THE STANDARD MODEL OF LEPTONS AS A PURELY VECTORIAL THEORY

(revised and augmented version)

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Abstract. We propose a way to reconcile the Standard Model of leptons with a purely vectorial theory. The observed neutrino is predicted to be massless. The unobservability of its partner and the $V-A$ structure of the weak currents are given the same origin.

PACS: 12.15.Cc 12.15.Mm 13.10.+q 14.60.Gh 14.80.Gt

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1 Introduction

The Standard Model \[1\] is the presently accepted description of the leptonic interactions. Its most puzzling aspects stay anyhow the \( V - A \) structure of the weak currents and the experimental absence of a right-handed neutrino. We connect here both phenomena: the structure of the weak currents is related to the unobservability of one (Majorana) component of the neutrino which gets an infinite mass; the other component, corresponding to the observed neutrino, is strictly massless. This arises in a purely vectorial theory endowed with a composite scalar multiplet.

We describe here an anomaly-free leptonic sector. We can forget about the quarks since we make the hadronic sector anomaly-free too in refs.\[2, 3\]; in the same works the gauge boson mass generation is presented as a hadronic phenomenon.

2 The leptonic Lagrangian

We only consider here 1 family of leptons, and we will not question the \( e - \mu - \tau \) universality. Let \( \Omega \) be the leptonic doublet

\[
\Omega = \begin{pmatrix} \ell^- \\ \nu \end{pmatrix},
\]

(1)

with hypercharge \( Y = Q - T^3 = -1/2 \). The starting Lagrangian is

\[
\mathcal{L} = i \bar{\Omega} \gamma^\mu (\partial_\mu - ig' B_\mu Y - ig \vec{W}_\mu \vec{T}) \Omega - m \bar{\Omega} \Omega.
\]

(2)

\( \vec{T} = \vec{\tau}/2 \), with \( \vec{\tau} \) the Pauli matrices. The interaction being purely vectorial, the mass term is gauge invariant and there is no anomaly.

We introduce the 2 Majorana neutrinos

\[
\begin{cases}
\chi = \nu_L + (\nu_L)^c, \\
\omega = \nu_R + (\nu_R)^c.
\end{cases}
\]

(3)

The superscript “c” means “charge-conjugate”. In terms of \( \chi \) and \( \omega \), \( \mathcal{L} \) can be written (we forget hereafter the ‘’-‘’ superscript for \( \ell \))

\[
\mathcal{L} = i \bar{\ell} \gamma^\mu \partial_\mu \ell + \frac{i}{2} \bar{\chi} \gamma^\mu \partial_\mu \chi + \frac{i}{2} \bar{\omega} \gamma^\mu \partial_\mu \omega
\]

\[
+ (\bar{\ell}, \chi) \gamma^\mu (g'B_\mu Y + g \vec{W}_\mu \vec{T}) \frac{1-\gamma^5}{2} \begin{pmatrix} \ell \\ \chi \end{pmatrix} + (\bar{\ell}, \omega) \gamma^\mu (g'B_\mu Y + g \vec{W}_\mu \vec{T}) \frac{1+\gamma^5}{2} \begin{pmatrix} \ell \\ \omega \end{pmatrix}
\]

\[
- \frac{1}{2} m (\bar{\chi} \omega + \bar{\omega} \chi + 2 \bar{\ell} \ell).
\]

(4)
3 Introducing a scalar triplet

We introduce a scalar triplet $\Delta$ of composite fields, with leptonic number 2, according to

$$\Delta = \begin{pmatrix} \Delta^0 \\ \Delta^- \\ \Delta^{--} \end{pmatrix} = \frac{\rho}{\nu^3} \begin{pmatrix} \overline{\omega_L} \omega_R \\ \frac{1}{\sqrt{2}}(\overline{\omega_L} \ell_R + (\ell_R)^c \omega_R) \\ \frac{1}{\sqrt{2}}(\overline{\ell_R}^c \ell_R) \end{pmatrix}.$$

It is a triplet of $SU(2)$ (right). Its hermitian conjugate is

$$\Delta = \begin{pmatrix} \Delta^0 \\ \Delta^+ \\ \Delta^{++} \end{pmatrix} = \frac{\rho}{\nu^3} \begin{pmatrix} \overline{\omega_R} \omega_L \\ \frac{1}{\sqrt{2}}(\overline{\ell_R} \omega_L + \overline{\omega_R} (\ell_R)^c) \\ \overline{\ell_R} (\ell_R)^c \end{pmatrix}.$$

