Extending the canonical thermodynamic model: inclusion of hypernuclei

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Abstract

The canonical thermodynamic model has been used frequently to describe the disassembly of hot nuclear matter consisting of neutrons and protons. Such matter is formed in intermediate energy heavy ion collisions. Here we extend the method to include, in addition to neutrons and protons, Λ particles. This allows us to include productions of hypernuclei in intermediate energy heavy ion collisions. We can easily predict average mass number of hypernuclei produced and values of relative cross-sections of different $^{A}\Lambda z$ nuclei. Computation of absolute cross-section was not attempted at this stage and will require much more detailed work.

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I. INTRODUCTION

The canonical thermodynamic model (CTM) as applied to the nuclear multifragmentation problem addresses the following scenario. Assume that because of collisions we have a finite piece of nuclear matter which is heated and begins to expand. During the expansion the nucleus will break up into many fragments (composites). In the expanded volume, the nuclear interaction between different composites can be neglected and the long range Coulomb force can be absorbed in a suitable approximation. The partitioning of this hot piece of matter according to the availability of phase space can be calculated exactly (but numerically) in CTM. Many applications of this model have been made and compared to experimental data [1]. The model is very similar to the statistical multifragmentation model (SMM) developed in Copenhagen [2]. SMM is more general but requires complicated Monte-Carlo simulations. In typical physical situations the two models give very similar results [3].

In usual situations, the piece of nuclear matter has neutrons and protons, that is, it is a two-component system. Initially CTM was formulated for one kind of particle [4] and already many interesting properties like phase transition could be studied. Subsequent to the extension of CTM to two kinds of particles [5], many applications of the model to compare with experimental data were made [1, 6]. The objective of this paper is to extend CTM to three-component systems. While this, in general, is interesting, it can also be useful for calculations in an area of current interest. I refer here to the production of hyperons (usually Λ) in heavy ion reaction in the 1GeV/A to 2 GeV/A beam energy range. The Λ’s can get attached to nuclei turning them into composites with three species. The conventional thinking is this. The Λ particle is produced in the participating zone, i.e., the region of violent collisions. The produced Λ’s have an extended rapidity distribution and some of these can be absorbed in the much colder spectator parts. These will form hypernuclei. Those absorbed in the projectile fragment (PLF) can be more easily studied experimentally because they emerge in the forward direction.

This idea has been recently used to study the production of hypernuclei in a recent paper using the SMM model [7]. Our work closely follows the same physics, however, using a different and what we believe a much easier prescription. In addition our focus is different and we emphasize other aspects. The question of hypernucleus production in heavy ion reaction was already looked at in detail more than twenty years ago [8]. The authors used
a coalescence model. Both the break up of a PLF into composites and the absorption of the Λ used coalescence. The coalescence approach has been revived in a much more ambitious calculation recently [9]. Intuitively the coalescence model is appealing but for a satisfactory formulation of the PLF breaking up into many composites there are many very difficult details which need to be worked out. Certainly there are some points of similarities between the thermodynamic model for multifragmentation and production of composites by coalescence. For the production of the deuteron, the simplest composite, the two models were compared [10]. But that study also shows that for a heavier fragment (say 12C) the two routes become impossible to disentangle from each other. However, there is at least one argument in favour of the thermodynamic model (both SMM and CTM). They have been widely used for composite production and enjoyed very good success [1, 2].

The physics ansatz for the calculation reported in the earlier work [7] and the present work is the same. The Λ particle (particles) which arrive at the PLF interact strongly with the nucleons. Thus fragments can be calculated as in normal prescriptions. Hypernuclei as well as normal (non-strange) composites will be formed. The model gives definitive predictions as the following sections will show. Experiments can vindicate or contradict these predictions.

II. MATHEMATICAL DETAILS

The case of two-component system (neutrons and protons) have been dealt with in many places including [1]. The generalisation to three components is straightforward.

