Off-axis magnetic fields of a circular loop and a solenoid for the electromagnetical induction of a magnetic pendulum

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Keywords: magnetic field, off-axis circular loop, off-axis solenoid, analytic solutions, mathematica simulation

Abstract
In this study, two types of approximate analytic functions for the off-axis magnetic field $\vec{B}(r, \theta)$ of a circular loop and a finite-length solenoid are presented. The derived analytic functions reduce to a well-known magnetic field formula with respect to the vertical axis of the circular loop and the solenoid when $\theta = 0$. In addition, we investigated two types of $\vec{B}(r, \theta)$ within the approximate conditions through a simulation performed using Wolfram Mathematica. The derived analytic functions can be used to determine the magnetic field $\vec{B}(r, \theta)$ at arbitrary points with large $r$ and small $\theta$ around a circular loop and a solenoid. They are helpful for investigating the electromagnetic induction that can be attributed to a magnet swinging over a coil or a solenoid.

1. Introduction

In a circular loop and a solenoid, a magnetic field is generated when a current flows through them. A refined formula is derived in university textbooks for obtaining the magnetic field on the central axis of a circular loop and a finite-length solenoid [1, 2]. In particular, the magnetic field on the central axis is uniform when the solenoid is infinitely long [1, 2]. A recent study considered a magnet as a solenoid and theoretically calculated the magnetic force between a magnet and a solenoid [3]. Although the magnetic fields produced by circular loops and solenoids have been studied in previous works [4–6], majority of the studies have investigated magnetic fields on the vertical central axis, namely, the $z$ axis of the cylindrical coordinate. Knowledge regarding the magnetic field at a point away from the solenoid axis is useful for determining the electromotive force induced on the solenoid by a magnet moving around the solenoid and the magnetic force between the magnet and the solenoid. Accordingly, theoretical formulas for magnetic fields on the off-axis of a solenoid are required. Previous studies have used the Biot–Savart law [7], magnetic scalar potential [8], or magnetic vector potential [9] to determine the off-axis magnetic field. However, the analytic function involving entirely elementary functions is not available because the formulas obtained in these studies contain 1st and 2nd elliptic integrals or Heuman lambda function. Therefore, off-axis magnetic fields of a solenoid cannot be estimated without the numerical calculating or high order approximation [7–9]. The analytic functions consisting of entirely elementary functions will be helpful for undergraduate students to evaluate the physical quantities. For instance, in calculating the magnetic force acting on a magnet traversing a current-flowing coil, the elementary functions for the magnetic field of the coil are more convenient than the functions involving elliptic integrals.

Recently, we derived some formulas for the magnetic force between a magnet and a solenoid based on the mutual inductance effect [10]. In this study, $\vec{B}(\rho, z)$ in the cylindrical coordinate was required for estimating the magnetic force between a magnet and solenoid because the center of the magnet position is on the vertical axis of a solenoid. $\vec{B}(\rho, z)$ is also useful for estimating the magnetic force acting on a magnet to oscillate vertically over a solenoid. In our previous study [11], approximate analytic functions consisting of entirely elementary functions were derived for the magnetic field at arbitrary points around a solenoid. In addition, the $\vec{B}(\rho, z)$ at arbitrary points around a solenoid were estimated via a simulation using Wolfram Mathematica within an approximate limit.

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Meanwhile, we introduced an experiment [12] for the electromagnetic induction law, using a magnetic pendulum as seen in figure 1. In this experiment, whenever a magnet traverses over a rectangular coil with $N$ turns, the waveform of the induction current as shown in figure 2 is observed. In this study, considering a relative motion between a magnet and a rectangular coil as illustrated in figure 3, we tried to estimate the induced voltage given by

$$\mathcal{E}(t) = -\frac{d\phi_{m}}{dt} = \oint_{\ell} N (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{\ell}$$  \hspace{1cm} (1)

where $\phi_{m}$ is the magnetic flux and $v_{p}$ is the velocity of the rectangular coil (in fact, the velocity of the magnet). In this case, the magnetic field $\mathbf{B}(r, \theta)$ of a magnet traversing the coil, expressed as

$$\mathbf{B}(r, \theta) = B_{r} \mathbf{\hat{r}} + B_{\theta} \mathbf{\hat{\theta}}$$  \hspace{1cm} (2)

is needed to estimate $\mathcal{E}(t)$. Therefore, $\mathcal{E}(t)$ is given by

$$\mathcal{E}(t) = N \oint_{\ell} [v_{p} \mathbf{\hat{\phi}} \times (B_{r} \mathbf{\hat{r}} + B_{\theta} \mathbf{\hat{\theta}})] \cdot d\mathbf{\ell}$$  \hspace{1cm} (3)

