Lambda Polarization in Peripheral Heavy Ion Collisions

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We predict the polarization of \(\Lambda\) and \(\bar{\Lambda}\) hyperons in peripheral heavy ion collisions at ultrarelativistic energy, based on the assumption of local thermodynamical equilibrium at freeze-out. The polarization vector is proportional to the curl of the inverse temperature four-vector field and its length, of the order of percents, is maximal for particle with moderately high momentum lying on the reaction plane. A selective measurement of these particles could make \(\Lambda\) polarization detectable.

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I. INTRODUCTION

In peripheral high energy heavy ion collisions the system has a large angular momentum \([1]\). It has been recently shown in hydrodynamical computation that this leads to a large shear and vorticity \([2]\). When the Quark-Gluon Plasma (QGP) is formed with low viscosity \([3]\), interesting new phenomena may occur like rotation \([4]\), Helmholtz instability (KHI) \([5, 6]\), or other turbulent phenomena \([7]\). Furthermore, the large angular momentum may manifest itself in the polarization of secondary produced particles \([1, 8, 9]\). Recently, a formula for the \(\Pi\) may manifest itself in the polarization of secondary phenomena \([7]\). Furthermore, the large angular momentum may manifest itself in the polarization of secondary produced particles \([1, 8, 9]\). Recently, a formula for the polarization of weakly interacting particles with spin 1/2 at local thermodynamical equilibrium has been found in Ref. \([10]\) based on the extension of the Cooper-Frye formula to particles with spin. Provided that spin degrees of freedom equilibrate locally, the polarization turns out to be proportional to the vorticity of the inverse temperature four-vector field and can thus be predicted in a full hydrodynamical calculation of the collision process ended by the Cooper-Frye freeze-out prescription.

Early measurements of the \(\Lambda\) hyperon polarization \([11]\), averaged over a significantly large centrality range, indicated relatively small values, with an upper bound \(|P_{\Lambda,\bar{\Lambda}}| \leq 0.02\) averaging over all azimuthal angles of \(\Lambda\) momentum. In this paper, we present a quantitative prediction of the \(\Lambda,\bar{\Lambda}\) polarization, within a specific hydrodynamical calculation, at different centralities and its momentum dependence. At top RHIC energy \((\sqrt{s_{NN}} = 200\text{ GeV})\), although the resulting polarization is of the order of 1-2\% on average, thus consistent with experimental bounds, it turns out to be the largest (around \(\approx 1-2\%\)) \(\Lambda\) polarization has been approached with different models (e.g. \([8, 13]\)). Recently, Ref. \([14]\) has considered the local polarization of fermions in the plasma phase induced by the chiral anomaly, thus far with an unspecified transferring mechanism to final hadrons. We stress that in our approach the polarization of the observable hadrons is a consequence of the paradigm of local thermodynamical equilibrium; to be effective, the chiral anomaly should induce a modification of the velocity and temperature fields at the freeze-out.

II. POLARIZATION

The \(\Lambda\) polarization in the participant centre-of-mass frame, as a function of its momentum, reads (in units \(c = K = 1\) \([10]\))

\[
\Pi_\mu(p) = \frac{\hbar c}{8m} \int \frac{d\Sigma_\lambda p^\lambda n_F(1 - n_F)\partial^\sigma \beta^\sigma}{d\Sigma_\lambda p^\lambda n_F},
\]

where \(\beta^\mu(x) = (1/T(x))\omega^\mu(x)\) is the inverse temperature four-vector field, \(n_F\) is the Fermi-Jüttner distribution of the \(\Lambda\), that is \(1/(e^{\beta(x)\cdot p - \xi(x)} + 1)\), being \(\xi(x) = \mu(x)/T(x)\) with \(\mu\) the relevant \(\Lambda\) chemical potential and \(p\) its four-momentum. Because at the temperatures typical of freeze-out \(\Lambda\) is quite dilute \((m_\Lambda \gg T)\), the Pauli blocking factor, \((1 - n_F)\), can be neglected in Eq. \([1]\).

very same formula, with the replacement \( \xi \rightarrow -\xi \) applies to \( \Lambda \), namely particles and antiparticles have the same polarization in the Boltzmann approximation.\(^1\)

The polarization vector is then proportional to the antisymmetric part of the gradient of the inverse temperature field, henceforth defined as thermal vorticity:

\[
\varpi^{\mu \nu} = \frac{1}{2}(\partial^\nu \beta^\mu - \partial^\mu \beta^\nu) \tag{2}
\]

The spatial part of the polarization vector \([1]\) gives rise to three terms:

\[
\Pi(p) = \frac{\hbar}{8m} \int \frac{d\Sigma_{p^\lambda} n_F (\nabla \times \beta)}{dV n_F} + \frac{\hbar \hat{p}}{8m} \times \frac{\int d\Sigma_{p^\lambda} n_F (\partial_\nu \beta + \nabla \beta^\nu)}{dV n_F} . \tag{3}
\]

The last two terms on the right hand side, involving polar vectors, should vanish because of the overall parity invariance (achieved combining symmetry by reflection with respect to the reaction plane of the two colliding nuclei and invariance by rotation of \( \pi \) around the axis orthogonal to the reaction plane). On the other hand, the first term, involving the spatial average of the curl of the \( \beta \) field, which is an axial vector, is not ought to vanish; in fact it is a vector aligned with the total angular momentum direction, which is orthogonal to the reaction plane (see Fig. 1). It should be pointed out that these formulae apply to primary particles emitted from a locally equilibrated source. Secondary \( \Lambda \)s emitted from these formulae apply to primary particles emitted from a locally equilibrated source. Secondary \( \Lambda \)s emitted from either strong or weak decays - most likely - will have a lower polarization inherited from their parent particles.

