Construction of a mathematical model for the extraction of mineral raw materials

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Abstract. This article discusses the Essentuki deposit, namely the Central area. The scheme of the field and the hydrogeological section are presented. The methodology of constructing a mathematical model for the extraction of mineral raw materials is shown. Namely, a flat-spatial mathematical model of the hydrolithospheric processes of the Essentuki groundwater deposit was recorded. In this case, the hydrolithospheric processes in aquifers are described by spatial models, and the processes in aquicludes by flat models. It also shows how to take into account the geological and hydrogeological structure of an object as much as possible when building a mathematical model. It is shown how the boundary conditions of the object are set. The physical parameters of the object and geometric data used in mathematical modeling are presented. The article presents the result of modeling the extraction of mineral raw materials from one layer. The stages of construction of control systems for the parameters of hydrolithospheric processes are considered.

1. Introduction
Taking into account the rapid development of human life, the careless use of resources leads to the fact that environmental problems, including water pollution, have now become too acute. And solving these environmental problems is the number one task today.

Groundwater is water that is located in the interlayer spaces of rocks, occurs in the upper layer of the earth's crust. Groundwater can be presented in any state of aggregation: liquid, solid and gaseous. Groundwater is tons of fluid [1].

An incorrect choice of the mode of extraction of mineral raw materials can lead to catastrophic consequences for the entire hydromineral base of the Caucasian Mineral Waters region, and for an individual deposit, in particular. Building a distributed control system capable of maintaining a stable mode of mineral water production is a way out of the current situation [2].

The construction of control systems for the parameters of hydrolithospheric processes is associated with the solution of the following problems [3]:
1. determination of the dynamic characteristics of the object under consideration;
2. approximation of dynamic characteristics using special models;
3. design of control systems.

But first of all, for the development of such a control system, an important role is played by mathematical modeling of the control object. [4].
Mathematical modeling of the extraction of mineral raw materials will allow environmentally safe and rational use of natural resources, diagnose the state of the hydrolithosphere, predict the development of technogenic processes in the hydrolithosphere and manage them.

2. Objects and research methods
Modeling the extraction of mineral raw materials will be considered on the example of the Essentuki deposit.

The Essentuki mineral water deposit is located in the central part of the Caucasian Mineral Waters region (figure 1) and stretches for a distance of about 17 km to the Byk and Verblud laccolithic mountains. The world fame of the Essentuki resort was brought by the mineral springs of the same name No. 4 and No. 17.

![Production wells of the Essentuki deposit.](image)

**Figure 1.** Hydrogeological section of the Essentuki deposit.

The Essentuki deposit has long been known for its medicinal and medicinal-table waters “Essentuki-17” and “Essentuki-4”, which are used for drinking in the drinking galleries of the same name located in the city's Healing Park. These are waters of chloride-hydrocarbonate sodium composition with a mineralization of 8-14 g/l, which are removed from a depth of 40-210 meters from the Paleogene aquifer of the Central section of the field by wells 17-bis, 41-bis and others. The central section is located directly in the city of Essentuki.

Two aquifers are exploited here - the Paleocene and the Upper Cretaceous.

The supply of highly conditioned, bacteriologically clean mineral waters from the Beshtaugorskoye deposit to the Essentuki resort made it possible to abandon the use of bacteriologically unfavorable waters of the spring No. 20, Gaazo-Ponomarevsky and well No. 505. Since 1996, mineral water from wells No. 66 and 2-B has been bottled as medicinal table water.

At present, hydrodynamics is monitored using an observational network, the location of which cannot be considered successful. Its information content is extremely low, it is covered by single wells, and is unable to characterize the situation in the field as a whole. In this case, it is possible to resort to mathematical modeling.

It is envisaged that the structure of the model and methods for implementing solutions will allow maximum consideration of the geological and hydrogeological structure of the object. This concerns issues related to the modeling of reservoirs with double porosity (porous-fractured type), complex tectonic boundaries (internal and external), taking into account the hydraulic relationship of aquifers of different hydrodynamic levels involved in the zone of technogenic impact. The model should provide the ability to predict the situation with a lead time of up to several hundred years. Taking into account the current state of the scientific and technical base, the most preferable are spatial and plane-spatial models, implemented by numerical methods on a computers according to various schemes. [5].
Several enterprises are involved in the extraction of hydromineral raw materials and, as a rule, they have separate wells located at a distance from each other. At the same time, the mutual influence of the wells is quite small. In the considered field, three types of narzans are extracted from different layers (general, dolomite and sulfate). In the article, we will limit ourselves to considering one layer from which the common narzan is extracted (figure 2).

