High-Utility Interval-Based Sequences

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Abstract. Sequential pattern mining is an interesting research area with a broad range of applications. Most prior research on sequential pattern mining has considered point-based data where events occur instantaneously. However, in many application domains, events persist over intervals of time of varying lengths. Furthermore, traditional frameworks for sequential pattern mining assume all events have the same weight or utility. This simplifying assumption neglects the opportunity to find informative patterns in terms of utilities, such as cost. To address these issues, we incorporate the concept of utility into interval-based sequences and define a framework to mine high utility patterns in interval-based sequences. In the proposed framework, the utility of events is considered while assuming multiple events can occur coincidentally and persist over varying periods of time. An algorithm named High Utility Interval-based Pattern Miner (HUIPMiner) is proposed and applied to real datasets. To achieve an efficient solution, HUIPMiner is augmented with a pruning strategy. Experimental results show that HUIPMiner is an effective solution to the problem of mining high utility interval-based sequences.

Keywords: High utility interval-based, utility mining, sequential mining, temporal pattern, event interval sequence

1 Introduction

Sequential pattern mining aims to find patterns from data recorded sequentially along with their time of occurrence. Depending on the application scenario, symbolic sequential data is categorized as either point-based or interval-based. Point-based data reflect scenarios in which events happen instantaneously or events are considered to have equal time intervals. Duration has no impact on extracting patterns for this type. Interval-based data reflect scenarios where events have unequal time intervals; here, duration plays an important role.

In many application domains, such as medicine [11], sensor technology [3], sign language [4], and motion capture [5], events persist over intervals of time of varying lengths, which results in complicated temporal relations among events. Thirteen possible temporal relations between a pair of event intervals were nicely categorized by Allen [6]. Some studies have been devoted to mining frequent sequential patterns from interval-based data and describing the temporal relations among the event intervals. Wu and Chen [7] presented a nonambiguous
representation of temporal data utilizing the beginning and ending time points of the events. By adapting the PrefixSpan [8], they proposed the TPrefixSpan algorithm to mine frequent temporal sequences. Chen et al. [9] proposed the co-occurrence representation to simplify the processing of complex relations among event intervals. They also proposed an algorithm named CTMiner to discover frequent time-interval based patterns in large databases.

The aforementioned work has focused on representations of temporal data and discovering frequent temporal patterns. However, frequent pattern mining (FPM) may not be the right solution to problems where the weight of patterns may be the major factor of interest and the frequency of patterns may be a minor factor. The weight of a pattern can be interpreted differently depending on the problem or scenario. For example, it may represent the profit or the cost that a business experiences when a particular pattern occurs. Some patterns of interest may have high weights but low frequencies. Thus, FPM may miss patterns that are infrequent but valuable. FPM may also extract too many frequent patterns that are low in weight. To address these problems, high utility pattern mining (HUPM) has emerged as an alternative to FPM. The goal of HUPM is to extract patterns from a dataset with utility no less than a user-specified minimum utility threshold.

Tackling the HUPM problem requires facing more challenges than FPM. The major FPM algorithms rely on the downward closure property (also known as the Apriori Property) [10] to perform efficiently. This property, which is utilized by most pruning strategies, states that if a pattern is frequent then all of its sub-patterns are frequent and if a pattern is infrequent all of its super-patterns are infrequent. However, this property does not hold in utility mining because the utilities of patterns are neither monotone nor anti-monotone [11]. As a result, the existing optimization approaches for FPM are not applicable to HUPM. To cope with this challenge, previous studies introduced several domain-dependent weighted downward closure properties, including the transaction-weighted downward closure property (TDCP) [12] for itemset pattern mining, the sequence-weighted downward closure property (SDCP) [13] for sequential pattern mining, and the episode-weighted downward closure property (EDCP) [14,15] for episode pattern mining.

Most prior studies on HUPM have been devoted to transactional data rather than sequential data. However, such studies do not address the problem of HUPM in interval-based sequences, which covers a wide range of applications mentioned above. Interval-based applications can be better described when the concept of utility is employed. For example, interval-based sequences commonly occur in businesses where different services or packages, which persist over time, are offered to customers. Providing informative patterns to policy makers is an essential task, especially in a competitive marketplace. Neglecting the fact that these services or packages have various utilities (or weights) results in misleading information. For instance, HUPM can be beneficial to telecommunication companies or insurance companies which sell products that last over varying periods of time at various costs.
To the best of our knowledge, Huang et al. [16] recently made the first attempt to mine interval-based sequence data for patterns based on utility. They suggested a method to discover the top-K high utility patterns (HUPs). Their approach consists of two main parts. It first discovers a set of frequent patterns, and then it extracts the top-K HUPs from the set. This indirect approach suffers from a major drawback. The set of frequent patterns may not contain all HUPs. Hence, the approach may miss some high utility but infrequent patterns and consequently, it may select low-ranked HUPs as the top-K HUPs.

