EVOLUTION OF ACCRETION DISKS AROUND MASSIVE BLACK HOLES: CONSTRAINTS FROM THE DEMOGRAPHY OF ACTIVE GALACTIC NUCLEI

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ABSTRACT

Observations have shown that the Eddington ratios (the ratio of the bolometric luminosity to the Eddington luminosity) in QSOs/active galactic nuclei (AGNs) cover a wide range. In this paper we connect the demography of AGNs obtained by the Sloan Digital Sky Survey with the accretion physics around massive black holes and propose that the diversity in the Eddington ratios is a natural result of the long-term evolution of accretion disks in AGNs. The observed accretion rate distribution of AGNs (with host galaxy velocity dispersion $\sigma \approx 70-200 \text{ km s}^{-1}$) in the nearby universe ($z < 0.3$) is consistent with the predictions of simple theoretical models in which the accretion rates evolve in a self-similar way. We also discuss the implications of the results for the issues related to self-gravitating disks, coevolution of galaxies and QSOs/AGNs, and the unification picture of AGNs.

Subject headings: accretion, accretion disks — black hole physics — galaxies: active — galaxies: evolution

1. INTRODUCTION

The past decade has seen great progress in measuring the mass of massive black holes (BHs) in both nearby normal galaxies and active galactic nuclei (AGNs)/QSOs (e.g., Magorrian et al. 1998; Richstone et al. 1998; Wandel et al. 1999; Kaspi et al. 2000). With reliably measured BH masses in AGNs/QSOs, their Eddington ratios (i.e., the ratio of the bolometric luminosity to the Eddington luminosity) can be estimated. Compared to the bolometric luminosity (or the accretion rate, given the mass-to-energy conversion efficiency $\eta$), the Eddington ratio (or dimensionless accretion rate) is a normalized parameter and more related to the accretion physics. Observations have shown that the Eddington ratios in AGNs/QSOs cover a wide range (at least 2–3 orders of magnitude) with an upper limit of about 1 (Woo & Urry 2002; McLure & Dunlop 2004). It is natural to ask what determines the diversity of the Eddington ratios. The simplest attempt to answer this question would be by first asking whether the diversity of the Eddington ratios is a consequence of the evolution of accretion processes onto BHs.

The questions raised above on the origin of the diversity of the Eddington ratios in AGNs/QSOs may have important implications for understanding the evolution of the AGN/QSO luminosity function (e.g., Schmidt & Green 1983; Boyle et al. 2000). Generally, the number density of AGNs/QSOs at a given redshift and luminosity/dimensionless accretion rate should be jointly determined by both the number densities of galaxies whose nuclear activities are triggered at earlier times and the follow-up nuclear luminosity/accretion rate evolution of individual AGNs/QSOs. The history of the triggering rate is related to the formation and evolution of galaxies (e.g., major mergers of galaxies) and may have a strong cosmic evolution (e.g., Kauffmann & Haehnelt 2000). The follow-up nuclear luminosity/accretion rate evolution of individual AGNs/QSOs is directly related to the evolution of the accretion disk around the BH and the fueling process onto the disk in their host galaxies. In this paper we extract the evolution of the accretion processes from the observational distribution of accretion rates in AGNs. This provides constraints on theoretical models of accretion disk evolution and also illuminates the origin of the diversity of the Eddington ratios.

To extract the information on the nuclear luminosity/accretion rate evolution from observations, statistical methods involving a large sample of AGNs are required, since a single AGN may only represent one specific period in a prolonged phase of nuclear activity. A large sample of AGNs with different ages will span all phases of this activity and allow us to extract information about evolution. In addition to age, other physical parameters may be important in determining how AGNs evolve, and a statistical method may help to clarify these. In § 2, we describe our method of extracting the evolution of accretion processes phenomenologically from the observational distribution of the accretion rates in nearby AGNs (see also the related methods in Yu & Lu 2004a, 2004b). We argue that results for nearby AGNs are not strongly affected by the uncertainty in the nuclear activity triggering history of these objects, because the evolution of the triggering rate at low redshifts ($z < 0.5$) is probably rather weak. Recent progress by the Sloan Digital Sky Survey (SDSS) on the demography of AGNs (Kauffmann et al. 2003b; Heckman et al. 2004) makes the application of the method practical. In § 3, we apply the method to a large sample of SDSS AGNs and get the phenomenological constraints on the evolution of accretion processes. We find that the constraints are consistent with theoretical expectations of the accretion disk evolution models shown in § 4. This consistency suggests that the diversity of Eddington ratios in AGNs is probably a natural result of the evolution of the accretion disks around massive BHs. In § 5, we also discuss the implications of the results for the long-term evolution of accretion disks, AGN unification models, coevolution models of galaxies and QSOs/AGNs, etc. Our conclusions are summarized in § 6.

2. METHODOLOGY

In this section we analyze how the accretion rate evolution of an AGN is incorporated into the accretion rate distribution of AGNs in the local universe.

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Observations have revealed that the central BH mass in nearby normal galaxies is tightly correlated with the velocity dispersion of the bulge components of the host galaxies (Ferrarese & Merritt 2000; Gebhardt et al. 2000; Tremaine et al. 2002). In this paper we take this correlation as a boundary condition for mass growth of BHs in AGNs, that is, the final BH masses of nearby AGNs after the nuclear activity, $M_f$, is assumed to follow the same correlation with velocity dispersion $\sigma$, since mass growth of BHs comes mainly from the nuclear activity phase (e.g., Yu & Tremaine 2002; Fabian 2004; Marconi et al. 2004). Here the velocity dispersion $\sigma$ (or the gravitational potential field in galactic scales) is assumed not to change significantly during and after the nuclear activity phase. This assumption has also been implicitly used in many physical models to interpret the correlation between BH mass and velocity dispersion in nearby normal galaxies (e.g., Silk & Rees 1998; Blandford 1999; Fabian 1999; Adams et al. 2001; Burkert & Silk 2001; King 2003; Murray et al. 2005; see also more discussions on the BH mass vs. velocity dispersion relation in AGNs in §5 and in Yu & Lu 2004b). We describe the correlation through a probability distribution function of $\log M_f$ at a given $\sigma$, $P(\log M_f | \sigma)$, and $P(\log M_f | \sigma) d \log M_f$ gives the probability that the logarithm of the BH mass of an AGN with velocity dispersion $\sigma$ is in the range $\log M_f - d \log M_f$. The mean log $M_f$ at a given $\sigma$ is denoted by $\langle \log M_f | \sigma \rangle = [8.13 + 4.02 \log (\sigma/200 \text{ km s}^{-1})]$. Tremaine et al. 2002). The intrinsic scatter of $P(\log M_f | \sigma)$ around $\langle \log M_f | \sigma \rangle$ is small enough that it is difficult to distinguish from measurement errors but is less than 0.27 dex (Tremaine et al. 2002). Given the bolometric luminosity of an AGN $L_{\text{bol}}$, the mass accretion rate is given by $M_{\text{bol}} \equiv L_{\text{bol}}/(c \epsilon^2)$, where $c$ is the speed of light. In practice, the $[O \text{ iii}]$ Eddington luminosity $L_{\text{Edd}}(\text{O iii})$ is often used as a tracer of AGN activity (e.g., see Heckman et al. 2004), and the bolometric luminosity of an AGN may be estimated from its observed $[O \text{ iii}]$ line luminosity, given its bolometric correction. $f_{\text{bol}} = L_{\text{bol}}/L_{[O \text{ iii}]}$. We may also define the accretion rate through the $[O \text{ iii}]$ line luminosity by $M_{O \text{ iii}} \equiv f_{\text{bol}} L_{[O \text{ iii}]}/(c \epsilon^2)$, where $f_{\text{bol}}$ is a factor representing the average bolometric correction between $L_{[O \text{ iii}]}$ and $L_{\text{bol}}$ and given through $f_{\text{bol}} = \langle \log f_{\text{bol}} \rangle$. The average of the logarithm of the bolometric correction $\langle \log f_{\text{bol}} \rangle$ may be obtained by giving the probability distribution function $P(\log L_{[O \text{ iii}]}, \log L_{\text{bol}})$, which is defined so that $P(\log L_{[O \text{ iii}]}, \log L_{\text{bol}}) d \log L_{[O \text{ iii}]} d \log L_{\text{bol}}$ is the probability that the logarithm of the $[O \text{ iii}]$ line luminosity in the range $\log L_{[O \text{ iii}]} - d \log L_{[O \text{ iii}]}$ at a given $L_{\text{bol}}$. In general, $M_{\text{bol}}$ and $M_{O \text{ iii}}$ are not the same, since the bolometric corrections are not the same for all the AGNs.

