Numerical Study of Aging Phenomena in Short-Ranged Spin Glasses

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(Received )

Aging phenomena of short-range Ising spin glass models have been investigated using Monte Carlo simulations. It is found that in the low-temperature spin-glass phase the mean domain size exhibits a crossover from a power-law growth associated with the critical fluctuation at the transition temperature to slower growth inherent in the low-temperature phase. The temperature dependence of the growth law of the domain size can be almost explained by this crossover. We also find that the spin-autocorrelation function in the quasi-equilibrium regime follows the expected scaling behavior from the droplet theory expressed in terms of the mean domain size and a characteristic length scale of droplet excitations.

§1. Introduction

Spin glasses (SG) exhibit characteristic slow dynamics below the SG transition temperature. Recently such slow dynamics, in particular aging phenomena, has been much attractive both in experimental and theoretical studies. Several attempts have been made so far to explain the slow dynamics. There are mainly two distinct phenomenological approaches; one is a phase-space approach, in which the dynamics is described by a diffusion in the hierarchically constructed phase space, inspired by the mean-field theory which suggests multi-valley structure of the phase space. The other is a real-space picture based on the scaling theory in which low-lying excitations are attributed to connected clusters reversed from one of two ground states. There has been, however, no satisfactory description from a microscopic model, except for the dynamical mean-field theory. In this paper we present the results on the non-equilibrium dynamics obtained by large-scale Monte Carlo (MC) simulations on short-range Edwards-Anderson (EA) Ising SG models. Our analyses of the obtained data are based on the droplet theory. We believe that, even when the phase-space approach may give us a correct description for the slow dynamics, what really occurs in the real space is also indispensable for thorough understandings of the aging phenomena.

Let us explain briefly the droplet theory. According to the theory, aging process is described by coarsening of domain walls, which is driven by successive flipping of thermally activated droplets. During isothermal aging up to waiting time \( t_w \) after quench, domains with the mean size \( R(t_w) \) separating different pure states have grown up. Within each domain, small droplets of size \( L \sim L(\tau) \ll R(t_w) \) are thermally fluctuating within a time scale of \( \tau \) as in equilibrium. The typical value
of their excitation gap $F^\text{typ}_L$ scales as

$$F^\text{typ}_L \sim \Upsilon (L/L_0)^\theta,$$

(1.1)

where $\Upsilon$ is the stiffness constant and $L_0$ is a microscopic length scale, and that of free-energy barrier $B^\text{typ}_L$ also scales as

$$B^\text{typ}_L \sim \Delta (L/L_0)^\psi,$$

(1.2)

where $\Delta$ is a characteristic free-energy scale.

As compared with the equilibrium, some droplets which touch the domain wall with the length scale $R(t_w)$ could reduce their excitation gap from (1.1). This effect is estimated by the droplet theory as the reduction of the averaged excitation gap which is given by

$$F^\text{typ}_{L,R} = \Upsilon_{\text{eff}} (L/L_0)^\theta,$$

(1.3)

with the effective stiffness constant

$$\Upsilon_{\text{eff}} = \Upsilon \left(1 - c_v (L/R)^{d-\theta}\right),$$

(1.4)

where $c_v$ is a numerical constant. Physical quantities such as the spin autocorrelation function are estimated in terms of such length scales by taking into account statistical weights of the droplet excitations appropriately. The growth law of the length scales of the domain $R(t_w)$ and the droplet $L(t)$ is given by

$$R(t), L(t) \sim \left(\frac{T}{\Delta} \ln(t/\tau_0)\right)^{1/\psi},$$

(1.5)

where $\tau_0$ is a microscopic time scale. One notices that the droplet theory for aging phenomena consists of two almost independent steps; the scaling argument based on the typical length scales of $R(t_w)$ and $L(t)$ and the growth law of these length scales. The main purpose of the present work is to test these two steps separately.

The present paper is organized as follows: in the next section after introducing the model system studied, time evolution of the length scale of domain wall is discussed. The results of spin-autocorrelation function in the quasi-equilibrium regime of isothermal aging are presented in Sect. 4.

