Multi-Compartment Vehicle Routing Problem to Find the Alternative Distribution Route of Petroleum Product Delivery

I F Febriandini¹, Yuniaristanto², W Sutopo³

¹ Laboratory Assistant Logistic System and Business, Industrial Engineering Department, Universitas Sebelas Maret, Indonesia
²,³ Research Group of Industrial Engineering and Techno-Economic, Industrial Engineering Department, Universitas Sebelas Maret, Indonesia

Email: izatulfitria23@gmail.com

Abstract. Distribution is an activity that the firm has to do to deliver their product to the customers. The product has to deliver to the customers on the right condition, at the right time, and with the minimal cost of delivery. This paper focuses to solve the planning distribution problem on petrol product delivery or Petrol Station Replenishment Problem. One of the ways to solve the planning distribution is VRP (Vehicle Routing Problem). VRP aims to find the minimal total cost delivery with some constraints depend on the factory. In this research, VRP is used by the distribution center to find the optimal route of distribution of petrol products. Using Multi-Compartment Vehicle Routing Problem (MC-VRP) that considers the demand for the petrol product, tank-truck capacity, distribution cost, and the number of the compartment. In this work, a mathematical formulation for MC-VRP is developed in the Mixed Integer Linear Programming (MILP) model. Using Ilog Cplex with branch and bound algorithm to solve this distribution problem with the minimum total distribution cost. There are 10 customers with different distances, different time deliveries and different travel costs in this research. The mathematic formulation produces the best alternative that is Rp 1,898,584 to deliver products from depot to the customers.

1. Introduction
Goods and service industries cannot be separated from distribution activities. Distribution activities are one of the strategy of the company to share their product to their customers. Distribution channels are a series of organizational participants who perform all the functions needed to deliver products or services from the company to the end customers [1].

One of the problems in the distribution process is selecting the distribution route or we mostly call it VRP (Vehicle Routing Problem). VRP itself has many variations in its development. In this paper, using one of the variants on the VRP, called MC-VRP (Multi-Compartment Vehicle Routing Problem). MC-VRP is the variant of the VRP that when selecting the route of the vehicle they enable to jointly transport product with different variances, thus it could reduce the number of visits to a store [2]. Martins also said that MC-VRP can be used to deliver many variations of the product, such as fuel distribution, grocery distribution (e.g. product with different temperature zones), and waste distribution (e.g. glass waste).

Another version of VRP is PSRP (Petrol Station Replenishment Problem). PSRP is a variation of VRP that studies the distribution of fuel products (Oil and Gas Company). The aim is to optimize the distribution of petroleum products to the petrol station (SPBU) using a limit number of a vehicle [3].
PSRP is a tool to find an alternative solution to distribute products from the company to its customers. This method already discussed in previous studies with different cases and different algorithms. There are so many kinds of the algorithm to do VRP, there are Tabu Search, Branch And Bound, Neighbourhood Search, Genetic Algorithms, etc.

Table 1. Previous Research.

| No | Author                          | VRP Variation                          | Algorithm                  |
|----|---------------------------------|----------------------------------------|----------------------------|
| 1  | Sevaux and Sorensen, 2008       | VRP-Cluster                            | Floyd-Warshall             |
| 2  | Surjandari, et.al, 2011         | VRP-Capacitated, Time Window, Multi-Compartment, Loading-Unloading | Tabu Search                |
| 3  | Benantar and Ouafi, 2012        | VRP- Time Window, Multi-Compartment    | Tabu Search                |
| 4  | Battarra, et.al, 2014           | VRP-Cluster, Capacitated               | Branch and Cut & Price     |
| 5  | Henke, et.al, 2015              | VRP-Capacitated, Multi-Compartment      | Neighborhood Search        |
| 6  | Coelho and Laporte, 2015        | VRP-Capacitated, Time Window, Multi-Compartment | Branch and Cut Algorithms |
| 7  | Belov and Slastnikov, 2017     | VRP-Capacitated                         | Ant Colony                 |
| 8  | Pop, et.al, 2018                | VRP-Cluster, Capacitated               | Genetic Algorithms         |
| 9  | Marc, et.al, 2018               | VRP-Cluster, Capacitated               | Genetic Algorithms         |
| 10 | Henke, et.al, 2018              | VRP-Cluster, Capacitated, Multi-Compartment | Branch and Cut Algorithms |

This problem is based on the real study case of the oil and gas depot owned by PT. Pertamina, one of the Oil and Gas Company in Indonesia. This research held in PT. Pertamina Terminal Bahan Bakar (Depot of the petrol product) located in Boyolali, Central Java.

