Highly magnetized region in pulsar wind nebulae and origin of the Crab gamma-ray flares

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ABSTRACT

The recently discovered gamma-ray flares from the Crab nebula are generally attributed to the magnetic energy release in a highly magnetized region within the nebula. I argue that such a region naturally arises in the polar region of the inner nebula. In pulsar winds, efficient dissipation of the Poynting flux into the plasma energy occur only in the equatorial belt where the energy is predominantly transferred by alternating fields. At high latitudes, the pulsar wind remains highly magnetized therefore the termination shock in the polar region is weak and the postshock flow remains relativistic. I study the structure of this flow and show that the flow at first expands and decelerates and then it converges and accelerates. In the converging part of the flow, the kink instability triggers the magnetic dissipation. The energy release zone occurs at the base of the observed jet. A specific turbulence of relativistically shrinking magnetic loops efficiently accelerates particles so that the synchrotron emission in the hundreds MeV band, both persistent and flaring, comes from this site.

Key words: acceleration of particles – pulsars: general – supernova remnants – ISM:individual:the Crab Nebula – (magnetohydrodynamics) MHD

1 INTRODUCTION

The unexpected discovery of strong, short gamma-ray flares from the Crab nebula (Tavani et al. 2011; Abdo et al. 2011) called attention to the processes that occur in pulsar wind nebulae once again. During these events, the flux in a few hundred MeV band grows significantly and varies at a time-scale of a half of a day. The observed properties of the flares place severe limits on possible models. The short time-scale implies a very compact emission region; the high energy of emitted photons is an evidence for an extremely efficient acceleration process. The underlying physical mechanism is generally attributed (Bednarek & Idec 2011; Uzdensky et al. 2011; Cerutti et al. 2012; Clausen-Brown & Lyutikov 2012; Sturrock & Aschwanden 2012) to a rapid magnetic energy release via, e.g., reconnection, which assumes a magnetically dominated region within the nebula.

Even though the pulsar winds are highly magnetized, it is widely believed that at the termination shock, the magnetization is already extremely small. This conclusion is based on the results of spherically or axisymmetric models (Rees & Gunn 1974; Kennel & Coroniti 1984; Komissarov & Lyubarsky 2004; Del Zanna et al. 2004), which demonstrate that the magnetic hoop stress could easily distort the nebula beyond the observational limits. Begelman (1998) argued that beyond the shock, the magnetic field is kink unstable and would not necessarily pinch the flow as much as would otherwise be supposed; this conjecture is supported by numerical simulations (Mizuno et al. 2011). In the equatorial belt, the wind magnetization might not be so unreasonably small as spherically and axisymmetric models suggest.

According to the pulsar wind theory, the Poynting flux could be converted to the plasma energy only via dissipation of variable fields (see, e.g., reviews by Arons (2007) and Kirk et al. (2009)). In the equatorial belt, the pulsar magnetic field changes polarity every half of period so that a striped wind is formed (Michel 1971; Coroniti 1990). The alternating field decays either already in the wind or at the termination shock (Lyubarsky & Kirk 2001; Kirk & Skjæraasen 2003; Lyubarsky 2003b; Petri & Lyubarsky 2007; Zenitani & Hoshino 2007; Sironi & Spitkovsky 2011) therefore in the equatorial belt, a weakly magnetized plasma is injected into the nebula; it is this plasma that forms a bright X-ray torus. At high latitudes, the magnetic field does not change sign. The obliquely rotating magnetosphere excites fast magnetosonic waves in this part of the wind (Bogovalov 2001), which decay via non-linear steepening and formation of multiple shocks (Lyubarsky 2003a), but the fraction of the energy transferred by the waves is not large therefore even after the waves decay, the flow remains highly magnetized.
this paper, I study the fate of the highly magnetized plasma injected into the nebula at high latitudes.

The paper is organized as follows. In the next section, I analyz the latera distribution of the pulsar wind parameters. In sect. 3, the equations governing the high latitude flow in the nebula are derived. In sect. 4, the structure of the flow is found. In sect. 5, I shortly discuss stability of the flow and mechanisms of the magnetic energy release and particle acceleration. Conclusions are presented in sect. 6.

