Fast ConvNets Using Group-wise Brain Damage

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Abstract
We revisit the idea of brain damage, i.e. the pruning of the coefficients of a neural network, and suggest how brain damage can be modified and used to speedup convolutional layers. The approach uses the fact that many efficient implementations reduce generalized convolutions to matrix multiplications. The suggested brain damage process prunes the convolutional kernel tensor in a group-wise fashion by adding group-sparsity regularization to the standard training process. After such group-wise pruning, convolutions can be reduced to multiplications of thinned dense matrices, which leads to speedup. In the comparison on AlexNet, the method achieves very competitive performance.

1 Introduction
In the original Optimal Brain Damage work \cite{26} of 25 years ago, LeCun et al. observed that a carefully designed “brain-damage” process can sparsify the coefficients of a multi-layer neural network very significantly while incurring minimal or no loss of the prediction accuracy. Such process resembles the biological learning processes in mammals, in whose brains the number of synapses peak during early childhood and is then reduced substantially in the process of synaptic pruning \cite{6}. The optimal brain damage algorithm and its variants, however, impose sparsity in an unstructured way. As a result, while a large number of parameters can be pruned, the attained level of sparsity in the network is usually insufficient to achieve substantial computational speedup.

These days, due to the overwhelming success of very big convolutional neural networks (ConvNets) \cite{24} on a variety of machine learning problems, the task of speeding up ConvNets has become a topic of active research and engineering. Generalized convolution, i.e. the operation of convolving a 4D kernel tensor with the stack of input maps in order to produce the stack of output maps, is at the core of ConvNets and also represents their speed bottleneck. Here, we present a simple approach that modifies the standard generalized convolution process by imposing structured “brain-damage” on the kernel tensor. We demonstrate that considerable speed-up of ConvNets can be obtained for a certain structure.

This structure is motivated by the observation that the majority of current implementations of generalized convolutions (including the most efficient one at the time of submission) \cite{7,13,20,9,35,30} compute generalized convolutions by reducing them to matrix multiplications (this reduction is also referred to as lowering, unrolling, or the im2col operation). While unstructured brain damage in a convolutional layer, i.e. shrinking some of the coefficients of the convolutional kernel tensor to zero, will make one of the factor matrices (the filter matrix) sparse, it will not make the overall multiplication run faster. Our idea therefore is to group together the entries of the convolutional tensor in a
certain fashion and to shrink such groups to zero in a coordinated way. By doing this, we can eliminate rows and columns from both factor matrices that are multiplied when convolution is reduced to matrix multiplication. This makes both of these matrices thinner (but still dense) and results in faster matrix multiplication. As we demonstrate in our experiments, considerable speed-ups comparable with other CNN acceleration techniques are attainable in this way. E.g., we show that group-wise brain damage can accelerate the bottleneck layers of AlexNet ('conv2' and 'conv3') by a factor of 5x each simultaneously, while incurring only modest (1.5%) loss of the prediction accuracy.

We demonstrate that conventional group sparsity regularizer [36] embedded into stochastic gradient descent minimization is able to accomplish group-wise brain damage efficiently. The use of group sparsity thus allows us to optimized receptive fields in the convolutional network. Our approach therefore makes the case for the natural idea of using structured sparsity as a simple way to optimize connectivity in deep architectures.

2 Related work

As discussed above, currently a lot of attention is on speeding up generalized convolutions. In parallel to the lowering-based approaches mentioned above, which reduce convolutions to matrix multiplications, several works investigate the use of fast Fourier transforms [29, 34]. Despite the theoretical appeal, the use of Fourier transforms has its own limitations (mostly related to memory usage) and most existing packages stick to the lowering approach, which at the moment of the submission is also used by the fastest implementation [30].

Alternatively, several recent works investigate various kinds of tensor factorization in order to break generalized convolution into a sequence of smaller convolutions with fewer parameters [16, 12, 23]. Using inexact low-rank factorizations within such approaches allows to obtain considerable speedup when low enough decomposition rank is used. Our approach is related to tensor-factorization approaches as we also seek to replace full convolution tensor with a tensor that has fewer parameters. Our approach however does not perform any sort of decomposition/factorization for the kernel tensor. Another distantly related approach is represented by a group of methods [1, 15, 31] that compress the initial large ConvNet into a smaller network with different architecture while trying to match the outputs of the two networks.

