COMPRESSING DEEP CNNS USING BASIS REPRESENTATION AND SPECTRAL FINE-TUNING

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ABSTRACT

We propose an efficient and straightforward method for compressing deep convolutional neural networks (CNNs) that uses basis filters to represent the convolutional layers, and optimizes the performance of the compressed network directly in the basis space. Specifically, any spatial convolution layer of the CNN can be replaced by two successive convolution layers: the first is a set of three-dimensional orthonormal basis filters, followed by a layer of one-dimensional filters that represents the original spatial filters in the basis space. We jointly fine-tune both the basis and the filter representation to directly mitigate any performance loss due to the truncation. Generality of the proposed approach is demonstrated by applying it to several well known deep CNN architectures and data sets for image classification and object detection. We also present the execution time and power usage at different compression levels on the Xavier Jetson AGX processor.

Index Terms— Basis representation, network compression, orthogonal filters

1. INTRODUCTION

While there has been a tremendous surge in convolutional neural networks and their applications in computer vision, relatively little is still understood about how information is learned and stored in the network. This is evidenced by the fact that researchers have successfully proposed different approaches for compressing a network after it has been trained, including techniques like pruning weights\textsuperscript{[1,2]}, assuming row-column separability and applying low rank approximations for computational gains\textsuperscript{[3,4]} and using basis representation\textsuperscript{[5,6,7,8,4]} to approximate the filter kernels. Although the results are impressive, existing compression techniques are not always easy to implement using standard deep learning tool boxes. In this paper, we are motivated by the observation that the original filters can be represented as weighted linear combinations of a set of 3D basis filters with one-dimensional weight kernels as shown in Figure 1. While compression is achieved by using fewer basis filters, we show that these basis filters can be jointly finetuned along with the weight kernels to compensate for any loss in performance due to truncation, and to thereby achieve state of the art results. The representation of spatial filters as a linear combination of orthogonal basis filters is also known as spectral decomposition, and the weights comprise the corresponding spectra of the filters. Since our approach updates the basis filters and their weighted contribution to the overall result, we refer to this process as Spectral Fine Tuning (SFT). Not only does SFT reduce the number of learnable parameters, but also the weights are statistically uncorrelated, and therefore adapt much faster than the conventional finetuning of spatial domain filters. Figure 2 shows the main steps in our proposed approach for network compression.

Low rank decomposition has been extensively used for neural network compression in the past, but our work is dif-
fers from previously published papers in the manner in which the filters are represented and finetuned using basis filters and weights. For example methods such as [4] enforce filter rank reduction while training the network, thereby making it more compressible than its normally trained counterpart. In contrast, [4] achieves compression by combining channel wise low rank approximation and with separable one-dimensional spatial filters. In [7] multiply-accumulate operations are reduced via constrained optimization. Low rank factorization and pruning are combined in [8] by cascading the low rank projections of filters in the current layer to the next layer. In comparison, our method is relatively straight forward (as depicted in Figure 2), and yet provides competitive results.

2. BASIS REPRESENTATION AND LEARNING

Consider the fundamental convolution operation in any given layer of a convolutional neural network. Assume that an input block of data \(x(m, n, l)\) (such as the activations or output of the previous layer) is convolved with a set of 3D filters \(h_k(m, n, l), k = 1 \ldots P\). The output \(y_k(m, n)\) can be expressed as \(y_k(m, n) = x(m, n, l) \ast h_k(m, n, l), 1 \leq k \leq p\) where * represents the convolution operation. The filters can be expressed as a linear combination of Q basis functions \(f_i(m, n, l), i = 1 \ldots Q\), such that \(h_k(m, n, l) = \sum_{i=1}^{Q} w_k(i) \cdot f_i(m, n, l)\) where \(w_k(i)\) are the weights of linear combination. Using this representation, the output can be expressed as

\[
y_k(m, n) = \sum_{i=1}^{Q} w_k(i) \cdot [x(m, n, l) \ast f_i(m, n, l)], \quad 1 \leq k \leq P
\]  

