Squark and slepton masses as probes of supersymmetric $SO(10)$ unification

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Abstract

We carry out an analysis of the non-universal supersymmetry breaking scalar masses arising in $SO(10)$ supersymmetric unification. By considering patterns of squark and slepton masses, we derive a set of sum rules for the sfermion masses which are independent of the manner in which $SO(10)$ breaks to the Standard Model gauge group via its $SU(5)$ subgroups. The phenomenology arising from such non-universality is unaffected by the symmetry breaking pattern, so long as the breaking occurs via any of the $SU(5)$ subgroups of the $SO(10)$ group.

Keywords: Supersymmetry, unification, renormalization group analysis, sum rules

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I. INTRODUCTION

Grand unification [1] of the standard model gauge group into a simple group with the matter and Higgs particles transforming under irreducible representations of this simple group is an attractive framework for pursuing physics beyond standard model (SM). On the other hand supersymmetry [2] is at present the only framework in which the Higgs sector of the Standard Model (SM), so crucial for its internal consistency, is natural. A much favored implementation of the idea of low energy supersymmetry (SUSY) is the Minimal Supersymmetric Standard Model (MSSM) [3], which is obtained by doubling the number of states of the SM, and introducing a second Higgs doublet (with the opposite hypercharge of the SM Higgs doublet) to generate masses for all the fermions and to cancel triangle gauge anomalies. The MSSM leads to a successful prediction [4] for the ratios of the three gauge couplings, with a unification scale of $M_G \simeq 10^{16}$ GeV and supersymmetric thresholds near the electroweak scale ($M_S \simeq 1$ TeV). The possibility of such a unification scale in the supersymmetric context would naturally suggest the framework of supersymmetric grand unification at the scale $M_G$. Furthermore, the presence of supersymmetry would also make the hierarchy between the disparate scales associated with the grand unification ($M_G$), and the weak scale $M_W \sim 100$ GeV technically natural. However, despite many years of dedicated experimental work, no direct evidence has been found to vindicate the idea of unification, especially its most heralded prediction of proton decay. All the present experimental data is, however, consistent with the idea of low energy supersymmetry with a grand unified scale being around $10^{16}$ GeV. It is therefore important to keep an open mind toward all possibilities, conventional and unconventional, in our search for unification of strong and electroweak interactions, since there is going to be a concerted effort to search for collider signatures of such possibilities at the next generation of experiments [5].

At the theoretical level, the gauge group $SU(3) \times SU(2) \times U(1)$ remains a completely unexplained feature of the standard model. However, it can be elegantly unified into simple groups like $SU(5)$ [6] or $SO(10)$ [7]. The $SO(10)$ group is particularly appealing, since it has a 16 dimensional representation which is large enough to accomodate an entire generation of standard model fermions, and can further accomodate a right-handed neutrino as well. The $SO(10)$ framework can readily be extended to include supersymmetry. Furthermore, a complex 10 dimensional representation of $SO(10)$ can be employed to accomodate the two Higgs doublets of the low energy minimal supersymmetric standard model.

Unlike the case of $SU(5)$, the rank of the group $SO(10)$ exceeds that of the standard model by 1. The breaking of the additional $U(1)$ factor in $SO(10)$ group leads to characteristic non-universal contributions to the otherwise universal soft SUSY breaking scalar masses of the minimal supersymmetric models. These additional corrections to the soft scalar masses are usually referred to as $D$--term contributions, and lead to non-universality for the soft scalar masses. In general $D$--term contributions to the SUSY breaking soft scalar masses arise whenever a gauge symmetry is spontaneously broken with a reduction of rank [8]. These $D$--term contributions have important phenomenological consequences at low energies as they allow one to reach certain regions of parameter space which are not otherwise accessible with universal boundary conditions [9–11]. Non-universality, and in particular the $D$--term contributions, may have a dramatic impact on the sum rules [12] satisfied by the squark and slepton masses. Such effects are likely to help distinguish between
different scenarios for breaking of grand unified symmetry at high energies [13–15].

