Sky Subtraction with Fiber Spectrographs

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Received 1994 May 20; accepted 1994 August 16

ABSTRACT. The sky-subtraction performance of multifiber spectrographs is discussed, analyzing in detail the case of the OPTOPUS system at the 3.6-m ESO telescope at La Silla. A standard technique, based on flat fields obtained with a uniformly illuminated screen on the dome, provides poor results. A new method has been developed, using the [O I] emission line at 5577 Å as a calibrator of the fiber transmittance, taking into account the diffuse light and the influence of each fiber on the adjacent ones, and correcting for the effects of the image distortions on the sky sampling. In this way the accuracy of the sky subtraction improves from 2%–8% to 1.3%–1.6%.

1. INTRODUCTION

Spectrographs fed by optical fibers allow one to observe many (from tens to hundreds) objects simultaneously. The higher the spatial density of the targets, the larger the advantage over “classic” single-slit spectroscopy; thus a considerable effort has been spent in extending fiber spectroscopy to fainter and fainter objects. To this end and in view of the application of this technique to telescopes of the 8–10-m class, the limitations imposed by the accuracy in the sky subtraction have become a key issue.

Various papers (Elston and Barden 1989; Wyse and Gilmore 1992; Mignoli and Cuby 1994) have discussed this point with the following results.

1) An accuracy of the sky subtraction in the range 1%–2% is considered good in practice, while, on theoretical grounds, a limiting accuracy of a percent or better is expected to be achievable.

2) Such a limitation is due to focal-ratio degradation of the fibers, internal scattered light, sky variations across the field (caused by blue galaxies, airglow, etc.), telecentricity effects (Wynne 1993), contamination from adjacent fibers (cross talk), and poor determination of the fiber transmittance and of the wavelength calibration.

Section 2 describes a “standard” approach to the sky subtraction and how to measure the accuracy of the method. In Sec. 3 the importance of the various sources of noise is analyzed. Section 4 outlines the basics of a new sky subtraction method and presents the comparison with the results of Sec. 3. Finally, Sec. 5 deals with further problems regarding the extraction of the spectra.

2. ESTABLISHING THE ACCURACY OF THE SKY SUBTRACTION

2.1 The Instrument

In the following we will analyze the sky subtraction performances of a multifiber spectrograph in the case of the OPTOPUS system operating at the 3.6-m telescope of ESO La Silla, described by Avila and D’Odorico (1993) and in the ESO Users Manual (Schwarz and Melnick 1993). The instrument consists of 50 fibers, manually plugged in a previously drilled metal template and feeding a B. & C. grating spectrograph. Each fiber has a diameter of 320 μm, corresponding to 2.3 arcsec on the sky. A typical raw CCD frame obtained with OPTOPUS with a scale of 9 arcsec/pixel (resolution of 21 Å) is shown in Fig. 1.

There is a fundamental difference between long-slit and multifiber spectrographs: In the former, the sky spectrum is sampled on both sides of the target; in the latter some fibers are dedicated to the various targets, while the sky is sampled by a suitable number of different fibers, each with its own transmittance. In a multifiber spectrograph, in addition to the standard operations applied for the reductions of long-slit data, to evaluate the “true sky” it is necessary to determine the transmittance of each fiber and the influence of each fiber on the adjacent ones (cross talk).

Together with the “astronomical” frames, a few sets of calibration frames are obtained: bias and dark exposures, dome flat fields, internal-lamp flat fields, and wavelength calibration exposures. After the bias and dark correction of the frames, individual spectra are extracted integrating the flux along the columns of each channel (7 pixels wide) centered by Gaussian fitting of the transversal profile. For each frame this produces as many monodimensional spectra as the fibers in use.

2.2 The “Standard” Method

Following a “standard” approach, in order to determine the relative transmittance of the fibers we have used the fluxes recorded in the flat-field spectra, obtained with diffuse light on the dome (the results obtained with an internal lamp are definitely worse).

