Evolution of the Velocity Ellipsoids in the Thin Disk of the Galaxy and the Radial Migration of Stars

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Abstract

Data from the revised Geneva–Copenhagen catalog are used to study the influence of radial migration of stars on the age dependences of parameters of the velocity ellipsoids for nearby stars in the thin disk of the Galaxy, assuming that the mean radii of the stellar orbits remain constant. It is demonstrated that precisely the radial migration of stars, together with the negative metallicity gradient in the thin disk, are responsible for the observed negative correlation between the metallicities and angular momenta of nearby stars, while the angular momenta of stars that were born at the same Galactocentric distances do not depend on either age or metallicity. The velocity components of the Sun relative to the Local Standard of Rest derived using data for stars born at the solar Galactocentric distance are $(U_\odot, V_\odot, W_\odot)_{LSR} = (5.1 \pm 0.4, 7.9 \pm 0.5, 7.7 \pm 0.2) \text{ km s}^{-1}$. The two coordinates of the apex of the solar motion remain equal to $\langle l_\odot \rangle = 70^\circ \pm 7^\circ$ and $\langle b_\odot \rangle = 41^\circ \pm 2^\circ$, within the errors. The indices for the power-law age dependences of the major, middle, and minor semi-axes become $0.26 \pm 0.04$, $0.32 \pm 0.03$, and $0.07 \pm 0.03$, respectively. As a result, with age, the velocity ellipsoid for thin-disk stars born at the solar Galactocentric distance increases only in the plane of the disk, remaining virtually constant in the perpendicular direction. Its shape remains far from equilibrium, and the direction of the major axis does not change with age: the ellipsoid vertex deviation remains constant and equal to zero within the errors $(\langle L \rangle = 0.7^\circ \pm 0.6^\circ)$, $(\langle B \rangle = 1.9^\circ \pm 1.1^\circ)$. Such a small increase in the velocity dispersion perpendicular to the Galactic plane with age can probably be explained by "heating" of the stellar system purely by spiral density waves, without a contribution from giant molecular clouds.

Keywords: kinematics of stars, velocity ellipsoids, thin disk of the Galaxy

Introduction

This study is a logical continuation of our earlier paper [1], where we used data from the revised Geneva–Copenhagen catalog [2] to investigate the age and metallicity dependences of parameters of the velocity ellipsoids for selected stars of the thin disk of the Galaxy. In particular, we showed that the velocity ellipsoid for solar-neighborhood thin-disk stars increases with age, only slightly circularizes its shape, turns slightly toward the direction of the Galactic center, and loses angular momentum. This behavior is in full agreement with the idea that the stellar velocity dispersions increase with age under the action of
various relaxation processes, while the subsystem itself approaches an equilibrium state. However, it was simultaneously demonstrated that the velocity ellipsoid for stars of mixed age increased with increasing metallicity, while displaying a weak tendency to increase its sphericity and turn toward the direction of the Galactic center. Thus, the velocity behaved in the same way as it did with variations in age; however the rotational velocity around the Galactic center monotonically increased with decreasing metallicity (1), rather than decreasing. This last effect was first described in [3], where it was discovered in an analysis of the mean tangential velocities and metallicities of nearby stars, and was called the "kinematics–metallicity paradox for thin-disk Galactic stars." In [1], we suggested that the metallicity dependences for the velocity ellipsoids inferred for solar-neighborhood, thin-disk stars could be due to the radial migration of stars.

