Dynamic of HNLS Solitons using Compact Split Step Padé Scheme

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Abstract: In this communication, we use a compact split step Padé scheme (CSSPS) to solve
the scalar higher-order nonlinear Schrödinger equation (HNLS) with higher-order linear and
nonlinear effects. The second part, consisting of two sections, the first section is dedicated to
the study numerically the stabilization of high-order solitons dynamic in optical fibers by
compensation or by the interplay of higher order nonlinearity - especially quintic nonlinearity-
and the self-steepening. In the second section we study also numerically the propagation of
conventional chirped or unchirped solitons in optical fibers with to the management of the non-
linearity, dispersion and loss (gain).

Keywords: higher order optical solitons, compact split step Padé scheme, higher-order
nonlinear Schrödinger equation (HNLS), quintic nonlinearity, dispersion managed.

1. Introduction:
Propagation of solitons in optical fibers has attracted a great attention of many authors [1-3]. The
balance between nonlinear Kerr effect and chromatic dispersion is the clue of the stability of optical
NLS solitons [2]. The propagation of ultra-short and ultra-intense optical solitons in optical fibers is
not so obvious and requires efficient and fast numerical methods in order to be investigated. At high
power higher order nonlinearities may alter the propagation of the solution especially with the
interplay of higher order dispersion effects such as third or fourth ordered dispersion. Quintic
nonlinearity [4] is the most important nonlinear effect due to the saturation of optical field [5] and
must be taken into account in the study of ultra-intense optical solitons. In this communication we use
a compact split step Padé scheme (CSSPS) [6] to solve the higher-order nonlinear Schrödinger
(HNLS) equation with power law nonlinearity [7,8] and higher order dispersion effects, in order to
study numerically the impact of the combined nonlinear effects, such self-steepening and quintic
nonlinearity on the propagation of the higher-order soliton in optical fibers, and the evolution of
conventional unchirped or chirped solitons in optical fibers with to the management of the non-
linearity, dispersion and loss (gain).
2. Theory:
The higher-order nonlinear Schrödinger HNLS equation with power law nonlinearity and higher order dispersion:
\[
\frac{\partial A(Z, T)}{\partial Z} = -\frac{\alpha}{2} A + i \sum_{n=2}^{N} \frac{i^n}{n!} \beta_n \frac{\partial^n A}{\partial T^n} + i \gamma \left( |A|^2 A + y |A|^2mA + \frac{i}{\omega_0} \frac{\partial}{\partial T} (|A|^2 A) - T_n A \frac{\partial |A|^2}{\partial T} \right) \tag{1}
\]
Where \( i = \sqrt{-1} \) and \( A(Z, T) \) is the slowly varying envelope of the optical pulse and \( T = t_{lab} - Z/v_g \) the temporal coordinate in retarded frame that moves at the group velocity \( v_g \) of the pulse and \( Z \) the spatial coordinate representing the distance of transmission fibers.

We first normalize equation (1) in the following way:

3. Numerical scheme
We first normalize equation (1) in the following way:
\[
\begin{align*}
t &= \frac{T}{T_0}, \\
Z &= \frac{Z}{L_D}, \\
L_D &= \frac{T_0^2}{\beta_2}, \\
L_D' &= \frac{T_0^3}{\beta_3}, \\
L_D'' &= \frac{T_0^4}{\beta_4}, \\
L_{NL} &= \frac{1}{\gamma P_0}, \\
\kappa &= \frac{1}{\gamma P_0^{m-1}}, \\
S &= \frac{1}{\omega_0 T_0}, \\
\tau_R &= \frac{T_R}{T_0}
\end{align*}
\]
Where \( T_0 \) is the initial pulse width, \( L_D, L_D', L_D'' \) and \( L_{NL} \) are respectively the dispersion lengths for the different orders and the nonlinear length. The parameters \( s \) and \( \tau_R \) govern the effects of self-steeping and Raman scattering respectively. Both of these effects are quite small for picoseconds pulses but must be considered for ultra-short pulses with \( T_0 < 0.1 \text{ps} \) [1]. In our simulation, we suppose the parameter (\( \alpha = 0 \)) for lossless fibers. We choose to rewrite the equation (1) in the form (2) because it is more convenient for the numerical calculations.

