How political parties adjust to fixed voter opinions

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Abstract

We propose a new version of the spatial model of voting. Platforms of five parties are evolving in a two-dimensional landscape of political issues so as to get maximal numbers of voters. For a Gaussian landscape the evolution leads to a spatially symmetric state, where the platform centers form a pentagon around the Gaussian peak. For a bimodal landscape the platforms located at different peaks get different numbers of voters.

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1 Introduction

Dynamics of public opinion is a central subject in political sciences. As votes can be described with numbers, research in this field belongs at least partially to the behavioral tradition in America or to the sociophysics in Europe. Indeed, opinion dynamics attracts attention of several authors in the Old Continent [1, 2, 3, 4, 5]. In these approaches, the process considered is that voters, convinced by other voters, change their opinions about parties. However, there has been also an opposite point of view, established in literature for 50 years [6]. According to this point, parties adopt political platforms in order to get maximal number of voters. Although it is clear that in reality both processes occur, it seems advisable to investigate the latter separately, as if opinions of voters remain constant in time. Such a search is our purpose here.

Actually, there is at least one argument that the variations of political platforms to meet the voter’s preferences are quicker than the changes of voter’s preferences. This argument is as follows: a standard voter is not economically motivated to optimize his performance. It is clear that one vote cannot change a political landscape. Then our political preferences are based rather on an identification with a given politician than on acceptance of his program. Programs are long, complicated and devious, whereas people can be qualified as fine or not in seconds [7]. As a result, many vote for candidates who look good on TV. On the contrary, politicians are strongly motivated to fight for voters. There, the difference between success and failure is equivalent to the difference between being Prime Minister and being unemployed. In this aspect, politicians can then be expected to be much smarter, better informed and quicker than voters. If this is so, the variations of platforms can be described with an assumption that the preferences of voters are constant in time. This is a sort
of adiabatic approximation. In fact, both processes occur: the platforms move and the voters change their opinion, but in many cases the characteristic time of the latter is longer. Once the platforms are established, most of voters have no choice but to vote for a platform prepared for them: workers for the left, business for the right, intellectuals for professors, young for greens etc. The coupling between well-defined platforms and clusterized groups of voters has an additional feedback formed by media: every reader finds a newspaper where things are presented according to his own opinion. To be precise, a platform should be understood as a set of issues which can serve as criteria for voters. A choice of these issues does depend on tradition and history. However, final output of a candidate appears to depend also on his/her charisma, age, sex, height and health which we are well-trained by the Darwinian evolution to evaluate.

In this perspective, the game between politicians and voters is no more equivalent to a time evolution of the statistical distribution of opinions on static issues. It is close rather to a deterministic search for herds of voters, unable to change their opinions. These herds form a political landscape in a multi-dimensional space of issues. The picture is known as spatial voting model. The deterministic character of the time evolution can however be relaxed by the incomplete knowledge on the public opinion. Indeed, much money is paid by governments to recognize the voters’ response for issues which could or could not be a basis of a winning platform. Here we adopt the approach of Kollman, Miller and Page, who simulated the evolution of platforms in a given landscape. In these works, the incomplete knowledge on the landscape was reflected by the time evolution rules, determined by the landscape only at the actual position of the platform. Here we use the same locality principle. On the other hand, we feel to continue the sociophysical tradition, asking for the probability distribution of votes. The aim of this paper is to investigate this distribution in a given landscape. Namely, we ask if there is any connection between the distribution of votes and the shape of the landscape.

In Section II the model is explained. The results are described in Section III and discussed in Section IV. Final conclusions close the text.

2 The model

In computer simulations, the incompleteness of knowledge was reflected by using one of three approaches: random search, local hill-climbing and genetic algorithm. As the results of these approaches are qualitatively the same, we feel free to use one of them, namely the hill-climbing algorithm. The model space of issues is limited here to two dimensions, $x$ and $y$. The criterion of selection of issues is that they should have a discriminative power. For example, slavery does not fulfil this criterion. On the contrary, this discriminative power cannot be too large; a woman who wants her husband to be Prime Minister cannot gather a party around this postulate. Still, a rich spectrum of possible between these two extremes.

Having the axes, one should be able to construct the landscape. Here again we encounter another eternal problem in social sciences: the scale. As it is known from the utility theory, scales do depend on the respondent, what makes the construction subjective. Various solutions of the problem can be
found in [13]. Here we intend to postulate that a landscape of an unbiased issue should be close to a Gaussian function, just by nature of statistics. By unbiased we mean that i) no abrupt changes of the opinion happened recently, ii) people are not personally engaged into an issue. If they are engaged, bimodal distributions are likely to appear [14].

