Effects of partial slip on micropolar fluid flow and heat transfer over a permeable shrinking sheet with thermal radiation and convective boundary condition

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Abstract. In the presence of thermal radiation, partial slip and convective boundary condition, micropolar fluid flow and heat transfer over a shrinking sheet has been studied. The problem is modeled as a mathematical formulation that involves a system of the partial differential equation. The similarity approach is adopted and self-similar ordinary differential equations are obtained and then those are solved numerically using the shooting method. The flow field is affected by the presence of physical parameters, such as micropolar parameter, wall mass transfer parameter and slip parameter whereas the temperature field is affected by thermal radiation, Biot number and Prandtl number. The skin friction, couple stress and heat transfer coefficients are tabulated and analyzed. The effects of the governing parameters on the velocity, angular and temperature profiles are illustrated graphically. Dual solutions of velocity, angular velocity and temperature are obtained for several values of the each parameter involved. Results shows that, the thickness of boundary layer decline as skin friction coefficient decreases for both solution as the velocity slip increases while for heat transfer coefficient, the physically increases of Biot number corresponding to increment of temperature distribution.

1. Introduction
The study of flow and heat transfer over a permeable shrinking sheet problem has received a great owing to its engineering applications, for instance in the extrusion of plastic sheet, glass blowing, crystal growing, and paper production. Non-newtonion fluids have a different kind of form that can be observed in our daily life such as animal blood, liquid crystal, exotic liquids and polymers etc. Last few decades, it has been a phenomenon and become very important to engineers and scientists working with hydrodynamic-fluid problems.

The study of flow and physical heat transfer micropolar fluids have been examined by the researchers due to the importance of various industrial applications. The micropolar fluid is a subclass of microfluid models that the pioneering work was made by Eringen [1]. However, an intensive investigation and extended Eringen’s work are conducted by many researcher [2-4] by considering different aspect. Damseh et al. [5] considered the effect of heat generation and first-order chemical reaction on micropolar fluids flows over a uniformly stretched permeable surface. Ishak [6] reported thermal boundary layer...
flow induced by linearly stretching sheet immersed in an incompressible micropolar fluid with radiation and constant surface temperature. He shows that the existence of radiation decreased the heat transfer rate at the surface. Ishak et al. [7] investigated on mixed convection two-dimensional boundary layer flow of a micropolar fluid near the stagnation point on a stretching vertical sheet. Das [8] examined the combined effects of thermophoresis and chemical reaction of a micropolar fluid over an inclined plate with variable fluid properties. Bhattacharyya et al. [9] analyzed micropolar fluid flow over a permeable shrinking sheet with radiation. Latiff [10] analyzed on forced bioconvection slip flow of a micropolar nanofluid past a stretching and shrinking sheet. Naveed et al. [11] studied the MHD flow of a micropolar fluid with thermal radiation over a curved stretching sheet. Recently, Bilal [12] discussed on Boundary layer flow of magneto-micropolar nanofluid flow in the presence of Hall and ion-slip using variable thermal diffusivity.

The motivation of the studies is to investigate the effects of the partial slip on micropolar flow and heat transfer over a permeable shrinking sheet with thermal radiation and convective boundary condition. The nonlinear ordinary differential equations obtained here are solved numerically by shooting method. The effects of governing parameters on the skin friction coefficients, wall couple stress coefficient and local Nusselt number are investigated and analysed.

2. Mathematical formulation
Consider the steady two-dimensional flow of micropolar fluid past a shrinking sheet in the presence of thermal radiation with partial slip and convective boundary condition. The steady state boundary layer equations for propose problem in the Cartesian coordinates are given by

\[
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0, \quad (1)
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left( v + \frac{\kappa}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \frac{\kappa \partial N}{\rho \partial y}, \quad (2)
\]

\[
u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = \frac{\gamma_1}{\rho j} \frac{\partial^2 N}{\partial y^2} - \frac{\kappa}{\rho j} \left( 2N + \frac{\partial u}{\partial y} \right), \quad (3)
\]

and

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa^*}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q}{\partial y}, \quad (4)
\]

subject to the following boundary conditions:

\[u = -cx + c^*[(\mu + k) \frac{\partial u}{\partial y} + kN], \quad v = v_w, \quad N = -m \frac{\partial u}{\partial y} - k \frac{\partial T}{\partial y} = h_w(T_w - T), \quad \text{at } y = 0, \quad (5)\]

\[u \rightarrow 0, \quad N \rightarrow 0, \quad T \rightarrow T_\infty \text{ as } y \rightarrow \infty. \quad (6)\]

