CMB low multipole alignments in the \(\Lambda\)CDM and dipolar models

L. Polastri,\(^a\) A. Gruppuso\(^{b,c}\) and P. Natoli\(^{a,b,d}\)

\(^a\)Dipartimento di Fisica e Scienze della Terra, Università degli Studi di Ferrara, via Giuseppe Saragat 1, I-44122 Ferrara, Italy
\(^b\)INAF-IASF Bologna, via Piero Gobetti 101, I-40129, Bologna, Italy
\(^c\)INFN, Sezione di Bologna, via Irnerio 46, I-40126 Bologna, Italy
\(^d\)INFN, Sezione di Ferrara, via Giuseppe Saragat 1, I-44122 Ferrara, Italy

E-mail: linda.polastri@student.unife.it, gruppuso@iasfbo.inaf.it, natoli@fe.infn.it

Received March 6, 2015
Accepted March 10, 2015
Published April 15, 2015

Abstract. The dipolar model [1] has attracted much interest because it may phenomenologically explain the CMB hemispherical power asymmetry found in the WMAP and Planck data. Since such a model explicitly breaks isotropy at large angular scales it is natural to wonder whether it can also explain other CMB directional anomalies. Focusing on the low \(\ell\) alignments and assuming \(\Lambda\)CDM, we confirm that the quadrupole/octupole and the dipole/quadrupole/octupole alignments are anomalous with a significance up to 99.9% C.L., for both WMAP and Planck data. Moreover, we show for the first time that such features are anomalous also in the dipolar model, roughly at the same level as in \(\Lambda\)CDM. We conclude that the dipolar model does not provide a better fit to the data than the \(\Lambda\)CDM.

Keywords: cosmological simulations, CMBR experiments, CMBR theory

ArXiv ePrint: 1503.01611
Cosmic microwave background, henceforth CMB, anisotropy observations (as well as other astrophysical and cosmological observations) can be described with just six parameters in the $\Lambda$CDM model. To date, no extension of this model has improved in a significant way the fit to the available data [2, 3]. It is impressive that all the huge amount of data arising from cosmological observations seem to suggest that such simple model is sufficient to describe the large scale universe we live in. However, observed features exist that are not very well explained by the $\Lambda$CDM model. This is the case of the largest CMB angular scales where so-called anomalies occur. These can be grossly divided in two classes: isotropic and anisotropic anomalies. Examples of the former are the lack of power at large angular scale [4–7], the lack of correlation in the two-point correlation function [8–12] and the so-called point-parity anomaly [7, 13–17]. In the latter we list the hemispherical power asymmetry [7, 18–25], the mirror-parity anomaly [7, 26–29], the cold spot [30–33] and the low $\ell$ alignments [34–42]. The significance of these anomalies is in general of the order of 2-3 $\sigma$, rarely more.

A key point is whether these anomalies can be ascribed to residual systematic contamination (of astrophysical or instrumental origin), or may hint to new physics. Since we now know that the CMB anomalies are consistently observed in both WMAP and Planck data, little room is left for the possibility that they are artificially created by residual systematic effects. The high quality level of foreground component separation performed by Planck [43] appears to rule out the case for residual foreground contamination unless there are unaccounted ingredient to the foreground model, see e.g. [44] for a possible candidate. The simplest explanation is that of statistical flukes; such line of reasoning is supported when properly accounting for multiplicity of tests also known as the “look-elsewhere effect” [45]. However, the number of these features, the fact that not all of them are related one another in an obvious manner and their almost exclusive occurrence at large angular scales motivate
the quest for a (possibly unifying) explanation even if the individual statistical significance
is not very high.¹

In the current paper we focus on the low $\ell$ alignments, namely the unlikely alignments
between the quadrupole and the octupole, as well as the dipole with both of the former. In the
light of several foreground cleaned CMB maps released by both WMAP and Planck, we aim at
assessing the statistical significance of these features. In so doing, we test not only the ΛCDM
model, but also the so called dipolar model. The latter is a phenomenological model which
has been invoked to explain the already mentioned power hemispherical asymmetry [7, 9, 20–
22, 46]. The dipolar model [1] consists of a particular mechanism for breaking the isotropy
on the large-angle CMB fluctuations. The model is described by:

$$\left(\frac{\Delta T}{T}\right)_{\text{mod}}(\hat{n}) = (1 + A\hat{n} \cdot \hat{p}) \left(\frac{\Delta T}{T}\right)_{\text{iso}}(\hat{n}),$$

where $\hat{n}$ is the observed direction, $(\Delta T/T)_{\text{mod}}$ is the observed (and modulated) CMB tem-
perature fluctuations, $(\Delta T/T)_{\text{iso}}$ is the usual isotropic CMB pattern, $A$ is the amplitude
of the dipole modulation and $\hat{p}$ is a given direction. In [22] it is found that $A = 0.07 \pm 0.022,$
statistically significant at $\sim 3\sigma$ and the direction $\hat{p}$ is given by $(l, b) = (224^\circ, -22^\circ) \pm 22^\circ$ in
Galactic coordinates, significant at $\sim 3.3\sigma,$ see also [20] for previous results.

