NUCLEON SPIN STRUCTURE FUNCTIONS

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The review of the nucleon spin structure functions problems is presented. The perturbative QCD predictions for the small $x$ behaviour of the nucleon spin structure functions is discussed. The role of the resummation of the $\ln^2 1/x$ terms is emphasised. Predictions for the nonsinglet structure function $g_1$ in case of a flat as well as a dynamical input are given. The so called ‘spin crisis’ in the context of both theoretical and experimental future hopes are also briefly discussed.

PACS numbers: 12.38 Bx

1. Introduction

Since 1988, when the famous EMC experiment [1] provided surprising results, the polarised deep inelastic lepton-nucleon scattering (DIS) became very interesting from experimental as well as theoretical point of view. This experiment, in which longitudinally polarised muons scattered on longitudinally polarised protons, brought the conclusion that quarks are carrying only a small part of the proton spin projection in the polarised proton. This result called ‘spin crisis’ is still a challenge for theoretical and experimental research. The main questions should be answered are: how the nucleon spin is distributed among its constituents: quarks and gluons and how the dynamics of these constituent interactions depend on spin. Solutions of these problems may be found within perturbative QCD because they involve hard and semihard (short-distance) processes. From experimental point of view there are many projects, which provide or will provide a mechanism to probe the spin properties of nucleon and should be a crucial test of QCD. The most important of these projects are experiments in SLAC, DESY (HERMES, HERA), CERN (SMC) (with polarised proton, deuteron and $\text{He}^3$ targets), CERN (COMPASS) (with polarised muon beams and longitudinally polarised hydrogen ($\text{NH}_3$) and deuteron ($\text{Li}^6\text{D}$) targets. The
main goal of all these experiments is to measure the nucleon spin structure functions \( g_1(x, Q^2) \) and \( g_2(x, Q^2) \). It can be done by measurement of the cross section asymmetry, where two considered cases correspond to antiparallel or parallel spin orientation of the longitudinally polarised lepton (\( \mu, e \)) and nucleon (\( p, n, d \)) respectively. Recently the experiment data has allowed to investigate the nucleon spin structure in the large range of the kinematical variables: Bjorken \( x \) and \( Q^2 \). The most interesting, both theoretically and phenomenologically, is the region of small \( x \). Theoretical understanding of the small \( x \) (\( x \sim 10^{-3} \) and less) behaviour of the polarised nucleon structure function enables the correct estimation of \( \Gamma_1 \) momenta in sum rules. It is very important because present experimental data do not cover however the whole very small \( x \) region and the only way (at present) to know the nucleon spin structure completely is extrapolation of large and medium \( x \) results into the small \( x \) region through the theoretical QCD analysis. From the other side, future polarised experiments in HERA [2] will enable spin DIS investigations in the very small \( x \) region: \( x \sim 10^{-4} \) and less. Then theoretical predictions would be verified by the experiment. These future spin experiments would be a crucial test of theoretical analysis. Description of the nucleon spin structure function \( g_1 \) within perturbative QCD for small \( x \) can be done in different frames (in LO, NLO, \( \ln 1/x \), \( \ln^2 1/x \) etc. approximations) giving different results for \( g_1 \) in this region. Thus the future comparison of theoretical and experimental results could be definitive.

In the next section we shall briefly remind the sum rules and the 'spin crisis' problem. In section 3 we shall discuss the polarised structure functions of nucleon in the small Bjorken \( x \) region. We shall emphasise the \( \ln^2 1/x \) resummation which is significant in this region. In point 4 the nonsinglet \( g_1^{NS}(x, Q^2) \) predictions are presented. We show LO and unified LO+\( \ln^2 1/x \) resummation results in case of a flat (nondynamical) and a dynamical input parametrisation as well. We compare our results with recent SMC data. Finally in conclusions we shall briefly discuss future experimental hopes and possible scenario of solving the spin crisis problem.

2. Sum rules and the 'spin crisis'

There are four basic nucleon structure functions: \( F_1, F_2 \) for a spin independent case and \( g_1, g_2 \) for a polarised one, which characterise the pointlike interaction between virtual hard photon (Compton scattering) of \( Q^2 \gg \Lambda^2 \) and hadron constituents - partons in the deep inelastic lepton - hadron scattering. In the parton model \( F_1, F_2 \) and \( g_1 \) have a very simple form and interpretation. Thus

\[
F_2(x) = x \sum_{i=u,d,s,..} e_i^2 [q_{i+}(x) + q_{i-}(x)]
\]

(2.1)
where $e_i$ is a charge of the i-flavour quark, $q_{i+}(x)$ ($q_{i-}(x)$) is the density distribution function of the i-quark with the spin parallel (antiparallel) to the parent nucleon. In the Bjorken limit

$$F_2(x) = 2xF_1(x)$$  \hspace{1cm} (2.2)$$

$$g_1(x) = \frac{1}{2} \sum_{i=u,d,s,..} e_i^2 \Delta q_i(x)$$  \hspace{1cm} (2.3)$$

$$\Delta q_i(x) = q_{i+}(x) - q_{i-}(x)$$  \hspace{1cm} (2.4)$$

Function $g_1(x,Q^2)$ is connected with the helicity of the nucleon (i.e. spin projection on the momentum direction). Thus the integral

$$\langle \Delta q_i \rangle = \int_0^1 \Delta q_i(x)dx$$  \hspace{1cm} (2.5)$$

is simply a part of the nucleon helicity, carried by a quark of i-flavour (i=u,d,s,..). The second spin dependent structure function $g_2(x,Q^2)$, related to the transverse spin polarisation of the nucleon has no simple meaning in the parton model. The main goal in the deep inelastic lepton - nucleon scattering experiments with polarised both the lepton and the nucleon particles is to find the spin dependent structure function of nucleon $g_1(x,Q^2)$. This measurement of $g_1(x,Q^2)$ provides the knowledge how the spin of the nucleon is distributed among the partons: valence quarks $u_v,d_v$, sea quarks $q_{sea}$ and gluons $g$. The experimental measurement of $g_1(x,Q^2)$ is based on the measurement of the cross section asymmetry factor $A$ [13]:

$$A = \frac{\sigma(++) - \sigma(+-)}{\sigma(++) + \sigma(+-)}$$  \hspace{1cm} (2.6)$$

where $\sigma(++)$, $\sigma(+-)$ correspond to the cases when the spins of longitudinally polarised lepton ($\mu$ or $e$) and nucleon ($p$ or $n$ or $d$) are parallel or antiparallel. Finally, $g_1(x,Q^2)$ can be determined through the relation

$$g_1(x,Q^2) = \frac{F_2(x,Q^2)A_1(x,Q^2)}{2x(1 + R)}$$  \hspace{1cm} (2.7)$$

where

$$A_1 = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}}$$  \hspace{1cm} (2.8)$$

and

$$R = \frac{\sigma_L}{\sigma_T}$$  \hspace{1cm} (2.9)$$
The cross section $\sigma_{3/2}$ and $\sigma_{1/2}$ correspond to the helicities $3/2$ and $1/2$ of the absorbed virtual photon. $\sigma_L$ and $\sigma_T$ are the absorption cross sections of longitudinal and transverse virtual photons in the polarised scattering. Through polarised DIS experiments with different targets: proton, neutron or deuteron ones, it is possible to combine the $g_1$ results for these different cases and hence to find out the spin dependent distribution functions of partons ($\Delta u_v$, $\Delta d_v$, $\Delta q_{\text{sea}}$, $\Delta g$) in the nucleon. The measurement of $g_1$ function enables also verification of the sum rules, which play a very important role as a test of QCD. The most significant of them are Bjorken and Ellis-Jaffe sum rules [7]. Both sum rules are related with the nucleon spin structure functions and their estimation requires the knowledge of first moments of $g_1$ for proton, neutron and deuteron $\Gamma^p_1(Q^2)$, $\Gamma^n_1(Q^2)$, $\Gamma^d_1(Q^2)$:

$$\Gamma^i_1(Q^2) = \int_0^1 g^i_1(x, Q^2) dx \quad (2.10)$$

To test experimentally the Bjorken and Ellis-Jaffe sum rules it is necessary to know function $g_1(x, Q^2)$ for certain $Q^2$ and in the entire range of $x$: $x \in (0; 1)$. This data collection of $g_1$ for arbitrary values of $Q^2$ and $x$ variables is however impossible because of technical constraint in the experiments. The broad ranges of $x$ and $Q^2$ (where $Q^2 = -q^2$, $q$ is the four momentum transfer between lepton and hadron and $W^2 = Q^2(1/x - 1)$ is total CM energy squared) cannot be reached independently. The accessible at present kinematical region in the polarised fixed target HERMES experiment in HERA is $0.004 < x < 1$, $0.2 < Q^2 < 20 \text{ GeV}^2$ [26]. It must be however emphasised, that HERA will extend the present region by two orders of magnitude both in $x$ and $Q^2$: $x$ down to $6 \cdot 10^{-5}$ and $Q^2$ up to $2 \cdot 10^4 \text{ GeV}^2$. It will take place in the future experiment with polarised electron and proton beams [8]. Hence physicists hope for a new very interesting field of experimental investigations. Broader range of $x$ and $Q^2$ in the future polarised DIS experiments in HERA will enable more precise knowledge of the proton spin structure and the test of QCD predictions for the polarised structure functions of the nucleon. The accessible region of $x$ and $Q^2$ in future polarised experiments in HERA is roughly presented in Fig.1 [8]. Perturbative QCD provides methods for theoretical analysis of structure functions for $Q^2 \gg \Lambda^2$ ($\Lambda \sim 200 \text{ MeV}$). When the coupling constant of strong interactions $\alpha_s(Q^2)$ becomes large ($\alpha_s \sim 1$) perturbative calculations are not applicable. QCD as a theory with asymptotic freedom improves the naive parton model by permission for a weak $Q^2$ dependence of structure functions $F_1$, $F_2$, $g_1$, $g_2$. This scaling violation is caused by strong interactions between partons (quarks and gluons). The larger $Q^2$ (i.e. smaller distance between partons) the weaker interaction between partons. Pertur-
bative QCD enables $Q^2$ evolution of structure functions, where the values of the structure functions at some fixed scale $Q^2 = Q_0^2$ are taken from the experiment and used as an input in evolution equations. As it was already mentioned above, the most important test of QCD in the spin dependent DIS phenomena is verification of Bjorken (BSR) and Ellis-Jaffe (EJSR) sum rules. BSR is related to the isovector axial current SU(3) flavour symmetry in the nucleon $\beta$ decay while EJSR results from the octet axial current SU(3) symmetry in the $\beta$ decay of hyperons. In the parton model BSR and EJSR have forms:

