Cooperative Robustness to Static Disorder

G. Luca Celardo,1,2 Giulio G. Giusti,1,3 and Fausto Borgonovi1,2

1 Dipartimento di Matematica e Fisica and Interdisciplinary Laboratories for Advanced Materials Physics, Università Cattolica del Sacro Cuore, via Musei 41, I-25121 Brescia, Italy
2 Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, via Bassi 6, I-27100, Pavia, Italy
3 International Research Center on Mathematics & Mechanics of Complex Systems, via XIX marzo 1, I-04012 Cisterna di Latina, Italy

(Dated: March 31, 2014)

We analyze a 1-d ring structure composed of many two level systems, in the limit where only one excitation is present. The two level systems are coupled to a common environment, where the excitation can be lost, which induces super and subradiant behavior, an example of cooperative quantum coherent effect. We consider time independent random fluctuations of the excitation energies. This static disorder, also called inhomogeneous broadening in literature, induces Anderson localization and is able to quench Superradiance. While, for weak opening, Superradiance is quenched at the same critical disorder at which the states of the closed system localizes, for strong opening, this occurs at a disorder strength proportional to the system size, showing robustness of cooperativity to disorder.

PACS numbers: 71.35.-y, 72.15.Rn, 05.60.Gg

I. INTRODUCTION

Since the discovery that quantum coherences might have a functional role in biological systems even at room temperature [1,5], there has been great interest in understanding how coherences can be maintained and used under the influence of different environments with competing effects. In particular much of recent research focused on one-dimensional nanostructures, due to their relevance to molecular aggregates, such as the J-aggregates [6], natural photosynthetic systems [7], bio-engineered devices for photon sensing [8] and light-harvesting systems [9].

Here we focus on a ring-like structure of two level systems coupled with nearest neighbor tunneling amplitudes which has been recently considered in literature [7–12]. Usually, under low light intensity, in many natural photosynthetic systems or in ultra-precise photon sensors the single excitation approximation can be used. In this case the system is equivalent to a tight binding model where one particle can hop from site to site, see Fig. 1.

Most photosynthetic organisms contain ring-like chlorophyll molecular aggregates in their light-harvesting complexes, which are called LHI and LHII [13]. These complexes have the purpose to absorb light and to transfer the excitations to other structures or to a central core absorber, the reaction center, where charge separation, necessary in the next steps of photosynthesis, occurs. These complexes are subject to the effects of different environments: i) dissipative, where the excitation can be lost ii) proteic, which induces static or dynamical disorder. The efficiency of excitation transfer can be determined only through a comprehensive analysis of the effects due to the interplay of all those environments.

Here, in particular, we consider a system subject to the influence of both a dissipative environment, characterized by a single decay channel where the excitation can be lost, and a static disorder. The first environment can be thought of as a model for the coupling of a molecular aggregate to the electromagnetic field [11] (loss of excitation by recombination), and for the coupling of the molecular aggregate to a central core absorber (loss of excitation by trapping). For many molecular aggregates, the single channel approximation is appropriate to describe the coupling with the electromagnetic field, since the wavelength of the absorbed light is much larger than the system size (natural complexes such as LHI, LHII typically span few tens of nanometers, while the wavelength of the involved photon is hundreds of nanometers). Moreover, it can be considered also a good approximation for the coupling to a central core absorber, modeled for instance as a semi-infinite one-dimensional lead.

The second environment consists of a proteic bath, in which photosynthetic complexes are embedded, that induces fluctuations in the sites energies. The fluctuations which occur on a time scale much larger than the time scale of the dynamics are usually described as static disorder. By static disorder we mean position dependent, but time independent, fluctuations of the site energies. The case of time dependent fluctuations of site energies has been considered in a separate paper [13].

It is well known that, when many sites are all coupled to the same channel, we can have a superradiant behavior [15]. Superradiance implies the existence of some states with a cooperatively enhanced decay rate (i.e. proportional to the number of sites). Superradiance comes always together with Subradiance, that is the existence of states with a cooperatively suppressed decay rate (i.e. smaller than the single site decay rate).

Though originally discovered in the context of atomic clouds interacting with the electromagnetic field [10], in presence of many excitations, Superradiance was soon recognized to be a general phenomenon in open quantum systems [15] under the conditions of coherent coupling.
with a dissipative environment. Most importantly, it can also occur in presence of a single excitation (the super of Superradiance [17]), entailing a purely quantum effect.

It is important to stress that this phenomenon is not limited to the interaction with an electromagnetic field, but it can happen due to the interaction with any continuum of states characterizing an external environment. For instance, it has been shown to strongly affect energy transport when the external environment is a continuum of scattering states [18].

The functional role that Superradiance might have in natural photosynthetic systems has been discussed in many publications [21, 22, 23, 24], and experimentally observed in molecular aggregates [25, 26]. Superradiance (or Supertransfer) is also thought to play an important role w.r.t. the transfer of excitation to the central core absorber [5], and its effects on the efficiency of energy transport in suprasynthetic molecular aggregates have been recently analyzed [22, 23].

