Critical Evaluation of Identified Flow Curves Using Homogeneous and Heterogeneous Solutions for Compression Test

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Abstract

Background The cylindrical profile model (CPM) is commonly used to convert the load–displacement measurement from the axis-symmetric compression test (ACT) to the flow curve. The model ignores the barrelling of the sample which results in serious flow curve distortion and underestimation of the effective stress and strain. To minimize these, a new solution of ACT with heterogeneous deformation is proposed in this work as an alternative to CPM.

Method Representative point is introduced in this new approach for better utilization of the new solution. Some key attributes of a typical flow curve are also considered to explore the sensitivity of the identified flow curve to the heterogeneous deformation. Sample flow curves are identified based on the new solution and compared with that of CPM.

Results It was found that CPM underestimates the maximum effective stress, strain and strain rate. Significant deviations were found between the reference flow curves and those of CPM. The model is unable to appropriately account for the rate-dependent behaviour of material and work hardening variations in the samples. Therefore, given its non-realistic homogeneous effective strain and strain rate in the sample, the CPM based flow curves are questionable. The new ACT solution provided more reliable flow curves than those of the CPM.

Conclusions Considering the heterogeneous deformation at the centre of the sample, the new ACT solution provided flow curves that for strains below 0.4 closely resemble the reference curves obtained from the finite element model. Thus, the new ACT solution reduces the serious limitations of CPM and provides less error in the study of the hot deformation phenomena (e.g. recovery and recrystallization). Further recommendations were also given to limit the deviations in the identified flow curve.

Keywords Heterogeneous deformation · Flow curve identification · Underestimated strain · Non-uniform strain rate

Introduction

Axisymmetric Compression Test (ACT) is extensively used to design, optimize and characterize hot and cold flow behaviours of materials and the phenomena associated with them. From an application point of view, thermo-mechanical processing of metals relies heavily on the identified flow curves. Key phenomena that describe the hot deformation of the material, their identified parameters and constants are generally described as mathematical models. Example phenomena include static and dynamic recovery, recrystallisation. The evaluation and calibration of these, are generally performed indirectly via the measured flow stress. Flow stresses are also measured indirectly based on the directly measured load-displacement of the test samples. Therefore, the validity of such mathematical models is strictly linked to that of the identified flow curves. As such, the reliability and accuracy of the identified flow curves are extremely important for a meaningful interpretation of the phenomena and a transformation of the bench-scale observations to the industrial processes.

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A 6-7% barrelling is inevitable during an ACT under a typical lubrication condition. This is partly due to a hard to control and a varying friction condition at the anvil-sample interface. The maximum effective strain and strain rate developed during a heterogeneous compression test, induced by barrelling, is almost twice that predicted by commonly used simplistic solutions of the test. To identify the flow stress behaviour of the material, the most commonly used solutions of the ACT are incapable of including the barrelling and the variations on the effective strain and strain rate in the sample.

In the available literature, only a few studies focus on the heterogeneous deformation during the ACT and its impact on identified flow stress. Bennett et al. [1] presented a comparative numerical-experimental study of error in the flow curve identified using the ACT. Employing a Norton-Hoff material model as the reference in their coupled thermomechanical finite element simulation, they compared the predictions with the identified flow curve for an isothermal axisymmetric solution of the ACT data obtained from Gleeble® physical simulator. Throughout the test, variable errors of 20% in amplitude were found in the experimentally identified flow curves. Even more, errors emerge when the strain and the friction factor exceed the upper and lower limits of 0.8 and 0.2, respectively. However, they did not investigate the flow curve distortion and had no suggestion to avoid or minimize the errors. Consarnau and Whisler [2] introduced a kinematic study to identify material flow behaviour during the compression test. They applied the conservation of momentum and mass while assuming contact friction on the surfaces to account for the barrelling and heterogeneous deformation. The transferability of their derivations to the real deformation scenarios can be very limited due to some assumptions and significant simplifications to solve the governing equations. Based on Hill’s general solution of the compression test, Chen and Chen [3] obtained a solution similar to that of Avitzur [4]. The solution excluded the shear deformation in its model and thus in its effective stress, strain and strain rate.

To include the barrelling in the identified effective stress, Ettouney and Hardt [5] derived a bulge correction factor. It included Bridgman’s correction factor for effective stress at the post-necking during the tensile test [6]. They applied the factor to the calculated ideal (no friction) flow stress. No correction factor for strain or strain rate was suggested in their approach [5]. This results in a major disadvantage from a microstructural correlation perspective when the intention is to correlate the structure with the deformation and deformation rate. To evaluate the heterogeneity of deformation during ACT, Ye et al. [7] conducted a systematic numerical study. They also demonstrated significant error when correlating the microstructural observations at the sample centre to the strain and strain rate estimated based on the available solutions of the test. Their study is limited to the case studies of the heterogeneous deformation without any recommendation to minimize it.

Evans and Scharning [8] performed a large number of numerical cases. They suggested that the test produces systematic and similar errors in their identified hot flow curves for many alloys. They argued that due to sufficiently similar properties of the investigated alloys, the generalization of the observed trend (e.g. the pattern of deformation heterogeneity in the sample) can be explained. They concluded that friction and geometry significantly influence the heterogeneities during deformation. Also, strain and strain rates were found to play an intermediate important role while temperature has the least influence on the heterogeneous deformation. Similar suggestions were made by Fardi et al. [9]. However, none of the above studies evaluated the distortion of the identified flow curve while they only investigated the error in the amplitude of flow stresses.

