Sub-threshold $\phi$-meson yield in central $^{58}\text{Ni}+^{58}\text{Ni}$ collisions

A. Mangiarotti$^{f,1}$, N. Herrmann$^g$, P. R. Maurenzig$^f$, A. Gobbi$^d$, R. Kotte$^e$, J. Kecskemeti$^b$, Y. Leifels$^d$, J. P. Alard$^c$, A. Andronic$^d$, R. Averbeck$^d$, V. Barret$^c$, Z. Basrak$^m$, N. Bastid$^c$, I. Belyaev$^h$, A. Bendarag$^c$, G. Berek$^b$, R. Čaplar$^m$, P. Crochet$^e$, A. Devismes$^d$, P. Dupieux$^c$, M. Dželalija$^m$, Ch. Finck$^d$, Z. Fodor$^b$, Y. Grishkin$^h$, O. Hartmann$^d$, K. D. Hildenbrand$^d$, B. Hong$^j$, Y. J. Kim$^j$, M. Kirejczyk$^{\ell}$, P. Koczoń$^d$, M. Korolija$^m$, T. Kress$^d$, R. Kutsche$^d$, A. Lebedev$^h$, V. Manko$^i$, M. Merschmeyer$^g$, D. Moisa$^a$, A. Nianine$^i$, W. Neubert$^e$, D. Pelte$^g$, M. Petrovici$^a$, C. Plettner$^e$, F. Rami$^k$, W. Reisdorf$^d$, B. de Schauenburg$^k$, D. Schüll$^d$, Z. Seres$^b$, B. Sikora$^{\ell}$, K. S. Sim$^j$, V. Simion$^a$, K. Siwek-Wilczyńska$^{\ell}$, V. Smolyankin$^h$, M. Stockmeier$^g$, G. Stoicea$^a$, M. Vasiliev$^i$, P. Wagner$^k$, K. Wiśniewski$^g$, D. Wohlfarth$^e$, I. Yushmanov$^i$ and A. Zhilin$^h$

$^a$National Institute for Nuclear Physics and Engineering, Bucharest, Romania
$^b$KFKI Research Institute for Particle and Nuclear Physics, Budapest, Hungary
$^c$Laboratoire de Physique Corpusculaire, IN2P3/CNRS and Université Blaise Pascal, Clermont-Ferrand, France
$^d$Gesellschaft für Schwerionenforschung, Darmstadt, Germany
$^e$Forschungszentrum Rossendorf, Dresden, Germany
$^f$Università di Firenze and Sezione INFN, Firenze, Italy
$^g$Physikalisches Institut der Universität Heidelberg, Heidelberg, Germany
$^h$Institute for Theoretical and Experimental Physics, Moscow, Russia
$^i$Kurchatov Institute, Moscow, Russia
$^j$Korea University, Seoul, South Korea
$^{\ell}$Institute of Experimental Physics, Warsaw University, Poland
$^k$Institut de Recherches Subatomiques, IN2P3-CNRS and Université Louis Pasteur, Strasbourg, France
$m$Rudjer Boskovic Institute, Zagreb, Croatia
Abstract

The $\phi$-meson production cross section is measured for the first time at a sub-threshold energy of 1.93 AGeV in $^{58}$Ni+$^{58}$Ni central collisions. The $\phi$ data were obtained within the acceptance of the CDC/Barrel subsystem of FOPI. For a sample of $4.7 \cdot 10^6$ central events, after background subtraction, 23 candidates were observed. Extensive GEANT simulations of the detector performance are shown in a thorough comparison to the real response, aiming at a good understanding of the apparatus and at a trustable determination of the efficiencies, production probability and possible systematic errors. A filter procedure is elaborated, which is meant to facilitate the comparison of any theoretical calculation or new data with the current ones. How to extrapolate the present value to a $\phi$-meson cross section in $4\pi$ is also discussed. This result on pseudo-vector mesons can now be compared to existing experimental knowledge for the same reaction at the same incident energy for various outgoing channels, $K^+$ and $K^-$ included. A significant fraction (at least 20%) of the $K^-$-mesons is originating in the decay of the $\phi$, supporting the statement that the two channels are strongly correlated.

Key words: heavy ion collision, SIS, sub-threshold meson production, pseudo-vector mesons, $\phi$-meson

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1 Introduction

Relativistic heavy ion collisions offer a unique possibility to probe experimentally hot and dense nuclear matter: at bombarding energies of $1 - 2$ AGeV baryonic densities of $2 - 3$ times the saturation value $\rho_0$ at zero temperature are reached, however the condition lasts merely about 15 fm/c and the degree of equilibration established in such a short interval is open to questions [1]. A long standing goal of the study of central reactions in the beam energy range $1 - 10$ AGeV is the determination of the compressibility $\kappa$ of nuclear matter at non-zero temperature and high density. Unfortunately, almost all the proposed observables sensitive to $\kappa$ have at some point been discovered unreliable. Many forms of collective motion (flow [2]) were thought to manifest simply the effect of the compressibility of nuclear matter, but were later proven to be strongly affected also by the momentum dependent part of the mean field so that the former one can not be simply disentangled [3,4,5] (in ref. [5] a reasonable estimate of $165 < \kappa < 220$ MeV was derived from the

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1 Corresponding author, address: Largo E.Fermi 2, 50125 Firenze, Italy Tel.: +39 055 2307787 FAX: +39 055 229330 email: mangiaro@fi.infn.it
Recently elliptic flow has been proposed as a possible test quantity [6]. Pion production was believed [7,8,9] to be another good observable to extract \( \kappa \); but the high absorption in nuclear matter makes it susceptible not only to the compression phase and maximum density but also to the details of the expansion [10,11,12]. Finally, Aichelin and Ko [13] proposed sub-threshold \( K^+ \)-production as a promising probe: in fact strangeness is conserved in strong interactions and \( K^+ \) absorption is negligible in nuclear matter (strangeness exchange is present but at equilibrium should be balanced) and hence the yield is insensitive to the expansion phase. Because of the Pauli blocking the direct nucleon-nucleon reaction, that could happen with the help of the Fermi momentum, is almost suppressed and at sub-threshold energies the dominant creation mechanism is a two step one. The conclusion has been confirmed experimentally by the KaoS collaboration measuring the variation in the yield with the number of participants to the central collision [14,15,16]. The production probability is then tightly bound to the maximum density reached in the compression phase.

However such apparently clear situation is obscured by the possibility of in-medium effects on the \( K^+ \), \( K^- \) and in general meson properties. Despite the small alteration of the \( K^+ \) mass in the nuclear environment, the experimental determination of the EOS is rendered difficult [17]. Studying the relation between system size and \( K^+ \)-production with high quality data, in order to distinguish between the in-medium and fire-ball density (i.e. EOS) contributions, has accumulated an evidence in favor of a soft equation of state of nuclear matter (\( \kappa \approx 200 \text{ MeV} \)) [18].

Nevertheless the subject of the possible influences of the surrounding nuclear medium on the meson masses is interesting by its own, with important implications in astrophysics [19,20,21,22]. For example, eventually the \( K^- \) condensation in neutron stars [23,24,25] could lead to a softening of the equation of state of nuclear matter above 3 times the saturation density \( \rho_0 \). The evolution of supernova explosions is then altered causing a core with \( 1.5 - 2 \) solar masses to collapse into a black hole rather than to form a neutron star [20,21,22].

In particular it is in general well accepted that in nuclear matter at equilibrium increasing temperature and/or density will lower the so called quark condensate \( \langle q\bar{q} \rangle \) thus partially restoring the chiral symmetry. However, while the prediction of the consequences on pseudo-scalar mesons (like the \( K^+ \) and \( K^- \)) is robust to different model frameworks [26], the situation appears disputed for pseudo-vector mesons (like the \( \phi \)) more exposed to problems in the theoretical description [27,28,29,30,31,32]. Additionally the \( \phi \)-meson yield is coupled to the sum of the \( K^+ \) and \( K^- \) mass. If, as it is currently hypothesized, the reduction of the latter can be as high as \( \approx 100 \text{ MeV} \) for \( \rho = 3 \rho_0 \), the corresponding decay width in the nuclear medium is diminished increasing the absorption probability. Actually Pisarski and independently Shuryak and
collaborators [33,34,35] originally proposed to study the kaon in-medium properties from an enhancement or suppression of the \( \phi \)-production probability in heavy ion collisions, implicitly considering the modification of the \( \phi \) mass itself less important. Hartnack, Oeschler and Aichelin have suggested that the \( K^- \) yield is almost insensitive to its own in-medium mass, but is instead strongly linked to the \( K^+ \) one by the secondary interaction \( K^- B \leftrightarrow \Lambda(\Sigma)\pi \) [36]. All these arguments indicate the strong interconnection between the \( K^+, K^- \) and \( \phi \)-channels and assess the impossibility of studying each of them separately. The present result is the missing piece to complete the already available knowledge on \( K^+ \) and \( K^- \)-mesons [37,38,39] for the same reaction and beam-energy.

Besides, due to \( \omega - \phi \) mixing, the \( \phi \)-meson wave function is dominated by the \( s\sigma \) component and so its propagation in nuclear matter is interesting for complementing the informations gained from hadrons with open strangeness \((K^+, K^-, K^0_S, \Lambda)\) [40,37,41,39,42].

Recently the parameterization of the \( NN \) in-medium cross section has been shown to be the major element of uncertainty for the calculation of the charged kaons production probability. As a matter of fact the disagreement between the Giessen group [43] and Nantes group [4,44] transport codes, quite dissimilar on a physical basis, was found to be marginal, under such respect, when a common parameterization is supplied [36]. It is also comparable to the predicted variation between the in-medium and no in-medium effect scenarios.

The long awaited measurement of both the elementary processes \( pp \rightarrow K^- X \) [45,46] and \( pp \rightarrow \phi pp \) [47,48] near threshold allows to nail down in a critical region the freedom in the cross section parameterizations necessary for theoretical models, consistently reducing ambiguities.

