Attaining Doppler Precision of 10 cm s$^{-1}$ with a Lock-in Amplified Spectrometer

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ABSTRACT. We explore the radial velocity performance benefits of coupling starlight to a fast-scanning interferometer and a fast-readout spectrometer with zero readout noise. By rapidly scanning an interferometer, we can decouple wavelength calibration errors from precise radial velocity measurements, exploiting the advantages of lock-in amplification. In a Bayesian framework, we investigate the correlation between wavelength calibration errors and resulting radial velocity errors. We construct an end-to-end simulation of this approach to address the feasibility of achieving 10 cm s$^{-1}$ radial velocity precision on a typical Sun-like star using existing, 5 m-class telescopes. We find that such a precision can be reached in a single night, opening up possibilities for ground-based detections of Earth-Sun analog systems.

Online material: color figures

1. INTRODUCTION

Recent results from NASA’s Kepler mission indicate that Sun-like stars are teeming with rocky exoplanets (Koch et al. 2010; Borucki et al. 2011; Howard et al. 2012; Batalla et al. 2013; Petigura et al. 2013). According to statistics derived from Kepler’s discoveries, and consistent with prior exoplanet searches, ground-based radial velocity and transit surveys targeting nearby stars are approaching the performance edge of discovering hundreds of exoplanets suitable for detailed atmospheric studies (e.g., Deming et al. 2009; Howard et al. 2010; Dressing & Charbonneau 2013; Berta et al. 2013). NASA’s future Transiting Exoplanet Survey Satellite (TESS) will likely find those rocky planets that transit nearby stars in short-period orbits (Ricker 2014). TESS will determine the radii of exoplanets transiting nearby stars. With precise radial velocity measurements of the TESS discoveries, the exoplanet masses, mean densities, and surface gravities can be determined, thereby constraining their interior structures and atmospheric characteristics.

The future of exoplanet science is therefore promising; however, the precision needed to detect nearby rocky planets with radial velocities (RVs), and measure the masses of those found to transit with TESS, is daunting. For example, the Earth introduces an 8.9 cm s$^{-1}$ semi-amplitude Doppler reflex motion on the Sun. To date, the smallest semi-amplitude radial velocity measured on a star is 51 cm s$^{-1}$ on αCentauri Bb, indicating the presence of an exoplanet with a minimum mass of 1.13 $M_{\oplus}$ and a semimajor axis of 0.04 AU (Dumusque et al. 2012). The planet’s discovery has yet to be confirmed by independent instruments or techniques, so α Centauri Bb is widely regarded as an exoplanet candidate. Nevertheless, the measurement indicates the state-of-the-art in stellar radial velocity precision. The detection of α Centauri Bb requires both astrophysical and instrumental noise corrections. Known astrophysical noise sources, such as stellar oscillation modes, granulation, and activity signals, can be individually removed through careful choices of exposure times and observing cadence, as well as modeling each effect separately (Dumusque et al. 2011). Pepe et al. (2011) describe how instrumental noise sources in HARPS, such as changes to temperature, pressure, and illumination, are addressed through high-cadence, long-time baseline observations of nearby, slowly rotating stars with little known activity. To this end, HARPS observed HD 85512 for 7.5 yr, obtaining 185 RV data points and discovering a 3.5 $M_{\oplus}$ planet. The standard deviation of the data residuals after subtracting the model of the planet was measured to be 0.75 m s$^{-1}$. While this value could encompass uncharacterized astrophysical noises, 0.75 m s$^{-1}$ is taken to be representative of the instrumental noise in HARPS. In order to reduce the state-of-the-art RV semi-amplitude from 51 cm s$^{-1}$ (αCentauri Bb) to 8.9 cm s$^{-1}$ (Earth-Sun analog), these instrumental noise sources must be understood and eliminated.

