Generating the curvature perturbation without an inflaton

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We present a mechanism for the origin of the large-scale curvature perturbation in our Universe by the late decay of a massive scalar field, the curvaton. The curvaton is light during a period of cosmological inflation, when it acquires a perturbation with an almost scale-invariant spectrum. This corresponds initially to an isocurvature density perturbation, which generates the curvature perturbation after inflation when the curvaton density becomes a significant fraction of the total. The isocurvature density perturbation disappears if the curvaton completely decays into thermalised radiation. Any residual isocurvature perturbation is 100\% correlated with the curvature. The same mechanism can also generate the curvature perturbation in pre big bang/ekpyrotic models, provided that the curvaton has a suitable non-canonical kinetic term so as to generate a flat spectrum.

Introduction. It is now widely accepted that the dominant cause of structure in the Universe is a spatial curvature perturbation \(\delta\). This perturbation is present on cosmological scales a few Hubble times before these scales enter the horizon, at which stage it is time-independent with an almost flat spectrum. One of the main objectives of theoretical cosmology is to understand its origin.

The usual assumption is that the curvature perturbation originates during inflation, from the quantum fluctuation of the slowly-rolling inflaton field. As cosmological scales leave the horizon, the quantum fluctuation is converted to a classical gaussian perturbation with an almost flat spectrum, generating immediately the required curvature perturbation which is constant until the approach of horizon entry. This idea has the advantage that the prediction for the spectrum is independent of what goes on between the end of inflation and horizon entry \(\delta\). The spectrum depends only on the form of the potential and on the theory of gravity during inflation (usually taken to be Einstein gravity), providing therefore a direct probe of conditions during this era. On the other hand, the demand that inflation should produce the curvature perturbation in this particular way is very restrictive, ruling out or disfavouring several otherwise attractive models of inflation.

In this note we point out that the primordial curvature perturbation may have a completely different origin, namely the quantum fluctuation during inflation of a light scalar field which is not the slowly-rolling inflaton, and need have nothing to do with the fields driving inflation. We call this field the curvaton. The curvaton creates the curvature perturbation in two separate stages. First, its quantum fluctuation during inflation is converted at horizon exit to a classical perturbation with a flat spectrum. Then, after inflation, the perturbation in the curvaton field is converted into a curvature perturbation. In contrast with the usual mechanism, the generation of curvature by the curvaton requires no assumption about the nature of inflation, beyond the requirement that (if the curvaton has a canonical kinetic term) the Hubble parameter is practically constant. Instead, it requires certain properties of the curvaton and of the cosmology after inflation so that the required curvature perturbation will be generated. We shall explore the simplest setup, consisting of the following sequence of events. First, the curvaton field starts to oscillate during some radiation-dominated era, so that it constitutes matter with an isocurvature density perturbation. Secondly, the oscillation persists for many Hubble times so that a significant curvature perturbation is generated. Finally, before neutrino decoupling, the curvaton decays and the curvature perturbation remains constant until the approach of horizon entry.

We shall show that under these conditions the quantum fluctuation of the curvaton during inflation is converted into a curvature perturbation after decay according to the formula

\[
\zeta \sim r\delta
\]

where \(\delta\) is the isocurvature fractional density perturbation in the curvaton before it decays and \(r\) is the fraction of the final radiation that the decay produces.

The mechanism that we are describing may succinctly be described as the conversion of an isocurvature perturbation into a curvature perturbation. It was actually discovered more than a decade ago by Mollerach \(\delta\), who corrected the prevailing misconception that no conversion would occur. At the time the conversion was regarded as a negative feature, because the focus was on finding a good mechanism for generating an isocurvature perturbation. For this reason, and also because cosmology involving late-decaying scalars was not considered to be very likely, the conversion mechanism has received lit-
tle attention\textsuperscript{[5]} The situation now is very different. On the observational side, we know that the curvature perturbation provides the principle origin of structure. On the theoretical side, late-decaying scalars are routinely invoked in cosmology, and are ubiquitous in extensions of the Standard Model of particle physics. Also, one is now aware of inflation models whose only defect is their failure to generate the curvature perturbation from the inflaton, some of which will be mentioned later.

