Online Self-compensation for Enhanced the Scale Factor Stability of a Micromachined Gyroscope

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Abstract. In this paper, an online self-compensation control scheme for micromachined gyroscope has been presented to eliminate the scale factor drift due to temperature influence. Firstly, the error sources of scale factor have been analyzed. According the analysis results, a novel control scheme which contains three loops has been proposed: a phase-locked loop of driving mode is to drive the proof mass oscillation in its' resonant frequency, an AGC loop of driving mode is to keep a constant value of the drive amplitude, an additional scale factor error online detection and cancellation loop is to keep the scale factor stable. A digital hardware prototype has been implemented to perform the precision loop control and self-compensation loop. Scale factor of the gyroscope has been measured in a temperature-controlled turntable. Experiment results show that the scale factor drift is -3.5% to 5.2% over the temperature range of -45°C to +80°C without the self-compensation loop, while the scale factor drift decrease to -0.009% to 0.15% after the self-compensation loop is applied.

1. Introduction
The performance improvement of micromachined gyroscope is greatly depend on its' environmental adaptability, especially the temperature dependency of the bias and scale factor. The temperature drift error of a micromachined gyroscope may come from the defects of structure, difference of material properties, inner air pressure change, error of detection and control circuit etc. An effective way to reduce the influence of those errors is to apply precision close loop control on the movement of gyroscope sensing elements. Although the errors from forward channel can be eliminated, the changes out of the loop still influence the stability of the bias and scale factor of a gyroscope. Considering the test cost, temperature compensation for scale factor is more expensive than the same for bias. According the performance of the available MEMS gyroscopes from commercial market, the scale factor drift over temperature range is normally large than 1% (typical value). The application range of those devices will be limited without temperature compensation of scale factor. In this sense, it is quite important to develop a low cost way to greatly improve the scale factor stability [2, 3].

2. System model and scale factor error analysis

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The dynamic model of a micromachined Coriolis vibratory gyroscope can be described as a two-freedom oscillation mass-spring-damp system [1]. As shown in Fig. 1, $O_{x_a}y_a$ is inertial reference coordinate system and $O_{x_b}y_b$ is vector coordinate system.

![Figure 1. Dynamic model of a micromachined gyroscope.](image)

The ideal time-domain model of gyroscope can be expressed by

\[
\begin{align*}
\ddot{x}(t) + \frac{1}{Q_1} \omega_{n_1} \dot{x}(t) - 2 \dot{\theta} \ddot{y}(t) - \ddot{\theta} y(t) - \dot{\theta}^2 x(t) + \omega_{n_1}^2 x(t) &= f_{ex}(t) \\
\ddot{y}(t) + \frac{1}{Q_2} \omega_{n_2} \dot{y}(t) + 2 \dot{\theta} \ddot{x}(t) + \ddot{\theta} x(t) - \dot{\theta}^2 y(t) + \omega_{n_2}^2 y(t) &= f_{ey}(t)
\end{align*}
\]  

(1)

where $x(t)$ and $y(t)$ are the displacement of drive mode (primary mode) and sense mode (secondary mode) respectively. $Q_1$ and $Q_2$ are the quality factors of two modes. $\omega_{n_1}$ and $\omega_{n_2}$ are the natural frequency of two modes. $\theta$ is the rotation angle. $f_{ex}(t)$ and $f_{ey}(t)$ are generalized electro-static force of drive mode and sense mode respectively.

In general case, a micromachined gyroscope is always working in primary close loop mode which the sensing mass is driven in its’ resonant frequency of drive mode and the oscillation amplitude is equal to desired value $A_0$. In the primary close loop mode, the oscillation amplitude of primary mode is only influenced by the error of detection circuit. Therefore the oscillation of primary can be expressed by

\[
x(t) = A_0(1 + k_{e_1}) \sin \omega_{n_1} t
\]  

(2)

where $k_{e_1}$ is the gain error of primary detection circuit.