Based on the observation, TBBM Boyolali delivers the product to their customers, we call it SPBU (petrol station) every day. But, before the company delivers its product, they need to make distribution planning. Distribution planning is about the route of distribution and the delivery schedule. They have to make the distribution planning every day, based on the demand of the customers. So this research aims to minimize the total cost of distribution to deliver the petrol product.

2. Methodology

TBBM Boyolali delivers its product into 3 zones. Three of them be distinguished based on the distance from customers to the depot. Zone 1 for consumers with a distance of fewer than 30 km, zone 2 for customers between 30 to 60 km, and the last is zone 3 is for customers with a distance more than 60 km from the depot location. Figure 1 describes the distribution zones of TBBM Boyolali.

The company determines the shipping route in line 1, which is delivered in zone 2 at 01.00 - 09.00, followed by shipping on line 2, which is in zone 1 at 09.00-15.00, and the last one in line 3, which is delivered in zone 3 at 15.00 - end.
Table 2. Distribution Zones.

| No | ZONE I |       | ZONE II |       | ZONE III |       |
|----|--------|-------|--------|-------|----------|-------|
|    | Area   | Total | Area   | Total | Area     | Total |
| 1  | Boyolali | 15    | Salatiga | 12  | Sragen   | 1     |
| 2  | Salatiga | 3     | Semarang | 3   | Magetan  | 16    |
| 3  | Surakarta | 24    | Sragen   | 24  | Wonogiri | 14    |
| 4  | Karanganyar | 12   | Klaten   | 8   | Purwodadi/Blora | 13   |
| 5  | Sukoharjo | 15    | Karanganyar | 8 | Salatiga | 1     |
| 6  | Klaten   | 16    | Sukoharjo | 3  | Ngawi    | 16    |
| 7  | Sragen   | 1     |         |     | Pacitan  | 5     |
|    | TOTAL    | 86    |         |     | Semarang | 14    |

The business process of petrol product distribution in TBBM Boyolali are:

SPBU (gas stations) that run out of stock will order via SMS (Short Message Service) to Boyolali TBBM Sales Services one day before petrol products are sent to the gas station. The Sales Service will send the results of the product order recapitulation to the Distribution Division. Distribution Division then makes a planning distribution that will be given to Patra Niaga. PT Patra Niaga was launched in 2004 to focus on the downstream oil and gas (O&G) business [Patra Niaga]. Distribution Division will make a different distribution planning every day, this depends on the requests that go to the Sales service the day before.

The product sent by TBBM Boyolali are RON 88, RON 95, RON 95 and bio solar. There are 241 customers (SPBU/ Petrol Station) around the 3 zones. 103 tank trucks have a capacity of 16 kiloliters, 24 kiloliters, and 32 kiloliters in TBBM Boyolali explained in table 3. Whereas to meet consumer demand every day, TBBM Boyolali uses around 95 tank truck, the rest of the tank cars are not operating (maintenance). Each compartment has 8,000 liters on its capacity.

Table 3. Several vehicles and their capacity.

| Capacity (kiloliter) | Number of Compartment | Number of Vehicles | Total Compartment | Total Capacities |
|----------------------|-----------------------|-------------------|-------------------|-----------------|
| 16                   | 2                     | 30                | 32                | 480             |
| 24                   | 3                     | 52                | 72                | 1248            |
| 32                   | 4                     | 21                | 128               | 672             |
| TOTAL                | 103                   | 232               | 2,400             |                 |
All of these vehicles are leased from the Patra Niaga Company, which is a subsidiary of PT. Pertamina. Patra Niaga only focuses on the distribution of petrol products. Each truck has a different number of compartments, depending on the capacity of the vehicle. Each compartment has a capacity of 8 kiloliters. So, consumers who want to order petrol products must order with multiple capacities of 8 kiloliters.

2.1. Mathematic Formulation

Assumptions:
- The delivery cost per kilometer is known and constant.
- Delivery cost Rp 2,920,- per kilometer [3].
- Several trips can be assigned to the same vehicle.
- Only study 10 samples of location in zone 3, from 241 real customer location.
- Only use 5 tank trucks with capacity: 32 kiloliters.
- SPBU (gas station) only order the amount of fuel that is a multiple of 8 kiloliters (according to the capacity of each compartment).

Model. In this paper, a mathematical formulation for MC-VRP is developed in the Mixed Integer Linear Programming (MILP) model. The model below is written based on the research conducted by Benantar 2012 [4] and 2016 [5] which was conducted in the petroleum industry.

The MC-VRP problem is formulated by a multi-product, single depot, with time windows.

