Aircraft Inflight Icing Detection Based on Statistical Decision Theory

D Ding1,2,*, W Q Qian1 and Q Wang1

1 Computational Aerodynamics Institute, China Aerodynamics Research and Development Centre, Mianyang, 621000, China
2 Research supported by CARDC Fundamental and Frontier Technology Research Fund

*E-mail: dingdi1981@cardc.cn

Abstract. Aircraft inflight icing detection is very important for aviation flight safety. The ice alarm signal given by detectors can help the pilot to actuate the ice protection system. The application of a Neyman Pearson (NP) based statistical decision approach in icing detection is discussed in this paper. First, the flight dynamics for icing aircraft is modelled, the dynamic behaviour of an airbus influenced by ice accreting on the leading edge of wings is analysed. Then the residuals which are directly affected by the aerodynamic parameters are presented and their generation way is given. The Generalized Likelihood Ratio Test (GLRT) method is utilized in detecting the changes of residuals inflight, its performance is assessed by a flight scenario with wing ice. The results show that the fault of wing icing can be detected by every residual under different window sizes. By comparing the results of the residuals, it can be concluded that the small window size will degrade the detection performance with the longer alarming time delay and the lower probability of detection; appropriately increasing the window size can reduce the alarming time delay; diagnosing by residual of normal overload is more reliable than by the other two residuals.

1. Introduction
Aircraft in-flight icing is one of the most severe threats to flight safety, and sometimes it even leads to fatal accidents and casualties. Based on statistic data given by the American Safety Advisor, from 1990 to 2000, there are 12% of all the flight accidents which resulted from adverse weather conditions that occurred in icing and 92% of the ice induced accidents that took place in in-flight icing [1]. Thus, the icing detection and protection technologies become more and more important in the aviation industry. The ice protection system usually needs the ice alarm signal to actuate the de-ice or anti-ice devices.

There are two main means to detect the alarm signal: the direct detection using icing detectors or sensors [2-3], and the indirect detection using mathematical algorithms. Although many aircrafts have utilized the direct detection technology successfully, there still is a strong demand to develop detection algorithms as an auxiliary means for the insufficient or incapable usage of the direct detection method.

Since the concept of Smart Icing System [4] presented by NASA icing research group, the studies on indirect detection methods have been developed rapidly. There are three main types of algorithms for aircraft in-flight icing detection: the parameter identification method, the data-based modelling method, and the observer-based detection method. The parameter identification method estimates the flight status and the changing aerodynamic parameters jointly by filtering algorithms, such as Extended
Kalman Filter (EKF) [5-6], $H_\infty$ filter [6-8] and Multiple Model Adaptive Estimator (MMAE) [9-10]. The parameterized ice accretion process can be estimated by these filters or estimators and can be used for giving icing alarm or for reconfiguring the flight control [11-13] to ensure the aircraft stability in icing conditions. The data-based modelling method models the correlations between flight status and the detailed icing information, such as ice-influenced parameters, icing severity factor, position and time. Caliskan [13] presented a Neural Network (NN) method for identifying icing parameters from the measured state data, Schuchard [14] also developed an artificial NN method to detect the presence of icing and classify its severity, Dong [15-16] utilized a probabilistic NN to decide the icing location. For icing detection, data-based modelling methods can obtain as many icing details as parameter identification methods, but the problems of the two approaches, like the over-fitting and generalization problems of data-based modelling methods and the parameter identifiability and accuracy problems of identification methods, limit the usages of the two methods in some circumstances.

On the other hand, the observer-based detection method is more effective in giving an icing alarm. There are two applications of this method: the model-based fault diagnosis technique and the statistical diagnosis technique. The model-based fault diagnosis technique models the aircraft failure (including actuator [17], sensor [18], structure and icing [19-21]) and then detects this failure by estimators. The statistical diagnosis technique uses hypothesis testing theory to detect the changes in the observed flight data. This diagnosis approach does not require a priori statistical characteristics of the faults and has low computational cost. The innovation approach [12,22-23] and the Neyman Pearson based statistical change detection approach [24] are the commonly used statistical diagnosis techniques for aircraft icing detection, and the former one directly uses the hypothesis testing on the innovation sequence to determine the changes of the aircraft dynamics, its performance relies on the predetermined confidence coefficient. However, the NP based diagnosis technique has a predictable performance under the NP theory, and is proved to be the optimal test subject to a constant probability of false alarm.