In the simplest scenario of an isochronous (\( t = \text{const.} \)) freeze-out at a given stage of the fluid dynamical expansion, according to Cooper-Frye prescription \( d\Sigma_{p^\lambda} \rightarrow dV \varepsilon, \varepsilon = p^0 \) being the \( \Lambda \)'s energy. In this case, the above formula simplifies to:

\[
\Pi(p) = \frac{\hbar}{8m} \int \frac{dV n_F (\nabla \times \beta)}{dV n_F} . \tag{4}
\]

The \( \Lambda \) polarization is usually determined by measuring the angular distribution of the decay protons, which, in the \( \Lambda \) rest frame is given by:

\[
\frac{1}{N} \frac{dN}{d\Omega} = \frac{1}{4\pi} (1 + \alpha \Pi_0 \cdot \hat{p}^*)
\]

where \( \alpha = 0.647 \), \( \Pi_0 \) is the polarization vector and \( \hat{p}^* \) is the direction of the decay proton, both in the \( \Lambda \)'s rest frame. The vector \( \Pi_0 \) can thus be obtained by Lorentz boosting to this frame the one in Eq. \([3]\):

\[
\Pi_0(p) = \Pi(p) - \frac{p}{\varepsilon(\varepsilon + m)} \Pi(p) \cdot p \tag{5}
\]

where \((\varepsilon, p)\) is \( \Lambda \) four-momentum and \( m \) its mass. One can readily realize that \( |\Pi_0| \leq |\Pi| \) and equality is achieved only if either \( p = 0 \) (non-relativistic limit) or when \( \mathbf{p} \cdot \Pi = 0 \). In both cases one has \( \Pi_0(p) = \Pi(p) \).

The above finding implies that maximal proper polarization of \( \Lambda \) with finite momentum is achieved when they are transversely polarized. Thus, if \( \Pi \) is directed along the total angular momentum (\(-y \) direction in Fig. 1), \( \Lambda \)'s having maximal polarization are those with momentum in the reaction plane or those with vanishing polar angle \( \theta \) (normally undetectable) and, in this case, their proper polarization vector is aligned with the total angular momentum.

\[\text{FIG. 1. (Color online) Sketch of a peripheral heavy ion collisions at high energy. The } \Lambda \text{ polarization points essentially into the direction of the total angular momentum (\(-y \) of the interaction region, orthogonal to the reaction plane. As with the largest polarization are emitted into the (xz) reaction plane.}\]

III. HYDRODYNAMICAL CALCULATION

The goal of the hydrodynamic calculation is to evaluate the thermal vorticity \([2]\) at the freeze-out. In this work we calculate it by using the Particle in Cell (PIC) fluid dynamic model, which provides us with the space-time development of the flow of the QGP. The freeze-out is enforced by means of the Cooper-Frye prescription at a fixed laboratory time \( t \), such that the average temperature is \( \approx 180 \text{ MeV} \) (see below). In comparison with Ref. \([2]\), only the relativistic case is considered.

For computational purposes, it is convenient to absorb the \( \hbar \) constant into \( \hat{\beta}^\mu \) and redefine thermal vorticity as:

\[
\varpi^{\mu \nu} = \frac{1}{2}(\partial^\nu \hat{\beta}^\mu - \partial^\mu \hat{\beta}^\nu), \tag{6}
\]

where \( \hat{\beta}^\mu \equiv \hbar \beta^\mu \). Thereby, \( \varpi \) becomes dimensionless. Note that in the thermal vorticity definition there is no projection of the derivatives transverse to the flow (the operator \( \nabla_\mu = \partial_\mu - u_\mu u_\nu \partial^\nu \)), unlike in the usual definition of the vorticity of the four-velocity field.
We present in Fig. 2 the $zx$ component of the thermal vorticity weighted with the energy density in the cell (that is $Ω_{zx}(\text{cell}) = ω_{zx}(\text{cell})/\epsilon_{\text{cell}}$) when the likewise weighted average temperature is 180 MeV, hence close to the freeze-out. The weighting with the energy density of the cell is described in detail in Ref. [2].

From Fig. 2 it can be seen, that at the last time step presented, in the reaction plane we have already an extended area occupied by matter. In case of peripheral reactions the multiplicity is relatively small, hence fluctuations in the reaction plane are considerable. In the relativistic case the outer edges show larger vorticity and random fluctuations are still strong. The average vorticity is smaller for the smaller impact parameters and it has positive value in the center and negative value at the edges.

