Microstates of D1-D5(-P) black holes as interacting D-branes

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In our previous study \cite{1}, we figured out that the thermodynamics of the near extremal black p-branes can be explained as the collective motions of gravitationally interacting elementary p-branes (the p-soup proposal). We test this proposal in the near-extremal D1-D5 and D1-D5-P black holes and show that their thermodynamics also can be explained in a similar fashion, i.e. via the collective motions of the interacting elementary D1-branes and D5-branes (and waves). It may imply that the microscopic origins of these intersecting black branes and the black p-brane are explained in the unified picture. We also argue the relation between the p-soup proposal and the conformal field theory calculations of the D1-D5(-P) black holes in superstring theory.

\textbf{Introduction.} — One of the most remarkable achievements in the superstring theory is the microscopic computations of several classes of the (near-)extremal black hole entropies initiated by the work of Strominger and Vafa \textsuperscript{2}. (See reviews \textsuperscript{3–6}.) These results provide the microscopic descriptions of these black holes, and they are the strong evidences that the superstring theory works as the quantum gravity at the non-perturbative level. However these studies have been mainly developed in the intersecting black branes, especially in the D1-D5 system \textsuperscript{7}, and it is the outstanding problem whether string theory can explain the thermodynamics of other black holes.

Recently, it has been shown that the thermodynamics of the near-extremal black p-branes in supergravity may be explained by an effective theory of gravitationally interacting elementary p-branes \textsuperscript{1}. (Related studies have been done in \textsuperscript{8–14}.) The elementary branes may compose a bound state at low temperature due to the strong gravitational force, and, by using the virial theorem, we can estimate the energy and entropy of the bound state as functions of physical parameters: gravitational coupling, brane tension, the number of the elementary branes and temperature. Then they parametrically agree with those of the corresponding black brane. Also the size of the bound state agrees with the size of the event horizon of the near-extremal black brane. (Interestingly we are naturally able to reproduce the \pi dependence too.) We call this model ‘warm p-soup’ \textsuperscript{1}, since the bound state is strongly coupled.

The p-soup model works general near extremal black p-branes including branes in the superstring theory, e.g. Dp, Mp, F1 and NS5-branes \textsuperscript{1,3,15}. Then it is natural to ask whether the p-soup model can explain the intersecting black branes. In this letter, we will study the near-extremal D1-D5(-P) system and show that indeed the p-soup model may work. It may imply that the microscopic origins of the intersecting black branes and black p-branes are explained in the unified way. We will also compare this result and the conformal field theory calculations of the D1-D5(-P) system in string theory \textsuperscript{7}.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
 & 1 & 2 & 3 & 4 & (5) \tabularnewline
\hline
Q\textsubscript{1} D1 & - & - & - & - & - \\
\hline
Q\textsubscript{5} D5 & - & - & - & - & - \\
\hline
N P & - & - & - & - & - \\
\hline
\end{tabular}
\caption{The brane configuration of the D1-D5-P system. The configuration of D1-D5 system is the same one with N = 0. We take x\textsubscript{5} as the S\textsuperscript{1} coordinate with the period 2\pi R.}
\end{table}

\textbf{D1-D5 system.} — To study the D1-D5 black brane, we consider IIB superstring theory compactified on S\textsuperscript{1} \times T\textsuperscript{4} and put Q\textsubscript{1} D1-branes and Q\textsubscript{5} D5-branes winding on S\textsuperscript{3} and S\textsuperscript{1} \times T\textsuperscript{4} respectively. (See Table \textbf{I}) We take the size of T\textsuperscript{4} small and assume that the D1-branes uniformly spread over there.

