Anomalous thermal escape in Josephson systems perturbed by microwaves

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We investigate, by experiments and numerical simulations, thermal activation processes of Josephson tunnel junctions in the presence of microwave radiation. When the applied signal resonates with the Josephson plasma frequency oscillations, the switching current may become multi-valued temperature ranges both below and above the classical to quantum crossover temperature. Switching current distributions are obtained both experimentally and numerically at temperatures both near and far above the quantum crossover temperature. Plots of the switching currents traced as a function of the applied signal frequency show very good agreement with a simple anharmonic theory for Josephson resonance frequency as a function of bias current. Throughout, experimental results and direct numerical simulations of the corresponding thermally driven classical Josephson junction model show very good agreement.

1 Introduction

The Josephson tunnel junction is a physical system very well studied due to its simplicity and nonlinearity [1]. Statistical properties of Josephson junctions have been another subject of intense investigation through, e.g., measurements of the escape statistics from the zero-voltage state, successfully confirming consistency with the classic Kramers model for thermal activation from a potential well [2, 3]. Escape measurements represent a powerful tool for probing the nature of the underlying potential well, and applying an ac field to a low-temperature system has been reported to produce anomalous switching distributions with two, or more, distinct dc bias currents for which switching is likely. These measurements have been interpreted as a signature of the ac field aiding the population of multiple quantum levels in a junction, thereby leading to enhancement of the switching probability for bias currents for which the corresponding quantum levels match the energy.
of the microwave photons. Work performed in this direction appeared first in the literature two decades ago[4, 5]. These results have significantly attracted interest toward Josephson junction systems as possible basic elements in the field of quantum coherence and quantum computing[6, 7, 8, 9, 10], and more recently other investigations have further indicated that the application of microwaves may not be the only condition under which level quantization can be observed in Josephson junctions [11].

Within the framework of this research topic we recently reported experimental measurements conducted on a Josephson junction, operated well above the so-called quantum transition temperature $T^*$, and direct numerical simulations of the classical pendulum model, parameterized to mimic the experimental device[12]. It was found that multi-peaked switching distributions are not unique to the quantum regime (below $T^* = \hbar \omega_0 / 2\pi k_B$), and, in fact, are manifested with the same features and under the same conditions in the classical regime as has been previously reported for low temperature measurements below $T^*$. With the present paper we wish to contribute an anharmonic theory that accurately captures the bias current values of the observed resonant peaks in the switching distributions as a function of the applied frequency of the microwave field. We demonstrate agreement between the theory, direct numerical simulations, and experimental measurements for direct resonances as well as harmonic and sub-harmonic resonances at temperatures both well above and near $T^*$.

![Fig. 1. Sketch of the physical phenomenon under investigation: a driven oscillation energy $E_{ac}$ superimposed onto thermal excitations, may cause a particle to escape a washboard potential.](image-url)
Figure 1 illustrates the process under investigation: in the classical one-degree-of-freedom single-particle washboard potential of the Josephson junction [4], thermal excitations (shaded in the sketch) of energy $k_B T$ and the energy $E_{ac}$ of forced oscillations due to microwave radiation, can cause the particle to escape from the potential well. This process can be traced by sweeping the current-voltage characteristics of the Josephson junction periodically. Escape from the potential well corresponds to an abrupt transition from the top of the Josephson-current zero-voltage state to a non-zero voltage state. The statistics of the switching events, in the absence of time-varying perturbations, have been shown to be consistent with Kramers’ model [2] for thermal escape from a one-dimensional potential. Since the thermal equilibrium Kramers model does not include the effect of non-equilibrium force terms, the results of the switching events generated by the presence of a microwave radiation on a Josephson junction can be investigated, in a thermal regime, only by a direct numerical simulation of the governing equations (RSCJ model) [1].

