$V - A$ Constraint on a Product of $R$-parity Violating Couplings

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Abstract

We study in the framework of $R$-parity violating supersymmetric theories the effect of $R$-parity violation due to the operator $L_i L_j E_k$ on the $(V - A)$ structure of the muon decay. The precisely measured muon decay parameters can constrain a product of $R$-parity violating couplings: $|\lambda_{232}\lambda_{131}| < 0.022$ at the 90% CL, which is complementary to the previous limits obtained by the $e-\mu$ universality in $\tau$ decay.
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I. INTRODUCTION

$R$-parity violation is introduced into the minimal supersymmetric standard model through additional terms in the superpotential:

$$W_{R} = \lambda_{ijk} L_i L_j E_k + \lambda'_{ijk} L_i Q_j D_k + \lambda''_{ijk} U_i D_j D_k + \mu_i L_i H_u,$$  \hspace{1cm} (1)

where $L, E, Q, U, D, H_u$ are the superfields, $i, j, k$ are family indices, and $\lambda, \lambda'$ and $\lambda''$ are the $R$-parity violating (RPV) couplings. The last term $\mu_i L_i H_u$ can be rotated away by a redefinition of the lepton field. (We neglect the effects of a possible soft-breaking bilinear term.) The operators $LLE$ and $LQD$ violate lepton number while $UDD$ violates baryon number. A prominent constraint coming from proton decay requires either $\lambda'$ or $\lambda''$ to be zero. Moreover, these RPV couplings would violate a number of existing data. The present limits on these RPV couplings are listed in recent reviews [1], where the limits are obtained by assuming only one nonzero coupling at a time. Constraints on products of RPV couplings have also been calculated for proton stability, lepton-family-number violating processes, and flavor-changing neutral current (FCNC) processes [2].

In this note, we are primarily interested in the $\lambda LLE$ term. We point out that the precise measurement on the $(V-A)$ structure in $\mu$ decay puts an additional constraint on a product of $\lambda$s, namely $\lambda_{131}\lambda_{232}$. As will be shown later, using the $e-\mu$ universality in $\tau$ decay to put limits on $\lambda_{33k}$ might run into the danger if several couplings coexist; in particular, when $|\lambda_{13k}|$ and $|\lambda_{23k}|$ are approximately equal their contributions to $R_\tau$ cancel. Thus, in this case the $e-\mu$ universality cannot effectively constrain the $\lambda$s; however, the constraint on the product of the two $\lambda$s, $\lambda_{131}\lambda_{232}$, from the $(V-A)$ structure in $\mu$ decay remains useful. Note that all previous constraints on products of RPV couplings come from FCNC processes or lepton-family-number violating processes; here the $(V-A)$ structure in $\mu$ decay does not involve any of these.

The $(V-A)$ structure has been tested in a number of processes, e.g., $\pi$ decay, $\tau$ decay, $\mu$ decay, of which the $\mu$ decay was measured to a very high precision. In the following, we will
use the µ decay parameters to constrain the product $|\lambda_{131}\lambda_{232}|$. We find that our new limit is complementary to the previous limit obtained using the e-µ universality in the $\tau$ decay: $\tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau$, $\tau^- \rightarrow \mu^-\bar{\nu}_\mu\nu_\tau$. We shall first describe the general structure of the µ decay. Next, we shall derive the effect of the RPV couplings and obtain the upper limit on the $\lambda$s. Finally, we comment on the constraint coming from the high energy process $e^+e^- \rightarrow \mu^+\mu^-$ and conclude.

