Influences of neutron star parameters on evolutions of different types of pulsar; evolutions of anomalous X-ray pulsars, soft gamma repeaters and dim isolated thermal neutron stars on the P-˙P diagram

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Abstract

Influences of the mass, moment of inertia, rotation, absence of stability in the atmosphere and some other parameters of neutron stars on the evolution of pulsars are examined. It is shown that the locations and evolutions of soft gamma repeaters, anomalous X-ray pulsars and other types of pulsar on the period versus period derivative diagram
can be explained adopting values of $B<10^{14}$ G for these objects. This approach gives the possibility to explain many properties of different types of pulsar.

Key words: neutron star, pulsar, AXP, SGR, evolution

1 Introduction

Soft gamma repeaters (SGRs) and anomalous X-ray pulsars (AXPs) have been observed in the last about 35 years, but their important role in astrophysics has been understood only about 10 years ago and intensive observations and theoretical research have since been done on these exotic objects. Later, some other types of neutron star have been identified. These are dim radio quiet neutron stars (DRQNSs), which are connected to supernova remnants (SNRs), and the single neutron stars with very long spin periods which are named as dim isolated thermal neutron stars (DITNSs). It is possible to find many data on these sources given in Mereghetti & Stella (1995), Mereghetti et al. (2002), Becker & Aschenbach (2002), and Guseinov et al. (2003a, 2003b).

The existence of single neutron stars with large values of $P>5$ s and $\dot{P}>5\times10^{-13}$ s/s was not expected in pulsar astronomy. In addition to this, the inverse value of the ratio of the X-ray luminosity ($L_x$) to the rate of rotational energy loss ($L_x/\dot{E}>10$) for AXPs and SGRs and the $\gamma$-ray bursts observed from SGRs were unexpected phenomena (Thompson & Duncan 1995; Mereghetti 2001). Today, it is natural to put AXPs and SGRs together as one type of young neutron star as done in the magnetar model.

It is easy and a usually chosen way to explain $\gamma$-ray bursts by recombination of the magnetic field and this is indeed a more reliable way. On the other hand, the initial magnetic field $B\sim10^{14}$-$10^{15}$ G may easily explain the ages of AXPs and SGRs and also the burst energy and the time characteristics of the bursts together with the temperatures and luminosities of these objects (Thompson & Duncan 1995; Mereghetti et al. 2002). This approach was also important to expand the limit of the physical idea and the application used in astrophysics.

The processes of steady accretion onto neutron stars have begun to be examined since $\sim40$ years ago and the idea of accretion was successfully used to predict binary X-ray sources together with their main properties
(Zeldovich & Guseinov 1966; Shklovskii 1967; Guseinov 1970; Bisnovatyi-Kogan & Komberg 1974; Lipunov 1992). Naturally, the theory of accretion from fall-back matter (Chatterjee et al. 2000) was used together with neutron star activity to explain the nature of AXPs/SGRs (Rothschild et al. 2002a, 2002b, but see also Tagieva et al. 2003).

In this work, we analyse possible evolutions of different types of single pulsar on the P-\(\dot{P}\) diagram in detail taking into consideration the possibility of the absence of stability for different masses of neutron stars. We consider AXPs/SGRs and DITNSs to be neutron stars with initial magnetic field \(\geq 10^{13}\) G.

2 Influences of the value of neutron star mass on electrodynamical parameters of pulsars

There are relations between mass and radius of neutron star which are constructed using models with different equations of state. Maximum mass values of the theoretical mass-radius relations lie in the interval 1.5-2.2 M\(_{\odot}\) for which the radius value changes in the interval 10-12 km (Bejger & Haensel 2003).

Moment of inertia (I) of a star is equal to \(\alpha MR^2\), where \(\alpha\) is a coefficient which depends on the mass density (the total energy density) with respect to radius, M the mass and R the radius of the star. From the relations given in Bejger & Haensel (2003) and Ravenhall & Pethick (1994) it is seen that the coefficient changes monotonically with respect to mass and adopting a radius of 11 km it takes values of 0.37 and 0.44 (corresponding to 1.5 M\(_{\odot}\) and 2.2 M\(_{\odot}\) respectively) for neutron stars. As seen, the value of \(\alpha\) weakly depends on the equation of state and on the mass of the compact star. The value of \(\alpha\) slowly decreases with the diminishing of the compactness. Using these values we have estimated: \(I\approx 1.3\times10^{45}\) gr cm\(^2\) for \(\alpha=0.37\) and \(M=1.5\) M\(_{\odot}\).

