Entropic model and optimization of a refrigeration machine

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Abstract. The refrigeration machines are operating upon various reverse cycles. However, Blanchard, then Goth and Feidt have shown in the 1980s that unlike engines, there is no natural optimum in working fluid temperature, since these temperatures are not bounded in principle. This paper adds complements to the modeling of refrigeration machines on examples of machines derived from the Carnot one, endoreversible or not (model according to Chambadal; model according to Curzon – Ahlborn). The consequences on the machine operation optimization in the presence or absence of additional constraints are reported. The main constraints observed in the literature are (1) imposed refrigeration load, (2) imposed energy consumption, or (3) imposed coefficient of performance (COP). The present work reveals that while Chambadal model does not provide an optimal solution when constraints are not considered, the Curzon-Ahlborn model shows that working fluid temperatures depend on the heat transfer laws at the source and sink, but also on the extensity transfer entropy at the source, \(\Delta S\), taken as reference and accounting for the existence of the cycle. The results emphasize an optimal distribution of the physical properties of finite dimensions at the source and sink, as function of \(\Delta S\). This new result is the outcome of a sensitivity study. Extensions are under development.

1. Introduction

The equilibrium Thermodynamics has been for a long time the reference modelling for reverse cycle machines through the Carnot Coefficient of Performance (COP) defined as the useful effect divided by the work expenses. The well-known upper-bound of the refrigerating machine COP is:

\[
COP = \frac{T_{CS}}{T_{HS} - T_{CS}}
\]

where \(T_{CS}\) is the cold source temperature and \(T_{HS}\) is the hot source temperature.

These considerations have been overcome during the 1980s with the work of Blanchard [1], then Goth and Feidt [2], showing that there is no natural optimum in working fluid temperatures since they are outside of the source and sink temperatures.

The present paper proposes to complete and renew the modelling of reverse cycle machines by developing precise configurations that extend the Carnot one [3, 4]. In the literature, the endoreversible situation is well studied. Various objectives have been studied [5-12], mainly:

- the useful effect (refrigerating load), \(Q_C\), aiming to its maximization;
- the work expenses, \(W\), aiming to its minimization.

These optimizations are performed with or without constraints (for example, imposed refrigeration load, imposed energy consumption, imposed COP, see papers [13, 14]).
The present paper is a total reconsideration from a new point of view relating the heat source and sink heat transfer to the internal cycled fluid through $\Delta S_C$, the extracted entropy at the source. This reference entropy appears as a necessary condition for the cycle existence and the coupling of the source and sink. Consequences of this modelling are carefully examined on some classical cases.

Section 2 of the paper explores the Chambadal and Curzon-Ahlborn models of the machines. The influence of irreversibilities appears gradually through internal or total entropy productions, but also through irreversibility degrees introduced by Novikov [15], and irreversibility ratios introduced by Ibrahim et. al. [16].

Section 3 reports on optimization of the refrigeration machine focusing on minimum work expenses. It is shown that the Chambadal model optimum coincides with the equilibrium thermodynamics one whatever the model refinement is used (basic one, linear heat transfer at the source). When moving to the Curzon-Ahlborn model (section 3.3), it appears that the external temperatures are related to the finite dimensions of the machine ($G_H$, $G_C$) that have to be optimally allocated.

Conclusions and perspectives are discussed in the last section.

2. Chambadal’s model versus Curzon-Ahlborn’s model of a refrigeration machine

2.1. Cycles representation in $T$–$S$ diagram

The difference between the two machines is illustrated on figure 1.

For both machines, the cold side heat reservoir is a thermostat of temperature $T_{CS}$. The heat transfer at the cold source, $Q_C$, is done due to the temperature difference between the source and the working fluid, $\Delta T_C = T_{CS} - T_C$. Conversely, for the Chambadal’s model, the heat transfer at the hot sink is considered perfect (reversible), at temperature $T_{HS} = T_0$ ($T_0$, ambient temperature, figure 1a).

Figure 1b corresponds to Curzon-Ahlborn’s model that, in addition, considers the temperature difference $\Delta T_H = T_H - T_{HS}$, necessary for the heat removal to the environment.

