The renormalization of the effective gauge theory with spontaneous symmetry breaking: the $SU(2) \times U(1)$ case

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We formulate the electroweak chiral Lagrangian in its mass eigenstates, and study the its one-loop renormalization and provide its renormalization group equations to the same order, so as to complete it as the low energy effective theory of the standard model below a few TeV. In order to make our computation consistent, we have provided a modified power counting rule to estimate the contributions of higher loop and higher operators. As one of the application of its renormalization group equations, we analyze the solution to the effects of the Higgs scalar. We find that similar to the $SU(2)$ case, that the triple anomalous couplings are sensitive to the quartic couplings (here $\alpha_5$). While the quadratic anomalous couplings are not sensitive, due to the large leading contributions and the accidental cancellation. The differences in the triple anomalous couplings between the direct method and renormalization group equation method are well within the detection power of the LHC and LC, if the Higgs scalar is not too heavy (say, 300 or 400 GeV). We also suggest a new mechanism to generate the negative $S$ parameter through the radiative corrections of the anomalous couplings. Comparison of the renormalization group equation method and direct methods is provided in the full theory, the standard model, to reveal the basic differences of them. The problem of the unitarity violation is also addressed for our assumption in the modified power counting rule.

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I. INTRODUCTION

The effective field theory (EFT) method \cite{1-3} is a universal and powerful theoretical framework for us to understand the laws and rules of the nature. According to its spirit \cite{2, 3}, the practical ingredients of this method should at least include the following two basic constituents: 1) the most important dynamic degrees of freedom (DOFs) and 2) the most important interactions among these DOFs. For some cases where quantum corrections are considerable, we need the third ingredient, 3) the renormalization group equations (RGEs), which is indispensable to efficiently sum up large logarithms, like in B physics. 4) The only remnants of the higher dynamics are reflected by the initial conditions of the anomalous couplings (ACs) at the matching scale, which can affect low energy phenomenologies when the RGEs develop from the ultraviolet cutoff (the matching scale) of the effective theory down to the infrared cutoff (below which some of the low energy DOFs of the effective theory will decouple and the effective description will break down, and a new effective theory should be introduced).

There are two important effective theories, the QCD chiral Lagrangian \cite{4} and the electroweak chiral Lagrangian (EWCL) \cite{5, 6}. These two theories are suitable to describe hadronic dynamics from 100 MeV to 1 GeV ($4\pi v_0$, with $v_0 = 92$ MeV) and electroweak dynamics from 90 GeV to 2.5 TeV ($4\pi v_0$, with $v_0 = 246$ GeV), respectively. The RGEs of the Chiral perturbation theory (ChPT) have been studied up to $O(p^6)$ order \cite{7}. While the RGEs of the EWCL is still lack.

The RGEs of the EWCL are necessary due to the fact that most of the ACs (the anomalous in ACs means the deviation from the requirement of the renormalizability, since after integrating out the heavy DOF, the divergences generated by low energy DOF can not be canceled out so that the anomalous operators (AOs) must be introduced) are induced by loop processes in the standard model (SM) and are small, so that if the ACs from new physics are not small, the radiative corrections of low energy DOF might be relatively important. Since we do not know the actual mechanism of the electroweak symmetry breaking, and the ACs can be considerable large, as in the not too heavy Higgs case we consider below. The present experimental constraint on the ACs \cite{8} can be summarized and estimated as

\[
(a_1 a_8 \beta) \sim O(0.01), \ (a_2 a_3 a_9) \sim O(0.1), \ (a_4 a_5 a_6 a_7 a_a) \sim O(1),
\]

which indicates that the radiative corrections from the permitted ACs could be large. Another pure theoretical reason for us to consider the RGEs of the EWCL is that up to the $O(p^4)$, the 11 extra operators belong to the marginal operators in the Wilsonian
renormalization method [9, 10], we just want to know the behavior of these ACs under the drive of quantum fluctuations. More practically, considering the fact that the new machines, the LHC and future linear colliders (TESLA or JLC), can increase their measurement precision up to two orders than the LEP, it is urgent to upgrade the theoretical prediction precisions of the electroweak theory to the same order. Furthermore, once these machines will start to run, it is quite necessary for the experimentalists to conduct their analysis in a universal and model-independent way, then the RGEs can meet this end, and can provide a powerful tool to comparatively study most of the new physics candidates. So, deriving the RGEs of EWCL and taking into account the radiative corrections of low energy DOF become important, at least should be as important as to consider the contributions of operators at $O(p^6)$ order.

The study of the EWCL is started from quite early time, and can be traced back to the references [5], where the author began to study the independent operator set of the EWCL up to $O(p^4)$, and in 1993, the authors of [6], A. Appliquist and G. H. Wu, established the relations of the ACs in the EWCL with the usual precision test quantities [11]. The ACs in various models have been derived in [12]. The authors of the reference [13] also extended the EWCL to include one light Higgs, and recently, the reference [14] explored the light Higgs case to the precision test constraints. In 1996, the authors of [15], H. J. He, Y. P. Kuang, and C. P. Yuan, by using the equivalence theorem [16] and the modified power counting rule, qualitatively and quasi-quantitative estimated the detection power of several possible new machines to these effective operators. The authors of [17] have extended the bosonic EWCL to the fermionic part. Recently, the authors [18] have explored to formulate EWCL in the partial mass eigenstates. Several groups even have studied the complete set of operators of $O(p^6)$, and their possible effects in the LHC and LC are also analyzed [19].

People expect that these operators up to $O(p^6)$ might be important in LHC and LC. Considering the possible situation for the measurement at the LHC and LC, the ACs can be determined at 200 GeV, 500 GeV, and 1 TeV, for instance. The values of these couplings might be definitely measured different considering the precision that these machines can reach, just like that $\alpha_{em} = 1/137$ at $m_e$ and $\alpha_{em} = 1/128$ at $m_Z$. Then it is urgent for us to understand the underlying reason for these differences.

There are two obstacles for us to study the renormalization of the EWCL and derive its RGEs. 1) The non-linear effective gauge theory is a non-renormalizable theory (Non-renormalizability here means that there will be the tower of infinite divergence structure and quartic divergences even at one-loop level), how to consider its renormalization order by order? 2) There are so many interaction vertices, how to efficiently evaluate the
loop contributions of those large amount of Feynman diagrams? We have shown the basic conceptions in our paper [20] as to overcome the first obstacle in the framework of effective theory, and the second one by using the background field method [21] (BFM). In our previous work [22], by using the related conceptions and tools, we have found that when the quartic ACs are large (In the Higgs model, corresponding to the case that the Higgs is not too heavy), the contributions of low energy dynamic DOF can be quite significant, and the predictions of the RGE method and the direct method (DM) (where the mass squared terms are dropped due to the fact that in the decoupling limit those terms can be safely neglected) are quite different. So it is natural and logic for us to examine these properties in the EWCL case.

In this paper, we will study the one-loop renormalization of the EWCL and derive its RGEs. In order to systematically and consistently control the contributions of the higher loop and higher operators, we provide a modified power counting rule. Considering the fact that the complete RGEs are very complicated (we will provide in our next paper [23]), here we only provide the simplified but workable ones. But we would like to emphasize that our numerical analysis is based on the complete RGEs and is reliable. We will use the conceptions developed in our previous works. The basic computational skills include the BFM [21], the Stueckelberg transformation [24], the Schwinger proper time and heat kernel method [25], and the covariant short distance expansion technology [26]. With these conceptions and methods, we will extract the desired one loop RGEs of EWCL, so, theoretically, as to complete it as a realistic theoretical framework to describe the SM below a few TeV. We will also examine the Higgs’ contribution to the low energy precision test parameters. We find that similar to the $SU(2)$ case, the triple gauge vertices are quite sensitive to the large quartic ACs. While the quadratic ones are not sensitive for the EWCL of Higgs model, and the underlying reason is due to the large leading contributions and the accidental cancellation between $\alpha_2$ and $\alpha_3$. For those cases where there is no such cancellation, the quadratic vertices might also be sensitive to the large quartic ACs couplings. We also compare the predictions given by the DM. The results given by these two methods are quite different in the case of EWCL of Higgs model, and the differences are well within the detection power of the LHC and LC. The basic reason for the differences are revealed in the full theory, the SM. We also address the problem of the unitarity violation related with our modified power counting rule.

For the conventions of the computation below, we would like to emphasize that, in order to avoid inconsistency, all formula are provided in the Euclidean space, including the partition functional from the beginning.

This paper is organized as follows. In Sec. II, the bosons sector of the standard model
is introduced. In Sec. III, the EWCL $\mathcal{L}_{\text{ew}}$ up to $O(p^4)$ are introduced, we introduce a new basis of the mass eigenstates for the effective operators. In Sec. IV, the renormalization of the SM in BFM is conducted to provide a reference for the effective ones. In Sec. V, we use the BFM to extract the quadratic terms of quantum fields, to evaluate the logarithm and trace, and to construct the RGEs. In Sec. VI, the ACs of the EWCL of the Higgs model is analyzed and the Higgs’ contribution to electroweak precision test parameters are studied. We end this paper with several discussions and conclusions. The appendix is devoted to provide the related matrices of the quadratic terms of the standard form.

II. THE STANDARD MODEL

The Lagrangian of the standard $SU(2) \times U(1)$ gauge theory can be formulated as

$$\mathcal{L} = -H_1 - H_2 - (D\phi)^\dagger \cdot (D\phi) + \mu^2 \phi^\dagger \phi - \frac{\lambda}{4} (\phi^\dagger \phi)^2 + \mathcal{L}_\psi + \mathcal{L}_{\text{Yukawa}},$$

$$H_1 = \frac{1}{4} W_{\mu\nu} W^{\mu\nu},$$

$$H_2 = \frac{1}{4} B_{\mu\nu} B^{\mu\nu},$$

where the $W$ and $B$ are the vector bosons of $SU_L(2)$ and $U_Y(1)$ gauge groups, respectively. The $\phi$ is the Higgs field, a weak doublet scalar. The $\mu^2$ and $\lambda$ are two variables of the Higgs potential, which determine the spontaneous breaking of symmetry. The $\mathcal{L}_\psi$ and $\mathcal{L}_{\text{Yukawa}}$ are the standard gauge interactions of Fermions and the Yukawa coupling between the Higgs field and Fermions, respectively. For the sake of simplicity, the interactions of Fermions are neglected below. And the relevant definitions are listed below

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + gf^{abc} W^b_\mu W^c_\nu,$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu,$$

$$D_\mu \phi = \partial_\mu \phi - ig W^a_\mu T^a - iy_\phi g' B_\mu T^3 \phi,$$

$$\phi^\dagger = (\phi_1^\dagger, \phi_2^\dagger),$$

where $T^a$ are the generators of the Lie algebra of $SU(2)$ gauge group, and $a = 1, 2, 3$. The $Y$ charge of the field $\phi$ and $y_\phi = -1$. The $g$ and $g'$ are the couplings of the corresponding gauge interactions, respectively.

The spontaneous symmetry breaking is induced by the positive mass square $\mu^2$ in the Higgs potential. The vacuum expectation value of Higgs field is solved from the Higgs...
potential as $|\langle \phi \rangle| = v/\sqrt{2}$. And by eating the corresponding Goldstone bosons, the vector bosons $W$ and $Z$ obtain their masses, while the vector bosons $A$ of the unbroken $U(1)_{em}$ gauge symmetry are still massless. The Lagrangian given in Eq. (2) with the Higgs mechanism can be reformulated in its nonlinear form by changing the variable $\phi$

$$\phi = \frac{1}{\sqrt{2}}(v + h)U, \quad U = \exp \left( \frac{2i\xi^a T^a}{v} \right), \quad v = 2\sqrt{\frac{u^2}{\lambda}},$$

(9)

where the $h$ is the Higgs scalar, $v$ is the vacuum expectation value. The $U$ is a phase factor, and the $\xi^a$, $a = 1, 2, 3$ are the corresponding Goldstone bosons as prescribed by the Goldstone theorem.

As we know, the change of variables in Eq. (9) induces a determinant factor in the functional integral

$$Z = \int D\mu^a Dw D\xi^b \exp \left( -S[W, h, \xi] \right) \det \left( 1 + \frac{1}{v}h \right) \delta(x - y).$$

(10)

and correspondingly modifies the Lagrangian density to

$$L = -H_1 - H_2 - \frac{(v + h)^2}{4} tr[DU^\dagger \cdot DU]$$

$$-\frac{1}{2} \partial h \cdot \partial h + \frac{\mu^2}{2} (v + h)^2 - \frac{\lambda}{16} (v + h)^4 - \delta^4(0) ln \left( 1 + \frac{1}{v}h \right).$$

(11)

As pointed out by several references [27], this determinant containing quartic divergences is indispensable and crucial to cancel exactly the quartic divergences brought into by the longitudinal part of vector bosons, and is important in verifying the renormalizability of the Higgs model in the U-gauge and the equivalence of U-gauge to other gauges.

III. THE EFFECTIVE LAGRANGIAN UP TO $O(P^4)$ (THE RELEVANT AND MARGINAL OPERATORS)

The most general effective Lagrangian $L_{EW}$, which respects the Lorenz invariance, the $SU(2) \times U(1)$ gauge symmetry, and the discrete symmetries (the charge, parity, and the combined CP symmetries), can be formulated as

$$L_{EW} = L_{EW}^{\mu^2} + L_{EW}^{\nu^4} + \cdots + L_{qd}$$

(12)

$$L_{EW}^{\mu^2} = L_B,$$

(13)

$$L_{EW}^{\nu^4} = \beta L_0 + \sum_{i=1}^a \alpha_i L_i$$

(14)
where

\[ \mathcal{L}_B = -H_1 - H_2 + \mathcal{L}_{WZ}, \]
\[ \mathcal{L}_{WZ} = \frac{v^2}{4} tr(V \cdot V) = -\frac{v^2}{8} (G^2 Z \cdot Z + 2g^2 W^+ \cdot W^-), \]

where \( G \) is defined as \( G = \sqrt{g^2 + g'^2} \). After using the relations of Lie algebra and the classic equation of motion to eliminate the redundant operators, the complete Lagrangian \( \mathcal{L}_{EW}^{p4} \) includes the following independent operators [5, 6]:

\[ \mathcal{L}_0 = \frac{v^2}{4} [tr(\mathcal{T}V_\mu)]^2, \]
\[ \mathcal{L}_1 = i\frac{gg'}{2} B_{\mu\nu} tr(\mathcal{T}W^{\mu\nu}), \]
\[ \mathcal{L}_2 = i\frac{g'}{2} B_{\mu\nu} tr(\mathcal{T}[V^\mu, V^\nu]), \]
\[ \mathcal{L}_3 = ig tr(W_{\mu\nu}[V^\mu, V^\nu]), \]
\[ \mathcal{L}_4 = [tr(V_\mu V_\nu)]^2, \]
\[ \mathcal{L}_5 = [tr(V_\mu V^\mu)]^2, \]
\[ \mathcal{L}_6 = tr(V_\mu V_\nu) tr(\mathcal{T}V^\mu) tr(\mathcal{T}V^\nu), \]
\[ \mathcal{L}_7 = tr(V^\mu V_\mu) [tr(\mathcal{T}V^\nu)]^2, \]
\[ \mathcal{L}_8 = \frac{g^2}{4} [tr(\mathcal{T}W_{\mu\nu})]^2, \]
\[ \mathcal{L}_9 = i\frac{g}{2} tr(\mathcal{T}W_{\mu\nu}) tr(\mathcal{T}[V^\mu, V^\nu]), \]
\[ \mathcal{L}_a = [tr(\mathcal{T}V^\mu) tr(\mathcal{T}V_\nu)]^2. \]

where the auxiliary variable \( V_\mu \) and \( \mathcal{T} \) is defined as

\[ V_\mu = U^\dagger (\partial_\mu - iW^\mu T^a) U + iB_\mu T^3. \]
\[ \mathcal{T} = 2U^\dagger T^3 U = U^\dagger \tau^3 U, \]

with the \( \tau^3 \) is the third Pauli matrices. The operators \( H_1, H_2, \) and \( \mathcal{L}_i, i = 1, \cdots, a \) contribute the kinetic, trilinear, and quartic interactions. While operators \( \mathcal{L}_{WZ} \) and \( \mathcal{L}_0 \) contribute to the mass terms.

The effective Lagrangian \( \mathcal{L}_{EW} \) is invariant under the following local chiral transformation

\[ U \rightarrow g_L U g_R^\dagger, \]
\[ W_\mu \rightarrow g_L W_\mu g_R^\dagger + ig_L \partial_\mu g_R^\dagger, \]

\[ \mathcal{L}_E = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_6 + \mathcal{L}_7 + \mathcal{L}_8 + \mathcal{L}_9 + \mathcal{L}_a. \]
\[ W_{\mu\nu} \rightarrow g_L W_{\mu\nu} g_L^\dagger, \]
\[ B_\mu \rightarrow B_\mu + ig_R \partial_\mu g_R^\dagger, \]
\[ B_{\mu\nu} \rightarrow B_{\mu\nu}, \]  

(20)

where the gauge transformation factor \( g_L \) and \( g_R \) are defined as
\[ g_L = \exp \left\{ i \alpha L T^a \right\}, \quad g_R = \exp \left\{ i \beta R T^3 \right\}. \]

(21)

The ACs \( \alpha_i \) form the effective parameter space, when the effective scale \( \mu \) runs from its ultraviolet cutoff down to its infrared cutoff, each theory will depict a characteristic curve in this space. The initial conditions of the ACs at the ultraviolet cutoff (the matching scale of the effective theory and the full theory) reflect the remnant of high energy dynamics, while the effects of the heavy DOF to the low energy dynamics can be solved out from the corresponding RGEs. By measuring the ACs at different energy scales, we can extract important information of the possible underlying theory and induce the actual mechanism of the spontaneous symmetry breaking.

The Higgs model (a full and renormalizable theory) given in Eq. (11) can be effectively described by the effective Lagrangian \( \mathcal{L}_{EW} \) if the Higgs field is heavy and integrated out. The equation of motion of the Higgs field \( h \) can express it into the low energy DOFs, which reads
\[ h = -\frac{v}{2m_0} tr(DU^\dagger \cdot DU) + \cdots, \]
\[ = \frac{v^2}{2m_0^2} tr(V \cdot V) + \cdots, \]
\[ m_0^2 = \frac{1}{2} \lambda v^2, \]

(22)

(23)

where \( m_0 \) is the mass of Higgs boson. The omitted terms contain at least four covariant partials and belong to operators \( O(p^6) \).

At the matching scale, after matching the full theory and the effective theory by integrating out the heavy Higgs scalar, we get the following initial condition for the ACs
\[ \beta(m_0) = 0, \quad \alpha_5(m_0) = \frac{v^2}{8m_0^2} = \frac{1}{4\lambda}, \quad \alpha_i(m_0) = 0, \quad i \neq 5, \]

(24)

Since the couplings of these AOs are dimensionless, naively from the power law we expect that the complete Lagrangian with both relevant and marginal operators is renormalizable. But, as well known, the longitudinal part of the propagators of vector bosons will invalidate this power counting law. Here we emphasize that the terms in the \( \mathcal{L}_{qcd} \)
make it possible in practical computation to simply and consistently discard those quartic divergences, and make it possible to renormalize the effective Lagrangian order by order.

The above set of operator is formulated in the interaction eigenstates, and below we introduce an equivalent basis (in U-gauge) represented in its mass eigenstates \( \mathcal{A}, \mathcal{W}, \mathcal{Z} \) (these particles are the ones detected in experimental facilities), which read

\[
L_{EW} = -\sum_{i=1}^{4} C_i O_i + \sum_{i=5}^{c} C_i O_i - O_{MW} - \rho O_{MZ},
\]

(25)

\[
\begin{align*}
O_1 &= \frac{1}{4} A_{\mu\nu} A^{\mu\nu}, \\
O_2 &= \frac{1}{2} A_{\mu\nu} Z^{\mu\nu}, \\
O_3 &= \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu}, \\
O_4 &= \frac{1}{2} W^{+\mu} W^{-\mu}, \\
O_5 &= \frac{i}{2} A_{\mu\nu} W^{+\mu} W^{-\nu}, \\
O_6 &= \frac{i}{2} Z_{\mu\nu} W^{+\mu} W^{-\nu}, \\
O_7 &= \frac{i}{2} \left( W^{+\mu} Z^{\mu\nu} W^{-\nu} - W^{-\mu} Z^{\mu\nu} W^{+\nu} \right), \\
O_8 &= Z \cdot Z W^+ \cdot W^- , \\
O_9 &= Z \cdot W^+ Z \cdot W^- , \\
O_a &= Z \cdot Z Z \cdot Z , \\
O_b &= W^+ \cdot W^- W^+ \cdot W^- , \\
O_c &= W^+ \cdot W^+ W^- \cdot W^- , \\
O_{MW} &= \frac{v^2}{4} W^+ \cdot W^- , \\
O_{MZ} &= \frac{v^2}{8} Z \cdot Z .
\end{align*}
\]

(26)

where the first 12 contribute to the kinetic, triple, and quartic interactions, and the last two contribute to the masses of vector bosons. This set of operators are all expressed in the mass eigenstates, while the operators given in [18] are only partially expressed in the mass eigenstates, where the authors have only chosen \( Z \) and \( W^\pm \), and used \( B \) (which is not mass eigenstates). That’s why we call their basis as operators in partial mass eigenstates. And the relevant definitions are given as

\[
A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu ,
\]
\[ Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu, \]
\[ W^\pm_{\mu\nu} = d_\mu W^\pm_\nu - d_\nu W^\pm_\mu, \]
\[ d_\mu W^\pm_\nu = \partial_\mu W^\pm_\nu \mp ieA_\mu W^\pm_\nu. \quad (27) \]

The fundamental relations between the mass eigenstates and the weak interaction eigenstates are determined as
\[
A = \sin \theta_W W^3 + \cos \theta_W B, \quad Z = -\cos \theta_W W^3 + \sin \theta_W B,
\]
\[
W^+ = \frac{1}{\sqrt{2}}(W^1 - iW^2), \quad W^- = \frac{1}{\sqrt{2}}(W^1 + iW^2),
\]
\[
e = \frac{g'g}{G}, \quad \tan \theta_W = \frac{g'}{g}, \quad (28)
\]

where the \( \theta_W \) is also called the Weinberg angle. There are also some important relations, which we list below
\[
\tilde{W}^+_{\mu\nu} = W^+_{\mu\nu} - ig^2 G F^+_{\mu\nu}, \quad (29)
\]
\[
\tilde{W}^-_{\mu\nu} = W^-_{\mu\nu} - ig^2 G F^-_{\mu\nu}, \quad (30)
\]
\[
\tilde{W}^3_{\mu\nu} = g' G A_{\mu\nu} - \frac{g'}{G} Z_{\mu\nu} - ig F^Z_{\mu\nu}, \quad (31)
\]
\[
B_{\mu\nu} = \frac{g}{G} A_{\mu\nu} + \frac{g'}{G} Z_{\mu\nu}, \quad (32)
\]

where the definitions of \( F^+_{\mu\nu}, F^-_{\mu\nu}, \) and \( F^Z_{\mu\nu} \), read
\[
F^+_{\mu\nu} = W^+_{\mu\nu} Z_\nu - W^+_{\nu\nu} Z_\mu, \quad (33)
\]
\[
F^-_{\mu\nu} = Z_\mu W^-_{\nu\nu} - Z_\nu W^-_{\mu\nu}, \quad (34)
\]
\[
F^Z_{\mu\nu} = W^+_{\mu\nu} W^-_{\nu\nu} - W^+_{\nu\nu} W^-_{\mu\nu}. \quad (35)
\]

These 14 operators are linearly independent with each other, and are equivalent with those 14 bosonic terms in the effective Lagrangian given in Eq. (12).

