Comprehensive evaluation of urban livability

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Abstract. This article is based on the comprehensive evaluation of urban livability. This article selects the indicators to measure the urban livability, and uses the factor analysis method to reasonably screen the indicators, and establishes an improved BP neural network evaluation model to obtain the livability of different cities in Shaanxi Province. Monte Carlo simulation is used to analyze the impact of different indicators on urban livability. Then this article comprehensively considers the uncertain factors, establishes a predictive model of stochastic differential equations, and establishes a dynamic evaluation model to reassess the livability of cities.

1. Introduction
Urban livability is not only one of the hot topics in the current urban scientific research field, but also the focus of Chinese government and urban residents. Building a livable city has become an important goal of China's urban development at this stage. It is of great significance to improving the quality of life of urban residents, improving urban functions and improving the efficiency of urban operations. Although there are many urban livability evaluation systems, these objective evaluations mainly use statistical data to pay attention to the livability characteristics of urban unit scales. The objective evaluation of urban living environment for urban entity livable elements still needs to be supplemented. Besides, the impact of uncertainty on the livability of cities in these evaluation systems has almost never been considered. Therefore, for many cities in Shaanxi Province, a variety of stability and instability factors can be used to evaluate the livability of cities, which can provide reference for the city construction and people's living choice in Shaanxi Province.

2. Urban livability evaluation model
By establishing a factor analysis model, the information of the original variables can be reflected by a small number of uncorrelated integrated variables[1]. These few comprehensive variables are called common factors.

2.1. Establishment and Solution of Factor Analysis Model
(1) Selection of indicators
The preliminary selection results of the indicators in this paper are as follows[2]:

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Figure 1. Initial selection of indicators.

(2) Factor Analysis Model to achieve dimensionality reduction screening of indicators

① Data collection
The article takes some cities in China in 2011 as the main research object. We select one of the top ten cities in urbanization process randomly from each province and collect the values of each index.

② Data standardization
Make data dimensionless and the calculation formula is

\[ Z = \frac{X - \bar{X}}{\delta} \]

③ Establishment of factor analysis model

The factor analysis model is a linear combination of each indicator \( X_i \) with \( m (m \leq n) \) common factors \( F_1, F_2, \ldots, F_m \).

\[
\begin{align*}
F_1 &= a_{1,1}X_1 + a_{1,2}X_2 + \cdots + a_{1,n}X_n \\
F_2 &= a_{2,1}X_1 + a_{2,2}X_2 + \cdots + a_{2,n}X_n \\
& \vdots \\
F_m &= a_{m,1}X_1 + a_{m,2}X_2 + \cdots + a_{m,n}X_n
\end{align*}
\]

➢ Factor analysis feasibility analysis
The premise of using factor analysis is that there is a strong correlation between the original indicators. This paper applies the KMO test to determine whether the sample is suitable for factor analysis. The numerical value of the collected data is substituted into the above formula, and finally the KMO test value is 0.806, so the selected initial index is in accordance with the condition for establishing
the factor analysis model.

➢ Determine the number of indicators after screening

Using the eigenvalues as the retention principal component standard, the scree plot is as follows:

![Scree plot](image)

Figure 2. Scree plot.

It can be seen from the above figure that the eigenvalues of factor 4 and factor 5 are very similar, and the eigenvalues with factor number greater than 5 are very slow, so it is feasible to select five common factors.

(3) Screening indicator results

In order to make the attribution of the factors of different variables more clear, the orthogonal rotation process is performed to obtain the rotation component matrix, and the load of 23 initial indicators on the five screening indicators can be obtained by the matrix. The maximum load on which common factor the indicator has, indicates that this common factor has the greatest impact on the initial indicator.

Finally, we filter the original 23 indicators into 5 indicators. $F_1, F_2, F_3, F_4$ are respectively called “per capita possession of urban resources”, “city comprehensive strength”, “environmental construction intensity”, “educational development level”, “educational investment”.

2.2. Establishment and Solution of improved BP Neural Network Model

This paper selects the improved BP neural network model based on factor analysis, which not only avoids the manual determination of the subjectivity of the weights of various indicators, but also makes the results of different urban evaluation schemes have certain comparability.

BP neural network consists of three layers: the input layer, the hidden layer, and the output layer. In this article, the input layer is 5 indicators for measuring livable cities, and the output layer is the evaluation value of livable quality of cities. Then we construct a 3-layer neural network. It is found that when the number of hidden layers is 15, the convergence effect is better. After repeated training of sample data, the neural network with accurate nonlinear mapping between input and output is obtained.

The final average score of the 7 cities obtained from 100 solutions is as follows:
Figure 3. Average livability score for 7 cities.

