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**Anisotropic Elasticity for Soils: A Synthesis of Some Key Issues**

**Abstract:** Traditionally, the elastic response of soils has been assumed to be isotropic. Natural soils are, however, more likely to exhibit anisotropic response. For example, sedimentary soils, which are typically deposited under gravity, possess different properties in the direction of deposition as opposed to the planes normal to this direction. This paper synthesizes several key issues related to anisotropic elastic material idealizations for soils. Emphasis is placed on transversely isotropic (“cross-anisotropic”) elastic material idealizations.

**Keywords:** elasticity, isotropy, anisotropy, orthotropic, transversely isotropic.

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1. **Introduction.** Traditionally, the elastic response of soils has been assumed to be isotropic. This was primarily done for two reasons. First, was a desire not to overly complicate analytical formulations. Second, was the lack of suitable experimental apparatus to measure the elastic constants necessary to characterize the anisotropic elastic response of soils. Over the last 35 or so years, the latter constraint has been significantly lessened, as substantial progress has been made in the development of experimental apparatus and techniques that facilitate measurement of the aforementioned elastic constants. Such measurements confirm that soils indeed exhibit elastic response, albeit at low strain levels, and that this response is typically anisotropic. Consequently, anisotropic elastic material idealizations for soils have become significantly more tractable.

This paper reviews some key issues related to anisotropic elastic material idealizations for soils. Although some of these issues have been discussed in previous papers, missing from the earlier documents was any attempt to synthesize these issues. Such a synthesis is presented in this paper.

2. **Elastic Deformations in Soils**

In an elastic material, the state of stress is a function only of the current state of deformation; it does not depend on the history of straining or loading. When loaded, an elastic material stores 100% of the energy due to deformation (i.e., strain energy). Upon removal of the applied loading, the material releases 100% of the stored energy and returns to its initial state; no permanent deformation is realized. Finally, the response is rate-independent; i.e., the rate at which the loading is applied has no effect upon the material response.

Elastic response is seemingly ambiguous for highly nonlinear materials such as soils [10]. However, under certain conditions, the behavior of soils is very nearly elastic. For example, Hardin and Black [6] found that at very small axial strain levels (i.e., less than 0.01%), dynamic loading tests on normally consolidated clays exhibited only very small hysteretic damping, thus indicating nearly elastic response. Subsequent experimental studies, performed in the 1980’s and 90’s, showed that the small-strain behavior of soils is generally linear elastic and time- and rate-independent [25, 11, 7, 26]. Based on the results of very careful experiments performed on a variety of geomaterials, Tatsuoka et al. [27] subsequently concluded that such materials exhibit “imperfect elasticity,” even at strains less than 0.001%. Consequently geomaterials were considered to exhibit “quasi-elastic” response that was essentially rate-independent and nearly linear. This was consistent with the earlier findings of Hicher [7]. In summary, the exact “threshold” strain value, below which geomaterials behave truly elastically, is still debatable. Nevertheless, elastic material characteristics for such materials are commonly considered to be applicable for strains smaller than 0.001% for uncemented soils [25, 11, 7].

In granular soils elastic deformations are attributed primarily to the distortion of individual particles. This occurs at relatively low levels of loading. At higher load levels the particles will move relative to one another, resulting in a permanent (inelastic) deformation. If the load level is particularly high, the particle may also begin to crush.
In cohesive soils subjected to changes in effective stress conditions, both the shear and normal forces at points of interparticle contact undergo changes in magnitude. These changes produce an elastic bending of particles, as well as a relaxation of previously bent or distorted particles or particle clusters. Both of these phenomena occur without slippage or breakage of interparticle bonds and result in an instantaneous elastic deformation of the macroelement.

3. The Issue of Elastic Isotropy

Limited experimental results on several different sands indicate isotropic behavior upon unloading, even when the strains during loading indicated anisotropic behavior [23]. A similar conclusion was reached by Krizek [12], who presented results of unconfined compression tests on sedimented specimens of kaolin clay with different degrees of inherent anisotropy. Results for sensitive clays studied by Wong and Mitchell [28] also showed nearly isotropic elastic behavior. The associated plastic stress-strain relations were, however, anisotropic. Citing the above results for sands and clays, Lade and Nelson [14] concluded that although microscopic elastic behavior of geomaterials is randomly anisotropic and non-homogeneous, such materials can be considered as macroscopically homogeneous and isotropic. This is particularly true for remolded laboratory soil samples.

For their characterization, isotropic materials require the values of two material constants. Traditionally, the bulk modulus ($K$) and shear modulus ($G$), or the elastic (Young’s) modulus ($E$) and Poisson’s ratio ($\nu$) have been used to characterize isotropic elastic materials. Experimental results for geomaterials indicate that they generally exhibit nonlinear elastic response, with $K$, $G$, and $E$ being primarily dependent on the 1) state of stress, 2) density (or void ratio), and 3) stress history [6]. The importance of this nonlinearity has been generally recognized and a variety of models, possessing varying degrees of complexity, have been proposed [9, 10, 14, 7].

4. Anisotropic Elastic Material Idealizations

Natural soils are more likely to exhibit anisotropic response. For example, sedimentary soils, which are typically deposited under gravity, possess different properties in the direction of deposition as opposed to the planes normal to this direction. For a general homogeneous, anisotropic linear elastic (Hookian) material, in the absence of initial strains and stresses, the constitutive relations, in “direct” vector-matrix form, are given by

$$\delta \varepsilon = A \delta \sigma'$$

where $A$ is a symmetric ($N_{rough} \times N_{rough}$) matrix of compliance coefficients characterizing the material, $\delta \varepsilon$ and $\delta \sigma'$ are ($N_{rough} \times 1$) vectors of infinitesimal elastic strain and effective stress increments, respectively, and $N_{rough}$ is the number of stress and strain components (for three-dimensional analyses, $N_{rough} = 6$; for torsionless axisymmetry, $N_{rough} = 4$; for plane strain analyses, $N_{rough} = 3$.).

For three-dimensional analyses,

$$\delta \varepsilon = \begin{bmatrix} \delta \varepsilon_{11} & \delta \varepsilon_{22} & \delta \varepsilon_{33} & \delta \gamma_{12} & \delta \gamma_{13} & \delta \gamma_{23} \end{bmatrix}^T$$

$$\delta \sigma' = \begin{bmatrix} \delta \sigma'_{11} & \delta \sigma'_{22} & \delta \sigma'_{33} & \delta \sigma'_{12} & \delta \sigma'_{13} & \delta \sigma'_{23} \end{bmatrix}^T$$

where $\gamma_{12}$, $\gamma_{13}$, and $\gamma_{23}$ are engineering shear strains, and the superscript $T$ denotes the operation of vector transposition.

Written in “inverse” “direct” vector-matrix form, the constitutive relations are are given by generalized Hooke’s law; viz.,

$$\delta \sigma' = D \delta \varepsilon$$

where $D$, which is the inverse of $A$, represents the symmetric ($N_{rough} \times N_{rough}$) matrix of elastic moduli.