Given an AGN with nuclear activity triggered at cosmic time $t_t$ and with final BH mass $M_f$, we describe the evolution of its accretion rate by $M_{\text{bol}}(\tau, t_t, M_f)$, where $\tau = t - t_t$ is the physical time passed since the triggering of the nuclear activity. The total time that the AGN will spend with the logarithm of its accretion rate in the range $\log M_{\text{bol}} - d \log M_{\text{bol}}$ at cosmic time $t_t \equiv t(z = z_t) \leq t \leq t(z = 0) \equiv t_0$ is given by (see eq. [15] in Yu & Lu 2004a)

$$T(\dot{M}_{\text{bol}}, t_t, M_f) d \log \dot{M}_{\text{bol}} \equiv \sum_k \frac{(\dot{M}_{\text{bol}} \ln 10) d \log \dot{M}_{\text{bol}}}{dM_{\text{bol}}(\tau, t_t, M_f)/d\tau |_{\tau = \tau_k}},$$

(1)

where $\tau_k (k = 1, 2, \ldots)$ are the solutions of the equation $M_{\text{bol}}(\tau, t_t, M_f) = \dot{M}_{\text{bol}}$ that satisfy $t_t \leq \tau_k \leq t_0$ and $T(\dot{M}_{\text{bol}}, t_t, M_f) = 0$ if no such solutions exist. In this paper, the redshift $z_t$ is set to 0.3 and AGNs with $z \leq z_t$ are taken as nearby AGNs. We define the function $\phi(M_{\text{bol}}, M_f, \sigma, t)$ so that $\phi(M_{\text{bol}}, M_f, \sigma, t) d \log \dot{M}_{\text{bol}} d \log M_f d \log \sigma$ gives the comoving number density of AGNs at redshift $z$ with the logarithms of the accretion rate, the final BH mass, and the velocity dispersion in the ranges $\log M_{\text{bol}} \rightarrow \log M_{\text{bol}} + d \log M_{\text{bol}}$, $\log M_f \rightarrow \log M_f + d \log M_f$, and $\log \sigma \rightarrow \log \sigma + d \log \sigma$. Given the evolution of the nuclear activity triggering rate $N(t_t, M_f, \sigma)$, which is defined so that $N(t(t_t, M_f, \sigma) dt d \log M_f d \log \sigma$ is the comoving number density of BHs with host galaxy nuclear activity triggered at cosmic time $t_t \rightarrow t_t + dt$ and with the logarithms of its final BH mass and velocity dispersion in the ranges $\log M_f \rightarrow \log M_f + d \log M_f$ and $\log \sigma \rightarrow \log \sigma + d \log \sigma$, we have the time integral of $\phi(M_{\text{bol}}, M_f, \sigma, t)$ given by (see eq. [10] in Yu & Lu 2004a)

$$\int_0^t \phi(M_{\text{bol}}, M_f, \sigma, t) dt = \int_0^t N(t_t, M_f, \sigma) T(\dot{M}_{\text{bol}}, t_t, M_f) dt_t = \mathbb{N}(\dot{M}_{\text{bol}}, M_f, \sigma) \mathbb{T}(\dot{M}_{\text{bol}}, M_f),$$

(2)

where

$$\mathbb{N}(\dot{M}_{\text{bol}}, M_f, \sigma) \equiv \int_0^t N(t_t, M_f, \sigma) T(\dot{M}_{\text{bol}}, t_t, M_f) dt_t,$$

(3)

and

$$\mathbb{T}(\dot{M}_{\text{bol}}, M_f) \equiv \int_0^t T(\dot{M}_{\text{bol}}, t_t, M_f) dt_t.$$  

(4)

By multiplying equation (2) by $P(\log L_{[O \text{ iii}]} | L_{\text{bol}})$ and then integrating it over $\log M_{\text{bol}}$ and $\log M_f$, we have

$$\int_0^t \Phi(\dot{m}_{[O \text{ iii}]}, \sigma, t) dt = \int d \log M_f
\times \int d \log \dot{M}_{\text{bol}} P(\log L_{[O \text{ iii}]} | L_{\text{bol}}) \mathbb{N}(\dot{M}_{\text{bol}}, M_f, \sigma) \mathbb{T}(\dot{M}_{\text{bol}}, M_f),$$

(5)

where

$$\Phi(\dot{m}_{[O \text{ iii}]}, \sigma, t) \equiv \int d \log M_f \int d \log \dot{M}_{\text{bol}} P(\log L_{[O \text{ iii}]} | L_{\text{bol}}) \phi(\dot{M}_{\text{bol}}, M_f, \sigma, t),$$

(6)

