Progress towards Nash-Williams’ Conjecture on Triangle Decompositions

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Partitioning the edges of a graph into edge disjoint triangles forms a triangle decomposition of the graph. A famous conjecture by Nash-Williams from 1970 asserts that any sufficiently large, triangle divisible graph on $n$ vertices with minimum degree at least $0.75 \ n$ admits a triangle decomposition. In the light of recent results, the fractional version of this problem is of central importance. A fractional triangle decomposition is an assignment of non-negative weights to each triangle in a graph such that the sum of the weights along each edge is precisely one.

We show that for any graph on $n$ vertices with minimum degree at least $0.827327 \ n$ admits a fractional triangle decomposition. Combined with results of Barber, Kühn, Lo, and Osthus, this implies that for every sufficiently large triangle divisible graph on $n$ vertices with minimum degree at least $0.82733 \ n$ admits a triangle decomposition. This is a significant improvement over the previous asymptotic result of Dross showing the existence of fractional triangle decompositions of sufficiently large graphs with minimum degree more than $0.9 \ n$. This is joint work with Luke Postle.