A Comparison of Simulated and Analytic Major Merger Counts

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ABSTRACT

We use large volume, high resolution, N-body simulations of 3 different ΛCDM models, with different clustering strengths, to generate dark matter halo merging histories. Over the reliable range of halo masses, roughly galaxy groups to rich clusters of galaxies, we quantify the number density of major mergers for two different time intervals and compare with analytic predictions based on the extended Press-Schechter (1974) theory.

Key words: cosmology:theory – methods:numerical –dark matter: merging histories

1 INTRODUCTION

It is now widely accepted that the observed large-scale structure in the universe formed hierarchically, through the process of gravitational amplification of small initial perturbations in a universe with predominantly cold dark matter (CDM). Within this “bottom-up” paradigm, larger structures form by the merging of smaller units, resulting in a dynamic and continually evolving matter distribution. Multiple lines of evidence support this scenario, suggesting that clusters and groups are still forming in the present universe, and that the idealization of a relaxed, virialized structure is somewhat unrealistic. Thus the merger history is of fundamental importance in determining not simply the details of structure formation but also in many cases the global properties of clustered objects under consideration.

Among merger events there are two extreme limits: major mergers and accretion. In a major merger, the masses of the progenitor halos are comparable, and consequently their
interaction results in a halo which is dynamically disrupted for some time after the merger. In contrast accretion is the merger of a small halo with a much larger halo, which is generally not severely disrupted by the process. We shall focus throughout on major mergers of objects “in the field” (which are expected to differ from major mergers of sub-haloes within a larger halo, e.g. galaxies in a galaxy cluster, Cavaliere & Menci (1997)).

The dynamical disruption caused by a major merger is expected to have many observational consequences (for an extensive list see Roettiger et al. [1996]). For mergers of groups and clusters, these include among other things: inducing scatter in IGM temperature (Mathiesen & Evrard [2000]), increasing and then decreasing star formation rates (e.g. Fujita et al. [1999]), for galaxy size halo consequences see Barnes and Hernquist (1992, and references therein), disrupting cooling flows (e.g. Allen et al. [1999]) and producing non-thermal radiation (Blasi 2000).

For all of these reasons there has been intense study of mergers analytically, numerically and observationally. On the theoretical side these range from in depth studies of specific mergers (Huss, Jain & Steinmetz [1999] to properties of mergers as an ensemble, their rates, consequences for formation times, etc., e.g. in Bond et al. [1991], Lacey & Cole [1993; 1994], Kitayama & Suto [1996a; 1996b], Tormen [1998], Somerville et al. [2000], Percival & Miller [1999]. This paper is an instance of the latter, that is, a study of the bulk statistical properties of major mergers in the field. Specifically the question we address is: What is the number density of objects, at a given time and mass, that have undergone a major merger within the last 0.5 or 2.5 Gyr? Part of our aim is to address this question with several different methods, to further understand our answers and to investigate the accuracy of some analytic estimates based on “extended” Press-Schechter ([1974]; hereafter PS) theory (see below).

The outline of the paper is as follows. We discuss the simulations in §2 and the numerical merger counts in §3. A review of some of the relevant analytic work is given in §4 and these estimates are compared with our numerical results in §5. Some technical details are relegated to an Appendix.

2 SIMULATIONS

We have chosen to focus on high mass objects, roughly galaxy groups to cluster scales, where we believe that the evolution is dominated by gravity and thus relatively inexpensive
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to compute numerically and under reasonable theoretical control. Our choice of time-scales is motivated by the fact that 0.5 Gyr is close to that relevant for star formation caused by mergers (e.g. as noted in Fujita et al. (1999) and for galaxies argued by Percival & Miller (2000) using luminosity curves in Bruzual & Charlot (1993)) while the larger time interval, 2.5 Gyr, is closer to those expected for cluster relaxation processes (e.g. see Mathiesen & Evrard (2000)).

We ran three simulations of a ΛCDM model with a high resolution N-body code (Bagla 1999). The cosmological models all assumed \( \Omega_m = 0.3 \) and \( \Omega_\Lambda = 0.7 \) but differed in \( z = 0 \) clustering strengths. We generated one realization for each of \( \sigma_8 = 0.8, 1.0 \) and 1.2, where \( \sigma_8^2 \) is the variance of the matter fluctuations in top-hat spheres of radius \( 8h^{-1}\text{Mpc} \). Each simulation employed \( 128^3 \) dark matter particles in a box of side \( 256h^{-1}\text{Mpc} \), i.e. a particle mass of \( 6.7 \times 10^{11}h^{-1}\text{M}_\odot \), with a Plummer force law with softening length \( 200h^{-1}\text{kpc} \). The initial conditions were generated by displacing particles from a regular grid, using the Zel’dovich approximation, with the initial redshift was chosen so that the maximum displacement was (slightly) smaller than the mean interparticle spacing due to the large volume of the simulation. Time steps were chosen to be a small fraction of the shortest dynamical time of any particle in the simulation. The initial redshift and number of steps ranged from \( z_{\text{init}} = 9 \) and 445 time steps for \( \sigma_8 = 0.8 \), to \( z_{\text{init}} = 14 \) and 635 time steps for \( \sigma_8 = 1.2 \). Further details (transfer function used, etc.) are given in the Appendix.

The large volume of the simulations provides good statistics on the high-mass end of the mass function, where we have focused our attention. We have checked that the resulting mass functions scale as expected over the range of masses and redshifts which shall be of interest in this work. In our merger counts we do notice some possible effects at early times for low mass halos in the \( \sigma_8 = 0.8 \) simulation, and these would be consistent with residuals from our grid initial conditions and limited numerical resolution. We shall comment further on this in §5.

2.1 Group catalogues

The full particle distribution was dumped every 0.5 Gyr starting from \( z = 2.16 \). For each output we generated a group catalogue using one of two group finders. We used HOP (Eisenstein & Hut 1998) for all three runs. For comparison, as well as to make explicit contact with
other work, we additionally used the friends-of-friends algorithm (FOF\textsuperscript{⋆}, Davis et al. (1985)) for the $\sigma_8 = 1$ run. From these catalogues we were able to construct merger trees (see below) back to $z = 1.86$ for 0.5 Gyr intervals and back to redshift $z = 1.11$ for 2.5 Gyr intervals.