While the opening to an external dissipative environment induces Superradiance, static disorder induces eigenstates localization [24] and acts against Superradiance. The interplay of Superradiance and Localization has been studied in various papers [12, 25, 27]. In particular, in [12] the case of weak opening has been analyzed for one-dimensional systems, while in [26] the case of strong opening in one-dimensional and three-dimensional Anderson models were considered. In this paper we aim to study both the regimes of weak and strong opening and their effects on localization in one-dimensional nanostructures.

Though it is easy to imagine that Superradiance and disorder have competing effects w.r.t. the efficiency of energy absorption and transfer, a deeper analysis is necessary to fully understand their action. For instance, disorder decreases the efficiency of the suprasynthetic states in absorbing light or in transferring excitations, but, in the meanwhile, it allows for energy absorption and transfer from subradiant states. Thus, for these states, disorder is useful to enhance efficiency. The latter effect is strongly related to the enhancement of efficiency due to noise: the so called EMAQT, discussed in [28, 29]. EMAQT constitutes a general effect in quantum networks, even if its relation with Subradiance has never been stressed up to now, to the best of our knowledge. The identification of a critical disorder strength at which Superradiance is quenched will be the main goal of this paper.

II. THE MODEL

We considered a 1-d chain of sites with periodic boundary conditions, arranged to form a ring-like structure, as shown in Fig. 1. Static disorder is introduced in the standard way, letting the site energies $\epsilon_i$ vary randomly, so that we end up with the following Hamiltonian (1-d periodic Anderson model [24]):

$$H_0 = \sum_{i=1}^{N} \epsilon_i \langle i | j \rangle - \Omega \sum_{\langle i,j \rangle} (\langle j | i \rangle + \langle i | j \rangle),$$

where the summation index $\langle i, j \rangle$ runs over the pairs of nearest-neighbor sites, $\epsilon_i$ are random variables uniformly distributed in $[-W/2,+W/2]$, $W$ is the strength of the disorder, and $\Omega > 0$ is the tunneling transition amplitude. For zero disorder, $W = 0$, the eigenvalues,

$$E_q = -2\Omega \cos \frac{2\pi q}{N}$$

with $q = 1,...,N$, and the eigenstates, $|\psi_q\rangle$, of the system can be computed exactly. The components of the $|\psi_q\rangle$ eigenstate on the site basis $|s\rangle$ are given by

$$\langle s | \psi_q \rangle = \frac{1}{\sqrt{N}} \cos \frac{2\pi sq}{N}$$

for $q = 1,...,N/2,N$, and

$$\langle s | \psi_q \rangle = \frac{1}{\sqrt{N}} \sin \frac{2\pi s(N-q)}{N}$$

for $q = N/2 + 1,...,N - 1$. Let us also notice that the ground state ($q = N, E_N = -2\Omega$) is fully symmetric and extended in the site basis:

$$|\psi_N\rangle = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} |k\rangle.$$

The 1-d Anderson model can be “opened” by allowing the particle to escape the system from any site into the same continuum channel. This situation of “coherent dissipation”, can be met in many systems and it has been recently considered in [26], where it has been shown to give rise to the following effective non-Hermitian Hamiltonian, see also [30]:

$$(H_{\text{eff}})_{ij} = (H_0)_{ij} - \frac{i}{2} \sum_{c} A_i^c (A_j^c)^* \equiv (H_0)_{ij} - \frac{i\gamma}{2} Q_{ij},$$

where $H_0$ is the Anderson Hamiltonian, Eq. (1), and $A_i^c$ are the transition amplitudes from the discrete states $i$ to the continuum channels $c$. In our case, we have a single decay channel, $c = 1$, and equal couplings, $A_i^1 = \sqrt{\gamma}$, so that $Q_{ij} = 1 \forall i, j$. The fact that $Q$ is a full matrix implies that an effective long-range hopping is present in the system due to the coupling to a common
dissipative environment. Long-range hopping is usually expected to destroy Anderson localization, but this is not the case in the present model, as explained below and in the discussion in [26].

The quantum evolution is given by the operator,

\[ U = e^{-iH_{\text{eff}}t/\hbar} \]

which is non-unitary, and gives rise to a loss of probability in the decay channel. The complex eigenvalues of \( H_{\text{eff}} \) can be written as \( E_r - i \Gamma_r/2 \), where \( \Gamma_r \) represent the decay widths of the eigenstates.

Due to its specific structure, the operator \( Q \) has only one eigenstate with a non zero eigenvalue: this is the fully extended state with eigenvalue equal to \( N \). This eigenstate also corresponds to the ground state of \( H_0 \), given in Eq. (3). All the other eigenstates of \( Q \) are degenerate with null eigenvalue and, for \( W = 0 \), since \([Q, H_0] = 0 \), they can be chosen to match the eigenstates \(|\psi_q\rangle \), \( q < N \), of \( H_0 \). This implies that, for \( W = 0 \), only the state \(|\psi_N\rangle \), Eq. (3), has a non-vanishing decay width equal to the total decay width of the system: \( \Gamma_N = N\gamma \). Note that the size dependence of the decay width is the hallmark of the cooperative nature of Superradiance. This is the superradiant state. All the other states with zero decay width are called subradiant. Importantly, the superradiant effect explains the strong dependence on the initial state of the efficiency of energy transfer to a central core absorber, as discussed in Ref. [10].

