A ROTATED SKYRMION AND
PIONS-NUCLEON INTERACTIONS

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Abstract

It was shown by careful consideration of the time–dependent classic solution for the rotated skyrmion, that such solution at large distances has radiational component similar to the Lienart-Viehartz retarded potential in Electrodynamics. We consider such solution as stationary phase configuration of the pion field in path integrals for the correlators of $n$ isovector axial currents $J_{A,i}^{\mu}$ and two nucleon currents $J_{N}$. $n$ pion – nucleon amplitudes are extracted from these correlators. The careful account of the asymptotics of this stationary phase configuration of the pion field (rotated skyrmion) in path integral leads to nonzero contribution of this classical part of the total pion field to pion-nucleon amplitudes. In result, this approach correctly reproduce pions – nucleon amplitudes(with the Born – diagrams contributions, too).

1. Introduction

The skyrmion or skyrmion-like models of a nucleon has a many attractive features and was discussed very much (see review [1], for example). Among them the most powerful are chiral quark models, inspired by QCD.

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We will consider pions-nucleon interactions in the framework of the $SU(2)$-- effective chiral QCD model [2]. In this model the radius of SBCI is much smaller than the radius of confinement $R_{con}$.

It was shown that the interaction of valence quarks with pion field in nucleon confines them at the distances much smaller than $R_{con}$ (chiral confinement) [2]. The color confinement forces can be accounted as a perturbation in this case.

The Lagrangian of the effective chiral QCD has the form of the nonlinear quark $\sigma$--model describing interacting quark $\psi$ and isovector pion $\vec{\phi}$ fields without kinetic term for the pion field:

$$L = \bar{\psi}(i\gamma \cdot \partial + M_q \exp(i\gamma_5 \vec{\tau} \vec{\phi})))\psi.$$ (1)

The quantity $M_q$ in fact is a function of the momentum $p$. This function $M_q(p)$ in the Euclid space can be approximated by $\Theta$-function and contains two main parameters: the effective quark mass $M_q(0)$ and a cutoff $p_0$, which is defined from the condition $M_q(p_0) = 0$. Integration over quark fields in the partition function generates, as usual, the effective action

$$S_{eff}[\phi|B=0] = \int dx L_p$$

for the states with baryon number $B = 0$, where Lagrangian

$$L_p = -3 \ln \det[i\gamma \cdot \partial + M_q \exp(i\gamma_5 \vec{\tau} \vec{\phi})] = \frac{1}{4} f^2 Tr L_\mu L^\mu + ...$$ (2)

Here $f$ is the charge pion decay constant, $L_\mu = U^+ \partial_\mu U$ and $SU(2)$--valued matrix $U$ can be represented as $U = e^{i\vec{\tau} \vec{\phi}}$.

The $M_q(0) = 340 Mev$ and $p_0 = 600 Mev$ were fixed from the well known quantities $f = 93 Mev$ and the electric charge radius of pion $r_\pi = (300 Mev)^{-1}$.

The calculation of the nucleon properties is related with the calculation of the correlator of the 3--quarks currents with the quantum numbers of the nucleons $J_N$. Integration over quarks in this correlator gives

$$\Pi_N(x, y) = \langle J_N(x) J_N^\dagger(y) \rangle = \int D\phi \prod_{i=1}^3 G_i(x, y|\phi) e^{i S_{eff}[\phi|B=0]}$$ (3)

Here $G_i(x, y|\phi)$ is the quark propagator in the external field $\phi$ and $S_{eff}[\phi|B = 0] - was defined in the Eq. (2)). So, the effective action for the states with
baryon number $B = 1$

$$S_{\text{eff}}[\phi, x, y | B = 1] = S_{\text{eff}}[\phi | B = 0] - iln \prod_{i=1}^{3} G_i(x, y | \phi). \quad (4)$$

For a stationary external pion field and in the Euclid space ($\vec{x} = \vec{y} = 0, x_4 = T \to \infty, y_4 = 0$)

$$G_i^E \sim \exp(-E_{\text{val}}[\phi] \cdot T), S_{\text{eff},E}[\phi | B = 0] = E_{\text{vac}}[\phi] \cdot T. \quad (5)$$

