Liquid-Droplet model, a possible solution for the rotation curve problem

F. Darabi *

Department of Physics, Azarbaijan University of Tarbiat Moallem, 53714-161 Tabriz, Iran

January 15, 2013

Abstract

A new model for the dynamical structure of galaxies is studied based on the so-called Liquid-Droplet model describing the structure of the nucleus. It is argued that in the galactic scale the inverse squared law of long range gravitational interaction may be replaced by a new law of gravitational interaction which is effectively short range while representing the characteristic features of Mach principle. Application of this new interaction at galactic scale gives the rotation curve in good agreement with observations. Then, the Virial theorem for this kind of gravitational interaction is developed and also the Tully-Fisher relation is obtained. Finally, a physical explanation is given for the so called constant acceleration in the MOND as the effective gravitational strength of

*f.darabi@azaruniv.edu
galaxies.

Keywords: Liquid-droplet model, galaxy, rotation curve
1 Introduction

It is well known that classical Newtonian dynamics fails on galactic scales. There is astronomical and cosmological evidence for a discrepancy between the dynamically measured mass-to-light ratio of any system and the minimum mass-to-light ratios that are compatible with our understanding of stars, of galaxies, of groups and clusters of galaxies, and of superclusters. Observations on the rotation curves have turn out that galaxies are not rotating in the same manner as the Solar System. If the orbits of the stars are governed solely by gravitational force, it was expected that stars at the outer edge of the disc would have a much lower orbital velocity than those near the middle. In fact, by the Virial theorem the total kinetic energy should be half the total gravitational binding energy of the galaxies. Experimentally, however, the total kinetic energy is found to be much greater than predicted by the Virial theorem. Galactic rotation curves, which illustrate the velocity of rotation versus the distance from the galactic center, cannot be explained by only the visible matter. This suggests that either a large portion of the mass of galaxies was contained in the relatively dark galactic halo or Newtonian dynamics does not apply universally.

The dark matter proposal is mostly referred to Zwicky [1] who gave the first empirical evidence for the existence of the unknown type of matter that takes part in the galactic scale only by its gravitational action. He found that the motion of the galaxies of the clusters induced by the gravitational field of the cluster can only be explained by the assumption of dark matter in addition to the matter of the sum of the observed galaxies. Later, It was demonstrated that dark matter is not only an exotic property of clusters but can also be found in single galaxies to explain their flat rotation curves.
The second proposal results in the modified Newtonian dynamics (MOND), proposed by Milgrom, based on a modification of Newton’s second law of motion [2]. This well known law states that an object of mass \( m \) subject to a force \( F \) undergoes an acceleration \( a \) by the simple equation \( F = ma \). However, it has never been verified for extremely small accelerations which are happening at the scale of galaxies. The modification proposed by Milgrom was the following

\[
F = m \mu \left( \frac{a}{a_0} \right) a, 
\]

where \( a_0 = 1.2 \times 10^{-10} \text{ms}^{-2} \) is a proposed new constant. The acceleration \( a \) is usually much greater than \( a_0 \) for all physical effects in everyday life, therefore \( \mu (a/a_0) = 1 \) and \( F = ma \) as usual. However, at the galactic scale outside the central bulge where \( a \sim a_0 \) we have the modified dynamics \( F = m (\frac{a^2}{a_0}) \). Using this new law of dynamics for the gravitational force \( F = \frac{GmM}{r^2} \) we obtain

\[
\frac{GM}{r^2} = \left( \frac{a^2}{a_0^2} \right), 
\]

which results in the constant rotational velocity

\[
v^2 = \sqrt{GMa_0}. 
\]

Another interesting model in this direction has been recently proposed by Sanders. In this model, it is assumed that gravitational attraction force becomes more like \( 1/r \) beyond some galactic scale[3]. A test particle at a distance \( r \) from a large mass \( M \) is subject to the acceleration

\[
a = \frac{GM}{r^2} g(r/r_0), 
\]
where $G$ is the Newtonian constant, $r_0$ is of the order of the sizes of galaxies and $g(r/r_0)$ is a function with the asymptotic behavior

$$g(r/r_0) = \begin{cases} 
1 & \text{if } r \gg r_0 \\
1/r_0 & \text{if } r \ll r_0.
\end{cases} \quad (6)$$