Eliminating the effects of instrumental noise on radial velocity measurements requires ever more precise wavelength calibration techniques. Historically, iodine absorption cells and thorium-argon (ThAr) lamps have been the principal wavelength calibration tools. Starlight passing through an iodine gas cell results in a dense series of iodine absorption lines...
superimposed onto the stellar spectrum. This method has been implemented on HIRES (High-Resolution Echelle Spectrometer), where planet semi-amplitudes as small as \(1.89 \text{ m s}^{-1}\) have been detected (HD 156668b; Howard et al. 2011). The small number of absorption lines at red and near-IR wavelengths, however, limits the use of iodine cell calibration for redder stars. Furthermore, the technique requires complex modeling of the combined iodine/stellar spectrum, which in turn requires high signal-to-noise data (Lovis & Pepe 2007). As an alternative to the iodine cell approach, fiber-fed spectrographs have used ThAr lamps as a simultaneous wavelength reference (Baranne et al. 1996). Starlight and ThAr lamp light are fed into the spectrograph through separate fibers such that their spectra are simultaneously recorded onto the detector. While ThAr provides more lines in the near-IR than iodine, the nonuniformity in line spacing and long-term variability of ThAr sources limits the technique.

Recently, laser frequency combs (LFCs) have provided large numbers of equally spaced, stable lines over a wide range of wavelengths (e.g., Murphy et al. 2007; Steinmetz et al. 2008). LFCs fix the phase of standing waves inside a laser cavity such that the waves periodically interfere constructively, producing bursts of light. The time between bursts can be accurately controlled using an atomic clock. In the frequency domain, the laser provides more lines in the near-IR than iodine, the nonuniformity in line spacing and long-term variability of ThAr sources limits the technique.

An experimental LFC installed on HARPS in 2010 has yielded unprecedented RV stability over short timescales (Wilken et al. et al. 2012). The HARPS spectrograph is fed by two multimode fibers, or channels; one channel is coupled to starlight, and the other to light from the LFC. In Wilken et al. et al. (2012), the HARPS team fed LFC light through both fibers and differenced the two channels to measure the instrumental drift. By optimally binning 20–30 s exposures for 4 minutes, the limiting RV precision due to instabilities between the two channels was found to be $2.5 \text{ cm s}^{-1}$. Between November of 2010 and January of 2011, however, synchronous drift in the channels induced a standard deviation of $34 \text{ cm s}^{-1}$ between data taken in the two time periods. While planned changes to the environmental conditions surrounding HARPS made this $34 \text{ cm s}^{-1}$ drift measurement a slight underestimate of the long-term stability, the two channels nevertheless experienced long-term drifts larger than the $2.5 \text{ cm s}^{-1}$ short-term limiting RV precision. These drifts are thought to be due to uncontrolled, long timescale instrumental changes.

An innovative approach to acquiring Doppler measurements of stars was proposed by Erskine (2003). By introducing a Michelson or Mach-Zender interferometer into the optical path before a traditional spectrometer, Erskine (2003) suggested high Doppler precision could be achieved with instrumental fluctuations minimized. The technique, called externally dispersed interferometry (EDI), requires that the optical path difference in the interferometer be modulated, either spatially or temporally. He suggested both a spatial modulation, by tilting one of the interferometer cavity mirrors, and a temporal modulation, by physically moving one of the cavity mirrors. The phases of the resulting fringe patterns, whether temporal or spatial, are highly sensitive to changes in the Doppler velocity of the target star. As noted by Erskine (2003), the modulation of a spectrum into a fringe pattern provides a robust way of eliminating systematic effects, such as fixed-pattern noise, in the resulting radial velocity measurements.

Since then, multiple groups have implemented EDI systems. Ge et al. et al. (2006) and Mahadevan et al. (2008) describe an instrument called the Exoplanet Tracker (ET), which uses a tilted cavity mirror without temporal modulation. van Eyken et al. (2010) describe the theory and implementation of ET and a similar instrument called the Multiobject APO Radial Velocity Exoplanet Large-area Survey (MARVELS). Hajian et al. (2007) describe the dispersed Fourier Transform Spectrometer (dFTS), designed to acquire high-resolution spectra rather than measure precise radial velocities, also achieved by Erskine et al. (2003). And finally, Muirhead et al. (2011) describe the TripleSpec Exoplanet Discovery Instrument (TEDI), which used temporal, rather than spatial, modulation to map out the fringes.

