Leptophilic Dark Matter at Linear Colliders

P. S. Bhupal Dev

Department of Physics and McDonnell Center for the Space Sciences, Washington University,
St. Louis, MO 63130, USA
E-mail: bdev@wustl.edu

We discuss model-independent collider constraints on the effective couplings of leptophilic dark matter (LDM), considering its production at a future electron-positron linear collider, with both polarized and unpolarized beam options, in the mono-photon and mono-\(Z\) channels. We show that the future collider constraints are largely complementary to the direct and indirect detection limits on LDM, and can potentially provide the best-ever LDM sensitivity in the low-mass regime (below \(\sim 10\) GeV).

Keywords: Dark Matter, Effective Field Theory, Lepton Collider

1. Introduction

Many of the existing experimental constraints on dark matter (DM) crucially rely on the DM interactions with nucleons, and therefore, can be largely weakened if the DM predominantly interacts with the Standard Model (SM) leptons, but not quarks at tree-level. Such leptophilic DM (LDM) could arise naturally in many beyond the Standard Model (BSM) scenarios\(^{1,2}\), some of which could even explain various experimental anomalies, such as the muon anomalous magnetic moment\(^23\), DAMA/LIBRA annual modulation\(^24\), anomalous cosmic ray positron excess\(^25,26\), the galactic center gamma-ray excess\(^27,28\), and XENON1T electron excess\(^30\). Dedicated searches for LDM in direct detection\(^31,32\) and beam dump\(^33,34\) experiments have also been discussed.

In this proceeding based on Ref.\(^36\) we focus on the LDM searches at lepton colliders, which are complementary to the direct and indirect detection searches. We adopt an effective field theory (EFT) approach, which has been widely used in the context of collider searches for DM following the early works of Refs.\(^37,38\). The same interactions responsible for DM pair-annihilation in the early universe leading to their thermal freeze-out guarantee their direct production at colliders, as long as kinematically allowed. This will give a characteristic mono-\(X\) signature, where the large missing transverse momentum carried away by the DM pair is balanced by a visible sector particle \(X\) (which can be either a photon, jet, \(W\), \(Z\), or Higgs, depending on the model) emitted from an initial, intermediate or final state (see Refs.\(^46,47\) for reviews). Specifically, the mono-jet signature has become emblematic for LHC DM searches\(^48,49\). However, for an LDM with loop-suppressed interactions to the SM quarks, the hadron colliders like the LHC are not expected to provide a better limit than the existing constraints from indirect searches, such as from AMS-02\(^52,53\), at least within the EFT framework with contact interactions.
On the other hand, lepton colliders provide an ideal testing ground for the direct production of LDM and its subsequent detection via either mono-photon \cite{42,45,54-63} or mono-\(Z\) \cite{64-67} signatures. We go beyond the existing literature and perform a comprehensive and comparative study of both mono-photon and mono-\(Z\) signatures of LDM at future \(e^+e^-\) colliders in a model-independent, EFT approach \cite{36}. Our analysis is generically applicable to all future \(e^+e^-\) colliders, such as the ILC \cite{68}, CLIC \cite{69}, CEPC \cite{70} and FCC-ee \cite{71}, but for concreteness, we have taken the \(\sqrt{s} = 1\) TeV ILC as our case study for numerical simulations. We also assume the DM to be fermionic and limit ourselves to the dimension-6 operators, but taking into consideration all possible dimension-6 operators of scalar-pseudoscalar (S-P), vector-axialvector (V-A) and tensor-axialtensor (T-AT) type as applicable for the most general DM-electron coupling. Within the minimal EFT approach, the only relevant degrees of freedom in our analysis are the DM mass and an effective cut-off scale \(\Lambda\) which determines the strength of the four-Fermi operators. This enables us to derive model-independent ILC sensitivities on LDM in the \((m_\chi, \Lambda)\) plane in both mono-photon and mono-\(Z\) (leptonic and hadronic) channels, after taking into account all relevant backgrounds and systematic uncertainties. We consider both unpolarized and polarized beam options \cite{68,72}, and find that with the proper choice of polarizations for the \(e^-\) and \(e^+\) beams (which depends on the operator type), the DM sensitivities could be significantly enhanced.

2. Effective operators

Our primary assumptions are (i) the DM particle \(\chi\) couples directly only to the SM leptons but not to the quarks (hence leptophilic), and (ii) the energy scale of the associated new physics is large compared to the collider energies under consideration, thus allowing us to integrate out the heavy mediators and parametrize the DM-SM interactions using effective higher-dimensional operators. For concreteness, we assume that the DM particles are Dirac fermions, and therefore, the leading-order DM-SM interactions are the dimension-six four-Fermi interactions, with the most-general effective Lagrangian given by \(\chi\) \cite{37,23}:

\[
\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2} \sum_j \left( \bar{\chi} \Gamma_j^\ell \chi \right) \left( \bar{\ell} \Gamma_j^\ell \ell \right),
\]

where \(\Lambda\) is the cut-off scale for the EFT description and the index \(j\) corresponds to different Lorentz structures, as shown below. Since our main focus is on \(e^+e^-\) colliders, we will just set \(\ell = e\) in Eq. 1 and assume this to be the only leading-order coupling, but our discussion below could be easily extended to other cases, e.g. future muon colliders \cite{73} by setting \(\ell = \mu\).

