Optimization of helicopter blade section based on multi-objective optimization algorithm

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Abstract. This paper aims to find a better performance multi-objective optimization algorithm to optimize the helicopter blade section structure, so that the optimized blade section characteristics are better. Optimization objectives include the section quality linear density of the blade and the section stiffness characteristics of the blade. In this paper, three multi-objective optimization algorithms including non-dominated sorting genetic algorithm II (NSGA-II), non-dominated sorting genetic algorithm III (NSGA-III) and multi-objective evolutionary algorithm based on decomposition (MOEA/D) algorithm are used to optimize the blade section. The performance evaluation results of the three algorithms are evaluated by inverted generation distance (IGD). The calculation results show that the MOEA/D algorithm has better convergence, uniformity and extensiveness, and the comprehensive performance is better than other algorithms in high-dimensional multi-objective optimization. After optimization by the algorithm, the quality linear density of the section is reduced by 1.8%, the torsional stiffness of section is increased by 2.1%, the shimming stiffness of section is decreased by 1.9%, the waving stiffness of section is decreased by 1.7%, and the tensile stiffness is increased by 0.9%.

1. Introduction
Among the various components of the helicopter, the rotor system is the key moving part of the helicopter [1]. It consists of blades and hubs. The rotation of the blades around the center of the hub produces the helicopter's lift, forward force and maneuvering force, thus achieving helicopter flight. The structure of the blade directly affects the flight quality, reliability, safety and other performance of the helicopter. The performance of blade’s design has a very important impact on the performance of the helicopter. Compared with other structures, the blade structure dynamics problem (the problem of blade structure characteristics) is particularly prominent. In addition, because it works under continuous elastic vibration, it causes rotor fatigue problems and helicopter vibration problems that are
highly valued in helicopter development. Therefore, the optimization design of the blade structure is very important in the development of the helicopter. The optimization of the section characteristics of the blade is an important optimization goal. The section characteristics of the blade mainly include the quality linear density of the section, the torsional stiffness of the section, the tensile stiffness of the section, the waving stiffness of section, and shimming stiffness of section. There are mainly direct analytical methods and finite element analysis methods for the analysis of section characteristics of blade. This paper uses direct analytical method [2] to construct the section analysis model of blade. In the optimization, the section characteristics model is taken as the optimization target and the geometric parameters of the components in section of blade are used as design variables to optimize the section of the blade.

For blade structure optimization, the commonly used optimization methods can be divided into the following three types [3]: numerical optimization algorithm, direct search method and global exploration method. The numerical optimization algorithm includes MMFD [4] (Modified Method of Feasible Directions) algorithm and NLPQL [5] (Sequence Quadratic Programming) algorithm, which are mainly used to solve problems with nonlinear and continuous characteristics. The advantage of this method is that it can effectively explore the local area around the initial design point. The disadvantage is that it has a strong dependence on the initial design point and is easy to fall into the local optimal solution. The direct search method includes the Hooke-Jeeves algorithm [6], etc., which has the advantage of being able to efficiently search for the area around the initial point, and can search for a broader design space than the numerical optimization algorithm. The disadvantage is that the dependence on the initial design point is relatively strong. And it is possible to fall into the local optimal solution. The global exploration method mainly includes multi-objective optimization algorithms such as genetic/evolutionary algorithms [7,8] and simulated annealing algorithms [7]. The advantage is that there is a large probability to find the global optimal solution. The disadvantage is that the computational complexity is relatively large. In the optimization design, it is necessary to fully consider the characteristics of the actual optimization problem and select the appropriate optimization algorithm.

