Accelerator performance analysis of the Fermilab Muon Campus

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Fermilab is dedicated to hosting world-class experiments in search of new physics that will operate in the coming years. The Muon g-2 Experiment is one such experiment that will determine with unprecedented precision the muon anomalous magnetic moment, which offers an important test of the Standard Model. We describe in this study the accelerator facility that will deliver a muon beam to this experiment. We first present the lattice design that allows for efficient capture, transport, and delivery of polarized muon beams. We then numerically examine its performance by simulating pion production in the target, muon collection by the downstream beam line optics, as well as transport of muon polarization. We finally establish the conditions required for the safe removal of unwanted secondary particles that minimizes contamination of the final beam.

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I. INTRODUCTION

Measurement of the muon anomalous magnetic moment, $\alpha_\mu = (g_\mu - 2)/2$, provides an important test of the Standard Model (SM) [1]. The Brookhaven Muon g-2 Experiment measured a discrepancy of more than 3 standard deviations compared to the SM prediction [2]. As this discrepancy hints at the possibility of new physics, efforts have been made to improve both the theoretical [3] and experimental [4,5] precision of $\alpha_\mu$.

The Fermilab Muon g-2 Experiment [6] is a next generation experiment mainly motivated by this discrepancy. The goal of the experiment is a fourfold improvement in the measured precision that will reduce the experimental uncertainty to 140 parts per billion (ppb) [7]. One of the key requirements to achieve this goal is a factor of 20 increase in recorded muon decays compared to the Brookhaven experiment [8].

A combination of beam lines that are part of the so-called Muon Campus [9] have been designed to deliver a muon beam with sufficient intensity. The sequence starts at the booster [10], where short batches of 8 GeV protons are delivered to the recycler ring [11]. An rf system separates a batch into four tighter bunches of $10^{12}$ protons each. These bunches are extracted one at a time and directed to a target station, which is tuned to collect 3.1 GeV/c positive secondary particles. Pions and their daughter muons are transported along a 280 m channel and then injected into the repurposed antiproton debuncher ring [12,13], where they make several revolutions before being kicked into a final beam line that terminates at the entrance of the muon storage ring that is used by the Muon g-2 Experiment.

The statistical uncertainty of the measurement scales as $1/\sqrt{NP^2}$ [14], where $P$ is the beam polarization and $N$ is the number of muons, and the muon storage ring has a narrow momentum acceptance of less than $\pm0.5\%$. It is therefore essential to maximize polarized muon transmission along the Muon Campus, while minimizing the growth of the beam emittance. Moreover, the delivery of a clean muon beam that has a pion contaminant fraction below $10^{-5}$, with no protons present, is another key requirement, as these hadrons can cause a large hadronic “flash” at injection that paralyzes detector systems and leads to baseline shifts on a slowly decaying background. The present accelerator scheme, however, has a series of bending sections, elevation changes as well as complicated injection and extraction regions. Such beam line sections, if not well controlled, can degrade the beam quality substantially [15]. The three interesting questions then are, what is the overall transmission and momentum spread of positive muons, what is the level of contamination from protons and pions, and what is the degree of polarization of secondaries.
the delivered beam. A key requirement to resolve these issues is the ability to accurately evaluate the performance of the Muon Campus beam lines.

The main goal of this paper is to present the baseline lattice for the Muon Campus and evaluate its performance numerically. In order to accomplish this, we follow a sequence of steps. First, we present a lattice design to efficiently capture, transport and deliver a muon beam from the production target towards the storage ring of the Muon g-2 Experiment. Then, we numerically examine its performance using G4BEAMLINE [16], a GeANT4 [17] based code that fully incorporates all basic physical processes such as muon decay and spin precession. We initiate our tracking at the muon production target and present a detailed end-to-end simulation. In order to support our findings, we compare our numerical results against an exponential-decay estimate and find good agreement. Our study suggests that the Muon Campus has the potential to deliver highly polarized muons at a rate of $\sim 8 \times 10^5 \mu^+$ per bunch.

The outline of this paper is as follows: In Sec. II, we give an overview of the Muon Campus and provide details of the design parameters of all beam lines. In Sec. III we report the results from our simulations modeling the aforementioned channel. Finally, we present our conclusions in Sec. IV.

II. LATTICE DESIGN AND OPTICS

To enhance cost savings, the existing tunnel enclosures and beam lines from the Antiproton Source [13] were largely reused for operation of the Muon Campus. In addition, stochastic cooling components and other infrastructure no longer needed in the debuncher ring were removed in order to improve its aperture and the ring was renamed delivery ring (DR).

