Dynamic damping of vibrations of technical object with two degrees of freedom

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Abstract. Approach to the solution of problems of dynamic damping for the technical object with two degrees of freedom on the elastic supports is developed. Such tasks are typical for the dynamics of technological vibrating machines, machining machine tools and vehicles. The purpose of the study is to justify the possibility of obtaining regimes of simultaneous dynamic damping of oscillations in two coordinates. The achievement of the goal is based on the use of special devices for the transformation of motion, introduced parallel to the elastic element. The dynamic effect is provided by the possibility of changing the relationships between the reduced masses of devices for transforming motion. The method of structural mathematical modeling is used, in which the mechanical oscillatory system is compared, taking into account the principle of dynamic analogies, the dynamically equivalent structural diagram of the automatic control system. The concept of transfer functions of systems interpartial relations and generalized ideas about the partial frequencies and frequencies dynamic damping is applied. The concept of a frequency diagram that determines the mutual distribution of graphs of frequency characteristics in the interaction of the elements of the system is introduced.

Introduction

Dynamic damping of oscillations as the effect of local balancing of the elements of a technical system or an object under the action of vibrational loads has been sufficiently well studied, which is reflected in the works [1 ÷ 3]. Design and technical solutions that implement solutions for adjustment of the active vibration absorbers are known [4, 5]. These are applied in practice in solving the problems of improving the reliability and operation of various machines and equipment. [6].

At the same time, in most cases dynamic damping of oscillations is realized as a local effect created at one characteristic point of the object and a certain frequency of an external disturbance.

The idea of the proposed method is to use the fact that in a system with two degrees of freedom under the kinematic form of external action (for example, base vibration), the inputs of two partial systems are simultaneously excited. In this case, the system acquires, in contrast to the usual situation, additional dynamic capabilities.

1. Some general provisions; justification of the design scheme

1. The schematic diagram of the technical object is a solid body on elastic supports \((k_1, k_2)\) with parallel input devices for transformation of motion (UPD). Such devices can be realized with the help of non-self-locking screw mechanisms, in which there are nut-flywheels with reduced masses \(L_1\) and \(L_2\) [2, 7, 8]. The schematic diagram is shown in Figure 1. The motion of a rigid body with mass inertial
parameters \( M \) and \( J \) is determined by the coordinates \( y_1 \) and \( y_2 \). The parameters \( a, b, c \) reflect the geometric parameters of the system \((a = \frac{l_2}{l_1 + l_2}, \ b = \frac{l_1}{l_1 + l_2}, \ c = \frac{1}{l_1 + l_2})\), where \( l_1, l_2 \) are the distances from the center of gravity to the ends of the solid.

External influences are harmonic oscillations of the supporting surface \( z_1(t), z_2(t) \).

![Figure 1](image1.png)

**Figure 1.** The schematic diagram of a technical object with two degrees of freedom under kinematic perturbation

The initial mechanical oscillatory system (Figure 1) is used to construct a system of linear differential equations based on Lagrange's equations of the second kind, which is transformed by Laplace under zero initial conditions. The technology of constructing mathematical models is described in detail in [3, 7, 8]. The block diagram of the partial system with main coupling and feedback and characteristics of external excitation is shown in Figure 2.

![Figure 2](image2.png)

**Figure 2.** The structural mathematical model of the initial system in Figure 1 with kinematic perturbation: The notation \( p = j \omega \) is a complex variable \((J = \sqrt{-1})\); the \( \leftrightarrow \) icon above the variable means its Laplace image [9, 10]

2. Using the block diagram in Figure 2, let us determine the transfer functions of the system; let us take \( z_1 = z_2 = z \), then:

\[
W_1(p) = \frac{\frac{\ddot{y}_1}{p}}{\frac{z}{p}} = \frac{(L_1 p^2 + k_1)[(MB^2 + Jc^2 + L_2) p^2 + k_2] + (\alpha d_1 p^2 + k_1)(Jc^2 - Mab) p^2}{A_0(p)}, \tag{1}
\]

\[
W_2(p) = \frac{\frac{\ddot{y}_2}{p}}{\frac{z}{p}} = \frac{(L_2 p^2 + k_2)[(Ma^2 + Jc^2 + L_1) p^2 + k_1] + (\alpha d_1 p^2 + k_1)(Jc^2 - Mab) p^2}{A_0(p)}, \tag{2}
\]

whereat

\[
A_0(p) = [(Ma^2 + Jc^2 + L_1) p^2 + k_1] \cdot [(MB^2 + Jc^2 + \alpha d_1) p^2 + k_2] - [(Jc^2 - Mab) p^2]^2\tag{3}
\]

– characteristic frequency equation.

