Large-scale Polarization of the Cosmic Microwave Background Radiation

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The anisotropy and polarization of the cosmic microwave background radiation (CMBR) induced by the scalar and tensor metric perturbations are computed in the long-wavelength limit. It is found that the large-scale polarization of CMBR induced by the decaying tensor mode can reach a few percents. This is different from the scalar or inflation-induced tensor mode, whereas the polarization is at least two orders of magnitude lower. The effect of matter re-ionization on CMBR is also considered. We conclude that measuring the polarization of CMBR on large-angular scales can probe the ionization history of the early Universe, and test different cosmological models.

PACS numbers: 98.70.Vc, 04.30.+x, 98.80.Cq
The recent detection of large-angular-scale temperature quadrupole anisotropy in the cosmic microwave background radiation (CMBR) by the DMR aboard the COBE satellite opens a window to our understanding of physics associated with the initial conditions of the early Universe. The most important conclusion to be drawn from the COBE data on the temperature correlation function is that they are consistent with a scale-invariant spectrum of primordial density perturbations predicted by inflationary cosmology.

There are two sources of CMBR anisotropy, namely, the density perturbations (scalar mode) and the primordial gravitational waves (tensor mode). As density perturbations and gravity waves enter the horizon, they induce large-scale temperature anisotropies in the CMBR via the Sachs-Wolfe (SW) effect. It is generally believed that the scalar mode can account for the CMBR anisotropy, which is regarded as evidence for the seeds of galaxies formation.

Recently, it was argued that the anisotropy might be dominated by the tensor mode, and was showed that the tensor-mode dominance actually occurs in certain inflation models. In Ref. [7], the authors suggested that by comparing large- and small-scale anisotropy measurements, one can separate the scalar- and tensor-mode contributions. At small-angular scales (≈ 1°), the tensor-mode contribution to the CMBR anisotropy decreases relative to the scalar mode. A preliminary measurement of the CMBR anisotropy on 1° have indicated a low value when assuming a scalar-mode contribution. If the result should hold true, then this may be an evidence for the tensor-mode dominance. However, an alternative explanation is that late matter re-ionization may wash out the small-scale anisotropy too.

A potential method for distinguishing tensor mode from re-ionization scenario is to measure the polarization component of the CMBR on small-angular scales: re-ionization leads to much greater polarization. Anisotropic radiation acquires linear polarization when it is scattered with free electrons (Thomson scattering). The effect of Thomson scattering on the polarization of CMBR for the scalar-mode and tensor-mode cases can be found in Refs. [10] and [11] respectively. Ref. [10] showed that roughly 10% of CMBR anisotropy is polarized on angular scales less than 1°. Ref. [11] showed that tensor mode induces large polarization at small angular scales if primordial spectrum extends to small scales. In this paper we estimate the polarization to anisotropy ratio of CMBR due to long-wavelength
scalar and tensor modes in an universe with and without a late re-ionization era. The result shows that polarization measurements of CMBR on large-angular scales can probe the reionization history of the early Universe and possibly test different cosmological models.

To study how polarized photons propagate in the Universe, one need to solve the equation of transfer for photons. The general formalism for the subject of radiative transfer is given in Ref. [12]. In general, arbitrarily polarized photons are characterized by four Stokes parameters, \( n = (n_l, n_r, n_u, n_v) \), where \( n = n_l + n_r \) is the distribution function for photons with \( l \) and \( r \) denoting two directions at right angle to each other.

We shall use the units \( c = \hbar = 1 \) throughout. The metric that we use is of the flat Robertson-Walker form

\[
ds^2 = a^2(\eta) \left(d\eta^2 - dx^2\right),
\]

where \( a(\eta) \) and \( d\eta = dt/a(t) \) are the scale factor and conformal time respectively. Then, it follows that, for a matter-dominated universe,

\[
p = 0, \quad a(\eta) = \frac{2\eta^2}{H_0}, \quad \rho = \frac{3H_0^2}{8\pi G\eta_0^6}, \quad \eta_0 = 1,
\]

where \( p, \rho, \) and \( H_0 \) are the pressure, energy density, and present Hubble constant respectively. Note that we normalize the conformal time to unity today. In this metric, \( \Omega_{\text{total}} = \Omega_{\text{DM}} + \Omega_B = 1 \), where \( \Omega_{\text{DM}} \) and \( \Omega_B \) denote respectively the dark and baryonic matter.