The multifragmentation of the system we study has a given number of baryons $A$, charges $Z$ and strangeness number $H$. This will break up into composites with mass $a$, charge $z$ and $h$ number of Λ particles. The canonical partition function of the system $Q_{A,Z,H}$ is given by the following equation. Once the partition function is known, observables can be calculated.

$$Q_{A,Z,H} = \sum \prod \left( \frac{\omega_{a,z,h}}{n_{a,z,h}!} \right)^{n_{a,z,h}}$$

(1)

Here $\omega_{a,z,h}$ is the partition function of one composite which has mass number $a$, charge number $z$ and $h$ hyperons (here Λ’s) and $n_{a,z,h}$ is the number of such composites in a given channel. The sum over channels in eq.(1) is very large and each channel must satisfy

$$\sum a n_{a,z,h} = A$$
\[\begin{align*}
\sum zn_{a,z,h} &= Z \\
\sum hn_{a,z,h} &= H
\end{align*}\] (2)

Proceeding further, we have

\[\langle n_{a,z,h} \rangle = \frac{1}{Q_{A,Z,H}} \sum \prod n_{a,z,h} \frac{(\omega_{a,z,h})^{n_{a,z,h}}}{n_{a,z,h}!} \] (3)

which readily leads to

\[\langle n_{a,z,h} \rangle = \frac{1}{Q_{A,Z,H}} \omega_{a,z,h} Q_{A-a,Z-z,H-h} \] (4)

Since \( \sum a \langle n_{a,z,h} \rangle = A \) we have

\[\sum a \frac{1}{Q_{A,Z,H}} \omega_{a,z,h} Q_{A-a,Z-z,H-h} = A \] (5)

which immediately leads to a recurrence relation which can be used to calculate the many particle partition function:

\[Q_{A,Z,H} = \frac{1}{A} \sum_{a=1}^{A} a \omega_{a,z,h} Q_{A-a,Z-z,H-h} \] (6)

It is obvious other formulae similar to the one above exist:

\[Q_{A,Z,H} = \frac{1}{H} \sum h \omega_{a,z,h} Q_{A-a,Z-z,H-h} \] (7)

The above equations are general. In this paper we do numerical calculations for the cases \( H=1 \) and \( H=2 \). The composites considered have either \( h=0 \) (non-strange composites) or \( h=1 \). For the case \( H=2 \) this means that in a given channel there will be two composites each with \( h=1 \). A more general treatment would include composites with two \( \Lambda \)'s. To complete the story we need to write down the specific expressions for \( \omega_{a,z,h} \) that we use.

The one particle partition function is a product two parts:

\[\omega_{a,z,h} = z_{\text{kin}}(a, z, h)z_{\text{int}}(a, z, h)\] (8)

The kinetic part is given by

\[z_{\text{kin}}(a, z, h) = \frac{V}{h^3}(2\pi MT)^{3/2}\] (9)

where \( M \) is the mass of the composite: \( M = (a - h)m_n + hm_\Lambda \). Here \( m_n \) is the nucleon mass (we use 938 MeV) and \( m_\Lambda \) is the \( \Lambda \) mass (we use 1116 MeV). For low mass nuclei,
we use experimental values to construct $z_{\text{int}}$ and for higher masses a liquid-drop formula is used. The neutron, proton and Λ particle are taken as fundamental blocks and so $z_{1,0,0} = z_{1,1,0} = z_{1,0,1} = 2$ (spin degacy). For deuteron, triton, $^3$He and $^4$He we use $z_{\text{int}}(a, z, 0) = (2s_{a,z,0} + 1) \exp(-e_{a,z,0}(gr)/T)$ where $e_{a,z,0}$ is the ground state energy and $(2s_{a,z,0} + 1)$ is the experimental spin degeneracy of the ground state. Contributions to the $z_{\text{int}}$ from excited states are left out for these low mass nuclei. Similarly experimental data are used for $^3\Lambda$H, $^4\Lambda$H, $^4$ΛHe and $^5$ΛHe.

For heavier nuclei ($h=0$ or 1), a liquid-drop formula is used for ground state energy. This formula is taken from [7]. All energies are in MeV.

$$e_{a,z,h} = -16a + \sigma(T)a^{2/3} + 0.72z^2/(a^{1/3}) + 25(a-h-2z)^2/(a-h) - 10.68h + 21.27h/(a^{1/3}) \quad (10)$$

Here $\sigma(T)$ is temperature dependent surface tension: $\sigma(T) = 18\sqrt{T^2-t^2}/T^2$. A comparative study of the above binding energy formula can be found in [7]. This formula also defines the drip lines. We include all nuclei within drip lines in constructing the partition function.