Transformation relations of the masses operators of these two independent sets are determined as
\[
\mathcal{L}_{WZ} = O_{MW} + O_{MZ}, \quad \mathcal{L}_0 = -2O_{MZ}. \quad (36)
\]

And the relations of rest operators read
\[ H_1 = -\frac{1}{2G^2} \left( -2g^2O_1 + 2gg'O_2 - 2g^2O_3 - 2G^2O_4 + 4Gg'O_5 \\
-4g^2GO_6 + 4g^2GO_7 - 2g^4O_8 + 2g^4O_9 - g^2G^2O_b + g^2G^2O_c \right), \]
\[ H_2 = -\frac{1}{G^2} \left( -g^2O_1 - gg'O_2 - g^2O_3 \right), \]
\[ L_1 = -\frac{1}{G^2} \left( -2g^2g'O_1 + g^3g'O_2 - gg^3O_2 \\
+2g^2g^3O_3 + 2g^3Gg'O_5 + 2g^2Gg^2O_6 \right), \]
\[ L_2 = \frac{1}{G} \left( -2g^3g'O_5 - 2g^2g^2O_6 \right), \]
\[ L_3 = -2g^3g'O_5 + 2g^4O_6 - 2g^2GO_7 + 2g^4O_8 - 2g^4O_9 + g^4O_b - g^4O_c, \]
\[ L_4 = \frac{1}{4} \left( 4g^2G^2O_9 + G^4O_a + 2g^4O_b + 2g^4O_c \right), \]
\[ L_5 = \frac{1}{4} \left( 4g^2G^2O_8 + G^4O_a + 4g^4O_b \right), \]
\[ L_6 = \frac{1}{2} \left( 2g^2G^2O_9 + G^4O_a \right), \]
\[ L_7 = \frac{1}{2} \left( 2g^2G^2O_8 + G^4O_a \right), \]
\[ L_8 = \frac{g^2}{2G^2} \left( 2g^2O_1 - 2gg'(O_2 + 2GO_5) \\
+g^2(2O_3 + G(4O_6 + GO_b - GO_c)) \right), \]
\[ L_9 = \frac{g^3}{G} \left( -gO_5 + 2gO_6 + gGO_b - gGO_c \right), \]
\[ L_a = \frac{G^4}{2}O_a. \]  

(37)

The reverse relations among these operators, which are quite helpful for us to extract the standard structures of the EWCL given in Eq. (17), read

\[ O_{M_Z} = -\frac{1}{2}L_0, \]
\[ O_{M_W} = L_{WZ} + \frac{1}{2}L_0, \]
\[ O_1 = \frac{1}{g^2G^2} \left( g^4H_2 + g^2(L_1 + L_2) \\
+g^2(-L_4 + L_5 + L_6 - L_7 + L_8 - L_9) \right), \]
\[ O_2 = \frac{1}{gG^2g'} \left[ g^2(2g^2H_2 - L_1 + L_2) \\
+g^2(L_1 - L_2 + 2L_4 - L_5 - L_6 + L_7 - L_8 + L_9) \right], \]
One of the advantages of the set of operators given in Eq. (38) is that it is helpful to discuss different symmetry breaking patterns. By setting all terms with $Z$ vanished we get the pattern SU(2) → U(1) [28]; by setting all terms with $A$ vanished we get the pattern SU(2) breaks to a global U(1) if the mass of $Z$ different from that of $W^\pm$. In our paper [22], we have studied the pattern that a local SU(2) breaks to a global SU(2). For the different symmetry breaking patterns, the corresponding RGEs can be obtained by taking the limits to eliminate some of the ACs from the effective Lagrangian.

The $\rho$ is related to $\beta$ as

$$\beta = \frac{\rho - 1}{2}. \quad (39)$$

The relations of ECs between $\alpha_i$ and $C_i$ are determined as

$$\alpha_1 = -\frac{C_1}{G^2} + \frac{g C_2}{G^2 g'} - \frac{g' C_2}{g G^2} + \frac{C_3}{G^2}, \quad (40)$$

$$\alpha_2 = \frac{C_1}{G^2} - \frac{g C_2}{G^2 g'} + \frac{g' C_2}{g G^2} - \frac{C_3}{G^2} - \frac{C_5}{2 g G g'} - \frac{C_6}{2 g^2 G}, \quad (41)$$

$$\alpha_3 = \frac{C_4}{G^2} - \frac{C_7}{2 g^2 G}, \quad (42)$$

$$\alpha_4 = \frac{g^2 C_1}{g^2 G^2} - \frac{2 g' C_2}{g G^2} + \frac{C_3}{G^2} - \frac{g' C_5}{g^3 G} + \frac{C_6}{g^2 G} + \frac{2 C_c}{g^4}, \quad (43)$$
The inverse relations between the ACs $\alpha_i$ and the ECs $C_i$ read

\[ \alpha_5 = -\frac{g^2 C_1}{g^2 G^2} + \frac{2g' C_2}{g^2 G} - \frac{C_3}{G^2} + \frac{g' C_5}{g^2 G} - \frac{C_6}{g^2 G} + \frac{C_b}{g^4} - \frac{C_e}{g^4}, \]  

(44)

\[ \alpha_6 = -\frac{g^2 C_1}{g^2 G^2} + \frac{2g' C_2}{g^2 G} - \frac{C_3}{G^2} + \frac{g^2 C_4}{G^4} + \frac{g' C_5}{g^2 G} - \frac{C_6}{g^2 G} - \frac{C_7}{G^3} + \frac{C_9}{g^2 G^2} - \frac{C_8}{g^4} + \frac{2C_c}{g^4}, \]  

(45)

\[ \alpha_7 = -\frac{g^2 C_1}{g^2 G^2} + \frac{2g' C_2}{g^2 G} - \frac{C_3}{G^2} + \frac{g^2 C_4}{G^4} - \frac{g' C_5}{g^2 G} + \frac{C_6}{g^2 G} + \frac{C_7}{G^3} + \frac{C_8}{g^2 G^2} - \frac{C_b}{g^4} + \frac{C_c}{g^4}, \]  

(46)

\[ \alpha_8 = -\frac{g^2 C_1}{g^2 G^2} + \frac{2g' C_2}{g^2 G} - \frac{C_3}{G^2} + \frac{C_4}{G^2} + \frac{g' C_5}{g^2 G} - \frac{C_6}{2g^2 G} + \frac{C_7}{2g^2 G}, \]  

(47)

\[ \alpha_9 = \frac{g^2 C_1}{g^2 G^2} - \frac{2g' C_2}{g^2 G} + \frac{C_3}{G^2} - \frac{C_4}{G^2} - \frac{g' C_5}{2g^2 G} + \frac{C_6}{2g^2 G} + \frac{C_7}{2g^2 G}, \]  

(48)

\[ \alpha_a = -\frac{C_8}{g^2 G^2} - \frac{C_9}{g^2 G^2} + \frac{2C_a}{G^4} + \frac{C_b}{2g^4} + \frac{C_c}{2g^4}. \]  

(49)

The inverse relations between the ACs $\alpha_i$ and the ECs $C_i$ read

\[ C_1 = 1 - \frac{g^2 g'^2}{G^2} (2\alpha_1 + \alpha_8), \]  

(50)

\[ C_2 = \frac{g^2 g'}{G^2} (\alpha_1 g^2 - \alpha_1 g'^2 + \alpha_8 g^2), \]  

(51)

\[ C_3 = 1 - \frac{g^2}{G^2} (\alpha_8 g^2 - 2\alpha_1 g^2), \]  

(52)

\[ C_4 = 1, \]  

(53)

\[ C_5 = \frac{2gg'}{G} \left( 1 - (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_8 + \alpha_9)g^2 \right), \]  

(54)

\[ C_6 = -\frac{2g^2}{G} \left( 1 - (\alpha_3 + \alpha_8 + \alpha_9)g^2 + (\alpha_1 + \alpha_2)g^2 \right), \]  

(55)

\[ C_7 = \frac{2g^2}{G} \left( 1 - \alpha_3 G^2 \right), \]  

(56)

\[ C_8 = -\frac{g^4}{G^2} + 2\alpha_3 g^4 + (\alpha_5 + \alpha_7)g^2 G^2, \]  

(57)

\[ C_9 = \frac{g^4}{G^2} - 2\alpha_3 g^4 + (\alpha_4 + \alpha_6)g^2 G^2, \]  

(58)

\[ C_a = \frac{G^4}{4} (\alpha_4 + \alpha_5 + 2\alpha_6 + 2\alpha_7 + 2\alpha_a), \]  

(59)

\[ C_b = -\frac{g^2}{2} + \frac{g^4}{2} (2\alpha_3 + \alpha_4 + 2\alpha_5 + \alpha_8 + 2\alpha_9), \]  

(60)

\[ C_c = \frac{g^2}{2} - \frac{g^4}{2} (2\alpha_3 - \alpha_4 + \alpha_8 + 2\alpha_9). \]  

(61)
There is an equation among $C_1$, $C_2$, and $C_3$, which reads

$$G^2 = C_1 g^2 + 2g g' C_2 + C_3 g'^2.$$  

(62)

Such a relation indicates that there are only two of these three parameters are free. The coupling $C_4$ is a fixed constant. It is worthy to mention that the factors before the gauge kinetic terms of vector field $A$ and $Z$ are not unit. For a large $\alpha_1$ and $\alpha_8$, with large couplings $g$ and $g'$, $C_1$ and $C_3$ might be very small (it is possible for some strong couplings theories), which in effect is equivalent to the strong coupling of $A$ and $Z$ after normalizing the kinetic terms. The ECs $C_i$ contain not only the contributions of the SM, but also those of the ACs. When $\alpha_i$ vanish, these $C_i$ reduce to their values of the SM at tree level without including the contribution of Higgs.

There is a remarkable feature, that the ACs $\alpha_i$ always appear in the combination with $g^2, g'^2, G^2$. Such a feature will be quite helpful for us to establish the modified power counting rule.

Thus, there are 12 free parameters in $C_i$, including $g$ and $g'$ (please remember $G^2 = g^2 + g'^2$). While there are also 12 free parameters in $\alpha_i$, including $g$ and $g'$. Such a fact also indicates that these two bases are equivalent.

The $C_i$ are just the ECs of the effective vertices among vector bosons when we calculate the $S-$matrix by using the traditional Feynman diagram method in the mass eigenstates, and the effects of the ACs of the EWCL and the background of the SM have been taken into account in these ECs, as shown in Eq. (50—61).

IV. THE ONE LOOP RENORMALIZATION OF THE SM

In the spirit of the background field gauge quantization [21], we can decompose the Goldstone field into the classic part $\bar{U}$ and quantum part $\xi$ as

$$U \to \bar{U} \hat{U}, \quad \hat{U} = \exp\left\{ \frac{i2\xi}{(v+h)} \right\}. \quad (63)$$

To parameterize the quantum Goldstone field in the above form is to simplify the presentation of the standard form of quadratic terms. The vector fields in the mass eigenstates are split as

$$V_\mu \to \bar{V}_\mu + \hat{V}_\mu, \quad (64)$$

where $\bar{V}_\mu$ represents the classic background vector fields and $\hat{V}_\mu$ represents the quantum vector fields.
By using the Stueckelberg transformation \[24\] for the background vector fields,
\[
W^a \rightarrow U^\dagger W U + i U^\dagger \partial U, \quad B \rightarrow B, \quad \hat{W}^a \rightarrow U^\dagger \hat{W} U, \quad \hat{B} \rightarrow \hat{B},
\]
(65)
So the background Goldstone fields can completely be absorbed by redefining the background vector fields, and will not appear in the one-loop effective Lagrangian. The Stueckelberg fields is invariant under the gauge transformation of the background gauge fields, such a property guarantees that the following computation is gauge invariant from the beginning if we can express all effective vertices into the Stueckelberg fields. After the loop calculation, by using the inverse Stueckelberg transformation, the Lagrangian can be restored to the form represented by its low energy DOFs.

Similarly, the Higgs scalar is split as
\[
h = \bar{h} + \hat{h}.
\]
(66)

We would like to comment on the relations between the interaction and mass eigenstates. The mass eigenstates should be understood as the combination of the Stueckelberg fields, and read
\[
Z = - \cos \theta_W W^s_3 + \sin \theta_W B, \quad A = \sin \theta_W W^s_3 + \cos \theta_W B,
\]
\[
W^+ = \frac{1}{\sqrt{2}}(W^{s_1} - iW^{s_2}), \quad W^- = \frac{1}{\sqrt{2}}(W^{s_1} + iW^{s_2}).
\]
(67)
Then by using these relations and the definition of Stueckelberg transformation, we can formulate the set of independent operators in the mass eigenstates in U-gauge back to that of the EWCL in the interaction eigenstates.

The equation of motion of the background vector fields is determined as
\[
D_\mu \bar{W}^{\mu\mu} = -\sigma_{0,VV}V^\nu,
\]
(68)
with $\bar{W}^{\mu\nu,T} = \{A^{\mu\nu} + ieF_{Z}^{\mu\nu}, Z^{\mu\nu} - i\frac{g^2}{G}F_{Z}^{\mu\nu}, \bar{W}^{+,\mu\nu}, \bar{W}^{-,\mu\nu}\}$. The EOM of vector bosons derives the following relations
\[
\partial \ln (v + \bar{h}) \cdot Z = -\frac{1}{2} \partial \cdot Z, \quad (69)
\]
\[
\partial \ln (v + \bar{h}) \cdot W^+ = -\frac{1}{2} d \cdot W^+ + i \frac{g^2}{2 G} Z \cdot W^+, \quad (70)
\]
\[
\partial \ln (v + \bar{h}) \cdot W^- = -\frac{1}{2} d \cdot W^- - i \frac{g^2}{2 G} Z \cdot W^-.
\]
(71)
The equation of motion of the background Higgs field is given as
\[ \partial^2 \mathcal{H} = (v + \mathcal{H}) \left[ \frac{G^2}{4} Z \cdot Z + \frac{g^2}{2} W^+ \cdot W^- - \mu^2 + \frac{\lambda}{4} (v + \mathcal{H})^2 \right]. \] (72)

The gauge fixing term for the quantum fields are chosen as below in order to make the quadratic terms have the standard form

\[ \mathcal{L}_{GF,A} = -\frac{1}{2} (\partial \cdot \hat{A} - ie(\hat{W}^- \cdot \hat{W}^+ - \hat{W}^+ \cdot \hat{W}^-))^2, \] (73)

\[ \mathcal{L}_{GF,Z} = -\frac{1}{2} (\partial \cdot \hat{Z} - \frac{1}{2} G(v + \mathcal{H}) \xi_Z + i\frac{g^2}{G} (\hat{W}^- \cdot \hat{W}^+ - \hat{W}^+ \cdot \hat{W}^-))^2, \] (74)

\[ \mathcal{L}_{GF,W} = -(d \cdot \hat{W}^+ + \frac{1}{2} g(v + \mathcal{H}) \xi^+_W + i\frac{g^2}{G} \hat{Z} \cdot \hat{W}^+ - i\frac{g^2}{G} \hat{W}^+ \cdot \hat{Z} + ie \hat{W}^+ \cdot \hat{A}) \]

\[ (d \cdot \hat{W}^- + \frac{1}{2} g(v + \mathcal{H}) \xi^+_W - i\frac{g^2}{G} \hat{Z} \cdot \hat{W}^- - i\frac{g^2}{G} \hat{W}^- \cdot \hat{Z} - ie \hat{W}^- \cdot \hat{A}). \] (75)

Compared with the Dyson-Feynman method, the number of diagrams in BFM [21] for the loop corrections can be greatly reduced. Another remarkable advantage is that, in the BFM, each step of calculation is manifestly gauge covariant with reference to the gauge transformation of the background gauge field, and the Ward identities have been naturally incorporated in the calculation procedure. The method is quite powerful to deal with the theories with many vertices, gravity and the nonlinear effective gauge theories (the EWCL given below), for instance. At the same time, the freedom for choosing the different gauges for the classic and quantum gauge field makes the procedure of calculation simple. The Schwinger proper time and heat kernel method [25] in per se is the Feynman integral. Combining with the covariant short distance expansion [26] in the coordinate space, these methods can considerably simplify the loop calculation. In the next subsections, we will use these concepts and methods to help us to extract the RGEs of the EWCL.

Several groups of authors have conducted the renormalization of the SM in the BFM [29]. Different from their procedures which are performed in the momentum space, here we conduct our calculation in the coordinate space and we only consider up to the one loop renormalization ( For using the BFM to consider the two loop renormalization, please refer to the literature of C. Lee in [26] and [32] ). Our purpose here is to check our method and to provide a comparison to the renormalization of the effective one given in the next section.

A. The quadratic forms of the one-loop Lagrangian

We can cast the quadratic terms of the one-loop Lagrangian into its standard form, as prescribed in [22], which read
The mass matrices have the form
\[ \sigma_{\mu,\nu} \xi \xi = \mathcal{D}_{\mu,\nu} \xi \xi + \mathcal{C}_{\sigma,\nu} \xi \xi. \]
and the components read
\[ \sigma_{\mu,\nu} \xi \xi = \mathcal{D}_{\mu,\nu} \xi \xi + \mathcal{C}_{\sigma,\nu} \xi \xi. \]
where \( V^\dagger = (A, Z, W^-, W^+) \) and \( \xi^\dagger = (\xi_z, \xi^- , \xi^+) \), the covariant differential operators
\[ \square_{VV} = D_{2,\alpha} \xi \xi + \sigma_{\nu,\mu} \xi \xi + \sigma_{0,\nu} \xi \xi, \]
\[ \square_{h} = \partial + \Gamma \xi, \]
and the gauge connection of vector bosons \( \Gamma V \) is defined as
\[ \Gamma_{V,\mu} = \begin{pmatrix}
0 & 0 & i e W^-_\mu & -i e W^+_\mu \\
0 & 0 & -i g^G W^-_\mu & i g^G W^+_\mu \\
i e W^+_\mu & -i g^G W^-_\mu & -i e A_\mu & i g^G Z_\mu \\
-i e W^-_\mu & i g^G W^+_\mu & i e A_\mu & -i g^G Z_\mu
\end{pmatrix}, \]
The gauge connection of Goldstone bosons \( \Gamma \xi \) is defined as
\[ \Gamma_{\xi,\mu} = \begin{pmatrix}
0 & i \frac{g}{2} W^-_\mu & -i \frac{g}{2} W^+_\mu \\
i \frac{g}{2} W^+_\mu & -i e A_\mu & 0 \\
-i \frac{g}{2} W^-_\mu & 0 & i e A_\mu
\end{pmatrix}. \]
The mass matrices have the form \( \sigma_{0,V,V} = \text{dia}\{0, G^2(v + \bar{h})^2/4, g^2(v + \bar{h})^2/4, g^2(v + \bar{h})^2/4\} \)
and \( \sigma_{0,\xi,\xi} = \text{dia}\{G^2(v + \bar{h})^2/4, g^2(v + \bar{h})^2/4, g^2(v + \bar{h})^2/4\} \).
The matrix \( \sigma_{2,V,V} \) is given below as
\[ \sigma_{\mu,\nu}^{2,V,V} = \begin{pmatrix}\sigma_{2,AA} & \sigma_{2,AZ} & \sigma_{2,AW^+} & \sigma_{2,AW^-} \\
\sigma_{2,ZA} & \sigma_{2,ZZ} & \sigma_{2,ZW^+} & \sigma_{2,ZW^-} \\
\sigma_{2,W^- A} & \sigma_{2,W^- Z} & \sigma_{2,W^- W^+} & \sigma_{2,W^- W^-} \\
\sigma_{2,W^+ A} & \sigma_{2,W^+ Z} & \sigma_{2,W^+ W^+} & \sigma_{2,W^+ W^-}\end{pmatrix}, \]
and the components read
\[ \sigma_{2,AA} = \sigma_{2,AZ} = \sigma_{2,ZA} = \sigma_{2,ZZ} = 0, \]
\[ \sigma_{2,AW^+} = -\sigma_{2,W^+ A} = 2i e W^-_{\mu,\nu} , \]
\[ \sigma_{2,AW}^{\mu} = -\sigma_{2,W-A}^{\mu} = -2ieW^{+\mu}, \]
\[ \sigma_{2,W}^{\mu+} = -\sigma_{2,W}^{\mu} = -2ig^2/G \tilde{W}^{-\mu}, \]
\[ \sigma_{2,W}^{\mu-} = -\sigma_{2,W}^{\mu} = 2igZW^{+\mu}, \]
\[ \sigma_{2,W-W} = -\sigma_{2,W} = 2igW_{3\mu}, \]
\[ \sigma_{2,W+W} = \sigma_{2,W-W} = 0. \] (83)

The matrix \( \sigma_{2,\xi\xi} \) is given as
\[ \sigma_{2,\xi\xi}^{ij} = \begin{pmatrix} \sigma_{2,\xi\xi} & \sigma_{2,\xi\xi} & \sigma_{2,\xi\xi} \\ \sigma_{2,\xi\xi} & \sigma_{2,\xi\xi} & \sigma_{2,\xi\xi} \\ \sigma_{2,\xi\xi} & \sigma_{2,\xi\xi} & \sigma_{2,\xi\xi} \end{pmatrix}, \]
and its components read
\[ \sigma_{2,\xi\xi}^{\xi\xi} = \frac{\lambda}{4} [v^2 - (v + \overline{h})^2] - \frac{G^2}{4} Z \cdot Z, \]
\[ \sigma_{2,\xi\xi}^{\xi+} = -g^2/4 W^{-} \cdot W^{-}, \]
\[ \sigma_{2,\xi\xi}^{\xi-} = -g^2/4 W^{+} \cdot W^{+}, \]
\[ \sigma_{2,\xi\xi}^{\xi+} = \sigma_{2,\xi\xi}^{\xi+} = \frac{gG}{4} W^{-} \cdot Z, \]
\[ \sigma_{2,\xi\xi}^{\xi-} = \sigma_{2,\xi\xi}^{\xi-} = \frac{gG}{4} W^{+} \cdot Z, \]
\[ \sigma_{2,\xi\xi}^{\xi+} = \sigma_{2,\xi\xi}^{\xi-} = \frac{\lambda}{4} [v^2 - (v + \overline{h})^2] - \frac{g^2}{4} W^{+} \cdot W^{-}. \] (84)

We have used the equation of motion of the background Higgs given in Eq. (72) in this step, which is reflected by the terms proportional to \( \lambda \) in \( \sigma_{2,\xi\xi}^{\xi\xi}, \sigma_{2,\xi\xi}^{\xi+}, \) and \( \sigma_{2,\xi\xi}^{\xi-}. \)

The \( \sigma_{hh} \) is determined as
\[ \sigma_{hh} = -\frac{1}{4} (G^2 Z \cdot Z + 2g^2 W^{-} \cdot W^{-}) + \frac{\lambda}{4} v^2 - \frac{3}{4} \lambda (v + \overline{h})^2. \] (85)

The mixing terms between the vector and Goldstone bosons are determined as
\[ X_{\xi}^{\mu,\alpha} = \begin{pmatrix} 0 & -i2g^2/4 (v + \overline{h}) W^{-\mu} & \frac{ig^2}{4} (v + \overline{h}) W^{+\mu} \\ \frac{G\partial^{\mu} h}{i2} & \frac{i2g}{2} (g^2 - g^2')(v + \overline{h}) W^{-\mu} & -i2g^2/4 (v + \overline{h}) W^{+\mu} \\ -i2g/2 (v + \overline{h}) W^{+\mu} & -g\partial^{\mu} h - \frac{1}{2} igG (v + \overline{h}) Z^{\mu} & 0 \\ \frac{ig^2}{2} (v + \overline{h}) W^{-\mu} & 0 & -g\partial^{\mu} h + \frac{1}{2} igG (v + \overline{h}) Z^{\mu} \end{pmatrix}, \]
while the matrix $\tilde{X}_\xi^{\mu,aj}$ is just the rearrangement of the $\tilde{X}_\xi^{\mu,aj}$, and here we do not rewrite it. The mixing terms between vector and Higgs bosons are determined as

$$\tilde{X}_h^{\mu,a} = \{0, -\frac{1}{2} G^2 (v + \overline{h}) Z^\mu, -\frac{1}{2} g^2 (v + \overline{h}) W^+ \cdot \mu, -\frac{1}{2} g^2 (v + \overline{h}) W^- \cdot \mu\}, \quad (86)$$

The mixing terms $\tilde{X}_h^{\mu,a}$ is the rearrangement of the $\tilde{X}_h^{\mu,a}$. The mixing terms between Higgs and Goldstone bosons are determined as

$$X_{h\xi}^{\alpha,i} = \{-G Z^\alpha, g W^- \cdot \alpha, g W^+ \cdot \alpha\}, \quad (87)$$

$$X_{i,h\xi,0} = \{-\frac{g}{2} \partial \cdot Z, \frac{g}{2} d \cdot W^-, -\frac{1}{2} \left(\frac{g^2}{G} - G\right) W^- \cdot Z, \frac{g}{2} d \cdot W^+ + \frac{1}{2} \left(\frac{g^2}{G} - G\right) W^+ \cdot Z\} \quad (88)$$

The terms $X_{\xi h}^{\alpha,i}$ and $X_{\xi h,0}^{i}$ are omitted here.

