In this paper, $E_6$ and especially $E_7$ GUT are considered in the F-theory setting in view of the free fermionic construction of the $4D$ heterotic string. In particular, the NAHE-Based LRS model of [29,30] is revisited as an illustration where the starting point was taken to be the $N = 4$, $E_7 \times E_7 \times SO(16)$ which through the use of boundary condition basis vectors is reduced to obtain the flipped $SO(10)$ GUT symmetry.

We also seek to extend the results of [32] in the case of the flipped $SU(5)$ to home in on the flipped $SO(10)$ vacua in the Horáva-Witten theory where the $E_8$ gauge group on the observable sector decomposes as $E_8 \supset E_6 \times SU(3)$ with $E_6$ being the gauge group of the effective field theory. We find for the $E_6$ GUT symmetry, solutions of type A and solutions of type B where the Hirzebruch surfaces are considered for the base contrary to [32] where flipped $SU(5)$ vacua were studied and only solutions of type B were found. Moreover, no solutions are found in the case of the base being the del Pezzo surfaces. Furthermore, this goes hand in hand with the heterotic, low-energy string-derived effective model discussed in [2,3].
1 Introduction

To break GUT symmetries in F-theory [8–10] to construct viable models, one can use Wilson lines [15, 22] or introduce a $U(1)$ flux corresponding to a fractional line bundle [16–21].

The basic ingredient for model building in F-theory is a space-time filling seven-brane which wraps a four-dimensional internal subspace of the six internal directions of the compactification where each lower-dimensional subspace provides an important model building element as can be seen from Table 1. An Abelian or a non-Abelian gauge flux of the rank greater than two can then be turned on in the bulk to break the gauge group [16].

| Dimension | Ingredient           | Complex Codimension | Enhancements |
|-----------|----------------------|---------------------|--------------|
| 8D        | Gauge Theory         | 1                   | -            |
| 6D        | Matter               | 2                   | Rank 1       |
| 4D        | Yukawa Couplings     | 3                   | Rank 2       |

Table 1: The key elements of model building in F-theory.

The aim of this paper is to present a study of the flipped $SO(10)$ model embedded completely in the $E_6$ and $E_7$ GUT but with a different accommodation of the SM representations in the $27$ of $E_6$ in string-derived, heterotic low-energy effective models constructed in the free fermionic formulation.

1.1 F-Theory $E_6$

\[ E_8 \supset E_6 \times SU(3)_\perp \]

with

\[ 248 \rightarrow (78, 1) + (1, 8) + (27, 3) + (\overline{27}, \overline{3}) \]

where the inhomogeneous Tate form for $E_6$ is given by

\[ x^3 - y^2 + b_1 xyz + b_2 x^2 z^2 + b_3 yz^2 + b_4 xz^3 + b_6 z^5 = 0. \]

Here, the $SU(3)_\perp$ factor is considered as the group ‘perpendicular’ to the $E_6$ GUT divisor. In what follows assume semi-local approach where the $E_6$ representations transform non-trivially under the $SU(3)_\perp$. In the spectral cover approach the $E_6$ representations are distinguished by the weights $t_{1,2,3}$ of the $SU(3)_\perp$ Cartan subalgebra subject to the traceless condition

\[ \sum_{i=1}^{3} t_i = 0 \]
while the $SU(3)_\perp$ adjoint decomposes into singlets. All the various $E_6$ breaking patterns

\begin{align*}
(1a) \quad E_6 & \rightarrow SO(10) \times U(1) \rightarrow SU(5) \times U(1)^2 & (1.1) \\
(1b) \quad E_6 & \rightarrow SO(10) \times U(1) \rightarrow SU(4) \times SU(2) \times SU(2) \times U(1) & (1.2) \\
(2a) \quad E_6 & \rightarrow SU(6) \times SU(2) \rightarrow SU(5) \times SU(2) \times U(1) & (1.3) \\
(2b) \quad E_6 & \rightarrow SU(6) \times SU(2) \rightarrow SU(4) \times SU(2) \times SU(2) \times U(1) & (1.4) \\
(2c) \quad E_6 & \rightarrow SU(6) \times SU(2) \rightarrow SU(3) \times SU(3) \times SU(2) \times U(1) & (1.5) \\
(3) \quad E_6 & \rightarrow SU(3) \times SU(3) \times SU(3) & (1.6)
\end{align*}

reduce to one of the two extended MSSM models of rank 6

\begin{align*}
E_6 & \rightarrow SU(3) \times SU(2) \times [U(1)^3] \\
E_6 & \rightarrow SU(3) \times SU(2) \times [SU(2) \times U(1)^2]
\end{align*}

which are equivalent up to linear transformations.\footnote{see \cite{13,14} for example.}

