Solution of the two-mode quantum Rabi model using extended squeezed states

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Abstract – The two-mode quantum Rabi model with bilinear coupling is studied using extended squeezed states. We derive $G$-functions for each Bargmann index $q$. They share a common structure with the $G$-function of the one-photon and two-photon quantum Rabi models. The regular spectrum is given by zeros of the $G$-function while the conditions for the presence of doubly degenerate (exceptional) eigenvalues are obtained in closed form through the lifting property. The simple singularity structure of the $G$-function allows to draw conclusions about the distribution of eigenvalues along the real axis and to understand the spectral collapse phenomenon when the coupling reaches a critical value.

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Introduction. – The Quantum Rabi model (QRM) describes a two-level system (qubit) coupled to a cavity electromagnetic mode (an oscillator) [1,2], a minimalist paradigm of matter-light interactions with applications in numerous fields ranging from quantum optics to quantum information science and condensed-matter physics [3]. Its Hamiltonian reads

$$H_R = \Delta \sigma_z + \omega a^\dagger a + g(a^\dagger + a)\sigma_x. \quad (1)$$

The Pauli matrices $\sigma_{x,z}$ describe the two-level system and $a$ ($a^\dagger$) denote the annihilation (creation) operators of the bosonic mode. Simple as it appears, the solution of the QRM is nontrivial and it was not clear whether it can be obtained analytically. Recently it was shown that the QRM is not only exactly solvable but integrable [4], using Bargmann-space methods [5]. A function $G_R(E)$ was derived, whose zeros yield the so-called regular spectrum, i.e. $G_R(E_n) = 0$ entails $E_n \in \text{spec}(H_R)$. This $G$-function can be written explicitly in terms of confluent Heun functions [6]. $G_R(E)$ was then recovered with the simpler formalism of extended coherent states [7]. These results have stimulated extensive research in the QRM and related models [8–21].

It is well known in quantum optics [22] that two-mode squeezed states are important since several devices produce correlated light at two frequencies. Recently, a quantum memory for light, a key element for the realization of future quantum information networks, was constructed employing a set of displaced two-mode squeezed states with an unconditionally high fidelity that exceeds the classical benchmark [23]. Two-mode squeezed states have been prepared also in cavity optomechanics via reservoir engineering [24]. There are many hints to potential applications of a single qubit coupled simultaneously to two different light modes in quantum information technology, e.g. for the implementation of fast beam splitters [25]. Such a system may also be realized in various solid-state devices [26–28], and especially in circuit QED [29].

One analytically tractable form of the two-mode generalization of the QRM couples the qubit bilinearly to the two cavity modes, thereby creating squeezed states. This model will be studied in the next section. We derive a $G$-function in analogy to the single-mode case and obtain regular and exceptional spectra analytically. Most important, we can derive the overall features of the spectrum...
from the pole structure of the \(G\)-function alone, without
the need for numerical evaluation, and explain in this way
the collapse of the discrete spectrum to a continuum which
happens in this model as in the QRM with quadratic cou-
pling to a single mode \([8,30]\).

Two-mode Rabi model. — The Hamiltonian for the
two-mode QRM \([31]\) reads

\[
H_{\text{tm}} = \Delta \sigma_z + \omega \left( a_1^\dagger a_1 + a_2^\dagger a_2 \right) + g \left( a_1^\dagger a_2 + a_1 a_2 \right) \sigma_x,
\]

where \(g\) is the coupling strength and \(a_i^\dagger\) and \(a_i\) are the creation and annihilation operators for the quantized fields in two cavities \(i = 1, 2\). The energy scale is defined by setting \(\omega = 1\).

A rotation around the \(y\)-axis through the angle \(\pi/2\) yields the Hamiltonian in spin-boson form,

\[
\text{see eq. (3) above}
\]

which will be convenient in the following.