In equation (1), assume that $2m\dot{\theta} \ddot{y}(t)$, $m\dot{\theta} \dot{y}(t)$, $m\dot{\theta}^2 x(t)$, $m\dot{\theta} \dot{x}(t)$, $m\dot{\theta}^2 y(t)$ are small enough and can be neglected. The error of secondary detection circuit is also taken into account. Then the scale factor of a micromachined gyroscope can be expressed by

\[
SF\bigg|_0 = \frac{\frac{2(1 + k_{e_1})(1 + k_{e_2})A_0 \omega_{n_1}}{\sqrt{(\omega_{n_2}^2 - \omega_{n_1}^2)^2 + (\omega_{n_2} \omega_{n_1} / Q_2)^2}}}{(1 + k_{e_1})A_0 \omega_{n_1}}
\]  

(3)

where $k_{e_2}$ is the gain error of secondary detection circuit. $SF\bigg|_0$ is the scale factor in secondary open loop mode.

As the stiffness of structure is alternated by environment temperature due to the temperature dependency of Young’s modulus, the nature frequency $\omega_{n_1}$ and $\omega_{n_2}$ are also changed with temperature.
change. The gain error $k_{e_1}$ and $k_{e_2}$ of detection circuit are influenced by the temperature dependency of the electronic components and the parasitic parameters. The temperature dependency of gas damping (also influenced by air pressure \cite{5,6}) and structure material damping can influence the quality factor $Q_1$ and $Q_2$. All the factors show that scale factor of a micromachined gyroscope is strongly related to environment temperature. In this case, a secondary close loop (also called force rebalance loop \cite{4}) can be introduced to reduce the temperature influence of $Q_2$, $k_{e_2}$ and $\omega_{n_2}$. In force rebalance loop mode, the scale factor of a gyroscope can be simply expressed by

$$SF_c = 2(1 + k_{e_1})A_0\omega_{n_1}$$  \hspace{1cm} (4)

3. **Principal of online compensation for scale factor**

According equation (4), when a gyroscope is working in primary close loop and force rebalance loop mode, the main errors of scale factor come from temperature drift of the drive detection circuit and nature frequency $\omega_{n_1}$. The key of online self-compensation is to construct a reference signal which can observe the drift of Yong’s modulus and the detection circuit error. In simplified model, the movement of drive mode can be considered as a second order system. In this case, a reference sine signal is added to drive mode to force the sensing mass oscillation. Therefore the errors of scale factor can be observed. Assume expression of the reference oscillation is $x_r(t) = A_r \sin \omega_r t$, where $A_r$ is the amplitude of reference sine oscillation and $\omega_r$ is the frequency of reference sine oscillation. Under the important assumption that the gain error of detection circuit to signals with frequency of $\omega_r$ and $\omega_{n_1}$ is equal, then the amplitude of reference signal can be expressed by

$$A_r = \frac{(1 + k_{e_1})K_{D_{DRV}}V_T}{\sqrt{\left(\omega_{n_1}^2 - \omega_r^2\right)^2 + \left(\frac{\omega_{n_1}^2}{\omega_{n_1}^2 + Q_1^2}\right)^2}}$$  \hspace{1cm} (5)

where $V_T$ is the reference drive voltage. $K_{D_{DRV}}$ is the conversion coefficient between drive voltage and drive force.

In a small area for similar and assuming $\omega_r \ll \omega_{n_1}$ and $Q>50$, the expressions of $A_r$ versus the temperature dependency of Yong’s modulus and quality factor can be expressed by

$$A_r(T)_{\xi_1} \approx \left[1 + \kappa_{\xi_1}(T - T_0)\right] \frac{(1 + k_{e_1})K_{D_{DRV}}V_T}{\omega_{n_1}^2(T_0)}$$  \hspace{1cm} (6)

$$A_r(T)_{\xi_3} \approx \left[1 - \kappa_{\xi_3}(T - T_0)\right] \frac{\omega_r^2(1 + k_{e_1})K_{D_{DRV}}V_T}{\omega_{n_1}^2Q_1^2(T_0)}$$  \hspace{1cm} (7)

where $\kappa_{\xi_1}$ and $\kappa_{\xi_3}$ are the temperature coefficients of material Yong’s modulus and system damping separately. $A_r(T)_{\xi_1}$ and $A_r(T)_{\xi_3}$ are the amplitude expressions versus the temperature drift of Yong’s modulus and damping separately.

