Wiggly cosmic strings accrete dark energy

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This paper deals with a study of the cylindrically symmetric accretion of dark energy with equation of state \( p = w \rho \) onto wiggly straight cosmic strings. We have obtained that when \( w > -1 \) the linear energy density in the string core gradually increases tending to a finite maximum value as time increases for all considered dark energy models. On the regime where the dominant energy condition is violated all such models predict a steady decreasing of the linear energy density of the cosmic strings as phantom energy is being accreted. The final state of the string after such an accretion process is a wiggleless defect. It is argued however that if accretion of phantom energy would proceed by successive quantum steps then the defect would continue losing linear energy density until a minimum nonzero value which can be quite smaller than that corresponding to the unperturbed string.

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I. INTRODUCTION

Cosmic strings are known as topological defects that occur in theories with spontaneous symmetry breaking of a local U(1) gauge symmetry which were formed during phase transitions in the early universe. They are trapped infinitely long very thin tubes filled with a previous false-vacuum phase characterized by a energy density per unit length \( \mu_0 \sim \sigma^2 \), with \( T_0 \) the string tension and \( \sigma \) the symmetry breaking scale, immersed in the true-vacuum phase created after the phase transition \([1]\). Typically, cosmic strings have been hypothesized as the seeds for the seeds for ultra galaxy formation \([2]\) or as the cosmic sites where primordial inflation took place \([3]\). At any event, among the different theoretical objects that are thought to have populated the universe at some previous or current periods, which also include black holes with distinct sizes, Lorentzian wormholes or ringholes, etc., cosmic strings are the sole objects whose existence has been confirmed in the laboratory \([4]\). Therefore, the great interest that cosmic strings raised when they were first introduced in cosmology has remained alive all the way up to now. Two general kind of cosmic strings have been so far considered, straight strings and string loops \([1]\). In the present paper we shall investigate how distinct forms of dark energy can be accreted by cosmic strings. We shall restrict ourselves to consider only straight cosmic strings. These are usually described by a static space-time exterior metric, first derived by Vilenkin \([5]\)

\[
ds^2 = -dt^2 + dr^2 + dz^2 + (1 - 8G\mu_0)r^2d\phi^2. \tag{1.1}
\]

By defining a new cylindrical angular coordinate \( \phi' = (1 - 8G\mu_0)\phi \), it can immediately be seen that this metric corresponds to a flat spacetime with a conical singularity that is associated with a deficit angle given by \( \Delta = 8\pi G\mu_0 \).

However, an incoming or outgoing energy flux due to dark energy accretion is no longer strictly possible for an exterior locally flat metric like that of a motionless straight string having no wiggles \([1]\). In fact, a motionless string with no wiggles \([1]\) cannot accrete anything that is motionless and homogeneous around it -in particular it could not accrete dark energy. Thus, if we want to consider accretion of dark energy onto cosmic strings we need these string to be perturbed by wiggles. In that case the exterior string metric can no longer be given by the locally flat line element \((1.1)\), as wiggle-induced variations of the string mass per unit length and tension would convert these quantities into space-time dependent functions, \( \mu \) and \( T \), with the state equation \( \mu T = \mu_0^2 \) and \( \mu > T \), whose values can initially be considered to be very similar to each other and therefore also very similar to their unperturbed counterparts in the linear approximation \([1]\). The linearized wiggly string metric reads

\[
ds^2 = \left[ 1 + 4G(\mu - T)\ln \frac{r}{r_0} \right] dt^2 + dr^2 + \left[ 1 - 4G(\mu - T)\ln \frac{r}{r_0} \right] dz^2 + (1 - 4G(\mu + T))r^2d\phi^2, \tag{1.2}
\]
which, contrary to metric (1.1), produces a non-vanishing Newtonian potential. In this case the deficit angle is given by $4\pi G(\mu + T)$.

