Proton Stability and Small Neutrino Mass in String Inspired $E_6$ Models

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Abstract

We propose a new possibility to realize simultaneously the sufficient proton stability and the interesting structure of neutrino mass matrix in superstring inspired $E_6$ models. In this model the leptons and Higgs fields are assigned to a fundamental representation $27$ in the different way among generations. Two pairs of Higgs doublets naturally remain light from three generation ones by imposing certain discrete symmetries, although all extra color triplets become sufficiently heavy. Under these symmetries suitable $\mu$-terms to bring appropriate vacuum expectation values are prepared and the dangerous FCNC is avoidable. Some related phenomena to this model, especially, the structure of neutrino mass matrix are also discussed.
1 Introduction

The unification of interactions is a very fascinating idea. Although it brings many remarkable successes in supersymmetric models [1, 2], it also causes some difficulties because of the strong constraints due to its unified group structure. The existence of extra light color triplets generally causes serious problem to the unified models since it makes proton unstable [1]. This problem occurs because extra color triplets are contained in the same multiplets together with the light doublet Higgs fields. How to resolve this difficulty is one of the almost common issues of the realistic unified model buildings [1, 3].

It is well known that the same problem often annoys superstring inspired models too, although there are not necessarily the above mentioned multiplet structure. In the realistic model buildings the existence of discrete symmetries and/or intermediate scales is often assumed in order to decouple these dangerous color triplet fields from ordinary quarks and leptons in the low energy world.

In superstring inspired $E_6$ models the same problem occurs. The full contents of 27 of $E_6$ remain massless in the low energy effective theory, although their multiplet structure is lost by the symmetry breaking due to the existence of the background fields on the extra dimensions [4]. A fundamental representation 27 contains extra color 3 and $3^*$ fields and the above mentioned triplet-doublet splitting problem appears. Usually it is assumed the existence of an intermediate scale to make these extra color triplets heavy enough providing suitable discrete symmetries [5]. However, if we adopt such schemes it becomes difficult to give favorably small masses to neutrinos [5, 4, 4]. In string inspired $E_6$ models with an intermediate scale the proton stability and small neutrino mass seem not to be so easily reconciled.

In this paper in the certain type of string inspired $E_6$ models we propose a new possibility that small neutrino masses are successfully introduced, although the proton stability is guaranteed by making extra color triplet fields heavy. The essential point of this scenario is the use of the freedom of the field assignment in 27 of $E_6$. We will show how our scenario works in a very simple example and also discuss its phenomenological features briefly.

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1 There are other type of models with no intermediate scale [6]. In such models extra color triplets are kept light but proton decay is forbidden by discrete symmetries. Small neutrino mass is induced by the loop effects [6, 9].
A fundamental representation 27 of $E_6$ contains one generation quarks, leptons and two singlets ($16 + 1$ of $SO(10)$), a pair of Higgs doublets and a conjugate pair of extra color triplets ($10$ of $SO(10)$) as shown in Table 1. There is usually the extended gauge structure other than the standard model gauge group $G_{SM} = SU(3) \times SU(2) \times U(1)$. In the following discussion we consider the models whose gauge group is $G_{SM} \times U(1)^2$. Two $G_{SM}$ singlets have these extra $U(1)$ charges. The gauge invariant superpotential for 27 chiral superfields can be written by using the field notation presented in Table 1 as

$$W = \lambda_{1}^{ijk} A_i A_j E_k + \lambda_{2}^{ijk} A_i B_j F_k + \lambda_{3}^{ijk} A_i C_j G_k + \lambda_{4}^{ijk} A_i D_j H_k$$

$$+ \lambda_{5}^{ijk} B_i C_j D_k + \lambda_{6}^{ijk} B_i E_j I_k + \lambda_{7}^{ijk} C_i E_j J_k + \lambda_{8}^{ijk} D_i E_j K_k$$

$$+ \lambda_{9}^{ijk} F_i G_j K_k + \lambda_{10}^{ijk} F_i H_j J_k + \lambda_{11}^{ijk} G_i H_j I_k + \cdots,$$

where indices $i, j$ and $k$ represent the generation. The ellipses stand for nonrenormalizable terms. As is seen from Table 1, each pair of $(C, D), (G, H)$ and $(J, K)$ has the same quantum numbers of $G_{SM}$, respectively. This means that there remains the freedom how to assign the physical fields to them if suitable phenomenological conditions are satisfied. Usually the assignment is adopted so as to guarantee the existence of the following necessary terms in $W$:

$$Q_i \bar{U}_j H^2, \quad Q_i \bar{D}_j H^1, \quad L_i \bar{E}_j H^1, \quad S H^1 H^2.$$  \hspace{1cm} (2)

The first three terms are Yukawa couplings which induce quarks and charged leptons mass. The last one brings so-called $\mu$-term after a singlet $S$ gets a suitable vacuum expectation value(VEV). In the conventional assignment[5, 6], moreover, the same assignment is assumed to be applied to all three generations. However, in principle, there is no necessity for such a field assignment. We can adopt different ones for each generation. In fact, such an unconventional assignment has been proposed in ref.[10] within no intermediate scale models. As pointed out in it, there appear some novel phenomena associated with such assignments, for example, extra color triplets remain light, the neutrino mass appears at one loop level, the extra $U(1)$ interaction loses its universality among generations and so on. However, if suitable $U(1)$ interaction loses its universality among generations and so on. However, if suitable discrete symmetries are imposed and parameters are also appropriately chosen, then they can be consistent with all experimental constraints at the present stage.