With the liquid-drop formula we also include the contribution to $z_{\text{int}}(a, z, h)$ coming from the excited states. This gives a multiplicative factor $= \exp(r(T)Ta/\epsilon_0)$ where we have introduced a correction term $r(T) = \frac{12}{12+4}$ to the expression used in [2]. This slows down the increase of $z_{\text{int}}(a, z, h)$ due to excited states as $T$ increases. Reasons for this correction can be found in [5, 11] although for the temperature range used in this paper the correction is not important.

We also incorporate the effects of the long-range Coulomb force in the Wigner-Seitz approximation [2].

We have used eq.(7) to compute the partition functions. If the PLF which absorbs the Λ has mass number $A$ and proton number $Z$, we first calculate all the relevant partion functions for $H=0$ first. This requires calculating upto $Q_{A,Z,0}$. We then calculate, for $H=1$ partition functions upto $Q_{A+1,Z,1}$. We can then proceed to calculate for $H=2$ upto $Q_{A+2,Z,2}$ and so on.

III. RESULTS FOR $H=1$

We assume one Λ is captured in the projectile like fragment (PLF). The PLF breaks up into various fragments. In an event one of these fragments will contain the Λ particle, the rest
of the fragments will have \( h = 0 \). There is also a probability that the \( \Lambda \) remains unattached, i.e., after break-up it emerges as a free \( \Lambda \). There is also another extreme possibility (this requires very low temperature in the PLF) that the \( \Lambda \) gets attached to the entire PLF which does not break up. In such an event the number of composites with \( h = 0 \) is zero. The average over all events give the average multiplicity of all composites, with \( h = 0 \) and \( h = 1 \) (eq.(4))

We will show results for \( \Lambda \) captured by a system of \( A = 100, Z = 40 \) and \( A = 200, Z = 80 \). These are the same systems considered in [7]. The results for \( \langle n_{a,z,h}\rangle \) depend quite sensitively on the temperature and less so on the assumed freeze-out density. Except in one case, all the results shown use freeze-out density to be one-third normal density. Past experiences have shown [12, 13] that a freeze-out density of one-third normal density gives better results for disassembly of PLF than, for example, the value of one-sixth normal density which is more appropriate for the participating zone. Again from past experiences, temperatures in the range 5 to 10 MeV are considered to be appropriate.

In Fig.1 we show results for \( A = 100, Z = 40 \) at a low temperature of \( T = 4 \) MeV. In order to display the results easily we sum over the charge and plot \( \langle n_{a,h}\rangle = \sum_z \langle n_{a,z,h}\rangle \). Note that for this choice of temperature, the average mass number of the hypernucleus formed is very high, about 95. The multiplicity of non-strange composites is low (1.24). The average mass of non-strange composite is about 5 and the average charge is about 1.7. We thus have a curious situation. The non-strange part is a gas with very few particles and the strange part of matter is a liquid since in heavy ion physics, a large blob of matter is attributed to be the liquid part. While this aspect of hybrid liquid-gas co-existence may lead to an interesting study, our focus here will be the population of hypernuclei.

For brevity we do not show the population of composites at this temperature for a system of \( A = 200, Z = 80 \). There are remarkable similarities in the shapes of the the curves, but the differences are also significant and the curve for \( A = 200 \) can not be scaled onto the curve for \( A = 100 \). At higher temperature, however, one can guess the results for \( A = 200 \) knowing, for example, the results for \( A = 100 \).

Fig. 2 shows the graph of \( \langle n_{a,h}\rangle \) at 8 MeV temperature for both \( A = 100, Z = 40 \) and the system double its size. The important feature which allows one to scale the results of one system to another is this. At this temperature the relative population of \( \langle n_{a,h}\rangle \) drops off rapidly with \( a \) so that the population beyond, say, \( a = 40 \) can be ignored. For composites with
\( h = 1 \), both the systems \( A = 100 \) and \( A = 200 \) are virtually the same: for both, \( \sum_{a=1}^{40} \langle n_{a,1} \rangle = 1 \) and hence it is possible to have the same value of \( \langle n_{a,1} \rangle \) for the the two systems. The graphs for \( h = 1 \) bear this out. But for \( h = 0 \), \( \sum_{a=1}^{40} a \langle n_{a,0} \rangle \) have to add up to different numbers. For \( A = 100 \) they have to add up to \( [101 - \langle a(h = 1) \rangle] \) and for \( A = 200 \) they have to add up to \( [201 - \langle a(h = 1) \rangle] \). The simplest ansatz is that \( \langle n_{a,1} \rangle \) for \( A = 200 \) is larger than the corresponding quantity for \( A = 100 \) by the ratio \( [201 - \langle a(h = 1) \rangle] / [101 - \langle a(h = 1) \rangle] \approx 2 \). Fig. 2 shows this to be approximately correct.

