Abstract. This methodological note examines the epistemological foundations of hysteresis with particular reference to applications to economic systems. The economy principles of Ernst Mach are advocated and used in this assessment.

1. Introduction
Since the term “hysteresis” was first coined by James Alfred Ewing to describe the effects observed on iron and steel wires when they were magnetised when twisted [1], hysteresis effects have been observed or postulated to exist in a wide range of non-ferromagnetic contexts ranging from soil moisture [2] to migration flows [3]. The assessor for the Royal Society of London paper in which Ewing coined the term “hysteresis”, Sir William Thompson (Lord Kelvin), suggested that the more familiar phrase “effects of retentiveness” be used instead. Ewing stuck to his new term, arguing that similar effects would be found in a far wider range of contexts than ferromagnetism, see [4]. Subsequent work across the physical, natural and social sciences has attested to Ewing’s prescience, but also raised a challenge to received theories in the subject areas affected. Does the postulation of hysteresis, in an ontological, epistemological, mathematical or metaphorical guise, represent an improvement on the received wisdom? Are there rational grounds for supposing that the hysteresis-based explanations are in some sense to be preferred to ahysteretic explanations? The task of this note is to ask how this type of methodological question may be addressed.

Hysteresis was first coined to describe experimental observations at a macroscopic level. Major subsequent innovations include the Preisach model of the ferromagnetic behaviour at the micro level that could produce the macroscopic hysteresis observed [5], the Krasnosel’skii and Pokrovskii mathematical formalisation of hysteresis as a general systems property [6], and a diaspora of applications some metaphorical distance from the ferromagnetic homeland, see [7].

A cleavage between analytic statements, which are independent of matters of fact, and synthetic statements, grounded in fact, would suggest a need to appraise hysteresis differently according to the ontological, mathematical, metaphorical or epistemological context in which the term is used. If, however, we accept Quine’s “two dogmas of empiricism” demolition, the analytic-synthetic distinction is at best blurred and it is “nonsense, and the root of much nonsense, to speak of a linguistic component and a factual component in the truth of any individual statement – total science, mathematical and natural and human, is underdetermined by experience” ([8], pp. 42, 45). On this view ontological, and by extension metaphorical, issues are difficult to distinguish from epistemological ones. So the ontological issue, raised by
Leibniz, of whether action at a distance is possible, [9], which arises in relation to hysteresis because of the selective, erasable memory property, is blurred with the epistemological issue of whether action-via-memory explanations can provide “better” explanations than those without. Similarly applying a hysteresis metaphor derived from ferromagnetics to, say, voting behaviour raises issues of appropriateness to a political context which cannot be disentangled from whether the hysteresis account of voting behaviour provides a “better” explanation than one without. In terms of hysteresis in its guise as a mathematical construct, the blurring, though less stark, involves a sceptical rather than formalist approach to mathematical proofs. In line with the Lakatos reconstruction of the mathematical literature subsequent to Euler’s \( V - E + F = 2 \) conjecture regarding polyhedra [10], where \( V \) is vertices, \( E \) is edges and \( F \) is faces, the impact of the formalisation of hysteresis as a general systems property, and the appraisal of the subsequent mathematical innovations, can be seen as involving proofs and counter-examples that are difficult to distinguish from epistemological conjectures and refutations.

If hysteresis-inspired innovations are considered in epistemological terms a basic issue is whether there is some logic underlying scientific method that can guide an appraisal. After discussing the problems encountered when trying to articulate such a logic this note considers the conventionalist account of science, focussing on the epistemology of science proposed by Mach. This Machian framework is then used to appraise hysteresis-based explanations of economic phenomena.