A particularly interesting result from Muirhead et al. (2011) was the achievement of $30 \text{ m s}^{-1}$ of rms radial velocity performance without any calibration of the spectrometer point-spread function (PSF). Typically, precise radial velocity spectrometers involve a significant amount of effort to stabilize the spectrometer PSF because uncalibrated asymmetries in the PSF are degenerate with wavelength calibration errors and result in spurious radial velocity measurements on the target star. Muirhead et al. (2011) show that the reduced importance of wavelength calibration is a direct result of the radial velocity measurement being encoded in the phase of a sinusoid varying within each pixel, rather than the change in flux across several pixels, as is the case for traditional spectroscopy as well as for ET and MARVELS, which use spatial modulation across pixels rather than temporal modulation within a pixel.

However, a significant challenge to temporal modulation techniques involves effects from readout noise. The robustness against errors in wavelength calibration is due to modulation taking place on short timescales and the PSF fluctuations taking place over longer timescales. PSF fluctuations or wavelength calibration errors that occur over the same timescale as the modulation are not removed. Fast scanning is therefore desired. Readout noise in large format detectors limits the speed at which one can scan and read out the detector in the spectrometer. In Muirhead et al. (2011), the authors took 30 s exposures to ensure that the dominant noise source was photon noise, rather than detector readout noise.

Recent developments in large format, high frame rate, and low-readout noise detectors motivate studies of temporal modulation at very high frame rates. Electron multiplying charge-coupled devices (EMCCDs) and complementary metal-oxide-semiconductor (CMOS) detectors can be manufactured and
operated to have inherently low readout noise ($<1 \, \text{e}^-/\text{pixel}^{-1}$) and the ability to expose and read out at very high frame rates (100 megapixels $\text{s}^{-1}$). With a high enough gain enabled, EMCCDs have effectively zero readout noise. These detectors are becoming commonplace in astronomical instruments (e.g., Dhillon & Marsh 2001; Baranec et al. 2013), motivating a study of the advantages of fast-modulation EDI for ultraprecise Doppler velocimetry.

In this article, we show that the act of modulating a spectrum in time effectively decouples wavelength calibration errors from radial velocity measurements. Therefore, the nature of the radial velocity measurement is fundamentally different than that obtained from conventional spectroscopy, where wavelength calibration completely determines the resulting radial velocity measurement. Instead, the radial velocity precision relies upon precise knowledge of the interferometer positions.\(^5\) We augment previous theoretical studies of the EDI technique (e.g., Erskine 2003; van Eyken et al. 2010) by specifically introducing and recovering wavelength calibration errors into simulated data. We also investigate the effects of interferometer position errors on radial velocity precision. Finally, we simulate the performance of a time-varying interferometric system on a medium-sized telescope to determine the feasibility of an Earth–Sun analog detection.

2. THEOREY

Following the discussion in Erskine (2003) and also Muirhead et al. (2011), we derive the relationship between a temporally modulated spectrum and a radial velocity measurement. We use this relationship to demonstrate the decoupling of wavelength calibration errors and RV measurements.

The intensity of the stellar spectrum recorded by the detector depends on both the wavelength and the interferometer delay. The interferometer delay is assumed to vary sinusoidally with time, therefore acting as a sinusoidal transmission comb. For a wavenumber, $\nu$, and delay, $\tau$, the measured intensity is given by

$$I_{\nu, \tau} = |S_{\nu}(1 + \cos(2\pi \nu \tau))| \ast R_{\nu},$$

where $S_{\nu}$ is the intrinsic stellar spectrum, $(1 + \cos(2\pi \nu \tau))$ represents the transmission comb, $R_{\nu}$ is the spectrograph line-spread function, and $\ast$ represents the convolution. While most practical interferometer configurations will result in two output beams, we assume here that the beams have been combined (e.g., by placing a spectrograph and detector at each output and combining the intensities in postprocessing).

