Transformation optical designs for wave collimators, flat lenses and right-angle bends

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Abstract. The transformation optics technique is applied to design three novel devices—a wave collimator, far-zone and near-zone focusing flat optical lenses and a right-angle bend for propagating beam fields. The structures presented in this paper are all two-dimensional (2D), however, the transformation optics design methodologies can be easily extended to develop 3D versions of these optical devices. The required values of the permittivity and the permeability tensors are derived for each of the three devices considered here. Furthermore, the functional performance of each device is verified using full-wave electromagnetic simulations. A wave collimator consists of a 2D rectangular cylinder where the fields (cylindrical waves) radiated by an embedded line source emerge normal to the top and bottom planar interfaces thereby producing highly directive collimated fields. Next, a far-zone focusing lens for a 2D line source is created by transforming the equi-amplitude equi-phase contour to a planar surface. It is also demonstrated that by aligning two far-zone focusing flat lenses in a back-to-back configuration, a near-zone focusing lens is obtained. Finally, a 2D square cylindrical volume is transformed into a cylinder with a fan-shaped cross section to design a right-angle bend device for propagating beam fields.
1. Introduction

The transformation electromagnetics/optics technique [1]–[3] provides electromagnetic/optical device designers with an unprecedented ability to manipulate and control the behaviour of wave phenomena. The device design is obtained via a spatial coordinate transformation from the original free space to the transformed space, where the compression and dilation of space in different coordinate directions are interpreted as appropriate scalings of the material parameters (i.e. the permittivity and the permeability tensors). This interpretation is possible due to the transform-invariant nature of Maxwell’s equations [4, 5] under such coordinate transformations.

The most widely studied transformation electromagnetics/optics device to date has been the electromagnetic/optical cloak of invisibility [3, 6, 7]. In a typical cloak design based on the transformation technique, a specific volume of free space is transformed into a shell-type region that has the same external boundary as the original volume. Ideally, any object located inside the cloak is not detectable by measurement at any position outside the cloak. Any incident external field is guided smoothly around the cloaked region such that the field after having emerged from the cloak is exactly the same as if it had just passed through free space. Many aspects of cloaks have been studied and reported since they were first introduced. An optical cloak design with a reduced material parameter set conceived to avoid the use of any magnetic materials in the optical regime was presented in [8] and nonlinear transformations were introduced for the radial coordinates in [9]. The effects of imperfections on cloaking performance were analysed in [10, 11]. Kwon and Werner [12, 13] utilize a spatial transformation technique to design an important class of two-dimensional (2D) electromagnetic cloaks that do not possess rotational symmetry around the longitudinal axis. Application of cloaks in the RF regime as effective shielding devices which enable improved antenna performance (input impedance and radiation pattern) in highly scattering multiple-antenna environments was suggested in [14]. The transformation technique was also used in [15] to design a 2D device that rotates the propagation direction by 90°. Rahm et al introduced designs for a square cloak and a cylindrical wave concentrator [16]. The transformation approach was also proposed for possible use in the design of acoustic cloaking shells [17, 18].

Employing the same principle of coordinate transformation, designs for a reflectionless beam shifter and a beam divider were presented in [19]. Unlike in cloak designs, discontinuities in the transformations along the domain boundaries were allowed. This class of transformations, known as ‘finite embedded transformations,’ provide a generalized framework for the development of novel optical device designs that may not be possible to obtain by using continuous transformations, such as have been employed in cloaks. By using a transformation...
that linearly shifts the space within a rectangular slab in the \( \hat{y} \)-direction, a narrow beam propagating in the \( +\hat{x} \)-direction was shown to be linearly translated while passing through the beam shifter. In addition, the use of separate shifting transformations, one shifting to the top half and the other shifting to the bottom half of a rectangular region, has led to a beam splitter design. For both of these designs, the interfaces at the entry to and exit from the device were found to be reflectionless.