The paper is organized as follows. Section 2 is the bulk of this paper. In particular in
section 2.1 we discuss the state of the art of the CMB anomalous alignments and describe the
used data set. In section 2.2 we introduce the methodology employed, based on the multipole
vectors formalism. We set forth the estimators adopted in section 2.3 and present our data
analysis pipeline, employed both for real data and realistic simulations in section 2.4. We
present our results in section 3 while section 4 is reserved for conclusions.

2 CMB low $\ell$ alignments

2.1 State of the art and employed data set

The occurrence of the anomalous alignments in the large angle CMB pattern has been noted
since the very first appearance of the WMAP data [47]. Using a different methodology, it
was confirmed [34] that the quadrupole and the octupole are unlikely aligned in the WMAP
ILC 1 year data (see also [48] for a similar and independent analysis). It was later shown [8]
that the quadrupole/octupole unlikely alignment is still present in the WMAP ILC 3 year
map at 99.6% C.L.. Moreover in the same paper a correlation between quadrupole, octupole
and dipole was found with a significance of 99.7%. The quadrupole/octupole alignment has
also been studied in the Planck data [7], where similar conclusions were drawn although with
slightly lower significance. The WMAP ILC 7 and 9 year maps are analyzed in [42] where it is
reported that the quadruple/octupole alignment occurs with probability 0.327% and 0.511%,
respectively. In the same paper, it has been pointed out that Planck and WMAP data are
much in better agreement after the application of the Doppler boosting correction [49], that
is, the distortion of the CMB anisotropy pattern induced by the proper motion of the observer
with respect to CMB rest frame.

In this paper we analyse CMB maps from both WMAP and Planck. For WMAP we
consider three releases of ILC (Internal Linear Combination of the multi-frequency) maps

¹Note that such significance is largely dominated by cosmic variance in the underlying ΛCDM model
assumed.
Figure 1. The $\ell = 2$ (upper panel) and $\ell = 3$ (lower panel) contributions to the Planck SMICA map.

of the CMB sky [50], namely we use WMAP ILC 5 year [51], WMAP ILC 7 year [52] and WMAP ILC 9 [53]. See also [54] for further details about the ILC method. While for the Planck satellite we use two maps of the 2013 cosmological release of data [43]: SMICA, Spectral Matching Independent Component Analysis, [55], that implements a parametric approach for foreground reduction in the harmonic domain, and NILC, which employs a spherical needlet version of the ILC algorithm [57].

The alignments are visually illustrated in figure 1 where $\ell = 2$ and $\ell = 3$ of the Planck SMICA map are shown as a representative case.

\footnote{In fact we consider an inpainted SMICA map which has been produced by replacing the masked pixels with a constrained Gaussian realization obtained by the method described in [56].}
2.2 Multipoles vectors

It is customary to expand CMB anisotropy maps into spherical harmonics. However in the context of multipole alignments, it is very convenient to use an alternative and completely equivalent representation, namely multipole (or Maxwell) vectors expansion \cite{34, 42, 48}. The fundamental idea is that the information contained in each set of (complex) $a_{\ell m}$ coefficients for any integer $m = -\ell, \ldots, \ell$, can be recast in \ell unit (real) vectors $\hat{v}_i$ and one (real) amplitude $A^{(i)}$:

$$a_{\ell m} \rightarrow A^{(i)} \hat{v}_i \ldots \hat{v}_\ell. \quad (2.1)$$

In fact, we note that strictly speaking the term vector is improper here because we should rather speak of axes or directions. This happens because the association given in eq. (2.1) is defined up to a “global” sign.

The main advantage of this formalism is that it is much easier to build quantities invariant under rotation from multipole vectors rather than from $a_{\ell m}$. The latter is rather an important point because we will make use in the following of estimators based on rotation invariant quantities. Unfortunately no closed analytical expression for eq. (2.1) is available. Therefore, numerical routines must be used to build the vectors. Further details and properties can be found in \cite{34, 42, 48}.

2.3 Estimators

We build eight estimators, all defined in the interval $[0, 1]$ \cite{34, 36, 38, 42}. Of these, six are for the quadrupole/octupole alignment:

$$S = \frac{1}{3} \sum_{j=1}^{3} |\hat{q} \cdot o_j|, \quad (2.2)$$

$$T = 1 - \frac{1}{3} \sum_{j=1}^{3} (1 - |\hat{q} \cdot o_j|)^2, \quad (2.3)$$

$$S_{23} = \frac{1}{3} \sum_{j=1}^{3} |q \cdot o_j|, \quad (2.4)$$

$$T_{23} = 1 - \frac{1}{3} \sum_{j=1}^{3} (1 - |q \cdot o_j|)^2, \quad (2.5)$$

$$\hat{S}_{23} = \frac{1}{3} \sum_{j=1}^{3} |\hat{q} \cdot \hat{o}_j|, \quad (2.6)$$

$$\hat{T}_{23} = 1 - \frac{1}{3} \sum_{j=1}^{3} (1 - |\hat{q} \cdot \hat{o}_j|)^2, \quad (2.7)$$

and two for the dipole/quadrupole/octupole alignment:

$$DQOS = \frac{1}{4} \left( |q \cdot d| + |o_1 \cdot d| + |o_2 \cdot d| + |o_3 \cdot d| \right), \quad (2.8)$$

$$DQOT = 1 - \frac{1}{4} \left[(1 - |q \cdot d|)^2 + (1 - |o_1 \cdot d|)^2 + (1 - |o_2 \cdot d|)^2 + (1 - |o_3 \cdot d|)^2 \right]. \quad (2.9)$$
In the above equations, the symbol \( \hat{\text{\textbullet}} \) denotes the unit vector, and the area vectors \( q \) and \( o_j \) are defined via the following vector products:

\[
q = q_{21} \times q_{22},
\]

