Infrared Behavior of the Gluon Propagator in Lattice Landau Gauge

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Abstract

We evaluate numerically the four-momentum-space gluon propagator in lattice Landau gauge, for pure SU(2) lattice gauge theory with periodic boundary conditions. Simulations are done in the strong-coupling regime, namely at $\beta = 0, 0.8, 1.2$ and 1.6, and for lattice sizes up to $30^4, 16^4, 20^4$ and $24^4$ respectively. In the limit of large lattice volume, we observe in all cases a gluon propagator decreasing as the momentum goes to zero. This counter-intuitive result has a straightforward interpretation as resulting from the proximity of the so-called first Gribov horizon in infrared directions.

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1. A non-perturbative investigation of QCD in the infrared limit is necessary in order to get an understanding of crucial phenomena such as color confinement and hadronization. In particular, the infrared behavior of the gluon propagator can be directly related \[1\] to the behavior of the Wilson loop at large separations, and to the existence of an area law. Of course in developing non-perturbative techniques for QCD, one has to deal with the redundant gauge degrees of freedom. The standard gauge-fixing technique is based on the assumption that a gauge-fixing condition can be found which uniquely determines a gauge field on each gauge orbit. The correctness of this procedure was questioned by Gribov \[2\]. He showed that the Coulomb and the Landau gauges do not fix the gauge fields uniquely, namely there exist gauge-related field configurations (Gribov copies) which all satisfy the gauge condition. Gribov copies affect also lattice numerical simulations \[3\]. In fact, the lattice Landau (or Coulomb) gauge is imposed by minimizing a functional which usually displays several relative minima (lattice Gribov copies).

In order to get rid of the problem of spurious gauge copies, Gribov proposed \[2\] the use of additional gauge conditions. In particular he restricted the physical configuration space to the region \(\Omega\) of transverse configurations (\(i.e. \partial \cdot A = 0\)), for which the Faddeev-Popov operator \(\mathcal{M}[A] \equiv -\nabla \cdot D[A]\) is nonnegative. This region is delimited by the so-called first Gribov horizon, defined as the set of configurations for which the smallest, non-trivial eigenvalue \(1\) of the Faddeev-Popov operator is zero. We now know that \(\Omega\) is not free of Gribov copies and that the physical configuration space has to be identified with the so-called fundamental modular region \(\Omega\).\[4, 5\]. Nevertheless, the region \(\Omega\) is of interest in numerical simulations. In fact, configurations which satisfy the usual lattice Landau gauge condition belong to this region.

The restriction of the path integral, which defines the partition function, to the region \(\Omega\) implies a rigorous inequality \[6, 7\] for the Fourier components of the gluon field. From this inequality, which is a consequence only of the positiveness of the Faddeev-Popov operator, it follows \[3, 6\] that the region \(\Omega\) is bounded by a certain ellipsoid \(\Theta\). This bound implies proximity of the first Gribov horizon in infrared directions, and consequent suppression of the low-momentum components of the gauge field, a result already noted by

\[1\] The Faddeev-Popov operator has a trivial null eigenvalue, corresponding to a constant eigenvector.
Gribov in reference \[2\]. This bound also causes a strong suppression of the gluon propagator in the infrared limit. More precisely, Zwanziger proved \[7\] that, in the infinite-volume limit, the gluon propagator is less singular than \(k^{-2}\) in the infrared limit and that, very likely, it does vanish as rapidly as \(k^2\). A gluon propagator that vanishes as \(k^2\) in the infrared limit was also found (under certain hypotheses) by Gribov \[2\]. In particular, he obtained \(D(k) = k^2/(k^4 + \gamma)\). The momentum scale \(\gamma\) appears when the restriction of the physical configuration space to the region \(\Omega\) is imposed. This propagator agrees with the zeroth-order perturbative prediction \(k^{-2}\) at large momenta, but gives a null propagator at \(k = 0\). A propagator which is a generalization of the one obtained by Gribov has also been introduced in reference \[8\] as an Ansatz for a non-perturbative solution of the Schwinger-Dyson equation. Let us notice that these results are in complete contradiction with the \(k^{-4}\) singularity obtained for the gluon propagator when the Schwinger-Dyson equation is approximately solved in the infrared limit \[1\].