Dark matter as the manifestation of Mach principle has also been considered as one of the solutions for the dark matter problem. According to Mach principle [4] the distant mass distribution of the universe has been considered as being responsible for generating the local inertial properties of the close material bodies. Borzeszkowski and Treder have shown that the dark matter problem may be solved by a theory of Einstein-Mayer type [5]. The field equations of this gravitational theory contain hidden matter terms, where the existence of hidden matter is inferred solely from its gravitational effects. In the nonrelativistic mechanical approximation, the field equations provide an inertia-free mechanics where the inertial mass of a body is induced by the gravitational action of the cosmic masses. From the Newtonian point of view, this mechanics shows that the effective gravitational mass of astrophysical objects depends on $r$ such that one expects the existence of new type of matter, the so called dark matter. In what follows we will introduce a new model for the dynamical structure of a galaxy, based on a model for the structure of the nucleus, as an alternative to dark matter and MOND to explain the rotation curve.

## 2 Liquid-Droplet model for the nucleus

In nuclear physics, Weizsäcker’s formula or the semi-empirical mass formula (SEMF) is used to approximate the mass and various features of atomic nucleus [6]. The theoretical basis of
this formula is the liquid drop model. This model in nuclear physics, first proposed by George Gamow and developed by Niels Bohr and John Archibald Wheeler, treats the nucleus as a drop of incompressible nuclear fluid which is made of nucleons (protons and neutrons) binding together by the strong nuclear force. It predicts the binding energy of a nucleus in terms of the numbers of protons and neutrons it contains. This statement manifests in the semi-empirical mass formula which has five terms on its right hand side. These correspond to i) volume binding energy of all the nucleons inside the nucleus, ii) surface binding energy of all the nucleons on the surface of nucleus, iii) the electrostatic mutual repulsion of the protons, iv) an asymmetry term and v) a pairing term (both derivable from quantum mechanical considerations).

The first two terms concern about the attractive strong nuclear force. We shall focus on these two terms and leave the other terms irrelevant for our purposes. Assuming an approximately constant density for the nucleus one can calculate the nuclear radius by using that density as if the nucleus were a drop of a uniform liquid. The liquid droplet model of the nucleus takes into account the fact that the forces on the nucleons on the surface is different from those associated with the interior ones. This is something similar to taking into account of surface tension as a contributor to the energy of a tiny liquid drop.

In the case of volume binding energy, when an assembly of nucleons of the same size is packed together into the smallest volume, each interior nucleon has a certain number of other nucleons in close contact with it. Actually, the number of pairs that can be taken from A particles is $A(A - 1)/2$, so one may expect a term for binding energy proportional to $A^2$ interacting pairs. However, the strong force has a very limited range, and a given nucleon may only interact strongly with its nearest neighbors. In other words, the nuclear
force is saturated and each nucleon contributes an almost constant energy to the binding of the nucleus. Therefore, the number of pairs of particles that effectively interact is roughly proportional to $A$ and not $A^2$, and the nuclear binding energy is proportional to the volume of the nucleus. The volume term suggests that each nucleon interacts with a constant number of surrounding nucleons, independent of $A$.

