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A semi-analytical model for the study of the evolution of a microwave plasma channel

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Abstract. A simplified electrodynamic (non-electrostatic) approach is developed to study the evolution of a single microwave (MW) plasma channel (plasmoid) forming in the above breakdown electric field of linearly polarized MW beam (or beams). The results of numerical calculations in air within the pressure range \( P = 60 - 140 \) Torr based on this approach coupled with a sufficiently complete system of plasma chemical reactions, diffusion and photoionization are presented. Quasi-stationary values of the main MW plasmoid characteristics are estimated. The estimated values of the plasma density, the length of the plasmoid and its dipole moment agree well with the experimental results.

1. Introduction

In the problems of plasma aerodynamics and plasma-assisted combustion, a MW channel is considered as an object transforming the energy of the electromagnetic radiation into the gas energy [1,2]. The plasma channel is formed as a result of the development of a freely localized (away from surfaces) non-equilibrium MW discharge in the above breakdown electric field of a linearly polarized wave beam (or beams) at high pressure under the condition \( \nu \gg \omega \) (\( \nu \) is an effective collision frequency of electrons with molecules, \( \omega \) is the electromagnetic wave angular frequency). The investigation of the system MW radiation-plasmoid-gas requires a self-consistent solution of Maxwell's equations, plasmachemical kinetics equations and gasdynamics equations. The characteristic time scale at which the gas density changes noticeably is longer than the channel formation time. This fact makes it possible to use the following simplified scheme of the investigation.

The stage of a plasmoid formation under the condition of constant gas density. Three main phases of the plasma channel formation in the above breakdown electric field are revealed both theoretically [3] and experimentally [4]. (i) The development of an electron avalanche in an incident electric field. (ii) The MW channel-streamer elongation in mutually opposite directions in parallel to the incident electric field up to the length comparable to the wavelength, \( \lambda \), of the incident radiation. (iii) At the quasi-stationary phase, the dimensions of the plasmoid change very slightly [4], the plasma density, the electric field inside a plasmoid, the total charge and current as well as the absorbed power reach quasi stationary levels [3]. It should be emphasized that the plasmoid accumulates its energy at the quasi-stationary phase.

The gasdynamic stage. At this stage, the accumulated energy can be used as an input parameter in the equations of gasdynamics.

Here, we present a semi-analytical self-consistent model which makes it possible to investigate the space-time evolution of a plasmoid under the condition of constant gas density (first stage). The model
is based on a simplifying assumption that allows the research to be carried out within one-dimensional approximation. The main feature of the model is the analytical expression for the electric field on the axis of the plasma channel. The model contains one undefined parameter - the characteristic radius of the streamer - that can be estimated on the basis of the discharge photos. The model has been applied to a freely localized self-sustained microwave discharge developing in the form of a single thin plasma channel in the above breakdown electric field of linearly polarized microwave beams in air within the pressure range of \( P = 60 - 140 \text{ Torr} \). We have estimated quasi-stationary values of the main MW plasmoid characteristics (the plasma density, the absorbed power). The estimated values of the plasma density, the length of the plasmoid and its dipole moment agree well with the experimental results.

2. Model

A plasmoid is in a high-frequency electric field \( \text{Re}\{\mathbf{E}_p(\mathbf{r}) \exp(-i\omega t)\} \) with components \( \mathbf{E}_0 = (0,0,0) \). Its longitudinal (along the incident electric field \( \mathbf{E}_0 \)) dimension is less than the wavelength and characteristic transverse dimensions are much less than \( \lambda \). The coordinate origin is placed at the plasmoid center. We assume that the variations in the amplitude of the electric and magnetic fields, the averaged over a period, \( T = 2\pi/\omega \), plasma density, the characteristic dimensions of the plasmoid are insignificant for the cycle \( T \).