Before ending this introduction, we need to emphasise that the curvaton can produce a curvature perturbation \textit{without} any accompanying isocurvature perturbation at late times. This is the reason why we include in our setup the requirement that the curvaton decays before neutrino decoupling. If it decays later, the curvature perturbation may be accompanied by a significant isocurvature neutrino perturbation as discussed recently by Hu \textsuperscript{[6]}.

\textbf{The curvature perturbation.} The spatial curvature perturbation is of interest only on comoving scales much bigger than the Hubble scale (super-horizon scales). To define it one has to specify a foliation of spacetime into spacelike hypersurfaces (slicing), and the most convenient choice is the slicing of uniform energy density (or the slicing orthogonal to comoving worldlines, which is practically the same on super-horizon scales). The curvature perturbation on uniform-density slices \textsuperscript{[7]} is given by the metric perturbation $\zeta$, defined with a suitable coordinate choice by the line element

$$
\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)(1 + 2\zeta)\,\mathrm{d}^3 x.
$$

On cosmological scales, the spectrum $P_\zeta$ of $\zeta$ at the approach of horizon entry is almost flat, with magnitude of order $10^{-10}$.

The time-dependence of $\zeta$ on large scales is given by

$$
\dot{\zeta} = -\frac{H}{\rho + P}\delta P_{\text{nad}},
$$

where $H \equiv \dot{a}/a$ is the Hubble parameter, $\rho$ is the energy density, $P$ is the pressure and $\delta P_{\text{nad}}$ is the pressure perturbation on uniform-density slices (the non-adiabatic pressure perturbation).

In the usual scenario where $\zeta$ is generated during inflation through the perturbation of a single-component inflaton field, it becomes practically time-independent soon after horizon exit and remains so until the approach of horizon entry. The mechanism that we are proposing starts instead with a negligible curvature perturbation, which is generated later through a non-adiabatic pressure perturbation associated with the curvaton perturbation.

\textit{The curvaton field.} The curvaton field $\sigma$ lives in an unperturbed Robertson-Walker spacetime characterised by the line element

$$
\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j
$$

and its lagrangian is

$$
\mathcal{L}_\sigma = \frac{1}{2}\dot{\sigma}^2 - \frac{1}{2}(\nabla\sigma)^2 - V(\sigma)
$$

The potential $V$ depends of course on all scalar fields but we exhibit only the dependence on $\sigma$ which is assumed to have no significant coupling to the fields driving inflation.

The initial era for our discussion is the one which begins several Hubble times before the observable Universe leaves the horizon during an inflationary phase, and ends several Hubble times after the smallest cosmological scale leaves the horizon. During this era, we assume that the Hubble parameter $H \equiv \dot{a}/a$ is almost constant, that is

$$
\epsilon_H \equiv -\dot{H}/H^2 \ll 1.
$$

In the usual slow-roll paradigm with Einstein gravity, $2\epsilon_H \simeq (M_P V'/V)^2$ where $V(\phi)$ is the inflationary potential, $\phi$ is the slowly rolling inflaton and $M_P = 2 \times 10^{18}$ GeV is the reduced Planck mass. However for our mechanism we need not assume any specific paradigm for inflation.

We assume that the curvature perturbations is negligible during inflation. For slow-roll inflation with Einstein gravity this requires $H \lesssim 10^{-5}\epsilon_H^{1/2}M_P$ or $V^{1/2} \lesssim 10^{-2}\epsilon_H^{1/2}M_P$. In any case, an inflation model with Einstein gravity requires $H \lesssim 10^{-5}M_P$ from the cosmic microwave background limit on gravitational waves.

We write at any given time

$$
\sigma(\mathbf{x}) = \sigma + \delta\sigma(\mathbf{x})
$$

(Throughout we adopt the convenient notation that the absence of an argument denotes the unperturbed quantity.) Like any cosmological quantity the spatial dependence of $\delta\sigma$ can be Fourier-expanded in a comoving box much larger than the observable Universe, but it is unnecessary and in fact undesirable for the box to be indefinitely large. Failure to limit the box size leads among other things to an indefinitely large fluctuation for any quantity with flat spectrum. It is a source of confusion in the usual case of inflaton-generated curvature and the same would be true for curvaton-generated curvature.

