On the Optimality and Predictability
of Cultural Markets with Social Influence

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Abstract

Social influence is ubiquitous in cultural markets, from book recommendations in Amazon, to song popularities in iTunes and the ranking of newspaper articles in the online edition of the New York Times to mention only a few. Yet social influence is often presented in a bad light, often because it supposedly increases market unpredictability.

Here we study a model of trial-offer markets, in which participants try products and later decide whether to purchase. We consider a simple policy which ranks the products by quality when presenting them to market participants. We show that, in this setting, market efficiency always benefits from social influence. Moreover, we prove that the market converges almost surely to a monopoly for the product of highest quality, making the market both predictable and asymptotically optimal. Computational experiments confirm that the quality ranking policy identifies “blockbusters” in reasonable time, outperforms other policies, and is highly predictable.

These results indicate that social influence does not necessarily increase market unpredictability. The outcome really depends on how social influence is used.

1 Introduction

Social influence is ubiquitous in cultural markets. From book recommendations in Amazon, to song popularities in iTunes, and article rankings in the online version of the New York Times, social influence has become a critical aspect of the customer experience. Yet there is considerable debate about its benefits and its effects on the market.

In their seminal paper [13], Salganik, Dodds, and Watts argued that social influence makes markets more unpredictable, providing an explanation about why prediction in cultural markets is a wicked problem. To investigate this hypothesis experimentally, they...
created an artificial music market called the MusicLab. Participants in the MusicLab were presented a list of unknown songs from unknown bands, each song being described by its name and band. The participants were divided into two groups exposed to two different experimental conditions: the independent condition and the social influence condition. In the first group (independent condition), participants were provided with no additional information about the songs. Each participant would decide which song to listen to from a random list. After listening to a song, the participant had the opportunity to download it. In the second group (social influence condition), each participant was provided with an additional information: The number of times the song was downloaded by earlier participants. Moreover, these participants were presented with a list ordered by the number of downloads. Additionally, to investigate the impact of social influence, participants in the second group were distributed in eight “worlds” evolving completely independently. In particular, participants in one world had no visibility about the downloads and the rankings in the other worlds.

The MusicLab is a trial-offer market that provides an experimental testbed for measuring the unpredictability of cultural markets. By observing the evolution of different worlds given the same initial conditions, the MusicLab provides unique insights on the impact of social influence and the resulting unpredictability. In particular, Salganik et al suggested that social influence contributes to both unpredictability and inequality of success, with follow-up experiments confirming these initial findings [15, 14, 11, 18] (see also [5] for a perspective on the MusicLab paper).

In recent work [2], Abeliuk et al reconsidered the MusicLab setting by exploiting the generative model that reproduces the experimental data [6]. They showed that social influence always benefits market efficiency, if participants are presented with the optimal ranking, i.e., the ranking that maximizes the probability of a download at each step. Computational experiments also showed that the optimal ranking significantly outperforms the ranking in which products are displayed by decreasing popularity (known as popularity ranking).

In this paper, we analyze the properties of a simple ranking policy for trial-offer cultural markets: the quality ranking that simply orders the products by quality. We show a number of positive results about the quality ranking, including the fact that social influence always benefits market efficiency when it is used and that the benefits of social influence and position bias [8] are cumulative. Most importantly, we show that, under the quality ranking, the market almost surely converges to a monopoly, assigning the entire market share to the product of highest quality. As a corollary, this implies that the market is optimal asymptotically. These results are robust and do not depend on specific values for the appeal and quality of the products. Moreover, computational experiments using the generative model of the MusicLab [6] indicate a fast convergence to the asymptotic predictions: The quality ranking quickly assigns the largest proportion of the market share to the product of highest quality and outperforms other ranking policies. Also, there is only minor unpredictability in the market under the quality ranking policy.
Our results provide a sharp contrast with the conclusions in [13]. Under the quality ranking policy, the market is entirely predictable: It converges to “blockbusters” and this convergence is due to social influence. In other words, it is not social influence that makes markets unpredictable: Unpredictability depends on how social influence is used.

It is also important to emphasize that the benefits of popularity, the social influence signal used in this paper, have been validated experimentally. Indeed, two recent studies [4, 19] were conducted to understand the relative importance of the different social signals on consumer behaviour. The first study [4] surveyed the downloads of more than 500,000 from the Android marketplace Google play, while the second study [19] conducted an online experiment (n = 168) where participants were asked to rank hotels based on the number of reviews and the average rating. Both studies arrived at the same conclusion, namely that the popularity signal (i.e., social preference) has a much stronger impact than the rating signal (i.e., a state preference). These two studies provide evidence that quantitative models with popularity as the social signal are of significant importance.

The relationships between quality, popularity, and appeal have also been studied experimentally. Stoddard [16] studied the relationship between the intrinsic article quality and its popularity in the social news sites Reddit and Hacker news. The author proposed a Poisson regression model to estimate the demand for an article based upon its quality, past views and age among others. The results obtained after an estimation of each intrinsic article from these social news site showed that the most popular articles are typically the articles with the highest quality. Another study of social influence was carried out by [17]. The authors conducted a field experiment which showed that popularity information may benefit products with narrow appeal significantly more than those with a broad appeal.

The rest of this paper is organized as follows. Section 2 introduces trial-offer markets and Section 3 surveys the ranking policies used in this paper. Section 4 presents the main properties of the quality ranking, including the benefits of position bias and social influence and the asymptotic convergence. Section 5 describes the computational experiments and Section 6 presents the final discussion. All the proofs are in the appendix.