There are other typical models with the extended abelian gauge structure and an intermediate scale in the string inspired $E_6$ models. In this kind of models the extra
fields generally have very different features in comparison with ones in \cite{10} because of the existence of intermediate scale. It is interesting and also useful to study what effects are induced by such unconventional assignments in the models with an intermediate scale from the viewpoint of the model building. In the followings, we will show that in the intermediate scale models it is possible to prohibit the fast proton decay due to the triplet-doublet splitting and also give neutrinos small masses in the very simple way. It should be noted that in our model these features are largely dependent on the introduction of the intermediate scale, which is the crucial difference from the model in ref.\cite{10}. In particular, the introduction of an intermediate scale makes it possible to present the neutrino mass matrix with the large Majorana mass of a right handed neutrino. In this neutrino mass matrix the seesaw mechanism\cite{15} works and then the neutrino sector can realize the wide range mass scales by the collaboration with the loop effects. These mass scales may simultaneously solve the neutrino problems\cite{11, 12, 13}, which have recently attracted much interests of many authors. This feature cannot be seen in the model of ref.\cite{10} in which only loop effects induce the neutrino masses.

2 A model with an unconventional field assignment

We consider a simple example of models which satisfy some properties which we require. This model is characterized by two features. One is the unconventional field assignment and the other is the existence of a massless singlet which associates with its conjugate field. The latter makes it possible to introduce an intermediate scale as explained in the next part. The field assignment of our model is presented in Table 1. We take the same field assignment for the first two generations but the third generation is assigned in the different way.

As is well known in the Wilson loop breaking mechanism in superstring theories\cite{4}, there can generally exist massless conjugate pairs of chiral superfields ($R, \bar{R}$) other than 27\cite{5, 6}. Here $R$ represents some components of 27 given in Table 1. Taking this fact into account, we assume the existence of a conjugate pair of $G_{SM}$ singlet chiral superfields

\footnotesize
2 There are some arguments that the existence of the intermediate scale is phenomenologically unfavorable\cite{8}. In our model, however, there are no such problems. The excessive entropy production associated with flat directions is the common problem in superstring models\cite{14}.

\normalsize
(J,J) and represent them as (J,J)\(^3\).

The superpotential W can be divided into a (J,J) independent part \(W_0\) and a dependent part \(W_J\). They are written down explicitly by using the physical fields notation given in Table 1,

\[
W = W_0 + W_J,
\]

\[
W_0 = \lambda_{ij}^{i} Q_i Q_j g_k + \lambda_{ij}^{j} Q_i \bar{U}_j H_k^2 + \lambda_{ij}^{(a3)} Q_i \bar{g}_j \left( \frac{L_{\alpha}}{H_3^1} \right) + \lambda^{(a3)}_{i} Q_i \bar{D}_j \left( \frac{H_3^1}{L_3} \right)
\]

\[
+ \lambda_{ij}^{i} \bar{U}_i \bar{g}_j \bar{D}_k + \lambda_{ij}^{j} \bar{U}_i \bar{g}_j \bar{E}_k + \lambda_{ij}^{(a3)} \bar{g}_i g_j \left( \frac{S_{\alpha}}{\bar{N}_3} \right) + \lambda^{(a3)}_{i} \bar{D}_i g_j \left( \frac{\bar{N}_3}{S_3} \right)
\]

\[
+ \lambda_{ij}^{(a3)(\beta 3)} h_i \left( \frac{L_{\alpha}}{H_3^1} \right) \left( \frac{\bar{N}_3}{S_3} \right) + \lambda_{ij}^{(a3)(\beta 3)} h_i \left( \frac{H_3^1}{L_3} \right) \left( \frac{\bar{N}_3}{S_3} \right)
\]

\[
+\lambda_{ij}^{(a3)(\beta 3)} \left( \frac{L_{\alpha}}{H_3^1} \right) \left( \frac{H_3^1}{L_3} \right) \bar{E}_i + ..., \tag{4}
\]

\[
W_J = \lambda_{ij}^{(a3)} \bar{g}_i g_j J + \lambda_{ij}^{(a3)} \bar{h} \left( \frac{H_3^1}{L_3} \right) \bar{J} + \lambda_{ij}^{(a3)} \bar{J} \left( \frac{S_3}{\bar{N}_3} \right) \bar{J} \left( \frac{S_3}{\bar{N}_3} \right)
\]

\[
+ \frac{\lambda_{ij}^{(a3)} \bar{J} \bar{J}}{M_{\text{pl}}} \left[ J \bar{J} \right]^{n} + ..., \tag{5}
\]

where \(n\) corresponds to the dimension of the lowest order gauge invariant allowed non-renormalizable term which contains \(J\) and \(\bar{J}\). The ellipses represent higher order non-renormalizable terms. The indices \(\alpha\) and \(\beta\) stand for the first and second generations.