In this work the data from FOPI restricted to the CDC/Barrel subsystem on \( \phi \)-meson yield in \(^{58}\text{Ni} + ^{58}\text{Ni}\) at 1.93 AGeV will be introduced. From within the limited experimental acceptance, a 4\( \pi \) extrapolation can exclusively be done once the shape and characteristics of the source are assumed. The channels and thresholds for \( \phi \)-meson creation are not too different from that of a \( K^+ K^- \) pair and, as recalled, they are strongly related: our best estimate of the \( \phi / K^- \) ratio for central collisions will be discussed. A significant part of the \( K^- \), at least 20\%, seems to arise from the decay of the \( \phi \). Preliminary hints of sub-threshold \( \phi \)-meson production have already been reported using both the CDC/Barrel [49] and the Helitron/Forward Wall [50] combinations, here the emphasis is on the extraction of a quantitative estimate of their occurrence probability in central collision.
2 Data analysis

2.1 The FOPI detector

The FOPI detector is a modular system for fixed target experiments at the SIS accelerator of GSI-Darmstadt. The four subdetectors (CDC, Barrel, Helitron and Forward-Wall [51,52]) are coupled two by two (CDC/Barrel and Helitron/Forward-Wall) to provide momentum, energy-loss and time of flight information in the polar angles $39^\circ < \vartheta_{\text{lab}} < 133^\circ$ and $7^\circ < \vartheta_{\text{lab}} < 28^\circ$, respectively. The coverage in the azimuthal angle $\varphi$ is almost complete, except for the Plastic Barrel. The reference polar coordinate system of the current work has the beam direction (i.e. $z$-axis) as opening direction and the target as origin for the polar angle $\vartheta_{\text{lab}}$. The azimuthal angle $\varphi$ lies in the plane orthogonal to the beam axis containing the target position (i.e. $xy$-plane).

The Barrel and the CDC are operated in a magnetic field of 0.6 T produced by a super-conducting coil. The Helitron is a vector chamber and together with the Forward-Wall is placed in the fringe field of the solenoid in the forward hemisphere. In the ongoing analysis the CDC/Barrel subsystem alone is involved and in what follows we focus on it.

The CDC is a drift chamber of the Jet type [53,54,55,56,57,58] divided in 16 sectors with projective geometry for the forward edge (inner radius 20.7 cm and length 75 cm, outer radius 80.1 cm and length 192.5 cm, see table 1) each equipped with 60 sense wires in the median plane. Such choice has the advantage of allowing a more uniform charge collection and multiplication field together with a reduction of the drift length by a factor of 2, still keeping an equal amount of electronic channels. However, the effective readout comes on cost of the left-right ambiguity for every hit that for tracking purposes needs to be considered in a pair of spatial positions of which just one corresponds to the correct localization. To partly remedy the problem the sectors are tilted by $8^\circ$ with respect to the ideal radial symmetry around the target so that only one of the two mirror images points to the latter. In addition, the sense wires are staggered [53,54,55,56,57,58] by $\pm 200 \ \mu$m about the nominal sense wire plane giving an independent criterion for the distinction of real and mirror tracks. The gas mixture chosen is 88% Ar, 2% $CH_4$ and 10% iso-butane at atmospheric pressure.

In Jet chambers the transverse position of the hit in the $xy$-plane is deduced from the cell number and measured drift time, while that along the $z$-direction by the charge division between the ends of the wire: the physical basis is quite different for the two coordinates and also the physical mechanisms imposing the actual limits to the resolutions are not the same; as a consequence the first
is known much better than the second. Such feature is taken into account in the processing algorithm where the hits candidate to form a track are identified initially in the $xy$-projection of the event [54] requiring the distance from the nearest neighbor to be less than $\Delta_{xy\text{hit}}$ (see table 2). In the successive step the position along $z$ of the hits pertaining to a track is examined and all those further apart from the closest one of $\Delta_{z\text{hit}}$ (see table 2) are rejected as not belonging to it. For each particle the transverse momentum component $p_t$ over charge $Z$ (in units of the elementary charge) can be obtained by the inverse $k$ of the radius of the arc of circle fitted through the $xy$-projection of the trajectory, as

$$
\frac{p_t}{Z} = \frac{0.3 \, B \text{ GeV/c T}^{-1} \text{m}^{-1}}{k} = \frac{0.18 \text{ GeV/c m}^{-1}}{k},
$$

where in the last expression we substituted $B = 0.6$ Tesla for the magnetic field of FOPI. From the angle $\vartheta_{\text{lab}}$, delivered by the straight line fitted to the $z$-coordinate of the hits, the total momentum $p$ over charge $Z$ and its components are found

$$
\frac{p}{Z} = \frac{p_t}{\sin \vartheta_{\text{lab}}},
$$

The error on $p$ is

$$
\frac{\sigma_p^2}{p^2} = \frac{\sigma_{p_t}^2}{p_t^2} + \cot^2(\vartheta_{\text{lab}}) \sigma_{\vartheta_{\text{lab}}}^2
= \frac{\sigma_k^2}{k^2} + \cot^2(\vartheta_{\text{lab}}) \sigma_{\vartheta_{\text{lab}}}^2
$$

where the two autonomous contributions are apparent: $\sigma_k$, coming from the determination of the radius of curvature of the track in the transverse plane and, $\sigma_{\vartheta_{\text{lab}}}$, from the reconstruction of the emission angle $\vartheta_{\text{lab}}$. In the CDC the physical origins of $\sigma_k$ and $\sigma_{\vartheta_{\text{lab}}}$ are quite dissimilar as we have already discussed. The condition $2/k > R_{\text{min}}$ for the trajectory to enter the CDC can be reexpressed with eq. (1) as $p_t > 0.019 \text{ GeV/c}$.

The particle identification is based on a double technique allowing a redundancy very powerful to reject the background. Once the momentum is known, the charge over mass ratio can be determined (and hence the species assigned) if the velocity is measured.

Exploiting the dependence of the mean energy-loss on the velocity, and not on the momentum, of the particle sweeping through the detection medium (i.e. the CDC gas), using an empirical parameterization, a mass named $C\text{mass}$ can be extracted. In fig. 1 (upper two panels) the truncated mean energy-loss [54,56] (after correction for the $\vartheta$ angle of incidence in each cell evaluated...
from the track fit) is shown versus momentum: the branches corresponding to \( \pi \), \( K \) and \( p \) are discernible and separable. Since when the Barrel is reached the track length inside the CDC and hence the number of sampling points for the energy-loss distribution are constrained, only such cases are considered in fig. 1, besides it will be the final requirement of the present analysis. The continuous curves, also drawn, refer to the empirical parameterization defining the regions where the various species can be identified.

An uncorrelated information on the mass is derived for the ones that survive up to the Barrel by the time of flight and the length of the extrapolated trajectory fitted to the CDC hits (a condition is imposed on the difference \( \Delta \varphi \) in \( \varphi \)-angle and \( \Delta z \) in \( z \)-coordinate between the measured and expected impact position in the Barrel, see table 2). In this case the velocity is determined directly. The Barrel consists of 180 bars of scintillator of width 3.25 cm, depth 4.05 cm and length 240 cm arranged in 30 modules (the space for 2 further ones is occupied by the CDC support structure) disposed in a cylindrical fashion at a radius of \( R_{\text{Bar}} = 111.5 \) cm coaxially with the \( z \)-axis and covering the \( \vartheta_{\text{lab}} \) angles reported in table 1. Due to the missing modules and to the inter-module and inter-bar spacings also the azimuthal coverage is incomplete. The scintillators are readout with photo-multipliers located outside the FOPI magnet through long light guides. The condition \( 2/k > R_{\text{Bar}} \) for the Barrel to be reached can be reexpressed with eq. (1) as \( p_t > 0.10 \) GeV/c. In table 1 we have summarized the acceptances of the CDC and Barrel subdetectors of FOPI. In fig. 1 (lower two panels) the velocity is displayed versus momentum and again the branches corresponding to \( \pi \), \( K \) and \( p \) are distinguishable. The continuous curves are the well known relativistic expression in the nominal cases. A mass named \( B_{\text{mass}} \) is calculated as

\[
B_{\text{mass}} = \frac{p}{c \beta} \sqrt{1 - \beta^2}.
\]  

The \( C_{\text{mass}} \) is limited at high momentum from the relativistic rise of the energy-loss as a function of \( p \): above \( \approx 0.6 \) GeV/c the pion and kaon region merge and contamination is unavoidable (see fig. 1 a and b).

Also \( B_{\text{mass}} \) is limited, for kaons, below \( \approx 0.6 \) GeV/c by the finite time resolution of the Barrel (\( \approx 400 \) ps) joined to the divergence in the relativistic momentum-velocity relation (see fig. 1 c).

The advantage of the redundant simultaneous measurement of both \( C_{\text{mass}} \) and \( B_{\text{mass}} \), is illustrated in fig. 2 representing one mass versus the other: a structure corresponding to the \( K^+ \) is clearly visible, the \( K^- \) are less well separated owing to their small abundance. To perceive an idea of the difficulty of such task, let us remind that in the acceptance for 1 proton there are roughly \( \approx 10^{-1} \) pions, \( \approx 10^{-3} \) \( K^+ \) and \( \approx 10^{-5} \) \( K^- \). The superior quality of \( B_{\text{mass}} \)
over $C_{mass}$ can be deduced from the deformation of the distribution along the latter. For this reason they will not be treated alike in the final identification cuts.

2.2 Event selection

The online event centrality selection is based on the detected charged particle multiplicity in the Forward-Wall, $M$, obtained with fast analog electronics [51]. A minimum bias trigger is defined when more than 4 are found, and a central one when $M > M_c$. During event acquisition, the quantity $M_c$ is set so that the ratio between the minimum bias and central triggers corresponds to about 10% [51]. In particular we focus on the $7.7 \cdot 10^6$ central collisions collected with the FOPI detector in 1995 for the $^{58}\text{Ni} + ^{58}\text{Ni}$ reaction at 1.93 AGeV beam kinetic energy. After imposing constraints on the energy deposited in the start counter, for the purpose of optimizing the time resolution, and on the primary interaction vertex position in both the transverse $xy$-plane and $z$-direction, for the purpose of reducing background from interactions not produced in the target, a sample of $4.7 \cdot 10^6$ events is left. In the more accurate offline reprocessing a certain jitter is observed in the multiplicity threshold for the central trigger caused by the non-ideal behavior of the analog electronics with which it is implemented (see fig. 3). The similarity of a selection on minimum multiplicity $M$ to an upper limit on impact parameter $b(M)$ is straightforward in the sharp cut-off hypothesis assuming a monotonic relation between $M$ and $b$. Adopting a simple geometrical model the central collision condition cited through this work can be seen to be equivalent to $0 < b < \approx 3.3$ fm or to the most central $\sigma_c \approx 350$ mb (i.e. 12%) of the geometrical reaction cross section $\sigma_{geo} \approx 2.9$ b [38]. A rather simple comparison is possible for the yield per central event between our experimental finding and transport codes once the the prediction is studied as a function of centrality. For the theoretical result $P_{th}(b)$ an appropriate mean $\langle P_{th} \rangle$ over $b$ in the range $0 < b < 3.3$ fm is to be computed taking as a weight the ratio of the differential reaction cross section $d\sigma_R/db$ to the integrated one $\sigma_c$

$$ \langle P_{th} \rangle = \frac{2}{b_{max}} \int_0^{b_{max}} P_{th}(b) b \, db , $$

where $b_{max}$ is 3.3 fm in the case of FOPI.
2.3 \( \Phi \)-meson identification

The \( \phi \)-meson is identified through its decay channel in a \( K^+ \) and a \( K^- \) \((BR = 49.1\%)\). The charged kaons are pre-selected mainly with a window in both \( C_{mass} \) and \( B_{mass} \) and an upper momentum limit. A data sub-sample is generated with kinematic and other quantities. For calculating the invariant mass of the couple, the common nominal value of the two species is used. The combinatorial background is reconstructed via the event-mixing technique [59,60]: at the same time an independent data sub-sample is generated regrouping in turn each kaon from the current event together with the opposite charged partner from a stack of depth 20 of previous ones. The latter are sorted with respect to the amount of particles detected in the Forward-Wall (i.e. impact parameter, see sec. 2.2). The stack can be filled with central collisions where a \( K^+ \) or a \( K^- \) is pre-identified or where both a \( K^+ \) and a \( K^- \) are pre-identified. The momentum vectors from the stack are rotated around the beam axis so that their reaction plane has the right current orientation. The population size in the event-mixing is easily made much larger (2 orders of magnitude) than in the data, in such fashion the statistical error contribution to the outcome for the \( \phi \)-candidates is negligible.

