Stable vortex loops in two-species BECs

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Abstract
We consider the creation of stable, stationary closed vortex loops, analogues of vortons in dense matter and superconducting cosmic strings in cold atom BECs. We explore the parameter region where these solutions are likely to exist and comment on methods to create them experimentally.

1. Introduction

The existence of vortex solutions is one of the characteristics of superfluidity. They are seen in superfluid $^4$He as well as in Bose–Einstein condensates in cold atom traps [1]. Superfluid vortices are also the main factors in the leading theory of rotation and ‘glitches’ observed in rotating neutron stars (pulsars) [2]; superfluid vortices are supported in both the hadron and quark matter phases at high density. Similar structures are also likely to appear in grand unified theories (GUT) of physics beyond the standard model of particle physics. In that context, vortex filaments in the early universe might serve as seeds around which galaxies could form [3].

As long as energetic considerations prevent the condensate field from leaving the manifold that minimizes the energy, the stability of superfluid vortices is guaranteed by their topological properties. Vortices may decay and/or join but not simply disappear. The velocity circulation around a vortex is characterized by a phase which wraps axially around the vortex. Because the field must be single-valued, this phase must go through an integer number of cycles and thus cannot change continuously. Discontinuous changes through quantum tunnelling or thermal activation are usually negligible.

Vortices with opposite circulation can annihilate one another. This phenomenon seems to preclude the possibility of stable closed vortex loops, because the opposite sides of a closed loop form a vortex/antivortex pair that can annihilate and, in fact, the energetics of the system favours annihilation. Because a vortex has a tension $T$ along its length, the energy of a vortex loop is proportional to its length: $E \sim 2\pi RT$, where $R$ is the radius of the loop. This energy is reduced as the loop shrinks. When the opposite sides of the loop come closer than their thicknesses, they can annihilate. Single-species vortex loops can only be stabilized by the Magnus force, which will sustain a loop if it is moving at a specific velocity in a direction perpendicular to its plane.

It has been pointed out in the context of relativistic models that a stable closed vortex loop can exist if two species are competing to condense [5]. Qualitatively, the mechanism is straightforward. Suppose in some region there is a bulk where the first species was condensed, and there is a vortex in that medium. In that vortex’s core the condensate of species 1 vanishes. If the two species repel each other it will be energetically favourable for the second species to condense in the core of the vortices of species 1 rather than occupy the same space as the first species. Suppose that the vortex of species 1 forms a closed loop and, in addition, there is a non-zero vorticity of species 2 along the core of the loop (see figure 1). The energy cost related to the vorticity $l$ of species 2 is proportional to $2\pi R(l/R)^2 \sim l^2/R$, which scales like the kinetic energy of species 2 inside the vortex. Because this contribution increases as the loop shrinks, there will always be some nonzero radius where the total energy $E \sim TR+I^2/R$ has a minimum. Thus, if this equilibrium radius is big enough to preclude annihilation, the second species provides a stabilization mechanism for vortex loops.

If the particle condensing in the interior of the vortex is charged, the vortex will be superconducting and solutions of this kind are known as superconducting strings [6] in order to distinguish them from superconducting flux tubes (Abrikosov vortices) where the superconducting region is outside of the vortex. Closed loops of superfluid vortices stabilized by the mechanism sketched above are known as ‘vortons’. Vortons can arise in GUT-scale models during the early universe [7]. Quark matter, which can exist in the core of neutron stars, also provides the ingredients for the existence of vortons, the role of the two competing species played by quasiparticle...
excitations of the colour superconducting ground state with quantum numbers of neutral and charged kaons [8–10, 4].

The purpose of this paper is to study the possibility that stable vortons can exist in cold atomic traps with two-species BECs. Two-species BECs are known to support Skyrmions (see, for example, [11, 12]). They form a class of solutions related to vortons with the special property of having a combination of the condensates of the two species constant throughout space.

