Modelling with Self-Organising Maps and Data Envelopment Analysis: A Case Study in Educational Evaluation

Lidia Angulo Meza, Luiz Biondi Neto, Luana Carneiro Brandão, Fernando do Valle Silva Andrade, João Carlos Correia Baptista Soares de Mello and Pedro Henrique Gouvêa Coelho

Universidade Federal Fluminense and Universidade do Estado do Rio de Janeiro
Brazil

1. Introduction

In this chapter we deal with a problem of educational evaluation. We deal with an organization for distance education in the State of Rio de Janeiro, Brazil. This organization is the centre for distance undergraduate education in the Rio de Janeiro State (CEDERJ for the name in Portuguese). Although CEDERJ provides a wide set of undergraduate courses we focus ourselves on the Mathematics undergraduate course. The choice of this course is due to the fact that it exists since the very beginning of the CEDERJ.

We do not intend to evaluate distance undergraduate education itself. That is, we will not compare results from distance undergraduate education with results from in situ undergraduate education. Instead, we will compare distance education with itself, thus meaning we will evaluate some thirteen centres of distance education, all of them belonging to the CEDERJ. We want to determine the best managerial practices and the most favourable regions to inaugurate new CEDERJ centres.

The comparison hereabove mentioned takes into account how many students finish the course in each centre, how many students have began the course and the proxy for the resources employed in each centre. In the present chapter, we only consider graduates as outputs because graduating students is the main target of CEDERJ, while producing researches have low priority.

In order to perform this evaluation, we will use a non parametric technique known as Data Envelopment Analysis – DEA. Initially developed by Charnes et al (1978), this technique deals with productive units, called Decision Making Units (DMUs). The DMUs use the same inputs to produce the same outputs and the DMUs set must be homogenous, i.e. they must work in similar environmental conditions. It is important to notice that these DMUs are not necessarily units involved in a productive or manufacture process, but they can be entity using resources (inputs) to generate some kind of products (outputs).

In our case, the homogenous conditions are not verified since CEDERJ centres are located in different regions of the Rio de Janeiro State with different socio economical conditions that cannot be considered in the evaluation. So, in order to perform a DEA evaluation, we need...
to separate the centres in homogenous clusters according to their environmental conditions. To do that, we use the Kohonen self-organizing maps to cluster the centres. This is done taking into account some environmental variables.

After the clustering of the centres, we perform a DEA evaluation inside each cluster and overall DEA evaluation using an handicap index to compare the heterogeneous DMUs. We also identify the efficient centre and the benchmarks for the inefficient ones.

As mentioned above, this chapter deals with Data Envelopment Analysis and Kohonen Self Organizing Maps. The self-organising maps are a special case of neural networks. There are already some papers dealing with the use of Neural Networks and Data Envelopment Analysis altogether. For instance, Samoilenko and Osei-Bryson (2010) use Neural Networks and DEA to determine if the differences among efficiency scores are due to environmental variables or the management process. The use of Neural Network for clustering and benchmarking container terminals was done by Sharma and Yu (2009). Also Churilov and Flitman (2006) used Kohonen self-organizing maps to cluster countries participating of the Olympics and then using DEA for producing a new ranking of participating teams. Emrouznejad and Shale (2009) and Biondi Neto et al. (2004) used the back propagation neural network algorithm to accelerate computations in DEA. Çelebi and Bayraktar (2008) used Neural Networks to estimate missing information for suppliers evaluation using DEA.

This chapter is organized as follows; in the next two sections we briefly present the fundamentals of Data Envelopment Analysis (DEA) and Kohonen Neural Networks. In each of these sections we also present a brief bibliographical review of each one in the area of interest in this chapter, educational evaluation. In section 4, we present our case study, the CEDERJ distance undergraduate centres. Kohonen maps are used to cluster and DEA to evaluate the CEDERJ centres. Finally we present some conclusions, our acknowledgments and the references.

2. The fundamentals of data envelopment analysis

Data Envelopment Analysis – DEA was initially developed by Charnes et al. (1978) for school evaluation. This is a linear programming method to compute Decision Making Units – DMUs comparative efficiencies whenever financial matters are neither the only ones to take into consideration nor even the dominant ones. A DMU relative efficiency is defined as the ratio of the weighted sum of its outputs to the weighted sum of its inputs. Contrary to traditional multi-criteria decision aid models there is no arbitrary decision-maker that chooses the weights to be assigned to each weighing coefficient. These obtain instead from the very mathematical model. To do so, a fractional programming problem is solved to assign to each DMU the weights that maximize its efficiency. The weights are thus different for each unit and they are the most advantageous for the unit. So the DEA approach avoids the criticism from unit managers whose evaluation was not good that the weights were biased.

DEA models can take into account different scales of operation. When that happens the model is called BCC (Banker et al., 1984). When efficiency is measured taking no account of scale effects, the model is called CCR (Charnes et al., 1978). The formulation for the previously linearized fractional programming problem is shown in (1) for the DEA CCR (Cooper et al., 2000, Seiford, 1996).

