Generative design of biosimilar structures

Stepanyan I V
Mechanical Engineering Research Institute of the Russian Academy of Sciences (IMASH RAN) 4, M. Kharitonyevskiy Pereulok, 101990 Moscow, the Russian Federation.

neurocomp.pro@gmail.com

Abstract. The article discusses ideas at the junction of various paradigms for the synthesis of new structures and shapes, where part of the calculation processes is based on computer technology using non-Euclidean geometry. Algorithms for generating bio-similar surfaces, parts, and textures that can be used in industry, in particular in additive technologies are given. The article has the descriptions of equations based on hyper-complex systems and conditions of the analyticity of functions. The algorithms for generating biosimilar surfaces can be used in generative design. While determining the parameters of hypercomplex systems and assessing the mechanical characteristics of the final products require additional research, the relative simplicity of mathematical algorithms for generating structures and shapes with non-Euclidean geometry opens up the possibility of applying the proposed method in various fields.

1. Introduction
The scientific community is gaining ground in the importance of discussing ideas at the junction of various approaches for the synthesis of new materials and structures and the analysis of methods for their application. Light industry materials and structures with high strength characteristics and complex geometry are in demand in the industry. Currently, production technologies mainly use Euclidean geometry. However, all living things have a non-Euclidean metric and fractal form. At the same time, living systems are based on effective mechano-biological solutions that make sure their survival during evolution. In this regard, biosimilar materials and designs are used in architecture, they are in demand for the development of new forms and structural elements of anthropomorphic biosimilar robots, they are necessary for medical purposes (for bioprosthetics and three-dimensional printing of tissues and internal organs), in machine mechanics, agriculture and other areas.

Bio-similarity allows not only to increase strength while reducing the weight of the structure but also to expand functionality. For example, bone tissue due to its porous structure has strength and lightness, while the complex shape of the joint surface provides ample mobility. In connection with the foregoing, the use of non-Euclidean geometry and fractal principles is proposed for generative design. The aim of this work is to show the mathematical foundations of the synthesis of biosimilar objects with a complex shape and internal structure.
2. Related works
The curved surfaces that characterize living organisms are usually close to two-dimensional and are well described by differential Riemannian geometry [1]. In the book "Imaginary Geometry" [2] P. A. Florensky (1882-1937) drew attention to the special mathematical properties of duality. He managed to interpret imaginary quantities without leaving the initial premises of analytic geometry on a plane with the interpretation of two-dimensional images on curved surfaces. The mathematical properties of two-dimensional space make it possible to obtain a variety of shapes and surfaces.

In the work [3] S.V. Petukhov showed that the geometry of living organisms can be described using a non-Euclidean metric, and the genetic foundations of living matter are well described by hypercomplex numbers, which are finite-dimensional algebras over the field of real numbers. Algebras of hypercomplex numbers allow describing objects with non-Euclidean geometry and are suitable for describing biomechanical structures used by nature.

Fractal theory is widely using in engineering and biotechnology [4]. While the adsorptive fouling in membrane bioreactors (MBRs) is highly dependent of the surface morphology, little progress has been made on modeling biocake layer surface morphology. In [5] a method, which combined static light scattering method for fractal dimension (Df) measurement with fractal method represented by the modified two-variable Weierstrass-Mandelbrot function, was proposed to model biocake layer surface in a MBR. The constructed biocake layer surface by the proposed method was quite close to the real surface, showing the feasibility of the proposed method. It was found that Df was the critical factor affecting surface morphology, while other factors exerted moderate or minor effects on the roughness of biocake layer. The study [6] evaluated attachment of a 30-nm nanoparticle to and detachment from fractal surfaces by calculating Derjaguin-Landau-Verwey-Overbeek (DLVO) interaction energies in three-dimensional space using the surface element integration technique. The fractal surfaces were generated using the Weierstrass-Mandelbrot function. Results show that theoretical findings can explain various experimental observations in the literature, and can have important utility to development of water filtration techniques in engineered systems and to assessment of environmental risks of nanoparticles.

