A Minimal Type II Seesaw Model

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We propose a minimal type II seesaw model by introducing only one right-handed neutrino besides the $SU(2)_L$ triplet Higgs to the standard model. In the usual type II seesaw models with several right-handed neutrinos, the contributions of the right-handed neutrinos and the triplet Higgs to the CP asymmetry, which stems from the decay of the lightest right-handed neutrino, are proportional to their respective contributions to the light neutrino mass matrix. However, in our minimal type II seesaw model, this CP asymmetry is just given by the one-loop vertex correction involving the triplet Higgs, even though the contribution of the triplet Higgs does not dominate the light neutrino masses. For illustration, the Fritzsch-type lepton mass matrices are considered.

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Recent neutrino oscillation experiments have provided us with very convincing evidence that neutrinos are massive and lepton flavors are mixed. In the three-neutrino mixing scheme, a global analysis of experimental data yields: two independent neutrino mass-squared differences $\Delta m^2_{21} = (7.2 \sim 8.9) \times 10^{-5} \text{eV}^2$, $\Delta m^2_{32} = (1.7 \sim 3.3) \times 10^{-3} \text{eV}^2$ and three mixing angles $\theta_{12} < 38^\circ$, $36^\circ < \theta_{23} < 54^\circ$, $\theta_{13} < 10^\circ$ at the 99% confidence level.

On the other hand, the cosmological baryon asymmetry, which is characterized by the ratio of baryon to photon number densities, has also been measured by the WMAP experiment to a very good precision:

$$\eta_B \equiv \frac{n_B}{n_\gamma} = (6.1 \pm 0.2) \times 10^{-10}.$$  

Sometimes the baryon asymmetry is also represented by $Y_B \equiv n_B/n_s = \eta_B/7.04 \simeq (8.4 \sim 8.9) \times 10^{-13}$ with $s$ being the entropy density.

Leptogenesis is now an attractive scenario to simultaneously explain the neutrino oscillation phenomena and the baryon asymmetry through the famous seesaw mechanism. In the ordinary type I seesaw models, the leptogenesis scenario can be realized by introducing two or more right-handed neutrinos with heavy Majorana masses to the $SU(2)_L \times U(1)_Y$ standard model. Another interesting alternative is to consider the so-called type II seesaw models, in which the $SU(2)_L$ triplet Higgs, besides the right-handed neutrinos, can also contribute to the light neutrino masses as well as the baryon asymmetry generation.

In this paper, we propose a minimal type II seesaw model by extending the standard model with only one right-handed neutrino in addition to the $SU(2)_L$ triplet Higgs.\(^1\) The CP asymmetry in the decay of the right-handed neutrino just arises from the interference between the tree level diagram and the one-loop vertex correction involving the triplet Higgs. At the same time, the light neutrino mass matrix can receive comparable contributions from the triplet Higgs and the right-handed neutrino. For illustration, we will consider Fritzsch-type lepton mass matrices with five texture zeroes in the specific discussions and calculations. It will be shown that our model can simultaneously explain the neutrino properties and the baryon asymmetry.

It is convenient to start with our discussions from the general type II seesaw models:

$$- \mathcal{L} = M^2_3 \text{Tr} \Delta^2_L + g_{\alpha \beta} \bar{\psi}^\alpha L \phi^\alpha \tau_2 \Delta_L \phi_L^\beta - \frac{1}{2} \mu \phi^T i \tau_2 \Delta_L \phi + \frac{1}{2} M_i \bar{\nu}_R \nu_R + \gamma_{\alpha \iota} \bar{\psi}^\alpha L \nu_{R\iota} \phi + h.c.$$  

$$= M^2_3 \text{Tr} \Delta^2_L + \frac{1}{2} M_i \bar{\nu}_L N_i + g_{\alpha \beta} \bar{\psi}^\alpha L \phi^\beta \tau_2 \Delta_L \phi_L + \frac{1}{2} \mu \phi^T i \tau_2 \Delta_L \phi + h.c.$$  

