Using smart meters for checking the topology and power flow calculation of a secondary distribution network

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Abstract. The paper analyzes options of using smart meters for power flow calculation and for assessing the state of a real three-phase four-wire secondary distribution network based on measurements of average values of active and reactive power and of voltages. The work is based on the authors’ research on allocation of measurements to ensure secondary distribution network observability and on selection of the most efficient method for linear and non-linear state estimation. The paper illustrates solution of a problem on identification composition of load nodes in the phases and reveals challenges related to voltage account in the neutral wire and in its grounding.

Introduction

The first experience in studying the operation conditions of primary and secondary distribution networks (DN) of medium and low voltage consisted in calculation of energy losses using the deterministic method [1] whose efficiency was confirmed by numerous practical calculations.

Power consumption data were obtained using power meters records at a primary substation, permitted power of consumers in the secondary network, and the number of consumption phases. Total produced power was determined using power supplied to the primary network that was distributed in proportion to the rated power of secondary transformers and adjusted with account of power losses in transformers and in the primary network. Total load of each feeder was evenly distributed between load nodes or according to the requested consumers’ demand.

A primary network was modeled using a single-line equivalent circuit, and a three-phase four-wire equivalent circuit proposed in [2] was used for the secondary network that allowed power flow calculation in the phase coordinates. Unlike Carson method [3] where three equations are constructed for the feeder section that connect phase voltages to the nodal currents, method [2] in the general case requires constructing five equations corresponding to three phases, a neutral wire and grounding. Use of five equations instead of three is caused by the fact that parameters of phase wires in method [2] do not include corrective additives that take into account the impact of ground (ground is assumed to be an ideal conductor) and of a neutral wire.

Modeling of a secondary distribution network revealed significant differences between its characteristics and characteristics of a high voltage network.

1. A secondary distribution network operates as an open network, power flows in the feeders being directed from a distribution substation to the load nodes. This allows power flow calculation using a backward-forward method [4] following the network graph with detailed representation of models of DN elements, which is similar to direct and reverse run in the universal Gaussian method.

2. Loads in the secondary network can be single-phase, two-phase and three-phase ones that results in asymmetry loads under symmetry of parameters of the network equivalent circuit.

3. Imaginary components of voltage drops in the feeder sections are practically equal to zero, which is due to high impedance ration R/X and small lengths of individual sections.

4. Strong dependence of voltages both on reactive and active power is an important feature of the network.

5. Peculiarities of a three-phase four-wire network that are due to the impact of current running in the neutral wire and voltage drop in the phase wires caused by this current are also noteworthy.

Ref. [5] mentions that currents and voltages in the non-grounded neutral wire are the higher the greater the asymmetry of phase loads; the current curve profile in this case notably differs from the profile of voltage curve. Current in the neutral wire in the general case can exceed current in phases. Current drop in the neutral wire is due to its numerous groundings that allow distribution of this current between a neutral wire and
ground in inverse proportion to their resistances. So far as current in the neutral wire effects phase voltages, then groundings and asymmetry loads have an impact on the operation condition of an electric network.

Ref. [6] shows that current in the neutral wire and voltage drop in it cause voltage change in the nodes of phase wires which is due to connection of loads between phase and zero wires, rather than between a phase wire and ground. It should be noted that voltages in the phase wires are measured in reference to the neutral wire rather than in reference to the ground. Numerous groundings of a neutral wire approximate voltage between a phase and a neutral wire to the voltage between a phase and a ground, still, they do not ensure equality to the voltage zero in the neutral wire and do not allow us to neglect it when calculating power flows in the network with asymmetry loads.

Conversion of a passive network into an active one is primarily due to introduction of uncontrollable sources of renewable generation that redirect power flows during the day. Accurate and reliable state estimation is the main method that ensures flexibility of such a transition. As compared to the state estimation using synchronized vector measurements (PMU) in the HV transmission networks, state estimation of a distribution network is a new more complicated task that requires detailed study. Moreover, synchronized PMU measurements are not used in the distribution networks due to their high cost.

Data on the state of a LV distribution network can be obtained by smart meters. In the distribution networks abroad an Advanced Metering Infrastructure (AMI) [7] connecting a distribution network with a communication network and with a modern metering structure is responsible for data transfer from a Smart Meter to the data acquisition system.

Automated System for Commercial Accounting of Electric Energy (ASCAEE) [8] is a Russian analogue of AMI. Data from consumers’ smart meters and from the balance meter installed on the LV side of a secondary substation are transmitted to the upper level of ASCAEE. Along with consumed energy, these meters allow measurements of average values of active and reactive power, magnitudes of currents and voltages. These measurements are used for state estimation of a secondary distribution network and ensure its gradual transition from a passive network to an active one.