It should be pointed out that while the standard velocity field vorticity rapidly decreases with expansion [2], thermal vorticity decrease is much slower and at some peripheral points it even increases. This is due to the fact that the matter cools during the expansion, so the temperature in the denominator of $\beta_\mu$ decreases compensating for the decrease of velocity field vorticity with time.

One should also mention that, in our calculation, hydrodynamical evolution starts after a dynamical longitudinal expansion based on collective Yang-Mills dynamics. The initial longitudinal size of the system is about $2 \times 4$ fm, so the hydro process starts $\approx 4$ fm/c after the penetration of the two Lorentz contracted nuclei. Consequently the configuration in Fig. 2 follows the interpenetration by about 8.5 fm/c, which is the time at which the energy density weighted average temperature is 180 MeV (see above).

![FIG. 2. (Color online) The energy density weighted thermal vorticity, $Ω_{zx}(x, z)$ of the inverse temperature four-vector field $β_\mu$ (see text for definition) calculated for all $[x-z]$ layers at $t=4.75$ fm/c, corresponding to an energy density weighted temperature of 180 MeV. The collision energy is $\sqrt{s_{NN}} = 200$ GeV, $b = 0.7 b_{\text{max}}$. The cell size is $dx = dy = dz = 0.4375$ fm, while the average weighted vorticity is $⟨Ω_{zx}⟩ = 0.0453$.](image-url)
and antiparticle states, a non-vanishing \( \bar{\Lambda} \) polarization. Since our predicted polarization applies to both particle and antiparticle states, a non-vanishing \( \bar{\Lambda} \) polarization is consistent with zero. Notice that in NN collisions only \( \Lambda \) are found to be polarized whereas \( \bar{\Lambda} \)'s have a polarization consistent with zero.

We figure out that in NN collisions only \( \Lambda \)'s are found to be polarized whereas \( \bar{\Lambda} \)'s have a polarization consistent with zero. Since our predicted polarization applies to both particle and antiparticle states, a non-vanishing \( \Lambda \) polarization would be free from this background. Nevertheless, we figure out that the NN background can be neglected also for \( \Lambda \) particle. Indeed, experimental observations show that \( \Lambda \)'s polarization scales with \( x_F \equiv 2p/\sqrt{s} \) [10], being \( p \) its momentum in the NN centre-of-mass frame and that its magnitude strongly increases with \( x_F \) [17]. At very low \( x_F \), where our calculation is performed (with \( y < 1 \) and \( p_T \) up to 6 GeV, at the LHC energy scale of 1 TeV we have \( x_F \simeq 0.07 \)), the observed trend [18] indicates an approximate (generous) maximal polarization of 5% for \( p_T \) up to few GeVs. In order to estimate the impact of this background on the hydrodynamically originated polarization, one should estimate the number of single NN collisions in the corona as a function of the number of participants nucleons \( N_P \) in peripheral nuclear collisions. A calculation carried out by one of the authors [19] with Glauber Monte-Carlo model at \( \sqrt{s_{NN}} = 200 \) GeV shows that for peripheral collisions with \( N_P \simeq 100 \) the number of nucleons undergoing single collisions in the corona is \( N_{PC} \simeq 30 \). According to STAR measurement [20], for \( N_P \simeq 100 \), at midrapidity the \( \Lambda \) multiplicity is approximately 3.6 \( N_P \) times the one in pp collisions at the same energy. Therefore, the fraction of \( \Lambda \) coming from NN collisions with respect to the total production at \( N_P = 100 \) can be estimated to be (see also eq. (2) in ref. [19]) \( (N_{PC}/2)/(3.6N_P) = 15/360 \simeq 0.042 \). This implies that at top RHIC energy, and even more so at LHC energy where the fraction of corona collisions is lower, at most only about 4% of \( \Lambda \) hyperons come from NN collisions, and that their contribution to the measured polarization, at very low \( x_F \), is at most \( 0.04 \times 0.05 = 0.002 \), far below the signal level.

\section{Conclusions}

In conclusion, we have predicted the polarization of \( \Lambda \) hyperons in relativistic heavy ion collisions at RHIC energy and its momentum dependence. Our calculation did not include the polarization of secondary \( \Lambda \)'s from decays of resonances or \( \Xi \)s which, most likely, will tend to dilute the signal. Still, the polarization value may reach sizeable and detectable values of several percents for momenta of some GeV’s directed along the reaction plane. While the average value is predicted to be of the order of 1-2%, in agreement with the experimental bound previously set at RHIC with about \( 10^7 \) minimum bias Au-Au events [11], with the much larger statistics (at least a factor of 30) collected by RHIC in later runs [21] the momentum differential measurement of \( \Lambda \) and \( \Lambda \) polarization in the direction along the reaction plane and at the participant c.m. should be feasible. We are also going to carry out similar calculations for the larger LHC energy.

The observation of a polarization arising from this thermo-mechanical effect of equipartition of angular momentum and in agreement with the predicted kinematic features would be a striking confirmation of the achievement of local thermodynamical equilibrium (for the spin degrees of freedom too) of the matter created in relativistic heavy ion collisions. It would also indicate that significant vorticity and circulation predicted in [4] may persist up to the freeze-out.
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