Work-hardening behavior prediction model of arbitrary reloading process based on material crystallographic structure

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Abstract. In the case of complex forming process that receives a secondary loading after the primary loading, it is necessary to appropriately represent the work-hardening state according to the forming history. Namely, it is necessary to accurately model complex forming process including reversal loading (Bauschinger effect) and orthogonal loading (cross-hardening). In this study, a composite anisotropic hardening expression based on the crystallographic structure using the finite element polycrystal model is proposed. Numerical investigations are conducted to consider the effectiveness of the proposed model. In which, an alternation rate is introduced to capture the activity of the slip systems to support the idea behind the proposed model. In addition, flow curves of the secondary loading are produced by the proposed model, which is compared with the conventional model.

1. Introduction
The importance of material models for improving the accuracy of forming simulation is increasing. However, since multi-path forming has increased to realize complex shape, achieving accurate simulation has become difficult. Therefore, it is necessary to construct a model that can handle not only monotonic loading but also complex loading histories including reverse and cross-loadings. In such a case, a kinematic hardening model is often used[1][2][3], but there are problems such as insufficient consideration of the microstructure of the material. In these models, many parameters are used to achieve better fit to the experimental data; however, some of them do not have explicit physical meaning. Therefore, if these parameters are determined by multivariable optimization technique, it is difficult to confirm that the obtained solution is the globally optimized solution. The other problem is that a back stress is frequently used to express the reduction of the yield stress at the secondary loading phase know as Bauschinger effect; however, this type of model cannot be applied to arbitrary reloading process because another hardening mechanism could be activated when the reloading path is deviated from pure reverse.

The authors have studied for the purpose of establishing a composite anisotropic hardening expression based on the crystallographic structure of polycrystalline materials. In this paper, a verification of the proposed work-hardening model of the arbitrary load path is conducted. This paper also describes the results of numerical analysis using the finite element polycrystal model.
2. Representation of a general reloading process

In this study, two-stage loading paths are considered; namely, there are secondary loading paths subsequently continued after the primal monotonic loading. The following quantity is used as an indicator of different strain paths[4]:

$$\beta = \frac{D_1 : D_2}{||D_1||||D_2||}$$  (1)

where $D_1$ and $D_2$ are the strain rate tensors under the primary loading and the secondary loading, respectively. In this way, switching of an arbitrary load path can be expressed. Any reloading processes ($-1 < \beta < 0$) is called a general reloading process. It is important to define a representation of the work-hardening curve in this general reloading process in the sheet forming process. The general reloading process is characterized by the strain change parameter $\beta$. It can be considered that Bauschinger effect appears most remarkably at the reversal loading ($\beta = -1$), and the cross-hardening is caused most apparently at the orthogonal loading ($\beta = 0$). When $\beta$ changes from $-1$ to $0$, the work-hardening curve would gradually change. Therefore, the general work-hardening curve can be represented by a combined curve of the Bausinger curve and the cross-hardening curve with the weighting factor according to the value of $\beta$.

The reload path for the preload path is represented by the angle $\alpha$ between the preload deviatoric stress tensor $\sigma_P'$ and the reload deviatoric stress tensor $\sigma_R'$, which is expressed as

$$\cos \alpha = \frac{\sigma_P' : \sigma_R'}{||\sigma_P'||||\sigma_R'||},$$  (2)

where $\alpha = 0, \pi/2, \pi$ represent the continued preloading, orthogonal reloading and reverse reloading, respectively. Thus, the general work-hardening curve $H_R(\int d\varepsilon^P)$ is expressed as

$$H_R(\int d\varepsilon^P; \alpha) = H_C(\int d\varepsilon^P)\sin^2 \alpha + H_B(\int d\varepsilon^P)\cos^2 \alpha, \quad \left( \frac{\pi}{2} < \alpha < \pi \right),$$  (3)

where $H_C$ represents a cross-hardening curve and $H_B$ represents a Bauschinger curve, respectively. If an expression such as Eq. (3) can be established, once a Bauschinger and cross-hardening curves are obtained, the work-hardening curve for the general reloading paths can be predicted as an arbitrary curve between them by designating $\alpha$.

The Bauschinger effect is considered to be caused by the reversal of the slip direction while the active slip system remains unchanged, on the other hand, the cross-hardening is considered to be caused by the alternation of the active slip system. If the correlation between the change of the alternation ratio of slip systems and $\sin^2 \alpha$ is shown, the proposed model is physically valid. Here, an alternation ratio, which is explained in section 4, means a ratio of slip systems whose activeness is switched at the load-path change.

3. Description of models that express reverse and orthogonal loadings

The crystal plasticity code used in this study is based on a successive integration scheme with implicit finite element method, which is called FEPM (finite element polycrystal model [5]). To express Bauschinger effect and cross-hardening effect, two extra models were introduced by Takahashi. In this study, these models were implemented to compute the proposed model. In this section, these models are briefly explained.