This paper focuses on the issue of giving an icing alarm in aircraft flight, the NP based diagnosis technique is chosen because of its detection efficiency and predictable performance. The diagnosis residuals of overloads and angular accelerations are discussed, and their generation way is considered in this paper. A Generalized Likelihood Ratio Test method is utilized to detect the changes of residuals caused by the variation of any aerodynamic parameter. Then an airbus inflight wing icing scenario is assumed and used to assess the performance of the icing detector. Different window sizes are considered for diagnosing the residual, the icing alarming time and the probability of detection are concerned about for the inflight detection performance assessment.

2. Aircraft icing dynamics

2.1. Icing research airbus

An airbus model is specifically presented for the aircraft icing research by the research group. The prototype has the similar aerodynamic and flight dynamic characteristics as Boeing 737 and Airbus A320. Figure 1 shows the configuration structure of the airbus, and Table 1 lists its physical parameters.

Figure 1. Icing research airbus 3D structure.  

Figure 2. Severe wing icing.
The aerodynamic characteristics of the airbus are calculated by computational fluid dynamics (CFD) tool. RANS and multi-block structured grid are used to obtain the longitudinal aerodynamic data under clean and severe wing icing configuration (Figure 2). Then a polynomial equation (1) is used to fit the longitudinal characteristics, where \( C_D \), \( C_L \), and \( C_M \) represent drag, lift and pitch moment coefficients respectively, \( \alpha \) is the angle of attack (AOA), \( q \) is the pitch rate, \( \delta_e \) is the elevator angle. The aircraft’s stability and control derivatives in two configurations are compared in Table 2. Ignoring the small influence of wing icing on \( C_{D0} \), \( C_{L0} \), and \( C_{M0} \) in this table, the other longitudinal derivatives are influenced by ice accretion. The fitted results indicate wing icing increases drag coefficients, and decreases lift and pitch moment coefficients and control surface efficiency.

\[
C_D = C_{D0} + C_{Da}\alpha \\
C_L = C_{L0} + C_{La}\alpha + C_{Lae}\delta_e \\
C_M = C_{M0} + C_{Ma}\alpha + C_{Mae}\alpha + C_{Mq}q + C_{Me}\delta_e 
\]  

(1)

Table 1. Parameters of icing research airbus.

| Aircraft Parameters | Values       |
|---------------------|--------------|
| Mass \( m \)        | 72000 kg     |
| Wing reference area \( S \) | 124 m²      |
| Wing span \( l \)   | 34.1 m       |
| Mean aerodynamic chord \( b_A \) | 4.15 m     |
| Moment of inertia \( I_{xx} \) | 1658755 Kg×m² |
| Moment of inertia \( I_{yy} \) | 2392630 Kg×m² |
| Moment of inertia \( I_{zz} \) | 3846326 Kg×m² |

Table 2. Longitudinal derivatives of icing research airbus in clean and severe iced configurations.

| Longitudinal Derivatives | Clean      | Severe Iced |
|--------------------------|------------|-------------|
| \( C_{D0} \)             | 0.0277     | 0.0277      |
| \( C_{Da} \)             | 0.1069     | 0.7701      |
| \( C_{Lae} \)            | 0          | 0.0149      |
| \( C_{L0} \)             | 0.1415     | 0.1415      |
| \( C_{La} \)             | 6.2326     | 3.0637      |
| \( C_{Lae} \)            | 0.448      | 0.2848      |
| \( C_{M0} \)             | 0.0353     | 0.0353      |
| \( C_{Ma} \)             | -1.7873    | -1.2762     |
| \( C_{Ma} \)             | -13.7477   | -13.0491    |
| \( C_{Mq} \)             | -41.513    | -39.4878    |
| \( C_{Me} \)             | -1.9035    | -1.3476     |