2 PRELIMINARY CONSIDERATIONS

The pulsar wind is highly anisotropic; one can conveniently adopt the latera distribution of the Poynting flux in the monopole wind (Michel 1973; Bogovalov 1999)

\[ F = F_0 \sin^2 \theta, \]  

where \( F \) is the energy injected per unit time and unit solid angle, \( \theta \) the polar angle. Most of the energy is transferred in the equatorial belt where the flow has a structure of the striped wind. If the pulsar inclination angle is \( \alpha \), the striped wind zone is formed at the polar angles exceeding

\[ \theta_0 = \frac{\pi}{2} - \alpha. \]  

Since the alternating field decays, a weakly magnetized plasma is injected into the nebula in the equatorial belt.

At high latitudes, \( \theta < \theta_0 \), the magnetic field does not change polarity; the obliquely rotating magnetosphere just excites fast magnetosonic waves propagating outwards (Bogovalov 2001). These waves eventually decay via non-linear steepening and formation of multiple shocks (Lyubarsky 2003a) therefore within the nebula, only the mean field survives, the energy of variable fields being converted into the plasma energy. One can calculate the final (after the waves decay) wind magnetization assuming that the wave has a sine-like form such that the magnetic structure is locally presented as

\[ B = B_0 [1 + \xi \sin(\Omega r)], \]  

where \( \Omega \) is the pulsar angular velocity, \( \xi < 1 \) the relative amplitude of the wave, \( B_0 \) the slowly varying (\( \propto 1/r \)) mean field. Throughout the paper, the speed of light is taken to be unity. In the relativistic wind, the total Poynting flux is

\[ F = \frac{\langle B^2 \rangle}{4\pi} = \frac{B_0^2}{4\pi} \left( 1 + \frac{\xi^2}{2} \right). \]  

After the waves decay, the Poynting flux decreases to

\[ F = \frac{\langle B^2 \rangle}{4\pi} = \frac{B_0^2}{4\pi}, \]  

the residual being converted into the plasma energy. The plasma magnetization, defined as the ratio of the Poynting to the plasma energy fluxes, after the wave decay is found as (one can safely neglect the initial plasma energy)

\[ \sigma = \frac{\sigma}{F} = \frac{2}{\xi^2}. \]  

One sees that after the waves decay, the wind magnetization at high latitudes, \( \theta < \theta_0 \), remains large, \( \sigma > 2 \).

The lateral dependance of the wind magnetization is very important because it leads to the observed disc-jet dichotomy in PWNe (Lyubarsky 2002): the disc is formed by the weakly magnetized plasma from the striped part of the wind whereas the high latitude flow is collimated by the magnetic hoop stress into a jet-like feature. In simulations of the PWN structure (Komissarov 

\[ \sigma = \frac{\sigma}{p} = \frac{2}{\xi^2}. \]  

Inasmuch as the energy flux in the wind decreases with latitude, the shock is elongated. In all previous works, the termination shock was assumed to be strong everywhere; then the distance from the pulsar to the shock at high latitudes is estimated as \( z = (1/2)\theta^2 a \) (Lyubarsky 2002). Now I assume that at \( \theta < \theta_0 \), the flow is highly magnetized and therefore it could be terminated only at a weak shock, which means that the postshock flow remains relativistic and radial. Then the shock should arise even closer to the pulsar in order to provide enough space for the flow to be adjusted to the external pressure.

The postshock flow could be matched to the external medium if it is causally connected. In fact the role of the termination shock is just to make the flow causally connected. It is quite possible that a single shock is unable to do the job; then a few shocks arise. For example, in weakly magnetized models, the high latitude flow passes two shocks: a (highly oblique) termination shock and then a rim shock (Komissarov 

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Let the flow with the Lorentz factor $\gamma$ subtend the angle $\theta \gg 1/\gamma$. Then one can consider the flow as being composed from magnetic loops moving along the axis and expanding. In the frame moving together with the loop upwards with the velocity $c\cos\theta$, the loop expands relativistically with the Lorentz factor $\gamma_{\text{comp}} = \theta\gamma$. The flow remains causally connected provided the expansion velocity exceeds the fast magnetosonic velocity. In high-$\sigma$ flows, the last is close to the speed of light, the corresponding Lorentz factor being $\gamma_{\text{ms}} = \sqrt{\sigma}$. Then the causality condition, $\gamma_{\text{comp}} < \gamma_{\text{ms}}$, is written as

$$\theta\gamma < \sqrt{\sigma}. \tag{8}$$

Note that the flow could remain super-magnetosonic and even a highly super-magnetosonic if the flow opening angle is small. However, if the condition (8) is satisfied, the flow could be adjusted to the outer boundary conditions (e.g., Bogovalov (1992)). Below I assume that the injected radial flow already satisfies this condition.

