The structure of cold neutron star with a quark core within the MIT and NJL models

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Abstract

Neutron star due to their high interior matter density are expected to be composed of a quark core, a mixed quark-hadron matter, and a layer of hadronic matter. Thus, in this paper, we compute the equation of state of these parts of neutron star to evaluate its structure properties. We use two models for describing EOS of quark matter, NJL and MIT bag models, and employ three approaches in this work. A density dependent bag constant satisfy the quark confinement in the simple MIT bag model. We also study the interaction behavior of quarks, firstly one gluon exchange within MIT bag model and the secondly dynamical mass will be held as effective interaction that roles between particles. Density dependence of quark mass is obtained from NJL self consistent model. NJL model is a effective manner for justify the chiral symmetry. Applying the Gibbs conditions the equation of state of the quarks and hadrons mixed phase is obtained. Since the hadronic matter is under the influence of strong force of nucleons, we calculate the equation of state of this phase using a powerful variational many-body technique. Finally, we calculate the mass and radius of a cold neutron star with a quark core by numerically solving the TOV equation. To check our used EOS, we compare our results with the recent observational data. Our results are in a good agreement with some observed compact objects such as $SAXJ1748.9-2021$, $4U1608-52$ and $VelaX-1$. 
Keywords: neutron star, quark core, quark matter, MIT bag model, NJL model, gravitational mass, radius.

I. INTRODUCTION

Neutron stars are placed in category of compact objects, and the interior matter of them can reach a density much greater than the normal nuclear saturation density. Therefore, these astrophysical objects are best laboratory and a unique environment to probe the properties of dense matter. Studying these stars is one of main problems in physics. In high densities, hadrons dissolve to quarks, and a phase transition is happen from hadronic matter to quark matter. Many years ago, the presence of quark matter in neutron stars has been suggested by Ivanenko[1], Itoh[2] and Collins[3]. There are up, down, and strange quarks in the quark matter, and this strange matter is a fermi gas which the other quarks because of their high masses do not appear in this part. Since all the hadrons do not converge to quarks simultaneously, it is expected to exist a mixed phase of quarks and hadrons at finite range of density that the energy is lower than that of quark and hadron matters.

Historically Glendenning was the first who pointed to the neutrality charge of two phases in mixed phase[4]. In mixed phase, we study the transition from a hadron phase to quark phase using the Gibbs conditions. Since the existence of mixed phase of quarks and hadrons affects the properties of neutron star, we consider the neutron stars to be composed of a quark matter core, a mixed phase of quarks and hadrons and a layer of hadrons. There is a high uncertainty in equation of state (EOS) of quark matter. Usually, two more efficient models are used to study deconfined quark matter, the MIT bag model[5, 6] and the Nambu-Jona-Lasinio (NJL) model[7]. The total energy density in MIT bag model is the sum of the kinetic energy of free quarks and a bag constant $B$ that is nonperturbative energy shift. We will be search influence of one-gluon-exchange in MIT bag model. There is asymptotically interaction among quarks at high densities. This interaction can show up by one gluon exchange. Therefore, in addition of $B$, we add another term to EOS that is identified by $\alpha$ the QCD coupling constant[8–10]. In MIT bag model, the mass of quarks is constant, but in the second method, NJL, mass of quarks depends on density that is considered as the effective interaction of quarks. Historically first time the NJL model is presented in two papers from Nambu and Jona-Lasinio in 1961[11, 12]. In spite of MIT bag model, the
NJL model does not have confinement, but it justifies chiral symmetry. But at high density matter like quark matter, although the confinement and chiral symmetry are in the least importance, both models treat to be similar [13].

In recent years, we have applied the MIT bag model to investigate the cold and hot strange quark star. For example we consider MIT bag model with a density dependent bag constant for a hot strange star in [14], and found that the mass and radius of strange star decreases when temperature increases. We have also found that a higher mass and radius will be obtained for a density dependent bag constant compare to a fixed bag constant. In [15, 16] using MIT bag model with fixed bag constant, we considered the stability of spin polarized quark star in a strong magnetic field compared to unpolarized case, and calculated the structure of this star at zero and finite temperatures. Also We have used MIT bag model with a density dependent bag constant to calculate the structure of spin polarized strange star in presence of magnetic field at zero and finite temperatures [17, 18]. We have also applied NJL model to calculate the equation of state of quark matter [19].