Symmetries. The Hamiltonian eq. (2) exhibits a con-
tinuous \(U(1)\)-symmetry similar to the Jaynes-Cummings
model \([2]\); it is invariant under the transformation

\[
a_1 \rightarrow e^{i\phi} a_1, \quad a_1^\dagger \rightarrow e^{-i\phi} a_1^\dagger,
\]

\[
a_2 \rightarrow e^{-i\phi} a_2, \quad a_2^\dagger \rightarrow e^{i\phi} a_2^\dagger
\]

for arbitrary \(0 \leq \phi < 2\pi\). This symmetry is generated by the operator \(\hat{C} = a_1^\dagger a_1 - a_2^\dagger a_2\), which commutes with \(H_{\text{tm}}\). Accordingly, there exist infinitely many invariant subspaces \(\mathcal{H}_m\) labeled by the eigenvalue \(m \in \mathbb{Z}\) of \(\hat{C}\). Confinement to a single \(\mathcal{H}_m\) effectively eliminates one of the bosonic modes. Conforming to the usual terminol-
ogy \([31]\), we set \(m = 2q - 1\) and call \(q\) the Bargmann index of \(\mathcal{H}_m\). In the following we consider \(m \geq 0\) (\(m < 0\) ex-
changes modes 1 and 2), i.e. \(q\) runs over positive multiples
of \(1/2\). Besides the continuous symmetry acting only in
the bosonic subspace, \(\mathcal{H}_m\) possesses also a discrete sym-
metry \(\hat{P}\) which takes the form \(\hat{P} = \sigma_z \exp(i\pi a_2^\dagger a_2)\). We have \(\hat{P} H_{\text{tm}} \hat{P}^\dagger = H_{\text{tm}}\) and \(\hat{P}^2 = 1\). \(\hat{P}\) generates therefore a \(Z_2\)-symmetry (parity). It has eigenvalues \(\pm 1\), entai-
ing a further decay of \(H_{2q-1}\) into two invariant subspaces \(\mathcal{H}_{2q-1}\), labeled by the parity quantum number. These two symmetries together render the two-mode QRM eq. (2) integrable according to the level-labeling criterion \([4,20]\).

Bogoliubov transformations. In analogy with the two-
photon QRM \([7]\), we may perform a bosonic Bogoliubov transformation to new operators \(b_j, b_j^\dagger, j = 1, 2\), which in

the present case mixes the two modes,

\[
b_1 = u a_1 + v a_2, \quad b_1^\dagger = u a_1^\dagger + v a_2^\dagger,
\]

\[
b_2 = u a_2 + v a_1, \quad b_2^\dagger = u a_2^\dagger + v a_1^\dagger.
\]

The \(b_j\) satisfy bosonic commutation relations if \(u^2 - v^2 = 1\). Because

\[
\hat{C} \equiv a_1^\dagger a_1 - a_2^\dagger a_2 = \hat{C}_b \equiv b_1^\dagger b_1 - b_2^\dagger b_2,
\]

this transformation leaves each subspace \(\mathcal{H}_{2q-1}\) invariant. With the choice

\[
u = \sqrt{\frac{1+\beta}{2\beta}}, \quad v = \sqrt{\frac{1-\beta}{2\beta}},
\]

where \(\beta = \sqrt{1-g^2}\), the upper left block of the
Hamiltonian matrix eq. (3) becomes

\[
H_{0}^{11} = \beta \left( b_1^\dagger b_1 + b_2^\dagger b_2 + 1 \right) - 1,
\]

which no longer contains the squeezing terms.

Alternatively, we could transform to operators \(c_j, c_j^\dagger\) via

\[
c_1 = u a_1 - v a_2, \quad c_1^\dagger = u a_1^\dagger - v a_2^\dagger,
\]

\[
c_2 = u a_2 - v a_1, \quad c_2^\dagger = u a_2^\dagger - v a_1^\dagger,
\]

and \(u, v\) from eq. (6). The lower right block of eq. (3) becomes now

\[
H_{c}^{22} = \beta \left( c_1^\dagger c_1 + c_2^\dagger c_2 + 1 \right) - 1,
\]

and is free from squeezing terms.