$\dot{m}_{[O \text{ iii}]} \equiv M_{[O \text{ iii}]} / L_{\text{Edd}}(M_f | \sigma)$ is the normalized accretion rate, $M_f | \sigma) = 10^{(\log M_f | \sigma)}$, $M_{\text{Edd}}(M_f | \sigma) \equiv L_{\text{Edd}}(M_f | \sigma)/(c \epsilon^2)$, and $L_{[O \text{ iii}]}(M_f | \sigma)$ is the Eddington luminosity of a BH with mass $M_f | \sigma)$. Note that here the definition of the normalized accretion rate is a little different from the conventionally defined Eddington ratio in that $M_{[O \text{ iii}]}$ may not be the same as $\dot{M}_{[O \text{ iii}]}$, and the Eddington luminosity used here is for the average final BH mass, rather than for the BH mass that is powering the nuclear activity in an AGN. If the bolometric correction of each AGN in the sample and its BH mass that is powering the nuclear activity are obtained from future observations, we may construct the methods for the distribution of the Eddington ratios as above and perform a consistency check on the results.
We can further define the time-averaged accretion rate and velocity distribution function of nearby AGNs (with \( z < z_i \)) by
\[
\bar{\Phi}(\dot{m}(\sigma), \sigma) \equiv \frac{\int_0^\infty \Phi(\dot{m}(\sigma), \sigma, t) \, dt}{t_0 - t_i}.
\] (7)

Below we assume that the triggering rate of nearby AGNs \( N(t_i, M_i, \sigma) = \dot{n}(t_i, \sigma) P(\log M_i | \sigma) \), where \( \dot{n}(t_i, \sigma) = \int d \log M_i N(t_i, M_i, \sigma) \) is independent of \( M_i \). For further assumptions about \( \dot{n}(t_i, \sigma) \) is independent of \( t_i \), we have \( N(\dot{M}_{\text{bol}}, M_i, \sigma) = N(t_i, M_i, \sigma) \); then by combining equations (5) and (7), we have
\[
\bar{\Phi}(\dot{m}(\sigma), \sigma) = n(\sigma) \int d \log M_i P(\log M_i | \sigma) \times \frac{\int_0^\infty \dot{\bar{M}}_{\text{bol}} P(\log L_{3000} | L_{\text{bol}}) \bar{T}(\dot{M}_{\text{bol}}, M_i) \, dt}{t_0 - t_i}.
\] (8)

Below we also assume that \( \dot{\bar{M}}_{\text{bol}}(\tau, t_i, M_i) \) is independent of the cosmic time \( t_i \), and according to equations (1) and (4), we have
\[
\bar{T}(\dot{M}_{\text{bol}}, M_f) = \sum_k \frac{\dot{\bar{M}}_{\text{bol}} \ln 10}{d\dot{\bar{M}}_{\text{bol}}(\tau, M_f) / d\tau} \mid_{\tau = \tau_k} \, dt_k,
\] (9)

where \( \tau_k \) (\( k = 1, 2, \ldots \)) are the solutions of the equation \( \dot{\bar{M}}_{\text{bol}}(M_f, \tau) = \dot{\bar{M}}_{\text{bol}} \) that satisfy \( 0 \leq \tau_k \leq t_0 \). If all the solutions \( \tau_k < t_1 \) (\( k = 1, 2, \ldots \)), which is satisfied by the assumed models and its best-fit parameters shown in Fig. 2 below, we have \( \dot{\bar{M}}_{\text{bol}}(M_f, \tau) \propto t_0 - t_i, \dot{\bar{m}}(\sigma, \sigma) = \Phi(\dot{m}(\sigma), \sigma, \tau) \) at \( \tau(t) \leq z_i \), and \( \dot{\bar{m}}(\sigma, \sigma, \tau) \) is independent of \( \tau \). In reality AGNs may have a high triggering rate at high redshifts; however, AGNs triggered at high redshifts may have become faint enough at the current epoch to move out of the range of the accretion rates of interest in this paper. For example, using the accretion rate evolution model assumed below (e.g., [10] and [11]) and the best-fit parameters obtained in § 3, only those AGNs whose nuclear activities are triggered at \( z < 0.5 \) may have an accretion rate high enough at \( z < z_i \) to be within the range shown in Figure 1a below. As an additional check, we have assumed that the triggering rate is proportional to the major merger rate of galaxies, e.g., \( n(t_i, \sigma) \propto [1 + z(t_i)]^{\nu} \) with \( \nu = 0.51 \pm 0.28 \) (Lin et al. 2004), and our results are not significantly affected.

It is plausible to assume that the growth of a BH involves two phases after the nuclear activity is triggered on (Yu & Lu 2004a; see also Small & Blandford 1992). In the first (or “demand limited”) phase, there is plenty of material to fuel BH growth. However, not all of the available material can contribute to the BH growth immediately, and the BH growth is limited by the Eddington luminosity. We assume that the BH accretes with the Eddington luminosity for a period \( \tau_f \) and denote the BH mass at cosmic time \( t_f = t_i + \tau_f \) by \( M_f \). We assume the mass-to-energy conversion efficiency \( \epsilon \) to be constant. Thus, the accretion rate in the first phase increases with time as follows:
\[
\dot{\bar{M}}_{\text{bol}}(\tau) = \dot{\bar{M}}_{\text{Edd}}(M_f) \exp \left( \frac{\tau - \tau_f}{\tau_{\text{Sp}}} \right), \quad 0 < \tau < \tau_f,
\] (10)

where \( \tau_{\text{Sp}} = 4.5 \times 10^7 \epsilon /(0.1(1 - \epsilon)) \) yr is the Salpeter time [the time for a BH radiating at the Eddington luminosity to 5-fold in mass; we set \( \epsilon(1 - \epsilon) = 0.1 \) below]. With the decline of material supply, the BH growth enters into the second (or “supply limited”) phase, and the nuclear luminosity is expected to decline below the Eddington luminosity. Below we assume that the accretion rate in the second phase evolves as follows:
\[
\dot{\bar{M}}_{\text{bol}}(\tau) = \dot{\bar{M}}_{\text{Edd}}(M_f) \left( \frac{\tau - \tau_f + \tau_D}{\tau_D} \right)^{-\gamma}, \quad \tau \geq \tau_f,
\] (11)

where \( \dot{\bar{M}}_{\text{bol}}(\tau) \propto \tau^{-\gamma} \) at \( \tau - \tau_f \gg \tau_D \) and \( \tau_D \) is the characteristic transition timescale from the first phase to the power-law declining stage of the accretion rate. We assume that the two phases appear only once for an AGN. The final BH mass obtained after the nuclear activity is given by
\[
M_f = M_i + \int_{\tau_f}^{\infty} (1 - \epsilon) \dot{\bar{M}}_{\text{bol}}(\tau) \, d\tau = \left[ 1 + \frac{\tau_D}{(\gamma - 1)\tau_{\text{Sp}}} \right] M_i,
\] (12)

where the efficiency \( \epsilon \) is assumed to be the same as the efficiency in the first phase. Here we do not consider that the accretion process may transit to a radiatively inefficient mode when the accretion rate becomes low enough (i.e., when the Eddington ratio is \( \leq 0.001 \); Narayan & Yi 1994; Blandford & Begelman 1999). Such rates are in any case below the accretion rate range studied in this paper (see Fig. 1). For simplicity, below we assume that the timescales \( \tau_f \) and \( \tau_D \) are independent of \( M_i \).