A complete set of Lorentz-invariant operators consists of scalar (S), pseudoscalar (P), vector (V), axial-vector (A), tensor (T) and axial-tensor (AT) currents.
We classify them as follows:

- **S-P type** : \( \Gamma_\chi = c_\chi^S + ic_\chi^P \gamma_5 \), \( \Gamma_e = c_e^S + ic_e^P \gamma_5 \).
- **V-A type** : \( \Gamma_\mu = (c_\chi^V + c_\chi^A) \gamma^\mu \), \( \Gamma_{e\mu} = (c_e^V + c_e^A) \gamma_5 \gamma^\mu \).
- **T-AT type** : \( \Gamma_{\mu\nu} = (c_\chi^T + ic_\chi^{AT}) \gamma_5 \sigma_{\mu\nu} \), \( \Gamma_{e\mu\nu} = \sigma_{\mu\nu} \).

where \( \sigma_{\mu\nu} = i/2[\gamma_\mu, \gamma_\nu] \) is the spin tensor and \( c^\chi,e \) are dimensionless, real couplings.

For simplicity, in Eq. (1) we have used a common cut-off scale \( \Lambda \) for all Lorentz structures. Furthermore, in our subsequent numerical analysis, we will consider one type of operator at a time, by setting the corresponding couplings \( c^\chi,e = 1 \) without loss of generality and all other couplings equal to zero, unless otherwise specified. For instance, setting \( c_\chi^S = c_\chi^P = c_e^S = c_e^P = 1 \) and all other couplings equal to zero gives us the (S+P)-type operator, which we will simply refer to as the **SP**-type in the following discussion. Similarly, we will denote the case \( c_\chi^V = c_\chi^A = c_e^V = c_e^A = 1 \) simply as the **VA**-type, and \( c_\chi^T = c_\chi^{AT} = 1 \) as the **TAT**-type for presenting our numerical results in the \( (m_\chi, \Lambda) \) plane. For other choices of the couplings, our results for the sensitivity on \( \Lambda \) can be easily scaled accordingly.

We will impose a theoretical limit of \( \Lambda > \sqrt{s} \) for the EFT validity. For relatively larger DM mass, we must also have \( \Lambda > 2m_\chi \) in order to describe DM pair annihilation by the EFT. In fact, using \( \Lambda = 2m_\chi \) induces 100% error in the EFT prediction for s-channel UV completions. Therefore, we will use \( \Lambda > \max\{\sqrt{s}, 3m_\chi\} \) as a conservative lower bound to ensure the validity of our EFT approach.

### 3. Mono-photon channel

For the mono-photon signal \( e^+e^- \rightarrow \chi\chi\gamma \), the \( \chi \)'s will contribute to the missing transverse energy at the detector. The dominant irreducible SM background to this process comes from neutrino pair production with an associated ISR photon, i.e. \( e^+e^- \rightarrow \nu\nu\gamma \). Since neutrinos are practically indistinguishable from DMs on an event-by-event basis, the majority of this background survives the event selection cuts. However, as we will show later, this background is highly polarization-dependent, and therefore, can be significantly reduced by the proper choice of polarized beams, without affecting the signal much.

Apart from the neutrino background, any SM process with a single photon in the final state can contribute to the total background if all other visible particles escape detected. The SM processes containing either jets or charged particles are relatively easy to distinguish from a DM event, so their contribution to the total background is negligible. The only exception is the **Bhabha scattering** process associated with an extra photon (either from initial or final state radiation), i.e., \( e^+e^- \rightarrow e^+e^-\gamma \), which has a large cross section, is polarization-independent, and can significantly contribute to the total background whenever the final-state electrons and positrons go undetected, e.g. along beam pipes. In our following analysis, we consider both neutrino and radiative Bhabha backgrounds.
Fig. 1. Variation of mono-photon signal cross-section with the DM mass (left) and the cut-off scale (right) at $\sqrt{s} = 1$ TeV ILC. The solid, dashed and dotted lines are for the SP, VA and TAT-type operators respectively. In the left panel, the red, green and blue curves respectively correspond to different values of the cut-off scale $\Lambda = 1$ TeV, 3 TeV and 5 TeV, while in the right panel, they correspond to different values of the DM mass $m_\chi = 100$ GeV, 250 GeV and 450 GeV.

3.1. Cross-sections

The cross-sections for the mono-photon signal $e^+e^- \rightarrow \chi \chi \gamma$ and the radiative neutrino background $e^+e^- \rightarrow \nu \bar{\nu} \gamma$ at $\sqrt{s} = 1$ TeV ILC are estimated using CalcHEP with proper implementation of ISR and beamsstrahlung effects, which significantly affect the width and position of the neutrino $Z$-resonance. For this purpose, the EFT Lagrangian (1) is implemented in FeynRules to generate the CHO library required for CalcHEP. To avoid collinear and infrared divergences, we limit the phase space in the event generation with the following cuts on the outgoing photon energy $E_\gamma$ and its polar angle $\theta_\gamma$:

$$8 \text{ GeV} < E_\gamma < 500 \text{ GeV}, \quad |\cos \theta_\gamma| \leq 0.995. \quad (3)$$

The radiative Bhabha scattering events are generated using WHIZARD (to better handle the singularities) with the same set of cuts as in Eq. (3) to the matrix element photon (i.e., excluding the ISR and beamsstrahlung photons). Also, some additional cuts are implemented for the Bhabha process to take care of the soft and collinear divergences:

$$M_{e^+e^-} < 2m_e, \quad M_{\text{cut},e^+e^-} < 5 \text{ GeV}, \quad P_T^\gamma > 1 \text{ GeV}, \quad \Delta R_{e^+\gamma} > 0.2, \quad \Delta R_{e^-,\gamma} > 0.4. \quad (4)$$

After generating the signal and background events, we perform a fast detector simulation of the SiD detector of ILC using Delphes3 with the configuration card validated in Ref. The variations of the unpolarized signal cross section as a function of the DM mass and the cut-off scale are shown in Fig. left and right panels respectively for all three operator types, namely, SP (solid), VA (dashed) and TAT (dotted)-type. We find that the cross-section is the smallest (largest) for the SP (TAT)-type operator at any given DM mass. In the left panel, the sudden drop in the cross-section as $m_\chi$ approaches $\sqrt{s}/2$ is due to phase-space suppression. Otherwise, for smaller DM masses, the cross-section for a given operator type and
Table 1. Mono-photon background and signal cross-sections with different beam polarizations at $\sqrt{s} = 1$ TeV. For the signal, we have fixed $m_\chi = 100$ GeV and $\Lambda = 3$ TeV. The numbers in bold highlight the optimal polarization choice for a given operator type.