The (meta)stability of vorton solutions is analysed in the first part through the calculation of the energy of the vorton as a function of its radius and thickness. We first show that in some circumstances, the equilibrium radius of the vorton is on the scale of microns and is reasonably larger than its thickness. We then comment on possible ways of actually creating vortons experimentally.

2. Stability and equilibrium properties of vortons

Consider a system of two species of spinless bosons. In the dilute limit, where the interparticle distances are typically larger than the scattering lengths, this system can be described by the Hamiltonian

\[ H = \frac{\hbar^2}{2M_1} |\nabla \phi_1|^2 + \frac{\hbar^2}{2M_2} |\nabla \phi_2|^2 + V(\phi_1, \phi_2), \]

where

\[ V(\phi_1, \phi_2) = \frac{1}{2} \left( \frac{8\pi \hbar^2}{M_1} |\phi_1|^4 + \frac{8\pi \hbar^2}{M_2} |\phi_2|^4 + \frac{2\pi \hbar^2}{M_{12}} |\phi_1|^2 |\phi_2|^2 - \mu_1 |\phi_1|^2 \right). \]

Here, \( \phi_1 \) and \( \phi_2 \) are the second quantized fields annihilating species 1 and 2, while \( a_i \) and \( M_i \) are the scattering lengths and masses of the two species \( i = 1, 2 \). \( a_{12} \) is the interspecies scattering length and \( M_{12} \) is the reduced mass of the two species. The chemical potential of species 1 is given, in the absence of species 2 and at leading order in the diluteness expansion, by

\[ \mu_1 = \frac{8\pi \hbar^2 n_1 a_1}{M_1}, \]

where \( n_1 \) is the asymptotic density of the first species. We will always work in situations where species 1 forms a large bulk that effectively forms a reservoir of species 1. We will minimize the energy for a fixed value of \( \mu_1 \), given by (3). We do not include a chemical potential term for species 2; instead we will work with a large and fixed number of particles of that type. We will be interested in the phase separation regime \( 4a_1 a_2 / M_1 M_2 < (a_{12} / M_{12})^2 \), where the interspecies repulsion encourages the two species to stay at separate points in space [13]. This phase separation confines the particles of species 2 to the vorton core, fixing that species’ total number.

Bose–Einstein condensation is described by a non-vanishing matrix element, the field operators \( \phi_{1,2} \). At low densities \( (n_{1,2} / a_1, a_2, a_{12}) \), the mean field approximation is valid and the matrix element (also denoted by \( \phi_{1,2} \)) satisfies the classical equations of motion, that is, the Gross–Pitaevski equations. For a straight vortex of only the first species, \( \phi_1 \) is of the form

\[ \phi_1(r, \theta, z) = f_1(r) e^{i\theta}, \]

where \( r, \theta, \) and \( z \) are cylindrical coordinates, \( f_1(r) = 0 \) and \( f_1(r \to \infty) = \sqrt{M_1} \). To keep \( \phi_1 \) single valued the winding number \( j \) must be an integer. We always use \( j = 1 \), as vortices with larger \( j \) are unstable and split into \( j \) vortices with winding number 1, but keeping general \( j \) is useful for studying the energy of a vorton. The precise profile \( f_1(r) \) can be obtained by solving the Gross–Pitaevski equations or, equivalently, minimizing the energy. Detailed and numerically intensive analyses of Skyrmion stability solve the Gross–Pitaevski and demonstrate dynamical stability of those field configurations [14, 15]. Here we are satisfied with stability arguments and finding the equilibrium radii for vortons without such costly computation.