Nowadays cosmology, on the other hand, relies mainly on the idea that the total energy of the current universe and possibly that of the early universe (that is the two cosmic periods known to show accelerating expansion) is dominated by some form of the so-called dark energy [6]. It is therefore of interest to investigate the effects that dark energy may cause in cosmic strings. Following the recent studies performed on black holes [7,8], one can actually suppose that dark energy can also be accreted onto a cosmic string, inducing some variation in its energy density per unit length $\mu$. This work aims at considering the effects that the accretion of dark energy may have in the fate of wiggly straight cosmic string in an accelerating universe. We shall represent dark energy as a perfect fluid characterized by a negative parameter $-1/3 > w = p/\rho$ (with $p$ the pressure and $\rho$ the energy density) filling a Friedmann-Robertson-Walker universe whose scale factor is given by [9]

$$a(t) = a_0 \left(1 + \frac{3}{2(1 + w)}C^{1/2}(t - t_0)\right)^{2/[3(1 + w)]}, \ (1.3)$$

where $C = 8\pi G\rho_0/3$ and we have taken for the energy density $\rho = \rho_0 a^{-3(1 + w)}$, with $\rho_0$ an integration constant, if we adopt a general quintessence model. It can be readily seen that, whereas the universe enters a steady regime of accelerating expansion which keeps its energy density per unit length $\mu$ steady, the universe would expand along super-accelerated patterns that drive it to a singularity at a finite time in the future at which everything -even the elementary particles - loses any independent, local behavior by its own to be ripped apart under the sole influence of the global phantom cosmological law. This singularity has been dubbed the big rip [11] and takes place at a time

$$t_* = t_0 + \frac{2}{3(|w| - 1)C^{1/2}}. \ (1.4)$$

Such a rather weird behavior takes also place when the other main contender model for dark energy, that is to say the K-essence model [12], is assumed to dominate. In fact, if $w < -1$ we obtain in this case [13]

$$a(t) \propto (t - t_*)^{-2\beta/[3(1 - \beta)]}, \ 0 < \beta < 1, \ (1.5)$$

where $t_*$ again represents the time for the big rip which is an arbitrary parameter in this case.

Of particular interest is the scenario in which we consider that the dark energy is given in terms of a generalized Chaplygin gas having an equation of state $p = -A\rho^{-\alpha}$, with $A > 0$ and $\alpha$ a parameter [14]. In this case the cosmic time $t$ relates to the scale factor by the more complicated expression [15]

$$t - t_0 = \frac{2}{3(1 + 2\alpha)}\frac{A^{\frac{2}{1 + 3\alpha}}}{\sqrt{CA^{1/(1 + \alpha)}\rho_0}} \times F \left(1, \frac{1 + 2\alpha}{2(1 + \alpha)}; \frac{3 + 4\alpha}{2(1 + \alpha)}; 1 + \frac{B}{A}a^{-3(1 + \alpha)}\right), \ (1.6)$$

with $F$ a hypergeometric function and $B = (\rho^{1 + \alpha} - A)a_0^{3(1 + \alpha)}$. It can be seen that even in the phantom energy regime a generalized Chaplygin gas does not lead to any big rip singularity in the future [15,16], but it always drives a steady regular accelerating expansion for the universe.

In this paper we use a formalism which is able to encompass the accretion of dark energy described by any of the above models onto wiggly straight cosmic strings. We obtain that as quintessence or K-essence dark energy is accreted onto a perturbed straight cosmic string the energy density per unit length of this string either progressively increases up to a constant finite value if $w > -1$, or steadily decreases down to the unperturbed value first and might then enter a region where quantum accretion makes it reach a minimum value, quite before the occurrence of the big rip singularity if $w < -1$. The behavior of the strings when they accrete Chaplygin gas is similar: their energy density per unit length also progressively either increases or decreases toward an extremal value, depending on whether the dominant energy condition is satisfied or violated.

The paper can be outlined as follows. In Sec. II we present the general formalism for the accretion of dark energy onto straight wiggly cosmic strings and obtain a general rate equation for the string core energy density per unit length, $\mu$, in terms of the internal dark energy, and apply such a formalism to quintessence and K-essence cosmological fields, so as to the generalized Chaplygin gas model. Approximate expressions of $\mu$ as a function of time for the first two dark energy models are also derived, both for $w = p/\rho > -1$ and $w = p/\rho < -1$, analyzing the corresponding evolution of the cosmic strings. Finally we conclude and add some further comments in Sec. III.