| Process type | Unpolarized cross-section (fb) | Polarization $P(e^-, e^+)$ | Polarized cross-section (fb) |
|--------------|-------------------------------|-------------------------------|-----------------------------|
| $\nu\nu\gamma$ | 4782 | (80, 0), (80, 20), (80, 30) | 1116, 1268, 1393, 67920, 67909, 67809 |
| $e^- e^+\gamma$ | 68439 | (80, 0), (80, 20), (80, 30) | 67920, 67909, 67809, 66439 |
| SP-type | 25.5 | (80, 0), (80, 20), (80, 30) | 25.5, 29.6, 31.6 |
| VA-type | 34.3 | (80, 0), (80, 20), (80, 30) | 61.7, 49.4, 43.2 |
| TAT-type | 36.5 | (80, 0), (80, 20), (80, 30) | 36.5, 42.3, 45.2 |

As for the background, we find that the neutrino background cross section at $\sqrt{s} = 1$ TeV is 4.8 pb, while the radiative Bhabha background is 68.4 pb (though it is substantially reduced after the baseline selection). On the other hand, the DM signal cross section is found to be much smaller, as shown in Table 1 for a benchmark DM mass of $m_\chi = 100$ GeV and the cut-off scale $\Lambda = 3$ TeV.

3.2. Effect of polarization

One important advantage of lepton colliders is that the incoming beams can be polarized. This helps to reduce the neutrino background considerably, as shown in Table 1. To utilize the full advantage of the beam polarization, we investigate the effect of different choices of polarization on the signal and background. At the ILC, the baseline design foresees at least 80% electron beam polarization at the interaction point, whereas the positron beam can be polarized up to 30% for the undulator positron source (up to 60% may be possible with the addition of a photon collimator). For comparison, we show our results for three different nominal absolute values of polarization: $|P(e^-, e^+)| = (80, 0), (80, 20)$ and (80, 30). In each case, we can also have four different polarization configurations, namely, $\text{sign}(P(e^-), P(e^+)) = (+, +), (+, -), (-, +)$ and $(-, -)$, where $+$ and $-$ denote the right- and left-handed helicities respectively.

In Table 1, we show the effect of different schemes of polarizations and helicity orientations on the mono-photon signal and background cross-sections. It is clear that the radiative Bhabha background remains almost unchanged. On the other hand, electron beam polarization is very effective in reducing the neutrino back-
ground, as a 80% right-handed electron beam can reduce the neutrino background to 23% of the unpolarized case, even without any polarization on the positron beam. The effect is further enhanced by a left-handed positron beam. We see that for 20% and 30% left-handed positron beam polarization, the neutrino background is reduced to 20% and 18% of its unpolarized value, respectively.

The signals are also affected to some extent by beam polarization and the optimal helicity configuration depends on the operator type. For SP- and TAT-type operators we see no effect of electron-beam polarization, but a 20% (30%) right-handed positron beam can enhance the signal by 16% (24%). The VA-type signal, on the other hand, prefers the (+, −) helicity configuration – the same choice for which the neutrino background is minimized. With the (+80%, −30%) configuration, the VA-type signal is enhanced by a factor of 2.3, whereas the (+80%, +30%) configuration enhances it by a modest 26%.

Overall, although the (+80%, −30%) configuration minimizes the background the most, looking at the different signal to background ratio, we find that the (+80%, +30%) configuration is the best for the SP- and TAT-type operators. For direct comparison between the results for different operators, we choose to work with the (+80%, +30%) configuration democratically for all the operator types, unless otherwise specified.

3.3. Cut-based analysis

Now we analyze various kinematic distributions and perform a cut-based analysis to optimize the signal-to-background ratio. This of course depends on the DM mass, so in Table 2 we list three benchmark points (BPs) with \( m_\chi = 100 \text{ GeV}, \) 250 GeV and 350 GeV respectively, and present the corresponding selection cuts optimized for each case. Here we fix \( \Lambda = 3 \text{ TeV} \) for illustration, but in the next subsection, we will vary both \( m_\chi \) and \( \Lambda \) to obtain the 3σ sensitivity limits. As for the choice of the DM mass values, since it was seen from Figure 1 that the signal cross-sections are barely sensitive to the DM mass up to around 100 GeV, our BP1 essentially captures the light DM scenario. Similarly, our BP3 is chosen moderately close to the kinematic limit of \( \sqrt{s}/2 \) (going too close to \( \sqrt{s}/2 \) will result in cross-section values too low too low to give sizable event counts after all the selection cuts). The BP2 is chosen for an intermediate mass DM in between BP1 and BP2.

We define our mono-photon signals by those events that pass through the baseline selection criteria as defined below, in addition to the cuts given in Eq. (3):

\[
E_\gamma > 10 \text{ GeV, } |\eta_\gamma| < 2.45 \text{ and } P_T^{\text{miss}} > 10 \text{ GeV},
\]

where the hardest photon in an event is considered as the signal photon. For the radiative Bhabha background, we define the selection criteria for electrons as \( P_{T,e} > 10 \text{ GeV, } |\eta_e| < 2.5, \) and have kept only those events which contain no electrons (and positrons) passing these criteria, which means they have escaped detection. After implementing these baseline selection cuts, we find that the signal...
Table 2. Mono-photon selection cuts for different BPs across all operator types.

