Catalog of Wide Binary, Trinary and Quaternary Candidates from the Gaia Data Release 2 (Region $|b| > 25^\circ$)

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Abstract

The occurrence of multiple stars, dominantly binaries, is studied using the Gaia-ESA DR2 catalog. We apply the optimized statistical method that we previously developed for the analysis of 2D patterns. The field of stars is divided into a mosaic of small pieces that represent a statistical set for analysis. Specifically, data input is represented by a grid of circles (events) with radius $0.02\text{°}$ covering the sky in the field of galactic latitude $|b| > 25^\circ$. The criteria for selecting candidates for multiple stars are based on two parameters: angular separation and collinearity of proper motion. Radial separation, due to limited accuracy, is used only as a weaker supplementary constraint. Due attention is paid to the accurate calculation of the background, which is a necessary input for evaluating the quality of the candidates. Our selection algorithm generates the catalog of candidates: 900,842 binaries, 5282 trinaries, and 30 quaternaries.

Unified Astronomy Thesaurus concepts: Multiple stars (1081)

1. Introduction

It is believed that statistics of binaries and multiple stars can provide deeper insights into the formation and evolution of galaxies. Wide binaries may serve as a sensitive probe of the Galactic gravitational potential. Some recent studies suggest that very wide binaries may provide data on the presence of dark matter in the Galaxy (Peñarrubia et al. 2016; Hernandez 2019; Pittordis & Sutherland 2019). At present, an extremely rich source of data on stars of our Galaxy is provided by the Gaia-ESA mission and collected in the catalog DR2 (Gaia Collaboration et al. 2018). The release of the next and more expanded full version (DR3) is expected in the first half of 2022. Several authors have recently studied various aspects of binaries using the Gaia DR1 (Oelkers et al. 2017; Oh et al. 2017), DR2 (Godoy-Rivera & Chanamé 2018; El-Badry & Rix 2018; Ziegler et al. 2018; Jiménez-Esteban et al. 2019; González-Santamaría et al. 2020; Hartman & Lépine 2020; Sapozhnikov et al. 2020; Tian et al. 2020), and EDR3 (Early Data Release 3; El-Badry et al. 2021; Gaia Collaboration et al. 2021) catalogs. Other important articles examining wide binaries before Gaia include, for example, Caballero (2009) and Close et al. (1990).

In our previous study (Zavada & Piška 2018, 2020) we developed and applied statistical methods for the identification of binaries and multiple stars with an accurate estimate of the background. The resulting catalog is based on the 3D event analysis and contains about $8 \times 10^7$ binary candidates at a distance of up to $\lesssim 340 \text{pc}$. This catalog involves the binary stars whose total separation does not exceed the event sphere diameter, which was $4 \text{pc}$. Because determining the radial separation $Z_{ij}$ can have a significant error, we have been selecting the binary stars in the events based on the distribution of the projected distances, $\Delta_{ij}$, the error of which is only slightly affected by the parallax error. At the same time, the error of parallax and radial distance causes that the constraint on radial separation $Z_{ij} \leq 4 \text{pc}$ can exclude a large number of true binary stars. In the present study, we solve this drawback of 3D events by a more general procedure. The procedure combines the analysis of angular separations $d_{ij}$ of sources inside 2D events with the information on the radial separation $Z_{ij}$. Simultaneously, we apply the condition of approximate collinearity of proper motion here as before. The result of the procedure is the selection of candidates for the binary with the defined probability that it is the true binary and not a random background.

Our 2D analysis is described in Section 2 and consists of several parts. The basic notions, which our method deals with are defined in Section 2.1. Input data from the sector of the Gaia catalog are defined in Section 2.2. In the next Section 2.3 we analyze the probability of the binary depending on the angular separation and collinearity of proper motion of its components. This analysis sets the rules for the selection of binary candidates for our new catalog. Using parallax, we can define the distance of binary and the projection of the absolute separation, which is more important for physics than just the angular separation. In Section 2.4 we show that distributions of the projection and correlations with proper motion allow us to obtain information about binary orbit. In Section 2.5 we show the results of our analysis on the occurrence of trinaries and quaternaries. Candidates with a high degree of reliability are listed in the electronic catalog. Its structure and content are described in Section 3. The comparison with the other catalogs (Jiménez-Esteban et al. 2019; Zavada & Piška 2020; Hartman & Lépine 2020; El-Badry et al. 2021) is done in Section 4. The last Section 5 is devoted to the summary and concluding remarks.

2. Statistical Analysis of 2D Patterns

2.1. Methodology

The method of 2D analysis is described in detail in our previous paper Zavada & Piška (2020), so here we repeat only basic notions. The data for analysis are represented by the grid...
of circles with patterns of stars covering a defined region of sky (Figure 1).

The input data for generating the grid are given in the galactic reference frame. So, the position \( \mathbf{L} \) of a source is defined by spherical coordinates \( \mathbf{L}, l, \) and \( b \) (distance from the Sun, galactic longitude and latitude):

\[
\mathbf{L} = L \mathbf{n}; \quad \mathbf{n} = (\cos b \cos l, \cos b \sin l, \sin b),
\]

\[-\frac{\pi}{2} \leq b \leq \frac{\pi}{2}, \quad -\pi < l \leq \pi.
\] (1)

In the center of each circle we define local orthonormal frame defined by the basis:

\[
\mathbf{k}_r = \mathbf{n}_0 = (\cos b_0 \cos l_0, \cos b_0 \sin l_0, \sin b_0),
\]

\[
\mathbf{k}_l = (-\sin l_0, \cos l_0, 0),
\]

\[
\mathbf{k}_b = (-\sin b_0 \cos l_0, -\sin b_0 \sin l_0, \cos b_0),
\] (2)

where \( \mathbf{k}_r = \mathbf{n}_0(b_0, l_0) \) defines angular position of the circle center. Unit vector \( \mathbf{k}_r \) is perpendicular to \( \mathbf{k}_l \) and has direction of increasing \( l \). Unit vector \( \mathbf{k}_b \) is defined as \( \mathbf{k}_b = \mathbf{k}_r \times \mathbf{k}_l \) and has direction of increasing \( b \). Vector \( \mathbf{k}_r \) has radial direction, perpendicular vectors \( \mathbf{k}_b \) and \( \mathbf{k}_l \) lies in the transverse plain. The stars inside the circle of small radius \( \rho_2 \) satisfy

\[
| \mathbf{n}_i - \mathbf{n}_0 | \leq \rho_2, \quad i = 1, \ldots M
\] (3)

or

\[
\{x_i, y_i \}; \quad x_i^2 + y_i^2 \leq \rho_2^2, \quad i = 1, \ldots M,
\] (4)

where \( \{x_i, y_i \} \) are local rectangular coordinates defined by the basis (2):

\[
x_i = n_i^1 k_i, \quad y_i = n_i^2 k_b; \quad n_i^1 = n_i - n_0.
\] (5)

