The electron–phonon interaction from fundamental local gauge symmetries in solids

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Received 9 July 2013, revised 17 November 2013
Accepted for publication 25 November 2013
Published 23 December 2013

Abstract
The elastic properties of solids are described in close analogy with General Relativity, by locally gauging the translational group of space–time. Electron interactions with the crystal lattice are thus generated by enforcing full gauge invariance, with the introduction of a gauge field. Elementary excitations are associated with the local gauge, contrasting to the usual interpretation as Goldstone bosons emerging from global symmetry breaking. In the linear limit of the theory, the gauge field displays elastic waves, that we identify with acoustic phonons, when the field is quantized. Coupling with the electronic part of the system yields the standard electron–phonon interaction. If spin–orbit effects are included, unusual couplings emerge between the strain field and the electronic spin current, leading to novel physics that may be relevant for spintronic applications.

Keywords: nonlinear elasticity, general relativity, gauge symmetry, non-Abelian gauge fields, electron–phonon interaction, spin currents
PACS numbers: 11.15.\(±\)q, 11.30.\(±\)j, 04.20.\(±\)q, 63.20.K\(±\), 72.25.\(±\)b

1. Introduction
In the physics of condensed matter, the concept of collective excitations gives rise to a large number of quasi-particles associated with the quanta of these excitations. Some well known

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examples, among the most important, are the vibrational modes of the atomic lattice, whose quanta are called phonons, and spin waves whose quanta are magnons [1]. There are two prominent and, in many respects, inequivalent ways to get the physics of collective excitations: (i) by means of the so-called emergence principle, and (ii) by invoking gauge symmetry. For the former, the detailed interactions between ‘truly’ elementary particles are considered at the very beginning, leading to the ‘emergence’ of their collective excitations and the quantum field describing it. For the latter, which is well known to high energy and particle physicists, it is the gauge symmetry principle which will tell us about the allowed interactions and what kinds of particles can be considered as elementary [2].

Historically, in condensed matter physics, the most relevant collective interactions were firstly obtained via the emergence principle. But important cooperative phenomena, such as superfluidity and superconductivity, are now recognized as gauge theories, and the gauge principle is being currently used with great success in the condensed matter province [3]. This way, new visions of the field are opened, including symmetry breaking and generation of mass via the Higgs mechanism, concepts that show the unified character of physical laws. This fact leads Weinberg to state literally that ‘a superconductor is simply a material in which the electromagnetic gauge invariance is spontaneously broken’ [2].

In a different road, but following the same paradigm, spin interactions of non-relativistic Pauli electrons were obtained by considering the full gauge symmetry group of the Pauli–Schrödinger theory [4], with no explicit reference to the Dirac equation. Within the same framework, Dartora and Cabrera showed that the spin-current density operator can be correctly defined using gauge invariance and Noether’s theorem [5]. The above quantity has fundamental relevance in the field of spintronics [6–8], allowing to explain several interesting phenomena that include the spin Hall effect and the spin-transfer torque mechanism [9–11]. The authors have also studied magnetic excitations (magnons) following a similar approach, gauging the $SU(2)$ group [14].

Many open questions remain to be elucidated in this cross field between condensed matter and high energy physics. In particular, the theory of gravity has resisted a successful quantization, a problem which is believed to be relevant at very high energy scale. However, some realistic and exotic scenarios can be explored and tested in the low energy regime of condensed matter. For instance, the study of electronic transport in graphene has shown that, under certain conditions, a slowly moving electron behaves as a relativistic Dirac electron in a space–time structure that mimics gravity theory in a lower dimensional space [12, 13]. In another direction, artificial metric spaces have been created in optical systems to simulate exotic space–time geometries [15]. Acoustic ‘black holes’ display many of the properties that are attributed to black holes of general relativity, as shown by Unruh [16]. Striking parallels are formulated between the physics of quantum liquids and particle physics and cosmology [17]. Topological defects in graphene have been described by gauge fields which couple to the Dirac equation for Dirac electrons [18, 19]. All the above facts suggest that some condensed matter systems can be engineered with a prescribed set of gauge symmetries, allowing to test several aspects of high energy physics and gravity theory, otherwise impossible to do within current technological limitations.