To reject mirror or erroneous tracks, three general cuts to ensure good quality are imposed: on \( d_0 \) (the minimum distance in the transverse \( xy \)-plane of the trajectory from the reconstructed event vertex position), on \( z_0 \) (the displacement from the target of the point where the helix crosses the \( z \)-axis) and on \( n_{hit} \) (number of hits): \(|d_0| < 0.5 \text{ cm}, |z_0| < 10 \text{ cm} \) and \( n_{hits} > 30 \) (see table 2). A difference between \( d_0 \) and \( z_0 \) should be noted: while the target thickness is small and the initial collision \( z \)-coordinate is well defined, the corresponding statement is not true for the beam spot transverse dimension and \( d_0 \) refers to the \( xy \)-position extracted from the data. By the way, the acceptance windows on \( d_0 \) and \( z_0 \) also help to reduce the background of pions coming from secondary vertices. All tracks fulfilling the \( d_0 \) and \( z_0 \) selections are then refitted forcing their origin in the event estimated first interaction point. This new helix is then the base for calculating the momentum and for performing the extrapolation to the Barrel necessary to evaluate the matching condition and the length of flight. In the present situation the procedure can not rise any problem since all the \( \phi \)-mesons are decaying almost immediately outside the fireball (with \( c\tau = 44.5 \text{ fm} \) and \( p = 1 \text{ GeV/c} \) the mean length traveled is 43.7 fm) and hence they are all well inside the target. It is of course not accomplished in those studies, like on \( \Lambda \)-baryon production, in which detection of a vertex apart from the primary reaction one has to be exploited [40,42].

The final cuts (see table 2) \( p_{lab} < 0.6 \text{ GeV/c}, C_{mass} > 0.37 \text{ GeV} \) (for pion background suppression, see fig. 2 a and b), \( C_{mass} < 0.8 \text{ GeV} \) (for proton background suppression, see fig. 2 a) and \(|B_{mass} - 0.494| < 0.1 \text{ GeV} \) are then
applied for particles of both charges to the two sub-samples (data and event-mixing) and the $K^+ K^-$ invariant mass distributions are constructed (as shown in fig. 4). The normalization is provided integrating the spectra aside from the $\phi$-peak ($m_{inv} > 1.06$ GeV/c). The combinatorial background drawn with the lines enclosing one standard deviation in fig. 4 is obtained filling the mixing stack with events containing either a $K^+$ or a $K^-$, the other, represented with the standard error bars, when the mixing stack is filled from events containing a $K^+$ and a $K^-$. The first is much more regular and will be employed in the subsequent analysis. The second has still instabilities of non statistical origin indicating that, although the whole sample is just 30% smaller, the poorer class from which it is drawn (i.e. the same element of the stack is reused more often) makes the destruction of the correlations harder. It can be considered solely for estimating a kind of maximum systematic error due to self correlation induced by the limited population adopted for mixing (such source would not be important in an analysis with much more candidates).

After subtraction, the number of events in the $\phi$-peak (defined from the end-points at 1.01 and 1.03 GeV where the remainder of the signal and the background becomes negative) with its statistical error is of $23 \pm 7$. In a previous analysis \[61\], done with older calibrations, a result of 22 was found. From this difference and from the difference obtained subtracting both backgrounds a systematic error of $\pm 2$ or 9% is estimated.

3 Simulation

The $(23 \pm 7 \pm 2)$ $\phi$-mesons given in sec. 2.3, are of little utility for a comparison with future experiments or with theoretical calculations. An accurate determination of the detection efficiency by simulation is needed to deduce a cross section within the phase space coverage (or in $4\pi$).

In the present section we will always assume that all $\phi$-mesons are decaying through the $K^+ K^-$-channel. The appropriate branching ratio will be included in sec. 4 for deducing the production probability.

The overall efficiency $\epsilon_{\text{tot}}$ to measure in our device such particle out of $4\pi$ can be splitted into two terms

$$\epsilon_{\text{tot}} = \epsilon_{\text{max}} \epsilon_{\text{det}},$$

where $\epsilon_{\text{max}}$ is defined as the efficiency of measuring a $\phi$ within the geometrical and momentum acceptance of a ”perfect” apparatus plus the effect of kaon decay in flight during their path to the Barrel and $\epsilon_{\text{det}}$ contains the further
reduction introduced by detector limitations and by the constraints necessary for a clean identification (vertex, matching and mass cuts).

Information on the $\phi$ distribution in $4\pi$ cannot be obtained in the current effort because the phase space region covered by the CDC/Barrel subsystem around mid-rapidity is small. For an extrapolation into the full solid angle of the cross section extracted in the acceptance, one has to rely upon a $4\pi$ shape.

The advantage of the factorization (6) is that while $\epsilon_{\text{max}}$ depends essentially on the initial phase space distribution in $4\pi$, $\epsilon_{\text{det}}$ does not, within a good approximation. Such case will be demonstrated to hold in a representative example in sec. 3.2 and 3.3 where the ”effective” temperature of a one source thermal model is varied. Thus the comparison between our experiment and new ones or different calculations is greatly facilitated. For any elaborated study delivering the entire $\phi$ population, it is sufficient to apply a simple ”filter” procedure (see appendix A). This takes into account the detector geometry and momentum cut as well as the kaon decay, neglecting first all complicated details of the apparatus performance, which can be considered separately through the constant coefficient $\epsilon_{\text{det}}$, determined once for all with the necessary knowledge of FOPI in sec. 3.3.

3.1 The CDC/Barrel maximum acceptance

In the study developed here the simple hypothesis of a thermal source controlled by only one parameter (i.e. the temperature $T$) is adopted. A more refined model would be the Siemens and Rasmussen [62] formula, containing a temperature $T_{\text{SR}} = 92$ MeV and a collective expansion with velocity $\beta_{\text{flow}} = 0.3$. The whole idea behind this description of a central collision is that $T_{\text{SR}}$ and $\beta_{\text{flow}}$ are common for all hadrons and so their values can be supplied by a previous study done with the FOPI detector about $p$, $d$ and $\pi^-$ from the same $^{58}\text{Ni}+^{58}\text{Ni}$ reaction at the same beam-energy of 1.93 AGeV [63]. However, as long as just the $\phi$-meson is concerned and within the necessary accuracies, the source can be approximated with a pure thermal one with $T = 127$ MeV. The influence of the dynamics can then be reabsorbed in the temperature $T$, that becomes distinct from the real one $T_{\text{SR}}$ (and species associated). Transport model calculations generally follow a similar trend: the shape of the phase space distributions can be considered thermal within the present statistical and systematic errors, but the effective temperatures are lower, typically $70 - 90$ MeV. We can than in rough approximation reduce our uncertainty on the source to a thermal source where the temperature is unknown by a wide margin.

To investigate the change in CDC and Barrel acceptances with temperature
$T$ in the interval $130 - 70$ MeV, four GEANT simulations (see table 3) were undertaken. However the detector response description and event reconstruction stages are not involved in the extraction of such estimates and solely the FOPI geometry implementation and particle transport part of the code were exploited. The results are summarized in table 3 with and without kaon decay enabled. The column corresponding to $\epsilon_{\text{max}}$ is the third from the end: a total excursion with temperature by a factor of $\approx 4$ is found. This clearly illustrates the impossibility of giving one cumulative number and the necessity of the decomposition (6) in order to allow a comparison with the predictions.

It is important to realize that the smallness of $\epsilon_{\text{max}}$ derives from the localization of the phase space covered by the CDC/Barrel around the target rapidities. In fig. 5 a the $p_t/m$ versus normalized rapidity $y$ plot is shown for the $K^+$ and $K^-$ mesons reaching the Barrel and originating in the decay of the $\phi$ emitted by a thermal source with 130 MeV temperature. As can be seen, the relevant cuts are the Barrel forward $\vartheta_{\text{lab}}$ angle limit of $39.2^\circ$ and the maximum momentum allowance of 0.6 GeV/c (see table 1). In fig. 5 b the $p_t/m \div y$ representation is used for the primary quantities of the $\phi$-mesons whose $K^+$ and $K^-$ decayed into the range of acceptance. Its maximum is still well away from the mid-rapidity region. The dot-dashed curves, also in panel b, correspond to the most probable momentum for a relativistic Maxwell-Boltzmann $\phi$ emission law centered at $y = 0$. The portion of the diagram accessible with the CDC/Barrel subdetector combination is outside this line indicating that exclusively the tail of the distribution is probed. This becomes even more pronounced if the source temperature is lowered, indicating why $\epsilon_{\text{max}}$ is reduced under such conditions.

3.2 The detection efficiency estimation from GEANT

As already mentioned $\epsilon_{\text{det}}$ contains the adjoined contribution to the efficiency, in the phase space acceptance and after kaon decay, of the detector behavior. For its determination a precise reproduction of the apparatus response is necessary. The current section is devoted to the discussion of the quality of the environment implemented in the simulation. This is important to judge upon the reliability of the conclusions derived from the latter.