In this work we systematically consider the $D-$ term non-universality that is generated in $SO(10)$ unification. This is the simplest of grand unification models where a $D-$ term non-universality is inevitable. Indeed, since $SO(10)$ contains $SU(5) \times U(1)$ as one its “natural” subgroups, one common route for breaking $SO(10)$ is via $SO(10) \rightarrow SU(5) \times U(1)_Z$, with $SU(5)$ subsequently breaking via $SU(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_X$. However, in the breaking of $SO(10)$ via $SU(5)$, there exist two different ways of embedding $SU(5)$ into $SU(10)$. In the “conventional” embedding [7] via $SU(5)$, the hypercharge generator $Y$ is identified with the generator $X$ of $U(1)_X$. On the other hand, in the “flipped” [16–18] embedding the hypercharge generator is identified with a linear combination of the generators $X$ and $Z$. We shall study in detail the consequences of these two different embeddings of the SM gauge group in $SO(10)$. We should mention here that apart from the “natural” subgroup $SU(5) \times U(1)$, the group $SO(10)$ also has “natural” subgroup $SO(6) \times SO(4)$. Since $SO(6)$ is isomorphic to $SU(4)$ and $SO(4)$ is isomorphic to $SU(2) \times SU(2)$, $SO(10)$ contains the group $SU(4) \times SU(2) \times SU(2)$. We shall not consider this pattern of breaking of $SO(10)$ in this paper.

The plan of this paper is as follows. In Sec. II, we consider in detail the the $D-$term non-universality in supersymmetric $SO(10)$ unification. We shall review the embedding of the standard model gauge group in the $SO(10)$ group. We shall identify the hypercharge generator $Y$ for conventional and flipped embeddings, and the orthogonal generator $Y^\perp$ whose eigenvalues determines the $D-$ term contributions at the unification scale. In Sec. III, we write down the renormalization group equations for the sfermion masses and their solutions, and derive sum rules for these masses in generality for the embeddings of the SM gauge group in the $SO(10)$ group. For the conventional embedding of $SO(10)$, it turns out that our sum rules are same as those that were discovered for $SU(5)$ unification with non-universal masses [15]. We trace the origin of this to the fact that the $D$-term non-universality in the conventional embedding of $SO(10)$ arises in such a manner that it is degenerate within the $10$ and $5^*$ representations of $SU(5)$. On the other hand we find the result that the sparticle sum rules for flipped embedding, where the $D-$ term non-universality is not expected to have a simple pattern, are identical to the case of conventional embedding. By considering the nature of the embedding of the conventional and flipped scenarios is some detail we establish why this is the case. A further surprising aspect we find is that the individual masses entering the sum rules also do not depend on the manner in which the SM is embedded into $SO(10)$.

In Sec. IV, we present results for the sparticle spectrum based on numerical studies and our conclusions. We note that since $D$ term contributions to the scalar masses at the unification scale [10] has received considerable attention, our results show that it is immaterial whether the embedding is the conventional or the flipped one. One input we have employed is the requirement that the fine tuning problem inherent in the embedding of MSSM in $SO(10)$ [19] be alleviated, in both the conventional and flipped scenarios, by the $D$ term contribution, thereby enabling us to fix the sign of this contribution. To our knowledge, this is the first time that this has been pointed out in the context of flipped unification. Noting that the fine tuning problem is further alleviated by the effects of the right-handed neutrino Yukawa coupling to the renormalization group flow from the unification scale $M_G$ to a scale $M_R \sim 10^{14.5}$ GeV, a scale inspired by solar neutrino physics [20], it is possible to
render supersymmetric $SO(10)$ unification into a technically natural and viable candidate
for a complete theory, with unique signatures.

II. CONVENTIONAL AND FLIPPED EMBEDDING OF THE SM IN $SO(10)$

We begin by noting a crucial difference between the $SO(10)$ GUT model and the $SU(5)$
GUT model. Whereas the embedding of the SM gauge group into a unified gauge group
is unique for $SU(5)$, there are two different ways in which the hypercharge group can be
embedded in $SO(10)$. These different embeddings may lead to different predictions for the
low energy physics.

