On Efficient DCT Type-I Based Low Complexity Channel Estimation for Uplink NB-IoT Systems

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ABSTRACT Channel estimation is a challenging and timely issue for the 3rd Generation Partnership Project (3GPP) standardized low power wide-area network technology named narrowband Internet of Things (NB-IoT). Channel estimation is crucial to achieve extended radio coverage, energy efficiency, coherent detection, and channel equalization for signal repetition dominated NB-IoT uplink transmission. The NB-IoT inherits simplified baseband radio frequency processing, physical channels, reference signal structure, and numerology from existing Long Term Evolution (LTE) systems to save power and costs. Thus, channel estimation methods extensively employed in LTE systems may not be applied to the NB-IoT uplink systems. In this paper, efficient discrete cosine transform type-I (DCT-I)-based transform-domain channel estimation approaches are proposed by modifying the original definition of DCT-I. The proposed methods can mitigate the problems experienced in the discrete Fourier transform (DFT)-based channel estimation, such as signal aliasing error, and border effect. The proposed approaches improve channel estimation precision by reducing signal distortion from the high-frequency region in the time-domain when non-sample-spaced path delays exist in multipath fading channels. Signal aliasing error experienced from the virtual subcarriers is also minimized with the anticipated schemes. The proposed methods are applied on simple least squares (LS) estimates in time-domain to eliminate estimation noise. The viability of the proposed estimators is verified as compared to the conventional LS, DFT-based de-noising LS, and standard DCT-I based methods through extensive numerical simulations. Based on the numerical simulations, the proposed estimators show better mean square error and bit error rate performances than their competitors in extremely low coverage conditions.

INDEX TERMS Channel estimation, Internet of Things (IoT), Long Term Evolution (LTE), narrowband Internet of Things (NB-IoT), orthogonal frequency-division multiplexing (OFDM), signal repetition.

I. INTRODUCTION Narrowband Internet of Things (NB-IoT) is an emerging and extremely narrowband cellular-based low power wide-area network (LPWAN) technology [1]–[5], which was introduced by the 3rd Generation Partnership Project (3GPP) as a representative technology in release-13 for providing massive connectivity of geographically distributed IoT devices to the Internet. The NB-IoT will be a strategic technology in the 5th generation (5G) new radio networks, and a crucial driver in future semiconductor industry [6]. The NB-IoT was standardized with the objectives, such as dense deployments (52547 devices per cell), extended coverage, low data-rates, low delay sensitivity, and communication services with high
power efficiency (10 years and beyond battery life), to enable the IoT framework [1], [6]–[9].

In NB-IoT systems, three deployment options are allowed, such as in-band, guard-band, and standalone [10]. The simplified radio frequency (RF) baseband processing, and numerology from the current Long Term Evolution (LTE) standards can be reused for deploying NB-IoT technology with in-band and guard-band modes of operation. One physical resource block (PRB) which corresponds to 180 kHz system bandwidth from LTE carrier will be shared for in-band and guard-band operations. Therefore, these simplified features and reuse of existing physical setup and resources of the LTE network enliven rapid and low-cost NB-IoT deployment. A small chunk of 200 kHz bandwidth from the Global System for Mobile Communications (GSM) carriers will be employed to implement standalone mode of operation. The NB-IoT deployment scenarios [5] are depicted in Fig. 1. One can refer to [1] and [5] for in-depth insights of the NB-IoT systems.

The repetition of identical signal (i.e., user data and the related control signaling) transmission is allowed as a key technique for NB-IoT systems to achieve wider coverage for the IoT devices that are in extreme coverage regime. Authors in [11] investigated that channel estimation (CE) error significantly affects on the uplink coverage performance gain from signal repetitions. The CE is crucial, and a prerequisite for achieving wide-area coverage since the expected operating signal-to-noise ratio (SNR) for NB-IoT is extremely low (≪0 dB). Furthermore, a novel reference signal sequence also called narrowband demodulation reference signal (NDMRS) is designed for uplink channel estimation. Therefore, the various CE techniques in the literature [12]–[17] extensively used in multicarrier communication systems may be no longer pertinent for NB-IoT systems. In this respect, a novel/modified efficient CE technique is necessarily required that is simple and efficient to meet the design objectives [11] on coverage enhancement, IoT device costs and power consumption, but robust enough to ensure transmission reliability under deficient coverage regime.

Consequently, the discrete cosine transform (DCT) type-I (DCT-I) can be used in CE application of uplink NB-IoT system for its excellent energy concentration, frequency compaction, and low-complexity features [18]–[21]. The DCT-I based CE methods can eradicate the problems experienced in discrete Fourier transform (DFT)-based CE techniques, such as signal aliasing error, and border effect owing to considerable virtual subcarriers at the spectrum extremities. The DCT-I based CE schemes reduce estimation noise in the transform-domain from high-frequency region [22], leads to high CE precision as well as system performance improvement as compared to the DFT-based methods in low SNR environments. The background of the DCT-I based transform-domain CE for NB-IoT uplink systems is provided in the following section.

We then propose efficient NDMRS-aided low-complexity CE approaches by modifying the original definition of DCT-I for LTE-based NB-IoT uplink transmission. However, the compatibility issue is the primary concern of using the DCT-based CE within the 3GPP LTE-based...
NB-IoT standards. Fortunately, the 3GPP standard [23] in release-14 reserved several modulations and coding schemes (MCS) and offered to comprise new baseband signaling formats for NB-IoT systems; this provides the opportunity to implement non-standardized signal processing techniques. Thus, the DCT-based CE methods can be readily implemented by only updating the software on top of the LTE base station (BS) receiver. The current work concentrates on uplink CE and its related signal processing, which are within the LTE BS receiver. Hence, low complexity and low power consumption of each NB-IoT user equipment (NB-IoT UE) are achieved, since it is only responsible for generating the single-carrier frequency-division multiple access (SC-FDMA) signal and transmitting the signal to a BS. To the best of our knowledge, CE based on the DCT-I domain is not investigated yet specifically for NB-IoT uplink transmission. A summary of key contributions and novelties of the work is as follows:

- The LTE-based uplink NB-IoT system model is provided, including the reference signal generation and mapping process in an resource unit (RU), and the received signal model.
- Efficient DCT-I based transform-domain CE approaches are proposed by modifying the original definition of DCT-I to effectively estimate the channel response as well as equalize and detect the repeatedly transmitted NB-IoT UE signals.
- The system performance of the proposed estimators is evaluated and compared with the state-of-art methods in terms of channel mean square error (MSE) and bit error rate (BER) through rigorous link-level computer simulations following the 3GPP release-15 uplink NB-IoT standards [23], [24]. A comprehensive computational complexity analysis of the algorithms is also provided.