Simple scaling arguments imply that the solution \( f_1(r) \) will change from \( f_1 = 0 \) to \( f_1 = \sqrt{M_1} \) over a distance of order \( \delta \approx 1 / \sqrt{8\pi n_1 a_1} \), the string thickness. The toroidal geometry of the vorton makes an analogous calculation a little involved. We can, however, bypass most of the difficulty by assuming that the radius \( R \) of the vorton is much larger than its thickness \( \delta \). In this case, we can compute the energy of the vorton by computing the energy per length (that is, the tension) of a straight vortex and multiplying it by the length \( 2\pi R \). This approximation neglects the energy associated with the curvature of the vorton. For a straight vortex of the first species with an internal current of the second, the mean-field solution has the form

\[ \phi_1 = f_1(r) e^{i\theta}, \]
\[ \phi_2 = f_2(r) e^{i\phi_2}. \]

To encode the fact that we wish to consider a vortex loop of radius \( R \), we impose periodic boundary conditions in \( z \), identifying \( z = \pm 2\pi R \), so that \( z \) spans the arclength of the loop. The continuity of the phase \( \phi_2 \) implies that \( k = l / R \), with integer \( l \).

Plugging the functional forms in (5) with \( k = l / R \) into the Hamiltonian given in (1), the total energy of a vorton of a radius \( R \) is

\[ E \approx 2\pi R \int dr 2\pi r \left[ \frac{\hbar^2}{2M_1} \left( \frac{\partial f_1}{\partial r} \right)^2 + \frac{l^2}{R^2} f_1(r)^2 \right]
+ \frac{\hbar^2}{2M_2} \left( \frac{\partial f_2}{\partial r} \right)^2 + \frac{l^2}{R^2} f_2^2
+ V(f_1, f_2) + \frac{1}{2} \frac{8\pi \hbar^2}{M_1} n_1^2 - \mu_1 |\phi_1|^2. \]
which becomes more accurate as the ratio \( R/\delta \) grows. One might worry that including curvature corrections might encourage the vorton’s radius to be smaller. Brief consideration alleviates this fear: the configuration we consider when formulating (6) has no gradients of \( \phi_1 \) along the vorton’s length. Curvature effects will make those terms nonzero, and the nonuniformity of \( \phi_1 \) will provide a considerable potential barrier. Therefore, we expect that the calculation which included curvature effects would find an \( R \) larger than the \( R \) we find from considering the energy expression in (6).

In (6), we measure the energy in relation to the homogeneous ground state \( j = 0, f_1 = \sqrt{\pi} \), \( f_2 = 0 \). These parameters correspond to no vorton at all. Any vorton will have a greater energy, and thus must be at best metastable.

We now argue that vortons are indeed long-lived. If a vorton is initially created with non-zero number \( N_2 \) particles of species 2 in its interior, we hope they remain trapped there, or the stabilizing energy that scales inversely with \( R \) will disappear, causing the vorton to collapse. Escaping the vorton would require these particles to go through a large region, from the vorton to the border of the trap, where species 1 is condensed, and that is energetically expensive. In fact, the tunnelling through a macroscopic region is exponentially small and can be made arbitrarily small by making the trap larger. The rate of this decay mechanism will be dependent on the details of the geometry/size of the trap but is certainly smaller than the usual decay by two or three particle collisions that set a limit to the lifetime of BECs in atomic traps. One could also imagine a bubble of species 2 detaching from the vorton and moving towards the edge of the bulk, but that costs an amount of energy proportional to the area of the bubble due to its surface tension. These escape mechanisms are exponentially suppressed and, for all practical effects \( N_2 \) is a conserved quantity that is confined to the vorton’s interior. To implement this conservation, when we minimize the energy, we keep

\[
N_2 = 2\pi R \int dr \, 2\pi r [f_2(r)]^2
\]

fixed. Yet another way the vorton can collapse is by having its radius to shrink to zero. The stability against this mode can be checked by minimizing the (static) energy against variations of \( R \). To summarize, we find our vorton candidates by numerically minimizing the energy in (6) in relation to \( f_1(r) \), \( f_2(r) \) and \( R \), while keeping the parameters \( N_2, n_1, a_1, a_2, a_{12}, M_1 \) and \( M_2 \) fixed. We have eliminated \( \mu_1 \), and henceforth take \( j = 1 \). A typical example of the profile functions \( f_1(r) \) and \( f_2(r) \) resulting from this minimization is shown in figure 2.

