Preparing the bound instance of quantum entanglement

J. DiGuglielmo,1 A. Samblowski,1 B. Hage,1 C. Pineda,2,3 J. Eisert,4,3 and R. Schnabel1

1Institut für Gravitationsphysik, Leibniz Universität Hannover, 30167 Hannover, Germany
2Instituto de Física, Universidad Nacional Autónoma de México, México
3Institute of Physics and Astronomy, University of Potsdam, 14476 Potsdam, Germany
4Dahlem Center for Complex Quantum Systems, Freie Universität Berlin, 14195 Berlin, Germany
(Dated: February 22, 2012)

Among the possibly most intriguing aspects of quantum entanglement is that it comes in “free” and “bound” instances. Bound entangled states require entangled states in preparation but, once realized, no free entanglement and therefore no pure maximally entangled pairs can locally be regained. Their existence hence certifies an intrinsic irreversibility of entanglement in nature and suggests a connection with thermodynamics. In this work, we present a first experimental unconditional preparation and detection of a bound entangled state of light. We consider continuous-variable entanglement, use convex optimization to identify regimes rendering its bound character well certifiable, and realize an experiment that continuously produced a distributed bound entangled state with an extraordinary and unprecedented significance of more than ten standard deviations away from both separability and distillability. Our results show that the approach chosen allows for the efficient and precise preparation of multi-mode entangled states of light with various applications in quantum information, quantum state engineering and high precision metrology.

PACS numbers: 03.67.Mn, 03.67.-a, 03.65.Ta, 03.65.Ud

The preparation of complex multi-mode entangled states of light distributed to two or more parties is a necessary starting point for applications in quantum information processing [1] [2] as well as for fundamental physics research. An aggressively pursued example of the latter is the preparation of the bound instance of entanglement, a type of entanglement that can only exist in higher-dimensional or multi-mode quantum states [3]. Bound entanglement is fundamentally interesting since, in contrast to “free” entanglement, it can not be distilled to form fewer copies of more strongly entangled pure states [3] by any local device allowed by the rules of quantum mechanics. This irreversible character has triggered entire theoretical research programmes [4], in particular by linking entanglement theory to a thermodynamical picture, with this irreversibility reminiscent of—but being inequivalent with—the second law of thermodynamics [5]. In order to investigate such connections both new theoretical as well as experimental means of constructing multi-mode states must be innovated.

In recent years, great progress in information processing, metrology and fundamental research has actually been achieved in the photon counting (discrete variable, DV) regime using postselection [1]. States of light are the optimal system for entanglement distribution because they propagate fast and can preserve their coherence over long distances. Postselection means that the measurement outcome of the detectors which characterizes the quantum state is also used to select the state, conditioned on certain measurement outcomes. In such an approach, conditional applications are possible, however, an unconditional application of the states in downstream experiments is conceptually not possible. Another limitation that any postselected architecture will eventually face is that without challenging prescriptions of measurement, quantum memories and conditional feedforward, the preparation (post-selection) efficiency will exponentially decay with an increasing number of modes. In parallel to postselected architectures of light, unconditional platforms for research in quantum information have been developed which build on the detection of position and momentum like variables having a continuous spectrum and a Gaussian statistics. In such platforms the preparation efficiency of one mode is identical to the preparation efficiency of N modes. In the past, this continuous variable (CV) platform has been used to demonstrate the Einstein-Podolsky-Rosen (EPR) paradox [6] and unconditional quantum teleportation [7]. Recently, the CV platform has been extended to investigate multimode entangled states [8]; however, the significance of their nonclassical properties have typically been smaller compared to their postselected counterparts.

In this work, we demonstrate the continuous unconditional preparation of one of the rarest types of multi-mode entangled states — bipartite bound entangled states — using the CV platform. The property of bound entanglement is verified by four downstream balanced homodyne detectors with a detection efficiency of almost unity. Alternatively, our setup can make available bound entangled states for any downstream application. The bound entanglement is generated with unprecedented significance, i.e., with state preparation error bars small with respect to the distance to the free entanglement regime and with respect to the distance to the separability regime. Our result is achieved by the convex optimization of state preparation parameters, and by introducing the experimental techniques of single-sideband quantum state control and classical generation of hot squeezed states.

