Investigating the characteristics of the development of the boundary layer motion of locally nonequilibrium systems

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Abstract. The method of integral momentum transfer relations has been extended to polymer systems with a locally nonequilibrium relaxation microstructure. The influence of the locally non-equilibrium transfer of the impulse flux on the characteristics of the development of their boundary-layer motion is analyzed.

Introduction
The method of integral relations is often used in the analysis of the integral characteristics of boundary layer motion, resistance and heat and mass transfer of ordinary structureless media [1,2]. This is due to the mathematical difficulties that arise in the joint integration of nonlinear differential equations of boundary layer motion and heat and mass transfer, despite the simplest (instantaneous and local equilibrium) connections that close the dynamic equation with respect to the flow and the associated forces.

Locally nonequilibrium media and their systems are endowed with a complex internal polymer microstructure and, as a result, have special physical and chemical properties [3-5]. Depending on the deformation conditions (flow geometry), they exhibit nonlinear-viscous properties, partially store the energy, supplied from outside in the form of changes in the components of the normal stress tensor, and relax the stress state, exhibiting non-instantaneous and locally nonequilibrium [6].

Applied problems on the influence of locally nonequilibrium transfer processes on the boundary layer movement and heat and mass transfer arise in the technological processes of polymer coatings of products from corrosion by immersion, in flows, when protecting the surface from icing, in the initial sections in the channels of extrusion molding, as well as when obtaining and modifying stratified flow solutions and melts of polymers and their systems.

The importance of the problem is complemented by the fact that when obtaining a high-quality adhesive coating and a uniform distribution of modifying micro and nano-additives with a predicted microstructure, it is necessary to have parameters for matching the technological process time and the relaxation time of polymer systems.

In contrast to structureless media, the problem due to relaxation phenomena in polymer systems is additionally complicated by the fact that the systems under consideration respond nonlinearly to external disturbances. In addition, due to delayed relaxation phenomena, they respond immediately in time and nonlocally, that is, nonequilibrium even in a small differential volume (point) of the medium, the deformation state of which depends on the state of the nearest medium[6]. This difference arises at the level of rheological closure of the law of conservation of momentum (momentum) of the elementary volume of the system.
The regularities of motion in the case of an external flow of non-equilibrium media with a complex internal structure around bodies have been insufficiently studied. The problem lies, as noted, in the closure of the fundamental conservation laws by the equation of state of the medium, in the development of engineering methods for calculating the transfer of momentum, which allows analyzing the influence of structural and relaxation properties on the transfer processes.

The main goal of the work is to develop an applied research method for the study of quasi-stationary boundary-layer processes of transfer of locally nonequilibrium polymer media and their systems and to establish patterns and analyze the influence of locally nonequilibrium processes of momentum transfer on the development and characteristics of their boundary layer motion.

2. Integral ratio of impulses of boundary layer motion of locally nonequilibrium systems.

The starting point of this work is the analysis of the fundamental system, which is not closed with respect to flows and coupled forces, and the dynamic equations for the stationary boundary layer motion of locally nonequilibrium polymer media and their systems.

In the boundary layer theory, the well-known estimates [1,2,7] of the terms included in the dynamic equations with respect to the layer length and thickness of development lead to a system of equations of two-dimensional stationary boundary layer motion in the form:

\[
\begin{align*}
\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) &= \frac{\partial P}{\partial x} + \frac{\partial P_{xx}}{\partial x} + \frac{\partial P_{xy}}{\partial y} \\
\frac{\partial P}{\partial y} + \frac{\partial P_{yy}}{\partial y} &= 0 \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0
\end{align*}
\]

(1)

The flow field in the outer region of the streamlined body with velocity \( u_0 \), as is usually assumed in the theory of boundary layer motion, is described by the solution of the Euler equation.

Differentiating the second equation of system (1) with respect to \( x \), from the first two equations we have:

\[
\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{\partial P_{xx}}{\partial x} + \frac{\partial P_{yy}}{\partial y} (P_{xx} - P_{yy})
\]

(2)

Equation (2) was obtained taking into account the concepts of pressure continuity and constancy of velocity of the gradientless boundary layer motion, zero value of the velocity gradient, shear stress gradient, and gradient of the difference between the normal components of the deviator of the stress tensor in the outer region.

Equation (2) is not closed with respect to the kinematic characteristics of the flow and requires the introduction of the rheological equation of state. However, already in this form it establishes that the forces of inertia in the boundary layer are balanced by the gradients of shear stresses and the first difference of the normal components of the deviator of the stress tensor [7,8].

It should be noted that, in contrast to ordinary structureless fluids, for locally nonequilibrium systems, the value of \( P \) in (1) is not a hydrostatic pressure. This is an arbitrary scalar quantity that cannot be regarded as an isotropic pressure, since the trace of the stress tensor for the media under consideration and their systems is not equal to zero. In equation (2), this uncertainty is eliminated by the manifestation of the difference \( P_{xx} - P_{yy} \) of the normal components of the stress tensor.

