Mechanical resonators having low damping rate are widely employed for sensing and timing applications [1]. At sufficiently low temperatures such devices may allow the experimental exploration of the crossover from classical to quantum mechanics [2–8]. Commonly, the observation of non-classical effects in such experiments is possible only when the damping rate [9] of the mechanical resonator is sufficiently low. Mechanical resonators made of superconductors are widely employed in such low-temperature experiments. In this study we experimentally investigate the effect of superconductivity on the damping rate of a mechanical resonator made of aluminum near its normal-to-superconducting phase transition.

The damping rate can be measured by coupling the mechanical resonator under study to a displacement detector. In general, a variety of different mechanisms may contribute to the total mechanical damping rate. In order to isolate the effect of superconductivity it is important to employ a method of displacement detection that is unaffected by the normal-to-superconducting phase transition. In addition, systematic errors in the measured damping rate due to back-reaction effects originating from the coupling between the mechanical resonator and the displacement detector have to be kept at a sufficiently low level.

In our setup (see fig. 1) we employ the so-called optomechanical cavity configuration [10–12], in which displacement detection is performed by coupling the mechanical resonator to an electromagnetic cavity. While many of the previous studies of superconducting mechanical resonators have employed such a configuration with a superconducting microwave cavity [4,6,7,13–21], our setup, which is based on a cavity in the optical band, allows displacement detection that is unaffected by the phase transition occurring in the mechanical resonator under study, which, in-turn, allows isolating the effect of superconductivity on the mechanical damping. Moreover, back-reaction effects are suppressed by employing a relatively low driving power to the optical cavity (see discussion below). The magnetomotive scheme has been employed for displacement detection in ref. [22] in order to study the temperature dependence of linear and nonlinear damping rates of a mechanical resonator made of aluminum near the normal to superconducting phase transition. The results indicate a significant reduction in the damping rate in the superconducting phase [22].

In our setup the optomechanical cavity is formed between two mirrors, a stationary fiber Bragg grating (FBG)
mirror and a movable mirror made of a mechanical resonator in the shape of a trampoline supported by four beams (see fig. 1(a)). A graded index fiber (GIF) spliced to the end of the single mode fiber (SMF) is employed for focusing. A cryogenic piezoelectric three-axis positioning system having sub-nanometer resolution is employed for manipulating the position of the optical fiber. A photolithography process is used to pattern a t\textsubscript{Al} = 200 nm thick aluminum layer on top of a t\textsubscript{SiN} = 100 nm thick silicon nitride layer into the shape of a 100 × 100 µm\textsuperscript{2} trampoline with four supporting beams (see fig. 1(b)). Details of the fabrication process can be found elsewhere [23]. Measurements are performed in a dilution refrigerator at a pressure well below 2 × 10\textsuperscript{-5} mbar. Another device on the same wafer, which is made in the shape of a microwave microstrip resonator, allows characterizing the surface resistance of the aluminum layer [23].

A tunable laser operating near the Bragg wavelength of the FBG together with an external attenuator are employed to excite the optical cavity. The optical power reflected off the cavity is measured by a photodetector (PD), which is connected to a network analyzer (NA). Actuation is performed by applying an alternating voltage (with a direct voltage offset) between the trampoline and a stationary electrode positioned 200 µm below it.

The fundamental mechanical mode is characterized by its frequency ω\textsubscript{m}/2π and damping rate γ\textsubscript{m}. Both parameters can be extracted from the resonance lineshape of the measured NA signal S\textsubscript{NA} vs. angular driving frequency ω\textsubscript{NA} with a fixed driving amplitude (see fig. 1(c)). In the regime of linear response S\textsubscript{NA}(ω\textsubscript{NA}) is expected to be given by

\[ S\textsubscript{NA}(ω\textsubscript{NA}) = \frac{S\textsubscript{NA,R}}{1 + (\frac{ω\textsubscript{NA} - ω\textsubscript{m}}{γ\textsubscript{m}})^2}, \]  

where S\textsubscript{NA,R} is the value of S\textsubscript{NA}(ω\textsubscript{NA}) at resonance, i.e. when ω\textsubscript{NA} = ω\textsubscript{m}.