For the above reasons, we formalize the problem of the mining of high utility interval-based patterns (HUIPs) from sequences and propose a new framework to solve this problem. The major contributions of this work are as follows: 1) We propose the coincidence eventset representation (CER) to represent interval-based events. 2) We incorporate both internal and external utilities into interval-based sequences and propose an algorithm called HUIPMiner to mine all high utility sequential patterns from interval-based sequences; 3) We introduce the L-sequence-weighted downward closure property (LDCP), which is used in our pruning strategy and utilize LDCP in HUIPMiner to reduce the search space and identify high utility sequential patterns efficiently; and 4) We report on experiments that show the proposed framework and algorithm are able to discover all high utility patterns from interval-based data even with a low minimum utility threshold.

The rest of the paper is organized as follows. Section 2 provides background and preliminaries. It then proposes a framework of interval-based sequence utility and finally it formulates the problem of mining high utility interval-based sequential patterns. Section 3 presents the details of the HUIPMiner algorithm and the pruning strategy. Experimental results on real datasets and evaluation are given in Section 4. Section 5 presents conclusions and future work.

2 Problem Statement

Let \( \sum = \{A, B, \ldots\} \) denote a finite alphabet. A triple \( e = (l, b, f) \), where \( l \in \sum \) is the event label, \( b \in \mathbb{N} \) is the beginning time, and \( f \in \mathbb{N} \) is the finishing time \( (b < f) \), is called an event-interval. An event-interval sequence or \( E \)-sequence \( s = \langle e_1, e_2, \ldots, e_n \rangle \) is a list of \( n \) event intervals ordered based on beginning time in ascending order. If event-intervals have equal beginning times, then they are ordered lexicographically by their labels. The size of \( E \)-sequence \( s \), denoted as \( |s| = n \), is the number of event-intervals in \( s \). A database \( D \) that consists of set of tuples \( (sid, s) \), where \( sid \) is a unique identifier of \( s \), is called an \( E \)-sequence database. Table 2.1 depicts an \( E \)-sequence database consisting of four \( E \)-sequences with identifiers 1 to 4.

**Definition 1.** Given an \( E \)-sequence \( s = \langle (l_1, b_1, f_1), (l_2, b_2, f_2), \ldots, (l_n, b_n, f_n) \rangle \), the multiset \( T = \{b_1, f_1, b_2, f_2, \ldots, b_n, f_n\} \) consists of all time points corresponding to sequence \( s \). If we sort \( T \) in ascending order and eliminate redundant elements, we can derive a sequence \( T_s = \langle t_1, t_2, \ldots, t_m \rangle \), where \( t_k \in T, t_k < t_{k+1} \).
Table 2.1: Example of an E-sequence database

| sid | Event Label | Beginning Time | Finishing Time | Event Sequence |
|-----|-------------|----------------|---------------|----------------|
| 1   | A           | 8              | 16            | A              |
|     | B           | 18             | 21            | B              |
|     | C           | 24             | 28            | C              |
|     | E           | 25             | 27            | E              |
| 2   | A           | 1              | 5             | A              |
|     | C           | 8              | 14            | C              |
|     | E           | 9              | 12            | E              |
|     | F           | 9              | 12            | F              |
| 3   | B           | 6              | 12            | B              |
|     | A           | 7              | 14            | A              |
|     | C           | 14             | 20            | C              |
|     | E           | 16             | 18            | E              |
| 4   | B           | 2              | 7             | B              |
|     | A           | 5              | 10            | A              |
|     | D           | 5              | 12            | D              |
|     | C           | 16             | 22            | C              |
|     | E           | 18             | 20            | E              |

$T_s$ is called the *E-sequence unique time points* of $s$. We denote the number of elements in $T_s$ by $|T_s| = m$.

**Definition 2.** Let $s = \langle (l_1, b_1, f_1), ..., (l_j, b_j, f_j), ..., (l_n, b_n, f_n) \rangle$ be an E-sequence. A function $\Phi_s : \mathbb{N} \times \mathbb{N} \rightarrow 2^\Sigma$ is defined as:

$$\Phi_s(t_p, t_q) = \{l_j \mid (l_j, b_j, f_j) \in s \land (b_j \leq t_p) \land (t_q \leq f_j)\}$$  \hspace{1cm} (1)

where $1 \leq j \leq n$ and $t_p < t_q$. Given an E-sequence $s$ with corresponding E-sequence unique time points $T_s = \langle t_1, t_2, ..., t_m \rangle$, a coincidence $c_k$ is defined as $\Phi_s(t_k, t_{k+1})$ where $t_k, t_{k+1} \in T_s$, $1 \leq k \leq m-1$, are two consecutive time points. The duration $\lambda_k$ of coincidence $c_k$ is $t_{k+1} - t_k$. The size of a coincidence is the number of event labels in the coincidence.

For example, the E-sequence unique time points of $s_2$ in Table 2.1 is $T_{s_2} = \{1, 5, 8, 9, 12, 14\}$. Coincidence $c_4 = \Phi_{s_2}(9, 12) = \{C, E, F\}$, $\lambda_4 = 3$ and $|c_4| = 3$.

**Definition 3.** A coincidence label sequence, or *L-sequence* $L = \langle c_1, c_2, ..., c_g \rangle$ is an ordered list of $g$ coincidences. An L-sequence is called a $K$-L-sequence, iff there are exactly $K$ coincidences in the L-sequence. We define the size of an L-sequence, denoted $Z$, to be the maximum size of any coincidences in the L-sequence.