2 Trial-Offer Markets

This paper considers a trial-offer market in which participants can try a product before deciding to buy it. Such models are now pervasive in online cultural markets (e.g., books and songs). The market is composed of n products and each product i ∈ {1, . . . , n} is characterized by two values:

1. Its appeal A of which represents the inherent preference of trying product i;

2. Its quality q which represents the conditional probability of purchasing product i given that it was tried.
Each market participant is presented with a product list $\pi$: she then tries a product $s$ in $\pi$ and decides whether to purchase $s$ with a certain probability. The product list is a permutation of $\{1, \ldots, n\}$ and each position $p$ in the list is characterized by its visibility $v_p > 0$ which is the inherent probability of trying a product in position $p$. Since the list $\pi$ is a bijection from positions to products, its inverse is well-defined and is called a ranking. Rankings are denoted by the letter $\sigma$ in the following, $\pi_i$ denotes the product in position $i$ of the list $\pi$, and $\sigma_i$ denotes the position of product $i$ in the ranking $\sigma$. Hence $v_{\sigma_i}$ denotes the visibility of the position of product $i$.

This paper examines a number of questions about these markets, including

- What is the best way to allocate the products to positions?
- Is it beneficial to display a social signal, e.g., the number of past purchases, to customers?
- Is the market predictable?

The primary objective is to maximize the market efficiency, i.e., the expected number of purchases. Note also that the higher this objective is, the lower the probability that consumers try a product but then decide not to purchase it. Thus, if we interpret this last action as an inefficiency, maximising the expected efficiency of the market also minimizes unproductive trials.

**Static Market** In a static market, the probability of trying product $i$ given a list $\sigma$ is

$$p_{i}(\sigma) = \frac{v_{\sigma_i} A_i}{\sum_{j=1}^{n} v_{\sigma_j} A_j}$$

and the static market optimization problem consists of finding a ranking $\sigma$ maximizing the expected number of purchases, i.e.,

$$\max_{\sigma \in S_n} \sum_{i=1}^{n} p_{i}(\sigma) \ q_i \tag{1}$$

where $S_n$ represents the symmetry group over $\{1, \ldots, n\}$.

Observe that consumer choice preferences for trying the products are essentially modelled as a discrete choice model based on a multinomial logit\textsuperscript{[9]} in which product utilities are affected by their position. Abeliuk et al.\textsuperscript{[1]} studied a generalization of the maximization problem\textsuperscript{[1]} where it is not required to show all products to consumers. The authors interpreted the problem as a optimal assortment problem with position-bias and they proved that it can be solved in polynomial time.
Dynamic Market with Social Influence  The paper also considers a dynamic market where the appeal evolves over time according to a social influence signal. Given a social signal \( d = (d_1, \ldots, d_n) \), the appeal of product \( i \) becomes \( A_i + d_i \) and the probability of trying product \( i \) given a list \( \sigma \) becomes

\[
p_i(\sigma, d) = \frac{v_{\sigma_i}(A_i + d_i)}{\sum_{j=1}^n v_{\sigma_j}(A_j + d_j)}.
\]

The dynamic market uses the number of purchases \( d_{i,t} \) of product \( i \) at a time \( t \) as the social signal. Hence, in a dynamic market with \( T \) steps, the expected number of purchases is defined by the following recurrence:

\[
u_t(d) = \max_{\sigma \in S_n} \sum_{i=1}^n P_i(\sigma, d)(q_i(1 + u_{t+1}(d_{i \leftarrow d_{i} + 1}))) + (1 - q_i)u_{t+1}(d)) \quad (t \in 1..T)
\]

\[
u_{T+1}(d) = 0
\]

where \( d[i \leftarrow v] \) represents the vector \( d \) where element \( i \) has been replaced by \( v \). The value \( u_1((0, \ldots, 0)) \) denotes the optimal expected profit over the time horizon.

Observe that the probability of trying a product depends on its position in the list, its appeal, and its number of purchases at time \( t \). As the market evolves over time, the number of purchases dominates the appeal of the product and the trying probability of a product becomes its market share. Note also that a dynamic market with no social influence simply amounts to solving the static market optimization problem repeatedly. This is called the independent condition in the rest of the paper.

In the following, without loss of generality, we assume that the qualities and visibilities are non-increasing, i.e.,

\[
q_1 \geq q_2 \geq \cdots \geq q_n
\]

and

\[
v_1 \geq v_2 \geq \cdots \geq v_n.
\]

We also assume that the qualities and visibilities are known. In practical situations, the product qualities are obviously unknown. However, the quality of the products can be recovered accurately and quickly, either before or during the market execution \footnote{[2]}. We use

\[
a_{i,t} = A_i + d_{i,t}
\]

to denote the appeal of product \( i \) under social influence at step \( t \). When the step \( t \) is not relevant, we omit it and use \( a_i \) instead for simplicity. Finally, also for simplicity, we sometimes omit the range of indices in aggregate operators when they range over the products.
3 Rankings Policies

This paper focuses on the quality ranking which simply orders the products by quality, assigning the product of highest quality to the most visible position and so on. In other words, with the above assumptions, a quality ranking $\sigma$ satisfies $\sigma_i = i$ ($1 \leq i \leq n$).