The total energy of such a state consists of contributions from the 3 valence quarks ($3E_{\text{val}}$) and polarization of quark vacuum ($E_{\text{vac}}$). Both of them are functionals of the external pion field. Integration in the saddle-point approximation over pion field can be reduced to the minimization of the total energy over some suitable trial fields $\phi$:

$$M_0^N = \min_{\phi} (3E_{\text{val}}[\phi] + E_{\text{vac}}[\phi]) \quad (6)$$

The Skyrme model teaches us to choose this field in the form of the chiral soliton (skyrmion):

$$U_0(\vec{x}) = \exp i\vec{n}\phi_0, \quad \vec{n} = \vec{r}/r,$$

$$\phi_0(0) = \pi, \phi_0(r) \sim -3g_A/(8\pi f^2 r^2), (r \to \infty). \quad (7)$$

Asymptotic of $\phi_0$ is in fact the Goldberger-Treiman relation, where $g_A$ is the axial charge of the nucleon. This soliton field configuration has unit topological number $\mathbb{Z}$. The simplest trial function is $\mathbb{Z}$:

$$\phi_0 = 2 \arctan(\frac{r_0}{r})^2 \quad (8)$$

In the Effective chiral QCD model valence quarks in the nucleon are confined in the region of order $0.5 fm$ due to interaction effects with their own pion field (chiral confinement). The properties of the nucleon in this case resemble chiral soliton – skyrmion very much.

It is useful to compare this and Skyrme models. The Lagrangian of Skyrme model has the form:

$$L_{sk} = \frac{1}{4} f^2 Tr L_{\mu} L^\mu + \frac{1}{32 e^2} Tr [L_{\mu}, L_{\nu}]^2, \quad (9)$$
The energy of the trial field configuration (8) is \( E = A f^2 r_0 + B/(e^2 r_0) \). The first term just coincides with the \( E_{\text{vac}} \), and the second one simulates \( E_{\text{val}} \).

The main difference between Skyrme and this models is that Skyrme model action is the same for the pion and nucleon sectors. In contrast, the action of the effective chiral QCD model has different forms for these sectors because the nucleon sector has an additional contribution of valence quarks.

2. Time-dependent classic solution for the rotated skyrmion

We will consider classic time-dependent solution of the chiral fields equations of \( S_{\text{eff}}[\phi|B = 1] \). We suppose that static equations of motion has topologically nontrivial solution – skyrmion \( U_0(\vec{x}) \). We suppose that the time dependence for the rotated skyrmion at the small distances \( x \sim r_0 \) (\( r_0 \) was defined at Eq. (8)) can be introduced into the static solution \( U_0(\vec{x}) \) accordingly Ansatz:

\[
U(\vec{x}, t) = A(t)U_0(\vec{x})A^+(t).
\]

\( SU(2) \) matrix \( A(t) \) has a meaning of the collective coordinate of the rotation of the skyrmion as a whole in the isospace (rigid rotation) with the frequency \( \vec{\Omega} = -iS\partial A(t)A^+(t)\vec{\tau} \). We suppose that \( \Omega r_0 << 1 \) (slow rotation).

It is clear that the solution of the chiral field equations in the static case is equivalent to the minimization of the classic mass \( M_s \) over parameter \( r_0 \) of the suitable skyrmion-like trial function the Eqs. (7), (8). The parameter \( r_0 \) has clear meaning of the size of the skyrmion. Shall we consider the

\[
U(\vec{x}, t) = \exp i\vec{r}\vec{\phi}(\vec{x}, t) = \exp i\tau_i R_{ij}(t)n_j \phi_0(x)
\]

at large distances in more detail. At large distances \( \Omega^{-1} \gg x \gg r \) we can neglect by nonlinearity of the pion fields and

\[
\phi_i(\vec{x}, t) = R_{ij}(t)n_j (r/x)^2
\]

On other hand the equation for the \( \vec{\phi}(\vec{x}, t) \) at large distances has the form of the usual wave equation:

\[
(\Delta - \partial^2/\partial t^2)\phi_i(\vec{x}, t) = 0
\]
It is clear that we can improve the asymptotic (10) for the distances \( x \sim \Omega^{-1} \) to take it in the form similar to the retarded Lienart-Viehart potentials:

\[
\phi_i(\vec{x}, t) = -\partial_j R_{ij}(t - x)(r/x)
\]  

(12)

As usual the account of retarded time (which is the time of the propagation of field from source to the point \( \vec{x} \)) leads to the \( 1/x \) dependence of fields \( \phi_i(\vec{x}, t) \) at large distances. This component of fields can be named as radiating one.

