Anomalous neutral gauge boson interactions and simplified models

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Trilinear $Z$ boson interactions are sensitive probes both of new sources of $CP$ violation in physics beyond the Standard Model and of new particle thresholds. Measurements of trilinear $Z$ interactions are typically interpreted in the frameworks of anomalous couplings and effective field theory, both of which require care in interpretation. To obtain a quantitative picture of the power of these measurements when interpreted in a TeV-scale context, we investigate the anatomy of $ZZZ$ interactions and consider two minimal and perturbative simplified models which induce such interactions through new scalar and fermion loops at the weak scale. We show that both threshold and non-threshold effects often are small compared to the sensitivity of the LHC, while the increased sensitivity of a future lepton collider should allow us to constrain such scenarios complementary to direct searches at hadron colliders.

I. INTRODUCTION

The discovery of the Higgs boson at the Large Hadron Collider has not revealed conclusive hints towards new phenomena beyond the Standard Model (BSM) and the nature of the mechanism of electroweak symmetry breaking remains elusive. Taking this lack of BSM physics at face value, the high energy physics community has moved towards studying the Standard Model (SM) as a low-energy effective field theory (EFT), paving the way towards comprehensive data analyses in a dimension six extended SM-EFT framework \cite{1,2}. As the EFT parameterization of our ignorance towards BSM physics includes all possible UV completions of the SM, this suggests that collider processes can receive corrections from multiple and competing EFT terms which can potentially introduce issues when calculating these processes perturbatively.

$Z$-boson pair production \cite{3,4} and $Z+2$ jet production via weak boson fusion \cite{5,6} are standard candles that inform both SM and BSM interpretations of LHC measurements. In particular, they are sensitive to new particle thresholds as well as the presence of a high scale-induced $ZZZ$ interaction, which is a sign of $CP$-violation beyond that within the SM. The presence of a new source of $CP$ violation is required to explain the baryon asymmetry of the universe \cite{7,8,9}, and links the collider phenomenology of the $ZZZ$ vertex to baryogenesis. Interactions which induce such couplings exist in a range of models \cite{10,11,12,13}. Accordingly, such trilinear gauge couplings have been searched for by the ATLAS and CMS Collaborations \cite{14,15,16,17,18,19}, which have placed constraints on their existence using an approach based on anomalous couplings.

The anomalous coupling approach is not ideal from a theoretical perspective. It is not gauge invariant from an electroweak point-of-view, and leads to violation of unitarity bounds at LHC energies. This unitarity violation is often overcome through the use of momentum dependent form factors (as used by ATLAS in \cite{17} for instance, following \cite{21,22}). However, these form factors are themselves not well motivated \cite{23}. A more robust approach is to use effective field theory (EFT), by adding gauge-invariant higher-dimensional operators to the SM Lagrangian. Indeed, there have been a number of recent studies of the phenomenology of trilinear gauge couplings from this perspective \cite{24,25}. Perturbative unitarity violation remains a possibility in such an approach. However, as matching to concrete UV scenarios becomes possible beyond the limitations of form factors, violating perturbative unitarity bounds translates into a non-existing constraint for perturbative UV scenarios at the high scale, and therefore does not limit the use of EFT as a mediator between theories at different scales.

Effective field theories work best when there is a clear hierarchy between the energy scales being probed experimentally and the fields which have been integrated out. This is not always the case for the parameter space of interest in UV complete models. On the one hand, theories with new sources of $CP$ violation in the context of electroweak baryogenesis generally require new fields with masses close to the electroweak scale (see e.g. \cite{10,11,12,13}). On the other hand, for extended fermion sectors, as predicted for instance in scenarios of partial compositeness \cite{26,27}. $ZZZ$ interactions can be sourced at one-loop by non-diagonal $Z$ couplings to top-quark or lepton partners. Both cases can imply marked changes in collider observables, as amplitudes become imaginary when virtual particles are able to go on-shell.

It is the purpose of this paper to bridge the gap between anomalous couplings/EFTs and UV complete
Theories by studying (gauge-invariant) simplified models \cite{30}. These have proved of great utility in searches for supersymmetry \cite{30,31} and dark matter production \cite{32–34} at the LHC. This allows us to gauge the reported constraints from LHC precision measurements in a more realistic context relevant for TeV scale physics.

If a simplified model is also renormalisable, it opens up the possibility to correlate oblique electroweak precision \cite{35–37} measurements (for earlier EFT-related work see \cite{38–40}) with the potential sensitivity of \( \mathcal{Z}Z \mathcal{Z} \) measurements without direct sensitivity to unspecified UV cut-offs, whose role is taken over by the physical mass scales of a concrete UV simplified theory. The merit of simplified models is hence two-fold: Firstly, they provide a minimal interface that captures both resonant and non-resonant features of an BSM-motivated scenario. Secondly, they provide a framework to critically assess the sensitivity reach of colliders, allowing a direct comparison of different collider concepts within a consistent theoretical framework. The price paid for this level of predictability is that we are limited to perturbative theories, which possess a well-defined approach to renormalisation and power-counting of interactions.

