The role of the $S_{31} \pi N$ partial wave in the three-nucleon force

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Abstract

We re-examine the contribution of the $\pi N S_{31}$ channel to the three-nucleon force for an energy dependent separable potential. Despite the lack of cancellation between the $S_{31}$ and the $P_{11}$ and $P_{33}$ channels, the total contribution of the three-nucleon force to the triton binding is still small.
The very small contribution of the three-body force to the triton binding energy as recently reported by Saito and Afnan (SA) \cite{1,2} was attributed to: (i) The energy dependence of the $\pi N$ amplitude in the subthreshold region. (ii) The cancellation between the repulsive $S_{31}$ $\pi N$ partial wave and the attractive $P_{11}$ and $P_{33}$ partial waves. (iii) The very soft $\pi NN$ form factor for the pion emission and absorption vertices. In this brief report we re-examine in more detail the role of the $S_{31}$ $\pi N$ amplitude in the cancellation between the contribution from the different partial waves within the framework of a separable potential.

In the previous studies \cite{1,2} the $\pi N$ potentials \cite{3,4} used were constructed to fit the scattering data with potential application to $\pi d$ scattering. Here, we will concentrate on the subthreshold behavior of the amplitude for a rank one separable potential with the aim of examining how this behavior effects the contribution of the $S_{31}$ partial wave to the three-nucleon force.

To get a better understanding of the role of the subthreshold behavior of the amplitude for a rank one separable potential, we recall that the commonly used potentials $v(k, k')$ are of the form

$$v(k, k') = g(k)\lambda g(k'),$$  \hspace{1cm} (1)

where $g(k)$ is a form factor and $\lambda$ is the strength of the potential. The off-energy-shell t-matrix $T(k, k'; E)$, is determined by solving the Lippmann-Schwinger equation

$$T(k, k'; E) = v(k, k') + \int_0^\infty dq q^2 v(k, q)G(q; E)T(q, k'; E),$$  \hspace{1cm} (2)

where the $\pi N$ propagator $G(q; E)$, is given as

$$G(q; E) = \frac{1}{E - (q^2/2m_N + m_N + \sqrt{q^2 + m_N^2})}.\hspace{1cm} (3)$$

By solving Eq.\,(2) for the above separable potential, the resultant off-shell t-matrix takes the simple form

$$T(k, k'; E) = g(k)\tau(E)g(k'),$$  \hspace{1cm} (4)
where the energy dependence of the off-shell t-matrix is given by

\[ \tau(E) = \frac{1}{1/\lambda - \langle g|G(E)|g \rangle}, \]  

with

\[ \langle g|G(E)|g \rangle = \int_0^\infty dq q^2 g(q) G(q; E) g(q). \]  

This t-matrix, \( T(k, k'; E) \), has a specific energy dependence in the subthreshold region, which is most simply illustrated by considering the derivative of \( \tau(E) \) with respect to \( E \), i.e.,

\[ \frac{d\tau(E)}{dE} = -[\tau(E)]^2 \langle g|G^2(E)|g \rangle. \]  

Since \( \tau(E) \) is real in the subthreshold region, \( d\tau(E)/dE \) is negative. As a result, the t-matrix has negative slope as a function of \( E \) for both attractive and repulsive partial waves.

For the \( S_{31} \pi N \) potential used previously [1,2], the value of \( \tau(E) \) increases, and approaches \( \lambda = +1 \) as \( E \to \infty \). To reduce the possible cancellation between the repulsive \( S_{31} \) and the attractive \( \pi N \) partial waves, we propose to introduce a parameterization of the \( S_{31} \) potential that is energy dependent, and in this way change the energy dependence of the subthreshold amplitude within the framework of a rank one separable potential.

This energy dependence in the potential is introduced by replacing the strength of the potential \( \lambda \), by \( c/(E - M) \) where \( c \) and \( M \) are parameters of the potential. This allows \( \tau(E) \) to deviate from the condition dictated by Eq. (7). For the form factor \( g(k) \), we use

\[ g(k) = \frac{1}{\sqrt{\omega_k}} \left( \frac{1}{k^2 + \alpha_1^2} + \frac{\beta k^2}{(k^2 + \alpha_2^2)^2} \right), \]  

where \( \omega_k = \sqrt{k^2 + m_\pi^2} \). The factor of \( \frac{1}{\sqrt{\omega_k}} \) is introduced to maintain consistency with the relativistic treatment of the pion.

