A Two-Stage Optimization Scheme of Fuel Consumption and Drivability for Plug-In HEVs

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Abstract: This paper addresses an energy consumption optimization problem with consideration of drivability improvement for series-parallel plug-in hybrid electric vehicles. Consumption of fuel and electricity are chosen as the cost function of the optimization, and the acceleration rate is involved in the constraint for improving drivability. Then, a two-stage optimization scheme is proposed, which consists of the long-term off-line optimization and the short-term on-line optimization. In the former, the entire route is dealt with as previously known, and the optimal mode switching and engine power are provided by solving the optimization problem, and in the latter, the short-term local optimization in the fashion of receding horizon is used to obtain optimality regarding the actual route situation where the vehicle-to-everything-based driver’s demand prediction is involved instead of requiring previous information of the future horizon. The proposed controller was evaluated in the case of a long driving situation including the city and the motorway that required a high-power acceleration. The performance was improved in consideration of the drivability, and the fuel economy was optimized in comparison to the charge depleting and charge sustaining strategy.

Key Words: receding horizon control, fuel economy, real-time optimization, plug-in hybrid electric vehicles (PHEVs).

1. Introduction

Hybrid electric vehicles (HEVs) are in widespread use due to high energy efficiency. A key technology for the HEVs is an energy management control strategy of the electric power from the battery and the internal combustion engine. The main motivation of the energy management is energy efficiency, emission, driving performance, drivability, battery deterioration, and battery safety. In addition, plug-in hybrid electric vehicles (PHEVs) have a high-power, large-capacity battery and have been expected as an enhancement of vehicle electrification. Numerous pieces of literature about the energy management of HEVs and PHEVs have been published. Wirasingha et al. [1] have reviewed the control strategy for PHEVs. Control strategies are classified into two categories, a rule-based controller [2],[3] and an optimization-based controller [4]–[6]. The rule-based approach is easy to implement, fast executable in real-time applications, and reliable for safety concerns. A large percentage of the energy management concept for the series production PHEVs is the charge-depleting, charge-sustaining (CDCS) strategy [7],[8]. However, it is difficult to find efficient rules for the complex hybrid system, and the rule-based approach will take much time to calibrate control parameters. Therefore, much attention has been focused on the optimization-based approach. Dynamic programming (DP) and model predictive control (MPC) are kinds of the optimization control scheme in the automotive practice.

Meanwhile, to use the future velocity profile of the targeted vehicle in optimization has been reported in several pieces of literature. For example, trip-oriented energy management using static traffic information and some surrounding traffic conditions is reported [4],[9],[10]. Macro traffic flow can be represented by a stochastic distribution. The traffic flow is affected by driving route, time zone, traffic congestion, accidents, and constructions [11]–[13]. A short-term vehicle’s velocity trajectory can be predicted using the vehicle’s velocity history and the information provided by the intelligent transport systems (ITS) and on-board sensors such as radar, camera, and the global positioning system (GPS) [14],[15]. The behavior of the driver is affected by psychological and physiological matters and the surrounding traffic environment. The driver’s characteristics consist of his/her skill, personality, and conditions. It is difficult to include all influential factors into the prediction of the driver’s behavior. Therefore, the short-term velocity prediction must have a fuzziness, and the optimization-based approach is dealt with as a stochastic optimal control [16]–[18].

In general, the main objective of the optimization problem for HEVs/PHEVs is to minimize fuel consumption. However, state of charge (SOC) should also be controlled in a range due to its deterioration. SOC of the battery and vehicle velocity are often dealt with from the point of view of the state variables. In addition, necessary acceleration force to satisfy the driver’s demand is realized using some assistance from the battery. Therefore, a constrained optimization problem for a driving cycle is constructed.

The optimal control is classified into the global optimization intended for the entire driving cycle and the local optimization intended for the short-term driving cycle. They are also called non-causal (offline) and causal (real-time) [1],[19],[20]. The global optimization can minimize the stage cost and the terminal cost and satisfy the terminal state constraint for the entire driving cycle. However, it is difficult to react to the real-time change of the traffic environment due to its high calculation effort. Therefore, PHEVs also need the local optimization algorithm on-line to determine the control input [21]. For exam-
ple, the equivalent consumption minimization strategy (ECMS) based on an instantaneous optimal control (IOC) strategy has developed [22]. The IOC implements the optimization using only the current state information and realizes an on-line real-time calculation strategy. The key feature of ECMS is to minimize the instantaneous cost function that consists of the fuel consumption and converted electric power with an equivalent factor. The electric power changes the SOC of the battery. It is known that ECMS can acquire optimal control results similar to the global optimization when the equivalent factor is set appropriately [23]. That is, the equivalent factor plays a key role in the minimization of the fuel consumption and the control of SOC for the entire driving cycle. Numerous pieces of literature have discussed the equivalent factor of ECMS. In [12], the equivalent factor of ECMS is identified using statistic velocity information. In [24], the equivalent factor is defined based on the efficiency of the battery charge and discharge, and the efficiency of the electric motor. In these studies, a fixed velocity trajectory is used to adapt the equivalent factor. In [25], the velocity trajectory is predicted deterministically using ITS and GPS. The future trajectory of charge/discharge energy rate is calculated, and the equivalent factor is adapted depending on the charge/discharge rate. In other words, the equivalent factor should be set depending on the driving cycle. When the optimal energy management is implemented in the real world, the factor should be adapted flexibly.