This work is organised as follows: we first discuss the anomalous coupling parameterisation of a tree-level \( \mathcal{Z}Z \mathcal{Z} \) vertex and consider the size of the one-loop induced effect from additional scalars and fermions, using a 2HDM simplified models to probe the sensitivity of \( \mathcal{Z}Z \mathcal{Z} \) interactions. Accordingly, we consider scenarios in which these interactions are generated from UV-finite one-loop corrections. In this sense it is hard to gauge whether the limit is a result of perturbative unitarity or really relates to the lack of new physics, which could be well-described by perturbative means. The most stringent constraints are derived in this manner, namely, Ref. \cite{20},

\begin{equation}
-0.0012 < f_4^Z < 0.0010, \\
-0.0010 < f_5^Z < 0.0013.
\end{equation}

Comparison of these results with similar limits obtained at LEP, e.g. recent L3 results \cite{46},

\begin{equation}
-0.48 < f_4^Z < 0.46, \\
-0.36 < f_5^Z < 1.03.
\end{equation}

indicates how much the kinematic coverage feeds into the constraints once potential energy-dependencies are not considered. Identifying \( f_4^Z \) and \( f_5^Z \) with effective operators we can cast these constraints into new physics scales in the context of an effective field theory.

Beginning with \( f_4^Z \) we follow \cite{49} which finds that three different dimension–eight operators contribute to \( f_4^Z \) (there is no contribution at dimension–six):

\begin{equation}
f_4^Z = \frac{M_Z^2 v^2}{2c_W s_W} \left( c_W^2 c_{WW} + 2c_W s_W c_{BB} + 4s_W^2 c_{BB} \right) \Lambda^4,
\end{equation}

where \( s_W^2 = 1 - c_W^2 = \sin^2 \theta_W \) is the Weinberg angle, and the Wilson coefficients \( c_{WW}, c_{BB}, \) and \( c_{BB} \) correspond to

*We focus on vectorlike leptons as constraints on coloured particles such as top partners \cite{41} are substantially stronger than for vectorlike leptons \cite{42}.
to the effective operators,

\[ \mathcal{O}_{WW} = iH^\dagger W_{\mu \nu} W^{\mu \nu} \{ D_\mu, D_\nu \} H, \]

\[ \mathcal{O}_{BW} = iH^\dagger B_{\mu \nu} W^{\mu \nu} \{ D_\mu, D_\nu \} H, \]

\[ \mathcal{O}_{BB} = iH^\dagger B_{\mu \nu} B^{\mu \nu} \{ D_\mu, D_\nu \} H. \]

If we assume only one of the operators is generated by a new UV complete model with a Wilson coefficient \( c_i \sim 1 \), we can infer the scale of new physics from the maximum allowed size of \( f_z^Z \) given above. We find the lowest scale of new physics (NP) corresponds to the operator \( \mathcal{O}_{BB} \) giving a scale \( \Lambda \sim 680 \) GeV. Since this is a loop generated effect, if we take instead a loop-suppressed Wilson coefficient \( c_i \sim 1/(16\pi^2) \) we find a lowest scale of \( \Lambda_{\text{NP}} \sim 190 \) GeV.

Next we can connect \( f_z^Z \) with a dimension twelve operator, which was recently identified in \[^{[50]}\].

\[ \mathcal{O}_{5Z} = \frac{c_{5Z}}{\Lambda^4} (H^\dagger D_\mu H)^2 (H^\dagger D_\nu H)^2 + \text{h.c.}. \]  

This operator will generate a \( Z^3 \partial \phi \) vertex at tree level when expanded, which allows a \( ZZZ \) contribution to be induced at one loop. Therefore we take \( c_{5Z} \sim 1 \), assuming a loop suppression factor in the IR theory, and using the bounds in Eq. \[^{[3]}\] we find a NP scale corresponding to \( \Lambda_{\text{NP}} \sim 200 \) GeV. It is important to note that generic UV complete models typically generate more than one effective operator so these derived scales should be taken as a guide only \[^{[51],[52]}\].

Given this we see that despite the seemingly strong constraints the experiments have placed on \( f_z^Z \), they do not indicate strong constraints on the mass scale of new physics. Given the relatively low constraints it is possible that the new degrees of freedom are propagating and an EFT or constant form factor approach is not appropriate. We thus adopt a more UV-complete perspective testing extended scalar and fermion sectors, which also covers potentially large threshold effects.

In the following we will consider the full one-loop expressions \[^{[1]}\] for the \( ZZZ \) vertex in the SM and different New Physics scenarios, and discuss the expected size of the contribution of the \( ZZZ \) vertex to the \( pp \to ZZ \) and \( pp \to Zjj \) processes at the 13 TeV LHC and future linear colliders. We will specifically focus on the potential effects of thresholds that might provide a sensitive probe of new physics. It should be stressed however, that the LHC cross sections provided in this work should be understood as approximations to the full electroweak corrections in these modified scenarios in the sense that we add finite contributions which are loop-induced and have no leading-order counterpart.\(^{[1]}\). For the case of the LHC, full NLO electroweak corrections have been provided recently in Refs. \[^{[57],[60]}\].