Due to symmetry, in their most general form, both $A$ and $D$ contain 21 independent coefficients that characterize the elastic material. The prospect of determining values for 21 material coefficients from experimental results is a formidable task. Fortunately, however, most of the important engineering materials possess some internal structure that exhibits certain symmetries that reduce the number of independent elastic coefficients required to characterize the material. For the present development, it is expedient to first consider an orthotropic material idealization.

4.1 Orthotropic Elastic Idealizations

Consider a material through each point of which pass three mutually perpendicular planes of elastic symmetry. If similar planes are parallel at all points in the material, then taking the $(x_1, x_2, x_3) \equiv$
(x, y, z) coordinate axes normal to these planes (i.e., along the principal directions) it follows that there should be no interaction between the various shear components or between the shear and normal components. Consequently, the compliance matrix has the following entries [15]:

\[
A = \begin{bmatrix}
\frac{1}{E_1} & -\nu_{21}/E_2 & -\nu_{31}/E_3 & 0 & 0 & 0 \\
-\nu_{12}/E_1 & \frac{1}{E_2} & -\nu_{32}/E_3 & 0 & 0 & 0 \\
-\nu_{13}/E_1 & -\nu_{23}/E_2 & \frac{1}{E_3} & 0 & 0 & 0 \\
0 & 0 & 0 & 1/G_{12} & 0 & 0 \\
0 & 0 & 0 & 0 & 1/G_{13} & 0 \\
0 & 0 & 0 & 0 & 0 & 1/G_{23}
\end{bmatrix}
\]  

(3)

The material constants appearing in equation (3) are defined as follows: \(E_1, E_2,\) and \(E_3\) are elastic moduli associated with tension or compression in the material coordinate direction \(x_1, x_2,\) and \(x_3,\) respectively. These moduli are obtained under drained conditions; they are thus defined in terms of effective stress. The \(G_{ij}\) is the elastic shear modulus that relates the shear stress \(\sigma_{ij}\) to the shear strain \(\gamma_{ij},\) where no summation on repeated indices is implied. Finally, \(\nu_{ij}\) is the Poisson’s ratio that is equal to the ratio of the lateral contraction in the \(x_j\) material coordinate direction resulting from a uniaxial extension in the \(x_i\) coordinate direction [15].

Symmetry of \(A\) implies that \(\nu_{21}/E_2 = \nu_{12}/E_1, \nu_{21}/E_2 = \nu_{12}/E_1,\) and \(\nu_{32}/E_3 = \nu_{23}/E_2.\) Thus, only nine of the twelve elastic constants entering equation (3) are independent; viz,

\[
E_1, E_2, E_3, \nu_{12}, \nu_{13}, \nu_{23}, G_{12}, G_{13}, G_{23}
\]

4.2 Transversely Isotropic Idealizations

Due to the manner in which natural soils are deposited, it is logical to expect them to exhibit approximately transversely isotropic (or “cross-anisotropic”) response. While this realization is not new [3, 22, 2, 5, 24], the lack of suitable experimental apparatus to accurately measure the five elastic constants associated with transverse isotropy has, in the past, precluded the use of such idealizations. More recently [8, 17, 13, 1, 21], substantial progress has been made in experimental techniques that facilitate the measurement of the aforementioned elastic constants.

Through all points of a transversely isotropic material there pass parallel planes of elastic symmetry in which all directions are elastically equivalent (i.e., planes of isotropy). Thus at each point there exists one principal direction and an infinite number of principal directions in a plane normal to the first direction [15]. Assume that the local material axes \((\tilde{x}_1, \tilde{x}_2, \tilde{x}_3) \equiv (\tilde{x}, \tilde{y}, \tilde{z})\) coincide with the global \(x, y\) and \(z\) coordinate axes (Figure 1). Furthermore, assume that the global \(x\)-axis is taken normal to the planes of isotropy, with the global \(y\) and \(z\) axes directed arbitrarily in such planes.

**Figure 1** – Schematic illustration of an element of transversely isotropic material.
In light of this definition of a transversely isotropic material, and in relation to the orthotropic elastic constants given in equation (3), the following elastic constants are defined: \( E_1 \equiv E_n \), where \( E_n \) is the elastic modulus for compression or tension in a direction normal to the plane of isotropy, and \( E_2 = E_3 = E_t \), where \( E_t \) is the elastic modulus for compression or tension in the plane of isotropy (i.e., in a direction tangential to the \( x \)-axis). Since the \( y - z \) plane is a plane of isotropy, \( \nu_{21} = \nu_{31} \equiv \nu_{nt} \), where \( \nu_{nt} \) is the Poisson’s ratio characterizing the lateral contraction normal to the plane of isotropy when tension is applied in the plane. The modulus \( G_{12} = G_{13} \equiv G_{nt} \) is associated with shearing involving \( \gamma_{12} \) and \( \gamma_{13} \). Finally, \( G_{23} \equiv G_{tt} \) characterizes shearing in the plane of isotropy. It is given by \( 1/G_{tt} = 2(1 + \nu_{tt})/E_t \), from which it is evident that \( G_{tt} \) is thus not an independent material constant.

From equation (3), symmetry of \( A \) requires that \( A_{23} = A_{32} \), giving \( \nu_{32}/E_3 = \nu_{23}/E_2 \). Since \( E_2 = E_3 \), it follows that \( \nu_{32} = \nu_{23} \equiv \nu_{tt} \), where \( \nu_{tt} \) is the Poisson’s ratio characterizing transverse contraction in the plane of isotropy when tension is applied in the same plane.

Symmetry considerations also require that \( A_{12} = A_{21} \) and \( A_{13} = A_{31} \), giving \( \nu_{12}/E_1 = \nu_{21}/E_1 \) and \( \nu_{31}/E_3 = \nu_{31}/E_1 \). Since \( \nu_{31} = \nu_{21} \), it follows that now \( \nu_{13}/E_1 = \nu_{23}/E_1 \), thus giving \( \nu_{12} = \nu_{13} = \nu_{nt} \). Here \( \nu_{nt} \) is the Poisson’s ratio characterizing the lateral contraction in the plane of isotropy when tension is applied normal to the plane.

When the global \( x \)-axis is taken normal to the planes of isotropy, a transversely isotropic material is thus characterized by the values of five material constants, namely:

\[
E_t, E_n, \nu_{nt}, \nu_{tt}, G_{nt}
\]

The compliance matrix given by equation (3) thus becomes

\[
A = \begin{bmatrix}
1/E_n & -\nu_{tn}/E_t & -\nu_{tn}/E_t & 0 & 0 & 0 \\
-\nu_{tn}/E_n & 1/E_t & -\nu_{tt}/E_t & 0 & 0 & 0 \\
-\nu_{nt}/E_n & -\nu_{tt}/E_t & 1/E_t & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1/G_{nt} & 0 \\
0 & 0 & 0 & 0 & 0 & 2(1 + \nu_{tt})/E_t \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

where \( \nu_{tn} = \nu_{nt}(E_t/E_n) \).

