Study on thin plates under thermal action

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Abstract. Under the Hamilton system, the dual form governing equations of thin plate problems are established using displacements and stresses as the basic variables, and the problem is accordingly transformed into finding the eigenvalues and eigensolutions by adopting the method of separation of variables. Furthermore, solutions for zero eigenvalues and non-zero eigenvalues are systematic studied. The study shows that zero eigensolutions are composed of all the overall deformation solutions such as the traditional tension and bending problems, while non-zero eigensolutions include the torsion and bending groups characterized by local deformations.

1. Introduction
When the thermal expansion is caused by the temperature change, the thermal stress will be generated inside the material structure, and the larger deformation will occur [1-3]. The thermal deformation and thermal stress seriously affect the normal use of the structure. Because of the particularity of the temperature load, increasing the thickness of the plate and shell structure uniformly does not necessarily reduce the thermal deformation and thermal stress of the structure, so the research on the optimal design of the thermal structure needs special experience and theory [4].

Hamiltonian system method is a direct solution method in the field of mechanics. In recent years, more and more attention has been paid to this method and it has been successfully applied in elasticity [5]. However, due to the existence of energy dissipation, viscoelastic problems are non conservative systems, and Hamiltonian system method can not be directly applied.

According to the properties of symplectic system and integral transformation, this paper takes the plane viscoelastic problem as a breakthrough point, and reduces the original problem to the problem of Saint Venant solutions and the problem of non-zero eigenvalue eigensolution reflecting local effect. The symplectic orthogonal normalization relation between the eigensolutions is extended from the phase space to the time domain, so that the problem can be solved directly in the symplectic eigenvalue eigensolution space in the time domain, and the unnecessary difficulties caused by repeated use of Laplace Inverse transform are overcome. At the same time, according to the expansion method of symplectic eigensolutions, a set of solution methods for solving nonhomogeneous equations and boundary conditions are given.

2. Governing equations and solution method
Viscoelastic materials are time and temperature dependent materials and the Hamiltonian function described by dual variable can be expressed as
\[ \mathbf{H}(\mathbf{q}, \mathbf{p}) = \mathbf{p}^T \mathbf{q} - L(\mathbf{q}, \mathbf{p}) \]  

(1)

in which \( L \) is Lagrange energy density function. In the temperature field, the constitutive relationship can be expressed as follows:

\[ S_q + \beta \frac{\partial S}{\partial t} = 2\alpha e_q + 2\frac{\partial e_q}{\partial t}, \quad \sigma = 2\gamma e - cT \quad (\alpha\beta \leq 1) \]

(2)

where \( \alpha, \beta, \gamma \) are parameters of the material, \( c = \theta / G, \theta \) is the coefficient of thermal expansion, and \( T \) is temperature. Eq.(2) can describe the stress-strain relationships of Maxwell model, Kelvin model and the standard linear solid model.

The dual variable is

\[ \tilde{\mathbf{p}} = \frac{\partial L}{\partial \mathbf{q}} = \begin{bmatrix} \bar{\sigma}_x \\ \bar{\tau}_{xy} \end{bmatrix} = \begin{bmatrix} (\lambda + 2G)\bar{u} + \lambda \frac{\partial \bar{v}}{\partial y} + cT \\ G \left( \frac{\partial \bar{u}}{\partial y} + \bar{v} \right) \end{bmatrix} \]

(3)

The dual equation in the Laplace domain equivalent to the original basic control equation is obtained by theoretical derivation is

\[ \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{\sigma} \\ \tilde{\tau} \end{bmatrix} = \begin{bmatrix} 0 & -E \frac{\partial}{\partial y} & a_2 & 0 \\ -\frac{\partial}{\partial y} & 0 & 0 & a_1 \\ 0 & 0 & 0 & -\frac{\partial}{\partial y} \\ 0 & -\frac{\partial^2}{\partial y^2} & -v \frac{\partial}{\partial y} & 0 \end{bmatrix} \begin{bmatrix} \bar{u} \\ \bar{v} \\ \bar{\sigma} \\ \bar{\tau} \end{bmatrix} + \begin{bmatrix} T \\ 0 \\ \bar{g}_1 \\ \bar{g}_2 + \frac{\partial T}{\partial y} \end{bmatrix} \]

(4)

in which \( \bar{g}_1 \) and \( \bar{g}_2 \) are body forces. The lateral condition is

\[ E \frac{\partial \bar{v}}{\partial y} + v \bar{\sigma} = \bar{\sigma}_0^+ + cT, \quad \bar{\tau} = \tau_0^+ \quad (y = \pm 1) \]

(5)

For the convenience of analysis, the dual Eq. (4) is simplified as

\[ \tilde{\psi} = \mathbf{H} \tilde{\psi} + \tilde{\mathbf{f}} \]

(6)

It can be seen from the above equation that the temperature function only appears in the form of non-homogeneous term in the expression of stress-strain relationship and lateral condition, so the temperature condition only affects the special solution of non-homogeneous equation. In order to solve the nonhomogeneous problem caused by the temperature condition, the nonhomogeneous lateral condition can be transformed into homogeneous form, so we construct a new set of basic dual variables

\[ \bar{u}' = \bar{u}, \]
\[ \bar{v}' = \bar{v}, \]
\[ \bar{\sigma}' = \bar{\sigma} - \frac{1}{2}\left[ (1+y)\bar{\sigma}_0^+ + (1-y)\bar{\sigma}_0^- \right] - cT \]
\[ \bar{\tau}' = \bar{\tau} - \frac{1}{2}\left[ (1+y)\bar{\tau}_0^+ + (1-y)\bar{\tau}_0^- \right] \]

(7)
Under the new dual variable, the basic governing equation (4) is transformed into homogeneous equation:

\[ \dot{\psi} = \mathbf{H} \psi + \mathbf{\Gamma} \]  

(8)

So the temperature condition problem is transformed into finding a special solution of nonhomogeneous equation.

3. Numerical example

Fig. 1 and Fig. 2 respectively describe the creep behavior of Kelvin model and standard linear solid model in the constant temperature field under simple tension. The results show that the viscoelastic material has a strong dependence on the change of temperature. Even in the constant temperature field, the change of temperature conditions also has a significant impact on the stress and deformation of the material.

4. Conclusion

Viscoelastic materials have obvious dependence on time and temperature conditions. Based on the Laplace integral transformation, the basic equations of thermoviscoelastic problems can be transformed into nonhomogeneous boundary conditions and nonhomogeneous equations. By using this technique, the creep properties of viscoelastic materials are discussed.

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