As we know, a neutron star with a quark core is called hybrid star. In our previous works, we have obtained the structure of hybrid star by MIT bag model with fixed and density dependent bag constant model at zero temperature [20] and at finite temperature [21, 22]. In those works, we have considered the simplest version of the MIT bag model. In the present work, we intend to develop our previous calculations by considering the effects of one gluon exchange for the quark matter in a neutron star with a quark core. We also use the NJL method in this work. NJL model is an effective theory and a good choice for the studying the chiral quark and diquark condensates. Chiral symmetry and its breakdown in vacuum are the basic property of NJL model. The outline of our work is as follows: In section II we calculate the equation of state of three mentioned phases of the neutron star matter. Then using this equation of state, we determine the mass and radius of neutron star with a quark core in section III.

II. EQUATION OF STATE OF A HYBRID NEUTRON STAR

Here, we determine EOS of different part of neutron star: a hadron phase, quark phase and a mixed phase of quarks and hadrons respectively.
A. Hadron Phase

We consider the lowest order constrained variational (LOCV) many-body method for hadron phase \[23–26\]. By considering a many body trail wave function such as $\psi = F\phi$ and some calculations, the cluster expansion of the energy functional is gotten \[28\],

$$ E([f]) = \frac{1}{A} \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle} = E_1 + E_2 + E_3 + \cdots, \quad (1) $$

where $F = S \prod_{i<j} f(ij)$ is an A-body correlation operator ($f(ij)$ is two-body correlation function and $S$ leads to a symmetric product) and $\phi$ is the slater determinate of A noninteracting nucleons. We consider the first two terms in above equation, the one-body term $E_1 = \sum_{i=1,2,3} \frac{k_i^2}{2m_i} \rho_i$, and the two-body term $E_2 = \frac{1}{2A} \sum_{ij} <ij|\nu(12)|ij-ji>$. In these relations, $\rho_i$ is the nucleon density, $\rho = \rho_p + \rho_n$ is the total density and $k_i = (3\pi^2 \rho_i)^{1/3}$. Here $\nu = -\frac{k^2}{2m}(f(12),[\nabla_{12}^2, f(12)]) + f(12)V(12)f(12)$ is the nucleonic effective potential ($V(12)$ is the nuclear potential). See reference \[24\] for full nuclear matter calculations.

B. Quark Phase

We employ MIT bag model and NJL model which are two well-known and efficient models for describing the characteristics of deconfined quark matter.

1. The MIT Bag Model

Quark matter is a fermi gas which composed of deconfined up, down, and strange quarks. Therefore the total energy is given by

$$ \mathcal{E}_{tot} = \mathcal{E}_u + \mathcal{E}_d + \mathcal{E}_s + B. \quad (2) $$

In equation (2) $\mathcal{E}_i$ is

$$ \mathcal{E}_i = \frac{3m_i^4}{8\pi^2} \left[ x_i(2x_i^2 + 1)(\sqrt{1 + x_i^2} - \sinh^{-1} x_i) \right] - \alpha_e \frac{m_i^4}{\pi^3} \left[ x_i^4 - \frac{3}{2}[x_i(\sqrt{1 + x_i^2} - \sinh^{-1} x_i)]^2 \right], \quad (3) $$

