I. INTRODUCTION

Field theories defined on noncommutative spaces have attracted considerable attention during the last few years [1]. This is largely due to the study of open string theory in background NS-NS $B$-field. For constant $B$-field, a number of papers [2-8] by different authors have revealed that the quantization procedure would lead to noncommutativity at the boundary, i.e. D-branes in the perturbative string picture, though there are some discrepancies [8] on the way in which coordinates on the D-branes noncommute.

For string theory to be a fundamental theory, it is reasonable to require that the description of string theory should be background independent. Of course the background independence cannot be realized in a verbose way, but there is a large library of string dualities to which we can resort to realize in a lose sense the background independence. Moreover, in the study of noncommutative gauge theories, Seiberg and Witten [9] found some famous maps between 1) a noncommutative field theory with a small noncommutative parameter and a commutative one and 2) one noncommutative field theory with another with different noncommutative parameters. However, there are some ambiguities in the Seiberg-Witten maps, some of which can be resolved [10] by field-dependent gauge transformations and field redefinitions, whilst others cannot [11].

In an earlier paper [8], we studied the problem of quantization of open string in flat spacetime background with constant $B$-field, with the aim to resolve the discrepancies between different works. The approach we used is a direct modification of the Poisson brackets at the boundaries, following the principle of locality [12]. To our astonishment, the equations determining the modifications are under determined (2 equations for 3 unknowns), and we found an infinite number of consistent Poisson structures, each leads to a different quantization scheme. In particular, all previous results on the same problem are special cases of our result, and the discrepancies among them are just a choice of Poisson structure from our result. What is more remarkable is that, in our result, there is a particular Poisson structure which does not lead to noncommutativity at the boundaries upon quantization. This means that in the presence of constant background $B$-field, open string can be quantized without introducing noncommutativity on the D-branes.

In this Letter, we shall consider the Poisson structure for open string in generic spacetime metric and background $B$-field using the same method as in our previous paper [8]. Unlike the case with constant $B$-field, the equations determining the modified Poisson brackets are over determined and no general solution to them is known to exist. However, there is a simple solution which, upon quantization, gives rise to vanishing commutators between coordinates at the open string boundaries. In other words, the $D$-branes corresponding to our solution are still commutative, in spite of the presence of the generic background $B$-field. It should be remarked that the same problem has already been studied by Ho and Yeh in [13], which contains very different result. However, the result of [13] is based on the assumption that the spacetime coordinates $X^i$ depend only linearly on the world sheet coordinate $\sigma$, i.e. $X^i = x^i + p^i \sigma$, which is not valid in general, and it is not known whether the Poisson brackets (and consequently the commutators after quantization) obtained there obey Jacobi identities or not.

II. POISSON STRUCTURE FOR OPEN STRING IN GENERIC BACKGROUND

Now let us consider the problem in detail. The bosonic part of the world sheet action for open string theory reads,

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{-g} \left[ g^{ab} G_{ij}(X) \partial_a X^i \partial_b X^j + 2\pi\alpha' B_{ij}(X) \varepsilon^{ab} \partial_a X^i \partial_b X^j + \alpha' \Phi(X) R^{(2)} \right],$$

where $G_{ij}(X)$, $B_{ij}(X)$ and $\Phi(X)$ are, respectively, the background metric, antisymmetric tensor and dilaton fields, which are the only fields in the NS-NS sector. The world sheet metric $(g^{ab})$ is chosen to have the signature $(1, -1)$, and $\varepsilon^{01} = 1 = -\varepsilon^{10}$. It is commonly known that a quantum string theory cannot exist under arbitrary background fields $G_{ij}(X)$, $B_{ij}(X)$ and $\Phi(X)$. To ensure quantum consistency, only those background fields which have quantum Weyl invariance are allowed. This requirement induces certain constraints on the choice of the background fields, which, to first order in $\alpha'$, are given by the vanishing of the beta functional. A crucial observation is that, the vanishing conditions for the beta
functionals are differential equations in spacetime only which do not involve any world sheet derivatives. Therefore, we assume throughout this Letter that the background fields \( G_{ij}(X), B_{ij}(X) \) and \( \Phi(X) \) depend only on the spacetime coordinates \( X^i \) and subject to the vanishing conditions for the beta functionals. Such background fields are referred to as generic backgrounds as compared to the constant backgrounds. We assume also that the field \( B_{ij} \) is spacetime-filling and invertible, otherwise we shall be considering only the subset of the spacetime in which \( B_{ij} \) is invertible.