(1)

The key observation is that the Q convolution terms \(z_i(m, n) = x(m, n, l) \ast f_i(m, n, l)\) need to be computed only once, and they are common to all P outputs \(y_k(m, n)\). These can be stacked together to form the 3D intermediate result \(z(m, n, i)\) while the weights can be treated as \(1 \times 1 \times Q\) filter \(w_k(i)\). Therefore, the outputs \(y_k(m, n)\) are simply the convolution of two, i.e

\[
y_k(m, n) = w_k(i) \ast z(m, n, i)
\]  

(2)

We refer to this construct using two successive convolutions as BasisConv.

Compression of pretrained Networks and Spectral Weights

Fine Tuning: SFT is the simultaneous learning of \(w_k(i)\) and \(f_i(m, n, l)\) in Equation (1) (across all layers of the network) to mitigate overall performance loss due the choice of \(Q < P\). We now discuss how the basis filters \(f_i(m, n, l)\) and spectral weights \(w_k(i)\) are initialized, and the orthogonality criteria introduced in the loss function to ensure that the basis representation condition is preserved.

It is well known that eigen decomposition results in a compact basis that minimizes the reconstruction error achieved by a linear combination of basis functions. We therefore initially choose \(f_i(m, n, l)\) as the eigen filters that represent the sub-space in which the original filters \(h_k(m, n, l)\) lie. The method for obtaining these is also well-known and straightforward. Basically, we define the \(LD^2 \times 1\) dimensional vector \(h_k\) as a vectorized representation of \(h_k(m, n, l)\), and construct the matrix \(A = [h_1, h_2, \ldots, h_P]\) with \(h_k\) as its columns. The eigenvectors of \(AA^T\) represent the sub-space of the filters, and satisfy the relation \(A^Tf_i = \lambda_i f_i\), where \(f_i\) are the eigenvectors, and \(\lambda_i\) are the corresponding eigenvalues. The eigen filter \(f_i(m, n, l)\) is readily obtained by re-ordering the elements of the eigenvector \(f_i\) into a \(D \times D \times L\) array. It should be noted that these matrix-vector manipulations do not alter the inherent tensor structure and relations between the individual elements of the 3D filters. We select a small subset of eigenvectors which correspond to the largest Q eigen-values that best represent the dominant coordinates of the filters’ subspace using the metric \(t = \frac{\sum_{i=1}^{Q} \lambda_i}{\sum_{i=1}^{P} \lambda_i}\) to choose Q such that most of the relevant information is retained in the selected eigen-vectors. Using this criteria, Q is chosen for each layer such that it results in no more than 3% drop in overall performance.

The decomposition of the filters \(h_k\) of any given layer of the network can be succinctly expressed in matrix vector notation by defining \(F = [f_1, f_2, \ldots, f_Q]\) (i.e. the matrix of eigenvectors of the filters for that layer) so that \(h_k = Fw_k\) and \(w_k = [w_k(1) w_k(2) \ldots w_k(Q)]^T\) is a \(Q \times 1\) vector of weights. Since \(F^TF = I\) (i.e. the identity matrix), the weights are ea-
The elements of the weight vectors are statistically uncorrelated. This is easy to show by noting that the fact that the elements of the weight vectors are statistically uncorrelated. This is partly due to the attributes to the overall learning process in an uncorrelated manner. Thus far, we have discussed fine-tuning of the weights while holding the basis filters fixed as eigen-vectors of the original spatial filters. This is partly due to the appearance of the eigen values \( \lambda_i \) as its diagonal elements. Hence, \( \mathbf{W}^T \mathbf{W} = \mathbf{\Delta} \) is a diagonal matrix whose columns are the weight vectors associated with the filters in the corresponding columns of \( \mathbf{A} \). Therefore, \( \mathbf{A} \mathbf{\Delta}^T = \mathbf{F} \mathbf{\Delta} \mathbf{F}^T \). It is also true that \( \mathbf{A} \mathbf{\Delta}^T = \mathbf{F} \mathbf{\Delta} \mathbf{F}^T \) where \( \mathbf{\Delta} \) is a diagonal matrix with the eigenvalues \( \lambda_i \) as its diagonal elements. Hence, \( \mathbf{W}^T \mathbf{F}^T \mathbf{W} = \mathbf{\Delta} \) is a diagonal matrix and \( \sum_{k=1}^{P} w_k(i)^2 = \lambda_i \), and \( \sum_{k=1}^{P} w_k(i)w_k(j) = 0 \). Therefore the convergences of each element of \( \mathbf{w}_k \) is statistically uncorrelated with the behavior of the other elements, which implies that the corresponding eigen vector also contributes to the overall learning process in an uncorrelated manner. Thus far, we have discussed fine-tuning of the weights while holding the basis filters fixed as eigen-vectors of the original filters. However refining the basis filters can improve performance of the compressed network. To finetune the basis filters, we must ensure that they remain an orthonormal set (so that the weights continue to be a spectral representation in the new basis). This is achieved by including the following term in the overall cost function

