Monte Carlo simulation of magnetic multilayered structures with giant magnetoresistance effects

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Abstract. Description of giant magnetoresistance effects in magnetic multilayered structures with the use of the anisotropic Heisenberg model for determination of magnetic properties of thin ferromagnetic films forming these structures is given. Monte Carlo simulations of magnetic properties for structures, which are constructed from two ferromagnetic films divided by nonmagnetic film, are carried out. The temperature and magnetic field dependencies are considered for ferromagnetic and antiferromagnetic configurations of these structures. The calculation of the magnetoresistance is carried out for different thicknesses of the ferromagnetic films. It was shown, that the obtained temperature dependence for the magnetoresistance is agreed very well with experimental results, measured for the magnetic multilayered structures similar to structures, which are considered in our investigations.

The physics of ultrathin magnetic films with thickness from one and two to several tens of monolayers appears as a direction of intensive research over the past two decades (for a review see [1]). This heightened interest of scientists is caused by quite a number of unique properties of films different from properties of bulk materials, that defines an importance of these objects for the new facilities of fundamental physics of magnetism, surface physics and their practical applications [2, 3]. Thus, the ultrathin films of magnetic metals and alloys are used as the building blocks for magnetic multilayer structures. These artificial created magnetic superlattices has become of great interest in wide range of applications based on the phenomena of the giant magnetoresistance (GMR) [4–6] and the tunneling magnetoresistance (TMR) [7–9].

The multilayer structures with GMR effects consist of the ferromagnetic layers separated by nonmagnetic layers. The thickness of these nonmagnetic metal layers are selected in such a way that the long-range and oscillating RKKY exchange interaction between the spins of the ferromagnetic layers is effective antiferromagnetic. Through this interaction, the magnetizations of the adjacent ferromagnetic layers are oriented opposite to each other. When this structure is placed in an external magnetic field, the magnetization of layers begin to orient in parallel, that leads to a significant change in electrical resistance. Devices based on the GMR effect are widely used as read heads of hard disks, memory devices, sensors, etc [2,10,11].

Achievements in the development of technology give possibility now to receive a high-quality ultrathin films and multilayer coatings on the basis of the magnetic transition metals Fe, Co, and Ni [1]. Investigation of the nature of magnetism in ultrathin films and multilayer structures has a large fundamental interest because the dimensional dependence of the magnetic characteristics which demonstrate the transition from the specific bulk values for films with thickness of several tens monolayers ($d \geq 10$ nm) to the two-dimensional values in films with thickness less than 4-6 monolayers ( $d \leq 1 - 2$ nm) [12, 13]. The magnetic properties of ultrathin films and
superstructures sensitive to the effects of anisotropy generated by the crystal field of the substrate or by the nonmagnetic layers. Therefore, the physical properties of ultrathin films on the basis of Fe, Co and Ni can be described by the anisotropic Heisenberg model.

At present time, the statistical Monte Carlo methods have proved as successful methods for simulation behavior and describing the physical properties of various magnetic systems with different dimensionality with identifying their features during phase transitions [15]. The Monte Carlo study of dimensional crossover effects in the critical properties of multilayer Heisenberg films have been performed in works [16, 17]. For films with different thicknesses it was taken into account the effect of anisotropy generated by the crystal field of the substrate. The values of calculated critical exponents demonstrated a dimensional crossover from two-dimensional to three-dimensional properties of the films with increase in number of monolayers.

In this paper we apply the methodology and results of the multilayer Heisenberg films studies obtained in [16, 17] to Monte-Carlo simulations and calculation of magnetic properties in magnetic multilayer and spin valve sandwich structures. At first, data of the microscopic spin configurations of ferromagnetic films were used for the calculation of the magnetoresistance temperature dependence for these structures with different thicknesses of the ferromagnetic films.

For the statistical Monte Carlo description of the Heisenberg ferromagnetic films we introduce the Hamiltonian of the spin system in the form:

\[
H = -J \sum_{<i,j>} [(1 - \Delta(N))(S_x^i S_x^j + S_y^i S_y^j) + S_z^i S_z^j] - h \sum_i S_i, \tag{1}
\]

where \( J > 0 \) characterizes the short-range exchange interaction between the spins \( S_i \) fixed in the sites of lattice; \( S_i = (S_x^i, S_y^i, S_z^i) \) is introduced as a three-dimensional unit vector, \( h \) is the external magnetic field, \( \Delta \) is an anisotropy parameter. The dependence of the anisotropy parameter \( \Delta(N) \) from the film thickness \( N \) is selected in accordance with the results in articles [16, 17].