Let us also notice that these features are independent of the \( \gamma \) value: for any \( \gamma > 0 \), due to the symmetry present in our system, we have a maximum degree of Superradiance. This kind of geometrically induced subradiant subspaces are usually called in literature trapping free subspaces [29]. Such situation is by no means typical, indeed, as discussed in many papers [15, 31], maximal Superradiance is usually reached only above a critical coupling strength. This fact might indicate a relation between structure and function in natural complexes, suggesting the use of ring-like structures to exploit the superradiant behavior.

III. SUPERRADIANCE AND ANALYSIS OF DECAY WIDTHS

The origin of Superradiance lies in the fact that the excitation can be coherently spread over several sites, thus inducing a cooperative effect. On the other hand, static disorder can destroy Superradiance, since it induces Localization, which implies that excitons are localized on one site only, thus hindering cooperativity. The interplay of Localization and Superradiance has been studied in literature due to their widespread interest: static disorder is present in natural systems as well as in artificial devices due to the coupling of the sites with different local environments. The main question we want to address here is whether a critical disorder exists at which Superradiance and, thus, cooperativity are destroyed. We stress here that, while Superradiance has been studied mainly w.r.t. the electromagnetic field, it also occurs in the rate of energy release to a central core absorber: Superradiance can be exploited not only to create superabsorbing structures, but also to allow for supertransfer of the excitation to a reaction center.

The presence of static disorder on the site energies breaks the symmetry of the system under site perturbations, inducing the width of the superradiant state to decrease and the widths of the subradiant states to increase (the total decay width \( N\gamma \) does not depend on the degree of disorder \( W \), so that all of the eigenstates can decay into the continuum channel.

The effect of static disorder on the decay widths has been analyzed in Fig. 2 where the width of the superradiant state and the average width of all subradiant states are shown as a function of the disorder strength, \( W \). As one can see, for small disorder, the effect on subradiant states is much more evident than that on the superradiant state. For large disorder, all of the widths approach the value \( \gamma \), corresponding to the decay width of an isolated site. In this regime there is no collective behavior anymore and Superradiance is completely destroyed.

For small disorder, one can use perturbation theory (see Appendix A) to obtain the mean decay width of the \( N - 1 \) smallest widths:

\[
\langle \Gamma_{\text{sub}} \rangle = \frac{\gamma W^2}{48\Omega^2(N-1)} \sum_{q=1}^{N-1} \left[ \cos \left( \frac{2\pi q}{N} \right)^2 \right] + \frac{N^2\gamma^2}{16\Omega^2} \right]^{-1}.
\]

The sum in Eq. (5) can be well approximated in different
parameter regimes to give (see Appendix B)

\[
(G_{\text{sub}}) = \begin{cases} 
\gamma W^2 N^3 & \text{for } N\gamma/2 \ll \delta E_{\text{min}} \\
96\pi^2 \Omega^2 & \text{for } \delta E_{\text{min}} \ll N\gamma/2 \ll 2\Omega \\
\gamma^{1/2} W^2 & \text{for } \delta E_{\text{min}} \ll N\gamma/2 \ll 2\Omega \\
12\Omega^{1/2} N^{3/2} & \text{for } N\gamma/2 \gg 2\Omega .
\end{cases}
\]  

(6)

The different regimes shown above can be understood if we consider that

\[
\delta E_{\text{min}} = E_{N-1} - E_N \simeq 4\Omega^2 N^2/

\text{is the minimal nearest neighbor energy distance, see Eq. (2). In Ref. [12] a perturbative result was obtained in the regime of very weak opening, } N\gamma/2 \ll \delta E_{\text{min}}, \text{ which agrees with our findings. As one can see from Eq. (6), the average subradiant width always increases as } W^2, \text{ but the dependence on the system size at fixed disorder strength, } W, \text{ changes with the degree of opening: for very weak opening, the widths increase with } N_\text{and } \gamma, \text{ whereas, for very strong opening, they decrease with } N \text{ and } \gamma.

In Fig. 2 the perturbative expression is shown as a dashed line and agrees very well with numerical data. From Eq. (5) one can define a critical disorder strength, } W_{\text{cr}}, \text{ for which Superradiance is quenched, given by the condition

\[
\langle G_{\text{sub}}(W_{\text{cr}}) \rangle = \gamma,
\]

from which one gets

\[
W_{\text{cr}} = \sqrt[3]{96\pi^2 \Omega N^{-3/2}} \text{ for } N\gamma/2 \ll \delta E_{\text{min}} \\
\sqrt[3]{6} \gamma N^{-1/4} \text{ for } \delta E_{\text{min}} \ll N\gamma/2 \ll 2\Omega \\
\sqrt[3]{3} N\gamma \text{ for } N\gamma/2 \gg 2\Omega .
\]

(7)

(8)

Note that though the validity of Eq. (7) has been shown only in one case, see Fig. 2, we checked that it gives an excellent estimate of the disorder at which Superradiance is quenched in every regime. To confirm that the value of } W_{\text{cr}} \text{ computed above can be considered as the value at which ST occurs, we computed the variance of the decay width. Indeed, it is well known [31] that at the ST the variance of the widths has a maximum. The results of such a comparison are presented in Fig. 3 showing a good agreement between the two estimates of the ST.