II. DARK ENERGY ACCRECTION ONTO WIGGLY STRAIGHT COSMIC STRINGS

We shall consider next how the general accretion theory can be applied to the case in which dark energy is accreted onto wiggly cosmic strings. We shall generally follow the procedure put forward by Babichev, Dokuchaev and Eroshchenko [7] for the case of Schwarzschild black holes, generalizing it to the case of straight wiggly cosmic strings. Thus, we start by integrating the energy-momentum conservation law by using the exterior metric (1.2). Although for metric (1.1) there are only two non-vanishing components of the Christoffel symbols,
\[ \Gamma_{\theta\theta}^r = -(1 - 8G\mu) r \text{ and } \Gamma_{\theta\theta}^\rho = 1/r, \] when the string is perturbed with wiggles there will be twenty one generally non-vanishing components of the Christoffel symbols which make the calculation to follow more complicated. For a cylindrical symmetry we then have from the time-component of the conservation law of the energy-momentum tensor, \( T_{\mu\nu}^\rho = 0, \)

\[ \sqrt{\mu u} \sqrt{1 - h_{00}} \sqrt{1 - b(1 + h_{00}) \sqrt{u^2 - 1}(p + \rho)} = C, \]

where

\[ h_{00} = 4G(\mu - T) \ln (r/r_0) \]

\[ b = 4G(\mu + T), \]

with \( r_0 \) and \( C \) integration constants and \( u = dr/ds \).

After integrating the conservation law for the energy-momentum tensor projected onto the four-velocity, \( u_\mu T_{\mu\nu}^\rho = 0, \) we also obtain

\[ ur \sqrt{\mu(1 - h_{00}^2)(1 - b)} e^{f_{\rho\nu}} \frac{d}{ds} A = 0, \]

where we have taken into account that \( u \) should be positive for incoming energy flux in this case, and \( A \) is a positive constant. From Eqs. (2.1) and (2.4) we can then get

\[ \sqrt{(u^2 - 1)(1 - h_{00})}(p + \rho) e^{-f_{\rho\nu}} \frac{d}{ds} C_2 = 0, \]

in which the constant \( C_2 \) can be expressed as \( C_2 = C/A = A[\rho_\infty + p(\rho_\infty)], \) with \( A > 0 \) a constant, for the cylindrical symmetry used.

By integrating now the momentum density \( T^\rho_\mu \) over the circular length element of the cylinder we can obtain the rate of change of the energy per unit length of the wiggly cosmic string, so that

\[ \dot{\mu} = - \int_0^{2\pi} r T_{\rho\phi}^\mu d\phi = \int_0^{2\pi} r(p + \rho)(1 + h_{00}) \frac{dt}{ds} \frac{dr}{ds} d\phi. \]

Using the property \( \sqrt{1 + h_{00} dt} = \sqrt{\frac{dr^2}{ds^2} - 1} ds \) stemming from the cylindrical symmetry being used and Eqs. (2.5) and (2.6), we finally derive the relevant rate equation for the energy density of a wiggly cosmic string

\[ \dot{\mu} = \frac{2\pi A[\rho_\infty + p(\rho_\infty)]}{\mu(1 - b)(1 - h_{00}^2)}, \]

with \( A = A \hat{A} > 0 \) a constant. Therefore, one has the following integral expression that governs the evolution of the wiggled mass per unit length of the cosmic string

\[ \int_{\mu_0}^{\mu} \sqrt{\mu(1 - b)(1 - h_{00}^2)} d\mu = 2\pi A \int_{t_0}^{t} [\rho_\infty + p(\rho_\infty)] dt. \]

It is worth noticing that the above expressions restrict by themselves the interval along which the quantity \( \mu \) is allowed to vary on its real values. In fact, one can derive the two conditions

\[ \mu_0 < \frac{1}{8G} \]

\[ 1 - \sqrt{1 - 64G^2 \mu_0^2} < \mu < \frac{1 + \sqrt{1 - 64G^2 \mu_0^2}}{8G}. \]

Condition (2.9) expresses nothing but the impossibility for an supermassive wiggleless cosmic string to reach a linear energy density larger than nearly 1/G. Even though the concepts of radius and mass per unit length for a source like the string core are not unambiguously defined [17], specially in the presence of an interacting dark energy fluid, at the extreme supermassive case \( \mu = 1/8G \) one would expect the string to no longer exist because it then corresponded to the situation where all the exterior broken phase is collapsed into the core, leaving a pure false-vacuum phase in which the picture of a cosmic string with a core region of trapped is lost [18]. When the string is wiggled then condition (2.9) reflects into condition (2.10) by which it is seen that a wiggly cosmic string cannot exceed a given maximum value or be less than a given minimum nonzero value. If a cosmic string has the extreme supermassive linear mass density, then it cannot be wiggled nor accrete any kind of dark energy.