From the large distances rotated skyrmion can be considered as rotated dipole.

In general case time-dependent solution must have radiating component and such skyrmion have to lost his energy by emitting of the pions. The reason to have stable solution for the nucleon is the same as in atom physics. It is well known that nucleon will be the lowest fermion state of the quantum rotating skyrmion. So, we have the time dependent solution \( \phi_i(\vec{x}, t) \) in the form:

at \( 0 < x < r \ (r_0 << r << \Omega^{-1}) \)

\[
\phi_i(\vec{x}, t) = R_{ij}(t) n_j 2 \arctan \frac{r_0}{2 x^2}
\]  

and at \( x >> r \)

\[
\phi_i(\vec{x}, t) = -\partial_j R_{ij}(t - x) \frac{r}{x}
\]  

(13)

Let us calculate the action by using the Eq. (13):

\[
S = \int_{t_i}^{t_f} dt \int_{x<r} d^3x L(\phi_i(\vec{x}, t)) = R_{ij}(t)n_j \phi_0(x)) + f^2/2 \int_{x>r} d^3x \partial_\mu \phi_i(\vec{x}, t) \partial_\mu \phi_i^*(\vec{x}, t) =
\]

\[
\int_{t_i}^{t_f} dt [(I(\vec{\Omega})^2/2 - M_s)_r + f^2/2 \int_{x>r} d^3x \partial_\mu (\phi_i(\vec{x}, t) \partial_\mu \phi_i^*(\vec{x}, t))]
\]  

(14)

We calculated the former integral by accounting of the Eq. of motion at large
distances and by the integration by part. For this part of the action we have

\[
f^2/2 \int_{t_i}^{t_f} \int_{x>r} d^3x \partial_\mu (\phi_i(\vec{x},t) \partial_\mu \phi_i^*(\vec{x},t)) =
\]

\[
f^2/2(\int_{x>r} d^3x (\phi_i(\vec{x},t) \partial_0 \phi_i^*(\vec{x},t)))|_{t_f} - \int_{t_i}^{t_f} dt \int_{(x=r)} d\vec{n} x^2 (\phi_i(\vec{x},t) \partial_\mu \phi_i^*(\vec{x},t))) = O(1/r)
\]

(15)

The terms of order \(O(1/r)\) reduce the dependence of first term in the action \(S\) on \(r\).

It is easy to calculate the energy emitted by rotated skyrmion in the unit of time by calculation of the energy-momentum tensor

\[
T_{\mu\nu} = f^2/2(Tr \partial_\mu U \partial_\nu U^+ - 1/2g_{\mu\nu}Tr \partial_\alpha U \partial_\alpha U^+) + O(\partial^4).
\]

At \(x >> r\)

\[
T_{00} = f^2/2(\partial_0 \phi_i \partial_0 \phi_i^+ + \partial_j \phi_i \partial_j \phi_i^+) , T_{0j} = f^2 \partial_0 \phi_i \partial_j \phi_i^+ \\
\int_{S(x=r)} dS_m T_{0m} = -\alpha f^2(2\Omega^4(t-r) + 2(\partial_0 \Omega)^2(t-r) + 3\partial_0(\Omega(t-r))^2)
\]

In fact we need components \(T_{0n}\) at large distances to calculate \(dE/dt = \int_{S(x=r)} dS_n T_{0n}\).

Energy at the distances \(x > r\):

\[
\Delta E(t) = \int_{x>r} dx \alpha f^2(2\Omega^4(t-r) + 2(\partial_0 \Omega)^2(t-r) + 3\partial_0(\Omega(t-r))^2)
\]

it is nonzero if \(t > r\)

(Another formula for the calculation of the same quantity is \(\Delta E(t) = \int_{r}^{t} dt' f^2 r^2 d\vec{n}_m T_{0m}\)

and give the same answer). We can neglect it because \(\Delta E(t) \sim O(\Omega^4)\) and we suppose that \(\partial_0 \Omega << \Omega^2\), \(\Omega r_0 << 1\).