Well

![Well diagram](image)

**Figure 2.** Field scheme.

The mathematical model of the object under consideration is given in [5].

Let us write down a plane-spatial mathematical model of hydrolithospheric processes of the Essentuki groundwater deposit. In this case, the hydrolithospheric processes in aquifers will be described by spatial models, and the processes in aquicludes by flat models [6].

**Ground water**

\[
\frac{\partial h_1(x, y, z_1, \tau)}{\partial \tau} = k_{1,x} \frac{\partial^2 h_1(x, y, z_1, \tau)}{\partial x^2} + k_{1,y} \frac{\partial^2 h_1(x, y, z_1, \tau)}{\partial y^2} + k_{1,z} \frac{\partial^2 h_1(x, y, z_1, \tau)}{\partial z_1^2};
\]

\[
0 < x < L_x; 0 < y < L_y; 0 < z_1 < L_{z_1}
\]

**Common narzan**

\[
\frac{\partial H_2(x, y, z_2, \tau)}{\partial \tau} = \frac{1}{\eta_z} \left( k_{2,x} \frac{\partial^2 H_2(x, y, z_2, \tau)}{\partial x^2} + k_{2,y} \frac{\partial^2 H_2(x, y, z_2, \tau)}{\partial y^2} + k_{2,z} \frac{\partial^2 H_2(x, y, z_2, \tau)}{\partial z_2^2} \right) +
\]

\[
+ V(\tau) \cdot \delta(x_0, y_0, z_0);
\]

\[
0 < x < L_x; 0 < y < L_y; 0 < z_2 < L_{z_2}
\]

**Boundary conditions (Darcy conditions)** between the layers are given in the form

**Groundwater - upper layer**

\[
h_1(x, y, L_{z_1}, \tau) = h_1(x, y, L_{z_1}, \tau) + b_1 \cdot \left( H_2(x, y, 0, \tau) - h_1(x, y, L_{z_1}, \tau) \right),
\]
\[ H_2(x, y, 0, \tau) = H_2(x, y, 0, \tau) - b_1 \cdot \left( H_2(x, y, 0, \tau) - h_1(x, y, L_z, \tau) \right). \]

where \( b_1 = 0.00003 \text{ day}^{-1} \) – overflow parameter.

Lower formation boundary: \( \partial H_2(x, y, L_x, \tau)/\partial z = 0 \)

Side faces: \( h_1(0, y, z_1, \tau) = h_{1,0} ; H_2(0, y, z_2, \tau) = H_{2,01}, \)

\( \partial h_1(L_x, y, z_1, \tau)/\partial x = 0 ; H_2(L_x, y, z_2, \tau) = H_{2,02}. \)

\[ h_1(x, 0, z_1, \tau) = h_1(x, L_y, z_1, \tau) = h_{1,0}, \]

When forming boundary conditions along the y coordinate, we assume that the thickness of the reservoir is such that disturbances from intake wells do not affect the state of the reservoir at the boundary points:

\[ H_2(x, 0, z_2, \tau) = H_2(x, L_y, z_2, \tau) = H_{2,01} - (H_{2,01} - H_{2,02}) \cdot x/L_x, \quad (3) \]

(where in the equation 3) \( h_{1,0} = 0 \) (0<z<L_z), \( H_{2,01} = 85 \text{m}, \) \( H_{2,02} = 65 \text{m} \) - initial states of undisturbed groundwater and reservoir.

Comment: In the considered mathematical model of the object, there is no component F1, x, which takes into account the flow rate in the 1st aquifer.

Studies show that the dynamic characteristics of the control object will be the same if:

1. In the boundary conditions (at \( y = 0 \) and \( y = L_y \)), we take into account the level difference (\( H_{2,01} = 85 \text{m}, \) \( H_{2,02} = 65 \text{m} \)), While the main equation will lack F1, x;

2. In the boundary conditions (for \( y = 0 \) and \( y = L_y \)), we set the same level, and add the velocity to the main equation \( F_1(x) = k_{2,x} (H_{2,01} - H_{2,02})/L_x. \)

Both options for describing mathematical models of hydro lithospheric processes are used in solving practical problems.