The circles in the grid are, in fact, spherical caps. However, their radius \( \rho_2 \) will be so small (\( \geq 3.5 \times 10^{-2} \) rad), that the caps can be reliably considered as flat circles.

A set of stars defined by Equation (4) is called the event. We define the pair angular separations

\[
x_{ij} = |x_j - x_i|; \quad y_{ij} = |y_j - y_i|;
\]

\[
d_{ij} = \sqrt{x_{ij}^2 + y_{ij}^2}; \quad i, j = 1, 2, \ldots M,
\] (6)

\[
\hat{\xi} = \frac{d_{ij}}{2\rho_2}; \quad 0 \leq \hat{\xi} \leq 1,
\] (7)

where \( \rho_2 \) is the angular radius of the events. Distribution of separations of randomly distributed sources inside the circle does not depend on \( M \) and is given by formula

\[
q(\hat{\xi}) = \frac{16\hat{\xi}}{\pi} (\arccos \hat{\xi} - \hat{\xi} \sqrt{1 - \hat{\xi}^2}),
\] (8)

which was proved in (Zavada & Piška 2020). This curve is important for subtraction of random background.

Before practical use, we add a few general remarks:

1. Of course, the grid in Figure 1 can be rectangular only locally. In fact, we work with circles with the same radius aligned only in rows with a constant galactic latitude.
2. The radius \( \rho_2 \) must be set to be significantly larger than a typical angular separation of true binary. At the same time, it must be so small that the distribution of stars within the event can be considered random and uniform. In Section 2.3 these conditions will be explained in more detail. We will also explain that the obtained results are not sensitive to the exact setting of \( \rho_2 \).
3. Events with too high multiplicity, \( M \), in which various higher dense structures may dominate, are excluded from processing.
4. The circular shape of the events is chosen due to a simple but accurate formula (8) for calculating a random background. Another shape would lead to a more complex formula depending on other shape parameters (triangle, square, orientation ...). On the other hand, the grid of circle events only partially covers the sky. The complete set of binaries is obtained in Section 3 from several shifted grids.

2.2. Input Data

The parameters of the events we work with are listed in Table 1. By symbol \( \mathbf{R} \) we denote the union of both regions in table:

\[
\mathbf{R} = \mathbf{R}_1 \cup \mathbf{R}_2.
\] (9)

In Figure 2 we have shown distribution of the distances of the stars in the table from the Sun (origin of the galactic reference frame). If the parallax \( p \) is given in angular units as \( (= \, 1') \), then the corresponding distance is

\[
L[pc] = \frac{1}{p[as]}.
\] (10)

The figure demonstrates the scale of distances measured by Gaia. This histogram also suggests the convention valid for all histograms \( P(x) \) in this paper: ordinate represents the number of entries of variable \( x \) of given value into the bins of the width specified in the figure caption.
Table 1

| 2D Region: \( \times b (\text{deg}^2) \) | \( \rho_2 (\text{as}) \) | \( \langle L \rangle (\text{pc}) \) | \( \langle M \rangle \) | \( N_p \) | \( N_b \) | \( N_{\text{tot}} \) |
|----------------------------------------|----------------|----------------|------------|--------|--------|----------|
| \((-180, 180) \times (\pm 45, \pm 90)\) | 72 | 1807 | 3.86 | 5,887,737 | 22,083,670 | 30,643,238 |
| \((-180, 180) \times (\pm 25, \pm 45)\) | 72 | 2134 | 7.86 | 6,985,043 | 53,043,629 | 80,653,496 |

Note. Only sources with positive parallax and in distance \(<15,000\,\text{pc}\) are taken into account and only events \(2 \leq M \leq 25\) are accepted for present analysis. \(N_p\) is number of sources after these cuts, and \(N_{\text{tot}}\) is total number of sources with positive parallax in \(R_{1,2}\).

Stars in the sky are divided into the circle regions (events) of angular radius \(\rho_2\) and in Figure 3 we show distribution of the star multiplicities in these events. The distribution of stars in region \(R_1\) is roughly homogenous, so the multiplicity distribution has a nearly Poisson shape

\[
P(M) = \frac{\lambda^M}{M!} \exp(-\lambda); \quad \lambda = \langle M \rangle.
\]

Since region \(R_2\) shows greater density fluctuations resulting in a broader distribution than Poisson, we fail to make its fit. This region is dominated by an inhomogeneous stellar density due to patchy interstellar extinction, the presence of open clusters, and the imprint of the Galactic spiral pattern.

\[
\mu_\alpha^B (\equiv \mu_\alpha \cos \delta), \quad \mu_\delta
\]

in directions of the R.A. and decl. in the ICRS. So the corresponding transverse 2D velocity \(U\) is given as

\[
U = Lu, \quad u = (\mu_\alpha^B, \mu_\delta), \quad u = |u|.
\]

\[
\alpha_{ij} = \arccos\frac{u_i \cdot u_j}{|u_i| |u_j|}.
\]

Our selection of binaries is based on Figure 4 obtained from all events in the table. The peak in the domain of small separations \(d_{ij}\) and small angles \(\alpha_{ij}\) represents a clear signature of binaries (panel a). The domain of the peak can be approximately defined by the conditions

\[
d_{ij} \lesssim 0.5 \text{as}, \quad \alpha_{ij} \lesssim 15^\circ.
\]

The second condition was applied also in our former 3D analysis. Panel (b) shows distribution \(P(\alpha_{ij})\) in the band \(d_{ij} \lesssim 15\,\text{as}\), similarly in panel (c) we have distribution \(P(d_{ij})\) for \(\alpha_{ij} \lesssim 15^\circ\). Recall that the stars separated by \(d_{ij} \lesssim 0.5 \text{as}\) are missing due to current resolution limit in the DR2 data set as stated in (Arenou et al. 2018).