| BP1 | BP2 | BP3 |
|-----|-----|-----|
| **Definition** | $m_\chi = 100$ GeV, $\Lambda = 3$ TeV | $m_\chi = 250$ GeV, $\Lambda = 3$ TeV | $m_\chi = 350$ GeV, $\Lambda = 3$ TeV |
| Baseline selection | $E_\gamma > 10$ GeV, $|p_\gamma| < 2.45$, $P_T^{miss} > 10$ GeV | $E_\gamma < 450$ GeV | $E_\gamma < 250$ GeV |
| **SP-type** Cut-1 | $E_\gamma < 450$ GeV | $E_\gamma < 340$ GeV | $E_\gamma < 250$ GeV |
| Cut-2 | $|p_\gamma| < 1.6$ | $|p_\gamma| < 1.7$ | $|p_\gamma| < 1.7$ |
| Cut-3 | $P_T^{miss} < 450$ GeV | $P_T^{miss} < 340$ GeV | $P_T^{miss} < 240$ GeV |
| Cut-4 | $P_T^{miss} < 1.3$ | $P_T^{miss} < 1.2$ | $P_T^{miss} < 1.2$ |
| Cut-5 | $1.1 < \Delta R_{e,MET} < 4.5$ | $1.1 < \Delta R_{e,MET} < 4.5$ | $1.1 < \Delta R_{e,MET} < 4.5$ |
| **VA-type** Cut-1 | $E_\gamma < 440$ GeV | $E_\gamma < 350$ GeV | $E_\gamma < 250$ GeV |
| Cut-2 | $|p_\gamma| < 1.7$ | $|p_\gamma| < 1.7$ | $|p_\gamma| < 1.7$ |
| Cut-3 | $P_T^{miss} < 400$ GeV | $P_T^{miss} < 340$ GeV | $P_T^{miss} < 250$ GeV |
| Cut-4 | $P_T^{miss} < 1.2$ | $P_T^{miss} < 1.2$ | $P_T^{miss} < 1.2$ |
| Cut-5 | $1.1 < \Delta R_{e,MET} < 4.5$ | $1.1 < \Delta R_{e,MET} < 4.5$ | $1.1 < \Delta R_{e,MET} < 4.5$ |
| **TAT-type** Cut-1 | $E_\gamma < 460$ GeV | $E_\gamma < 360$ GeV | $E_\gamma < 230$ GeV |
| Cut-2 | $|p_\gamma| < 1.7$ | $|p_\gamma| < 1.7$ | $|p_\gamma| < 1.7$ |
| Cut-3 | $P_T^{miss} < 450$ GeV | $P_T^{miss} < 350$ GeV | $P_T^{miss} < 230$ GeV |
| Cut-4 | $P_T^{miss} < 1.2$ | $P_T^{miss} < 1.2$ | $P_T^{miss} < 1.2$ |
| Cut-5 | $1.1 < \Delta R_{e,MET} < 4.4$ | $1.1 < \Delta R_{e,MET} < 4.4$ | $1.1 < \Delta R_{e,MET} < 4.4$ |

and the neutrino background are reduced to about 60% of their original values in Table 1. Similarly, the actual Bhabha-induced background relevant for our signal is found to be only about 13% of its original value quoted in Table 1 after the baseline selection cuts, taking into account only the missed electron events. To further enhance our signal-to-background ratio, we then examine the signal versus background distributions of some relevant kinematic variables and devise further cuts, which are dynamic with respect to different BPs, as summarized in Table 2. See Ref. 36 for details.

Even after implementing the baseline and analysis cuts 1 through 5, the neutrino background can only be reduced to about 40% of its original value in Table 1. Similarly, the radiative Bhabha background, although substantially reduced to about 4% of its original value in Table 1, after the baseline selection and analysis cuts, still remains sizable and comparable to the neutrino background. However, an electromagnetic calorimeter in the very forward direction of the beamline (BeamCal) can further suppress the Bhabha background to the per mille level. To properly incorporate the effect of BeamCal, we have used the selection efficiencies obtained from a full detector simulation performed in Ref. 61 by modeling the complete instrumented region in a realistic way. According to this analysis, the selection efficiency of the Bhabha background after the BeamCal veto only is 2.7%, while that of the neutrino background is between 98% and 99.6%. As for the DM signal, we expect it to be basically unaffected (just like the neutrino background) by the BeamCal veto, as it does not contain highly energetic charged particles in the longitudinal direction.
Table 3. Signal significance in the mono-photon channel for the three BPs at $\sqrt{s} = 1$ TeV. The values in parenthesis correspond to 1% background systematic uncertainty.

| Operator type | Signal significance for $L_{\text{int}} = 1000\text{fb}^{-1}$ | Unpolarized beams | Polarized beams |
|---------------|-------------------------------------------------------------|-------------------|-----------------|
|               | BP-1 BP-2 BP-3 | BP-1 BP-2 BP-3 |
| SP-type       | 8.1 (0.6) 5.8 (0.4) 3.5 (0.3) | 18.1 (2.4) 13.0 (1.7) 7.8 (1.0) |
| VA-type       | 10.9 (0.8) 8.5 (0.6) 5.6 (0.4) | 24.9 (3.2) 19.4 (2.5) 12.9 (1.7) |
| TAT-type      | 11.8 (0.8) 10.8 (0.8) 8.5 (0.6) | 26.2 (3.5) 24.1 (3.2) 19.2 (2.6) |

For the polarized case, after the baseline selection cuts, the Bhabha background remains almost same as in the unpolarized case. The neutrino background, on the other hand, is significantly reduced in the polarized case to about 28% of its unpolarized value. The other cut efficiencies are also slightly better for the neutrino background in the polarized case.

As for the signals, from Table 1 we see that the TAT-type operator has the largest cross section to start with, both for the unpolarized as well as for the (+80%, +30%) polarized cases. Even after the baseline selection and the specialized cuts discussed above, the TAT-type signal retains the largest efficiency among the three types. This will be reflected in our signal significance results below.