Finally, it is interesting to estimate the value of } W_{\text{cr}} \text{ for the photosynthetic complexes, LHI and LHII. In this case } \Omega \approx 600 \text{ cm}^{-1} \text{ and } N = 32, 16 [12, 13], \text{ respectively. The coupling to the central core absorber and with the electromagnetic field is very weak w.r.t. the energy scale of } \Omega, \text{ we can thus safely assume that we are in a regime } N\gamma/2 \ll \delta E_{\text{min}}. \text{ We can thus use } W_{\text{cr}} = \sqrt[3]{96\pi^2 \Omega N^{-3/2}} \text{ for LHI complexes [12, 21], and their values is also quantitatively compatible with the estimated ranges of values for static disorder in natural photosynthetic complexes, ranging from 100 - 600 cm}^{-1} \text{ for LHI complexes [12, 21] to 600 - 1400 cm}^{-1} \text{ for LHII complexes [13, 21] (Note that in the above references gaussian static disorder is considered with standard deviation } \sigma. \text{ Thus in our case of uniform disorder, we have } W = \sqrt[3]{6\gamma N}. \text{ This might suggest that natural photosynthetic complexes operate close to the ST.}
by means of the participation ratio

$$PR = \left< \frac{\sum_i |\langle i | \psi \rangle|^2}{\sum_i |\langle i | \psi \rangle|^2} \right>,$$

of the eigenstates $|\psi\rangle$ of $H_{ds}$ given in Eq. (4), where $\left< \ldots \right>$ stands for the ensemble average over realizations of the static disorder. The $PR$ is widely used to characterize localization properties [34] and it clearly satisfies the bounds $1 \leq PR \leq N$. For extended states, it increases proportionally to the system size, $N$, while, for localized states, it is independent of $N$.

Our aim is to compare the disorder strength at which Superradiance is quenched, $W_{cr}$, see Eq. (7), with the one at which the states localize, $W_d$, see Eq. (10). To do that, we analyze separately the $PR$ of the superradiant state (the state with the maximum decay width), and the average $PR$ of the other $N-1$ states as a function of disorder strength, $W$.

The typical behavior of the $PR$ as a function of the disorder strength, $W$, has been analyzed in Fig. 5 in two different situations: for weak opening such that $N\gamma / 2 \ll \delta E_{\text{min}}$ (upper panel), and for strong opening such that $N\gamma / 2 \gg 2\Omega$ (lower panel). In both cases, the $PR$ of the superradiant state decreases roughly at the ST, as given by $W_{cr}$, while the $PR$ of subradiant states decreases roughly at $W_d$.

To be more quantitative one can define an empirical disorder strength for the superradiant state ($W_{SR}$) and subradiant states ($W_{sub}$) as the one at which the $PR$ decreases by 3% of its value at $W = 0$. To highlight the peculiar effects due to opening, these results should be somehow compared with those of the closed system. For the closed system we cannot define superradiant and subradiant states, but, since the localization length depends on the energy level, we can compare states of the open system with states of the closed system having the same real energy. In particular, the superradiant state is compared with the ground state of the closed system.

Results are shown in Fig. 5 for the superradiant state (upper panel) and for the subradiant ones (lower panel) as a function of the system size, $N$. In this Figure we fix $\gamma$ and, by varying $N$, we switch from the weak opening regime (for small $N$ values) to the strong opening regime (for large $N$). The $N_{cr}$ value which separates the two different regimes can be estimated from Eq. (4), and has been indicated as a dashed vertical line in both panels.

The behavior of super and subradiant states is very different. As one can see, the opening does not modify the behavior of the subradiant states if compared with the behavior of the closed system (see lower panel). Differently, the opening strongly modifies the behavior of superradiant states in the strong opening regime (see Fig. 5 upper panel). Indeed, the disorder strength necessary to decrease by 3% the $PR$ decreases with the system size up to $N_{cr}$, following the behavior of the closed system, whereas, for $N > N_{cr}$, it increases with the system size, and departs from behavior of the closed system.

It is interesting to observe that the disorder strength,
where we chose $|\psi_N\rangle$, see Eq. (3), which is the disorder strength quenching Superradiance (compare full line with symbols in upper panel of Fig. 5). This can be explained by considering that the decay width of a state is given by its superposition with the fully extended state $|\psi_{N}\rangle$, see Eq. (3). Therefore, when a state localizes its decay width is consequently quenched.

Summarizing we can conclude that:

- the disorder strength necessary to localize the superradiant states is the same of the corresponding value for the closed system. In particular, it is $\propto N^{-1/2}$, as in the closed system, in agreement with the fact that for non-edge states, $\xi \approx 100/W^2$ [33];

- for the superradiant states the proportionality between $W_{SR}$ and $W_{cr}$ implies that $W_{SR} \propto N^{-3/2}$ for weak opening and $W_{SR} \propto N$ for large opening;

- the dependence on $N^{-3/2}$ found for the disorder strength necessary to quench Superradiance scales as the disorder strength necessary to localize edge states. Indeed, at the edges of the energy band of the closed systems, the localization length, $\xi(W) \propto W^{-1/3}$ [22]. Taking into account, Eq. (10), this implies that the disorder strength necessary to localize edges states is $\propto N^{-3/2}$. This indicates once more that quenching of Superradiance due to disorder can be fully monitored by looking at the localization properties of the superradiant state.