\[ \text{Figure 1.} \quad \text{The model of the multilayer structure (a) which consists of two ferromagnetic films separated by nonmagnetic metal film; } L \text{ and } N \text{ are linear sizes of films, } J_1, J_2 \text{ are the exchange integrals. Model of the spin valve structure (b) which consists of two ferromagnetic films separated by non-magnetic metal film, and a layer of antiferromagnetic adjacented to one of the ferromagnetic films; } L, N \text{ and } N_{AF} \text{ are linear sizes of films, } J_1, J_2, J_3, J_4 \text{ are the exchange integrals.} \]

At first stage, we considered in this paper the multilayer structure which consists of two ferromagnetic films separated by a nonmagnetic metal film (Fig. 1). The simulations were performed for the films with linear sizes \( L \times L \times N \) with applied periodic boundary conditions in the film plane. \( L \times L \) gives a number of spins in each layer, and \( N \) is the number of monolayers in the thin film. The value of the exchange integral that determines the interaction of neighboring spins in the ferromagnetic film set as \( J_1 = 1 \), the interaction between the films \( J_2 = -0.1 \). The temperature \( T \) in this system are measured in units of exchange integral \( J_1 \). The spin configurations of the Heisenberg ferromagnetic films in the magnetic structure (Fig. 1) are updated using the Swendsen-Wang cluster algorithm [18].
Ni soft ferromagnetic material such as (Cu, Ag or Au) with thicknesses of 1.5 - 5 nm. The ferromagnetic layers are produced from a threelayer structures through the use of a nonmagnetic conductive film from some noble metals the construction of read heads for hard disks. In the spin-valve structures, the exchange coupling spin-valve structures were first introduced in 1991 [19] and are now widely used, for example, in magnetization in the film by exchange coupling. Such structures are called as spin-valves. The ferromagnetic films is coupled with an antiferromagnetic metal, which constrains the direction of the film gains strength on the contrary.

Figure 2. The temperature dependence of the staggered magnetization $m^{(stg)}$ and its projections $m^{x,y}_{stg}$ and $m^{xy}_{stg}$ for threelayer structure with linear sizes of films $N = 3$, $L = 32$ for values of the external magnetic field $h = 0$ (a), $h_z = 0.4$ (b).

We measured the magnetization of films $m_{1,2}$ and their components $m_z$ and $m_{xy}$, which are determined by the relations:

$$m = \left\langle \frac{1}{N_s} \left[ \sum_{\alpha \in \{x,y,z\}} \left( \sum_{i=1}^{N_s} S_i^\alpha \right)^2 \right]^{1/2} \right\rangle,$$

$$m_z = \left\langle \frac{1}{N_s} \sum_{i=1}^{N_s} S_i^z \right\rangle, \quad m_{xy} = \left\langle \frac{1}{N_s} \left[ \left( \sum_{i=1}^{N_s} S_i^x \right)^2 + \left( \sum_{i=1}^{N_s} S_i^y \right)^2 \right]^{1/2} \right\rangle,$$

where $N_s = NL^2$ is a total number of spins in film, angle brackets denote the statistical averaging.

In the case of absence of external magnetic field $h$, the antiferromagnetic configuration is realized in threelayer structure, therefore for description of the magnetic ordering in the system we measured the staggered magnetization $m^{(stg)} = | m_1 - m_2 |$ and projections of the staggered magnetization $m^{stg}_z$ and $m^{stg}_{xy}$.

We have calculated the temperature dependence of the staggered magnetization for threelayer structures with different thicknesses $N$ of the ferromagnetic films in the range of the $N = 1 \div 30$ monolayers. As an example, we give in Fig. 2a the resulting temperature dependence of the staggered magnetization and its projections for the films with the sizes of $N = 3$, $L = 32$ in the absence of a magnetic field. Also, we have carried out the study of the influence of an external magnetic field $h_z$ on the magnetic characteristics of the same structure. The graphs presented in Fig. 2b show that under the influence of an external magnetic field the $z$-component of staggered magnetization is weakened, but the the $xy$-component of staggered magnetization in plane of the film gains strength on the contrary.