The unperturbed curvaton field satisfies

$$
\ddot{\sigma} + 3H\dot{\sigma} + V_\sigma = 0,
$$

where $H \equiv \dot{a}/a$ is the Hubble parameter, and a subscript $\sigma$ denotes $\partial/\partial\sigma$.

We are interested in the perturbation $\delta\sigma(\mathbf{k})$, where $\mathbf{k}$ denotes the comoving momentum. It is conveniently defined on the spatially-flat slicing. In general, the scalar
field perturbations on this slicing satisfy to first order the set of coupled equations [10]

\[ \delta \phi_{i} + 3H \delta \phi_{i} + \frac{k^{2}}{a^{2}} \delta \phi_{i} + \sum_{j} \left[ V_{i,j} - \frac{1}{M^{2}_{\text{P}} a^{3}} \left( \frac{a^{3}}{H} \delta \phi_{j} \right) \right] \delta \phi_{j} = 0 \]  

(9)

For simplicity we assume that \( \sigma \) is sufficiently decoupled from the other perturbations that the latter can be ignored, leading to

\[ \delta \sigma_{k} + 3H \delta \sigma_{k} + (\langle k/a \rangle^{2} + V_{\sigma \sigma}) \delta \sigma_{k} = 0. \]  

(10)

We assume that the curvaton potential is sufficiently flat during inflation,

\[ |V_{\sigma \sigma}| \ll H^{2}, \]  

(11)

and that on cosmological scales each Fourier component is in the vacuum state well before horizon exit. The vacuum fluctuation then causes a classical perturbation \( \delta \sigma_{k} \) well after horizon exit, which satisfies Eq. (10) with negligible gradient term,

\[ \delta \sigma_{k} + 3H \delta \sigma_{k} + V_{\sigma \sigma} \delta \sigma_{k} = 0. \]  

(12)

The perturbation is gaussian, and in the limit where Eq. (11) is very well satisfied its spectrum given by

\[ P_{\delta \sigma}^{\frac{1}{2}} = \frac{H_{*}}{2\pi}. \]  

(13)

The star denotes the epoch of horizon exit, \( k = a_{*}H_{*} \), and, by virtue of Eq. (6), \( P_{\sigma} \) is almost flat. To be more precise, the spectral tilt of the perturbation is given by

\[ n_{\sigma} = \frac{d \ln P_{\sigma}}{d \ln k} = \frac{2}{H_{*}^{2}} \left( \frac{2}{3} \frac{V_{\sigma \sigma}}{H_{*}^{2}} \right). \]  

(14)

Oscillating phase. We now move on to the epoch when the curvaton field starts to oscillate around the minimum of its potential. We suppose that the oscillation starts during some radiation-dominated era. It may not be the one in which nucleosynthesis occurs, but if it is we require that the oscillation starts well before cosmological scales enter the horizon. We assume that Einstein gravity is valid from the oscillation time onwards, so that the total energy density is \( \rho = 3H^{2}M^{2}_{\text{P}} \).

We assume that the curvaton continues to satisfy Eq. (6), and that its perturbation continues to satisfy Eq. (12), after inflation. Assuming that the potential \( V(\sigma) \) is quadratic, \( V = m^{2}\sigma^{2}/2 \), then oscillations start at the epoch \( H \sim m \). Also, Eq. (12) for \( \sigma \) and Eq. (6) for \( \delta \sigma \) are then the same and the ratio \( \delta \sigma/\sigma \) remains fixed on super-horizon scales. This assumption can easily be relaxed, and in particular one can handle the situation where the curvaton may initially be near a maximum of the potential [11]. Any evolution of \( \delta \sigma \) on super-horizon scales leads to an overall scale-independent factor which will not spoil the flatness of the spectrum. Even if the potential is not quadratic at the onset of oscillation, it will become practically quadratic after a few Hubble times as the oscillation amplitude decreases.

We are interested in the curvaton energy density

\[ \rho_{\sigma}(x) = \rho_{\sigma} + \delta \rho_{\sigma}(x) \]  

(15)

and in the density contrast

\[ \delta \equiv \frac{\delta \rho_{\sigma}}{\langle \rho_{\sigma} \rangle} \]  

(16)

Since the spatial gradients are negligible on super-horizon scales, the oscillation is harmonic at each point in space and

\[ \rho_{\sigma}(x) = \frac{1}{2}m^{2}\sigma^{2}(x), \]  

(17)

where \( \sigma(x) \) is the amplitude of the oscillation.