To explore the importance of such barrelling and its impact on the flow curve characterization, a virtual experiment is devised in this work. This setup includes the employment of a series of flow curves in a numerical model of ACT and the extraction of their load-displacement outputs. The selected material in the numerical model is an austenitic stainless-steel sample deformed at 750 °C with four strain rates of 0.1, 10, 30, 50 and 100 s⁻¹. The numerically calculated load-displacement data were subsequently converted to the flow curves using three different models with and without barrelling and compared with the reference flow curves obtained from [10]. The three models included:

1. A cylindrical profile model with constant friction factor (CPM-CFF, no-barrelling) [11] and
2. A cylindrical profile model with the Coulomb friction coefficient (CPM-CFC, no-barrelling)
3. An exponential profile model (EPM, with barrelling)

The first two models are being extensively used in the current literature and their solution predicts homogeneous strain and strain rate in the entire sample. The third one involves a recently developed model for ACT that allows estimating the heterogeneous deformation and the friction factor via its closed-form solution. The third model is referred to as the “exponential profile model” (EPM). Based on a single set of the barrelling-load-deformation test data and the closed-form solution of the model, one can estimate the strain and strain rate changes in the sample, instantaneous friction, and eventually can identify the material’s flow response. The identified flow data using these three models are compared against their corresponding references, the merits and relative errors of each model.
Methodology for the Evaluation

Scope of the Distortion Assessment While there exist several mathematical models of the flow curve, a typical curve can be characterised based on some key attributes. Example key attributes include the ultimate stress, slope, peak strain and ultimate strain.

New Definitions It is required to introduce some new definitions, mostly geometric, to understand the methodology used in this work and also to understand the framework utilized here to explore the heterogeneous deformation of material inside anACT sample. These are listed below.

Ultimate Stress It is the last effective stress in the curve identified at the end of ACT.

Slope The slope of the curve in the plastic region is an indicator of the work hardening rate.

Peak Strain A value close to 70-90% of the strain at which dynamic recrystallization starts [13].

Ultimate Strain The maximum effective strain induced in a given material point at the end of the test is referred to as the forming limit of the point, $\varepsilon_{\text{max}}$.

Ideal Flow Curve An ideal flow curve is that obtained by converting the test data using the exact solution; we assume that there exists an exact solution for the test that is not limited by simplifying assumptions.

Comparison Between Homogeneous and Heterogeneous Deformation

Deformation during ACT is assumed homogeneous when strain or strain rate is uniformly distributed in the sample. Alternatively, the deformation is called heterogeneous when strain or strain rate is distributed non-uniformly in the sample. Several test conditions were considered with friction, non-ideal geometrical alignments, material imperfections, etc., which contribute to a heterogeneous deformation in the sample during the real test. It is very difficult to include the conditions in an ACT mathematical model and even more difficult to solve. Consequently, the existing models deviate from the real deformation process. Given the heterogeneous nature of the real deformation in a barrelled sample, various material points experience various effective strains and effective strain rates in the sample. Therefore, the flow curves obtained at various points were not identical. Thus, the following “Tracing point” approach and new definitions are employed in this work to investigate the importance of such deviations on the identified flow curves.

Tracing Point A material point utilized to trace the flow curve, is called a tracing point; a tracing point is a selected virtual material point that represents the location and motion of the material point; which is traced continuously as the sample deforms.

The Representative Tracing Point It is desirable, for example, to monitor the flow curve of the sample at a point that allows the largest possible forming limit. To identify the most suitable tracing point to characterise material behaviour (e.g. the forming limit), one can select a few tracing points within the sample and identify the point where the traced flow curve closely resembles the ideal flow curve. This point is called the representative tracing point.

Given a set of load-displacement curves and a mathematical model that represents a heterogeneous distribution of the effective strain and strain rate in the sample, one can trace the flow curve at several material points in the deforming body. Each of these points experiences various strains and strain rates and therefore various flow stresses. Also, these flow curves deviate from their corresponding ideal flow curve at the same point due to the limitations of the utilized non-ideal model. We aim to choose a representative point as the material point that has a minimum flow distortion for a given flow conversion technique with a heterogeneous deformation model. Thus, the calculated effective strain and strain rate at the representative point based on the simple ACT theories have a low deviation from the ideal flow curve that was obtained at the same point. In the subsequent sections, deformation in five selected points in the sample is examined to evaluate how well a nominated point complies with the definition of the representative point.

Selected Tracing Points

An arbitrary tracing point \( q_i(r, z) \) together with five specific tracing points \( q_1(0, 0.5H), q_2(0, z_2), q_3(0, z_3), q_4(0, 0) \) and \( q_5(0.5D, 0) \) are illustrated in Fig. 1. To find the representative point, deformation at these points are studied analytically, numerically and the flow curves developed at each point will be compared in this work. The selected points shown in Fig. 1 represent separate references and comparison points to evaluate the sensitivity of a typical ACT data conversion to the heterogeneous deformation. It also justifies the appropriateness of a nominated tracing point as the representative point. To achieve these, a series of simulated experiments are carried out in this work.
barrelled sample diameters at the mid and top planes, respectively. The quadratic profile radius, \( R(z, H) \) varies from \( 0.5D \) in the sample mid-plane to \( 0.5d \) at the sample top-plane, thus the profile radius becomes:

\[
R(z, H) = 0.5D - \frac{2(D - d)z^2}{H^2}
\]

(2)

Extensive numerical modelling by the authors, not presented here, indicated that the quadratic profile is a reasonably close approximation of the real profile. Upon the onset of the foldover, the quadratic profile deviates from the real one at the vicinity of the point shared by the free surface and platen-sample profile. The foldover effect \([14, 15]\) is not included in this study.

The incompressibility principle between the initial and current geometry of the sample can be used to correlate \( D \) and \( d \). The correlation becomes:

\[
d = \frac{\sqrt{5} \sqrt{H(9D_0^2H_0 - 4D^2H) - 2DH}}{3H}
\]

(3)

Equation (3) makes it more convenient to represent the real profile by the quadratic profile since it is now based on the measurement of the mid-plane diameter \( D \) only; one notes that the commonly available test rigs cannot measure the instantaneous diameters \( D \) and \( d \).

Foldover is a transient process that affects the sample’s exact profile (see for example \([14]\)). Our careful numerical examination of the profile changes show that the quadratic profile is a reasonably precise representation of the real profile until the onset of the foldover. Before the foldover, one can reliably use Equation (3) to estimate \( d \) instead of measuring it. Upon the onset of foldover, the sample’s top edge deforms (blunts) locally and the quadratic profile and real profile deviate locally at the vicinity of the edge. Profile modelling after the foldover is beyond the scope of current work and has been studied by the authors to be published soon.