The parameters of this \( S_{31} \) potential have been adjusted to fit the scattering length and phase shift up to 400 MeV pion energy. The resultant parameters are: \( c = -0.0230136, \beta = 14.1251, M = 1589.375 \) MeV, \( \alpha_1 = 120.178 \) MeV and \( \alpha_2 = 240.863 \) MeV. In Fig. 1 we compare the phase shifts for this potential (solid line) with the phase shifts predicted by
Thomas [3] and used in the previous calculation [1,2] (dotted line). Also included are the phase shift based on the analysis of Höhler et al. [3] (dots). Clearly the two sets of phase shifts are almost identical and consistent with the Höhler et al. analysis. The scattering length for this new potential of $a_3 = -0.101 \, m_{\pi}^{-1}$ is in better agreement with the empirical value of $a_3 = -0.101 \pm 0.004 \, m_{\pi}^{-1}$ [3] when compared with the scattering length of $a_3 = -0.091 \, m_{\pi}^{-1}$ for the potential of Ref. [3].

In Fig. 2 we compare the subthreshold amplitude $T(k = 0, k' = 0; E)$ as function of the energy $E$ for this potential (solid line), with that used in Ref. [1,2] (dotted line). Here we note that the t-matrix for this new potential has the positive slope and approaches zero as the energy decreases while that of Ref. [1,2] has a negative slope. As a result, we expect the contribution of the $S_{31}$ channel to the three-nucleon force to be reduced, and the total contribution from the $\pi - \pi$ three-body force enhanced in comparison with the results reported previously [1,2]. In Fig. 3 we compare the corresponding form factor for the $S_{31}$ potential (solid line) with that used in our previous calculation (dotted line). This new form factor is considerably softer than that previously used. However, it is important to note that this form factor is not the $\pi NN$ form factor associated with the pion production and absorption vertices. In fact, this form factor determines the off-shell behavior of the amplitude in the $S_{31}$ channel, and to that extent is not constrained by the $\pi N$ scattering data, i.e. the phase shifts.

To examine the effect of changes to the subthreshold $\pi N$ amplitude in the $S_{31}$ channel on the three-body force, we have calculated the contribution of this partial wave to the three-nucleon force following the same procedure as in Ref. [1,2], i.e., we use first order perturbation theory. The triton wave function used is obtained from the solution of the Faddeev equation with the Paris-Ernst-Shakin-Thaler (PEST) potential [7,8]. The $\pi NN$ form factor used is obtained from the $P_{11}$ potential $PJ$ of Ref. [4]. The resultant three-nucleon force contribution is presented in Table I. As we have expected, the contribution of the $S_{31}$ partial wave is suppressed due to the smaller value of the t-matrix and the softer form factor. Here we note that the contribution of the $S_{31}$ channel to the three-body force
gets larger as we fix the energy in the amplitude first at $E = m_N$ and then at $E = m_N + m_\pi$. This is due to the positive slope of $\tau(E)$ for this amplitude. Although this reduction in the contribution of the $S_{31}$ amplitude has suppressed the cancellation between the repulsive and attractive partial waves, and has given a considerably more attractive contribution to the three-body force, the overall contribution from the $P_{11}$ and $P_{33}$ channels of -8.8 and -16.0 KeV is still too small to bridge the gap between the experimental binding energy of 8.45 MeV and the commonly reported value for many of the realistic potentials of 7.7 MeV [9]. This could be due to the overall energy dependence of the $\pi N$ amplitude and the range of the $\pi NN$ form factor. The important result of this investigation is the fact that there is considerable model dependence in the analytic continuation of the $\pi N$ amplitude in the $S_{31}$ channel from the physical region to the subthreshold region, and this introduces an element of uncertainty in the determination of the three-nucleon force.

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FIGURES

FIG. 1. The resultant $\pi N$ phase shift. The solid line is obtained in this report and that used in Ref. [1,2] for the dotted line. The empirical data are from Ref. [5].

FIG. 2. The energy dependence of the $\pi N$ t-matrix, $T(k, k'; E)$, taking $k = k' = 0$, for the $S_{31}$ channel. The solid line is obtained using the potential reported in this work, while the dotted curve is that used in Ref. [1,2].

FIG. 3. The solid line is the form factor $g(k)$, reported in this work, while the dotted curve is that used in Ref. [1,2].
TABLES

TABLE I. The contribution of $S_{31}$ channel to the three-nucleon force are listed in the second and third columns. The resultant three-body force contribution (3BF) to the triton binding are listed in the fourth and fifth columns. We obtained those values by replacing only the $S_{31}$ contributions of Table 10 of Ref.[2]. The second line is calculated taking the full energy dependence. The third and fourth lines are calculated by fixing the energy of $\tau(E)$ at $E = m_N$ and $E = m_N + m_\pi$, respectively. All energies are in keV.

| $S_{31}$ contribution | Ref. [1,2] | This work | Ref. [1,2] | This work |
|-----------------------|------------|-----------|------------|-----------|
| $\tau(E)$             | 31.7       | 4.1       | -2.3       | -29.9     |
| $\tau(E = m_N)$       | 23.5       | 5.3       | -23.1      | -41.3     |
| $\tau(E = m_N + m_\pi)$ | 19.5   | 5.9       | -55.5      | -69.1     |
Fig. 1
Fig. 2
Fig. 3