\( (2.10) \)

\[
o_1 = o_{32} \times o_{33},
\]

\( (2.11) \)

\[
o_2 = o_{33} \times o_{31},
\]

\( (2.12) \)

\[
o_3 = o_{31} \times o_{32},
\]

\( (2.13) \)

where \( q_{2j} \) (with \( j = 1, 2 \)) represent the two multipole vectors associated to the quadrupole and \( o_{3i} \) (with \( i = 1, 2, 3 \)) represent the three multipole vectors associated to the octupole. The vector \( d \) represents the dipole direction which reads \((l, b) = (263.99, 48.26)\) in Galactic coordinates. Note the presence of the absolute values in the definition of the estimators in eqs. (2.2)–(2.9) which is due to the fact that multipole vectors define directions, i.e. they are headless vectors, see section 2.2.

The estimators introduced in eqs. (2.2)–(2.9) can be divided in “S” and “T” statistics as denoted by the labels. They measure “distance” from a situation of complete misalignment, i.e. orthogonality, which is associated to zero in both cases, whereas complete alignment, i.e. parallelism, is represented by the value 1. However, the “S” estimators weight the cosine contributions from the scalar product linearly while the “T” estimators weight it quadratically. Note that in principle these two sets do contain different statistical information but we anticipate that they provide very similar results [41].

### 2.4 Simulations pipeline and observed data analysis

We perform \( 10^5 \) Monte Carlo simulations, extracting \( a_{l m} \) coefficients from the Planck 2013 \( \Lambda\)CDM fiducial model.\(^3\) For each realization, we transform to multipole vectors employing the publicly available code written by Copi et al. \(^4\), whose use is acknowledged here.\(^5\) Then, for each of the performed realizations we compute the eight estimators defined in eqs. (2.2)–(2.9). We therefore can build the empirical distributions of the estimators in the \( \Lambda\)CDM model, see green histograms in figure 2. For the dipolar model, our pipeline flows in a similar way. The only difference is that once the \( a_{l m} \) are drawn, we transform them to a real space map, i.e. \( \Delta T/T|_{\text{iso}} \), and use eq. (1.1) to compute \( \Delta T/T|_{\text{mod}} \). We then go back to harmonic space, i.e.

\[
\Delta T/T|_{\text{mod}} \rightarrow a_{l m}^{\text{mod}},
\]

and use these \( a_{l m}^{\text{mod}} \) to compute the multipole vectors. Once this is repeated \( 10^5 \) times, we can build the eight empirical distributions of the considered estimators in the dipolar model, see the red histograms in figure 2.

Of course the same estimators are evaluated for five observed CMB maps, see section 2.1. These values are represented by the vertical lines in figure 2: WMAP ILC 5 in blue, WMAP ILC 7 in pink, WMAP ILC 9 in balck, Planck 2013 NILC in cyan and Planck 2013 SMICA in magenta. In fact before evaluating these numbers, we have applied a “boost correction” to the observed \( a_{l m} \) coefficients. This is necessary because the observed quadrupole is slightly affected by the motion of the satellite with respect to the CMB rest frame. The details of this correction for every multipole \( \ell \) are given in the next subsection.

---

\(^3\)We have tested that the particular model chosen is irrelevant.

\(^4\)See http://www.phys.cwru.edu/projects/mpvectors/.

---
2.4.1 Boost correction

It is possible to show, see e.g. [58, 59], that the spherical harmonic coefficients, $a_{\ell m}^{RF}$, observed in the CMB rest frame (hereafter $S_{\text{cmb}}$) are related to the spherical harmonic coefficients, $a_{\ell m}'$ defined in a frame $S$ which is moving in the $\hat{z}$ direction at velocity $v$ with respect to $S_{\text{cmb}}$, in the following way

$$a_{\ell m}' = \sum_{\ell'=0}^{\infty} a_{\ell m}^{RF} I_{m}^{\ell'}(v), \quad (2.14)$$

where no sum on $m$ is understood and where the $I_{m}^{\ell'}(v)$ is defined as

$$I_{m}^{\ell'}(v) = \int_{-1}^{1} \frac{1}{2\pi} \sqrt{\frac{1 - v^2}{1 + vx}} \tilde{P}_{m}^{\ell'}(x) \tilde{P}_{m}^{\ell'} \left( \frac{x + v}{1 + vx} \right) dx, \quad (2.15)$$

with the $\tilde{P}_{m}^{\ell'}$ functions defined through the Legendre polynomial $P_{m}^{\ell'}$ as

$$\tilde{P}_{m}^{\ell'} = \sqrt{\frac{2\ell + 1 (\ell - m)!}{4\pi (\ell + 1)!}} P_{m}^{\ell'.} \quad (2.16)$$

In fact we need to invert eq. (2.14) and “deboost” the WMAP and Planck observations. This can be done using the following orthonormality relation

$$\sum_{\ell'} I_{m}^{\ell_{1}} I_{m}^{\ell_{2}} = \delta_{\ell_{1}\ell_{2}}, \quad (2.17)$$

and considering that

$$I_{m}^{\ell}(v) = I_{m}^{\ell}(-v). \quad (2.18)$$

Therefore one finds

$$a_{\ell m}^{RF} = \sum_{\ell'} a_{\ell m}' I_{m}^{\ell'}(-v). \quad (2.19)$$

In practice, only $\ell = 2$ has to be corrected by this kinematic term. For this multipole, the typical correction is roughly around $10 - 30\%$. For $\ell \geq 3$ this effect is completely negligible. For the octupole the maximum deviation is computed to be of the order of $0.1\%$. See appendix A where explicit values are reported.

