Three-body Description of $2n$-Halo and Unbound $2n$-Systems: $^{22}$C and $^{26}$O

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We study the two-neutron correlations in the ground state of the weakly-bound two-neutron halo nucleus $^{22}$C sitting at the edge of the neutron-drip line and also in the unbound nucleus $^{26}$O sitting beyond the neutron dripline. For the present study, we employ a three-body (core + $n$ + $n$) structure model designed for describing the two-neutron halo system by explicit inclusion of unbound continuum states of the subsystem (core + $n$). We use either a density-independent or a density-dependent contact-delta interaction to describe the neutron-neutron interaction and its strength is varied to fix the binding energy. We report the configuration mixing in the ground state of these systems for different choices of pairing interactions.

**KEYWORDS:** Two-neutron halo, Two-neutron unbound system, Two-neutron correlations.

1. Introduction

The new generation of radioactive ion beam facilities around the various parts of the globe has provided the access to the neutron-rich side of the nuclear chart. Due to this, there has been a rapidly increasing interest in the physics of the two-neutron (2n) halo nuclei sitting right on the top of neutron driplines and decays of 2n-unbound systems beyond the neutron dripline. These systems demand a three-body (3b) description with proper treatment of continuum, the conventional shell-model assumptions being insufficient. The well established 2n-halo $^6$He has long history of studies in 3b-framework and thus it can be safely remarked that the understanding of 3b-dynamics is fairly established for p-shell nuclei, whereas for the s-d shell nuclei still the situation is in progress stage [1]. In the present study we consider two different s-d shell nuclei, the 2n-halo $^{22}$C and the 2n-unbound system $^{26}$O. Very recently a high precision measurement of the interaction cross-section for $^{22}$C was made on a carbon target at 235 MeV/nucleon [2] and also the unbound nucleus $^{26}$O has been investigated, using invariant-mass spectroscopy [3] at RIKEN. The structural spectroscopy of the two-body subsystem plays a vital role in the understanding the 3b-system. These high precision measurements and the sensitivity of the structural spectroscopy of subsystem with the structure of 3b-system (core+$n$+$n$), are the motivation for selecting these nuclei for the present study. We have studied the pairing collectivity in the ground state of the 2n-halo $^{22}$C and in the the 2n-unbound system $^{26}$O. For this study we have used our recently implemented 3b-structure model (core+$n$+$n$) for the ground and continuum states of the 2n-halo nuclei [4–6]. We have explored the role of different pairing interactions such as density independent (DI) contact-delta pairing interaction and density dependent (DD) contact-delta pairing interaction with the configuration mixing in the ground-state of these systems.
2. Model Formulation

In our approach we consider a 3b-system consisting of an inert core nucleus and two valence neutrons, which is specified by Hamiltonian

$$H = -\frac{\hbar^2}{2\mu} \sum_{i=1}^{2} \nabla_i^2 + \sum_{i=1}^{2} V_{\text{core}+n}(\vec{r}_i) + V_{12}(\vec{r}_1, \vec{r}_2)$$

(1)

where $\mu = A_c m_N / (A_c + 1)$ is the reduced mass, and $m_N$ and $A_c$ are the nucleon mass and mass number of the core nucleus, respectively. The recoil term is neglected in the present study, as $A_c = 20$ and 24 are large enough to ignore it. $V_{\text{core}+n}$ is the core-$n$ potential and $V_{12}$ is $n$-$n$ potential. The neutron single-particle bound $s$-, $p$-, $d$- and $f$-wave continuum states of the subsystem $^{21}\text{C}$ and and $^{25}\text{O}$ are calculated in a simple shell model picture for the converged model parameter, bin width ($\Delta E = 0.1$), by using the Dirac delta normalization and are checked with a more refined phase-shift analysis. These core + $n$ continuum wave functions are used to construct the two-particle states of the core + $n$ + $n$ system by proper angular momentum couplings. We use a density-independent (DI) contact-delta pairing interaction for simplicity, and its strength is the parameter which will be fixed to reproduce the ground-state energy. We have also used a density-dependent (DD) contact-delta pairing interaction. For a detailed formulation one can refer to [4–6].

3. Two-body unbound subsystems (core + $n$)

The investigation of the two-body (core + $n$) subsystem is crucial in understanding the three-body system (core + $n$ + $n$). The interaction of the core with the valence neutron ($n$) plays a vital role in the binding mechanism of the core + $n$ + $n$ system. The elementary concern over the choice of a core + $n$ potential is the scarce experimental information about the core-neutron systems. We employ the following core + $n$ potential

$$V_{\text{core}+n} = \left( V_0' + V_{ls} l' \right) \frac{d}{r} \frac{1}{1 + \exp\left(\frac{r-R_c}{a}\right)},$$

(2)

where $R_c = r_0 A_c^{1/3}$ with $r_0$ and $a$ are the radius and diffuseness parameter of the Woods-Saxon potential.