\[
J_f = \frac{\alpha}{Q} \sum_{i=1}^{Q} (1 - f_i^T f_i)^2 + \frac{2(1 - \alpha)}{Q(Q-1)} \sum_{i=1}^{Q} \sum_{j=i+1}^{Q} (f_i^T f_j)^2 \tag{3}
\]

where \( f_i \) are the individual basis filters, \( Q \) is the number of such filters in a given layer, and \( \alpha \) is a positive number between 1 and 0. The first term in \( J_f \) causes the \( L2 \) norm of \( f_i, i = 1,...,Q \) to be as close to unity as possible, while the second term ensures the filters remain orthogonal. The parameter \( \alpha \) can be set to 0.5 to equally emphasize both term, or selected as necessary to balance the trade-off between the two terms. Finally, \( f_i(m, n, l) \) and \( w_k(i) \) are both jointly fine-tuned to minimize the sum of \( J_f \) and the cost function (such as cross-entropy loss) used for training the original network.

### 3. EXPERIMENTS

**Data Sets and CNN Architectures:** We performed our experiments on various publicly available datasets. These include image classification datasets CIFAR10, CIFAR100, ImageNet and object detection dataset MS-COCO.

**VGG16 on ImageNet:** We tested our method on VGG16 pretrained on ImageNet by compressing the model to 3x, 4x and 5x speedup ratios. Results of this experiment are presented in Table 1. Compared with other methods our basis VGG16 achieves better Top-5 accuracy on all speedup ratios.

**CIFAR-100 with Different Networks:** Table 2 shows the results of compressing four different networks using the proposed method, and their performance on the CIFAR-100 data set. For instance, compressing DenseNet190 reduces FLOPS by 81%, and the number of parameters by 73% with a minor loss of 0.55% in performance accuracy. Similar gains are also noted for other architectures.

**Resnet50 on ImageNet:** We also tested our method on Resnet with 50 convolution layers, pretrained on ImageNet by compressing the model at two different levels of compression: 51.28% and 61.68%. Results of this experiment and comparisons with other methods are presented in Table 3. When compared with existing state of art methods like IMP [14], CP [10], FPGM [17], GBN [19], LFPC [20] and HRank [21] our model (A) achieves better accuracy for the similar levels of compression. Our second configuration (B) achieves best compression both in terms of FLOPs and trainable parameters, while keeping Top-1 and Top-5 accuracy comparable to other methods.

**Resnet56 on CIFAR10:** For CIFAR10 dataset we tested our method on Resnet56 model and compared it with other similar methods. Comparative Results are shown in Table 4.

**Results on Object Detection:** To demonstrate our method’s effectiveness irrespective of the model architecture, dataset and the objective function, we tested it on object detection problem. We used 2 predefined variants of YOLOv3 [24] object detector pretrained on MS-COCO dataset. These variants are YOLOv3-tiny [24], which contains 13 convolution layers and a much deeper model YOLOv3-416 [24], which contains 75 convolution layers. For YOLOv3-tiny our method was able to achieve 51.26% reduction in FLOPs and 67.73% reduction in model parameters with 29.6 MAP while the MAP for baseline model is 32.9. We also compressed YOLOv3-416 to 2x and 3x speedup ratios. These compressed models attain 53.83 and 55.34 MAP score respectively, as compared to the baseline model which has 55.40 MAP.

