The analysis of the implementation of research-based learning on the students combinatorial thinking skills in solving a resolving perfect dominating set problem

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Abstract. The Indonesian government gives freedom to citizens to get a quality education, this will enable students to develop their potential in a more targeted way. The purpose of implementing research-based learning is to see the combinatorial thinking ability of students in completing the resolving perfect dominating set. The research method used a combination method (qualitative and quantitative) research methods involving 30 students divided into two classes, namely the control class 16 students and the experiment class 14 students. The implementation of research based learning was carried out in both classes, but there were differences in the implementation where the experiment class was given the developed tools, but not in the control class. Analysis of the results of the homogeneity test on the pre-test items showed that the two classes were homogeneous with a significant value of 0.171 $\geq$ 0.05, and the analysis of the results of the independent sample t-test on post test result showed that the sig (2-tailed) value is 0.007 ($p \leq 0.05$) so its significant. The results that have been obtained in this study can be concluded that there is an increase in students' combinatorial thinking skills in solving resolving perfect dominating set in research based learning.

1. Introduction
The Indonesian government has given freedom to every citizen to obtain quality education which will make education in Indonesia more developed in various aspects. The government's effort will run well if there is support of education actors who participate in upgrading the quality of education. At the moment the government is focusing on student-centered learning patterns where students become learning subjects. One of learning models that gives students freedom in building knowledge based on research steps is a research based learning model.

Research based learning models raise knowledge with the following stages, namely formulating hypotheses, collecting data to be used, analyzing data that have been obtained, making conclusions, then compiling reports [1]. Research based learning is the integration of theoretical knowledge with suitability in data collection and analysis procedures to examine, verify/study the phenomenon of an event [2]. Research-based learning models have been widely studied, including Syaibani [3] applying a research based learning model to analyze students' creative thinking skills, Yudha [4] applying a research based learning model to analyze 21st century students' creative and innovative skills, and Wardani [5] applying a research based learning model to analyze its effect on student conjecturing skills.
Teaching and learning activities are closely related to students’ thinking processes. The thinking abilities of students are very diverse, one of which is the ability to think combinatorial where students are able to consider many possibilities in certain situations. Combinatorial thinking skills are able to train students in estimating, generalizing, and being able to think systematically in solving mathematical problems [6]. Combine and combinatoric thinking are different things, combinatorial thinking is a process to find alternatives in solving discrete problems, however combinatoric is the science of mathematics that discusses certain topics/studies that are taught and discussed in depth to students.

The indicator of combinatorial thinking in this study uses the modified results [7]. The following indicators and sub-indicators of combinatorial thinking are presented in table 1.

| Indicator | Sub-Indicator |
|-----------|---------------|
| Identified several cases | a. Identifying the properties/characteristics of the problems |
| Recognize the pattern of all cases | b. Implementing several cases |
| Generalizing all cases | a. Identifying the pattern of problem solving |
| | b. Expanding the pattern of problem solving obtained |
| | a. Applying mathematical symbolization |
| Proving mathematically | c. Calculating cardinality |
| | c. Developing algorithm |
| | a. Performing argument calculations |
| | b. Testing algorithm |
| | c. Developing the seed |
| | d. Testing the seed |
| | e. Applying inductive, deductive, and qualitative proof |
| Considering other combinatorial problems | a. Performing Interpretation |
| | b. Proposing an open problem |
| | c. Identifying new combinatorial problems |
| | d. Finding potential applications |

Mathematical problems, especially in graphic theory, have an important role in solving problems in everyday life. The application of graph theory itself can be used in scheduling subjects, fingerprint recognition, social network analysis, analysis of the morphology of city roads, to modeling a problem to facilitate problem solving. In general, graph theory is always undergoing development, discussions about dominating sets also have new discussions, namely locating dominating set, efficiency dominating set, perfect dominating set and many more. This discussion was further extended to the resolving perfect dominating set, which is the concept of combining the perfect dominating set and resolving set. This is what makes graph theory will always experience developments in the renewal of research that will be carried out. Graph definition can be seen in [8,9].

The concept of dominating set number denoted \( \gamma(G) \) independently introduced by Haynes [10], and the first time to introduce the resolving dominating set is [11,12]. A set of vertex \( D \subseteq V \) is called a dominating set of \( G \) if each vertex of \( G \) is dominated by at least one member of \( D \). A graph is called the resolving perfect dominating set if conforms to the concept of the of the perfect dominating set and resolving set. Perfect dominating set, that is if each vertex of a graph \( G \) is dominated by exactly one vertex in \( D \) (dominator) [13], while resolving set is an ordered set for each dominator vertex having a different representation of \( W \) where \( r(v|W) \neq r(u|W) \). The ordered set \( W_{rp} \subseteq V(G) \) is called the resolving perfect dominating set on graph \( G \) if \( W_{rp} \) is the resolute set and perfect dominating set of graph \( G \). The minimum cardinality of the resolving perfect dominating set is called the resolving perfect...
dominating number which is denoted by \( \gamma_{rp}(G) \). The following is an explanation to make it clearer about the definition related to the perfect dominating set.

