Highlights

The Optics of Language-Integrated Query*

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- Getter, Affine Fold and Fold optics are lifted into Optica, a query language for LINQ
- XQuery and SQL queries derived from non-standard optic representations
- Optics as a higher-level interface over comprehension-based query languages
- Typed tagless-final encoding in Scala of the Optica type system and semantics
The Optics of Language-Integrated Query

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Abstract
Monadic comprehensions reign over the realm of language-integrated query (LINQ), and for good reasons. Indeed, comprehensions are tightly integrated with general purpose programming languages and close enough to common query languages, such as SQL, to guarantee their translation into effective queries. Comprehensions also support features for writing reusable and composable queries, such as the handling of nested data and the use of functional abstractions.

In parallel to these developments, optics have emerged in recent years as the technology of choice to write programs that manipulate complex data structures with nested components. Optic abstractions are easily composable and, in principle, permit both data access and updates. This paper attempts to exploit the notion of optic for LINQ as a higher-level language that complements comprehension-based approaches.

In order to do this, we lift a restricted subset of optics, namely getters, affine folds and folds, into a full-blown DSL. The type system of the resulting \textit{language of optics}, that we have named \textit{Optica}, distills their compositional properties, whereas its denotational semantics is given by standard optics. This formal specification of the concept of optic enables the definition of non-standard optic representations beyond van Laarhoven, profunctor optics, etc. In particular, the paper demonstrates that a restricted subset of XQuery can be understood as an optic representation; it introduces Triplets, a non-standard semantic domain to normalize optic expressions and facilitate the generation of SQL queries; and it describes how to generate comprehension-based queries from optic expressions, thus showing that both approaches can coexist.

Despite the limited expressiveness of optics in relation to comprehensions, results are encouraging enough to anticipate the convenience and feasibility of extending existing comprehension-based libraries for LINQ in the functional ecosystem, with optic capabilities. In order to show this potential, the paper also describes S-Optica, a Scala implementation of Optica using the tagless-final approach.

Keywords:
optics, language-integrated query, type systems, comprehensions, typed tagless-final, Scala
1. Introduction

The research field of language-integrated query (LINQ [11, 2, 3]) aims at alleviating the impedance mismatch problem [4, 5] that commonly originates in software systems where general-purpose programming languages, on the one hand, and query languages, on the other, need to interoperate. The problem manifests itself in the form of maintenance, reliability and security problems, which are essentially due to the mismatches of programming paradigms and data models endorsed by the interacting languages. In order to tackle this issue, the LINQ research field favors a domain-specific language (DSL)-based approach [6]. From this perspective, the programmer does not simply inject query expressions in the general-purpose language as plain strings, a practice which is a well-known source of bugs and injection attack problems; on the contrary, she uses a DSL which ensures that the query is well-formed, correctly typed, and moreover, that it helps to overcome the conceptual gap between both the general-purpose and the query language.

Indeed, not every DSL can be given the seal of approval from a language-integrated perspective. For instance, we may embed SQL in a host language like Scala [7] to attain the stated demand of type safety and yet, the disparity between the flat and nested computational models from both languages would not be reduced in the slightest. This is, without a shadow of a doubt, a necessary step in order to generate well-formed SQL queries. Scala libraries such as Doobie [8], which focus on this specific issue, are worthwhile. However, to properly bridge the impedance mismatch gap, we need DSLs at a higher level of abstraction: close enough to general-purpose languages, yet specific enough to allow for the efficient generation of queries for a wide range of querying languages [9]. Since early in its foundation, the LINQ research field has exploited monadic comprehensions [10, 11] as its DSL of choice for this purpose. The basic insight was originally introduced in [12], and then developed by the Nested Relational Calculus (NRC) [13, 14], which provides the foundation of query languages based on comprehensions. NRC subsumes much work on LINQ theories and systems such as in Kleisli [15], Links [16], Microsoft’s LINQ [2, 17], Database Supported Haskell (DSH) [18], T-LINQ [3], QUEΛ [19] and SQUR [20].

In essence, the purpose of research on LINQ is borrowing the comprehension syntax that in-memory data structures such as lists, sets, bags, and other bulk types enjoy in order to express queries at a generic monadic level. To this aim, bulk types are lifted into a proper DSL that abstracts away its characteristic in-memory representation but still allows to express queries using comprehension syntax. In some cases, as in Kleisli, Links and Microsoft’s LINQ, this lifting mechanism is a primitive part of the general-purpose language itself. In the case of more conventional, functional programming (FP) languages, such as Scala, F#, OCaml, Haskell, etc., the DSL may be embedded in the host language using one of the several techniques that FP offers for this purpose: typed tagless-final [21], generalized algebraic data types (GADTs) [22] or quoted domain-specific languages (QDSLs) [23]. For instance, quotation is used to embed the T-LINQ language in F# [3], while typed tagless-final inspires the design of QUEA, in OCaml. Similarly, Quill [24] is a QDSL heavily inspired by T-LINQ, which is embedded in Scala.

To illustrate the use of comprehensions in LINQ, we consider the data structures in column “comprehensions” of Table 1, implemented in the Scala programming language – our language of choice throughout this paper. According to this model, a couple consists of two members, first

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1 The entities of a flat model are either base types or classes with no nested collections. Otherwise, we say that the model is nested.

2 Appendix A offers a brief account of the major Scala features that are used in this paper.
and second, to which we refer by their names; besides names, each person has also an age. Given
a list of couples and a list of people, we may obtain the names of those partners who occupy
the first position and are under 50 years of age by using a list comprehension, as query under50_a
shows. Now, using Slick [25] or Quill [24], two well-known libraries of the Scala ecosystem,
we may express the very same query in a generic way. To be precise, we call a query generic if
it can be efficiently run (e.g. via appropriate compilation) against data stores of different sorts:
in-memory, external relational databases, non-relational stores such as XML/JSON files, etc. For
instance, the query under50_b in Table 1 is a generic query implemented in Quill. Being generic,
this query may be compiled to different targets according to the mappings between Scala case
classes and database schemas that the Quill framework supports (as of this writing Cassandra’s
CQL [26] and SQL). For instance, the resulting SQL expression generated for this query by Quill
would be as follows:

```sql
SELECT w.name
FROM Couple AS c INNER JOIN Person AS w ON c.fst = w.name
WHERE w.age < 50
```

We have illustrated the use of comprehensions with a simple example of the so-called flat-flat
query, i.e. a query that receives and returns flat types. These queries are significant because rela-
tional databases cannot handle nested results. For instance, we cannot write an SQL query that
returns rows in which a field contains a list of values. This demonstrates a significant mismatch
between SQL and programming languages, where nested data models are customary. More-
ever, comprehension queries, being founded on NRC, can perfectly well handle nested data,
which might lead us to think about wasted expressiveness. In fact, the opposite is true: several
conservativity results show that we can certainly use nested data as intermediate values in com-
prehensions [27], even in the presence of parameterized queries (i.e. using lambda expressions)
[16] and still be able to generate normalized queries which do not use nested data in any way. We
can even accommodate flat-nested queries through several flat-flat normalized queries by using
techniques for query shredding [28]. In sum, comprehensions are exceptionally good from a
LINQ perspective: well integrated to a wide range of programming languages and close enough
to relational databases in order to generate effective queries.
In spite of this, we find three major problems or inconveniences in the current use of comprehensions for LINQ. First, comprehensions can only express retrieval queries but updates are equally important. This is acknowledged as an open problem in the LINQ field \[9\]. Second, the use of nested data and functional abstraction in comprehension-based languages such as Links/T-LINQ undoubtedly helps in obtaining more compositional queries \[3\]. However, this is done at the expense of a complex re-writing machinery, specially in the case of QDSLs like T-LINQ. Alternative approaches based on normalization-by-evaluation \[20\] ameliorate this problem, but the support for compositional queries is nevertheless limited. Basically, this is due to the fact that comprehensions are nearer to the point-wise notation exemplified by the relational calculus than to pure relational algebra and have to deal with variable (re)namings, freshness and scope. Functionally, both formalisms are equally expressive, but the point-free combinators of relational algebra are arguably more flexible \[29\]. This flexibility naturally derives in more modular queries, which directly impacts non-functional concerns such as reuse and change tolerance \[30\] \[31\]. Third, there are potential querying infrastructures which are essentially hierarchical rather than relational, such as NoSQL document-oriented databases, which build upon nested data sources in JSON, XML or YAML. The translation of queries at the programming language level into these infrastructures may benefit from a more primitive, algebraic querying model, which is hierarchical by nature.

This paper attempts to show that optics \[32\] may play this role in the realm of language-integrated query, and it supports a pure algebraic approach to LINQ that may potentially be extended for dealing with updates. Indeed, optics, also known as functional references, are abstractions that select parts which are contextualized within a whole, which provide methods to access and/or update the values that they are selecting. Since the first appearance of lens \[33\], arguably the most prominent optic, a rich catalog of optics has emerged \[34\]. They can compose with each other, with a very few exceptions, so that they can seamlessly produce quite complex transformations over immutable nested data structures. Indeed, they have become an essential companion for the functional programmer, as evidenced by the growing popularity of optic libraries in the functional ecosystem \[35\] \[36\]. In sum, we may say that optics are the de-facto standard to manipulating nested data in a point-free, algebraic style; they are as ubiquitous as comprehensions in functional programming languages and, more importantly, the most common variants are explicitly designed to handle both read and write accessors.

How do we use optics as a higher-level language to express generic queries? How do these optic-based queries relate to generic comprehension queries? How do we translate optic expressions to SQL/NoSQL query languages directly? To answer these questions, we may follow the same strategy that is illustrated in Table 1 for comprehension queries; by using lenses \[33\], traversals \[37\], folds, and other optic abstractions, we can query and update immutable data structures quite naturally; why not borrow this very same syntax to express generic queries that can be interpreted over different target storage systems? For instance, in column “optics” of Table 1 an alternative nested model is defined for the couples example, where keys in Couple are replaced by full-blown Person values. Then, query under50_c, derived from optic under50 Fl, which in turn is composed from several optics (the fold couples, and getters her, age and name), offers an alternative formulation to the comprehension query under50_a. What we are looking for is a generic optic-based query, analogous to the comprehension-based query under50_b. In essence, we need to lift optics into a full-fledged DSL.

Contributions and outline. This paper sets out to define the language of optics which we have dubbed Optica. We aim at showing that Optica may play the role of an effective query language
for LINQ, alone and in combination with comprehension queries, albeit at the expense of limited expressiveness. In general, we aim to prove that optics offer a fruitful abstraction for LINQ, and restrict our attention to proving the feasibility of this approach on a selected subset of optic abstractions and domain examples. In particular, these are our contributions:

- A review of concrete optics from the mindset of LINQ. We show how to exploit a subset of standard optic abstractions and their combinators in order to express compositional queries (Sect. 2). We focus exclusively on read-only optics, i.e. those which select parts from the whole but do not write back in the data structure, namely, getters, affine folds and folds. This allows us to focus on a tractable subset of optics, and to prepare the ground to tackle more ambitious problems in future work, such as the modeling of updates in LINQ.

- A formal specification of read-only optics in terms of Optica, a full-blown DSL. The syntax and type system of the language formalize their compositional and querying features in an abstract way. Its denotational semantics is given by concrete optics themselves (Sect. 3). We show how to implement generic queries over abstract optic models by using Optica in a declarative way, once and for all.

- The abstract specification of read-only optics provided by Optica enables the definition of alternative, non-standard optic representations. We provide three major Optica interpretations which attempt to show the capabilities of Optica as a general query language for LINQ:
  - An XQuery interpretation, that allows us to directly translate Optica queries into XQuery [38] expressions (Sect. 4). This aims at showing the adequacy of Optica for dealing with common data sources of NoSQL document-oriented databases.
  - A SQL interpretation, that generates SQL queries from Optica expressions (Sect. 5). This non-standard denotational semantics builds upon Triplets, a semantic domain which normalizes the optic expression in order to facilitate its direct translation to SQL. The proposed semantics works similarly to the normalization-by-evaluation approach of SQR [20]. The major difference is that SQR consists of a relational calculus whereas we work over optics, which are more akin to relational algebra.
  - A T-LINQ interpretation, that generates comprehension queries. This non-standard semantics is aimed at showing how to use Optica as a higher-level language for nested data in conjunction with comprehension-based languages (Sect. 6).

- S-Optica, an embedded DSL implementation of Optica in Scala using the tagless-final approach (Sect. 7). This implementation is intended as a proof-of-concept for illustrating how to implement the formal Optica type system and semantics in a common, general purpose programming language of the software industry. It also aims at providing an example of the tagless final approach, as well as serving as a reference for extending existing libraries for LINQ with optic capabilities.

As can be seen, Sects. 2-7 contain the bulk of the paper. Section 8 discusses related work and limitations of the approach. Finally, section 9 concludes and points towards current and future work. The Scala library that accompanies this paper is publicly available on a Github repository [39].
2. Querying with Optics

This section introduces three different kinds of read-only optics: getters, affine folds and folds \[3\] together with their main combinators, where we use Scala as the vehicle to implement them. In essence, read-only optics are just views without updates, and hence they are not subject to the familiar optic laws \[34\]. They are not as widespread as their siblings with updating capabilities (namely, lenses, affine traversals and traversals), given that selecting nested fields from immutable data structures is usually a trivial task. Nonetheless, they exhibit the same compositional features and patterns as the rest of optics, and will thus allow us to illustrate the general declarative querying style advocated by them. The abstractions and examples that we put forward in this section will be used throughout the paper.

2.1. Getters, Affine Folds and Folds

First of all, it is worth noting that we choose the concrete optic representation, where the notions of whole and part are clear, in order to make definitions easier to understand. There are other representations, such as van Laarhoven \[37\] or profunctor \[32\], that implement optic composition in a remarkably elegant way, whose signatures, however, are not as easy to approach for an outsider.\[5\]

**Definition 1 (Getter).** A getter consists of a function that selects a single part from the whole.\[6\]

We encode it in Scala as follows:

```scala
case class Getter[S, A](get: S => A)
```

The type parameters \( S \) and \( A \) will consistently serve as the whole and the part, respectively, throughout the different optic definitions.

There are several getter combinators that will be used frequently in the text that have been collected in the companion object for `Getter`, which is shown in Fig. 1. The `andThen` method combines getters that are selecting nested values in order to produce a new getter that selects a deeply nested value. The getter `id` is the neutral component under the `andThen` composition, where whole and part do coincide. The `fork` combinator is required if we wish to put different parts together. The `like` Combinator selects a constant part which is taken as parameter, where the whole is ignored. The remaining combinators essentially lift arithmetic operations into functions that take getters selecting operands as parameters and produce a getter that selects the operation result.

**Remark 1.** We assume \(\gg\) and \(\ggg\) as infix versions of `andThen` and `fork`, respectively, where the last symbol has precedence over the first. Similarly, we will overload the operators `===`, `>` and `-` as infix versions of `equal`, `greaterThan` and `subtract`, respectively. Last, we will use the postfix expression `p.not` as an alias for `not(p)`.

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\[3\] We ignore other read-only optics such as `fold1`, since they do not add value in the particular examples that we have selected for this paper.

\[4\] Appendix A provides a Scala cheat sheet that describes the most fundamental Scala abstractions in the context of this work.

\[5\] As evidenced by the jokes around this topic in the functional programming community https://pbs.twimg.com/media/CypY7S1W8AAvqwl.jpg

\[6\] A concrete lens is basically a getter plus a function to update the whole from a new version of the part.
object Getter {
  def id[A]: Getter[A, A] = Getter(a => a)
  def andThen[S, A, B](u: Getter[S, A], d: Getter[A, B]): Getter[S, B] = Getter(s => d.get(u.get(s)))
  def fork[S, A, B](l: Getter[S, A], r: Getter[S, B]): Getter[S, (A, B)] = Getter(s => (l.get(s), r.get(s)))
  def like[S, A](a: A): Getter[S, A] = Getter(const(a))
  def not[S](b: Getter[S, Boolean]): Getter[S, Boolean] = b ≫ Getter(!_)
  def equal[S, A: Equal](x: Getter[S, A], y: Getter[S, A]): Getter[S, Boolean] = Getter(s => x.get(s) === y.get(s))
  def greaterThan[S](x: Getter[S, Int], y: Getter[S, Int]): Getter[S, Boolean] = Getter(s => x.get(s) > y.get(s))
  def subtract[S](x: Getter[S, Int], y: Getter[S, Int]): Getter[S, Int] = Getter(s => x.get(s) - y.get(s))
}

Figure 1: Getter Combinators.

Remark 2. Fork-like optic composition (we will also refer to it as horizontal composition\(^7\)) is not widespread in the folklore. Indeed, it is not possible to implement it in a safe way for most optics. For example, an analogous implementation for composing lenses horizontally would violate lens laws\(^40\) when both lenses select the very same part.

Definition 2 (AffineFold). An affine fold consists of a function that selects at most one part from the whole. We encode it as follows:

```scala
case class AffineFold[S, A](preview: S => Option[A])
```

We could see this optic as a simplification of an affine traversal, where we omit the updating part.

Once again, we have packaged several affine fold combinators in Fig.\(^2\). The identity affine fold simply selects the whole value and wraps it in a Some case. The andThen combinator selects the innermost value just in case both optics \(u\) and \(d\) denote defined values. Otherwise, it will select nothing. We implement this functionality in terms of the Option monad using for-comprehension syntax. Finally, we consider filtered as an interesting builder of affine folds, which declares the same types for whole and part. It just discards the value (returning None) in case it is actually pointing to something and the input predicate does not hold for it.

Remark 3. It is worth emphasizing that the predicate that filtered takes as parameter is a getter itself. This is unusual in folklore libraries, where a plain lambda expression is taken as an argument instead. However, predicates can be perfectly understood as queries (getters, in particular). We will exploit this idea in the next section to avoid introducing lambda terms in the Optica DSL.

\(^7\)And consequently, we will refer to andThen as vertical composition.
object AffineFold {

  def id[A]: AffineFold[A, A] = AffineFold(a => Some(a))

  def andThen[A, B, C](u: AffineFold[A, B],
                          d: AffineFold[B, C]): AffineFold[A, C] =
    AffineFold(s =>
      for {
        b ← u.preview(s)
        c ← d.preview(b)
      } yield c)

  def filtered[S](p: Getter[S, Boolean]): AffineFold[S, S] =
    AffineFold(s => if (p.get(s)) Some(s) else None)

  implicit def toAf[S, A](g: Getter[S, A]): AffineFold[S, A]
    = AffineFold(s => Some(g.get(s)))
}

Figure 2: Affine Fold Combinators.

Remark 4. One of the major benefits of optics is that they compose heterogeneously\(^8\) in other words, it is possible to combine getters, affine folds and folds among them. To put it simply, we can turn getters into affine folds and we can turn affine folds into folds. An example of such casting is shown in Fig. 2 (\(\text{toAf}\)), where we find it implemented as an implicit converter. Thereby, the Scala compiler applies the conversion automatically when it detects a getter where an affine fold is expected instead.

Definition 3 (Fold). A fold consists of an optic that selects a (possibly empty) sequence of parts from the whole.

case class Fold[S, A](getAll: S => List[A])

We could see this optic as a simplification of a traversal\(^9\), where we omit the updating part.

As usual, we have packaged the fold primitives in the corresponding companion object, which can be found in Fig. 3. The implementation of \(\text{id}\) and \(\text{andThen}\) is basically the same as the one we showed for affine folds, the difference being that we work with the \(\text{List}\) monad instead of the \(\text{Option}\) one\(^4\). The \(\text{nonEmpty}\) method takes a fold as a parameter and produces a getter that checks whether the number of selected parts is equal to zero. The remaining combinators (\(\text{empty}\), \(\text{all}\), \(\text{any}\) and \(\text{elem}\)) are just derived definitions, which are implemented in terms of other combinators, where we assume that object-oriented \(\text{dot}\) syntax is available. For instance, \(\text{nonEmpty(fl)}\) becomes \(\text{fl.nonEmpty}\) and \(\text{all(fl)}(p)\) becomes \(\text{fl.all(p)}\).