When the oscillation starts, the mean-square perturbation of \( \sigma \) is given by

\[ \langle (\delta \sigma)^{2} \rangle = \int_{k_{\text{min}}}^{k_{\text{max}}} P_{\sigma}(k) \frac{dk}{k}. \]  

(18)

As discussed in [13], the short distance cutoff at the epoch when the oscillation starts is \( k_{\text{max}} \sim (aH) \) where a tilde denotes this epoch, sub-horizon modes having red-shifted away. Also, since we are working in a box not too much bigger than the the observable Universe the long distance cutoff is \( k_{\text{min}} \sim a_{0}H_{0} \). Assuming that \( P_{\sigma} \) is flat this gives the estimate

\[ \frac{\langle (\delta \sigma)^{2} \rangle}{\sigma^{2}} = \left( \frac{H_{*}}{2\pi \sigma_{*}} \right)^{2} \ln(k_{\text{max}}/k_{\text{min}}) \sim (H_{*}/\sigma_{*})^{2}. \]  

(19)

(If \( P_{\sigma} \) increases dramatically on small scales, the estimate has to be increased appropriately.)

If \( H_{*} \ll \sigma_{*} \), the field perturbation is small and

\[ \delta = 2\frac{\delta \sigma}{\sigma} \]  

(20)

This is a time-independent gaussian perturbation with a flat spectrum given by

\[ P_{\delta \sigma}^{\frac{1}{2}} = 2P_{\sigma}^{\frac{1}{2}}/\sigma = \frac{H_{*}}{\pi \sigma_{*}} \ll 1. \]  

(21)

In the opposite regime, \( H_{*} \gg \sigma_{*} \), the perturbation is bigger than the unperturbed value and

\[ \delta = \frac{\langle (\delta \sigma)^{2} \rangle}{\langle (\delta \sigma)^{2} \rangle}. \]  

(22)

This is again time-independent, but is now the square of a gaussian quantity (a \( \chi^{2} \) quantity). Its spectrum is flat up to logarithms [11].
\[ P_\delta(k) = 4 \ln(k/k_{\text{min}}) \frac{P_\sigma(k_{\text{min}})^2}{(\langle \delta \sigma \rangle)^2} \sim 1. \]  

We shall show that the curvature perturbation is a multiple of \( \delta \), which means that it is gaussian in the regime \( H_* \ll \sigma_* \), but a \( \chi^2 \) non-gaussian quantity in the opposite regime. A \( \chi^2 \) curvature perturbation is strongly forbidden by observation \([13]\), which therefore requires

\[ H_* \ll \sigma_* . \]  

**Generating the curvature perturbation** Once the curvaton field starts to oscillate the energy density becomes a mixture of matter (the curvaton) and radiation. According to Eq. (3), the generation of the curvature perturbation begins at that point, because the pressure perturbation corresponding to this mixture is non-adiabatic. It ends when the pressure perturbation again becomes adiabatic, which is at the epoch of curvaton matter domination, or the epoch of curvaton decay, whichever is earlier.

The curvature perturbation finally generated could be precisely calculated from Eq. (3) knowing the decay rate \( \Gamma \) of the curvaton, but for an estimate it is enough to assume that the decay occurs instantaneously at the epoch \( H = \Gamma \). In that case one can avoid the use of Eq. (3) altogether by considering separately the curvature perturbations \( \zeta_r \) and \( \zeta_\sigma \) on respectively slices of uniform radiation and matter density. These are separately conserved \([8]\) as the radiation and matter are perfect non-interacting fluids. The curvature perturbations are given by

\[
\zeta = -H \frac{\delta \rho}{\rho}, \\
\zeta_r = -H \frac{\delta \rho_r}{\rho_r} = \frac{1}{4} \frac{\delta \rho_r}{\rho_r}, \\
\zeta_\sigma = -H \frac{\delta \rho_\sigma}{\rho_\sigma} = \frac{1}{3} \frac{\delta \rho_\sigma}{\rho_\sigma} = \frac{1}{3} \delta ,
\]

where the density perturbations are defined on the flat slicing of spacetime. (Note that the constancy of \( \zeta_\sigma \) is equivalent to the constancy of \( \delta \) for the oscillating field which we noted earlier.) Using these results the curvature perturbation is

\[
\zeta = \frac{4 \rho_r \zeta_r + 3 \rho_\sigma \zeta_\sigma}{4 \rho_r + 3 \rho_\sigma} .
\]