2.2. Aircraft icing flight dynamics

The longitudinal flight dynamics considering ice accretion process for aircraft is discussed. The motion and rotation equations (2) are established in the velocity and the body coordinate respectively.
\[ V = \frac{q_v S}{m} C_D + \frac{P_x}{m} + g_z, \]
\[ \dot{V} = -\frac{q_v S}{m} \cos \beta C_L + \frac{P_x}{m} \cos \beta + \frac{g_z}{V \cos \beta} - \frac{1}{\cos \beta} \left( p \cos \alpha \sin \beta - q \cos \beta + r \sin \alpha \sin \beta \right) \]
\[ \dot{q} = \frac{q_v S b_h}{I_{yy}} C_{m}^d + \frac{M_x}{I_{yy}} + \frac{I_{zz} - I_{yy}}{I_{yy} I_{yy}} p r - \frac{I_{zz}}{I_{yy}} \left( p^2 - r^2 \right) \]
\[ \dot{\theta} = q \cos \phi - r \sin \phi \]
\[ h = V \cos \alpha \cos \beta \sin \theta - V \sin \beta \cos \theta \sin \phi - V \sin \alpha \cos \beta \cos \theta \cos \phi \]

where \( V \) is the aircraft velocity, \( \beta \) is the angle of sideslip, \( p, q, \) and \( r \) represents roll, pitch and yaw rates respectively, \( \theta \) is pitch angle, \( \phi \) is roll angle, \( h \) is aircraft height, \( q_v \) is dynamic pressure, \( S \) is wing reference area, \( b_h \) is the mean aerodynamic chord, \( m \) is the aircraft mass, \( P_x \) and \( P_z \) represent engine thrusts, \( M_t \) is the thrust moment, \( g_z \) and \( g_x \) are the gravitational acceleration components.

The changes of drag, lift and pitch moment coefficients during ice accretion process are needed to be modelled for analysing the icing influence on aircraft flight dynamics. Bragg [4] of Illinois University presented a mathematical model to describe the icing influence on aerodynamic derivatives. It has already been utilized in the development of Ice Management System (IMS) and aircraft icing online detection [7,9,16]. This model is not an accurate icing influence model, but it can indicate the variation trend of the aerodynamic derivatives during ice accretion. The icing influence on aerodynamic derivatives can be expressed by:

\[ C_{\alpha}^{\text{iced}} = C_{\alpha}^{\text{clean}} \left( 1 + K_C \eta_{\text{ice}} \right) \]

\( C_{\alpha} \) represents arbitrary aerodynamic derivative, the superscript “iced” indicates the iced derivative, and “clean” indicates the clean derivative. \( K_C \) represents the coefficient slope which depends on the modified parameter. \( \eta_{\text{ice}} \) is the icing severity factor, with \( \eta_{\text{ice}}=0 \) denoting a clean configuration, and \( \eta_{\text{ice}}=1 \) denoting wing iced configuration. The curve of \( \eta_{\text{ice}} \) represents the icing influence on aerodynamic derivative. Melody [7] gave a continuous accretion model of ice over time. The ice accretion rate is considered as a function of both atmospheric conditions and the amount of ice already accreted. The differential equation is given by:

\[ \dot{\eta}_{\text{ice}} = N_1 \left( 1 + N_2 \eta_{\text{ice}} \right) d_q \]

\[ d_q(t) = \frac{1}{2} \left[ 1 - \cos \left( \frac{2 \pi t}{T_{\text{cl}} / 2} \right) \right] \]

\[ N_1 = \frac{2}{\eta_{\text{Ice}}(T_{\text{cl}})} \ln \left[ 1 + N_2 \eta_{\text{Ice}}(T_{\text{cl}}) \right] \]

\[ N_2 = \frac{\eta_{\text{Ice}}(T_{\text{cl}}) - 2 \eta_{\text{Ice}}(T_{\text{cl}} / 2)}{\eta_{\text{Ice}}(T_{\text{cl}} / 2)} \]