The implementation of \(\text{elem}\) might deserve further explanation. Since we favor getters over plain functions as predicates (as stated in Remark 3), we need to use optic abstractions and combinators to build them. The following is a derivation where we start with an implementation that we consider natural —which requires lambda expressions— and we end up with the standing implementation —where lambda expressions are removed.

\(^8\)With a very few exceptions, which are beyond the scope of this paper.
\(^9\)Similarly, we can have getters following the same pattern by using the \(\text{id}\) monad, but we avoid doing this for brevity.
object Fold {
    def id[A]: Fold[A, A] = Fold(a => List(a))
    def andThen[A, B, C](u: Fold[A, B], d: Fold[B, C]): Fold[A, C] =
        Fold(s =>
            for {
                b ← u.getAll(s)
                c ← d.getAll(b)
            } yield c)
    def nonEmpty[S, A](fl: Fold[S, A]): Getter[S, Boolean] =
        Getter(fl.getAll(_).nonEmpty) /* List.nonEmpty */
    def empty[S, A](fl: Fold[S, A]): Getter[S, Boolean] =
        fl.nonEmpty.not
    def all[S, A](fl: Fold[S, A])(p: Getter[A, Boolean]): Getter[S, Boolean] =
        (fl ≪ filtered(p.not)).empty
    def any[S, A](fl: Fold[S, A])(p: Getter[A, Boolean]): Getter[S, Boolean] =
        fl.all(p.not).not
    def elem[S, A: Equal](fl: Fold[S, A])(a: A): Getter[S, Boolean] =
        fl.any(id === like(a))
    implicit def to[A](a: AffineFold[S, A]): Fold[S, A] =
        Fold(s => a.preview(s).toList)
}

Figure 3: Fold Combinators.

fl.any(Getter(s => s === a))
  ≃ [definition of 'id' getter]
fl.any(Getter(s => id.get(s) === a))
  ≃ [definition of 'like' getter]
fl.any(Getter(s => id.get(s) === like(a).get(s))
  ≃ [definition of 'equal' getter]
fl.any(id === like(a))

Note that === is overloaded. In fact, the occurrence of this method in the last line corresponds to the equal Combinator from getters, while the rest refer to the standard comparison method from the Equal type class.

Remark 5. As we have seen throughout this section, read-only optics are essentially functions that select parts from the whole, but we have introduced them as separated definitions. This distinction between functions and optics turns out to be central in this work, since Optica expressions denoting one or the other can be evaluated in very different ways, as we will see through Sects. 3-6.

2.2. Composing Optics and Running Queries

Once we have seen several standard combinators and some interesting features from optics, we will exercise them to illustrate the querying style and common patterns advocated by optics. For this task, we have selected two examples from \[3\]\(^{10}\) which will be used throughout the paper.

\(^{10}\)The first example has been slightly updated in order to adapt it to today’s society.
2.2.1. Couples Example

This example extends the one which was introduced in Table 1. Remember that it consists of a simple relation of couples, where the name and age of each person forming them is also supplied:

```scala
type Couples = List[Couple]
case class Couple(fst: Person, snd: Person)
case class Person(name: String, age: Int)
```

The associated data structures are defined following a nested, rather than a relational approach, i.e. couples contain a full person value, rather than a person key. This distinction becomes essential in Sect. 6. Once we have defined the model, we provide CoupleModel-specific optics to select relevant parts from the domain.

```scala
object CoupleModel {
  val couples: Fold[Couples, Couple] = Fold(identity)
  val fst: Getter[Couple, Person] = Getter(_.fst)
  val snd: Getter[Couple, Person] = Getter(_.snd)
  val name: Getter[Person, String] = Getter(_.name)
  val age: Getter[Person, Int] = Getter(_.age)
}
```

Basically, and for this particular example, we supply a getter for each field, where whole and part correspond to data and field types, respectively. The Scala placeholder syntax is used in these definitions. There is also a simple fold that we can use to select each couple from Couples, that we see as the root type in the nested model.

**Remark 6.** The examples that are presented in this paper do not include affine folds as part of the domain models, but they could be helpful to model optional values. For instance, we could use them to consider an optional address field associated to each person.

Now, we can use the standard optics defined in the previous section and the specific optics defined for this domain to compose new ones. For instance, the following fold selects the name and age difference from all those couples where the first member is older than the second one.

```scala
val differencesFl: Fold[Couples, (String, Int)] =
  couples ≫
  filtered((fst ≫ age) > (snd ≫ age))
  ≫
  (fst ≫ name) ***
  ((fst ≫ age) - (snd ≫ age))
```

Firstly, we use `couples` as an entry point and we use `filtered` to remove the couples in which the age of the first member is not greater than the age of the second one. Right after filtering, we select the name of the first member and we put it together with the age difference, by means of `***`, to determine the subparts that the optic is selecting.

Once we have defined the fold, we need to generate the query that selects the corresponding information from the immutable structure, i.e. a function that takes the couples as argument and returns the matching values. For this task, we simply use `getAll`.

```scala
val differences: Couples => List[(String, Int)] =
  differencesFl.getAll
```

If we feed this query with the same data that was used in the original example, we should expect the same result.

```scala
val data: Couples = List(
  Couple(Person("Alex", 60), Person("Bert", 55)),
  Couple(Person("Cora", 33), Person("Demi", 31)),
  Couple(Person("Eric", 21), Person("Fred", 60)))
val res: List[(String, Int)] = differences(data)
// res: List[(String, Int)] = List((Alex,5), (Cora,2))
```
The comment in the last line of the snippet shows the value that we get in `res` when we run the query with the original data. As expected, it indicates that Alex and Cora are older than their mates by 5 and 2 years, respectively.

### 2.2.2. Organization Example

We move on to the next example, where our model is an organization which is formed by employees. In addition, each employee has a set of tasks that she is able to perform.

```scala
type Org = List[Department]
case class Department(dpt: String, employees: List[Employee])
case class Employee(emp: String, tasks: List[Task])
case class Task(tsk: String)

Once again, we supply `OrgModel`-specific optics to select relevant parts from the domain:

```scala
object OrgModel {
  val departments: Fold[Org, Department] = Fold(identity)
  val dpt: Getter[Department, String] = Getter(_.dpt)
  val employees: Fold[Department, Employee] = Fold(_.employees)
  val emp: Getter[Employee, String] = Getter(_.emp)
  val tasks: Fold[Employee, Task] = Fold(_.tasks)
  val tsk: Getter[Task, String] = Getter(_.tsk)
}
```

In this case, we find several fields containing lists, thus, we provide folds instead of getters to deal with sequences of parts. Now, we compose a fold to select the name of those departments where all employees know how to `abstract`.

```scala
def expertiseFl: Fold[Org, String] = 
  departments ≫ filtered(employees.all(tasks.elem("abstract"))) ≫ dpt
```

This expression refers first to all departments, and then it filters the ones where all employees contain the task "abstract". Finally, it selects their textual identifier (`dpt`). Once the fold is defined, we produce the query to obtain the selected departments:

```scala
def expertise: Org => List[String] =
  expertiseFl.getAll
```

Once more, we feed the query with the original organization’s data.

```scala
val data: Org = List(
  Department("Product", List(
    Employee("Alex", List(Task("build"))),
    Employee("Bert", List(Task("build"))))),
  Department("Quality", List.empty),
  Department("Research", List(
    Employee("Cora", List(Task("abstract"), Task("build"), Task("design"))media, Task("call"), Task("design"))))),
  Department("Sales", List(
    Employee("Demi", List(Task("abstract"), Task("design"))),
    Employee("Eric", List(Task("abstract"), Task("call"), Task("design"))))),
  Department("Fred", List(Task("call"))))

val res: List[String] = expertise(data)
// res: List[String] = List(Quality, Research)
```

The resulting value shows that the departments of **Quality** and **Research** are the only ones where all employees are able to `abstract`.

The general pattern should be clear now. Firstly, we define the involved data types in the model and supply specific optics to select their parts. Secondly, we use these optics and the standard ones to express more sophisticated selectors in a modular and elegant way. Finally, we run the optic method with an initial whole to produce the expected query. As it can be seen, the approach is eminently declarative: the aspects of composing the desired optic and running it are completely decoupled.
Remark 7. We will refer to \texttt{get}, \texttt{preview} and \texttt{getAll} as the \textit{queries} derived from their corresponding optic types. Read-only optics just supply basic reading queries, in a one-to-one mapping. Thereby, the separation of concerns between optic expressions and generated queries is not as clear as in other optics. For instance, lenses broaden their catalog of derived queries with queries to read, replace or even modify the part that the lens is selecting. We further discuss the implications of this insight in Sect. 8.

3. Optica Core

The last section has introduced optics using their so-called \textit{concrete} representation, but the same querying style is actually supported by other isomorphic representations, such as van Laarhoven or profunctor optics. This section aims at \textit{specifying} the concepts of getters, affine folds and folds, in a generic way, independently of any particular \textit{representation}. The resulting formalization is a domain-specific language that we have named \textit{Optica}, which directly supports the optic querying style, and potentially allows for non-standard optic representations that generate queries in XQuery or SQL, for instance, as the next sections will show.

This section will first introduce the syntax and type system of Optica, where standard primitives and combinators are declared. Secondly, we will show how to provide a generic version of the models and queries that we have seen in Sect. 2.2. Finally, we will present the standard semantics that we can use as an interpretation to deal with immutable data structures: they recover concrete optics and queries, as introduced in the last section.

3.1. Syntax and Type System

We introduce the syntax of Optica in Fig. 4. The upper part contains the model types (natural numbers, boolean, string and product), optic types (getters, affine folds and folds) and query types (selection functions). The second part shows the set of expressions that form the language, which are defined in close correspondence with their concrete counterparts, introduced in the previous section. Despite that, there are several aspects which deserve further explanation:
```python
def empty fl = nonEmpty fl \Rightarrow not
def all fl p = empty (fl \Rightarrow filtered (not p))
def any fl p = not (all fl (not p))
def elem fl u = any fl (id == like u)
```

Figure 5: Optica derived definitions

- Constants are not valid query expressions on their own. As we will see later, we use `like` as the mechanism to represent literals in the language. By doing so, we can reuse optic combinators for constants, improving the language composability.
- The formal syntax avoids the object-oriented `.` notation, idiomatic in Scala, and favours the prefix notation, as is usual practice in related work.
- The methods `all`, `any`, `elem` and `empty` are not included as syntax primitives. Instead, they are introduced as derived definitions, as can be seen in Fig. 5.
- At present, query expressions are atomic, i.e. it is not possible to compose several of them.

The type system is presented in Fig. 6, where \( \alpha \), \( \beta \) and \( \gamma \) represent model types (see Fig. 4). Unlike T-LINQ [9] or QUEA [20], Optica does not introduce terms for variables. Thereby, its type rules are slightly simplified, since they omit the characteristic `\( \Gamma \vdash \)` prefix. They are structured into four groups, corresponding to getters, affine folds, folds, and their derived queries. The only optic constructors are `id`, and `like`, which allow us to form new optic expressions from scratch. The remaining combinators should be straightforward, since they exactly correspond with the ones introduced in the companion objects for getters, affine folds and folds of Sect. 2.1

In regard to queries, note that their type rules do not proceed from the companion objects, but from the case class definitions of concrete optics themselves. Their formalization leads to introducing functions as a new semantic domain for Optica. However, note that the part of the language corresponding to optics is purely first-order, i.e. no lambdas are needed in order to create optic expressions.

3.2. Core Extensions and Generic Queries

As we have seen in Sect. 2, we defined domain optics to model the couple and organization examples. Now, we want to do the same, but in a general way. To do so, we need to extend the core language from Sect. 3.1 with new primitives, specific to the particular domain. We present them, along with their associated type rules, in Fig. 7. As can be seen, it introduces the entity types (`Couple` and `Person`) and a term for each optic introduced in Sect. 2.2.1. The type rules just determine the type associated to each term, i.e. the kind, whole and part associated to each optic.

Once we have defined the Optica language primitives (where we place the standard optics and combinators) and the domain extension (where we find the structure of the domain data

\[^{12}\text{With the exception of those combinators, like}\ any,\ all,\ elem\ \text{and}\ empty,\ \text{which can be defined compositionally in terms of other combinators and do not need to access the internal optic representation.}\]
Figure 6: Optica type system
model in terms of specific optics), we should be able to provide generic domain queries. We adapt differences (Sect. 2.2.1) as follows:

**Definition 4.** The generic versions for \texttt{differencesFl} (optic expression) and \texttt{differences} (query expression) are implemented as follows, in terms of the Optica and couple-specific primitives.

\[
\texttt{def differencesFl} = \\
\texttt{couples} \gg (\texttt{filtered} ((\texttt{fst} \gg \texttt{name}) > (\texttt{snd} \gg \texttt{name}))) \\
\texttt{toFl} ((\texttt{fst} \gg \texttt{name}) +++ ((\texttt{fst} \gg \texttt{age}) – (\texttt{snd} \gg \texttt{age})))
\]

\[
\texttt{def differences} = \\
\texttt{getAll differencesFl}
\]

The implementation of the generic versions of \texttt{differencesFl} and \texttt{differences} are basically the same as the ones we introduced in Sect. 2.2.1—modulo the differences that we listed in Sect. 3.1 and the fact that the invocations to casting methods such as \texttt{toFl} and \texttt{toAf} are made explicit.

We can carry out the same exercise for the organization example (Sect. 2.2.2). Once again, we introduce a language extension containing the entity types and terms associated to this example (Fig. 8). Once we do that, we are able to introduce a generic counterpart for the expertise query.
\[ T[N] = \text{Int} \]
\[ T[B] = \text{Boolean} \]
\[ T[S] = \text{String} \]
\[ T[(\alpha, \beta)] = (T[\alpha], T[\beta]) \]
\[ T[\text{getter } \alpha \beta] = \text{Getter}[T[\alpha], T[\beta]] \]
\[ T[\text{affine } \alpha \beta] = \text{Affine}[T[\alpha], T[\beta]] \]
\[ T[\text{fold } \alpha \beta] = \text{Fold}[T[\alpha], T[\beta]] \]
\[ T[\alpha \rightarrow \beta] = T[\alpha] \Rightarrow T[\beta] \]
\[ T[\alpha \rightarrow \text{option } \beta] = T[\alpha] \Rightarrow \text{Option}[T[\beta]] \]
\[ T[\alpha \rightarrow \text{list } \beta] = T[\alpha] \Rightarrow \text{List}[T[\beta]] \]

Figure 9: Optica semantic domains

**Definition 5.** The generic versions for `expertiseFl` (optic expression) and `expertise` (query expression) are implemented as follows, in terms of the Optica and organization-specific primitives.

```python
def expertiseFl =
    departments >> to_d (filtered (all employees (elem (tasks >> to_tk (toAff tsk)) 'abstract')) >> toAf dpt)
```

```python
def expertise =
    getAll expertiseFl
```

At this point, we have defined generic queries which are not coupled to any particular querying infrastructure. In the rest of the paper, we will show how to reuse such queries for generating in-memory, XQuery, SQL and comprehension-based queries.

### 3.3. Standard Semantics

In defining a new language, it is common practice to start with its syntax and type system, and then proceed to define its semantics. In our case, we have proceeded in reverse: we started with the intended semantics (optics and queries) and created an abstract syntax and type system which mimic its structure. Therefore, what is new in this section is how to formalize the connection between the syntax and type system of Optica and concrete optics, its intended semantics. For this task, we provide semantic functions \( T \) (Fig. 9) and \( E \) (Fig. 10). The first maps Optica types to their corresponding semantic domains. The second maps an expression of type \( t \) to an element of the semantic domain \( T(t) \). As can be seen, \( T \) just maps types to their Scala counterparts.\(^{12}\)

Given this scenario, the implementation of \( E \) turns out to be trivial. In fact, we just translate the Optica expressions into their Scala analogues from Sect. 2. We use \( \oplus \) to unify the different binary combinators (\( \cdot \), \(-\), etc.).

We also need to take into account the evaluation of the extended versions of the language, where terms specific to each example are introduced. For instance, Fig. 11 shows the semantic

---

\(^{12}\)Scala does not include a standard type for natural numbers. Instead of supplying them on our own, we prefer to choose the standard \( \text{Int} \) type for simplicity.
\[ E[id : \text{getter } a] = \text{Getter.id} \]
\[ E[g \gg h : \text{getter } a \gamma] = \text{Getter.andThen}(E[g : \text{getter } a \beta], E[h : \text{getter } \beta \gamma]) \]
\[ E[g \ast h : \text{getter } a (\beta, \gamma)] = \text{Getter.fork}(E[g : \text{getter } a \beta], E[h : \text{getter } a \gamma]) \]
\[ E[\text{like } b : \text{getter } a] = \text{Getter.like}(E[b : \beta]) \]
\[ E[\text{not } g : \text{getter } a] = \text{Getter.not}(E[g : \text{getter } \beta \beta]) \]
\[ E[g \otimes h : \text{getter } a \delta] = \text{Getter.} \oplus(E[g : \text{getter } a \beta], E[h : \text{getter } a \gamma]) \]
\[ E[id, \text{affine } a] = \text{AffineFold.id} \]
\[ E[g \gg h, \text{affine } a \gamma] = \text{AffineFold.andThen}(E[g : \text{affine } a \beta], E[h : \text{affine } \beta \gamma]) \]
\[ E[\text{filtered } p : \text{affine } a] = \text{AffineFold.filtered}(E[p : \text{getter } \beta \beta]) \]
\[ E[to \eta g, \text{affine } a \beta] = \text{AffineFold.to}(E[g : \text{getter } a \beta]) \]
\[ E[\text{id, fold } a] = \text{Fold.id} \]
\[ E[g \gg h, \text{fold } a \gamma] = \text{Fold.andThen}(E[g : \text{fold } a \beta], E[h : \text{fold } \beta \gamma]) \]
\[ E[\text{nonEmpty } g : \text{fold } a] = \text{Fold.nonEmpty}(E[g : \text{fold } a \beta]) \]
\[ E[to \omega g, \text{fold } a \beta] = \text{Fold.to}(E[g : \text{affine } a \beta]) \]
\[ E[\text{get } g : a \rightarrow \beta] = E[g : \text{getter } a \beta].\text{get} \]
\[ E[\text{preview } g : a \rightarrow \text{option } \beta] = E[g : \text{affine } a \beta].\text{preview} \]
\[ E[\text{getAll } g : a \rightarrow \text{list } \beta] = E[g : \text{fold } a \beta].\text{getAll} \]

Figure 10: Optica standard semantics

\[ T[Couples] = \text{Couples} \]
\[ T[\text{Couple}] = \text{Couple} \]
\[ T[\text{Person}] = \text{Person} \]
\[ E[\text{couples : fold } \text{Couples } \text{Couple}] = \text{CoupleModel.couples} \]
\[ E[\text{fst : getter } \text{Couple } \text{Person}] = \text{CoupleModel.fst} \]
\[ E[\text{snd : getter } \text{Couple } \text{Person}] = \text{CoupleModel.snd} \]
\[ E[\text{name : getter } \text{Person } S] = \text{CoupleModel.name} \]
\[ E[\text{age : getter } \text{Person } N] = \text{CoupleModel.age} \]

Figure 11: Semantic domains and standard semantics for couples extension
domains and the evaluation of each term from the couples example extension. It is also trivial, since it just maps the domain-specific terms to the concrete optics from Sect. 2.2.1. The corresponding instance for the organization extension follows the very same pattern and we omit it for brevity. Once we have defined the standard semantics for all terms, we should be able to translate generic queries into plain functions, by means of $E$. We evaluate $\text{differences}$ (Def. 4) as follows:

\[
\text{def differencesR : Couples } \Rightarrow \text{List}[\{\text{String, Int}\} ] = \\
E[\text{differences : Couples } \rightarrow \text{list } (S, N)]
\]

As can be seen, the resulting value is a Scala function that works with immutable data structures. Finally, $\text{expertise}$ (Def. 5) is evaluated in this way:

\[
\text{def expertiseR : Org } \Rightarrow \text{List}[\text{String}] = \\
E[\text{expertise : Org } \rightarrow \text{list } S]
\]

It recovers a Scala function which selects the corresponding department names. These functions are exactly the same as their counterparts from Sect. 2.

4. XQuery

So far, we have seen that optics allow us to manipulate immutable data structures in a modular and elegant way, and that concrete optics can be lifted into the Optica language, a full-blown DSL. The standard semantics of Optica is given in terms of concrete optics; however, this is not significant from the point of view of language-integrated query. The state of real applications is mostly handled through SQL and NoSQL databases, web services, etc.; therefore, this section and the following will show how to reuse Optica expressions in order to generate queries for external data sources beyond in-memory data structures. In particular, this section shows how getters, affine folds and folds from the Optica DSL can actually be given a non-standard representation in terms of XQuery expressions. Prior to that, we will manually adapt the $\text{differences}$ and $\text{expertise}$ queries and corresponding models into the XML/XQuery setting [38] in an idiomatic way. This will serve as a point of reference to implement the aforementioned semantics. In this sense, there are several assumptions that we need to make in order to map optics and XML models, which are described subsequently.

4.1. XML/XQuery Background

Accommodating objects into XML types is not a trivial task [41]. Figure [12] shows a possible way of encoding the state of the couples example in an XML document. It contains a root element $\text{xml}$ where couples hang from as $\text{couple}$ elements, which in turn contain subelements for the first ($\text{fst}$) and second ($\text{snd}$) members that form the couple. Finally, $\text{name}$ and $\text{age}$ are simple tags that contain primitive attributes.

Usually, an XML document is accompanied by an XSD schema, which is essential to validate the information that we place in the document. The schema associated to the couple document can be found in [Appendix B.1]. Among other things, it prevents us from defining people without a $\text{name}$ element, placing non numerical values as $\text{age}$ values, and defining several $\text{fst}$ elements inside a couple. Later, we will see that it is important to take this schema into account while implementing queries.

Now, we would like to produce an XQuery expression, analogous to $\text{differences}$. It should be able to collect the name and age difference of all people who occupy the first position in the
couple and are older than their mates. Since we do not want to calculate a single value for this query, like a number or a boolean, the results should be presented as a sequence of nodes, i.e. the output is also an XML tree. For example, this is the output of the query we are looking for:

```
<tuple>
  <one><name>Alex</name></one>
  <two>5</two>
</tuple>
<tuple>
  <one><name>Cora</name></one>
  <two>2</two>
</tuple>
```

We pair values by means of a contrived `tuple` element, which contains `one` and `two` projection subelements, where data is finally stored. Once we know the output that we want to produce, we show the XQuery expression that we could use to generate it:

```
/xml/couple[fst/age > snd/age]/<tuple>
  <one>{fst/name}</one>
  <two>{fst/age - snd/age}</two>
</tuple>
```

We describe its main components in the following paragraphs:

- One of the most fundamental queries is `/`, which grants access to the so-called *document* node, which can be seen as the entry point in the document. Since XML documents are essentially nested data structures, XQuery provides concise syntax to access nested elements. For example, `/xml/couple` selects all elements `couple` that are hanging from an element `xml` which in turn should be accessible from the document node.

- XQuery does provide filters to enrich queries, which are placed inside square brackets. For example, `[fst/age > snd/age]` is a filter that we apply over `/xml/couple` to discard the couples in which the age of the first member is not greater than the age of the second one. The operator `>` is able to extract the inner value of these elements and interpret them as numbers. In fact, this should be safe, if we take into account the XML schema.

---

13 We broke the query into several lines for readability, but note that certain interpreters require a single line in order to produce a valid result.
• XQuery supports XML interpolation to enrich its results. It serves us to provide the structure that we need in order to put pairs of values together. It is worth mentioning that this is the only feature from XQuery which is not available in XPath, among the ones we use in this work.

Now, we could adapt the organization example, along with the expertise query. Figure 13 shows the XML document where we adapt the information from the original example. This document is valid according to the schema that we have placed in Appendix B.2. As we already showed, expertise returns the name of the departments where all employees are able to abstract. Once again, we need to return a node sequence since we could find many departments matching the criteria. Thereby, the output that the query should produce might be:

```
<xml>
  <dpt>Quality</dpt>
  <dpt>Research</dpt>
</xml>
```

Producing such a query, analogous to expertise, is by no means straightforward since there is no standard XQuery method to check if all elements that are hanging from a certain context do satisfy a predicate. Fortunately, we can implement the desired behavior in terms of simpler primitives, as we show in the following query:

```
/xml/department[not(employee[not(task[tsk = "abstract"])]))]/dpt
```

The query produces the expected results, although it is difficult to read due primarily to the combination of filters and negations. This query uses new XQuery features, that we describe next:

• There are several invocations to the not function. This is just the negation function that we could find in many programming languages, but it adds extra functionality beyond negating booleans. Namely, it also produces true if the argument corresponds to a non-empty sequence of elements, and false if the argument corresponds to an empty one.

• We find a new operator =, which corresponds to equality. The first operand in the equality is a tag, and thus its value is extracted to do the comparison. The second operand is a string literal. Beyond strings, XQuery provides literals for other basic types, like numbers or booleans.

Finally, we would like to introduce a new element that we have deliberately ignored so far since it was not present in the queries. It is the self axis and it refers to the current context. In XQuery, it is represented as a . (dot). This self notion is neutral under nested access. For example, .//couple//tst/, is equivalent to couple/tst. We will need this self notion afterwards, in order to implement the non-standard semantics.

4.2. XQuery Non-standard Semantics

We come back to our objective of turning Optica expressions into XQuery expressions. For this task, we will use \( E^{\text{xml}} \) as the semantic function that assigns well-typed Optica expression their denotations. Prior to that, we need to choose \( T^{\text{xml}} \), which maps Optica types to semantic domains for this infrastructure. Since we aim at generating XQuery expressions, it seems reasonable to use \( XQuery \) as the semantic domain for query types. We also need to identify the semantic domain for optic types. Although this might sound contrived at this point, we adopt the very
<xml>
  <department>
    <dpt>Product</dpt>
    <employee>
      <emp>Alex</emp>
      <task><tsk>build</tsk></task>
    </employee>
    <employee>
      <emp>Bert</emp>
      <task><tsk>build</tsk></task>
    </employee>
  </department>
  <department>
    <dpt>Quality</dpt>
  </department>
  <department>
    <dpt>Research</dpt>
    <employee>
      <emp>Cora</emp>
      <task><tsk>abstract</tsk></task>
      <task><tsk>build</tsk></task>
      <task><tsk>design</tsk></task>
    </employee>
    <employee>
      <emp>Demi</emp>
      <task><tsk>abstract</tsk></task>
      <task><tsk>design</tsk></task>
    </employee>
    <employee>
      <emp>Eric</emp>
      <task><tsk>abstract</tsk></task>
      <task><tsk>call</tsk></task>
      <task><tsk>design</tsk></task>
    </employee>
  </department>
  <department>
    <dpt>Sales</dpt>
    <employee>
      <emp>Fred</emp>
      <task><tsk>call</tsk></task>
    </employee>
  </department>
</xml>

Figure 13: Organization deployed as XML.
\[ \mathcal{E}^{xml}[\cdot] :: XQuery \]

\[
\begin{align*}
\mathcal{E}^{xml}[\text{couples : fold Couples Couple}] &= \text{couple} \\
\mathcal{E}^{xml}[\text{fst : getter Couple Person}] &= \text{fst} \\
\mathcal{E}^{xml}[\text{snd : getter Couple Person}] &= \text{snd} \\
\mathcal{E}^{xml}[\text{name : getter Person S}] &= \text{name} \\
\mathcal{E}^{xml}[\text{age : getter Person N}] &= \text{age}
\end{align*}
\]

Figure 14: XQuery non-standard semantics for couples extension

same semantic domain as the one that we have embraced for queries. Therefore, we define \( T^{xml} \) as follows:

\[ T^{xml}[t] = XQuery \]

In fact, regardless of the input type, it will always evaluate to an XQuery expression. Remark 8 will shed some light on this decision. The rest of the section revolves around the details of \( \mathcal{E}^{xml} \) and discusses the results.

4.2.1. Evaluating domain primitives

Before presenting the implementation of \( \mathcal{E}^{xml} \), there are several assumptions about the adaptation of the Optic models into XML schemas that need to be made, where we basically adopt the same conventions that we have seen in Sect. 4.1. Firstly, we will assume that all information is hanging from an xml element, which acts as the root of the XML document. Secondly, we will assume that every optic corresponds to an XML element, where the optic kind determines the cardinality. Finally, optics that select base types are adapted as simple type elements containing a value with the corresponding base type; optics that select domain entity types are adapted as elements with complex type, since they nest other elements in turn. Each of the previous conventions are assumed in the XSD schemas that can be found in the appendix.

Now, we have all the ingredients that we need to provide the implementation of the XQuery evaluator. Given its simplicity, we will start with the evaluation of the extended terms for the couples example that we have collected in Fig. 14. As we have said in the previous paragraph, optics correspond to XML elements, and thus we represent them as mere element selection. Indeed, optic names are good candidates as tag names. However, we need to adjust the plural names of folds into the singular, like in couple, since this information is supplied as individual elements. The evaluation for the organization model should be straightforward now and does not add any value; therefore, we do not show it.

4.2.2. Evaluating core primitives

The evaluations for the core combinators are collected in Fig. 15. We start with the combinators for getters. Firstly, \( \gg g \) is translated as nested access, where the evaluations of the composing expressions \( g \) and \( h \) are tied together. For \( *** \) we use the XML interpolation, where the evaluation of the composing expressions is placed in the corresponding projection elements. Finally, \( id_g \) is interpreted as a self reference (.), which is neutral under composition. Now, we move on to standard getter constructions, beginning with like. Since it produces constant optics, whose part does not depend on the surrounding whole, we decide to map them to XQuery liter-
$\mathcal{E}^{\text{xml}}[\_]$ :: XQuery

$\mathcal{E}^{\text{xml}}[\text{id}]: \text{getter } a \ a] = \_.$

$\mathcal{E}^{\text{xml}}[g \gg h]: \text{getter } a \ y] = \mathcal{E}^{\text{xml}}[g : \text{getter } a \ b] / \mathcal{E}^{\text{xml}}[h : \text{getter } b \ y]$

$\mathcal{E}^{\text{xml}}[g \ast h]: \text{getter } (a, b)] = \prec \text{tuple} >$

$\mathcal{E}^{\text{xml}}[\text{id}]: \text{getter } a \ a] = \_.$

$\mathcal{E}^{\text{xml}}[\text{like } b]: \text{getter } a \ b] = b$

$\mathcal{E}^{\text{xml}}[\text{not } g]: \text{getter } a \ 0] = \_.$

$\mathcal{E}^{\text{xml}}[g \oplus h]: \text{getter } a \ y] = (\mathcal{E}^{\text{xml}}[g : \text{getter } a \ b] \oplus \mathcal{E}^{\text{xml}}[h : \text{getter } b \ y])$

$\mathcal{E}^{\text{xml}}[\text{id}]: \text{affine } a \ a] = \_.$

$\mathcal{E}^{\text{xml}}[g \gg h]: \text{affine } a \ y] = \mathcal{E}^{\text{xml}}[g : \text{affine } a \ b] / \mathcal{E}^{\text{xml}}[h : \text{affine } b \ y]$

$\mathcal{E}^{\text{xml}}[\text{filtered } p]: \text{affine } a \ a] = [\mathcal{E}^{\text{xml}}[p : \text{affine } a \ b]]$

$\mathcal{E}^{\text{xml}}[\text{to } g]: \text{affine } a \ b] = \mathcal{E}^{\text{xml}}[g : \text{getter } a \ b]$

$\mathcal{E}^{\text{xml}}[\text{id}]: \text{fold } a \ a] = \_.$

$\mathcal{E}^{\text{xml}}[g \gg h]: \text{fold } a \ y] = \mathcal{E}^{\text{xml}}[g : \text{fold } a \ b] / \mathcal{E}^{\text{xml}}[h : \text{fold } b \ y]$

$\mathcal{E}^{\text{xml}}[\text{nonEmpty } g]: \text{getter } a \ 1] = \_.$

$\mathcal{E}^{\text{xml}}[\text{to } g]: \text{fold } a \ b] = \mathcal{E}^{\text{xml}}[g : \text{getter } a \ b]$

$\mathcal{E}^{\text{xml}}[\text{get } g]: \alpha \rightarrow \beta] = /\text{xml}/\mathcal{E}^{\text{xml}}[g : \text{getter } a \ b]$

$\mathcal{E}^{\text{xml}}[\text{preview } g]: \alpha \rightarrow \text{option } \beta] = /\text{xml}/\mathcal{E}^{\text{xml}}[g : \text{affine } a \ b]$

$\mathcal{E}^{\text{xml}}[\text{getAll } g]: \alpha \rightarrow \text{list } \beta] = /\text{xml}/\mathcal{E}^{\text{xml}}[g : \text{fold } a \ b]$

Figure 15: XQuery non-standard semantics
Next, we can see that \texttt{not} is interpreted as the function \texttt{not}, and binary combinators, which are unified by the symbol \texttt{⊕}, are interpreted as the corresponding XQuery operations.

Moving on to affine folds, we find that the composition and identity primitives share the same implementation as the ones we have seen for getters. This situation—which also occurs in fold combinators—demonstrates that we do not make a difference between semantic domains in the interpretation. In fact, if we understand \textit{XQuery} as a representation of an affine fold, it is natural that we can also use it as a representation of a getter, and the implementation of \texttt{toGet} confirms this intuition. This module also contains \textit{filtered}. Since we have a filtering mechanism available in XQuery, we simply interpret this primitive into square brackets ([]), passing the semantics of the predicate getter as an argument to it.

Finally, we present the fold related method \texttt{nonEmpty}. In this particular case, we need to adapt any fold into a getter that selects a boolean. Luckily, XQuery provides a function \texttt{exists} which turns XQuery expressions into booleans. It does it by checking that the result produced by the query is not empty. It might have been noticed that \texttt{exists} was not even mentioned in the background section. In fact, it was not necessary since the \texttt{not} function does the trick by turning an expression denoting a sequence into a boolean. In particular, \texttt{not(exists[sg])} (where \texttt{sg} denotes a sequence of elements) is equivalent to \texttt{not(sg)}. However, while evaluating, we do not know whether the \texttt{exists} invocation denoted by \texttt{nonEmpty} will be consumed by a function like \texttt{not} (denoted by another expression), and thus we need to invoke \texttt{exists} explicitly\footnote{There are different implementation techniques in the literature that we could use to optimize this kind of situation, but we ignore them for brevity.}.

As final notes, we must say that interpreting optic expressions like \texttt{differencesFl} (Def. \ref{f:diffFl}) or \texttt{expertiseFl} (Def. \ref{f:expertiseFl}) leads to relative queries, i.e. queries that do not start with / and which are relative to the current context. Those queries are valid XQuery expressions, but they will not produce any results if we run them against the XML document which contains the whole hierarchy. Fortunately, we could easily compose such relative queries with the ones generated by external models to produce queries over more complex domains. Leaving this possibility aside, the next section shows the final refinement that we need to perform in order to obtain the expected XQuery expressions.

### 4.2.3. Target queries and results

The evaluation of query expressions from Optica can be found at the bottom of Fig.\ref{fig:evalExample}. Since both optic and query types denote an XQuery expression, the semantics of query expressions is almost direct. The only caveat is that \texttt{get}, \texttt{preview} and \texttt{getAll} prepend the /\texttt{xml} fragment to the relative query obtained in the optic representation. This is just a consequence of one of the assumptions that we made when adopting XML, where we have stated that an \texttt{<xml/>} root element was necessary by convention. Thereby, we take the opportunity to prepend it here.

At this point, with the required evaluations at hand, we should be able to recover the target queries. As a result, \texttt{E\textsuperscript{xml}[differences]} provides the following XQuery expression:

```xml
/xml/couple[fst/age > snd/age]/<tuple>
  <one>{fst/name}</one>
  <two>{fst/age - snd/age}</two>
</tuple>
```

The resulting query is exactly the same as the one that we have introduced in Sect.\ref{sect:evalExample}. We also supply the output provided by \texttt{E\textsuperscript{xml}[expertise]}:

\textsuperscript{14}The notation \texttt{b} just indicates the adaptation of \texttt{b} into an XQuery literal.
\textsuperscript{15}There are different implementation techniques in the literature that we could use to optimize this kind of situation, but we ignore them for brevity.
We can see a self-reference (dot) when comparing the task with the literal ‘abstract’, which is a consequence of the particular implementation of elem that we have presented in Sect. 3.1 (we use the identity getter to refer to the predicate parameter while invoking any). Besides, we find redundant invocations to exist. If we ignore these minor differences, the query is essentially the same as the one that we presented at the beginning of this section, and therefore it produces the very same output. The accompanying implementation in Scala of the XQuery interpreter contains tests that show the right behavior of these queries, where we use BaseX as the XQuery engine to access XML documents.

Remark 8. As we have seen throughout this section, both optic and query types are evaluated into the same semantic domain XQuery. Indeed, if we leave interpolation facilities aside, this is essentially an interpretation into XPath, which is just a language to select parts from an XML document, just like optics select parts from immutable data structures. In this sense, it is only natural that XQuery can behave as a non-standard optic representation.

5. SQL

SQL is a query language for relational data sources which greatly differs from the hierarchical nature of both XML and optic models. Nevertheless, this section will show that we can generate SQL statements from Optica expressions. Firstly, we manually adapt the couple and organization examples into the SQL setting to better understand the kind of queries that we want to produce. Then, we will present the SQL non-standard semantics of Optica and the assumptions that we build upon in order to automatically generate analogous queries to the ones that we have obtained manually.

5.1. SQL Background

As opposed to XML, relational databases are organized around flat data sources. As a consequence, we face the object-relational impedance mismatch [5] when trying to accommodate the object models underlying optics into the relational setting. Fortunately, there are patterns that we can embrace to approach this task, like the Foreign Key Aggregation or the Foreign Key Association patterns [42]. We take them as a reference and propose the following tables to adapt the couples example model that we introduced in 2.2.1:

\[
\text{CREATE TABLE Person (}
\text{name varchar(255) PRIMARY KEY,}
\text{age int NOT NULL)};
\]

\[
\text{CREATE TABLE Couple (}
\text{fst varchar(255) NOT NULL,}
\text{snd varchar(255) NOT NULL,}
\text{FOREIGN KEY (fst) REFERENCES Person(name),}
\text{FOREIGN KEY (snd) REFERENCES Person(name)});
\]

Once again, these invocations could be removed from the resulting query by means of annotations, as in [19], but we wanted to keep the interpretation compositional in order to make it simpler.

[18]http://basex.org/baseX/xquery/
As can be seen, case classes are adapted as tables and their attributes are adapted as columns. Once again, as we have seen in the XQuery interpretation, it is necessary to distinguish between attributes which contain base types and attributes containing other entities. In fact, attributes that refer to entities require pointers to establish the precise connections between the adapted tables, following the Foreign Key Aggregation pattern. We assume figures `[16a]` and `[16b]` as the initial state for these tables, where the columns in `Couple` are clearly selecting names from `Person`.

Previously, we have seen that the adaptation of `differences` in the XML setting produced XML as output. We are now dealing with SQL tables, where the output of a statement is a table itself. Thereby, we would expect Fig. `[16c]` as the result of executing the adaptation of `differences`. In particular, we could produce such output with the following query:

```sql
SELECT w.name, w.age - m.age
FROM Couple c
INNER JOIN Person w ON c.fst = w.name
INNER JOIN Person m ON c.snd = m.name
WHERE w.age > m.age;
```

This statement is clearly separated in three major sections. First, we describe `FROM`, which builds the raw table that the other parts use to gather information from. This table is created by joining the table `Couple` with two occurrences of table `Person`, thereby incorporating the information from the couple members `fst` and `snd`. Variables `c`, `w` and `m` allow us to refer to these three tables. Second, the `WHERE` clause introduces filters that are applied over the compound table to discard the rows that do not match the criteria: those where the age of the first member is not greater than the age of the second one. Last, the `SELECT` clause indicates the columns that we are interested in: the name of the first member and the age difference.

Now, we move on to the organization example. First of all, we create tables for departments, employees and tasks following the same adaptation pattern:

```sql
CREATE TABLE Department {
  dpt varchar(255) PRIMARY KEY
};

CREATE TABLE Employee {
  emp varchar(255) PRIMARY KEY,
  dpt varchar(255) NOT NULL,
  FOREIGN KEY (dpt) REFERENCES Department(dpt)
};

CREATE TABLE Task {
  tsk varchar(255) NOT NULL,
  emp varchar(255) NOT NULL,
  FOREIGN KEY (emp) REFERENCES Employee(emp)
};
```

All components in the previous statements should be familiar at this point, but there is an important change in the way we configure foreign keys. As we have seen in the couples example,
getters selecting entities were mapped into a column containing a foreign key. However, the organization example contains multivalued attributes, like employees or tasks, that should not be adapted as a single column. For this situation we adopt the Foreign Key Association pattern. We assume that these tables have been populated with the data in figures 17a, 17b and 17c.

As we have seen before, Quality and Research are the departments where all employees are able to abstract; therefore, the adaptation of expertise should produce Fig. 17d as a result. We propose the following query to generate it:

```sql
SELECT d.dpt
FROM Department AS d
WHERE NOT (EXISTS (SELECT e.*
    FROM Employee AS e
    WHERE NOT (EXISTS (SELECT t.*
        FROM Task AS t
        WHERE (t.tsk = 'abstract') AND (e.emp = t.emp))
    AND (d.dpt = e.dpt))));
```

Reading this query is by no means trivial. Fortunately, it shares the same pattern as the query that we have seen while adapting expertise in the XQuery setting. In fact, EXISTS is a function that returns true as long as the nested statement produces non-empty results. If we combine it with NOT to negate predicates, we can check if all rows satisfy a condition. Beyond the noise generated by this pattern, there are additional filters which manifest relations between nested and outer variables that introduce even more complexity in the picture.

5.2. SQL Non-standard Semantics

Peculiarities of SQL have been known for a long time now [43]. As Date states in connection with certain aspects of SQL, “there is so much confusion in this area that it is difficult to criticize it coherently”. Part of the problem resides in that the formal definition of SQL was produced after the fact, where many academic considerations were neglected. Consequently, the language does have its weak points, where the lack of orthogonality becomes a central issue. Although many deficiencies have been remedied in the last decades, the obtrusive syntax of the SELECT statement remains a problem. For example, despite the fact that relational algebra combinators may appear in any order, the rigid structure of SELECT statements might demand the programmer to recast a relational algebra expression that is considered natural (like UNION (tabexp1, tabexp2)) into a semantically equivalent form, compliant with the SQL
standard (like \( \text{SELECT} \ldots \text{FROM} \ldots \text{WHERE} \ldots \text{UNION} \text{SELECT} \ldots \text{FROM} \ldots \text{WHERE} \ldots \)). Fortunately, \([16]\) supplies a list of syntactic rules which we can use to rewrite any expression from an ordinary impure functional programming language into its SQL form. Optica expressions share with relational algebra the purely compositional character of algebraic expressions; hence, they also require a set of transformations before being able to be translated to SQL queries. These transformations will not be carried out on the optic expression itself, but through a new semantic domain which plays the role of an intermediate expression that can be directly translated to SQL.

\[
\begin{align*}
T^{\text{sql}}[\text{getter } \alpha \beta] &= \text{Triplet} \rightarrow \text{Triplet} \\
T^{\text{sql}}[\text{affine } \alpha \beta] &= \text{Triplet} \rightarrow \text{Triplet} \\
T^{\text{sql}}[\text{fold } \alpha \beta] &= \text{Triplet} \rightarrow \text{Triplet} \\
T^{\text{sql}}[\alpha \rightarrow \text{list } \beta] &= (\mathcal{S} \rightarrow \mathcal{S}) \rightarrow \text{SQL} \\
T^{\text{sql}}[\text{N}] &= \text{Fragment} \\
T^{\text{sql}}[\text{B}] &= \text{Fragment} \\
T^{\text{sql}}[\text{S}] &= \text{Fragment}
\end{align*}
\]

Figure 18: SQL semantic domains

Accordingly, the new semantic domains defined by the semantic function \( T^{\text{sql}} \) are shown in Fig. 18. Firstly, all optic types are mapped to a Triplet endofunction. Triplets are the intermediate expressions which lie between optic and SQL expressions, whose major purpose is to reconcile the main disagreements among them. Secondly, since we aim at generating SQL statements, the semantic domain associated to query types is, as expected, an SQL expression. However, it is required to supply a function \( (\mathcal{S} \rightarrow \mathcal{S}) \) that maps relational table names to the column name which corresponds to the primary key —information that is not contemplated by the optic model—in order to produce SQL statements. It is important to remark that the types of the queries get and preview are ignored here. Later on we will explain why this partiality is needed. Finally, base types are mapped to triplet fragments, i.e. their evaluation will be used to form triplets.

For the rest of the section we will proceed as usual, introducing the semantic function \( E^{\text{sql}} \), which is responsible for evaluating domain, optic and query terms, and we will conclude discussing the results. Prior to that, we find it essential to describe the details about the intermediate structure Triplet.

5.2.1. Triplet: motivation and details

As we have just seen, a SQL select statement exhibits a remarkable separation of concerns, where selection and filtering, although sharing syntax, belong to different query clauses. This separation requires a unifying mechanism to refer to the very same item from both clauses. SQL solves this problem by means of variables declared in the FROM clause, which are accessible from the SELECT and WHERE scopes.

This way of representing queries in SQL contrasts with its optic counterpart. In Optica, the aspects of selection and filtering may appear anywhere within the expression. Moreover, the information required by these components does not need to be collected in a single FROM component, but specified on demand. In optic expressions, there is no need for variables either, since it is the context where two optics appear that determines whether they are selecting the same item or not. For example, consider the following optic expression\(^\text{18}\) where we find two

---

\( ^{18} \)This query is only a less direct way of implementing query under50 from Sect. [1]
Figure 19: Triplet generated for differencesFl.

couples ≫ filtered (fst ≫ age < 50) ≫ fst ≫ name

Despite having one of them surrounded by filtered, we can see that both of them are selecting the very same person. Furthermore, note that the information required by the filtering expression (the age of the first member) is collected within the predicate scope and not shared globally.

Triplet is the data structure that we use as an intermediary to alleviate the aforementioned disagreements. Its main objective is to segregate the three different aspects, which are evident in a SELECT statement, from an Optica expression. In particular, a triplet is made of three components which correspond to the SELECT, FROM and WHERE clauses, respectively. We present an informal view of this concept in Fig.19 where we represent the triplet associated to the expression differencesFl (Def. [4]). A triplet may be considered as a structured optic whose actual focus is determined through three components:

- The middle component determines the potential focus of the optic. In particular, Fig. [19] shows this component as a trie whose edges are optics focusing on entity types (not base types). Its elements are sequences of optics that represent a vertical composition, e.g. the sequence made of the primitive fold couples and getter fst represents the fold couples ≫ fst. The figure labels each node with a distinct name that refers to the entities of that unique path. In this example we potentially refer to the list of couples (c) and two lists of people: its first (w) and second (m) members. The nodes of the trie, colored in black, and its associated names can be reused in the left and right components.

- The right component further constrains the potential collections of entities identified by the entity trie, by imposing conditions over them. In the example, there is just one condition that restricts the collection of couples (and, consequently, its dependent collections of people) to those where her age (w) is greater than his (m). These conditions are represented in terms of directed graphs whose edges are optics or binary combinators, like >, which make two different paths converge. Note that red nodes form restriction graphs.

[1]https://en.wikipedia.org/wiki/Trie
\[
\begin{align*}
t &::= (s, f, w) \\
s &::= (e, e, \ldots, e) \\
f &::= \text{/ } | \text{insert } \hat{p} \text{ } f \\
w &::= [e, e, \ldots, e] \\
e &::= \text{like } c \mid \text{not } e \mid e > e \mid e == e \mid e - e \mid \hat{p} \mid \text{ optic } \mid \text{nonEmpty } t \\
\hat{p} &::= (\text{optic}, \text{optic}, \ldots, \text{optic})
\end{align*}
\]

Figure 20: Triplet syntax

- Last, the left component defines the actual selection of the overall optic by selecting in sequence certain collections from the entity trie, and possibly by further refining them through additional optic expressions selecting base values. In the example, we select her name and the age difference of the couple (which will be greater than 0) according to the constraints which were imposed by the right component. Selections are represented using the same graphs as in the constrain component, but nodes forming them are colored in blue.

We formalize the notion of triplet in Fig. 20 through its associated syntax. As we have pointed out previously, the middle component is just a trie whose keys are primitive optic expressions focusing on entities. Thus, the elements stored in the trie are sequences of such expressions, which we will refer to as paths (\(\hat{p}\)). Entity tries may be the empty trie, /, or the result of inserting a new path, \(\text{insert } \hat{p} \text{ } f\). The left and right components, \(s\) and \(w\), are a sequence and a set of expressions (\(e\)), respectively. Repeated restrictions in \(w\) are redundant and their ordering is irrelevant—that is why a set is chosen. Expressions \(e\) are very similar to those from Optica (like, not, >, etc.), but there are a few major changes that deserve further explanation. Essentially, expressions do not include vertical composition as such; instead, if the vertical composition selects an entity, it is simply represented through a path from the entity trie. Otherwise, if it selects a base type, it is represented as the projection of an attribute from a path, as in \(\hat{p} \text{ optic}\). For instance, the Optica expression \(\text{couples } \gg \text{fst}\) would denote the path (\(\text{couples, fst}\)), while the expression \(\text{couples } \gg \text{fst } \gg \text{name}\) would denote the projection (\(\text{couples, fst, name}\)). Horizontal composition is also unneeded since the left component is able to collect a sequence of single selections. Finally, expressions also contain a \text{nonEmpty} term, where we keep a snapshot of a triplet that is later used to produce nested queries. Section 5.2.4, where we formalize the precise correspondence between triplets and SQL, will show that a \text{nonEmpty} term is eventually translated into an \text{EXISTS} operator.

At this point, we might consider using \text{Triplet} as the chosen optic representation. However, composing the different triplets generated by optic subexpressions turns out to be a clumsy task. Instead, we would like to use a representation with better compositional guarantees. In this sense, it is more convenient to use a triplet endofunction so that each subexpression can describe the precise transformation that it performs over the input triplet when it is composed through vertical composition\(^{21}\). This is how we obtain \text{Triplet } \rightarrow \text{Triplet}, the chosen semantic domain for optic types. We illustrate the idea behind this function in Fig. 21 which shows the evolution of the

\(^{20}\) We will use hats, as in \(\hat{p}\), to emphasize the terms which correspond to paths.

\(^{21}\) This is reminiscent of the functional representation of difference lists, where concatenation is implemented in terms of plain composition, and the list is recovered by passing the empty list as input. In this case, the analogues of the empty list and concatenation are the empty triplet (Def. 6) and vertical composition.
resulting triplet for the differencesFl query, starting from the empty triplet. The arcs in this figure are labelled by the optic subexpressions that identify the applied transformations. As expected, the last triplet in the chain corresponds to the structure that we presented in Fig. 19. We will detail these steps throughout the next sections while presenting the $E^{opt}$ definition.

5.2.2. Evaluating domain primitives

This section provides the semantics of primitive optics from the domain syntax, such as couplings, $fst$, etc., in terms of the precise transformations that they carry out over an input Triplet. Their formalization can be found in Fig. 22 using the couples domain for illustration purposes. Note that $\sim$ represents the concatenation of sequences. Before explaining this formalization, we will describe the occurrences of domain primitives in the particular example shown in Fig. 21 as well as in Fig. 24 where Step b) is shown in detail.

- Step a) shows the changes introduced by the term couples. This is a very special case since it takes the initial triplet as input. As can be seen, the new changes consist of introducing the new path in the trie and selecting it in the left component. Bear in mind that we can only introduce optics selecting domain entities in the trie, like couples, that selects a sequence of Couple entities.

- Step b) contains more domain terms in the predicate. In particular, Step b1) (Fig. 24) shows the changes introduced by $fst$ when it is applied to a triplet that focuses on couples. Since this optic focuses on entities and the input triplet is not empty, the result is a triplet that extends the entity trie by appending the new optic to the couples path, changing its focus (i.e. the left part of the triplet) to the new path $w$.

- Step b3) represents the changes introduced by age. In this case, we deal with an optic selecting a base type $\mathbb{N}$. Thereby, it cannot be introduced in the trie. Instead, we refine the focus of the input triplet which becomes a projection to the new optic.

Fortunately, the behaviour of the first and second cases can be factorized, as long as we contemplate the following definition for the empty triplet:

**Definition 6.** We formalize the empty triplet as the one that contains a single selection $\hat{()}$, an empty trie and an empty set of restrictions.

$$empty = ((\hat{()}), /, \emptyset)$$

The empty sequence () is used in tries to refer to its root, and thereby, we use it as the initial path in the left component.

However, our formalization must take into account the distinction between optics selecting entities and optics selecting base types. Figure 22 introduces functions $base$ and $entity$ for this task, where $\mapsto$ just represents the standard “maps to” notation from functions. They take the optic expression as parameter and they produce triplet endofunctions as result. The rest of the implementation should be straightforward, given the previous explanations. We do not show the evaluation of the organization terms since they follow the very same pattern, exploiting entity and base.
Figure 21: Triplet evolution for differencesFl
5.2.3. Evaluating core primitives

This section specifies the triplet transformations that are associated to the Optica core combinators which can be found in Fig. 23. Before delving into the semantics of the getter, affine fold and fold combinators, we will introduce the next definitions, which will be useful for ensuring the consistency of the formalization:

**Definition 7.** Given \( e : \text{optic } \alpha \beta \), where \( \text{optic} \in \{ \text{getter, affine, fold} \} \), a triplet \( t \) is a valid input for \( e \) if one of the following conditions holds:

1. \( t = \text{empty} \)
2. \( t = \mathcal{E}_{opt}[ \text{fold \ Couples \ Couple} ] \), for some \( e \), \( t \) such that \( t \) is a valid input for \( e \)

Basically, an input triplet is valid for a given optic \( e \) if it is the empty triplet (Def. 6), or if it is the result obtained from evaluating an optic expression \( e \) with a valid input, where the ‘part’ type of optic \( e \) coincides with the ‘whole’ type from \( e \).

**Definition 8.** A singleton model type is either a base type or a domain type, i.e. it is the result of discarding product types from Optica model types.

**Proposition 1.** Let \( e : \text{optic } \alpha \beta \), where \( \text{optic} \in \{ \text{getter, affine, fold} \} \), \( \beta \in \text{singleton model type} \), and \( t \) a valid input for \( e \); then:

\[
((s), \ldots) = \mathcal{E}_{opt}[ \text{e : optic } \alpha \beta ] \ t
\]

The proposition states that, given a valid input, the result from evaluating an optic that selects a singleton model type always returns a single selection \( s \). This can be easily proven by induction since all combinators producing optics that select singleton model types do generate a single expression as left component, according to Fig. 23. In fact, this proposition turns out to be necessary to consider that the implementations of base and entity (Fig. 22) are well-defined.

**Getters.** First, we describe the implementation of \( \gg_{\alpha \beta} \). As Fig. 21 suggests, vertical composition should be evaluated as the chaining of transformations, i.e. as function composition. Consequently, \( \mathcal{id}_{\alpha \beta} \) is implemented as the identity function, meaning no transformation at all.

---

\[ \mathcal{E}_{opt}[ \rightarrow \rightarrow \alpha \beta ] = \mathcal{id} \]

**Figure 22:** Triplet non-standard semantics for couples extension
\( E^n[\_ : \text{op} a \beta \text{ where } \text{op} \in \{\text{getter, affine, fold}\}] \) :: Triplet → Triplet

\[
E^n[\text{id}_a : \text{getter a a}] = t \mapsto t
\]

\[
E^n[\text{g \gg h : getter a }\gamma] = E^n[h : \text{getter }\beta \gamma] \cdot E^n[g : \text{getter a }\beta]
\]

\[
E^n[\text{g \gg g : getter a }\beta, \gamma] = t \mapsto (s_1 \mapsto (s_2, f_2 \cup f_1), w_1 \cup w_2)
\]

\[
E^n[\text{like b : getter a }\beta] = (_, f, w) \mapsto ((\text{like }\beta, b), f, w)
\]

\[
E^n[\text{not g : getter a }\beta] = ((b, f, w) \mapsto ((\text{not } b), f, w)) \cdot E^n[g : \text{getter a }\beta]
\]

\[
E^n[\text{g \gg h : getter a }\beta] = t \mapsto ((b_1, f_1, w_1) = E^n[g : \text{getter a }\beta] t \text{ and } (b_2, f_2, w_2) = E^n[f : \text{getter }\gamma] t)
\]

\[
E^n[\text{id}_{\text{aff}} : \text{affine a a}] = t \mapsto t
\]

\[
E^n[\text{g \gg h : affine a }\gamma] = E^n[h : \text{affine }\beta \gamma] \cdot E^n[g : \text{affine a }\beta]
\]

\[
E^n[\text{filtered g : affine a a}] = (s, f, w) \mapsto (s, f, (b) \cup w)
\]

\[
E^n[\text{to}_{\text{aff}} g : \text{affine a }\beta] = E^n[g : \text{getter a }\beta]
\]

\[
E^n[\text{id}_{\text{fold}} : \text{fold a a}] = t \mapsto t
\]

\[
E^n[\text{g \gg h : fold a }\gamma] = E^n[h : \text{fold }\beta \gamma] \cdot E^n[g : \text{fold a }\beta]
\]

\[
E^n[\text{nonEmpty g : getter a }\beta] = (s, f, w) \mapsto ((\text{nonEmpty }E^n[g : \text{fold a }\beta](s, f, \varnothing)), f, w)
\]

\[
E^n[\text{to}_{\text{fold}} a : \text{fold a }\beta] = E^n[a : \text{affine a }\beta]
\]

Figure 23: Triplet non-standard semantics
Figure 24: Filtered step in detail
Step e) (Fig. 21) shows an example of horizontal composition (**∗∗∗**), where a pair of diverging triplets are somehow combined. In this special case, the changes are only reflected in the left component since the middle and right components are exactly the same in both triplets. The evaluation of **∗∗∗** supplies the input triplet $t$ as an argument to the evaluations of $g$ and $h$, which results in a pair of diverging triplets, as those in the illustration. To carry out the combination of them, we concatenate the selections $s_1$ and $s_2$, we merge the tries $f_1$ and $f_2$ (▽), and we make the union of the sets of restrictions $w_1$ and $w_2$. Both the union of sets and the merging of tries are idempotent operations.

Next, we find like and not as examples of unary standard combinators which just update the left component of the input triplet. The former ignores the previous selection and replaces it by the constant value. The latter transforms the triplet by applying the operation over the current selection. The evaluation demands a single expression as input, where we rely on Prop. [1]. Moreover, the Optica type system guarantees that such an expression represents an optic selecting a boolean.

Finally, Step b5) (Fig. 24) represents a binary combinator. The situation is very similar to that of **∗∗∗**. However, instead of concatenating the selections, their single components are fused into the corresponding expression. The evaluation of this term assumes that the triplets which derive from the evaluations of $g$ and $h$ contain singleton selections. Once again, we rely on Prop. [1] since all the binary combinators that we can find in Optica take base types as operands.

**Affine Folds.** As in the XQuery evaluation, composition and identity primitives are exactly the same as those we have just presented for getters. In addition, $\text{to}_x$ only returns the evaluation of its argument. The same concept will apply to folds. Consequently, there only remains filtered. As we have seen before, Step b) (Fig. 21) represents this combinator, which was further detailed in Fig. 24, given its complexity. This figure shows an inner box that describes the triplet evolution specified by the predicate, which starts from the same input triplet as the filtered whole expression. The rest of the evolution inside the box should be straightforward now. However, it has yet to be explained how to move from the result of Step b5) to the final result of Step b). Informally, what happens in this example is that the selection of the whole expression does by no means change, which seems meaningful since the filter expression should not change the focus; the left component of the inner expression represents the predicate, which becomes a new constraint in the right component of the resulting triplet; last, the middle component remains unchanged in this particular case.

The evaluation of filtered in Fig. 23 formalizes the previous intuitions. Firstly, the overall input triplet is passed as argument to the evaluation of the predicate. Its right component is reset to the empty set and the triplet generated by the predicate is expected to contain an empty set of restrictions, since getters are unable to update the restriction component of the triplet. In the resulting triplet, the selection $s$ passes as is, while the restriction that was selected in the predicate is appended to the existing ones in $w$. Finally, note that the new entity tree results from the inner triplet $f_1$, since new paths may have been created internally.

**Folds.** Lastly, we present the interpretation of nonEmpty, which introduces a significant difference in comparison with the rest of combinators: it takes a fold as parameter. The evaluation of folds is problematic since they lead to the introduction of nested queries in this infrastructure, as we will see later. This is the reason why we use the nonempty term from Fig. 20 here, which

\[22\] Incidentally, this step is better to illustrate the result of merging two tries.
basically stores the triplet resulting from the evaluation of the fold over the input triplet (after resetting its constraints, since they will not be necessary). This resulting triplet will typically extend the entity trie with new paths, but these will not be propagated into the overall resulting triplet since they are considered private to the inner scope. As a result, the entity trie of the nonempty expression, as well as its constraints, are kept unchanged in the overall triplet.

**Remark 9.** An Optica expression is always translatable into a triplet endofunction, as evidenced by the total implementation of \( \mathcal{E}^{sql} \), where Prop. 1 has proven essential. In fact, this evaluation just consists on moving things around to adapt Optica expressions to the triplet configuration. Unfortunately, translating triplets into SQL statements is a partial process, as described in the following section.

### 5.2.4. Target queries and results

We have designed triplets to be easily translatable into `SELECT` statements. This is clearly evidenced in Fig. 25, where we compare the triplet generated for `differencesFl` (Def. 4) and the expected SQL query that we presented in the background (Sect. 5.1) for the same example. While the translation of the expressions in the left and right components is straightforward, the generation of the `FROM` clause from the middle component requires further explanation. We present the formal translation of Optica query expressions into SQL in Fig. 26. What first calls our attention is the absence of translations for `get` and `preview`. In fact, it is only possible to produce a SQL statement from `getAll`. As suggested in Prop. 3, the translation of triplets into SQL statements is a partial process.

**Preconditions.** We describe the precise conditions that an Optica query should satisfy in order to produce a valid SQL statement:

1. The optic selected type, i.e. its ‘part’, is a flat type. For instance, `couples` is not translatable into SQL, since it selects `Couple` as part, which contains nested references to the entity `Person`. The expression `couples >> fst` is valid, since `Person` does not contain further nested data structures: name and age are plain values.
2. The expression cannot contain a fold selecting a base type. For example, `departments >> employees >> tasks` is valid since all the involved folds do select entity types.
3. The original kind (ignoring castings) of the leftmost expression forming a query has to be a fold. For example, `couples >> fst >> name` is translatable into SQL (it starts with the `couples` fold) while `fst >> name` is not (it starts with the `fst` getter). Thereby, `get` and `preview` are omitted, since getter or affine fold expressions do not satisfy such condition.

We will further motivate each limitation in the following paragraphs, where the whole process of generating SQL statements from triplets is described.

Since we aim at turning triplets into SQL expressions, the very first step is to produce a triplet. We achieve this by evaluating the fold expression that accompanies `getAll` and supplying the empty triplet (Def. 6) to the resulting function. Then, we need to refine the entity trie of the obtained triplet by assigning fresh names for each of its paths (which the evaluation function \( \mathcal{E}^{sql} \) does not generate). Last, we pass the refined triplet as argument to the actual translator (sql). Besides the triplet, note that this function receives an additional argument, `loca`, that specifies

---

23It is important to note that an error should be raised if any of them is not satisfied.
the path which actually determines the scope of the query to be generated. This is necessary since the sql function will be used to translate both the whole SQL query, and the inner queries of nonEmpty expressions. In this very first invocation, we aim at translating the whole triplet; thus, we pass as local the top of the entity trie, which represents the common prefix of every path of the trie.

The sql function delegates the generation of each clause of the whole SELECT statement into the corresponding functions select, from and where. Moreover, it calls an additional function, where+, whose purpose will be motivated later on. The results obtained from each function are concatenated to form the final query. Note that parentheses and brackets are discarded in the result, they are simply introduced to delimit the arguments supplied to each function. In particular, an invocation surrounded by brackets informs that the invocation may be omitted, taking into account the accompanying conditions. We describe the generation process of each clause in the next paragraphs, where we will make frequent use of the following additional definitions:

- \( \rho.\text{top} \): The key which starts every path of the entity trie, if any
- \( \rho_1.\text{local}(\rho_2) \): The local path of \( \rho_2 \) which is extended by \( \rho_1 \), if any
- \( \rho(\hat{p}) \): The name assigned to the given path in the refined trie
- \( \hat{p}.\text{last} \): The key which finishes the given path
- \( \hat{p}.\text{up} \): The second to last key of the given path
- \( \text{optic.name} \): The name of the given optic primitive
- \( \text{optic.kind} \): The kind of optic: getter, affine fold or fold
- \( \text{optic.whole} \): The type of the whole entity that the optic inspects
- \( \text{optic.part} \): The type of focus to which the optic points to (an entity or base type)
- \( \text{pk(type)} \): The primary key of the relational table associated to the given type

With a little abuse of notation, we will omit the last attribute in path expressions, as in \( \hat{p}.\text{name} \), instead of writing the more verbose \( \hat{p}.\text{last.name} \).

**Select clause.** The select function generates the SELECT clause by separating the result of translating each expression with commas. We describe the translation of the different types of expressions in the following lines:

- The translation of a path \( \hat{x} \) simply refers to all the columns of the table corresponding to that path, which was assigned by the fresh function. Also note that the path must refer to an entity with no further nested entities (Precondition 1). Otherwise, the query output would not contain all the data required to reassemble the entity, i.e. this work does not support query shredding yet.

- The translation of a projection \( \hat{x}.\text{base} \) is basically the same, but we find an interesting restriction here. SQL does not support multivalued columns, and therefore we cannot use a fold to project values (Precondition 2).

- The translation of nonEmpty is given in terms of EXISTS, which contains a nested SQL expression. Thereby, we invoke the sql generator recursively. Before doing this, we need first to generate fresh names for the trie of the nonEmpty expression and to merge it with the outer entity trie. Second, we need to calculate the right local path and to pass it to the sql generator.

---

24The combinator \( \triangleright \) merges tries, keeping the names from the left when it finds conflicting paths.
• The evaluation of the rest of expressions should be straightforward since they just adapt operators and literals into their SQL form.

Remark 10. None of the optic models associated to the guiding examples include affine folds in their definitions. In the particular case of the SQL interpretation, such optics are assumed as fields which may contain a \texttt{NULL} value, i.e. as \textit{nullable} table columns.

\textbf{Where clause.} We continue with the \texttt{WHERE} clause given its similarity with the \texttt{SELECT} clause, which is generated by the \texttt{where} and \texttt{where}+ functions. The former is quite similar to \texttt{select} since it basically delegates the evaluation of the restriction expressions, although it uses \texttt{AND} as delimiter for the results. The evaluation of expressions is exactly the same as the one that we have introduced in the previous paragraph. Note that \texttt{where} produces \texttt{WHERE True} if the set of restrictions is empty. Concerning \texttt{where}+, this function is responsible for appending the precise connection between nested and outer variables, which were introduced at the very end of Sect. 5.1. We will explain it together with the discussion of the generation of the \texttt{FROM} clause in the next paragraph.

\textbf{From clause.} Before venturing into the \texttt{from} function, there are a few conditions that the generator should preserve. Firstly, it is assumed that \texttt{p.top} must refer to a \textit{fold} (Precondition 3), since we need an entry point in the hierarchical tables. This means that we can only translate expressions that start with a fold, like \texttt{differencesFl} (Def. 3) —which starts with \texttt{couples}— or \texttt{expertiseFl} (Def. 5) —which starts with \textit{departments}. Secondly, the invocation to \texttt{from} is omitted if \texttt{local} is not defined, since this indicates that the current query is not introducing new variables, and therefore no \texttt{FROM} clause is required.

As expected, the \texttt{from} function prepares the \texttt{FROM} clause. It selects the ‘part’ type from \texttt{local} as the initial table. Then, it produces an \texttt{INNER JOIN} expression for each element hanging from it. This is the reason why tries contain nothing more than entities, since they correspond to relational tables. In general, the complexity associated to these definitions is due to the choice and implementation of the corresponding \texttt{FOREIGN KEY} patterns (Sect. 5.1).

![Figure 25: From triplet to SQL](image)
\( \mathcal{E}_{SQL}[\text{getAll } g : \alpha \rightarrow \text{list } \beta] \) :: \((\mathcal{S} \rightarrow \mathcal{S}) \rightarrow \text{SQL}\)

\[ \mathcal{E}_{SQL}[\text{getAll } g : \alpha \rightarrow \text{list } \beta] \quad \text{pk} \]

\( \text{sql} \ (s, \rho, w) \text{ pk} \) = \( (\text{select } s \rho \text{ pk}) \) \( \text{from pk} = \text{sql}(s, \rho, w) \text{ pk} \) \( \rho = \text{fresh } f \) \( \rho.\top \text{p is defined} \)

\( \text{select} \ (e_1, e_2, \ldots, e_n) \ \rho \ \text{ pk} \) = \( \text{SELECT} \ expr \ e_1 \ \rho \ \text{ pk} , \ expr \ e_2 \ \rho \ \text{ pk} , \ldots , \ expr \ e_n \ \rho \ \text{ pk} \)

\( \text{expr} \ \hat{x} \ \rho \ \text{ pk} \) = \( \rho(\hat{x}).* \) \( \text{where} \ 
\hat{x}.\text{last.part} \in \text{flat types} \)

\( \text{expr} \ \hat{x}.\text{optic} \ \rho \ \text{ pk} \) = \( \rho(\hat{x}).(\text{optic.name}) \) \( \text{where} \ 
\text{optic.part} \in \text{base types} \) \( \text{and} \ 
\text{optic.kind} \in \{\text{getter, affine}\} \)

\( \text{expr} \ (t \oplus u) \ \rho \ \text{ pk} \) = \( \text{expr} \ t \ \rho \ \text{ pk} \oplus \text{expr} \ u \ \rho \ \text{ pk} \)

\( \text{expr} \ (\text{not } e) \ \rho \ \text{ pk} \) = \( \text{NOT} ( \text{expr} \ e \ \rho \ \text{ pk} ) \)

\( \text{expr} \ (\text{like } a) \ \rho \ \text{ pk} \) = \( a \)

\( \text{expr} \ (\text{nonEmpty}(s, f, w)) \ \rho \ \text{ pk} \) = \( \text{EXISTS} ( \text{sql} (s, \rho', w) \text{ pk } \rho.\text{local}(\rho') ) \) \( \text{where} \ 
\rho' = \rho \triangleleft \text{fresh } f \)

where \( \emptyset \ \rho \ - \) = \( \text{WHERE} \ True \)

where \( \{e_1, e_2, \ldots, e_n\} \ \rho \ \text{ pk} \) = \( \text{WHERE} \ \text{expr} \ e_1 \ \rho \ \text{ pk} \ \text{AND} \ \text{expr} \ e_2 \ \rho \ \text{ pk} \ \text{AND} \ \ldots \ \text{AND} \ \text{expr} \ e_n \ \rho \ \text{ pk} \)

where \( \rho \ \text{ pk } \text{lo}c\text{al} \) = \( \text{AND} \ \rho(\text{lo}c\text{al}.\text{key}) = \rho(\text{lo}c\text{al}.\text{up}.\text{key}) \text{ where} \ 
\text{key} = \rho(\text{lo}c\text{al}.\text{whole}) \)

from \( \rho \ \text{ pk } \text{lo}c\text{al} \) = \( \text{FROM} \ \text{lo}c\text{al}.\text{part} \ AS \ \rho(\text{lo}c\text{al}) \text{ joins} \) \( \text{where} \ 
\text{joins} = \{\text{eqjoin } \hat{x} \ \rho \ \text{ pk} \ | \ \hat{x} \in \rho, \ \hat{x} = \text{lo}c\text{al} \ominus \hat{y} \ \text{for some} \ \hat{y} \neq ()\} \)

\( \text{eqjoin } \hat{x} \ \rho \ \text{ pk} \) = \( \text{INNER JOIN} \ \hat{x}.\text{part} \ AS \ \rho(\hat{x}) \text{ cond} \) \( \text{where} \ 
\text{cond} = \begin{cases} \text{USING} \ \rho(\hat{x}.\text{whole}) & \text{if} \ \hat{x}.\text{kind} = \text{fold} \\ \text{ON} \ \rho(\hat{x}.\text{up}).(\hat{x}.\text{up}.\text{name}) = \rho(\hat{x}).(\rho(\hat{x}.\text{part})) & \text{otherwise} \end{cases} \)

Figure 26: SQL generation
Once we have implemented $E_{sql}$, we can use it to translate generic queries into SQL statements. For differences (Def. 4) we get:

```python
def differencesSQL : SQL =
    $E_{sql}$[differences : Couples $\rightarrow$ list(S, N)) (Person $\rightsquigarrow$ name)
```

and we adapt expertise (Def. 5) as follows:

```python
def expertiseSQL : SQL =
    $E_{sql}$[expertise : Org $\rightarrow$ list(S) (Department $\rightsquigarrow$ dpt, Employee $\rightsquigarrow$ emp)
```

Unlike other evaluators, $E_{sql}$ requires the relation of primary keys for the involved tables as an additional argument, since this information is not contemplated in the optic model. We use the notation $(t_0 \rightsquigarrow k_0, t_1 \rightsquigarrow k_1, ..., t_n \rightsquigarrow k_n)$ to build such argument. For instance, the primary key associated to the table Person is the column name. If we ignore variable names, the SQL statements which are generated by the previous definitions are exactly the same as those that were introduced in Sect. 5.1.

As can be inferred from Fig. 26, the evaluation of getAll always leads to a SQL select statement, unless an error condition is present. The resulting query does not contain nested subqueries, beyond the ones that emerge in the context of exists (due to the nonEmpty term). The FROM clause uses INNER JOINS as the means to navigate downwards the tables in the model. Besides the previous elements, the evaluator just produces expressions with basic functions, operators and literals; no additional SQL features are required.

Clearly, the SQL semantics are not as neat as those associated the XML infrastructure (Sect. 4), since they require a non-trivial normalization into triplets prior to generating the SQL statement. Besides, such generation is partial, and thus triplets must meet certain conditions to guarantee a correct translation. Fortunately, as we will see in the next section, we can take a different path towards the generation of SQL where we can rely on existing work on language-integrated query. Despite this fact, in Sect. 8 we will discuss why the triplet normalization is still relevant.

6. T-LINQ

This section introduces Optica as a higher level language that we can interpret into comprehensions. In particular, we generate T-LINQ [3] queries from optic expressions, essentially following a similar translation implemented in the Links language in [44]. By doing this, we demonstrate that the compositional style embraced by optics can be fruitfully exploited in order to generate comprehension expressions automatically. Moreover, we open the possibility of delegating the arduous task of generating SQL statements from optic expressions, described in the previous section, to the existing translation and normalization techniques of T-LINQ. As usual, we supply a brief background on the querying language, T-LINQ, and then we show the non-standard semantics that is needed in order to produce the corresponding comprehension-based queries.

6.1. Background

In order to manually adapt the expertise query (Def. 5) as a T-LINQ expression, we will first review the difference between a relational (or flattened) model and a nested model. Figure 27

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This section omits the couples example for the sake of brevity. We select expertise over differences since we consider it to be more challenging.
shows the nested (NestedOrg) and flat models (Org) for the organization example from [3], as T-LINQ records. Note that Org differs from the nested version NestedOrg in the type of their fields, since it contains textual values which act as foreign keys to refer to the corresponding entities. In fact, the second version has a strong correspondence with the SQL tables that we introduced in Sect 5.1. Cheney et al show the quoted expression that adapts the flattened model into the nested one (Fig. 28), where %org splices the database representation (<>database("Org")>). In particular, the programmer understands such representation as a list of entities from the relational model; therefore, she can use the widespread notation of list comprehensions to implement the desired queries, where filtering (if ... then) and mapping (yield) features are also available. Figure 29 shows the implementation of the expertise query in terms of T-LINQ, which builds upon the flattened model. Later on, we will see that the nested model becomes essential in the evaluation of Optica expressions, where we will try to produce an equivalent for expertiseTlinq from the evaluation of expertise.