**Figure 1.** \( \{b, e, f\} \) is perfect dominating set with \( \gamma_p(G) = 3 \).

**Figure 2.** Metric dimension.

**Figure 3.** Resolving perfect dominating set with \( \gamma_{rp}(G) = 3 \).

Learning instrument can support success in learning activities, one of which is research based learning. Learning instrument in higher education are the existence of student worksheet, learning activity tests, and monographs [14,15]. A worksheet given to students containing assignments, instructions, and work steps. The research activity test contains a pre-test and post-test which are given to students before and after learning. The monographs contain books that cover a specific topic.

In this study, students were requested to labelling the graph and the edges of the graph, determine the perfect dominating set, determine the resolving perfect dominating set, and determine the function of the vertex representation in the resolving perfect dominating set of the graph built on the definition and research based learning measures.

2. **Research Methods**

This type of research is using a combination research or a combination of qualitative and quantitative research methods or commonly known as mixed methods. This research will use a sequential exploratory design which is a combination research with the first stage of collecting and analyzing qualitative data, the second stage is collecting and analyzing quantitative data to make conclusions from the research results in the initial phase. Mixed methods have a very important influence in understanding problems in research if quantitative and qualitative methods are not used together then they are not accurate enough to understand the problems in research.

The implementation of this study aims to determine the effect of research based learning models on students' thinking abilities, especially combinatorial thinking skills in the resolving perfect dominating set study. Research implementation at the beginning and end of learning in the experiment and the control class is to provide tests (pre-test and post-test) to students before and after learning are carried out. The purpose of being given a pre-test is to determine the initial abilities of students, while the goal is given a post-test, which is to see the effect of the treatment given. The treatment in question is at the core of the implementation of learning where the experiment and control classes carry out research based learning, but for the experiment class they are given the tools developed (student worksheets) and the control class is not given.

The data analysis that was carried out for the first time was a quantitative analysis by performing the homogeneity test, normality test, and independent sample t-test. Then, analyzing qualitative data to see statistical data on the application of learning tools that have been developed, qualitative data analysis uses descriptive and inferential statistics to find information related to the acquisition of the mean, frequency, and standard deviation. Comparison of the means in the two classes using the independent sample t-test with a significant difference at the 0.05 level. Qualitative analysis is concerned with student activities, particularly in identifying cases, identifying patterns of all cases, generalizing all cases,
proving mathematically, and considering other combinatorial problems. Combinatorial thinking skill levels can be seen in Table 2.

### Table 2. Level of combinatorial thinking skills.

| Combinatorial Thinking Skill                  | Score |
|-----------------------------------------------|-------|
| Level 1 (very low)                            | 0-8   |
| Level 2 (low)                                 | 9-16  |
| Level 3 (intermediate)                        | 17-28 |
| Level 4 (high)                                | 29-48 |
| Level 5 (very high)                           | 49-64 |

2.1 Population

This research was applied to mathematics education students, University of Jember, Indonesia. The sample of this study consisted of two classes, namely the control class totaling 16 students, and the experiment class totaling 14 students.

2.2 Research Instrument

The instruments in this study consisted of a test (pre-test, post-test), observation and interviews. Figure 4 shows the model of a combination method consisting of the stages of preliminary study (qualitative research), analysis of combinatorial thinking skills and the application of research based learning (quantitative research), a portrait of the phase of combinatorial thinking skills in students in completing a resolving perfect dominating number study (qualitative research). The following is an explanation of the research procedure which can be seen in Figure 4.

![Figure 4. Research procedure of the mixed method model.](image)

2.3 Task

The tasks that will be done by students are pre-test, post-test, and student worksheets which are equipped with indicators to measuring students’ combinatorial thinking skills. The pre test and post test will be given to students in the control and experiment classes, while the student work sheets are only provided students in the experiment class. The pre-test will be given to students before they get the material to be
given, for the post-test will be given after the learning has ended (after the material has been delivered). Students are asked to understand and implement the basic concepts of the perfect dominating set and resolving set, recognize the resolving perfect dominating set pattern and expand the graph, label the graph and determine its cardinality, make the resolving perfect dominating set function on a graph and test the function, finally the student is able to describes the flow of the tests given. The following assignments given to students can be seen in Figure 5.

Figure 5. Labeling graph $B_t^4 \bowtie P_2$.

Cardinality of graph $B_t^4 \bowtie P_2$

\[
V(B_t^4 \bowtie P_2) = \left\{x_i; i = 1, 2 \right\} \cup \left\{y_i; 1 \leq i \leq 4 \right\}
\]

\[|V(B_t^4 \bowtie P_2)| = 12\]

\[
E(B_t^4 \bowtie P_2) = \left\{x_i, x_{i+1}, i = 1 \right\} \cup \left\{x_i, y_i, i = 1, 2 \right\} \cup \left\{y_i, y_{i+1}, i = 1, 2 \right\} \cup \left\{x_i, y_j; i = 1, 2; 1 \leq j \leq n \right\}
\]

\[|E(B_t^4 \bowtie P_2)| = 15\]

Figure 6. Resolving perfect dominating set on graph $B_t^4 \bowtie P_2$.