In engineering practice like the blade optimization, the optimization problems encountered are mostly multi-objective optimization problems—allowing multiple sub-goals to achieve optimal problems at the same time. In most cases, these sub-goals are often mutually exclusive, and improvements in one sub-goal may lead to deterioration of other sub-goals. In response to the above problems, designers generally compromise the goals and try to optimize each sub-goal. In recent years, researchers have proposed many excellent multi-objective optimization algorithms, including NSGA-II (non-dominated sorting genetic algorithm II), NSGA-III (non-dominated sorting genetic algorithm III), MOEA/D (multi-objective evolutionary algorithm based on decomposition) and other algorithms to solve the multi-objective optimization problem. NSGA-II [9] was proposed by Kalyanmoy Deb et al. in 2002. It is an improvement of the NSGA. Compared with NSGA, the improvement of NSGA-II has two main points: (1) Propose a fast non-dominated sorting, making Pareto The time complexity of dominating sorting is reduced. (2) A crowded distance is proposed to measure the distribution of solutions, and based on this selection of suitable individuals in the population. At present, NSGA-II is a mainstream multi-objective optimization algorithm. In particular, its framework based on Pareto dominance relationship has been adopted by many algorithms, such as NSGA-III, etc. At the same time, NSGA-II is good for low-dimensional multi-objective optimization
problems, but for high-dimensional multi-objective optimization problems, the first problem is that the selection pressure is too small due to its Pareto dominance relationship, and secondly That is, the crowded distance is not applicable in high-dimensional space, and the computational complexity is also relatively high. NSGAIII [10] was proposed by Kalyanmoy Deb in 2014. The algorithm adapted the order of crowding to maintain population diversity by introducing widely distributed reference points. MOEA/D [11] was proposed by Mr. Zhang Qingfu and others in 2008. MOEA/D converts a multi-objective optimization problem into multiple scalar quantum problems, and each sub-problem consists of a uniformly distributed weight vector. For each new solution generated, the solution near the subproblem is replaced based on the aggregate function.

In the optimization of the blade section in this paper, there are five optimization objectives, which belong to the high-dimensional multi-objective optimization problem. In this paper, We will use inverse generation distance [12] (IGD) to compare the effects of NSGA-II, NSGA-III and MOEA/D multi-objective optimization algorithms on blade section optimization to find a more suitable solution to the problem proposed in the paper. The optimized blade section structure is obtained by the better algorithm, and the improvement of the section characteristics of the optimized blade is calculated.

2. Modelling of blade section characteristics

2.1. Parametric description of blade section

This paper mainly studies the section structure of C-spar blade. Fig.1 is the schematic diagram of the C-spar blade section. It consists of four parts: skin, spar, beam and trailing edge.

![Figure 1. Schematic diagram of the C-spar blade section.](image)

For parametric description of blade section structure, we need to define the type of airfoil selected and the chord length of the airfoil (C). On this basis, four components are parameterized. For the skin, this paper uses the form of full skin layer, so it only needs to define the thickness (h, i=1,…,n) and quantity of the layer (n). For C-spar, trailing edges and beam, their parametric descriptions are shown in Fig. 2, Fig. 3, Fig. 4.

As shown in fig.2. The section of C-spar consists of inner and outer contour lines. The outer contour line can be obtained by cutting the inner airfoil contour of the skin layer by the points P_a and P_d. P_a is the point on the outer contour of the spar, and its chordwise distance to the leading edge point is Y_a. P_d is the lower end point of the outer contour of the spar, and its chordwise distance to the leading edge point is Y_d. The inner contour line can be determined by the coordinates of the Pu point, the Pd point, and the intermediate point {P_1, P_2, ..., P_n, ..., P_m}. Therefore, the section parameters of the spar can be described by Y_a, Y_d, Y_p and Z_p (i = 1, 2, ..., m).
As shown in fig.3. The profile of the beam can be represented by the chordwise distances of the four endpoints to the leading edge point \{Y_1, Y_2, Y_3, Y_4, Y_5\} and the thickness \Delta of the beam. As shown in fig.4. The section of the trailing edge is described by the two parameters of the lower edge endpoint \(P_u'\) to the leading edge point chordwise distance \(Y_F\) and the upper edge endpoint \(P_d'\) to the leading edge point chordwise distance \(Y_E\).

2.2 Modelling of blade section characteristics

The blade section characteristics are determined by the airfoil of the blade section, the distribution of the components, and the material properties of the component. It mainly includes section quality linear density \(M_p\), tensile stiffness \(E_s\), waving stiffness \(E_{ib}\), pendulum stiffness \(E_T\) and torsional stiffness \(\Gamma\). This paper uses direct analytical method[2] which proposed by Yang jianling to calculate the section characteristics of the C-spar blade.

Torsional stiffness of the Blade section \(\Gamma_{TOT}\) is mainly provided by torsion boxes consisting of skins (or consisting of skins and beam) and spar. In engineering applications, the torsional stiffness of the blade section is approximately equal to the sum of the torsional stiffness of the torsion box \(\Gamma\) and the torsional stiffness of the beam section \(\Gamma_{Spar}\).

\[
\Gamma_{TOT} = \Gamma + \Gamma_{Spar} \tag{1}
\]

Calculation formula for section torsional stiffness of single torsion box:

\[
\Gamma = \frac{4\Omega_i}{\alpha_i} \tag{2}
\]