where

$$ x_i = \frac{k_F^{(i)}}{m_i}, \quad (4) $$
Here, $k_F^{(i)} = (\rho_i \pi^2)^{1/3}$, $m_i$ and $\rho_i$ are the mass and baryon density of quark $i$, respectively, and $\alpha_c$ is the QCD coupling constant. We consider three values for $\alpha$: $\alpha = 0$ (MIT bag model without interaction), $\alpha = 0.16$ and $\alpha = 0.5$. Although these amount of $\alpha$ are small and perturbative, they show an appropriate range of quark interaction and are in the selection range of Farhi and Jaffe work [27]. In equation (2), $B$ is a density dependent bag constant which satisfies the quark confinement in MIT bag model. We consider a Gaussian form $B(\rho) = B_\infty + (B_0 - B_\infty) \exp \left[-\beta (\rho/\rho_0)^2\right]$, where $B_0 = B(\rho = 0) = 400 MeV/fm^3$, $\beta$ is a numerical parameter equal to $\rho_0 = 0.17 fm^{-3}$ and $B_\infty$ is a free parameter which is determined by using the experimental data reported in the CERN SPS. [20, 31]. Now, using the energy density from Eq. (2), the EOS of quark matter in the MIT bag model is obtained,

$$P(\rho) = \rho \frac{\partial E}{\partial \rho} - E. \quad (5)$$

2. The NJL Model

In NJL model, The dynamical mass is held the effective interaction between particles. In this method, we adopt a lagrangian similar to that given in reference [19], as follow,

$$\mathcal{L} = \bar{q} (i\gamma^\mu \partial_\mu - \hat{m}_0) q + G \sum_{k=0}^{8} \left[ (\bar{q} \lambda_k q)^2 + (\bar{q}i\gamma_5 \lambda_k q)^2 \right]$$

$$- K \left[ det_f(\bar{q}(1 + \gamma_5)q) + det_f(\bar{q}(1 - \gamma_5)q) \right], \quad (6)$$

where $q$ is the field of quarks with three flavors and three colors, $\hat{m}_0 = diag(m^0_u, m^0_d, m^0_s)$ in flavor space, and $\lambda_k (0 \leq k \leq 8)$ are the flavor matrices. Restoring chiral symmetry, breaking is indicated with a ultra-violet cut-off. We employ parameters of reference [29, 30] as follow $\Lambda = 602.3 MeV$, $GA^2 = 1.835\Lambda$ and $KA^2 = 12.36$. $G$ and $K$ are coupling strength.

The dynamical mass is calculated by

$$m_i = m_0^i - 4G \langle \bar{q}_i q_i \rangle + 2K \langle \bar{q}_j q_j \rangle \langle \bar{q}_k q_k \rangle, \quad (7)$$

$$\langle \bar{q}_i q_i \rangle = -\frac{3}{\pi^2} \int_{p_{Fi}}^{\Lambda} p^2 dp \frac{m_i^2}{\sqrt{m_i^2 + p^2}}, \quad (8)$$

$$p_{Fi} = (\pi^2 \rho_i)^{1/3}. \quad (9)$$
At this stage we will determine EOS of quark matter in NJL model,

\[ P = \sum_{i=u,d,s} n_i \sqrt{p_i^2 + m_i^2} - \mathcal{E}, \quad (10) \]

where

\[ \mathcal{E} = \sum_{i=u,d,s} \frac{3}{\pi^2} \int_0^{p_F} p^2 dp \sqrt{p^2 + m_i^2} - (B - B_0). \quad (11) \]

In equation (11), \( B \) is the bag pressure [13] which is consequence of interaction,

\[ B = \sum_{i=u,d,s} \left[ \frac{3}{\pi^2} \int_0^{\Lambda} p^2 dp \left( \sqrt{p^2 + m_i^2} - \sqrt{p^2 + m_0^2} \right) - 2G \langle \bar{q}_i q_i \rangle > + 4K \langle \bar{u} u \rangle > < \bar{d} d > < \bar{s} s >. \quad (12) \]

C. Mixed phase

There is a mixed phase of quarks and hadrons within a finite range of density. In this phase, we apply the Gibbs condition. According to this equilibrium condition, the pressures and chemical potentials of both quark and hadron phases are equal [4],

\[ \mu_Q^Q = \mu_H^H, \quad (13) \]

\[ \mu_Q^Q = \mu_H^H, \quad (14) \]

where \( \mu_Q^Q (\mu_H^H) \) and \( \mu_Q^P (\mu_H^P) \) are the neutrons (protons) chemical potential in nucleonic and quark part in the mixed phase, respectively. Using above equations, we can determine the charge density of quarks and hadrons. \( \chi \), which is the volume fraction occupied by quarks is determined by considering the global charge neutrality. Then baryon and total energy densities in mixed phase could be calculated.

