Fuzzy inventory models with partial backordering for deteriorating items under stochastic inflationary conditions: Comparative comparison of the modeling methods

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Abstract: This article presents two deteriorating inventory models with constant demand and deterioration rates to determine optimal ordering policy under inflation and partial backlogging with respect to two modeling methods, the average annual cost method and the discounted cost method. Minimizing the total inventory costs over an infinite time horizon is the objective function of the models. Here, the unit purchasing cost is in the uncertain environment and assumed as a fuzzy number. The fuzzy models are solved by the fuzzy non-linear programming method with a numerical example in the GAMS software. The two mentioned modeling methods have been compared to each other in this study. The obtained results of this comparison will be useful for the inventory managers to make a better decision.

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PUBLIC INTEREST STATEMENT
One of the most popular topics in industrial engineering is related to inventory control models. Therefore, these models considerably are studied by many researchers. The average annual cost and the discounted cost methods are two well-known modeling methods of the inventory models. Due to the mentioned modeling methods, two inflationary inventory models with partial backordering for deteriorating products are developed in this article. Determining exact value for some parameters of the inventory problems is impossible Hence, here, unit purchasing cost is considered as a fuzzy number. In order to solve the fuzzy models, the fuzzy non-linear programming method is used. The original goal of the study is comparison the derived inventory models of above two modeling methods. Thus, finally, the fuzzy models of these methods are compared to each other using a sensitivity analysis over all parameters.
A sensitivity analysis of the wide range of the problem parameters of the derived models to illustrate the theoretical results.

Subjects: Operations Research; Production Systems; Supply Chain Management

Keywords: inventory; the average annual cost method; the discounted cost method; partial backordering; fuzzy

1. Introduction
The inflation and time value of money have significant effects on the inventory system costs, especially in developing countries. Thus, these factors must be considered in modeling inventory costs. Buzacott (1975) presented an inflationary economic order quantity (EOQ) model with a constant inflation rate. Bierman and Thomas (1977) studied an inventory model under inflation and discount. Misra (1979) presented a discounted cost model with considering the time value of money and different inflation rates for the inventory costs. Hariga and Ben-Daya (1996) dealt with the inventory replenishment problem with linearly time-varying demand in a fixed planning horizon under inflationary conditions. Dey, Mondal, and Maiti (2008) investigated two-storage inventory problems with finite time horizon under inflation and time value of money where demand rate is dynamic. Sarkar and Moon (2011) proposed an inflationary economic production quantity (EPQ) model with stochastic demand in an imperfect production system. Mousavi, Hajipour, Niaki, and Alikar (2013) developed a mixed binary integer mathematical programming model for a multi-item multi-period inventory control problem with discounts, time value of money, and inflation.

Unlike the above cases, a few researches considered the inflation rate as a random variable. For example, Horowitz (2000) studied an economic order quantity model under stochastic inflationary conditions. Mirazadeh, Ghami, and Esfahani (2011) presented an inventory model with stochastic inflation rate for multiple items with budget constraint.

Some products of the inventory system like fruits, vegetables, meat, perfumes, etc. are gradually deteriorated during their storage period. Therefore, many researchers considerably studied inventory models with perishable items in the recent years. Dave and Patel (1981) discussed a deteriorating inventory model with time-proportional demand where shortages are not allowable. Chung and Tsai (2001) proposed an inventory model for deteriorating items under time value of money. Yang (2004) studied the deteriorating two-warehouse inventory problem with constant demand rate under inflation and shortages. Moon, Giri, and Ko (2005) presented the EOQ models for ameliorating/deteriorating items considering the inflation effect. Tripathy and Mishra (2010) studied the deteriorating inventory model with assumptions of the Weibull distribution for deterioration rate, quadratic demand, and permissible delay in payments. Prasad and Mukherjee (2016) developed a deteriorating inventory model with stock- and time-dependent demand considering a two-parameter Weibull distribution for the deterioration rate. Duan, Cao, and Huo (2018) investigated a joint dynamic pricing and production problem for deteriorating products in a finite time horizon.

In real life, backlogging is not accepted by some customers and so demands are usually lost for certain types of products (e.g. foods, pharmaceutical, and others) during the shortage period. Papachristos and Skouri (2000) considered the deteriorating inventory model with exponential partial backordering where the demand rate is time-varying. Teng and Yang (2004) studied the EOQ models with partial backlogging when demand and purchase cost are fluctuating with time. Chern, Yang, Teng, and Papachristos (2008) discussed deteriorating inventory models with assumptions of time-varying demand, partial backordering, and inflation. Sana (2010) presented an EOQ model with partial backordering for deteriorating items under time-varying deterioration rate and price-dependent demand in an infinite time horizon. Yang (2012) presented a two-warehouse inventory model under partial backlogging and inflation with considering the three-parameter Weibull distribution for deterioration rate.
Mishra, Singh, and Kumar (2013) considered a deteriorating inventory model with partial backlogging where the demand rate is time-dependent and holding cost is assumed to be time-varying. Mousavi, Sadeghi, Niaki, and Tavana (2016) developed a seasonal multi-product multi-period inventory model with partial backordering and all-unit discount. Khalilpourazari, Pasandideh, and Niaki (2016) presented a multi-product EPQ model under partial backordering with fixed and linear costs. San-José, Sicilia, González-De-la-Rosa, and Febles-Acosta (2017) studied an inventory model with partial backlogging under power demand pattern. Chakraborty, Jana, and Roy (2018) studied two-warehouse inventory models with ramp type demand rate, partial backlogging, and three-parameter Weibull distribution for deterioration rate under inflationary conditions. Rastogi and Singh (2018) presented a deteriorating production inventory model with partial backordering under inflation where the demand rate is dependent to selling price and the production rate is a function of occurring demand. Singh, Kumar, and Yadav (2018) investigated a deteriorating two-storage inventory model with time, selling price, and advertisement dependent demand under backlogging.

Moreover, a series of inventory models have been studied in a fuzzy environment. Katagiri and Ishii (2002) investigated a deteriorating inventory model with fuzzy shortage costs. Maity and Maiti (2008) studied a multi-item production inventory system with deteriorating items and dynamic demand under fuzzy inflation and time discounting. Samal and Pratihar (2014) developed an inventory model in a fuzzy environment where holding, ordering, and backorder costs are represented by fuzzy numbers. Pal, Mahapatra, and Samanta (2015) investigated an EPQ model for deteriorating items with ramp type demand rate, inflation, and shortages under fuzziness. Sharmila and Uthayakumar (2015) presented a deteriorating inventory model with power demand pattern under fully backlogged. Here, all related inventory parameters have been considered as fuzzy variables. Kazemi, Olugu, Abdul-Rashid, and Ghazilla (2016) developed a fuzzy EOQ model with backordering by modeling forgetting effect in setting the fuzzy parameters. Mohanty and Tripathy (2017) presented a fuzzy inventory model for deteriorating items with exponentially decreasing demand under partial backlogging. Indrajitsingha, Samanta, and Misra (2018) proposed an EOQ model of deteriorating items with stock-dependent demand in fuzzy environment considering demand and deterioration rates, holding and deterioration costs as triangular fuzzy numbers. Shaikh, Bhunia, Cárdenas-Barrón, Sahoo, and Tiwari (2018) studied a deteriorating inventory model with assumptions of selling price and advertisement dependent demand, back ordering, permissible delay in payments, and fuzziness.

The average annual cost and the discounted cost methods are known as two methods of modeling the inventory system which determines the optimal value of decision variables (time and quantities of order) by minimizing total average annual cost and the present value of total future costs, respectively. Since the obtained optimal values of two mentioned modeling methods may differ to each other, the comparison of these methods is very important. The existing difference between related modeling methods is affected on the final decision of the manager. Based on the results of the comparing process, the decision-maker is able to access the best system analysis. Hadley (1964) and Mirazadeh (2011) compared these methods to each other under various conditions.

This study extends Mirazadeh’s work (2011) to develop two deteriorating inventory models under stochastic inflationary conditions by considering assumptions of internal and external inflation rates, partial backordering, and fuzziness. Therefore, in the present article, two fuzzy inventory models with partial backordering are developed using the average annual cost and the discounted cost methods. Then, in order to determine the best method for system analysis, the models of these modeling methods are compared to each other.
The rest of this study is organized as follows. Section 2 introduces the used assumptions and notations to formulate the models of this paper. In Section 3, the inventory models using the average annual cost and the discounted cost methods are derived. In Section 4, the proposed inventory models in fuzzy environment are surveyed. Then, in Section 5, a numerical example is employed to illustrate the solution procedure of the fuzzy models. In Section 6, by performing a sensitivity analysis on the parameters, two mentioned methods are compared to each other. In Section 7, a conclusion and some proposal cases for future researches are presented.