Dominator on the perfect Dominating Set of graph $B_t^4 \bowtie P_2$, namely $D(B_t^4 \bowtie P_2) = \{x_1, x_2, y_1, y_2, y_3, y_4\}$. $W$ is the resolving set of graph $G$ if $r(u|W) \neq r(v|W)$ for each vertex $u, v \in V$ where $u \neq v$.

\[
W(B_t^4 \bowtie P_2) = \{x_1, x_2, y_1, y_2, y_3, y_4\}
\]

\[
r(x_1|W) = (0, 1, 1, 1, 1, 1)
\]

\[
r(x_2|W) = (1, 0, 1, 1, 1, 1)
\]
r(y_1|W) = (1,1,0,2,2,2) 
 r(y_2|W) = (1,1,2,0,2,2) 
 r(y_3|W) = (1,1,2,2,0,2) 
 r(y_4|W) = (1,1,2,2,2,0) 
 r(x_{1,1}|W) = (1,2,2,2,2,2) 
 r(x_{2,1}|W) = (2,1,2,2,2,2) 
 r(y_{1,1}|W) = (2,2,1,3,3,3) 
 r(y_{2,1}|W) = (2,2,3,1,3,3) 
 r(y_{3,1}|W) = (2,2,3,3,1,3) 
 r(y_{4,1}|W) = (2,2,3,3,3,1)

It has been obtained that 
 r(x_1|W) \neq r(x_2|W) \neq r(y_1|W) \neq r(y_2|W) \neq r(y_3|W) \neq r(y_4|W) \neq r(x_{1,1}|W) \neq r(x_{2,1}|W) \neq r(y_{1,1}|W) \neq r(y_{2,1}|W) \neq r(y_{3,1}|W) \neq r(y_{4,1}|W),

so it can be concluded that 
 W(B_{t_1} \triangleright P_2) = \{x_1, x_2, y_1, y_2, y_3, y_4\} 
 is the resolving set of B_{t_1} \triangleright P_2 
 because \ r(u|W) \neq r(v|W). 
 So, the resolving dominating number on the graph 
 B_{t_1} \triangleright P_2 
 is \ r_{rp}(B_{t_1} \triangleright P_2) = 6.

Representation of vertex titik \ v \in \ G 
 with respect to 
 W(B_{t_1} \triangleright P_2) = \{x_1, x_2, y_1, y_2, y_3, y_4\} 
 can be written as a repeating function which can be seen in table 3 as follows:

Table 3. Representation of the vertex \ v \in V(B_{t_1} \triangleright P_2) 
 respect to \ W = x_i, y_i, x_{i,1}, y_{i,1}.

| V      | r(V|W)                              | Condition |
|--------|-------------------------------------|-----------|
| x_i    | \(\ldots, 1, 0, 1, \ldots\)_{n-i}  | \(i \leq i \leq n, n \geq 3\) |
| y_i    | \(1,1,2, \ldots, 2, 0, 2, \ldots\)_{n-i} | \(i \leq i \leq n, n \geq 3\) |
| x_{i,1} | \(\ldots, 2, 1, 2, \ldots\)_{n-i} | \(i = 1, 2, n \geq 3\) |
| y_{i,1} | \(2,2,3, \ldots, 3, 1, 3, \ldots\)_{n-i} | \(i \leq i \leq n, n \geq 3\) |

Because it produces different representations of vertex, \ W 
 is the resolving perfect dominating set and we get that 
 \ r_{rp}(B_{t_1} \triangleright P_2) = n + 2.

Hypothesis testing in this study uses a significant level of 5%. 
 The following hypotheses will be tested:

H_0 : There is no effect of research based learning model of combinatorial skills in students 
 H_1 : There is an effect of research based learning model of combinatorial skills in students

The test criteria H_0 is accepted if the probability value is \geq 0.05 then H_0 is accepted, and

- If the \ p_{value} < 0.05 then H_0 is rejected, and H_1 is accepted
- If the \ p_{value} \geq 0.05 then H_0 is accepted and H_1 is rejected

3. Research Finding

This study used two classes, including experiment and control classes using qualitative and quantitative methods. 
Students' combinatorial thinking skills can be identified through qualitative methods. 
This research was conducted after the implementation of reliability and validity tests on the test instruments 
that will be tried out on students. The purpose of conducting this test is to determine the accuracy of a 
measurement instrument in performing the measurement function.

After implementing the pre test in both experiment and control classes, the next step is to provide 
learning by applying a research based learning model in both classes. However, in the implementation 
of learning there is a different treatment, where the experiment class is given a student worksheet or
device being developed, but the control class is not given a student worksheet. Following are the results of the analysis that has been obtained, the data analysis in this study uses the SPSS application.

### 3.1 Instrument Validation

Research based learning tools have the aim to determine the impact of combinatorial thinking skills possessed by students in learning materials resolving perfect dominating set. In the initial research, the method used was a qualitative method of testing the validity and reliability of the test instruments that would be tested on students. The following is an explanation and the results of the reliability and validity tests that have been carried out, can be seen in Table 4.