In Figure 5 we have shown distribution of radial separations \(Z_{ij}\) defined as

\[
Z_{ij} = |L_i - L_j| \quad (16)
\]

and obtained with the use of parallax in Equation (10). Panels (a) and (b) show distribution of pairs outside and inside the window (Equation (15)), respectively. The sharp peak at small radial separations in the second panel is expected for binaries, the tail with greater separations corresponds partly to the background pairs and partly to the true binaries having large errors of parallaxes. Alternative distributions \(P(\Delta L/L)\), where

\[
\Delta L \approx Z_{ij}, \quad L = \frac{L_i + L_j}{2}
\]

are shown in Figures 6(a), (b).

Equation (10) implies

\[
\frac{\Delta L}{L} \approx \frac{\Delta \rho}{\rho}. \quad (18)
\]

In Figure 6(c) we show distribution \(P(\Delta \rho/\rho)\) obtained directly from the Gaia data, where the parallax of the source is accompanied by its estimated error. Our \(\Delta L, L\) related to binaries are also obtained from parallaxes; however, our estimate of relative error of the parallax is independent and can serve as a cross-check. We observe a noticeable similarity between the distributions \(b, c\). A smaller difference can arise because the window of binaries also contains a background with random radial separations.

2.3. Selection of Binaries and Calculation of Background

In the next we will work with the numbers: \(n_2\) —number of (true) binaries, \(b_2\) —number of background pairs, and \(N_2\) —total number of pairs; \(N_2 = n_2 + b_2\). (19)

These numbers are related to given sample of pairs defined by the corresponding cuts. One cannot decide if the pair is a binary or background, but we can find out the probability that the pair is binary \(n_2/N_2\) or background \(b_2/N_2\). Obviously, in the domain of the peak in Figure 4(a) the probability of binaries \(n_2/N_2\) is high.

Radial separation \(Z_{ij}\) can be used to further increase the probability of binaries in the peak, but due to low accuracy,
the cut must be set judiciously. Too strict cut generates a cleaner sample of binaries (higher ratio \( n_2/N_2 \)) but more binaries are excluded. And conversely, too soft cut preserves more binaries, but at the price of the higher background and lower ratio \( n_2/N_2 \).

The area \( d \times \alpha \) of Figure 4(a) can be divided into four the windows \( A, B, C, \) and \( D \) (like in Figure 8):

\[
A = \langle 0, d_\ell \rangle \times \langle 0, \alpha_c \rangle; \quad d_\ell = 15 \text{ as}, \quad \alpha_c = 15^\circ,
\]  

\[ (20) \]
which is the domain of the peak with a high population of binaries defined by Equation (15). The remaining windows are

\[ \begin{align*}
B &= \langle d_e, d_{\max} \rangle \times \langle 0, \alpha_c \rangle, \\
C &= \langle d_e, d_{\max} \rangle \times \langle \alpha_c, 180 \rangle, \\
D &= \langle 0, d_e \rangle \times \langle \alpha_c, 180 \rangle.
\end{align*} \]

(21)

Distribution \( P(d_{ij}) \) can be represented by its normalized form

\[ Q(\hat{\xi}) = \frac{1}{N_2} P(d_{ij}); \quad \hat{\xi} = \frac{d_{ij}}{2 \rho_2}. \]

(22)

The random background is described by the normalized function \( q(\hat{\xi}) \) defined by Equation (8). Both distributions are shown in Figure 7(a) for region \( R_1 \). Due to equal normalization, an excess in the peak of binaries in distribution \( P \) is compensated by the lack of pairs in the region of background. Assuming that for \( d_{ij} \geq 2/3d_{\max} \) or \( \hat{\xi} \geq 2/3 \) distribution \( P \) involves only background, we renormalize \( q \) correspondingly:

\[ q(\hat{\xi}) \rightarrow q'(\hat{\xi}) = \gamma q(\hat{\xi}); \quad \gamma = \frac{\int_{\hat{\xi} \geq 2/3} Q(\hat{\xi}) d\hat{\xi}}{\int_{\hat{\xi} \geq 2/3} q(\hat{\xi}) d\hat{\xi}}. \]

(23)

In Figure 7(b) we have shown distribution \( Q(\hat{\xi}) \) together with the renormalized background \( q'(\hat{\xi}) \). The same distributions for region \( R_2 \) is shown in panel c. Obviously, the distribution of binaries is given by their difference

\[ P_{\text{bin}}(\hat{\xi}) = N_2(Q(\hat{\xi}) - q'(\hat{\xi})). \]

(24)

It is now clear from these figures why the diameter of events \( 2\rho_2 \) is chosen much greater than typical separation of the binaries \( d_{\text{bin}} \) (the parameter \( \hat{\xi}_{\text{bin}} = d_{\text{bin}}/2\rho_2 \approx 0.1 \) in the figures). The larger diameter allows us to more accurately determine the weight of the background curve to be subtracted. At the same time we observe the distribution of separations is perfectly random outside the region of binary peak.

\[ \beta(\hat{\xi}) = \frac{P_{\text{bin}}(\hat{\xi})}{N_2 Q(\hat{\xi})} = 1 - \frac{\gamma q(\hat{\xi})}{Q(\hat{\xi})}, \]

(25)

where one can replace \( \hat{\xi} \rightarrow d_{ij} = 2\rho_2 \hat{\xi} \). Examples of the function \( \beta \) will be given below. The shape of the background distribution \( q \) depends on \( \hat{\xi} \) only. In the background, we can see an increase or peak at low \( \hat{\xi} \), which is a signal of the presence of binaries. An additional selection of pairs with small \( \alpha_{ij} \) or a small radial separation \( Z_{ij} \) increases the dimension of the peak on the background, but the shape of the background curve does not change. For instance, Figures 7(b) and (c) correspond to \( \alpha_{ij} \leq 15^\circ \). The reason is simple: in the sky without binaries (only background can be observed) there is no correlation between angular separation \( d_{ij} \) and the parameters \( \alpha_{ij} \) or \( Z_{ij} \).