Our approach is also related to methods that use structured sparsity [36, 32, 18] to discover optimal architectures of certain machine learners, e.g. to discover the optimal structure of a graphical model [17] or the optimal receptive fields in the two layered image classifier [19]. On the other hand, since our approach effectively learns receptive fields within a ConvNet, it can be related to other receptive field learning approaches, e.g. [10, 28].

The combination of sparsity and deep learning has been investigated within several unsupervised approaches such as sparse autoencoders [3, 5] and sparse deep belief networks [27]. We also note two reports that use some form of sparsification of deep feedforward networks and appeared in the recent months as we were developing our approach. Similarly to [26], the work [11] uses sparsification to reduce the number of parameters in the memory-bound scenario. Their goal is thus to save memory rather than to attain acceleration. In the report of [14], the output of the convolution is computed at a sparsified set of locations with the gaps being filled by interpolation. This approach does not sparsify the convolutional kernel and is therefore different from the group-wise brain damage approach we suggest here.

3 Group-Sparse Convolutions

Below, we discuss the reduction from generalized convolution to matrix multiplication [7] and introduce the notation along the way. We then explain the group-sparse convolution idea. Generalized convolution within a convolutional layer transforms an input stack of $S$ maps of size $W' \times H'$, which can be treated as a three-dimensional tensor (array) $U_{a/hs}$, into an output stack of $T$ maps of size $W'' \times H''$ which form a three-dimensional tensor $V_{a/hlt}$. The exact relation between $W', H'$ and
Figure 1: Standard Generalized Convolution (top) vs. Generalized Convolution after Group-wise Brain Damage (bottom). In both cases, we show the diagram for two input maps \((S = 2)\), blue-green color coding). We highlight three output maps \(t_1, t_2, t_3\) color-coded red-orange-yellow, and we also highlight two spatial locations \(l_1\) and \(l_2\). In both cases, the output map stack is obtained by reshaping the product of the filter matrix and the patch matrix. In the standard case, the filters and the patches sampled during the formation of the patch matrix are dense. After group-wise brain damage, both the filters and the patch sampling patterns are group-sparse (one sparsity pattern per input map), which results in much thinner filter and patch matrices and thus leads to much faster matrix multiplication/convolution.

\(W'', H''\) depends on the padding and stride settings within the layer, and our approach can handle any padding/striding settings seamlessly. The transformation is defined by the following formula:

\[
V(x, y, t) = \sum_{s=1}^{S} \sum_{i=1}^{d} K(i, j, s, t) \cdot U(x+i-\frac{d+1}{2}, y+j-\frac{d+1}{2}, s)
\] (1)

Here, \(K\) is a four-dimensional kernel tensor of size \(d \times d \times S \times T\) with the first two dimensions corresponding to the spatial dimensions, the third dimension corresponding to input maps, the fourth dimension corresponding to output maps. The spatial width and height of the kernel are denoted as \(d\) (for simplicity, we assume square shaped kernels and even \(d\)).

The implementation of (1) constitutes the speed bottleneck for ConvNets. In [8], it was suggested to reduce the computation of all entries of \(V\) to the multiplication of two large and dense matrices. The reduction allows to use highly optimized implementations of dense matrix multiplications (e.g. variants of BLAS [4] libraries) that have been developed over many years for all possible computing architectures. The reduction proceeds as follows:

- The kernel tensor \(K\) is reshaped into the filter matrix \(F\) of size \(T \times d^2 S\), where the \(t\)-th row corresponds to a sequence of \(S\) 2D filters \(K(\cdot, \cdot, s, t)\) reshaped in a row-wise fashion into row vectors.
- The input map stack \(V\) is reshaped into the patch matrix \(P\) of size \(d^2 S \times W''H''\), where the \(l\)-th column corresponds to a certain output location \(l = (x, y)\) and is stacked from the \(S\) patches extracted from \(S\) input maps, all centered at this location and reshaped in a row-wise fashion into column vectors.
The filter matrix $F$ is multiplied by the patch matrix $P$ resulting in a matrix $\tilde{V}$ of size $T \times W''H''$ that contains all the elements of $V$ (each column corresponds to a certain location and contains the values of this location in the $T$ output maps). The multiplication implements (1) exactly, as each row-by-column product within the multiplication corresponds to one instance of the computation (1) for certain $(x, y, t)$. The output tensor (map stack) $V$ can be obtained from $\tilde{V}$ by reshaping.