We can impose the conditions

$$\langle \overline{\omega_L} \omega_R \rangle = \langle \overline{\omega_R} \omega_L \rangle = \nu^3 \neq 0,$$

or, equivalently

$$\langle \Delta^0 \rangle = \langle \overline{\Delta^0} \rangle = \rho.$$

Indeed, as soon as the mass of $\omega$ is not identically 0, the electroweak fluctuations depicted in fig.1 have no reason to vanish.

Fig.1: contributions to the vacuum expectation value of the $\Delta^0$ boson

The scalars being made up with fermions, the path integrations over those degrees of freedom cannot be performed independently; we thus introduce (like in the hadronic sector) constraints that we exponentiate into the effective Lagrangian ($\Lambda$ is an arbitrary mass scale)

$$\mathcal{L}_c = \lim_{\beta \to 0} -\frac{\Lambda^2}{\beta^2} \left[ (\Delta^0 - \frac{\rho}{\nu^3} \overline{\omega_L} \omega_R) (\overline{\Delta^0} - \frac{\rho}{\nu^3} \overline{\omega_R} \omega_L) + (\Delta^- - \frac{\rho}{\nu^3} \overline{\omega_L} \ell_R + (\ell_R)^c \omega_R) (\Delta^{++} - \frac{\rho}{\nu^3} \overline{\ell_R}^c \ell_R) \right]$$

(9)
4 Effective 4-leptons couplings; the mass eigenstates

The equations (4) and (9) yield a “see-saw” mechanism \[5\] in the neutrino sector. Indeed, when \(\langle \Delta^0 \rangle = \rho\), \(\mathcal{L}_c\) gives the \(\omega\) neutrino an infinite bare Majorana mass,

\[
M_0 = -\frac{\Lambda^2 \rho^2}{\beta \nu^3},
\]

(10)

the \(\chi\) neutrino a vanishing (Majorana) mass, and a finite Dirac mass connects \(\chi\) and \(\omega\). We have to diagonalize the mass matrix to get the mass eigenstates, (see for example \[8\]); they are the Majorana neutrinos \(\chi\) and \(\omega\) themselves, and correspond to mass eigenvalues 0 and \(\infty\) respectively. The charged lepton keeps its Dirac mass \(m\).

However, 4-fermions couplings may alter the mass spectrum, together with being an obstacle for renormalizability. We propose to build a reshuffled perturbative expansion based not on the ‘bare’ 4-fermions couplings occurring in \(\mathcal{L}_c\), but rather on effective couplings obtained by resumming infinite series of ‘ladder’ diagrams as proposed by Nambu and Jona-Lasinio \[6\]. We however differ from them by bare couplings and a bare fermion mass both infinite; this makes the effective couplings vanish with \(\beta\), and the “see-saw” mechanism above stay unaltered.

In all figures below, the \(L, R\)’s at the vertices stand for the projectors \((1 - \gamma_5)/2\) and \((1 + \gamma_5)/2\).

4.1 The 4\(\omega\) coupling and the \(\omega\) mass

Let \(\zeta_{\omega\omega}(q^2)\) be the 4\(\omega\) effective coupling defined by resumming the geometric series depicted in fig. 2:

\[
\begin{align*}
\omega & \quad \zeta_{\omega\omega} \quad \omega \\
L \quad x \quad R & \quad = \quad L \quad \langle \omega\rangle \quad R
\end{align*}
\]

When \(\langle \omega\rangle \neq 0\), \(\zeta_{\omega\omega}\) contributes to the \(\omega\) mass according to fig. 3 (the \(L, R\) projectors forbid fig. 4 with \(\ell\ell\omega\omega\) coupling to contribute).

\[
\begin{align*}
M & \quad = \quad M_0 \\
\omega & \quad \zeta_{\omega\omega} \quad \omega \\
\omega & \quad \omega \quad \omega
\end{align*}
\]

\[
\begin{align*}
\omega & \quad B \quad \omega \\
\omega & \quad \zeta_{\omega\omega} \quad \omega
\end{align*}
\]