\[ \chi \left( \frac{2}{3} \rho_u - \frac{1}{3} \rho_d - \frac{1}{3} \rho_s \right) + (1 - \chi) \rho_B - \rho_e = 0, \quad (15) \]

\[ \rho_B = \chi \rho_Q + (1 - \chi) \rho_H, \quad (16) \]

\[ \mathcal{E}_{MP} = \chi \mathcal{E}_{QP} + (1 - \chi) \mathcal{E}_{HP}. \quad (17) \]

See reference [38] for detail of calculations regarding the EOS of mixed phase.

At this stage we can determine EOS of neutron star with a quark core using the results of proceeding sections. We consider three approaches for the quark matter (in the quark
FIG. 1: EOS for the hybrid neutron star. The results of our calculations in different models of quark matter have been plotted.

phase and mixed phase); simple MIT bag model (model 1), MIT bag model by considering the quark interaction (model 2) and NJL model (model 3). EOS results for neutron star (corresponding to different models for the quark matter calculations) are given in Fig. 1. This figure shows that the difference between the equation of state in model 1 and model 3 is substantially lower than those of other mentioned models. However, we see that the equation of state of neutron star becomes stiffer when coupling constant is higher, particularly at high densities. This shows the importance of interaction at higher densities.

III. STRUCTURE PROPERTIES OF THE HYBRID NEUTRON STAR

Using the obtained EOS in previous section, the structure of neutron star with a quark core can be calculated. Before this work, we investigate the energy and stability conditions for our results. For this purpose at first, we fit a polynomial function for the equations of
state in Fig. 1 as,

\[ P = \sum_{i=1}^{7} a_i \xi^{7-i} \]  \hspace{1cm} (18)

The coefficients \( a_i \) have been given in Table III. We use this relation to justify the energy

TABLE I: Different coefficients presented in Eq. (18) for our applied models.

| Model   | \( a_1 \times 10^{-57} \) | \( a_2 \times 10^{-40} \) | \( a_3 \times 10^{-25} \) | \( a_4 \times 10^{-10} \) | \( a_5 \times 10^6 \) | \( a_6 \times 10^{21} \) | \( a_7 \times 10^{35} \) |
|---------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
| model 1 | 1.194                     | -0.2467                   | 2.011                     | -8.123                    | 1.656                     | -1.201                    | 2.915                     |
| model 2; \( \alpha = 0.16 \) | 6.487                     | -1.279                    | 9.852                     | -37.23                    | 7.090                     | -4.962                    | 10.80                     |
| model 2; \( \alpha = 0.5 \) | 9.400                     | -1.912                    | 15.11                     | -58.19                    | 11.20                     | -6.819                    | 11.27                     |
| model 3 | 1.285                     | -0.2407                   | 1.793                     | -6.630                    | 1.209                     | -0.7027                   | 1.321                     |

and stability conditions as follows.

A. Energy conditions

Energy conditions in the center of neutron star include the null energy condition (NEC), weak energy condition (WEC), strong energy condition (SEC) and dominant energy condition (DEC). These condition are expressed as follow,

\[ NEC \rightarrow P_c + \xi_c \geq 0, \]  \hspace{1cm} (19)

\[ WEC \rightarrow P_c + \xi_c \geq 0 \hspace{0.5cm} \& \hspace{0.5cm} \xi_c \geq 0, \]  \hspace{1cm} (20)

\[ SEC \rightarrow P_c + \xi_c \geq 0 \hspace{0.5cm} \& \hspace{0.5cm} 3P_c + \xi_c \geq 0, \]  \hspace{1cm} (21)

\[ DEC \rightarrow \xi_c > |P_c|, \]  \hspace{1cm} (22)

where \( \xi_c \) is the energy density and \( P_c \) is the pressure at the center of star. Results of the above conditions for our equations of state are given in Table III A. We have found that our equations of state satisfy mentioned energy conditions, except the dominant energy condition for model 2 with \( \alpha = 0.5 \) in which the central pressure is very high with respect to the other models.
TABLE II: Energy conditions for hybrid neutron star for applied models.