\[ BSR : \quad a_3 = g_A \]  

(2.11)

where

\[ a_3 = \int_0^1 (\Delta u + \Delta \bar{u} - \Delta d - \Delta \bar{d}) dx \]  

(2.12)

and $g_A \approx 1.257$ is $\beta$ decay structure constant for neutron;

\[ EJSR : \quad a_8 = 3F - D \]  

(2.13)
where
\[ a_8 = \int_0^1 g_{1_{\text{octet}}}(x)dx = \int_0^1 (\Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} - 2\Delta s - 2\Delta \bar{s})dx \quad (2.14) \]

\( \Delta q(q = u, d, s, \ldots) \) are polarised distribution functions of quarks and antiquarks (2.4) and \( F, D \) are axial coupling constants, estimated from the weak \( \beta \) decays of hyperons: \( 3F - D \approx 0.579 \). Taking into account the QCD corrections one can obtain:

\[ a_{0,3,8} \rightarrow a_{0,3,8}(1 - \text{corr}(\alpha)) \quad (2.15) \]

where \( \text{corr}(\alpha) \) is just a QCD correction, calculated in a perturbative way [13]:

\[ \text{corr}(\alpha) \approx (\alpha_s/\pi) + 3.58(\alpha_s/\pi)^2 + 20.22(\alpha_s/\pi)^3 + 130(\alpha_s/\pi)^4 \quad (2.16) \]

\( \alpha_s(Q^2) \) is the running coupling constant of strong interaction and in LO approximation has a form

\[ \alpha_s(Q^2) = \frac{12\pi}{(33 - 2N_f)\ln \frac{Q^2}{\Lambda^2}} \quad (2.17) \]

where \( \Lambda = 232 \text{ MeV} \) is a scale parameter of QCD. \( a_0 \) in (2.15) denotes the singlet part of axial current:

\[ a_0 = \int_0^1 g_{1_{\text{singlet}}}(x)dx = \int_0^1 \Delta \Sigma dx \quad (2.18) \]

For three flavours of quarks \((N_f = 3)\) \( \Delta \Sigma \) is given by

\[ \Delta q_S \equiv \Delta \Sigma \equiv \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s} \quad (2.19) \]

Thus, taking into account QCD corrections (2.15) one can rewrite the BSR (2.11) in the form

\[ \Gamma_1^p - \Gamma_1^n \equiv \int_0^1 (g_1^p - g_1^n)dx = \frac{1}{6}g_A(1 - \text{corr}(\alpha)) \quad (2.20) \]

Bjorken and Ellis-Jaffe sum rules imply the following expressions for first moments of spin structure functions \( g_1 \) of the nucleon:

\[ \Gamma_1^p \equiv \int_0^1 g_1^p(x)dx = \left( \frac{a_3}{12} + \frac{a_8}{36} + \frac{a_0}{9} \right)(1 - \text{corr}(\alpha)) - \frac{N_f}{18\pi}\alpha_s(Q^2)\langle \Delta g(Q^2) \rangle \quad (2.21) \]
\[ \Gamma_1^n \equiv \int_0^1 g_1^n(x) dx = \left( -\frac{a_3}{12} + \frac{a_8}{36} + \frac{a_9}{9} \right) (1 - \text{corr}(\alpha)) - \frac{N_f}{18\pi} \alpha_s(Q^2) \langle \Delta g(Q^2) \rangle \]

(2.22)

\[ \Gamma_1^d \equiv (1 - \frac{3}{2} \omega_D) \int_0^1 g_1^d(x) dx = \left[ \left( \frac{a_8}{36} + \frac{a_9}{9} \right) (1 - \text{corr}(\alpha)) \right. \]

\[ \left. - \frac{N_f}{18\pi} \alpha_s(Q^2) \langle \Delta g(Q^2) \rangle \right] (1 - \frac{3}{2} \omega_D) \]

(2.23)

where \( \Delta g \) is the spin dependent distribution function of gluons and \( \langle \Delta g \rangle \) is defined similarly as for quarks (2.5). Parameter \( \omega_D \sim 0.058 \) denotes probability that deuteron is in D state. QCD corrections \( \alpha_s^n \) in formulae (2.20)-(2.23) enable us to compare theoretical perturbative QCD predictions for the structure function \( g_1 \) and its first moment \( \Gamma_1 \) with the experimental results (2.7). Thus one can easily read from (2.20), (2.16) and (2.17) that BSR at \( Q^2 = 10 \text{ GeV}^2 \) for \( \Lambda = 200 \text{ MeV} \) up to \( \alpha_s^4 \) correction gives

**THEORY** : \( \Gamma_1^p - \Gamma_1^n = 0.185 \) \hspace{1cm} (2.24)

This result is in a good agreement with the most recent SMC and SLAC measurements [13],[14], which yield for \( Q^2 = 10 \text{ GeV}^2 \) the quantity

**EXPERIMENT** : \( \Gamma_1^p - \Gamma_1^n = 0.195 \pm 0.029 \) \hspace{1cm} (2.25)

It means that the Bjorken sum rule BSR is fulfilled within the experimental errors. For the Ellis-Jaffe sum rule EJSR however, there is a significant disagreement between theory and experiment. Theoretical value for EJSR from the simplest version of (2.21), where the gluon contribution is neglected and the strange sea quarks are unpolarised \( (\Delta s = 0) \) gives [13]

**THEORY** : \( \Gamma_1^p(Q^2 = 10.7 \text{ GeV}^2) = 0.171 \) \hspace{1cm} (2.26)

while the EMC result [1] is

**EXPERIMENT** : \( \Gamma_1^p(Q^2 = 10.7 \text{ GeV}^2) = \\
= 0.126 \pm 0.010(\text{statist}) \pm 0.015(\text{system}) \) \hspace{1cm} (2.27)

Comparing (2.26) and (2.27) one can see, that the EJSR prediction disagrees with experimental data by 2.6 standard deviations. From the latest experimental analyses [11], [12],[14] it can be read that BSR is always fulfilled within the experimental errors while EJSR is broken at the level of about 3 standard deviations. This fact is confirmed by many polarised DIS
experiments: SLAC in CERN with the polarised electron beam scattered on polarised proton (p), neutron (n) or deuteron (d) targets, SMC at CERN with muons $\mu^+$ and p,d targets and HERMES at DESY experiments with positrons $e^+$ and n,p,d targets. The characteristic of kinematic variables for experimental groups EMC, SLAC, SMC and HERMES is presented below in Tab.1 [12]. The experimental data of $g_1$, $\Gamma_1$ and $\langle \Delta q \rangle$ (2.5) are widely

| Experiment | Beam | Lepton energy E (GeV) | Smallest $x$ $Q^2 > 1$ (GeV$^2$) | Average $Q^2$ (GeV$^2$) | Nucleon target |
|------------|------|-----------------------|----------------------------------|--------------------------|----------------|
| EMC        | $\mu^+$ | 100-200               | 0.01                             | 10.7                     | p              |
| SMC        | $\mu^+$ | 100-190               | 0.003                            | 10                       | p,d            |
| SLAC E-142 | $e^-$  | 19-25                 | 0.03                             | 2                        | n              |
| SLAC E-143 | $e^-$  | 10-29                 | 0.029                            | 3                        | p,d            |
| SLAC E-154 | $e^-$  | 48.3                  | 0.014                            | 5                        | n              |
| SLAC E-155 | $e^-$  | 48.3                  | 0.014                            | 5                        | p,d            |
| HERMES     | $e^+$  | 27.5                  | 0.023                            | 2.3                      | n,p,d          |

Table 1. The kinematic variables in DIS for different experimental groups

reviewed e.g. in [6],[16]. The main problem nowadays is to find out how the spin of the proton is distributed among partons: valence quarks, sea quarks and gluons. The part of the proton spin carried by the parton $p$, where $p$ denotes $u_{val}$, $d_{val}$, $u_{sea}$, $d_{sea}$, $s_{sea}$, antiquarks, $g$ is given by (2.28).

$$\langle \Delta p \rangle = \int_0^1 \Delta p \, dx$$

(2.28)

and

$$\Delta q = \Delta q_{val} + \Delta q_{sea}$$

(2.29)

$$\Delta q_{sea} = \Delta \bar{q}_{sea} \equiv \Delta \bar{q}$$

(2.30)

hence

$$\Delta q_{val} = \Delta q - \Delta \bar{q}$$

(2.31)

The method of the extraction $\Delta p$ and then the important quantity $\langle \Delta p \rangle$ from experimental data is as follows:

1. Polarised distribution functions $\Delta p(x, Q^2)$ are parametrised at the given low scale $Q_0^2$ (e.g. $Q_0^2 = 1$ GeV$^2$) in the form:

$$\Delta p(x, Q_0^2) = N_{\eta f} x^{\alpha f} (1 - x)^{\beta f} (1 + \gamma_f x^{\delta_f})$$

(2.32)
where

\[ \eta_f = \frac{1}{N_f} \int_0^1 \Delta p(x, Q^2_0) \, dx \]  \hspace{1cm} (2.33)

\( N \) is a normalisation factor and \( \alpha_f, \beta_f, \gamma_f, \delta_f \) are parameters.

