Density probability distribution functions of diffuse gas in the Milky Way

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ABSTRACT

In a search for the signature of turbulence in the diffuse interstellar medium (ISM) in gas density distributions, we determined the probability distribution functions (PDFs) of the average volume densities of the diffuse gas. The densities were derived from dispersion measures and H I column densities towards pulsars and stars at known distances. The PDFs of the average densities of the diffuse ionized gas (DIG) and the diffuse atomic gas are close to lognormal, especially when lines of sight at |b| < 5° and |b| ≥ 5° are considered separately. The PDF of ⟨nH⟩ at high |b| is twice as wide as that at low |b|. The width of the PDF of the DIG is about 30 per cent smaller than that of the warm H I at the same latitudes. The results reported here provide strong support for the existence of a lognormal density PDF in the diffuse ISM, consistent with a turbulent origin of density structure in the diffuse gas.

Key words: turbulence – ISM: structure.

1 INTRODUCTION

Simulations of the interstellar medium (ISM) have shown that, if isothermal turbulence is shaping the structure of the medium, the density distribution becomes lognormal (Elmegreen & Scalo 2004, and references therein). However, Madsen, Reynolds & Haffner (2006) found large temperature differences between lines of sight (LOS) through the warm ionized medium, which seem inconsistent with an isothermal gas. Also in the magnetohydrodynamic (MHD) simulations of de Avillez & Breitschwerdt (2005) and Wada & Norman (2007), the ISM became non-isothermal. The latter authors showed that for a large enough volume, and for a long enough simulation run, the physical processes causing the density variations in the ISM in a galactic disc can be regarded as random and independent events. Therefore, the probability distribution function (PDF) of log(density) becomes Gaussian and the density PDF lognormal, although the medium is not isothermal. The medium is inhomogeneous on a local scale, but in a quasi-steady state on a global scale.

The shape of the gas density PDF can be an important component in theories of star formation. Elmegreen (2002) showed that a Schmidt-type power-law relation between the star formation rate per unit area and the gas surface density can be deduced if the density PDF is lognormal and star formation occurs above a threshold density; the dispersion of the lognormal density PDF is a key parameter in the model. However, from the existing simulations it is not yet clear whether a lognormal density PDF in galaxies is universal and which factors determine their shape (Wada & Norman 2007).

Little observational evidence exists to test the results of the simulations. Wada, Spaans & Kim (2000) showed that the luminosity function of the H I column density in the Large Magellanic Cloud is lognormal. Recently, Hill et al. (2007, 2008) derived a lognormal distribution of the emission measures perpendicular to the Galactic plane of the diffuse ionized gas (DIG) in the Milky Way, observed in the Wisconsin Hα Mapper survey (Haffner et al. 2003) above Galactic latitudes of 10°. They note that emission measures towards classical H II regions also fit a lognormal distribution, with different parameters. Furthermore, Tabatabaei (2008) found a lognormal distribution of the emission measures derived from an extinction-corrected Hα map of the nearby galaxy M33 (Tabatabaei et al. 2007). Gaustad & Van Buren (1993) plotted the distribution of the local volume density of dust near stars within 400 pc of the Sun, which is also consistent with a lognormal distribution. In this Letter, we present the PDFs of average volume densities of the DIG and the diffuse atomic gas in the Milky Way, and show that they are consistent with lognormal distributions as well.

2 BASICS AND DATA

We investigated the density PDFs of the DIG and of diffuse atomic hydrogen gas (H I) in the solar neighbourhood.

Various volume densities of the DIG can be obtained from the dispersion measure DM (in cm−3 pc) and emission measure EM (in cm−6 pc) towards a pulsar at known distance D (in pc) from the relations:

\[ DM = \int_0^D n_e(l) dl = \langle n_e \rangle D = N_e F_\alpha D, \] (1)
\[ E_M = \int_0^D n_e^2(l) dl = \langle n_e^2 \rangle D = N_c^2 F_d D, \]  
where \( n_e(l) \) is the local electron density at point \( l \) along the LOS, \( \langle n_e \rangle \) and \( \langle n_e^2 \rangle \) are averages along \( D \) and \( F_d \) is the fraction of the LOS in clouds of average density \( N_c \) (see fig. 1 in Berkhujsen, Mitra & Müller 2006). The final equality in equation (2) is only valid when the average density of every cloud \( N_c \) along the LOS is the same: then \( \langle n_e^2 \rangle = \langle n_e \rangle F_d = N_c^2 F_d \). Thus, \( N_c \) and \( F_d \) are crude approximations of the true average cloud density and filling factor along a LOS, but given the large number of different LOS in our sample their mean and dispersion are reasonable estimators of \( N_c \) and \( F_d \).

