Hierarchical structure of stock price fluctuations in financial markets

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Abstract. The financial market and turbulence have been broadly compared on account of the same quantitative methods and several common stylized facts they share. In this paper, the She–Leveque (SL) hierarchy, proposed to explain the anomalous scaling exponents deviating from Kolmogorov monofractal scaling of the velocity fluctuation in fluid turbulence, is applied to study and quantify the hierarchical structure of stock price fluctuations in financial markets. We therefore observed certain interesting results: (i) the hierarchical structure related to multifractal scaling generally presents in all the stock price fluctuations we investigated. (ii) The quantitatively statistical parameters that describe SL hierarchy are different between developed financial markets and emerging ones, distinctively. (iii) For the high-frequency stock price fluctuation, the hierarchical structure varies with different time periods. All these results provide a novel analogy in turbulence and financial market dynamics and an insight to deeply understand multifractality in financial markets.

Keywords: scaling in socio-economic systems
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1. Introduction

The financial market has been deemed as a complex system due to the large number of interacting individuals. However, according to the association between the interactions and stock movements, the study of time series of stock prices provides an entry to explore the intrinsic interacting mechanisms of stock markets for experts in the scientific community. Bachelier firstly assumed the evolution of stock prices as Brownian motion, the simplest stochastic processes [1], while advanced investigation by Mandelbrot pointed out the returns of cotton prices did not follow the Gaussian distribution, but rather a Lévy stable distribution [2]. Furthermore, a fatter tail of the return distribution than the Lévy stable distribution was observed by Mantegna and Stanley when analyzing the S&P 500 index constructed from a series of stock prices [3]. Since then, the stylized facts, namely the statistical features of financial time series in global stock markets are investigated by both economists and physicists [4]–[7]. Several stylized facts are revealed comprehensively and empirically, such as the fat tail in the return distribution and long-range volatility correlation [8]–[13], and simple agent-based models are successfully raised to reproduce and explain these statistical features [14]–[18].

As the statistical techniques to quantitatively measure the time series of stock prices have been commonly adopted in turbulence for a long while, the analogies in the statistical properties between turbulence and stock market dynamics have been surveyed since 1996 by Ghashghaie et al, who presented the similarities in probability densities of foreign exchange rate changes and the velocity difference of turbulence flow depending on the time delay and spatial separation respectively, as well as intermittency and cascade dynamics [19].

Further study by Mantegna and Stanley revealed that there was intermittency in both processes, yet the shapes of probability density functions are different, when they systematically compared the statistical features of the S&P 500 index with velocity of turbulent air [20, 21]. Inspired by this parallel analysis, a multifractal process is available to model the time series of stock price on account of the exhibition of intermittency and...
There are plenty of works describing multifractal structure in financial markets, such as the generalized Hurst exponent, one of the most popular methods, having been widely used to quantify the multifractality of time series of stock prices by observing the dependence of the scaling exponent $H(q)$ on order $q$ [27]. However, few of them have been engaged in exploiting the intrinsic understanding of this phenomenon in stock markets. While in turbulent fluid flows, the analogous anomalous scaling exponent deviating from monofractality is explained by the SL hierarchy, which is advanced by She and Leveque after Kolmogorov and his collaborators improving the linear scaling law [28, 29]. The SL hierarchy implies a scaling model with only three parameters, and thus could help to fully exploit the intrinsic evolutional mechanics in time series of stock prices as well. In this work, we therefore formulate the SL hierarchy from turbulence to investigate the time series of stock prices from developed to emerging financial markets, and find the hierarchical structure associated with the multifractal scaling generally rooted in them. The SL hierarchy provides extended interpretation of the fractal properties in time series of stock prices and simplifies the description with fewer parameters of the multifractal measurement, from which statistical parameters of various values are apparently distinguished from developed to emerging financial markets. Besides, a relevance between the hierarchical structure and different periods of the financial market is observed in high-frequency time series of stock prices.

In the following section, we first introduce the financial time series to be analyzed, and then the theory of SL hierarchy. In section 3, we will give experimental results on the hierarchical structure of stock price fluctuation in financial markets. Finally, the conclusion is given in section 4.