In the case of surface binding energy however, each nucleon at the surface of a nucleus interacts with fewer other (interior and surface) nucleons than one in the interior of the nucleus. Therefore, the binding energy associated with each surface nucleon is less than that of the interior one. In fact, the surface term makes correction to the volume term by its negative contribution as follows

$$B = a_V A - a_S A^{2/3}, \quad (7)$$

where $B$ accounts for the binding energy and $a_V, a_S$ are some appropriate constants with the same order of magnitude. One may focus more on the volume term to make maximum use of its physical justification. The physics behind this term in the case of a real liquid-droplet is as follows: To evaporate one liquid-droplet we have to use a definite heat energy as $Q_v M_m A$ where $Q_v$ accounts for the vaporization latent heat, $M_m$ is the mass of each molecule and $A$ is the total number of molecules in the droplet. This heat energy is necessary to overcome all the attractive interactions between the molecules. Hence, it is exactly equal to the binding energy of the droplet. This justifies the first term as

$$B_V = Q_v M_m A = a_V A \quad (8)$$

which results in the identification $a_V = Q_v M_m$. Therefore, we come to the conclusion that the binding energy per each molecule, namely $(B/A)$, corresponding to the volume term is
independent of the total number of molecules $A$. This has a simple reason: the number of nearest neighbours of each interior molecule is independent of the total size of the system (droplet), then $(B/A)$ becomes independent of $A$. This is characteristic feature of every system in which the range of interaction between the constituent particles is short in comparison with the size of the system. Of course, this feature undergoes a surface correction due to the finite size of the system according to (7). Moreover, if we consider the repulsive coulomb force between the protons, then its negative contribution proportional to $Z^2$ ($Z$ is the atomic number) affects the ratio $(B/A)$ in a more complicate way. One may add also the contributions of an asymmetry term and a pairing term. In general, all of these contributions to the volume term except for the pairing term are negative.

The result is that the curve of $(B/A)$ with respect to $A$ is almost flat except for the small and large values of $A$. The decrease for small values of $A$, is due to the fact that the full binding of each nucleon is accomplished when that nucleon is fully encircled by other neighbor nucleons. However, this does not apply for the surface nucleons; then for light nucleus having larger portion of surface nucleons the binding energy for each nucleon in the nucleus is decreased. On the other hand, the decrease for large values of $A$ results from the coulomb repulsive force between each protons which is proportional to $Z^2$ and hence increases more rapidly than $A$, resulting in a decrease of binding energy per each nucleon. It is common to associate the almost flat portion of the curve with the saturation property of the nuclear force which itself is emerged due to the short range of this force. If this force would not be saturated and the nuclear force would be a long range force, then each nucleon would interact with all other nucleons with a term proportional to the size of the nucleus or $A^2$. The more large size of
the nucleus, the more stabilized system, a result which is not favored by the nature. Hence, it seems the saturation property or short range of the nuclear force is related to the fact that nature avoids the structures with unlimited large size.

3 Liquid-Droplet model for a typical galaxy

To model the structure of the galaxy on the basis of Liquid-Droplet model we need to revisit the concept of the range of gravitational interaction in a large size structure of a galaxy. In fact, both the range of an interaction and the size of structure in which such interaction is governed are deeply related to the causal structure of space-time. Newton believed in the action of distance so that every gravitational effect, whatsoever far from a point, can be felt immediately at that point. The concept of a local gravitational field obeying the Lorentz invariance however changed this view: no gravitational effect at a large distance can be experienced at a point unless after a time interval that a signal with light velocity takes to enter the future light cone at that point. Even, in the Machian viewpoint where it is assumed that each gravitational constituent feels gravitationally all other constituents, the concept of local gravitational field obeying the Lorentz invariance is considered. This lorentz invariant causal structure then pose a crucial limit on the effective range of gravitational interaction in a large size structure. In fact, it is reasonable to assume that the effective range of any interaction in an structure is modeled by the size of that structure. For example, the nucleus with its small size determines an effective short range of Yukawa nuclear force. A deeper analysis on the basis of causal space-time structure may reveal that the large size of a galaxy determines the effective range for gravitational interaction inside this structure. Suppose, in a similar way that one calculates
the binding energy in a nucleus by Liquid-Droplet model, we calculate the binding energy in a galaxy. In the case of nucleus, we learned that each nucleon has full interaction just with neighbouring nucleons, due to short range of nuclear interaction. In the case of galaxy, we know gravitational interaction has long range. But, if we vaporize a small portion of a galaxy to free all the stellar mass in this portion from their mutual gravitational interactions, then all other stellar mass far enough from this portion of the galaxy will not feel this vaporization until its signal (with light velocity) can be reached by them. This exactly means that each local portion of a galaxy has an effective short range interaction compared with the large size of the whole galaxy. In other words, each small portion of the galaxy has just effective short range gravitational interaction with its limited neighborhood portions due to the limited velocity of light. This feature in the galaxy is similar to the feature in a massive nucleus where the range of effective nuclear force between close nucleons is short in comparison to the size of the nucleus. Therefore, if we accept in the context of Liquid-Droplet model that the effective range of gravitational interaction in a galactic structure is modeled by the effective size and mass of that structure, (an example of which is the short range nuclear force in the nucleus), then we may propose a model of gravitational interaction for the galaxy as follows