3 Results

Our results are shown in figure 2. We evaluate the level of anomaly comparing the histograms with the observed values, i.e. the vertical bars in figure 2. We consider both the $\Lambda$CDM and dipolar, and for each analyzed CMB map, i.e. WMAP ILC 5, WMAP ILC 7, WMAP ILC 9, Planck 2013 NILC and Planck 2013 SMICA.

At the price of a slight inaccuracy in terminology, we define the probability to exceed, henceforth PTE, as the number of the simulated counts that have the value of the considered estimator smaller that the observed value. These values are reported in table 1 and the PTEs are provided in table 2 and in table 3.

A few comments are in order. First, the empirical histograms for $\Lambda$CDM and dipolar model are very similar. This means it is not easy to distinguish between the two models on basis of the observed alignments. Second, all vertical lines are very close to each other.
Figure 2. S statistic for the upper row and T statistic for the lower row. Green histograms for the empirical distribution of the considered estimators in ΛCDM and red for the dipolar model. From left to right we consider $S$, $S_{23}$, $\hat{S}_{23}$ and $DQOS$ in the first row and similarly $T$, $T_{23}$, $\hat{T}_{23}$ and $DQOT$ in the second row. Vertical lines are for the observed estimators (already boost-corrected): WMAP ILC 5 in blue, WMAP ILC 7 in pink, WMAP ILC 9 in black, Planck 2013 NILC in cyan and Planck 2013 SMICA in magenta. In each panel we show the counts in the y-axis and the estimator in the x-axis.

Table 1. Values of the estimators extracted from the WMAP and Planck CMB maps.

| Estimator | WMAP ILC 5 yr | WMAP ILC 7 yr | WMAP ILC 9 yr | Planck SMICA | Planck NILC |
|-----------|---------------|---------------|---------------|--------------|-------------|
| $S$       | 0.799         | 0.804         | 0.807         | 0.794        | 0.804       |
| $T$       | 0.959         | 0.962         | 0.963         | 0.956        | 0.962       |
| $S_{23}$  | 0.776         | 0.783         | 0.788         | 0.718        | 0.697       |
| $T_{23}$  | 0.949         | 0.953         | 0.955         | 0.919        | 0.928       |
| $\hat{S}_{23}$ | 0.869     | 0.877         | 0.884         | 0.859        | 0.877       |
| $\hat{T}_{23}$ | 0.982     | 0.984         | 0.986         | 0.979        | 0.985       |
| $DQOS$    | 0.789         | 0.792         | 0.799         | 0.774        | 0.776       |
| $DQOT$    | 0.940         | 0.943         | 0.946         | 0.936        | 0.944       |

This means that at large angular scale in temperature the CMB maps obtained with two different experiments and with three different methods are very similar in terms of phases. Third, all vertical bars, for all the considered estimators, stand in the right hand part of the histograms. This means that data tend to show alignments of the considered low multipoles. The significance of these alignments is in general larger than 99%, with few cases at the level of 98 – 99%, and can be as large as 99.9% in selected cases.

4 Conclusion

We have tested the CMB quadrupole/octupole and dipole/quadrupole/octupole alignments for several foreground cleaned products for both WMAP (5, 7 and 9 year data) as well as Planck 2013 data. Specifically, we have considered the WMAP ILC products for the several
Table 2. Percentage of anomaly for the quadrupole/octupole alignment, for all analysed estimators ($S$, $T$, $S_{23}$, $T_{23}$, $\hat{S}_{23}$ and $\hat{T}_{23}$) for the WMAP data (WMAP ILC 5 yr, WMAP ILC 7 yr and WMAP ILC 9 yr) and for the Planck data (Planck SMICA and Planck NILC).

| Estimator | $\Lambda$CDM Dipolar | $\Lambda$CDM Dipolar | $\Lambda$CDM Dipolar | $\Lambda$CDM Dipolar | $\Lambda$CDM Dipolar | $\Lambda$CDM Dipolar | $\Lambda$CDM Dipolar |
|-----------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| $S$       | 99.647                | 99.640                | 99.701                | 99.701                | 99.750                | 99.731                | 99.581                | 99.578                | 99.704                | 99.707                |
| $T$       | 99.828                | 99.832                | 99.856                | 99.866                | 99.873                | 99.880                | 99.775                | 99.769                | 99.856                | 99.866                |
| $S_{23}$  | 99.722                | 99.724                | 99.793                | 99.791                | 99.838                | 99.830                | 98.649                | 98.606                | 97.990                | 97.951                |
| $T_{23}$  | 99.863                | 99.868                | 99.892                | 99.891                | 99.905                | 99.906                | 99.217                | 99.207                | 98.861                | 98.833                |
| $\hat{S}_{23}$ | 98.355          | 98.308                | 98.569                | 98.539                | 98.689                | 98.676                | 98.128                | 98.089                | 98.550                | 98.523                |
| $\hat{T}_{23}$ | 98.654          | 98.646                | 98.839                | 98.802                | 98.901                | 98.881                | 98.420                | 98.379                | 98.839                | 98.806                |