To reduce the saturation field in threelayer magnetic structure, usually an one of the ferromagnetic films is coupled with an antiferromagnetic metal, which constrains the direction of magnetization in the film by exchange coupling. Such structures are called as spin-valves. The spin-valve structures were first introduced in 1991 [19] and are now widely used, for example, in the construction of read heads for hard disks. In the spin-valve structures, the exchange coupling between the ferromagnetic layers is selected ferromagnetic and are weakened in comparison with threelayer structures through the use of a nonmagnetic conductive film from some noble metals (Cu, Ag or Au) with thicknesses of 1.5 - 5 nm. The ferromagnetic layers are produced from a soft ferromagnetic material such as $Ni_{80}Fe_{20}$ (permalloy) with 1.5-5 nm thick, at that one of these layers is coupled with a layer of antiferromagnetic alloy.
Figure 3. The dependence of the staggered magnetization on the external magnetic field $h_z$ for threelayer structure (a) and spin-valve structure (b) at temperature $T = 1.2$ and for linear sizes of the ferromagnetic films $N = 3$, $L = 32$.  

The materials which are used for the antiferromagnetic film that are alloys FeMn, NiO, TbCo, IrMn. As an example, the spin-valve sandwich Ni$_{80}$Fe$_{20}$/Cu/Ni$_{80}$Fe$_{20}$/FeMn has the property that the magnetization of insulated copper layer Ni$_{80}$Fe$_{20}$ is free to rotate under the influence of an external magnetic field with respect to the magnetization Ni$_{80}$Fe$_{20}$ layer, magnetization of which is constrained by exchange coupling to the antiferromagnet FeMn.  

We calculated the magnetization in spin-valve, for which the structure is shown in Fig. 1 b and is characterized by addition of the antiferromagnetic film with a constant of exchange interaction $J_3 = -2.0$, fixing the orientation of the magnetization in a coupled ferromagnetic film due to the exchange interaction with the constant $J_4 = 0.1$. From comparison of the graphs in Figs. 3a and 3b, it is seen that with the increase of the external magnetic field the $z$-component of the staggered magnetization for spin-valve decreases faster than for threelayer structure. Henceforward, we calculate the magnetoresistance for an multilayer structures which is introduced by relation

$$\delta h = \frac{R_{AP} - R_P}{R_P},$$  

where $R_{AP}$ is the resistance of structure when the magnetization of adjacent ferromagnetic layers are aligned antiparallel, and $R_P$ is the resistance of structure for parallel orientation of the magnetization of ferromagnetic layers. For the threelayer structure with antiferromagnetic exchange coupling of adjacent ferromagnetic layers, the $R_{AP}$ characterizes the resistance of structure without an external magnetic field, and the $R_P$ is the resistance in field $h$ above the ferromagnetic layer saturation field $h_s$. The situation for the spin-valve structure is reversed, and, namely, the parallel orientation of the films magnetization with resistance $R_P$ is realized in the absence of a magnetic field, and the antiparallel ordering with $R_{AP}$ is in the magnetic field $h$, but with $h \ll h_s$.  

The multilayer structure can connect in current line by two ways for measurements of resistance. For measurements of the current in plane (CIP), the current conduction is realized along layers and the electrodes are situated on a single side of structure. For measurements of the current perpendicular to plane (CPP), the current conduction is realized perpendicular layers of superstructure and the electrodes are situated on both sides of structure. It was shown [20, 21] that CPP magnetoresistance is characterized by larger values than the CIP magnetoresistance approximately to twice as much and becomes more competitive as device size shrinks. Regardless of the fact that the the CPP way of measurements is complicated for technical realization, now this method has a great practical interest because of the CPP magnetoresistive sensors.
Figure 4. Equivalent scheme for determination of the resistance for antiparallel $R_{AP}$ and parallel $R_P$ configurations of the layers magnetizaion in threelayer magnetic structure.

demonstrate more sensitivity than CIP-MR sensors [2]. At present paper we present the Monte Carlo realization of calculation of the CPP magnetoresistance.

Let us denote the resistance of an ferromagnetic film for two groups of electrons with spins up and down as $R_\uparrow$ and $R_\downarrow$, respectively, and use for calculation of the magnetoresistance $\delta_h$ of threelayer structure the simple two-current Mott model [22] for description of the resistance of different conductance channels [23]. This model suggests the conservation of orientation of electron spin moments during penetration of structure and better corresponds to measurement of the CPP magnetoresistance. The model was developed in papers [24–26] for description of the CPP-GMR in multilayer magnetic structures. Within the bounds of two-current model we neglect by scattering of current carriers from interfaces and by resistance of the nonmagnetic metal in comparison with resistance of the ferromagnetic metal. The total resistance of threelayer structure for the antiparallel configuration which is realized in the absence of a magnetic field is determined in concordance with equivalent resistor scheme in Fig. 4 by relation $R_{AP} = (R_\uparrow + R_\downarrow)/2$. The parallel configuration of the threelayer structure for the magnetic field $h \geq h_s$ is characterized by the resistance in the form $R_P = (2R_\uparrow R_\downarrow)/(R_\uparrow + R_\downarrow)$. Consequently, the magnetoresistance of threelayer structure is determined by relation

