Two-dimensions finite-difference heat transfer study for a single borehole heat storage system during a seasonal charge-discharge cycle

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Abstract. Borehole heat exchangers are used to remove and/or add heat from/into the ground in different applications involving low-temperature heat usage, such as residential or social buildings heating by means of heat pumps. In such types of applications, seasonal heat storage is necessary since during the warm season an additional solar system is used to charge heat into the ground via the borehole system in order to regenerate the heat source for usage during the cold season. Heat transfer in the ground by means of the borehole results in both radial and axial propagations of the perturbation (ground heating/cooling). The magnitude of this perturbation and the added/removed heat are important parameters of the process that must be evaluated. The theoretical model presented in the paper uses the decomposition finite difference technique to determine the temperature distribution in the ground at different moments (1, 2, 4, and 6 months) during a charge/discharge cycle of the heat storage. The numerical results are necessary in order to evaluate the penetration depth of the thermal perturbation along the radial and longitudinal axes and the amount of stored/extracted heat.

1. Introduction
The continuous growth of the energy demand of humankind in order to satisfy its mounting necessities requires burning enormous amounts of fossil fuels. The consequence is the environment’s degradation which is about to reach the threshold of irreversibility. Phenomena such mountain glaciers retraction and polar caps melting, increasingly unpredictable weather patterns, etc. rise serious concerns regarding the alteration of living conditions at a global scale.

One solution to overcome this drawback is to use alternate forms of energy, such as renewables. Since the ground contains huge amounts of quasi-isothermal, low-temperature heat at depths less than a few tens of meters, a feasible idea might be to use the ground as a heat source for heat pumps - called Ground-Source Heat Pumps (GSHPs) - in residential heating applications. This solution only applies to countries with no geothermal potential; privileged countries such as Iceland or New Zealand [1] benefit from high-temperature ground heat due to volcanism and therefore heat pumps are not necessary. Most of the countries with volcanic activity on their territory use geothermal energy to generate power: 24 of them report that 15-22% of the power is obtained from geothermal energy [1], whereas 82 countries use directly this energy for residential heating [2, 3].

Direct utilization of geothermal heat represented at the end of 2014 a total 70,329 MWt of installed thermal power covering a consumption of 163,287 GWh/yr, distributed as follows: 55.3% ground-
source heat pumps, 20.3% bathing and swimming, 15.9% space heating, and the rest for other purposes (greenhouse heating, industrial process heating, snow melting, etc.) [2].

Although the technology dates from the 1940’s [4], the interest in this solution was very weak until the increase of the greenhouse gases effects on the global climate made necessary a radically different approach in obtaining energy (for power and heating).

GSHPs are increasingly used in residential heating and domestic hot water production applications in regions or countries with low geothermal potential where only the heat naturally contained in the ground is available, at temperatures in the range 8 … 20°C. Presently, 50,000 units are installed annually in the USA, with a capacity of around 9,600 MWht [1]. One of the iconic examples is a 19.6 MW heating/15.8 MW cooling unit for 100 apartments and 89,000 m² of office surface area (161,650 m² in total) which supplies heat for Galt House Hotel (Louisville, Kentucky), thus saving 47 percent of the necessary energy, representing $25,000 per month [2]. In Europe, one must mention Mining Engineering Building of the National Technical University of Athens, which is heated and cooled by a GSHP, as a result from a pilot project from 1993, accomplished with Swiss support [4].

There are three configurations for the underground heat exchangers [5, 6]: horizontal loop, spiral loop, and vertical loop (borehole). When the available surface area is small, the third solution is recommended. A specific advantage of this configuration is the fact that below a depth of 15-20 meters, the ground temperature remains practically constant all year long, which provides a quasi-isothermal heat reservoir. The depth of the borehole is usually in the range 23 … 100 m [6].

The paper deals with the investigation of the conduction heat transfer in the ground during injection followed by extraction of the heat by a single borehole working as a heat exchanger for a ground heat storage unit. A six-month operation is supposed for each of the two modes (injection or charging, respectively removal or discharging), roughly representing the warm and the cold seasons respectively. Heat is stored in the ground during the warm season, and is removed when there is a heat demand in order to operate a heat pump for residential heating purposes.