The less the time interval between measurements in case of coincidence of time interval for load measurements, the higher the accuracy of power flow calculation based on ASCAEE data. Nevertheless, smart meter measurements are, as a rule, non-synchronized due to errors in adjustment of a clock recording the time of measurements, and due to time delay of data transmission by communication channels.

Errors in the data (obtained from specialized geographic systems) on connection of low-voltage feeders to transformers as well as errors in the data on network topology that may vary in emergencies or during scheduled maintenances, shall also be avoided. Data on the length of conductors and their parameters need checking and, probably, clarification prior to the state estimation procedure.

Data on connection of loads to feeder phases is of primary importance and, as a rule, needs special study. This information is needed for modeling each phase of a distribution network, for determining the composition of load nodes in it and for specifying the measured values of power and voltage in the nodes for further calculation of power flows, power losses, and for state estimation.

**Phase identification**

There are several approaches to phase identification, the most interesting, promising and simple one being the method proposed in [9] that consists in simultaneous measurement during a long period of time, \( t = 1, \ldots, n \), where \( n \) – the number of voltage magnitude observations \( U_{i_1}(t), U_{i_2}(t), U_{i_3}(t) \) in the power node with known phases \( a, b, c \), and voltage magnitude \( U_{i_j}(t) \) in the phase \( x \) of a load node \( i \), that needs identification.

Phase \( x \) is identified by maximum value of correlation coefficients between two vectors for measuring the voltage magnitudes in nodes 1 and \( i - U_{i_1}(t) \) and \( U_{i_2}(t) \) and \( U_{i_3}(t) \) \( U_{i_2}(t) \) and \( U_{i_3}(t) \). If the maximum correlation coefficient corresponds to the first pair \( U_{i_1}(t) \) and \( U_{i_2}(t) \), then \( x \) corresponds to phase \( a \).

It is noteworthy that not only measurements of a specially organized experimental study can be used for this method of identification, but hourly daily measurements of average values of voltage magnitudes, and daily instant measurements of voltage within one or several months as well. On the assumption that voltage measurements in nodes 1 and \( i \) are normally distributed random values, the coefficient of mutual correlation for them is determined by:

\[
R_{i_{i_1}i_{i_2}} = \frac{\sum_{t=1}^{n} (U_{i_1}(t) - \bar{U}_{i_1}) (U_{i_2}(t) - \bar{U}_{i_2})}{\sqrt{\sum_{t=1}^{n} (U_{i_1}(t) - \bar{U}_{i_1})^2 \sum_{t=1}^{n} (U_{i_2}(t) - \bar{U}_{i_2})^2}}.
\] (1)

**State estimation of a three-phase four-wire distribution network**

Assuming that topology of a secondary distribution network that includes information on the composition of load nodes in each phase and parameters of elements of a distribution network scheme are known, let us consider the main stages of its state estimation using records of meters that include active \( z_{a,b,c} \) and reactive \( z_{0}^{a,b,c} \) nodal powers and voltage magnitudes \( z_{a,b,c} \) in phases \( a, b, \) and \( c \).

Let the problem of state estimation of a secondary distribution network with a zero wire be written similarly to the problem considered in [2] for calculation of power flow in a three-phase four-wire distribution network, and let a non-linear system of equations for measurements be solved using the simple iteration method [10]. State
variables would include real \( u_{i}^{a,b,c} \), \( u_{i}^{*} \), \( w_{i}^{a,b,c} \) and imaginary components \( u_{i}^{a,b,c} \), \( u_{i}^{*} \), \( w_{i}^{a,b,c} \) of the vectors of phase voltages, neutral wire voltages, and ground voltages for the grounded neutral wire, where \( i \) is the node number.

State estimation procedure in the simple iteration method consists of two steps repeated iteratively. On the first step of the first iteration we determine pseudo measurements \( \bar{Z}_{a_{i}} + j \bar{Z}_{b_{i}} \) of nodal currents in each phase and current in the neutral wire using measured values of active and reactive power and initial approximations of voltage magnitudes, and currents in the ground in case of a grounded neutral wire

