Research Article

New Rough Approximations Based on E-Neighborhoods

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Received 7 December 2020; Revised 15 January 2021; Accepted 30 January 2021; Published 2 March 2021

Academic Editor: Abdel-Haleem Abdel-Aty

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This paper puts forward some rough approximations which are motivated from topology. Given a subset \( R \subseteq U \times U \), we can use 8 types of E-neighborhoods to construct approximations of an arbitrary \( X \subseteq U \) on the one hand. On the other hand, we can also construct approximations relying on a topology which is induced by an E-neighborhood. Properties of these approximations and relationships between them are studied. For convenience of use, we also give some useful and easy-to-understand examples and make a comparison between our approximations and those in the published literature.

1. Introduction and Preliminaries

The problem of imperfect knowledge became a crucial issue for computer scientists, especially in the area of information system and artificial intelligence [1, 2]. There are various approaches to manipulate and understand imperfect knowledge, among which is rough set theory. Rough set was proposed by Pawlak [3, 4] in 1982 which has been generalized in many ways [5–13]. What we are concerned about are those methods whose ideas are motivated from topology, for example, methods constructing the lower and upper approximations by using different kinds of neighborhoods, such as right and left neighborhoods [10, 14], minimal right neighborhoods [15], and intersection and union neighborhoods (see [16, 17]). In fact, a combination of rough set theory and topological theory became the main goal of many studies (see [18–25]).

The present paper is a continuation to these works where we initiate new types of neighborhoods (namely, \( E_j \)-neighborhoods) to give lower and upper approximations of an arbitrary set \( X \) directly or indirectly. In Section 2, we apply \( N_j \)-neighborhoods to establish the concepts of \( E_j \)-neighborhoods and discuss the main properties. With the help of examples, we show the relationships between them as well as with \( N_j \)-neighborhoods and \( P_j \)-adhesion neighborhoods. In Section 3, we formulate and study the concepts of \( E_j \)-lower and \( E_j \)-upper approximations, \( E_j \)-boundary region, \( E_j \)-positive and \( E_j \)-negative regions, and \( E_j \)-accuracy measure of a subset and make comparisons between them with respect to different types of \( j \). In Section 4, we study the previous concepts from a topological view and explore their main properties. In Section 5, we give some conclusions and make a plan for future works.

Now, we recall some basic properties and results of rough set theory, particularly those related to some types of neighborhood systems.

Definition 1 (see [3, 5, 16])

(1) A subset \( R \subseteq U \times U \) (also called a binary relation on \( U \)) is said to be an equivalence relation if it is reflexive (i.e., \( (v, v) \in R \) for each \( v \in U \)), symmetric (i.e., \( (u, v) \in R \) if \( (v, u) \in R \)), and transitive (i.e., \( (u, w) \in R \) whenever \( (u, v) \in R \) and \( (v, w) \in R \)). It is said to be a preorder (or quasi-order) if it is reflexive and transitive. It is said to be a partial order if it is an antisymmetric (i.e., \( u = v \) whenever \( (u, v) \in R \) and \( (v, u) \in R \)).
Proposition 1 (see [3, 4] for a special case). The lower approximations and the upper approximations have the following properties: \( |E, F| \cup \{E_i\}_{i=1}^{2^l} = \mathbb{R}^l, \) the power set of \( U \):

1. \( R(\emptyset) = R(\emptyset) = \emptyset \) and \( R(U) = R(U) = U. \)
2. \( R(E) \subseteq E \subseteq R(E) \) and \( R(E) \subseteq \{R(E')\}^l. \)
3. \( R(U) = U \cup \cap E \subseteq R(E) \) and \( R(U) = U \cup \cap E \subseteq R(E) \). Particularly, \( R(E) \subseteq R(F) \) and \( R(E) \subseteq R(F) \) if \( E \subseteq F. \)
4. \( R[R(E)] = R(E) \) and \( R[R(E)] = R(E). \)

Definition 2. A subset \( \mathcal{J} \subseteq 2^U \) is called a topology on \( U \) (and \( (U, \mathcal{J}) \) is called a topological space) if it is closed under arbitrary union and finite intersection. A topology satisfying that every open set is also closed is called a clopen topology. We will use \( A^c \) to denote the interior of \( A \) (i.e., the union of all open sets that are contained in \( A \)) and \( A^c \) to denote the closure of \( A \) (i.e., the intersection of all closed sets containing \( A \)) in this paper.