Effective Tracing Point \( q_{\text{eff}} \) In our search for the representative tracing point, we define a new point, \( q_{\text{eff}} \). The imaginary point, effective point, is defined at \((r = 0, z = z_{\text{eff}})\) as the intersection of a plane parallel to the mid-plane with \( z \) axis where the radius of sample profile (according to its quadratic profile Equation (2)) is equal to that of effective radius in Equation (1). The tracing point is shown as \( q_{\text{eff}} \). As shown in Fig. 1, \( D \) and \( d \) are the

\[
q_{\text{eff}} = \left( 0, z_{\text{eff}} \right)
\]

(4)

\( q_{\text{eff}} \) can be related to other geometrical parameters as:

\[
R = 0.5D_0 \sqrt{\frac{H_0}{H_n}} = 0.5D - \frac{2(D - d)z_{\text{eff}}^2}{H^2}
\]

(4)

\( \frac{H_0}{H_n} \) is a two-dimensional curve that represents the variation of the sample radius as a function of \( z \) and \( H \). As shown in Fig. 1, \( D \) and \( d \) are the

\[
f\text{eff} < 2^{-\frac{1}{3}}\frac{d}{R}
\]

(5)

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This profile is used to simplify the analysis of the test by ignoring the shear-induced barrelled profile. The definition in Equation (1) allows maintaining the incompressibility condition for a given sample height \( H \) by replacing the real barrelled profile with a virtual cylindrical profile.

Virtual Cylindrical Profile The effective radius, \( \bar{R} \), is defined as:

\[
\bar{R} = R(H) = 0.5D_0 \sqrt{\frac{H_0}{H}}
\]

(1)

where \( D_0 \) and \( H_0 \) are the initial sample diameter and height, respectively. This profile is used to simplify the analysis of the test by ignoring the shear-induced barrelled profile. The definition in Equation (1) allows maintaining the incompressibility condition for a given sample height \( H \) by replacing the real barrelled profile with a virtual cylindrical profile.

Virtual Quadratic Profile The virtual quadratic profile, shown using a dashed red line in Fig. 1, is a second-order curve that represents the variation of the sample radius as a function of \( z \) and \( H \). As shown in Fig. 1, \( D \) and \( d \) are the
Thus, the axial coordinate of this point \( z_{\text{eff}} \) can be found by combining Equations (3) and (4):

\[
z_{\text{eff}} = z_{\text{eff}}(H) = H \left[ \frac{1.5\left(DH - D_0\sqrt{H_0H}\right)}{10DH - 2\sqrt{3} \sqrt{H(9D_0^2H_0 - 4D^2H)}} \right]
\]

(5)

**Homogeneous Deformation in ACT Sample**

A consequence of ignoring the barrelling in an ACT model is a simplistic deformation prediction with a homogenous strain and strain rate in the entire sample. The models which ignore the barrelling are referred to as Cylindrical Profile Models (CPM) in this article. The effective strain rate, strain and stress for the CPM are briefly presented next.

**Effective Strain Rate and Strain for CPM**

Ignoring the barrelling, homogenous strain rate at the five selected points \( q_1 \) to \( q_5 \) can be estimated in a sample as:

\[
\dot{\epsilon}_i = \dot{\epsilon}_{\text{CPM}} = \frac{H}{H}
\]

for \( i = 1 \) to \( 5 \)  \hspace{1cm} (6)

and their homogeneous effective strain can be estimated as:

\[
\varepsilon_i = \varepsilon_{\text{CPM}} = \frac{-1}{H} \int_{H_0}^{H} \dot{\varepsilon}dH = \ln H_0 - \ln H
\]

for \( i = 1 \) to \( 5 \) \hspace{1cm} (7)

**Effective Stress for CPM**

To identify CPM effective stress in the sample, one requires simplifying assumptions both on the barrelling and the friction between the anvil-sample interface. Two commonly used CPM solutions are considered here that include:

1. Constant Friction Factor (CPM-CFF)
2. Coulomb Friction Coefficient (CPM-CFC)

**Flow Stress for CFF** Both CFF and CFC solutions of stress rely on a slab analysis of the sample that involves the radial balance of the forces on a differentially thick slab of material. The CFF solution [4] assumes that the friction at the sample-anvil interface can be expressed by a constant friction factor \( m \). The solution predicts that a homogenous effective stress \( \tilde{\sigma}_{\text{CFF}} \) develops in the sample. Therefore, for a given stage of deformation, CFF’s effective stress does not change in the sample and for all the selected tracting points \( q_1 \) to \( q_5 \) can be identically expressed as:

\[
\tilde{\sigma}_i = \tilde{\sigma}_{\text{CFF}} = \frac{F}{\pi R \left( 1 + \frac{2mR}{3\sqrt{3}H} \right)} \quad \text{for} \quad i = 1 \text{ to } 5
\]

(8)

**Flow Stress for CFC** The frictional conditions on the top and bottom faces of the sample can also be described by a constant coefficient of Coulomb friction. Given average pressure at the interface, \( P_{\text{ave}} \), the correlation between \( \mu \) and \( m \) can be found as:

\[
\mu P_{\text{ave}} = \frac{m\tilde{\sigma}}{\sqrt{3}}
\]

(9)

The CPM slab solution is carried out by assuming a sliding Coulomb Coefficient of Friction \( \mu \) at the anvil-sample interface [12]:

\[
\tilde{\sigma}_i = \tilde{\sigma}_{\text{CFC}} = \frac{2F\mu^2}{H\pi \left( He + \frac{2m}{3\sqrt{3}} - 2R\mu - H \right)} \quad \text{for} \quad i = 1 \text{ to } 5
\]

(10)

The solution predicts that, for a given stage of the test, the same effective stress \( \tilde{\sigma}_{\text{CFC}} \) develops at all selected tracting points \( q_1 \) to \( q_5 \).

