The QCD Equation of state and critical end-point estimates at $O(\mu_B^6)$

Sayantan Sharma

Brookhaven National Laboratory, Upton, New York 11973

[For Bielefeld-BNL-CCNU collaboration]

Abstract

We present results for the QCD Equation of State at non-zero chemical potentials corresponding to the conserved charges in QCD using Taylor expansion up to sixth order in the baryon number, electric charge and strangeness chemical potentials. The latter two are constrained by the strangeness neutrality and a fixed electric charge to baryon number ratio. In our calculations, we use the Highly Improved Staggered Quarks (HISQ) discretization scheme at physical quark masses and at different values of the lattice spacings to control lattice cut-off effects. Furthermore we calculate the pressure along lines of constant energy density, which serve as proxies for the freeze-out conditions and discuss their dependence on $\mu_B$, which is necessary for hydrodynamic modelling near freezeout. We also provide an estimate of the radius of convergence of the Taylor series from the 6th order coefficients which provides a new constraint on the location of the critical end-point in the $T-\mu_B$ plane of the QCD phase diagram.

Keywords: Finite temperature and density Quantum Chromodynamics, Quark-gluon plasma, critical point

1. Introduction

There has been a considerable progress in the understanding of the phase diagram of strongly interacting matter described by Quantum Chromodynamics (QCD), much of it driven from the lattice studies. The Equation of state (EoS) relating the energy density and pressure to temperature is known very precisely from lattice studies in the continuum limit for vanishing baryon density [1, 2] and has been extended to finite baryon densities as well [3, 4, 5, 6, 7, 8]. At low temperatures the thermodynamic quantities are found to be in quite good agreement with the hadron resonance gas (HRG) model approximation, although systematic deviations have been observed, which may be due to the existence of additional resonances which are not taken into account in HRG model calculations based on experimentally measured resonances [9, 10]. Beyond the fundamental understanding of the non-perturbative aspects of QCD medium at finite temperature, the EoS at vanishing chemical potentials is an essential input into the hydrodynamic modeling of hot and dense matter created in heavy ion collisions at the LHC and the highest RHIC beam energies. However BES run I and the upcoming BES-II experiments at RHIC will be probing hot and dense media characterized by $0 \leq \mu_B/T \leq 3$ if indeed thermalization is achieved under experimental conditions. Thus lattice input of
the EoS at non-vanishing baryon number, strangeness and electric charge chemical potentials is a crucial theoretical input for the upcoming experimental efforts. Due to the sign problem a direct calculation of the EoS at non-zero ($\mu_B$, $\mu_Q$, $\mu_S$) using lattice techniques is unfortunately not yet possible. This problem can be circumvented at small enough densities by performing Taylor expansion of the thermodynamic quantities \cite{13,11}. Moreover the presence of non-analyticities in the phase diagram e.g., a possible critical end-point, can be \cite{12,13} also located from the radius of convergence of the Taylor series of the baryon number fluctuation \cite{14}. In order to have a control on the Taylor expansion for pressure for a wide range of $\mu_B$ and to study its radius of convergence, it is important to measure the higher order expansion coefficients which are simply the generalized baryon number susceptibilities.

In this proceedings we report our latest results for the Taylor coefficients of pressure and baryon number density up to sixth order in $\mu_B$ \cite{15}. This enables us to calculate the EoS in the continuum limit for $\mu_B/T < 2.5$ which is expected to describe heavy ion collisions at centre of mass energies as low as $\sqrt{s} \sim 11 GeV$ \cite{16}. We also provide a new estimate for the location of the critical end-point in the QCD phase diagram.

2. The QCD Equation of state in QCD under strangeness neutral conditions

To describe the medium created in heavy ion collisions one has to impose strangeness neutrality condition i.e., $n_S = 0$. Moreover for most colliding heavy ions, there is a constraint on the net baryon to charged species i.e., $n_Q/n_B = 0.4$ \cite{17}. These conditions constrain the values of $\mu_B$, $\mu_Q$, $\mu_S$, that enter into the calculations of the EoS. We expand $\mu_S$ and $\mu_Q$ as a series in $\mu_B$ so that the derivatives of the partition function with respect to $\mu_S$ and $\mu_Q$ can be expressed in terms of $\mu_B$. The details of the derivation of thermodynamic quantities like pressure, energy density for the constrained case are given in \cite{15}.