§2. Model and Method

We focus on the four-dimensional (4D) Ising SG model, because its static critical properties have been established in the sense that a SG phase transition occurs at finite temperature with a rigid order parameter. The 4D EA Ising SG model is defined by Hamiltonian,

$$\mathcal{H} = - \sum_{\langle ij \rangle} J_{ij} S_i S_j,$$

(2.1)

where the sum runs over nearest-neighbor sites and the Ising variables $S_i$ are defined on a hypercubic lattice with periodic boundary conditions. The interactions are
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bimodal variables \((J_{ij} = \pm 1)\) distributed randomly with equal probability. The simulation method is the standard single-spin-flip Monte Carlo (MC) method using two-sublattice dynamics with the heat-bath transition probability. Using the MC method, we have simulated aging processes after a rapid quench from \(T = \infty\) to the SG phase. The system size studied is mainly \(L = 24\), while the size \(L = 32\) is partly studied. We have found no significant difference in the data of \(L = 24\) and 32.

§3. Growth Law of Domain Size

In order to extract a length scale characterizing the growth of ordering, we calculate spatial replica correlation function in off-equilibrium, defined as

\[
G(r, t) = \sum_i \langle S_i^{(\alpha)}(t)S_i^{(\beta)}(t)S_{i+r}^{(\alpha)}(t)S_{i+r}^{(\beta)}(t) \rangle, \tag{3.1}
\]

where \(\alpha\) and \(\beta\) denote the replica indices which are updated independently and with different initial spin configurations. The bracket \(\langle \cdot \cdot \cdot \rangle\) denotes the average over independent bond realizations. In our simulations, only one MC sequence is performed for each random bond configuration. We extract the mean domain size \(R(t)\) by directly fitting \(G(r, t)\) to an exponential form.

In Fig. 1 we show time dependence of \(R(t)\) at the SG transition temperature \(T_c(\approx 2.0J)\) and below. Just at \(T_c\), the length scale is expected to grow as a power law with the dynamical critical exponent \(z\), \(R(t) \sim t^{1/z}\), irrespective of the physical picture underlying the ordered phase. As expected, it is found that the length scale follows a power law. The exponent \(z\) is estimated to be 4.98(5), which is consistent with that of the previous work.\(^6\) Below \(T_c\), \(R(t)\) grows with time slower and slower as temperature decreases. We try to see crossover between the critical fluctuation and slow dynamics inherent in the low-temperature phase. In off-equilibrium both at the critical and the off-critical temperatures, the length scale would exhibit a power law in short length and time regime where the critical fluctuation dominates the dynamics. We assume that such microscopic length \(R_0\) and time \(\tau_0\) are the correlation length and time associated with critical fluctuation in equilibrium, respectively. Thus, we propose a scaling form

\[
R(t)/R_0 = g(t/\tau_0), \tag{3.2}
\]

with \(R_0 = |T - T_c|^{-\nu}\) and \(\tau_0 = |T - T_c|^{-2\nu}\). The scaling plot of \(R(t)\) is shown in Fig. 2. As expected from a standard scaling theory of critical phenomena, the scaling function \(g(x)\) for smaller \(x\) exhibits a power law \(x^{1/z}\) associated with the critical temperature. We find a significant deviation from the power law at longer times, suggesting that the characteristic slow dynamics of the SG phase takes place there. In fact, the functional form is not incompatible with a power law of \(\ln(t)\) predicted by the droplet theory (1.5). It is noted that the strong temperature dependence of \(R(t)\) shown in Fig. 1 can be almost explained by introducing the microscopic units \(R_0\) and \(\tau_0\) associated with the critical fluctuation, while it is not sure whether the asymptotic form at longer times is also described by a universal scaling function or not.
§4. Scaling analysis in quasi-equilibrium

Recent studies have revealed that the off-equilibrium dynamics in the SG is separated into two characteristic time regimes. One is a short-time regime, called “quasi-equilibrium regime”, and the other is a long-time regime, called “aging regime”. A typical observable to see such two time regimes is the spin auto-correlation function

$$C(\tau; t_w) = \frac{1}{N} \sum_i (S_i(t_w)S_i(\tau+t_w)),$$  \hspace{1cm} (4.1)

where $t_w$ denotes a waiting time after the rapid quench. Our interest is in its behavior in the quasi-equilibrium regime, namely $t_w \gg \tau$. Based on the droplet argument using the effective stiffness constant (1.4), the behavior of $C(\tau; t_w)$ is explicitly given by

$$C(\tau; t_w) = C_{eq}(\tau) + c \frac{T}{(L(\tau)/L_0)^\theta} \left( \frac{L(\tau)}{R(t_w)} \right)^{d-\theta} + \cdots,$$  \hspace{1cm} (4.2)

with the equilibrium part $C_{eq}(\tau)$ \(^4\)

$$C_{eq}(\tau) = q_{EA} + \frac{A}{(L(\tau)/L_0)^\theta},$$  \hspace{1cm} (4.3)

where $q_{EA}$ is the EA order parameter and $c$ and $A$ are numerical constants. This gives us an extrapolation form of the large $\tau$ limit, namely a way of determining the EA order parameter. An empirical form $C(t) = q_{EA} + a/t^\alpha$ has been used frequently for estimating $q_{EA}$ from the spin autocorrelation function. \(^8\) This is true only if the time dependence of length scale $L(t)$ in (4.3) is a power law. As seen in the last section, however, the observed length scale $R(t)$ in this model exhibits the crossover from the critical power law to slower growth at large $t$.