Since the background fields we have chosen to study preserve conformal symmetry, we may use this symmetry to choose a specific gauge for the world sheet metric, i.e. the flat gauge under which the two dimensional Ricci scalar \( R^{(2)} \) vanishes and the metric \( g^{ab} \) reads \( (g^{ab}) = \text{diag}(1, -1) \). Under this choice, the action may be rewritten as

\[
S = \frac{1}{4\pi\alpha} \int d^2\sigma \left[ g^{ab} G_{ij}(X) \partial_a X^i \partial_b X^j + 2\pi\alpha' B_{ij}(X) \varepsilon^{ab} \partial_a X^i \partial_b X^j \right],
\]

in which the dilatation field \( \Phi(X) \) simply decouples.

The variation of (1) yields the equations of motion

\[
\partial^a \partial_a X^k + \Gamma^k_{ij}(X) g^{ab} \partial_b X^i \partial_b X^j = 0
\]

and the mixed boundary conditions

\[
\left[ G_{ik}(X) \partial_a X^i + 2\pi\alpha' B_{kj}(X) \partial_a X^j \right] \big|_{\sigma=0, \pi} = 0, \quad (2)
\]

where

\[
\Gamma^k_{ij}(X) = \frac{1}{2} G^{kl}(X) \left( \frac{\delta G_{ij}}{\delta X^l} + \frac{\delta G_{il}}{\delta X^j} - \frac{\delta G_{lj}}{\delta X^i} \right)
\]

is the spacetime connection and

\[
B_{ij}(X) = \frac{\delta B_{ij}}{\delta X^i} + \frac{\delta B_{jl}}{\delta X^i} + \frac{\delta B_{li}}{\delta X^j}
\]

is the cyclic spacetime derivative (the 3-form field strength) of the background field \( B \).

The canonical conjugate momenta are defined in the usual way,

\[
P_i = \frac{\delta L}{\delta \partial_a X^i} = \frac{1}{2\pi\alpha} \left[ G_{ij}(X) \partial_a X^j + 2\pi\alpha' B_{ij}(X) \partial_a X^j \right],
\]

so that \( \partial_a X^k \) can be expressed in terms of \( P_i \) and \( X^j \) as

\[
\partial_a X^k = 2\pi\alpha' G^{ki}(X) \left[ P_i - B_{kj}(X) \partial_a X^j \right]. \tag{3}
\]

In the absence of boundary conditions, the naive Poisson brackets hold for the world sheet string theory, i.e.

\[
\{ X^i(\sigma), X^j(\sigma') \} = \{ P_i(\sigma), P_j(\sigma') \} = 0, \quad (4)
\]

\[
\{ X^i(\sigma), P_j(\sigma') \} = \delta^i_j \delta(\sigma - \sigma'). \tag{5}
\]

However, the appearance of boundary conditions (2) makes the naive Poisson structure (4) and (5) inconsistent at the end points of the open string, and one has to modify (4) and (5) to get a consistent Hamiltonian formalism of the world sheet theory.

Since the inconsistency only appears at the string end points, one needs to modify the naive Poisson structure only at these end points by locality. The most general form of the modified Poisson brackets read

\[
\{ X^i(\sigma), X^j(\sigma') \} = (A_L)_i^j \delta(\sigma - \sigma') + (A_R)_i^j \delta(2\pi - \sigma - \sigma'), \tag{6}
\]

\[
\{ X^i(\sigma), P_j(\sigma') \} = \delta^i_j \delta(\sigma - \sigma'), \tag{7}
\]

\[
\{ P_i(\sigma), P_j(\sigma') \} = (C_L)_i^j \delta(\sigma + \sigma') + (C_R)_i^j \delta(2\pi - \sigma - \sigma'), \tag{8}
\]

where \( A_{L,R}, B_{L,R} \) and \( C_{L,R} \) are assumed to be some operators which may act on the variable \( \sigma \), and \( A_{L,R}, C_{L,R} \) are antisymmetric under \( i \leftrightarrow j \). The first term on the right hand side of (7) has to be there since we need to keep the bulk equations of motion unchanged. Our main task will be the determination of the operators \( A_{L,R}, B_{L,R}, C_{L,R} \), so that the equations (6)-(8) defines a consistent Poisson structure for the world sheet theory of the open string.