With this accuracy we can consider classic rotated skyrmion as stable stationary state. In this case the quantization of this object is well defined procedure. We will follow to the method of Ref. [11], where was considered the similar problem but with \(U(1)\) internal symmetry. The straightforward generalization of this results means in our case that we must to minimize over variational parameter \(r_0\) the expression for the total energy of rotated
skyrmion with account of the kinetic energy of the rotation of the skyrmion and the contributions of the small quantum transverse (to zero-mode $A$) fluctuations. In this case the parameter $r_0$ has meaning of the size of a rotated skyrmion.

$$
\min E_{\text{tot}}[r_0] = M_s[r_0] + j^2/2I[r_0] + 
\text{(contributions of the quantum transverse fluctuations)}
$$

(16)

Quantization of the rotational motion over $A(t)$ leads to the spectrum of the quantum rotator

$$
E = M_s + T(T + 1)/2I
$$

with $T = S$ ($S$ is a spin, $T$–isospin and $I$ – classical part of the moment of inertia). Wave functions are the Wigner $D$– functions:

$$
W.F. = D_{S,T}^{S=T}(A).
$$

(17)

The nucleon is identified with the lowest fermion state with $S = T = \frac{1}{2}$.

3. Pions – nucleon interactions in the Effective Chiral QCD model

From the general point of view any $n$ pions – nucleon amplitude can be extracted from the appropriate correlator of $n$ isovector axial currents $J_{\mu}^{A,i}$ (or their divergencies) and two nucleon currents $J_N$.

$$
A_{\mu_1...\mu_n}(p_1, p_2, q_1, ..., q_n) = \int d^4y_1d^4y_2d^4x_1...d^4x_n \exp i(q_1x_1 + ... + q_nx_n - p_1y_1 + p_2y_2)
$$

$$
\langle T J_N^+(y_1)J_N(y_2)J_{\mu_1}^{A,i}(x_1)...J_{\mu_n}^{A,j}(x_n) \rangle
$$

(18)

(In the calculations of this part of the section we will use Minkowsky space formulation). The $n$ pions – nucleon amplitude $A_n(p, q)$ is a residue in the mass shell poles of the correlator (13). For the calculation of this correlator we introduce external axial-vector field $A_{\mu} = \tau_i/2A_{\mu}^i(x)$. The interaction of quarks with this field can be introduced by substitution: $\partial_{\mu} \Rightarrow \partial_{\mu} - i\gamma_5A_{\mu}$ in the Lagrangian (1).
In this case, we calculate the correlator in Eq. (18) by using the formula:

$$\langle T J^+_{N}(y_1) J_N(y_2) J^{A,1}_\mu(x_1) ... J^{A,n}_\mu(x_n) \rangle = \frac{\delta}{\delta A^{1}_{\mu}(x_1)} ... \frac{\delta}{\delta A^{n}_{\mu}(x_n)} \langle T J^+_{N}(y_1) J_N(y_2) \rangle |_{A=0},$$

(19)

where \(\langle T J^+_{N}(y_1) J_N(y_2) \rangle_A\) is the correlator of the 3 – quarks currents with the quantum numbers of the nucleons \(J_N\) in the presence of the external axial-vector field \(A_{\mu}\). Integration over quarks in this correlator gives

$$\langle T J^+_{N}(y_1) J_N(y_2) \rangle_A = \int D\phi e^{iS_{eff}[\phi, y_1, y_2 | B=1]} \int D\phi e^{iS_{eff}[\phi | B=0]},$$

(20)

where effective action in presence of external axial field \(A\) (see also Eq.(3))

$$S_{eff}[\phi, y_1, y_2 | B=1] = -\frac{1}{2} \ln \prod_{i=1}^{N_c} G_i(y_1, y_2 | \phi) + S_{eff}[\phi, A | B=0]$$

(21)

In order to have the effective action in the presence of the external axial-vector field \(A\) we must replace in \(S_{eff}[\phi, y_1, y_2 | B=1]\) \(\partial_{\mu} U\) by \(\partial_{\mu} U + A_{\mu} U + U A_{\mu}\) etc. [10].