Most analyses of metalworking processes have been performed using \( \mu \). However, when dealing with hot deformation, the use of \( m \) provides a better representation for the mix sticking and sliding conditions at the interface. It can be shown that Coulomb’s constant friction factor, \( \mu \), can be correlated to \( m \) as:

\[
\mu = \frac{3mH}{3H\sqrt{3 + 2mR}}
\]

(11)

therefore, the flow stress \( \tilde{\sigma}_{\text{CFC}} \) can also be found as a function of \( m \).

**Deformation Rate Control for a Constant Effective Strain Rate**

Material flow studies are typically based on constant strain rate curves. To produce the data for a constant strain rate test, most ACT rigs control the movement of their anvil with time \((H - t)\) based on the following expression:

\[
H = H_0e^{-\tilde{\sigma}_{\text{CPM}}}
\]

(12)

Equation (12) is derived from Equation (6). It indicates that for a constant effective strain rate deformation test, the initial height of the sample, \( H_0 \), should be reduced to its current height, \( H \), exponentially. It assumes a homogenous effective strain and strain rate development in the sample during the test.
Heterogeneous Deformation in the ACT Sample

A recent model for heterogeneous deformation during ACT, Exponential Profile Model (EPM), is briefly summarized next. The summary presents expressions to calculate the variations of EPM effective strain rate, strain and its effective stress in the sample.

EPM Effective Strain Rate and Strain

The effective plastic strain rate distribution [16] in the sample can be estimated using EPM solution as:

$$\dot{\varepsilon}(r, z) = \frac{2H}{\sqrt{3}H^2} \left[ (1 - 2BH)^2r^2 + 3(BH - 4z) + 2z \right]^2$$

(13)

the barrelling parameter $B$ in Equation (13) links EPM axial and radial components of the velocity field. Details on the calculation of $B$ can be found in our earlier work [17].

It has been shown [16] that effective strain distribution using EPM in the ACT sample is:

$$\bar{\varepsilon}(r, z) = \sqrt{\frac{2}{3}} \sqrt{6 \left( \frac{(H_0 - H)(BH_0 + 2z)}{H_0 H} + 4Bz(\ln H - \ln H_0) \right)^2 + 2r^2 \left( \frac{H_0 - H}{H_0 H} + 2B(\ln H - \ln H_0) \right)^2}$$

(14)

Equation (14) is a key relationship to compute the effective strain rate distribution (and hence to interpret microstructure distribution in ACT sample) and to convert its raw data to the flow curves for a given tracing point. Converting the test data by EPM, the five selected points ($q_1$ to $q_5$) are chosen as the evaluation points to obtain material flow responses corresponding to each tracing point.

EPM Friction Factor

Even though the friction coefficient increases with deformation as the barrelling escalates, an ACT flow data conversion using CFF or CFC requires an average estimation of a priori known $m$ or $\mu$, respectively. The EPM’s friction theory [18] offers two advantages: there is no need for constant $m$ or $\mu$ as inputs for flow stress calculations; an instantaneous and varying friction factor $m$ can be calculated during the test based on the geometry of the barreled sample.

EPM Flow Stress; A Multi-Layer Approach

To convert the ACT data into the flow stress, we proposed an averaging scheme to account for the barreled geometry. For the half sample shown in Fig. 2, $N$ and $j$ are the total number of layers and the layer number, respectively.

It has been shown in [19] that EPM’s average flow stress becomes:

$$\bar{\sigma} = \frac{NF}{\sum_{j=1}^{N} \pi R^2_j \left( 1 + \frac{2m}{3\sqrt{3}H} \left( \frac{R}{R_j} \right) R_j \right)}$$

(15)

The above equation has been verified for a special case in which the barrelling does not occur; a special case of zero barrelling ($R_j = R$) in which Equation (15) reduces to the CPM-CFF formula Equation (8). The "work hardening heterogeneity" in the barreled sample is not considered in the current work as it is beyond the scope of this work.

In "Homogeneous Deformation in ACT Sample" and the current section, we present two models that assume homogeneous and heterogeneous deformation in the ACT sample, respectively. While the latter is more realistic, however, both solutions have to be evaluated and compared based on a reference model solution. In "Finite Element Reference Model/Solution", we present the construction and utilization of a finite element (FE) model for ACT case studies which serve as virtual experiments and provide the reference solutions. In the virtual experiments employed in this study, the rate control expressed by Equation (12) was enforced in its finite element solution of ACT. The chosen tracing points allow compact closed-form expressions to
estimate EPM effective strain, strain rate and stress which can be easily compared with their counterpart solutions based on CPM and the finite element (FE) reference model ("Finite Element Reference Model/Solution").

**EPM Stress, Strain, Strain Rate Evaluation at \( q_1 \) to \( q_5 \)**

**Point \( q_1 \)** Point \( q_1(0,0.5H) \) is located along the sample centreline and is considered as a candidate representative point. From an analytical point of view, point \( q_1 \) is a convenient point to evaluate the deformation parameters due to the ease of tracing its coordinates during the test. The heterogeneous effective strain and strain rate at this point can be found by simplifying Equations (13) and (14):

\[
\varepsilon_{q_1} = \frac{2H}{H}(1 - BH) \tag{16}
\]

and

\[
\dot{\varepsilon}_{q_1} = 2 \left( 1 - BH - \frac{H}{H_0} + BH \ln(H - \ln H_0) \right) \tag{17}
\]

where \( B \) is the EPM barrelling parameter [20].