The existence of radial migration of stars was first proposed by Grenon [4], in order to explain certain properties of nearby stars. (Since we are considering here not systematic variations in the positions of stars, but only fluctuations in their positions in non-circular orbits, it would be more correct to call these radial motions of stars, however, we have retained the established term from the literature.) According to modern concepts, the orbits of stars born from interstellar material moving in circular orbits increase their eccentricity with time due to interactions with perturbations in the gravitational potential of the Galaxy. However, it is believed that the mean radii of their orbits should remain essentially constant in this process, reflecting the Galactocentric distance of their birth [5, 6]. As a result, some fraction of stars currently located near the Sun have migrated there from other Galactocentric distances [7], and this can distort the parameters of the velocity ellipsoids for nearby stars. If we suppose that stars of any metallicity in the thin disk of the Galaxy form from interstellar material that moves in circular orbits, the orbits for stars of different metallicities should, on average, vary in roughly the same way with age. However, it turned out that the semi-axes of the velocity ellipsoids for nearby stars systematically (although only slightly) increased with decreasing metallicity, while the deviation of the vertex—the direction of the solar motion—and the angular momentum of the stars varied quite strongly with metallicity [1, Fig. 4]. It appears that precisely the radial migration of stars could lead to metallicity dependences for the parameters of the velocity ellipsoids for nearby stars, since there is a negative radial metallicity gradient in the thin disk (see, for example, [8]).

The current study is concerned with a detailed verification of the influence of the radial migration of stars on the parameters of the velocity ellipsoids for nearby stars, using the same sample as in our previous work.

OBSERVATIONAL DATA.

We will now briefly summarize the main aspects of how our sample of stars was constructed. We used data from the new version of the Geneva–Copenhagen survey [2], which contains the ages, metallicities, spatial-velocity components, and Galactic-orbit elements for \( \approx 14000 \) F-G dwarfs and subgiants brighter than \( V \approx 8.5 \). (This same group continued the process of refining the stellar parameters in the catalog (see [9]), but the resulting changes in the distances, temperatures, and ages of the stars were smaller than in the first revision, and, according to those authors, the features of the corresponding age–metallicity dependences remained virtually the same. The newest version of the catalog is not yet available.) We first eliminated binary stars, very evolved stars \( \delta M_V > 3 \), and stars with uncertain ages \( \epsilon_t > \pm 3 \) billion years) from the sample. Moreover, to avoid boundary
effects, we restricted our range of effective temperatures to 5200–7000 K. Thin-disk stars were selected according to our modification of the method [1], first proposed in [10], which calculates the probability of membership of stars to the thin-disk and thick-disk subsystems based on their spatial-velocity components. We determined these probabilities using our refined values [1] of the Geneva–Copenhagen survey data for the dispersions of each of the three spatial-velocity components ($\sigma_U$, $\sigma_V$, $\sigma_W$), the mean rotational velocities ($V_{rot}$), and the relative populations of stars in the two disk subsystems at the solar Galactocentric distance. Our kinematic criterion proved to be in good agreement with the chemical compositions of the stars: using this criterion naturally minimizes the numbers of stars with high relative abundances of magnesium in the thin disk and with low relative abundances of magnesium in the thick disk. Further, we removed members of the most numerous moving groups from the sample: Sirius, the Hyades, the Pleiades, Coma Berenices, and $\xi$ Hercule. The resulting sample contains 4549 single F–G dwarfs and subgiants of the thin disk of the Galaxy lying within $\approx 150$ pc of the Sun.

**DEPENDENCE OF THE VELOCITY ELLIPSOIDS FOR STARS OF MIXED AGES AND METALLICITIES ON THEIR MEAN ORBITAL RADI.**

Let us first investigate how the velocity ellipsoids for stars of mixed ages and metallicities depend on the mean radii of their orbits. (The mean orbital radius is $R_m = (R_a + R_p)/2$, where $R_a$ and $R_p$ are the apogalactic and perigalactic orbital radii; we take the distance of the Sun from the Galactic center to be $R_0 = 8.0$ kpc.) We separated our sample into 12 subgroups with equal numbers of stars in narrow bands of $R_m$ and calculated the parameters of the velocity ellipsoids for each subgroup. We calculated the velocity ellipsoids and the solar velocity relative to the local centroids using the formulas of Ogorodnikov [11], and the errors in these parameters using the formulas of Parenago [12]. The corresponding dependences are shown in Fig. 1. The first two plots in this figure show that neither the semi-axes nor their ratios depend significantly on $R_m$: none of these correlations are significant ($P > 5\%$). The probabilities that the correlations in Fig. 1c arose by chance, $P_L < 1\%$ and $P_B > 5\%$, indicate that the $R_m$ dependence of the vertex deviations in longitude is highly significant, while the dependence of the vertex deviations in latitude is not significant. Although the vertex coordinates display a clear dependence on $R_m$, their deviations from zero nowhere exceed $3^\circ$; their mean values are ($\langle L \rangle = 0.7^\circ \pm 0.4^\circ$), ($\langle B \rangle = 0.4^\circ \pm 0.3^\circ$), indicating that these deviations are insignificant. We checked this result by dividing the entire sample into four groups in metallicity and constructing analogous plots for each group. The slopes of the $R_m$ dependences for the two coordinates of the vertex deviations appear random, and the subgroups of stars with different metallicities do not show any systematic behavior. Thus, we conclude that the vertex deviations for stars with different $R_m$ values are the same and equal to zero within the errors.