For our model to be physically relevant, we expect the yield \( \langle n_{a,h} \rangle \) to be proportional to \( \sigma(a, h) \) although the model, at the moment, is not capable of providing the value of the proportionality constant. The average value: \( \langle a(h = 1) \rangle = \sum_a a \langle n_{a,1} \rangle / \sum_a \langle n_{a,1} \rangle = \sum_a a \langle n_{a,1} \rangle \) is a useful quantity and is predicted to be the average value of the mass number of the hypernuclei measured in experiment. This is plotted in Fig. 3 as a function of temperature both for \( A = 100 \) and \( A = 200 \) calculated at one-third the nuclear density (graphs labelled 1 and 3 respectively). Curves labelled 2 and 4 refer to the cases when \( H = 2 \) and we deal with them in the next section. In the figure we also plot the value of \( \langle a(h = 1) \rangle \) if this is calculated at a lower one-sixth normal density for \( A = 100 \) (graph labelled 5). Several comments can be made. Assuming that the PLF temperature is in the expected 6 MeV to 10 MeV range the average mass number of hypernuclei should be in 20 to 7 range. Secondly this value is insensitive to the PLF mass number so long as it is reasonably large. We can also use the graph to state that if the temperature is above 6 MeV the grand canonical model can give a dependable estimate but if the temperature is significantly lower, say 5 MeV, grand canonical calculation can be in significant error. As expected if a lower value for the freeze-out density is used the predicted value for \( \langle a(h = 1) \rangle \) is lowered.

A more detailed plot of yields for \( \Lambda z \) for \( z \) in the range 1 to 6 and all relevant \( a \)'s is given in Fig.4. There are two curves for each \( z \). Let us concentrate on the lower curves. These belong to the case considered here, i.e., \( H = 1 \). Although we have drawn this this for \( A = 100, Z = 40 \) for \( T = 8 \) MeV it is virtually unchanged for \( A = 200, Z = 80 \). The reasons were already given. These plots provide a very stringent test of the model as these yields are proportional to experimental cross-sections.
IV. RESULTS FOR H=2

We consider now $A = 100, Z = 40$ and $A = 200, Z = 80$ but these systems have captured two $\Lambda$'s rather than one. How do we expect the results to change? Fig.5 compares the yields of the composites with two $\Lambda$'s entrapped in $A = 100$ at 8 MeV with the already studied case of one $\Lambda$ in $A = 100$ at 8 MeV. For $H=2$ the number of hypernuclei (and also the number of free $\Lambda$'s) is doubled with only very small changes in the number of non-strange composites. We can understand why this happens following a similar chain of arguments as presented in the previous section. The reason for this correspondence is that at temperature 8 MeV there are only insignificant number of composites beyond $a=40$.

The average value of $\langle a(h = 1) \rangle$ as a function of temperature is shown in Fig.3 for $H=2$. Curve 2 is for $A=100, H=2$ and curve 1 is for $A=100, H=1$. As explained above, for $T > 7$ the average value $\langle a(h = 1) \rangle$ will be very close but at lower temperature (i.e., $T = 4 MeV$) the situation is very different. For $H=1$ there is a very large hypernucleus containing most of the nucleons (Fig.1) but for $H=2$ there are two hypernuclei thus they will together share the bulk of the nucleons. Thus the average value of $\langle a(h = 1) \rangle$ will drop to about half the value obtained for $H=1$. In Fig.3 curve 4 is for $H=2$ in a system with $A=200$, curve 3 is for $H=1$ in a system with $A=200$.