Here $\psi_\alpha = (\nu_\alpha, l_\alpha)^T$ ($\alpha = e, \mu, \tau$), $\phi = (\phi^0, \phi^+, \phi^-)^T$ are the lepton and the Higgs doublets, while

$$\Delta_L = \left( \begin{array}{ccc} \delta^+ & \delta^+ & \delta^+ \\ \frac{1}{\sqrt{2}} \delta^0 & -\frac{1}{\sqrt{2}} \delta^0 \\ \frac{1}{\sqrt{2}} \delta^0 & \frac{1}{\sqrt{2}} \delta^0 \end{array} \right)$$

is the triplet Higgs. $\nu_{Ri}$ ($i = 1, ..., d$) are the right-handed neutrinos, and $N_i = \nu_{Ri} + \nu^c_{Ei}$ is defined as the heavy Majorana neutrinos. We have conveniently chosen the basis, in which the Majorana mass matrix of right-handed neutrinos is diagonal, i.e. $M = \text{Diag}\{M_1, ..., M_d\}$. Obviously, $g$ is generally a complex $3 \times 3$ matrix. In the usual type II seesaw models, there are several right-handed neutrinos, i.e. $d \geq 2$. However, only one right-handed neutrino and hence three Yukawa couplings $y_{\alpha i}$ ($\alpha = e, \mu, \tau$) occur in our minimal type II seesaw model with $d = 1$.

\(^1\) Here the minimal is in the sense of the number of new particles compared with the conventional type II seesaw model.
As $v$ is the type II seesaw mass term. $\nu_M$ is the correction involving the heavy Majorana neutrinos, while $1. \text{ The first two diagrams are the self-energy and vertex correction involving the heavy Majorana neutrinos, while}

$$M_\nu = -y^* \frac{1}{M} y v^2 + 2g v_L = M_\nu^I + M_\nu^II,$$  \quad (3)

where $M_\nu^I$ is the ordinary type I seesaw mass term, $M_\nu^II$ is the type II seesaw mass term. $v = 174 \text{GeV}, v_L \approx \mu^* v^2/M_{\Delta}^*$ are the vacuum expectation values of $\phi$ and $\Delta_L$, respectively. It is easy to see that $v_L$ is naturally seesaw suppressed if $\Delta_L$ is very heavy.

The CP asymmetry $\varepsilon_{N_i}$ from the decay of the heavy Majorana neutrinos $N_i$ is given by the interference of the ordinary tree level decay with the three diagrams of Fig. 1. The first two diagrams are the self-energy and vertex correction involving the heavy Majorana neutrinos, while the third is due to the contribution from the triplet Higgs. Under the assumption that $M_1 \ll M_2, ..., M_d, M_\Delta$, the CP asymmetry $\varepsilon_{N_i}$ can be simplified as:

$$\varepsilon_{N_i} = \varepsilon_{N_i}^N + \varepsilon_{N_i}^A,$$  \quad (4)

$$\varepsilon_{N_i}^N \approx \frac{3}{16\pi^2} \frac{M_1}{v^2} \sum_{\alpha \beta} \Im \left[ y^\nu_{\alpha \beta} y^\nu_{\alpha \beta}^* \left( M_\nu^II \right)_{\alpha \beta} \right],$$  \quad (5)

$$\varepsilon_{N_i}^A \approx \frac{3}{16\pi^2} \frac{M_1}{v^2} \sum_{\alpha \beta} \Im \left[ y^\nu_{\alpha \beta} y^\nu_{\alpha \beta}^* \left( M_\nu^II \right)_{\alpha \beta} \right].$$  \quad (6)

It is shown that the contributions of $N_i$ (diagrams (a) and (b) in Fig. 1) and $\Delta_L$ (diagram (c) in Fig. 1) to $\varepsilon_{N_i}$ are proportional to their respective contributions to the light neutrino mass matrix $\tilde{M}$. Therefore, the contribution from the triplet Higgs to the above CP asymmetry can be neglected in the limit $M_\nu^II \ll M_\nu^I$. However, this analysis is invalid in our minimal type II seesaw model, since there is just one heavy Majorana neutrino, the one-loop vertex correction involving the triplet Higgs is the sole source of this CP asymmetry and we always obtain

$$\varepsilon_{N_i} = \varepsilon_{N_i}^A,$$  \quad (7)

even though the contribution of $\Delta_L$ does not dominate the light neutrino mass.