Notations of parameters used for the tank trucks assignment and trip route are as follow:

- \( N \): Number of customers
  - \( N = 10 \)
- \( V \): \( N \cup \{0\} \)
  - Where 0 represent the depot
  - Number of vehicle \( k \) that can be used
  - \( K = 5 \)
- \( P \): Number of product
  - \( P = 3 \) \{Premium, Pertamax, Peralite\}
- \( Q \): Number of compartment
  - \( Q = 3 \)
- \( d_{ip} \): The demand of customer \( i \) for the product \( p \)
- \( C_k \): The capacity of vehicle \( k \)
- \( c_{qk} \): The capacity of compartment \( q \) for the vehicle \( k \)
- \( d_{ij} \): Distance between customer \( i \) and customer \( j \)
- \( t_{ijk} \): Travel time of vehicle \( k \) from customer \( i \) to customer \( j \)
- \( a_i \): Earliest start time for servicing the customer \( i \)
- \( b_i \): Latest start time for servicing the customer \( i \)
- \( C_{ij} \): The cost spent by the vehicle \( k \) from customer \( i \) to customer \( j \)

Then, the decision variable are:

- \( x_{ijk} = \begin{cases} 1: & \text{if } i \text{ precede } j \text{ in the route of vehicle } k \\ 0: & \text{otherwise} \end{cases} \)

- \( y_{ikp} = \begin{cases} 1: & \text{if and only if customer } i \text{ receives product } p \text{ from vehicle } k \\ 0: & \text{otherwise} \end{cases} \)

\( S_{ik} = \) Specifies the arrival time at \( i \) when serviced by vehicle \( k \). If vehicle \( k \) does not service \( i \), \( s_{ik} \) has no meaning and consequently, its value is considered irrelevant.
So, the mathematic formulation is:

Objective Function:

\[
\min \sum_{k \in \{K_1 \cup K_2\}} \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ijk}
\] (1)

The objective function in formula (1) is to minimize the total cost of distribution of all routes.

\[
\sum_{i \in V, i \neq h} x_{ihk} - \sum_{i \in V, j \neq h} x_{hjk} = 0, \quad h \in N, \quad k \in \{K_1 \cup K_2\}
\] (2)

\[
\sum_{j \in V, i \neq j} \sum_{k \in K} x_{ijk} \leq 1, \quad i \in N
\] (3)

\[
\sum_{i \in V, i \neq j} \sum_{k \in K} x_{ijk} \leq 1, \quad j \in N
\] (4)

\[
\sum_{j \in N} x_{j0k} = 1, \quad k \in \{K_1 \cup K_2\}
\] (5)

\[
\sum_{i \in N} x_{i0k} = 1, \quad k \in \{K_1 \cup K_2\}
\] (6)

\[
\sum_{j \in N} d_{j} y_{jkp} = c_{k}^{q}, \quad k \in \{K_1 \cup K_2\}, \quad q \in \{Q_1 \cup Q_2\}, \quad p \in P
\] (7)

\[
y_{jkp} = \sum_{i \in V} x_{ijk}, \quad j \in N, \quad k \in \{K_1 \cup K_2\}, \quad p \in P
\] (8)

\[
s_{ik} + t_{i} + t_{ij} - M_{ij}(1 - x_{ijk}) \leq s_{jk}, \quad i \in N, j \in V, i \neq j, \quad k \in \{K_1 \cup K_2\}
\] (9)

\[
a_{i} \leq s_{ik} \leq b_{i}, \quad i \in V, \quad k \in K
\] (10)

\[
x_{ijk} \in \{0,1\}, \quad i, j \in V, \quad k \in \{K_1 \cup K_2\}
\] (11)

\[
y_{ipk} \in \{0,1\}, \quad i \in V, \quad k \in \{K_1 \cup K_2\}, \quad p \in P
\] (12)

\[
s_{ik} \geq 0, \quad i \in N, \quad k \in \{K_1 \cup K_2\}
\] (13)

Constraint (2) states that the vehicle arrived at each customer has to leave it. Constraint (3) and (4) states that the vehicle will not return to the previous customer. Constraint (5) shows that the start of the route for each vehicle must be depot and (6) and the end of the route must be depot. Constraint (7) ensure that the capacity of the compartment is not exceeded. Constraint (8) shows that if customer i is not visited by vehicle k, so the value of \(y_{jkp}\) is zero. Constraint (9) represents the relationship between the starting time of the service to one customer and the departure time from its predecessor. \(M\) is an arbitrarily large value. And constraint (10) states that the service takes place at each customer concerning the time window. Decision variables stated in constraint (11), (12) and (13).