Some series-parallel hybrid systems have two electric motors and several operation modes [22],[8]. These hybrid systems have a mechanical path and an electric path that are shifted by operation mode control. The system efficiency and the available system power depend on the operation modes. In industrial practice, it is important to construct the optimization problem that considers the fuel economy, drivability, and performance. How to use the hybrid system in energy management is still a challenging issue.

This paper will address the energy management and power management of series-parallel PHEVs using the MPC. In this study, the departure place and destination are known, and the vehicle’s velocity profile of the driving route is defined. The main contributions of this paper are as follows. A problem formulation for real-time optimization including fuel economy, drivability, and performance is proposed in which a two-dimension nonlinear dynamic model coordinated by SOC and vehicle speed is taken into account with physical constraints. And this study introduces long-term optimization in off-line and short-term optimization in on-line. In the long-term optimization, the equivalent factor and hybrid operation mode trajectory are determined by the dynamic programming as the SOC state variable. In the industrial point of view, engine speed behavior as drivability and short time-dependent battery suppression for charge and discharge are considered as constraints. The future vehicle velocity trajectory for a few seconds is predicted and used for MPC. The control input variable and state variables are selected reasonably for the long-term and short-term optimization problems to reduced calculation costs. The proposed controller is demonstrated by a well-verified simulation environment. The verified scene is the autobahn highway in Germany where vehicles are requested high-power in the case of overtaking and high-speed cruising. Therefore, novel energy management is necessary.

This paper is organized as follows. Section 2 describes the modeling of the PHEV system. In Section 3, an optimization problem is constructed. In Section 4, the modeling of the short-term vehicle’s velocity is discussed. In Section 5, optimization problems are reconstructed to the long-term optimization and the short-term optimization, and the proposed controller is explained. In Section 6, simulation-based verification of the proposed controller is implemented, and Section 7 concludes this paper.

2. Modeling of the HEV System

In this section, the mathematical control-oriented model is discussed. Vehicle speed dynamics is represented as follows:

\[ M\ddot{v} = F_{\text{trac}} - F_{\text{load}} - F_{\text{brake}}, \]  
\[ F_{\text{load}} = F_{\text{aero}} + F_r + F_g, \]
\[ F_{\text{aero}} = \frac{1}{2} \rho AC_\text{d}v^2, \]
\[ F_r = Mg \mu \sin \theta, \]
\[ F_g = Mg \sin \theta, \]
\[ v = \omega_w R_w, \]

where \( M \) is the vehicle weight, \( v \) is the vehicle speed, \( F_{\text{trac}} \) is the total tractive force generated by the powertrain at the wheel-road interface, and \( F_{\text{brake}} \) is the mechanical braking force. The running resistance \( F_{\text{load}} \) consists of the aerodynamic drag force \( F_{\text{aero}} \), the gravity force \( F_g \) based on the load gradient \( \theta \), and the rolling resistance force \( F_r \). The aerodynamic drag \( F_{\text{aero}} \) is nonlinear in the quadratic equation of vehicle speed as shown in (3); \( \omega_w \) and \( R_w \) denote the wheel rotation speed and the radius of the wheel, respectively. (1) can be transformed into power balance by multiplying both sides by vehicle speed \( v \) as follows:

\[ Mv\ddot{v} = P_{\text{trac}} - P_{\text{load}} - P_{\text{brake}}, \]

where \( P_{\text{trac}} \), \( P_{\text{load}} \), and \( P_{\text{brake}} \) denote the total tractive power, the running resistance power, and the mechanical braking power. Theoretically, the right hand side of (7) corresponds to the driver’s demand power \( P_{\text{dem}} \).