### 2. A Logic of Scientific Method?

Laws, often named after the great scientist who discovered them, are pervasive in the language of science. The implicit suggestion is that the universe obeys such laws, but that it takes an exceptional scientist to discover them. “Nature and nature’s laws lay hid in the night, God said let there be Newton and all was light” was Pope’s epitaph to Newton. The problem is that the “truths” embodied in laws often change when laws are qualified, modified or are superseded by new laws. What we accept as laws today tends not to be what the scientific community previously accepted as laws. And there are, of course, disagreements about which laws should be accepted. The process of law revision can be cumulative, with new laws encompassing previous laws as special cases, but it is not unusual for there to be contradictions between rival laws or variants thereof. This raises the question of whether there is some logic of scientific method that could be used to arbitrate in the virtual court in which the claims of rival laws are assessed.

Induction is one such logic, espoused by Newton amongst others. Laws, or scientific theories, are based on observations, the theories being general laws covering the specific observations. There are several problems with this account of the logic underlying scientific theories. Hume pointed out that there are no logical grounds for expecting regularities in the relationships observed between phenomena in the past to recur. There may be a psychological propensity to expect such a recurrence, but that is not a matter of logic. Observations, also, are theory dependent. For example, the observation of Barkhausen effects in electromagnetic fields relies, inter alia, on some theory of molecular structure. So the idea that theories can be some generalisations of theory-independent observations is not tenable.

The hypothetico-deductive account of scientific method starts with general theories whose implications are to be tested by comparing them with the actual observational evidence. The Vienna Circle \((H \rightarrow I) \land O \rightarrow H\) version of hypothetico-deductivism, where \(H\) is hypothesis, \(I\) is implication and \(O\) is observation, the symbol \(\land\) means “conjoined with” and the arrow means “implies”, is logically invalid. Just because the implications of a hypothesis are observed to be the case does not imply that the hypothesis is true: true implications can follow from false premises. Logical empiricism accepts that hypotheses cannot be proved to be true, but argues that it is possible to talk meaningfully about the probabilities associated with hypotheses. There are problems with this position, the various paradoxes of confirmation being to the fore, such
as the paradox of the ravens or the “gruesome” paradox of Goodman [11]. We paraphrase the latter paradox as follows. \( H_1 \) says that all ferric metals are affected by hysteresis, \( H_2 \) says that this is the case for ferric metals observed up to now, but not in the future. Despite having radically different implications about the future, on the basis of the evidence available up to now we would have \( P(H_1) = P(H_2) \), where \( P(\ldots) \) is some epistemological probability function.

The falsificationism of Popper switches the hypothetico-deductive logic to the arrow of falsity rather than truth. So we have \((H \rightarrow I) \land \neg O \rightarrow \neg H\) where the \( \neg \) means “not”. A central problem is that it is virtually impossible to test a target hypothesis on its own, there inevitably being auxiliary hypothesis involved. As the Duhem-Quine thesis points out [8], the non-separability of the individual hypotheses involved in producing testable implications means that any refutations will be unfocussed, the arrow of falsity pointing at the theoretical whole rather than at any specific individual hypotheses. The Lakatos variant of falsificationism addresses this problem by regarding scientific research programmes as a whole as being the theoretical entities subjected to falsification. Thus we have the logic \((T \rightarrow I) \land \neg O \rightarrow \neg T\), where \( T \) is the research programme. Amongst other problems, that of the theory-dependence of observations also applies here. The seemingly “objective” evidence that could challenge research programmes is, to a degree, dependent on the theories that are to be tested. It is difficult to see how an account of the logic putatively underlying scientific method can circumvent this problem.

3. Conventions and Mach

If the theory-fact distinction is a matter of convention it is plausible to think of science more in terms of conventions than tight logical relationships between theories and facts. In the rhetoric account of the epistemology of science the issue is one of distinguishing between good and bad scientific arguments, honesty, clarity and tolerance being amongst the qualities required to distinguish the wheat from the chaff. In Kuhn’s account scientific theories are regarded as paradigms containing beliefs about the world as well as hypotheses, techniques and so on. During periods of “normal” science anomalous “facts” are explained away within the paradigm. It is only during intermittent periods of “revolutionary” science that accumulations of empirical anomalies challenge accepted paradigms and new paradigms, involving new beliefs about facts, emerge. Of the various conventionalist accounts of science that of Mach is particularly interesting in that it gives an account of how facts and theories emerge and interact. This might illuminate any different theory and fact guises in which hysteresis appears.