The measured damping rate γ\textsubscript{m} vs. temperature T, which is extracted from the NA data using eq. (1), is indicated by the crosses in fig. 2. The same procedure yields the mode's frequency ω\textsubscript{m}/2π = 432.318 kHz, which is found to be almost a constant for the range of temperatures explored in this measurement (between 0.5 K and 1.3 K). As can be seen from fig. 2, the measured damping rate γ\textsubscript{m} sharply drops as the temperature T is lowered below the value of 1.1 K. Note that at the same temperature of 1.1 K the resonance of the microwave microstrip resonator becomes visible, indicating thus that the critical temperature is T\textsubscript{c} = 1.1 K, as expected from high quality aluminum layer. The data seen in fig. 2 is obtained by setting the laser power that is injected into the optical cavity to the value P\textsubscript{L} = 3 × 10\textsuperscript{-8} W.

In general, heating due to optical absorption by the aluminum layer may give rise to a systematic error in the measurement of γ\textsubscript{m}. Two possible mechanisms are discussed below. The first one is due to back-reaction originating from the bolometric optomechanical coupling [24,25], which gives rise to a shift in the effective value of γ\textsubscript{m} denoted by γ\textsubscript{m,ba}. The magnitude of γ\textsubscript{m,ba} can be roughly estimated using the relation γ\textsubscript{m,ba}/γ\textsubscript{m} ≃ P\textsubscript{H}/P\textsubscript{LT}, where P\textsubscript{H} is the laser power that is employed for the measurement of γ\textsubscript{m} and P\textsubscript{LT} is the laser power at the threshold of self-excited oscillation [26]. For the same cavity tuning, for which the data seen in fig. 2 is obtained, self-excited oscillation occurs at a threshold power given by P\textsubscript{LT} = 4 × 10\textsuperscript{-3} P\textsubscript{L}, and thus the effect of back-reaction can be safely disregarded.

The other possible source of a systematic error originates from temperature rise due to optical absorption. Due to the low heat conductance of both superconducting aluminum and silicon nitride, this mechanism imposes a severe upper limit upon the allowed values of laser power. The heating power is given by P\textsubscript{H} = ζ H β L / (1 - R\textsubscript{C}) P\textsubscript{L}, where ζ is the absorption coefficient (for aluminum ζ = 0.03), β is the cavity finesse, R\textsubscript{C} is the cavity reflectivity, and P\textsubscript{L} is the laser power [27]. For the measurement of γ\textsubscript{m} that is presented in fig. 2 the optical cavity is tuned to have finesse of β\textsubscript{L} = 1.8 and reflectivity of R\textsubscript{C} = 0.32, and thus for this measurement P\textsubscript{H} = 1.1 × 10\textsuperscript{-9} W.

The temperature rise ΔT due to optical absorption is estimated by ΔT = P\textsubscript{H}/K\textsubscript{b}, where K\textsubscript{b} is the thermal conductance of each of the four nominally identical beams that support the trampoline. The thermal conductance K\textsubscript{b} is given by K\textsubscript{b} = (l\textsubscript{Al}κ\textsubscript{Al} + l\textsubscript{SiN}κ\textsubscript{SiN})(w\textsubscript{b}/l\textsubscript{b}), where κ\textsubscript{Al} (κ\textsubscript{SiN}) is the thermal conductivity of aluminum (silicon nitride), and where w\textsubscript{b}/l\textsubscript{b} = 0.5 is the ratio between width and length of the beams. For the lowest value of 0.5 K in the temperature range seen in fig. 2 the thermal conductivities are estimated to be κ\textsubscript{Al} ≃ 4 × 10\textsuperscript{-2} W K\textsuperscript{-1} m\textsuperscript{-1} for aluminum [28] and κ\textsubscript{SiN} ≃ 2 × 10\textsuperscript{-2} W K\textsuperscript{-1} m\textsuperscript{-1} for silicon nitride [29,30]. For these values the estimated temperature rise is ΔT = 0.06 K. For temperatures below 0.5 K the temperature rise ΔT becomes even larger since both κ\textsubscript{Al} and κ\textsubscript{SiN} rapidly drop at low temperatures, and therefore no reliable measurements can be obtained unless P\textsubscript{H}
Damping in a superconducting mechanical resonator