In this paper, three novel optical devices based on the transformation optics design technique are presented. Firstly, a design is introduced in section 2 for a flat wave collimator which effectively converts cylindrical waves produced by an embedded line source to plane waves. For a line source radiating cylindrical waves, a circular cylindrical region is transformed into a rectangular slab such that plane waves emerge from the top and bottom faces of the slab. Next, the design of a far-zone focusing flat lens for a line source is presented in section 3, where a circular arc centred at the source location is transformed into a planar interface such that the original cylindrical phasefronts are converted into planar ones. Moreover, by concatenating two far-zone focusing lenses in a back-to-back configuration, a near-zone focusing lens is obtained. Section 4 presents a right-angle bend design that rotates the direction of propagation of an incident beam field by 90°. In this case, the appropriate transformation maps a square cylindrical region into a fan-shaped cylindrical region such that an electromagnetic beam propagating along a straight-line path in the original space is smoothly bent into a circular arc in the transformed space. Finally, section 5 summarizes the significant contributions of this paper.

2. Wave collimator

In transformation optics, the coordinate transformation is interpreted as anisotropic expansion and compression of the original space. In a transformation from the unprimed coordinates \((x_1, x_2, x_3) = (x, y, z)\) in free space to the primed coordinates \((x'_1, x'_2, x'_3) = (x', y', z')\), the permittivity tensor \(\epsilon'\) and the permeability tensor \(\mu'\) in the \((x'_1, x'_2, x'_3)\) system are written as [4, 5]

\[
\epsilon' = \mu' = \frac{\mathbf{A A}^\text{T}}{\det(\mathbf{A})},
\]

where the elements of the matrix \(\mathbf{A}\) are given by

\[
A_{ij} = \frac{\partial x'_i}{\partial x_j}, \quad i, j = 1, 2, 3.
\]

To create a wave collimator for a line source embedded inside, consider the spatial transformation described in figure 1, where the cross sectional geometries in the \(x-y\)-plane are shown for 2D structures that are infinite and have no variation in the \(\pm z\)-directions. This mapping transforms the circular domain of \(\rho = \sqrt{x^2 + y^2} \leq a\) in figure 1(a) into the rectangular domain described by \(|x'| \leq w, |y'| \leq l\) in figure 1(b). When a line source is located at the coordinate origin in the original space, the contours of equal phase and equal amplitude for the radiated fields are a family of concentric circles centred at the origin, shown as the green dashed lines in figure 1(a). If a circle centred at the origin is transformed into planar interfaces in the new system, the equivalent source along these interfaces will radiate in-phase and create propagating waves with planar phasefronts.

There are infinitely many transformations that can be invoked to achieve this particular mapping. In [20], Jiang et al employed a transformation that maps a circular cylindrical domain
to a square cylindrical domain as the basis for a cylindrical-to-plane-wave conversion device. An embedded line source was shown to radiate four distinct beams in the far zone of the device. A different coordinate transformation from a circular to a rectangular domain presented in [21] was also employed to achieve directive radiation patterns normal to a planar conformal surface for an embedded point source. In this study, we choose the following transformation

\[ x' = \frac{wx}{a}, \quad (3a) \]
\[ y' = \frac{ly}{\sqrt{a^2 - x'^2}}, \quad (3b) \]
\[ z' = z, \quad (3c) \]

for the region \( \rho \leq a \). The original circular geometry is stretched linearly in the \( \pm \hat{x} \)-direction from the range \( |x| \leq a \) to \( |x'| \leq w \) according to \((3a)\). In the \( \pm \hat{y} \)-directions, the coordinates are stretched linearly with respect to \( y \) such that the circle defined by \( \rho = a \) is mapped to the straight line segments given by \( |x'| \leq w, y' = \pm l \). Finally, there is no compression, expansion or translation applied in the \( \pm \hat{z} \)-directions.