Here \(u\) and \(v\) are velocities along the \(x\) and \(y\) direction, respectively. \(T\) is the fluid temperature, \(N\) is the microrotation vector normal to \(xy\)-plane, \(j\) is the microinertia density, \(\kappa\) is the vortex viscosity (gyro-viscosity), \(\mu\) is the dynamic viscosity, \(\rho\) is the density, \(\gamma_1\) is the spin gradient viscosity and \(\kappa^*\) is the thermal conductivity. We note that \(m\) is a constant such that \(0 \leq m \leq 1\). It is noted that \(m = 0\) indicates \(N = 0\) which is no-spin condition. The case \(m = 1/2\) represent the weak concentration of microelements. The case corresponding to \(m = 1\) is used for the modelling of turbulent flows. By succeeding the research of many researcher, we assumed assuming that:
\[ \gamma_1 = \left( \mu + \frac{K}{2} \right) j = \mu \left( 1 + \frac{K}{2} \right) j, \]  

(7)

where \( K = \kappa / \mu \) is the micropolar parameter.

Now regarding the approximation of Rosseland for radiation according to Basant et al [13], the radiative heat flux is simplified as 

\[ q_r = (4\sigma / 3k) \partial T^2 / \partial y, \]  

where \( \sigma \) is the Stefan–Boltzmann constant, \( k \) is the absorption coefficient. It is assume that the temperature differences within the flow such as the term \( T^4 \) can be expressed as a linear function of temperature and by expanding \( T^4 \) in a Taylor’s series about \( T_\infty \), neglecting higher order terms, we obtain, 

\[ T^4 = 4T_\infty^3 T - 3T_\infty^4. \]

Therefore, equation (4) reduces to:

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa^*}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma T_\infty^3}{3k \rho c_p} \frac{\partial^2 T}{\partial y^2}, \]  

(8)

It is assumed that the temperature differences within the flow such the term \( T^4 \).

The following dimensionless variables are introduced as follows:

\[ n = (cv)^{1/2}, \psi = (cv)^{1/2} x f(\eta), N = cx \left( \frac{c}{v} \right)^{1/2} h(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \]  

(9)

where \( \eta \) is the similarity variable and \( \psi \) is the stream function defined in the usual way to identically satisfied equation (1) as \( u = \frac{\partial \psi}{\partial y} \) and \( v = - \frac{\partial \psi}{\partial x} \).

On substituting equation (9) into equations (2) and (3) along with boundary conditions (8) are then transform into the following nonlinear ordinary differential equations:

\[ (1 + K) f'''' - f f''' + f'^2 + Kh' = 0, \]  

(10)

\[ (1 + K/2) h'' + f' h - f'' h + K(2h + f'') = 0, \]  

(11)

\[ (3R + 4) \theta'' + 3R Pr f \theta' = 0, \]  

(12)

subject to boundary conditions

\[ f(\eta) = S, f'(\eta) = -1 + \alpha(1 + K)f''(0), h(\eta) = -mf''(\eta), \]  
\[ \theta(\eta) = -\gamma(1 - \theta(0)) \text{ at } \eta = 0, \]  

(13)

\[ f'(\eta) \to 0, h(\eta) \to 0, \theta(\eta) \to 0 \text{ as } \eta \to \infty, \]  

(14)

where differentiation with respect to \( \eta \) is signified by (') \( Pr = \mu c_p / \kappa^* \) is the Prandtl number and \( R = \kappa k_1 / 4 \sigma T_\infty^3 \) is the thermal radiation parameter, \( S = -v_w / (cv)^{1/2} \) is the constant mass transfer parameter with \( S > 0 \) corresponds to suction and \( S < 0 \) corresponds to injection, \( \alpha \) is velocity slip parameter and \( \gamma \) is Biot number.