\[
\frac{\partial u}{\partial z} = i \alpha_2 \frac{\partial^2 u}{\partial t^2} + \alpha_3 \frac{\partial^3 u}{\partial t^3} + i \alpha_4 \frac{\partial^4 u}{\partial t^4} + i \left[ \alpha_5 |u|^2 u + \alpha_6 |u|^{2m}u - i \alpha_7 \frac{\partial}{\partial t} (|u|^2 u) + \alpha_8 u \frac{\partial |u|^2}{\partial t} \right]
\]
\[
\frac{\partial u}{\partial z} = \mathcal{L} u + \mathcal{N} u \tag{2}
\]

Where:
\[
\begin{align*}
\alpha_2 &= -\frac{\text{sgn}(\beta_2)}{2}, \\
\alpha_3 &= \frac{\text{sgn}(\beta_3)}{6} \frac{L_D}{L_D'}, \\
\alpha_4 &= \frac{\text{sgn}(\beta_4)}{24} \frac{L_D}{L_D'}, \\
\alpha_5 &= \frac{L_D}{L_{NL}}, \\
\alpha_6 &= \frac{L_D}{L_{NL}}, \\
\alpha_7 &= -\frac{s L_D}{L_{NL}}, \\
\alpha_8 &= -\frac{s \tau_R L_{NL}}{L_{NL}}
\end{align*}
\]

This equation (2) can be written formally in the form:

\[
\frac{\partial u}{\partial z} = (\mathcal{L} + \mathcal{N}) u \tag{3}
\]

Where \( \mathcal{L} \) is a linear operator that accounts for all the linear effects, and \( \mathcal{N} \) is a nonlinear operator that governs the effect of all the fiber nonlinearities.

\[
\mathcal{L} u = \left( i \alpha_2 \frac{\partial^2 u}{\partial t^2} + \alpha_3 \frac{\partial^3 u}{\partial t^3} + i \alpha_4 \frac{\partial^4 u}{\partial t^4} \right) u
\]

\[
\mathcal{N} u = i \alpha_5 |u|^2 u + i \alpha_6 |u|^{2m}u + \alpha_7 \frac{\partial}{\partial t} (|u|^2 u) + i \alpha_8 u \frac{\partial |u|^2}{\partial t}
\]

The exact solution is approximated by solving for the half step size \( \Delta z / 2 \) separately and alternatively the purely linear equation.
And purely nonlinear equation

$$\frac{\partial u}{\partial z} = \hat{L} u \quad (3 - c)$$

The solution of one sub problem is employed as an initial condition for the next sub problem. The validity of this approximation is discussed in reference [1] using Baker–Hausdorff formula. The linear equation is solved using the compact Padé scheme algorithm an implicit scheme that is unconditionally stable and well adapted for the different derivative orders for more detail see for example [6]. Whereas the nonlinear equation is solved using a fourth order Runge-Kutta scheme (RK4) that satisfies the CFL condition.

4. Simulation and discussion

4.1. Study of the combined effects self-steepening and quintic nonlinearity on the propagation of high order soliton in optical fiber.

The remarkable effect of the self-steepening on the propagation of higher order-solitons in optical fibers is the breakup of such solitons into their constituents, a phenomenon referred to as soliton fission [9]. In the first case we study numerically by means of (CSSPS) method the impact of the self-steepening governed by the parameter $\alpha_7$ in the equation (2). In the second case we study the impact of combined effects the self-steepening and the quintic effect governed by the parameter $\alpha_6$ in the equation (2) on the propagation of higher order-solitons. Pulse evolution inside fibers is then governed by equation (4):

$$\frac{\partial u}{\partial z} = i \alpha_2 \frac{\partial^2 u}{\partial t^2} + i \left[ \alpha_5 |u|^2 u + \alpha_6 |u|^{2m} u - i \alpha_7 \frac{\partial}{\partial t} (|u|^2 u) \right] \quad (4)$$

Figure 1, shows this comportment for a second order soliton with $\alpha_7 = 0.2$. We note that both solitons propagate at different speeds, as a result, they separate from each other, and the separation increases in a linear manner as a function of distance this result is in good agreement by comparison with literature [10]. Figure 2, shows the temporal and amplitude evolution of a second order soliton ($N = 2$) over ten dispersion lengths. One can conclude that the simultaneous presence of the self-steepening effect and quintic effect allows the high-order soliton to keep its shape longer without significant breaking or deformation on a longer propagation distance, when the quintic parameter increases. It can be concluded as that the existence of the quintic effect plays an important role in the removal of the breakup caused by the self-steepening effect. This is almost similar to the phenomenon of cancellation effects caused by the dispersion of the group velocity (GVD) by the Kerr effect (SPM) for a fundamental soliton.
4.2 Study of conventional unchirped or chirped soliton propagation in optical fiber in the presence of management of dispersion, nonlinearity and gain (loss).