From the data we have known the numbers of pairs in four windows defined above: \( N_2^A, N_2^B, N_2^C, \) and \( N_2^D \). The number \( N_2^A \) is called the number of binary candidates. According to an assumption above, we have also numbers of background pairs obtained for \( \hat{\xi} \geq 2/3 \) in domains

\[ \langle 0, \alpha_c \rangle \rightarrow N_2^A, \quad \langle \alpha_c, 180 \rangle \rightarrow N_2^B. \]

(26)
If we denote

\[ w(x) = \int_{-\xi}^{\xi} q(\xi) \, d\xi, \]  

(27)

then we can calculate the numbers of background pairs in the windows \( A-D \) as

\[ b_2^A = N_2^A \frac{1 - w(d_c)}{w(2/3)}, \quad b_2^B = N_2^B \frac{w(d_c)}{w(2/3)}, \]

\[ b_2^C = n_2^C \frac{w(d_c)}{w(2/3)}, \quad b_2^D = n_2^D \frac{1 - w(d_c)}{w(2/3)}. \]  

(28)

Therefore, the numbers of binaries in the respective windows read

\[ n_2^A = N_2^A - b_2^A, \quad n_2^B = N_2^B - b_2^B, \]

\[ n_2^C = N_2^C - b_2^C, \quad n_2^D = N_2^D - b_2^D. \]  

(29)

These numbers, obtained in both regions \( R_1 \) and \( R_2 \) under different conditions are given in Figure 8. The numbers in upper panels \( A, B \) (the set “All events”) follow from panels (b) and (c) in Figure 7. The peaks of binaries are evident; however, the area under the background curve is also considerable. Correspondingly, the quality ratios \( \beta^A = n_2^A/N_2^A \) (in the second row of the panels \( A \)), representing the probability of the true binary in window \( A \) are not satisfactory, particularly for the region \( R_2 \). Parameter \( \beta^A \) can be increased by applying the additional cut on radial separation \( \Delta L \). The condition

\[ \Delta L \leq \Delta L_{\text{max}} = 500 \text{ pc} \]  

(30)

applied in the region \( R_1 \) gives “enriched” sample with the quality ratio \( \beta^A = 0.77 \) (left middle panel). In the region \( R_2 \) the situation is more complicated. The density of stars is higher, so the background grows. Moreover, the average density varies with longitude. So, we have divided the region into three subregions with different \( \Delta L_{\text{max}} \) cuts, which give a similar \( \beta^A \) in these subregions. Their definition is shown in Table 2. These cuts give resulting \( \beta^A \) and other parameters related to \( R_2 \) listed in right middle panel. One could further squeeze the cuts to obtain a cleaner sample of binaries (higher ratio \( n_2^A/N_2^A \)); however, at the price that more true binaries are excluded (\( n_2^A \) is smaller). This is illustrated by the numbers in upper and middle yellow panels in the figure. The cuts exclude not only background pairs but also true binaries in window \( A \). It is due to a rather poor accuracy of radial separation that we have shown in Figure 5(b). If we had an accurate radial separation, then a suitable cut would suppress only background and preserve true binaries.

The situation is more favorable with brighter stars. The lower panels in Figure 8 show that for binaries of magnitude \( G \leq 15 \) the quality ratio \( \beta^A \) is very good even without any cut on radial separation. The quality of this sample is illustrated also by Figure 9, where we observe the high binary peaks with low background. In any case, our criterion is based on the resulting ratio, \( \beta^A \), so we can accept candidates with greater parallax uncertainty if \( \beta^A \) is greater than requested. In other words, if the signature from \( d_i \) and \( \alpha_i \) is sufficient, then the parallax is not important.

Equivalently, abundance of binaries depending on their separation is defined by \( \beta \) function in Equation (25). In Figures 10(a)–(c) we have shown this function for some subsets of regions \( R_1 \) and \( R_2 \) defined by Figure 8. Figure 10 demonstrates that higher \( \beta^A \) could be also reached by squeezing the parameter \( d_i \) in window \( A \) defined by Equation (20). Note, the effectively wider peak for magnitude \( G \leq 15 \) is due to a lower level of background pairs, which can be seen from the comparison of Figure 9 with Figures 7(b) and (c).

We have shown that the function \( \beta \) is a key to determining the quality and quantity of binaries in the selected subset. It is, therefore, necessary to verify that this function does not change with the choice of the purely technical parameter \( p_2 \). Figure 10(d) shows the function \( \beta \) calculated for the subsample of enriched exposition with event radius \( p_2 = 36 \) as (half the radius of standard events). The functions in Figures 10(b) and (d) agree perfectly. A similar agreement was also verified for Figures 12–14 below.

What is the effect of input data uncertainties on the selection quality? The initial distribution \( P(d_{ij}, \alpha_{ij}) \) in Figure 4 consists of two parts

\[ P(d_{ij}, \alpha_{ij}) = P_{\text{bin}}(d_{ij}, \alpha_{ij}) + P_{\text{bg}}(d_{ij}, \alpha_{ij}), \]  

(31)

corresponding to the true binaries and the background. The shape of the distribution \( P_{\text{bin}}(d_{ij}, \alpha_{ij}) \) is affected by the measurement errors. In general, these errors cause a widening of this distribution, but its integral \( n_2 \) does not change, as suggested in Figure 11. However, the errors will affect the value of the quality ratio

\[ \beta^p = \frac{n_2}{n_2 + b_2} \]  

(32)
in the domain of the peak. As suggested in the figure, wider \( P_{\text{bin}} \) means larger \( b_2 \), which implies a smaller ratio \( \beta^p \). In this
way, the measurement errors of the selection parameters directly affect the $\beta$ function, larger errors mean worse $\beta$. At the same time, larger errors expand the space for selection. A similar result follows from the algorithm in (Halbwachs 1986) for the selection of common proper motion pairs.