According to the accretion rate evolution model assumed above and equations (1)–(6), \( \dot{\bar{M}}(\dot{m}(\sigma), \sigma) \) is roughly constant with \( \log \dot{m}(\sigma) \) if most AGNs are in the first phase and is proportional to \( \dot{m}(\sigma) \) if most AGNs are at the late power-law declining stage of the accretion rate [here \( P(\log M_f | \sigma) \) and \( P(\log L_{3000} | L_{\text{bol}}) \) are assumed to follow Gaussian distributions; see also § 3]. If the scatter of the distributions \( P(\log M_f | \sigma) \) is sufficiently small and the bolometric correction \( f_{\text{bol}} \) is roughly constant, \( \dot{m}(\sigma) \) has a maximum given by
\[
\dot{m}_{\text{max}} \equiv \left( 1 + \frac{\tau_D}{\gamma - 1} \right)^{-1}
\] (13)

when the AGN is transiting from the first phase to the second phase.

3. APPLICATION TO OBSERVATIONS

The SDSS has obtained a large sample of 33,589 narrow-line AGNs in the local universe. Details of the sample selection and the derivation of the AGN properties used in this paper (\( L_{3000}(\sigma), \sigma \), and \( \mu_\sigma \) [the stellar surface mass density]) are given in Kauffmann et al. (2003a, 2003b) and Heckman et al. (2004). In this section we fit the accretion rate evolution models assumed in § 2 (eqs. [10] and [11]) to these observed AGNs. (Note that type I Seyfert AGNs are not included in this sample because for these objects the AGN itself may have a significant impact at least on the estimate of the velocity dispersion. Possible effects on our conclusions of including type I Seyfert I galaxies are discussed at the end of this section.)

We consider AGNs with \( L_{3000} \geq L_{\text{bol}} \), \( \sigma \geq 70 \text{ km s}^{-1} \), and \( \mu_\sigma \geq 3 \times 10^9 \text{ M}_\odot \text{ kpc}^{-2} \). A sample of fainter AGNs is incomplete because at larger distances, the fiber samples more of the stellar light of the host galaxy, and weak emission lines become difficult to detect. The lower limit on

5 See http://www.mpa-garching.mpg.de/SDSS/Data/agncatalogue.html.
velocity dispersion is set by the instrumental resolution of the SDSS spectrograph. The galaxies with lower stellar surface mass densities are mainly disk dominated (Kauffmann et al. 2003a), and their velocity dispersions may be substantially affected by the disk components. (Note that the quantity \( L_{[O\text{III}]} \) used here is not extinction corrected. The extinction effect may be implicitly included in the scatter of the bolometric correction used in the calculations below. See more discussions in Heckman et al. [2004].) We get the observational distribution \( \Phi_{\text{obs}}(\tilde{m}_{[O\text{III}]}, \sigma) \) \( (j = 1, 2, \ldots, k = 1, 2, \ldots) \) by counting the AGNs in such bins: (1) the bin of the velocity dispersions is \( [\log \sigma_j, \log \sigma_{j+1}] \), where \( \log (\sigma_j/\sigma_{\text{min}}) = (j - 1) \Delta \log \sigma \) and the bin size \( \Delta \log \sigma = 0.05 \) dex; and (2) the bin of the normalized \([O\text{III}]\) line accretion rate is \( [\log \tilde{m}_{[O\text{III]}}, k, \log \tilde{m}_{[O\text{III}]} + \sigma_{\text{min}}] \), where \( \log (\tilde{m}_{[O\text{III}]} + \sigma_{\text{min}}/\tilde{m}_{[O\text{III}]}) = (k - 1) \Delta \log \tilde{m}_{[O\text{III}]} \), \( \log \tilde{m}_{[O\text{III}]} = \log L_{[O\text{III}]} - \log L_{\text{Edd}}(M_f) + \log f_{\text{bol}} \), and the bin size \( \Delta \log \tilde{m}_{[O\text{III}]} = \Delta \log \sigma d \log [M_f/(\sigma)/d \log \sigma \approx 0.20 \) dex. Each observed AGN has a weighted factor \( 1/V_{\text{max},j} \), where \( V_{\text{max},j} \) is the volume over which the object would

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**FIG. 1.**—Distribution of the normalized accretion rate \( \tilde{m}_{[O\text{III}]} \) in AGNs \((z < 0.3)\). The difference between \( \tilde{m}_{[O\text{III}]} \) and the conventionally defined Eddington ratio is described in § 2. Different colors represent different velocity dispersion bins \([\log \sigma_j, \log \sigma_{j+1}]\) \( (j = 1, 2, \ldots) \) with \( \sigma_1 = 70 \) km s\(^{-1}\) and bin size \( \Delta \log \sigma = 0.05 \) dex. In (a), from top to bottom, the colors black, red, green, blue, cyan, magenta, black, blue, and green correspond to \( j = 1, 2, \ldots, 9 \), respectively (\( \sigma_{\text{min}} \approx 200 \) km s\(^{-1}\)). In (b), the colors red, cyan, magenta, black, blue, and green correspond to \( j = 10, 11, \ldots, 15 \), respectively. The sum \( \sum_j \) in the normalization of the distribution is over \( j = 1, 2, \ldots, 9 \) in (a) and over \( j = 1, 2, \ldots, 15 \) in (b). The observational distribution obtained from nearby type II Seyfert galaxies are drawn as points \((\triangle, \times, \square, \bullet, \ast, \#)\) at the middle point of each \( [\log \tilde{m}_{[O\text{III}]}] \) bin, and their error bars are shown by dotted lines. For open triangles, only one AGN is in each bin; for open squares, only two AGNs are in each bin. The falloff of crosses at the low-\( \dot{m} \) end is caused by the luminosity cut at \( L_{[O\text{III}]} = L_{[O\text{III}]-\text{min}} \). The binned model distributions (only for \( j = 1, 2, \ldots, 9 \) in a), obtained with \((\gamma_{\text{bol}}, \gamma_{\text{bol}}, \gamma_{\text{bol}}) = (1, 1.26, 3.1)\), are connected by solid curves (for clarity, the binned distributions are not drawn in the form of a histogram). The observational distribution at the accretion rates \( 10^{-3} \leq \tilde{m}_{[O\text{III}]} \leq 0.1 \) is consistent with a power-law distribution. The tilted dashed lines are the reference lines for the power-law distribution with a slope of \(-1/\gamma\). The horizontal dashed lines are the reference lines illustrating a flat distribution predicted by an exponential declining form of the accretion rate evolution in the second phase of the accretion model. See details in § 3.
have been detectable (Schmidt 1968). The quantity \( \Phi_{\text{abs}}^{i,k}(m_{\text{obs}}, \sigma) \) is obtained by summing up the weighted factors \( \left( \sum_{k} 1/P_{\text{max},k} \right) \) of AGNs in each bin, and its error \( \delta \Phi_{\text{abs}}^{i,k} \) is estimated by \( \left( \sum_{k} 1/P_{\text{max},k}^2 \right)^{1/2} \) (Fig. 1, points and their dotted error bars). Note that the realistic error may not be well represented by the estimate above if the number of AGNs in a bin is small. In the fit we do not use the points with only one (Fig. 1, open triangles) or two (open squares) AGNs in their bins. In each velocity dispersion bin, the falloff of \( \Phi_{\text{abs}}^{i,k}(m_{\text{obs}}, \sigma) \) at the low accretion rate end (Fig. 1, crosses) is caused by the luminosity cut at \( L_{\text{obs}} = L_{\text{min}} \), and these crosses are not used in the fit, either.