2. Assumptions and notations
In this section, the required assumptions and notations of the models are presented.

2.1. Assumptions
The inventory models are developed based on the following assumptions:

1. The models are studied in an infinite time horizon.
2. Shortages are allowed and partially backlogged.
3. Due to the internal (company) and external (general economic) inflation rates, the inventory costs are divided into two classes. The inflation rates are assumed as random variables with known distributions.
4. The deterioration, backorder, and demand rates are known and constant.
5. The replenishment is instantaneous and the initial inventory is zero.
6. The interest rate is greater than the internal and external inflation rates.
7. In both models, the unit external purchasing cost is considered as a fuzzy number and the membership function of the unit purchasing cost $C$ is as follows:

$$
\mu(C) = \begin{cases} 
1 & \text{for } C \geq C_0 \\
1 - \frac{C - C_0}{P_1} & \text{for } C_0 - P_1 \leq C \leq C_0 \\
0 & \text{for } C \leq C_0 - P_1
\end{cases}
$$

where $C_0$ and $P_1$ represent the initial value of the unit purchasing cost and its maximum permissible tolerance, respectively.

2.2. Notations
The used notations of this paper are summarized as follows:

- $j$: Index of inventory cycles ($j = 1, 2, \ldots, \infty$)
- $A$: The ordering cost per order ($$/order)
- $D$: The demand rate per unit per time (units/year)
- $r$: The interest rate ($$/$/year)
- $\beta$: Percentage of backordered demands during shortage period ($0 \leq \beta \leq 1$)
- $\theta$: The constant deterioration rate ($0 \leq \theta \leq 1$)
- $i_m$: The internal ($m = 1$) and external ($m = 2$) inflation rates
- $f(i_m)$: The probability density function of $i_m$
The unit internal \((m=1)\) and external \((m=2)\) holding costs at time zero \(($/\text{unit/year})\)

\(C_2m\) The unit internal \((m=1)\) and external \((m=2)\) backorder costs at time zero \(($/\text{unit/year})\)

\(C_2'\) The unit internal \((m=1)\) and external \((m=2)\) lost sale costs at time zero \(($/\text{unit/year})\)

\(C\) The unit external purchase cost \(($/\text{unit})\)

\(\tilde{\mathcal{C}}\) The fuzzy unit external purchase cost \(($/\text{unit})\)

\(T\) The interval of the time between two sequential orders

\(K\) A part of the inventory cycle with positive inventory level \((0 \leq K \leq 1)\)

\(EAC(K,T)\) The total average annual cost

\(EDC(K,T)\) The total discounted cost

The other required notations are defined later.

3. Formulation of the models

According to the assumptions cited above, the inventory system is studied under an infinite time horizon so as this planning horizon divided into equal parts with length \(T\). Each inventory cycle, \(T\), includes two parts, namely, the positive inventory level and shortage level. \((0 \leq K \leq 1)\) is a part of the inventory cycle with positive inventory level. Based on Figure 1, the positive inventory level gradually decreases due to deterioration and demands. The inventory system is faced with

Figure 1. Graphical representation of the inventory system.
shortage at time \((j + K - 1)T\). During shortage period, \([(j + k - 1)T, jT]\), there is no deterioration and shortage level linearly increases with respect to the demand rate. Here, shortages are accumulated until \(jT\) and partially back ordered.

The components of the total inventory cost include ordering, purchasing, holding, and shortage costs that are modeled by the average annual cost and the discounted cost methods. Considering the internal and external inflation rates, these costs are divided into two cost classes, the ordering cost and purchasing cost placed in the internal and external cost classes, respectively. The holding and shortage costs change with respect to both inflation rates and hence these costs are members of both classes.

3.1. The average annual cost model
Referring to Mirzazadeh (2011), the total average annual cost under two internal and external inflation rates is formulated as follows:

- The average annual ordering cost:
  \[
  EACR(K, T) = E[1 + i_1(1 - T)/A]/T = [1 + \mu_1(1 - T)/2]A/T
  \]  

- The average annual purchasing cost:
  \[
  EACP(K, T) = C \left[ \frac{D(e^{\theta KT} - 1)}{\theta} + DT(1 - K) \right] [1 + \mu_2(1 - T)/2]/T
  \]  

- The average annual holding cost:
  \[
  EACH(K, T) = D \left( \frac{\theta KT e^{\theta KT} - e^{\theta KT} + 1}{\theta^2 T} \right) \sum_{m=1}^{2} (C_{2m}[1 + \mu_m(1 - T)/2])
  \]  

- The average annual shortage cost:
  Considering assumption of the partial backordering, the shortage cost includes two parts, backorder cost and lost sale cost that are formulated as follows:

  - Backorder cost:
    \[
    BO = \theta \frac{DT(1 - K)^2}{2} \sum_{m=1}^{2} (C_{2m}[1 + \mu_m(1 - T)/2])
    \]
  
  - Lost sale cost:
    \[
    LS = (1 - \theta) \frac{DT(1 - K)^2}{2} \sum_{m=1}^{2} (C_{2m}[1 + \mu_m(1 - T)/2])
    \]

So,
\[
EACS(K, T) = BO + LS
\]  

Hence, the total average annual cost equals to:
\[
EAC(K, T) = EACR + EACP + EACH + EACS
\]  

3.2. The discounted cost model
Referring to Mirzazadeh (2011), the present value of the total cost over the infinite planning horizon under two inflation rates is also obtained as follows:

The present value of the total ordering cost:
EDCR(K, T) = E \left[ A \sum_{j=0}^{\infty} e^{-j(r-h)T} \right] = A E \left[ \frac{1}{1 - e^{-(r-h)T}} \right] \quad (8)

The present value of the total purchasing cost:

EDCP(K, T) = CD \theta \left( e^{\theta KT} - 1 \right) E \left[ \frac{1}{1 - e^{-(r-h)T}} \right] + CD(1 - K) \left[ E \left[ \frac{1}{1 - e^{-(r-h)T}} \right] - 1 \right] \quad (9)

The present value of the total holding cost:

EDCH(K, T) = \sum_{m=1}^{2} C_{2m}D \left[ \frac{1 + e^{\theta(r-l_m)KT}(KT(\theta - (r - l_m)) - 1))}{(\theta - (r - l_m))^2(1 - e^{-(r-l_m)T})} \right] \quad (10)

The present value of the total shortage cost:

Similarly, each part of the shortage cost equals to:

Backorder cost:

BO = \sum_{m=1}^{2} C_{2m}D \left[ \frac{e^{-(r-l_m)T} + ((1 - k)(r - l_m)T - 1)e^{(r-l_m)KT}}{(r - l_m)^2(1 - e^{-(r-l_m)T})} \right] \quad (11)

Lost sale cost:

LS = \sum_{m=1}^{2} C_{2m}D(1 - \theta)E \left[ \frac{e^{-(r-l_m)T} + ((1 - k)(r - l_m)T - 1)e^{(r-l_m)KT}}{(r - l_m)^2(1 - e^{-(r-l_m)T})} \right] \quad (12)

So,

EDCS(K, T) = BO + LS \quad (13)

Hence,

EDC(K, T) = EDCR + EDCP + EDCH + EDCS \quad (14)

4. The fuzzy models and solution procedure

In reality, the inventory costs are consciously fluctuating due to various reasons. Therefore, the parameters of the unit inventory cost are vaguely and related values of these parameters cannot be exactly determined. In this section, the above introduced models are investigated in fuzzy environment by considering the unit purchasing cost C as a fuzzy number \( \tilde{C} \). Hence, the objective functions are rewritten in the average annual cost model and the discounted cost model as follows:

Fuzzy average annual cost model:

\[ \text{Min} \ EAC \left( K, T, \tilde{C} \right) = EACR + EACP + EACH + EACS \quad (15) \]

Fuzzy discounted cost model:

\[ \text{Min} \ EDC \left( K, T, \tilde{C} \right) = EDCR + EDCP + EDCH + EDCS \quad (16) \]

4.1. Fuzzy non-linear programming problem (FNLP)

A non-linear programming problem with fuzzy objective function and fuzzy co-efficient is formulated as follows:
where $\bar{y} = \bar{C}_T$ is the fuzzy co-efficient vector of $g_0$. Based on the fuzzy set theory, the fuzzy objective function and co-efficient are defined by their related member functions. Here, $\mu_0$ and $\mu_y$ are considered as the non-increasing continuous linear membership functions of fuzzy objective and co-efficient vector $\bar{y}$ of the objective function $g_0$, respectively. These relationship functions are as follows:

$$
\mu_0(g_0(x)) = \begin{cases} 
1 & \text{for } g_0(x) < Z_0 \\
1 - \frac{g_0(x) - Z_0}{P_0} & \text{for } Z_0 \leq g_0(x) \leq Z_0 + P_0 \\
0 & \text{for } g_0(x) > Z_0 + P_0 
\end{cases}
$$

$$
\mu_y(u) = \begin{cases} 
1 & \text{for } u > C_0 \\
1 - \frac{u - C_0}{P_1} & \text{for } C_0 - P_1 \leq u \leq C_0 \\
0 & \text{for } u < C_0 - P_1 
\end{cases}
$$

Using max-min operator, the fuzzy model given by Equation (17) is transformed to the crisp model as follows:

$$
\text{Max } \alpha
$$

s.t.