In terms of \(b\)-operators, the Hamiltonian eq. (3) reads

\[
H_b = \begin{pmatrix}
H_{0}^{11} & -\Delta \\
-\Delta & H_{0}^{22}
\end{pmatrix},
\]

with

\[
H_{c}^{22} = \frac{1+g^2}{\beta} \left( b_1^\dagger b_1 + b_2^\dagger b_2 \right) - \frac{2\beta}{\beta} \left( b_1^\dagger b_2 + b_2^\dagger b_1 \right)
\]

\[
- \frac{1}{\beta} \left( \beta + \beta^2 - 2 \right).
\]

Only the lower diagonal block contains terms of the form
\(b_1^\dagger b_1^\dagger, b_1 b_2\).

We confine the analysis now to the space \(\mathcal{H}_{2q-1}\) and
define three sets of mutually orthogonal vectors \(\{|n\rangle\}^2\),
each forming an orthonormal basis for \(\mathcal{H}_{2q-1}\).

\[
|n\rangle^2 = \frac{(s_j)^{n+2q-1}}{\sqrt{(n+2q-1)!n!}} |0\rangle, \quad n = 0, 1, 2, \ldots
\]
for \( s = a, b, c \). \(|0\rangle_s\) denotes the normalized vacuum state [32] belonging to the operators of type \( s \), satisfying \( s_1|0\rangle_s = s_2|0\rangle_s = 0 \) in \( \mathcal{H} \). The corresponding vacuum state in \( \mathcal{H}_{2q-1} \) is \(|0\rangle \equiv \left((2q-1)! \right)^{-1/2} s_1^{2q-1} |0\rangle_s \). We call \(|0\rangle \) the two-mode extended squeezed states (ESS) and the set \{\(|n\rangle \)\} the \( s \)-basis for \( \mathcal{H}_{2q-1} \). Later we shall use the overlaps

\[
q(\langle n|)_b = (-1)^n q(\langle 0|)^2 = \left(1 - \frac{v^2}{u^2}\right)^n \sqrt{\frac{(n+2q-1)!}{n!(2q-1)!}},
\tag{12}
\]

between the vacuum of the \( a \)-basis and arbitrary states in the \( b \)-basis.

**G-functions.** We drop the index \( q \) from now on and consider the Schrödinger equation \( H_{sb}|\psi\rangle = E|\psi\rangle \). We make an ansatz for \(|\psi\rangle\) using the \( b \)-basis

\[
|\psi\rangle_b = \frac{\sum_{n=0}^{\infty} e_n \sqrt{(n+2q-1)!n!} |n\rangle_b}{\sum_{n=0}^{\infty} f_n \sqrt{(n+2q-1)!n!} |n\rangle_b}.
\tag{13}
\]

With eq. (9) we obtain the following recurrence relation for the coefficients \( e_n, f_n \):

\[
e_n = \frac{\Delta}{2\beta(n+q) - 1 - E f_n},
\tag{14}
\]

\[
\Delta e_n = \left(\frac{1 + g^2}{\beta} (n+q) - 1 - E \right) f_n - \frac{2g}{\beta} \left[ f_{n-1} + (n+1)(n+2q) f_{n+1}\right].
\tag{15}
\]

Note that eq. (14) expresses \( e_n \) in terms of \( f_n \), so that a linear three-term recurrence relation is obtained for the \( f_n \),

\[
f_{n+1} = \frac{(1 + g^2)(n+q) - \beta^2(q+x) + \frac{\beta^2}{4(1-x)}}{g(n+1)(n+2q)} f_n - \frac{f_{n-1}}{(n+1)(n+2q)},
\tag{16}
\]

where we have defined the spectral parameter \( x = (E+1)/(2\beta) - q \).

The initial conditions for the recurrence eq. (16) are \( f_1 = 0 \) and \( f_0 = r_b \), where \( r_b \) may be used to normalize \(|\psi\rangle_b\) if this is possible. Of course, \(|\psi\rangle_b\) will be only normalizable if \( E \) coincides with an eigenvalue of \( H_{sb} \).