To determine the values of the operators \( A_{L,R}, B_{L,R}, C_{L,R} \), we now reformulate the boundary conditions (2) into the following boundary constraints, in which the world sheet time derivatives \( \partial_e X^i \) are expressed in terms of \( P_i \) and \( X^j \) by use of (3):

\[
(G_L)_i^j \equiv \lim_{\epsilon \to 0^+} \int_0^\pi d\sigma \delta(\sigma - \epsilon) [\partial_e X^i]
\]

\[
(2\pi\alpha')^2 B_{ij}(X) (P_m - B_{mk}(X) \partial_e X^k) \right] \approx 0, \tag{9}
\]

\[
(G_R)_i^j \equiv \lim_{\epsilon \to 0^+} \int_0^\pi d\sigma \delta(\pi - \epsilon - \sigma) [\partial_e X^i]
\]

\[
(2\pi\alpha')^2 B_{ij}(X) (P_m - B_{mk}(X) \partial_e X^k) \right] \approx 0. \tag{10}
\]

One should notice that the order of limitation and integration in the last two equations cannot be reverted in order that the boundary constraints \( (G_L)_i^j \) and \( (G_R)_i^j \) are actually equivalent to the original boundary conditions.

Through some straightforward calculations with the help of (6)-(8), we can obtain the following Poisson brack-
\(\{(G_L)^i, X^j(\sigma')\} = \lim_{\epsilon \to 0^+} \int_0^\pi d\sigma \delta(\sigma - \epsilon)\)
\[
\times \left[ \delta_n^i \partial_\sigma + (2\pi \alpha')^2 \left( \frac{\delta B_{im}^n}{\delta X^m} P_m - B_{im}^n B_{mk} \delta_k^i \partial_\sigma \right) \right]
\left\{ X^n(\sigma), X^j(\sigma') \right\}
+ \lim_{\epsilon \to 0^+} \int_0^\pi d\sigma \delta(\sigma - \epsilon) (2 \pi \alpha')^2 B^{im} \{ P_m(\sigma), X^j(\sigma') \}
= \left( (2 \pi \alpha')^2 ((\mathcal{P} - \chi)L) + (I - (2 \pi \alpha')^2 B^2) A_L \partial_\sigma, \right.
\left. - (2 \pi \alpha')^2 B(I + B_L) \partial_\sigma, \right) \left\{ \{ P_m(\sigma), X^j(\sigma') \} \right\}
+ \lim_{\epsilon \to 0^+} \int_0^\pi d\sigma \delta(\sigma - \epsilon) (2 \pi \alpha')^2 B^{im} \{ P_m(\sigma), X^j(\sigma') \}
= \left( (2 \pi \alpha')^2 B^2 - I \right) \left( \{ P_m(\sigma), X^j(\sigma') \} \right)
+ \left( \{ (\mathcal{P} - \chi)L + B_L \} \right)^i \delta(\sigma'). \tag{11} \]

Analogously, we have
\[
\{(G_L)^i, P_j(\sigma')\} = \lim_{\epsilon \to 0^+} \int_0^\pi d\sigma \delta(\sigma - \epsilon)\]
\[
\times \left[ \delta_n^i \partial_\sigma + (2\pi \alpha')^2 \left( \frac{\delta B_{im}^n}{\delta X^m} P_m - B_{im}^n B_{mk} \delta_k^i \partial_\sigma \right) \right]
\left\{ X^n(\sigma), P_j(\sigma') \right\}
+ \lim_{\epsilon \to 0^+} \int_0^\pi d\sigma \delta(\sigma - \epsilon) (2 \pi \alpha')^2 B^{im} \{ P_m(\sigma), P_j(\sigma') \}
= \left( (2 \pi \alpha')^2 B^2 - I \right) \left( \{ P_m(\sigma), P_j(\sigma') \} \right)
+ \left( \{ (\mathcal{P} - \chi)L + B_L \} \right)^i \delta(\sigma'). \tag{12} \]

With the replacements \(A_L \to A_R, B_L \to B_R, C_L \to C_R\) and \(\delta(\sigma' - \epsilon) \to \delta(\pi - \epsilon - \sigma')\) in (11) and (12), we can get the similar Poisson brackets for \(G_R\).