For model (1) with \( n \) DMUs, \( m \) inputs and \( s \) outputs, let \( h_0 \) be the efficiency of DMU \( o \) being studied; let \( x_{ik} \) be \( i \) input of DMU \( k \); let \( y_{jk} \) be \( j \) output of DMU \( k \); let \( v_i \) be the weight assigned
to \( i \) input; let \( u_j \) be the weight assigned to \( j \) output. This model must be solved for each DMU.

\[
\begin{align*}
\min & \sum_{i=1}^{m} v_i x_{io} \\
\text{st} & \\
\sum_{j=1}^{s} u_j y_{jo} &= 1 \\
\sum_{j=1}^{s} u_j y_{jk} - \sum_{i=1}^{m} v_i x_{ik} &\leq 0, \quad k = 1, \ldots, n \\
u_j, v_i &\geq 0 \quad \forall x, y
\end{align*}
\]

(1)

Evaluating governmental institutions, such as CEDERJ and other educational institutions, is difficult mainly because of the price regulation and subventions, what generally leads to distortion (Abbott & Doucouliagos, 2003). However, DEA does not require pricing, and this is why it is broadly used for this type of evaluations.

DEA has been widely used in educational evaluation. For instance, Abbott & Doucouliagos (2003) measured technical efficiency in the Australian university system. They considered as outputs many variables referring to research and teaching. Abramo et al (2008) evaluated Italian universities, concerning basically scientific production.

The first authors went through analysis using various combinations of inputs and outputs, because the choice of the variables can greatly influence how DMUs are ranked, which is similar to what is done the process of variable selection in the present paper. The seconds also verify the importance of choosing the right variables, by comparing the final results with analysis of sensitivity, and observing how different they are.

Abbott & Doucouliagos (2003) introduce the concept of benchmarking as one of DEA strengths, though neither of the articles actually calculates it. Finding benchmarks and anti-benchmarks is important for the study’s applicability, since it is the first step to improving the inefficient DMUs. These authors also propose clustering the universities, according to the aspects of tradition and location (urban or not), which in their work, does not significantly affect results.

A more comprehensive review of DEA in education can be found in Soares de Mello et al (2006).

### 3. Fundamentals of Kohonen maps

The human brain organizes information in a logic way. The cortex has billions of neurons with billions of synaptic connections among them involving nearly all brain. The brain is orderly divided in subsections including: motor cortex, somatosensory cortex, visual cortex, auditory cortex. The sensory inputs are orderly mapped to those cortex areas (Kohonen, 2001, Haykin, 1999, Bishop, 1995).

It seems that some of these cells are trained in a supervised way and others in a supervised and self-organized way.

A paramount aspect of the self-organized networks is motivated by the organization of the human brain in regions in such a way that the sensory inputs are represented by
topologically organized maps. The Kohonen self-organizing map emulates that unsupervised learning in a simple and elegant way and also taking into account the neuron neighbourhood (Mitra et al., 2002).

The topographic map development principle according to Kohonen (2001) is as follows: “The space location of an output neuron in a topographic map corresponds to a particular domain or feature of data drawn from the input space.”

From that principle came up two feature mapping models: the Willshaw (1969) and Willshaw and von der Malsburg (1976) model, having strong neurobiological motivations, and the Kohonen (2001) model, not as close to neurobiology as the previous one but enabling a simple computing treatment stressing the essential characteristics of the brain maps. Moreover, the Kohonen model depicted in Figure 1 yields a low input dimension.

Fig. 1. Kohonen Self-Organizing Map

Another way to characterize a SOM (self-organizing maps) is shown in Figure 2. In that case, it is easily seen that each neuron receives identical input set information.

Fig. 2. Another way to represent Kohonen maps
The SOMs are Artificial Neural Networks (ANN) special structures in a grid form that work in a similar way of the human brain as far as the information organization is concerned, and are based on competitive learning. The most used SOM is the topologically interconnected two-dimensional, where the neurons are represented by rectangular, hexagonal and random grid knots of neighbour neurons. Higher dimensional maps can also be modelled. In Figure 3 one can see the neuron position in a (8X8) hexagonal representation.

![Hexagonal neuron positions](image)

**Fig. 3. Hexagonal neuron positions**

In order to analyze the competitive process, let us suppose that the input space is m-dimensional and that \( X \) represent a random input pattern (Haykin, 1999) such that one can write

\[
X = [x_1, x_2, x_3, ..., x_m]^T
\]  

(2)

Assuming the weight vector \( W \) of each neuron has the same dimension as that of the input space, for a given neuron \( j \) of a total of \( l \) neurons, the weight vector can be written as

\[
W_j = [w_{j1}, w_{j2}, w_{j3}, ..., w_{jm}]^T, \quad j = 1, 2, 3, ..., l
\]  

(3)

For each input vector, the scalar product is evaluated in order to find the \( X \) vector which is closest to the weight vector \( W \). By comparison, the maximum scalar product as defined in (4) is chosen, representing the location in which the topological neighbourhood of excited neurons should be centred,

\[
\max (W_j^T X), \quad j = 1, 2, 3, ..., l
\]  

(4)

Maximizing the scalar product in (4) is equivalent to minimize the Euclidian distance between \( X \) and \( W \). Figure 4 shows that the less the Euclidian distance the more approximation between \( X \) and \( W \).

