A speed and departure time optimization algorithm for the Pollution-Routing Problem

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Abstract
We propose a new speed and departure time optimization algorithm for the Pollution-Routing Problem (PRP) which runs in quadratic time. This algorithm is embedded into an iterated local search-based metaheuristic to achieve a combined speed, scheduling and routing optimization. Extensive computational experiments are conducted on classic PRP benchmark instances. Allowing delayed departure times from the depot significantly increases the search space, and contributes to reduce CO$_2$ emissions by 8.31% on the considered instances.

1 Introduction
The Pollution-Routing Problem (PRP) is a variant of the Vehicle Routing Problem with environmental considerations, introduced in Bektaş and Laporte (2011). The objective is to minimize operational and environmental costs subject to capacity and hard time-window constraints. In the PRP, the costs are mainly based on fuel consumption, which depends on many factors (e.g. distance traveled, vehicle load, level of CO$_2$ emissions, etc.), and driver wages. To our knowledge, the PRP was addressed by Demir et al. (2012), Kramer et al. (2015) and Dabia et al. (2014). Demir et al. (2012) proposed an adaptative large neighborhood search heuristic,
while Kramer et al. (2015) developed a matheuristic, integrating an iterated local search with a set-partitioning formulation. More recently, Dabia et al. (2014) put forward a branch-and-price algorithm for the PRP.

Vehicle-speed decisions play an important role in the PRP, since they do not only affect the total cost, but also the travel times between the locations, with a large impact on time-window feasibility. For this reason, most algorithms for the PRP perform – at regular times during the search – an optimization of vehicle speeds for the current routes. The resulting sub-problem, called speed optimization problem (SOP), seeks to find the most cost-efficient arc speeds on a given route while respecting arrival time constraints at each customer. In the considered vehicle routing application, the objective function is non-linear, based on fuel consumption and work duration.

Several speed-optimization algorithms are available in the literature (Demir et al. 2012; Kramer et al. 2015; Norstad et al. 2011; Hvattum et al. 2013). These algorithms run in quadratic time, consider identical cost/speed functions for each arc, and assume that the departure time at the origin is fixed. Considering the departure time as a decision variable may yield a significant cost reduction, but also increase resolution complexity. In Dabia et al. (2014), the departure time from the first customer is optimized by means of a golden section search, within a dynamic programming for column generation. Finally, Vidal et al. (2014) showed that the optimal solution of the SOP with arc-dependent speed/cost functions can be achieved by solving a hierarchy of resource allocation problems with nested constraints. Other works on SOP can be found in Norlund and Gribkovskaia (2013), Psaraftis and Kontovas (2014) and Lipp and Boyd (2014).

This article contributes to the resolution of difficult vehicle routing variants with speed and departure time optimization. We propose a simple polynomial algorithm for the SOP, which runs in quadratic time and also optimizes the departure time from the depot. This speed optimization algorithm is then embedded into a vehicle routing matheuristic to produce optimized routing plans. Computational experiments are performed on the PRPLIB instances presented in Demir et al. (2012) and Kramer et al. (2015), to assess the impact of departure time optimization. The results highlight very significant routing cost reduction, of 8.31% on average, when comparing to the best known solutions of the original PRP. The computation time of the new metaheuristic remains comparable to current state-of-the-art methods despite the fact that it deals with a more general problem.

2 Problem Description

The PRP is defined as follows. Let $G = (\mathcal{V}, \mathcal{A})$ be a complete and directed graph with a set $\mathcal{V} = \{0, 1, 2, \ldots, n\}$ of vertices and a set $\mathcal{A} = \{(i, j) : i, j \in \mathcal{V}, i \neq j\}$ of arcs. Vertices $\mathcal{V}' = \{1, 2, \ldots, n\}$ correspond to the customers, while vertex $\{0\}$ represents the depot. A set $K = \{1, 2, \ldots, m\}$ of vehicles with capacity $Q$ is available to serve the customers. Each customer
A speed and departure time optimization algorithm for the PRP

Kramer, R.; Maculan, N.; Subramanian, A.; Vidal, T.

Let \( i \in \mathcal{V} \) have a non-negative demand \( q_i \), specified time-window interval \([a_i, b_i]\) to be served and a service time \( \tau_i \). Demand, departure time and service time from depot are assumed to be zero. Each arc \((i, j) \in \mathcal{A}\) represents a travel possibility from node \( i \) to \( j \) for a distance \( d_{ij} \), which can be traveled with any speed \( v_{ij} \) in the interval \([v_{\text{min}}, v_{\text{max}}]\).