$$U(r) = -G \left[ k(r)m \right][Nm] / R,$$

(9)

where $U$ is the effective gravitational potential in a galaxy containing $N$ gravitational constituents of average mass $m$, $R$ is the radius (effective size) of the galactic bulge, $G$ is the gravitational constant, and $k(r)$ is the $r$-dependent number of neighbor constituents for each individual constituent interacting with them.

This form of gravitational potential may be partly interpreted as Machian. We know the
mass is a positive quantity and it is not possible to screen the gravitational effects. Therefore, we assume that in any two (close) body process in the galaxy the extra influence of all other matter distributed over the effective size of the galaxy should be taken into account. Accordingly, we can replace the whole gravitational effect of all effective interactions at the position of a constituent of mass $m$ by the effect of a spherical shell of the *effective* mass $M$ and the effective radius $R$. This shell acts like a gravitational Faraday cage inside of which a gravitational potential exists

$$\Phi = -\frac{GM}{R}.$$  (10)

From the Liquid-Droplet model, we shall assume that each constituent gravitationally interacts with some limited neighbor constituents $k$. But, this number will depend on the distance $r$ of that constituents from the center of the galaxy, as we shall explain later. By this kind of interaction we mean a *local* interaction. On the other hand, we assume a Machian (or nonlocal) interaction, as explained above, contributing to $\Phi$ along with the local interaction. Therefore, if we consider the contribution of both local and nonlocal interactions to the gravitational potential (10) as the manifestation of effective mass $M$, and compare it with the gravitational potential energy (9) through $U = m\Phi$, then we obtain the effective mass $M$ as an *active* gravitational mass

$$M(r) = k(r)Nm.$$  (11)

This is a key feature of the present model in which the active gravitational mass is different from the additive mass of the galaxy, namely $Nm$. Moreover, we may associate the mean contribution $-Gm^2/R$ to each pair of constituents so that the potential energy at the location $r$ will contain $k(r)N$ terms where $k(r)$ and $N$ account for the local and nonlocal sectors of
interactions, respectively.

We expect that the linear dependence of $U$ on $N$ will result in the saturation of the gravitational force, similar to the saturation of the nuclear force in the nucleus. In fact, similar to the volume binding energy of a nucleus which is calculated using the Liquid-Droplet model, the volume binding energy of the galaxy is given

$$B_V = a_V N,$$

where the vaporization latent heat $a_V$ per each constituent in the galactic bulge is given by

$$a_V = \int_0^R G \frac{k(r)m^2}{R} dr.$$  

(13)

4 Rotation curve

In this section, we study the rotation curve of a typical galaxy upon the Liquid-Droplet model. For each gravitational constituent located at a distance $r$ from the center of galaxy having equilibrium state, one may approximately relate the order of magnitudes of the kinetic and potential energies as

$$T \sim |U(r)|.$$  

(14)

Now, we introduce the very important role of the variable $k$ in the potential energy $U$. In the Liquid-Droplet model, $k$ is the number of neighbor constituents interacting effectively with the individual constituent in consideration. In the case of a real Liquid droplet or a nucleus this number is a constant which is determined by the geometric shape of packed molecules or nucleons. However, this is not the case for the galaxy where there are large voids between the constituents. Assuming a spherical symmetry for the high density mass distribution in the
central galactic bulge, it is easy to realize that the number of neighbor constituents around each constituent \(i\) located in a volume element grows with the distance \(r\) similar to the volume element \(dV = 4\pi r^2 dr\). Therefore, in the central galactic bulge we have

\[
k_{in} = \alpha r_i^2,
\]

where \(r_i\) is the distance of the \(i^{th}\) constituent from the center of galaxy and \(\alpha\) is a constant.

