Equation of state at high densities and modern compact star observations

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Abstract. Recently, observations of compact stars have provided new data of high accuracy which put strong constraints on the high-density behaviour of the equation of state of strongly interacting matter otherwise not accessible in terrestrial laboratories. The evidence for neutron stars with high mass ($M = 2.1\pm0.2\,M_\odot$ for PSR J0751+1807) and large radii ($R > 12$ km for RX J1856-3754) rules out soft equations of state and has provoked a debate whether the occurrence of quark matter in compact stars can be excluded as well. In this contribution it is shown that modern quantum field theoretical approaches to quark matter including color superconductivity and a vector meanfield allow a microscopic description of hybrid stars which fulfill the new, strong constraints. The deconfinement transition in the resulting stiff hybrid equation of state is weakly first order so that signals of it have to be expected due to specific changes in transport properties governing the rotational and cooling evolution caused by the color superconductivity of quark matter. A similar conclusion holds for the investigation of quark deconfinement in future generations of nucleus-nucleus collision experiments at low temperatures and high baryon densities such as CBM @ FAIR.

1. Introduction: modern compact star observations

Compact stars provide natural laboratories for the exploration of baryonic matter at high densities, well exceeding in their centres the nuclear saturation density of $n_0 = 0.16\,\text{fm}^{-3}$, where nuclear matter properties can be calibrated in terrestrial experiments with atomic nuclei. Recently, results from observations of compact star properties have been reported which have the potential to provide serious constraints for the nuclear equation of state.
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(EoS), see [1] and references therein. In particular, the high mass of \( M = 2.1 \pm 0.2 \, M_\odot \) for the pulsar J0751+1807 in a neutron star - white dwarf binary system [2] and the large radius of \( R > 12 \, \text{km} \) for the isolated neutron star RX J1856.5-3754 (shorthand: RX J1856) [3] point to a stiff equation of state at high densities. Measurements of high masses are also reported for compact stars in low-mass X-ray binaries (LMXBs) as, e.g., \( M = 2.0 \pm 0.1 \, M_\odot \) for the compact object in 4U 1636-536 [4]. For another LMXB, EXO 0748-676, constraints for the mass \( M \geq 2.10 \pm 0.28 \, M_\odot \) and the radius \( R \geq 13.8 \pm 0.18 \, \text{km} \) have been reported [5]. The status of these data is, however, unclear since the observation of a gravitational redshift \( z = 0.35 \) in the X-ray burst spectra [6] could not be confirmed thereafter despite numerous attempts.

Measurements of rotation periods below \( \sim 1 \, \text{ms} \) as discussed for XTE J1739-285 [7], on the other hand, would disfavor too large objects corresponding to a stiff EoS and would thus leave only a tiny window of very massive stars in the mass-radius plane [8, 9] for a theory of compact star matter to fulfill all above mentioned constraints.

In the left panel of Fig. 1 we show these modern observational constraints for masses and mass-radius relationships together with solutions of the Tolman-Oppenheimer-Volkoff (TOV) equations for a set of eight hadronic EoS classified in three groups: (i) relativistic mean-field (RMF) approaches with non-linear (NL) self-interactions of the \( \sigma \) meson [10]. In NL\( \rho \) the isovector part of the interaction is described only by a \( \rho \) meson, while the set NL\( \rho \delta \) also includes a scalar isovector meson \( \delta \) that is usually neglected in RMF models [11]; (ii) RMF models with density dependent couplings and masses are represented here by four different models from two classes, where in the first one density dependent meson couplings are modeled so that a number of properties of finite nuclei (binding energies, charge and diffraction radii, surface thicknesses, neutron skin in \( ^{208}\text{Pb} \), spin-orbit splittings) can be fitted [12]. D\(^3\)C has in addition a derivative
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coupling leading to momentum-dependent nucleon self-energies and DD-F4 is modeled such that the flow constraint [13] from heavy-ion collisions is fulfilled. The second class of these models is motivated by the Brown-Rho scaling assumption [14] that not only the nucleon mass but also the meson masses should decrease with increasing density. In the KVR and KVOR models [15] these dependences were related to a nonlinear scaling function of the $\sigma$-meson field such that the EoS of symmetric nuclear matter and pure neutron matter below four times the saturation density coincide with those of the Urbana-Argonne group [16]. In this way the latter approach builds a bridge between the phenomenological RMF models and (iii) microscopic EoS built on realistic nucleon-nucleon forces. Besides the variational approaches (APR [16], WFF [17], FPS [18]) such ab-initio approaches to nuclear matter are provided, e.g., by the relativistic Dirac-Brueckner-Hartree-Fock (DBHF) [19] and the nonrelativistic Brueckner-Bethe-Goldstone [20] approaches. Stiff EoS like D$^3$C, DD-F4, BBG and DBHF fulfill the demanding constraints for a large radius and mass, while the softer ones like NL$\rho$ don’t. It is interesting to note that the flow constraint [13] shown in the right panel of Fig. 1 sets limits to the tolerable stiffness: it excludes the D$^3$C EoS and demonstrates that DD-F4, BBG and DBHF become too stiff at high densities above $\sim 0.55$ fm$^{-3}$. For a detailed discussion, see Ref. [1].

A key question asked in investigating the structure of matter at high densities is whether the phase transition to quark matter can occur inside compact stars. In Ref. [5], Özel has debated that the new constraints reported above would exclude quark matter in compact star interiors reasoning that it would entail an intolerable softening of the EoS. Alford et al. [21] have given a few counter examples demonstrating that quark matter cannot be excluded. In the following section we discuss a recently developed chiral quark model [22] which is in accord with the modern constraints, see also [23].