Since these processes are loop-induced and electroweak in nature, their effects can be small at the LHC; for the more promising case of the lepton models we will therefore also discuss the expectations at a future lepton collider where increased sensitivity will open the possibility to probe such new states through their modified electroweak corrections.

### A. Within the Standard Model

We first consider the generation of the \( ZZZ \) vertex at one-loop within the SM. Most loop-contributions cancel exactly and the only contributions which do not vanish at one-loop are those from intermediate fermions in the loop \[^{[33],[35],[61]}\]. We can understand this behavior by considering the SM current which couples the \( Z \)-boson to the other fields of the SM.

Beginning with the pure gauge currents we consider the gauge kinetic term for the \( SU(2) \) fields,

\[ \mathcal{L} = -\frac{1}{4} W_{\mu \nu} W^\mu \nu. \]

Expanding in terms of \( W_{\mu \nu} \equiv \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g_W c_\nu \epsilon^{ijk} W_\mu^j W_\nu^k \), where \( g_W \) is the \( SU(2)_L \) gauge coupling, and keeping only terms containing only a single \( Z \) and two \( W^\pm \) we find:

\[ \mathcal{L}_Z = -i g_{CW} \left[ \partial_\mu Z_\nu (W^\mu_\nu W^-_\nu - W^+_\nu W^-_\nu) \right. \]

\[ + Z_\nu (W^\mu_\nu \partial_\nu W^+_\mu - W^+_\mu \partial_\nu W^\mu_\nu) \]

\[ \left. + Z_\nu (W^\mu_\nu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^\mu_\nu) \right]. \]

Noting that for \(+ \leftrightarrow -\) the current changes by an overall sign, i.e. \( \mathcal{L}_Z \leftrightarrow -\mathcal{L}_Z \), we conclude that the \( ZZZ \) vertex generated at one loop by intermediate \( W^\pm \) vanishes due to Furry’s theorem \[^{[62]}\]. That is, the loop with intermediate \( W^+ \) has the opposite sign of that with intermediate \( W^- \) and they cancel.

Moving on to the contribution of fermions and Higgs bosons, we begin with the covariant derivative. The covariant derivative which acts on a field \( \phi \) in the fundamental representation of \( SU(2)_L \) with hypercharge \( Y \) (ignoring couplings to the gluons) is given by

\[ D_\mu \phi = (\partial_\mu - igW^a_\mu \tau^a - igY B_\mu) \phi, \]

where \( \tau^a = \sigma^a/2 \), and \( \sigma^a \) are the Pauli matrices. Considering the coupling of the \( Z \) to the Higgs, recalling \( Y_H = 1/2 \), and employing the unitary gauge for simplicity we have

\[ D_\mu H = \left( \partial_\mu + igZ_\mu \right) \frac{(v+h)}{\sqrt{2}} \phi. \]
The gauge kinetic term for the Higgs, keeping only the terms coupling $h$ to $Z$ is

$$\mathcal{L}_{ZH} = \frac{ig}{4c_W} h (\partial_\mu h) Z^\mu - \frac{ig}{4c_W} Z_\mu h (\partial^\mu h) = 0.$$  \hspace{1cm} (14)$$

Since this vanishes there is no $Zh^2$ coupling in the SM and the $Z^3$ one-loop vertex with an internal Higgs may not be formed. A more general analysis including Goldstone bosons reveals that while the neutral component also vanishes identically, the charged component does not. The current for the charged component changes sign under substitution $\chi^+ \leftrightarrow \chi^-$, so as in the case of $W^\pm$ there will be no contribution due to Furry’s theorem.

We are therefore left only with the possibility of a contribution due to the fermions. The gauge–kinetic term of the fermions of the Standard Model is

$$\mathcal{L} = i\bar{Q}_i \gamma_\mu Q_i + i\bar{u}_{R,i} \gamma_\mu u_{R,i} + i\bar{d}_{R,i} \gamma_\mu d_{R,i} + i\bar{\ell}_{R,i} \gamma_\mu \ell_{R,i}.$$  \hspace{1cm} (15)

Taking $Y_Q = 1/6$, $Y_{u_R} = 2/3$, and $Y_{d_R} = -1/3$, and considering only terms involving the $Z$-boson we can write in terms of four-component spinors

$$\mathcal{L}_{qZ} = \frac{g}{2c_W} \bar{u} \gamma^\mu \left[ \left( \frac{1}{2} - \frac{4}{3}s^2_W \right) 1 - \frac{1}{2} \gamma_5 \right] u \ Z^\mu$$

$$+ \frac{g}{2c_W} \bar{d} \gamma^\mu \left[ \left( \frac{2}{3}s^2_W - \frac{1}{2} \right) 1 + \frac{1}{2} \gamma_5 \right] d \ Z^\mu.$$  \hspace{1cm} (16)

We do not discuss the leptons in further detail as their SM contributions are sub-dominant to the quarks.

Recalling that the vector current $J^\mu_V = \bar{\psi} \gamma^\mu \psi$ changes sign under charge conjugation Furry’s theorem implies that there is no contribution to the $ZZZ$ coupling from three insertions of the vector part of the current (i.e $V^3$).