Our results are based on the lattice QCD calculation with 2+1 flavors using Highly Improved Staggered Quark (HISQ) discretization scheme. We have chosen different lattices of extent $N^3 \times N_t$, where $N_t = 6, 8, 12, 16$ and fixed $N_t = 4N_t$, in order to be in the thermodynamic limit. The strange quark mass are always tuned to its physical value. The light quark masses are chosen such that the Goldstone pion mass $m_\pi = 140$ MeV for $T \leq 175$ MeV. At high temperatures since the thermodynamic quantities are not sensitive to the choice of the light quark masses we have chosen heavier than physical light quark mass which corresponds to $m_\pi = 160$ MeV. For further technical details see Ref. \cite{15}. We expand the pressure in the constrained case as a series in $\mu_B/T$, as

$$ P(T,\mu_B)/T^4 = \frac{P(T,\mu_B = 0)}{T^4} + P_2 \left( \frac{\mu_B}{T} \right)^2 + P_4 \left( \frac{\mu_B}{T} \right)^4 + P_6 \left( \frac{\mu_B}{T} \right)^6 + \ldots, \quad P_n = \frac{1}{n!} \frac{\partial^n \ln Z_{QCD}}{\partial (\mu_B/T)^n}. \quad (1) $$

The computational complexity of the lattice calculation of the coefficients $P_n$ increases with each order $n$. For $n = 2, 4$ we use the conventional method of introducing $\mu_B$ which is known to cancel any unphysical divergences explicitly for $n \leq 4$ \cite{18}. However for $n \geq 4$ we adopt a new method of introducing $\mu$.
which significantly reduces the computational costs [19] for these higher order $P_n$ without introducing any potentially divergent term [20]. Our results for Taylor coefficients for pressure are shown in the left and central panels of Fig. 1 for different lattice spacings. The gray band represents the continuum extrapolated results obtained from our $N_c = 6,8$ data. For $P_6$ we observe a deviation from the HRG values already for $T \geq 150$ MeV. For $P_8$ the central values are systematically below the HRG estimates for $T > 145$ MeV however given the current errors we cannot make a conclusive statement. However with the high statistics data, we can clearly observe a negative dip in $P_8$ immediately in the vicinity and above the chiral crossover region given by $T_c = 154(9)$ MeV [21]. This is a genuine signature of the non-perturbative nature of the QCD medium just above deconfinement which cannot be reproduced within Hard Thermal Loop perturbation theory [22]. Our results are in good agreement with the continuum estimates obtained from imaginary $\mu$ techniques using analytic continuation [23-24].

Using these estimates of $P_n$, we have calculated the pressure in QCD for different values of $\mu_B/T$ relevant for RHIC BES experiments, the continuum estimates of which are shown in the rightmost panel of Fig. 1. For $T > 170$ MeV we infer that it is sufficient to consider terms up to $O(\mu_B^6)$ for calculating pressure for a wide range of $\mu_B/T \leq 2.5$ due to convergence of the results at this order compared to the results obtained from truncation at $O(\mu_B^4)$. For $T < 165$ MeV, the convergence is still fairly good limited only due to the large errors in the $P_6$ data. For the net baryon number density shown in the left panel of Fig. 2 and terms up to $O(\mu_B^6)$ are not sufficient for its estimation for $\mu_B/T > 2$ as we do not find a convergence of the results from different orders of truncation. This is due to the fact that the higher order Taylor coefficients contribute with a larger numerical prefactor in the baryon number density compared to pressure. Hence a precise calculation of the sixth order coefficient and extending to further higher orders are needed for estimating the baryon number density for $\mu_B/T > 2$. In summary we now have continuum extrapolated results for the EoS for $\mu_B/T \leq 2$ for a wide range of temperatures. A parameterization of the pressure and number densities as a function of $T_c/T$ for the constrained case are available, for more details see [15].

3. The critical end-point estimates up to $O(\mu_B^4)$

The net baryon-number susceptibility, $\chi_B^2$ is expected to diverge at the critical end-point, so the ratios of the successive terms in its Taylor series, would give an estimate for the radius of convergence. Having obtained the coefficients of the Taylor series for pressure, we can calculate estimates for the radius of convergence as $r_n^{\chi} = \left[ \frac{2n(2n-1)P_n}{P_{2n+1}(2n+2)(2n+1)} \right]^{1/2}$, but for $\mu_S = \mu_Q = 0$. These estimates will converge to the true radius of convergence in the limit $n \to \infty$. Our results for $r_n^{\chi}$ are shown in the right panel of Fig. 2. The upper boundary of the excluded region is obtained by considering the upper values of errors in our $P_6/P_4$ data. Our results are consistent with the very recent results for these ratios obtained from calculations with an imaginary $\mu_B$ [25], where the estimates of $r_4^{\chi}$ are larger than our current lower bound. Estimators calculated.
based on a reweighting technique \cite{26} as well as from Taylor series expansion in 2-flavor QCD \cite{27,28} are consistently lower than our bounds. However one has to note that all the lattice results are at fixed lattice spacing which leads to the present differences, which hopefully would agree in the continuum limit.