### 3.4. Signal significance

After implementing all the cuts mentioned above, we calculate the final signal significance for our benchmark scenarios using the definition

$$\text{Sig} = \frac{S}{\sqrt{S+B+(\epsilon B)^2}},$$

where $S$ and $B$ are the number of signal and total background events respectively for a given integrated luminosity, and $\epsilon$ is the background systematic uncertainty. Our results are given in Table 3 for the three BPs. We show the numbers for an ideal case with zero systematics and also for a more realistic case with 1% systematics, i.e. with $\epsilon = 0.01$ (in parentheses). The results are significantly weakened in the latter case because of the relatively large background compared to the signal.

From Table 3 we see that the significance enhances as we go to lower DM mass regions, as expected because of kinematic reasons. Operator-wise we see that TAT and VA-type operators perform better than the SP-type. We also find substantial (around 50%) increase in significance on application of optimal beam polarization.

Going beyond the three BPs, we now vary the DM mass and calculate the signal significance following the same cut-based analysis procedure outlined above. Our results for the 3$\sigma$ sensitivity contours in the $(m_\chi, \Lambda)$ plane are shown in Figure 2 for all the operator types. The solid (dashed) contours are for the unpolarized (optimally polarized) case, and the blue (green) contours are assuming zero (1%) background systematics. The shaded regions are excluded by various constraints. First of all, for $\Lambda < \max\{\sqrt{s}/2, 3m_\chi\}$, our EFT framework is not valid (cf. Sec. 2).
Fig. 2. 3σ sensitivity contours in the mono-photon channel for the SP (left), VA (middle) and TAT (right)-type operators with unpolarized (solid lines) and polarized (dashed lines) $e^+e^-$ beams at $\sqrt{s} = 1$ TeV center-of-mass energy and with $L_{\text{int}} = 1000$ fb$^{-1}$ integrated luminosity. The blue (green) contours are assuming zero (1%) background systematics. The various shaded regions are excluded by direct detection (XENON1T, PANDAX-4T), indirect detection (Fermi-LAT, AMS), astrophysics (SN1987A) and cosmology (CMB) constraints. In the shaded region below $\Lambda = \max\{\sqrt{s}/2, 3m_\chi\}$, our EFT framework is not valid. Along the dot-dashed line, the observed DM relic density is reproduced for a thermal DM assuming only DM-electron effective coupling. This is shown by the navy blue-shaded regions in Fig. 2. For $\sqrt{s} = 1$ TeV as considered here, this EFT validity limit supersedes the previous LEP limit$^{32}$. The same effective operator given in Eq. (1) also gives rise to DM scattering with electrons $\chi e^- \rightarrow \chi e^-$. The exact analytic expressions for these cross sections in our EFT framework can be found in Appendix C of Ref.82 for all the operator types. Comparing these with the experimental upper limits on $\sigma_{\chi e}$ from dedicated direct detection experiments$^{31,32}$, we can derive a lower limit on the cut-off scale $\Lambda$ as a function of the DM mass $m_\chi$. However, the current best limit on $\sigma_{\chi e}$ from XENON1T is at the level of $\mathcal{O}(10^{-39})$ cm$^2$ which translates into a very weak bound on $\Lambda$ and is not relevant for our study. Even the future ambitious proposals like DARKSPHERE can only reach up to $\mathcal{O}(10^{-42})$ cm$^2$ still 5 orders of magnitude weaker than that needed to probe a TeV-scale $\Lambda$ value.

However, more stringent limits can be derived from DM-nucleon scattering searches. Even for an LDM as in our case, DM-nucleon couplings are necessarily induced at loop level from photon exchange between virtual leptons and the quarks. In fact, as shown in Ref.37, the loop-induced DM-nucleon scattering almost always dominates over the DM-electron scattering. The analytic expressions for the one and two-loop DM-nucleon scattering cross sections can be found in Ref.37. We have translated the experimental upper limits from XENON1T$^{84}$ and PANDAX-4T$^{85}$ onto the $(m_\chi, \Lambda)$ plane, as shown by the yellow and grey-shaded regions respectively in Fig. 2. Note that these limits are only applicable for the vector and tensor lepton currents, i.e. $\Gamma_\ell = \gamma_\mu$, $\sigma_{\mu\nu}$ in Eq. (1). For the scalar lepton current, $\Gamma_\ell = 1$, the one-loop DM-nucleon coupling vanishes, and one has to go to two loops which is suppressed by $\alpha_{\text{em}}^2$, for the S-S type coupling and $\alpha_{\text{em}}^2 v^2$ (where $v \sim 10^{-3}$ is the DM velocity) for the P-S type coupling. In contrast, for pseudo-scalar and axial-vector lepton currents, i.e. $\Gamma_\ell = \gamma_5$, $\gamma_\mu\gamma_5$, the DM-nucleon coupling vanishes to all orders. Therefore, we have not shown the XENON1T and
PANDAX-4T limits for the SP-type operator on the top left panel of Fig. 2.

The same effective operator given in Eq. (1) also enables DM annihilation into electrons $\chi \chi \rightarrow e^+ e^-$. The exact analytic expressions for these cross sections in our EFT framework can be found in Appendix C of Ref. [82] for all the operator types. Using these, we calculate the thermal-averaged cross section times relative velocity $\langle \sigma v \rangle$ which goes as $m_\chi^2 / \Lambda^4$ and compare it with the existing indirect detection upper limits on $\langle \sigma v \rangle$ in the $e^+ e^-$ channel to put a lower bound on $\Lambda$ as a function of the DM mass. This is shown in Fig. 2 by the red and brown-shaded regions respectively for the Fermi-LAT[86] and AMS-02[53] constraints on $\langle \sigma v \rangle$. Similar constraints on $\langle \sigma v \rangle$ can be derived using CMB anisotropies[86], which is shown by the cyan-shaded region in Fig. 2, assuming an s-wave annihilation (for p-wave annihilation, the CMB bound will be much weaker).