The neutron star mass values which are estimated from binary systems have a sharp peak at 1.4 M\(_{\odot}\) (see e.g. Tagieva et al. 2000; Lyne & Graham-Smith 1998; Charles & Coe 2003). It is not possible to measure the mass of single neutron stars, but one may assume that single pulsars also have masses about 1.4 M\(_{\odot}\). Please note that, on the period versus period derivative (P-\(\dot{P}\))
diagram of pulsars, the lines of constant effective magnetic field (B) and constant characteristic time (τ), which are found using P and ˙P values, are shown assuming R=10 km and I=10^{45} gr cm^2 (see e.g. Lyne & Graham-Smith 1998) for the case of pulsars in vacuum which have pure magnetic dipole radiation (n=3, where n is the braking index). The lines of constant rate of rotational energy loss (˙E) are also shown on the P-˙P diagram (Figure 1). Even if there exist considerable differences in the masses of neutron stars, the influences of these differences on B, τ and ˙E are neglected. Actually, in some cases the real B value must be less than the effective B value found by assuming the case of pure magnetic dipole radiation. Likewise, the characteristic time must be different than the real age in some cases and moreover the values of τ may be different for two pulsars with the same values of P and ˙P. On the other hand, in contrast to the cases of B and τ, the ˙E values found from P and ˙P values are always the actual values of pulsars if we neglect the small differences in the values of I for different pulsars. This is true also because of the very fast change in ˙E along the evolutionary tracks of pulsars.

The maximum values of the binding energy (defect mass) correspond to the maxima of the mass versus central density relations. In these relations, the part on the right of the maximum (i.e. large central density part) corresponds to the unstable states of neutron star, whereas, the part on the left of the maximum corresponds to the stable states of neutron star and the most stable neutron star states are at and just on the left side of the maximum (Zeldovich & Novikov 1971). The value of 1.4 M_⊙ found from observations is close to the lowest value (1.5 M_⊙) of the interval of the maximum mass given above. So, the neutron star models with maximum mass values ~1.5-1.6 M_⊙ are reliable, whereas, the probability of the existence of neutron stars with masses ~2-2.2 M_⊙ is very low. In this case, the I value corresponding to the maximum mass value must be adopted as ~(1.3-1.4)×10^{45} gr cm^2 and this is very close to the commonly adopted value of 10^{45} gr cm^2. It is necessary to note that even rapidly rotating neutron stars with various different equations of state based on general theory of relativity may have additional masses only up to about 10%. Influence on the radius and especially on the equatorial region is slightly larger in such cases (Gurovich & Guseinov 1965).

Let us consider a 0.7-0.8 M_⊙ neutron star and a 1.5 M_⊙ neutron star both of which have the same value of the component of dipole magnetic field B perpendicular to the rotation axis and both of which are located in vacuum.
Magnetic dipole radiation of neutron star (Lipunov 1992)

\[ L = \frac{2 \mu^2 \omega^4}{3 c^3} \sin^2 \beta = \frac{1}{3} \frac{B^2 R^6 \omega^4}{c^3} \sin^2 \beta \]  

(1)

(where \( \mu \) is the magnetic moment, \( \omega \) the angular velocity, \( c \) the speed of light, and \( \beta \) the angle between the rotation axis and the axis of the magnetic field) leads to braking of the pulsar spin. If we equate the magnetic dipole luminosity (Eqn.1) to the rate of rotational energy loss of neutron stars which have rigid body rotation given as

\[ \dot{E} = \frac{4 \pi^2 I \dot{P}}{P^3} \]  

(2)

then we get

\[ \dot{P} \propto \frac{B^2 R^4}{MP} . \]  

(3)

As seen from the equation, for the same values of \( B \) and \( P \), the \( \dot{P} \) value of a 0.7-0.8 \( M_\odot \) neutron star is about 3-4 times larger than the \( \dot{P} \) value of a 1.5 \( M_\odot \) neutron star considering also the possible increase in the radius value up to a factor of \( \sim 1.2 \) when we reduce the mass value. So, the \( \dot{E} \) value of a 1.5 \( M_\odot \) neutron star must roughly be a factor of \( \sim 2.5 \) less than the \( \dot{E} \) value of a 0.7-0.8 \( M_\odot \) neutron star for a given period value.