If the branes are static, this configuration is BPS and no forces work. However if they are moving, the interactions arise. We estimate the effective action of this interacting brane system. If the branes are well separated, the gravitational interactions may dominate. We can read off these gravitational interactions between the branes from the probe D1-brane action in the extremal D1-D5 brane background \textsuperscript{4},

\begin{equation}
S_{D1}^{\text{probe}} = -m_1 \int dt \left( \frac{1}{H_1} \sqrt{1 - H_1 H_5 \dot{v}^2} - \left( \frac{1}{H_1} - 1 \right) \right),
\end{equation}

\begin{equation}
H_1 = 1 + \frac{r_1^2}{r_2^2}, \quad H_5 = 1 + \frac{r_2^2}{r_{\text{5}}^2},
\end{equation}

\begin{equation}
r_1^2 = \frac{4m_1 G_5 Q_1}{\pi}, \quad r_2^2 = \frac{4m_5 G_5 Q_5}{\pi}.
\end{equation}

Here we have taken the radius of the S\textsuperscript{1} as R, and assumed that R is small and the probe D1-brane depends on the time t only \textsuperscript{21}. \vec{r} and \vec{v} = \partial_t \vec{r} are the position and the velocity of the D1-brane in the (non-compact) 4 dimensional space. G\textsubscript{5} \equiv 4\pi g\textsubscript{s}^2 \alpha'\textsuperscript{4}/V\textsubscript{4}R is the 5-dimensional Newton constant where g\textsubscript{s} and \alpha' are the string coupling and the Regge parameter and V\textsubscript{4} is the volume of T\textsuperscript{4}. m\textsubscript{1} and m\textsubscript{5} are masses of single D1 and...
D5-brane defined by
\[ m_1 \equiv \frac{R}{g_s \alpha'}, \quad m_5 \equiv \frac{R V_4}{(2\pi)^3 g_s \alpha'^3}. \] (2)

Now we consider the situation \( r^2 \ll r_1^2, r_5^2 \). (We will later see that this corresponds to the near extremal limit in gravity.) Then the probe action \( \mathcal{L}_{D1} \) is reduced to
\[
S_{D1}^{\text{probe}} = \int dt \left( \frac{2G_5 m_1 m_5 Q_5}{\pi^2} \dot{\vec{r}}^2 + \frac{8G_5 m_1^2 Q_1 (G_5 m_5 Q_5)}{\pi^3 r^6} \dot{\vec{r}}^4 + \sum_{n=3}^{\infty} L_n^{\text{probe}} \right). \] (3)

Here we have omitted the rest mass term. Note that, under the assumption \( r^2 \ll r_1^2, r_5^2 \), the interaction terms dominate and even the ordinary kinetic terms \( m_1 \dot{\vec{r}}^2 / 2 \) can be neglected. The first and second terms arise from the single and three graviton exchanges, respectively. \( L_n^{\text{probe}} \) describes the \( 2n \) graviton exchange interaction
\[
L_n^{\text{probe}} = \frac{3n!}{n!} \frac{(G_5 m_1 m_5 Q_5)^{n-1}}{\pi^{2n}} \vec{v}^{2n}. \] (4)

Similarly we calculate the probe action for a D5-brane in the same D1-D5 brane background in case \( r^2 \ll r_1^2, r_5^2 \)
\[
S_{D5}^{\text{probe}} = \int dt \left( \frac{2G_5 m_1 m_5 Q_5}{\pi^2} \dot{\vec{r}}^2 + \frac{8G_5 m_1^2 Q_1 (G_5 m_5 Q_5)}{\pi^3 r^6} \dot{\vec{r}}^4 + \cdots \right). \] (5)

From the probe actions \( \mathcal{L}_{D1} \) and \( \mathcal{L}_{D5} \), we can speculate the effective action of separated \( Q_1 \) D1-branes and \( Q_5 \) D5-branes for \( r^2 \ll r_1^2, r_5^2 \), as
\[
S_{D1D5} = \sum_{n=1}^{\infty} L_n, \quad \mathcal{L}_1 = \sum_{i=1}^{Q_1} \sum_{j=1}^{Q_5} \frac{2G_5 m_1 m_3 \vec{v}_{ij}^2}{\pi \vec{r}_{ij}}, \quad \mathcal{L}_2 = \sum_{i=1}^{Q_1} \sum_{j=1}^{Q_5} \sum_{k=1}^{Q_5} \frac{G_5^2 m_1^2 m_5^2}{\pi^3} \frac{\vec{v}_{ij}^4}{r_{ij}^2 r_{ik}^2 r_{jk}^2} + \cdots. \] (6)