The remainder of this paper is structured as following sections: Section II presents the literature survey of DCT-I based CE techniques. The reference signal structure and system model of uplink NB-IoT are provided in Section III. The state-of-art and proposed CE techniques are given in Sections IV and V, respectively. A linear interpolation method is given in Section VI. Numerical results and computational complexity analysis are presented in Section VII, and the conclusion of the paper is drawn in Section VIII.

**Notations:** In this correspondence, the time (frequency)-domain vectors/matrices are denoted by the boldface lowercase (uppercase) symbols. The statistical expectation, and Euclidean norm will be represented with the operators $E[\cdot]$ and $\| \cdot \|$, respectively.

## II. RELATED WORK

The CE methods for NB-IoT can be categorized in two ways, namely frequency-domain and transform-domain.

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1 The RU was introduced in release-13 [24] as a fundamental resource grid for transmitting user data/reference signal. Note that the RU is completely distinct from the current LTE systems.

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Frequency-domain CE approaches, particularly the least squares (LS) [11], [25], maximum likelihood (ML) [26], and linear minimum mean square error (LMMSE) [5], [12] estimators, can be used. The LS is the most straightforward CE approach among them but it exhibits severe performance degradation due to irreducible Gaussian distribution noise. The LMMSE method reveals better performance at the expense of high computational cost since it requires prior information of the channel auto-covariance matrix and noise variance. It also contributes to computational complexity owing to matrix inversion in real-time. In recent years, several efforts provided by the researchers to minimize the complexity of the LMMSE algorithm [27]–[29]. Besides, the LS and LMMSE estimators also modified by removing the insignificant channel taps in time-domain for orthogonal frequency-division multiplexing (OFDM)/SC-FDMA systems [30], [31]. However, these modified versions of LS and LMMSE approaches still possess high computational complexity in practical applications. In the transform-domain, especially the DFT-based approaches, have the capability to reduce high-frequency distortions in time-domain as well as improve estimation precision with low complexity. Many researchers investigated the transform-domain channel estimation for the present LTE/LTE-Advanced (LTE-A) systems to detecting signals coherently at the receiver [32]–[35]. Authors in [36] employed the DFT-based CE to attain the channel frequency response (CFR) of the LS estimate in OFDM systems. A new DFT-based technique presented in [37] by detecting significant channel taps adaptively in OFDM systems for improving the MSE performance. Furthermore, an enhanced DFT-based CE algorithm by employing the frequency-domain weighting technique proposed and investigated for LTE-A uplink in [38]. However, these enormous collections of CE methods may be invalidated in uplink NB-IoT transmission, because the LTE and NB-IoT signals will not be multiplexed as normal LTE transmission [39] due to imbalanced RF baseband processing and sampling rate.

In the literature, the research related to channel estimation is inadequate since the NB-IoT system is still in its early stage of development for 5G IoT framework. Authors in [5] reported significant coverage performance improvement of uplink NB-IoT system by exploiting modified LS and LMMSE algorithms with considerable computational costs. Note that the NB-IoT supports only phase-shift keying modulation schemes [24]. Therefore, the optimal LMMSE method has no considerable impact on uplink NB-IoT receiver performance [11]. Furthermore, the precise information of the channel covariance matrix is very crucial but it is very difficult to obtain at a low SNR environment (i.e., $\text{SNR} \ll 0 \, \text{dB}$).

Recently, researchers in [40] proposed efficient DFT-based transform-domain CE methods, such as random sorting LS (RS-LS) and the de-noising variant of LS (D-LS), for NB-IoT uplink systems. These algorithms revealed significant performance improvement in low SNR environments, but the main concern is high computational cost.
The border effect is also our concern for using the DFT-based estimators when the number of allocated user subcarriers is not identical with the DFT size [33], [41]. Obviously, the DFT size would be larger than the allocated user subcarriers (e.g., 1, 3, 6, or 12-tones) in uplink NB-IoT systems to avoid signal aliasing in practice. Also, signal aliasing error can be occurred owing to the channel impulse response (CIR) leakage at the low-frequency region. Besides, the MSE error floor can be yielded due to incorrect channel delay spread. One can suppress these real problems by using the DCT-domain CE as an alternative to the DFT-based approach. Many researchers in [42]–[45] studied the DCT type-II based CE approaches for traditional OFDM/SC-FDMA systems. The work [21] investigated DCT-I based LS and LMMSE estimators for CE applications in LTE uplink systems, but the high computational complexity makes such estimators unsuitable for uplink NB-IoT systems. Besides, the works on NB-IoT, researchers have also studied other emerging technologies such as non-orthogonal multiple access, backscatter communication, vehicle to everything, cognitive radio, blockchain, artificial intelligence, and physical layer security [46]–[51].

III. NB-IoT UPLINK SYSTEM MODEL

The reference signal (NDMRS) structures and their mapping processes within NB-IoT uplink subframe in an RU are given below for both multi-tone and single-tone operations. The NB-IoT uplink received signal model is also presented. Note that the single-tone operation is mandatory in NB-IoT uplink transmission as the 3GPP standards.

