On the Quantum Aspects of Geophysics

F. Darabi*
Department of Physics, Azarbaijan University of Tarbiat Moallem, 53714-161, Tabriz, Iran.

Abstract

We introduce a simple quantum mechanical justification for the formation of folded mountains. It is very appealing to develop this idea to a theory of Quantum Geophysics.

PACS: 03.65.Ca, 91.45.Dh.

Quantum mechanics is the basis of our knowledge about the microscopic world. The basic element in this theory is the wave function $\Psi(\vec{r}, t)$ which, in principle, is assumed to include all the physical information about a quantum mechanical system, and its dynamics is governed by the Schrödinger equation. However, the domain to which one may apply the rules of quantum mechanics is not limited to the microscopic world. Quantum cosmology is an outstanding indication of this claim. In this theory, the whole universe, as a single system, is described by a wave function $\Psi(h_{ij}, \phi)$ which is believed to incorporate all possible three geometries and matter field configurations. This wave function satisfies a constraint called Wheeler-DeWitt equation which plays the role of Schrödinger equation. One of the major tasks in quantum cosmology is to look for a good correspondence between the quantum cosmological predictions and the classical solutions for the geometry and matter field configurations. This is called the classical limit of quantum cosmology. In both quantum mechanics and quantum cosmology the quantity $|\Psi|^2$ plays a key important role in the sense that it provides us with the probability of finding a definite configuration. Especially, in the quantum cosmology $|\Psi|^2$ may be used to investigate the correspondence between the solutions of Wheeler-DeWitt equation, in one hand, and the classical paths, on the other hand. One is then tempted to try to test the applicability of quantum mechanics to other macroscopic systems. In this regard, one may pay attention to the Earth, as a single system, and consider the formation of mountains on its crust.

Plate tectonics is a relatively new theory that has revolutionized the way geologists think about the Earth. Since the mid-1960s, the plate tectonic theory has been rigorously and successfully tested. According to this theory, the surface of the Earth is broken into a series of relatively thin, but large plates. The size and position of these plates change over time. The fastest plates e.g. Pacific Plates, move at over 10cm a year. The African plate moves about 25 mm per year, whereas the Australian plate moves about 60 mm per year. The edges of these plates, where they move against each other, are sites of intense geologic activities, such

*e-mail: f.darabi@azaruniv.edu
as earthquakes, volcanoes, and mountain building. Folded mountains are the most common type of mountain. They are created by tectonic plates pushing against each other. This creates intense pressure. Therefore, the only direction for these mountains to move is up. The formation of folded mountains on Earth’s surface can be compared to the process of wrinkling a flexible rug on a floor. As horizontal pressure is applied along the edge of a rug, folds appear perpendicular to the direction of pressure. On Earth, as horizontal pressure is applied to rocks, ridges and valleys form perpendicular to the direction of the pressure.

According to quantum theory, every mass $m$, with momentum $p$ has a wavelength $\lambda = \frac{h}{p}$, where $h$ is the planck constant. In quantum cosmology the universe is simply assumed to be a particle in a definite potential and one usually takes the units so that $h = 1$. This is because people are usually interested in quantum cosmology to describe the birth of the universe and find the correct boundary conditions to investigate properly the singularity problems in general relativity, and the value of $h$ is not of prime importance. In other words, to the extent we are dealing with the characteristic behavior of the wave function of the universe, namely $|\Psi|^2$, we may ignore $h$.

It is therefore appealing to assume a relation $\lambda \sim p^{-1}$ to describe the formation of folded mountains on the basis of wave-particle interpretation of quantum mechanics. Suppose a particle with total energy $E = mgy_0$ is subjected to the following one-dimensional gravitational potential