```csharp
def nestedOrg = <*>
    for d in %org.departments do
        yield {dpt = d.dpt, employees =
            for e in %org.employees do
                if d.dpt = e.dpt then
                    yield {emp = e.emp, employees =
                        for t in %org.tasks do
                            if e.emp = t.emp then
                                yield {tsk = t.tsk} }} @>
```

Figure 28: From flat to nested organization model

---

26 T-LINQ does support a compositional style, where analogous combinators for all, any, etc. could be supplied [3] Sect. 3.2. Using these combinators and the nested version of the organizational model, NestedOrg, the expertiseTlinq could be written more concisely. Then, and thanks to its normalization engine, the query could be rewritten to its equivalent version over the relational model.
def expertiseTlinq = <@
for d in %org.departments do
  if not exists
    for e in %org.employees do
      if d.dpt = e.dpt ∧ not exists
        for t in %org.tasks do
          if e.emp = t.emp ∧ t.tsk = "abstract" then yield t.tsk
        then yield e.emp
      then yield d.dpt @>

Figure 29: T-LINQ analogous for expertise

6.2. T-LINQ non-standard Semantics

As usual, we provide $E^{\text{tlinq}}$ in order to evaluate Optica expressions into T-LINQ expressions. Prior to this, we need to determine the semantic domains for this evaluation by means of $T^{\text{tlinq}}$, which is shown in Fig. 30. As expected, this semantic function maps Optica types to T-LINQ representation types ($\text{Repr}$). In particular it just relies on an auxiliary function $T^{aux}$ and wraps the resulting type with $\text{Expr}$. The implementation of $T^{aux}$ is direct for base types, whereas tuples are represented as records. Concerning query types, its representation is also straightforward since functions are directly supported by T-LINQ, although we map $\text{option}$ to $\text{list}$, since such datatype is not contemplated by T-LINQ. Last, optic types are simply represented by the query type they generate. The next sections present the semantic domains for domain types, the implementation of $E^{\text{tlinq}}$ and discusses the final results.

$$T^{\text{tlinq}}[t] = \text{Expr}<T^{\text{aux}}[t] >$$

$$T^{\text{aux}}[\text{N}] = \text{int}$$
$$T^{\text{aux}}[\text{B}] = \text{bool}$$
$$T^{\text{aux}}[\text{S}] = \text{string}$$
$$T^{\text{aux}}[(\alpha,\beta)] = [\text{1} : T^{\text{aux}}[\alpha], \text{2} : T^{\text{aux}}[\beta]]$$
$$T^{\text{aux}}[\alpha \to \beta] = T^{\text{aux}}[\alpha] \to T^{\text{aux}}[\beta]$$
$$T^{\text{aux}}[\alpha \to \text{option}\beta] = T^{\text{aux}}[\alpha] \to \text{list} T^{\text{aux}}[\beta]$$
$$T^{\text{aux}}[\alpha \to \text{list}\beta] = T^{\text{aux}}[\alpha] \to \text{list} T^{\text{aux}}[\beta]$$
$$T^{\text{aux}}[\text{getter} \alpha \beta] = T^{\text{aux}}[\alpha \to \beta]$$
$$T^{\text{aux}}[\text{affine} \alpha \beta] = T^{\text{aux}}[\alpha \to \text{option}\beta]$$
$$T^{\text{aux}}[\text{fold} \alpha \beta] = T^{\text{aux}}[\alpha \to \text{list}\beta]$$

Figure 30: Semantic domains of the T-LINQ evaluation

6.2.1. Evaluating domain and core primitives

This section introduces the evaluation of domain and core terms into T-LINQ expressions. As we have already seen, all domain terms represent optic expressions, and thus they have to be adapted as functions. Figure 31 shows the semantic domains (by extending $T^{\text{aux}}$) and the
evaluation of the terms in the organization example. Note how the organization types are mapped
to the corresponding nested (instead of relational) types. This aspect will be relevant later on
while generating the target queries. Back to the evaluation of terms, we can see that this is
essentially a T-LINQ adaptation of the code that we presented in \textit{OrgModel} (Sect. 2.2.2) where we
used lambda expressions to build concrete optics.

\[
\begin{align*}
&T^{\text{aux}}[\text{Org}] = \text{NestedOrg} \\
&T^{\text{aux}}[\text{Department}] = \text{NestedDepartment} \\
&T^{\text{aux}}[\text{Employee}] = \text{NestedEmployee} \\
&T^{\text{aux}}[\text{Task}] = \text{Task} \\
\end{align*}
\]

\[
\begin{align*}
&\text{Etlinq[departments : fold Org Department]} = <@ \text{fun}(ds) \rightarrow ds @> \\
&\text{Etlinq[dpt : getter Department S]} = <@ \text{fun}(d) \rightarrow d.dpt @> \\
&\text{Etlinq[employees : fold Department Employee]} = <@ \text{fun}(es) \rightarrow es @> \\
&\text{Etlinq[emp : getter Employee S]} = <@ \text{fun}(e) \rightarrow e.emp @> \\
&\text{Etlinq[tsk : fold Employee Task]} = <@ \text{fun}(ts) \rightarrow ts @> \\
&\text{Etlinq[t : getter Task S]} = <@ \text{fun}(t) \rightarrow t.tsk @>
\end{align*}
\]

Figure 31: T-LINQ semantic domains and non-standard semantics for organization extension

The evaluation of core combinators (Fig. 32) also shares a strong resemblance with those
that we have seen for their concrete counterparts in Sect. 2.1. In essence, the difference lies
in the fact that concrete optics build directly upon the type system of Scala and the T-LINQ
interpretation on its own type system. Thus, the evaluation of \textit{∅∅∅} creates a T-LINQ lambda
expression using T-LINQ records instead of using the lambda expressions and products of Scala.
Similarly, \textit{∅∅∅} and \textit{∅∅∅} implement composition by using directly the primitives of T-LINQ,
whereas the implementation of this combinator in concrete optics is based upon the standard
Scala implementation.

6.2.2. Target queries and results
The last step towards the generation of final queries is supplying the non-standard seman-
tics for queries, which are shown in Fig. 33. This step is trivial since they share the very same
semantic domain as their associated optics; therefore, we just need to evaluate their optic argu-
ment. However, and in order to produce the final queries, there is a non-negligible disagreement
that we need to address: the T-LINQ expressions which are generated by \textit{∅∅∅} refer to entities
from the nested model, as introduced by Fig. 31. To resolve this mismatch, we need to reconcile
the relational model with the nested model, so that we can use \textit{nestedOrg} (Fig. 28) for the task.
Thereby, we just supply the nested data to the T-LINQ lambda expression generated from the
Optica expression:

\[
\text{def expertiseTlinq} = <@ \text{Etlinq[expertise : Org \rightarrow list S]} \text{\%nestedOrg @}>
\]

This produces an alternative implementation of the query which was presented in Fig. 29. How-
ever, the T-LINQ expression generated by the new version is much more difficult to read and less
efficient, given the complexity introduced by \textit{nestedOrg}. Fortunately, this is not a problem, since
both queries share the very same normal form, and consequently, they produce the same SQL
statement.
\[ E_{\text{term}}[\text{id} : \text{getter }\alpha \alpha] = <\oplus \text{fun}(a) \rightarrow a @> \]
\[ E_{\text{term}}[g \gg h : \text{getter }\alpha \gamma] = <\oplus \text{fun}(a) \rightarrow %E_{\text{term}}[h : \text{getter }\beta \gamma] (%E_{\text{term}}[g : \text{getter }\alpha \beta] a) @> \]
\[ E_{\text{term}}[g \gg Saints h : \text{getter }\alpha (\beta, \gamma)] = <\oplus \text{fun}(a) \rightarrow \{1 = %E_{\text{term}}[g : \text{getter }\alpha \beta] a, 2 = %E_{\text{term}}[h : \text{getter }\alpha \gamma] a\} @> \]
\[ E_{\text{term}}[\text{like } b : \text{getter }\alpha \beta] = <\oplus \text{fun}(a) \rightarrow %E_{\text{term}}[\text{getter }\alpha \beta] a @> \]
\[ E_{\text{term}}[\text{filtered } p : \text{getter }\alpha \beta] = <\oplus \text{fun}(a) \rightarrow %E_{\text{term}}[g : \text{getter }\alpha \beta] a \oplus %E_{\text{term}}[h : \text{getter }\alpha \gamma] a @> \]

Figure 32: T-LINQ non-standard semantics for optic terms
\[ \mathcal{E}^{\text{linq}}[\text{get } g : \alpha \to \beta] = \mathcal{E}^{\text{linq}}[g : \text{getter } \alpha \beta] \]

\[ \mathcal{E}^{\text{linq}}[\text{preview } g : \alpha \to \text{option } \beta] = \mathcal{E}^{\text{linq}}[g : \text{affine } \alpha \beta] \]

\[ \mathcal{E}^{\text{linq}}[\text{getAll } g : \alpha \to \text{list } \beta] = \mathcal{E}^{\text{linq}}[g : \text{fold } \alpha \beta] \]

Figure 33: T-LINQ non-standard semantics for query terms

7. S-Optica: Optica as a Scala library

This section aims at implementing the Optica DSL in Scala. The resulting library (which we call S-Optica) is provided as a proof-of-concept of the feasibility of extending existing libraries for LINQ, especially those based on comprehensions with optic capabilities. We will show in detail the S-Optica implementation of the syntax and type system of Optica, as well as its standard semantics. The reader may want to look into the accompanying sources for more information about the S-Optica implementation of the interpreters for XQuery, SQL and T-LINQ. The S-Optica implementation is also intended to serve as an illustration of the tagless-final style \[21\] that we have chosen in order to implement our DSL.

7.1. Syntax and type system

In the tagless-final style, the syntax and type system of a typed DSL is implemented through a type constructor class\[27\] which represents the class of representations, or possible interpretations, of that DSL. This type class does not need to be a single, monolithic module, but it is usually decomposed into different type classes which encode different aspects of the DSL. In our case, the division of classes has taken into consideration the structure of optics and combinators that we followed in Sect. 2, and the difference between optic and query types as introduced in Sect. 3.1. Accordingly, Fig. 34 shows the syntax and semantics of the Optica fragment corresponding to getters, affine folds and folds; Fig. 35 shows the implementation of the fragment of queries, as well as the overall Optica type class. Some comments on the implementation follow below:

- The primitive combinators of the different types of optics, getters, affine folds and folds, are implemented in their respective modules. Those which are not primitive, but can be defined in terms of other combinators, namely, \texttt{any}, \texttt{all}, \texttt{elem} and \texttt{empty}, are defined in the \texttt{OpticCom} type class.

- The implementation of these derived combinators benefits from the same syntactic enhancements that we assumed in Sect. 2.1. In fact, their implementation is literally the same as that for concrete optics shown in Fig. 3. The differences simply lie in their signatures and the intended semantics: whereas the implementations of Fig. 3 only work for concrete optics, the implementations of Fig. 34 work for any optic representation \texttt{Repr[\_\_]}.

Thus, we may instantiate this class in order to work with concrete optics, or any other standard representation such as van Laarhoven or profunctor optics; of course, we may also instantiate this class in order to work with XQuery, TripletFun or T-LINQ, since these are legitimate read-only optic representations as have been shown throughout the paper.

\[27\] We supply a brief tour of how to encode type (constructor) classes in Scala in Appendix A.1.
• We actually used concrete optic types in the signatures of these type classes, i.e. the types `Getter[_,_]`, `Affine[_,_]`, etc. (to which the signatures refer to) are exactly those defined in Sect. 2.1. How can these signatures work for any representation, then? The reason is simply that these combinators do not receive and return plain concrete optics types, but their representations: the `empty` Combinator does not receive a concrete fold, but anything that counts as a fold optic. Concrete types thus behave mostly as phantom types, which specify the abstract semantic domains of the language and aid in the definition of its type system.

• The query types of Optica correspond in the tagless-final style to observations [19]. These can be understood as the standard interpretations that we demand from any representation of the implemented DSL. This matches perfectly with the distinction between optics and query types: for instance, we will always want a `getAll` interpretation from a fold program, irrespective of the optic representation. In the tagless-final style, we commonly assign different type constructors to DSL expressions and observations. Thus, in Fig. 35 we use `Repr[_]` and `Obs[_]` for optic and query representations, respectively. This is actually equivalent to having two different DSLs, one for optics and another for queries, into which optics are compiled.

• Base types of Optica also enjoy a different representation. As the implementation of the `like` combinator shows, base values are represented using the very type system of the host language, i.e. Scala. Thus, its representation is not `Repr[_]` nor `Obs[_]`, but the identity type constructor. This representation for base types is also common practice in tagless-final style.

• To avoid overloading the `like` method for the different base types, `Int`, `String`, and `Boolean`, we use the GADT `Base`, whose object instances are marked as implicits, thereby enabling the context bound syntax in Scala. The `Base` GADT is also declared in those signatures that depend on the `like` combinator, namely, `elem`, and the combinator `equal`.

7.2. Domain queries

In order to write domain queries, we need to extend the syntax and type system of the Optica language, as we have seen in Sect. 3.2. Quoting from [47], “extensibility is the strong suite of the tagless-final embedding”; therefore, this task should be easy. Indeed, we simply need to declare a new type class where we have a component containing an entry for each domain optic in the model, as shown in Fig. 36. The types `Couples`, `Person`, etc., are immutable data structures which mostly behave as phantom types and aid in the extension of the type system of the language.

Once we have the core and domain primitives available, we should be able to implement generic optic expressions by declaring both dependencies, the combinators of the Optica API and the domain model (note that observations are not needed to write pure optic expressions):

---

28We would run into problems, however, if the target optic representation does not also use itself the Scala types for representing base types.

29This constraint could be slightly lifted since we may want to compare not only base, but model types in general (cf. Fig. 4).
trait GetterCom[Repr[_]] {
  def id[S]: Repr[Getter[S, S]]
  def andThen[S, A, B](u: Repr[Getter[S, A]], d: Repr[Getter[A, B]]): Repr[Getter[S, B]]
  def fork[S, A, B](l: Repr[Getter[S, A]], r: Repr[Getter[S, B]]): Repr[Getter[S, (A, B)]]
  def like[S, A: Base](a: A): Repr[Getter[S, A]]
  def not[S](b: Repr[Getter[S, Boolean]]): Repr[Getter[S, Boolean]]
  def equal[S, A: Base](x: Repr[Getter[S, A]], y: Repr[Getter[S, A]]): Repr[Getter[S, Boolean]]
  def greaterThan[S](x: Repr[Getter[S, Int]], y: Repr[Getter[S, Int]]): Repr[Getter[S, Boolean]]
  def subtract[S](x: Repr[Getter[S, Int]], y: Repr[Getter[S, Int]]): Repr[Getter[S, Int]]
}

trait AffineFoldCom[Repr[_]] {
  def id[S]: Repr[AffineFold[S, S]]
  def andThen[S, A, B](u: Repr[AffineFold[S, A]], d: Repr[AffineFold[A, B]]): Repr[AffineFold[S, B]]
  def filtered[S](p: Repr[Getter[S, Boolean]]): Repr[AffineFold[S, S]]
  def to[A](gt: Repr[Getter[S, A]]): Repr[AffineFold[S, A]]
}

trait FoldCom[Repr[_]] {
  def id[S]: Repr[Fold[S, S]]
  def andThen[S, A, B](u: Repr[Fold[S, A]], d: Repr[Fold[A, B]]): Repr[Fold[S, B]]
  def nonEmpty[S, A](fl: Repr[Fold[S, A]]): Repr[Getter[S, Boolean]]
  def to[A](afl: Repr[AffineFold[S, A]]): Repr[Fold[S, A]]
}

trait OpticaCom[Repr[_]] extends GetterCom[Repr] with AffineFoldCom[Repr] with FoldCom[Repr] {
  def empty[S, A](fl: Repr[Fold[S, A]]): Repr[Getter[S, Boolean]] = fl.isEmpty.not
  def all[S, A](fl: Repr[Fold[S, A]]): Repr[Getter[S, Boolean]] = fl.isEmpty.filter(p => p.isEmpty).isEmpty
  def any[S, A](fl: Repr[Fold[S, A]]): Repr[Getter[S, Boolean]] = fl.isEmpty.filter(p => p.isEmpty).not
  def elem[S, A: Base](fl: Repr[Fold[S, A]])(a: A): Repr[Getter[S, Boolean]] = fl.isEmpty.any(id == like(a))
}

Figure 34: OpticaCom semantics (optic combinators).
trait GetterQuery[Repr[_], Obs[_]] {  
def get[S, A](gt: Repr[Getter[S, A]]): Obs[S => A]
}

trait AffineFoldQuery[Repr[_], Obs[_]] {  
def preview[S, A](af: Repr[AffineFold[S, A]]): Obs[S => Option[A]]
}

trait FoldQuery[Repr[_], Obs[_]] {  
def getAll[S, A](fl: Repr[Fold[S, A]]): Obs[S => List[A]]
}

trait Optica[Repr[_], Obs[_]] extends OpticaCom[Repr]
  with GetterQuery[Repr, Obs]
  with AffineFoldQuery[Repr, Obs]
  with Fold[Repr, Obs]

Figure 35: Optica symantics (generic combinators and queries).

trait CoupleModel[Repr[_]] {  
def couples: Repr[Fold[Couples, Couple]]  
def fst: Repr[Getter[Couple, Person]]  
def snd: Repr[Getter[Couple, Person]]  
def name: Repr[Getter[Person, String]]  
def age: Repr[Getter[Person, Int]]
}

Figure 36: Couple domain symantics

def differencesFl[Repr[_]](implicit O: OpticaCom[Repr], M: CoupleModel[Repr]): Repr[Fold[Couples, (String, Int)]] =
couples >>= filtered((fst >>= age) > (snd >>= age)) >>=
{fst >>= name} * ((fst >>= age) - (snd >>= age))

and generic query expressions (in this occasion, we pass the whole Optica type class, which includes the queries):
def differences[Repr[_], Obs[_]](implicit O: Optica[Repr, Obs], M: CoupleModel[Repr]): Obs[Couples => List[(String, Int)]] =
differencesFl.getAll

As can be seen, the required primitives are injected using the Scala implicit mechanism. In contrast with Def. 4, this version remarks the aforementioned existence of different representations for optics and queries, as evidenced by the result types. Scala implicits are also exploited by the library to omit invocations to casting methods, although the required syntax is not shown for brevity.

7.3. Standard semantics

Type classes in the tagless-final style are commonly named Symantics, a portmanteau of ‘syntax’ and ‘semantics’, to emphasise the fact that the same abstraction serves a double purpose: the type class declaration defines the syntax and type system of the language, whereas type class instances provide its semantics. The standard semantics of the language is no exception, and for this purpose we greatly benefit from having reused the standard semantic domains at the syntactic
level: simply use the identity type lambda $\lambda [x \mapsto x]$ for both the Repr and Obs parameters, and map each primitive into its concrete counterpart.

We can find the interpretation that supplies the standard semantics of Optica in Fig. 37. In particular, it is represented by the singleton object $R$, which is also a common name for meta-circular interpretations in the tagless-final style. We follow the very same pattern to instantiate the couple domain terms, as we show in Fig. 38.

Now we can use the standard semantics to evaluate generic queries, and to re-implement, in a modular way, the ad-hoc functions that deal with immutable data structures. For instance:

```java
val differencesR: Couples => List[(String, Int)] =
  differences[λ [x => x], λ [x => x]](R, CoupleModelR)
```

As can be seen, we have specified the standard representation types for optics and queries alongside the associated evidences. Fortunately, they could be inferred implicitly, as shown in this alternative and preferred version.