Along the dot-dashed line in Fig. 2, the observed relic density can be reproduced for a DM. In principle, the region to the left and above of this line is disfavored for a thermal DM, because in this region $\langle \sigma v \rangle$ is smaller than the observed value of $\sim (2 - 5) \times 10^{-26} \text{cm}^3\text{sec}^{-1}$ (depending on the DM mass[87]), which leads to an overabundance of DM, since $\Omega_\chi h^2 \propto 1 / \langle \sigma v \rangle$. However, this problem can be circumvented by either opening up additional leptonic annihilation channels (like $\mu^+ \mu^-$, $\tau^+ \tau^-$ and $\nu \bar{\nu}$) or even going beyond the DM paradigm and invoking e.g., the freeze-in mechanism[88]. This will not affect our main results, since the collider phenomenology discussed here only depends on the DM coupling to electrons.

Also shown in Fig. 2 is the supernova constraint, which excludes the magenta-shaded region from consideration of energy-loss and optical depth criteria from the observation of SN1987A[82]. Here we have used an average supernova core temperature of 30 MeV. Note that the supernova bound is only applicable for DM mass below $\sim 200$ MeV or so, and for a certain range of $\Lambda$ values, above which the DM particles cannot be efficiently produced in the supernova core, and below which they will no longer free-stream.

From Fig. 2, we find that in spite of a large irreducible background, the accessible range of the cut-off scale $\Lambda$ at $\sqrt{s} = 1$ TeV ILC looks quite promising in the mono-photon channel, especially for low mass DM, where the collider sensitivity is almost flat, whereas the existing direct and indirect detection constraints are much weaker. This complementarity makes the collider searches for DM very promising. With unpolarized beams, the $3\sigma$-reach for the SP-type operator can be up to 3.9 TeV, while for the VA and TAT-type operators, it can be up to 4.2 TeV. With optimally polarized beams, i.e. with $(+80\%, +30\%)$ for the SP and TAT-types and $(+80\%, -30\%)$ for the VA type, the sensitivity reaches can be extended to 4.8 TeV (SP), 6.5 TeV (VA) and 5.3 TeV (TAT), as shown in Fig. 2.

4. Mono-$Z$ channel

In addition to the mono-photon channel discussed in the previous section, another useful channel for LDM search at lepton colliders is the mono-$Z$ channel, where the
$Z$-boson is emitted from one of the initial states. Depending on the subsequent decay of the $Z$-boson to either leptonic or hadronic final states, we perform a dedicated cut-based signal and background analysis, as discussed below.

4.1. Leptonic mode

For the leptonic decay of the $Z$-boson, we examine the process $e^+e^- \rightarrow \chi \chi Z (\rightarrow \ell^- \ell^+)$.

4.1.1. Unpolarized and polarized cross-sections

For the signal and background simulation, we generated the UFO library for our EFT framework using FeynRules and then generated events for both signal and background using MadGraph 5 with the following basic baseline cuts:

$$P_T(\ell) > 10 \text{ GeV}, \quad |\eta_\ell| \leq 2.5, \quad \Delta R_{\ell\ell} \geq 0.4.$$ (7)

For the signal, the $Z$-bosons are decayed into the charged lepton pairs via the MadSpin package which is implemented in MadGraph 5, to take care of the spin-correlation effects of the lepton pairs. A fast detector simulation to these events is done using Delphes 3 with the same configuration card as in Sec. 3.1.

With unpolarized beams, we find that the neutrino background cross section at $\sqrt{s} = 1$ TeV is 420.5 fb, whereas the DM signal cross section is much smaller, as shown in Table 4 for a benchmark DM mass of $m_\chi = 100$ GeV and the cut-off scale $\Lambda = 3$ TeV.

The neutrino background can be reduced to 28% of its original value by making the electron beam +80% polarized, and further reduced to 21% of its original value by additionally making the positron beam −30% polarized. The (+80%, −30%) polarization configuration also enhances the VA-type signal by a factor of 2.4. However, the (+80%, +30%) configuration is better for the SP and TAT-type signals. For ease of comparison between different operator types, we choose to work with the (+80%, +30%) configuration democratically for all operator types, as well as for the background, unless otherwise specified.

4.1.2. Cut-based analysis

We define our signals by those events that pass through the baseline selection criteria as defined below: $P_T(\ell) > 20 \text{ GeV}, \quad |\eta_\ell| < 2.45$, where the $Z$-boson is reconstructed by the condition that all final state lepton-pairs are oppositely charged and of same flavor (OSSF).

Other selection criteria are dynamic with respect to different BPs. We have taken the same three BPs as in the mono-photon case to probe different regions of the parameter space, namely, BP1 essentially represents all light
Table 4. Comparison of the leptonic mono-$Z$ background and signal cross-sections for different choices of beam polarization for $m_{\chi} = 100$ GeV and $\Lambda = 3$ TeV at $\sqrt{s} = 1$ TeV ILC. The numbers in bold highlight the optimal polarization choice for a given operator type.

| Process type | Unpolarized cross-section (fb) | Polarization $P(e^-, e^+)$ | Polarized cross-section (fb) |
|--------------|--------------------------------|-----------------------------|------------------------------|
|              |                                | $P(e^-, e^+)$               | $(+,+) \quad (+,-) \quad (-,+)$ |
| $\nu\tau^-\ell^+$ | 420                            | $\psi = (80, 0)$             | 116 116 723 723               |
|               |                                | $\psi = (80, 20)$            | 135 98 856 590                |
|               |                                | $\psi = (80, 30)$            | 145 88 926 524                |
| SP-Type       | 0.28                           | $\psi = (80, 0)$             | 0.26 0.26 0.25 0.25           |
|               |                                | $\psi = (80, 20)$            | 0.29 0.22 0.21 0.29           |
|               |                                | $\psi = (80, 30)$            | 0.32 0.19 0.19 0.32           |
| VA-Type       | 0.08                           | $\psi = (80, 0)$             | 0.15 0.15 0.02 0.02           |
|               |                                | $\psi = (80, 20)$            | 0.12 0.18 0.01 0.02           |
|               |                                | $\psi = (80, 30)$            | 0.11 0.19 0.01 0.02           |
| TAT-Type      | 0.68                           | $\psi = (80, 0)$             | 0.62 0.62 0.62 0.62           |
|               |                                | $\psi = (80, 20)$            | 0.72 0.52 0.52 0.72           |
|               |                                | $\psi = (80, 30)$            | 0.77 0.47 0.47 0.77           |