In this paper, we follow the gauge paradigm, suggesting that interactions in condensed matter systems are consequence of local gauge invariance and follow from general symmetry principles. This assumption serves as a guidance to study the motion of electrons in a solid. Mutual interactions result from local symmetry properties, with the gauge field describing the elementary excitations of the solid. We found that a similar procedure was employed in the past, to formulate gravitation as a gauge invariant theory under local Lorentz and Poincaré transformations. After the seminal papers by Utiyama and Kibble [20], gauge theories
of gravitation develop into a prolific field [21]4, which includes teleparallel gravity as an important case [22]. The concept of teleparallelism was created by Einstein in 1928 in order to incorporate spin into General Relativity [23]. In teleparallel gravity, the affine connection is generalized to include an antisymmetric part, called torsion, while the curvature vanishes identically. In contrast, standard Einstein General Relativity can be considered as a gauge theory with curvature and no torsion. One can think of a profusion of gauge theories for the mixed case, with non vanishing torsion and curvature. It came to the authors’ surprise that teleparallel gravity can be interpreted as a gauge theory for the translation group, with the so-called tetrad field [22] as the gauge potentials5. This is strikingly similar to our approach to introduce gauge invariance in solid state physics. In fact, we proceed by requiring invariance under local space–time translations in our condensed matter system, leading in a natural way to the introduction of gauge fields, to account for elastic properties of continuous matter. Outcomes of our procedure are twofold: (i) coupling of electrons with the gauge field are generated straightforwardly, with electrons being scattered by elastic fluctuations. In the linear regime, this interaction is identified with the standard electron–phonon coupling. When the electron dynamics include spin–orbit effects, we found that the elastic field couples with the spin current, inducing novel magneto-elastic phenomena as a reflection of gauge symmetries. We discuss several interesting scenarios that may be relevant in the field of spintronics; (ii) the dynamics of the free gauge field mimics the weak limit of Einstein’s theory of gravity, with equations that are similar in many respects to Einstein’s field equations. Hopefully, one may get interesting insights from this analogy, considering that the theory of elasticity can be quantized, at least in the linear regime, with phonons as the field quanta. This topic deserves a full study by itself in future investigations. In this paper, we will concentrate on the electronic interactions generated by the gauge field.

The content of this paper is organized as follows: in the next section, we develop the general theory, imposing gauge invariance under general space–time dependent translations. Our electronic system is described by a simple Lagrangian density, with a free-electron structure. Local symmetry requires the introduction of a gauge field, associated with the elastic properties of the solid. A dominant vector phonon field is obtained in the linear regime, for translations which are of space-like character. In section 3, we illustrate the linear regime for the unidimensional case. The vector phonon field is quantized yielding phonon quasi-particles, and we identify the electron–phonon coupling comparing with standard solid state results. In section 4, relying on spin–orbit effects and gauge invariance properties, we predict a novel interaction which couples the elastic field with the electronic spin and which may be important for spin transport in a variety of systems. Finally, in the last section a few comments and conclusion are added. In particular, we speculate over possible scenarios for superconductivity, when pairing is induced by the electron–phonon interaction, in the presence of strong magneto-elastic effects. We suggest that our discussion is relevant for the physics of a class of iron-based superconductors [24, 25].