Analytical methods are very difficult to use to study the unsteady conduction heat transfer within the ground (and only in very simple situations - one-dimension heat propagation in bodies with simple geometric shapes, constant thermo-physical properties of the heat transfer medium, and Dirichlet boundary conditions) [7, 8]. For instance, one-dimension unsteady heat conduction in long cylinders involves the use of Bessel functions, which are very difficult to manipulate especially when it comes to write the boundary conditions [8].

With respect to the considered mathematical model for a single borehole, Bandos et al. [9], Lamarche and Beauchamp [10], use the finite-source model to determine the effect of vertical temperature variations, and Philippe et al. [11], compare the results provided by three analytical methods: the infinite line source, the infinite cylindrical source and the finite line source respectively.

The actual phenomenon of heat propagation across the ground involve two- or three- dimension unsteady conduction heat transfer with Neumann boundary conditions, making the analytical approach almost impossible to use, due to the complications associated with finding solutions for the differential equations that describe the theoretical model. A different, more pragmatic, less accurate, approach involves the use of numerical techniques, among which one of the most popular is the finite difference method [12], [13], [14]. Due to the difficulties arisen by the analytical model involving two-dimension unsteady conduction heat transfer in the studied case, the approach presented in the paper makes use of the finite differences technique.

When modelling a two-dimension heat conduction, two variations of the finite difference technique can be used: the explicit finite difference method (EFD), which allows the calculation of the present time temperature in the current node, respectively the implicit scheme (IFD), where three nodal temperatures are simultaneously considered at the present time step. The EFD determines the unknown temperature in a sequential manner, but the magnitude of the time step is drastically limited in order to secure the stability of the scheme. In the EFD such a constraint does not operate the scheme being unconditionally stable, but the method involves the necessity to solve a set of linear algebraic equations corresponding to the total number of nodes [14, 15].
The paper deals with the use of the decomposition technique (DT) of the IFD scheme in two dimensions, in cylindrical coordinates, for a cycle of heat injection in the ground followed by heat removal from the ground by a single borehole, with an intermediate “standby” time gap when no heat transfer from the borehole occurs.

2. The physical model
A long hollow cylinder of length \( L \), inner radius \( R_0 \), and outer radius \( R_d \) represents the ground domain, initially isothermal at temperature \( T_\infty \). The ground material is supposed to be homogenous and its physical properties (density \( \rho \), thermal conductivity \( \lambda \), specific heat \( c \), and thermal diffusivity \( a \)) are not temperature-dependent. Figure 1 shows the schematic of half the domain, due to the symmetrical configuration of the cylindrical geometry.

During the charging stage of the cycle, a fluid of temperature \( T_H > T_\infty \) flows along the vertical tube (representing the borehole), transferring heat by convection (convection heat transfer coefficient \( k_r \)) to the cylindrical domain that represents the ground. The latter is supposed to have adiabatic boundaries on its outer and bottom surfaces, whereas the frontal surface (in contact with air) allows the convection heat transfer between the ground and the atmospheric air to occur (convection heat transfer coefficient: \( k_z \)).

During the heat removal stage (discharge), a cold fluid of temperature \( T_L \), lower than \( T_\infty \), flows along the central channel (tube) that represents the borehole, cooling the cylindrical domain (the ground). Heat is removed by convection with the heat transfer coefficient \( k_z \).

The essence of the decomposition approach is described in [15]. During a time step of the procedure, the two-dimension conduction heat transfer is split into two one-dimension heat transfer processes that occur sequentially: first in radial direction (intermediate stage), and then axially (final stage). The order of the two one-dimension processes for a time step is arbitrary. The final temperatures at the end of the previous (superscript \( p-1 \)) time step \( \Delta \tau \) are the start temperatures (known) of the intermediate time step (superscript \( i \)). The intermediate temperatures resulting from the
application of the decomposition procedure in its first (intermediate) stage are the input temperatures for the second (final) one the output of which is represented by the final temperature (superscript \(p\)).

