Breakage Characteristics of Spherical Gypsum Particles under Three-point Contact

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Abstract. Coarse-grained soil is widely used in various civil engineering. Particle breakage has a certain influence on its mechanical properties and engineering application. To study the breakage characteristics of coarse-grained particles under lateral pressure, the breakage characteristics of spherical gypsum particles under three-point contact were studied. The particle breakage experiments of spherical gypsum particles under three-point contact were carried out using Rock Rheological Testing System, combined with associated auxiliary fixtures. The contact mechanical properties and breakage characteristics of spherical gypsum particles were studied from the aspects of breakage process, breakage form, mass distribution after breakage, cone core and so on. It is found that under the condition of three-point contact, the spherical gypsum particles undergo integral fracture along the contact point line controlled by tensile stress. With the increase of lateral contact force, the vertical contact force decreases gradually when breakage occurs. The cone core was formed at the contact point when the lateral stress is high. The size of the cone core mainly depends on the contact force of the corresponding contact point, and is less affected by the contact force at other contact points. The stress state of the contact is analyzed and the breakage criterion in the case of three-point contact is established based on the Coulomb-Moore Criterion. It is proved that the breakage criterion is effective.

Keywords. spherical gypsum particle; three-point contact; breakage; cone core; breakage criterion

1. Introduction
Coarse-grained soil is widely used in civil engineering cases, such as embankment, port and roadbed, because of its high strength, high bearing capacity, good permeability and so on. However, in the case of high stress, coarse-grained soil will be broken. Particle breakage has a certain influence on the mechanical properties and engineering applications of coarse-grained soil. It is of great significance to study the breakage characteristics of coarse-grained soil for a wide range of natural and industrial processes. Many scholars have studied the mechanical properties of coarse-grained soil through theoretical analysis [1–4], laboratory experiments [5–7] and numerical simulation [8–12]. It is found
that the breakage characteristics and mechanism of coarse-grained soil are very complex, and the breakage of coarse-grained soil is affected by many factors, such as particle material, shape, stress state and so on.

Scholars in China and other countries have achieved some success in the research on the breakage characteristics of coarse-grained materials under different lateral pressures. Wei et al. [13] studied the particle breakage law of a coarse-grained material under constant pressure consolidation, peak value and different stress levels through a triaxial particle breakage experiment. It was concluded that the particle breakage rate increases rapidly with the increase of stress level under the same confining pressure, and that normalization treatment could be carried out between particle fragmentation rate and stress level under different confining pressure. Wang et al. [14] analyzed the particle breakage of rockfill through indoor large-scale uniaxial consolidation experiment and triaxial shear experiment. The experimental results showed that the breakage rate of particles increases continuously and tends to be stable with the increase of confining pressure, and the relationship between them could be expressed by a power function. Riccardo et al. [15] studied the influence of the contact location distribution of cylindrical particles on the particle strength and fracture pattern by the drop weight crushing experiment. However, the research on the effect of contact position anisotropy on particle breakage properties is still in its infancy.

To study the breakage characteristics of the coarse-grained particles under lateral pressure, the particle breakage experiments of spherical gypsum particles under three-point contact were carried out in this paper. The breakage experiment of spherical gypsum particles in the case of three-point contact was carried out by using Rock Rheological Testing System, combined with the related auxiliary fixture. It is mainly analyzed from the aspects of breakage process, breakage shape, mass distribution after breaking, cone core and so on. The contact mechanical properties and breakage characteristics of spherical gypsum particles under three-point contact were studied comprehensively. Finally, based on the Mohr-Coulomb criterion, the breakage criterion of spherical gypsum particles in the case of three-point contact is established.

2. Experimental study

2.1. Experimental material

In this paper, high-strength gypsum was selected as the experimental material, which has the characteristics of fine particle size, simple proportioning, convenient plasticity, short curing time, low maintenance requirements and so on. Spherical particles with a diameter of 50 mm were used as experiment specimens. The method of mold pouring was adopted in specimen production. The sample was vibrated and compacted during pouring, and the bubbles inside the sample were eliminated as far as possible. The specimens were demoulded 30 minutes after pouring and then cured for 14 days at room temperature and indoor drying environment. The main mechanical properties of the specimens were determined by uniaxial compression and splitting experiments as shown in table 1.