2. The phonon field as a gauge field

It can be thought that electrons actually moving under the influence of a crystalline potential live in a world whose background geometry is provided by the crystal lattice. The background metric is the analogous of a local Minkowskian metric in general relativity and corresponds

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4 see also [3], Vol. II, Part IV.
5 We are grateful to an anonymous referee, who called our attention concerning gauge theories of gravitation and teleparallel gravity.
to what the electron effectively feels by freezing all the crystal lattice ions in their exact equilibrium positions. Excitations, such as vibrations of the lattice and displacements of the ions, are ‘felt’ by the electrons as fluctuations of the background metric. This is done in close parallel with the idea that the gravitational field can be considered as a gauge field, by locally gauging Lorentz transformations (with local coordinate dependent space–time translations and rotations)  [26]. In the present case, we want to associate a gauge field with elastic deformations in a solid. In order to achieve this goal, we must impose invariance of the electron Lagrangian density under coordinate dependent translations. Infinitesimal transformation of this kind involves rigid-body displacements (constant translations) and deformations with an associated strain field. Considering the non-relativistic theory as our first approximation, the Lagrangian involves rigid-body displacements (constant translations) and deformations with an associated strain field. In the present case, we want to associate a gauge field with elastic deformations (local coordinate dependent space–time translations and parallel with the idea that the gravitational field can be considered as a gauge field, by locally gauging Lorentz transformations (with local coordinate dependent space–time translations and rotations)  [26]. In the present case, we want to associate a gauge field with elastic deformations in a solid. In order to achieve this goal, we must impose invariance of the electron Lagrangian density under coordinate dependent translations. Infinitesimal transformation of this kind involves rigid-body displacements (constant translations) and deformations with an associated strain field. Considering the non-relativistic theory as our first approximation, the Lagrangian involves rigid-body displacements (constant translations) and deformations with an associated strain field. In the present case, we want to associate a gauge field with elastic deformations (local coordinate dependent space–time translations and

\[ L = i \psi^\dagger \frac{\partial \psi}{\partial t} - \frac{1}{2m^*} \nabla \psi^\dagger \cdot \nabla \psi, \]

where \( \psi \) is a Pauli spinor describing the electron and \( m^* \) is the electronic effective mass. Apart from \( m^* \), the Lagrangian density (1) represents a free electron in the lattice. In the long wavelength limit of the theory, the crystalline lattice is ‘seen’ as a uniform mass distribution. In passing, we notice that a common viewpoint in the description of collective excitations of a crystalline lattice, assumes a symmetry breaking from a continuous translational symmetry 

to a discrete subgroup, corresponding to translations between equivalent points in different primitive cells. A manifest consequence of the discrete nature of crystalline translations is the Bloch theorem, which ensures that a translation \( x \rightarrow x + \mathbf{a} \), being \( \mathbf{a} \) an appropriate lattice translation vector, acts on the electron wavefunction \( \psi \) by means of the translation operator \( \tau(\mathbf{a}) = \exp[-i\mathbf{a} \cdot \mathbf{p}] \), where \( \mathbf{p} = -i\nabla \) is the momentum operator. The transformed wavefunction \( \psi' = \tau(\mathbf{a})\psi = \exp[-i\mathbf{a} \cdot \mathbf{p}]\psi \) differs from \( \psi \) only by a phase factor of the form \( \exp[-i\mathbf{k} \cdot \mathbf{a}] \). This concept is extended here, and we associate a gauge field to slight distortions of the crystal lattice, inducing a space–time dependent translation of the electronic wavefunction. Instead of a translation by a lattice vector \( \mathbf{a} \), we subject the Pauli spinor \( \psi \) to an infinitesimal translation \( \delta \mathbf{a}(\mathbf{x}, t) \) in space–time, as follows:

\[ \psi' = \tau(\delta \mathbf{a}(\mathbf{x}, t))\psi = [1 - i\mathbf{a}(\mathbf{x}, t) \cdot \mathbf{p}]\psi, \]