Figure 1 shows the hybrid system that has two electric motors, one engine, and one battery pack [8],[26]. In this study, the hybrid system can be operated in three traction modes and one deceleration mode explained below. In the electric vehicle (EV) mode, the electric motor is connected to the wheels and driving energy is supplied by the battery. In the HEV mode, the electric motor is driven by electric energy generated by the electric generator and supplied by the battery. The engine serves as power assistance for the electric motor. In the engine drive mode, a clutch located between the engine and wheels is engaged, and the vehicle is propelled by the engine output and the electric motor assistance. In the following subsection, the modeling of the powertrain system is discussed.

2.1 HEV Mode

The power balance equation in the HEV mode is shown in (8). The traction power is supplied from both the engine power...
and the charge/discharge battery power $P_{\text{batt}}$. When $P_{\text{batt}}$ is negative, a part of the power generated by the engine is stored to the battery. When $P_{\text{batt}} \geq 0$, we have

$$P_{\text{true}} = \left( P_{\text{eng}} \cdot \eta_{\text{gen}} + P_{\text{batt}} \cdot \eta_{\text{batt}} \right) \cdot \eta_{\text{mot}}$$

(8)

When $P_{\text{batt}} < 0$, we have

$$P_{\text{true}} = \left( P_{\text{eng}} \cdot \eta_{\text{gen}} + \frac{P_{\text{batt}}}{\eta_{\text{batt}}} \right) \cdot \eta_{\text{mot}}$$

(9)

where $\eta_{\text{gen}}$, $\eta_{\text{batt}}$, and $\eta_{\text{mot}}$ denote the transformation efficiency from the engine power to the electric power by the electric generator, the efficiency of the charge and discharge in the battery, and the transformation efficiency from the electric power to the mechanical power by the electric motor, respectively. The engine power is generated by the engine torque $T_{\text{eng}}$ and speed $\omega_{\text{eng}}$ as shown in (10). In general, $T_{\text{eng}}$ and $\omega_{\text{eng}}$ are controlled in the low brake specific fuel consumption (BSFC) area. In the optimization-based control strategy, $T_{\text{eng}}$ and $\omega_{\text{eng}}$ will be arbitrarily controlled by the optimization results of the cost function and constraint:

$$P_{\text{eng}} = \omega_{\text{eng}} \cdot T_{\text{eng}} \cdot \frac{2\pi}{60 \cdot 1000}$$

(10)

### 2.2 Engine Drive Mode

The power balance equation in the engine drive mode is shown in (11). As is in the HEV mode, the traction power is supplied from both $P_{\text{eng}}$ and $P_{\text{batt}}$. When $P_{\text{batt}}$ is negative, a part of the power generated by the engine is stored to the battery. Because the clutch in Fig. 1 is engaged, the engine is connected to the wheels. When $P_{\text{batt}} \geq 0$, we have

$$P_{\text{true}} = P_{\text{eng}} \cdot \eta_{\text{cl}} + P_{\text{batt}} \cdot \eta_{\text{batt}} \cdot \eta_{\text{mot}}$$

(11)

When $P_{\text{batt}} < 0$, we have

$$P_{\text{true}} = P_{\text{eng}} \cdot \eta_{\text{cl}} + \frac{P_{\text{batt}}}{\eta_{\text{batt}}} \cdot \eta_{\text{mot}}$$

(12)

where $\eta_{\text{cl}}$ denotes the transmission efficiency of the mechanical path from the engine to the wheels. The efficiency $\eta_{\text{cl}}$ is higher than $\eta_{\text{gen}} \cdot \eta_{\text{mot}}$ because the energy conversion between mechanical energy and electric energy causes much energy losses. The engine speed $\omega_{\text{eng}}$ is physically constrained in (13).

$$\omega_{\text{eng}} = \frac{i_{\text{g}}}{R_{\text{e}}} \cdot \frac{1000}{2\pi \cdot 60}$$

(13)

where $i_{\text{g}}$ denotes the overall gear ratio from the engine to the wheels.

### 2.3 EV Mode

The power balance equation in the EV mode is shown in (14). The whole traction power is supplied from the battery, and the engine is stopped. When the $P_{\text{batt}}$ is negative, the powertrain decelerates the vehicle. When $P_{\text{batt}} \geq 0$, we have

$$P_{\text{true}} = P_{\text{batt}} \cdot \eta_{\text{batt}} \cdot \eta_{\text{mot}}$$

(14)

When $P_{\text{batt}} < 0$, we have

$$P_{\text{true}} = \frac{P_{\text{batt}}}{\eta_{\text{batt}} \cdot \eta_{\text{mot}}}$$

(15)

$$P_{\text{eng}} = 0, \omega_{\text{eng}} = 0.$$  

(16)