At the same time, the integrated indices for the ellipsoids demonstrate quite significant dependences. The essentially linear and very substantial decrease in the V component of the solar velocity with increasing $R_m$ ($P < 1\%$) is especially striking. The dependence of the U component on $R_m$ is also highly significant, though its slope is appreciably smaller. Since the solar motion reflects the motion of the corresponding centroids, this means that the angular momentum of the stars decreases with the mean radii of their orbits. This makes sense: stars located closer to the apogalactic radius of their orbits rotate about
the Galactic center more slowly at a given Galactocentric distance. The $U$ component also decreases here. Since we are dealing with a statistical ensemble and the $U$ and $V$ components vary differently, the direction toward the apex of the solar motion also depends on the orbital phase at which the majority of stars in each $R_m$ subgroup are located. Both the apex-coordinate dependences are highly significant ($P \ll 1\%$), but are not shown in Fig. 1 because they are non-linear: the longitude of the apex is roughly constant for small orbits, then begins to grow sharply after crossing the mean orbital radius for the solar Galactocentric distance. The apex latitude also begins to deviate sharply from a constant value near this radius. This behavior is due to the fact that, near the solar orbital radius, all three components of the solar velocity are roughly equal to each other in magnitude, while the ratios of their values reverse with further increase in $R_m$. We checked the behavior of the $R_m$ dependences of all ellipsoid parameters by dividing the sample into four metallicity groups. These all demonstrate virtually identical dependences for all parameters. Even the direction of the vertex in latitude exhibits similar fluctuations at high values of the mean orbital radii in all the metallicity groups.

Using our linear regression fits for each component of the solar motion in Fig. 1d, we can determine the components of the solar motion relative to the local centroid (i.e., the Local Standard of Rest, LST) for stars that were born at the Galactocentric distance corresponding to the current position of the Sun. These were found to be $(U_\odot, V_\odot, W_\odot)_{LSR} = (5.1 \pm 0.4, 7.9 \pm 0.5, 7.7 \pm 0.2) \text{ km s}^{-1}$.

Thus, we see that the tangential component of the solar velocity relative to the corresponding centroid, and, as a consequence, the longitude of the apex of the solar motion, depend most strongly on the mean orbital radius, while the deviation of the major semiaxes of the ellipsoid from the direction toward the Galactic center does not vary significantly. Thus, the angular momenta of stars located in the solar neighborhood are directly dependent on the mean radii of their orbits. Let us now consider the metallicity dependences for these two most significant ellipsoid parameters in Fig. 1 for stars with narrow ranges of mean orbital radii.

**METALLICITY DEPENDENCE FOR THE APEX OF THE SOLAR MOTION AND ANGULAR MOMENTUM FOR STARS WITH DIFFERENT MEAN ORBITAL RADII**

