A wide-template and high-accuracy data transfer method for unstructured adjoint-based grid adaptation

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Abstract. Grid adaptation is an important way to improve the identification of flow field and the accuracy of aerodynamic characteristics. Adjoint-based error estimation and grid adaptation method focuses on improving aerodynamic characteristics by optimizing local grids areas which are more sensitive to output functions. The data transfer between different meshes during the iteration of adjoint-based grid adaptation is the basis of accurate error estimation and grid adaptation. The traditional vertex-based method, however, is not suitable for cell-centred scheme finite volume method. A new wide-template and high-accuracy data transfer method for unstructured adjoint-based grid adaptation is proposed in this paper. Numerical results show that the proposed method is more accurate than traditional node-based method when transferring discontinuous flow field containing shock waves in cell-centred scheme finite volume method, and the accuracy of aerodynamic characteristics is obviously improved after grid adaptation.

1. Introduction

Numerical simulations have become indispensable components of the design process in aerospace industry due to their potential to provide valuable physical data thereby reducing the need for costly wind tunnel testing. Computational accuracy mainly depends on numerical scheme and mesh quality. Traditionally, Computational Fluid Dynamics (CFD) researchers can get high-quality mesh by two methods, one is to refine the all grid areas uniformly, another is to predict the flow features and then refine the related grid areas. Definitely the uniform refined grid can improve computation accuracy, however, the fine mesh may cost too much computational resource and time. Besides, it requires rich practical experiences to predict the flow features, especially for some new and complicated configurations.

Grid adaptation is a strategy to minimize the cost of a computational simulation while achieve high accuracy [1]. There are two main grid adaptation methods, one is feature-based grid adaption, which focuses on typical flow features such as shock waves and vertexes [2]. Feature-based grid adaption method detects the flow features and refines the related grid regions to improve the identification of flow field. Some features detectors are employed to identify grid areas where there locate some flow features, for example, weighted gradient detector and normal Mach number detector are employed to capture shock waves, and entropy increase detector and eigenvalue detector are employed to capture vertexes [3].
On the other hand, the majority of engineering CFD analysis and design applications focus on output functions, such as coefficient of lift, drag, and momentum. Feature-based grid adaption, however, in some cases, is not helpful to improve the accuracy of output functions, refining the grid areas may have minimal effect on output functions, such as the downstream shock wave [4]. Another promising grid adaptation method for engineering applications is to optimize the grid areas where the grids are contributing to the accuracy of output functions. Many efforts have been done on adjoint-based grid adaptation and error correction. Park [5-7] applied these techniques to three-dimensional isotropy unstructured grids. Park and Aftosmis [8] developed an adjoint-based grid adaptation method on structured grids, and applied this method to low boom configuration analysis with FUN3D solver. Oliver and James [9-10] combined adjoint-based adaptation with high-order numerical scheme, which obviously improved the accuracy of output function. Yamaleev and Diskin [11-12] applied those methods to two-dimensional unsteady problems on unstructured grids.

Error estimation is the basis of adjoint-based grid adaptation, and a uniform refined embedded mesh is created to estimate the error of current grid in grid adaptation iterations. However it is expensive to solve the flow equations and adjoint equations on uniform refined mesh. A compromised way is to reconstruct the flow and adjoint of refined mesh according to the flow and adjoint of current mesh. So the data transfer method plays an important role in adjoint-based error estimation and grid adaptation.

This paper aims at developing a wide-template and high-accuracy data transfer method for unstructured adjoint-based grid adaptation based on cell-centred scheme finite volume method. Numerical results show that the proposed method is more accurate than traditional node-based method when transferring discontinuous flow field containing shock waves in cell-centred scheme finite volume method.

2. Numerical Method
The numerical simulations here are conducted by the China Aerodynamics Research & Development Center (CARDC) NNW-FlowStar software [13-14], an unstructured finite volume cell-center CFD solver. Flowstar has attended several drag prediction workshops(DPW) and high-lift prediction workshops(HLWP) organized by AIAA and NASA, and it behaved as good as some other software such as Fluent, such as Fluent, FUN3D, CFL3D, CFD++ [15].