For each sky spectrum we have calculated

2 In the present observations a CCD ESO TEK 571×520, RON=8.8e−, GAIN=1.7e−/ADU, pixels of 27 μm was used.
3 The term “channel” in the following will indicate the subimage of the CCD frame containing the signal coming from a given fiber.

Based on observations collected at the European Southern Observatory, La Silla, Chile.

3 Telecentricity: Effect due to the misalignment between the axis of the optical fibers and the light beam axis coming from the telescope output pupil.
Fig. 1—A typical raw CCD frame obtained with OPTOPUS (in this case, frame C). Notice the 22nd fiber from the left, strongly exposed at the ambient light. The "anomalous" fiber discussed in the text lies immediately next to the right.

\[
\langle \text{SKY}_i \rangle = \frac{\langle \text{SKY spectrum}_i \rangle}{\langle FF_i \rangle},
\]

where, the symbol \(i\) indicates the channel (it identifies the spectrum and the fiber, too); the symbols \(\langle \text{SKY spectrum}_i \rangle\) and \(\langle FF_i \rangle\) are the values of the total flux into the channel between two given pixels \(y_1\) and \(y_2\) along the dispersion axis (e.g., 50–450) for the sky and the flat-field spectra, respectively; the symbol \(\langle FF \rangle\) is the average flux of the observed flat fields \(\langle FF_i \rangle\); and \(\langle \text{SKY}_i \rangle\) represents the sky flux estimated from the \(i\)th fiber.

We can average over all the \(\langle \text{SKY}_i \rangle\), obtaining \(\langle \text{SKY} \rangle\) and compute

\[
\langle E_{\text{sky}} \rangle_i = \frac{\langle \text{SKY}_i \rangle}{\langle \text{SKY} \rangle} - 1.
\]

We can adopt the root-mean-square of \(E_{\text{sky}}\) as an estimate of the accuracy of the sky subtraction. Such a procedure is implicitly based on a number of hypotheses.

(i) The shape of the spectral transmittance \(T(\lambda)\) of the fibers is the same for all the fibers:

\[
T(\lambda_i)/T(\lambda_j) = f(\lambda)
\]

for each fiber \(i\) and \(j\).

(ii) The influence (cross talk) of each fiber on the adjacent ones is negligible and there is no stray light into the spectrograph.

(iii) The variations of the sky due to blue galaxies and airglow are negligible.

The results obtained for some frames are shown in Table 1. Two frames for each field were obtained sequentially. The number of fibers dedicated to the sky varies between 14 and 36.

As listed in Table 1, the accuracy of the sky subtraction is greatly variable (2%–8%) from case to case, showing that the flat-field-based procedures provide typically poor and in any case unreliable results.

### 2.3 The “5577 Method”—An Improved Standard Method

In the spectral range accessible in our observations there are a number of sky emission lines, the most intense of which is the oxygen line \([O I]\) at 5577.4 \(\text{Å}\).

The \([O I]\) line has the necessary qualifications to be used as an estimator of the fiber transmittance. In fact:

1. its flux is sufficient to reduce the shot noise always below 0.6% and typically at 0.45% (with a telescope of 3.6 m, a dispersion of 9 \(\text{Å/pixel}\) and 40 min of exposure time);
2. at the faint magnitudes of interest in the present analysis; it is easily measurable both in the object and sky spectra;
3. it is subject to the same variations of the instrumental transmittance as the object flux (e.g., differential fiber flexures during the exposure, that should not, however, cause variations in the transmittance in a properly constructed system).

On the other hand there are also a few drawbacks:

4. it represents the fiber transmittance only at 5577 \(\text{Å}\);
5. it occupies only a small area on the CCD and consequently a "bad" pixel (a defective pixel, a cosmic-ray hit, etc.) can lead to a loss of accuracy;
6. its availability is not guaranteed for all the configurations of the spectrograph.

A subtle distinction between the "5577" and the "flat-field" methods has to be noted: While the variance of the latter is due only to errors in the calibration of the fiber transmittances, the variance of the former is due also to spatial sky variations. In this way the "5577" method potentially provides a more accurate sky subtraction.