We observe the autocorrelation function $C(\tau; t_w)$ at $T/J = 1.2$ well below $T_c$ and check the scaling form (4.2) by making use of the $R(t)$ estimated through $G(r, t)$ in the last section. According to the droplet theory, both $R(t_w)$ and $L(\tau)$ exhibit...
the same time dependence. Also, the microscopic length scale $L_0$ is assumed to be the same as $R_0$. For fixed $\tau$, the autocorrelation function is expressed as a function of $R(t_w)$. Using the estimated value of $\theta (= 0.82)$, a simple linear fitting gives the equilibrium autocorrelation function $C_{eq}(\tau)$ in the large $t_w$ limit. Collecting $C_{eq}(\tau)$ for each $\tau$ thus extracted, we confirm directly the droplet prediction (4.3) as shown in Fig 3 and determine the equilibrium EA order parameter. The value of $q_{EA}$ estimated to be $0.58(1)$ is compatible with the recent estimation from static MC simulation.

Next we discuss correction to the equilibrium limit. The expression (4.2) suggests that the correction term $\Delta C(\tau; t_w) = C(\tau; t_w) - C_{eq}(\tau)$ multiplied by $L^\theta(\tau)$ becomes only a function of $L(\tau)/R(t_w)$. As shown in Fig. 4, we confirm this scaling prediction. For the limit of $L(\tau)/R(t_w) \ll 1$, the scaling function shows $(L(\tau)/R(t_w))^{d-\theta}$ consistent with (4.2).

![Fig. 3. Equilibrium autocorrelation function extracted to the large $t_w$ limit. The line represents a fitting according to the expression (4.3).](image1)

![Fig. 4. Scaling plot of the correction term of $C(\tau; t_w)$. The line has an expected slope $d-\theta$ for small $L(\tau)/R(t_w)$ limit.](image2)

§5. Discussion and Summary

Let us compare the present results with those obtained in 3D Ising SG models. In three dimensions, results on the growth law by numerical simulations as well as experiments are well fitted to a power law as

$$R(t) \sim t^{1/z(T)},$$

(5.1)

where the exponent $1/z(T)$ is proportional to temperature $T$ and continuously connects with the dynamical critical exponent $z$ at $T_c$. It is not clear yet how to interpret physically such a power law with temperature-dependent exponent $1/z(T)$. One of the possibilities is the crossover observed in the present work on 4D Ising SG model. Certainly it is worth examining the crossover effect of the critical fluctuation in the 3D model.

On the other hand, the scaling argument in terms of the length scales $R(t_w)$ and $L(\tau)$ successfully explains the aging behavior of the correlation function in the
quasi-equilibrium regime also in three dimensions. Recently it has been confirmed not only in the isothermal but also in temperature-shift aging processes,\textsuperscript{15}) which are basic experimental procedures frequently used.

To conclude, we have investigated non-equilibrium dynamics after the temperature quench from infinity to the SG phase in the 4D Ising SG model using Monte Carlo simulations. We have studied the growth law of the mean domain size by analyzing time evolution of the spatial replica correlation functions. We have found that the growth law shows a crossover from the critical regime to the low-temperature one and its main temperature dependence within time range of our simulation can be explained by this crossover.

We have also analyzed the spin autocorrelation function in the quasi-equilibrium regime. The off-equilibrium correction of the correlation function to its equilibrium limit, namely, violation of the time translational invariance, in the quasi-equilibrium limit can be explained by the scaling argument in terms of the characteristic length scales $R(t_w)$ and $L(\tau)$, as observed already in the 3D Ising SG model.\textsuperscript{7}) These results strongly suggest that the analysis consisting of the two almost independent steps is promising for understanding the aging phenomena in low-dimensional SG systems.

Acknowledgements

The present simulations has been performed on Fujitsu VPP-500/40 at the Supercomputer Center, Institute for Solid State Physics, the University of Tokyo.

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