The first ever generation of bound entangled states was claimed in 2009 [9]. This work used photon counting and postselection, however, the data presented did not support this claim, an issue which has been addressed in a comment, see Ref. [10]. In Ref. [11] a DV nuclear magnetic resonance state
whose density matrix has a small contribution of bound entanglement has been observed. Such a state has been called a “pseudo-bound entangled state”. Very recently, the actual first bound entangled states have been generated in two experiments, both on the basis of discrete variables. In Ref. [12] bipartite bound entangled states of trapped ions have been verified by the unconditional detection of resonance fluorescence. In Ref. [13] the first bound entangled states of light have been generated, albeit of multipartite and not of bipartite nature. Similar to Ref. [9], photon counting and postselection have been used. An unconditional application of the distributed entanglement in a downstream experiment is hence not possible. This is now made possible in our work, with a significance of the presence of bound entanglement. Since the studied states interfere very nearby. Optimal entanglement witnesses can be efficiently constructed for Gaussian states [10], yet to maximize the distance of an optimal hyperplane separating separable states to the boundary of non-distillable states—hence maximizing robustness of a prepartion—is a non-convex difficult problem. What is more, a reasonable compromise with the preparation complexity has to be found, with a surprisingly simple feasible scheme being shown in Fig. 1.

We now present the measures required for verifying the presence of bound entanglement. Since the studied states are Gaussian they are fully described by their first—which will not play a role here—and second moments, specified by the covariance matrix of a state \( \rho \) [17–19]. We define a set of quadratures for each optical mode given by \( \hat{x}_j = (\hat{a}_j + \hat{a}_j^\dagger)/2^{1/2} \) and \( \hat{p}_j = -i(\hat{a}_j - \hat{a}_j^\dagger)/2^{1/2} \) where \( \hat{a}_j, \hat{a}_j^\dagger \) are the annihilation and creation operators, respectively. Collecting these \( 2n \) coordinates in a vector \( \vec{O} = (\vec{x}, \vec{p}) \), we can write the commutation relations as \( [\hat{O}_j, \hat{O}_k] = i\sigma_{j,k} \), where \( h = 1 \) and is a matrix \( \sigma \) often known as symplectic matrix. The second moments are embodied in the \( 2n \times 2n \) covariance matrix

\[
\gamma_{j,k} = 2\text{Re} \text{tr} \left( \rho (\hat{O}_j - d_j)(\hat{O}_k - d_k) \right),
\]

with \( d_j = \text{tr}(\rho \hat{O}_j) \), giving rise to a real-valued symmetric matrix \( \gamma \), see supplementary material.

Verification of bipartite bound entanglement requires showing that the state is entangled (inseparable) with respect to a bipartition of the modes and that the state remains positive under partial transposition \([3, 14, 15]\) proving that the state is not distillable. The state is said to be entangled if no physical covariance matrices \( \gamma_A \) and \( \gamma_B \) exist of states in modes \( A \) and \( B \), respectively, so real matrices satisfying \( \gamma_A \gamma_B \geq -i\sigma \), such that \( \gamma_A \geq \gamma_A \oplus \gamma_B \). This idea suggests a natural entanglement measure \([20]\) for Gaussian states, defined as the solution of

\[
E(\gamma) = 1 - \max_x \gamma_A \gamma_B \geq -i\sigma.
\]

Non-distillability can be tested by evaluating the partial transposition of a state \([3]\) which physically reflects time reversal. For covariance matrices, partial transposition amounts to changing the sign of momentum coordinates or by applying the operation \( \gamma^T = M\gamma M \), where \( M = (1, 1, 1, 1, -1, 1, -1) \), with \( a = -1 \) in all momentum coordinates belonging to \( B \). A covariance matrix \( \gamma \) is said to be PPT if its partial transpose is again a legitimate covariance matrix, or equivalently, \( \gamma^T + i\sigma \geq 0 \). A measure as to the quantitative extent a state is PPT can be taken to be the minimum eigenvalue of this matrix,

\[
P(\gamma) = \min \text{eig}(\gamma^T + i\sigma).
\]

The continuity of the eigenvalues with respect to variations in the matrix are enough to guarantee that the measure is meaningful. A strictly positive value of \( P(\gamma) \) unambiguously certifies that the state is not distillable.