### 3 EQUATIONS DESCRIBING THE HIGH-LATITUDE FLOW

Let us for a while consider an axially symmetric flow. One can conveniently make the projections of this equation on the direction of the flow, $\mathbf{l}$, and on the normal to the flow lines, $\mathbf{n}$.

The transfield equation (14) to the form

$$\eta\mu v |\nabla\Psi| = \frac{R}{\sigma} \left[ \frac{\rho}{\rho}\gamma^2 v + \frac{E}{4\pi} \rho \nabla \cdot \mathbf{B} \right] = \mu(\Psi); \tag{16}$$

which yields, with account of Eqs. (10), (11) and (12),

$$\eta v = \frac{\sigma - \rho}{\rho}. \tag{17}$$

Note that the magnetization parameter $\sigma$, defined as the ratio of the Poynting to the matter energy flux, is presented as

$$\sigma = \frac{h - \rho}{\rho\gamma}. \tag{18}$$

Now making use of Eqs. (10), (11) and (12) one reduces the transfield equation (14) to the form

$$\frac{\eta v}{2\pi R} = \frac{1}{\sigma} \left( \frac{\rho}{\gamma^2 - 1} \right)^{1/3} \nabla \cdot \mathbf{B} = \nabla \cdot \mathbf{p}; \tag{19}$$

In the relativistically hot medium, $p \propto \rho^{4/3}$, $h = 4p/\rho$. With account of the continuity and Bernoulli equations, one can write

$$\frac{p}{\rho} = \left( \frac{\rho_{\text{in}} \gamma_{\text{in}}}{\rho} \frac{\mu - \rho}{\gamma_{\text{in}}} \right)^{4/3} \tag{20};$$

$$\frac{h}{\rho_{\text{in}}} = \left( \frac{\rho_{\text{in}} \gamma_{\text{in}}}{\rho} \frac{\mu - \rho}{\gamma_{\text{in}}} \right)^{1/3}; \tag{21}$$

where the index “in” is referred to the plasma parameters at the inlet of the flow.

Equations (17), (19), (20) and (21) form a complete set of equations. They should be complemented by appropriate boundary conditions. If the flow is confined by the pressure of the external medium, $p_0$, the pressure balance condition should be satisfied at the boundary:

$$\frac{1}{\sigma} \left( \frac{B^2 - E^2}{\gamma^2 - 1} + p \right) = p_0. \tag{22}$$

Making use of Eqs. (10) and (11), one writes the boundary condition in the form

$$\left\{ \frac{|\nabla\Psi|^2}{8\pi(\gamma^2 - 1)} + p \right\} = p_0. \tag{23}$$

where $\psi_0$ is the potential of the last flow line. The boundary condition at the axis of the flow is just $\Psi(r = 0) = 0$.

The integrals of motion $\mu$ and $\eta$ are determined by the structure of the pulsar wind. The flow in the wind is radial.
so that $\Psi$, $\eta$ and $\mu$ depend only on the polar angle $\theta$. Since the wind is highly relativistic, one can present the Poynting flux assuming $B = E = |\nabla \Psi|$, which yields

$$F = \frac{EB}{4\pi}D^2 = \frac{1}{4\pi} \left( \frac{\partial \Psi}{\partial \theta} \right)^2. \quad (24)$$

Here $D$ is the radial distance from the pulsar. Comparing this expression with the angular distribution of the Poynting flux in the pulsar wind (1), one finds the angular distribution of the electric potential in the wind:

$$\Psi = \sqrt{4\pi F_0 (1 - \cos \theta)}. \quad (25)$$

Within the wind, one can write Equation (17) for the energy integral as (note that the wind is cold)

$$\mu = \gamma_0 + \frac{\sin \theta \partial \Psi}{2\eta \partial \theta}, \quad (26)$$

where $\gamma_0$ is the Lorentz factor of the pulsar wind near the axis. Eliminating $\theta$ with the aid of Equation (24), one can write $\mu(\Psi)$ close to the axis in the form

$$\mu(\Psi) = \gamma_0 - \frac{\Psi}{\eta}, \quad (27)$$

which represents in fact the first two terms in the universal expansion of $\mu(\Psi)$ in small $\Psi$ (Lyubarsky 2009). The first term represents the plasma energy flux whereas the second the Poynting flux. Since the pulsar wind is Poynting dominated, the plasma term could play the role only extremely close to the axis. We are interested in the Poynting dominated domain of the wind, $\Psi \gg \eta \gamma_0$; therefore we will neglect the first term Equation (27).