A. THE NDMRS SEQUENCE GENERATION AND MAPPING

The reference signal (RS) sequence $\tilde{s}_u(m)$ can be engendered for single-tone transmission (i.e., an RU comprises only one subcarrier) as [24]:

$$\tilde{s}_u(m) = \frac{1}{\sqrt{2}}(1 + j)(1 - 2 c(m))w(m \bmod 16),$$

$$0 \leq m \leq P_{\text{slots}}^{\text{RU}} N_{\text{RU}} - 1 \quad (1)$$

The Gold sequence with length-31 is employed to generate the sequence $c(m)$ (i.e., pseudo-random). A unit impulse function of length-31 is used to initialize the first sequence. At the beginning of narrowband physical uplink shared channel (NPUSCH) transmission, the second sequence would be initialized by the seed $c_{\text{init}} = 35$. The symbols $P$, $N_{\text{slots}}^{\text{RU}}$, and $N_{\text{RU}}$ represent NPUSCH/pilot repetitions, scheduled uplink slots within an RU, and employed RUs, respectively. The parameter $w(m)$ is taken as [24], and the base sequence index $u$ is attained with $u = N_{\text{ID}} \bmod 16$, where $N_{\text{ID}} \in [0, 1, \ldots, 503]$ denotes the identity of the narrowband physical radio cell. Then, the pilot sequence without enabling sequence/group hopping for NPUSCH format-1 is given by $s_u(m) = \tilde{s}_u(m)$.

The NDMRS sequence is generated using the cyclic shift (CS) of a base sequence for multi-tone operation as [24]:

$$s_u(m) = \exp(j \delta m) \exp(j \phi(m)) \pi / 4), \quad 0 \leq m \leq M_{\text{RU}}^{\text{RU}} - 1 \quad (2)$$

where $\phi(m)$ is specified as [24] to the considered NB-IoT UE subchannels $M_{\text{RU}}^{\text{RU}}$ in an RU. The CS $\delta$ is characterized as [21] for $M_{\text{RU}}^{\text{RU}} = 3$ and 6, and $\delta = 0$ for $M_{\text{RU}}^{\text{RU}} = 12$. The CS preserves the orthogonality between different NB-IoT UEs signals. The higher layer signaling is assumed to be not enabled; then, the quantity $u$ can be attained as:

$$u = \begin{cases} 
N_{\text{ID}} \bmod 12 & \text{for } M_{\text{RU}}^{\text{RU}} = 3 \\
N_{\text{ID}} \bmod 14 & \text{for } M_{\text{RU}}^{\text{RU}} = 6 \\
N_{\text{ID}} \bmod 30 & \text{for } M_{\text{RU}}^{\text{RU}} = 12 
\end{cases} \quad (3)$$

The NDMRS sequence generation is followed by the DFT operation; then, the pilot symbols transmit together with the user data symbols within the identical uplink subframe. Hence, the employed subcarrier(s) set would be the same for both NDMRS and NPUSCH transmissions.

The subcarrier index set $\{k, k + 1, \ldots, M_{\text{PRB}}^{\text{PRB}} - 1\}$ is considered for the NB-IoT signal transmission, which does not overlap with the set of LTE subcarrier index. Here, $k$ and $M_{\text{PRB}}^{\text{PRB}}$ are the first subcarrier index and contiguous subcarriers in a PRB, respectively. The following SC-FDMA symbols are specified as [24] for the transmission of NDMRS in an NB-IoT uplink subframe:

$$l = \begin{cases} 
4, 11 & \text{for } \Delta f = 3.75 \text{ kHz} \\
3, 10 & \text{for } \Delta f = 15 \text{ kHz} 
\end{cases} \quad (4)$$

where $\Delta f$ represents the subcarrier spacing and $l$ corresponds to the symbol index. The remaining SC-FDMA symbols within the same subframe will be used for NPUSCH transmission. The pilot patterns (e.g., localized/block-type subcarrier mapping [24], [40]) within an NB-IoT uplink subframe of an RU for single-tone (3.75 kHz subchannel spacing), and multi-tone (e.g., 12-subchannels with 15 kHz subcarrier spacing) transmissions are depicted in Figs. 2 (a) and 2 (b), respectively. The mapping of NPUSCH and NDMRS for single-tone transmission using 15 kHz subcarrier spacing is identical as the multi-tone transmission.

B. RECEIVED SIGNAL MODEL OF UPLINK NB-IoT SYSTEM

The NB-IoT uplink baseband signaling is depicted in Fig. 3. The traditional OFDM is considered as a baseband modulation for the NB-IoT downlink transmission. The FDMA is employed in NB-IoT uplink single-tone transmission, while the DFT-spreading OFDM systems also known as SC-FDMA is used for multi-tone operation owing to reduced peak-to-average power ratio (PAPR) [5], [24]. Without loss of generality, one PRB (i.e., 12 subcarriers correspond to 180 kHz) is assumed to be a system bandwidth from LTE carrier for transmitting the NB-IoT signal.

In NB-IoT uplink, the NB-IoT device transmits user data (or pilot) repeatedly $P$ times using allocated $M_{\text{RU}}^{\text{RU}}$ contiguous subcarriers in an RU to achieve wide-area coverage. Let us consider a block of mapped user data/pilot symbols represented as $S = [S(0), S(1), \ldots, S(M_{\text{RU}}^{\text{RU}} - 1)]^T$ in frequency-domain is modulated on a set of orthogonal subcarriers. The NB-IoT signal will be sampled with a very
low sampling frequency (e.g., 1.92 MHz) to reduce power consumption and device costs. Assuming \( M \gg M_{RU} \) for some positive integers to avoid signal aliasing error and the sampling frequency \( F_s = 1 / T_s = M \Delta f \), where \( M \) is the DFT size and \( T_s \) denotes the sampling duration. The discrete-time baseband NB-IoT signal in time-domain can be represented by implementing an \( M \)-point IDFT operation as:

\[
s(m) = \frac{1}{\sqrt{M}} \sum_{k=0}^{M_{RU}-1} S(k) \exp\left(\frac{j2\pi mk}{M}\right)
\]

\[
= \frac{1}{\sqrt{M}} \sum_{k=0}^{M-1} S'(k) \exp\left(\frac{j2\pi mk}{M}\right), \quad 0 \leq m \leq M-1
\]  