The conjugate pair \((J,J)\) has an opposite charge of the extra \(U(1)\) and then, as is well known, there is a D-flat direction \(|\langle J \rangle| = |\langle \bar{J} \rangle|\). If the negative soft squared mass \(m^2_S\) for the scalar component of \(J\) is induced as a result of supersymmetry breaking and also the radiative effects due to the Yukawa couplings \(\lambda_{ij}^{(a3)}\) and \(\lambda_{ij}^{(a3)}\), VEVs of \(J\) and \(\bar{J}\) will be produced through the \(\lambda_{ij}^{(a3)}\) term in \(W_J\) as follows\[\text{4} \, \text{5}\],

\[
|\langle J \rangle| = |\langle \bar{J} \rangle| \sim \left( \lambda_{ij}^{(a3)} \right)^{-1} M_{\text{pl}}^{2n-3} m_S \frac{1}{\sqrt{\lambda_{ij}^{(a3)}}}. \tag{6}
\]

\[\text{3}\] The systematic study of this kind of spectrum has been done in ref.\[\text{4}\]. From its results it is found that two different types of singlets (for example, \(J\) and \(K\)) can not be massless with its conjugate fields simultaneously, at least in the case that the gauge structure is \(G_{\text{SM}} \times U(1)^2\). This is because Wilson loop can not be constructed to be orthogonal to both \(J\) and \(K\).
If Yukawa couplings $\lambda_{ij}^{(23)}$ and $\lambda_{(23)3}^{(a3)}$ in $W_1$ are suitably arranged, all extra color triplets $g_i$, $\bar{g}_i$ and only one pair of Higgs doublets $(H_1^1, H_2^2)$ become heavy due to these VEVs. For example, if $n = 3$ and $\lambda_{33}^{(2)} = O(1)$, $|\langle J \rangle|$ becomes large enough as $\sim 10^{15}$ GeV.

Thus the proton decay process mediated by $g_i$ and $\bar{g}_i$ can be sufficiently suppressed. Moreover, through the $\lambda_{12}^{(a3)}$ term $S_\alpha$ and $\bar{N}_3$ can get the mass of order of $\frac{|\langle \alpha \rangle|^2}{m_\alpha}$.

As discussed later, $N_3$ plays a role of the heavy right handed neutrino. For $S_\alpha$ we assume that only $S_1$ becomes super heavy. This assumption is also related to the neutrino mass production. It should be noted that these phenomena can simultaneously occur because of the unconventional field assignment adopted here.

Now we can write down the effective superpotential $W_{\text{light}}$ of light fields,

$$W_{\text{light}} = W_1 + W_2 + W_3,$$

$$W_1 = \lambda_{ij}^{(23)} Q_i U_j \left( \frac{H_2^2}{H_3^2} \right) + \lambda_{ij}^{(2)} Q_i \bar{D}_j H_2^1 + \lambda_{11}^{(2a)} L_\alpha H_2^1 \bar{E}_i + \lambda_{11}^{(2a)} L_\alpha H_3^1 \bar{E}_i$$

$$+ \lambda_{9}^{(23)33} \left( \frac{H_2^2}{H_3^2} \right) H_3^1 S_3 + \lambda_{10}^{(23)22} \left( \frac{H_2^2}{H_3^2} \right) H_2^1 S_2 + \lambda_{10}^{(23)33} \left( \frac{H_2^2}{H_3^2} \right) L_3 \bar{N}_3,$$

$$W_2 = \lambda_{ij}^{(23)} Q_i \bar{D}_j L_3 + \lambda_{11}^{(2a)} L_\alpha L_3 \bar{E}_i + \lambda_{11}^{(2a)} H_3^1 H_2^1 \bar{E}_i,$$

$$W_3 = \lambda_{9}^{(23)\alpha \beta} \left( \frac{H_2^2}{H_3^2} \right) L_\alpha \bar{N}_3 + \lambda_{9}^{(23)3} \left( \frac{H_2^2}{H_3^2} \right) L_\alpha S_3 + \lambda_{9}^{(23)3} \left( \frac{H_2^2}{H_3^2} \right) H_3^1 \bar{N}_3$$

$$+ \lambda_{10}^{(23)32} \left( \frac{H_2^2}{H_3^2} \right) L_3 S_2 + \lambda_{10}^{(23)23} \left( \frac{H_2^2}{H_3^2} \right) H_2^1 \bar{N}_3,$$

where we add the terms relevant to $\bar{N}_3$ although it is heavy. All necessary terms which induce quark and charged lepton masses and $\mu$-terms are contained in $W_1$. Typical $R$-

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4 As discussed later, in order to introduce the suitable neutrino mass structure we need two pair of light Higgs doublets. The possibility of these arrangements of Yukawa couplings will be justified by the later argument of discrete symmetries.

