Cosmological perturbations in a generalized gravity including tachyonic condensation

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We present unified ways of handling the cosmological perturbations in a class of gravity theory covered by a general action in eq. (1). This gravity includes our previous generalized $f(\phi, R)$ gravity and the gravity theory motivated by the tachyonic condensation. We present general prescription to derive the power spectra generated from vacuum quantum fluctuations in the slow-roll inflation era. An application is made to a slow-roll inflation based on the tachyonic condensation with an exponential potential.

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I. INTRODUCTION

In our present paradigm of physical cosmology, the observed large-scale cosmic structures and the anisotropies of the cosmic microwave background (CMB) are regarded as small deviations from the spatially homogeneous and isotropic Friedmann world model $\mathbb{R}$. In such a paradigm the structures in the large-scale limit and in the early stage of the evolution are assumed to be linear deviations from the background world model $\mathbb{R}$. Although the observations are consistent with the perturbed Friedmann world model, these, however, do not necessarily constrain the underlying gravity theory (and the matter content) to be the Einstein one. Generalized forms of gravity appear in variety of situations involving the quantum aspects of the gravity theory and the low energy limits of the unified theories of gravity with other fundamental forces. Thus, it is likely that the early stages of the universe were governed by the gravity more general than Einstein one.

We have been studying the cosmological perturbations in the so called $f(\phi, R)$ gravity theory which includes diverse generalized gravity theories known in the literature as cases. In this work, motivated by the recent interests on the action based on the tachyonic condensation $\mathbb{R}$, and also by a previous study in the context of “k-inflation” $\mathbb{R}$, we extend our study to a more general form of gravity presented in eq. (1). Section II presents the classical evolutions in a unified form. Section III presents the quantum generation process and the generated power spectra under the slow-roll assumption and others. Section IV is an application a tachyonic slow-roll inflation. We set $c \equiv 1 \equiv \hbar$.

II. GRAVITY

We consider an action

$$S = \int d^4 x \sqrt{-g} \left[ \frac{1}{2} f(R, \phi, X) + L_m \right],$$

where $X \equiv \frac{1}{2} \dot{\phi}^2 \phi, c$, and $f$ is a general algebraic function of $R, \phi$ and $X$. This action includes the following gravity theories as cases. (1) A minimally coupled scalar field: $f = \frac{1}{8\pi G} R - 2X - 2V(\phi)$. (2) $f(\phi, R)$ gravity: $f = \tilde{f}(\phi, R) - 2\omega(\phi)X - 2V(\phi)$. (3) $p(\phi, X)$ gravity: $f = \frac{1}{8\pi G} R + 2p(\phi, X)$. (4) Tachyonic condensation: $f = \frac{1}{8\pi G} R - 2V(\phi)\sqrt{1 + 2X}$.

The gravitational field equation and the equation of motion become

$$G_{ab} = \frac{1}{F} \left[ T_{ab}^{(m)} + \frac{1}{2} (f - FR) g_{ab} + F_{a:c} g_{b:c} \right] - \frac{1}{2} f, \cdot X, a, \phi, b \cdot \equiv 8\pi G T_{ab},$$

$$f, \cdot X, \cdot c, \cdot = f, \cdot \phi,$$

$$T_{(m)a:b}^{(m)} = 0,$$

where $F = f, \cdot \phi$. $T_{a:b}$ is the effective energy-momentum tensor, and $T_{(m)a:b}^{(m)}$ is the energy-momentum tensor of additional matters.

III. CLASSICAL PERTURBATIONS

We consider the Friedmann background with the scalar- and the tensor-type perturbations. Our metric convention follows Bardeen’s $\mathbb{R}$

$$ds^2 = -a^2 (1 + 2\alpha) \eta^2 - 2a^2 \beta, \cdot \eta^\alpha dx^\alpha + a^2 \left[ g^{(3)}_{\alpha, \beta} (1 + 2\varphi) + 2T_{\gamma, \alpha; \beta} + 2C_{\alpha, \beta} \right] dx^\alpha dx^\beta.$$  \hspace{1cm} (5)

The energy-momentum tensor is decomposed as

$$T_0^0 = - (\bar{\rho} + \delta \rho), \hspace{0.5cm} T_0^\alpha = - (\mu + \rho) v, \cdot \alpha / k,$$

$$T_\beta^\alpha = (\bar{p} + \delta p) \delta_\beta^\alpha + \left( \frac{1}{k^2} \nabla^{(3)}_\alpha \nabla^{(3)}_\beta + \frac{1}{3} \delta_\beta^\alpha \right) \pi^{(3)} + \pi_\beta^\alpha.$$  \hspace{1.0cm} (6)

A vertical bar $|$ and $\nabla^{(3)}_\alpha$ are the covariant derivatives based on $g^{(3)}_{\alpha, \beta}$.