Here \( \vec{r}_{ij} \) and \( \vec{v}_{ij} \) denote the relative position and relative velocity of the \( i \)-th and \( j \)-th branes. \( \mathcal{L}_2 \) is predicted from the second terms of the probe actions \( \mathcal{L}_{D1} \) and \( \mathcal{L}_{D5} \), and represents the three graviton exchange interactions among two D1-branes and two D5-branes. The precise expressions for these interactions cannot be determined from the probe actions, and we would need to solve the multi-body problem in a similar manner to [16]. However we will not use the precise expression for \( \mathcal{L}_2 \) in the following arguments and leave this issue for future works.

\( \sim \) in this article denotes equality including dependence on physical parameters but also including all factors of \( r \). Corresponding to the term \( \mathcal{L}_{D1} \) in the probe action, the effective action would have the following interactions
\[
L_n \sim \sum_{i_1, i_2, \ldots, i_n} \sum_{j_1, j_2, \ldots, j_n} \left( G_5^{2n-1} m_1^m m_5^n \prod_{k=2}^{n} \prod_{l=1}^{n} r_{i_k j_l}^{-2} r_{i_k l}^{-2} \vec{v}^{2n} + \cdots \right), \] (7)

which represent the \( 2n \) graviton exchange among \( n \) D1-branes and \( n \) D5-branes. Again the precise expressions for these interactions cannot be determined from the probe actions.

From now, we estimate the dynamics of this interacting brane system \( \mathcal{L}_1 \). We first assume that the branes are bound due to the interactions, and the branes satisfy
\[
\vec{v}_{ij} \sim v, \quad \vec{r}_{ij} \sim r. \] (8)

Here \( v \) and \( r \) are the characteristic scales of the velocity and size of the branes in the bound state which do not depend on the species of the branes. (Note that if the branes are dilute \( r \gg r_1, r_5 \) and the interactions \( \mathcal{L}_1 \) are weak, the kinetic terms \( m_1 \vec{v}_{ij}^2 \) and \( m_5 \vec{v}_{ij}^2 \) cannot be ignored. If it happens, we cannot assume that these scales do not depend on the species of the branes because their masses are different.) Then we can estimate the scales of the terms in the Lagrangian \( \mathcal{L}_1 \) as
\[
L_1 \sim \frac{G_5 Q_1 Q_5 m_1 m_5 v^3}{\pi r^2}, \quad L_2 \sim \frac{G_5^3 Q_1^2 Q_5^2 m_1^2 m_5^2 v^4}{\pi^3 r^6}. \] (9)

Here the virial theorem may imply that these terms are balanced at the same order in the bound state as \( L_1 \sim L_2 \), and this leads to the relation
\[
v^2 \sim \frac{Q_1 Q_5 m_1 m_5 G_5^2}{Q_1 Q_5 m_1 m_5 G_5^2}. \] (10)

Note that, at this scale, the higher order interactions \( L_n \) also become the same order. It means that the branes are strongly coupled in the bound state. For this reason, we called such a bound state as ‘warm p-soup’ in Ref. [1].

By substituting the relation \( \mathcal{L}_1 \) to the Lagrangian \( \mathcal{L}_1 \), we estimate the free energy of the system as
\[
F \sim L_1 \sim \frac{\pi v^2}{G_5}. \] (11)

In order to consider the entropy of this system, we further assume that the velocity are characterized by the temperature of the system through
\[
v = \partial_T r = \pi T r. \] (12)

In Ref. [3], it has been argued that this assumption is related to a scaling symmetry of the effective action and
π dependence has also been discussed. Then, from (10), we obtain the relation between the size of the bound state and the temperature
\[ r \sim T G_5 \sqrt{Q_1 Q_5 m_1 m_5}. \] (13)
By substituting this relation into the free energy (11), we estimate the entropy of the bound state as
\[ S_{\text{entropy}} = -\frac{\partial F}{\partial T} \sim \pi m_1 m_3 Q_1 Q_5 G_5 T. \] (14)

