Test calculation of steel I-beams with non-standart cross-section at restrained torsion

Tatiana Dmitrieva¹, Alexey Lukin² and Ulambayar Khukhuede³

¹Irkutsk National Research Technical University, 664074, Lermontova str., 83, Irkutsk, Russia
²Samara State Technical University, 443110, Molodogvardeyskaya str., 244, Samara, Russia

Abstract: Solved the test problem to determine the stress-strain state of a steel I-beam with non-symmetrical flat web in conditions of restrained torsion. In the first version, the problem is solved analytically using the formulas of the mechanics of materials. The second version of the calculation is performed numerically in «ANSYS 14.5» software package based on the spatial model, where finite elements of shell type are used. A comparative analysis of numerical solution with an analytical calculation revealed the adequacy of the FE model of shell type.

1. Introduction

Thin-walled rods of an open profile are widely used in the construction of buildings and structures, due to the simplicity of their manufacture and ease of operation compared to rods of a closed profile.

The founder of the theory of calculating thin-walled rods should be considered the world-famous mechanic S.P. Timoshenko, who turned to one of the problems of restrained torsion as early as 1905-1906. When considering the issue of overall stability of an I-beam, he investigated the
effect of torsion and derived a formula for the angle of twist of cantilever beam with fixed support, which he tested experimentally [1, 2].

The general theory of the calculation of thin-walled rods was created by Professor V.Z. Vlasov and published in his book [3], where the issues of strength, stability and vibrations are covered in detail. V.Z. Vlasov created a complete theory of calculating thin-walled rods of an open profile.

Further development of this theory and its application to particular problems can be traced in the works [1, 4-10]. In 2005, V.I. Slivker [11] proposed a semi-shear theory that takes into account part of shear deformations in the middle surface of the walls of rods, caused by the action of sectorial forces. This theory allows the use of a single algorithm for the calculation of thin-walled structures made from rods of open, closed and combined profiles, which is important for numerical implementation of the algorithm in the form of the finite element method [12, 13]. Thin-walled rods in steel structures can work in conditions of bending with torsion. The calculation of the stress-strain state in bending, coupled with the curvature of the cross section as a result of torsion, is a difficult task. To make an informed choice of parameters of a structure, it is necessary to study in detail its behavior under these conditions.

The purpose of this article is to substantiate the applicability of the shell finite element model to the calculation of thin-walled rod structures of an open profile under the action of bending with torsion. To achieve this goal, to decide two tasks are required. First, using known formulas of mechanics of materials describing the stress-strain state of a thin-walled rod of an open profile, it is necessary to perform an analytical calculation. The next task is a numerical calculation based on the spatial model in «ANSYS 14.5» software package, where shell-type finite elements are used. The results obtained numerically allow us to conclude about the applicability of the used FE model.

2 Calculation of I-beam with non-standard cross-section with flat web

Consider test calculation of I-beam of non-symmetrical section with flat thin wall under restrained torsion with a bend, which is caused by the action of an eccentric applied concentrated force \( P \) (Fig. 1).
2.1 Analytical calculation

Let's give analytical expressions for stiffness and strength parameters of calculation model of the beam at restrained torsion.

Static moment of area about the axis ZI:

$$S_{ZI} = \int y \, dA,$$

where, $y$ - the perpendicular distance to the axis $ZI$ (Fig. 2,a).

Moments of inertia:

$$I_y = \int z^2 \, dA,$$
$$I_z = \int y^2 \, dA,$$

where, $z$ and $y$ - distances to axes $OY$ and $OZ$.

Sectorial-linear static moment about the $Y$ axis:

$$S_{\omega_B} = \int \omega_B \, z \, dA = \int \omega_B \, z \, t_w \, dl,$$

where, $\omega_B$ - sectorial area, which is defined by following formula:

$$\omega_B = \int_0^L r \, dl.$$

Moment of inertia at pure torsion:

$$I_k = \frac{\alpha}{3} (2Br_z^3 + ht^3).$$
where, \( B \) - flange width, \( t_f \) - flange thickness, \( h \) - height of web, \( t_w \) - web thickness, the coefficient \( \alpha \) for I-beams is assumed to be 1.2.

Sectorial moment of inertia:

\[
I_{\omega} = \int \left( \alpha_0^2 (t_f + t_w) \right) dl ,
\]

(7)

Flexure-torsion characteristics \( K \):

\[
K = \sqrt{\frac{GI_x}{EI_{\omega}}} ,
\]

(8)

where, \( E \) and \( G \) are modulus of elasticity and shear modulus.