Now, the integration in the left-hand-side of Eq. (2.8) appears to be very difficult to perform and, in fact, we have been unable to obtain an integrated expression from it in closed form. Nevertheless, in the physically relevant cases that \( \mu \) is very close to \( \mu_0 \) and/or \( r \) is very close to \( r_0 \), that term can be integrated to approximately give

\[ \int_{\mu_i}^{\mu} \frac{\mu(1 - b)(1 - h_{00}^2)}{16G} \simeq I(\mu) = \frac{8G\mu - 1}{16G} \sqrt{-4G^2 \mu^2 + \mu - 4G\mu_0^2} \]

\[ + \frac{64G^2 \mu_0^2 - 1}{64G^{3/2}} \arcsin \left( \frac{1 - 8G\mu}{\sqrt{1 - 64G^2 \mu_0^2}} \right). \]

The integration of the right-hand-side of Eq. (2.8) will be performed in what follows for the distinct dark energy models considered in the Introduction.

### A. Quintessence and K-Essence

Starting with the equation of state \( p = w\rho \), where \( w \) is assumed constant, we can use the conservation of cosmic energy to finally derive

\[ \rho = \rho_0 \left( \frac{a_0}{a} \right)^{3(1+w)}, \]
with \( \rho_0 \) and \( a_0 \) constants. Hence
\[
2\pi A \int_{t_0}^{t} [\rho_{\infty} + p(\rho_{\infty})] \, dt = 2\pi A (1 + w) \rho_0 a_0 \int_{t_0}^{t} \, dt a^{-3(1+w)}. \tag{2.13}
\]
We then have for the scale factor (1.3) corresponding to a general flat quintessence universe
\[
t = t_0 + \frac{I(\mu)}{(1 + w) \left( 2\pi A \rho_0 - \frac{3}{4} G^{1/2} I(\mu) \right)}, \tag{2.14}
\]
where \( I(\mu) \) is defined in Eq. (2.11). This is a parametric equation from which one can obtain how the energy per unit length of a wiggled cosmic string evolves in the accelerating universe. Thus, if \( w > -1 \) we see that the string energy in the core will progressively increases from its initial value \( \mu_i \), tending to the maximum value
\[
\mu_{\text{max}} = 1 + \sqrt{1 - 64G^2 \mu_i^2}.
\]
The larger \( w \) the shorter the time required by the accretion process to make the string to reach \( \mu_{\text{max}} \). If \( w < -1 \), i.e. if we are in the phantom regime, then the linear energy density in the string core will rapidly decreases from its initial value down to recover its unperturbed value at \( \mu_0 \). The smaller \( w \) the shorter the time taken by the system to reach the value \( \mu_0 \). As the string is approaching that value the gravitational potential should be getting on smaller and smaller values to finally vanish at \( \mu_0 \), so that the classical accretion process will stop at that point. Such a behavior is also checked to occur in the case that phantom K-energy is accreted.

An interesting question is however posed in the two considered kinds of phantom energy. Even though the classical, continuous accretion process must only proceed down to \( \mu_0 \), if we assumed that phantom energy accretion would proceed by discrete steps, then the limit at \( \mu_0 \) should be overtaken and the linear energy density of the string core would continue decreasing below \( \mu_0 \) as the phantom energy was being accreted. We would reach in this way a regime where \( T > \mu \) which would end when \( \mu \) reached the minimum value
\[
\mu_{\text{min}} = 1 - \sqrt{1 - 64G^2 \mu_0^2},
\]
which would never vanish provided \( \mu_0 > 0 \). The spacetime metric of the cosmic string given by Eq. (1.2) would then exchange the values between the \( tt \) and \( zz \) components, as in this case \( \mu < T \).

