A method to rescale experimental data with dependence on $Q^2$ for DVCS process

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ABSTRACT

We analyze the procedure for rescaling the DVCS cross section data collected with different invariant mass, $W$, of the virtual photon–proton system. We suggest a method which makes the rescaling more functional to conduct statistical analysis on overall data. The study can be applied to rescale data collected with different photon virtuality $Q^2$. We also show a dependence on $Q^2$ for the $\delta$ parameter used to describe the cross section as a function of $W$.

Introduction

The models used to interpret the Deeply Virtual Compton Scattering (DVCS) process are compared with an overall set of experimental results. This is obtained from several data sets collected at different energies. As shown in Fig. 1, the DVCS is the diffractive scattering of a virtual photon ($\gamma^*$) off a proton (p), i.e. $\gamma^* p \rightarrow \gamma p$ where $\gamma$ denotes the outgoing photon. The integrated cross section can be reported [1] as a simple function:

$$\sigma(Q^2, W) \propto W^n \left( \frac{1}{Q^2} \right)^n,$$

where $W$ is the invariant mass of the $\gamma^* p$ system and $Q^2$ is the virtuality of the photon. $\delta$ and $n$ are parameters obtained from fits to experimental data, by keeping fixed respectively the $Q^2$-value or the $W$-value. The cross section as a function of $Q^2$, or $W$, is measured by H1 and ZEUS experiments [1,2]. The overall set of experimental results is determined by a procedure consisting in rescaling the values of the raw data measured at different energies. This is possible by applying some factors that are used also for the error analysis. As an example, the ZEUS Collaboration [2] data, taken at $W = 89$ GeV and $Q^2 = 9.6$ GeV$^2$, were rescaled to the H1 Collaboration [1] data, taken at $W = 82$ GeV and $Q^2 = 8$ GeV$^2$, with $\delta$ and $n$ respectively fixed to values 0.75 and 1.54 [3]. In this case, the procedure for rescaling depends on the definition of appropriate normalization factors, which are indicated hereafter with $\varepsilon$. In particular, $\varepsilon_2$ represents the normalization factor when we consider the cross section, $\sigma(W)$, as a function of $W$ with fixed $Q^2$; $\varepsilon_W$ represents the normalization factor when considering the cross section, $\sigma(Q^2)$, as a function of $Q^2$ with fixed $W$. Using the Eq. (1) we have

$$\sigma_r(W) = \sigma_r(Q^2, W) \sigma_r(W) = \frac{(Q^2_r)^{\delta_r}}{(Q^2_s)^{\delta_s}} \sigma_r(W),$$

$$\sigma_r(Q^2) = \frac{\sigma_r(Q^2_s, W_s)}{\sigma_r(Q^2_s, W_s)} \sigma_r(Q^2) = \frac{(W_s)^{\delta_r}}{(W_d)^{\delta_r}} \sigma_r(Q^2),$$

where the subscripts “dr”, “$s$” and “r” respectively denote the data to be rescaled, those considered in scale and the rescaled data. The previous relations allow us to obtain the following formulas:

$$\varepsilon_2^r = \frac{(Q^2_r)^{\delta_r}}{(Q^2_s)^{\delta_s}},$$

$$\varepsilon_W^r = \frac{(W_s)^{\delta_r}}{(W_r)^{\delta_r}}.$$

In the “standard” procedure the equalities $\delta_d = \delta_s = \delta$ and $n_d = n_s = n$ are considered valid [3], whereby the normalization factor $\varepsilon_2^r$, for $\sigma_d(W) \rightarrow \sigma_r(W)$, is given by the ratio between $(Q^2_s)^{\delta_s}$ and $(Q^2_r)^{\delta_r}$ and the normalization factor $\varepsilon_W^r$, for $\sigma_d(Q^2) \rightarrow \sigma_r(Q^2)$, is given by the ratio between $(W_s)^{\delta_r}$ and $(W_d)^{\delta_r}$. Considering $\delta = 0.77$ and $n = 1.54$ [1], the ZEUS measurements are rescaled to the $Q^2$ and $W$ values of the H1 measurements through following expressions:

$$\sigma_r(W) = \varepsilon_2^r \sigma_d(W) \simeq 1.3242 \sigma_d(W),$$

$$\sigma_r(Q^2) = \varepsilon_W^r \sigma_d(Q^2) \simeq 0.9389 \sigma_d(Q^2).$$

Footnotes

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This is an open access article under the CC BY-NC-SA license (http://creativecommons.org/licenses/by-nc-sa/3.0/).
As shown in Fig. 2, where we illustrate the effect of the procedure for rescaling the cross section as function of \(Q^2\), the rescaled ZEUS data are roughly moved over the H1 data. To highlight this feature, we fit lines to data in order to catch the general trend of the two data series which allows us to compare the trends of two data sets.