Using \((3a)-(3c)\) in \((1)\) and \((2)\), one obtains the material parameters of the wave collimator in terms of the transformed system coordinates as

\[ \epsilon'_{xx} = \frac{\sqrt{w^2 - x'^2}}{l}, \quad (4a) \]
\[ \epsilon'_{yy} = \frac{\epsilon'_{xx} x' y'}{l \sqrt{w^2 - x'^2}}, \quad (4b) \]
\[ \epsilon'_{yy} = \frac{x'^2 y^2}{l (w^2 - x'^2)^{3/2}} + \frac{l}{\sqrt{w^2 - x'^2}}, \quad (4c) \]
\[ \epsilon'_{zz} = \frac{a^2 \sqrt{w^2 - x'^2}}{w^2 l}. \quad (4d) \]
Figure 2. The material parameters of the wave collimator design: (a) \( \epsilon'_{xx} \), (b) \( \epsilon'_{xy} \), (c) \( \epsilon'_{yy} \) and (d) \( \epsilon'_{zz} \). The geometry of the collimator design is specified by \( a = 0.4 \text{ m}, l = 0.05 \text{ m}, \) and \( w = 0.4 \text{ m} \).

Figure 3. Snapshots of the total electric field distribution due to a line source located at the coordinate origin: (a) the line source radiating in free space and (b) the line source embedded in the wave collimator.

and \( \epsilon'_{xz} = \epsilon'_{zx} = \epsilon'_{yz} = \epsilon'_{zy} = 0 \). For an example transformation with the geometrical parameter values given by \( a = 0.4 \text{ m}, l = 0.05 \text{ m} \) and \( w = 0.4 \text{ m} \), the material parameters of the transformation optical wave collimator are plotted in figure 2. The values of \( \epsilon'_{xx} \) and \( \epsilon'_{zz} \) in figures 2(a) and (d) range from 0 to \( w/l \) and from 0 to \( a^2/wl \), respectively, where both maximum values are equal to 8 for the design example considered here. Theoretically, the range of \( \epsilon'_{xy} \) varies from \(-\infty\) to \( \infty\) as a function of position, but only values that lie between \(-10\) and \(10\) are considered in figure 2(b). The permittivity tensor element \( \epsilon'_{yy} \) takes on values from \( l/w \) to \( \infty \) in figure 2(c). The reason for the infinite values associated with \( \epsilon'_{xy} \) and \( \epsilon'_{yy} \) is that isolated points corresponding to \( x = \pm a, y = 0 \) in the original space are stretched and mapped into a finite line segment \( x' = \pm w, |y'| \leq l \) in the transformed space.

Figure 3 demonstrates the performance of the wave collimator via numerical simulations. Snapshots of the \( \hat{z} \)-directed total electric field are shown for a time-harmonic electric line
Figure 4. Spatial coordinate transformation used for the 2D far-zone focusing lens design: (a) the original coordinate system and (b) the transformed system.

source at 3 GHz radiating at the coordinate origin in free space as well as for the same line source embedded inside the collimator in figures 3(a) and (b), respectively. At the frequency of operation, the collimator measures $8\lambda \times 1\lambda$ in terms of the free space wavelength $\lambda$. The field distributions are obtained using the commercial electromagnetic simulation package COMSOL Multiphysics, which is based on a finite element analysis technique. In figure 3(a), the line source radiates cylindrical waves into free space. The boundary of the flat collimator to be applied is outlined in black. When the source is embedded inside the flat collimator, waves with planar equi-phase surfaces emerge from the top and bottom faces of the collimator as demonstrated in figure 3(b), therefore producing highly collimated beams in the $\pm \hat{y}$-directions.

It is noted that the phasefront contours of the waves emerging from the collimator are not perfectly planar and several small ripples can be observed in figure 3(b). These ripples are attributed to diffraction from the edges of the slab at $x' = \pm w$ and an imperfect impedance match at the boundary between the collimator and free space. Nevertheless, the performance of the collimator is remarkably good. It is anticipated that the application of other transformations may lead to improved designs which exhibit a reduced amount of phasefront irregularities. Such designs are currently under investigation.

