Simulation of oil whirl for geared rotor journal bearing system

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Abstract. This study developed a dynamic model of gear-pair system, involving nonlinear oil film force of hydrodynamic bearings and the gear mesh force with backlash. Based on this model, numerical simulations are made about instability led by the oil whirl in geared rotor journal bearing system. By comparing responses between single rotor journal bearing system and geared rotor system, evolution of oil whirl from individual rotor to adjacent geared rotors is examined. The results show that dynamic mesh force can transfer the excitations from the adjacent rotors, making different influences on the stability for individual rotors. The stabilities of the geared rotors are not independent with each other, but coupled by the mesh force. This coupling can be accompanied with tooth separation.

1. Introduction

Geared rotor bearing systems are widely used as power transmission mechanism, among which oil journal bearings are usually adopted in high speed and heavy loaded applications, such as marine reduction gearbox, special power plant and multi-stages centrifugal compressor. Oil whirl phenomenon induced by the nonlinearity of the hydrodynamic bearing force leads to a large amplitude vibration. With the increasing demand for quiet and reliable transmissions, these forms of instability should be sufficiently studied and avoided.

Among numerous studies on gear system dynamics in the past decades, lumped mass models are prevalent. Descriptions for the motion of the gear system become more complete, including purely torsional motion [1], coupled torsional-lateral motion [2] and three-dimensional motion [3]. Plenty of nonlinear factors are taken into consideration, including time-varying mesh stiffness, tooth clearance, and bearing clearance [4-6], bearing position error [7], localized defects and housing stiffness [8], based on which rich nonlinear phenomena are analyzed such as jump, bifurcation, chaos [9]. Generally, journal bearing forces are usually introduced with linearized oil film force models [10] or nonlinear force models [11, 12], which are derived from Reynolds equation based on different simplifications. These models introduce strong nonlinear factors to the rotor systems and makes descriptions about the instability caused by oil whirl [13, 14]. For geared systems, Hamad and Seireg [10] calculated the whirl orbits and the stability of a pinion-gear system supported on oil film bearings with a linearized bearing force model. Kishor and Gupta [15, 16] introduced the nonlinear hydrodynamic bearing force model.
and presented effects of parameters for gears and bearings. Fargère and Velex [17] studied the dynamic interactions in gears-hydrodynamic journal bearing systems by a refined model and compared with experiment results. Liu et al. [18] studied the interactions between journal bearing clearance and backlash in spur gear system and investigated the oil whirl and tooth wedging with bearing position errors.

When adopting the nonlinear oil film force model expressions, some studies [19, 20] neglected the differences caused by opposite rotations of the geared rotors. Further, the development of oil whirl from individual rotor to adjacent geared rotors, is still not been thoroughly analyzed. Aiming at oil whirls, this paper makes a simulation about the dynamic response for a gear pair system supported on oil-film bearings, where Capone’s bearing force model [12] is adopted. By investigating oil whirls accrued on individual rotors, the evolution of the instability in the gear pair are analyzed.

2. Mathematic model

A lumped parameter model of a spur gear pair supported on cylindrical journal bearing is illustrated in figure 1. Only planar and torsional motions are discussed, $x_p, y_p$ and $x_g, y_g$ are lateral displacements, and $\theta_p, \theta_g$ for the torsional. The subscript “p” and “g” refer to the pinion and the gear, respectively.

![Figure 1. Gear pair journal bearing system.](Image)

2.1. Mesh force

Mesh deflection is the function depending on displacements of points $A$ and $B$ on the base circles. On the drive-side, it is given as

$$
\delta_d = x_g \sin \alpha + y_g \cos \alpha + r^b \theta_g - x_p \sin \alpha - y_p \cos \alpha + r^p \theta_p
$$

(1)

where $\alpha$ is the pressure angle, $r^b_g, r^b_p$ are the radius of base circle for the pinion and the gear, respectively.

Considering the separation, the expression of contact force on drive-side is given by

$$
f_d = k_m h_d \delta_d,
$$

(2)

where $k_m$ is the mesh stiffness, which is simplified as constant.