The “trick” of splitting a two-dimension heat conduction process into two two-dimension ones results in a significant simplification, because one only must deal with very simple finite difference equations, which are much easier to manipulate.

The heat conduction equation written corresponding to the direction of propagation is [7], [8]:

- radial propagation of the conduction heat transfer (in cylindrical coordinates):

\[
\frac{\partial \theta}{\partial \tau} = \frac{\lambda}{\rho c} \left( \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2} \right)
\]  

(1)

- axial propagation (in Cartesian coordinates):

\[
\frac{\partial \theta}{\partial \tau} = \frac{\lambda}{\rho c} \frac{\partial^2 \theta}{\partial z^2}
\]

(2)

Since the model deals with a two stages-cycle governed by different values of the heat transfer fluid temperature, it is necessary to define a unique criterion when considering both heat removal from the ground (discharging stage) and heat injection into the ground (charging stage). Therefore, a dimensionless temperature \(\theta\) is defined with respect to the hot fluid temperature \(T_H\) and the temperature \(T_\infty\) of the initially isothermal ground:

\[
\theta = \frac{T - T_\infty}{T_H - T_\infty}
\]

(3)

As a result of the above description of the domain, the boundary conditions can be written as:

- at \(r = R_0\):

\[
k \theta_0 = -\lambda \left( \frac{\partial \theta}{\partial r} \right)_{r=R_0}
\]

(4)

where \(\theta_0\) represents the dimensionless temperature of the central channel’s surface.

- at \(z = 0\):

\[
k \left( \theta_a - \theta_0 \right) = -\lambda \left( \frac{\partial \theta}{\partial z} \right)_{z=0}
\]

(5)

where \(\theta_a\) represents the dimensionless temperature of atmospheric air above the frontal surface of the domain.

- at \(r = R_d\):

\[
\left( \frac{\partial \theta}{\partial r} \right)_{r=R_d} = 0
\]

(6)

- at \(z = L\):

\[
\left( \frac{\partial \theta}{\partial z} \right)_{z=L} = 0
\]

(7)

3. Finite difference approach

A rectangular mesh has been attached to the cylindrical domain, consisting of \(N_r\) nodes in the radial direction (radial step: \(h_r\)) and \(N_z\) nodes in the axial direction (axial step: \(h_z\)) – see figure 2.

The heat transfer equation (1) can be rewritten in finite difference form for a time step \(\Delta \tau\), by using the finite difference operator \(D^2\) as follows [13], [14]:

\[
\frac{\theta_{m,n}^p - \theta_{m,n}^{p-1}}{\Delta \tau} = a D_r^2 \theta_{m,n}^p
\]

(8)
where \( a = \frac{\lambda}{\rho c} \) stands for the thermal diffusivity of the ground, \( \Delta \tau \) for the time step, and \( p \) for the current time step number.

The operator \( D^2_r \) can be expressed as [16]:

\[
D^2_r \theta_{m,n} = \frac{1}{h^2_r} \left( \frac{c_m}{c_m} \theta_{m-1,n} + \frac{c_m}{c_m} \theta_{m+1,n} + \frac{c_m + 1}{c_m} \theta_{m,n} \right)
\]  
\[\text{(9)}\]

where:

\[
c_m = 2 \left( N \frac{R_0}{R_0 - R} + m \right)
\]  
\[\text{(10)}\]

Figure 2. The rectangular mesh attached to the domain.

Equation (2) can be written in finite difference form as:

\[
\theta_{m,n}^{p+1} - \theta_{m,n}^{p} = aD^2_r \theta_{m,n}^p
\]  
\[\text{(11)}\]

where

\[
D^2_z \theta_{m,n}^p = \frac{1}{h^2_z} \left( \theta_{m,n+1}^p - 2 \theta_{m,n}^p + \theta_{m,n-1}^p \right)
\]  
\[\text{(12)}\]

The convergence and stability criteria for the two directions of propagation of the decomposition technique are:

\[
\alpha_r = \frac{a \Delta \tau}{h^2_r} \quad \text{(radial direction)}
\]  
\[\text{(13)}\]

\[
\alpha_z = \frac{a \Delta \tau}{h^2_z} \quad \text{(axial direction)}
\]  
\[\text{(14)}\]