Equation (2) is the starting point for deriving the integral boundary layer ratio of impulses. Omitting the characteristic calculations known for ordinary media [1,2,9], we obtain an equation for a gradientless flow of pulses in the form:

\[
\rho u_0^2 (d\delta^*/dx) = P_{CT}(x) - d/dx \int_0^\delta (P_{xx} - P_{yy})dy
\]

(3)

Equation (3) contains the pulse loss thickness \( \delta^* = \delta \int_0^1 \omega(1 - \omega) d\xi \) is pulse loss thickness, and \( \xi = \gamma/\delta(x) \) which is the dimensionless coordinate, divided by the thickness of the boundary layer \( \delta(x) \). The quantity \( \omega = u/u_0 \), where \( u_0 \) is the velocity of the flow unperturbed by the wall; \( \rho \) is the density of the medium; \( P_{CT}(x) \) is the shear stress on the wall (\( y=0 \)) variable along the longitudinal coordinate \( x \), and \( P_{xx} - P_{yy} \) is the first difference of the normal components of the stress tensor deviator.

Thus, the problem is reduced to closing the ratio of the momentum flux, establishing the connection between the components of the stress tensor and the components of the strain rate tensor, which are the main kinematic characteristics of the flow. That is, the problem is reduced to the
establishment of the rheological equation of state between the flows and the coupled forces in relation to the integral ratio of momenta for polymer media and their systems.

3. The rheological equation of state for the boundary layer transfer of the impulse flux of locally nonequilibrium systems.

At present, phenomenological and structural-statistical methods and approaches are being developed for the closure of fundamental conservation laws. In [10,11], within the framework of extended nonequilibrium statistical thermodynamics for a class of polymer media, a non-instantaneous and non-local process of transfer of a momentum flux of polymer media with a relaxation microstructure in the field of shear, entropy, and diffusion forces was modeled. The relations for the dynamic components of the stress tensor of the pulse flow with the coupled forces are obtained in the form of:

\[
P_{xy} = \frac{\varepsilon G\varepsilon t}{2}(1 - \exp(-2\varepsilon^{-1}t))
\]

\[
P_{xx} - P_{yy} = \frac{\varepsilon G^2\varepsilon^2}{2}[1 - \exp(-2\varepsilon^{-1}t) - 2\varepsilon^{-1}t \exp(-2\varepsilon^{-1}t)]
\]

The analysis of the space-time relations of equations 4 and 5 (Figs. 1 and 2) reveals the areas of influence of the shear and the first difference of the normal components of the stress tensor depending on the properties of the medium and the conditions of uniform deformation \(G \equiv \frac{du}{dy}\). The criterion for the non-instantaneous influence is the Deborah number (the ratio of the relaxation time \(\varepsilon\) to the characteristic time of the process \(De = \varepsilon t\)), and locally, the non-equilibrium influence is the Weissenberg number (\(We = G\varepsilon\)). The value \(0.5\varepsilon\varepsilon = \mu\) is the dynamic viscosity, and the quantity \(\varepsilon\) is the elastic modulus. Equations (4 and 5) also lead to the fundamental conclusion that the variable non-Newtonian viscosity of the medium (dependence on the shear rate) is due to relaxation phenomena, which determines the processes of momentum transfer aimed at neutralizing external disturbances.

The areas of influence of local nonequilibrium are established in accordance with (5), (Fig. 2). At \(We > 2\), dissipative rheological flows are not determined only by the gradients of the corresponding transfer potential, as for ordinary structureless media, but are already the solutions of an evolutionary equation describing the process of relaxation of the stress state parameters of a nonequilibrium polymer system to their local equilibrium values [11].

![Figure1](image)

Figure1. Influence of De and We on the magnitude and nature of the change in the shear component of the stress tensor in a locally nonequilibrium process of momentum transfer.
Figure 2. Influence of De and We on the magnitude and nature of changes in the first difference of the normal components of the stress tensor in a locally nonequilibrium process of momentum transfer.

It follows from relations (4 and 5) that under the conditions of a quasi-stationary shear characteristic of the integral boundary layer class of problems (3), the stressed state of a medium with a locally nonequilibrium relaxation microstructure has the form:

$$\frac{P_{xx}-P_{yy}}{P_{xy}} = G \alpha$$

(6)

4. Influence of the locally nonequilibrium process of momentum transfer on the characteristics of the development of boundary layer motion.

In the method of integral relations, it is usually assumed that the velocity field is approximated across the boundary layer, followed by the determination of its thickness and the corresponding characteristics of the flow along the length of its development. The distribution of velocities is given from the boundary conditions of the flow [1,2,9]. In this case, an assumption is made about the affinity of the velocity profiles on a flat plate streamlined in the longitudinal direction.