Similarly, the mesh force on the back-sides is given as follow

$$
f_h = k_m h_b (\delta_h - b_0)
$$

$$
\delta_h = x_g \sin \alpha - y_g \cos \alpha - r^h \theta_g - x_p \sin \alpha + y_p \cos \alpha - r^p \theta_p
$$

(3)

$$
h_b = \begin{cases} 
1 & \delta_h > b_0 \\
0 & \delta_h < b_0 
\end{cases}
$$

where $b_0$ is the initial backlash of the gear pair.
2.2. Bearing force

The analytical oil film force model by Capone [13] is adopted for journal bearing force. For a cylindrical journal bearing as shown in figure 2(a), the fluid film pressure distribution is governed by Reynolds Equations, simplified under short-length bearing assumption.

\[ \left( \frac{R}{L} \right)^2 \frac{\partial}{\partial \zeta} \left( \frac{R}{L} \frac{\partial p}{\partial \zeta} \right) = \bar{x} \sin \theta_b - \bar{y} \cos \theta_b - 2(\hat{x} \cos \theta_b + \hat{y} \sin \theta_b) \]  

(4)

where \( R \) and \( L \) are the journal radius and bearing length, \( \theta_b \) is the circumferential coordinate start from \( ox \) direction, and \( \bar{z} = z / L \) is dimensionless axial coordinate. Similarly, \( \bar{x} = x / C, \bar{y} = y / C \) are dimensionless coordinates of the shaft center (\( C \) is the radial clearance of the bearing). \( \hat{x} \) is the derivative of \( x \) with respect to dimensionless time(\( t = \omega \tau \)). \( \mu \) is the hydrodynamic viscosity of the fluid, \( \omega \) is the angular frequency of rotation.

Introducing boundary conditions \( p=0 \), at \( z = \pm L/2 \), the pressure distribution is gotten by the integral along \( \theta_b \) and \( z \) direction.

\[ \begin{aligned}
    \{ f_{b,x} \\ f_{b,y} \} &= f^\circ \left( \bar{x}, \bar{y}, \hat{x}, \hat{y} \right) = -\frac{\sqrt{\left( \bar{x} - 2\hat{x} \right)^2 + \left( \bar{y} + 2\hat{y} \right)^2}}{(1 - \bar{x}^2 - \bar{y}^2)} \sigma \left[ 3\pi V - G \sin \beta_b - 2S \cos \beta_b \right] \\
    &= 3\pi G \cos \beta_b - 2S \sin \beta_b
\end{aligned} \]  

(5)

Where, the symbol \( V, G, S, \sigma, \beta_b \) are expressed as follows.

\[ G = \int_{\theta_b}^{\beta+\pi} \frac{d\theta_b}{1 - \bar{x} \cos \theta_b + \bar{y} \sin \theta_b} = \frac{2}{1 - \bar{x}^2 - \bar{y}^2} \left[ \frac{\pi}{2} + \arctan \frac{\bar{y} \cos \beta_b - \bar{x} \sin \beta_b}{\left( 1 - \bar{x}^2 - \bar{y}^2 \right)^{1/2}} \right] \]  

(6)

\[ V = \frac{2 + (\bar{y} \cos \beta_b - \bar{x} \sin \beta_b)G}{1 - \bar{x}^2 - \bar{y}^2}, S = \frac{\bar{x} \cos \beta_b + \bar{y} \sin \beta_b}{1 - (\bar{x} \cos \beta_b + \bar{y} \sin \beta_b)^2}, \sigma = \mu \omega RL \left( \frac{R}{C} \right)^2 \left( \frac{L}{2R} \right)^2 \]

And \( \beta_b \) is a correction for half-Sommerfeld boundary condition[9]. The positive pressure interval is \([\beta_b, \beta_b + \pi]\) along the circumferential direction.

\[ \beta_b = \arctan \frac{\bar{y} + 2\hat{y}}{\bar{x} - 2\hat{x}} - \pi \frac{\text{sign} \left( \frac{\bar{y} + 2\hat{y}}{\bar{x} - 2\hat{y}} \right)}{2} \frac{\pi}{2} \frac{\text{sign} \left( \frac{\bar{y} + 2\hat{y}}{\bar{x} - 2\hat{y}} \right)}{2} \]