5. Volumetric Strain

Assuming infinitesimal kinematics, the elastic volumetric strain increment for an anisotropic elastic material described by equation (1) is

\[
\delta \varepsilon_v^e = \delta \varepsilon_{11}^e + \delta \varepsilon_{22}^e + \delta \varepsilon_{33}^e \\
= (A_{11} + A_{21} + A_{31}) \delta \sigma_{11}' + (A_{12} + A_{22} + A_{32}) \delta \sigma_{22}' + (A_{13} + A_{23} + A_{33}) \delta \sigma_{33}'
\]

Substituting the requisite compliance entries from equation (3) into equation (5), the elastic volumetric strain increment for an orthotropic elastic material idealization is

\[
\delta \varepsilon_v^e = \frac{1}{E_1} (1 - \nu_{12} - \nu_{13}) \delta \sigma_{11}' + \frac{1}{E_2} (1 - \nu_{21} - \nu_{23}) \delta \sigma_{22}' + \frac{1}{E_3} (1 - \nu_{31} - \nu_{32}) \delta \sigma_{33}'
\]

For the transversely isotropic elastic material idealization defined by equation (4), equation (6) reduces to

\[
\delta \varepsilon_v^e = \frac{1}{E_n} (1 - 2\nu_{nt}) \delta \sigma_{11}' + \frac{1}{E_t} (1 - \nu_{tn} - \nu_{tt}) \delta \sigma_{22}' + \frac{1}{E_t} (1 - \nu_{tt} - \nu_{tt}) \delta \sigma_{33}'
\]

For the special case of isotropic compression, \( \delta \sigma_{11}' = \delta \sigma_{22}' = \delta \sigma_{33}' \). The elastic volumetric strain increment for an orthotropic elastic material idealization is thus

\[
\delta \varepsilon_v^e = \left[ \frac{1}{E_1} (1 - \nu_{12} - \nu_{13}) + \frac{1}{E_2} (1 - \nu_{21} - \nu_{23}) + \frac{1}{E_3} (1 - \nu_{31} - \nu_{32}) \right] \delta \sigma_{11}'
\]
For a transversely isotropic elastic material subjected to a state of isotropic compression, equation (6) reduces to

$$\delta \varepsilon_v^c = \left[ \frac{1}{E_n} (1 - 2 \nu_{nt}) + \frac{1}{E_t} (1 - 2 \nu_{tt}) + \frac{1}{E_t} (1 - 2 \nu_{tt}) \right] \delta \sigma'_{11} = \frac{1}{K} \delta \sigma'_{11}$$

(9)

where \( K \) is a generalized bulk modulus.

For completeness, note that for an isotropic elastic material, \( E_1 = E_2 = E_3 = E \), \( \nu_{12} = \nu_{13} = \nu_{23} = \nu_{31} = \nu_{32} = \nu \). Equation (8) thus reduces to

$$\delta \varepsilon_v^c = \frac{1}{E} \left[ 3(1 - 2\nu) \right] \delta \sigma'_{11} = \frac{1}{K} \delta \sigma'_{11}$$

where \( K \) is now equal to the “drained” elastic bulk modulus \( K' = E/3(1 - 2\nu) \).

6. Undrained Conditions

For a saturated anisotropic elastic soil with incompressible fluid and solid phase, under undrained conditions, equation (5) becomes

$$\delta \varepsilon_v^c = \delta \varepsilon_{11}^c + \delta \varepsilon_{22}^c + \delta \varepsilon_{33}^c = 0$$

(10)

For an orthotropic elastic material idealization, equation (6) gives

$$\frac{1}{E_1} (1 - \nu_{12} - \nu_{13}) \delta \sigma'_{11} + \frac{1}{E_2} (1 - \nu_{21} - \nu_{23}) \delta \sigma'_{22} + \frac{1}{E_3} (1 - \nu_{31} - \nu_{32}) \delta \sigma'_{33} = 0$$

(11)

Since the normal effective stress increments \( \delta \sigma'_{11} \), \( \delta \sigma'_{22} \) and \( \delta \sigma'_{33} \) are, in general, non-zero, and since \( E_1 > 0 \), \( E_2 > 0 \) and \( E_3 > 0 \), the kinematic constraint of zero volume imposes the following constraints on the Poisson’s ratios associated with an orthotropic material idealization:

$$(1 - \nu_{12} - \nu_{13}) = 0 \quad ; \quad (1 - \nu_{21} - \nu_{23}) = 0 \quad ; \quad (1 - \nu_{31} - \nu_{32}) = 0$$

(12)

For a transversely isotropic elastic material idealization, equation (7) gives

$$\frac{1}{E_n} (1 - 2\nu_{nt}) \delta \sigma'_{11} + \frac{1}{E_t} (1 - \nu_{tn} - \nu_{tt}) \delta \sigma'_{22} + \frac{1}{E_t} (1 - \nu_{tn} - \nu_{tt}) \delta \sigma'_{33} = 0$$

(13)

Since the normal effective stress increments \( \delta \sigma'_{11} \), \( \delta \sigma'_{22} \) and \( \delta \sigma'_{33} \) are, in general, non-zero, and since \( E_n > 0 \) and \( E_t > 0 \), the kinematic constraint of zero volume imposes the following constraints on the Poisson’s ratios associated with a transversely isotropic material idealization:

$$\frac{1}{E_n} (1 - 2\nu_{nt}) = 0 \quad ; \quad \frac{1}{E_t} (1 - \nu_{tn} - \nu_{tt}) = 0$$

(14)

Since \( E_n > 0 \), the first of the constraint equations (14) gives \( \nu_{nt} = 1/2 \). Using this value, along with the relation \( \nu_{tn} = E_t \nu_{nt}/E_n \), which is due to the symmetry of \( A \) (recall equation 4), gives

$$\nu_{tn} = \frac{1}{2} \left( \frac{E_t}{E_n} \right)$$

(15)

Substituting equation (15) into the second constraint equation (14), and recalling that \( E_t > 0 \), gives

$$\nu_{tt} = 1 - \frac{1}{2} \left( \frac{E_t}{E_n} \right)$$

(16)

Lempriere [?] showed that \( \nu_{tt} \) must be in the range \( -1 < \nu_{tt} < 1 \). Combining this fact with equation (16), the admissible range for the ratio of these elastic moduli is thus \( 0 < (E_t/E_n) < 4 \).

In summary, since \( \nu_{nt} = 1/2 \) and \( \nu_{tt} \) is computed using equation (16), the number of independent material constants associated with a transversely isotropic elastic idealization under undrained conditions thus reduces from five to three; i.e., \( E_n \), \( E_t \) and \( G_{nt} \).