The quality ranking contrasts with the popularity ranking which was used in [13] to show the unpredictability caused by social influence in cultural markets. At iteration $k$, the popularity ranking orders the products by the number of purchases $d_{i,k}$. Note that purchases do not necessarily reflect the inherent quality of the products, since they depend on how many times the products were tried.

The performance ranking was proposed in [2] to show the benefits of social influence in cultural markets. The performance ranking maximizes the expected number of purchases at each iteration, exploiting all the available information globally, i.e., the appeal, the visibility, the purchases, and the quality of the products. More precisely, the performance ranking at step $k$ produces a list $\sigma_k^*$ defined as

$$\sigma_k^* = \arg\max_{\sigma \in S_n} \sum_{i=1}^{n} p_i(\sigma, d_k) \cdot q_i$$

where $d_k = (d_{1,k}, \ldots, d_{n,k})$ is the social influence signal at step $k$. The performance ranking uses the probability $p_i(\sigma)$ of trying products $i$ at iteration $k$ given ranking $\sigma$, as well as the quality $q_i$ of product $i$. The performance ranking can be computed in strongly polynomial time and the resulting policy is scalable to large markets [2].

In the rest of this paper, Q-RANK, D-RANK, and P-RANK denote the policies using the quality, popularity, and performance rankings respectively. The policies are also annotated with SI or IN to denote whether they are used under the social influence or the independent condition. For instance, P-RANK(SI) denotes the policy that uses the performance ranking under the social influence condition, while P-RANK(IN) denotes the policy using the performance ranking under the independent condition. We also use RANK-RANK to denote the policy that simply presents a random order at each period.

Under the independent condition, the optimization problem is the same at each iteration as mentioned earlier. Since the performance ranking maximizes the expected purchases at each iteration, it dominates all other policies under the independent condition.

**Proposition 3.1** (Optimality of Performance Ranking). P-RANK(IN) is the optimal policy under the independent condition.

The following example shows that the performance ranking is not optimal in the dynamic market.

**Example 3.1.** Consider the following instance with 3 products:

- **Visibilities:** $v_1 = 0.7$, $v_2 = 0.2$, and $v_3 = 0.01$. 

• Qualities: \( q_1 = 0.8, \ q_2 = 0.5, \) and \( q_3 = 0.1. \)

• Appeals: \( A_1 = 0.01, \ A_2 = 0.1, \) and \( A_3 = 0.9. \)

In the first step, the performance ranking is \( \sigma^* = [2, 1, 3], \) The expected number of purchases at time 1 given ranking \( \sigma^* \) is

\[
\lambda^* = \sum_i \frac{v_{\sigma^*_i} A_i}{\sum_j v_{\sigma^*_j} A_j} q_i = 0.463.
\]

The probability that product \( i \) is purchased in time 1 is given by

\[
P^*_i = \frac{v_{\sigma^*_i} A_i}{\sum_j v_{\sigma^*_j} A_j} q_i
\]

which yields

\[
P^*_1 = 0.0198, \ P^*_2 = 0.432, \ P^*_3 = 0.0111.
\]

At time 2, there are four possible states, depending upon whether some product was purchased at time 1. Let \( s_0 \) be the state where no product was purchased and let \( s_i \ (i \in \{1, 2, 3\}) \) be the state where product \( i \) was purchased at time 0. The performance rankings for each state are: \( \sigma^*(s_1) = [1, 2, 3] \) and \( \sigma^*(s_0) = \sigma^*(s_2) = \sigma^*(s_3) = [2, 1, 3]. \) The expected numbers of purchases for each state are thus given by

\[
\lambda^*(s_0) = 0.463, \ \lambda^*(s_1) = 0.783, \ \lambda^*(s_2) = 0.496, \ \lambda^*(s_3) = 0.423.
\]

Therefore, the expected number of purchases of the performance ranking for the first two steps is

\[
\lambda^* + \sum_{i=1}^3 P^*_i \cdot \lambda^*(s_i) + \left(1 - \sum_{i=1}^3 P^*_i\right) \lambda^*(s_0) = 0.946,
\]

where the first term is the expected purchases at time 1 and the second and third terms are the expected number of purchases at time 2 given state \( s_i \) times the probability of state \( s_i. \)

Consider now the quality ranking which corresponds to the permutation \( \sigma^q = [1, 2, 3] \) which is fixed for all possible states. The expected number of purchases for ranking \( \sigma^q \) in each of the four states are

\[
\lambda^q(s_0) = 0.458, \ \lambda^q(s_1) = 0.783, \ \lambda^q(s_2) = 0.494, \ \lambda^q(s_3) = 0.380.
\]

The probabilities that product \( i \) is purchased at time 1 are given by

\[
P^q_1 = 0.156, \ P^q_2 = 0.278, \ P^q_3 = 0.025.
\]

As a result, the expected performance of the quality ranking in two periods is 0.975, outperforming the performance ranking by about 3% in this case.
4 The Properties of the Quality Ranking

This section describes the main properties of the quality ranking.

The Benefits of Position Bias  We first show that position bias always increases the expected number of purchases when quality ranking is used.

