High Energy Afterglow Emission from Giant Flares of Soft Gamma-Ray Repeaters: The Case of the 2004 December 27 Event from SGR 1806-20

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ABSTRACT

We discuss the high energy afterglow emission (including high energy photons, neutrinos and cosmic rays) following the 2004 December 27 Giant Flare from SGR 1806-20. If the initial outflow is relativistic with a bulk Lorentz factor $\Gamma_0 \sim$ tens, the high-energy tail of the synchrotron emission from electrons in the forward shock region gives rise to a prominent sub-GeV emission, if the electron spectrum is hard enough and if the initial Lorentz factor is high enough. This signal could serve as a diagnosis of the initial Lorentz factor of the giant flare outflow. This component is potentially detectable by GLAST if a similar giant flare occurs in the GLAST era. With the available 10 MeV data, we constrain that $\Gamma_0 < 50$ if the electron distribution is a single power law. For a broken power law distribution of electrons, a higher $\Gamma_0$ is allowed. At energies higher than 1 GeV, the flux is lower because of a high energy cut off of the synchrotron emission component. The synchrotron self-Compton emission component and the inverse Compton scattering component off the photons in the giant flare oscillation tail are also considered, but they are found not significant given a moderate $\Gamma_0$ (e.g. $\leq 10$). The forward shock also accelerates cosmic rays to the maximum energy $10^{17}$ eV, and generate neutrinos with a typical energy $10^{14}$ eV through photomeson interaction with the X-ray tail photons. However, they are too weak to be detectable.

Key words: satrs: neutron -- stars: winds, outflows -- hydrodynamics -- Gamma Rays: bursts -- acceleration of particles -- elementary particles

1 INTRODUCTION

The Soft Gamma-ray Repeater (SGR) 1806-20 lies in the Galactic plane, at a distance of about $D_L \approx 15.1$ kpc (Corbel & Eikenberry 2004; cf. Cameron et al. 2005). A giant flare originated from it on 2004 Dec. 27 is the brightest extrastellar transient event ever recorded (e.g., Hurley et al. 2005; Palmer et al. 2005). Radio follow-ups have resulted in detections of its afterglow (e.g., Cameron et al. 2005; Gaensler et al. 2005). Thanks to its brightness, an amazing variety of the data, including the source size, shape, polarization and flux at multi-frequencies as a function of time, have been collected (e.g., Gaensler et al. 2005; Cameron et al. 2005; Gelfand et al. 2005). Even so, our understanding of the outflow is still in dispute. For example, the earliest afterglow data obtained so far is about 7 days after the Giant flare. At this epoch, even an initially relativistic outflow has been decelerated to the Newtonian phase by the interstellar medium (ISM). As a result, whether the outflow is relativistic initially (e.g. Wang et al. 2005; Dai et al. 2005) or not (e.g. Gelfand et al. 2005; Granot et al. 2005) is uncertain. In principle, similar to the Gamma-ray Burst case (e.g., Krolik & Pier 1991), if the spectrum of the giant flare is nonthermal, a lower bound of the initial Lorentz factor $\Gamma_0 \sim$ tens can be derived from the so-called “compactness argument” (e.g., Huang et al. 1998; Thompson & Duncan 2001; Nakar, Piran & Sari 2005; Ioka et al. 2005). Observationally, the giant flare spectrum may be thermal (Hurley et al. 2005) or nonthermal (Palmer et al. 2005), so that $\Gamma_0$ could not be constrained well.

In order to understand the dynamical evolution of the outflow better, early multi-wavelength (including optical and hard $\gamma-$ray band) observations are highly needed. The early optical emission has already been calculated in Cheng & Wang (2003) and Wang et al. (2005). In this work, we
focus on the high energy afterglow emission, including sub-GeV photons (see §2), high energy neutrinos and cosmic rays (see §3). High energy neutrinos from magnetars in the quiescent state have been discussed by Zhang et al. (2003). Assuming the internal shock mechanism, the neutrino, cosmic ray and TeV photon emission accompanying the prompt giant flare have been discussed recently (Ioka et al. 2005; Asano et al. 2005; Halzen et al. 2005).