Figure 1 displays a schematic layout of the Fermilab Muon Campus, where 8 GeV protons are transported via the M1-line to an Inconel target at AP0 for the Muon g-2 Experiment. A secondary beam from the target is collected using a lithium lens, where the positively charged particles with 3.1 GeV/$c$ ($\pm 10\%$) are selected by using a bending magnet. The secondary beam leaves the target station and travels through the M2- and M3-line, which is designed to capture as many 3.1 GeV/$c$ muons as possible from the pion decays. The beam is then injected into the DR, where essentially all remaining pions decay into muons after several revolutions. The DR is also used to separate muons in time from the heavier protons. A kicker is then used to remove the protons, where the muon beam is extracted into the M4-line. The beam then continues along the M5-line which terminates just upstream of the entrance to the muon storage ring, where the storage ring entrance is taken as the simulation end point for this study.

Figure 2(a) shows a schematic layout of the target station, which consists of five main devices: pion production target, lithium lens, collimator, pulsed magnet, and beam dump. The primary proton beam interacts with the target to produce positive secondaries that are focused by the lithium lens and then are momentum selected via the downstream pulsed dipole magnet (PMAG). The target [19]...
The primary purpose of the 505 m circumference DR is to provide sufficient time for most pions to decay into muons. The original debuncher [12,13] lattice design is used with just a few modifications [6], where Fig. 3(c) displays a schematic layout wherein the injection and extraction beam lines are omitted for simplicity. A salient feature of the DR is that it has a periodicity of 3, where the basic arrangement of the ring includes three long dispersion-free straight sections together with three arc sections having mirror symmetry in each of the three

| Parameter                      | Value   |
|--------------------------------|---------|
| Intensity per pulse            | $10^{12}$ |
| Total POT per cycle            | $16 \times 10^{12}$ |
| Number of pulses per cycle     | 16      |
| Cycle length (s)               | 1.4 s   |
| Primary momentum (GeV/c)       | 8.89    |
| Beam size at target (mm)       | 0.15    |
The magnet structure consists of 57 approximately 60° phase-advance FODO cells, where the beam line’s natural chromaticity is compensated by two families of sextupoles bracketing all arc quads. These arcs have a dispersion function maximum of 2.4 m and transverse beta function maxima of approximately 15 m, along with the typical tunes of 9.79 horizontal and 9.77 vertical. Figure 4(a) presents the optical functions along the DR that are calculated with MAD-X.

Injection from the M3-line and extraction to the M4-line takes place in the same straight section with the latter happening in the downstream half, where Fig. 3(b) shows a schematic layout of the extraction scheme [6]. Two kicker magnets (EKIK1, EKIK2) upstream of quad D2Q3 are first used to kick the beam off of the closed orbit, and then a combination of three vertical bending magnets (ELAM, ECMAG, EDWA), in conjunction with an intervening quadrupole (D2Q5), bend the beam upward to a final elevation of 0.81 m above the DR. The beam bends upward into the M5-line after traversing 30 m in the M4-line, and continues towards the muon storage ring. The section of the M4-line between the DR and the beginning of the M5-line must also be able to transport 8 GeV protons, as it is shared with the Mu2e Experiment [18]. Figure 4(b) shows the beta and dispersion functions along the M4- and M5-line as calculated by MAD-X, where $S = 0$ corresponds to the front edge of the ECMAG magnet and the beam line terminates 0.30 m upstream of the entrance to the muon storage ring. The M5-line is 100 m long and includes a 27.1° horizontal bend string at $S = 46.5$ m that provides the proper entry position and angle into the muon storage ring, and right before the end of the M5-line there is a strong-focusing and tunable final focus section using four quadrupole magnets at $S = 119.8$ m that provides optical matching to the storage ring. The FODO section between the horizontal bend string and final focus section is designed to transport beam with minimal losses.

### III. TRACKING STUDIES

The performance of the Muon Campus beam lines is simulated using G4BEAMLINE, a scripting tool for the Geant4 Monte Carlo program. The majority of the Muon Campus quadrupoles have special vacuum chambers that conform to the poles [21] in order to extend the aperture in the horizontal and vertical planes, where these...
The evolution of secondary particles as a function of distance along the M2- and M3-line is shown in Fig. 5, where the number of muons and pions are shown with the momentum ranges of 3.1 \( \text{GeV}/c \) ± 2% and ±0.5%. A glance of Fig. 5 reveals a rapid loss of secondary particles within the first 10 m of the channel. This fact is not surprising since the particles produced at the target have a very wide momentum spread which extends all the way to 7 \( \text{GeV}/c \). On the other hand, PMAG selects particles only near 3.1 \( \text{GeV}/c \). As a result, a significant number of particles are lost from the momentum selection process. Note further that the population of muons drops roughly by a factor of 2 near \( S = 8.7 \text{ m} \). This is also the location of the first quadrupole magnet of M2, suggesting that collimation is an additional factor of particle loss in the first few meters of the beam line.