**Theorem 4.1.** Position bias increases the expected number of purchases under the quality-ranking policy, i.e., for all visibilities \( v_i \), appeals \( a_i \), and qualities \( q_i \) (\( 1 \leq i \leq n \)), we have

\[
\frac{\sum_i v_i a_i q_i}{\sum_j v_j a_j} \geq \frac{\sum_i a_i q_i}{\sum_j a_j}.
\]

The Benefits of Social Influence  An important question in cultural markets is whether the revelation of past purchases to consumers improves market efficiency. This section shows that, under the quality ranking policy, the expected number of purchases in the studied trial-offer model increases when past purchases are revealed. This indicates that both social influence and position bias improve the market efficiency and their benefits are cumulative. The proof uses a lemma from [2] specifying a sufficient condition for a ranking to benefit from social influence.

**Theorem 4.2.** The expected number of purchases is non-decreasing over time for the quality ranking under social influence.

Asymptotic Behavior of the Quality Ranking  We now show the key result of the paper: The trial-offer market becomes a monopoly for the best product when the quality ranking is used at each step. As a consequence, the quality ranking is optimal asymptotically since the best product has the highest probability to be purchased. The result also indicates that trial-offer markets are predictable asymptotically. To prove the result, we first characterize the probability that the next purchase is product \( i \). Then we reduce a trial-offer market with two products to a generalized Pólya scheme, a urn and ball model with replacement. The result is then generalized to many products.

The first result characterizes the probability of the next purchase.

**Lemma 4.1.** The probability \( p_i \) that the next purchase (after any number of steps) is product \( i \) is

\[
p_i = \frac{v_i a_i q_i}{n} \frac{1}{\sum_{j=1}^n v_j a_j q_j}.
\]

Consider first a trial-offer market with two products. Since the steps in which no product is purchased can be ignored, by Lemma 4.1 we can use the following variables (\( 1 \leq j \leq 2 \)) to specify the market:

\[
X_{j,t} \doteq a_{j,t} q_j
\]  
(2)
where $\hat{q}_j = v_j q_j$.

The trial-offer market can thus be modeled as generalized Pólya scheme \[12\], where $X_{j,t}$ represents the number of balls of type $j$ at step $t$ and $Z_{j,t}$ is the probability that a ball of type $j$ is drawn at step $t$. Since $X_{j,t+1} = X_{j,t} + \hat{q}_j$ if product $j$ is purchased, the generalized Pólya scheme add $\hat{q}_j$ balls, each time product $j$ is purchased. As a result, the Pólya scheme uses the replacement matrix

\[
\begin{pmatrix}
\hat{q}_1 & 0 \\
0 & \hat{q}_2
\end{pmatrix}.
\]

This Pólya scheme is known to converge almost surely to a monopoly whenever $\hat{q}_1 \neq \hat{q}_2$ (see Theorem 6 in \[12\], whose proof uses stochastic approximation techniques \[7\]).

**Lemma 4.2** (Monopoly of a 2-Product Market). Consider a Trial-Offer model with two products of quality $q_1$ and $q_2$ ($q_1 > q_2$). The market converges almost surely to a monopoly for product 1.

This lemma can be generalized when there are many products but only two values of $\hat{q}_i$.

**Lemma 4.3.** Consider a market with $m$ products such that $\hat{q}_1 = \ldots = \hat{q}_j = \hat{q}^+$ and $\hat{q}_{j+1}, \ldots, \hat{q}_n = \hat{q}^-$ ($1 \leq j \leq n$) with $\hat{q}^+ > \hat{q}^-$. Then, the market share of products $j+1, \ldots, n$ converges almost surely to zero.

We are now in position to prove the main result of this section.

**Theorem 4.3** (Monopoly of Trial-Offer Markets). Consider a trial-offer markets where $q_1 > \ldots > q_n$. Then the market converges almost surely to a monopoly for product 1.

This result has some key corollaries. In particular, the quality-ranking is optimal asymptotically, since only the best-quality song is left.

**Corollary 4.4.** The quality-ranking is asymptotically optimal in trial-offer markets.

It is also important to point out that the market is completely predictable at the limit.

For completeness, it is useful to consider the case where the products have the same quality $q$ and there are no position bias. In this case, the market share of each product converges almost surely to a random variable following a beta-distribution. We prove this result for the case of two products. By Lemma 4.1 the probability that the next purchase is product $i$ becomes

\[
p_i = \frac{a_i}{\sum_{j=1}^n a_j},
\]
and the replacement matrix becomes the diagonal matrix $q^{n \times n}$. Now

$$Z_{i,k} = \frac{a_{i,k}}{a_{1,k} + a_{2,k}}.$$

is a Martingale sequence. Indeed,

$$
\begin{align*}
\mathbb{E}_k[Z_{i,k+1}] &= \frac{a_{i,k} + q}{a_{1,k} + a_{2,k} + q} Z_{i,k} + \frac{a_{i,k}}{a_{1,k} + a_{2,k} + q} (1 - Z_{i,k}) \\
&= \left( \frac{a_{i,k} + q}{a_{1,k} + a_{2,k} + q} \right) \frac{a_{1,k}}{a_{1,k} + a_{2,k}} + \left( \frac{a_{i,k}}{a_{1,k} + a_{2,k} + q} \right) \frac{a_{2,k}}{a_{1,k} + a_{2,k} + q} \\
&= \frac{a_{i,k}}{a_{1,k} + a_{2,k}} = Z_{i,k}
\end{align*}
$$

As a result, the market is equivalent to a classical Pólya-Eggenberger urn model where the variable $Z_{i,k}$ converges almost surely to a random variable obeying a beta distribution (Theorem 3.2 in [10]).

