Device-independent certification of a nonprojective qubit measurement

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Quantum measurements on a two-level system can have more than two independent outcomes, and in this case, the measurement cannot be projective. Measurements of this general type are essential to an operational approach to quantum theory, but so far, the nonprojective character of a measurement could only be verified experimentally by already assuming a specific quantum model of parts of the experimental setup. Here, we overcome this restriction by using a device-independent approach. In an experiment on pairs of polarization-entangled photonic qubits, we violate a Bell inequality on qubits—however high—can always be explained by locally selecting from binary quantum measurements on a single system, as the outcomes of measurements on a single system can always be explained in terms of a hidden variable model where all are either 0 or 1, where depends on the preparation of the system. The situation changes when considering the correlations between independent measurements on an entangled system [5], but still, a violation of a conventional Bell inequality on qubits—however high—can always be explained by locally selecting from binary quantum measurements [9]. Yet, there are specialized Bell-like inequalities, where qubit measurements with more than three outcomes outperform the maximal violation attainable when only binary measurements are considered [10]. An analysis of this advantage reveals that this effect is very small and would require an overall visibility of more than

The qubit is the abstract notion for any system which can be modeled in quantum theory by a two-level system. In such a system, any observable has at most two eigenvalues and hence any projective measurement can have at most two outcomes. Still, a qubit allows for a finite number of different two-outcome measurements, the value of which, in general, cannot be known to the observer beforehand, but rather follows a binomial distribution. In quantum information theory, additional properties reflecting this binary structure have been revealed, e.g., the information capacity of a qubit is one classical bit, even when using entangled qubits [1]. Nonetheless, the properties of a qubit sometimes break with the binary structure, e.g., transferring the quantum state of a qubit is only possible with the communication of two classical bits and the help of entanglement [2]. Moreover, it is well-known that general quantum measurements can be nonprojective and have more than two irreducible outcomes [3]. The most general quantum measurement with outcomes is described by positive semidefinite, possibly nonprojective, operators with . The number of outcomes is reducible, if it is possible to write so that are measurements for each , is a probability distribution over , and for each there is at least one such that . Nonprojective measurements found first applications in quantum information processing in the context of the discrimination of nonorthogonal quantum states. Ivanovic [4] found that it is possible to discriminate two pure qubit states without error even if the two states are nonorthogonal, but at the cost of allowing a third measurement outcome that indicates a failure of the discrimination procedure. The strategy with the lowest failure probability can be shown to be an irreducible three-outcome measurement [5]. Also recently, nonprojective measurements proved to be essential in purely information theoretical tasks like improving randomness certification [6].
have been a measurement composed of binary quantum measurements on whatever quantum system and selected by the measurement apparatus, as shown in Fig. 1(b).

Since projective measurements on a qubit necessarily are binary or trivial, a violation of $I < 1.2711$ certifies the implementation of a nonprojective measurement. This requires, however, that the system at Alice’s laboratory is actually a qubit, which is manifestly the case in our experimental set-up, as we explain below. In addition, this assertion of Alice’s system being a qubit, can also be verified in a device-independent way by measuring the violation of the Clauser–Horne–Shimony–Holt (CHSH) Bell inequality [12]. If this violation is maximal, the joint state has to be a maximally entangled qubit-qubit state [13–15], independently of what measurement apparatuses are used. If the observed value for the CHSH violation deviates by $\epsilon$ from the maximum $2\sqrt{2} - 2$, the state must still have a fidelity of at least $1 - 2\epsilon$ with a maximally entangled qubit-qubit state [16]. A description of the system in the corresponding qubit-qubit-space is hence accurate up to $2\epsilon$.

