CHARGINOS AND NEUTRALINOS AT $e^+e^-$ LINEAR COLLIDERS

JAN KALINOWSKI

Institute of Theoretical Physics, Warsaw University
Hoża 69, 00681 Warsaw, Poland

It is shown how the fundamental gaugino and higgsino parameters of the chargino and neutralino system in the MSSM can completely be determined in high-precision experiments at $e^+e^-$ linear colliders even if in their initial phase only the light charginos $\tilde{\chi}_1^\pm$ and the light neutralinos $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ are kinematically accessible.

1 Introduction

In the Minimal Supersymmetric Standard Model (MSSM) the light chargino $\tilde{\chi}_1^\pm$ and the two light neutralinos $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ are expected to be among the lightest supersymmetric particles. The lightest neutralino $\tilde{\chi}_1^0$ is commonly assumed to be the lightest SUSY particle (LSP), stable in the $R$-parity conserving case, and invisible. The chargino and neutralino masses are given in terms of the fundamental parameters in the gaugino/higgsino sector, the U(1) and SU(2) gaugino masses $M_1$ and $M_2$, the higgsino mass $\mu$, and the ratio of Higgs vacuum expectation values $\tan \beta$. If $|\mu| < |M_1|, |M_2|$, the light chargino and light neutralinos are higgsino-dominated and $m_{\tilde{\chi}_1^\pm} \sim m_{\tilde{\chi}_1^0} \sim m_{\tilde{\chi}_2^0}$. In the opposite, gaugino-dominated case, $m_{\tilde{\chi}_1^\pm} \sim m_{\tilde{\chi}_1^0} \sim 2m_{\tilde{\chi}_2^0}$ is expected e.g. if the gaugino masses unify at high scale in supergravity inspired scenarios. Thus $\tilde{\chi}_1^\pm$ and $\tilde{\chi}_2^0$ cannot be much heavier than the LSP $\tilde{\chi}_1^0$. Therefore one can envisage a scenario in which in the initial phase of future $e^+e^-$ colliders, only the $\tilde{\chi}_1^\pm$, $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ states will be accessed kinematically with all other supersymmetric particles being too heavy to be produced. The question then arises, which we address here, to what extent the fundamental SUSY parameters of the gaugino/higgsino sector can be reconstructed from the initially limited experimental input.

It has been demonstrated in the literature that if all chargino and neutralino states are accessible experimentally, the measurement of their masses and production cross sections with polarized $e^+e^-$ beams allows us to derive the parameters $M_1$, $M_2$, $\mu$ and $\tan \beta$ analytically at
Here I will report how these parameters can nevertheless be determined with the limited experimental input available form light charginos and light neutralinos produced in pairs in $e^+e^-$ collisions

$$e^+e^- \rightarrow \tilde{\chi}^+_1 \tilde{\chi}^-_1, \quad \tilde{\chi}^0_1 \tilde{\chi}^0_2$$

with visible final states.

## 2 The Chargino System

Defining the mixing angles in the unitary matrices diagonalizing the chargino mass matrix by $\phi_L$ and $\phi_R$ for the left- and right-chiral fields, the fundamental SUSY parameters $M_2$, $|\mu|$, $\cos \Phi_\mu$ and $\tan \beta$ can be derived from the chargino masses and $c_{2L,2R} = \cos 2\phi_{L,R}$,

$$M_2 = m_W \sqrt{\Sigma - \Delta (c_{2L} + c_{2R})} \quad (2)$$

$$|\mu| = m_W \sqrt{\Sigma + \Delta (c_{2L} + c_{2R})} \quad (3)$$

$$\cos \Phi_\mu = \frac{\Delta^2 (2 - c_{2L}^2 - c_{2R}^2) - \Sigma}{\sqrt{[1 - \Delta^2 (c_{2L} - c_{2R})]^2}} \quad (4)$$

$$\tan \beta = \frac{\sqrt{[1 - \Delta (c_{2L} - c_{2R})]/[1 + \Delta (c_{2L} - c_{2R})]} \quad (5)$$

where $\Sigma = (m_{\tilde{\chi}^\pm_1}^2 + m_{\tilde{\chi}^\pm_2}^2 - 2m_{\tilde{\nu}_e}^2)/2m_W^2$ and $\Delta = (m_{\tilde{\chi}^\pm_2}^2 - m_{\tilde{\chi}^\pm_1}^2)/4m_W^2$. The $\cos 2\phi_{L,R}$ can be determined uniquely from the measurement of cross sections $\sigma(e^+e^- \rightarrow \tilde{\chi}^+_1 \tilde{\chi}^-_1)$ at one energy with polarized beams including transverse beam polarization, or else if only longitudinal beam polarization is available, they are measured at two different incoming energies.

