Possible cosmological implications of a time varying fine structure constant.

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Abstract

Webb et al. \cite{2} experimental results on the fine structure “constant” variation with the age of the Universe, is here analyzed. By using the experimental data on the fine-structure “constant”’s variation with the age of the Universe, and Dirac’s LNH (Large Number Hypothesis), we find how should vary the total number of nucleons in the Universe, the speed of light, Newton’s gravitational “constant” and the energy density, and we make an estimate on the deceleration parameter, finding that the Universe would be accelerating, just as Supernovae observations have concluded.

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POSSIBLE COSMOLOGICAL IMPLICATIONS OF A TIME VARYING FINE STRUCTURE CONSTANT.
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In the two landmarking letters, Webb et al. [1] and Webb et al. [2] have provided experimental data on quasars that span 23\% to 87\% of the age of the Universe, finding a variation in the fine structure constant, given by $\Delta \alpha / \alpha \cong -0.72 \times 10^{-5}$. We shall show that this result is coherent with other cosmological facts.

One can ask whether this variation could be caused by a varying speed of light. Gomide [18] has long ago studied cosmological models with varying $c$, and/or varying $\varepsilon_0$. Dirac’s LNH (Large Numbers Hypothesis) [9][10][11] was included in his framework. Barrow [6] and Barrow and Magueijo [19] have studied how a time varying $c$ would explain both, the $\dot{\alpha} \neq 0$ and the Supernovae observations. We refer to their papers for further information. Our framework will presently be different from those references.

If we perceive the present Universe as having constant deceleration parameter, $q$, like, for instance, Einstein-de Sitter’s Universe, where $q = 1/2 = \text{const.}$, we may use Berman’s[3][4] formula for the Hubble’s parameter,

$$H = \frac{1}{mt} = \frac{1}{1 + q} t^{-1}$$

where

$$H = \frac{\dot{R}}{R}$$

and

$$q = -\frac{\ddot{R}R}{R^2}$$

Overdots are for time-derivatives, and $R$ is the scale-factor in Robertson Walker’s metric,

$$ds^2 = dt^2 - \frac{R^2(t)}{(1 + kr^2)^2}(dx^2 + dy^2 + dz^2)$$

where $k = 0, \pm 1$ is the tricurvature.

We may now express Webb’s et al.’s[2] experimental result as:

$$\left( \frac{\dot{\alpha}}{\alpha} \right)_{\text{exp}} \simeq -1.1 \times 10^{-5} H (1 + q)$$

We define
\[ \alpha \equiv \frac{e^2}{\hbar c} \]  

(6)

and we shall suppose that the time-variation of \( \alpha \) is caused by a varying speed of light, as in Barrow’s paper’s[5][6][7].

From (6) we find,

\[ \frac{\dot{\alpha}}{\alpha} = -\frac{\dot{c}}{c}. \]  

(7)

If we suppose that the speed of light varies with a power-law of time, say:

\[ c = At^n \]  

(8)  

\( (A = \text{const.}) \); we find, from the above experimental values,

\[ n = 1.1 \times 10^{-5}. \]  

(9)

We see that the speed of light varies slowly with the age of the Universe, as Berman and Trevisan [8] have shown elsewhere, for another model.

Now, let us consider Dirac’s large number’s hypothesis[9][10][11]. The number of nucleons in the Universe is called \( N \), and we find roughly that both the ratio of the Hubble’s length and the classical electronic radius and the ratio of electrostatic and gravitational interactions between a proton and a electron are of order \( \sqrt{N} \), with \( N \sim 10^{80} \), i.e.

\[ \frac{cH^{-1}}{m_ec^2} \approx \sqrt{N} \]  

(10)

\[ \frac{e^2}{Gm_p m_e} \approx \sqrt{N} \]  

(11)

\[ \frac{\rho(cH^{-1})^3}{m_p} \approx N \]  

(12)

Dirac’s LNH can be presented by the assumption that \( N \) varies with the age of the Universe, so that a present “large” number \( N \) only means that the Universe is “old”.

When we plug law (1) into (10)-(11)-(12), we find that,

\[ \rho \propto t^{-2} \sqrt{N} \]  

(13)

\[ G \propto (N)^{-1} \]  

(14)
\[ c \propto (N)^{\frac{1}{2}} t^{-\frac{1}{3}} \]  

In order to accommodate (15) with (8) and (9), we assume that

\[ N \propto t^{2.0001} \]  

and, then,

\[ G \propto t^{-1.0005} \]  

\[ \rho \propto t^{-0.99995} \]  

This “solves” Dirac’s LNH, for the experimental found fine-structure “constant’s” time-variation. We find \( \dot{G}/G \simeq 1.0t^{-1} \simeq 1.0H(1 + q) \) while it has been found under Lunar laser ranging and Viking radar measurements by Williams et al. [12] and Reasenberg [13], that \( \dot{G}/G = \sigma H \) with \( |\sigma| < 0.6 \). Will [14][15] comments that these two kinds of measurements give the best limits on \( \sigma \)[20]. This means that \( -0.4 > q > -1.6 \). A negative \( q \) is a good result because of Supernovae observations [16]. We have thus shown that \( \Delta \alpha/\alpha \) should really be negative, for a positive result could mean a positive deceleration parameter. As a bonus, we found how \( N, G, \rho \), and the speed of light may vary with the age of the Universe, in order to be in accordance with LNH and Webb et al’s results.

In the model presented here, \( \alpha = \alpha(t) \), because \( c = c(t) \). It is important to remark that the electric permittivity of the vacuum, as well as its magnetic permeability, and Planck’s constant, are thought really constants; other possibility was devised by Berman and Trevisan [17]. It is important to elaborate different models and make some predictions in order to decide among them. In fact a Superunification theory will only survive in case that such evolution of the constants values with the age of the Universe will be explained by this theory.

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