In this study, the vehicle deceleration is always implemented by the EV mode. In the case that the driver’s demand power $P_{\text{dem}}$ is positive, $P_{\text{brake}}$ is 0. When the $P_{\text{true}}$ can not satisfy the driver’s deceleration demand, $P_{\text{brake}}$ is controlled to compensate the driver’s demand. The mechanical brake power $P_{\text{brake}}$ is consumed by a thermal energy loss:

$$P_{\text{brake}} = P_{\text{dem}} - P_{\text{load}} = \frac{P_{\text{batt}}}{\eta_{\text{batt}} \cdot \eta_{\text{mot}}}.$$ (17)

### 2.4 Engine

The engine model considers the work efficiency in a steady state. BSFC is measured in the engine test bed and expressed as a map function of engine torque $T_{\text{eng}}$ and speed $\omega_{\text{eng}}$. The fuel consumption $\dot{m}_f$ is calculated using (19).

$$\text{BSFC} = \text{MAP(}\omega_{\text{eng}}, T_{\text{eng}}\text{)},$$

(18)

$$\dot{m}_f = \frac{\text{BSFC} \cdot P_{\text{eng}}}{3600}.$$ (19)

### 2.5 Battery

The battery power is characterized by the following dynamic equation in terms of SOC [6],[16]:

$$S \dot{O}C = \frac{-U_{\text{oc}} + \sqrt{U_{\text{oc}}^2 - 4R_b P_{\text{batt}}}}{2Q_{\text{max}} R_b},$$

(20)

where $U_{\text{oc}}$, $R_b$, and $Q_{\text{max}}$ denote the open-circuit voltage, the internal resistance, the maximum battery capacity, respectively.

### 3. Control Problem

The main control objective is to minimize the fuel consumption $\dot{m}_f$ during the prediction horizon. The fuel consumption is a function of the state $x$ and the control input $u$. To consider the electrical energy consumption from the battery as cost function, the idea of ECMS is introduced [24],[27],[28].

$$\min J = \int_{T_{\text{f}}} \left[ \dot{m}_f(x,u) + \lambda \cdot P_{\text{batt}} \right] dt,$$

(21)

$$x = [x_1 \ x_2]^T = [v \ \text{SOC}]^T,$$

(22)

$$u(\tau) = [M(\tau) \ P_{\text{eng}}(\tau) \ \omega_{\text{eng}}(\tau)]^T$$

(23)

subject to

$$\text{SOC}_{\text{min}} \leq x_2(\tau) \leq \text{SOC}_{\text{max}},$$

(24)

$$P_{\text{batt,min}}(x_2) \leq P_{\text{batt}}(\tau) \leq P_{\text{batt,max}}(x_2),$$

(25)
\[
\omega_{\text{eng,min}} \leq \omega_{\text{eng}}(\tau) \leq \omega_{\text{eng,max}}, \quad (26)
\]
\[
P_{\text{eng,min}}(\omega_{\text{eng}}(\tau)) \leq P_{\text{eng}}(\tau) \leq P_{\text{eng,max}}(\omega_{\text{eng}}(\tau)), \quad (27)
\]
where the control inputs are \( M(\tau), P_{\text{eng}}(\tau), \) and \( \omega_{\text{eng}}(\tau) \). The operation mode is denoted by \( M \in [0, 1, 2, 3] \). When the velocity is low, the EV mode or the HEV mode should be selected due to a minimum engine speed constraint. Thereafter, when the vehicle’s velocity is very high, the engine drive mode can generate high acceleration force. When SOC is low, \( P_{\text{batt}} \) will be limited at a low level. Therefore, the control of the state variables \( x_t \) affects the driver’s demand constraint.

The tractable power \( P_{\text{raw}} \) is constrained to satisfy the driver’s acceleration or deceleration (positive and negative value) demand. The maximum tractable power of the powertrain used in this study depends on the operation mode and SOC as shown in Fig. 2. When the vehicle’s velocity is very high, the engine drive mode can generate high acceleration force. When SOC is low, the acceleration force is reduced because the available battery discharge is low. On the other hand, when the vehicle’s velocity is low, the EV mode or the HEV mode should be selected due to a minimum engine speed constraint. Therefore, an appropriate selection of the operation mode and SOC control is necessary to satisfy the driver’s acceleration demand. The above constraints of the driving force generated and the operation mode by the powertrain hardware are common in the industrial practice [8].