Mach’s account of science [12] was designed to be able to encompass not only his well-known work in physics but also his research on the physiology of senses. The starting point is one of reality being a non-repeating mosaic of elemental qualities. As Heraclitus is reported, in Plato’s Cratylus dialogue, to have said, “you do not put your foot into the same river more than once”. Memory plays a key role in economising on the effort required to place some sort of order on, or reveal disorder in, this Heraclitean flux. So, for example, memory recalls element types such as a “red” colour and can associate this with “fading of light” in a “not-completely occluded sky” to allow a “red sunset” to be recognised. By using conventions or classifications memory can pave the way for science by distilling regularities from non-recurrent elemental qualities. So memory is in effect producing low level theories to aid comprehension of the flux of otherwise unique facts.

Science, according to Mach, proceeds by applying an epistemological economy principle to the ontological classifications or associations that are now treated as “facts”. So, for example, Snell’s law is an attempt to provide an economical or compendious rule covering different types of light refraction. “It is the object of science to replace, or save, experiences by the reproduction and anticipation of facts in thought - science itself - may be regarded as a minimal problem, consisting of the completest possible presentment of facts with the least possible expenditure of thought” ([13], pp. 577, 586). This allies the “entities are not to be multiplied without need”
common sense of Ockham’s razor to “the completest possible presentment of facts”. It was for such economy of thought reasons that Mach was reluctant to accept the use of “atoms” in the theoretical language of physics, and, much more presciently, reformulated Newtonian mechanics without the theoretical concepts of absolute motion, space and time [13].

This Machian account of science has interesting implications for scientific theories involving hysteresis. A first issue is whether hysteresis is a fact or a theory. When first coined the term referred to what was seen as a fact. “I have found it convenient and even necessary to employ a new term, which merely designates this peculiar action without implying any theory as to its cause – hysteresis occurs when there are two qualities \( M \) and \( N \) such that the cyclic variations of \( N \) cause cyclic variations of \( M \) and the changes of \( M \) lag behind those of \( N \) . . . the value of \( M \) at any point of the operation depends not only on the value of \( N \), but on all the preceding changes (and particularly on the immediately preceding changes) of \( N \) . . .” ([14], pp. 524-526) In this original guise hysteresis is a “fact”, whereas in later guises hysteresis becomes a “theory”, [5, 6], illustrating the Machian shift from ontological to epistemological economy in the compendious classifications or theories that constitute science. The key question is whether the “facts” that hysteresis theories can explain are endemic, or whether they are special cases that could be explained with less expenditure of thought without invoking hysteresis by some more compendious theory.

A second issue illuminated is the Leibniz-type ontological objection to hysteresis as involving action at a temporal distance, effects being posited to remain after causes are removed. In the Machian world the way sensations appear in the present involves memory processes that are inherent in perception. So there is plausible, memory-driven, ontological process whereby the past can affect the present. At the epistemological level we would expect hysteresis, Mach’s economy principle implying that the more compendious scientific rules or theories that emerge are palimpsests that encompass the less compendious rules of past science.

A third aspect of hysteresis that fits in reasonably well with the Machian view of science is the selective, erasable nature of the memory proposed in theories of systems with hysteresis. The unique elemental qualities that are sensed are ordered by ontological and then epistemological classification, so if the classification rules change the perceptions can change. It is uncontroversial to point out that memory is subject to embellishment and forgetfulness, but the traditional view was that long-term memory is “hard-wired” and permanent, if sometimes difficult to retrieve. Research in neuroscience has challenged this view, suggesting that the act of recall can lead to a reconfiguration of long-term memory, facilitated by the short half-lives of the proteins in brain cells and synapses [15]. Systems with hysteresis are postulated to contain a memory of, and be influenced by, the non-dominated extremum values of the perturbations affecting the system [6]. So, large perturbations in input variables can wipe out the memory of previous perturbations that were smaller. Work in neuroscience suggests that memory processes are malleable enough for this to happen, though it remains to be seen if memory reconfigurations of the precise type suggested by hysteresis can be identified at the level of the individual brain. The intriguing possibility here is that the brain might be representable as a system with hysteresis. In a social science context this would suggest that behaviour could be represented in terms of hysteresis at both the level of the individual agent and at the aggregate level. In the physical sciences the challenge is to see if such memory processes can be identified at molecular, atomic or sub-atomic levels as well as at the aggregate level.

