Direct Evidence for Synchronization in Japanese Business Cycles

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Abstract
We have analyzed the Indices of Industrial Production (Seasonal Adjustment Monthly Index) for a long period of 240 months (January 1988 to December 2007) in order to develop a deeper understanding of economic shocks. Angular frequencies estimated using the Hilbert transform are almost identical for the 16 industrial sectors in the indices. Moreover, partial phase locking was observed for the sectors. These are direct evidence of the synchronization present in Japanese business cycles. We also showed that the phase time series carries information on economic shocks. Common shock and individual shocks are separated using phase time series. The former dominates the economic shock in all of 1992, 1998 and 2001. The obtained results suggest that the business cycle may be described as dynamics of coupled limit cycle oscillators exposed to random individual shocks.

Keywords: business cycle, synchronization, Hilbert transform.

1. Introduction
Business cycles have been defined as a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises (Burns and Mitchell, 1964). A cycle consists of expansions occurring at roughly the same time in many economic activities, followed by similar general recessions, contractions, and revivals that merge into the expansion phase of the next cycle; a sequence of change that is recurrent but not periodic. Durationwise, one business cycle may vary from more than one year to 10 or 12 years, and it is not divisible into shorter cycles of similar character with amplitudes approximating their own.

Business cycles have a long history of theoretical studies (Yoshikawa, 2000; Aoki and Yoshikawa, 2007; Granger and Hatanaka, 1964). Samuelson (1939) showed

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that the second-order ordinary linear differential equation based on a multiplier and accelerator could generate the cycle in the gross domestic product (GDP). Hickes, by introducing a “ceiling” and a “floor” into such a model, demonstrated that the cycle is sustainable (Hickes, 1950). In contrast, some other theorists focused on the non-linearity. Goodwin (1951) introduced a nonlinear accelerator in order to generate a sustainable cycle. Kaldor (1940) captured the business cycles as a limit cycle. Subsequently, the idea of nonlinearity evolved in application of the chaos theory (Lorenz, 1989; Sterman and Mosekilde, 1994). It should be noted that the earlier empirical studies were reported mainly for US business cycles (Burns and Mitchell, 1964; Tinbergen, 1939).

In neoclassical economics, predominant causes of business cycles are considered as real changes in the supply side, such as innovation in production technology and a shift in fiscal policy, instead of nominal demand change due to a shift in financial policy. The most influential theory among the neoclassical economists is the real business cycle theory (Kydland and Prescott, 1982), whose essential feature is to treat shocks due to technological innovations as the most important cause of business cycles, as outlined below.

If we deal with the Cobb-Douglas production function

\[ Y = bK^\alpha L^{1-\alpha}, \quad (1) \]

the Solow residual

\[ \frac{\Delta b}{b} = \frac{\Delta Y}{Y} - \alpha \frac{\Delta K}{K} - (1-\alpha) \frac{\Delta L}{L} \quad (2) \]

is considered as a shock due to technological innovations. Here, \( Y, K, L, \alpha, \) and \( b \) are total production, capital input, labor input, output elasticities of capital, and total factor productivity, respectively. The occurrence of technological innovation could be treated as a random process. Therefore the Solow residual is emulated as a moving summation of random shocks, following Slutsky’s explanation (Slutsky, 1937). Figure 1 depicts the moving summation of random numbers generating cyclical fluctuations. The policy implication of the real business cycle theory is that the aggregate fluctuation is interpreted as rational or optimal, and any shift in financial policy has no effect on business cycles.

It is noted here that business cycles with a period of four to six years are usually considered as caused by adjustments in stock, such as inventory stock. Trajectories in the two-dimensional plane of the GDP growth rate and changes in inventory suggest the existence of a limit cycle in business cycles. This limit cycle is interpreted to be due to adjustments in inventory stock.

In this paper, we explored the alternative view for business cycles in order to develop a deeper understanding of economic shocks. Assume that two pendulum clocks, representing industrial sectors, with slightly different oscillation periods are hang on a beam. The pendulums of these two clocks oscillate with the same
period and two clocks show the same time (Huygens, 1966). When we move one pendulum clock to another beam, oscillation periods are not same, and they gradually deviates. A pendulum clock is considered a limit cycle oscillator, which receives energy from the rest of the system and releases energy and entropy to the rest of the system during oscillation. The observed oscillation with the same period is induced by the interaction between the two pendulum clocks mediated by the beam. This phenomenon is called synchronization, or entrainment (Kuramoto, 1984; Ikeda et al., 2012).