3.1. Backlash model

When a reverse load such as compression is applied after tension, the Bauschinger effect occurs in which the flow stress of the reverse load decreases. This effect can be expressed by FEPM
using the backlash model [6]. The Bauschinger curve has a stagnation in flow stress, which seems to correspond to the process of collapse and reconstruction of the cell structure of dislocation. In the larger strain region, it can be considered that the primal loading curve is parallel to the Bauschinger curve with a same gradient. The amount of difference of these two curves can be considered as a preparation period for resuming work-hardening in the reverse load direction due to collapse and reconstruction of the cell structure after stress reversal. Based on the analogy to the backlash of mechanical gears, this amount is called the backlash quantity. The Bauschinger effect can be expressed by introducing such a concept of backlash into a crystallographic work-hardening mechanism.

First, a strain \( \gamma_a \) which is effective to work-hardening in each slip system is considered. As shown in Fig.1, \( \gamma_a \) increases with shear strain \( \gamma \) at the beginning of loading, but when the stress is reversed at point A, a non-hardening region, i.e. backlash region, appears. The backlash region is represented by \( \gamma_b \). Let \( \gamma_a|_{\text{max}} \) be the largest effective strain \( \gamma_a \) that a slip system has ever experienced till then, and \( \gamma_b \) is defined as a function of this value, which is given by the following equation.

\[
\gamma_b = B(\gamma_a|_{\text{max}}).
\]  

If \( \gamma_a \) is constant in the backlash region, the stress drop due to the Bauschinger effect cannot be expressed. Then, as shown in Fig.1, \( \gamma_a \) decreases slightly. An arbitrary position in the backlash region is presented by a parameter \( \xi \).

\[
\xi = 2\frac{\gamma - \gamma_c}{\gamma_b},
\]  

where \( \xi \) takes a value between \(-1 \) and \(+1 \) in the backlash region, \( \gamma_c \) is the midpoint of the backlash region. Then, the work-hardening rule is formulated as follows.

**1.** \( |\xi| = 1 \) (work-hardened state)

\[
\dot{\gamma}_a = |\gamma|.
\]  

**2.** \( |\xi| < 1 \) (backlash region)

\[
\dot{\gamma}_a = |\gamma| \frac{\text{sign}(\tau) + \lambda}{1 + \lambda},
\]  

where \( \lambda = 1 (\gamma_a|_{\text{max}} < 0.04), \lambda = 0 (\gamma_a|_{\text{max}} > 0.04) \). This equation is defined to express softening of the flow curve during backlash region.

By incorporating this model, it is possible to express the Bauschinger effect using a finite element polycrystal model.

### 3.2. Maximum forest dislocation model

In the case of an orthogonal loading, it is observed that the initial flow stress temporarily rises at the secondary yielding. This phenomenon is called cross-hardening, which is considered to be caused by an abrupt change of active slip systems. In order to express this cross-hardening, a maximum forest dislocation model [7] was introduced.

First, the slip planes are designated by \( p = 1 \sim 4 \), the slip directions are represented by \( q = 1 \sim 3 \), and the slip systems are represented by \( r = 3(p - 1) + q = 1 \sim 12 \). The strain of each slip system is represented by \( \gamma^{(p,q)} \) or \( \gamma^{(r)} \). The dislocation accumulated around the obstacle on the slip plane \( (p) \) is expressed by the following equation.

\[
\overline{\gamma}^{(p)}_a = \sum_q \gamma^{(p,q)}_a.
\]
For example, consider the slip surface $p = 1$. $\gamma^{(2)}_a$, $\gamma^{(3)}_a$, $\gamma^{(4)}$ can be interpreted as a forest dislocation for slip plane $p = 1$. Thus, it is assumed that the flow stress $k^{(1)}$ depends on the sum of them. As an improved model, it is assumed that the largest value of these forest dislocations controls the flow stress. Therefore, the critical yielding shear stress $k^{(p)}$ is represented by

$$ k^{(p)} = H(\Gamma^{(p)}) \text{ where } \Gamma^{(p)} = \max \gamma^{(p')}_a \quad (p' \neq p). $$

4. Introducing an alternation rate

The alternation rate 1: $\rho_1(\alpha)$ and the alternation rate 2: $\rho_2(\alpha)$, which represent the switching ratio of the active slip systems during primary and secondary loadings, are defined as follows:

$$ \rho_1(\alpha) = \frac{N_{10}}{N_{10} + N_{11}}, \quad \rho_2(\alpha) = \frac{N_{01}}{N_{01} + N_{11}}. $$

Here, $N$ represents the number of slip systems (or slip planes), subscripts 0 and 1 mean active and inactive, respectively. For example, $N_{10}$ means the number of slip systems (planes) that are active slip systems (planes) under the primary load and subsequently become inactive under the secondary load. In other words, the alternation rate 1 indicates the ratio of the slip system (plane) that was active under the primary load and became inactive under the secondary load. In this study, two definitions are used to investigate which expression captures the phenomenon better.