### 1.2 A String-Derived Low-Energy Effective Model

The string-derived model presented in \cite{1–6} was constructed in the free fermionic formulation \cite{7} of the four-dimensional heterotic string. The details along with the the massless spectrum and the superpotential can be found in \cite{1} and therefore omitted here. It was shown that the space-time vector bosons are obtained solely from the untwisted sector and generate the observable and hidden gauge symmetries:

\begin{align*}
\text{observable} : \quad & SO(6) \times SO(4) \times U(1)_1 \times U(1)_2 \times U(1)_3 \\
\text{hidden} : \quad & SO(4)^2 \times SO(8).
\end{align*}

Under the decomposition $E_6 \rightarrow SO(10) \times U(1)_\zeta$, following from Table \ref{table2} the fundamental representation of $E_6$ decomposes as follows

$$27 \rightarrow 16_{+1/2} + 10_{-1} + 1_{+2}.$$
Table 2: The massless spectrum where the charges are displayed in the normalisation used in free fermionic heterotic string-derived models.

The $E_6$ GUT symmetry can be broken following [11–14] as

\[
E_6 \rightarrow SO(10) \times U(1)_{\zeta} \\
\rightarrow [SU(5) \times U(1)_{\zeta'}] \times U(1)_{\zeta} \\
\rightarrow [SU(3) \times SU(2) \times U(1)_{\zeta''}] \times U(1)_{\zeta'} \times U(1)_{\zeta}
\]

and the SM representations are accommodated in the 27 of $E_6$ as

\[
27 = \begin{cases} 
16_{+\frac{1}{2}} & F_L + F_R = (Q, u^c, d^c, L, e^c, N) \\
\rightarrow (Q, u^c) + (d^c_L) + N \\
10_{-1} & D + H \\
1_{+2} & S \rightarrow S
\end{cases}
\]
1.2.1 The $E_7$ Enhancement: A Word

The enhancement $E_6 \to E_7$ where the extra matter transforms in the $27$ representation of $E_6$ as can be determined from the branching rule

$$133 \to 78_0 \oplus 1_0 \oplus 27_{+1} \oplus 27_{-1}$$

where the subscripts denote the $U(1)$ charges under the decomposition

$$E_7 \to E_6 \times U(1).$$

2 F-Theory $E_7$

$$E_8 \supset E_7 \times SU(2)_{\perp}$$

with

$$248 \to (133, 1) \oplus (1, 3) \oplus (56, 2)$$

where the inhomogeneous Tate form for $E_7$ is given by

$$x^3 - y^2 + b_1 xyz + b_2 x^2 z^2 + b_3 yz^3 + b_4 xz^3 + b_6 z^5 = 0.$$  

2.1 The $E_7$ Gauge Enhancements And Breaking Patterns

$$\Delta = z^9 \left[ -1024 b_4^3 + ((b_1^2 + 4 b_2)^2 - 96 b_1 b_3) b_4^2 + 72(b_1^2 + 4 b_2) b_4 b_6 - 432 b_6^2) z + \mathcal{O}(z^2) \right]$$

|               | deg($\Delta$) | Type | Gauge Group | Object Equation |
|---------------|---------------|------|-------------|----------------|
| GUT           | 9             | $E_7$| $E_7$       | $S: z = 0$     |
| Matter Curve  | 10            | $E_8$| $E_8$       | $b_4 = 0$      |

Table 3: The $E_7$ gauge enhancements. There are no interaction terms, and conclude that an $E_7$ GUT is not possible if we are only interested in the generic enhancements. For completeness $SU(5)$ and $SO(10)$ gauge enhancements can be found in Appendix A.
However, we are interested in the breaking patterns of $E_7$ which following [31] are found to be

$$(1a) \ E_7 \rightarrow E_6 \times U(1) \rightarrow SO(10) \times U(1)^2 \quad (2.1)$$

$$(1b) \ E_7 \rightarrow E_6 \times U(1) \rightarrow SU(6) \times SU(2) \times U(1) \quad (2.2)$$

$$(1c) \ E_7 \rightarrow E_6 \times U(1) \rightarrow SU(3) \times SU(3) \times SU(3) \times U(1) \quad (2.3)$$

$$(2a) \ E_7 \rightarrow SU(6) \times SU(3) \rightarrow SU(5) \times SU(3) \times U(1) \quad (2.4)$$