2. From QCD evolution equations (see Appendix A) (A.1)-(A.3) one can get in NLO approximation \( \Delta p(x, Q^2) \) functions for \( (x, Q^2) \) set, available in experiments.

3. Using the spin dependent distribution functions \( \Delta p \), one can calculate the spin structure function \( g_1(x, Q^2) \) from the formula:

\[
g_1(x, Q^2) = \frac{1}{2} \langle e^2 \rangle \int_x^1 \frac{dy}{y} \left[ C_S^q(x/y, \alpha_s(Q^2)) \Delta \Sigma(y, Q^2) + 2N_f C_g^q(x/y, \alpha_s(Q^2)) \Delta g(y, Q^2) + C_{qNS}^q(x/y, \alpha_s(Q^2)) \Delta q_{NS}(y, Q^2) \right] \]  \hspace{1cm} (2.34)

where

\[
\langle e^2 \rangle = \frac{1}{N_f} \sum_k e_k^2 \]  \hspace{1cm} (2.35)

\( \Delta q_S \equiv \Delta \Sigma \) (2.19) is singlet and \( \Delta q_{NS} \) is the nonsinglet polarised distribution function of quarks:

\[
\Delta q_{NS} \equiv \sum_{i=1}^{N_f} \left( \frac{e_i^2}{\langle e^2 \rangle} - 1 \right) (\Delta q_i + \Delta \bar{q}_i) \]  \hspace{1cm} (2.36)

4. Now one can compare the calculated \( g_1 \) (2.34) with experimental data for \( g_1 \) (2.7) and fit free parameters \( \alpha_f, \beta_f, \gamma_f, \delta_f \) of input parametrisations (2.32) in such a way to minimise the \( \chi^2 \) for all experimental points \( (x, Q^2) \). In this way it was found, as it was already mentioned above, that all experiments confirm the validity of the Bjorken sum rule while the Ellis-Jaffe sum rule is violated. After extraction of spin dependent distribution functions \( \Delta p \) from experimental data it was possible to determine the value \( \int \Delta \Sigma(x, Q^2) \, dx \), which is a part of the proton spin carried by quarks. The result was surprising because it turned out that quarks carry only a very small part of the proton spin:

\[
\int_0^1 \Delta \Sigma(x, Q^2) \, dx \approx 0.17 \pm 0.17 \]  \hspace{1cm} (2.37)
and the strange quarks carry much larger part of the proton spin that it follows from the simple parton model:

\[ \int_{0}^{1} \Delta s(x, Q^2) dx \approx -0.14 \]  

This result known since EMC experiments [1] as a "spin crisis" problem should be correctly solved within QCD. Theoretical analysis should provide a proper interpretation of the EMC results. The spin problem is in fact not the "spin crisis" but the problem of understanding how the nucleon spin is composed of parton spins? Disagreement of experimental data with theoretical predictions, emerging in EJSR violation, shows that the simple quark model is definitely inadequate to the proper description of the nucleon spin structure. The "spin crisis" has arisen because experimental results did not confirm the naive quark model in which the proton spin is carried only by valence quarks i.e.:

\[ \int_{0}^{1} \Delta \Sigma dx = \int_{0}^{1} (\Delta u_{val} + \Delta d_{val}) dx = 1 \]  

In this model it was assumed that strange sea quarks as well as gluons do not or almost do not contribute to the nucleon spin. Using this assumption, the input distribution functions (2.32) were parametrised and then used in the fitting of experimental data. So to remove the spin problem it is necessary to revise the previous knowledge about the spin distribution in the nucleon. It would be helpful with this aim to consider the following facts and assumptions [6]:

1. The anomalous dimension of the first moment of gluons is equal to -1:

\[ \int_{0}^{1} \Delta g(x, Q^2) dx \sim \ln Q^2 \text{ for } Q^2 \to \infty \]  

It means that

\[ \lim_{Q^2 \to \infty} \left[ \alpha_s(Q^2) \int_{0}^{1} \Delta g(x, Q^2) dx \right] = \text{const} \]  

and gluons give a nonvanishing correction of order \( \alpha_s \) to EJSR (2.21)-(2.23), when \( Q^2 \to \infty \) (opposed to the first moment of quarks, which the anomalous dimension is zero). This gluon anomalous dimension via evolution equations
(A.1)-(A.3) changes also the spin dependent distribution functions of quarks.

2. Gluons can carry a very large part of the nucleon spin. If we assume that

\[ \int_{0}^{1} \Delta g(x, Q^2 = 10)dx \approx 2 \]  

then [13]:

\[ \int_{0}^{1} \Delta s(x, Q^2)dx = -0.06 \pm 0.06 \]  

\[ \int_{0}^{1} \Delta \Sigma(x, Q^2)dx = 0.42 \pm 0.17 \]  

3. The polarised sea quarks distribution function is large and negative, so it cancels most of the valence quark contribution to the nucleon spin.

4. The orbital momenta of the nucleon constituents \( L_z(Q^2) \) is large and it is a lacking part of the nucleon spin, according to the total nucleon spin law conservation:

\[ \frac{1}{2} \langle \Delta \Sigma \rangle + \langle \Delta g \rangle + L_z(Q^2) = \frac{1}{2} \]  

The problem of the "spin crisis" is still open. Its solution requires most of all the knowledge about polarised distribution functions of gluons. Theoretical analyses QCD as well as future polarised experiments at HERA [8], COMPASS [15] and RHIC [21] should better determine the gluon spin function behaviour, particularly in the small Bjorken \( x \) region, what is a crucial point to overcome the "spin crisis".

3. Spin structure functions in the small Bjorken \( x \) region

Determination of the nucleon spin structure functions in the small Bjorken \( x \) region is very important from both theoretical and experimental point of view. Because of technical limit, present experiments do not give any information about small \( x \) region \( (x \sim 10^{-4}, 10^{-5}) \) and therefore there are still the uncertainties in the determination of parton distribution functions (in particular the gluons) in this region. Theoretical analyses, based on the perturbative QCD, allow to calculate the nucleon structure function within some approximations \( (Q^2 \text{LO}, Q^2 \text{NLO}, \ln 1/x \text{ etc.}) \). Choice of some particular approximation depends of course on the region of its application and the basic criterion is agreement of the theoretical predictions with experimental data. Thus the small \( x \) behaviour of the nucleon spin structure functions implied by QCD can be via sum rules (BSR, EJSR) tested experimentally.
Moreover, the aim of the QCD analysis is to yield an adequate, compact description of the nucleon structure functions in the whole range of $x$: for the values of $x$, which are available in experiment and which are not as well.

The small $x$ region is also a challenge for QCD analysis, because theoretical predictions of the structure function $g_1^p(x, Q^2)$ at low $x$ are relevant for the future polarised HERA measurements [8].

Structure functions for small $x$ and fixed $Q^2$ are connected with the virtual Compton scattering total cross-section at very high energy $W^2 \to \infty$:

$$W^2 = Q^2 \left( \frac{1}{x} - 1 \right) \quad (3.1)$$

where $W$ is the total CM energy. The small value of $x$ ($x \to 0$) corresponds by definition to the Regge limit and therefore the small $x$ behaviour of structure functions can be described using the Regge pole exchange model [2]. The Regge theory predicts, that spin dependent structure functions $g_1^{p,n,d}$ in the small $x$ region behave as

$$g_1^{p,n,d} \sim x^{-\alpha} \quad (3.2)$$

where $\alpha$ denotes the axial vector meson trajectory and lies in limits:

$$-0.5 \leq \alpha \leq 0 \quad (3.3)$$

The experimental data from HERA confirm such a Regge behaviour of structure functions (3.2) but only in the low $Q^2$ region $Q^2 \leq \Lambda^2 \ (\Lambda^2 \approx 200 \text{ MeV})$ i.e. in the region, where the perturbative methods are not applicable. At larger $Q^2$, because of parton interaction, the structure functions undergo the $Q^2$ evolution and their behaviour, implied by perturbative QCD is more singular than that, coming from the Regge picture. This fact is also in agreement with experiments of unpolarised as well as polarised DIS. It is well known at present, that for $x \to 0$ the Regge behaviour $x^{-\alpha}$ ($-0.5 \leq \alpha \leq 0$) is less singular than the perturbative QCD predictions for all of parton distributions except unpolarised, nonsinglet (valence) quarks $q_{NS}$.