The formal solution \(|\psi\rangle\) of \( H_{sb}|\psi\rangle = E|\psi\rangle \) can also be expanded in the \( c \)-basis as

\[
|\psi\rangle_c = \frac{\sum_{n=0}^{\infty} (-1)^n w_n \sqrt{(n+2q-1)!n!} |n\rangle_c}{\sum_{n=0}^{\infty} (-1)^n z_n \sqrt{(n+2q-1)!n!} |n\rangle_c}.
\tag{17}
\]

It turns out that the \( w_n \) and \( z_n \) satisfy the same relations eqs. (14) and (15) as \( f_n \) and \( e_n \). Thus,

\[
w_n = r f_n, \quad z_n = r e_n,
\tag{18}
\]

with some constant \( r \). Now we assume the state \(|\psi\rangle\) to be the unique eigenvector of \( H_{sb} \) belonging to energy \( E \), i.e. \(|\psi\rangle_b = |\psi\rangle_c \). It follows,

\[
\sum_{n=0}^{\infty} e_n \sqrt{(n+2q-1)!n!} |n\rangle_b = r \sum_{n=0}^{\infty} (-1)^n f_n \sqrt{(n+2q-1)!n!} |n\rangle_c,
\tag{19}
\]

\[
\sum_{n=0}^{\infty} f_n \sqrt{(n+2q-1)!n!} |n\rangle_b = r \sum_{n=0}^{\infty} (-1)^n e_n \sqrt{(n+2q-1)!n!} |n\rangle_c.
\tag{20}
\]

Projecting both sides of eqs. (19), (20) onto the vacuum of the \( a \)-basis \(|0\rangle \) yields

\[
\Gamma_c(x) = r \Gamma_f(x),
\tag{21}
\]

\[
\Gamma_f(x) = \sum_{n=0}^{\infty} f_n(x)(n+2q-1)! \left(\frac{v}{w}\right)^n,
\tag{22}
\]

\[
\Gamma_c(x) = \sum_{n=0}^{\infty} e_n(x)(n+2q-1)! \left(\frac{v}{w}\right)^n,
\tag{23}
\]

where eq. (12) has been used. Eliminating \( r \) from eq. (21) leads to \( \Gamma_f(x) \pm \Gamma_c(x) = 0 \). With eq. (14), we can define two \( G \)-functions,

\[
G_{\pm}^q(x) \equiv \Gamma_f(x) \pm \Gamma_c(x) = \sum_{n=0}^{\infty} f_n(x) \left[1 \pm \frac{\Delta}{2\beta(n-x)} \right] (n+2q-1)! \left(\frac{v}{w}\right)^n,
\tag{24}
\]

each corresponding to a subspace of \( \mathcal{H}_{2q-1} \) with fixed parity. If \( x_n \) satisfies \( G_{\pm}^q(x_n) = 0 \), we conclude that \( E_{n} = 2\beta(x_n + q) - 1 \) is a non-degenerate eigenvalue of \( H_{sb} \).

The reason for the equivalence of the recurrence relations for \{\( e_n, f_n \)\} and \{\( z_n, w_n \)\}, which made the foregoing arguments possible, is the discrete \( z_0 \)-symmetry of the two-mode QRM, just as for the single-mode, linearly coupled model [4,7]. However, while the representation in the Bargmann space provides a mathematically rigorous justification of the method in the latter case [33], the present analysis using ESS needs to be rederived by imposing the proper normalization conditions on the eigenstates of \( H_{sb} \) in the space \( \mathcal{H}_{2q-1} \), which will be the subject of future work.

**Regular spectrum.** We plot the \( G \)-functions in fig. 1 for the case of \( \Delta = 0.35 \) and \( q = 0.5 \) and 0.95 for some values of \( q \). The zeros \( x_n \) of the functions \( G_{\pm}^q(x) \) yield the eigenvalues \( E_n \) of the regular spectrum, depending on the parameters \( q, \Delta, g \). Figure 2 displays the spectrum of the two-mode QRM as a function of \( g \) for qubit splitting \( \Delta = 0.35 \). We find complete agreement with numerical diagonalization in truncated Hilbert spaces of sufficiently high dimension.

This is to be expected, because the three-term recurrence relation eq. (16) allows for minimal and dominant
that singularities as a function of known. To the contrary, the as the singularity structure of the continued fraction is not about the features of the spectrum is lost in this approach \(H(34)\). However, all qualitative information equivalent to truncate the model on a finite-dimensional.

