Abstract. Metrological studies for particle image velocimetry (PIV) are recent in literature. An attempt to evaluate the uncertainty quantifications (UQ) of the PIV velocity field are in evidence. Therefore, a short review on main sources of uncertainty in PIV and available methodologies for its quantification are presented. In addition, the potential of some mathematical techniques, coming from the area of geometric mechanics and control, that could interest the fluids UQ community are highlighted as good possibilities.

“We must measure what is measurable and make measurable what cannot be measured” (Galileo)
1.2. Sources of Uncertainty
From the metrological perspective, PIV is still in its infancy. Uncertainty sources galore: in processing, they come from illumination, camera positioning and data recording. Seeding requires a mix of "art and science" and it is impossible to make it reproducible. Moreover, postprocessing has an "original sin": the estimated velocity at a image pixel comes from statistical correlations of a group of nearby particles motion. In addition, most commercial softwares extrapolate measurements to non-seeded areas in ways not detailed in the manuals. Seeds, being material particles, do not move precisely along the flow. There is a transversal component, due to several physical effects [3]. This is a serious issue specially for particle tracking velocimetry.

2. Review of Uncertainty Quantification effort in Particle Image Velocimetry

2.1. The 2011 meeting in Las Vegas
Probably the first concerted attempt to discuss uncertainty quantification in particle image velocimetry was the meeting held in May 11-13, 2011, Las Vegas, organized by Barton Smith (Utah State) and Pavlos Vlachos (Virginia Tech). The event website contains slides/notes of fifteen presentations on uncertainty quantification for PIV and a short course by Coleman and Steele about their UQ book [4]. We now summarize some of the aspects discussed in that meeting, aimed to "establish the foundations for developing a comprehensive PIV uncertainty estimation methodology". The determination of the measurements accuracy with PIV was the main discussion topic in this meeting. The uncertainty sources analysis have been approached in numerous speeches. The Ronald J. Adrian presentation described an attempt to develop a systematic framework for the development and application of the analysis schemes for detections of the uncertainty sources on experiments with PIV [1]. The following presentations have described studies with different methods to detect the uncertainty sources. As such, experiments with complementary measurements, the use of CFD solutions for verification and evaluation of the auto-correlation used in postprocessing. Hence, this event shows the need of more investigations in this research field.

2.2. Measurement Science and Technology
MST recent issue 25:8 ([5], 2014) contains a whole section devoted to current developments in particle image velocimetry. Besides the focus on characterizing uncertainties sources, hardware and system reliability were also discussed. More questions were posed than answers: in fact, new challenges for UQ are coming from recent experimental techniques, such as tomographic PIV. For instance, an artifact that haunts experimentalists are so called "ghost particles". Curiously, these are easier to detect and eliminate in time resolved image sequences. As for hardware, the uncertainties of CCD and CMOS sensors (charge-coupled devices and complementary metal-oxide semiconductors) were addressed in the article by Abdelsalam et al, specially how individual pixels behave. From the quantitative perspective, an exciting development in this MST issue and other recent sources is the possibility to estimate dynamic quantities (accelerations, pressures) from the velocity measurement. These questions may give rise to interesting mathematical developments.

2.3. Towards an automatized UQ for PIV
As far as we know, only one research group, that leaded by Barton L. Smith in Utah has already proposed a thorough methodology for UQ in PIV [6]. Their work could serve as a departure point for further advances, leading to metrologically standardized procedures as advocated in GUM. The Timmins master’s thesis describes a method to estimate the PIV uncertainty [6].
Initially, the relationship between error sources and their velocity measurements is investigated. Once the error source an "uncertainty surface" is provided for the PIV algorithm used. After PIV processing, it’s possible to measure the value of each of these parameters and estimate the uncertainty of each vector. This methodology was described to be general and adapted to any PIV analysis.

3. Mathematical Tools

3.1. Probabilistic Methods
We mentioned that seeding is not reproducible, but this does not prevent metrologists to think of a fluid flow measurement as a realization from a probabilistic ensemble of admissible initial conditions. In that sense, the practical need of doing repeated PIV recordings becomes an experimental feature. We refer to [7, 8] for data assimilation in fluids, where spectral methods are currently the state of the art. Such probabilistic methods fit well with the current tendency of GUM to move towards Bayesian statistics. We have looked into the literature of gaussian processes\(^4\) and we believe there is an enormous potential there for uncertainty quantification, as an optimal distribution compatible with the available data comes also with error bars, albeit conditioned to a given probability of lying there.

We finish this brief report presenting three prospective mathematical techniques coming from geometric mechanics and control, and the planetary sciences community.

3.2. Observers
Observer theory gives life to Galileo’s motto that opens our assessment. It is well now in geometric control [9]. It allows estimating parameters and dynamic variables not directly measurable in an experiment. The observer ODE or PDE adds artificial terms to the model equation so that it converges exponentially fast to a desired trajectory. Observed measurements \(y(t)\) are fed into the observer equation. The desired quantities appear in the output \(x(t)\), which is “best” fit in a Bayesian philosophy. In metrology the observer method is not widely disseminated. The only reference we found is Ruhm [10], where he describes it as a “model-based measurement”. Observer theory has been already used for fluids [11, 12]. Its use in PIV was advocated by Rouchon [13].

3.3. Data assimilation
In a series of papers, starting around 2010, Edriss Titi and collaborators have developed a research program which combines ideas from the geophysical and the control theory communities [14, 15, 16, 17, 18]. In a nutshell, consider a dynamical systems where partial information is known about its state, observed from an initial time to the present. This information is used to ’gently nudge’ (say, by a fictitious forcing term in the right hand side the Navier Stokes equations) the system solution to the best full initial condition at current time. Evidently data assimilation has the same flavor as the observer method.

3.4. “Jetlets”
In a nutshell, a k-jetlet is a local kth degree polynomial that approximates the flow near a particle. Exact solutions of regularized Euler equations can be constructed from a finite set of particles, which follow the corresponding flow field. This flow is automatically volume preserving and can explicitly be numerically simulated [19, 20]. For metrology, jetlets could become a novel way to reconstruct the velocity field from a finite set of measurements of seed positions and velocities.

\(^4\) http://www.gaussianprocess.org
4. Conclusion
The particle image velocimetry have been used as a powerful tool to investigate fluid dynamics behavior. Nevertheless, the PIV metrological approach began in the end of last decade. Therefore, many issues need to be investigated. The uncertainty sources and its quantification are a new scientific brand in this research area. In this paper, different types of mathematical techniques coming from geometric mechanics and control and planetary science community were briefly presented indicating new ways to approach uncertainty quantification for particle image velocimetry.

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