and require the invariance of the Lagrangian density (1) under such symmetry transformation. By letting the translation operator \( \tau \) depend on space–time coordinates, we raise the status of the transformation to a local gauge symmetry, and (2) is a generalization of the Bloch theorem for such a case. In the non-relativistic limit, the velocity of light is set as being infinite, and we have to introduce a velocity scale \( c_s \) relevant for the physics of the elastic media. This will allow us to preserve the covariant notation of relativistic physics and to construct a close analogue of the general relativity theory, with space–time coordinates \( x^\mu = (c_s t, \mathbf{x}) \). Within this modified Minkowskian metric, we interpret \( c_s \) as being the velocity of signals in the elastic ‘world’. More on this later. Now that the metric is defined, we follow the gauge prescription [27]. In order to preserve the invariance of the lagrangian density (1) under the local transformations (2), we must introduce gauge fields. Ordinary derivatives \( \partial_\mu \equiv \partial / \partial x^\mu \) have to be replaced by the covariant forms \( D_\mu \) defined as \( D_\mu = \partial_\mu + g W_\mu \), \( W_\mu \) being the gauge potentials and \( g \) the coupling constant of the theory. The gauge potentials are written in terms of the infinitesimal generators of space–time translations, as

\[ W_\mu = -i R_{\mu \nu} p^\nu, \]

where \( p^\nu = i \partial^\nu \) is the four-momentum operator, and \( R_{\mu \nu} = R_{\nu \mu} \) is a symmetric second rank tensor. Einstein summation convention over repeated indices is implied and is to be used...
throughout this paper. Our choice of symmetric connections follows by analogy with standard General Relativity, but other choices are possible, including torsion, which is associated to the antisymmetric part of the connection. In gauge theories of gravity, torsion appears as an independent gauge field, which couples to the intrinsic spin of elementary particles [22]. In our condensed matter model, we will keep symmetric potentials. The role played by torsion is an interesting problem, and will be studied elsewhere. In Einstein’s theory of gravity, the potentials \( R_{\mu\nu} \) are directly related to the metric tensor, at least in the weak gravitational field regime [28, 29], but here we identify them as the tensor components of the elasticity field. The dynamic laws of \( R_{\mu\nu} \) are similar to Einstein’s field equations and follows from the free field lagrangian density

\[
\mathcal{L}_{GF} = -\frac{1}{4} G_{\mu\nu\rho} G^{\mu\nu\rho} = -\frac{1}{4} \text{Tr}[G_{\mu\nu} G^{\mu\nu}].
\]

with the elasticity fields \( G_{\mu\nu} \) being obtained from the commutator of the covariant derivatives [27]

\[
gG_{\mu\nu} = i[D_{\mu}, D_{\nu}] = i(D_{\mu}D_{\nu} - D_{\nu}D_{\mu})
\]

\[
= ig(\partial_{\mu}W_{\nu} - \partial_{\nu}W_{\mu}) + ig^{2}[W_{\mu}, W_{\nu}]
\]

\[
= g G_{\mu\nu\beta} p^{\beta}.
\]

In relation (4), the symbol Tr means taking the trace over the vector space of the field. The commutator \([W_{\mu}, W_{\nu}]\) does not vanish, and the gauge is non-Abelian. Also, the theory is obviously nonlinear and invariant under the gauge transformation \( R'_{\mu\nu} = R_{\mu\nu} - \partial_{\mu}a_{\nu} - \partial_{\nu}a_{\mu} \), being \( a_{\mu} \) an arbitrary four-vector. An immediate consequence of the definition (5) is that the fields \( G_{\mu\nu} \) satisfy the Bianchi identities,

\[
D_{\mu} G_{\nu\lambda} + D_{\lambda} G_{\mu\nu} + D_{\nu} G_{\lambda\mu} = 0.
\]

In the spirit of a linear theory, the lagrangian density of the elasticity fields becomes

\[
\mathcal{L}_{GF} = -\frac{1}{4}[\partial_{\mu}R_{\nu\rho} \partial^{\mu}R^{\nu\rho} - \partial_{\mu}R_{\mu\nu} \partial^{\mu}R^{\mu\nu}].
\]