$$
g_0(x, \mu_y^{-1}(\alpha)) \leq \mu_0^{-1}(\alpha), \ x \geq 0, \alpha \in [0, 1] \tag{18}
$$

where

$$
\mu_y^{-1}(\alpha) = [C_0 - (1 - \alpha)P_1]
$$

$$
\mu_0^{-1}(\alpha) = Z_0 + (1 - \alpha)P_0
$$

For the fuzzy models given by Equations (15) and (16), the membership functions of the fuzzy objective function and unit purchasing cost are defined as follows:

$$
\mu_C(u) = \begin{cases} 
1 & \text{for } u > C_0 \\
1 - \frac{u - C_0}{P_1} & \text{for } C_0 - P_1 \leq u \leq C_0 \\
0 & \text{for } u < C_0 - P_1 
\end{cases}
$$

$$
\mu_0(\text{EAC}(K, T)) = \begin{cases} 
1 & \text{for } \text{EAC}(K, T) < Z_0 \\
1 - \frac{\text{EAC}(K, T) - Z_0}{P_0} & \text{for } Z_0 \leq \text{EAC}(K, T) \leq Z_0 + P_0 \\
0 & \text{for } \text{EAC}(K, T) > Z_0 + P_0 
\end{cases}
$$

$$
\mu_0(\text{EDC}(K, T)) = \begin{cases} 
1 & \text{for } \text{EDC}(K, T) < Z_0 \\
1 - \frac{\text{EDC}(K, T) - Z_0}{P_0} & \text{for } Z_0 \leq \text{EDC}(K, T) \leq Z_0 + P_0 \\
0 & \text{for } \text{EDC}(K, T) > Z_0 + P_0 
\end{cases}
$$

where $K$ and $T$ are positive variables of the problem, $C_0$, $Z_0$, and $Z_0^d$ are the initial values of the unit purchasing cost and objectives goal of the average annual cost and the discounted cost models, respectively, and $P_1$, $P_0$, and $P_0^d$ are their respective tolerances.

Hence, the fuzzy models given by Equations (15) and (16) with respect to Equation (18) reduce to the following form, respectively:
For the fuzzy average annual cost model:

\[
\text{Max } \alpha \\
\text{s.t.} \\
\text{EAC}(K, T, \alpha) \leq Z_0^\alpha + (1 - \alpha)P_0^\alpha, K, T \geq 0, \alpha \in [0, 1]
\]  \hspace{1cm} (19)

where

\[
\text{EAC}(K, T, \alpha) = \left[ 1 + \mu_1(1 - T)/2 \right] A/T \\
+ \left[ C_0 - (1 - \alpha)P_1 \right] \frac{D(e^{K(T - t_1)})}{\theta} + DT(1 - K) \left[ 1 + \mu_2(1 - T)/2 \right]/T \\
+ D \left( \frac{\theta K T e^{K(T - t_1)} - e^{K(T - t_1)}}{\theta^2 T} \right) \frac{1}{2} \sum_{m=1}^{2} \left[ C_{1m}[1 + \mu_m(1 - T)/2] \right] \\
+ \theta \frac{DT(1 - K)^2}{2} \sum_{m=1}^{2} \left[ C_{2m}[1 + \mu_m(1 - T)/2] \right] \\
+ (1 - \theta) \frac{DT(1 - K)^2}{2} \sum_{m=1}^{2} \left[ C_{2m}[1 + \mu_m(1 - T)/2] \right]
\]

Similarly, in the fuzzy discounted cost model:

\[
\text{Max } \alpha \\
\text{s.t.} \\
\text{EDC}(K, T, \alpha) \leq Z_0^\alpha + (1 - \alpha)P_0^\alpha, K, T \geq 0, \alpha \in [0, 1]
\]  \hspace{1cm} (20)

where

\[
\text{EDC}(K, T, \alpha) = A \frac{1}{1 - e^{-(r - i_1)T}} \left[ C_0 - (1 - \alpha)P_1 \right] \frac{D(e^{K(T - t_1)})}{\theta} \left[ 1 - e^{-(r - i_1)T} \right]/(1 - 1/e^{-(r - i_1)T}) \\
+ \left[ C_0 - (1 - \alpha)P_1 \right] DT(1 - K) \left[ 1 - e^{-(r - i_1)T} \right]/(1 - 1/e^{-(r - i_1)T}) \\
+ \sum_{m=1}^{2} \left[ C_{1m}D \left[ 1 + e^{(K - (r - i_m))T} (K \theta - (r - i_m) - 1) \right] \frac{(\theta - (r - i_m)^2)(1 - e^{-(r - i_m)T})}{(r - i_m)^2(1 - e^{-(r - i_m)T})} \right] \\
+ \sum_{m=1}^{2} \left[ C_{2m}D \left[ e^{-((r - i_m)T + ((K - (r - i_m))T - 1) e^{(r - i_m)K(T - t_1)})/(r - i_m)^2(1 - e^{-(r - i_m)T})} \right] \right] \\
+ \sum_{m=1}^{2} \left[ C_{2m}D \left[ e^{-((r - i_m)T + ((K - (r - i_m))T - 1) e^{(r - i_m)K(T - t_1)})/(r - i_m)^2(1 - e^{-(r - i_m)T})} \right] \right]
\]

5. Numerical example
In this section, the crisp and fuzzy models of the average annual cost and the discounted cost methods are described by the following numerical example:

Let \( D = 1200 \) units/year, \( A = \$90/\text{order} \), \( C_{11} = \$2/\text{unit/year} \), \( C_{12} = \$4/\text{unit/year} \), \( C_{21} = \$8/\text{unit/year} \), \( C_{22} = \$6/\text{unit/year} \), \( C_{21} = \$10/\text{unit/year} \), \( C_{22} = \$8/\text{unit/year} \), \( C = \$28/\text{unit} \), \( r = 0.25/\text{year} \), \( \beta = 0.7 \), \( \theta = 0.25 \).
Table 1. Optimal solution

| Model            | Average annual cost method | Discounted cost method |
|------------------|----------------------------|------------------------|
|                  | $K$ | $T$ | $EAC^*$ | $K$ | $T$ | $EDC^*$ |
| Crisp model      | 0.532 | 0.213 | 36,834.273 | 0.392 | 0.099 | 847,821.282 |
| Fuzzy model      | 0.579 | 0.198 | 25,337.597 | 0.458 | 0.102 | 579,478.607 |

Here, the probability density functions of the internal and external inflation rates are considered as the normal distribution with means of $\mu_1 = 0.09$ and $\mu_2 = 0.14$, standard deviations of $\sigma_1 = 0.04$ and $\sigma_2 = 0.06$, respectively.

Also, parameter values of the fuzzy models are equal to $C_0 = \$28/unit$, $P_1 = 18$, $P_0^d = 23003.504$, $P_0^r = 536862.806$, $Z_0^e = 13830.769$, and $Z_0^d = 310958.476$.

Considering these values, the optimal solutions of the crisp and fuzzy models have been shown in Table 1.

6. Comparison of the fuzzy models
In the above presented models, $T_o$ and $T_d$ as the interval of the time between two sequential orders, $K_o$ and $K_d$, as a part of the inventory cycle with positive inventory level are considered as two decision variables which are obtained by solving the fuzzy average annual cost model and the fuzzy discounted cost model, respectively. $EAC$ and $EDC$ represent the optimal values of the cost functions in the fuzzy average annual cost model and the fuzzy discounted cost model, respectively. With respect to assumption of the fuzzy unit purchasing cost, $C_{EAC}$ and $C_{EDC}$ represent the optimal values of the related fuzzy parameter in the average annual cost and the discounted cost models, respectively. Considering the optimal value of the unit purchasing cost $C_{EDC}$ in the fuzzy discounted model, $EDC_0$ is computed by replacing $T_o$ and $K_o$ in equation (14). According to Table 2, the comparison between the optimal solutions of the fuzzy models is performed using a sensitivity analysis on the problem parameters. Here, $T_o/T_d$, $K_o/K_d$, and $EDC_0/EDC_d$ are considered as criteria of the system analysis.

The related main conclusions are summarized as follows:

1. $T_o/T_d$ has high sensitivity to the changes in $C_{11}$, $C_{12}$, $C_{21}$, $D$, $\theta$, $C_0$, $\mu_1$, $\mu_2$, $\sigma_2$, $r$, and $P_1$. It has moderate sensitivity to $A$, $C_{22}$, and $Z_0^e$, and $Z_0^d$, and is insensitive to the changes in $C_{22}$ and $\theta$.

2. $K_o/K_d$ has high sensitivity to the parameters $C_{12}$, $C_{22}$, $D$, $C_0$, $\mu_1$, $\mu_2$, $r$, $P_0^d$, and $Z_0^d$, and $Z_0^d$, it has moderate sensitivity to $C_{11}$, $C_{12}$, $C_{21}$, $\theta$, $\sigma_1$, $\sigma_2$, $P_1$, $P_0^r$, and $Z_0^r$, and has slight sensitivity to $A$.

3. $EDC_0/EDC_d$ increases as the parameters $A$, $C_{11}$, $C_{12}$, $C_{22}$, $C_{22}$, $\mu_2$, $\sigma_2$, and $Z_0^d$ increase and conversely decreases as the parameters $D$, $C_0$, $C_{11}$, $\theta$, $r$, $\mu_1$, and $\sigma_1$ increase. $EDC_0/EDC_d$ has only a moderate sensitivity to the parameters $r$, $\mu_1$, and $\mu_2$. Based on the obtained results, these modeling methods do not significantly differ to each other on their inventory costs. In other words, speed of increasing the inventory system’s costs in the average annual cost method in comparison with the discounted cost method is very low and, indeed, amount of imposed cost into the inventory system is approximately same to each other. Hence, the manager can analyze the inventory system using both methods.