2 HIGH ENERGY PHOTON EMISSION

We first take \( \Gamma_0 = 10 \) as the typical Lorentz factor of the flow to do sample calculations. The effect of varying \( \Gamma_0 \) will be discussed later. The isotropic energy of the outflow is taken as \( E_{iso} \sim 10^{46} \text{ergs} \). In the following analytical discussion, we assume that the shocked electrons distributes as a single power-law \( \frac{dn}{d\gamma} \propto \gamma^{-p} \) for \( \gamma_m < \gamma < \gamma_M \), where \( p \sim 2.5 \), \( \gamma_M \sim 10^4 B^{-1/2} \) (\( B \) is the shock generated magnetic field strength, see equation (4)). Wang et al. (2005) find that a broken power-law distribution of electrons, i.e., \( \frac{dn}{d\gamma} \propto \gamma^{-p_1} \) for \( \gamma_m < \gamma < \gamma_0 \) and \( \frac{dn}{d\gamma} \propto \gamma^{-p_2} \) for \( \gamma_0 < \gamma < \gamma_M \), is required to interpret the chromatic radio afterglow lightcurve steepening around day 9. We therefore also include such a possibility in the numerical calculations (see §2.3).

With the standard parameters, the relativistic outflow is decelerated by the ISM in a timescale

\[
\tau_{dec} \approx 300 s \left( \frac{E_{iso,46}}{10^{46}} \right)^{1/3} \Gamma_0^{8/3},
\]

(1)

after which the ejecta moves with the Lorentz factor (for \( \Gamma > 1/\theta_0 \))

\[
\Gamma \approx 5.8 \frac{E_{iso,46}}{10^{46}} n_0^{-1/8} \tau_{dec}^{3/8} \Theta_{obs,3},
\]

(2)

where \( n \) is the number density of the ISM, \( \Theta_{obs} \) is the observer time in unit of seconds. Throughout the work, we adopt the convention \( Q = 10^4 \) using cgs units.

As usual, we assume \( \epsilon_e \) and \( \epsilon_B \) as the shock energy equipartition parameters for the shock accelerated electrons and the magnetic fields, respectively. The minimum electron Lorentz factor reads

\[
\gamma_m \approx 184 C_\rho \epsilon_e^{-0.5} (\Gamma - 1),
\]

(3)

where \( C_\rho = 3(p-2)/(p-1) \). The strength of shock generated magnetic fields can be estimated as

\[
B' \approx 3.9 \times 10^{-2} \text{Gauss} \left( \frac{E_{iso,46}}{10^{46}} \right)^{1/2} \Gamma_0^{1/2} \Theta_{obs,3}^{1/2} (\Gamma - 1)^{1/2}.
\]

(4)

Throughout the work, the superscript ‘ represents the parameter measured in the comoving frame of the ejecta.

2.1 Inverse Compton Radiation

A soft thermal X-ray tail emission modulated by the magnetar period is typically detected after a giant flare hard spike. For the Dec. 27 event from SGR 1806-20, such a tail lasts for \( T_{t_{tail}} \sim 300 \) with a typical photon energy \( \epsilon_X \sim 30 \text{keV} \) (e.g. Mazets et al. 2005) and a luminosity \( L_X \sim 2 \times 10^{34} \text{ergs s}^{-1} (\Theta_{obs,50}^{-3})^{-1} \). For \( \Theta_{obs} < T_{t_{tail}} \), besides the synchrotron and synchrotron-self-Compton cooling processes (see 2.2 for detail), the electrons in the shocked region are also cooled by inverse Compton (IC) scattering off these X-ray tail photons.

Since \( T_{t_{tail}} \) is comparable to \( \tau_{dec} \), the ejecta has not decelerated significantly, i.e., \( \Gamma \sim \Gamma_0 \). In the comoving frame of the ejecta, the energy density of the X-ray tail reads

\[
U_X \approx \frac{L_X}{4\pi R^2 c \tau_{dec}^2} \approx 0.27 \text{ ergs cm}^{-3} L_{X,43} R_{15}^{-2} \Gamma_0^{-2},
\]

(5)

where \( R \) is the radial distance of the forward-shock front from the central source. On the other hand, the magnetic energy density generated in the forward shock front reads

\[
U_B \approx 6 \times 10^{-3} \text{ ergs cm}^{-3} \epsilon_{B,-2} R_{15}^2 n_0.
\]

(6)