**Implementation On Edge Processors:** We also measured the GPU speed up of our method on Nvidia Jetson Nano by compressing the VGG16 to reduce the FLOPs by 50% and 66%. Our method achieves 35.6% and 45.3% speedup respectively. Table 5 shows the inference time, power and energy consumption of VGG16 on the NVIDIA Jetson AGX Xavier (trained on ImageNet) using compression factors of 2x, 3x, 4x and 5x speedup ratios.
| Model     | FLOPs (Billions) | Params (Billions) | # of Filters | Accuracy (%) |
|-----------|------------------|-------------------|--------------|--------------|
|           | CNN BasisNet ↓%  | CNN BasisNet ↓%  |               |              |
| Alexnet   | 0.03 76.20       | 2.47 72.33        | 1152         | 43.87        |
| VGG16     | 0.63 84.44       | 14.71 88.68       | 4224         | 68.72        |
| ResNet10  | 0.51 66.85       | 1.73 63.41        | 4144         | 71.99        |
| Densenet190 | 18.61 81.52     | 25.60 72.73       | 20290        | 82.83        |

Table 2: Results of compressing four different networks on CIFAR100 shows significant FLOPs and parameter reduction is achieved with minimal loss of accuracy. [%] denotes percent decrease in the metric compared to the baseline model.

| Method     | FLOPs ↓% | Params ↓% | Accuracy (%) |
|------------|----------|-----------|--------------|
| SFP [13]   | 41.80    | -         | 74.61 92.06  |
| IMP [14]   | 45.00    | 51.48     | 74.50 -      |
| CP [10]    | 50.00    | -         | - 90.80     |
| LFC [15]   | 50.00    | -         | 73.40 91.40 |
| ELR [11]   | 50.00    | -         | - 91.20     |
| GDP [16]   | 51.30    | -         | 71.89 90.719|
| FPGM [17]  | 53.50    | -         | 74.83 92.32 |
| DCP [18]   | 55.76    | 51.45     | 74.95 92.32 |
| GBN [19]   | 55.06    | 53.40     | 75.18 92.41 |
| LFPC [20]  | 60.80    | -         | 74.46 92.04 |
| HRank [21] | 62.10    | 46.13     | 71.98 91.01 |
| Alvarez et. al. [6] | -       | 27.0     | 75.2 91.01   |
| TRPI+Nu [3] | -       | 44.0     | 74.06 92.07 |

Table 3: Results for ResNet-50 pretrained on ImageNet. Our compression configuration (A) achieves highest Top-1 and Top-5 accuracy at comparable level of compression.

4x and 5x. We see that total energy use can be reduced in half with less than 1% loss in accuracy. Thus, we verified that the compressed model not only runs faster, but also requires less energy when implemented in edge processors that operate with limited resources such as GPU memory and power. It should be noted that we did not implement our method in C++ but used the python based Pytorch instead. We believe that an optimized C++ cuda implementation can achieve even better performance on GPUs.

4. SUMMARY AND DISCUSSIONS

We have presented a straightforward yet effective method for compressing deep CNNs that outperforms most other state of the art techniques in terms of reduction in FLOPS and number of parameters with negligible performance loss. Despite the overwhelming interest in neural network compression in recent years, there is still a lack of standardized definitions for metrics such as FLOPs. Some methods choose to compress the network to arbitrary reduction in FLOPS and report the model’s accuracy, while others compress the network in such a way that accuracy of the resulting model is within a certain range of base model. The inference time of the compressed model is an important metric that should be considered, but introduces its own complexities due to dependencies on libraries and hardware platform. We also note that power utilization is an important consideration for edge processors, and should be included as a metric for comparing compressed networks.

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