For example, $\langle \{B\} \{A, B\} \{A\} \rangle$ is a 3-L-sequence because it has 3 coincidences and its size is 2 because the maximum size of the coincidences in it is $max\{1, 2, 1\} = 2$.

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Table 2.2: C-sequence database corresponding to the E-sequences in Table 2.1

| sid | C-sequence                                      |
|-----|-------------------------------------------------|
| 1   | ⟨(A, 8)(∅, 2)(B, 3)(∅, 3)(C, 1){(C, E), 2}(C, 1)⟩ |
| 2   | ⟨(A, 4)(∅, 3)(C, 1){(C, E, F), 3}(C, 2)⟩       |
| 3   | ⟨(B, 1){(A, B), 5}(A, 2)(C, 2){(C, E), 2}(C, 2)⟩ |
| 4   | ⟨(B, 3){(A, B, D), 2}{(A, D), 3}(D, 2)(∅, 4)(C, 2){(C, E, F), 2}(C, 2)⟩ |

2.1 The Coincidence Eventset Representation (CER)

The representations proposed in previous studies, such as [7,9], do not store the durations of intervals. These approaches transform each event interval into a point-based representation encompassing only temporal relations. Although these formats are described as unambiguous, they actually leave an ambiguity with respect to duration. It is true that the temporal relations among intervals can be mapped one-to-one to the temporal sequence by these representations, but the duration for which these relations persist is ignored. Consequently, it is impossible to reverse the process and reconstruct the original E-sequence if we receive one of these representations. In this section, we address this limitation by incorporating the duration of intervals into a new representation called the coincidence eventset representation (CER).

Definition 4. Given a coincidence $c_k$ in E-sequence $s$, a coincidence eventset, or C-eventset, is denoted $\sigma_k$ and defined as an ordered pair consisting of the coincidence $c_k$ and the corresponding coincidence duration $\lambda_k$, i.e.:

$$\sigma_k = (c_k, \lambda_k)$$

For brevity, the braces are omitted if $c_k$ in C-eventset $\sigma_k$ has only one event label, which we refer as a C-event. A coincidence eventset sequence, or C-sequence, is an ordered list of C-eventsets, which is defined as $C = \langle \sigma_1...\sigma_{m-1} \rangle$, where $m = |T_s|$. A C-sequence database $\delta$ consists of a set of tuples \((\text{sid}, C)\), where sid is a unique identifier of $C$.

For example, the E-sequences in the database shown in Table 2.1 can be represented by the CER to give the C-sequences shown in Table 2.2. We denote the sid = 1 C-sequence as $C_{s1}$; other C-sequences are numbered accordingly. The “∅” symbol is used to distinguish disjoint event intervals. A “∅” indicates a gap between two event intervals, whereas the lack of a “∅” indicates that the two event intervals are adjacent. It can be seen that CER incorporates the durations of the event intervals into the representation.

Definition 5. Given two C-eventsets $\sigma_a = (c_a, \lambda_a)$ and $\sigma_b = (c_b, \lambda_b)$, $\sigma_b$ contains $\sigma_a$, which is denoted $\sigma_a \subseteq \sigma_b$, iff $c_a \subseteq c_b \wedge \lambda_a = \lambda_b$. Given two C-sequences $C = \langle \sigma_1\sigma_2...\sigma_n \rangle$ and $C' = \langle \sigma'_1\sigma'_2...\sigma'_n \rangle$, we say $C$ is a C-subsequence of $C'$, denoted $C \subseteq C'$, iff there exist integers $1 \leq j_1 \leq j_2 \leq ... \leq j_n \leq n'$ such that $\sigma_k \subseteq \sigma'_k$ for $1 \leq k \leq n$. Given a C-sequence $C = \langle \sigma_1\sigma_2...\sigma_n \rangle = \langle (c_1, \lambda_1)(c_2, \lambda_2)...(c_n, \lambda_n) \rangle$
and an L-sequence $L = \langle c'_1 c'_2 \ldots c'_m \rangle$, $C$ matches $L$, denoted as $C \sim L$, iff $n = m$ and $c_k = c'_k$ for $1 \leq k \leq n$.

For example, $\langle (A, 2) \rangle$, $\langle (A, 2) (A, 3) \rangle$, and $\langle \{(A, B, D), 2 \} \rangle$, are C-subsequences of C-sequence $C_{s_4}$, while $\langle \{(A, F), 2 \} \rangle$ and $\langle (A, 2) (D, 5) \rangle$ are not. It is possible that multiple C-subsequences of a C-sequence match a given L-sequence. For example, if we want to find all C-subsequences of $C_{s_4}$ in Table 2.2 that match the L-sequence $\langle A \rangle$, we obtain $\langle (A, 2) \rangle$ in the second C-eventset and $\langle (A, 3) \rangle$ in the third C-eventset.