**New procedure**

The analysis of the two fits shown in Fig. 2 evidences that the rescaled ZEUS data tend to remain higher than those of H1; therefore it seems that the “standard” procedure described above does not rescale the ZEUS experimental data to those of H1. In particular, data points for \(Q^2 = 55\) GeV\(^2\) are not superimposed, although they are consistent within the error bars. Indeed, we would expect that, after rescaling, the data will be superimposed when they refer to the same value of \(Q^2\). In this regard, we might consider an alternative rescaling procedure by normalizing the ZEUS data to those of H1 and using the following normalization factor:

\[
\frac{\sigma}{\sigma_0} = \frac{\sigma_w(Q^2 = 55\) GeV\(^2\))}{\sigma_0(Q^2 = 55\) GeV\(^2\))} = \frac{0.15}{0.20} = \frac{3}{4} \tag{8}
\]

where 0.15 and 0.20 are the cross section values measured by ZEUS and H1 experiments at the same value of \(Q^2\). Fig. 3 shows the ZEUS data rescaled according to Eq. (8). As the previous figures show, the changing of normalization factor has given a better approximation of rescaled ZEUS data to those of H1. However, Fig. 3 shows clearly that it is not possible to conduct statistical analysis on overall data. In the current study we suggest that the rescaling procedure should be based on a comparison of the trend determined by fit to the rescaled ZEUS data with the trend determined by fit to the H1 data. i.e. the characteristic parameters of both fits must have similar values since the fitting curves must be proximate. This observation is physically correct because the process is the same for both Collaborations, although the data are collected at different \(Q^2\) and \(W\) values. In effect, if the ZEUS and H1 data were taken at the same energies, we would expect similar values for the characteristic parameters of the fits. This consideration is the basis of any rescaling procedure. Also to avoid experimenter’s bias, we suggest to consider the trend of the fits to the data rather than data points itself. Therefore it is necessary to redefine another normalization factor, which we indicate with \(\zeta_w\). The latter can be determined by varying the value of \(\zeta_w\) until there is a sound agreement on characteristic parameters of fits, as previously highlighted. So we find \(\zeta_w = 0.67\), value for which the parameters of fits describe the same curve as the Fig. 4 shows. In this case, the fit on overall data gives a n-value compatible with that obtained by the H1 Collaboration, i.e. \(n = 1.54 \pm 0.09 \pm 0.04\) [1], where the first error is statistical, the second systematic.

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**Fig. 1.** Diagram illustrating the DVCS. The process is accessed through the reaction \(ep \rightarrow e'p\) [1,2,7,8].

**Fig. 2.** DVCS cross section \(\sigma(\gamma'p \rightarrow \gamma p)\) as a function of \(Q^2\) for \(W = 82\) GeV (\(|t| < 1.0\) GeV\(^2\)), where \(t\) is the four momentum transfer squared at the proton vertex). The error bars represent the statistical and systematic uncertainties added in quadrature. The experimental data collected by the ZEUS Collaboration [2] have been rescaled to those collected by the H1 Collaboration [1] using Eq. (7), where \(\zeta_w \approx 0.9389\).

**Fig. 3.** DVCS cross section \(\sigma(\gamma'p \rightarrow \gamma p)\) as a function of \(Q^2\) for \(W = 82\) GeV (\(|t| < 1.0\) GeV\(^2\)). The error bars represent the statistical and systematic uncertainties added in quadrature. The experimental data collected by the ZEUS Collaboration [2] have been rescaled to those collected by the H1 Collaboration [1] using the normalization factor \(\zeta_w = 0.67\) determined in “New procedure.”

