Research on Dynamic Assignment Problem of Subway Based on High and Low Peak Time Sharing Pricing

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Abstract: Although the subway greatly relieves traffic jams, the phenomenon of congestion within the subway during the peak period continues to increase. From the perspective of traffic demand management, subway time-of-day pricing can relieve peak congestion, shifting part of the passenger flow during the peak period to a low peak period, thereby improving transportation efficiency. In this paper, the dynamic flow allocation model based on timetable is studied. The timeliness of passenger route selection, the influence of congestion on passenger flow, the factors of ticket price and the penalty coefficient of early to late arrival are considered. Through the characteristics of subway operation and the method of high and low peak time-sharing pricing, the subway space-time network is established, and the influence of subway time-sharing ticket price on the passenger flow is discussed emphatically. Finally, an example is given to verify the model and algorithm of this paper.

1. Introduction

Urban subway is an efficient public transportation. It has the characteristics of rapidity, punctuality, and large capacity, effectively alleviating traffic congestion and other issues. However, at the same time, the phenomenon of subway congestion has been continuously aggravated during the peak period. In order to alleviate this phenomenon, this paper will discuss the impact of fare on passenger flow regulation from the perspective of high and low peak-time pricing. Early studies by domestic and foreign scholars on metro fare and allocation were mostly focused on multi-ticket system and multiple modes of public transport. Rare research consider the combination of fare and allocation. In recent decades, domestic and foreign scholars have increasingly studied urban subways, but most of them basically draw on the urban subway distribution of road traffic networks [1]. However, these studies did not consider the time dynamics of the subway. At present, the dynamic allocation of subway based on timetable has become a trend of research and development [2]. However, this paper will discuss the effect of fares on passenger flow distribution in combination with time-table-based subway dynamic allocation and high and low peak time sharing pricing.

2. Network expansion based on timetable and fare selection

The spatial topology of subways and the time-varying passenger flow are fully considered in this paper. Based on train timetables, each physical site is expanded on the original subway space network to form a subway dynamic network. The core of building a spatio-temporal network is to load timetables on the spatial network. In other words, trains of different trains pass through each station at different times. The space-time network mainly includes space-time nodes and space-time arcs. In this paper, space-time arcs include online arcs, running arcs, transfer arcs, parking arcs, and lower net arcs. Each space-time path consists of a space-time node and these five arcs. In order to consider different passengers'
different choices of space-time paths under different fare policies, two types of spatio-temporal arcs, the online arc and the lower network arc, are added to indicate the choice of different travel times between passengers in the same OD pair.

Figure 1 depicts a passenger travel path that includes a transfer station in a static form. The figure includes two subway lines, like line1 and line2. The set of collections and road segments in the defined network are N and A, respectively. N includes the passenger departure and destination and various physical stations of the subway. A includes the arcs and arcs under which the passengers make decisions, and the running sections of the subway between the various stations.

![Subway path network](image)

Figure 1. Subway path network

The metro spatiotemporal network diagram can be represented as G(V,E,L,T), where V is represented as a set of nodes in the spatiotemporal network, E is represented as a set of arcs in the spatiotemporal network, L is represented as a set of lines, and T is represented as subway access Station time point collection. In general, a physical site _i(iεN)_ can be expanded into (n+1) nodes in the spatiotemporal network, where _t=0, 1, ..., n(iεV, iεL)_ denoted at the _i_-th station on the route. According to the metro space-time network, train schedules and fares, the time-space network diagram considering the fares is obtained. As shown in Figure 2.

![Metro Space-time Network Diagram Considering Fare](image)

Figure 2. Metro Space-time Network Diagram Considering Fare

### 3. Path cost function

In order to better simulate the path selection behavior of passengers, it is necessary to define a generalized cost for each path, which reflects the comprehensive cost of passengers choosing the path. Here, in addition to the different fares at the high and low peak hours, the broad range of travel expenses of passengers includes crowded costs, transfer fees, walking time, and parking fees caused by congestion resulting from too many people in the train. This article adds online arcs and lower network arcs. The online arc indicates that the passengers will choose which time zone to travel on the same physical starting point under the premise of understanding the fare of different time periods, while the lower network arc represents the same end point that the passengers finally arrived in different time periods. Since the fare of the same OD is the same for the same time period, and the fare is not the same in different time periods, so the fare is put into the online arc for consideration.