(7)

\[ \text{Figure 2.} \text{ Cylindrical journal bearing: (a) Depict of the oil force model; (b) Mirror symmetry when considering an opposite rotation direction of the shaft journal.} \]

Note that oil force expressions in Eqs. (5-7) are only valid when rotation direction is anticlockwise. A transformation is necessary when the rotation is clockwise. Such a transformation is available according to the symmetry, as shown in figure 2(b). A symmetrical whirling velocity \((\hat{x}',\hat{y}')\) about the
eccentricity line $O_iO_j$, together with an opposite rotation direction, leads to a symmetrical oil film force $\{f_{b,x}, f_{b,y}\}^T$. Thus, the derivation of the oil film force with clockwise rotation including the following steps:

a) Calculate the symmetrical velocity to transform the problem to a traditional clockwise rotation form

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = [T] \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}, \quad [T] = \begin{bmatrix} x^2 - y^2 & 2\sqrt{x^2 + y^2} \\ x^2 + y^2 & (x + y)^2 \\ (x + y)^2 & -2x^2 \\ x^2 + y^2 & x^2 + y^2 \end{bmatrix} \quad (8)$$

b) Calculate the oil film force $\{f_{b,x}, f_{b,y}\}^T$, according to Eq. (5).

c) Make the symmetry of force $\{f_{b,x}, f_{b,y}\}^T = [T] \{f'_{b,x}, f'_{b,y}\}^T$.

Summarizing the derivation above, apply it in gear-pair systems

$$\begin{bmatrix} f'_{b,x} \\ f'_{b,y} \end{bmatrix} = \mathbf{f}_b(\mathbf{x}, \mathbf{y}, \mathbf{x'}, \mathbf{y'}) \begin{bmatrix} f_{b,x} \\ f_{b,y} \end{bmatrix} = [T] \mathbf{f}_b(\mathbf{x}, \mathbf{y}, \mathbf{x'}, \mathbf{y'}) \quad (9)$$

where, $\mathbf{x}, \mathbf{y} = \{x_i/C_i, y_i/C_i\}$, $\mathbf{x'}, \mathbf{y'} = \{x_j/C_j, y_j/C_j\}$. Define $i = g, j = p$ when the rotation of the gear is anti-clockwise, otherwise $i = p, j = g$, while the rotation frequency $\omega$ remains positive.

2.3. System equations

Considering unbalance excitations, the dynamics equations for the system illustrated in figure 3(a) can be written as

$$\begin{align*}
m_x \ddot{x} + k_x h_x \delta_x \sin \alpha + k_m h_b (\delta_b - b_0) \sin \alpha &= f_{u,x}^g + f_{b,x}^g \\
m_y \ddot{y} + k_m h_a \delta_a \cos \alpha - k_m h_b (\delta_b - b_0) \cos \alpha &= m_g \dot{\omega} + f_{u,y}^g + f_{b,y}^g \\
f_x \ddot{\theta}_x + \zeta_x \dot{\theta}_x + \kappa_x \theta_x + k_m r_h \delta_a - r k_m h_b (\delta_b - b_0) &= f_{u,\theta}^g \\
m_p \ddot{x}_p - k_m h_b \delta_b \sin \alpha - k_m h_b (\delta_b - b_0) \sin \alpha &= f_{u,x}^p + f_{b,x}^p \\
m_p \ddot{y}_p - k_m h_a \delta_a \cos \alpha + k_m h_b (\delta_b - b_0) \cos \alpha &= m_p \dot{\omega} + f_{u,y}^p + f_{b,y}^p \\
f_p \ddot{\theta}_p + \zeta_p \dot{\theta}_p + \kappa_p \theta_p + k_m r_h \delta_a - r k_m h_b (\delta_b - b_0) &= f_{u,\theta}^p \quad (10)
\end{align*}$$

Figure 3. Investigated geared rotor system and single rotor systems.
where \( m_i \) and \( \kappa_i, \zeta_i \) are the mass, torsional stiffness and damping coefficients of the gear and the pinion, \( g = -9.8 \text{m/s}^2 \) is the gravity acceleration, \( f_b^i \) and \( f_{torq}^i \) are the bearing force and input torque. \( f_u^i \) is the unbalance excitation force written as
\[
\begin{align*}
  f_{u,x}^i &= m_i \varepsilon_i \omega_i^2 \cos \omega t, \\
  f_{u,y}^i &= m_i \varepsilon_i \omega_i^2 \sin \omega t \\
\end{align*}
\]
(\( \varepsilon_i \) is the eccentricity of the mass center).