It is easy to foresee some qualitative dependences before doing the numerical minimization. For instance, the larger the species 2 vorticity \( l \), the larger \( R \) and \( R/\delta \) should be. A larger value of \( a_{12} \) increases interspecies repulsion and shrinks the region where both species coexist. This tendency should quickly saturate, as the coexistence region becomes negligible and \( a_{12} \) becomes effectively infinite. We can also see that if we increase \( n_1 \) by a factor of \( \eta \) and reduce the number of particles of species 2 by a factor of \( \sqrt{\eta} \), both the radius and thickness decrease by a factor of \( \sqrt{\eta} \) and therefore \( R/\delta \) remains unchanged.

![Figure 2](image)

**Figure 2.** The radial density profile of the two species \(^7\)Li and \(^87\)Rb and \( l = 5 \). The remaining parameters are given in the first row of table 1.

| \( M_1/M_2 \) | \( a_{1}/a_0 \) | \( a_{2}/a_0 \) | \( a_{12}/a_0 \) | \( n_1 \) (cm\(^{-3}\)) | \( N_2 \) | \( R/\delta \) | \( R \) (\( \mu \)m) |
|---|---|---|---|---|---|---|---|
| 12 | 100 | 40 | 5000 | 4 \times 10^{12} | 2 \times 10^6 | 2.3 | 7.9 |
| 12 | 100 | 40 | 5000 | 4 \times 10^{12} | 2 \times 10^6 | 2.5 | 19 |
| 12 | 100 | 40 | 5000 | 4 \times 10^{12} | 2 \times 10^6 | 2.5 | 19 |
| 2.12 | 100 | 85.5 | 5000 | 4 \times 10^{13} | 2 \times 10^4 | 1.7 | 5.3 |
| 2.12 | 100 | 85.5 | 5000 | 4 \times 10^{13} | 2 \times 10^4 | 1.9 | 12 |
| 2.12 | 100 | 85.5 | 5000 | 4 \times 10^{13} | 2 \times 10^4 | 1.9 | 2.8 |

On the other hand, the dependence of \( R/\delta \) on some parameters is harder to anticipate. For example, as \( M_1/M_2 \) grows, the terms in the energy describing species 2 (and its interaction with species 1) grow. Because species 2 exerts a pressure counterbalanced by the bulk, this increase of the contribution from species 2 should increase both \( R \) and \( \delta \), so it is difficult to know in advance if their ratio will grow or shrink. Numerical studies indicate that the ratio \( R/\delta \) increases with \( M_1/M_2 \). We also observe an increase of \( R/\delta \) with increasing \( a_1 \) and decreasing \( a_2 \).

To make our detailed discussion more concrete, we study the vortons formed when the roles of species 1 and 2 are played by the \( |F = 1, m_F = 1 \rangle \) hyperfine state of \(^87\)Rb and the \( |1, 1 \rangle \) state of \(^7\)Li, respectively. The interspecies scattering length can be tuned through the use of Feshbach resonance [16]. A naive estimate of the thickness of the vorton \( \delta \) is simply the thickness of a \(^87\)Rb vortex, which is given by the healing length \( 1/\sqrt{8\pi n_1 a_1} \), an order-of-magnitude estimate supported by our numerical calculations. For definiteness, we pick the parameters which correspond to the first row of table 1 and estimate the healing length to be 0.45 \( \mu \)m, while \( \delta = 3.4 \mu \)m. The ratio of these two quantities is roughly 7.5, which is seen in figure 2.