II. MUON DECAY PARAMETERS

The muon decay and the inverse muon decay at low energy can be conveniently parameterized in terms of amplitudes $g^\gamma_{e\mu}$ and the Fermi Constant $G_F$, using the matrix element

$$\frac{4G_F}{\sqrt{2}} \sum_{\gamma=V,S,T} \sum_{\epsilon,\mu=L,R} g^\gamma_{e\mu} \langle \bar{e}_\epsilon | \Gamma^\gamma | (\nu_e)_n \rangle \langle (\nu_\mu)_m | \Gamma | \mu_\mu \rangle ,$$

where $\gamma = V, S, T$ denotes a vector, scalar, or tensor interaction, $\epsilon, \mu$ denote the chirality of the electron and muon, respectively, and the chiralities $n$ and $m$ of $\nu_e$ and $\nu_\mu$ are determined by $\gamma, \epsilon, \mu$. In the standard model, the $(V - A)$ requires $g^V_{LL} = 1$ and others equal zero. The rate, energy and angular distributions, and polarization can be affected by these $g^\gamma_{e\mu}$. In the rest frame of the muon, the energy and angular distribution is given by the Michel spectrum:

$$\frac{d^2\Gamma}{dx d\cos \theta} \sim \left\{ 3(1 - x) + \frac{2\rho}{3} (4x - 3) \mp \xi \cos \theta \left[ 1 - x + \frac{2\delta}{x} (4x - 3) \right] \right\} x^2 ,$$

where $\rho, \xi, \delta$ are functions of $g^\gamma_{e\mu}$. The measurements of $\rho, \xi, \delta$ can constrain various combinations of $g^\gamma_{e\mu}$. In order to determine the amplitudes $g^\gamma_{e\mu}$ uniquely, Fetscher et al. introduced four probabilities $Q_{e\mu}(\epsilon, \mu = L, R)$ for the decay of a µ-handed muon into a $\epsilon$-handed electron:

$$Q_{e\mu} = \frac{1}{4} \left| g^S_{e\mu} \right|^2 + \left| g^V_{e\mu} \right|^2 + 3(1 - \delta_{e\mu}) \left| g^T_{e\mu} \right|^2 .$$
The $Q_{LL}$ is constrained to be very close to unity, while others very close to zero. The current limits on $g_{\gamma \epsilon,\mu}$ are summarized in the Particle Data Book [4]. The ones that are relevant to our analysis are

$$|g^{S}_{RR}| < 0.066, \quad |g^{V}_{LL}| > 0.96$$

at the 90% CL.

**III. EFFECT OF $R$ PARITY VIOLATION**

With the term $\frac{1}{2} \lambda_{ijk} L_i L_j \overline{E}_k$ in the superpotential the Lagrangian is given by

$$\mathcal{L} = \lambda_{ijk} \left\{ \overline{e}^c_k \overline{E}_k \right\} \nu_i L + \overline{e}^c_k \nu_i L_j \overline{\nu}_j L - \overline{e}^c_k \nu_i L_{i} \overline{\nu}_j L_j \right\} + h.c.$$ (6)

There are two possible diagrams contributing to the muon decay. The first one is via an exchange of $\overline{\tau}_L$ and the amplitude is given by

$$\mathcal{L}_1 = -\frac{\lambda_{131} \lambda^*_{232}}{m_{\tau_L}^2} \left( \overline{\nu}_{\epsilon L} \right) \left( \nu_{\mu L} \mu_R \right).$$ (7)

This amplitude contributes to $g^{S}_{RR}$ as follows

$$\delta \left( g^{S}_{RR} \right) = -\frac{\sqrt{2}}{4G_F} \frac{\lambda_{131} \lambda^*_{232}}{m_{\tau_L}^2}.$$

Note that this contribution has a different helicity structure as the SM $(V - A)$ amplitude and, therefore, the experimental limit on the $(V - A)$ structure can effectively constrain the product $|\lambda_{131} \lambda_{232}|$. Using Eqs. (6) and (7) we obtain at the 90% CL, for $m_{\tau_L} = 100$ GeV,

$$|\lambda_{131} \lambda_{232}| < 0.022.$$ (9)

The second one is via an exchange of $\overline{e}_R, \overline{\mu}_R$, or $\overline{\tau}_R$. The amplitude is given by

$$\mathcal{L}_2 = -\sum_{k=1}^{3} \frac{\left| \lambda_{12k} \right|^2}{2m_{\ell_k}^2} \left( \overline{e}_L \gamma^\mu \nu_{\epsilon L} \right) \left( \overline{\nu}_{\mu L} \gamma_\mu \mu_L \right).$$ (10)