Let \( \mathcal{J}_R = \{A \subseteq 2^U : [x] \subseteq A (\forall x \in A)\}. \) Then, \( (U, \mathcal{J}_R) \) is a topological space, \( R(X) = X^c, \) \( \mathcal{J}_R(X) = X^c, \) and \( [x] \) is the smallest open neighborhood of \( x (\forall x \in U). \) This inspires many people to define the lower approximation and the upper approximation by neighborhoods; actually, our approach in this paper is also motivated from topology. In the following, we write \( J = \{r, l, (r), (l), i, u, (r), (l)\}. \)

Definition 3 (see [10, 14, 16]). Let \( R \subseteq U^2 \) and \( j \in J. \)

1. The 8 kinds of \( j \)-neighborhoods are defined as follows: \( N_j(v) = \{w \in U : wRv\}, \) \( N_l(v) = \{w \in U : wRv\} \)
2. The triple \( (U, R, \lambda_j) \) is called a \( j \)-neighborhood system (in brief, \( j \)-NS), where \( \lambda_j \) is a mapping from \( U \) to \( 2^U \) which associated each \( v \in U \) with a \( j \)-neighborhood

Definition 4 (see [6]). Let \( R \subseteq U^2. \) The \( j \)-adhesion neighborhoods are defined as follows:

(1) \( P_j(v) = \{w \in U : N_j(w) = N_j(v)\} \)
2. Complexity

2. \( E \)-Neighborhoods

In this section, we introduce the notions of \( E \)-neighborhoods using \( j \)-neighborhoods and study their properties.

Definition 5. Let \( R \subseteq U^2. \) The \( E \)-neighborhoods are defined as follows:

1. \( E_i(x) = \{y \in U : N_i(y) \cap N_i(x) \neq \emptyset\} \)
2. \( E_(o) (x) = \{y \in U : N_(o) (y) \cap N_(o) (x) \neq \emptyset\} \)
3. \( E_(j) (x) = E_(j) (x) \cap E_(j) (x) \)
4. \( E_(w) (x) = E_(w) (x) \cap E_(w) (x) \)

We give the following example to illustrate how we calculate different types of neighborhoods. Also, we will benefit from this example to clarify some obtained results.

Example 1. Let \( U = \{v, w, x, y\} \) and \( R = \{(v, v), (y, y), (v, x), (v, y), (y, w), (w, y)\}. \) Then, the \( j \)-neighborhoods and \( E \)-neighborhoods of a point are as in Table 1.

Theorem 1. \( E \)-neighborhoods have the following properties \( (R \subseteq U^2, v \in U): \)

1. \( E_i(v) \subseteq E_i(v) \cup E_i(v) \subseteq E_i(v) \subseteq E_i(v) \)
2. \( E_(o) (v) \cap E_(o) (v) \subseteq E_(o) (v) \cap E_(o) (v) \subseteq E_(o) (v) \)
3. \( v \in E_j(v) \) if \( x \in E_j(v) \) \( j \in J. \)
4. \( E_i(v) \subseteq E_i(v) \cup E_i(v) \subseteq E_i(v) \subseteq E_i(v) \)
5. \( E_i(v) \subseteq E_i(v) \cup E_i(v) \subseteq E_i(v) \subseteq E_i(v) \)
6. \( j \in J. \)
7. \( E_i(v) \subseteq E_i(v) \subseteq E_i(v) \subseteq E_i(v) \)
8. \( f \in E_j(v) \) if \( v \in E_j(v) \) \( f \in E_j(v) \) \( j \in J. \)
9. \( j \in J. \)
10. \( j \in J. \)
Proof. (3) Obviously, \( v \in E_j(x) \Leftrightarrow N_j(v) \cap N_j(x) \neq \emptyset \Leftrightarrow x \in E_j(v) \) for each \( j \in \{r, u, \angle, \langle, \rangle \} \). Then, \( v \in E_j(x) \Leftrightarrow x \in E_j(v) \) for each \( j \in \{i, u, \langle, \rangle, u' \} \).