To the background order, eq. (2) gives

$$\frac{1}{8\pi G} R - 2V(\phi)\sqrt{1 + 2X} = 0,$$

$$\frac{1}{8\pi G} \delta R + 2\omega(\phi) \sqrt{1 + 2X} = 0,$$

$$\frac{1}{8\pi G} \delta R - 2V(\phi) \sqrt{1 + 2X} = 0,$$
\( H^2 = \frac{8\pi G}{3} \mu - \frac{K}{a^2}, \quad \dot{H} = -4\pi G (\mu + p) + \frac{K}{a^2}, \) \( \tag{7} \)

where \( H \equiv \frac{\dot{a}}{a} \) and an overdot denotes a time derivative based on \( t \) with \( dt \equiv ad\eta \). We also have \( R = 6(2H^2 + \dot{H} + \frac{K}{a^2}) \). The effective fluid quantities are

\[
8\pi G \mu = \frac{1}{F} \left[ \mu^{(m)} - \frac{1}{2} (f - FR) - \frac{1}{2} f \cdot \dot{\phi}^2 - 3HF \right], \quad 8\pi G p = \frac{1}{F} \left[ p^{(m)} + \frac{1}{2} (f - FR) + \dot{F} + 2HF \right], \quad \tag{8}
\]

where we have \( X = -\frac{1}{2} \dot{\phi}^2 \). To the background order eq. \( \mathcal{B} \) gives

\[
\frac{1}{a^3} \left( a^3 f \cdot \phi \right)' + f \cdot \phi = 0. \quad \tag{9}
\]

Perturbed set of equations can be derived similarly. The perturbed set of equations in Einstein gravity based on our convention in eqs. \( \mathcal{B} \) is presented in \( \mathcal{B}^* \). These equations are valid even in our gravity theory if we reinterpret the fluid quantities as the effective ones. The perturbed order effective fluid quantities can be easily read by comparing eq. \( \mathcal{B} \) with eq. \( \mathcal{B} \).

For the scalar-type perturbation we ignore the presence of additional fluid, thus \( T^{(m)}_{\mu \nu} = 0 \). In the following we consider two general situations: (i) \( F = F(\phi) \) and \( K = 0 \), and (ii) \( F = \frac{1}{a^2} \), but general \( K \). We introduce the Field-Shepley combination \( \mathcal{B} \)

\[
\Phi \equiv \varphi \delta \phi - \frac{K/a^2}{4\pi G(\mu + p)} \varphi_\chi, \quad \tag{10}
\]

where \( \varphi_\delta \equiv \varphi - (H/\dot{\phi}) \delta \phi, \quad \varphi_\chi \equiv \varphi - \dot{H} \chi \), \( \chi = a(\beta + \alpha \gamma) \) is a spatially gauge-invariant combination \( \mathcal{B} \).

(i) In the first case, perturbed parts of eq. \( \mathcal{B} \) can be combined to give a closed form of second-order differential equation for \( \varphi_\delta \mathcal{B} \)

\[
\frac{1}{a^3 Q} \left( a^3 Q \varphi_\delta \right)' + c_A^2 k^2 a^2 \varphi_\delta = 0, \quad \tag{12}
\]

\( Q \equiv \frac{3\dot{c}_A^2}{4} + f \cdot X + 2f \cdot XX^2 \equiv \frac{\dot{\phi}^2}{H^2} Z, \)

\( c_A^2 \equiv \left( 1 + \frac{2f \cdot XX^2}{3\dot{c}_A^2 + f \cdot XX} \right)^{-1} \).

For \( f \cdot X = -2\omega(\dot{\phi}) \) we recover the result derived in the \( f(\phi, R) \) gravity theory \([1] \).