3. Flat lenses for far-zone and near-zone focusing

In section 2, a transformation optical device was demonstrated that effectively converts cylindrical waves, produced by an embedded line source, to plane waves. The same design principle can be applied to produce highly directive radiation for cases where a source is located outside the optical device. Consider a line source radiating at the coordinate origin in the original coordinate system shown in figure 4(a). In addition, figure 4(a) shows the cross-sectional area of a semi-circular region having radius $a$ centred at the origin, which is specified by $\rho \leq a$, $y \geq g$. If the arc portion of the original boundary, which is a contour of equal amplitude and equal phase for the radiated cylindrical field, is transformed into a planar interface, then the plane waves will emerge and an image of the source will form in the far field. One such transformation is given by

$$x' = x,$$  \hspace{1cm} (5a)

$$y' = \frac{1}{\sqrt{a^2 - x^2 - g}} (y - g) + g,$$  \hspace{1cm} (5b)

$$z' = z.$$  \hspace{1cm} (5c)

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Figure 5. The material parameters of the far-zone focusing lens: (a) $\epsilon'_{xx}$, (b) $\epsilon'_{xy}$, (c) $\epsilon'_{yy}$ and (d) $\epsilon'_{zz}$. The geometry of the design is specified by $w = 0.4$ m, $g = 0.05$ m and $l = 0.1$ m.

The widths of the original and the transformed regions remain the same at $2w$, where $w = \sqrt{a^2 - g^2}$. Moreover, the thickness of the device is equal to $l$. For this device, the original space is stretched or compressed only in the $y'$-direction. It can be easily seen from (5b) that the arc $\sqrt{x^2 + y^2} = a$ in the original space is mapped to the line $y' = g + l$ in the transformed space. The straight portion of the boundary $|x| \leq w$, $y = g$ remains unchanged through the transformation.

The material parameters of the device obtained by using (5a)–(5c) in conjunction with (1) and (2) are found to be

\begin{align}
\epsilon'_{xx} &= \frac{\sqrt{a^2 - x'^2} - g}{l}, \\
\epsilon'_{xy} &= \epsilon'_{yx} = \frac{x'(y' - g)}{l\sqrt{a^2 - x'^2}}, \\
\epsilon'_{yy} &= \frac{1}{\sqrt{a^2 - x'^2} - g} \left[ \frac{x'^2(y' - g)^2}{l(a^2 - x'^2)} + l \right], \\
\epsilon'_{zz} &= \frac{\sqrt{a^2 - x'^2} - g}{l}
\end{align}

and all other elements of $\epsilon'$ are equal to zero. For a far-zone focusing lens design with dimensions $w = 0.4$ m, $g = 0.05$ m, and $l = 0.1$ m, the material parameters are plotted over the cross section of the rectangular cylindrical device in Figure 5. Both $\epsilon'_{xx}$ and $\epsilon'_{zz}$ are functions of $x'$ only, and their values range from zero at $x' = \pm w$ to $(a - g)/l$ at $x' = 0$, which is equal to 3.5 for the design example considered here. Unlike in the case of the wave collimator, the values of $\epsilon'_{xy} = \epsilon'_{yx}$ for the flat lens design do not diverge because the numerator in (6b) never becomes zero. The minimum and the maximum values are $\pm w/g$ realized at $x' = \pm w$ and $y' = g + l$. On the other hand, the range of values required for $\epsilon'_{yy}$ is from zero at $x' = 0$ to infinity at $x' = \pm w$. 

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Figure 6. Snapshots of the total electric field distribution due to an electric line source located at the origin: (a) the line source radiating in free space and (b) the line source radiating in the presence of the far-zone focusing lens.

Full-wave simulation results for the $\hat{z}$-directed total electric field distribution due to a line source radiating at 3 GHz are compared in figure 6 for two configurations. Figure 6(a) shows the free space radiation case, whereas figure 6(b) shows how the radiation is modified when a far-zone focusing lens with material parameters described by (6a)–(6d) is placed in front of the source. It is observed that the portion of the cylindrical incident field which is captured by the lens is effectively converted into plane waves.

Considering that the far-zone focusing lens converts diverging cylindrical waves into planar waves, the reverse operation should also hold true due to reciprocity. In other words, the same lens under a plane wave illumination at normal incidence should form an image at the focal point, which corresponds to the location of the line source in the original radiation problem. This is indeed supported by the COMSOL simulations (results not shown). The field plot under the aforementioned plane wave scattering case reveals a very similar result to that shown in figure 6(b).