$$(2b) \ E_7 \rightarrow SU(6) \times SU(3) \rightarrow SU(4) \times SU(3) \times SU(2) \times U(1) \quad (2.5)$$

$$(3) \ E_7 \rightarrow SO(12) \times SU(2) \rightarrow SO(10) \times SU(2) \times U(1) \quad (2.6)$$

$$(4) \ E_7 \rightarrow SU(8) \quad (2.7)$$

where (2.1) is of key interest since under the decomposition

$$E_7 \rightarrow E_6 \times U(1)$$

$$133 \rightarrow 78_0 \oplus 1_0 \oplus 27_{+1} \oplus 27_{-1}$$

where the subscript denotes the $U(1)_\delta$ charge and under further decomposition

$$E_6 \rightarrow SO(10) \times U(1)$$

$$27 = 16_{+1/2} + 10_{-1} + 1_{+2},$$

$$27 = 16_{-1/2} + 10_{+1} + 1_{-2},$$

where the subscript denotes the $U(1)_{\zeta}$ charge.

3 The NAHE-Based Models: A Discussion

The NAHE set consists of five basis vectors:

$$1 = \{\psi^{1,2}_{\mu}, \chi_{1,6}, y_{1,6}, \omega_{1,6}, \bar{y}_{1,6}, \bar{\omega}_{1,6}, \bar{\psi}_{1,5}, \bar{\eta}_{1,3}, \bar{\phi}_{1,8}\},$$

$$S = \{\psi^{1,2}_{\mu}, \chi_{1,6}\},$$

$$b_1 = \{\psi^{1,2}_{\mu}, \chi_{1,2}, y_{3,6}, \bar{y}_{3,6}, \bar{\psi}_{1,5}, \bar{\eta}\},$$

$$b_2 = \{\psi^{1,2}_{\mu}, \chi_{3,4}, y_{1,2}, \omega_{5,6}, \bar{y}_{1,2}, \bar{\omega}_{5,6}, \bar{\psi}_{1,5}, \bar{\eta}^2\},$$

$$b_3 = \{\psi^{1,2}_{\mu}, \chi_{5,6}, \omega_{1,4}, \bar{\omega}_{1,4}, \bar{\psi}_{1,5}, \bar{\eta}^3\}.$$

The basis vectors $1$ and $S$, generate a model with the $SO(44)$ gauge symmetry and $N = 4$ space–time supersymmetry. The vectors $b_i$ for $i = 1, 2, 3$ correspond to the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold twists. The vector $b_1$ breaks the $SO(44)$ gauge group to $SO(28) \times SO(16)$ and the $N = 4$ space–time supersymmetry to $N = 2$. The vector
$b_2$ then reduces the group to $SO(10) \times SO(22) \times SO(6)^2$ gauge group and the $N = 2$ supersymmetry is further reduced to $N = 1$. Furthermore, the basis vector $b_3$ gives the decomposition $SO(10) \times SO(16)_1 \times SO(6)^3$ where we fix the GGSO projection coefficient in order to preserve the $N = 1$ space–time supersymmetry. Moreover, the sector, $\xi$, given by the linear combination

$$\xi = 1 + b_1 + b_2 + b_3 \equiv \{\phi^{1,\ldots,8}\}$$

together with the $NS$–sector form the adjoint representation of $E_8$ thereby enhancing the $SO(16)_1$. As a result, we obtain

$$SO(10) \times E_8 \times SO(6)^3$$

as the gauge group with $N = 1$ space–time supersymmetry at the NAHE level.

There are two classes of $SO(10)$ breakings found in the literature using the free fermionic construction of the heterotic string:

- Flipped $SU(5)$ [23], Pati-Salam [24], and Standard-like models [25–28];
- Left-right symmetric [29] and $SU(4) \times SU(2) \times U(1)$ models [30].

We are interested not in the former, but in the later case where the starting point is the $N = 4$, $E_7 \times E_7 \times SO(16)$ obtained by the use of the basis

$$B = \{1, S, x, 2\gamma\}$$

where

$$x = \{\bar{\psi}^{1,\ldots,5}, \bar{\eta}^{1,2,3}\}$$

and

$$2\gamma = \{\bar{\psi}^{1,2,3}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,8}\}$$

and with the following choice of one-loop GGSO projection phases

$$\begin{pmatrix}
1 & S & x & 2\gamma \\
1 & 1 & -1 & -1 \\
1 & 1 & -1 & -1 \\
1 & -1 & -1 & 1 \\
1 & -1 & -1 & 1
\end{pmatrix}$$

is reduced to

$$SO(10) \times U(1)_\zeta \times U(1)_\delta \times U(1) \times SO(4)^3 \times SO(16)$$

by introducing basis vectors $b_i$ for $i = 1, 2$ corresponding to the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold twists.
4  $E_6$ From Heterotic M-Theory

A brief study of the existence of solutions with three generations and $E_6$ observable gauge group in the case of compactification on the torus-fibred Calabi-Yau over Hirzebruch surfaces with some remarks about the case where the base is taken to be the del Pezzo surfaces.