In order that the new Poisson brackets \(\{ G_L, G_R \} \) be consistent with the boundary conditions \(\{ G_L, B_L \} \), the Poisson brackets (11) and (12) have to vanish. This leads to the following operator equations
\[
(2\pi \alpha')^2 ((\mathcal{P} - \chi)L + (I - (2 \pi \alpha')^2 B^2) A_L) \partial_\sigma, \]
\[
- (2 \pi \alpha')^2 B(I + B_L) \partial_\sigma, \]
\[
= 0, \tag{13} \]
\[
(2 \pi \alpha')^2 ((\mathcal{P} - \chi)L + B_L) \partial_\sigma, \]
\[
= 0, \tag{14} \]

where the action of the operators are right-associative. In particular, \(\partial_\sigma\) acts not only on the operators next to it, but also on any other quantities to which the left hand side of the complete equation may be applied. Once the last two equations (13) and (14) are satisfied, the boundary constraints \(G_L, G_R\) will Poisson-commute with everything in the phase space and hence they can be set equal to zero without causing any inconsistency.

However, contrary to the case with constant background \(B\)-field, we cannot finish the story upon getting the two equations (13–14). We must also require that the modified Poisson brackets \(\{ G_L, G_R \} \) satisfy Jacobi identity. As mentioned just now, since the boundary constraints \(G_L, G_R\) Poisson-commute with everything in the phase space once (13–14) are satisfied, we cannot expect to get anything new with \(G_L, G_R\) inserted into Jacobi identities. But there are other instances of Jacobi identities which need a check. In fact, there are totally 4 instances to be checked: the Jacobi identities for \(\{ X^i, X^j, P_k \}, \{ X^i, J^j, P_k \}, \{ X^i, X^j, P_k \}\) and for \(\{ P_i, P_j, P_k \}\). The first two of these read
\[
\{ X^i(\sigma), X^j(\sigma'), P_k(\sigma) \} + \{ X^j(\sigma), P_k(\sigma') \} \delta(\sigma - \sigma' - \epsilon) = 0, \tag{15} \]
\[
\{ X^i(\sigma), P_j(\sigma'), P_k(\sigma') \} + \{ P_j(\sigma'), P_k(\sigma') \} \delta(\sigma - \sigma' - \epsilon) = 0. \tag{16} \]

Straightforward calculations using (6–8) shows that (15) and (16) are equivalent to the following equations at the left end of the open string,
\[
(\mathcal{A}_L)^{im}(\sigma + \sigma'') \frac{\delta[(\mathcal{B}_L)^j \delta(\sigma' + \sigma'\prime)]}{\delta X^m(\sigma')} \]
\[
+ \frac{\delta^m \delta(\sigma - \sigma''') + (\mathcal{B}_L)^m \delta(\sigma + \sigma''')} \delta[(\mathcal{B}_L)^j \delta(\sigma' + \sigma''')] \delta P_m(\sigma''') \]
\[
- (\mathcal{A}_L)^{im} \delta(\sigma + \sigma'') \delta[(\mathcal{B}_L)^j \delta(\sigma' + \sigma'\prime)] \frac{\delta P_m(\sigma''')}{\delta X^m(\sigma')} \]
\[
+ \frac{\delta^m \delta(\sigma - \sigma''') + (\mathcal{B}_L)^m \delta(\sigma + \sigma''')} \delta[(\mathcal{A}_L)^{ij} \delta(\sigma + \sigma'')] \delta X^m(\sigma'') \]
\[
+ (\mathcal{C}_L)^{km} \delta(\sigma' + \sigma'\prime) \frac{\delta[(\mathcal{A}_L)^{ij} \delta(\sigma + \sigma'')]}{\delta P_m(\sigma''')} = 0, \tag{17} \]
\[
(\mathcal{A}_L)^{im}(\sigma + \sigma'') \frac{\delta[(\mathcal{C}_L)^{jk} \delta(\sigma' + \sigma'\prime)]}{\delta X^m(\sigma')} \]
\[
+ \frac{\delta^m \delta(\sigma - \sigma''') + (\mathcal{B}_L)^m \delta(\sigma + \sigma''')} \delta[(\mathcal{C}_L)^{jk} \delta(\sigma' + \sigma'\prime)] \delta P_m(\sigma''') \]
\[
+ \frac{\delta^m \delta(\sigma - \sigma''') + (\mathcal{B}_L)^m \delta(\sigma + \sigma''')} \delta[(\mathcal{B}_L)^k \delta(\sigma + \sigma'')] \delta X^m(\sigma'') \]
\[
+ (\mathcal{C}_L)^{km} \delta(\sigma' + \sigma'\prime) \frac{\delta[(\mathcal{B}_L)^{ij} \delta(\sigma + \sigma'')]}{\delta P_m(\sigma''')} = 0 \tag{18} \]

and similar ones at the right end of the open string with \(A_L, B_L, C_L \leftrightarrow A_R, B_R, C_R\), and \(\sigma, \sigma' \leftrightarrow \pi - \sigma, \pi - \sigma'\) in all the \(\delta\)-functions.