Before the oscillation begins, \( \zeta = \zeta_r \) which we are supposing is negligible. It follows that

\[
\zeta = \frac{\rho_\sigma}{4 \rho_r + 3 \rho_\sigma} \delta .
\]

This calculation applies until the curvaton decays, after which \( \zeta \) is constant. If the curvaton dominates the energy density before decay, the final value of \( \zeta \) is

\[
\zeta = \frac{1}{3} \delta .
\]

In the opposite case, the curvaton density just before decay is some fraction \( r < 1 \) of the radiation density. Making the approximation \( r \ll 1 \),

\[
r \approx \frac{1}{6} \left( \frac{\sigma_*}{M_P} \right)^2 \left( \frac{m}{\Gamma} \right)^{1/2} ,
\]

and the final curvature perturbation is

\[
\zeta = \frac{1}{4} r \delta .
\]

Dropping the prefactors \( \frac{1}{3} \) and \( \frac{1}{4} \), the spectrum of the curvature perturbation is given in the Gaussian regime, \( H_* \ll \sigma_* \), by

\[
P_\zeta^\perp \approx \frac{H_*}{\pi \sigma_*} .
\]

The scale dependence of the spectrum is the same as that of \( \delta \sigma \). From Eqs. (13) and (14) its spectral index \( n \) is given by

\[
n - 1 = \frac{\Delta \ln P_\zeta}{\Delta \ln k} = 2 \frac{\dot{H}_*}{H_*^2} + \frac{2}{3} \frac{m^2}{H_*^2} .
\]

If we relax the demand that the potential is quadratic, \( m^2 \) is replaced by the effective mass-squared \( m_*^2 = (V_{\sigma \sigma})_* \), which can be either positive or negative. The observational constraint at 95\% confidence level is \( n = 0.93 \pm 0.13 \) \([13]\), which requires

\[
|m_*^2|/H_*^2 \lesssim 0.1 .
\]

The completely non-gaussian regime \( H_* \gg \sigma_* \) is strongly forbidden by observation \([13]\), but the intermediate regime is allowed provided that the non-gaussian component is small. From Eq. (17), the curvature perturbation in that case is of the form \([14]\)

\[
\zeta = \frac{r}{4} \left[ 2 \frac{\delta \sigma}{\sigma} + \left( \frac{\delta \sigma}{\sigma} \right)^2 \right] ,
\]

with

\[
P^{\perp}_{\delta \sigma/\sigma} = \frac{H_*}{2 \pi \sigma_*} .
\]

and, since the first term dominates

\[
r \approx 10^{-5} / P^{\perp}_{\delta \sigma/\sigma} .
\]

Observational constraints on non-gaussianity can place strong upper limits on the small ratio \( H_*/\sigma_* \), which it would be interesting to evaluate.
An isocurvature density perturbation? At the epoch when perturbations first become observable as cosmological scales approach the horizon, the curvature perturbation seems to be the dominant cause of structure but it may not be the only one. In particular, there may be an isocurvature perturbation (one present on slices of uniform total energy density) in the density of one or more of the constituents of the Universe relative (conventionally) to the photon density. In the standard picture the constituents are the photon plus (i) the cold dark matter (ii) the baryon and (iii) the three neutrinos of the Standard Model which have travelled freely since they fell out of equilibrium shortly before nucleosynthesis. There could be an isocurvature density perturbation in any or all of these three components. As described for instance in [14–16], these isocurvature perturbations could be a significant fraction of the total as far as present microwave background observations are concerned, though the PLANCK satellite will rule out (or detect) them at something like the 10% level if their spectrum is flat. Going beyond the standard picture, the dark matter might have non-trivial properties and there may be neutrinos or other free-streaming matter with a non-thermal momentum distribution.