(ii) In the second case, perturbed parts of eq. \( \mathcal{B} \) can be combined to give

\[
\Phi = \frac{H^2}{4\pi G(\mu + p) a} \left( \frac{a}{H} \varphi_\chi \right)' \quad \tag{14}
\]

\[
\Phi = -\frac{H c_A^2 k^2}{4\pi G a^2} \varphi_\chi, \quad \tag{15}
\]

where \( \mu + p = -\frac{1}{2} f \cdot \dot{\phi}^2 = f \cdot X, \) and

\[
c_A^2 \equiv c_X^2 - \frac{c_A^2 k^2}{\mu + p}, \quad \tag{16}
\]

Equations \( \mathcal{B}, (14,15) \) were derived by Garriga and Mukhanov; see eqs. \( (21,22) \) in \( \mathcal{B} \). Equations \( \mathcal{B}, (14,15) \) can be combined to give

\[
\frac{1}{a^3 Q} \left( a^3 Q \varphi_\delta \right)' + c_A^2 k^2 a^2 \varphi_\delta = 0, \quad \tag{17}
\]

\[
\frac{\mu + p}{H} \left[ \frac{H^2}{(\mu + p) a} \left( \frac{a}{H} \varphi_\chi \right)' \right] + c_A^2 k^2 a^2 \varphi_\chi = 0. \quad \tag{18}
\]

Using

\[
v \equiv z \Phi, \quad u \equiv \frac{\varphi_\chi}{\sqrt{\mu + p}}, \quad z \equiv a\sqrt{Q} = \frac{1}{c_A} \tilde{z}, \quad \tag{19}
\]

eqqs. \( \mathcal{B}, (14,15) \) become the well known equations \( \mathcal{B}, (14,15) \)

\[
u'' + \left( c_A^2 k^2 - \frac{u''}{z}\right) v = 0, \quad \tag{20}
\]

\[
u'' + \left( c_A^2 k^2 - \frac{(1/\tilde{z})''}{1/\tilde{z}}\right) u = 0, \quad \tag{21}
\]

where a prime indicates a time derivative based on \( \eta \). Equation \( \mathcal{B}, (14,15) \) is valid for the first case in eq. \( \mathcal{B}, (12) \) as well.

In the large-scale limit, with \( z''/z \gg c_A^2 k^2 \) and \( \tilde{z}(1/\tilde{z})'' \gg c_A^2 k^2 \), we have exact solutions

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\* See eqs. (43)-(50) in \( \mathcal{B} \). We have \( \epsilon \equiv \delta \mu, \pi = \delta p, \Psi \equiv -\dot{\Psi}(\mu + p)v, \) and \( \sigma = \frac{a^2}{\mu p}(\dot{\psi}) \).

\† See the paragraph containing eq. (36) in \( \mathcal{B} \).

\‡ \( \varphi_\delta \) is the same \( \varphi \) in the uniform-field gauge \( (\delta \phi \equiv 0) \) \([1] \).

\§ \( \varphi_\chi \) is the same as \( \phi \) in the zero-shear gauge \( (\chi \equiv 0) \) \([1] \), and is the same as \( \Phi_H \) which is often called the Bardeen potential \([2] \).

\# The procedure is exactly the same as the one used to derive eqs. \( (32,33) \) in \( \mathcal{B} \).
\[ \Phi = C(x) - D(x) \int_0^t \frac{dt}{a(x)^2}, \]  
(22)

\[ \varphi_x = 4\pi G \frac{H}{a} \left[ C(x) \int_0^t \frac{z^2}{a} dt + \frac{1}{k^2} D(x) \right]. \]  
(23)

Ignoring the transient solution (which is the D-mode in expanding phases) we have a temporally conserved behavior for \( \Phi \)

\[ \Phi(x, t) = C(x), \]  
(24)

For the tensor-type perturbation, for the general action in eq. (14), we have

\[ \ddot{C}_\beta + \left( 3H + \frac{\dot{F}}{F} \right) \dot{C}_\beta + \frac{k^2 + 2K}{a^2} C_\beta = \frac{1}{F} \pi^{(m) a}_{\beta}, \]  
(25)

which is the same as eq. (111) in [9] based on \( f(\phi, R) \) gravity. Thus, the presence of general algebraic complication of \( X \) in eq. (14) has no effect on the tensor-type perturbation. Also, eq. (23) can be written as in eqs. (17, 22). In such cases we have \( \Phi = F_{\beta}, \) \( Q = F \equiv Z/(8\pi G), \) \( c_A^2 = 1, \) thus \( z = a/\sqrt{F}, \) and eqs. (23, 24) also remain valid.

The vector-type perturbation of additionally present fluid(s) is described by eq. (13) which is not affected by the generalized nature of the gravity theory in eq. (1).