(5)

where

\[
\hat{S}(k) = \begin{cases} 
S(k), & k = 0, 1, \ldots, M_{RU} - 1 \\
0, & k = M_{RU}, \ldots, M - 1.
\end{cases}
\]  

(6)

The signal \( \hat{S}(k) \) in (6) is a zero-padded form of the mapped signal \( S(k) \), which is transmitted by the \( k \)-th subchannel. The discrete-time index is denoted by \( m \). The scaling factor \( \frac{1}{\sqrt{M}} \) is added here to accomplish a unitary transform. A chunk of each transmitted symbol called cyclic prefix (CP) is used to evade inter-symbol-interference (ISI) among adjacent SC-FDMA symbols and preserves orthogonality between the subcarriers. The length of CP, \( L_{cp} \) is taken longer than the channel maximum delay spread. The received discrete-time signal \( \tilde{s}(m) \) after CP insertion will be propagated via a wireless multipath fading channel. The frequency-domain received NB-IoT signal (or pilot sequence) after CP removal and an \( M \)-point DFT operation can then be represented as:

\[
R(k) = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} \tilde{s}(m) \exp\left(-\frac{j2\pi km}{M}\right)
\]

\[
R(k) = \hat{S}(k)H(k) + V(k), \quad 0 \leq k \leq M - 1.
\]  

(7)

The frequency-domain channel response is denoted as \( H(k) \) with the length \( L_{ch} \) and \( V(k) \) is considered as additive noise with Gaussian distribution (i.e., additive white Gaussian noise (AWGN)) process \( L(0, \sigma_v^2) \) with \( L_0/2 \) power spectral density. Furthermore, the CFR \( H(k) \) can also be realized by the CIR of wireless multipath fading channel in each SC-FDMA symbol as:

\[
H(k) = \sum_{p=0}^{L_{ch}-1} h(p) \exp\left(-\frac{j2\pi pk}{M}\right), \quad 0 \leq k \leq M - 1
\]  

(8)

where \( h(p) \) corresponds to the \( p \)-th multipath channel response in time-domain. Therefore, the estimated
frequency-domain signal after performing CE and equalization at the LTE-BS receiver can be attained as:

\[
\hat{S}(k) = \Psi(k)R(k) = \zeta \hat{H}^{\text{Final}}(k)R(k), \quad 0 \leq k \leq M - 1.
\] (9)

The final estimated CFR denoted here as \( \hat{H}^{\text{Final}}(k) \) is obtained for the entire resource grid after applying the CE techniques and time dimensional linear interpolation [40]. The effective channel equalization coefficients \( \Psi(k) \) in (9), which are computed as [52]. The equalizer in [52] is an efficient one-tap frequency-domain channel equalization scheme specific for LTE-based uplink NB-IoT systems that strives in multiservice regime which having mismatched RF baseband processing and sampling rate between the transmitter and receiver. Specifically, the equalization coefficients in [52] are not equal to the direct DFT of the channel response (i.e., \( \hat{H}^{\text{Final}}(k) \)). In actual fact, the effective channel equalization coefficients \( \Psi(k) \) in (9) is the DFT of CIR with phase shift \( \zeta \) (i.e., \( \zeta \hat{H}^{\text{Final}}(k) \)). Therefore, the received NB-IoT signal can be equalized efficiently with the equalizer [52] in uplink NB-IoT receiver.

IV. CHANNEL ESTIMATION WITH STATE-OF-ART METHODS

The reference signal sequence (NDMRS) is assumed to be known both to the NB-IoT UE and LTE BS. The hopping of user data and NDMRS is not taken into account in our current study for the sake of simplicity.

A. CHANNEL ESTIMATION WITH CLASSICAL LS ALGORITHM

The initial CFR with LS CE is obtained by reducing the channel MSE as:

\[
\gamma^{\text{LS}}(k) = \left\| R(k) - \hat{S}(k)H(k) \right\|^2, \quad 0 \leq k \leq M - 1 \quad (10)
\]

The primary superimposed CFR within an uplink subframe can be estimated at the pilot frequencies (e.g., \( \hat{H}_4(K) \) and \( \hat{H}_{11}(K) \) for 3.75 kHz subchannel spacing as in Fig. 2 (a)) by exploiting the conventional LS criterion [27] as:

\[
\hat{H}^{\text{LS}}(k) = \hat{S}^{-1}(k)R(k) = H(k) + H_\zeta(k), \quad 0 \leq k \leq M - 1 \quad (11)
\]

where \( H(k) \) is the exact CFR and \( H_\zeta(k) = \hat{S}^{-1}(k)V(k) \) refers to the frequency-domain estimation noise with Gaussian distribution. Obviously, the estimation noise raises with the noise power intensification. Therefore, the scope of LS estimation noise suppression using the original LS method is limited without increasing the power of the reference signal.

B. TRANSFORM DOMAIN CHANNEL ESTIMATION WITH DFT-BASED D-LS (DFT-D-LS) METHOD

The LS estimation error \( H_\zeta(k) \) (11) is assumed to be uncorrelated Gaussian noise. Thus, the time-domain version (i.e., after applying \( M \)-point IDFT operation on \( H_\zeta(k) \)) of estimation noise denoted by \( h_\zeta(m) \) as in [40] will be spread uniformly over all channel paths. The maximum channel energy is concentrated within only a few channel taps [37]. To minimize LS estimation error, the scalar filter \( \psi(m) \) is used as multiplication with the LS CIRs represented by \( \hat{h}^{\text{LS}}(m) \). Then, the DFT-D-LS estimated CIR by applying the filtering operation [40] can be obtained as:

\[
\hat{h}^{\text{DFT-D-LS}}(m) = \psi(m)\hat{h}^{\text{LS}}(m) \quad (12)
\]