In $SO(10)$ grand unification, all the matter particles of one family of the Standard
Model (SM) together with a right handed neutrino belong to the spinor representation $16$. Each such spinor representation $16$ can be decomposed under the maximal subgroup
$SO(10) \supset SU(5) \times U(1)_Z$ as

\[
16 = 5^*_{+3} + 10_{+1} + 1_{-5},
\]

\[
10 = 5_{+2} + 5^*_{-2}.
\]

Furthermore under $SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_X$, we have the decomposition

\[
5 = (3, 1)_{-2} + (1, 2)_{3},
\]

\[
5^* = (3^*, 1)_{2} + (1, 2)_{-3},
\]

\[
10 = (3, 2)_{1} + (3^*, 1)_{-4} + (1, 1)_{6},
\]

\[
1 = (1, 1)_{0}.
\]

We note that $U(1)_X$, which is the subgroup of $SU(5)$, is not identical with the $U(1)_Y$ of the
SM at this stage. Note also that each $16$ includes two pairs of $(3^*, 1)$ and $(1, 1)$.

In order to identify the hypercharge group, we consider the decomposition $SO(10) \supset
SU(5) \times U(1)_Z \supset SU(3)_C \times SU(2)_L \times U(1)_X \times U(1)_Z$. Therefore, the hypercharge $U(1)_Y$
must be a linear combination of $U(1)_X$ and $U(1)_Z$, i.e. $U(1)_Y \subset U(1)_X \times U(1)_Z$. Thus,
there are two ways to define the hypercharge generator of the SM:

1. $Y = X$,

2. $Y = -\frac{1}{5}(X + 6Z),$

upto an overall normalization factor. The first case corresponds to the the Georgi-Glashow
model [6], whereas the second identification of hypercharge corresponds to the flipped
case [16,17].

In the first case the $U(1)$ generator of $SO(10)$ that is orthogonal to $Y$ and the diagonal
generators of $SU(3)_C$ and $SU(2)_L$ is

\[
Y^\perp = -Z,
\]

(4)
whereas in the flipped case we have for the orthogonal generator\(^1\)

\[
Y^\perp = \frac{-4X + Z}{5},
\]  

We will comment on the normalization in the following. After a suitable identification of the fields lying in the relevant representations of \(SO(10)\), the effect of \(SO(10)\) breaking at the unification scale leads to \(D\)-term non-universality, which is computed in terms of the eigenvalues of the operator \(Y^\perp\) on the fields which will discuss in the next section.

**III. SOLUTIONS OF RENORMALIZATION GROUP EQUATIONS AND SUM RULES**

For the squarks and sleptons of the first and second family(light generations), the renormalization group (RG) equations for the soft scalar masses are given by

\[
16\pi^2 \frac{dm_{Q_L}^2}{dt} = -\frac{32}{3} g_3^2 M_3^2 - 6 g_2^2 M_2^2 - \frac{2}{15} g_1^2 M_1^2 + \frac{1}{5} g_1^2 S,
\]  

\[
16\pi^2 \frac{dm_{u_R}^2}{dt} = -\frac{32}{3} g_3^2 M_3^2 - \frac{32}{15} g_1^2 M_1^2 - \frac{4}{5} g_1^2 S,
\]  

\[
16\pi^2 \frac{dm_{d_R}^2}{dt} = -\frac{32}{3} g_3^2 M_3^2 - \frac{8}{15} g_1^2 M_1^2 + \frac{2}{5} g_1^2 S,
\]  

\[
16\pi^2 \frac{dm_{L_L}^2}{dt} = -6 g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 - \frac{3}{5} g_1^2 S,
\]  

\[
16\pi^2 \frac{dm_{e_R}^2}{dt} = -\frac{24}{5} g_1^2 M_1^2 + \frac{6}{5} g_1^2 S,
\]

where \(t \equiv \ln(Q/Q_0)\), with \(Q_0\) being some initial large scale; \(M_{3,2,1}\) are the running gaugino masses, \(g_{3,2,1}\) are the usual gauge couplings associated with the SM gauge group, and

\[
S \equiv \text{Tr}(Y m^2) = m_{H_u}^2 - m_{H_d}^2 + \sum_{\text{families}} (m_{Q_L}^2 - 2 m_{u_R}^2 + m_{d_R}^2 - m_{L_L}^2 + m_{e_R}^2).
\]  