**B. Evaluating the traces and logarithms**

By diagonalizing the quantum fields, we can integrate the quadratic terms of the Lagrangian by using the Gaussian integral. And the $\mathcal{L}_{1\text{-loop}}$ can be expressed as the traces and logarithms

$$S_{1\text{-loop}} = Tr \Box_{\xi\xi} - \frac{1}{2} \left[ Tr \ln \Box_{VV} + Tr \ln \Box_{\xi\xi}' + Tr \ln \Box_{hh}'' \right], \quad (89)$$

where

$$\Box_{\xi\xi}' = \Box_{\xi\xi}' - \tilde{X}_\xi \Box_{VV}^{-1} \tilde{X}_\xi, \quad (90)$$

$$\Box_{hh}' = \Box_{hh} - \tilde{X}_h \Box_{VV}^{-1} \tilde{X}_h, \quad (91)$$

$$\Box_{hh}'' = \Box_{hh}' - X_{h\xi} \Box_{\xi\xi}^{-1} X_{\xi h}, \quad (92)$$

$$X_{h\xi}' = X_{h\xi} - \tilde{X}_h \Box_{VV}^{-1} \tilde{X}_\xi, \quad (93)$$

$$X_{\xi h}' = X_{\xi h} - \tilde{X}_\xi \Box_{VV}^{-1} \tilde{X}_h, \quad (94)$$

Expanding the $Tr \ln \Box_{\xi\xi}'$ and $Tr \ln \Box_{hh}''$ with the following relations

$$Tr \ln \Box_{\xi\xi}' = Tr \ln \Box_{\xi\xi} + Tr \ln (1 - \tilde{X}_\xi \Box_{VV}^{-1} \tilde{X}_\xi \Box_{\xi\xi}^{-1}), \quad (95)$$

$$Tr \ln \Box_{hh}'' = Tr \ln \Box_{hh}' + Tr \ln (1 - X_{h\xi} \Box_{\xi\xi}^{-1} X_{\xi h} \Box_{hh}'^{-1}). \quad (96)$$

Since we consider the renormalization, so we are only interested in those divergent terms, which can be expressed as
\[
\int_x \mathcal{L}_{1-loop} = Tr \Box \bar{\psi} - \frac{1}{2} \left[ Tr \ln \Box VV + Tr \ln \Box \xi \xi + Tr \ln \Box hh \\
- Tr(\hat{X}_\xi \Box VV \hat{X}_\xi \Box^{-1} - \hat{X}_h \Box VV \hat{X}_h \Box^{-1}) \\
- Tr(\hat{X}_h \Box \xi \xi \Box^{-1} \hat{X}_h \Box^{-1}) \\
- \frac{1}{2} Tr(\hat{X}_h \Box \xi \xi \Box^{-1} \hat{X}_h \Box^{-1} \hat{X}_h \Box \xi \xi \Box^{-1} \hat{X}_h \Box^{-1}) + \cdots \right]. \tag{97}
\]

Due to the property of the \( Tr \), the above equation is independent of the sequence of integrating-out quantum fields. The omitted terms are finite and will not contribute to the one-loop divergence structures.

C. Counter terms

To evaluate the traces in the Eq. (97), we use the Schwinger proper time and heat kernel method [25] with the covariant short distance expansion technique [26]. The detailed calculation steps are omitted here. By using the heat kernel method directly, we have the following divergence structures from the contributions of \( Tr \ln \Box \) in the Eq. (97)

\[
\frac{1}{2} \tilde{\epsilon} Tr \ln \Box VV = -\frac{20}{3} H_1 - \frac{(2g^2 + G^2)}{16} (v + \bar{h})^4, \tag{98}
\]
\[
\frac{1}{2} \tilde{\epsilon} Tr \ln \Box \xi \xi = +\frac{g^2}{12} H_1 + \frac{g^2}{12} H_2 + \frac{1}{12} \mathcal{L}_1 - \frac{1}{24} \mathcal{L}_2 - \frac{1}{24} \mathcal{L}_3 \\
- \frac{1}{12} \mathcal{L}_4 + \frac{1}{48} \mathcal{L}_5 - \frac{G^2 g'^2}{32} (v + \bar{h})^2 Z \cdot Z \\
+ \frac{1}{32} [\lambda v^2 - (g^2 + \lambda)(v + \bar{h})^2] (G^2 Z \cdot Z + 2g^2 W^+ \cdot W^-) \\
+ \frac{1}{32} [3\lambda + (2g^2 + G^2)] \lambda v^2 (v + \bar{h})^2 \\
- \frac{1}{64} [(2g^4 + G^4) + 2(2g^2 + G^2) \lambda + 12\lambda^2](v + \bar{h})^4 \\
- \frac{3}{64} \lambda^2 v^4, \tag{99}
\]
\[
\frac{1}{2} \tilde{\epsilon} Tr \ln \Box hh = -\frac{1}{16} L_5 + \frac{1}{32} [v^2 - 3(v + \bar{h})^2] (G^2 Z \cdot Z + 2g^2 W^+ \cdot W^-) \\
+ \frac{3}{32} \lambda^2 v^2 (v + \bar{h})^2 - \frac{9}{64} \lambda^2 (v + \bar{h})^4 - \frac{1}{16} \lambda^2 v^4, \tag{100}
\]
\[
-\tilde{\epsilon} Tr \ln \Box \bar{\psi} = -\frac{2}{3} H_1 + \frac{G^2 + 2g^2}{32} (v + \bar{h})^4, \tag{101}
\]

where \( 1/\tilde{\epsilon} = i/16\pi^2 (2/\epsilon - \gamma_E + \ln(4\pi^2)) \), \( \gamma_E \) is the Euler constant, and \( \epsilon = 4 - d \). The terms \( (v + \bar{h})^4 \) in the \( Tr \ln \Box \xi \xi \) comes from the EOM of the background Higgs field Eq. (72). The divergence terms from the mixing terms with two propagators are given as
The divergences of the four propagators term is given as
\[
-\frac{\bar{\epsilon}}{2} Tr(\hat{X}_\xi \square^{-1}_\nu \hat{X}_\xi \square^{-1}_\xi) = \frac{g^2 g^2}{8} Z \cdot Z(v + \bar{\nu})^2
- \frac{g^2 + G^2}{8} (v + \bar{\nu})^2 (G^2 Z \cdot Z + 2 g^2 W^+ \cdot W^-)
- \frac{1}{2} (2g^2 + G^2) \partial \bar{\nu} \cdot \partial \bar{\nu},
\] (102)

\[
-\frac{\bar{\epsilon}}{2} Tr(\hat{X}_h \square^{-1}_\nu X_h \square^{-1}_{hh}) = -\frac{g^2 g^2}{8} Z \cdot Z(v + \bar{h})^2
- \frac{g^2}{8} (v + \bar{h})^2 (G^2 Z \cdot Z + 2 g^2 W^+ \cdot W^-),
\] (103)

\[
-\frac{\bar{\epsilon}}{2} Tr(X_{h\xi} \square^{-1}_\nu X_{\xi h} \square^{-1}_{hh}) = -\frac{1}{2} (t_{BB1} + t_{BB2} + t_{BC} + t_{CC}),
\] (104)

\[
t_{BB1} = -\frac{g^2}{6} H_1 - \frac{g^2}{6} H_2 + \frac{1}{6} L_1 + \frac{1}{6} L_2 + \frac{1}{6} L_3 + \frac{1}{6} L_4 - \frac{1}{6} L_5
- \frac{1}{2} \left( \frac{g^2}{G^2} - 1 \right)^2 L_6 + \frac{1}{2} \left( \frac{g^2}{G^2} - 1 \right]^2 L_a
- \frac{G^2}{4} (\partial \cdot Z)^2 - \frac{g^2}{2} (d \cdot W^+) (d \cdot W^-)
- \frac{g^2 g^2}{G} [(d \cdot W^+) (W^- \cdot Z) - (d \cdot W^-) (W^+ \cdot Z)],
\] (105)

\[
t_{BB2} = -\frac{1}{4} \frac{G^2}{2} (\partial \cdot Z)^2 + g^2 (d \cdot W^+) (d \cdot W^-)
\] (106)

\[
t_{BC} = (\frac{g^2}{G^2} - 1)^2 L_6 - (\frac{g^2}{G^2} - 1)^2 L_a
+ \frac{G^2}{2} (\partial \cdot Z)^2 + g^2 (d \cdot W^+) (d \cdot W^-)
- \frac{g^2 g^2}{G} [(d \cdot W^+) (W^- \cdot Z) - (d \cdot W^-) (W^+ \cdot Z)],
\] (107)

\[
t_{CC} = -\frac{1}{2} \left( \frac{g^2}{G^2} - 1 \right)^2 L_6 + \frac{1}{2} \left( \frac{g^2}{G^2} - 1 \right]^2 L_a
- \frac{G^2}{4} (\partial \cdot Z)^2 - \frac{g^2}{2} (d \cdot W^+) (d \cdot W^-)
+ \frac{g^2 g^2}{2G} [(d \cdot W^+) (W^- \cdot Z) - (d \cdot W^-) (W^+ \cdot Z)].
\] (108)

The divergences of the four propagators term is given as
\[
-\frac{\bar{\epsilon}}{4} Tr(X_{h\xi} \square^{-1}_\nu X_{\xi h} \square^{-1}_{hh}) = -\frac{1}{12} L_4 - \frac{1}{24} L_5,
\] (109)

The sum over all contributions yields the following total divergence structures as
\[ \tilde{\epsilon} D_{\text{tot}} = -\frac{43}{6} g^2 H_1 + \frac{1}{6} g^2 H_2 - \frac{2g^2 + G^2}{8}(v + \bar{h})^2(G^2 Z \cdot Z + 2g^2 W^+ \cdot W^-) - \frac{2g^2 + G^2}{2} \partial h \cdot \partial \bar{h} + \frac{1}{32}(6\lambda + 2g^2 + G^2)\lambda v^2(v + \bar{h})^2 - \frac{1}{64}[(6g^4 + 3G^4) + (2g^2 + G^2)\lambda + 12\lambda^2](v + \bar{h})^4 - \frac{1}{16}\lambda^2 v^4. \]

The \( D_{\text{tot}} \) just indicates that the extra divergences just cancel out exactly with each other, and even the terms like \((\partial Z)^2\) will not appear in the total divergence structures. No gauge fixing term of the background fields should be added to the Lagrangian, and the equations of motion are just sufficient.

The coefficients of \( H_1 \) and \( H_2 \) have the correct value which contribute to the \( \beta \) functions of the gauge couplings \( g \) and \( g' \), respectively. The coefficients of the terms \((v + \bar{h})^2(G^2 Z \cdot Z + 2g^2 W^+ \cdot W^-)/8 \) and \( \partial h \cdot \partial \bar{h}/2 \) are equal, and such a fact is not accidental and should be the requirement of renormalizability. If we reformulate the Lagrangian in its linear form, the combination of these two terms just yields the term \((D\phi)^\dagger \cdot (D\phi)\). From the requirement of renormalizability of the theory, we know that coefficients of divergences of these two terms should be the same. The last constant divergences just contribute to the unobservable vacuum, and can be dropped out. This constant will also appear in other computational methods, and is not the special feature of BFM.

Due to the parameterization in the Eq. (63), we have found that there is no quartic divergences in the one-loop Lagrangian, contrary to the expectation of [30]. As matter of fact, had the authors of the references [30] used the EOM of the background Higgs to sum the quartic divergences and higher divergence structures, they would have gotten the same result as given by us. In other words, such a fact is independent of the parameterization of quantum Goldstone bosons.

Another remarkable feature is that the EOM of the background Higgs makes the counter term of the quartic coupling of Higgs potential in BFM different than that calculated in the usual Feynman diagrams in linear representation. This difference is caused directly by the term \( \sigma_{\xi \xi} \), and might be one of the special characters of the BFM.

To extract the divergences, we have used the following relation

\[ H_{\mu \nu}SF_{+, \mu \nu} = 8iO_6 + 4iO_7 + 4Z \cdot W^+ d \cdot W^- - 4Z \cdot W^- d \cdot W^+ + H_{\mu \nu}SF_{-, \mu \nu}, \]
V. THE ONE LOOP RENORMALIZATION OF THE EWCL

In this section we will conduct the one loop renormalization of the EWCL, by taking the limit that all ACs vanish, we can check the following calculation with the renormalization of the SM, as provided in the above section. Before the actual computation, we would like to establish a modified power counting rule, in order to control higher corrections from higher loops and higher dimension operators.

A. The modified power counting rule in EWCL

Before establishing our power counting rule, we would like to make a brief review on the framework of the hadronic chiral perturbation theory.

In the usual ChPT approach to low-energy hadronic, the chiral Lagrangian is organized as an expansion in powers of momenta \( p^2 \)

\[
L^{\text{eff}} = L_2 + L_4 + L_6 + ... \tag{112}
\]

Each term \( L_n \), in turn, is given by a certain number of operators \( O_i^{(n)} \) with low-energy constants \( l_i^{(n)} \) that, a priori, are determined by the underlying theory:

\[
L_n = \sum_i l_i^{(n)} O_i^{(n)}. \tag{113}
\]

The general expectation of the importance of an operator is that the lower order it belongs, the more importance of it. Therefore, in the ChPT, the \( L_2 \) is the most important operator, and determines the propagators of massless Goldstone bosons and the scattering interaction at tree level, which can be expressed as \( c_2 \frac{p^4}{v^2} \), \( (c_2 \) is a \( O(1) \) constant). At one-loop level, the scattering amplitude will get the radiative corrections from the loop with two of this vertex and with internal lines of Goldstones. While after dropping the divergences of the loop integral, we get the finite one-loop contribution of this interaction can be expressed as \( \alpha \frac{1}{(4\pi)^2} \frac{\Delta^4}{v^4} \), \( (\alpha \) is a constant factor determined by the loop and \( c_2 \), which is of order 1). Such a contribution has the same momentum power with those of operators in the \( L_4 \), which can be expressed as \( \alpha_0 \frac{\Delta^4}{v^4} \).

In the ChPT, coincidentally (not necessary for general chiral perturbation theories), \( \alpha_0 \), as determined from low energy phenomenologies, like hadronic scattering and decay processes, etc, is of the order \( \frac{1}{(4\pi)^2} \) [31].

So, if we go to further higher order, say two-loop order, then we should include three parts of contributions, 1) the two loop contributions of pure \( O(p^2) \) vertices, 2) the one-loop
contribution with one $O(p^2)$ vertex and one $O(p^4)$ vertex, and 3) the tree level contribution of $O(p^6)$. The first part can be expressed as $\beta_2 \frac{1}{(4\pi)^2} \frac{p^2}{v^2}$, the second part can be expressed as $\beta_1 \frac{1}{(4\pi)^2} \frac{p^2}{v^2}$, and the third part can be expressed as $\beta_0 \frac{p^6}{v^6}$. The first part contains two loop suppression factor $\frac{1}{(4\pi)^2}$, while the second part contains only one loop suppression factor $\frac{1}{(4\pi)^2}$. But due to the fact that the $\beta_1$ is determined by both $c_4$ and $\alpha_0$, so not only on the momentum power, but also on the magnitude order controlled by the loop factors, the second parts will share the same importance as the first parts. We also expect that, coincidently (not necessary for general chiral perturbation theories), the $\beta_0$ will have a magnitude like $\frac{1}{(4\pi)^2}$. So that we expect that such a standard power counting rule will hold at any a specified higher order.

But for the EWCL, it seems not easy to take into account the radiative corrections of low energy quantum DOFs (which should include both the massive vector boson and its corresponding Goldstone).

The first difficulty is more manifest when we represent the EGT in their unitary gauge. The propagator of massive vector bosons can be expressed as

$$i\Delta^{\mu\nu} = i\Delta_T^{\mu\nu} + i\Delta_L^{\mu\nu},$$

$$\Delta_T^{\mu\nu} = \frac{1}{k^2 - m_V^2} \left( -g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2} \right),$$

$$\Delta_L^{\mu\nu} = \frac{1}{m_V^2} \frac{k^\mu k^\nu}{k^2},$$

where $\Delta_T$ and $\Delta_L$ represent the transverse and longitudinal parts, respectively. The longitudinal part of the propagator can bring into quartic divergences and lead to the well-known bad ultraviolet behavior. Two direct consequences of this fact are 1) that the quartic divergences will appear in radiative corrections and 2) that low dimensions operators can induce the infinite number of divergences of higher dimension operators, even at one-loop level. In a renormalizable theory, the Higgs model for instance, these two problems do not exist. The quartic divergences produced by the low energy DOF just cancel exactly with those produced by the Higgs scalar, and no extra divergence structure will appear.

The second difficulty, which is related with the first difficulty, is about the counting rule. In the gauge theories with spontaneous symmetry breaking, the marginal interaction vertices are proportional to ECs (expressed in both gauge couplings and ACs, as given in (61)), not to the momentum power $\frac{p^2}{v^2}$ as in the hadronic ChPT. Then by direct evaluating the Feynman diagrams, radiative corrections the ACs (which are determined at matching scale by the ultraviolet dynamics and there is no reason to assume that they must be as
small as $\frac{1}{(4\pi)^2}$, are of the $\frac{1}{(4\pi)^2}$, not as $\frac{1}{(4\pi)^4}$ as expected from the SPCR in the hadronic ChPT. So the native power counting rule is not proper to be used in this case. While we know, in order to collect and reliably estimate the contributions of higher orders (say, those of higher loops and higher dimension operators) in terms of magnitude, a power counting rule is needed. So to find a consistent power counting rule for this case is necessary.

As we know, for the EGT with spontaneous symmetry breaking mechanism, we have at least two ways to collect and classify operators.

The first way is to collect operators in terms of their dimensions, (not by the momentum power $\frac{p^2}{v^2}$, as in the above case). We can formulate the EWCL in the unitary gauge, then restore the low energy DOFs with the inverse Stueckelberg transformation, while the ECs are regarded as free parameters, of which the magnitude at the ultraviolet cutoff is determined by the underlying dynamics and the matching conditions. In the most general assumption, we regard these ACs are of order $O(1)$. Then, according to the Wilsonian renormalization scheme, EOs can be classified into three groups: the relevant operators, marginal operators, and irrelevant operators. The relevant operators have mass dimensions less than the dimension of space-time, and have ECs with positive mass power. The marginal operators have the same dimensions of that of space-time, and have massless ECs. The irrelevant operators have dimensions larger than the dimensions of space-time, and have couplings with negative mass power. By study the running of the ECs, we can determine the importance of operators, which is controlled by the strength of their corresponding ECs. The couplings of the relevant operators will be dependent on the ultraviolet cutoff $\Lambda = UV$ in positive powers; those of the marginal operators will be logarithmically dependent on the $\Lambda = UV$; while those of the irrelevant operators will be dependent on the $\Lambda = UV$ in negative powers. If the $\Lambda = UV$ is large enough, the irrelevant operators will become unimportant, and the relevant and marginal operators will mainly determine the low energy dynamics. Such a conclusion is based on the most general analysis of the behavior of RGEs without assuming the smallness of the ECs of irrelevant operators, as shown in [9, 10]. So we can truncate the infinite operator towers permitted in the EGT to a specified order. While for the quartic divergences, we can use the dimension regularization method, and simply discard them.

As we know, the groups of relevant and marginal operators include both the renormalizable operators and AOs up to $O(p^4)$. Meanwhile, in the general cases, the relative importance of an operator might be quite different and is determined by the relative magnitude of its EC. For instance, if the coupling is zero or much much smaller, in principle, we can drop its contributions and regard it as higher order corrections; while if the
coupling is much much lager than others, this operator should be definitely important to the low energy dynamics. Then we should classify it as a lower order operator, \textit{i.e.} to increase its importance for our consideration. So a practical and realistic power counting rule must based on the actual information of the relative magnitude of the ECs.

The second way is to mimic the ChPT by classifying according to the momentum power. In this way, up to \( O(p^4) \), without regarding any information on the magnitude of the ECs, these operators are divided into two groups, the renormalizable ones are classified to \( p^2 \) order, while the anomalous ones are classified as \( p^4 \) order. In this way, to classify the gauge kinetic terms into \( O(p^2) \) is somewhat ambiguous to the momentum power counting rule. But it is unlikely not to include the kinetic terms in this \( p^2 \). So the dimensionless gauge couplings have to be set to have momentum power. To classify the rest of marginal operators in the group of \( O(p^4) \), such a counting rule, borrowed from the hadronic ChPT, implicitly assumes that the strength of their couplings should be of order \( \frac{1}{(4\pi)^2} \).

We would like to point out that such assumptions are too strong for the a general EGT. In the framework of EFT, the magnitudes of the couplings of an operator is determined at the matching scale. There is no reason to expect that the ACs must be such small. Since at the matching scale, those ACs are determined by the ultraviolet dynamics, and can get contributions from either the tree level or loop level, or both. The magnitude of these ACs is related with both the actual value of the matching scale and the underlying dynamics. As we know, the ACs can receive the tree level contributions, like in the Higgs model we show in the numerical analysis, in the left-right hand model, etc. Furthermore, even determined at loop level, if the ultraviolet dynamics are strong coupling case, like in the Technicolor models, these ACs can be estimated as \( \frac{1}{(4\pi)^2} \frac{g_s^2}{g_w^2} \). If \( g_s \) is much larger than \( g_w \), the ACs might still be one or two order larger than the expectation of SPCR in the hadronic ChPT.