is further reduced. However, no significant reduction of $P_H$ is possible in our setup due to noise, and consequently reliable data far below 0.5 K cannot be obtained. Note, however, that above 0.5 K no significant change in the damping rate is obtained when the measurements are repeated with a laser power two times higher, verifying thus the damping rate is obtained when the measurements are repeated.

To account for the experimental results the measured damping rate $\gamma_m$ is compared with theory. Let $\gamma_m = \gamma_e + \gamma_S$, where $\gamma_e$ denotes the electronic contribution to the total rate of mechanical damping $\gamma_m$, and $\gamma_S$ denotes the contribution of all other mechanisms. Well below the critical temperature the phonon energy $\hbar\omega_{\text{ph}}$ becomes far smaller than the superconducting energy gap $\Delta$, and consequently the electronic contribution $\gamma_e$ is expected to drop well below its value in the normal phase, which is denoted by $\gamma_N$. The solid line seen in fig. 2 represents the calculated value of $\gamma_m$ obtained from the following expression:

$$\gamma_m = \frac{2\gamma_N}{1 + \exp \frac{\Delta(T)}{k_B T}} + \gamma_S,$$  

(2)

where the fitting parameters $\gamma_N$, $\gamma_S$ and $T_c$ are taken to be given by $\gamma_S/2\pi = 3.9$ Hz, $\gamma_N/2\pi = 1.0$ Hz and $T_c = 1.1$ K. The temperature dependence of $\gamma_S$ is assumed to be negligibly small in the experimentally explored range between 0.5 K and 1.3 K. The first term in eq. (2) represents the temperature dependence of ultrasonic attenuation according to the Bardeen Cooper Schrieffer (BCS) theory [31,32], which is obtained using the golden rule formula in the limit where $\hbar\omega_{\text{ph}} \ll \Delta$. The temperature-dependent energy gap $\Delta(T)$ is found by numerically solving the BCS gap equation [33]

$$\nu = \int_0^\infty dx \frac{\tanh \left(\frac{\xi\Delta(x)}{T}\right)}{\sqrt{1 + x^2}},$$  

(3)

where $\nu = 1/gD_0$ is the inverse interaction strength with $g$ being the electron-phonon coupling coefficient and $D_0$ being the density of states per unit volume, $\delta = \Delta/\Delta_0$ is the normalized gap with $\Delta_0$ being the zero temperature gap, $\tau = T/T_c$ is the normalized temperature, and the number $\xi$ is given by $\xi = \pi/2e^{C_E}$ with $C_E \approx 0.577$ being Euler’s constant. As can be seen from fig. 2, fair agreement between data and theory is obtained. A similar theoretical approach has been employed before to successfully account for the results of a measurement of a contact-less friction between a superconducting niobium film and a cantilever [34] (see also [35]).

The rate $\gamma_N$ has been calculated in ref. [36] for the case where the electron mean-free path is greater than the wavelength of the oscillating acoustic mode. However, this assumption is not valid for our device. When the electron mean-free path is shorter than the acoustic wavelength the rate $\gamma_N$ can be roughly estimated using Stokes’ law of sound attenuation [37,38], which relates this rate to the electronic viscosity [39]. For the case of an acoustic wave having angular frequency $\omega_p$ propagating in a bulk aluminum the rate according to this approach is given by

$$\gamma_{N,\text{bulk}} = \frac{2\eta_{AI}c^2}{3\rho_{AI} c_{AI}},$$  

(4)