In the Poynting dominated domain, $E^2/4\gamma^2 \gg \eta p$; therefore one can also neglect the pressure term in the transfield equation (17) and in the boundary condition (20). Since the flow is relativistic, one can also neglect unity as compared with $\gamma^2$ in the right-hand side and take $v = 1$ in the left-hand side of the transfield equation. Now the transfield equation takes the form

$$\frac{\Psi \nabla |\nabla \Psi|}{R} = \frac{1}{4r} \hat{n} \cdot \nabla \left( \frac{r^2 \nabla |\nabla \Psi|^2}{\gamma^2} \right). \quad (28)$$

In the same approximation, Equations (20) and (21) are reduced to

$$h = h_0 \left( \frac{\rho}{p_{in}} \right)^{1/4} = h_0 \left( \frac{r_0^2 \gamma_{in} \mu}{r^2 \gamma} \right)^{1/3}. \quad (29)$$

4 THE HIGH-LATITUDE FLOW; AXISYMMETRIC CASE

In principle, the set of equations (17), (28) and (29) could be reduced to a single equation for $\Psi$ just by expressing $\gamma$ via $\Psi$ from Equations (17) and (29) and substituting the result into Equation (28). The problem is that in Poynting dominated flows, $\gamma$ is presented in the Bernoulli equation (17) as a small difference of two large terms, which makes the obtained equation for $\Psi$ inappropriate for approximate solution. One can circumvent this difficulty (Lyubarsky 2004) noticing that $\gamma$ could be easily found from the transfield equation (28) provided the shape of the magnetic surfaces, $\Psi(r, z)$, is known. An important point is that, in this case, an extra accuracy is generally not necessary because in the transfield equation, $\gamma$ is not presented as a difference of large terms. A special care should be taken only if the flow becomes nearly radial because the curvature of the flux surfaces (the left-hand side of Equation (28)) is determined in this case by small deviations of the flow lines from the straight lines.

Let us for a while neglect corrections of the order $h\gamma/\mu$ (i.e. of the order of $1/\sigma$) to the shape of the flux line; validity of this approximation will be checked a posteriori. Then the Bernoulli Equation (17) is reduced, with account of Equation (27), to

$$r |\nabla \Psi| = 2 \Psi, \quad (30)$$

which could be considered as an equation for $\Psi$.

One can find an analytical solution to the set of equations (28) and (30) assuming that the flow subtends a small polar angle so that $z \gg r$. In this case one can take $\hat{n} \cdot \nabla = \partial/\partial r$. When looking for the shape of the magnetic surfaces, one can conveniently use the unknown function $r(\Psi, z)$ instead of $\Psi(r, z)$. Then, e.g.,

$$E = |\nabla \Psi| \approx \frac{\partial \Psi}{\partial r} = \left( \frac{\partial r}{\partial \Psi} \right)^{-1}. \quad (31)$$

In the same approximation, the curvature radius may be presented as (note that $R$ is defined to be positive for concave surfaces)

$$\frac{1}{R} = - \frac{\partial^2 r}{\partial z^2}. \quad (32)$$

Now the transfield equation (28) and the Bernoulli equation (30) are reduced to

$$- \Psi \frac{\partial^2 r}{\partial \Psi^2} = \frac{\partial}{\partial \Psi} \left( \frac{\Psi^2}{\gamma^2} \right), \quad (33)$$

$$2 \Psi \frac{\partial r}{\partial \Psi} = r, \quad (34)$$

correspondingly. With account of Equation (28) the boundary condition (20) takes the form

$$\frac{\Psi^2}{(r^2)^2 \Psi - \Psi_0^2} = 2 \pi p_0, \quad (35)$$

The general solution to Equation (34) may be presented as

$$r = R(z) \sqrt{\frac{\Psi}{\Psi_0}}, \quad (36)$$

where $R(z)$ is an arbitrary function, which is in fact the cylindrical radius of the flow. In order to find the function $R(z)$, let us substitute Eq. (36) into the left-hand side of Eq. (33) and integrate obtained equation from 0 to $\Psi$.