Based on our theoretical parameter search our final experimental setup is realized as shown in Fig. 1. In total three optical parameter amplifiers (OPAs), three phase-gates, consisting of a beam splitter and a piezo mounted mirror, and a vacuum mode are utilized as the base setup. The four homodyne detectors are only necessary for the verification of bound entanglement but not for its preparation. We set our OPAs to produce the minimum and maximum vacuum noise normal-
modes), using the methods described in Ref. [21]. Indeed, the passive optics following the sources can no longer alter the eigenvalues of $\sigma \gamma$, which also define the degrees of squeezing and mixedness of the state. Hot squeezing therefore appears to give rise to a necessary ingredient of quantum and classical correlations in order to create robust bound entangled states. We demonstrate that the same state can also be prepared in a purely classical way by applying a local random displacement on the phase quadrature of a vacuum mode while parametrically amplifying the state’s amplitude quadrature. The stationary random phase modulation is produced by using an EOM (electro-optical modulator) driven with the output from a homodyne detector measuring shot noise. The amplitude modulation is generated by operating OPA$_1$ in Fig. 1 in amplification mode, effectively anti-squeezing the amplitude quadrature and deamplifying the thermal noise phase quadrature of the input state. In principle the random amplitude noise of the first input mode can also be provided by a second homodyne detector and an amplitude modulator, thereby replacing the parametric OPA$_1$ device. We note that pseudo-random numbers could be insufficient in this scheme since they could introduce artificial correlations and a non-stationary noise into the final state.

In order to hit the tiny regions in parameter space where bound entanglement does exist we introduce to our setup a new technique for precisely controlling phase-gates at arbitrary angles. This method relies on an optical single-sideband scheme (see supplementary material) that can be used to arbitrarily and independently set the working point of both a phase-gate network and multiple homodyne detectors. This scheme reduces setting the relative phase between interfering modes to selecting the electronic demodulation phase used in the control loop. A portion of the light leaving the phase-gates, PG1-3 in Fig. 1, is redirected to control photodetectors. We are able to derive a strong error-signal by tapping pseudo-randomly sampled from the total $10^4$ bootstrap correlation matrices, showing that they are significantly far away from the boundary of covariance matrices allowed by the uncertainty principle.

The four balanced homodyne detectors are used for the full tomographic reconstruction of the covariance matrix. The results of the reconstruction are used to evaluate two characteristics of the state: namely, its entanglement $E$ (2) and its PPTness $P$ Eq. [3]. In order to build the statistics of these characteristics we first continuously recorded 4 million data points from the amplitude and phase quadratures of each mode. Using the bootstrapping method, we then randomly sampled from the total 4 million points, with uniform distribution, points that were different, and produced a series of covariance matrices from which the entanglement, PPT and physical properties were calculated. Our results are represented in Fig. 2 by the black points. The cross corresponds to the average state inferred from the total data set. The ab-

**FIG. 1.** Experimental setup: The experiment is composed of three optical parametric amplifiers (OPA$_{1-3}$), three actively controlled piezo mounted mirrors forming phase-gates (PG1 – 3) and four homodyne detectors which are independent of the preparation. The inset shows the construction of an OPA as a non-linear crystal inside a resonator producing a spatial TEM$_{00}$ (transverse electro-magnetic) mode. The bound entangled state is obtained through the bipartite splitting such that Alice and Bob each possess two of the four modes.

**FIG. 2.** Experimental results: The state measured after 4 million sets of raw quadrature data points yields the entanglement $E$ and non-distillability $P$ indicated by the red cross. Other $10^4$ points are obtained by bootstrapping the original 4 million data points and show that we are $16\sigma$ away from separability and $46\sigma$ away from distillability. In the inset we depict the minimum eigenvalue of $\gamma + i \sigma$ of each of the $10^4$ bootstrapped correlation matrices, showing that they are significantly far away from the boundary of covariance matrices allowed by the uncertainty principle.
Nexactly the same efficiency as detecting ample, one squeezed mode with one homodyne detector has number of entangled modes. That is to say, detecting, for ex-
form the state preparation efficiency does not depend on the 
ous variable platform for the precise engineering of complex 
pictures of entanglement be studied experimentally. 