**IV. SLOW-ROLL INFLATION**

As in [13] the quantum generation process can be presented in a unified form. From eq. (17) we can construct the perturbed action

\[ \delta^2 S = \frac{1}{2} \int a^3 Q \left( \dot{\Phi}^2 - c_A^2 \frac{1}{a^2} \Phi \gamma \Phi \right) dt d^3 x, \]  
(26)

which is valid for both the scalar-type and tensor-type perturbations in a unified form. The rest of the canonical quantization process is straightforward, see [14]. Under an ansatz

\[ z''/z = n/n_s^2, \quad c_A^2 = \text{constant}, \]  
(27)

where \( n = n_s, \) \( n_t \) for the two perturbation types, the mode function has an exact solution in terms of the Hankel functions, see eq. (24) in [14]. The power spectrum based on the vacuum expectation value of \( \Phi \) can be constructed as in eq. (26) of [14], and in the large-scale limit we have[1]

\[ \left| P_{\Phi}^{1/2} \right|_{LS} = \frac{H}{2\pi a H |\eta|} \frac{\Gamma(\nu)}{\Gamma(3/2)} \frac{(k|\eta|)}{2} \frac{1}{c^\nu_{\tilde{A}} \sqrt{\eta}} \]  
(28)

where \( \nu \equiv \sqrt{n + 1/4}. \) We can read the spectral indices

\[ n_S - 1 = 3 - 4n_s + 1, \quad n_T = 3 - 4n_t + 1. \]  
(29)

We introduce the slow-roll parameters [11]

\[ \epsilon_1 \equiv \frac{\dot{H}}{H^2}, \quad \epsilon_2 \equiv \frac{\dot{\phi}}{H \phi}, \quad \epsilon_3 \equiv \frac{\dot{F}}{2 HF}, \quad \epsilon_4 \equiv \frac{\dot{E}}{2 HE}, \]

\[ E \equiv F \left( \frac{3\ddot{F}}{2\dot{\phi}^2} - \frac{1}{2} {f}_x - f_{,XX} X \right). \]  
(30)

Compared with the Einstein gravity in [16] we have two additional parameters \( \epsilon_3 \) and \( \epsilon_4 \) for the scalar-type perturbation which reflect the effects of additional parameters \( F(\equiv f, R) \) and \( f, X \) in our generalized gravity; for the tensor-type perturbation we have only one additional parameter \( \epsilon_3 \) from \( F. \) Compared with [11] the only difference occurs in our definition of \( E \) which includes the \( f(\phi, R) \) gravity in [11] as a case. Using our present definition of \( \epsilon_i \)'s our unified analyses made in eqs. (30-32) of [11] remain valid.

To the first-order in the slow-roll parameters, i.e., assuming

\[ \epsilon_i = 0, \quad |\epsilon_i| \ll 1, \]  
(31)

we can derive

\[ P_{\phi \delta \phi}^{1/2} = \frac{\pi}{|\phi|} P_{\delta \phi}^{1/2} |_{LS} = \frac{H^2}{2\pi |\phi| \sqrt{Z_s}} \left( 1 + \epsilon_1 \right) \]

\[ + \left[ \gamma_1 \ln (k|\eta|) \right] (2\epsilon_2 - \epsilon_3 + \epsilon_4) \right) e^{-\gamma_3}, \]  
(32)

\[ P_{\phi \delta \phi}^{1/2} = \sqrt{16\pi G} \frac{H}{2\pi \sqrt{Z_s}} \left( 1 + \epsilon_1 \right) \]

\[ + \left[ \gamma_1 \ln (k|\eta|) \right] (\epsilon_1 - \epsilon_3) \right), \]  
(33)

where \( \gamma_1 \equiv \gamma_E + \ln 2 - 2 = -0.7296 \ldots, \) with \( \gamma_E \) the Euler constant. We have

\[ Z_s = \frac{E/F}{(1 + \epsilon_3)^2}, \quad Z_t = 8\pi GF, \]  
(34)

where \( Z \)'s become unity in Einstein gravity. Thus, besides \( \epsilon_1, \) the scalar-type perturbation is affected by \( \epsilon_2, \epsilon_3 \) and \( \epsilon_4 \) (thus, \( f, \phi, F \) and \( f, X \)), whereas the tensor-type perturbation is affected by \( \epsilon_3 \) (thus, \( F \)) only. The spectral indices of the scalar and tensor-type perturbations in eq. (3) become

\[ n_S - 1 = 2(2\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4), \quad n_T = 2(\epsilon_1 - \epsilon_3). \]  
(35)