5 This possibility has already suggested in [3]. However, the conventional field assignment was used there so that the extra color triplets could not be heavy. There is a proposal [17] to introduce two D-flat directions, each of which is responsible for the heavy mass of extra color triplets and right handed neutrinos, respectively. Such two D-flat directions, however, seem to be difficult to exist simultaneously within the framework of usual Wilson loop breaking as mentioned in footnote 3.

6 We will discuss the consistency of this assumption related to the discrete symmetries later.

7 Although $S_1$ can play the same role as $\bar{N}_3$ at this stage, we distinguish them here. This treatment will be also be justified by the introduction of discrete symmetries.
parity violating terms are contained in $W_2$. In $W_3$ phenomenologically dangerous terms are included. They should be forbidden by suitable discrete symmetries since they cause the unwanted masses and mixings among neutral fermions after doublet Higgses get VEVs. It is the most interesting problem what kind of neutrino masses are induced in this $W_{\text{light}}$. It also gives the most important criterion to introduce the discrete symmetries for select terms from $W_{\text{light}}$.

3 Discrete symmetries and neutrino masses

In this section we examine the discrete symmetries and their relation to the neutrino masses and set up our model definitely. In order to introduce the discrete symmetries we will impose the following conditions:

(i) all necessary terms in $W_1$ are kept as invariant ones,

(ii) to avoid the FCNC problem in the quark sector we require that only $H_2^2$ couples to the up-quark sector

$$
\lambda_{ij}^{32} = 0,
$$

(iii) $W_2$ which includes usual R-parity violating terms is forbidden

$$
\lambda_{ij}^{43} = \lambda_{11}^{3i} = \lambda_{11}^{2i} = 0,
$$

(iv) all extra color triplets can become heavy through the couplings with $J$ in $W_J$

$$
\lambda_i^{3j} \neq 0,
$$

(v) only $S_2$ in the singlets ($S_\alpha, N_3$) and two pairs of doublet Higgses ($H_2^1, H_2^2$), ($H_3^1, H_3^2$) remain massless so that they are forbidden to couple with $J$ and $\tilde{J}$

$$
\lambda_{10}^{3(23)J} = \lambda_{10}^{3(23)\tilde{J}} = \lambda_{12}^{2(3)} = 0.
$$

Under these conditions we will find out appropriate discrete symmetries and study their results. Before giving such an example, it will be useful to present some features of the terms in $W_{\text{light}}$ relevant to the neutrino masses.

As seen from eq.(5), in the present model only the third generation neutrino $N_3$ gets the large Majorana mass and then the seesaw mechanism\[13\] works for Dirac masses.
related to $\bar{N}_3$. From this point of view, to avoid large left handed neutrino masses it is necessary to impose

$$\lambda^{(23)\alpha\beta}_9 = \lambda^{(23)\alpha 3}_9 = \lambda^{(23)32}_{10} = 0. \quad (15)$$

Under these assumptions $L_3\bar{N}_3$ Dirac mass and $\bar{N}_3\bar{N}_3$ large Majorana mass alone exist at tree level. However, due to the radiative corrections based on the remaining interactions of $W_{\text{light}}$, $L_\alpha\bar{N}_3$ Dirac masses and $N_\alpha\bar{N}_3$, $\bar{N}_\alpha\bar{N}_\beta$ Majorana masses are induced through the one loop diagram shown in Figs.1-3. If we assume that soft supersymmetry breaking parameters take the universal value $O(m_3/2)$, their magnitudes are roughly estimated as$[8, 9]$, 

$$m \sim \frac{A}{32\pi^2}\lambda^{(23)30}_{11}\lambda^{(23)33}_{10} m_\tau \quad (\text{for } L_\alpha\bar{N}_3),$$

$$m \sim \frac{A}{32\pi^2}\lambda^{(23)30}_9\lambda^{(23)23}_{10} m_{\tilde{H}} \quad (\text{for } \bar{N}_\alpha\bar{N}_3),$$

$$m \sim \frac{A}{32\pi^2}\lambda^{(23)30}_9\lambda^{(23)3\beta}_{10} m_{\tilde{H}} \quad (\text{for } \bar{N}_\alpha\bar{N}_\beta), \quad (16)$$