V. DYNAMICS OF THE SURVIVAL PROBABILITY

In this section we aim at studying how the time evolution of the survival probability $P(t)$ (that is the probability of finding the excitation in the system, initially prepared in the state $|\psi_0\rangle$) is modified by the presence of static disorder. That quantity is given by

$$P(t) = \sum_{k=1}^{N} |\langle k | e^{-iH_{tot}t/\hbar} | \psi_0 \rangle|^2,$$

where we chose $|\psi_0\rangle = |\psi_N\rangle$, the fully extended superradiant state of Eq. (3), for our first analysis.

For $W = 0$ and we clearly have $P(t) = e^{-N\gamma t}$. For $W \neq 0$ the truly superradiant state does not coincide with the fully extended state anymore, and it should be written as a superposition of superradiant and subradiant states. Using first order perturbation theory (in the disorder strength) we can derive an approximate expression for $P(t)$ valid for small time, see Appendix A Eq. (A8):

$$P(t) \approx c_1 e^{-N\gamma t} + (1 - c_1) e^{-\Gamma_{\text{sub}}^{\text{max}} t},$$

Eq. (13) takes into account only the superradiant decay and the fastest subradiant decay, $\Gamma_{\text{sub}}^{\text{max}}$, which can be computed from Eq. (A6) given in Appendix A, setting $q = N - 1$.

In Fig. 6 we compared numerical data for $P(t)$ with the analytical expression given in Eq. (13): the agreement is excellent for small disorder, where the decay is well approximated by a bi-exponential function. From Eq. (13) it is also possible to compute the time at which a change in the decay occurs, $t^*$, by equating the two terms on the r.h.s. of Eq. (13), obtaining

$$t^* = \frac{1}{(N\gamma - \Gamma_{\text{sub}}^{\text{max}}) \log \left( \frac{c_1}{1 - c_1} \right)}.$$  

That instant, for one value of the disorder strength, is shown with an arrow in Fig. 6. Note that $t^*$ can be considered as the time up to which the decay is superradiant. As the disorder increases, $t^*$ goes to zero and the decay of the extended state becomes similar to the decay of independent sites, i.e. $P(t) = e^{-\gamma t}$.

The generality of our results can be assessed by observing that the critical disorder at which Superradiance is destroyed is an important threshold for the whole system dynamics and not only for the superradiant state. To this end let us consider as initial state a random superposition of site states

$$|\psi_0\rangle = \sum_{k=1}^{N} c_k |k\rangle,$$
$c_k$ being random complex coefficients such that $\sum_{k=1}^{N} |c_k|^2 = 1$. For such initial states, we compute the survival probability $P(t)$, for one realization of disorder. By changing the random initial state and the random diagonal disorder, we can consider the average survival probability $\langle P(t) \rangle$ and defined the decay time $\tau \equiv 1/\Gamma$ as

$$\langle P(\tau) \rangle = 1/e.$$ 

Results are shown in Fig. 7. The agreement between the analytical expression, Eq. (13), and numerical data is very good for different sets of parameters. In other words, the disorder strength necessary to quench Superradiance is also a valid tool in estimating the time scale of the survival probability associated with generic random initial conditions.

The problem of computing the survival probability of the superradiant state in presence of inhomogeneous broadening was also considered in [23] for $N$ two-levels systems. A bi-exponential behavior was numerically found for any excitation number. On the other hand, even if we considered only one excitation, we were able to confirm their results and also to give an approximate analytical expression for the survival probability $P(t)$.

VI. CONCLUSIONS

We analyzed the interplay of Superradiance, induced by the coupling to a common decay channel, and Localization, induced by static disorder, in 1-d ring-like molecular aggregates. We have shown that, for zero disorder, these structures are in a superradiant regime for any value of the coupling strength to an external dissipative environment. Above a critical disorder strength super-radiant effects decrease until, for very large disorder, all states decay independently with a common width $\gamma$, and cooperativity is completely lost. Our main purpose was
to determine the critical disorder at which Superradiance is hindered. Using a perturbative approach we determined analytically such critical disorder and we related it firstly with the localization properties of superradiant and subradiant states and then to the system dynamics. We found that Superradiance can be quenched by disorder in different ways, depending on the regime entailing either weak or strong coupling to the continuum. These regimes are triggered by the parameter $N\gamma/4\Omega$, which represents the ratio between the coupling strength to the continuum, $\gamma$, and the unperturbed mean level spacing in absence of disorder, $4\Omega/N$. When this ratio is small, i.e. $N\gamma/4\Omega \ll 2(\pi/N)^2$, the critical disorder is independent of the coupling strength with the external environment and it is determined only by the parameters of the molecular chain. In this regime, the critical disorder decreases with the size of the system, but, for large system size $N \to \infty$, such a regime becomes less and less feasible, since it implies the condition $\gamma \ll 8\pi\Omega/N^3 \to 0$.