The construction discussed above has proven to be highly successful and is used in the majority of modern ConvNets “backends”, e.g. [8, 13, 20, 9, 35, 30]. Our key idea is to train ConvNets with sparse convolutional kernels that are consistent with this construction. Such consistency can be achieved if the sparsity patterns are aligned in a certain way. Formally, group-wise brain damage introduces a sparsity pattern $Q_s$ for every input map $s \in 1 \ldots S$. The sparsity pattern is defined as a subset of the full spatial $d$-by-$d$ grid, i.e. $Q_s \subset \{1 \ldots d\} \otimes \{1 \ldots d\}$.

The convolutional operation then becomes a slight modification of (1):

$$V(x, y, t) = \sum_{s=1}^{S} \sum_{(i,j) \in Q_s} K(i, j, s, t) \cdot U(x+i-d+1, y+j-d+1, s)$$

(2)

The reduction of (2) is an almost straightforward replication of the procedure [8]. The only modifications are (Figure 1):

- When the filter matrix is assembled, each 2D filter $K(:,: ,s, t)$ is reshaped into a row-vector of length $|Q_s|$ by including only non-zero elements. The filter matrix thus becomes of size $T \times \sum_{s=1}^{S} |Q_s|$.

- When the patch matrix is assembled, each 2D patch at location $l = (x, y)$ in map $S$ is reshaped into a column vector of size $|Q_s|$ by sampling the input map $U(:,: ,s)$ sparsely at locations $(x+i-d+1, y+j-d+1)$, where $(i, j) \in Q_s$. The patch matrix thus becomes of size $\sum_{s=1}^{S} |Q_s| \times W''H''$.

As a result of this modification, the multiplication of two dense matrices of sizes $T \times d^2S$ and $d^2S \times W''H''$ is replaced by the multiplication of two dense matrices of sizes $T \times \sum_{s=1}^{S} |Q_s|$ and $\sum_{s=1}^{S} |Q_s| \times W''H''$, which results in the $d^2S / \sum_{s=1}^{S} |Q_s|$-times reduction in the number of scalar operations. In our experiments with the reference implementation of [21] the wall-clock reduction in the convolution time between the original implementation and the group-sparse convolution was almost matching the “theoretical” speed-up factor (Figure 2).
4 Attaining Group Sparsity

In principle, it is possible to pick sparsity patterns in advance and to learn a ConvNet with group sparse convolutions from scratch. We, however, adopted a data-driven approach that allows to identify optimal sparsity patterns from data. For a certain kernel matrix $K$, for a certain spatial location $(i,j)$ in a 2D filter $(i,j) \in \{1 \ldots d\} \otimes \{1 \ldots d\}$, for a certain input map $s \in 1..S$, let us define $K_{ijs} = K(i,j,s,:)$ to be a $T$-dimensional vector that includes the kernel values corresponding to this input map and this location and all possible output maps.

1. **Regularized learning.** Train a ConvNet with a standard (dense) convolutions, using a strong sparsity-inducing regularization on the kernel tensors for the layers that need to be sped up.

2. **Group-wise brain damage.** For each layer, define a desired level of sparsity $\tau$ (the proportion of non-zeros after brain damage). Compare the $l_2$-norms of all vectors $K_{ijs}$ for all possible $(i,j)$ and $s$ in that layer, and set the $(1 - \tau)d^2S$ out of $d^2S$ of all such vectors to zero (picking those with the smallest $l_2$-norms). This will result in a certain sparsity pattern $Q_s$ for each input map, so that $\sum_{s=1}^{S} |Q_s|/d^2S = \tau$, and an approximate acceleration of the layer by the factor $1/\tau$.

