Finite element analysis of the bending moment-curvature of the double-layered graded concrete beam

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Abstract. The use of functionally graded concrete in structural member provides a beneficial prospect since it promises the increase of beam stiffness with less of construction cost. In structural analysis, concrete stiffness plays role in determining the performance of beam elements in resisting deflection due to the working loads. In this study, the behaviour of the graded concrete beam is continuously investigated. The bending moment-curvature relationship of the graded concrete beam is conducted numerically to understand the level of ductility and the failure mode compared with the conventional beam. A singly reinforced graded concrete beam is modelled using a non-linear simulation program, Strand7. The result of the analysis is then evaluated in accordance with the Eurocode. The graded concrete beam is expected to exhibit a reliable behaviour of bending moment – curvature so that the graded concrete beam is possible for advanced application as structural elements.

1. Introduction

Graded concrete is one of advanced technologies in construction that combines multiple concrete mix with different strength in a structural element to optimise construction cost. Preceding researches show that the utilisation of graded concrete can significantly minimise the construction cost to 50 per cent due to the reduction of cementitious material used in reinforced concrete members [1-6]. Research findings on previous studies also stated that the graded concrete is also capable to maintain material stiffness and to improve the structural serviceability [7-9]. Unfortunately, these researches were only limited to basic research and require continuous investigation on applied research.

Ductility is a parameter that should be concerned on the structural design besides the stiffness. The greater the ductility of the structural member, the more resistant the elements to withstand the working load [10]. In structures, the building members should be designed to undergo a ductile failure rather than a brittle failure. A brittle failure should be avoided due to the risk of human safety when the failure happens. A ductile failure, meanwhile, allows elements to undergo a greater deflection and improve the safety factor of the members. The brittle failure is commonly found in structural elements using the high-strength concrete so it is recommended to improve the reinforcement details to increase its ductility [11]. Therefore, this paper aims to investigate the structural behaviour of the use of graded concrete in beam element. The study is conducted numerically using non-linear analysis program, Strand7.
2. Reinforced concrete beam design

Three models of reinforced concrete beams with a cross-sectional dimension of 120 x 240 mm and 2 metres of net-spans are situated on simple supports setup, and are designed to undergo the four-point bending test. Each beam model is designed with a uniform concrete strength of 20.75 MPa and 41.5 MPa, and with a graded concrete which combines both concrete mixes. The models are then labelled with U250 for the beam with 20.75 MPa concrete strength, U500 for the 41.5 MPa, and G500-250 for the graded concrete beam. For the graded concrete beam, the high concrete strength is positioned in the upper layer, while the lower concrete strength is placed at the bottom. This approach is carried out in accordance with recommendations from previous research stated that the strength of concrete in tensile fibres is diminished during analysis, while the strength of the concrete is utilized in the compressive fibres. All beam models use a single reinforcing bar with a yield stress of 410 MPa. The highest concrete strength and the steel yield stress are used as inputs in determining the minimum reinforcement ratio. From the calculation, it resulted that the reinforcement ratio = 0.004 or equivalent to 11.4 mm in diameter. Details of the graded concrete beam design are shown in Figure 1.

![Graded concrete beam design](image)

**Figure 1.** Graded concrete beam design

3. Finite element modelling

3.1. Material inputs

In the modelling of reinforced concrete beams, the constitutive material properties are prepared as basic data in finite element analysis. The behaviour of concrete under tensile and compressive conditions is represented in a stress-strain table recommended by CEB FIB 2010 - European code [12], while the behaviour of steel is modelled in a bilinear graph showing the yield and peak point of both tensile and compressive conditions. The modelling of steel and concrete strain stress is shown in Figure 2 – 3. The properties of concrete and steel materials are then defined in detail in the programme. Steel reinforcement is modelled as a beam element with an elastic modulus of 200,000 MPa. The steel is considered a full elastic material. The cross-sectional geometry of rebar is determined according to the diameter of 11.4 mm. Concrete is modelled in 2D-plane-stress and is considered as an isotropic material, where the properties of the concrete material will be constant in all directions during the loading. The initial modulus of elasticity of concrete is set in accordance with CEB-FIB 2010 recommendations. Concrete material is considered to behave non-linear elastic. Failure criteria are evaluated based on max-stress criterion.
Figure 2. Stress-strain relationship of 20.75 MPa and 41.5 MPa concrete strength

Figure 3. Stress-strain relationship of steel rebar (f_y = 410 MPa)