So we regard that, in order to be more realistic and be consistent with the EFT method as a general and universal method, we should abandon the second way of the classification of the operators, and before knowing the actual information on the magnitude of the ACs (equivalently, the underlying theories), we will treat all relevant and marginal AOs as operators in \( O(p^2) \) order by implicitly assuming all these ACs are of \( O(1) \) (This assumption is a more general one, and the assumption of the second way of classification is only one of its specific cases). So we modify the momentum power counting rule to include the ECs of all AOs in the Eqs. (17—17) \( \alpha_i \) as momentum \( p^{-2} \), like coefficient of the gauge kinetic terms \( 1/g^2 \). And in this way, when extracting the Feynman rules directly from the Lagrangian given in , the combination of \( g^2 \alpha_i \) in the trilinear and quartic couplings
is regarded as of $O(p^0) \sim O(1)$. Thus, this modified power counting rule will possess the powerful potent of the SPCR, and can be applied to estimate and control the contributions of higher loop and higher dimension operators, just like in the hadronic ChPT.

With this modified power counting rule in mind, below we will study the renormalization of the EWCL up to relevant and marginal operators and derive the one-loop RGE of its ECs.

**B. The gauge fixing terms in the BFM**

The equations of motion of the mass eigenstates are determined as

$$D'_\mu \tilde{W}^{\mu \nu} = \sigma_{0, VV} V^\nu,$$  \hspace{1cm} (117)

These EOM, in effect, act as the gauge fixing of the background fields, and can derive the following relations

$$\partial \cdot Z = 0, \quad d \cdot W^{\pm} = \pm i \frac{ccw}{2} Z \cdot W^{\pm},$$

$$ccw = -e \frac{C_2}{4C_1} - \frac{C_5 C_2}{8 C_1} + \frac{C_6}{8} + \frac{C_7}{8}. \hspace{1cm} (118)$$

In the background field gauge, the covariant gauge fixing term for the quantum fields can be chosen as

$$\mathcal{L}_{GF,A} = -\frac{g_A}{2} (\partial \cdot \hat{A} + f_{AZ} \partial \cdot \hat{Z} - i f_{AW} (\hat{W}^{-} \cdot \hat{W}^{+} - \hat{W}^{+} \cdot \hat{W}^{-}))^2,$$

$$\mathcal{L}_{GF,Z} = -\frac{g_Z}{2} (\partial \cdot \hat{Z} + f_{Z\xi} \xi_Z - i f_{ZW} (\hat{W}^{-} \cdot \hat{W}^{+} - \hat{W}^{+} \cdot \hat{W}^{-}))^2,$$

$$\mathcal{L}_{GF,W} = -g_W (d \cdot \hat{W}^{+} + f_{W\xi} \xi_W + i f_{WZ} Z \cdot \hat{W}^{+} + i p_{WZ} W^+ \cdot \hat{Z} + i p_{WA} W^+ \cdot \hat{A})$$

$$\left( d \cdot \hat{W}^{-} + f_{W\xi} \xi_W^{-} - i f_{WZ} Z \cdot \hat{W}^{-} - i p_{WZ} W^- \cdot \hat{Z} - i p_{WA} W^- \cdot \hat{A} \right), \hspace{1cm} (121)$$

where the parameters in these gauge fixing terms are determined by requiring the quadratic terms of Lagrangian has the standard form specified in Eq. (139), and they read

$$f_{Z\xi} = \frac{\rho}{g_Z} \frac{G_W}{2},$$

$$f_{W\xi} = -\frac{1}{g_W} \frac{g^v}{2},$$

$$g_A = C_1,$$  \hspace{1cm} (124)

$$f_{AZ} = \frac{C_2}{C_1},$$  \hspace{1cm} (125)
\[ g_{Z} = C_{3} - \frac{C_{2}^{2}}{C_{1}}, \quad (126) \]
\[ g_{W} = 1, \quad (127) \]
\[ p_{WA} = \frac{C_{5}}{2}, \quad (128) \]
\[ f_{AW} = \frac{g'_{G} C_{1}}{G}, \quad (129) \]
\[ p_{WZ} = \frac{C_{6}}{2}, \quad (130) \]
\[ f_{ZW} = -\frac{C_{7}}{2g_{Z}} - \frac{1}{g_{Z}} \frac{g'_{G} C_{2}}{G C_{1}}, \quad (131) \]
\[ f_{WZ} = \frac{C_{7}}{2}, \quad (132) \]

When all the ACs are set to vanish, these parameters will reduce to those of the SM.

**C. The quadratic terms of the effective Lagrangian**

Thus, after diagonalizing and normalizing the variables, we can collect the quadratic terms of quantum boson fields in the following standard form

\[
\mathcal{L}_{\text{quad}} = \frac{1}{2} \hat{V}_{\mu}^{\dagger} \Box_{\mu}^{V} \hat{V}_{\mu} + \frac{1}{2} \xi^{i} \Box_{\xi}^{ij} \xi^{j} + \tilde{c}^{a} \Box_{\tilde{c}c}^{ab} c^{b},
\]
\[ + \frac{1}{2} \hat{V}_{\mu}^{\dagger, a} \Gamma_{\mu}^{V} \xi^{j} + \frac{1}{2} \xi^{i, a} \Gamma_{\xi}^{V} \hat{V}_{\nu}, \quad (133) \]
\[ \Box_{\mu}^{V} = D_{\mu}^{V} g^{\mu \nu} + \sigma_{\mu \nu, V} + \sigma_{2 \nu, V}, \quad (134) \]
\[ \Box_{\xi}^{ij} = D_{ij}^{\nu, \xi} + X^{\alpha, i} d_{\alpha, i}^{j} + X^{\alpha \beta, i} d_{\alpha, i}^{j} d_{\beta}^{j}, \quad (135) \]
\[ \Box_{\xi}^{ij} = d_{ij}^{\nu, \xi} + \sigma_{0, \xi}^{ij} + \sigma_{2, \xi}^{ij} + \sigma_{4, \xi}^{ij}, \quad (136) \]
\[ \Box_{\tilde{c}c}^{ab} = D_{ab}^{V} + \sigma_{ab, V}, \quad (137) \]
\[ \Gamma_{\mu}^{V} = \tilde{\Gamma}_{\mu, V} \rightleftharpoons X_{\alpha, a} D_{a}^{\nu, \alpha} D^{a} + X_{0, a} D^{a, \alpha} + X_{01} D^{a} + X_{03} + \partial_{a} X_{03}, \quad (138) \]
\[ \Gamma_{\xi} = \tilde{\Gamma}_{\xi, \xi} \rightleftharpoons X_{\alpha, \beta} D_{\alpha, a}^{\nu} D_{a}^{\beta, \nu} + X_{0, a} D_{a}^{\nu, \beta} + X_{01} D^{\nu} + X_{03} + \partial_{a} X_{03}, \quad (139) \]

where \( V^{\dagger} = (A, Z, W^{-}, W^{+}) \) and \( \xi^{\dagger} = (\xi_{Z}, \xi^{-}, \xi^{+}) \). And the covariant differential operators \( D = \partial + \Gamma_{V} \) and \( d = \partial + \Gamma_{\xi} \). The gauge connections \( \Gamma_{V} \) and \( \Gamma_{\xi} \) are defined as

\[
\Gamma_{V} = X_{V}^{T} \Gamma_{V} X_{V}, \quad \Gamma_{\xi} = X_{\xi}^{T} \Gamma_{\xi} X_{\xi}, \quad (140) \]

where the matrices \( X_{V} \) and \( X_{\xi} \) are determined by the matrices \( W_{V} \) and \( W_{\xi} \) by
\[ X_V^T W_V X_V = 1_{4 \times 4}, \quad X_\xi^T W_\xi X_\xi = 1_{3 \times 3}. \] (141)

The matrices \( W_V \) and \( W_\xi \) will be presented below.

About the matrices given in Eq. (139), due to the fact that \( A \) is still massless and has no the corresponding Goldstone bosons, the number of vector bosons and of Goldstone is different. So we deliberately present the indices of the Goldstone and vector bosons with different letters.

The related quantities given in (139) are defined as

\[ \sigma_{2,\nu,ab}^{\mu\nu} = -\partial \cdot \Gamma_{V,\gamma} g^{\mu\nu} - \Gamma_{V,\gamma}^{\alpha c} \cdot \Gamma_{V,\gamma}^{\beta b} g^{\mu\nu} + \tilde{\sigma}_{2,VV}^{\mu\nu,ab}, \] (142)

\[ \sigma_{2,\xi}^{ij} = -\partial \cdot \Gamma_{\xi}^{ij} \Gamma_{\xi}^{ik} + \tilde{\sigma}_{2,\xi}^{ij}, \] (143)

\[ X_{\alpha,ij}^{\beta,ij} = -\tilde{\Gamma}_{\alpha,ij}^{\beta,ij}, \] (144)

\[ X_{\alpha,ij}^{\beta,ij} = \tilde{X}_{\alpha,ij}^{\beta,ij} - \partial_{\beta} \tilde{X}_{\alpha,ij}^{\beta,ij} + 2 \tilde{S}_{\alpha,\beta} g^{ij} \tilde{\Gamma}_{\xi}^{ji}, \] (145)

\[ \sigma_{4,\xi}^{ij} = \tilde{X}_{4}^{ij} + \tilde{S}_{\alpha,\beta} (\partial_{\beta} \Gamma_{\xi}^{kj} - \Gamma_{\xi}^{ki} \Gamma_{\xi}^{ji}) \] (146)

\[ \tilde{X}_{\alpha,ai}^{\mu,ai} = -\tilde{\xi}_{\alpha,ai}^{\mu,ai}, \] (147)

\[ \tilde{X}_{\alpha,ai}^{\mu,ai} = \tilde{X}_{1,\alpha,ai}^{\mu,ai} - \tilde{X}_{2,\alpha,ai}^{\mu,ai} - \partial_{\beta} \tilde{X}_{\alpha,ai}^{\mu,ai} g^{\alpha\alpha'} + 2 \tilde{S}_{\alpha,\beta} \Gamma_{\xi}^{ij} g^{ij}, \] (148)

\[ \tilde{X}_{01}^{\mu,ai} = \tilde{X}_{01}^{\mu,ai}, \] (149)

\[ \tilde{X}_{03Z}^{\mu,ai} = \tilde{X}_{03Z}^{\mu,ai} + \tilde{S}_{\alpha,\beta} \Gamma_{\xi}^{ij} \Gamma_{\xi}^{ji} \] (150)

\[ \tilde{X}_{03Y}^{\nu,i} = -\tilde{X}_{2,\nu,i}^{\nu,i}, \] (151)

\[ \tilde{X}_{\alpha,\beta}^{\nu,i} = -\tilde{\xi}_{\alpha,\beta}^{\nu,i}, \] (152)

\[ \tilde{X}_{\alpha,ia}^{\nu,i} = \tilde{X}_{2,\alpha,ia}^{\nu,i} - \tilde{X}_{1,\alpha,ia}^{\nu,i} - \partial_{\beta} \tilde{X}_{\alpha,ia}^{\nu,i} g^{\alpha\alpha'} + 2 \tilde{S}_{\alpha,\beta} \Gamma_{\xi}^{ij} g^{ij}, \] (153)

\[ \tilde{X}_{01}^{\nu,i} = \tilde{X}_{01}^{\nu,i}, \] (154)

\[ \tilde{X}_{03Z}^{\nu,i} = \tilde{X}_{03Z}^{\nu,i} + \tilde{S}_{\alpha,\beta} \Gamma_{\xi}^{ij} \Gamma_{\xi}^{ji} \] (155)

\[ \tilde{X}_{03Y}^{\nu,i} = -\tilde{X}_{1,\nu,i}^{\nu,i}, \] (156)

The tilded quantities given in the above equations are different the ones given below, and these two have the relation as \( \tilde{X}_{\xi}^{\text{above}} = X_{\xi}^{\dagger} \tilde{X}_{\xi}^{\text{below}} X_{\xi} \), and \( \tilde{X}_{V}^{\text{above}} = X_{V}^{\dagger} \tilde{X}_{V}^{\text{below}} X_{V} \).

The tilded quantities are determined by the following pre-standard forms prescribed in [32]
The matrix \( W^{ab} \) determines the mixing and normalization of the quantum vector boson fields, and reads
\[
W_{V}^{ab} = \begin{pmatrix}
C_1 & C_2 & 0 & 0 \\
C_2 & C_3 & 0 & 0 \\
0 & 0 & C_4 & 0 \\
0 & 0 & 0 & C_4
\end{pmatrix},
\]

The matrix \( W_{\xi}^{ij} \) determines the mixing and normalization of Goldstone particles, and reads
\[
W_{\xi}^{ij} = \begin{pmatrix}
\rho & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}.
\]

The \( \tilde{\Gamma}_{V}^{\alpha,ab} \) will determine the gauge connection of the covariant differential operators \( D \), which have the following form
\[
\tilde{\Gamma}_{V}^{\alpha,ab} = \begin{pmatrix}
0 & 0 & ic_{AW}W^{\alpha,-} & -ic_{AW}W^{\alpha,+} \\
0 & 0 & ic_{ZW}W^{\alpha,-} & -ic_{ZW}W^{\alpha,+} \\
ic_{AW}W^{\alpha,+} & ic_{ZW}W^{\alpha,+} & -ieA^{\alpha} + iC_{W^{+} - Z^{\alpha}} & 0 \\
ic_{AW}W^{\alpha,-} & -ic_{ZW}W^{\alpha,-} & 0 & ieA^{\alpha} - iC_{W^{+} - Z^{\alpha}}
\end{pmatrix},
\]

where \( c_{aw} = e/2 + C_5/4 \), \( c_{zw} = (C_6 - C_7)/4 \), and \( c_{W^{+} - W^{-}} = C_7/2 \). The \( \tilde{\Gamma}_{\xi} \) is defined as
\[
\tilde{\Gamma}_{\xi}^{\alpha,ij} = \begin{pmatrix}
0 & i\frac{g}{2}W^{\alpha,-} & -i\frac{g}{2}W^{\alpha,+} \\
i\frac{g}{2}W^{\alpha,+} & -ieA^{\alpha} + ic_{W^{+} - Z^{\alpha}} & 0 \\
i\frac{g}{2}W^{\alpha,-} & 0 & ieA^{\alpha} - ic_{W^{+} - Z^{\alpha}}
\end{pmatrix},
\]

where the coefficient \( c_{W^{+} - W^{-}} = \rho G/2 - g'^2/G \).
The mass matrices have the form $\tilde{\sigma}_{0,VV} = \text{dia}\{0, -\rho G^2 v^2/4, -g^2 v^2/4, -g^2 v^2/4\}$ and $\tilde{\sigma}_{0,\xi\xi} = \text{dia}\{-\rho^2 G^2 v^2/4cZ', -g^2 v^2/4, -g^2 v^2/4\}$.

From the pre-standard form, we can define covariant differentials, as in the [32], and collect all quadratic terms into the standard form. We provide all matrices in the appendix.

**D. The Schwinger proper time and the heat kernel method**

Since the one-loop effective Lagrangian is Gaussian, we can use the functional integral to integrate out all quantum fields, i.e. to take into account the contributions of quantum corrections or quantum fluctuations of low energy DOFs. And their contributions to the $L_{\text{eff}}$ can be elegantly and concisely expressed

$$\int_x L_{1-\text{loop}} = \text{Tr} \left( \frac{1}{2} \sum_{i,j} (\text{Tr} \ln \tilde{\sigma}_{ij} - \text{Tr} \ln \tilde{\sigma}_{ij} + \text{Tr} \ln (1 - \tilde{X}^\dagger_{ij} \tilde{X}_{ij})) \right).$$

To extract the desired divergence structures, we need to expand this compact expression. The expansion of logarithm is simply expressed by the following formula

$$\langle x|\ln(1 - X)|y \rangle = -\langle x|X|y \rangle - \frac{1}{2} \langle x|X.X|y \rangle - \frac{1}{3} \langle x|X.X.X|y \rangle - \frac{1}{4} \langle x|X.X.X.X|y \rangle + \ldots,$$

and here the $X$ should be understood as an operator (a matrix) which acts on the quantum states of the right side.

To evaluate the trace, we will use the Schwinger proper time method and the heat kernel method [25]. In this method, the standard propagators can be expressed as

$$\langle x|\tilde{\sigma}_{ij}^{-1,ab}|y \rangle = \int_0^\infty \frac{d\tau}{(4\pi\tau)^{\frac{d}{2}}} \exp\{-\epsilon_F \tau\} \exp\left(-\frac{z^2}{4\tau}\right) H^\mu_{ij,ab}(x, y; \tau),$$

$$\langle x|\tilde{\sigma}_{ij}^{-1,ab}|y \rangle = \int_0^\infty \frac{d\tau}{(4\pi\tau)^{\frac{d}{2}}} \exp\{-\epsilon_F \tau\} \exp\left(-\frac{z^2}{4\tau}\right) H^\mu_{ij,ab}(x, y; \tau),$$

where the $\epsilon_F$ is the Feynman prescription which will be taken to vanish, and $z = y - x$. The integral over the proper time $\tau$ and the factor $1/(4\pi\tau)^{\frac{d}{2}} \exp(-z^2/(4\tau))$ conspire to separate the divergent part of the propagator. And the $H(x, y; \tau)$ is analytic with reference to the arguments $z$ and $\tau$, which means that $H(x, y; \tau)$ can be analytically expanded with reference to both $z$ and $\tau$. Then we have
\[ H(x, y; \tau) = H_0(x, y) + H_1(x, y)\tau + H_2(x, y)\tau^2 + \cdots, \]  
\[ H_i(x, y) = H_i(x, y)|_{x=y} + z^\alpha \partial_\alpha H_i(x, y)|_{x=y} + \frac{1}{2}z^\alpha z^\beta \partial_\alpha \partial_\beta H_i(x, y)|_{x=y} + \cdots \]  
where \( H_0(x, y), H_1(x, y), \) and, \( H_2(x, y) \) are the Seeley-De Witt coefficients. The coefficient \( H_0(x, y) \) is the Wilson phase factor, which indicates the phase change of a quantum state from the point \( x \) to the point \( y \) and reads
\[ H_0(x, y) = \exp \int_x^y \Gamma(z) \cdot dz, \]  
where \( \Gamma(z) \) is the affine connection defined on the coordinate point \( z \). Higher order coefficients are determined by the lower ones by the following recurrence relation
\[ (1 + n + z^\mu D_{\mu, x})H_{n+1}(x, y) + (D_x^2 + \sigma)H_n(x, y) = 0. \]  
All these Seeley-De Witt coefficients are gauge covariant with respect to the gauge transformation.

The divergence counting rule of the integral over the coordinate space \( x \) and the proper time \( \tau \) can be established as
\[ [z^\mu]_d = 1, \quad [\tau]_d = -2, \]  
It is easy to evaluate the \( Tr \ln \Box_V V \) and \( Tr \Box_{\bar{\epsilon}c} \) by directly using the result of the heat kernel method, which reads
\[ \varepsilon Tr \ln \Box_V V = \int_x \left[ tr[\sigma_0,V V \sigma_2,V V] + \frac{8}{3} \left( \frac{1}{4} \Gamma_{V,\mu\nu} \Gamma_{V,\mu\nu} \right) \right. \]  
\[ + \frac{1}{2} \left. tr[\sigma_2,V V \sigma_2,V V] \right], \]  
\[ \varepsilon Tr \ln \Box_{\bar{\epsilon}c} = \int_x \left[ \frac{2}{3} \left( \frac{1}{4} \Gamma_{V,\mu\nu} \Gamma_{V,\mu\nu} \right) \right], \]  
from these results, to extract the divergences of quadratic and logarithm is straightforward.

