Hadron Resonance Gas with Repulsive Interactions and Fluctuations of Conserved Charges

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Abstract

We discuss the role of repulsive baryon-baryon interactions in a hadron gas using relativistic virial expansion and repulsive mean field approaches. The fluctuations of the baryon number as well as strangeness-baryon correlations are calculated in the hadron resonance gas with repulsive interactions and compared with the recent lattice QCD results. In particular, we calculate the difference between the second and fourth order fluctuations and correlations of baryon number and strangeness, that have been proposed as probes of deconfinement. We show that for not too high temperatures these differences could be understood in terms of repulsive interactions.
I. INTRODUCTION

Fluctuations and correlations of conserved charges, e.g. baryon number \(B\), electric charge \(Q\) and strangeness \(S\) have been studied in lattice QCD for some time now. The reason is that they are sensitive probes of deconfinement and can also be used to calculate thermodynamic quantities at non-zero baryon density via Taylor expansion (see Refs. \[1,2\] for recent reviews and references therein). At sufficiently low temperatures QCD thermodynamics is expected to be fairly well described by a gas of non-interacting hadrons and hadron resonances, by so-called hadron resonance gas (HRG) model \[3\]. This picture naturally emerges from the S-matrix based relativistic virial expansion, where the interactions are manifested as the phase shifts of two particle scattering \[4–7\]. In pion-pion and pion-nucleon interactions the repulsive part associated with the negative phase shifts, is largely canceled by parts of the positive phase shifts associated with attractive interactions. The effect of the remaining attractive interactions on thermodynamics, can be well approximated as a contribution of free resonances with zero widths \[5\], although some differential observables may require explicit treatment of the interactions \[8\].

As the temperature increases, particle densities increase, and the virial expansion only up to second virial coefficient becomes less and less reliable. To establish the validity of the HRG model at temperatures close to the QCD transition temperature requires a detailed comparison with the results from lattice QCD. Early comparisons have been discussed in Refs. \[9–13\], where, however, large cutoff effects and/or unphysical quark masses made a detailed comparison difficult (see e.g. Ref. \[13\]). In the past several years the fluctuations and correlations of conserved charges have been studied on the lattice using stout and highly improved staggered quark (HISQ) actions, and physical quark masses \[14–27\]. These lattice formulations significantly reduce the cutoff effects. As the result the comparison between the lattice results and HRG have become straightforward. Second order fluctuations and correlations seem to agree reasonably well with the HRG model. However, higher order fluctuations show deviations from the HRG model close to the transition temperature. In Ref. \[18\] it was argued that the apparent breakdown of HRG when describing certain differences of fourth and second order fluctuations and correlations is a signal of deconfinement. On the other hand, it has been recently shown that the repulsive interactions modeled by excluded volume can have significant effect on thermodynamic observables, in particular on higher order fluctuations \[28–30\]. The role of repulsive interaction in the context of statistical hadronization has also been discussed, see e.g. Ref. \[31\].

The aim of this paper is to study the effect of repulsive baryon-baryon interactions using the S-matrix based virial expansion and the repulsive mean field approach. In this paper we will calculate the fluctuations and correlations of conserved charges defined as

\[
\chi^X_n = \left. T^n \frac{\partial^n p(T, \mu_X)}{\partial \mu_X^n} / T^4 \right|_{\mu_X = 0},
\]

\[
\chi^{XY}_{nm} = \left. T^{n+m} \frac{\partial^{n+m} p(T, \mu_X, \mu_Y)}{\partial \mu_X^n \partial \mu_Y^m} / T^4 \right|_{\mu_X = 0, \mu_Y = 0}.
\]

Here \(X = B, Q, S\), i.e. we consider fluctuations and correlations of conserved charges corresponding to baryon number, electric charge and strangeness. It may not be easy to disentangle the effects of repulsive interactions from other medium effects such as in-medium mass shift and broadening of width. Therefore it is useful to study the differences of fluctuations and correlations, which are not affected by the latter effects. In particular we evaluate
II. REPULSIVE INTERACTION IN NUCLEON GAS