In order to the better understanding, graphical representation of the changes in the ratio $EDC_0/EDC_d$ for parameters $D$, $u_1$, $u_2$, and $A$ is shown in Figure 2.
Figure 2. Graphical representation of the changes in the ratio $\frac{\text{EDC}_a}{\text{EDC}_d}$. 

(a) $D$

(b) $u_1$

(c) $u_2$

(d) $A$
Table 2. Numerical comparison

|       | -0.9 | -0.5 | -0.2 | -0.1 | 0   | 0.1 | 0.2 | 0.5 | 0.9 |
|-------|------|------|------|------|-----|-----|-----|-----|-----|
| D     |      |      |      |      |     |     |     |     |     |
| $T_a$ | 0.767| 0.296| 0.225| 0.211| 0.198| 0.188| 0.179| 0.158| 0.139|
| $T_d$ | 0.307| 0.141| 0.113| 0.107| 0.102| 0.098| 0.094| 0.086| 0.077|
| $T_a/T_d$ | 2.493| 2.09929| 1.9911| 1.9719| 1.9411| 1.9183| 1.9042| 1.8372| 1.8051|
| $k_a$ | 0.490| 0.542| 0.566| 0.573| 0.579| 0.584| 0.589| 0.602| 0.616|
| $k_d$ | 0.348| 0.409| 0.441| 0.449| 0.458| 0.465| 0.473| 0.491| 0.511|
| $k_a/k_d$ | 1.4080| 1.3251| 1.2834| 1.2761| 1.2641| 1.2559| 1.2452| 1.2260| 1.2054|
| $C_{EAC}$ | 35.092| 25.516| 21.16| 20.026| 19.064| 18.082| 17.236| 15.112| 12.988|
| $C_{EDC}$ | 34.786| 25.408| 21.124| 20.008| 19.008| 18.082| 17.254| 15.166| 13.06|
| $EAC_a$ | 47.56915| 17.012.905| 22.570.575| 24.027.886| 25.337.597| 26.521.032| 27.595.594| 30.297.899| 33.022.714|
| $EDC_a$ | 108.327.268| 388.336.98| 515.850.20| 549.348.92| 579.478.60| 606.723.38| 631.478.76| 693.808.98| 756.776.37|
| $EDC_a$ | 111.099.884| 392.260.36| 519.750.35| 553.493.84| 583.673.13| 610.808.99| 635.640.43| 698.064.75| 761.380.21|
| $EDC_{a/EDC_{d}}$ | 1.02559| 1.01010| 1.007560| 1.007545| 1.007238| 1.006733| 1.006590| 1.006133| 1.006083|
| A     |      |      |      |      |     |     |     |     |     |
| $T_a$ | 0.063| 0.140| 0.177| 0.188| 0.198| 0.208| 0.217| 0.243| 0.274|
| $T_d$ | 0.032| 0.072| 0.092| 0.097| 0.102| 0.107| 0.112| 0.125| 0.141|
| $T_a/T_d$ | 1.9687| 1.9444| 1.9239| 1.9381| 1.9411| 1.9439| 1.9375| 1.944| 1.9432|
| $k_a$ | 0.581| 0.579| 0.579| 0.579| 0.579| 0.578| 0.578| 0.578| 0.577|
| $k_d$ | 0.458| 0.458| 0.458| 0.458| 0.458| 0.458| 0.458| 0.458| 0.458|
| $k_a/k_d$ | 1.2685| 1.2641| 1.2641| 1.2641| 1.2641| 1.2620| 1.2620| 1.2620| 1.2598|
| $C_{EAC}$ | 19.252| 19.108| 19.036| 19.018| 19.018| 18.982| 18.964| 18.91| 18.856|
| $C_{EDC}$ | 19.162| 19.072| 19.018| 19.018| 19.018| 18.982| 18.982| 18.946| 18.91|
| $EAC_a$ | 25.015.143| 25.199.713| 25.287.938| 25.313.464| 25.337.597| 25.360.539| 25.382.451| 25.443.154| 25.515.201|
| $EDC_a$ | 57.455.148| 577.367.42| 578.717.17| 579.108.42| 579.478.60| 579.830.81| 580.167.43| 581.101.27| 582.212.05|
| $EDC_a$ | 575.864.827| 580.378.19| 582.239.18| 583.264.93| 583.673.13| 584.048.36| 584.906.04| 586.264.76| 588.075.53|
| $EDC_{a/EDC_{d}}$ | 1.002277| 1.005214| 1.006085| 1.007177| 1.007238| 1.007273| 1.008167| 1.008885| 1.010071|

(Continued)
Table 2. (Continued)

|     | -0.9 | -0.5 | -0.2 | -0.1 | 0    | 0.1  | 0.2  | 0.5  | 0.9  |
|-----|------|------|------|------|------|------|------|------|------|
| C_{11} | T_a  | 0.218| 0.208| 0.202| 0.2  | 0.198| 0.197| 0.195| 0.190| 0.185|
|     | T_d  | 0.104| 0.103| 0.103| 0.103| 0.102| 0.102| 0.102| 0.102| 0.101|
|     | T_a/T_d | 2.096 | 2.019 | 1.961 | 1.941 | 1.941 | 1.933 | 1.917 | 1.862 | 1.831 |
|     | k_a  | 0.621| 0.601| 0.587| 0.583| 0.579| 0.574| 0.570| 0.557| 0.542|
|     | k_d  | 0.473| 0.466| 0.461| 0.459| 0.458| 0.456| 0.455| 0.450| 0.444|
|     | k_a/k_d | 1.312 | 1.289 | 1.273 | 1.270 | 1.264 | 1.258 | 1.257 | 1.237 | 1.207 |
|     | C_{EAC} | 19.036 | 19.018 | 19 | 19 | 19 | 19 | 19 | 19 | 19 |
|     | C_{EDC} | 19 | 19 | 19 | 19 | 19 | 19 | 19 | 19 | 19 |
|     | E_{AC} | 25,295.001 | 25,315.191 | 25,328.973 | 25,333.338 | 25,337.597 | 25,341.753 | 25,345.812 | 25,357.438 | 25,371.779 |
|     | E_{DC} | 579,379.238 | 579,424.350 | 579,457.180 | 579,467.940 | 579,478.600 | 579,489.180 | 579,499.660 | 579,503.600 | 579,570.680 |
|     | E_{DC} | 584,837.203 | 584,222.410 | 583,878.600 | 583,775.600 | 583,673.130 | 583,613.700 | 583,512.450 | 583,255.370 | 583,037.840 |
|     | E_{DC}/E_{AC} | 1.0094 | 1.0082 | 1.0076 | 1.0074 | 1.0072 | 1.0071 | 1.0069 | 1.0064 | 1.0059 |
| C_{12} | T_a  | 0.252| 0.221| 0.206| 0.202| 0.198| 0.195| 0.192| 0.184| 0.175|
|     | T_d  | 0.114| 0.108| 0.104| 0.103| 0.102| 0.102| 0.101| 0.099| 0.096|
|     | T_a/T_d | 2.210 | 2.046 | 1.980 | 1.961 | 1.941 | 1.911 | 1.900 | 1.858 | 1.822 |
|     | k_a  | 0.672| 0.627| 0.597| 0.588| 0.579| 0.570| 0.561| 0.537| 0.508|
|     | k_d  | 0.566| 0.512| 0.478| 0.468| 0.458| 0.448| 0.439| 0.414| 0.385|
|     | k_a/k_d | 1.187 | 1.246 | 1.248 | 1.256 | 1.264 | 1.273 | 1.277 | 1.297 | 1.319 |
|     | C_{EAC} | 19.072 | 19.036 | 19.018 | 19 | 19 | 19 | 19 | 19 | 19 |
|     | C_{EDC} | 19 | 19 | 19 | 19 | 19 | 19 | 19 | 19 | 19 |
|     | E_{AC} | 25,237.126 | 25,288.547 | 25,319.548 | 25,328.803 | 25,333.597 | 25,345.968 | 25,357.837 | 25,401.049 | 25,401.049 |
|     | E_{DC} | 578,721.958 | 579,108.760 | 579,342.780 | 579,412.490 | 579,478.600 | 579,541.400 | 579,764.100 | 579,949.910 | 579,949.910 |
|     | E_{DC} | 583,526.527 | 583,811.570 | 583,519.100 | 583,609.090 | 583,673.130 | 583,770.030 | 583,845.620 | 583,521.370 | 583,734.730 |
|     | E_{DC}/E_{AC} | 1.0083 | 1.0081 | 1.0072 | 1.0072 | 1.0072 | 1.0073 | 1.0073 | 1.0065 | 1.0065 |