There are two basic approaches of perturbative analysis of unpolarised structure functions in the small $x$ region: a resummation of the GLAP $Q^2$ logarithms [3],[10],[16]:

$$\sum_{n,m} [\alpha_s(Q^2)]^n (\ln Q^2)^m \quad (3.4)$$

or a resummation of $1/x$ logarithms BFKL [4]:

$$\sum_{n,m} [\alpha_s(Q^2)]^n (\ln \frac{1}{x})^m \quad (3.5)$$
In the latest experimental data analyses of DIS the standard approach is based on the QCD evolution equations within next-to leading (NLO) approximation [3],[10]. This approach is appropriate for polarised as well as unpolarised nucleon structure functions. It has to be emphasised, that the changes, which appear by the transition from the LO approach to the NLO one are different in the case of polarised and unpolarised structure functions. Splitting functions $P_{ij}^{(0)}(x)$ and $\Delta P_{ij}^{(0)}(x)$, governing the evolution of quark and gluon distribution functions in the LO approximation have a form (A.7)-(A.10) in the spin independent case and a form (A.11)-(A.14) in the spin dependent case respectively. In the small $x$ limit $x \to 0$ they take a LO form:

$$P_{qq}^{(0)}(x) \sim \frac{4}{3}; \quad P_{qG}^{(0)}(x) \sim \frac{1}{2}; \quad P_{Gq}^{(0)}(x) \sim \frac{8}{3x}; \quad P_{GG}^{(0)}(x) \sim \frac{3}{x};$$  

(3.6)

$$\Delta P_{qq}^{(0)}(x) \sim \frac{4}{3}; \quad \Delta P_{qG}^{(0)}(x) \sim \frac{1}{2}; \quad \Delta P_{Gq}^{(0)}(x) \sim \frac{8}{3}; \quad \Delta P_{GG}^{(0)}(x) \sim 9$$  

(3.7)

In the NLO approximation, functions $P_{ij}^{(1)}(x)$ and $\Delta P_{ij}^{(1)}(x)$ have the following small $x$ behaviour [16],[17]:

$$P_{qq}^{(1)}(x) \sim \frac{1}{x}; \quad P_{qG}^{(1)}(x) \sim \frac{1}{x}; \quad P_{Gq}^{(1)}(x) \sim \frac{1}{x}; \quad P_{GG}^{(1)}(x) \sim \frac{1}{x};$$  

(3.8)

$$\Delta P_{qq}^{(1)}(x), \quad \Delta P_{qG}^{(1)}(x), \quad \Delta P_{Gq}^{(1)}(x), \quad \Delta P_{GG}^{(1)}(x) \sim a + b \ln x + c \ln^2 x$$  

(3.9)

where $a,b,c$ are constants. From the comparison of (3.6)-(3.9) one can read an interesting information about the small $x$ behaviour of polarised and unpolarised nucleon structure functions. First: comparing (3.6) with (3.8) i.e. spin independent $P_{ij}(x)$ function in LO and NLO approximations, one can notice, that the singular terms $1/x$ appear in $P_{qq}^{(1)}(x)$ and $P_{qG}^{(1)}(x)$ while they do not in $P_{qq}^{(0)}(x)$ and $P_{qG}^{(0)}(x)$ respectively. This means that the evolution of unpolarised, fermion distribution functions at small $x$ is completely dominated by the NLO terms in the perturbative calculation. Secondly: the singular $1/x$ terms in $P_{Gq}^{(1)}(x)$ and $P_{GG}^{(1)}(x)$ are opposite in sign with respect to the corresponding LO terms: $P_{Gq}^{(0)}(x)$ and $P_{GG}^{(0)}(x)$. Thus, the rapid growth of the unpolarised gluon distribution function at small $x$, well known from LO analyses, will be dampened by the NLO contribution. Thirdly: all spin dependent splitting functions $\Delta P_{ij}^{(1)}(x)$ are nonsingular (without $1/x$ terms). (An exact form of $\Delta P_{ij}^{(1)}(x)$ is given e.g. in [16].) It means, in general, that in the small $x$ region spin dependent parton distribution functions are by a factor of $x$ less singular than the corresponding unpolarised
distributions. Knowledge of the nucleon structure function at small $x$ is extremely important: $g_1(x, Q^2)$ results for this region enter into integrals for moments $\Gamma_1^{p,n,d}$ (2.10) and hence into the Bjorken and Ellis-Jaffe sum rules (2.20)-(2.23). From the other side, lack of the very small $x$ ($x \leq 10^{-4}$) experimental data causes that the all knowledge of structure functions in limit small $x$ ($x \to 0$) comes almost entirely from theoretical QCD analyses. Therefore perturbative (and nonperturbative as well) QCD predictions in the small $x$ region are at present of great importance.

Evolution equations GLAP for unpolarised gluon distribution functions $g(x,Q^2)$ and unpolarised singlet quark distributions $\Sigma(x,Q^2)$

$$
\Sigma(x,Q^2) = \sum_{i=1}^{N_f} [q_i(x,Q^2) + \bar{q}_i(x,Q^2)]
$$

(3.10)

lead to the following behaviour [18]:

$$
xg(x,Q^2) \sim \sigma^{-1/2} e^{2\gamma \sigma - \delta} (1 + \sum_{i=1}^{n} \epsilon^i \rho^{i+1} \alpha_s)
$$

(3.11)

$$
x\Sigma(x,Q^2) \sim \rho^{-1} \sigma^{-1/2} e^{2\gamma \sigma - \delta} (1 + \sum_{i=1}^{n} \epsilon^i \rho^{i+1} \alpha_s)
$$

(3.12)

where $\xi$, $\zeta$, $\sigma$, $\rho$ are given as:

$$
\xi = \ln\left(\frac{x_0}{x}\right); \quad \zeta = \ln\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right); \quad \sigma = \sqrt{\xi \zeta}; \quad \rho = \sqrt{\xi/\zeta}
$$

(3.13)

$n = 0$ in the summation (3.11) and (3.12) corresponds to LO while $n = 1$ to NLO approximations. Remaining parton distributions i.e. $p = q_{NS}, \Delta q_{NS}, \Delta \Sigma, \Delta g$ behaves in the small $x$ region like:

$$
p(x,Q^2) \sim \sigma^{-1/2} e^{2\gamma} \zeta (1 + \sum_{i=1}^{n} \epsilon^i \rho^{2i+1} \alpha_s)
$$

(3.14)

One can read from (3.11), (3.12) and (3.14), that spin dependent parton distribution functions $x\Delta q_{NS}, x\Delta \Sigma, x\Delta g$ and the unpolarised nonsinglet quark distribution $xq_{NS}$ are by a factor of $x$ less singular than the unpolarised distribution $xg$ and $x\Sigma$. Moreover, in the case of $p$ function ($p = q_{NS}, \Delta q_{NS}, \Delta \Sigma, \Delta g$) (3.14) higher order corrections are more important than in the $xg$ (3.11) and $x\Sigma$ (3.12) case because of appearance $\rho^{2i+1}$ terms in (3.14) instead of $\rho^{i+1}$ in (3.11) and (3.12). It must be also emphasised, that the small $x$ behaviour of parton distributions strongly depends on the
input parametrisation $p(x,Q_0^2)$ (2.32). When it is nonsingular, then the singular small $x$ behaviour of parton distributions is fully generated by $Q^2$ evolution and has a form (3.11)-(3.14). Whereas in a case of singular input parametrisation e.g.

$$
\Delta g(x,Q_0^2) \sim x^\alpha (1-x)^\beta (1+\gamma x^\delta)
$$

(3.15)

where $\alpha = -0.5$, $\beta = 4$, $\gamma = 3$, $\delta = 1$, singular $x^\alpha$ small $x$ behaviour of $\Delta g$ distribution will survive $Q^2$ QCD evolution and will be leading towards the singular terms, generated perturbatively. Choosing the input parametrisation (2.32), experimental data must be taken into account. As a start scale for the GLAP evolution equations $Q_0^2 = 1 \text{ GeV}^2$ is assumed at present. This value is a limit where from the one side ($Q^2 < Q_0^2$) experimental measurements confirm the Regge theory of DIS, and from the other side $Q^2 > Q_0^2$ it is a starting point for methods of perturbative QCD ($Q^2 \geq 1 \text{ GeV}^2$). There are two approaches dependent on the choice of the input parametrisation. Either we assume Regge behaviour (3.2) of parton distributions at small $x$ and then their shape at small $x$ and larger $Q^2$ $Q^2 > Q_0^2$ is fully implied by NLO QCD evolution (3.11)-(3.14) or we take singular input parametrisations (but no more singular than it results from experimental data i.e. $\Delta q_{NS}(x,Q^2) \leq x^{-0.5}$), which will survive LO and NLO GLAP evolution. The Regge theory predicts the following behaviour of parton distributions at small $x$ and $Q^2 \leq 1 \text{ GeV}^2$:

$$
x \Sigma \sim \text{const (Pomeron)}
$$

$$
q_{NS} \sim x^{-0.5} \text{ (Reggeon } A_2 : \rho - \omega);
$$

$$
\Delta \Sigma, \Delta q_{NS} \sim x^0 \div x^{0.5} \text{ (Reggeon } A_1 \text{)}
$$