If we divide the aggregate business cycles into industrial sectors, fluctuation of industrial sectors must be synchronized with the same oscillation period; otherwise the aggregate business cycles will not be observed. Here, it is noted that the division into industrial sectors is regarded as a macroscopic resolution. The present work is an empirical study of synchronization in Japanese business cycles using data from the Indices of Industrial Production.

2. Empirical Analysis
2.1 Data
We analyzed data from the Indices of Industrial Production (Seasonal Adjustment Monthly Index) for a long period of 240 months (monthly data from January 1988 to December 2007). The database includes indices of production, shipment, and inventory for 16 industrial sectors: steel industry (s1), non-ferrous metal industry (s2), metal products industry (s3), machinery industry (s4), transportation equipment industry (s5), precision machinery industry (s6), ceramic products industry (s7), chemical industry (s8), petroleum and coal products industry (s9), plastic products industry (s10), paper and pulp industry (s11), textile industry (s12), food industry (s13), other industries (s14), mining industry (s15), and electric machinery industry (s16). The analysis performed in the present study employed just the indices of production for all 16 sectors.
2.2 Stationarization

Stationarization is the important reprocessing step of time series analysis. First, log returns of the indices were calculated for the 16 industrial sectors. Stationarity of these time series were then confirmed using the unit root test. The time series of log returns of the indices are shown for the sectors in Fig. 2. Here, the length of the time series is $L = 239$.

Fourier series expansion of the time series of log returns of the indices was then calculated, and some high- and low-frequency components were removed. The remaining Fourier components are described later.

2.3 Hilbert transform

Suppose we have two oscillators, with phases of $\theta_1(t)$ and $\theta_2(t)$, respectively. Synchronization is defined as the phases locking $n\theta_1(t) + m\theta_2(t) = \text{const.}$, where $n$ and $m$ are integers, while the amplitudes can be different. In a limited case of $n = -m$ and const. $= 0$, discussion with a correlation coefficient is suitable. However, in the case of $n = -m$ and const. $\neq 0$, where the phase difference means a delay, direct evaluation of the phase is adequate, instead of the correlation coefficient. This is because the correlation coefficient $\rho$ varies depending on the delay $\delta$. For example,
for the trigonometric function with the period of oscillation equal to $2\pi$, we have $\rho=1$ for $\delta=0$, $\rho=0$ for $\delta=\pi/2$, and $\rho=-1$ for $\delta=\pi$. This simple example illustrates that the correlation coefficient is not suitable for the case of the phases locking where we have phase difference or delay.

The Hilbert transform of a continuous time series $x(t)$ is defined by Eq. (3) (Gabor, 1946; Rosenblum et al., 1996),

$$ y(t) = H[x(t)] = \frac{1}{\pi} \text{PV} \int_{-\infty}^{\infty} \frac{x(s)}{1-s} \, ds, $$

where PV represents the Cauchy principal value. A complex time series is obtained by adopting the time series $y(t)$ as an imaginary part. Consequently, a phase time series $\theta(t)$ is obtained using Eq. (4),

$$ z(t) = x(t) + iy(t) = A(t) \exp[i\theta(t)]. $$

Here, $i$ is the unit imaginary number defined by $i^2 = -1$. The following example may help readers understand the idea of the Hilbert transform. Suppose the time series $x(t)$ is a cosine function $x(t) = \cos(\omega t)$, then the Hilbert transform of $x(t)$ will be $y(t) = H[\cos(\omega t)] = \sin(\omega t)$. Similarly, for a sine function $x(t) = \sin(\omega t)$, the Hilbert transform will be $y(t) = H[\sin(\omega t)] = -\cos(\omega t)$. Using the Euler’s formula $z(t) = \cos(\omega t) + i\sin(\omega t) = A(t) \exp[i\theta(t)]$, we can calculate the phase time series $\theta(t)$.