2.1. Adjoint Equations
Output function \( f \) can be expressed as a function of flow solution \( f=f(U) \), the chain rule for the output function is:

\[
\frac{\partial f}{\partial U} = \left( \frac{\partial f}{\partial R} \right)^T \frac{\partial R}{\partial U} \tag{1}
\]

Where \( R \) is the flow equation residual, adjoint variable \( \Psi \) is defined as the output function’s sensitivity to flow equation residual:

\[
\Psi \equiv \frac{\partial f}{\partial R} \tag{2}
\]

Adjoint equations can be expressed as the combination of equations (1) and (2):

\[
\left( \frac{\partial R}{\partial U} \right)^T \Psi = \left( \frac{\partial f}{\partial U} \right)^T \tag{3}
\]

The solve methods of adjoint equations is integrated in Flowstar, and LU-SGS scheme and multi-grid algorithm are employed to solve adjoint equations [16-17].
2.2. Error Correction

Consider two different meshes, a coarse mesh \( \Omega_c \) and a fine mesh \( \Omega_h \), with a solution on the coarse mesh \( \Omega_c \), it is desirable to predict the output function on the mesh \( \Omega_h \), the flow solution on coarse grid \( U^c_h \) can be interpolated to fine grid to compute the output function on fine mesh, and the prediction can be computed with the adjoint variables. The detailed derivation of adjoint-based adaption and error correction is available in Refs [5] and [9], following are some important results.

Prediction of output function on the fine mesh can be expressed as:

\[
 f_{est} \approx f_h(U^c_h) - \Psi^T_h R_h(U^c_h)
\]

(4)

where \( f_{est} \) is the predicted output function, \( f_h(U^c_h) \) is the output function computed by interpolation flow solution, \( \Psi^T_h R_h(U^c_h) \) is an error correction of output function, \( U^c_h \) and \( \Psi^c_h \) are flow solution and adjoint variables interpolated from coarse mesh to fine mesh, and \( R_h(U^c_h) \) is residual of flow equations computed by \( U^c_h \) on fine mesh. Two kinds of interpolation method, linear interpolation and quadratic interpolation, are considered in this study, thus yield the linear correction and quadratic correction for output function:

\[
 f_{est}^L = f_h(U^c_h) - (\Psi^T_h) R_h(U^c_h)
\]

\[
 f_{est}^Q = f_h(U^c_h) - (\Psi^Q_h) R_h(U^c_h)
\]

(5)

Where \( L \) and \( Q \) represent linear interpolation and quadratic interpolation.

There are still remaining errors after correction which can be expressed as:

\[
 f_{est} - f_h(U_h) = (\Psi_h - \Psi^c_h) R_h(U^c_h)
\]

\[
 f_{est} - f_h(U_h) = (U_h - U^c_h) R_h(\Psi^c_h)
\]

(6)

Where \( U_h \) and \( \Psi_h \) are flow solution and adjoint variables on fine mesh, which would cost much computational resource, they can be replaced by quadratic interpolated flow solution and adjoint variables, then these two kinds of remaining errors can be expressed as:

\[
 (\Psi_h - \Psi^c_h) R_h(U^c_h) = (\Psi^Q_h - \Psi^L_h) R_h(U^c_h)
\]

\[
 (U_h - U^c_h) R_h(\Psi^c_h) = (U^Q_h - U^L_h) R_h(\Psi^c_h)
\]

(7)

The average of the absolute values of the two remaining errors yields the adaption parameter, which represents the upper bound of remaining error in each cell:

\[
 \epsilon = \frac{1}{2} \left[ \left| R_h(\Psi^Q_h) \right| \left| U^Q_h - U^L_h \right| + \left| \Psi^Q_h - \Psi^L_h \right| \left| R_h(U^Q_h) \right| \right]
\]

(8)

2.3. Data Transfer Method

A uniform refined embedded mesh is created to compute the adaption parameter and error estimation. Several methods are considered to construct the embedded mesh, an original tetrahedron can be divide into two, four or eight interior tetrahedrons (Fig.1). Among all those methods the one-eight is employed in this study, the four new tetrahedrons constructed at the corners of the original tetrahedron have the same shape as the original tetrahedron, thus the new fine grids keep the quality as original grids. Besides, the new mesh after grid adaptation must be optimized, Laplacian method is employed to smooth the grids in this study [18].
The flow variables and adjoint variables are required to be interpolated to the embedded mesh to compute the error correction and adaption parameter, each variable is interpolated in two methods, linear interpolation and higher order interpolation. The interpolation method in most published literatures is based on grid nodes and employs cell-vertex scheme finite volume method (Fig.2).