For our study we started from the well proven software package GEANT (version 3.21 distributed with CERNlib version 99) written at CERN [64]. In particular this frame comprises very powerful routines for the description of the detector geometry (only the CDC, Barrel and the magnet, return iron yoke, CDC/Barrel support structures and target passive volumes have been implemented for the ongoing attempt). Once the material of each volume and its characteristics have been specified the standard package routines are capable of calculating the particle transport in the magnetic field throughout
the detector including the decay with the appropriate lifetimes and branching ratios and the interactions with the penetrated media. Each decay product can then in turn be transported and let decay until they are all outside the interesting regions. If a volume is specified as sensitive (e.g. the CDC gas or the Barrel scintillator) hits are generated with the corresponding positions and energy-loss values.

The remaining parts under the user responsibility (except the geometrical and material descriptions) are the event generator and the hit processing routine necessary to build the final output.

With the thermal source a class of simulations, indicated in the following as “single-\(\phi\)”, has been run. However the realistic track multiplicity of the full nuclear Ni+Ni event, originating the \(\phi\)-meson, will certainly reduce the efficiency. In fact, due to the low granularity of the Barrel, one or more of the 180 scintillator bars can be crossed by multiple particles in the same event. In such case a sole improper position is recovered and both hits are discarded in the CDC/Barrel matching phase. If one of them corresponds to a kaon from the \(\phi\) decay, the \(\phi\) itself is lost. The CDC behavior too is not expected to be sampled correctly under the simplified situation with just two kaons not involving noise hits together with intersecting tracks and their mirror images. Each of the preceding reasons of inefficiency is affected by the distribution of the emitted \(\phi\)-meson azimuthal angle with respect to the reaction plane where trajectories tend to concentrate. To take into account as much as possible the effect of the environment surrounding the paths of the \(K^+\) and \(K^-\)-mesons created by the \(\phi\) decay, an other class of simulations, named in the following as “embedded-\(\phi\)”, was accomplished. In it, to the thermally generated \(\phi\) a whole IQMD Ni+Ni central collision \( (b = 0) \) was superimposed. The orientation of the reaction plane, with respect to the \(\phi\) azimuthal emission angle, was changed at random. We remind that the version of IQMD [44] employed contains no kaons, but only pions, nucleons and heavier fragments.

In the hit processing routines a description of the front-end electronics has been embodied with the associated noise. For the CDC to derive from the information provided by GEANT (position and energy-loss) the contents of the 100 Mhz Flash ADC memories, four steps are necessary. First, for all tracks in a sector the charge corresponding to the given energy-loss is distributed with fluctuations along the path within each individual drift cell between electron-ion clusters. The latter are afterwards propagated under the position dependent influence of the electric and magnetic fields [54] up to the sense wire. For
every cluster a generic pulse shape at each of the ends is also delivered. Second, all the pulses arriving at one specific readout channel are added together time bin wise without regarding the particle that produced them (in such way a cross-talk between different tracks in a sector is introduced). Third, the noise is added in an uncorrelated manner, separately for each wire, and in a correlated one, injecting charge simultaneously into all wires of one sector (as it was found to occur in the experiment). In both cases a Gaussian distribution is assumed. Finally, the sum is digitized with a 8 bit nonlinear (10 bits effective) ADC accuracy. For the Barrel the time and energy digitized information is derived on the basis of a constant propagation velocity and attenuation length in the scintillator bars and light guides. A Gaussian smearing is added both in time and pulse height to include experimental resolutions. Double hits in the same scintillator bar are treated in a fashion that the fastest signal reaching the photomultiplier defines the time tag (independently for both ends) and the pulse heights are summed. Now the events are in a form compatible to be fed as input to the tracking and reconstruction code with which data are elaborated.

3.2.1 The basic input resolutions as compared to the experiment

Since the characteristics of the detection system can not be probed by a well defined beam hitting into the acceptance of the CDC, the performance of the Monte-Carlo digitization routines has to be monitored and tuned from the response to usual tracks. For those, of course, the real transverse momentum $p_t$ or $\theta_{\text{lab}}$ angle are not known. To remain as close as possible to the basic physical resolutions connected to how the chamber operates we have decided to compare the mean rms of the track fit residuals in the transverse plane $\text{RMS}_{xy}$, the mean rms of the track fit residuals in the $z$-coordinate $\text{RMS}_z$ and the mean rms of the truncated energy-loss distribution $\text{RMS}_{dE/dx}$. The results are displayed in fig. 6 for $K^+$ and $K^-$-mesons from the single-$\phi$ simulation and $K^+$ alone from the Ni+Ni collisions due to limitations in $K^-$ statistics.

For the data a set of identified kaons, defined as in the $\phi$-meson analysis, was selected (see table 2). It has to be stressed that at a fixed $p$ the energy-loss for unlike particles is not the same and the CDC behavior is correspondingly changed; so not all of them together, but exclusively $K^+$-mesons, should be compared with the calculation.

In the case of the Monte-Carlo only kaons known to have reached the Barrel sensitive volumes during the GEANT transport phase and with generated momentum less than 0.6 GeV/c were considered. Because at most two tracks per event are present, the position of the primary collision vertex in the transverse $xy$-plane can not be pinpointed as in the data; rather the refit is always forced to the origin where the initial position of all the reaction products is located.
As default $d_0$ and $z_0$ are taken from the free fit. Such procedure, adopted here and in what follows, is expected to reproduce best the experimental situation where the number of CDC tracks is far greater ($\approx 30$) and hence the determination of the event vertex $xy$-coordinates is accurate. Note that the true conditions correspond more to the embedded-$\phi$ scenario.

To gain statistics a 130 MeV thermal source was used and the kaon decay inhibited. A comparison with a temperature of 70 MeV or with the kaon decay enabled has exhibited negligible differences below the systematic ones between data and simulation (this is the case also for the observables studied in the next section).

The agreement is quite satisfactory for RMS$_{xy}$ and RMS$_{dE/dx}$ (see fig. 6 a and c). There are indications that the increase of RMS$_z$ with $p_t$ is too weak in the simulation (see fig. 6 b). The stronger rise in the real detector is probably to be ascribed to a form of common high frequency noise induced in all wires, known to be relevant in this type of chambers [56], but not handled properly in the CDC digitization routine developed for GEANT. In fact while its repercussions are relatively negligible for RMS$_{xy}$, being the transverse position deduced from the drift time, they are rather important for RMS$_z$ where the coordinate is determined from the charge division.

The discrepancies between the values of RMS$_{xy}$ and RMS$_z$ in the Monte-Carlo and the data are small with respect to the much larger cuts $\Delta_{xy\,\text{hit}}$ and $\Delta_{z\,\text{hit}}$ controlling the tracker behavior and so no readjustment in particular of the latter was done for the analysis of the simulation (see table 2).

Once RMS$_{xy}$ and RMS$_z$ have been fixed, the CDC accuracy in transverse momentum RMS$_{p_t}$ and in $\vartheta_{\text{lab}}$ angle RMS$_{\vartheta}$ can be investigated with the Monte-Carlo incorporating the description of the detector response. Energy-loss, energy-loss straggling and multiple scattering in the passive and active volumes of the CDC/Barrel combination were accounted profiting of the description of the physical processes embodied in the GEANT software package [64]. The results are summarized in fig. 7 both for the mean systematic shifts and the resolution of the reconstructed quantities.

For $p_t$ a maximum systematic variation of $\pm 1\%$ is seen. The low $p_t$ distortion in fig. 7 a is to be attributed to the energy-loss in the target and chamber entrance materials, as is proven by the simulation where these passive volumes were removed. The dissimilar behavior between positive and negative particles in the high $p_t$ region (see fig. 7 a) is due to the sectors tilting that introduces a systematic asymmetry. In fig. 7 b the $p_t$ resolution is also compared with what is expected from the analytical formulas of Gluckstern [65,66] considering the multiple scattering in the CDC gas (dashed line) and the transverse hit position measurement error RMS$_{xy}$ of 400 $\mu$m, as deduced from fig. 6 a,
and under the vertex constraint (dot-dashed line). The quadratic addition of the two contributions (continuous line) is quite near to what is obtained from the Monte-Carlo environment without target and chamber entrance materials, after tracking. It can be concluded that the multiple scattering in the chamber gas as well as in the passive volumes is the dominant limitation in the extraction of transverse momentum through the entire $p_t$ range of interest. In the real experiment, if trajectories need to be merged between adjacent sectors, the situation is expected to be worsened by other problems, not included in the digitization routines.

For the $\vartheta_{\text{lab}}$ angle, the systematic effects can be neglected as compared to the corresponding accuracy. The worsening of RMS$_\vartheta$ toward $90^\circ$ can be understood as a $z$-coordinate constant resolution (not function of $z$ itself) which translates into an uncertainty in $\vartheta_{\text{lab}}$ angle increasing toward $\vartheta_{\text{lab}} = 90^\circ$.

The performance of the Barrel is less critical to model as it contains basically nothing else but the time resolution (being the position determined by the time difference) and the outcome will be supplied in the next section in terms of the final quantities.

3.2.2 The shapes of the distributions associated to the selection cuts

The comparison and tuning of the basic resolutions RMS$_{xy}$, RMS$_{z}$ and RMS$_{dE/dx}$ is a prerequisite, but for the estimation of the detection efficiency the more complex derived observables, necessary for kaon selection in the $\phi$-meson production evaluation, have to be studied: $d_0$, $z_0$, CDC/Barrel matching in $z$-coordinate $\Delta_z$ and in $\phi$-angle $\Delta_{\phi}$, $C_{\text{mass}}$ and $B_{\text{mass}}$. For each of these, attention should be devoted to both the mean and the width. In all cases the $K^+$ and $K^-$ from the single-$\phi$ and the $K^+$ from the Ni+Ni FOPI data have been chosen. In the simulation, again the sample of particles known to have reached the Barrel sensitive volumes during the GEANT transport phase was always taken (even for variables like $d_0$) and the initial momentum was limited to 0.6 GeV/c. The values of the data refer to kaons identified with the cuts of the $\phi$ analysis (see table 2). Only the constraint relevant for the quantity under consideration was removed to see the shape of the distribution. A Gaussian fit was found satisfactory in all cases except the CDC-Barrel matching in $\phi$-angle, where the influence of the scintillator bar finite width is present. Let us discuss all of them in turn.

For $d_0$ and $z_0$ the results of the comparison are given in fig. 8 a and b, respectively. The mean and width of $d_0$ are in good agreement, the sole discrepancy is in the bump starting at $-0.5$ cm in the simulation. The feature is connected to the problem that parts of the mirror tracks are sometimes also misinterpreted as real new ones. In the Monte-Carlo it happens in 0.07% and 3% of
the cases for the images of $K^+$ and $K^-$-mesons, respectively (with a statistics of 75000 tracks). The already mentioned tilting of the sectors renders the situation asymmetric and the misidentification is not equal for positive and negative particles. The bump is not visible in the data for a simple reason of statistics: the numbers of $K^+$ and $K^-$ in the acceptance are the same when exclusively the $\phi$-meson decay is assumed as a source, however in the experiment the first is greater than the second by 2 orders of magnitude [39] and so the incomplete rejection of the image of negative kaon trajectories at the level of a few percent contributes a $10^{-4}$ background for positive ones.

