Shot Noise in Tunneling through a Quantum Dot Array

G. Kießlich,* (a), A. Wacker (a), E. Schöll (a), A. Nauen (b), F. Hohls (b), and R. J. Haug (b)

(a) Institut für Theoretische Physik, Technische Universität Berlin, Hardenbergstr. 36, D-10623 Berlin, Germany
(b) Institut für Festkörperphysik, Universität Hannover, Appelstr. 2, D-30167 Hannover, Germany

*Corresponding author; Fax: +49-(0)30-314-21130, kieslich@physik.tu-berlin.de

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Abstract  The shot noise suppression in a sample containing a layer of self-assembled InAs quantum dots has been investigated experimentally and theoretically. The observation of a non-monotonic dependence of the Fano factor on the bias voltage in a regime where only few quantum dot ground states contribute to the tunneling current is analyzed by a master equation model. Under the assumption of tunneling through states without Coulomb interaction this behaviour can be qualitatively reproduced by an analytical expression.

Introduction  Shot noise measurements provide a sensitive tool for probing transport properties of charged particles in mesoscopic systems, e.g. tunneling through semiconductor heterostructures, which are not available by conductance measurements alone. The dynamic correlations between individual tunneling events can reveal details of the potential shape and the effects of electron-electron interaction [1]. Up to frequencies f of the inverse transit time of carriers the spectral power density of the current noise is frequency-independent: \( S(f) \approx S(0) \). For an uncorrelated flow of electrons (Poisson statistics of individual tunneling events) this value is proportional to the elementary charge e and the stationary current I: \( S_P(0) = 2eI \). A reduction of this value refers to negative correlations in the current (sub-Poissonian noise), e.g. caused by the Pauli exclusion principle or repulsive Coulomb interactions, quantified by the Fano factor \( \alpha = S(0)/S_P \leq 1 \). In double-barrier resonant tunneling structures this value is given by the ratio of the tunneling rates \( \Gamma_E/C \) of emitter/collector barrier respectively: \( \alpha = (\Gamma_E^2 + \Gamma_C^2)/(\Gamma_E + \Gamma_C)^2 \). In [4] a master equation model was applied in order to describe double-barrier tunneling through a metallic quantum dot (QD) including the Coulomb interaction which results in the Coulomb Blockade effect. In the plateau regions of the Coulomb staircase the shot noise was Poissonian whereas the Fano factor shows dips in the region of steps which is quantitatively confirmed in the experiment [5]. For a semiconductor QD the shot noise in the nonlinear bias regime was studied in [6] by a nonequilibrium Green’s function technique. They find in the plateau region a suppressed Fano factor and at the steps an enhancement of \( \alpha \) still below the Poissonian value \( \alpha = 1 \).

Here, we use a master equation model to investigate the shot noise in tunneling through electrostatically coupled semiconductor QDs. The distinction between the effect of the Pauli exclusion principle and Coulomb interaction on the shot noise suppression will be shown. We apply the theoretical result to a combined shot noise and current measurement of tunneling through a layer of self-assembled QDs [7].

Experiment  A layer of self-assembled InAs QDs (10-15 nm diameter, 3 nm height [10]) is embedded in a GaAs-AlAs-GaAs tunneling structure (Fig. 1a). The plane of QDs is sandwiched between two AlAs barriers of nominally 4 nm and 6 nm thickness. About one million QDs are placed randomly on the area of an etched diode structure of 40\( \mu \)m\( \times \)40\( \mu \)m.
A 15 nm undoped GaAs spacer layer and a GaAs buffer with graded doping on both sides of the resonant tunneling structure provide three-dimensional emitter and collector contacts. Connection to the active layer is realized by annealed Au/Ge/Ni/Au contacts. The respective conduction band edge and the Fermi energy $E_F = 13.6$ meV for zero bias is depicted in Fig. 1b. By applying a bias voltage $V$ the current through the sample is measured (upper trace in Fig. 1c). The current-voltage characteristic shows steps which are assumed to be caused by selection of resonant single QD ground states by single-electron tunneling [10, 11, 12]. The displayed bias direction corresponds to tunneling of electrons from the bottom to the top of the pyramidal QDs. We have performed noise measurements in a frequency range from 0 to 100 kHz. Above the cut-off frequency for $1/f$-noise of around 20 kHz we observe frequency independent noise spectra. The temperature was 1.6 K. The Fano factor $\alpha$ (lower trace in Fig. 1c) shows an average suppression $\alpha \approx 0.8$ which can be used for the determination of the thickness of the collector barrier [9]. Furthermore, a non-monotonic behaviour of $\alpha$ is visible where the maxima correspond to steps in the current-voltage characteristic as indicated by arrows in Fig. 1c. This feature we address in the next section.