First we would like to study the role of repulsive interactions in the gas of nucleons at temperature $T = 1/\beta$. The most natural way to do this is to consider the virial expansion. In this case the nucleon pressure can be written as

$$p(T, \mu) = p_0(T) \cosh(\beta \mu) + 2b_2(T)T \cosh(2\beta \mu). \quad (2.1)$$

Here

$$p_0(T) = \frac{4M^2T^2}{\pi^2}K_2(\beta M) \quad (2.2)$$

is the pressure of free nucleon gas at zero chemical potential and the second virial coefficient can be written as

$$b_2(T) = \frac{2T}{\pi^3} \int_0^\infty dE \left( \frac{ME}{2} + M^2 \right)K_2 \left( 2\beta \sqrt{\frac{ME}{2} + M^2} \right) \frac{1}{4i} \text{Tr} \left[ S \frac{dS}{dE} - \frac{dS^\dagger}{dE} S \right]. \quad (2.3)$$

with $S$ being the scattering S-matrix and $E$ is the kinetic energy in the lab frame. Furthermore, $M$ is the nucleon mass and $K_2(x)$ is the Bessel function of second kind. The nucleon-nucleon ($NN$) interactions break the simple factorisation of the pressure into temperature dependent and $\mu$-dependent parts. As the result $\chi_2^B - \chi_4^B$ is not zero as in the case of non-interacting HRG. Even if the effect of $NN$ interactions is small for the pressure when $\mu = 0$, it could be significant for higher order fluctuations as each derivative in $\mu$ will multiply $b_2$ by factor two. Because of the exponential suppression of $K_2(x)$ at large values of the argument, the virial coefficient $b_2$ is very small for the nucleon gas. Therefore, it makes sense to introduce the reduced virial coefficient

$$\bar{b}_2(T) = \frac{2Tb_2(T)}{p_0(T)K_2(\beta M)}. \quad (2.4)$$

The pressure can now be written as

$$p(T, \mu) = p_0(T)(\cosh(\beta \mu) + \bar{b}_2(T)K_2(\beta M) \cosh(2\beta \mu)). \quad (2.5)$$

To evaluate $b_2(T)$ we need to know the S-matrix for the $NN$ scattering. Through the partial wave analysis we have a good parametrisation of the elastic part of the S-matrix, however, the inelastic part of the S-matrix is not known. The inelastic channels open up for $E > 280$ MeV and become significant for $E > 400$ MeV, and their importance increases with the energy. We estimate $b_2(T)$ using the elastic part of the S-matrix and try to include the effects of the inelastic channel as a systematic uncertainty.

The elastic S-matrix is block diagonal with matrix elements $S_J$, that are $2 \times 2$ matrices for each value of angular momentum $J$. In the so-called BASQUE parametrisation $S_J$ has diagonal elements

$$S_{\pm} = \cos^2 \rho_\pm^J \cos 2\epsilon^J \exp(2i\delta_\pm^J) \quad (2.6)$$
corresponding to orbital angular momenta \( L = J \pm 1 \), and off-diagonal elements

\[
S_0 = i \cos \rho^I_+ \cos \rho^I_- \sin 2 \epsilon^I \exp(i(\delta^I_+ + \delta^I_- + \phi^I)).
\] (2.7)

Here \( \delta^I_\pm \) are the phase shifts corresponding to angular momentum \( J \). The parameters \( \rho^I_\pm \) describe the inelasticity of the collisions, while \( \epsilon^I \) and \( \phi^I \) are the elastic and inelastic mixing parameters of \( L = J \pm 1 \) states. For \( E < 280 \) MeV the parameters \( \rho^I_\pm \) and \( \phi^I \) are zero. In this case

\[
\frac{1}{4i} \text{Tr} \left[ S^I \frac{dS}{dE} - \frac{dS^{\dagger}}{dE} S \right] = \sum_{s=\pm} \sum_J (2J+1) \left( \frac{d\delta^I_{s=0}}{dE} + 3 \frac{d\delta^I_{s=1}}{dE} \right),
\] (2.8)

where we distinguish the isospin zero \((I = 0)\) and isospin one \((I = 1)\) channels in the nucleon-nucleon system. If the parameters \( \rho^I_\pm \) and \( \phi^I \) are different from zero, the above equation will become complex, leading to complex value of \( b_2(T) \), which is clearly unphysical. The reason for this problem is that \( S_J \) is not unitary. If the inelastic channels were included the unitarity would be restored, the imaginary terms in the above equation would drop out and the derivative of inelastic phase shift would appear. This is easy to see for the simple case when the S-matrix has one elastic and one inelastic channel \([7]\). In the following we will set the parameters \( \rho^I_\pm \) and \( \phi \) to zero and use Eq. \([2.8]\) for all energies to evaluate \( b_2 \).

