Universal dissipation scaling for non-equilibrium turbulence

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It is experimentally shown that the non-classical high Reynolds number energy dissipation behaviour, \( C_e \equiv \varepsilon L / u^3 = f(Re_M) / Re_L \), observed during the decay of fractal square grid-generated turbulence is also manifested in decaying turbulence originating from various regular grids. For sufficiently high values of the global Reynolds numbers \( Re_M \), \( f(Re_M) \sim Re_M \).

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In recent papers describing the wind tunnel turbulence generated by fractal square grids [1, 2] it was shown that the turbulent kinetic energy dissipation rate, \( \varepsilon \), at moderately high Reynolds numbers does not follow the expected scaling \( \varepsilon L / u^3 \approx C_e \approx \text{const} \) (where \( L \) is the longitudinal integral length-scale and \( u \) the streamwise r.m.s. velocity). Instead [1, 2] found that \( C_e = f(Re_M) / Re_L \) during the turbulence decay where \( f(Re_M) \) is an increasing function of \( Re_M = U_\infty M / \nu \), a global Reynolds number based on a length-scale \( M \) characteristic of the grid, and where \( Re_L = uL/\nu \) is a local, downstream position dependent, Reynolds number (\( \nu \) is the kinematic viscosity and \( U_\infty \) the inflow velocity). This behaviour is accompanied by a well-defined power-law energy spectrum (with exponent close to Kolmogorov’s -5/3) over a broad range of length-scales and is therefore caused by a physically different underlying phenomenon than the well-known low Reynolds number law \( C_e \sim Re_L^{-1} \).

Evidence of such a non-classical behaviour is significant due to the central role the empirical law \( C_e \approx \text{const} \) has on most, if not all, models and theories of both homogeneous and inhomogeneous turbulence [3–6]. Clearly, one should expect the existing models to inadequately describe turbulent flows (or regions thereof) not obeying the \( C_e \approx \text{const} \) scaling and consequently fail in their predictions of transport phenomena (energy transfer, dissipation, particle dispersion, scalar diffusion, etc...). Most importantly, it challenges our understanding of turbulence phenomena in general, nevertheless providing a starting point for its study as well.

In this Letter we report results which show that this non-classical behaviour is in fact more general than previously thought and is not exceptional to the very special class of inflow conditions defined by fractal square grids. Hence this non-classical behaviour is of general scientific and engineering significance and therefore of much greater importance.

In the present experiments we compare turbulence generated by three different regular square-mesh grids (RG230, RG115 and RG60) with the turbulence generated by the fractal square grid (FSG) of [1] (see Fig. 1 and table I). Our aim is to investigate the origin for the non-classical dissipation behaviour of the FSGs. The dimensions of RG230 are purposefully similar to those of the largest square on the FSG. This allows a ceteris paribus comparison between RG230 and FSG in two respects: (i) comparable inflow Reynolds numbers \( Re_M \) for similar inflow velocities if \( M \) is taken to be the side-length of the largest square on the grid (see Fig. 1) and (ii) comparable distance from the grid where the wakes of the RG230 bars meet and where the wakes of the FSG largest bars meet. Starting from any one of our grids, the turbulent kinetic energy increases as one moves downstream along the tunnel’s centreline and reaches a peak at a streamwise distance \( x_{\text{peak}} \) from the grid beyond which the turbulence decays [1, 2, 7]. This distance \( x_{\text{peak}} \) is closely related to the distance from the grid where the wakes (largest wakes in the case of FSG) meet. Indeed, [2] introduced the wake interaction length-scale \( x_\ast = M^2 / t_0 \) where \( t_0 \) is the lateral thickness of the largest bars (see Fig. 1) and showed that \( x_{\text{peak}} \) scales with \( x_\ast \) in the case of FSGs. Subsequently, [1] showed that \( x_{\text{peak}} / x_\ast \) took comparable values for RGs and SFGs, a point which the experiments reported in this Letter allow us to confirm (see table I). The length-scales \( x_{\text{peak}} \) and \( x_\ast \) turn out to be paramount for a meaningful comparison between grids.