Flowstar solver employs the cell-centred scheme finite volume method and it would be very complicated for cell-centred method to use vertex-based method (Fig.3).
A wide-template polynomial data transfer method which suits for cell-centred scheme is proposed in this study. Linear interpolation is based on an assumption that the variables are linear-distributed in original cell \( k \), and meet the linear equation:

\[
q = a + bx + cy + dz
\]  

(9)

There are four unknown parameters, suppose the variables in all the \( m \) donor cells which share the same node with cell \( k \) also meet this linear equation (Fig. 2), thus generates \( m \) linear equations:

\[
q_i = a + bx_i + cy_i + dz_i, i = 1 \sim m
\]  

(10)

The four parameters can be computed by least-squares method. The \( m \) donor cells have different weight which depends on how far the donor cell is from cell \( k \), the closer the donor cell is from cell \( k \), the greater weight it has. Thus the weight of donor cell can be computed as:

\[
wr = 1 / l^2
\]  

(11)

Where \( l \) is the distance between the center of donor cell and cell \( k \). The linear interpolation can be computed with the four parameter and coordinates of embedded grid.

Also, the higher order interpolation is based on an assumption that the variables are quadratic-distributed in original cell \( k \), and meet the quadratic equation:

\[
q = a + bx + cy + dz + ex^2 + fy^2 + gz^2 + hxy + lxz + pyz
\]  

(12)

There are ten unknown parameters, similar to linear interpolation, suppose the variables in all the \( m \) donor cells also meet this quadratic equation, thus generates \( m \) quadratic equations:

\[
q_j = a + bx_j + cy_j + dz_j + ex_j^2 + fy_j^2 + gz_j^2 + hx_jy_j + lx_jz_j + my_jz_j, j = 1 \sim m
\]  

(13)
3. Results and Discussion
The wide-template data transfer method is validated by several standard mathematic functions and a transonic flow field, and is applied to adjoint-based grid adaptation. The results show that this method is more accurate than vertex-based method, and is suitable for cell-centred scheme finite volume method.

3.1. quadratic function

\[ f(x) = 3x^2 + 100 \]  
(14)

Second-order discretization is used in most CFD flow solvers, so a quadratic function and M6-wing are employed to validate the wide-template data transfer method. Fig. 5 shows the comparison of vertex-based method and wide-template data transfer method. It can be seen that both two methods can transfer the data accurately.

3.2. quintic equation

\[ f(x) = 3x^5 + 100 \]  
(15)

A quintic function is employed to examine the capacity of the wide-template data transfer method to transfer the high-order functions. Fig. 6 shows the comparison of vertex-based method and wide-template data transfer method. It can be seen that both two methods can transfer the high-order function accurately.

3.3. discontinuous flow field
There exists a shock wave on M6-wing at Ma=0.84 and \( \alpha = 3^\circ \), and this case is employed to examine the capacity of the wide-template data transfer method to transfer discontinuous flow field. Fig. 7 shows

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**Figure 5. Validation of quadratic function**

(a) Vertex-based method  
(b) Wide-template method  
(c) Theoretical value

**Figure 6. Validation of quintic function**

(a) Vertex-based method  
(b) Wide-template method  
(c) Theoretical value
the comparison of vertex-based method and wide-template data transfer method. There is a $\lambda$ shock wave on the wing, it can be seen that the proposed method transfers the shock wave flow field more accurately, and the shock wave get a high resolution by proposed method.

![Figure 7. Validation of quintic function](image)

**Figure 7. Validation of quintic function**

3.4. **Error estimation and grid adaptation**

The NACA0012 airfoil is employed to validate error estimation and grid adaptation, and the computational condition is set as:

$$M_w = 0.8, \quad \alpha = 1.25^\circ, \quad T_w = 279.3 K, \quad Re = 9 \times 10^6$$

Output function is the coefficient of drag $C_d$, and the error tolerance is 10% of coefficient of drag which is computed in former adaption iteration.

Fig.8 shows the grid and pressure distribution before and after adaption, the grids around the shock waves, the leading edge and trailing edge are obvious refined after adaption, and shock waves are more accurate after grid adaptation. Fig 9 shows the process of drag adaption, the coefficient of drag meets the expected value with much less grids, and the accuracy of drag is greatly improved with grid adaptation and error correction.

![Figure 8. Grid and pressure of NACA0012](image)

**Figure 8. Grid and pressure of NACA0012**

![Figure 9. Drag adaption of NACA0012 airfoil](image)

**Figure 9. Drag adaption of NACA0012 airfoil**

4. **Conclusion**

A wide-template data transfer method for unstructured adjoint-based grid adaptation is proposed in this paper, which is more accurate than traditional node-based method when transferring discontinuous flow
field containing shock waves in cell-centred scheme finite volume method. And the error estimation and grid adaptation results show that the accuracy of aerodynamic characteristics is obviously improved after grid adaptation.

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