In the rest frame of the shocked electrons with a random Lorentz factor \( \gamma_e \), the energy of the thermal tail \( \gamma_e \epsilon_X / \Gamma \) is much larger than \( m_e c^2 \), so that the Klein-Nishina correction is important. For convience, we define \( x \equiv \gamma_e \epsilon_X / \Gamma m_e c^2 \geq \gamma_e / 177 \). In the Klein-Nishina limit, \( \sigma_{IC} = A(x) \sigma_T \), where \( A(x) \approx \frac{3}{2} \left( \frac{x}{1 + x} \right) \left( \frac{1}{1 + x} - \ln(1 + 2x) \right) + \frac{x}{2} \ln(1 + 2x) - \frac{1}{1 + x} \gamma^2 \), with the asymptotic limits \( A(x) \sim 1 - 2 \frac{x}{2} \) for \( x \ll 1 \), and \( A(x) \approx \frac{1}{2} x^{-1} \ln(2x + 1) \) for \( x \gg 1 \) (e.g. Rybicki & Lightman 1979).

For illustration, we take \( t_{obs} = T_{t_{tail}} \), at which \( R \approx 2 \times 10^{15} \text{cm} \) \( \Gamma T_{t_{tail}} / 2.5 \). For \( \gamma_e = \gamma_m \), we have \( x = 10.8 \), \( A(x = 10.8) \approx 0.1 \). The IC scattering is therefore in the extreme Klein-Nishina limit, and the typical IC photon energy can be well approximated by

\[
\frac{h \nu_{\text{IC}}}{\epsilon_e} \approx h \Gamma_0 \gamma_m m_e c^2 \approx \epsilon_e (p - 2)/(p - 1) (\Gamma - 1) \Gamma_0 m_e c^2 \approx 9 \text{GeV} \epsilon_{e,-5} \Gamma_0. \]

(7)

The IC optical depth is

\[
\tau \sim A(10.8) \sigma_T n R / 3 \times 4 \times 10^{-11} n_0 R_{15.26},
\]

(8)

so that the 10 GeV photon luminosity can be estimated by

\[
L_{10 \text{GeV}} \sim \tau (L_X / \epsilon_X) (\epsilon_{\text{IC}} / h \nu_{\text{IC}}) \sim 4.3 \times 10^{47} \text{ ergs s}^{-1} \epsilon_{e,-5} n_0 R_{15.26}^{1/2} \Theta_{obs,2.5}^{1/2},
\]

(9)

where \( h \) is the Planck constant. For \( t_{obs} \leq T_{t_{tail}} \), \( \Gamma \sim \Gamma_0 \), \( R \propto t_{obs} \), we have \( L_{10 \text{GeV}} \propto R_{t_{obs}}^{-2} \propto t_{obs}^{1/2} \). We can then estimate the total number of the photons detectable by the Gamma-Ray Large Area Telescope (GLAST)\(^1\) in construction

\[
N_{t_{obs}} (10 \text{GeV}) \sim \frac{A_{GLAST}}{4 \pi D_E^2} \int_0^{T_{t_{tail}}} \frac{L_{10 \text{GeV}} dt_{obs}}{10 \text{GeV}} \sim 0.03 \tau_{t_{tail,2.5}} n_0 (D_L / 15.1 \text{ kpc})^{-2},
\]

(10)

where \( A_{GLAST} \approx 8000 \text{ cm}^2 \) is the effective area of the GLAST. Since usually at least 5 photons are needed to claim a detection (e.g. Zhang & Mészáros 2001 and references therein), the above predicted \( N_{t_{obs}} \) is well below the threshold of GLAST. This component is undetectable for an energetic giant flare similar to the recent one even for a much closer SGR, for example, SGR 1900+14.