For illustration, we simply assume the Fritzsch-type textures $^{10}$ of $M_\nu^II$ and the charged lepton mass matrix $M_\nu$:

$$M_\nu^II = v_L \begin{pmatrix} 0 & C_{\nu e} e^{i\alpha_{\nu e}} & 0 \\ C_{\nu e} e^{i\alpha_{\nu e}} & 0 & B_{\nu e} e^{i\beta_{\nu e}} \\ 0 & B_{\nu e} e^{i\beta_{\nu e}} & A_{\nu e} e^{i\gamma_{\nu e}} \end{pmatrix},$$  \quad (8)

and

$$M_\nu = v \begin{pmatrix} 0 & C_{\nu l} e^{i\alpha_{\nu l}} & 0 \\ C_{\nu l} e^{i\alpha_{\nu l}} & 0 & B_{\nu l} e^{i\beta_{\nu l}} \\ 0 & B_{\nu l} e^{i\beta_{\nu l}} & A_{\nu l} e^{i\gamma_{\nu l}} \end{pmatrix},$$  \quad (9)

where $A_{\nu e}, B_{\nu e}$ and $C_{\nu e}$ are real and positive. In addition, we set

$$y = iy_0 (0, r, 1)^T,$$  \quad (10)

where the imaginary unit $i$ has been inserted to cancel the minus sign in front of the type I term for convenience. Substituting Eqs. 8$^{10}$ and 11$^{11}$ into Eq. 6, we can obtain the effective neutrino mass matrix

$$M_\nu = m_0 \begin{pmatrix} 0 & \tilde{C} c^{i\alpha_{\nu e}} & 0 \\ \tilde{C} c^{i\alpha_{\nu e}} & r^2 & r + \tilde{B} e^{i\beta_{\nu e}} \\ 0 & r + \tilde{B} e^{i\beta_{\nu e}} & 1 + \tilde{A} e^{i\gamma_{\nu e}} \end{pmatrix},$$  \quad (11)

where $m_0 \equiv v^2 y_0^2 / M_1$ and $A \equiv v_L A_{\nu e} / m_0$, likewise for $\tilde{B}$ and $\tilde{C}$. The strategies to diagonalize lepton mass matrices of the form in Eqs. 9$^{10}$ and 11$^{11}$ can be found in Refs. 10$^{10}$ and 11$^{11}$. For simplicity, we adopt two assumptions:
\( r = \sqrt{m_2/m_0} \) and \( \arg (1 + \hat{A} e^{i\gamma_{\nu}}) = 2 \arg (r + \hat{B} e^{i\beta_{\nu}}) \), and then obtain

\[
\hat{A} = \left[ \frac{(m_3 - m_1)^2}{m_0^2} - \sin^2 \gamma_{\nu} \right]^{1/2} - \cos \gamma_{\nu},
\]

\[
\hat{B} = \left[ \frac{m_3 m_4 (m_3 - m_1 - m_2)}{m_0^2 (m_3 - m_1)} - r^2 \sin^2 \beta_{\nu} \right]^{1/2}
\]

\[
- r \cos \beta_{\nu},
\]

\[
\hat{C} = \left[ \frac{m_1 m_2 m_3}{m_0^2 (m_3 - m_1)} \right]^{1/2}
\]

and

\[
A_l = (m_\tau - m_\mu + m_e),
\]