7. Plane Strain Conditions

Consider a transverse isotropic material idealization with the global \( x \)-axis again taken normal to the planes of isotropy. The \( z \)-axis is chosen to coincide with the “long” direction of the material. As such, \( \gamma_{13} = \gamma_{23} = \varepsilon_{33} = 0 \). From the third of equations (4),

$$\varepsilon_{33} = -\frac{\nu_{nt}}{E_n} \sigma'_{11} - \frac{\nu_{tt}}{E_t} \sigma'_{22} + \frac{1}{E_t} \sigma'_{33} = 0 \quad \Rightarrow \quad \sigma'_{33} = E_t \left( \frac{\nu_{nt}}{E_n} \sigma'_{11} + \frac{\nu_{tt}}{E_t} \sigma'_{22} \right)$$

(17)
Substituting equation (17) into the first two of equations (4) gives the following constitutive relations:

\[
\begin{bmatrix}
\delta \varepsilon_{11}^e \\
\delta \varepsilon_{22}^e \\
\delta \gamma_{12}^e
\end{bmatrix} = \frac{1}{E_t} \begin{bmatrix}
n(1 - \nu_n \nu_t) & -\nu_n (1 + \nu_t) & 0 \\
-\nu_n (1 + \nu_t) & 1 - (\nu_t)^2 & 0 \\
0 & 0 & 1/G_{nt}
\end{bmatrix} \begin{bmatrix}
\delta \sigma'_{11} \\
\delta \sigma'_{22} \\
\delta \sigma'_{12}
\end{bmatrix}
\]

(18)

8. Axisymmetric Triaxial Conditions

Under axisymmetric triaxial conditions, only principal stresses are applied to a sample; all shear stresses and shear strains are thus zero. As such, in writing the constitutive relations, only the leading principal 3 by 3 sub-matrix of \( \mathbf{A} \) in equation (4) need be considered; viz.,

\[
\begin{bmatrix}
\delta \varepsilon_{11}^e \\
\delta \varepsilon_{22}^e \\
\delta \varepsilon_{33}^e
\end{bmatrix} = \frac{1}{E_t} \begin{bmatrix}
1/E_n & -\nu_n/E_t & -\nu_n/E_t \\
-\nu_n/E_n & 1/E_t & -\nu_t/E_t \\
-\nu_n/E_n & -\nu_t/E_t & 1/E_t
\end{bmatrix} \begin{bmatrix}
\delta \sigma'_{11} \\
\delta \sigma'_{12} \\
\delta \sigma'_{13}
\end{bmatrix}
\]

(19)

Letting \( n = E_t/E_n \), and recalling that due to the symmetry of \( \mathbf{A} \), \( \nu_{nt}/E_n = \nu_{tn}/E_t \), equation (19) is commonly re-written as

\[
\begin{bmatrix}
\delta \varepsilon_{11}^e \\
\delta \varepsilon_{22}^e \\
\delta \varepsilon_{33}^e
\end{bmatrix} = \frac{1}{E_t} \begin{bmatrix}
n & -\nu_n & -\nu_n \\
-\nu_n & 1 & -\nu_t \\
-\nu_n & -\nu_t & 1
\end{bmatrix} \begin{bmatrix}
\delta \sigma'_{11} \\
\delta \sigma'_{22} \\
\delta \sigma'_{33}
\end{bmatrix}
\]

(20)

Constitutive relations for specific axisymmetric triaxial stress states are next derived. Recalling the definition of the mean normal effective stress \( p' \) and the deviatoric effective stress \( q \); viz.,

\[
\delta p' = \frac{\delta \sigma'_{11} + 2\delta \sigma'_{33}}{3}, \quad q = \delta \sigma'_{11} - \delta \sigma'_{33}
\]

Solving these two equations for \( \delta \sigma'_{11} \) and \( \delta \sigma'_{33} \) gives

\[
\delta \sigma'_{11} = \delta p' + \frac{2}{3}q, \quad \delta \sigma'_{33} = \delta p' - \frac{1}{3}q
\]

(21)

8.1 “Vertical” Specimens with \( \delta \sigma'_{11} > \delta \sigma'_{22} = \delta \sigma'_{33} \)

This case is shown in Figure 3. The elastic strain increments are \( \delta \varepsilon_{11}^e = \delta \varepsilon_{22}^e = \delta \varepsilon_{33}^e \), \( \delta \varepsilon_{22}^e = \delta \varepsilon_{33}^e \), with \( \delta \varepsilon_{22}^e = \delta \varepsilon_{33}^e \). The associated effective stress increments are \( \delta \sigma'_{11} = \delta \sigma'_{11}, \delta \sigma'_{22} = \delta \sigma'_{22}, \) and \( \delta \sigma'_{33} = \delta \sigma'_{33}, \) with \( \delta \sigma'_{22} = \delta \sigma'_{33} \).
Substituting equations (21) for $\delta\sigma'_{1}$ and $\delta\sigma'_{3}$ into equations (20) gives, after some manipulation, the elastic strain increments in terms of $\delta p'$ and $\delta q$; viz.,

\[
\delta \varepsilon_{11}^e \equiv \delta \varepsilon_{1}^e = \frac{1}{E_t} \left( n \delta \sigma'_{1} - 2\nu_t \nu_n \delta \sigma'_{3} \right) \frac{1}{E_t} \left( (n - 2\nu_t) \delta p' + \frac{2}{3} (\nu_t - \nu_n) \delta q \right) \quad (22)
\]

\[
\delta \varepsilon_{22}^e \equiv \delta \varepsilon_{2}^e = \frac{1}{E_t} \left( -\nu_t \delta \sigma'_{1} + (1 - \nu_t) \delta \sigma'_{3} \right) \frac{1}{E_t} \left( (1 - \nu_t - \nu_n) \delta p' + \frac{1}{3} (\nu_t - 2\nu_n - 1) \delta q \right) \quad (23)
\]

\[
\delta \varepsilon_{33}^e \equiv \delta \varepsilon_{3}^e = \frac{1}{E_t} \left( -\nu_t \delta \sigma'_{1} + (1 - \nu_t) \delta \sigma'_{3} \right) \frac{1}{E_t} \left( (1 - \nu_t - \nu_n) \delta p' + \frac{1}{3} (\nu_t - 2\nu_n - 1) \delta q \right) \quad (24)
\]

where, as expected, $\delta \varepsilon_{2}^e = \delta \varepsilon_{3}^e$.