For the calculation of the correlator (19) near pion’s mass-shell region it is enough to account such a substitution only in the kinetic term in the Eq.(2). In Eq. (19) we can replace \(\delta/\delta A^{i}_{\mu}(x)\) by \((-2i)\delta/\delta \partial_{\mu} \phi^i(x)\). It is clear, that in the result we have

$$A_{\mu_1...\mu_n}(p_1, p_2, q_1, ..., q_n) = \int d^4 x_1 d^4 y_2 d^4 x \exp i(q_1 x_1 + ... + q_n x_n - p_1 y_1 + p_2 y_2)$$

$$\frac{\int D\phi e^{iS_{eff}[\phi, y_1, y_2 | B=1]} \int D\phi e^{iS_{eff}[\phi | B=0]}}{\int D\phi e^{iS_{eff}[\phi | B=0]}},$$

(22)

A straightforward calculation of the Eq. (22) leads to the following formula for \(A_n(p, q)\)

$$A_n(p, q) = \int d^4 x_1 ... d^4 x_n d^4 y_2 \exp i(q_1 x_1 + ... + q_n x_n - p_1 y_1 + p_2 y_2)$$

$$\frac{\int D\phi e^{iS_{eff}[\phi, y_1, y_2 | B=1]} \int D\phi e^{iS_{eff}[\phi | B=0]}}{\int D\phi e^{iS_{eff}[\phi | B=0]}},$$

(23)
The stationary phase method is suitable for the calculations of Eq. (23). It is clear from previous section that in the numerator stationary phase configuration of pion field is Eq. (13). In the denominator it is \( \vec{\phi} = 0 \). The measure in the path integral in the numerator consist of from measure of the collective translations \( R(t) \) and rotations \( A(t) \) and transverse to these modes the quantum fluctuations \( \vec{\phi}^q \). Total field

\[
\vec{\phi}(x) = \vec{\phi}^s(x) + \vec{\phi}^q(x),
\]

\( \phi^s \) is accounted the translations \( R(t) \) too and finally has a form

\[
\phi^s_i(x) = R_{ij}(t) n_j 2 \arctan \frac{r_0}{2(\rho(t))^2} \quad \text{at} \quad 0 < \rho < r
\]

and

\[
\phi^s_i(x) = -\partial_j R_{ij}(\tau) \frac{r_0^2}{\rho(\tau)}, \quad \text{at} \quad \rho >> r.
\]

Here \( \rho(t) = x - R(t) \) and \( \tau = t - \rho(\tau) \) is retarded time.

The expansion of the numerator of the Eq. (23) around \( \vec{\phi}^s \) over \( \vec{\phi}^q \) leads to two main parts of amplitude \( A_n(p, q) \). The first one contains only \( \vec{\phi}^s \) and second one – leading order over \( \vec{\phi}^q \). Accordingly it the measure and the effective action in Eqs. (22), (23) can be written as

\[
D\phi = D\vec{R}(t) DA(t) D\phi^q(\vec{x}, t),
\]

\[
S_{\text{eff}}[\phi, y_1, y_2|B = 1] = S_s + S_q.
\]

Here

\[
S_s = \int_{y_{1,0}}^{y_{2,0}} dt (-M_s + M_s \vec{V}^2/2 + I \vec{\Omega}^2/2),
\]

\[
S_q = \int d^4x tr \phi^q(x) 1/2 \hat{L}[\phi_s] \phi^q(x) + O(\phi^3_q),
\]

where \( M_s \) and \( I \) are classical parts of the nucleon mass and moment of inertia; \( \vec{V} = d\vec{R}/dt \) and \( \vec{\Omega} \) are nucleon velocity and nucleon angular velocity; \( \hat{L}[\phi_s] \) – the quadratic form for the quantum field \( \phi^q \).