Calculation formula for section torsional stiffness of double torsion box:

\[
\Gamma = \frac{4(\Omega_2\alpha_1 + \Omega_1\alpha_2 + (\Omega_1 + \Omega_2)\alpha^* \alpha^*)}{\alpha_1\alpha_2 + \alpha_1\alpha^* + \alpha_2\alpha^*} \tag{3}
\]

In Eq.(2) and Eq.(3), \(\Omega_i\) is the area enclosed by the torsion box. \(\alpha_i\) is the ratio of the perimeter and thickness of the skin layer constituting the torsion box \(i (i = 1, 2)\). \(\alpha^*\) is the ratio of the perimeter and the thickness of the beam region in the torsion box.
The blade C-spar can usually be regarded as solid beams, so the torsional stiffness calculation formula\(^I\) of the solid beam can be applied to obtain the torsional stiffness of the spar section:

\[
\Gamma_{\text{spar}} = \frac{G \cdot S^4}{4\pi^2 I}
\]  

(4)

In Eq.(4), \(S\) is the cross-sectional area of the spar, and \(I\) is the moment of inertia of the spar. They are all obtained by the blade geometry parameter extraction algorithm mentioned later; and \(G\) is the shear modulus of material.

Calculation formula of waving stiffness \(E_B\) and shimmy stiffness \(E_T\) in a Cartesian coordinate system:

\[
E_B = \sum_{e=1}^{E} E_e \cdot Z_N^2 \cdot S_e
\]  

(5)

\[
E_T = \sum_{e=1}^{E} E_e \cdot Y_N^2 \cdot S_e
\]  

(6)

Tensile stiffness \(E_s\) is an important parameter for strength checking. Its physical meaning refers to the ability of an object to resist tensile deformation. The calculation of the tensile stiffness of the blade section in the project is also calculated by accumulating the tensile stiffness of the components of the blade section:

\[
E_s = \sum_{e=1}^{E} E_e \cdot S_e
\]  

(7)

The quality linear density \(M_p\) of the blade section is a parameter that can describe the mass distribution of blade and calculate the blade weight. The blade is a combination of components which made up of a variety of different materials, so the quality linear density of the blade section is obtained by algebraic superposition of the mass linear density of the components:

\[
M_p = \sum_{e=1}^{E} \rho_e \cdot S_e
\]  

(8)

In Eq.(5-8), \(Y_N\) and \(Z_N\) represent the coordinates of the center of gravity of the components in the Cartesian coordinate system. \(E_e\) is the Young's modulus of the \(e\)-th component material. \(P_e\) is the density of the \(e\)-th component. \(S_e\) is the area of the \(e\)-th component material

3. Optimization of blade section characteristics

3.1. Optimization model

In this paper, there are five main objective of section optimization in this article:

- Minimize \(M_p(X)\) — finding the minimum of mass linear density of section
- Maximize \(\Gamma_{\text{TOT}}(X)\) — finding the maximum of torsional stiffness of section
- Maximize \(E_s(X)\) — finding the maximum of tensile stiffness of section
- Minimize \(E_T(X)\) — finding the minimum of shimmy stiffness of section
- Minimize \(E_B(X)\) — finding the minimum of waving stiffness of section

In this paper, based on the previous parametric description of the blade profile, the design variables are selected from the parameters defined. The results are as follows:

Design variable: \(X=[C \ h_i \ Y_a \ Y_d \ Y_f^p \ Z_f^p \ Y_u \’ \ Y_d^’ \ Y_1 \ Y_2 \ Y_3 \ Y_4 \ \Delta]\)
Blade section optimization is generally performed given the specific parameters required. However, it is difficult to ensure that the blade profile characteristics are equal to the given value during design. Therefore, according to the design experience (the tensile stiffness and torsional stiffness are as large as possible, mass linear density, the shimmy stiffness and the waving stiffness are as small as possible) and the given values, the following inequalities are obtained. The combination of constraints:

\[
\begin{align*}
G_1(x) &= M_p \geq \bar{M}_p \\
G_2(x) &= \Gamma_{TOT} \geq \bar{\Gamma}_{TOT} \\
G_3(x) &= E_B \leq \bar{E}_B \\
G_4(x) &= E_T \leq \bar{E}_T \\
G_5(x) &= E_s \geq \bar{E}_s
\end{align*}
\]