Thus, $v_{p}$ and $\mathbf{B}(r, \theta)$ are a prerequisite of estimating $\mathcal{E}(t)$. $v_{p}$ is well known is the textbook [13] as follows:

$$v_{p} = \sqrt{2gL (\cos \phi - \cos \phi_{0})}$$  \hspace{1cm} (4)

where $\ell$ is the length of the pendulum, $\phi$ and $\phi_{0}$ are the angle at a time and the initial angle, respectively.

But, $\mathbf{B}(r, \theta)$ of the magnet is not introduced in textbooks. A magnet can be considered as a solenoid [10]. The formulas for off-axis magnetic fields of a solenoid contains the special functions such as 1st, the 2nd elliptic integrals and Heuman lambda functions [7–9]. It is impossible to estimate $\mathcal{E}(t)$, without numerical calculations, substituting these special functions with $B_{r}$ and $B_{\theta}$ in equation (3). Moreover, for the undergraduate students making an experiment for the electromagnetic induction law, more simpler formulas without special functions for $B_{r}$ and $B_{\theta}$ will be needed.

Thus, the functions with analytic solutions for off-axis magnetic fields $\mathbf{B}(r, \theta)$can be useful to study the electromagnetic induction that can be attributed to a magnet swinging over a coil or a solenoid. Currently, we have no simple and analytic $\mathbf{B}(r, \theta)$ that can be considered for studying the magnetic field effects such as voltage pulse via electromagnetic induction [14]. Therefore, we obtained two types of approximate analytic functions $\mathbf{B}(r, \theta)$ not involving special functions for off-axis magnetic fields of a circular loop and a solenoid in this study.
The theoretical approach used to calculate $B_r(r, \theta)$ at an arbitrary point of the off-axis of a circular loop and solenoid is as follows. First, based on the results of our previous work \cite{11}, we present two types of approximate analytic functions $B_r(r, \theta)$ for a circular loop and subsequently integrating them, approximate analytic functions $B_r(r, \theta)$ of a solenoid. Second, the derived magnetic field formula was validated by comparing the magnetic fields on the central $z$ axis obtained from the formula with those obtained for the $z$ axis using a renowned magnetic field formula. Finally, a simulation was performed using Wolfram Mathematica to calculate

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure2.png}
\caption{The waveform of the induced voltage as function of time \cite{12}.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure3.png}
\caption{A simple pendulum motion based on the relative motion between a magnet and a rectangular coil \cite{12}.}
\end{figure}
the magnetic fields $\vec{B}(r, \theta)$ at arbitrary points around a circular loop and a solenoid for comparing two types of approximate analytic functions.

2.1. Magnetic fields $\vec{B}(r, \theta)$ at arbitrary points $(r, \theta)$ around a circular loop

The vector potential $A$ of a circular loop is expressed as [11]

$$A_r(r, \theta) = \frac{\mu_0 I}{4} \left( \frac{a^2 r \sin \theta}{(r^2 + a^2 + 2ar \sin \theta)^{3/2}} \right)$$

(5)

under the conditions $r \gg a, a \gg r$, or $\sin \theta \approx 0$. The conditions $r \gg a, a \gg r$, or $\sin \theta \approx 0$ are used for the approximation of the 1st and the 2nd elliptic integrals involved in $A_r(r, \theta)$ [15]. It is note that the approximate conditions generally hold good for the system (figure 1) considered in this study.