Then, the equation of transfer for an arbitrarily polarized photon is governed by the collisional Boltzmann equation,

\[
\left( \frac{\partial}{\partial \eta} + \hat{e} \cdot \frac{\partial}{\partial \mathbf{x}} \right) n = -\frac{d\nu}{d\eta} \frac{\partial n}{\partial \nu} - \sigma_T N_e a \left[ n - \frac{1}{4\pi} \int_{-1}^{1} \int_0^{2\pi} P(\mu, \phi, \mu', \phi') n d\mu d\phi' \right],
\]

where \( \sigma_T \) is the Thomson scattering cross section, \( N_e \) is the number of free electrons per unit volume, \( (\mu = \cos \theta, \phi) \) are the polar angles of the propagation direction \( \hat{e} \) of the photon with a comoving frequency \( \nu \), and \( P \) is the phase matrix for Thomson scattering given in Ref. [12].

The first term on the right-hand side of Eq. (3) describes how the frequency of a photon is shifted by metric fluctuations. The second term accounts for the Thomson scattering effect on photon polarizations. For an universe with the metric (1), the solutions of the perturbed
Einstein field equation are given in [4]. In the matter-dominated epoch, the solution for the spatial part of the metric perturbations is given by

$$h_{ij} = \frac{1}{\eta} \frac{\partial}{\partial \eta} \left( \frac{D_{ij}}{\eta} \right) - 2 \left( \frac{8}{\eta^3} - \nabla^2 \right) (C_{i,j} + C_{j,i}) + \frac{A_{ij}}{\eta^3} + \eta_{ij}B - \frac{\eta^2}{10} B_{ij}$$

(4)

up to a gauge transformation, and the matter density fluctuation

$$\delta \rho = \frac{H_0^2}{32\pi G} \nabla^2 \left( \frac{6A}{\eta^9} - \frac{3B}{5\eta^4} \right),$$

(5)

where the functions $A(x)$ and $B(x)$, $C_i(x)$ and $D_{ij}(x, \eta)$ correspond to the scalar, vector and tensor metric perturbations respectively. In the presence of the metric perturbations, the rate of change of the photon frequency is given by the SW formula

$$\frac{1}{\nu} \frac{d\nu}{d\eta} = \frac{1}{2} \frac{\partial h_{ij}}{\partial \eta} e^i e^j - \frac{\partial h_{ij0}}{\partial \eta} e^j.$$

(6)

At a sufficiently early epoch (onset of matter-dominated era at redshifts $z \leq 10^4$, say), photon radiation is assumed to be a blackbody and unpolarized. Afterwards, the photons interact with matter through Thomson scatterings, and the subsequent evolution for photon distribution function is determined by Eq. (3). Solutions for Eq. (3) which are relevant to the present consideration have been obtained in two cases: the first case is that the frequency shift is owing to weak gravitational waves (corresponding to the $D_{ij}$ functions in Eq. (4)) [11], and the second due to anisotropic expansion of an universe [9,13]. It was found that in both cases the polarization to anisotropy ratio of CMBR on large-angular scales is about a few percents. If matter re-ionization occurs subsequent to the hydrogen recombination, the ratio will be much enhanced in the anisotropic model [13]. It is realized that the case of unequal Hubble expansion along three axes is equivalent to the infinite-wavelength limit of the tensor-mode case up to an overall normalization factor, which is determined by the magnitude of the gravitational wave [11]. Also, it is equivalent to an axisymmetric universe up to a spherical harmonic function [13]. Below we will consider the effect of a generic scalar perturbation to the large-scale anisotropy and polarization of CMBR, i.e., setting $D_{ij} = C_i = 0$ (hence $h_{j0} = 0$) in Eq. (4). Then, we will consider the large-scale anisotropy and polarization of CMBR due to the tensor perturbation (corresponding to $D_{ij}$) generated in an inflationary cosmology.