The infrared behavior of the gluon propagator in lattice Landau gauge has been the subject of relatively few numerical studies \[10, 11, 12, 13, 14\]. In some cases \[12, 14\] results seem to indicate that the gluon propagator is finite at zero momentum and in the infinite-volume limit. This is in agreement with Zwanziger’s prediction of a gluon propagator less singular than \(k^{-2}\). Recently, a gluon propagator evaluated avoiding Gribov copies \[13\] has been successfully fitted by a Gribov-like formula \[15\]. Finally, in references \[16, 17\], it was checked that the influence of Gribov copies on the gluon propagator (Gribov noise) is of the order of magnitude of the numerical accuracy. In particular, this seems to be the case even at small values of the coupling \(\beta\), namely in the strong-coupling regime, where the number of Gribov copies is higher and Gribov noise, if present, should be larger and more easily detectable.

In references \[16, 17\] it was also observed, at \(\beta = 0\), that the gluon propagator decreases as the momentum goes to zero. A similar behavior seemed to be present also at higher values of \(\beta\), and in particular at \(\beta = 1.6\) and for lattice volume \(V = 24^4\). In the last case, however, the behavior is not as clear as that at \(\beta = 0\), due to the limited statistics (only 7 configurations). In the present work we extend the analysis done in references \[16, 17\], and we would like to obtain a conclusive result for the infrared behavior of the gluon propagator in the strong-coupling regime. Let us notice that Zwanziger’s predictions \[3, 4\] for the gluon propagator are \(\beta\)-independent: in fact, they
are derived only from the positiveness of the Faddeev-Popov operator when the lattice Landau gauge is imposed. Thus, results in the strong-coupling regime are a valid test for those predictions. Of course, once the behavior of the gluon propagator is clarified in this regime, we should try to extend the results to larger values of $\beta$, possibly up to the scaling region.

2. We consider a standard Wilson action for $SU(2)$ lattice gauge theory in 4 dimensions with periodic boundary conditions. For notation and details about numerical simulations we refer to [16, 17]. The only difference will be that here we consider lattices with different sizes $N_\mu$ in the different directions. Let us recall that the gluon propagator, which will be evaluated in lattice Landau gauge [10, 18], is defined, in momentum space, as

$$D(0) \equiv \frac{1}{12V} \sum_{\mu,a} \left\langle \left( \sum_A A_\mu^a(x) \right)^2 \right\rangle$$

$$D(k) \equiv \frac{1}{9V} \sum_{\mu,a} \left\langle \left\{ \left( \sum_A A_\mu^a(x) \cos(2\pi k \cdot x) \right)^2 + \left( \sum_A A_\mu^a(x) \sin(2\pi k \cdot x) \right)^2 \right\} \right\rangle.$$

In Table 1 we report, for each pair $(\beta, N)$, the parameters used for the simulations. Only for the lattice volume $V = 8^4$, at $\beta = 0.8$ and 1.6, and for $V = 24^4$, at $\beta = 1.6$, we used the data reported in reference [17, average “fm”]. Computations were performed on the IBM SP2 at the Cornell Theory Center, on several IBM RS-6000/250–340 workstations at New York University, on a IBM RS-6000/550E workstation at the University of Rome “La Sapienza” [3] and on an ALPHAnstation 255 at the ZiF-Center in Bielefeld.

In Figure 1 we plot the data for the gluon propagator as a function of the

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2. Here $k$ has components $k_\mu N_\mu = 0, 1, \ldots, N_\mu - 1$. As in reference [17], we evaluate the gluon propagator by considering only values of $k$ with three of the four components equal to zero, namely $k = (0, 0, 0, k_4)$.

3. I thank S.Petrarca and B.Taglienti for kindly offering me the access to this machine.
square of the lattice momentum

\[ p^2(k) \equiv 4 \sum_{\mu} \sin^2 (\pi k_\mu) = 4 \sin^2 (\pi k_4) \]  

(3)

for different lattice sizes, at \( \beta = 0, 0.8, 1.2 \) and 1.6 respectively. Finally, in Table 2 we report the data for \( D(0) \) at \( \beta = 0 \) as a function of the lattice volume.

Data at \( \beta = 0 \) confirm previous results [16, 17]: the propagator is decreasing (more or less monotonically) as \( k \) decreases. At \( \beta > 0 \) the propagator is also decreasing in the infrared limit, at least for values of \( p^2 \) smaller than a turn-over value \( p_{to}^2 \). Clearly \( p_{to}^2 \) is \( \beta \)-dependent. Using, for each \( \beta \), the results for the largest lattice volume available we obtain: \( p_{to}^2 \approx 1.4 \) at \( \beta = 0.8 \), \( p_{to}^2 \approx 1 \) at \( \beta = 1.2 \), and \( p_{to}^2 \approx 0.6 \) at \( \beta = 1.6 \).