| Material   | Compressive strength (MPa) | Tensile strength (MPa) | Elastic modulus (GPa) | Possion ratio | Cohesion (MPa) | Internal friction angle |
|------------|---------------------------|------------------------|-----------------------|---------------|----------------|-----------------------|
| gypsum     | 47.2                      | 2.839                  | 16.7                  | 0.33          | 5.788          | 1.917                 |

2.2. Experimental instrument

The loading platform used in this paper is Rock Rheological Testing System, which is located in the Key Laboratory of the Ministry of Geotechnical and Underground Engineering Education of Tongji University, as shown in figure 1. It can provide stable servo control and displacement control under biaxial loading conditions with control and measurement accuracy of 0.001 mm and 5 N respectively. In the three-point normal contact loading experiment of gypsum particles, to make the contact form of each direction the same, each contact point was loaded in the form of spherical gypsum particles, and
the vertical and transverse contact points were connected in the same plane. To achieve the above purpose, the auxiliary fixture required for the experiment was designed separately. The design drawing and model physical drawing are shown in figure 2. The spherical gypsum particles in the middle were the object of experimental study.

![Figure 1. Rock rheological testing system.](image1)

![Figure 2. Design drawing and actual installation of auxiliary fixture: (a) Design drawing; (b) Actual installation.](image2)

2.3. Experiment scheme

2.3.1. Experiment Groups. In the three-point contact experiment, five different lateral force levels were set, and there were three parallel experiments under each level, numbered from 3-1 to 3-15. Among them, the lateral force was set to different levels according to the breakage force $R_c$ of the previous single point contact experiment [11], and the value of $R_c$ was 6.849 kN. For example, the lateral force selected in No. 3-1 is $0.1R_c$, which was 0.1 times of the $R_c$. The experiment groups are shown in table 2.

| Experiment No. | Lateral force |
|----------------|---------------|
| 3-1~3-3        | $0.1R_c$      |
| 3-4~3-6        | $0.3R_c$      |
| 3-7~3-9        | $0.5R_c$      |
| 3-10~3-12      | $0.7R_c$      |
| 3-13~3-15      | $0.9R_c$      |

2.3.2. Experiment process. (1) Filled the sample. Fixed the upper and lower balls in the center first, then the other two balls in the lateral concave bases. The left and right concave bases were centering and fixed by centering the screw. (2) Applied preload. The fixture was installed on the loading platform of Rock Rheological Experimenting System. Started the loading device and applied the preload around 0.2kN. Removed the centering screw after the preload had been applied. (3) Applied load. Started the horizontal loading device and controlled the loading speed by displacement. The horizontal normal load was applied at a speed of 0.002mm/s. When the control load reached the
predetermined value, the load was stopped and maintained. Then started the vertical loading device and applied the normal load at a speed of 0.002mm/s. Stopped loading until the sample was destroyed.

3. Analysis of experimental results

3.1. Breakage mechanical properties

Through experimental observation, the breakage process of the gypsum particle under three-point normal contacts is as follows. Firstly, the displacement increases continuously with the progress of loading. The local breakages occur at the contacts, which is extruded inward to form the conical compaction zone. When the volumetric strain develops to a certain extent, the gypsum spherical particle is overall broken. Penetrating cracks are formed, and most of the cracks develop along the line of the contact point. The spherical gypsum particle cracks into two pieces, but due to the restraint, the overall bearing capacity is not completely lost. This breakage is called the first breakage in this paper. As the load continues to increase, the contact area between the upper and lower particles continues to increase. The cracks formed by the whole breakage expand and the vertical contact force also increases again, even exceeding the contact force of the first breakage. When the volumetric strain accumulates to a certain extent, the spherical gypsum particles are broken for the second time along the other direction. In the subsequent loading process, the contact force can not exceed the contact force corresponding to the secondary breakage, which is called the peak contact force.

Figure 3 shows the force-displacement curve under different levels of lateral contact force. It can be seen that the force-displacement curves of three-point contact experiments under different lateral force levels are similar. The vertical stiffness of the particles is the same before the first breakage. After the first breakage, although the peak load is discrete, the form of the force-displacement curve is still close to each other. By comparing the force-displacement curves under different lateral contact forces, it is not difficult to find that with the increase of lateral contact forces, the vertical contact forces of the first breakage decrease in turn. On the other hand, the law of peak contact force is not obvious, and the average value fluctuates between 12~13kN. The main reason is that the stress acting on the particles is redistributed after the first breakage. The form of redistribution is affected by many factors, including the shape of the first breakage, the internal pores of the particles and so on, so the results show great discreteness. This paper mainly studies the breaking law of spherical gypsum particles during the first breakage, so the peak contact force is only qualitatively discussed.