(4) Step 1: since \( R \) is reflexive, \( \bigcap_{x \in N_j(x)} N_r(v) \subseteq N_r(v) \) and \( \bigcap_{x \in N_j(x)} N_r(x) \subseteq N_r(x) \). This implies that \( E_j(v) \subseteq E_r(v) \) and \( E_j(v) \subseteq E_i(v) \). Consequently, \( E_j(v) \subseteq E_j(v) \) and \( E_j(v) \subseteq E_j(v) \).

Step 2: let \( x \in N_j(v) \). By reflexivity of \( R \), we have \( \{x\} \subseteq N_j(x) \cap N_j(x) \). Therefore, \( x \in E_j(v) \); thus, \( N_j(v) \subseteq E_j(v) \). Also, let \( x \in E_j(v) \). Then, \( N_j(x) \subseteq N_j(v) \). Since \( R \) is reflexive, \( N_j(x) \cap N_j(x) \neq \emptyset \). Therefore, \( x \in E_j(v) \); thus, \( P_j(v) \subseteq E_j(v) \). Hence, we obtain the desired result.

(5) Since \( R \) is symmetric, \( N_r(v) = N_i(v) \). Therefore, \( N_r(v) \cap N_i(z) \neq \emptyset \Rightarrow N_r(v) \cap N_i(z) \neq \emptyset \); thus, \( E_j(v) = E_i(v) \). Consequently, \( E_j(v) = E_i(v) = E_r(v) \). Similarly, \( E_j(v) = E_j(v) = E_j(v) = E_j(v) \).

(6) Let \( a \in N_j(v) \). Then, \( xR \). For each \( N_j(x) \) containing \( v \), we have \( xR \). Since \( R \) is transitive, \( xR \). Therefore, \( a \in N_j(x) \); thus, \( a \in N_j(v) \).

(7) It follows from (4) and the fact \( v \in E_j(x) \Leftrightarrow N_j(v) \cap N_j(x) \neq \emptyset \Leftrightarrow x \in E_j(v) \) (\( j \in \{r, u, \langle, \rangle, u' \} \)).

(8) Step 1: we prove the case \( j = r \). Let \( x \in E_r(v) \). Then, \( v \in E_r(x) \cap N_r(x) \neq \emptyset \), i.e., there exists \( z \in N_r(x) \cap N_r(x) \), \( xRz \), and \( vRz \). Since \( R \) is symmetric and transitive, \( vRx \). Therefore, \( x \in N_r(v) \); thus, \( E_r(v) \subseteq N_r(v) \).

Step 2: since \( R \) is symmetric, \( E_r(v) = E_r(v) = E_r(v) \) by (5). We only prove the case \( j = I \). Let \( v \in E_i(u) \). Then, \( N_i(v) \cap N_i(u) \neq \emptyset \); therefore, there is \( a \in U \) such that \( aRv \) and \( bRu \). Now, let \( x \in E_i(v) \). Then, \( N_i(x) \cap N_i(u) \neq \emptyset \), i.e., there exists a \( b \in U \) such that \( bRx \) and \( bRu \). Note that \( aRb \); this means that \( aRx \); consequently, \( a \in N_i(v) \); also, \( a \in N_i(u) \); thus, \( N_i(v) \cap N_i(u) \neq \emptyset \). Hence, \( x \in E_i(u) \), as required.