We assume a density perturbation wave
\[ \delta \rho = \delta_0 \rho e^{i k \cdot x} \]  

(7)

at the present time \( \eta_0 = 1 \) with comoving wave vector \( \mathbf{k} \). It would induce a small amount of anisotropy and polarization in the CMBR. Thus we expand the photon distribution function as

\[ n = n_0 + \frac{1}{2} n_0 n_1(\eta) e^{i k \cdot x}, \]  

(8)

where \( n_0 \) is the blackbody distribution function, \( n_0 = \frac{1}{2} n_0(1,1,0,0) \) denotes unpolarized photons, and \( n_1 \) is a small fluctuation. Choosing \( \mathbf{k} \) along with the z-axis, and making use of the axial symmetry of the problem, we obtain from Eq. (3) the equation of transfer for the reduced perturbation \( n_1 = (n_{1l}, n_{1r}) \) \( (n_{1u} = n_{1v} = 0) \) as

\[ \left( \frac{\partial}{\partial \eta} + i k \mu \right) n_1 = -\frac{\partial \ln n_0}{\partial \ln \nu} \frac{1}{\nu} \frac{d \nu}{d \eta} e^{-i k \cdot x} \left( \begin{array}{c} 1 \\ 1 \end{array} \right) \]

\[-\sigma T e \left[ n_1 - \frac{3}{8} \int_{-1}^{1} \left( \begin{array}{c} 2(1 - \mu^2)(1 - \mu'^2) + \mu^2 \mu'^2 \\ \mu^2 \mu'^2 \end{array} \right) n_1 d \mu' \right], \]  

(9)

where \( \gamma \equiv \partial \ln n_0 / \partial \ln \nu = -1 \) in the Rayleigh-Jeans region.

To evaluate \( \nu^{-1} d \nu / d \eta \) in Eq. (9), we separate two cases: \( A \neq 0 \) and \( B = 0 \); \( B \neq 0 \) and \( A = 0 \), corresponding to \( \delta \rho / \rho \) which decreases and increases with time respectively. The increasing mode is believed to be responsible for the structure formation of the Universe.

From Eqs. (4)-(7), we find that

\[ \frac{1}{\nu} \frac{d \nu}{d \eta} = \begin{cases} -\frac{3 \delta \rho}{\rho_0 \eta^2} \mu^2 e^{i k \cdot x} & \text{for } A \neq 0; \\ \frac{2 \delta \rho}{\rho_0} \eta \mu^2 e^{i k \cdot x} & \text{for } B \neq 0, \end{cases} \]  

(10)

where \( \rho_0 = 3H_0^2/(8\pi G) \) is the present average energy density. In either case, Eq. (9) is too complicated to be solved. However, by taking infinite-wavelength limit, i.e., \( k \to 0 \), one could extract simple solutions from Eq. (9). These solutions are indeed good estimates of the anisotropy and polarization of CMBR on large-angular scales (see below).

By substituting Eq. (10) in Eq. (9), taking \( k \to 0 \) limit (i.e., neglecting the term \( i k \mu n_1 \)), and performing mode-decomposition,

\[ n_1 = \alpha(\mu^2 - \frac{1}{3}) \left( \begin{array}{c} 1 \\ 1 \end{array} \right) + \beta(1 - \mu^2) \left( \begin{array}{c} 1 \\ -1 \end{array} \right), \]  

(11)
where $\alpha$ and $\beta$ are respectively the anisotropy and polarization of the cosmic radiation at time $\eta$ (in essence, $\alpha$ represents the temperature anisotropy, $\delta T/T$, and $\beta$ represents the fractional temperature difference between two orthogonal directions along the line of sight), we obtain

$$\frac{d\alpha}{d\eta} = -\gamma \Delta(\eta) - \sigma_T N_e a \left( \frac{9}{10} \alpha + \frac{3}{5} \beta \right), \quad (12)$$

$$\frac{d\beta}{d\eta} = -\sigma_T N_e a \left( \frac{1}{10} \alpha + \frac{2}{5} \beta \right), \quad (13)$$

where, in terms of redshift $z$,

$$\Delta(\eta) = \frac{\Delta(z)}{2\eta^2} \equiv \Delta(z) = \begin{cases} -\frac{3}{2} \frac{\delta_0 \rho}{\rho_0} (1 + z)^3 & A \neq 0 ; \\ \frac{2}{3} \frac{\delta_0 \rho}{\rho_0} (1 + z)^{\frac{1}{2}} & B \neq 0 . \end{cases} \quad (14)$$