The complex amplitude of the total electric field can be written in the integral form [5]

\[
\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0(\mathbf{r}) + \mathbf{E}_p(\mathbf{r}, t),
\]

(1)

where \( \mathbf{E}_p \) is the slowly time varying (\( \partial \mathbf{E}_p \ll \omega \mathbf{E}_p \)) complex amplitude of the plasma response electric field produced by electron current and space charges

\[
\mathbf{E}(\mathbf{r}, t) \approx iE_c(t)\sigma_e(t)\Psi(\mathbf{r}, t),
\]

(2)

\[
E_c \approx \frac{\varepsilon_0}{1-i\sigma_e \Psi_c},
\]

(3)

\[
\Psi_x = -\frac{k^3}{4\pi} \int dV'f(\mathbf{r}', t) G(\kappa)\eta_x\eta_x(1+i\kappa^{-1} - 3\kappa^{-2}),
\]

\[
\Psi_z = \frac{k^3}{4\pi} \int dV'f(\mathbf{r}', t) G(\kappa)[1 + i\kappa^{-1} - \kappa^{-2} - \eta_x^2(1 + 3i\kappa^{-1} - 3\kappa^{-2})].
\]

\( E_c, \sigma_e, \Psi_c = E_z, \Psi_z(\mathbf{r} = 0), \sigma = \sigma/\omega \varepsilon_0, \sigma = \varepsilon N_e/m(\nu - i\omega) \) is the high-frequency complex electrical conductivity, \( N_e \) is an averaged over the period \( T \) electron density, \( \varepsilon \) is the electron charge, \( m \) is the electron mass, \( \varepsilon_0 \) is the permittivity of free space, \( k = \frac{2\pi}{\lambda}, f = \sigma/\sigma_c, G = \exp(\frac{ik}{\kappa}), \kappa = k|\mathbf{r} - \mathbf{r}'|, \eta = (\mathbf{r} - \mathbf{r}')/|\mathbf{r} - \mathbf{r}'| \).

Everywhere below, we will assume the condition \( \theta < 1/\left(\theta = \omega/\nu\right) \) to be met.

The results of the experiments [4-9] indicate that a plasma channel is nearly ellipsoid. This allows us to consider the interaction of an electromagnetic wave with a channel, which averaged over a microwave cycle electron density is distributed as follows

\[
N_e(\mathbf{r}, t) = N_{\text{vis}}(t)f(t, \xi(t)), 0 \leq \xi \leq 1,
\]

(4)

where \( \xi = \sqrt{(r/R_{\text{max}})^2 + (z/Z_{\text{max}})^2}, r = \sqrt{x^2 + y^2}, R_{\text{max}} \) and \( Z_{\text{max}} \) are the minor and major semi-axes of the prolate spheroid, respectively. Within the scope of our model \( Z_{\text{max}} \geq \lambda, Z_{\text{max}} = \text{const} \)

\( R_{\text{max}}(t) = Z_{\text{max}}/L(t) \). The value \( L \) comes from the experiment:

\[
L(t) = \frac{R_{\text{vis}}(t)}{R_{\text{vis}}(t)},
\]

(5)

where \( Z_{\text{vis}}, R_{\text{vis}} \) are the visible dimensions of the plasmoid in the photos. We assume that the visible volume is localized inside the spheroid with

\[
N_e(r \leq R_{\text{vis}}z \leq Z_{\text{vis}}) \geq 10^{12} \text{ cm}^{-3}.
\]

The value \( Z_{\text{vis}}(t) \) is computed. The assumption (4) transforms the 2D problem into the 1D one. Based on this assumption, we can consider the self-consistent evolution of the electron density and longitudinal electric field, \( E_z(z, t) \), along only \( z \)-axis.