2 Theory

The RSCJ model reads,

$$\frac{\hbar C}{2e} \frac{d^2 \varphi}{dt^2} + \frac{\hbar}{2e R} \frac{d\varphi}{dt} + I_c \sin \varphi = I_{dc} + I_{ac} \sin \omega dt + N(t) . \quad (1)$$

Here, $\varphi$ is the phase difference of the quantum mechanical wave functions of the superconductors defining the Josephson junction, $C$ is the magnitude of junction capacitance, $R$ is the model shunting resistance, and $I_c$ is the critical current, while $I_{dc}$ and $I_{ac} \sin \omega dt$ represent, respectively, the continuous and alternating bias current flowing through the junction. The term $N(t)$ represents the thermal noise-current due to the resistor $R$ given by the thermodynamic dissipation-fluctuation relationship [13]

$$\langle N(t) \rangle = 0 \quad (2)$$

$$\langle N(t) N(t') \rangle = 2 \frac{k_B T}{R} \delta(t - t') , \quad (3)$$

with $T$ being the temperature. The symbol, $\delta(t - t')$, is the Dirac delta function. Current and time are usually normalized respectively to the Josephson critical current $I_c$ and to $\omega_0^{-1}$, where $\omega_0 = \sqrt{2e I_c / \hbar C}$ is the Josephson plasma frequency. With this normalization, the coefficient of the first-order phase derivative becomes the normalized dissipation $\alpha = \hbar \omega_0 / 2e RL_c$. It is also convenient to scale the energies to the Josephson energy $E_J = I_c \hbar / 2e = I_c \Phi_0 / 2\pi$, where $\Phi_0 = \hbar / 2e = 2.07 \cdot 10^{-15} Wb$ is the flux-quantum. Thus, the set of equations (1-3) can be expressed in normalized form as
\[ \ddot{\phi} + \alpha \dot{\phi} + \sin \phi = \eta_0 \sin \Omega_d \tau + n(\tau) \tag{4} \]

\[ \langle n(\tau) \rangle = 0 \tag{5} \]

\[ \langle n(\tau) n(\tau') \rangle = 2 \chi \delta(\tau - \tau') \tag{6} \]

where \( \theta = \frac{k_B T}{E} \) is the normalized temperature. The normalized dc and ac currents are \( \eta_0 = \frac{I_0}{I_c} \) and \( \eta_d = \frac{I_d}{I_c} \), respectively.

For small-amplitude oscillations around a stable (zero-voltage) energetic minimum we obtain the standard relationship between resonance frequency and bias current,

\[ \Omega_p = \sqrt{1 - \eta^2} \tag{7} \]

where we have omitted the dissipative contribution to the resonance frequency. However, this linear resonance is not directly relevant for the dynamics leading to anomalous resonant switching. Looking at Figure 1 it is obvious that a switching event will arise from probing the anharmonic region of the potential near the local energetic maximum, and we therefore must anticipate a depression of the resonance frequency at these large amplitudes.

In order to quantify this notion, we will adopt the following ansatz,

\[ \phi = \phi_0 + \psi \tag{8} \]

where \( \phi_0 \) is a constant and \( \psi \) represents oscillatory motion. Inserting this ansatz into equation (4) (for \( \theta = 0 \)) yields,

\[ \ddot{\psi} + \sin \phi_0 \cos \psi + \cos \phi_0 \sin \psi = \eta + \eta_d \sin \Omega_d \tau - \alpha \dot{\psi} \tag{9} \]

Making the single-mode assumption, \( \psi = a \sin(\Omega_d \tau + \kappa) \), \( \kappa \) being some constant phase, we obtain the following

\[ \sin \phi_0 = \frac{\eta}{J_0(a)} \tag{10} \]

\[ \Omega_{res} = \sqrt{\frac{2J_1(a)}{a \sqrt{1 - \left( \frac{\eta}{J_0(a)} \right)^2}}}, \tag{11} \]

the functions, \( J_n \), being the zero’s order Bessel function of the first kind. Notice that \( \Omega_{res} \to \Omega_p \) for \( a \to 0 \). Since multi-peaked switching distributions must require some switching events to happen near the resonance and others to happen for larger bias currents, we can estimate that the amplitude \( a \) must be approximately given by

\[ a \approx \pi - 2 \sin^{-1} \frac{\eta}{J_0(a)} \tag{12} \]

which represents the phase distance from the energetic minimum to the saddle point. Approximating \( J_0(a) \) by its Taylor expansion, we can arrive at the
simple expression between the oscillation amplitude and the applied bias current $\eta$,

$$a \approx \sqrt{\frac{4}{3} [1 - \eta]}.$$  \hspace{1cm} (13)

Inserting this approximate expression into (11) gives an explicit relationship between the anharmonic resonance and the bias current, relevant for the bias current location of the anomalous secondary peak in the switching distribution.