The width of the calculated $z_0$ distribution is about 4 times as wide as measured. The reason is an underestimation of the correlated electronic noise pick-up that adds pulse height coherently into all preamplifier channels on one side of the CDC. Due to the projective geometry of the chamber this causes a strong distortion of the angle while the intercept with the beam axis is relatively stable. Since we operate with very wide windows on $z_0$ no attempt was made to fine tune its behavior. The cut at $\pm 10$ cm in the true conditions does not discard any good track but reduces the background from fake ones and from trajectories originated by secondary vertices, correspondingly the permitted range in the processing of GEANT output was opened to $\pm 20$ cm.

The expected shift of the mean as well as the precision for the CDC-Barrel difference in both $z$-coordinate and $\varphi$-angle ($\Delta_\varphi$ and $\Delta_z$, respectively) are gathered together with those of the data in fig. 9. For $\Delta_z$, the systematic displacement from 0 of the latter (see fig. 9 a) is motivated by the difficulty of calibrating the longitudinal coordinate in the real experiment, however it is less important than the corresponding standard deviation by at least a factor of $\approx 2$. The accuracy in $\Delta_z$ is somewhat better in the Monte-Carlo (see fig. 9 b), so the limits of the allowed region in its analysis were rescaled in the ratio $6/8$ from $\pm 25$ cm to $\pm 19$ cm (see table 2). In the simulation, the systematic oscillating shift in $\varphi$-angle matching is imputable to a small inconsistency in the description of the CDC geometry, it is anyhow $\approx 3$ times less than the resolution. The rms of $\Delta_\varphi$ is dominated by the shadow of the scintillator bar physical size and is hence easily reproduced.

The comparisons of mean position and width of the $Cmass$ and $Bmass$ distributions are shown in fig. 10 in momentum bins of 0.1 GeV/c up to 0.6 GeV/c. As dimensionless fractions $Cmass/m_K^+$ and $Bmass/m_K^+$, where $m_K$ is the nominal charged kaon mass, were preferred to the bare $Cmass$ and $Bmass$, respectively. With this choice both the mean and the standard deviation $\sigma$ can be expressed in percentage. All trends are similar. For $Cmass$ the systematic shift (see fig. 10 a) reflects how well the employed empirical parameterization is adapted to the energy-loss versus momentum dependence. It is a function of the calibration constants and was tailored to follow as closely as possible the experimental one (obtained in a far more complex environment
with many particle species and hence subject to more restrictions than in the Monte-Carlo). A residual uncertainty at the highest momenta is possible between the 10% of the simulation and the 5% that could be extrapolated from the data with a straight line. The width in \( C_{mass} \) (see fig. 10 b) is dominated by the RMS\(_{dE/dx} \), found to be in good accordance (see fig. 6 c), it is then not surprising that the measurements are appropriately modeled. However substantial extrapolation of the experimental points is necessary to compare the resolution to the simulation at the highest allowed momenta and as a consequence a margin of variation between the 8% constant value of the first and the 13% of the second is possible. For \( B_{mass} \) less freedom is available since no empirical parameterization is needed and the discrepancy in mean position (see fig. 10 c) could not be reconciliated, it is anyhow at worst of 2%, much less than the associated precision. Its accuracy is dominated by the scintillator Barrel time of flight resolution (found to be 400 ps), again good agreement exists with the data (see fig. 10 d), but substantial extension is needed at the highest momentum and a discordance remains from the 10% predicted at 0.6 GeV/c and the 8% that could be extrapolated with a straight line from the latter.

The estimated possible maximum discrepancies in mean or width (for all relevant cases the distributions are Gaussian both in the data and the simulation so that the standard deviation \( \sigma \) can be adopted) discussed in detail in this paragraph are summarized in table B.1. They will be the base to estimate the associated systematic uncertainty in the extraction of \( \epsilon_{det} \) (see appendix B).

The accepted ranges in the analysis of the GEANT and real results are collected in table 2.

In the Monte-Carlo the contribution of each cut can be sorted out within the statistical accuracy of \( \pm 1\% \). The exclusion of hit multiplicities lower then 30 causes no decrease and a reduction of 97% on the negative and positive track numbers, respectively, owing to the more frequent rejection of \( K^- \) mirror images, that has already been discussed in connection with the \( d_0 \) distribution shape. The efficiency of the \( d_0 \) selection itself, for the same problem, is also charge asymmetric being 95% for positive and 98% for negative tracks. The \( z_0 \) and \( C_{mass} \) allowed windows do not leave out anything. The requirements on \( \Delta_\varphi \) and \( \Delta_z \) are strongly correlated, so that once the \( \varphi \)-angles selection is imposed, not much more is eliminated by the one on the \( z \)-coordinate difference. They will be lumped together as a single condition in the following. The match, \( B_{mass} \) and momentum cut all have an efficiency near 97 – 98%. The previous three figures have then to be squared to take into account the positive and negative kaon identification in every event to know the impact on the pair (i.e. \( \phi \)) population. The 3% missing after matching can not be ascribed to the \( \Delta_\varphi \) and \( \Delta_z \) cuts, that correspond to more than 3 \( \sigma \), but rather to the scintillator efficiency. Although for a path impinging along the normal to the
bar entrance surface, the depth of 4.05 cm is more than enough to generate a light signal above threshold in 100% of the cases, it is nevertheless not true if the trajectories are curved by the magnetic field, and edge effects become important. It should be noticed that the width is 3.25 cm, to be compared with the already mentioned depth.

A contamination at the level of (1.3 ± 0.2)% of the $K^+ K^-$-couples with both momenta below 0.6 GeV/c from the part of the kaon spectrum above 0.6 GeV/c is found with the present resolution of the CDC in the simulation.

3.3 Final efficiency

The position and width of the $K^+ K^-$ invariant mass peak are determined only by the $p_t$ and $\vartheta_{\text{lab}}$ angle systematic shifts and resolutions because, as already mentioned, once a particle is selected as a kaon candidate the nominal mass is assumed. If the peaks are fitted with Gaussian in the region of interest (1.01 – 1.03 GeV, as utilized for extracting the number of $\phi$-mesons in sec. 2.3) the mean is found to be $1.022 \pm 0.003$ GeV (data) and $1.020 \pm 0.003$ GeV (simulation) and the standard deviation $\sigma$ to be $5.8 \pm 1.4$ MeV (data) and $4.7 \pm 0.3$ (simulation). However Lorentzian tails outside the fit region are distinguishable solely in the Monte-Carlo; the experimental spectrum is too much altered by the lack of statistics and the presence of the background. To grasp an idea of how looks the comparison to the $\phi$-meson intrinsic $\Gamma$ of 4.43 MeV, the FWHM of the Gaussians can be used: they are a factor of $\approx 3$ larger. The agreement between the real and expected invariant mass resolution confirms that the broadening should be attributed to the CDC momentum uncertainty. Since the first and second moments are compatible in the measurements and the simulation, for calculating the fraction of identified $\phi$ an identical binning is performed and the counts in the same 4 consecutive channels are added together. From the Monte-Carlo it can be estimated that $(10 \pm 3)$% of the events are lost in such a way.

Adopting the previous procedure the last two columns of table 3 have been obtained. The detection efficiency of the CDC/Barrel subsystem of FOPI for the $\phi$, independently of the temperature in the considered range within one standard deviation, amounts to

$$\epsilon_{\text{det}} = 70\% \pm 1\% .$$  \hspace{1cm} (7)

The total efficiency with a thermal source of a temperature of 130 MeV is expected to be

$$\epsilon_{\text{tot}} = (0.95 \pm 0.02)\%$$  \hspace{1cm} (8)
and with a temperature of 70 MeV to be

\[ \epsilon_{\text{tot}} = (0.24 \pm 0.01)\% . \]  

(9)

Up to now a single-\( \phi \) has been used to derive the figures supplied in eqs. (7) through (9), but to really arrive at the final ones the additional bearing of the whole nuclear event surrounding the \( \phi \)-meson has to be included. The lack in CDC/Barrel matching, due to the double hits in the latter, can be directly estimated from the data. From the simulation the expected Barrel detection efficiency for one charged kaon in the appropriate polar angle acceptance is (84 \( \pm 2 \))\%, owing to the azimuthal coverage and edge effects. It is difficult to compare safely with the experimental corresponding one, because \( K^+ \) and \( K^- \) can be cleanly tagged only with \( B\text{mass} \) and because of the low statistics.

We opt to refer to a sample of well sorted protons in the CDC, in a similar momentum range of 0.4–0.6 GeV/c, although the energy-loss is not alike. The measured value is then of (85 \( \pm 2 \))\% for few-track events, in good agreement with the prediction. On the other hand, for multiplicities, typically associated with \( \phi \)-candidates, it drops to (81 \( \pm 2 \))\%. We introduce a correction to \( \epsilon_{\text{det}} \) given by the square of their ratio

\[ \epsilon_{\text{mtrk}} = (91 \pm 3)\% . \]  

(10)

This is just a lower limit on the reduction in the reconstructed fraction induced by the presence of all the collision products, it is nevertheless very well compatible to the embedded-\( \phi \) GEANT outcome of (92 \( \pm 5 \))\% obtained with a thermal source of \( \phi \) at a temperature of 130 MeV superimposed on IQMD Ni+Ni events [61]. Such agreement suggests that double hits in the Barrel are the main origin of failure caused by a realistic multiplicity. Instead changes in behavior of the CDC tracking code seem to be minor, confirming its reliability.

The maximum systematic error for the extraction of the efficiency from the simulation is estimated to be 40\% comprehensive of all contributions: different behavior of each cut in the Monte-Carlo environment and in the data, small discrepancies in the geometry of the real and the simulated detector and the influence of the nuclear event surrounding the \( \phi \) (see appendix B).