However, the axial current $J^\mu_A = \bar{\psi} \gamma^\mu \gamma_5 \psi$ does not change sign under charge conjugation. Therefore we may expect contributions from $V^2A$ and $A^2$, but not from $A^2V$ or $V^3$.

Employing FeynRules [63, 64] (with output in the UFO format [65]), FORMCALC [66, 67], and MADGRAPH5 [68, 69] we generate cross sections for the SM ZZ production process, the same but including the one loop result for only the top quark, and the same excluding the top quark but including the bottom quark one loop result. The results are summarised in Table 1. Below these results we include the results for only the $t$-quark loop squared and only the $b$-quark contribution squared. When we consider both the $t$- and $b$- quark loops we find they destructively interfere to give a smaller overall cross section.\(^{\S}\) The reason for this behavior is easiest to see in

\(^{\S}\)While this cancellation is expected from the quantum numbers of the $SU(2)$ fermion doublets in the limit $p^2 > m_t^2$, the individual fermion contributions are UV-finite.

| Processes | $ZZ$ | $Zjj$ |
|-----------|------|------|
| SM tree only | 8.869 ± 0.001 | 4641 ± 15 |
| SM tree + $t$ | 8.867 ± 0.002 | – |
| SM tree + $b$ | 8.887 ± 0.002 | – |
| SM tree + $t + b$ | 8.873 ± 0.002 | – |
| $t$-loop squared | 0.006 ± 7.6 · 10^{-8} | 0.003 ± 1 · 10^{-7} |
| $b$-loop squared | 0.018 ± 2.2 · 10^{-8} | 0.005 ± 1 · 10^{-5} |
| $t+b$-loops squared | 0.004 ± 4.8 · 10^{-8} | 0.0009 ± 2 · 10^{-6} |

**TABLE I:** Cross sections in picobarns for Standard Model $pp \to ZZ$ and $pp \to Zjj$ at $\sqrt{s} = 13$ TeV. “SM tree only” is the SM tree level process only. “tree + $q$” is the tree level result plus the one loop $ZZZ$ vertex for a given quark $q$, while “$t+t+b$” is for both quarks simultaneously. “$q$-loop” squared is only the contribution due to the one loop $ZZZ$ vertex calculation squared, “$t+b$-loops” allows for interference between the two contributions to the loop. As mentioned in the text the $W$ and $h$ bosons do not contribute to the loop process. In the case of $Zjj$ the tree level cross section is so large we don’t include the tree+1–loop results.

The case of the coupling $A^3$, taking the axial coupling of the $Z$ to the up–quark cubed we find $-g^3/(64c_W^2)$ while that for the down–quark is $+g^3/(64c_W^2)$. Considering the $V^2A$ coupling we find that these terms differ in sign, and also in magnitude by a factor of approximately $1 - 4s^2_W/3$.

The one loop calculation differs only in the coupling and mass of the quark running in the loop, and therefore we expect destructive interference.

From Table 1 we are able to see that in the SM the effect of the one–loop $ZZZ$ coupling (including both the top and bottom contributions) is at most a 0.01% deviation for the $ZZ$ process and will be hidden by the large QCD contributions and comparably large uncertainties associated with the cross section for $pp \to Zjj$. Therefore within the SM it is unlikely any progress can be made on measuring the one loop triple–$Z$ process at the LHC.

### B. Scalars and the CP–violating 2HDM

We consider the addition of new scalars and fermions which couple to the $Z$-boson. New vector resonances may also contribute, but since we focus on perturbative completions in this work, we will not focus on them any further. We note that some discussion of vector resonances can be found in [43]. In this subsection we consider the effect of extended scalar sectors, focusing on the 2HDM as a particular example, and move on to additional fermions in Sec. [HC]

The simplest extended scalar sectors involve the addition of one new $N$–plet of $SU(2)_L$ with some hypercharge $Y$. The $U(1)_Q$ charge of a particular component of the new scalar is given by

$$Q = T^3 + Y,$$  \hspace{1cm} (17)

where $T^3$ is the diagonal generator of $SU(2)_L$ in the $N$–dimensional representation. There will be only one neutral component of the $N$–plet for a given hypercharge.
For example, for a real scalar in the $N$ representation of $SU(2)_L$, we expect $(N - 1)/2$ charged scalars and one $CP$–even neutral scalar\(^*\). For any complex scalar we expect either $(N - 1)$ charged scalars and one $CP$–even and one $CP$–odd neutral scalar, or $N$ charged scalars, depending on the hypercharge assignment.

The additional charged scalars will be subject to Furry’s theorem and will not contribute to the $ZZZ$ loop. One might expect contributions of the neutral components to vanish similar to Eq. (14). However, since there are additional neutral scalars in the Lagrangian, mixing effects allow this issue to be evaded. We will consider the 2HDM as an example of extended scalar sectors. Since, as mentioned above, all extended scalar sectors with a single new $N$–plet of $SU(2)_L$ will have at most one additional $CP$–even and one $CP$–odd neutral scalar, we take the 2HDM and its phenomenology to be representative of all models in this class.