In our numerical analysis we use the elastic phase shifts from the SM16 partial wave analysis \([33]\). We also use SP07 partial wave analysis \([34]\) as well as an old analysis from Ref. \([35]\). The differences arising from the use of different partial wave analyses are small compared to other uncertainties of our calculations. For \( E > 10 \) MeV the effects of Coulomb interactions are small, so the \( I = 1 \) phase shifts are taken from \( pp \) scattering data, while the \( I = 0 \) phase shifts are taken from the \( np \) scattering data. At lower energies the electromagnetic effects are important and there is a difference between \( pp \) phase shifts and \( I = 1 \) \( np \) phase shifts. Since in our study we do not include electromagnetic interactions for \( E < 10 \) MeV we use the phase shifts from \( np \) scattering data for both \( I = 0 \) and \( I = 1 \) channels. Here it is sufficient to consider the lowest partial waves \((^1S_0 \text{ for } I = 1 \text{ and } ^3S_1 \text{ for } I = 0)\). Finally to obtain the correct threshold behaviour we use effective range expansion for the S-wave \( np \) phase shifts: \( \cot \delta^I = -1/(a_{k1}) + r^0_{k1}/2 \), with \( a_{I=1} = -23.7 \) fm and \( r^0_{I=1} = 2.76 \) fm for \( I = 1 \) \([36]\), and \( a_{I=0} = 5.4194 \) fm and \( r^0_{I=0} = 1.7536 \) fm \([37]\). We checked that the effective range expansion with the above parameters matches smoothly to SM16 analysis for \( E \) of about few MeV. We note that there is a large cancellation between the contributions of \( I = 0 \) and \( I = 1 \) channels to \( b_2 \) at low energies. This is due to different sign of the scattering length \( a_{I} \) in these two channels and unnaturally large value of \( a_{I=0} \). At high energies the derivative of the sum of all the phase shifts is negative, which is reflective the repulsive hard core \( NN \) interactions.

Finally we need to estimate the uncertainty in \( b_2 \) due to the inelastic channels. For this we consider the ratio of the inelastic to total \( pp \) cross-section from SM16 partial wave analysis. The inelastic cross-section is very small for \( E < 400 \) MeV. For \( 400 \) MeV \( < E < 500 \) MeV the inelastic cross-section is about 10% of the total cross-section. For \( 500 \) MeV \( < E < 600 \) MeV, \( 600 \) MeV \( < E < 800 \) MeV and \( E > 800 \) MeV the inelastic cross-section is about 25%, 40% and 50% of the total cross-section, respectively. Therefore, we estimate that the uncertainty in \( b_2 \) that comes from the energy range \( 400 \) – \( 500 \) MeV, \( 500 \) – \( 600 \), \( 600 \) – \( 800 \) MeV and \( > 800 \) MeV is 20%, 50%, 80% and 100%, respectively. Here we tried to be conservative and assumed that the contribution of the (unknown) inelastic phase shifts to \( b_2 \) is by a factor two larger than to the total cross-section. Our numerical result for the reduced virial coefficient and its uncertainty is shown in Fig. 1.
So far we only considered nucleon-nucleon interactions. Nucleons can also interact with anti-nucleons. Much less is known about the interactions between the nucleons and anti-nucleons, but one may expect that these interactions are significant as well. Fortunately, the nucleon anti-nucleon interactions give a contribution to the pressure, which is independent of the chemical potential. Therefore, these interactions will not affect the fluctuations and correlations that is the main focus of this paper.

