

Next, we show temperature distribution in x-y plane in figure 5. Figures 5(a)-(d), and figures 5(e)-(h) show the temperature distributions for smaller and larger superconducting plates, respectively. The external magnetic field is $H_a = 5 \times 10^7 \text{kG}$ for (a) and (e), $H_a = 15 \times 10^7 \text{kG}$ for (b) and (f), $H_a = 20 \times 10^7 \text{kG}$ for (c) and (g), $H_a = 25 \times 10^7 \text{kG}$ for (d) and (h). These temperature distributions are dynamically depicted in each magnetic field, which is gradually increased.

**Figure 3.** Three-dimensional current density for smaller superconducting plate

**Figure 4.** Current density in x direction; (a) and (b) the superconductor size is $8 \times 16 \times 0.2 \mu \text{m}^3$, (c) and (d) the superconductor size is $80 \times 160 \times 2 \mu \text{m}^3$
We find temperature rise at corners in superconducting plate in figures 5(b) and (f). In figure 5(d), where the external magnetic field is $H_a = 25 \times 10^7$ kG, the superconductivity is completely destroyed in the smaller superconducting plate since temperature exceeds critical temperature 39 K. On the other hand, in figure 5(h), superconductivity is destroyed only along edges in the larger superconducting plate at same magnetic field.

We can explain the change of current density and temperature rise on critical state in these results, as follows. Current density is large at corners of superconducting plate where temperature begins to rise. After that, heat propagates isotropically, the temperature rises around short sides because of short propagation distance. In the smaller superconducting plate, when the temperature exceeds the critical temperature 39 K, the superconductivity is destroyed and the current density becomes zero. While in the larger superconducting plate, the current density has finite value and superconductivity is only partially destroyed. But these results are a little strange. Considering the effect of demagnetization, the magnetic field locally becomes large at the edge of the superconducting plate and vortices are likely to penetrate. So larger superconducting plate generally lose superconductivity than smaller one. In order to solve this contradiction, we should consider following point: Total amounts of current flowing inside of both of the superconductors are same, but for smaller superconductor, the current density becomes larger. Therefore, the heat generation also increases, and the smaller the superconducting plate, superconductivity is easily destroyed. So, in this model, as we take the heat generation by current into account, these results are reasonable.

Figure 5. Temperature distribution in $x$-$y$ plane. Sizes of superconductors are $8 \times 16 \times 0.2$ $\mu$m$^3$ for (a), (b), (c) and (d) and $80 \times 160 \times 2$ $\mu$m$^3$ for (e), (f), (g) and (h). The external field is $H_a = 5 \times 10^7$ kG for (a) and (e), $H_a = 15 \times 10^7$ kG for (b) and (f), $H_a = 20 \times 10^7$ kG for (c) and (g), $H_a = 25 \times 10^7$ kG for (d) and (h).
4. Summary and Future
We have studied critical state for superconducting plates. Solving the heat transport equation and the Maxwell equation with current-voltage relation for the superconductor by using three-dimensional finite element method, we obtained current density and temperature distribution. We found the effect of heat generation of superconductivity in small system for critical state and destruction of superconductivity. There is a size dependence of temperature rise, and superconducting state in smaller system is destroyed easier than that larger one.

In this study in order to obtain maximum thermal effects on the critical state, we consider a situation with the magnetic field up to $H_a = 25 \times 10^7 \text{kG}$, which is much higher than today’s status experimental techniques. But even for lower magnetic field, similar but weak effects will appear, we think. Such effects on the critical state as well as vortex avalanches are future problems.

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