A nonperturbative vacuum state of the GUT theory on the observable sector is specified by a set of M-theory 5-branes wrapping a holomorphic 2-cycle on the three-fold. These 5-branes are then described by a 4-form cohomology class $[W]$ satisfying the anomaly-cancellation condition being Poincaré-dual to an effective cohomology class of $H_2(X,\mathbb{Z})$ expressed as

$$[W] = \sigma_* (\omega) + c(F-N) + dN$$

where $c$ and $d$ are integers and $\omega$ is a class in the base manifold $B$ and $\sigma_* (\omega)$ is the pushforward to $X$ under $\sigma$ [33].

5  The Hirzebruch Surfaces $F_r$

In this section, we consider the rules for constructing realistic, viable vacua with $E_6$ GUT symmetry where the base manifold $B$ are taken to be the Hirzebruch surfaces, $F_r$. We arrive at the following conditions modified for the $E_6$ observable gauge group [32][33]:

5.1 The Semistability Condition

The semistability condition offers a choice: either

$$\lambda \in \mathbb{Z}$$

and

$$s \text{ even}, \quad e - r \text{ even}$$

or

$$\lambda = \frac{2m - 1}{2}, \quad m \text{ even}, \quad r \text{ even}.$$ 

5.2 The Involution Conditions

The involution conditions are

$$\sum_i \kappa_i = \eta \cdot c_1(B = F_r) = 2e + 2s - rs.$$
5.3 The Effectiveness Condition

The effectiveness condition boils down to

\[ s \geq 24, \text{ and } 12r + 24 \geq e \]

with

\[ \sum_i \kappa_i^2 \leq 100 + \frac{9}{4\lambda} - 9\lambda \]

and

\[ \sum_i \kappa_i^2 \leq 4 + \frac{9}{4\lambda} - 9\lambda + \sum_i \kappa_i. \]

5.4 The Commutant Condition

The commutant condition for \( E_6 \) becomes

\[ \eta \geq 3c_1 \]

which implies that

\[ s \geq 6, \text{ and } e \geq 3r + 6. \]

5.5 The Three Family Condition

The three family condition reads

\[ -rs^2 + 3rs + 2es - 6e - 6s = \frac{6}{\lambda}. \]

Solving the three family condition for \( e \) assuming that the value of \( s \) is known leads to

\[ e(r; \lambda) = \frac{1}{2s - 6} \left( rs^2 - 3rs + 6s + \frac{6}{\lambda} \right). \]
5.6 Space of Solutions of Type A

In this class

\[ s \text{ even, } e - r \text{ even, } \lambda = \pm 1, \pm 3. \]

| \( s \) | \( e(r; \lambda) \) |
|---|---|
| 6 | \( 3r + 6 + \frac{1}{\lambda} \in \mathbb{Z} \)  
\( \lambda = \pm 1 \) |
| 8 | \( 4r + \frac{24}{5} + \frac{3}{5\lambda} \notin \mathbb{Z} \) |
| 10 | \( 5r + \frac{30}{7} + \frac{3}{7\lambda} \notin \mathbb{Z} \) |
| 12 | \( 6r + 4 + \frac{1}{3\lambda} \notin \mathbb{Z} \) |
| 14 | \( 7r + \frac{42}{11} + \frac{3}{11\lambda} \notin \mathbb{Z} \) |
| 16 | \( 8r + \frac{48}{13} + \frac{3}{13\lambda} \notin \mathbb{Z} \) |
| 18 | \( 9r + \frac{18}{5} + \frac{1}{5\lambda} \notin \mathbb{Z} \) |
| 20 | \( 10r + \frac{60}{17} + \frac{3}{17\lambda} \notin \mathbb{Z} \) |
| 22 | \( 11r + \frac{66}{19} + \frac{3}{19\lambda} \notin \mathbb{Z} \) |
| 24 | \( 12r + \frac{24}{7} + \frac{1}{7\lambda} \notin \mathbb{Z} \) |

Table 4: Solutions for \( e(r; \lambda) \) for each value of \( 6 \leq s \leq 24 \) and the corresponding appropriate choice for \( \lambda \) contd.