The ranking is obvious. Xi'an has an average score of 5.27 or so, which is the most livable among the seven cities. The livability of Xianyang and Baoji are very close. The city with the lowest livability is Tongchuan, and the livability score is only 2.02.

2.3. Comprehensive analysis of urban livability results
Since Xi'an, Xianyang and Baoji have the highest scores, the comprehensive analysis is carried out by taking these three cities as examples. The five factor scores of the three cities are calculated. The results are as follows:

![Factor Score Chart]

As can be seen from the figure above, Xi'an is much ahead of Baoji and Xianyang in terms of per capita urban resources, comprehensive urban strength, environmental construction and investment in science and education, and is the most livable among the seven cities. However, Baoji and Xianyang are relatively comprehensive in terms of five indicators, and there is little difference in each indicator between the two cities. The comprehensive strength of these two cities is relatively strong. They have become a livable city second only to Xi'an.

2.4. Multi-parameter sensitivity analysis
The sensitivity analysis of multiple parameters based on Monte Carlo\cite{4} simulation is selected in this article. The specific steps are as follows:

(1) Select the test parameters, that is, select the evaluation indicators that need to be analyzed.
(2) Set the fluctuation range of the five indicators.
(3) For each selected indicator, five uniformly distributed independent random numbers are generated within the range of values of the five indicators;
(4) Using the generated five random numbers to solve the problem based on the above neural network evaluation model, calculate the corresponding city livability score;
(5) The deviation of the simulation result from the actual result is measured by the objective function value, and the objective function value is expressed by the sum of the squares of the error between the analog value and the measured value. Calculated as follows:

$$f = \sum_{i=1}^{N} [y_a(i) - y_c(i)]^2$$
If the simulated objective function value is less than the "subjective indicator", it is considered "acceptable", otherwise it is "unacceptable", taking 33%, 50%, and 66% of the N objective function values obtained by the simulation. The objective function value of the locus is taken as the "subjective indicator". The distribution of "acceptable" and "unacceptable" index values is obtained after 10,000 times of simulation.

(6) Evaluate the sensitivity of each indicator: compare the distribution of the two sets of indicators and make a cumulative frequency curve. The measured values are obtained by simulating the median value range of each indicator. The degree of separation of the two cumulative frequency curves of the indicator represents the sensitivity of the indicator. The calculation method is as follows:

\[
SD = 1 - \frac{\sum_{i=1}^{N} (Y_i - \bar{Y})^2}{\sum_{i=1}^{N} (Y_i - \bar{Y})^2}
\]

According to the multi-parameter sensitivity analysis model based on Monte Carlo simulation, the cumulative frequency curve of five indicators is obtained, and the cumulative frequency curve of per capita urban resources and urban comprehensive strength is selected, as shown below:

![Cumulative frequency curve of two sets of index values.](image)

The greater the degree of separation between the "acceptable curve" and the "unacceptable curve", the greater the sensitivity of the indicator. It can be directly seen from the above figure that the separation of the two curves of "per capita possession of urban resources" is relatively large, that is, the sensitivity of the indicator is relatively large. The sensitivity of "urban comprehensive strength" is relatively small.

We calculate the degree of separation SD of the two curves for each indicator. The result is as follows:

| parameter | \( F_1 \) | \( F_2 \) | \( F_3 \) | \( F_4 \) | \( F_5 \) |
|-----------|-----------|-----------|-----------|-----------|-----------|
| SD        | 0.9853    | 0.1917    | 0.7044    | 0.2265    | 0.2144    |

The closer the SD value is to 1, the less sensitive the parameter is. Therefore, the following conclusions can be drawn:

The sensitivity of the five indicators from large to small is the per capita possession of urban resources, environmental construction intensity, educational development level, science and education investment, and the comprehensive strength of the city. That is to say, the per capita possession of urban resources and environmental construction intensity of a city is a relatively important indicator, which has greatly affected the livability of the city.

3. Dynamic comprehensive evaluation model of urban livability

Under the influence of uncertain factors, the indicators for measuring the livability of cities are not only related to the current indicators, but also have an important relationship with the time changes. Therefore,
we establish a relationship between indicators and changes in time, and then evaluate and analyze the livability of the city.

3.1. Selection of uncertain factors

According to the reality, there are three uncertain factors that have great influence on the livability of cities:

① Sudden natural disaster $Y_1$: A sudden natural disaster will cause the value of $F_1, F_2, F_3, F_4$ to drop rapidly in a short period of time.

② Market economy fluctuations $Y_2$: When the market economy fluctuates, it will affect the value of $F_1, F_2$ to some extent.

③ Adjustment of macro policy $Y_3$: It can change the environmental construction intensity of the city and the investment in science and education, so the value of $F_1$ and $F_5$ will fluctuate.