can be tested, as well as the applicability of thermodynamical 
noise [23] and the ineffectiveness of distillation schemes [24] 
ence on bound entangled states due to photon loss and phase 
be kilometers apart using optical fibers [22]. The decoher-
ence on bound entangled states due to photon loss and phase 
noise [23] and the ineffectiveness of distillation schemes [24] 
can be tested, as well as the applicability of thermodynamical 
pictures of entanglement be studied experimentally. 
Our results clearly exemplify the potential of the continu-
ous variable platform for the precise engineering of complex 

multi-mode states of light. We underline that using this plat-
form the state preparation efficiency does not depend on the 
number of entangled modes. That is to say, detecting, for ex-
ample, one squeezed mode with one homodyne detector has 
exactly the same efficiency as detecting N squeezed states 
with N homodyne detectors simultaneously. Furthermore, we 
estimate our total quantum detection efficient to be between 
90-95% being already considered in the preparation of bound 
extangle. Alternatively, this loss could be mapped di-
rectly onto the measured state by inclusion of neutral density 
filters, and verification with perfect detectors would reveal the 
same statistics as depicted in Fig. 2.

We believe that the precise and unconditional preparation of (bi-partite) bound entangled states of light demonstrated 
uplifts the theoretical and experimental research on the link 
between entanglement theory and statistical physics. From 
a more general and also technological perspective, the high 
efficiency and the high degree of control in multimode 
quantum state preparation achieved certainly promotes the 
application of the unconditional continuous variable platform 
for the preparation of quantum states of light for fundamental 
research as well as quantum metrology.

This work has been supported by the EU (QESSENCE, 
MINOS, COMPAS), the EURYI, the grant UNAM-PAPIIT 
IN117310 and by the Centre for Quantum Engineering and 
Space-Time Research, QUEST. We acknowledge discussions 
with P. Hyllus and M. Lewenstein at an early stage of this 
project.

Appendix A: Heisenberg uncertainty and entanglement criteria

Explicitly, for n modes the symplectic matrix σ reads as

\[ \sigma = \bigoplus_{j=1}^{n} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \] (4)

The Heisenberg uncertainty relation, expressed in terms of the 
covariance matrix [17], is given by

\[ \gamma + i \sigma \geq 0. \] (5)

Such operator valued inequalities \( A \geq B \) for Hermitian \( A \) 
and \( B \) always refer to operator ordering, meaning that the real 
eigenvalues of \( A - B \) are non-negative. The entanglement 
measure \( E \) for covariance matrices defined in the main text in-
deed indicates entanglement in states [20], and for two modes this is essentially nothing but the familiar negativity [25].

In the discussion of the main text we show that the spectrum 
of \( \gamma + i \sigma \) is bounded from below by \( \varepsilon > 0 \), hence manifesting 
the Heisenberg uncertainty principle. It is worth mentioning 
that this also means that the smallest symplectic eigenvalue 
\( s_1(\gamma) \) of \( \gamma \) is bounded away from 1. In the experiment, we 
also test whether the reconstructed covariance matrix satisfies 
the Heisenberg uncertainty relation as this is a test if the ma-
trix corresponds to a physical state. Unphysical states might 
occur if the error bars of the quantum state preparation or the 
tomographic characterization are too large.

Identifying robust bound entangled states

The relative volume of bound entangled states compared to 
all states is very small under every reasonable measure, and 
any verification as pursued here necessarily requires a careful 
analysis as to what parameter set is most suitable. In this sub-
section, we report techniques that have been used to identify 
regimes of robust bound entangled states.
Consider a general 4 mode correlation matrix, expressed in bipartite normal form \[27\]:

\[
\gamma = \begin{pmatrix}
\lambda_1 & 0 & 0 & 0 & \lambda_5 & 0 & \lambda_9 & \lambda_{10} \\
0 & \lambda_1 & 0 & 0 & 0 & \lambda_6 & \lambda_{11} & \lambda_{12} \\
0 & 0 & \lambda_2 & 0 & \lambda_{13} & \lambda_{14} & \lambda_7 & 0 \\
0 & 0 & 0 & \lambda_2 & \lambda_{15} & \lambda_{16} & 0 & \lambda_8 \\
\lambda_5 & 0 & \lambda_{13} & \lambda_{15} & \lambda_3 & 0 & 0 & 0 \\
0 & \lambda_6 & \lambda_{14} & \lambda_{16} & 0 & \lambda_3 & 0 & 0 \\
\lambda_9 & \lambda_{11} & \lambda_7 & 0 & 0 & 0 & \lambda_4 & 0 \\
\lambda_{10} & \lambda_{12} & 0 & \lambda_8 & 0 & 0 & 0 & \lambda_4
\end{pmatrix}.
\] (6)