(Continued)
|     |      | -0.9 | -0.5 | -0.2 | -0.1 | 0    | 0.1  | 0.2  | 0.5  | 0.9  |
|-----|------|------|------|------|------|------|------|------|------|------|
| $C_{21}$ | $T_a$ | 0.236 | 0.215 | 0.204 | 0.201 | 0.198 | 0.196 | 0.193 | 0.187 | 0.181 |
|      | $T_d$ | 0.109 | 0.106 | 0.104 | 0.103 | 0.102 | 0.102 | 0.101 | 0.1    | 0.098 |
|      | $T_a/T_d$ | 2.165 | 2.028 | 1.961 | 1.954 | 1.941 | 1.9215 | 1.9108 | 1.87   | 1.8469 |
|      | $k_a$ | 0.479 | 0.529 | 0.560 | 0.569 | 0.579 | 0.587 | 0.596 | 0.619 | 0.646 |
|      | $k_d$ | 0.406 | 0.430 | 0.447 | 0.452 | 0.458 | 0.463 | 0.468 | 0.483 | 0.501 |
|      | $k_a/k_d$ | 1.1798 | 1.2302 | 1.2527 | 1.2588 | 1.2641 | 1.2678 | 1.2735 | 1.2815 | 1.2894 |
| $C_{EAC}$ | 19.054 | 19.018 | 19   | 19   | 19   | 19   | 19   | 18.982 | 18.982 | 18.964 |
| $C_{EDC}$ | 19.018 | 19   | 19   | 19   | 19   | 19   | 19   | 18.982 | 18.982 | 18.964 |
| $EAC_a$ | 25,263.478 | 25,301.934 | 25,324.609 | 25,331.290 | 25,337.597 | 25,343.563 | 25,349.216 | 25,364.536 | 25,381.836 |
| $EDC_a$ | 579,059.469 | 579,258.47 | 579,394.00 | 579,436.84 | 579,478.60 | 579,519.32 | 579,559.04 | 579,672.54 | 579,811.95 |
| $EAC_d$ | 583,893.582 | 583,397.86 | 583,546.02 | 583,604.84 | 583,673.13 | 583,766.32 | 583,813.19 | 583,507.69 | 583,846.84 |
| $EDC_d$ | 578,328.181 | 1.007146 | 1.007166 | 1.007238 | 1.007328 | 1.007340 | 1.006616 | 1.006958 |
| $C_{22}$ | $T_a$ | 0.223 | 0.210 | 0.203 | 0.2 | 0.198 | 0.196 | 0.195 | 0.190 | 0.184 |
|      | $T_d$ | 0.122 | 0.111 | 0.105 | 0.104 | 0.102 | 0.101 | 0.1 | 0.097 | 0.094 |
|      | $T_a/T_d$ | 1.8278 | 1.8918 | 1.9333 | 1.9230 | 1.9411 | 1.9405 | 1.95 | 1.9567 | 1.9574 |
|      | $k_a$ | 0.507 | 0.541 | 0.564 | 0.572 | 0.579 | 0.585 | 0.592 | 0.610 | 0.632 |
|      | $k_d$ | 0.323 | 0.390 | 0.433 | 0.445 | 0.458 | 0.469 | 0.481 | 0.512 | 0.548 |
|      | $k_a/k_d$ | 1.5696 | 1.3871 | 1.3025 | 1.2853 | 1.2641 | 1.2473 | 1.2307 | 1.1914 | 1.1532 |
| $C_{EAC}$ | 19.036 | 19.018 | 19   | 19   | 19   | 19   | 19   | 18.982 | 18.982 | 18.964 |
| $C_{EDC}$ | 19.018 | 19   | 19   | 19   | 19   | 19   | 19   | 18.982 | 18.982 | 18.964 |
| $EAC_a$ | 25,285.288 | 25,311.358 | 25,327.805 | 25,332.807 | 25,337.597 | 25,342.187 | 25,346.592 | 25,358.799 | 25,373.073 |
| $EDC_a$ | 578,328.181 | 578,928.46 | 579,278.98 | 579,381.77 | 579,478.60 | 579,570.02 | 579,656.47 | 579,889.97 | 580,152.20 |
| $EAC_d$ | 582,362.854 | 583,044.23 | 583,273.64 | 583,461.59 | 583,673.13 | 583,856.40 | 584,098.20 | 584,056.69 | 584,596.57 |
| $EDC_d$ | 578,328.181 | 1.007109 | 1.006895 | 1.007041 | 1.007238 | 1.007395 | 1.007662 | 1.007185 | 1.007660 | (Continued)
Table 2. (Continued)

| $C_{2j}$ | $-0.9$ | $-0.5$ | $-0.2$ | $-0.1$ | $0$ | $0.1$ | $0.2$ | $0.5$ | $0.9$ |
|----------|--------|--------|--------|--------|-----|--------|--------|--------|--------|
| $T_o$    | 0.214  | 0.206  | 0.201  | 0.2    | 0.198 | 0.197  | 0.196  | 0.192  | 0.188  |
| $T_d$    | 0.106  | 0.104  | 0.103  | 0.103  | 0.102 | 0.102  | 0.102  | 0.101  | 0.1    |
| $T_o/T_d$| 2.0188 | 1.9807 | 1.9514 | 1.9417 | 1.9411| 1.9313 | 1.9215 | 1.9009 | 1.88   |
| $k_o$    | 0.531  | 0.553  | 0.569  | 0.574  | 0.579 | 0.583  | 0.588  | 0.601  | 0.618  |
| $k_d$    | 0.431  | 0.443  | 0.452  | 0.455  | 0.458 | 0.461  | 0.463  | 0.472  | 0.482  |
| $k_o/k_d$| 1.2320 | 1.2483 | 1.2588 | 1.2615 | 1.2641| 1.2646 | 1.2699 | 1.2733 | 1.2821 |
| $C_{EAC}$| 19.018 | 19.018 | 19     | 19     | 19   | 19     | 19     | 19     | 18.982 |
| $C_{EDC}$| 19     | 19     | 19     | 19     | 19   | 19     | 19     | 19     | 18.982 |
| $EAC_o$  | 25,303.409 | 25,319.845 | 25,330.825 | 25,334.262 | 25,337.597 | 25,340.833 | 25,343.977 | 25,352.887 | 25,363.687 |
| $EDC_o$  | 579,266.851 | 579,364.28 | 579,433.82 | 579,456.36 | 579,478.60 | 579,500.54 | 579,522.19 | 579,585.43 | 579,666.01 |
| $EAC_d$  | 583,391.440 | 583,489.91 | 583,597.57 | 583,663.92 | 583,723.49 | 583,787.15 | 583,886.51 | 583,538.29 |
| $EDC_d$  | 583,391.440 | 583,489.91 | 583,597.57 | 583,663.92 | 583,723.49 | 583,787.15 | 583,886.51 | 583,538.29 |
| $EAC_o/EAC_d$ | 1.0071203 | 1.0071209 | 1.007185 | 1.007261 | 1.007397 | 1.007420 | 1.006680 |
| $EDC_o/EDC_d$ | 1.0071203 | 1.0071209 | 1.007185 | 1.007261 | 1.007397 | 1.007420 | 1.006680 |

(Continued)
Table 2. (Continued)

| θ   | $T_a$ | $T_d$ | $T_a/T_d$ | $k_a$ | $k_d$ | $k_a/k_d$ | $C_{EAC}$ | $C_{EDC}$ | $EAC_a$ | $EAC_d$ | $EAC_a/EAC_d$ | C/3_EAC | $EAC_a$ | $EAC_d$ | $EAC_a/EAC_d$ | C/3_EAC | $EAC_a$ | $EAC_d$ | $EAC_a/EAC_d$ | C/3_EAC |
|-----|-------|-------|-----------|-------|-------|-----------|-----------|-----------|--------|--------|----------------|--------|--------|--------|----------------|--------|--------|--------|----------------|--------|
| 0.9 | 0.279 | 0.119 | 2.3445    | 0.699 | 0.594 | 1.1767    | 19.108    | 19.036    | 25,204.847 | 578,511.629 | 1.009841 | 19.108 | 25,204.847 | 578,511.629 | 1.009841 | 19.108 | 25,204.847 | 578,511.629 | 1.009841 |
| 0.5 | 0.208 | 0.105 | 2.1009    | 0.601 | 0.482 | 1.2468    | 19.018    | 19.036    | 25,315.378 | 579,314.10  | 1.007209 | 19.018 | 25,315.378 | 579,314.10  | 1.007209 | 19.018 | 25,315.378 | 579,314.10  | 1.007209 |
| 0.2 | 0.198 | 0.104 | 2.9809    | 0.590 | 0.470 | 1.2641    | 19.018    | 19.036    | 25,337.597 | 579,478.60  | 1.007252 | 19.018 | 25,337.597 | 579,478.60  | 1.007252 | 19.018 | 25,337.597 | 579,478.60  | 1.007252 |
| 0.1 | 0.194 | 0.102 | 1.9519    | 0.579 | 0.458 | 1.2775    | 19.018    | 19.036    | 25,357.267 | 579,623.97  | 1.007252 | 19.018 | 25,357.267 | 579,623.97  | 1.007252 | 19.018 | 25,357.267 | 579,623.97  | 1.007252 |
| 0.0 | 0.190 | 0.101 | 1.9411    | 0.557 | 0.446 | 1.3004    | 19.018    | 19.036    | 25,382.96  | 579,813.04  | 1.007252 | 19.018 | 25,382.96  | 579,813.04  | 1.007252 | 19.018 | 25,382.96  | 579,813.04  | 1.007252 |
| −0.1| 0.181 | 0.098 | 1.9207    | 0.528 | 0.406 | 1.3279    | 19.018    | 19.036    | 25,411.786 | 580,022.96  | 1.007252 | 19.018 | 25,411.786 | 580,022.96  | 1.007252 | 19.018 | 25,411.786 | 580,022.96  | 1.007252 |
| −0.2| 0.171 | 0.095 | 1.8469    | 0.494 | 0.372 | 1.8       | 19.018    | 19.036    | 25,437.718 | 581,282.97  | 1.007252 | 19.018 | 25,437.718 | 581,282.97  | 1.007252 | 19.018 | 25,437.718 | 581,282.97  | 1.007252 |
| −0.5| 0.167 | 0.092 | 1.771     | 0.452 | 0.330 | 1.306     | 19.018    | 19.036    | 25,478.60  | 581,544.98  | 1.007252 | 19.018 | 25,478.60  | 581,544.98  | 1.007252 | 19.018 | 25,478.60  | 581,544.98  | 1.007252 |
| −0.9| 0.161 | 0.087 | 1.394     | 0.447 | 0.300 | 1.2906    | 19.018    | 19.036    | 25,520.948 | 25,320.948  | 1.007252 | 19.018 | 25,520.948 | 25,320.948  | 1.007252 | 19.018 | 25,520.948 | 25,320.948  | 1.007252 |