If we use the observational values of \( P \) and \( \dot{P} \), the component of the effective magnetic field perpendicular to the spin-axis is determined as

\[ B = \sqrt{\frac{3 c^3 I \dot{P} \dot{P}}{8 \pi^2 R^6}} . \]  

(4)

This value of \( B \) is equal to the real value if all the other mechanisms for the rotational energy loss are negligible compared to the magnetic dipole radiation.

The characteristic time of a neutron star which is defined as

\[ \tau = \frac{P}{(n - 1) \dot{P}} \]  

(5)

(where \( n \) is the braking index) does not depend on the parameters of neutron star, which may change in time, nor on any conditions in the environment if
n is assumed to be constant. In the case of pulsars which rotate as rigid body and which are in hydrodynamical equilibrium in vacuum with pure magnetic dipole radiation \((n=3)\), if the initial spin period \((P_0)\) is negligible compared to \(P\), then \(\tau\) is equal to the real age of the pulsar. If \(n=\)constant along all the evolutionary tracks, then the age of the pulsar is

\[
t = \frac{P}{(n-1)\dot{P}}(1 - (\frac{P_0}{P})^{n-1}).
\]  

(6)

In principle, the value of \(n\) may change continuously along the evolutionary track. As \(\tau\) shows the character of the period change at every point along the trajectory, it is a local characteristic and expression (5) always gives the real value of \(\tau\) if we use the real value of \(n\) at each point of the evolutionary track for each pulsar.

There will be a difference of a factor of about 3-4 between the \(\tau\) values of a 1.5 \(M_\odot\) neutron star and a 0.7-0.8 \(M_\odot\) neutron star because of the larger value of \(\dot{P}\) for smaller value of mass for the neutron star if both of the neutron stars have the same magnetic field.

Let us consider two neutron stars which have the same dipole magnetic field but different masses under the following assumption. Both of these rigidly rotating neutron stars have the same rotational period, but the less massive one with mass \(M_2\) has 10 times larger \(\dot{P}\) compared to the other. In such a case, \(\tau_2=0.1\tau_1\) from expression (5). If we estimate value of the ratio \(E_2/E_1\) from Figure 1, we find that \(E_2=10E_1\). Actually, if \(\dot{P}_2=10\dot{P}_1\) then \(E_2>E_1\) but not 10 times, because the ratio \(E_2/E_1\) depends on the equation of state which is used in the constructed model of neutron star. If the neutron star does not rotate as rigid body, then the dependence on the total value of rotational moment becomes stronger.

In Figure 1, the locations of \(\tau=\)constant lines do not change if \(n=\)constant as seen from equation (5), but the locations of and the distances between the lines of \(B=\)constant and \(\dot{E}=\)constant must change as seen from the dependences (2) and (4). The change in mass values and the relation between the mass and the radius lead to a change in the value of \(I\) and a change in the relation between \(\tau\) and the real age. It is well known that for the neutron stars which are in hydrodynamical equilibrium, both the coefficient in the expression \(I=\alpha MR^2\) and the value of \(R\) depend very weakly on the equation of state. But these quantities may considerably be different when the neu-
tron star is far from equilibrium, has a large rotational moment and magnetic field strength, and/or does not rotate as rigid body.

Electric field intensity which is generated under the rotation of a neutron star which has pure magnetic dipole radiation and around which no plasma exists may roughly be given as (Lipunov 1992)

\[ E_{el} \sim \frac{BR}{P}. \] (7)

Therefore, for equal values of P and B, the star with a larger radius (smaller mass) must have a larger value of \( E_{el} \). The exact expression for \( E_{el} \) may be found only in simple model approximations. But it is enough to know the simple relation (7) for our purposes.

Using equation (4) we get

\[ E_{el} \sim \frac{I^{\frac{3}{2}}}{R^{2}}\left(\frac{\dot{P}}{P}\right)^{\frac{1}{2}}. \] (8)

Therefore, the \( E_{el} \) value of a 1.5 M\(_\odot\) neutron star must be a factor of about 1.1-1.2 less than the \( E_{el} \) value of a 0.7-0.8 M\(_\odot\) neutron star. It must be noted that, in actuality, there can be some plasma around the neutron star. There may be jets also. These can distort the magnetic field so that the expression for \( E_{el} \) can be different.