Since it may be unfamiliar, we briefly review the operation of HOP. For technical details the reader is referred to Eisenstein & Hut (1998). HOP finds groups by first assigning each particle a density, and then “hopping” to neighbors with a higher density. Each particle belongs to the same class as its densest neighbor, and in this way each particle is assigned to a local density peak. To correct for the possibility of local density maxima causing groups to fragment, groups are merged if the bridge between them exceeds some chosen density threshold. There are 6 parameters one must choose for HOP: we used the default values for $N_{\text{merge}}$ and $N_{\text{hop}}$ (number of neighbors to look at when searching for a boundary and the densest neighbor) but took a lower value of the number of particles averaged over when calculating the density ($n_d = 8$ rather than 64). The results were similar for $n_d = 8$ and $n_d = 16$, with the higher $n_d$ excluding small groups. We excluded all groups of fewer than 8 particles. The remaining three parameters are all density thresholds. Eisenstein & Hut (1998) claim the method works best if the thresholds are in the ratio $1 : 2.5 : 3$, so we took $\delta_{\text{outer}} = 50$, $\delta_{\text{saddle}} = 125$ and $\delta_{\text{peak}} = 150$. These thresholds correspond to: the required density for a particle to be in a group, the minimum boundary density between two groups for them to be merged, and the minimum central density for a group to be independently viable. We experimented with these thresholds and found negligible differences in the final catalogues. At redshift zero there were between 18,000 and 19,000 groups (see Fig. 1).

The Friends-of-Friends algorithm is much better known. It has only a single free parameter: the linking length $b$, usually given in units of the mean interparticle spacing. FOF defines groups of particles which are each separated by less than $b$ from at least one other member of the group. Roughly speaking a FOF group consists of all particles within an iso-density contour of $b^{-3}$ of the background matter density. We follow convention and choose $b = 0.2$ (corresponding to a local overdensity 125). For the $\sigma_8 = 1$ case studied here, some notable differences for merger counts and number counts were found, shown in Fig. 2. For halos with more than 50 FOF particles, FOF halos had on average about 90 per cent of the particles of the corresponding HOP halos. FOF did find a larger total number halos, but these extra halos were at very low mass.

\textsuperscript{⋆} The implementation used here is from [http://www-hpcc.astro.washington.edu/tools/FOF/](http://www-hpcc.astro.washington.edu/tools/FOF/).
Both of these group finders do not identify substructure within large halos. This is not a particular problem for our purposes since we are interested primarily in mergers which take place in the field. Mergers within clustered environments are expected to behave differently (Cavaliere & Menci 1997) and have been studied by other groups, e.g. Gottloeber, Klypin & Kravtsov (2000).

2.2 Merger trees

The definition of a major merger is somewhat arbitrary as there is some dependence on what physical consequence is of interest. In this paper we define major mergers to be mergers in which the portions of the two largest halos that become part of the final halo have a mass ratio between 1:1 and 1:5.† We compute the number of mergers within a given time interval, a very interesting quantity from an observational and theoretical standpoint. Note that this is an inclusive rate – the number of mergers is not given directly by a merger rate since in a given time interval, halos can merge and then accrete, or accrete, merge, accrete, or merely merge, etc. We count any halo which has a major merger in the given time interval, regardless of what else occurred to it during the interval.

Having obtained the group catalogues for each output we constructed a merger tree for each simulation. For each time and for each (parent) group, we identified the (daughter) group membership of all the constituent particles at the next output time. The parent group was then considered a predecessor of the daughter group if more than 8 parent-group particles belonged to the daughter group at the next output time. For 0.5 Gyr merger trees, four of these “daughter groups” were considered for each predecessor group. This resulted in missing very few of the daughter groups – less than twenty total daughter groups went uncounted for the combined 22 outputs times three values of $\sigma_8$ for HOP (there were $\gg 10,000$ groups for most time steps). For the 2.5 Gyr merger trees, a more conservative maximum of 15 daughters was recorded for each predecessor group and no predecessor group had more than this number.

† Note that this definition also allows an unbound collection of particles from a parent halo to count as a predecessor for the purposes of defining a merger. Quantifying how closely this criterion restricts one to a resulting dynamically disrupted halo would be very interesting. We thank the referee for emphasizing this issue.
3 NUMERICAL MERGER COUNTS

The runs and merger tree construction described above resulted in halo evolution and merger counts with the following properties. We found, in agreement with Tormen (1998), that about 20 per cent of the mass of the halos was lost as the halos evolve. In fact some groups, in particular many of those close to the minimum group size, disappeared completely between time steps. While this could be partly due to our finite force resolution, it also highlights the ambiguity inherent in halo identifications using these group finders.

We anticipate that small mass predecessors may be undercounted, especially at earlier times. For this reason we only present results for halos with more than 10 times the minimum group size, or \(5 \times 10^{13} \, h^{-1}M_{\odot}\).\(^\dagger\) When predecessor sizes are compared for the purpose of identifying a major merger (i.e. a ratio 1 : 1 to 1 : 5), the number of particles from the predecessor group that go into the final group, not the number of total particles in the predecessor group are considered.

The number density of halos and of merger counts (within 0.5 Gyrs) at redshift zero for the three values of \(\sigma_8\) (using HOP), are given in Fig. 1. The smooth curve going through the upper set of points (the number density) is the PS fit discussed in the next section. As we can see, the PS theory provides an adequate fit to the simulation results, as has been noted before by numerous authors (Efstathiou et al. 1988; Efstathiou & Rees 1988; White, Efstathiou & Frenk 1993; Lacey & Cole 1994; Eke, Cole & Frenk 1996; Gross et al. 1998; Governato et al. 1999). The lower curves correspond to two of the analytic fits, also discussed in the next section, as well as a quadratic fit to \(\log M/(h^{-1}M_{\odot})\) and \(\ln(1 + z)\) for all times.

Many groups have found dependence on group finder of their results, e.g. comparing SO(178) and FOF with different linking lengths (e.g. Lacey and Cole (1994), Tormen (1998)). Our FOF merger trees differed from those for halos identified with HOP, just as the total numbers of halos differed. The comparison between the HOP and FOF groups and merger trees can be seen in Fig. 2.