The traveling speed is directly proportional to the environmental costs and inversely proportional to the operational costs, respectively. Therefore, the PRP aims at determining a speed matrix \((v)_{ij}\) for the arcs and a set of routes \( \mathbf{R} \) (such that \(|\mathbf{R}| \leq m\) to serve all customers while minimizing environmental and operational costs, satisfying the constraints of the well known vehicle routing problem with time windows, that is, each route must start and ends at the depot, every customer must be visited within its time-window and the sum of customers demand of each route cannot exceed the vehicle capacity.

Let \( \sigma = (\sigma_1, \sigma_2, \ldots, \sigma_{|\sigma|}) \) be a route, \( f_{\sigma_i, \sigma_{i+1}} \) be the vehicle load on arc \((\sigma_i, \sigma_{i+1})\) and \( t_{\sigma_i} \) be the arrival time at customer \( \sigma_i \). The environmental cost is proportional to the fuel consumption, which is computed according to Equation (1), where \( w_1, w_2, w_3, w_4 \) are parameters based on fuel properties, vehicle and network characteristics. On the other hand, the operational costs (driver salaries) are proportional to route duration. Defining \( \omega_{\text{fc}} \) as the fuel cost per liter and \( \omega_{\text{fd}} \) as the driving cost per second, the overall PRP cost – to be minimized – can be computed as in Equation (2).

\[
F_{\sigma_i, \sigma_{i+1}}^v (v_{\sigma_i, \sigma_{i+1}}) = d_{\sigma_i, \sigma_{i+1}} \left( \frac{w_1}{v_{\sigma_i, \sigma_{i+1}}} + w_2 + w_3 f_{\sigma_i, \sigma_{i+1}} + w_4 v_{\sigma_i, \sigma_{i+1}}^2 \right) \tag{1}
\]

\[
Z_{\text{PRP}}(\mathbf{R}, \mathbf{v}) = \sum_{\sigma \in \mathbf{R}} \left( \omega_{\text{fc}} \sum_{i=1}^{\sigma_{|\sigma|}-1} F_{\sigma_i, \sigma_{i+1}}^v (v_{\sigma_i, \sigma_{i+1}}) + \omega_{\text{fd}} (t_{\sigma_{|\sigma|}} - t_{\sigma_1}) \right) \tag{2}
\]

Notice that the operational cost is computed based only on the time \( t_{\sigma_{|\sigma|}} \) in which the vehicle returns to depot (i.e. the departure time from the depot is equal to zero). When a late departure time from the depot is allowed, the PRP cost function is defined as in Equation (3).

\[
Z_{\text{PRP}}(\mathbf{R}, \mathbf{v}) = \sum_{\sigma \in \mathbf{R}} \left( \omega_{\text{fc}} \sum_{i=1}^{\sigma_{|\sigma|}-1} F_{\sigma_i, \sigma_{i+1}}^v (v_{\sigma_i, \sigma_{i+1}}) + \omega_{\text{fd}} (t_{\sigma_{|\sigma|}} - t_{\sigma_1}) \right) \tag{3}
\]

3 The proposed speed and departure time optimization algorithm

The fuel consumption per distance unit \( F_{\sigma_i, \sigma_{i+1}}^v (v_{\sigma_i, \sigma_{i+1}}) \) is a convex function. The speed value \( v^*_{\text{fc}} \) that minimizes fuel costs is given in Equation (4). Similarly, for any arc \((\sigma_i, \sigma_{i+1})\), assuming that there is no waiting time in the route after \( \sigma_i \), the speed value \( v^*_{\text{fd}} \) that minimizes fuel and
A speed and departure time optimization algorithm for the PRP

Kramer, R.; Maculan, N.; Subramanian, A.; Vidal, T.

driver costs is expressed in Equation (5). Both values, \( v^* \) and \( v_{PD}^* \), are independent of the arc under consideration.

\[
\frac{dE^v}{dv^*_{\sigma_i\sigma_{i+1}}}(v^*) = 0 \iff v^*_\sigma = \left( \frac{w_1}{2w_4} \right)^{1/3} \quad (4)
\]

\[
v_{PD}^* = \left( \frac{w_{PD} + w_1}{2w_4} \right)^{1/3} \quad (5)
\]

Given a route \( \sigma \), the speed and departure time optimization problem consists of finding the departure time from the first location (i.e., the depot when dealing with vehicle routing problems) and the optimal speeds for each arc while respecting customers’ time-windows. In order to solve this problem, we propose a recursive algorithm that extends those presented in Demir et al. (2012); Kramer et al. (2015); Hvattum et al. (2013).