| NS+Quark Core | $\varepsilon_c\left(10^{14}\text{gr/cm}^3\right)$ | $P_c\left(10^{14}\text{gr/cm}^3\right)$ | NEC | WEC | SEC | DEC |
|---------------|---------------------------------|---------------------------------|-----|-----|-----|-----|
| model 1       | 25.8                            | 7.79                            | √   | √   | √   | √   |
| model 2 ; $\alpha = 0.16$ | 18.25                           | 17.8                            | √   | √   | √   | √   |
| model 2 ; $\alpha = 0.5$   | 14.05                           | 28.9                            | √   | √   | ×   |     |
| model 3       | 22.9                            | 5.56                            | √   | √   | √   | √   |

B. Stability

According to the stability condition, an equation of state is physically acceptable when the corresponding obtained velocity of sound ($v$) be less than the light’s velocity ($c$) \[32, 33\]. Thus the stability condition is \[0 \leq v^2 = \left(\frac{dP}{d\varepsilon}\right) \leq c^2\]. By Using Eq. 18 we have computed $\frac{v^2}{c^2}$ versus density which has been given in Fig. [2] It is evident that the stability condition is satisfied by the our calculated EOS of neutron star with quark core for models 1 and 3. So these two models are suitable for determining the structure of a hybrid neutron star. The other models do not obey the stability condition, and we can’t use their equations of state in the structure calculations.

C. Properties of the neutron star with a quark core

We use TOV equation to calculate the structure of star. This equation is determined by Tolman-Oppenheimer-Volkoff (TOV) \[34-36\].

\[
\frac{dP}{dr} = -\frac{G \left[ \frac{P}{c^2} + \varepsilon \right] \left[ m + \frac{4\pi r^4 P}{c^2} \right]}{r^2 \left[ 1 - \frac{2Gm}{rc^2} \right]}, \tag{23}
\]

\[
\frac{dm}{dr} = 4\pi r^2 \varepsilon \tag{24}
\]

where $P$ is the pressure and $\varepsilon$ is the energy density. In our calculations for the neutron star, we consider the following equation of state: up to the density $0.05 fm^{-3}$, we use the data of Baym calculations \[37\]. For the hadron, quark, and mixed phases, we consider our equations of state obtained in the previous sections. Numerically integrating the TOV equation for
a given equation of state, the mass and radius of the neutron star with a quark core is determined. Our results are as follows.

The gravitational mass of hybrid neutron star (a neutron star with a quark core) have been presented in Fig. 3 versus the central mass density. The mass-radius relation has been also shown in Fig. 4 for this star for different models. For comparison, we have also brought the results for a neutron star without a quark matter [38] in Figs. 3 and 4. Results show that in the NJL model with a density dependent mass of quarks, the maximum mass is lower than that of other models. The structure properties of neutron star in the cases without and with a quark and have been given in Table III C for different models. In table III C, it is seen that when we consider a quark core for the neutron star in both considered models, the maximum mass decreases and radius increases. This is because the equation of state becomes softer when we consider a quark matter in the core of neutron star. We have obtained the lowest amount of mass in NJL model (mass = 1.75M⊙), but it is nearly close to that of MIT bag model with zero coupling constant (mass = 1.8M⊙). The calculated
FIG. 3: Gravitational mass versus the central mass density for the neutron star with a quark core for different models. The results for a neutron star without quark core (NS) have been also given for comparison.