**Point \( q_2 \) and \( q_3 \)** Two arbitrary tracing points \( q_2(z_2,0) \) and \( q_3(z_3,0) \) are located on the sample centreline \((0 < z_3 < z_2 < 0.5H)\). Having known their strain and strain rates, one can estimate (interpolate or extrapolate) the EPM strain and strain rate at any other point on the sample centreline including \( q_{\text{eff}} \). From Equations (13) and (14), the effective strain rate and strain at these two points can be expressed as:

\[
\dot{\varepsilon}_{q_i} = \frac{2H}{H^2} (BH(H - 4z_i) + 2z_i) \quad i = 2 \text{ and } 3 \tag{18}
\]

\[
\varepsilon_{q_i} = 2 \left( \frac{(H_0 - H)(BH_0H + 2z_i)}{H_0H} + 4Bz_i \ln H - \ln H_0 \right) \quad i = 2 \text{ and } 3 \tag{19}
\]

The evaluated strain rate, strain and stress at two points are used later in this work, to estimate those at \( z_{\text{eff}} \), as a representative point candidate.

**Point \( q_4 \)** The centre of the test sample, is also a convenient point from an analytical perspective since both cylindrical coordinates \( r - z \) are zero. This point is most commonly used by researchers and material scientists as the point from which the microstructures of the deformed sample is extracted. Expressions for effective strain rate and strain at \( q_4 \) are simplified based on Equations (13) and (14) as follows:

\[
\dot{\varepsilon}_{q_4} = 2BH \tag{20}
\]

and

\[
\varepsilon_{q_4} = 2B(H_0 - H) \tag{21}
\]

**Point \( q_5 \)** The tracing point \( q_5(0,0.5D) \) in Fig. 1 belongs to the profile. This is contrary to all other tracing points \( q_1 \) to \( q_4 \) that is located on the sample centreline. The candidacy can be justified due to the low sensitivity of the strain and strain rate to the \( r \) coordinate of the sample (see for example [20]). Substituting the points coordinate in Equations (13) and (14):

\[
\dot{\varepsilon}_{q_5} = \frac{2H}{\sqrt{3H^2}} \sqrt{3B^2H^4 + 0.25(D - 2BDH)^2} \tag{22}
\]

and

\[
\varepsilon_{q_5} = \sqrt{\frac{2}{3}} \left[ 6B^2(H - H_0)^2 + 0.5D^2 \left( \frac{1}{H} - \frac{1}{H_0} + 2B(\ln H - \ln H_0) \right) \right]^2 \tag{23}
\]

**Point \( q_{\text{eff}} \)** The effective strain, strain rate and stress can be easily evaluated at the effective representative point \( q_{\text{eff}} \) for CPM and EPM based on their relationships presented in "Homogeneous Deformation in ACT Sample" and "Heterogeneous Deformation in the ACT Sample", respectively.

EPM effective strain and strain rate evaluated at the proposed tracing points are different (see Equations (16) to (23)). However, the solution from EPM for effective stress presented by Equation (15) is an average value that is uniformly distributed over the entire sample [19]; the average flow stress at \( q_1 \) to \( q_5 \) and \( q_{\text{eff}} \) can be expressed as:

\[
\bar{\sigma}_{q_i} = \bar{\sigma}(F,m,R_j,H) = \bar{\sigma}_{q_{\text{eff}}} \quad i = 1, 2, 3, 4 \text{ and } 5 \tag{24}
\]

A finite element model is also considered as the reference model for comparison purposes, which is presented in "Finite Element Reference Model/Solution".

**Finite Element Reference Model/Solution**

To evaluate the heterogeneity of the effective strain, strain rate and stress in the deformed sample, SFTC-DEFORM Premier software was used in this work to construct and solve the FE model of ACT for a series of ACT cases. Further details of these are provided next.
Sample Geometry and Anvil Speed  Due to the axisymmetric nature of the problem, only a quarter of each test sample and the upper anvil were modelled using the coordinate system shown in Fig. 1. Also, a rigid upper anvil was assumed. Several test scenarios were considered with the initial sample diameter and height of $D_0 = 10\text{mm}$, $H_0 = 16\text{mm}$ and $H_f = 8\text{mm}$ with varying total number of deformation steps $N_{\text{steps}}$ depending on the deformation rate of each scenario. The current deformation step, $i_{\text{step}}$, was related to the sample height $H$ according to $H = 16 - \frac{8}{(N_{\text{steps}}-1)}(i_{\text{step}} - 10)$. Also a variable ram velocity of $\dot{H} = H\ddot{e}_{\text{CPM}}$ was introduced as the velocity boundary condition to achieve a constant average effective strain rate, $\ddot{e}_{\text{CPM}}$, in the sample during the entire deformation. The sample dimensions, effective strain rates and friction factors for each test are listed in Table 1. Average effective strain rates of $\ddot{e}_{\text{CPM}} = 0.1, 1, 10, 30, 50$ and $100\text{s}^{-1}$ were used for the virtual experiments and their corresponding load–displacement data were extracted.

Barrelling parameter $B$ was calculated [17] based on the instantaneous values of the mid and top-plane diameters $D$ and $d$; the mid plane diameter $D$ was extracted from the deformed mesh geometry data and the top-plane diameter $d$ for each intermediate step was estimated using Equation (3). Alternatively, one may use a transducer ring [21], to record the intermediate diameters concurrently with $H$.

Elements and Meshes  Axisymmetric iso-parametric 4 node quadrilateral elements were used. The FE model consists of 384 nodes and 343 elements to discretise a quarter of the test sample (Fig. 1).

Friction  For each simulation, a constant value of friction factor $m$, was chosen (see Table 1) and the calculated elemental and nodal results were extracted from the FE solution. These comprised of nodal coordinates, velocities, calculated forces and elemental connectivity matrix. Also, the von-Mises effective strains $\ddot{e}$, strain rates $\dot{e}$ and stresses $\ddot{\sigma}$ were evaluated at each element centroid and recorded.

Virtual Measurements  As explained in "Methodology for the Evaluation", the software and its FE model of ACT serve as a virtual lab to produce pseudo data. The produced deformation scenarios were recorded for every 10 steps.