\[
\begin{bmatrix}
\bar{Z}_{a_{i}} + j \bar{Z}_{b_{i}} \\
\bar{Z}_{a_{i}} + j \bar{Z}_{b_{i}} \\
\bar{Z}_{a_{i}} + j \bar{Z}_{b_{i}} \\
\bar{Z}_{a_{i}} + j \bar{Z}_{b_{i}} \\
\bar{Z}_{a_{i}} + j \bar{Z}_{b_{i}} \\
\bar{Z}_{a_{i}} + j \bar{Z}_{b_{i}} \\
\end{bmatrix} = \begin{pmatrix}
\bar{U}_{i}^{a} - j \bar{w}_{i}^{a} \\
\bar{U}_{i}^{b} - j \bar{w}_{i}^{b} \\
\bar{U}_{i}^{c} - j \bar{w}_{i}^{c} \\
\bar{U}_{i}^{n} \\
\bar{U}_{i}^{n} \\
\bar{U}_{i}^{n} \\
\end{pmatrix} ,
\]

where, according to [2], \( Z_{a_{i}} = Z_{a_{i}} + Z_{a_{i}} \) equals to the sum of ground impedance \( Z_{a_{i}} \) and grounding impedance \( Z_{a_{i}} \), \( Z_{a_{i}} \) is impedance of neutral line at node \( i \), \( a_{1} = 1 - j \sqrt{3}/2 \), \( a_{2} = -1 - j \sqrt{3}/2 \).

In equation (2) we assumed that admittances of shunt elements of phase wires and that of a neutral wire are equal to zero.

At subsequent iterations, instead of initial approximations of voltages \( U_{i}^{a,b,c} \), the estimates of phase voltage vectors \( U_{i}^{a,b,c} = \begin{pmatrix}
\bar{U}_{i}^{a} - j \bar{w}_{i}^{a} \\
\bar{U}_{i}^{b} - j \bar{w}_{i}^{b} \\
\bar{U}_{i}^{c} - j \bar{w}_{i}^{c} \\
\end{pmatrix} \) obtained at a previous iteration and refined taking into account the voltages in the neutral wire \( U_{i}^{n} \) are inserted into denominators of the first three equations

\[
U_{i}^{a,b,c} = U_{i}^{a,b,c} - \bar{U}_{i}^{a,b,c}.
\]

Linear equations of measurements (4) corresponding to pseudo measurements of active and reactive components of currents and real components of voltages are specified as equal to measured voltage magnitudes [10]:

\[
\begin{pmatrix}
\bar{Z}_{a_{i}} + j \bar{Z}_{b_{i}} \\
\bar{Z}_{a_{i}} + j \bar{Z}_{b_{i}} \\
\bar{Z}_{a_{i}} + j \bar{Z}_{b_{i}} \\
\bar{Z}_{a_{i}} + j \bar{Z}_{b_{i}} \\
\bar{Z}_{a_{i}} + j \bar{Z}_{b_{i}} \\
\bar{Z}_{a_{i}} + j \bar{Z}_{b_{i}} \\
\end{pmatrix} = \begin{pmatrix}
\bar{U}_{i}^{a} - j \bar{w}_{i}^{a} \\
\bar{U}_{i}^{b} - j \bar{w}_{i}^{b} \\
\bar{U}_{i}^{c} - j \bar{w}_{i}^{c} \\
\bar{U}_{i}^{n} \\
\bar{U}_{i}^{n} \\
\bar{U}_{i}^{n} \\
\end{pmatrix} ,
\]

where \( Z_{a_{i}} \) and \( Z_{a_{i}} \) are nodal admittance matrices; \( I_{a_{i}} \) and \( 0_{a_{i}} \) are identity and zero matrices.

If all the mutual resistances between phase wires, neutral wire and ground are assumed to be equal to zero, equations (4) can be solved at each iteration either simultaneously or for each phase, for neutral wire and for ground separately.

In the general form the system (4) can be put down as:

\[
R^{-1} H \cdot u = R^{-1} Z ,
\]

where \( R \) is diagonal matrix whose elements are equal to the variances of measurement errors.

Under availability of redundant measurements the \( R^{-1} H \) is over determined matrix, therefore, for solving (5) the weighted least-squares method is used

\[
J(u) = (Z - Hu)^T R^{-1} (Z - Hu) .
\]

**Results of calculations on the example of an actual distribution network**

Let us first illustrate the preparation of data on referring the loads to phases of a real main feeder of a distribution network to nine nodes of which 24 private houses are connected (numbered from 12 to 35 in Fig. 1).

![Fig. 1. Scheme of the main feeder of a distribution network](image)

Electric power consumed by five three-phase loads 12, 13, 23, 30, and 35 of houses is measured by three-phase meters MIR C-04, and electric power consumed by remaining single-phase consumers is measured by MIR C-05 meters. A balance three-phase meter MIR C-07 whose connection phases are known is installed in the power node of the feeder. All the meters are installed on the supports of the main feeder, e.g., in node 3 there are three meters that measure consumed power of 12-14 houses and power losses in branches to those houses.