From equations (5) and (8) we get

\[ E_{el} \sim I^{\frac{3}{2}}R^{-2}\tau^{-\frac{1}{2}} \] (9)

so that, using logarithmic scale, the average constant \( E_{el} \) lines will be parallel to constant \( \tau \) lines on the P-\( \dot{P} \) diagram for fixed values of I and R.

3 Which changes may take place on the P-\( \dot{P} \) diagram under the change of neutron star parameters?

The lines of constant B, \( \tau \) and \( \dot{E} \) pass through with given parameters as displayed in Figure 1 and with fixed intervals next to each other in the ideal case. The 'ideal case' means that the neutron stars are assumed to have \( \sim 1 \)
M⊙ (under the chosen equation of state), having pure dipole magnetic field with constant values, rotating as rigid body, and being in hydrodynamical equilibrium. Actually, we can not expect that all neutron stars have the same mass and radius, have similar configurations for the magnetic field, locate in similar environments nor they are in hydrodynamical equilibrium. Some of them can reach the hydrodynamical equilibrium state in short time and some in relatively long time. The conditions mentioned above can be different especially for young neutron stars and in this case we can expect them to have very different evolutionary tracks on the P-˙P diagram.

We can estimate only a value of ˙E close to the exact value from the position of the pulsar on the P-˙P diagram. Two pulsars which are located very close to each other on the P-˙P diagram may have different values of τ, B, Eel and age as seen from expressions (5), (4), (8) and (6).

In general, the neutron star model with R=10 km and I=10^{45} gr cm^2 is used and therefore the lines of B=constant and ˙E=constant on the P-˙P diagram are constructed by using equations

\[ B = \left( \frac{3c^3 I}{8\pi^2 R^6} \right)^{\frac{1}{2}} (P\dot{P})^{\frac{1}{2}} = 3.2 \times 10^{19} (P\dot{P})^{\frac{1}{2}} \text{ G} \]  

(10)

and

\[ \dot{E} = 4\pi^2 I \frac{\dot{P}}{P^3} = 3.94 \times 10^{46} \frac{\dot{P}}{P^3} \text{ erg/s} \]  

(11)

(Lyne & Graham-Smith 1998). For neutron stars with masses about 0.7 M⊙, it is possible to use more reliable values of R=12 km and I=7.2×10^{44} gr cm^2 instead of the very simple values of R and I, though in principle there will be no significant change. In this case, instead of the expressions (10) and (11), we have

\[ B = 1.58 \times 10^{19} (P\dot{P})^{\frac{1}{2}} \text{ G} \]  

(12)

and

\[ \dot{E} = 2.84 \times 10^{46} \frac{\dot{P}}{P^3} \text{ erg/s} \]  

(13)

similar to the expressions (10) and (11) which are always used for the very compact and stable neutron stars. The relations (10) and (11) are also applicable for dominant number of neutron stars which have masses 1-1.5 M⊙.

If we use the expressions (12) and (13) for the P-˙P diagram, the locations of τ=constant lines do not change, the lines of ˙E=constant practically do
not change either, but the lines of $B=constant$ shift down by a factor of 2. Therefore, a more accurate definition of the places of these lines on the $P-\dot{P}$ diagram does not have an essential role and can not help us in solving the important problems about the origins and evolutions of different types of neutron star.

As seen from Figure 1, bulk of the neutron stars are born with magnetic field similar to the effective field of Crab and Vela pulsars ($10^{12}$-$10^{13}$ G). We can assume that about 20% of neutron stars are born with masses about half of the 1.4 $M_\odot$ which is the maximum value in the number-mass distribution of neutron stars in binary systems (Lyne & Graham-Smith 1998; Tagieva et al. 2000; Charles & Coe 2003). These neutron stars may have $R=13$ km as found from the soft equations of state of rotating neutron stars (see Bejger & Haensel 2003). In this case, $\dot{P}$ value of such a pulsar must be about 5-6 times larger than the $\dot{P}$ of a normal radio pulsar (similar to Crab) if the dipole magnetic field and the $P$ values of these pulsars are the same (see equation (3)). The small mass neutron stars with such values of $\dot{P}$ must also have about 5-6 times smaller value of $\tau$ compared to the bulk group of pulsars with the same value of $P$. As seen from Figure 1, there will be no problem in explaining the position of DITNS RXJ0720.4-3125 on the $P-\dot{P}$ diagram by using such an approach (i.e. considering such objects as neutron stars with smaller masses). This approach can be useful in explaining most of the DITNSs. Most of these sources may have values of $P<10$ s and $\dot{P}<10^{-12}$ s/s.