3. Results and discussion

The nonlinear ordinary differential equations (10) - (12) subject to the boundary conditions (13) and (14) were solved numerically using the shooting method. Numerical solutions for effects of partial slip on boundary layer flow in micropolar fluid in the presence of thermal radiation and convective boundary
condition are reported in this section. The results are presented graphically in figures 1 to 7. The values of the skin-friction coefficient $f''(0)$, wall couple stress coefficient $h'(0)$ and local Nusselt number $-\theta'(0)$ for various governing parameter are obtained. The effect of velocity slip parameter and Biot number for different values of suction parameter are given in table (1) and (2) respectively. The physical behavior of controlling parameters, like, suction parameter, radiation parameter, Prandtl number, Biot number and velocity slip parameter on momentum and thermal boundary layers as well as on friction, angular and heat coefficients are presented graphically through Figures 1 to 8. In these studied, dual solutions are obtained for some specific values of suction $S$ in which profiles of velocity, angular and temperature satisfy the boundary condition at infinity asymptotically.

Figure 1 presented the skin-friction coefficient, $f''(0)$ for various values of velocity slip. It is clearly observed that the effect of slip parameter, $\alpha$ is to decrease the skin friction coefficient for both first and second solutions. This portrayed that the growing values of velocity slips parameter, $\alpha$, diminish the boundary layer thickness. Meanwhile, for $h'(0)$ in figure 2, we can see that the first solution decreases and second solution increases with increases values of velocity slips parameter, $\alpha$. For the variation of velocity slip parameter $\alpha$ and Biot number $\gamma$, the heat transfer coefficient $-\theta'(0)$ increases for both first solution and second solution, it is observed in figure 3 and figure 4 respectively. However, in figure 4 shows the behavior of Biot number, $\gamma$, on temperature distribution and we can say that thermal boundary layer thickness is more prominent for higher values of Biot number. Strong heat transfer coefficient which results in enhancement of temperature distribution is due to physically increment values of Biot number, $\gamma$.

The effect of velocity slip and Biot number on the velocity, the angular and temperature profiles is illustrated in figures 5 to 8. The dual velocity profiles show that the velocity decreases with increasing magnitude of $\alpha$ in first solution and conversely for second solution it increase. In figure 5 we can see that $f'(\eta)$ decreases on increasing $\alpha$ for first solution but $f'(\eta)$ decreases with $\alpha$ for second solution. This is because when slip occurs, fluid velocity near the sheet is no longer equal to the velocity of the shrinking sheet and consequently fluid velocity decreases on increasing velocity slip factor.

However, for angular velocity profile shown in figure 6 shows that, both first and second solutions increases. This is due to increasing velocity slip and the boundary layer thickness increases. For temperature profiles in figure 7, the first solution decreases while the second solution increases with increasing $\alpha$. Figure 8 represents the graph of temperature profile for differents values of Biot number parameter, $\gamma$. It is interesting to observe that the effect of $\gamma$ is to increase the temperature distribution in the thermal boundary layer. This show that the results in enhancement of temperature distribution due to strong heat transfer coefficient with the increases of Biot number, $\gamma$.

4. Conclusions
In the present work, the effects of slip velocity and boundary convective condition on the flow and heat transfer of a micropolar fluid over a permeable shrinking sheet with thermal radiation have been studied. Results revealed that dual solutions are possible up to a certain range of the suction parameter. Dual solutions of velocity, angular velocity and temperature are obtained for different values of the governing parameter involved. Results show that, the thickness of boundary layer decline as skin friction coefficient decreases for both solution as the velocity slip increases while for heat transfer coefficient, the physically increases of Biot number corresponding to increment of temperature distribution.