3. **Fine-tuning.** Optionally, the resulting ConvNet can be “fine-tuned” by carrying on training with the fixed sparsity patterns $Q_s$ and without regularization.

In the experiments, we compared the two standard sparsity-inducing regularizers, namely the $l_1$-norm regularizer $\Omega_1(K) = \lambda \sum_{i,j,s,t} |K(i,j,s,t)|$ that does not induce any structure in the non-zero pattern as well as the group-sparsity regularizer using $l_{2,1}$-norm:

$$\Omega_{2,1}(K) = \lambda \sum_{i,j,s} \|K_{ijs}\| = \lambda \sum_{i,j,s} \sqrt{\sum_{t=1}^{T} K(i,j,s,t)^2}$$

that shrinks the groups corresponding to $K_{ijs}$ to zero in a coordinated fashion [36, 32] and results in a sparsified kernel tensor with the zero entries following the pattern that we need.

In the experiments, we took the simplest approach to ConvNet training with $l_{2,1}$ regularization and simply added the gradient of (3), i.e.:

$$\frac{\partial \Omega_{2,1}(K)}{\partial K(i,j,s,t)} = \lambda \frac{K(i,j,s,t)}{\sqrt{\sum_{z=1}^{T} K(i,j,s,z)^2}}$$

(4)

to the stochastic gradient at each iteration.

5 Experiments

**Implementation details.** Our implementation is based on Caffe [21] and modifies their original convolution, which is implemented as two subsequent layers (the im2col-layer that forms the patch matrix and the multiplication layer). To implement the group-sparse convolution we focused on the forward propagation step, although we can potentially accelerate back-prop in the same way, when the sparsity patterns $Q_s$ are fixed, e.g. during the fine-tuning step. Naturally, we only needed to modify the im2col-layer, so that it can fill in the patch matrix while following a certain sparsity patterns.

We conducted experiments with two types of architectures. The small-scale experiments are on the classical LeNet-architecture [25] (based on Caffe reimplementation), while the larger scale experiments are on the AlexNet-architecture [22] (again, Caffe reimplementation).

**Overall protocol.** The amount of sparsity naturally depends on the weight of the regularizer during training. Different desired levels of sparsity $\tau$ can require different regularizer strength $\lambda$ to obtain the best results. We therefore adopted the following protocol. We first do regularized learning of a set of models while varying the regularizer weight (with large ten-fold increments). For a certain desired sparsity level $\tau$ we pick the model that yields the optimal performance (on the validation...
Figure 3: Accuracy vs. density level on MNIST dataset (LeNet architecture) after group-wise brain damage (no fine-tuning). – the curves for different $\lambda$ (and the upper envelope) for the $l_{2,1}$ regularizer, – same for the $l_1$ regularizer. Right – the envelopes (and the curve for brain-damaged model trained without the regularizer). The curves are produced by averaging over ten runs and demonstrate the advantage of using group-sparsity regularizer.

set) after brain damage. We report the results by plotting the classification accuracy as a function of $\tau$. The resulting curve is thus the upper envelope of accuracy-vs-$\tau$ curves that are obtained for individual regularization strengths. Optionally (for the AlexNet) case, we do fine-tuning of optimal models for certain $\tau$.

**MNIST Results.** We trained the LeNet architecture on the MNIST dataset from random initialization while varying the regularizer ($l_1$, $l_{2,1}$, or training without regularization) and the regularization strength $\lambda$. Group-wise brain damage was applied to all convolutional layers with the same sparsity level $\tau$ (note that this strategy might be suboptimal). The results (without the fine-tuning of weights) are shown in Figure 3. The rightmost plot shows the comparison of the $l_1$-envelope, $l_{2,1}$-envelope, and the performance of the group-wise brain damage applied to the network trained without sparsity-inducing regularizer. The use of group-sparsity regularization boosts the performance of group-wise brain damage very considerably. Twenty-fold acceleration of convolutional layers can be obtained while keeping the error low ($2.1\%$, reduced to $1.71\%$ after fine-tuning). Using $l_1$-regularizer followed by optimal brain damage works worse than $l_{2,1}$-regularizer, but is still much better than applying group-wise brain damage to a network trained without sparsity-inducing regularizer.