The above observation lets us design a flat lens that forms an image of the source close to the lens interface on the opposite side in the near zone. Specifically, if two far-zone focusing lenses are arranged in a back-to-back configuration in front of a source, the first lens will transform the diverging cylindrical waves into planar waves, and the second lens will convert the planar waves into converging cylindrical waves to form an image in the near field. The performance of the near-zone focusing lenses is shown in figure 7 for two different configurations. Two far-zone focusing lens designs of the type shown in figures 5 and 6 are employed in both configurations for the numerical experiment, and snapshots of the total electric field distributions are shown for a line source radiating at the coordinate origin.

In figure 7(a), the near-zone focusing lens is composed of two far-zone focusing lenses placed directly back-to-back. This figure clearly illustrates the phasefronts of the waves inside the device smoothly changing from diverging cylindrical waves, through planar phasefronts, to converging cylindrical waves to form an image in the near zone directly above the lens. Since the phasefront is planar once the cylindrical waves radiated by the line source pass through the first far-zone focusing lens, the distance between the two far-zone focusing lenses that comprise the near-zone focusing lens can be arbitrary in principle. In figure 7(b), the two far-zones focusing lenses are separated by 0.1 m, which is equal to one wavelength. A planar phasefront between...
Figure 7. Near-zone focusing lens designs realized by concatenating two far-zone focusing lenses in a back-to-back configuration: (a) two far-zone focusing lenses connected without separation, (b) two lenses with 0.1 m separation between them. An electric line source located at the coordinate origin radiates at 3 GHz.

Figure 8. Spatial coordinate transformation used for the 2D right-angle bend design: (a) the original coordinate system and (b) the transformed system.

the two far-zone focusing lenses can be clearly seen. The qualities of the images formed by the two different lens configurations considered in figure 7 are approximately the same.

4. Beam propagation through a right-angle bend

Novel designs for a parallel beam shifter and a beam divider were presented in [19] based on the application of finite embedded transformations. For an incoming narrow beam incident either normally or at an angle to the interface, it was shown that the entire beam could be translated in one direction or divided (i.e. split) into two beams with narrower widths. In this section, we apply the embedded transformation to a square cylindrical volume to rotate the direction of beam propagation by 90°. We start with the coordinate transformation illustrated in figure 8. A 2D cylinder with a square cross section of size $w \times w$ is shown in figure 8(a). This is transformed into a cylinder with a fan-shaped cross section as shown in figure 8(b).
Consequently, as a narrow beam propagates in the \(+\hat{x}\)-direction in the original system, the same beam entering the device in the transformed system will undergo a rotation through 90° and emerge propagating in the \(−\hat{y}'\)-direction.

The appropriate coordinate transformation may be written as

\[
\begin{align*}
\rho' &= y, \\
\phi' &= \frac{\pi}{2w}(w - x), \\
\z' &= z,
\end{align*}
\]  

(7a)–(7c)

where \(\rho' = \sqrt{x^2 + y^2}\) and \(\phi' = \tan^{-1}(y'/x')\). The Cartesian coordinates in the transformed systems can be retrieved from \(x' = \rho'\cos\phi'\) and \(y' = \rho'\sin\phi'\). Under the transformation given in (7a)–(7c), constant-\(x\) lines are mapped into constant-\(\phi'\) radial lines, and constant-\(y\) lines are mapped into constant-\(\rho'\) curves. The corresponding material parameters in the primed system are found from (1) to (2) as

\[
\begin{align*}
\epsilon'_{xx} &= \frac{\pi \rho'}{2w} \sin^2\phi' + \frac{2w}{\pi \rho'} \cos^2\phi', \\
\epsilon'_{xy} &= \epsilon'_{yx} = \sin\phi' \cos\phi' \left( \frac{2w}{\pi \rho'} - \frac{\pi \rho'}{2w} \right), \\
\epsilon'_{yy} &= \frac{\pi \rho'}{2w} \cos^2\phi' + \frac{2w}{\pi \rho'} \sin^2\phi', \\
\epsilon'_{zz} &= \frac{2w}{\pi \rho'}.
\end{align*}
\]  

(8a)–(8d)

The material parameters of the right-angle bend are plotted in figure 9 for an example design with \(w = 0.3\) m. All material parameters approach infinity at the origin because the line segment corresponding to \(0 \leq x \leq w\) and \(y = 0\) is mapped to a single point in the transformed system. The minimum values of \(\epsilon'_{xx}\) and \(\epsilon'_{yy}\) are zero at the origin when approached from the directions \(\phi' = \pi/2\) and 0, respectively, as shown in figures 9(a) and (c). The minimum value of \(\epsilon'_{yy}\) is equal to \((2/\pi - \pi/2)/2\) realized at \(\rho' = w\) and \(\phi' = \pi/4\). \(\epsilon'_{zz}\) is a function of \(\rho'\) only and its minimum value is equal to \(2/\pi\) at \(\rho' = w\).