3.2. Optimization method

According to the characteristics of blade optimization, it is very important to select the appropriate solution method for the optimization of the blade section structure. In the optimization of the blade section, there are many design variables, the numerical changes are discrete, and the constraints have contradictory characteristics. For example, an increase in torsional stiffness causes an increase in the waving stiffness, while in the blade design it is desirable to have a greater torsional stiffness and a lower waving stiffness. In addition, there is a mutual constraint relationship between the blade optimization targets, which belongs to the multi-objective optimization problem. Therefore, it is necessary to choose a multi-objective optimization algorithm that can adapt to discrete design variables and constraint associations. At present, more applications in blade optimization are NSGA-II algorithms. This algorithm is good for low-dimensional multi-objective optimization problems, but for high-dimensional multi-objective optimization problems, it first faces the problem of too small selection pressure due to its Pareto dominance relationship. Secondly, the crowded distance is not applicable in high-dimensional space, and the computational complexity is also relatively high. Because there are five optimization goals in this paper, it belongs to the high-dimensional multi-objective optimization problem. It has the following three questions:

- As the number of optimization targets increases, the proportion of non-dominated solutions in the population also increases, which leads to a slow search process;
- For high-dimensional target space, the computational complexity of indicators for maintaining diversity is too high, and the neighboring elements of the solution are difficult to find;
- For high-dimensional target spaces, the search ability of the recombination operator is too inefficient.

Therefore, in order to solve the above problem, a predefined target search can be used, which can be divided into two types: the first one is a predefined set of search directions spanning the entire pareto front, and the representative of this method is MOEA/D; The other is to redefine multiple reference points, such as the algorithm NSGAIII.

In this paper, we use NSGA-II, NSGA-III, and MOEA/D algorithms to optimize the profile. We evaluate the optimization effects of the three algorithms through the evaluation indicators of the
multi-objective optimization algorithm to find the optimal optimization algorithm. The basic method of blade profile optimization is shown in the fig.5.

![Figure 5. Blade section optimization method.](image)

First you need to enter design variables and divide the population according to the design variables. The blade geometry feature extraction algorithm is used to calculate the geometric feature parameters of the component, such as area S, perimeter L, and moment of inertia I. This step translates the input design variable parameters into parameters that can be used to calculate the profile characteristics of the blade. The mathematical model of the blade profile is then used as the optimization target and constraints, and optimized by the multi-objective optimization algorithm. Finally, the optimized results are output.

The section feature extraction algorithm mainly considers the section of each component of the blade as a complex polygon, and converts the component area, arc length, moment of inertia, and extraction of the center of gravity into polygon area, arc length, moment of inertia and center of gravity. Each side of the polygon is determined by the set of imported airfoil coordinate points and the component geometry.

4. Result and discussion
This paper selects the OA212 as airfoil for optimization, the chord length is 350mm, and the airfoil contains four components: skin, C-spar, beam and trailing edge. In order to reduce the amount of calculation, the number of skin layers is n=1, and the number of intermediate points in the inner contour of the beam is m=1. In order to compare the optimized effects, the profile geometry parameter values must be set before optimization, as shown in table.1.
**Table 1.** Pre-optimization parameter list.

| Component name | Component parameter | material | ply number | h/mm | ply type |
|----------------|----------------------|----------|------------|------|----------|
| skin           |                      | C3228    | 1          | 1    | full coverage |
| spar           |                      | V3233    | 40         | 30   | Y1/mm, Y2/mm, Y3/mm, Y4/mm, Z1p/mm |
| beam           |                      | V3233    | 150        | 180  | 180       | 180       | 1 |
| trailing edge  |                      | V3233    | 280        | 280  | 1 |

The optimization goal of section optimization is to minimize the mass line density, maximize torsional stiffness, maximize tensile stiffness, minimize shimmy stiffness, and minimize waving stiffness. The constraint conditions for section optimization are shown in Table 2.

**Table 2.** Blade section optimization constraints.

| Constraint | $E_S$ (N) | $E_B$ (N·m²) | $E_T$ (N·m²) | $\Gamma_{TOT}$ (N·m²) |
|------------|-----------|--------------|--------------|----------------------|
| Range      | $\geq 8 \times 10^7$ | $(1.5 \times 10^4, 2 \times 10^4)$ | $(5 \times 10^5, 6 \times 10^5)$ | $\geq 2 \times 10^4$ |

The value range of the design variables is shown in Table 3.