Our discussion of the 2HDM will follow the work \[46, 47\] which discusses the $ZZZ$ vertex resulting from the $CP$–violating 2HDM. The 2HDM scalar potential is

\[
V = - \frac{m_{11}^2}{2} |\Phi_1|^2 - \frac{m_{22}^2}{2} |\Phi_2|^2 - \frac{1}{2} \left( m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right) + \frac{\lambda_5}{2} |\Phi_1|^4 + \frac{\lambda_6}{2} |\Phi_2|^4 + \frac{\lambda_7}{2} |\Phi_1|^2 |\Phi_2|^2 + \left( \frac{\lambda_8}{2} (\Phi_1^\dagger \Phi_2) + \frac{1}{2} \lambda_7 (\Phi_1^\dagger \Phi_2) \right) + \frac{1}{2} \lambda_7 (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right).
\]

Possible complex parameters of this Lagrangian include

\[
\{ m_{12}, \lambda_5, \lambda_6, \lambda_7, e^{i\xi} \},
\]

where $\xi$ is the relative phase between the vevs of $\Phi_1$ and $\Phi_2$. Of these complex parameters an overall $SU(2)$ rephasing of the scalar potential may remove two phases leaving a total of three complex parameters in the model \[46\].

Expanding $\Phi_1$ and $\Phi_2$ in terms of their component fields and vacuum expectation values $v_i$,

\[
\Phi_i(x) = e^{i\xi_i} \left( \begin{array}{c} \phi_i^+(x) \\ (v_i + \eta_i(x) + i\chi_i(x))/\sqrt{2} \end{array} \right)
\]

results in mixing between the various components. In order to obtain the physical states we first rotate to a basis with massless Goldstone boson fields $G^0$ and $G^\pm$, and massive physical states $\eta_1$ and $H^\pm$ via

\[
\begin{pmatrix} G^0 \\ \eta_3 \end{pmatrix} = \begin{pmatrix} v_1/v & v_2/v \\ -v_2/v & v_1/v \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix},
\]

and

\[
\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \begin{pmatrix} v_1/v & v_2/v \\ -v_2/v & v_1/v \end{pmatrix} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}.
\]

If $CP$ is conserved there is mixing between the two $CP$–even scalars $\eta_1$ and $\eta_2$, but not with the $CP$–odd $\eta_3$ state. The theory has couplings $Z\eta_1\eta_3$ and $Z\eta_2\eta_3$ between the $Z$ and a $CP$–even and the $CP$–odd scalar, as well as $ZZ\eta_1$ and $ZZ\eta_2$ couplings. However these are still insufficient to generate the $ZZZ$ vertex at one loop as there is no $Z\eta_1\eta_2$ or $Z\eta_1\eta_3$ coupling. For any real $N$–plet, or complex $N$–plet without $CP$–violation there is no contribution to the $ZZZ$ vertex at one–loop.

However, for a $CP$–violating 2HDM there will generally be mixing between the three neutral components $\eta_i$. The mass matrix for the neutral states is diagonalised by an orthogonal mixing matrix $R$,

\[
\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix},
\]

where the $H_i$ are the physically propagating states. After this rotation there are three scalars of mixed $CP$. The Lagrangian coupling the scalars to the $Z$–boson now has the couplings $ZH_iH_j$ (for $i \neq j$), $ZH_iG^0$, and $Z^2H_i$. There are also quartic interactions between two scalars and two gauge bosons, however diagrams involving this coupling are identically zero. Using these interactions one can construct all of the diagrams in Fig. 1 which contribute to the $ZZZ$ vertex at one loop in the 2HDM (in addition to the SM contributions previously discussed). We note that our calculation agrees with the results recently obtained by \[50\].

With this framework in hand we are now free to proceed to calculate the loops and simulate their effects as in Sec. II A. We begin by noting that all diagrams (except the bubble which is identically zero) in Fig. 1 share the common prefactor \[46, 47\]:

\[
c_{ZZZ} \equiv \frac{\frac{\lambda_3}{2} (R_{11} v_1 + R_{12} v_2)(R_{21} v_1 + R_{22} v_2)(R_{31} v_1 + R_{32} v_2)}{v^3}.
\]

We consider $c_{ZZZ}$ as a function of the mixing angles $\theta_i$ in the matrix $R$ and the vevs $v_i$, and maximise its value subject to $v_1^2 + v_2^2 = (246 \text{ GeV})^2$ and $-\pi/2 \leq \theta_i \leq \pi/2$. We find that $c_{\max} \approx 0.19$ We assume this maximal value and simulate the size of the one–loop contribution to both the $ZZ$ and $Zjj$ processes. As the effects are too small to distinguish from the SM contribution we only give here
FIG. 1: The four graphs involving scalars contributing to the ZZZ vertex in the Two Higgs Doublet Model. Additional contributions come from the SM fermions as discussed in Sec. II A. In all diagrams $i$, $j$, and $k$ must be different. The bubble diagram (bottom right) is identically zero, but included for completeness. We agree with the relative sign of the diagram involving the $Z$ in the loop that was recently corrected in [20].