Subsequently, the expected CFR estimates \( \hat{H}^{\text{DFT-D-LS}}(k) \) are obtained by applying an \( M \)-point DFT operation as:

\[
\hat{H}^{\text{DFT-D-LS}}(k) = \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} \hat{h}^{\text{DFT-D-LS}}(m) \exp \left( -j2\pi km \right) \quad (13)
\]

We assume that the energy of the channel with the DFT-D-LS method [40] is almost zero/negligible within the smearing area. The channel energy of \( h(m) \) (corresponds to the IDFT output of \( H(k) \) in (11)) would be smeared uniformly over all channel paths, and first few taps contain maximum channel energy [27]. Therefore, these channel taps contain only estimation noise in the energy smearing region. The DFT-D-LS estimator coefficients, represented by \( \psi^{\text{DFT-D-LS}}(m) \) using the specified smearing zone of the channel can be obtained as [40]:

\[
\psi^{\text{DFT-D-LS}}(m) = \begin{cases} 0 & \text{if } m \in \Phi \\ 1 & \text{if } m \notin \Phi. \end{cases} \quad (14)
\]

The parameter \( \Phi \) can also be characterized as \( \Phi = \{ m : \Delta + \Lambda, \ldots, M - \Lambda - 1 \} \), where \( \Delta \) corresponds to the channel delay spread with equal in length of CP \( L_p \). Thus, the energy concentration zone can be specified as \( m \notin \Phi \). The significant channel taps within the energy smearing region is denoted by the parameter \( \Lambda \), which is assumed to be truncated during noise elimination process. The feasible values of the parameter can be selected as [27]. Note that a trade-off subsists between the better estimation noise reduction and low estimation accuracy. One can refer to [40] for an in-depth look at the DFT-D-LS estimation process.

C. TRANSFORM DOMAIN CHANNEL ESTIMATION WITH CLASSICAL DCT-I BASED METHOD

The DCT-I based approach can eradicate signal aliasing error experienced from the DFT-based estimators. The effect of discontinuous edge (i.e., border effect) is also smoothed since the data is symmetric by a mirror extension. We choose DCT-I for narrow-bandwidth and low-complexity NB-IoT systems among the available eight types [45], [53]. Besides, authors in [21] presented two attractive features of DCT-I for low-complexity and narrowband applications.

Since the maximum channel power is concentrated in the low-frequency components [37], the DCT-I based estimator aims to reduce only the noise power from \( M - L_{ch} \) samples

2Note that the DFT assumes a periodic extension. Due to the effect of discontinuous edge (i.e., border effect) at \( H(k) \), a considerable energy smearing effect to \( h(m) \) will be occurred.
of the CIR estimates that exists outside of the exact CIR within the first $L_{ch}$ samples. The length $L_{ch}$ of CIR is also likely to be shorter than $L_{sc}$ and significantly smaller over the DFT size $M$; thus, the significant CIR is belonged to the first $L_{ch}$ samples as well as $M_{sc}^{RU}$ user subcarriers. Now, the original definition of DCT-I [54] is implemented in LS CFR estimates at the pilot frequencies. The estimated CIR can be obtained by using an $M$-point inverse DCT-I (IDCT-I) operation on CFR estimates $\hat{H}^{LS}(k)$ provided by (11). Thus, we have CIR based on the DCT-I estimator as:

$$
\hat{h}^{DCT-I}(m) = \sqrt{\frac{2}{M}} \beta_m \sum_{k=0}^{M_{sc}^{RU}-1} \beta_k \hat{H}^{LS}(k) \cos \frac{\pi km}{M}
$$

$$
= \sqrt{\frac{2}{M}} \beta_m \sum_{k=0}^{M-1} \beta_k \hat{H}^{LS}(k) \cos \frac{\pi km}{M}
$$

$$
0 \leq m \leq M_{sc}^{RU} - 1 \quad \text{(or} \quad M - 1 \text{if length extended to} \quad M) \tag{15}
$$

where $\beta_m$ and $\beta_k$ represent the DCT-I coefficients, which retain two different values depending on the values of $m$ and $k$, respectively. Assuming $\beta_m = \beta_k$ for the sake of simplicity; then, the DCT-I coefficients, denoted by $\beta_z$ can be specified as:

$$
\beta_z = \begin{cases} 
1 & \text{if} \quad z = 0 \text{ or } M_{sc}^{RU} - 1 \text{ (or} \quad M - 1 \text{ if the length is} \quad M) \\
\sqrt{2} & \text{otherwise} 
\end{cases} \tag{16}
$$

It is seen from the DCT-I computation and multipath channel characteristics that the effective CIR in (15) is concentrated at lower-order components (i.e., the first $M_{sc}^{RU}$ subcarriers) since the null subcarriers exist at the frequency extremities. Thus, the rest of the subcarriers contain only noise. The channel response beyond $M_{sc}^{RU}$ is not exactly zero, but can be treated as insignificant; this feature attracts for using DCT instead of DFT. Therefore, effective CIR estimates using DCT-I based estimator can be represented by considering only the first $M_{sc}^{RU}$ subcarriers as:

$$
\hat{h}(m) = \begin{cases} 
\hat{H}^{DCT-I}(m) & \text{if} \quad 0 \leq m \leq M_{sc}^{RU} - 1 \\
0 & M_{sc}^{RU} \leq m \leq M - 1. \tag{17}
\end{cases}
$$

Finally, the desired CFR can be estimated by implementing an $M$-point DCT-I operation on $\hat{h}(m)$, which is the same as the IDCT-I due to its involuntary property [21] as:

$$
\hat{H}^{DCT-I}(k) = \sqrt{\frac{2}{M}} \beta_k \sum_{m=0}^{M-1} \hat{h}(m) \cos \frac{\pi km}{M}, \quad 0 \leq k \leq M - 1 \tag{18}
$$

The estimated CFR in (18) is denoted by $\hat{H}^{DCT-I}(k)$. The true CFR $H(k)$ in (11) cannot be obtained precisely since the DCT-I shifts the time-domain data as compared to the DFT. As a result, the original low-throughput data of uplink NB-IoT systems would not remain the same after applying IDCT-I operation. Therefore, an approximation of channel response is obtained as:

$$
H(k) \simeq \hat{H}^{DCT-I}(k), \quad 0 \leq k \leq M - 1 \tag{19}
$$

Observing from (19), it is seen that the original DCT-I definition is not quite perfect for estimating channel response accurately. Consequently, the definition of DCT-I needs to be modified for efficient CE applications.