1. In the distribution $P(d_{ij}, \alpha_{ij})$ in Figure 4 we observe the peak on a smooth background. The peak is a sign of the presence of binaries. We take the edges of this peak, which are defined by Equation (15), as the limits of our selection of binary candidates.

2. The number of true binaries can be accurately determined by subtracting the random background as shown in Figures 7(b) and (c). The background curve is given by Equation (8). This curve corresponds to the random distribution of sources within a circle. That is why we work with circular events and with the distribution of angular separations of the pairs.

3. The background level in the peak domain can be further reduced by the cuts on radial separation $\Delta L_{\text{max}}$.

4. The cuts $\alpha_c$ and $\Delta L_{\text{max}}$ reduce the background level, but does not change the shape of the corresponding background curve $q(\xi)$. This distribution, after the renormalization (Equation (23)), is used for calculation of the probabilistic function $\beta$, which defines the selection quality.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
[l [deg]] & $(-30, +30)$ & $(+30, \pm 90)$ & $(90, 270)$ \\
\hline
$\Delta L_{\text{max}}$ [pc] & 50 & 100 & 400 \\
\hline
\end{tabular}
\caption{Cuts on Radial Separation in Galactic Longitude Subregions of Region $R_2$}
\end{table}
2.4. Projected Absolute Separations, Periods, and Masses of Binaries

In Figure 12 we have shown an important correlation in the peak of binaries, where angular separation \( d_{ij} \), as one would expect, strongly correlate with the projected distance \( \Delta_{ij} \) that is calculated as

\[
\Delta_{ij} = \frac{d_{ij} (L_i + L_j)}{2}.
\]

Distribution of \( \Delta_{ij} \) in the region of peak (windows A in Figure 8) is shown in Figure 13. In panels (b), (c) we have shown distributions from subsets with higher rate of binaries. We observe that

\[
\Delta_{ij} \lesssim 0.1 \, \text{pc},
\]

which confirms our former result from 3D analysis. In panel (a) we observe presence of pairs, which contradicts this constraint. It is due to high rate of background pairs in this set. Note that nearby binaries with wider separation can be outside of our acceptance window A. In fact only candidates with separation

\[
\Delta_{ij} \lesssim d_{ij} L \approx 7 \times 10^{-5} L,
\]

where \( L \) is average distance of the pair, are accepted. Similarly as before (Zavada & Piška 2020), we can estimate the projection of the orbital velocity of the binary as

\[
\nu_{ij} = |\mathbf{u}_i - \mathbf{u}_j| L_i + L_j
\]

From the plot in Figure 14(a), we can roughly estimate averages of the orbital periods and total masses of the binary systems. The binaries are accumulated in a peak very similar to that obtained earlier in 3D analysis. A very similar plot (with much lower statistics) can be obtained also for the subset of binaries with magnitude \( G \approx 15 \). If we take the sources roughly in the domain of half width of the maximum (panels b and c)

\[
\Delta_{ij} \lesssim 0.01 \, \text{pc}, \quad \nu_{ij} \lesssim 2.5 \, \text{kms}^{-1}
\]
then in the approach described in (Zavada & Píška 2020) we now obtain
\[
\langle T \rangle \approx 4.2 \times 10^4 \text{yr}, \quad \langle M_{\text{tot}} \rangle \approx 0.65 M_\odot,
\] (38)

which is comparable with the previous approximate estimate \((8 \times 10^4 \text{yr}, 0.8 M_\odot)\) obtained in the cited paper. These numbers can also be compared with the results of the sample of thirty common proper motion pairs, which were discussed in Duquennoy & Mayor (1991; Table 4). These pairs represent wide binaries in the solar neighborhood, for which the table lists the parameters \(\log T\) and masses \(M_{1i}\) and \(M_{2i}\). We have calculated the mean values
\[
\langle T \rangle = 10^{\langle \log T \rangle} \approx 2.6 \times 10^4 \text{yr},
\]
\[
\langle M_{\text{tot}} \rangle = \langle M_{1i} + M_{2i} \rangle \approx 1.6 M_\odot,
\] (39)

which are also well comparable with our estimates above.

2.5. Wide Trinaries and Quaternaries

So far we have assumed the excess of close pairs (window \(A\)) is due to only binaries, which is correct only in a first approximation. A more detailed analysis indicates also a limited presence of multiple systems, trinaries, and
To identify their candidates, we will generalize our criterion of binary candidates. For each pair in the candidate multiple system we require the same Equation (20) as for separation of binaries. Three stars create three separations, four stars create six separations. In Table 3 we have shown statistics of events involving just one isolated trinary or quaternary candidate \( N_{m} \). The random background events \( b_{m} \) are obtained from the real events in which the local coordinates \( \{ x_i, y_i \} \) are replaced by random positions inside the same events. The difference \( N_{m} - b_{m} \) represents the estimate of number of the true multiple systems \( n_{m} \). The presence of multiple systems implies that the number of binary candidates \( N_{2} \) should be reduced by

\[
\Delta N_{2} = 3N_{3} + 6N_{4}.
\]

For each triplet of stars inside the event we define the triangle separation as

\[
d_{ijk} = \max(d_{ij}, d_{jk}, d_{ki}).
\]

If \( \alpha \beta \) are subscripts of the most separated pair, then the components of triangle separation are defined as

\[
x_{ijk} = |x_{\alpha} - x_{\beta}|, \quad y_{ijk} = |y_{\alpha} - y_{\beta}|.
\]

Distributions of these parameters are shown in Figure 15 together with the distributions corresponding to the background, which is generated by randomly distributed sources in the event. The high level of the background under the slight trinary peak at \( d_{ijk} \lesssim 20 \) as (upper panel left) is due to primarily by binaries. Each binary generates also excess of triangle separations. It is illustrated in Figures 16 and 17. In the first figure (left panel) we show the simulated distribution of separations of random sources with the admixture of extra close pairs. The parameters of this admixture (probability of close pairs and width of normal distribution of their separations) are set to reproduce distributions in the real events (right panel). Distributions of triangle separations obtained from the same set are shown in Figure 17. The last figure explains a high background under the trinary peak in upper panels of Figure 15.