The potential (9) is then rewritten as follows

\[
U(r_i) = -Ar_i^2,
\]

where \(A = \alpha G N m^2 / R\) is another constant. By substituting (16) into the equation (13) we obtain

\[
\frac{1}{2} m v_i^2 \sim Ar_i^2,
\]

which leads to the familiar result in that section of rotation curve which corresponds to the mass distribution in the galactic bulge, as

\[
v_i \sim r_i.
\]

For the outside of the central bulge where the mass is smoothly distributed with very low density (compared with the bulge) over each volume element, it is plausible to assume that each constituent located in the middle of a volume element is surrounded by some of the other interacting constituents whose number is a very slow function of the distance \(r_i\). A suitable choice is

\[
k_{out} = \ln r_i,
\]

for which we obtain

\[
U(r_i) = -B \ln r_i,
\]
\[ \frac{1}{2}mv_i^2 \sim B \ln r_i, \]  
\[ (21) \]

where \( B = GNm^2/R \), or

\[ v_i \sim \sqrt{\ln r_i}. \]  
\[ (22) \]

This result gives an almost flat rotation curve for the outside of the central bulge, in complete agreement with current observations.

We may repeat the calculations in a more dynamical way to obtain the almost same results by resorting to Newton’s second law. In the case of galactic bulge the Newton’s law becomes

\[ \frac{mv_i^2}{r_i} = - \frac{dU}{dr_i} = 2Ar_i, \]  
\[ (23) \]

which gives the velocity proportional to the distance as

\[ v_i = \sqrt{\frac{2A}{m}r_i}. \]  
\[ (24) \]

For the outside of the central bulge the Newton’s law becomes

\[ \frac{mv_i^2}{r_i} = - \frac{dU}{dr_i} = B, \]  
\[ (25) \]

which gives a constant velocity

\[ v_i = \sqrt{\frac{B}{m}}. \]  
\[ (26) \]

In order to match the velocities at \( r_i = R \) we need to equate the followings

\[ \sqrt{\frac{2A}{m}R} = \sqrt{\frac{B}{m}}, \]  
\[ (27) \]

which results in

\[ \alpha = \frac{1}{2R^2}. \]  
\[ (28) \]
Substituting this into Eq.(15) leads to the important result that each constituent inside the central bulge is at most \( r_i = R \) effectively interacting with one half of another neighbor constituent. This shows that in the Liquid-Droplet model each constituent is \textit{locally} interacting with a fraction \( k \) of other neighbor individual constituent, while globally interacts with \( N \) distant constituent. This fraction is very small close to the center of galaxy but approaches to “one half” at the radius \( R \) of the bulge.

Note that all the above results come out due to our special and simple assumptions about the forms of the quantity \( k(r) \). It is worth noting that the quantity \( k(r) \) inside the galactic bulge seems to be almost the same for all galaxies, but outside the galactic bulge it may depend on some characteristic features of each galaxy like its shape, size, ellipticity, mass distribution, etc. Therefore, it is expected that in this model the rotation curve for the outside galactic bulge is not universally flat for all the galaxies, rather it differs for different galaxies according to their characteristic features; a result which is observed in some galaxies.