**Theory** In order to calculate the Fano factor $\alpha$ and the stationary tunneling current $I$ through two Coulomb interacting non-degenerate QD states we apply a master equation for the occupation probabilities: $P(t) = M \cdot P(t)$ with $P(t) = (P_{00}(t), P_{01}(t), P_{10}(t), P_{11}(t))^T$. In the matrix elements of $M$ the rates $\Gamma_{E/C}^i$ ($i = 1, 2$) for tunneling from the emitter/collector into the $i$-th QD state and vice versa enter. The occupation of the contacts is treated in local equilibrium by Fermi functions $f_E = (1 + \exp((E_i - \eta V)/(k_B T)))^{-1}$ and $f_C^U = (1 + \exp((E_i + U - \eta V)/(k_B T)))^{-1}$ (with energy $E_i$ of $i$-th QD state, Coulomb interaction energy $U$, voltage drop $\eta$ across emitter barrier, and bias voltage $V$). We use $f_C^V = f_C^U = 0$ assuming $eV \gg k_B T$. In the derivation of an expression for the spectral power density we follow the lines of Ref. [4]. For the time evolution of the occupation probabilities one defines the propagator $\mathbf{T}(t) \equiv \exp(\mathbf{Mt})$ such that $P(t) = \mathbf{T}(t) \cdot P(0)$. The stationary occupation probability is obtained by $\mathbf{M} \cdot P^0 = 0$. With the current operator for tunneling
through the collector barrier $j_C$, the stationary current reads

$$I = \sum_i (j_{iC} \cdot P_0)_i.$$  

(1)

The Fourier transform of the current auto-correlation function defines the frequency independent spectral power density: $S(0) = 4 \int_0^\infty dt (\langle i(t)i(0) \rangle - I^2)$). This leads to the Fano factor

$$\alpha = \frac{S(0)}{2eI} = 1 + \frac{2}{eI} \int_0^\infty dt \left[ \sum_i (j_{iC} \cdot T(t) \cdot j_{iC} \cdot P_0)_i - I^2 \right].$$  

(2)

In Fig. 2a the results of Eqs. (1) and (2) for different Coulomb interaction strengths $U$ are shown. In the current (upper trace of Fig. 2a) two steps occur due to the different resonances of the QD states with the emitter Fermi energy ($U = 0$). The corresponding Fano factor $\alpha$ (lower trace of Fig. 2a) equals one below the first resonance. As a consequence of the Pauli exclusion principle the shot noise becomes suppressed for tunneling through the non-interacting state $i$. The corresponding Fano factor is expressed by $\alpha_i = 1 - \frac{2r}{\gamma_i + 2r} f_E$. For biases below the second current step, only tunneling through state $i = 1$ is possible and thus $\alpha = \alpha_1$. At the resonance of the second QD state the Fano factor has a peak which is also caused by the Pauli exclusion principle: when the state $i = 2$ becomes resonant with thermal excited electrons of the emitter contact an additional transport channel opens. As $f_E^2 \ll 1$, we find $\alpha_2 \approx 1$, i.e. uncorrelated tunneling events close to the onset. For tunneling through an arbitrary number of non-interacting QD states the current is $I = \sum_i I_i$ and we obtain the Fano factor

Figure 2: (a) upper trace: Calculated current-voltage characteristic (1) for two QD states; lower trace: Corresponding Fano factor; $E_1 = 0$, $E_2 = 2$ meV, $T = 1.6$ K, $\eta = 0.3$, $\gamma_i = 7.8$ ($i = 1, 2$). (b) solid lines: theory see text; dashed curve in upper picture and crosses in lower picture: experimental data, see shaded region in Fig. 1c.
\[ \alpha = \sum I_i \alpha_i \]  
with  
\[ I_i = \Gamma_i \left( \frac{1}{1 + \gamma_i} \right) f^K \]  
(3)

Thus \( \alpha \) increases at the onset. Further increasing the bias voltage enhances \( f^K \) towards 1 and the Fano factor \( \alpha_2 \) decreases. Consequently, \( \alpha \) decreases as well.

With a finite Coulomb interaction \( U \) a third step occurs in the current due to the resonance of the double occupied state. For \( U \approx k_B T \) there is hardly any difference in the current compared to the non-interacting case. But the Fano factor changes drastically: the peak decreases and a dip arises caused by Coulomb correlations. Hence, even though the influence of a small Coulomb interaction does not affect the current, the shot noise reacts very sensitively.

Now we apply the theoretical results to the experiment and consider the Fano factor peak in the shaded region of Fig. 1c. Under the assumption of non-interacting QD states we fitted the current step (upper trace in Fig. 2b) by \( I(V) = I_1 + I_2 f^E_2 + I_3 f^E_3 \) where we extracted the following parameter: \( I_1 = 3.21 \) nA, \( I_2 = 0.8 \) nA, \( I_3 = 1.07 \) nA, \( E_2 - E_F^* = 45.86 \) meV, \( E_3 - E_F^* = 46.43 \) meV, \( \eta = 0.25 \), \( T = 1.6 \) K. For the fit of the Fano Factor we used Eq. (3) with \( \gamma_i = 7.4 \) (\( i = 1, 2 \)), \( I_1/I_3 = 3 \), and \( I_2/I_3 = 9.375 \). The latter current ratio differs significantly from the values for the current fit. In spite of this discrepancy which is not clarified yet at least the agreement in the peak shape is intriguing and confirms the applicability of the derived expression (3) on similar experiments.

**Conclusion** We investigated the bias dependence of shot noise of tunneling through self-assembled QDs focusing on the Fano factor modulation observed in our experiment. Using a master equation approach we find that the Fano factor peaks at the current steps are caused by the Pauli exclusion principle. Neglecting Coulomb interaction we derived an analytical expression for the Fano factor which gives good qualitative agreement with the experiment. The Coulomb interaction \( U \approx k_B T \) leads to an additional dip in the Fano factor.

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