3. Result and Discussion
There are 10 customers and using 5 vehicles in this research. Each customer located in Semarang, Central Java, Indonesia. Each customer has their code to describe the petrol station. There are:

Table 4. List of customers.

| No. | Code of Petrol Station | Address                                      |
|-----|------------------------|----------------------------------------------|
| 1   | 44.505.01             | Jl. Raya Klepu Karangjati Ungaran            |
| 2   | 44.505.02             | Jl. Diponegoro Ungaran                      |
| 3   | 44.505.03             | Kel. Karangjati Kec. Bregas                 |
| 4   | 44.505.04             | Ds. Jatijajar Dk. Tegal Panas Bergas        |
| 5   | 44.505.05             | Ds. Randugunting Kec. Klepu                 |
| 6   | 44.505.06             | Jl. A Yani Kel. Ungaran                    |
| 7   | 44.505.07             | Jl. Karangjati Pringapus Ungaran           |
| 8   | 44.505.08             | Jl. Diponegoro 204                          |
| 9   | 44.505.09             | Jl. Halmahera Kel. Gendangan Kec. Ungaran  |
| 10  | 44.506.01             | Ds. Harjosari Bawen                         |

The input data that been used for this research are distance, time deliveries and cost of delivery from the depot to each customer. Distance is obtained from the real distance by google maps. The distance matrix from each customer shows in table 5.

Table 5. The distance for each customer and depot.

| DEPOT | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1     | 78  | 67  | 61  | 61  | 58  | 65  | 62  | 65  | 67  | 57  |
| 2     | 67  | 5   | 7   | 8   | 11  | 2   | 7   | 1   | 2   | 10  |
| 3     | 61  | 7   | -   | 3   | 6   | 7   | 2   | 7   | 5   | 5   |
| 4     | 61  | 4   | 8   | 3   | -   | 2   | 11  | 5   | 8   | 10  |
| 5     | 58  | 6   | 11  | 6   | 2   | -   | 13  | 6   | 11  | 1   |
| 6     | 65  | 7   | 2   | 7   | 11  | 13  | -   | 8   | 1   | 19  |
| 7     | 62  | 1   | 7   | 2   | 5   | 6   | 8   | -   | 7   | 7   |
| 8     | 65  | 6   | 1   | 7   | 8   | 11  | 1   | 7   | -   | 3   |
| 9     | 67  | 6   | 2   | 5   | 10  | 11  | 19  | 7   | 3   | -   |
| 10    | 57  | 6   | 10  | 5   | 2   | 1   | 11  | 5   | 10  | 10  |

From the distance, we can calculate the cost per km by multiple the distance with traveling cost per kilometer. Table 6 shows the travel cost from depot to 10 customers.

Table 6. The travel cost for each customer and depot.

| DEPOT | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0     | 22863 | 19593 | 17870 | 17899 | 16819 | 19009 | 17987 | 19096 | 19418 | 16673 |
| 1     | 6     | 2     | 4     | 6     | 2     | 2     | 2     | 8     | 0     | 2     |
| 2     | 22863 | 19593 | 17870 | 17899 | 16819 | 19009 | 17987 | 19096 | 19418 | 16673 |
| 1     | 0     | 14600 | 2920  | 11096 | 17812 | 21608 | 4088  | 18688 | 17228 | 16644 |
| 2     | 14600 | 0     | 20440 | 24528 | 31244 | 7008  | 21024 | 4088  | 6424  | 30076 |
The vehicle needs 40 minutes to load all of the products to the compartments, then the vehicle needs 40 minutes to unload the product to each petrol station. To minimize the total cost of delivery, this problem solved by Ilog Cplex 12.7.1 with branch and cut algorithm. Using 5 available vehicles with 4 compartments or 32,000 liter capacity. Each compartment has 8,000 liter capacity. Each compartment carries only one kind of product.