The radius in NJL model is bigger than that of MIT bag model. We have also brought some observational data of neutron star candidates in table III C to compare our results with these data. The mass of a neutron star with a quark core which we have been determined in this paper is closed to the mass of \textit{SAXJ1748.9 − 2021} \cite{43}, \textit{4U1608 − 52} \cite{44}, and \textit{VelaX − 1} \cite{44}. Also the calculated radius of neutron star with a quark core in two models are in a good agreement with the radius of the mentioned observed compact objects. However, the mass and radius do not agree with the recent observational data of for pulsar \textit{PSRJ1614 − 2230} \cite{42} with \(M = 1.97 \pm 0.04 M_\odot\), while our result of neutron star without quark core \cite{38} has a good agreement with the mass of this pulsar. But the calculated radius is smaller than the radius of pulsar \textit{PSRJ1614 − 2230}.

Also we review the works of several authors who have researched on the properties of hybrid neutron star. The results of their models are presented briefly in table III C. In this
FIG. 4: Mass-radius relation for the neutron star with a quark core for different models. The results of neutron star without quark core (NS) have been also given for comparison.

In the table, $x_\sigma$ is the hyperon coupling constant and $G_2$ is one of the EOS parameters within the field correlated method [44]. We can see that the results of reference [44] with $x_\sigma = 0.6$, and $G_2 = 0.012$ and 0.016 ($M = 1.73M_\odot$ and $1.8M_\odot$) are in a good agreement with our calculated mass. In table III C, $\sigma$ and $G_v$ are the surface tension and the vector coupling constant, respectively [45]. Mass values in that paper are about $2M_\odot$, therefore they have a good agreement with PSRJ1614 − 2230, while they do not have agreement with our work. In table III C, ESCO8 is a model for determining hyperon coupling constant and $\beta$ is a parameter which is the same with the one in bag constant formula, $B$, in our work [46]. The obtained masses in this reference is higher than our calculated result. The authors in all these papers have parameterized the EOS in order to get $M \approx 2.0M_\odot$ which has been recently observed for PSRJ1614 − 2230. Here, we can conclude that if we want to get the mass of a hybrid star to be about the mass of pulsar PSRJ1614 − 2230 with the mass $M = 1.97 \pm 0.04M_\odot$, we should modify the equation of state that used in this work, or we
TABLE III: Structure properties of neutron star without (NS) and with (NS+Q) a quark core for different models. The observational data have been also presented for comparison.

| Compact object | $M_{max} \ (M_{\odot})$ | $R \ (km)$ |
|----------------|-------------------------|------------|
| NS            | 1.98                    | 9.8        |
| NS+Q: model 1 | 1.8                     | 10         |
| NS+Q: model 3 | 1.75                    | 10.4       |
| $4U1820 - 30$ $[39]$ | $1.58 \pm 0.6$ | $9.1 \pm 0.4$ |
| $PSRJ1903 + 0327$ $[40]$ | $1.667 \pm 0.021$ | $9.438 km$ |
| $PSRJ1614 - 2230$ $[41]$ | $1.97 \pm 0.04$ | $13 \pm 2 km$ |
| $SAXJ1748.9 - 2021$ $[42]$ | $1.78 \pm 0.3$ | $8.18 \pm 1.62 km$ |
| $4U1608 - 52$ $[43]$ | $1.74 \pm 0.14$ | $9.3 \pm 1 km$ |
| $VelaX - 1$ $[43]$ | $1.77 \pm 0.08$ | $9.56 km$ |

should use another computational method to calculate the structure of a hybrid star.

IV. SUMMARY AND CONCLUSION

Since the neutron stars are one of the compact objects with high density, this idea raises that there is a deconfined quark matter in these stars. Thus, here we considered a crust of hadronic matter, a mixed phase of quark and hadronic matters and a quark matter in the core of neutron star. In this work, we calculated the EOS for quark matter phase of neutron star in three models, simple MIT bag model, MIT bag model including one gluon exchange correction with two different coupling constants and NJL model with a density dependent mass for quarks. For hadronic matter phase, we chose a variational method (LOCV). For the mixed phase, the hadron-quark phase transition was modeled by the Gibbs constructions. After calculation of EOS, we studied the energy and stability conditions. We found that when the interaction of quarks is given by the one gluon exchange, the equation of state doesn’t satisfy these conditions. Therefor in this case EOS isn’t suitable for calculation of structure of this star. Using determined equation of state and solving Tolman-Oppenheimer-Volkof (TOV) equations, we computed the structure of a neutron star with a quark core.
(hybrid star) in two models 1 and 3. We saw that our result for the maximum mass of neutron star with a quark core agrees with the observed mass for SAXJ1748.9 – 2021, 4U1608 – 52 and VelaX – 1. However, that is not in a good agreement with the recent observational data for PSRJ1614 – 2230. While, we found that our result for the maximum mass of neutron star without quark matter is in agreement to the mass of this object. Here, it can be concluded that our equation of state should be modified in order to get some good agreements with the new observational data.