Using the paradigm of Fatou-Julia iteration, at [7] iterations to map fractals accompanied with a criterion to ensure that the image is again a fractal developed. The characterization of fractals as trajectory points is an important step toward a better understanding of the link between chaos and fractal geometry and would be helpful to enhance and widen the scope of their applications in physics and engineering. In [8] authors present a novel fractal coding method with the block classification scheme based on a shared domain block pool. A block pool is called dictionary and is constructed from fractal Julia sets. The image is encoded by searching the best matching domain block with the same BTC (Block Truncation Coding) value in the dictionary. The experimental results show that the scheme is competent both in encoding speed and in reconstruction quality. Particularly for large images, the proposed method can avoid excessive growth of the computational complexity compared with the traditional fractal coding algorithm.

3. Material and methods
Fractal sets constructed using the algebra of complex numbers are Mandelbrot and Julia sets [9-13]. In [14], we obtained generalizations of these sets for various hypercomplex systems. Since generalized algebraic fractal sets have a high degree of diversity [15-17], we performed a series of experiments for the algorithmic generation of biosimilar objects with non-Euclidean geometry.

Iterations of complex analytic mappings serve as a source of various fractal structures on a two-dimensional surface. Nonlinear math mapping:

\[ z = z^2 + c, \quad (1) \]

where c is some constant complex number, contains incredibly complex dynamics (see examples in [9]). Our computational experiments showed that biosimilarities are characterized by generalizations
of this mapping. There are several ways to consider mappings of a more general form. One can consider polynomial or transcendental mappings. For them, there is a well-developed theory [10], where Julia sets acquire more symmetries.

The synthesis of new forms of structures and materials with a biosimilar structure can be associated with a departure from the complex analyticity of equation (1) [11]. The visual images of generalized analogs of the Mandelbrot and Julia sets are qualitatively different from the originals, which is especially evident in the example of the “burning ship” mapping [12]. In a more general context, complex analytic mappings and a “burning ship” are special cases of two-dimensional real mappings obtained from complex ones by extracting the real and imaginary parts. In this form, mapping (1) has the form:

\[ x = x^2 - y^2 + \Re(c) \] (2)
\[ y = 2xy + \Im(c) \]. (3)

and in the "burning ship" equation (2) is the same, and (3) becomes

\[ y = 2|xy| + \Im(c) \]. (4)

4. Results and discussion
Formally, such mappings are distinguished by fulfilling the Cauchy – Riemann conditions for them, which leads to the impossibility of the appearance or disappearance of fixed points when changing the characteristic c of the mapping (1). The subject of the article [11] was a study of how these properties are lost when moving away from complex analyticity. For this, the mapping was used.

\[ z = z^2 + c + a \], (5)

coinciding with (1) for \( a = 0 \) and nonanalytical for \( a \neq 0 \). Bearing in mind further generalizations of the mapping (5), we realized a more general mapping

\[ x = x^2 - y^2 + \Re(c) + ax \] (6)
\[ y = 2xy + \Im(c) - by \]. (7)

Julia sets of generalized mappings are structures that can be used in generative design (Figure 1).
Complex analytic dynamics is not the only example of two-dimensional real mappings with pronounced features of their Julia sets. Another way of generating the mappings is the application of formulas (1-7) for various types of hypercomplex systems.

5. Conclusion
The method of hypercomplex synthesis of biosimilar structures and materials involves the choice of a variant of hypercomplex mapping and its scope. Mapping options define the shape of the resulting structure. The choice of the domain of definition of the mapping allows us to expand the variety of forms due to the fractal properties of sets.

The properties of two-dimensional mappings with a sufficiently small step of characteristic changes allow the generation of surfaces with pronounced nonlinear properties. The result of each conversion can be additive printed in the form of the corresponding product layer. A small step of changing mapping parameters allows printing three-dimensional products with a fairly smooth surface shape. Changing the parameters of the selected hypercomplex mapping allows adjusting the geometry of forms and structures, as well as the structure of their internal content. In some cases, within the structure, it is possible to create a tree-like structure of branching cavities by analogy with the blood circulatory system (from large arteries to small capillaries). This circumstance makes the described technologies promising for biomedical purposes. The same circumstance allows designing materials with mechanical features and resistance to complex influences due to its complex internal structure. In general, the proposed paradigm “erodes” the connection between construction and material, combining these objects into a single technological solution.

The above algorithms for generating biosimilar surfaces, parts, and textures can be used in generative design, including additive technologies. While determining the parameters of hypercomplex systems and assessing the mechanical characteristics of the final products need more research, the relative simplicity of mathematical algorithms for generating structures and shapes with non-Euclidean geometry open up the possibility of applying the proposed methodology in various fields.

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