#### 3.1 Running arc costs

Considering the influence of subway compartments congestion on passenger travel, the degree of congestion will change when the passenger flow changes. Specifically, when the passenger gradually
exceeds the number of seats in the vehicle and the content of the vehicle, discomfort increases and the crowding coefficient is expressed as [3]:

\[ p(x_{(i,j)}) = \begin{cases} 
0 & x_{(i,j)} < Z \\
\alpha(x_{(i,j)} - Z)/Z & Z \leq x_{(i,j)} < C \\
\alpha(x_{(i,j)} - Z)/Z + \beta(x_{(i,j)} - C)/C & x_{(i,j)} \geq C 
\end{cases} \] (1)

Among them, \( x_{(i,j)} \) represents the number of passengers on the running arc \((i, j)\), \( Z \) and \( C \) represent the number of seats on the subway and the maximum capacity of the subway, \( \alpha \) and \( \beta \) are corresponding parameters, which can be calculated by actual data. The running time \( c_{ij} \) on the running arc is fixed. Taking into account the subway running time and car congestion, the cost of the passenger on the running arcs of the train is:

\[ T_{ij} = c_{ij}(1 + p(x_{(i,j)})) \] (2)

### 3.2 Parking arc costs

The cost of a train’s parking arc is similar to that of a running arc. Considering the passenger’s congestion cost and the train’s stop time at the station, the parking arc costs are:

\[ T_{ij} = c_{ij}(1 + p(x_{(i,j)})) \] (3)

### 3.3 Transfer arc costs

The transfer fee in the space-time network mainly considers the transfer time and the number of transfers. Every additional passenger transfer increases the additional perceived cost, assuming that \( \omega \) is used to increase the additional cost of a transfer. \( t_{ij} \) is the difference between the departure time of the train after the transfer and the arrival time of the train before the transfer. The transfer arc costs can be expressed as:

\[ T_{ij} = \theta \cdot t_{ij} + \omega \] (4)

### 3.4 Penalty fee

Passengers choosing high and low peak travel will have early to late travel costs, so set a penalty fee. And it will be loaded from the early evening to the penalty fee to the lower net arc.

\[ T_{ij} = \eta \cdot \max\{0, \Delta t\} \] (5)

Among them, \( \eta \) is the penalty coefficient, \( \eta_1 \) is the penalty for late arrival, and \( \eta_2 \) is the early penalty. It is the difference between the expected arrival time interval and the actual arrival time of the travelers.

### 3.5 Fares

To describe the high and low peak hour pricing, the fare is loaded onto the arc of the Internet. Assume that the peak interval is \([t_1, t_2]\). When the subway is running during this time period, the fare adjustment for the running segment is:

\[ p_{rs}^{k} = p_{ij} + \varepsilon \] (6)

Among them, \( \varepsilon > 0 \) is the fare adjustment amount during peak hours, and the fare adjustment factor \( \varepsilon < 0 \) during the low peak period.

To sum up, it can be defined that in the space-time network, the total route travel fee of the kth space-time path selected by the passenger in any OD pair r-s is:

\[ C_{rs}^{k} = \sum_{i} \sum_{j} (\lambda_1 \cdot T_{ij} \delta_{ij,k}^{rs} + \lambda_2 \cdot p_{rs}^{k}) \] (7)

Where \( \lambda \) is the time-valued coefficient, \( p_{rs}^{k} \) is the fare of the kth space-time path of OD to r-s, \( \delta_{ij,k}^{rs} \) is the associated parameter. If \( \delta_{ij,k}^{rs} = 1 \), it means that the spatiotemporal arc i-j belongs to the k-th spatio-temporal path of the OD pair r-s. Otherwise, \( \delta_{ij,k}^{rs} = 0 \).

### 4. Equilibrium assignment model and algorithm based on space-time network
For the problem of dynamic allocation of metro space-time network, it is transformed into a simple static allocation problem through the space-time path. The following is a dynamic user equilibrium model based on space-time network, described by the following mathematical model[4].

\[
\min Z(x) = \sum_{i,j} f_{ij}(x) c_{ij}(x) dx.
\]  

The constraints are: \(\{\sum_k f_{k}^{rs} = q^{rs}, \forall r, s; f_{k}^{rs} \geq 0, \forall r, s, k; x_{(i,j)} = \sum_{rs} \sum_{k} f_{k}^{rs} \cdot \delta_{ij,k}, \forall i, j\}\). Where \(c_{ij}(x)\) represents the cost of each spatiotemporal arc.

The above model is a minima problem with linear equality constraints and non-negative constraints. Its equivalence and uniqueness are proved in reference [5]. This article uses the MSA algorithm [5] to solve the user equilibrium model. The specific steps of the algorithm are as follows:

Step 1. Initialization. It is assumed that the flow of each spatiotemporal arc in the space-time network is zero. The minimum cost path is obtained according to the improved Dijkstra algorithm. Then all-no-all allocation is performed, and the traffic \(x_{(i,j)}\) on each spatio-temporal arc is obtained. Let the number of iterations \(n=1\).

Step 2. Update the impedance. The cost function of each path is recalculated according to the current flow on each time and space arc obtained in step 1.

Step 3. Find a workable direction. According to the current path costs, the new Dijkstra algorithm is used to search for the new minimum cost path, and all-no-all allocation is performed to obtain traffic \(y_{(i,j)}\) on each spatio-temporal arc.