To construct the indicated dependences, we divided the initial sample into four groups with roughly equal numbers of stars separated by $R_m$ values of 7.37, 7.72, and 8.10 kpc, then divided each into seven subgroups in metallicity. The left and right plots in Fig. 2 present the metallicity dependences of $V$ and the longitude of the apex of the solar motion relative to the corresponding centroids for each interval of $R_m$. The dependences of the angular momentum are virtually flat for all orbital radii. [Although significant correlations are found in some cases (see the captions next to the dependences), their slopes vary chaotically with variations in the mean orbital radii of the stars, testifying that these dependences are due to selection effects associated with the appreciable widths of the $R_m$ intervals.] With regard to the longitude of the apex, in contrast to the total sample of thin-disk stars, for which we observe an appreciable growth in $L$ with increasing metallicity [1, Fig. 4c], we can see everywhere in Fig. 2b clear dependences showing the opposite tendency. In all intervals with $R_m < R_0$, the dependences are essentially horizontal, and only in the intervals with the largest mean orbital radii are the slopes substantial, demonstrating the large width
of the total interval considered. It is striking that, for the subgroups of any metallicity, the longitude of the apex of the solar motion is oriented in the opposite direction for stars with larger and smaller mean orbital radii. This can be understood if the angular momenta of stars with larger orbits are larger at the solar Galactocentric distance, since they are located further from their apogalactic distances, where the tangential velocity component is minimum.

We also traced the metallicity dependences of the other parameters of the velocity ellipsoids for stars in narrow intervals of \( R_m \) (not shown in the figure in order to conserve space). It turned out that the dependences for parameters of a given type do not depend on the size of the stellar orbits. Only the semi-axes increase (slightly) with decreasing metallicity within each group in \( R_m \). However, this can be explained by the fairly large widths of the intervals in \( R_m \), especially in the group with the smallest orbital radii.

Thus, we conclude that no metallicity dependence is observed for the velocity ellipsoids of stars having the same mean orbital radii. Let us now consider how the velocity ellipsoids for stars born within narrow ranges of Galactocentric distance depend on age, and whether they will display such substantial variations of the angular momentum with age.

**AGE DEPENDENCES OF THE VELOCITY ELLIPSOIDS FOR STARS WITH VARIOUS MEAN ORBITAL RADIi**

To construct these dependences, we divided the original sample into four groups in \( R_m \) with roughly equal numbers of stars separated by 7.37, 7.72, and 8.10 kpc, and then divided each of these into 12 subgroups in age. Since the defining parameters of the velocity ellipsoids are their semi-axes and the components of the solar motion, we present here only these, and only briefly describe the behavior of the secondary parameters. The first row of plots in Fig. 3 presents the age dependences of the ellipsoid semi-axes \( \sigma_i(t) \) for all four groups. All the dependences were fit with power laws of the form \( \sigma_i \sim (t)^{\gamma} \), where \( \sigma_i \) is the dispersion of the corresponding velocity component and \( t \) the ages of the stars. The corresponding power-law indices are indicated in the figure next to each curve. The correlation coefficients are not presented, since all the dependences are highly significant \( (P \ll 1\%) \). Let us consider the variations of the power-law indices for a particular type of dependence in all the plots. For example, the power-law index for the semi-major axis displays a weak tendency to increase with increasing mean stellar radius: the values of \( \gamma_1 \) for the first two groups are significantly smaller than those at the right-hand side of the plots. The power-law index for the second semi-axis does not display any systematic trend, but the contrast between the high values for the two middle groups and small values for the two end groups is striking. We can see that the difference in the power-law indices came about due to the large semi-axes of the ellipsoids for the subgroups with small ages in the end plots, while the semi-axes are roughly the same size for the subgroups with larger ages, for all intervals of \( R_m \). The origin of this is the absence of stars with unperturbed initial orbits in the two end groups, due to selection effects. In contrast to the middle semi-axes, the minor semi-axes demonstrate very small and roughly equal power-law indices for both groups with intermediate mean orbital radii, while both end groups have \( \gamma_3 \) values roughly twice as high, with this difference lying beyond the errors (see captions on the plots). Here, the semi-axes for the end groups were found to be larger than those for the middle groups due to the same selection effects, for the entire age interval. Overall, we can see that the power-law indices for the age dependences of the ellipsoid semi-axes also depend on the mean orbital radii of the stars and the sizes of the \( R_m \) intervals; therefore, to derive
parameters of the velocity ellipsoids undistorted by selection effects, we must select only stars with mean orbital radii that are close to the current solar Galactocentric distance.