where $m_\tau$ and $m_{\tilde{H}}$ are the masses of tau and corresponding charginos. The soft breaking A-terms are parametrized as $Am_3/2$. In the present model there are fruitful structures in the Higgs sector. It should be noted that as its result there may be the tree level contributions to these masses which are not explicitly presented here. This can be easily seen by replacing the Higgs internal lines into their VEVs in Figs.1$\sim$3. However, their relative largeness completely depends on the values of soft supersymmetry breaking parameters and Higgs VEVs. At one-loop level $L_i\bar{N}_\alpha$ Dirac masses and $L_iL_j$ Majorana masses are not induced under the present assumptions. On the other hand, Yukawa couplings $\lambda^{(23)3\alpha}_{0}$ and $\lambda^{(23)23}_{10}$ induce the mixings between Higgsinos and right handed neutrinos. These mixings can largely affect the $N_\alpha$ Majorana masses, although their effects on the $N_3$ Majorana mass are negligible. In order for such effects to be of order $10^{-1}$ eV, the relevant Yukawa couplings $\lambda^{(23)3\alpha}_{0}$ should be less than $O(10^{-4})$ if we take the Higgsino masses as $\sim 100$ GeV.

Now we look for the discrete symmetries which satisfy the conditions (i) $\sim$ (v). These discrete symmetries should not be broken by the VEVs of $\mathcal{J}$ and $\bar{\mathcal{J}}$ so that they must not have their charges. Taking account of these, we can find a simple but interesting example of such discrete symmetries and $W_{\text{light}}$ invariant under it. Such an example is
$Z_2 \times Z_2 \times Z_n \ (n \geq 3)$ and the charge assignment for each field is\footnote{We systematically searched such solutions as satisfying conditions (i) \sim (v) within $Z_2 \times Z_2 \times Z_n$ type discrete symmetries providing that the quark sector transforms as simple as possible under them. The promising solution is very restricted and the following one seems to be almost unique. It seems to be difficult that the condition (15) is satisfied, simultaneously. An unwanted term $\lambda_9^{3\alpha} H_3^2 L \alpha S_3$ cannot be forbidden only by the present discrete symmetry. Although we may need to impose more complicated discrete symmetry to prohibit it, we only assume here that this Yukawa coupling is accidentally zero. If it happens, it can be checked that the dangerous mixings induced by its existence are not caused by the one-loop effects.}

\[ H^1_2(-1,1,\alpha), \ H^1_3(1,-1,\alpha), \ H^1_3(1,1,\alpha\beta^{-1}), \ S_2(-1,-1,\alpha^{-1}), \]
\[ S_3(1,1,\alpha^{-1} \beta), \ L_\alpha(-1,1,\beta^{-1}), \ L_\beta(-1,-1,\alpha^{-1}), \]
\[ E_i(-1,-1,\beta), \ N_\alpha(-1,1,\alpha^{-2} \beta), \ N_\beta(1,-1,1), \ D_i(1,-1,1), \] (17)

where we represent the charges of each field as $(p, q, r)$ where $p$, $q$ and $r$ are charges of $Z_2$’s and $Z_n$. Nontrivial $Z_n$ elements $\alpha$ and $\beta$ satisfy $\alpha^n = \beta^n = 1$. All other fields in Table 1 including $\mathcal{J}$ and $\bar{\mathcal{J}}$ are invariant under this discrete symmetry. The superpotential $W_{\text{light}}$ of light fields can be written as

\[ W_{\text{light}} = \lambda_2^{ij} Q_i \bar{U}_j H^2_2 + \lambda_4^{ij} Q_i \bar{D}_j H^1_2 + \lambda_9^{\alpha 2i} L_\alpha H^1_2 \bar{E}_i + \lambda_9^{33} L_3 H^1_3 \bar{E}_i + \lambda_9^{33} H^2_2 H^1_3 S_3 + \lambda_9^{32} H^2_3 H^1_2 S_2 + \lambda_9^{33} H^2_3 L_3 N_3,
\]
\[ + \lambda_9^{33} H^2_3 H^1_3 N_\alpha + \lambda_9^{32} H^2_3 H^1_2 \bar{N}_3. \] (18)

It is noticeable that this superpotential contains all necessary terms and some nice features. Although there are two Higgs doublet pairs $(H^1_2, H^2_2)$ and $(H^1_3, H^2_3)$ which can be expected to get VEVs through the existence of $\mu$-term couplings $\lambda_9^{233}$ and $\lambda_9^{322}$, up and down quarks couple a different Higgs field respectively and then the large FCNC in the quark sector can be avoidable as the minimal supersymmetric standard model (MSSM). Although the structure of $W_{\text{light}}$ is similar to the one of the MSSM, the differences from the MSSM appear in lepton and Higgsino sectors. The charged leptons have Yukawa couplings with two Higgs fields and there are two types of $\mu$-terms. The detailed study of these aspects is beyond the scope of this paper but the further investigation of FCNC in the charged lepton sector and also the phenomenology of Higgsino sector will be necessary.