Other metrics such as Minkowski, Manhatten, Hamming, Hausdorf, Tanimoto coefficients and angle between vectors could also be used (Kohonen, 2001, Haykin, 1999, Michie et al., 1994).
Fig. 4. Minimization of Euclidian Distance

The closest neuron to the input vector \( X \), given by (5), is called the winner neuron whose index is \( V(X) \), where

\[
V(X) = \min_{j=1,2,3,\ldots,L} \| X - W_j \|
\]  

(5)

By means of a competitive process, a continuous input space pattern can be mapped into a discrete output space of neurons.

In the cooperative process, the winner neuron locates the centre of a topological neighbourhood of cooperating neurons, which is biologically defined by the existence of interactive lateral connections in a cluster of biological neural cells. So the active winner, the winner one, tends to strongly stimulate its closest neighbour neurons and weakly the farthest ones. It is apparent that the topological neighbourhood concerned to the winner neuron decreases with increasing lateral distance.

It is essential to find a topological neighbourhood function \( s_{N_{j,V(X)}} \) that be independent from the winner neuron location written in (5). That neighbourhood function should represent the topological neighbourhood centred in the winner neuron, indexed by \( V \), having as closest lateral neighbours, a group of excited neurons and cooperative ones from which a representative can be chosen which is denominated \( j \) neuron. The lateral distance, \( D_{j,V} \), between the winner neuron indexed, by \( V \), and the excited neuron, indexed by \( j \) can be written as in (6) (Haykin, 1999).

\[
N_{j,V(X)} = \exp \left( -\frac{D_{j,V}^2}{2\sigma^2} \right)
\]  

(6)

where \( \sigma \) is the neighbourhood width.

The topological neighbourhood function \( A_{N_{j,V(X)}} \) shown in Fig. 5 should have the following properties (Mitra et al., 2002, Haykin, 1999):

- Be symmetric relative to the point of maximum, characterized by the winner neuron, indexed by \( V(X) \), for which \( D_{j,V} = 0 \).
- When \( D_{j,V} \) goes to \( \pm \infty \), the magnitude of the topological neighbourhood function monotonically decreases, tending towards zero.

The more dependent the lateral distance \( D_{j,V} \) be, the greater will be the cooperation among the neighbourhood neurons. So, for a two-dimensional output grid, the lateral distance can be defined as in (7), for which the discrete vector \( \rho_j \) represents the position of the excited neuron, and \( \rho_V \) the position of the neuron that won the competition.
Another point to be considered is that the topological neighbourhood should decrease with discrete time \(n\). In order to accomplish that, the width \(\sigma\) of the topological neighbourhood \(N_{j,V(X)}\) should decrease in time. That could be achieved if the width of the topological neighbourhood decreases in time. The width could be written as in (8) where \(\sigma_0\) represents the initial value of the neighbourhood width and \(\tau_1\) a time constant. Usually \(\sigma_0\) is adjusted to have the same value as the grid ratio, i.e. \(\tau_1=1000/\log \sigma_0\).

\[
D_{j,V} = \sqrt{(\phi_j - \phi_V)^2} 
\]

(7)

The expression of the topological neighbourhood in time can be written as

\[
N_{j,V(X)}(n) = \exp \left( -\frac{D_{j,V}^2}{2\sigma^2(n)} \right), \quad n = 0, 1, 2, 3, \ldots 
\]

(9)

The adaptive process is the last phase of the self-organizing map procedure and during this phase the adjustment of the connection weights of the neurons are carried out. In order the network succeed in the self-organization task, it is necessary the weights \(W_j\) of the excited \(j\) neuron be updated relatively to the input vector \(X\). Due to the connection changes that happen in one direction, the Hebb rule can not be used in the same way as in the supervised learning that would lead the weights to saturation. For
that, a slight change is done in the Hebb rule, including a new term \( g(y_j) \) \( W_j \) called forgetting term, in which \( W_j \) is the vector weight of the excited \( j \) neuron and \( g(y_j) \) is a positive scalar function of the output \( y_j \) of neuron \( j \). The only requirement imposed on the function \( g(y_j) \) is that the constant term in the Taylor series expansion of \( g(y_j) \) be zero, so that \( g(y_j) = 0 \) for \( y_j = 0 \). Given such a function, the change to the weight vector of the excited neuron \( j \) in the grid can be written as in (9) where \( \eta \) is the learning rate parameter.