The 16 parameters can be seen as describing a manifold in \(\mathbb{R}^{16}\). We sample uniformly the hypercube \([-1/2, 1/2]^{16}\) until we get a bound entangled state. The evaluation of the \(P\) measure for a given covariance matrix \(\gamma\) amounts to solving an eigenvalue problem, that of the degree of entanglement \(E\) to solving a semi-definite problem. In practice, the latter problem can also be performed in the dual space of witnesses that are quadratic polynomials in the quadratures, as explored in Ref. [16]. Once an instance is found we construct a random walk in order to improve the robustness, by (i) displacements in the direction of the axis by a small amount \(\Delta \lambda = 0.01\) and (ii) rotations, by the same angle, in each of the \(16 \times 15/2\) two dimensional Cartesian planes; the new covariance matrix is accepted if the new corresponding state has a larger degree of entanglement and is more significantly a PPT state as measured by \(E\) and \(P\), respectively. The most suitable state found (after several hundred hours of computer time), as quantified by the biggest value of \(\min\{E(\gamma), P(\gamma)\}\), is characterized by an entanglement value of \(E(\gamma) = 0.054\) and \(P(\gamma) = 0.132\), giving an idea of the limiting values that one can achieve.

However, experimentally it is too expensive to engineer a state with an arbitrary correlation matrix. We thus construct a circuit which, starting from a product of noisy Gaussian single mode states, can produce bound entangled states, but is simple enough to be producible in the lab with available technology. A (non-unique) example of such a circuit is plotted in Fig. 1. The resulting scheme is a result of a variation within the given parameterized family of circuits – again using a random walk approach as described above, but now on the physically feasible set of covariance matrices by parametrizing each of the optical components – maximizing the statistical significance of being bound entangled by running semi-definite problems in each step. Afterwards we filter the results allowing only those which require achievable values of squeezing at the input and which only require a single mode with hot squeezing, as this is also a precious resource that, at the moment, can only be input in a single mode. Within the resulting states we choose the most robust according to the aforementioned criteria.

Appendix B: Details of the experiment

The three OPAs used to produce the underlying quadrature squeezing at sideband frequency of 6.4 MHz were constructed from a type I non-critically phase-matched MgO:LiNbO\(_3\) crystal inside a standing wave resonator, similar to the design that previously has been used in Ref. [26]. They were pumped with approximately 100 mW of green light at 532 nm each resulting in a classical gain of about 5. The length of the OPA cavity as well as the phase of the second harmonic pump beam were controlled using radio-frequency modulation/demodulation techniques.

Balanced homodyne detection was performed on each of the four modes in order to reconstruct the \(8 \times 8\) covariance matrix. The optical local oscillator was filtered through a three mirror ring cavity operated in high finesse mode resulting in a linewidth of 55 kHz. The detector difference currents were electronically mixed with a 6.4 MHz local oscillator and low-pass filtered with a 400 kHz bandwidth. The dark noise separation from shot noise was measured to be more than 10 dB for each detector. The raw data was acquired using a 14 bit National Instruments DAQ-card and in total eight measurement settings including the shot noise measurement were required in order to reconstruct the covariance matrix.

The hot squeezed states were generated by randomly phase modulating the control beam used to set the length of the OPA cavity at the squeezing sideband frequency, 6.4 MHz, and locking the OPA cavity in amplification. This produces phase squeezed states whose smallest quadrature can be controlled by varying the strength of the random noise modulated on the control field and whose amplitude quadrature is controlled by the degree of classical gain.