The interferometer delay $\tau$ can be expressed as the sum of a constant bulk delay, $\tau_0$, and a smaller time-varying phase shift $\Delta \tau$. Taking $\tau = \tau_0 + \Delta \tau$, and applying trigonometric identities, the measured intensity can be rewritten as:

$$I_{\nu, \tau_0 + \Delta \tau} = A_{\nu} + B_{\nu} \cos(2\pi \Delta \nu) - C_{\nu} \sin(2\pi \Delta \nu).$$

The coefficients $A_{\nu}$, $B_{\nu}$, and $C_{\nu}$ are defined as

$$A_{\nu} = S_{\nu} \ast R_{\nu},$$
$$B_{\nu} = [S_{\nu} \cos(2\pi \tau_0 \nu)] \ast R_{\nu},$$
$$C_{\nu} = [S_{\nu} \sin(2\pi \tau_0 \nu)] \ast R_{\nu},$$

where $A_{\nu}$ is the nonmodulated spectrum, while $B_{\nu}$ and $C_{\nu}$ describe the interference between the stellar spectrum and the transmission comb. These coefficients can be determined by varying the interferometer delay $\Delta \tau$ and observing the intensity $I_{\nu, \tau_0 + \Delta \tau}$. A data set, or “scan,” is a list of intensities at a range of $\Delta \tau$ and $\nu$ values. In order to compare the fitted coefficients between two scans, $B_{\nu}$ and $C_{\nu}$ are combined into a “complex visibility”:

$$B_{\nu} - iC_{\nu} = [S_{\nu} e^{-i2\pi \tau_0 \nu}] \ast R_{\nu}.$$  

The complex visibility is used to compare two scans, where one has been Doppler-shifted by $\Delta \nu = (\Delta RV/c) \nu$. The values of $\Delta \nu$ and $1/\tau_0$ are assumed to be small compared to a resolution element of the spectrograph. By applying the Fourier convolution and shift theorems, the Doppler-shifted “epoch” complex visibility $B_{\nu}^0 - iC_{\nu}^0$ and the unshifted “template” visibility $B_{\nu}^1 - iC_{\nu}^1$ are related by an exponential phase:

$$B_{\nu}^0 - iC_{\nu}^0 = [B_{\nu}^0 - iC_{\nu}^0] e^{-i2\pi \nu / \Delta \nu} = [B_{\nu}^0 - iC_{\nu}^0] e^{-i2\pi \nu (\Delta RV/c) / \nu}.$$  

For clarity, the discussion above applies to small radial velocity shifts only, due to the use of the Fourier shift theorem. On-sky observations, however, will be affected by the Earth’s motion relative to the barycenter of the solar system, producing RV shifts on the order of 10 km s$^{-1}$. Muirhead et al. (2011) describe the detailed derivation of the relationship between the epoch and template complex visibilities, arriving at the following generalization of equation (7):

$$B_{\nu}^1 - iC_{\nu}^1 = e^{i2\pi \nu (\Delta RV/c)/\nu} [B_{\nu}^{01} - iC_{\nu}^{01}]_{\nu = \nu + \Delta \nu}.$$  

Therefore, a Doppler shift changes the phase of the complex visibility (for example, an Earth–Sun analog would result in a phase shift of $3.4 \times 10^{-5}$ radians). A wavelength calibration error reassigns the phase values to a different wavelength grid without changing the value of the phase itself. The radial velocity measurements are therefore decoupled from wavelength calibration changes; instead, the RV precision relies upon precise knowledge of the interferometer positions, described by $\Delta \tau$ and $\nu$.

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\(^5\) A possible implementation could involve a combination of fine path-length control with precision piezoelectrics and fine sensing using a copropagating laser with exceptional wavelength stability.
§ 4 describes the effects of wavelength calibration and interferometer position errors in detail.

3. SIMULATION ARCHITECTURE

In order to quantify instrumental contributions to radial velocity precision, we construct an end-to-end simulation of the radial velocity reconstruction process described in § 2. The flux entering the instrument is modeled using a simulated solar spectrum (Coelho et al. 2007). We multiply the stellar spectrum by the interferometer comb, which is modeled as a sinusoidal transmission comb as a function of wavenumber $\nu$ and interferometer delay $\tau$ (see eq. [1]). The result is therefore a series of spectra multiplied by transmission combs of different interferometer delays.