DM region, BP3 represents the region close to the kinematic limit of $\sqrt{s}/2 - m_Z$, whereas BP2 captures the intermediate DM mass region.

After implementing the baseline selection cuts, we find that the background is reduced to about 40% of its original value in Table 4 for the unpolarized (polarized) case, whereas the signals are reduced to about 60%-70% of their original values. We then consider various kinematic distributions for the signal and background, and devise some specialized selection cuts\(^\text{36}\). We find that after applying all these cuts, we can still retain about 35%-45% of the signal, whereas the background is reduced to below percent level of the original values given in Table 4.

4.1.3. Results

After implementing all these cuts, we calculate the final signal significance for the three BPs using Eq. (6). Our results are given in Table 5 for an integrated luminosity of $L_{\text{int}} = 1000$ fb\(^{-1}\). We see that as we go higher up in the DM mass the signal significance drops. We also find that the best-performing operator type is the TAT-type, for which more than 97% of the background events are removed after all the selection cuts. For the signal we retain 58% – 61% of the events, although for BP3 only 48% remains. The SP-type operator also gives good results, where we retain 50% – 66% of the signal across BPs and polarization choices, while removing more than 96% of the background events. Even for VA-type we retain more than 50% of the signal events and are able to cut down the background event yields to 11%. We also notice the positive effect of the beam polarization by which we achieve an enhancement of signal significance by more than 2 times compared to the ones with unpolarized beams. For VA-type though the significance can be further increased for the polarized beam case by choosing the left-handed positron beams as is evident from Table 4.

Going beyond the three BPs, we now vary the DM mass and present the $3\sigma$
Table 5. Signal significance in the mono-$Z$ leptonic channel at $\sqrt{s} = 1$ TeV and $L_{\text{int}} = 1000$ fb$^{-1}$. The values in the parenthesis correspond to 1% background systematic uncertainty.

| Operator Type | Signal significance for $L_{\text{int}} = 1000$ fb$^{-1}$ |
|---------------|--------------------------------------------------------|
|               | Unpolarized beams | Polarized beams |
|               | BP-1 | BP-2 | BP-3 | BP-1 | BP-2 | BP-3 |
| SP-type       | 1.7 (1.3) | 0.7 (0.5) | 0.1 (0.1) | 3.8 (3.6) | 1.7 (1.5) | 0.3 (0.3) |
| VA-type       | 0.2 (0.1) | 0.1 (0.1) | 0.1 (0.1) | 0.5 (0.4) | 0.4 (0.3) | 0.2 (0.2) |
| TAT-type      | 4.5 (3.9) | 2.4 (1.9) | 0.6 (0.5) | 9.4 (9.1) | 5.4 (5.1) | 1.3 (1.3) |

Fig. 3. $3\sigma$ sensitivity contours in the mono-$Z$ leptonic channel. Labels are same as in Fig. 2 for all the operators. The labels and shaded regions are the same as in the mono-photon case (cf. Fig. 2). We see that the accessible range of the cut-off scale $\Lambda$ for the unpolarized beams can reach up to 3.2 TeV for the TAT-type operator, whereas for the SP and VA-type, it can reach up to 2.6 TeV and 1.6 TeV respectively. But with the application of optimally polarized beams as discussed earlier, we see an increase by about 25% of the $3\sigma$ reach on the $\Lambda$ scale, up to 3.2 TeV, 2.5 TeV and 4 TeV for the for SP, VA and TAT-type operators, respectively.

4.2. Hadronic mode

Next we study $e^+e^- \rightarrow \chi\chi Z(\rightarrow jj)$, where $j \equiv u, d, c, s, b$ quarks. The relevant SM background processes for this channel are $e^+e^- \rightarrow \nu\bar{\nu}jj$ and $e^+e^- \rightarrow jj\ell\nu$ (with one charged lepton escaping the detector) where the jets and leptons in the final state can come from any possible source (not necessarily from an on-shell $Z$).

4.2.1. Unpolarized and polarized cross-sections

We use the same UFO library as before which is implemented using FeynRules and simulate the events for the signal and backgrounds via MadGraph with the following basic cuts to the parameter space:

$$P_T(j, \ell) > 10 \text{ GeV}, \quad |\eta_j| \leq 3.0, \quad |\eta_\ell| \leq 2.5, \quad \Delta R_{jj,\ell\ell} \geq 0.4. \quad (8)$$

For the signals, as in the leptonic case, the on-shell $Z$-bosons are decayed into the pairs of jets using the MadSpin package, implemented in MadGraph 5. Both
the signal and background samples are hadronized using Pythia 8.2 and then the final state jets are reconstructed with with anti-$k_T$ clustering algorithm with a minimum $P_T$ of 10 GeV and a cone radius ($R$) of 0.4 using the FastJet package. The fast detector simulation to these events are done using Delphes 3 with the same configuration card as discussed in Sec. 3.1.