\[
B_l = \left[ \frac{(m_\mu - m_e)(m_\tau - m_\mu)(m_e + m_\tau)}{(m_\tau - m_\mu + m_e)} \right]^{1/2},
\]

\[
C_l = \left[ \frac{m_e m_\mu m_\tau}{(m_\tau - m_\mu + m_e)} \right]^{1/2}.
\] \hspace{1cm} (12)

and

\[
A_l = (m_\tau - m_\mu + m_e),
\]

\[
B_l = \left[ \frac{(m_\mu - m_e)(m_\tau - m_\mu)(m_e + m_\tau)}{(m_\tau - m_\mu + m_e)} \right]^{1/2},
\]

\[
C_l = \left[ \frac{m_e m_\mu m_\tau}{(m_\tau - m_\mu + m_e)} \right]^{1/2}.
\] \hspace{1cm} (13)

The Maki-Nakagawa-Sakata (MNS) mixing matrix is \( V = U_l^T U_\nu \), where \( U_l^T \) and \( U_\nu \) are the unitary matrices used to diagonalize the lepton mass matrices: \( U_l^T M_l U_l^* = \text{Diag} \{ m_e, m_\mu, m_\tau \} \) and \( U_\nu^T M_\nu U_\nu^* = \text{Diag} \{ m_1, m_2, m_3 \} \).

Taking the neutrino oscillation experimental data at the 99% confidence level as input\(^2\), the allowed regions of three mixing angles and Jarlskog invariant \( J \equiv \text{Im} (V_{11}^* V_{22} V_{12}^* V_{21}^*) \) in our model are shown in Figs. 2 and 3. The phase parameters, exactly combinations of \( \alpha_l, \beta_l, \gamma_l \), are all varying in the range \([0, 2\pi)\). From Fig. 3, we can see that the smallest mixing angle \( \theta_{13} \) is larger than \( 1^\circ \) and the maximal value of CP-violation parameter \( J \) can reach \( 2.5\% \), which are to be strictly tested in the future neutrino oscillation experiments. Here we remark that the Fritzsch-type lepton mass matrices can serve as a good example to illustrate the features of the minimal type II seesaw model. For more phenomenological discussions about lepton mass matrices in the type-II seesaw models, see [12].

We now calculate the produced baryon asymmetry in the minimal type II seesaw model. Assuming the heavy Majorana neutrino \( N_1 \) is much lighter than the triplet Higgs \( \Delta_3 \), the final lepton asymmetry, which is partially converted to the baryon asymmetry via the sphaleron process [14], will mainly come from the decay of \( N_1 \). We

\(^2\) For simplicity, we assume the normal mass hierarchy of light neutrinos in our numerical analysis.
predicted value for the final baryon asymmetry is 

\[ \varepsilon_{N_1} = -4.1 \times 10^{-6} \]

\[ M_1 = 4 \times 10^{11} \text{ GeV} \]
\[ m_1 = 8.6 \times 10^{-3} \text{ eV} \]

\[ Y_N = 2 \times 10^{-3} \]
\[ Y_B = 28/79 Y_{B-L} \]

FIG. 4: The solutions of the Boltzmann equations. \( Y_{X}^{(\text{eq})} \) is the (equilibrium) number of heavy Majorana neutrino, while \( Y_B = 28/79 Y_{B-L} \) denotes the baryon. In the numerical calculations, we have taken \( M_1 = 4 \times 10^{11} \text{ GeV} \), \( m_1 = 8.6 \times 10^{-3} \text{ eV} \), \( \Delta m_{21}^2 = 8.3 \times 10^{-5} \text{ eV}^2 \), \( \Delta m_{32}^2 = 2.3 \times 10^{-3} \text{ eV}^2 \), \( y_0 = 0.02 \), \( r = 0.64 \) and \( (\beta_3, \gamma_3) = (-100^\circ, -36^\circ) \). The predicted value for the final baryon asymmetry is \( Y_B \approx 8.5 \times 10^{-11} \).

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