5. Numerical study of the proposed model

5.1. Verification using the alternation rate

After applying a pre-strain of 0.1, reverse loading ($\alpha = \pi$), 45-degree shear loading ($\alpha = 5\pi/6$), and orthogonal loading ($\alpha = \pi/2$) were applied. As an example, Fig. 2 shows the change of the alternation rate under the reverse loading case. Alternation rates exhibit rapid reduction and converge to the value around 0.2. This means that switching of active slip system occurred at a relatively lower degree. This calculation was also conducted for the other cases, as shown in Fig.3, and the converged alternation rates are collected, resulted in the relationship as follows.

$$ \rho(\pi) < \rho\left(\frac{5\pi}{6}\right) < \rho\left(\frac{\pi}{2}\right). $$
From this magnitude relation, as described above, the change of the active slip system shows the transition according to the change of the load path. Namely, this result state that the alternation rate increases from the minimum value that is observed in reverse loading as the angle $\alpha$ increases toward the maximum value that is observed in orthogonal loading. Therefore, at least from the viewpoint of the switching of the active slip system, the physical validity of the proposed model is shown.

Figure 2. Alternation rate of reverse loading.

Figure 3. Alternation rate of orthogonal loading.

5.2. Generation of secondary loading stress-strain curves

If a model that can express the general reloading process can be established, it is possible to predict any work-hardening curve between orthogonal and reverse reloading. The necessary data are flow curves of reverse and orthogonal loading. To verify the proposed model, a material data of DC06 is used and the results are compared with those of the Teodosiu-Hu (TH) model that is recognized as a reference curve in these analyses [3]. Note that the implemented FEPM model is based on the material structure of FCC (face-centered crystal) but DC06 is a BCC (body-centered crystal) material. Therefore, the following simulations only serves as qualitative investigation; however, the proposed concept is assumed to be effective for any crystalline metals.

Figures 4, 5, 6, and 7 show the verification results of the model for each load path. Prestrain of 0.1 and 0.2 are applied and represented by solid and dashed curves, respectively. As shown in Figs.4 and 5, typical drop and overshoot of flow stress are observed in both TH model and the proposed model. In each case, the qualitative trend represents the material response well, and the results are quantitatively similar. Figs.6 and 7 show examples of work-hardening curve prediction based the proposed model, which represent similar ones to the reference curve. Although further investigation and parameter adjustment are necessary, it is considered that the effectiveness of the proposed model could be shown.

6. Conclusion

A novel work-hardening model that can express arbitrary loading paths has been proposed. This model requires only Bauschinger and cross-hardening curves for the initial input, and an arbitrary reloading process can be predicted by designating an angle between the primary and secondary loading paths. This concept was confirmed by using the alternation rate that can represent how much slip systems change its activity. In this paper, numerical analyses of the proposed model for predicting the work-hardening behavior in the arbitrary secondary reloading process was performed using a finite element polycrystal model, and valid results were obtained.
Figure 4. Comparison of shear stress-strain curve of FEPM and Teodosiu-Hu for reverse loading.

Figure 5. Comparison of shear stress-strain curve of FEPM and Teodosiu-Hu for cross loading.

Figure 6. Comparison of shear stress-strain curve of FEPM, Teodosiu-Hu and proposed model for $\alpha = 5\pi/6$ loading.

Figure 7. Comparison of shear stress-strain curve of FEPM, Teodosiu-Hu and proposed model for $\alpha = 2\pi/3$ loading.

References
[1] Yoshida, F., Uemori, T. 2002 A model of large-strain cyclic plasticity describing the Bauschinger effect and workhardening stagnation International Journal of Plasticity, 18, 661–686.
[2] Chaboche, J. L. 1991 On some modification of kinematic hardening to improve the description of ratcheting effects International Journal of Plasticity, 7, 661–678.
[3] Haddadi, H., Bouvier, S., Banu, M., Maier, C., and Teodosiu, C. 2006 Towards an accurate description of the anisotropic behavior of sheet metals under large plastic deformations: Modelling, numerical analysis and identification International Journal of Plasticity, 22, 2226–2271.
[4] Schmitt, J. H., Aernoudt, E., and Baudelet, B., 1985 Yield loci for polycrystalline metals without texture Materials Science Engineering, 75, 13–20.
[5] Takahashi, H., Motohashi, H., Tokuda, M., and Abe, T. 1994 Elastic-plastic finite element polycrystal model International Journal of Plasticity, 10, 63–80.
[6] Takahashi, H. and Shiono, I. 1991 Backlash model for large deformation behavior of aluminum under torsional cyclic loading International Journal of Plasticity, 7, 199–217.
[7] Takahashi, H., Fujiwara, K., and Nakagawa, T. 1998 Multiple-slip work-hardening model in crystals with application to torsion-tension behaviors of aluminum tubes International Journal of Plasticity, 14(6), 489–509.