### 5 Virial theorem

For a system of \( N \) point particles, the virial \( G \) satisfies the following equation [7]

\[
\frac{dG}{dt} = 2T + \sum_{i=1}^{N} \mathbf{F}_i \cdot \mathbf{r}_i,\tag{29}
\]

where \( T \) is the total kinetic energy of the system, \( \mathbf{F}_i \) is the net force on the \( i^{th} \) particle at the position \( \mathbf{r}_i \), and \( G \) is defined as

\[
G = \sum_{i=1}^{N} \mathbf{P}_i \cdot \mathbf{r}_i.\tag{30}
\]
We have realized that in the Liquid-Droplet model, the total force on each particle $i$ is acted by a fraction of only a neighbor particle. Moreover, this fraction depends on the distance of that particle from the center of galaxy. Therefore, the mutual interaction between all particles which is usually considered in a gravitational system has now been replaced by this new type of limited interaction. If we assume the force to be derived from a potential energy, then we should accept that this potential is just a function of the distance of each particle from the center of galaxy $r_i$ rather than the relative distance $r_{ji}$ between the particles, namely $F_i = -\nabla_{r_i} U$. Therefore, we have

$$\sum_{i=1}^{N} F_i \cdot r_i = - \sum_{i=1}^{N} \frac{dU}{dr_i} r_i. \quad (31)$$

Hence, Eq.(29) becomes

$$\frac{dG}{dt} = 2T - \sum_{i=1}^{N} \frac{dU}{dr_i} r_i. \quad (32)$$

For the central galactic bulge with $U_{in} \sim r_i^2$ we obtain

$$\frac{dG}{dt} = 2T - 2 \sum_{i=1}^{N_{in}} U_{in}(r_i) = 2T - 2U_{in}, \quad (33)$$

where $U_{in}$ is the total potential energy of the system in the galactic bulge consisting of $N_{in}$ particles. On the other hand, outside the central bulge of the galaxy consisting of $N_{out}$ particles with $U_{out} = -B \ln r_i$, we have

$$\frac{dG}{dt} = 2T + \sum_{i=1}^{N_{out}} B = 2T + N_{out} B. \quad (34)$$

The virial theorem states that the time average $\langle dG/dt \rangle$ vanishes, hence we obtain

$$\langle T \rangle_{in} = \langle U_T \rangle_{in}, \quad (35)$$
for the inside, and

$$\langle T \rangle_{\text{out}} = -\frac{1}{2} N_{\text{out}} B, \quad (36)$$

for the outside of the galactic bulge. Therefore, the total time average of the kinetic energy of the whole galaxy is obtained as

$$\langle T \rangle_{\text{tot}} = \langle U_T \rangle_{\text{in}} - \frac{1}{2} N_{\text{out}} B \quad (37)$$

$$= \frac{G N m^2}{R} \left( \sum_i \langle k_{\text{in}} \rangle - \frac{1}{2} N_{\text{out}} \right).$$

By interpreting $N_{\text{out}} B$ as $\langle U_T \rangle_{\text{out}}$, we obtain

$$\langle E \rangle_{\text{in}} = \langle T \rangle_{\text{in}} + \langle U_T \rangle_{\text{in}} = 2 \langle T \rangle_{\text{in}}, \quad (38)$$

$$\langle E \rangle_{\text{out}} = \langle T \rangle_{\text{out}} + \langle U_T \rangle_{\text{out}} = -\langle T \rangle_{\text{out}}, \quad (39)$$

$$\langle E \rangle_{\text{tot}} = 2 \langle T \rangle_{\text{in}} - \langle T \rangle_{\text{out}}. \quad (40)$$

In a stable gravitationally bound system we require $\langle E \rangle_{\text{tot}} < 0$ which leads to

$$\langle T \rangle_{\text{in}} < \frac{1}{2} \langle T \rangle_{\text{out}}. \quad (41)$$

This is an interesting result which states that the total average kinetic energy in the galactic bulge is less than the half total average kinetic energy in the outside of galactic bulge.