We can estimate \( N_2 \) by simply multiplying the cross-section of the vorton, its length and the density of the lithium, \( N_2 \approx (\pi \delta^2) (2\pi R) n_2 \). As an example, for lithium and rubidium, if the density of lithium is roughly the same as the density of rubidium \( (10^{13} \text{/cc}) \), and we want \( R \) of the order of 10 \( \mu \)m with an \( R/\delta \) of 2.5, then \( N_2 \) should be of the order of 10^4. We show some example situations in table 1, where we list the results for the Li-Rb \((M_1/M_2 = 12)\) vorton and the K-Rb \((M_1/M_2 = 2.12)\) vorton for some realistic values of \( n_1 \) and \( N_2 \). These results indicate that the vorton has only a
moderate sensitivity to the number of particles present. It also indicates that the $R/\delta$ ratio is not large, making the corrections to the approximation we used in computing its energy sizable. However, as argued, the curvature effects increase $R/\delta$, so the existence of stable vortons is nonetheless supported by this analysis.

3. Production mechanisms

Up to now we have discussed the properties of static vortons. We now briefly comment on two possible ways to actually create them experimentally.

3.1. Raman scattering and Gauss–Laguerre beams

A possible way to engineer a vorton in cold atoms traps suggested by Porto [20] involves three ingredients.

First, a closed-vortex loop of $^{87}$Rb can be created by the use of two counterpropagating light beams, each one with a central circular region with one frequency and an outer annular region with another frequency, as shown in figure 3. In the left-moving beam, the inner frequency $\omega_a$ is chosen to match the $D_1$ line of rubidium. The outer frequency $\omega_b$ is chosen to match a transition from the same excited state to another hyperfine state of $^{87}$Rb. In the counterpropagating beam, the frequencies $\omega_a$ and $\omega_b$ are exchanged. Thus, rubidium atoms located in the inner region of the beams will absorb a photon with momentum $\hbar k = \hbar \omega_a/c$ and emit a photon in the opposite direction with nearly the same momentum, due to the stimulation of the second beam. The net effect is that the $^{87}$Rb atoms in the inner region acquire a momentum $2\hbar k$. In the annular outer region of the beams, the emission and absorption are reversed and the atoms acquire a momentum $-2\hbar k$.

At the boundary separating the two regions, the shearing of the $^{87}$Rb encourages vortex loop formation. The recoil energy of the rubidium atom, for optical photons, is of the order of 10 $\mu$K so heating may be an issue. This problem might be circumvented by having only a small fraction of the atoms go through the absorption–emission process. Further scattering of these atoms will distribute its energy and momentum among nearby atoms.

The second ingredient is that the creation of the rubidium loop is accompanied by a change in magnetic field to a value close to the Feshbach resonance between the rubidium and lithium atoms. Such a tuning will entice the system to phase separate, so that the lithium will seek locations with little rubidium: either the boundary of the bulk or the interior of the rubidium vortices.

Finally, the lithium atoms must acquire a net angular momentum around the vorton before it has time to collapse. This can be accomplished with the use of a Gauss–Laguerre beam [21]. The small values of $l \lesssim 5$ considered in the numerical examples in table 1 were motivated by the limitations of the Gauss–Laguerre beam technique. With improved Gauss–Laguerre techniques, a higher $l$ and larger $R/\delta$ could be achieved.

3.2. The Kibble–Zurek mechanism

At high temperatures, the phase of the bosonic fields $\phi_1$ and $\phi_2$ at different points in space is uncorrelated. At low temperatures, in the superfluid state, the phase has long-range correlations. However, following a rapid quench, the phases do not have enough time to correlate and defects form at the regions where different phases meet. This process (the Kibble–Zurek mechanism [22, 23]) was demonstrated to create regular vortices in $^{87}$Rb [24]. In the present case, two ingredients should be added. Vorticity needs to be given to the $^7$Li using standard methods and a change of the external magnetic field to the Feshbach resonance, in order to impose phase separation.

Any rubidium vortex generated by the standard Kibble–Zurek mechanism will be filled with lithium atoms and any vorticity of the lithium enclosed by the vortex loop will be conserved and will guarantee the stability of the vorton.