3.2. Describe the process of change of indicators

(1) Assume that the degree of damage to the indicator caused by natural disasters in time is:

$$ D = dt \times P \times (S - Q) $$

where $S$ is the disaster destruction rate, $Q$ is the repair rate after the disaster, $P$ is the probability of disaster occurrence.

In the process of describing natural disasters, the following indicators are defined:

It is generally believed that the degree of damage $D$ of natural disasters in $dt$ time is directly proportional to the value of the index. When a city develops faster, the construction is more perfect, that is, when the livability is high, the degree of damage to natural disasters in a short period of time is greater, measured by the following formula:

$$ S = dt \times X_i \times k_1 $$

It is generally believed that the ability $Q$ of the city to repair itself during the time $dt$ is directly proportional to the value of the indicator. When a city’s comprehensive strength is strong, the speed of repair after a natural disaster is faster, which can be measured by the following formula:

$$ Q = dt \times X_i \times k_2 $$

(2) The degree of perturbation of various measures by markets and policies $R$ is:

$$ R = dt \times T \times V $$

where $T$ is the probability of disturbance occurring and $V$ is the degree of influence of disturbance on the indicator.

The disaster occurs randomly or does not occur at a certain moment, and the pros and cons of the indicator are random, and the probability $T$ of the disturbance is:

$$ T = \begin{cases} 
1 & \text{Negative disturbance} \\
0 & \text{No disturbance} \\
1 & \text{Positive disturbance} 
\end{cases} $$

It is generally believed that when a city's economic development is more rapid, it will be more disturbed by the market and policy measures in a short period of time. It can be measured by the following formula:

$$ V = dt \times X_i \times k_3 $$

3.3. Establishment of stochastic differential equations

Taking factor as an example, the change in time from to can be written as:

$$ X_3(t + \Delta t) - X_3(t) = aX(t)(1 - X(t)/c)\Delta t - P(t)X(t)(k_1 - k_2)\Delta t $$
where, \( a \) is a constant and \( c \) is the maximum value when the index reaches stability.

In the form of a differential equation:

\[
\frac{dX_3}{dt} = X_3 \left(1 - \frac{X_3}{c}\right) - PX_3k
\]

Similarly, the differential equations for all five indicators are

\[
\begin{align*}
\frac{dX_1}{dt} &= aX_1 \left(1 - \frac{X_1}{c_1}\right) - PX_1k + X_1(T_k + T_2 k, i, j) \\
\frac{dX_2}{dt} &= aX_2 \left(1 - \frac{X_2}{c_2}\right) + T_1X_2k, i, j \\
\frac{dX_3}{dt} &= aX_3 \left(1 - \frac{X_3}{c_3}\right) - PX_3k \\
\frac{dX_4}{dt} &= aX_4 \left(1 - \frac{X_4}{c_4}\right) \\
\frac{dX_5}{dt} &= aX_5 \left(1 - \frac{X_5}{c_5}\right) + T_2X_5k, i, j
\end{align*}
\]

3.4. Establishment of dynamic comprehensive evaluation model

In the multi-index dynamic comprehensive evaluation, the time-series stereo data can be regarded as composed of three-dimensional data composed of indicators, evaluation objects and time. In this paper, the quadratic weighted evaluation method is used to obtain the comprehensive evaluation value of the evaluated object at a certain time from the selected comprehensive evaluation model.

The second weighted evaluation method adopts the method of two weighted synthesis. The first weighted synthesis is to highlight the important role of each evaluation index at different times. The second weighted synthesis is based on the first weighted synthesis, and then highlight the role of time.

That is, the weight coefficient \( \psi_j(t_k) \) \((j = 1,2,\cdots,m; k = 1,2,\cdots,p)\) of the index \( x_i \) at time \( t_k(k = 1,2,\cdots,p) \) is given, and the comprehensive evaluation value \( y_i(t_k) \) of the system (or the evaluated object) \( S_i \) at time \( t_k \) is calculated by the selected comprehensive evaluation model.

In order to reflect the impact of timing on the system, the TOWA (or TOWGA) operator is introduced to define the final evaluation result as:

\[
h_i = F(t_k, y_i(t_k), y_i(t_2), \cdots, y_i(t_n), y_i(t_p)) = \sum_{i=1}^{p} w_k b_{ikw} (i = 1,2,\cdots,n)
\]

\[
(\text{or } h_i = G(t_k, y_i(t_k), y_i(t_2), \cdots, y_i(t_n), y_i(t_p)) = \prod_{i=1}^{p} b_i w_k (i = 1,2,\cdots,n))
\]

where \( h_i \) is the final evaluation value of the system, \( W = (w_1, w_2,\cdots,w_p)^T \) is the time weight vector, and \( b_{ikw} \) is \( \psi_j(t_k) (k = 1,2,\cdots,p) \) in the TOWA pair corresponding to the \( k-th \) time.

