The strength of the hull on a high longitudinal wave

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Abstract. The paper presents the method for calculating the strength of the ship vessel on the longitudinal wave, based on the modern theory of the beam and computer mathematics. It consists of two parts, which are the determination of hydrodynamic loads on the hull during the passage of a wave and the calculation of the stress-strain state caused by the ship vessel as a beam. The main feature of the technique is that the loads and deflection are considered simultaneously, which was made possible by computer mathematics. The developed method allows; increasing the accuracy of calculations of ship hulls for durability, has a relatively low cost, and does not require expensive software.

Keywords: Hull strength, Deformation, Beam bending, Hydrodynamic loads, Computer mathematics, Boundary-value problem.

Introduction

The problem of the overall longitudinal strength of ship hulls is still actual. On the longitudinal wave, the hull of the ship bends under the action of gravity and hydrodynamic loads. In the cross-sections of the hull, bending moments, shear forces and stresses from a common bend arise.

Modern methods of calculating the total longitudinal strength in Russia, proposed by the authors [1-4, 11-14, 17, 18], based on experimental data obtained during the test vessels models in pools. Equations, which are used, do not take into account the redistribution of loads during the deformation of the hull. The basic equations are presented in the second part of the rules of the Maritime Register [5].

New techniques based on the finite element method (FEA) are quite expensive and do not take into account the redistribution of loads during bending of the body [6-7].

The development of mathematical modeling with the use of computer mathematics has opened up a huge scope for solving complex engineering problems [9-10]. The Mathcad [8] package used in the work is very effective and is best combined with classical approaches in the language of formulas.

The purpose of the work is to create an engineering method for calculating the strength of a ship vessel on a high longitudinal wave taking into account the hydrodynamic forces, based on the modern theory of beams and computer mathematics. The main feature of this technique is that hydraulic and hydrodynamic loads and deflection are considered simultaneously. This became possible thanks to computer math.

The practical significance of the work is in the ability to predict deformations and stresses of the ship hull under the action of a longitudinal wave without experimental data and the use of expensive commercial programs, which will reduce the cost of designing ships.
The Determination of Hydrodynamic Loads

The body section is symmetrical with respect to the Cartesian axis \( y \) and is given by the equation:

\[
z = z(y), \quad 0 \leq y \leq H .
\]  

(1)

![Figure 1. Body section](image)

Distance \( h \) in fig. 1 is the draft of the ship, on the submerged part of the circuit \( 0 \leq y \leq h \). At each point of the contour of the section, we have the ort of the normal, the components which are found using differential geometry:

\[
n_x dy + n_z dz = 0, \quad n_y = -z'(y)/\sqrt{1+z'^2} .
\]  

(2)

Each section has geometrical characteristics: static moment \( S \) and moment of inertia \( I_z \).

The buoyant force of pressure per unit length of the ship as a beam, when there is no wave and hull deflection, is:

\[
q_0(h) = -2 \int_0^h p(y)n_y \sqrt{1+z'^2} dy = 2\gamma \int_0^h (h-y)z'dy .
\]  

(3)

To account for hydrodynamic forces, the wave coefficient \( c_w \) is used, taking into account the load and acceleration on the hull from the sea. The coefficient is determined depending on the length of the ship vessel:

\[
c_w = 0,0856L \quad \text{at} \ L \leq 90 \text{ m},
\]

\[
c_w = 10,75 - \left(\frac{300 - L}{100}\right)^{3/2} \quad \text{at} \ 90 \leq L \leq 300 \text{ m},
\]  

(4)

\[
c_w = 10,75 \quad \text{at} \ 300 \leq L \leq 350 \text{ m}.
\]

Theoretical and practical studies show that the pressure of water under the top and under the sole of the wave is different from the hydrostatic. The pressure increases under the base of the wave. This is due to the orbital movement of water particles in the wave. This effect is named Smith [15].

The Smith effect is taken into account by multiplying the ordinate in the olfactory profile by the coefficient \( \kappa \leq 1 \), called the Smith Amendment.
Hydrodynamic forces arise due to the changes in the field due to the presence of the ship vessel itself. The ship vessel violates the circular motion of particles, contributes to the formation of secondary surface waves, etc. This effect is called diffraction and is taken into account by the diffraction correction:

\[ \chi_0 = 1 - \varphi \alpha \frac{\pi^2 BL}{4L \lambda}, \]  

(5)

where \( B \) is the width of the ship vessel amidships, \( L \) is the length of the ship vessel, \( \alpha \) is the coefficient of completeness of the waterline on calm water, \( \lambda \) is wavelength and \( \varphi \) is coefficient of longitudinal completeness.

With both amendments, we have the estimated wave height:

\[ h_{ef} = \kappa \chi_0 h \leq h. \]  

(6)

The wave profile is determined by the function \( Y(x,t) \) from Cartesian coordinates \( 0 \leq x \leq l \) (along the ship) and time \( t \). Deflection of the same hull as beams denote \( u(x,t) \) (both functions are positive with upward direction). The intensity of the distributed load, taking into account the hydrodynamic forces, is equal to:

\[ q(x,t,u) = q c \varphi w [Y(x,t) - u] - f(x). \]  

(7)

Added static load \( f = \rho g \) from the weight of the ship (\( \rho \) is the linear mass and \( g \) is the gravity acceleration).

Note that the dependence (7) is nonlinear. This almost eliminates the possibility of an analytical calculation of the deflection. However, modern computer mathematics (Mathead [8]) removes almost all the difficulties.