The set-up of our experiment is shown in Fig. 2. Degenerate 810 nm photon pairs, with orthogonal polarisations, are produced from spontaneous parametric down-conversion (SPDC) in a bulk type-II nonlinear periodically poled potassium titanyl phosphate (PPKTP) 20 mm long crystal. The crystal is pumped by a single-longitudinal mode continuous wave 405 nm laser with 1 mW of optical power. We resort to an ultra-bright source architecture, where the type-II nonlinear crystal is placed inside an intrinsically phase-stable Sagnac interferometer [17–19]. This interferometer is composed of two laser mirrors, a half-wave plate (HWP) and a polarizing beamsplitter cube (PBS). HWP and PBS are both dual-wavelength with anti-reflection coatings at 405 nm and 810 nm. The fast axis of the HWP is set at 45 degree with respect to the horizontal, such that down-converted photons are generated in the clockwise and counter-clockwise directions. The clockwise and counter-clockwise propagating modes overlap inside the polarizing beamsplitter and, by properly adjusting the pump beam polarization mode, the two-photon state emerging at the output ports is $|\psi^+\rangle = (|HV\rangle + |VH\rangle)/\sqrt{2}$, where $|H\rangle$ ($|V\rangle$) denotes the horizontal (vertical) polarization of a down-converted photon. Due to the phase-matching conditions, there may be entanglement between other degrees of freedom of the generated photons, or coupling between the polarization and the momentum of these photons that would compromise the quality of the polarization entanglement. To avoid this we add extra spectral and spatial filtering. To remove the remaining laser light we adopt a series of dichroic mirrors followed by a long-pass color glass filter. Then, Semrock high-quality (peak transmission greater than 90%) narrow bandpass (full-width-half-maximum of 0.5 nm) interference filters centered at 810 nm are used to ensure that phase-matching

 FIG. 1. Testing correlations that cannot be explained in terms of binary measurements. (a) Scheme of the test performed. Pairs of entangled systems are sent to Alice’s and Bob’s laboratories (represented by boxes with yellow buttons at the top and lights of different color in the side). In each laboratory, one system is submitted to a measurement (represented by the yellow button pressed) and produces an outcome (represented by a light flashing). All possible measurements have two outcomes, except for Alice’s measurement $x = 3$ which has three outcomes (represented by lights of different color, green for 0, red for 1, and blue for 2). (b) Discarded scenario. Our experiment excludes that the outcomes of Alice’s measurement $x = 3$ are produced by a measurement apparatus that selects one out of three binary quantum measurements with outcomes 0/1, 1/2, or 2/0 (represented by three coins with green/red, red/blue, and blue/green sides, respectively).  

0.992 [9–11].

Here we introduce an inequality where this threshold is lowered to 0.9845, enabling the device-independent certification of a nonbinary measurement on a qubit. We consider a bipartite scenario, cf. Fig. 1(a), where one party, Alice, chooses one among four measurements $x = 0, 1, 2, 3$ while the other party, Bob, chooses one among three measurements $y = 0, 1, 2$. All measurements have two outcomes, $a = 0, 1$ and $b = 0, 1$, except Alice’s measurement $x = 3$, which has three outcomes, $a = 0, 1, 2$. We denote by $P(ab|x y)$ the probability for outcome $a$ and $b$ when the setting $x$ and $y$ were chosen and consider the expression

$$I = P(00|00) + P(00|11) + P(00|22) - P(00|01) - P(00|12) - P(00|20) - P(00|30) - P(10|31) - P(20|32).$$

(1)

When restricted to binary quantum measurements, not necessarily on a qubit, then the value of $I$ is upper bounded by 1.2711. Without this restriction, the maximal quantum value of $I$ is $3\sqrt{3}/4 \approx 1.2990$ and can be achieved for two qubits using a maximally entangled state. Thus, an experiment violating the inequality $I < 1.2711$ proves that Alice’s measurement $x = 3$ cannot
FIG. 2. Experimental set-up. A PPKTP nonlinear crystal placed into a phase-stable Sagnac interferometer is pumped by a single mode laser operating at 405 nm to produce pairs of polarization-entangled photons at 810 nm. The quarter-wave plate QWP and the half-wave plate HWP are used to control the polarization mode of the pump beam. Dichroic mirrors (D) and longpass color filters are used to remove the pump beam light. The generated photons are then sent to Alice and Bob through single-mode fibers (SMF). Alice (Bob) can choose among three different binary measurements (depicted in blue boxes) labeled by $x = 0, 1, 2$ ($y = 0, 1, 2$). These measurements are performed using a set of a QWP, a HWP, and a PBS. Besides, Alice also performs a three-outcome measurement $x = 3$ using a polarization based two-path Sagnac interferometer (depicted in the Alice’s violet box). The elements of the three-outcome qubit measurement are defined by HWP$_r$, HWP$_t$, and HWP$_o$. The coincidence counts between Alice’s and Bob’s detectors are recorded using a coincidence electronics unit based on a field programmable gate array device.