To estimate the accuracy of the sky subtraction we simply replace the flat-field flux \(\langle FF \rangle\) with the \([O I]\) flux \(\langle F5577 \rangle\), evaluated by fitting the line profile along the dispersion direction with a suitable function (a Gaussian turns out to be good approximation), to limit the influence of possible bad pixels. The results are shown in Table 2.

The accuracy of the sky subtraction improves (to 2.1%–3.0%) except for frame C (shown in Fig. 1) where an "anomalous" fiber is present (see below); if this fiber is rejected, the accuracy becomes 3.2% for this frame, too. Above all the reliability of the results is greatly improved.


3. ANALYSIS OF THE RESULTS

We have investigated various hypotheses to justify the poor results provided by the “flat-field” method.

(1) Diffusion and reflections into the spectrograph.

(2) Cross talk among fibers.

(3) Change from fiber to fiber of the shape of the spectral transmittance $T(\lambda)$.

(4) Saturation and/or nonlinearity effects of the CCD.

(5) Unknown dimensional dependent factors: Possible correlations of the $E_{sky}$ with the number of the fiber $i$, with the position of the fibers on the focal plane of the telescope, “azimuth” on the OPTOPUS template.

None of the effects listed in items 3, 4, and 5 turned out to be sufficient to explain the improvement between the “flat-field” and the “5577” method (for example, Table 3 shows the effect of the variation of the transmittance as a function of the wavelength range estimated with dome flat fields; effects 4 and 5 have been found to be even smaller).

3.1 The Sky Concentration

Let us address the effects of internal diffusion and reflections: We have grouped these phenomena under the name sky concentration or, in the following, SC. To extract the SC it is useful to study particular frames denominated monofiber flats in which only one fiber has been exposed to the dome or internal-lamp light. By two-dimensional fitting with suitable functions it is possible to evaluate the SC contribution. Comparing a number of monofiber flats obtained exposing different fibers, it has been possible to deduce the following properties of the SC.

(1) It is independent of the particular fiber exposed.

(2) It has rotational symmetry.

Table 3

| Frame | Pixels range (pixels) | Wavelength range (Å) | RMS $S_{\text{sky}}$ (%) |
|-------|-----------------------|----------------------|--------------------------|
| A     | 50-150                | 4200-5100            | 8.9                      |
|       | 150-250               | 5100-6000            | 4.1                      |
|       | 250-350               | 6000-6800            | 3.8                      |
|       | 350-450               | 6800-7700            | 4.5                      |
| B     | 50-150                | 4200-5100            | 5.7                      |
|       | 150-250               | 5100-6000            | 5.4                      |
|       | 250-350               | 6000-6800            | 5.5                      |
|       | 350-450               | 6800-7700            | 5.9                      |

The possible improvement is negligible and the application of the flat-field method to the SC corrected frames confirms this deduction. On the contrary, applying the “5577”
procedure to the SC corrected frames, the sky subtraction accuracy improves to 1.6%–2.4% (Table 3).

3.2 The Cross-Talk Influence

After removing the SC from the monofiber flats, we have analyzed the extended broad tails in the transversal profile of the monofiber flats (Fig. 3). We have called transversal profile, or TP, a trace along a direction perpendicular to the direction of the dispersion (in our case approximately the same as the y axis).

A first-order argument as in the case of the SC shows that the improvement in accuracy on $E_{\text{Sky}}$ is negligible also in this case for frames with fibers exposed at roughly the same level.

We concluded therefore that the source of unreliability of the “flat-field” method has to be ascribed to telecentricity effects and/or nonuniformity of the flat-field light source.

4. THE INFLUENCE OF EACH FIBER ON THE ADJACENT ONES

What are the effects of the cross-talk (in the following point spread function background or PSFB) on the “5577” method?