The product of the stellar spectrum and interferometer transmission comb is convolved with the spectrograph instrument profile (IP). The spectrograph IP is modeled as a normal distribution with a width corresponding to a spectrograph resolution of $R = 10,000$. The resolution was chosen to be low in order to avoid resolving the interferometer comb; in order for the phase of the interference fringes to correspond to a radial velocity change, the comb must remain unresolved. The sampling is assumed to be 4 pixels per resolution element. The result is a stellar spectrum recorded at each interferometer step (Fig. 1). The final data product can be represented by a three-dimensional map of interferometer delay, wavelength, and observed intensity (Fig. 2).

It is clear from the axes of Figure 2 that wavelength calibration and radial velocity information are separated; the $y$-axis represents time (the time-varying position of the interferometer), while the $x$-axis represents detector pixels (the wavelength information introduced by the spectrograph). A radial velocity shift will therefore result in a vertical shift as the phase of the interference fringes changes, whereas a change in wavelength calibration will result in a global horizontal shift as the spectrum’s wavelength is reassigned.

Each vertical cut through Figure 2 represents the intensity as a function of phase delay at a given wavenumber described by equation (2). Noise is added to the intensity at each phase delay by adding values drawn from a Gaussian distribution with a mean of zero and a standard deviation given by the spectrum’s mean divided by the desired signal-to-noise ratio. For clarity, we refer to this as Poisson noise, but our simplifications (Gaussian statistics with a constant standard deviation) should be noted. The $A_{\nu}$, $B_{\nu}$, and $C_{\nu}$ coefficients are then fit to each vertical cut, defining the complex visibilities (eq. [6]).

In order to measure a radial velocity shift, two data sets, each represented by a figure like Figure 2, are produced: A template spectrum and an epoch spectrum with an RV shift and wavelength calibration error. The template spectrum is assumed to have infinite signal-to-noise, due to either a long on-sky integration or the use of a simulated spectrum. The template spectrum is multiplied by a complex phasor, as in equation (8), containing a test RV shift, and is interpolated onto a new wavelength grid to represent the effect of the wavelength calibration error. This modified template spectrum and the originally shifted epoch spectrum are then compared using a $\chi^2$ test that treats the real and imaginary parts of the complex visibilities separately:

$$-\frac{\chi^2}{2} = \sum_{\nu} (B_{\nu} - B_{\nu}^t)^2 + (C_{\nu} - C_{\nu}^t)^2,$$  \hspace{1cm} (9)

where $B_{\nu}^t$ and $C_{\nu}^t$ describe the epoch spectrum, and $B_{\nu}$ and $C_{\nu}$ describe the modified template spectrum. Because the likelihood function is defined as

$$L = e^{-\chi^2/2},$$  \hspace{1cm} (10)
equation (9) is considered to be $\log(L)$. We compute $\log(L)$ on a grid of radial velocity and wavelength calibration error test points and find the points that minimize equation (9) by parabola fitting. By computing many such solutions for independent realizations of the noise, we construct histograms of radial velocity and wavelength calibration error solutions that are well described by Gaussian statistics. We define the standard deviation of 100 radial velocity solutions to be the “radial velocity precision.” By fitting a two-dimensional (2D) Gaussian distribution

![Image](image_url)
to a 2D histogram of several thousand radial velocity and wavelength calibration error solutions, we compute error ellipses demonstrating the relationship between the two parameters (Fig. 3). In contrast to traditional RV reconstruction methods, the contours show an elliptical shape, demonstrating that the RV and wavelength calibration error measurements are not highly correlated.

4. ERROR BUDGET

The simulation described in § 3 is used to analyze the effects of Poisson noise, wavelength calibration errors, and interferometer position errors. The assumptions used in the simulation are listed in Table 1. We also note that all radial velocity and wavelength calibration error solutions discussed in this section were generated using a spectral bandwidth of $\Delta \lambda = 88 \, \text{Å}$, while the FWHM of the $V$-band filter is $\Delta \lambda = 880 \, \text{Å}$. We chose this smaller bandwidth to accommodate the available computing resources. Because the number of spectral lines increases with the square root of the bandwidth; however, we can represent the $\Delta \lambda = 880 \, \text{Å}$ radial velocity precision by dividing the $\Delta \lambda = 88 \, \text{Å}$ radial velocity precision by $\sqrt{10}$. The text and figures below refer to radial velocity precisions that have been modified by this factor.

