Experimental Detection of Entanglement with Polarized Photons

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Abstract

We report on the first experimental realization of the entanglement witness for polarization entangled photons. It represents a recently discovered significant quantum information protocol which is based on few local measurements. The present demonstration has been applied to the so-called Werner states, a family of ”mixed” quantum states that include both entangled and non entangled states. These states have been generated by a novel high brilliance source of entanglement which allows to continuously tune the degree of mixedness.
One of the main issues of modern technology is the manipulation of information, its transmission, processing, storing, and computing, with an increasingly high demand of speed, reliability and security. Quantum Physics has recently opened the way to the realization of radically new information-processing devices, with the possibility of guaranteed secure cryptographic communications, and of huge speedups of some computational tasks. In this respect quantum entanglement represents the basis of the exponential parallelism of future quantum computers [1], of quantum teleportation [2–5] and of some kinds of cryptographic communications [6,7]. In practical realizations, however, entanglement is degraded by decoherence and dissipation processes that result from unavoidable couplings with the environment. Since entanglement is an expensive resource—it cannot be distributed between distant parties by classical communication means—it becomes crucial to be able to detect it efficiently, with the minimum number of measurements. Several methods have been proposed to assess the presence of entanglement for different types of quantum systems [8–13]. In particular, the so-called method of ”entanglement witness” is a simple and efficient protocol that uses only a few local measurements [13]. In the present paper we report the first experimental implementation of an entanglement witness for polarization-entangled photons. The entangled photon state is generated by a new general method of spontaneous parametric downconversion (SPDC) and the entanglement is detected by only three independent quantum measurements.

Precisely, we implement experimentally the method of Ref. [10] for a pair of polarized photons that can be in any of the so-called Werner states [14], a family of ”mixed” quantum states that include both entangled and non entangled states. The key resource of our apparatus consists of a new, universal high-brilliance source of bi-partite entangled states spanning the Hilbert space $H_1 \otimes H_2$, where $\text{dim}(H_1) = \text{dim}(H_2) = 2$. These states can be generated as either “pure” or ”mixed”, with a complete control of the degree of mixedeness [15,16]. In particular, here we generate the Werner states

$$
\rho_W = p|\Psi_-\rangle\langle \Psi_-| + \frac{1-p}{4} \mathbb{1}_4 ,
$$

(1)
which are mixtures with probability $p \in [0, 1]$ of the maximally chaotic state $\frac{1}{4} \mathbb{1}_4$ ($\mathbb{1}_4$ is the four dimensional identity operator) and of the maximally entangled "singlet" state:

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}(|HV\rangle - |VH\rangle),$$

(2)

where $|HV\rangle \equiv |H\rangle_1 \otimes |V\rangle_2$ denotes the state of the two photons, the symbols $H$ and $V$ representing horizontal and vertical linear polarizations, respectively. The method to establish whether a state is entangled or not is based on the concept of entanglement witness [17,18]. According to this framework a state $\rho$ is entangled if and only if there exists a Hermitian operator $W$—so called *entanglement witness*—which has positive expectation value $\text{Tr}[W\rho_{\text{sep}}] \geq 0$ for all separable states $\rho_{\text{sep}}$, nevertheless has negative expectation value $\text{Tr}[W\rho] < 0$ on our state $\rho$. For pairs of qubits—the two-level quantum systems of quantum information—in our case the polarized photons, also the non positivity of the partial transposition of the state $\rho$ gives a necessary and sufficient criterion for entanglement [19,20]. In this case simple ways to construct entanglement witnesses are known [18]. The Werner states (2) which are tested in our experiment, are particularly appropriate for entanglement detection, because they include both entangled ($p > 1/3$) and separable ($p \leq 1/3$) states.

The detection method proposed in [10] in the case of Werner states expressed by (1) gives the following *entanglement witness* operator

$$W = \frac{1}{2}(|HH\rangle\langle HH| + |VV\rangle\langle VV| + |DD\rangle\langle DD|$$

$$+ |FF\rangle\langle FF| - |LR\rangle\langle LR| - |RL\rangle\langle RL|),$$

(3)

where $|D\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$ and $|F\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$ denote diagonally polarized single photon states, while $|L\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle)$ and $|R\rangle = \frac{1}{\sqrt{2}}(|H\rangle - i|V\rangle)$ correspond to the left and right circular polarization states. The above operator can be locally measured by choosing correlated measurement settings, that allow detection of the linear, diagonal and circular polarization for both photons. It represents the most efficient witness, since it involves the minimum number of local measurements.

The experimental apparatus is shown in Fig. 1. A Type I, .5mm thick, $\beta$-barium-borate (BBO) crystal is excited by a $V$-polarized cw Ar$^+$ laser beam ($\lambda_p = 363.8\text{nm}$) with
wavevector $-k_p$, i.e. directed towards the left in Fig.1. The two degenerate ($\lambda = 727.6nm$) SPDC photons have common $H$-polarization, and are emitted with equal probability, over a corresponding pair of wavevectors belonging to the surface of a cone with axis $k_p$. The emitted radiation and the laser beam are then back-reflected by a spherical mirror $M$ with curvature radius $R = 15\text{cm}$, highly reflecting both $\lambda$ and $\lambda_p$, placed at a distance $d = R$ from the crystal. A zero-order $\lambda/4$ waveplate placed between $M$ and the BBO intercepts twice both back-reflected $\lambda$ and $\lambda_p$ beams and then rotates by $\pi/2$ the polarization of the back-reflected photons with wavelength $\lambda$ while leaving in its original polarization state the back-reflected pump beam $\lambda_p \approx 2\lambda$. The back-reflected laser beam excites an identical albeit distinct downconversion process with emission of a new radiation cone directed towards the right in Fig.1 with axis $k_p$. In this way each pair originally generated towards the left in Fig. 1 is made, by optical back-reflection and a unitary polarization-flipping transformation, "in principle indistinguishable" with another pair originally generated towards the right and carrying the state $|HH\rangle$. The state of the overall radiation, resulting from the two overlapping indistinguishable cones, is then expressed by the pure entangled Bell-state:

$$|\Phi\rangle = \frac{1}{\sqrt{2}} (|HH\rangle + e^{i\phi}|VV\rangle)$$ (4)

with phase ($0 \leq \phi \leq \pi$) reliably controlled by micrometric displacements $\Delta d$ of $M$ along $k_p$. A positive lens transforms the overall emission conical distribution into a cylindrical one with axis $k_p$, whose transverse circular section identified the "Entanglement-ring". Each couple of points symmetrically opposed through the center of the ring are then correlated by quantum entanglement. An annular mask with diameter $D = 1.5\text{cm}$ and width $\delta = .07\text{cm}$ provides an accurate spatial selection of the ring. This is divided in two equal portions along a vertical axis by a prism-like two-mirror system and detected by two independent silicon-avalanche photodiodes, mod. SPCM-AQR14 at sites $A$ and $B$. Typically, two equal interference filters, placed in front of the $A$ and $B$ detectors, with bandwidth $\Delta \lambda = 6\text{nm}$, determine the coherence-time of the emitted photons: $\tau_{coh} \approx 140 \text{ fsec}$. More than 4000 coincidences per second are detected for a pump power $P_p \approx 100\text{mW}$. 


Werner states (1) are generated by selecting a convenient patchwork technique which implies the following steps [16]: [i] Making reference to the original source-state expressed by Eq. (4), the singlet state $|\Psi_-\rangle$ is easily obtained by inserting a zero-order $\lambda/2$ waveplate in front of detector $B$. [ii] A anti-reflection coated glass-plate $G$, 200$\mu$m thick, inserted between $M$ and BBO with a variable transverse position $\Delta x$, introduces a decohering fixed time-delay $\Delta t > \tau_{coh}$ that spoils the indistinguishability of the intercepted portions of the overlapping quantum-interfering radiation cones: Fig.1, inset. As a consequence, all nondiagonal elements of $\rho_W$ given by the surface sectors $B + C$ of the Entanglement-ring, the ones optically intercepted by $G$, are set to zero while the non intercepted sector $A$ expresses the singlet contribution to $\rho_W$. [iii] A $\lambda/2$ waveplate is inserted in the semi-cylindrical photon distribution reflected by the beam-splitting prism towards the detector $A$. Its position is carefully adjusted in order to intercepts half of the $B + C$ sector, i.e. by making $B = C$. Note that only half of the ring which is represented in Fig.1 inset needs to be intercepted by the optical plates, in virtue of the EPR nonlocality. In summary, the sector $A$ contributes to $\rho_W$ with probability $p$ with the pure state $|\Psi_-\rangle \langle \Psi_-|$, the sector $B + C = 2B$ with probability $1 - p$ with the mixture:

$$\frac{1}{4} \left[ |HV\rangle \langle HV| + |VH\rangle \langle VH| + |HH\rangle \langle HH| + |VV\rangle \langle VV| \right]$$ (5)

and the probability $p$ can be easily varied over its full range of values, going from $p = 0$ ($\rho_W \equiv \frac{1}{4} \mathbb{I}_4$) to $p = 1$ ($\rho_W \equiv |\Psi_-\rangle \langle \Psi_-|$).

In order to detect whether the produced Werner state is entangled or not, the expectation value of the witness operator (3) has been measured, by performing local correlated measurements on each arm of the linear, diagonal and circular polarization. The polarization of the detected photon is selected by means of a sequence of $\lambda/4$ and $\lambda/2$ waveplates, and a polarization beam splitter. Several different values of the singlet weight $p$ have been tested in the experiment. Each measurement corresponding to the different probabilities of Eq. (3) which contribute to evaluate the entanglement witness has been performed with an average of 30 sec. The relative results are shown in Fig. 2, where the experimental expectation
value of $W$ is plotted as a function of $p$, with the corresponding error bars. The agreement of the experimental results with the theoretical prediction $\text{Tr}[W \rho_W] = (1 - 3p)/4$ appears very good. The experimental results verify the transition between separable and entangled Werner states, occurring at $p = 1/3$, $\text{Tr}[W \rho_W] = 0$, as expected.

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**Figure Captions**

FIGURE 1. Scheme of the experimental apparatus. The polarization entangled photons are generated by spontaneous parametric down conversion in a nonlinear BBO crystal, and are detected by two silicon-avalanche photodiodes preceded by polarization analyzers. In this experiment we exploit a novel geometry for collecting down-converted radiation, which results in a very bright source of entangled photons, with also the advantage of complete freedom in choosing the two-photon state. Inset: partition of the Entanglement-ring into the spatial contributions of the emitted pair distribution to an output Werner-state.

FIGURE 2. Experimental results of entanglement detection for Werner states. These states range from the pure singlet at weight $p = 1$ to the totally chaotic state at $p = 0$ (see 2, with the transition between entangled and separable states at $p = 1/3$. The entanglement is detected by the witness observable given in 3. The straight line corresponds to the theoretical prediction. The experimental results verify the transition between separable and entangled Werner states, occurring at zero-witness at $p = 1/3$ (dotted line).
Witness vs. Singlet weight