One can consider the number of mergers within a given recent time interval, taken here to be 0.5 Gyr, as a function of redshift, as is shown in Fig. 3. One can see that the number of high mass halos decreases as one goes back to earlier times, and that simultaneously the

\(^\dagger\) This was because one could find in the 0.5 Gyr merger trees major mergers with a second predecessor as small as 1/10 the size of the final mass. For the 2.5 Gyr merger trees of order 20 of the second predecessors were smaller than this fraction for medium range masses, for each value of \(\sigma_8\), summed over all times.
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Figure 1. Number density of groups and those with recent (within 0.5 Gyr) major mergers at \( z = 0 \) found by HOP, for three values of \( \sigma_8 \). The triangles are the simulations, the upper (smooth) curve is the PS prediction (Eq. 1) with \( \delta_c = 1.48 \). The lower curves are quadratic fits as a function of log mass and \( \ln(1 + z) \). The dotted line is the fit to the simulation results, the dashed line corresponds to the fit to the “direct” calculation, Eq. (8), and the dot-dashed line corresponds to the “jump” estimate given in Eqs. (10). One halo in the entire simulation volume corresponds to a density of \( 6 \times 10^{-8} (h^{-1} \text{Mpc})^3 \).

The fraction of total halos of any given mass which have undergone a major merger within a recent fixed time interval increases.

A related quantity, the merger counts\(^\dagger\) as a function of redshift at fixed mass, is plotted in Fig. 4. It appears that, for lower mass objects, the number of merger counts within 0.5 Gyr.

\(^\dagger\) The merger rate, rather than counts, as a function of redshift was studied by e.g. Percival & Miller (1999), Percival, Miller & Ballinger (1999), along with consequences such as star formation.
Gyrs first increases with look-back time and then decreases. For higher mass objects the number merely decreases, which may be due to the lack of available high mass objects at earlier times. The trend is similar for the 2.5 Gyrs merger counts, however for $\sigma_8 = 1.2$ and low mass ($10^{13.6} h^{-1} M_\odot$) it appears that there is not yet any evidence for a high redshift decline in counts.

In the next section some analytic estimates of number of major mergers within a given interval will be made using (extended) PS theory.

4 ANALYTIC ESTIMATES

There has been much theoretical work on major mergers and on mergers more generally. The most popular, and successful, semi-analytic formalism for predicting merger rates and associated quantities is PS theory and its extensions (Bond et al. 1991; Bond & Myers 1996; Bower 1991; Lacey & Cole 1993; Lacey & Cole 1994; Kauffmann & White 1993; Kitayama & Suto 1996a; Somerville & Kolatt 1999; Tormen 1998; Sheth 1995; Sheth 1998; Sheth & Lemson 1999), although there have also been merger calculations done in other frameworks (Carlberg 1990; Blain & Longair 1993; Cavaliere, Colafrancesco & Menci 1992; Cavaliere & Menci 1997). Extended PS provides the necessary ingredients for calculating the quantity of interest here: the number of halos of a given mass which have had a major merger within
Figure 3. Number density of groups and major mergers within 0.5 Gyrs at \( z = 0.0, 0.53, 0.98, 1.42 \) for \( \sigma_8 = 1 \). The curves are as in Fig. 1, i.e., the upper (smooth) curves again are PS predictions for number densities with \( \delta_c = 1.48 \) and the lower curves are quadratic fitting functions (as a function of log mass and \( \ln(1 + z) \)) to merger counts in section 3: the dotted line is the fit to the simulation results, the dashed line corresponds to the fit to the “direct” calculation, Eq. (8), and the dot-dashed line corresponds to the “jump” estimates given in Eqs. (12,10). One halo in the entire simulation volume corresponds to a density of \( 6 \times 10^{-8} (h^{-1} \text{Mpc})^3 \).

some given time frame at some given epoch, which is to be compared with the simulations described above.

Press-Schechter and extended PS theory provide extremely useful characterizations of the distributions of mass and the growth of structure. They are computationally inexpensive and exhibit clearly the dependence on cosmological parameters. Their use has been widespread because, for many quantities (e.g. in Bond et al. (1991), Lacey & Cole (1993);
Figure 4. Simulated major merger counts as a function of redshift for 4 different masses. The number of recent mergers increase in time and then decrease, perhaps due to the fewer numbers of halos at earlier times. The time period is 0.5 Gyrs for the left panel and 2.5 Gyrs for the right panel.

Given the simplistic assumptions going into the PS theory, it is intriguing that the agreement with numerical simulations is so good. Many of the assumptions are known to be incorrect in detail: spherical collapse (Sheth, Mo & Tormen 1999), the monotonic growth of halos (e.g. Tormen 1998), and the association of initial density peaks with final halos (Carlberg 1990; Katz, Quinn & Gelb 1993; Frenk et al. 1988). Hence, while PS is very useful and captures something essential about the process of structure formation, its failures (Gross et al. 1998; Sheth & Tormen 1999; Jenkins et al. 2000) can tell us something very interesting as well. For example, extending the picture of spherical collapse to ellipsoidal collapse improves agreement with the mass function from numerical simulations (Monaco 1997a; Monaco 1997b; Lee & Shandarin 1998; Sheth, Mo & Tormen 1999). It is to be hoped that finding where and how extended PS theory fails to predict mergers will shed further light on this issue.

We briefly review some background from the theory of PS and extended PS which we shall make use of below. More detailed definitions are in the Appendix.
4.1 Comoving number density of spherical collapsed systems

The basic prediction of PS theory is the comoving number density of virialized halos with mass in the range \((M, M + dM)\) at time \(t\). The PS prediction is

\[
N_{PS}(M, t)\,dM = \frac{1}{\sqrt{2\pi}} \frac{\rho_0}{M} \frac{\delta_c(t)}{\sigma^3(M)} \left| \frac{d\sigma^2(M)}{dM} \right| \exp \left[ -\frac{\delta_c^2(t)}{2\sigma^2(M)} \right] dM .
\]  

(1)

Here \(\rho_0\) is the comoving, mean mass density of the universe, for matter density \(\Omega_m\), \(\delta_c(t)\) is the threshold density contrast for spherical collapse at time \(t\) and \(\sigma^2(M)\) is the variance of the matter fluctuations smoothed over a region of radius \(R\) corresponding to mass \(M = 4\pi R^3 \rho_0 / 3\). For the functional form of \(\delta_c(t)\) and a fit to \(\sigma^2(R)\) for the model under consideration see the Appendix.