As \( \beta \) increases the behavior of the gluon propagator depends strongly on the lattice volume. This is evident at \( \beta = 1.6 \). In fact, for the smallest lattice volume \( V = 8^4 \), we obtain a gluon propagator increasing (monotonically) as \( k \) decreases. On the contrary, for \( V = 24^4 \), the gluon propagator clearly decreases as \( p^2 \) goes to zero.

Let us notice that, at high momenta, there are very small finite-size effects, at all values of \( \beta \). The situation is completely different in the small-momenta sector, as already stressed for the case \( \beta = 1.6 \), and as can be observed in Figure 4 also for \( \beta = 0.8 \) and 1.2. The value \( D(0) \) of the gluon propagator at zero momentum is also very volume-dependent (see Table 2 for \( \beta = 0 \), and Figure 4 for the other values of the coupling). In particular, at \( \beta = 0 \), as the volume increases, we obtain an initial monotonic decrease of the value of \( D(0) \); for \( V = 18^3 \times 36 \) this value unexpectedly jumps up, and then starts again to decrease monotonically.

Finally, for \( \beta = 0.8 \) and 1.6,
the value of the zero-momentum gluon propagator decreases (more or less monotonically) as $V$ increases. These results suggest a finite value for $D(0)$ in the infinite-volume limit, in agreement with references [12, 14]. From our data it is not clear if this value would be zero or a strictly positive constant. Therefore, the possibility of a zero value for $D(0)$ in the infinite-volume limit is not ruled out.

We think that our data for the gluon propagator are very interesting. The prediction [2, 5, 7] of a propagator decreasing as the momentum goes to zero is clearly verified numerically for several values $\beta$, even if only in the strong-coupling regime. Moreover, it appears that the lattice size at which this behavior for the gluon propagator starts to be observed increases with the coupling. Of course we should extend our simulations to higher values of $\beta$ and to larger volumes. This is, at the moment, beyond the limits of our computational resources.

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9 This was expected since Zwanziger’s predictions [5, 7] for the gluon propagator are valid only in the infinite-volume limit.
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Table 1: The pairs \((\beta, V)\) used for the simulations, the number of configurations, the number of sweeps used for thermalization, the number of sweeps between two consecutive configurations used for collecting our data, and the parameter \(p\) used by the stochastic overrelaxation algorithm.
Table 2: The zero four-momentum gluon propagator $D(0)$ [see eq. (1)] as a function of the lattice volume $V$ at $\beta = 0$. Notice that $16^3 \times 32 \approx 20^4$ and that $20^3 \times 40 \approx 24^4$. 

| $V$     | $D(0)$       |
|---------|--------------|
| $12^4$  | 0.154(0.007) |
| $16^4$  | 0.149(0.006) |
| $16^3 \times 32$ | 0.143(0.005) |
| $20^4$  | 0.142(0.005) |
| $18^3 \times 36$ | 0.160(0.005) |
| $20^3 \times 40$ | 0.153(0.006) |
| $24^4$  | 0.153(0.004) |
| $28^4$  | 0.146(0.005) |
| $24^3 \times 48$ | 0.144(0.007) |
| $30^4$  | 0.136(0.007) |
Figure 1: Plot of the gluon propagator $D(k)$ [see eqs. (1) and (2)] as a function of the square of the lattice momentum $p^2(k)$ [see eq. (3)] for lattice volumes: (a) $V = 28^4$ (□), $V = 24^3 \times 48$ (◇) and $V = 30^4$ (*), at $\beta = 0$; (b) $V = 8^4$ (□), $V = 12^4$ (◇) and $V = 16^4$ (*), at $\beta = 0.8$; (c) $V = 12^4$ (□), $V = 16^4$ (◇) and $V = 20^4$ (*), at $\beta = 1.2$; (d) $V = 8^4$ (□), $V = 16^4$ (◇) and $V = 24^4$ (*), at $\beta = 1.6$. In all our runs we set $k = (0,0,0,k_4)$. Since we use periodic boundary conditions, only data for $k_4 \leq 1/2$ are reported here. Error bars are one standard deviation.