Initial positions of the platforms are selected randomly, with uniform distribution. The number of platforms is arbitrary, but this choice is supported by some common sense. Parties which get small amounts of votes do not enter into parliaments in many countries. Moreover, their results in our simulation would be probably distorted by statistical errors of the order of their yields. The time evolution is governed by the principle of hill-climbing: if a party can get more votes by a shift of the position of its platform, the shift is done. We note that this algorithm was checked in [10] to produce similar results as the random-search algorithm and the genetic algorithm. The length of steps in the space of issues is arbitrary, but small with respect to a characteristic length of the landscape variation. Our algorithm is equivalent to a differential equation

\[ \frac{\delta y_i}{\delta t} = \nabla y w_i(y) \]  \hspace{1cm} (1)

where \( w = (w_1, ..., w_5) \) is the number of votes gained by \( i \)-th party at position \( y_i \) in two-dimensional space of issues. The number of votes of \( i \)-th party is calculated from its position in the space of issues,

\[ w_i = \int d^2x \rho(x) g(y_i - x)[1 - \frac{1}{N} \sum_j g(y_j - x)] \]  \hspace{1cm} (2)

where the function \( g(x) \) describes the profile of votes as dependent on the position of the platform. Here it is selected to be also Gaussian, with the width \( \sigma \) set as \( 2^{-1/2} \). The second term under the integral describes the interaction between the parties, which is repulsive; it can be more beneficial for a party to explore voters in an area where other parties are not active, even if the number of voters is somewhat smaller there. From the point of view of a physicist, the defined system is analogous to five interacting overdamped particles, looking for local equilibria in a potential minimum. The potential is the landscape with inverted sign. The resulting set of equations of motion for \( i = 1, ..., 5 \) is

\[ \frac{dy_i}{dt} = -\frac{2y_i}{1 + 2\sigma^2} I(i) + \sum_j \frac{2y_i + 4\sigma^2(y_i - y_j)}{N(1 + 4\sigma^2)} J(i, j) \]  \hspace{1cm} (3)

where \( I(i), J(i, j) \) are scalar quantities

\[ I(i) = \frac{1}{\pi(1 + 2\sigma^2)} \exp \left( -\frac{y_i^2}{1 + 2\sigma^2} \right) \]  \hspace{1cm} (4)

and

\[ J(i, j) = \frac{1}{\pi(1 + 4\sigma^2)} \exp \left( -\frac{y_i^2 + y_j^2 + 2\sigma^2(y_i - y_j)^2}{1 + 4\sigma^2} \right) \]  \hspace{1cm} (5)

and \( N = 5 \) is the number of parties. The term with \( J(i, j) \) describes the repulsion between parties \( i \) and \( j \). We keep \( J(i, i) = 0 \). Equation (3) is solved numerically. In general, Eq. 1 reduces to a differential equation, provided that
the product of functions under the integral (2) can be approximated by a linear combination of products of Gaussian functions and polynomial functions.

3 Results and discussion

First we consider the landscape which is a single Gaussian function with its maximum at the coordination centre. We performed the calculations for several sets of initial positions of the platforms in the two-dimensional space of issues. For large values of $\sigma$, the emerging result is always the same: the centers of the platforms tend to equidistant positions on a circle, formed around the peak of the Gaussian peak of the landscape. Example of the trajectories is shown in Fig. 1. Even if the initial position of one party is on the top of the peak, i.e. in the center of coordinates, this party gets down the peak and finally is placed on the circle. In stable equilibrium, the yields $w_i$ of all the parties become equal. This kind of symmetry should appear for any number of parties; we checked that it is true for $N = 2$.

In principle, it could be expected that one party, initially closest to the centre, will be able to fix its platform there before the other parties. On the contrary to this expectations, a central platform placed at the peak gets down and moves to a position equivalent to those of other platforms. The memory of the initial state is lost except the angular coordinates of the parties. In Fig. 2 we show final positions of the platforms, reached from several random initial positions.

If the width of the landscape peak $\sigma$ is small enough, it becomes worthwhile for the parties to occupy the top of the peak even if shared with platforms of other parties. We can apply the stability analysis to investigate the stability of the situation when all platforms are situated at the peak top. For two parties, the result is analytical: for $\sigma > ((1 + 2^{1/2})/2)^{1/2} \approx 1.1$, the point $y_i = 0$ is not stable. This means, that the coexistence at the top is not fruitful. For five parties, the critical value of $\sigma$ is about 1.18. However, even fairly below this value the time evolution of the platforms is very slow near the top, and the above stability is hard to be evaluated from the numerical solution.

It is clear that the obtained circular symmetry must vanish if the landscape is not symmetric. As it was recognized in [11], the ability of a platform to get an optimal position decreases with the landscape ruggedness. In particular, for a bimodal landscape it is obvious that the hill-climbing algorithm traps some platforms at a peak which is maybe more occupied and therefore less favorable.