In order to compare the simulations to analytic PS estimates, \(\delta_c\) must be specified. The spherical top-hat collapse model \([\text{Peacock 1999}, \text{Liddle & Lyth 2000}]\) predicts that \(\delta_c \approx 1.69\) with a small cosmology dependence. We choose to regard \(\delta_c\) as a free parameter in the PS theory and adjust it to get the best agreement with the simulations for a given choice of group finder. The groups found by HOP resulted in a set of halo mass distributions fit within 40 per cent (better than 20 per cent most of the time) by PS predictions for \(M > 10^{13} h^{-1} M_\odot\) if \(\delta_c = 1.48\) was taken. For FOF the best fit is obtained with \(\delta_c = 1.64\), closer to the spherical top-hat prediction.

4.2 Conditional probabilities

The random walk model inherent in the PS theory can be extended to describe conditional probabilities: given a point in space that ends up in a halo of mass \(M_2 > M_1\) at \(t_2 > t_1\), the probability that it was in a halo of mass \(M_1\) at \(t_1\) is:

\[
P_1(M_1, t_1|M_2, t_2)\,dM_1 = \frac{1}{\sqrt{2\pi}} \frac{\delta_c - \delta_c}{\sigma^2} \left| \frac{d\sigma^2}{dM_1} \right| \exp \left[ -\frac{(\delta_c - \delta_c)^2}{2(\sigma^2 - \sigma^2)} \right] dM_1 .
\]

(2)

where \(\delta_c = \delta_c(t_2)\), \(\sigma_2 = \sigma_2(M_2)\), etc. Given that a point starts in a halo of mass \(M_1\) at \(t_1\), the reverse probability that it ends up in a halo of mass \(M_2\) at \(t_2\) is

\[
P_2(M_2, t_2|M_1, t_1)\,dM_2 = \frac{1}{\sqrt{2\pi}} \frac{\delta_c}{\delta_c} \left[ \frac{\sigma_1^2}{\sigma_1^2 - \sigma_2^2} \right] \left| \frac{d\sigma_2^2}{dM_2} \right| \exp \left[ -\frac{\sigma_2^2\delta_c - \sigma_1^2\delta_c^2}{2\sigma_1^2 \sigma_2^2 (\sigma_1^2 - \sigma_2^2)} \right] dM_2 .
\]

(3)

4.3 Merger rates

Merger rates are calculated as a limiting case of the conditional probabilities above. The probability of a mass change over a small time is
\[ P_{1,2}(M + \Delta M, t \pm \Delta t|M, t) \sim \frac{dP_{1,2}(M \rightarrow M + \Delta M; t)}{dt} \Delta t \] (4)

if \( \Delta t \) is small enough. (Note that \( P_{1,2}(M + \Delta M, t|M, t) = 0 \). One might think of a large mass change in a small time as coming from the addition of one halo, i.e. a merger (Lacey & Cole [1993]). With this interpretation one gets the

- Rate at which a point in a halo of mass \( M_1 \) is incorporated into a halo of mass \( M_2 \) at \( t \):
  \[
  \frac{dP_1(M_1 \rightarrow M_2; t)}{dt} dM_1 = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \frac{1}{\sqrt{2\pi} (\sigma_1^2 - \sigma_2^2)^{3/2}} \left[ -\frac{d\delta_c(t)}{dt} \right] \frac{d\sigma_1^2}{dM_1} dM_1 . \] (5)

- Rate at which a halo of mass \( M_2 \) is formed from addition of mass to a halo of mass \( M_1 \) at \( t \), per point:
  \[
  \frac{dP_2(M_1 \rightarrow M_2; t)}{dt} dM_2 = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \frac{1}{\sqrt{2\pi} (\sigma_2^2 - \sigma_1^2)^{3/2}} \left[ -\frac{d\delta_c(t)}{dt} \right] \frac{d\sigma_2^2}{dM_2} \exp \left[ -\delta_c(t)^2 \left( \frac{1}{2\sigma_2^2} - \frac{1}{2\sigma_1^2} \right) \right] dM_2 \] (6)

These bulk probabilities can be combined to calculate the number of halos of a given final mass that have undergone a major merger (i.e. have predecessors with a given ratio) within a certain time interval, with the latter chosen to correspond to the relaxation process of interest. (We note that a modified version of the PS formalism that differentiates between (instantaneous) major mergers and accretion has been developed in [Raig, Gonzalez-Casado & Salvador-Sole1998; Salvador-Sole, Solanes & Manrique1998] as well.)

Heuristically, one can count the major mergers which have occurred at a specific time for halos of a given final mass, whether or not they accreted mass before or after the merger, by taking the

- starting # of halos
  \[ \times \] merger prob. per unit time
  \[ \times \] prob. to end up in final mass range .

This is for a specific time and must be integrated to study a time interval. Plugging in the probabilities and integrating over time this becomes

\[
N_{\text{mer, direct}}(M_2, t_1, t_2) dM_2 = \int_{t_1}^{t_2} dt \int d(M_1 + \Delta M_1) \int dM_1 N_{PS}(M_1, t) \times \frac{dP_1}{dt}(M_1 \rightarrow M_1 + \Delta M_1; t) P_2(M_2, t_2|M_1 + \Delta M_1, t) . \] (8)

This will be referred to as the “direct” estimate.\footnote{After the bulk of the work in this note was completed, the authors became aware of a similar expression in [Cavaliere, Menci & Tozzi 1999].} In words, this is the probability that at some time \( t \) in a given time interval of size \( t_2 - t_1 \) a starting halo mass of mass \( M_1 \) increases...
its mass instantaneously by $\Delta M_1$, times the probability that the resultant $M_1 + \Delta M_1$ mass ends up, at $t_2$, in a halo of mass of $M_2$. Taking $M_1$ to be the mass of the larger of the predecessors, the range of integration for $\Delta M_1$ (the mass of the second largest predecessor) is determined by the chosen merger mass ratio. This allows for the possibility of a merger happening any time between $t_1$ and $t_2$, with accretion before or afterwards, which is in some sense the key difference between this and the work of Carlberg (1990) as formulated by Lacey & Cole (1993). Another differences is the range of integration for the masses taken.