4 Some problems about AXPs/SGRs and DITNSs

Both the magnetar model and the accretion from fall-back matter model have some considerable successes in explaining some properties of AXPs/SGRs as mentioned briefly in the Introduction. We suggest to bring the successful parts of both approaches together and in addition to this we propose a way to explain the properties of these objects based on the intrinsic parameters (different mass, different structure, absence of hydrodynamical equilibrium etc.). Guseinov et al. (2003a, 2003b) analyse the data of AXPs/SGRs and suggest the initial magnetic field to be $3\times10^{13}$-$10^{14}$ G by examining also the activity of young neutron stars. From the analysis of the X-ray properties and the birth rates of different types of neutron star (Guseinov et al. 2003c), a possibility arises to step back from the idea of very high magnetic field.
In Figure 1, all the 9 radio pulsars with distances up to 3.5 kpc from the Sun which are connected to SNRs and have values of $\tau \leq 5 \times 10^4$ yr are represented. Among these 9 pulsars, SNR shell and pulsar wind nebula (PWN) have not been observed for 4 and 3 of them, respectively (Green 2001; Guseinov et al. 2003d). The ages of SNRs in many cases are in agreement with the $\tau$ values of the pulsars connected to them in the error limits and their ages in some cases may practically be as large as $3 \times 10^4$ yr with the exception of the pair pulsar J0205+6449 – SNR G130.7+3.1. The $\tau$ value of this pulsar is $5.4 \times 10^3$ yr, but the SNR connected to this pulsar may be a historical remnant with an age of $\sim 800$ yr (Lorimer et al. 1998).

Since $P=0.066$ s for this pulsar and as the remnant is F-type (i.e. a reliable age estimation can not be made for the SNR), the existing discrepancy can be removed. Therefore, it is not necessary to discuss the differences between the values of $\tau$ and the real ages for this group of pulsars when discussing the average characteristics. However, more reliable larger values of age can be adopted based on the data of the SNRs which are connected to this group of radio pulsars: $t \sim 3 \times 10^4$ yr (see Guseinov et al. 2003e and the references therein). SNR G8.7-0.1 has such a large age of about $3 \times 10^4$ yr (Finley & Ogelman 1994) and this SNR is connected to pulsar J1803-2137 (Frail et al. 1994). In general, the positions of these 9 radio pulsars on the P-$\dot{P}$ diagram do not raise any questions. All of them are located in the belt $B=(2.5-10) \times 10^{12}$ G.

There are 7 SNRs with ages up to $2 \times 10^4$ yr which contain DRQNSs in the same region of 3.5 kpc from the Sun (Guseinov et al. 2003c, 2003d). The locations of 2 of them, 1E1207.4-5209 and RXJ0002+6246, on the P-$\dot{P}$ diagram are known with small errors (see Figure 1). For all the others, even the value of $P$ is not known that we can not find the positions of them on the P-$\dot{P}$ diagram. On the other hand, we know that PWN exists only around the pulsars which have $\dot{E}>3 \times 10^{35}$ erg/s and $L_{2-10keV}>10^{33}$ erg/s (Guseinov et al. 2003d). Since DRQNSs do not have such large values of $L_{2-10keV}$ and no PWN has been found around them, they must be located on the right hand side of $\dot{E}=3 \times 10^{35}$ erg/s line in Figure 1. Therefore, the values of $\tau$ for DRQNSs, on average, can be about one order (maybe more) of magnitude larger than their real ages. It follows from the numbers and ages of the SNRs which are connected to radio pulsars and DRQNSs that the birth rates of these objects are approximately the same. According to Guseinov et al. (2003c), the birth rate of DRQNSs near the Sun is about half of the birth
rate of all pulsars. These DRQNSs may directly be born at their present locations on the P-\(\dot{P}\) diagram or they may come to these positions in very short time after the birth. There is no data directly showing if pulsars can be born with initial periods greater than 0.1 s or not.