The thermal tail photons would be also scattered by the electrons accelerated by the reverse shock. The reverse shock is expected sub-relativistic. At \( t_{dec} \), \( \gamma_{734} \sim 1.2 \), where \( \gamma_{734} \) is the Lorentz factor of shocked region relative to initial unshocked outflow. Therefore, for the electrons accelerated by the reverse shock, one has \( \gamma_m = \epsilon_e [(p - 2)/(p - 1)](m_p / m_e)(\gamma_{734} - 1) \sim 37 \) by assuming the same parameters as in the forward shock region. Therefore \( \epsilon_e \) will be scattered

\(^1\) http://glast.gsfc.nasa.gov/
to an energy $\sim \gamma_{m}^{2}e_{x} \sim 30$MeV. According to Eqs.(9) and (10), the detected number of photons essentially depends on the IC optical depth $\tau$ and is independent on the typical energy of the photons. We can then estimate the total number of the IC photons from the reverse shock region by comparing that in the forward shock region. First, the IC is now in the Thomson regime, i.e. $\sigma_{T} \approx \sigma_{T}$. Second, the total number of electrons contained in the reverse shock region is about $\Gamma_{0}$ times that in the forward shock region. The expected total number of the 30 MeV photons is therefore

$$N_{\text{tot}}(30\text{MeV}) \sim \frac{\Gamma_{0}}{A(x = 10.8)} N_{\text{tot}}(10\text{GeV}) \sim 3. \quad (11)$$

The actual value should be smaller since the timescale of having a strong reverse shock could be shorter than $T_{\text{trail}}$. Although this $\sim 30$MeV reverse shock component is more prominent than the $\sim 10$ GeV forward shock component, it is undetectable by GLAST, either.

### 2.2 Synchrotron and Synchrotron-self-Compton Radiation

For the forward shock emission, the cooling frequency $\nu_{c}$, the typical synchrotron frequency $\nu_{\text{ms}}$, and the maximum spectral flux $F_{\nu, \text{ms}}$ read (e.g. Cheng & Wang 2003; Wang et al. 2005)

$$\nu_{c} = 3.1 \times 10^{19} \text{Hz} \, E_{\text{iso}, 46}^{-1/2} B_{-2}^{-3/2} \Gamma_{0}^{-1/2} (1 + Y)^{-2}, \quad (12)$$

$$\nu_{\text{ms}} = 2.4 \times 10^{12} \text{Hz} \, C_{p}^{-1} E_{\text{iso}, 46}^{1/2} B_{-2}^{-2} \epsilon_{-0.5}^{-2} \approx (1 + Y)^{-2}, \quad (13)$$

$$F_{\nu, \text{ms}} = 474 \text{Jy} \, E_{\text{iso}, 46}^{1/2} B_{-2}^{-1} \Gamma_{0}^{-1/2} \frac{D_{L}}{15.1 \text{Mpc}}^{-2}, \quad (14)$$

where $Y$ is the inverse Compton parameter, which can be estimated by $Y \leq \left[ 1 + \sqrt{1 + 4 \epsilon_{6} e_{6}/e_{6}} \right]$ (e.g. Sari & Esin 2001), where $x = \min[1, 2.67(\gamma_{m}/\gamma_{0})^{(p-2)}]$ is the radiation coefficient of the shocked electrons (see equation (A8) of Fan, Zhang & Wei (2005a)), and $\gamma_{c}$ is the electron cooling Lorentz factor

$$\gamma_{c} \approx 7.7 \times 10^{6} \frac{1}{(1 + Y) \, \Gamma B_{2}^{2} \tau_{\text{obs}}}. \quad (15)$$

Notice that only the synchrotron self-Compton is considered. The IC component discussed in §2.1 is in the extreme K-N regime at $\gamma_{c}$, giving a very small contribution to the IC optical depth (see §2.3). The synchrotron-self-Component (SSC) luminosity ($L_{\text{SSC}}$) could be estimated through the $Y$ parameter, i.e. $Y = L_{\text{SSC}}/L_{\text{syn}}$. This results in the maximum SSC spectral flux ($t_{\text{dsec}} < t_{\text{obs}} < t_{j}$)

$$F_{\nu, \text{SSC}} \approx Y^{-p-3} \gamma_{m}^{2} F_{\nu, \text{ms}} \quad (18)$$

where $\nu_{\text{SSC}}^{\text{obs}}$ is the typical SSC frequency

$$\nu_{\text{SSC}}^{\text{obs}} \approx \frac{\gamma_{m}^{2} \nu_{c}}{\tau_{\text{ms}}^{\text{obs}}} = 1.9 \times 10^{18} \text{Hz} \, C_{p}^{-1/2} \epsilon_{-0.5}^{-2} \approx (1 + Y)^{-2} \frac{D_{L}}{15.1 \text{Mpc}}^{-2}, \quad (19)$$