2. Materials and methods

2.1. Data sets

The composition of experimental data includes two parts. One is the daily close prices of 7 stock indices selected from American, European, and Chinese financial markets [30]. Specifically, the stock indices in American financial markets consist of the Dow Jones Industrial Average (DJIA), Standard & Poor 500 (S&P500), and Nasdaq composite (NASDAQ), all of which range from 4 January 1986 to 6 September 2011, including the same 6475 data points. From European financial markets we selected the DAX and FTSE 100 indices, the corresponding durations of which start from 11 November 1990 (DAX) and 6 January 1986 (FTSE) and both end at 6 September 2011, leading to data lengths of 5256 (DAX) and 6486 (FTSE), respectively. The remaining two daily indices are the Shanghai Composite Index (SCI) and Hang Seng Index (HSI) of Hong Kong from the Chinese financial markets. The duration of the SCI with 2989 daily stock prices is from 4 January 2000 to 6 September 2011, while the HSI with a size of 6133 ranges from 31 December 1986 to 6 September 2011. Besides the daily-frequency time series, we also investigated the minute-to-minute prices of the HSI in order to study the effect of time changes on the hierarchical structure. The high-frequency time series of 16,663 HSI stock prices step over 3 years, from 3 January 1994 to 31 December 1996.
2.2. Method

For a given time series of stock prices \( s(t) \), the return is defined as \( r(t, \tau) = s(t + \tau) - s(t) \), where \( \tau \) is the time scale. In a range of \( \tau \), the SL hierarchy suggests there is a relationship between moments of different orders,

\[
\begin{bmatrix}
X_{p+2}(\tau) \\
X_{p+1}(\tau)
\end{bmatrix}
= A_p \begin{bmatrix}
X_{p+1}(\tau) \\
X_p(\tau)
\end{bmatrix}^{\beta} [X^\infty(\tau)]^{1-\beta},
\]

where \( X_p(\tau) = \langle |r(t, \tau)|^p \rangle \) is the \( p \)th order moment of volatility \( |r(t, \tau)| \) denoted by a \( p \)th order structure function and \( \langle \cdot \rangle \) indicates the statistical average. \( X^\infty(\tau) = \lim_{p \to \infty} X_{p+1}(\tau)/X_p(\tau) \) suggests that \( X^\infty(\tau) \) is dominated by large volatility and therefore describes the largest amplitude fluctuations in the time series of stock prices. The parameter \( \beta \) characterizing the SL hierarchy is in the interval \((0, 1)\) and \( A_p \) is a proportionality coefficient as a function of \( p \).

Recalling the scaling law \( X_p(\tau) \sim \tau^{\xi(p)} \) derived from the structure function \([22]\), equation (1) implies the scaling model \([28, 31]\)

\[
\xi(p) = h_0 p + C (1 - \beta^p),
\]

where \( h_0 \) and \( C \) are two other parameters of the SL hierarchical model. According to equations (1) and (2), \( X^\infty(\tau) \sim \tau^{h_0} \) is obtained. Also, the nonlinear dependence of \( \xi(p) \) on \( p \) indicates a multifractal scaling, hence the deviation from the monofractal scaling is mainly characterized by \( \beta \) and \( C \). In particular, \( \beta \) measures the degree of multifractality, and the monofractal structure of the stock price fluctuation will be achieved when \( \beta \to 1 \).

The parameter \( C \) initially represents the codimension of the set of the largest amplitude fluctuation of the flow for the multifractal description of fluid turbulence \([28]\). Ching et al introduced it for the first time to study the heat rate variability and advanced a comprehension that a larger (smaller) occurring probability of large amplitude fluctuations is in correspondence with a smaller (larger) value of \( C \) \([31]\), which is helpful to interpret the stock price fluctuations.

Based on the above theoretical analysis, the procedure to check whether SL hierarchical structure exists in the stock price variation is displayed as follows \([32]\):

(1) It is assumed that there is a scaling property with a power-law relationship between normalized structure functions

\[
\frac{X_p(\tau)}{X_n(\tau)} \sim \left\{ \frac{X_q(\tau)}{X_n(\tau)} \right\}^{\rho_n(p,q)}
\]

which is known as the generalized extended self-similarity in fluid turbulence \([33, 34]\).