In the approximation (4) the expression for the form-factor \( \Psi_z \) is

\[
\Psi_z(z, t) = \Phi(z, t) + \sum_{m=0}^{\infty} \Psi_m(z, t),
\]

(7)

where \( z = z/Z_{\text{max}} \).
\[
\Phi(z,t) = -f(z,t) + \frac{2\mu^4}{t^2} \int_0^1 dz \left[ \frac{f(z,\xi, t)}{(z^2-\xi^2)^2} \right], \quad \mu = L/\sqrt{L^2 - 1}, \\
\Psi_m(z,t) = \frac{(k^2x)^2}{m!(m+2)} \int_0^1 d\xi \xi^{m-1} f(\xi, t) \Gamma_m(\frac{z}{\xi}), \\
\Gamma_m(\frac{z}{\xi}) = \int_1^1 dx \left[ \frac{(m-1)(1-x^2)}{2z^2} U^{m-3} \right], \quad U = \left( x^2 - 2\mu^2 \frac{z^2}{\xi^2} + \frac{\mu^2}{z^2} + \left( \frac{z}{\xi} \right)^2 \right).
\]

Here we used the representation for the parameter \(\Psi_z\) obtained by expanding function \(G\) in series in powers of \(kr\).

The averaged over a MW cycle electron number density in air follows the equation
\[
\partial_t N_e = Q_{ion} + Q_{ass} - Q_{rec} - Q_{at} + Q_{det} + Q_{ph}\, + \, \nabla \cdot (D_{eff} \nabla N_e), \quad (8)
\]
where \(Q_{ion}, Q_{ass}, Q_{rec}, Q_{at}, Q_{det}, Q_{ph}\) are contributions of the impact ionization, associative ionization \([10],\) recombination, attachment, detachment from negative ions and photoionization \([11]\) processes, respectively, \(D_{eff}\) is the effective diffusion coefficient. The effective diffusion coefficient is given by \([12]\)
\[
D_{eff} = \frac{C_D D_e + D_a}{1 + C_D},
\]
where \(D_{e}, D_{a}\) are the free and ambipolar diffusion coefficients, respectively, \(C_D = \nu_i \tau_M, \tau_M(z,t) = \varepsilon_0/\sigma(z,t)\) is the local Maxwell’s relaxation time.

The kinetic block of the model in air includes the system of processes that describe the change in the densities of the main neutral and charged components of the nitrogen–oxygen mixture and the vibrational excitation of the gas. Nine types of positive and negative ions are taken into account: \(O_2^+, O_2^+, N_2^+, N_2^+, NO^+, O^-, N_2^-, O_2^-, O_2^-\). The system of ion-molecular reactions \([10]\) was taken as the basis. In addition, the reactions involving the following neutral particles were included in the model:
\[
N_2(X^1\Sigma_g^+), N_2(A^3\Pi_u), N_2(B^3\Pi_g), N_2(C^3\Pi_u), N_2(a^4\Sigma_g^+), N(4S), N(2D), N(2P), NO, O_2(X^3\Sigma_g), O_2(a^1\Delta_g), O(3P), O(1D), O(1S) \quad [10, 13].
\]
In the conditions under consideration, the particles are assumed to be excited mainly by electron impact from the ground electronic state.

Using this model we considered the main features of the development of a plasmoid in air within the ranges of discharge parameters \(P = 60 - 140\) Torr, \(E_{eff} = (1.1 \div 1/3)E_{br}\), \(E_{eff} = |E_0|/\sqrt{2(1+\theta^2)}, E_{br}\) is the breakdown electric field, \(E_{eff} = 40\) V·cm\(^{-1}\)·Torr\(^{-1}\) and obtained the quasi stationary values of its basic characteristics.