The resulting flux at $h\nu_{\text{obs}} = 0.1 \text{GeV}$ reads

$$F_{\nu, \text{SSC}} = \frac{F_{\nu, \text{SSC}}^{\text{ms}}}{\nu_{\text{SSC}}^{\text{obs}}} \frac{\nu_{\text{ms}}^{\text{obs}}}{\nu_{\text{SSC}}^{\text{obs}}} \approx 2.2 \times 10^{-8} \text{ergs cm}^{-2} \text{GeV}^{-1} (1 + Y)^{-2} \epsilon_{-0.5}^{-2} \approx (1 + Y)^{-2} (13 - 3p)/8 \approx (13 - 3p)/8 \frac{D_{L}}{15.1 \text{Mpc}}^{-2} (h\nu_{\text{obs}}/0.1 \text{GeV})^{-2}. \quad (20)$$

For $h\nu_{\text{obs}} \leq 0.1 \text{GeV}$, this radiation component is much weaker than the synchrotron component. Beyond the synchrotron cutoff at $h\nu_{\text{obs}} \sim 1 \text{GeV}$ the SSC component dominates, but it is well below the GLAST threshold.

### 2.3 Numerical Results

Similar to Huang et al. (2000) and Cheng & Wang (2003), we have calculated the dynamical evolution of the ejecta (see Fig.1) and the accompanying high energy photon emission (see Fig.2) numerically.

As shown in Fig. 1, the jet half-opening angle increases with time rapidly. With sideways expansion, the evolution of the jet half opening angle could be written as (e.g. Huang et al. 2000) $d\theta/dt_{\text{obs}} = c_{\theta}(\Gamma + \sqrt{\Gamma^{2} - 1})/R$, where $c_{\theta} \approx \sqrt{4(5 + 3)(\Gamma^{2} - 1)/3(4(5 + 3)^{2} - 1)}/3$ is the local sound speed. For $\Gamma \gg 1$, this could be approximated as $d\theta/dt_{\text{obs}} \approx 1.2/(2\Gamma t_{\text{obs}})$. It is apparent that the sideways expansion of the jet is very important from the very beginning of the dynamical evolution if the initial Lorentz factor is as small as 10. As a result, there is no jet break in the $(\Gamma - 1)$ lightcurve (Fig.1) the energy flux lightcurve (Fig.2). This is different from the case of ultra-relativistic GRB outflows, in which the sideways expansion is important only at later times. One conclusion drawn from Fig.1 is that the ejecta accounting for the radio afterglow is nearly isotropic, which matches the observations well (e.g. Cameron et al. 2005; Galensker et al. 2005).
According to Fig. 2 where \( \Gamma_0 = 10 \) is adopted, we can see that the predicted energy flux in the 0.05-0.15 GeV band is above the GLAST sensitivity (thick dashed line), especially when a single power-law electron energy distribution (thin dashed line) is adopted. If the electron distribution is a broken power law (solid line), the detectability by GLAST is only marginal. In the energy band above 1 GeV, the predicted SSC energy flux (thin dash-dotted line) is always below the GLAST sensitivity (thick dash-dotted line), so that it is undetectable.

In Fig. 3, we investigate the dependence of the predicted 0.05-0.15 GeV energy flux (only the synchrotron radiation component is taken into account) on \( \Gamma_0 \). The general trend is that a higher \( \Gamma_0 \) leads to a stronger sub-GeV emission. For \( \Gamma_0 \sim \) tens, regardless of the distribution of the shocked electrons (single power law or broken power law), the predicted fluxes are all below the GLAST. For \( \Gamma_0 \sim \) a few, only the single power law distribution model can yield to marginally observable 0.05-0.15 GeV photon emission.

In principle, a measurement of the sub-GeV flux in the GLAST era could serve as a diagnosis of the initial Lorentz factor of the outflow. For SGR 1806-60, available data already gives interesting constraints. According to Matzets et al. (2005), the time averaged energy flux in the \( \epsilon_\gamma \sim 10 \) MeV band could be estimated as 

\[
\frac{dN}{d\epsilon_\gamma} \sim 1.6 \times 10^{-6} \text{ergs cm}^{-2} \text{s}^{-1},
\]

where \( dN/d\epsilon_\gamma \sim 10^{-5} \text{photon cm}^{-2} \text{s}^{-1} \text{keV}^{-1} \) is the photon number spectrum of the tail emission at 10 MeV. Comparing with our numerical results presented in Fig. 3, the single power law distribution model with \( \Gamma_0 = 50 \) is already above the observed level\(^2\). A Lorentz factor \( \Gamma_0 \geq 50 \) is allowed only when a broken power law distribution of the electrons is assumed.