(3.16)

Experimental analyses [5],[19] are in a good agreement with (3.16) for $Q^2 \leq 1 \text{ GeV}^2$ and also for larger $Q^2$, after taking into account the perturbative effects. Investigating small $x$ region within perturbative methods, one should include all of those terms in $C(x,Q^2)$ and $P_{ij}(x,\alpha_s)$, which remarkably influence the shape of the nucleon structure functions and what will be soon verified experimentally. It has been lately noticed [20],[24],[25] that the spin dependent structure function $g_1$ in the small $x$ region is dominated by $\ln^2(1/x)$ terms. These contributions correspond to the ladder diagrams with quark and gluon exchanges along the ladder - cf Fig.2. The contribution of non-ladder diagrams to the nonsinglet spin dependent structure function is negligible. Thus the behaviour of the spin dependent nucleon structure functions at small $x$ is expected to be governed by leading double logarithmic terms of type $\alpha_s^n \ln^{2n}(x)$. These terms must be resummed in the coefficients and splitting functions $P_{ij}(x,\alpha_s^2)$. The resummation of the
double logarithmic terms \( \ln^2 x \) in the limit of a very small \( x \) \( (x \to 0) \) is given by the following equations [20]:

\[
f_{NS}(x, k^2) = f_{NS}^{(0)}(x, k^2) + \frac{\alpha_s}{2\pi} \int \frac{dz}{z} \int \frac{dk^2}{k^2} \Delta P_{qq}^{(0)}(z) f_{NS}(\frac{x}{z}, k^2) \quad (3.17)
\]

\[
f_{S}(x, k^2) = f_{S}^{(0)}(x, k^2) + \frac{\alpha_s}{2\pi} \int \frac{dz}{z} \int \frac{dk^2}{k^2} \Delta P_{qG}^{(0)}(z) f_{S}(\frac{x}{z}, k^2)
\times \left[ \Delta P_{Gq}^{(0)}(z) f_{S}(\frac{x}{z}, k^2) + \Delta P_{Gq}^{(0)}(z) f_{g}(\frac{x}{z}, k^2) \right] \quad (3.18)
\]

\[
f_{g}(x, k^2) = f_{g}^{(0)}(x, k^2) + \frac{\alpha_s}{2\pi} \int \frac{dz}{z} \int \frac{dk^2}{k^2} \Delta P_{Gq}^{(0)}(z) f_{S}(\frac{x}{z}, k^2)
\times \left[ \Delta P_{Gq}^{(0)}(z) f_{S}(\frac{x}{z}, k^2) + \Delta P_{Gq}^{(0)}(z) f_{g}(\frac{x}{z}, k^2) \right] \quad (3.19)
\]
where $\Delta p_{ij}^{(0)}(z)$ are LO approximation at $z \to 0$ and have a form (3.7). Unintegrated distributions $f$ in equations (3.17)-(3.19) are related to the corresponding polarised distributions $\Delta p(x, Q^2)$ via

$$
\Delta p(x, Q^2) = \Delta p^{(0)}(x) + \int_{k_0^2}^{W^2} \frac{dk^2}{k^2} f(x' = x(1 + k^2/Q^2), k^2) \quad (3.20)
$$

$$
\Delta p^{(0)}(x) = \int_{0}^{k_0^2} \frac{dk^2}{k^2} f(x, k^2) \quad (3.21)
$$

$k^2$ is the transverse momentum squared of the parton. The inhomogeneous term $f^{(0)}(x, k^2)$ in the nonsinglet case (3.17) has a form

$$
f^{(0)}_{NS}(x, k^2) = \frac{\alpha_s(k^2)}{2\pi} \left[ \frac{1}{3} \int_x^1 \frac{dz}{z} (1 + z^2) \Delta p^{(0)}(x/z) - 2z \Delta p^{(0)}(x) \right]
$$

$$
+ \frac{\alpha_s(k^2)}{2\pi} \left[ 2 + \frac{8}{3} \ln(1 - x) \right] \Delta p^{(0)}(x) \quad (3.22)
$$

where the nonperturbative part of the distribution $\Delta p^{(0)}(x)$ is parametrised on the basis of the experimental data at the small values of $Q^2$ ($Q^2 < 1 \text{ GeV}^2$). This nonperturbative parametrisation is given by

$$
\Delta p^{(0)}_{NS}(x) = N(1 - x)^{\eta} \quad (3.23)
$$

Parameter $\eta$ in the valence quark case ($\Delta q_{NS}$) is equal to 3, $g_A = 1.257$ is the axial vector coupling and $N$ is the normalisation constant, in the nonsinglet case determined from the Bjorken sum rule (2.11), (2.12), which can be written as:

$$
\int_0^1 g_1^{NS}(x, Q_0^2) \, dx = \int_0^1 (g_1^p - g_1^n)(x, Q_0^2) \, dx = \frac{1}{6} g_A \quad (3.24)
$$

The source of the double logarithmic terms $\ln^2 x$ in $g_1(x, Q^2)$ is the double integration in the formula for function $f(x, k^2)$:

$$
f(x, k^2) \sim \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \frac{k^2/z}{k_0^2} \int_{k_0^2}^{d k^2} \frac{d k^2}{k^2} \quad (3.25)
$$
where the upper limit in the integral over the transverse momentum $k'^2$ is $z$-dependent ($= k^2/z$). Thus, double logarithmic terms come from the integration over the longitudinal momentum fraction $z$ together with the integration over $k^2$ with $z$-dependent upper limit:

$$f(x, k^2) \sim \ln^2(1/x) = \ln^2 x$$

Equations (3.17)-(3.19) in the case of the fixed coupling constant $\alpha_s$ can be solved analytically [20]. These equations generate singular small $x$ behaviour of the polarised parton distributions and hence of the spin dependent structure function $g_1$ i.e.:

$$g_1^{NS}(x, Q^2) \sim x^{-\lambda_{NS}}; \quad g_1^{S}(x, Q^2) \sim x^{-\lambda_{S}}; \quad \Delta g(x, Q^2) \sim x^{-\lambda_{S}}$$

where $g_1^{NS} = g_1^p - g_1^n$, $g_1^{S} = g_1^p + g_1^n$, $\Delta g$ is the polarised gluon distribution and exponents $\lambda_i$ have forms:

$$\lambda_{NS} = 2\sqrt{\frac{\alpha_s}{2\pi} \Delta P_{qq}^{(0)}(x)}; \quad \lambda_{S} = 2\sqrt{\frac{\alpha_s}{2\pi} \gamma^+}$$

$$\gamma^+ = \frac{1}{2} [\Delta P_{qq}^{(0)}(x) + \Delta P_{GG}^{(0)}(x)] + \sqrt{[\Delta P_{qq}^{(0)}(x) - \Delta P_{GG}^{(0)}(x)]^2 + 4\Delta P_{qq}^{(0)}(x) \Delta P_{GG}^{(0)}(x)}$$

As it has already been mentioned above, the singular small $x$ behaviour of the polarised structure function (3.27) become leading only in the case of the nonsingular input parametrisation (2.32) e.g. for the simple parametrisation (3.23). Because only unpolarised, nonsinglet parton distributions $q_{NS}$ have Regge small $x$ behaviour $x^{-0.5}$ more singular than that, implied by QCD evolution, the shape of all spin dependent distributions is mostly governed by QCD evolution. Thus the leading small $x$ behaviour of polarised nucleon structure functions is (3.27) and moreover singlet structure functions dominate over the nonsinglet ones ($\lambda_{S} > \lambda_{NS}$). For example, at $Q^2 = 10 \text{ GeV}^2$, $N_f = 3$ and scale parameter $\Lambda = 232 \text{ MeV}$, the corresponding values of $\lambda_i$ are: $\lambda_{S} = 1.4$, $\lambda_{NS} = 0.48$ ($\alpha_s = 0.27$, $\gamma^+ = 11.7$). It must be pointed out, that the equations (3.17)-(3.19) can be applied only for very small $x$. They are based on the approximation, that for very small $x$ splitting functions $\Delta P_{ij}^{(0)}(x \to 0)$ are given by (3.7). At large and moderately small values of $x$ this approach is no more adequate. In the region of “not too small $x$”, which can be explored experimentally, in theoretical QCD analyses one should use the GLAP (LO or NLO) equations with complete $\Delta P_{ij}(z)$ functions. Combining the standard LO GLAP approach with the double $\ln^2 x$ resummation,
it is possible on the one hand to guarantee an agreement of QCD predictions with experimental data in the large and moderately small \( x \) and on the other hand to generate the singular small \( x \) shape of polarised structure functions, governed by \( \ln^2 x \) terms. With this aim the equations (3.17)-(3.19) should be extended to include complete splitting functions \( \Delta P_{ij}(z) \) and not only their approximations (3.7) for \( z \to 0 \). In this way one can obtain system of equations, containing both LO GLAP evolution and the double logarithmic \( \ln^2 x \) effects at small \( x \). Analyses of such unified GLAP LO + \( \ln^2 x \) approach are presented in [20]. On the basis of this interesting method we give in the next chapter the predictions for the \( g_1^{NS} \) function in the case of nonsingular as well as singular input parametrisation \( \Delta p_{NS}(x,Q_0^2) \).