3.1 $^{21}\text{C}$

For $^{21}\text{C}$, not much is known beyond that it is unbound. The only available experimental study using the single-proton removal reaction reported the limit to the scattering length $|a_0| < 2.8$ fm and due to the low statistics of this experimental data at low energies, the possibility of low-lying resonance states can not be ruled out [7]. In the view of exploring the sensitivity of the core-$n$ potential to the possible resonances and configuration mixing in the ground state of $^{22}\text{C}$, very recently we examined in detail the four different potential sets (for details see text and Table 1 of Ref. [6]). Here we will discuss the results corresponding to potential Set-1, which is adopted from the literature [9], because of its acceptance by different 3-body models leading to good explanation of the observed properties for $^{22}\text{C}$. The subshell closure of the neutron number 14 is assumed for the core configuration given by $(0s_{1/2})^2(0p_{3/2})^4(0d_{5/2})^6$. The seven valence neutron continuum orbits, i.e., $s_{1/2}, d_{3/2}, f_{7/2}, p_{3/2}, f_{5/2}, p_{1/2}$ and $d_{5/2}$ are considered in the present calculations for $^{21}\text{C}$.

3.2 $^{25}\text{O}$

In the recent measurement conducted at RIKEN [3], along with high accuracy measurement of ground state of $^{26}\text{O}$, they have also reported the $d_{3/2}$ resonance state at 749(10) keV with width of 88(6) keV for $^{25}\text{O}$. This information will serve as input for fixing the core+$n$ potential parameters. For $^{25}\text{O}$, we adopt the same value for the diffuseness parameter ($a$) and the radius parameter ($r_0$), as in Ref. [10]. For the Wood-Saxon depth parameter ($V_0$) and the strength of spin-orbit potential ($V_{ls}$) parameter tabulated in Table I, we use the information for the energy of unbound $d_{3/2}$ state. Our parameters for $V_0$ and $V_{ls}$ are consistent with the one reported in Ref. [10]. The neutron number 16 is assumed for the core configuration given by $(0s_{1/2})^2(0p_{3/2})^4(0p_{1/2})^2(0d_{5/2})^6(1s_{1/2})^2$. The three
valence neutron continuum orbits, i.e., $d_{3/2}$, $p_{3/2}$ and $f_{7/2}$ are considered in the present calculations for $^{25}\text{O}$.

4. Results and Discussions

The 3b-model with two non-interacting particles in the above single-particle levels of $^{21}\text{C}$ and $^{25}\text{O}$ produces different parity states, when two neutrons are placed in different unbound orbits. The seven configurations, $(s_{1/2})^2$, $(p_{1/2})^2$, $(p_{3/2})^2$, $(d_{3/2})^2$, $(d_{5/2})^2$, $(f_{5/2})^2$ and $(f_{7/2})^2$ couple to $J^p = 0^+$ for $^{22}\text{C}$ and three configurations $(d_{3/2})^2$, $(p_{3/2})^2$ and $(f_{7/2})^2$ couple to $J^p = 0^+$ for $^{26}\text{O}$. In the 3b-calculations, along with the core-n potential the other important ingredient is the $n-n$ interaction. An attractive contact-delta pairing interaction is used, $g_0(\vec{r}_1 - \vec{r}_2)$ for simplicity, with the only adjustable parameter being $g$. We also use the DD contact-delta pairing interaction to explore the role of different pairing interactions. In the DD contact-delta pairing interaction (defined by Eq. (8) of Ref. [6]), the strength of the DI part is given as $v_0 = 2\pi^2\hbar^2m_n\frac{2a_{nn}}{n-2ka_{nn}}$, where $a_{nn}$ is the scattering length for the free neutron-neutron scattering and $k_c$ is related to the cutoff energy, $e_c$, as $k_c = \sqrt{\frac{m_ne_c}{\hbar^2}}$. We use $a_{nn} = 15\text{ fm}$ and $e_c = 30\text{ MeV}$ [10], which leads to $v_0 = 857.2\text{ MeV fm}^3$. For the parameters of the DD part, we determine them so as to fix the ground-state energy of $^{22}\text{C}$ and $^{26}\text{O}$, $E = -0.140\text{ MeV}$ [8] and $0.018\text{ MeV}$ [3] respectively. The values of the parameters that we employ are $R_p = 1.25\times A^{1/3}_c$, $A_c = 20, 24$, $a = 0.65\text{ fm}$ and $v_p = 591.55$ and $1058.70\text{ MeV fm}^3$ for $^{22}\text{C}$ and $^{26}\text{O}$ respectively. We found that configuration mixing in the ground state of $^{22}\text{C}$ does not change much with the choice of $n-n$ interaction. We present the numbers for our potential Set 1 in Table II which are consistent with results of Ref. [9], and the same behavior is observed for the other sets [6]. Whereas for the $^{26}\text{O}$ case our results report different magnitude of configuration mixing for the different interactions and the results corresponding to the DD part that are consistent with the results of Ref. [10], where they have also used the DD interaction. One possible reason for this difference is that $^{26}\text{O}$ is unbound by 18 keV whereas $^{22}\text{C}$ is bound by 0.140 MeV. In order to reach a final conclusion we need more analysis, which will be reported elsewhere.