**Numerical results**

The initial stage of the development lasts as long as a plasma cloud is transparent to an incident electromagnetic radiation. The duration of the initial stage, \(t_0\), for \(P > 50\) Torr, \(E_{eff} < 45\) V·cm\(^{-1}\)·Torr\(^{-1}\), and the corresponding maximum electron density at the cloud center, \(N_{ec0}(t_0)\), are given by \([5]\)
\[
t_0 \approx \frac{1}{\nu_1^{eff}(E_{eff}/P)} \left( \frac{E_{eff}}{\sigma_0(t_0)/n_0(t_0)} \right), \quad N_{ec0} \approx \frac{5 \times 10^{11}P[\text{Torr}]}{\lambda[\text{cm}]}, \quad (9)
\]
where \(\nu_1^{eff} = \nu_i - \nu_a\), \(\nu_a\) is the attachment frequency, \(n_z = \mu \ln(L + \sqrt{L^2 - 1})/(L^2 - 1)\) is the depolarization factor in the \(z\) direction \((n_z = 1/3\) for a sphere\). We assume that by the end of the initial stage the profile \(f(\xi, t_0)\) is Gaussian with the radius \(r_0\). A rough, order-of-magnitude estimation of this radius from above is
\[
r_0 < \sqrt{D_{eff}t_0}. \quad (10)
\]

Running ahead, we will note that the streamer dynamics and its basic quantitative characteristics, at the final stage of the evolution, are virtually independent of the choice of the parameter \(r_0\).

The photos of the streamer evolution demonstrate that the visible radius is proportional to the visible length, when \(Z_{vis} \geq Z_{vis0} \geq 0.05\) cm. On the basis of this result as well as the approximation from \([3]\) in our computations we used the following expression
\[ R_{\text{vis}} \approx \begin{cases} r_0(Z_{\text{vis}}/r_0)^b, & r_0 \leq Z_{\text{vis}} \leq Z_{\text{vis}0}, \\ 0.2Z_{\text{vis}}, & Z_{\text{vis}} \geq Z_{\text{vis}0} \end{cases} \quad [3] \]

where \( b = \ln(0.2Z_{\text{vis}0}/r_0)/\ln(Z_{\text{vis}0}/r_0) \). Note that the sizes \( R_{\text{vis}}, Z_{\text{vis}} \) depend slightly upon air pressure within the range \( P = 60 - 140 \) Torr. Under our experimental conditions, a plasma channel develops in the focal region of the standing wave, where the electric field spatial distribution can be approximated by the expression

\[ E_{oz}(r) = \text{Re} \left\{ \exp(-i\omega t) \left[ A_+ \exp \left( ikz - \left( \frac{y^2 + z^2}{a_f^2} \right) \right) + A_- \exp(-ikz) \right] \right\} \quad [12] \]

with \( a_f \approx 0.7\lambda, \lambda = 2.3 \) cm, \( A_- \approx 0.1A_+ \).

The simulation results at times \( t > t_0 \) (the streamer and quasi stationary phases are under the investigation) are presented in figures 1–5.

Figure 1. Axial profiles of (a) electron density, (b) axial component of the normalized electric field amplitude plotted from the streamer center (one half of the streamer is represented). Time moments \( \tau = 50 - 350 \) ns (1–7), step 50 ns. \( E_{br} \) is the breakdown level of the electric field. Air \( P = 100 \) Torr, \( E_{eff} = 1.2E_{br} \).

Figure 1 shows a typical picture of the evolution of electron density profiles and the amplitude of the electric field along the \( z \) axis. As follows from this figure, at \( t < 250 \) ns, the maximal magnitude of the amplitude of the electric field increases along with the plasma density. In the central region the
amplitude of the electric field slowly decreases. With increasing of the plasma density and dimensions of the streamer, at $t > 250$ ns, the role of the electric field produced by the current flowing along the streamer significantly growth. This field tends to compensate the field of the space charges both in the central area and at the tips of the streamer [3]. As a result, the ionization waves are sharply slowed down, when the length of the streamer is about $0.7\lambda$. Later, at $t > 300$ ns, the electrostatic field of the space charges can no longer ensure further noticeable elongation of the plasmoid. Finally, at $t > 350$ ns, the state of an ionization-recombination quasi equilibrium reaches. In this state, the plasma density and the amplitude of the electric field reach their quasi-stationary levels.

Figure 2 Contour lines of the electron density $N_e = 10^{11}$ cm$^{-3}$ and $N_e = 10^{13}$ cm$^{-3}$ (dots). Time moments are 200 ns (1), 250 ns (2), 350 ns (3).