An equivalent way of writing the above is

$$\int dt \int d(M_1+\Delta M_1) \int dM_1 \frac{dP_1}{dt}(M_1 \rightarrow M_1+\Delta M_1; t) P_1(M_1+\Delta M_1, t \mid M_2, t_2) N_{PS}(M_2, t_2) \frac{M_2}{M_1} \tag{9}$$

The limits on the integrals are as follows. The time integral is taken to correspond to the time interval of interest (0.5 or 2.5 Gyr), and the range of $\Delta M_1$ is determined by the mass ratios required for a major merger. So for our case, with ratios 1 : 1 to 1 : 5, $M_1/5 \leq \Delta M_1 \leq \text{Min}(M_f - M_1, M_1)$. For the $dM_1$ integral, the chosen mass ratio fixes the upper limit to be $5M_2/6$. The lower limit is not obvious, but it turns out that the dependence is rather weak. By considering the simulations one finds that $M_1 \in [M_2/3, 5M_2/6]$ will cover most of the range of largest predecessor masses for 0.5 Gyr. This was also used for the 2.5 Gyr run. To use this formula more generally one would like to be able to choose this lower limit without relying upon simulations. Changing this lower limit for the 0.5 Gyr case between $M_2/2.5$ and $M_2/3.5$ produced indistinguishable results. To evaluate Eq. (8), two of the three integrals can be done exactly, but the last integral over $M_1$ seems to require numerical integration (see Appendix). A quadratic fit to the corresponding $\log N_{\text{mer}}(> M)$, for $\delta_c = 1.48$ as a function of $\log(M)$ and $y = \ln(1 + z)$ is shown as the dashed line in Figs. [1] and [8].

It should be noted that even within the PS description this expression is only approximate. If a halo has two jumps before getting to the final mass and they are both a sizeable fraction if its mass, they may be counted twice. In principle one could bound this effect by calculating the probability of two jumps occurring using a generalization of Eq. (8). One will also double count the number of mergers where the two initial masses are identical, but this

The factor $M_2/M_1$ is due to the one to one correspondence (by assumption) between initial halos of mass $M_1$ and final number of halos. If a halo of mass $M_1$ ends up in a halo of mass $M_2$ then all the mass in the $M_1$ halo ends up in the halo of mass $M_2$. So one can count the number of initial, and thus final, halos by taking the mass fraction and dividing by $M_1$ to get the corresponding number of halos.

⋆⋆ We thank the referee for suggesting we calculate this dependence.
is expected to be a small number. Note that the distribution of mergers with a given mass change should be independent of the initial mass, using the random walk interpretation of the PS formulae (Percival, Miller & Peacock 2000). However, for a major merger the given mass change of interest depends on the mass of the initial halo, and in addition the total number of mergers depends on the number of initial halos (which depends on both time and mass), consequently some dependence on mass and era is expected.

There are other approximate expressions which can be considered. Here we focus on one which has a particularly simple origin, in order to see how well it captures general trends and the quantity of interest. This is to use a jump to $M$ instantaneously as an estimate of merger counts. Saying that the largest mass component of the two halo merger has at least half the final mass and allowing it to jump by a mass in the range $(\frac{M}{2}, \frac{5}{6}M)$ corresponds to taking

$$N_{\text{mer,jump}}(M, t) \equiv \int_{\frac{1}{2}M}^{\frac{5}{6}M} dM_1 \frac{dP_1(M_1 \rightarrow M,t)}{dt} \frac{M}{M_1} N_{\text{PS}}(M,t)$$

(10)

This will be referred to as the “jump” estimate. Requiring the largest predecessor to have mass only $\geq\frac{M}{2}$ avoids counting changes in both components of a major merger as distinct mergers (except in the case, probably rare, where both initial masses are exactly equal to half the final mass). However, there is some under-counting expected because, as noted earlier, the simulations showed that the largest predecessor could sometimes be as little as $1/3$ the final mass. This expression only counts the masses which come exactly to a given mass at the given time, rather than any masses which might have merged and then later accreted to reach the given mass, or those which have reached this mass and then accreted out of the mass range. As the time dependence factors out in front, the only difference between 2.5 Gyrs and 0.5 Gyrs is the multiplication by an overall factor of 5 to change time units.

This turns out to be a simple quantity since for the range of interest the effect of the factor $M/M_1$ in Eq. (10) gives just an overall prefactor of approximately 1.44:

$$N_{\text{mer,jump}}(M, t) \simeq 1.44 \int_{\frac{1}{2}M}^{\frac{5}{6}M} dM_1 \frac{dP_1(M_1 \rightarrow M,t)}{dt} \frac{M}{M_1} N_{\text{PS}}(M,t)$$

$$\simeq 1.44 \sqrt{\frac{2}{\pi}} \left( \frac{1}{\sigma^2(\frac{3}{5}M) - \sigma^2(M)} - \frac{1}{\sigma^2(\frac{1}{2}M) - \sigma^2(M)} \right) \left[ -\frac{d\delta_c(t)}{dt} \right] N_{\text{PS}}(M,t).$$

(11)

Another approximation which is sometimes used as the formation rate (e.g. Kitayama and Suto (1996a))$^{††}$

$^{††}$ There is no factor of $M/M_1$ here because the number of initial halos is not in one to one correspondence with the number of final halos. For example, a huge number of infinitesimally sized halos can go to form one final halo.
\[
N_{\text{mer}_{\text{form}}}(M, t; M/2) = \int_0^{M/2} dM_1 \frac{dP_2(M_1 \to M; t)}{dt} N_{PS}(M, t)
\]
\[
= \sqrt{\frac{2}{\pi}} \sqrt{\frac{1}{\sigma^2(M/2) - \sigma^2(M)}} \left[ -\frac{d\delta_{c}(0)}{dt} \right] N_{PS}(M, t).
\]

which turns out (as seen by numerical integration in the regime of interest) to also be close to the “jump” estimate when multiplied by 1.44:

\[
N_{\text{mer}_{\text{jump}}} \sim 1.44 N_{\text{mer}_{\text{form}}}.
\]

The “jump” estimate is somewhat larger, by about 4 per cent, for masses \(\sim 5 \times 10^{13} h^{-1} M_\odot\) and \(\sim 9\) per cent larger for masses \(\sim 1.5 \times 10^{15} h^{-1} M_\odot\). In both of these, one is only looking at mergers going up to the final mass of interest, while in reality accretion should be taking halos both in and out of the range of interest. These quantities will turn out to work surprisingly well, perhaps because of some averaging out from the accretion effects.