**Table 4.** The result of validity.

| Correlations | S01     | S02     | S03     | S04     | TOTAL |
|--------------|---------|---------|---------|---------|-------|
| Pearson Correlation | 1   | .403*   | .344    | .166    | .758**|
| Sig. (2-tailed) |        | .027    | .062    | .379    | .000  |
| N            | 30     | 30      | 30      | 30      | 30    |
| Pearson Correlation | .403* | 1       | .059    | .308    | .755**|
| Sig. (2-tailed) | .027  | .758    | .098    | .000    |       |
| N            | 30     | 30      | 30      | 30      | 30    |
| Pearson Correlation | .344  | .059    | 1       | -.355   | .424* |
| Sig. (2-tailed) | .062  | .758    | .054    | .020    |       |
| N            | 30     | 30      | 30      | 30      | 30    |
| Pearson Correlation | .166  | .308    | -.355   | 1       | .478**|
| Sig. (2-tailed) | .379  | .098    | .054    | .008    |       |
| N            | 30     | 30      | 30      | 30      | 30    |
| Pearson Correlation | .758**| .755**  | .424*   | .478**  | 1     |
| Sig. (2-tailed) | .000  | .000    | .020    | .008    |       |
| N            | 30     | 30      | 30      | 30      | 30    |

* Correlation is significant at the 0.05 level (2-tailed).

** Correlation is significant at the 0.01 level (2-tailed).

Table 4 explains that the value of $r_{count}$ in problem 1 is 0.758, $r_{count}$ in problem 2 is 0.755, $r_{count}$ in problem 3 is 0.424, $r_{count}$ in problem 4 is 0.478, while $r_{table}$ for $n = 30$ is . So it can be concluded that $r_{count}$ on questions 1 until 5 > $r_{table}$ then all questions in that question are valid. Based on table 5, the reliability value is 0.398 and $r_{table}$ from the 0.05 significant level with $dk = N − 2 = 28$, $r_{table} = 0.3610$. The conclusion is $r_{count} > r_{table}$ thus the instrument can be said to be valid.

**Table 5.** Test of the results of reliability.

| Reliability Statistics | Cronbach's Alpha | N of Items |
|------------------------|------------------|------------|
|                        | .398             | 4          |

Data analysis consisted of quantitative and qualitative data analysis, the t-test was carried out on quantitative data analysis, then interviews, observations and ordinal data were carried out on qualitative data analysis. Quantitative and qualitative data analysis using descriptive and inferential statistical data. Statistical data are obtained from the average value, standard deviation, and frequency, while inferential data is obtained through homogeneity, normality, and free tests between control and experimental classes in research-based learning. The objective of using an independent sample is to compare the control and the experiment class with a significance value of difference at the 0.05 level.
3.2 Result
In the initial research, it was found that the pre-test results were carried out in the experiment and control classes to determine the combined abilities possessed by students. Based on the results of the pre-test conducted on 16 students in the control class, it was found that 27% of the students were at the level (1) very low, 18% of the students were at level (2) low, 14% of the students were at the level (3) intermediate, 23% of students were at the level (4) high, and 18% of the students were at the level (5) very high. The result of each class can be seen in Figure 7 and Figure 8.

![Figure 7. The distribution of pre-test of control class on each indicator of combinatorial skill.](image)

![Figure 8. The distribution of pre-test of experiment class on each indicator of combinatorial skill.](image)

While the results of the pre-test conducted on 14 students in the experiment class, it was found that 29% of students were at the level (1) very low, 18% of the students were at the level (2) low, 14% of the students were at the level (3) intermediate, 18% of students were at the level (4) high, and 21% of the students were at the level (5) very high.

The next step is to analyze the control class and experiment class data using quantitative methods obtained from the pre-test and post-test using the SPSS application. The statistical tests to be conducted are normality test, homogeneity test, and independent test. The first step to analyzing the pretest data is to test the normality of the two classes (control class and experiment class) to determine the distribution...
of data in the two classes with normal distribution or not. Based on the Kolmogorov-Smirnov Table 6, the significance value that has been obtained is 0.200 ≥ 0.05, it can be concluded that the pretest data from the two classes is normally distributed.

**Table 6.** The results of normality test of pre-test in the control and experiment class.

| One-Sample Kolmogorov-Smirnov Test | Unstandardize Residual |
|-----------------------------------|------------------------|
| N                                 | 30                     |
| Normal Parameters \(^{a,b}\)       | Mean .0000000          |
|                                   | Std. Deviation 6.95968333 |
| Most Extreme Differences          | Absolute .095           |
|                                   | Positive .095           |
|                                   | Negative -.079          |
| Test Statistic                    | .095                   |
| Asymp. Sig. (2-tailed)            | .200\(^{c,d}\)          |

a. Test distribution is Normal.
b. Calculated from data.
c. Lilliefors Significance Correction.
d. This is a lower bound of the true significance.

The second step was to test the homogeneity of two class control and experiment classes using the independent t-test. Table 7 shows the results of the independent t-test, the significance of the value is obtained 0.171 where the significant value obtained is greater than 0.05, it can be concluded that there is no difference between the means pre-test data in the control and the experiment class, that means the pre-test data obtained is homogeneous.

**Table 7.** The homogenity results of the pre-test in the control and experiment classes.

| Test of Homogeneity of Variances | PRETEST |
|----------------------------------|---------|
| Levene Statistic                 | df1 1   |
|                                 | df2 28  |
|                                 | Sig .171 |

Combinatorial thinking ability of the control class

**Figure 9.** The distribution of post-test of control class on each indicator of combinatorial skill.
Furthermore, implementing the post-test, post-test is carried out after the pre-test has been carried out by giving a different action to the class. The implementation of the post-test aims to obtain a description of the abilities that have been achieved by students after the delivery of material carried out by the educator. The number of post-test questions given to students was 4 items containing questions about basic graphs and resolving perfect dominating set material which contained indicators of combinatorial thinking skills. The results of the post-test carried out on 16 students in the control class, it was found that 24% of the students were at the level (1) very low, 18% of the students were at the level (2) low, 17% of the students were at the level (3) intermediate, 23% of students are at level (4) high, and 18% of students are at level (5) very high. The result of each class can be seen in Figure 9 and Figure 10.