V. TRANSFORM DOMAIN CHANNEL ESTIMATION WITH MODIFIED DCT-I (MDCT-I) METHOD

In this section, we define three options of DCT-I, which can be utilized for efficient CE in uplink NB-IoT systems. The proposed MDCT-I CE algorithm is structured as in Fig. 4. We follow the three-step procedure to perform CE using each of these MDCT-I approaches as:

(i) The proposed MDCT-I based CE can be realized by employing any fast DCT algorithms reported
in [55], [56], which have lower computational complexity compared to the fast DFT algorithms. The CIRs using three MDCT-I approaches can be estimated by taking an M-point MDCT-I operation, where the definition option is denoted here as the subscript of CIR \( \hat{h}_{(i)}^{MDCT-I}(m) \), \( i = 1, 2, 3 \). Thus, we have

\[
\begin{align*}
\hat{h}_{(1)}^{MDCT-I}(m) &= \frac{2}{M} \beta_m \sum_{k=0}^{M_{RU}-1} \beta_k \hat{H}^{LS}(k) \cos \frac{\pi km}{M}, \\
\hat{h}_{(2)}^{MDCT-I}(m) &= \frac{2}{M} \bar{\beta}_m \sum_{k=0}^{M_{RU}-1} \beta_k \hat{H}^{LS}(k) \cos \frac{\pi km}{M} , \\
\hat{h}_{(3)}^{MDCT-I}(m) &= \frac{2}{M} \beta_m \sum_{k=0}^{M_{RU}-1} \hat{H}^{LS}(k) \cos \frac{\pi km}{M} ,
\end{align*}
\]

where the employed conventional DCT-I coefficients \( \beta_z \) are provided by (16), and the MDCT-I coefficients, represented by \( \bar{\beta}_z \), can be defined as:

\[
\bar{\beta}_z = \begin{cases} 
1 & z = 0 \\
\frac{1}{2} & \text{or } M_{RU} - 1 \text{ or } M - 1 \text{ if the length is } M \\
1 & \text{otherwise}
\end{cases}
\]

(21)

The DCT-I coefficient \( \beta_z \) given by (16) or the MDCT-I coefficient \( \bar{\beta}_z \) in (21) modifies the original definition of DCT-I to achieving the high-ranking estimation precision for the signal transmitted by NB-IoT UEs that are in extreme coverage environments.

(iii) The CIRs \( \hat{h}_i(m) \) employing MDCT-I approaches by considering only significant/allocated user subcarrier(s) \( M_{RU} \) in the transform-domain, where the CIRs are treated as zero at the null subcarriers. Thus, we have

\[
\hat{h}_{(i)}(m) = \begin{cases} 
\hat{h}_{(i)}^{MDCT-I}(m), & 0 \leq m \leq M_{RU} \text{ if } \hat{h}_{(i)}^{MDCT-I}(m) \\
0, & \text{otherwise}
\end{cases}
\]

(22)

Observing (22), an error floor may be exhibited with the MDCT-I approaches at high SNR due to excluding/ignoring the channel taps beyond the NB-IoT UE subcarriers. However, the error floor with the MDCT-I approaches at high SNR is not significant because the typical operating SNR is likely to be \( < 0 \text{ dB} \). Due to the continuation of non-ignorable CIRs \( \hat{h}_{(i)}^{MDCT-I}(m) \) in (20) at the null subcarriers, the MDCT-I techniques can be biased with an error floor at the high SNR regime.

(ii) Finally, the desired channel CFR can be estimated by means of \( M \)-point MDCT-I operation on \( \hat{h}_{(i)}(m) \) as

\[
\hat{H}_{(i)}^{MDCT-I}(k) = \frac{1}{M} \sum_{m=0}^{M-1} \hat{h}_{(i)}(m) \cos \frac{\pi km}{M},
\]

\[
\hat{H}_{(1)}^{MDCT-I}(k) = \beta_k \sum_{m=0}^{M-1} \hat{h}_{(1)}(m) \cos \frac{\pi km}{M},
\]

\[
\hat{H}_{(2)}^{MDCT-I}(k) = \bar{\beta}_k \sum_{m=0}^{M-1} \hat{h}_{(2)}(m) \cos \frac{\pi km}{M},
\]

\[
\hat{H}_{(3)}^{MDCT-I}(k) = \sum_{m=0}^{M-1} \hat{h}_{(3)}(m) \cos \frac{\pi km}{M},
\]

(23)

We can see that the three variants of the proposed MDCT-I are basically the same. One can use any of these definitions to get the desired channel response by satisfying the fundamental requirements of any channel estimator. However, the estimation precision of each method of the proposed MDCT-I approaches might be different due to the variation of the DCT-I and MDCT-I coefficients defined in (16) and (21), respectively. The DCT-I coefficient \( \beta_z \) or the MDCT-I coefficient \( \bar{\beta}_z \) provides the best estimated initial CFR \( \hat{H}_{(i)}^{DCT-I}(0) \) or final CFR \( \hat{H}_{(i)}^{DCT-I}(M) \) to achieve the precise channel estimation at the LTE BS receiver. Thus, the estimated CFRs \( \hat{H}_{(i)}^{MDCT-I}(k) \) can be obtained by exploiting any form of MDCT-I definitions. The estimated CFRs with the MDCT-I approaches have identical property; and then, we can write as:

\[
\hat{H}_{(i)}^{MDCT-I}(k) = H(k), \quad 0 \leq k \leq M - 1
\]

(24)

The proposed MDCT-I channel estimators show that all estimated CFRs are within the true CFRs interval represented in (24), which is the fundamental requirement of any channel estimator. This feature makes it efficient and different from the extensively used conventional DCT-I and DFT-based approaches in the LTE/LTE-A systems, resulting in the suitable application of such methods for CE in uplink NB-IoT systems. However, the estimated CFR might be different in practical scenarios due to noise in null subcarriers beyond the assigned NB-IoT UE subcarriers.