Let us consider the simplest amplitude \( A_1(p, q) \):

\[
A_1(p, q) = \int d^4x d^4y_1 d^4y_2 \exp i(qx - p_1 y_1 + p_2 y_2)
q_1^2(M_N^2 - p_1^2)(M_N^2 - p_2^2) \int D\vec{R}(t) DA(t) D\phi^q(\vec{x}, t) \phi^s_i(x) \exp i(S_s + S_q)
\]

(25)
Integration over $\phi^q$ leads to the quantum correction of the nucleon mass. Integration over the nucleon position $\vec{R}(t)$ and orientation $A(t)$ provide us with nucleon propagator:

$$G(t_2, R_2) = A(t_2); t_1, R_1 = R(t_1), A_1 = A(t_1)) = \sum_{T,T_3} \langle R_2, A_2 | T, T_3 \rangle \langle T, T_3 | R_1, A_1 \rangle \int d^3 \vec{p}/(2\pi)^3 \exp i(\vec{p}(\vec{R}_2 - \vec{R}_1) - (M_s + \vec{p}^2/2M_s + T(T + 1)/2I)(t_2 - t_1)$$

The Eq. (25) can be written in the form:

$$A_1(p, q) = \int d^4x d^4y_1 d^4y_2 \exp i(qx - p_1y_1 + p_2y_2)q_1^2(M_N^2 - p_1^2)(M_N^2 - p_2^2)G(y_{2,0}, \vec{y}_2, A_2; \tau(x_0), \vec{x}, A)\phi^s(x)G(\tau(x_0), \vec{x}, A; y_{1,0}, \vec{y}_1, A_1)$$

(26)

It is important feature of Eq. (24) that we used the classical field $\phi^s(x)$ in the form of the retarded Lienard–Viechard potential. By this reason in Eq. (26) retarded time $\tau = x_0 - |\vec{x} - \vec{R}(\tau)|$ is used. We perform the integrations in this Eq. by natural introducing of the new variables: $\tau$ instead of $x_0$ and $\vec{z} = \vec{x} - \vec{R}(\tau)$. Nucleon propagators cancel the factors $(M_N^2 - p_{1,2}^2)$, integrations over $\vec{x}$ and $\tau$ provide energy-momentum conservation $\delta$ -function. At last, the integral over $\vec{z}$ has the form:

$$\vec{\tau}\vec{q}\Phi(q) = \int d^3\vec{z}\exp i(q\vec{z} - q_1, 0|\vec{z})\phi_0(\vec{z}),$$

(27)

where $\phi_0(\vec{z})$ is given by Eq. (7). It is clear that in the limit $q^2 \to 0$ it is sufficient to save asymptotic of $\phi_0$ Eq. (7) only due to $q^2$ factor in Eq.(26).

In this case we have an usual answer for the pion – nucleon vertices

$$\Gamma(A) = 3g_A/4f Tr(\tau_a A\tau_b A^+ q_b)$$

(28)

It is clear from Eq. (27) that the off – mass shell (nonzero $q^2$) pion – nucleon form – factor is given by the formula

$$F_{\pi NN}(q^2) = 2f/(3g_A)(q^2\Phi(q^2))$$

(29)

Certainly, this form-factor is unphysical and depend on the definition of the off-mass shell continuation, as it is clear from the previous discussion of the
correlator, Eq. (22) and the amplitudes, Eq. (23). Now, it is rather easy to discuss pion-nucleon scattering. In the pre-exponent factor in the general Eq.(23) we have the contribution from $\phi_s(x_1)\phi_s(x_2)$ and from $\phi^q(x_1)\phi^q(x_2)$ terms (it is clear that mixed terms $\phi^q(x_1)\phi_s(x_2)$ and $\phi_s(x_1)\phi^q(x_2)$ contributions are zero).

We can repeat and extend of the above calculations of the contributions of the $\phi^s$ for two stage at every point $x_1$ and $x_2$. Time ordering leads to two type of the contributions – $s$–channel and $u$–channel poles. We would have naturally it in the form of the Born pole-diagrams with sequence of the intermediate different rotated states of nucleon ($S = T = 1/2, 3/2, ...$).

The calculation of the contribution of the $\phi^q(x_1)\phi^q(x_2)$ term to the pion–nucleon scattering amplitude is related with the calculation of the inverse operator $\hat{L}^{-1}[\phi^s]$ (see e.g. [9]). This one is usual potential scattering problem with

$$\hat{L}[\phi^s] = \delta^2 S_{eff}[\phi|B = 1]/\delta^2\phi|_{\phi^s}.$$

Accordingly Ref.[2] $S_{eff}[\phi|B = 1]$ and $\hat{L}[\phi^s]$ consequently get the short ranged contribution from valence quarks term ($< 0.5 fm$) and long ranged one from vacuum polarization term ($> 0.5 fm$).

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