4. Summary and Outlook

We now have continuum extrapolated results for the QCD EoS for $\mu_B/T \leq 2$ under the constraints that are realized in heavy-ion experiments using Taylor expansion upto $O(\mu_B^6)$ . Furthermore we have measured different estimators for the radius of convergence of the net baryon number fluctuation, which allows us to rule out the existence of a critical end-point in the QCD phase diagram for $\mu_B/T \leq 2$ and $145 \leq T \leq 155$ MeV. This is consistent with the fact that we find no non-analyticities in the Taylor expansion of pressure for these values of $\mu_B/T$. In order to extend our results to $\mu_B/T > 2$ to be accessible in the BES II experiments at RHIC and provide more robust bounds on the location of the critical end-point we need to calculate the eighth order Taylor coefficient for pressure, which is well within our reach.

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References

[1] S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg and K. K. Szabo, Phys. Lett. B 730 99 (2014) [arXiv:1309.5258 [hep-lat]].
[2] A. Bazavov et al. [HotQCD Collaboration], Phys. Rev. D 90, 094503 (2014) [arXiv:1407.6387 [hep-lat]].
[3] C. R. Allton et al., Phys. Rev. D 66 074507 (2002) [hep-lat/0204010].
[4] R. V. Gavai and S. Gupta, Phys. Rev. D 68, 034506 (2003) [hep-lat/0303013].
[5] C. R. Allton et al., Phys. Rev. D 68, 014507 (2003) [hep-lat/0305007].
[6] C. R. Allton et al., Phys. Rev. D 71, 054508 (2005) [hep-lat/0501030].
[7] S. Ejiri, F. Karsch, E. Laermann and C. Schmidt, Phys. Rev. D 73, 054506 (2006) [hep-lat/0512040].
[8] S. Borsanyi et al., JHEP 1208 (2012) 053 [arXiv:1204.6710 [hep-lat]].
[9] A. Majumder and B. Müller, Phys. Rev. Lett. 105, 252002 (2010) [arXiv:1008.1747 [hep-ph]].
[10] A. Bazavov et al., Phys. Rev. Lett. 113, 072001 (2014) [arXiv:1404.6511 [hep-lat]].
[11] R. V. Gavai and S. Gupta, Phys. Rev. D 64 074506 (2001) [hep-lat/0103013].
[12] M. Asakawa, K. Yazaki, Nucl. Phys. A 504, 668 (1989).
[13] A. M. Halasz et al., Phys. Rev. D 58, 096007 (1998).
[14] R. V. Gavai and S. Gupta, Phys. Rev. D 71, 114014 (2005) [hep-lat/0412035].
[15] A. Bazavov et al., Phys. Rev. D 95, no. 5, 054504 (2017) [arXiv:1701.04325 [hep-lat]].
[16] J. Cleymans, H. Oeschler, K. Redlich and S. Wheaton, Phys. Rev. C 73, 034905 (2006).
[17] A. Bazavov et al., Phys. Rev. Lett. 109, 192302 (2012) [arXiv:1208.1220 [hep-lat]].
[18] P. Hasenfratz and F. Karsch, Phys. Lett. 125B, 308 (1983).
[19] R. V. Gavai and S. Sharma, Phys. Rev. D 85, 054508 (2012) [arXiv:1112.5428 [hep-lat]].
[20] R. V. Gavai and S. Sharma, Phys. Lett. B 749, 8 (2015) [arXiv:1406.0474 [hep-lat]].
[21] A. Bazavov et al., Phys. Rev. D 85 054503 (2012) [arXiv:1111.1710 [hep-lat]].
[22] N. Haque et al., JHEP 1405, 027 (2014) [arXiv:1402.6907 [hep-ph]].
[23] J. Gunther, R. Bellwied, S. Borsanyi, Z. Fodor, S. D. Katz, A. Pasztor and C. Ratti, arXiv:1607.02493 [hep-lat].
[24] Sz. Borsanyi et al., Proceedings of Quark Matter 2017. 
[25] M. D’Elia, G. Gagliardi and F. Sanfilippo, arXiv:1611.08285 [hep-lat]. 
[26] Z. Fodor and S. D. Katz, JHEP 0404, 050 (2004) [hep-lat/0402006].
[27] S. Datta, R. V. Gavai and S. Gupta, PoS LATTICE 2013, 202 (2014).
[28] S. Datta, R. V. Gavai and S. Gupta, arXiv:1612.06673 [hep-lat].