After a series of manipulations [16], equation (9) becomes:

\[
-\theta_{m-1,n}^{p} + \sigma_r \theta_{m,n}^{p} - \gamma_r \theta_{m+1,n}^{p} = \beta_r
\]  
\[\text{(15)}\]

where:
\[
\sigma_{r,m} = \frac{c_m}{c_m - 1} \left(2 + \frac{1}{\alpha_r}\right)
\]

(\ref{eq:16})

\[
\gamma_{r,m} = \frac{c_m + 1}{c_m - 1}
\]

(\ref{eq:17})

\[
\beta_{r,m} = \frac{c_m}{c_m - 1} \frac{\theta_{m,n}^{p-1}}{\theta_{m,n}^p}
\]

(\ref{eq:18})

In a similar manner \cite{16}, one obtains the finite difference form of equation (\ref{eq:12}):

\[-\theta_{m,n}^{p} + \sigma_{\xi} \theta_{m,n}^{p} - \theta_{m,n+1}^{p} = \beta_{\xi,n}^{p}
\]

(\ref{eq:19})

where:

\[
\sigma_{\xi} = 2 + \frac{1}{\alpha_{\xi}}
\]

(\ref{eq:20})

\[
\beta_{\xi,n} = \frac{1}{\alpha_{\xi}} \theta_{m,n}^{p}
\]

(\ref{eq:21})

The temperatures on the boundaries of the domain result from the boundary conditions written in the finite difference form.

The decomposition technique, as described in \cite{15}, involves the use, during a time step $\Delta \tau$, in a successive manner, of two one-dimension implicit finite difference schemes corresponding to the two directions of propagation (radial and axial respectively - the order is arbitrary). From the physical standpoint, the two-dimension heat propagation is firstly “decomposed” in a radial propagation by inserting thermal insulating barriers (separated by a space step) between the nodes, parallel to the direction of propagation thus preventing heat from diffusing in the normal direction. This stage is followed by the axial heat propagation stage by removing the insulating barriers and placing them parallel to the axial propagation and thus forcing heat to propagate in this direction.

\section{Results and discussion}

The decomposition method described above applied to the finite difference technique has been used to study a single borehole ground thermal storage system consisting of a 20 m long tube 0.25 m across and a surrounding ground region of radius $R_d = 5.125$ m. The temperatures were: $T_H = 100^\circ$C, $T_c = 10^\circ$C, and $T_L = 5^\circ$C. The thermo-physical properties of the ground were: $\lambda = 0.25$ Wm$^{-1}$K$^{-1}$, $a = 0.18 \times 10^{-6}$ m$^2$s$^{-1}$. The convection heat transfer coefficients were $k_t = 1000$ Wm$^{-2}$K$^{-1}$ and $k_r = 10$ Wm$^{-2}$K$^{-1}$ respectively. The mesh attached to the cylindrical domain had $N_r = 50$ and $N_z = 50$ nodes respectively.

A computer code was written for the algorithm of the decomposition technique and was run in order to obtain the nodal temperatures at 30, 60, 120, and 180 days respectively for the two stages of the heat storage cycle. The computer code had also the option to list the stored heat’s evolution during each stage. The final nodal temperatures supplied by running the computer code written for the charging (heat injection) stage as well as the other necessary data have been stored in a data file, which has been used to initiate the computer code for the discharge (heat removal) stage.

The output data supplied by the computer codes for the charging, respectively the discharge stages of the cycle have been used to generate two families of 3-D plots: wireframe and colour contour.

The plots corresponding to the charging stage are displayed in figures 3 through 10.

One can notice that the heat diffusion is very slow: the penetration radius is about 10 nodes after 60 days and about 15 nodes after 180 days. This can be explained by the low conductivity of the ground material.