The first term in equation (10) is the Hebbian term and the second the forgetting (Kohonen, 2001, Haykin, 1999, Bishop, 1995).

\[
\Delta W_j = \eta y_j X - g(y_j) W_j
\]

In order to satisfy the requirement, a linear function for \( g(y_j) \) is chosen as

\[
g(y_j) = \eta y_j
\]

Using \( y_j = N_{j,V(X)} \), equation (10) can be written as (12) as

\[
\Delta W_j = \eta N_{j,V(X)} (X - W_j)
\]

Using discrete-time notation a weight updating equation can be written which applies to all neurons that are within the topographic neighbourhood equation of the winner neuron (Kohonen, 2001, Haykin, 1999),

\[
W_j(n+1) = W_j(n) + \eta(n)N_{j,V(X)}(n)(X - W_j(n))
\]

In (13) the learning rate parameter changes each iteration, with an initial value around 0.1 and decreasing with increasing discrete-time \( n \) up to values above 0.01 (Mitra et al., 2002). To that end, equation (14) is written in which \( \eta \) decays exponentially and \( \tau_2 \) is another time-constant of the SOM algorithm. For the fulfilment of the requirements one could choose for instance, \( \eta_0 = 0.1 \) and \( \tau_2 = 1000 \).

\[
\eta(n) = \eta_0 \exp \left(-\frac{n}{\tau_2}\right), \quad n = 0, 1, 2, 3, ...
\]

Self-organizing maps have been widely used in many fields. For instance, regarding the subject of the present chapter, Kohonen networks have been used in education for peer identification process in business schools (re)accreditation process (Kiang et al., 2009) and to determine students' specific preferences for school websites (Cooper & Burns, 2007). In the Brazilian Rio de Janeiro state self-organized maps were used to cluster cities according to characteristics of electrical consumption (Biondi Neto et al., 2007). Then the self-organizing maps will be used to cluster CEDERJ distance education centres, in order to perform a DEA evaluation.

### 4. Distance learning in Rio de Janeiro: The CEDERJ

One of CEDERJ's main target is to contribute with the geographic expansion of undergraduate public education. This is also one of the targets of public universities in general. A second main target is to grant access to undergraduate education for those who
are not able to study in regular hours, usually because of work. Finally, developing the state’s high school teachers and offering vacancies in graduate courses are also targets to be achieved. We can notice that many of these are similar to UAB’s aims, which is a consequence of the fact that CEDERJ is part of the UAB system.

In CEDERJ, students have direct contact with tutors, who are of great importance (Soares de Mello, 2003) for they are responsible for helping students with their subjects as well as their motivation. Its pedagogical program is based on advances in the area of information and communication technologies, but also offers practical classes in laboratories. Students receive printed and digital material, which includes videos, animations, interactivity with tutors, teachers, other students and guests. This whole environment helps creating knowledge.

Its expansion in terms of number of local centres and types of courses brings up the need to evaluate CEDERJ globally, since the system consumes public resources, and also locally, in order to reduce eventual differences.

Gomes Junior et al (2008) evaluated CEDERJ courses using the so called elementary multi-criteria evaluation, Condorcet, Copeland and Borda (Roy & Bouyssou, 1993). The authors point out that there is an apparent relation between regions wealth and its position in the final ranking; and a reverse relation between the number of regular universities and the local centre’s position. In the present study, these variables should be considered when clustering the local centres.

Menezes (2007) made a scientific investigation on distance education, focusing on CEDERJ, analysing how new information and communication technologies impact on time and space organization.

There are many other studies on CEDERJ, yet they are mostly qualitative. Qualitative literature allows different interpretations, and it might become clearer with measurable facts. Our goal is with this quantitative approach to complement the existent qualitative literature, with no intention to replace it.

5. Evaluation of CEDERJ with DEA and Kohonen maps

The DMUs being evaluated in the present research are the local centres that offer Mathematics undergraduate course, therefore each of the following variables are related to the Math course in each local centre.

AI – Number of students enrolled in the course in a certain semester (input)
NT – Number of tutors in the first semester of 2009 (input) proxy for the resources used in the centre.
AF – Number of students that graduated in the first semester of 2009 (output)

There are other professionals, besides tutors, involved in the CEDERJ system, such as those responsible for preparing the material. However, the Math material is the same in every local centre, so these professionals should be attributed to each course, not to each local centre.

Seeking the semester that should be used for the first input, a process of variable selection is carried out because, according to Thanassoulis (1996), the group of variables used in the analysis can have great impact on its result. Therefore, in this evaluation process, variables are selected in a way that inputs better explain outputs and that less DMUs have maximum efficiency.
This process has been performed on the work of Andrade et al (2009) and it aimed to obtain a set of values for the AI variable, considering 1st and 2nd semesters of 2005 (1/2005 and 2/2005, respectively) and 1st semester of 2006 (1/2006). Since the graduation semester is the 1st one of 2009 (1/2009) and that the Math course has eight semesters of duration, it would be normal to use the number of enrolled students in 2/2005. Nevertheless, students may anticipate or postpone their graduation and therefore another semester might be chosen as the one that better explains the outputs. If 1/2005 is chosen, for example, it means that the majority of students postpone their graduation.