(Continued)
Table 2. (Continued)

| $r$ | $T_a$ | $T_d$ | $T_{a/d}$ | $k_a$ | $k_d$ | $k_a/k_d$ | $C_{EAC}$ | $C_{EDC}$ |
|-----|-------|-------|-----------|-------|-------|-----------|-----------|-----------|
| 0.1 | 0.196 | 0.083 | 2.3614    | 0.576 | 0.430 | 1.3395    | 19        | 19        |
| 0.2 | 0.197 | 0.097 | 2.0412    | 0.578 | 0.439 | 1.3143    | 19        | 19        |
| 0.3 | 0.198 | 0.098 | 1.9411    | 0.579 | 0.458 | 1.2759    | 19        | 19        |
| 0.4 | 0.199 | 0.102 | 1.8773    | 0.579 | 0.463 | 1.2505    | 19        | 19        |
| 0.5 | 0.201 | 0.106 | 1.8256    | 0.579 | 0.469 | 1.2245    | 19        | 19        |
| 0.6 | 0.203 | 0.109 | 1.7873    | 0.579 | 0.492 | 1.1888    | 19        | 19        |
| 0.7 | 0.208 | 0.111 | 1.7414    | 0.580 | 0.515 | 1.1521    | 19        | 19        |
| 0.8 | 0.212 | 0.114 | 1.6964    | 0.581 | 0.531 | 1.1182    | 19        | 19        |
| 0.9 | 0.219 | 0.116 | 1.6515    | 0.582 | 0.549 | 1.0855    | 19        | 19        |

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| $\mu_2$ | $T_a$ | $T_d$ | $T_a/T_d$ | $k_a$ | $k_d$ | $k_a/k_d$ | $C_{EAC}$ | $C_{EDC}$ | $EAC_{a}$ | $EAC_{d}$ | $EAC_{a}/EAC_{d}$ | $\sigma_a$ | $T_a$ | $T_d$ | $T_a/T_d$ | $k_a$ | $k_d$ | $k_a/k_d$ | $C_{EAC}$ | $C_{EDC}$ | $EAC_{a}$ | $EAC_{d}$ | $EAC_{a}/EAC_{d}$ | $\sigma_a$ |
|--------|-------|-------|-----------|-------|-------|-----------|-----------|-----------|-----------|-----------|------------------|-----------|-------|-------|-----------|-------|-------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| $-0.9$ | 0.158 | 0.160 | 0.9875 | 0.582 | 0.4 | 1.455 | 19.486 | 30.7 | 24,719.656 | 231,302.049 | 1.002927 | 1.00198 | 0.106 | 1.8679 | 0.579 | 1.2478 | 19 | 19 | 1.003141 | 1.005306 | 1.004394 | 1.007238 | 1.005306 | 1.01842 | 1.006578 | 1.006448 |
| $-0.5$ | 0.173 | 0.143 | 1.0797 | 0.580 | 0.432 | 1.4043 | 19.252 | 27.91 | 25,001.388 | 314,788.06 | 1.003141 | 1.003141 | 0.105 | 1.8857 | 0.579 | 1.2559 | 19 | 19 | 1.003141 | 1.005306 | 1.004394 | 1.007238 | 1.005306 | 1.01842 | 1.006578 | 1.006448 |
| $-0.2$ | 0.187 | 0.123 | 1.5203 | 0.579 | 0.432 | 1.3402 | 19.108 | 23.932 | 25,205.348 | 434,154.00 | 1.005306 | 1.005306 | 0.103 | 1.9223 | 0.579 | 1.2614 | 19 | 19 | 1.005306 | 1.005306 | 1.005306 | 1.005306 | 1.005306 | 1.01842 | 1.006578 | 1.006448 |
| $-0.1$ | 0.192 | 0.114 | 1.6842 | 0.579 | 0.442 | 1.3099 | 19.054 | 23.926 | 25,337.597 | 497,642.00 | 1.007238 | 1.007238 | 0.102 | 1.9411 | 0.579 | 1.2664 | 19 | 19 | 1.007238 | 1.007238 | 1.007238 | 1.007238 | 1.007238 | 1.01842 | 1.006578 | 1.006448 |
| $0$    | 0.198 | 0.102 | 1.6941 | 0.579 | 0.442 | 1.2641 | 19.054 | 23.926 | 25,337.597 | 497,642.00 | 1.007238 | 1.007238 | 0.102 | 1.9411 | 0.579 | 1.2664 | 19 | 19 | 1.007238 | 1.007238 | 1.007238 | 1.007238 | 1.007238 | 1.01842 | 1.006578 | 1.006448 |
| $0.1$  | 0.205 | 0.088 | 2.3295 | 0.578 | 0.442 | 1.2641 | 19.054 | 23.926 | 25,337.597 | 497,642.00 | 1.007238 | 1.007238 | 0.102 | 1.9411 | 0.579 | 1.2664 | 19 | 19 | 1.007238 | 1.007238 | 1.007238 | 1.007238 | 1.007238 | 1.01842 | 1.006578 | 1.006448 |
| $0.2$  | 0.212 | 0.065 | 3.2615 | 0.578 | 0.442 | 1.2641 | 19.054 | 23.926 | 25,337.597 | 497,642.00 | 1.007238 | 1.007238 | 0.102 | 1.9411 | 0.579 | 1.2664 | 19 | 19 | 1.007238 | 1.007238 | 1.007238 | 1.007238 | 1.007238 | 1.01842 | 1.006578 | 1.006448 |
| $0.5$  | 0.205 | 0.088 | 2.3295 | 0.578 | 0.442 | 1.2641 | 19.054 | 23.926 | 25,337.597 | 497,642.00 | 1.007238 | 1.007238 | 0.102 | 1.9411 | 0.579 | 1.2664 | 19 | 19 | 1.007238 | 1.007238 | 1.007238 | 1.007238 | 1.007238 | 1.01842 | 1.006578 | 1.006448 |
| $0.9$  | 0.212 | 0.065 | 3.2615 | 0.578 | 0.442 | 1.2641 | 19.054 | 23.926 | 25,337.597 | 497,642.00 | 1.007238 | 1.007238 | 0.102 | 1.9411 | 0.579 | 1.2664 | 19 | 19 | 1.007238 | 1.007238 | 1.007238 | 1.007238 | 1.007238 | 1.01842 | 1.006578 | 1.006448 |
| $\sigma_2$ | $T_a$ | $T_d$ | $T_a/T_d$ | $k_a$ | $k_d$ | $k_a/k_d$ | $C_{EAC}$ | $C_{EDC}$ | $EAC_a$ | $EAC_d$ | $EAC_a/EAC_d$ | EDCa | EDCd | EDCa/EDCd | EDCa | EDCd | EDCa/EDCd | EDCa | EDCd | EDCa/EDCd |
|------------|-------|-------|---------|------|------|--------|--------|--------|-------|-------|----------|------|------|----------|------|------|----------|------|------|----------|
| -0.9       | 0.198 | 0.116 | 1.706     | 0.579 | 0.439 | 1.3189 | 19.45  | 22.438 | 2337.597 | 476,723.513 | 1.005567 | 1.6548 | 0.659    | 1.1342 | 6.4   | 2025.001 | 203,476.445 | 205,333.590 | 1.009127 |
| -0.5       | 0.198 | 0.111 | 1.7837   | 0.579 | 0.446 | 1.2982 | 19.882 | 21.07   | 2537.597 | 517,448.11 | 1.005907 | 1.7850 | 0.621    | 1.1919 | 6.4   | 16,388.752 | 370,649.82  | 373,634.42  | 1.008052  |
| -0.2       | 0.198 | 0.106 | 1.8679   | 0.579 | 0.453 | 1.2781 | 19.45  | 19.882  | 2537.597 | 552,942.62 | 1.006459 | 1.875  | 0.595    | 1.2370 | 6.4   | 21,758.806 | 459,962.10  | 499,354.90  | 1.006840  |
| -0.1       | 0.198 | 0.104 | 1.9038   | 0.579 | 0.455 | 1.2725 | 19.45  | 19.882  | 2537.597 | 565,896.06 | 1.007238 | 1.9411 | 0.579    | 1.2641 | 6.4   | 23,548.324 | 579,478.60  | 541,818.15  | 1.007616  |
| 0          | 0.198 | 0.102 | 1.9411   | 0.579 | 0.458 | 1.2641 | 19.45  | 19.882  | 2537.597 | 593,737.20 | 1.007567 | 2      | 0.579    | 1.2505 | 6.4   | 25,337.597 | 608,723.61  | 625,571.44  | 1.007238  |
| 0.1        | 0.198 | 0.101 | 1.9603   | 0.579 | 0.461 | 1.2505 | 19.45  | 19.882  | 2537.597 | 613,110.65 | 1.008124 | 2      | 0.579    | 1.2215 | 6.4   | 27,126.627 | 658,653.05  | 667,458.27  | 1.007616  |
| 0.2        | 0.198 | 0.099 | 2.1290   | 0.579 | 0.463 | 1.2505 | 19.45  | 19.882  | 2537.597 | 664,004.19 | 1.01006  | 2      | 0.579    | 1.1768 | 6.4   | 28,915.415 | 739,703.52  | 793,706.03  | 1.007616  |
| 0.5        | 0.198 | 0.093 | 2.3855   | 0.579 | 0.474 | 1.1768 | 19.45  | 19.882  | 2537.597 | 684,004.19 | 1.01006  | 2      | 0.579    | 1.1768 | 6.4   | 34,280.327 | 739,703.52  | 793,706.03  | 1.007616  |
| 0.9        | 0.198 | 0.083 | 2.3855   | 0.579 | 0.474 | 1.1768 | 19.45  | 19.882  | 2537.597 | 684,004.19 | 1.01006  | 2      | 0.579    | 1.1768 | 6.4   | 46,012.617 | 1,062,254.5 | 1,070,253.5 | 1.007616  |