**Combinatorial thinking ability of the experiment class**

![Combinatorial thinking ability of the experiment class](image)

**Figure 10.** The chart of post-test of experiment class on each indicator of combinatorial skill.

While the results of the post-test conducted on 14 students in the experiment class, it was found that 13% of students were at the level (1) very low, 18% of the students were at the level (2) low, 20% of the students were at the level (3) intermediate, 18% of students were at the level (4) high, and 31% of the students were at level (5) very high.

Table 8 shows that the results of the post-test analysis of normality test in the control and experiment class are normally distributed with a significant value 0.200, where the significance value is more than 0.05. Table 9 shows that the average value obtained from the control class is 30.563 and the experiment class is 39.643, so it is found that the average value in the control class is lower than the experiment class.

**Table 8.** The results of normality test of post-test in the control and experiment class.

| One-Sample Kolmogorov-Smirnov Test | Unstandardized Residual |
|------------------------------------|-------------------------|
| N                                  | 30                      |
| Normal Parametersa,b                | Mean                    |
|                                    | Std. Deviation          |
| Most Extreme Differences           | Absolute                |
|                                    | Positive                |
|                                    | Negative                |
| Test Statistic                     | Asymp. Sig. (2-tailed)  |
| a. Test distribution is Normal     | .200a,d                 |

a, b, c, d: See Table 8.
b. Calculated from data.
c. Lilliefors Significance Correction.
d. This is a lower bound of the true significance.

Table 9. The mean results of the post-test in the control and experiment class.

| Group Statistics | GROUP   | N  | Mean   | Std. Deviation | Std. Error Mean |
|------------------|---------|----|--------|----------------|-----------------|
| Learning outcomes| Experiment | 14 | 39.643 | 6.8793         | 1.8386          |
|                  | Control  | 16 | 30.563 | 4.2421         | 1.0605          |

The independent test results have been obtained by sig. (2-tailed) 0.000 < 0.05, so it can be concluded that the results of the post-test between the two classes, namely the control and the experiment class, differed significantly after the implementation of research based learning. Table 10 shows the independent test results of post-test. In the post test results of the normality, homogeneity, and independent t-test using SPSS, the results obtained were that there was no significant difference in the control and experimental classes, meaning that $H_1$ was accepted. So it can be concluded that the control and experimental classes have a significant difference in the post-test results after the student worksheets are applied to research learning.

Table 10. The independent test results of the post-test in the control and experiment class.

| Independent Samples Test | Levene's Test for Equality of Variances | t-test for Equality of Means |
|--------------------------|----------------------------------------|-----------------------------|
|                          | F           | t     | df | Sig. (2-tailed) | Mean Difference | Std. Error Difference | 95% Confidence Interval of the Difference |
|                          |            |       |    |                |                |                  | Lower | Upper |
| Learning Outcomes        | Equal variances assumed | 2.770 | .107 | .4413 | 28 | .000 | 9.0804 | 2.0576 | 4.8655 | 13.2952 |
|                          | Equal variances not assumed | 4.278 | 21.0 | 69 | .000 | 9.0804 | 2.1225 | 4.6672 | 13.4935 |

Figure 11. The results of student activity in the experiment class.
Student activities carried out during research based learning in the experiment class are shown in Figure 11 with the results that 17% of students are very active, 31% of students are active, 33% of students are less active, and 19% of students are inactive. So it can be concluded that learning in the classroom has positive results.

### 3.3 Phase Portrait

The phase portrait of this study was taken to describe the flow of students' work in completing the test on the resolving perfect dominating set of graphs that have been obtained. The purpose of the student job analysis is to obtain information in conducting interviews, so that the answers from students match the results of the answers. At this stage, we chose four subjects to serve as illustrations, namely as follows:

The graph resulting from student (1) work can be seen in Figure 12. Student (1) uses a star graph (Sn) to determine the resolving perfect dominating set. After determining the graph, student (1) determines the vertex label by mentioning vertices y1, y2, y3,…, yn and x. Determining the label of the vertex makes it easier to determine the perfect dominating set. The next step, student (1) determines the perfect dominating set, and checks the result of determining the perfect dominating set, but because student (1) does not double-check the result of determining the perfect dominating set, so the graph on his work does not meet the requirements of the perfect dominating set. In student (1) work, a function of the vertex representation against W is displayed, where $W = y_i$ is the resolving set if it produces a different vertex representation, the displayed results have been obtained are $r(x|W) = r(y_i|W)$ then W is taken by the student is not a resolvable set, because it does not meet the resolving set.