Fig.2: effective 4-\(\omega\) coupling

Fig.3: the effective \(\omega\) mass
Figs. 2,3 yield the two coupled equations

\[ \zeta_{\omega\omega}(q^2) = \frac{\zeta_{\omega\omega}^0}{1 - \zeta_{\omega\omega}^0 A_{\omega\omega}(q^2, M)}, \tag{11} \]

and

\[ M = M_0 - B \zeta_{\omega\omega}(0), \tag{12} \]

where \( \zeta_{\omega\omega}^0 \) is the bare \( \omega \) coupling appearing in \( \mathcal{L}_c \), behaving like \( \beta^{-1} \)

\[ \zeta_{\omega\omega}^0 = \frac{\Lambda^2 \rho^2}{2\beta\nu^6} = \frac{M_0}{2\nu^3}. \tag{13} \]

\( A_{\omega\omega}(q^2, M) \) is the 1-loop \( \omega \) fermionic bubble, and \( B = \langle \omega\omega \rangle = 2\nu^3 \). Notice that the effective coupling at 0 momentum transfer is involved in (12).

\( B \) being finite, \( M = M_0 \) is a solution of (12) as soon as \( \zeta_{\omega\omega}(0) \) goes to 0; this is precisely the case as seen from (11) since \( A(0, M_0) \) involves a term \( b M_0^2 + \cdots \) (see for example [7]). It corresponds to an effective coupling

\[ \lim_{\beta \to 0} \zeta_{\omega\omega}(q^2) = -\frac{1}{a q^2 + b M_0^2}, \tag{14} \]

vanishing like \( \beta^2 \) when \( \beta \to 0 \). (\( a \) and \( b \) in the formulas above are numerical coefficients.)

4.2 The \( \ell\ell\omega\omega \) coupling

We define similarly the effective (\( \ell\ell\omega\omega \)) coupling by the series depicted in fig. 5. The same argumentation shows that it behaves like \( (-)A_{\ell\omega}(q^2, m, M_0)^{-1} \), where \( A_{\ell\omega}(q^2, m, M_0) \) is the fermionic loop involving one \( \ell \) and one \( \omega \), and thus vanishes like \( \beta^2 \).
4.3 The 4\(\ell\) coupling

The situation here is more delicate since the same type of resummation as above leads to an effective coupling behaving like a polynomial of degree 0 in \(\beta\), the fermionic loop involving no infinitely massive particle. However, if we go beyond the Nambu-Jona-Lasinio approximation, which corresponds, in the language of [6] to propagating \(\Delta\) bound states (see figs. 2, 5), we can show that the presence of \(\ell\ell\omega\omega\) couplings, by introducing \(\omega\) loops, makes the bare 4\(\ell\) coupling \(\zeta^0_{\ell\ell}\) exactly cancel with the four series analogous to that of fig. 6, which writes

\[
\tilde{\zeta}_{\ell\ell} = \zeta^0_{\ell\omega} \frac{A(\omega, \omega)}{1 - A(\omega, \omega)} \zeta^0_{\ell\omega} \approx -\frac{\zeta^0_{\ell\omega}}{\zeta^0_{\omega\omega}} = -\frac{1}{4} \zeta^0_{\omega\omega} = -\frac{1}{4} \zeta^0_{\ell\ell}.
\]  

(15)

The \(\zeta^0_{\ell\omega}\) and \(\zeta^0_{\omega\omega}\) are the bare \(\ell\ell\omega\omega\) and 4\(\omega\) couplings of \(\mathcal{L}_c\); \(A(\omega, \omega)\) is the fermionic \(\omega\)-loop involved here.

\[
\tilde{\zeta}_{\ell\ell} = \zeta^0_{\ell\omega} \frac{A(\omega, \omega)}{1 - A(\omega, \omega)} \zeta^0_{\ell\omega} \approx -\frac{\zeta^0_{\ell\omega}}{\zeta^0_{\omega\omega}} = -\frac{1}{4} \zeta^0_{\omega\omega} = -\frac{1}{4} \zeta^0_{\ell\ell}.
\]

(15)

Fig.6: effective 4–l coupling cancelling the bare one

Owing to the four similar series yielding the same result which can be built from the four \(\ell\ell\omega\omega\) couplings of \(\mathcal{L}_c\), we have

\[
\tilde{\zeta}_{\ell\ell} + \zeta^0_{\ell\ell} = 0.
\]

(16)

This is only to be taken as an indication that further cancellations are expected than those in the Nambu-Jona-Lasinio approximation, to which we will return for the rest of the paper.