Step 4. Update the traffic. Calculate the new spatio-temporal arc flow according to the following formula:

\[
x_{(i,j)}^{n+1} = x_{(i,j)}^{n} + \left(y_{(i,j)}^{n} - x_{(i,j)}^{n}\right)/n, \forall (i, j)
\]

Step 5. Convergence judgment. If satisfied

\[
\left(\sum_{(i,j)} \left(x_{(i,j)}^{n+1} - x_{(i,j)}^{n}\right)^2 / \sum_{a} x_{(i,j)}^{n}\right) \leq \varepsilon
\]

The algorithm ends; otherwise, let \(n=n+1\), go to step 2.

In the space-time network, the traditional shortest path search algorithm does not consider the transfer constraint conditions of the transfer station node. Therefore, this paper improves on the Dijkstra algorithm. The steps are as follows:

Step 1. Permanently label the starting point \(r\) with \(U\), and let \(U_r=0\); Give other points a temporary label \(R\), \(R_i = +\infty\). U and R represent the generalized costs of the node. Let the number of transfers \(e=0\).

Step 2. Let \(i\) be the point for which the permanent label \(U\) has just been obtained. Look for all points in the set that have the \(R\) label and set it to \(j\):

1. If \(i\) is the starting point, then it is judged whether \(R_j \geq U_i + p_k^i\) holds, and if so, let \(R_j = U_i + p_k^i, e_j = e_i = 0\);
2. If \(i\) is not a starting point and does not need to transfer, determine whether \(R_j \geq U_i + T_{ij}\) holds. If it is, let \(R_j = U_i + T_{ij}, e_j = e_i + 1\);
3. If \(i\) is a transfer point and \(e_i \leq 3\), determine whether \(R_j \geq U_i + T_{ij}\) holds. If it is, let \(R_j = U_i + T_{ij}, e_j = e_i + 1\).

Step 3. Check all the points with \(R\) labels, give them a permanent label \(U\) with the smallest \(R\) value, set it to \(i\), let \(R_i = U_i\), if \(i\) is not the end, return to step 2; otherwise, the algorithm ends.

5. Analysis of examples

The example will use the simple subway network shown in Figure 1 to illustrate the effect of fare on passenger flow distribution in this paper. The timetable information has been identified in Figure 2 above. In order to consider the path containing secondary transfer, this article adds a morning and evening peak. Among them, the paths start at 6:40 and 6:50 corresponding to early low peaks, and at 7:00 and 7:10 corresponding to early morning peaks. Assume that the same OD demand is 1000, the initial fare is ¥5, and the path cost parameters \(\alpha, \beta, 0, \omega, \eta, \lambda_1\) and \(\lambda_2\) are as shown in Table 1.
Table 1. Parameter presets

| Parameter | α | β | Z | C | θ | ω | η₁ | η₂ | λ₁ | λ₂ |
|-----------|---|---|---|---|---|---|----|----|----|----|
| value     | 0.1 | 2.0 | 80 | 150 | 1.3 | 4 | 0.6 | 0.3 | 0.3 | 1.2 |

Based on the above OD demand and corresponding parameters, the above-mentioned equalized allocation model and algorithm are used to average the passenger flow distribution of the subway. The calculation can obtain the number of passengers in each interval and the passenger flow and expenses on each route.

Firstly, the convergence of the algorithm is analyzed. According to the spatio-temporal network diagram shown in Fig. 2, the convergence of the algorithm is proved by the flow rate changes of some space-time arcs such as 7:11-7:16 and 7:10-7:32 for each iteration. As can be seen from Figure 3, the MSA algorithm has a good convergence for solving the equilibrium assignment problem.

![Algorithm convergence](image)

Figure 3. Algorithm convergence

Since the effect of fare on passenger flow is mainly discussed in the article, the flow on the arc is mainly considered in the flow distribution of spatio-temporal arc. According to the model, the flow distribution results are shown in Table 2.

Table 2. Effect of Fare Adjustment on Flow Distribution

| Adjust the fare                  | O-7:10 | O-7:00 | O-6:50 | O-6:40 |
|----------------------------------|--------|--------|--------|--------|
| 0(not adjusted)                  | 272    | 271    | 236    | 221    |
| -¥1.5(low peak price reduction)  | 255    | 256    | 251    | 238    |
| +¥1.5, -¥1.5(peak price increase, low peak price reduction) | 240    | 249    | 260    | 247    |

As can be seen from Table 2, comparing the price without adjusting the fare, adjusting the fare has an impact on the passenger flow. Among them, peak price increase measures with low peak price cuts have the greatest effect on flow regulation; followed by low peak price reductions, which will attract some passengers to the low peak period. This proves that the high and low peak time-share pricing based on the dynamic space-time network of subways is effective for passenger flow regulation and is feasible.

6. Conclusion

In this paper, a subway dynamic network considering the fare factor is constructed based on the subway space-time network, the path cost is defined, and a user equilibrium model for describing the fare is established. Based on this, an improved shortest path search algorithm is proposed. Finally, an algorithm is used to demonstrate the convergence and feasibility of the algorithm. At the same time, the regulation effect of fare on passenger flow is proved from the perspective of metro space-time network.

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