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TABLE IV: Structure properties of neutron star with a quark core is calculated by other authors. Our results have been also given for comparison.

| Reference  | Quark Phase model | Hadron Phase model | Mixed Phase condition | used parameter | used parameter | $M_{\text{max}}$ ($M_\odot$) | $R$ (km) |
|------------|-------------------|--------------------|-----------------------|----------------|----------------|-----------------------------|---------|
| present work | MIT Bag Model | LOCV | Gibbs | $x_\sigma = 0$ | $G_2 = 0.006$ | 1.44 | 9.54 |
| present work | NJL Model | LOCV | Gibbs | $x_\sigma = 0$ | $G_2 = 0.012$ | 1.89 | 12.55 |
| present work | NJL Model | LOCV | Gibbs | $x_\sigma = 0$ | $G_2 = 0.016$ | 2.05 | 12.66 |
| present work | NJL Model | LOCV | Gibbs | $x_\sigma = 0.8$ | $G_2 = 0.006$ | 1.44 | 9.52 |
| present work | NJL Model | LOCV | Gibbs | $x_\sigma = 0.8$ | $G_2 = 0.012$ | 1.89 | 12.53 |
| present work | NJL Model | LOCV | Gibbs | $x_\sigma = 0.8$ | $G_2 = 0.016$ | 2.04 | 12.40 |
| present work | NJL Model | LOCV | Gibbs | $x_\sigma = 0.6$ | $G_2 = 0.006$ | 1.43 | 9.51 |
| present work | NJL Model | LOCV | Gibbs | $x_\sigma = 0.6$ | $G_2 = 0.012$ | 1.73 | 12.00 |
| present work | NJL Model | LOCV | Gibbs | $x_\sigma = 0.6$ | $G_2 = 0.016$ | 1.8 | 11.49 |
| [43] Field correlator Method | Relativistic Mean Field Model | Gibbs | $\sigma = 0$ | $G_v = 0$ | 1.91 | 13.09 |
| | | | $\sigma = 0$ | $G_v = 0.2G_s$ | 2.05 | 13.00 |
| | | | $\sigma = 0$ | $G_v = 0.4G_s$ | 2.13 | 12.77 |
| | | | $\sigma = 10$ | $G_v = 0$ | 1.94 | 13.3 |
| | | | $\sigma = 10$ | $G_v = 0.2G_s$ | 2.08 | 13.01 |
| | | | $\sigma = 10$ | $G_v = 0.4G_s$ | 2.15 | 12.77 |
| | | | $\sigma = 40$ | $G_v = 0$ | 2.00 | 13.37 |
| | | | $\sigma = 40$ | $G_v = 0.2G_s$ | 2.11 | 13.03 |
| | | | $\sigma = 40$ | $G_v = 0.4G_s$ | 2.17 | 12.67 |
| [44] NJL Model | Relativistic Mean Field Model | Gibbs | $\beta = 0.000$ | $\beta = 0.000$ | 2.029 | 11.13 |
| | | | $\beta = 0.025$ | $\beta = 0.025$ | 2.003 | 11.56 |
| | | | $\beta = 0.050$ | $\beta = 0.050$ | 1.958 | 11.71 |
| | | | $\beta = 0.100$ | $\beta = 0.100$ | 1.896 | 11.63 |
| | | | $\beta = 0.150$ | $\beta = 0.150$ | 1.866 | 11.5 |
| | | | $\beta = 0.200$ | $\beta = 0.200$ | 1.853 | 11.41 |