4.1. Poisson Noise

In the absence of all instrumental or astrophysical noise sources, the radial velocity precision decreases as $1/(S/N)$ (Fig. 4). Under these conditions, a signal-to-noise ratio (S/N) of about 94 per spectrograph resolution element is required to reach a radial velocity precision of $10 \, \text{cm s}^{-1}$. § 4.2 and 4.3 below describe how instrumental noise sources cause the RV precision to deviate from this ideal case.

4.2. Wavelength Calibration Errors

Figure 5 shows that the radial velocity precision changes by millimeters per second per kilometers per second of wavelength calibration error for high signal-to-noise ratios. The black line in Figure 5 represents the 1:1 correspondence between wavelength calibration and radial velocity using conventional RV reconstruction methods. We have therefore demonstrated that by temporally varying a stellar spectrum, the dependence of the radial velocity precision on a consistent, correct wavelength solution is reduced.

4.3. Interferometer Position Errors

This robustness against wavelength calibration errors comes at the cost of precise knowledge of the interferometer position. The interferometer position is described by a constant “bulk” offset and a much smaller time-varying phase shift ($\tau_0$ and $\Delta \tau$, respectively, in eq. [2]).
Equation (8) shows that errors in estimation of the bulk delay produce proportional errors in the RV precision. Therefore, the same error in a bulk delay measurement will result in poorer RV precision for stars with larger radial velocities. In order to calculate the smallest required bulk delay measurement error, we must consider the target stars with the largest radial velocities. Barycentric motion produces radial velocity differences of up to 60 km s\(^{-1}\). In order to reach an RV precision of 10 cm s\(^{-1}\) with a typical bulk delay of 2 cm, the maximum bulk delay measurement error is \((\frac{10}{60 \text{ km s}^{-1}}) 2 \text{ cm} = 33 \text{ nm}\). State-of-the-art piezoelectric stages are capable of subnanometer rms measurement accuracy (e.g., Samuele et al. 2007), so we conclude that the bulk delay can be adequately measured to provide 10 cm s\(^{-1}\) precision on the stars with the largest radial velocities.

Errors in the phase step estimations, however, impose more strict metrology requirements. Figure 6 shows the radial velocity precision plotted against the standard deviation of the phase step error. The horizontal dotted lines represent the Poisson-limited RV precision for each S/N. In the phase-step-error-limited regions, the RV precisions approach the limiting S/N = \(\infty\) condition, shown by the black line. For S/N = 100, \(\Delta \tau\) must be known to less than 1 nm to reach the Poisson-limited regime. Because the phase step errors are assumed to be uncorrelated, the total number of phase steps affects the final RV precision. In this simulation, we chose 200 steps (Table 1). The RV precisions shown in this plot were calculated using a spectral bandwidth of \(\Delta \lambda = 88 \text{ Å}\) and were divided by \(\sqrt{10}\) to reflect the RV precisions associated with the full V-band bandwidth of 880 Å. See the electronic edition of the PASP for a color version of this figure.
5. FEASIBILITY OF AN EARTH-SUN ANALOG MEASUREMENT

We have shown that the modulation of a stellar spectrum in time decouples wavelength calibration errors from RV measurements, while coupling interferometer position errors to RV measurements. Given these new constraints, we now address the feasibility of reaching a radial velocity precision of 8.9 cm s\(^{-1}\) on a G-type star using an exiting telescope. Our assumed bandpass, throughput, interferometer control requirement, and target star magnitude are summarized in Table 2 and described in detail below.

Operating in the visible (\(\approx 500-600 \text{ nm}\)) while observing Sun-like stars has the dual advantages of covering the peak of the G star blackbody function while reducing contamination due to telluric lines compared to the infrared. A visible bandpass, however, may come at the cost of throughput. In order to minimize the effects of interferometer position errors, calibration and starlight should be on a common path in the interferometer and spectrograph. One possible approach would be to feed the instrument with a single-mode fiber, containing both starlight and light from a calibration source (e.g., a frequency-stabilized laser). In this way, modulation of the calibration light, imprinted on the science data itself, will measure the changes in the optical path difference of the interferometer.