Integrating $\alpha$ and $\beta$ in Eqs. (12) and (13) with respect to $z$, we obtain their present values

$$\alpha = -\gamma \int_0^\infty \Delta(z) \left[ \frac{6}{7} e^{-\tau(z)} + \frac{1}{7} e^{\frac{3}{2m} \tau(z)} \right] (1 + z)^{-3/2} dz, \quad (15)$$

$$\beta = -\gamma \int_0^\infty \Delta(z) \left[ \frac{1}{7} e^{-\tau(z)} - \frac{1}{7} e^{\frac{3}{2m} \tau(z)} \right] (1 + z)^{-3/2} dz, \quad (16)$$

where $\Delta(z)$ is given in Eq. (14) and the optical depth is given by

$$\tau(z) = \tau_0 \int_0^z \chi_e(z') \sqrt{1 + z'} dz' ; \quad \chi_e \equiv \frac{N_e}{N_B} , \quad \tau_0 = \frac{\sigma_T \rho_c \Omega_B}{H_0 m_B} , \quad (17)$$

where $\chi_e$ is the degree of ionization, $m_B$ and $N_B$ are respectively the proton mass and number density, $\rho_c$ is the critical energy density, and $\Omega_B \equiv \rho_B/\rho_c$. Henceforth, we assume $H_0 = 100 \text{ km sec}^{-1} \text{Mpc}^{-1}$ with $h = 1$. Then, $\tau_0 = 0.0696 \Omega_B h$.

To evaluate $\alpha$ and $\beta$ in Eqs. (15) and (16), we need to determine the optical depth which is determined by the dynamics of hydrogen recombination and the history of re-ionization. Let us first assume an instantaneous recombination: $\chi_e = 1$ for $z \geq z_r$ and $\chi_e = 0$ for $z < z_r$, where $z_r$ is the redshift at which hydrogen recombination occurs; $z_r = 1350, 1450, 1500$ for $\Omega_B = 0.01, 0.1, 1$ respectively. Express $\alpha = \pm \alpha_0 \gamma \frac{\delta_0 \rho}{\rho_0}$, where the plus (minus) sign corresponds to the $A \neq 0$ ($B \neq 0$) case.

For $A \neq 0$ we obtain from Eqs. (14)-(17), $\alpha_0 \simeq 5 \times 10^4$ and $\beta/\alpha \simeq 10^{-2} - 10^{-4}$ for $\Omega_B$ from 0.01 to 1. In this case, $\alpha \simeq -0.5$ when assuming a horizon-sized density perturbation with amplitude $\delta_0 \rho/\rho_0 \simeq 10^{-5}$, which is too big when compared to observational data. However,
the $A \neq 0$ case is equivalent to (i) an axisymmetric universe up to a numerical factor, and (ii) that of an infinitely long decaying gravitational wave up to a numerical factor times a spherical harmonic function. Nevertheless, all the three cases have the same $\beta/\alpha$ ratio. The reasons for the above equivalences are the following. For an axisymmetric universe with the present shear given by $\Delta H_0$, the rate of change of the photon frequency is given by

$$
\frac{1}{\nu} \frac{d\nu}{d\eta} = \frac{2\Delta H_0}{H_0} \frac{1}{\eta^4} \left( \mu^2 - \frac{1}{3} \right),
$$

which is similar to the $A \neq 0$ case in Eq. (10) ($k \to 0$) up to an irrelevant monopole term. In the presence of a monochromatic gravitational wave with comoving wave vector $k$ and polarization $\lambda = +$ mode, it was shown that

$$
\frac{1}{\nu} \frac{d\nu}{d\eta} = \frac{1}{2} (1 - \mu^2) \cos 2\phi \ e^{-i k \cdot x} \frac{d}{d\eta} (h \ e^{i k \eta}) ; \ h = \frac{i k f}{\eta^2} \left( 1 + \frac{i}{k \eta} \right),
$$

where $f$ is a constant wave amplitude. Taking the $k \to 0$ limit, Eq. (19) is similar to the $A \neq 0$ case in Eq. (10). For such wave, the mode decomposition is given by

$$
\mathbf{n}_1 = \frac{\alpha}{2} (1 - \mu^2) \cos 2\phi \begin{pmatrix} 1 & (1 + \mu^2) \cos 2\phi \\ 0 & -(1 + \mu^2) \cos 2\phi \end{pmatrix} + \frac{\beta}{2} \begin{pmatrix} 1 & 4 \mu \sin 2\phi \\ 0 & 4 \mu \sin 2\phi \end{pmatrix}.
$$

For $B \neq 0$, we find $\alpha_0 \simeq 1$ and $\beta/\alpha \simeq 10^{-5} - 10^{-7}$ for $\Omega_B$ from 0.01 to 1. Then, $\alpha \simeq \delta_0 \rho/\rho_0 \simeq 10^{-5}$ which is the well-known SW contribution to the large-scale anisotropy of CMBR. $\beta$ is below $10^{-10}$ which is consistent with the values of lower multipole moments that were given in Ref. [10].

