Bandgap engineering and defect modes in photonic crystals with rotated hexagonal holes

Aaron Matthews\textsuperscript{1}, Sergei F. Mingaleev\textsuperscript{1,2}, and Yuri S. Kivshar\textsuperscript{1}

\textsuperscript{1}Nonlinear Physics Group and Center for Ultra-high bandwidth Devices for Optical Systems (CUDOS), Research School of Physical Sciences and Engineering, Australian National University, Canberra ACT 0200, Australia
\textsuperscript{2}Institut f"{u}r Theorie der Kondensierten Materie, Universit"{a}t Karlsruhe, 76128 Karlsruhe, Germany

We study the bandgap structure of two-dimensional photonic crystals created by a triangular lattice of rotated hexagonal holes, and explore the effects of the reduced symmetry in the unit-cell geometry on the value of the absolute bandgap and the frequencies of localized defect modes. We reveal that a maximum absolute bandgap for this structure is achieved for an intermediate rotation angle of the holes. This angle depends on the radius of the holes and the refractive index of the background material. We also study the properties of the defect modes created by missing holes, and discuss the mode tunability in such structures.

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During recent years we observe a rapidly growing interest in the design and fabrication of novel types of photonic-crystal structures possessing large absolute band gaps. The study of various geometries of three-dimensional photonic crystals and different ways to enlarge their absolute bandgaps is a key issue of the physics of periodic dielectric structures where different polarizations of light are coupled together and, therefore, the existence of an absolute bandgap is crucially important for various applications of three-dimensional photonic crystals in optics (see, e.g., Ref. \cite{Qui03} and references therein).

However, for two-dimensional (2D) photonic crystals Maxwell’s equations are known to decouple effectively for two polarizations, so that the study of a particular photonic bandgap of such periodic structures can be carried out independently for both \(E\)-polarized and \(H\)-polarized electromagnetic waves. Accordingly, two types of photonic bandgaps are usually distinguished to exist for different polarizations. When the bandgaps for two different polarizations overlap, they create the combined bandgap known as an absolute bandgap.

One of the main reasons to enlarge the absolute bandgaps and to study the bandgap properties of 2D periodic photonic structures is an attractive possibility of creating novel types of tunable waveguides and circuits for applications of photonic crystals in integrated optics. In particular, the existence of a large absolute bandgap would allow to design the waveguides in planar structures which can support propagating modes of both polarizations in the same frequency domain.

Up to now, a number of different approaches has been suggested for the enlargement of the absolute bandgaps of 2D photonic-crystal structures. One of these approaches relies on the idea of using the photonic crystals created by a two-dimensional lattice of non-circular holes and exploring the symmetry-reducing properties of 2D photonic crystals for the bandgap enlargement \cite{Qui01,Hup07}. In particular, Wang \textit{et al.} \cite{Wan01} demonstrated that the absolute bandgap becomes maximal in the case of air holes of the same symmetry as the lattice symmetry. For example, the bandgap takes a maximum value for a triangular lattice of rotated hexagonal holes. However, Qui \textit{et al.} \cite{Qui01} suggested that the bandgap as large as that demonstrated in Ref. \cite{Wan01} can be achieved in 2D photonic crystals created by air holes of more complex, non-circular shape without any rotation.

In spite of many studies of the absolute bandgaps of photonic crystals created by a lattice of non-circular holes, no specific applications of such large bandgaps have been discussed and demonstrated. In particular, the usual statement that large absolute bandgaps should be useful for functioning of photonic circuits is not well-grounded. As a matter of fact, there is no solid reason to exploit the absolute bandgaps in two-dimensional structures unless photonic-crystal circuits guide light of both polarizations, but the study of defect modes and waveguides in such structures is still incomplete or missing.

The purpose of this paper is twofold. First, we explore further the concept of the enlargement of the absolute bandgaps of 2D photonic crystals through reducing symmetry in the unit-cell geometry taking as an example a 2D dielectric periodic structure created by a triangular lattice of rotated hexagonal holes. In particular, we reveal that in such 2D structures large absolute bandgaps can be achieved for an intermediate value of the rotation angle of the hexagonal holes. Second, we demonstrate that, by using 2D photonic crystals with a large absolute bandgap, it is possible to create defects which can support localized modes for both polarizations of light, so that the waveguides based on such defects can guide light of both polarizations as well.