V. DISCUSSION

We have given a detailed description of what happens once the PLF captures the produced $\Lambda$ particle. How $\Lambda$'s are produced in the violent collision zone and the probability of arrival both temporally and positionally at PLF are not described here. This will depend strongly on the experiment: for example, the case of say, $^{197}$Au hitting $^{12}$C will have to be treated differently from that of Sn on Sn collisions. We hope to embark upon this aspect in future. We have looked at statistical aspects only. This can be investigated more easily using the canonical thermodynamic model.

The calculations here looked at productions of hypernuclei in the PLF. The technique can also be applied in the participant zone. In the participant zone the temperature will be higher. Also the freeze-out density is expected to be lower. As an example if we use freeze-out density 1/6-th of the normal density, $A = 100, Z = 40$ and temperature 18 MeV
the average value of $a$ for $h=1$ is 2.55. We produce more single $\Lambda$’s than hypernuclei. Heavier hypernuclei are not favoured at high temperature.

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FIG. 1: For $A = 100, Z = 40$ CTM results for average yields of composites $\langle n_{a,h} \rangle$ for $h=0$ (solid line) and $h=1$ (dashed line). The freeze-out density is $1/3$ of normal density. The case shown uses a temperature of 4 MeV. Note that the yields first drop off but then rise again (specially for $h=1$ case). This is a case of liquid-gas co-existence. The average value of multiplicity for non-strange composites $\langle m(h = 0) \rangle$ is 1.24 and the average mass number $\langle a(h = 1) \rangle$ of composites with one $\Lambda$ is 94.9. These values are strongly dependent on the temperature as shown in Fig2.
FIG. 2: Again we plot average yields $\langle n_{a,h} \rangle$ at 1/3-rd normal density but now at temperature $T=8$ MeV. We show results for both $A=100, Z=40$ and $A=200, Z=80$. Note that the pattern of yields have completely changed from that at temperature 4 MeV. The yields fall off rapidly with $a$. They do not further rise again. Under such conditions the population of $h=1$ composites will be remarkably same for both $A=100$ and 200. This is explained more fully in the text. However, the yields of $h=0$ for $A=100$ (shown by a solid line) and $A=200$ (dashed curve) can be expected to differ by roughly a constant factor.
FIG. 3: The average mass number of hypernuclei for five different scenarios. If the average mass number of hypernuclei is measured in heavy ion reactions these numbers can be directly compared. The average mass is a function of the temperature in the PLF. The temperature range of 6 to 10 MeV might be most relevant. Curve 1 is for PLF with $A = 100, Z = 40, H = 1$ (one Λ absorbed). The freeze-out density is one-third normal density. Curve 5 is the same system but a smaller freeze-out density (1/6 th normal density): shows how the average value $\langle n_{a,1} \rangle$ changes with freeze-out density. Except for curve 5 all other curves use 1/3-rd normal density. Curve 3 has $A = 200, Z = 80, H=1$. Note that except at low temperature the average value $\langle n_{a,1} \rangle$ does not distinguish much between systems with $A=100$ and $A=200$. Curves 2 and 4 are drawn to show how results differ when two Λ’s rather than one are absorbed by the PLF. Curve 2 has $A = 100, Z = 40, H = 2$. Curve 4 has $A = 200, Z = 80, H=2$. Note that above 6 MeV temperature curves computed for freeze-out density 1/3-rd give very similar result. For low temperatures the $H=2$ gives about half the value from what is obtained for $H=1$. See text for an explanation.
FIG. 4: For $A = 100, Z = 40$ we plot $\langle n_{a,z,1} \rangle$ for a range of $z$’s and relevant $a$’s for $H=1$ (lower curves) and $H=2$ (upper curves). Calculations done at 1/3-rd normal nuclear density and temperature 8 MeV. These yields should be proportional to the measured cross-sections $\sigma(a, z, 1)$. For $A = 200, Z = 80$, the results are almost the same.
FIG. 5: Plots of $\langle n_{a,h} \rangle$ for $h=1$ and $h=0$. The absorbing system is $A = 100, Z = 40$. We show results for both $H=1$ and 2. For $H=2$ the yields of hypernuclei are nearly a factor of 2 higher than the case with $H=1$. Populations of normal composites ($h=0$) do not alter much between the two cases.