```java
val differencesR: Couples => List[(String, Int)] = differences
```

The resulting function is extensionally equal to $\text{differences}$ from Sect. 2.2.1. The implementation of the rest of interpretations in this article (XQuery, SQL and T-LINQ) follows the same principles. Interested readers can find a README file in the companion sources [39] which briefly describes the library structure and supplies links to the aforementioned interpreters —and other relevant modules.

8. Discussion

The language of optics.

One the most prominent sought-after features of optics is modularity, i.e. the capacity of creating optics for compound data structures out of simpler optics for their parts. This is specially emphasized in the framework of profunctor optics [32], where optic composition builds upon plain function composition and enables straightforward combinations of isos, prisms, lenses, affine traversals, and traversals. The profunctor representation is particularly convenient to implement (and even reveal) the compositional structure of the different varieties of optics, but, in essence, this structure is also enjoyed by concrete optics, van Laarhoven optics, etc. Modularity is a feature of the language of optics, rather than of any particular representation. This paper has shown, albeit for a very restricted subset of optics (getters, affine folds and folds), that this compositional structure of optics can be encoded in the type system of a formal language, that we have named Optica. The denotational semantics of this language was given in terms of concrete optics but any other isomorphic representation, such as profunctor optics, may have served as well.

Now, the specification of Optica includes not only the compositional features of read-only optics but also, and significantly, their characteristic queries. Taking into account this non-compositional character of optics is essential as soon as we tackle the extension of Optica with new varieties of optics. For instance, the major difference between folds and traversals is not found in their compositional properties, but in the queries that they must support: besides $\text{getAll}$, traversals must also support a $\text{putAll}$ query to replace the content of the elements that they are selecting.

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30This is the common case in which standard semantic domains do not eventually behave as phantom types.
trait RGetterCom extends GetterCom[λ x => x] {  def id[S] = Getter.id  def andThen[S, A, B](u: Getter[S, A], d: Getter[A, B]) = Getter.andThen(u, d)  def fork[S, A, B](l: Getter[S, A], r: Getter[S, B]) = Getter.fork(l, r)  def like[S, A: Base](a: A) = Getter.like(a)  def not[S](b: Getter[S, Boolean]) = Getter.not(b)  def eq[S, A: Base](x: Getter[S, A], y: Getter[S, A]) = Getter.eq(x, y)  def gt[S](x: Getter[S, Int], y: Getter[S, Int]) = Getter.gt(x, y)  def sub[S](x: Getter[S, Int], y: Getter[S, Int]) = Getter.sub(x, y)}

trait RAffineFoldCom extends AffineFoldCom[λ x => x] {  def id[S] = AffineFold.id  def andThen[S, A, B](u: AffineFold[S, A], d: AffineFold[A, B]) = AffineFold.andThen(u, d)  def filtered[S](p: Getter[S, Boolean]) = AffineFold.filtered(p)  def as[S, A](gt: Getter[S, A]) = gt}

trait RFoldCom extends FoldCom[λ x => x] {  def id[S] = Fold.id  def andThen[S, A, B](u: Fold[S, A], d: Fold[A, B]) = Fold.andThen(u, d)  def nonEmpty[S, A](fl: Fold[S, A]) = fl.nonEmpty  def as[S, A](af: AffineFold[S, A]) = af}

trait RGetterAct extends GetterAct[λ x => x, λ x => x] {  def get[S, A](gt: Getter[S, A]) = gt.get}

trait RAffineFoldAct extends AffineFoldAct[λ x => x, λ x => x] {  def preview[S, A](af: AffineFold[S, A]) = af.preview}

trait RFoldAct extends FoldAct[λ x => x, λ x => x] {  def getAll[S, A](fl: Fold[S, A]) = fl.getAll}

implicit object R extends Optica[λ x => x, λ x => x]  with RGetterCom  with RGetterAct  with RAffineFoldCom  with RAffineFoldAct  with RFoldCom  with RFoldAct

Figure 37: Optica standard semantics.

implicit object CoupleExampleR extends CoupleModel[λ x => x] {  val couples = CoupleModel.couples  val fst = CoupleModel.fst  val snd = CoupleModel.snd  val name = CoupleModel.name  val age = CoupleModel.age}

Figure 38: Couple domain standard semantics
On the implementation side, we have found the typed tagless-final approach especially suitable in order to encode this separation of concerns between declarative optic combinators and their intended queries. In essence, it closely corresponds to the difference between representations of the DSL and their observations or interpreters. Another essential feature from the tagless-final pattern that we plan to profit from is extensibility. In particular, new optics will be added to S-Optica through their own type classes (as we have done for getters, affine folds and folds) so that we can fully reuse old queries without recompiling sources.

Optics versus comprehensions

Optics can be seamlessly combined with comprehensions, as shown in Sect. 6. Indeed, by using the T-LINQ interpreter of Optica we can freely mix optic expressions with general comprehension queries. In this way, optics may play within comprehensions a similar role to that which is played by XPath within XQuery [44]. In the following paragraphs, we discuss the basic trade-off between expressiveness and modularity of comprehension and Optica queries, so as to better appreciate their role in the LINQ landscape.

The separation of concerns between declaratively selecting parts of a data structure and building a variety of queries related to those parts is the cornerstone of optics. In this regard, the LINQ approach based on comprehensions focuses on the query building side and, commonly, on constructing queries of a simple kind: retrieval queries denoting a multiset (the semantic domain for queries on QUEA [19], T-LINQ [3], NRC [13], etc.). The optics approach is, hence, potentially more modular. For instance, a representation of traversals intended for SQL should allow us to generate both select and update statements for the queries getAll and putAll, respectively. We plan to deal with this extension and its trade-offs with expressiveness in future versions of Optica.

We can still claim further modularity advantages of optics over comprehensions. Basically, these are due to the fact that optics provide a language which is more akin to relational algebra than the calculus approach that monads provide for comprehensions [29]. Arguably, the support for functional abstraction and intermediate nested data of comprehensions languages and systems such as Links, T-LINQ or DSH offer leads also to highly compositional queries. We can find examples, however, where the difference in style manifests itself. For instance, this is the query that remains to complete Table 1, in the style of S-Optica:

```scala
def under50_d(Repr[_], Obs[_])(implicit O: Optica[Repr, Obs], M: CoupleModel[Repr]): Obs[Couples] => List[String] =
(couples ≫ fst ≫ filtered (age < 50) ≫ name).getAll
```

which we compare to an analogous query using the Scala implementation of T-LINQ:

```scala
def under50_e(Repr[_])(couples: Repr[Couples])(implicit Q: Tlinq[Repr], N: CoupleNested[Repr]): Repr[List[String]] =
foreach(couples)(c => where (c.fst.age < 50) (yields (c.fst.name)))
```

31 DSH, in particular, comes with an extensive catalog of list-processing combinators: [https://github.com/ulricha/dsh/blob/master/src/Database/DSH/Frontend/Externals.hs](https://github.com/ulricha/dsh/blob/master/src/Database/DSH/Frontend/Externals.hs)

32 Indeed, our version of the expertise query in Sect. 6 is no more simple than the equivalent version using nested data in [19].

33 Tlinq[_[_]] provides the tagless-final implementation in Scala of T-LINQ, that we have used to implement the corresponding interpreter for S-Optica. The role of CoupleNested in the sample query is similar to the OrgNested model in Sect. 6.3.
As this example shows, in adopting the language of optics, modularity is improved in several respects. First, as we have mentioned earlier, the query is actually composed of two major parts: the optic expression, which declares what to select from, and the query expression, which actually specifies the kind of query to be executed over the selection. Second, the optic expression is unaware of variables and builds upon finer grained and reusable modules, such as couples, \texttt{fst}, \texttt{age} and \texttt{name}. This results in pure algebraic queries that are arguably more simple to compose and maintain. In essence, we are building out of simpler optics in a purely compositional style, and deriving queries in one shot.

The downside of the optics approach in relation to comprehensions, at least in the current version of Optica, is its limited expressiveness. Indeed, variables are fruitfully exploited in comprehensions to express arbitrary joins (e.g. cyclic) whereas optic queries appear to move only downwards from the root of the hierarchy. Relational models are more general than nested models, providing the programmer with better navigation tools \cite{48}, and therefore not every model is expressible in Optica. Take the couple model as an example. We assume that each person is hanging from a couple, so that we can find them by diving into the couple fields \texttt{fst/snd}. However, the relational model is able to supply more entries for people who do not necessarily form a couple. To alleviate this problem, we may introduce a new fold \texttt{people} besides the existing \texttt{couples}, sharing a virtual root type as source. The connections between \texttt{people} and \texttt{fst/snd} would still be unclear in the optic model; therefore, new mechanisms should be introduced in order to establish the precise relationship among them. We leave for future work a more precise investigation of the compared expressiveness of the comprehension and optic languages, as well as the extension of Optica with already supported features in comprehension languages like grouping, aggregation and order-by queries \cite{19, 20, 49}.

\textbf{Optics as a general query language.}

The role of optics in LINQ expands beyond its combined use with comprehensions. By lifting optics into a full-fledged DSL, we have opened the door to non-standard interpretations that directly translate the language of optics to data accessors for alternative representations beyond immutable data structures. For instance, we have provided an interpretation to turn Optica queries into XQuery expressions where we have seen that the connection among them is straightforward, leading to a compositional interpreter. The translation ignores the XQuery FLWOR syntax and basically focuses on XPath features. Indeed, we understand XPath as a language to select parts from an XML document, which makes it a perfect example of optic representation. Moreover, since XPath does not provide the means to update an XML document, it also fits perfectly with read-only optics such as getters, affine folds and folds.

It might be worth mentioning that synergies among optics and XML are by no means new. In fact, prominent optic libraries are extended with modules to cope with XML\footnote{https://hackage.haskell.org/package/xml-lens-0.1.6.3/docs/Text-XML-Lens.html} or JSON documents, even packaged as domain-specific query languages, such as JsonPath\footnote{https://github.com/julien-truffaut/jsonpath.pres}. In these projects, standard optics facilitate the definition of these DSLs for querying JSON or XML documents. Nevertheless, our approach is radically different since we provide a general optic language in order to build generic optics which may be translated over those DSLs (JsonPath, XQuery, etc.).

Our approach also differs from others where the process is reversed and a translation of XPath expressions into a general query language based on comprehensions is performed \cite{3}. In
this spirit, we could also relate this paper with SilkRoute [50], where the database administrator exposes the database application data in terms of a public XML view, and external components issue XQuery application queries against it. Then, the framework translates them into one or more SQL statements and collects the results from the inner database, which are returned as XML documents. One of the key aspects of SilkRoute is view forests, a concept that is exploited by the framework to separate the XML structure from its computation. On its part, Optica exposes a hierarchy of domain optics that external components may use to compose optic expressions, as application queries. In addition, we could understand view forests as a kind of optic since they also select parts from the underlying database. However, Optica is more general considering that the same application queries can be reused against different targets, and not only SQL.

Nevertheless, SQL is the primary target of classical LINQ with comprehensions, and we have also provided a non-standard SQL interpreter for Optica. Commonly, comprehension-based queries need to be flattened in order to guarantee a good performance: the naive translation to SQL is not optimal since it typically leads to nested subqueries. Moreover, translations of flat-flat queries to SQL are guaranteed to be total and to avoid the problem of query avalanche [16]. In systems like Links or T-LINQ, these guarantees are even statically checked. In Sect. 5.2.4, our translation to SQL attains similar guarantees concerning the type of generated queries, which are absent of subqueries, beyond those that are generated by EXISTS, which are unavoidable. However, failures in query generation are signalled at run-time rather than at compile time. We plan to solve this limitation in future work by using the optimization techniques that the tagless-final approach offers [19]. Our translation process resembles the denotational approach of SQUR [20] and Links [51] rather than the rewriting approach followed in [16, 3, 19]. In particular, we use an intermediate language TripletFun to decouple the filtering, selection and collection aspects of the final SQL query. We differ from SQUR, however, because the ultimate translation to SQL is performed directly from this non-standard semantic domain rather than from a normalized optic query. We plan to incorporate normalization and partial evaluation in future work, which will be convenient as soon as we extend the language with projections first and second, in correspondence with the fork combinator.

Given the existing translation to comprehensions from Optica and the established results concerning the generation of SQL from comprehensions the usefulness of TripletFun for this purpose is certainly relative. However, this demonstrates an instance of optic representation in the relational setting, which we believe to have the potential of being very useful when we extend our results for optics with updating capabilities. In this light, we intend to exploit the very same TripletFun representation to generate both SELECT and UPDATE statements. Moreover, the TripletFun interpreter represents an example of complex translation using an intermediate optic representation, which resembles the denotational approach of [20] but performed in the algebraic setting of optics rather than in the relational calculus of comprehensions. This semantic style may serve as a reference for similar complex interpreters, e.g. for NoSQL databases such as MongoDB [52].

**Optica versus ORMs and LINQ libraries**

Connections between optics and databases are widespread. As a matter of fact, lenses emerged in this context [33] under the umbrella of bidirectional programming. We remark [33] as a recent work in this field, where a practical approach to the view update problem is introduced by means of the so-called incremental relational lenses. Although we still do not know if extending Optica will lead us to contemplate views in the non-standard SQL semantics, we find this research essential to deal with updating optics in an effective way.
S-Optica and object-relational mappers (ORMs), like Hibernate, pursue similar goals: they aim at working with data in persistent stores as if it were plain in-memory data. However, S-Optica uses the language of optics while ORMs try to remain as close as possible to the customary object-oriented style. These are other relevant differences:

- S-Optica does not stick to relational databases as its preferred target infrastructure. In fact, Sect. 3 and Sect. 4 show that in-memory immutable data structures and XML files are also potential sources of information.

- S-Optica is eminently declarative. Indeed, S-Optica queries are simply values that do not produce side effects on their own. This contrasts with ORMs, where it requires a huge understanding of the particular ORM to identify which queries are being launched at any time. The declarative style of S-Optica enables compositionality as well as the possibility to introduce further optimizations.

- S-Optica queries are expressive and well-typed. Many ORMs introduce contrived additional languages to express queries, and their expressions are usually presented as plain strings, so that errors are not detected at compile time.

- ORMs usually consider the notion of object as the smallest granularity concept to deal with, while S-Optica supports queries that select very specific parts from the whole data.

- ORMs are able to write data back to the store, while this feature is future work in Optica.

In general, ORMs have been used for a long time and they are consequently very mature, while Optica is still an experimental and limited library. However, it already solves many of the problems that are deep-rooted in the ORM approach. The Scala libraries Quill [24] and Slick [25] are arguably the most similar frameworks to S-Optica. The former is strongly inspired by T-LINQ [3] and it therefore follows the same theoretical principles. The major benefit with regard to the original P-LINQ, the F# implementation of T-LINQ, is the ability of Quill to produce the final queries at compile-time, exploiting the Scala metaprogramming facilities [54]. Unfortunately, although Quill provides a `flatMap` method for the type constructor `Query`, it apparently lacks an implementation for `point` which is required to translate some Optica queries into Quill expressions. Slick is similar to Quill, but it does not build upon a theoretical language like T-LINQ. In any case, both Quill and Slick map relational models in Scala in a direct way, i.e. as flat data models, whereas S-Optica works with nested data models and has to solve a bigger impedance mismatch. On the other hand, although Quill and Slick support updates and deletes, they do this with ad-hoc languages that escape the collection-like interface. Optica should be able to supply a standard interface in order to support writes by introducing additional optics. As a final note, we want to recall that, as Sect. 6 points out, Optica should not be seen as a competitor but as a complement for these libraries, since optics and comprehensions were shown to be compatible.

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36Strictly speaking, values are objects in Scala whereas S-Optica queries are polymorphic methods. These may be easily turned into values by using a Church encoding representation.

37For instance, the S-Optica query `(like 1).getAll`.

38A comparison between Quill and Slick (written by Quill's author) is provided here: https://github.com/getquill/quill/blob/master/SLICK.md.
9. Conclusions

This paper has attempted to demonstrate that optics embrace a much wider range of representations beyond concrete, van Laarhoven, profunctor optics and other isomorphic acquaintances. We have shown, for instance, that a restricted subset of XQuery can be properly understood as an optic representation, i.e. as an abstraction whose essential purpose is to allow us to select parts from a data source by using powerful combinators, declaratively, and derive queries from those selectors. From this standpoint, data sources of optic representations may range far beyond general immutable structures: they might be XML documents, as in the case of XQuery, or relational databases. In fact, we have also shown how to derive SQL queries from TripletFun, an optic representation that endorses the separation of concerns between selection, filtering and collection aspects, which characterizes SQL \texttt{SELECT} statements. Strictly speaking, we may say that SQL is not an optic but a query language which is translatable from an optic representation. In future work, we aim at testing the generality of the language of optics through the generation of other effective, idiomatic translations into a diverse range of querying infrastructures. We will particularly pay attention to technologies that are more recent than XQuery, with a clear bias towards nested data models such as document-oriented NoSQL databases like MongoDB \cite{52} \cite{52}, and languages like GraphQL \cite{55}.

We put forward Optica, a full-fledged DSL, to specify what all these representations have in common, i.e. the concept of optic itself. Technically speaking, the type system of Optica encodes the compositional and querying features of getters, affine folds and folds, independently of any particular representation; concrete optics provide the semantic domains for its standard denotational semantics; and XQuery, TripletFun and T-LINQ represent semantic domains for non-standard optic representations. Currently, Optica only pays attention to a very restricted set of optics, namely getters, affine folds and folds. In future work, we will contemplate other optics like lenses, affine traversals or traversals, as well as additional combinators that populate de-facto libraries like Haskell lens and Monocle. This will force us to also pay attention to the laws (e.g. the get-set law of lenses) that the intended queries of optics must comply with. We think optic algebras \cite{56} will be instrumental in that formalization.

The ultimate goal behind this quest for the language of optics has been to show that optics can play a significant role in the theory and practice of language-integrated query. In particular, we have demonstrated how optics can be used as a high-level language in order to derive comprehension queries, the most common approach in the LINQ field nowadays. This has the advantage of allowing programmers to exploit optics, the de-facto standard for dealing with hierarchical data structures, in their LINQ developments. Additionally, the XQuery and SQL interpretations have also shown that the language of optics is general enough to cope with LINQ systems independently from comprehensions. However, in the case of SQL, this is done at the expense of a more limited expressiveness since joins are not currently supported. We plan to investigate possible extensions to Optica based on the compositional encoding of equijoins in \cite{29}. We also plan to investigate future interpretations of Optica into declarative query languages such as Datalog \cite{57} and description logics \cite{58}, as well as its connection with recent developments in comprehension-related languages based on monoids \cite{59}. We also think that Optica in its current shape has a great potential to deal with modern warehouse technologies aimed at data analytics, where updates are not customary.

Optics show a potential to cope not only with retrieval queries but also with updates, a kind of query that is commonly unaddressed in theoretical accounts but patently necessary in practice. This paper lays the foundation to engage with this issue in future work. On the one hand,
extending the syntax and type system of Optica (and S-Optica) with new optic types and combinators is trivial. On the other hand, the feasibility of introducing updates in the interpretation is subject to limitations of the particular infrastructure. For example, XQuery does not support updates (although there are extensions that deal with them [60]), and thus the evaluation of optics with updating capabilities would be partial. As for SQL, it does support updates, but there is a tradeoff with expressiveness: not all relational queries can be updatable views, which introduces a new level of partiality. Whether triplets need to be extended in order to accomodate updates is something that requires further research. Lastly, we are very optimistic about the potential of updates in modern technologies based on nested models [52, 55], where we have carried out several simple experiments with positive results.

Finally, we have implemented a proof-of-concept of Optica and its interpreters in the Scala library S-Optica by using the tagless-final style. Optica is thus implemented as a type class: the class of optic representations and their intended queries. Beyond the generic queries that we have used to guide the explanations, we have tested S-Optica with other queries around the same domains and with new domains that were extracted from the official documentation of Monocle, Slick and Quill. These examples are located in an experimental branch of S-Optica that will be available as soon as the library matures. In this sense, we intend to profit from the many improvements in the imminent release of Scala 3.0, particularly in regard to type classes and metaprogramming facilities [39] with a new implementation of Optica in Dotty [61]. Similar implementations may have also been developed in other languages that support the tagless-final approach, such as Haskell or OCaml. In any case, the results that have been obtained are encouraging enough to anticipate the feasibility of extending existing comprehension-based libraries in these languages for LINQ, with optic capabilities.

Appendix A. Scala Background

This section aims at providing a brief background of those Scala features that we use in this paper. First, Table A.2 supplies a cheat sheet where we can find examples and short descriptions of the abstractions and constructions that we consider to be more relevant in the particular context of this work. As can be seen, some of them are specific to Scala but there are other concepts which are widespread in the functional programming community, where we just want to show how to encode them in this language. Second, we describe the general pattern to encode type classes [62] in Scala [63].

Appendix A.1. Encoding Type Classes in Scala

In Scala, we can use traits to define new type classes. For instance, we encode Functor as follows:

```scala
trait Functor[F[_]] { 
  def fmap[A, B](fa: F[A])(f: A => B): F[B] 
}
```

The trait itself is parameterized with a type constructor `F[_]`; therefore, this is a type constructor class. It declares the `fmap` method, which is parameterized with concrete type parameters `A` and `B` and value parameters `fa` and `f` (organized in two sections [40]). As can be seen, function types

---

[39] <https://dotty.epfl.ch/docs/reference/metaprogramming/toc.html>

[40] Scala supports definitions with multiple groups of value parameters, delimited by parentheses. In this particular situation, the separation turns out to be helpful to improve type inference while invoking the method.
| Abstraction                | Code Example                                                                 | Description                                                                 |
|---------------------------|-----------------------------------------------------------------------------|-----------------------------------------------------------------------------|
| algebraic data type       | **sealed abstract class** `Option[A]`                                        | ADTs are implemented using object inheritance. The example shows `Option`, which is also known as Maybe. |
| case class                | **case class** `Person`                                                        | Defines a class with special features, like construction and observation facilities. |
|                           | `name: String, age: Int`                                                      |                                                                            |
| companion object          | **trait Person**                                                             | Module that serve as a companion to a class or trait with the very same name. We use it to supply class members and provide implicit definitions, like conversors and type class instances. |
|                           | **object Person**                                                            |                                                                            |
| for comprehension         | `for { i ← List(1, 2); j ← List(3, 4); yield i + j } // res: List[Int] = List(4,5,5,6)` | Syntactic sugar for `flatMap, map, etc. Analogous for Haskell's do-notation. |
| function type             | `val f: Int => Boolean = i => i > 0`                                        | Function types are represented with arrows separating domain and codomain. Lambda expressions follow a similar syntax, where the arrow separates parameter and function body. |
| implicit resolution       | `def isum( x: Int)(Implicit y: Int): Int = x + y`                             | Family of techniques to let the compiler infer certain parameters automatically. In the example, `i` is implicitly passed as second argument to `isum`. |
|                           | `implicit val i: Int = 3`                                                     |                                                                            |
|                           | `isum(1) // res: Int = 4`                                                     |                                                                            |
| partial function syntax   | `val f: Option[Int] => Boolean = { case None => true case Some(_) => false }` | Special syntax for those situations where we want to produce an anonymous (potentially partial) function that requires pattern matching on its parameter. |
| placeholder syntax        | `val inc: Int => Int = i => i + 1`                                           | Syntax for lambda expressions where we refer to the parameter as `._`; consequently, there is no need to name it. Both `inc` and `inc2` (placeholder syntax) are equivalent. |
|                           | `val inc2: Int => Int = _ + 1`                                                |                                                                            |
| trait                     | **trait Person {**                                                          | Similar to Java interfaces (as they enable multiple inheritance), but traits support partial implementation of members. |
|                           | `def name: String = "John"`                                                 |                                                                            |
|                           | `def age: Int                  `                                              |                                                                            |
| type parameter            | **trait List[A]**                                                            | Types that are taken as parameters by class or method definitions. It is required a special notation if we expect higher kinded types, like `Repr`. |
|                           | **trait Symantics[Repr[_]]**                                                |                                                                            |
| type lambda               | `λ[ x => x ]`                                                                | Notation enabled by the `kind-projector` compiler plugin to produce anonymous type functions of the kind `x → x`. |
|                           | `λ[ x => Int ]`                                                              |                                                                            |
|                           | `λ[ x => Option[x] ]`                                                        |                                                                            |

Table A.2: Scala Cheat Sheet.
are represented with the arrow \(\Rightarrow\). Now, we could follow the same pattern to provide other type classes, like `Pointed`:

```
trait Pointed[F[_]] {  
  def point[A](a: A): F[A]
}
```

Or `Bind`:

```
trait Bind[F[_]] {  
  def bind[A, B](fa: F[A])(f: A => F[B]): F[B]
}
```

which follow the very same pattern. The previous definitions form the building blocks of `Monad`. Thereby, we could compose them to provide the corresponding type class:

```
trait Monad[F[_]] extends Functor[F] with Pointed[F] with Bind[F] {  
  def fmap[A, B](fa: F[A])(f: A => B) = bind(fa)(a => point(f(a)))
}
```

Here, we exploit the multiple inheritance mechanism provided by Scala to mix the involved traits. At this point, we should be able to implement for `fmap` in terms of `bind` and `point`, once and for all. It is common practice to deploy type class instances in the type class companion object since the Scala compiler will search for instances in this module, among other places. For example, this is the `Monad` companion object, where we have placed the monad instance for `Option` (`Maybe` in Haskell).

```
object Monad {  
  implicit object OptionMonad extends Monad[Option] {  
    def point[A](a: A) = Some(a)  
    def bind[A, B](fa: Option[A])(f: A => Option[B]) = fa match {  
      case None => None  
      case Some(a) => f(a)  
    }
  }
}
```

There are several alternatives to supply an instance. In this occasion, we have decided to implement it as an object `OptionMonad` which is declared with the `implicit` modifier, so that the compiler could find it `implicitly` if necessary. The implementation of `point` and `bind` turns out to be trivial. Once we have defined a type class, we could implement derived functionality. For instance, we could define the typical `join` method.

```
def join[F[_], A](ffa: F[F[A]])(implicit M: Monad[F]): F[A] = M.bind(mma)(identity)
```

This method requires an implicit evidence of `Monad` for `F`, which is used in the implementation to invoke `bind`. Now, we could use the `Option` instance to flatten a nested optional value by means of `join`.

```
join[Option, Int](Option(Option(3)))(Monad.OptionMonad)  
// res: Option[Int] = Some(3)
```

Here, we manually supply the type parameters and the monad evidence. Fortunately, the Scala compiler is able to infer them; therefore, the next version is preferred.

```
join(Option(Option(3)))  
// res: Option[Int] = Some(3)
```

As a final remark, note that a monad instance subsumes instances for the rest of type classes that form it, e.g. `OptionMonad` is also an `Option` instance for `Pointed`.

59
Appendix B. XML Schemas

Appendix B.1. Couple XSD

```xml
<?xml version="1.0" encoding="UTF-8"?>
<xs:schema xmlns:xs="http://www.w3.org/2001/XMLSchema">
  <xs:element name="xml">
    <xs:complexType>
      <xs:sequence>
        <xs:element name="couple" minOccurs="0" maxOccurs="unbounded">
          <xs:complexType>
            <xs:sequence>
              <xs:element name="fst">
                <xs:complexType>
                  <xs:sequence>
                    <xs:element name="name" type="xs:string"/>
                    <xs:element name="age" type="xs:positiveInteger"/>
                  </xs:sequence>
                </xs:complexType>
              </xs:element>
              <xs:element name="snd">
                <xs:complexType>
                  <xs:sequence>
                    <xs:element name="name" type="xs:string"/>
                    <xs:element name="age" type="xs:positiveInteger"/>
                  </xs:sequence>
                </xs:complexType>
              </xs:element>
            </xs:sequence>
          </xs:complexType>
        </xs:element>
      </xs:sequence>
    </xs:complexType>
  </xs:element>
</xs:schema>
```

Appendix B.2. Organization XSD

```xml
<?xml version="1.0" encoding="UTF-8"?>
<xs:schema xmlns:xs="http://www.w3.org/2001/XMLSchema">
  <xs:element name="xml">
    <xs:complexType>
      <xs:sequence>
        <xs:element name="department" minOccurs="0" maxOccurs="unbounded">
          <xs:complexType>
            <xs:sequence>
              <xs:element name="dpt" type="xs:string"/>
              <xs:element name="employee" minOccurs="0" maxOccurs="unbounded">
                <xs:complexType>
                  <xs:sequence>
                    <xs:element name="emp" type="xs:string"/>
                    <xs:element name="task" minOccurs="0" maxOccurs="unbounded" type="xs:string"/>
                  </xs:sequence>
                </xs:complexType>
              </xs:element>
            </xs:sequence>
          </xs:complexType>
        </xs:element>
      </xs:sequence>
    </xs:complexType>
  </xs:element>
</xs:schema>
```

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References

[1] V. Tannen, P. Buneman, L. Wong, Naturally embedded query languages, in: J. Biskup, R. Hull (Eds.), Database Theory - ICDT’92, 4th International Conference, Berlin, Germany, October 14-16, 1992, Proceedings, Vol. 646 of Lecture Notes in Computer Science, Springer, 1992, pp. 140–154. doi:10.1007/3-540-56039-4_48

[2] E. Meijer, B. Beckman, G. M. Bierman, LINQ: reconciling object, relations and XML in the .NET framework, in: S. Chaudhuri, V. Hristidis, N. Polyzotis (Eds.), Proceedings of the ACM SIGMOD International Conference on Management of Data, Chicago, Illinois, USA, June 27-29, 2006, ACM, 2006, p. 706. doi:10.1145/11422473.1142552

[3] J. Cheney, S. Lindley, P. Wadler, A practical theory of language-integrated query, in: Proceedings of the 18th ACM SIGPLAN International Conference on Functional Programming, ICFP ’13, ACM, New York, NY, USA, 2013, pp. 403–416. doi:10.1145/2500365.2500586

[4] G. Copeland, D. Maier, Making Smalltalk a database system, in: ACM Sigmod Record, Vol. 14, ACM, 1984, pp. 316–325.

[5] C. Ireland, D. Bowers, M. Newton, K. Waugh, A classification of object-relational impedance mismatch, in: First International Conference on Advances in Databases, Knowledge, and Data Applications, IEEE, 2009, pp. 36–43.

[6] P. Hudak, Building domain-specific embedded languages, ACM Comput. Surv. 28 (4es) (Dec. 1996). doi:10.1145/242224.242477

[7] M. Odersky, P. Altherr, V. Cremet, B. Emir, S. Maneth, S. Micheloud, N. Mihaylov, M. Schinz, E. Stenman, M. Zenger, An overview of the Scala programming language, Tech. rep. (2004).

[8] R. Norris, Doobie - a functional JDBC layer for Scala [cited 14/6/2019]. URL https://tpolecat.github.io/doobie/

[9] J. Cheney, Language-integrated query: state of the art and open problems, in: L. Dayn’es, G. Fletcher, W. S. Han (Eds.), NII Shonan Meeting Report No. 2017-6, 2017. doi:10.1145/2588555.2612186. URL https://shonan.nii.ac.jp/archives/seminar/098/wp-content/uploads/sites/149/2017/02/cheney-shonan-linq.pdf

[10] P. Wadler, Monads for functional programming, in: International School on Advanced Functional Programming, Springer, 1995, pp. 24–52.

[11] P. Trinder, P. Wadler, Improving list comprehension database queries, in: Fourth IEEE Region 10 International Conference TENCON, 1989, pp. 186–192. doi:10.1109/TENCON.1989.176921

[12] P. Buneman, L. Libkin, D. Suciu, V. Tannen, L. Wong, Comprehension syntax, SIGMOD Record 23 (1) (1994) 87–96. doi:10.1145/131550.131564

[13] P. Buneman, S. A. Naqvi, V. Tannen, L. Wong, Principles of programming with complex objects and collection types, Theor. Comput. Sci. 149 (1) (1995) 3–48. doi:10.1016/0304-3975(95)00024-Q

[14] L. Wong, Kleisli, a functional query system, J. Funct. Program. 10 (1) (2000) 19–56. doi:10.1017/S0956796899003585

[15] E. Cooper, The script-writer’s dream: How to write great SQL in your own language, and be sure it will succeed, in: P. Gardner, F. Geerts (Eds.), Database Programming Languages - DBPL 2009, 12th International Symposium, Lyon, France, August 24, 2009, Proceedings, Vol. 5708 of Lecture Notes in Computer Science, Springer, 2009, pp. 36–51. doi:10.1007/978-3-642-03793-3_3

[16] D. Syme, Leveraging .NET meta-programming components from F#: integrated queries and interoperable heterogeneous execution, in: A. Kennedy, F. Pottier (Eds.), Proceedings of the ACM Workshop on ML, 2006, Portland, Oregon, USA, September 16, 2006, ACM, 2006, pp. 43–54. doi:10.1145/114574139876.1159848

[17] A. Ulrich, G. Giorgidze, J. Weijers, N. Schweinsberg, DSH: Database supported Haskell, http://hackage.haskell.org/package/DSH (2000–2004).

[18] K. Suzuki, O. Kisel’ev, Y. Kameyama, Finally, safely-extensible and efficient language-integrated query, in: Proceedings of the 2016 ACM SIGPLAN Workshop on Partial Evaluation and Program Manipulation, PEPM ’16, ACM, New York, NY, USA, 2016, pp. 37–48. doi:10.1145/2847538.2847542

[19] O. Kisel’ev, T. Katsushina, Sound and efficient language-integrated query - maintaining the ORDER, in: B. E. Chang (Ed.), Programming Languages and Systems - 15th Asian Symposium, APLAS 2017, Suzhou, China, November 27-29, 2017, Proceedings, Vol. 10695 of Lecture Notes in Computer Science, Springer, 2017, pp. 364–383. doi:10.1007/978-3-319-71237-6_18
[21] O. Kiselyov, Typed tagless final interpreters, in: Generic and Indexed Programming, Springer, 2012, pp. 130–174.
[22] H. Xi, C. Chen, G. Chen, Guarded recursive datatype constructors, in: Proceedings of the 30th ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages, POPL ’03, ACM, New York, NY, USA, 2003, pp. 224–235. doi:10.1145/604131.604150
[23] S. Najd, S. Lindley, P. Wadler, Everything old is new again: quoted domain-specific languages, in: M. Erwig, T. Rompf (Eds.), Proceedings of the 2016 ACM SIGPLAN Workshop on Partial Evaluation and Program Manipulation, PEPM 2016, St. Petersburg, FL, USA, January 20 - 22, 2016, ACM, 2016, pp. 25–36. doi:10.1145/2847538.2847561
URL http://dl.acm.org/citation.cfm?id=2847538
[24] F. W. Brasil, Quill - compile-time language integrated queries for Scala [cited 14/6/2019]. URL https://getquill.io
[25] Lightbend, Slick - functional relational mapping for Scala [cited 14/6/2019]. URL http://slick.lightbend.com/
[26] Apache Software Foundation, The Cassandra query language. URL http://cassandra.apache.org/doc/4.0/cql/
[27] L. Wong, Normal forms and conservative extension properties for query languages over collection types, J. Comput. Syst. Sci. 52 (3) (1996) 495–505. doi:10.1006/jcss.1996.0037
[28] J. Cheney, S. Lindley, P. Wadler, Query shredding: efficient relational evaluation of queries over nested multisets, in: Proceedings of the 2014 ACM SIGMOD international conference on Management of data, ACM, 2014, pp. 1027–1038. doi:10.1145/2588558.2612186
[29] J. Gibbons, F. Henglein, R. Hinze, N. Wu, Relational algebra by way of adjunctions, PACMPL 2 (ICFP) (2018) 86:1–86:28. doi:10.1145/3236781
[30] D. L. Parnas, On the criteria to be used in decomposing systems into modules, Commun. ACM 15 (12) (1972) 1053–1058. doi:10.1145/361598.361623
[31] J. Hughes, Why functional programming matters, The computer journal 32 (2) (1989) 98–107.
[32] M. Pickering, N. Wu, J. Gibbons, Profunctor optics: Modular data accessors, Art, Science, and Engineering of Programming 1 (2) (4 2017). doi:10.22152/programming-journal.org/2017/1/7
[33] J. N. Foster, M. B. Greenwald, J. T. Moore, A. Schmitt, Combinators for bidirectional tree transformations: A linguistic approach to the view-update problem, ACM Trans. Program. Lang. Syst. 29 (3) (May 2007). doi:10.1145/1232420.1232424
[34] O. Grenrus, Glassery (Apr 2018) [cited 14/6/2019]. URL http://oleg.fi/gists/posts/2017-04-18-glassery.html
[35] E. Kmett, Lens - lenses, folds, and traversals [cited 14/6/2019]. URL https://github.com/ekmett/lens
[36] J. Truffaut, Monocle - optics library for Scala [cited 14/6/2019]. URL http://julien-truffaut.github.io/Monocle/
[37] P. Wadler, XQuery: A typed functional language for querying XML, in: International School on Advanced Functional Programming, Springer, 2002, pp. 188–212.
[38] R. O’Connor, Functor is to Lens as Applicative is to Biplate: Introducing Multiplate, CoRR abs/1103.2841 (2011). arXiv:1103.2841
[39] C. W. Bachman, The programmer as navigator, Commun. ACM 16 (11) (1973) 653658. doi:10.1145/355611.362534
[49] T. Katsushima, O. Kiselyov, Language-integrated query with ordering, grouping and outer joins (poster paper), in: U. P. Schultz, J. Yallop (Eds.), Proceedings of the 2017 ACM SIGPLAN Workshop on Partial Evaluation and Program Manipulation, PEPM 2017, Paris, France, January 18-20, 2017, ACM, 2017, pp. 123–124. doi:10.1145/3018882.
URL http://dl.acm.org/citation.cfm?id=3018893

[50] M. Fernández, Y. Kadiyska, D. Suciu, A. Morishima, W.-C. Tan, Silkroute: A framework for publishing relational data in XML, ACM Transactions on Database Systems (TODS) 27 (4) (2002) 438–493.

[51] S. Lindley, J. Cheney, [Row-based effect types for database integration] in: B. C. Pierce (Ed.), Proceedings of TLDI 2012: The Seventh ACM SIGPLAN Workshop on Types in Languages Design and Implementation, Philadelphia, PA, USA, Saturday, January 28, 2012, ACM, 2012, pp. 91–102. doi:10.1145/2103786.2103798
URL http://dl.acm.org/citation.cfm?id=2103786

[52] MongoDB, Inc., MongoDB, https://www.mongodb.com/ (2019).

[53] R. Horn, R. Perera, J. Cheney, Incremental relational lenses, Proc. ACM Program. Lang. 2 (ICFP) (2018) 74:1–74:30 [doi:10.1145/3236769]

[54] E. Burmako, Scala macros: let our powers combine!: on how rich syntax and static types work with metaprogramming, in: Proceedings of the 4th Workshop on Scala, ACM, 2013, p. 3.

[55] O. Hartig, J. Pérez, An initial analysis of Facebook’s GraphQL language, in: AMW 2017 11th Alberto Mendelzon International Workshop on Foundations of Data Management and the Web, Montevideo, Uruguay, June 7-9, 2017, Vol. 1912, Juan Reutter, Divesh Srivastava, 2017.

[56] J. López-González, J. M. Serrano, Towards optic-based algebraic theories: The case of lenses, in: International Symposium on Trends in Functional Programming, Springer, 2018, pp. 74–93.

[57] S. Ceri, G. Gottlob, L. Tanca, What you always wanted to know about Datalog (and never dared to ask), IEEE transactions on knowledge and data engineering 1 (1) (1989) 146–166.

[58] M. Krotzsch, F. Simancik, I. Horrocks, [A description logic primer] CoRR abs/1201.4089 (2013). arXiv:1201.4089
URL https://arxiv.org/pdf/1201.4089

[59] L. Fegaras, An algebra for distributed big data analytics, Journal of Functional Programming 27 (2017) e27. doi:10.1017/S0956796817000193

[60] M. Benedikt, J. Cheney, Semantics, types and effects for XML updates, in: P. Gardner, F. Geerts (Eds.), Database Programming Languages, Springer Berlin Heidelberg, Berlin, Heidelberg, 2009, pp. 1–17.

[61] M. Odersky, D. Petrashko, G. Martres, et al., The Dotty project [cited 18/6/2019]. URL https://github.com/lampepfl/dotty

[62] P. Wadler, S. Blott, How to make ad-hoc polymorphism less ad hoc, in: Proceedings of the 16th ACM SIGPLAN-SIGACT symposium on Principles of programming languages, ACM, 1989, pp. 60–76.

[63] B. C. Oliveira, A. Moors, M. Odersky, Type classes as objects and implicits, in: Proceedings of the ACM International Conference on Object Oriented Programming Systems Languages and Applications, OOPSLA ’10, ACM, New York, NY, USA, 2010, pp. 341–360. doi:10.1145/1869455.1869489