(10) Step 1: by (4) and (8), \( E_i(v) = N_i(v) \) and \( E_i(v) \subseteq E_i(v) \). It remains to prove \( E_i(v) \subseteq E_i(v) \). Since \( R \) is symmetric, \( E_i(v) = E_i(v) = E_i(v) \) by (5). Let \( x \in E_i(v) \). Then, \( N_i(x) \cap N_i(u) \neq \emptyset \). Since \( R \) is equivalence, \( N_i(x) = N_i(v) \). Therefore, \( x \in P_i(v) \); consequently, \( E_i(v) \subseteq P_i(v) \). Thus, \( P_i(v) = E_i(v) \) is proved.

Step 2: we only prove the case \( j = I \). Let \( v \in E_i(u) \). Then, \( N_i(v) \cap N_i(u) \neq \emptyset \). Since \( R \) is equivalence, \( N_i(v) = N_i(u) \neq \emptyset \). Therefore, \( E_i(v) = E_i(v) \). Conversely, \( E_i(v) = E_i(v) \). Since \( R \) is reflexive, \( v \in N_i(v) \).

Hence, \( v \in E_i(u) \).

3. Rough Approximations Using E-Neighborhoods Directly

Let \( (U, R, \lambda) \) be \( j \)-NS and \( \mathcal{R}_j = \{E_j(v) : v \in U\} \) \( (j \in J) \). We devote this section to formulate the following concepts using E-neighborhoods directly: \( \mathcal{R}_j \)-lower approximation, \( \mathcal{R}_j \)-upper approximation, \( \mathcal{R}_j \)-boundary region, \( \mathcal{R}_j \)-positive region, \( \mathcal{R}_j \)-negative region, and \( \mathcal{R}_j \)-accuracy measure of a subset \( X \). We will also illustrate the relationships between them and reveal the main properties with the help of examples.

Definition 6. Let \( X \subseteq U \) and \( j \in J \). Then, \( \mathcal{R}_j^+ (X) = \{x \in U : E_j(x) \subseteq X\} \) and \( \mathcal{R}_j^- (X) = \{x \in U : E_j(x) \subseteq X \} \) are called \( \mathcal{R}_j \)-lower approximation and \( \mathcal{R}_j \)-upper approximation, respectively; \( \mathcal{R}_j (X) = \mathcal{R}_j^+ (X) \cap \mathcal{R}_j^- (X) \), POS \( \mathcal{R}_j (X) = \mathcal{R}_j^- (X) \), and NEG \( \mathcal{R}_j (X) = U - \mathcal{R}_j^+ (X) \) are called \( \mathcal{R}_j \)-boundary, \( \mathcal{R}_j \)-positive, and \( \mathcal{R}_j \)-negative regions of \( X \), respectively; \( \mathcal{M}_j (X) = (|\mathcal{R}_j^+ (X) \cap X|)/|\mathcal{R}_j^+ (X) \cap X| \) is called the \( \mathcal{R}_j \)-accuracy measure of \( X \) (if \( 0 < |\mathcal{R}_j^+ (X) \cap X| < \aleph_0 \)), where \( |X| \) denotes the cardinality of a set \( X \) and \( \aleph_0 \) denotes the cardinality of the set of natural numbers.

Theorem 2. \( \mathcal{R}_j \)-approximations have the following properties (\( R \subseteq U^3, \{E, F\} \cup \{E_{j, i}\} \subseteq 2^U \), \( j \in J \)):

(1) \( \mathcal{R}_j^+(\emptyset) = \emptyset \subseteq \mathcal{R}_j^- (\emptyset) \).