Another way to include the repulsive interaction is via a repulsive mean field. In this approach it is assumed that the repulsive interactions lead to shifts in the single particle energies by $U = K n_b$ and $\bar{U} = K \bar{n}_b$ for nucleons and anti-nucleons, respectively [38, 39]. Here $n_b$ and $\bar{n}_b$ are the densities of nucleons and anti-nucleons defined as

$$n_b = 4 \int \frac{d^3p}{(2\pi)^3} e^{-\beta(E_p - \mu + U)}, \quad \bar{n}_b = 4 \int \frac{d^3p}{(2\pi)^3} e^{-\beta(E_p + \mu + \bar{U})}, \quad E_p^2 = p^2 + M^2,$$

with $\mu$ being the chemical potential corresponding to the net nucleon density. We use Boltzmann approximation because the nucleon mass is much larger than the temperature. The phenomenological parameter $K$ characterises the strength of the repulsive interactions and can be related to the integral of the $NN$ potential over the spatial volume [38, 39].

The presence of the short distance repulsive core in the $NN$ potential implies that $K > 0$. Requiring, that $\partial p/\partial \mu$ should give the net nucleon density, i.e $n_b - \bar{n}_b$ one obtains the following expression for the pressure [38, 39]

$$p(T, \mu) = T(n_b + \bar{n}_b) + \frac{K}{2}(n_b^2 + \bar{n}_b^2).$$

In principle Eq. (2.9) should be solved self-consistently to obtain $n_b$ ($\bar{n}_b$). However, for temperatures below the QCD transition temperature $n_b$ ($\bar{n}_b$) is small, and for typical phenomenological values of $K$, e.g. $K = 450$ MeV fm$^3$ [40], $\beta U$ is small too. For example even for $T = 175$ MeV we get $\beta U = 0.077$. Therefore we can expand the exponential in the equations for $n_b$ and $\bar{n}_b$, and the factor $(1 + n_b)^{-1}$ and $(1 + \bar{n}_b)^{-1}$ when solving $n_b$ and $\bar{n}_b$, and write

$$n_b = n_b^0(1 - \beta Kn_b^0), \quad \bar{n}_b = \bar{n}_b^0(1 - \beta K\bar{n}_b^0),$$
with \( n_b^0 \) and \( \bar{n}_b^0 \) being the free nucleon and anti-nucleon densities. With this the pressure can be written in terms of \( n_b^0 \) and \( \bar{n}_b^0 \) as follows:

\[
p(T, \mu) = T(n_b^0 + \bar{n}_b^0) - \frac{K}{2} \left( (n_b^0)^2 + (\bar{n}_b^0)^2 \right).
\]

(2.12)

Taking into account that \( n_b^0 = 2TM^2/\pi^2K_2(\beta M)e^{\beta \mu} \) and \( \bar{n}_b^0 = 2TM^2/\pi^2K_2(\beta M)e^{-\beta \mu} \) we finally get

\[
p(T, \mu) = \frac{4T^2M^2}{\pi^2}K_2(\beta M) \cosh(\beta \mu) - 4K\frac{T^2M^4}{\pi^4}K_2^2(\beta M) \cosh(2\beta \mu)
\]

(2.13)

The structure of the above equation is very similar to the one obtained in the virial expansion. The correction to the free gas result is negative and the factorisation of the pressure in \( T \)-dependent part and \( \mu \) dependent part does not hold. Comparing the above result with the virial expansion result one can determine the value of \( K \) at some temperature. To estimate the relative size of the second term in the above equation we write

\[
p(T, \mu) = p_0(T)(\cosh(\beta \mu) - \frac{KM^2}{\pi^2}K_2(\beta M) \cosh(2\beta \mu)).
\]

(2.14)

Comparing this equation with Eq. (2.5) we see that \(-KM^2/\pi^2\) corresponds to the reduced virial coefficient \( \bar{b}_2(T) \). Therefore, in Fig. 1 we show this combination for the previously used phenomenological value \( K = 450 \text{ MeV fm}^3 \). At low temperatures \(-\bar{b}_2(T)\) is significantly smaller than \( KM^2/\pi^2 \). However, at the highest temperatures the two agree. We stress again that the smallness of \(-\bar{b}_2\) comes from the cancellation of positive and negative contributions in the \( I = 0 \) and \( I = 1 \) channels. Such cancellation is a somewhat accidental feature of the NN interactions and may not be present for other baryons. For these reasons we will use the value \( K = 450 \text{ MeV fm}^3 \) in what follows.