For the contributions of terms \( Tr \ln \Box_{\bar{\xi} \bar{\xi}} \) and \( Tr \ln \left( 1 - \bar{X}^\mu \Box_V^{-1} \bar{X} \bar{X}^\mu \Box_{\xi, \xi}^{-1} \right) \), it needs somewhat labor. Below we list some crucial steps of calculations. The first two relations are about the action of the covariant differential on the propagators, which read
\[ \langle x|D^a_{\alpha}|\Box^{-1,ab}_{V;\mu\nu}|y \rangle = \int d\lambda \frac{1}{(4\pi \lambda)^{d/2}} \exp\{-\epsilon_F \tau\} \exp\left(-\frac{z^2}{4\lambda}\right) \]  
\[ \times \left( \frac{z^\alpha}{2\lambda} + D_\alpha \right) H(x, y; \lambda), \]  
\[ \langle x|D^a_{\alpha}D^b_{\beta}|\Box^{-1,ab}_{V;\mu\nu}|y \rangle = \int d\lambda \frac{1}{(4\pi \lambda)^{d/2}} \exp\{-\epsilon_F \tau\} \exp\left(-\frac{z^2}{4\lambda}\right) \]  
\[ \times \left[ \frac{z^\alpha z^\beta}{4\lambda^2} - \frac{g_{\alpha \beta}}{2} + \frac{1}{2\lambda} (z_\alpha D_\beta + z_\beta D_\alpha) + D_\alpha D_\beta \right] H(x, y; \lambda) \]  
(171)
The second relations are about the Short distance expansion, which make it possible to covariantly expand the external fields over the coordinate space, and are defined as

\[
\begin{align*}
\tilde{X} D \tilde{X} &= \tilde{X} \partial \tilde{X} + \tilde{X} \Gamma_{W \tilde{X}} \tilde{X} - \tilde{X} \tilde{X} \Gamma_{\xi}, \\
\tilde{X} D D \tilde{X} &= \tilde{X} \partial \partial \tilde{X} + \tilde{X} \Gamma_{W \tilde{X}} \tilde{X} + \tilde{X} \tilde{X} \Gamma_{\xi} \Gamma_{\xi} - 2 \tilde{X} \Gamma_{W \tilde{X}} \Gamma_{\xi} \\
&+ 2 \tilde{X} \Gamma_{W \tilde{X}} \partial \tilde{X} + 2 \tilde{X} \partial \partial \tilde{X} \Gamma_{\xi} + \tilde{X} \partial \Gamma_{W \tilde{X}} \tilde{X} - \tilde{X} \tilde{X} \partial \Gamma_{\xi}.
\end{align*}
\]  

(172)

The rest of corresponding relevant integrals are based on these two relations given in Eq. (171) and Eq. (172). So that after dropping those quartic divergences and only keeping terms up to \(O(p^4)\), we can get the following results about the logarithm and trace of the terms \(Tr \ln \Box_{\xi \xi} \) and \(Tr \ln \left(1 - \tilde{X} \Box_{V;\mu} \tilde{X} \Box_{\xi \xi}^{-1}\right)\)

\[
\begin{align*}
Tr \ln \Box_{\xi \xi} &= \int_x \left[ tr[\sigma_0, \xi \xi \sigma_2, \xi \xi] + tr[\sigma_0, \xi \xi \sigma_4, \xi \xi] \\
&+ \frac{2}{3} \left( \frac{1}{4} \Gamma_{\xi \mu \nu} \Gamma_{\xi \nu \mu} \right) + \frac{1}{2} tr[\sigma_2, \xi \xi \sigma_2, \xi \xi] \right], \quad (173)
\end{align*}
\]

\[
\begin{align*}
Tr \ln \left(1 - \tilde{X} \Box_{V;\mu} \tilde{X} \Box_{\xi \xi}^{-1}\right) &= \frac{1}{\epsilon} \int_x (p4t + p3t + p2t). \quad (174)
\end{align*}
\]

The \(\Gamma_{\mu \nu}\) is the field strength tensor corresponding to the affine connection \(\Gamma_{\mu}\). We have used the dimension regularization and the modified minimal subtraction scheme to extract the divergent structures in this step. The \(p4t\) represents the contributions of four propagators \(tr(\tilde{X} \Box_{W;\mu} \tilde{X} \Box_{\xi \xi}^{-1} \tilde{X} \Box_{W;\nu} \tilde{X} \Box_{\xi \xi}^{-1})\), which reads

\[
p4t = \frac{g_{\mu \nu} g_{\alpha \beta'}}{6} \left[ \frac{g_{\alpha \beta' \beta''}}{4} tr[2 \tilde{X}_{\alpha \beta'} \tilde{X}_{\alpha' \beta',} \tilde{X}_{\alpha' \beta',}] \\
+ 2 \tilde{X}_{\alpha \beta'} \tilde{X}_{\alpha' \beta',} + \tilde{X}_{\alpha \beta'} \tilde{X}_{\alpha' \beta',} + \tilde{X}_{\alpha \beta'} \tilde{X}_{\alpha' \beta',} + \tilde{X}_{\alpha \beta'} \tilde{X}_{\alpha' \beta',} + \tilde{X}_{\alpha \beta'} \tilde{X}_{\alpha' \beta',} \right] \\
- \frac{g_{\alpha \beta' \beta''}}{16} tr[\tilde{X}_{\alpha \beta'} \sigma_{0, \nu} \tilde{X}_{\alpha' \beta',} \tilde{X}_{\alpha' \beta',} + \tilde{X}_{\alpha \beta'} \sigma_{0, \nu} \tilde{X}_{\alpha' \beta',} \tilde{X}_{\alpha' \beta',} \\
+ \tilde{X}_{\alpha \beta'} \sigma_{\xi, \xi} \tilde{X}_{\alpha' \beta',} \tilde{X}_{\alpha' \beta',} + \tilde{X}_{\alpha \beta'} \sigma_{\xi, \xi} \tilde{X}_{\alpha' \beta',} \tilde{X}_{\alpha' \beta',} \\
+ \tilde{X}_{\alpha \beta'} \sigma_{\xi, \xi} \tilde{X}_{\alpha' \beta',} \tilde{X}_{\alpha' \beta',} + \tilde{X}_{\alpha \beta'} \sigma_{\xi, \xi} \tilde{X}_{\alpha' \beta',} \tilde{X}_{\alpha' \beta',} \\
+ \tilde{X}_{\alpha \beta'} \sigma_{\xi, \xi} \tilde{X}_{\alpha' \beta',} \tilde{X}_{\alpha' \beta',} + \tilde{X}_{\alpha \beta'} \sigma_{\xi, \xi} \tilde{X}_{\alpha' \beta',} \tilde{X}_{\alpha' \beta',} \right]. \quad (175)
\]

The \(p3t\) represents the contributions of three propagators \(tr(\tilde{X} \Box_{W;\mu} \tilde{X} \Box_{\xi \xi}^{-1} \tilde{X} \Box_{W;\nu} \tilde{X} \Box_{\xi \xi}^{-1})\), which reads

\[
p3t = \frac{1}{24} g_{\alpha \beta' \beta''} g_{\mu \nu} \left[ \tilde{X}_{\alpha \beta'} \sigma_{0, \nu} \tilde{X}_{\alpha' \beta',} + \tilde{X}_{\alpha \beta'} \sigma_{0, \nu} \tilde{X}_{\alpha' \beta',} \\
+ \tilde{X}_{\alpha \beta'} \sigma_{\xi, \xi} \tilde{X}_{\alpha' \beta',} + \tilde{X}_{\alpha \beta'} \sigma_{\xi, \xi} \tilde{X}_{\alpha' \beta',} \right].
\]
Further divided into six groups:

\[ + \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \sigma_{0, \xi} + \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \sigma_{0, \xi} \]

\[ - \frac{1}{4} g_{\alpha \beta} g_{\mu \nu} tr[\dot{X}_{01} \dot{X}_{01} \dot{X}_{01}] . \]  

(176)

The p2t represents the contributions of two propagators \( tr(\dot{X} \otimes W X \otimes \xi) \), which can be further divided into six groups:

\[ p2t = t_{AA} + t_{AB} + t_{AC} + t_{BB} + t_{BC} + t_{CC} , \]  

(177)

\[ t_{AA} = \frac{g_{\nu \mu}}{8} \left( \frac{g^{\alpha \beta \gamma \delta \eta}}{6} - \frac{2 g^{\alpha \beta} g^{\gamma \delta \eta}}{3} - \frac{g^{\alpha \beta \gamma} g^{\delta \eta}}{3} + g^{\alpha \beta} g^{\gamma \delta} \right) tr[\dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \sigma_{0, V V} D_\delta D_\gamma X_{\alpha \beta} \gamma \delta \sigma_{0, \xi}] \]

\[ + \frac{g_{\nu \mu}}{8} \left( \frac{g^{\alpha \beta \gamma \delta \eta}}{6} - \frac{g^{\alpha \beta \gamma \delta} \eta}{3} - \frac{2 g^{\alpha \beta \gamma} g^{\delta \eta}}{3} + g^{\alpha \beta \gamma \delta} \right) tr[\dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \sigma_{0, V V} D_\delta D_\gamma X_{\alpha \beta} \gamma \delta \sigma_{0, \xi}] \]

\[ + \frac{g^{\alpha \beta \gamma \delta \eta}}{24} \left[ g_{\mu \nu} tr[\dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \sigma_{0, V V} X_{\alpha \beta} \gamma \delta \sigma_{0, \xi}] + g_{\mu \nu} g_{\nu \sigma} tr[\dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \sigma_{0, V V} X_{\alpha \beta} \gamma \delta \sigma_{0, \xi}] \right] \]

\[ + \frac{g^{\alpha \beta \gamma \delta \eta}}{12} \left[ g_{\mu \nu} tr[\dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \sigma_{0, V V} X_{\alpha \beta} \gamma \delta \sigma_{0, \xi}] + g_{\mu \nu} g_{\nu \sigma} tr[\dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \sigma_{0, V V} X_{\alpha \beta} \gamma \delta \sigma_{0, \xi}] \right] , \]  

(178)

\[ t_{AB} = \frac{g_{\mu \nu} g_{\nu \mu}}{4} \left[ (g^{\alpha \beta \gamma \delta \eta} - \frac{1}{3} g^{\alpha \beta \gamma \delta \eta}) tr[\dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \sigma_{0, V V} D_\delta D_\gamma X_{\alpha \beta} \gamma \delta \sigma_{0, \xi}] \right] \]

\[ + (g^{\alpha \beta \gamma \delta \eta} - \frac{2}{3} g^{\alpha \beta \gamma \delta \eta}) tr[\dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \sigma_{0, V V} D_\delta D_\gamma X_{\alpha \beta} \gamma \delta \sigma_{0, \xi}] , \]

(179)

\[ t_{AC} = - \frac{g_{\mu \nu} g_{\nu \mu}}{4} tr[\dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \sigma_{0, V V} X_{\alpha \beta} \sigma_{0, \xi}] \]

\[ - \partial_{\alpha} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \sigma_{0, V V} X_{\alpha \beta} \sigma_{0, \xi} + X_{\alpha \beta} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \sigma_{0, \xi} \sigma_{0, V V} \partial_{\alpha} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \sigma_{0, \xi} \]

\[ + X_{\alpha \beta} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \sigma_{0, \xi} \sigma_{0, V V} X_{\alpha \beta} \sigma_{0, \xi} \sigma_{0, V V} \partial_{\alpha} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \sigma_{0, \xi} \]

\[ + \frac{1}{4} g^{\alpha \beta \gamma \delta} tr[\dot{g}_{\mu \nu} \dot{g}_{\nu \mu} \dot{g}_{\mu \nu} \dot{g}_{\nu \mu} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \sigma_{0, \xi}] \]

\[ + \frac{1}{6} g^{\alpha \beta \gamma \delta} tr[\dot{g}_{\mu \nu} \dot{g}_{\nu \mu} \dot{g}_{\mu \nu} \dot{g}_{\nu \mu} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \dot{X}_{01} \sigma_{0, \xi}] , \]  

(180)

\[ t_{BB} = \frac{g_{\nu \mu} g_{\nu \mu}}{4} tr[\dot{X} \dot{X} \sigma_{0, V V} X_{\alpha \beta} \sigma_{0, \xi}] , \]

(181)

\[ t_{BC} = \frac{g_{\alpha \beta} g_{\mu \nu} g_{\nu \mu} tr[\dot{X} \dot{X} \dot{X} \sigma_{0, V V} X_{\alpha \beta} \sigma_{0, \xi}] , \]  

(182)

where the trace is to sum over the group indices, and the high rank tensors \( g^{\alpha \beta \gamma \delta} \) and \( g^{\alpha \beta \gamma \delta \mu} \) are symmetric on all indices and defined as

\[ g^{\alpha \beta \gamma \delta} = g^{\alpha \beta} g^{\gamma \delta} + g^{\alpha \gamma} g^{\beta \delta} + g^{\alpha \delta} g^{\beta \gamma} , \]

(183)

\[ g^{\alpha \beta \gamma \delta \mu} = g^{\alpha \beta} g^{\gamma \delta \mu} + g^{\alpha \gamma} g^{\beta \delta \mu} + g^{\alpha \delta} g^{\beta \gamma \mu} + g^{\alpha \mu} g^{\beta \gamma \delta} + g^{\alpha \nu} g^{\beta \gamma \delta \mu} . \]  

(184)
The formula given in Eq. (175—182), can reduce to the ones given the $SU(2)$ case [22].

E. The renormalization group equations

Substitute the relevant matrices in the appendix A into the formula given in Eqs. (175—182) and after some tedious algebraic manipulation, we can extract out the complete RGEs of the system, which will be provided in our next paper with including the effective Higgs sector [23].

To derive the RGEs, we have taken into account the contributions of Fermions in the SM to the gauge couplings $g$ and $g'$. For the sake of simplicity, we do not consider the fact that top quarks are much heavier than other fermions (b quarks), so there is no contribution of fermions to the ACs. Strictly speaking, we should consider several possible cases for the EWCL of Higgs: 1) Higgs scalar is heavier than top quarks, 2) Higgs scalar is as heavy as top quarks, and 3) Higgs scalar is lighter than top quarks. We will conduct such a study in the future.

The complete RGEs, which are expressed in the ECs $C_i$, are quite complicated and difficult to understand. We will provide in our next paper. But here we would like to provide a simplifies version to extract some helpful features about the RGEs. We can expand the effective couplings $C_i$ in $\alpha_i g^2(\alpha_i g'^2)$ and neglect those higher powers of $\alpha_i g^2(\alpha_i g'^2)$, and get the following simplified version of RGEs

\[
8\pi^2 \frac{dg}{dt} \approx \frac{g^3}{2} \left\{ -\frac{10}{3} + \frac{1}{12} + \alpha_1 g^2 + \frac{7\alpha_2 g^2}{6} \right. \\
+ \frac{\alpha_3}{2\rho} \left[ 3g^2 \rho + g^2 (1 + 31\rho) \right] - 2\alpha_8 g^2 + \frac{9\alpha_9 g^2}{2} \left\}, \right.	ag{185}
\]

\[
8\pi^2 \frac{dg'}{dt} \approx \frac{g'^3}{2} \left\{ \frac{13}{2} - \frac{\rho}{3} + \frac{\rho^2}{12} + 2\alpha_1 g^2 - \alpha_2 g^2 (-4 + \rho) + \frac{7\alpha_3 g^2}{3} \right\}, \tag{186}
\]

\[
8\pi^2 \frac{d\alpha_1}{dt} \approx \frac{\rho}{6} - \frac{\rho^2}{12} + 2\alpha_1 g^2 + \frac{\alpha_2 g^2 (6 + \rho)}{2} - \frac{\alpha_3 g^2 (22 + 3\rho)}{6} \right. \\
+ \alpha_8 g^2 - \frac{\alpha_9 g^2 (-4 + \rho)}{2}, \tag{187}
\]

\[
8\pi^2 \frac{d\alpha_2}{dt} \approx -\frac{1}{24} + \frac{1}{12\rho} - \frac{\rho}{6} + \frac{\rho^2}{12} - \frac{\alpha_1 g^2}{2} \\
+ \frac{1}{24} \left[ \alpha_2 \left( -12g^2 (-1 + \rho) + g^2 (46 + \rho) \right) \right] \\
+ \frac{1}{24\rho} \left[ \alpha_3 \left( 24g^2 \rho + g^2 \left( -17 + 47\rho + \rho^2 \right) \right) \right] \\
+ \frac{1}{4} \left[ \alpha_4 \left( -2g^2 + g^2 (-4 + \rho) \right) \right]
\]

35
\[
\begin{align*}
8\pi^2 \frac{d\alpha_3}{dt} &\approx \frac{1}{24} - \frac{1}{12\rho} + \frac{1}{4}[\alpha_1 g^2 (-2 + \rho)] + \frac{1}{6}[\alpha_2 g^2 (-7 + 2\rho)] \\
&\quad + \frac{1}{24\rho}[\alpha_3 (-6g^2 (-1 - 8\rho + \rho^2) + g^2 (-1 + 110\rho + 14\rho^2))] \\
&\quad - \frac{1}{8\rho}[\alpha_4 (5g^2 \rho + g^2 (1 + 13\rho))] + \frac{1}{4}[\alpha_5 (g^2 + 13g^2 \rho + g^2 \rho)] \\
&\quad - \frac{1}{8\rho} \frac{8}{\alpha_6 (5g^2 \rho + g^2 (1 + 13\rho))} + \frac{1}{4}[\alpha_7 (g^2 \rho + g^2 (1 + 13\rho))] \\
&\quad + \alpha_8 g^2 \rho + \frac{1}{6}[\alpha_9 (12g^2 + g^2 (-3 + 2\rho))], \tag{188}
\end{align*}
\]

\[
\begin{align*}
8\pi^2 \frac{d\alpha_4}{dt} &\approx \frac{1}{12} - \frac{1}{12\rho^2} - \frac{\rho^2}{12} + \frac{1}{24}[\alpha_2 g^2 (7 - 2\rho^2)] \\
&\quad - \frac{1}{24\rho}[\alpha_3 g^2 (-41 + 42\rho + 2\rho^2)] \\
&\quad + \frac{1}{2\rho}[\alpha_4 (g^2 (-2 + 11\rho) + g^2 (2 + 9\rho - \rho^2))] \\
&\quad + \frac{1}{2}[\alpha_5 (3g^2 + g^2 (3 + \rho))] + \frac{1}{\rho}[\alpha_6 (g^2 (-1 + \rho) + g^2 (1 + 2\rho))] \\
&\quad + 2\alpha_7 g^2 + \frac{1}{24\rho}[\alpha_9 g^2 (7 + 80\rho - 2\rho^2)], \tag{189}
\end{align*}
\]

\[
\begin{align*}
8\pi^2 \frac{d\alpha_5}{dt} &\approx -\frac{1}{12} - \frac{1}{24\rho^2} + \frac{\rho^2}{12} + \frac{1}{12\rho}[\alpha_2 g^2 (-2 + \rho^2)] \\
&\quad + \frac{1}{24\rho}[\alpha_3 g^2 (-11 + 18\rho + 2\rho^2)] \\
&\quad + \frac{1}{2}[\alpha_4 (-5g^2 \rho + g^2 (-1 - 2\rho + \rho^2))] + \alpha_5 \left(-g^2 (-1 + \rho) - \frac{g^2}{\rho}\right) \\
&\quad - \frac{1}{2}[\alpha_6 (g^2 + 2g^2)] + \frac{1}{\rho}\left[\alpha_7 (g^2 - g^2) (1 + 2\rho)\right] \\
&\quad + \frac{1}{12\rho}[\alpha_9 g^2 (-2 - 40\rho + \rho^2)], \tag{190}
\end{align*}
\]

\[
\begin{align*}
8\pi^2 \frac{d\alpha_6}{dt} &\approx \frac{1}{12\rho^2} - \frac{1}{12\rho} - \frac{\rho^2}{12} \\
&\quad - \frac{1}{24\rho}[\alpha_2 (-24g^2 (-2 + \rho) \rho + g^2 (7 - 20\rho + 6\rho^2 + \rho^3))] \\
&\quad + \frac{1}{48\rho}[\alpha_3 g^2 (-62 + 53\rho + 44\rho^2 - 2\rho^3)] \\
&\quad + \frac{1}{4}[\alpha_4 (g^2 (-16 + \rho) \rho + g^2 (-2 - 3\rho + 7\rho^2))] \tag{191}
\end{align*}
\]
\[ 8\pi d\alpha \approx \frac{1}{2}\rho \left[ \alpha_5 (g^2 (9 - 2\rho) \rho + 2g^2 (-1 - 3\rho + \rho^2)) \right] + \frac{1}{4\rho} \left[ \alpha_6 (g^2 (-4 + 2\rho + \rho^2) - g^2 (-2 + 9\rho + \rho^2)) \right] + \frac{1}{2\rho} \left[ \alpha_7 (g^2 (2 + 11\rho - 2\rho^2) + 2g^2 (-2 - 2\rho + \rho^2)) \right] \]

\[ - \frac{1}{24\rho} \left[ \alpha_9 g^2 (7 + 92\rho + 2\rho^2 + \rho^3) \right] + \frac{1}{\rho} \left[ 2\alpha_a (-g^2 + g^2 (1 + 2\rho)) \right], \quad (192) \]

\[ 8\pi^2 \frac{d\alpha_7}{dt} \approx \frac{1}{24\rho^2} + \frac{1}{12\rho} - \frac{\rho^2}{24} - \frac{\rho^2}{12} \]

\[ + \frac{1}{48\rho} \left[ \alpha_2 (-48g^2 (-2 + \rho) \rho + g^2 (8 - 40\rho + 12\rho^2 + 11\rho^3)) \right] + \frac{1}{48\rho} \left[ \alpha_3 g^2 (17 - 56\rho - 8\rho^2 + 11\rho^3) \right] \]

\[ + \frac{1}{4\rho} \left[ \alpha_4 (g^2 (11 - 6\rho) \rho + g^2 (-2 + 11\rho + 2\rho^2)) \right] + \frac{1}{2\rho} \left[ \alpha_5 (g^2 \rho (1 + 2\rho) + g^2 (1 + 3\rho)) \right] \]

\[ + \frac{1}{4\rho} \left[ \alpha_6 (-2g^2 \rho (3 + \rho) + g^2 (-4 + 5\rho + 2\rho^2)) \right] + \frac{1}{2\rho} \left[ \alpha_7 (7g^2 \rho + g^2 (-1 - 17\rho + 2\rho^2)) \right] \]

\[ + \frac{1}{48\rho} \left[ \alpha_9 g^2 (8 + 136\rho + 28\rho^2 + 11\rho^3) \right] - \frac{1}{\rho} \left[ \alpha_a (g^2 + g^2 (-1 + \rho)) \right], \quad (193) \]

\[ 8\pi^2 \frac{d\alpha_8}{dt} \approx -\frac{1}{12\rho} + \frac{\rho^2}{12} - \alpha_1 g^2 - \frac{7\alpha_2 g^2}{6} \]

\[ + \frac{1}{2\rho} \left[ \alpha_3 (-3g^2 \rho + g^2 (-1 - \rho + 2\rho^2)) \right] + 6\alpha_8 g^2 + \frac{1}{2} \left[ \alpha_9 g^2 (3 + 2\rho) \right], \quad (194) \]

\[ 8\pi^2 \frac{d\alpha_9}{dt} \approx \frac{1}{12\rho} - \frac{\rho^2}{12} - \frac{1}{4} \left[ \alpha_1 g^2 (-2 + \rho) \right] + \alpha_2 \left( g^2 + \frac{g^2 (4 - 9\rho)}{24} \right) \]

\[ + \frac{1}{8\rho} \left[ \alpha_3 (2g^2 \rho^2 + g^2 (4 - 21\rho - 9\rho^2)) \right] \]

\[ + \frac{1}{8\rho} \left[ \alpha_4 (9g^2 \rho + g^2 (1 + \rho - 2\rho^2)) \right] \]

\[ + \frac{1}{4\rho} \left[ \alpha_5 (3g^2 \rho + g^2 (-1 - \rho + 2\rho^2)) \right] \]

\[ + \frac{1}{8\rho} \left[ \alpha_6 (9g^2 \rho + g^2 (1 + 13\rho)) \right] \]

\[ - \frac{1}{4\rho} \left[ \alpha_7 (-3g^2 \rho + g^2 (1 + 13\rho)) \right] - \frac{1}{4} \left[ \alpha_8 g^2 (2 + \rho) \right] \]
\[
8\pi^2 \frac{d\alpha_a}{dt} \approx \frac{-1}{16\rho^2} \frac{\rho}{8} - \frac{\rho^4}{16} + \frac{1}{16\rho} \left[ \alpha_2 g'^2 \left( 1 - 3\rho^3 \right) \right]
\]
\[
- \frac{1}{16\rho} \left[ \alpha_3 g^2 \left( -5 + 7\rho - 4\rho^2 + \rho^3 \right) \right]
\]
\[
- \frac{1}{4\rho} \left[ \alpha_4 \left( g^2 \left( -1 + \rho + \rho^2 \right) + g'^2 \left( -4 + \rho + 3\rho^2 \right) \right) \right]
\]
\[
+ \alpha_5 \left( -3g^2 + \frac{g'^2}{\rho} - \frac{g^2}{\rho} \right)
\]
\[
+ \frac{1}{4\rho} \left[ \alpha_6 \left( g^2 \left( 10 - 7\rho - 3\rho^2 \right) + g'^2 \left( -4 + 10\rho + 7\rho^2 \right) \right) \right]
\]
\[
+ \frac{1}{2\rho} \left[ \alpha_7 \left( g^2 \left( 5 - 3\rho - 2\rho^2 \right) + 2g'^2 \left( -1 + 2\rho + \rho^2 \right) \right) \right]
\]
\[
+ \frac{1}{16\rho} \left[ \alpha_9 g^2 \left( 1 + 16\rho - 8\rho^2 - 3\rho^3 \right) \right]
\]
\[
+ \frac{1}{\rho} \left[ \alpha_a \left( 3g^2 + g'^2 \left( -3 - 11\rho + 2\rho^2 \right) \right) \right],
\]
\[
8\pi^2 \frac{dv}{dt} \approx \frac{v}{2} \left\{ \frac{-3g^2}{4} + \frac{3g'^2}{4} + \frac{g^2}{4\rho} - \frac{g'^2}{\rho} \right\},
\]
\[
8\pi^2 \frac{d\rho}{dt} \approx \frac{-9g^2}{4} + \frac{3g^2\rho}{4} - \frac{3g'^2\rho}{4} + \frac{3g^2\rho^2}{2}.
\]

One feature about the RGEs given in Eqs. (185—198) is that quartic ACs (\(\alpha_4, \alpha_5, \alpha_6, \alpha_7,\) and \(\alpha_a\)) will not contribute directly to the \(\beta\) functions of quadratic ACs (\(\alpha_1\) and \(\alpha_8\)), and the contribution is mediated by the triple ACs (\(\alpha_2, \alpha_3,\) and \(\alpha_9\)), and vice versa (such a statement only exist in the approximation we have used). Similarly, quartic ACs do not contribute directly to the \(\beta\) function of gauge couplings \(g\) and \(g'\).