**Table 3.** Range of design variables.

| variable | $h$ (mm) | $Y_u$, $Y_d$ (mm) | $Y_1p$ (mm) | $Z_1p$ (mm) | $Y_1$, $Y_2$, $Y_3$, $Y_4$ (mm) | $\Delta$ (mm) | $Y_u^\prime$, $Y_d^\prime$ (mm) |
|----------|---------|-----------------|------------|----------|-------------------------------|--------|----------------|
| Range    | (0, 1)  | (10, 70)        | (-10, 10)  | (100, 250) | (0.5, 1)                      | (280, 320) | |

NSGA-I, NSGA-III, and MOEA/D were used as optimization algorithms for optimization. For high-dimensional multi-objective optimization, it is difficult to show its pareto frontier, so we evaluate the optimization effect by the convergence performance and the distribution performance of the solution set.

The Inverted Generational Distance (IGD) is a comprehensive performance evaluation indicator. It evaluates the convergence performance and distribution performance of the algorithm by calculating the minimum distance between each point on the real Pareto frontier surface and the individual set obtained by the algorithm. The smaller the value, the better the comprehensive performance (convergence performance and distribution performance) of the algorithm.

In this paper, we set the population size to 400 and the evolutionary algebra to 2000, and use three algorithms to optimize the blade profile. Finally, we get the evolution algebra-IGD value relationship shown in Fig.6-8.
The test of the MOEA/D algorithm is decreasing faster, followed by NSGA-III algorithm, and last one is NSGA-II algorithm. It indicates that MOEA/D algorithm can shorten the distance between the resulting solution and the real solution faster. In addition, it can be seen from the IGD values of 2000th generation that the IGD values (NSGA-II, NSGA-III, and MOEA/D) are 3.53×10^{-5}, 3.43×10^{-5} and 3.21×10^{-5} respectively. The IGD value of MOEA/D is smaller, indicating that the obtained solution set is closer to the real Pareto frontier, the distribution of the solution set is more uniform, and the number of solution sets is also more abundant.

Since the multi-objective evolution/genetic algorithm has different solution sets for each calculation, in order to test the above conclusions, we repeat the experiment 30 times and calculate the mean and standard deviation of the IGD values obtained by each algorithm. We get the table as shown in Table 4.

| Table 4. IGD mean and standard deviation of NSGA-II, NSGA-III, MOEA/D. |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| Population size | evolutionary algebra | Run times | Average value of IGD | Standard deviation of IGD |
| NSGA-II | 400 | 2000 | 30 | 3.51×10^{-5} | 8.16×10^{-3} |
| NSGA-III | 400 | 2000 | 30 | 3.42×10^{-5} | 7.01×10^{-3} |
| MOEA/D | 400 | 2000 | 30 | 3.27×10^{-5} | 7.51×10^{-3} |

The grayed-out part of the table 4 is the average and standard deviation of the IGD obtained after optimizing the section 30 times using the MOEA/D algorithm. The average IGD of MOEA/D is the smallest among the three algorithms. This proves the previous conclusion——MOEA/D can get better optimization results.

The optimized design variables obtained by the MOEA/D algorithm are shown in Table 5.

| Table 5. Pre-optimized and optimized design variables. |
|------------|------------|------------|------------|------------|-------------|-------------|-------------|
| /mm | h | Y_u | Y_d | Y_p | Z_p | Y_1 | Y_2 | Y_3 | Y_4 | A | Y_a | Y_d |
| before | 1 | 40 | 30 | 36 | 0 | 150 | 180 | 150 | 180 | 1 | 280 | 280 |
| after | 0.62 | 45.9 | 45.2 | 30.2 | 0 | 191 | 221 | 191 | 221 | 0.7 | 288.3 | 287.7 |

According to the model of blade section characteristic, the parameters of the blade profile characteristics before and after optimization of the blade are calculated. The change situation of optimized section characteristics are obtained, as shown in Table 6.

| Table 6. rate of change of blade section characteristic. |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| section characteristic | Mp (kg • m) | Es (N) | E_{th} (N • m²) | E_{th} (N • m²) | \Gamma_{TOT} (N • m²) |
| -1.8% | 1.3% | -1.7% | -1.9% | 2.1% |
5. Conclusion
Through the IGD evaluation index, the optimization effects of NSGA-II, NSGA-III and MOEA/D algorithms in the optimization of blade profile were compared. The results show that the NSGA-II algorithm performs poorly in the high-dimensional multi-objective optimization problem, while the NSGA-III and MOEA/D algorithms show better comprehensive performance (convergence performance, diversity performance). Among them, MOEA/D works best.

After using the optimized design variables to calculate the blade structure characteristics, the results show that the section quality linear density is reduced by 1.8%, the section torsional stiffness is increased by 2.1%, the tensile stiffness is increased by 1.3%, and the swing stiffness and the shimmy stiffness are respectively reduced. 1.9% and 1.7%.

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