Given a finite element solution, the evaluation of the effective stress, strain and strain rate at an arbitrary tracing point is not straightforward. This can be better understood with the use of $q_{\text{eff}}$. One notes that $z_{\text{eff}}$ is a function of $D$ and $H$ and therefore its position changes continuously along the centreline. Also, we note that for a given FE mesh, $z_{\text{eff}}$ does not necessarily coincide with an existing node in the mesh. For the $z_{\text{eff}}$ to falls between two of the nodes, one has to perform an interpolation/extrapolation of the values for two adjacent points to $q_{\text{eff}}$.

Constitutive Behaviour of the Material  The previous numerical study indicated that the’s deformed mesh of the sample was not highly sensitive to the material type and temperature during deformation [9]. In all FE simulations, AISI-304 stainless steel samples at 700 °C isothermal conditions were used from the software library (DEFORM material library, Scientific forming technology corporation(SFTC), Columbus, Ohio) [22] include both strain and rate hardening behaviour. The following constitutive model for AISI 304 was utilized in the FE model:

$$\ddot{\sigma} = 219.54\dot{e}^{-0.334\epsilon_0^0.08} + 329.98$$

The model in Equation (25) was deliberately chosen to represent work hardening behaviour without a capacity to demonstrate a dynamic recovery or recrystallization. The flow curves will also be identified by the CPM and EPM using the extracted virtual/pseudo experimental data from the FE solutions. These will be compared with the real flow behaviour employed in the FE model to generate the virtual compression test data. The tracing point at which the two compared flow behaviours comply the best will be nominated as the representative point. The identified flow curves also expected to reveal the work hardening behaviour otherwise the solutions will become unqualified to study materials hot deformation behaviour using the ACT solutions.

| Strain rate, $\ddot{e}_{\text{CPM}}$ [s$^{-1}$] | $H_0$ | $D_0$ | $H_f$ | $D_f$ | $m$ |
|-------------------------------------------|------|------|------|------|-----|
| 0.1                                      | 15.14 | 9.98 | 5.47 | 17.37 | 0.4 |
| 1                                        | 15.12 | 9.98 | 5.15 | 17.33 | 0.4 |
| 10                                       | 15.11 | 9.98 | 5.12 | 17.85 | 0.6 |
| 30                                       | 15.16 | 9.98 | 5.13 | 17.46 | 0.6 |
| 50                                       | 14.92 | 10.04 | 5.47 | 17.37 | 0.5 |
| 100                                      | 15.14 | 9.98 | 5.42 | 17.45 | 0.65 |

Subscripts $i$ and $f$ indicate the initial and final measured height $H$ or diameter $D$, respectively.
Results

According to the FE simulations, the real ACT deformation paths at different points and their traced material flow curves are different due to friction-induced barrelling and heterogeneity. That is, each tracing point experiences a different deformation path, work hardening and rate hardening path. To evaluate the CPM and EPM flow curves at each tracing point, they were identified based on the load-deformation data provided by the FE solutions; the inputs needed for CPM and EPM were taken from their corresponding FE solutions. The required inputs to identify CPM and EPM flow curves are the initial geometry (H0 and D0), friction factor m and the deformation data (F - H - D - B). The FE flow curves at the specified tracing points were also extracted from their FE solutions. Subsequently, the identified CPM’s and EPM’s flow curves were compared with their corresponding FE (reference) flow curves to evaluate the reliability of the identified flow curves in terms of the key flow curve attributes including the ultimate stress, slope, peak strain and ultimate strain. The slope of the curves in the plastic region will be classified according to their dominant work hardening (WH) or work softening due to dynamic recovery (DRV) and/or dynamic recrystallization (DRX) [23].

Flow Curve Comparisons

Errors in the flow curves identified by simplified CPM solutions is a well-known issue. However, due to the lack of a more reliable solution to covert the ACT data to the flow curve, CPM solutions and their identified flow curves have been used far beyond their limits; the errors have been overlooked in many material hot behaviour studies such as ACT-based SRX and DRX characterizations. One has to note that the choice of the tracing point has no impact on the error since the CPM solutions provide only a homogeneous estimation of the effective strain rate, strain and stress which are independent of the position of the tracing point. However, the proper choice of tracing point can minimize the error when a non-homogeneous solution such as EPM is used to convert the ACT data to flow stress curves. Therefore, assuming the FE flow curve as the reference, the proposed tracing points have been examined in this section to find the most appropriate tracing point after comparing their corresponding EPM identified flow curves with those of CPM.

We noticed that the average effective strain rates for the finite element flow curves at q1 deviate significantly from their CPM corresponding values; e.g. for q1, the corresponding average strain rate based on the FE solution was \( \dot{\varepsilon}_{FE} = 0.004 \). To resolve this issue and to allow a reasonable comparison, flow curves from FE simulation at \( q_2 \) were chosen as the reference. This produced finite element average effective strain rates of \( \dot{\varepsilon}_{FE} = 0.11, 1, 8.4, 29.9, 42.5 \) and 96.4 s\(^{-1}\) in comparison with the average CPM strain rates of \( \dot{\varepsilon}_{CPM} = 0.1, 1, 10, 30, 50 \) and 100 s\(^{-1}\), respectively. Also, due to a similar issue with that of \( q_1 \), as explained above, \( q_2 \) was used as the FE reference point when EPM and CPM flow curves were evaluated at \( q_{eff} \).

Figure 3 compares the CPM and EPM flow curves traced at \( q_1 \) with that of the prediction from the FE model (i.e. from the nearby tracing point \( q_2 \) as explained above). In the FE model, the speed of the anvil was described according to Equation (12), for each case (see "Deformation Rate Control for a Constant Effective Strain Rate").

A flow curve with a constant and homogeneous effective strain rate \( \dot{\varepsilon}_{CPM} \) can only be assumed when an un-realistic homogeneous CPM scenario is considered in which an anvil speed is controlled based on \( H = H_0 e^{-r t_{CPM}} \).

It can be seen that \( \dot{\varepsilon}_{eff} \) is a function of \( D \) and \( H \) (see Equation (5)) and therefore its position changes continuously along the centreline. To compare the effective strain, strain rate and stress in the finite element solutions with those of \( q_{eff} \) based on CPM and EPM, the FE flow curves at \( q_2 \) was taken as the reference as its average strain rates is close to those of CPM and EPM.