Table 3. Percentage of anomaly for the dipole/quadrupole/octupole alignment, for all analysed estimators (DQO$_S$ and DQO$_T$) for the WMAP data (WMAP ILC 5 yr, WMAP ILC 7 yr and WMAP ILC 9 yr) and for the Planck data (Planck SMICA and Planck NILC).

| Estimator | $\Lambda$CDM Dipolar | $\Lambda$CDM Dipolar | $\Lambda$CDM Dipolar | $\Lambda$CDM Dipolar | $\Lambda$CDM Dipolar | $\Lambda$CDM Dipolar | $\Lambda$CDM Dipolar | $\Lambda$CDM Dipolar |
|-----------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| DQO$_S$   | 99.803                | 99.796                | 99.829                | 99.823                | 99.872                | 99.865                | 99.672                | 99.662                | 99.687                | 99.681                |
| DQO$_T$   | 99.776                | 99.779                | 99.810                | 99.808                | 99.859                | 99.851                | 99.725                | 99.728                | 99.825                | 99.823                |

year releases and Planck NILC and SMICA maps. We have used a total of eight estimators based on the multipole vector formalism, two for the dipole/quadrupole/octupole and six for quadrupole/octupole alignments. All these estimators are supported by a large Monte Carlo of $10^5$ independent maps. We report that all the data combinations and all the estimators we have tested exhibit anomalous alignments for both combinations of multipoles considered, typically at the 98%-99% level, and up to 99.9% in selected cases. The consistent pattern for the alignments observed in both WMAP and Planck strongly disfavours an origin of the effect related to unaccounted instrumental systematics. The wide frequency leverage of the Planck data (30 to 353 GHz), weakens considerably the case for residual foreground emission. The fact that we find consistent results also among different foreground separation procedures (SMICA, NILC and WMAP’s ILC) makes this conclusion stronger. We have also investigated the possibility that the phenomenological dipolar model may provide a better framework for the existence of the observed alignments with respect to plain $\Lambda$CDM. This possibility is, in principle, intriguing because the dipolar model has gathered some success in accounting for other anomalies, e.g. the hemispherical asymmetry. We report negative findings on this last issue: the dipolar model does not seem to be able to accommodate for the existence of anomalies significantly better than $\Lambda$CDM.

Acknowledgments

We are grateful to Bruce Partridge for valuable comments. We acknowledge the use of the publicly available code for the multipole vectors decomposition (http://www.phys.cwru.edu/projects/mpvectors/) described in [34]. We also acknowledge the use of the HEALPix package.
Table 4. \(a_{\ell m}\) for \(\ell = 2\) and \(\ell = 3\) (no correction applied). Units: \(\mu\)K.

| \(a_{20}\) | \(a_{21}\) | \(a_{22}\) | \(a_{30}\) | \(a_{31}\) | \(a_{32}\) | \(a_{33}\) |
|----------|----------|----------|----------|----------|----------|----------|
| WMAP ILC 5 yr | 12.350 | -1.087+6.069i | -14.211-17.858i | -6.449 | -12.733+2.443i | 22.019+0.698i | -11.813+33.393i |
| WMAP ILC 7 yr | 11.771 | -0.771+6.215i | -14.120-17.941i | -6.479 | -12.191+2.026i | 21.999+0.591i | -11.709+33.554i |
| WMAP ILC 9 yr | 12.563 | -1.727+6.209i | -13.846-18.017i | -6.449 | -12.191+2.026i | 21.999+0.591i | -11.709+33.554i |
| Planck 2013 SMICA | 13.089 | -1.530+2.497i | -15.503-17.091i | -5.959 | -12.841+1.671i | 22.086+1.670i | -12.465+29.402i |
| Planck 2013 NILC | 13.512 | -1.375+1.722i | -13.564-16.325i | -6.117 | -9.547+1.896i | 22.242+1.875i | -12.914+28.346i |

Table 5. De-boosted \(a_{\ell m}\) for \(\ell = 2\) and \(\ell = 3\). Units: \(\mu\)K.

| \(a_{20}\) | \(a_{21}\) | \(a_{22}\) | \(a_{30}\) | \(a_{31}\) | \(a_{32}\) | \(a_{33}\) |
|----------|----------|----------|----------|----------|----------|----------|
| WMAP ILC 5 yr | 10.882 | -1.385+8.716i | -13.076-17.619i | -6.458 | -12.750+2.477i | 22.066+0.726i | -11.767+33.351i |
| WMAP ILC 7 yr | 10.304 | -1.068+8.861i | -12.985-17.701i | -6.486 | -12.209+2.059i | 22.046+0.619i | -11.664+33.523i |
| WMAP ILC 9 yr | 11.095 | -2.023+8.854i | -12.710-17.777i | -6.855 | -11.288+1.614i | 21.903+0.562i | -12.017+32.811i |
| Planck 2013 SMICA | 11.622 | -1.830+6.543i | -14.363-16.852i | -5.964 | -12.857+1.709i | 22.139+1.696i | -12.421+29.362i |
| Planck 2013 NILC | 12.046 | -1.670+4.368i | -12.423-16.086i | -6.122 | -9.563+1.935i | 22.291+1.900i | -12.873+28.301i |