There has been related analytic work using extended PS to calculate “formation times.” These differ from the merger counts under study here. A few differing formation time definitions are used, e.g. when half the mass in the halo has the halo has assembled (Lacey & Cole 1993; Kitayama & Suto 1996a) or when the assembly of mass in the halo ends (Blain & Longair 1993; Sasaki 1994; Percival & Miller 1999). For example, a major merger by our definition could occur after half of the halo mass assembled, e.g. with a halo of half the final mass merging with a halo of one third the final mass. Analytic and numerical formation times have been compared, e.g. by Tormen (1998).

5 COMPARISONS

The analytic formulae in section §3 for the number of mergers, the “direct” and “jump” calculations (Eqs. 8, 10) in the previous section, were compared with the simulation results described in section §2 for mass starting at \(5 \times 10^{13} h^{-1} M_\odot\) and above. As mentioned earlier this minimum mass was chosen because a major merger could have a smaller predecessor of mass 1/10 the final mass, and so the minimum final mass was taken to be 10 times the minimum halo mass. The two analytic estimates differed from each other and consequently had different success in fitting the simulation results. The goodness of fit also depended on the time interval (0.5 or 2.5 Gyrs) considered and (very weakly) on the group finder used.

Comparison with Eq. (9), the “direct” estimate, will be made first. The analytic expressions were integrated numerically and then fit quadratically (in \(\log(M)\) and \(\ln(1 + z)\)) and
these quadratic fits were compared with the simulations. A quadratic best fit to the simulations directly, for all masses and redshifts, could vary from the binned simulation values by up to 40 per cent in the 0.5 Gyr runs – thus exact agreement with any smooth curve is not expected to be better than this. For 0.5 Gyrs and using HOP, the quadratic fit to the “direct” formula was in broad agreement with that found in the simulations. The analytic calculation tended to over-predict the number of low mass \( (M < 10^{14} h^{-1} M_\odot) \) halos which had major mergers, and this occurred more often at early times and lower \( \sigma_8 \), being within a factor of three in the worst case and within 40 per cent much of the time. This may be in part due to the known tendency of PS to overestimate the number of halos, and thus predecessors for major mergers, at low mass (Gross et al. (1998), Somerville et al. (2000)). A competing effect is the “loss” of small halos in the simulations. Another factor may be the residual effects of the initial conditions, again strongest at early times and smallest \( \sigma_8 \).

For mass \( \sim 10^{14} M_\odot \) the “direct” prediction was close, over-predicting for lower mass and under-predicting for higher mass. (For example, for \( \sigma_8 = 0.8 \) and the 0.5 Gyr time frame, the predicted “direct” merger counts for final halos of mass \( M < 10^{14} \) were too high by more than 40 per cent down to \( z \sim 1 \).) For the 2.5 Gyr runs, the agreement at low mass was a lot worse between the “direct” estimate and the simulations, with the number of mergers predicted for mass less that \( 10^{14} h^{-1} M_\odot \) over-predicted by 40 per cent for almost all times and values of \( \sigma_8 \). The fit improved with increasing \( \sigma_8 \).

Surprisingly, the “jump” estimate (Eq. 10) worked quite well. The “jump” estimate was unbiased for 0.5 Gyr but for 2.5 Gyr tended to overestimate the number of mergers. As noted, the “jump” estimate was slightly larger than 1.44 times the formation rate based estimate and closer to the simulations for 0.5 Gyrs. Note that the “jump” and formation rate based estimates have time dependence only in an overall factor out front, taking the time units to be the interval of interest. In particular, for the two intervals chosen here, 0.5 Gyrs, and 2.5 Gyrs, the predictions only differ by a factor of 5. This scaling was not observed in the simulations, which differed by a smaller amount. Thus if the “jump” or formation rate based estimate works for one time, it will not work very well for the other. We found that it worked well for 0.5 Gyrs (i.e. within 40 per cent most of the time) but not for 2.5 Gyrs (consistently too high, centered at an overprediction of counts by \( \sim 40 \) per cent).

The percentage deviation for the two time intervals and the “direct” and “jump” estimates, for \( \sigma_8 = 1 \), is shown in Fig. 5. Each line corresponds to a different time step, and horizontal lines mark 0, 20 and 40 per cent deviation from 0. Large spikes at high mass
Figure 5. “Direct” and “jump” predicted merger counts vs. simulations: Eqs. (8, 10) for 0.5 and 2.5 Gyrs. The horizontal lines indicate 0, 20 and 40 per cent deviations between the simulation and the analytic estimate. The simulation counts have been smoothed over a range of .05 in log $M$ before comparing with the analytic expressions.

are often due to small number statistics (which are worse at early times). For the “jump” calculation, the trends for the fits (within 40 per cent for 0.5 Gyrs and centered at 40 per cent too high for 2.5 Gyrs) were the similar for the other values of $\sigma_8$.

In Fig. 6, the RMS deviation of the calculated versus simulated counts and the average deviation are shown as a function of redshift for all three values of $\sigma_8$ for both the “direct” and “jump” predictions.

Comparing the HOP and FOF group finders for the $\sigma_8 = 1$ case, some notable differences
for merger counts and number counts were noted, as mentioned earlier. However, the analytic estimates worked similarly for both group finders when the appropriate $\delta_c$ was used.

In conclusion, it seemed that both analytic estimates gave comparable numbers of mergers to what was found in the simulations. The “jump” estimate worked better than the “direct” estimate in matching the shape of the curve $N_{\text{mer}}(> M)$, but over-predicts counts for all masses for longer times. It is of clear interest to pursue this, but doing so will require a larger mass and redshift range and larger simulations.

It is also interesting to try to find some estimate of the merger fraction from the sim-
ulations alone, similar to “universal” fitting functions for numbers of halos as functions of various parameter combinations which have been found to have a particularly simple form (e.g. Sheth-Tormen (1999), Jenkins (2000)). We found that, using \( \nu = \delta_c / \sigma(M, z) \), the combinations \( \frac{N_{\text{mer}}(>M)}{N(>M)} \sim \nu (1+z)^{1.5(1)} \) for 0.5(2.5) Gyrs and \( \frac{N_{\text{mer}}(M)}{N(M)} \sim \nu (1+z)^{1.5} \) seem to be promising possible scalings. As the relatively small number of mergers makes these quantities very noisy, bigger simulations and better statistics will allow further study of this question.