**Fig. 4.** DVCS cross section \(\sigma(\gamma'p \rightarrow \gamma p)\) as a function of \(Q^2\) for \(W = 82\) GeV (\(|t| < 1.0\) GeV\(^2\)). The error bars represent the statistical and systematic uncertainties added in quadrature. The experimental data collected by the ZEUS Collaboration [2] have been rescaled to those collected by the H1 Collaboration [1] using the normalization factor \(\zeta_w = 0.67\) determined in “New procedure.”
New normalization factor for fixed $W$

Adopting the following power-type function

$$\sigma(Q^2) = a \times \left[ \frac{1}{Q^2} \right]^n$$

(9)

and by performing a fit on H1 data, we have $a = 83.47 \pm 10.96$ and $n = 1.54 \pm 0.06$, with reduced chi-square $\chi^2$/d.o.f. = 0.15; these parameters are compatible with those calculated by performing a fit on the rescaled ZEUS data using $\zeta_W$ factor: $a_{\zeta_W} = 80.99 \pm 8.71$ and $n_{\zeta_W} = 1.53 \pm 0.04$, with $\chi^2$/d.o.f. = 0.26. Furthermore, if we use the factor $\zeta_W$ for the rescaling procedure, the fit on rescaled ZEUS data gives $a_{\zeta_W} = 113.50 \pm 12.21$, which is inconsistent with $a_r$. Hence we must introduce the factor $\zeta_W$ and reject the standard procedure. It is possible to move from $\zeta_W$ to $\zeta_W$ by applying the following formula:

$$\zeta_W = \frac{a_{\zeta_W}}{a_r} \equiv \Xi_W \zeta_W,$$

(10)

where we introduce the factor $\Xi_W$. This one may show a $W$ dependence which could not be considered taking only the factor $\zeta_W$. The value of $\Xi_W$ is found from Eq. (10):

$$\Xi_W = \frac{a_r}{a_{\zeta_W}} \approx 0.71.$$  

(11)

We might ask if $\Xi_W$ can be determined using the ratio between the $W$ energies with which the H1 and ZEUS Collaborations have performed their measurements. Actually this event happens when we raise the ratio to the fourth power:

$$\Xi_W \approx \left( \frac{W_s}{W_d} \right)^4 = \left( \frac{82}{89} \right)^4 = 0.72.$$  

(12)

Hence, in order to make the rescaling procedure more efficient in statistical terms, it is necessary to replace $\zeta_W$ with the following normalization factor:

$$\zeta_W' = \left( \frac{W_s}{W_d} \right)^{4.7} = \left( \frac{82}{89} \right)^{4.7} = 0.6766,$$

(13)

where we use $\delta = 0.77$ [1]. Since the $\zeta_W'$ value is approximately equal to $\zeta_W$, the curve in Fig. 4 represents approximately the fit of the power function to ZEUS data rescaled by the $\zeta_W'$ factor. If we fit the overall data using the function of Eq. 9, we obtain $a = 84.21 \pm 9.06$ and $n = 1.54 \pm 0.04$, with $\chi^2$/d.o.f. = 0.26; these parameters are clearly compatible with those obtained by the fit to the H1 data.

Method for rescaling the DVCS data collected at different energies

Introducing the following function

$$P(Q, W) = W^{4.13} \times \left( \frac{1}{Q^2} \right)^n,$$

(14)

Eq. (1) can be written as

$$\sigma(Q^2, W) \propto \frac{1}{W^\gamma} P(Q, W),$$

(15)

whereby, according the rescaling procedure here proposed, we have to carry out the ratio between the quantities $P_r$ and $P_{dr}$:

$$\sigma_r(W) = \frac{P_r(Q_r, W)}{P_{dr}(Q_r, W)} \sigma_{dr}(W),$$

(16)

Table 1

| $Q^2$ (GeV$^2$) | $\delta$ | Reference |
|-----------------|----------|-----------|
| 2.4             | 0.44 $\pm$ 0.19 | ZEUS 1999–2000 [7] |
| 3.2             | 0.52 $\pm$ 0.09 | ZEUS 1999–2000 [7] |
| 4               | 0.69 $\pm$ 0.32 $\pm$ 0.17 | H1 1996–1997 [1] |
| 6.2             | 0.75 $\pm$ 0.17 | ZEUS 1996–2000 [7] |
| 8               | 0.81 $\pm$ 0.34 $\pm$ 0.22 | H1 1999–2000 [1] |
| 8               | 0.61 $\pm$ 0.10 $\pm$ 0.15 | H1 2004–2007 [8] |
| 9.6             | 0.75 $\pm$ 0.15 $\pm$ 0.06 | ZEUS 1996–2000 [2] |
| 9.9             | 0.84 $\pm$ 0.18 | ZEUS 1996–2000 [7] |
| 15.5            | 0.61 $\pm$ 0.13 $\pm$ 0.13 | H1 2004–2007 [8] |
| 18              | 0.76 $\pm$ 0.22 | ZEUS 1996–2000 [7] |
| 25              | 0.90 $\pm$ 0.36 $\pm$ 0.27 | H1 2004–2007 [8] |

Fig. 5. $\delta$ parameter as a function of $Q^2$. The experimental values are given in Table 1. Two fits are shown. The dotted lines indicate the error bands.