(http://healpix.sourceforge.net), see [60]. Some results presented in this papers are based on observations obtained with Planck (http://www.esa.int/Planck), an ESA science mission with instruments and contributions directly funded by ESA Member States, NASA, and Canada. Moreover, we acknowledge the use of the Legacy Archive for Microwave Background Data Analysis (LAMBDA), part of the High Energy Astrophysics Science Archive Center (HEASARC). HEASARC/LAMBDA is a service of the Astrophysics Science Division at the NASA Goddard Space Flight Center. Work supported by ASI through ASI/INAF Agreement I/072/09/0 for the Planck LFI Activity of Phase E2.

A Impact of the boost correction

In table 4 and 5 we report the \(a_{\ell m}\) for quadrupole and octupole without and with de-boosting correction respectively.

References

[1] C. Gordon, W. Hu, D. Huterer and T.M. Crawford, \textit{Spontaneous isotropy breaking: a mechanism for CMB multipole alignments}, \textit{Phys. Rev. D} 72 (2005) 103002 [astro-ph/0509301] [INSPIRE].

[2] WMAP collaboration, G. Hinshaw et al., \textit{Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results}, \textit{Astrophys. J. Suppl.} 208 (2013) 19 [arXiv:1212.5226] [INSPIRE].

[3] PLANCK collaboration, P.A.R. Ade et al., \textit{Planck 2013 results. XVI. Cosmological parameters}, \textit{Astron. Astrophys.} 571 (2014) A16 [arXiv:1303.5076] [INSPIRE].

[4] C. Monteserin, R.B.B. Barreiro, P. Vielva, E. Martínez-González, M.P. Hobson et al., \textit{A low CMB variance in the WMAP data}, \textit{Mon. Not. Roy. Astron. Soc.} 387 (2008) 209 [arXiv:0706.4289] [INSPIRE].

[5] M. Cruz, P. Vielva, E. Martínez-González and R.B.B. Barreiro, \textit{Anomalous variance in the WMAP data and Galactic foreground residuals}, \textit{Mon. Not. Roy. Astron. Soc.} 412 (2011) 2383 [arXiv:1005.1264] [INSPIRE].
[6] A. Gruppuso, P. Natoli, F. Paci, F. Finelli, D. Molinari et al., Low Variance at large scales of WMAP 9 year data, *JCAP* **07** (2013) 047 [arXiv:1304.5493] [INSPIRE].

[7] PLANCK collaboration, P.A.R. Ade et al., Planck 2013 results. XXIII. Isotropy and statistics of the CMB, *Astron. Astrophys.* **571** (2014) A23 [arXiv:1303.5083] [INSPIRE].

[8] C.J. Copi, D. Huterer, D.J. Schwarz and G.D. Starkman, The Uncorrelated Universe: Statistical Anisotropy and the Vanishing Angular Correlation Function in WMAP Years 1-3, *Phys. Rev.* **D 75** (2007) 023507 [astro-ph/0605135] [INSPIRE].

[9] A. Bernui, T. Villela, C.A. Wuensche, R. Leonardi and I. Ferreira, On the CMB large-scales angular correlations, *Astron. Astrophys.* **454** (2006) 409 [astro-ph/0601593] [INSPIRE].

[10] C.J. Copi, D. Huterer, D.J. Schwarz and G.D. Starkman, No large-angle correlations on the non-Galactic microwave sky, *Mon. Not. Roy. Astron. Soc.* **399** (2009) 295 [arXiv:0808.3767] [INSPIRE].

[11] A. Gruppuso, Two-point Correlation Function of WMAP 9 year data, *Mon. Not. Roy. Astron. Soc.* **437** (2014) 2076 [arXiv:1310.2822] [INSPIRE].

[12] C.J. Copi, D. Huterer, D.J. Schwarz and G.D. Starkman, Lack of large-angle TT correlations persists in WMAP and Planck, *arXiv:1310.3831* [INSPIRE].

[13] K. Land and J. Magueijo, Is the Universe odd?, *Phys. Rev. D* **72** (2005) 101302 [astro-ph/0507289] [INSPIRE].

[14] J. Kim and P. Naselsky, Anomalous parity asymmetry of the Wilkinson Microwave Anisotropy Probe power spectrum data at low multipoles, *Astrophys. J.* **714** (2010) L265 [arXiv:1001.4613] [INSPIRE].

[15] J. Kim and P. Naselsky, Anomalous parity asymmetry of WMAP power spectrum data at low multipoles: is it cosmological or systematics?, *Phys. Rev. D* **82** (2010) 063002 [arXiv:1002.0148] [INSPIRE].

[16] A. Gruppuso, F. Finelli, P. Natoli, F. Paci, P. Cabella et al., New constraints on Parity Symmetry from a re-analysis of the WMAP-7 low resolution power spectra, *Mon. Not. Roy. Astron. Soc.* **411** (2011) 1445 [arXiv:1006.1979] [INSPIRE].

[17] P.K. Aluri and P. Jain, Parity Asymmetry in the CMBR Temperature Power Spectrum, *Mon. Not. Roy. Astron. Soc.* **419** (2012) 3378 [arXiv:1108.5894] [INSPIRE].