The semi-axis ratios in all $R_m$ groups are essentially independent of age, but the sequence of ratios of the middle to the major and minor to the major further from each other than those for the two end groups, where the influence of selection effects is manifest. All the age dependences of the vertex deviations have only low significance ($P \geq 5\%$), and sometimes change their sign between neighboring subgroups, testifying that their origin is most likely selection effects.

The bottom row of plots in Fig. 3 presents the age dependences for the integrated parameters of the ellipsoids for the same groups of stars in narrow intervals of the mean orbital radii. The dependences for the $U$ component of the solar velocity relative to the corresponding centroids do not display systematic variations in their slopes. Moreover, correlations with low significance ($P > 5\%$) are observed in three plots, while the substantial slope of the dependence in Fig. 3g is due exclusively to one distant point in the plot, and the variation of the $U$ component occur here in the vicinity of zero velocity. The slope for the age dependence of the $V$ component displays a systematic decrease with increasing mean orbital radius (which becomes even more negative for the group with the largest orbits). An appreciable variation of the angular momentum with age is observed only in the group with $R_m < 7.37$, and is due to the fact that the mean orbital radii in this group differ substantially from the solar Galactocentric distance and are contained in a wide range, so that only stars with large orbital eccentricities reach the solar neighborhood. The slopes in other groups are negligibly small. The systematic nature of the behavior of the slopes testifies that the mean angular momentum for a general collection of stars with strictly equal mean orbital radii should not depend on the age. The figure also shows that the slopes for the $W$ component are small, vary sign chaotically, and have low significance. The slopes for the age dependences of both coordinates of the apex likewise are small and statistically insignificant. In summary, we can conclude that all three components of the solar velocity relative to the centroids for stars with mixed metallicity and age are roughly the same for stars that are all born at the same distances from the Galactic center.

Since the age dependences of the ellipsoids in Fig. 3 are not always determined with certainty, we verified these results by shifting the boundaries between the groups several times, right to $\Delta R_m = \pm 0.3$, and again constructing the dependences. Our conclusions above are consistent with all these additional relations. In particular, the insignificant, small negative slope of the $V$ component for stars with larger orbits (i.e., in the open $R_m$ interval) systematically increases with the distance of the orbit for the lower boundary of the range from the solar radius.

Thus, nearby stars with mixed metallicity demonstrate different age dependences for their velocity ellipsoids for different ranges of mean orbital radii. This can be fully explained as an increase in the sample of a deficit of stars with orbits that are close to circular when the difference $|R_\odot - R_m|$ is increased for the sample. Let us now investigate how these dependences behave for stars with different metallicities.
AGE DEPENDENCE OF THE VELOCITY ELLIPSOIDS
FOR STARS OF VARIOUS METALLICITIES BORN AT
VARIOUS GALACTOCENTRIC DISTANCES

We have already described the systematic variations of the velocity ellipsoids with metallicity, and here consider only the variations of the ages dependences for stars of different metallicities with $R_m$. To construct Fig. 4, we divided the sample stars into two groups with equal numbers, divided by the metallicity $[Fe/H] = -0.13$ (which corresponds to the maximum of the metallicity distribution of the disk stars in our sample), then subdivided each of these into two groups with different mean orbital radii, separated by $R_m = 7.70$ (which is roughly the position of the maximum in the $R_m$ distribution for disk stars).

The low-metallicity group that was closest to the Galactic center (Fig. 3a) and the high-metallicity group that was furthest from the Galactic center (Fig. 3d) contained fewer stars due to the negative metallicity gradient in the thin disk. The figure shows the behavior of only certain parameters of the ellipsoids, namely, the semi-axes and the components of the solar motion. In spite of the relatively low number of stars in the groups and the large uncertainties in the coefficients for the linear fits in ($log\sigma_i$, $logt$) coordinates, we can trace certain tendencies in the power-law indices for the age dependences for stars with different metallicities in the first row of plots in Fig. 4. In particular, we observe significantly higher power-law indices for the semi-major axes for the low-metallicity group with large orbits than for the corresponding group with small orbits. This arose because the youngest stars in the low-metallicity group that is close to the Galactic center have larger semi-major axes than does the distant low-metallicity group. High-metallicity stars also display a similar difference in their power-law indices, although this difference formally lies within the errors. Here, this difference arose due to the small semi-axes of the subgroup with large ages in the near high-metallicity group. We also note higher power-law indices for the middle semi-axes for the subgroup with larger orbital radii, with this difference lying beyond the errors for the high-metallicity stars. Both distant groups demonstrate significantly lower power-law indices for the semi-minor axes than the two close groups. Any possible differences in the age dependences for the ratios of the semi-axes and the vertex directions (not presented in the figure) are dwarfed by the errors. We suggest that the differences in the dispersions for the subgroups of different ages with different mean orbital radii arise purely due to selection effects, which affect stars with different metallicities somewhat differently, but lead to similar results.