where $\eta_{AI} = 2.7$ g cm$^{-3}$ and $c_{AI} = 5.1 \times 10^3$ m s$^{-1}$ are the mass density and the speed of sound, respectively, of aluminum, and where $\eta_{AI}$ is the electronic viscosity of aluminum. The electronic viscosity can be expressed as $\eta_{AI} = (2/3)\eta_{AI}\tau_{AI}(\epsilon_{AI})$ where $\eta_{AI}$ is the density of free electrons, $\tau_{AI}$ is the scattering relaxation time, and $\epsilon_{AI}$ is the averaged kinetic energy (see eq. (43.8) in [40]). At low temperatures $\epsilon_{AI} = 3e_Fc_{AI}/5$, where $e_Fc_{AI}$ is the Fermi energy. Using the values $\eta_{AI} = 18.1 \times 10^{-22}$ cm$^{-3}$, $\tau_{AI} = 6.5 \times 10^{-14}$ s and $e_Fc_{AI} = 11.7$ eV [41] one obtains $\gamma_{N,\text{bulk}} = 0.6$ Hz. This rough estimate, which disregards confinement, yields a value that is of the same order of magnitude as the value of $\gamma_N/\tau = 1.0$ Hz that has been obtained above from the fitting of the data with eq. (2).

In summary, the contribution of free electrons to the damping rate of an aluminum mechanical resonator is measured. The behavior near the normal to superconducting phase transition is compared with a prediction based on the BCS theory. The relative contribution of free electrons in the normal state to the total damping rate of the resonator under study is found to be $\gamma_N/(\gamma_N + \gamma_S) = 0.20$.

***

This work was supported by the Israel Science Foundation, the Binational Science Foundation, the Security Research Foundation at Technion and the Russell Berrie Nanotechnology Institute.

REFERENCES

[1] Erinci K. L. and Roukes M. L., Rev. Sci. Instrum., 76 (2005) 061101.
[2] Meyrueis Pierre, Ann. Phys. (Berlin), 525 (2013) 215.
[3] Poot M. and van der Zant H. S. J., Phys. Rep., 511 (2012) 273.
[4] O’Connell A. D., Hofheinz M., Ansmann M., Bialczak Radoslaw C., Lenander M., Neeley Erik Luceroand M., Sank D., Wang H., Weides M., Wenner J., Martinis John M. and Cleland A. N., Nature, 464 (2010) 697.
[5] Pikovskii Igor, Vanner Michael R., Aspelmeyer Markus, Kim M. S. and Brukner Caslaw, Nat. Phys., 8 (2012) 393.
[6] Weinstein A. J., Lei C. U., Wollman E. E., Suh J., Metelmann A., Clerk A. A. and Schwab K. C., Phys. Rev. X, 4 (2014) 041003.
[7] Teufel J. D., Donner Toblas, Li Dale, Harlow J. W., Allman M. S., Cicak K., Sirois A. J., Whittaker Jed D., Lehnert K. W. and Simmonds Raymond W., Nature, 475 (2011) 359.
[8] Chan Jasper, Mayer Alegre T. P., Safavi-Naeini Amir H., Hill Jeff T., Krause Alex, Gröblacher Simon, Aspelmeyer Markus and Painter Oskar, Nature, 478 (2011) 89.

[9] Unterreittmeier Quirin P., Faust Thomas and Kotthaus Jörg P., Phys. Rev. Lett., 105 (2010) 027205.

[10] Braginsky V. B. and Manukin A. B., Zh. Eksp. Teor. Fiz., 52 (1967) 986; Sov. Phys. JETP, 25 (1967) 653.

[11] Aspelmeyer Markus, Kippenberg Tobias J. and Marquardt Florian, Rev. Mod. Phys., 86 (2014) 1391.

[12] Metcalfe Michael, Appl. Phys. Rev., 1 (2014) 031105.

[13] Teufel J. D., Li Dale, Allman M. S., Cicak K., Sirois A. J., Whittaker J. D. and Simmonds R. W., Nature, 471 (2011) 204.

[14] Palomaki T. A., Teufel J. D., Simmonds R. W. and Lehnert K. W., Science, 342 (2013) 710.