To solve the problem using Ilog Cplex, we need to insert the mathematic formulation such as decision variables, constraints, and objective function into the software. If 0 represent the depot, so the result is:

Table 7. Alternatives solution in Cplex.

| Vehicle | Route      | Code Delivery | Time Windows         |
|---------|------------|---------------|----------------------|
| 1       | 0 – 5 – 10 – 0 | Depot – 44.505.05 – 44.506.01 – Depot | 15.00 – 16.27 – 17.29 – 19.56 |
| 2       | 0 – 3 – 4 – 0   | Depot – 44.505.03 – 44.505.04 – Depot | 15.00 – 16.30 – 17.39 – 20.02 |
| 3       | 0 – 1 – 7 – 0   | Depot – 44.505.01 – 44.505.07 – Depot | 15.00 – 16.18 – 17.27 – 19.53 |
| 4       | 0 – 9 – 2 – 0   | Depot – 44.505.09 – 44.505.02 – Depot | 15.00 – 16.38 – 18.19 – 20.30 |
| 5       | 0 – 8 – 6 – 0   | Depot – 44.505.08 – 44.505.06 – Depot | 15.00 – 17.00 – 18.18 – 20.50 |

Based on table 7, the best cost is Rp 1,898,584. The result of the mathematic formulation produces 5 routes from 5 available vehicles. For example, vehicle 1 must distribute the petrol product to customer 44.505.05 and 44.506.01. The vehicle will start shipping at 15.00, then at 16.27 the vehicle will arrive in point 44.505.05. Then the vehicle will unload the product and then continue to deliver the product to the last customer 44.506.01, and arrive at 17.29. After unloading, the vehicle must come back to the depot and arrive at 19.56.

4. Conclusion
Pertamina TBBM Boyolali should distribute their product everyday based on the customer need. From table 7, we knew that the best cost for this research is Rp 1,898,584. Using 5 available vehicles with the result of each vehicle are vehicle 1: Depot – 44.505.05 – 44.506.01 – Depot, vehicle 2: Depot – 44.505.03 – 44.505.04 – Depot, vehicle 3: Depot – 44.505.01 – 44.505.07 – Depot, vehicle 4: Depot – 44.505.09 – 44.505.02 – Depot, and vehicle 5: Depot – 44.505.08 – 44.505.06 – Depot. So the company could minimize their distribution cost for all routes by choosing the best route. This model could be applied for all routes, so the company could reduce the distribution cost.

Acknowledgments
This research is supported by the Institute for Research and Community Service, Universitas Sebelas Maret with Hibah Penelitian Unggulan Terapan UNS (PUT-UNS) Research Program (Contract No. 516/UN27.21/PP/2019).

5. References
[1] Tjiptono, F. Pemasaran Jasa – Prinsip, Penerapan, dan Penelitian, Andi Offset, Yogyakarta. (2014).
[2] Martins, S., et.al. Product-Oriented Time Window Assignment for a Multi-Compartment Vehicle Routing Problem. European Journal of Operation Research. (2019)

[3] Surjandari, I., et.al. Petrol Delivery Assignment with Multi-Product, Multi-Depot, Split Deliveries and Time Windows. International Journal of Modeling and Optimization, Vol. 1, No, 5, p. 375-379. (2011).

[4] Benantar, A. and Ouafi, R. Optimization of Vehicle Routes: An Application to Logistic and Transport of the Fuel Distribution. 9th International Conference on Modeling, Optimization and Simulation, Bordeaux France. (2012).

[5] Benantar, A., Ouafi, R., & Boukachour, J. A petrol station replenishment problem: new variant and formulation. Logist. Res. (pp. 1-18). Springerlink.com. (2016).

[6] Sevaux, M., & Sorensen, K. Hamiltonian Paths in Large Clustered Routing Problem. EU / MEeting 2008, (pp. 1-7). Troyes, France. (2008).

[7] Battara, M., Edorgan, G., & Vigo, D. Exact Algorithms for the Clustered Vehicle Routing Problem. Operation Research, 58-71. (2014).

[8] Henke, T., Speranza, M. G., & Wascher, G. The multi-compartment vehicle routing problem with flexible compartment sizes. European Journal of Operational Research, 730-743. (2015).

[9] Coelho, L. C., & Laporte, G. Classification, Models and Exact Algorithms for Multi-Compartment Delivery Problems. European Journal of Operational Research, 854-864. (2015).

[10] Belov, A., & Slastnikov, S. Modeling of Multi Depot Vehicle Routing Problem for Petroleum Products. Lobachevskii Journal of Mathematics, 884-887. (2017).

[11] Pop, P., & Chira, C. A Hybrid Approach based on Genetic Algorithms for Solving the Clustered Vehicle Routing Problem. IEEE Congress on Evolutionary Computation (CEC), 1421-1426. (2014).

[12] Marc, A. H., Fuksz, L., Pop, P. C., & Danciulescu, D. A decomposition-based method for solving the clustered vehicle routing problem. UK: Oxford University Press. (2017).

[13] Henke, T., Speranza, M. G., & Wascher, G. A branch-and-cut algorithm for the multi-compartment vehicle routing problem with flexible compartment sizes. Annals of Operations Research (pp. 1-18). Germany: Springer. (2018).