![Figure 3](image3.png)  ![Figure 4](image4.png)

**Figure 3.** Force-displacement curve of three-point contact breakage experiment under different lateral contact forces

**Figure 4.** The relationship between vertical contact force and lateral contact force

According to the statistics of the breakage force of spherical gypsum particles, it is found that the contact force of the first breakage decreases with the increase of lateral contact force. The quantitative relationship between the two can be described as shown in Figure 4:

\[ F_1 = -0.306F_2 + 7.632 \]  

(1)
Among them, $F_1$ and $F_2$ are vertical and lateral contact forces respectively. In terms of breaking morphology, the whole breakage of spherical gypsum particles is the splitting breakage controlled by tensile stress. The splitting planes are mostly planes determined by three contact points. Both vertical contact force and lateral contact force cause tensile stress components in the same direction in this plane, and they have the effect of superposition. Therefore, as the lateral contact force increases, the vertical contact force decreases.

3.2. Breakage form

Figure 5 shows the broken morphology of gypsum particles under different lateral force contact conditions. It can be found that the breakage rules of three-point contact experiments under different lateral contact forces are similar. That is, the local breakage occurs at the contact points first, then the whole penetrating crack occurs, and the whole breakage shows splitting form. The main differences are as follows: (1) Under the condition of low lateral force, there are no local breakages near the lateral contact points, so the penetrating crack does not strictly pass through the lateral contact points, but develops near the lateral contact points. However, under the condition of medium and high lateral forces, local breakage also occurs at the lateral contact points, and all the penetrating cracks pass through the contact points. (2) At higher lateral forces, more fragmental debris is produced in the same direction as the first penetrating crack. (3) Most of the spherical gypsum samples are broken into 3-4 pieces, which are more likely to be broken into multiple pieces under the action of higher lateral contact forces.

![Figure 5](image)

**Figure 5.** The broken morphology of gypsum particles under different lateral force contact conditions: (a) No. 3-2; (b) No. 3-5; (c) No.3-8; (d) No.3-10; (e) No.3-13.

According to the statistics of the fine particles whose particle size is less than 1/50 of the initial particle size, it is defined as the debris rate. The results show that the ratio of debris mass to the total mass of broken gypsum particles is high, ranging from 4.47% to 14.12%, and the debris rate increases significantly with the increase of lateral contact force. The relationship between the debris rate and the lateral contact force can be approximately described by (2):

$$\delta = 0.0131F_2 + 0.0459$$

where $F_2$ is the lateral contact force, $\delta$ is the rate of detritus.

3.3. Cone core analysis

When the spherical gypsum particles are broken under the condition of three-point contact, the cone core will be formed at the top contact point and the lateral contact point. When the lateral contact force level is lower (0.1$R_c$), as shown in figure 6(a), the cone core is formed only at the top contact after breakage. Because the lateral contact force is relatively small, there is no local breakage at the lateral contact points. When the level of lateral contact force is moderate (0.3–0.5$R_c$), as shown in figure 6(b),
two or three cone cores are formed after contact breakage, that is, one cone core at the top contact point and one or two cone cores at lateral contact points. However, the volume of the lateral cone core is smaller than that of the top, indicating that although there is local breakage at the lateral contact point, the degree of local breakage is significantly lower than that of the top. When the level of lateral contact force is higher (0.7~0.9Rc), as shown in figure 6(c), a cone core is formed at each of the three contact points, indicating that local breakage occurs at all three contact points.

![Figure 6](image)  
**Figure 6.** The formation of cone cores under the different lateral contact forces: (a) 0.1Rc; (b) 0.3~0.5Rc; (c) 0.7~0.9Rc.

According to the statistical results of the conical core size, there is little correlation between the size of the cone core and the lateral force, but there is a certain correlation between the size of the cone core and the vertical contact force. This further demonstrates that the generation of the cone core is the local fracture under the control of stress concentration. Due to the stress diffusion, the stress near the top cone core caused by lateral contact force is already very small, which has little effect on the shape and size of the cone core.