[15] Yanai S., Singh V., Yuan M., Gely M., Bosman S. J. and Steele G. A., Mechanical dissipation in more superconducting metal drums, arXiv:1608.01871 (2016).

[16] Andrews R. W., Reed A. P., Cicak K., Teufel J. D. and Lehnert K. W., Nat. Commun., 6 (2015) 10021.

[17] Lecocq F., Clark J. B., Simmonds R. W., Aumentado J. and Teufel J. D., Phys. Rev. Lett., 116 (2016) 043601.

[18] Suh J., Shaw M. D., LeDuc H. G., Weinstein A. J. and Schwab K. C., Nano Lett., 12 (2012) 6260.

[19] Wollman E. E., Lewis T. Jr., Weinstein A. J., Suh J., Kronwald A., Marquardt F., Clerk A. A. and Schwab K. C., Science, 349 (2015) 952.

[20] Pirkkalainen J.-M., Damskågg E., Brandt M., Massel F. and Sillanpää M. A., Phys. Rev. Lett., 115 (2015) 243601.

[21] Suchoi Oren, Shlomi Keren, Ella Lior and Buks Eval, Phys. Rev. A, 91 (2015) 043829.

[22] Lulla K. J., Defoort Martial, Blanc Christophe, Bourgeois Olivier and Collin Eddy, Phys. Rev. Lett., 110 (2013) 177206.

[23] Suchoi Oren, Ella Lior, Shtempluk Oleg and Buks Eval, Phys. Rev. A, 90 (2014) 033818.

[24] Metzger C., Ludwig M., Neuenhahn C., Orlt J., Favero I., Karrai K. and Marquardt F., Phys. Rev. Lett., 101 (2008) 133903.

[25] Zaitsev Stav, Pandey Ashok K., Shtempluk Oleg and Buks Eval, Phys. Rev. E, 84 (2011) 046605.

[26] Dhayalan Yuvaraj, Baskin Illya, Shlomi Keren and Buks Eval, Phys. Rev. Lett., 112 (2014) 210403.

[27] Shlomi Keren, Yuvaraj D., Baskin Illya, Suchoi Oren, Winik Roni and Buks Eval, Phys. Rev. E, 91 (2015) 032910.

[28] Feschenko A. V., Saira O.-P., Peltonen J. T. and Pekola J. P., Thermal conductance of nb thin films at sub-kelvin temperatures, arXiv:1609.06519 (2016).

[29] Zink B. L. and Hellman F., Solid State Commun., 129 (2004) 199.

[30] Woodcraft Adam L., Sudiwala Rashmi V., Waki Elley, Bhatia Ravinder S., Bock James J. and Turner Anthony D., Physica B: Condens. Matter, 284 (2000) 1968.

[31] Persson B. N. J., Solid State Commun., 115 (2000) 145.

[32] Claiborne Lewis T. Jr. and Morse Robert W., Phys. Rev., 136 (1964) A893.

[33] Bardeen John, Cooper Leon N. and Schrieffer John Robert, Phys. Rev., 108 (1957) 1175.

[34] Kienel Marcin, Gnecco Enrico, Gysin Urs, Marot Laurent, Rast Simon and Meyer Ernst, Nat. Mater., 10 (2011) 119.

[35] Dayo A., Alnasrallah W. and Krim J., Phys. Rev. Lett., 80 (1998) 1690.

[36] Lindenfeld Ze’ev and Lifshitz Ron, Phys. Rev. B, 87 (2013) 085448.

[37] Stokes G. G., Trans. Cambridge Philos. Soc., 8 (1845) 287.

[38] Kittel C., Rep. Prog. Phys., 11 (1947) 205.

[39] Heinisch Martin, Voglhuber-Brunnmaier Thomas, Reichel Erwin K., Dufour Isabelle and Jakoby Bernhard, Sens. Actuators A: Phys., 220 (2014) 76.

[40] Kittel Charles, Elementary Statistical Physics (Courier Corporation) 2004.

[41] Ashcroft Neil W. and David N. Mermin, Solid State Physics (Saunders College, Philadelphia) 1976.