4. The Economy of Hysteresis Theories in Economics

Economics is often upbraided as not being a science, or as not being a “hard” science, because of the debatable nature of many economic “facts”. So a Machian view of science, in which what are regarded as “facts” depends on the classification devices or theories used, can at least explain how this is the case in economics, and indeed explain how the harder sciences are not immune
from such problems. For example, the term “unemployment” first came into use in the English language in the 1880s. It is perhaps more than coincidence that this was at a time when Hobson and others were postulating that workers could be “involuntarily” without work and surveys came to be conducted to try and determine the extent of such unemployment - earlier data relied on trades union records of members out of work and receiving benefits from the unions. Similar theory-dependence can be discerned in a wide range of economic data, such as theories of utility maximisation influencing the appropriate price index used to measure inflation, or theories of inequality aversion having an influence on absolute or relative measures of poverty.

A second area in which a Machian epistemology throws favourable light on hysteresis theories in economics regards the heterogeneous representation of economic agents. In the mainstream dynamic, stochastic, general equilibrium (DSGE) model in macroeconomics the obvious differences between economic agents are suppressed by assuming the existence of a single representative agent. As Solow puts it, this involves a “. . . macroeconomy that is deduced from a model in which a single immortal consumer-worker-owner maximises a perfectly conventional time-additive utility function over an infinite horizon, under perfect foresight or rational expectations, and in an institutional and technological environment that favours universal price-taking behaviour – it is taken as an advantage that the same model applies in the short run, the long run and every run with no awkward shifting of gears” ([16], p.243). The contrast with hysteresis macroeconomic models which represent economic agents as hysterons characterised by different propensities to respond to aggregate economic shocks, is stark. The hysteresis models contain a far more complete “presentment of the facts” of heterogeneity between economic agents, between those who can borrow from high street banks and those who cannot, those who are in secure jobs, insecure jobs and without work - indeed the DSGE model does not give a serious account of unemployment - and so on. A standard defence of “representative agent” assumptions in DSGE and other models is as a simplification necessary to deal with the problem of aggregating over economic agents to explain aggregate variables such as GDP and inflation. Hysteresis models, however, deal with the aggregation problem whilst respecting the heterogeneity of economic agents, the testable implication being a selective, erasable memory of aggregate economic shocks to interest rates, oil prices and so on. To date hysteresis models have not used the formal structures of intertemporal utility maximisation subject to budget constraints used in the DSGE model, but, as the above quotation from Solow suggests, much of this apparatus is inconsistent with the “facts” of economic behaviour at the micro level. There is no inherent difficulty in incorporating models of maximising behaviour into hysteresis models of heterogeneous economic agents ([17] for example), though that could violate the Machian economy principle of only employing theoretical entities when necessary for the “completest presentment of the facts”.