The broadness of the TP obviously indicates that also the PSF of the instrument has long tails at low flux levels. In principle, it is possible to deconvolve the frame with a given PSF but problems arise (Brault and White 1971), due mainly to the type of sampling and to the boundary conditions required (the signal does not go to zero within the boundaries of the CCD). We have preferred to produce a simplified method by which we can subtract the cross-talk effect among the fibers. We assume that the PSF is a sum of functions of which only the first component (the main, PSF_0) is not responsible for the cross talk. We can write

$$\text{PSF}(r) = \text{PSF}_0(r) + \text{PSF}_1(r).$$

The PSF is assumed with a rotational symmetry. In fact, all the emission lines of the helium lamp can be fitted by a two-dimensional Gaussian with the same standard deviation ($\sigma_x = \sigma_y = 1.24 \pm 0.05$ pixels with the dispersion due mainly to differences among fibers), independent of the wavelength and the position of the channel on the CCD frame. In this case, the PSF can be written as

$$\text{PSF}(r) = \text{PSF}_0(r) + \text{PSF}_1(r).$$

The parameters of TP, and PSF_1 Gaussian functions are given in Table 4.
way we can make use of the classical convolution theorem (Andrews and Hunt 1977). The observed TP is the result of the convolution of the spectrum with the PSF:

$$TP(x) = \text{Spectrum} \ast \text{PSF}_{y=\text{constant}} = TP_0(x) + TP_1(x),$$

where $TP_0(x)$, $TP_1(x)$ are due to PSF$_{0}(r)$ and PSF$_{1}(r)$, respectively. It is useful to see that only the study of PSF$_0$ is immediately to the right of a strongly exposed fiber, whose

$$TP_1(x) = \sum_k H_k e^{-x^2/2\sigma_k^2},$$

where $H_k$ and $\sigma_k$ are the amplitude and the standard deviations of the Gaussian functions, respectively.

If we hypothesize a PSF$_1$ as sum of Gaussian functions, also $TP_1$ is a sum of Gaussian functions (as it is observed) with the same standard deviations.$^5$

$$\text{PSF}_1(r) = \sum_k A_k e^{-r^2/2\sigma_k^2}. $$

According to the convolution theorem, the PSFB expression in the pixel space is

$$F(x_p, y_p) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} PSF(t, u - y) J(y) \, dy,$$

where $J(y)$ is the flux distribution of the “true,” ideal spectrum, $x_p$, $y_p$ are the coordinates of the pixel $P$, and $F(x_p, y_p)$ represents the observed flux at the pixel $P$, which becomes with a first-order development (for $\sigma_k > 1$) and assuming the monofiber flat positioned at $x = 0$:

$$F(x_p, y_p) = \sum_k A_k e^{-x^2/2\sigma_k^2} \sum_i j(y_i) e^{-(y_p - y_i)^2/2\sigma_y^2},$$

where $j(y_i)$ is the observed flux of the spectrum at the pixel coordinate $y_i$.

For a given $y_p = \text{constant}$ we obtain a particular $TP_1(x_p)$. The coefficients $A_k$, $H_k$ are related by

$$A_k = \frac{H_k}{\sum_i j(y_i) e^{-(y_p - y_i)^2/2\sigma_y^2}}.$$

In practice only three Gaussian functions are necessary. In this way from the observed TP at a given $y_p$ it is possible to estimate by Gaussian fitting the $\sigma_k$ and the $H_k$ parameters (note that the $H_k$ depend on $y_p$) from which the $A_k$ (independent from $y_p$) are obtained (Table 4). Then it is possible to estimate the overall background due to the PSF$_1$ by convolving an artificial frame (in which the flux contained in each channel has been concentrated in one column corresponding to the $x$ position of the center of the channel), representing a rough approximation of a deconvolved image, with the PSF$_1$ (the main responsible for the cross talk). This background PSFB (Fig. 4) can now be subtracted from the SC corrected frame:

$$II = I - \text{PSFB},$$

where $II$ and $I$ represent, respectively, the final frame and the SC corrected frame.