1 Formally speaking, one could not integrate from 0 since the solution (36) was obtained only in the Poynting dominated domain, where the first term in the full Bernoulli equation (17) is neglected. Therefore this solution becomes invalid close enough to the axis where the flow ceases to be Poynting dominated (the Poynting flux goes to zero at the axis, see Equation (27)). However, the integrands grow with $\Psi$ so that the lower boundary condition could be continued to zero. For more details, see Lyubarsky (2009).
\[ \frac{\Psi_0^2 R}{3} \frac{d^2 R}{dz^2} = -\frac{\Psi_0^2}{(\gamma^2)_{\Psi=\Psi_0}}. \]  

(37)

Making use of the boundary condition (35), one gets a simple equation for \( R \):

\[ \frac{d^2 R}{dz^2} + \beta R = 0, \]  

(38)

where

\[ \beta = \frac{6\pi p_0}{\Psi_0^2}. \]  

(39)

Equation (38) is a partial case of the governing equation that determines all the properties of highly magnetized flows (Lyubarsky 2009).

Solution to Equation (38) describes the postshock flow at high latitudes where the wind remains highly magnetized. Let the high magnetization flow subtend a small polar angle \( \theta_0 \). Then one can write, with account of Equations (7) and (25),

\[ \beta = \frac{6}{\theta_0^2 a^2}. \]  

(40)

Since the shock is weak, the flow just beyond the shock remains nearly radial so that if the flow line \( \Psi = \Psi_0 \) enters the shock at the distance \( z_0 \) from the pulsar, the solution to Equation (38) should satisfy the conditions: \( R(z_0) = z_0\theta_0; R'(z_0) = \theta_0 \). Such a solution is simply

\[ R = \frac{\theta_0}{\sqrt{\beta}} \sin \sqrt{\beta} z = \frac{\theta_0^3 a}{\sqrt{6}} \sin \frac{\sqrt{6} z}{\theta_0^3 a}. \]  

(41)

Note that the solution is independent of the position of the shock, \( z_0 \). This is because the postshock flow remains anyway radial. At the end of this section, I present an estimate for \( z_0 \).

The Lorentz factor of the flow is obtained by substituting the solution (36) and (41) into Equation (33) and integrating from zero to \( \Psi \); this yields

\[ \gamma = \left( \frac{3\Psi_0}{\Psi} \right) \left( \theta_0 \sin \frac{\sqrt{6} z}{\theta_0^3 a} \right)^{-1}. \]  

(42)

One sees that initially the flow expands and decelerates and then converges and accelerates. Substituting the obtained solutions into Equation (20), one finds

\[ h \propto \left( \sin \frac{\sqrt{6} z}{\theta_0^3 a} \right)^{-1/3}, \]  

(43)

which means that the flow is cooled when it expands and heated again when it converges.

The flow is maximally expanded at the distance

\[ z_1 = \frac{\pi}{2\sqrt{6}} \theta_0^2 a. \]  

(44)

At this distance, the radius and the Lorentz factor of the flow are

\[ R_1 = \frac{2}{\pi} \theta_0 z_1 = \frac{\theta_0^3}{\sqrt{6}} a; \quad \gamma_1 = \frac{\sqrt{3}}{\theta_0}. \]  

(45)

obtained; this solution is matched smoothly with the solution in the Poynting dominated domain.

Note that if the flow were weakly magnetized so that the termination shock were strong, the shock would arise at the distance \( z = (1/2)\theta_0^3 a \sim z_1 \) (Lyubarsky 2002), roughly equal to \( z_1 \). In a highly magnetized flow, the shock is much closer to the pulsar.

The above solution was obtained by neglecting the \( h\gamma \) term in the Bernoulli Equation (17). This means that the expression (36) for the shape of the flow lines, where \( R(z) \) satisfies the equation (38), is valid to within \( h\gamma/\mu = 1/\sigma \ll 1 \). This approximation fails if the flow lines are nearly straight because in this case, the curvature of the flow lines (the lefthand side of the transfield Equation (39)) is determined by small deviations from the straight line so that even small corrections to the flow line shape could not be neglected. In our solution, this happens near the origin of the flow, \( z \approx 0 \), and near the converging point, \( z \approx 2z_1 \).