A third area where Machian principles throw favourable light on hysteresis models is in the representation of the memory of shocks affecting economic systems. Mainstream economic models, such as the DSGE model, contain a short-term but no long-term memory of such shocks. At the level of the individual economic agent this is inconsistent with the neuroscientific evidence on long-term memory, the key issue being how cognitive and affective processes affect the relative impact of short and long-term memory, or anticipation, on economic behaviour rather than complete erasure of long-term memory. At the aggregate level the mainstream models imply that the major shocks associated with wars, political revolutions, famines, financial crises and so on are eventually forgotten. This is inconsistent with a wide variety of evidence, such as that many women who joined the non-domestic workforce during the 1939-45 World War remained as participants in this type of work long after the war was over. The evidence on recovery from the recessions that often emerge after financial crises, wars and so on is particularly interesting in this respect. Cerra and Saxena [18] used World Bank data for 192 countries 1960-2001 to investigate the speed and extent of the subsequent recoveries from recessions. Most of the recoveries were
reasonably steep, output regaining its pre-recession level within three years of the recession trough in most of the episodes studied. The recoveries, however, left the economies in question below the pre-recession trend value for output, implying that the crises had a lasting effect. This is consistent with the memory properties of hysteresis models, but not with the no long-term memory property of mainstream models. An interesting implication is that the low growth rates experienced in many countries in sub-Saharan Africa and elsewhere may, inter alia, reflect the historical legacy of “one damned crisis after another”. Some tentative econometric evidence is available to support the selective, erasable memory property in relation to unemployment [19], though a major problem is that many economic time series are too short to contain enough extremum values for these tests to have much power against alternative specifications. Unit root representations of economic time series imply that all past shocks are remembered. Hysteresis models imply that major shocks are remembered until superseded by bigger shocks, and so lie between the devil of no long-term memory and the deep blue sea of more or less infinite memory with no forgetting.

A fourth area of note is the consistency of the Machian view with the practice, if not the preaching, of economics regarding “stylised facts”. In the preaching of mainstream economics the “facts” that should be used to test the implications of economic theories are the raw data, over or across time. The Machian account of ontological economy tells us that a “good” representation of reality is one that gives “the completest possible presentment of facts with the least possible expenditure of thought”. So “stylised facts” can obey the economy principle. In practice, economists often use consistency or inconsistency with stylised facts as evidential grounds for supporting or criticising theories. So, for example, a theory of consumption may be criticised or supported by the “excess sensitivity of consumption to current income”, where the “excess” is in relation to the implications of permanent income or life cycle theories of consumption; efficient market theories in finance are criticised for being inconsistent with “non-Gaussian” features of asset market returns, such as fat tails and clustered volatility; stylised puzzles, such as the “equity premium puzzle”, are testimony to how the raw data are subject to filtering in a manner not inconsistent with Machian principles. If nothing else, Mach’s epistemology can allow economists to live at greater ease with important aspects of their practice when appraising the evidential merits of their theories. Indeed, arguably the most eminent economist of the last seventy years has endorsed Mach as the philosopher of science whose precepts correspond most closely to his working practice [20, 21].

5. Some Limitations

The memory properties of hysteresis models of economic behaviour are interesting and consistent with many stylised facts about the behaviour of economic systems. There is, however, a lack of quantitative precision in the accounts of how perturbations in input shocks affect economic outcomes. At the level of the economic agent it will be interesting to see if the imaging techniques of neuroscience can detect the specific memory properties postulated in hysteresis accounts of the neural processes underlying economic decision taking. At the aggregate level the problem lies in finding data sets rich enough in extremum values to allow greater precision and power in tests for the existence of the memory of the non-dominated extremum values of input shocks postulated. The high frequency data sets available for financial time series offer an interesting area for research in this respect.

A related limitation is the lack of information on the values of the switching points involved in Preisach-type models of hysteresis. The procedure used in economic models is to assume some statistical distribution for the weight function of switching points across economic agents, and check for sensitivity to alternative distributional assumptions. Neuroscience might be able to provide measures of increases in brain activity as switching points are approached and decisions to change behaviour are activated. Experimental economics can also be used to see if the
switching behaviour postulated is observed in simulated laboratory conditions. There is a rich research agenda here, including testing for asymmetry in reactions to gains and losses, and for evidence of revision of switching points in response to revealed outcomes as the experiments are repeated.