Recalling equation (5), and using equations (22) to (24), the elastic volumetric strain increment is

\[
\delta \varepsilon_v^e = \delta \varepsilon_{11}^e + \delta \varepsilon_{22}^e + \delta \varepsilon_{33}^e = \delta \varepsilon_{1}^e + 2\delta \varepsilon_{3}^e = \frac{1}{E_t} \left( n - 2\nu_t \nu_n \right) \left( (n - 2\nu_t) \delta \sigma'_{1} + 2(1 - \nu_t - \nu_n) \delta \sigma'_{3} \right) \quad (25)
\]

For the special case of isotropic compression, $\delta \sigma'_{1} = \delta \sigma'_{2} = \delta \sigma'_{3}$. equation (25) then becomes

\[
\delta \varepsilon_v^e = \frac{1}{E_t} \left( n + 2(1 - 2\nu_t - \nu_n) \right) \delta p' \quad (25')
\]

The axial distortional strain increment is given by

\[
\delta \varepsilon_s^e = \frac{2}{3} (\delta \varepsilon_{1}^e - \delta \varepsilon_{3}^e) = \frac{2}{3} (\delta \varepsilon_{11}^e - \delta \varepsilon_{33}^e) = \frac{2}{3E_t} \left( (n + \nu_t) \delta \sigma'_{1} + (\nu_t - 2\nu_n - 1) \delta \sigma'_{3} \right) \quad (26)
\]

For the special case of isotropic compression, $\delta q = 0$. Equation (26) thus becomes

\[
\delta \varepsilon_s^e = \frac{2}{3E_t} \left( n - \nu_t - \nu_n \right) \delta p' \quad (26')
\]

indicating that, because of anisotropy, $\delta \varepsilon_s^e \neq 0$ even though the effective stress state is isotropic.
Following the example of Graham and Houlsby [5], the elastic stress-strain are written in vector-matrix form, giving

\[
\begin{align*}
\{\delta \varepsilon_v^e\} = \left[\begin{array}{cc}
1/K^* & 1/J^* \\
1/J^* & 1/3G^*
\end{array}\right] \{\delta p'\} \\
\{\delta \varepsilon_s^e\} = \left[\begin{array}{c}
\delta q
\end{array}\right]
\end{align*}
\]

(27)

where, in light of equations (25) and (26),

\[
\begin{align*}
1/K^* = \frac{1}{E_t}\left[n + 2(1 - 2\nu_{tn} - \nu_{tt})\right] \\
1/J^* = \frac{2}{3E_t}\left[1 - \nu_{tn} + \nu_{tt} - 1\right] \\
1/3G^* = \frac{2}{9E_t}\left[2(n + 2\nu_{tn}) - \nu_{tt} + 1\right]
\end{align*}
\]

(28) (29) (30)

As a check, consider an isotropic material idealization. Now \(\nu_{tn} = \nu_{tt} \equiv \nu\), \(E_t \equiv E\) and \(n = 1\). Equations (28) to (30) reduce to

\[
\begin{align*}
1/K^* = \frac{1}{E}\left[1 + 2 - 4\nu - 2\nu\right] = \frac{3}{E}(1 - 2\nu) & \Rightarrow K^* = \frac{E}{3(1 - 2\nu)} \equiv K' \\
1/J = \frac{2}{3E}\left[1 - \nu + \nu - 1\right] = 0 \\
1/3G^* = \frac{2}{9E}\left[2(1 + 2\nu) - \nu + 1\right] = \frac{2(1 + \nu)}{3E} & \Rightarrow G^* = \frac{E}{2(1 + \nu)} \equiv G
\end{align*}
\]

where \(K'\) is the “drained” elastic bulk modulus, and \(G\) is the elastic shear modulus. The “usual” results for isotropic elasticity are thus obtained; viz., \(\delta \varepsilon_v^e = \delta p'/K'\) and \(\delta \varepsilon_s^e = \delta q/(3G)\).

Recalling the definition of the increment in octahedral shear strain, written in terms of principal strains [4]; viz.,

\[
\delta \gamma_{oct}^2 = \frac{1}{9} \left[(\delta \varepsilon_1 - \delta \varepsilon_2)^2 + (\delta \varepsilon_2 - \delta \varepsilon_3)^2 + (\delta \varepsilon_3 - \delta \varepsilon_1)^2\right]
\]

(31)

using equations (22) to (24), it is instructive to compute the following three elastic incremental principal strain differences:

\[
\begin{align*}
\delta \varepsilon_1^e - \delta \varepsilon_2^e &= \delta \varepsilon_{11}^e - \delta \varepsilon_{22}^e = \frac{1}{E_t}\left[(n - \nu_{tn} + \nu_{tt} - 1)\delta p' + \frac{1}{3}(2n + 4\nu_{tn} - \nu_{tt} + 1)\delta q\right] \\
\delta \varepsilon_2^e - \delta \varepsilon_3^e &= \delta \varepsilon_{22}^e - \delta \varepsilon_{33}^e = 0 \\
\delta \varepsilon_3^e - \delta \varepsilon_1^e &= \delta \varepsilon_{33}^e - \delta \varepsilon_{11}^e = -(\delta \varepsilon_1^e - \delta \varepsilon_2^e)
\end{align*}
\]

For the special case of isotropic compression, \(\delta q = 0\). Thus,

\[
\delta \varepsilon_1^e - \delta \varepsilon_2^e = -(\delta \varepsilon_1^e - \delta \varepsilon_2^e) = \frac{1}{E_t}\left[(n - \nu_{tn} + \nu_{tt} - 1)\delta p'\right]
\]

indicating that, because of anisotropy, \(\delta \gamma_{oct}^e \neq 0\) even though the effective stress state is isotropic. Only for the case of material isotropy \((\nu_{tn} = \nu_{tt} \equiv \nu\), \(E_t \equiv E\), \(n = 1\)) will \(\delta \gamma_{oct} = 0\).

8.2 “Horizontal” Specimens with \(\delta \sigma_{33}^e > \delta \sigma_{11}^e = \delta \sigma_{22}^e\)

This case is shown in Figure 4. The elastic strain increments are \(\delta \varepsilon_1^e \equiv \delta \varepsilon_{33}^e\), \(\delta \varepsilon_2^e \equiv \delta \varepsilon_{11}^e\), \(\delta \varepsilon_3^e \equiv \delta \varepsilon_{22}^e\). The associated effective stress increments are \(\delta \sigma_1^e \equiv \delta \sigma_{33}^e\), \(\delta \sigma_2^e \equiv \delta \sigma_{11}^e\), and \(\delta \sigma_3^e \equiv \delta \sigma_{22}^e\), with \(\delta \sigma_2^e = \delta \sigma_3^e\).

Substituting equations (21) for \(\delta \sigma_1^t\) and \(\delta \sigma_3^t\) into equations (20) gives, after some manipulation, the elastic strain increments in terms of \(\delta p'\) and \(\delta q\); viz.,

\[
\begin{align*}
\delta \varepsilon_{11}^e &= \delta \varepsilon_{22}^e = \frac{1}{E_t}\left[-\nu_{tn} \delta \sigma_1^e + (n - \nu_{tn})\delta \sigma_3^e\right] = \frac{1}{E_t}\left[(n - 2\nu_{tn})\delta p' - \frac{1}{3}(n + \nu_{tn})\delta q\right] \\
\delta \varepsilon_{12}^e &= \delta \varepsilon_{21}^e = \frac{1}{E_t}\left[-\nu_{tn} \delta \sigma_2^e + (n - \nu_{tn})\delta \sigma_3^e\right] = \frac{1}{E_t}\left[(n - 2\nu_{tn})\delta p' - \frac{1}{3}(n + \nu_{tn})\delta q\right]
\end{align*}
\]