Using the fact that
\[
\frac{\delta}{\delta X^m(\sigma'')} \delta(\sigma + \sigma'') = 0
\]
and that $\delta(\sigma + \sigma')\delta(\sigma' + \sigma'') = \delta(\sigma + \sigma')\delta(\sigma' - \sigma'')$ is non-zero only at $\sigma = \sigma' = \sigma'' = 0$ if $\sigma, \sigma', \sigma''$ are all non-negative, we may drop all the $\delta$-function dependencies in (17) and (18) and get (now the equations at both ends negative, we may drop all the $\delta$'s)

$$
(A_{L,R})^{ij} \partial_i \left( [B_{L,R}]^j_k \right) + \left[ \partial_i + (B_{L,R})^j_i \right] \frac{\delta [B_{L,R}]^j_k}{\delta P_m} \\
- (A_{L,R})^{ij} \partial_i \left( [B_{L,R}]^k_j \right) - \left[ \partial_j + (B_{L,R})^k_j \right] \frac{\delta [B_{L,R}]^k_j}{\delta P_m} \\
- \left[ \partial_m + (B_{L,R})^m_j \right] \frac{\delta [A_{L,R}]^{ij}}{\delta X^m} + \left[ \partial_m + (B_{L,R})^i_m \right] \frac{\delta [A_{L,R}]^{ij}}{\delta X^m} = 0,
$$

(19)

By similar arguments, we get the following equations from the other two instances of Jacobi identities involving three $X$'s or three $P$'s, which are also required to hold at both ends of the open string,

$$
\frac{\delta (A_{L,R})^{ij}}{\delta X^m} (A_{L,R})^{mk} - \frac{\delta (A_{L,R})^{ij}}{\delta P_m} \left[ \partial_m + (B_{L,R})^m_j \right] \\
+ \frac{\delta (A_{L,R})^{jk}}{\delta X^m} (A_{L,R})^{mi} - \frac{\delta (A_{L,R})^{jk}}{\delta P_m} \left[ \partial_m + (B_{L,R})^m_i \right] \\
+ \frac{\delta (A_{L,R})^{ki}}{\delta X^m} (A_{L,R})^{mj} - \frac{\delta (A_{L,R})^{ki}}{\delta P_m} \left[ \partial_m + (B_{L,R})^m_j \right] = 0,
$$

(20)

$$
\frac{\delta (C_{L,R})^{ij}}{\delta X^m} (C_{L,R})^{km} - \frac{\delta (C_{L,R})^{ij}}{\delta P_m} \left[ \partial_m + (B_{L,R})^m_j \right] \\
+ \frac{\delta (C_{L,R})^{jk}}{\delta X^m} (C_{L,R})^{mi} - \frac{\delta (C_{L,R})^{jk}}{\delta P_m} \left[ \partial_m + (B_{L,R})^m_i \right] \\
+ \frac{\delta (C_{L,R})^{ki}}{\delta X^m} (C_{L,R})^{mj} - \frac{\delta (C_{L,R})^{ki}}{\delta P_m} \left[ \partial_m + (B_{L,R})^m_j \right] = 0.
$$

(21)

It is remarkable that the form of the above Poisson structure coincides with the $(A_{L,R})^{ij} = 0$ solution in our previous paper [8], though the background fields $G_{ij}$ and $B_{ij}$ are now both varying with spacetime. Therefore, we may call the Poisson brackets (23)- (25) the “background independent” Poisson structure for the open string theory. It is also worth mentioning that the Poisson brackets (23)- (25) does not depend on the detailed choice of the background fields, therefore they are also valid for general nonlinear sigma models defined by (11) with the fields $G_{ij}(X)$ and $B_{ij}(X)$ not necessarily preserving conformal symmetry. Let us remark that the Poisson structure (23)- (25) is an exact Poisson structure for any background fields $G_{ij}$ and $B_{ij}$, unlike the approximate result of (13), which works only for a non-oscillating string or a rigid rod.