(2) \( \mathcal{R}_j^+(U) \subseteq U = \mathcal{R}_j^- (U) \).
Example 2. For j-NS $(U, R, \lambda_j)$ in Example 1, the $R_j$-lower approximations and $R_j$-upper approximations $(j \in \{r, l, i, u\})$, the $R_j$-accuracy measure, the lower approximations $X_-$ and upper approximations $X_+$ in the sense of [6] for $P_{c,p}$, and the lower approximations $X^-$ and upper approximations $X^+$ in the sense of [10] for $N$, are given in Table 2 (where $Y = \{v, w, y\}$ and $Z = \{v, x, y\}$). From Table 2, we can see the approximations in this paper and those in [6] (resp., [10]) are incomparable.

4. Rough Approximations Induced by $E$-Topologies

We will construct rough approximations using $E$-neighborhoods indirectly in this section. Let $(U, R, \lambda_j)$ be $j$-NS $j \in J$. We first employ $E_j$-neighborhoods to generate a topology $J_{R,j}$ (called an $E_j$-topology) and then call the interior $J_{R,j}^-$ ($X$) and the closure $J_{R,j}^+$ ($X$) of a subset $X \subseteq U$ the $J_{R,j}$-lower approximation and $J_{R,j}$-upper approximation of $X$, respectively. These kinds of approximations are compared with those in Section 3.

Theorem 3. For $j \in J$,

1. $J_{R,j} = \{A \in 2^U : E_j(x) \subseteq A \ (\forall x \in A)\}$ is a topology on $U$ satisfying $U - A \in J_{R,j}$ whenever $A \in J_{R,j}$ ($j \in J$).
2. Both $J_{R,N,j} = \{A \in 2^U : N_j(x) \subseteq A \ (\forall x \in A)\}$ and $J_{R,P,j} = \{A \in 2^U : P_j(x) \subseteq A \ (\forall x \in A)\}$ are topologies on $U$ ($j \in J$).
3. $J_{R,N} \subseteq J_{R,j} \subseteq J_{R,N,j} \cup J_{R,j} \subseteq J_{R,j}^+$.
4. $J_{R,(0)} \subseteq J_{R,(0)} \subseteq J_{R,(0)} \cup J_{R,(0)} \subseteq J_{R,(0)}$.
5. If $R$ is reflexive, then $J_{R,R,j} \subseteq J_{R,j}$ ($j \in J$).
6. If $R$ is serial, then $J_{R,R,j} \subseteq J_{R,j}$ ($j \in \{r, l, i, u\}$).
7. If $R$ is an equivalence relation, then $J_{R,j}$ is constant for all $j \in J$.

Proof. We only prove (1). Obviously, $J_{R,j}$ is a topology on $U$. For each $A \in J_{R,j}$ and each $x \in U - A$, we need to prove $E_j(x) \subseteq U - A$. Without loss of generality, we assume $E_j(x) \neq \emptyset$. Suppose $x \in E_j(x) \cap A$. Then, $x \in E_j(x)$ (by Theorem 1 (3)) and $E_j(x) \subseteq J_{R,j}$, and thus, $x \in A$. This is a contradiction. Therefore, $E_j(x) \cap A = \emptyset$, i.e., $E_j(x) \subseteq U - A$.

Definition 7. Let $X \subseteq U$ and $j \in J$. Then, $J_{R,j}^-(X) = X^-$ (the interior of $X$ in $(U, J_{R,j})$) and $J_{R,j}^+(X) = X^+$ (the closure of $X$ in $(U, J_{R,j})$) are called the $J_{R,j}$-lower approximation and $J_{R,j}$-upper approximation, respectively; $J_{R,j}^-(X) - J_{R,j}^+(X)$, $J_{R,j}^-(X)$, and $U - J_{R,j}^+(X)$ are called $J_{R,j}$-boundary, $J_{R,j}$-positive, and $J_{R,j}$-negative regions of $X$, respectively; $\mathcal{M}_{J_{R,j}}(X) = (|J_{R,j}^-(X)|/|J_{R,j}^+(X)|)$ is called the $J_{R,j}$-accuracy measure of $X$ (if $0 < |J_{R,j}^-(X)| < \aleph_0$).