To test the performance of the right-angle bend design, a Gaussian beam is employed as the incident field. A \(\hat{z}\)-directed 2D Gaussian beam incident field \(E' = \hat{z}E'_z(x', y')\) may be expressed as [22]

\[
E'_z(x', y') = \frac{W_0}{W(x')} e^{-\left[\frac{(x'-x_r)^2}{w(x')^2}\right]} e^{-j\left[k(x'-x_r) + \frac{2\pi(x'-x_r)^2}{\lambda w(x')} - \tan^{-1}\frac{y'}{x'}\right]},
\]  

(9)

where \(k = 2\pi/\lambda\) is the free-space wavenumber, and an \(\exp(j\omega t)\) time convention is assumed and suppressed. The radius of curvature \(R(x')\) and the spot size \(W(x')\) are given by

\[
R(x') = (x' - x_r) \left[ 1 + \left(\frac{x_0}{x'}\right)^2 \right],
\]  

(10)

\[
W(x') = W_0 \sqrt{1 + \left(\frac{x' - x_r}{x_0}\right)^2},
\]  

(11)

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Figure 9. The material parameters of the right-angle bend: (a) $\epsilon'_{xx}$, (b) $\epsilon'_{xy}$, (c) $\epsilon'_{yy}$ and (d) $\epsilon'_{zz}$. The side length of the device is specified by $w = 0.3$ m.

Figure 10. The right-angle bend design: (a) a snapshot of the $\hat{z}$-directed total electric field with a Gaussian beam incident on the device at a right angle and (b) normalized power flow.

The parameter $x_0$ is defined as $x_0 = kW_0^2/2$, where $W_0$ denotes the minimum waist of the beam. In addition, $(x_r, y_r)$ represents a reference point for the beam expression. In figure 10, a 3 GHz Gaussian beam propagating in the $+\hat{x}'$-direction is incident on the right-angle bend. The dimension of the device is given by $w = 0.3$ m, and the minimum waist is assumed to be $W_0 = 0.05$ m. Finally, the reference point for the beam is taken to be $(x_r, y_r) = (0.15$ m, $0.15$ m). In the COMSOL simulation, an impressed magnetic surface current was placed on the surface $x' = -0.3$ m in order to radiate the Gaussian beam into the simulation domain.

It is clearly observed in figure 10(a) that the propagation direction of the incident Gaussian beam is rotated by 90° in the $-\hat{\phi}'$-direction. There is no reflection at both the entry plane along $x' = 0$ and at the exit plane along $y' = 0$. Figure 10(b) shows the normalized power flow within
the simulation domain. The power density changes along the propagation trajectory due to the variable nature of the beam waist. Virtually all the incident power is redirected into the $-\hat{y}'$-direction.

5. Conclusion

The recently introduced transformation optics design principle was employed in this paper to develop three novel electromagnetic devices—a wave collimator, far-zone and near-zone focusing flat lenses and a right-angle bend. In the collimator and lens designs, circular contours of cylindrical waves are transformed into planar boundaries such that waves with planar phasefronts would emerge in the transformed space. A near-zone focusing lens was obtained by concatenating two far-zone focusing lenses in a back-to-back configuration. Finally, a 2D square cylinder was transformed into a fan-shaped cylinder to create a device capable of bending a propagating beam through a right angle with no reflection loss.

In this paper, only a few possibilities for novel optical device designs utilizing the transformation optics technique were presented. The unprecedented design flexibility provided by the transformation methodology promises to yield a wide variety of new devices that exhibit an unprecedented ability to manipulate and control electromagnetic/optical waves in ways not previously possible.

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