**Theorem 4.4.** Consider a market with two products of the same quality and equal visibilities. The market share of product 1 converges almost surely to a random variable obeying a beta distribution with parameters $A_1/q$ and $A_2/q$.

**Static Performance of Quality Ranking** The previous section has shown that the quality-ranking is optimal asymptotically in the dynamic market. We also know that the performance ranking is optimal for the static market. This section shows that the quality ranking has some performance guarantees even for the static market. In particular, it presents a bound on the performance of the quality ranking which depends only the visibility coefficients. The bound is also tight.

**Theorem 4.5** (Static Performance Bound). The quality ranking is an $\alpha$-approximation of the static market optimization problem, where $\alpha = v_1/v_n$. Moreover, the $\alpha$-approximation of the quality ranking is tight.

## 5 Computational Experiments

The previous sections have derived theoretical results for the quality ranking. In particular, they have shown that social influence and position bias always increase the expected performance of the quality ranking. These results hold for all possible values of the product quality, appeal, and visibility inputs. In addition, the results have provided strong static and asymptotic guarantees for the quality ranking. This section aims at illustrating these
results on settings that model the MusicLAB experiments discussed in [13, 6, 2]. As mentioned in the introduction, the MusicLAB is a trial-offer market where participants can try a song and then decide to download it. It is important to emphasize that the generative model of the MusicLAB [6] uses the same model for consumer choice preferences.

The Experimental Setting  The experimental setting uses an agent-based simulation to emulate the MusicLAB. Each simulation consists of $N$ iterations and, at each iteration $t$,

1. the simulator randomly selects a song $i$ according to the probabilities $p_i(\sigma, d)$, where $\sigma$ is the ranking proposed by the policy under evaluation and $d$ is the social influence signal;

2. the simulator randomly determines, with probability $q_i$, whether selected song $i$ is downloaded; In the case of a download, the simulator increases the social influence signal for song $i$, i.e., $d_{i,t+1} = d_{i,t} + 1$. Otherwise, $d_{i,t+1} = d_{i,t}$.

Every $r$ iterations, a new list $\sigma$ is computed using one of the ranking policies described above. For instance, in the social influence condition of the original MusicLAB experiments, the policy ranks the songs in decreasing order of download counts, i.e., the D-RANK policy. The parameter $r \geq 1$ is called the refresh rate.

The experimental setting, which aims at being close to the MusicLAB experiments, considers 50 songs and simulations with 20,000 steps. The songs are displayed in a single column. The analysis in [6] indicated that participants are more likely to try songs higher in the list. More precisely, the visibility decreases with the list position, except for a slight increase at the bottom positions. Figure 1 depicts the visibility profile based on these guidelines, which is used in all computational experiments. Under this setting, the
Figure 2: The Quality $q_i$ (blue) and Appeal $A_i$ (red) of song $i$ in the four settings. In the first setting (top left), the quality and the appeal were chosen independently according to a Gaussian distribution. The second setting (top right) explores an extreme case where the appeal is negatively correlated with the quality used in setting 1. In the third setting (bottom left), the quality and the appeal were chosen independently according to a uniform distribution. The fourth setting (bottom right) explores an extreme case where the appeal is negatively correlated with the quality used in setting 3.

(worst-case) approximation factor of the quality ranking (see Theorem 4.5) is $\alpha = v_1/v_n = 0.8/0.2 = 4$.

The paper also uses four settings for the quality and appeal of each product, which are depicted in Figure 2:

1. In the first setting, the quality and the appeal were chosen independently according to a Gaussian distribution normalized to fit between 0 and 1.

2. The second setting explores an extreme case where the appeal is negatively correlated with quality. The quality of each product is the same as in the first setting but the appeal is chosen such that the sum of appeal and quality is exactly 1.

3. In the third setting, the quality and the appeal were chosen independently according to a uniform distribution.

4. The fourth setting also explores an extreme case where the appeal is negatively cor-
related with quality. The quality of each product is the same as in the third setting but the appeal is chosen such that the sum of appeal and quality is exactly 1.

The experimental results were obtained by averaging the results of $W = 400$ simulations.

**Performance of the Market** Figure 3 depicts computational results on the expected number of purchases for the various rankings and settings. They highlight two significant findings:

1. The quality ranking exhibits a similar performance to the performance ranking and provides substantial gains in expected downloads compared to the download (purchase) and random rankings. On settings with a negative correlations between appeal
and quality, the quality ranking performs better than the performance ranking.

2. The benefits of social influence and position bias are complementary and cumulative as predicted by the theoretical results. Both are significant in terms of the expected performance of the market.

Predictability of the Market  Figures 4 and 5 depict computational results on the predictability of the market under various ranking policies. The figures plot the number of downloads of each song for the 400 experiments. In the plots, the songs are ranked by increasing quality from left to right on the x-axis. Each dot in the plot shows the number of downloads of a song in one of the 400 experiments. Figure 4 presents the results for the first setting where the quality and the appeal were chosen independently according to a Gaussian distribution. Figure 5 presents the result for the second setting where the appeal is negatively correlated with quality and the quality of each song is chosen according to a Gaussian distribution.