4. Predictions for the nonsinglet spin structure function \( g_1 \)

The small \( x \) behaviour of both nonsinglet and singlet spin dependent structure functions \( g_1^{NS}(x,Q^2) \) and \( g_1^S(x,Q^2) \) is governed by the double logarithmic terms \( \alpha_s^n \ln^{2n}(x) \) [20], [24],[25]. But in contrast to the singlet polarised function, for the nonsinglet one the contribution of nonladder diagrams is negligible. Thus we should consider only ladder diagrams with quark (antiquark) exchange, Fig.2. Hence the nonsinglet part of the polarised structure function \( g_1 \) has a form:

\[
g_1^{NS}(x,Q^2) = g_1^p(x,Q^2) - g_1^n(x,Q^2)
\]

where \( g_1^p \) and \( g_1^n \) are spin dependent structure functions of proton and neutron respectively. According to (2.3), (2.4), (2.29)-(2.31) one can obtain for the colour number \( N_c = 3 \):

\[
g_1^p = \frac{2}{9} \Delta u + \frac{1}{18} \Delta d + \frac{5}{18} \Delta \bar{u} + \frac{1}{9} \Delta \bar{s}
\]

\[
g_1^n = \frac{1}{18} \Delta u + \frac{2}{9} \Delta d + \frac{5}{18} \Delta \bar{u} + \frac{1}{9} \Delta \bar{s}
\]

and hence

\[
g_1^{NS} = \frac{1}{6} (\Delta u_{val} - \Delta d_{val}) = \frac{1}{6} (\Delta u - \Delta d)
\]

The simple form of \( g_1^{NS} \) (4.4) results from the assumption of SU(3) flavour symmetry:

\[
\Delta \bar{u} = \Delta \bar{d}
\]

and hence all of gluon and sea quark contributions from the proton and the neutron structure function cancel mutually. This feature that the small \( x \) behaviour of the spin dependent nonsinglet structure function is governed by the double logarithmic terms \( \alpha_s^n \ln^{2n}(x) \) is very important from the point of
view of small $x$ QCD analysis. This is different from the case of unpolarised nonsinglet structure functions $F_{2}^{NS}$, where the small $x$ behaviour of $F_{2}$, generated by the $\alpha_{s}^{2}\ln^{2n}(x)$ terms, is dominated by the nonperturbative contribution of $A_{2}$ Regge pole. For $g_{1}^{NS}$ the relevant $A_{1}$ Regge pole has low intercept $\alpha_{NS}(0) \leq 0$ and for small $x$ in the Regge limit one has:

$$g_{1}^{NS}(x, Q^{2}) \sim x^{-\alpha_{NS}(0)}$$

(4.6)

Thus the Regge behaviour of the spin dependent structure functions is unstable against the resummation of the $\ln^{2}x$ terms, which generate more singular $x$ shape than that (4.6) with $\alpha_{NS}(0) \leq 0$. Therefore the measurement of the nonsinglet spin dependent structure function can be a very important test of the QCD perturbative analyses in the small $x$ region. In our numerical analysis we follow [20] and [25]. Solving the unified equation incorporating GLAP $Q^{2}$ evolution and the $\ln^{2}x$ resummation we get the results for the nonsinglet polarised structure function $g_{1}^{NS}(x, Q^{2})$ in the perturbative region $Q^{2} > Q_{0}^{2}$ for different values of $x \in (0; 1)$. This equation taking into account both GLAP evolution and $\ln^{2}x$ effects for $g_{1}^{NS}$ function has a form [20],[25]:

$$f(x, k^{2}) = f^{(0)}(x, k^{2}) + \frac{2\alpha_{s}(k^{2})}{3\pi} \int_{x}^{k^{2}/x} \frac{dz}{z} \int_{k_{0}^{2}}^{k^{2}} \frac{dk^{2}}{k^{2}} f\left(\frac{x}{z}, k^{2}\right)$$

$$+ \frac{\alpha_{s}(k^{2})}{2\pi} \int_{k_{0}^{2}}^{k^{2}} \frac{dk^{2}}{k^{2}} \left[ \frac{1}{3} \int_{x}^{1} \frac{dz}{z} (z + z^{2}) f(x/z, k^{2}) - 2zf(x, k^{2}) \right]$$

$$+ \left( \frac{1}{2} + \frac{8}{3} \ln(1-x) \right) f(x, k^{2})]$$

(4.7)

where

$$f^{(0)}(x, k^{2}) = \frac{\alpha_{s}(k^{2})}{2\pi} \left[ \frac{4}{3} \int_{x}^{1} \frac{dz}{z} (1 + z^{2}) g_{1}^{(0)}(x/z, k^{2}) - 2zg_{1}^{(0)}(x) \right]$$

$$+ \left( \frac{1}{2} + \frac{8}{3} \ln(1-x) \right) g_{1}^{(0)}(x)$$

(4.8)

$$g_{1}(x, Q^{2}) = g_{1}^{(0)}(x) + \int_{k_{0}^{2}}^{Q^{2}(1/x-1)} \frac{dk^{2}}{k^{2}} \left( x + \frac{k^{2}}{Q^{2}} \right) f\left( x, \frac{k^{2}}{Q^{2}} \right)$$

(4.9)

and

$$g_{1}^{(0)}(x) = \int_{0}^{k_{0}^{2}} \frac{dk^{2}}{k^{2}} f(x, k^{2})$$

(4.10)
Comparing (4.7)-(4.10) with (3.17), (3.20)-(3.22) it is clear that $\Delta P_{qq}(x)$ in (4.7) has a full GLAP form instead of its approximation for $x \to 0$ in (3.17) and $g_{1NS}^p$ plays simply the role of $\Delta p$ from (3.20). We solve eq.(4.7) using different parametrisations of $g_{1NS}^0(x)$: the simple one, implied by Regge behaviour of $g_{1NS}$ in nonperturbative region

$$g_{1NS}^0(x) \equiv g_{1NS}^p(x,Q_0^2) = N(1 - x)^3$$ (4.11)

and two dynamical inputs: GRSV (Glück, Reya, Stratmann, Vogelsang) [22] and GS (Gehrmann, Stirling) [23]. The nonsinglet spin dependent structure function must satisfy the Bjorken sum rule (2.11), (2.12) independently of the value of $Q^2$. This means that for any $Q^2$, the first moment of $g_{1NS}$ must be equal to $1/6g_A$ similarly to the case of the low scale $Q_0^2$ (3.24):

$$\langle g_{1NS}(x,Q^2) \rangle \equiv \frac{1}{0} g_{1NS}(x,Q^2)dx = \frac{1}{0} (g_{1p}^n - g_{1n}^p)(x,Q^2)dx = \frac{1}{6}g_A = 0.2095$$ (4.12)

This condition implies the proper normalisation constants $N$ in all of input parametrisations. Thus the constant $N$ in (4.11), found from the Bjorken sum rule is equal to $2/3g_A = 0.838$ (we set $g_A = 1.257$) and the Regge nonsingular input (4.11) takes a form:

$$REGGE : \quad g_{1NS}^N(x,Q_0^2) = \frac{2}{3}g_A(1 - x)^3 = 0.838(1 - x)^3$$ (4.13)

The Regge behaviour of structure functions at small $x$, as it was mentioned above, has been confirmed by HERA experiments in the low $Q^2$ region ($Q^2 < 1$ GeV$^2$). Therefore the choice of the Regge input allows to unite the nonperturbative origin with QCD perturbative analysis starting at $Q_0^2 \sim 1$ GeV$^2$. In this way, assuming the Regge (flat, nonsingular) behaviour of structure functions at low $Q^2$ scale \textit{i.e.} $Q_0^2 = 1$ GeV$^2$, we expect that the singular small $x$ behaviour of polarised structure functions is completely generated by QCD evolution, involving NLO or even (as in our case) GLAP+$\ln^2 x$ approach. This analysis, based on the Regge input (4.13), is however one of two main possible scenarios, describing the small $x$ behaviour of spin structure functions. The second is to allow steeper (more singular) inputs of structure functions at $Q_0^2$, what intensifies more the growth of structure functions as $x \to 0$ implied by QCD. The only constraint on these two scenarios is consistency of their predictions with experimental data. In our analysis of the $g_{1NS}$ structure function we consider dynamical inputs proposed by GRSV [22] and GS [23]. These inputs result from a global analysis of all available recently deep inelastic polarised structure function data [14].
Our calculations incorporating both GLAP evolution and resummation of the $\ln^2 x$ terms are based on the LO fitted inputs. In such a way the spin dependent nonsinglet structure function $g_{NS}^1$ has an input form:

$$G R S V \ : \ g_{NS}^1(x, Q_0^2 = 1 \, \text{GeV}^2) = 0.327 x^{-0.267} (1 - 0.583 x^{0.175} + 1.723 x$$
$$+ 3.436 x^{3/2}) (1 - x)^{3.486} + 0.027 x^{-0.624} (1 + 1.195 x^{0.529}$$
$$+ 6.164 x + 2.726 x^{3/2}) (1 - x)^{4.215} \quad (4.14)$$