The single-sideband was generated by overlapping the output of a second laser operating at around 1064 nm with the bright output of OPA1. The beams were phase-locked at a beat frequency of 15 MHz resulting in a field that corresponds to both a phase and amplitude modulation. The beat was detected by directing approximately 1% of the phase-gate outputs to photodetectors placed behind the phase-gates as well as in each homodyne detector. The relative phase between the carriers at both the phase-gates and the homodyne detectors could then be set to an arbitrary phase simply by changing the demodulation phase of the electronic local oscillator. We estimate a phase sensitivity at each phase-gate to be approximately 2 deg.

Appendix C: Discussion of the Gaussian character of the state

The findings of our work show with a remarkable and unprecedented statistical significance that the second moments are certified to be those of Gaussian bound entangled states. What is more, a perfectly Gaussian statistics is clearly expected due to the underlying physical mechanisms of our setup, i.e., parametric amplification of vacuum states, their superposition on beam splitters, time-independent linear losses and phase shifts, and finally balanced homodyne detection. In this subsection, however, we highlight the extent to which the states are indeed Gaussian states, and carefully discuss some theoretical issues associated with the certification of bound
entanglement for infinite-dimensional quantum systems.

In the course of our work we have taken time series data from the homodyning measurements and have not only estimated the second moments (leading to the covariance matrix specified in the main text), but in fact all moments. For each moment, we have tested for the Gaussian character of the state. Indeed, such studies of Gaussianity are interesting in their own right. More precisely, for our experiment, we have computed Q-Q-plots for a measured distribution against a Gaussian one, comparing these two probability distributions by plotting their quantiles against each other. Fig. 3 shows a representative sample Q-Q-plot of this kind. In each case the Gaussian character is verified to a remarkable level of statistical significance. We also performed a $\chi^2$ test which also confirmed normality. Hence, one can conclude that the prepared is indeed Gaussian to a very high degree of accuracy.

Having said that, strictly speaking, one may argue that in infinite-dimensional Hilbert spaces, the free-entangled states are (trace-norm) dense in state space [28, 29]. Hence, in the vicinity of every bound entangled Gaussian state one can find a free entangled state that is operationally indistinguishable. This is as such no surprise at all: In the same way, it is true that arbitrarily close to any separable state there is a free entangled state for continuous-variable systems. However, in quantitative terms, the degree of free entanglement will indeed be negligible [30]. That is to say, the measured mean energy and the closeness to Gaussian states readily give rise to rigorous bounds to the distillable entanglement of the unknown state. Therefore, even in quantitative terms, one can falsify the proposition that a significant distillable entanglement is found – again strengthening the claim made in the main text.

FIG. 3. Q-Q-plot of the sample quantiles of a measured 4 million point data set versus the theoretical one for a Gaussian distribution based on the measured second moments (red dots). For comparison we also give a Q-Q-plot by plotting quasi-random numbers from a perfect Gaussian distribution (black dots). Such a Q-Q-plot depicts the $q$-th quantile of one distribution against the $q$-th quantile of the other. The diagrams show that the prepared states of our work are Gaussian to a very high level of significance.

[1] J.-W. Pan, C. Simon, C. Brukner, and A. Zeilinger, Nature 410, 1067 (2001); P. Walther et al., Nature 434, 169 (2005). W. Wieczorek et al., Phys. Rev. Lett. 101, 010503 (2008); M. Aspelmeyer and J. Eiscrt, Nature 455, 180 (2008); W.-B. Gao et al., arXiv:1004.4162

[2] J. L. O’Brien, A. Furusawa, J. Vuckovic, Nature Photonics 3, 687 (2009); K. Banaszek, R. Demkowicz-Dobrzanski, and I. A. Walmsley, Nature Photonics 3, 673 (2009); J. C. F. Matthews, A. Politi, D. Boumeau, and J. L. O’Brien, arXiv:1005.5119

[3] M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. Lett. 80, 5239 (1998).

[4] R. Horodecki, P. Horodecki, and M. Horodecki, Rev. Mod. Phys. 81, 865 (2009).

[5] M. Horodecki, J. Oppenheim, and R. Horodecki, Phys. Rev. Lett. 89, 240403 (2002); F. G. S. L. Brandao and M. B. Plenio, Nature Physics 4, 873 (2008).

[6] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. Lett. 47, 777 (1935); Z. Y. Ou, S. F. Pereira, H. J. Kimble, and K. C. Peng, Phys. Rev. Lett. 68, 3663 (1992).