Although 24 local centres offer the Math course, only 13 have had graduates in 1/2009. Therefore only these 13 centres can be considered in the model, otherwise, results might be distorted because of the zero output. Besides the 24 centres, other four centres offer math tutorials – not the whole course, only tutorials. These, however, are not considered in this work.

According to the process of variable selection demonstrated in Andrade et al (2009), the semester chosen for the number of students enrolled in the course in a certain semester (AI) is 2/2005.

Another point to be considered is that local centres are subjected to different social, environmental and structural realities (Gomes Junior et al., 2008). This is important because in order to use DEA and compare DMUs, we should guarantee that they are homogeneous. The CEDERJ centres are located in regions with socio-economic characteristics very different among them. So, the DMUs are clearly non homogenous. If we try to use DEA with the complete set of centres we will have a DEA model with non homogenous DMU. This is a well-know pitfall in DEA (Dyson et al., 2001). So, we must be divided into clusters with homogeneous characteristics before using DEA. Afterwards, a homogenisation process will be carried out to perform an overall evaluation.

5.1 Clustering the DMUs
For the clustering of the CEDERJ centres we used the Kohonen self-organizing maps. The variables used were:

- The number of vacancies as a proxy to the size of the centre.
- The ratio of the candidates per vacancies for the Maths undergraduate course as a proxy to the cognitive level of the students enrolled in the course.
- The city’s Human Development Index (HDI) as a proxy for the socio economical characteristics of the city.

The number of semesters since the opening of the centre as a proxy to the maturity of the centre.

Different configurations for the Kohonen Maps were tested. We used grids with the (6x6), (4x4), (3x4), (3x3), (3x2) representations. Of all the clusters obtained we selected the one that did not let a centre isolated, which allows a better condition to perform an efficiency analysis using Data Envelopment Analysis. This was achieved using a grid with the (3x3) and the (3x2) representations, with the same clustering. The final clustering is shown in Table 1.

We obtained four clusters, with the mentioned representation, that contain centres with similar characteristics regarding size, students level, centre’s maturity and socio-economical characteristics as explain previously.
Cluster  | Centre
---------|--------
1st Cluster | Volta Redonda
            | Paracambi
            | Petrópolis
2nd Cluster | Angra dos Reis
            | São Pedro da Aldeia
            | Saquarema
            | Três Rios
            | Campo Grande
3rd Cluster | Macaé
            | Piraí
            | São Fidelis
4th Cluster | Cantagalo
            | Itaperuna

Table 1. Centres Clustering

5.2 Evaluation in each cluster
Once the clustering process is finished we performed the evaluation inside each cluster. We use the CCR output oriented model shown in section 2. The data, for the three variables considered, and the results for each one of the four clusters can be found in Tables 2, 3, 4 and 5.

| Centre      | Inputs |         |        | Efficiency Index (%) |
|-------------|--------|---------|--------|----------------------|
|             | AI     | NT      | AF     |                      |
| Volta Redonda | 99     | 10      | 10     | 80.80                |
| Paracambi    | 72     | 7       | 9      | 100.00               |
| Petrópolis   | 79     | 8       | 1      | 10.00                |

Table 2. Efficiency Index for the Centres in cluster 1
In these tables we can see that we obtained exactly one efficient centre in each cluster. This shows that despite having few DMUs in each cluster, DEA had success in obtaining a ranking in each cluster.

We can also observe that there are notorious differences among the efficiency indexes in the same cluster. A large proportion of centres are less than 50% efficient. This is not usual in DEA.

| Centre            | Inputs | Output | Efficiency Index (%) |
|-------------------|--------|--------|----------------------|
|                   | AI     | NT     | AF                   |
| Angra dos Reis    | 60     | 6      | 8                    | 100.00 |
| São Pedro da Aldeia | 20   | 5      | 1                    | 37.50  |
| Saquarema         | 62     | 6      | 4                    | 50.00  |
| Três Rios         | 60     | 8      | 3                    | 37.50  |
| Campo Grande      | 62     | 6      | 1                    | 12.50  |

Table 3. Efficiency Index for the Centres in cluster 2

| Centre            | Inputs | Output | Efficiency Index (%) |
|-------------------|--------|--------|----------------------|
|                   | AI     | NT     | AF                   |
| Macaé             | 29     | 6      | 3                    | 50.00  |
| Piraí             | 23     | 6      | 6                    | 100.00 |
| São Fidelis       | 61     | 6      | 2                    | 33.33  |

Table 4. Efficiency Index for the Centres in cluster 3

| Center            | Inputs | Output | Efficiency Index (%) |
|-------------------|--------|--------|----------------------|
|                   | AI     | NT     | AF                   |
| Cantagalo         | 40     | 7      | 2                    | 50.00  |
| Itaperuna         | 36     | 7      | 4                    | 100.00 |

Table 5. Efficiency Index for the Centers in cluster 4
5.3 Clusters evaluation
In performing the clustering and DEA evaluation in each cluster we take into account the differences in the environmental conditions of the centres. Now we are going to perform a DEA evaluation with the efficient centres of each cluster. Such centres are representative of the best managerial practices for each environmental condition. As was done previously, we used the CCR output oriented DEA model. The results of the evaluation of the four centres can be found in Table 6.