The single-mode fiber coupling efficiency is determined primarily by the Strehl ratio of the associated adaptive optics system. Single-mode fibers are inherently diffraction-limited, and require diffraction-limited beams delivered from a telescope in order to have any consequential coupling: Seeing-limited telescopes will not couple nearly enough starlight for a Doppler survey of nearby stars (Shaklan & Roddier 1988). Recently, however, advanced adaptive optics (AO) systems have reported significant Strehl ratios at visible wavelengths. For example, the Robo-AO system on the Palomar 60 inch (1.5 m) telescope has achieved a Strehl ratio of 18% in \(i\) band (Baranec et al. 2012). The Palm-3000 adaptive optics system on the Palomar 200 inch (5.1 m) telescope is designed to achieve high Strehl ratios at visible wavelengths. Palm-3000 is predicted to achieve 30% Strehl in \(V\)-band, equivalent to a 95 nm rms wavefront error (Dekany et al. 2013). Researchers at the University of Arizona recently commissioned a visible-light adaptive optics system (VisAO) on the 6.5 m Magellan Clay Telescope, demonstrating a 43% Strehl ratio, or 149 nm rms wavefront error, in \(Y\) band (Close et al. 2013). For the purposes of this study, we estimate the total sky-to-detector throughput to be 7.4% by multiplying the ideal fiber coupling efficiency of 82% by a Strehl ratio of 30%, based on the performance of Palm-3000 at visible wavelengths and a spectrograph efficiency of 30%.

As described in § 2, instrumental noise varying more slowly than the interferometer’s phase steps is rejected. For maximum noise rejection, the interferometer should therefore scan as quickly as possible. The interferometer’s speed is then limited by detector frame rates and read noise. The lowest noise detectors at optical wavelengths, CCDs, have significant readout times of seconds to minutes. This limits the rate at which we can modulate an optical signal to the point that lock-in amplification is effectively pointless with most CCD detectors. However, the development of high frame rate, low read noise EMCCDs and CMOS detectors opens up the possibility of using lock-in amplification at optical wavelengths. Assuming a detector read noise of 2 \(e^-\) pixel\(^{-1}\), gain = 1, and 4 pixels per resolution element, the detector must measure 64 photons per resolution element (S/N = 8) to remain well within the photon-noise-limited regime. The exposure time necessary to reach this minimum S/N will place an upper limit on the scanning speed.

Taking a series of scans can reduce normally distributed noise due to interferometer position errors. The number of scans is chosen such that the desired RV precision is achieved under realistic position error conditions. State-of-the-art piezoelectric stages can be controlled to less than 1 nm rms (e.g., Samuele et al. [2007] controlled the Physik Instrumente P-752 flexure stage to 0.4 nm rms), so we choose a 1 nm control requirement. In order to reach 10 cm s\(^{-1}\) with phase step errors of 1 nm rms and bulk delay errors of 10% \(\tau_0\), we require 139 scans, each with S/N = 8.

We now choose a target star magnitude and telescope size to determine the observing time required to take 139 such scans. To choose a representative target star magnitude, we constructed a histogram of the 100 brightest G stars. We choose \(m_v = 8.5\) as a representative magnitude from this sample.

The Palomar 200 inch telescope would take about 0.074 s to reach S/N = 8 per resolution element, given the assumptions in

| Parameter | Value | Units | Comments |
|-----------|-------|-------|----------|
| Bandpass  | 507–595 | nm | V-band FWHM |
| Sky-to-detector throughput | 7.4 | % | Representative of a single-mode fiber coupled to a 30% Strehl input beam and a spectrograph with 30% efficiency |
| Interferometer position control requirement | 1 | nm | Conservative requirement based on 0.4 nm state-of-the-art piezoelectric parameters |
| Stellar visual magnitude | 8.5 | \(m_v\) | Representative of the mode of the visual magnitudes of the 100 brightest G stars |
| Minimum S/N per resolution element | 8 | | Based on minimum S/N to avoid the read-noise-limited regime |
Tables 1 and 2. For 139 scans, each with 200 0.074 s phase steps, the 200 inch telescope would require about 0.57 hr to reach a radial velocity precision of 10 cm s\(^{-1}\). It is therefore possible to reach the radial velocity precision necessary to detect Earth-like planets around Sun-like stars using existing, 5 m-class telescopes.