The equivalent factor \( \lambda \) converts the electric power to the fuel consumption in the cost function. When \( \lambda \) is low value, the discharge and charge of the battery are underestimated. Therefore, EV mode or the assistance from the battery decreases the cost function. On the other hand, when \( \lambda \) is high value, the battery charge using the engine power decreases the cost function. It is difficult to set a constant value of \( \lambda \). However, as shown in (28), an available value range of \( \lambda \) can be considered [29]:

\[
\frac{1}{Q_{\text{hov}}} \leq \lambda \leq \frac{1}{Q_{\text{hov}}} \cdot \frac{\eta_{\text{eng}} \cdot \eta_{\text{batt}} \cdot \eta_{\text{mot}}}{\bar{\eta}_{\text{eng}}}, \quad (28)
\]
where \( Q_{\text{hov}} \) denotes the lower heating value and is equal to 44.4 MJ/kg. The average values of the efficiency are denoted by \( \eta_{\text{eng}}, \eta_{\text{batt}}, \) and \( \eta_{\text{mot}} \), respectively. To optimize the efficiency of the system, the equivalent factor \( \lambda \) should be optimized for the entire driving cycle. In this study, \( \lambda \) is determined by dynamic programming and stored in a space-domain dependent function. Section 5.1 discusses the detail.

In general, the usage of deeper depth of discharge (DOD) causes the battery deterioration. The permitted battery charge and discharge power are constrained to guarantee the endurance reliability. In this study, the maximum charge and discharge power of the battery is controlled based on the continuous charge and discharge limitation using the simple time-dependent boundary functions \( B_c \) and \( B_d \) as shown in (29) and (30).

For the battery charge, when \( P_{\text{batt}}(t) \leq P_{\text{cont,batt,min}}(x_t) \), we have

\[
\frac{dB_c}{dt} = \kappa_c \left( P_{\text{cont,batt,min}}(x_t) - P_{\text{batt}}(t) \right) \left( 1 - \frac{P_{\text{cont,batt,min}}(x_t)}{P_{\text{batt}}(t)} \right), \quad (29)
\]

For the battery discharge, when \( P_{\text{batt}}(t) \geq P_{\text{cont,batt,max}}(x_t) \), we have

\[
\frac{dB_d}{dt} = \kappa_d \left( P_{\text{cont,batt,max}}(x_t) - P_{\text{batt}}(t) \right) \left( 1 - \frac{P_{\text{cont,batt,max}}(x_t)}{P_{\text{batt}}(t)} \right), \quad (30)
\]
where \( P_{\text{cont,batt,min}} \) and \( P_{\text{cont,batt,max}} \) denote the specification of the continuous charge and discharge limitation depending on the SOC, respectively. \( \kappa_c \) and \( \kappa_d \) denote the charge restriction coefficient and the discharge restriction coefficient, respectively, which are identified using an industrial detailed battery model.

4. Short-Term Velocity Prediction

This section discusses how to predict the ego vehicle’s velocity trajectory a few seconds ahead. First, the vehicle velocity model is established with machine learning, and the vehicle velocity at the next discrete time \( v_{t+1} \) is predicted. In this study, a Gaussian process regression (GPR) model is used to predict the ego vehicle velocity. GPR is a stochastic non-parametric kernel based model and can represent a non-linearity behavior. In addition, even relatively small training data is valid for the modeling. The structure of GPR is shown in (31). The \( n \) length training data \( [(x_i, y_i); i = 1, 2, \ldots, n] \) are collected from the vehicle simulation of a driver in advance, where \( x_i \in \mathbb{R}^d \) is a \( d \)-dimensional predictor vector, \( y_i \in \mathbb{R} \) is observed output, \( f(x_i) \) is a predictive distribution, \( h(x_i) \) is a basis function, \( \beta \) is a weight factor vector, and \( \sigma^2 \) is variance. The output \( y_i \) is the pedal action \( v_{t+1} \). When a new output \( y^* \) is predicted with new predictor \( x^* \), a new variance \( \sigma^2 + \beta \) is also predicted depending on the predictor \( x^* \).

\[
P(y_i|f(x, x_i), \beta) = N(y_i|h(x_i)^T \beta + f(x_i), \sigma^2). \quad (31)
\]

The inputs for the prediction of the ego vehicle’s velocity are the following.

![Fig. 2 The available acceleration rate on flat road in the case of the operation mode (EV mode, HEV mode, engine drive mode).](image)
1. \( v \): ego vehicle’s velocity history
2. \( vl \): regulatory speed limit
3. \( dv_p \): relative speed to predecessor vehicle
4. \( d \): relative distance to the predecessor vehicle, or the stop sign or the red light
5. \( sl \): slope of the road

If more complex modeling is necessary, the below inputs will be considered.