As observed in this Table, two centres were efficient, Angra dos Reis and Piraí. The least efficient of the four was Itaperuna.

| Centre            | Efficiency index (%) |
|-------------------|----------------------|
| Paracambi         | 96.43                |
| Angra dos Reis    | 100.00               |
| Piraí             | 100.00               |
| Itaperuna         | 53.37                |

Table 6. Evaluation of the efficient centres

We can say that the efficient centres, thus, efficient clusters, are so because of them being regions with accelerated development based on tourism, oil and industry in general. The students in these clusters have no other options for undergraduate courses other than the long distance centres of CEDERJ.

In the city of Itaperuna is from cluster 4, which contains an underdeveloped region of the northwest Rio de Janeiro state. This region has an improving number of high schools but still of poor quality.

The first cluster, represented by Paracambi, is composed by very developed cities. These cities are close to in situ centres of high quality undergraduate courses. This condition nullifies the existence of potential good students, because the mostly preferred the in situ courses. This fact justifies its efficiency index in the group form by the efficient centres in each cluster.

The efficient centres are located in developed regions with good students but not with significant number of in situ courses.

Therefore, we may suppose that the differences among those centres are due only to the environmental aspects, as the centres have the best managerial practices in their clusters. So it is possible to use the efficiency in Table 6 to evaluate the environmental conditions of the cluster represented by each centre. The efficiencies will be used as a handicap factor for each cluster.

5.4 Overall evaluation
Taking into account the differences between clusters, we perform an homogenisation of the centres, to be able to compare all of them in one single cluster. This is done by multiplying the inputs (number of students enrolled in the course in the 2nd semester of 2005 and number of tutors) of each centre times the efficiency obtained by their representative in
Table 6. We consider that the efficiency index obtained by each representative centre in Table 6 acts has a handicap factor. This methodology is inspired by the sports handicapping system for competitions with disabled athletes (Percy & Scarf, 2008, Percy & Warner, 2009). The data used and the efficiency obtained using the CCR output oriented DEA model are shown in Table 7.

In this Table we can observed that, as expected, the efficiency centres in the original clusters are still efficient. We may now compare centres of different clusters. One of the lowest overall efficient is the centre of Campo Grande. This centre is located in a poor region of a reach city, Rio de Janeiro. This may indicate a problem in clustering this centre. Furthermore, there are a lot of in situ undergraduate courses surrounding Campo Grande. As explained before those factors are not favourable to a centre. The Petrópolis centre, with the lowest efficiency, is in a rich city and very close, less than one hour driving, of the major campus of the main Brazilian university. Due to the fact that distance education is not yet well know and the nearness of a prestigious university, many students prefer to travel to the in situ courses. The city of São Pedro da Aldeia is in a summer vacations region, many people living in Rio de Janeiro have a summer house in this city. Often, it occurs that some students obtain a vacancy in the centre of São Pedro da Aldeia, profiting from the fact of of having a house in the city and later they enrol in an in situ course in Rio de Janeiro, abandoning the long distance course in São Pedro de Aldeia. This explains the lower efficiency.

| Centre                | Input          | Output        | Efficiency Index (%) |
|-----------------------|----------------|---------------|----------------------|
|                       | AI  | NT  | AF   |                      |
| Volta Redonda         | 95,4643 | 9,64286 | 10  | 78.09                |
| Paracambi             | 69,4286 | 6,75  | 9   | 100.00               |
| Petrópolis            | 76,1786 | 7,71429 | 1   | 9.77                 |
| Angra dos Reis        | 60  | 6   | 8   | 100.00               |
| São Pedro da Aldeia   | 20  | 5   | 1   | 9.82                 |
| Saquarema             | 62  | 6   | 4   | 50.00                |
| Três Rios             | 60  | 8   | 3   | 31.30                |
| Campo Grande          | 62  | 6   | 1   | 12.50                |
| Macaé                 | 29  | 6   | 3   | 47.44                |
| Piraí                 | 23  | 6   | 6   | 100.00               |
| São Fidelis           | 61  | 6   | 2   | 25.00                |
| Cantagalo             | 21,36 | 3,738 | 2   | 48.60                |
| Itaperuna             | 19,224 | 3,738 | 4   | 100.00               |

Table 7. Homogenized data and overall efficiency index
We also perform an analysis of benchmarks for the inefficient centres. The benchmarks of an inefficient centre give the managerial guidelines to achieve the efficient levels in inputs or outputs. These are depicted in Table 8.