This work initially used the distribution of not the kinematic, but the dynamic characteristics of the flow over the cross section of the boundary layer by a cubic parabola [12]. The advantage of this approximate approach is that it does not depend on the rheological properties of the medium under study. The values of the coefficients are determined from the boundary conditions of the problem $\xi = 0$, $\bar{P}_{xy} = 1$; $\xi = 1$, $\bar{P}_{xy} = 0$.

$$\bar{P}_{xy} = \frac{P_{xy}(x)}{P_{CT}(x)} = 1 - 3\xi^2 + 2\xi^3$$

(7)

The distribution of the longitudinal component of the velocity vector is determined from the quasi-stationary ($t \to \infty$) relation of the system of equations (4) and (7):

$$\omega = \frac{u}{u_0} = \frac{2P_{CT}(x)}{\delta \varepsilon} u_0 \int_0^\xi \bar{P}_{xy} d\xi$$

(8)

Therefore, from the boundary conditions $\xi=1$, $\omega=1$, we find the relation for $P_{CT}(x)$ and the distribution for the velocity profile across the boundary layer:

$$\omega = 2\xi - 2\xi^3 + \xi^4$$

(9)

Substituting the corresponding values of quantities in relation (3), at $\delta^{**} = 0,1175\delta$, awe arrive at a closed momentum equation for changing the thickness of the boundary layer along the longitudinal coordinate $x$:

$$\frac{d\delta}{dx} + \frac{12.65 \nu \delta^{-1}}{dx} = \frac{17.02}{\delta u_0} = 0$$

(10)
where \( v = \mu / \rho \) is the kinematic medium viscosity.

The solution to equation (10), considering \( x = 1, \delta = 0 \), has the form:

\[
\delta^2 - 25.3 \nu \varepsilon \ln(\delta) - \frac{34.04 \nu}{u_0(x+1)} = 0 \tag{11}
\]

From equation (11) it follows that there is a class of locally nonequilibrium media for which the inertial forces of equation (2) are balanced simultaneously by tangential and normal stresses (Fig. 2, the number \( \text{We} > 5 \)), and the boundary layer does not develop from the leading critical edge of the flow, but at some distance from it.

The development of the layer from the leading-edge \( x = 0 \) of the flow deceleration can occur after the completion of dissipative (\( \text{G P}_{xy} \)) relaxation from a quasi-solid to some fluid state of stresses in the flow (the number \( \text{We} < 5 \)). In this case, the last term in equation (2) is no longer dominant, and the inertial forces are balanced by the shear stresses and partly by the first difference of normal stresses.

It should be noted that this relaxation process of unwanted elastic turbulence into the pre-inlet region of the forming channel [13] can be observed in the pre-inlet chambers of the channels of the forming elements in the extrusion technology, the source of which is their leading edges of the flow deceleration.

In accordance with relation (6) and constancy of \( \text{We} \), to study the influence of the locally nonequilibrium process of momentum transfer on the characteristics of the boundary layer motion, starting from the leading edge of the plate, it is convenient to use the cubic parabola approximation (7).

For a locally nonequilibrium flow, when the last term in equation (2) is not dominant (\( 2 < \text{We} < 5 \)) from relation (2) we have:

\[
\delta^2 + \frac{17.02 \delta \text{We}}{R e} - \frac{34.04 X}{R e} = 0 \tag{12}
\]

From equation (12) we find the development of the boundary layer thickness along the plate of length \( L \), that is \( X = x / L \), depending on the numbers \( R e = u_0 L / v \) and \( \text{We} \). The criterion \( \text{We} / R e \), which depends only on the rheological properties of the medium and, essentially, on the geometry of the flow, for the first time offered in [8] based on the analysis of dimensions, is proposed for systematic use. In equation (12), \( \delta \) is dimensioned by the length \( L \).

The change in the thickness of the boundary layer along the flow, considering \( \delta = 0 \) at \( X = 0 \), has the form:

\[
\delta(X) = 8.51 \text{We} / R e \left[\left(1 + \frac{0.47 X \text{Re}}{\text{We}^2}\right)^{0.5} - 1\right] \tag{13}
\]

For the case of a locally equilibrium flow (\( \text{We} / R e \to 0 \)), from Eq. (12) we have changes of the thickness of the boundary layer, shear stress and local resistance in the form [12]:

\[
\delta(x) = 5.83 \left(\frac{u_0}{u_0}\right)^{0.5} \tag{14}
\]

\[
P_{\text{CT}}(X) = \frac{\mu u_0}{\delta(x)\left( \frac{d\varepsilon}{dt} \right)_{\text{CT}}} = 0.343 \left(\frac{\mu u_0}{x}\right)^{0.5} \tag{15}
\]

Local resistance coefficient in section \( x \):

\[
C_x = \frac{2P_{\text{CT}}(x)}{\rho u_0^2} = 0.686 (v / \nu u_0)^{0.5} \tag{15}
\]

5. Conclusion

The method of integral relations has been extended to the class of polymer media and their systems with a locally nonequilibrium relaxation microstructure. The problem of stationary flow around a plate has been considered. The characteristics of nonequilibrium influence on the development of the boundary layer have been established and analyzed for the identified areas of locally nonequilibrium deformation. The development of the boundary layer depends on the properties and essentially on the geometry of the flow. In locally nonequilibrium systems with a relaxation microstructure, the transfer processes are characterized not only by boundary potentials, but also by relaxation phenomena in subsystems with a delay to the equilibrium state, affecting the development of the boundary layer.
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