**Figure 15.** Distributions of angular separations \( x_{ijk}, y_{ijk} \) and \( d_{ijk} \) in trinary candidates. Upper panels show results from the region \( R \) (enriched exposition). The lower panels represent background. Unit: \( x_{ijk}, y_{ijk}, d_{ijk} \) as. Binning: \( 2.88 \times 2.88 \times 0.288 \) as.

**Table 3**

Statistics of Trinary and Quaternary Candidates \( (m = 3,4) \) in Regions \( R_{1} \) and \( R_{2} \)

| Region | \( m \) | \( N_{m} \) | \( b_{m} \) | \( n_{m} \) | \( \beta \) | \( N_{m}^{\beta} \) | \( b_{m}^{\beta} \) | \( n_{m}^{\beta} \) | \( \beta \) |
|--------|------|-------|-------|------|------|-------|-------|-------|------|
| \( R_{1} \) | 3 | 1724 | 137 | 1587 | 0.921 | 53 | 2 | 51 | 0.96 |
| | 4 | 12 | 0 | 12 | 1. | 1 | 0 | 1 | 1. |
| \( R_{2} \) | 3 | 1946 | 147 | 1789 | 0.924 | 119 | 5 | 114 | 0.96 |
| | 4 | 10 | 0 | 10 | 1. | 0 | 0 | 0 | 0 |
| \( \Sigma \) | 3 | 3670 | 284 | 3386 | 0.923 | 172 | 7 | 165 | 0.96 |
| | 4 | 22 | 0 | 22 | 1. | 1 | 0 | 1 | 1. |

**Note.** This shows the numbers of candidates, estimated background, and true trinaries and quaternaries \( (N_{m}^{\beta}, b_{m}^{\beta}, n_{m}^{\beta}), \beta = n_{m}^{A} / N_{m}^{A} \).
Figure 8 and Table 3 show that in the window \( A (R) \) of enriched exposition we have 511,998 binaries, 3404 trinaries, and 22 quaternaries. However, the total numbers are greater. The estimate based on the all events window \( A – D \) in upper panels of Figure 8 gives the total number of binaries roughly \( 1.5 \times 10^6 \). However, as we discussed in \( Zavada & Piška 2020 \), part of them \( \text{mainly in windows } B, C \) may be an image of widening pairs that were less separated but weakly bound in the past.

3. Catalog

In this section, we describe the catalog of the multiple star candidates, which are selected with the use of the events defined by Table 1. For the present version of the catalog, we accept only the candidates from window \( A(R) \) of the enriched exposition defined in the middle part of Figure 8. So, the selected pairs satisfy the conditions of angular separation, collinearity, and radial separation:

\[
d \leq 15 \text{ as}, \quad \alpha \leq 15^\circ, \quad \Delta L \leq \Delta L_{\text{max}},
\]

where \( \Delta L_{\text{max}} \) is defined in Equation (30) for region \( R_1 \) and in Table 2 for \( R_2 \). We record:

1. Binary candidates: separate pairs that satisfies Equation (43).
2. Trinary candidates: separate triplets of sources, where each pair (three in total) satisfies Equation (43).
3. Quaternary candidates: separate quaternions of sources, where each pair (six in total) satisfies Equation (43).

Examples of possible patterns in window \( A \) generated in one event are symbolically shown in Figure 18. We skip the events with empty window \( A \). Pairs with the bar meet Equation (43), pairs without the bar do not. Now we work with the patterns, where each source has at least one bar. According to the above definition, diagrams represent the candidates in the event: one binary (a), two binaries (b), one trinary (c), one quaternary (d). The rate of other combinations of candidates is negligible. Combinations that do not correspond to the defined candidates, such as panels (e) and (f), also have very little weight. We skip them and accept only candidates a-d into the catalog. The circle events cover only part of the sky (corresponding to the fraction

![Figure 16](image1.png)

Figure 16. Distributions of angular separations \( d_{ij} \). Left: random sources with the admixture of close pairs. Right: real event data from region \( R \) (enriched exposition). Unit: \( d_{ij}[\text{as}] \). Binning: 0.288 as.

![Figure 17](image2.png)

Figure 17. Distributions of angular triangle separations \( x_{ijk}, y_{ijk} \) and \( d_{ijk} \) for the same set of simulated events used in the previous figure. Unit: \( x_{ijk}, y_{ijk}, d_{ijk}[\text{as}] \). Binning: 2.88 as \( \times \) 2.88 as, 0.288 as.

![Figure 18](image3.png)

Figure 18. Examples of patterns relevant to the selection (rejection) of candidates for multiple stars.
\(\pi/4\). We also lose candidates between neighboring events when the pairs are split between the neighbors. To recover these losses, we work with modified coverage:

(i) The event circles of radius 72 as are replaced by squares of edge 144 as with no gaps between them. In each square, we search for multiple star candidates.

(ii) The procedure is repeated with the same squares centered in the corners and the edge centers of the former squares (we have four grids in total). Then the search results are merged, the summary data are given in Table 4. The quality ratios \(\beta = n^A_m / N^A_m\) are taken from the analysis of circle events with well defined background (Figure 8 and Table 3). Now we can use them to estimate \(n^A_m\) and corresponding statistical errors

\[
\Delta n^A_m = \beta \sqrt{N^A_m}.
\]

It is seen the number of multiple systems decreases rapidly with their multiplicity:

\[
\frac{n^A_3}{n^A_2} \approx 0.7\%, \quad \frac{n^A_4}{n^A_3} \approx 0.6\%.
\]

In Figure 19 we have shown distribution of distances of all stars accepted to the catalog. Note that selection criteria exclude preferably the most distant sources from distribution in Figure 2. Obviously, the candidates of higher quality (but lower quantity) can be obtained from the catalog by reselection with more strict cuts than defined by Equation (43).

The catalog of selected candidates is represented by a matrix, which is defined as follows. Each row represents one star and there are the following data in the columns:

1–2: group ID and group size (\(n = 2, 3, 4\)) to match stars with the group they belong to.
3–96: copy of the original entry for the star from Gaia-DR2 catalog, according to the documentation.
97–98: minimum and maximum angular separation of the star from other stars in the group [as].
99–100: minimum and maximum projected physical separation (33) of the star from other stars in the group [pc].