$$G S \ : \ g_{NS}^1(x, Q_0^2 = 4 \, \text{GeV}^2) = 0.29 x^{-0.422} (1 + 9.38 x - 4.26 \sqrt{x})$$
$$\times (1 - x)^{3.73} + 0.196 x^{-0.334} (1 + 10.46 x - 5.10 \sqrt{x}) (1 - x)^{4.73} \quad (4.15)$$

for details see Appendix B. All numerical calculations have been performed in C code on PC computer under LINUX system. Our numerical results for $g_{NS}^1$ based on Regge (4.13), GRSV (4.14) and GS (4.15) input parametrisations are presented in Figs.3-7. In Fig.3 we plot different input parametrisations $g_{NS}^1(x, Q_0^2)$. Figs.4,5 show the nonsinglet function $g_{NS}^1$ after evolution to $Q^2 = 10 \, \text{GeV}^2$ for these different parametrisations (Regge, GRSV, GS) and Figs.6,7 present the function $6 x g_{NS}^1(x, Q_0^2)$ at $Q^2 = 10 \, \text{GeV}^2$ also for different inputs $g_{NS}^1(x, Q_0^2)$. In all of Figs.4-7 pure GLAP evolution is compared with double logarithmic $\ln^2 x$ effects at small $x$. Additionally, in Figs.5-7 we compare our numerical results with recent SMC (1997) data [14]. Contributions $6 \langle g_{NS}^1 \rangle$ (4.12) and $6 \Delta I(x_a, x_b, Q^2)$

$$\Delta I(x_a, x_b, Q^2) \equiv \int_{x_a}^{x_b} g_{NS}^1(x, Q^2) dx \quad (4.16)$$

to the Bjorken sum rule at $Q^2 = 10 \, \text{GeV}^2$ together with experimental SMC values are presented in Tab.2: *) means the extrapolation of experimental data to low $x$ and **) is the integral over the measured range of $x$. From Figs.4-7 one can read that the double logarithmic $\ln^2 x$ effects are very significant for $x \leq 10^{-2}$. Besides, as it has been expected, the growth of the nonsinglet proton spin structure function $g_{NS}^1$ in the very small $x$ region is much steeper for dynamical parametrisations (GRSV or GS) than for the Regge one. The comparison of our theoretical model with experimental data in Tab.2 and Figs.5-7 yields the conclusion that all of the theoretical predictions for different parametrisations (Regge, GRSV, GS) and incorporating pure LO GLAP QCD evolution as well as LO GLAP evolution with $\ln^2 x$ effects are in a good agreement with experimental data within statistical errors. Unfortunately, the most interesting $x$ region is still nonavailable for experiment. So the problem, which QCD approach is the most adequate for the description of small $x$ physics in the polarised deep-inelastic scattering of particles remains unsolved.
**Table 2.** Theoretical contributions $6\Delta I(x_a, x_b, Q^2)$ and their experimental SMC values

| Parametrisation | $6\Delta I$ | $6\Delta I$ | $6\Delta I$ |
|----------------|-------------|-------------|-------------|
|                | $(0, 1, Q^2)$ | $(0, 0.003, Q^2)$ | $(0, 0.003, 0.7, Q^2)$ |
| Input          | 1.257       | 0.0150      | 1.232       |
| REGGE LO GLAP  | 1.255       | 0.0342      | 1.219       |
| LO GLAP + ln^2 x | 1.249    | 0.0493      | 1.198       |
| Input          | 1.257       | 0.0786      | 1.194       |
| GRSV LO GLAP   | 1.249       | 0.107       | 1.171       |
| LO GLAP + ln^2 x | 1.242  | 0.119       | 1.153       |
| Input          | 1.257       | 0.123       | 1.160       |
| GS LO GLAP     | 1.253       | 0.134       | 1.151       |
| LO GLAP + ln^2 x | 1.247  | 0.142       | 1.139       |
| EXPERIMENT     | 1.29±0.24   | **0.09±0.09** | **1.20±0.24** |

Fig. 3. Input parametrisations of the nonsingled spin structure function of the proton $g_1^{NS}(x, Q^2)$: REGEE (4.13) - dotted line; GRSV (4.14) - solid line; GS (4.15) - dashed line.
Fig. 4. $g_1^{NS}$ at $Q^2 = 10$ GeV$^2$ based on inputs: REGGE - dotted, GRSV - solid, GS - dashed. For each pair of lines the pure LO GLAP prediction lies below the LO GLAP+$\ln^2 x$ one.

Fig. 5. $g_1^{NS}$ at $Q^2 = 10$ GeV$^2$; similarly as in Fig.4 but for the measurable experimentally region of $x$. Squares show the recent SMC data 1997 [14].
Fig. 6. Function $6xg_1^{NS}$ at $Q^2 = 10$ GeV$^2$. Predictions based on the REGGE input - dotted and GRSV - solid. LO GLAP above LO GLAP+$\ln^2 x$ at $x = 0.2$. SMC 1997 data with statistical errors are shown.

Fig. 7. LO GLAP+$\ln^2 x$ predictions for function $6xg_1^{NS}$ at $Q^2 = 10$ GeV$^2$, based on input parametrisations: REGGE - dotted, GRSV - solid, GS - dashed. Plots are compared with SMC 1997 data.
5. Summary and conclusions

In this paper the main theoretical and experimental problems in nucleon spin structure physics have been briefly reviewed. The results of current experiments are deviation of the Ellis-Jaffe sum rule and validity of the Bjorken sum rule. This causes that the question "how is the spin of the nucleon made out of partons?" is still open. The great puzzle are experimental results which violating the Ellis-Jaffe sum rule imply that only a very small part of the spin of the proton is carried by quarks. So where is the nucleon spin? Maybe gluons take a large fraction of the nucleon spin? Or maybe the spin of the proton is "hidden" in orbital angular momentum of quarks and gluons? Maybe at last the solution of the spin problem lies in the small $x$ physics and the lacking spin of the nucleon is hidden in the unmeasured very small $x$ region. The answer the above questions will be possible thanks to the progress in theoretical and experimental research in the small $x$ physics. Perturbative QCD analysis, based on GLAP evolution equations is in a good agreement with experimental data. This agreement concern unpolarised and polarised structure functions of the nucleon $F_1$, $F_2$, $g_1$ within NLO approximation in the large and the moderately small Bjorken $x$ region. Unfortunately, practically lack of the experimental measurements in the very small $x$ region ($x \leq 10^{-3}$) makes the satisfactory verification of the theoretical QCD predictions in this region impossible. Knowledge of the behaviour of the nucleon spin structure functions when $x \to 0$ is crucial in the determination of Bjorken and Ellis-Jaffe sum rules i.e. in overcoming the "spin crisis". Understanding of the small $x$ physics in the polarised DIS processes requires to take into account all of these perturbative QCD effects which become significant in the small $x$ region and which could be verified by future experiments. Present QCD analyses, based on the GLAP LO or NLO $Q^2$ evolution seem to be incomplete when $x \to 0$. The growth of the unpolarised as well polarised structure functions of the nucleon in the small $x$ region is governed by leading double logarithmic terms of the form $\alpha_s^n \ln^{2n}(x)$, generated by ladder diagrams with quark and gluon exchange. This singular behaviour of the structure functions at low $x$, implied by $\ln^2 x$ terms, is however better visible in the polarised case. For unpolarised, nonsinglet structure functions of the nucleon the QCD evolution behaviour at small $x$ is screened by the leading Regge contribution. Therefore the spin dependent structure functions of the nucleon are a sensitive test of the perturbative QCD analyses in the low $x$ region. Our numerical analyses incorporating the LO GLAP evolution and the $\ln^2 x$ effects at small $x$ show, that the growth of the nonsinglet polarised structure function of the nucleon $g_1^{NS}$, implied by $\ln^2 x$ terms, is significant for $x \leq 10^{-2}$. Our predictions for $g_1^{NS}$ are in a good agreement with the
recent SMC data for small $x$ region ($x \sim 10^{-3}$). The contribution from the
low $x$ region ($x \leq 0.003$) to the Bjorken sum rule is found to be around 4%
(for Regge input $g_{1}^{NS}(x, Q^2_0)$) and 10% (for dynamical inputs) of the value
of the sum. Theoretical predictions for $g_{1}^{p,n,d}$, taking into account the $\ln^2 x$
resummation effects will be in the future verified experimentally. There are
a few hopeful experimental projects of the investigation of the nucleon’s spin
structure. One of these is the HERMES experiment (start in 1995) located
in HERA at DESY with a fixed polarised H,D or $^3$He target and a longitudi-
dinally polarised positron beam of 27.5 GeV [26]. The accessible kinematic
range is $0.004 < x < 1$ and $0.2 < Q^2 < 20 \text{ GeV}^2$. The HERMES experiment
allows a direct measurement of the polarised quark distributions for individual
flavours also $g_{1}^{p,n,d}(x, Q^2)$ and even $g_2(x, Q^2)$. The question of the gluon
polarisation is also addressed experimentally. The polarised gluon distribu-
tion $\Delta g(x, Q^2)$ may play a crucial role in understanding of the nucleon
spin structure. The measurement of $\Delta g(x, Q^2)$ in the charm production via
photon-gluon fusion process $\gamma^* g \rightarrow c\bar{c}$ will be possible at COMPASS exper-
iment at CERN [15]. In this project the polarised muons will be scattered
on polarised proton and deuteron targets. The energy of the muon beam
will be of 100 GeV and 200 GeV and the Bjorken $x$ region $x > 0.02$. The
COMPASS measurements are expected to start in 2000. A very important
program which will test many elements of QCD in the perturbative as well
as in the nonperturbative region is RHIC spin project at Brookhaven [21].
This program with polarised proton-proton collider will start in 2000 and
will allow for a measurement of the polarised gluon density via heavy quark
production ($gg \rightarrow Q\bar{Q}$) or via direct photon production ($gg \rightarrow \gamma q$). Finally,
a very promising experimental project in high energy spin physics is HERA
[8]. The polarisation of the proton and electron beams at $\sqrt{s} = 300 \text{ GeV}$
will enable to measure the structure function $g_1(x, Q^2)$ and spin dependent
quark distributions $\Delta q_f(x, Q^2)$ at very low $x$ ($x \sim 10^{-5}$). From polarised
di-jet production it will be possible to determinate the polarised gluon dis-
tribution $\Delta g(x, Q^2)$ for the region $0.002 < x < 0.2$. Additionally in HERA,
a program of polarised proton-proton collisions is proposed. This high en-
ergy proton-proton scattering will allow via $J/\psi$ production for the direct
determination of the gluon function $\Delta g(x, Q^2)$. The new HERA with po-
larised experiments and the largely extended kinematical region of $x$ and
$Q^2$ will contribute a lot to our understanding of high energy spin physics.
The problem of the spin structure of the nucleon is nowadays one of the
most important challenges for theory and experiment as well.
Acknowledgements