Physical parameters of the object:

- \( h_1 \) – the pressure in the water table;
- \( H_2 \) - the pressure in the studied aquifer;

the coefficients correspond to certain coordinates:

\[ k_{1,x} = 0.198 \text{ m/day}, k_{1,y} = 0.196 \text{ m/day}, k_{1,z} = 0. \text{ m/day}, \]
\[ k_{2,x} = 0.146 \text{ m/day}, k_{2,y} = 0.148 \text{ m/day}, k_{2,z} = 0.024 \text{ m/day}; \]
\[ \eta = 0.000581/\text{m}. \] – reservoir elasticity;
\( V(\tau) \) – pressure drop caused by the impact (flow rate) of a production well; \( \delta(x_0, y_0, z_0) \) - function equal to one if \( x = x_0, y = y_0, z = z_0 \), and equal to zero in other cases; \( x, y, z \) – spatial coordinates; \( \tau \) – time.

Geometric data of the field: \( L_x = 350 \text{m}, \) \( L_y = 300 \text{m}, \) \( L_z1 = 45 \text{m}, \) \( L_z2 = 170 \text{m}. \)

When modelling a control object, we will assume that the number of sampling points in coordinates:
\( x \to (N_x = 13), y \to (N_y = 21), z \to (N_z1 = 9, N_z2 = 9). \) Accordingly, the sampling steps were specified in the form:

\[ \Delta x = L_x/(N_x - 1); \Delta y = L_y/(N_y - 1); \]
\[ \Delta z1 = L_z1/(N_z1 - 1); \Delta z2 = L_z2/(N_z2 - 1); \]

The detail of the breakdown of the model was determined, on the one hand, by the availability of information on exploration and production areas within its limits, on the other hand, by the possibility of modeling individual stages of operation and experimental hydrogeological work in the Central area of the Essentuki field. Therefore, the mesh of the model is uneven.
The production well is carrying out the intake of hydromineral raw materials “to the point” $N_0=7$, $N_{u0}=11$, $N_{z20}=5$.

The input influence on the control object is the production well flow rate $Q(\tau)$, which is related to the function $V(\tau)$ by the following relation [7, 8]

$$V(\tau) = K \cdot Q(\tau) \quad (4)$$

There is a working well, which is used to extract hydromineral raw materials. In this case, the flow rate of the well under consideration is 1000 m$^3$/day. Let's carry out an experiment on a real object - we will jump-increase the flow rate by 100 m$^3$/day. $(1000+100) = 1.01 \cdot 1000 = 1010$ m$^3$/day $(1000 \cdot 100) = 0.0011574$ m$^3$/sec.)

3. Results and discussions

The decrease in the level in the steady state in the area of the location of the intake device of the well was 2 m.

As the main working hypothesis on the model, the assumptions were taken of the presence of a crossflow into the Upper Jurassic, and in the areas of its pinching out directly into the Upper Cretaceous horizon of water-gas fluid from the basement along local tectonic zones within the Kavminvodsky intrusive-dome rise, in the central part of which the Essentuki field is located. From the basement, through local tectonic faults, mainly carbon dioxide comes, and the water component in the balance of mineral water deposits is provided by the natural resources of the Lower Cretaceous and Upper Jurassic aquifers, formed within the entire Mineral Vody artesian basin. Since it was not possible to model this complex and little-studied process in this case, such a concept, conditional for the real natural situation, but acceptable for the model, was introduced as “inflow from the basement”.

In this regard, water exchange across the lower boundary as a whole over the area was not considered, but along the fault zones surrounding the laccoliths, an additional inflow from the basement was modeled, which was schematized by the task $\Gamma Y-1$ (H= const).

As part of this work, the results of drilling and testing of prospecting, exploration and production wells for underground waters, as well as a number of wells for exploratory oil drilling, were collected and summarized. Based on these materials, a set of maps was constructed for the main aquifers (horizons) of the Mineralovodsk artesian basin (MAB), including structural maps of the thickness of water-bearing sediments, hydroisopieses, filtration parameters, etc. The constructed maps were taken as a basis for creating a numerical regional geofiltration model MAB, which included an inset model for the area of the Essentuki mineral water deposit.

As part of the work, the previously developed inset model was detailed and refined both by the number of wells and by long-term data on the groundwater regime. At the same time, new monitoring data and experimental hydrogeological works for 2005-07 were used to justify and calibrate the model.

In the area of the detailed and refined model of the area of the Essentuki mineral water deposit, which completely coincides with the area of the hydrogeological map of the area.