The quality of our catalog can be easily verified for brighter binaries, where the DR2 data includes radial velocities. The catalog contains 6469 such pairs, only 16 of them involves a star with magnitude \(G > 15\). In Figure 20 we show the correlation of both velocities, which confirms the common radial motion of the pairs. This result agrees very well with the same correlation obtained from the DR2 catalog in Andrews et al. (2018) and Sapozhnikov et al. (2020).

### 4. Comparison with Other Catalogs

The catalogs of wide binaries (Hartman & Lépine 2020; Zavada & Piška 2020; Jiménez-Esteban et al. 2019; El-Badry et al. 2021) and our present one are based on the selection defined by three parameters measured by the Gaia angular position (separation), proper motion, and parallax. However, catalogs differ in the choice of algorithm, which processes these variables. To illustrate, we will compare the algorithm used in this study (A1) with the algorithm used for the recent catalog (El-Badry et al. 2021) containing a comparable amount of binaries (A2).

(i) Angular separation: this parameter is measured with a high accuracy. The presence of binaries generates a clear peak in its distribution at small separations. The random background in the events is exactly described by Equation (8). The peak in angular separations is the basic signature of binaries in the algorithm A1.

From angular separation one can, with the use of parallax, calculate projected separation. Both parameters are strongly correlated. The cut on projected separations \(\Delta \leq 1\) pc is applied in the algorithm A2. However, the question of the background is here more complex, see discussion below.

(ii) Proper motion: in algorithm A2, the difference of velocities \(v_{ij}\) defined in Equation (36) is limited by the cut

\[
v_{ij} \leq v_{\text{max}}.
\]
Algorithm A1 is based on Equation (15), which define the domain of the peak with the high population of binaries (Figure 4). The algorithm includes an accurate calculation of the background under the peak. What are the velocities $v_{ij}$ inside and outside this peak? The answer is given in Figure 21.

The left panel shows the distribution of velocities for all pairs within the event, the peak at small $v_{ij}$ corresponds to the binaries. The right panel shows the same distribution for the pairs in the peak domain. Corresponding pairs are mostly binaries, so the large velocities are strongly suppressed. The panel represents the distribution of the orbital motion projection. One can admit, the tail of distribution may still be contaminated with some background pairs. Alternatively, instead of our cut

$$\alpha_{ij} \leq \alpha_{\text{max}}$$  \hspace{1cm} (47)

it would be possible to set the cut (Equation (46)). In the present study, we preferred cut of Equation (47), because the angle $\alpha_{ij}$ does not depend on the parallax, which has a large measurement error. For future analysis of the Gaia DR3 data, in which a more accurate parallax measurement is expected, we plan to try replacing the cut of Equation (47) with that of Equation (46), which can more effectively suppress the background.

(ii) Parallax: the parallax $p$ or the distance $L = 1/p$ are determined with a large error, $\Delta L$ is usually larger than the expected binary dimension. Large radial separation does not necessarily mean that the pair is not true binary. However, a smaller radial separation (or difference of parallaxes) increases the probability that the pair is true binary. Therefore the algorithm A1 requires limited radial separation defined in Equation (30) and Table 2. Similarly, the A2 requires limited difference of parallaxes.

Finally, both algorithms similarly reject denser clusters of sources: event multiplicity $M > 25$ (A1) and number of neighbors $> 30$ (A2). In general, algorithms have a similar philosophy but differ in technical details.

A very important technical step is to define random background (A1) or equivalently the chance alignments (A2). In this respect, the two algorithms differ significantly. Let’s make a comparison.

A1: the distribution of separations of random sources inside a circle is given exactly by formula (8). The shape of this curve does not depend on other parameters such as magnitude, the direction of proper motion or the parallax. The binary peak can be separated from the background described by this curve, see Section 2.3 and examples in Figures 7 and 9. The normalized background curve is

$$q(\xi) \approx 8\xi + ...$$  \hspace{1cm} (48)

for small $\xi$ (peak region). The probability of finding binary in a selected subset is given by function $\beta$ defined in Equation (25), examples are shown in Figure 10.

A2: from the Gaia EDR3 data input, there are produced two files involving pairs with seven parameters $x = \{\text{angular separation, distance, parallax difference uncertainty, ...}\}$:

1. Catalog of candidates.
2. Catalog of chance alignments (shifted catalog).

The densities of pairs $N_{\text{candidate}}(x)$ and $N_{\text{chance align}}(x)$ in the seven-dimensional parameter space are approximated by the Gaussian kernel density estimates. The ratio of these approximations

$$R(x) = \frac{N_{\text{chance align}}(x)}{N_{\text{candidate}}(x)}$$  \hspace{1cm} (49)

is the parameter, which provides classification of the quality of candidates: the low $R$ means a high probability that the candidate is true binary, as illustrated in Figure 5 in El-Badry et al. (2021). In the right panel of this figure, we observe: the lower $R$ (and higher quality of the candidate) means stronger suppression of more separated pairs. It is the result, which correlates with the shape of probabilistic function $\beta$ in A1. However, the ratio $R$ of both approximations does not strictly mean probability, which the authors admit.
We will compare the contents of the catalogs listed in Table 5. We define three types of binary candidates for any two compared catalogs $A_i, A_j$:

1. Unique—the candidate appears in only one catalog $N_{ij}^{\text{uniq}}$.
2. Identical—the candidate appears in both catalogs $N_{ij}^{\text{ident}}$.
3. Indefinite—cannot be decided, for example, two candidates from two catalogs have only one common star, or the pair in one catalog is part of a greater system in another one.