The improvement of the accuracy (from 1.6%–2.4% to 1.2%–1.6%, see Table 2) depends strongly on the distribution of fluxes among the fibers of the original frame. The “anomalous” fiber in frame C turns out not to be “anomalous” any longer. As shown in Figs. 1 and 5 this fiber lies immediately to the right of a strongly exposed fiber, whose cross talk was responsible for the “anomaly.”

Unfortunately, the lack of information about the light flux outside the CCD frame causes a PSFB correction not entirely satisfactory, especially in the red part of the spectra. The variations of the sky subtraction accuracy as a function of the wavelength range are shown in Table 5.

### 5. THE EFFECTS OF IMAGE DISTORTIONS ON THE SKY SAMPLING

With the “5577” method some defects of the “standard” approach have been eliminated. However, the subtraction of the sky from the object spectrum leaves embarrassingly large oscillations in the observed flux in correspondence to strong emission lines and bands. This problem strongly affects the red part of many spectra, making them in practice unusable beyond 7000 Å. The usual spectra extraction procedure consists in the following steps:

(i) Spectra extraction, optimizing the signal-to-noise ratio, e.g., the procedure EXTRA/LONG in the MIDAS package (Banse et al., 1983).

(ii) Calibration in wavelength of the spectra (sky and object spectra), rebinning with a constant wavelength step.

(iii) Mean-sky evaluation.

(iv) Sky subtraction.

(v) Absolute flux calibration.

The second step partially removes the image distortions but introduces secondary effects: Not only the rebinning deteriorates the signal-to-noise ratio but also small errors in the wavelength calibration of each fiber give origin to large errors in the subtraction of the narrow sky emission lines. To overcome this problem we have developed a procedure

| Frame | Pixels range (pixels) | Wavelength range (Å) | RMS (%) |
|-------|----------------------|----------------------|---------|
| A     | 50-150               | 4200-5100            | 1.7     |
|       | 150-250              | 5100-6000            | 0.7     |
|       | 250-350              | 6000-6800            | 2.3     |
|       | 350-450              | 6800-7700            | 3.0     |
| B     | 50-150               | 4200-5100            | 2.3     |
|       | 150-250              | 5100-6000            | 1.7     |
|       | 250-350              | 6000-6800            | 2.3     |
|       | 350-450              | 6800-7700            | 3.2     |
| C     | 50-150               | 4200-5100            | 2.4     |
|       | 150-250              | 5100-6000            | 1.2     |
|       | 250-350              | 6000-6800            | 2.0     |
|       | 350-450              | 6800-7700            | 2.9     |
| D     | 50-150               | 4200-5100            | 1.9     |
|       | 150-250              | 5100-6000            | 0.6     |
|       | 250-350              | 6000-6800            | 1.6     |
|       | 350-450              | 6800-7700            | 1.9     |
which tries to map the image distortions in the pixel space. Using the Seidel distortion theory (Jenkins and White 1957):

\[ p = p_0 + E p_0^3 + o(5), \]

where, \( p \) is the distance of a point from the optical axis on the object plane; \( p_0 \) is the distance of a point from the optical axis on the image plane, \( E \) is the distortion coefficient of the optical system, and \( o(5) \) represents fifth-order terms in \( p_0 \) which are neglected in the third-order theory.

In an undistorted coordinate system the expression for a spectrum is

\[ X = \text{constant}. \]

Its expression, in a first-order development, in the distorted coordinate system (the observed one) is

\[ x = a_i + \beta_i y + \gamma_i y^2, \]

where

\[ \begin{align*}
\alpha_i &= Q_i + a - mb \\
\beta_i &= m - 2bQE \\
\gamma_i &= QE
\end{align*} \]

where \( Q_i \) are the \( x \) positions of the spectra at \( y = 0 \), \( m \) represents the tangent of the angle describing the misalignment of the \( y \) columns of the CCD with respect to dispersion direction, and \( a \) and \( b \) are the \( x \) and \( y \) coordinates of the optical axis on the CCD frame.