The proposed approach can be generally viewed as a divide-and-conquer strategy, which iteratively solves a relaxed speed optimization problem obtained by ignoring time windows at intermediate destinations. If the resulting solution satisfies all constraints, then it is returned. Otherwise the customer \( p \) with maximum time-window violation is identified and its arrival time is set to its closest feasible value. Fixing this decision variable creates two sub-problems which are recursively solved (Alg. 1, Line 20-21).

Algorithm 1 is first applied on a given route \( \sigma \) by setting the “start” \( s \) to 1 and the “end” \( e \) to \( |\sigma| \). We recall that the departure time and return time to the depot are considered as decision variables. In this first iteration, still, the departure time is set to the earliest possible value \( t_{\sigma_1} = a_{\sigma_1} \) (Alg. 1, Line 6). This decision will be revised later on. From this starting time, the arrival time at each customer is derived as in Kramer et al. (2015): at first, the arrival time at the last customer when traveling at speed \( v_{PD}^* \) is determined and, in case of violation, updated to its closest time-window bound (Alg. 1, Line 8). Such arrival time leads to a reference speed \( v_{ref} \) on the route (Alg. 1, Line 11) which will be used to compute the arrival time at each customer as well as the maximum time-window violation (Alg. 1 Lines 12-17).

In case of violation, two subproblems will be recursively solved. Any subproblem starting at the depot is now solved without fixing the departure time to \( a_{\sigma_1} \) (as opposed to the method presented in Kramer et al. (2015)). Indeed, the arrival time to the last customer of this subproblem is already fixed. It is thus possible to evaluate the reference speed “backwards”, deriving the best departure time at the depot (Alg. 1, Line 10), and the arrival time at each customer. The other sub-problems are solved in a similar manner. For these cases, the starting date – and possibly the arrival date to the last customer or depot – are already set. The recursion is repeated until no violation is found.

Finally, once arrival times \( t_{\sigma_i} \) are known for all customers \( i = 1, \ldots, |\sigma| \), the associated speeds are revised in such a way that any speed below \( v^*_\sigma \) is replaced by \( v^*_\sigma \) and a waiting time (Alg. 1 Lines 22-24).

Figure 1 shows an execution example of the presented algorithm in a route involving seven
nodes. The horizontal lines represent the customers and the brackets their corresponding time windows. Bullet points indicate the arrival times. The best departure and arrival times of this example are depicted in Figure 1.e.

Algorithm 1 Speed and Departure Time Optimization Algorithm — SDTOA

1: Procedure SDTOA($\sigma$, $s$, $e$)
2: $p \leftarrow$ violation $\leftarrow$ maxViolation $\leftarrow$ 0
3: $D \leftarrow \sum_{i=1}^{e-1} d_{\sigma_i, \sigma_{i+1}}$
4: $T \leftarrow \sum_{i=1}^{e-1} \tau_{\sigma_i}$
5: if $s = 1$ and $e = |\sigma|$ then
6:   $t_{\sigma_1} = a_{\sigma_1}$
7: if $e = |\sigma|$ then
8:   $t_{\sigma_e} = \min\{\max\{a_{\sigma_e}, t_{\sigma_e} + D/v_{ref} + T\}, b_{\sigma_e}\}$
9: if $s = 1$ then
10:  $t_{\sigma_1} = \min\{\max\{a_{\sigma_1}, t_{\sigma_1} - D/v_{ref} - T\}, b_{\sigma_1}\}$
11: $v_{ref} \leftarrow D/(t_{\sigma_e} - t_{\sigma_s} - T)$
12: for $i = s + 1 \ldots e$ do
13:   $t_{\sigma_i} = t_{\sigma_{i-1}} + \tau_{\sigma_{i-1}} + d_{\sigma_{i-1}, \sigma_i}/v_{ref}$
14:   violation $\leftarrow \max\{0, t_{\sigma_i} - b_{\sigma_i}, a_{\sigma_i} - t_{\sigma_i}\}$
15: if violation $> maxViolation$ then
16:   maxViolation $= violation$
17: $p = i$
18: if maxViolation $> 0$ then
19:   $t_{\sigma_p} = \min\{\max\{a_{\sigma_p}, t_{\sigma_p}\}, b_{\sigma_p}\}$
20: SDTOA($\sigma$, $s$, $p$)
21: SDTOA($\sigma$, $p$, $e$)
22: if $s = 1$ and $e = |\sigma|$ then
23: for $i = 2 \ldots |\sigma|$ do
24:   $v_{\sigma_{i-1}, \sigma_i} = \max\{d_{\sigma_{i-1}, \sigma_i}/(t_{\sigma_i} - t_{\sigma_{i-1}} - \tau_{\sigma_{i-1}}), v_{\sigma_i}^\star\}$

Figure 1: Computing departure time and arrival times with SDTOA

4 Computational Results

The proposed algorithm was integrated into the ILS-SP-SOA matheuristic of Kramer et al. (2015), which makes use of an adaptive speed matrix through the search to keep track of speed
A speed and departure time optimization algorithm for the PRP

Kramer, R.; Maculan, N.; Subramanian, A.; Vidal, T.

decisions. Instead of SOA, the SDTOA is executed on each route associated to a local optimal solution found by the ILS, returning routes with optimized speeds and departure times. These routes are stored in a pool that are further combined together by means of integer programming over a set partitioning (SP) formulation.