The computational results are compelling and confirms the theory. Figure 4 shows
Figure 5: The Distribution of Download Versus Song Qualities (Second Setting). The songs on the x-axis are ranked by increasing quality from left to right. Each dot is the number of downloads of a product in one of the 400 experiments.

that the best song always receives the most downloads in the quality ranking (with social influence) and that the variance in its number of downloads across the experiments is very small. The performance ranking (with social influence) also performs well but the best song does not necessarily receive the most downloads and the variance in its download is significantly larger. The download ranking is highly unpredictable, while the random ranking is highly predictable as one would expect. It is also interesting to note that these observations continue to hold even when the appeal is negatively correlated with quality, as Figure 5 indicates. The contrast between the popularity ranking used in [13] and the quality ranking in this setting is particularly striking.

In summary, the computational results confirm that, under the quality ranking policy, the market is highly predictable. The song with the highest quality emerges and the variance in its number of downloads is very small.
6 Discussion

This paper studied trial-offer cultural markets, which are ubiquitous in our societies and involve products such as books, songs, videos, clothes, and even newspaper articles. In these markets, participants are presented with products in a certain ranking. They can then try the products before deciding whether to purchase them. Social influence signals are widely used in such settings and help promote popular products to maximize market efficiency. However, it has been argued that social influence makes these markets unpredictable [13]. As a result, social influence is often presented in a negative light.

In this paper, we have reconsidered this conventional wisdom. We have shown that, when products are presented to participants in a way that reflects quality, the market is both efficient and predictable. In particular, a quality ranking tends to a monopoly for the product of highest quality, making the market both optimal and predictable asymptotically. Moreover, we have also shown that both social influence and position bias improve market efficiency. These results are robust and do not depend on the particular values for the appeal and quality of the products. In addition, computational experiments using the generative model of the MUSICLAB show that there is a fast convergence to our asymptotic theoretical results about the performance and the predictability of the quality ranking. These results are in sharp contrast with the popularity ranking studied in [13], where the products are ranked by the strength of the social influence signals. In these conditions, the market is indeed unpredictable and less efficient. There are some important lessons to draw from these results.

- It appears that unpredictability is not an inherent property of social influence: Whether a market is predictable or not really depends on how social influence is used.

- With the quality ranking, it is not hard to predict popularity as the market will converge to a monopoly. The computational experiments also show that “blockbusters” are quickly identified, even when the appeal is negatively correlated with quality.

- Whenever the quality of the products can be approximated easily (e.g., by running some experiments with a small number of participants or learning it on the fly as suggested in [2]), the quality ranking is the policy of choice: It is predictable and optimal asymptotically and is comparable, or outperforms, the performance ranking, i.e., greedy policy that continuously optimizes the ranking based on the appeal, the quality, and the social influence signal of each product.

- With the quality ranking, a high-quality product overcomes a poor appeal but the opposite does not hold. A product of poor or average quality never become popular even with a great appeal. Obviously, aligning appeal and quality makes the market more efficient but it does not change the asymptotic outcome.

It is also interesting to contrast our results with the study in [3] which uses a different model for consumer choice preferences. In their model, the probability to purchase a product is
given by

\[ p_i(\phi) = \frac{e^{J\phi_i + q_i}}{\sum_{j=1}^{n} e^{J\phi_j + q_j}} \]

where \( J \) is a constant and \( \phi_i \) denotes the market share of product \( i \). With this choice model, with \( J \) is large, monopoly occurs and “eventually any of the products can get the largest market share, so the number of equilibria is \( n \)” \[3\]. This comes from the fact that the social influence signal is much stronger in this model, illustrating again that it is how social influence is used, not social influence per se, which makes markets unpredictable and less efficient. In other words, the popularity ranking reinforces the social signal too much while, in the model from \[3\], the social signal may be too strong. Recall also that the choice model used in this paper was shown to reproduce the original experiments of the MUSICLAB \[6\].

Overall, these results paint a very different picture of the benefits and weaknesses of social influence. Social influence does not make the market unpredictable: When used with the quality ranking, it creates a predictable market for our trial-offer model. Our current goal is to obtain a broader understanding of social influence by generalizing these results to different types of markets (e.g., markets with heterogeneous customers) and different types of social signals.

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Proofs

Proof of Theorem 4.1. Let \( \lambda = \frac{\sum_i v_i a_i q_i}{\sum_j v_j a_j} \) be the expected number of purchases for the quality ranking. We have

\[
\sum_i v_i a_i (q_i - \lambda) = 0.
\]

Consider the index \( k \) such that \( (q_k - \lambda) \geq 0 \) and \( (q_{k+1} - \lambda) < 0 \). Since \( v_1 \geq \ldots \geq v_n \), we have

\[
\sum_{i=1}^k v_i a_i (q_i - \lambda) + \sum_{i=k+1}^n v_i a_i (q_i - \lambda) \leq \sum_i v_i a_i (q_i - \lambda) = 0
\]

and, since \( v_k \geq 0 \),

\[
\sum_{i=1}^n a_i (q_i - \lambda) \leq 0.
\]

It follows that \( \lambda \geq \frac{\sum_{i=1}^n a_i q_i}{\sum_{i=1}^n a_i} \).