There are of course important differences between the four grids used here, for example different values of blockage ratio \( \sigma \) (ratio between the blocking area of the grid and the area of the tunnel’s test section) and different values of \( x_\ast \) (see table I). These differences cause differences in various mean flow and turbulence profiles across the tunnel section. However, they have no bearing on our main finding that the outstanding behaviour previously found in FSG-generated turbulence is also present in turbulence generated by regular grids for a region whose extent is determined by \( x_\ast \). Beyond this region, in the one case (RG60) where we can reach sufficiently far beyond it as a result of the wind tunnel’s test section being much longer than \( x_\ast \), we find the classical behaviour \( C_e \approx \text{const} \) provided the Reynolds number is sufficiently high.

The experimental apparatus described in [1] was repeated for the present experiments with the length of the 0.46m x 0.46m-wide test section shortened from \( \approx 4.5m \) to \( \approx 3.5m \) to match the extent of the longitudinal traverse mechanism. We also installed a grid at the entrance of the diffuser to maintain a slight overpressure across
TABLE I. Details of turbulence-generating grids; d is the longitudinal thickness of the bars.

| Grid     | M     | t₀   | d   | σ   | xₚ/ xₚ | xₚ/ xₚ |
|----------|-------|------|-----|-----|--------|--------|
| RG230 mono-planar | 230   | 20   | 6   | 17  | 2.65   | 0.63   |
| RG115 mono-planar | 115   | 10   | 3.2 | 17  | 1.32   | 0.63   |
| RG60 bi-planar     | 60    | 10   | 10  | 32  | 0.36   | ≈ 0.4² |
| FSG mono-planar    | 237.7 | 19.2 | 5   | 25  | 2.94   | 0.45   |

* Taken from measurements of a very similar grid.

The test section. All data are recorded with one- and two-component hot-wire anemometers operated at constant temperature. The main data are recorded with two in-house etched Pt-(10%)Rh single-wire (SW) sensors, SW1 and SW2, having sensing lengths of lₚ = 0.5mm and lₚ = 0.2mm and wire diameters of dₚ = 2.5µm and dₚ = 1µm, respectively. A Dantec 55P51 cross-wire (XW) with lₚ = 1.0mm and dₚ = 5µm is also used to record basic isotropy statistics. The spatial resolution of the measurements, quantified by lₚ/η (η ≡ (ν²/ε)¹/₄ is the Kolmogorov microscale; the isotropic estimate of dissipation ε = 15ν(du/dx)² is used), is given in table II for the furthermost up- and downstream locations and for the different inflow velocities. We repeated the electronic tests to confirm that the maximum unattenuated frequency response of the SWs was at least kη = 1 (k is the wavenumber). The data acquisition and processing methodologies are also similar to those described in [1].

An exception is that we use, for simplicity, the classical Taylor’s frozen field hypothesis to convert temporal into spatially varying signals, although we checked that this does not meaningfully affect the results.

This Letter’s new data are recorded along the centre-line in the lee of each of our four grids (Fig. 1 and tables I and II). Data recorded between a grid and its corresponding xₚ are excluded (see caption of table II) as we confine our study to decaying turbulence. In these decay regions, u/v (where v is the r.m.s. lateral velocity) is typically between 1.2 and 1.1 and the ratio of the mean square of the lateral turbulence velocity derivative with respect to the streamwise coordinate x to the mean square of the streamwise turbulence velocity derivative with respect to x takes values between 1.4 and 1.6. Both ratios vary by less than 5% along the streamwise extent of our records. Note that xₚ is about as long as half the wind tunnel’s extent in the cases of RG230 and FSG (see table I). The RG60 was investigated in [1] where it was shown that for sufficiently high inflow velocities the dissipation followed a convincing C₂ ≈ const during decay far downstream. We repeat those measurements using a higher resolution sensor (SW2) and include recordings much closer to the grid (table II).

First, we compare the dissipation scalings of the decaying turbulence originating from RG230 and FSG. The

TABLE II. Overview of the experimental results. xₘ₄ & xₘ₅ are the first and last measurement locations corresponding to 0.48xₚ & 1.09xₚ, 0.64xₚ & 1.19xₚ, 0.61xₚ & 2.38xₚ, and 0.72xₚ & 8.75xₚ for FSG, RG230, RG115 and RG60, respectively. Probe SW1 is used for the measurements of the first two grids and SW2 for the last two.