(32)
\[ \delta \sigma_{33} = \delta \sigma_1 \]
\[ \delta \sigma_{11} = \delta \sigma_2 \]
\[ \delta \sigma_{22} = \delta \sigma_3 \]

Figure 4 – Transversely isotropic material under axisymmetric triaxial conditions: “horizontal” specimen with \( \delta \sigma'_{11} > \delta \sigma'_{22} \).

\[
\delta \varepsilon^e_{22} \equiv \delta \varepsilon_3^e = \frac{1}{E_t} \left( -\nu_{tt} \delta \sigma'_1 + (1 - \nu_{tt}) \delta \sigma'_3 \right) = \frac{1}{E_t} \left[ (1 - \nu_{tn} - \nu_{tt}) \delta p' + \frac{1}{3} \left( \nu_{tn} - 2\nu_{tt} - 1 \right) \delta q \right]
\]

\[
\delta \varepsilon^e_{33} \equiv \delta \varepsilon_1^e = \frac{1}{E_t} \left[ \delta \sigma'_1 - (\nu_{nn} + \nu_{tt}) \delta \sigma'_3 \right] = \frac{1}{E_t} \left[ (1 - \nu_{tn} - \nu_{tt}) \delta p' + \frac{1}{3} \left( \nu_{tn} + \nu_{tt} + 2 \right) \delta q \right]
\]

From equations (32) and (33) it is evident that although \( \delta \sigma'_{11} = \delta \sigma'_{22} \), \( \delta \varepsilon^e_{11} \neq \delta \varepsilon^e_{22} \) due to anisotropy. The condition \( \delta \varepsilon^e_{11} = \delta \varepsilon^e_{22} \) will, however, be realized if the material is isotropic (i.e., for \( n = 1, E_t \equiv E \), and \( \nu_{tn} = \nu_{tt} \equiv \nu \)).

Recalling equation (5), and using equations (32) to (34), the elastic volumetric strain increment is

\[
\delta \varepsilon^e_v = \delta \varepsilon^e_{11} + \delta \varepsilon^e_{22} + \delta \varepsilon^e_{33} = \delta \varepsilon^e_1 + \delta \varepsilon^e_2 + \delta \varepsilon^e_3 = \frac{1}{E_t} \left[ (1 - \nu_{tn} - \nu_{tt}) \delta \sigma'_1 + (n - 3\nu_{tn} - \nu_{tt} + 1) \delta \sigma'_3 \right]
\]

\[
= \frac{1}{E_t} \left\{ n + 2 \left( 1 - 2\nu_{tn} - \nu_{tt} \right) \delta p' + \frac{1}{3} \left( 1 + \nu_{nn} - \nu_{tt} - n \right) \delta q \right\}
\]

(35)

For the special case of isotropic compression, \( \delta q = 0 \). Equation (35) then becomes

\[
\delta \varepsilon^e_v = \frac{1}{E_t} \left[ n + 2 \left( 1 - 2\nu_{tn} - \nu_{tt} \right) \right] \delta p'
\]

The axial distorsional strain increment is given by

\[
\delta \varepsilon^e_s = \frac{2(1 + \nu_{tt})}{3E_t} \left[ \delta \sigma'_1 - \delta \sigma'_3 \right] = \frac{2(1 + \nu_{tt})}{3E_t} \delta q
\]

(36)

For the special case of isotropic compression, \( \delta q = 0 \Rightarrow \delta \varepsilon^e_s = 0 \).

The elastic stress-strain are next written in vector-matrix form, giving

\[
\begin{bmatrix} \delta \varepsilon^e_v \\ \delta \varepsilon^e_s \end{bmatrix} = \begin{bmatrix} 1/K^* & 1/J_1^* \\ 1/J_2^* & 1/3G^* \end{bmatrix} \begin{bmatrix} \delta p' \\ \delta q \end{bmatrix}
\]

(37)

where, in light of equations (35) and (36),

\[
\frac{1}{K^*} = \frac{1}{E_t} \left[ n + 2 \left( 1 - 2\nu_{tn} - \nu_{tt} \right) \right]
\]

(38)

\[
\frac{1}{J_1^*} = \frac{1}{3E_t} \left[ (1 + \nu_{nn} - \nu_{tt} - n) \right] \quad \frac{1}{J_2^*} = 0
\]

(39)

\[
\frac{1}{3G^*} = \frac{2(1 + \nu_{tt})}{3E_t}
\]

(40)
For an isotropic material idealization, equations (38) to (40) reduce to

\[ \frac{1}{K^*} = \frac{1}{E} \left[ 1 + 2(1 - 2\nu - \nu) \right] = \frac{3}{E} (1 - 2\nu) \Rightarrow K^* = \frac{E}{3(1 - 2\nu)} = K' \]

\[ \frac{1}{J'_t} = \frac{1}{3E_t} \left[ (1 + \nu - \nu - 1) \right] = 0 \]

\[ \frac{1}{3G^*} = \frac{2(1 + \nu)}{3E} \Rightarrow 3G^* = \frac{3E}{2(1 + \nu)} = 3G \]

where \( K' \) and \( G \) are again the “drained” elastic bulk and shear modulus, respectively.

Using equations (32) to (34), it is instructive to compute the three elastic incremental principal strain differences entering equation (31) for the increment in octahedral shear strain; viz.,

\[ \delta\varepsilon' - \delta\varepsilon'_2 = \delta\varepsilon'_{33} - \delta\varepsilon'_{11} = \frac{1}{E_t} \left[ (1 + \nu_{tn} - \nu_{tt} - n)\delta\sigma' + \frac{1}{3}(2 + 2\nu_{tn} + \nu_{tt} + n)\delta q \right] \]

\[ \delta\varepsilon'_2 - \delta\varepsilon'_3 = \delta\varepsilon'_{11} - \delta\varepsilon'_{22} = \frac{1}{E_t} \left[ (\nu_{tt} - \nu_{tn} + n - 1)\delta\sigma' + \frac{1}{3}(1 - 2\nu_{tn} + 2\nu_{tt} - n)\delta q \right] \]

\[ \delta\varepsilon'_3 - \delta\varepsilon'_1 = \delta\varepsilon'_{22} - \delta\varepsilon'_{33} = \frac{1}{E_t} \left[ -(1 + \nu_{tt})\delta q \right] \]

For the special case of isotropic compression, \( \delta q = 0 \). Thus,

\[ \delta\varepsilon'_1 - \delta\varepsilon'_2 = -(\delta\varepsilon'_2 - \delta\varepsilon'_3) = \frac{1}{E_t} \left[ (1 + \nu_{tn} - \nu_{tt} - n)\delta\sigma' \right] \]

indicating that, because of anisotropy, \( \delta\gamma_{oct} \neq 0 \) even though the effective stress state is isotropic. Only for the case of material isotropy (\( \nu_{tn} = \nu_{tt} = \nu \), \( E_t = E \), \( n = 1 \)) will \( \delta\gamma_{oct} = 0 \).