The results of the comparison are shown in Table 6. In the section above the diagonal, there are the numbers $N_{ij}^{\text{uniq}}$. In the catalog $A_2$, only such pairs can be used for comparison, where the IDs of both stars appear also in the Gaia-DR2 data. We perform the comparison of $A_2$ only with our catalog $A_1$, $|b| > 25^\circ$, so the other corresponding places in the table are empty ($\times$). Unique candidates are based on different selection conditions in $A_1$ and $A_2$. For example, in $A_2$, the unique part is generated mostly by candidates with greater angular separation ($d > 15$ as) than is accepted in $A_1$. On the other hand, $A_1$ imposes much weaker constraints on parallaxes, which generate its unique part. That is why in Figure 19 we observe many candidates more distant than 1000 pc (upper limit in $A_2$). The occurrence of trinaries and quaternaries is analyzed only in $A_1$. So, the content of both independent catalogs is partly complementary and partly identical.

Our current analysis is limited to $|b| > 25^\circ$ because we have verified that in the dense region $|b| < 25^\circ$, the efficiency of selection (ratio $\beta^3$) based on conditions (15) from the DR2 catalog is even lower than in the region $R_2$. Additional cuts on radial separation (similar to Table 2) are not sufficiently effective. With the expected DR3 data release, where higher accuracy of astrometric data (mainly of the parallax) is assumed, we plan to recalculate the selection of multiple stars in the full angular range (4$\pi$).

The content of our previous catalog $A_4$ has already been compared with the $A_5$, which contains 3055 binaries with magnitude $G \leq 13$ (Zavada & Piška 2020). Comparison $A_1$ with $A_5$ implies that only 381 of the $A_5$ candidates belong to $A_1$ region $|b| > 25^\circ$ and only about 60 of which meet selection criterion $d \leq 15$ as. As shown in Table 6, similar comparisons were made also for other catalogs. In general, the only partial overlap of different catalogs is due to mainly different cuts in selections algorithms.

For example, the content of present catalog $A_1$ is compared with our previous version $A_4$, where the binary candidates inside the surrounding cubic region (400 pc)$^3$ are recorded. The candidates from the previous catalog that meet the conditions

$$|b| > 25^\circ, d \leq 15 \text{ as}$$

should be present in the new catalog as well. There are 25,604 such candidates and we succeeded 24,949 of them to identify with binaries of the new catalog. So, the misidentification rate is small, $\approx 2.6\%$. It can occur, for example, with the diagram in Figure 18(e) that is excluded for the new catalog, but still one pair can meet the criteria of the previous one. At the same time, within the part of the cubic region that is common to both catalogs, the number of binary candidates of the new catalog is 79,771. It is $79,771/25,604 \approx 3$ times more, than the candidates in the previous one. This ratio proves high efficiency of the optimized method applied in the present analysis.

Our new catalog $A_1$ covers the region $|b| > 25^\circ$ up to the distance $\leq 3000$ pc and contains about 10 times more candidates than the previous $A_4$, which covered full 4$\pi$ geometry, but only up to $\leq 340$ pc. A total of almost $10^6$ candidates are recorded in both catalogs. For more detailed comparison of the catalogs $A_1, A_2, A_3, A_4, A_5, A_6$, which are based on the Gaia-DR2 data, we have created the merged catalog. This catalog is the list of pairs consisting of four items: order number of binary, mask 103456 denoting origin from $A_1$, $A_3$, $A_4$, $A_5$, $A_6$, and two DR2 sources ID. The catalogs $A_1$, $A_4$ and the merged one are available in the csv form on the website https://www.fzu.cz/~piska/Catalogue/.

### 5. Summary and Conclusion

With the use of the optimized statistical method for analysis of 2D patterns we studied occurrence of wide multiple stars: binaries, trinaries, and quaternaries. The candidates are selected using the astrometric data collected in the Gaia-DR2 catalog. So, we have studied the pairs with angular separation wider than $\approx 0.5$ as, which is present Gaia lower limit for resolution of two sources. Our present analysis covers the region of galactic latitude $|b| > 25^\circ$ and radial distance $L \leq 15,000$ pc. In this space we have analyzed about $1.3 \times 10^7$ circle events of angular diameter 144 as involving $7.5 \times 10^7$ sources. The circle shape is advantageous for calculation of the random background. The total number of processed sources with positive parallax is about $1.2 \times 10^8$.

The analysis is focused on two basic parameters related to any pair of sources in the multiple systems: angular separation $d$ and collinearity $\alpha$ of their proper motion. Distribution of these parameters is compared with distributions generated by

| Table 6 |
| --- |
| Comparison of Catalogs A1–A6 Shows the Numbers of Identical $N_{ij}^{\text{ident}}$ (above Diagonal) and Unique $N_{ij}^{\text{uniq}} \setminus N_{ij}^{\text{ident}}$ (Below Diagonal) Candidates |
| A1 | A2 | A3 | A4 | A5 | A6 |
|---|---|---|---|---|---|
| A1 | 343,302 | 39,031 | 24,949 | 59 | 2862 |
| A2 | 151,738 \(\times\) 555,691 | x | x | x | x |
| A3 | 54,530 \(\times\) 861,469 | x | 28,312 | 85 | 7290 |
| A4 | 54,707 \(\times\) 874,969 | x | 51,811 \(\times\) 65,148 | 108 | 7937 |
| A5 | 2979 \(\times\) 900,765 | x | 2967 \(\times\) 93,810 | 2937 \(\times\) 80,442 | 11 |
| A6 | 7115 \(\times\) 897,980 | x | 2282 \(\times\) 85,742 | 2040 \(\times\) 72,693 | 9966 \(\times\) 3015 |
the random background. The domain of the clear peak of binaries is limited by Equation (15) that serve as the cut for the selection of candidates. The exact knowledge of the background allows us to define probabilistic parameter $\beta$ representing the quality of candidates. Additional condition, which is required for the radial separation $\Delta L$, improves quality of candidates. After this selection (enriched exposition) the candidates are recorded in the attached catalog. Total numbers of candidates of wide multiple stars in the region $|b| > 25^\circ$ are shown in Table 4. The catalog is compared with some other catalogs of wide binaries selected from the Gaia data.

We have also shown that the results of the present 2D analysis are fully consistent with our previous 3D analysis of Gaia-DR2 data. We confirm that the projection of wide binary orbit approximately meets Equation (34). The average period and mass of the wide binary systems in Equation (38) are very similar to our previous estimates based on 3D analysis.

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