Note that figures 1a and 1b suppose endoreversible cycles, and ignore the heat losses. Furthermore, one can see on fig. 1 that basically there are no natural bounds at the temperatures $T_H$ and $T_C$. This fact is a characteristic of the reverse cycle machine [1, 2].
2.2. The basics of the thermodynamic model

The basics are valid for the two types of refrigeration machines presented in section 1 and also for the heat pumps of the two same types.

The energy balance of these machines when no heat losses are considered is expressed as:

\[ |W| = |Q_H| - Q_C \]  

(1)

where \(W\) is the mechanical energy consumed in the cycle, \(Q_H\) is the heat removal from the cycle hot side (and useful effect for a heat pump), and \(Q_C\) is the heat transferred to the cycle cold side that represents the useful effect for the refrigeration machine.

Generally, the heat transfers to and from the cycle can be differently expressed by using:

- the entropic form

\[ Q_H = T_H \Delta S_H \]  

(2)

\[ Q_C = T_C \Delta S_C \]  

(3)

- the heat transfer law involving a physical dimension \(G\) that represents a thermal transmittance, a general transfer law depending on temperature \(T_S\) (of hot or cold source), and the cycled fluid temperature \(T\), leading to

\[ Q_H = G_H \cdot f_H(T_{HS}, T_H) \]  

(4)

\[ Q_C = G_C \cdot f_C(T_{CS}, T_C) \]  

(5)

Moreover, the entropy balance of the irreversible cycle of whatever machine is written as:

\[ \Delta S_C + \Delta S_I = \Delta S_H \]  

(6)

where \(\Delta S\) represents the entropy production on the cycle, \(\Delta S_C\) and \(\Delta S_H\) are the transfer entropies at the cold and hot side respectively.

Note that a dissymmetry appears between refrigeration machine and heat pump regarding the entropy. Indeed, the input entropy is \(\Delta S_C\) for the refrigeration machine, while \(\Delta S_H\) is the one for the heat pump. Therefore, only the refrigeration machine approach will be presented hereafter, for which the reference entropy will be considered \(\Delta S = \Delta S_C\) (simplified notation).

Note that the following inequality exits: \(\Delta S_C < \Delta S_C = \Delta S < \Delta S_H < \Delta S_{HS}\)

By combining equations (2), (3) and (6) in equation (1) leads to:

\[ |W| = (T_H - T_C) \Delta S + T_H \Delta S_I \]  

(7)

The machine irreversibility rises the mechanical work consumption.

One considers also the coefficient of performance COP, as a measurement of the energetic efficiency of the refrigeration machine, as:

\[ COP = \frac{Q_C}{|W|} \]  

(8)

By combining equations (1) and (8) leads to:

\[ \frac{1}{COP} = \left| \frac{Q_H}{Q_C} \right| - 1 \]  

(9)

The above equation is expressed as a function of temperatures and entropy variation when equations (2), (3), and (6) are replaced in equation (9):

\[ \frac{1}{COP} = \left( \frac{T_H}{T_C} - 1 \right) + \frac{T_H}{T_C} \frac{\Delta S_I}{\Delta S} \]  

(10)

Note that the first term in the right side of equation (10) corresponds to the reverse of the COP of the endoreversible refrigeration machine \((\Delta S_I = 0)\). The second term accounts for the machine irreversibilities that decrease the real COP. Moreover, the first ratio of intensive terms, namely of
temperature, is related to thermal transfers, while the second ratio of extensive type, \( \frac{\Delta S_I}{\Delta S} \), corresponds to the irreversibility degree introduced by Novikov \cite{15}. It corresponds as well to the entropic ratio considered by Ibrahim \cite{16} for the thermal engines. For the refrigeration machine, this ratio is expressed as:

\[
I_C = \frac{\Delta S_I}{\Delta S_C} = 1 + \frac{\Delta S_I}{\Delta S}
\] (11)

Note that other definitions are possible, and that there is a strong link between the ratio and the irreversibility degree.