Further examination of Fig. 5 indicates a substantial beam loss at \( S = 160.0 \text{ m} \), which corresponds to the location of one of the two 9.25° horizontal bending magnets in the M3-line. While for \( \Delta p/p = \pm 2\% \) and \( \Delta p/p = \pm 0.5\% \) there is no pion loss at \( S = 160.0 \text{ m} \), the daughter muon loss is nearly 40% for both momentum acceptances suggesting that the performance deterioration for muons is primarily caused by the transverse aperture rather than the momentum cut. We estimate the effective transverse beam emittance, from the second-order moments, upstream of the horizontal bend as 38 mm-mrad for pions and 120 mm-mrad for muons. As the Muon Campus lines have a 40 mm-mrad transverse acceptance [23], we conclude that the nearly 40% muon loss is from collimation due to the muon beam emittance being 3 times larger than the lattice acceptance.

The simulation finds that the beam is dominated by secondary protons at the end of the M3-line (\( S_{\text{CMAG}} \approx 280.0 \text{ m} \)), where the distribution of muons arriving at the CMAG has an average momentum of 3.1 \( \text{GeV}/c \) with one standard deviation of 2.1%. The number of pions surpasses the number of muons by an order of magnitude, which indicates the potential for collecting more muons further downstream. There are 93,960 positive pions that survive when tracking \( 1.17 \times 10^6 \) pions from the end of the lithium lens to the end of the M3-line and turning off pion decay (dashed line), while there are 18,698 positive pions and 2680 positive muons that survive when turning on pion decay. The triangle in Fig. 5 corresponds to the exponential decay expression \( \tau_{\pi} = 933 \text{ ns} \), which agrees very well with our simulations, where \( \tau_{\pi} = 26 \text{ ns} \) is the average rest frame pion lifetime, \( \gamma \) is the Lorentz factor, \( N_0 \) is the number of pions without decays, and \( \tau_{\text{CMAG}} = 933 \text{ ns} \) is the time required for the beam to reach the CMAG.

Figure 6 shows the number of secondary particles as a function of the number of turns in the DR. Protons outnumber both muons and pions by at least 2 orders of magnitude, which is unsurprising as a considerable number of secondary protons travel downstream of the target and apertures have been included in our G4BEAMLINE simulation model. The beam-target interactions are modeled with MARS [22] by assuming \( 10^6 \) incident protons on the target (POT), where a particle distribution is then created 0.772 m upstream of the PMAG and propagated using G4BEAMLINE. The input distribution in the G4BEAMLINE simulation contains \( 1.25 \times 10^{-2} \) protons and \( 1.17 \times 10^{-3} \) positive pions per POT. Note that any muons produced within the lithium lens volume are excluded from the downstream simulation, since they contribute less than 3% to the total delivered muons at the entrance of the muon storage ring used by the Muon g-2 Experiment. Table II summarizes the number of secondary particles at different locations along the Muon Campus beam lines.

![Diagram](image-url)
note that all pions decay to muons after the third turn. The pion population decreases substantially, and eventually drops below 10^{-9} per POT after three turns. Conversely, there is only a 3% muon loss after each turn, which is close to the expected 2.6% muon decay loss after each turn for the 3.1 GeV/c muons that have a 64 μs lab frame lifetime. Finally, the rms transverse beam emittance remains practically unchanged with 14.1 mm. mrad horizontal and 15.1 mm. mrad vertical values.

Figure 7 gives the muon transmission along the M4- and M5-line after four revolutions in the DR for several different momentum cuts, where in a fashion similar to Fig. 4(b), the upstream face of the ECMAG is at S = 0 and the simulation terminates at the end of the M5-line. We found that > 85% of the muons reach the end of the line, where the primary sources of muon loss are collimation in the vertical extraction section and horizontal bend along the

FIG. 5. The simulated performance along the M2- and M3-line, where the evolution of secondary particles is shown as a function of distance along the channel for different momentum acceptances. The dashed line shows the performance of pions when decays are not included. The final number of pions is 1.87 × 10^{-5} per POT at the end of the channel (S = 280 m), which is in agreement with our theoretical findings (triangle). One should note that S = 0 is the same point as in Fig. 2(b).

FIG. 6. The simulated performance along the DR, where the evolution of secondary particles is given as a function of the number of turns for different momentum acceptances. One should note that all pions decay to muons after the third turn.