Another remarkable feature is that the contributions of the quartic ACs dominate the \(\beta\) functions of the triple and quartic ACs and those of the triple ACs dominate those of quadratic ACs, if we estimate the contributions of ACs by using the experimental constraints given in Eq. (1).

The third feature is that the ACs in the \(\beta\) functions always appear with gauge couplings (\(g^2\) and \(g'^2\)) in the form \(\alpha_i g^2 (g'^2)\), and this fact is related with the parameterization of the ACs in Eq. (17). Actually, according to our modified power counting rule, the higher power of these combinations should also belong to \(O(p^0)\) and should appear at the \(\beta\) functions. Here just for the sake of simplicity and in order to qualitatively understand the running behavior of the ACs, we have intentionally omitted higher power terms. The complete RGEs will be provided in our coming paper. However, the numerical analysis is
based on the complete RGEs.

To get a negative $S$ parameter, large positive initial $\alpha_1$ at the matching scale is one possible solution, as many new physics models propose. Here, based on the RGEs of the $\alpha_1$, we regard it is possible to generate a negative $S$ through the radiative corrections. As indicated by the $\beta$ function of $\alpha_1$, such a mechanism is possible either for a large $\rho$ (which make the leading contributions smaller or even negative) or for large triple ACs, especially the $\alpha_2$ and $\alpha_3$ due to their terms have a factor 3 compared with $\alpha_8$ and $\alpha_9$ (only if they have different signs, and $\alpha_2$ is negative). But considering the fact that $\rho$ is constrained by the $T$ parameter, and according to the $\beta$ function of $\rho$, we conclude that a reasonable way is large triple ACs. It is no hard to find solutions to the negative $S$ parameter at $m_Z$ with positive initial value at the matching scale in the parameter space, and the region for such a solution is quite large when considering the present experimental limits on the ACs given in Eq. (1). Such a new mechanism might be a good news to some technicolor models, where the theoretical predictions on $S$ contradict with the present experimental measurement.

In the case when all ACs are of order $\frac{1}{(4\pi)^2}$, we can neglect those terms related with $\alpha_i$. In this limit, our result should reduce to the result of the DM, which has the following form

\begin{align*}
8\pi^2 \frac{dg}{dt} &= -\frac{13}{4} g^3, \\
8\pi^2 \frac{dg'}{dt} &= \frac{25}{4} g'^3, \\
8\pi^2 \frac{d\alpha_1}{dt} &= \frac{1}{12}, \\
8\pi^2 \frac{d\alpha_2}{dt} &= -\frac{1}{24}, \\
8\pi^2 \frac{d\alpha_3}{dt} &= -\frac{1}{24}, \\
8\pi^2 \frac{d\alpha_4}{dt} &= \frac{1}{12}, \\
8\pi^2 \frac{d\alpha_5}{dt} &= -\frac{1}{24}, \\
8\pi^2 \frac{dv}{dt} &= -\frac{3}{8} (2g^2 + g'^2) v, \\
8\pi^2 \frac{d\rho}{dt} &= \frac{3}{4} g'^2,
\end{align*}

while the rest of ACs vanish and do not develop, according to the calculation of this method. These constants in the $\beta$ functions of $\alpha_i$’s come from the contribution of Goldstone bosons.
Below are several comments on the RGEs up to $O(p^4)$. 1) The RGEs complete the EWCL as an effective field theory method to effectively describe the SM (without Higgs) below TeV, since the RGEs are one of the basic ingredients of the effective field method. 2) Although the RGE method is essentially equivalent to the calculations of infinite Feynman diagrams by keeping to the leading logarithm results, it simplifies the calculation of loop processes and makes the matching procedure much easier. 3) Not like the DM, the RGE method includes the contributions of all low energy DOFs, not only those of Goldstone bosons, but also those of vector bosons and those of mixing terms of these two kinds of bosons as well. As we have shown in the $SU(2)$ case [22], the last two kinds of contributions might make the predictions of these method complete different. 4) The RGEs also provide a new powerful tool for the comparative study on the possible new physics near TeV region through the study of vector boson scatterings. The effects of the heavy DOF to the low energy physics are quite transparent in this method, only the masses (which determines the matching scale) and the couplings with the low energy DOF play a part. Compared with the traditional procedure (to formulate Feynman rules, construct Feynman diagrams, calculate Feynman integral, renormalize the full theory, and extract radiative corrections), the RGEs have integrated these necessary steps into a powerful and ease-to-use chip.

About the simplified-version RGEs given in Eqs. (185—198), we would like to point out that in the effective Lagrangian of the Higgs model, the AC $\alpha_5$ can reach $O(1)$. So the terms contain $\alpha_5$ can make the behavior of the ACs quite different from the predictions made by the DM\textsuperscript{1}, as we will show in the next section.

\textsuperscript{1}Most of the past works have considered how to extract the non-decoupling contributions of heavy Higgs. Therefore the tree-level contribution from the exchange of heavy Higgs should vanish in the decoupling limit. Their results indeed have discarded the tree-level contribution and are consistent with their assumptions. Since in the decoupling limit, the terms proportional to $v^2/m_H^2$ indeed can be safely neglected. (The result given by M. J. Herrero and E. R. Morales has correctly included the tree level contribution.). But unfortunately, people use those results to consider the Higgs effects even for the light Higgs case (say 200-600Gev), where $v^2/m_H^2$ is not a small number which can be safely neglected, and its radiative correction is not small as expected from the NPC rule.
VI. NUMERICAL ANALYSIS: THE APPLICATION OF THE RGE METHOD TO THE HIGGS EFFECTS

To solve the differential equations of RGEs (the numerical analysis in this section is conducted by using the complete RGEs, while the simplified-version ones given in Eqs. (185—198) can provide a qualitative understanding to the behaviors of the ACs. Quantitatively, the differences of results made by the complete and the simplified are neglectable, at least for the below case.), we need some basic inputs, which are also called the boundary conditions. The boundary conditions of the RGEs given in Eqs. (185—198) contain two classes: 1) the class of the low energy boundary conditions, where the parameters $g$, $g'$, and $v_0$ are fixed from the experimental values. We take the following experimental inputs

$$m_Z = 91.18 \text{ GeV}, \quad m_W = 80.33 \text{ GeV}, \quad \sin \theta_W(m_Z) = 0.2312, \quad \alpha_e(m_Z) = \frac{1}{128},$$

with these inputs, according to the definition of $g$, $g'$, and $v_0$, we have

$$g(m_Z) = 0.65, \quad g'(m_Z) = 0.36, \quad v_0(m_Z) = 246.708 \text{ GeV}.$$  

2) the class of the matching-scale boundary conditions, where the initial values of the ACs $\alpha_i$ are input by integrating out the heavy DOF and matching the full theory with the effective theory. For the heavy Higgs case, at the tree-level the matching procedure yields

$$\alpha_5(m_0) = \frac{v^2}{8m_0^2} = \frac{1}{4\lambda}, \quad \rho(m_0) = 1,$$

while the rest of ACs vanish. For the RGE of DM, we will set all ACs vanish expect that $\rho(m_0) = 1$, in order to exaggeratingly demonstrate the differences of these two methods.

The relations between the ACs and the precision test parameters, the quadratic vertices ($S$, $T$, and $U$) and the triple gauge vertices are determined as [6,33]

$$S = -16\pi\alpha_1,$$

$$T = \frac{\rho - 1}{\alpha_{em}},$$

$$U = -16\alpha_8,$$

$$g_1^Z - 1 = \frac{1}{c^2 - s^2} \frac{\rho - 1}{2} + \frac{1}{c^2(c^2 - s^2)} e^2 \alpha_1 + \frac{1}{s^2 c^2} e^2 \alpha_3,$$

$$g_1^1 - 1 = 0,$$

$$k_Z - 1 = \frac{1}{c^2 - s^2} \frac{\rho - 1}{2} + \frac{1}{c^2(c^2 - s^2)} e^2 \alpha_1.$$
\[ + \frac{1}{c^2} e^2 (\alpha_1 - \alpha_2) + \frac{1}{s^2} e^2 (\alpha_3 - \alpha_8 + \alpha_9), \quad (216) \]

\[ k_\gamma - 1 = \frac{1}{s^2} e^2 (-\alpha_1 + \alpha_2 + \alpha_3 - \alpha_8 + \alpha_9), \quad (217) \]

Below we conduct a comparative study on the predictions of the RGE method and the DM in the effective Lagrangian of the Higgs model.

As we know, the Higgs scalar’s effects include both the decoupled mass square suppressed part as shown in Eq. (24), and the nondecoupling logarithm part as shown explicitly in the RGEs of the DM. So we consider the following four cases to trace the change of roles of these two competing parts: 1) the light scalar case, with \( m_0 = 150 \) GeV, where the decoupling mass-square-suppressed part dominates; 2) the mediate heavy scalar case, with \( m_0 = 300 \) GeV, where the decoupling mass-square-suppressed part dominates; 3) the not too heavy scalar case, with \( m_0 = 450 \) GeV, where both contributions are important; 4) the very massive scalar case, with \( m_0 = 900 \) GeV, where the nondecoupling logarithm part dominates.

Fig. 1 is devoted to the first case, Fig. 2 to the second case, Fig. 3 to the third case, and Fig. 4 to the third case. We also scan the region with the mass of Higgs taken from 120 GeV to 420 GeV, and the result is given in Fig. 5 In Fig. 6, we compare the predictions of these two method to the ACs and to the precision test parameters, 1) the quadratic vertices parameters, \( S \) and \( T \) (the \( U \) parameter is quite small in both methods); 2) the triple gauge vertices, \( g_\gamma^2 - 1 \), \( k_Z - 1 \), and \( k_\gamma - 1 \).

Due to the large contributions of the leading terms and the fact that quartic ACs do not contribute to their \( \beta \) functions, the gauge couplings \( g \) and \( g' \) are quite dull to the effects of the ACs. So there is no viewable difference between these two methods, and we omit the running of these two gauge couplings.

The differences of the AC \( \alpha_1 \) in these two methods are small in these three cases we consider. The underlying reason is due to the large leading contributions in its \textit{beta} function, and due to the cancellation of the terms \( \alpha_2 \) and \( \alpha_3 \) (since \( \alpha_2 \) and \( \alpha_3 \) have the same signs).

The differences of these two methods are dramatic in the ACs \( \alpha_2, \alpha_3, \) and \( \alpha_4 \), when Higgs scalar is far from its decoupling limit, similar to the \( SU(2) \) case [22]. The differences of the AC \( \alpha_5 \) is the most dramatic, since the tree-level contribution is much larger than the loop corrections. As revealed in these figures, the heavier the Higgs, the smaller the differences between these ACs.

For the ACs \( \alpha_6, \alpha_7, \alpha_8, \alpha_9, \) and \( \alpha_a \), due to the fact that the DM predicts a vanished value, the differences are at the order of \( 10^{-4} \) in the first case (except \( \alpha_8 \) which is \( 10^{-6} \)), at \( 10^{-5} \) in the second, third and fourth cases (except \( \alpha_8 \) which is \( 10^{-7} \)). As the result
of the accidental symmetry—the custodial symmetry, the $\rho$ is always near the unit in all
these three cases.

When the Higgs scalar is far lighter than its decoupling limit, the differences between
these two methods are much more dramatic, as shown in Fig. 5. The tendency that the
differences between these two methods become smaller when the Higgs scalar is taken
heavier has been vividly outlined.

Such differences between these two methods can not be meaningfully detected in LHC
and LC for the $a_1$, $a_6$, $a_7$, $a_8$, $a_9$, and $a_{a}$, but can be hopefully detected for the $a_2$,
$\alpha_3$, $a_4$, and $a_5$ when the Higgs scalar is light. The detection power of these machines is
estimated as

\begin{align}
(a_1 \ a_8 \ \beta) & \sim O(0.001) - O(0.0001), \\
(a_2 \ a_3 \ a_9) & \sim O(0.01) - O(0.001), \\
(a_4 \ a_5 \ a_6 \ a_7 \ a_{a}) & \sim O(0.1) - O(0.01). \\
\end{align}

From Fig. 6, we see that from the triple gauge vertices, if the Higgs scalar is relatively
light (say 200 or 400 GeV or so), it is definitely possible for the experiments to distinguish
the predictions of these two methods.

VII. DISCUSSIONS AND CONCLUSIONS

In this paper we have formulated the EWCL in its mass eigenstates, and provided the
relations between the complete basis of the weak-interaction eigenstates and the one of
the mass eigenstates. We have modified the naive power rule in order to reliably extract
the large contributions of AOs and to control higher order contributions. We also have
studied the one-loop renormalization of the EWCL and derived its RGEs to the same
order. Theoretically, the RGEs complete the EWCL as an effective theory to describe the
SM below a few TeV. We have studied the EWCl of the Higgs model and have found that
after taking into account the contributions of Goldstone bosons, those of vector bosons,
and those of mixing terms between these two kinds of bosons as well, the effects of Higgs
can yield quite large triple ACs when the Higgs scalar is light.

It is helpful to compare the result of the RGE method and the DM with the underly-
ing renormalizable theory, the SM, to see why the trilinear couplings have such different
behavior in these two methods in the lower energy region. The terms proportional to
quartic couplings in the beta function of trilinear couplings, is equivalent to the contri-
bution of the diagram (in unitary gauge) given in Fig. 7. While in the SM, this diagram
corresponds to the two diagrams (in unitary gauge) given in Fig. 8. By contracting the Higgs line to a point, the Feynman diagrams in Fig. 8 reduce to the diagram in the Fig. 7. In the SM, these two kinds of diagrams will contribute to the trilinear couplings in both divergent and non-divergent terms. The divergent part will contribute to both a small finite constant term and a term with log dependence on the mass of Higgs, while the non-divergent part is proportional to $\frac{m_W^2}{m_H^2}$. The RGE method has correctly taken into account both the log term and the non-divergent part, but has missed the small finite constant term (by using the one-loop matching conditions and two loop RGEs, this small finite term will be taken into account); while the DM has assumed that the $m_H^2$ is much larger than $m_W^2$, so in this method the non-divergent part is neglected in order to be consistent with its assumption, and only the log terms and the small finite constant are taken into account. In the case when Higgs is not too heavy, the non-divergent terms might be important, as the RGE method has revealed. In the case when Higgs is heavy, i.e. approaching to the decoupling limit, then only the log terms play the major part in the trilinear couplings, then both these two method yield the almost same prediction, with a small finite constant terms as difference. That is the reason why these two effective theory methods give quite different predictions on the behavior of trilinear couplings, when the Higgs is not too heavy. In the case when Higgs is relative light, the higher order operators (say, some $O(p^6)$ operators) might be also important, and contributions from these operators will explain the difference between the result of RGE and the exact computation in the underlying theory. According to the direct computation, the relative ratio of the higher order operators contribution over the part computed by the RGE method can be approximately expressed as $\ln(1 + 2m_W^2/m_H^2) - 1$, and considering the fact that such a ratio is quite small, so we claim that the RGE method has efficiently summed over the most important effects.

In the SM, the AC $\alpha_5$ is tightly related with Higgs mass. While in the nonstandard Higgs model [34], the ACs will have a lack dependence on the Higgs mass at the matching scale,

$$\alpha_5 = \frac{k_s^2}{4\lambda}, \quad \alpha_7 = \frac{k_sk'_s}{2\lambda}, \quad \alpha_a = \frac{k'_s}{4\lambda}, \quad \alpha_i(i \neq 5, 7, a) = 0. \quad (219)$$

With the RGEs, we can careful explore the effects of quartic couplings to low energy dynamics, in this model.

The RGEs will greatly simplify the procedure for us to study the effects of the new physics beyond the SM, due to the fact that the tree-level matching conditions are enough for the calculation while the one-loop contributions have been efficiently summed up by the RGEs. As the applications of the RGEs, it is easier to comparatively study the
possible effects to the vector bosonic sector of the SM which might come from various possible new physics candidates, SUSY models, TC models, or ED models.

To establish the modified power counting rule and to derive the RGEs, we have assumed that all ACs are of $O(1)$. By assuming ACs of $O(1)$, the EL might be limited in the realistic application, due to the fact that the amplitude of the longitudinal components of vector bosons (Goldstones) scattering processes at higher energy regions might violet the unitarity condition once the momentum of vector boson goes a little higher than the mass of vector bosons. However, considering the fact that the parameter space of the effective theories should be composed by both the ultraviolet cutoff $\Lambda_{UV}$ and the ACs at that scale, the condition for the violation of unitarity just imposes a helpful correlation on the matching scale and the magnitude of ACs. If the magnitude of ACs is smaller, then the $\Lambda_{UV}$ can be larger, vice versa. So our assumption and the RGE method have relatively more flexibility to match with an unknown underlying theory from $m_W$ to $4\pi v$ than the specific assumption by assuming ACs are tiny and the cutoff is at $4\pi v$. As the matter of fact, for the case when the ACs is large, before the unitarity condition is actually violated, new particles or new resonances might have been found. Therefore, new effective theories should be formulated to include new particles, and new RGEs should be derived.

In order to simplify the calculations, we have not included the contributions of the Fermion, especially the RGEs of the case with the top quarks much heavier than the bottom quarks and the EW symmetry is further broken from the fermionic part, where the anomaly term and terms violating $C$, $P$, and $CP$ might be important after taking into account radiative corrections.

The real world might prefer a light Higgs, as predicted in the SUSY models. So it seems necessary to take the Higgs scalar as one of the basic blocks of the SM, and consider the corresponding EWCL with not only the effective bosonic sector but also the effective Higgs sector [13]. We will consider such a case in our future works [23].

The one-loop RGEs still can not reach to the precision which can be approached by the future’s colliders, we will provide two-loop RGEs of EWCL in our future works [23].

**ACKNOWLEDGMENTS**

One of the author, Q. S. Yan, would like to thank Professor C. D. Lü in the theory division of IHEP of CAS for helpful discussion, and Dr. H. J. He for his helpful informations on the related references. And special thanks to Prof. Y. P. Kuang and Prof. Q. Wang from physics department of Tsinghua university, for their kind help to ascertain some
important points and to improve the representation of this paper. To manipulate the algebraic calculation and extract the $\beta$ functions of RGEs, we have used the FeynCalc [35].

The work of Q. S. Yan is supported by the Chinese Postdoctoral Science Foundation and the CAS K. C. Wong Postdoctoral Research Award Foundation. The work of D. S. Du is supported by the National Natural Science Foundation of China.

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**VIII. APPENDIX: THE RELATED MATRICES**

In this appendix, we provide the related matrices of the quadratic terms of the $\mathcal{L}_{\text{1-loop}}$. The basic definitions include, 1) the physics fields $W^+, W^-, Z$, and $A$ from the interaction eigenstates:

\begin{align}
W^+ &= \frac{1}{\sqrt{2}} (W^1 - iW^2), \quad W^- = \frac{1}{\sqrt{2}} (W^1 + iW^2), \\
Z &= \frac{1}{2} (\sin \theta_W B - \cos \theta_W W^3), \quad A = \frac{1}{2} (\cos \theta_W B + \sin \theta_W W^3),
\end{align}

(220)

2) the Lie algebra generator matrices:

\begin{align}
T^+ &= \frac{1}{\sqrt{2}} (T^1 + iT^2), \quad T^- = \frac{1}{\sqrt{2}} (T^1 - iT^2), \\
[T^+, T^3] &= -T^+, \quad [T^-, T^3] = T^-, \\
[T^+, T^-] &= T^3, \quad tr[T^+ T^-] = tr[T^3 T^3] = \frac{1}{2}.
\end{align}

(221)

There are some definitions to simplify the expressions:

\begin{align}
H_{\mu\nu}^+ &= d^\mu W^\nu + d^\nu W^\mu, \quad H_{\nu\mu}^- = d^\mu W^- + d^\nu W^- , \\
H^\mu_Z &= \partial^\mu Z^\nu + \partial^\nu Z^\mu, \quad SF_{\mu\nu}^+ = W_+^\mu Z^\nu + W_+^\nu Z^\mu , \\
SF_{\mu\nu}^- = Z^\mu W^- + Z^\nu W^- , \quad SF_{\mu\nu}^Z = W_+^\mu W^- + W_+^\nu W^- , \\
SW_{\mu\nu}^+ &= W_+^\mu W_+^\nu, \quad SW_{\mu\nu}^Z = W_+^\mu W^- , \\
SZ_{\mu\nu}^Z &= Z^\mu Z^\nu, \quad SFP = 2W_+ \cdot Z,
\end{align}
\[ SFM = 2W_\cdot Z, \quad SFZ = 2W_\cdot W, \]
\[ OW^+ = W_\cdot W, \quad OW^- = W_\cdot W, \quad OZZ = Z_\cdot Z. \]  

(222)

Below are those related matrices of the quadratic terms. The field strength of \( \Gamma_V \) is given as

\[
\Gamma_{V\nu}^{\mu} = \left( \begin{array}{cccc}
\Gamma_{AA}^{\mu\nu} & \Gamma_{AZ}^{\mu\nu} & \Gamma_{AW}^{\mu\nu} & \Gamma_{AW}^{\mu\nu} \\
\Gamma_{ZA}^{\mu\nu} & \Gamma_{ZZ}^{\mu\nu} & \Gamma_{ZW}^{\mu\nu} & \Gamma_{ZW}^{\mu\nu} \\
\Gamma_{W-A}^{\mu\nu} & \Gamma_{W-Z}^{\mu\nu} & \Gamma_{W-W}^{\mu\nu} & \Gamma_{W-W}^{\mu\nu} \\
\Gamma_{W+A}^{\mu\nu} & \Gamma_{W+Z}^{\mu\nu} & \Gamma_{W+W}^{\mu\nu} & \Gamma_{W+W}^{\mu\nu}
\end{array} \right)
\]

and its components read

\[
\Gamma_{AA}^{\mu\nu} = \Gamma_{AZ}^{\mu\nu} = \Gamma_{ZZ}^{\mu\nu} = \Gamma_{ZZ}^{\mu\nu} = 0, \\
\Gamma_{AW}^{\mu\nu} = \frac{i(2e + C_5)}{4\sqrt{C_1}} W_{\mu\nu}^{\nu} + \frac{(2e + C_5)C_7}{8\sqrt{C_1}} F_{\mu\nu}^{\nu}, \\
\Gamma_{ZW}^{\mu\nu} = \frac{-iC_2 + 2eC_5 + C_1(-C_6 + C_7)}{4\sqrt{g_z C_1}} W_{\mu\nu}^{\nu} \\
\frac{-C_7}{8\sqrt{g_z C_1}} \frac{2eC_2 + 2eC_5 + C_1(-C_6 + C_7)}{F_{\mu\nu}^{\nu}}, \\
\Gamma_{W-W}^{\mu\nu} = -ieA_{\mu\nu}^{\nu} + \frac{ieC_7}{2} Z_{\mu\nu}^{\nu} \\
\frac{-g_z C_1 (2e + C_5)^2 + (2eC_2 + C_5 C_6 + C_1(-C_6 + C_7))^2}{16g_z C_1^2} F_{\mu\nu}^{\nu}, \\
\Gamma_{AW}^{\mu\nu} = \Gamma_{AW}^{\mu\nu}, \quad \Gamma_{ZW}^{\mu\nu} = \Gamma_{ZW}^{\mu\nu}, \quad \Gamma_{W-A}^{\mu\nu} = -\Gamma_{AW}^{\mu\nu}, \\
\Gamma_{W-A}^{\mu\nu} = -\Gamma_{AW}^{\mu\nu}, \quad \Gamma_{W-W}^{\mu\nu} = \Gamma_{W-W}^{\mu\nu}, \quad \Gamma_{W-W}^{\mu\nu} = -\Gamma_{W-W}^{\mu\nu} = 0.
\]  