This amplitude contributes to $g^{V}_{LL}$.
\[ \delta \left( g_{LL}^V \right) = - \frac{\sqrt{2}}{4G_F} \sum_{k=1}^{3} \frac{|\lambda_{12k}|^2}{2m_{\tilde{\ell}_k R}^2}. \]  

(11)

This \( \mathcal{L}_2 \) has the same helicity structure as the SM \((V - A)\) amplitude and, therefore, the \((V - A)\) structure cannot constrain \(|\lambda_{12k}|^2\), but the total rate should be able to do so (similar to the analysis in [3].) However, it was shown [8] that the \(e - \mu - \tau\) universality is also able to constrain \(|\lambda_{12k}|^2\) to a very small value.

Recall that the previous constraints on \(\lambda_{13k}\) and \(\lambda_{23k}\) came from the \(e - \mu\) universality in \(\tau\) decay:

\[ R_{\tau} \equiv \frac{\Gamma(\tau \to e\bar{\nu})}{\Gamma(\tau \to \mu\bar{\nu})} = R_{\tau}^{SM} \left[ 1 + \frac{1}{2\sqrt{2}G_F} \sum_{k=1}^{3} \left( \frac{|\lambda_{13k}|^2}{m_{\tilde{\ell}_k R}^2} - \frac{|\lambda_{23k}|^2}{m_{\tilde{\ell}_k R}^2} \right) \right]. \]  

(12)

The constraint on each \(\lambda_{3k}\) was obtained from the experimental value of \(R_{\tau} = 1.0006 \pm 0.0103\) assuming only one \(\lambda\) nonzero at a time. The limit was \(|\lambda_{3k}| < 0.076\) at 90\% CL for \(m_{\tilde{\ell}_k R} = 100\) GeV and \(i = 1, 2\) and \(k = 1, 2, 3\). The danger of this limit can be seen from Eq. (12). When \(|\lambda_{13k}| \approx |\lambda_{23k}|\) their contributions to \(R_{\tau}\) cancel and, therefore, the limits on \(|\lambda_{3k}|\) are no longer valid. Physically, if the partial widths of the tau into muon and electron increased with the same amount, the \(e - \mu\) universality in the tau decay could not constrain the \(\lambda_s\).

The importance of our result in Eq. (9) can be appreciated immediately. Even in the scenario where \(|\lambda_{131}| \approx |\lambda_{232}|\) (the \(e - \mu\) universality in tau decay is not useful anymore) our result in Eq. (9) can constrain them effectively to be \(|\lambda_{131}| = |\lambda_{232}| < 0.15\) at 90\% CL. Of course, when \(\lambda_{131}\) is very different from \(\lambda_{232}\) the limit from \(e - \mu\) universality is more restrictive.

Actually, we can combine our result in Eq. (9), the limit from \(e - \mu\) universality in \(\tau\) decay, and the limit from \(e - \mu - \tau\) universality in \(\Gamma(\tau \to \mu\bar{\nu})/\Gamma(\mu \to e\bar{\nu})\) (which

\[ \text{In the reviews [1] only the 1\sigma results are given. In order for a direct comparison with our result we convert their limits to 1.65\sigma level, i.e., 90\% CL. In the PDB only 90\% CL upper bounds on } g_{1\mu}^{\gamma*} \text{ are listed.} \]
constrains $\sum[|\lambda_{12k}|^2 - |\lambda_{23k}|^2])$, as well as other constraints coming from the lepton-family-number violating processes and FCNC processes [2], e.g., $\tau \rightarrow 3e$ constrains $|\lambda_{121}\lambda_{123}|, |\lambda_{131}\lambda_{133}|, |\lambda_{231}\lambda_{121}|$, $\mu \rightarrow 3e$ constrains $|\lambda_{121}\lambda_{122}|, |\lambda_{131}\lambda_{132}|, |\lambda_{231}\lambda_{131}|$, etc. These constraints on products of two different $\lambda$s are complementary to the constraints obtained by the $e-\mu-\tau$ universality. In principle, we can perform a combined statistical analysis using all these constraints to find a global set of constraints on all these RPV couplings with correlations.