We estimate the model distribution of \( \Phi_{\text{mod}}^{i,k} (m_{\text{obs}}, \sigma) \) by first using equation (8) to obtain \( \Phi_{\text{mod}} (m_{\text{obs}}, \sigma) \) and then binning the distribution and normalizing it by setting \( \Phi_{\text{mod}}^{i,k} (m_{\text{obs}}, \sigma) = \sum_{i} \Phi_{\text{obs}}^{i,k} (m_{\text{obs}}, \sigma) \), where the sums are taken over the bins with \( \Phi_{\text{obs}}^{i,k} (m_{\text{obs}}, \sigma) \neq 0 \) (but excluding the bins of crosses and open points shown in Fig. 1; similarly for the sum in \( \chi^2 \) below). We assume that \( P(\log M_f/\sigma) \) follows a Gaussian distribution with an intrinsic scatter of 0.2 dex around \( \langle \log M_f/\sigma \rangle \). (The value of the scatter is set with some degree of arbitrariness, since as mentioned in § 2 the intrinsic scatter is difficult to distinguish from measurement errors but is less than 0.27 dex [Tremaine et al. 2002]. See also discussions below.) The distribution of \( P(\log L_{\text{obs}}/L_{\text{bol}}) \) is assumed to be independent of \( L_{\text{bol}} \) and also follow a Gaussian distribution with \( \langle \log L_{\text{obs}}/L_{\text{bol}} \rangle \sim \log (1/3500) \) and a scatter of \( -0.38 \) dex (Heckman et al. 2004). We use the \( \chi^2 \) statistic to fit the model distribution to the observational distribution of \( \Phi_{\text{obs}}^{i,k} (m_{\text{obs}}, \sigma) \) (\( j = 1, 2, \ldots, k = 1, 2, \ldots \)). The best-fit parameters are obtained by minimizing \( \chi^2 = \sum_{i,j,k} (\log \Phi_{\text{mod}}^{i,k} - \log \Phi_{\text{obs}}^{i,k})^2 / \sigma^2 \). We use \( P_{\chi^2} \) to denote the probability of \( \chi^2 \) being higher by chance if the data were drawn from the model. Our a priori definition of an acceptable fit is one with \( P_{\chi^2} > 0.01 \). For different \( \tau_D/\tau_{\text{Sp}} = 5, 1, 0.1, \) and 0, we obtain the best fits for parameters \( (\gamma, \tau_D/\tau_{\text{Sp}}) \). We find that if the observational distribution only with \( \sigma_1 \leq \sigma \leq \sigma_{10} \approx 200 \text{ km s}^{-1} \) the range of the black hole mass \( M_f/\sigma \) of the nine bins of the velocity dispersion is \( 2 \times 10^{8} - 1.3 \times 10^{9} \text{ M}_\odot \); see Fig. 1a) is included in the fit, all the best fits are acceptable with \( P_{\chi^2} > 0.02 \); if the observational distribution only with \( \sigma_1 \leq \sigma \leq \sigma_0 \approx 176 \text{ km s}^{-1} \) is included in the fit, the best-fit parameters are not affected significantly but with an improved \( P_{\chi^2} \approx 0.07 - 0.10 \); and if adding the observational distribution with higher velocity dispersions \( \sigma \geq \sigma_{10} \) (see Fig. 1b) in the fit, no fits are acceptable (since the distribution may be complicated by some other factors, not solely by the accretion evolution, which is further discussed below).

For the observational distribution with \( \sigma_1 \leq \sigma \leq \sigma_{10} \), the contour plots of confidence levels for the best-fit parameters are shown in Figure 2. The obtained best-fit parameters (Fig. 2, crosses) are insensitive to \( \tau_D \) and the average of the best fits over four cases of \( \gamma \tau_{D}/\tau_{\text{Sp}} \approx (1.26, 3.1) \). We take the errors of the fit parameters as \( \delta \gamma \approx \pm 0.1 \) and \( \delta \tau_{D}/\tau_{\text{Sp}} \approx \pm 1 \), which roughly correspond to the 68% confidence contours (i.e., \( \chi^2 \approx \chi^2_{\text{min}} = 2.30 \) shown in Figure 2). We show the model distribution \( \Phi_{\text{mod}}^{i,k} (m_{\text{obs}}, \sigma) \) (\( j = 1, 2, \ldots, 9 \)) obtained with \( (\gamma \tau_{D}/\tau_{\text{Sp}}, \chi^2) \approx (1.26, 3.1) \) as solid lines in Figure 1a.

As shown in Figure 1a, the observational accretion rate distribution at \( 10^{-3} \leq m_{\text{obs}} \leq 0.1 \) is well consistent with a power-law distribution, and the parameter \( \gamma \) is mainly determined by the slope of the distribution \( \approx -1/\gamma \). If assuming an exponentially declining form of the accretion rate evolution in the second phase of the accretion model instead of the power-law declining form, all fits will be unacceptable. This can be easily understood, since the exponentially declining form predicts a flat accretion rate distribution (see eq. [1]), as illustrated by the horizontal dashed line in Figure 1 (assuming no significant cosmic evolution of the nuclear activity triggering rate).

The best-fit parameter \( \tau_D \) is mainly determined by the observed turnover of the power-law distribution of the accretion rates at the high-rate end \( m_{\text{obs}} \approx 0.1 \); see also \( m_{\text{max}} \) in eq. [13]). In Figure 1, although there exist AGNs with \( \log m_{\text{obs}} \approx \log m_{\text{max}} \approx -1.1 \), the existence of these AGNs does not necessarily mean that they are accreting at a super-Eddington rate with \( M_{\text{bol}} > M_{\text{Edd}}(M_{\text{BH}}) \). [Here \( M_{\text{Edd}}(M_{\text{BH}}) \) is the conventionally defined Eddington accretion rate for the BH mass \( M_{\text{BH}} \) that is powering the nuclear activity in the AGN, not the final mass \( M_f \) after the nuclear activity.] In the accretion evolution model used here, we always have \( M_{\text{bol}} < M_{\text{Edd}}(M_{\text{BH}}) \). The existence of \( m_{\text{max}}, m_{\text{obs}} \approx \log m_{\text{max}} \approx -1.1 \) is caused by the nonzero scatter of the \( M_f - \sigma \) relation and the bolometric correction. The best-fit parameter \( \tau_D \) may be affected by the values of the scatter in the \( M_f - \sigma \) relation and the bolometric correction. For example, if the intrinsic scatter of \( P(\log M_f/\sigma) \) is 0.27 dex, the best fits are \( (\gamma, \tau_D/\tau_{\text{Sp}}) \approx (1.24, 3.5) \); if the intrinsic scatter is 0, no fits are acceptable, but the best fits for the observational distribution with \( \sigma_1 \leq \sigma \leq \sigma_{10} \) are \( (\gamma, \tau_D/\tau_{\text{Sp}}) \approx (1.23, 2.1) \). Precise understanding of the evolution of accretion processes would require precise measurements of this scatter (see also Yu & Lu 2004a).