On the other hand, for strong opening, $N\gamma/4\Omega \gg 1$, the critical disorder increases with both the size of the system and the coupling strength with the external environment. This is in agreement with the results recently found in the companion paper [14], where the same ring structure and subradiant states and then to the system dynam-ics. We found that Superradiance can be quenched by disorder in different ways, depending on the regime entailing either weak or strong coupling to the continuum. These regimes are triggered by the parameter $N\gamma/4\Omega$, which represents the ratio between the coupling strength to the continuum, $\gamma$, and the unperturbed mean level spacing in absence of disorder, $4\Omega/N$. When this ratio is small, i.e. $N\gamma/4\Omega \ll 2(\pi/N)^2$, the critical disorder is independent of the coupling strength with the external environment and it is determined only by the parameters of the molecular chain. In this regime, the critical disorder decreases with the size of the system, but, for large system size $N \to \infty$, such a regime becomes less and less feasible, since it implies the condition $\gamma \ll 8\pi\Omega/N^3 \to 0$.

On the other hand, for strong opening, $N\gamma/4\Omega \gg 1$, the critical disorder increases with both the size of the system and the coupling strength with the external environment. This is in agreement with the results recently found in the companion paper [14], where the same ring structure and subradiant states and then to the system dynam-ics. We found that Superradiance can be quenched by disorder in different ways, depending on the regime entailing either weak or strong coupling to the continuum. These regimes are triggered by the parameter $N\gamma/4\Omega$, which represents the ratio between the coupling strength to the continuum, $\gamma$, and the unperturbed mean level spacing in absence of disorder, $4\Omega/N$. When this ratio is small, i.e. $N\gamma/4\Omega \ll 2(\pi/N)^2$, the critical disorder is independent of the coupling strength with the external environment and it is determined only by the parameters of the molecular chain. In this regime, the critical disorder decreases with the size of the system, but, for large system size $N \to \infty$, such a regime becomes less and less feasible, since it implies the condition $\gamma \ll 8\pi\Omega/N^3 \to 0$.

As for the relation between Localization and Superradiance, we have demonstrated that the critical disorder at which Superradiance is suppressed is close to the disorder at which superradiant states localize [26]. Specifically, we found that, for weak opening, $N\gamma/4\Omega \ll 2(\pi/N)^2$, Superradiance is quenched at the same disorder at which the edge state of the closed system, with real energy equal to that of the superradiant state, localizes. Moreover, for strong opening, $N\gamma/4\Omega \gg 1$, Superradiance is a manifestation of cooperative robustness to disorder, in that the superradiant state localizes at a disorder strength (proportional to the system size) much larger than the one needed to localize the corresponding edge state of the closed system. As for subradiant states, in any regime, they begin their process of localization at the same dis-

Acknowledgments. We would like to thank R. Kaiser and B. Sterzi for providing many useful discussions.

Appendix A: Decay widths, a perturbative approach

Perturbation theory is applied to the symmetric unperturbed Hamiltonian, $H_0 = -i\gamma Q/2$, in order to find the critical disorder strength, $W_{cr}$, at which Superradiance is destroyed. Let us rewrite the Hamiltonian given in

$$H_{\text{eff}} = H_0 - \frac{i\gamma}{2} Q = H^{tb} - (i/2)\gamma Q + D$$

where $H^{tb}$ is the tight binding Hamiltonian, Eq. (1), in absence of disorder and $D = \sum_i \epsilon_i |i\rangle \langle i|$ is a diagonal matrix which contains the disordered site energies $\epsilon_i$.

It is necessary to define the non-Hermitian “bra” as the transposed of a ket

$$\langle \langle \psi | := (|\psi\rangle)^\dagger,$$

while the standard bra is the adjoint

$$\langle \psi | := (|\psi\rangle)^\dagger.$$

Indeed, given the right eigenvectors of a symmetric Hamiltonian $|\psi\rangle$, the left eigenvectors are $\langle \langle \psi |$, that is

$$H |\psi\rangle = E_i |\psi\rangle, \quad \text{and} \quad \langle \langle \psi | H = E_i \langle \langle \psi |,$$

and we have the biorthogonality condition

$$\langle \langle \psi_i | \psi_j \rangle = \delta_{ij}.$$

Clearly, for real eigenstates we have $\langle \langle \psi | = |\psi\rangle$.

Matrix elements of the operators defined above, in the site basis $\{ |s\rangle, s = 1, \ldots, N \}$, are given by

$$D_{ss} = \epsilon_s, \quad H_0^{tb} = H_0^{tb} = -\Omega, \quad Q_{ss+1} = 1, \quad \text{(A1)}$$

with $r, s = 1, \ldots, N$. In Eq. (A1) $\Omega > 0$, and $-W < \epsilon_s < W$, are independent identically distributed random variables with mean 0 and variance $W^2/12$.