Now we compare the obtained results with the D1-D5 black hole [15]. In the near extremal regime, the black hole thermodynamics tells us,
\[ F = \frac{\pi r_H^2}{8 G_5}, \] (15)
\[ S_{\text{entropy}} = 16 \pi m_1 m_3 G_5 Q_1 Q_5 T, \] (16)
\[ r_H = 8 G_5 T \sqrt{m_1 m_5 Q_1 Q_5}. \] (17)
Here \( r_H \) is the location of the horizon. Therefore, if we identify the size of the bound state \( r \) with the horizon \( r_H \), our result (15), (16) and (17) reproduce the parameter dependences of the black hole thermodynamics including \( \pi \). (\( r_H \) depends on the coordinate and we have argued what coordinate is natural in [1].) This agreement may indicate that the interacting D1 and D5-branes described by the effective action [6] provide the microscopical origin of the D1-D5 black hole thermodynamics. We should emphasize that the free energy (15) has been reproduced without imposing the assumption (12).

Finally we comment on the assumption \( r^2 \ll r_1^2, r_5^2 \) which we have used when we consider the effective action [6]. At the scale (19), this relation becomes \( T \ll 1/r_1, 1/r_5 \) and this is the near extremal limit in supergravity [4]. Thus our analysis is valid when we consider the near extremal black holes, according to supergravity, a phase transition related to the Gregory-Laflamme transition along the \( S^1 \) occurs around \( r_H \sim \alpha' / R \) [17], hence through (17),
\[ T_{\text{GL}} \sim \frac{1}{g_s \alpha' R} \sqrt{\frac{V_4}{Q_1 Q_5}}. \] (18)
Therefore our moduli theory results may be valid in the region \( T_{\text{GL}} < T \ll 1/r_1, 1/r_5 \).

**D1-D5-P system.**— We apply the similar analysis to the D1-D5-P system. We consider the same brane configuration to the D1-D5 system but now add momentum \( N/R \) along \( S^1 \). (See Table I)

To derive the effective theory of this system, we consider the gravitational interactions among the branes and the gravitational waves which carry the momentum \( 1/R \) along \( S^1 \). First we look at the probe D1-brane in the extremal D1-D5-P background [4]
\[ S_{\text{D1}} = -\frac{R}{g_s \alpha'} \int dt \left( \frac{1}{H_1} \sqrt{1 - H_1 H_5 H_p R^2} - \left( \frac{1}{H_1} - 1 \right) \right), \]
\[ H_p = 1 + \frac{r_p^2}{r^2}, \quad r_p^2 = \frac{4 G_5 N}{\pi R}. \] (19)

Then, by repeating the arguments in the previous section, we can estimate the effective theory for the branes and waves. For \( r \ll r_1, r_5, r_p \), the action is estimated as
\[ S_{\text{D1DSP}} = \int dt \sum_{n=1}^{\infty} L_n, \]
\[ L_1 \sim \sum_{i=1}^{Q_1} \sum_{j=1}^{Q_5} \sum_{k=1}^{G_5} \frac{G_5 m_1 m_5}{\pi R} \frac{\pi r_H^2}{r_H^2 r_i^2 r_j^2 r_k^2} + \cdots, \]
\[ L_2 \sim \sum_{i,j,k} \sum_{m,n=1}^{G_5} \frac{G_5^2 m_1^2 m_5^2}{\pi^2 R^2} \frac{\pi r_H^4}{r_H^4 r_i^4 r_j^4 r_k^4} + \cdots, \]
\[ L_n \sim \sum_{i_1,...,i_n j_1,...j_n k_1,...k_n} \frac{G_5^{3n-1} m_1^m m_5^n}{\pi^{2n} R^n} \prod_{a=2}^{n} \prod_{b=1}^{k_n} \prod_{c=2}^{i_{n+a}} \prod_{d=1}^{j_{n+a}} \prod_{e=1}^{k_{n+a}} \frac{1}{r_{i_{a+1}}^2 r_{i_{a+1}}^2 r_{j_{a+1}}^2 r_{j_{a+1}}^2 r_{k_{a+1}}^2 r_{k_{a+1}}^2} \cdot \] (20)

\( L_n \) describes the interactions among \( n \) D1-branes, \( n \) D5-branes and \( n \) waves through the exchanges of the \( 3n-1 \) gravitons. Although these schematic expressions can be predicted from the probe action, we need to solve the multi-body problem in the supergravity to determine the precise expressions. Note that the interactions shown in the D1-D5 action (11) exist in this system too, but they are subdominant in the limit \( r \ll r_1, r_5, r_p \) and they have been omitted here.