If only the light charginos $\tilde{\chi}^\pm_1$ can be produced, the mass of $\tilde{\chi}^\pm_2$ remains unknown. Then it depends on the CP properties of the higgsino sector whether the parameters of eqs.(2-5) can be determined or not in the chargino system alone.

(i) If $\mu$ is real, as suggested by the electric dipole moments, eq.(1) can be exploited to determine $m_{\tilde{\chi}^\pm_2}$ from $\cos \Phi_\mu = \pm 1$ up to at most a two-fold ambiguity.

(ii) If $\mu$ is complex, the parameters in eqs.(2-5) cannot be determined in the chargino sector alone. However, they can be calculated as functions of the unknown heavy chargino mass $m_{\tilde{\chi}^\pm_2}$.

Acturally, there are two classes of solutions corresponding to the two values $\Phi_\mu$ and $(2\pi - \Phi_\mu)$ for the phase of the higgsino mass parameter, i.e. the sign of $\sin \Phi_\mu$. Although the heavy chargino mass is still unknown, its range is bounded from above by eq.(3), and from below by not observing the heavy chargino in mixed light-heavy pair production

$$\frac{1}{2} \sqrt{s} - m_{\tilde{\chi}^\pm_1} \leq m_{\tilde{\chi}^\pm_2} \leq (m_{\tilde{\chi}^\pm_1}^2 + 4m_W^2)/|\cos 2\phi_L - \cos 2\phi_R|^{1/2} \quad (6)$$

To resolve the two-fold ambiguity in case (i), and to fix the heavy chargino mass in case (ii) other observables are needed. It is interesting to note that in solving both cases the mixed-pair $\tilde{\chi}^0_1 \tilde{\chi}^0_2$ production process can be used since at the same time the $U(1)$ gaugino mass parameter $M_1$ can be determined.

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*For precision measurements, however, loop corrections will have to be included bringing in all soft SUSY breaking parameters and ultimately one will have to rely on a global fit to all experimental data.

*The cross sections for processes depend on the sneutrino and selectron masses which we assume, for the sake of simplicity, to be measured elsewhere.

*This phase ambiguity can be resolved by measuring the sign of CP–odd observables, e.g. associated with normal $\tilde{\chi}^0_2$ polarization in $\tilde{\chi}^+_1 \tilde{\chi}^-_2$ pair production.
3 The Neutralino System

The symmetric neutralino mass matrix is diagonalized by a unitary matrix, defined such that the mass eigenvalues of the four Majorana fields \( \tilde{\chi}_i^0 \) are positive. The squared mass eigenvalues of the neutralinos are solutions of the characteristic equations

\[
m^{2\chi_i^0} - a m^{6\chi_i^0} + b m^{4\chi_i^0} - c m^{2\chi_i^0} + d = 0 \quad \text{for} \quad i = 1, 2, 3, 4
\]

with the invariants \( a, b, c \) and \( d \) given by the gaugino mass parameters \( M_2 \) and \( M_1 \), and the higgsino mass parameter \( \mu \), i.e. the moduli \( M_2, |M_1|, |\mu| \) and the phases \( \Phi_1, \Phi_\mu \). Since \( a, b, c \) and \( d \) are binomials of \( \Re M_1 = |M_1| \cos \Phi_1 \) and \( \Im M_1 = |M_1| \sin \Phi_1 \), the characteristic equation (7) for each neutralino mass has the form

\[
(\Re M_1)^2 + (\Im M_1)^2 + u_i \Re M_1 + v_i \Im M_1 = w_i \quad \text{for} \quad i = 1, 2, 3, 4
\]

The coefficients \( u_i \), \( v_i \) and \( w_i \) are functions of the parameters \( M_2, |\mu|, \Phi_\mu, \tan \beta \) and the mass eigenvalue \( m^{2\chi_i^0} \) for fixed \( i \). The coefficient \( v_i \) is necessarily proportional to \( \sin \Phi_\mu \) because physical neutralino masses are CP–even: \( \sin \Phi_\mu = 0 \) implies a sign ambiguity in the CP–odd quantity \( \Im M_1 \), i.e. in \( \sin \Phi_1 \). Therefore the characteristic equation (8) defines a circle in the \( \{ \Re M_1, \Im M_1 \} \) plane for each neutralino mass \( m^{2\chi_i^0} \). With two known light neutralino masses \( m^{2\chi_1^0} \) and \( m^{2\chi_2^0} \), we have two circles which cross at two points, see left panel of Fig. 1.

Now we are in position to solve cases (i) and (ii) discussed above.

(i) For \( \mu \) real, the two–fold ambiguity for \( M_2, \mu \) and \( \tan \beta \) can be resolved by checking which combination provides two crossing circles, i.e. is consistent with the measured neutralino masses and production cross sections. At the same time \( |M_1| \) is determined with a remaining two–fold ambiguity for \( \Im M_1 \).