$$\delta_h = \frac{(R_\uparrow - R_\downarrow)^2}{4R_\uparrow R_\downarrow} = \frac{(J_\uparrow - J_\downarrow)^2}{4J_\uparrow J_\downarrow}, \quad (4)$$

where $J_{\uparrow, \downarrow} = e n_{\uparrow, \downarrow} \langle V_{\uparrow, \downarrow} \rangle$ is the current density. Here, $n_{\uparrow, \downarrow}$ is the density of electrons with $z$-components of spin moment equal $+1/2$ and $-1/2$, $n = n_\uparrow + n_\downarrow$ is the total electron density, $\langle V_{\uparrow, \downarrow} \rangle$ are the averaged velocity of electrons with corresponding spin projections. The electron densities with spin up and down can be expressed through the magnetization of film $n_{\uparrow, \downarrow}/n = (1 \pm m)/2$ determined in process of the Monte Carlo simulation. The averaged electron velocity $\langle V_{\uparrow, \downarrow} \rangle$ can be expressed through an electron mobility and the external electric field intensity $E$, and after that through a probability of electron jump in unit time from $i$-cell to a neighbouring cell in the direction of electric field [27] with averaging over all film cells:

$$\langle V_{\uparrow, \downarrow} \rangle = \mu_{\uparrow, \downarrow} E = \frac{e}{m} E \left\langle \exp \left( - \frac{\Delta E_{i,\uparrow} \downarrow}{T} \right) \right\rangle, \quad (5)$$

where $\mu$ is the electron mobility, $\Delta E_i$ characterizes the change of system energy connected with electron jump from $i$-cell to a neighbouring cell. $E_{i,\uparrow \downarrow}$ is determined by relation

$$E_{i,\uparrow \downarrow} = \mp J_i \left[ \sum_{j \neq i} S^z_j (n_{j,\uparrow} - n_{j,\downarrow}) + S^z_i (n_{i,\uparrow} - n_{i,\downarrow}) \right], \quad (6)$$

where the summation is realized of cells $j$ neighbouring to $i$ cell. In relation (6) for $E_{i,\uparrow \downarrow}$ it was realized the fundamental idea of Nevill Mott [22], according to which an electrons of different spin bands realize a current only if their spin projection coincides with direction of the local magnetization of system, characterized at this case by spin $S^z_j$ of the cell $j$. 

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**Figure 4.** Equivalent scheme for determination of the resistance for antiparallel $R_{AP}$ and parallel $R_P$ configurations of the layers magnetizaion in threelayer magnetic structure.
Figure 5. Temperature dependence of the magnetoresistance $\delta_h$ for threelayer structure (a) and spin-valve (b) with different thicknesses $N$ of the ferromagnetic films

On basis of above presented relations we calculated the temperature dependence of magnetoresistance for the threelayer structure with different thicknesses $N$ of ferromagnetic films. The procedure of calculation consists of the following steps: first step is connected with Monte Carlo simulations of an magnetic structure in equilibrium state at temperature $T$ with determination of the magnetization of ferromagnetic films $m_1$ and $m_2$, which gives possibility to calculate the electron densities $n_{T,\downarrow}$ for film cells; at second step the average electron velocities $\langle V_{T,\downarrow} \rangle$ and the current densities $J_{\uparrow,\downarrow}$ are calculated under relations (5)-(6) subject to spin configuration, realized at given time of simulation, and averaged over Monte Carlo steps at times of equilibrium state simulation; at last step the calculation of the magnetoresistance is carried out under relation (4).