[7] A. Furusawa et al., Science 282, 706 (1998); W. P. Bowen et al., Phys. Rev. A 67, 032302 (2003).

[8] G. Glöckl et al., Phys. Rev. A 68, 012319 (2003); A. M. Lance, T. Symul, W. P. Bowen, B. C. Sanders, and P. K. Lam, Phys. Rev. Lett. 92, 177903 (2004); A. S. Villar, L. S. Cruz, K. N. Cassemiro, M. Martinelli, and P. Nussenzveig, Phys. Rev. Lett. 95, 243603 (2005); X. Su et al., Phys. Rev. Lett. 98, 070502 (2007); M. Yukawa, R. Ukai, P. van Loock, and A. Furusawa, Phys. Rev. A 78, 012301 (2008); S. L. W. Midgley, A. S. Bradley, O. Pfister, and M. K. Olsen, arXiv:1002.2019

[9] E. Amselem and M. Bourennane, Nature Physics 5, 748 (2009).

[10] J. Lavoie, R. Kaltenbaek, M. Piani, K. J. Resch, Nature Physics 6, 827 (2010).

[11] H. Kampermann, D. Bruss, X. Peng, and D. Suter, Phys. Rev. A 81, 040304(R) (2010).

[12] J. T. Barreiro et al., Nature Phys. doi:10.1038/nphys1781 (2010).

[13] J. Lavoie, R. Kaltenbaek, M. Piani, and K. J. Resch, Phys. Rev. Lett. 105, 130501 (2010).

[14] R. F. Werner and M. M. Wolf, Phys. Rev. Lett. 86, 3658 (2001).

[15] G. Giedke, L.-M. Duan, P. Zoller, and J. I. Cirac, J. Quant. Inf. Comp. 1, 79 (2001).

[16] P. Hyllus and J. Eisert, New J. Phys. 8, 51 (2006).

[17] R. Simon, E. C. G. Sudarshan, and N. Mukunda, Phys. Rev. A 36, 3868 (1987).

[18] J. Eisert and M. Plenio, Int. J. Quant. Inf. 1, 479 (2003).

[19] S. L. Braunstein and P. van Loock, Rev. Mod. Phys. 77, 513 (2005).

[20] G. Giedke and J. I. Cirac, Phys. Rev. A 66, 032316 (2002).

[21] See appendix.

[22] M. Mehmet et al., Opt. Lett. 35, 1665 (2010).

[23] A. Franzén, B. Hage, J. DiGuglielmo, J. Fiurasek, and R. Schnabel, Phys. Rev. Lett. 97, 150505 (2006).

[24] B. Hage et al., Nature Physics 4, 915 (2008).
[25] J. Eisert, PhD thesis (Potsdam, February 2001); G. Vidal and R. F. Werner, Phys. Rev. A 65, 032314 (2002); M. B. Plenio, Phys. Rev. Lett. 95, 090503 (2005).
[26] S. Chelkowski, H. Vahlbruch, K. Danzmann, and R. Schnabel, Phys. Rev. A 75, 043814 (2007).
[27] A. Ferraro, S. Olivares, and M. G. A. Paris, Gaussian states in quantum information (Bibliopolis, Napoli, 2005).
[28] R. Clifton and H. Halvorson, Phys. Rev. A 61, 012108 (2000); H. Halvorson and R. Clifton, J. Math. Phys. 41, 1711 (2000).
[29] J. Eisert, C. Simon, and M. B. Plenio, J. Phys. A 35, 3911 (2002).
[30] This can be seen as follows: Bounds of Gaussianity of a state as above imply that $\|\rho - \rho_G\|_1 \leq \varepsilon$ for a small $\varepsilon$, where $\rho_G$ is the bound entangled Gaussian state with known covariance matrix and $\|\cdot\|_1$ denotes the trace-norm. In turn, it has been shown in Ref. [29] that the relative entropy of entanglement $E_R$ with respect to separable states, defined as the infimum $E_R(\rho) = \inf_\omega S(\rho|\omega)$ over separable states $\omega$, is trace-norm continuous for states with finite mean energy. What is more, $E_D(\rho) \leq E_R(\rho)$, so the relative entropy of entanglement is always an upper bound to the distillable entanglement $E_D$. This gives a bound to the distillable entanglement.