(2) In the case of SL hierarchy, the exponent \( \rho_n(p, q) \) in equation (3) is connected only with \( \beta \) of the scaling model,

\[
\rho_n(p, q) = \frac{n(1 - \beta^p) - p(1 - \beta^n)}{n(1 - \beta^q) - q(1 - \beta^n)}.
\]

(3) Fixing the values of \( n \) and \( q \), and gradually increasing \( p \) with step \( \delta p = 0.2 \), equation (4) is transformed into a new formulation,

\[
\Delta \rho_n(p + \delta p, q) = \beta^{\delta p} \Delta \rho_n(p, q) - \frac{\delta p(1 - \beta^n)(1 - \beta^{\delta p})}{n(1 - \beta^q) - q(1 - \beta^n)},
\]

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3. Empirical results

Before the procedure that verifies the validity of the SL hierarchy in stock price fluctuations, we first study the scaling properties of $X_p(\tau) \sim \tau^{\xi(p)}$ with a series of order $p$. By computing the $p$th moments of $X_p(\tau)$ as a function of time scale $\tau$, we can derive the scaling exponents $\xi(p)$. To clarify the scaling relationship, we take the stock price fluctuations of S&P 500 and DJIA in American financial markets as examples (see in figure 1), and show that the integer moments of $X_p(\tau)$ with $p = 1–5$ linearly increase with time scales $\tau$, which suggests that the multifractal scaling of stock price fluctuations is indeed true.

It has been argued that the empirical tails of financial datasets are a power law with an exponent around 4 [35], which theoretically induces diverging moments with $p \geq 3$. However, in our case of finite-size datasets, the moments of orders within our consideration of absolute returns (or volatility) are finite although the distributions of returns have fat tails. It should also be noted that for finite-size time series, an apparent multifractality may be produced due to a fat-tail distribution of return or the long-range nature of the volatility correlations [36]. Kantelhardt et al [37] has proved that both of them are involved in empirical time series since the multifractality gets weaker after shuffling time series. Nevertheless, Barunik et al [38] recently report that the multifractality is mainly a consequence of the fat-tail distribution of return and the time correlation weakens the measured multifractality. Taken together, in the stock price fluctuations we analyzed, the multifractal structure may be connected with both the long-range correlations and the fat tails.

Based on the scaling properties of $X_p(\tau) \sim \tau^{\xi(p)}$, we can depict the dependence of $\xi(p)$ on $p$. In figure 2, it is observed that for all stock price fluctuations $\xi(p)$ increases with respect to $p$ with varying degrees of nonlinearity, in particular the curve of SCI...
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Figure 2. The dependence of $\xi(p)$ on $p$. The nonlinear increase of $\xi(p)$ with respect to $p$ shows the existence of multifractal scaling and a diverse degree of multifractality for all stock price fluctuations. The dot–dash line with a slope of 0.6 is a reference for the curves.

is distinctively different from the others. This result suggests that multifractal scaling efficiently exists, yet the degrees of multifractality are diverse on the basis of the scaling model of equation (2).

As the theoretical analysis of scaling properties was affirmed by the nonlinear relationships between $\xi(p)$ and $p$ presented above, we were motivated to further perform the procedure of equations (3)–(5) to check the validity of SL hierarchical structures in the 7 stock price fluctuations. It is achieved through drawing the scatter plots of $\Delta \rho_n(p+\delta p, q)$ versus $\Delta \rho_n(p, q)$, where a series of values of $\Delta \rho_n(p, q)$ can be obtained after the exponents of $\rho_n(p, q)$ are estimated from equation (3) with various $p$. Figure 3 presents the typical scatter plots only of DJIA and SCI with $\delta p = 0.2$ because the other 5 scatter plots show similar features. The slopes observed from the these scatter plots are in correspondence to the theoretical analysis of equation (5), which demonstrates that the SL hierarchical structure indeed is rooted in stock price fluctuations. In addition, the scaling property is repeatedly checked by employing a variety of $n$ and $q$, as shown in figure 3, from which we testify that the slopes of $\Delta \rho_n(p+\delta p, q)$ versus $\Delta \rho_n(p, q)$ are insensitive to the parameters $n$ and $q$. Besides, it should be noticed that the curves are shifted by tuning the offsets by 0.5.