Clearly, the space of solutions of type A contains exactly one vacua over the
Hirzebruch surfaces for any allowed value of \( r \).

### 5.7 Space of Solutions of Type B

In this class

\[
\lambda = \pm \frac{1}{2}, \pm \frac{3}{2}.
\]

We will only consider the cases where integer solutions to the three family equation are found. These are given by

\[
s = 6, \quad e \left( r; \lambda = \pm \frac{1}{2} \right) = 3r + 6 + \frac{1}{\lambda} = 3r + 6 \pm 2.
\]

### 5.8 Explicit Expressions

For \( B = F_r \)

\[
\omega = (24 - s)S + (12r + 24 - e)E
\]

with

\[
c = c_2 + \left( \frac{1}{24} (n^3 - n) + 11 \right) c_1^2 - \frac{1}{2} \left( \lambda^2 - \frac{1}{4} \right) n\eta(\eta - nc_1) - \sum_i \kappa_i^2
\]

\[
d = c_2 + \left( \frac{1}{24} (n^3 - n) - 1 \right) c_1^2 - \frac{1}{2} \left( \lambda^2 - \frac{1}{4} \right) n\eta(\eta - nc_1) + \sum_i \kappa_i - \sum_i \kappa_i^2
\]

where

\[
c_1^2 = 8, \quad \eta^2 = -rs^2 + 2es, \quad \eta c_1 = -rs + 2e + 2s
\]

and \( c_1 \) denotes the first Chern class and \( c_2 \) denotes the second Chern class.

### 6 del Pezzo Surfaces \( dP_N \): A Word

We very briefly remark on the results. From [32] it can be seen that the first Chern class can be defined as

\[
c_1(dP_N) = \sum_{i=1}^N M_i
\]
with the three family condition being

\[-2 \sum_{i=1}^{N} m_i^2 + n \sum_{i \neq j} m_i m_j - 2n \sum_{i=1}^{N} m_i = \frac{6}{\lambda}\]

which for \( \lambda = \pm 1, \pm 3 \) will not admit any integer solutions and therefore no vacua exists.

7 Conclusion

In this paper, we looked at the \( E_6 \) and \( E_7 \) GUTs in view of the free fermionic construction of the \( 4D \) heterotic string. An illustrative example, that of the NAHE-Based LRS model of \[29,30\] was briefly reviewed where the starting point was taken to be the \( N = 4, E_7 \times E_7 \times SO(16) \) which was reduced to obtain the flipped \( SO(10) \) GUT symmetry albeit with a different accommodation of the SM representations in the \( 27 \) of \( E_6 \).

In light of \[2,3\], we also considered the existence of solutions with three generations and \( E_6 \) observable gauge group. By use of Wilson lines the \( E_6 \) GUT symmetry can be broken to \( SO(10) \times U(1) \). We found solutions of type A and solutions of type B which follow from the semistability condition, one of crucial ingredient when constructing realistic vacua contrary to \[32\] where flipped \( SU(5) \) vacua were studied and only solutions of type B were found. It was also shown that no solutions can be found in the case of the base being the del Pezzo surfaces.

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A The Gauge Enhancements

A.1 \( SU(5) \)

\[ \Delta = -z^5 [P_{10}^4 P_5 + zP_{10}^2 (8b_4 P_5 + P_{10} R) + z^2 (16b_3 b_4^2 + P_{10} Q) + \mathcal{O}(z^3)] \]

\[ P_{10} = b_5 \]
\[ P_5 = b_3^2 b_4 - b_2 b_3 b_5 + b_1 b_5^2 \]
\[ R = 4b_1 b_4 b_5 - b_3^2 - b_2^2 b_5 \]
\[
\Delta = -16b_2^3b_3^2z^7 + \left( -27b_3^4 - 8b_1^2b_2^2b_3^2 + 72b_2b_4b_3^2 \\
+ 4b_1b_2(9b_3^2 + 4b_2b_4)b_3 + 16b_2^2(b_4^2 - 4b_2b_6) \right)z^8
\]

\[
+ \mathcal{O}(z^9)
\]

\[
= z^7 \left[ -16b_2^3b_3^2 + \left( -27b_3^4 - 8b_1^2b_2^2b_3^2 + 72b_2b_4b_3^2 \\
+ 4b_1b_2(9b_3^2 + 4b_2b_4)b_3 + 16b_2^2(b_4^2 - 4b_2b_6) \right) \\
+ \mathcal{O}(z^2) \right]
\]
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