I would like to thank Jan Kwieciński for a great help and critical remarks during preparing this work. I am also grateful to Andrzej Kotlorz for useful discussions about numerical problems.

Appendix A

GLAP evolution of polarised quark and gluon distribution functions in the nucleon

In perturbative QCD distribution functions of partons $q(x,Q^2)$, $g(x,Q^2)$, $\Delta q(x,Q^2)$, $\Delta g(x,Q^2)$ evolve with $Q^2$. This evolution is described by GLAP \cite{3},\cite{9} equations and it is assumed that for polarised functions $\Delta p(x,Q^2)$ the evolution equations have the same form like for unpolarised functions $p(x,Q^2)$:

$$
\frac{d}{dt}[\Delta q_{NS}(x,t)] = \frac{\alpha_s(t)}{2\pi} \Delta P_{qq} \otimes \Delta q_{NS}(x,t) \quad (A.1)
$$

$$
\frac{d}{dt}[\Delta q_S(x,t)] = \frac{\alpha_s(t)}{2\pi} [\Delta P_{qq} \otimes \Delta q_S(x,t) + 2N_f \Delta P_{qG} \otimes \Delta g(x,t)] \quad (A.2)
$$

$$
\frac{d}{dt}[\Delta g(x,t)] = \frac{\alpha_s(t)}{2\pi} [\Delta P_{Gq} \otimes \Delta q_S(x,t) + 2N_f \Delta P_{GG} \otimes \Delta g(x,t)] \quad (A.3)
$$

where $t = \ln(Q^2/\Lambda^2)$ and

$$
P(x) \otimes q(x,t) \equiv \int_0^1 \frac{dz}{z} P(z) q\left(\frac{x}{z},t\right) \quad (A.4)
$$

Functions $C(x,\alpha_s)$ and $P_{ij}(x,\alpha_s)$ are calculated in leading LO approximation or with the next correction to LO i.e. in NLO approximation with respect to coupling constant $\alpha_s$:

$$
LO: \quad P_{ij}(x,\alpha_s) = \alpha_s P_{ij}^{(0)}(x) \quad (A.5)
$$

$$
NLO: \quad P_{ij}(x,\alpha_s) = \alpha_s P_{ij}^{(0)}(x) + \alpha_s^2 P_{ij}^{(1)}(x) \quad (A.6)
$$

Functions $P_{ij}(x)$ for the unpolarised case are different from those $\Delta P_{ij}(x)$ for the polarised one \cite{10}. In LO they have a form:

$$
P_{qq}^{(0)}(x) = \frac{4}{3} \frac{1 + x^2}{(1-x)_+} + 2\delta(1-x) \quad (A.7)
$$

$$
P_{qG}^{(0)}(x) = \frac{1}{2} (x^2 + (1-x)^2) \quad (A.8)
$$
\[ P^{(0)}_{Gq}(x) = \frac{4}{3} \frac{1 + (1 - x)^2}{x} \]  
(A.9)

\[ P^{(0)}_{GG}(x) = 3 \left[ \frac{x}{(1 - x)_+} + \frac{1 - x}{x} + x(1 - x) + \frac{3}{4} \delta(1 - x) \right] \]  
(A.10)

and

\[ \Delta P^{(0)}_{qq}(x) = \frac{4}{3} \frac{1 + x^2}{(1 - x)_+} \]  
(A.11)

\[ \Delta P^{(0)}_{qG}(x) = \frac{1}{2} (2x - 1) \]  
(A.12)

\[ \Delta P^{(0)}_{Gq}(x) = \frac{4}{3} (2 - x) \]  
(A.13)

\[ \Delta P^{(0)}_{GG}(x) = 3 \left[ \frac{1 + x^4}{(1 - x)_+} + (3 - 3x + x^2 + x^3) - \frac{7}{12} \delta(1 - x) \right] \]  
(A.14)

where \((1 - x)_+\) is defined as:

\[ \int_0^1 \frac{f(x)dx}{(1 - x)_+} \equiv \int_0^1 \frac{f(x) - f(1)}{(1 - x)}dx \]  
(A.15)

**Appendix B**

*Dynamical input parametrisations of the nonsinglet polarised structure function \(g_1^{NS}\)*

In our calculations we adopt GRSV (Glück, Reya, Stratmann, Vogelsang) [22] and GS (Gehrmann, Stirling) [23] parametrisations of valence quarks \(\Delta q_{\text{val}}\). We assume SU(3) flavour symmetric scenario, where

\[ \Delta \bar{u} = \Delta \bar{d} \]  
(B.1)

This assumption leads to formula (4.4):

\[ g_1^{NS} \equiv g_1^u - g_1^n = \frac{1}{6} (\Delta u - \Delta d) = \frac{1}{6} (\Delta u_{\text{val}} - \Delta d_{\text{val}}) \]  
(B.2)

Input parametrisation of \(\Delta u_{\text{val}}\) and \(\Delta d_{\text{val}}\) have a general form:

**GRSV**: \[ \Delta q_{\text{val}} = N x^{a_2} x^{a_1-1} (1 + A x^b + B x + C x^{3/2})(1 - x)^D \]  
(B.3)

**GS**: \[ \Delta q_{\text{val}} = N' x^{d'-1} (1 + \gamma x + \rho \sqrt{x})(1 - x)^{D'} \]  
(B.4)
where $N, N'$ are normalisation factors, implied by Bjorken and Ellis-Jaffe sum rules (2.11)-(2.14). These sum rules for input scale $Q_0^2$ can be read as

$$ a_3 = \int_0^1 (\Delta u_{val} - \Delta d_{val}) dx = 1.257 \quad (B.5) $$

$$ a_8 = \int_0^1 (\Delta u_{val} + \Delta d_{val}) dx = 0.579 \quad (B.6) $$

(B.5) and (B.6) give immediately

$$ \int_0^1 \Delta u_{val} dx = 0.918 \quad (B.7) $$

$$ \int_0^1 \Delta d_{val} dx = -0.339 \quad (B.8) $$

what allows to find $N, N'$ factors. The full set of input parameters for GRSV and GS distributions is as follows:

**GRSV:**

$Q_0^2 = 1$ GeV$^2$, $\Lambda_{QCD} = 232$ MeV

- for $\Delta u_{val}$: $N = 1.964, a_1 = 0.573, a_2 = 0.16, b = 0.175$,
  $A = -0.583, B = 1.723, C = 3.436, D = 3.486$
- for $\Delta d_{val}$: $N = -0.162, a_1 = 0.376, a_2 = 0, b = 0.529$,
  $A = 1.195, B = 6.164, C = 2.726, D = 4.215$

**GS:**

$Q_0^2 = 4$ GeV$^2$, $\Lambda_{QCD} = 200$ MeV

- for $\Delta u_{val}$: $N' = 1.741, a' = 0.578, \gamma = 9.38, \rho = -4.26, D' = 3.73$
- for $\Delta d_{val}$: $N' = -1.176, a' = 0.666, \gamma = 10.46, \rho = -5.10, D' = 4.73$

In both GRSV and GS inputs we employ the LO fits. Thus the input parametrisations have final forms:

**GRSV:**

$$ \Delta u_{val} = 1.964x^{-0.267}(1-0.583x^{0.175} + 1.723x + 3.436x^{3/2})(1-x)^{3.486} \quad (B.9) $$
$$
\Delta d_{val} = -0.162 x^{-0.624} (1 + 1.195x^{0.529} + 6.164x + 2.726x^{3/2})(1 - x)^{4.215} \tag{B.10}$$

GS:

$$
\Delta u_{val} = 1.741 x^{-0.422} (1 + 9.38x - 4.26\sqrt{x})(1 - x)^{3.73} \tag{B.11}
$$

$$
\Delta d_{val} = -1.176 x^{-0.334} (1 + 10.46x - 5.10\sqrt{x})(1 - x)^{4.73} \tag{B.12}
$$

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