The second row of plots in Fig. 4 shows that the age dependences of the $U$ and $W$ velocity components relative to the corresponding centroids do not display systematic differences. Moreover, the probability that these apparent correlations arose by chance is $P \gg 5\%$. At the same time, the tangential components of the velocity in both close subgroups turned out to be strongly age dependent (for both, $P \ll 1\%$) and nearly identical. The age dependences of these components for the groups of stars with large $R_m$ are not significant; the metal-rich group even displays a slightly negative slope.

Thus, the stars of different metallicity demonstrate the same variations for the age dependences of the velocity ellipsoids with increasing orbital radius, within the errors.
AGE DEPENDENCIES OF THE VELOCITY ELLIPSOIDS FOR STARS BORN AT THE SOLAR GALACTOCENTRIC DISTANCE

Thus, our results provide evidence that the radial migration of stars distorts the age dependences of the velocity ellipsoids of stars currently located near the Sun. Therefore, it is more correct to investigate the age dependences for stars with approximately the same mean orbital radii that were born at the solar Galactocentric distance. For this, we restricted our initial sample to single, nearby, thin-disk stars with mean orbital radii $7.7 < R_m < 8.4$ kpc, since the catalog distance of the Sun from the Galactic center is taken to be $8.0$ kpc. The resulting sample contains 1853 stars. Selection effects acting against stars with circular orbits are virtually absent from the sample, although there is likely some deficit of stars with orbits that are strongly elongated and go far from the Galactic plane, since stars in such orbits will spend a large fraction of their time near the most distant points in the orbits. To construct Fig. 5 and follow the age dependences of the velocity ellipsoids, we divided the sample into 17 subgroups with 109 stars in each. The approximated dependences $\sigma_i(t)$ and captions in the plots in Fig. 5a show that the power-law indices for all the semi-axes are appreciably different from those obtained for all thin-disk stars in the solar neighborhood (see below). In particular, the values of $\gamma$ for the major and middle semi-axes increased slightly, becoming $0.26 \pm 0.04$ and $0.32 \pm 0.03$, respectively. At the same time, the power-law index for the minor semi-axis strongly decreased, becoming equal to zero within $3\sigma$ ($\gamma_3 = 0.07 \pm 0.03$); i.e., the velocity dispersion in the direction perpendicular to the Galactic plane is essentially independent of age. Recall that this index was always appreciably larger, and not smaller, than the other two semi-axes for the entire sample of nearby, thin-disk stars (see, for example, [1, 9, 13, 14]). The power-law index for the age dependence of the total residual velocity remained virtually the same: $\sigma_{V_{res}} = 0.26 \pm 0.03$. (Recall that the power-law indices for the age dependences of the semi-axes and total residual velocity dispersion for all nearby, thin-disk stars were $0.22 \pm 0.03, 0.26 \pm 0.02, 0.27 \pm 0.02$, and $0.24 \pm 0.02$, respectively [1]).