Due to the practical absence of a hydraulic connection between the surface and ground waters of the Lower Cretaceous and Upper Cretaceous aquifers, their levels were set by setting the boundaries of the 1st kind along the contours of the relief, i.e. averaged values per model block.

The infiltration feeding on the model in the initial version was taken in accordance with the map of the underground runoff modules of the active water exchange zone at a level of 30 mm / year (~ 1 l / s * km$^2$).

In the process of modeling, the value of infiltration nutrition was adjusted and amounted to: from 40 mm / year - in the vicinity of laccolithic mountains to 20 mm / year - in the western part of the area. Moreover, we immediately note that the value of the given infiltration does not have a significant effect on the levels of the main aquifers exploited at the estimated Essentuki mineral water deposit.

It should also be noted that the available information on the area is very uneven and characterizes mainly the exploration and exploitation areas (mainly the Central area). Horizontal $k$ by layers were
originally calculated in accordance with the maps of the conductivity coefficients and the thickness of the selected layers, and were refined when solving inverse hydrodynamic problems.

The main selection of parameters was carried out by selecting the value of the vertical filtration coefficient, mainly in the separating low-permeability strata. Analyzing the results of the assessment of geofiltration parameters, the highest values of horizontal filtration coefficients in general over the area (0.198-0.196 m / day) are characteristic of the Lower Cretaceous aquifer, and have the highest values in the Central area. Note that in this case we are talking about the calculated (model) filtration coefficient related to the total capacity of the selected aquifer, while in reality water inflows provide separate intervals with higher filtration parameters. The indicated values correspond to values of water conductivity from 1 to 4 m² / day. Directly around Mount Zheleznaya, in a limited area, there is an increase in the filtration coefficient of water-bearing rocks in the Upper Cretaceous horizon up to 0.1-0.5 m / day.

The model values of the filtration parameters correspond to the results of hydrogeological testing in the studied areas, and in the absence of data on the permeability of the reservoirs, they were selected on the model in the process of solving inverse problems.

To solve a series of predictive problems in relation to the existing technological scheme for the exploitation of mineral waters in the area of the Essentuki deposit and in its Central area, it is necessary to set on the developed numerical model a water withdrawal corresponding to the Central area of its modern resource potential, assessed by the hydraulic method or according to the approved operational reserves of mineral waters.

Qualitative indicators of groundwater are associated with operating modes and climatic factors [9]. Well disturbance leads to disruption of the existing water balance. In this case, both the lateral and vertical flows change their intensity, which is reflected in the change in the initial mineralization. These changes are not dynamic, since the filtration processes are characterized by low speeds, and stabilization occurs after a rather long time. In this case, the features of the geological and hydrogeological structure and the nature of the disturbance of the system play an important role. In some cases, transient processes are completed relatively quickly, forming one level of mineralization. In others, on the contrary, the processes are strongly extended in time and have a different level of salt content. So, if pumping is carried out only from the Lower Valanginian horizon, then due to the presence of hydraulic connection with both the Tithonian and the Upper Valanginian, the salinity in the well will be established depending on the ratio of two multidirectional flows that have large differences in salinity. If pumping is carried out from the Upper and Lower Valanginian simultaneously, then the formation of the chemical composition in the Valanginian horizons will be determined mainly by the ascending flow from the Tithonian. That is, the graphs of the transient process for the described cases will differ significantly. Even after complete stabilization of the elements of the underground flow, mineralization will undergo changes for some time due to the presence of vertical and areal hydrogeochemical zoning. In general, the nature of the change in mineralization can be described by an exponential relationship of the form [10]:

\[ M(t) = M_0 + \alpha Q_{cp} \times (1 - \exp(-t/T)) \]  

(5)

where: \( \alpha \) – amplification factor; \( T \) – time characterizing the speed of the transition process; \( M_0 \) – some initial value of mineralization; \( Q_{cp} \) – average pumping rate; \( t \) - the current time from the start of the disturbance.

All of the above parameters are determined empirically based on experimental data.

4. Conclusion

Therefore, it is necessary to develop measures to maximize the preservation of the historically established diversity (in terms of temperature, carbon dioxide content, etc.) of numerous medicinal mineral springs of the Essentuki resort, for which it is necessary to minimize the technogenic impact on the conditions of their formation, to strengthen control over their hydrodynamic and gas-hydrochemical state and sanitary protection.
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