In order to find limits of applicability of the solution, let us present the shape of the flow lines as (cf. Equation (40))

\[ r = R(r) \sqrt{\frac{\Psi}{\Psi_0}} (1 + \delta), \]  

(46)

where \( \delta(\Psi, z) \) describes a small correction to the shape of the flux surfaces due to a nonzero \( h\gamma/\mu \). Substituting this expression into the full Bernoulli Equation (17) and expanding, with account of Equation (21) and (41), in small \( \delta \), one gets

\[ h\gamma = \frac{2\Psi^2}{\eta} \frac{\partial \delta}{\partial \Psi}, \]  

(47)

which implies \( \delta \sim h\gamma/\mu = 1/\sigma \). One can neglect this correction provided the contribution to the curvature of the flow line due to this correction,

\[ \left| \frac{\partial^2 R \delta}{\partial z^2} \right| \sim \frac{R \delta}{\sigma z^2}, \]  

(48)

remains less than \( d^2 R/dz^2 \). According to equation (38), the last is just equal to \( -\beta R \) therefore the condition that one can neglect the corrections of the order of \( 1/\sigma \) to the shape of the flow line, which is in fact the validity condition for the solution described by Equations (36) and (41), is written as

\[ z \gg \frac{1}{\sqrt{\sigma} \beta} = \frac{\theta_0^3}{\sqrt{6} \sigma} a = \frac{2}{\pi \sqrt{\sigma}} z_1. \]  

(49)

Taking into account that according to Equation (42), the Lorentz factor of the flow at small \( z \) is presented as \( \gamma = \theta_0 \gamma / (\sqrt{2} z) \), one sees that this condition may be written as \( \theta_0 \gamma \ll \sqrt{\sigma} \), which is in fact the condition (3) of the causal connection of the flow. Therefore our solution is valid only in the domain of the causal connection of the flow.

The full structure of the flow could be formally obtained by matching the above solution with the free pulsar wind by inserting shocks and making use of the shock jump conditions. I do not address here the entire problem but just assume that shock(s) make the flow marginally causally connected, after which my solution is valid. This conjecture could be justified by contradiction: if the flow remains causally disconnected (even having passed a shock), it could not be adjusted to the outer conditions and therefore more shocks must arise; if the flow is causally connected, it is smoothly adjusted to the outer boundary conditions according to the above solution. Therefore one could expect
that the shock(s) just make the flow marginally connected, after which the above solution works. This means, according to the estimate (49), that the shock(s) arise approximately at the distance

$$z_0 = \frac{\theta_0^2}{\sqrt{\sigma}} \approx \frac{z_1}{\sqrt{\sigma}}. \quad (50)$$

According to this solution, the flow is focused to a point \(z = 2z_1\) at the axis. Close enough to this point, just like close to the origin, the solution becomes invalid because the flow lines become nearly straight. But what seems to be more important is that such a converging flow ceases to be axisymmetric because of development of instabilities. This issue is discussed in the next section.

### 5 INSTABILITIES, TURBULENCE AND PARTICLE ACCELERATION

The axisymmetric flow considered in the previous section could not remain axisymmetric. The flow is composed from magnetic loops disconnected one from another therefore they could easily come apart. In the volume of the flow, the axisymmetry could be destroyed by the kink instability (Beckman 1998; Mizuno et al. 2011). Near the boundary of the flow, the Kelvin-Helmholtz instability (Begelman 1990) develops. In relativistic flows, these instabilities develop slowly because of relativistic time delay. However, if the flow converges, even small perturbations eventually destroy the regular structure. If two converging loops initially shifted one with respect to another by a displacement much less than their radius, the distortion becomes strong when the radius approaches the initial displacement. Therefore when the axisymmetric flow is focused into a point at the axis, the magnetic loops come apart close enough to the converging point so that a specific turbulence of shrinking magnetic loops emerges.

Taking into account that the hoop stress within the loop is not counterbalanced by either the poloidal magnetic field or the plasma pressure, the loops shrink with relativistic velocity, \(\gamma \sim \sqrt{\sigma}\), until the plasma energy reaches the magnetic energy. Therefore one concludes that the energy of the high latitude flow is released in a very small region close to the converging point, \(z = 2z_1\). In this picture, the observed jet (Weisskopf et al. 2000) begins in the vicinity of this point. One can expect also an efficient particle acceleration because in such a turbulence, the electric and magnetic fields are nearly equal.