As a particular case, we will consider only translations in space coordinates, but not in the time coordinate, corresponding to a transformation \([x' = x + \delta a(x, t), t' = t]\), allowing us to write the covariant derivatives as follows:

\[
D_{t} \equiv \frac{D}{Dt} = \frac{\partial}{\partial t} - ig_{0} R \cdot p \equiv \frac{\partial}{\partial t} - ig_{0} R \cdot p
\]

\[
D_{i} \equiv \frac{D}{Dx^{i}} = \frac{\partial}{\partial x^{i}} - ig R_{i} \cdot p.
\]

where \( R \) and \( R_{i} \) are gauge vector potentials related to the phonon and strain fields to be specified later, and \( g_{0} = c_{s} g \) and \( g \) are the corresponding coupling constants. The full gauge invariant lagrangian density is then given by

\[
\mathcal{L} = i\bar{\psi} \left( D_{t} \psi - \frac{1}{2m^{*}}(D_{i} \psi)^{\dagger} (D_{i} \psi) \right) + \mathcal{L}_{GF}.
\]

Note that the field \( R_{\mu\nu} \) is of quadrupolar character, as in general relativity. For space-like translations, with \( g \ll g_{0} \), the coupling with the electronic part is dominated by the vector field \( R \), which we identify with the phonon vector field \( \phi(x, t) \). This corresponds to lattice displacements in the linear limit, with three polarization degrees of freedom. In the general case, all components of the field are coupled and phonons will be scattered by fluctuations of the strain field. For the strong nonlinear regime, identity of phonons may be lost. In the next section we will make our calculation more explicit using the (1+1)-dimensional case as an illustrative example.
3. The unidimensional example

For sake of simplicity, suppose that space is one-dimensional. In such a case, the Lagrangian density of the theory takes the form

\[ \mathcal{L} = \mathcal{L}_{GF} + i \bar{\psi} \frac{\partial \psi}{\partial t} - \frac{1}{2m^*} \left( (\partial_x - gR \partial_x) \bar{\psi} \psi - 1 \right) \]

(10)

The quantity \( R \) is actually the off-diagonal component of the tensor \( R_{\mu\nu} \), with \( R = R_{01} = R_{10} \), while \( R_x = R_{11} \). The linearized version of the free gauge Lagrangian density now takes the form

\[ \mathcal{L}_{GF} = - \frac{1}{2} \left[ (\partial^0 R_x)^2 + (\partial_x R)^2 - 2 (\partial^0 R) (\partial_x R) \right] \]

(11)

yielding the Hamiltonian density of the gauge field in a straightforward way

\[ \mathcal{H}_{GF} = \frac{1}{2} \left[ c_s^2 (\partial R_x) \frac{\partial R_x}{\partial t} + (\partial_x R)^2 - c_s^2 (\partial^0 R_x) \frac{\partial R_x}{\partial t} \right] \]

(12)

The fields can be decoupled if we fix the gauge with the condition

\[ \frac{\partial R_x}{\partial t} = 0, \]

showing that \( R_x \) plays the role of a static strain field. As discussed above, we identify the field \( R_x \) with the phonon field \( \phi \), with longitudinal polarization for the one-dimensional line. The Hamiltonian density is rewritten in the form

\[ \hat{\mathcal{H}}_{FG} = \int d^3x \mathcal{H}_{FG} = \sum_q \omega_q a_q^* a_q \]

Notice that the dispersion relation \( \omega_q = c_s |q| \) here obtained, is a typical feature of long wavelength acoustic phonons. Since the free gauge field emulates Einstein theory of gravity, this unidimensional condensed matter system can be used to study particular aspects of gravity and how it can be quantized. As a rewarding point in this example, we get the standard electron–phonon interaction, once the electron field is second-quantized. If we only consider the coupling with the \( R \) field in (10), the \( \bar{\psi} g_0 R \partial_t \psi \) term is not Hermitian. Proper symmetrization leads to

\[ \mathcal{L}_I = - \frac{1}{2} g_0 \bar{\psi} (\partial_t R) \psi, \]

(14)
i.e. the interaction is given in terms of the ‘dilation operator’ ∂xR, as shown for example in [1].