The \(U(1)_Y\) gauge coupling \(g_1\) (and \(\alpha_1\)) is taken to be in a GUT normalization throughout this paper. The quantity \(S\) evolves according to

\[
\frac{dS}{dt} = \frac{66 \alpha_1}{5} \frac{\alpha_1}{4\pi} S
\]

which has the solution

\[
S(t) = S(t_G) \left( \frac{\alpha_1(t)}{\alpha_1(t_G)} \right).
\]

\(^1\)In order to determine this, one must work with a set of normalized generators. The explicit normalization factors are adapted from the expressions given in ref. [17].
We note that if $S = 0$ at the initial scale, which would be the case if all the soft sfermion and Higgs masses are same, then the RG evolution will maintain it to be zero at all scales.

The solution for the renormalization group equations (6)–(10) can then be written as

\begin{align*}
m_{u_L}^2 &= m_{Q_L}(t_G) + C_3 + C_2 + \frac{1}{36} C_1 + \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) M_Z^2 \cos(2\beta) + \frac{1}{5} K, \\
m_{d_L}^2 &= m_{Q_L}(t_G) + C_3 + C_2 + \frac{1}{36} C_1 + \left(-\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \right) M_Z^2 \cos(2\beta) + \frac{1}{5} K, \\
m_{u_R}^2 &= m_{Q_R}(t_G) + C_3 + C_2 + \frac{4}{3} C_1 + \frac{2}{3} \sin^2 \theta_W M_Z^2 \cos(2\beta) - \frac{4}{5} K, \\
m_{d_R}^2 &= m_{Q_R}(t_G) + C_3 + \frac{1}{9} C_1 - \frac{1}{3} \sin^2 \theta_W M_Z^2 \cos(2\beta) + \frac{2}{5} K, \\
m_{e_L}^2 &= m_{L_L}(t_G) + C_2 + \frac{1}{4} C_1 + \left(\frac{1}{2} + \sin^2 \theta_W \right) M_Z^2 \cos(2\beta) - \frac{3}{5} K, \\
m_{e_R}^2 &= m_{L_L}(t_G) + C_2 + \frac{1}{4} C_1 + \frac{1}{2} M_Z^2 \cos(2\beta) - \frac{3}{5} K, \\
m_{\tilde{e}_R}^2 &= m_{\tilde{e}_R}(t_G) + C_1 - \sin^2 \theta_W M_Z^2 \cos(2\beta) + \frac{6}{5} K,
\end{align*}

where $C_1$, $C_2$ and $C_3$ are given by

\begin{align}
C_i(t) &= \frac{a_i}{2\pi^2} \int_t^{t_G} g_i(t)^2 M_i(t)^2, \quad i = 1, 2, 3, \\
a_1 &= \frac{3}{5}, \quad a_2 = \frac{3}{4}, \quad a_3 = \frac{4}{5},
\end{align}

and

\begin{align}
K &= \frac{1}{16\pi^2} \int_t^{t_G} g_1^2(t) S(t) \, dt = \frac{1}{2a_1} S(t) \left[ 1 - \frac{a_1(t_G)}{a_1(t)} \right],
\end{align}

is the contribution of the non-universality parameter $S$ to the sfermion masses, and $b_1 = -33/5$.

We now proceed to determine the combinations of sfermion masses that would satisfy sum rules which are independent of the scalar masses at $t_G$, and the $D$– term contribution. We first consider the conventional case. To this end, we note that the right hand side of the equations given above that determine the sfermion masses contain a term proportional to $Y_i K$, which is proportional to $Y_i(\text{Tr} Y m^2)$, and the index $i$ runs over all the sfermions, $i = \tilde{Q}_L, \tilde{L}_L, \tilde{u}_R, \tilde{d}_R, \tilde{e}_R$. We are interested in finding those combinations of squared sfermion masses where this contribution proportional to $K$ vanishes. Defining this combination as $\sum_i \kappa_i m_i^2$, where the constants $\kappa_i$ are to be determined, and we have the condition

\begin{equation}
\sum_i \kappa_i Y_i = 0.
\end{equation}

We now recall the boundary condition at the GUT scale which reads $m_0^2 = m_0^2 + Y_i^t g_i^2 D$, where $m_0$ denotes a generic universal soft mass parameter for the sfermion. For the combination of squark and slepton masses where no unknown parameters enter, we require that
the universal mass term contribution as well as the D term contribution vanish. These conditions translate into the equations

\[ \sum_i \kappa_i = 0, \]  
\[ \sum_i \kappa_i Y_i^\perp = 0. \]  