The two-mode QRM based on continued fractions, which is \(\Delta=0\), solution. corresponds therefore almost to a non-degenerate exceptional solutions like the QRM \([4]\). One can show \([31]\) that only the unique minimal solution is normalizable and corresponds therefore to a point of the discrete spectrum if the energy is chosen such that it matches the initial conditions of \((16)\). It is therefore possible to derive a \(G\)-function for the two-mode QRM based on continued fractions, which is equivalent to truncate the model on a finite-dimensional Hilbert space \([34]\). However, all qualitative information about the features of the spectrum is lost in this approach as the singularity structure of the continued fraction is not known. To the contrary, the \(G\)-functions \((23)\) have known singularities as a function of \(E\): One sees from eq. \((16)\) that \(f_{n+1}(x)\) has a simple pole at \(x=n\) and from \((23)\) we deduce that \(G_n^\pm (x)\) has simple poles at \(x=0,1,2,\ldots\). The \(x_n^\pm\), where \(G_n^\pm (x)\) vanishes, must be located in between these poles; the number varies between zero and two, but because the poles are simple, there can be no adjacent intervals which are both devoid of zeros or contain both two zeros. This feature follows for \(\Delta \ll 1, g\) from eq. \((16)\). The zeros of \(G_n^\pm (x)\) are therefore smoothly distributed between the equidistant poles, i.e. \(x_{n+1}^\pm -x_n^\pm \approx 1\) (for fixed parity). The distance between poles of \(G_n^\pm (E) = G_n^\pm (x(q,E))\) on the energy axis depends on \(q\): If \(x_{n+1}^{\text{pole}} = n, x_n^{\text{pole}} = n+1\), we find for \(E_{n+1}^{\text{pole}} - E_n^{\text{pole}} = 2\beta = 2\sqrt{1-g^2}\). If \(g\) approaches the critical value \(g_c = 1\) from below, the distance between the poles of \(G_n^\pm (E)\) goes to zero and it follows \(E_{n+1}^\pm - E_n^\pm \approx 2\sqrt{1-g}\). This is the spectral collapse observed in previous numerical investigations of the two-photon QRM \([7,8,30,35]\), which behaves similar to the two-mode model in this respect. At the critical point \(g = 1\), the spacing between adjacent discrete energy levels vanishes and the spectrum becomes continuous —accordingly no normalizable states exist anymore. Because there cannot be adjacent empty intervals between poles of \(G_n^\pm (E)\), the ground state at \(g_c\) is not separated from the continuous spectrum by a finite gap. It is also clear that the density of states goes to a constant for \(g \to g_c\). The number of bosonic excitations in low-lying states grows very fast in approaching the critical coupling \([30,35]\), which makes numerical diagonalization more and more difficult. In our approach, the qualitative properties of the collapse can be deduced very easily by a simple analysis of the singularity structure of the \(G\)-functions.

**Exceptional spectrum.** Koc et al. \([36]\) have obtained isolated and doubly degenerate solutions for the QRM, the quasi-exact Juddian solutions \([37]\). We have explicitly excluded degenerate eigenstates in the derivation of the \(G\)-functions pertaining to the regular spectrum. It follows that the Juddian solutions belong to the exceptional spectrum. It can be analyzed along precisely the same lines as for the QRM \([4]\). For special values of the model parameters \(g\) and \(\Delta\), there are eigenvalues which do not
correspond to zeros of $G^g_\pm(x)$; the corresponding energy has the value $E^{exc} = 2\beta(n+q) - 1$. A sufficient condition for the occurrence of this eigenvalue is

$$f_n^g(x_n) = 0,$$  \hspace{1cm} (24)

which provides a condition on the model parameters $g$ and $\Delta$. They occur when the pole of $G^g_\pm(x)$ at $x = n$ is lifted because the corresponding numerator in eq. (23) vanishes. The conditions for all $n$ can be obtained from the recurrence eq. (16). For example, the conditions for $n = 1$, respectively $n = 2$ read,

$$ (2q + 1)g^2 + \Delta^2 - 1 = 0, \hspace{1cm} (25) $$

$$ \left(4qg^2 - 6\beta^2 + 4 + \frac{\Delta^2}{2}\right) = \left(4q - 4\beta^2 (1 + q) + \frac{\Delta^2}{4}\right) - 8g^2q = 0. \hspace{1cm} (26) $$