2.2 Utility

Let each event label $l \in \sum_i$ be associated with a value, called the external utility, which is denoted as $p(l)$, such that $p : \sum \rightarrow \mathbb{R}_{\geq 0}$. The external utility of an event label may correspond to any value of interest, such as the unit profit or cost, that is associated with the event label. The values shown in Table 2.3 are used in the following examples as the external utilities associated with the C-sequence database shown in Table 2.2.

| Event label | A | B | C | D | E | F | δ |
|-------------|---|---|---|---|---|---|---|
| External utility | 1 | 2 | 1 | 3 | 2 | 1 | 0 |

| Event label | A | B | C | D | E | F | δ |
|-------------|---|---|---|---|---|---|---|
| External utility | 1 | 2 | 1 | 3 | 2 | 1 | 0 |

Let the utility of a C-event $(l, \lambda)$ be $u(l, \lambda) = p(l) \times \lambda$. The utility of a C-eventset $\sigma = \{(l_1, l_2, \ldots, l_n), \lambda\}$ is defined as: $u_\sigma(\sigma) = \sum_{i=1}^{n} u(l_i, \lambda)$. The utility of a C-sequence $C = \langle \sigma_1 \sigma_2 \ldots \sigma_m \rangle$ is defined as: $u(C) = \sum_{i=1}^{m} u(\sigma_i)$. Therefore, the utility of the C-sequence database $\delta = \{ \langle sid_1, C_{s_1} \rangle, \langle sid_2, C_{s_2} \rangle, \ldots, \langle sid_r, C_{s_r} \rangle \}$ is defined as: $u(\delta) = \sum_{i=1}^{r} u(C_{s_i})$. For example, the utility of C-sequence $C_{s_3} = \langle (B, 1) \{(A, B), 5 \} (A, 2) (C, 2) \{(C, E), 2 \} (C, 2) \rangle$ is $u(C_{s_3}) = 1 \times 2 + 5 \times (1 + 2) + 2 \times 1 + 2 \times 1 + 2 = 39$, and the utility of the C-sequence database $\delta$ in Table 2.2 is $u(\delta) = u(C_{s_1}) + u(C_{s_2}) + u(C_{s_3}) + u(C_{s_4}) = 22 + 19 + 29 + 46 = 116$.

**Definition 6.** The maximum utility of $k$ C-eventsets in a C-sequence is defined as: $u_{\text{max}}(C, k) = \max \{ u(C') \mid C' \subseteq C \land |C'| \leq k \}$. Note: In the name of the $u_{\text{max}}$ function, the “$k$” is part of the name rather than a parameter.

For example, the maximum utility of 2 C-eventsets in C-sequence $C_{s_3} = \langle (B, 1) \{(A, B), 5 \} (A, 2) (C, 2) \{(C, E), 2 \} (C, 2) \rangle$ is $u_{\text{max}}(C_{s_3}, 2) = u(C_{s_3}, 2) = u(\{(\{A, B\}, 5 \}\{(C, E), 2 \})) = 15 + 6 = 21$.

**Definition 7.** Given a C-sequence database $\delta$ and an L-sequence $L = \langle c_1 c_2 \ldots c_n \rangle$, the utility of $L$ in C-sequence $C = \langle \sigma_1 \sigma_2 \ldots \sigma_m \rangle \in \delta$ is defined as a utility set:

$$u(L, C) = \bigcup_{C' \sim L \land C' \subseteq C} u_{\text{max}}(C')$$

(3)
The utility of \( L \) in \( \delta \) is also a utility set:

\[
\mathit{u}_L(L) = \bigcup_{C \in \delta} \mathit{u}_L(L, C) \quad (4)
\]

For example, consider \( L \)-sequence \( L = \langle \{B\}\{A\} \rangle \). The utility of \( L \) in \( C_{s_3} \) shown in Table 2.2 is \( \mathit{u}_L(L, C_{s_3}) = \{ \mathit{u}_L(\langle \{B,1\}\{A,5\} \rangle), \mathit{u}_L(\langle \{B,1\}\{A,2\} \rangle), \mathit{u}_L(\langle \{B,5\}\{A,2\} \rangle) \} = \{7,4,12\} \), and thus the utility of \( L \) in \( \delta \) is \( \mathit{u}_L(L) = \{ \mathit{u}_L(L, C_{s_3}), \mathit{u}_L(L, C_{s_4}) \} = \{\{7,4,12\}, \{8,9,7\}\} \). From this example, one can see that an \( L \)-sequence may have multiple utility values associated with it, unlike a sequence in frequent sequential pattern mining.

### 2.3 High Utility Interval-based Pattern Mining

**Definition 8.** The maximum utility of an \( L \)-sequence \( L \) in \( C \)-sequence database \( \delta \) is defined as \( \mathit{u}_{\text{max}}(L) \):

\[
\mathit{u}_{\text{max}}(L) = \sum_{C \in \delta} \max(\mathit{u}_L(L, C)) \quad (5)
\]

For example, the maximum utility of an \( L \)-sequence \( L = \langle \{B\}\{A\} \rangle \) in \( C \)-sequence database \( \delta \) shown in Table 2.2 is \( \mathit{u}_{\text{max}}(L) = 0 + 0 + 12 + 9 = 21 \).