6 CONCLUSIONS

In this note we have concentrated on the question of how many halos of a given mass and time have had a major merger (ratio of two largest predecessors ranging from 1 : 1 to 1 : 5) within the last 0.5 and 2.5 Gyrs, for the mass range \( 5 \times 10^{13} h^{-1} M_\odot \sim 10^{15} h^{-1} M_\odot \). Numerical simulations for 3 values of \( \sigma_8 \) in a ΛCDM cosmology were compared with two analytical estimates. These counts can be related to observational processes with 0.5 Gyr and 2.5 Gyr relaxation times. For 0.5 Gyr, the “jump” analytic estimate, Eq. (10), was closest to the simulations, although the “direct” estimate, Eq. (8), was close for low enough redshift and high enough \( \sigma_8 \). Ratios of major merger counts were also considered. The main conclusion is that the PS estimates of merger counts, a very interesting quantity from the observational point of view, roughly agree with simulations. This suggests that future study of this approach may be fruitful. Many extensions and refinements are possible, such as using larger simulations, going back to higher redshifts, and varying parameters such as time intervals and mass ratios. The dynamical disruption caused by the major merger may be further characterized by including additional information about the predecessors and daughter halos. For instance, our criteria do not discriminate between the transfer of a bound subhalo or the stripping of outer (perhaps spatially very far apart and dynamically unrelated) particles of a parent halo to a daughter halo. The former would be expected to be more likely to cause more dynamical disruption. In addition, the analytic calculations may be able to use the more accurate fitting functions for masses (Sheth and Tormen (1999), Jenkins et al (2000)), if these approaches can be generalized to extended PS conditional probabilities.

\(^{\dagger}\) In terms of the often used parameter \( M^* \), \( M = M^* \leftrightarrow \nu = 1 \).

\(^{\ddagger}\) We thank the anonymous referee for suggesting we introduce this distinction.
APPENDIX A: PRESS-SCHECHTER AND EXTENDED PRESS-SCHECHTER

A1 Notation

In numerical simulations it appears that many properties of the final density field are present in the initial conditions and are simply “sharpened” by the non-linear amplification of gravity. Press Schechter (1974) theory utilizes this in an essential way, and couples it with the theory of non-linear evolution of a uniform, spherical overdensity embedded in a homogeneous universe. For this reason we need to be able to predict the scale- and time-dependence of density fluctuations in linear theory.

We shall convert between mass and length scales using a top-hat filter in real space, i.e. the mass associated with a smoothing scale $R$ is $M = (4\pi/3)\pi R^3 \rho_0$ where $\rho_0$ is the comoving mean mass density of the Universe,

$$\rho_0 = 2.7755 \times 10^{11} \Omega_m (h^{-1}M_{\odot})(h^{-1}\text{Mpc})^{-3}.$$  \hspace{1cm} (A1)

We assume that the initial fluctuations were Gaussian with zero mean. The variance of the fluctuations, smoothed over a region of mass $M$, is given by

$$\sigma^2(M) \equiv \sigma^2(M, z = 0) = \int_0^\infty \frac{dk}{k} W^2(kR(M)) \Delta^2_\delta(k) \hspace{1cm} (A2)$$

where $W(kR)$ is the window function for top-hat filtering

$$W(kR) = 3 \left( \frac{\sin(kR)}{(kR)^3} - \frac{\cos(kR)}{(kR)^2} \right). \hspace{1cm} (A3)$$

The power spectrum (today) is given by

$$\Delta^2_\delta(k) \equiv \frac{k^3 P(k)}{2\pi^2} = \delta^2_H(k) \frac{k^4}{H_0^2} T^2(k) \hspace{1cm} (A4)$$

which uses the primordial density spectrum $\delta_H(k) \sim k^{n-1}$, for $n \sim 1$. We take $n = 1$. For
the transfer function in both the calculations and the simulations we used (Efstathiou et al. (1992))

\[ T(k) = \left[ 1 + (ak + (bk)^{3/2} + (ck)^2)^\nu \right]^{-1/\nu} \]  

where

\[ a = (6.4/\Gamma)h^{-1}\text{Mpc}, \quad b = (3/\Gamma)h^{-1}\text{Mpc}, \quad c = (1.7/\Gamma)h^{-1}\text{Mpc}, \quad \nu = 1.13 \]  

(A6)

In practice \( \sigma^2(M) \) was calculated up to an overall constant which is fixed by the choice of \( \sigma_8 \). A good fit near \( R = 8h^{-1}\text{Mpc} \) is

\[ \log_{10} \left( \frac{\sigma^2(R)}{\sigma_8^2} \right) = -1.4936 \log R_8 - 0.46554(\log R_8)^2 - 0.0982478(\log R_8)^3 \]  

where \( R_8 = (R/8h^{-1}\text{Mpc}) \). This is a fit to numerical integration between \( 0.27 \leq \log R \leq 1.49 \) which is better than \( \sim 1 \) per cent.

Collapsed halos are taken to be regions in the linear density field with density greater than some critical density contrast, \( \delta_c \), when smoothed on a scale \( R \). In practice we take account of the linear growth by holding the variance fixed and reducing the density threshold

\[ \delta_c(t(z)) = (1 + z) \frac{g(\Omega_0)}{g(\Omega(z))} \delta_c. \]  

We found \( \delta_c(z = 0) = 1.48 \) was a good fit to the mass distribution of the halos found by HOP (Eisenstein & Hut 1998) and \( \delta_c(z = 0) = 1.64 \) a good fit for those found with FOF (Davis et al. 1985). The growth rate can be approximated as (Carroll, Press & Turner 1992; Viana & Liddle 1999)

\[ g(\Omega(z)) = \frac{5}{2} \Omega \left[ \frac{1}{70} + \frac{209}{140} \Omega - \frac{\Omega^2}{140} + \frac{\Omega^4}{7} \right]^{-1} \]  

(A9)

and

\[ \Omega(z) = \frac{(1 + z)^3}{(1 - \Omega_m + (1 + z)^3 \Omega_m)} \]  

(A10)

for a flat \( \Lambda \) universe.