III. DISCUSSIONS ON QUANTIZATION

Having obtained a consistent set of Poisson brackets (23)- (25) for the open string theory, we now come to the step for quantization. The Poisson brackets (23)- (25) are both linear and can be changed into canonical equal time commutators via the substitution $\{ , \} \rightarrow -i[ , ]$, i.e.

$$
[X'(\sigma), X'(\sigma')] = 0,
$$

(26)

$$
[X'(\sigma), P_j(\sigma')] = \delta_j^i \left[ \delta(\sigma - \sigma') - \delta(\sigma + \sigma') - \delta(2\pi - \sigma - \sigma') \right].
$$

(27)

The last one of the Poisson brackets, eq. (26), is in general nonlinear in the spacetime coordinates $X'$, therefore to change this last Poisson bracket into commutators we need to pay some more care on the operator ordering on the right hand side. Fortunately, since the right hand side of (26) depends only on $X'$, and we have seen from (26) that the operators $X'$ commute among themselves.
at equal world sheet time, the operator ordering problem on the right hand side of (26) can be easily resolved. The quantized form of (26) is then given as

\[
[P_i(\sigma), P_j(\sigma')] = i \frac{2}{(2\pi \alpha')^2} \times : (G + 2\pi \alpha' B)_{ik} (B^{-1})^{kl} (G - 2\pi \alpha' B)_{lj} : \\
\times \partial_{\sigma'} [\delta(\sigma + \sigma') + \delta(2\pi - \sigma - \sigma')].
\] (28)

The equations (26)-(28) constitute the set of fundamental commutators for the quantized open string theory in generic background. We can see that the coordinates at the ends of the strings are free of noncommutativity, thanks to (26). Moreover, the commutators between the coordinates and momenta at the boundaries are also vanishing (see eq. (27) at \( \sigma = \sigma' = 0, \pi \)). The only noncommutativity to appear in the quantum theory is in between the momenta, due to (28). It then follows from the standard Heisenberg equations that the world sheet time derivatives \( \partial_\sigma X^i \) at \( \sigma = 0, \pi \) will be identically zero, implying that the branes to which the string ends are attached can only be D-branes, even if classically the boundary conditions appear to be mixed. Though we cannot say that the above quantization scheme is the only possible one for open string in generic background, we can at least conclude that the spacetime noncommutativity at the boundary D-branes can be avoided, and should be avoided, since noncommutative boundaries would in general spoil the stability of the D-branes, i.e. \( \partial_\tau X^i \) can become nonzero at \( \sigma = 0, \pi \). That spacial noncommutative boundaries could be avoided for open string in generic background might also be helpful in settling the embarrassing ambiguities in the Seiberg-Witten maps which are so far unresolved [11].

Before ending this Letter, we should mention that we are not attempting to make a systematic description for the quantum theory of open strings in a generic background. For that purpose we should have gone done into details on the treatment of Virasoro constraints in the bulk (which is a consequence of the bulk equation of motion for the world sheet metric) as well as the analysis on the physical spectrum. However, since the Virasoro constraints are constraints purely in the bulk because on the boundaries the variations of the world sheet metric should simply be vanishing, giving rise to no constraints over the world sheet fields at all, our results on the boundary commutators should not be affected by the bulk Virasoro constraints. On the other hand, the analysis of physical spectrum in the presence of generic background is a very difficult task if not impossible, because such an analysis depends crucially on the mode expansions for the solutions of bulk fields, the form of which is not known unless the explicit form of the background fields is given. We therefore simply skip these steps by stressing once again that the boundary commutators between the world sheet fields will not be affected by such analysis.

After the completion of this work, we are informed by the authors of [14] and [15] about their works. These works also considered the problem of quantization for open string in curved metric and non-constant B-fields, and both of them give noncommutative results. However, the validity of Jacobi identities are not checked in [14], while in [15], it is explicitly stated that the results are based on the violation of Jacobi identities. Our results in this Letter is on the complete contrary: we showed that the spacial noncommutativity on the string ends does not necessarily appear, and that when the string ends are commutative, there is no violation of Jacobi identities.

Acknowledgement

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[1] The literature on noncommutative field theories is by now quite extensive and out of our means to list. A simple search in the hep-th archive with the word “noncommutative” in the title area results in 808 hits, as of 27 May, 2003.

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