2. Phase transition to quark matter: masquerade problem

The thermodynamics of the deconfined quark matter phase is described within a three-flavor quark model of Nambu–Jona-Lasinio (NJL) type, with a mean-field thermodynamic potential given by

$$
\Omega_{MF}(T, \mu) = \frac{1}{8G_S} \left[ \sum_{i=u,d,s} (m_i^* - m_i)^2 - \frac{2}{\eta_N} (2\omega_0^2 + \phi_0^2) + \frac{2}{\eta_D} \sum_{A=2,5,7} |\Delta_{AA}|^2 \right] - \int \frac{d^3p}{(2\pi)^3} \sum_{a=1}^{18} \left[ E_a + 2T \ln \left( 1 + e^{-E_a/T} \right) \right] + \Omega_t - \Omega_0 .
$$

Here, $\Omega_0$ is the thermodynamic potential for electrons and muons, and the divergent term $\Omega_0$ is subtracted in order to assure zero pressure and energy density in vacuum ($T = \mu = 0$). The quasiparticle dispersion relations, $E_a(p)$, are the eigenvalues of the hermitean matrix

\[
\mathcal{M} = \begin{bmatrix}
-\vec{\gamma} \cdot \vec{p} - \hat{m}^* + \gamma^0 \hat{\mu}^* & i\gamma_5 C \tau_A \lambda_A \Delta_{AA} \\
-Ci\gamma_5 \tau_A \lambda_A \Delta_{AA}^* & -\vec{\gamma}^T \cdot \vec{p} + \hat{m}^* - \gamma^0 \hat{\mu}^*
\end{bmatrix},
\]
in color, flavor, Dirac, and Nambu-Gorkov space. Here, $\Delta_{AA}$ are the diquark gaps. $\hat{m}^*$ is the diagonal renormalized mass matrix and $\hat{\mu}^*$ the renormalized chemical potential matrix, 

$$
\hat{\mu}^* = \text{diag}(\mu_u - G_S \eta_V \omega_0, \mu_d - G_S \eta_V \omega_0, \mu_s - G_S \eta_V \phi_0)
$$

The gaps and the renormalized masses are determined by minimization of the mean-field thermodynamic potential [11]. We have to obey constraints of charge neutrality which depend on the application we consider. In the (approximately) isospin symmetric situation of a heavy-ion collision, the color charges are neutralized, while the electric charge in general is non-zero. For matter in $\beta$-equilibrium in compact stars, also the global electric charge neutrality has to be fulfilled. For further details, see [24, 25, 26, 27].

We consider $\eta_D$ as a free parameter of the quark matter model, to be tuned with the present phenomenological constraints on the high-density EoS. Similarly, the relation between the coupling in the scalar and vector meson channels, $\eta_V$, is considered as a free parameter of the model. The remaining degrees of freedom are fixed according to the NJL model parameterization in table I of [28], where a fit to low-energy phenomenological results has been made.

As a relativistic unified description of quark-hadron matter, naturally including a description of the phase transition, is not available yet, we apply here the so-called two-phase description, being aware of its limitations. For the description of the nuclear matter phase we choose the DBHF approach and the transition to the quark matter phase given above is obtained by a Maxwell construction. In the right panel of Fig. 2 it can be seen that the necessary softening of the high density EoS in accordance with the flow constraint is obtained for a vector coupling of $\eta_V = 0.5$ whereas an appropriate deconfinement density is obtained for a strong diquark coupling in the range $\eta_D = 1.02 - 1.03$. The resulting phase transition is weakly first order with an almost negligible density jump. Applying this hybrid EoS with so defined free parameters under compact star conditions a sequence of hybrid star configurations is obtained which fulfills all modern constraints, see the left panel of Fig. 2. In that figure we also indicate the minimal mass $M_{DU}$ for which the central density reaches a value allowing the fast direct Urca (DU) cooling process in DBHF neutron star matter to occur, leading to problems with cooling phenomenology [29,30]. Note that for a strong diquark coupling $\eta_D = 1.03$, the critical density for quark deconfinement is low enough to prevent the hadronic direct Urca (DU) cooling problem by an early onset of quark matter. For the given hybrid EoS, there is a long sequence of stable hybrid stars with two-flavor superconducting (2SC) quark matter, before the occurrence of the strange quark flavor and the simultaneous on set of the color-flavor-locking (CFL) phase renders the star gravitationally unstable. Comparing the hybrid star sequences with the purely hadronic DBHF ones one can conclude that the former ‘masquerade’ themselves as neutron stars [31] by having very similar mechanical properties. To unmask the neutron star interior might therefore require observables based on transport properties, strongly modified from normal due to the color superconductivity. It has been suggested to base tests of the structure of matter at high densities on analyses of the cooling behavior [30,32,33] or the stability of fastly rotating stars against r-modes [34,35]. It has turned out that for these phenomena
the fine tuning of color superconductivity in quark matter is an essential ingredient.

3. Conclusions

The evidence for neutron stars with high mass ($M = 2.1 \pm 0.2 \, M_\odot$ for PSR J0751+1807) and large radii ($R > 12 \, \text{km}$ for RX J1856-3754) rules out soft equations of state and has provoked a debate whether the occurrence of quark matter in compact stars can be excluded as well. In this contribution it is shown that modern quantum field theoretical approaches to quark matter including color superconductivity and a vector meanfield allow a microscopic description of hybrid stars which fulfill the new, strong constraints. The deconfinement transition in the resulting stiff hybrid equation of state is weakly first order so that signals of it have to be expected due to specific changes in transport properties governing the rotational and cooling evolution caused by the color superconductivity of quark matter. A similar conclusion holds for the investigation of quark deconfinement in future generations of nucleus-nucleus collision experiments at low temperatures and high baryon densities [36], such as CBM @ FAIR.

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