**ODE System and Its Solution**

A beam is a straight bending rod. Direct the Cartesian \( x \)-axis along the beam through the centers of gravity of the sections. Let the beam section be symmetrical about the axis \( y \), and bending occurs in the plane \( x, y \).

The shape of the ship hull will take a cylindrical and constant cross section.

Then we have the following system of four ordinary differential equations (ODE):

\[ Q' + q = 0, \quad M' + Q = 0, \quad \theta' = AM, \quad u' = 0. \]  

(8)

Here the prime means the derivative with respect to \( x \), \( Q \) represents the shear force, \( M \) is the bending moment, \( \theta \) designated to the angle of rotation of the section, \( u \) is the deflection, \( A \) is the Flexural compliance and \( q \) is the distributed load per unit length.

The first two equations express the balance of forces and moments for the beam element. The third and fourth equations are related to the formulas of the theory of elasticity:

\[ u_x(x, y, z) = u(x), \quad u_y(x, y, z) = -\theta(x)y, \]

\[ \varepsilon_x = \partial_x u_x = -\theta' y = \sigma_x / E, \quad 2\varepsilon_{xy} = \partial_x u_y + \partial_y u_x = 0 \Rightarrow \]

\[ M = -\int_F y \sigma_x \, dF = EI \theta', \quad \theta = u', \]  

(9)

\[ I = \int_F (y - y_0)^2 \, dF. \]

Here \( u_x, u_y \) are displacements (in a flat statement), \( \varepsilon_x, \varepsilon_{xy}, \sigma_x \) are components of strain and stress tensors and \( E \) is Young’s module. Integrals are taken over the section \( F \).
The system reduces to a well-known fourth-order equation:

$$A^{-1}u^IV = q.$$  

Boundary conditions at free ends without load are: $u^0 = 0, u''' = 0$.

We have a boundary value problem with ODE and boundary conditions. The problem can be solved in the Mathcad package using the shooting method implemented by the built-in functions $sbval - Rkadapt$.

The calculation is done for the ship vessel with the following parameters: $L = 130 \text{ m}, \ H = 10 \text{ m}, \ I_s = 6,3 \text{ m}^4$, wave height given sinusoid, $Y_m = 3 \text{ m}, \ \lambda = L, \ c_w = 8$. The material of the hull is steel, so we have the following graphs of the distribution of the shearing force ($Q(x)$), bending moment ($M(x)$), deflection ($u(x)$) and precipitation ($Y(x)-u(x)$), stress on the deck ($\sigma(x,H)$), presented in fig.2.
Dynamic Body Bend

Wave load on the hull of the vessel and the bending it causes are dynamic processes with a dependence on the coordinates and time. For deflection, we have the following nonlinear partial differential equation [9]:

\[ \Gamma[u] \equiv (a(x)u^*)^* + \rho(x)i\ddot{u} - c_uq_0[Y(x,t) - u] + f(x) = 0. \]  \hspace{1cm} (11)

The boundary conditions for (11) are the same as above in the absence of forces and moments. Initial conditions: the equality of zero deflection and its speed \( u = \dot{u} = 0 \).

Until recently, solution of the (11) was not available. However, even now, with the advent of computer mathematics, the problem remains rather difficult. To construct an approximate solution, we use the Galerkin method [16].

Differential equation (11) is equivalent to the following variation equation:
\[ \int_0^L \Gamma[u] \delta u \, dx = 0, \quad (12) \]

and at the ends \( u' = (au')' = 0 \). Approximate solution will be in the form of:

\[ u(x, t) = \sum_{i=1}^n U_i(t) \varphi_i(x); \quad \delta u = \sum_i \varphi_i \delta U_i; \quad x = 0, L; \quad \varphi_i'' = (a\varphi_i')' = 0 \quad (13) \]

Coordinate functions \( \varphi_i \) set by us. Functions \( U_i(t) \) is unknown variable, for them we derive the ODE. From (13) we obtain the nonlinear system of ODE:

\[ \sum_i (m_i \ddot{U}_i + c_i U_i) = f_k - G_k(U_i, t); \quad m_i \equiv \int_0^L \varphi_i \varphi_i \, dx, \quad c_i \equiv \int_0^L (a\varphi_i')' \varphi_i \, dx, \quad (14) \]

\[ f_k \equiv \int_0^L f \varphi_k \, dx, \quad G_k(U_i, t) \equiv \int_0^L q_k[Y(x, t) - \sum_i U_i \varphi_i] \varphi_i \, dx \]

The following coordinate functions are suitable for solving:

\[ \varphi_1(x) = \left( \frac{x}{L} \right) \left( 1 - \frac{x}{L} \right)^2, \quad \varphi_2(x) = 1, \quad \varphi_3(x) = \frac{x}{L}. \quad (15) \]

Having solved the system (14) by the Runge-Kutta method, we find the stresses as shown in fig. 3.

![Figure 3](image-url)  
**Figure 3** Fig.3. Stress on the mid-frame

**Results**

1. A method has been developed for calculating the deformation and strength of a ship hull on a longitudinal wave, based on modern beam theory and computer mathematics.
2. The function of hydrodynamic load, non-linearly dependent on the deflection and wave height, is derived.
3. The ODE system was compiled, a boundary-value problem was posed, and calculation of the stress-strain state caused by the ship vessel as a beam was done.
Conclusion

The developed technique has a low cost and high speed of work, compared with large packages of 3D modeling and calculation. The technique will allow the use of experimental data to predict the strength of the hull in the specified modes of navigation.

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