\( T_{\text{cl}} \) is the ice accretion time. When \( T_{\text{cl}}, \eta_{\text{Ice}}(T_{\text{cl}} / 2) \) and \( \eta_{\text{Ice}}(T_{\text{cl}}) \) are given, the curve of arbitrary aerodynamic derivative varying over time can be described by equations (3)–(6). The parameters of \( \eta_{\text{Ice}}(T_{\text{cl}} / 2) \) and \( \eta_{\text{Ice}}(T_{\text{cl}}) \) denote the different ice accretion rates.

2.3. Analysis of icing influence on flight dynamics

A steady-level cruise scenario with ice accreting on the leading edge of wings is studied to analyse the ice influence on dynamic behaviour. Assuming the aircraft maintains a steady-level cruise on the trimmed status, the engine thrusts are used to keep the aircraft stable in the absence of a control system.
The flight status and the ice accretion parameters of this scenario are shown in Table 3. The process noises are considered and assumed to be the independent vertical and horizontal acceleration perturbations. White Gaussian noise with the intensity of $[0.1 \text{m/s}, 0.1/V_0]^T$ is used for both $V$ and $\alpha$, and the other flight status is assumed to have zero process noise.

The aircraft icing flight dynamics model is used to analyse the dynamic behaviour influenced by ice accretion. Figure 3 depicts the simulated flight states of velocity, AOA, pitch rate and height. The results indicate wing ice would raise the aircraft nose, reduce the speed, increase the height and cause the periodic vibration of the aircraft flight states.

| Flight States          | Values   |
|------------------------|----------|
| Height $H$             | 5000 m   |
| Velocity $V$           | 0.3 Ma   |
| Trimmed elevator angle $d_e$ | 1.06 deg |
| Angle of attack $\alpha$ | 0 deg    |
| Pitch angle $\theta$  | 0 deg    |
| Pitch rate $q$         | 0 deg/s  |
| Ice occurring time $t_{\text{ice}}$ | 200 s    |
| Ice accretion time $T_{\text{cld}}$ | 200 s    |
| Ice accretion rate $\eta_{\text{ice}}(T_{\text{cld}})$ | 1.0      |
| Ice accretion rate $\eta_{\text{ice}}(T_{\text{cld}}/2)$ | 0.7      |

**Table 3.** Flight status and ice accretion parameters of simulation scenario.

3. Aircraft icing statistical diagnosis approach

3.1. Generalized likelihood ratio test

For the realistic problem, the probability density function (PDF) of a signal will be partly unknown due to some unknown parameters. The GLRT method estimates the unknown parameters by Maximum Likelihood estimators (MLEs) to solve these problems [25]. The detection problem can be mathematically expressed as:

$$H_0: x[n] = w[n], \quad n = 0, 1, \ldots, N-1$$

$$H_1: x[n] = A + w[n], \quad n = 0, 1, \ldots, N-1$$

(7)
Where $x$ denotes the testing signal, $A$ is unknown with $-\infty < A < \infty$ and $w[n]$ is white Gaussian noise with unknown variance $\sigma^2$. $N$ is the window size. The $\mathcal{H}_0$ hypothesis describes the case where the unknown parameter $A$ equals zero, whereas the alternative hypothesis $\mathcal{H}_1$ describes the case where the unknown parameter has an offset from zero. Meanwhile, the signal contains noise with unknown statistical characteristics in both hypotheses.