The two particle density of $^{22}\text{C}$ and $^{26}\text{O}$ as a function of two radial coordinates, $r_1$ and $r_2$, for valence neutrons, and the angle between them, $\theta_{12}$ in the LS-coupling scheme is calculated by following Refs. [5, 10]. The distribution at smaller and larger $\theta_{12}$ are referred to as “di-neutron” and “cigar-like” configurations, respectively. One can see in Fig. 1 that the two-particle density is well concentrated around $\theta_{12} \leq 90^\circ$, which is the clear indication of the di-neutron correlation and the di-neutron component has a relatively higher density in comparison to the small cigar-like component for both $^{22}\text{C}$ and $^{26}\text{O}$. The reflection of dominance of $s$-component in ground state of $^{22}\text{C}$ can be seen in left panel of Fig. 1 showing extended di-neutron component in comparison to $^{26}\text{O}$ (in right panel of Fig. 1), which has sharper dineutron component due to the mixing of $l > 0$ components in its ground-state.

5. Conclusions

In the present study we present the emergence of bound 2$n$-halo ground state of $^{22}\text{C}$ from the coupling of seven unbound $spd f$-waves in the continuum of $^{21}\text{C}$ and 2$n$-unbound ground state of $^{26}\text{O}$ from the coupling of three unbound $pdf$-waves in the continuum of $^{25}\text{O}$ due to presence of

| $lj$   | $r_0$(fm) | $a$(fm) | $V_0$(MeV) | $V_0$(MeV) | $E_0$(MeV) | $T$(MeV) |
|-------|----------|---------|------------|------------|------------|----------|
| $d_{3/2}$ | -44.10   | 22.84   | 0.740      | 0.086      |            |          |
| $p_{3/2}$ | 1.25     | 0.72    | -48.67     | 22.84      | 0.570      | 1.382    |
| $f_{7/2}$ | -44.10   | 22.84   | 2.440      | 0.206      | 2.026      |          |

Table I. Parameter sets of the core-n potential for $l = 1, 2, 3$ states of a $^{24}\text{O} + n$ system. The possible resonances with resonance energy $E_R$ and decay width $\Gamma$ in MeV are also tabulated.
Table II. Components of the ground state ($0^+$) of $^{22}$C and $^{26}$O, with model parameter energy cut $E_{\text{cut}}$. For $^{22}$C we have used the potential Set 1 of Table I of [6] for the shallow case with the ground-state energy, $-0.140$ MeV and for $^{26}$O we have used the potential tabulated in Table I. In last column of the table, the comparison has been made with the Ref. [9] for $^{22}$C and Ref. [10] for $^{26}$O.

| System | $E_{\text{cut}}$ (MeV) | $l/j$ | DI | DD | Reference |
|--------|-----------------------|------|----|----|-----------|
|        |                       |      |    |    | Present work |
| $^{22}$C | 5                    | $(s_1/2)^2$ | 0.923 | 0.899 | 0.915 |
|         |                       | $(p_1/2)^2$ | 0.004 | 0.008 | 0.009 |
|         |                       | $(s_1/2)^2$ | 0.022 | 0.029 | 0.024 |
|         |                       | $(d_3/2)^2$ | 0.045 | 0.047 | 0.033 |
|         |                       | $(d_5/2)^2$ | 0.001 | 0.005 | 0.003 |
|         |                       | $(f_5/2)^2$ | 0.0003 | 0.001 | 0.003 |
|         |                       | $(f_7/2)^2$ | 0.003 | 0.007 | 0.007 |
| $^{26}$O | 10                   | $(d_3/2)^2$ | 0.798 | 0.643 | 0.661 |
|         |                       | $(p_3/2)^2$ | 0.024 | 0.088 | 0.105 |
|         |                       | $(f_1/2)^2$ | 0.178 | 0.268 | 0.183 |

Fig. 1. Two-particle density for the ground state of $^{22}$C (left-panel) and $^{26}$O (right-panel) as a function $r_1 = r_2 = r$ and the opening angle between the valence neutrons $\theta_{12}$ for settings mentioned in caption of Table II.

Pairing interaction. Contribution of different configurations has been presented along with the 2n-correlations. More investigations are needed to comment on the role of different pairing interactions and to explain in detail the role of each wavefunction component.

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