Highest normal stress at torsional with the bend:

\[
\sigma = \sigma_u + \sigma_\omega = \frac{M_{y_{\text{max}}}}{W_y} + \frac{B_{\text{max}}}{W_\omega} ,
\]

(9)

where, \( M_{y_{\text{max}}} \) - maximum bending moment, \( W_y \) - the moment of resistance about axis \( Y \), \( B_{\text{max}} \) - highest value of bimoment obtained in fixed support at \( x=L \), \( W_\omega \) - sectorial moment of resistance.

\[
M_{y_{\text{max}}} = P \cdot L ,
\]

(10)

\[
W_y = \frac{I_y}{Z} ,
\]

(11)

\[
B_{\text{max}} = \frac{M_c \cdot \text{sh}(KL)}{K \cdot \text{ch}(KL)} ,
\]

(12)

\[
W_\omega = \frac{I_\omega}{\omega_{\text{max}}} ,
\]

(13)

where, \( P \) - the applied load, \( L \) - the span length, \( M_c \) - torsion moment acting on the free end of the rod, \( K \) - flexure-torsion characteristics, \( \text{sh}(KL) \) and \( \text{ch}(KL) \) - hyperbolic functions, \( \omega_{\text{max}} \) - main sectorial area.

\[
M_c = P \cdot e ,
\]

(14)

where, \( e \) - the eccentricity.

Shear stress in the dangerous section of the rod:

\[
\tau = \tau_u + \tau_{M_c} + \tau_{M_0} = \frac{Q \cdot S_Y^{\text{ov}}}{} + \frac{M_0 S_{\omega}^{\text{ov}}}{I_{\omega} I_f} + \frac{M_0}{I_K} I_f ,
\]

(15)

where, \( \tau_u \) - shear stresses from bending, \( \tau_{M_c} \) - shear stresses from flexure-torsion moment, \( \tau_{M_0} \) - shear stresses from pure torsion, \( S_Y^{\text{ov}} \) - static moment of severed part of the section about the axis \( Y \), \( S_{\omega}^{\text{ov}} \) - sectorial static moment.
of severed part of the section, $M_\alpha$ - flexure-torsion moment, $M_0$ - moment of pure torsion.

$$M_0 = GLK\theta = \frac{Ma(ch(KL) - ch(Kx))}{ch(KL)},$$

(16)

$$M_\alpha = -EI\theta = \frac{Mc(ch(Kx))}{ch(KL)},$$

(17)

Expression for the angle of twist:

$$\theta = B_{\alpha_0} \frac{1 - ch(Kx)}{GLK} + M_{\kappa_0} \frac{x}{K}.$$

(18)

We obtain the numerical values of parameters from the list (1-18) for given beam geometry given in Table 1 and in Fig.2.

**Table 1.** Initial data.

| Parameters of the cross section | Calculated data |
|---------------------------------|----------------|
| **H, mm** | **B, mm** | **h, mm** | **t_w, mm** | **t_f, mm** | **E, GPa** | **G, GPa** | **P, kN** | **L, mm** |
| 400 | 300 | 386 | 372 | 12 | 14 | 200 | 76.9 | 20 | 3200 |

**Figure 2.** Cross section to the calculation: a) gravity center coordinates, b) finding the eccentricity and coordinates of the center of the bend (the point $A$).

Obtained geometric characteristics of the beam are given in Table 2.


Table 2. Results of calculations.

| $A$, cm$^2$ | $S_{Z1}$, cm$^3$ | $I_y$, cm$^4$ | $I_z$, cm$^4$ | $S_{o}$, cm$^3$ | $I_K$, cm$^4$ | $I_{o}$, cm$^4$ | $K$, m$^{-1}$ |
|------------|-----------------|---------------|---------------|----------------|---------------|---------------|-------------|
| 128,64     | -504            | 36218,4       | 6663,44       | 183864,25      | 91,6          | 2475805       | 0,37        |

2.2 Automated calculation

The results reliability of spatial model was confirmed by solving the problem of analyzing the FE, by performing the numerical calculation of the beam using «ANSYS 14.5» software package. FE model of the beam is shown in Fig. 3. The results of numerical study are shown in Table 3.

![Finite element model of the calculated beam.](image)

2.3 Results evaluation of analytical and numerical calculation

Let’s compare maximum values of normal and shear stresses, calculated in the beam using expressions (9), (15), with similar numerical values obtained in «ANSYS 14.5» software package.
### Table 3. Comparative results of calculations.

| №  | Calculation methods                      | $\sigma_{\text{max}}, \text{MPa}$ | $\tau_{\text{max}}, \text{MPa}$ | $\theta$ |
|----|------------------------------------------|---------------------------------|---------------------------------|--------|
| 1  | Analytical solution                      | 113                             | 61,4                            | 2,3    |
| 2  | Solution in «ANSYS 14.5» for element SHELL181 | 114,85                         | 64,6                            | 2,5    |
| 3  | Calculation error, $\epsilon, \%$      | 1,64                            | 5,21                            | 8,70   |

Based on comparison of results obtained, where the error is less than 5%, it can be concluded about the adequacy of adopted shell FE model when calculating beam on torsion with bend.

### 3 Conclusion

The results obtained in a numerical and analytical calculation confirmed the applicability of the shell FE model for the calculation of beams during torsional bending with a sufficient degree of accuracy. This study substantiates the possibility of using the shell FE model for the calculation of beams of complex geometry of variable cross section, when the calculation by the rod scheme is difficult.

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