The relation between $\mathcal{M}_{J_{R,j}}(X)$ and $\mathcal{M}_{J_{R,j}}(X) (j \in J)$ is given by the following.

Theorem 4. $\mathcal{M}_{J_{R,j}}(X) \leq \mathcal{M}_{J_{R,j}}(X)$ for each $j \in J$ (if $0 < |J_{R,j}^-(X)| < \aleph_0$).

Proof

(i) Step 1: for each $z \in J_{R,j}^-(X)$, we have $z \in J_{R,j}^-(X)$ (because $J_{R,j}^-(X) = X^-$ and $E_j(z) \subseteq X^+ \subseteq X$), and thus, $J_{R,j}^-(X) \subseteq J_{R,j}^-(X) \cap X$, which implies $|J_{R,j}^-(X)| \leq |J_{R,j}^-(X) \cap X|$.

(ii) Step 2: let $z \in J_{R,j}^+(X) \cup X$. If $z \in X$, then $z \in J_{R,j}^+(X)$ (because $J_{R,j}^+(X) = X^+$) such that $v \in E_j(z)$ and $v \in X$. Consequently, for any $V \in J_{R,j}$ containing $z$, we have $v \in V$. Therefore, $V \cap X \neq \emptyset$, and thus, $z \in J_{R,j}^+(X)$ (because $J_{R,j}^+(X) = X^+$). It follows that $J_{R,j}^+(X) \subseteq J_{R,j}^+(X)$, and thus,

$$\frac{1}{|J_{R,j}^+(X)|} \leq \frac{1}{|J_{R,j}^+(X) \cap X|}$$

From (1) and (2), we can see that $\mathcal{M}_{J_{R,j}}(X) = (|J_{R,j}^-(X)|/|J_{R,j}^+(X)|) \leq (|J_{R,j}^-(X) \cup X|/|J_{R,j}^+(X) \cup X|) = \mathcal{M}_{J_{R,j}}(X)$.

Example 3

(i) Now, we exemplify an application of rough approximations introduced in this paper. Let $X = \{x_1, x_2, \ldots, x_{50}\}$ be a group of people who have just reached Xian Yang Airport (but are not allowed to outbound station) from two countries (by two planes involving 200 people denoted by a set
of each person contacting

Assume that

To ensure the safety of this meeting,

U, R, . . .

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In the upcoming works, we will study new types of

Motivated by topology, this article has initiated two new rough approximations by introducing a new class of neighborhood systems (called Ej-neighborhoods) using j-neighborhoods. We have probed the main features and formulated the concepts of Ej-lower and Ej-upper approximations and Ej-accuracy measure which are related to the contact of one induced from different types of j and compared them. We complete this work by studying these concepts from a topological view and comparing them. In all comparisons, we obtain higher accurate approximations in the case of j = i.

In the upcoming works, we will study new types of neighborhoods in rough set theory and use them to define a

\( j \in \{r, l, i, u\} \) and the \( M_{j, l} \)-accuracy measure are given in Table 3.

5. Concluding Remarks

Motivated by topology, this article has initiated two new rough approximations by introducing a new class of neighborhood systems (called Ej-neighborhoods) using j-neighborhoods. We have probed the main features and formulated the concepts of Ej-lower and Ej-upper approximations and Ej-accuracy measure which are related to the contact of one induced from different types of j and compared them. We complete this work by studying these concepts from a topological view and comparing them. In all comparisons, we obtain higher accurate approximations in the case of j = i.

In the upcoming works, we will study new types of neighborhoods in rough set theory and use them to define a
topological structure. Also, we will investigate the $E_j$-neighborhoods and approximations on the fuzzy rough set motivated from fuzzy control problems and fractional-order nonlinear systems.

**Data Availability**

The data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this paper.

**Acknowledgments**

This work was supported by the National Natural Science Foundation of China under Grant nos. 11771263 and 61967001.

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