For the scale independent Harrison-Zel'dovich (\( n_S - 1 \) \( \approx 0 \approx n_T \)) spectra [17] the CMB quadrupole anisotropy becomes

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[1] For \( \nu = 0 \) we have an additional \( 2 \ln (c_A k|\eta|) \) factor. For the gravitational we should consider additional \( \sqrt{2} \) factor [13].
\( (a_2^2) = (a_2^2)_{.S} + (a_2^2)_{.T} = \frac{\pi}{75} \varphi_{.s} + 7.74 \frac{1}{5} \frac{3}{2} P_{\alpha s}, \) \( (36) \)

which is valid for \( K = 0 = \Lambda \). The four-year COBE-DMR data give \((a_2^2) \simeq 1.1 \times 10^{-10}, [14]\). From eqs. \([32,33,34,37]\) the ratio between two types of perturbations \( r_2 \equiv (a_2^2)_{.T} / (a_2^2)_{.S} \) becomes

\[
\begin{align*}
  r_2 &= 13.8 \times 4 \pi G \frac{\dot{\phi}^2}{H^2} \left| \frac{Z_0}{Z_t} \right| c_{2\nu}^2 \\
  &= 13.8 \frac{1}{(1 + \epsilon_3)^2} \left( (1 - \epsilon_3)(1 + \epsilon_3) + \frac{\epsilon_3}{H^2} \right) c_{2\nu}^2 \\
  &\simeq 13.8 |1 - \epsilon_3| c_A \\
  &\simeq 6.92 |n_T| c_A, \quad (37)
\end{align*}
\]

\( r_2 = 13.8 \times 4 \pi G \frac{\dot{\phi}^2}{H^2} \left| \frac{Z_0}{Z_t} \right| c_{2\nu}^2 \)
\( = 13.8 \frac{1}{(1 + \epsilon_3)^2} \left( (1 - \epsilon_3)(1 + \epsilon_3) + \frac{\epsilon_3}{H^2} \right) c_{2\nu}^2 \\
\simeq 13.8 |1 - \epsilon_3| c_A \\
\simeq 6.92 |n_T| c_A, \quad (37)
\]

where in the last two steps we used the slow-roll conditions in eq. \[31\]. In the limit of Einstein gravity we have \( r_2 = -13.8 \epsilon_3 = -6.92 n_T \) which is independent of \( V \) and is known as a consistency relation. The \( c_A \) factor differs from the Einstein gravity for \( p(\phi, X) \) gravity noticed in \[14\]. For the \( f(\phi, R) \) gravity we have \( c_A^2 = 1 \).

V. TACHYONIC CONDENSATION

The recently popular tachyonic condensation is a case of our gravity with a form \( f = \frac{1}{2} \alpha \sum R - 2 \sqrt{1 + 2 \phi} \): if based on the string theory, we should regard the field in this action as being written in the unit where the string theory is relevant. We have

\[
Q = \frac{\dot{\phi}^2}{H^2} \left( 1 - \dot{\phi}^2 \right)^{3/2}, \quad c_A^2 = 1 - \dot{\phi}^2. \quad (38)
\]

Equations \( \[20\] \) and \( \[21\] \) in this case were derived in eq. \( \[17\] \) of \[19\] and in eq. \( \[44\] \) of \[22\], respectively. We have \( \epsilon_3 = 0 \) and \( E = \frac{\epsilon_3}{\alpha} (1 - \dot{\phi}^2)^{-3/2} \).

Assuming a set of slow-roll conditions \( \dot{\phi} \ll 3H \dot{\phi} \) and \( \dot{\phi}^2 \ll 1 \), and under an ansatz \( V \equiv V_0 e^{-\alpha \phi} [21] \), from eqs. \( \[19\] \) for \( K = 0 \) we have \[22\]

\[
\phi = -\frac{2}{\alpha} \ln \left( C - \frac{\sqrt{3} \alpha^2 M_{pl}^2}{6 \sqrt{V_0}} \right), \quad a \propto e^{\sqrt{V_0} M_{pl} \sqrt{24} \dot{\phi}^2}, \quad (39)
\]

where \( M_{pl}^2 \equiv 1 / (8 \pi G) \). If we set \( t_i = 0 \), we have \( C = e^{-\alpha \phi} \), and \( V_i = V_0 C^2 \). For \( t \simeq t_i \) we have an accelerated expansion stage. In such a situation we have the slow-roll conditions in eq. \( \[31\] \) are well met, with the result:

\[
\epsilon_1 = -\epsilon_2 = \epsilon_4 = -\frac{\alpha^2 M_{pl}^2}{2 V_1}, \quad \epsilon_3 = 0. \quad (40)
\]