We can solve these three constraint equations for the five mass parameters, and in general there is a family of solutions. We arbitrarily fix the coefficient of the right handed selectron in the sum rules to be \(-1\), and ask for integer values for the parameter \(\kappa_{\tilde{Q}}\) for the squark doublet. We then arrive at the following two linearly independent sum rules:

\[ 2m_{\tilde{Q}}^2 - m_{\tilde{u}_R}^2 - m_{\tilde{e}_R}^2 = (C_3 + 2C_2 - \frac{25}{18}C_1), \]  
\[ m_{\tilde{Q}}^2 + m_{\tilde{d}_R}^2 - m_{\tilde{e}_R}^2 - m_L^2 = (2C_3 - \frac{10}{9}C_1), \]  

where we have used the notation

\[ m_{\tilde{Q}}^2 = \frac{1}{2}(m_{\tilde{u}_L}^2 + m_{\tilde{d}_L}^2), \quad m_L^2 = \frac{1}{2}(m_{\tilde{e}_L}^2 + m_{\tilde{\nu}_L}^2). \]

The essential point is that we have for the conventional embedding \(Y = X\) and \(Y^\perp = -Z\), and have solved the constraint equations (24) and (26). Furthermore, the solutions we obtain for flipped embedding is exactly the same. This follows from the fact that the flipped embedding is a special case of the solutions of the constraint equations for the general case \(Y = aX + bZ\) and \(Y^\perp = cX + dZ\), where \(a, b, c, d\) are arbitrary constants. This is not entirely unexpected, since in each of the \(SU(5)\) symmetry breaking chains the \(D\)-term contribution is the same, as each depends only on the gauge contribution. This is one of the main results of this paper. It explicitly demonstrates that despite the possibility that the model loses its uniqueness in the choice of embedding, the gauge symmetry guarantees that certain relations are preserved.

The two sum rules expressed above, which are valid only when the symmetry breaking takes place via \(SU(5)\) or flipped \(SU(5)\), denote two linearly independent sum rules with integer coefficients. The particular choice we have presented is similar to the one in [15]. We have chosen to present our sum rules in this manner in order to bring out this feature. The reason for our ability to reproduce those sum rules is that the generic non-universality that was considered in [15] for the masses of the \(10\) and \(5^*\) has the same structure as is generated by \(D\)-terms in \(SO(10)\) unification in the conventional case, where the non-universality is now explicitly given by considering the eigenvalues of the corresponding \(Y^\perp:\)

\[ m_{\tilde{Q}_L}(t_G) = m_{\tilde{u}_R}(t_G) = m_{\tilde{e}_R}(t_G) = m_{16}^2 + g_{10}^2D, \]
\[ m_{\tilde{L}_L}(t_G) = m_{\tilde{d}_R}(t_G) = m_{16}^2 - 3g_{10}^2D, \]
\[ m_{H_u}(t_G) = m_{10}^2 - 2g_{10}^2D, \]
\[ m_{H_d}(t_G) = m_{10}^2 + 2g_{10}^2D, \]  

at the \(SO(10)\) unification scale \(M_G\). Here \(m_{16}\) and \(m_{10}\) are the common soft scalar masses, corresponding to the \(16\) and \(10\) dimensional representations, respectively of \(SO(10)\), at the
We note here that in the breaking of $SO(10)$ the rank is reduced by one, and hence the $D$-term contribution to the soft masses is expressed by a single parameter $D$.