Finally, we note that the first quantum correction to the pressure of the nucleon gas is

\[-M^2T^2/\pi^2K_2(2\beta M) \cosh(2\beta \mu)\].

It has the same dependence on \( \mu \) as the contribution of repulsive interactions but is about 20 times smaller. Therefore, it will be neglected in the following analysis.

### III. REPULSIVE MEAN FIELD IN MULTI-COMPONENT HADRON GAS AND FLUCTUATIONS OF CONSERVED CHARGES

It is straightforward to generalise the above repulsive mean field approach to multi-component hadron gas. The baryon density is written as

\[
n_B(T, \mu_B, \mu_S, \mu_Q) = \frac{T}{2\pi^2} \sum_i g_i M_i^2 K_2(\beta M_i)e^{\beta \mu_i,\text{eff}},
\]

(3.1)

where \( M_i \) is the mass of the \( i^{th} \) baryon and \( g_i \) is the corresponding degeneracy factor. Furthermore, the effective chemical potential of the \( i^{th} \) baryon is given by

\[
\mu_{i,\text{eff}} = \sum_j q_{ij} \mu_j - Kn_B,
\]

(3.2)
with \((q_i^1, q_i^2, q_i^3) = (B_i, S_i, Q_i)\) being the baryon number, strangeness and electric charge of the \(i\)th baryon. Here we assumed that the repulsive interaction is the same for all baryons. This is clearly an oversimplification. While lattice calculations indicate that repulsive core in the central potential is similar for many baryon combinations (e.g. \(NN, \Lambda N, \Lambda \Lambda\), etc.), there are some differences \([41]\). The hyperon nucleon and hyperon-hyperon interactions have been studied also in chiral effective theory \([42, 43]\). It has been found that these interactions are dominantly repulsive but different from nucleon-nucleon interactions. However, we do not have sufficient information about baryon-baryon interactions to come up with a more sophisticated mean field model. Replacing \(\mu_{i, \text{eff}}\) in Eq. (3.1) by \(\bar{\mu}_{i, \text{eff}} = \sum_j q_i^j \mu_j - K \bar{n}_B\), we obtain the density of anti-baryons, \(\bar{n}_B\). Note that \(q_i^j = -q_i^j\). Expanding the exponential to leading order in \(K\) as in the previous section for the baryon and antibaryon densities, and again requiring that \(\partial p/\partial \mu_B = n_B - \bar{n}_B\), we obtain

\[
p_B(T, \mu_B, \mu_S, \mu_Q) = T(n_B^0 + \bar{n}_B^0) - \frac{K}{2} \left( (n_B^0)^2 + (\bar{n}_B^0)^2 \right) ,
\]

where \(n_B^0\) and \(\bar{n}_B^0\) are the free baryon and anti-baryon densities. The pressure of the free baryon gas can be decomposed into partial baryonic pressure of strangeness one, strangeness two, and strangeness three baryons, and the same is true for anti-baryons. Therefore, we write

\[
p_B(T, \mu_B, \mu_S, \mu_Q) = \tilde{p}_B(\mu_S, \mu_Q) e^{\beta \mu_B} + \tilde{p}_B(-\mu_S, -\mu_Q) e^{-\beta \mu_B}
\]

and

\[
\tilde{p}_B(\mu_S, \mu_Q) = p_{B^0}^S + p_{B^1}^S e^{-\beta \mu_S} + p_{B^2}^S e^{-2\beta \mu_S} + p_{B^3}^S e^{-3\beta \mu_S},
\]

and \(p_{B^k}^S\) denotes the contribution of \(S = -k\) baryons to the free pressure at zero chemical potentials. With this it is straightforward to get the baryon number fluctuations and baryon-strangeness correlations

\[
\chi_n^B = \chi_n^{B(0)} - 2^n \beta^4 K \left( N_B^0 \right)^2 , \quad (n \text{ even})
\]

\[
\chi_{n_1}^{B^S} = \chi_{n_1}^{B^S(0)} + 2^{n+1} \beta^5 K N_B^0 (p_{B^1}^S + 2p_{B^2}^S + 3p_{B^3}^S) . \quad (n \text{ odd})
\]

Here

\[
N_B^0(T) = \frac{T}{2\pi^2} \sum_i g_i M_i^2 K_2(\beta M_i)
\]

and the subscript ”\(^0\)” in the above equation refers to the non-interacting HRG.