FIG. 7. The simulated performance along the M4-and M5-line, where the evolution of muons is given as a function of distance along the channel for different momentum acceptances.

|          | p, all | π^+, all | μ^+, all | μ^+, Δp/p = ±2% | μ^+, Δp/p = ±0.5% |
|----------|--------|----------|----------|-----------------|-----------------|
| End of M3| 1.37 × 10^{-4} | 1.87 × 10^{-5} | 2.68 × 10^{-6} | 1.19 × 10^{-6} | 3.26 × 10^{-7} |
| DR (Turn 1) | 8.02 × 10^{-5} | 5.06 × 10^{-7} | 9.50 × 10^{-7} | 8.40 × 10^{-7} | 2.59 × 10^{-7} |
| DR (Turn 2) | 7.94 × 10^{-5} | 2.72 × 10^{-8} | 9.15 × 10^{-7} | 8.16 × 10^{-7} | 2.54 × 10^{-7} |
| DR (Turn 3) | 7.89 × 10^{-5} | 1.93 × 10^{-9} | 8.83 × 10^{-7} | 7.89 × 10^{-7} | 2.47 × 10^{-7} |
| DR (Turn 4) | 7.88 × 10^{-5} | <10^{-9} | 8.54 × 10^{-7} | 7.65 × 10^{-7} | 2.39 × 10^{-7} |
| End of M5 | <10^{-9} | 7.80 × 10^{-7} | 6.80 × 10^{-7} | 2.08 × 10^{-7} |
M4- and M5-line. The total number of muons at the end of the M5-line is \( \sim 8 \times 10^{-7} \) per POT or \( 8 \times 10^5 \) per proton bunch on target (per fill), while the horizontal and vertical rms beam emittances are 14.0 mm-mrad and 15.0 mm-mrad respectively. Our simulation also successfully captures the muon momentum distribution spread, where Fig. 8 shows that the muon distribution at the end of the M5-line has an average momentum of 3.09 GeV/c with one standard deviation of 1.20%. Based on the published parameters in Refs. [6,24], \( 1.095 \times 10^{14} \) muons within \( \Delta p/p = \pm 2.0 \% \) at the end of the M5-line are required for the collection of the required Muon g-2 Experiment statistics [25,26]. From the simulation results of Fig. 7, the number of muons within \( \Delta p/p = \pm 2\% \) is \( 6.8 \times 10^{-7} \) per POT or \( 6.8 \times 10^5 \) per fill. Given that 16 fills occur in 1.4 s (see Table I), the Muon Campus has the potential to deliver the desired muons within about a year of running time. Moreover, the performance of the Muon Campus beam...
lines was modeled with BMAD, an independent simulation code that also included muon decay. Both BMAD and G4BEAMLINE revealed good agreement (within 5%) for the predicted number of muons at the end of M5.

Polarized muons are obtained from the weak decays of in-flight pions: \( \pi^+ \rightarrow \mu^+ + \nu_\mu \). The daughter muons have a very wide momentum spectrum in the lab frame that ranges from around one-half of the pion momentum (backward decays) to slightly greater than the pion momentum (forward decays), where forward and backward refers to the center-of-mass frame muon direction relative to the Lorentz boost between the frames. Figure 9 in combination with Table III show the muon polarization evolution along different locations within the Muon Campus. Numerical simulation predicts that 70% of the pions have decayed into muons when the beam reaches the end of the M3-line, where only forward muon decays are eventually selected given the narrow \( \Delta p/p = \pm 2\% \) momentum acceptance of the channel. The muon beam has an average longitudinal polarization of 96% when it reaches the DR [Fig. 9(a)], where the polarization is negative due to the positive muon spin and momentum having opposite directions from parity violation. We find good agreement [28] when comparing our simulation results to the theoretical expressions given by Combley and Picasso [29]. The beam experiences a vertical magnetic field when it circulates in the DR, which causes the muon spins to precess in the horizontal plane [Fig. 9(b)]. The average polarization is almost equal in both horizontal and longitudinal directions after four revolutions.

| Location        | \( \langle P_x \rangle \) | \( \langle P_z \rangle \) |
|-----------------|--------------------------|--------------------------|
| End of M3       | −0.01                    | −0.96                    |
| DR (Turn 2)     | −0.38                    | −0.88                    |
| DR (Turn 4)     | −0.71                    | −0.65                    |
| End of M5       | −0.72                    | −0.64                    |

The tracking of muon and proton longitudinal distributions along the DR: (a) entering, (b) after one turn in, (c) after three turns in, and (d) after four turns in the DR. The separation of muon and proton bunches increases by 75 ns after each turn. This plot indicates that four revolutions are required to safely remove the proton contamination, because of the \(~ 180 \) ns rise time for the proton removal kicker magnets. One should note that the histograms are normalized so that the area sum is equal to 1.
pion contamination and four turns is sufficient for separating secondary protons from the muons. Finally, both pion and muon simulated decay rates are found to have good agreement with the exponential decay law. We conclude from our numerical findings that the Muon Campus beam lines can deliver the designed beam parameters [6] for the Muon g-2 Experiment.

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