the size of the loop contribution squared to demonstrate how small the effect is. Scanning over $M_{H_3} \in (250, 1250)$ GeV and $M_{H_2} < M_{H_3} \in (500, 1500)$ GeV the maximal values of the cross section for both processes are:

$$
\sigma_{ZZ} \sim \mathcal{O}(10^{-3}) \text{ fb}, \\
\sigma_{Zjj} \sim \mathcal{O}(10^{-2}) \text{ fb}. 
$$

(25)

Imposing WBF cuts [10] reduces the $Zjj$ cross section by a further order of magnitude. These values are too small to be probed at the LHC in particular as the associated QCD uncertainty is at the level of $3\%$ for $Z$ pair production total cross section of $\sim 17$ pb. Percent-level corrections can be expected from non-resonant modified electroweak corrections (see below) but these have multiple sources and do not pinpoint a ZZZ vertex. Therefore, the 2HDM is not the optimal UV complete motivation for studying the loop induced ZZZ vertex. Indeed inspecting [10], while cross sections are not provided but elements of the amplitude are discussed, we would expect similar cross sections since [10] shows

$$
f_4^Z \sim 10^{-4} \delta ,
$$

(26)

where $\delta = 1 - (v_1 R_{11} + v_2 R_{12})/v \lesssim 5\%$ [17]. This is significantly below the constraint from CMS in Eq. [3], a constraint which itself may be relaxed by a more realistic treatment of the high $p_T$ part of phase-space as mentioned in the beginning of this section.

As we have previously argued any scalar sector extended by at most one complex $N$–plet of $SU(2)_L$ only allows for at most two new neutral scalars. In any such scenario the new scalar will contribute a similarly negligible contribution to the $ZZZ$ coupling. That is to say, any extension of the SM scalar sector by a complex $N$–plet will generate a negligible contribution to the $ZZZ$ coupling at one loop and therefore may be disregarded phenomenologically for studies of the $ZZZ$ coupling.

C. Simplified Fermionic Models

In the light of the previous discussion, the first fermionic simplified model that one could consider is a single fermion with axial $U(1)_Y$ couplings

$$
L_\psi = i\bar{\psi}\gamma^\mu(\partial_\mu - ig^f B_\mu\gamma_5)\psi - m_\psi\bar{\psi}\psi \quad (27)
$$

which leads to a QED charge for the field $\psi$ of $\alpha$ after rotating the hypercharge gauge field $B$ to the gauge boson mass basis. Such a state decouples leading to vanishing oblique corrections. However, the one-loop $AZZ$ and $ZZZ$ interactions which are relevant for Drell Yan production as well as VBF $Zjj$ production also vanish and such a model does not lead to an interesting new physics signal for our purposes.

One can extend this model to a vectorlike $SU(2)_L$ doublet with vectorial and axial couplings

$$
L_\psi = i\bar{\Psi}\gamma^\mu(\partial_\mu -igt^a W^a_\mu\{\alpha + 5\beta\})\Psi - m_\psi\bar{\Psi}\Psi \quad (28)
$$

with $\Psi = (\ell_1, \ell_2)^T$. Then, $SU(2)$ gauge invariance requires the identification of the vectorlike masses $m_{\ell_1} = m_{\ell_2}$.\footnote{Violating this identification induces spurious UV singularities in $T \sim (m_{\ell_1}^2 - m_{\ell_2}^2)(\alpha^2 - 5\beta^2)$.} For such a model the electroweak oblique corrections can be calculated without UV cut-off sensitivity. While $S, T$ vanish identically, constraints can be derived from the dimension eight parameter $U$. Inputting the constraints from [71], we find that the bounds are not tight enough to limit the parameter space $(\alpha, \beta, m_\psi)$ for the region where $S, T, U$ can be trusted, i.e. $m_\psi > m_Z$. However, it is exactly the choice of $m_{\ell_1} = m_{\ell_2}$ that decouples the $\Psi$ contribution from the anomalous gauge interactions and again such a scenario would not leave noticeable ZZZ interactions.

The only way to include sensitivity to thresholds while keeping the possibility to compare to oblique electroweak corrections is by introducing additional “chiral” masses through the Higgs mechanism on top of vectorlike masses. The effects discussed in the context of the third SM family of quarks can then be lifted to a higher mass scale and comparably large non-diagonal $Z$ couplings of the fermions in the mass basis can be induced in principle. We take this as motivation to consider a fourth generation of vectorlike leptons as another minimal and concrete BSM scenario with potential sensitivity to $ZZZ$ interactions.
Measurements. Such scenarios have been discussed in the context of $H \rightarrow \gamma \gamma$ measurements \cite{72} and they provide an avenue to raise the mass of the lightest Higgs boson in models of weak-scale supersymmetry, since the mass correction from new vectorlike supermultiplets will be positive if the fermions are lighter than their scalar mass correction from new vectorlike supermultiplets will be positive if the fermions are lighter than their scalar partner. The mass spectrum is determined by the vectorlike mass terms and Yukawa couplings given by

$$-\mathcal{L}_{\text{mass}} \supset m_{\nu L}^2\nu_L^c\nu_L + m_e\bar{e}_L^c\nu_L + m_{\nu_R}^2\nu_R^c\nu_R + \text{h.c.}$$

$$+ Y^c_{\nu}(\bar{H})\nu_L^c + Y^c_{e}(\bar{H})\epsilon_L^c + \text{h.c.}$$

$$+ Y^c_{\nu}(\bar{H})\nu_R^c + Y^c_{e}(\bar{H})\epsilon_R^c + \text{h.c.}. \quad (29)$$