To determine the effects of non-instantaneous recombination, we approximate the history of ionization by,

$$
\chi_e = \begin{cases} 
3.2 \times 10^{-4} \Omega_B^{-1} & z < 900, \\
5.68 \times 10^{6} \Omega_B^{-1} z^{-1} e^{-14453 z} & 900 < z < 1500, \\
1 & z > 1500,
\end{cases}
$$

where we assume that a residual ionization remains for $z < 900$. For $900 < z < 1500$, we use the results given in Ref. [14] which take into account the possibility of two-photon decay of the hydrogen 2S level during the process of recombination. For $z > 1500$, we assume a complete ionization (an exact treatment would require using the Saha equation). Plugging
Eq. (21) in Eq. (17) and then using Eqs. (14)-(16), we find that the degree of anisotropy, on comparing with the instantaneous case, remains approximately the same, whereas the degree of polarization can possibly greatly enhanced. Being weakly dependent on the value of $\Omega_B$, the ratio $\beta/\alpha$ increases to about $-4\%$ for the case of $A \neq 0$ and $-10^{-4}$ for that of $B \neq 0$. These results are shown in Figs. 1 and 2. Our value of the degree of polarization for the $A \neq 0$ case is consistent with the $2\%$ result that was given in Ref. [11]. To understand our results for the non-instantaneous case, we examine the value of the optical depth evaluated at the beginning of hydrogen recombination. This value denotes the number of Thomson scatterings between epochs $z = z_r$ and $z = 0$. We find that $\tau(z = 1500) \simeq 90$, which is much larger than one. This explains why the polarization is sensitive to the recombination dynamics.

Finally, we consider the effects of matter re-ionization subsequent to the recombination epoch. This re-ionization of matter may attribute to the reheating by radiation released from protogalaxies during galaxy formation. To demonstrate this, we approximate the ionization fraction by $\chi_e = 1$ for $z > z_r$; 0 for $z_h < z < z_r$; 1 for $z < z_h$, where $z_h$ is the redshift at which the re-ionization occurs. Note that the case of instantaneous recombination is equivalent to setting $z_h = 0$. Using Eqs. (14)-(17) and varying $z_h$ from 10 to 100, we find that the polarization, being very sensitive to the re-ionization, becomes much larger (see Fig. 2). To understand why the polarization curves all suddenly change around $z_h = 10$, we evaluate the optical depth at $z = 10$. We find that $\tau(10) \simeq 1.6\Omega_B$. This explains the jumps for all the cases except the $B \neq 0$ and $\Omega_B = 0.01$ case. However, in this case the polarization before recombination is extremely small, even a small amount of scatterings during the re-ionization epoch can induce an increase in the polarization over its prerecombination value.

As for the anisotropy $\alpha_0$, it is found that it remains almost unchanged in the case of $B \neq 0$ and it is diminished in that of $A \neq 0$ (see Fig. 1). As mentioned above, the $A \neq 0$ case is equivalent to an axisymmetric universe, this decreasing behavior agrees with the results given in Ref. [13].

We are now to justify the infinite-wavelength approximation in our calculations. Since horizon-sized waves dominate the contributions to the anisotropy and hence polarization of CMBR on large-angular scales via the SW effect either for the scalar [15] or tensor mode.
Thus, one should compare the Thomson scattering mean-free path, $(\sigma_T N_e a)^{-1}$, with the wavelength of horizon-sized density perturbations, $k_{\text{hor}}^{-1}$, at the moment when the contributions to the integrals in Eqs. (15) and (16) are dominant. If the former is much smaller than the latter, then the term $i k \mu n_1$ in Eq. (9) can be neglected. For example, in the case of instantaneous recombination, the polarization $\beta$ receives contribution mainly from $z \geq z_r$. For $\Omega_B = 0.01$ and $z_r = 1350$, $\sigma_T N_e a \geq 2500$ for $z \geq z_r$ which is much bigger than $k_{\text{hor}} \simeq 1$. For other values of $\Omega_B$ and $z_h$, we find that $\sigma_T N_e a$ is at least comparable to or much bigger than 1. Thus, we conclude that taking the infinite-wavelength limit is an efficient approximation in calculating the lower multipole moments, especially the quadrupole moment. However, the finite-wavelength correction is of interest for intermediate-scale calculation.