Maximum Achievable Effective Strain by ACT

It can be seen in Figs. 3 to 4 that CPM and EPM have underestimated the magnitude of the flow stress in all cases. Up to effective strains of 0.4, EPM estimated maximum effective strains, however, closely resemble those of the reference FE compared to those of CPM. An example comparison for the point \( q_2 \) are shown in Fig. 4. Beyond this point, all predictions become poor. Also, EPM’s error depends strongly on strain rate. For example: at a strain rate 0.1/s the model is poor after an equivalent strain of ~0.5. Increasing the strain rate to 1/s suddenly improves the flow predictions but the model becomes poor again at a strain rate of 10/s. The CPM model should not be used any strain while the new model is reliable only up to equivalent strains of 0.4.

CPM based solutions consider only an average strain during the test as they neglect the shear strain. As a result, the strain calculated by these methods becomes significantly lower than the maximum strain produced in the sample. A key advantage of EPM is that it can evaluate the induced strain and strain rate at different positions in the sample, notably the effective strain \( \dot{\varepsilon}_{eff} \) in its centre (\( r = 0, z = 0 \)). A simple comparison between the CPM based solutions with that of EPM can be made by evaluating the ratio between the induced effective strains from the two models for a similar instantaneous height in the sample as defined by Equation (26). The ratio between the EPM representative effective strains of EPM to CPM becomes:
A plot of Equation (26) is shown in Fig. 5. It demonstrates that when the instantaneous height of the sample reduces from 15.50 to 5.65 mm, barrelling parameter, \( B \), increases from 0.038 to 0.110 (see [17] for the details) and the ratio of \( \frac{H_0}{H} \) increases from 1.2 to 2.1. This result suggests that the effective strain at the sample centre estimated by EPM is 2.1 times larger than that found by the CPM for the same amount of loading or displacement. The increase in the formability limit of the identified flow curve based on EPM is quite significant as it allows a significantly larger range of deformation to be studied with the test without a need to increase the test stroke. This effect can also be used to investigate a full range of comparison between flow curves obtained by compression test and the hot torsion test which is typically chosen for its ability to produce larger strains.

The finite element simulations employed in this work did not include fracture in the sample. This is because the hot deformation of metals is generally limited by the available stroke of the sample. The simulated deformation started at \( H_0 = 16 \text{mm} \) and stopped at \( H = 5.5 \text{mm} \). Given the heterogeneous nature of the deformation in the sample, the very same height reduction induces different instantaneous effective strains in different tracing points (\( q_1 \) to \( q_5 \)) as shown in the Figs. 3–4.

### Discussions

#### Strain and Strain Rate Heterogeneity

Since ACT’s real process deforms the sample heterogeneously, it can never result in a constant effective strain rate deformation path anywhere in the sample. Thus, variations...
of the effective strain rate during ACT are inevitable irrespective of the chosen tracing point. Therefore, the tracing point in the FE solution should be chosen carefully, such that its average strain rate to near that of the CPM to make the flow curves more comparable. As an example, it can be seen in Fig. 3 that for \( \dot{\varepsilon}_{\text{CPM}} = 50 \, \text{s}^{-1} \), the CPM average strain rate deviates from that of FE (traced at \( q_2 \)) up to 34% and the EPM average strain rate deviates from that of FE up to 13%. The comparisons with the average effective strain rate in the FE and EPM traced flow curves enable the estimation of the errors due to the simplifying assumptions.

Due to the limitations in the flow curve comparison outlined above, it is not surprising to see the poor agreement of the flow curves shown in Figs. 3 to 4. Neither EPM flow curves nor those of CPM provides an acceptable flow curve whose attributes (e.g. the ultimate stress, slope, peak strain and ultimate strain) represent accurately and reliably those in the reference FE flow curve. A quantitative assessment of the attributes from each curve and also the area under the curve was conducted to rank the “goodness of fit” of CPM and EPM flow curves. The assessments (not shown here) indicated that EPM and CPM-CFC flow curves nearly produce the same goodness of fit compared to the FEM flow curves which were better than those of CPM-CFF. The importance of the errors in the identified flow curves and the usefulness of the identified flow curves depend on their intended application and usage (e.g. formability, maximum rolling load, deformation energy and thermomechanical processing).

Irrespective of the chosen tracing point, according to Figs. 3–4, both EPM and CPM provide a poor representation of the reference flow curves ranging from bad (CPM-CFC and EPM) to worse (CPM-CFF). The hot flow curves show high sensitivity to the deformation rate and that both...
CPM and EPM are incapable to properly account for the stain rate effects. For strains below 0.4 in Fig. 6, one can notice that EPM flow curves identified at $q_4$, at centre of the sample, provide a better compliance with those of FEM compared to those of other methods (most notably in terms of their slope).

It has to be noted that none of the investigated methods provide a fully satisfactory compliance with the reference flow curve. From a scientific perspective, neither of the methods are reliable for indirect measurement of stress and their second-tier stress-related phenomenon. However, the flow curve is also the basis for far more simple estimations such as the process design and tool sizing. Therefore, these simplistic methods still offer some advantages for such applications.