**Definition 9.** An \( L \)-sequence \( L \) is a high utility interval-based pattern iff its maximum utility is no less than a user-specified minimum utility threshold \( \xi \). Formally: \( \mathit{u}_{\text{max}}(L) \geq \xi \iff L \) is a high utility interval-based pattern.

**Problem I:** Given a user-specified minimum utility threshold \( \xi \), an \( E \)-sequence database \( D \), and external utilities for event labels, the problem of high utility interval-based mining is to discover all \( L \)-sequences such that their utilities are at least \( \xi \). By specifying the maximum length and size of the \( L \)-sequence, Problem I can be specialized to give **Problem II**, which is to discover all \( L \)-sequences with lengths and sizes of at most \( K \) and \( Z \), respectively, such that their utilities are at least \( \xi \).

### 3 The HUIPMiner Algorithm

In this section, we propose the \textit{HUIPMiner} algorithm to mine high utility interval-based patterns. HUIPMiner is composed of two phases in which each iteration generates a special type of candidates of a certain length. We also obtain the \( L \)-sequence-weighted downward closure property (LDCP) (Theorem 11), which is similar to the sequence-weighted downward closure property (SDCP) [13]. LDCP is utilized in the proposed pruning strategy to avoid generating unpromising \( L \)-sequence candidates. LDCP has an advantage over SDCP since it reduces the size of the search space by using a tighter upper bound, which we present in Definition [10].
Definition 10. \((\text{LWU}_k)\) The L-sequence-weighted utilization of an L-sequence w.r.t. a maximum length \(k\) is defined as:

\[
\text{LWU}_k(L) = \sum_{C' \sim L, C' \subseteq C \land C \in \delta} u_{\text{max}}(C, k)
\]

For example, the L-sequence-weighted utilization of \(L = \langle \{B\} \{A\} \rangle\) w.r.t. the maximum length \(k = 2\) in the C-sequence database shown in Table 2.2 is \(\text{LWU}_2((\{B\} \{A\})) = 0 + 0 + 21 + 24 = 45\).

Theorem 1 (L-sequence-weighted downward closure property). Given a C-sequence database \(\delta\) and two L-sequences \(L\) and \(L'\), where \(L \subseteq L'\) and \(|L'| \leq k\), then

\[
\text{LWU}_k(L') \leq \text{LWU}_k(L)
\]

Proof. Let \(\alpha\) and \(\beta\) be two C-subsequences that match the L-sequences \(L\) and \(L'\), respectively. Since \(L \subseteq L'\), then \(\alpha \subseteq \beta\). Let \(Q' \in \delta\) be the set of all C-sequences containing \(\beta\) and \(Q \in \delta\) be the set of all C-sequences containing \(\alpha\). Since \(\alpha \subseteq \beta\), then \(Q\) must be a superset of \(Q'\), that is, \(Q \supseteq Q'\). Therefore, we infer

\[
\sum_{\beta \sim L \land \beta \subseteq C' \land C' \in Q'} u_{\text{max}}(C', k) \leq \sum_{\alpha \sim L \land \alpha \subseteq C \land C \in Q} u_{\text{max}}(C, k)
\]

and equivalently we derive \(\text{LWU}_k(L') \leq \text{LWU}_k(L)\).

Algorithm 1 shows the main procedure of the HUIPMiner algorithm. The inputs are: (1) a C-sequence database \(\delta\), (2) a minimum utility threshold \(\xi\), (3) a maximum pattern length \(K \geq 1\), and (4) a maximum pattern size \(Z \geq 1\). The output includes all high utility interval-based patterns. The algorithm has two phases, a coincident phase to obtain high utility coincidence patterns (L-sequences with lengths equal to 1) and a serial phase to obtain high utility serial patterns (L-sequences with lengths greater than 1).

3.1 The Coincident Phase

The coincident phase, which is the first phase of HUIPMiner (Lines 1-13), generates coincidence candidates by concatenating event labels.

Definition 11. Let \(c = \{l_1, l_2, ..., l_n\}\) and \(c' = \{l'_1, l'_2, ..., l'_{m}\}\) be two coincidences. The coincident concatenation of \(c\) and \(c'\) is the ordinal sum of the coincidences and is defined as coincident-concat\((c, c') = (c \cup c', \leq) = c \oplus c'\).

For example, coincident-concat\((\{A, B\}, \{A, C\}) = \{A, B, C\}\).

In the first round of this phase, all event labels are considered as coincidence candidates with a size of 1 (Line 1). Then, the algorithm searches each C-sequence to find matches to these candidates. Next, it calculates the maximum utility \(u_{\text{max}}\) and L-sequence-weighted utilization \(\text{LWU}_k\) of each candidate. If \(u_{\text{max}}\) for a candidate is no less than the given threshold \(\xi\), then the candidate
is classified as a high utility coincident pattern and placed in set HUCP. For example, suppose we want to find all HUIPs of Table 2.2 when the threshold is 14, the maximum size of a coincidence $Z$ is 2, and the maximum length of an L-sequence $K$ is 2. For simplicity, suppose all event labels have equal external utilities of 1. Table 3.1 shows the coincidence candidates of size 1 and their maximum utilities and L-sequence-weighted utilizations, which are denoted LWU$_2$. At the end of the first round, $\{A\}$ is the only candidate that is added into HUCP because $u_{\text{max}}(\{A\}) \geq 14$.

| Candidate | $\{A\}$ | $\{B\}$ | $\{C\}$ | $\{D\}$ | $\{E\}$ | $\{F\}$ |
|-----------|----------|----------|----------|----------|----------|----------|
| $u_{\text{max}}$ | 20 | 11 | 9 | 3 | 9 | 3 |
| LWU$_2$ | 51 | 38 | 51 | 12 | 51 | 13 |

Before the next round is started, coincidence candidates of size 2 are generated. In order to avoid generating too many candidates, we present a pruning strategy, which is based on the following definition.