The numerical results confirm that the choice of the cylindrical domain’s radius was good. This choice is justified by the fact that a semi-infinite domain in the radial direction is a hypothesis that cannot be taken into account for the numerical model, since the finite difference approach cannot operate with infinite values. For this reason, the only choice left is to consider a boundary of finite...
radius at which an adiabatic boundary condition must be imposed. If this radius is long enough, the heat perturbation induced by the presence of the central channel (tube) which adds or removes heat cannot reach the outer boundary.

**Figure 3.** Wireframe plot of the temperatures at charging after 30 days.

**Figure 4.** Contour plot of the temperatures at charging after 30 days.

**Figure 5.** Wireframe plot of the temperatures at charging after 60 days.

**Figure 6.** Colour contour plot of the temperatures at charging after 60 days.

**Figure 7.** Wireframe plot of the temperatures at charging after 120 days.

**Figure 8.** Colour contour plot of the temperatures at charging after 120 days.
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Figure 9. Wireframe plot of the temperatures at charging after 180 days.

Figure 10. Colour contour plot of the temperatures at charging after 180 days.

As for the bottom surface of the cylindrical domain, the adiabatic boundary hypothesis is sufficiently accurate, because the end effects of the central tube are negligible.

The discharge stage of the yearly cycle represents 180 days. Figures 11 through 18 display the temperature plots in 3-D (wireframe) and colour contour respectively for elapsed periods of 30, 60, 120, and 180 days.

Figure 11. Wireframe plot of the temperatures during discharging after 30 days.

Figure 12. Colour contour plot of the temperatures during discharging after 30 days.

Figure 13. Wireframe plot of the temperatures during discharging after 60 days.

Figure 14. Colour contour plot of the temperatures during discharging after 60 days.
The change of the temperature distribution across the domain is very noticeable during discharging, along with the temperature decrease due to the heat removal. The colour contour plots (figures 12, 14, 16, and 18) highlight a warmer core (in red and orange) as a result of the concurrent heat removal by both the cold fluid flowing through the central channel and by the atmospheric air. The evolution of this central core is seemingly not in accordance with an expected shrinking as the result of the cooling of the ground. However, this discrepancy can be explained by the different temperature ranges allocated to every conventional colour at different elapsed times by the data analysis and graphing software used to generate the graphs displayed in the paper.

Negative dimensionless temperatures of the $z$-axis of the 3-D wireframe plots are explained by the fact that the ground is cooled in some regions (rendered more visible on the colour contour plots) to temperatures that are less than the initial temperature $T_\infty$ of the ground.

Between the two stages of the storage cycle, we have considered a five days “standby” period, during which there is no heat transfer whatsoever except the heat exchange with atmospheric air.

The evolution of the stored heat during the charging stage of the heat storage is shown on figure 19 and on figure 20 for the discharging stage respectively. The end value for the charging period differs from the start value for the discharging one because of the time gap of five days between the two stages, when some heat is lost to the atmospheric air across the frontal surface of the domain.

After the discharging stage ends, a small amount of the stored heat remains in the ground.
5. Conclusions

Borehole heat storage systems represent a promising solution to store heat in the ground and to remove it in order to make it useful for ground-source heat pumps (GSHPs). This approach improves the performance of heat pumps used to provide heat and domestic hot water for residential applications.

Such a system adds heat into the ground during the warm season when this is available via solar collector heat conversion systems and extract it in order to operate the heat pump during the cold season.

In order to determine the amount of heat that can be accumulated in the ground with a borehole ground heat storage system, a study must be performed to evaluate the temperature distribution at every moment. A practical approach is to use the finite-difference technique for the two-dimension unsteady conduction heat transfer in the cylindrical setting representing the central tube and the ground.

The decomposition technique has been applied to a two-dimension implicit finite-difference scheme used to calculate the temperature distribution across a cylindrical domain that represents the ground around a central tube along which alternately flow hot and cold fluids respectively that transfer heat to/from the ground.

The numerical results obtained by running a computer software developed by the authors show that the penetration radius of the thermal perturbation during the charging stage is small because of the low thermal conductivity of the ground.

The computer software also allowed to determine the amount of heat stored in the ground at every moment and thus to provide data for future developments involving the calculation of the ground heat pump performance.

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