We consider a 2D photonic crystal created by a triangular lattice of hexagonal holes assuming an arbitrary rotation of the hole relative to the lattice symmetry axis, as shown in Fig. \ref{fig:rotated_holes}. The photonic band gap is calculated by solving Maxwell’s equations, by means of the plane-wave expansion method \cite{Qui01}. Two examples of the photonic bandgap structure of such a photonic crystal...
hexagonal holes in the material with intermediate (critical) angle of rotation; in the case of the follows. First, the maximal bandgap is achieved for some of the holes. In brief, our findings can be summarized as case, the bandgap depends strongly on the rotation angle lar holes to reduce the rotational symmetry. In this latter concept of local symmetry reduction in the unit cell of hexagon orientation.

is shown in Figs.\[1\]b,c], for the particular cases of the hole rotation, i.e. for \( \theta = 0 \) and \( \theta = 30^\circ \), respectively. Our main goal here is to study the effect of the hole rotation on the value of the (lowest-order) absolute band gap. The results can naturally be compared with the bandgap spectra of the 2D structures created by circular holes, as well as with the case when the hexagonal holes are not rotated, see Fig.\[1\]b).

In Fig.\[2\] we show the dependence of the normalized width of the absolute bandgap for two types of triangular lattices created by circular and hexagonal holes, as the functions of the normalized radius, \( r/a \). We make this type of comparison for different values of the rotation angle of the hexagonal lattices, such that the rotation for the hexagonal holes is chosen to maximize the absolute bandgap at a given radius. In all the cases we confirm that the hexagonal lattices possess a larger absolute bandgap, and this bandgap depends strongly on the hexagon orientation.

As a matter of fact, we observe that the absolute bandgap can be enlarged dramatically by employing the concept of local symmetry reduction in the unit cell of the structure, and using hexagonal holes instead of circular holes to reduce the rotational symmetry. In this latter case, the bandgap depends strongly on the rotation angle of the holes. In brief, our findings can be summarized as follows. First, the maximal bandgap is achieved for some intermediate (critical) angle of rotation; in the case of the hexagonal holes in the material with \( \epsilon = 12 \), this critical angle is close to the value \( 24^\circ \). Second, the critical angle \( \theta_{cr} \) depends on the value of the dielectric constant \( \epsilon \) of the photonic-crystal material, but it varies slowly near this intermediate value. Figures\[3\]a,b) show the maximum value of the absolute bandgap \( \Delta \omega/\omega \) and the variation of the critical angle \( \theta_{cr} \) of the rotated hexagonal holes on the value of the dielectric permittivity \( \epsilon \) of the photonic crystal material. It is therefore clear that this critical rotation angle is not a fundamental constant of the structure, but it is defined by some effective geometry and material properties corresponding to the maximum effect of the local symmetry reducing in the unit-cell geometry, similar to the effects discussed in Ref.\[10\] for a complete different problem. Moreover, the absolute bandgap varies dramatically for relatively small values of \( \epsilon \), whereas it saturates for large \( \epsilon \), and the critical angle approaches the value \( 21^\circ \).

As we mentioned above, in the case of 2D periodic dielectric structures, both \( E \)- and \( H \)-polarization components of the electromagnetic field are decoupled. In this case, the existence of large absolute bandgaps can be useful to support localized modes for both polarizations.
The frequencies vary with a change of the rotation angle shown in Fig. 4(b). From the results presented in Fig. 4(b) it follows that the frequencies of the defect modes inside the absolute bandgap are almost constant for all angles.

Our results show that the similar features are observed for the waveguides created in the lower-symmetry photonic crystals possessing a large absolute photonic bandgap: the possibility to enlarge the bandgap in a 2D photonic structures created by rotated hexagonal holes. Moreover, the frequencies of the defect modes which can be supported by missing holes in such structures do not vary much with the hole rotation and changing the value of the absolute bandgap.

In conclusion, we have presented the results of engineering of the absolute bandgaps in 2D photonic crystals for photonic-circuit applications, for the example of a photonic crystal created by a triangular lattice of hexagonal holes. We have revealed that the maximum value of the absolute bandgap in this structure can be achieved when all hexagonal holes are rotated by a finite angle, and this rotation angle takes an intermediate value between the values corresponding to the simplest symmetries of the lattice. The critical rotation angle depends on the parameters of photonic crystals, but it is shown to be almost constant for large values of the dielectric permittivity. Moreover, the frequencies of the defect modes which can be supported by missing holes in such structures do not vary much with the hole rotation and changing the value of the absolute bandgap.

We believe that our results will be important for the design of the photonic-crystal waveguides and circuits supporting propagating guided modes of both polarizations in the same frequency domain, as well as for the study of nonlinear waveguides where the intensity-induced coupling between the modes of different polarization can be controlled through the rotation of holes or non-circular defects. In addition, we believe that our results can be useful for a design of novel types of coupled-resonator optical waveguides with the properties which can be tuned by using rotated hexagonal holes instead of circular holes.

In addition, we would like to mention that large absolute bandgaps can be very useful for exploring nonlinear properties of photonic crystals, when the intensity-dependent refractive index of the waveguides can be employed to couple the field polarizations. In this case, we can create nonlinear photonic-crystal circuits where one polarization is used for the signal transmission, whereas the other polarization is employed for controlling the signal propagation, in order to realize switching between different transmission regimes, etc.

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