4 \( \phi \)-meson production yield

We are now in a position to deduce the \( \phi \)-meson yield in central Ni+Ni collisions. Let us focus on the phase space portion relevant for the detector subsystem (see table 1) to initially avoid the uncertainty in the source temperature. Using the number of \( \phi \)-candidates of 23 (see sec. 2.3) obtained from \( 4.7 \cdot 10^6 \)
central events (see sec. 2.2), the 70% detection efficiency (see eq. (7)) diminished under the realistic track multiplicity (see eq. (10)) and taking into account the partial width of the $\phi$ into the $K^+ K^-$-channel of 49.1%, we find the $\phi$-production probability in the CDC/Barrel acceptance per central event ($0 < b < \approx 3.3$ fm) $P_{\text{acc}} = (1.6 \pm 0.5 \pm 0.8) \cdot 10^{-5}$. The first error is statistical and the second systematic, calculated with the quadratic and linear sum, respectively, of the contributions of the data (see sec. 2.3) and the simulation (see sec. 3.3 and appendix B).

An extrapolation to a full azimuthal angle coverage of the Barrel and no edge effects can be performed with $\epsilon^*$ (see table 1 and appendix A): this benefits that it is independent from the chosen source temperature

$$P_{2\pi} = P_{\text{acc}}/\epsilon^* = (1.9 \pm 0.6 \pm 0.95) \cdot 10^{-5},$$

where the errors have an equal meaning as in the previous $P_{\text{acc}}$. Paying attention to the kaon decay in flight (as described in appendix A), $P_{2\pi}$ can be immediately compared with theoretical predictions within the $\theta_{\text{lab}}$ angle and total momentum acceptances.

If the assumption of a source temperature of 130 MeV is made (see sec. 3.1), for the yield estimated in the full solid angle, it is found

$$P_{4\pi} @ 130 \text{ MeV} = P_{\text{acc}}/\epsilon_{\text{max}@130\text{MeV}} = (1.2 \pm 0.4 \pm 0.6) \cdot 10^{-3}.$$

Finally for a source temperature of 70 MeV (see sec. 3.1), the same quantity of eq. (12) amounts instead to

$$P_{4\pi} @ 70 \text{ MeV} = P_{\text{acc}}/\epsilon_{\text{max}@70\text{MeV}} = (4.5 \pm 1.4 \pm 2.2) \cdot 10^{-3}.$$

The errors in eqs. (12) and (13) have been derived like in eq. (11).

The very preliminary result announced earlier in [49] is, when compared to eq. (12), smaller by a factor of $\approx 2.5$: this has to be ascribed to the previous version of the Monte-Carlo, less accurate than the current one and not yet tuned through a careful comparison with the real response as in sec. 3.2.1 and 3.2.2. We underline that the actual size of candidates sample has not been changed with respect to both [49] and [61]. It is the extracted efficiency that was different, demonstrating the importance of a sound study about the agreement between simulation and experiment for the observables necessary to sort out the kaons and showing how appropriate it is to estimate a maximum possible systematic errors (see appendix B) as attempted in eq. (11) through (13).
The $\phi/K^-$ ratio extrapolated to the full solid angle can now be given. The $K^-$-mesons production probability in $4\pi$ can be deduced to be $(2.1\pm0.4)\cdot10^{-3}$, using the rapidity distribution measured by KaoS [38] also for $^{58}\text{Ni}+^{58}\text{Ni}$ at 1.93 AGeV and for central collisions, but with the KaoS experiment centrality selection ($0 < b < 4.4$ fm) and assuming a Gaussian shape. Scaling this value with the mean number of participants as $A^{1.8\pm0.3}_{\text{part}}$ [67], the corresponding one to the FOPI experiment central trigger is $(2.6\pm0.5)\cdot10^{-3}$. If the last procedure is applied to $K^+$-mesons consistency is found between the KaoS data [38] and those of FOPI [37]. The $\phi/K^-$-ratio is then $(0.44\pm0.16\pm0.22)$ and $(1.7\pm0.6\pm0.85)$ at 130 MeV and 70 MeV temperature, respectively. Hence referring to the highest, but still reasonable of the two, and minding at the branching ratio, it can be established that at least $\approx 20\%$ of the $K^-$-mesons arise from the decay of an intermediate $\phi$.

5 Comparison with theoretical models

The first detailed theoretical study of $\phi$-meson yield in the Ni+Ni system at 1.93 GeV was that of Chung, Li and Ko [68,69,70]. Their relativistic transport model RVUU describes the rapidity distributions of $p$ and $\pi$ data and the collective flow reasonably well. For the $\phi$-meson production, the calculation foresees four scenarios: i) no in-medium effects, ii) in-medium effects only on the kaon masses, iii) in-medium effects on the kaon masses and through this on the $\phi$ decay width and iv) in-medium effects also on the $\phi$-meson mass. The parameterization of [28] is taken for the in-medium modification of the $\phi$-meson mass together with the elementary cross sections from their own model [68]. Within this frame the dominant channel for $\phi$-production is $\pi N \rightarrow \phi N$. The sensitivity was assessed relative to the free scenario i) for the $K^-$-meson to be a factor of 3 more in case ii), iii) and iv) and for the $\phi$ to be a factor of 2 reduction in case iii) and 10% less in case iv). Their expectation for the $\phi$-meson occurrence probability in central collisions is $2.3\cdot10^{-4}$ in case iv). It is extracted from [69] with the appropriate mean over $b$ (eq. (5)). Assuming a $\phi N$ elastic cross section of 0.56 mb (their preferred choice) and of 8.3 mb (the maximum compatible with the $\phi$-meson photo-production data) the deduced final $\phi$ kinetic energy distribution exhibits an inverse slope of 110 and 130 MeV, respectively. Adopting a temperature of 110 MeV for extrapolation and comparison to the experiment, the model underpredicts the data by 6 times. Such discrepancy is, however, not too surprising when viewing the large uncertainties in the parameterization, close to threshold, of the elementary cross sections.

Recently Barz, Zétényi, Wolf and Kämpfer [71,72] have been engaged in a new effort, profiting of the measurements now available for the elementary $pp \rightarrow \phi pp$ reaction at $\sqrt{s} - \sqrt{s_{\text{th}}} = 83$ MeV [48]. It is based mainly on
their previous work [73] about the role of elementary three body interactions, where the intermediate particles can be off-shell. It is found that in current microscopic transport models a small error is done considering successive two body collisions, where every particle is on-shell, if all the relevant intermediate steps are included. They point at the $B\rho$ and $\pi\rho$-channels as important ones, neglected in any preceding investigation, and develop their own predictions starting from this remark. The in-medium effects are accounted for the $\phi$ alone with the same result of [28] as in the calculations exposed before [69]. The awaited $\phi$-production probability for central collisions (9% of the total cross section) is $1.7 \cdot 10^{-3}$. The authors apply the FOPI CDC/Barrel geometrical acceptance filter and obtain a value of $2.7 \cdot 10^{-5}$ in reasonable agreement with the experimental finding of $1.9 \cdot 10^{-5}$ (see eq. (11)). It is however not clear how well the implementation elaborated for the moment, can explain the global experimental observables. In addition it remains to be seen how in-medium modifications of the kaon properties change the picture, since the $\phi$ yield is coupled to the sum of the $K^+$ and $K^-$ masses through its decay width. More work is needed to compare carefully their theory with the experimental achievement reported here.

A completely different approach is that of the thermal models [74,75,76] where, assuming a chemical equilibrium before freeze-out, the abundances can be predicted (not the yields themselves). Particular care is to be exercised in the treatment of conservation laws like baryon number, electric charge and strangeness. In the thermodynamic limit both canonical and grand canonical formulations are equivalent, but for small systems, however, the discrepancies are large. In fact in general all these quantum numbers should be handled in the canonical treatment (implying exact conservation) as has been proven necessary for strangeness, while for baryon number and electric charge the grand canonical one (implying conservation in mean) is adequate [74]. If also strangeness is described with the latter, the data in the SIS energy regime are not matched. Determining the temperature and baryonic chemical potential from the $d/p$, $\pi^+/p$ and $K^+/\pi^-$ relative experimental intensities, the maximum of the $\phi/K^-$ one, that can be accommodated, is of the order of 0.1 [76]. The last value is compatible with the figures of sec. 4, within the large systematic and statistical errors, only if a temperature of at least 130 MeV is hypothesized. This situation illustrates the limitations of the actual small acceptance measurement where the $\phi$ source is not well characterized and of this class of models where the species kinetic temperatures are poorly predictable.
6 Conclusions

A first experimental production probability of sub-threshold $\phi$-meson in heavy ion collisions has been obtained. Before solely $K^+$ and $K^-$ data were available. With a pseudo-vector meson a new sector of the theoretical machinery, proposed to describe in-medium effects, has been tested. It supports the emerging awareness of a strong interconnection between the $K^+$, $K^-$ and $\phi$-channels.

Here only the data gained from the CDC/Barrel subsystem are discussed; the effort necessary to make a quantitative statement for the probability of occurrence in central collisions is described. The creation of a $\phi$-meson is a very rare event at sub-threshold energies: out of $4.7 \cdot 10^6$ events in the sample of central collisions, barely 23 candidates were found after background subtraction. Despite the poor statistics, a signal to background ratio of $\approx 1$ could be achieved when applying appropriate cuts.

An important part of the effort has been the estimation of the efficiency that was splitted into two parts: one, $\epsilon_{\text{max}}$, due to the geometrical acceptance, allowed momentum range and $K^+$ and $K^-$ decay in flight and a second, $\epsilon_{\text{det}}$, originating in the identification requirements necessary to decrease the background. While $\epsilon_{\text{max}}$ is between 1.35% and 0.36% (depending on the source temperature), showing that the greatest loss in $\phi$ events is to be attributed to the localization of the phase space covered by the detector around target rapidities, $\epsilon_{\text{det}}$ is 70% indicating the validity of the cuts chosen in reducing the background without rejecting too many good events.

Furthermore $\epsilon_{\text{det}}$ was proven to be insensitive, within the statistical error of the simulations, to the source temperature (and it could be argued even from the shape of a moderately anisotropic one); on the contrary $\epsilon_{\text{max}}$ varies by a factor of 4 when the temperature changes from 70 to 130 MeV. The comparison with calculations or new results is hence greatly simplified: in fact while $\epsilon_{\text{max}}$ can easily be evaluated along the guidelines of appendix A by other groups, $\epsilon_{\text{det}}$ can exclusively be extracted from a complete simulation of the detector response, requiring a specific knowledge of the apparatus; it has been deduced once for all in the present work.

Future confrontations will benefit from a thorough discussion of the possible sources of systematic error: a value of 40% and 9% has been given for simulation and data, respectively.

This measurement suggests that, near threshold, a large fraction of the $K^-$-mesons (at least 20%) arise from the decay of an intermediate $\phi$-meson. As a consequence it would be important to verify, both experimentally and theoretically, to what extent the $K^-$ phase space distribution and the $\phi/K^-$-ratio can be contaminated from the $\phi$-production channel. The discrepancies with
the predictions based on microscopic transport codes can partially be ascribed to insufficient knowledge of the elementary cross sections. A possible role of intermediate $\rho$-mesons production for the $\phi$ yield appears to be an interesting aspect of investigation. The $\phi/K^-$-ratio seems to be hardly compatible with thermal models.