Another description would consist of reasoning on the system (composed of the machine and the source and sink). The reader will easily find the energy consumption:

\[
|W| = T_{HS}(\Delta S + \Delta S_S) - T_{CS} \cdot \Delta S
\] (12)

and the reverse COP:

\[
\frac{1}{COP} = \left( \frac{T_{HS}}{T_{CS}} - 1 \right) + \frac{T_{HS}}{T_{CS}} \cdot \frac{\Delta S_S}{\Delta S}
\] (13)

One can see in equation (13) the inverse of the reversible COP in the first term of the second member, then the term of total energy degradation, showing this time, the irreversibility degree of the system, \( d_{IS} = \Delta S_S / \Delta S \).

### 3. Optimization options of the refrigeration machine by using the Chambadal’s model

The study of the refrigeration machine reveals three main objectives leading the optimization, namely, maximum of useful effect, \( Q_C \), minimum of energy consumption, \( W \), and maximum of COP. Nevertheless, from an environmental point of view, minimum heat rejection, \( Q_H \), is required.

To these typically engineering objectives corresponds an objective of the physicist, namely, the minimum of entropy production, that covers at least two aspects:

- entropy production of the machine, \( \Delta S_I \);
- entropy production of the system, \( \Delta S_S \).

These optimization aspects are summarized in table 1.

#### Table 1. Essential examples of objective functions.

| Objective function            | Refrigeration machine RM | Heat pump HP |
|-------------------------------|--------------------------|--------------|
| Useful Effect                 | Max \( Q_C \)             | Max|\( Q_H \)| |
| U. E.                         |                          |              |
| Energy Consumption            | min |\( W \)|         | min |\( W \)| |
| E. C.                         |                          |              |
| Energy Efficiency             | \( COP_{RM} = \frac{Q_C}{|W|} \) | \( COP_{HP} = \frac{|Q_H|}{|W|} \) |
| E. E.                         |                          |              |
| Reject and Environment        | min |\( Q_H \)|         | Max \( Q_C \) | charge, no reject |
| R                             |                          |              |
| Physical: entropy             | min \( \Delta S_I \)     | min \( \Delta S_I \) |
| S                             | min \( \Delta S_S \)     | min \( \Delta S_S \) |
| others                        |                          |              |
|                               |                          | (with or without constraint) |

Further, the paper will focus on the most common objective function, namely the search for the minimum of \( W \), if it exists.
3.1. Refrigeration machine according to Chambadal, in the entropic model only

The combination of equations (1), (2), (3) and (6) allows to express the mechanical work as:

$$|W| = T_0 (ΔS + ΔS_I) - T_C \cdot ΔS$$

(14)

For $ΔS$ as parameter of the model, other than zero so that the cycle exists, one can see that $W$ is a decreasing function of $T_C$, whose minimum is zero when $T_C = T_0$ (thermodynamic equilibrium and machine absence).

3.2. Refrigeration machine according to Chambadal, in the entropic model with a linear heat transfer law at the source

The model of section 3.1 is now completed by the heat transfer equation at the source:

$$Q_C = G_C (T_{CS} - T_C)$$

(15)

By combining equations (15) and (3), the expression of the reference heat transfer entropy results as a function of $T_C$:

$$ΔS = G_C \left( \frac{T_{CS}}{T_C} - 1 \right)$$

(16)

Furthermore, equations (14) and (15) replaced in equation (16) lead to:

$$|W| = G_C \left( \frac{(T_0 - T_C)(T_{CS} - T_C)}{T_C} + T_0 \cdot ΔS_I \right)$$

(17)

The derivation with respect to $T_C$ confirms the decrease of $W$ with $T_C$, whatever $T_0$, $T_{CS}$, $G_C$ and $ΔS$ are. This confirms the non-existence of a minimum of energy consumption for the irreversible refrigeration machine, contrarily to the engine, even if $ΔS_I$ is a function of $T_C$ (various cases are available near the authors). Moreover, $Q_C$ and $Q_H$ are also monotonous functions decreasing with $T_C$.

3.3. Refrigeration machine according to Curzon – Ahlborn, on an entropic basis

The model is still the one presented in section 3.2 further completed by the heat transfer equation at the machine hot side (see figure 1b):

$$Q_H = G_H (T_H - T_{HS})$$

(18)

However, the original bias of the model is to maintain as parameter the reference entropy, $ΔS$, that ensures the cycle existence condition and consequently, of the machine.