**ImageNet Results.** In the case of the AlexNet architecture, we initialize with the original (undamaged) model as initialization. We focus on the use of $l_{2,1}$ regularizer as a clearly superior to $l_1$-regularizer based on theoretical considerations and MNIST experiments. As the computational bottleneck are in the second and the third convolutional layers with the second layer being the slowest one, we limit group-wise brain damage to either both of these layers (with the same $\tau$) or to the second layer alone. We use relatively small number of iterations ($10,000$ minibatches) for each $\lambda$, and more iterations ($250,000$ minibatches) during fine-tuning.

The results prior to fine-tuning are given in Figure 3, and the results with fine-tuning for some of the sparsity levels are in Table 1. We also compare to the results of state-of-the-art ConvNet acceleration methods based on tensor decompositions [12, 16, 23]. At least in terms of the speedup relative to the base implementation, our method outperforms them. In addition, for the case of a single-layer sparsification, we show the obtained sparsity patterns $Q_s$ in Figure 5.

6 Discussion

We have presented an approach to speeding up ConvNets by the group-wise brain damage process that achieves the acceleration by sparsifying the convolution operations. The approach takes into account the way generalized convolutions are reduced to matrix multiplications, and prune the entries of the convolution kernel in a groupwise fashion. The exact sparsity patterns can be learned from
Figure 4: The accuracy on ImageNet (ILSVR) for damaged AlexNet architecture (no fine-tuning) vs sparsity level. Left – damaging the second layer only, Right – damaging the second and the third layers. Note that fine-tuning improves the results considerably (Table 1).

| method               | sparsity | speed-up | accuracy after damage | accuracy after finetune | drop after finetune |
|----------------------|----------|----------|-----------------------|-------------------------|---------------------|
| **Second convolutional layer speedup**               |          |          |                       |                         |                     |
| Ours                 | 0.1      | 10x      | 8.5                   | 55.70                   | 1.13                |
| Ours                 | 0.2      | 5x       | 34.9                  | 56.40                   | 0.43                |
| Ours                 | 0.3      | 3.33x    | 44.4                  | 56.72                   | 0.11                |
| Ours                 | 0.4      | 2.5x     | 47.6                  | 56.92                   | -0.09               |
| Denton et al. [12]   |          |          |                       |                         | 1%                  |
| Lebedev et al. [23]  |          |          |                       |                         | 1%                  |
| Jaderberg et al. [16] |         |          |                       |                         | 1%                  |
| **Second+third convolutional layers speedup**         |          |          |                       |                         |                     |
| Ours                 | 0.2      | 5x       | 8.4                   | 55.33                   | 1.50                |
| Ours                 | 0.3      | 3.33x    | 35.0                  | 55.66                   | 1.17                |
| Ours                 | 0.5      | 2x       | 43.7                  | 56.26                   | 0.57                |

Table 1: Results of our method (for various sparsity levels) with and without fine-tuning alongside tensor-decomposition based methods (note: the results for [16] are reproduced from [23]). When followed by fine-tuning, group-wise brain damage obtains state-of-the-art results.

Figure 5: The sparsity patterns obtained by group-wise brain damage on the second convolutional layer of AlexNet for different sparsity levels. Nonzero weights are shown in white. In general, group-wise brain damage shrinks the receptive fields towards the center and tends to make them circular.
data using group-sparsity regularization. When applied after learning with such regularization and followed by fine-tuning, group-wise brain damage obtains state-of-the-art performance for speeding up ConvNets.

Aside from the immediate practical value, the proposed approach also makes the case for the use of sparse learning for discovering optimal network architectures. In our case, group-sparse regularizer allows the model to discover optimal receptive fields (Figure 5). It is interesting to see that the optimization process decided to shrink the receptive fields towards the center compared to the full version (which is consistent with the findings in [33]). Perhaps, even more interesting is to see that in general, the learning process decided to make the receptive fields roughly circular. Also, there were no cases (at least for these levels of sparsity) where the learning process would have decided to erase a filter all together, thus effectively pruning a certain map away from the input stack. Such pruning can however be explicitly encouraged within our approach using hierarchical group-sparsity regularizers [2, 18] and lead to additional speed-up.

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