The developments of analytical methods for flow curve identification have stopped abruptly and prematurely in the last 30 years. This is partly due to the advent of finite element methods since ~1940 and their potentials to solve complex non-linear boundary value plasticity problems. Examples of these include inverse methods and finite element model updating (FEMU) to identify material flow curves. However, the numerically based inverse methods failed to provide adequate and practical techniques to identify flow curves since the flow curves are needed as an input to conduct a numerical study (see for example [24–26] and [27])

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**Table 2** Distortion of EPM and CPM flow curves at different tracing points compared to their references; in all cases, the identified dominant behaviour by EPM and CPM are wrong due to the distortions

| $\dot{\varepsilon}_\text{CPM}$ | Average strain rate | Dominant behaviour |
|-----------------------------|---------------------|--------------------|
|                             | 1 10 30 50 100      | 1 10 30 50 100     |
| Traced at $q_4$, Fig. 3     | FE                  | DRV DRV DRV DRV DRV |
| EPM                         | 0.11 1 8.4 29.9 37.5 | 96.4 DRV DRV DRV DRV |
| CPM-CFF                     | 0.10 1 10.0 30.0 50.0 | 100.0 WH DRX DRX +WH DRX |
| CPM-CFC                     | 0.10 1 10.0 30.0 50.0 | 100.0 DRX DRX +WH DRX |
| Traced at $q_{eff}$, Fig. 4  | FE                  | DRV DRV DRV WH DRX |
| EPM                         | 0.10 1 7.9 31.6 44.5 | 73.5 DRX +WH DRX +WH DRX |
| CPM-CFF                     | 0.10 1 10.0 30.0 50.0 | 100.0 WH DRX DRX +WH DRX |
| CPM-CFC                     | 0.10 1 10.0 30.0 50.0 | 100.0 DRX +WH DRX +WH DRX |
| Traced at $q_5$, Fig. 5     | FE                  | DRV DRV DRX DRX DRX |
| EPM                         | 0.19 1 11.3 33.1 57.5 | 115.4 DRX +WH DRX +WH DRX |
| CPM-CFF                     | 0.10 1 10.0 30.0 50.0 | 100.0 WH DRX DRX +WH DRX |
| CPM-CFC                     | 0.10 1 10.0 30.0 50.0 | 100.0 DRX +WH DRX +WH DRX |
| Traced at $q_5$, Fig. 6     | FE                  | WH WH WH WH WH |
| EPM                         | 0.12 1 11.5 33.2 58.2 | 117.2 DRX +WH DRX +WH DRX |
| CPM-CFF                     | 0.10 1 10.0 30.0 50.0 | 100.0 DRX +WH DRX +WH DRX |
| CPM-CFC                     | 0.10 1 10.0 30.0 50.0 | 100.0 DRX +WH DRX +WH DRX |

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**Fig. 7** Tracing at $q_4$, simulation speed control, $H = H_0 e^{-\dot{\varepsilon} t}$
in which the serious limitations of the methods have been highlighted). Despite the inadequacy of the available analytical methods for flow curve identification, several research groups have employed the highly questionable flow curves for indirect measurement of complex hypotheses on the deformation phenomenology, deformation mechanisms, texture evolution etc. Although recent developments of indirect stress measurement techniques based on both analytical and numerical-inverse methods are promising, they require further developments to provide reliable and accurate flow curves. More details on these can be found in a recent review paper [28].

Flow Curve Distortion

The identified flow curves are generally considered as an indirect indication of the hot working behaviour of alloys that is a consequence of microstructural changes: the generation of dislocations, work hardening (WH), the rearrangement of dislocations, their self-annihilation, and their absorption by grain boundaries, the nucleation and growth of new grains, DRV and DRX [23].

Therefore, the “distortion” of all identified CPM and EPM flow curves compared to the reference ones, see Figs. 3 to 6, is considered to be the most critical ramifications of employing the simplified models. These are summarized in Table 2 to emphasise the important finding: it is worth noting that all FE flow curves demonstrate DRV at $q_1$ and $q_{eff}$ (i.e. in Figs. 3 and 7). They show DRX and WH at $q_4$ and $q_5$, respectively (Figs. 6 and 8). However, CPM and EPM identified flow curves represent DRX behaviour. In some cases, the stress increases after peak strain due to friction at a large strain.

The flow curve distortion issue poses some serious questions on the validity of DRX studies/measurements which rely heavily on the ACT results and typically employ CPM or EPM to interpret the test results.

Conclusions

Available solutions for the Axisymmetric Compression Test (ACT), namely cylindrical profile model (CPM, homogeneous) and Exponential Profile Model (EPM, heterogeneous) were critically evaluated in this work. Key simplifications in the solutions are the neglect of barrelling in CPM and variations of work and rate hardening in both CPM and EPM. To assess the impact of these simplifications on the identified flow curves, a series of finite element-based virtual experiments were performed. These enabled the comparison of the identified flow curves by CPM and EPM with their corresponding reference ones.

The extend of heterogeneity in the deformed sample was explored first. This was done by choosing four tracing points in different locations and comparing their corresponding identified flow curves, based on the CPM and EPM, with the reference flow curves employed in the virtual experiments. A monotonic flow curve, incapable of representing dynamic recovery and recrystallization, was employed in a finite element model of the ACT next and their load-displacement outputs were taken as inputs in the CPM (homogeneous) and the EPM (heterogeneous) solutions of the test. Based on the available solutions and the test data, the material flow curves were identified at the chosen tracing points. Irrespective of the chosen solution, neither of the identified flow curves matched fully with the reference ones. Also, despite the monotonic nature of the reference material behaviour; the identified hot deformation behaviours were quite misleading ranging from work hardening to dynamic recovery (DRV) and dynamic recrystallization (DRX).

This poses serious questions on the validity of the mathematical models which are commonly used to characterize the hot deformation behaviour of materials in the literature that heavily rely on the available solutions of ACT to study static recrystallization, dynamic recrystallization and metadynamic recrystallization. It was shown that the CPM solutions fail to incorporate maximum strain of ACT in their identified flow curve; their identified flow curve offers an effective strain range that is significantly narrower than that obtained by the EPM. To adopt the sample centre as the representative tracing point and to use EPM solution to identify the material flow curve offer a simple but limited remedy to account for the heterogeneities of deformation in the ACT sample; EPM should not be used beyond an effective strain of 0.4.

Fig. 8 Comparison of $\varepsilon_{CPM}/\varepsilon_{EPM}$ and barrelling parameter $B$ vs. instantaneous height, $H$
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Data Availability The raw/processed data required to reproduce these findings cannot be shared at this time as the data also forms part of an ongoing study.

Declarations

Competing Interest The authors declare no conflict of interest.

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