Combining equations (1) and (2) we have
\[ N_c = \frac{E_M}{D^2}; \]
\[ F_d = \frac{D^2}{E_M} \langle n_e \rangle \]

The LOS filling factor \( F_d \) approximates the volume filling factor \( F_v \) if there are several clouds along the LOS. As this will generally be the case, we take \( F_d = F_v \).

We used the densities derived from two pulsar samples that were originally selected for studies of the volume-filling factor of the DIG.

(i) 34 pulsars at observed distances known to better than 50 per cent, collected by Berkhujsen & Müller (2008). The pulsar distances are in the range \( 0.1 < D < 9.5 \) kpc, with a mean distance of 2.4 kpc and a standard deviation of 2.9 kpc (the spread is large because 21 pulsars lie at \( D < 2 \) kpc).

(ii) 157 pulsars with distances obtained from DM and the model of the distribution of free electrons in the MW of Cordes & Lazio (2002), collected by Berkhujsen et al. (2006). These pulsars lie in the range \( 0.1 < D < 6 \) kpc, with mean distance 1.7 kpc and standard deviation 1.0 kpc.

Apart from six pulsars in the small sample, all pulsars are located at \( |b| \geq 5^\circ \) in order to ensure that the LOS towards the pulsars probe the DIG and not denser H II regions. The dispersion measures were taken from the catalogue of Manchester et al. (2005). The emission measures in the direction of the pulsars were obtained from Hα surveys (Finkbeiner et al. 2002; Haffner et al. 2003) and corrected for extinction (Diplas & Savage 1994b; Dickinson, Davies & Davis 2003) as well as for Hα emission originating beyond the pulsars. We refer to the work of Berkhujsen et al. (2006, hereafter called BMM) and Berkhujsen & Müller (2008) for further details.

Diplas & Savage (1994b) studied the scale height of the diffuse dust and the diffuse atomic gas using 393 stars in the Galaxy. We calculated the average H I volume density, \( \langle n_{HI} \rangle \) (in \( \text{cm}^{-3} \)), from the column density \( N(HI) \) (in \( \text{cm}^{-2} \)), corrected for contributions from the star, and the distance to the star as given in table 1 of Diplas & Savage (1994a):
\[ N(HI) = \int_0^D n_{HI}(l) dl = \langle n_{HI} \rangle D, \]

where \( n_{HI}(l) \) is the local electron density at point \( l \) along the LOS, \( \langle n_{HI} \rangle \) are averages along \( D \) and \( n_{HI} \) is the local volume density at distance \( l \) along the LOS. We removed 18 stars from the sample with denser clouds in their LOS indicated in fig. 9 of Diplas & Savage (1994b), leaving the data towards 375 stars for analysis. The stars in this sample have distances in the range \( 0.1 < D < 11 \) kpc, with a mean distance of 2.2 kpc and standard deviation 1.7 kpc.

3 RESULTS

3.1 Density PDFs of the DIG

In Fig. 1(a), we present the PDF of the mean density in clouds, \( N_c \), for the sample of 34 pulsars. As the volume filling factor is (anti-) correlated with \( N_c \) (see BMM), we show the PDF of \( F_v \) for the same sample in Fig. 1(b). In log space, both PDFs are consistent with a Gaussian distribution, which is equivalent to a lognormal distribution in linear space. The PDFs have about the same dispersion, \( \sigma \) (see Table 1). The positions of the maxima, \( \mu = \log(\text{density of maximum}) \), correspond to \( N_c = 0.19 \pm 0.02 \text{ cm}^{-3} \) and \( F_v = 0.078 \pm 0.006 \), which represent the centre of gravity in the \( F_v - N_c \) plot in fig. 6 of Berkhujsen & Müller (2008). This sample is rather small, with low counts \( N_c \) in the histogram bins and (probably) overestimated Poisson errors \( \delta_i = \sqrt{N_c} \) leading to rather small reduced-\( \chi^2 \) statistics (Table 1). Therefore, we also calculated the PDFs of \( N_c \) and \( F_v \) for the much larger sample of BMM, which are shown in Figs 1(c) and (d). Both are well fitted by Gaussians of widths that are nearly identical to those of the small sample (see Table 1). The positions of the maxima are at \( N_c = 0.19 \pm 0.01 \text{ cm}^{-3} \) and \( F_v = 0.115 \pm 0.005 \), corresponding to the centre of gravity in the \( F_v - N_c \) plot of BMM (their fig. 11). The good agreement between the PDFs of the two samples indicates that the statistical results on \( F_v \) and \( N_c \) of BMM are not influenced by the model distances and statistical absorption corrections that they used.