All exceptional solutions corresponding to condition (24) are doubly degenerate, because the pole is lifted both in $G^g_+(x)$ and $G^g_-(x)$, they correspond therefore to crossing points between states with different parity in the spectrum. As shown in fig. 2, the first crossing point at $g = 0.696$ coincides with the value obtained from eq. (25) for $\Delta = 0.35$, $q = 1/2$.

These doubly degenerate states are quasi-exact, i.e. they can also be obtained by a polynomial ansatz for the wave function [31]. However, not all exceptional eigenvalues are obtained as solutions of (24). Under certain conditions, the pole may be lifted only in $G^g_+(x)$ and not in $G^g_-(x)$ (or vice versa). These eigenvalues are therefore not doubly degenerate but belong nevertheless to the exceptional spectrum; they occur here as in the QRM [38]. One example is shown in fig. 1 for $q = 3/2$ and $g = 0.95$.

Conclusions. – We have derived $G$-functions for the two-mode QRM by using ESS for each Bargmann and parity index. The zeros of the $G$-functions $G^g_\pm(x)$ determine the regular spectra. Doubly degenerate eigenvalues are given by the conditions for lifting a pole in $G^g_\pm(x)$ at integer values of $x$, which can be given in closed form. Moreover, the known pole structure of $G^g_\pm(x)$ allows us to derive the shrinking average level spacing as one approaches the critical point $g = 1$ (spectral collapse) analytically, whereas any numerical approach is bound to fail in the region close to the critical point due to the presence of highly excited states even at low energy.

Our result adds the two-mode QRM to a list of models having a $G$-function with general structure

$$ G^g_\pm(x) = \sum_{n=0}^{\infty} f_n(x) \left[ 1 \pm \frac{\Delta}{F(g)(n-x)} \right] L_n(g), \hspace{1cm} (27) $$

where $f_n(x)$ is determined recursively. In the QRM [4],

$$ F(g) = 1, $$

$$ L_n(g) = g^n, $$

$$ E = x - g^2. $$

and $f_n(x)$ is determined by eq. (5) in [4]. For the present two-mode QRM we have

$$ F(g) = 2\sqrt{1 - g^2}, $$

$$ L_n(g) = (n + 2q - 1)! \left(\frac{v}{u}\right)^n, $$

$$ E = 2\sqrt{1 - g^2}(x + q) - 1. $$

and $f_n(x)$ is determined by eq. (16) for each $q$. The two-photon QRM [7] fits into this general scheme as well

$$ F(g) = 2\sqrt{1 - 4g^2}, $$

$$ L_n(g) = \left[2(n + q - \frac{1}{2})\right]! \left(\frac{v}{u}\right)^n, $$

$$ E = 2\sqrt{1 - 4g^2}(x + q) - \frac{1}{2}, $$

where the Bargmann index $q = \frac{1}{4}, \frac{3}{4}$, and $f_n(x)$ is determined by eq. (41) in [7]. In terms of $q$, $f_n(x)$ reads

$$ f_n^{(q)}(x) = \left(1 + 4q^2\right)(n + q) - \beta^2(x + q) - \frac{\Delta^2}{4(n-2)} f_n^{(q)} $$

$$ f_n^{(q)}(x) = \frac{(n + q + \frac{1}{4})}{4(n + q + \frac{3}{4})} \left(n + q + \frac{3}{4}\right). \hspace{1cm} (28) $$

The list may be expanded by other related models in the future. All qubit-cavity models possessing a compact $G$-function of the type (27) share two common properties: Isolated doubly degenerate eigenstates with energy $x(E) = n$ and regular spectra given by zeros of a $G$-function which is a linear combination of two formal solutions. Both properties have their root in the discrete $\mathbb{Z}_2$-symmetry present in all models belonging to this class, rendering them integrable.

** * **

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