3 Experiments and Simulations

Experiments were performed on Josephson tunnel junctions fabricated according to classical Nb-NbAlO$_x$-Nb procedures [14]. The samples had very good current-voltage characteristics and magnetic field diffraction patterns. The junctions were cooled in a $^3$He refrigerator (Oxford Instruments Heliox system), providing temperatures down to 360mK. Microwave radiation, brought to the chip-holders by a coax cable, was coupled capacitively to the junctions, and the junction had a maximum critical current of $I_c = 143 \mu A$ and a total capacitance of $6 \text{pF}$ from which we estimate a plasma frequency of $\omega_0/2\pi = 42.5 \text{GHz}$. From this value of the plasma frequency the classical to quantum crossover temperature [15] $T^* = (\hbar \omega_0)/(2\pi k_B) = 325 \text{mK}$ between classical thermal and quantum mechanical behavior can be estimated. The sweep rate of the continuous current $I_{dc}$ was $\dot{I}_{dc} = 800 \text{mA/s}$, and we verified that the experiment was being conducted in adiabatic conditions [11]. The junction has a Josephson energy $E_J = 46.4 \cdot 10^{-21} \text{J}$ in the temperature range from 370mK to 1.6K, and effective resistance $R = 74 \Omega$. Evaluation of the dissipation parameter was based on the hysteresis of the current-voltage characteristics of the junctions [1]. We show data for two temperatures, $T \approx 388 \text{mK}$ and $T = 1.6 \text{K}$.

Figure 2 shows experimentally obtained results at $T = 1.6 \text{K}$ [12]. The lower frames of the figure displays the switching distributions in bias current at different microwave frequencies. The top frame shows the relationship between the normalized current, for which the switching distributions have their resonant peak, and the applied frequency (normalized to the junction plasma frequency). Each black marker represents one of the switching distributions. Also shown in figure 2 is the linear resonance of equation (7), shown as a dashed curve, and the anharmonic resonance of equations (11) and (13), shown as a solid curve. The agreement between the experimental measurements and the anharmonic theory of the classical model is near perfect for the available data points, and we emphasize that the theoretical model of equations (11) and (13) has no fitted parameters to adjust in the comparison. Thus, the consistent depression of the experimental data relative to a linear resonance
consideration, observed in Ref. [12], should be expected and not give rise to re-fitting the critical current or the plasma resonance frequency.

**Fig. 2.** Experimentally obtained switching distributions, $\rho(\eta)$, for the microwave-driven junction obtained for increasing values of the drive frequency. The frequency data points in the uppermost plot are relative to the position of the secondary peak in the plots. Temperature is $T = 1.6K$, and bias sweep rate is $\dot{I} = 800mA/s$. Dashed curve in uppermost graph represents the linear plasma resonance of (7), while the solid curve represents the anharmonic resonance of (11) and (13).