Conditions are achieved with the horizontal and vertical polarization modes at degenerated frequencies.

The indistinguishability of the photon pair modes ("HV" and "VH") is guaranteed by coupling the generated down-converted photons into single mode fibers. These fibers implement a spatial mode filtering of the down-converted light, destroying any residual spatial entanglement or polarization-momentum coupling. To maximize the source’s spectral brightness, we resort to a numerical model [20]. In our case, the beam waist $w_p$ of the pump beam, and $w_{SPDC}$ of the selected down-converted modes, at the center of the PPKTP crystal, are adjusted by using a 20 cm focal length lens ($L_1$) and 10X objective lenses. The optimal condition for maximal photon-par yield is obtained when $w_{SPDC} = \sqrt{2} w_p$, with $w_p = 40 \mu m$. The observed source spectral brightness was 410000 photon pairs (s mW nm)$^{-1}$. The quality of the polarization entanglement generated at the source site was measured by observing a mean two-photon visibility of $0.987 \pm 0.002$ while measuring over the logical and diagonal polarization bases.

Due to the demand of a high overall visibility we built a coincidence electronics based on a field programmable gate array platform and capable of implementing up to 1 ns coincidence windows, thus reducing the probability for an accidental coincidence count to less than 0.00025. Therefore, the evaluation of the data does not require a separate treatment for accidental coincidence counts. The down-converted photons are registered using PerkinElmer single-photon avalanche detectors with an overall detection efficiency of 15%. We account for this by including the assumption into our analysis that the detected coincides are a fair sample from the set of all photon pairs.

Alice’s and Bob’s binary measurements are implemented using a set composed of a quarter wave plate (QWP), a HWP, and a PBS for each party, cf. Fig. 2. A high-quality film polarizer is also used in front of the detectors (not shown for sake of clarity) to obtain a total extinction ratio of the polarizers equal to $10^7$:$1$. Therefore, in our experiment the two-photon visibility is not upper limited by the polarization contrast of our measurement apparatuses. Alice’s three-outcome measurement $x = 3$ is implemented using the propagation modes of Alice’s down-converted photon. With this purpose, Alice’s photons are sent, after displacing a removable mirror, through a polarization based two-path Sagnac interferometer. The propagation modes of a photon within this interferometer are not co-propagating and depend on its polarization state. This allows for conditional polar-
The measurement settings were implemented independently for Alice and Bob, justifying the assumption that Alice’s measurements also act independently of Bob’s measurement setting $y$ and vice versa. Hence, any explanation in terms of binary measurements on an arbitrary quantum system is excluded by 8.7 standard deviations, which corresponds to a p-value of $1.6 \times 10^{-18}$.

In order to prove that Alice’s measurement $x = 3$ is a nonprojective quantum measurement, we need also to verify that Alice’s system can be properly described as a qubit. We rely here on two complementary arguments. First, one can resort to the design of the experiment where the source is designed to produce entanglement in polarization, i.e., qubit-qubit entanglement. Second, we measured the CHSH correlations with our set-up and observed a violation of $2\sqrt{2} - 2 - \epsilon$ with $\epsilon = 0.0253 \pm 0.0014$ and hence the fidelity with a maximally entangled qubit-qubit state is guaranteed in a device-independent way to be at least 0.9351 within 3 standard deviations [16].

Note, that we measured the CHSH correlations using the same source and the same measurement set-up as we used for the measurement of $I$—except that different angles at the HWPs are adopted. Except for some ubiquitous adversary ad-hoc models we can hence conclude that also in the measurement of $I$, the fidelity of the state with a maximally entangled qubit-qubit state is at least 0.9351. Notice that the estimate for the fidelity is pessimistic since imperfections in the measurement apparatuses reduce the CHSH violation and therefore lower the bound on the fidelity. Still, in an alternative explanation where 93.51\% of the times binary measurement was used, a bound of $I < 0.9351 \times 1.2711 + 0.0649 \times 3\sqrt{3}/4 < 1.2730$ would have to be obeyed, which is clearly violated in the experiment.