4. Breakage criterion under three-point contact

Hiramatsu et al. [16] obtained the vertical stress distribution along the loading direction in rock sphere under point load through test and theoretical calculation. That is, compressive stress is induced near the loading point and tensile stress is induced in other areas. The maximum tensile stress can be expressed in the form of equation (3).

\[
\sigma_{t_{\text{max}}} = 1.4 \frac{F_0}{\pi a^2},
\]

where \( \sigma_{t_{\text{max}}} \) is the maximum tensile stress caused by point load, \( F_0 \) is the magnitude of point load, and \( a \) is the radius of spherical particles.

The overall breakage of the spherical gypsum particle is the splitting breakage controlled by tensile stress. Cracks begin to develop near the contact point. Therefore, studying the stress state near the contact point is the key to establish the breakage criterion of spherical gypsum particles under three-point contact. Figure 7 shows the stress state near the contact point under three-point contact. \( F_1 \) is the vertical contact force and \( F_2 \) is the lateral contact force, where \( F_2 \) is perpendicular to the paper surface. At this point, the stress state near the contact point can be roughly decomposed into two components. They are the vertical compressive stress component \( \sigma_c \) caused by \( F_1 \) and the horizontal tensile stress \( \sigma_t \) caused by \( F_2 \), respectively. The maximum tensile stress caused by the superposition of the two is the cause of the crack between the conical nucleus and the particle, which leads to the overall breakage of the particle.

\( \sigma_c \) is the compressive stress generated by \( F_1 \) near the contact point, which is close to the contact point and can be approximately expressed by the following formula (4).

\[
\sigma_c = \frac{F_1}{A}
\]

where \( F_1 \) is the vertical contact force and \( A \) is the average contact area of single point contact experiment with a value of \( 1.605 \times 10^{-4} \text{m}^2 \).

\( \sigma_t \) is the tensile stress caused by \( F_2 \) near the contact point of \( F_1 \). Since the tensile stress is not on the extension line of \( F_2 \), considering the stress diffusion effect, it can be approximately expressed as the following formula (5) according to the formula (3).
\[ \sigma_t = m \cdot 1.4 \frac{F_2}{2\pi a^2} \]  

where \( F_2 \) is the lateral contact force, \( m \) is the stress diffusion coefficient, and \( a \) is the radius of the spherical particle.

**Figure 7.** Stress state near the contact points under three-point contact: (a) Loading state of the gypsum particle; (b) Stress state of the top contact point (\( F_2 \) is perpendicular to the surface of the paper); (c) Specific conditions of stress state.

The stress state near the top contact point can be described in figure 7(c). The breakage criteria for the three-point contact is established by combining the Coulomb-Moore criterion:

\[ \frac{\sigma_c + \sigma_t}{2} \cdot \sin \varphi - \frac{\sigma_c - \sigma_t}{2} + c \cdot \cos \varphi = 0 \]  

Based on the experiment results of No.3-1 to No.3-9, \( m \) is calculated by using the simultaneous equations (4-4), (4-5) and (4-8), and the average value is taken as the estimated value of \( m \):

\[ m = 0.297 \]  

To verify the validity of the above fracture criteria, the results of 3-10 to 3-15 groups of experiments are substituted for the fracture criteria. Comparing the contact force calculated by the model with the actual contact force, the results in table 3 show that the two are very close, which proves the validity of the criterion.

| Experiment No. | Calculated tensile stress \( \sigma_t \)(MPa) | Calculated compressive stress \( \sigma_c \)(MPa) | Calculated normal contact force \( F_1 \)(kN) | Experimental contact force \( F_1 \)(kN) | Deviation |
|----------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|----------|
| 3-10           | -0.510                          | 38.751                          | 6.220                           | 6.190                           | -0.49%   |
| 3-11           | -0.500                          | 38.929                          | 6.249                           | 6.671                           | 6.75%    |
| 3-12           | -0.520                          | 38.587                          | 6.194                           | 5.331                           | -13.93%  |
| 3-13           | -0.661                          | 36.240                          | 5.817                           | 5.944                           | 2.18%    |
| 3-14           | -0.657                          | 36.307                          | 5.828                           | 5.565                           | -4.52%   |
| 3-15           | -0.660                          | 36.266                          | 5.822                           | 5.965                           | 2.46%    |