![Graph Star (Sn)](image)

$$V(S_n) = \{\{x\} \cup \{y_i; 1 \leq i \leq n\}\}$$

$$|V(S_n)| = n + 1$$

$$E(S_n) = \{xy_i; 1 \leq i \leq n\}$$

$$|E(S_n)| = n$$

$$D(S_n) = \{yn_1, y2, ..., y_i, ..., y_{n-1}, y_n\}$$

$$\gamma(S_n) = n$$

**Representation of vertex $v \in V(G)$ respect to $W = y_1, y_2, ..., y_i, ..., y_{n-1}, y_n$**

$$r(x|W) = \{1,1,1,...,1\}$$

$$r(y_i|W) = \{2,2,...,2,0,2,...,2,2\}_{i-1}$$

**Figure 12.** Graph result of student 1.

Students (1) have reached level (2) low on combinatorial thinking skills. The analysis results show that student (1) is only able to apply the mathematical symbolization shown in the calculation of cardinality on the graph used, and the concept of determining the vertex representation to other vertices, student (1) has not been able to understand the perfect dominating set and resolving set, so the resolving perfect The determining set was not found in the work result. The results of the analysis that has been carried out on student (1) work it can be concluded that the work is not the resolving perfect dominating set, because it does not meet all the requirements of the perfect dominating set and resolving set. The following is a portrait of the student (1) phase in Figure 13.

The results of the phase portrait in figure 13 show that the combinatorial thinking process in students (1) in solving the resolving perfect dominating set problem always jumps, starting from 1a then jumping to 2b, after passing 1b student (1) jumping to 3a, jumping again to 3c, and jumping to 4b. The results of interviews that have been conducted with student 1 are as follows:

**Interviewer** : After seeing the problems given, what did you do first?

**Student 1** : The first thing I did was look for a graph
Interviewer: Then after finding the graph, what else do you do?
Student 1: I started to draw the graph, know the name of the graph, determine the sum of the edges and vertices of the graph
Interviewer: How about the labeling, do you also label the graph?
Student 1: Yes, I labeled graphs with symbols $x, y_1, y_2, y_3, \ldots, y_n$
Interviewer: Is the graph you got the resolving perfect?
Student 1: Not sure, because I have trouble finding a graph that meets the requirements of the resolving perfect dominating set so I do it like that

**Figure 13.** The results of phase portrait student 1.

Resolving Perfect Dominating Number

**Prism Graph**

$|V| = 6$

$|E| = 9$

$D = \{x_1, x_2, x_3\}$

$\gamma_r = 3$

$r(x_1|W) = (0,1,1)$

$r(x_2|W) = (1,0,1)$

$r(x_3|W) = (1,1,0)$

$r(x_4|W) = (1,2,2)$

$r(x_5|W) = (2,1,2)$

$r(x_6|W) = (2,2,1)$

**Figure 14.** Graph result of student 2.

The graph of the results of student (2) work can be seen in Figure 14. Students (2) use a prism graph to determine the resolving perfect dominating set. After determining the graph, student (2) determines the vertex label by mentioning the vertices $x_1, x_2, x_3, x_4, x_5, \text{and} x_6$. Determining the vertex label will make it easier to determine the perfect dominating set, but the vertex label used is less simple so that if you use the graph up to the not order $n$ it will be difficult to determine the function of the vertex representation. The next step, determine the perfect dominating set, and recheck the result of determining the perfect dominating set. The student (2) work graph has met the requirements of the perfect dominating set. Student (2) also displays the function of the vertex representation against $W$, where $W= x_1, x_2, x_3$, which are the resolving set if they produce different vertex representations. The
results shown have been obtained that are \( r(x_1|W) \neq r(x_2|W) \neq r(x_3|W) \neq r(x_4|W) \neq r(x_5|W) \neq r(x_6|W) \) then \( W \) which taken by the student (2) is a resolving set, because it meets the resolving set requirements. Students (2) are able to prove the vertex representation, and be able to explain the flow of the process. Students (2) have reached intermediate level (3) in combinatorial thinking skills. The analysis results show that student (2) was able to apply the mathematical symbolization shown in the cardinality calculation on the graph used, and the concept of determining the vertex representation of the other vertices, student (2) was also able to understand the perfect dominating set and resolving set, so that the resolving perfect The dominating set was found in the result even though the function graph used did not span to the order n. The results of the analysis carried out on student (2) work can be concluded that the work is the resolving perfect dominating set, because it has met all the requirements for the perfect dominating set and resolving set. The following is a portrait of the student phase (2) in Figure 15.

![Figure 15](image-url)

**Figure 15.** The results of phase portrait student 2.

The results of the phase portrait in Figure 15 show that the combinatorial thinking process in students (2) in solving the problem of the resolving perfect dominating set has jumped from 4a to 4c. The results of interviews that have been conducted with student 2 are as follows:

- **Interviewer:** After seeing the problems given, what did you do first?
- **Student 2:** Finding a single graph, then taking the initiative to perform operations on both graphs.

- **Interviewer:** Then after finding the graph, what else do you do?
- **Student 2:** I drew the graph of the result of the operation, labeled it, and didn't forget looking for the cardinality of the vertice and their sides.

- **Interviewer:** Is the graph you got the resolving perfect?
- **Student 2:** Yes, because it has met the requirements for the resolving perfect dominating set.

- **Interviewer:** How did you determine the perfect dominating set? And determine the resolving perfect dominating set?
- **Student 2:** I tried the dominating ones that match the terms of the resolving perfect dominating set, and tested them. If it is appropriate, I test the vertex representation at all vertice in the graph.