In Fig. 3, the trajectories of the time series are shown in the complex plane, where log returns of the indices $x_s(s=1, \cdots, L)$ and the Hilbert transform $y_s = H[x_s]$ ($s=1, \cdots, L$) are used as the horizontal and vertical axis, respectively, for the 16 sectors. Here, log returns of the indices $x_s(s=1, \cdots, L)$ are written as the discrete Fourier expansion as follows.

$$ x_s = \frac{1}{\sqrt{L}} \sum_{r=1}^{L} \left( A_r \cos \frac{2\pi rs}{L} + B_r \sin \frac{2\pi rs}{L} \right). $$

Then the time series $y_s$ is calculated using Fourier coefficients $A_r$ and $B_r$ in Eq. (5),

$$ y_s = \frac{1}{\sqrt{L}} \sum_{r=1}^{L} \left( A_r H \left[ \cos \frac{2\pi rs}{L} \right] + B_r H \left[ \sin \frac{2\pi rs}{L} \right] \right) $$$$ = \frac{1}{\sqrt{L}} \sum_{r=1}^{L} \left( A_r \sin \frac{2\pi rs}{L} - B_r \cos \frac{2\pi rs}{L} \right). $$

Here, we have a relation for the Fourier coefficient; $A_k = A_{L-k+1}$ and $B_k = B_{L-k+1}$. For the calculation method of the Hilbert transform actually used in this paper, refer to Appendix A.

By applying a band-path filter, the Fourier components of the oscillation period from 24 to 80 months ($r=3$ to 10, 230 to 237) were included in graphs of Fig. 3. Some irregular rotational movement was observed due to the non periodic nature of the business cycles.
The time series of the phase $\theta(t)$ was obtained using Eq. (4) for the 16 sectors. Figure 4 depicts the time series of the phase $\theta(t)$ for the sectors. Fourier components of the oscillation period from 24 to 80 months were included in these plots. We observed the linear trend of rotational movement with some fluctuation for sectors. The irregular rotational movement caused the small jumps of phases in Fig. 4, and notably, trajectories passed near the origin of the plane, which is ob-

![Fig. 3. From the top left to the bottom right in a transverse direction, the trajectories of the time series are shown in the complex plane, where the horizontal axis $x$ and vertical axis $y=H[x]$, for the 16 sectors: steel industry (s1), non-ferrous metal industry (s2), metal products industry (s3), machinery industry (s4), transportation equipment industry (s5), precision machinery industry (s6), ceramic products industry (s7), chemical industry (s8), petroleum and coal products industry (s9), plastic products industry (s10), paper and pulp industry (s11), textile industry (s12), food industry (s13), other industries (s14), mining industry (s15), and electric machinery industry (s16). Fourier components of the oscillation period from 24 to 80 months were included in these plots.](image-url)
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3. Discussion

3.1 Frequency entrainment

Frequency entrainment and phase locking are expected to be observed as direct evidence of the synchronization. The angular frequency $\omega_i$ and the intercept $\tilde{\theta}_i$ are estimated by the least squares fitting the time series of phase $\theta_i(t)$ using Eq. (7),

$$\theta_i(t) = \omega_i t + \tilde{\theta}_i,$$

where $i$ indicates the industrial sector. Actually, the intercept $\tilde{\theta}_i$ fluctuates over time due to economic shocks. In the least squares fitting, the average value of the intercept $\tilde{\theta}_i$ is estimated. The estimated angular frequencies $\omega_i$ for all 16 sectors are plotted for the Fourier components: 18–80 months and 24–80 months in Fig. 5, where we observed that the estimated angular frequencies $\omega_i$ are entrained to be almost identical for the sectors, and this means that frequency entrainment

![Fig. 4. From the top left to the bottom right in a transverse direction, the time series of the phase $\theta(t)$ are shown for the 16 sectors: steel industry (s1), non-ferrous metal industry (s2), metal products industry (s3), machinery industry (s4), transportation equipment industry (s5), precision machinery industry (s6), ceramic products industry (s7), chemical industry (s8), petroleum and coal products industry (s9), plastic products industry (s10), paper and pulp industry (s11), textile industry (s12), food industry (s13), other industries (s14), mining industry (s15), and electric machinery industry (s16). Fourier components of the oscillation period from 24 to 80 months were included in these plots.](image-url)
is observed. When the higher frequency Fourier components are included in the analysis, the deviations of the estimated angular frequencies $\omega_i$ among the sectors are expected to gradually increase. The increase in the deviations can be explained by the presence of larger jumps of phases due to the trajectories passing near the origin of the complex plane for the higher frequency Fourier components.