The graphs of calculated temperature dependence of the magnetoresistance are presented in Fig. 5a for the threelayer structure with different thicknesses $N$ of the magnetic films. The obtained results demonstrate that for the low temperature with $T = 1$ the structure with film thicknesses in interval $N = 1 \div 11$ is characterized by sequential increase of the magnetoresistance from value $\delta_h = 4.5\%$ for $N = 1$ before value $\delta_h = 112\%$ for $N = 11$ and by fast falling down of $\delta_h(T)$ when temperature is increased. For films with thicknesses in interval $N = 13 \div 20$, the magnetoresistance for temperature $T = 1$ begin to be decreased with increasing thickness $N$, but the temperature falling down of magnetoresistance becomes more flat. However, for structures with $N \leq 11$ and $N \geq 25$ the magnetoresistance is characterized by fast falling down practically before null in the same interval of temperatures. Note that the observable changes of the magnetoresistance with increasing thickness of ferromagnetic films $N$ correspond to dimensional changes of the critical temperature of ferromagnetic phase transition in Heisenberg films revealed in papers [16, 17] and agree with results of the experimental investigations of dimensional phenomena in the ultrathin films Fe, Co, and Ni on substrates from nonmagnetic metals [1]. In accordance with papers [16, 17], the films with thicknesses in interval $N = 13 \div 20$ in which the magnetoresistance takes on a maximal values with very slow temperature falling down correspond to Heisenberg films demonstrating the critical behavior of the 3D Ising model.

The graph of calculated temperature dependence of the magnetoresistance $\delta_h$ for spin-valve structure is presented in Fig. 5b with the same thicknesses $N$ of the ferromagnetic films. Comparison of data in Fig. 5a and b shows that the values of $\delta_h$ for spin-valve structure with the same thickness $N$ rather less but generally comparable with values $\delta_h$ for multilayer structure and agree with their temperature dependence trend, described above.

The graph of calculated magnetoresistance is given in Fig. 6a for the multilayer structure from the iron films with thickness $N = 10$ with determination of temperature scale through the
value of exchange integral $J_1 \approx 2 \cdot 10^{-14}$ erg corresponding to the iron. In the same Fig. 6a the results of experimental measurements of the CPP-MR are presented for the multilayer structure Fe(001)/Cr(001) with thickness of iron films equal 3 nm corresponding to $N = 10$ [21]. One can see that the calculated temperature dependence of the magnetoresistance agrees very well with experimental data.

Recently, a highly promising material for the use in spintronic devices have been allocated the Co$_2$ based Heusler alloys Co$_2$FeAl$_x$Si$_{1-x}$, characterized by the highest Curie temperatures ($T_c = 1170$ K, Co$_2$FeAl and $T_c = 1100$ K, Co$_2$FeSi) and by very high values of electron spin polarization at the Fermi surface (close to 100 % at $x \approx 0.5$), because it have been identified their half-metallic properties in ferromagnetic state [28]. In paper [29], it were presented the results that a magnetic tunnel junction (MTJ) showing 832 % tunneling magnetoresistance (TMR) at 2 K and 386 % at room temperature has been realized using the Heusler compound Co$_2$FeAl$_{0.5}$Si$_{0.5}$. These alloys are also recognized as very promising for production of read heads of ultrahigh density magnetic recording systems and magnetic sensors on basis of the CPP-GMR spin-valve structures because of the intrinsically low resistance of CPP-GMR devices with application of these alloys [30]. This property is suitable for achieving high-speed readout.

We used in this study the developed methodology for the calculation of the magnetoresistance for spin-valve structure with ferromagnetic layers from Heusler alloy Co$_2$FeAl$_{0.5}$Si$_{0.5}$ (CFAS). The graph of calculated temperature dependence of the magnetoresistance $\delta_h$ for spin-valve structure is presented in Fig. 6b for ferromagnetic layers with thickness $N = 23$ monolayers with determination of temperature scale through the value of exchange integral $J_1 \approx 2.2 \cdot 10^{-14}$ erg, corresponding to the Curie temperature of the CFAS alloy $T_c \approx 1140$ K. The same figure shows the results of experimental measurements of the temperature dependence of the CPP magnetoresistance for the spin-valve structure CFAS (20 nm) / Ag (5 nm) / CFAS (5 nm) / IrMn (10 nm) with an antiferromagnetic layer of an alloy Ir$_{22}$Mn$_{78}$, carried out in [30]. For this asymmetrical configuration of the spin-valve with a free CFAS layer thickness 20 nm and a layer CFAS with a fixed orientation of the magnetization with thickness 5 nm, we realized the calculation of magnetoresistance for symmetric spin-valve with a thickness of the ferromagnetic layers equal to the average value 12.5 nm corresponding to $N \approx 23$ CFAS alloy. The graphs in Fig. 6b show that calculated temperature dependence of the magnetoresistance by Monte Carlo method for this spin-valve structure is in good agreement with the experimental results [30].