**B. Generalized Chaplygin gas**

We shall derive now the expression for the rate \( \dot{\mu} \) in the case of a generalized Chaplygin gas. We start with the expression for the energy density
\[
\rho = \left( A_{ch} + \frac{B}{a^{3(1+\alpha)}} \right)^{1/(1+\alpha)}, \tag{2.15}
\]
which has been obtained by integrating the cosmic conservation law for the case of the equation of state of a generalized Chaplygin gas, that is \( p = -A_{ch}/\rho^\alpha \). Now, from the Friedmann equation we can get
\[
\dot{a} = \sqrt{\frac{8\pi G}{3}} a(t) \left( A_{ch} + \frac{B}{a^{3(1+\alpha)}} \right)^{1/(2(1+\alpha))}. \tag{2.16}
\]
Hence, from Eq. (2.11) it can be obtained
\[
I(\mu) = B A \sqrt{\frac{3\pi}{2G}} \int_{a_0}^{a} \, da \left( A_{ch} + \frac{B}{a^{3(1+\alpha)}} \right)^{(2\alpha+1)/[2(1+\alpha)]} = -\frac{2\pi A}{3G} \left( A_{ch} + \frac{B}{a^{3(1+\alpha)}} \right)^{1/[2(1+\alpha)]} - \sqrt{\rho_0}. \tag{2.17}
\]
It follows
\[
a^{3(1+\alpha)} = \frac{B}{\left( \sqrt{\rho_0} - \sqrt{\frac{3G}{2\pi A} I(\mu)} \right)^{2(1+\alpha)}} - A_{ch}. \tag{2.18}
\]
Again in this case the setting of a constant \( B > 0 \) implies a progressive increase of \( \mu \) with \( \alpha \) up to a maximum given by \( \mu_{\text{max}} \), and the assumption of a constant \( B < 0 \) (phantom) leads to a decrease of \( \mu \) with \( \alpha \) down to \( \mu_0 \) classically or to \( \mu_{\text{min}} \) if the Chaplygin phantom energy is supposed to be accreted in discrete steps.

**III. CONCLUSIONS AND FURTHER COMMENTS**

While cosmic strings have a long tradition and incidence in theoretical cosmology, the introduction of cosmic dark energy has taken place quite more recently though not with less incidence or surprise. Perhaps therefore their potential mutual relations and interactions have not been so far considered. This paper is a first step in the task of studying the effects that the presence of dark energy may have in the fate of cosmic string in an accelerating universe. We have restricted ourselves here to just looking at an approximate model describing how straight wiggly cosmic strings accrete dark energy during the accelerating expansion of the universe, leaving for future publications the accurate treatment for both wiggly straight strings and the similar accretion onto circular strings, so as the kinematic effects that the acceleration of the universe may have on the shape and size of any cosmic strings. A generalized description has first been thus built up and then adapted to the case of the cylindrically symmetric accretion of dark energy onto straight cosmic strings. That description is based on the integration of the conservation laws for the energy-momentum...
tensor and its projection on four-velocity using the exterior geometry of a wiggly cosmic string. We have considered the dark energy accretion onto straight cosmic strings using several scalar field models for the cosmological vacuum, namely quintessence and K-essence field models with equation of state \( p = \omega \rho \), and a general-ized model of Chaplygin gas with the unusual equation of state \( p = -\frac{\alpha}{\rho^\alpha} \). An rate equation for the energy density per unit length of the strings has been in this way derived and finally integrated for each of these dark energy models. This ultimately leads to the prediction that, whereas when the energy density of the cosmic vacuum decreases with time the linear energy density of the straight strings progressively increases as the universe grows bigger for all dark energy models, if the energy density of the universe grows with expansion, inducing a universal violation of the dominant energy condition, the stringy energy density steadily decreases. That energy density dropping makes the strings to eventually become free of wiggles to get thereafter on a quantum accretion regime where the string energy density reaches finally a minimum nonzero value, before the occurrence of any future big rip singularities.

It appears that the current value of the parameter \( \omega \) in the equation of state of the universe may be less than \(-1\). So, one could be tempted to think that the above evolution of cosmic strings leading eventually to the formation of exotic topological defects with negative-wiggles perturbations would be inescapable. However, having now \( \omega < -1 \) (provided this turns out to be definitively the case most favoured by observations) does not guarantee at all that the phantom regime will endure in the future. In fact, most general descriptions of quintessence field are based on tracking models where the parameter \( \omega \) is time dependent [19] and, therefore, it could well be that what is now less than \(-1\) would later turn out to be greater than \(-1\), so making the cosmic string evolution predicted by our constant-\(w\) models inapplicable in the far future. Nevertheless, the initial string evolution implied by our phantom models looks as being probable. That behaviour by itself would still be important enough for a variety of subjects. But even such a behaviour would not be guaranteed as phantom fields are characterized by Lagrangians containing negative kinetic terms which have very weird properties and lead to unwanted instabilities [20] making the whole phantom scenario problematic.