**Definition 12.** A coincidence candidate $c$ is promising iff LWU$_k(c) \geq \xi$. Otherwise it is unpromising.

**Property.** Let $a$ be an unpromising coincidence candidate and $a'$ be a coincidence. Any superset produced by coincident-concat($a, a'$) is of low utility.

**Rationale.** Property 1 holds by the LDCP property (Theorem 1).

**Pruning strategy.** Discard all unpromising coincidence candidates. If the LWU$_k$ value of a candidate is less than $\xi$, the candidate will be discarded since it is unpromising. If the LWU$_k$ value of a candidate is no less than $\xi$, the candidate is promising and thus it will be added to set $P$, the set of promising candidates for the current run. The HUIPMiner algorithm also extracts the unique elements of the promising candidates (Line 10). Before the algorithm performs the next round, $P$ is added into WUCP, which is the set of all weighted utilization coincident patterns with sizes up to $Z$. WUCP is later used in the serial phase. In our example, the algorithm prunes (discards) $\{D\}$ and $\{F\}$ in the first round because their LWU$_2$ values are less than 14. Therefore, $\{D\}$ and $\{F\}$ will not be involved in generating candidates for the second round. $\{A\}$, $\{B\}$, $\{C\}$ and $\{E\}$ are identified as promising candidates and added into $P$. Then, coincidence candidates of size 2 are generated for the next round by calling the $C$candidate procedure and sending $P$ and the unique elements as input arguments (Definition 11). The algorithm repeats this procedure until it reaches $Z$ or no more candidates can be generated. At the end of this phase, the algorithm has found all high utility coincident patterns and stored them in HUCP; it has also found all weighted utilization coincident patterns of maximum size $Z$ such that LWU$_k$ is no less than $\xi$ and stored them in WUCP. In the serial phase, WUCP is used to find the serial patterns.
3.2 The Serial Phase

In the serial phase, the second phase of HUIPMiner (Lines 14-27), serial candidates are generated by concatenating the weighted utilization coincident patterns found in the first phase.

**Definition 13.** Let \( L = \langle c_1, c_2, ..., c_n \rangle \) and \( L' = \langle c'_1, c'_2, ..., c'_m \rangle \) be two \( L \)-sequences. The **serial concatenation** of \( L \) and \( L' \) is defined as serial-concat\((L, L')\) = \( \langle c_1, c_2, ..., c_n, c'_1, c'_2, ..., c'_m \rangle \).

For example, the serial concatenation of two \( L \)-sequences \( L = \langle \{A, B\}, \{A, C\} \rangle \) and \( L' = \langle \{E\}, \{D, C, F\} \rangle \) is \( L'' = \langle \{A, B\}, \{A, C\}, \{E\}, \{D, C, F\} \rangle \).

In the first round of this phase, all serial \( L \)-sequence candidates of length 2 are generated. For this purpose, each coincident pattern \( w \) in WUCP is used to generate serial \( L \)-sequence candidates that start with \( w \) as the first coincidence of the \( L \)-sequence. This is done by calling the `Scandidate` procedure and sending \( w \) and WUCP as input arguments (Definition 13). Then, the algorithm searches each C-sequence in the C-sequence database to find matches to serial \( L \)-sequence candidates. The search for matches in this phase is more challenging than the search in the coincidence phase. It requires that the order of the coincidences also be taken into account. Therefore, it adds more complexity as the length of the \( L \)-sequence increases. After matches are found, as in the coincidence phase, the algorithm calculates \( u_{\text{max}} \) and \( \text{LWU}_k \) of every serial candidate. If \( u_{\text{max}} \) for a candidate \( l \) is no less than the given threshold \( \xi \), then \( l \) is classified as a high utility serial pattern (HUSP). If \( \text{LWU}_k \) for a serial candidate \( l \) is no less than threshold \( \xi \), then \( l \) is added into the set of promising candidates \( P \). In order to generate longer serial candidates, the algorithm extracts the unique coincidences located at the \( k^{th} \) position of the candidate (last coincidence) and stores them in \( \text{NewL} \). Next, `Scandidate` procedure generates serial candidates of length 3 for the next round by serially concatenating \( P \) and \( \text{NewL} \). The algorithm repeats these steps until it reaches the maximum length of patterns \( K \) or no more candidates can be generated. At the end of this phase, the algorithm has found all high utility serial patterns with lengths up to \( K \) and stored them in HUSP. After the serial phase ends, the high utility coincident and serial patterns are sent to the output.