In Fig. 2, we present the PDFs of the average densities \( \langle n_c \rangle \) and \( \langle n_{HI}^2 \rangle \) for both samples, all of which are well described by a lognormal distribution.

Note that we write \( F_d, F_v \) and \( N_c \) where Berkhujsen et al. (2006) used \( f_d, f_v \) and \( h_c \).

We do not consider the PDFs of the column densities DM, EM and \( N(HI) \) because they are influenced by the distributions of the distances to the pulsars and stars in the samples.

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distribution. The dispersion in \( n_e \) is smaller than the dispersions in \( F_v \) and \( N_c \) due to their (anti-) correlation: \( F_v \) and \( N_c \) are not independent random variables. Note that the dispersion of the PDF of \( n_e \) of the BMM sample is about half that of the small sample. As BMM used distances to the pulsars derived from the NE2001 model of Cordes & Lazio (2002), \( n_e \) = DM/D returns the densities of the model. The small dispersion reflects the fact that the model is much smoother than the density variations in the real ISM measured for the small sample. The dispersion in \( n_e \) is larger than that of \( n_e \) as the intrinsic spread in EM is much larger than in DM (see BMM; Berkhuijsen & Müller 2008) while the distances used to calculate \( n_e \) and \( n_e \) are the same.

It is interesting to compare our data with the results of the magneto-hydrodynamic simulations of the ISM in the solar neighbourhood made by de Avillez & Breitschwerdt (2005). Their Fig. 7 shows the density PDFs of five temperature regimes that developed after about 400 Myr. The curve for 8000 < \( T_e \) < 16000 K, which is most applicable to the DIG, closely resembles a lognormal with maximum at \( \log(n_0) = -0.75 \) and dispersion 0.52. The lognormal distribution extends over a much larger density range [at least \(-2.5 < \log(n_0) < 1.2 \)] than our observations of \( N_c \). [\(-2 < \log(N_c) < 0 \) in Fig. 1]. The density of the maximum of 0.18 cm\(^{-3}\) agrees well with that of \( N_c = 0.19 \pm 0.02 \) cm\(^{-3}\) derived by us (see Table 1), but the dispersion is about 70 per cent larger. This could be due to the larger temperature range of this component in de Avillez & Breitschwerdt (2005) compared to the 6000–10000 K observed for the DIG (Madsen et al. 2006).

We conclude that the PDFs of the electron densities and filling factors in the DIG in the solar neighbourhood are lognormal as is expected for a turbulent ISM from numerical simulations.

### 3.2 Density PDFs of diffuse \( \text{H} \) \( \text{I} \)

In Fig. 3(a), we present the PDF of the average volume density of \( \text{H}_2 \), \( \langle n_\text{H}_2 \rangle \), for the full sample of 375 stars of Diplas & Savage (1994a). Above \( \log(n_{\text{H}_2}) \sim -1 \), the distribution is approximately lognormal, but there is a clear excess at lower densities reflected in the large reduced-\( \chi^2 \) statistic (see Table 2). Because low densities can be expected away from the Galactic plane, we calculated the PDFs for the latitude ranges \( |b| < 5^\circ \) and \( |b| \geq 5^\circ \) separately, as shown in Figs 3(b) and (d). Both distributions have a lognormal shape but they are shifted with respect to each other: the maximum of the low-\( |b| \) sample is at \( \langle n_{\text{H}_2} \rangle = 0.30 \pm 0.02 \) cm\(^{-3}\) and that of the high-\( |b| \) sample at \( \langle n_{\text{H}_2} \rangle = 0.16 \pm 0.01 \) cm\(^{-3}\) (see Table 2). The latter sample clearly causes the low-density excess in Fig. 3(a).