(ii) For \( \mu \) complex, the position of two circles in the \( \{ \Re M_1, \Im M_1 \} \) plane, and therefore their crossings, will migrate as functions of the unknown heavy chargino mass, see right panel of Fig. 1. By comparing the predicted with the measured pair–production cross sections \( \sigma_L \{ \tilde{\chi}_1^0 \tilde{\chi}_2^0 \} \) and \( \sigma_R \{ \tilde{\chi}_1^0 \tilde{\chi}_2^0 \} \), a unique solution, for both the parameters \( m^{2\chi_3^0} \) and \( \Re M_1, \Im M_1 \) can be found.

As a result, the additional measurement of the cross sections leads to a unique solution for \( m^{2\chi_3^0} \) and subsequently to a unique solution for \( M_1, M_2, \mu \) and \( \tan \beta \) (assuming that the discrete CP ambiguity is resolved, see footnote c).

4 Extracting the Fundamental Parameters

The strategy described above is illustrated in Fig. 1. It has been worked out for a single reference point for a CP non–invariant extension of the MSSM, compatible with all experimental constraints,

\[
\text{RP} : (|M_1|, M_2, |\mu|; \Phi_1, \Phi_\mu; \tan \beta) = (100.5 \, \text{GeV}, 190.8 \, \text{GeV}, 365.1 \, \text{GeV}; \frac{\pi}{3}, \frac{\pi}{8}; 10)
\]

These fundamental parameters generate the following light chargino and neutralino masses,

\[
m^{2\chi_1^\pm} = 176.0 \, \text{GeV}, \quad m^{2\chi_1^0} = 98.7 \, \text{GeV}, \quad m^{2\chi_2^0} = 176.3 \, \text{GeV}
\]

while the heavy masses are given by

\[
m^{2\chi_3^0} = 389.3 \, \text{GeV}, \quad m^{2\chi_4^0} = 371.8 \, \text{GeV}, \quad m^{2\chi_4^0} = 388.2 \, \text{GeV}
\]

The cross sections depend on the sneutrino and selectron masses which we assume, for the sake of simplicity, to be measured in threshold scans

\[
m_{\tilde{\nu}_L} = 192.8 \, \text{GeV}, \quad m_{\tilde{\epsilon}_L} = 208.7 \, \text{GeV}, \quad m_{\tilde{\epsilon}_R} = 144.1 \, \text{GeV}
\]

The cross sections for chargino and neutralino pair–production with polarized beams,

\[
\sigma_L \{ \tilde{\chi}_1^+ \tilde{\chi}_1^- \} = 679.5 \, \text{fb} \quad \sigma_R \{ \tilde{\chi}_1^+ \tilde{\chi}_1^- \} = 1.04 \, \text{fb}
\]

\[
\sigma_L \{ \tilde{\chi}_1^0 \tilde{\chi}_2^0 \} = 327.9 \, \text{fb} \quad \sigma_R \{ \tilde{\chi}_1^0 \tilde{\chi}_2^0 \} = 16.4 \, \text{fb}
\]
The contours of two measured neutralino masses $m_{\tilde{\chi}^0_1}$ and $m_{\tilde{\chi}^0_2}$ in the \{Re$M_1$, Im$M_1$\} plane. The parameter set \{${M_2 = 190.8 \text{ GeV; } |\mu| = 365.1 \text{ GeV; } \Phi_\mu = \pi/8; \tan \beta = 10}$\}, corresponding to the point (3), is assumed to be known from the chargino sector. Right: Migration of the crossing points in the \{Re$M_1$, Im$M_1$\} plane, as functions of the heavy chargino mass (the small open circles are spaced by 5 GeV). The unique solution is determined by the measurement of the pair–production cross sections (marked by the black dot). From ref. 2.

at $\sqrt{s} = 500$ GeV are sufficiently large (between $\sim 7 \times 10^5$ and $1 \times 10^3$ events for $\tilde{\chi}_1^+ \tilde{\chi}_1^-$ and $\tilde{\chi}_1^0 \tilde{\chi}_2^0$ can be expected at TESLA), allowing the analysis of the properties of the chargino $\tilde{\chi}_1^\pm$ and the neutralinos $\tilde{\chi}_{1,2}^0$ with high precision.

To summarize. If only the light chargino and the two light neutralinos can be accessed kinematically in the initial phase of $e^+e^-$ linear colliders, measurements of their masses and production cross sections with polarized beams, and $\tilde{\chi}_1^0$ polarization in the process $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$, allow us to perform a complete and precise analysis of the basic MSSM parameters in the gaugino/higgsino sector: ($M_1, M_2; \mu; \tan \beta$). For more details and references, see ref. 3.

Acknowledgments

I would like to thank Organisers for inviting me to Moriond and for financial support. I am grateful to S.Y. Choi, G. Moortgat–Pick and P.M. Zerwas for many stimulating discussions. Work supported in part by the KBN Grant No. 5 P03B 119 20 (2001-2002).

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