With unpolarized beams, we find the neutrino-pair background is 798 fb, whereas the $jj\ell\nu$ background is 1186 fb. On the other hand, the DM signal is only at a few fb level, as shown in Table 6 for a benchmark DM mass of $m_\chi = 100$ GeV and the cut-off scale $\Lambda = 3$ TeV. We then examine different choices of beam-polarization on both the event samples for this channel, as shown in Table 6. We find that both backgrounds are polarization-dependent and fall off significantly for right-handed electron beam and with increasing degree of polarization. We choose the polarization configuration $P(e^-, e^+) = (80\%, +30\%)$ democratically over all the operator types.

4.2.2. Cut-based analysis

After obtaining the signal and background cross-sections as reported in Table 6, we proceed with our cut-based analysis to optimize the signal significance. We select the events that contain at least two jets with the following transverse momentum and pseudorapidity requirements: $P_{T,j} > 20$ GeV, $|\eta_j| < 2.45$. The hardest two jets are required to reconstruct the $Z$-boson. Further selection cuts are applied some of which depend on the DM mass. So, as in the leptonic channel, we have taken the same three BPs with varying DM mass and impose dynamic cuts.
Table 7. Signal significances of the mono-$Z$ hadronic channel at $\sqrt{s} = 1$ TeV and $L_{\text{int}} = 1000$ fb$^{-1}$. The values in parenthesis correspond to 1% background systematic uncertainty.

| Operator types | Signal significance for $L_{\text{int}} = 1000$ fb$^{-1}$ |
|----------------|----------------------------------------------------------|
|                | Unpolarized beams | Polarized Beam |
|                | BP-1   | BP-2 | BP-3 | BP-1 | BP-2 | BP-3 |
| SP-type        | 4.7 (3.3) | 1.5 (1.0) | 0.3 (0.2) | 10.3 (9.1) | 3.6 (3.1) | 0.8 (0.6) |
| VA-type        | 0.4 (0.2) | 0.3 (0.1) | 0.1 (0.1) | 0.2 (0.1) | 0.1 (0.1) | 0.1 (0.04) |
| TAT-type       | 14.2 (10.4) | 5.8 (3.7) | 1.2 (0.7) | 27.7 (25.6) | 12.9 (11.1) | 2.8 (2.3) |

Fig. 4. 3σ sensitivity contours in the mono-$Z$ hadronic channel. Labels are same as in Fig. 2.

4.2.3. Results

The signal significances calculated using Eq. (6) are tabulated in Table 7. We see similar behavior for the different BPs as in the previously discussed channels, i.e. enhanced signal significance with decreasing mass of the DM. The selection cuts are most efficient for SP- and TAT-type operators. For BP-1 we remove more than 98% of the background events while keeping at least 21% of the signal events for the two operator types, yielding a large signal significance especially for the TAT-type operator with polarized beams.

Varying the DM mass, we display the 3σ sensitivity contours for all three operators in Fig. 4. It is clear that the TAT-type operator has the best sensitivity, which reaches up to 4.2 TeV with unpolarized beams and 5.2 TeV with optimally polarized beams. The SP-type operator also has a sensitivity comparable to the mono-photon channel, and can reach up to 3.4 (4.2) TeV with unpolarized (polarized) beams. The VA-type operator has a modest sensitivity in this channel, only up to 1.7 (2.7) TeV with unpolarized (polarized) beams.

5. Conclusion

We have explored the physics potential of the future $e^+e^-$ colliders in probing such leptophilic DM in a model-independent way. As a case study, we have taken the $\sqrt{s} = 1$ TeV ILC with an integrated luminosity of 1000 fb$^{-1}$ and have analyzed the pair-production of fermionic DM using leptophilic dimension-6 operators of all possible bilinear structures, namely, scalar-pseudoscalar, vector-axialvector and tensor-axialtensor. We have performed a detailed cut-based analysis for each of
Table 8. Summary of our results for the 3σ sensitivity reach of the cut-off scale Λ in the three different channels discussed in the text. Here we have fixed the DM mass at 1 GeV. The numbers in parentheses are with 1% background systematics. The numbers in bold show the highest Λ value that can be probed for a given operator.

| Process type | Beam configuration | 3σ sensitivity reach of Λ (TeV) |
|--------------|--------------------|---------------------------------|
|              | SP                 | VA                             | TAT                            |
| Mono-γ       | Unpolarized        | 3.91 (1.99)                    | 4.19 (2.14)                    | 4.25 (2.17)                  |
|              | Polarized          | 4.84 (2.81)                    | 6.49 (3.94)                    | 5.28 (3.08)                  |
| Mono-Z       | leptonic Unpolarized | 2.62 (2.42)                    | 1.57 (1.36)                    | 3.22 (3.00)                  |
|              | Polarized          | 4.21 (3.16)                    | 2.75 (2.57)                    | 4.22 (3.38)                  |
| Mono-Z       | hadronic Unpolarized | 3.38 (2.79)                    | 1.74 (1.39)                    | 4.22 (3.38)                  |
|              | Polarized          | 4.21 (3.87)                    | 2.75 (2.57)                    | 5.25 (4.71)                  |

these operators in three different channels based on the tagged particle, namely, mono-photon, mono-Z leptonic and hadronic.

We have taken into account one of the most important and powerful features of lepton colliders, i.e., the possibility of beam-polarization with different degrees of polarization and helicity orientations. We find that the sign($P(e^-), P(e^+)) = (+, +)$ beam configuration is optimal for the SP and TAT-type operators, while the (+, −) configuration is better for probing the VA-type operators. The maximum value of the cut-off scale Λ that can be probed in each channel at 3σ is given in Table 8.

We find that without any systematics, the mono-photon channel provides the best sensitivity across all operator types, while in presence of background systematic effects, the mono-Z hadronic channel provides better sensitivity for the SP and TAT-type operators.

We also demonstrate the complementarity of our lepton collider study with other existing direct and indirect detection searches for LDM (cf. Figures 2, 3 and 4). In particular, we show that lepton colliders will be able to provide the best-ever sensitivity in the still unexplored light DM regime.

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