**8.3 “Horizontal” Specimens with \( \delta\sigma'_{22} > \delta\sigma'_{33} = \delta\sigma'_{11} \)**

This case is shown in Figure 5. The elastic strain increments are \( \delta\varepsilon'_1 \equiv \delta\varepsilon'_{33} \), \( \delta\varepsilon'_2 \equiv \delta\varepsilon'_{11} \), \( \delta\varepsilon'_3 \equiv \delta\varepsilon'_{22} \). The associated effective stress increments are \( \delta\sigma'_1 \equiv \delta\sigma'_{33} \), \( \delta\sigma'_2 \equiv \delta\sigma'_{11} \), and \( \delta\sigma'_3 \equiv \delta\sigma'_{22} \), with \( \delta\sigma'_2 = \delta\sigma'_3 \).

\[ \delta\sigma'_{22} = \delta\sigma_1 \]

\[ \delta\sigma'_{33} = \delta\sigma_2 \]

\[ \delta\sigma'_{11} = \delta\sigma_3 \]

**Figure 5** – Transversely isotropic material under axisymmetric triaxial conditions: “horizontal” specimen with \( \delta\sigma'_{22} > \delta\sigma'_{33} = \delta\sigma'_{11} \).

Substituting equations (21) for \( \delta\sigma'_1 \) and \( \delta\sigma'_3 \) into equations (20) gives, after some manipulation, the elastic strain increments in terms of \( \delta\sigma' \) and \( \delta q \); viz.,

\[ \delta\varepsilon'_{11} = \delta\varepsilon'_3 = \frac{1}{E_t} \left[ -\nu_{tn}\delta\sigma'_1 + (n - \nu_{tn})\delta\sigma'_3 \right] = \frac{1}{E_t} \left[ (n - 2\nu_{tn})\delta p' - \frac{1}{3}(n + \nu_{tn})\delta q \right] \]

(41)

\[ \delta\varepsilon'_{22} = \delta\varepsilon'_1 = \frac{1}{E_t} \left( \delta\sigma'_1 - (\nu_{tn} + \nu_{tt})\delta\sigma'_3 \right) = \frac{1}{E_t} \left[ (1 - \nu_{tn} - \nu_{tt})\delta p' + \frac{1}{3}(\nu_{tn} - 2\nu_{tt} + 2)\delta q \right] \]

(42)
For the special case of isotropic compression, equation (45) becomes
\[ \delta \varepsilon_{33} = \frac{1}{E_t} \left[ -\nu_t \delta \sigma_1 + (1 - \nu_t n) \delta \sigma_3 \right] = \frac{1}{E_t} \left[ (1 - \nu_t n - \nu_t) \delta p' - \frac{1}{3} (1 - \nu_t n + 2 \nu_t) \delta q \right] \] (43)

From equations (41) and (43) it is evident that although \( \delta \sigma_{11} = \delta \sigma_{33} \), \( \delta \varepsilon_{11} \neq \delta \varepsilon_{33} \) due to anisotropy. The condition \( \delta \varepsilon_{11}^* = \delta \varepsilon_{33}^* \) will, however, be realized if the material is isotropic (i.e., for \( n = 1 \), \( E_t = E \), and \( \nu_t = \nu_t = \nu \)).

The elastic volumetric strain increment is
\[ \delta \varepsilon_v = \delta \varepsilon_{11} + \delta \varepsilon_{22} + \delta \varepsilon_{33} = \delta \varepsilon_1 + \delta \varepsilon_2 + \delta \varepsilon_3 = \frac{1}{E_t} \left[ (1 - \nu_t n - \nu_t) \delta \sigma_1 + (n - 3 \nu_t n - \nu_t + 1) \delta \sigma_3 \right] \]
\[ = \frac{1}{E_t} \left[ \left( n + 2(1 - 2 \nu_t n - \nu_t) \right) \delta p' + \frac{1}{3} (1 + \nu_t n - \nu_t - n) \delta q \right] \] (44)

For the special case of isotropic compression, \( \delta q = 0 \). Equation (44) then becomes
\[ \delta \varepsilon_v = \frac{1}{E_t} \left[ n + 2(1 - 2 \nu_t n - \nu_t) \right] \delta p' \]

The axial distortion strain increment is given by
\[ \delta \varepsilon_s = \frac{2}{3 E_t} \left[ (1 + \nu_t n) \delta \sigma_1 - (n + \nu_t) \delta \sigma_3 \right] = \frac{2}{3 E_t} \left[ (1 + \nu_t n - \nu_t - n) \delta p' + \frac{1}{3} (2 + 2 \nu_t n + \nu_t + n) \delta q \right] \] (45)

For the special case of isotropic compression, equation (45) becomes
\[ \delta \varepsilon_s = \frac{2}{3 E_t} \left[ (1 + \nu_t n - \nu_t - n) \delta p' \right] \]

The elastic stress-strain are next written in vector-matrix form, giving
\[ \begin{pmatrix} \delta \varepsilon_1 \\ \delta \varepsilon_2 \\ \delta \varepsilon_3 \end{pmatrix} = \begin{pmatrix} 1/K^* & 1/J_1^* \\ 1/J_1^* & 1/3 G^* \end{pmatrix} \begin{pmatrix} \delta p' \\ \delta q \end{pmatrix} \] (46)

where, in light of equations (44) and (45),
\[ \frac{1}{K^*} = \frac{1}{E_t} \left[ n + 2(1 - 2 \nu_t n - \nu_t) \right] \] (47)
\[ \frac{1}{J_1^*} = \frac{1}{3 E_t} (1 + \nu_t n - \nu_t - n) \] (48)
\[ \frac{1}{J_2^*} = \frac{2}{3 E_t} (1 + \nu_t n - \nu_t - n) = \frac{2}{J_1^*} \] (49)
\[ \frac{1}{3 G^*} = \frac{2}{9 E_t} (2 + 2 \nu_t n + \nu_t + n) \] (50)

The expression for \( 1/K^* \) given by equation (47) is identical to that for the other two configurations considered in Sections 8.1 and 8.2 (recall equations (28) and (38)).