The dispersion of the PDF of the high-\( |b| \) sample is twice that of the low-\( |b| \) sample. It is not clear whether this is a real difference or due to selection effects in the low-\( |b| \) sample. Stars at low Galactic latitudes can only be seen through holes between the many dust clouds and the low latitude sample may be biased towards lower
The dispersion in density will narrow by a factor of $\sim 2$ if the number of cells increases by a factor of $\sim 5$.

Another interesting difference exists between the dispersions of the sample of warm HI at $|b| \geq 5^\circ$ and that of $\langle n_1 \rangle$ of the DIG (small sample), which is at the same latitudes. The temperatures of the two components are similar and if the ionized and atomic gas are well mixed, one would expect their dispersions to be the same. However, the dispersion of the DIG sample, 0.22 $\pm$ 0.01, is about 30 per cent smaller than that of the warm HI sample, 0.33 $\pm$ 0.03 (see Tables 1 and 2). A plausible explanation for the difference, which is also consistent with the higher density of the maximum in the diffuse $\langle n_1 \rangle$ PDF, is that low density regions are more readily ionized than higher density gas and that the average degree of ionization of the diffuse gas is substantially lower than 50 per cent. We estimate the degree of ionization to be about 14 per cent, using the densities for $\langle n_1 \rangle$ (small sample) and $\langle n_{HI} \rangle$ in Tables 1 and 2, consistent with the results of Berkhuijsen et al. (2006, their fig. 13) for a mean height above the mid-plane of about 500 pc. Alternatively, the DIG could have a higher mean temperature than the warm HI but with a smaller temperature range; in the simulations of de Avillez & Breitschwerdt (2005) high-temperature gas indeed has a lower median density and smaller dispersion.

Several groups have noted a link between the rms-Mach number $M$ and the dispersion of the gas density PDF in isothermal numerical simulations (e.g. Padoan, Jones & Nordlund 1997; Passot & Vázquez-Semadeni 1998; Ostriker, Stone & Gammie 2001). Although the DIG is not isothermal (Madsen et al. 2006), to a first approximation it can be considered so because the sound speed scales as $c_s \sim T^{1/2}$ and the observed temperature range of $6000 < T < 10000$ K corresponds to only a 30 per cent difference in $c_s$. Then, using the formula $\sigma^2 = \ln(1 + \beta^2 M^2)$ with $\beta \approx 0.5$ given by Padoan et al. (1997), and a typical value of $\sigma = 0.3 = \sigma_{HI}/\ln(10)$ from our results for the DIG and warm HI, we obtain $M \simeq 1.6$. Hill et al. (2008) compared their observed Emission Measure PDFs for the DIG with PDFs derived from isothermal MHD turbulence simulations to find reasonable agreement for $1.4 < M < 2.4$, consistent with our estimated value. A typical DIG temperature of $T \approx 8000$ K gives $c_s \approx 12.5$ km s$^{-1}$ and $M \simeq 1.6$ would then require turbulent velocities of $v \approx 20$ km s$^{-1}$.

While the global-disc simulations of Wada & Norman (2007) produced lognormal density PDFs, their dispersions are about 4 times greater than the dispersions we have found. This may be related to the average gas densities in their simulations: $\langle n_{HI} \rangle \sim 1$ cm$^{-3}$ compared to for example $\langle n_{HI} \rangle \sim 0.3$ cm$^{-3}$ for our total HI sample.

We may draw the following conclusions from the discussion of our results.

(i) The density PDF of the diffuse ISM cannot be fitted by one lognormal.

(ii) The density PDFs of the diffuse ISM in the disc ($|b| < 5^\circ$) and away from the disc ($|b| \geq 5^\circ$) are lognormal, but the positions of their maxima and the dispersions differ.

(iii) Several effects seem to influence the shape of the PDF. An increase of the number of clouds/cells along the LOS causes a decrease in the dispersion and a shift of the maximum to higher densities. On the other hand, an increase in the average density (or decrease in the mean temperature) increases the dispersion as well as the density of the maximum.