Our result shows that three-outcome nonprojective measurements can produce strictly stronger correlations between two qubits than projective two-outcome measurements on any quantum system and, therefore, that nature cannot be described in terms of binary quantum tests, not even when these tests are performed on two-level quantum systems. Quantum theory predicts also qubit-qubit correlations that cannot be explained as produced by three-outcome measurements. Observing them requires an overall visibility above 0.9927, which is beyond what is currently feasible in our set-up. Further theoretical and experimental efforts will be needed to identify and produce qubit correlations which can only be explained by four-outcome nonprojective measurements. This will be the farthest we can go, as qubit correlations can always be accounted that way [21].

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Maximal value of $I$ for binary quantum measurements.

To obtain the bound for $I$ while considering only binary quantum measurements, we note that $I$ contains the chained Bell inequality $^{[22, 23]} I_{\text{chain}} \leq 1$ with three settings, where

$$I_{\text{chain}} = P(00|00) + P(00|11) + P(00|22) - P(00|01) - P(00|12) - P(00|20). \quad (2)$$

The remainder, $I - I_{\text{chain}} = -P(00|30) - P(10|31) - P(20|32)$ only involves correlations of Alice’s three-outcome measurement $x = 3$. There are three possibilities for replacing Alice’s measurement $x = 3$ by a binary measurement, by omitting $a = 0$, $a = 1$, or $a = 2$. Taking into account the permutation symmetry of $I$, all of them are equivalent to $I = I_{\text{chain}} - P(00|30) - P(10|31)$. We used the Navascués–Pironio–Acín (NPA) hierarchy $^{[24]}$ to obtain an upper bound on the maximal value $I'$. Running level 2 of the hierarchy, we obtain $1.271045$ for this bound. Within the numerical precision, this value can be attained with a partially entangled qubit-qubit state showing that 1.2711 also corresponds to the maximal value of $I$ with binary qubit measurements.

Maximal value of $I$ for arbitrary quantum measurements.

An upper bound on the maximal value of $I$ attainable in quantum theory can be obtained by upper bounding $I_{\text{chain}}$ and the remainder $I - I_{\text{chain}}$ separately. The maximal value of $I_{\text{chain}}$ is $3\sqrt{3}/4$ and can be attained with a qubit-qubit maximally entangled state $^{[25]}$. On the other hand, by construction, $I - I_{\text{chain}}$ cannot be greater than zero since it only contains nonpositive terms. Put together, the maximal value of $I$ is upper bounded by $3\sqrt{3}/4$.

This value is tight and can be attained by preparing the qubit-qubit state $|\psi^+\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$ and choosing the following measurements: Alice’s binary measurements $x = 0, 1, 2$ are defined by $M_{0|x} = P(\alpha_x)$ and $M_{1|x} = \mathbb{1} - P(\alpha_x)$, with $P(\theta) = (\mathbb{1} + \sigma_x \cos \theta + \sigma_z \sin \theta)/2$, where $\sigma_x$ and $\sigma_z$ are Pauli matrices, and the angles are given by $\alpha_0 = 3\pi/2$, $\alpha_1 = \pi/6$, and $\alpha_2 = 5\pi/6$. Alice’s three-outcome measurement $x = 3$ is defined by $M_{a|x} = 2P(\gamma_a)/3$ for $a = 0, 1, 2$, with angles $\gamma_0 = 2\pi/3$, $\gamma_1 = 4\pi/3$, and $\gamma_2 = 0$. Bob’s measurements are defined by $M_{0|y} = P(-\gamma_y)$ and $M_{1|y} = \mathbb{1} - P(-\gamma_y)$ for $y = 0, 1, 2$.

Implementation of Alice’s three-outcome measurement.