4 Experiments

The HUIPMiner algorithm was implemented in C++11 and tested on a desktop computer with a 3.2GHz Intel Core 4 CPU and 32GB memory. We used four real-world datasets from various application domains in our experiments to evaluate the performance of HUIPMiner. The datasets include three publicly available datasets, namely Blocks [3], Auslan2 [3], ASL-BU [4], and a private dataset, called DS, obtained from our industrial partner. DS includes event labels corresponding to various services offered to customers. An E-sequence in
**Algorithm 1 HUIPMiner: High Utility Interval-based Pattern Miner**

**Input:** A C-sequence database $\delta$, minimum utility threshold $\xi$, maximum length $K \geq 1$, maximum size $Z \geq 1$

**Output:** All high utility interval-based patterns $HUIP$

1. Initialize the set of high utility coincident patterns $HUCP = \emptyset$, the set of weighted utilization coincident patterns $WUCP = \emptyset$, $z = 1$, and $C^z = \text{all event labels}$

2. while $z \leq Z$ and $C^z \neq \emptyset$ do

   3. $P = \emptyset$, $NewL = \emptyset$

   4. for each candidate $c$ in $C^z$ do

      5. Find $c$ in $\delta$ and Calculate $LWU_K(c)$

      6. if $u_{\text{max}}(c) \geq \xi$ then $HUCP = HUCP \cup c$

      7. if $LWU_K(c) \geq \xi$ then $P = P \cup c$

      8. NewL = $\{p \mid p \in c\}$

   9. $WUCP = WUCP \cup P$

   10. $z = z + 1$

11. $C^z = C_{\text{candidate}}(P, NewL)$

12. Initialize the set of high utility serial patterns $HUSP = \emptyset$ and $k = 2$

13. for each weighted utilization pattern $w$ in $WUCP$ do

14. $L^k = Scandidate(w, WUCP)$

15. while $k \leq K$ and $L^k \neq \emptyset$ do

16. $P = \emptyset$, $NewL = \emptyset$

17. for each candidate $l$ in $L^k$ do

18. Find $l$ in $\delta$ and Calculate $LWU_K(l)$

19. if $u_{\text{max}}(l) \geq \xi$ then $HUSP = HUSP \cup l$

20. if $LWU_K(l) \geq \xi$ then $P = P \cup l$

21. NewL = $\{k^{th} \text{ coincidence in } l\}$

22. $k = k + 1$

23. $L^k = Scandidate(P, NewL)$

24. $HUIP = HUCP \cup HUSP$

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this dataset represents a customer receiving services. The minimum, maximum and average external utilities associated with the event labels in DS are 10, 28, and 18, respectively. There are no external utilities associated with the public datasets. Therefore, we assume every event label in these datasets have an external utility of 1. The statistics of the datasets are summarized in Table 4.1.

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### 4.1 Performance Evaluation

We evaluate the performance of HUIPMiner on the four datasets in terms of their execution time and the number of extracted high utility patterns, while varying the minimum utility threshold $\xi$ and the maximum length of patterns $K$. These two evaluations are shown on a log-10 scale in Figure 4.1 and Figure 4.2, respectively. The execution time of HUIPMiner in seconds is shown on the left and the number of patterns discovered by HUIPMiner is presented on the
right of the two figures. The maximum size of patterns $Z$ is set to 5 in all experiments.

Figure 4.1 shows the evaluation of the HUIPMiner on the datasets while varying $\xi$ and keeping $K$ set to 4. The algorithm is able to discover a large number of HUIPs in a short time, especially for smaller datasets. For instance, the algorithm can extract more than 4500 HUIPs in about 60 seconds from Blocks under a low minimum utility. It is evident that as $\xi$ increases, the execution time drops exponentially and fewer patterns are discovered. This is especially well supported for larger datasets like ASL-BU and DS. Apart from the way that event intervals are distributed, the large number of event labels in ASL-BU are the major factor that contributes to high computational costs for extracting patterns. Similarly, the large number of E-sequences in DS requires more execution time to extract patterns from this dataset. The results also show that HUIPMiner is effective at finding patterns for small thresholds.

Figure 4.2 shows the evaluation of the HUIPMiner on the four datasets when $K$ is varied between 1 and 4. In these experiments, a small $\xi$ corresponding to each dataset is used to benchmark the algorithm. As shown in Figure 4.2, HUIPMiner discovers a high number of HUIPs from Blocks in a short time when $\xi$ is set to 0.02. The algorithm performs similarly on Auslan2 when $\xi = 0.01$. When the algorithm is applied to ASL-BU and DS, patterns are discovered at lower speeds than from the two other datasets, when the minimum thresholds are set to 0.1 and 0.05, respectively. As expected, $K$ plays an important role in determining both the execution time of the algorithm and the number of extracted patterns. As $K$ increases, the execution time increases and more patterns are discovered.

In general, the performance of the algorithm depends on the dataset characteristics (mentioned in Table 4.1) as well as the parameters used in the experiments ($Z$, $K$, $\xi$). The experiments show that HUIPMiner can successfully extract high utility patterns from datasets with different characteristics under various parameters setups.

