Identifying topological edge states in 2D optical lattices using light scattering

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Abstract. We recently proposed in a Letter [Phys. Rev. Lett. \textbf{108}, 255303] a novel scheme to detect topological edge states in an optical lattice, based on a generalization of Bragg spectroscopy. The scope of the present article is to provide a more detailed and pedagogical description of the system – the Hofstadter optical lattice – and probing method. We first show the existence of topological edge states, in an ultra-cold gas trapped in a 2D optical lattice and subjected to a synthetic magnetic field. The remarkable robustness of the edge states is verified for a variety of external confining potentials. Then, we describe a specific laser probe, made from two lasers in Laguerre-Gaussian modes, which captures unambiguous signatures of these edge states. In particular, the resulting Bragg spectra provide the dispersion relation of the edge states, establishing their chiral nature. In order to make the Bragg signal experimentally detectable, we introduce a “shelving method”, which simultaneously transfers angular momentum and changes the internal atomic state. This scheme allows to directly visualize the selected edge states on a dark background, offering an instructive view on topological insulating phases, not accessible in solid-state experiments.

1 Introduction

Ultra-cold atoms in highly controllable coupling fields constitute a novel experimental tool for studying the rich many-body physics arising in two dimensions \cite{1–4}. Motivated by the possibility of reaching interesting quantum phases, synthetic magnetic fields \cite{5} and spin-orbit couplings \cite{6–8} have been realized experimentally for neutral atoms. Today, the engineering of these synthetic gauge potentials opens an important path for the exploration of topological phases, such as quantum Hall (QH) states, topological insulators and superconductors, in the clean and versatile environment offered by cold-atom setups \cite{2, 3}.

For the last decades, these topological phases have gained the interest of the scientific community for their unique properties, such as quantized conductivities,
dissipationless transport and edge-states physics [9,10]. These impressively robust phenomena rely on an important concept, the so-called bulk-edge correspondence [11,12]. Topological phases of matter are characterized by robust, integer-valued topological invariants related to the bulk structure of the material. The bulk-edge correspondence stipulates that well-defined edge excitations localized near the boundaries of the system are associated to these topological invariants. Such edge excitations are of tremendous practical importance, as they usually carry some form of current protected against perturbations as long as the topological structure is preserved. As such, they are at the origin of the dissipationless transport observed for these phases. For example, in the QH effect taking place in 2D electronic systems, the topologically invariant Chern number [13,14] guarantees the presence of current-carrying edge states, and imposes their chirality [11].

Cold-atom realizations of topological phases therefore constitute a complementary, but also intrinsically appealing, playground to further deepen our understanding of these topological properties. However, the detectability of topological phases remains a fundamental issue in the cold-atom framework [15–35], where transport measurements constitute a possible, but very challenging task [36]. In this sense, alternative signatures of topological phases, together with novel experimental probes, have to be considered in this new context. Following this strategy, several schemes have been described to directly measure topological invariants, based on spin-resolved time-of-flight [27,33] and density measurements [19,28]. Alternatively, Bloch oscillations could also be performed to evaluate the Berry’s curvature in 2D atomic systems [30], which could then provide an estimation of the Chern number when integrated over the Brillouin zone.

Inspired by the bulk-edge correspondence, it has also been suggested that topological edge states could be directly probed [21,22,29,37,38]. For example, in the context of cold-atom QH insulators, a satisfactory signature of the non-trivial topological order would be obtained by probing the dispersion relation of QH edge states, thus demonstrating their chiral nature.

It is the aim of the present work to describe in detail such a realistic probe. We choose to analyze this detection scheme for an optical-lattice setup reproducing the Hofstadter model [39], which is one of the simplest tight-binding lattice models exhibiting non-trivial Chern numbers [13,14] and topological edge states [11]. The experimental realization of this model, using cold atoms in optical lattices, is currently in development in several laboratories [40–42], based on the proposals [43,44]. We believe that our detection scheme could easily be extended to any ultracold-atom setup emulating 2D topological phases.

2 The Hofstadter optical lattice and topological edge states

We start with a two-dimensional fermionic gas confined in a square optical lattice and subjected to a uniform synthetic magnetic field $B = B_1 z$ [43,44]. The Hamiltonian is taken to be

$$\hat{H}_0 = -J \sum_{m,n} \hat{c}_{m+1,n}^\dagger \hat{c}_{m,n} + e^{i2\pi \Phi m} \hat{c}_{m,n+1}^\dagger \hat{c}_{m,n} + \text{h.c.} + \sum_{m,n} V_{\text{conf}}(r) \hat{c}_{m,n}^\dagger \hat{c}_{m,n},$$

(1)

where $\hat{c}_{m,n}$ is the annihilation operator defined at lattice site $(m, n) \in \mathbb{Z}^2$ and where $J$ is the tunneling amplitude. The site indices $m, n = 1, \ldots N$ are related to the spatial coordinates through

$$(x, y) = a(m - N/2, n - N/2),$$