An isocurvature density perturbation may originate as the quantum fluctuation of a scalar field during inflation. However, such a field cannot be the inflaton, and in the usual scenario where the latter generates the curvature perturbation it is hard to see why the effect of an isocurvature perturbation should be big enough to be observable. A priori one expects that either it will be dominant, which is forbidden by observation, or else negligible. In contrast, if the curvature perturbation is generated by a curvaton field, that same field may also generate an isocurvature perturbation. In other words, the isocurvature density perturbation of the curvaton field might be converted into a mixture of a curvature and a correlated isocurvature perturbation.

A study of this possibility is outside the scope of the present paper, but we offer some brief comments. Consider first the case that the curvaton decays before neutrino decoupling. In that case the pre-existing radiation consists of photons and neutrinos which are now decoupled. In that case the curvaton decay will cause a perturbation in the relative abundance of neutrinos and photons, no matter whether it decays to photons or to neutrinos. In other words it will (conventionally) generate a neutrino isocurvature perturbation. At least if the curvaton decays to neutrinos, this isocurvature perturbation should be big enough to observe in the foreseeable future [6]. (Curvaton decay after nucleosynthesis can also significantly alter the epoch of matter-radiation equality, which again may be observable.)

A distinctive prediction of curvaton decay that generates both the curvature perturbation and an isocurvature perturbation at late times is that the two perturbations, arising from a single initial curvaton perturbation, must be completely correlated. Current microwave background data alone cannot rule out a significant contribution from an isocurvature perturbation correlated with the curvature perturbation [15,16] but future data will give much tighter constraints [14].

The curvaton as a flat direction We have still to consider the nature of the curvaton in the context of particle physics. In particular, we did not examine the fundamental assumption that the curvaton potential satisfies the flatness requirement Eq. (11).

Let us first suppose that the curvaton is a generic field, running over an indefinitely large range $\sigma > 0$ with the lower end the fixed point of a symmetry. The potential will be sufficiently flat only over some range $\sigma < \sigma_{\text{max}}$, beyond which it rises too steeply. If inflation lasts long enough we may expect $\sigma_*$ in our location to have equal probability of being anywhere in the range $0 < \sigma_* < \sigma_{\text{max}}$, leading to the rough estimate $\sigma_* \sim \sigma_{\text{max}}$.

For simplicity assume that the symmetry forbids odd powers of $\sigma$ in the potential. Adopting the usual paradigm of supersymmetry, the renormalizable (quadratic and quartic) terms of the potential can be eliminated at the level of global supersymmetry (the curvaton can be chosen as a flat direction in field space). However, at the supergravity level the typical effective mass-squared $m^2(t)$, of a generic field in the early Universe is of order $\pm H^2$, the true mass $m$ being relevant only after $H$ falls below $m$. This is marginally in conflict with the Eq. (11).

There will also be an infinite number of non-renormalizable terms, of the form $\lambda_d \Lambda^{4-d} \sigma^d$ with $d > 6$, where $\Lambda$ is the ultra-violet cutoff at or below the Planck scale. The generic couplings are $\lambda_d \sim 1$, and taking $d = 6$ the potential is sufficiently flat to satisfy Eq. (11) only in the regime $\sigma \lesssim \sqrt{\Lambda H}$. In that case one expects $\sigma_* \sim \sqrt{\Lambda H}$ and therefore from Eq. (11)

$$P_\zeta \sim \frac{r^2 H_*}{\Lambda}. \quad (39)$$

Alternatively, at least the first few non-renormalizable terms might be suppressed by a symmetry. In that case
one might have $\dot{\sigma} \sim M_P$ leading to the much smaller estimate

$$P_\zeta \sim v^2 \frac{H^2}{M_P^2}. \quad (40)$$

The actual value will probably lie between these extremes, and one sees that the observed value $P_\zeta \sim 10^{-10}$ can be achieved with quite reasonable values of the parameters. In contrast with the case where the curvature is generated by the inflaton, the prediction does not depend on the derivative of any potential and is in that respect under better control.

There is clearly quite a bit of uncertainty in this generic case. On the other hand, it has the advantage of occurring rather naturally in the context of particle physics. A recent example, which actually appeared when the present paper was almost written, is reference [17] in which $\sigma$ is the scalar super-partner of a right-handed neutrino. (Note though that this paper does not take account our isocurvature-curvature conversion mechanism, supposing instead that the decay of the sneutrino will set up only an isocurvature baryon density perturbation.)