In this Table we may observe that the three cities originally in cluster 1, Volta Redonda, Paracambi and Petropolis, have benchmarks outside their own cluster. In the original cluster 3, Sao Fidelis is the only centre that has not at least one benchmark inside its own cluster. All the efficient centres except Paracambi, are their own benchmarks. This fact vindicates that Paracambi is a weakly efficient centre. This means that in an overall evaluation that the number of students that graduated in the first semester of 2009 can be improved in comparison to other efficient centres.

| DMU              | Benchmarks          |
|------------------|---------------------|
| Volta Redonda    | Angra dos Reis; Piraí |
| Paracambi        | Angra dos Reis      |
| Petrópolis       | Angra dos Reis; Piraí |
| Angra dos Reis   | Angra dos Reis      |
| São Pedro da Aldeia | Angra dos Reis; Piraí |
| Saquarema        | Angra dos Reis      |
| Três Rios        | Angra dos Reis; Piraí |
| Campo Grande     | Angra dos Reis      |
| Macaé            | Angra dos Reis; Piraí |
| Piraí            | Piraí               |
| São Fidelis      | Angra dos Reis      |
| Cantagalo        | Angra dos Reis; Piraí |
| Itaperuna        | Itaperuna           |

Table 8. Benchmarks in the overall efficiency evaluation

6. Final comments

The main objective of this chapter was to perform the evaluation of the centres of distance undergraduate Math courses of the CEDERJ. This evaluation was carried out using Data Envelopment Analysis. A total of thirteen centres were evaluated, these having environmental differences among them. They were divided in four cluster using Kohonen self-organized maps according to the size of the centres, level of the centres, socio economical characteristics and maturity of the centres proxies. In each cluster, we performed a DEA analysis obtaining exactly one efficient centre for each cluster. Comparing the clusters we conclude that centres in very poor or very rich regions will probably have low efficiency.
We also performed an homogenisation of the centres in order to obtain and overall evaluation and a benchmark analysis. We observed that the majority of the centres have benchmarks outside their own cluster. The fact that a large number of centres have very little efficiency may indicate that we must refine the clustering process. A variable that seems to be important and may be used in future works is the distance of the centre to major campi of in situ courses universities. The distance between two distance centres may also be considered for the clustering process in future works.

It may also be useful to perform a time window analysis of the centres.

It worth noting that São Fidelis and Campo Grande are each one in a single cluster for almost all the Kohonen maps configurations. It only in the configuration used they are clustered with other centres. This may indicated that São Fidelis and Campo Grande have been under evaluated in this study. In the future we may study a new process to perform a fair evaluation to those two centres.

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8. References

Abbott, M. & Doucouliagos, C. (2003). The efficiency of Australian universities: A data envelopment analysis. *Economics of Education Review*, Vol. 22, No. 1, pp. 89-97, 0272-7757.

Abramo, G.; D’Angelo, C. A. & Pugini, F. (2008). The measurement of italian universities’ research productivity by a non parametric-bibliometric methodology. *Scientometrics*, Vol. 76, No. 2, pp. 225-244, 0138-9130.

Andrade, F. V. S.; Brandão, L. C. & Soares de Mello, J. C. C. B. (2009). Avaliação de um curso de matemática à distância com modelos DEA e seleção de variáveis. *Relatórios de Pesquisa em Engenharia de Produção da UFF*, Vol. 9, pp. 10, 1678-2399.

Banker, R. D.; Charnes, A. & Cooper, W. W. (1984). Some models for estimating technical scale inefficiencies in data envelopment analysis. *Management Science*, Vol. 30, No. 9, pp. 1078-1092, 0025-1909

Biondi Neto, L.; Coelho, P. H. G.; Soares De Mello, J. C. C. B. & Angulo Meza, L. (2007). Self-organizing maps for classification of the rio de janeiro state cities based on electrical energy consumption. *ICEIS 2007 - 9th International Conference on Enterprise Information Systems, Proceedings*, pp. 447-450, june, 2007, Funchal.

Biondi Neto, L.; Lins, M. P. E.; Gomes, E. G.; Soares de Mello, J. C. C. B. & Oliveira, F. S. (2004). Neural data envelopment analysis: a simulation. *International Journal of Industrial Engineering*, Vol. 11, pp. 14-24, 1072-4761.

Bishop, C. M. (1995). *Neural networks for pattern recognition*, Oxford University Press, 0-19-853864-2, New York.

Çelebi, D. & Bayraktar, D. (2008). An integrated neural network and data envelopment analysis for supplier evaluation under incomplete information. *Expert Systems with Applications*, Vol. 35, No. 4, pp. 1698-1710, 0957-4174.

Charnes, A.; Cooper, W. W. & Rhodes, E. (1978). Measuring the efficiency of decision-making units. *European Journal of Operational Research*, Vol. 2, pp. 429-444, 0377-2217.
Churilov, L. & Flitman, A. (2006). Towards fair ranking of olympics achievements: The case of Sydney 2000. *Computers and Operations Research*, Vol. 33, No. 7, pp. 2057-2082, 0305-0548.