The additional terms in this $W_{\text{light}}$ result in the interesting features in neutrino masses as noted in the previous part. Here we discuss the neutrino mass matrix in this model in
some details. At the stage of the first approximation the neutrino mass matrix induced from this $W_{\text{light}}$ can be written in the $(L_\alpha, L_3, \overline{N}_\alpha, \overline{N}_3)$ basis as,

$$
\begin{pmatrix}
0 & 0 & 0 & m_{\alpha 3} \\
0 & 0 & 0 & m_{33} \\
0 & 0 & M_{\alpha \beta} & M_{\alpha 3} \\
m_{\alpha 3} & m_{33} & M_{\alpha 3} & M_{33}
\end{pmatrix},
$$

(19)

where using formulae (16) each element is expressed as,

\begin{align*}
m_{33} & \sim \lambda^{333}_{10} \langle H^2_3 \rangle, \\
m_{\alpha 3} & \sim \text{max} \left[ \lambda_{11}^{23} \lambda_{10}^{333} \frac{\langle H^1_2 \rangle \langle H^2_3 \rangle}{m_{\tau}}, \frac{A}{32 \pi^2} \lambda_{11}^{23} \lambda_{10}^{333} m_{\tau} \right], \\
M_{\alpha 3} & \sim \text{max} \left[ \lambda_{9}^{33} \lambda_{10}^{223} \frac{\langle H^2_2 \rangle \langle H^2_3 \rangle}{m_{R}}, \frac{A}{32 \pi^2} \lambda_{9}^{33} \lambda_{10}^{223} m_{H} \right], \\
M_{\alpha \beta} & \sim \lambda_{9}^{333} \lambda_{9}^{33} \frac{\text{max}(\langle H^2_3 \rangle, \langle H^1_3 \rangle)^2}{m_{H}}, \\
M_{33} & \sim \lambda_{12}^{33} \frac{|\langle J \rangle|^2}{M_{\text{pl}}}.
\end{align*}

(20)

The tree level contributions are explicitly presented in $m_{\alpha 3}$ and $M_{\alpha 3}$. Although there is no contribution to $M_{\alpha \beta}$ from Fig.3, $M_{\alpha \beta}$ is induced as the result of the mixings with Higgsinos. These elements should satisfy the condition

$$
M_{\alpha \beta} \ll m_{\alpha 3}, m_{33}, M_{\alpha 3} \ll M_{33}.
$$

(21)

This type of matrix has three nonzero light mass eigenvalue other than $M_{33}$. Based on the analyses of solar neutrino\[^{11}\], atmospheric neutrino\[^{12}\] and various cosmological observations\[^{13}\], it seems to be preferable to consider that there are hierarchical three typical mass scales related to the light neutrino sector, that is, $\sim 10$ eV (dark matter), $\sim 10^{-1}$ eV (atmospheric neutrino) and $\sim 10^{-3}$ eV (solar neutrino) from the viewpoint of mass differences. If there is hierarchical structures in $M_{\alpha \beta}$, these appropriate mass scales will be induced in the light neutrino sector through the collaboration with the seesaw mechanism based on a right handed Majorana neutrino mass $M_{33}$ as,

$$
M_{11}(\sim 10^{-3} \text{ eV}) \ll M_{22}(\sim 10^{-1} \text{ eV}) \ll \frac{m_{33}^2}{M_{33}}(\sim 10 \text{ eV})
$$

(22)

In that case, for example, two flavor oscillations such as $\nu_e \leftrightarrow \overline{N}_1$ and $\nu_\mu \leftrightarrow \overline{N}_2$ may solve the solar and atmospheric neutrino problems and also $\nu_\tau$ will be a candidate of the dark
matter. For such an identification, the elements of eq.(19) should take the values

\[ m_{33} \sim 1 \text{ GeV}, \quad M_{23} \lesssim 10^{-1} \text{ GeV}, \quad M_{13} \lesssim 10^{-2} \text{ GeV} \]  

(23)

for \( M \sim 10^8 \text{ GeV} \). The arguments here are concentrated on the mass scales in the neutrino sector and the mixing angles are not discussed. For the study of mixing angles more complicated loop effects should be taken into account, which will fill the places of zero components in eq.(19). Such effects can be expected to be the same order as \( M_{\alpha\beta} \) or smaller than it. Thus the qualitative feature of the above arguments will not be changed. This subject, however, is beyond the scope of the present paper and we will not discuss it farther.