On the P-\(\dot{P}\) diagram (Figure 1) in the region \(B=(4-11)\times10^{11}\) G and \(\tau=(1-6)\times10^{5}\) yr, there are also 5 radio pulsars from which cooling X-ray radiation have been detected. The birth rate of these pulsars is 0.06 of the birth rate of all radio pulsars. This is about 0.04 of the combined birth rate of all the radio pulsars and the DRQNSs together (Guseinov et al. 2003c). Two of these sources, J1952+3252 and J0538+2817, are connected to S-type SNRs. We must add to this group also the radio pulsar J0659+1414 which is connected to a SNR and maybe the Geminga pulsar (see their positions on the P-\(\dot{P}\) diagram). In this case, the birth rate of pulsars with \(\tau>10^{5}\) yr and \(P<0.4\) s from which cooling X-ray radiation have been observed will be 0.084 of the birth rate of all pulsars. As seen, this value is very small. On the other hand, these pulsars avoid the belt \(B=10^{12}-10^{13}\) G. These data also show that it is necessary to give up the commonly used simple approach in order to be able to explain the pulsar evolution on the P-\(\dot{P}\) diagram. Naturally, future observations must lead to an increase in the number density of cooling pulsars. Note that, the ages are assumed to be equal to the values of \(\tau\) for these 5 pulsars in the cooling theories (Kaminker et al. 2002; Yakovlev et al. 2002).

Other than these sources, there exist many single neutron stars which radiate in soft X-ray band with spectra closer to the blackbody compared to cooling pulsars. Yet, luminosities of these sources are smaller (total \(L_x\simeq10^{29}-10^{32}\) erg/s), but their number density is very large. In spite of the large distances adopted by Guseinov et al. (2003c), their birth rate is not smaller than the birth rate of all pulsars and about half of the supernova explosion rate.

For 2 of these sources, RXJ0720.4-3125 and J1308.8+2127, the \(P\) values have been measured and unconfirmed \(P\) values are known for 2 other such sources. Note that values of \(\dot{P}\) were given for the first 2 sources (Zane et al. 2002; Hambaryan et al. 2002) but these values are uncertain. Since the \(P\) values of these 2 sources are reliable and their ages are not more than \(10^6\) yr, it is easy to understand their locations on the P-\(\dot{P}\) diagram for \(\dot{P}>10^{-14}\) s/s. But one faces with big difficulties against the high birth rate of such objects. We think that dominant number of DITNSs must have \(P<4-5\) s.
Understanding the existence of a relation between some of the DITNSs and AXPs/SGRs would be easy if the birth rate of DITNSs with large periods were in accordance with the birth rate of AXPs/SGRs, in error limits, which is about 60 times smaller than the supernova rate (Guseinov et al. 2003c). Since the birth rate of DITNSs is about 30 times larger than the birth rate of AXPs/SGRs, only a very small part of DITNSs, which have \( P > 6-8 \) s, may be related to AXPs/SGRs.

The known values of temperature and luminosity of DITNSs naturally limit the ages of these objects. As the cooling theories which we have used in the birth rate estimations do not have large uncertainties for neutron stars with masses very close to 1.4 \( M_\odot \), the ages of most of the DITNSs may be close to \( 10^6 \) yr if their masses are close to 1.4 \( M_\odot \). Taking this fact into account, we can begin to construct possible evolutionary tracks of DITNSs and AXPs/SGRs.

The data of the DITNSs may be not so reliable, but even in such a case we can not neglect the existence of some neutron stars, for example, with \( P > 5 \) s and \( \dot{P} > 4 \times 10^{-14} \) s/s for which the birth rate may be about 0.1 of the supernova rate. In this case, instead of accepting a bimodal number-magnetic field distribution for pulsars, we can say that neutron stars with \( \tau < 10^4 \) yr and \( B \sim 10^{13} \) G may transfer to the region of \( P > 5 \) s and \( \dot{P} > 4 \times 10^{-14} \) s/s mentioned above.

