The Natural Characteristics and Forced Response of O-shaped Plate Under Isotropic and Orthotropic Materials

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Abstract. In this paper, the O-shaped plate with different materials are used to analyze their natural frequencies at different boundaries and different plate thicknesses and the forced response under the action of point forces. By the improved Fourier series method, the discontinuity of coupling system at all boundaries in the overall solution domain is overcome. Through the energy principle and the Rayleigh-Ritz technology, the matrix equation of the system governing equations can be obtained. In addition, based on various materials, boundary conditions and thicknesses, some new results have been demonstrated, which may become the foundation for the future research.

1. Introduction

In recent years, O-shaped plate structure is widely used in real life. It is of great significance to study the natural characteristics and forced response of the O-shaped plate structure.

Some literatures about the coupled plate structures are listed here. Shen et al. [1] studied combination of rectangular thin isotropic plates. Li [2] used a scaling approach to analyzing the line-coupled plates at high frequencies. Tian et al. [3] introduced the vibration response of an L-shaped plate under a point force or a moment excitation. Du et al. [4] applied an analytical method to determine the vibrations of two plates which are generally supported along the boundary edges, and elastically coupled together at an arbitrary angle. Boisson et al. [5] discussed the relationship between factors and the vibration energy transmission characteristics. The above articles investigate different tools to consider the vibration characteristics. But the existing theoretical analysis of O-shaped coupled plate considers the bending moment between the plates, while ignoring the lateral shear, longitudinal and in-plane shearing effects in the plate. Studies have shown large deviations occur in the displacement response of the coupled plate structure using the modeling technique that does not include the in-plane vibration component. Therefore, it is very important to establish a more general vibration model.
In view of the above defects, the displacement admissible function is optimized based on the Fourier series method. At the end, the natural characteristics and forced response of the O-shaped coupling plate are discussed.

2. Theoretical Formulations

2.1. Description of the Model

As shown in figure 1, the O-shaped plate system is established to analyze the natural characteristics and forced response. The geometrical dimensions of the O-shaped plate system and the coordinate system used in this paper have been shown in figure 1. In this paper, the plate dimensions are \( a_1 = a_2 = a_3 = a_4 = b = 1 \) m. In table 1, we list four types of material properties.

![Figure 1. Analytical model.](image)

### Table 1. Four different types of material properties.

| Material | I | II | III | IV |
|----------|---|----|-----|----|
| Steel    | 216 | 71 | 185 | 39 |
| Aluminum | 216 | 71 | 10.5 | 8.4 |
| Graphite/epoxy | 7.3 | 4.2 |
| Glass/epoxy | 0.3 | 0.3 | 0.28 | 0.26 |
| \( \rho \) [kg/m\(^3\)] | 7800 | 2700 | 1600 | 1600 |

2.2. Energy Expressions of the O-shaped Plate System

In this paper, the energy equation will be used to describe the motion equation of the O-shaped plate system. The Lagrange equations of the system shown in figure 1 can be expressed as:

\[
L_{\text{plane}} = (U_{\text{bending}} + U_{\text{in-plane}} + U_{\text{coupling}}) - (T_{\text{bending}} + T_{\text{in-plane}}) - W_{\text{plane}}
\]

where \( U_{\text{bending}} \) (\( j = 1,2,3,4 \)) and \( U_{\text{in-plane}} \) represent respectively the total potential energy associated with bending vibration and the total potential energy associated with in-plane vibration in the \( j \)’th plate; \( T_{\text{bending}} \) and \( T_{\text{in-plane}} \) represent respectively the correspondence of the \( j \)’th plate kinetic energy of bending and in-plane vibration; \( U_{\text{coupling}} \) indicates the elastic potential energy stored in the coupling boundary spring. Take plate 1 as example, potential energy and kinetic energy can be expressed as:

\[
U_{\text{bending}} = \frac{D_1}{2} \int_{a}^{b} \left[ \left( \frac{\partial^2 u_1}{\partial x^2} \right)^2 + \left( \frac{\partial^2 u_1}{\partial y^2} \right)^2 \right] + 2(1-\mu) \left( \frac{\partial^2 w_1}{\partial x \partial y} \right)^2 \, dx \, dy + \frac{1}{2} \int_{a}^{b} \left[ k_{n_{11}} w_1^2 + k_{m_{11}} \left( \frac{\partial w_1}{\partial x} \right)^2 \right] \, dy
\]