In order to improve the knowledge on the experimental side, a more complete coverage of the acceptance is needed. Especially, the access to the region of low transverse momenta in the proximity of mid-rapidity is important for a reliable reproduction of the total yield of such rare particles produced in central heavy-ion collisions. A first step along this way is taken by the ongoing effort to include the Helitron/Forward Wall data [50]. In a near future the FOPI collaborations plans to accumulate more data with an increased statistics on $\phi$-production, when the undertaken upgrade program will be finished. On the other hand, the HADES collaboration will be able to detect the $\phi$-meson through the leptonic channel in search for direct evidences of in-medium effects. Both plans make the subject open for promising new developments.

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| $B$     | .6 Tesla |
|---------|----------|
| **CDC** |          |
| $R_{\text{max}}$ | 20.7 cm |
| $R_{\text{min}}$ | 80.1 cm |
| $\vartheta_{\text{min}}$ | 32.7 $^\circ$ |
| $\vartheta_{\text{max}}$ | 130 – 154 $^\circ$ |
| **Barrel** |          |
| $R_{\text{Bar}}$ | 111.5 cm |
| $\vartheta_{\text{min}}$ | 39.2 $^\circ$ |
| $\vartheta_{\text{max}}$ | 132.9 $^\circ$ |
| $z_{\text{min}}$ | $-103.5$ cm |
| $z_{\text{max}}$ | 136.5 cm |
| $\epsilon^*$ | $81 \pm 2.5$ % |
| **Momentum** |          |
| $p_{\text{max}}$ | 0.6 GeV/c |

Table 1
Geometrical acceptance of the CDC and the Barrel.
| Acceptance | Data GEANT |
|------------|-----------|
| $p$        | $< 0.6$ GeV/c $< 0.6$ GeV/c |

| Tracker | Data GEANT |
|---------|------------|
| $\Delta_{xy\,\text{hit}}$ | $\pm 0.11$ cm $\pm 0.11$ cm |
| $\Delta_{z\,\text{hit}}$ | $\pm 35$ cm $\pm 35$ cm |
| $\Delta_z$ | $\pm 25$ cm $\pm 19$ cm |
| $\Delta_{\phi}$ | $\pm 2^\circ$ $\pm 2^\circ$ |

| Analysis | Data GEANT |
|----------|------------|
| $n_{\text{hit}}$ | $> 30$ $> 30$ |
| $d_0$ | $\pm 0.5$ cm $\pm 0.5$ cm |
| $z_0$ | $\pm 10$ cm $\pm 20$ cm |
| $C_{\text{mass}}$ | $m > 0.37$ and $m < 0.8$ GeV $m > 0.37$ and $m < 0.8$ GeV |
| $B_{\text{mass}}$ | $|m - 0.494| < 0.1$ GeV $|m - 0.494| < 0.1$ GeV |

Table 2
Momentum, tracker and identification cuts used for the analysis of the data and of the simulation.
| T    | $\epsilon_{\text{CDC}}$ (%) | $\epsilon_{\text{Bar}}$ (%) | $\epsilon_{\text{max}}$ (%) | $\epsilon_{\text{Bar}}$ (%) | $\epsilon_{\text{max}}$ (%) | $\epsilon_{\text{tot}}$ (%) | $\epsilon_{\text{det}}$ (%) |
|------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| 130  | 17.03 ± 0.09                | 7.20 ± 0.06                 | 3.78 ± 0.04                 | 3.26 ± 0.04                 | 1.35 ± 0.02                 | .950 ± 0.02                 | 70 ± 2                      |
| 110  | 13.19 ± 0.07                | 5.17 ± 0.04                 | 2.93 ± 0.03                 | 2.26 ± 0.03                 | 1.07 ± 0.02                 | .751 ± .016                 | 70 ± 2                      |
| 90   | 9.36 ± 0.05                 | 3.20 ± 0.03                 | 2.02 ± 0.02                 | 1.31 ± 0.02                 | 0.67 ± 0.01                 | .475 ± .012                 | 71 ± 2                      |
| 70   | 5.53 ± 0.04                 | 1.63 ± 0.02                 | 1.10 ± 0.02                 | 0.64 ± 0.01                 | 0.36 ± 0.01                 | .242 ± .007                 | 67 ± 3                      |

Table 3
Geometrical $\epsilon_{\text{CDC}}$ and $\epsilon_{\text{Bar}}$, maximum $\epsilon_{\text{max}}$, total $\epsilon_{\text{tot}}$ and detection $\epsilon_{\text{det}}$ (see eq. (6)) efficiencies for four thermal source temperatures. The geometrical contribution is given for the CDC alone ($\epsilon_{\text{CDC}}$) and the CDC/Barrel combination ($\epsilon_{\text{Bar}}$). The latter $\epsilon_{\text{Bar}}$ includes the not complete azimuthal coverage of the Barrel and edge effects (see $\epsilon^{*}$ of table 1 and appendix A). If, within the acceptance of the CDC/Barrel subsystem ($\epsilon_{\text{Bar}}$), the upper momentum limit at 0.6 GeV/c is imposed, $\epsilon_{\text{max}}$ is obtained. For $\epsilon_{\text{Bar}}$ and $\epsilon_{\text{max}}$ the reduction due to the kaon decay in flight during their path to the Barrel is considered. All figures assume that the only open channel for the $\phi$-meson is the $K^{+}K^{-}$ one.
Fig. 1. Upper panels: logarithm of the truncated mean energy-loss $dE/dx$ versus momentum $p/Z$ for positively (panel a) and for negatively (panel b) charged particles reaching the Barrel. The dashed lines represent the empirical parameterizations used for the energy-loss versus momentum in the $Cmass$ reconstruction algorithm; the constants chosen are the same as those of the data analysis for the extraction of the $\phi$-candidates. Lower panels: momentum $p/Z$ versus velocity $v$ for positively (panel c) and for negatively (panel d) charged particles. The dashed lines represent the relativistic momentum versus velocity relation with the nominal masses.
Fig. 2. $C_{mass}$ versus $B_{mass}$ for positively (panel a) and negatively (panel b) charged particles with momentum less than 0.6 GeV/c. The area enclosed by the dashed line corresponds to the kaon identification cuts used in the $\phi$-meson analysis (see table 2).
Fig. 3. Charged particle multiplicity (PMUL) distributions corresponding to the Minimum Bias trigger (empty) and the Central trigger (shaded). The upper scale in impact parameter $b$ is derived with the assumption of a sharp cut-off geometrical model.
Fig. 4. Invariant mass spectra for identified $K^+ K^-$ pairs with the final cuts (see table 2). The histogram is the signal; the two lines enclose one standard deviation from the background reconstructed with the event-mixing technique when the mixing stack is filled with Ni+Ni collisions containing either a $K^+$ or a $K^-$-meson; the error bars refer to the background reconstructed when the mixing stack is filled with reactions containing both a $K^+$ and a $K^-$. 
Fig. 5. Transverse momentum over mass $p_t/m$ versus rapidity $y$ plots for the original $\phi$-meson (panel b) and for the $K^+$ and $K^-$-mesons produced in its decay (panel a). The GEANT simulation with a source temperature of 130 MeV and including the decay in flight of the kaons is used. The rapidity is renormalized such that $-1$ is the target, $+1$ is the projectile rapidity and 0 is mid-rapidity, where the $\phi$ source is located. The continuous and dashed lines in panel a and b illustrate the acceptance limits of the CDC/Barrel combination (compare with table 1). The dot-dashed curves in panel b represent the most probable momenta for a relativistic Maxwell-Boltzmann distribution of emitted $\phi$ centered at $y = 0$ and with a temperature of 130, 110, 90 and 70 MeV, respectively.
Fig. 6. Results obtained from the CDC digitization routine for the mean rms RMS_{xy} of the track fit residuals in the transverse plane (panel a), the mean rms RMS_{z} of the track fit residuals in the z-coordinate (panel b) and the mean rms RMS_{dE/dx} of the truncated energy-loss distribution (panel c). Note that in panel b K^{+} and K^{-}-mesons from the simulation lie on top of each other. The range in energy-loss of the data (500 – 1600 a.u.) corresponds to kaons with momenta ≈ 0.1 < p < 0.6 GeV/c.
Fig. 7. Transverse momentum $p_t$ and longitudinal angle $\theta_{lab}$ systematic shift and resolution from the Monte-Carlo. In addition to $K^+$ and $K^-$-mesons from the standard environment used in the present work, also $K^+$ from a simulation without target and chamber entrance materials are considered for comparison. In panel b the $p_t$ root mean square deviation is compared with what is expected from Gluckstern’s formulas [65] describing the effect of multiple scattering in the CDC gas (dashed line) and of a transverse hit position uncertainty of 400 $\mu$m (see fig. 6 a) under the vertex constraint (dot-dashed line). The two contributions are added quadratically (continuous line).
Fig. 8. Comparison of distributions in $d_0$ (panel a) and $z_0$ (panel b) of $K^+$-meson tracks. The solid histograms are from the data and the dashed ones from the simulation. In panel b a Gaussian fit is also shown.
Fig. 9. Comparison of the means and widths for CDC/Barrel matching in $z$-coordinate and $\varphi$-angle ($\Delta z$ and $\Delta \varphi$, respectively) between $K^+$ and $K^-$-mesons in the simulation and $K^+$ in the data.
Fig. 10. Comparison of means and widths of the non dimensional quantities $C_{mass}/m_K - 1$ and $B_{mass}/m_K - 1$ between $K^+$ and $K^-$-mesons in the simulation and $K^+$ in the data.
A Direct calculation of the geometrical acceptance