### 3. COSMIC RAYS AND NEUTRINOS

Below we estimate the maximum proton energy (\( \epsilon_p^M \)) accelerated by the forward shock. For simplicity, we only discuss the case where the electron distribution is a broken power law \( n_e \sim \epsilon_e^{-(2+p)} \) and the proton distribution is a single power law \( n_p \sim \epsilon_p^{-\Gamma_0} \). If the electron distribution is a broken power law \( n_e \sim \epsilon_e^{-(2+p)} \), the proton energy flux can be approximated as \( \epsilon_p^{-\Gamma_0} \), and the detectability by GLAST is only marginal. In the energy band above 1 GeV, the predicted SSC energy flux (thin dash-dotted line) is always below the GLAST sensitivity (thick dash-dotted line), so that it is undetectable.

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\[\epsilon_p^{-\Gamma_0} \approx 2.4 \times 10^{17} \text{eV} \quad (\Gamma_0 \geq 1)\]

(2) The comoving shock acceleration time \( t_a' \sim \epsilon_p/\Gamma_0 \) should be smaller than the comoving wind expansion time \( t_a \sim R/v_c \), which yields \( \epsilon_p^{M}(1) \sim cB'/R \).

\[\epsilon_p^{M}(1) \approx 2.4 \times 10^{17} \text{eV} \quad (\Gamma_0 \geq 1)\]

(2) The comoving proton synchrotron cooling timescale \( t_{cool} = \left[6\pi m_e^2 c^3 / \sigma T m_e^2 \Gamma_p^{-2} B'/R \right]^{-1/2} \) should be longer than the comoving acceleration timescale \( t_a' \), which results in

\[
\epsilon_p^{M}(2) \approx 2.6 \times 10^{21} \text{eV} \quad \Gamma_0 \geq 10
\]

(3) The comoving proton cooling timescale due to photomeson interaction should also be longer than the comoving acceleration timescale \( t_a' \). However, from equation (13), the typical frequency of the forward shock emission is too low to provide the target photons for photomeson interactions.

\[2 \text{ Our calculated energy flux is in the } \epsilon_\gamma = 50 - 150 \text{ MeV band. However, since } \nu_{\gamma \nu} F_{\gamma \nu} \times \nu_{\gamma \nu}^{(2+p)/2} \text{ very weakly depends on } \nu_{\gamma \nu}, \text{ the results could approximately apply to the } 10 \text{ MeV band as well.}\]

\[
T_{\text{delay}} \approx (c_B / D_L / c_{\text{CR}})^2 (l/c),
\]

where \( c_B \approx 10^{-6} \text{G} \) is the average magnetic field strength in the Galaxy, \( c_{\text{CR}} \) is the typical cosmic ray energy, and \( l \approx 10 - 100 \text{pc} \) is the correlation length of the magnetic field (e.g., Asano et al. 2005). One then gets \( T_{\text{delay}} \approx 6 \times 10^7 \text{yr} B_{\text{G}}^{-1} D_{\text{kpc}}^{-1} c_{\text{CR}}^{-1}(l/10 \text{pc})^{-1} (c_B / 10^{-6} \text{G})^{-1} \). As a result, these cosmic rays become a part of the cosmic ray background.

As shown in equation (13), the typical frequency of the forward shock is too low to provide the target photons for any photomeson interaction at the \( \Delta \)-resonance. The only interesting source of the neutrino emission is then the photomeson interaction during the early epoch when the X-ray tail overlaps with the shocked region. In the comoving frame of the ejecta, the thermal tail photons with energy \( \epsilon \approx \epsilon_X / T \) interact with the protons with energy

\[
\epsilon_p \sim 0.3 T^2 \text{GeV}/\epsilon_X \approx 10^{15} \text{eV} \quad \Gamma_0^2 (\epsilon_X/30 \text{keV})^{-1}.
\]
within the shock with the characteristic width $\Delta t$ estimated by the number of the
fraction of the energy converted to pions can be estimated. The thin dashed line and the solid line represent the flux in the energy range 0.05-0.15 GeV
and “B” denotes broken power law. Only synchrotron component is calculated since the SSC component is much dimmer. The thin dashed line represents the flux in the 1-300 GeV energy band. Only the SSC component is calculated since this is above the synchrotron cutoff energy. The thick dashed line and the thick dash-dotted line represent the GLAST sensitivity in the energy range 0.05-0.15 GeV. Except $\Gamma_0$, other parameters are the same as those taken in Fig.2.