(223)

The field strength of \( \Gamma_\xi \) is given as

\[
\Gamma_{\xi\nu}^{\mu} = \left( \begin{array}{cccc}
\Gamma_{\xi\xi}\xi\xi & \Gamma_{\xi\xi}\xi+ & \Gamma_{\xi\xi}\xi- \\
\Gamma_{\xi+\xi}\xi\xi & \Gamma_{\xi+}\xi+ & \Gamma_{\xi+}\xi- \\
\Gamma_{\xi-\xi}\xi\xi & \Gamma_{\xi-}\xi+ & \Gamma_{\xi-}\xi-
\end{array} \right)
\]

and its components read

\[
\Gamma_{\xi\xi}\xi\xi = \Gamma_{\xi+}\xi+ = \Gamma_{\xi-}\xi- = 0, \\
\Gamma_{\xi\xi\xi\xi}^{\mu\nu} = i\frac{g}{2\sqrt{\rho}} W_{\mu\nu}^{\nu} - \frac{-2e^2G + g^2G\rho}{4g\sqrt{\rho}} F_{\mu\nu}^{\nu}, \\
\Gamma_{\xi+\xi+}^{\mu\nu} = -ieA_{\mu\nu}^{\nu} + i\left( -\frac{e^2G}{g^2} + \frac{1}{2}G\rho \right) Z_{\mu\nu}^{\nu} - \frac{g^2}{4\rho} F_{\mu\nu}^{\nu}, \\
\Gamma_{\xi+\xi+}^{\mu\nu} = \Gamma_{\xi-}\xi+ = \Gamma_{\xi-}\xi-, \\
\Gamma_{\xi-}\xi- = -\Gamma_{\xi\xi}^{\mu\nu}, \quad \Gamma_{\xi+}\xi+ = -\Gamma_{\xi\xi}^{\mu\nu}. 
\]  

(224)
The \( \sigma_{VV}^{\mu\nu} = S\sigma_{VV}^{\mu\nu} + A\sigma_{VV}^{\mu\nu} \), and the symmetric matrix \( S\sigma_{VV}^{\mu\nu} \) is determined as

\[
S\sigma_{VV}^{\mu\nu} = \\
\begin{pmatrix}
S\sigma_{AA}^{\mu\nu} & S\sigma_{AZ}^{\mu\nu} & S\sigma_{AW+}^{\mu\nu} & S\sigma_{AW-}^{\mu\nu} \\
S\sigma_{ZA}^{\mu\nu} & S\sigma_{ZZ}^{\mu\nu} & S\sigma_{ZW+}^{\mu\nu} & S\sigma_{ZW-}^{\mu\nu} \\
S\sigma_{W-A}^{\mu\nu} & S\sigma_{W-Z}^{\mu\nu} & S\sigma_{W-W+}^{\mu\nu} & S\sigma_{W-W-}^{\mu\nu}
\end{pmatrix}
\]

and its components are listed as

\[
S\sigma_{AA}^{\mu\nu} = -\frac{1}{4C_1} \left( -4e^2 + C_5^2 \right) SF_Z^{\mu\nu} \\
+ \frac{1}{16C_1} \left( -12e^2 + 4eC_5 + C_5^2 \right) SF Zg^{\mu\nu},
\]

\[
S\sigma_{AZ}^{\mu\nu} = \frac{1}{4\sqrt{gZ}C_1} \left( -4e^2C_2 + C_5(C_2C_5 - C_1C_6) - 2eC_1C_7 \right) SF_Z^{\mu\nu} \\
+ \frac{1}{16\sqrt{gZ}C_1^2} \left[ 12e^2C_2 - C_5 (C_2C_5 + C_1(-C_6 + C_7)) \right] SF Zg^{\mu\nu},
\]

\[
S\sigma_{AW+}^{\mu\nu} = \frac{1}{8\sqrt{C_1}} ((2e - C_5) C_7) SF_{-\mu\nu} \\
+ \frac{1}{16\sqrt{C_1}} ((-2e + C_5) C_7) SF M g^{\mu\nu} \\
+ i \frac{1}{4\sqrt{C_1}} (2e - C_5) H_{-\mu\nu},
\]

\[
S\sigma_{ZZ}^{\mu\nu} = \frac{4C_a}{gZ} OZZ g^{\mu\nu} \\
+ \frac{1}{4gZC_1^2} \left( 4e^2C_2^2 - C_2^2C_5^2 + 2C_1C_2C_5C_6 + 4eC_1C_2C_7 - C_1^2(C_6^2 - 4C_6) \right) SF_Z^{\mu\nu} \\
+ \frac{1}{16gZC_1} \left[ -12e^2C_2^2 + C_2^2C_5^2 + 2C_1C_2C_5(-C_6 + C_7) \right] \\
+ 4eC_2(C_2C_5 - C_1(C_6 + 3C_7)) + C_1^2(C_6^2 - 2C_6C_7 + C_7^2 + 16C_8) \right] SF Zg^{\mu\nu} \\
+ \frac{8C_a}{gZ} SIZ^{\mu\nu},
\]

\[
S\sigma_{ZW+}^{\mu\nu} = \frac{1}{8gZC_1} \left( -2eC_2C_7 + C_2C_5C_7 + C_1(-C_6C_7 + 8C_8 + 4C_9) \right) SF_{\mu\nu} \\
+ \frac{1}{16\sqrt{gZ}C_1} \left( 2eC_2C_7 - C_2C_5C_7 + C_1(C_6C_7 - C_7^2 + 8C_9) \right) SF M g^{\mu\nu} \\
+ i \frac{1}{4\sqrt{gZ}C_1} (2gZf_{zw}C_1 + C_2C_5 - C_1C_6) H_{-\mu\nu},
\]

(225)
The antisymmetric matrix $W_{\mu \nu}$ means to change the field $W_{\mu \nu}$ to $W_{\mu \nu}^{\pm} = Z \mp g Z W_{\mu \nu}$, where $W_{\mu \nu}^{\pm} = S_{\sigma}^{\mu \nu} = S_{\sigma}^{\mu \nu -} = S_{\sigma}^{\mu \nu +}$.

(226) 

(227) 

(228) 

(229) 

(230) 

Here $\dagger$ means to change the field $W^{\pm} \to W^{\mp}$ and $i \to -i$.

The antisymmetric matrix $A_{\sigma}^{\mu \nu}$ is given as

$$A_{\sigma}^{\mu \nu} = \begin{pmatrix} A_{\sigma_{AA}}^{\mu \nu} & A_{\sigma_{AZ}}^{\mu \nu} & A_{\sigma_{AW+}}^{\mu \nu} & A_{\sigma_{AW-}}^{\mu \nu} \\ A_{\sigma_{ZA}}^{\mu \nu} & A_{\sigma_{ZZ}}^{\mu \nu} & A_{\sigma_{ZW+}}^{\mu \nu} & A_{\sigma_{ZW-}}^{\mu \nu} \\ A_{\sigma_{W-A}}^{\mu \nu} & A_{\sigma_{W-Z}}^{\mu \nu} & A_{\sigma_{W-W+}}^{\mu \nu} & A_{\sigma_{W-W-}}^{\mu \nu} \\ A_{\sigma_{W+A}}^{\mu \nu} & A_{\sigma_{W+Z}}^{\mu \nu} & A_{\sigma_{W-W+}}^{\mu \nu} & A_{\sigma_{W-W-}}^{\mu \nu} \end{pmatrix}$$

and its components are listed as

$$A_{\sigma_{AA}}^{\mu \nu} = 0, \quad A_{\sigma_{AZ}}^{\mu \nu} = 0,$$
$$A_{\sigma_{AW+}}^{\mu \nu} = \frac{1}{8\sqrt{C_1}} \left( (6e + C_5)C_7 \right) F_{\mu \nu}^{\pm} + i \frac{1}{4\sqrt{C_1}} (6e + C_5) W_{\mu \nu}^{\pm},$$
$$A_{\sigma_{ZW+}}^{\mu \nu} = 0,$$
$$A_{\sigma_{W-Z}}^{\mu \nu} = \frac{1}{8\sqrt{g Z C_1}} \left( -6e C_2 C_7 - C_2 C_5 C_7 + C_1 (C_6 C_7 + 8C_8 - 4C_9) \right) F_{\mu \nu}^{\pm} + i \frac{1}{4\sqrt{g Z C_1}} \left( 2g f_{zw} C_1 - 4e C_2 - C_2 C_5 + C_1 C_6 - 2C_1 C_7 \right) W_{\mu \nu}^{\pm},$$
$$A_{\sigma_{W+}}^{\mu \nu} = -\frac{ie}{2C_1} + C_b - 2C_c \left( -\frac{g f_{zw}^2}{2} \right) F_{\mu \nu}^Z - i \frac{C_6 - C_7}{2} Z_{\mu \nu}^{\pm},$$
$$A_{\sigma_{Z-}}^{\mu \nu} = 0, \quad A_{\sigma_{AW-}}^{\mu \nu} = A_{\sigma_{AW+}}^{\dagger \mu \nu}, \quad A_{\sigma_{Z+}}^{\mu \nu} = -A_{\sigma_{ZA}}^{\mu \nu}.$$
\[ A\sigma_{ZW}^{\mu\nu} = A\sigma_{ZW}^{\mu\nu} = A\sigma_{W-A}^{\mu\nu} = A\sigma_{W-Z}^{\mu\nu} = -A\sigma_{AW}^{\mu\nu}, \quad A\sigma_{W-Z}^{\mu\nu} = -A\sigma_{ZW}^{\mu\nu}, \]
\[ A\sigma_{W-A}^{\mu\nu} = -A\sigma_{AW}^{\mu\nu}, \quad A\sigma_{W+Z}^{\mu\nu} = -A\sigma_{ZW}^{\mu\nu}, \quad A\sigma_{W+W}^{\mu\nu} = -A\sigma_{W-W}^{\mu\nu}, \]
\[ A\sigma_{W+W}^{\mu\nu} = 0. \quad (231) \]

The symmetric \(\sigma_{2,\xi\xi}\) is given as

\[
\sigma_{2,\xi\xi} = \begin{pmatrix}
\sigma_{2,\xi_2\xi_2} & \sigma_{2,\xi_2\xi_1} & \sigma_{2,\xi_2\xi_1^-} \\
\sigma_{2,\xi_1\xi_2} & \sigma_{2,\xi_1^-\xi_1} & \sigma_{2,\xi_1^-\xi_1^-} \\
\sigma_{2,\xi_1^+\xi_2} & \sigma_{2,\xi_1^+\xi_1^+} & \sigma_{2,\xi_1^+\xi_1^-}
\end{pmatrix}
\]

and its matrix components are listed as

\[
\sigma_{2,\xi_2\xi_2} = \frac{g^2}{4\rho}SFZ, \quad \sigma_{2,\xi_2\xi_1} = \frac{gG\sqrt{\rho}}{8}SFZ, \quad (232)
\]
\[
\sigma_{2,\xi_1^-\xi_1} = \frac{G^2\rho^2}{4}OZZ + \frac{g^2}{8\rho}SFZ, \quad \sigma_{2,\xi_1^-\xi_1^-} = -\frac{g^2}{4\rho}OW^+, \quad (233)
\]
\[
\sigma_{2,\xi_1^+\xi_2} = \sigma_{2,\xi_1^+\xi_1^+} = \sigma_{2,\xi_1^+\xi_1^-}, \quad \sigma_{2,\xi_1^+\xi_1^-} = \sigma_{2,\xi_1^-\xi_1^-}, \quad \sigma_{2,\xi_1^+\xi_1^-} = \sigma_{2,\xi_1^-\xi_1^-}, \quad (234)
\]

One feature about the \(\sigma_{V,V}^{\mu\nu}\) and \(\sigma_{2,\xi\xi}\) is remarkable. There is no terms like \(\partial\cdot Z, d\cdot W^\pm\) in the components, the basic reason is due to the gauge fixing terms we have chosen in Eq. (121).

Considering the fact that only the diagonal components of the matrix \(\sigma_4\) contribute meaningfully to the renormalization up to \(O(p^4)\), we only list those we concern

\[
\rho v^2 \sigma_{4,\xi\xi\xi\xi} = H_1 \left( \frac{4g^2}{G^2} C_4 - \frac{2}{G^2} C_7 \right)
\]
\[
+ L_3 \left( \frac{-5g^2 - 4G^2}{G^4} C_4 + 22g^2 + G^2 g^2 G^2 C_7 + \frac{2}{g^2 G^2} C_8 - \frac{1}{g^2 G^2} C_9 \right)
\]
\[
+ L_5 \left( -4 \frac{g^2 - G^2}{g^2 G^2} C_1 - 8 \frac{e}{g^2 G} C_2 + 4 \frac{e}{G^2} C_3 - 4 \frac{e}{g^4} C_5 + \frac{4}{g^2 G^2} C_6 - \frac{8}{g^2} C_9 \right)
\]
\[
+ L_6 \left( -6 \frac{g^4}{G^6} C_4 + 6 \frac{g^2}{G^5} C_7 + 4 \frac{G}{G^4} C_8 - \frac{2}{G^4} C_9 \right)
\]
\[
+ L_7 \left( 4 \frac{g^2 - G^2}{g^2 G^2} C_1 + 8 \frac{e}{g^2 G} C_2 - 4 \frac{C_3}{G^2} C_3 + 2 \frac{3g^4 + 2g^2 G^2}{G^6} C_4 + 4 \frac{e}{g^4} C_5
\]
\[
- 4 \frac{g^2 G}{G^5} C_6 - 2 \frac{4g^2 + G^2}{G^5} C_7 - 4 \frac{g^2 + G^2}{g^2 G^4} C_8 - 2 \frac{3g^2 + 2G^2}{g^2 G^4} C_9 + 8 \frac{g^2}{G^4} C_9 \right)
\]
\[
+ L_8 \left( -4 \frac{1}{G^2} C_4 + 2 \frac{1}{g^2 G} C_7 \right)
\]
\[
+ L_9 \left( 5g^2 + 4G^2 \frac{G}{G^4} C_4 - 2 \frac{2g^2 + G^2}{g^2 G^3} C_7 - \frac{2}{g^2 G^2} C_8 + \frac{1}{G^2 G^2} C_9 \right)
\]
\begin{align}
v^2 \sigma_{4,\xi^-\xi^+} &= H_1 \left( 4 \frac{g^2 - G^2}{G^2} C_1 + 8 \frac{e}{G^2} C_2 - 4 \frac{g^2}{G^2} C_3 + 2 \frac{g^2 + 2G^2}{G^2 \rho} C_4 - \frac{1}{G \rho} C_7 \right) \\
&+ H_2 \left[ -4 \frac{(-g^2 + G^2)(-2 + \rho)}{G^2} C_1 + 4 \frac{e(2g^2 - G^2)(-2 + \rho)}{g^2 G} C_2 \\
&+ 4 \frac{(-g^2 + G^2)(-2 + \rho)}{G^2} C_3 + 2 \frac{e(-2 + \rho)}{g^2} C_5 - 2 \frac{(g^2 - G^2)(-2 + \rho)}{g^2 G} C_6 \right] \\
&+ \mathcal{L}_1 \left[ 2 \frac{2G^2 + 2g^2 \rho - G^2 \rho}{g^2 G^2} C_1 \right] \\
&- 2 \frac{1}{eG^5} \left( 2g^2 G^2 + 2 \frac{e^2 G^4}{g^2} + g^4 \rho + e^2 G^2 \rho - 3 \frac{e^2 G^4 \rho}{g^2} \right) C_2 \\
&- 4 \frac{\rho}{G^2} C_3 + \frac{1}{eg^2 G^2} \left( -2 \frac{e^2 G^2}{g^2} - g^2 \rho + \frac{e^2 G^2 \rho}{g^2} \right) C_5 - 2 \frac{(-1 + \rho)}{g^2 G} C_6 \\
&+ \mathcal{L}_2 \left[ \frac{6g^2 - 6G^2 - 5g^2 \rho + 3G^2 \rho}{g^2 G^2} C_1 + 2 \frac{1}{eG^3} \left( 6 \frac{e^2 G^2}{g^2} + g^2 \rho + 4 \frac{e^2 G^2 \rho}{g^2} \right) C_2 \\
&+ \mathcal{L}_3 \left[ \frac{(g^2 - G^2)^2}{g^2 G^4} C_1 + 8 \frac{e}{G^2} g^2 - g^2 G^2 + 4 \frac{g^2 + 2G^2}{G^2 \rho} C_3 + \frac{2 - 2 + \rho}{g^2 G} C_6 - 2 \frac{2 - 2 + \rho}{g^2 G} C_7 \right] \\
&+ \mathcal{L}_4 \left[ \frac{(g^2 - G^2)^2}{g^2 G^2} C_1 + 12 \frac{e}{g^4} \rho \right] C_2 - 6 \frac{\rho}{G^2} C_3 + 6 \frac{\rho}{g^4} C_5 \\
&- 6 \frac{\rho}{g^2 G} C_6 + 4 \frac{\rho}{g^4} C_b - 8 \frac{\rho}{g^4} C_c \\
&+ \mathcal{L}_5 \left[ -2 \frac{(g^2 - G^2)(1 + 3\rho^2)}{g^2 G^2 \rho} C_1 \\
&- 4 \frac{\rho}{g^2 G^2} \left( \frac{e G}{g} + 3 \frac{e G \rho^2}{g} \right) C_2 + 2 \frac{1 + 3\rho^2}{G^2 \rho} C_3 - \frac{2}{g^2 G \rho} \left( \frac{e G}{g} + 3 \frac{e G \rho^2}{g} \right) C_5 \\
&+ 2 \frac{1 + 3\rho^2}{G^2 \rho} C_6 - 4 \frac{1 + \rho^2}{g^4} C_b + 8 \frac{\rho}{g^4} C_c \\
&+ \mathcal{L}_6 \left[ -2 \frac{1}{g^2 G^6} \left( (g^2 - G^2)(2g^4 - 4g^2 G^2 + 2G^4 + 3G^4 \rho) \right) C_1 \\
&- 4 \frac{e}{g^2 G^5} \left( 2g^4 - 4g^2 G^2 + 2G^4 + 3G^4 \rho \right) C_2 \right]
\end{align}
$$+2 \frac{1}{G^6} \left( 2g^4 - 4g^2G^2 + 2G^4 + 3G^4 \rho \right) C_3 - 2 \frac{1}{G^6 \rho} \left( g^4 - 4g^4 \rho + 4g^2G^2 \rho \right) C_4$$

$$-2e \frac{1}{g^4G^4} \left( 2g^4 - 4g^2G^2 + 2G^4 + 3G^4 \rho \right) C_5$$

$$+2 \frac{1}{g^2G^5} \left( 2g^4 - 4g^2G^2 + 2G^4 + 3G^4 \rho \right) C_6$$

$$+ \frac{1}{G^5 \rho} \left( g^2 - 8g^2 \rho + 8G^2 \rho \right) C_7 - 8 \frac{1}{g^2G^3} \left( -g^2 + G^2 \right) C_9$$

$$-4 \frac{1}{g^4G^4} \left( -2g^4 + 4g^2G^2 - 2G^4 + G^4 \rho \right) C_6$$

$$+8 \frac{1}{g^4G^4} \left( 2g^4 - 4g^2G^2 + 2G^4 + G^4 \rho \right) C_c$$

$$+\mathcal{L}_7 \left[ 2 \left( g^2 - G^2 \right) \frac{1}{g^2G^6 \rho} \left( G^4 + 10g^4 \rho - 16g^2G^2 \rho + 6G^4 \rho + 4g^2G^2 \rho^2 + G^4 \rho^2 \right) C_1 \right]$$

$$+4e \frac{1}{g^4G^4 \rho} \left( G^4 + 10g^4 \rho - 16g^2G^2 \rho + 6G^4 \rho + 4g^2G^2 \rho^2 + G^4 \rho^2 \right) C_2$$

$$-2 \frac{1}{G^6 \rho} \left( G^4 + 10g^4 \rho - 16g^2G^2 \rho + 6G^4 \rho + 4g^2G^2 \rho^2 + G^4 \rho^2 \right) C_3$$

$$-2g^2 \frac{1}{G^6 \rho} \left( -g^2 - G^2 + 4g^2 \rho - 4G^2 \rho \right) C_4$$

$$+2e \frac{1}{g^4G^4 \rho} \left( G^4 + 8g^4 \rho - 14g^2G^2 \rho + 6G^4 \rho + 3g^2G^2 \rho^2 + G^4 \rho^2 \right) C_5$$

$$-2 \frac{1}{g^2G^5 \rho} \left( G^4 + 8g^4 \rho - 14g^2G^2 \rho + 6G^4 \rho + 3g^2G^2 \rho^2 + G^4 \rho^2 \right) C_6$$

$$+ \frac{1}{G^5 \rho} \left( -2g^2 - G^2 + 12g^2 \rho - 12G^2 \rho + 2G^2 \rho^2 \right) C_7$$

$$+2 \frac{1}{g^2G^5 \rho} \left( -G^2 + 8g^2 \rho - 8G^2 \rho + 2G^2 \rho^2 \right) C_8$$

$$+2 \frac{1}{g^2G^4 \rho} \left( g^2 - G^2 \right) C_9 + 4 \frac{1}{g^4G^4 \rho} \left( G^4 + 6g^4 \rho - 12g^2G^2 \rho + 6G^4 \rho \right)$$

$$+2g^2G^2 \rho^2 - G^4 \rho^2 \right) C_b - 8 \frac{1}{g^4C_c} \left[ \mathcal{L}_8 \left[ - \frac{4 \left( g^2 - G^2 \right) \left( 3 + \rho \right)}{g^2G^2} \right] C_1 - 8e \frac{3 + \rho}{g^2G} C_2 + 4 \frac{3 + \rho}{G^2} C_3 - 2 \frac{g^2 + 6G^2 \rho}{g^2G^2 \rho} C_4$$

$$-2 \frac{g^2}{g^4} C_5 + 2 \frac{\rho}{g^2G} C_6 + \frac{1}{g^2G \rho} C_7 \right]$$

$$+ \mathcal{L}_9 \left[ - \left( g^2 - G^2 \right) \frac{1}{g^2G^4} \left( 4g^2 - 8G^2 - 5G^2 \rho \right) C_1 \right]$$

$$-2e \frac{1}{g^2G^3} \left( 4g^2 - 8G^2 - 5G^2 \rho \right) C_2$$
\[
\sigma^{\xi+\xi^-} = \sigma^{\xi^-\xi+}.
\]