Since $[H^{tb}, Q] = 0$, it is convenient to study the whole system on the basis of eigenstates of $H^{tb}$, which are given by

$$\langle s |\psi_q\rangle = \frac{1}{\sqrt{N}} \sin \frac{2\pi sq}{N}$$

for $q = 1, \ldots, N/2, N$, and

$$\langle s |\psi_q\rangle = \frac{1}{\sqrt{N}} \cos \frac{2\pi s(N - q)}{N}$$

for $q = N/2 + 1, \ldots, N - 1$; with eigenvalues

$$\epsilon_q = -2\Omega \sin \frac{2\pi q}{N}.$$

The eigenvalues of the Hamiltonian, $H^{tb} - (i/2)\gamma Q$, are thus given by

$$\epsilon_q = -2\Omega \cos \frac{2\pi q}{N} - (iN\gamma/2)\delta_{qN}, \quad \text{(A2)}$$

that is, only the ground state acquires a decay width $N\gamma$. Such a state is called superradiant, and the others are subradiant. Notice that $|\psi_N\rangle$ and $|\psi_{N/2}\rangle$ are non-degenerate, while, for any $q = 1, \ldots, N/2 - 1$, $|\psi_q\rangle$ and $|\psi_{N-q}\rangle$ span a two-dimensional degenerate eigenspace.

When the disorder strength is turned on every state will get an eigenenergy with a negative imaginary part (decay width). Perturbation theory up to second order can be applied, for sufficiently small disorder strength, to
We finally define the critical disorder $W_{cr}$ as the one at which
\[ \langle \Gamma_{\text{sub}} \rangle = \gamma, \]
i.e. equals the single site decay width $\gamma$.

Let us now apply first order perturbation theory to the superradiant state. For $W = 0$, the superradiant state is given by the extended state, Eq. (3), while, for $W \neq 0$, we can write
\[ |SR \rangle \simeq \left( \frac{1}{\sqrt{N}} \right) \sum_{k=1}^{N} |k \rangle + \sum_{q \neq 0} \frac{D_{1,q}}{\epsilon_{1} - \epsilon_{q}} |\psi_{q} \rangle. \]

From this expression we can compute the probability to be in the superradiant state when starting from the extended state as
\[ c_{1} = \frac{1}{1 + \frac{W^{2}}{48\Omega^{2}N} \sum_{s=1}^{N-1} \left( 1 \cos(2\pi s/N) \right)^{2} + (\gamma N/4\Omega)^{2}}. \quad (A8) \]

Appendix B: Approximate formula for perturbative average width

Let us rewrite Eq. (A7) in the form
\[ \langle \Gamma_{\text{sub}} \rangle = \frac{\gamma W^{2}}{48\Omega^{2}} S_{a}, \quad (B1) \]

where we have defined
\[ S_{a} = \frac{1}{N-1} \sum_{q=1}^{N-1} \left( \cos \left( \frac{2\pi q}{N} \right) - a \right)^{2}, \quad (B2) \]

and
\[ a = \frac{N\gamma}{4\Omega}. \quad (B3) \]

Eq. (B2) can be put in integral form for sufficiently large $N$, that is $2\pi/N \ll 1$, as
\[ S_{a} = \frac{1}{2\pi} \int_{0}^{2\pi} dx \frac{1}{\left( \cos x - 1 \right)^{2} + a^{2}} \]
\[ = \frac{\left( 2a + 2\sqrt{4 + a^{2}} \right)^{1/2}}{a^{3/2} \sqrt{4 + a^{2}}}. \quad (B4) \]

It is easy to show that Eq. (B3) has two different limits, namely
\[ S_{a} = \begin{cases} 
1/(2a^{3/2}) & \text{for } a \ll 1 \\
1/a^{2} & \text{for } a \gg 1.
\end{cases} \quad (B5) \]

Nevertheless, substituting a sum with an integral works only for very large $N$. For small $N < 100$, or, in general for a sufficiently small $a$ value, it is more convenient approximating the sum with only two terms, namely those for which the denominator in Eq. (B2) is small. In detail, one has
\[ 1 - \cos(2\pi/N) \approx 2\pi^{2}/N^{2} \text{ for } a < a_{cr} = 2\pi^{2}/N^{2}. \quad (B6) \]

This implies that, in this regime, $S_{a} \approx N^{3}/2\pi^{4}$. On the other hand, for $a_{cr} < a < 1$ (for all those $N$ values for which $a_{cr} < 1$), one can approximate the sum with...
the integral and use the asymptotic behavior given in Eq. (B5). To summarize we have the following behavior:

$$S_a = \begin{cases} 
N^3/2\pi^4 & \text{for } a \ll 2\pi^2/N^2 \\
1/2a^{3/2} & \text{for } 2\pi^2/N^2 < a < 1 \\
1/a^2 & \text{for } a \gg 1.
\end{cases} \quad (B7)$$

These different regimes can be written in terms of physical parameters as follows:

$$\langle \sigma_{\text{sub}} \rangle = \begin{cases} 
\frac{\gamma W^2 N^3}{96\pi^4 \Omega^2} & \text{for } N^3 \gamma < 1 \\
\frac{\gamma W^2}{12\gamma 1/2 \Omega^{1/2} N^{3/2}} & \text{for } N^2 \gamma < 4 \Omega < 1 \\
\frac{W^2}{3N^2 \gamma} & \text{for } N^2 \gamma > 4 \Omega \gg 1.
\end{cases} \quad (B8)$$