Here $\bar{H} = ia^2 H^*$ and all coupling parameters are chosen to be real. All of the fields are singlets under SU(3)$_C$ and their SU(2)$_L \times U(1)_Y$ charges are given in Tab. 11. Unlike a new fermion generation with only Yukawa coupling-induced mass terms, the electroweak singlet mass terms allow the vectorlike fermions to decouple from electroweak precision constraints and on-shell Higgs observables \cite{72}.

After electroweak symmetry breaking the Lagrangian leads to 2 × 2 mixing matrices in the charged and neutral sectors:

$$-\mathcal{L}_{\text{mass}} \supset \left( \begin{array}{cc} \nu_{L}^c & \nu_{R}^c \\ \nu_{L}^c & \nu_{R}^c \end{array} \right) \frac{\epsilon_{L}}{\sqrt{2}} \frac{\epsilon_{R}}{\sqrt{2}}$$

$$+ \left( \begin{array}{cc} \nu_{L}^c & \nu_{R}^c \\ \nu_{L}^c & \nu_{R}^c \end{array} \right) \frac{\epsilon_{L}}{\sqrt{2}} \frac{\epsilon_{R}}{\sqrt{2}}$$

Rotating from the Lagrangian eigenstates $(\epsilon_{L}^c, \epsilon_{R}^c)$ to the mass eigenstates $(\epsilon_1, \epsilon_2)$ and $(\epsilon_1, \epsilon_2)$ will determine the relevant gauge interactions.

In the spirit of using simplified models to cross-relate different measurements some comments are in order. Since we do not mix the new lepton generation with the Standard Model leptons, there is in principle a parity symmetry protecting decays to the Standard Model which will make the lightest mass eigenstate stable on cosmological timescales and it will contribute a relic density. A charged exotic relic density should be avoided, so taking this effect at face value, we would have to require the lightest mass eigenstate to be $N_1/2$. A dark matter interpretation would however additionally have to avoid overclosing the universe while escaping direct detection constraints. These have been studied, with the addition of Majorana mass terms which split the $N_i$ into four mass eigenstates, in \cite{72} which found viable parts of parameter space, relying on the Xenon100 results \cite{76} as their most-constraining spin-independent limits. Xenon1T has recently improved these constraints by an order of magnitude \cite{77} compared to those used in this previous study, which forces the model to rely on co-annihilation between the lightest $N_i$ and $E_1$ and hence additionally requires one of the charged scalars to be close to mass degenerate with the $N_i$ which forms the relic density.

Since we do not want to constrain our parameter space to this extent and rather study it in a more general manner, while avoiding flavor-changing interactions involving $e$ and $\mu$, we will assume there is a small mixing with the third lepton generation in the Standard Model and the new vectorlike generation which can be ignored for the purpose of our calculations. This will avoid dark matter constraints completely and make decays of pair produced $E_i/N_i \rightarrow \tau/\nu_\tau + h/Z/W$ interesting direct signatures which can be looked for at colliders. Since LEP failed to find any such signatures this puts a robust lower bound on the lowest mass eigenstate $m_{\text{lightest}} > 104.5 \text{ GeV}$ which only can be avoided if the mixing would be fine-tuned to suppress the coupling of this state to the $Z$.

Higher masses are sensitive to direct searches at the LHC. These were studied in a phenomenological context in \cite{44} which found sensitivity at the LHC to masses up to 500 GeV with the full HL-LHC data set. However the two benchmarks models considered in their study correspond roughly to the SU(2)$_L$ singlet and doublet models of Eqs. (27)-(28) which have a simpler mass spectrum and interaction structure than the model we consider, and we can not easily recast their limits. To do so would require a propagation of mixing effects to both the production cross section and branching ratio calculations, taking into account new decays such as $E_1 \rightarrow N_i W$ which are absent in the mass degenerate case. Such a study is outside the scope of this paper, but based on the previous work we can reasonably expect the LHC to be sensitive to $m_{\text{lightest}}$ in the range of several hundred GeV.

More concrete constraints that relate to the generic modification of the electroweak sector due to the new
states can be imposed through oblique corrections that arise from the model of Tab. I and Eq. (29). The diagrams contributing to the $S, T, U$ parameters via the weak gauge boson polarisation functions are given in Fig. 2 and the resulting constraints on the model’s parameter space have been studied in Ref. [72, 73]. We scan the model over the relevant parameters in Eq. (29) and keep parameter points that are in agreement with the constraints of [71] at the 95% confidence level. In the following we will project these results onto the mass of the lightest state of Eq. (29) after diagonalisation.