A further limitation is that hysteresis models in economics to date have been concerned with the determination of just single endogenous variables. There are obvious interactions between aggregate economic variables such as output and inflation, so a shift towards vector hysteresis models in economics would be useful. This would allow the models to address important control problems, such as how to vary central bank repo interest rates in order to achieve inflation and output objectives. Output, and by extension inflation, retains a memory of the non-dominated extremum values of past values of input variables such as the repo interest rates set by central banks, so hysteresis models have the Keynesian feature that policy innovations can have lasting effects on the time path for output, in contrast to mainstream models in which the only lasting impact is on inflation. Work on providing mean-field approximations to heterogeneous agent simulation models of economic behaviour would also be useful to simplify the control specifications.

References

[1] Ewing J A 1881 On the production of transient electric currents in iron and steel conductors by twisting them when magnetised or by magnetising them when twisted, *Proceedings of the Royal Society of London* 33 21–23
[2] Flynn D, McNamara H, O’Kane P and Pokrovskii A 2006. Application of the Preisach model to soil-moisture hysteresis, in: *The Science of Hysteresis Vol. III*, eds. G. Bertotti and I.D. Majerczyk (Amsterdam: Elsevier) 689–744.
[3] Kemp M C and Wan J R Jnr 1974 Hysteresis of long-run equilibrium from realistic adjustment costs, in: *Trade, Stability and Macroeconomics*, eds. G. Horwich and P.A. Samuelson (New York: Academic Press) 175–189
[4] Ewing A W 1939 *The Man of Room 40: The Life of Sir Alfred Ewing* (London: Hutchinson)
[5] Preisach P 1938. ‚Uber die Magnetische Nachwirkung. Zeitschrift für Physik 94 277–302
[6] Krasnosel’ski M A and Pokrovskii A V 1989 *Systems With Hysteresis* (Berlin: Springer-Verlag)
[7] Bertotti G and Mayergoyz I D eds. 2006 *The Science of Hysteresis* Vols I-III (Amsterdam: Elsevier)
[8] Quine W V O 1980 *From a Logical Point of View* 2nd ed. (Cambridge MA: Harvard UP)
[9] Elster J 1976. A note on hysteresis in the social sciences *Synthese* 33 371–391
[10] Lakatos I 1979 *Proofs and Refutations: the Logic of Mathematical Discovery* (Cambridge UK: Cambridge UP)
[11] Goodman N 1983 *Fact, Fiction and Forecast* 4th ed., (Cambridge MA: Harvard UP)
[12] Banks E C 2004 The philosophical roots of Ernst Mach’s economy of thought *Synthese* 139 23–55
[13] Mach E 1883 *The Science of Mechanics* trans T.J. McCormack 1960 (La Sale, Open Court Publishing)
[14] Ewing J A 1885 Experimental researches in magnetism *Philosophical Transactions of the Royal Society of London* 176II 523–640
[15] McCrone J 2003 Not-so total recall *New Scientist* 13 May 26–29
[16] Solow R M 2008 The state of macroeconomics *J. of Economic Perspectives* 22.1 243–246
[17] Dixit A and Pindyck R S 1994 *Investment under Uncertainty* (New Jersey: Princeton UP)
[18] Cerra A and Saxena S C 2005 Growth dynamics: the myth of economic recovery. *IMF Working Paper WP/05/147* (Washington DC: IMF)
[19] Darby J, Cross R and Piscitelli L 2006. Hysteresis and Unemployment: a Preliminary Investigation, in: *The Science of Hysteresis* Vol. I, eds. G. Bertotti and I.D. Mayergoyz (Amsterdam: Elsevier) 667–699
[20] Samuelson P A 1992 My Life Philosophy: Policy Credos and Working Ways, in: *Eminent Economists: their Life Philosophies* ed. M. Szenberg (New York: Cambridge UP)
[21] Cross R 2006 Paul Samuelson’s Mach, in: *Samuelsonian Economics and the Twenty-First Century* Eds. M.Szenberg, L. Ramrattan and A.A. Gottesman (New York: Oxford UP) 330–341