IV. DISCUSSIONS

There will be future experiments on measuring the muon decay parameters with better sensitivity. A planned experiment TRIUMF-E614 [8] is scheduled to run and will have a sensitivity of $\rho, \delta, \xi$ down to $10^{-4}$. Such sensitivity on $\rho, \delta, \xi$ will be able to pin $|g_{RR}^S|$ down to $10^{-2}$, which would then give the limit on $|\lambda_{131}| = |\lambda_{232}|$:

$$|\lambda_{131}| = |\lambda_{232}| \lesssim 0.06 .$$

(13)

The product $|\lambda_{131}\lambda_{232}|$ can also be constrained by high energy experiments at $e^+e^-$ and $\mu^+\mu^-$ colliders [9–11]. The Lagrangian of Eq. (6) also contributes to the process $e^+e^- \rightarrow \mu^+\mu^-$ via $s$-channel exchanges of $\tilde{\nu}_{\tau L}$ and $\tilde{\nu}_{\tau L}^*$ (depending on the coupling, there could also be $t$-channel diagrams). The inclusion of scalar tau-sneutrino exchanges will affect both the cross section and the forward-backward asymmetry in muon-pair production. The change in cross section due to $s$-channel resonance production is given by

$$\delta\sigma = \frac{|\lambda_{131}\lambda_{232}|^2}{32\pi} \frac{s}{(s - m_{\tilde{\nu}_{\tau L}}^2)^2 + \Gamma_{\tilde{\nu}_{\tau L}}^2 m_{\tilde{\nu}_{\tau L}}^2} .$$

(14)

If the product of $\lambda$s is of an appreciable size and the mass of the scalar tau-sneutrino is below the energy of the machine, the LEP2 and the future NLC experiments should be able to see a prominent peak by scanning over the center-of-mass energy (LEP2 has effectively done that due to the initial state radiation); otherwise, the null result should be able to constrain
the product of $\lambda$s. If the mass of the scalar tau-sneutrino is above the center-of-mass energy of the machine, only the effect from the tail of the scalar tau-sneutrino can be seen and, therefore, the limit on $\lambda$s is much weaker. The L3 collaboration has recently published the 90% CL upper limit on $|\lambda_{131}| = |\lambda_{232}| \lesssim 0.04$ for $m_{\tilde{\nu}_L} = 110 - 170$ GeV by measuring the cross section and the forward-backward asymmetry in muon-pair production \cite{10}. The future experiment at the NLC can probe heavier scalar tau-sneutrino with $|\lambda_{131}\lambda_{232}|$ down to $10^{-4}$ level, assuming $\Gamma_{\tilde{\nu}_L}/m_{\tilde{\nu}_L} \sim 1\%$ and an integrated luminosity of $\sim 50$ fb$^{-1}$.

To conclude we have obtained a limit on $|\lambda_{131}\lambda_{232}| < 0.022$ (or $|\lambda_{131}| = |\lambda_{232}| < 0.15$) at the 90% CL from the $(V - A)$ measurement in the muon decay. Although this limit is not as good as the previous limits $|\lambda_{13k}|, |\lambda_{23k}| < 0.076$ for $m_{\tilde{\nu}_{kR}} = 100$ GeV at 90% CL obtained by the $e-\mu$ universality in $\tau$ decay, our limit is, however, very useful for the case when $|\lambda_{131}| \approx |\lambda_{232}|$, in which case the $e-\mu$ universality is satisfied no matter how large the $\lambda$s are. It should be noted that the scenario of several coexisting $R$-parity violating couplings is more complicated than the case previously examined in the literature, and one should extract as much information as possible from the existing experiments.

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