VI. TIME DIMENSIONAL LINEAR INTERPOLATION

A time dimensional (i.e., one dimensional (1D)) linear interpolation technique [40] is employed to carry out inter-slot interpolation through two NDMRS symbols as (4) within a subframe to attain the channel response of the entire uplink time-frequency grid. For the sake of simplicity, we assume \( \hat{H}_{Des.}(k) = \hat{H}^{LS}(k) = \hat{H}^{DFT-D-LS}(k) = \hat{H}_{(i)}^{DCT-I}(k) = \hat{H}_{(i)}^{MDCT-I}(k) \) for all employed CE techniques. Thus, the final estimated CFR \( \hat{H}_{Final}(k) \) can be computed for the whole OFDM symbols within a time-frequency grid of an uplink subframe as:

\[
\hat{H}_{Final}(k) = v_f \cdot \hat{H}_{Des.}(k) + (1 - v_f) \cdot \hat{H}_{Des.}(k)
\]

(25)

where \( v_f = (11 - l)/7 \) in (25) for 3.75 kHz subchannel bandwidth. The employed interpolation method in (25) is robust enough for limited mobility NB-IoT UEs, resulting in high interpolation precision in the low Doppler shift condition.

The channel estimates for 15 kHz subchannel spacing can be readily computed by considering only the mapped NDMRS symbols 3 and 10 instead of 4 and 11 within a subframe of an RU; then, \( v_f = (10 - l)/7 \) in (25).
VII. NUMERICAL SIMULATIONS AND COMPUTATIONAL COSTS ANALYSIS

The viability of the proposed CE approaches compared to the standard LS [27], DFT-D-LS [40], and original DCT-I [54] based CE techniques is verified through link-level Monte Carlo simulations. Furthermore, a comprehensive computational complexity analysis as compared to the state-of-art techniques is provided.

A. SIMULATION ASSUMPTION AND ENVIRONMENT SPECIFICATIONS

A single user (i.e., NB-IoT UE) and single-input single-output (SISO) configuration is considered to analyze the performance of different CE algorithms under the system context of the 3GPP NB-IoT specifications. The fundamental parameters employed to perform computer simulations are listed in Table 1, and referred to figures caption for a better understanding. The DFT size $M$ and the length of CP $L_{cp}$ are 512 and 16 samples, respectively. The sampling frequency 1.92 MHz and 7.68 MHz are assumed for 3.75 kHz and 15 kHz subcarrier spacings, respectively. The user subcarrier(s) $M_{RU}^{sc} = 1$ and 12 are taken into account for the analysis of system performance with single-tone and multi-tone operations, respectively.

Furthermore, the 3GPP [23] specified parameters are considered, such as MCS = 4, RU = 4, and TBS = 256 bits. To achieve wide-area coverage, a total of 64 times repeatedly transmitted same NB-IoT UE data/NDMRS are coherently combined before detecting the signal at the receiver for single-tone transmission with 3.75 kHz and 15 kHz subcarrier spacing (i.e., transmission periods are 8192 ms and 2048 ms). Moreover, a repetition of 64 times identical user
signal/NDMRS is combined for multi-tone (e.g., 12-tone) operation, which corresponds to 256 ms transmission time. The time/frequency synchronization is assumed to be perfect at the receiver, and the NPUSCH hopping is not adopted in this performance analysis.

**B. CHANNEL MSE PERFORMANCE ANALYSIS**

The average MSE is employed to validate the viability of the considered channel estimation techniques. The typical MSE is delineated here as the combination of the CE error with the channel modeling error owing to the exclusion of DCT-I extension coefficients. Thus, the average value of MSE considering the $P$-th repetition block can be obtained as:

$$
MSE = \frac{1}{mPZ} E \left[ \left\| \hat{H}_{\text{Final}}(k) - \hat{H}(k) \right\|^2 \right], \quad 0 \leq k \leq M - 1
$$

(26)

The plotted point of all simulation curves is obtained by averaging $Z = 10^4$ independent channel realizations. The simulation results for single-tone transmission (both 3.75 kHz

![SNR vs. MSE (12-tone, 15 kHz)](image-url)
and 15 kHz subchannels) using $\pi/2$-BPSK modulation are shown in Figs. 5 (a) and 5 (b), respectively. Observing Figs. 5 (a) and 5 (b), it can be seen that the DFT-D-LS method outperforms the conventional LS and classical DCT-I based estimators, but the main shortcoming is the high computational cost owing to the requirements of additional DFT/IDFT operations. It may also exhibit an irreducible error floor in high SNR regime because of ignoring the insignificant channel taps within the energy smearing area [40]. However, the proposed MDCT-I CE approaches exhibit improved MSE performance over their competitors by reducing the high-frequency distortion, border effect, and CIR leakage from the low-frequency region.

Figs. 6 (a) and 6 (b) show the channel MSE performance for single-tone operation (both 3.75 kHz and 15 kHz subcarrier spacings) using $\pi/4$-QPSK. The MSE performance of employed CE methods for multi-tone; for example, 12-tone operation using QPSK constellation is given in Fig. 7. Observing from Figs. 6 (a), 6 (b), and 7, the proposed approaches of MDCT-I also exhibit high-ranking MSE performance than other methods like LS, DFT-D-LS, and traditional DCT-I at a deficient SNR regime. In addition, to proving that the MDCT-I approaches are efficient and feasible for NB-IoT uplink transmission compared with the state-of-art schemes, the received SNRs, at the MSE of $10^{-3}$, for different subcarrier spacings, modulation techniques, modes of transmission, and CE techniques are noted, which is presented in Table 2. Based on the numerical results, the proposed estimators, especially MDCT-I approaches, exhibit outstanding performance for single-tone transmission with 3.75 kHz subchannel using $\pi/2$-BPSK modulation due to real-valued signals and low data-rates, which indicates the dense deployment of IoT devices in power and bandwidth limited scenarios. Furthermore, the proposed CE methods
C. SYSTEM BER PERFORMANCE ANALYSIS

The BER performance curves of uplink NB-IoT systems by implementing different CE methods for single-tone transmission (both 3.75 kHz and 15 kHz subchannels) employing $\pi/2$-BPSK and $\pi/4$-QPSK constellation schemes are shown in Figs. 8 and 9, respectively. We can see from simulation curves that all proposed approaches of the MDCT-I achieve significant SNR gain under the SNR $\ll 0$ dB as compared to other CE methods because the proposed approaches possess inherent energy concentration and spectral compaction features.