Proof of Theorem 4.2. Let \( \mathbb{E}[D_t] = \frac{\sum_i v_i a_i q_i}{\sum_i v_i a_i} = \lambda \) denote the expected number of purchases at time \( t \). The expected number of purchases in time \( t+1 \) conditional to time \( t \) is

\[
\mathbb{E}[D_{t+1}] = \sum_j \left( \frac{v_j a_j q_j}{\sum_i v_i a_i} \cdot \frac{\sum_{i \neq j} v_i a_i q_i + v_j (a_j + 1) q_i}{\sum_i v_i a_i + v_j (a_j + 1)} \right) + \left( 1 - \frac{\sum_i v_i a_i q_i}{\sum_i v_i a_i} \right) \cdot \frac{\sum_i v_i a_i q_i}{\sum_i v_i a_i}
\]

\[
= \sum_j \left( \frac{v_j a_j q_j}{\sum_i v_i a_i} \cdot \frac{\sum_i v_i a_i q_i + v_j q_j}{\sum_i v_i a_i + v_j} \right) + \left( 1 - \frac{\sum_j v_j a_j q_j}{\sum_i v_i a_i} \right) \cdot \lambda.
\]

We need to prove that

\[
\mathbb{E}[D_{t+1}] \geq \mathbb{E}[D_t],
\]

which amounts to showing that

\[
\sum_j \left( \frac{v_j a_j q_j}{\sum_i v_i a_i} \cdot \frac{\sum_i v_i a_i q_i + v_j q_j}{\sum_i v_i a_i + v_j} \right) + \left( 1 - \frac{\sum_j v_j a_j q_j}{\sum_i v_i a_i} \right) \cdot \lambda \geq \lambda.
\]

which reduces to proving

\[
\frac{1}{\sum_i v_i a_i} \sum_j \left[ \frac{v_j^2 a_j q_j}{\sum_i v_i a_i + v_j (q_j - \lambda)} \right] \geq 0
\]
or, equivalently,
\[
\sum_j \left( \frac{v_j^2 a_j q_j}{\sum_i v_i a_i + v_j} (q_j - \lambda) \right) \geq 0. 
\tag{10}
\]

Let \( k = \min\{i \in N | (q_i - \lambda) \geq 0\} \), i.e., the smallest index \( k \in N \), such that \( q_k \geq \lambda \). We have that
\[
\sum_{j=1}^n \left[ \frac{v_j^2 a_j q_j}{\sum_i v_i a_i + v_j} (q_j - \lambda) \right] = \sum_{j=1}^k \left[ \frac{v_j q_j}{\sum_i v_i a_i + v_j} a_j v_j (q_j - \lambda) \right] + \sum_{j=k+1}^n \left[ \frac{v_j q_j (q_j - \lambda)}{\sum_i v_i a_i + v_j} a_j v_j (q_j - \lambda) \right].
\]

By definition of \( k \), the terms in the summation on the left are positive and the terms in the summation on the right are negative. Moreover, for any \( c > 0 \) and \( v_i, v_j \geq 0 \), we have that
\[
\frac{v_i}{c + v_i} \geq \frac{v_j}{c + v_j} \iff (c + v_j) v_i \geq (c + v_i) v_j \iff cv_i \geq cv_j \iff v_i \geq v_j.
\]
Since \( v_1 \geq v_2 \geq \ldots \geq v_n \geq 0 \),
\[
\sum_i v_i a_i + v_1 \geq \sum_i v_i a_i + v_2 \geq \ldots \geq \sum_i v_i a_i + v_n. \tag{11}
\]

Moreover, since the quality ranking rankes the products by quality and \( q_1 \geq q_2 \geq \ldots \geq q_n \geq 0 \), Equation (11) and the definition of \( k \) implies that
\[
\forall i \leq k : \quad \frac{v_i q_i}{\sum_j v_j a_j + v_i} \geq \frac{v_k q_k}{\sum_j v_j a_j + v_k},
\]
\[
\forall i > k : \quad \frac{v_i q_i}{\sum_j v_j a_j + v_i} \leq \frac{v_k q_k}{\sum_j v_j a_j + v_k}.
\]
This observation, together with the fact that the left-hand (resp. right-hand) terms are positive (resp. negative), produces a lower bound to the right-hand side of Inequality (10):
\[
\sum_{j=1}^k \left[ \frac{v_j q_j}{\sum_i v_i a_i + v_j} a_j v_j (q_j - \lambda) \right] + \sum_{j=k+1}^n \left[ \frac{v_j q_j}{\sum_i v_i a_i + v_j} a_j v_j (q_j - \lambda) \right] \geq \frac{v_k q_k}{\sum_i v_i a_i + v_k} \sum_{j=1}^k [a_j v_j (q_j - \lambda)] + \frac{v_k q_k}{\sum_i v_i a_i + v_k} \sum_{j=k+1}^n [a_j v_j (q_j - \lambda)].
\]

Now, by definition of \( \lambda \),
\[
\lambda = \frac{\sum_{i=1}^n v_i a_i q_i}{\sum_{i=1}^n v_i a_i} \iff \lambda \sum_{i=1}^n v_i a_i = \sum_{i=1}^n v_i a_i q_i \iff \sum_{i=1}^n v_i a_i (q_i - \lambda) = 0.
\]

20
which implies that
\[
\sum_{i} v_{i} a_{i} + v_{k} \sum_{j=1}^{n} [a_{j}v_{j}(q_{j} - \lambda)] = 0
\]
concluding the proof. \qed