- **Interviewer:** What difficulties did you have after solving this problem?
- **Student 2:** I have a hard time determining the formula for the function from its vertex representation, so I just write the vertex representation like that.
The graph from student (3) work can be seen in Figure 16. Student (3) uses the graph resulting from the corona operation on the path and a circle graph symbolized by $C_n \triangleright P_2$ to determine the resolving perfect dominating set. After determining the graph, student (3) determines the vertex label by mentioning the vertices $x_i, x_{i,1}$. Determining the label of the vertex makes it easier to determine the perfect dominating set. The labeling done by student (3) is already very good, so that when using the graph it can be used up to the not order $n$ and makes it easier to determine the function of the vertex representation. The next step, determine the perfect dominating set, and recheck the result of determining the perfect dominating set. The graph of student (3) work has met the requirements of the perfect dominating set. Student (3) also displays the function of the vertex representation against $W$, where $W = \{x_i, 1 \leq i \leq n\}$ which is the resolving set if it produces different vertex representations. The results shown show that are $r(x_i|W) \neq r(x_{i,1}|W)$, where $1 \leq i \leq n$, then the $W$ taken by student 3 is the resolving sets, because it has met the resolving set requirements. Student (3) is able to prove the vertex representation, and is able to explain the flow of the process.

Students (3) have reached a high level (4) in combinatorial thinking skills. The analysis results show that student (3) is able to apply the mathematical symbolization shown in the calculation of cardinality to the graph used, and the concept of determining the vertex representation to other vertices, student (3) has also been able to understand the perfect dominating set and resolving set, so that the resolving perfect The dominating set has been found in the analysis of this work and the function in the vertex representation used extends to the nth order, but the function writing is still incomplete. The results of the analysis carried out on student (3) work can be concluded that the work is the resolving perfect dominating set, because it has met all the requirements for the perfect dominating set and resolving set. The following is a portrait of the student (3) phase in Figure 17.

The results of the phase portrait in Figure 17 show that the combinatorial thinking process in students (3) in solving the problem of the resolving perfect dominating set is in accordance with the order of the sub indicators. In section 3a students (3) are able to skip section 3b to occupy a section 3c, likewise in sections 4a to 4c, and 4c jump to 4e. In section 4a there is a repetition of 4a.

The results of interviews that have been conducted with student 3 are as follows:

Interviewer : After seeing the problems given, what did you do first?
Student 3 : Find two graphs, so that I can make two graphs which are obtained from the comb product operational.

Interviewer : Then after finding the graph, what else do you do?
Student 3 : I tried to draw the result of the graph, and labeled each vertex.
Interviewer  : Is the graph you got the resolving perfect dominating set?
Student 3  : At first I was still doubtful whether the graph has the resolving perfect dominating set or not, but after the perfect dominance test is done I think the graph is already appropriate.

Interviewer  : How did you determine the perfect dominating set? And determine the resolving perfect dominating set?
Student 3  : The first thing I did was try to find the perfect dominating set, if it met the predetermined conditions, I looked for the vertex representation, whether each vertex on the graph already has a different representation or not. If so, the graph I am using can be said to be the resolving perfect dominating set.

Interviewer  : What difficulties did you have after solving this problem?
Student 3  : The difficulty I experienced was when determining the general function of the vertex representation.

The results of phase portrait student 3.

The graph resulting from student work (4) can be seen in Figure 18. Students (4) use the graph resulting from the corona operation on the path and path graph symbolized by $P_n \odot P_2$ to determine the resolving perfect dominating set. After determining the graph, student (4) determines the vertex label by mentioning vertices $x_i$ and $y_i$. Determining the vertex label will make it easier to determine the perfect dominating set. The labeling done by student (4) is already very good, so that when using the graph it can be used up to the not order $n$ and makes it easier to determine the function of the vertex representation. The next step, determine the perfect dominating set, and recheck the result of determining the perfect dominating set. The student work graph (4) has met the requirements of the perfect dominating set. Student (4) also displays the function of the vertex representation against $W$, where $W = \{x_i\}$ which is the resolving set if it produces different vertex representations. The results shown have been obtained that are $r(x_i|W) \neq r(y_i|W)$, where $1 \leq i \leq n$ then the $W$ is taken by the student (4) is a resolving set, because it has met the requirements for a resolving set. Students (4) are able to prove vertex representations, be able to explain the flow of the process, and be able to find applications for resolving perfect dominating sets in everyday life.

Student (4) has reached a very high level (5) in combinatorial thinking skills. The analysis results show that student (4) is able to apply the mathematical symbolization shown in the cardinality calculation on the graph used, and the concept of determining the vertex representation to other vertices, student (4) has also been able to understand the perfect dominating set and resolving set, so that the resolving perfect The dominating set has been found in the results of this work and the function in the vertex representation used extends to the order n, but the function writing is still incomplete. The results of the analysis carried out on student work (4) can be concluded that the work is the resolving perfect
dominating set, because it has met all the requirements of the perfect dominating set and resolving set. The following is a portrait of the student phase (4) in Figure 19.

\[ P_n \odot P_2 \]

![Graph result of student 4](image)