The entropy of the time weight vector reflects the extent to which the weight contains information in the assembly process of the sample. Scientifically determining the time weight vector \( W = (w_1, w_2,\cdots,w_p)^T \) will be the key to obtaining a reasonable evaluation result.

First, define the entropy of the time weight vector as:

\[
I = -\sum_{k=1}^{p} w_k \ln w_k
\]

Define degree of time:

\[
\lambda = \sum_{k=1}^{p} \frac{p-k}{p-1} w_k
\]

Given the size of "degree of time" \( \lambda \), the time weight vector suitable for the aggregation of the sample is found by mining the information of the sample as much as possible and taking into account the difference information of the evaluated object in time series as the standard, so as to obtain more accurate evaluation results. The calculation is shown below:
### 3.5. Analysis of results of dynamic comprehensive evaluation model\cite{6}

(1) Prediction results of various indicators under uncertain factors

Taking Weinan City as an example, according to the differential equation, the change of each indicator over time is obtained, and the value of each indicator in the next 120 months is simulated. The changes of some indicators are shown in the figure below.

![Simulation of various indicators in the next 120 months.](image)

As shown in the above figure, the comprehensive strength of the city is related to two uncertain factors of natural disasters and market fluctuations. Under the compensation of two factors, the change of this index is more obvious. The intensity of environmental construction is only related to one factor of natural disasters. According to the above dynamic comprehensive evaluation model model, taking Weinan City as an example, after a weighting, the score change of Weinan City in the next ten years is as follows.

![Weinan City's score for the next ten years.](image)

From the above model, the values of the time weight vector are as follows:

\[ W = (0.0059, 0.0086, 0.0148, 0.0255, 0.0396, 0.0755, 0.0147, 0.2238, 0.6637, 0.8210) \]

Each city is sorted with the weight vector determined, and the final sort result is as follows:
Table 2. Sorting result based on two operators.

| City       | Yangling | Shangluo | Weinan | Xianyang | Baoji | Tongchuan | Xi'an |
|------------|----------|----------|--------|----------|-------|-----------|-------|
| Evaluation value | 0.1180   | 0.1695   | 0.1725 | 0.1756   | 0.1754| 0.1683    | 0.1849|
| Sort       | Xi'an>Xianyang>Baoji>Hainan>Shangluo>Tongchuan>Yangling |

Results based on TOWA operator

| City       | Yangling | Shangluo | Weinan | Xianyang | Baoji | Tongchuan | Xi'an |
|------------|----------|----------|--------|----------|-------|-----------|-------|
| Evaluation value | 0.0719   | 0.1092   | 0.1114 | 0.1138   | 0.1137| 0.1082    | 0.1231|
| Sort       | Xi'an>Xianyang>Baoji>Hainan>Shangluo>Tongchuan>Yangling |

The comparison of the ranking of the model with the scores of the neural network evaluation model is as follows:

![Figure 8. Comparison of scores before and after.](image)

As shown in the figure, the results of the two algorithms of the dynamic comprehensive evaluation model are the same. When comparing the rankings of the neural network evaluation model, it is found that the rankings of Xianyang and Baoji are reversed, and Yangling’s ranking has retreated several times. The analysis shows that this is because the size of Yangling City is relatively small, and it is affected by some uncertain factors, and the development is very unstable.

4. Conclusion

In this paper, the original 23 indexes are screened into 5 comprehensive indexes by factor analysis, and the livability evaluation system is established, and the livability ranking order of seven cities in Shaanxi Province is obtained. According to livability, these seven cities have been clustered. Xi'an has the largest scale and the highest livability, and it is a single class. Through sensitivity analysis, the sensitivity of the five indicators from large to small is the per capita possession of urban resources, environmental construction, the degree of education development, the investment in science and education, and the comprehensive strength of the city. Taking into account the impact of natural disasters, market policies and other uncertain factors, and taking Weinan City as an example, the changes of various indicators under uncertain factors are analyzed. Finally, the dynamic comprehensive evaluation model of urban livability is given, and the new city livability ranking is obtained. According to the evaluation information of this paper, it is concluded that there are problems such as insufficient development of education in Xi'an, and many policy suggestions are put forward for these problems.

The dynamic comprehensive evaluation system used in this paper is much more complicated than the static comprehensive evaluation system. The core problem is to reduce the dimension. This method can be widely used to solve dynamic comprehensive evaluation problems, such as economic system, dynamic parity of employee performance and so on.
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