It is important to note that \( \alpha \) is introduced as a coupling coefficient between the values of \( L_1 \) and \( L_2 \) \((L_2 = \alpha L_1)\). Within the framework of the developed method, the frequency of the regime of dynamic damping of oscillations along the coordinates is determined from the conditions “zeroing” of the numerators of the transfer functions (1) and (2).

The equation for finding frequency dynamic damping in kinematic excitation for the coordinate \( y_1 \) is written as:
In turn, for coordinate $y_2$ the following equation is obtained:

$$p^4 L_4 [\alpha M a^2 + J c^2 + L_4 + J c^2 - M a b] + p^3 [k_1 (M a^2 + J c^2 + \alpha L_4) + 
+ k_2 (L_4 + J c^2 - M a b)] + k_1 k_2 = 0.$$  \hspace{1cm} (4)

Plots of $\omega^2_{dyn}(\alpha)$ are shown in Figure 3 by solid line. The numerator of the transfer function (1) is used to plot graphs $\omega^2_{dyn}(\alpha)$. The value of the frequency $\omega^2_{dyn}(\alpha)$ depends on the ratio of connectedness $\alpha$. Since the frequency of dynamic oscillation damping of the coordinate $y_i$ is determined from the biquadratic frequency equation, then the graph of $\omega^2_{dyn}(\alpha)$ is shown in the diagram (Figure 3) two segments (or branches): the lower branch of the plot $\omega^2_{dyn}(\alpha)$ corresponds to the low frequency forms of movement. In turn, the graph of the frequency of dynamic oscillation damping $\omega^2_{2dyn}(\alpha)$ of the coordinate $y_i$ is determined from the frequency equation, based on the numerator of the transfer function (2). In the diagram (figure 3), both branches of the graph $\omega^2_{2dyn}(\alpha)$ are shown by a dotted line.

![Diagram of dynamic interaction modes](image-url)

**Figure 3.** Diagram of dynamic interaction modes ($L_1, L_2 = \alpha L_4$): definition of dependencies $\omega^2_{dyn}(\alpha)$, $\omega^2_{2dyn}(\alpha)$, $\omega^2_{nat}(\alpha)$, $\omega^2_{2nat}(\alpha)$, $n_1^2(\alpha)$, $n_2^2(\alpha)$

**II. Frequency diagram**

From the biquadratic equations for the coordinates $y_1$ and $y_2$, it follows that there will be two modes of dynamic damping of the oscillations for each coordinate. Determination of the frequency of dynamic damping of oscillations, at which simultaneous dynamic damping of oscillations will occur, is determined by the intersection of curves $\omega^2_{dyn}(\alpha)$ and $\omega^2_{2dyn}(\alpha)$ on the frequency diagram (Figure 4). Let us note that the partial, natural frequencies also depend on $\alpha$. For the model problem, the following parameters are used: $M = 7000$ kg, $J = 2000$ kgm$^2$, $k_1 = 1000$ kN/m, $k_2 = 2000$ kN/m, $a = 0.57$ m, $b = 0.43$ m, $L = 100$ kg.

In Figure 4 point (1) determines the frequency of dynamic damping of oscillations simultaneously with respect to two coordinates $y_1$ and $y_2$. The same point determines the value of the connectivity coefficient $\alpha$.

The question of controlling the parameter $\alpha$, that is, the coefficient of connectedness of the reduced masses of nuts-flywheels, is technically possible, but application of braking torque to nuts-flywheels is
required. Such moment with respect to each element $L_1$ and $L_2$, can be created by an appropriate control system.

**Figure 4.** The frequency diagram for determining the frequencies of dynamic damping of oscillations simultaneously in two coordinates $\bar{y}_1$ and $\bar{y}_2$

The frequency diagram in Figure 4, where $p, (1)$ determines the desired regime.

**Conclusion**
1. It is shown that under certain operating conditions of technical objects, the disturbance can be formed by simultaneous excitation of two partial systems. Under the conditions of compatibility of the action of disturbances, the introduction of relations between two force factors enables the linear formulation of the problem on the basis of the superposition principle to find the condition for simultaneous dynamic damping of oscillations with respect to two coordinates.
2. Taking into account the joint action of external disturbances, the regime of dynamic damping of oscillations can be considered in a generalized form, which correlates with the concepts of determining the corresponding frequencies from the condition of "zeroing" the numerators of the transfer functions.
3. It is shown that the frequencies of the regime of simultaneous dynamic damping of object oscillations are determined from the condition of simultaneous "zeroing" of the numerators of the transfer functions in two coordinates.
4. A method has been developed for determining the frequencies of dynamic damping of oscillations based on the use of frequency diagrams reflecting the complex character of the change in all frequencies of the system when the tuning parameters are changed.

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