We note from the above that $S(t_G) = -4g_{10}^2D$. The solution for $K$ is obtained by eliminating $C_1, C_2, C_3, m_{16}^2$, and $m_{10}^2$ from the sfermion mass equations. We get for the conventional embedding the result [15]:

$$K = \frac{1}{4}(m_Q^2 - 2m_U^2 + m_D^2 + m_E^2 - m_L^2 + \frac{10}{3}\sin^2\theta_W M_Z^2 \cos 2\beta).$$  \hspace{1cm} (33)

We now consider the eigenvalues corresponding to $Y_\perp$ for the flipped embedding. We find that the result is identical to that for the conventional embedding, as may be explicitly checked. Indeed, the normalization for $Y_\perp$ is chosen so as to obtain this result. The conclusion we draw is that the gauge symmetry of $SO(10)$ (i) protects the sum rules irrespective of the embedding; (ii) the explicit $D$-term contributions to the sfermion masses for the two embeddings are in fact identical. We note that (ii) implies (i), but not the other way round. The stronger result (ii) is of significance to the phenomenology of $SO(10)$ models with $D$-term corrections, which is the subject of the next section.

**IV. SPARTICLE SPECTRUM AND CONCLUSIONS**

In order to study the implications of the $D$-term non-universality for the sparticle spectrum, we have carried out numerical integration of the RG equations. We integrate the system of coupled renormalization group equations of the gauge couplings, Yukawa couplings, the gaugino masses, the scalar mass squared parameters and the soft trilinear couplings of the MSSM. We employ well-known results for the unification scale, the value of the unified gauge coupling constant and the unified Yukawa coupling for the heaviest generation. We present the results of numerical analysis that take into account the effects of the $D$-term non-universality on the boundary conditions for scalar mass squared parameters in the conventional and flipped scenarios. For all other parameters we assume universal boundary conditions. In other words, starting with values of the common gaugino mass ($M_{1/2}$), the scalar masses at the unification scale (parameterized by $m_0 = m_{16} = m_{10}$ and $g_{10}^2D$), trilinear couplings ($A$), and with the Yukawa couplings having a unified value $h$ at the unification scale, and $a_G$, we integrate the set of coupled differential equations down to the effective supersymmetry scale of $\sim 1$ TeV. Of the two minimization conditions for the Higgs potential, we recall that one can be written as

$$\frac{\mu_1^2 - \mu_2^2 \tan^2 \beta}{\tan^2 \beta - 1} = \frac{m_Z^2}{2}.$$  \hspace{1cm} (34)

Proceeding in the well-known fashion [21] of determining $\tan \beta$ from the accurately known value of the $\tau$-lepton mass, and inserting it into the equation above, and using the values of the Higgs mass squared parameters determined from the evolution of the parameters yields, from the relations $\mu_1^2 = m_{\tilde{H}_d}^2 + \mu^2$ and $\mu_2^2 = m_{\tilde{H}_u}^2 + \mu^2$, the parameter $\mu$.

We recall that for the universal case sufficiently large values of the common gaugino mass $M_{1/2}$ are required to ensure that the gluino is sufficiently heavy, and also fairly large values...
of $m_0 (< M_{1/2})$ are required to ensure that the neutralino is the LSP (in order to prevent the lightest slepton from becoming the LSP). An upper bound on $m_0$ ensues when we require a sufficiently large $m_A$ ($m_A = \mu_1^2 + \mu_2^2$). Keeping these features in mind, we study the effects of the $D-$term on the spectrum.

In our work we have chosen the sign of $D$ to be positive. This alleviates the problem of fine-tuning, inherent in $SO(10)$ unification, by allowing $m_{H_u}$ to evolve to values that are negative and larger in magnitude, compared to when the term is absent. (We also note here that once solar neutrino parameters are better determined, one would have to include the effects of the Yukawa neutrino coupling between $M_G$ and a scale $M_R \sim 10^{14.5}$ GeV which would also have an effect of alleviating the fine-tuning problem.) We begin with a simple illustration for choices of the parameters of the model that would guarantee radiative electroweak symmetry breaking, that the lightest neutralino is the LSP and that $m_A$ is significantly larger than $m_Z$. In Figs.1 and 2 we plot the evolution of the five sfermion mass squared parameters in a manner which is analogous to [13]. In Fig. 1(a) and 1(b) we illustrate the evolution of the mass parameters for the case with $D = 0$ for the two lightest generations and the heaviest generation, respectively. We have plotted the ratios $m_{\tilde{f}}^2/m_0^2$ vs. $t/\log(10)$, for all the sfermions of interest. We have illustrated this for a typical case of $M_{1/2} = 800$, $m_0 = 700$, $A_0 = 0$ in units of GeV and the common Yukawa coupling is taken to be $h_t = h_b = h_\tau = 2.0$. We now add a $D-$ term contribution with $\tilde{g}_{10}^2 D = 300$ GeV, which is a fairly large contribution. We illustrate this for the lighter generation (Fig. 2(a)) and for the heaviest generation (Fig. 2(b)). It is immediately apparent that for the right and left handed sleptons of the heaviest generation the renormalization group evolution compensates for the initial mass splitting at the GUT scale. As seen clearly in Fig. 2(b), the near degeneracy of the two slepton states in the absence of electroweak symmetry breaking makes the mixing between them, once $SU(2) \times U(1)$ is broken, significant. Indeed, it is important to observe the variation of the mass of the lightest slepton, since it has the tendency to become lighter than the lightest neutralino and to emerge as a candidate for the LSP, and is therefore not acceptable.