The curvaton as a pseudo-goldstone boson To achieve better control of the curvaton mechanism, one can suppose that $\sigma$ is a pseudo-goldstone boson, so that $\sigma \to \sigma + \text{const}$ under the action of some spontaneously broken global symmetry. In the limit where this symmetry is exact the potential would be exactly flat which makes the approximate flatness we require technically natural.

Also, in this case $\sigma$ runs over some finite range $0 < \sigma < v$ with periodic boundary conditions. If inflation lasts long enough, there is equal probability of finding $\sigma_*$ anywhere in this range so one expects $\sigma_* \sim v$. Even if it does not, no particular value is favoured and one has the same expectation.

As a simple example consider a complex field $\Sigma$ with a mexican-hat potential whose $U(1)$ symmetry is broken by non-renormalisable terms:

$$V(\Sigma) = (|\Sigma|^2 - v^2)^2 + \frac{1}{72 M_P^2} \left(2v^6 - \Sigma^6 - (\Sigma^*)^6\right). \quad (41)$$

Putting $\Sigma = ve^{i\sigma/v}$ this gives

$$V(\sigma) = \frac{v^6}{36 M_P^2} \left[1 - \cos \left(\frac{6\sigma}{v}\right)\right], \quad (42)$$

and hence

$$d^2V \left|\frac{d\sigma^2}{\sigma^2}\right| \leq \frac{v^4}{M_P^2}. \quad (43)$$

In order for the pseudo-goldstone boson $\sigma$ to acquire a spectrum of classical perturbations on super-horizon scales, while the radial field remains fixed in its vacuum manifold, $|\Sigma| = v$, we require

$$\frac{v^2}{M_P} \ll H_* \ll v. \quad (44)$$

This also ensures that variations in the local value of $\sigma$ has no effect on the background Hubble expansion.

After $N > v/H$ e-folds of inflation, the local value of the $\sigma$ field in the vacuum manifold is effectively randomised, so that we expect $\sigma_* \sim v$ and effective mass

$$m_*^2 \equiv (V_{\sigma\sigma})_* \sim v^2/M_P^2. \quad (45)$$

For simplicity we assume that $\sigma_*$ is close to a minimum so that the potential is quadratic.

The field begins to oscillate when $H \sim m_\sigma$ at which time $\rho_{\text{rad}}/\rho_\sigma \sim (v/M_P)^2 \ll 1$. Note that this is independent of the reheat temperature $T_{\text{rh}}$. The energy density of the oscillating $\sigma$ field comes to dominate over the energy density of the radiation when

$$H_{\text{eq}} \sim \frac{v^6}{M_P^2}. \quad (46)$$

On the other hand if we assume that $\sigma$ decays with only gravitational strength interactions, then the time of decay is given by

$$H_{\text{decay}} \sim \Gamma \sim \frac{m_\sigma^3}{M_P^2} \approx \frac{v^6}{M_P^2}. \quad (47)$$

(That this is before nucleosynthesis requires that $v > 10^{12}\text{GeV}$, i.e., $m_\sigma > 100\text{TeV}$.) Thus we generally expect $\rho_\sigma \sim \rho_{\text{rad}}$ at the time of decay, and hence from Eqs. (21) and (33)

$$P_\zeta \simeq \frac{H_*^2}{v^2}. \quad (48)$$

When the curvaton is a pseudo-goldstone boson, the flatness requirement Eq. (11) presents no problem since the flatness of the potential is protected by the global symmetry. The mass $\sim v$ of the radial field is not protected and in a generic supergravity theory one would expect that its effective value during inflation would receive contributions of order $H_*$. On the other hand, $\sigma_*$ in our part of the Universe is equally likely to lie anywhere in the range $0 < \sigma_* < v$ where the upper limit is the effective value of $v$ during inflation. Therefore, if $H_*$ is bigger than the true (vacuum) value of $v$, the Gaussianity constraint $H_* \ll \sigma_*$ represents a significant restriction on the supergravity theory.