1. predecessor vehicle speed
2. predecessor vehicle acceleration
3. relative speed to rear vehicle
4. distance to the next turn
5. radius of the next turn

Machine learning is performed using a fixed volume of time series data. In this study, it is assumed that the acquired sensor and traffic information are perfectly accurate. In order to increase the accuracy of prediction, these inputs for the prediction are edited into the input vector shown as in (32). The input vector consists of the historical vehicle’s velocity and the regulatory speed limit, relative speed to predecessor vehicle, relative distance to the front object, and slope of the road at the current time step \( t_k \). In this study, the historical velocity goes back to the seven steps ago.

\[
\mathbf{x}_t = \begin{bmatrix} v(t_{k-1}) & dv_p(t_k) & vl(t_k) & d(t_k) & sl(t_k) \end{bmatrix}.
\]

Figure 3 shows the prediction model for the velocity \( v(t_{k+1}) \). The predicted velocity \( v(t_{k+1}) \) is used as the input for prediction in the next time step. The relative distance from the object in front \( d \) is also updated depending on the predicted velocity. From the time step \( k \) to \( k + hp \), the ego vehicle’s velocity is predicted sequentially. Therefore, the longer prediction horizon is adopted, the more prediction error will accumulate. The appropriate length of the prediction horizon depends on the accuracy of the velocity prediction. The GPR model is identified using Statistics and Machine Learning Toolbox in MATLAB®.

Note that the proposed GPR model structure can be used for any driver and any driving environment because the inputs of the model are the information of the road, the traffic, and the historical velocity. It should be noted that prediction with the GPR model provides a probability distribution of the predicted signal. This means that if we use the expectation of the distribution as prediction, there is still uncertainty in the prediction. Therefore, when the actual value of the predicted signal is different from the expectation, the proposed control cannot guarantee the targeted optimality, even this situation happens with a small probability. This is a limitation when using GPR in practice.

Figure 4 shows the predicted vehicle’s velocity during prediction horizons and the actual velocity. The prediction horizon is 10 seconds. The predicted velocity shows high accuracy except for the scene of large velocity change. Figure 5 shows the histogram, the probability density function (PDF), and the cumulative density function (CDF) of prediction error that is evaluated for city and highway driving simulation. It can be observed that about 90% of CDF is ±10 km/h in this case.

5. Long-Term and Short-Term Optimization Problem

5.1 Long-Term Optimization Problem

The objective of the long-term optimization is to generate the trajectory of the SOC \( x_2 \), the operation mode \( M \), and the equivalent factor \( \lambda \) that minimize the fuel consumption for the entire driving cycle. The control input is \( M, \lambda, \) and the engine power \( P_{eng} \). In order to reduce the computation effort of the short-term optimization (on-line), the optimized \( M \) and \( \lambda \) are used as the fixed input of the short-term optimization.

\[
\min J_G = \phi_G(x_2^N) + \sum_{t=0}^{N-1} \{R_f + \lambda \cdot P_{batt}\},
\]

\[
x = [x_2]^T = [SOC]^T,
\]

\[
\phi_G(x_2^N) = [SOC_{end} - x_2^N],
\]

\[
u(t_k) = [M(t_k) \lambda(t_k) P_{eng}(t_k)]^T,
\]
subject to
\[ SO_{C\min} \leq x(t_k) \leq SO_{C\max}, \quad k = 0, 1, \ldots, N, \]
\[ P_{\text{batt,min}}(x_k) \leq P_{\text{batt}}(t_k) \leq P_{\text{batt,max}}(x_k), \]
\[ M(t_k) \in \{0, 1, 2, 3\}, \]
\[ \frac{1}{Q_{\text{inv}}} \leq \lambda(t_k) \leq \frac{1}{Q_{\text{inv}}}, \]
\[ r_{\min} \leq r_{g} \leq r_{\max}, \]
\[ P_{\text{eng,min}}(\omega_{\text{eng}}(t_k)) \leq P_{\text{eng}}(t_k) \leq P_{\text{eng,max}}(\omega_{\text{eng}}(t_k)), \]
where \( \Phi \) is the terminal cost of the SOC error, \( SO_{C\text{end}} \) is a constant value that is the same as the target value of \( CS \) mode in the CDCS strategy. The SOC is constrained by the minimum and maximum SOC limitation to prevent the over-charge and over-discharge. The electric power is constrained by the time-dependent bounds from the planned SOC. The engine speed \( \omega_{\text{eng}} \) during the prediction horizon is set with the linear relation to the velocity. It is assumed that the engine speed behavior like a traditional automatic transmission makes drivers feel good driveability. In this study, the engine speed is determined by the velocity and the control input \( r_{g} \) in (53), \( r_{g} \) is constant in the prediction horizon and constrained using the predicted ego vehicle velocity \( v_{\text{max},1+T_f} \) and \( v_{\text{min},1+T_f} \) shown in (54),(55). In addition, the introduction of the \( r_{g} \) makes it possible to determine the engine speed with only one variable and suppress the number of the control input.