In general, the differential cross section for the DVCS process, $d\sigma/dt$, can be expressed at high energies [5] as

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s} |\mathcal{M}|^2,$$

(17)

where the variable $t$, represented in Fig. 1, is the square of the four-momentum transferred at the proton vertex, $s$ is the squared center-of-mass energy of the incoming system, i.e. $s = W^2$, and $\mathcal{M}$ is the DVCS amplitude. If the $W$ dependence of the integrated cross section $\int (d\sigma/dt) dt$ is the same, over the relevant $W$ domain, as the $W$ dependence of the differential cross section $d\sigma/dt$ for $t = 0$, then the cross section can be expressed, as indicated in Eq. 15, in terms of $W^4$. These considerations suggest that the function $P$ is proportional to the integrated squared modulus of the DVCS amplitude. Therefore, according the Eq. 16 and Eq. 17, the ZEUS measurements can be rescaled to the values of the H1 measurements by performing the ratio between the integrated squared modulus of scattering amplitudes of the process studied in H1 and ZEUS experiments. In this manner it is interesting to note that the rescaling procedure depends essentially on the scattering amplitudes and that these latter contain all the information about the dynamics of the process.

1 The statistical analyses conducted here were performed using the program OriginPro [4].

2 See footnote 14 of Ref. [6].
Dependence of $\delta$ parameter on $Q^2$

In Table 1 we have collected the $\delta$-values calculated by ZEUS and H1 experiments [1,2,7,8]. Taking into account that several values are not within the error bars of other values, we consider the possibility of treating the $\delta$-parameter as function of $Q^2$, contrary to indications in the literature which state that $\delta$ is independent of $Q^2$ within the errors [8]. All the functions used to fit data of Table 1 exhibit a similar trend especially for low values of $Q^2$. In Fig. 5 two fits are shown: one is logarithmic-type, another one is power-type. The logarithmic-type curve is given by the following equation:

$$\delta(Q^2) = \delta_0 - \delta_1 \ln(Q^2 + \delta_2),$$  \hspace{1cm} (19)

where $\delta_0 = 0.5421 \pm 0.0768$, $\delta_1 = -0.0857 \pm 0.0389$ and $\delta_2 = -2.1511 \pm 0.5414$, with $\chi^2$/d.o.f. = 0.2497; the power-type curve is given by the following equation:

$$\delta(Q^2) = \delta' \left[ 1 - (Q^2)^{-\delta'} \right],$$  \hspace{1cm} (20)

where $\delta' = 0.8232 \pm 0.0887$ and $\delta_0' = 0.9137 \pm 0.2455$, with $\chi^2$/d.o.f. = 0.2051. Since $\delta$ is treated as a function of $Q^2$, we have that the factor $\epsilon_{W'}$ depends on $Q^2$:

$$\epsilon_{W'} = \left( \frac{W_v}{W_{\alpha}} \right)^{4+\delta'(Q^2)}.$$  \hspace{1cm} (21)

Fig. 6 shows the trend determined by fit to the ZEUS data, which are rescaled by using Eq. (21) and Eq. (19). This trend is superimposed to that determined by fit to the H1 data. In effect, by performing a fit to the rescaled ZEUS data, we have $a_1 = 83.74 \pm 8.99$ and $n_r = 1.54 \pm 0.04$, with $\chi^2$/d.o.f. = 0.26; these parameters are compatible with $a_1$ and $n_r$ obtained by performing a fit to the H1 data. It is interesting to note that the trends overlap although the visible dependence of $\delta$ on $Q^2$ has introduced a dependence of $\epsilon_{W'}$ on $Q^2$ in an independent manner with respect to the rescaling analysis conducted in “New normalization factor for fixed $W$”. Clearly, the growth, at low $Q^2$, and the flattening, at high $Q^2$, of $\delta$ do not fundamentally modify the rescaling procedure proposed in this paper.

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