Now we consider a landscape formed from two Gaussian peaks, placed in equal distances from the coordination centre. Their coordinates are $(0, c)$ and $(0, -c)$, and the latter is $3/2$ times higher. As chosen previously, $\sigma = 2.5$ for both peaks. In Fig. 3 we show how the final positions of five platform centers are distributed for various values of $c$. As we see, for short distance $2c$ between the peaks all parties are close to the higher peak. In this case, their yields $w_i$ are equal. As $c$ increases, we observe some kind of phase locking; a platform is located between the peaks and the repulsion between platforms leads them to fixed positions on some curve around the peak where $w_i = const$. Finally, when $c$ is large enough, the peaks can be considered as independent. In this case the peak selected by a platform does depend on the initial position of all the platforms. In principle, all partitions are possible, i.e. $(0 – 5), (1 – 4), (2 – 3)$,
Figure 1: Trajectories of platforms of five parties, starting from random initial positions. The density of points increases with time, because the velocity of platforms decreases. This reveals the direction of the trajectories, which is generally to the center. However, one of them (in the center of the lower part of the figure) changes the direction, repulsed by the others.

Figure 2: Positions of five platforms after some time, averaged over 100 random initial positions.
Figure 3: Positions of five platforms after some time, averaged over 1000 random initial positions for a) $c = 1.5$, b) $c = 3.0$, c) $c = 3.5$, d) $c = 5.5$. 
(3−2), (4−1) and (5−0). The weights of these partitions depend on the shape of the landscape, but it may be approximated by the binomial distribution. The numbers of voters $w_i$ do depend on the partition. It is best for a party to be only one occupying the maximum, even if this is the lower one.

4 Discussion

Some conclusions drawn from our results are at least not contradictory with a common experience. First, equilibrium positions of the platforms are to be in maximal possible distance. This makes an accordance between parties generically difficult, even if they are close to each other in their programs. Second, the strongest hostility can be expected between parties with neighboring platforms, because they fight for voters. Third, isolated maxima of the electoral landscape are expected to be willingly occupied even if these maxima are small. This is our contribution to an interpretation of extremist parties, which appear to be good ecological niches for some politicians. As a rule, the bosses of these parties are authoritative, as they are not forced to make compromises with neighbors - they have none. Fourth, the emerging picture is a convenient basis to investigate the response of parties for evolution of the electoral landscape. In particular, suppose that we take into account an increasing disappointment of voters with a ruling party. Their program is not executed or not fully executed, affairs disgrace their government and initial hope that their electoral victory will push the country into a prosperous future has no more support. To introduce these known facts to the spatial model, it is enough to reduce gradually the function $g$ of the ruling party, until its supremacy is lost. The effect is known as political pendulum. After several cycles, the process leads to a fragmentation of political scene, until new issues appear.

To conclude, in the spatial model of voting the positions of the political platforms is a part of the game, and it has not much to do with historical tradition of the parties. We note that this result cannot be obtained in a one-dimensional model, where the repulsion between parties prevents them to profit the same groups of voters. In a sense, our model could be applied to a problem of division of territories with herds of cattle between shepherds, or groups of buyers between companies. In all these problems, repulsion between shepherds (or companies or platforms) is a natural consequence of deficiency of resources in areas where two owners can met. Our results indicate that the positions of platforms display a kind of a collective optimization, where a supremacy of one party is unstable. Politically, the emerging system can be compared to an oligarchy, where the influence of each local ruler is taken into account by its neighbors.

We should note that there are also other politicians and parties who - for various reasons - do not try to get more votes. Obviously, their performance cannot be captured with the above description. However, it is only rarely that we can see them as winners of an election.

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References

[1] K Sznajd-Weron and J. Sznajd, Int. J. Mod. Phys. C 11, 1157 (2000).

[2] R. Hegselmann and M. Krause, Journal of Artificial Societies and Social Simulation 5, issue 3, paper 3 (2002).

[3] G. Deffuant, F. Amblard, G. Weisbuch and T. Faure, Journal of Artificial Societies and Social Simulation 5, issue 4, paper 1 (2002).

[4] D. Stauffer, Paper presented in 8-th Granada Seminar on Statistical and Computational Physics, 7-11 February 2005, Granada, Spain [physics/0503115].

[5] D. Stauffer, Journal of Artificial Societies and Social Simulation 5, issue 1, paper 4 (2002).

[6] A. Downs, An Economic Theory of Democracy, Harper and Brothers, New York 1957.

[7] D. M. Buss, Evolutionary Psychology. A New Science of Mind, Allyn and Bacon, Boston 1999.

[8] W. Riker, Liberalism against Populism, Freeman, San Francisco 1982.

[9] Reports of opinions of the author’s compatriots on the Polish Government are published each month by the Public Opinion Research Center (CBOS).

[10] K. Kollman, J. H. Miller and S. E. Page, Amer. Polit. Sci. Rev. 86 929 (1992).

[11] K. Kollman, J. H. Miller and S. E. Page, British J. Polit. Sci. 28 139 (1998).

[12] Ph. D. Straffin, Game Theory and Strategy, Math. Association of America, Washington 1983, and references cited therein.

[13] The Concise Oxford Dictionary of Sociology, Gordon Marshall (Ed.), Oxford UP 1998.

[14] K.Kulakowski, P.Gawronski and P.Gronek, Int. J. Modern Phys. C (2005), in print [physics/0501073].