Now we examine the possibility to realize this mass hierarchy (21)～(23) in the neutrino mass matrix (19) more concretely. Using formulae (20) to estimate the elements of this matrix, we take parameters in the following way. For Yukawa couplings \( \lambda_{10}^{223} \) and \( \lambda_{10}^{333} \) there is no phenomenological constraints and then we can take it as \( \sim O(1) \). Here it is useful to note that \( m_{33} \) depends on \( \langle H_3^2 \rangle \) which is irrelevant to the quarks and charged leptons masses and can be taken to be small enough as \( \sim 1 \text{ GeV} \). The consistency to the charged lepton mass eigenvalues requires that \( \lambda_{11}^{123} \lesssim 6 \times 10^{-4} \) and \( \lambda_{11}^{223} \lesssim 10^{-2} \) under the assumption of \( \langle H_2^2 \rangle \sim 50 \text{ GeV} \). As mentioned before, \( M_{\alpha\beta} \) comes from the mixing between \( \tilde{N}_\alpha \) and Higgsinos. These mixings bring the suitable contributions to \( \tilde{N}_\alpha \) Majorana mass. This requires \( \lambda_9^{331} \sim 10^{-5} \) and \( \lambda_9^{332} \sim 10^{-4} \). For these parameters the conditions (21)～(23) seems to be generally satisfied. As easily seen, however, the estimations of eqs.(20) are dependent on the assumptions of the soft supersymmetry breaking parameters. If we loose these assumptions and consider more general situations, their numerical estimations can rather largely change and the required conditions may also be changed to realize the suitable hierarchy. In any case we may have to consider the effects of some nonuniversal structure of soft supersymmetry breaking parameters[20] and also the multi-Higgses to know whether we can get the favorable mass hierarchy in the neutrino sector of this

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9 This rather small right handed neutrino mass can be realized by taking \( \lambda_{12}^{3j} \sim 10^{-3} \) even if \( \langle J \rangle \sim 10^{15} \text{ GeV} \) which can guarantee the proton longevity. Small value of \( \lambda_{12}^{3j} \) may be explained by the fact that in a certain type of string models nonrenormalizable terms are induced by nonperturbative effects[18].

10 In this type of neutrino mass matrix, the neutrino oscillation phenomena are studied in ref.[19].

11 This estimation also depends on a value of \( \langle H_3^1 \rangle \). However, in this model there are two Higgs doublet pairs and then it should take a rather small VEV.
model. It will be necessary to practice more quantitative study of this point providing the structure of soft supersymmetry breaking parameters.

4 Discussions and summary

We have studied that our present model works well in the triplet-doublet splitting and the small neutrino mass generation in addition that it has the similar structure to the MSSM. Now we order some other brief comments which should be added on its phenomenological features.

(1) It is well known that the existence of the light singlet fields is dangerous because their couplings to heavy fields can induce the vacuum instability through the tadpole diagram\[21\]. However, in our model the light singlets $S_2$ and $S_3$ can not couple to heavy fields and then this model is free from the tadpole problem. This decoupling of $S_2$ and $S_3$ from heavy fields are due to the discrete symmetry and the unconventional field assignment.

(2) In the string inspired $E_6$ models, there is no $E_6$ relation among Yukawa couplings\[4\]. This fact makes Yukawa couplings of extra color triplets $g$ and $\bar{g}$ with ordinary matters and the singlet $J$ to have less constraints than those in the usual supersymmetric GUTs. Because of the ambiguity caused by this looseness we can not definitely estimate the lower bound of the VEV $\langle J \rangle$ from the experimental bound of proton decay as done in refs.\[16\]. However, we can always set up $\langle J \rangle$ to escape its bound without bringing any other problems.

(3) The study of radiative symmetry breaking at $\sim$1 TeV scale based on the renormalization group equations is an interesting problem. In such a study experimental bounds including the top mass give us various informations on the parameters in the model, especially, on the extra $Z$ mass as found in \[7\]. As suggested in \[10\], there is the nonuniversal neutral current interaction due to the extra $Z$ in the present model. However, the extra $Z$ mass is expected to be $\gtrsim$ 1 TeV from the consistency with the primodial nucleosynthesis\[22\] because of the existence of extra light neutrino species. Although its observation will not be expected in near future, the consistency check of such extra $Z$ mass with radiative symmetry breaking is interesting.

(4) To make a realistic model on this line there remain many problems to be considered, for
example, the derivation of the realistic quark/lepton mass matrices. It will be necessary again to impose suitable discrete symmetries which act nontrivially on the quark sector to overcome this problem. One promising possibility is to give the Yukawa couplings in the quark sector in the following way,

$$\lambda^{ij}_{2} \left( \frac{J \bar{J}}{M_{pl}^2} \right)^{n_{ij}^U} Q_i \bar{U}_j H_2^2, \quad \lambda^{ij}_{4} \left( \frac{J \bar{J}}{M_{pl}^2} \right)^{n_{ij}^D} Q_i \bar{D}_j H_1^2,$$

(24)

where $n_{ij}^U$ and $n_{ij}^D$ are zero or positive integers and their values are determined by the discrete symmetries. This scheme is very similar to the proposal in [23]. However, the extra $U(1)$ invariance requires the appearance of $J \bar{J}$ pair in these formulae so that $\langle J \rangle$ should be rather large value $|\langle J \rangle| \sim 0.2 M_{pl}$ to realize the correct hierarchy under the assumption for the Yukawa couplings $\lambda \sim O(1)$.