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Table 1. Numerical values of skin friction coefficient $f''(0)$, couple stress coefficient $h'(0)$ and heat transfer coefficient $-\theta'(0)$ for different values of velocity slip parameter against wall mass transfer parameter by fixing $K = 0.1$, $Pr = 1.0$, $Q = 0$, $R = 1.0$, $n = 0.5$, $\gamma = 0.1$.

| $S$  | $\alpha$ | $f''(0)$ | $h'(0)$ | $-\theta'(0)$ |
|------|----------|----------|---------|---------------|
| 1.7397 | 0.4      | 0.6774   | 0.3266  | 0.0847        |
|       |          | (0.6552) | (0.3016) | (0.0841)      |
| 1.7418 |          | 0.6874   | 0.3385  | 0.0849        |
|       |          | (0.6401) | (0.2853) | (0.0839)      |
| 1.7438 |          | 0.6947   | 0.3474  | 0.0849        |
|       |          | (0.6320) | (0.2767) | (0.0838)      |
| 1.7459 |          | 0.7008   | 0.3549  | 0.0850        |
|       |          | (0.6258) | (0.2702) | (0.0837)      |
| 1.7479 |          | 0.7061   | 0.3616  | 0.0851        |
|       |          | (0.6205) | (0.2648) | (0.0837)      |
| 1.6885 | 0.5      | 0.6270   | 0.2999  | 0.0845        |
|       |          | (0.5997) | (0.2683) | (0.0837)      |
| 1.6908 |          | 0.6355   | 0.3102  | 0.0846        |
|       |          | (0.5888) | (0.2564) | (0.0836)      |
| 1.6931 |          | 0.6418   | 0.3182  | 0.0847        |
|       |          | (0.5819) | (0.2490) | (0.0835)      |
| 1.6954 |          | 0.6472   | 0.3250  | 0.0848        |
|       |          | (0.5764) | (0.2432) | (0.0834)      |
| 1.6977 |          | 0.6519   | 0.3312  | 0.0849        |
|       |          | (0.5717) | (0.2384) | (0.0833)      |
| 1.6457 | 0.6      | 0.5901   | 0.2850  | 0.0843        |
|       |          | (0.5458) | (0.2329) | (0.0832)      |
| 1.6565 |          | 0.6098   | 0.3111  | 0.0847        |
|       |          | (0.5256) | (0.2115) | (0.0829)      |
| 1.6674 |          | 0.6233   | 0.3300  | 0.0850        |
|       |          | (0.5127) | (0.1987) | (0.0828)      |
| 1.6783 |          | 0.6344   | 0.3461  | 0.0852        |
|       |          | (0.5026) | (0.1890) | (0.0826)      |
| 1.6891 |          | 0.6440   | 0.3606  | 0.0854        |
|       |          | (0.4939) | (0.1810) | (0.0826)      |

*SECOND SOLUTION*

Table 2. Numerical values of heat transfer coefficient $-\theta'(0)$ for different values of convective boundary condition parameter against wall mass transfer parameter by fixing $K = 0.1$, $Pr = 1.0$, $Q = 0$, $R = 1.0$, $n = 0.5$, $\alpha = 0.1$.

| $S$  | $\gamma$ | $-\theta'(0)$ |
|------|----------|---------------|
| 1.9505 | 0.4      | 0.2398        |
|       |          | (0.2352)      |
| 1.9604 |          | 0.2426        |
|       |          | (0.2325)      |
| 1.9703 |          | 0.2445        |
|       |          | (0.2314)      |
| 1.9802 |          | 0.2460        |
Figure 1. Skin friction coefficient $f''(0)$ with $S$ for different values of $\alpha$.
Figure 2. Couple stress coefficient $h'(0)$ with $S$ for different values of $\alpha$.

Figure 3. Heat transfer coefficient $-\theta'(0)$ with $S$ for different values of $\alpha$. 
Figure 4. Heat transfer coefficient $-\theta'(0)$ with $S$ for different values of $\gamma$.

Figure 5. Velocity profiles $f'(\eta)$ for different values of $\alpha$. 
Figure 6. Dual angular velocity profiles $h(\eta)$ for several values of $\alpha$.

Figure 7. Dual temperature profiles $\theta(\eta)$ for several values of $\alpha$. 
Figure 8. Temperature profiles $\theta(\eta)$ for several values of $\gamma$.

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