We estimate the free energy of this system by imposing the same assumptions to the D1-D5 case [4] and applying the virial theorem \( L_1 \sim L_2 \) to the effective action (20). Then we obtain
\[ \nu^2 \sim \frac{\pi r^6}{Q_1 Q_5 N G_5^2}, \quad F \sim L_1 \sim \frac{\pi r^2}{G_5}. \] (21)
In this derivation, we have used the relation \( m_1 m_5 / R = \pi / 4 G_5 \). To consider the temperature dependence, we further assume the relation (12) and obtain
\[ r^2 \sim G_5 T \sqrt{N Q_1 Q_5}, \] (22)
\[ S_{\text{entropy}} = -\frac{\partial F}{\partial T} \sim \pi \sqrt{N Q_1 Q_5}. \] (23)

Here we compare these results with the D1-D5-P black hole in the near extremal regime \( r_H \ll r_1, r_5, r_p \) [4]
\[ F = -\frac{\pi r_H^2}{4 G_5}, \] (24)
\[ r_H^2 = 8 G_5 T \sqrt{N Q_1 Q_5}, \] (25)
\[ S = 2 \pi \sqrt{Q_1 Q_5 N}. \] (26)
Therefore by identifying $r \sim r_H$, these results are consistent with our results including the $\pi$ dependence.

Our analysis is valid as far as $r^2 \ll r_H^2, r_0^2, r_p^2$ which correspond to the near extremal limit in the supergravity [4]. Contrast to the D1-D5 case, no phase transition would occur at low temperature, and our moduli theory calculations may be valid until zero temperature.

Discussions. We have studied that the D1-D5(-P) black hole thermodynamics can be explained by the effective theories of the gravitationally interacting elementary branes [6] and [20], using the approach of the p-soup model [1]. It is remarkable that these mechanisms are similar to that of the black $p$-branes and it suggests that we can understand the microstates of these distinct black holes by the p-soup model in the unified fashion.

The D1-D5(-P) system has been investigated through the conformal field theory (CFT) too and it reproduces the black hole thermodynamics exactly [4, 7]. Here we compare this computation and our analysis. In the CFT calculation, we assume that the branes are coincident in the transverse 4 dimensional space (Higgs branch). To retain the branes coincident, we need to turn on the NS-NS $B$-field. It means that the CFT calculation is done at the distinct point in the moduli space to the black holes ($B = 0$), and the agreement to the gravity would be due to the non-renormalization theorem. This point is different from the p-soup model where we treat the separated D-branes with $B = 0$. Thus the p-soup model may consider the system at the same point in the moduli space as the black holes. In this sense we might regard the p-soup model as a direct description of the black hole microstate.

The advantage of the CFT is that the exact calculation is possible. Thus, if we could understand the relation between the p-soup model and CFT calculation [22], it might give clues to achieve the exact analysis in the p-soup model. Especially, since the p-soup model may provide the unified description not only the D1-D5(-P) black holes but also the black $p$-branes, it might lead to the exact computations of the black $p$-brane microstates.

Finally we comment that the effective action [1] and [20] might be derived from the supersymmetric gauge theory on the D1-D5 branes. In cases of D-branes and M2-branes, we can calculate the single graviton exchange interaction from the one-loop calculation in the gauge theories [16, 18, 19] and we obtain a part of the effective action for these branes [13, 14]. Thus similar gauge theory calculations in the D1-D5 system may provide the effective action [1] and [20], and it may lead to the microscopic descriptions of the black holes in terms of the gauge theory.

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