The ratio of the middle to the major semi-axes in Fig 5b does not display appreciable variations with age (the probability that the formal correlation arose due to chance is $P \gg 5\%$), being, on average, equal to $\langle \sigma_2/\sigma_1 \rangle = 0.49 \pm 0.01$. This ratio is much smaller than in a stationary, rotating stellar system, when $\sigma_2/\sigma_1 = [-B/(A-B)]^{1/2}$; with $A = 13.7$ km s$^{-1}$ kpc and $B = -12.9$ km s$^{-1}$ kpc the "standard" values of the Oort constants ($A = 15$ and $B = -10$ km s$^{-1}$ kpc), the ratio of the semi-axes, 0.58, remains larger than the value we have obtained, beyond the errors. The ratios of the minor to major semi-axes show a significant ($P < 1\%$) but small decrease with age, since the dispersion of the $W$ velocity component depends only very weakly on age. Moreover, this ratio became very small, $\langle \sigma_3/\sigma_1 \rangle = 0.23 \pm 0.01$. Neither coordinate of the vertex deviation displays appreciable correlations in Fig. 5c ($P > 5\%$); they both pass through zero somewhere in the middle of the age range, and never take on values far from zero, even for young ages. Their mean values are $\langle L_\odot \rangle = 0.7^\circ \pm 0.5^\circ$ and $\langle B \rangle = 1.9^\circ \pm 1.1^\circ$.

The $U$ and $V$ components of the solar velocity in Fig. 5d display appreciable correlations ($P \approx 1\%$). The largest variations are observed for $U$; however, if we exclude two extreme points with the largest ages from the plot, the dependence essentially disappears, suggesting that this dependence is most likely due to selection effects associated with the wide range in $R_m$. Neither of the other two components display significant differences for their values at the opposite ends of the age interval. As a result, the mean values for $V$ for the stars in the
subgroups of different ages are roughly the same; i.e., their angular momentum does not depend on age. Due to the appreciable variations in $U$ and insignificant variations in $W$, a significant slope for the age dependences in Fig. 5e is observed only for the longitude of the apex of the solar motion ($P \approx 1\%$), but this is likely due to selection effects associated with the comparatively wide range in $R_m$ we have adopted. According to the data in Fig. 5e, the mean coordinates of the apex of the solar motion are $\langle l_\odot \rangle = 70^\circ \pm 7^\circ$ and $\langle b_\odot \rangle = 41^\circ \pm 2^\circ$.

To test the validity of our conclusions about the independence of metallicity of the parameters of the velocity ellipsoids for stars born near the solar Galactocentric distance, we divided sample described above into 11 subgroups in $[Fe/H]$. As usual, we calculated the velocity ellipsoids for each of these subgroups and constructed the corresponding dependences. Indeed, without exception, the velocity-ellipsoid parameters displayed an absence ($P \geq 5\%$) of any dependence on metallicity (the corresponding figure is not presented, in view of the absence of positive information).

**CONCLUSION**

Thus, excluding the influence of the radial migration of stars (i.e., selection effects) led to appreciable changes not only in the derived velocity-ellipsoid parameters for thin-disk stars, but also their age dependences. Our most significant result is the disappearance of the difference in the angular momenta for stars with different metallicities and ages. This means that, within the thin disk, the angular momentum of stars cannot be used as a statistical indicator of their ages. For stars near the disk, this quantity is more likely an indicator of the Galactocentric distance of their birth places: the rotational velocity about the Galactic center for disk stars near the solar Galactocentric distance decreases with decreasing mean orbital radius.

The components of the solar velocity relative to the LSR obtained via interpolation at the solar Galactocentric distance, $(U_\odot, V_\odot, W_\odot)_{LSR} = (5.1 \pm 0.4, 7.9 \pm 0.5, 7.7 \pm 0.2)$ km s$^{-1}$, differ slightly from the usually used values. For example, the values obtained in [16] based on the proper motions of 14 000 nearby dwarfs from the Hipparcos catalog are $(10.0 \pm 0.4, 5.3 \pm 0.6, 7.2 \pm 0.4)$ km s$^{-1}$, while those obtained in [17] using the data of the catalog [13] by extrapolating the dispersions of the residual velocities to zero are $(8.7 \pm 0.5, 6.2 \pm 2.2, 7.2 \pm 0.8)$ km s$^{-1}$. If the initial assumption of the constancy of them an orbital radii of stars is valid, the method for determining the motion of the Sun relative to the LSR used in the current study should be more trustworthy and free of systematic shifts, which are unavoidable in the case of extrapolation or the use of samples containing stars of diverse ages that were born at various distances from the Galactic center.