4.4 Summary: effective couplings and the mass eigenstates

A demonstration at all orders of the renormalizability of our model is beyond the scope of this work. We hope that the arguments given above have convinced the reader that the perturbative expansion is much better behaved that could be naively expected from the presence of 4-fermions couplings and that the special type of Nambu-Jona-Lasinio mechanism invoked deserves further investigation.

As announced, the “see-saw” mechanism exhibited at tree level has not been altered by the resummations above. The neutrino mass eigenstates are the Majorana \(\chi\) and \(\omega\) themselves, with respectively infinite and vanishing mass. The charged lepton keeps its Dirac mass \(m\), as contributions from 4-leptons couplings can only be of the type of fig. 4 and vanish.

5 The decoupling of the scalars

When introducing a kinetic term for \(\Delta\), the non-vanishing of the vacuum expectation value \(\langle \Delta^0 \rangle\) contributes to the mass of the Z and W gauge bosons (see for example [8]). If \(v\) if
the vacuum expectation value of the hadronic Higgs boson (see §3), we impose its role to be dominant, which yields the necessary condition

$$\rho \ll v$$  \hspace{1cm} (17)

consistent with an electroweak nature for \(\rho\) (see fig. 1), while that of \(v\) lies \textit{a priori} outside the realm of these interactions.

We shift in the usual way the neutral scalar field according to

$$\Delta^0 = \rho + \delta^0.$$  \hspace{1cm} (18)

From the expression of \(\mathcal{L}_c\), we see that the non-vanishing of \(\rho\) yields an infinite mass for \(\delta^0, \Delta^+, \Delta^{++}\) and their conjugates. All those fields will decouple and are undetectable. In the limit \(\beta \rightarrow 0\), the longitudinal degrees of freedom of the massive gauge bosons reduce to the hadronic triplet \(\Phi\) \[3, 9\].

6 The \(V - A\) theory

The (massless) \(\chi\) neutrino has the standard weak \(V - A\) couplings and we can identify it with the observed neutrino.

The \(\omega\) neutrino is infinitely massive and will never be produced as asymptotic state; we however expect renormalization effects through \(\omega\) loops \[10\]. They drastically affect the neutral weak couplings of the leptons, in a way that rebuilds their “standard” \(V - A\) structure. This result, non-trivial if one remembers that the original coupling is purely vectorial, is shown below.

To the bare (purely vectorial) couplings of \(W_\mu^3\) we must add the following diagrams

\begin{align*}
A &= \frac{i}{16\pi^2} \frac{M^2 \ln M^2 + \cdots}{A_{\ell\omega}(q^2, m, M)} \left( \frac{g^2}{2} \right) W_\mu^3 \gamma^\mu \frac{1 + \gamma_5}{2} \ell, \\
B &= -\frac{i}{16\pi^2} \frac{M^2 \ln M^2 + \cdots}{A_{\ell\omega}(q^2, m, M)} \left( \frac{g^2}{2} \right) W_\mu^3 \bar{\ell} \gamma^\mu \frac{1 - \gamma_5}{2} \ell^c, \\
A_{\ell\omega}(q^2, m, M) &= +\frac{i}{16\pi^2} 2M^2 \ln M^2 + \cdots.
\end{align*}

\hspace{1cm} (19)

In fig. 7, the 4-fermions vertex is the effective \((\ell\ell\omega\omega)\) coupling \(\zeta_{\ell\omega}\) obtained in 4.2, vanishing like \(\beta^2\). Because of this dependence in \(\beta\), diagrams similar to fig. 7 but with \(\omega\) replaced with \(\chi\) vanish. Those involving \(\omega\) do not because the \(\omega\) loop behaves like \(\beta^{-2}\). Using the \(\overline{MS}\) renormalization scheme \[11\], we indeed find, when \(M \rightarrow \infty\):