If the $\phi$-meson source is not spherical or the temperature is outside the range of table 3 (so that an interpolation can not be used) $\epsilon_{\text{max}}$ has to be calculated from the events generated by the assumed model. We briefly describe the filter procedure, with the related setup parameters, which is needed to compare future data or the prediction of a microscopic transport code with the $\phi$-production probability measured here in the CDC/Barrel acceptance. After the $\phi$ has decayed into a $K^+$ and a $K^-$-meson when both have a momentum smaller than 0.6 GeV/c, for each of them it is necessary to determine if it hits the nominal Barrel surface. The latter is defined as a cylinder located at a radius $R_{\text{Bar}}$ with a $z$ extension from $z_{\text{min}}$ to $z_{\text{max}}$. First for the inverse of the radius of curvature in the transverse plane $k$, that can be obtained from eq. (1), the condition $2/k > R_{\text{Bar}}$ has to be fulfilled. In this case the length $l_t$ of the transverse projection of the track from the origin to the Barrel can be calculated as

$$l_t = \frac{2}{k} \arcsin \left( \frac{R_{\text{Bar}}}{k} \frac{k}{2} \right)$$  \hspace{1cm} (A.1)

and the $z$-coordinate of the hit in the scintillator bar is

$$z = h l_t , \hspace{1cm} (A.2)$$

where $h$ is the helix pitch defined by $h = p_z/p_t$. Second the value given by (A.2) has to lie inside the interval $z_{\text{min}} \leq z \leq z_{\text{max}}$ with the $z_{\text{min}}$ and $z_{\text{max}}$ of table 1. To account for the possible decay in flight, each kaon must be weighted with a survival probability

$$\exp \left( -\frac{l m_K}{p_K c \tau_K} \right) , \hspace{1cm} (A.3)$$

where $m_K$, $\tau_K$, $p_K$ are the kaon mass, mean life and momentum, respectively and $l$ is the corresponding distance of flight to the Barrel

$$l = l_t \sqrt{1 + h^2} . \hspace{1cm} (A.4)$$

After the weighting with the product of the quantities (A.3) for each particle of the couple, the final outcome delivers $\epsilon^*_{\text{max}}$, where

$$\epsilon_{\text{max}} = \epsilon^*_{\text{max}} \cdot \epsilon^* . \hspace{1cm} (A.5)$$
In order to obtain $\epsilon_{\text{max}}$ of eq. (6) and table 3, a reduction factor $\epsilon^*$, comprehensive of both kaons, has to be applied to compensate for the incomplete azimuthal coverage of the Barrel due to the regions occupied by the CDC support structures and to the inter-module and inter-bar spacings. Because the depth of the scintillator bar is bigger than the width, edge effects are important at the level of a few percent (see sec. 3.2.2) and have to be included as well. The value of $\epsilon^*$ has been estimated through the ratio between the output of the filter just outlined and of GEANT where the geometry is defined accurately implementing separately each scintillator bar with its appropriate positions and dimensions. It turns out from the simulation that $\epsilon^*$ is independent from the source temperature in the considered range, within one standard deviation, and can be determined once for all as $(81 \pm 2.5)\%$ (see table 1). In a comparison of theory to the experiment one can benefit of eq. (11) and barely supply $\epsilon^*_{\text{max}}$ so that

$$P_{4\pi} = P_{2\pi}/\epsilon^*_{\text{max}}.$$  \hfill (A.6)

B  Estimate of the systematic errors

Our estimate of the efficiency is biased by systematic errors of distinct types: a non-identical behavior of every cut in the data and simulation, small geometrical discrepancies between the real detector and the simulated one, the influence of the nuclear event surrounding the $\phi$-meson. We discuss each of them ($d_0$, $z_0$, $B_{\text{mass}}$, $C_{\text{mass}}$, $\Delta_z$, $\Delta_\phi$, $n_{\text{hits}}$, upper momentum, Barrel geometry, multi-track environment) in turn.

Beginning with the $K^+K^-$-meson selection requirements, the ones on $d_0$ and $z_0$ are assumed to contribute negligible systematic distortions. As a matter of fact, the distribution of the first is well reproduced and the bump starting at $-0.5$ cm in the simulation is due to mirror tracks that have to be discarded (see sec. 3.2.2). Any eventual correction arising from the improper purge of such cases is assumed minor. The latter does not reject good events and is wide enough to accommodate the small discordances in the experimental and awaited shapes, once the condition imposed on $z_0$ in the analysis of GEANT is released as given in table 2.

The $B_{\text{mass}}$, $C_{\text{mass}}$ and $\Delta_z$ observables obey a Normal law with good approximation in the data and in the simulation, besides such assumption is legitimate for the estimation of systematic errors. As long as the real and calculated spectra have a Gaussian shape there could be only two sources of concern about the effect of a fixed accepted window: a difference in the mean or in the standard deviation $\sigma$. In truth if both are not quite similar the integral inside the same range is not equal, as can be easily obtained from the error
function. The discrepancy in the efficiency of a cut between the simulation and the data translates directly by twice the amount (once for each kaon of the pair) to the correctly identified fraction $\epsilon_{\text{det}}$. The change for variations in the mean or the $\sigma$ judged reasonable from our previous discussion (see sec. 3.2.2) is written up in the rightmost column of table B.1 in percentage of the number of events allowed by the selection. Since the domain of uncertainty quoted in mean or $\sigma$ are the maximum expected ones, an analogous statement is true also for the systematic error (in the same spirit for evaluating the effect of the shift in mean always the larger of the two cited $\sigma$’s is preferred). In the case of $C_{\text{mass}}$ the permitted region is widely asymmetric and solely the lower exclusion value matters, this peculiarity was accounted.

For $\Delta_z$, the width of the required interval used in the analysis of the data was rescaled for the simulation one. A small residual repercussion of the systematic discrepancy in the standard deviation is expected as compared to the others where the window is left untouched, here we will neglect it. Even if $\Delta_\phi$, partially reflecting the geometrical shadow of the scintillator bar, does not properly follow a Normal law, a Gaussian is not a too bad approximation for estimating the systematic error. It turns out to be very small, after all the cut is sufficiently wide, and it can be neglected (see table B.1). The two rms of $\Delta_\phi$ are well compatible with each other and no adjoined systematic distortions are considered from this side. The alteration in rejection caused by the dissimilarities in the shape of real and foreseen $\Delta_z$ and $\Delta_\phi$ distributions is in general not expected to be the major contribution to the systematic error in the matching ratio. In fact the corresponding accepted windows are $\approx 3\sigma$ wide and, as already discussed (see sec. 3.2.2), the origin of the loss is rather the scintillator efficiency coupled to the edge effects for curved tracks. However, if the matching fraction for single kaons predicted by the GEANT model $(84 \pm 2)\%$ and the measured one for well defined CDC protons tracks $(85 \pm 2)\%$ are compared (see sec. 3.3), the discordance of $1\%$ is not statistically meaningful. So no further contribution is introduced from the systematic limitations in the implementation of all the detailed processes affecting $\Delta_z$ and $\Delta_\phi$.

The invariant mass spectrum, being dominated by the CDC momentum resolution, is also approximately Gaussian (see sec. 3.3). No statistically significant disagreement exists between measurements and Monte-Carlo either for the mean or for the width, so the associated systematic error is assumed to be negligible.

The probability for a track to have an assigned number of hits does of course not follow a Normal law but is rather asymmetric with a tail towards low hit multiplicities (even for kaons reaching the Barrel) populated by fake tracks alone in the simulation and also by pion background in the experiment. It is difficult to quantify the reliability of the description of this quantity because it
depends on many subtle particularities of the detector response, so an estimate (see table B.1) equal to the remainder of what is rejected in the simulation (3%) and in the data (5.5%) was taken, it can not be exceed by the true discrepancy.

We interpret systematic errors as reasonable estimates of the possible displacement of the true result by effects not completely controllable and opt for a linear sum with the aim of arriving at trustable gap in which the final efficiency should lie. If the disagreement with a calculation is outside this, serious doubts should be shed on the theory itself. Adding all the discussed sources a maximum systematic error of 13% is found for one kaon and of 26% for the pair, i.e. for $\epsilon_{\text{det}}$.

The margin of uncertainty due to the upper momentum limitation was estimated varying the cut of $\pm 1\%$, that is roughly the systematic distortion of the momentum reconstruction (see fig. 7) and taking the biggest change (see table B.1) in the size of the selected $\phi$-sample (the source temperature was 130 MeV). Including this last contribution gives an expectation of 28% systematic error imputable to imperfect characterization of the kaon selection constraints behavior.

The difficulty in the determination of the geometrical acceptance can be reduced to the problem of knowing with enough precision the respective positions of the target and the Barrel, whose polar angle coverage in the forward direction is smaller than that of the CDC. Since the target is replaced after a set of runs, its displacement has to be deduced each time from relative measurements of the parts of its holder system, a maximum safety boundary of $\pm 1$ cm is appropriate. A simulation was run with a thermal source of $\phi$-mesons at 130 MeV temperature and the Barrel was moved of $\pm 1$ cm, the influence on $\epsilon_{\text{max}}$ is reported in table B.1.

For estimating the possible systematic mismatch in the reduction of detection efficiency by the multi-track environment, the difference in the total percentage of correctly identified events in the single-$\phi$ and embedded-$\phi$ Monte-Carlo is indicated in the rightmost raw of table B.1.

A maximum comprehensive systematic error for the extraction of the efficiency from the simulation of 40% is estimated.
| cut                        | data      | GEANT      | syst. error |
|----------------------------|-----------|------------|-------------|
| \(|B_{mass}/m_K - 1| < 20\%   | \langle m_b \rangle \approx 0\% | \langle m_b \rangle = -2\% | 0.45 \% |
| \sigma(m_b) \approx 8\%   | \sigma(m_b) = 10\% | 3.3 \% |
| \sigma(m_c) \approx 8\%   | \sigma(m_c) = 13\% | 2.6 \% |
| \Delta_z \leq 25\ cm     | \langle \Delta_z \rangle \approx 4\ cm | \langle \Delta_z \rangle = 0\ cm | 0.27 \% |
| \Delta_{\phi} \leq 2\ ^\circ | \langle \Delta_{\phi} \rangle \approx 0^\circ | \langle \Delta_{\phi} \rangle \pm .2^\circ | 0.06 \% |
| \(n_{hit} > 30\)          | rejection 5.5\% | rejection 3\% | 2.5 \% |
| \(p < 0.6\ \text{GeV}/c\) | variation of \pm 1\% @ \(T = 130\ \text{MeV}\) | 2.6 \% |
| Barrel geometry            | Barrel displacement of \pm 1\ cm @ \(T = 130\ \text{MeV}\) | 3.8 \% |
| multi-track env.           | embedded \phi simulation @ \(T = 130\ \text{MeV}\) | 8 \% |

**Table B.1**
Contributions to the systematic error. The values of the selection windows considered are those of the analysis of the data and the simulation (see table 2). For the first four cuts the mean and standard deviation \(\sigma\) quoted for the data are based on estimations of the maximum discrepancy and for the simulation are simply what is obtained at the highest permitted momentum of 0.6 GeV/c. From the difference between the measurements and the Monte-Carlo the systematic error on the cut effect is calculated assuming a Normal distribution and given as a percentage of the events allowed. For the last four contributions the estimation is adapted for each as explained in the text (see appendix B).
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