[18] H.K. Eriksen, F.K. Hansen, A.J. Banday, K.M. Gorski and P.B. Lilje, Asymmetries in the Cosmic Microwave Background anisotropy field, *Astrophys. J.* **605** (2004) 14 [Erratum ibid. 609 (2004) 1198] [astro-ph/0307507] [INSPIRE].

[19] F.K. Hansen, A.J. Banday and K.M. Gorski, Testing the cosmological principle of isotropy: local power spectrum estimates of the WMAP data, *Mon. Not. Roy. Astron. Soc.* **354** (2004) 641 [astro-ph/0404206] [INSPIRE].

[20] H.K. Eriksen, A.J. Banday, K.M. Gorski, F.K. Hansen and P.B. Lilje, Hemispherical power asymmetry in the three-year Wilkinson Microwave Anisotropy Probe sky maps, *Astrophys. J.* **660** (2007) L81 [astro-ph/0701089] [INSPIRE].

[21] F.K. Hansen, A.J. Banday, K.M. Gorski, H.K. Eriksen and P.B. Lilje, Power Asymmetry in Cosmic Microwave Background Fluctuations from Full Sky to Sub-degree Scales: Is the Universe Isotropic?, *Astrophys. J.* **704** (2009) 1448 [arXiv:0812.3795] [INSPIRE].

[22] J. Hoftuft, H.K. Eriksen, A.J. Banday, K.M. Gorski, F.K. Hansen et al., Increasing evidence for hemispherical power asymmetry in the five-year WMAP data, *Astrophys. J.* **699** (2009) 985 [arXiv:0903.1229] [INSPIRE].
[23] F. Paci, A. Gruppuso, F. Finelli, P. Cabella, A. De Rosa et al., *Power Asymmetries in the Cosmic Microwave Background Temperature and Polarization patterns*, *Mon. Not. Roy. Astron. Soc.* **407** (2010) 399 [arXiv:1002.4745] [SPIRE].

[24] F. Paci, A. Gruppuso, F. Finelli, A. De Rosa, N. Mandolesi et al., *Hemispherical power asymmetries in the WMAP 7-year low-resolution temperature and polarization maps*, *Mon. Not. Roy. Astron. Soc.* **434** (2013) 3071 [arXiv:1301.5195] [SPIRE].

[25] Y. Akrami et al., *Power asymmetry in WMAP and Planck temperature sky maps as measured by a local variance estimator*, *Astrophys. J.* **784** (2014) L42 [arXiv:1402.0870] [SPIRE].

[26] A. Ben-David, E.D. Kovetz and N. Itzhaki, *Parity in the CMB: Space Oddity*, *Astrophys. J.* **748** (2012) 39 [arXiv:1108.1702] [SPIRE].

[27] F. Finelli, A. Gruppuso, F. Paci and A.A. Starobinsky, *Searching for hidden mirror symmetries in CMB fluctuations from WMAP 7 year maps*, *JCAP* **07** (2012) 049 [arXiv:1111.5362] [SPIRE].

[28] A. Ben-David and E.D. Kovetz, *A close examination of cosmic microwave background mirror-parity after Planck*, *Mon. Not. Roy. Astron. Soc.* **445** (2014) 2116 [arXiv:1403.2104] [SPIRE].

[29] A. Rassat, J.-L. Starck, P. Paykari, F. Sureau and J. Bobin, *Planck CMB Anomalies: Astrophysical and Cosmological Secondary Effects and the Curse of Masking*, *JCAP* **08** (2014) 006 [arXiv:1405.1844] [SPIRE].

[30] P. Vielva, E. Martínez-González, R.B.B. Barreiro, J.L. Sanz and L. Cayón, *Detection of non-Gaussianity in the WMAP 1-year data using spherical wavelets*, *Astrophys. J.* **609** (2004) 22 [astro-ph/0310273] [SPIRE].

[31] M. Cruz, E. Martínez-González, P. Vielva and L. Cayón, *Detection of a non-Gaussian spot in WMAP*, *Mon. Not. Roy. Astron. Soc.* **356** (2005) 29 [astro-ph/0405341] [SPIRE].

[32] M. Cruz, M. Tucci, E. Martínez-González and P. Vielva, *The non-Gaussian cold spot in Wilkinson Microwave Anisotropy Probe: significance, morphology and foreground contribution*, *Mon. Not. Roy. Astron. Soc.* **369** (2006) 57 [astro-ph/0601427] [SPIRE].

[33] P. Vielva, *A Comprehensive overview of the Cold Spot*, *Adv. Astron.* **2010** (2010) 592094 [arXiv:1008.3051] [SPIRE].

[34] C.J. Copi, D. Huterer and G.D. Starkman, *Multipole vectors: a new representation of the CMB sky and evidence for statistical anisotropy or non-Gaussianity at 2 ≤ ℓ ≤ 8*, *Phys. Rev.* **D 70** (2004) 043515 [astro-ph/0310511] [SPIRE].

[35] K. Land and J. Magueijo, *The Multipole vectors of WMAP and their frames and invariants*, *Mon. Not. Roy. Astron. Soc.* **362** (2005) 838 [astro-ph/0502574] [SPIRE].