Some simplifying assumptions allow us to estimate the probability of forming a vorton during such a quench. First, we assume that the quench is sufficiently rapid such that the correlation length of the phases of each field is the same as before the quench. Second, we assume that there is little interaction between the two species before the quench (which can be enforced by tuning the interspecies scattering length to zero), but after the quench, lithium atoms immediately fall into the potential well generated by the core of rubidium vortices. The typical distance between rubidium vortices after the quench is given by the correlation length $\xi_1 \sim h/k \sim M_1 T / \sqrt{4\pi a_1 n_1}$, where $k$ is the typical momentum of a phonon, related to the typical energy of a phonon $\epsilon \sim T \sim ck$, where $c = \sqrt{4\pi a_1 n_1}/M_1$ is the speed of sound.

Since the orientation of the rubidium vortices is random, a fraction of order 1 of those vortices will connect to a vortex with the opposite orientation and form a vorton. In those cases, lithium atoms will be trapped in the core of the closed vortex and guarantee its stability as long as there is vorticity in the helium along the closed-vortex loop. We can estimate this vorticity by a standard argument [23]. The vorticity $l$ is the integral of the phase of $\phi_2$ along the vorton line

$$l = \int ds \cdot \nabla \phi_2,$$

where $\phi_2 = A_2 e^{i\theta_2}$. The phase of $\phi_2$ changes over a distance of order $\xi_2 \sim h/k \sim M_2 T / \sqrt{4\pi a_2 n_2}$, in exact analogy with $\phi_1$. If we assume that the phase of $\phi_2$ is uncorrelated in different regions apart by more than $\xi_2$, we expect the phase of $\phi_2$ to change between 0 and $\pi$ a number of times simply given by $2\pi R/\xi_2$. If the phase always changed in the same way, that
would be our estimate for $l$. However, since the phase can change either clockwise or counterclockwise, we expect $l$ to scale like a one-dimensional random walk,

$$l = \int ds \cdot \nabla \theta_2 \sim \sqrt{\frac{2\pi R}{\xi_2}},$$  \hspace{1cm} (9)

where $R$ is the vorton radius. The radius of the vorton is of order $R \sim \xi_1$ so we estimate

$$l \sim \sqrt{\frac{2\pi \xi_1}{\xi_2}} \approx \frac{2\pi M_1}{M_2} \left( \frac{a_{R \xi_2}}{a_{N_1}} \right)^{1/4}. \hspace{1cm} (10)$$

This estimate suggests that we should pick atoms with very different masses, and that for increased stability created by the Kibble–Zurek mechanism, one should begin with the light species at least as dense as the heavy species, but more dense if possible. We find that this heuristic estimate suggests, for the case of an equal density Rb/Li mixture and scattering lengths given by the first line of table 1, a vorticity of $l \approx 7$, a value similar to $l = 5$ which was considered in our stability analysis.

4. Summary and outlook

We considered a cold atom instantiation of vortons, two-species vortex loops. While vortex loops of a single species will collapse unless supported by the Magnus force, it is possible for vortons to achieve a stable equilibrium radius at rest with respect to the bulk. Vortons were theoretically studied in different contexts, but have never been observed experimentally. For realistic laboratory conditions, it should be possible to create vortons with radii of the order of micrometers, with $R/\delta$ large enough to provide a believable toroidal geometry.

Aside from the relations to astrophysical systems, such solitons are an interesting aspect of two-species BECs in their own right. It is an appealing notion to think that such a rich structure might be created and manipulated in the laboratory. The requisite two-species BEC cold-atom traps are a well-established technology, and many two-species pairs have been trapped (see e.g. [25–27]). While we have focused on a lithium–rubidium system for numerical results, in principle any two-species BEC supports such structures for some choice of vorticity and initial densities. Aside from computing equilibrium properties, we have discussed two possible mechanisms for crafting vortons in cold atom traps. Creating such objects would demonstrate their feasibility in that case of an equal density Rb/Li mixture and scattering lengths $\frac{a_{R \xi_2}}{a_{N_1}} = 7$, a value which was considered in our stability analysis.

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