In Ref. \([17]\) it was suggested that certain combinations of fluctuations and correlation of conserved charges can be used as indicators of deconfinement. In particular, the following two combinations

\[
\chi_3^B - \chi_1^B, \quad \text{and} \quad \chi_2^B - \chi_4^B
\]

have been suggested as measures of deconfinement in the light and strange hadron sectors, respectively. In non-interacting HRG these quantities are identically zero, while they have non-zero values for the ideal quark gas. The lattice results show that these quantities quickly rise above zero around the transition temperature and start approaching the ideal gas limit for \(T > 200\) MeV. This was interpreted as a transition from non-interacting hadron...
gas to quark gas \cite{17}. Therefore, it is interesting to see to what extent the increase in $\chi_{31}^{BS} - \chi_{11}^{BS}$, and $\chi_{2}^{B} - \chi_{4}^{B}$ around the transition temperature can be explained with the repulsive baryon interactions.

We calculated $\chi_{2}^{B} - \chi_{4}^{B}$, $\chi_{2}^{B} - \chi_{6}^{B}$ and $\chi_{31}^{BS} - \chi_{11}^{BS}$ in the HRG model with repulsive mean field using Eqs. (3.6) and (3.7). We considered only the contribution of ground state octet and decuplet baryons. The excited baryon states should appear as attractive (resonant) interactions in the hadron gas and thus, they are included in the non-interacting part of HRG. On the other hand, when resonances are interpreted as arising from attractive interactions, they lead to an increase in the density of ground state baryons \cite{6}. We leave creating a proper treatment of heavy resonances for a further study \cite{44}, and, as mentioned, concentrate here on the effects of ground state baryons and the lowest resonances.

As discussed before we use the value $K = 450$ MeV fm$^3$ in our numerical study. Our results are shown in Fig. 2 and compared with the lattice results obtained with HISQ action \cite{17,27} depicted with filled symbols. We also use the lattice results for $\chi_{2}^{B} - \chi_{6}^{B}$ obtained with stout action \cite{26} as well as continuum extrapolated results for $\chi_{2}^{B} - \chi_{4}^{B}$ from Ref. \cite{22}, depicted with open symbols. As expected the effect of the repulsive interactions is bigger for $\chi_{6}^{B}$ than for $\chi_{4}^{B}$. In our analysis so far we assumed that the density of baryons(anti-baryons) is small and therefore we kept the leading order term of the expansion in baryon density, i.e. the term proportional to $K$ (c.f. Eqs. (3.6) and (3.7)). As the temperature is increasing the number density of baryons and anti-baryons also increases and this expansion become less reliable. Therefore, we also calculated $\chi_{2}^{B} - \chi_{4}^{B}$ and $\chi_{2}^{B} - \chi_{6}^{B}$ using the unexpanded mean-field expressions and the results are shown in Fig. 2 as dashed lines. The difference between the expanded and un-expanded mean field results is significant at and above the crossover temperature. The full mean field result is below the lattice data. This problem could be cured by taking into account the effect of repulsive interactions for higher baryon resonances, although it is not clear to what extent the HRG model is reliable in this temperature region. Note, that using the full mean field result is more important for the higher order fluctuations and correlations than for the pressure since the effect of the repulsive interactions is enhanced by factor $2^n$ for the former (c.f. Eqs. (3.6) and (3.7)). In Ref. \cite{29} the decrease of $\chi_{4}^{B}/\chi_{2}^{B}$ from one was described in terms of HRG, where the repulsive interactions are modeled by excluded volume and good agreement with the lattice data was found. The increase in $\chi_{2}^{B} - \chi_{4}^{B}$ is equivalent to decrease of $\chi_{4}^{B}/\chi_{2}^{B}$ from unity, and thus our analysis confirms this result.