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[1] T.W.B. Kibble, J. Phys. A9, 1387 (1976); A. Vilenkin, Phys. Rep. 121, 236 (1985); A. Vilenkin and E.P.S. Shellard, Cosmic Strings and Other Topological Defects (Cambridge University Press, Cambridge, UK, 1994).

[2] R.H. Brandenberger, J. Phys. G: Nucl. Part. Phys. 15, 1 (1989); D.N. Vollick, Phys. Rev. D45, 1884 (1992).

[3] A.D. Linde and D.A. Linde, Phys. Rev. D50, 2456 (1994); A. Vilenkin, Phys. Rev. Lett. 72, 3137 (1994).

[4] T.W.B. Kibble, Testing Cosmological Defect Formation in the Laboratory, cond-mat/0111082 Proceedings of the Second European Conference on Vortex Matter in Superconductors, Crete (2001), to appear.

[5] A. Vilenkin, Phys. Rev. Lett. 46, 1169 (1981); Phys. Rev. D24, 2082 (1981).

[6] T. Padmanabhan, Dark Energy: The Cosmological Challenge of the Millennium, astro-ph/0411044 Current Science (to appear).

[7] E. Babichev, V. Dokuchaev and Yu. Erosenko, Phys. Rev. Lett. 93, 021102 (2004); P.F. González-Díaz and C.L. Sigüenza, Phys. Lett. B589, 78 (2004).

[8] P.F. González-Díaz and C.L. Sigüenza, Nucl. Phys. B697, 363 (2004).

[9] C. Wetterich, Nucl. Phys. B302, 668 (1988); J.C. Jackson and M. Dodgson, Mon. Not. R. Astron. Soc. 297, 923 (1998); J.C. Jackson, Mon. Not. R. Astron. Soc. 296, 619 (1998); R.R. Caldwell, R. Dave and P.J. Steinhardt, Phys. Rev. Lett. 80, 1582 (1998); L. Wang and P.J. Steinhardt, Astrophys. J. 508, 483 (1998); R.R. Caldwell and P.J. Steinhardt, Phys. Rev. D57, 6057(1998); G. Huey, L. Wang, R. Dave, R.R. Caldwell and P.J. Steinhardt, Phys. Rev. D59, 063005 (1999); P.F. González-Díaz, Phys. Rev. D62, 023513 (2000); J.D. Barrow, Phys. Lett. B180, 335 (1986); B235, 40 (1990); D. Bazcia and J. Jackiw, Ann. Phys. 270, 246 (1998).

[10] R.R. Caldwell, Phys. Lett. B545, 23 (2002).

[11] R.R. Caldwell, M. Kamionkowski and N.N. Weinberg, Phys. Rev. Lett. 91, 071301 (2003); S. Nojiri and S.D. Odintsov, Phys. Lett. B562, 147 (2003); L.P. Chimento and R. Lazkoz, Phys. Rev. Lett. 91, 211301 (2003); J.D. Barrow, Class. Quant. Grav. 21, L79 (2004); S. Nesseris and L. Perivolaropoulos, The Fate of Bound Systems in Phantom and Quintessence Cosmologies, astro-ph/0410309.

[12] C. Armendariz-Picón, T. Damour and V. Mukhanov, Phys. Lett. B458, 209 (1999); J. Garriga and V. Mukhanov, Phys. Lett. B458, 219 (1999).

[13] P.F. González-Díaz, Phys. Lett. B586, 1 (2004).

[14] P.F. González-Díaz, Phys. Lett. B586, 1 (2004).

[15] M. Bouhmadi and J.A. Jiménez Madrid, JCAP 0505 005 (2005).

[16] P.F. González-Díaz, Phys. Rev. D68, 021303 (2003).

[17] See for example, G.W. Gibbons, S.W. Hawking and T.
Vachaspati, *The Formation and Evolution of Cosmic Strings* (Cambridge University Press, Cambridge, UK, 1990).

[18] P. Laguna and D. Garfinkle, Phys. Rev. D40, 1011 (1989); M.E. Ortiz, Phys. Rev. D43, 2521 (1991).

[19] I. Zlatev, L. Wang and P.J. Steinhardt, Phys. Rev. Lett. 82, 896 (1999); P.J. Steinhardt, L. Wang and I. Zlatev, Phys. Rev. D59, 123504 (1999); P. Brax, J. Martin and A. Riazuelo, Phys. Rev. D62, 103505 (2000).

[20] S.M. Carroll, M. Hoffman and M. Trodden, Phys. Rev. D68, 023509 (2003).