(5) The right handed neutrino mass production through an abelian D-flat direction may be related to the inflation and also the primodial baryon number asymmetry as suggested in [24]. The study of this aspect is also necessary.

In summary we studied the unconventional field assignment in string inspired $E_6$ models with a conjugate pair of chiral superfields $(J, \bar{J})$ which are $G_{SM}$ singlets. Extra color triplets become heavy enough to guarantee the proton stability through VEVs of these singlets, although doublet Higgs fields are kept light. The massless fields sector can be almost the MSSM by imposing suitable discrete symmetries, except for neutralino, chargino and neutrino sectors. We showed that the interesting neutrino mass matrix can be derived in these models. These mass matrices may be able to explain the hierarchical masses appropriate for solar neutrino, atmospheric neutrino and dark matter. We consider here the restricted discrete symmetries which satisfy certain conditions. If we loose these conditions, there will be many other possibilities which may present fruitful neutrino mass structures and also other interesting phenomenology. Anyway, the possibility proposed here to prohibit the fast proton decay and give neutrinos small masses simultaneously seems to be worthy for further study.

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Table 1
The decomposition of $27$ of $E_6$ under $G_{\text{SM}}$ and the field assignment for them. All $U(1)$
charges are normalized as $\sum_{i \in 27} Q_i^2 = 5$.

| fields | $G_{\text{SM}}$ | $\frac{1}{\sqrt{10}}Q_\psi$ | $\frac{1}{\sqrt{6}}Q_\chi$ | $i = 1, 2$ | $i = 3$ |
|--------|-----------------|----------------|----------------|------------|----------|
| $A_i$  | $(3, 2)_{\frac{1}{6}}$ | 1 | −1 | $Q$ | $Q$ |
| $B_i$  | $(3^*, 1)_{\frac{-2}{3}}$ | 1 | −1 | $\bar{U}$ | $\bar{U}$ |
| $C_i$  | $(3^*, 1)_{\frac{1}{3}}$ | −2 | −2 | $\bar{g}$ | $\bar{g}$ |
| $D_i$  | $(3^*, 1)_{\frac{1}{4}}$ | 1 | 3 | $\bar{D}$ | $\bar{D}$ |
| $E_i$  | $(3, 1)_{\frac{-1}{3}}$ | −2 | 2 | $g$ | $g$ |
| $F_i$  | $(1, 2)_{\frac{1}{2}}$ | −2 | 2 | $H^2$ | $H^2$ |
| $G_i$  | $(1, 2)_{\frac{-1}{2}}$ | 1 | 3 | $L$ | $H^1$ |
| $H_i$  | $(1, 2)_{\frac{-1}{2}}$ | −2 | −2 | $H^1$ | $L$ |
| $I_i$  | $(1, 1)_{1}$ | 1 | −1 | $\bar{E}$ | $\bar{E}$ |
| $J_i$  | $(1, 1)_{0}$ | 4 | 0 | $S$ | $\bar{N}$ |
| $K_i$  | $(1, 1)_{0}$ | 1 | −5 | $\bar{N}$ | $S$ |
Figure Captions

Fig. 1
Supergraph of one-loop $L_\alpha \bar{N}_3$ Dirac neutrino mass. Either vertex of $\lambda_{10}^{(23)22}$ or $\lambda_{11}^{33i}$ should be understood as a soft supersymmetry breaking A-term.

Fig. 2
Supergraph of one-loop $\bar{N}_\alpha \bar{N}_3$ Majorana neutrino mass. Either vertex of $\lambda_9^{(23)22}$ or $\lambda_9^{(23)33}$ should be understood as a soft supersymmetry breaking A-term.

Figure 3
Supergraph of one-loop $\bar{N}_\alpha \bar{N}_\beta$ Majorana neutrino mass. Either vertex of $\lambda_9^{(23)33}$ should be understood as a soft supersymmetry breaking A-term.
\[ \langle S_2 \rangle \]
\[ H_2^1 \]
\[ \lambda_{10}^{(23)22} \]
\[ \lambda_{11}^{\alpha 2i} \]
\[ \lambda_{11}^{33i} \]
\[ \lambda_{11}^{(23)33} \]
\[ L_\alpha \]
\[ \bar{E}_i \]
\[ L_3 \]
\[ \langle H_3^1 \rangle \]

Fig. 1

\[ \langle S_3 \rangle \]
\[ H_3^1 \]
\[ \lambda_{10}^{(23)33} \]
\[ \lambda_{10}^{(23)23} \]
\[ \lambda_{10}^{(23)22} \]
\[ \lambda_{9}^{(23)3\alpha} \]
\[ \delta_{11}^{(23)\alpha} \]
\[ \langle S_2 \rangle \]
\[ \langle H_2^1 \rangle \]
\[ \langle H_2^2 \rangle \]
\[ \langle H_3^2 \rangle \]

Fig. 2
Fig. 3