By measuring \( \alpha_i \), \( \beta_i \), and \( \gamma_i \) for each spectrum, we can solve the system and obtain the parameters \( a \), \( b \), \( m \), and \( E \) (Table 6).

The observed \( x \) and \( y \) coordinates of a given pixel are mapped on the corrected \( Y \) by

\[ Y = y + mx - E (y - b) [(x - a)^2 + (y - b)^2], \]

where \( mx \) represents the effect of the rotation (typically <0.1 pixel, see Table 6) and \( E (y - b) [(x - a)^2 + (y - b)^2] \) the effect of distortion (it can easily exceed 1 pixel). Only the \( Y \) coordinate is actually relevant because it represents the position along the dispersion direction of the resulting spectrum. After this correction the extracted spectra (monodimensional) show different dispersion scales and zero points, which can be estimated using the wavelength calibration exposures. The behavior of the dispersion scales is shown in Fig. 6.

To bring all the spectra to the same wavelength scale and zero point it is necessary to introduce a further correction:

\[ Y_{ci} = \text{coef}_i Y + q_i, \]

where \( Y_{ci} \) indicates the corrected coordinate for the spectrum \( i \) and \( \text{coef}_i \) and \( q_i \) are the dispersion scales and the zeropoint shift values of the \( i \) spectrum, estimated by com-
paring the positions of He-Ar emission lines with a reference spectrum. From Fig. 6 it is possible to deduce that the effects due to the different scales imply displacements of the order of half a pixel.

To determine the residual shifts, essentially due to differences between the optical path of astronomical observations and He-Ar calibrations (Fig. 7, bottom panel), we can use a few emission sky lines (e.g., the [O i] 5577, 6300 Å) and carry out the final correction:

\[ Y_{ei} = Y_{ci} + r_i, \]

where \( Y_{ei} \) are the final corrected coordinates for the spectrum \( i \)th and \( r_i \) is the shift of the \( i \)th spectrum.

After all these corrections the sampling of each spectrum is different from the other ones. The union of all the sky normalized spectra (the SKY, spectra) allows the building of an “oversampled” global sky spectrum. By interpolation and rebinning of this “oversampled” spectrum we obtain an estimate of the sky in the reference system of wavelengths of each object spectrum.

The advantages of this sky subtraction technique are the drastic reduction of the oscillations in the sky subtracted spectra in correspondence of narrow emission features and the preservation of the true signal to noise ratio and of the independence of the flux of each pixel, allowing also a correct estimate of the S/N.

Typical results are shown in Figs. 8 and 9.

The procedures described in Secs. 4 and 5 are illustrated in the flow chart of Fig. 10, representing a set of procedures developed in the framework of the MIDAS image processing system (Banse et al. 1983).

6. CONCLUSIONS

The poor and unreliable sky subtraction often observed when operating a multifiber spectrograph depends strongly on the method used to estimate the transmittance of the fibers. A method based on the observed flux of the 5577 [O i] line has been shown to allow a considerable increase in the accuracy of the sky subtraction.

Further improvements are obtained when the effects of the internal scattered light and the influence of each fiber on the adjacent ones are determined and corrected.
For the OPTOPUS spectrograph, this allows to reach a sky subtraction accuracy of 1.2%-1.6%.

To obtain the maximum performance from the instrument it is necessary to develop optimized extraction and subtraction procedures that take into account also the effects of the image distortions on the sky sampling.

The overall improvement is important in view of the application of multifiber instruments on telescopes of 6-10 m diameter (VLT, KECK, etc.). With low dispersion (~10 Å/pix) and reasonably long exposures (>~1 hr) the main limitation to the S/N already for telescopes of the 4-m class is due to the sky subtraction, if its accuracy is limited to the 5%-8% range. In such a situation it would not be useful in terms of limiting flux to apply multifiber spectrographs on bigger telescopes. The techniques described in this paper demonstrate that this limit can be significantly pushed down, making worthwhile the construction of multifiber spectrographs for very large telescopes.

We thank P. Andreani, G. Avila, and S. D’Odorico for enlightening discussions and suggestions.

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