5. Conclusions

The particle breakage experiments of spherical gypsum particles under the condition of three-point contact were carried out. The breakage process, breakage form, mass distribution after breaking and cone core were studied. The contact mechanical properties and breakage characteristics of different lateral force levels are compared and analyzed. Finally, the breakage criterion for three-point contact can be established based on the Coulomb-Moore criterion. The main conclusions are as follows:

(1) In the case of three-point contact, the overall breakage of spherical gypsum particles is usually along the line of contact points, which is characterized by splitting breakage controlled by
tensile stress. As the lateral contact force increases, the vertical contact force decreases gradually. This is because both vertical and lateral contact forces produce tensile stresses on the plane defined by the contact points, which have an effect of superposition.

(2) When the level of lateral contact force is low, there are few cone cores near the lateral contact points. When the lateral force is high, the cone core is produced at each contact point. This phenomenon is the result of stress concentration. The shape and size of the cone core mainly depend on the corresponding contact force, which has little relationship with the contact force at other contact points.

(3) When the particles are overall broken, the cracks begin to develop near the contact point. The stress state near the contact point is the key to the overall breakage of the particles. The stress state of this area is analyzed, and the breakage criterion in the case of three-point contact can be established based on the Coulomb-Moore criterion. It is proved that the breakage criterion is effective.

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References
[1] Hardin B O 1985 Crushing of Soil Particles Journal of Geotechnical Engineering 111 1177–92
[2] Sukkarak R, Pramthawee P and Jongpradist P A modified elasto-plastic model with double yield surfaces and considering particle breakage for the settlement analysis of high rockfill dams KSCE J Civ Eng 21 734–45
[3] Alonso E E, Tapias M and Gili J 2015 A particle model for rockfill behaviour Geotechnique 1–20
[4] Niu Y, Li L, Zhang Y, Yu S and Zhou J 2021 Micromechanism of the Breakage of Two Spherical Gypsum Particles under Normal–Tangential Contact Conditions Applied Sciences 11 4039
[5] Zhou J, Zhang Y, Ouyang G and Zhao C Experimental Study on Particle Breakage Characteristics of Coarse-Grained Soil Under Normal and Tangential Force Proceedings of GeoShanghai 2018 International pp 754–64
[6] Cavarretta I, Coop M and O’eullivan C 2010 The influence of particle characteristics on the behaviour of coarse grained soils Geotechnique 60 413–23
[7] Lee K L and Farhoomand I 1967 Compressibility and crushing of granular soil in anisotropic triaxial compression Canadian geotechnical journal 4 68–86
[8] Yu F 2017 Characteristics of particle breakage of sand in triaxial shear Powder Technology 320 656–67
[9] Liu Y, Liu H and Mao H 2017 DEM investigation of the effect of intermediate principle stress on particle breakage of granular materials Computers and Geotechnics 84 58–67
[10] Xu M and Song E 2009 Numerical simulation of the shear behavior of rockfills Computers and Geotechnics 36 1259–64
[11] Yu S, Jia M, Zhou J, Zhao C and Li L 2019 Micro-Mechanism of Spherical Gypsum Particle Breakage under Ball–Plane Contact Condition Applied Sciences 9 4795
[12] Zhou J, Yu S C, Zhang J, Zhao C Experiment and Numerical Simulation on Contact and Breakage of Marble Particles Proceedings of GeoShanghai 2018 International pp 145–57
[13] Wei S, Zhu J G, Qian Q H and Li F 2009 Particle breakage of coarse-grained materials in triaxial tests Chinese Journal of Geotechnical Engineering 31 533–8
[14] Wang Z, Du Y, Su X, Shao l and Wang f 2013 Experimental study on partial breakage of rockfill material by considering lateral restraints Water Resource and Power 031 109–12
[15] Artoni R, Neveu A, Descantes Y and Richard P 2019 Effect of contact location on the crushing strength of aggregates Journal of the Mechanics and Physics of Solids 122 406–17
[16] Hiramatsu Y and Oka Y 1966 Determination of the tensile strength of rock by a compression test of an irregular test piece International Journal of Rock Mechanics and Mining Sciences &
Geomechanics Abstracts 3 89–90