3.2 Phase locking

Phase locking means that phase differences for all pairs of oscillators are constant. However, this condition is rarely seen to exactly satisfy the actual time series due to irregular fluctuation, i.e., economic shock. We introduce an indicator $\sigma(t)$ of the phase locking as

$$\sigma(t) = \left[ \frac{1}{16} \sum_{i=1}^{16} \left\{ \frac{d}{dt} (\theta_i(t) - \omega_i t) - \mu(t) \right\}^2 \right]^{1/2},$$

$$\mu(t) = \frac{1}{16} \sum_{i=1}^{16} \frac{d}{dt} (\theta_i(t) - \omega_i t).$$

In the case of frequency entrainment, Eqs. (8) and (9) become

$$\sigma(t) = \left[ \frac{1}{16} \sum_{i=1}^{16} \left\{ \tilde{\theta}_i(t) - \frac{1}{16} \sum_{j=1}^{16} \tilde{\theta}_j(t) \right\}^2 \right]^{1/2}.$$  

Here, temporal dependence of the intercept is explicitly expressed as $\tilde{\theta}_i(t)$. The indicator $\sigma(t)$ is equal to zero when the intercepts for all oscillators are the same. On the other hand, if the indicator $\sigma(t)$ satisfies the following relation, we call this partial phase locking,

$$\sigma(t) \ll \omega_i.$$

The estimated indicator of the phase locking $\sigma(t)$ is plotted in Fig. 6 for the Fourier components: 18–80 months and 24–80 months. Figure 6 depicts the indicator $\sigma(t)$ being much smaller than $\omega_i$ for most of the period. This means that partial phase locking is observed. As a consequence of this analysis, both frequency entrain-
3.3 Common shock and individual shock

The log returns of the indices $x_i$ are decomposed to the amplitude $A_i(t)$ and the phase $\theta_i(t)$ using Eqs. (3) and (4). An interesting question to ask is: “Which quantity carries the information about economic shocks, the amplitude $A_i(t)$ or the phase $\theta_i(t)$?” The averages of these quantities over the industrial sectors are written as:

$$\langle A(t) \rangle = \frac{1}{16} \sum_{i=1}^{16} A_i(t) = \frac{1}{16} \sum_{i=1}^{16} \frac{x_i(t)}{\cos \theta_i(t)},$$

$$\langle \cos \theta(t) \rangle = \frac{1}{16} \sum_{i=1}^{16} \cos \theta_i(t).$$

The average amplitudes $\langle A(t) \rangle$ and average phases $\langle \cos \theta(t) \rangle$ are shown in Figs. 7 and 8, respectively. These figures clearly show that the information about economic shocks is carried by the phase $\theta_i(t)$, not by the amplitude $A_i(t)$. In Japan, we experienced severe economic recessions in 1992, 1998, and 2001. The 1992 recession was due to the collapse of the bubble economy. In 1998, we had a banking crisis, during which some major banks in fact went bankrupt. The 2001 recession was due to the bursting of the information technology bubble. The average amplitudes $\langle A(t) \rangle$ in Fig. 7 have large values in all of 1992, 1998, and 2001. On the contrary, the average phases $\langle \cos \theta(t) \rangle$ in Fig. 8 show a sharp drop in all of three. It should be noted that the phase $\theta_i(t)$ is more sensitive to economic shock than the amplitude $A_i(t)$. Thus, it is adequate to interpret the average phases $\langle \cos \theta(t) \rangle$ and residual $\cos \theta_i(t) - \langle \cos \theta(t) \rangle$ as common shock and individual shock, respectively.