The three-outcome measurement is implemented by sending Alice’s down-converted photon through a polarization based two-path Sagnac interferometer, cf. Fig. 2. We write $|H\rangle$ ($|V\rangle$) for the horizontal (vertical) polarization. The mode entering the interferometer, rotating counter-clockwise and leaving for outcomes 0 and 1 is denoted by $|a\rangle$. $|b\rangle$ denotes the clockwise rotating mode leaving for outcome 3. In this way, the action of the PBS within the interferometer is given by

$$U_{\text{PBS}} = |H\rangle\langle H||a\rangle\langle a| + |b\rangle\langle b| + i|V\rangle\langle V||a\rangle\langle b| + |b\rangle\langle a|. \quad (3)$$

The actions of the HWP$_{\gamma}$ and HWP$_{\gamma'}$ of the interferometer in the transmitted and reflected mode, respectively, combine to $U_{\text{total}} = U_{\text{HWP}(\gamma)}|a\rangle\langle a| + U_{\text{HWP}(\gamma')}|b\rangle\langle b|$, where $U_{\text{HWP}(\gamma)}$ is the Jones matrix of a HWP whose fast axis is oriented at an angle $\gamma'$ with respect to the horizontal axis

$$U_{\text{HWP}(\gamma')} = \cos(2\gamma')(|H\rangle\langle H| - |V\rangle\langle V|) + \sin(2\gamma')(|V\rangle\langle H| + |H\rangle\langle V|). \quad (4)$$

Therefore, the Sagnac interferometer is described by $U_S = U_{\text{PBS}}U_{\text{total}}U_{\text{PBS}}$.

After the interferometer, the photon in mode $|a\rangle$ is transmitted through HWP$_x$ and an additional PBS. On the polarization degree of freedom, the three outcome modes 0, 1, and 2 are hence mediated by $|\psi\rangle \rightarrow A_k|\psi\rangle$ with the Kraus operators

$$A_0 = \langle b|U_{\text{PBS}}U_{\text{HWP}(\gamma_0')}|a\rangle\langle a|U_S|a\rangle, \quad A_1 = \langle a|U_{\text{PBS}}U_{\text{HWP}(\gamma_1')}|a\rangle\langle a|U_S|a\rangle, \quad A_2 = \langle b|U_S|a\rangle, \quad (5)$$

so that the implemented three-outcome measurement is given by $M_{k|3} = A_k|a\rangle$ for $k = 0, 1, 2$. The measurement required for a maximal violation of $I$ is achieved with $\gamma_0' = 0$, $\gamma_0' \approx 117.37^{\circ}$, and $\gamma_0' = 112.5^{\circ}$.

Qubit-qubit correlation inexplicable by three-outcome nonprojective measurements.

We consider a scenario where Alice chooses among the binary measurements $x = 0, 1, 2$ and the four-outcome measurement $x = 3$ and Bob chooses among the binary
measurement $y = 0, 1, 2, 3$. The expression

$$L = \beta_{el} - 8 \sum_{i=0}^{3} P(i, 0)[3, i],$$

(6)

has been used in Ref. [6] in the context of randomness extraction. The term $\beta_{el}$ was introduced by Bechmann-Pasquinucci and Gisin [26] in the Bell inequality $\beta_{el} \leq 6$, where

$$\beta_{el} = +P(10|02) + P(10|03) + P(10|11) + P(10|13)$$

$$+ P(10|21) + P(10|22) + 2P(00|00)$$

$$+ 2P(00|10) + 2P(00|20) + 4P(00|01)$$

$$+ 4P(00|12) + 4P(00|23) - 2P(10|00)$$

$$- 2P(10|10) - 2P(10|20) - 3P(00|02)$$

$$- 3P(00|03) - 3P(00|11) - 3P(00|13)$$

$$- 3P(00|21) - 3P(00|22).$$

(7)

Applying the methods developed in Section and Section, one finds, using the third level of the NPA-hierarchy, that the value of $L$ is upper bounded by 6.6876 for binary measurement and by 6.8489 for three-outcome measurements. Using four-outcome qubit measurements, $L$ can reach a value of $4\sqrt{3} > 6.9282$. Therefore, a verification of an irreducible four-outcome qubit measurements requires a visibility of 0.9928.

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