### 6 Tully-Fisher relation

The Tully-Fisher relation is an empirical relationship between the intrinsic luminosity (proportional to the stellar mass) of a spiral galaxy and its amplitude of its rotation curve (which sets
the total gravitational mass) [8]. It has been turned out that on large scales most astronomical systems like a spiral galaxy have much larger mass-to-light ratios than the central parts. This fact may be explained by the relation (11) as follows. In the present model, the gravitational mass is the active mass $M$. On the other hand, the luminosity which is proportional to the stellar mass becomes proportional to the additive mass $Nm$. Therefore, the mass-to-light ratio becomes proportional to

$$\frac{M}{Nm} = k(r). \quad (42)$$

Since in both the galactic bulge and outer region, $k(r)$ is an increasing function of the distance from the center of galaxy $r$ (see Eqs.(15), (19)), hence we have much larger mass-to-light ratios than the central parts which is in complete agreement with the Tully-Fisher relation.

From another point of view we may study the Tully-Fisher relation in the present model. We may rewrite the equation (26) as

$$v^4 = \left(\frac{G Nm}{R}\right)^2. \quad (43)$$

If we consider $R$ as the effective size of the galaxy and $Nm$ as the approximate stellar mass, then assuming a roughly constant mass density we may write $Nm \sim R^3$ or $R^2$ according to whether we assume a spherical shape or disk shape for the whole galaxy, respectively. Therefore, Eq.(43) reads

$$v^\alpha \sim Nm, \quad (44)$$

where $\alpha = 3$ or 4 corresponding to spherical or disk shapes for the galaxy, respectively. This is an interesting result which again accounts for the empirical Tully-Fisher relation.
7 Liquid-Droplet model and MOND

According to MOND, the rotational velocity is given by \( v = (GNma_0)^{\frac{1}{4}} \). However we find that in the present model this velocity is obtained by \( v = \left(\frac{GNm}{R}\right)^{\frac{1}{2}} \). Comparison of these two velocities leads to the following result

\[
a_0 = \frac{GNm}{R^2}.
\] (45)

This means the constant acceleration \( a_0 \) which has no clear physical explanation in the MOND is explained in the present model through (45) indicating that this acceleration is nothing but the effective gravitational strength of each galaxy. To justify this relation we consider for example the Andromeda galaxy whose stellar mass \( Nm \) and bulge radius \( R \) are approximately \( 7 \times 10^{11} SM \) and \( 10^5 LY \), respectively. Putting these values into (45) gives \( a_0 \sim 10^{-10} \) which is in good agreement with the value obtained by Milgrom. It seems similar calculation for other galaxies gives almost the same order of magnitude for the value of \( a_0 \).
8 Conclusion

The problem of rotation curves for the galaxies has been investigated in the context of a new model which may be considered as an alternative to the dark matter and MOND. This model is motivated by the so called Weizsäcker’s Liquid-Droplet model which had been used to describe the structure of the nucleus, according to which the nuclear force is saturated inside the nucleus to exhibit a short range force and a flat curve of binding energy. Similarly, we have tried to show that the reason for the flat rotation curve of a galaxy may have its origin in the fact that the gravitational force exhibits an effective local short range interaction in the galactic structure while representing characteristic features of Mach principle. This kind of gravitational interaction at the galactic scale gives the rotation curve in good agreement with observations. We have also developed the Virial theorem and also obtained the Tully-Fisher relation. Moreover, we have given a physical explanation for the so called constant acceleration in the MOND.

References

[1] F. Zwicky, Morphological Astronomy (Springer, Berlin, 1957).

[2] M. Milgrom, Astrophys. J. 270, 365 (1983).

[3] R. Sanders, Mod. Phys. Lett. A18, 1861 (2003).

[4] E. Mach, The Science of Mechanics (Open Court, La Salle, 1960), p. 267, transl. T. J. McCormack.
[5] H. H. v. Borzeszkowski and H. J. Treder; Foundations of Physics, Vol. 28, 273 (1998).

[6] C. F. Weizsäcker, To the theory of the core masses, Magazine for physics 96, 431 (1935).

[7] G. W. Collins, The Virial Theorem in Stellar Astrophysics (Pachart Press 1978), H. Goldstein, Classical Mechanics ,2nd ed. (Addison-Wesley, 1980).

[8] R. B. Tully, J. R. Fisher, Astronomy and Astrophysics, 54, 661 (1977).