Phases for meters connection were identified using data of ASCAAE protocols on daily measurements of 37 instant values of voltage magnitudes within two winter months (once at approximately the same time of a day) and measurements of average hourly values of voltage magnitudes in a spring and summer day.

Correlation coefficients were computed using MATLAB procedure *corrcoef* that along with determination of mutual correlation coefficients \( R \) gives information on probability \( P \) of their occurrence in the 95% default confidence range whose lower \( RLO \) and upper \( RUP \) boundaries are defined. Correlation is assumed to be valid if probability that the correlation coefficient exceeds the boundaries of a confidence range is less than 0.05.

Fig. 2 presents graphs of mutual correlation coefficients between each out of three measurements of voltage magnitudes for the sampling of instant measurements of voltage magnitudes for two winter months: 1 – in phase \( a \), 2 – in phase \( b \), 3 – in phase \( c \) of a power supply node with every (from 4 to 37) measurement of voltage magnitudes in the unknown phases of load nodes. Maximum positive values of
mutual correlation coefficients allowed determination of phases of each metering (from 4 to 37) shown in Fig. 1. Thus, maximum correlation coefficient for a three-phase meter related to node 12 corresponds to its first phase and phase $a$ of the power source; subsequent maximum correlation coefficients relate the second phase of the meter to phase $c$ of the power source, and the third phase of a meter to phase $b$.

Analysis of correlation coefficients for other samplings showed that all of them identify the same phases of load connections. But more certain assessment of load connection phases was obtained for daily hourly measurements of voltage magnitudes the major share of coefficient for which was in the strong correlation zone, and all the coefficients were relevant.

Fig. 2. Mutual correlation coefficients between measurements of voltages in phases $a$, $b$, $c$ in the power supply node and in the phases of each of the load nodes

Assuming that impedances of phase and neutral wires are set correctly, let us illustrate the process of state estimation in the distribution network (Fig. 1) for one of the operation conditions, for which 37 sets of phase measurements of average values of active and reactive power and of voltage magnitudes are known. Average values of power measurements (Fig. 3) and voltage magnitudes in phases with reference to the voltage in the neutral wire (Fig. 4) were determined for meters installed in the same node. Average value of measurements in phase $a$ were determined in nodes 10 and 11; in phase $b$ – in nodes 3, 8, 9, and in phase $c$ – in nodes 3, 8, 10, and 11.

State estimation was done for the basic set of measurements that included voltage measurement in power supply node 1 and measurements of active and reactive power in the load nodes. Fig. 5 shows current magnitudes in the phases and currents in the neutral wire caused by asymmetry loads that were obtained in the course of calculations. Estimates of voltages in phases with reference to ground that were obtained without account of voltages in the neutral wire, and voltages in the neutral wire with reference to ground are shown in Figs. 6 and 7.

Fig. 3. Measurements of active and reactive loads in the feeder phases

Fig. 4. Measurements of average values of voltage magnitudes in the feeder phases

Fig. 5. Currents in phases $I_a$, $I_b$, $I_c$ of the feeder and in the neutral wire $I_n$

Fig. 6. Voltages in phases $U_a$, $U_b$, $U_c$ of the feeder to ground
If voltage profiles in phases with reference to ground are similar, then voltage estimates in phases with reference to the neutral wire that were adjusted with account of voltages in the neutral wire (Fig. 8) differ from one another, especially at the end of the feeder. Estimates of phase voltages with reference to the neutral wire (Fig. 8) can be compared to measured voltage magnitudes and show that the maximum estimate error against the measurement is 1.5V.

Fig. 8. Voltage curves for phases a, b, c: measured ones (phase to neutral) – $U_{a1}$, $U_{b1}$, $U_{c1}$; calculated ones (phase to ground) – $U_{a2}$, $U_{b2}$, $U_{c2}$; refined ones with account of voltage in the neutral wire (phase to neutral) – $U_{a3}$, $U_{b3}$, $U_{c3}$

**Conclusion**

Comparison of the results of voltage assessment to their measured values demonstrates a small error of estimates that, hopefully, can be lowered by increasing the number of measurement samplings; there is no need to compute average voltage values in the nodes using mismatched measurements from several meters, and to clarify data on resistances of wires and groundings. Another important problem for the analyzed feeder is reduction of voltage asymmetry as the maximum coefficient of phase voltage asymmetry for measured voltage magnitudes made up 8%, and that for linear voltages made up 9%.

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