5 Discussion and Conclusions

In sections 2 and 3, we have discussed how the lines with constant values of \( \tau \), \( B \) and \( \dot{E} \) change on the P-\( \dot{P} \) diagram under the change of radius, moment of inertia and the component of the magnetic field perpendicular to the rotation axis of pulsar. If pulsars have considerably different, but constant, values of \( R \), \( I \) and real magnetic field component, then their evolution must be similar to what is known. But if these parameters change in time, then the values of the braking index at each point on the P-\( \dot{P} \) diagram can change and in such a case the evolutionary tracks must be very different and complicated. This gives us the possibility to use different parameters of neutron star (mass, magnetic field, structure, rotation, propeller mechanism, absence of the hydrodynamical equilibrium etc.) to understand the evolution on the P-\( \dot{P} \) diagram and the properties of different types of pulsar.
Today, we face with very different types of neutron star which are located on different parts of the P-˙P diagram (Figure 1). They may have different origins, structures and evolutionary tracks. There are not large differences between the locations of radio pulsars and DRQNSs on the P-˙P diagram and they do not create any significant problem in understanding the neutron star characteristics. But in the last about ten years AXPs/SGRs and comparably in small degree DITNSs created very difficult problems in astrophysics.

The locations of AXPs/SGRs on the P-˙P diagram rise a large astonishment, but it is very good that their number density in the Galaxy is very small and connected to this their birth rate is also very small. This makes it difficult to have necessary amount of observational data but also makes it easy to search ways to understand them. On the other hand, the γ-ray burst characteristic makes the problem more difficult. In the case of DITNSs, the large difficulties are related to the high birth rate of these objects. In this work, we do not discuss the difficulties of the magnetar and the accretion from fall back matter theories, which are known well. In the magnetar model, the existence of the bimodality in the number of pulsars versus the magnetic field distribution and the absence of a correlation between the increase in ˙P and the γ-ray burst activity are significant problems. In the accretion from fall back matter theory, the idea of accretion onto young neutron stars which may be very active does not seem to be realistic and it can not solve any principal problem about AXPs/SGRS without taking into account different possible activity of neutron stars as mentioned above. Therefore, by rejecting the idea of large magnetic field suggested in the magnetar model which is $10^{14}$-$10^{15}$ G, we exclude the bimodality and we accept that different types of neutron star have different values of parameter (e.g. different value of mass) as represented in the previous sections.

As known for single radio pulsars, the ratio of the X-ray radiation to the rate of rotational energy loss lies in the interval $3 \times 10^{-2}$-$10^{-5}$ (Becker & Trumper 1997; Possenti et al. 2002; Becker & Aschenbach 2002). The name 'anomalous' comes from the inverse value of $L_x/\dot{E}$ and DITNS RX J0720.4-3125 is similar to AXPs from this point of view (Kaplan et al. 2003; Zane et al. 2002). If we consider only $L_x/\dot{E}$ values, most of the DRQNSs and DITNSs must belong to inter-class objects between radio pulsars and AXPs/SGRs. Today, one can begin to investigate the differences and common properties of single neutron stars in detail. The data show the existence of differences not only in the values of the real magnetic field of neutron stars but also in
the masses and the internal structures. Neutron stars not only have different evolutionary tracks, but they may be born with very different values of $P$ and $\dot{P}$.

There exist models of stable neutron star with masses from $0.1 \, M_\odot$ up to $2 \, M_\odot$ (Bejger & Haensel 2003), but most of the neutron stars have masses of about $1.4 \, M_\odot$ (Charles & Coe 2003). Actually, this mass interval must be narrower; if the mass is $2 \, M_\odot$ the rest energy is $\sim 4 \times 10^{54}$ ergs and the binding energy including both the energy carried by neutrinos and the explosion energy is $\sim 10\%$ of the rest energy (Zeldovich & Novikov 1971). Naturally, the energy carried by the neutrino pairs is lower for the collapsed matter with smaller mass, but the total energy loss must always be large enough (Guseinov 1966) that such energy can not be produced to form a $0.1 \, M_\odot$ neutron star for which the binding energy is comparably very small. We must also take into account the oscillations which must take place under the formation of neutron stars and the masses (the pressure) which continue to fall onto their surfaces during the collapse. Under the collapse of progenitor stars, black holes may also be born that the newborn neutron stars must have the supply for stability against the overcompression. Neutron stars which have masses very close to the maximum mass can not resist against the matter falling on themselves. Therefore, the maximum masses of the existing neutron stars must be a little smaller than the maximum mass values found from the models and the minimum mass must be about $0.5 \, M_\odot$ which is the value we have adopted for AXP 1E1841-045 and AXP 1E2259+586 which are connected to SNRs. Fastly rotating neutron stars with such small masses may have about 1.3 times larger radius compared to the neutron stars with masses close to $1.4 \, M_\odot$.