For single rotor journal bearing system shown in figure 3(b,c), there are only three equations remain in Eq.(10), and items by mesh force are not included.

3. Numerical Simulation and Results

Oil whirl phenomenon exists in geared rotor system with journal bearings. To analyze the interaction between the nonlinear journal bearing forces and mesh forces, comparisons are made between responses of the single rotor journal bearing systems and geared rotor system. The parameters of pinion and gear are listed in table 1, where the pinion is the input wheel. All the bearings are the same, with parameters listed in table 2. Newmark time-step integration algorithm is adopted to solve the dynamic system governed by Eq.(10). For the nonlinearity of the oil film force, iterations are involved in every time step. Oil film forces are corrected according to the displacements and velocities gotten in the latest iteration step.

### Table 1. Gear system parameters.

| Description         | Pinion | Gear  |
|---------------------|--------|-------|
| Mass \( m_i \) kg  | 8.50   | 20.00 |
| Moment of inertial \( J_d \) kgm\(^2\) | 0.0125 | 0.1450 |
| Number of teeth     | 35     | 65    |
| Module              | 4      | 4     |
| Radius of base circle \( r^i_b \) m | 0.065  | 0.122 |
| Pressure angle \( \alpha \) deg | 20     | 20    |
| Torsional stiffness \( \kappa_i \) Nm/rad | -      | \( 1 \times 10^4 \) |
| Torsional damping \( \zeta_i \) Nms/rad | -      | \( 1 \times 10^2 \) |
| Input Torque \( f_{torq}^i \) Nm  | 26Nm   | -     |
| Mesh stiffness \( k_m \) N/m | \( 1 \times 10^8 \) |

### Table 2. Journal bearing parameters.

| Description         | Value  |
|---------------------|--------|
| length \( L \) m    | 0.025  |
| Radius of the shaft journal \( R \) m | 0.019  |
| Oil viscosity coefficient \( \mu \) Pas | 0.02   |
| Clearance ratio \( C/R \) | 3\%    |

3.1. Oil whirl in single rotor system

Without meshing pair, the pinion and the gear are treated as single rotors, as shown in figure 3(b,c). The dynamic response of the pinion with eccentricity of \( \varepsilon_p = 4 \times 10^{-5} \text{kgm} \) is calculated in the speed interval of \( 500 \text{r/min} \sim 30000 \text{r/min} \). Bifurcation and vibration spectra are shown in figure 4 and figure 5. A bifurcation is found since rotational speed of \( 23600 \text{r/min} \), 0.48 times of rotation frequency becomes predominant, which is shown as a typical oil whirl phenomenon. The orbits of the pinion in figure 6 show the enlargement of the vibration after oil whirl, the system becomes the unstable, where the red circle refers to the bearing clearance circle. The dynamic response of the gear with eccentricity of \( \varepsilon_g = 8 \times 10^{-4} \text{kgm} \) in the same speed interval. Figure 7 and figure 8 give the bifurcation and the
vibration spectra. Similarly, oil whirl emerges at the speed of 10600r/min, leading to 0.48 times of rotational frequencies and bifurcation.

![Figure 4](image4.png)  
**Figure 4.** Bifurcation of the pinion as a single rotor system.

![Figure 5](image5.png)  
**Figure 5.** Vibration spectra of the pinion as a single rotor system.

![Figure 6](image6.png)  
**Figure 6.** Typical orbits of the pinion before and after instability.

![Figure 7](image7.png)  
**Figure 7.** Bifurcation of the gear as a single rotor system.

![Figure 8](image8.png)  
**Figure 8.** Vibration spectra of the gear as a single rotor system.

3.2. Oil whirl in geared rotor system

For the geared rotors system, keeping the eccentricities of the rotor unchanged, with a static input torque of 26Nm acted on the pinion, the vibration responses are calculated in the input speed interval of 500r/min~ 30000r/min.