6. CONCLUSIONS

The current state-of-the-art radial velocity instruments are limited by their ability to maintain their system’s wavelength solution in the presence of slowly varying instrumental noise. Lock-in amplification, however, can suppress such long time-scale noise sources, while decoupling the effects of wavelength calibration errors from radial velocity precision. The simulations presented in this article indicate that lock-in amplified, externally dispersed interferometry is a possible path forward to reach the radial velocity precision necessary to detect Earth–Sun analog systems on existing, medium-sized telescopes.

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REFERENCES

Baranec, C., et al. 2012, Proc. SPIE, 8447, 844704
———. 2013, in Proc. Advanced Maui Optical and Space Surveillance Technologies Conf., Wailea, Maui, Hawaii, Sep. 10–13, ed. S. Ryan, The Maui Economic Development Board, ID E48
Baranne, A., et al. 1996, A&AS, 119, 373
Batalha, N. M., et al. 2013, ApJS, 204, 24
Berta, Z. K., Irwin, J., & Charbonneau, D. 2013, ApJ, 775, 91
Borucki, W. J., et al. 2011, ApJ, 736, 19
Close, L. M., et al. 2013, ApJ, 774, 94
Coelho, P., Bruzual, G., Charlot, S., Weiss, A., Barbuy, B., & Ferguson, J. W. 2007, MNRAS, 382, 498
Dekany, R., et al. 2013, ApJ, 776, 130
Deming, D., et al. 2009, PASP, 121, 952
Dhillon, V., & Marsh, T. 2001, NewA Rev., 45, 91
Dressing, C. D., & Charbonneau, D. 2013, ApJ, 767, 95
Dumusque, X., Udry, S., Lovis, C., Santos, N. C., & Monteiro, M. J. P. F. G. 2011, A&A, 525, A140
Dumusque, X., et al. 2012, Nature, 491, 207
Erskine, D. J. 2003, PASP, 115, 255
Erskine, D. J., Edelstein, J., Feuerstein, W. M., & Welsh, B. 2003, ApJ, 592, L 103
Ge, J., et al. 2006, ApJ, 648, 683
Hajian, A. R., et al. 2007, ApJ, 661, 616
Howard, A. W., et al. 2010, Science, 330, 653
———. 2011, ApJ, 726, 73
———. 2012, ApJS, 201, 15
Koch, D. G., et al. 2010, ApJ, 713, L 79
Lovis, C., & Pepe, F. 2007, A&A, 468, 1115
Mahadevan, S., van Eyken, J., Ge, J., DeWitt, C., Fleming, S. W., Cohen, R., Crepp, J., & Heuvel, A. V. 2008, ApJ, 678, 1505
Muirhead, P. S., et al. 2011, PASP, 123, 709
Murphy, M. T., et al. 2007, MNRAS, 380, 839
Pepe, F., Lovis, C., Ségransan, D., et al. 2011, A&A, 534, A58
Petigura, E. A., Howard, A. W., & Marcy, G. W. 2013, PNAS, 110, 19273
Ricker, G. R. 2014, J.AAVSO, 42, 234
Samuele, R., Wallace, J., Schmidtlin, E., Shao, M., Levine, B. M., & Fregoso, S. 2007, in 2007 IEEE Aerospace Conference (New York: IEEE), 1
Shaklan, S., & Roddier, F. 1988, Appl. Opt., 27, 2334
Steinmetz, T., et al. 2008, Science, 321, 1335
van Eyken, J. C., Ge, J., & Mahadevan, S. 2010, ApJS, 189, 156
Wilken, T., et al. 2012, Nature, 485, 611

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