As mentioned above, the observational sample used here does not include Seyfert I galaxies. The fraction of type II AGNs in the SDSS is a weak function of AGN luminosity, decreasing from \( -0 \) to \( 0.3 \) over the range \( L_{\text{obs}} \approx 10^{45.5} - 10^{5.8} L_{\odot} \) (Hao et al. 2005; Heckman et al. 2004). Thus, the fraction of AGNs with high accretion rates (e.g., at \( m_{\text{obs}} \approx 0.1 \); see Fig. 2) could be underestimated here, which mainly affects the estimate of \( \tau_D \). After including type I AGNs, the best-fit parameter \( \tau_D \) would decrease and \( \gamma \) would increase slightly, resulting in an increase in \( m_{\text{max}} \) (eq. [13]). If we assume that the dependence of the fraction of type II AGNs on accretion rate is the same as the dependence on luminosity, after including type I AGNs, we find that the distributions (at the high accretion rates) shown in Figure 1 move upward by at most \( 0.3 \) dex. The
value of $\tau_D/\tau_{SP}$ would then have to change from 3.1 to 2.2 in order to compensate.

The best-fit $\tau_D$ above is obtained using the number distribution of AGNs as a function of accretion rate. Generally, the timescale related to the length of the nuclear activity may also be constrained by comparing the number density ratio of AGNs to their remnants (e.g., Yu & Tremaine 2002). The predicted ratio of the number density of nearby AGNs to nearby inactive galaxies is affected by uncertainties in the triggering history of AGNs at high redshifts and hence does not help us obtain a more precise constraint.

As mentioned above, if adding the observational distribution with high velocity dispersion $\sigma \geq \sigma_{10} \simeq 200$ km s$^{-1}$ (see Fig. 1b) in the fit, no fits are acceptable. A full explanation for their detailed distribution would need to involve at least the following four factors. (1) These objects generally have relatively low normalized accretion rates, and thus their accretion rate distribution could be affected by the change of the efficiency $\epsilon$ due to the transition from the thin-disk accretion mode to the radiative inefficient accretion mode. (2) Host halos or galaxies of high-mass BHs generally have different major merger histories than those of low-mass BHs. The AGNs with high velocity dispersions generally have high BH masses, low normalized accretion rates, and probably long ages of the nuclear activities, and thus their distribution could be more likely to be affected by the cosmic evolution of the nuclear activity triggering history. (3) The low accretion rate in these objects (e.g., $\dot{m} \leq 10^{-3}$) may be simply caused by Bondi accretion. Note that the observational accretion rate distribution at the low accretion rate end shown in Figure 1b has indicated a steeper increase than the slope $-1/\gamma$. (4) The assumption that $\tau_D$ are independent of $M_f$ may be violated for a wide range of $M_f$.

4. COMPARISON WITH ACCRETION DISK EVOLUTION MODELS

The process that removes the angular momentum of gas in a galaxy and drives it into the very central region is still not well understood. It may require a hierarchy of mechanisms, handing off to one another as the material moves inward. Here we consider the “last stop” of the “handing off.” We hypothesize that some mechanism has moved gas to the central parsec in the galaxy, where it will then diffuse to the central BH through viscous transport (or, specifically, by the magnetorotational instability; e.g., Balbus & Hawley 1998) in an accretion disk. The surface mass density $\Sigma$ of an accretion disk with a central, dominant gravitating point mass and with viscosity $\nu$ evolves as a function of time $\tau$ and radius $R$ through the equation

$$\frac{\partial \Sigma}{\partial \tau} = \frac{3}{R} \frac{\partial}{\partial R} \left[ R^{1/2} \frac{\partial}{\partial R} \left( \nu \Sigma R^{1/2} \right) \right].$$

If the viscosity is of the form $\nu(R, \Sigma) \propto R^m \Sigma^n$ (where $m$ and $n$ are constants), this equation has solutions of self-similar form (Pringle 1981) as follows:

$$\frac{\Sigma(R, \tau)}{\Sigma_0} = \left( \frac{\tau}{\tau_0} \right)^{\eta} f \left( \frac{R}{R_0} \right) \left( \frac{\tau}{\tau_0} \right)^{\xi},$$

where the values of $\eta$ and $\xi$ are determined by the exponents $\nu(R, \Sigma)$ and also by the detailed boundary conditions of the accretion disk. Here $\Sigma_0, R_0,$ and $\tau_0$ are scale variables with arbitrary dimensions that satisfy $\tau_0 = R_0^2/\nu(R_0, \Sigma_0)$. The self-similar evolution of accretion disks has also been calculated using numerical simulations (e.g., Lin & Pringle 1987; Cannizzo et al. 1990; Pringle 1991). For an accretion disk with negligible torque at both the origin $R = 0$ and its outer boundary (so that its total angular momentum is conserved during its evolution), the rate at which the mass of the disk decreases is given by a power law of the form $M_f(\tau) = (dM_f/d\tau) \int 2\pi \Sigma R dR \propto \tau^{-(38+18\alpha+4\beta)/12+17\alpha+2\beta}$, where $a$ and $b$ are the exponents in the opacity law $\kappa(\rho, T) = \kappa_0 T^{a-b}$ (e.g., Cannizzo et al. 1990). If $a = b = 0$ (Thomson opacity), we have $\eta = -15/16, \xi = -3/8, f(x) = (28)^{-3/2} x^{-3/5}(1 - x^{7/5})^{-1/2}$ in equation (15), and the mass accretion rate evolves as

$$|\dot{M}_f(\tau)| = \frac{(16/3) M_{d,0}}{\tau_0} \left( \frac{\tau}{\tau_0} \right)^{-19/16},$$

where $M_{d,0} = (28)^{-3/2} (4\pi/7) R_0^2 \Sigma_0$, and the diffusion timescale of the disk

$$\tau_0 = \frac{R_0^2}{\nu(R_0, \Sigma_0)} = [(0.1-1.6) \times 10^8 \text{ yr}] \left( \frac{R_0}{0.3-1 \text{ pc}} \right)^{7/3} \times \left( \frac{M_{BH}}{10^7 M_\odot} \right)^{1/3} \left( \frac{\alpha}{0.1} \right)^{-4/3} \left( \frac{M_{d,0}}{10^7 M_\odot} \right)^{-2/3},$$

where $\alpha$ is the commonly used parameter related to the viscosity in the standard accretion disk model and $R_0$ may be taken as the size of the accretion disk or the location of its reservoir. The value of $\tau_0$ is sensitive to $R_0$, and the reference value of $R_0$ in equation (17) is roughly consistent with the observations of the accretion disks in several nearby AGNs (e.g., NGC 4258, Miyoshi et al. 1995; NGC 1068, Lodato & Bertin 2003; and NGC 3079, Kondratko et al. 2005). If $a = 1$ and $b = 7/2$ (Kramers opacity), we have $|\dot{M}_f| \propto \tau^{-5/4}$.