$$V(x) = \begin{cases} \infty & x \leq 0 \\ ax & x > 0 \end{cases}$$

where $a = \frac{E}{L}$ is a constant and $L$ is a length over which the particle is limited to its classical motion. Without solving the Schrödinger equation one may obtain the characteristic features of the particle’s wave function. At $x = 0$, the wave function is zero due to the infinite potential. At $x \approx 0$, the wavelength is small due to the high velocity of the particle and the amplitude is small, as well, due to the fact that the probability of observing this fast particle at $x \approx 0$ is small. As $x$ becomes larger the wavelength and the amplitude of the particle’s wave gets larger, as well, due to the fact that the velocity of the particle decreases with $x$. At $x = L$, the wave has its maximum wavelength and amplitude, because the region $x > L$ is classically forbidden for the particle, so the associated wave will lose the oscillating behavior and the amplitude starts decreasing. If we now evaluate $\Psi(x)$ and plot $|\Psi(x)|^2$ in the interval $0 \leq x \leq L$, as in Fig.1, we then surprisingly obtain almost the same general pattern of a folded mountain as viewed from its transverse cross section, starting from its flat point and ending at the top. We believe this similarity is not a coincidence. One may think that when two plates collide each other the velocity of their front boundaries decreases considerably whereas the whole plates keep their motion, as before. Therefore, as we move from inside the plates towards colliding boundaries the velocity of interior boundary layers decreases smoothly down to the velocity of the front boundaries. We now associate a wavelength to each layer which may be interpreted as a particle with varying velocity in the above potential. The wavelength of the interior boundary layers will be shorter and that of front boundaries will be longer due to the faster and slower motions of these layers, respectively. On the other hand, the amplitude $|\Psi|$ for the interior boundary layers are smaller and that of front boundaries are larger due to the same reason. We may now interpret the whole interior part of each plate as an infinite
potential and the boundary layers as a particle, with varying velocity, obeying the above linear potential. Each front boundary layer then represents a particle with total energy $E = mg y_0$ at a distance $L$ from the infinite potential. The associated wave of this layer will have the longest wavelength and the biggest amplitude; see the last wavelength at the right side in Fig.1, for a typical value $L = 200$. Combining these behaviors of two colliding front boundaries and taking into account $|\Psi|^2$ leads to a pattern which resembles the central wide and top portion of the folded mountain, see the central portion in Fig.2.

On the contrary, the last interior boundary layer of each plate represents a particle with total energy $E = \frac{p^2}{2m}$ approaching the infinite potential. The associated wave of these layers will have the shortest wavelength and vanishing amplitude; see the first wavelength at the left side in Fig.1. One may correspond this behavior, by taking $|\Psi|^2$, to the flat sides of the folded mountain, see Fig.2. In general, if we associate these wave properties, to each two-dimensional plate, as $\Psi_1$ and $\Psi_2$, then the folded mountains will be nothing but the manifestation of a wave interference phenomena, namely $|\Psi_1 + \Psi_2|^2$, which resembles the general pattern of these mountains. In this way, one may correspond $|\Psi|^2$ to the height of the mountain at a given point, provided we interpret $|\Psi|^2$ as the probability of finding the geologic matter at that point. The top point of a mountain is then a point for which $|\Psi|^2$ is maximum, namely, the probability of matter accumulation at this point is maximum.

In this regard, one may assume that all types of mountains, hills and valleys are produced due to the very complicate wave interference phenomena originating from the interior dynamics of hot mantle. This is an interesting realization of wave-particle duality at this large scale phenomena.

It seems, as in quantum mechanics (or quantum cosmology), that one may go beyond and look for a Schrödinger-like equation, in Geophysics, whose solution in principle may have complete information about the formation of all types of mountains, hills and valleys. If we define the state ket $|\alpha>$ to represent the whole crust of the Earth, one may then write

$$<\alpha|\alpha> = \int d^2X |<X|\alpha>|^2 = 1,$$

where $|<X|\alpha>|^2$ determines the shape of the crust at position $X$, on the two dimensional surface of the Earth.

References

[1] J. M. Bird, Plate tectonics, Washington. DC, American Geophysical Union, 986 (1980).
Figure captions

Fig. 1. $|\Psi|^2$ for a particle in the potential $V(x), 0 \leq x \leq L$.

Fig. 2. $|\Psi|^2$ for two particles in the piecewise potentials $V(x), 0 \leq x \leq L$ and $V(2L - x), L \leq x \leq 2L$. 
This figure "1.jpg" is available in "jpg" format from:

http://arxiv.org/ps/physics/0410060v1
This figure "2.jpg" is available in "jpg" format from:

http://arxiv.org/ps/physics/0410060v1