\[ \omega_{\text{eng}} = v(\tau) \cdot r_{g} = \frac{1000}{60 \cdot 2\pi R_{W}}, \]
\[ r_{\min} = \frac{\omega_{\text{eng,min}}}{v_{\text{min},k+hp}}, \]
\[ r_{\max} = \frac{\omega_{\text{eng,max}}}{v_{\text{max},k+hp}}. \]

The battery power is constrained by the time-dependent boundary functions \( B_{r} \) and \( B_{s} \) as well as the limitation from SOC. The prediction horizon is set as five steps with 2.0-seconds sampling rate in the MPC. Therefore, the prediction horizon is 10 seconds. Because the SOC variation of PHEVs is smaller than that of HEVs due to the large battery capacity, a long prediction horizon is necessary. When the prediction horizon is set as short, the MPC could not control the SOC state, and SOC is likely to be depleted.

6. Validation

This section evaluates the proposed control scheme with simulation. The long-term optimization is conducted beforehand, and the MPC using the result of the long-term optimization is validated. The intended vehicle is a C segment passenger car. The driving route includes city and motorway driving. In a part of the motorway the highest speed is limited to 120 km/h; otherwise, the speed limit is free like the autobahn in Germany. Figure 6 shows a sample velocity when the vehicle drives the route, the road gradient, and the traveled distance. In this study, the driver suppresses the velocity to 220 km/h that is the maximum speed of the vehicle. The vehicle simulation is conducted by CarMaker®. The driver determines the vehicle velocity according to the traffic environment. The plant model includes the energy losses of the electric motor, the generator motor, the boost converter, and the battery as look-up tables that are measured in steady-state condition, respectively. The engine model simulates the fuel consumption rate based on the BSFC data using \( T_{\text{eng}} \) and \( \omega_{\text{eng}} \). Therefore, the dynamics of the engine and the transmission inertia are not considered.

First, the driving route is defined, and the surrounding vehicles of the controlled vehicle are positioned at random. When
the preceding vehicle is slower than the ego vehicle, the driver tries to overtake the preceding vehicle. Therefore, the velocity profile of Fig. 6 is not constant, and high power is required for the overtaking. The road gradient is used to calculate vehicle speed dynamics. The total traveled distance is about 142 km. In this high power request and long driving situation, the appropriate energy management is required. To evaluate the proposed controller, the long-term optimization results using the dynamic programming and the CDCS strategy results are compared.

Figure 7 shows the simulation results for SOC and fuel consumption using the proposed controller. The SOC trajectory is shown with the reference trajectory from the dynamic programming and the results of the CDCS strategy. The reference trajectory is controlled at a high SOC level, and the MPC could track the reference SOC trajectory. On the other hand, the CDCS strategy decreases SOC during the early part of the driving, and SOC is maintained at a low level in the latter part. The fuel consumption using CDCS is lower than that using MPC in the early part due to usage of the battery power; however, the total fuel consumption is comparable to the case of the MPC. Table 1 shows the terminal SOC, fuel economy, and fuel optimality that is the relative fuel economy to the dynamic programming.

Figure 8 shows the driver’s demand acceleration power and the actual power. Basically, the proposed controller satisfied the driver’s demand. However, the actual acceleration power is not always enough. Because the available powertrain power depends on the SOC as discussed in Fig. 2, nevertheless, the driver’s demand power is determined by the intention of the acceleration. Battery power is restricted based on the SOC and the time-dependent limitation. Figure 9 shows the normalized frequency of the driver’s demand power undershooting (a) when the vehicle velocity is higher than 160 km/h, (b) when the vehicle velocity is slower than 160 km/h.

Table 1 Simulation result for fuel economy.

| Type     | Terminal SOC | Fuel (L/100 km) | Fuel optimality |
|----------|--------------|-----------------|-----------------|
| DP       | 0.12         | 9.86            | 1.00            |
| MPC      | 0.11         | 9.65            | 0.98            |
| CDCS     | 0.11         | 9.83            | 1.00            |

Fig. 6 The driving mode for the verification including the unlimited motorway.