Based on the relation $\vec{B} = \nabla \times \vec{A}$, the magnetic field components $B_r$, $B_\theta$, and $B_\phi$ of the magnetic field $\vec{B}$ in spherical coordinates can be expressed as [11]

$$B_r(r, \theta) = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_r(r, \theta))$$

$$= \frac{\mu_0 I}{4} a^2 \cos \theta \frac{(2r^2 + 2a^2 + ar \sin \theta)}{(r^2 + a^2 + 2ar \sin \theta)^{5/2}},$$

(6)

$$B_\theta(r, \theta) = -\frac{1}{r} \frac{\partial}{\partial r} (r A_r(r, \theta))$$

$$= \frac{\mu_0 I}{4} a^2 \sin \theta \frac{(r^2 - 2a^2 - ar \sin \theta)}{(r^2 + a^2 + 2ar \sin \theta)^{5/2}},$$

(7)

$$B_\phi = 0.$$  

(8)

If the solenoid is considered to be a combination of circular loops, integrating equations (6) and (7) along the $z$ axis yields the off-axis magnetic field of a solenoid.

Meanwhile, in case of $ar \sin \theta \approx 0$, equations (6) and (7) can be reduced to more simplified forms as follows:

$$B_r(r, \theta) = \frac{\mu_0 I}{2} a^2 \cos \theta \frac{1}{(r^2 + a^2)^{3/2}}$$

(9)

$$B_\theta(r, \theta) = \frac{\mu_0 I}{4} a^2 \sin \theta \frac{(r^2 - 2a^2)}{(r^2 + a^2)^{5/2}}$$

(10)

Equations (9) and (10) are also derived from earlier work when $ar \sin \theta \approx 0$ [15].

On the vertical axis ($\theta = 0, z = r \cos \theta = r, \rho = r \sin \theta = 0$), we obtain the following equation from equations (6) and (7) or equations (9) and (10).

$$B_z = \frac{\mu_0 I}{2} a^2 \frac{1}{(z^2 + a^2)^{3/2}}$$

(11)

$$B_\phi = 0.$$  

(12)

These are renowned formulas for obtaining the magnetic field on the vertical central axis of a single circular loop [16].

Within approximate limits, we will compare equations (6) and (7) with equations (9) and (10) because simpler formulas are required as approximate analytic functions. Moreover, as will be seen later, when we derive $B_r(r, \theta)$ and $B_\theta(r, \theta)$ for a solenoid by integrating equations (6) and (7), considerably complicated expressions are obtained because of the final term $2ar \sin \theta$ in the denominators.

2.2. Magnetic fields $\vec{B}(r, \theta)$ at arbitrary points $(r, \theta)$ around a solenoid

Figure 4 presents the magnetic field $\vec{B}(r, \theta)$ at an arbitrary point $(r, \theta)$ around a solenoid with $N$ turns. $\ell$ denotes the distance from the center of the solenoid to the field source. Because for small $\theta$, $r^\prime + \Delta \approx r + \Delta$ by substituting $r^\prime + \Delta = r + \ell \cos \theta$ and $\frac{N}{L} I d\ell (\equiv n I d\ell)$ into equations (6) and (7), we obtain [11]

$$dB_r(r, \theta) = \frac{\mu_0 n I}{4} a^2 \cos \theta \frac{[2(r + \ell \cos \theta)^2 + a(r + \ell \cos \theta) \sin \theta + 2a^2]}{[(r + \ell \cos \theta)^2 + 2a(r + \ell \cos \theta) \sin \theta + a^2]^{3/2}} d\ell$$

(13)
By replacing $r + \ell \cos \theta \equiv u$ and integrating equations (13) and (14) from $-L/2$ to $L/2$, we can derive

$$B_r(r, \theta) = \frac{\mu_0 n I}{4} [P + Q + R],$$

where

$$P = \left[ \frac{2p(2 \sin^2 \theta - 1) + 2a \sin \theta \cos \theta + 2(1 + \sin^2 \theta)(p + a \sin \theta)}{3 \cos^2 \theta s^{3/2}} \right] - \left[ \frac{2q(2 \sin^2 \theta - 1) + 2a \sin \theta \cos \theta + 2(1 + \sin^2 \theta)(q + a \sin \theta)}{3 \cos^2 \theta t^{1/2}} \right].$$

$$Q = \sin \theta \left[ \left( \frac{p \sin \theta + a}{3 \cos^2 \theta s^{3/2}} \right) + \frac{a \sin \theta (p + a \sin \theta)}{3a^2 \cos^3 \theta s^{1/2}} \right] + \left[ \frac{q \sin \theta (q + a \sin \theta)}{3 \cos^2 \theta t^{3/2}} + \frac{a \sin \theta (q + a \sin \theta)}{3a^2 \cos^4 \theta t^{1/2}} \right],$$

$$R = \left[ \frac{2(p + a \sin \theta)}{3 \cos^2 \theta s^{1/2}} \right] - \left[ \frac{2(q + a \sin \theta)}{3 \cos^2 \theta t^{1/2}} \right] - \left[ \frac{1}{2} \right].$$