The observed synchrotron spectrum from the Crab is extended up to \(\sim 100\) MeV, which implies particle acceleration on a time scale of the Larmor gyration period so that the accelerating electric field should be as strong as the magnetic field (Guilbert et al. 1983; de Jager et al. 1996). The recent discovery of daylong gamma ray flares in the band of a few hundred MeV (Tavani et al. 2011; Abdo et al. 2011; Striani et al. 2011; Vittorini et al. 2011; Bucher et al. 2012) poses even a larger challenge to the acceleration models (Bednarek & Idec 2011; Komissarov & Lyutikov 2011; Uzdensky et al. 2011; Bykov et al. 2012; Cerutti et al. 2012; Clausen-Brown & Lyutikov 2012; Sturrock & Aschwanden 2012). The relativistic turbulence could in principle resolve the problem. On the one hand, at relativistic turbulent velocities or, which is the same, at \(E \approx B\), even the second order Fermi process is efficient enough to permanently accelerate electrons up to the energies sufficient to emit 100 MeV synchrotron photons. On the other hand, random relativistic bulk motions could occasionally produce flares even in a higher energy band when a local radiating "knot" moves toward the observer (Yuan et al. 2011).

The efficient particle acceleration could occur not only via Fermi mechanism but also via reconnection (Uzdensky et al. 2011; Cerutti et al. 2012). When different magnetic loops slide one over another, strong field gradients, and therefore strong currents, arise. The magnetic reconnection comes into play when the current velocity approaches the speed of light. The corresponding scale, \(\delta \sim B/(8\pi\sigma P)\), could be expressed via the plasma multiplicity defined as the ratio of the plasma density to the Goldreich-Julian density in the pulsar magnetosphere, \(\kappa =\ encP/B\), where \(P\) is the pulsar period. Taking into account that within the magnetosphere, \(n \propto B\), whereas beyond the light cylinder, \(n \propto B/r\), one finds \(\delta \sim r/\kappa\). This scale may be considered as the dissipation scale of the turbulence; at this scale the magnetic energy is converted into the plasma energy.

The detailed analysis of the particle acceleration is beyond the scope of this paper; the above consideration just shows that a plausible site for the synchrotron emission in the hundreds MeV band, and in particular for gamma-ray flares, is the region close to the point \(z = 2z_1\) toward which the high latitude flow is focused. According to Equation (44), this region is at the axis of the system at the distance a few times less than the equatorial radius of the termination shock. In this region, the energy transferred by the strongly magnetized high latitude flow is released. This region may be identified with the base of the observed jet (Weisskopf et al. 2000).

### 6 CONCLUSIONS

In this paper, I considered the fate of the high latitude flow in the pulsar wind. This part of the wind remains strongly magnetized (see also Komissarov 2012) therefore it is terminated at a weak shock (or at a system of weak shocks), beyond which the flow is still radial and relativistic. The shock arises very close to the pulsar, much closer than if it were strong; the higher the magnetization of the flow, the closer the shock to the pulsar. Beyond the shock, the flow still expands but decelerates and eventually becomes to converge because the magnetic hoop stress is not counterbalanced by either the poloidal field or by the plasma pressure.

In the converging flow, magnetic energy is converted into the plasma energy therefore the plasma accelerates and heats. Even small perturbations due to, e.g., kink instability eventually destroy the converging flow so that magnetic loops come apart and then shrink independently of each other producing relativistic turbulence. Hence one can expect that the whole energy of the highly magnetized part of the pulsar wind is released in a very small region close to the converging point, which occurs on the axis of the system at the distance from the pulsar \(\sim \theta_0 a\), where \(\theta_0\) is the opening angle of the highly magnetized part of the wind, \(a\) the equatorial radius of the termination shock. I identify this region with the base of the observed jet. Relativistic turbulent mo-
tions in highly magnetized plasma imply $E \approx B$ so that in the energy release region, particles could be efficiently accelerated either via the second order Fermi mechanism or via the magnetic reconnection. Therefore the synchrotron gamma-ray emission in the hundreds MeV band, both persistent and flaring, could come from a small region at the base of the jet.

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