In order to verify some predictions of our model, we compared the experimental data with the results of numerical calculations. As follows from the experiments, (i) normalized full maximum channel length, $2Z_{vis}^{(max)}/\lambda$, is in the ranges of values $2Z_{vis}^{(max)}/\lambda \approx 0.6 \div 0.8$, and this length is weakly dependent on the pressure and amplitude of the incident field (our study); (ii) the quasi stationary value of the amplitude of the normalized dipole moment of the plasmoid, $|d_s^{(qs)}|$, $d_s = k^3 |d|/4\pi\varepsilon_0E_0$, $|d|$ is the dipole moment amplitude), is in the range of values $|d_s^{(qs)}| \approx 0.5 \div 0.7$, within the ranges of discharge parameters $P = 60 \div 140$ Torr, $E_{eff} = (1.1 \div 1.3)E_{br}$ [5]; (iii) the value of the electron density estimated in nitrogen by the spectroscopy method (Stark broadening of the hydrogen spectral line $H_\beta$) under the experimental conditions $\lambda = 4$ cm, $P = 50$ Torr [6] and $\lambda = 2$ cm, $P = 70$ Torr [14] are $N_e = 10^{14}$ cm$^{-3}$ and $N_e = 0.9 \cdot 10^{14}$ cm$^{-3}$, respectively.

Figure 3 shows time dependences of the normalized channel length (by the channel length we will mean the distance $2L_{ch}$ between the points at which the electric field amplitude is at a maximum) and amplitude of the normalized dipole moment of the plasmoid at different values of pressure and amplitude of the incident electric field. Figure 4 reflects the dynamics of the averaged over a MW cycle electron density at four axial points in the studied range of discharge parameters. In our opinion, the results of the experiments are in agreement with the predictions of our model.
Figure 3 The dynamics of the (a) normalized full plasma channel length, $2l_{ch}/\lambda$, and (b) amplitude of the normalized dipole moment, $|d_{z}|$, of the plasma channel in air for $E_{eff} = 1.1E_{br}$, (1) $P=60$ Torr, (2) $P=140$ Torr; $E_{eff} = 1.3E_{br}$, (3) $P=60$ Torr, (4) $P=140$ Torr.

Figure 4 The dynamics of the time-averaged electron density at the points $z/\lambda=0$ (1), 0.1 (2), 0.2 (3), 0.3 (4) for $E_{eff} = 1.1E_{br}$, (a) $P = 60$ Torr, (b) $P = 140$ Torr; $E_{eff} = 1.3E_{br}$, (c) $P = 60$, Torr, (d) $P = 140$ Torr.
The most important characteristics (for gasdynamic and plasma-assisted combustion applications) of a microwave plasmoid are the absorbed power (Joule losses)

\[ W_J = \frac{[dv|E|^2]}{2(1+\sigma^2)} = E_{\text{eff}}^2 \int dV \sigma \frac{E^2}{E_0^2} \]  

(13)

and the absorbed energy. As follows from expression (13) \( W_J \propto \left(\frac{P E_{\text{eff}}}{E_{br}}\right)^2 \). Figure 5 shows the evolution of the averaged over a MW cycle absorbed power and energy under the same conditions as in figure 4.

Summary

The semi-analytical self-consistent model is developed to study the space-time evolution of a single MW plasma channel forming in the above breakdown electric field of linearly polarized MW beam (or beams). The model is based on simplifying assumption that transforms the 2D problem into the 1D one. Using the model, we considered the main features of the development of a plasmoid in air within the ranges of discharge parameters \( P = 60 - 140 \) Torr, \( E_{\text{eff}} = (1.1 \div 1.3) E_{br} \) and obtained the quasi stationary values of the absorbed power and energy. The quasi stationary values of the plasma density, the length of the plasmoid and its dipole moment agree well with the experimental results.

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