Thus, eq. \( \[33\] \) gives

\[
n_s - 1 = 4 \epsilon_1, \quad n_T = 2 \epsilon_1, \quad (41)
\]

and eqs. \( \[32,34,37\] \) reduce to

\[
P_{\phi \phi_{.s}} = \frac{H^2}{2 \pi |\phi|} \sqrt{V} \simeq \frac{1}{2 \pi |\phi|} \frac{V_i}{M_{pl}^2}, \quad (42)
\]

\[
P_{\phi \phi_{.t}} = \frac{|\phi|}{16 \pi G} H \frac{V_i}{2 \pi \sqrt{V_i}} \frac{V_i}{M_{pl}^2}, \quad (43)
\]

\[
r_2 = 6.92 |n_T|. \quad (44)
\]

Therefore, if the seed structures were generated from the vacuum quantum fluctuation under such a slow-roll phase, the final spectra show that: (1) the spectra are nearly scale-invariant Harrison-Zel’dovich type, (2) the consistency relation is met, (3) the gravitational wave is suppressed, and (4) the CMB quadrupole requires

\[
\langle a_2^2 \rangle \simeq \frac{1}{12 \times \pi |\phi|} \frac{V_i}{M_{pl}^2} \approx 1.1 \times 10^{-10}. \quad (45)
\]

We have assumed that, firstly, the seed fluctuations were generated during the slow-roll inflation stage supported by the tachyonic condensation, and secondly, the tachyonic gravity stage was switched successfully to an ordinary big-bang stage while the fluctuations stay in the large-scale limit (see \[23\] for the reheating problem); in such a case the relatively growing \( C \)-mode fluctuation in eq. \( \[22\] \) survives as the same \( C \)-mode of the curvature fluctuation \( \Phi \) now supported by the Einstein gravity with ordinary matter. We have derived these results directly based on the generalized form of gravity theory whereas the previous analyses \[24,22\] were based on known formulation in Einstein gravity by using some field redefinition.

VI. DISCUSSIONS

We have presented unified ways of handling the cosmological perturbations in a class of gravity theory covered by an action in eq. \( \[1\] \). Section \( \[11\] \) presents the classical evolutions in a unified form, and eqs. \( \[23,24\] \) show the generated seed fluctuations of the quantum origin under an assumption in eq. \( \[27\] \). The rest of section \( \[15\] \) presents the general prescription to derive the power spectra generated under the slow-roll assumption, and section \( \[15\] \) is an application to a tachyonic slow-roll inflation.

We note that even in the gravity with additional stringy correction terms

\[
\xi(\phi) \left[ c_1 R_{GB}^2 + c_2 G^{ab} \phi_{.a} \phi_{.b} + c_3 \Box \phi_{.a} \phi_{.a} + c_4 (\phi_{.a} \phi_{.a})^2 \right], \quad g(\phi) R R, \quad (46)
\]

in the Lagrangian, where \( R_{GB}^2 \equiv R^{abcd} R_{abcd} - 4 R^{ab} R_{ab} + R^2 \) and \( RR \equiv \eta^{abcd} R_{ab} R_{cd} + \eta^{cd} R_{cd} + \eta^{ab} R_{ab} \). We still have eqs. \( \[12,20\] \) with more complicated \( Q \) and \( C_A^4, \[23\] \). Thus, the rest of the analyses made above can be applied similarly as well. \[23\]. Similar unified formulation also exists in the fluid context. \[14\]. We also have studied situation with \( R^{ab} R_{ab} \) term in the action \[26\], in which case the gravity becomes a fourth-order theory.
We would like to emphasize that our gravity theory in eq. (1) covers many of the modified gravity theory, and our assumption in eq. (27) is satisfied by most of the expansion stages (including diverse class of inflation scenarios available in analytic forms) considered in the literature, and we hope our slow-roll conditions in eq. (31) cover most of the specific slow-roll conditions in the inflation theories based on specific modified gravity theories. We emphasize, however, that the classical evolutions studied in section 11 are valid for the general cosmological situations governed by our action in eq. (1).

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