[36] L.R. Abramo, A. Bernui, I.S. Ferreira, T. Villela and C.A. Wuensche, *Alignment Tests for low CMB multipoles*, *Phys. Rev.* **D 74** (2006) 063506 [astro-ph/0604346] [SPIRE].

[37] K. Land and J. Magueijo, *Examination of Evidence for a Preferred Axis in the Cosmic Radiation Anisotropy*, *Phys. Rev. Lett.* **95** (2005) 071301 [astro-ph/0502237] [SPIRE].

[38] A. Gruppuso, C. Burigana and F. Finelli, *The impact of dipole straylight contamination on the alignment of low multipoles of CMB anisotropies*, *Mon. Not. Roy. Astron. Soc.* **376** (2007) 907 [astro-ph/0701295] [SPIRE].

[39] A. Gruppuso and C. Burigana, *Large scale alignment anomalies of CMB anisotropies: a new test for residuals applied to WMAP 5yr maps*, *JCAP* **08** (2009) 004 [arXiv:0907.1949] [SPIRE].

[40] A. Gruppuso and K.M. Gorski, *Large scale directional anomalies in the WMAP 5yr ILC map*, *JCAP* **03** (2010) 019 [arXiv:1002.3928] [SPIRE].
[41] C.J. Copi, D. Huterer, D.J. Schwarz and G.D. Starkman, Large angle anomalies in the CMB, *Adv. Astron.* 2010 (2010) 847541 [arXiv:1004.5602] [inSPIRE].

[42] C.J. Copi, D. Huterer, D.J. Schwarz and G.D. Starkman, Large-scale alignments from WMAP and Planck, *arXiv:1311.4562* [inSPIRE].

[43] PLANCK collaboration, P.A.R. Ade et al., Planck 2013 results. XII. Diffuse component separation, *Astron. Astrophys.* 571 (2014) A12 [arXiv:1303.5072] [inSPIRE].

[44] H. Liu, P. Mertsch and S. Sarkar, Fingerprints of Galactic Loop I on the Cosmic Microwave Background, *Astrophys. J.* 789 (2014) L29 [arXiv:1404.1899] [inSPIRE].

[45] E. Gross and O. Vitells, Trial factors or the look elsewhere effect in high energy physics, *Eur. Phys. J. C* 70 (2010) 525 [arXiv:1005.1891] [inSPIRE].

[46] A. Bernui, Anomalous CMB north-south asymmetry, *Phys. Rev. D* 78 (2008) 063531 [arXiv:0809.0934] [inSPIRE].

[47] A. de Oliveira-Costa, M. Tegmark, M. Zaldarriaga and A. Hamilton, The Significance of the largest scale CMB fluctuations in WMAP, *Phys. Rev. D* 69 (2004) 063516 [astro-ph/0307282] [inSPIRE].

[48] J.R. Weeks, Maxwell’s multipole vectors and the CMB, *astro-ph/0412231* [inSPIRE].

[49] PLANCK collaboration, N. Aghanim et al., Planck 2013 results. XXVII. Doppler boosting of the CMB: eppur si muove, *Astron. Astrophys.* 571 (2014) A27 [arXiv:1303.5087] [inSPIRE].

[50] WMAP collaboration, G. Hinshaw et al., Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Final Maps and Results, *Astrophys. J. Suppl.* 208 (2013) 20 [arXiv:1212.5225] [inSPIRE].

[51] WMAP collaboration, C.L. Bennett et al., Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Final Maps and Results, *Astrophys. J. Suppl.* 208 (2013) 20 [arXiv:1212.5225] [inSPIRE].

[52] H.K. Eriksen, A.J. Banday, K.M. Gorski and P.B. Lilje, Foreground removal by an internal linear combination method: limitations and implications, *Astrophys. J.* 612 (2004) 633 [astro-ph/0403098] [inSPIRE].

[53] J.-F. Cardoso, M. Martin, J. Delabrouille, M. Betoule and G. Patanchon, Component separation with flexible models. Application to the separation of astrophysical emissions, *arXiv:0803.1814* [inSPIRE].

[54] A. Benoit-Lévy, T. Dechelette, K. Benabed, et al., Full-sky CMB lensing reconstruction in presence of sky-cuts, *Astron. Astrophys.* 555 (2013) A37 [arXiv:1301.4145] [inSPIRE].

[55] J. Delabrouille, J.-F. Cardoso, M.L. Jeune, M. Betoule, G. Fay et al., A full sky, low foreground, high resolution CMB map from WMAP, *Astron. Astrophys.* 493 (2009) 835 [arXiv:0807.0773] [inSPIRE].

[56] A. Kosowsky and T. Kahniashvili, The Signature of Proper Motion in the Microwave Sky, *Phys. Rev. Lett.* 106 (2011) 191301 [arXiv:1007.4539] [inSPIRE].

[57] L. Amendola, R. Catena, I. Masina, A. Notari, M. Quartin et al., Measuring our peculiar velocity on the CMB with high-multipole off-diagonal correlations, *JCAP* 07 (2011) 027 [arXiv:1008.1183] [inSPIRE].
[60] K.M. Gorski, E. Hivon, A.J. Banday, B.D. Wandelt, F.K. Hansen et al., *HEALPix - A Framework for high resolution discretization and fast analysis of data distributed on the sphere*, *Astrophys. J.* 622 (2005) 759 [astro-ph/0409513] [InSPIRE].