Based on the scatter plots of $\Delta \rho_n(p+\delta p, q)$ versus $\Delta \rho_n(p, q)$, $\beta$ can be estimated from the slopes, in the light of equation (5). In figure 4, the values of $\beta$ for all 7 share price volatilities are presented, demonstrating that the SL hierarchical structure is compatible with the multifractal scaling. It is quite remarkable from the figure that $\beta$ is discriminable in respect to the development and geographical region of financial markets, which is reflected in the following details: the developed American and European financial markets oppose larger values of $\beta$, showing similar SL hierarchical structures of stock price fluctuations and much more tendency to monofractal scaling. In contrast, a lower value of $\beta$ for the emerging Chinese financial market suggests a greater irregularity of stock price fluctuations that may arise from a market economy that is defective. Finally, for the Hong Kong financial market, the value of $\beta$ falls in the middle because the geographical region of

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Figure 3. The scatter plots of \( \Delta \rho_n(p + \delta p, q) \) versus \( \Delta \rho_n(p, q) \) for stock price fluctuations of DJIA and SCI. We set \( \delta p = 0.2 \) for all plots. Other parameters for (a) and (b) are \( q = 1 \) and \( n = 1.6, 1.8, 2, 2.2, 2.4 \), while for (c) and (d) they are \( n = 2 \) and \( q = 0.6, 0.8, 1, 1.2, 1.4 \). These parameters increase in the direction of the arrow. Note that the slopes of \( \Delta \rho_n(p + \delta p, q) \) versus \( \Delta \rho_n(p, q) \) are evidently insensitive to the choice of \( n \) and \( q \).

Figure 4. Estimated values of \( \beta \) of stock price fluctuations in 7 financial markets. One can find that they are obviously discriminable in respect to the development and geographical region of financial markets.

Hong Kong is commonly affected by the Chinese domestic economy and global economic trends. The difference between developed and emerging markets can be in correspondence to the previous result reported by the generalized Hurst exponent [39, 40].

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To fully understand the SL hierarchy, She and Waymire (SW) introduced in [29] a multiplicative random cascade that consists of two dynamic components corresponding to scaling modeling of equation (2). It is illustrated that the first term $h_{0\beta}$ is associated with the basic component that generates the singular dynamics across continuous scales, while the second term $C(1 - \beta^p)$ correlates with another component that modulates the singular structure through the multiplication of $\beta$ in discrete steps. Utilizing the relationship between $\tau$ and $X^\infty$ declared in former analysis, it can be inferred how the terms in equation (2) really affect the scaling model [31, 32].

Since $X^\infty$ cannot be directly calculated from finite $p$, an alternative method is that we rewrite equation (1) as

$$X_p \sim [X^\infty(\tau)]^p \left\{ \frac{X_q(\tau)}{X^\infty(\tau)} \right\}^{\Gamma(p,q)}, \quad (6)$$

where $\Gamma(p, q) = (1 - \beta^p)/(1 - \beta^q)$ [31, 32]. With the distinct value of $\tau$ and $\tau_0$, equation (6) can give rise to a new formulation after some algebra, which is defined as

$$F_{p,q}(\tau, \tau_0) = \log_2 \left[ \frac{X^\infty(\tau)}{X^\infty(\tau_0)} \right] = \frac{\log_2[X_p(\tau)/X_p(\tau_0)] - \Gamma(p,q)\log_2[X_q(\tau)/X_q(\tau_0)]}{p - q\Gamma(p,q)}. \quad (7)$$

The existence of the invariant $F_{p,q}(\tau, \tau_0)$ in a range of $\tau$ and $\tau_0$ illustrates that $X^\infty(\tau)$ is $\tau$-independent. Herein we set $\tau_0 = 64$, and plot $F_{p,q}(\tau, \tau_0)$ of DJIA, DAX, SCI and HSI changing with $\tau$ on a linear–log scale respectively, at a set of $p$ with certain choices of $q$, as shown in figure 5. In this figure, different clusters of $\log_2(\tau)$-dependent $F_{p,q}(\tau, \tau_0)$ on the basis of different $q$ are shifted by a gradually tuning offset of 10, and the dash–dot line gives the basic line. It is worth noting that $F_{p,q}(\tau, \tau_0)$ is independent of $p$ and $q$, and what is more important, $F_{p,q}(\tau, \tau_0)$ is approximately consistent with zero in a range of $\tau$ (i.e., $\tau$-independent $X^\infty(\tau)$ in a range of $\tau$), therefore, the value $h_0 \sim 0$ in equation (2) is statistically ascertained and consequently $\xi(\tau) \sim C(1 - \beta^p)$. Similar results are also reached for other financial markets.