The impact of the new fermion loop contributions to the $ZZZ$ vertex on $ZZ$ and $Zjj$ production at the LHC, although bigger than in the two Higgs doublet model, is again small upon comparison with the SM. For instance using a parameter point that leads to relatively large modified electroweak corrections of $\sim 15\%$ and a lightest mass of $\sim 300$ GeV (see below) we find the representative values,

\[
\sigma_{ZZ} \lesssim \mathcal{O}(0.10) \text{ fb,} \\
\sigma_{Zjj} \lesssim \mathcal{O}(0.09) \text{ fb,}
\]

following again the tool chain described in Sec. II A. Compared to the Standard Model cross sections given in Tab. I it is clear that it would be challenging to probe this model using measurements of the $ZZZ$ vertex at the LHC.

Since the overall impact of the fermionic scenario is slightly more promising than the scalar model we have discussed previously, we can raise the question of whether this scenario can be constrained at all using indirect collider measurements. While the LHC is limited by systematic uncertainties eventually, this situation is vastly improved for a future lepton collider. At such a machine we can expect measurements of electroweak diboson production to reach subpercentage-level precision, which offers an opportunity to see the imprint of vectorlike leptons in $ZZ$ measurements. We have calculated the size of the vectorlike lepton contribution for demonstrative ILC and CLIC setups, again using a calculation based on FeyNArts, FormCalc and LoopTools, for a number of parameter points which are randomly distributed over the parameter space. The results are presented as a fraction of the Standard Model expectation in Fig. 3 for a SM expectation of $\sigma(e^+e^- \rightarrow ZZ) \simeq 1.0$ pb at $\sqrt{s} = 250$ GeV, including the electroweak correction expected in the SM (see e.g. [79, 80] LEP-era calculations).

We find that deviations of up to 10% for the total cross sections are possible, Fig. 3. These fall within the expected sensitivity of the ILC and CLIC proposals in particular and some parameter choices are already constrained by LEP searches, e.g., Ref. [88], beyond the oblique constraints from $S, T, U$. However, the bulk of parameter points is left unconstrained by these measurements but is likely to be constrained at future precision machines. Sizeable effects are also present for the case where the lightest mass eigenstate is too heavy to be pair produced directly, however the modifications to the cross section decouple for $m_{\text{lightest}} \gg m_Z$ as can be expected from the general arguments of [81]. While the challenging threshold results for the LHC do suggest that a plethora of new physics effects can still hide below the constraints of [29], precision $ZZ$ measurements can be employed to constrain the models of $ZZZ$ interactions even when direct LHC constraints are loose.

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**FIG. 3:** Relative size of the vectorlike lepton contribution to the $ZZ$ cross section with respect to the Standard Model expectation for centre-of-mass energies corresponding to the ILC and CLIC colliders. The parameter points are randomly distributed over the parameter space, and demonstrate that the cross section can vary by an order of magnitude for a fixed value of the lightest fermion mass eigenstate $m_{\text{lightest}}$. All points pass the $S, T, U$ constraints of Ref. [71] at the 95% confidence level. The statistical uncertainties on $ZZ$ measurements assuming the Standard Model using the leptonic and semi-leptonic final states and expected end-of-lifetime luminosities for the machines are also plotted to provide an estimate of the theoretical reach. We only consider statistical uncertainties. For comparison we also include the constraint from L3, $\sigma/\sigma_{\text{SM}} = 0.93 \pm 0.08(\text{stat}) \pm 0.06(\text{sys})$. As the $ZZ$ cross section decouples for larger invariant masses, the direct sensitivity at CLIC decreases.
III. SUMMARY AND CONCLUSIONS

Measurements of the electroweak sector of the Standard Model are well-motivated at the LHC as the high center-of-mass energy and luminosity allows us to test detailed predictions at unprecedented precision. Any deviations from the Standard Model expectation in the electroweak sector can also provide considerable insight into currently open problems by, for example, providing a source of sufficient CP-violation to explain the baryon asymmetry of the universe. In this paper we have investigated the use of simplified models to interpret measurements of the ZZZ vertex, which provide a more realistic and consistent theoretical framework than the commonly employed anomalous coupling and effective field theory approaches when the new physics is close to the weak scale. Indeed given the EFT arguments below Eq. (6) the experimental constraints are not sufficient to argue that the NP generating the ZZZ vertex is safely decoupling, a condition necessary for the use of form factors such as $f_{Z5}^2$ or the effective field theory framework. We have discussed the anatomy of the vertex and how it arises in the Standard Model at one-loop, and argued that the minimal simplified scenario which allows for new contributions from new scalar states at one-loop is given by a CP-violating 2HDM. We have also considered a minimal simplified scenario where the vertex arises from threshold contributions from new fermion loops, given by a new generation of vectorlike leptons. Our detailed analysis suggests LHC measurements of the ZZZ vertex are relatively insensitive to these scenarios once existing constraints are taken into account and electroweak thresholds are difficult to resolve from both the overall cross section contribution and QCD uncertainty perspective. However at a future lepton collider a ZZZ measurement could provide crucial new information, for example to confirm the vectorlike lepton nature of a new state discovered at the LHC.

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