We now turn to the inflation-induced tensor mode calculation. The calculation is similar to the case of decaying tensor mode. The rate of change of the photon frequency is given by Eq. (19) with $he^{ik\eta}$ replaced by $3A(k) j_1(k\eta)/(k\eta)$, where $A^2(k) = 8v/(3\pi)$ is the scale invariant spectrum, $j_1(k\eta)$ is the spherical Bessel function of order 1, and $v$ is a parameter depending on the inflation scale. By taking the infinite-wavelength limit, the rate of change of the photon frequency decreases to zero. This is expected since the amplitude of the tensor mode well outside the horizon is constant. Therefore, the above limiting approach fails in this case. However, as we have mentioned that horizon-sized waves dominate the contributions, we instead proceed the calculation by simply dropping the term $i k \mu n_1$ and taking $k = 2\pi$ in Eq. (9), and using $v = 4 \times 10^{-11}$. The results are shown in Figs. 1 and 2. The behavior of the results is very similar to that of the $B \neq 0$ case. This makes it difficult to distinguish the scalar mode from the inflation-induced tensor mode by measuring the large-scale anisotropy and polarization of CMBR. We have also carried a full numerical calculation of Eq. (9) by taking $k \simeq 1$ and found that dropping the term $i k \mu n_1$ does not make any significant difference.

In conclusion, we have calculated the large-scale anisotropy and polarization of CMBR due to the scalar and tensor metric perturbations, as well as the effects of the non-instantaneous recombination and matter re-ionization. The results are shown in Figs.1 and 2. The $\alpha_0$ (up to an overall normalization factor which is determined by the amplitude of the decaying tensor mode) and $\beta/\alpha$ values for the $A \neq 0$ case also represent respectively
the anisotropy and the polarization to anisotropy ratio due to the decaying tensor mode. Although it seems unlikely that measurable super-horizon sized decaying tensor modes could be generated in the early Universe, however, the large polarization due to this decaying-type perturbations or anisotropic Hubble expansion [16] remains an interesting possibility. It is shown that measuring the degree of polarization of CMBR on large-angular scales is a potential method for probing the ionization history of the early Universe. The linear polarization of CMBR was measured at 33 GHz over 11 declinations from 37°S to 63°N. Fitting the data to an axisymmetric model and spherical harmonics through the third order yields $|\beta| < 3 \times 10^{-5}$ and $|\beta| < 6 \times 10^{-5}$ respectively at 95% confidence level [17]. These upper limits can hardly be used to put any constraint on the present calculations. Currently, the linear antenna aboard COBE has collected data for the polarization component of CMBR which is being analysed. Any trace of polarization or a better limit will prove invaluable to our understanding of the early Universe.

Acknowledgements

This work was supported in part by the R.O.C. NSC Grant No. NSC82-0208-M-001-059, NSC83-0208-M-001-053 and NSC82-0208-M-001-131-T.
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Figure 1. Large-scale anisotropy of CMBR versus the redshift at which matter re-ionization occurs. Symbols $A$, $B$ and $T$ denote respectively the cases of $A \neq 0$, $B \neq 0$ and the inflation-induced tensor. For the cases of $A \neq 0$ and $B \neq 0$, $\alpha = \pm \alpha_0 \gamma \, \delta \rho / \rho_0$. For the tensor case, $\alpha = \gamma \alpha_0 \times 10^{-6}$. $z_h = 0$ corresponds to instantaneous recombination. Each solid triangle corresponds to non-instantaneous recombination for $\Omega_B = 1, 0.1$ and 0.01. Dashed, solid and dotted curves correspond respectively to $\Omega_B = 1, 0.1$ and 0.01.

Figure 2. Large-scale polarization to anisotropy ratio of CMBR. Symbols $A$, $B$ and $T$ denote respectively the cases of $A \neq 0$, $B \neq 0$ and the inflation-induced tensor. $z_h = 0$ corresponds to instantaneous recombination. Each solid triangle corresponds to non-instantaneous recombination for $\Omega_B = 1, 0.1$ and 0.01. Dashed, solid and dotted curves correspond respectively to $\Omega_B = 1, 0.1$ and 0.01.