\[
U_{\text{in-plane}} = \frac{G}{2} \int_{a}^{b} \int_{a}^{b} \left[ \left( \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} \right)^2 + \left( \frac{\partial u_1}{\partial y} - \frac{\partial v_1}{\partial x} \right)^2 \right] - 2(1-\mu) \left( \frac{\partial u_1}{\partial x} \frac{\partial v_1}{\partial y} \right)^2 \, dx \, dy + \frac{1}{2} \int_{a}^{b} \left[ k_{m_{11}} u_1^2 + k_{n_{11}} v_1^2 \right] \, dy
\]
\[ U_{\text{coupling}12} = \frac{1}{2} \int_0^b \left( K_{c12} \left( \frac{\partial w_1}{\partial x_1} \right)^2 + K_{c12} \left( \frac{\partial w_2}{\partial x_2} \right)^2 \right) \, dy \]
\[ + k_{c12} \left( u_1 \left| \gamma_{s-0} - u_2 \right| \gamma_{s-0} \cos \theta - w_2 \left| \gamma_{s-0} \right| \sin \theta \right)^2 \right) \, dy \]
\[ U_{\text{coupling}11} = \frac{1}{2} \int_0^b \left( K_{c41} \left( \frac{\partial w_1}{\partial x_1} \right)^2 + K_{c41} \left( \frac{\partial w_2}{\partial x_2} \right)^2 \right) \, dy \]
\[ + k_{c41} \left( u_1 \left| \gamma_{s-0} - u_2 \right| \gamma_{s-0} \cos \theta - w_2 \left| \gamma_{s-0} \right| \sin \theta \right)^2 \right) \, dy \]

where \( k_{b10}, k_{b11}, k_{b10}, k_{b11}, k_{c10}, k_{c11}, k_{c10}, k_{c11}, k_{c1}, k_{c2} \) and \( k_{c3} \) are the linear springs and \( K_{b10}, K_{b11}, K_{b11}, K_{c1} \) and \( K_{c} \) are the rotational springs. \( \rho \) and \( \mu \) represent mass density and Poisson's ratio. \( \omega \) is the circle frequency. \( D \) and \( G \) represent the bending stiffness and tensile stiffness of plate structure. \( \theta \) is the coupling angle between the two plates.

\[ T_{\text{bending}} + T_{\text{in-plane}} = \frac{1}{2} K_p h_1 \omega^2 \int_0^b \left( w_1^2 + u_1^2 + v_1^2 \right) dx dy \]

where \( K_p, M_p \) and \( F \) are the stiffness, mass and force matrices of the plate. \( \Gamma \) is the unknown Fourier coefficients vectors of the plate displacement.

3. Numerical Calculation and Analysis

3.1. Study on the Convergence Analysis

Because the Fourier series method is applied to the O-shaped plate system, it is necessary to study its convergence and accuracy. In table 2, the material was selected as material I, and the thickness of the plate was 8 mm. The boundary conditions of table 2 are that both the coupled edge and the uncoupled edge are fixed.
3.2. The Natural Characteristics of the O-shaped Plate System

In order to deepen the intuitive understanding of the vibration characteristics of O-shaped plate system, the displacement mode shapes of the O-shaped plate are given in figure 2. The material and dimensions are the same as those in table 2. The boundary condition of the coupling edge of coupling plate is the fixed support, and the remaining boundary conditions are free, simple and fixed, respectively, which are represented by the letters a, b, and c in figure 2. From the data given in the figure 2, we can know that the results of this paper are consistent with the results of finite element simulations. The more boundary conditions that are fixed, the higher the natural frequency of the O-shaped plate system. From the mode shapes, we can directly see the displacement distribution of the O-shaped plate at the natural frequencies. The mode shapes provide the basis for the analysis of the vibration characteristics of the structural system.

![Mode shapes](image1)

![Mode shapes](image2)

Figure 2. The mode shapes and natural frequency \( f \) [Hz] of the O-shaped plate system.

As shown in figure 3, we analyze effects of different thickness ratios. The A, B, C and D in the serial number represents plate thickness of 1, 2, 4, 8mm, respectively. The material, boundary condition and dimensions are the same as those in table 2. It can be concluded from the table that when the thickness of the four plates is the same, the natural frequency increases as the thickness of the plate increases. When the thicknesses of the four plates are not the same, as the thickness of the plate increases, the natural frequency increases but the increase at low frequencies is not very noticeable.
Figure 3. The natural frequency $f$ [Hz] of the O-shaped plate system under different plate thickness.

3.3. Response of the O-shaped Plate System
This section focuses on the influence of forces on the response of the O-shaped plate system. The geometric dimensions are the same as in figure 1. The plates are subjected to fixed boundary conditions. The point forces are applied on every plate at $(0.5,0.5)$ whose magnitude is $F=1N$. Figure 4 has shown the displacement response of O-shaped plate under different materials in the frequency range of 0-500 Hz. The first and third curves are the result of the present method. the second and fourth curves are the results of FEM. The results obtained by the present method and FEM match very well, which proves the correctness of the proposed method in predicting the vibration response of the O-shaped plate system. Form the response problem of the plate, for isotropic materials, it can be seen from the response curve that when the elastic modulus of the plate material and the density are smaller, the displacement of the plate is rather large. For orthotropic materials, the displacement of plates is different due to the different natural frequencies.

Figure 4. Responses of O-shaped plate system under different material and excitation of point forces.

4. Summary
In this paper, Fourier series method is applied to study the characteristics of O-shaped plate system. From the results of this paper, it can be seen the method of this paper is correct. The natural frequency will increase as the increase of the spring stiffness value. For the response problem of the isotropic plate, it can be seen from the response curve that when the elastic modulus of the plate material and the density are smaller, the displacement of the plate is rather large. For orthotropic materials, the displacement of the plates is different due to the different natural frequencies.
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