The GLRT can be used to distinguish between the two hypotheses. Based on the likelihood ratio between the probability of the two hypotheses, the test statistic for the problem in equation (7) decides $\mathcal{H}_1$ in a given window size of data if

$$L_0(x) = \frac{p(x; \hat{A}, \hat{\sigma}_1^2, \mathcal{H}_1)}{p(x; \hat{\sigma}_0^2, \mathcal{H}_0)} > \gamma$$

(8)

Where $\{\hat{A}, \hat{\sigma}_1^2\}$ is the MLE of the parameters $\{A, \sigma^2\}$ under $\mathcal{H}_1$ and $\hat{\sigma}_0^2$ is the MLE of the parameter $\sigma^2$ under $\mathcal{H}_0$. $\gamma$ denotes the threshold. With the MLEs the following test statistic can be derived and its asymptotic PDF is

$$T(x) = N \ln \left(1 + \frac{\bar{x}}{\hat{\sigma}_1^2}\right): \begin{cases} \chi^2_1 & \text{under } \mathcal{H}_0 \\ \chi^2_1(\lambda) & \text{under } \mathcal{H}_1 \end{cases}$$

(9)

Where the MLE of the unknown parameters can be written as

$$\hat{A} = \bar{x}$$

$$\hat{\sigma}_1^2 = \frac{1}{N} \sum_{n=0}^{N-1} (x[n] - \bar{x})^2$$

(10)

The noncentrality parameter $\lambda = NA^2/\sigma^2$ . Then the threshold can be determined according to the NP theorem, that is for a given $P_{FA}=\alpha$, the threshold $\gamma$ is found from equation (11) and maximizes the probability of detection $PD$ under $\mathcal{H}_1$. 

$$P_{FA} = \int_{\{x|x_L(x) > \gamma\}} p(x; \mathcal{H}_0) \, dx$$

(11)

Where $p(\cdot)$ is the probability distribution function of a given test statistic.

3.2. Residual analysis

Ice accreting on the leading edge of wings changes the aircraft structure and causes the variation of the aerodynamic parameters. Although the ice accretion causes the obviously periodic vibration of the aircraft flight states, it doesn’t mean the flight states are suitable for the statistical diagnosis. Because the state variables are the integral or multiple integral results of the changing aerodynamic parameters, which means the bias existing in state signals is not only related to the variation of the aerodynamic parameters, but also related to the noises and accumulative errors. Thus, the signal directly related to the changing aerodynamic parameters is needed for statistical diagnosis, such as the axial and normal overloads presented by Sorensen [24]. The overloads and angular accelerations are directly affected by aerodynamic parameters, their changes indicate the changing of forces and moments. If the engine forces and moments are all known, the overloads and angular accelerations are the perfect observing objects for icing detection. Although the icing statistical diagnosis based on overloads has already been proved to be efficient and effective, the feasibility of diagnosing by angular accelerations has not been discussed yet.

Figure 4 gives a scheme of generating diagnosis residuals for realizing the in-flight icing detection. In this scheme, the inputs, engine forces/moments and measurements of aircraft dynamic system are used to estimate the overloads and angular accelerations based on the clean aerodynamic characteristics, then the estimated values are compared with the measured values to generate the diagnosis residuals.
The estimation of overloads and angular accelerations is directly based on the dynamic equations, and for the problem in this paper, it can be calculated by equation (12).

\[
N_{x}^{est} = \left[ -q_{s}S \left( C_{D}^{clean} \cos \alpha - C_{L}^{clean} \sin \alpha \right) + P_{x} \right] \frac{m_{g0}}{N} \\
N_{z}^{est} = \left[ -q_{s}S \left( C_{D}^{clean} \sin \alpha + C_{L}^{clean} \cos \alpha \right) + P_{z} \right] \frac{m_{g0}}{N} \\
q^{est} = \frac{q_{s}Sb_{A}C_{M}^{clean}}{I_{yy}} + M_{y}
\]

(12)

Where \( g_{0} \) is the gravitational constant. If the ice accretion causes the aerodynamic coefficients displaying unexpected changes, a bias will be introduced into the residuals, and then the statistical diagnosis method can be used to detect the changes in the residual signals for giving an icing alarm.