Cooper, C. & Burns, A. (2007). Kohonen self-organizing feature maps as a means to benchmark college and university websites. *Journal of Science Education and Technology*, Vol. 16, No. 3, pp. 203-211, 1059-0145 (print version), 1573-1839 (electronic version).

Cooper, W. W.; Seiford, L. & Tone, K. (2000). Data envelopment analysis: A comprehensive text with models, applications, references and DEA-solver software, Kluwer, 0387452818, Boston.

Dyson, R. G.; Allen, R.; Camanho, A. S.; Podinovski, V. V.; Sarrico, C. S. & Shale, E. A. (2001). Pitfalls and protocols in DEA. *European Journal of Operational Research*, Vol. 132, No. 2, pp. 245-259, 0377-2217.

Emrouznejad, A. & Shale, E. (2009). A combined neural network and DEA for measuring efficiency of large scale datasets. *Computers and Industrial Engineering*, Vol. 56, No. 1, pp. 249-254, 0360-8352.

Gomes Junior, S. F.; Soares de Mello, J. C. C. B. & Soares de Mello, M. H. C. (2008). Utilização do método de Copeland para avaliação dos pólos regionais do CEDERJ. *Rio's international journal on sciences of industrial and systems engineering and management*, Vol. 2, No. 4, pp. 87-98, 1982-6443.

Haykin, S. (1999). *Neural networks: a comprehensive foundation*, Prentice Hall, 0132733501, New Jersey.

Kiang, M. Y.; Fisher, D. M.; Chen, J. C. V.; Fisher, S. A. & Chi, R. T. (2009). The application of SOM as a decision support tool to identify AACSB peer schools. *Decision Support Systems*, Vol. 47, No. 1, pp. 51-59, 0167-9236.

Kohonen, T. (2001). *Self-organizing maps*, Springer-Verlag, 3540679219, Berlin.

Menezes, E. P. (2007). A espacialidade e a temporalidade da educação a distância: O caso do CEDERJ/CECERJ. *13º Congresso Internacional de Educação a Distância*, september, 2007, Curitiba.

Michie, D.; Spiegelhalter, D. J. & Taylor, C. C. (1994). *Machine learning, neural and statistical classification*, Ellis Horwood, 013106360X, Chichester.

Mitra, P.; Murthy, C. A. & Pal, S. K. (2002). Unsupervised feature selection using feature similarity. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol. 24, No. 3, pp. 301-312

Percy, D. F. & Scarf, P. A. (2008). On the development of decision rules for bar quiz handicapping. *Journal of the Operational Research Society*, Vol. 59, No. 10, pp. 1406-1414, 0160-5682.

Percy, D. F. & Warner, D. B. (2009). Evaluating relative performances in disabled sports competitions. *IMA Journal Management Mathematics*, Vol. 20, No. 2, pp. 185-199, 1471-6798 (on line), 1471-678X (print).

Roy, B. & Bouyssou, D. (1993). *Aide multicritère à la décision: méthodes et cas*, Economica, Paris.

Samoilenko, S. & Osei-Bryson, K. M. (2010). Determining sources of relative inefficiency in heterogeneous samples: Methodology using Cluster Analysis, DEA and Neural Networks. *European Journal of Operational Research*, Vol. 206, No. 2, pp. 479-487, 0377-2217.
Seiford, L. M. (1996). Data envelopment analysis: The evolution of the state of the art (1978-1995). *Journal of Productivity Analysis, Vol. 7*, No. 2-3, pp. 99-137.

Sharma, M. J. & Yu, S. J. (2009). Performance based stratification and clustering for benchmarking of container terminals. *Expert Systems with Applications, Vol. 36*, No. 3 PART 1, pp. 5016-5022, 0957-4174.

Soares de Mello, J. C. B.; Gomes, E. G.; Angulo-Meza, L.; Soares de Mello, M. H. C. & Soares de Mello, A. J. R. (2006). Engineering Post-Graduate Programmes: A Quality and Productivity Analysis. *Studies in Educational Evaluation, Vol. 32*, pp. 136-152, 0191-491X.

Soares de Mello, M. H. C. (2003). Uma experiência presencial em EAD: o caso CEDERJ. . *XXVI CNMAC, september, 2003, São José do Rio Preto.*

Thanassoulis, E. (1996). Assessing the efficiency of schools with pupils of different ability using Data Envelopment Analysis. *Journal of the Operational Research Society, Vol. 47*, No. 1, pp. 84-97, 0160-5682.

Willshaw, D. J.; Buneman, O. P. & Longuet-Higgins, H. C. (1969). Non-holographic associative memory. *Nature, Vol. 222*, pp. 960-962, 0028-0836.

Willshaw, D. J. & Von der Malsburg, C. (1976). How patterned neural connections can be set up by self-organization. *Proceedings of the Royal Society of London Series B, Vol. 194*, pp. 431-445, 1471-2954.
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