Generally the gain error introduced by temperature dependency of damping is far small than the gain error introduced by temperature dependency of Yong’s modulus, i.e. $A_r(T)_{\xi_3} \ll A_r(T)_{\xi_1}$. In this way, the amplitude’s temperature drift of reference signal can be mainly expressed by equation (6) which shows the gain error of reference signal is directly proportion to the change of Yong’s modulus and error of detection circuit. Therefore the amplitude change of the reference signal can be used to observe the temperature drift of scale factor.
The scheme of online self-compensation in drive mode is shown in figure 2. Beside the primary traditional control loop which is composed of phase-locked loop and AGC loop, the scheme has an additional online self-compensation loop which includes the following 3 steps:

- Add a reference signal into the normal drive signal.
- Derive the amplitude of reference response signal through the phase sensitive detector by using reference signal and drive sense signal.
- According to the amplitude of reference response signal to adjust the desired value of AGC loop.

Figure 2. Schematic of Online Self-Compensation Loop.

4. Testing and results
As shown in figure 3, a micromachined gyroscope with digitalized control circuit has been implemented to verify this online self-compensation method. The gyroscope is placed inside a temperature-controlled turntable.

Figure 3. A testing sample of digital micromachined gyroscope.

The scale factor has been measured through -45°C to +80°C. The test result without compensation is shown in figure 4. The result shows that the scale factor change is about -3.5% to 5.2% over full temperature range without compensation. The scale factor with online self-compensation is shown in figure 5. It shows that the scale factor change is only -0.009% to 0.15%. The noise performance has also been measured. Without compensation the gyroscope output noise level is about 6.0°/h/√Hz while the noise level with self-compensation is about 6.1°/h/√Hz.
5. Conclusions
An online self-compensation scheme for enhanced the scale factor stability has been developed. The effectiveness of this method has been verified by a digital micromachined gyroscope. Experiment results show that the self-compensation scheme tremendously reduce the temperature drift of gyroscope. The temperature hysteretic effect of output caused by temperature gradient is also significantly inhibited due to the compensation loop works in real-time. Additionally, due to the whole loop of self-compensation is implemented by using digital circuit, signal generation and algorithm calculation can be achieved without additional hardware cost. This self-compensation technique has been proved to be a no-cost and highly efficient way to improve the stability of scale factor. Future work will focus on the technology of bias compensation.

References
[1] Shkel Andrei M, Horowitz Roberto, Seshia Ashwin A, Park Sungsu and Howe Roger T 1999 Proceedings of the 1999 American Control Conference. 3 2119
[2] Link T, ISimon, Trächtler M, Gaisser A, Braxmaier M, Manoli Y and Sandmaier H 2005 TRANSDUCERS '05. 1 401
[3] Gaisser Alexander, Gao Zhongyu, Zhou Bin, Zhang Rong and Chen Zhiyong 2006 J. Tsinghua Science and Technology 11 533
[4] Sung Woon-Tahk, Sung Sangkyung, Lee Jang Gyu and Kang Taesam 2007 Journal of Micromechanics and Microengineering. 17 1939

[5] Acar Cenk, Shkel Andrei M, Costlow Lynn and Madni Asad M 2005 Proceedings of IEEE Sensors. 2005 664

[6] Kulygin A, Schmida U and Seidel H 2007 J. Sensors and Actuators A: Physical. 145-146 52