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1 In a similar way, it has been pointed out [18] that making the inflaton a pseudo-Goldstone boson will keep a hybrid inflation potential sufficiently flat. The ‘natural inflation’ proposal [19] that the inflaton is a pseudo-Goldstone boson in a non-hybrid model does not keep the inflaton potential flat, because both the potential and its slope vanish in the limit of unbroken symmetry.
Pre big bang and ekpyrotic scenarios. In some models of the very early universe, such as the pre big bang scenario\textsuperscript{20}, there is essentially no curvature perturbation produced on large scales\textsuperscript{21,22} and these models have been largely discarded as possible sources for the origin of large-scale structure. Yet if they can produce an almost scale-invariant spectrum of isocurvature perturbations then they may generate a primordial adiabatic curvature perturbation on all scales after the pre-big-bang phase. Because the pre big bang ends with an explosive gravitational production of particle on small scales at energies approaching the Planck scale, the late production of entropy, such as from decaying massive fields, may be necessary in any case to avoid over-production of dangerous relics\textsuperscript{23}.

The evolution during a pre big bang\textsuperscript{20} or ekpyrotic\textsuperscript{24,26} phase is far from slow-roll in the Einstein frame ($|\dot{H}/H^2| > 1$) which leads to a steep blue spectrum of curvature perturbations during collapse\textsuperscript{27}. All massless moduli fields minimally coupled in the Einstein frame have blue spectra with spectral tilt $n_\sigma = 3$ in the pre big bang or $n_\sigma = 2$ in the ekpyrotic scenario\textsuperscript{28}. However it has previously been shown that a scale-invariant spectrum may be generated in axion fields in the pre big bang scenario\textsuperscript{29}. Axion-type fields have a non-minimal kinetic coupling to the dilaton field which yields a range of different scale dependences being determined by the symmetries of the sigma-model effective action and arbitrary constants of integration\textsuperscript{28,30}. If the axion remains decoupled during the uncertain transition from pre to post big bang era then these large-scale perturbations remain isocurvature perturbations at the start of the radiation dominated era\textsuperscript{31,32}.

Previous attempts to model structure formation in the pre big bang have assumed that these axions remain decoupled and effectively massless, only generating curvature perturbations when they re-enter the horizon\textsuperscript{33}. This latter model gives distinctive predictions for the spectrum of cmb anisotropies\textsuperscript{34}, but may be hard to reconcile with the latest observational data. However we have shown that an initial spectrum of axion perturbations can in fact generate curvature perturbations on super-horizon scales\textsuperscript{35}, as has previously been suggested in Refs.\textsuperscript{22,23}. If the axions acquire a non-perturbative mass and come to contribute a significant fraction to the total energy density before they decay in the early universe, then they can act as a curvaton field and generate a large-scale curvature perturbation long before horizon entry, indistinguishable from that produced in a conventional inflation model.

\footnote{Note added: This idea has also been studied in the context of pre big bang scenario by Enqvist and Sloth\textsuperscript{23} in a preprint that appeared while this work was being written up.}

Conclusions. We have drawn attention to the fact that the curvature perturbation in the Universe need not be generated by the quantum fluctuation of a slowly-rolling inflaton field, as is generally supposed. Instead it may be generated by the quantum fluctuation of a field that has nothing to do with the inflation model, which we have called the curvaton.

The curvaton mechanism for the generation of curvature perturbations makes it much easier to construct a viable model of inflation. Inflation need not be of the slow-roll variety, and even if it is there is no need for highly accurate slow-roll demanded by the observed spectral index $|n - 1| \lesssim 1$. For instance, extended inflation\textsuperscript{36} was ruled out by the predicted spectral index $n \lesssim 0.7$ by the first COBE result\textsuperscript{37} assuming the standard inflaton mechanism of curvature generation, but it becomes viable with the curvaton mechanism. A similar remark applies to modular inflation\textsuperscript{38}, which (at least typically) also predicts too low a spectral index\textsuperscript{39}. Another theoretically attractive model giving a too low spectral index is described in\textsuperscript{18}. One might even have inflation where the inflaton is not rolling at all, such as thermal inflation\textsuperscript{40}, though more than one bout of such inflation would be necessary since at least in the usual context of gravity-mediated supersymmetry breaking a single bout gives only of order 10 e-folds of inflation.

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