The age dependences for the velocity ellipsoids also changed appreciably. For example, the power-law indices for the age dependences of the major, middle, and minor semiaxes became equal to $0.26 \pm 0.04$, $0.32 \pm 0.03$, and $0.07 \pm 0.03$, respectively. Thus, as before, the values of $\gamma_1$ and $\gamma_2$ can be explained in a natural way by relaxation processes associated with stochastic spiral density waves [18]. The very small value of the power-law index for the velocity dispersion perpendicular to the Galactic plane suggests that "heating" by spiral density waves alone may be sufficient to explain this component as well. Recall that, to explain the large value of this index obtained earlier, we were led to invoke "heating" of the stars by molecular clouds [19], or even clusters of dark matter from disrupted satellite galaxies under the action of the tidal forces of our Galaxy [20]. Thus, we have shown that, with increasing age, the velocity ellipsoids for thin-disk stars
born at the solar Galactocentric distance increase only in the plane of the disk, while they remain virtually constant perpendicular to the disk. The shape of the ellipsoid remains always far from equilibrium, and the direction of its major semi-axis does not vary with age – the vertex deviation for the ellipsoid is constant and equal to zero within the errors $\langle L \rangle = 0.7^\circ \pm 0.6^\circ$ and $\langle B \rangle = 1.9^\circ \pm 1.1^\circ$. Both coordinates of the apex of the solar motion remain age independent within the errors, having values $\langle l_{\odot} \rangle = 70^\circ \pm 7^\circ$ and $\langle b_{\odot} \rangle = 41^\circ \pm 2^\circ$.

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Figure 1: Dependence on the mean orbital radius of (a) the semi-axes of the velocity ellipsoids and the total residual velocity, (b) the ratios of the ellipsoid semi-axes, (c) the vertex coordinates, (d) the components of the solar velocity relative to the corresponding centroids, and (e) the coordinates of the apex of the solar motion for nearby thin-disk stars. The solid lines are regression fits, near which are indicated the correlation coefficients and their uncertainties. No regression fits are shown in (e) due to the non-linear character of the dependences. Errors are presented everywhere, but in some cases, the formal parameter errors are smaller than the symbols.
Figure 2: Metallicity dependences for (a) the V component and (b) the longitude of the apex of the solar motion relative to the corresponding centroids for four groups of thin-disk stars in narrow ranges of mean orbital radii: 1 $R_m < 7.37$ kpc, 2 $7.37 < R_m < 7.72$ kpc, 3 $7.72 < R_m < 8.10$ kpc, 4 $R_m > 8.10$ kpc. Notation is the same as in Fig. 1.
Figure 3: Age dependences of the semi-axes of the velocity ellipsoids and the total residual velocity (upper row) and of the components of the solar velocity relative to the corresponding centroids (lower row) for thin-disk stars within four intervals of mean orbital radii (indicated above). The solid curves in the upper plots show power-law fits to the data points; the solid curves in the lower plots show linear fits, and the correlation coefficients and their errors are presented. The bars show the uncertainties in the corresponding quantities.
Figure 4: Age dependences for the velocity ellipsoid semi-axes and the total residual velocity (upper row) and for the components of the solar velocity relative to the corresponding centroids (second row) for thin-disk stars divided into two metallicity groups (indicated above) and for each group divided into two subgroups (indicated above). The captions and notation are as in Fig. 3.
Figure 5: Age dependences of the velocity ellipsoids for single F–G thin-disk stars with age uncertainties $\epsilon_t < \pm 3$ billion years, selected using our criterion and excluding stars in moving groups and in the narrow interval of mean orbital radii $7.70 < R_m < 8.40$ kpc. Presented are the age dependences for (a) the velocity-ellipsoid semi-axes and total residual velocity, (b) the ratios of the ellipsoid semi-axes, (c) the vertex coordinates, (d) the solar velocity components relative to the corresponding centroids, and (e) the coordinates of the apex of the solar motion. Notation and captions are as in Fig. 3.