The individual shocks $\cos \theta_i(t) - \langle \cos \theta(t) \rangle$ for each industrial sector are shown in Fig. 9. This depicts that we have many individual shocks all the time, and many of them seem to occur randomly. More noteworthy is the contraction of the individual shocks in 1992, 1998, and 2001, when we had severe economic recessions. All industries were exposed to the common shocks $\langle \cos \theta(t) \rangle$ during these periods.
We analyzed the Indices of Industrial Production (Seasonal Adjustment Monthly Index) for a long period of 240 months (January 1988 to December 2007) to develop a deeper understanding of economic shocks. The angular frequencies $\omega_i$ estimated using the Hilbert transform are almost identical for the 16 sectors in the indices. Moreover, the indicator of the phase locking $\sigma(t)$ shows that partial phase $\theta_i(t)$ is more sensitive to economic shock than the amplitude $A_i(t)$.

4. Summary
We analyzed the Indices of Industrial Production (Seasonal Adjustment Monthly Index) for a long period of 240 months (January 1988 to December 2007) to develop a deeper understanding of economic shocks. The angular frequencies $\omega_i$ estimated using the Hilbert transform are almost identical for the 16 sectors in the indices. Moreover, the indicator of the phase locking $\sigma(t)$ shows that partial phase $\theta_i(t)$ is more sensitive to economic shock than the amplitude $A_i(t)$.
locking is observed for the sectors. These are direct evidence of synchronization in Japanese business cycles. We also showed that the information on economic shock is carried by the phase time series. Common shock and individual shocks are separated using phase time series \( \theta_i(t) \). It is interpreted that the average phases \( \langle \cos \theta(t) \rangle \) and residual \( \cos \theta_i(t) - \langle \cos \theta(t) \rangle \) are common shock and individual shock, respectively. The former dominates the economic shocks in all of 1992, 1998 and 2001. In the same periods, the individual shock has contraction. We have many individual shocks all the time, and these seem to occur randomly. The obtained results suggest that business cycles may be described as dynamics of the coupled limit cycle oscillators exposed to random individual shocks.

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Appendix A: Discrete Hilbert Transform

We review the Hilbert transform for completeness in this appendix.

The discrete Fourier expansion of a (real) time series \{f_1, f_2, \ldots, f_M\} is given by the following:

\[
f_m = \frac{1}{\sqrt{M}} \sum_{k=1}^{M} \tilde{f}_k e^{i \frac{2\pi k}{M} m}, \quad \tilde{f}_k = \frac{1}{\sqrt{M}} \sum_{m=1}^{M} f_m e^{-i \frac{2\pi k}{M} m}
\]  

(A.1)

where \(\tilde{f}_k = \tilde{f}_{M-k}\) and \(\tilde{f}_M = \sum_{m=1}^{M} f_m / \sqrt{M}\)

is real.

For even \(M\), the Fourier expansion is written as follows:

\[
f_m = \frac{1}{M} \sum_{n=1}^{M} (1 + (-1)^{n+m}) f_n + \frac{2}{\sqrt{M}} \text{Re} \left[ \sum_{k=1}^{M/2-1} \tilde{f}_k e^{i \frac{2\pi k}{M} m} \right].
\]  

(A.2)

The Hilbert transform for this series is then defined as follows:

\[
f_m^{(1)} := \frac{2}{\sqrt{M}} \text{Im} \left[ \sum_{k=1}^{M/2-1} \tilde{f}_k e^{i \frac{2\pi k}{M} m} \right],
\]  

(A.3)

so that the oscillation part in the original time series now leads to rotation in the complex series \(\{f_m + i f_m^{(1)}\}\).

The following relation, which can be obtained by using Eq. (A.1) in the definition Eq. (A.3), is useful in the calculation:

\[
f_m^{(1)} = \frac{2}{M} \sum_{n=1}^{M} f_n \cot \left( \frac{\pi}{M} (m-n) \right).
\]  

(A.4)

For the odd \(M\), we have the following similarly to the even \(M\) case:
with,

\[ f_m^{(1)} = \frac{1}{M} \left[ \sum_{n=1 \atop n-m=\text{odd}}^{M} f_n \cot \left( \frac{\pi}{2M} (m-n) \right) - \sum_{n=1 \atop n-m=\text{even}}^{M} f_n \tan \left( \frac{\pi}{2M} (m-n) \right) \right]. \]  

(A.7)