Next, we expand the electron field in plane waves

\[ \psi = \sum_{k, \sigma} \frac{1}{\sqrt{L}} (a^{k+} c_{k\sigma} + a^{-k} c^\dagger_{k\sigma}), \]

where \( (c_{k\sigma}, c^\dagger_{k\sigma}) \) are fermionic destruction and creation operators of particles with wave number \( k \) and spin \( \sigma \). Comparison with well known results in solid state theory [31], allows us to identify the coupling constant of the linear theory as

\[ \frac{1}{2} g_0 \to \frac{1}{L} U(q), \]

where \( U(q) = \int dx e^{-iqx} U(x) \) is the Fourier transform of the lattice potential evaluated at the phonon wave number \( q \). For the long wavelength limit, \( q \approx 0 \), we get

\[ \frac{1}{2} g_0 = \langle U(x) \rangle, \]

where \( \langle U(x) \rangle \) is the mean value of the lattice potential.

We conclude this section by remarking that we have obtained the long wavelength phonon theory imposing gauge invariance of the electronic lagrangian density, in which the phonon field simply appears as the gauge field. That is, translational symmetry is of local character and not global, and the gauge field is introduced to ensure full invariance of the theory. In spite that we have resorted to crystalline structures to compare with well known solid state examples, it is evident that our approach is far more general and can be applied, in addition of crystals, to a plethora of ‘soft’ condensed matter systems. A contrasting point of view tells us that phonons emerge as Goldstone bosons associated to a spontaneous breaking of the global Galilean symmetry group. In this case, the continuous translational symmetry reduces to the discrete translational group of the crystalline lattice.

4. Coupling of elastic deformations with spin currents

The aim of this section is to demonstrate the existence of a coupling between the electronic spin and the strain field of the crystal lattice via the spin–orbit interaction, assuming that gauge invariance of the whole theory applies. This coupling has been included as an ad hoc interaction in several studies of strain effects in semiconductors [32, 33], under the name of ‘spin–orbital strain effects’. The interplay of strain, exchange, and spin–orbit coupling may also be responsible for spin polarization in graphene, as suggested by a recent calculation [34]. Below, we will show that the above coupling is of fundamental origin. Indeed, consideration of spin–orbit interaction leads to the electronic Hamiltonian density

\[ \mathcal{H}_{\text{SO}} = -\frac{i\mu_B}{4m} (\nabla \psi^\dagger \cdot \sigma \times E \psi - \psi^\dagger \sigma \times E \cdot \nabla \psi), \] (15)

where \( \mu_B \) is the Bohr magneton and \( E \) is the crystalline electric field (or any external applied electric field) [5]. Such effect is named ‘spin–orbit coupling’ because it has the same origin as the analogous spin–orbit effect \( -\lambda \sigma \cdot L \) present in atoms. It is apparent that the Hamiltonian density (15) is not gauge invariant under general translations, due to the presence of ordinary differential operators. In order to preserve the gauge invariance of the theory, one has to replace the ordinary derivatives \( \nabla \) by the covariant ones \( D \), yielding

\[ \tilde{\mathcal{H}}_{\text{SO}} = -\frac{i\mu_B}{4m} (\nabla \psi^\dagger \cdot \sigma \times E \psi - \psi^\dagger \sigma \times E \cdot \nabla \psi) \]
\[ + \frac{i\mu_B}{4m} g_{ij} \mu E_{ijR} [\partial_t (\psi^\dagger \sigma_i \psi - \psi^\dagger \sigma_i \partial_t \psi)], \] (16)
where repeated indices are to be summed over and \(g\) is the coupling constant defined previously. In the extra term appearing in (16), the electronic spin is coupled to the space-like elastic field \(R_{kl}\) and to the electric field of the strained lattice. It can be written in a more appealing form, if one defines the spin-current density as [5, 30]

\[
J_{il} = -\frac{\mu_B}{2m} \left[ (\partial_l \psi^\dagger) \sigma_i \psi - \psi^\dagger \sigma_i (\partial_l \psi) \right],
\]

thus allowing us to rewrite the additional term appearing in (16) in the following way:

\[
\mathcal{H}_{\text{SO}}' = -g^2 \varepsilon_{ijk} J_{il} R_{kl} E_j.
\] (17)