3.2. Beam modelling
The reinforced graded concrete beams are visualised in a full-model with suitable constraints and restraints. The concrete is modelled using quadrilateral elements – 4 nodes, and are then meshed with a dimension ratio close to 1 to avoid an ill condition during analysis. The rebar is approached using beam elements that are meshed in size with the plate width. Each rebar segment is connected to a nodal using the link element to provide strain compatibility between the concrete and the steel bar. The link used is a rigid type. The loading point at the top surface of the beam is approached using a displacement load of 1 mm in the direction of gravity load. The loading is carried out on 3 nodes to avoid an excessive loading stress at the loading point. The degree of freedom is adjusted to the node attributes menu to describe the joint and roll support conditions. The beam modelling of the graded concrete beam is shown in figure 4. The model is analysed using non-linear static loading. The material is considered to behave non-linear on the increment of loading. The displacement control (arc length) with an increase in 0.01 mm loading is defined. Program iterations are run until the analysis does not converge.
4. Results and discussion

4.1. Load – midspan deflection

The displacement readings in the middle of the span, the stress acting on the cross section, and the support reaction that works on each incremental load are generated from the program. The relationship of load-deflection in the middle of the span of the three models are shown in figure 5.

![Figure 5. Load – midspan deflection of beam](image)

The analysis shows that the models could not run the complete analysis until the ultimate failure occurs and only show structural behaviour in elastic conditions. From figure 5, the peak load and deflection of U250, U500, and G500-250 are (12987 kg; 0.29 mm), (20065 kg; 0.41 mm), and (10321 kg; 0.22 mm), respectively. The data shows that the G500-250 produces the smallest peak load and deflection compared to the other two models. This is due to the presence of a sharp transition of two layers of concrete strength so it causes a stress discontinuity in the model. The stress discontinuity causes the iteration terminated because the solution obtained does not converge. From the data, the graded concrete beam exhibits a higher structural stiffness compared to the U250. To magnify the analysis, the review is conducted on the loading level of 10000 kg. The G500-250 arises deflection of 0.21 mm, while the U500 shows 0.20 mm, and the U250 is 0.22 mm, so that the beam stiffness of G500-250; U500; and the U250 is 47619 kg/mm; 50000 kg/mm; and 45454 kg/mm, respectively. There is an increase in structural stiffness of 4.7% with the use of graded concrete in the beam elements so that the hypothesis in previous studies has been proven. As supporting data, the deflection that occurs in the G500-250 model is shown in figure 6.
4.2. Moment-curvature
The radius of curvature, neutral axis height, concrete compressive strain, and steel strain varies along the beam span and the increase of loading increment. The curvature of the element is the unit-length rotation, obtained by comparing the steel tensile strain and the concrete compressive strain to the effective height of the beam section on each loading stage. The steel and the concrete strain data were obtained by transposing the reading of stress to the strain value using a stress-strain table. This analysis is conducted in a spreadsheet program. Because the material is defined as an elastic material, the program cannot complete the analysis until the failure of materials reached. The concrete curvature moments of U250, U500 and G500-250 are shown in figure 7.

![Beam deflection of G500-250](image)

**Figure 6. Beam deflection of G500-250**

![Curvature vs Moment](image)

**Figure 7. Beam deflection of G500-250**

The analysis shows that the curvature generated by the G500-250 beam is the smallest compared to the two controlling beam models, U500 and U250. It is understood that the G500-250 produces the
minimum deflection on the level of the flexural moment. From herein, the application of graded concrete in the beam element can improve the serviceability level.

4.3. Stress distribution
Strand7 is able to visualise the stress distribution of the models. The stress distribution of concrete and steel is shown in figure 8. The purpose of inspecting the stress distribution is to confirm the validity of models. The figures show that the beams undergo deflection towards the negative Y-axis and is followed by the tensile stress on the bottom of the beam and the compressive stress on the upper side of the beam due to loading. Four-point bending test modelling causes the concentration of tensile stress on the downside fibre below the loading point. The link modelling between concrete element nodal and the reinforcement is also considered valid because of reinforcement experiences axial stress in resisting the tensile stress that occurs in the beam. The concentration of the tensile stress of the steel is located at the loading point so that it can be expected that the reinforcement will first fail. The direction and stress values of each beam model can be seen in figure 8.

![Stress distribution of G500-250](image)

**Figure 8. Stress distribution of G500-250**

5. Conclusion
From this study, it can be concluded that the initial development of finite element models for the graded concrete beam is considered robust in representing the structural behaviour. Based on the analysis result, the graded concrete beam is able to increase the structural stiffness up to 4.7%. The graded concrete beam is possibly reduced deflections that occur in the structural member indicated by a smaller curvature response to the working loads. Improvement of the beam model is required so that structural failure can be achieved by the ultimate condition. Significance tests should also be conducted on different size of meshing condition to obtain more accurate analysis result.

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