Fig. 7 Simulation result for velocity, SOC, and cumulated fuel mass using some controller types.

Fig. 8 The realization of the driver’s acceleration demand and battery power limitation.

Fig. 9 The normalized frequency of the driver’s demand power undershooting (a) when the vehicle velocity is higher than 160 km/h, (b) when the vehicle velocity is slower than 160 km/h.
with the relation to the velocity increase. In this case, the drivability is emphasized using (53), (54), (55).

Figure 11 shows the engine operation point and BSFC characteristic. In the CDCS strategy, the engine is operated on the lowest BSFC area. When SOC is low in the CS region, the engine torque becomes higher than the lowest BSFC torque in order to charge SOC. On the other hand, the proposed MPC uses not only the lowest BSFC region but also the maximum engine torque. The usage of the high engine torque makes it possible to maintain the SOC at a high level and satisfy both the fuel economy and the drivability.

Additional five sample simulations are conducted with some traffic participant cases because the proposed controller uses the GPR model that includes a stochastic manner. Figure 12 shows the simulation results for velocity and SOC in the five traffic participant cases. The driving route is common; however, the surrounding vehicles of the controlled vehicle are positioned at random. Therefore, the velocity and SOC result of each sample are different. And the driver model and the long-term optimization result are also common. Table 2 shows the terminal SOC, fuel economy, and fuel optimality in five traffic participant cases. The SOC is controlled appropriately, and the results of five samples show almost equal in fuel optimality.

7. Conclusion

This paper addressed the energy management and power management of series-parallel PHEVs. The optimization problem was formulated including fuel economy, drivability, performance, and the time-dependent battery limitation. The optimized control inputs affect the power balance and the efficiency of the HEV system. The two-stage optimization scheme was proposed, which consists of the long-term off-line optimization and the short-term on-line optimization. In the long-term optimization, it is assumed that the driving cycle is known. On the other hand, in short-term optimization, the MPC is conducted based on the driver’s demand tractable force. The cost function is based on the ECMS concept that has used for the instantaneous optimal control strategy. However, this study adopted MPC for the short-term optimization with the prediction horizon to consider the engine speed behavior as the drivability and the time-dependent battery power suppression. And the future velocity trajectory for a few seconds is predicted for the MPC. The proposed controller was evaluated in the case of a long driving situation including the city and the motorway that required high power acceleration. The fuel economy was optimized, and the performance was improved in consideration of the drivability.

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Appendix

Nomenclature

| Words   | Descriptions                        |
|---------|-------------------------------------|
| A       | Frontal projected area (m²)         |
| \( \alpha \) | Weighting factor (-)               |
| BSFC    | Low brake specific fuel consumption (g/kWh) |
| \( C_d \) | Drag coefficient (-)               |
| \( l_p \) | Gear ratio of transmission (-)      |
| \( l_{fg} \) | Final gear ratio of transmission (-) |
| M       | Vehicle weight (kg)                |
| \( M_o \) | Operation mode (-)                 |
| \( \dot{m}_f \) | Fuel consumption rate (g/s)        |
| \( R_w \) | Wheel radius (m)                   |
| \( F_{aero} \) | Aerodynamic drag force (N)        |
| \( F_{brake} \) | Mechanical braking force force (N) |
| \( F_{load} \) | Running resistance force (N)        |
| \( F_r \) | Gravity force (N)                  |
| \( F_r \) | Rolling resistance force (N)        |
| \( F_{trac} \) | Tractable force (N)                |
| \( P_{batt} \) | Battery power (kW)                |
| \( P_{brake} \) | Mechanical brake power (kW)        |
| \( P_{eng} \) | Engine power (kW)                  |
| \( P_{dem} \) | Driver’s demand power (kW)        |
| \( P_{load} \) | Road load power (kW)              |
| Q_{hv}  | Lower heating value (MJ/kg)        |
| \( \lambda \) | Equivalent factor (-)             |
| \( v \) | Vehicle velocity (km/h)            |
| \( \rho \) | Ambient density (kg/m³)            |
| \( \mu_r \) | Friction coefficient on the road (-) |
| SOC     | State of charge (-)                |
| \( \eta_{gen} \) | Efficiency of generator motor (-) |
| \( \eta_{batt} \) | Efficiency of battery charge and discharge (-) |
| \( \eta_{cl} \) | Efficiency of clutch (-)          |
| \( \eta_{mot} \) | Efficiency of tractable motor (-)  |
| \( \eta_t \) | Efficiency of transmission (-)     |
| \( w \) | Wheel rotation speed (rad/s)        |
| \( w_{eng} \) | Engine rotation speed (rad/s)      |

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