Note that the best-fit parameter $\gamma = 1.26 \pm 0.1$ for AGNs with $\sigma \simeq 70$–200 km s$^{-1}$ is consistent with the exponent $(-1.18$ or $-1.25)$ of $\tau$ in $M_f$ expected from the self-similar solutions above. The diffusion timescale $\tau_0$ in equation (17) is also roughly consistent with $\tau_D$ obtained from observations. These consistencies between the observations and the theoretical expectations above are an encouraging result. They support the hypothesis that the observed diversity in the Eddington ratios of AGNs results from an evolutionary sequence of accretion processes. They also suggest that the accretion disks of most Seyfert galaxies (with $\sigma \simeq 70$–200 km s$^{-1}$ and $m \simeq 10^{-3}$) are currently in the self-similar phase or are transiting toward this phase. It is interesting to note that the self-similar evolution of disk evolution is also supported by observations of accretion disks around T Tauri stars (Hartmann et al. 1998).

5. DISCUSSIONS

Despite the encouraging consistency above, the self-similar solution of the accretion disk evolution shown in § 4 is based on highly simplified accretion disk models. For example, the effects on the disk evolution by disk winds or by infalling material that is continuously deposited onto the disk are not considered. The role of possible disk instabilities is not considered, either. More complete investigations on the long-term evolution of accretion disks are needed.

There are also further issues to be considered.

Self-gravitating disks and star formation.—With the best-fit parameters obtained for the accretion evolution in § 3, the disk is self-gravitating in its early evolutionary stage and the central BH
mass becomes dominant in its late evolutionary stage. We note that this evolutionary pattern of the disk mass is probably compatible with current observations of the accretion disks in a few AGNs. For example, NGC 3079 (Kondratko et al. 2005) and NGC 1068 (Lodato & Bertin 2003) have Eddington ratios close to 1, which may correspond to the first demand-limited phase or a transition to the self-similar evolution of the second phase. Their accretion disks appear to be thick with masses larger than or comparable to their central BH masses. NGC 4258 has an Eddington ratio $\lesssim 0.01$ (Gammie et al. 1999 and references therein), so it may be in the late stage of self-similar evolution, and its accretion disk is thin with a mass that is much smaller than its BH mass (Miyoshi et al. 1995). The evolution of a self-gravitating disk may not exactly follow the simple prescription adopted in this paper. Recently, Fromang et al. (2004) simulated the evolution of self-gravitating tori around a massive BH and found that they quickly evolve to a dual structure with an inner, thin, Keplerian disk fed by an outer, thicker, self-gravitating disk (see also Bertin & Lodato 1999; Lodato & Rice 2004, 2005).

Further investigation of the long-term evolution of self-gravitating disks is crucial for understanding the transition to the self-similar phase. In addition, in a self-gravitating disk, star formation may be important because of gravitational instabilities that develop in the disk (Goodman 2003; Levin 2003). However, the growth of the central BH will consume a substantial fraction of the disk mass.

The fraction of the disk mass that goes into stars rather than the BH is worthy of further investigation.

The $M_f-\sigma$ relation in normal galaxies and the $M_{\text{BH}}-\sigma$ relation in AGNs.—As mentioned in § 2, in our model the $M_f-\sigma$ relation in normal galaxies is used as a boundary condition for BH mass growth in AGNs, and BH masses in AGNs are lower than the expectation from the $M_f-\sigma$ relation. In observations, some AGNs with sufficiently low Eddington ratios may be included in normal galaxies. As a self-consistency check for our model, here we show that the best-fit parameters for BH growth obtained in § 3 do not contradict the tightness of the $M_f-\sigma$ relation. For example, using equations (11) and (12), for $(\gamma, \tau_D) = (1.26, 3.1)$, when the Eddington ratio of an AGN declines to below $10^{-3}$, the logarithm of the ratio of its central BH mass to its final mass is $-0.18$ dex $\lesssim \log (M_{\text{BH}}/M_f) < 0$, which does not exceed the scatter of the $M_f-\sigma$ relation used in the model. In addition, considering that most of the supplying material may be blown away as outflows when a BH accretes material via the ADIOS (adiabatic inflow-outflow solutions) mode at low accretion rates (Blandford & Begelman 1999), the difference between the BH masses in AGNs with low Eddington ratios and their final masses will be smaller.

For the best fits $(\gamma, \tau_D) = (1.26, 3.1)$, BH masses in most AGNs ($\sigma \approx 70–200 \text{ km s}^{-1}$) with Eddington ratios in the range 1–0.1 are about 0.08–0.3 times their final mass $M_f$. (Note that here at a given velocity dispersion and luminosity, most AGNs are in the second evolutionary stage, rather than in the first stage in which the nuclear luminosity exponentially increases.) However, by studying 15 dwarf Seyfert I galaxies ($\sigma \approx 30–80 \text{ km s}^{-1}$), Barth et al. (2005) found that the BH masses in these AGNs, $M_{\text{BH}}$ (estimated from the empirical relations among the BH mass, the nonstellar 5100 Å continuum luminosity, the broad-line region radius, and the broad-line width in Kaspi et al. [2000]), are consistent with the $M_f-\sigma$ relation and that these AGNs have Eddington ratios in the range 0.1–1. Currently, it is hard to answer whether the evolution model in our paper contradicts the observations or not, because the answer should take into account at least the following uncertainties. (1) Usually, BH masses $M_{\text{BH}}$ used to derive the empirical relations in AGNs are estimated through the reverberation mapping technique with some assumptions on the kinematics of the broad-line emission clouds (e.g., Wandel et al. 1999; Kaspi et al. 2000). The reverberation mapping technique could either systematically overestimate or underestimate the BH mass by a factor of 3 or so (Krolik 2001). (2) It is possible that the sample in Barth et al. (2005) biases toward detection of AGNs with the highest mass black holes for a given $\sigma$. (3) The constraint on $\tau_D$ obtained in § 3 may not be strict (e.g., because of the exclusion of Seyfert I galaxies). If $\tau_D$ is smaller (e.g., $\approx 1\tau_\text{sh}$), the BH masses in most AGNs with Eddington ratios in the range 1–0.1 will be closer to their final mass (e.g., $\approx 0.2M_f–0.4M_f$). (4) The dwarf galaxies in Barth et al. (2005) are in a velocity dispersion range different from the SDSS AGNs and may not follow the same BH growth parameters.