The concept of dispersion management technique is to compensate these effects by using a periodic dispersion map of combining fibers with different characteristics [1]. However in real soliton application systems, the dispersion, nonlinearity, gain and loss are generally varied with the propagation distance. In the following we present the results of our study obtained by application of the (CSSPS) in the case of the propagation of conventional unchirped or chirped soliton in an optical fiber when the dispersion, non-linearity, gain and loss are generally varied with the propagation distance. In many references [11-12-13-14] intensive studies have been devoted to this situation. The Schrödinger equation model which governs the dynamics of solitons in this type of fiber is given by the equation (5):

$$i \frac{\partial A(Z, T)}{\partial Z} + \frac{1}{2} D(z) \frac{\partial^2 A}{\partial T^2} + \gamma(z)|A|^2A = i \alpha(z)A$$

(5)

Where $D(z)$ represents the variation of the dispersion of the group velocity (GVD) as a function of the distance, $\gamma(z)$ represents the variation of the Kerr non-linearity parameter and $\alpha(z)$ represent the variation of the gain (loss) as a function of propagation distance also. Note that the nonlinear Schrödinger equation with variable coefficients (6) is not integrable except for a few very special cases of combinations of variables carefully chosen $D(z), \gamma(z)$ and $\alpha(z)$ [15]. In our numerical simulations we considered that the variation of the dispersion of the group velocity (GVD) is given by [16].

$$D(z) = \frac{1}{D_0} \exp(\sigma z) \gamma(z), \quad \gamma(z) = \gamma_0 + \gamma_1 \sin(g z), \quad \alpha(z) = \frac{\sigma}{2}$$

(5-1)

The peak power of the initial pulse takes into account by the $D_0$ factor. $\gamma_0$, $\gamma_1$ and $g$ parameters represents the Kerr nonlinearity. We take $\sigma < 0$ for a system having a dispersion decreasing and loss (absorption), $\sigma > 0$ for the reverse case. The initial pulse injected in the form of a conventional chirped soliton given by equation [1]:

$$u(0, T) = \text{sech}(T) \exp\left(-\frac{i c T^2}{2}\right)$$

(5-2), Where $c$ is the chirp parameter. And for the others parameters we took the following values [14]: $\gamma_0 = 0, D_0 = 1; \gamma_1 = 1; g = 1$ (5-3), The results obtained are shown by curves (4) (5) and (6) below:

**Figure 04:** Evolution of the intensity profile of a conventional unchirped soliton with propagation distance

**Figure 05:** Evolution of the intensity profile of a conventional chirped soliton with propagation distance for $c = -0.10, \sigma = 0.050$. 
We noticed that the amplitude of a conventional unchirped solitons have an increase in a linear manner as a function of distance when the parameter \( \sigma \) is positive, in the opposite case \( \sigma \) negative the amplitude of a conventional unchirped solitons have an decrease in a linear manner as a function of distance. While the amplitude of the conventional chirped soliton oscillates (increase / decrease) depending on the module of the "chirped" parameter and it can be seen that the chirped soliton is obviously and periodically compressed and broadening with the increase of propagation distance. These results are in good agreement with the model presented in reference [17]

5-Conclusion

The compact split step Padé Scheme has been implemented to investigate the propagation of ultra-intense and ultra-short optical solitons under the effect of power law nonlinearity and high order dispersion effect. This scheme is more efficient, rapid and takes less memory space. It is also well adapted to the higher order derivatives. We applied this method for the case of combined effects such self-steepening quintic nonlinearity and concluded that the quintic effect plays an important role in the removal of the breakup caused by the self-steepening effect on the propagation of optical second-order solitons. In the presence of management of dispersion, nonlinearity and gain (loss) the amplitude of a conventional unchirped solitons have a decrease in a linear manner as a function of distance of propagation. While the amplitude of the conventional chirped soliton oscillates (increase / decrease) depending on the module of the "chirped" parameter.

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