The simulation scenario in Table 3 is utilized to analyse the residuals. Here the measurement noises are considered additionally. The white Gaussian noises are assumed, and the standard deviations are listed in Table 4 by referring to the measurement noises of A340 [12]. Figure 5 gives the three residuals of this simulation scenario. Except for the residual of \( N_{z} \), the ice influence can be barely seen from the other two residuals under the random noises. Then the normal distribution parameters under clean and iced intervals are estimated to illustrate the ice influence, the mean \( \mu \) and standard deviation \( \sigma \) are given in the figure respectively for the clean intervals of \([0, 200]\)s and iced intervals of \([400, 600]\)s. The results show the residuals have obvious offsets of mean values after the ice accretion, but the standard deviations are almost unaffected.

| Standard Deviations       | Values       |
|---------------------------|--------------|
| Overload \( N_{x} \)      | 0.01         |
| Overload \( N_{z} \)      | 0.01         |
| Dynamic pressure \( q_{\infty} \)| 1e-3 \( \text{kg/(m}^2\text{s}) \) |
| Angle of attack \( \alpha \)| 0.0056 \( \text{deg} \) |
| Pitch rate \( q \)        | 0.0069 \( \text{deg/s} \) |

3.3. Inflight icing detection performance assessment

Due to the lack of prior information on the noises, the inflight icing detection is a typical statistical decision problem with unknown parameters. The GLRT method is suitable for giving an inflight icing alarm. The assessment of GLRT performance in inflight icing detection is focused on giving a timely alarm. The three generated residuals are used to monitor the icing situation. The residual data segments of a given window size are successively taken out for statistical diagnosis. When the test statistic exceeds the threshold, the icing situation is decided.

The simulation residuals in Figure 5 are used to assess the performance of GLRT method. The trade-off between a high probability of detection \( P_{D} \) and a low probability of false alarm \( P_{FA} \) is needed to be concerned for GLRT diagnosis. In this paper a small \( P_{FA} \) of 1e-6 is used, then the right-tail probability of \( \chi_{1}^{2} \) distribution under \( H_{0} \) is used to calculate the threshold. For this problem, the threshold is constant and equals 23.93. The influence of window size is considered. Long time intervals are not good for alarming timely. Here the window size range of \([1, 40]\)s is chosen and the sampling frequency is 100Hz. The results of alarming time and probability of detection \( P_{D} \) under different window sizes are shown in Figure 6, in this figure the detection results of three residuals are compared. The results indicate the small window size will degrade the detection performance with the longer alarming time delay and the lower probability of detection. Detecting by the residual of \( N_{x} \) has the longer time delay than by the other two residuals. Diagnosing by the residual of \( N_{x} \) has higher probability of detection \( P_{D} \) than by the other two residuals. Increasing the window size reduces the alarming time to the range of \([270, 290]\)s by the residuals of \( N_{z} \) and \( dq/dt \). The results also show that diagnosing by residual \( N_{x} \) is more reliable than by
the other two residuals, it is probably because the aircraft wing ice causes the more obvious normal offset.

\[ \text{Figure 5. Diagnosis residuals analysis.} \]

\[ \text{Figure 6. Icing alarming time and probability of detection under different window sizes.} \]

4. Conclusion
This paper addresses the issue of detecting ice when it occurs in the aircraft flight. A NP based statistical decision approach is presented to give a quick alarm of icing. The dynamic behaviour under icing is modelled and analysed for the icing detector design. Then the axial and normal overloads and pitch angular acceleration which are directly affected by the aerodynamic parameters are considered to be the diagnosis residuals, and their generation way is discussed. The GLRT method is used in the inflight icing detection, its performance is assessed by a flight scenario of an airbus with ice accreting on the leading edge of wings. The results show the three residuals are all effective in icing detection. Due to the normal offset is more obvious, diagnosing by residual $N_z$ is more reliable than by the other two residuals.

The research in this paper is a preliminary work of introducing the statistical decision approach into inflight icing detection, the primary purpose of this paper is to verify the feasibility of this statistical decision approach. However, considering the complicated environment factors in real flight, there still have many studies to be done before applying the approach in real inflight icing detection.

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