### 4.2 Effect of pruning strategies

The computational benefits of the proposed pruning strategy is also evaluated. We compare our pruning strategy, which is based on the LDCP property, against
a pruning strategy based on the SDCP property and also against the execution of HUIPMiner when no pruning strategy is applied. Figure 4.3a shows the time for the strategies on Blocks dataset with $\xi = 0.02$. The LDCP based pruning strategy is a dominant winner on this dataset in comparison with no pruning. LDCP is also more efficient than SDCP, especially when the maximum length of patterns increases. This result is further supported in Fig 4.3b where LDCP is compared against SDCP on the Auslan2 dataset. Similar results were obtained with various values of $\xi$ and on other datasets.

5 Conclusions and Future Work

Mining sequential patterns from interval-based data is more challenging than mining from point-based data due to the existence of complex temporal relations among events. Seeking high utility patterns increases the complexity of the problem because the downward closure property does not hold. In this paper, we
proposed the coincidence eventset representation to express temporal relations among events along with the duration of events. This representation simplifies the description of complicated temporal relations without losing information. We incorporated the concept of utility into interval-based data and provided a novel framework for mining high utility interval-based sequential patterns. An effective algorithm named HUIPMiner was proposed to mine patterns. Furthermore, in order to mine the dataset faster, a pruning strategy based on LDCP was proposed to decrease the search space. Experimental evaluations have shown that HUIPMiner is able to identify patterns with low minimum utility.

Utility mining in interval-based sequential data could provide benefits in diverse applications. For instance, more industries could take advantage of the utility concept to model their monetary or non-monetary considerations. In medicine, alternatives for courses of treatment over a long period may have different utilities. Our approach could be applied to find high utility alternatives from records of many patients with long-lasting diseases. Similarly, managers could utilize the high utility patterns in making decisions about increasing profits based on many sequences of events with durations.

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References

[1] Patel, D., Hsu, W., Lee, M.L.: Mining relationships among interval-based events for classification. In: Proceedings of the 2008 ACM SIGMOD International Conference on Management of Data. SIGMOD ’08, New York, NY, USA, ACM (2008) 393–404
[2] Sheetrit, E., Nissim, N., Klimov, D., Shahar, Y.: Temporal Probabilistic Profiles for Sepsis Prediction in the ICU. In: Proceedings of the 25th ACM
SIGKDD International Conference on Knowledge Discovery & Data Mining, ACM (2019) 2961–2969

[3] Mörchen, F., Fradkin, D.: Robust mining of time intervals with semi-interval partial order patterns. In: Proceedings of the 2010 SIAM International Conference on Data Mining, SIAM (2010) 315–326

[4] Papapetrou, P., Kollios, G., Sclaroff, S., Gunopulos, D.: Mining frequent arrangements of temporal intervals. Knowledge and Information Systems 21(2) (2009) 133

[5] Liu, Y., Nie, L., Liu, L., Rosenblum, D.S.: From action to activity: Sensor-based activity recognition. Neurocomputing 181 (2016) 108–115

[6] Allen, J.F.: Maintaining knowledge about temporal intervals. Communications of the ACM 26(11) (1983) 832–843

[7] Wu, S.Y., Chen, Y.L.: Mining nonambiguous temporal patterns for interval-based events. IEEE Transactions on Knowledge and Data Engineering 19(6) (2007)

[8] Han, J., Pei, J., Mortazavi-Asl, B., Pinto, H., Chen, Q., Dayal, U., Hsu, M.: PrefixSpan: Mining sequential patterns efficiently by prefix-projected pattern growth. In: Proceedings of the 17th International Conference on Data Engineering. (2001) 215–224

[9] Chen, Y.C., Jiang, J.C., Peng, W.C., Lee, S.Y.: An efficient algorithm for mining time interval-based patterns in large database. In: Proceedings of the 19th ACM International Conference on Information and Knowledge Management, ACM (2010) 49–58

[10] Srikant, R., Agrawal, R.: Mining sequential patterns: Generalizations and performance improvements. Advances in Database Technology—EDBT’96 (1996) 1–17

[11] Yao, H., Hamilton, H.J.: Mining itemset utilities from transaction databases. Data & Knowledge Engineering 59(3) (2006) 603–626

[12] Ahmed, C.F., Tanbeer, S.K., Jeong, B.S.: A novel approach for mining high-utility sequential patterns in sequence databases. ETRI Journal 32(5) (2010) 676–686

[13] Yin, J., Zheng, Z., Cao, L.: USpan: An efficient algorithm for mining high utility sequential patterns. In: Proceedings of the 18th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, ACM (2012) 660–668

[14] Wu, C.W., Lin, Y.F., Yu, P.S., Tseng, V.S.: Mining high utility episodes in complex event sequences. In: Proceedings of the 19th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, ACM (2013) 536–544

[15] Fournier-Viger, P., Yang, P., Lin, J.C.W., Yun, U.: HUE-Span: Fast high utility episode mining. In: International Conference on Advanced Data Mining and Applications, Springer (2019) 169–184

[16] Huang, J.W., Jaysawal, B.P., Chen, K.Y., Wu, Y.B.: Mining frequent and top-k high utility time interval-based events with duration patterns. Knowledge and Information Systems (2019) 1–29