These protons lose $\sim 20\%$ of their energy at each $p\gamma$ interaction, dominated by the $\Delta$-resonance. Approximately, half of pions are charged and decay into high energy neutrinos $\pi^+ \rightarrow \mu^+ + \nu_\mu \rightarrow e^+ + \nu_e + \bar{\nu}_\mu + \nu_\mu$, with the energy distributed roughly equally among the decay products (e.g., Ioka et al. 2005). Therefore the neutrino energy is $\sim 5\%$ of the proton energy, i.e.,

$$\epsilon_\nu \sim 5 \times 10^{-14} \text{eV} \frac{1}{\Gamma_1^2 (\epsilon_\nu/30 \text{keV})^{-1}} \quad (25)$$

The comoving number density of the thermal photons at the radius $R \sim 10^{15}$ cm is

$$n_X \approx U_X/(\epsilon_X/\Gamma) \approx 5.5 \times 10^7 L_X A_3 R_{15}^{-2} \Gamma_1^{-1} (\epsilon_X/30 \text{keV})^{-1} \quad (26)$$

The fraction of the energy converted to pions can be estimated by the number of the $p - \gamma$ interactions occurring within the shock with the characteristic width $\Delta R \sim R/\Gamma$, i.e.

$$f_\pi \sim 0.2 n_X \frac{\sigma_\Delta}{\Gamma} R/\Gamma \approx 5.5 \times 10^{-7} L_X A_3 R_{15}^{-2} \Gamma_1^{-1} (\epsilon_X/30 \text{keV})^{-1} \quad (27)$$

where $\sigma_\Delta \sim 5 \times 10^{-28} \text{cm}^2$ is the cross section of the $\Delta$-resonance. For a neutrino detector with an area $A_{\text{det}} \sim 10^{10}$ cm$^2$, the expected event number is

$$N_\nu \sim P_{\nu \rightarrow \mu} f_\pi A_{\text{det}} E_{\nu} (32 \pi D_L^2 \epsilon_\nu) \sim 7 \times 10^{-5}, \quad (28)$$

where $P_{\nu \rightarrow \mu} \approx 3.5 \times 10^{-4} (\epsilon_\nu/10^{15} \text{eV})^{0.5}$ is the probability that a neutrino produces a detectable high energy muon for $\epsilon_\nu > 10^4 \text{TeV}$. We can see that the predicted neutrino number is well below the detection threshold of the most powerful neutrino detectors under construction. The main reason is that compared with GRBs, $f_\pi$ (Eq.[27]) is much smaller.

4 SUMMARY

We show that if a giant flare similar to the 2004 Dec.27 event happens in the GLAST era, a strong sub-GeV flare shortly after the flare (originated from the hard tail of the synchrotron emission from the forward shock region) should be detectable if the outflow is relativistic. A positive/negative detection in the sub-GeV band would then give a diagnosis of the initial Lorentz factor $\Gamma_0$. Although $\Gamma_0 > 50$ is allowed if the electron distribution is a broken power law, at higher energies (e.g. above 1 GeV), a cutoff of the synchrotron emission is expected. Neither the synchrotron self-Compton emission in
the forward shock region, nor the inverse Compton off the X-ray tail emission, could give a detectable flux for GLAST.

The forward shock is able to accelerate protons to an energy $\sim 10^{17}$ eV. But the time delay for these cosmic rays to reach us is very long, i.e. $\sim 10^9$ years. Neutrinos with an energy $10^{14}$ eV are also predicted, but the flux is too low to be detected. Therefore, for a giant flare similar to the Dec. 27 event taking place in the GLAST era, the most, and perhaps the only, interesting high energy afterglow emission is the bright sub-GeV photon emission lasting for thousands of seconds.

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