The \(sS\) matrix is a symmetric matrix about \(\xi\) Lorentz indices, and is given as

\[
u^2 sS^{\alpha\beta}_{\xi\xi} = \begin{pmatrix}
{sS}_{\xi\xi\xi\xi} & {sS}_{\xi\xi\xi+} & {sS}_{\xi\xi\xi^-} \\
{sS}_{\xi-\xi\xi} & {sS}_{\xi-\xi+} & {sS}_{\xi-\xi-} \\
{sS}_{\xi+\xi\xi} & {sS}_{\xi+\xi+} & {sS}_{\xi+\xi-}
\end{pmatrix}
\]

and its components read

\[
sS_{\xi\xi\xi\xi} = 16 \frac{C_a}{G^2} OZZ g^{\alpha\beta}
\]

\[
+ \frac{1}{G^3} \left( - e^2 GC_4 + g^2 (GC_4 - C_7) + GC_9 \right) SF_{Z}^{\alpha\beta} + 32 \frac{C_a}{G^2} S Z^{\alpha\beta},
\]

\[\text{Equation 236}\]

\[\text{Equation 237}\]
\[ s S_{\xi_2 \xi^+} = -\frac{1}{g G^2} \left( e^2 G C_4 + g^2 (-G C_4 + C_7) + G (2 C_8 + C_9) \right) S F^\alpha_\beta, \]  
(239)  
\[ s S_{\xi^- \xi^+} = \frac{2}{g^2 G} \left( e^2 G (C_1 - C_3) - e (2 g^2 C_2 + G C_5) \right. \]  
\[ + g^2 (B C_3 + C_6) + 2 G (B_6 + 2 C_c) \) \) S F^\alpha_\beta \]  
\[ + 4 \left( C_4 - \frac{e^2}{g^2} C_4 - \frac{1}{G} C_7 + \frac{1}{g^2} C_9 \right) S Z^\alpha_\beta, \]  
(240)  
\[ s S_{\xi^- \xi^-} = -\frac{4}{g G^2} \left( e^2 G (C_1 - C_3) - e (2 g^2 C_2 + G C_5) \right. \]  
\[ + g^2 (B C_3 + C_6) - 2 G B_6 \) \) S W^\alpha_\beta, \]  
(241)  
\[ s S_{\xi_2 \xi^-} = s S_{\xi_2}^\dagger, \quad s S_{\xi^- \xi^z} = s S_{\xi_2 \xi^-}, \quad s S_{\xi^\dagger} = s S_{\xi_2 \xi^-}; \]  
(242)  
The \( s A \) matrix is an antisymmetric matrix about its Lorentz indices, and is given as  
\[ v^2 s A^\alpha_\xi = \begin{pmatrix} s A_{\xi_2 \xi_2} & s A_{\xi_2 \xi^+} & s A_{\xi_2 \xi^-} \\ s A_{\xi^- \xi_2} & s A_{\xi^- \xi^+} & s A_{\xi^- \xi^-} \\ s A_{\xi^+ \xi_2} & s A_{\xi^+ \xi^+} & s A_{\xi^+ \xi^-} \end{pmatrix} \]  
and its components read  
\[ s A_{\xi_2 \xi_2} = 0, \]  
(243)  
\[ s A_{\xi_2 \xi^+} = -2i \frac{2 g^2 C_4 - G C_7}{g G^2} W^\alpha_\beta \]  
\[ + 2 \frac{1}{g G^2} \left( e^2 G C_4 + g^2 (G C_4 - 2 C_7) + G (-2 C_8 + C_9) \right) F^\alpha_\beta, \]  
(244)  
\[ s A_{\xi^- \xi^+} = -2i \frac{1}{g G^2} \left( -2 e G C_1 + 2 g^2 C_2 + G C_5 \right) A^{\alpha_\beta} \]  
\[ - 2i \frac{-2 e G C_2 + 2 g^2 C_3 + G C_6}{g^2 G} Z^\alpha_\beta \]  
\[ - 2 \frac{1}{g G^2} \left( e^2 G (C_1 - C_3) - 2 e (g^2 C_2 + G C_5) \right. \]  
\[ + g^2 (G C_3 + 2 C_6) - 2 G (B_6 - 2 C_c) \) \) F^\alpha_\beta, \]  
(245)  
\[ s A_{\xi^- \xi^-} = 0, \quad s A_{\xi_2 \xi^-} = s A_{\xi_2}^\dagger, \quad s A_{\xi^- \xi^z} = -s A_{\xi_2 \xi^-}; \]  
(246)  
The \( \tilde{S}^\mu_{\alpha_\beta} \) matrix is antisymmetric on \( \alpha \beta \), and is given as
\[ \tilde{S}_{\alpha\beta}^\mu = \begin{pmatrix} 0 & \text{ics}_{A\xi w} X_{\alpha\beta}^{-\mu} - \text{ics}_{A\xi w} X_{\alpha\beta}^{+\mu} \\ 0 & \text{ics}_{Z\xi w} X_{\alpha\beta}^{-\mu} - \text{ics}_{Z\xi w} X_{\alpha\beta}^{+\mu} \\ -\text{ics}_{W\xi z} X_{\alpha\beta}^{+\mu} - \text{ics}_{W\xi w} X_{\alpha\beta}^{0} & 0 \\ \text{ics}_{W\xi z} X_{\alpha\beta}^{-\mu} & 0 \end{pmatrix} \]

where the relevant definitions are

\[ X_{\alpha\beta}^{-\mu} = W_{\alpha}^{-\mu} g_{\beta} + W_{\beta}^{-\mu} g_{\alpha} - 2g_{\alpha\beta}W^{-\mu}, \]
\[ X_{\alpha\beta}^{+\mu} = W_{\alpha}^{+\mu} g_{\beta} + W_{\beta}^{+\mu} g_{\alpha} - 2g_{\alpha\beta}W^{+\mu}, \]
\[ X_{\alpha\beta}^{Z\mu} = Z_{\alpha} g_{\beta}^\mu + Z_{\beta} g_{\alpha}^\mu - 2g_{\alpha\beta}Z^\mu, \]
\[ v_{CS\xi w} = -\frac{-2eGC_1 + 2g^2C_2 + GC_5}{2gG}, \]
\[ v_{CSZ\xi w} = -\frac{-2eGC_2 + 2g^2C_3 + GC_6}{2gG}, \]
\[ v_{CSW\xi z} = -\frac{2g^2C_4 - GC_7}{2G^2}, \]
\[ v_{CSW\xi w} = \frac{2g^2C_4 - GC_7}{2gG}. \]

The \( \tilde{A}_{\alpha\beta}^\mu \) matrix is antisymmetric on \( \alpha\beta \), and is given as

\[ \tilde{A}_{\alpha\beta}^\mu = \begin{pmatrix} 0 & \text{ics}_{A\xi w} A_{\alpha\beta}^{-\mu} - \text{ics}_{A\xi w} A_{\alpha\beta}^{+\mu} \\ 0 & \text{ics}_{Z\xi w} A_{\alpha\beta}^{-\mu} - \text{ics}_{Z\xi w} A_{\alpha\beta}^{+\mu} \\ -\text{ics}_{W\xi z} A_{\alpha\beta}^{+\mu} - \text{ics}_{W\xi w} A_{\alpha\beta}^{0} & 0 \\ \text{ics}_{W\xi z} A_{\alpha\beta}^{-\mu} & 0 \end{pmatrix} \]

\[ A_{\alpha\beta}^{-\mu} = W_{\beta}^{-\mu} g_{\alpha} - W_{\alpha}^{-\mu} g_{\beta}, \]
\[ A_{\alpha\beta}^{+\mu} = W_{\beta}^{+\mu} g_{\alpha} - W_{\alpha}^{+\mu} g_{\beta}, \]
\[ A_{\alpha\beta}^{Z\mu} = Z_{\beta} g_{\alpha}^\mu - Z_{\alpha} g_{\beta}^\mu, \]

The \( \tilde{X}_{1}^{\mu\alpha} \) is a \( 4 \times 3 \) matrix

\[ v_{CS} \tilde{X}_{1}^{\mu\alpha} = \begin{pmatrix} X_{1, A\xi z}^{\mu\alpha} & X_{1, A\xi z}^{\mu\alpha} & X_{1, A\xi z}^{\mu\alpha} \\ X_{1, Z\xi z}^{\mu\alpha} & X_{1, Z\xi z}^{\mu\alpha} & X_{1, Z\xi z}^{\mu\alpha} \\ X_{1, W^{-}\xi z}^{\mu\alpha} & X_{1, W^{-}\xi z}^{\mu\alpha} & X_{1, W^{-}\xi z}^{\mu\alpha} \\ X_{1, W^{+}\xi z}^{\mu\alpha} & X_{1, W^{+}\xi z}^{\mu\alpha} & X_{1, W^{+}\xi z}^{\mu\alpha} \end{pmatrix}, \]

and its components read
\[ X_{1, A_k}^{\mu} = e^{2g^2C_4 - GC_7} SFZ g^{\alpha\mu} + e^{-2g^2C_4 + GC_7} SFZ g^{\alpha\mu}, \quad (257) \]
\[ X_{1, A_k}^{\mu} = e\left( -\frac{gC_4}{G} + \frac{C_7}{2g} \right) F_{\alpha\mu}^{\alpha} + e\left( \frac{gC_4}{G} - \frac{C_7}{2g} \right) SFZ g^{\alpha\mu} \]
\[ + e\left( -\frac{gC_4}{G} + \frac{C_7}{2g} \right) SFM g^{\alpha\mu}, \quad (258) \]
\[ X_{1, Z_{1}}^{\mu} = \frac{8C_6}{G} OZZ g^{\alpha\mu} + \frac{-g^2C_7 + 2GC_6}{G^2} SFZ g^{\alpha\mu} \]
\[ + \frac{(g^2C_7 + 2GC_6)}{G^2} SFZ g^{\alpha\mu} + \frac{16C_6}{G} SZ g^{\alpha\mu}, \quad (259) \]
\[ X_{1, Z_{1}}^{\mu} = \frac{3gC_7}{2G} \left( \frac{2C_8 - C_9}{g} \right) F_{\alpha\mu}^{\alpha} - \frac{g^2C_7 + 4GC_8 + 2GC_9}{2gG} SFZ g^{\alpha\mu} \]
\[ + \frac{(gC_7 - C_9)}{G} SFM g^{\alpha\mu} + i\left( \frac{2gC_4}{G} - \frac{C_7}{g} \right) W_{\alpha\mu}^{\alpha}, \quad (260) \]
\[ X_{1, W^{-}}^{\mu} = \frac{3g^2C_7 + 4GC_8 - 2GC_9}{2G^2} F_{\alpha\mu}^{\alpha} + \frac{g^2C_7 + 4GC_8 + 2GC_9}{2G^2} SFZ g^{\alpha\mu} \]
\[ - \frac{g^2C_7 - 2GC_9}{2G} SFMZ g^{\alpha\mu} + \frac{i}{G} (-2g^2C_4 + GCG) W_{\alpha\mu}^{\alpha}, \quad (261) \]
\[ X_{1, W^{-}}^{\mu} = -i\frac{-2eGC_1 + 2g^2C_2 + GC_5}{gG} \]
\[ + \frac{3eGC_5 - 3g^2C_6 + 4G(C_b - 2C_e)}{2gG} F_{\alpha\mu}^{\alpha} + \left( \frac{gC_7}{G} + \frac{2C_8}{g} \right) OZZ g^{\alpha\mu} \]
\[ + \frac{eGC_5 - g^2C_6 - 4G(C_b + 2C_e)}{2gG} SFZ g^{\alpha\mu} - \frac{eGC_5 - g^2C_6 + 4G(C_b)}{2gG} SFZ g^{\alpha\mu} \]
\[ + \left( \frac{gC_7}{G} - \frac{2C_9}{g} \right) Sz g^{\alpha\mu} \]
\[ - \frac{i}{gG} (-2eGC_2 + 2g^2C_3 + GC_6) Z g^{\alpha\mu}, \quad (262) \]
\[ X_{1, W^{-}}^{\mu} = \frac{1}{g} \left( eC_5 - \frac{g^2C_6}{G} + 4C_e \right) OW g^{\alpha\mu} \]
\[ + \frac{1}{g} \left( -eC_5 + \frac{g^2C_6}{G} - 4C_b \right) SW g^{\alpha\mu}, \quad (263) \]
\[ X_{1, A_k}^{\mu} = X_{1, A_k}^{\mu}, \quad X_{1, Z_{1}}^{\mu} = X_{1, Z_{1}}^{\mu}, \quad X_{1, W^{-}}^{\mu} = X_{1, W^{-}}^{\mu}, \quad X_{1, W^{+}}^{\mu} = X_{1, W^{+}}^{\mu}, \quad X_{1, W^{-}}^{\mu} = X_{1, W^{-}}^{\mu}, \quad (264) \]

The \( \tilde{X}_2^{\mu} \) is a 4 × 3 matrix.
The matrix

\[
v \tilde{X}_2^\mu = \begin{pmatrix}
X_{2, A\xi z}^\mu & X_{2, A\xi^+}^\mu & X_{2, A\xi^-}^\mu \\
X_{2, Z_\xi z}^\mu & X_{2, Z_\xi^+}^\mu & X_{2, Z_\xi^-}^\mu \\
X_{2, W^- \xi z}^\mu & X_{2, W^- \xi^+}^\mu & X_{2, W^- \xi^-}^\mu \\
X_{2, W^+ \xi z}^\mu & X_{2, W^+ \xi^+}^\mu & X_{2, W^+ \xi^-}^\mu
\end{pmatrix},
\]

and its components read

\[
X_{2, A\xi z}^\mu = 0,
\]
\[
X_{2, A\xi^+}^\mu = \frac{1}{gG} \left[ -4e^2 GC_2 - G^2 C_5 + g^2 (2GC_2 + C_5) + e \left( -4g^2 C_1 + G(2GC_1 + C_7) \right) \right] F_{-\mu}^{\alpha \mu} + i2 \frac{1}{gG} \left[ g^2 C_2 + eG(-C_1 + C_4) \right] W_{-\mu}^{\alpha \mu},
\]
\[
X_{2, Z_\xi z}^\mu = 0,
\]
\[
X_{2, Z_\xi^+}^\mu = \frac{1}{gG} \left[ 2e(-2g^2 + G^2)C_2 - 4e^2 GC_3 - G^2 C_6 + g^2 (2GC_3 + C_6 - C_7) \right] F_{-\mu}^{\alpha \mu} - 2i \frac{eGC_2 + g^2 (-C_3 + C_4)}{gG} W_{-\mu}^{\alpha \mu},
\]
\[
X_{2, W^- \xi z}^\mu = \left( -\frac{2g^2 C_1}{G} + C_7 \right) A_{+\mu}^{\alpha \mu},
\]
\[
X_{2, W^- \xi^+}^\mu = 2i \frac{g^2 C_2 + eG(-C_1 + C_4)}{gG} A_{+\mu}^{\alpha \mu} + \left( 2gC_4 - \frac{eC_5}{g} + \frac{gC_6}{G} \right) F_{Z\mu}^{\alpha \mu}
\]
\[
-2i \frac{1}{gG} \left[ eGC_2 + g^2 (-C_3 + C_4) \right] Z_{-\mu}^{\alpha \mu},
\]
\[
X_{2, W^- \xi^-}^\mu = 0, \quad X_{2, A\xi^-}^\mu = X_{2, A\xi^+}^{\mu \dagger}, \quad X_{2, Z_\xi^-}^\mu = X_{2, Z_\xi^+}^{\mu \dagger}, \quad X_{2, W^- \xi^-}^\mu = X_{2, W^+ \xi^-}^{\mu \dagger}, \quad X_{2, W^+ \xi^-}^\mu = X_{2, W^+ \xi^+}^{\mu \dagger}.
\]

The matrix \( \tilde{X}_{01}^\mu \) is given as

\[
\tilde{X}_{01}^\mu = \begin{pmatrix}
Z_{01, A\xi z}^\mu & Z_{01, A\xi^+}^\mu & Z_{01, A\xi^-}^\mu \\
Z_{01, Z_\xi z}^\mu & Z_{01, Z_\xi^+}^\mu & Z_{01, Z_\xi^-}^\mu \\
Z_{01, W^- \xi z}^\mu & Z_{01, W^- \xi^+}^\mu & Z_{01, W^- \xi^-}^\mu \\
Z_{01, W^+ \xi z}^\mu & Z_{01, W^+ \xi^+}^\mu & Z_{01, W^+ \xi^-}^\mu
\end{pmatrix},
\]

and its components read

\[
Z_{01, A\xi z}^\mu = 0, \quad Z_{01, A\xi^+}^\mu = -i \frac{g(2e + C_5)}{4} W_{-\mu}^\mu,
\]

\[
59
\]
\[ Z_{01, Z\xi}^\mu = 0, \quad Z_{01, Z\xi^+}^\mu = -\frac{i}{4g}(2e^2G + g^2C_\mu)\nu W^\mu, \]

\[ Z_{01, W^-\xi}^\mu = \frac{i}{2}f_{\xi\nu}G\rho\nu W^\mu, \quad (272) \]

\[ Z_{01, W^-\xi^+}^\mu = -\frac{i}{4g}(2e^2G + g^2C_\tau)\nu Z^\mu, \quad (273) \]

\[ Z_{01, W^-\xi^-}^\mu = 0, \quad Z_{01, A\xi^-}^\mu = Z_{01, A\xi^+}^\mu, \quad Z_{01, Z\xi^-}^\mu = Z_{01, Z\xi^+}^\mu, \quad (274) \]

\[ Z_{01, W^+\xi}^\mu = Z_{01, W^-\xi}^\mu, \quad Z_{01, W^+\xi^+}^\mu = Z_{01, W^-\xi^-}^\mu, \quad Z_{01, W^+\xi^-}^\mu = Z_{01, W^-\xi^+}^\mu, \quad (275) \]

The matrix \( \tilde{X}_{03}^\mu \) is given as

\[
\nu\tilde{X}_{01}^\mu = \begin{pmatrix}
Z_{03, A\xi^+}^\mu & Z_{03, A\xi^-}^\mu & Z_{03, A\xi^+}^\mu \\
Z_{03, Z\xi^+}^\mu & Z_{03, Z\xi^-}^\mu & Z_{03, Z\xi^-}^\mu \\
Z_{03, W^-\xi}^\mu & Z_{03, W^-\xi^+}^\mu & Z_{03, W^-\xi^-}^\mu \\
Z_{03, W^+\xi}^\mu & Z_{03, W^+\xi^+}^\mu & Z_{03, W^+\xi^-}^\mu
\end{pmatrix},
\]

and its components read

\[ Z_{03, A\xi^+}^\mu = 0, \quad (276) \]

\[ Z_{03, A\xi^-}^\mu = \frac{i}{2}eG^2C_8 + \frac{e^2C_9}{g}OZZ W^\mu \]

\[ + i\left(-\frac{2e^3}{g}C_1 + \frac{4g^5 + 2eG^2}{g^3C_2} - \frac{2e^2G}{C_3} - \frac{3}{2g}C_5 + \frac{2eG}{C_b}\right)SFZ W^\mu, \quad (277) \]

\[ Z_{03, Z\xi^+}^\mu = 0, \quad (278) \]

\[ Z_{03, Z\xi^-}^\mu = \frac{i}{2}eG^2C_8 + \frac{e^2G^3}{g}OZZ W^\mu \]

\[ + i\left(-2g^4 - 3g^5 + g^3C_1 - \frac{2eG}{C_2} - \frac{2g^2G}{C_3} - \frac{3g^3}{C_5} + \frac{2eG}{C_b}\right)SFZ W^\mu, \quad (279) \]

\[ Z_{03, W^-\xi}^\mu = 0, \quad (280) \]

\[ Z_{03, W^-\xi^+}^\mu = \frac{i}{2}eG^2C_8 + \frac{e^2G^3}{g}OZZ W^\mu \]

\[ + i\left(-\frac{2eG^2}{gG}C_5 + \frac{2g^3}{C_2} - \frac{2g^2G}{C_3} - \frac{g}{2C_7} + \frac{2eG}{C_b}\right)SFZ Z^\mu, \quad (281) \]

\[ Z_{03, W^-\xi^-}^\mu = 0, \quad Z_{03, A\xi^-}^\mu = Z_{03, A\xi^+}^\mu, \quad Z_{03, Z\xi^-}^\mu = Z_{03, Z\xi^+}^\mu, \quad (282) \]

\[ Z_{03, W^+\xi}^\mu = Z_{03, W^-\xi}^\mu, \quad Z_{03, W^+\xi^+}^\mu = Z_{03, W^-\xi^-}^\mu, \quad Z_{03, W^+\xi^-}^\mu = Z_{03, W^-\xi^+}^\mu, \quad (283) \]
In these tilded quantities, there are also no terms like $\partial \cdot Z$ and $d \cdot W^\pm$. The unbroken $U_{em}$ symmetry is explicit in these matrices.

Figures and Captions:
FIG. 1. The x axe is $t = \ln \frac{\mu}{m_\pi}$, and $m_\pi = 150$ GeV. The solid lines are for the RGE method, while the dashed lines for the DM. The fig1.a—fig1.j are for the ACs $\alpha_1 - \alpha_a$, respectively.
FIG. 2. The x axe is $t = \ln \frac{\mu}{m_Z}$, and $m_0 = 300$ GeV. The solid lines are for the RGE method, while the dashed lines for the DM. The fig2.a—fig2.j are for the ACs $\alpha_1—\alpha_0$, respectively.
FIG. 3. The x axe is \( t = \ln \frac{\mu}{m_Z} \), and \( m_0 = 450 \text{ GeV} \). The solid lines are for the RGE method, while the dashed lines for the DM. The fig3.a—fig3.j are for the ACs \( \alpha_1-\alpha_n \), respectively.
FIG. 4. The x axe is \( t = \ln \frac{\mu}{m_0} \), and \( m_0 = 900 \) GeV. The solid lines are for the RGE method, while the dashed lines for the DM. The fig4.a—fig4.j are for the ACs \( \alpha_1 - \alpha_a \), respectively.
FIG. 5. The $\alpha_i(m_Z)$ with the scanning of the Higgs mass from 120 GeV to 420 GeV. The solid lines are for the RGE method, while the dashed lines for the DM. The fig5.a—fig5.i are for the ACs $\alpha_1$—$\alpha_a$, respectively. The fig5.j is for $\rho$.

FIG. 6. The precision test parameters at $m_Z$ with the scanning of the Higgs mass from 120 GeV to 420 GeV. The solid lines are for the RGE method, while the dashed lines for the DM. The fig6.a is for the S-T plate, where the $S$ and $T$ are represented by the $x$ and $y$ axes, respectively. The fig6.b, fig6.c and fig6.d are for the triple gauge vertices $g_1^Z - 1$, $k_Z - 1$, and $k_y - 1$, respectively. While for these three figures, the $x$ axe is the varying of the mass of the Higgs.
FIG. 7. The related Feynman diagram (in unitary gauge) in the effective theory which contribute to the trilinear couplings.
FIG. 8. The related Feynman diagram (in unitary gauge) in the renormalizable theory which contribute to the trilinear couplings.