Cardinality
\[ V = \{(x_i; 1 \leq i \leq n) \cup (y_i; 1 \leq i \leq n)\} \]
\[ |V| = 2n \]
\[ E = \{(x_ix_{i+1}; 1 \leq i \leq n) \cup (x_iy_i; 1 \leq i \leq n)\} \]
\[ |E| = 2n - 1 \]
\[ W(G) = \{x_i; 1 \leq i \leq n\} \rightarrow Resolving\ Perfect\ Dominating\ set \]
\[ \gamma_{rP} = n \]

Representation of vertex \( x \in V(G) \) respect to \( S \)
\[ r(x_1|W) \]
\[ x_1 = (0,1,2, \ldots) \]
\[ x_2 = (1,0,1,2, \ldots) \]
\[ x_3 = (2,1,0,1,2, \ldots) \]
\[ x_4 = (3,2,1,0,1,2, \ldots) \]
\[ \vdots \]
\[ x_n = (\ldots,1,0,1, \ldots) \]
\[ y_1 = (1,2,3, \ldots) \]
\[ y_2 = (2,1,2,3, \ldots) \]
\[ y_3 = (3,2,1,2,3, \ldots) \]
\[ y_4 = (4,3,2,1,2,3,4, \ldots) \]
\[ \vdots \]
\[ y_n = (n-1, \ldots,2,1,2, \ldots, n+1) \]

**Figure 18.** Graph result of student 4.

The results of the phase portrait in Figure 19 show that the combinatorial thinking process in students (4) in solving the resolving perfect dominating set problem is in accordance to the order of the sub indicators. In part 1a students (4) are able to jump to the part 1b so that they occupy part 2a, as well as in sections 2b and 3a jump to 3c, 4a jump to 4c, 4c jump to 4e and there are repetitions at 4d to 5a and 4e to 5a.

The results of interviews that have been conducted with student 4 are as follows:

**Interviewer**: After seeing the problems given, what did you do first?
**Student 4**: I'm looking for a graph.

**Interviewer**: What graphs did you find?
**Student 4**: Helmet graph.

**Interviewer**: Then after finding the graph, what else do you do?
**Student 4**: I labeled and determined cardinality on the graph.

**Interviewer**: So what else did you do to get the perfect dominating set?
Student 4: I looked at the graph, and tried to find the perfect dominating according to the conditions.

Interviewer: After finding the perfect dominating number, can this graph be called the resolving perfect dominating set?

Student 4: After determining the perfect dominating set I still don't know whether this graph meets the resolving set requirements or not, because I haven't tested the vertex representation of the graph. However, after I tested the vertex representation at each vertex on the graph, I found that the graph can be said to be the resolving perfect dominating set because it fulfills the requirements of the perfect dominating set and resolving set.

Interviewer: What difficulties did you have after solving this problem?

Student 4: I am confused about determining the function formula from the vertex representation of the graph. But still I try not to know right or wrong.

Figure 19. The results of phase portrait student 4.

4. Discussion
Research based learning can be applied by educators or researchers into learning. It is hoped that students will get new experiences, curiosity will arise to solve a problem, and students will be more independent. In research based learning, students are also directed at thinking about combinatorial skills. Combinatorial thinking skills are skills in solving mathematical problems. These skills will make it easier for students to carry out investigations of several mathematical cases, especially in the field of graphs, be able to predict provisional guesses in each identical case, be able to prove the results of provisional conjectures by making hypotheses for identical cases on a graph, and able to connect different combinatoric problems with the previous case. This study was conducted to analyze the effect of research based research in improving students' combinatorial thinking skills in solving graph theory problems, especially the perfect dominating set.

This study shows that the results of the implementation of research based learning have increased the learning outcomes carried out by students, this can be seen in the results of students' post-tests. The post-test results obtained by students in the experiment class were better overall. The results of the post-test carried out on 16 students in the control class, it was found that 24% of the students were at the level (1) very low, 18% of the students were at the level (2) low, 17% of the students were at the level (3) intermediate, 23% of the students are at level (4) high, and 18% of students are at level (5) very high.

While the results of the post-test conducted on 14 students in the experiment class, it was found that 13% of students were at the level (1) very low, 18% of the students were at the level (2) low, 20% of the students were at the level (3) intermediate, 18% of students were at the level (4) high, and 31% of the students were at the level (5) very high.
In the independent test results obtained from the variant sig values. (2-tailed) $0.000 < 0.05$ so it can be concluded that the results of research conducted at the experiment and control class have differences after the application of research based learning, this is in accordance with research conducted by [16,17] where research based learning methods can be used assist students in meeting predetermined target values. The results of the research achieved by the experiment class show that research based learning objectives have an important role in solving mathematical problems in students.

5. Conclusions

The results of research that have been carried out in implementing research based learning obtained a significant influence on students' combinatorial thinking skills in the experiment class. This can be seen in the results of the post test in the experiment class showing an increase in students' scores and combinatorial thinking skills. Implementing research-based learning can help improve students' thinking skills, such as in the experimental class that got better grades because of implementing based learning. Based on the results that have been obtained, we recommend that future researchers implementation research based learning by developing different thinking skills in this study.

Acknowledgment

We thank a lot to the post-graduate department of Mathematics Education, Faculty of Teacher Training and Education, University of Jember, CEREBEL and CGANT research groups, as well as the reviewers who provided suggestions on this paper.

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