Proof of Lemma 4.1. The probability that product \( i \) is purchased in the first step is given by
\[
p_{i}^{1st} = \frac{v_{i} a_{i}}{\sum_{j=1}^{n} v_{j} a_{j}} q_{i}.
\]
The probability that product \( i \) is purchased in the second step and no product was purchased in the first step is given by
\[
p_{i}^{2nd} = \left( \frac{\sum_{j=1}^{n} v_{j} a_{j} (1 - q_{j})}{\sum_{j=1}^{n} v_{j} a_{j}} \right) \frac{v_{i} a_{i}}{\sum_{j=1}^{n} v_{j} a_{j}} q_{i}.
\]
More generally, the probability that product \( i \) is purchased in step \( m \) while no product was purchased in earlier steps is given by
\[
p_{i}^{mth} = \left( \frac{\sum_{j=1}^{n} v_{j} a_{j} (1 - q_{j})}{\sum_{j=1}^{n} v_{j} a_{j}} \right)^{m} \frac{v_{i} a_{i}}{\sum_{j=1}^{n} v_{j} a_{j}} q_{i}.
\]
Defining \( a = \frac{\sum_{j=1}^{n} v_{j} a_{j} q_{j}}{\sum_{j=1}^{n} v_{j} a_{j}} \), Equation 6 becomes
\[
p_{i}^{mth} = \left( 1 - a \right)^{m} \frac{v_{i} a_{i}}{\sum_{j=1}^{n} v_{j} a_{j}} q_{i}.
\]
Hence the probability that the next purchased product is product \( i \) is given by
\[
p_{i} = \sum_{m=0}^{\infty} \left( 1 - a \right)^{m} \frac{v_{i} a_{i}}{\sum_{j=1}^{n} v_{j} a_{j}} q_{i}.
\]
Since
\[
\sum_{m=0}^{\infty} \left( 1 - a \right)^{m} = \frac{1}{a},
\]
the probability that the next purchase is product $i$ is given by

$$p_i = \frac{v_i a_i q_i}{\sum_{j=1}^{n} v_j a_j q_j}.$$ 

**Proof of Lemma 4.3.** The probability of purchasing one of the first $j$ (resp. last $n - j$) product next is

$$\sum_{i=1}^{j} a_i \hat{q}^+ \quad (\text{resp.} \quad \sum_{i=j+1}^{n} a_i \hat{q}^-).$$

(12)

Hence, we can define a 2-product market in which product 1 has an appeal $a^+$ and a quality $\hat{q}^+$ and product 2 has an appeal $a^-$ and a quality $\hat{q}^-$, where

$$a^+ = \sum_{i=1}^{j} a_i \quad \text{and} \quad a^- = \sum_{i=j+1}^{n} a_i.$$  \hspace{1cm} (13)

By Lemma 4.2, this market converges almost surely to a monopoly for product 1. Hence the market share for products $j + 1, \ldots, n$ converges almost surely to zero.

**Proof of Theorem 4.3.** Consider the generalized Pólya scheme $S$ associated with the market with $n$ types of balls. Define a new generalized Pólya scheme $S'$ which is similar to $S$ except that $\hat{q}_1, \ldots, \hat{q}_{n-2}$ are replaced by $\hat{q}_{n-1}$. At the limit, the fraction of balls of type $n$ in scheme $S'$ cannot be smaller than in scheme $S$. Consider now the market that corresponds to $S'$. By Lemma 4.3, the market share of product $n$ converges almost surely to zero. The result follows from Lemma 4.2 after repeating the process for products $n-1, \ldots, 3$.

**Proof of Theorem 4.5.** Let $\sigma^*$ be the optimal sorting and $\lambda^*$ its expected number of purchases. We have

$$\lambda^* = \frac{\sum_{i} v_{\sigma^*} a_i q_i}{\sum_{j} v_{\sigma^*} a_j} \leq \frac{\sum_{i} v_1 a_i q_i}{\sum_{j} v_{n} a_j} = \alpha \frac{\sum_{i} a_i q_i}{\sum_{j} a_j}.$$ 

Let $\lambda^q$ be the expected number of purchases for the quality ranking, i.e.,

$$\lambda^q = \frac{\sum_{i} v_i a_i q_i}{\sum_{j} v_j a_j}.$$ 

By Theorem 4.1,

$$\lambda^q \geq \frac{\sum_{i} a_i q_i}{\sum_{j} a_j}.$$
Combining both bounds yields $\lambda^* \leq \alpha \lambda^q$.

We now show that the approximation is tight. Consider 3 products with qualities $q_1 = 1, q_2 = \epsilon, q_3 = 0$ and appeals $a_1 = 1, a_2 = x, a_3 = 0$ and let the visibilities be $v_1 = 1, v_2 = 1, v_3 < 1$. The quality ranking is $\sigma^q = (1, 2, 3)$ and the optimal performance ranking is $\sigma^* = (1, 3, 2)$. The expected number of purchases for the quality ranking is

$$\frac{1 + \epsilon x}{1 + x}$$

while it is

$$\frac{1 + \epsilon x \alpha}{1 + \alpha x}.$$ for the performance ranking. When $\epsilon$ tends to zero, the ratio between the performance and quality ranking becomes

$$\lim_{\epsilon \to 0} \frac{1 + v_3 \epsilon x}{1 + v_3 x} \frac{1 + x}{1 + x} = \frac{1 + x}{1 + v_3 x}.$$ 

Hence, when $x$ is large enough, the ratio is approximately $\alpha$, i.e.,

$$\frac{1 + x}{1 + v_3 x} \approx \frac{1}{v_3} = \frac{v_1}{v_3} = \alpha.$$ 

$\Box$