We tested the method on the instances suggested in Demir et al. (2012) (PRPLIB) and Kramer et al. (2015), ranging from 10 to 200 customers. The same values as in Demir et al. (2012) and Kramer et al. (2015) have been used for the objective function, that is: 

\[ w_1 = 1.01763908 \times 10^{-3}; \quad w_2 = 5.33605218 \times 10^{-5}; \quad w_3 = 8.40323178 \times 10^{-9}; \quad w_4 = 1.41223439 \times 10^{-7}; \]

\[ \omega_{fc} = 1.4 \frac{L}{l} \quad \omega_{fd} = 2.22222222 \times 10^{-3} \frac{L}{s}. \]

The algorithm was implemented in C++ and executed on an Intel Core i7 3.40 GHz processor with 16 GB of RAM, running under Linux Mint 13. The SP problems were solved using CPLEX 12.4. Only a single thread was used and the algorithm was run 10 times for each instance. Table 1 displays a summary of the results, reporting the average solution on ten runs. Results are compared with those of the current state-of-the-art Kramer et al. (2015), where late departure times from the depot are not allowed. Each line corresponds to averaged results on a set of 20 instances. The Gap(%) for each instance is computed as 100(\(Z - Z_{bks}\))/\(Z_{bks}\), where \(Z\) is the objective value of the solution and \(Z_{bks}\) is the value of the Best Known Solution (BKS) for the PRP without late departures from depot. The results show that when allowing departures times from the depot, one can obtain a considerable reduction on the total costs. More specifically, a reduction of 8.31% was achieved on average, and instances with tighter time-windows (sets B and C) tend to be more prone for solution quality improvement based on delays at the depot as can be seen in Figure 2.

Table 1: Results for the PRP with late departures from depot

| Instance | Avg. Cost | ILS-SP-SDTOA | BKS for PRP |
|----------|-----------|--------------|-------------|
| I         | CPU T(s)  | Best Cost   | Gap(%)      |
| 10-A      | 183.14    | 0.04         | 183.06      | -1.32      |
| 10-B      | 248.22    | 0.04         | 247.88      | -11.83     |
| 10-C      | 195.21    | 0.04         | 195.08      | -12.19     |
| 50-A      | 601.18    | 3.16         | 600.26      | -1.55      |
| 50-B      | 771.03    | 4.76         | 770.29      | -9.37      |
| 50-C      | 672.86    | 4.77         | 671.83      | -11.07     |
| 100-A     | 1105.12   | 36.29        | 1102.83     | -1.46      |
| 100-B     | 1386.35   | 69.57        | 1384.70     | -9.86      |
| 100-C     | 1228.35   | 61.13        | 1226.13     | -12.33     |
| 200-A     | 1951.74   | 340.71       | 1944.75     | -1.02      |
| 200-B     | 2345.41   | 782.76       | 2334.08     | -13.20     |
| 200-C     | 2114.28   | 482.29       | 2103.18     | -14.54     |
| Avg.      | 1066.91   | 148.79       | 1063.67     | -8.31      |

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A speed and departure time optimization algorithm for the PRP

Kramer, R.; Maculan, N.; Subramanian, A.; Vidal, T.

5 Conclusions

A new speed and departure time optimization algorithm (SDTOA) for the PRP has been presented. This algorithm is conceptually simple and requires a quadratic time as a function of the number of destinations. It was implemented and integrated in ILS-SP-SOA matheuristic of Kramer et al. (2015), where speed and departure time optimization occurs for each local minimum of the iterated local search. Our experimental results with this metaheuristic framework showed that allowing departure times from the depot can lead to very significant savings: 8.16% of fuel costs and driver wages in average for the considered benchmark instances.

Overall, integrated scheduling, speed control, and routing optimization techniques can help to reduce costs and environmental fingerprints in logistic systems and a variety of other industrial domains. The proposed methodology has contributed to efficiently address some open challenges such as joint departure time and speed optimization. Further research can now be focused on generalizing these methods to broader application classes. In particular, arc-dependent cost/speed functions are very relevant for ship speed optimization when traveling into locations with variable weather and sea conditions, but no efficient algorithm is currently known for this setting.

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Figure 2: Costs comparison
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