(Continued)
| $\rho$ | $T_a$ | $T_d$ | $T_a/T_d$ | $k_a$ | $k_d$ | $k_a/k_d$ | $C_{EAC}$ | $C_{EDC}$ | $EAC_a$ | $EDC_a$ | $EAC_d$ | $EDC_d$ | $EAC_a/EDC_a$ | $EDC_a/EDC_d$ |
|-------|-------|-------|-----------|--------|--------|-----------|-----------|-----------|---------|---------|---------|---------|----------------|----------------|
| 0.1   | 0.210 | 0.099 | 2.1212    | 0.540  | 0.403  | 1.3399    | 26.3638   | 26.3638   | 34,746  | 799,062 | 804,703 | 673,653 | 1007059        | 1987005        |
| 0.2   | 0.203 | 0.101 | 2.0099    | 0.562  | 0.434  | 1.2949    | 21.997    | 21.997    | 29,172  | 668,967 | 673,653 | 613,524 | 1007005        | 1987005        |
| 0.5   | 0.2   | 0.102 | 1.9607    | 0.573  | 0.450  | 1.2733    | 19.9936   | 19.9936   | 26,616  | 609,311 | 613,524 | 518,058 | 1006913        | 1987005        |
| 0.9   | 0.199 | 0.102 | 1.9509    | 0.576  | 0.454  | 1.2687    | 19.4626   | 19.4626   | 25,943  | 593,610 | 598,058 | 583,673 | 1006412        | 1987005        |
| 1     | 0.198 | 0.102 | 1.9411    | 0.579  | 0.458  | 1.2641    | 18.5752   | 18.5752   | 25,337  | 579,478 | 583,673 | 583,673 | 1006012        | 1987005        |
| 2     | 0.198 | 0.103 | 1.9223    | 0.581  | 0.461  | 1.2603    | 18.172    | 18.172    | 24,789  | 566,692 | 570,949 | 570,949 | 1005612        | 1987005        |
| 3     | 0.197 | 0.103 | 1.9126    | 0.583  | 0.464  | 1.2564    | 17.2      | 17.2      | 24,291  | 555,068 | 558,817 | 558,817 | 1005212        | 1987005        |
| 4     | 0.196 | 0.104 | 1.9029    | 0.589  | 0.473  | 1.2452    | 16.21     | 16.21     | 23,036  | 525,773 | 529,660 | 529,660 | 1004812        | 1987005        |
| 5     | 0.195 | 0.104 | 1.875     | 0.595  | 0.481  | 1.2370    | 15.16     | 15.16     | 21,766  | 496,141 | 499,685 | 499,685 | 1004412        | 1987005        |

(Continued)
Table 2. (Continued)

| $P_{r_0}^i$ | $-0.9$ | $-0.5$ | $-0.2$ | $-0.1$ | $0$ | $0.1$ | $0.2$ | $0.5$ | $0.9$ |
|------------|--------|--------|--------|--------|-----|--------|--------|-------|-------|
| $T_a$      | 0.190  | 0.195  | 0.197  | 0.198  | 0.199| 0.199  | 0.201  | 0.202  |
| $T_d$      | 0.102  | 0.102  | 0.102  | 0.102  | 0.102| 0.102  | 0.102  | 0.102  |
| $T_a/T_d$  | 1.8627 | 1.9117 | 1.9313 | 1.9411 | 1.9411| 1.9509 | 1.9509 | 1.9705 | 1.9803|
| $k_a$      | 0.623  | 0.596  | 0.584  | 0.581  | 0.579| 0.576  | 0.574  | 0.569  | 0.563 |
| $k_d$      | 0.458  | 0.458  | 0.458  | 0.458  | 0.458| 0.458  | 0.458  | 0.458  | 0.458 |
| $k_a/k_d$  | 1.3602 | 1.3013 | 1.2751 | 1.2685 | 1.2641| 1.2576 | 1.2532 | 1.2423 | 1.2292|
| $C_{sAC}$  | 11.638 | 15.994 | 17.992 | 18.514 | 19   | 19.432 | 19.81  | 20.8  | 21.79 |
| $C_{sDC}$  | 19     | 19     | 19     | 19     | 19   | 19     | 19     | 19     | 19     |
| $EAC_i$    | 15.922307 | 21.501628 | 24.059012 | 24.731964 | 25.337597 | 25.885526 | 26.383620 | 27.638706 | 28.908023 |
| $EDC_i$    | 579.478607 | 579.47860 | 579.47860 | 579.47860 | 579.47860 | 579.47860 | 579.47860 | 579.47860 | 579.47860 |
| $EAC_d$    | 583.923576 | 583.75513 | 583.68786 | 583.70133 | 583.67313 | 583.68790 | 583.66069 | 583.70669 | 583.68631 |
| $EDC_d$    | 1.007670 | 1.007379 | 1.007263 | 1.007287 | 1.007238 | 1.007216 | 1.007296 | 1.007261 |
| $EAC_{sAC}$| 14.302  | 16.39  | 17.956  | 18.478  | 19   | 19.522 | 20.044 | 21.61  | 23.698 |
| $EDC_{sAC}$| 439.528908 | 501.73538 | 548.38317 | 563.93118 | 579.47860 | 595.02544 | 610.57172 | 657.20730 | 719.38100 |
| $EAC_{sDC}$| 442.655004 | 505.32973 | 552.33577 | 568.00445 | 583.67313 | 599.34181 | 615.01049 | 662.01654 | 724.69126 |
| $EAC_{sAC}$| 1.007112 | 1.007163 | 1.007207 | 1.007222 | 1.007238 | 1.007254 | 1.007269 | 1.007317 | 1.007381 |