The competing effects described in the last conclusion will complicate the interpretation of PDFs observed for external galaxies.
Elmegreen (2002) and Wada & Norman (2007) have shown that the star formation rate in a galaxy is related to the shape of the density PDF if it is lognormal; the dispersion is an important parameter in this respect (Tassis 2007; Elmegreen 2008). A better understanding of the factors that influence the dispersion of the lognormal density PDF may be obtained from future simulations.

5 SUMMARY

Lognormal density PDFs have been found in recent numerical simulations of the ISM – both local, isothermal models (e.g. Ostriker et al. 2001; Vázquez-Semadeni & Garcia 2001; Kowal, Lazarian & Beresnyak 2007) and multiphase, global models (de Avillez & Breitschwerdt 2005; Wada & Norman 2007) – and have become an important component of theories of star formation (Elmegreen 2002; Tassis 2007; Elmegreen 2008). To date, there has been little observational data with which to compare density distributions produced by the simulations.

The results reported here provide strong support for the existence of a lognormal density PDF in the diffuse (i.e. average densities of $n < 1 \text{ cm}^{-3}$) ionized and neutral components of the ISM. In turn, the form of the PDFs is consistent with the small-scale structure of the diffuse ISM being controlled by turbulence. Future simulations should allow the calibration of the dispersion of the diffuse gas density PDF in terms of physically interesting parameters, such as the number of turbulent cells along the LOS.

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REFERENCES

Berkhuijsen E. M., Müller P., 2008, A&A, in press
Berkhuijsen E. M., Mitra D., Müller P., 2006, Astron. Nachr., 327, 82 (BMM)
Cordes J. M., Lazio T. W. J., 2002, preprint (astro-ph/0207156)
de Avillez M. A., Breitschwerdt D., 2005, A&A, 436, 585
Dickinson C., Davies R. D., Davis R. J., 2003, MNRAS, 341, 369
Diplas A., Savage B. D., 1994a, ApJS, 93, 211
Diplas A., Savage B. D., 1994b, ApJ, 427, 274
Elmegreen B. G., 2002, ApJ, 577, 206
Elmegreen B. G., 2008, ApJ, 672, 1006
Elmegreen B. G., Scalo J., 2004, ARA&A, 42, 211
Finkbeiner D. P., Schlegel D. J., Frank C., Heiles C., 2002, ApJ, 566, 898
Gaustad J. E., Van Buren D., 1993, PASP, 105, 1127
Haffner L. M., Reynolds R. J., Tufte S. L., Madsen G. J., Jaehnig K. P., Percival J. W., 2003, ApJS, 149, 405
Hill A. S., Reynolds R. J., Benjamin R. A., Haffner L. M., 2007, in Havercorn M., Goss W. M., eds, ASP Conf. Ser. Vol. 365, SINS–Small Ionized and Neutral Structures in the Diffuse Stellar Medium. Astron. Soc. Pac., San Francisco, p. 250
Hill A. S., Benjamin R. A., Kowal G., Reynolds R. J., Haffner L. M., Lazarian A., 2008, ApJ, in press (arXiv:0805.0155)
Kowal G., Lazarian A., Beresnyak A., 2007, ApJ, 658, 423
Madsen G. J., Reynolds R. J., Haffner L. M., 2006, ApJ, 652, 401
Manchester R. N., Hobbs G. B., Teob A., Hobbs M., 2005, AJ, 129, 1993
Ostriker E. C., Stone J. M., Gammie C. F., 2001, ApJ, 546, 980
Padoan P., Jones B. J., Nordlund Å P., 1997, ApJ, 474, 730
Passot T., Vázquez-Semadeni E., 1998, Phys. Rev. E, 58, 4501
Tabatabaei F. S., 2008, PhD thesis, Bonn Univ.
Tabatabaei F. S., Beck R., Krügel E., Krause M., Berkhuijsen E. M., Gordon K. D., Menten K., 2007, A&A, 475, 133
Tassis K., 2007, MNRAS, 382, 1317
Vázquez-Semadeni E., Garcia N., 2001, ApJ, 557, 727
Wada K., Norman C. A., 2007, ApJ, 660, 276
Wada K., Spaans M., Kim S., 2000, ApJ, 540, 797

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