The pulsars with $0.5 \, M_\odot$, which have the same real magnetic field as the massive pulsars with the same spin period, must have about 8 times larger value of $\dot{P}$ and smaller value of $\tau$ according to the expressions (3) and (5). They must also have a bit larger value of $E_{el}$ (see expression (7)). As seen from Figure 1, even a magnetic field of $\sim 3 \times 10^{13}$ G is small for such an explanation of the positions of all AXPs/SGRs on the $P$-$\dot{P}$ diagram for which the ages are not more than $\sim 3 \times 10^4$ yr. But the small mass model gives us several additional possibilities. If the magnetic field axis makes an angle close to $90^\circ$ with the rotational axis, then the atmosphere of the neutron star must be stretched in the direction of the magnetic field axis. The extended atmosphere and the high voltage make the gas easily flow along the magnetic
field lines and the propeller mechanism effectively works. From such an approach, it is easy for small mass neutron stars with magnetic fields $\geq 10^{13}$ G to come to the positions of AXPs/SGRs on the $P$-$\dot{P}$ diagram in short time. The extended low density atmosphere, which is not in stable state, may give many possibilities for different activity and also for recombination of the magnetic field. It is easier for the particles to be ejected from the extended hot atmosphere. In addition to the magnetic field energy, there exist rotational and gravitational energies which support the activity of AXPs/SGRs. The small mass pulsars must have considerably long cooling time and short characteristic time for the rotational energy loss. Therefore, the ratio $\frac{L}{E} > 1$ can easily be explained by this way.

In the given approach, it is not difficult to understand DITNSs, because they have very simple properties compared to AXPs/SGRs. The increase of the rotational period must accelerate the processes of contraction and coming to stability. These processes can lead to a considerable increase in the braking index. Therefore, many of the DITNSs must have $n > 3$. In most part of the lifetimes of these sources and AXPs/SGRs, they must evolve in the direction of decreasing $\dot{P}$. From this approach, it follows that DITNSs must be found in some nearby SNRs. Existence of ionized gaseous wind must promote to suppression of the radio pulsar phenomenon when their periods are still small. Particularly, large noise in the period and possibly the glitch phenomenon must also exist for the neutron stars with $P > 6$-8 s and $\dot{P} > 10^{-13}$ s/s.
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Figure Caption

Figure 1: Period versus period derivative diagram for different types of pulsar. The '+' signs denote the radio pulsars with d≤3.5 kpc which are connected to SNRs. The 'X' signs show the positions of the radio pulsars with d≤3.5 kpc and 10^5<τ<2×10^7 yr which have been detected in X-rays. The locations of 3 radio pulsars which have d≤3.5 kpc and τ<10^5 yr are shown with 'circles' to make a comparison between the birth rates (see text). DITNSs are represented with 'stars' and DRQNSs are displayed with 'empty squares'. The 'filled squares' show the positions of all AXPs/SGR in the Galaxy. Names of DITNSs, DRQNSs, 2 of the AXPs, and some of the radio pulsars are written. Constant lines of B = 10^{11-15} G, τ = 10^{3-9} yr, and ḳE = 10^{29}, 10^{32}, 10^{35}, 3×10^{35} and 10^{38} erg/s are shown. P=10 s line is also included (see text). This figure is taken from Guseinov et al. (2003c).
Period Derivative (s/s) vs. Period (s)

- E3
- E32
- E35
- E12
- E13
- E14
- 0002
- 0538
- 0659
- 0720
- 1952
- 2259
- 2337
- 3E35
- Geminga
- 1207.4
- 0142
- 0720
- 1308.8

Values include:
- 1e-16
- 1e-15
- 1e-14
- 1e-13
- 1e-12
- 1e-11
- 1e-10
- 0.1
- 1
- 10