| Grid     | Symbol | U∞    | Reₘ₄ | u/U∞(%) | Reₜ₅ | lₚ/η |
|----------|--------|-------|------|---------|------|------|
| FSG      | ★      | 15.0  | 237  | 9.7/5.0 | 385/249 | 4.8/3.0 |
|          | ⊙      | 17.5  | 277  | 4.8/3.0 | 418/275 | 5.5/3.5 |
| RG230    | ●      | 5.0   | 77   | 180/140 | 1.8/1.3 |
|          | □      | 10.0  | 153  | 261/200 | 2.9/2.2 |
|          | ◇      | 17.5  | 268  | 348/281 | 4.4/3.3 |
|          | ★      | 20.0  | 307  | 385/300 | 4.9/3.7 |
| RG115    | ●      | 20.0  | 153  | 6.9/2.7 | 255/160 | 2.3/1.1 |
|          | □      | 10.0  | 40   | 177/96  | 2.8/0.6 |
| RG60     | ▲      | 15.0  | 60   | 15/2.2  | 240/111 | 3.8/0.8 |
|          | △      | 20.0  | 80   | 290/135 | 4.7/1.0 |

FIG. 1. Turbulence generating grids. From left to right: FSG [1], RG230, RG115 and RG60.

FIG. 2. Longitudinal energy density spectra F₁₁ per wavenumber k of turbulence generated by RG230 for (black) U∞ = 20ms⁻¹, x/xₚ = 0.64, (dark grey) U∞ = 10ms⁻¹, x/xₚ = 0.64 and (light grey) U∞ = 5ms⁻¹, x/xₚ = 1.19.
Reynolds numbers $Re_\lambda \equiv u\lambda/\nu$ (where $\lambda$ is the Taylor microscale) at our measurement stations are given in table 1 and are all large enough for a significant separation to exist between the large, energy containing, eddies and the smallest dissipative eddies. Indeed, the scale separation at the highest Reynolds number is $L/\eta \approx 460$. The measured one-dimensional longitudinal energy spectra $F_{11}$ exhibit clear power-laws over more than a decade with an exponent close to Kolmogorov’s $-5/3$, at least for $Re_\lambda \gtrsim 2.3 \times 10^5$ and $Re_\lambda \gtrsim 250$ (see Fig. 2 where we only plot RG230 spectra for brevity and clarity: FSG spectra can be found in [1]). However, both for RG230 and SFG, the cornerstone assumption of turbulence theory, $C_\varepsilon \approx \text{const}$, does not hold in this region where the turbulence decays (between about 1.3m from the grid and the end of the test section) at these Reynolds numbers (see Fig. 3). Instead, for any fixed $Re_M$, $C_\varepsilon \sim Re_L^{-1}$ (as one moves along $x$) is a good qualitative approximation (in Fig. 3 each set of symbols corresponds to one $Re_M$ and one grid, see table 1 $Re_L$ decreases as $x$ increases). At the furthest downstream locations which correspond to the lowest $Re_L$ values for each $Re_M$ in Fig. 3 there is a slight departure from $C_\varepsilon \sim Re_L^{-1}$, probably due to far downstream test section confinement effects discussed in [1]. (In our records, $L$ reaches a maximum value smaller than $M/4$ at $x_{\text{max}}$ for all grids.) Note that the well-known relation $\varepsilon = 15\nu u^3/\lambda^2$ (e.g. [1]) and the definition of $C_\varepsilon$ imply $15(L/\lambda)^2 = C_\varepsilon Re_L$ and $15L/\lambda = C_\varepsilon Re_\lambda$ which means that $C_\varepsilon \sim Re_L^{-1}$ is equivalent to $C_\varepsilon \sim Re_\lambda^{-1}$ and that such $C_\varepsilon$ behaviour implies $L/\lambda \approx \text{const}$ during decay.

When, instead of keeping $Re_M$ fixed and varying $x$, we keep $x$ fixed and vary $Re_M$, we then find a very different dependence of $C_\varepsilon$ on Reynolds number; asymptotically independent of it for both RG230 and FSG as $Re_M$ increases. If we keep with the usual expectation that $C_\varepsilon$ is independent of $\nu$ at high enough $Re_M$ (which may be close to, but not exactly, true, see [3]), then these two different dependencies on Reynolds number can be reconciled by:

$$C_\varepsilon \propto \frac{Re_M}{Re_L} \propto \frac{Re_\lambda^{1/2}}{Re_\lambda}$$  \hspace{1cm} (1)

because $u/U_\infty$ and $L/M$ are independent of $Re_M$ to leading order at high enough Reynolds numbers. Note that $C_\varepsilon \propto Re_M/Re_L$ is equivalent to $L/\lambda \sim Re_\lambda^{1/2}$ and therefore to $C_\varepsilon \propto Re_\lambda^{1/2}/Re_\lambda$. This equation is fairly well supported by our data both for FSG and RG230 at $Re_\lambda \gtrsim 2.3 \times 10^5$ (Fig. 4) but with a grid-dependent constant of proportionality in [1].