Here, $p, q, s$, and $t$ denote $r + \frac{L}{2} \cos \theta$, $r - \frac{L}{2} \cos \theta$, $p^2 + 2pa \sin \theta + a^2$, and $q^2 + 2qa \sin \theta + a^2$, respectively. Equations (16), (17), and (18) are the revised expressions for $P$, $Q$, and $R$ in our previous work [11].

For the vertical central axis ($\theta = 0$, $z = r \cos \theta = r$, $\rho = r \sin \theta = 0$) of the solenoid, equation (15) is reduced to

$$B_r(r, \theta) \Rightarrow B_z(z, 0) = \frac{\mu_0 n I}{4} \left[ (P)_{r=x, \theta=0} + (Q)_{r=x, \theta=0} + (R)_{r=x, \theta=0} \right] = \frac{\mu_0 n I}{2} \left[ \frac{z + \frac{L}{2}}{\left( z + \frac{L}{2} \right)^2 + a^2} - \frac{z - \frac{L}{2}}{\left( z - \frac{L}{2} \right)^2 + a^2} \right].$$

which is a renowned formula for obtaining the magnetic field on the vertical central axis of a solenoid [16].

Similarly, equation (14) can be integrated using the integral table to obtain

$$B_\theta(r, \theta) = \frac{\mu_0 n I}{4} \tan \theta \left[ \frac{P}{2} - Q - R \right].$$

On the vertical central axis ($\theta = 0$, $z = r \cos \theta = r$, $\rho = r \sin \theta = 0$), equation (20) becomes 0 because $\tan \theta = 0$. 

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**Figure 4.** Magnetic field $\vec{B}(r, \theta)$ at arbitrary points $B(r, \theta)$ around a solenoid with $N$ turns [11].
which is well known in case of solenoids [16].

Using equations (15) and (20), we can obtain the magnetic field \( \vec{B}(r, \theta) \) as follows:

\[
\vec{B}(r, \theta) = B_r(r, \theta) \hat{r} + B_\theta(r, \theta) \hat{\theta},
\]

(22)

In equation (22), even if \( B_r(r, \theta) \) and \( B_\theta(r, \theta) \) are approximate analytic functions, they have considerably complicated expressions given by equations (15) and (20). Let’s obtain simpler \( B_r(r, \theta) \) and \( B_\theta(r, \theta) \) by integrating equations (9) and (10).

By substituting \( r + \Delta = r + \epsilon \cos \theta \) and \( N L d\epsilon = n I d\xi \) into equations (9) and (10), we obtain

\[
dB_r(r, \theta) = \frac{\mu_0 n I}{2} a^2 \cos \theta \left[ \frac{r + \frac{L}{2} \cos \theta}{\left( \left( r + \frac{L}{2} \cos \theta \right)^2 + a^2 \right)^{3/2}} - \frac{r - \frac{L}{2} \cos \theta}{\left( \left( r - \frac{L}{2} \cos \theta \right)^2 + a^2 \right)^{3/2}} \right],
\]

(23)

\[
dB_\theta(r, \theta) = \frac{\mu_0 n I}{4} \tan \theta \left[ \frac{r - \frac{L}{2} \cos \theta}{\left( \left( r - \frac{L}{2} \cos \theta \right)^2 + a^2 \right)^{3/2}} - \frac{r + \frac{L}{2} \cos \theta}{\left( \left( r + \frac{L}{2} \cos \theta \right)^2 + a^2 \right)^{3/2}} \right].
\]

(24)