A2 Formalism

The Press-Schechter formalism has been extended to describe histories of halos (Bond et al. (1991), Bond & Myers (1996), Bower (1991), Lacey and Cole (1993), (1994), Kitayama & Suto (1996a), Baugh et al. (1998), Somerville & Kolatt (1999), Tormen (1998)). A useful description can also found in the textbooks by Peacock (1999) and Liddle & Lyth (2000).

Press-Schechter and extended Press-Schechter theory have a heuristic random walk inter-
pretation based on consideration of trajectories. Each trajectory corresponds to the motion of a point in space under the addition of modes $\delta_k$ (i.e. filtering the density field around each point in space with filters of steadily increasing $k$). The arguments are rigorous when this filtering corresponds to a top hat filter in $k$-space, that is, adding $\delta_k$ for larger and larger values of $k$. This produces a random walk for each point $\delta = \sum_0^k \delta_k$. One can relate $k \rightarrow R, M \rightarrow \sigma$ and thus one talks about trajectories as a function of $M$ or $\sigma$. Those trajectories which have crossed some critical value of $\delta = \delta_c(t)$ at any given time are taken to be collapsed halos. In terms of the random walk picture, one can consider $\delta_c$ as an absorbing barrier and the mass function as given by the distribution of trajectories in $k$ as they are absorbed. The resulting number density is Eq. (11),

$$N_{PS}(M,t)dM = \frac{1}{\sqrt{2\pi} M \sigma^3(M)} \left| \frac{d\sigma^2(M)}{dM} \right| \exp \left[ -\frac{\delta_c^2(t)}{2\sigma^2(M)} \right] dM . \quad (A11)$$

This is the number of halos, for the amount of mass in halos of this mass (i.e. the number of trajectories), one multiplies this quantity by $M/\rho_0$.

This trajectory approach allows one to consider trajectories which have crossed the collapse threshold (absorbing barrier) at two different times, corresponding to being part of a halo of one mass at one time and another mass at another time, $P_1(M_1,t_1|M_2,t_2)$ and $P_2(M_2,t_2|M_1,t_1)$, in Eqs. (2,3).

These conditional probabilities can be multiplied together to get joint probabilities:

$$P_1(M_1,t_1|M_2,t_2)N_{PS}(M_2,t_2) \times \frac{M_2}{\rho_0} dM_1 dM_2 =$$

$$P_2(M_2,t_2|M_1,t_1)N_{PS}(M_1,t_1) \times \frac{M_1}{\rho_0} dM_1 dM_2 = P(M_1,t_1,M_2,t_2)dM_1dM_2$$

which obey, e.g.,

$$dM_1 \int dM_2 P_1(M_1,t_1|M_2,t_2)P_1(M_2,t_2|M_3,t_3) = P_1(M_1,t_1|M_3,t_3)dM_1 . \quad (A14)$$

(Sethi 1994).

### A3 Integration of merger rate

The starting integral is

A crucial identity for this is

$$\int_0^\infty du e^{-u^2} = \sqrt{\pi}$$

$$\int_0^\infty du e^{-u^2} - bu^2 = \sqrt{\pi} e^{-2\sqrt{ab}} \quad (A13)$$
A Comparison of Simulated and Analytic Major Merger Counts

\[ N_{\text{mer, direct}}(M_2, t_1, t_2) = \int_{t_1}^{t_2} dt \int d(M_1 + \Delta M_1) \int dM_1 N(M_1, t) \times \frac{dP}{dt}(M_1 \rightarrow M_1 + \Delta M_1; t) P_2(M_2, t_2 | M_1 + \Delta M_1, t). \]  

(A15)

Writing \( \sigma^2(M_1 + \Delta M_1) = \sigma_i^2 \) and integrating between times \( t_1 \) and \( t_2 \) corresponding to \( \delta_c(t_i) \equiv \delta_i \) one gets

\[ \int \frac{dM_1}{M_1} \frac{d\sigma_i^2}{d\sigma_i^2} \int \frac{d\sigma_i^2}{2\pi} \frac{1}{(\sigma_i^2 - \sigma_2^2)^{3/2}} \left( \frac{1}{\sigma_i^2 - \sigma_2^2} \right)^{1/2} \left[ 1 - e^{-\frac{(\delta_1 - \delta_2)^2}{2(\sigma_i^2 - \sigma_2^2)}} \right] \]  

\[ = \int_{x_1}^{x_2} dx \frac{1}{x^{3/2}(y-x)^{3/2}} \left[ 1 - e^{-A^2/(2x)} \right] \]  

(A16)

Consider the integral over \( \sigma_i^2 \):

\[ \int \frac{d\sigma_i^2}{2\pi} \frac{1}{(\sigma_i^2 - \sigma_2^2)^{3/2}} \left( \frac{1}{\sigma_i^2 - \sigma_2^2} \right)^{1/2} \left[ 1 - e^{-\frac{(\delta_1 - \delta_2)^2}{2(\sigma_i^2 - \sigma_2^2)}} \right] \]  

\[ = \int_{x_1}^{x_2} dx \frac{1}{x^{3/2}(y-x)^{3/2}} \left[ 1 - e^{-A^2/(2x)} \right] \]  

(A17)

where we have defined

\[ A = \delta_1 - \delta_2, \ x = \sigma_i^2 - \sigma_2^2, \ y = \sigma_1^2 - \sigma_2^2. \]  

(A18)

We then have

\[ \int_{x_1}^{x_2} dx \frac{1}{x^{1/2}(y-x)^{3/2}} = 2\sqrt{x_2} \frac{\sqrt{x_2} - 2\sqrt{x_1}}{y\sqrt{y - x_2} - 2\sqrt{y \sqrt{y - x_1}}} \]  

(A19)

For the second term we can further define \( z = 1/x \), to get

\[ + \int_{z_1}^{z_2} dz \ e^{-A^2/2z} (yz - 1)^{-3/2} = -\frac{1}{y^{3/2}} \left[ \frac{2e^{-A^2/2z}}{(z_2 - 1/y)^{1/2}} \right. \]  

\[ \left. - e^{-A^2/2y} 2\pi A (1 - \text{erf}\left[ A/2\sqrt{2}ight] (z_2 - 1/y)^{1/2}) \right] - (z_2 \rightarrow z_1) \]  

(A20)

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