Note in conclusion that in the present paper the Monte Carlo study of sandwich and
spin-valve magnetic structures with GMR effects is carried out with applying an anisotropic Heisenberg model to describe the magnetic properties of the thin ferromagnetic films forming these structures. For ferromagnetic and antiferromagnetic configuration given structures, we have received the dependence of the magnetic characteristics on the temperature and the external magnetic field. With the use of two-current model to describe the resistance of different conduction channels, we have obtained an expression for the magnetoresistance of multilayer magnetic structures, determined by the probabilities of the electron jumps along the crystal cells subject to their spin projections. For the first time, it was developed a methodology of determination of the magnetoresistance with the use of the Monte Carlo method and was calculated the magnetoresistance temperature dependence for the three-layer and spin-valve structures with different thicknesses of ferromagnetic films. It is shown that the calculated temperature dependencies of the magnetoresistance are in good agreement with the experimental results obtained for multilayer structure Fe (001)/Cr (001) [21] and the spin-valve structure CFAS/Ag/CFAS/IrMn [30] with the Co$_2$ based half-metallic Heusler alloy Co$_2$FeAl$_{0.5}$Si$_{0.5}$.

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References

[1] Vaz C A F, Bland J A C, Lauhoff G 2008 Rep. Prog. Phys. 71 056501
[2] Bland J A C, Heinrich B (Eds) Ultrathin Magnetic Structures: (Berlin: Springer) 1994 vol I p 350; 1994 vol II p 350; 2005 vol III p 318; 2005 vol IV p 257
[3] Ustinov V V, Milyaev M A, Naumova L I 2014 SPIN 04 144001
[4] Baibich M N, Broto J M, Fert A, Van Dau F N, Petroff F, Etienne P, Creuzet G, Friederich A, Chazelas J 1988 Phys. Rev. Lett. 61 2472
[5] Binash G, Grunberg P, Saurenbach F, Zinn W 1989 Phys. Rev. B 39 4828
[6] Barthelemy A, Fert A 1991 Phys. Rev. B 43 13124
[7] Juliere M 1975 Phys. Lett. A 54 225
[8] Miyazaki T, Tezuka N 1995 JMMM 139 L231
[9] Sousa R C, Sun J J, Soares V, Freitas P P, Kling A, Silva M F, Soares J C 1998 Appl. Phys. Lett. 73 3288
[10] Prinz G A 1999 JMMM 200 57
[11] Chappert C, Fert A, Van Dau F N 2007 Nature Mater. 6 813
[12] Li Y, Baberschke K 1992 Phys. Rev. Lett. 68 1208
[13] Huang F, Kief M T, Mankey G J, Willis R F 1994 Phys. Rev. B 49 3962
[14] Binder K, Landau D P 1976 Phys. Rev. B 13 1140
[15] Prudnikov V V, Vakilov A N, Prudnikov P V 2009 Phase transitions and their computer simulations (Moscow: Fizmatlit) p 224
[16] Prudnikov P V, Prudnikov V V, Medvedeva M A 2014 JETP Lett. 100 446
[17] Prudnikov P V, Prudnikov V V, Menshikova M A, Piskunova N I 2015 JMMM 387 77
[18] Wang J-S, Swendsen R H 1990 Physica A 167 565
[19] Dieny B, Speriosu V S, Parkin S S P, Gurney B A, Wilhoit D R, Mauri D 1991 Phys. Rev. B 43 1297
[20] Glig M A M, Bauer G E W 1997 Adv. Phys. 46 285
[21] Bass J, Pratt W P 1999 JMMM 200 274
[22] Mott N F 1936 Proc. Roy. Soc. (London) ser A 153 699
[23] Mathon J 1991 Contemp. Phys. 32 143
[24] Zhang S, Levy P M 1991 J. Appl. Phys. 69 4786
[25] Bauer G E W 1992 Phys. Rev. Lett. 69 1676
[26] Valet T, Fert A 1993 Phys. Rev. B 48 7099
[27] Prudnikov V V, Prudnikov P V, Mamonova M V 2014 Quantum-statistical theory of solid state (Omsk: OmsU press) p 492
[28] Kandpal H C, Fecher G H, Felser C 2007 J. Phys. D 40 1507
[29] Tezuka N, Ikeda N, Mitsuhashi F, Sugimoto S 2009 Appl. Phys. Lett. 94 162504
[30] Furubayashi T, Kodama K, Sukegawa H, Takahashi Y, Inomata K, Hono K 2008 Appl. Phys. Lett. 93 122507