In the case of multi-tone operation, e.g., 12-tone, the proposed MDCT-I estimators also exhibit an excellent BER performance improvement using only QPSK modulation, as shown in Fig. 10. The MDCT-I approaches exhibit an error floor at high SNR as expected, due to truncation of channel taps beyond NB-IoT UE subcarriers. These techniques are also biased in high SNR regime because the non-ignorable CIRs $\hat{h}_{MDCT-I}(m)$ in (20) continues at the null subcarriers, resulting in performance degradation. The raised shortcoming is not our concern for implementing the MDCT-I estimators in the uplink NB-IoT systems since it will be functioned within a very low SNR environment. Furthermore, the BER curves at very low SNR (e.g., $-15$ dB) seem to be converged for the instance of approximately 0.25 because of the high noise power than the signal power. It is noted that the received signal would be decoded even when the noise power is far greater than the signal power with the signal repetition technique of NB-IoT systems [5]. One can readily test the exact BER saturation level by considering further low SNR values. The received SNR gain comparison, at the BER of $10^{-2}$, for different CE techniques, subcarrier spacings, modulation formats, and modes of operation, is presented in Table 3 for more readability.

To summarize, it is seen from Table 2 and Table 3 that the approach-1 of the proposed MDCT-I techniques performs better than the other two MDCT-I approaches for all cases of MSE and BER evaluation for the modification of $\hat{h}_{DCT-I}(0)$ and $\hat{h}_{DCT-I}(M)$ by the original DCT-I and the MDCT-I coefficients $\beta_z$ and $\bar{\beta}_z$. The employed normalization factors of the MDCT-I approaches also have a significant impact on CE performance variations. Furthermore, the MSE and BER performances with the single-tone transmission are superior over the multi-tone operation, but the data-rates may be relatively low due to long transmission time. Because,
the single-tone operation requires long transmission time over the multi-tone operation to maintain the identical signal repetitions, as mentioned in subsection VII-A. Nevertheless, the proposed MDCT-I CE techniques with better system performance than their contestants explicate the viability of operation in the NB-IoT uplink receiver.

D. COMPUTATIONAL COMPLEXITY ANALYSIS

We deduce that the traditional LS CE method given by (11) needs complex multiplications to compute the CFR. The computational cost of the DFT-D-LS method [40] is mainly contributed from \( M \)-point IDFT and DFT operations. Henceforth, the additional complexity of DFT-D-LS scheme than the conventional LS technique [27] comprises \( 2M \log_2(M) \) complex additions and \( M \log_2(M) \) complex multiplications. Obviously, the computational cost of the DFT-D-LS CE method is higher than the conventional LS method for reducing the LS estimation error.

The computational cost of the \( M \)-point DCT-I estimator consists of \( M(M - 1) \) additions and \( M^2 \) multiplications. The complexity of the proposed MDCT-I estimators will also be the same as the standard DCT-I based channel estimator [54] since it only modifies the normalization factors and DCT-I coefficients. However, the proposed MDCT-I estimators exhibit significant system performance improvement than the original DCT-I estimator at the SNR < 0 dB. Furthermore, the proposed MDCT-I approaches can be readily implemented by employing any fast DCT algorithms [55], [56] with the computational complexity \( M \log_2(M) \), which has fewer computational steps than fast DFT algorithms. Table 4 shows a comprehensive computational cost comparison among different CE methods in terms of complex/real multiplication and addition operations. Therefore, the proposed MDCT-I transform-domain estimators require quite less computational complexity over the DFT-based estimators [40] because the DCT estimator performs only real arithmetic operations [18] instead of complex operations.

It is also seen from Table 4, the number of multiplication operations for both standard DCT-I and proposed MDCT-I estimators raises with the square of \( M \). The computational complexity of such algorithms is also tightly related to the employed technology. The number of maximum allocated subcarriers is \( M = \frac{M_{RU}}{2} = 12 \) for NB-IoT uplink systems; thus, the MDCT-I approaches can be feasible CE methods in the LTE BS receiver. Moreover, the channel is being estimated at the NB-IoT uplink receiver (i.e., LTE BS) where sufficient computing resources are available; hence, no substantial impact on power consumption as well as the NB-IoT UEs costs to enable the green IoT framework.

VIII. CONCLUSION

In this study, the signal aliasing and border effect issues caused by the DFT-based transform-domain CE techniques are addressed by employing the inherent energy concentration and frequency compaction properties of the proposed MDCT-I approaches. Three options of DCT-I are presented by modifying the original form of DCT-I for efficient CE application. The proposed MDCT-I CE approaches efficiently estimate channel response without inhabiting extra frequency components and contributing additional computational cost, compared with the conventional LS, DFT-D-LS, and standard DCT-I based CE methods. Furthermore, in comparison with the state-of-art CE methods, the effectiveness and viability of the proposed CE methods are examined by rigorous computer simulations. Numerical results indicate that the proposed estimators outperform their competitors regardless of evaluation metrics. Thus, the proposed approaches seem attractive for the NB-IoT uplink transmission. Furthermore, the use case of DCT-I is shown for efficient CE applications in any multicarrier-based extremely narrowband LPWAN technologies.

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