For an isotropic material idealization, equations (47) to (50) reduce to
\[ \frac{1}{K^*} = \frac{1}{E} \left[ 1 + 2(1 - 2 \nu - \nu) \right] = \frac{3}{E} (1 - 2 \nu) \quad \Rightarrow \quad K^* = \frac{E}{3(1 - 2 \nu)} = K' \]
\[ \frac{1}{J_1^*} = \frac{1}{J_2^*} = 0 \]
\[ \frac{1}{3 G^*} = \frac{2}{9 E} (2 + 2 \nu + \nu + 1) = \frac{2(1 + \nu)}{3 E} = \frac{1}{3 G} \]

where \( K' \) and \( G \) are again the “drained” elastic bulk and shear modulus, respectively.
Using equations (41) to (43), it is instructive to compute the three elastic incremental principal strain differences entering equation (31) for the increment in octahedral shear strain; viz.,
\[
\delta \varepsilon_1^e - \delta \varepsilon_2^e = \delta \varepsilon_{22}^e - \delta \varepsilon_{33}^e = \frac{1}{E_t} (1 + \nu_{tt}) \delta q
\]
\[
\delta \varepsilon_2^e - \delta \varepsilon_3^e = \delta \varepsilon_{33}^e - \delta \varepsilon_{11}^e = \frac{1}{E_t} \left[ (\nu_{tt} - \nu_{tt} - n + 1) \delta p' + \frac{1}{3} (2\nu_{tt} - 2\nu_{tt} + n - 1) \delta q \right]
\]
\[
\delta \varepsilon_3^e - \delta \varepsilon_1^e = \delta \varepsilon_{11}^e - \delta \varepsilon_{22}^e = \frac{1}{E_t} \left[ (-\nu_{tt} + \nu_{tt} + n - 1) \delta p' - \frac{1}{3} (2\nu_{tt} + \nu_{tt} + n + 2) \delta q \right]
\]
For the special case of isotropic compression, \( \delta q = 0 \). Thus,
\[
\delta \varepsilon_2^e - \delta \varepsilon_3^e = -(\delta \varepsilon_3^e - \delta \varepsilon_1^e) = \frac{1}{E_t} \left[ (\nu_{tt} - \nu_{tt} - n + 1) \delta p' \right]
\]
indicating that, because of anisotropy, \( \delta \gamma_{oct} \neq 0 \) even though the effective stress state is isotropic. Only for the case of material isotropy \( \nu_{tt} = \nu_{tt} \equiv \nu, E_t \equiv E, n = 1 \) will \( \delta \gamma_{oct} = 0 \).

9. Conclusions
Some key issues associated with anisotropic elastic material idealizations of soils have been presented in this paper. Although the discussion began with the orthotropic elastic idealizations, emphasis was placed on transversely isotropic (or “cross-anisotropic”) idealizations. The issues discussed for such idealizations facilitate the use of available closed-form solutions for homogeneous, transversely isotropic elastic geotechnical engineering problems [3, 18, 19, 20].

The inclusion of such idealizations into existing and new elastoplastic and/or elastoviscoplastic constitutive models for soils requires a specific analytical form for the anisotropic idealization, as well as suitable empirical expressions for the associated elastic material constants. These two topics are, however, beyond the scope of this paper, as they depend on the specific constitutive model being used.

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Анизотропное упругое деформирование грунтов обобщение некоторых ключевых вопросов

Аннотация: Традиционно эластичная реакция грунтов предполагалась быть изотропной. Природные грунты, однако, с большей вероятностью проявляют анизотропную реакцию. Например осадочные грунты, которые обычно осаждаются под действием силы тяжести, обладают различными свойствами в направлении осаждения в отличие от плоскостей, перпендикулярных этому направлению. В этой статье обобщается несколько ключевых вопросов, связанных с идеализацией анизотропных упругих материалов для грунтов. Акцент делается на поперечно-изотропных "поперечно-анизотропных" идеализациях упругого материала.

Ключевые слова: эластичность, изотропия, анизотропия, ортотропность, поперечно-анизотропность.
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"За публикацию в Вестнике ЕНУ ФИО автора"
Мақаланы рәсімдеу ұлғісі

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численное дифференцирование функций в контексте Компьютерного (вычислительного) поперечника

Аннотация: В рамках компьютерного (вычислительного) поперечника полностью решена задача приближенного дифференцирования функций, принадлежащих классам Соболева по неточной информации, полученной от произвольного конечного множества тригонометрических коэффициентов Фурье-Лебега дифференцируемой функции... [100-200 слов].

Ключевые слова приближенное дифференцирование, восстановление по неточной информации, предельная погрешность, компьютерный (вычислительный) поперечник. [6-8 слов/словосочетаний].

Введение

Текст введения...

Авторам не следует использовать нестандартные пакеты LaTeX (используйте их лишь в случае крайней необходимости)

Заголовок секции

1.1 Заголовок подсекции

Окружения.

Теорема 1. ...

Лемма 1. ...

Предложение 1. ...

Определение 1. ...

Следствие 1. ...

Замечание 1. ...

Теорема 2 (Темиргалиев Н. [2]). Текст теоремы.

Доказательство. Текст доказательства.

2. Формулы, таблицы, рисунки

\[ \delta_N(\varepsilon; D_N) Y \equiv \delta_N(\varepsilon; T; F; D_N) Y \equiv \inf_{(l^{(N)}, \varphi_N) \in D_N} \delta_N \left( \varepsilon; \left( l^{(N)}; \varphi_N \right) \right) Y, \quad (1) \]

где \( \delta_N \left( \varepsilon; \left( l^{(N)}, \varphi_N \right) \right) Y \equiv \delta_N(\varepsilon; T; F; \left( l^{(N)}, \varphi_N \right)) Y \equiv \sup_{f \in F} \left\| T f (\cdot) - \varphi_N \left( l^{(1)} f + \gamma^{(1)} N \varepsilon^{(1)} N, \ldots, l^{(N)} f + \gamma^{(N)} N \varepsilon^{(N)} N; \right) \right\| Y. \]

Таблицы, рисунки необходимо располагать после упоминания. С каждой иллюстрацией должна следовать надпись.

3. Ссылки и библиография
Table 3 – Название таблицы

| Простые  | Не простые |
|----------|------------|
| 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 | 4, 6, 8, 9, 10, 12, 14 |

Рисунок 22 – Название рисунка

Для ссылок на утверждения, формулы и т. п. можно использовать метки. Например, теорема 2,
Формула (1)
Для руководства по LATEX и в качестве примера оформления ссылок, см., например, Львовский С.М.
Набор и верстка в пакете LATEX. Москва: Космосинформ, 1994.
Список литературы оформляется следующим образом.

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Компьютерлiк (есептеуiш) диаметр мәнмәтiнiнде функцияларды санды дифференциалдау

Аннотация: Компьютерлiк (есептеуiш) диаметр мәнмәтiнiнде Соболев класында жататын функцияларды олардың
тригонометриялык Фурье-Лебег коэффициенттерiнi алынdan дәл емес а/uni049Bпарат бойынша жуы/uni049Bтау
есебi толы/uni0493ымен шешiлдi [100-200 c/uni04E9здер]

Т/uni04AFйiн с/uni04E9здер: жуы/uni049Bтап дифференциалдау, дәл емес а/uni049Bпарат бойынша жуы/uni049Bтау, шектiк /uni049Bателiк, Компьютерлiк
(есептеуiш) диаметр [6-8 c/uni04E9з/с/uni04E9з тiркестерi]

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Numerical differentiation of functions in the context of Computational (numerical) diameter

Abstract: The computational (numerical) diameter is used to completely solve the problem of approximate differentiation
of a function given inexact information in the form of an arbitrary finite set of trigonometric Fourier coefficients. [100-200 words]

Keywords: approximate differentiation, recovery from inexact information, limiting error, computational (numerical) di-
ameter, massive limiting error. [6-8 words/word combinations]
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