\begin{align*}
A &= \frac{i}{16\pi^2} \frac{M^2 \ln M^2 + \cdots}{A_{\ell\omega}(q^2, m, M)} \left( \frac{g^2}{2} \right) W_\mu^3 \gamma^\mu \frac{1 + \gamma_5}{2} \ell, \\
B &= -\frac{i}{16\pi^2} \frac{M^2 \ln M^2 + \cdots}{A_{\ell\omega}(q^2, m, M)} \left( \frac{g^2}{2} \right) W_\mu^3 \bar{\ell} \gamma^\mu \frac{1 - \gamma_5}{2} \ell^c, \\
A_{\ell\omega}(q^2, m, M) &= +\frac{i}{16\pi^2} 2M^2 \ln M^2 + \cdots.
\end{align*}
Using
\[ \bar{\ell} \gamma_{\mu} \frac{1 - \gamma_5}{2} \ell = -\bar{\ell} \gamma_{\mu} \frac{1 + \gamma_5}{2} \ell, \]
we see that fig. 7 contributes to a coupling
\[ + \frac{g}{2} W^3_{\mu} \bar{\ell} \gamma_{\mu} \frac{1 + \gamma_5}{2} \ell, \]
such that \( W^3_{\mu} \) couples finally to
\[ - \frac{g}{2} \bar{\ell} \gamma_{\mu} \ell + \frac{g}{2} \bar{\ell} \gamma_{\mu} \frac{1 + \gamma_5}{2} \ell = -\frac{g}{2} \bar{\ell} \gamma_{\mu} \frac{1 - \gamma_5}{2} \ell, \]
which is the “standard” \( V - A \) coupling.

Similarly, the coupling of \( B_{\mu} \) to the charged leptons gets affected by the same mechanism. However, it couples to \( \omega \) with a ‘−’ sign with respect to the corresponding coupling of \( W^3_{\mu} \) because the hypercharge of \( \omega \) is minus the corresponding value of \( T^3 \). This yields a final coupling of \( B_{\mu} \) to leptons
\[ - \frac{g'}{2} \bar{\ell} \gamma_{\mu} \ell - \frac{g'}{2} \bar{\ell} \gamma_{\mu} \frac{1 + \gamma_5}{2} \ell = -\frac{g'}{2} (2 \bar{\ell} \gamma_{\mu} \frac{1 + \gamma_5}{2} \ell + \bar{\ell} \gamma_{\mu} \frac{1 - \gamma_5}{2} \ell), \]
which is the usual coupling of the Standard Model, corresponding to an hypercharge \( -2 \) for right handed leptons and \( -1 \) for left-handed ones.

The charged couplings do not get modified with respect to their bare values since the diagram equivalent to fig. 7 depicted in fig. 8 involving an infinitely massive \( \omega \) behaves like \( \beta^2 M \ln M \) and so vanishes with \( \beta \).

\[ \text{Fig.8: the charged couplings do not get altered} \]

Ours is thus presently experimentally undistinguishable from the Standard Model.

7 Conclusion

This work constitutes the second step in our effort to give another status to the queer features of electroweak interactions than that of simple postulates [1]; this goes in particular through cutting the existing link between the hadronic and leptonic sectors.

The results that have been obtained are not trivial: they yield a natural origin for the “see-saw” mechanism, predict a massless observed neutrino and provide a link between the unobservability of its partner and the \( V - A \) structure of the weak currents.

From a more conceptual point of view, difficulties linked with Weyl fermions are serious enough to welcome the disappearance of anomalies as a neat progress. We also propose
a yet unexplored realization of the Nambu-Jona-Lasinio mechanism \cite{6}, where both the bare fermion mass and the bare 4-fermions coupling are infinite.

It is clear that forthcoming efforts should concern renormalizability, since the argumentation at the one-loop level is only a hint that our model behaves nicely in the ultraviolet regime.

The first step of this program is performed in \cite{2} for the abelian case and in \cite{3} for the non-abelian Standard Model: we show there how the (usual) Higgs boson and the quarks become unobservable, how the anomaly cancel and how the partners of the Higgs in the scalar multiplet are in exact correspondence with observed pseudoscalar mesons \cite{4}.

We clearly do not predict heaps of new particles, or new scales of interactions and rather favour economy as an attractive feature for a model.

**Acknowledgements:** it is a pleasure to thank P. Fayet for comments and advice.
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