In Ref. \cite{17} another combination of strangeness fluctuations and baryon-strangeness correlation has been considered, which is identically zero in the ideal HRG and approaches the free quark gas value at very high temperature, namely

$$v_2 = \frac{1}{3}(\chi_2^S - \chi_4^S) - 2\chi_{13}^{BS} - 4\chi_{22}^{BS} - 2\chi_{31}^{BS}. \quad (3.10)$$

We calculated $v_2$ in our HRG model with repulsive mean field. We find that it has different sign depending on the value of $K$ and the temperature, while lattice calculation shows that $v_2$ is positive and monotonically increases with the temperature. So the simplest mean field approach with the same mean-field for all baryons cannot describe this quantity, and the differences in the repulsive baryon interactions in strange and non-strange baryons are important here. This is contrary to the difference $\chi_{31}^{BS} - \chi_{11}^{BS}$ where the repulsive interactions in the different strangeness sectors contribute with the same sign. To understand $v_2$ in the framework of the hadron gas with repulsive interactions more information on baryon-baryon interactions in different strangeness sectors will be needed.
FIG. 2. The differences $\chi_{31}^{BS} - \chi_{11}^{BS}$, $\chi_2^B - \chi_4^B$ and $\chi_2^B - \chi_6^B$ calculated in the HRG model with repulsive mean field (dotted, solid and dashed lines) and in lattice QCD. The filled symbols correspond to lattice calculations of $\chi_2^B - \chi_4^B$ and $\chi_2^B - \chi_6^B$ with HISQ action on $32^3 \times 8$ lattices [27]. The open symbols correspond to lattice results on $\chi_2^B - \chi_4^B$ [22] as well as to lattice results on $\chi_2^B - \chi_6^B$ [26]. For $\chi_{31}^{BS} - \chi_{11}^{BS}$ the lattice results from Ref. [17] are used. The dashed lines correspond to the unexpanded mean field result (see text).

We also calculated the baryon electric charge correlations $\chi_{31}^{BQ}$ and $\chi_{11}^{BQ}$ in the repulsive mean field approach. The results are similar to the case of $\chi_{31}^{BS}$ and $\chi_{11}^{BS}$. In particular, $\chi_{31}^{BQ} - \chi_{11}^{BQ}$ increases with increasing temperature and the repulsive interactions between different baryons contribute with the same sign. Our results agree with the preliminary lattice results.

IV. CONCLUSIONS

In this paper we discussed the role of repulsive baryon interactions on the thermodynamics and fluctuations of conserved charges of hadronic matter using relativistic virial expansion and repulsive mean field approach. We showed that the two approaches lead to almost identical results. In particular the reduced virial coefficient $b_2(T)$ shows only a mild temperature dependence and corresponds to the combination $KM^2/\pi^2$ appearing in the repulsive mean field approach. The deviations from ideal HRG for higher order fluctuations and correlations of conserved charges can be naturally explained by the repulsive interactions. We pointed out that it is useful to study the effect of repulsive interactions in terms of the following differences: $\chi_{31}^{BS} - \chi_{11}^{BS}$, $\chi_2^B - \chi_4^B$ and $\chi_2^B - \chi_6^B$ since the ideal hadron resonance gas part drops out and thus the results are not sensitive to the hadron spectrum. This makes it easy to disentangle the effects of repulsive interactions from other effects such as missing states [19] and in-medium modifications of hadron properties. The size of the deviations from the ideal gas limit for these differences obtained in the simple mean field model is similar to that observed on the lattice, though the former has large uncertainties at and above the QCD crossover temperature. However, not all strangeness baryon correlations can be under-
stood within our simple mean field approach due to the fact that baryon-baryon interactions are different in different strangeness sectors. Therefore, in the future it will be important to refine the treatment of the repulsive interactions of strange baryons using information from lattice QCD and chiral effective theory \[42, 43\] to obtain a better description of the fluctuations and correlation of conserved charges. Nevertheless, it is clear that HRG with repulsive interactions is a useful approach for studying the contribution of baryons to the thermodynamics of hadronic matter at zero and not too high baryon density. It was shown in Ref. \[29\] that including repulsive interactions by excluded volume affects the equation of state and fluctuations of conserved charges improve the agreement with the lattice data. Along similar lines we plan to study the QCD equation of state and fluctuations of conserved charges at zero and non-zero baryon density using HRG model with repulsive mean field \[44\] and perform detailed comparisons to the available lattice results. We hope that this study will also clarify the range of applicability of the mean field model.

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