[1] G.S. Engel et al., Nature 446, 782 (2007).
[2] G. Panitchayangkoon et al., PNAS 107, 12766 (2010).
[3] M. Sarovar, A. Ishizaki, G.R. Fleming and K.B. Whaley, Nat. Phys. 6, 462 (2010).
[4] H. Hossein-Nejad and G. D. Scholes, New J. Phys. 12, 065045 (2010).
[5] J. Strumpfer, M. Sener, and K. Schulten. J. Phys. Chem. B 101, 7241 (1997).
[6] H. Fidder, J. Knoester and D. A. Wiersma, J. Chem. Phys. 95, 7880 (1991); J. Moll, S. Daelme, J. R. Durrant and D. A. Wiersma, J. Chem. Phys. 102, 6362 (1995).
[7] J. M. Moix, M. Khasin, and J. Cao, New J. Phys., 15, 085010 (2013).
[8] K. D. B. Higgins, S. C. Benjamin, T. M. Stace, G. J. Milburn, B. W. Lovett, E. M. Gauger, arXiv:1306.1483
[9] M. Sarovar, K. B. Whaley, New J. Phys. 15, 013030 (2013).
[10] A. Olaya-Castro, C. F. Lee, F. Fassioli Olsen, and N. F. Johnson, Phys. Rev. B 78, 085115 (2008).
[11] J. Grad, G. Hernandez, and S. Mukamel, Phys. Rev. A 37, 3835 (1988).
[12] F. C. Spano and S. Mukamel, J. Chem. Phys. 91, 683 (1989).
[13] T. Fukunaga, T. Ritz, A. Damjanovic, K. Schulten, J. Phys. Chem. B 101, 2019 (1997); X. Xu, A. Damjanovic, T. Ritz and K. Schulten, Proc. Natl. Acad. Sci. USA 95, 5935 (1998).
[14] G.L. Celardo, P. Poli, L. Lussardi, F. Borgenovi, arXiv:1403.5660
[15] R. H. Dicke, Phys. Rev. 93, 99 (1954).
[16] M. O. Scully and A. A. Svidzinsky, Science 328, 1239 (2010);
[17] G.L. Celardo and L. Kaplan, Phys. Rev. B 79, 155108 (2009); G. L. Celardo, A. M. Smith, S. Sorathia, V. G. Zelevinsky, R. A. Senkov, and L. Kaplan, Phys. Rev. B 82, 165437 (2010).
[18] S. Lloyd and M. Mohseni, New J. Phys. 12, 075020 (2010).
[19] G. D. Scholes, Chem. Phys. 275, 373 (2002).
[20] R. Monshouwer, M. Abrahamsson, F. van Mourik and R. van Grondelle, J. Phys. Chem. B 101, 7241 (1997).
[21] G.L.Celardo , F. Borgenovi, V.I. Tsifrinovich, M. Merkli and G.P. Berman, J. Phys. Chem. C 116, 22105 (2012).
[22] D. Ferrari, G. L. Celardo , G.P. Berman, R.T.Sayre, F. Borgenovi, J. Phys. Chem. C 118, 20 (2014).
[23] P. W. Anderson, Phys. Rev. 109, 1492 (1958).
[24] D. J. Heijls, V. A. Malyshev and J. Knoester, Phys. Rev. Lett. 95, 177402 (2005).
[25] G.L. Celardo, A. Biella, L. Kaplan, F. Borgenovi Fortschr. Phys. 61, No. 2-3, 250-260 (2013); A. Biella, F. Borgenovi, R. Kaiser, G.L. Celardo, Europhys. Lett. 103 57009 (2013).
[26] T.V. Shahbazyan, M.E. Raikh and Z. V. Vardeny, Phys. Rev. B 61, 13266 (2000).
[27] P. Rebentrost, M. Mohseni, I.Kassal, S. Lloyd and A. Aspuru-Guzik, New J. Phys. 11, 033003 (2009); P. Rebentrost, M. Mohseni and A. Aspuru-Guzik A, J. Phys. Chem. B 113, 9942 (2009).
[28] M. B. Plenio and S. F. Huelga, New J. Phys. 10, 113019 (2008); F. Caruso, A. W. Chin, A. Datta, S. F. Huelga and M. B. Plenio, J. Chem. Phys. 131, 105106 (2009).
[29] A. F. Sadreev and I. Rotter, J. Phys. A 36, 11413 (2003).
[30] G. L. Celardo, F. M. Izrailev, V. G. Zelevinsky, and G. P. Berman, Phys. Lett B 659, 170 (2008); G. L. Celardo, F. M. Izrailev, V. G. Zelevinsky, and G. P. Berman, Phys. Rev. E, 76, 031119 (2007); G. L. Celardo, S. Sorathia, F. M. Izrailev, V. G. Zelevinsky, and G. P. Berman, CP995, Nuclei and Mesoscopic Physics - WNMP 2007, ed. P. Danielewicz, P. Pieuch, and V. Zelevinsky.
[31] S. M. Vlaming, V. A. Malyshev, and J. Knoester, Phys. Rev. 79, 205121 (2009).
[32] F.M. Izrailev, A.A. Krokhin and N.M. Makarov, Phys.
Rep. 512, 125 (2012).

[34] A. Rodriguez et al., Phys. Rev. Lett. 90, 027404 (2003);
L. S. Levitov, Europhys. Lett., 9, 83 (1989).

[35] V. V. Temnov and U. Woggon, Phys. Rev. Lett. 95, 243602 (2005).