Figure 3 shows experimentally obtained results at $T = 388mK$, presented in the same manner as the data in figure 2. The agreement between the experimental measurements and the anharmonic theory of the classical model is again near perfect for the available data points. The resonance curves shown in figures 2 and 3 are identical, since we have not included the resonance dependence on the dissipation (since dissipation is very small) and since the measurements indicated that plasma resonance frequency and critical current were unchanged in the investigated temperature range. By comparing figures 2 and 3, we notice that there seem to be no qualitative (and hardly any quantitative) differences between the data obtained at the two very different temperatures, even though the data of figure 2 is acquired at $T \approx 5T_\ast$ and the data in figure 3 represent $T \approx 1.2T_\ast$. 
Numerical simulations of escape in a system described by equations (4)-(6) corresponding to the experiments with $\alpha = 0.00845$, $\theta = 115.4 \cdot 10^{-6}$, $4.76 \cdot 10^{-4}$, and continuous bias sweep rate $\frac{d\eta}{d\tau} = 2.1 \cdot 10^{-8}$ have also been conducted in order to investigate the purely classical dynamics in comparison with the experimental measurements. The parameters have been chosen in agreement with the experiments discussed above. Switching distributions (each corresponding to 1,000-10,000 events), obtained for different values of the normalized drive frequency and temperature, were obtained as a function of the continuous bias, and secondary resonant peaks in the distribution were easily obtained in the classical model by adjusting the simulated microwave amplitude for a given frequency.

Figure 4 shows both experimental measurements and direct numerical simulations of the resonant peak location as a function of applied microwave frequency at $T = 1.6K \approx 5T_\text{c}$. Experimental data are shown as box markers and numerically obtained data are shown as circles. As in figures 2 and 3, dashed curves represent the linear resonance (7) while the solid curves are generated from (11) and (13). Experimental data for the fundamental resonance (labeled $\Omega_p$) in the figure are the ones from figure 2. We clearly observe the close agreement between theory, experiment, and simulation. We
Fig. 4. The functional dependencies of the driving frequency upon the location of the secondary peak in $\rho(\eta)$ obtained for subharmonic and harmonic pumping. Circles represent numerical results and squares experimental data. Parameters are as given in Figure 2. Dashed curves represent the linear plasma resonance of (7), while the solid curves represent the anharmonic resonance of (11) and (13).

also present data for subharmonic resonances, and here too do we find very close agreement between simulation and experiment. The theoretical harmonic and subharmonic resonance curves are the ones of equations (7), (11), and (13) multiplied with the indicated fraction in the figure. This simple theory seems to also predict the sub-harmonic microwave induced resonant peak location very well.

We finally show, in figure 5, the data similar to the ones in figure 4 taken at $T = 388 m K \approx 1.2 T^\ast$. Also at this temperature do we observe very close agreement between experiment, simulation, and theory, amplifying the notion that microwave induced switching and anomalous switching distributions can be understood within a classical, thermal framework.

4 Conclusion

In conclusion, our theory, and experiments on ac-driven, thermal escape of a classical particle from a one-dimensional potential well have shown that resonant coupling (harmonic or subharmonic) between the applied microwaves and the plasma resonance frequency provides an enhanced opportunity for escape, and we have directly observed the signatures of such microwave-induced escape distributions in the form of anomalous multi-peaked escape statistics at two temperatures, $T = 388 m K \approx 1.2 T^\ast$ and $T = 1.6 K \approx 5 T^\ast$. The straightforward agreement between the classical hypothesis of anomalous distributions being directly produced by ac-induced anharmonic resonances, the
results of numerical simulations of the classical pendulum model of a Josephson junction, and actual Josephson junction experiments indicate a consistent interpretation of ac-induced anomalous multi-peaked switching distributions in the classical regime of Josephson junctions.

It is noted that previous experimental work on ac-induced escape distributions obtained at temperatures below $T_\ast$ is consistent with the observations presented here. Those experiments have produced ac-induced peaks in the observed switching distributions, and the relevant peaks are located alongside the expected classical plasma resonance curve, as we have also found here. An important observation is that the microwave-radiation frequency necessary for populating an excited quantum level ($\hbar \omega_d$) in a quantum oscillator coincides with the classical resonance frequency of the corresponding classical oscillator. Thus, the switching distributions obtained from classical and quantum mechanical oscillators may exhibit the same microwave-induced multi-peak signatures, which in the classical interpretation is merely due to resonant nonlinear effects. It is evident then that multi-peaked switching distributions are not a unique signature of quantum behavior in the ac-driven Josephson junction. We finally point out that similar anomalous (resonant) switching has been observed both experimentally [16] and theoretically [17] for single-fluxon behavior in long annular Josephson junctions in an external magnetic field.
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