Equation (1) may appear to clash with the fact that $C_\varepsilon$ is approximately independent of both $x$ and $Re_\lambda$ in the case of RG60 at distances greater than about 1.5m from that grid in a wind tunnel test section of exact same width as the present one (see Fig. 7 in [1]). This is a distance greater than about $4x_\ast$ from the grid because $x_\ast \approx 0.36m$ for RG60. However, (1) has so far been established for decaying turbulence originating from RG230 and FSG up to downstream distances of less than about $1.5x_\ast$ ($x_\ast$ takes much greater values for these grids, see table 1). It is therefore reasonable to investigate whether (1) and its equivalent relation $L/\lambda \sim Re_\lambda^{1/2}$ hold at distances below a few multiples of $x_\ast$ from the RG60 grid. In Fig. 5 we plot $L/\lambda$ as a function of the local Reynolds number $Re_\lambda$ for RG60 at different levels of $Re_M$. We find that $L/\lambda \approx \text{const}$ in the region between 0.72$x_\ast$ and 2$x_\ast$ (where $Re_\lambda$ takes the largest values) and that $L/\lambda$ and $Re_\lambda$ decay in exact proportion to each other (i.e. $L/\lambda \sim Re_\lambda$ which is equivalent to $C_\varepsilon = \text{const}$) at further
wise distances larger than 2 and $x$ than 50,

$$\frac{L}{\lambda} = \sqrt{f(Re_M)/15}$$

in the ranges of $x$ probed. However, $Re_M$ is too low for (1) to hold.

The present data and those of [1, 2] conspire to form the conclusion that, irrespective of the turbulence generating grid (Fig. 1) and for high enough $Re_M$,

$$\varepsilon \approx C_1 \frac{u^2 M}{L}$$

(2)

and equivalently $L/\lambda = \sqrt{C_1 Re_M/15}$ are acceptable approximations in the non-equilibrium decay region $x_{peak} < x < x_c$ where $x_c \approx 2x_*$ for RG60 and $C_1$ is a dimensionless constant which only depends on inlet/boundary geometry (type of fractal/regular grid, $\sigma$, etc.). We might expect $x_c$ to scale with $x_*$ for other grids as well, and the equilibrium dissipation scaling $\varepsilon \approx C_2 u^3/L$ (where $C_2$ is an inlet/boundary geometry-dependent dimensionless constant, see [8, 9]) to be recovered at $x > x_c$ for other grids too. However, our RG115, RG230 and FSG data and those of [1, 2] do not allow us to test these expectations, nor do they allow us to explore how $x_c/x_*$ may depend on inlet/boundary conditions. RG230 and FSG, in particular, act as magnifying lenses which make the non-equilibrium region to be longer than the entire tunnel section’s length. Equations (1) and (2), and more generally $C_\varepsilon = f(Re_M)/Re_L$ which also covers lower values of $Re_M$, are approximately true in the non-equilibrium region irrespective of flow/turbulence profile details which differ from grid to grid. The FSGs are magnifying lenses with added capabilities for tailoring flow and turbulence profiles which go beyond variations in $\sigma$.

Finally, it is important to stress that the energy spectrum has a well-defined power-law shape over nearly two decades with exponent close to -5/3 at the closest point to the grid that we sampled in the non-equilibrium region (Fig. 2). This power-law region becomes progressively narrower with an exponent progressively further away from -5/3 as $x$ increases. In the equilibrium region of RG60 where $\varepsilon \sim u^3/L$, the energy spectrum is far from Kolmogorov-shaped. This may just be a consequence of the low Reynolds numbers in the equilibrium region of our RG60 runs. But it is remarkable that a near-Kolmogorov power-law shaped energy spectrum does in fact appear well before the turbulence has had the time to reach equilibrium. A similar observation was made in [10] where near-Kolmogorov power-law energy spectra were reported in a cylinder wake within one cylinder diameter from the cylinder.

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