The aforementioned statement reaches a critical relationship, the parameter $C$ of the scaling model can be approximated by the statistically average of $\xi(\tau)/(1 - \beta^p)$ under the condition of various $p$. Figure 6 shows diverse values of $C$ for developed and emerging stock markets. In particular, HSI owns the lowest $C$, indicating the largest occurring probability of large stock price fluctuation, which agrees with the empirical observations. As an active banking center, the stock market of Hong Kong is easily impacted by both the geographic regional economy and the global economic trend, giving rise to a major occurrence of large stock price fluctuations.

So far, the results show the existence of SL hierarchical structure generally in the daily share prices, and for the sake of a deeper insight into the financial market, we turn to explore the high-frequency stock price fluctuation. The time series of minute-to-minute HSI stock prices from 1994 to 1996 are analyzed following the same procedure to calculate the parameters of the scaling model and also find the presence of SL hierarchical properties. Furthermore, in figure 7, the estimated values of $\beta$ are depicted for one year and the total duration, which changes for different years, implying the SL hierarchical structure varies for different financial periods. For example, in 1996, which mostly approaches the Asian

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Figure 5. $F_{p,q}(\tau,64)$ versus $\log_2(\tau)$ for various values of $p$ (superimposed) at certain choices of $q$. These curves from top to bottom correspond to $q = 1, 2, 1.6, 2, 2.4, 2.8$ and are shifted by tuning with an offset of 10. Both of them suggest that the $\tau$-independent $X^\infty(\tau)$ exists in a range of $\tau$.

Figure 6. Estimated values of $C$ for stock price fluctuations in 7 financial markets. One can find diverse values of $C$ for developed and emerging financial markets. In particular, HSI having the lowest $C$ means that large stock price fluctuations occur much more easily in this stock market.

financial crisis, the degree of multifractality with the smallest $\beta$ is the most irregular. Recently, this time-dependent multifractal scaling behavior, investigated by a generalized Hurst exponent over a weighted moving window, has been used to monitor unstable

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Figure 7. Estimated values of $\beta$ of stock price fluctuations for high-frequency HSI from a certain year and total duration. One can find a degree of multifractality of stock price fluctuations corresponding to the smallest $\beta$ in the year 1996, which mostly approaches the Asian financial crisis, is the most irregular.

periods in financial time series [41]. On the other hand, we cannot efficiently estimate the parameter $C$ because evidence of $\tau$-independent $X^\infty(\tau)$ is absent.

4. Conclusion

In conclusion, this work contributes to a new study on stock price fluctuations of 7 developed or emerging financial markets, in the first place to adopt the quantitative measurement of SL hierarchy, which was originally projected to characterize the derivation from Kolmogorov monofractal scaling of the velocity fluctuations in fluid turbulence. According to the three calculated parameters $h_0$, $\beta$ and $C$ of the scaling model, we verified that the SL hierarchical structure which is in connection with multifractal scaling generally exists in both daily and minute-frequency stock price fluctuations, considering $0 < \beta < 1$. However, the degree of multifractality differs between financial markets with different developmental levels and geographical regions, with diverse values of $\beta$ of their daily stock price volatilities. For instance, the developed American and European stock markets, corresponding to larger $\beta$, possess a lower degree of multifractality than emerging financial markets. Moreover, the occurrence of large stock price fluctuations is also smaller for these developed stock markets than for the HSI, taking the higher value of the parameter $C$ into account. We made further efforts to find that the SL structure alters as the time period of stock price fluctuations changes, by analyzing the minute-frequency in three certain years. These results increase the analogies between the turbulence and financial market dynamics, provide a profound understanding beyond the phenomenological description of multifractal scaling in stock price fluctuations, and thus may help us to better model the dynamic evolution of financial markets based on multiple cascade processes.

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