Figure 9 indicates the bifurcation of the pinion in \( y_p \) direction and \( \theta_p \) direction, and figure 10 indicates the bifurcation of the gear. According to the two figures, the system become unstable when
the input speed is larger than 20600r/min. In $y_i$ direction, the threshold speed of instability for the pinion is 24800r/min, the threshold input speed for the gear is 20600r/min (itself rotational speed of 11092r/min). After 24800r/min, points distribution in bifurcation map of the gear becomes narrower. For torsional vibration, the two rotors both become unstable since 20600r/min.

The amplitude-frequency curves of the two rotors are plotted in figure 11. For the lateral motion, a significant amplification of the pinion vibration amplitude can be seen at the interval of 20600r/min–24800r/min, and suppression at higher speeds. Referring to the motion in $y_s$ direction in figure 10(a), this suppression can be regarded as returning to a relatively stable status. The vibration amplitude curve of the gear rises rapidly after 24800r/min. In figure 11(b), torsional vibrations curves of the two rotors both rise after 20600r/min, and fluctuations occur at 24800r/min.

![Figure 9. Bifurcation of the high-speed rotor in gear system.](image1)

![Figure 10. Bifurcation of the low-speed rotor in gear system.](image2)

![Figure 11. Amplitude-frequency cure of the gear rotor under different input speeds.](image3)
Vibration spectra of the two rotors under input speed of 25000r/min are given in figure 12, when the pinion frequency is 416Hz. About half times of the frequency (202.8Hz) emerges in the spectra of the both two rotors. Besides, the spectra of the gear include itself frequency (224Hz) and half times of the frequency (109.7Hz).

Tooth separation due to the backlash is examined in figure 13. Compared with rolling-element bearings, larger bearing clearances exist in the journal bearing, the interaction between bearing clearances and backlash becomes significant under large-amplitude vibration conditions, even leading to a tooth separation. Figure 13 shows the statistic on the tooth separation status, it can be seen that that instability due to oil whirl can make great influence on the teeth contact status. It is notable that the threshold speed for tooth separation is 24800r/min, which coincides with the threshold speed for instability. It is illustrated that tooth separation is directly caused by the instability of the pinion.

![Figure 12](image-url)  
**Figure 12.** Response spectra of gears at input speed of 25 000/min.

3.3. Discussion

Comparisons are made between the results of single rotor system shown in figure 3(b,c) and geared rotor system in figure 3(a). The unstable speed intervals are listed in table 3. Because of the teeth number difference, the whole speed scope of the gear is 269r/min–13354r/min, corresponding to the pinion input speed scope of 500r/min–30000r/min.

![Figure 13](image-url)  
**Figure 13.** Tooth separation under different input speeds.
In table 3, for the same rotor in different systems, the threshold speeds of the lateral motion instability are closed, showing that the instability in the investigated geared system is mainly dominated by the individual rotor. However, the differences in torsional directions show the coupling between the two rotors. It can be deduced the following two factors contributes to the differences of the instable speed intervals between single rotors and geared rotors.

a) The bearing operation condition is altered, because static component of mesh force, which is due to the input torque load, contributes to the bearing loads. This changes the stability of the oil film, making different threshold speeds.

b) The dynamic component of the mesh force transfers excitations caused by the imbalance and oil whirl of the adjacent rotor. The coupling can be seen obviously in the $\theta_g$ and $\theta_s$ direction in figure 9(b) and figure 10(b). The suppression of the gear motion shown in figure 11(a) is induced to be caused by this coupling. It should be clarified that the instability in the system is not transferred from the adjacent rotor but caused by the individual unstable rotor itself according to the spectra in figure 11.

4. Conclusions
This paper developed a dynamic model of gear-pair system, involving nonlinear oil film force of hydrodynamic bearings and the gear mesh force with backlash. Based on this model, numerical simulations are made about instability led by the oil whirl in geared rotor journal bearing system. By comparing responses between single rotor journal bearing system and geared rotor system, the evolution of oil whirl from individual rotor to adjacent geared rotors is examined. The results show that dynamic mesh force can transfer the excitations caused by imbalance and oil whirl between the adjacent rotors, making different affecting on the stability of different motion directions. The stabilities of the geared rotors are not independent with each other, but coupled by the mesh force. The instability can be accompanied with tooth separation.

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