In addition, we note that the BH masses in AGNs measured through the reverberation mapping technique by Onken et al. (2004) are calibrated to follow the $M_f-\sigma$ relation in normal galaxies by multiplying them by a factor of 1.8. Onken et al. (2004) argue that the calibration factor could be caused by a nonisotropic velocity distribution of the broad-line emission clouds; however, an alternative explanation could be that there exists an offset between the $M_{\text{BH}}-\sigma$ relation in AGNs and the $M_f-\sigma$ relation in normal galaxies, which just represents BH mass growth during the nuclear activity phases (Yu & Lu 2004b). It is worthwhile to study whether such an offset really exists, which would provide strong constraints on BH growth models and the physical origin of the $M_f-\sigma$ relation.

Binary BHs (BBHs) and coevolution of galaxies and QSOs/AGNs.—In the current coevolution model of galaxies and QSOs (Kauffmann & Haehnelt 2000), the nuclear activity of galaxies is triggered by major mergers of galaxies. Mergers of galaxies may form a BBH if there exists a massive BH in each of the merging galaxies (Begelman et al. 1980; Yu 2002; Volonteri et al. 2003). The evolution of the accretion disk around a massive BBH may be different from that around one BH (e.g., because of different boundary conditions of the disk) and may depend on the orientation of the BBH orbital plane and BH spins. For example, studies of the time-dependent evolution of the accretion disk around the primary of a binary star with the outer disk radius tidally truncated by the secondary also yield self-similar solutions but with a steeper time dependence of the accretion rate $M_d \propto \tau^{-2.5}$ (Thomson opacity) or $\tau^{-3.3}$ (Kramers opacity) (Lipunova & Shakura 2000; see also a different self-similar solution for an external disk around a binary star by Pringle 1991). The consistency of the slope of the accretion rate function with the expectation of our simple model may suggest that galaxy mergers have not been sufficiently efficient in faint AGNs to form a large population of BBHs or that galaxy triaxiality or gas dynamics has caused BBHs to merge very quickly.

The AGN luminosity function, defined by the comoving number density of AGNs per unit luminosity at a given redshift, is proportional to $\int m^n \Phi(m, \sigma) d \log \sigma$ with $m \propto L/L_{\text{Edd}}[M_f(\sigma)]$. The luminosity function at the faint end, if fitted by a power-law form $\propto L^\alpha$, is expected to have the slope $\beta \lesssim -1 - 1/\gamma$ (i.e., $\beta \lesssim -1.85, -1.80, -1.40$, and $-1.30$ for $\gamma = 1.18, 1.25, 2.5$, and 3.3, respectively), where the symbol $\lesssim$ is expected instead of the symbol $\approx$ because the break of the power-law accretion rate distributions ($m \approx m_{\text{max}}$) at the high-rate end corresponds to different break luminosities for AGNs with different velocity dispersions, and thus the faint end of the luminosity function consists of not only AGNs with accretion rates lower than the break rate but also some with rates higher than the break rate, which may steepen the slope. According to the fit for the accretion rate distribution of nearby AGNs, the expectation for
their luminosity function at the faint end, $\beta \leq -1.8$, is consistent with the fit for the observational luminosity function by Hao et al. (2005). Observations of the QSO luminosity function indicate a relatively flat slope $\beta \simeq -1.45 \pm 0.03$ at the faint end (Richards et al. 2005), which might suggest the existence of BBHs in a significant fraction of QSOs, since $-1.8 < \beta < -1.3$. It is worthwhile incorporating the evolution of BBHs and the evolution of accretion disks in models for the coevolution of galaxies and QSOs/AGNs. One could then make joint predictions for the evolution of the QSO/AGN luminosity function as well as their accretion rate distribution function for comparison with observations.

Possible difference between AGNs and bright QSOs.—The luminosity evolution of bright QSOs ($L_{bol} \gtrsim 10^{46}$ erg s$^{-1}$) is constrained from observations through a relation between the BH mass function decreases exponentially at the high-mass end, a long characteristic timescale (see eq. [12]); and (2) because the local BH mass function is significantly different from the radiatively inefficient mode (see also Begelman & Celotti 1984). The existence of some AGNs whose spectra need to be explained through radiatively inefficient accretion modes (but with an accretion rate larger than the Bondi accretion rate) may thus also be explained as a consequence of the evolutionary sequence described in this paper. The distribution of the normalized accretion rates of nearby low-luminosity AGNs could provide constraints on the transition from the radiatively efficient accretion mode to the radiatively inefficient mode (see also Begelman & Celotti 2004).

Radio-loud versus radio-quiet.—AGNs/QSOs can be classified into two groups: radio-loud and radio-quiet. It has been a long-standing question whether all AGNs pass through a radio-loud phase or whether only certain objects develop jets and become radio-loud. Boroson (2002) shows that most radio-quiet QSOs have high Eddington ratios, while most radio-loud QSOs have low ratios. Since the accretion rate decreases with time as the accretion disk evolves, we propose that from the evolutionary point of view, a QSO/AGN appears radio-quiet at its early evolution stage and appears radio-loud at its late stage (of course, the detailed appearance is also complicated by orientation effects and possibly by BH spins).

6. CONCLUSIONS

In this paper we have connected the accretion rate distribution of nearby SDSS AGNs ($z < 0.3$) with the accretion physics around massive BHs. The observed distribution of the AGNs (with host galaxy velocity dispersion $\sigma \approx 70–200$ km s$^{-1}$) follows a power-law distribution at low accretion rates ($\dot{M}/\dot{M}_{\text{bol}} \approx 10^{-3}–0.1$), which is consistent with a scenario in which their mass accretion rates are declining with time in a power-law form ($\dot{M} \propto \tau^{-\gamma}$ with $\gamma = 1.26 \pm 0.1$) and the accretion process follows a self-similar evolutionary pattern, as simple theoretical models predict. The transition timescale of the accretion to the self-similar phase (estimated from the turnover of the observational power-law distribution at high accretion rates) is roughly consistent with the diffusion timescale of an accretion disk. The results suggest that the observed diversity in the Eddington ratios of AGNs ($\sim 10^{-2}–1$) is a natural result of the long-term evolution of accretion disks. Some other issues deserving of further investigation are discussed, such as a full explanation for the accretion rate distribution of AGNs with higher velocity dispersions.
(σ ≳ 200 km s⁻¹), the long-term evolution of accretion disks (including self-gravitating disks), the BH mass versus velocity dispersion relation in AGNs, the evolution of BBHs in QSOs/AGNs, the coevolution of galaxies and QSOs/AGNs, and the unification picture of AGNs.

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