(Continued)
|       | -0.9  | -0.5  | -0.2  | -0.1  | 0    | 0.1   | 0.2   | 0.5   | 0.9  |
|-------|-------|-------|-------|-------|------|-------|-------|-------|------|
| $P_d$ |       |       |       |       |      |       |       |       |      |
| $T_a$ | 0.198 | 0.198 | 0.198 | 0.198 | 0.198| 0.198 | 0.198 | 0.198 | 0.198|
| $T_d$ | 0.108 | 0.104 | 0.103 | 0.103 | 0.102| 0.102 | 0.102 | 0.102 | 0.101|
| $T_a/T_d$ | 1.8333 | 1.9038 | 1.9223 | 1.9223 | 1.9411 | 1.9411 | 1.9411 | 1.9603 |
| $k_a$ | 0.579 | 0.579 | 0.579 | 0.579 | 0.579 | 0.579 | 0.579 | 0.579 | 0.579 |
| $k_d$ | 0.524 | 0.483 | 0.466 | 0.462 | 0.458 | 0.454 | 0.451 | 0.443 | 0.436 |
| $k_a/k_d$ | 1.1049 | 1.1987 | 1.2424 | 1.2532 | 1.2641 | 1.2753 | 1.2838 | 1.3069 | 1.3279 |
| $C_{EAC}$ | 19 | 19 | 19 | 19 | 19 | 19 | 19 | 19 | 19 |
| $C_{ECD}$ | 11.638 | 15.994 | 17.992 | 18.532 | 19 | 19.432 | 19.81 | 20.8 | 21.79 |
| $EAC_g$ | 25,337.597 | 25,337.597 | 25,337.597 | 25,337.597 | 25,337.597 | 25,337.597 | 25,337.597 | 25,337.597 | 25,337.597 |
| $EDC_g$ | 359,770.301 | 489,968.20 | 549,643.43 | 565,346.42 | 579,478.60 | 592,264.39 | 603,887.39 | 633,175.30 | 662,796.43 |
| $EDC_a$ | 362,690.699 | 493,443.14 | 553,416.37 | 569,625.35 | 583,673.13 | 596,640.32 | 607,986.60 | 637,703.07 | 667,419.53 |
| $EDC_a/EDC_g$ | 1.008117 | 1.007092 | 1.006864 | 1.007568 | 1.007234 | 1.007388 | 1.006788 | 1.007150 | 1.006975 |
Here, the inventory formulated models assume $r > i_m$ (for $m = 1, 2$). In Table 2, by performing sensitivity analysis on the parameters, this assumption is violated for the row $r$ and columns $-0.9$, $-0.5$, and the row $u_2$ and columns $0.5$, $0.9$. In another word, the interest rate is lower than the inflation rates for the parameters $r$ and $u_2$ in these columns. Hariga and Ben-Daya (1996) proved that ordering is only done once over the time horizon if the inflation rate is greater than the interest rate. $\beta$ ($0 \leq \beta \leq 1$) is the percentage of backordered demands during shortage period. According to the above explanations, in Table 2, the row $r$ and columns $-0.9$, $-0.5$, and the row $u_2$ and columns $0.5$, $0.9$ and column $0.9$ of the parameter $\beta$ are not considered in the sensitivity analysis process.

7. Conclusion

Usually, a part of the demands that are faced with shortages lost. Therefore, in this paper, two deteriorating inventory models with partial backordering under stochastic inflationary conditions over an infinite time horizon have been developed with considering two modeling methods: (1) the average annual cost method and (2) the discounted cost method. In practice, due to the effects of the unstable inflation on the inventory costs, it is not possible to determine the exact value for parameters of the unit inventory costs. Hence, in these models, the unit purchasing cost has been considered as a fuzzy number. In order to solve the fuzzy models, the fuzzy non-linear programming method has been used. Then, the solution procedure has been illustrated by a numerical example. Also, the optimal values of the fuzzy models have been compared to each other using sensitivity analysis on all parameters. According to the obtained results, the ratio $EDC_{d}/EDC_{r}$ is only moderately sensitive to the changes in $r$, $\mu_1$, and $\mu_2$. Thus, the difference between the two derived fuzzy inventory models by these modeling methods is negligible.

As future researches, the comparison of two modeling methods can be extended with respect to the following cases:

(1) Considering the purchases costs under discount,
(2) Develop to the two-warehouse inventory models,
(3) Develop to the multi-item multi-objective inventory models in a fuzzy environment where the other parameters also can be assumed as fuzzy and fuzzy random variables.

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Correction
This article has been republished with minor changes. These changes do not impact the academic content of the article.

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Buzacott, J. A. (1975). Economic order quantities with inflation. Journal of the Operational Research Society, 26(3), 553–558. doi:10.1057/jors.1975.113
Chakraborty, D., Jana, D. K., & Roy, T. K. (2018). Two-warehouse partial backlogging inventory model with ramp type demand rate, three-parameter Weibull distribution deterioration under inflation and permissible delay in payments. Computers & Industrial Engineering, 123, 157–179. doi:10.1016/j.cie.2018.06.022
Chern, M. S., Yang, H. L., Teng, J. T., & Papachristos, S. (2008). Partial backlogging inventory lot-size models for deteriorating items with fluctuating demand under inflation. European Journal of Operational Research, 191(1), 127–141. doi:10.1016/j.ejor.2007.03.053
Chung, K. J., & Tsai, S. F. (2001). Inventory systems for deteriorating items with shortages and a linear trend in demand-taking account of time value. Computers & Operations Research, 28(9), 915–934. doi:10.1016/S0305-0548(00)00162-2
Dave, U., & Patel, L. K. (1983). T, (s, i) policy inventory model for deteriorating items with time proportional demand. Journal of the Operational Research Society, 33(2), 137–142.
Dey, J. K., Mondal, S. K., & Maiti, M. (2008). Two storage inventory problem with dynamic demand and interval valued lead-time over finite time horizon under inflation and time-value of money. European Journal
Mouni, I., Girf, B. C., & Ko, B. (2005). Economic order quantity models for ameliorating/deteriorating items under inflation and time discounting. European Journal of Operational Research, 162(3), 773–785. doi:10.1016/j.ejor.2003.09.025

Mousavi, S. M., Hajipour, V., Niaki, S. T. A., & Alikar, N. (2013). Optimizing multi-item multi-period inventory control system with discounted cash flow and inflation: two calibrated meta-heuristic algorithms. Applied Mathematical Modelling, 37(4), 2241–2256. doi:10.1016/j.apm.2012.05.019

Mousavi, S. M., Sadeghi, J., Niaki, S. T. A., & Tavana, M. (2016). A bi-objective inventory optimization model under inflation and discount using tuned Pareto-based algorithms: NSGA-II, NRGA, and MOPSO. Applied Soft Computing, 43, 57–72. doi:10.1016/j.asoc.2016.02.014

Pol, S., Mahapatra, G. S., & Samanta, G. P. (2015). A production inventory model for deteriorating item with ramp type demand allowing inflation and shortages under fuzziness. Economic Modelling, 46, 334–345. doi:10.1016/j.econmod.2014.12.031

Popachristos, S., & Skouri, K. (2000). An optimal replenishment policy for deteriorating items with time-varying demand and partial-exponential type-backlogging. Operations Research Letters, 27(4), 175–184. doi:10.1016/S0167-6377(00)00044-4

Prasad, K., & Mukherjee, B. (2016). Optimal inventory model under stock and time dependent demand for time varying deterioration rate with shortages. Annals of Operations Research, 243(1-2), 323–334. doi:10.1007/s10479-014-1759-3

Rastogi, M., & Singh, S. R. (2018). A production inventory model for deteriorating products with selling price dependent consumption rate and shortages under inflationary environment. International Journal of Procurement Management, 11(1), 36–52. doi:10.1504/IJPM.2018.088614

Samal, N. K., & Pratihar, D. K. (2014). Optimization of variable demand fuzzy economic order quantity inventory models without and with backordering. Computers & Industrial Engineering, 78, 146–162. doi:10.1016/j.cie.2014.10.006

Sana, S. S. (2010). Optimal selling price and lot-size with time varying deterioration and partial backlogging. Applied Mathematics and Computation, 217(1), 185–194. doi:10.1016/j.amc.2010.05.040

San-Jose, J. A., Sicilia, J., Gonzalez-De-la-Rosa, M., & Febles-Acosta, J. (2017). Optimal inventory policy under power demand pattern and partial backlogging. Applied Mathematical Modelling, 46, 618–630. doi:10.1016/j.apm.2017.01.082

Sarkar, B., & Moon, I. (2011). An EPQ model with inflation in an imperfect production system. Applied Mathematics and Computation, 217(13), 6159–6167. doi:10.1016/j.amc.2010.12.098

Shaikh, A. A., Bhunia, A. K., Cárdenas-Barrón, L. E., Sahoo, L., & Tiwari, S. (2018). A fuzzy inventory model for a deteriorating item with variable demand, permissible delay in payments and partial backlogging with Shortage Follows Inventory (SFI) policy. International Journal of Fuzzy Systems, 20(5), 1606–1623. doi:10.1007/s40815-018-0466-7

Sharmila, D., & Uthayakumar, R. (2015). Inventory model for deteriorating items involving fuzzy with shortages and exponential demand. International Journal of Supply and Operations Management, 2(3), 888–904.
Singh, R., Kumar, A., & Yadav, D. (2018). A two storage inventory model with variable demand and time dependent deterioration rate and with partial backlogging. Malaya Journal of Matematik (MJM), 2018(1), 35–40. doi:10.26637/MJM0S01/07

Teng, J. T., & Yang, H. L. (2004). Deterministic economic order quantity models with partial backlogging when demand and cost are fluctuating with time. Journal of the Operational Research Society, 55(5), 495–503. doi:10.1057/palgrave.jors.2601678

Tripathy, C. K., & Mishra, U. (2010). Ordering policy for Weibull deteriorating items for quadratic demand with permissible delay in payments. Applied Mathematical Science, 4(44), 2181–2191.

Yang, H. L. (2004). Two-warehouse inventory models for deteriorating items with shortages under inflation. European Journal of Operational Research, 157(2), 344–356. doi:10.1016/S0377-2217(03)00221-2

Yang, H. L. (2012). Two-warehouse partial backlogging inventory models with three-parameter Weibull distribution deterioration under inflation. International Journal of Production Economics, 138(1), 107–116. doi:10.1016/j.ijpe.2012.03.007