The combination of equations (3) and (15), then of equations (2) and (18) leads to the following expressions of the temperatures $T_C$ and $T_H$:

$$T_C = \frac{G_C T_{CS}}{G_C + ΔS}$$

(19)

$$T_H = \frac{G_H T_{HS}}{G_H (ΔS + ΔS_I)}$$

(20)

Then, the mechanical work expression can be derived as:

$$|W| = G_C \left( \frac{(T_0 - T_C)(T_{CS} - T_C)}{T_C} + T_0 \cdot ΔS_I \right)$$

(21)

with $ΔS$, $ΔS_I$, $T_{HS}$ and $T_{CS}$ as parameters, and $G_H$ and $G_C$ as variables of finite dimensions related by:

$$G_H + G_C = G_T$$

(22)

The variational method [17] applied to the model provides the optimal distribution of $G_H$ and $G_C$ leading to the minimum energy expenses:

$$G_C^* = \frac{ΔS [G_T (T_{CS} - (ΔS + ΔS_I)) (T_{HS} + \sqrt{T_{CS}})]}{(ΔS + ΔS_I) T_{HS} + ΔS_I T_{CS}}$$

(23)
\[
G_H^* = \frac{(\Delta S + \Delta S_i) G_T \sqrt{T_{HS} + \Delta S (\sqrt{T_{HS}} + \sqrt{T_{CS}})}}{(\Delta S + \Delta S_i) \sqrt{T_{HS}} + \Delta S (\sqrt{T_{HS}} + \sqrt{T_{CS}})}
\]  

(24)

By replacing these expressions in equation (21), the minimum of the consumed mechanical work results as:

\[
\min |W| = \frac{1}{G_T - \Delta S_i} \left\{ \sqrt{T_{HS}} (\Delta S + \Delta S_i) [G_T \sqrt{T_{HS}} + \Delta S (\sqrt{T_{HS}} + \sqrt{T_{CS}})] - \sqrt{T_{CS}} \Delta S \left[ G_T \sqrt{T_{CS}} - (\Delta S + \Delta S_i) (\sqrt{T_{HS}} + \sqrt{T_{CS}}) \right] \right\}
\]  

(25)

Note that equation (25) delivers the endoreversible case for \( \Delta S_i = 0 \). Also, it shows that \( \min W \) is an increasing function of \( \Delta S \), and a decreasing one of \( G_T \).

4. Conclusions and perspectives

The present work deals with an approach of a refrigeration machine operating upon an irreversible Carnot cycle.

The proposed originality is methodological, since it considers a reference heat transfer entropy \( \Delta S \), corresponding to the heat input in the refrigeration machine cycle.

After a brief review of the useful concepts to the refrigeration machine according to Equilibrium Thermodynamics, we consider the equivalent of the Chambadal engine transposed into refrigeration machine, by focusing the search on a minimum of energy supply to the machine.

It appears that the proposed model, as the one from Equilibrium Thermodynamics, does not lead to optimum.

Conversely, the use of a Curzon - Ahlborn type model shows that considering the reference transfer entropy, that ensures the existence of the cycle and therefore, of the machine, is sufficient to lead to a minimum. Thus, there is no need to introduce an additional constraint, i.e. the imposed useful effect, or other types, as done in the literature [13, 14]. This model provides an optimum design by the optimal distribution of a finite physical dimension \( G_i \).

One recalls that according to the model, the quantity \( G \) in steady state dynamic regime corresponds to:

- a heat transfer conductance, \( K_i = k_i \alpha_i \), at the source and sink;
- a product of the heat exchanger effectiveness, \( \varepsilon \), multiplied by the heat capacity rate of the external fluid, \( \dot{C}_i = \dot{m}_i c_{p,i} \), for an external transfer in sensible heat.

To the previous general case, one can add the typical specific cases where:

- the useful effect is imposed: \( Q_e = Q_0 \);
- the energy consumption is imposed: \( W = W_0 \);
- the machine COP is imposed: \( \text{COP} = \text{COP}_0 \).

These cases will be revisited in the future research, by looking for the particular values of the new variable \( \Delta S \), added to these constraints.

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