We have also plotted in Fig. 3 curves corresponding to important observables. We begin by observing the behaviour of $\mu$ which is seen to increase significantly as $D' \equiv g_{10}^2 D$ is increased. Note that the parameter $\mu$ is an important measurable quantity in the chargino spectrum. The pseudoscalar Higgs mass $m_A$ which is also plotted in the same figure is primarily responsible, apart from $\tan \beta$, for determining the properties of the entire Higgs spectrum of the model. We have not plotted the value of the mass of the lightest Higgs boson since in the regime of $m_A \gg m_Z$ and $\tan \beta \gg 1$, its value is a constant of about 120-125 GeV. It has been pointed out that one can view the rise of $m_A$ as a relaxation of fine tuning of the parameters in the model [15]. In Fig. 3 the smaller mass eigenvalue resulting from the slepton mass squared mixing matrix marked $\tilde{\tau}$ is plotted. Indeed it rises for a while until the mixing effects become pronounced and begins to fall. In particular, the value of $D' = 300$ GeV is where the lighter slepton becomes degenerate with the bino whose mass is $\sim 350$ GeV when $M_{1/2} = 800$ GeV. We have also varied the parameters of the model and find the trend to be maintained for reasonable variations.

To summarize, in this work we have considered what is perhaps the simplest example of the $D-$ term non-universality following from $SO(10)$ unification. We have obtained sum rules for sfermion masses for the conventional and flipped scenarios in a very general manner.
and the method may be easily adopted for scenarios based on other groups, e.g., $E_6$, etc. It is interesting to note that the same sum rules emerge in conventional and flipped embeddings. It turns out that the results are similar to that of non-universality of the type discussed in refs. [13,14]. The presence of $D-$ term non-universality assists in rendering the model to be technically natural.

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Figure Captions

Fig. 1 (a) Evolution of the ratio of the sfermion mass squared parameters to the universal mass squared parameter, as a function of the momentum scale, for the lightest generations. Here we have taken $D = 0$. The values of the other parameters are given in the text.

Fig. 1 (b) Evolution of the ratio of the sfermion mass squared parameters to the universal mass squared parameter, as a function of the momentum scale, for the heaviest generation. Here we have taken $D = 0$. The values of the other parameters are given in the text.

Fig. 2 (a) Evolution of the ratio of the sfermion mass squared parameters to the universal mass squared parameter, as a function of the momentum scale, for the lightest generations with $g_{\tilde{t}0}D = 300$ GeV. The values of the other parameters are given in the text.

Fig. 2 (b) Evolution of the ratio of the sfermion mass squared parameters to the universal mass squared parameter, as a function of the momentum scale, for the heaviest generation with $g_{\tilde{t}0}D = 300$ GeV. The values of the other parameters are given in the text.

Fig. 3 Values of mass parameters at the supersymmetry breaking scale, for the choice of parameters at the unification scale of $M_{1/2} = 800$, $m_0 = 700$, $A_0 = 0$, all in GeV, and $h_t = h_b = h_\tau = 2.0$, when $D'$ is varied from 0 to 400 GeV.
Fig. 2 (a)

Fig. 2 (b)
Fig. 3