By replacing \( r + \epsilon \cos \theta \equiv u \) and integrating equations (23) and (24) from \(-L/2 \) to \( L/2 \), we can easily derive

\[
B_r(r, \theta) = \frac{\mu_0 n I}{2} \left[ \frac{r + \frac{L}{2} \cos \theta}{\left( \left( r + \frac{L}{2} \cos \theta \right)^2 + a^2 \right)^{3/2}} - \frac{r - \frac{L}{2} \cos \theta}{\left( \left( r - \frac{L}{2} \cos \theta \right)^2 + a^2 \right)^{3/2}} \right],
\]

(25)

and

\[
B_\theta(r, \theta) = \frac{\mu_0 n I \tan \theta}{4} \left[ \frac{r - \frac{L}{2} \cos \theta}{\left( \left( r - \frac{L}{2} \cos \theta \right)^2 + a^2 \right)^{3/2}} - \frac{r + \frac{L}{2} \cos \theta}{\left( \left( r + \frac{L}{2} \cos \theta \right)^2 + a^2 \right)^{3/2}} \right].
\]

(26)

For the vertical central axis \( (\theta = 0, \ z = r \cos \theta = r, \ \rho = r \sin \theta = 0) \), equations (25) and (26) reduce the well-known equations (19) and (21), respectively.

Because equations (25) and (26) are quite simple compared with equations (15) and (20), we better use equations (25) and (26) as approximate analytic functions if they are valid for \( \vec{B}(r, \theta) \). Therefore, a comparison between equations (15) and (20) with equations (25) and (26) under approximate conditions is prerequisite. This is the major work conducted in this study.

3. Results and discussion

Under the approximation conditions \( r \gg a, \ a \ll r, \) or \( \sin \theta \approx 0, \) we compared equation (6) with equation (9) as well as equation (7) with equation (10) because we need simpler formulas to use them as approximate analytic functions for a circular loop. Similarly, we compared equations (15) and (20) with equations (25) and (26) under the same approximate conditions. For this, we performed a simulation using Wolfram Mathematica to determine \( B_r(r, \theta) \) and \( B_\theta(r, \theta) \) off-axes of a solenoid. The radius, length, number of turns, and applied current for a solenoid used in the simulation were \( a = 2.5 \) cm, \( L = 30 \) cm, \( N = 600, \) and \( I = 1 \) A, respectively. Before estimating \( B_r(r, \theta) \) and \( B_\theta(r, \theta) \) for a solenoid, we estimate \( B_r(r, \theta) \) and \( B_\theta(r, \theta) \) for a circular loop with \( a = 2.5 \) cm and \( L = 1 \) A using Wolfram Mathematica.

Figure 5 represents \( B_r(r) \) of a circular loop as functions of \( r \) for \( \theta = \pi/50 \) and \( \pi/20. \) As seen in figure 5, for both \( \theta = \pi/50 \) and \( \theta = \pi/20, \) \( B_r(r) \) estimated by equations (6) and (9) shows no difference in strength at small \( r \) and almost the same values at large \( r. \) This tendency appears prominently in case of small \( \theta. \) This means that equation (9) is another approximate function holding good for large \( r \) and small \( \theta. \)

Figure 6 illustrates \( B_\theta(r) \) of a circular loop as functions of \( r \) for \( \theta = \pi/50 \) and \( \pi/20. \) As shown in figure 6, for both \( \theta = \pi/50 \) and \( \theta = \pi/20, \) \( B_\theta(r) \) estimated by equations (7) and (10) shows almost the same values, especially at large \( r. \) This tendency can be clearly observed in case of small \( \theta. \) Thus, equation (10) is another approximate function holding good for large \( r \) and small \( \theta. \)

Figure 7 represents \( B_r(r) \) of a solenoid as a function of \( r \) for \( \theta = 0, \ \theta = \pi/50, \) and \( \pi/20. \) As shown in figure 7, \( B_r(r) \) estimated by equations (15) and (25) shows almost the same value as that obtained for \( \theta = \pi/50. \) \( B_r(r, \theta = \pi/50) \) shows almost the same curve as \( B_2(z) \) on the vertical axis of a solenoid. In figure 7, the curves corresponding to \( \theta = 0 \) represent \( B_2(z). \) This is a natural result because \( z = r \cos \theta \approx r \) in case of \( \theta \approx 0. \)
Even if \( B_r(r) \) shows a small difference for \( \theta = \pi/20 \), this discrepancy disappears at large \( r \). Therefore, we can use a simple equation (25) instead of a very complicated equation (15) at large \( r \) and small \( \theta \).