A coupling in the form \(J_{il} R_{kl}\) reveals the existence of an interaction between the spin-current density \(J_{il}\) and the elastic field \(R_{kl}\). The spin couples to the symmetric connection \(R_{kl}\) through the spin–orbit interaction, so this is not a direct coupling of the gauge fields with the spin. In the present contribution, we will not pursue the full quantization of the field \(R_{kl}\), that deserves a thorough study by itself, with intrinsic difficulties related to quantize a nonlinear field. Generically, we will call it as the ‘strain field’. At least some immediate physical consequences can be predicted, based on the above analysis: (i) electron spin-flip processes are allowed by scattering from strain field fluctuations, as given by relation (17). Thus, the thermal elastic field may also be responsible for spin decoherence in the process of spin transport in condensed matter; (ii) appropriate mechanical manipulations could be used to stimulate spin currents in the material or to control spin qubits in a quantum computer nanodevice, as for example the case shown in [35]; (iii) the newly discovered spin Seebeck effect [36] may also be related to the coupling (17). In a typical setup, a temperature gradient causes a spin ‘voltage’ across the sample, due to the generation of a spin current. In this novel phenomenon, thermal phonons, spin–orbit interaction and spin-currents effects are interrelated. A full theoretical explanation is still lacking, but we suggest that theoretical models based on (17) may yield some key clues in this scenario.

5. Conclusion

In summary, contrasting to the general understanding of phonons as being Goldstone bosons, in this paper we propose that phonons are related to a gauge field associated to local symmetry properties (gauge bosons). The application of Goldstone theorem is subdued by the gauge invariance principle, which provides us with a more general approach. In fact, the Goldstone boson is associated with the spontaneous breaking of a global symmetry, while gauge theories deal with local symmetry properties.

This local symmetry principle is applied to a condensed matter system consisting of electrons moving in a solid with elastic properties. We adopt for electrons a minimal model, that only includes the kinetic energy and the effective mass. In spite that infinitesimal generators of translations commute, the gauge field is not Abelian, due to the space–time dependent character of generalized translations. Electronic interactions are generated through couplings with the gauge field, when one requires full gauge invariance. The background medium is rigged with a space–time structure that mimics the Minkowskian metric, with the sound velocity playing the role of the velocity of light. The gauge field describes elastic properties of the solid and supports collective excitations in the form of elastic waves. The free field is in principle nonlinear, and emulates Einstein’s theory of General Relativity. When we consider space-like translations in the linear limit of the theory, a vector branch of the field decouples from the other components, forming the ‘phonon’ field. We elaborate this case in 1-dim, showing that the theory reduces to the long wavelength limit of acoustic phonons, with a dispersion relation of
the form $\omega_k = c_s k$, with $c_s$ being the sound velocity. The resulting electron–phonon interaction is full identified, comparing with standard treatments in solid state theory. Most important, novel predictions are obtained when we enforce gauge invariance on the spin–orbit interaction, revealing the existence of a coupling between the spin-current density and the elastic field. The interplay of elastic properties and spin transport is certainly a key question in the field of spintronics.

In closing this section, we comment on a possible pairing scenario for superconductivity, when the superconducting state is mediated by the electron–phonon interaction. If the spin–orbit coupling is not negligible, an interaction term such as (17) will introduce spin flip in the pairing mechanism, since phonons scattered by strain field fluctuations will couple with the spin current. As a result, Cooper pairs will acquire singlet and triplet admixtures. This will imply a mixed symmetry of the order parameter. Furthermore, coupling between elastic and magnetic fluctuations may open a road for enhancing the electron–phonon interaction in materials where the magnetoelastic effect plays an important role. We suggest that this discussion is relevant for the physics of the recently discovered iron-based superconductors [24, 25]. With unusual high transition temperatures, they exhibit a superconducting phase that is very close to structural and magnetic transitions, giving a strong hint that the above phenomena are interrelated [37]. Our theory proposes that the above connection is a fundamental trait that comes from general principles.

Acknowledgment

One of the authors (CAD) would like to thank the Brazilian agency CNPq for partial financial support.

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