Figure 8 represents \( B_r(\theta) \) of a solenoid as functions of \( \theta \) for \( r = a/10 \) and \( 10a \). As seen in figure 8, \( B_r(\theta) \) estimated by equations (15) and (25) shows almost the same values for \( r = 10a \). Even if \( B_r(\theta) \) shows a minor
difference in strength for large $\theta$ in case of $r = a/10$, this discrepancy disappears for small $\theta$. Therefore, a simpler equation (25) can be used instead of the complicated equation (15) for large $r$ and small $\theta$.

Figure 9 shows $B_\theta(r)$ of a solenoid as function of $r$ for $\theta = \pi/50$ and $\pi/20$. Similar to $B_\phi(r)$, as represented in figure 9, $B_\theta(r)$ values estimated by equations (20) and (26) show almost the same values for $\theta = \pi/50$. Even though $B_\theta(r)$ shows a small difference in strength for $\theta = \pi/20$, this discrepancy also disappears at large $r$. Therefore, a simpler equation (25) can be used instead of the complicated equation (20) at large $r$ and small $\theta$.

Figure 10 illustrates $B_\phi(\theta)$ of a solenoid as function of $\theta$ for $r = a/10$ and $10a$. Similar to $B_\phi(r)$, $B_\phi(\theta)$ estimated by equations (20) and (26) shows almost the same values for $r = 10a$. Even though $B_\phi(\theta)$ shows a minor difference in strength for large $\theta$ in case of $r = a/10$, this discrepancy disappears for small $\theta$. Therefore, a simpler equation (26) can be used instead of the complicated equation (20) for large $r$ and small $\theta$.

Thus, we present two types of useful analytical functions that can be used for determining the magnetic field $\vec{B}(r, \theta)$ at arbitrary points around a circular loop and a solenoid under the approximate conditions $r \gg a$, $a \gg r$, or $\sin \theta \approx 0$. Equations (6), (7), (9), (10) and (15), (20), (25), (26) are helpful to estimate the electromagnetic force acting on a magnet traversing a coil or a solenoid and to investigate the electromagnetic induction that can be attributed to a magnet swinging over a coil or a solenoid. Because equations (6) and (9), equations (7) and (10), equations (15) and (25), and equations (20) and (26) provide almost the same magnetic strengths, especially for large $r$ and small $\theta$, we can use simpler equations (9), (10), (25), and (26) to estimate $\vec{B}(r, \theta)$ in this region. Therefore, we can estimate $\dot{E}(r)$ of equation (1) using equations (9), (10), (25), and (26) without numerical calculations.
4. Conclusion

In this study, we present two types of approximate analytic functions for the magnetic field of a circular loop and a solenoid at arbitrary points under the approximate conditions $r \gg a$, $a \gg r$, or $\sin \theta \approx 0$. The derived functions of magnetic fields reduce to the well-known magnetic field formulas on the $z$ axis when $q = 0$, indicating the validity of the derived analytic functions. For both a circular loop and a solenoid, two types of approximate analytic functions $B_r(r, \theta)$ and $B_\theta(r, \theta)$ show a minor difference in strength. However, because the difference is negligible at large $r$ and small $\theta$, we can use simpler equations (9), (10), (25), and (26) to estimate $B(r, \theta)$ in this region.

In conclusion, we present limited but simple analytic functions for determining the magnetic field at arbitrary points $(r, \theta)$ in a circular loop and solenoid. The approximate analytic functions $B_r(r, \theta)$ and $B_\theta(r, \theta)$ that are newly obtained in this work consist of entirely elementary functions and are expected to be useful for undergraduate students to study electromagnetic induction that can be attributed to a permanent magnet traversing or swinging over a coil and a solenoid. Equations (9), (10), (25), and (26) obtained in this work is also used to estimate the magnetic forces acting on a magnet traversing a solenoid or magnetic forces and torques between two magnets placed on off axis. In the near future, we plan to conduct a study on a theoretical explanation for the experimental waveforms of a magnetic pendulum swinging over a coil or a solenoid, using equations (9), (10), (25), and (26).

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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