Two particle-two hole final states in quasi elastic neutrino-nucleus interactions

Omar Benhar$^1$, Alessandro Lovato$^{2,3}$, and Noemi Rocco$^{4,5}$

Center for Neutrino Physics,
Virginia Polytechnic Institute and State University,
Blacksburg, VA 24061, USA
2 Argonne Leadership Computing Facility,
Argonne National Laboratory, Argonne, IL 60439, USA
3 Physics Division, Argonne National Laboratory,
Argonne, IL-60439, USA
4 INFN, Sezione di Roma, I-00185 Roma, Italy
5 Department of Physics,
“Sapienza” Università di Roma, I-00185 Roma, Italy

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Experimental studies of neutrino-nucleus interactions carried out over the past decade have provided vast evidence of the inadequacy of the Relativistic Fermi Gas Model (RFGM), routinely employed in event generators for data analysis, to account for both the complexity of nuclear dynamics and the variety of reaction mechanisms – other than single nucleon knock out – contributing to the observed cross section.

A striking manifestation of the above problem is the large discrepancy between the predictions of Monte Carlo simulations and the double differential charged current quasi elastic (CCQE) cross section measured by the MiniBooNE collaboration using a carbon target.

As pointed out by the authors of Ref. [5], improving the treatment of nuclear effects, which is now acknowledged as one of the main sources of systematic uncertainty, will require the development of a comprehensive and consistent description of neutrino-nucleus interactions, validated through comparison to the large body of accurate electron-nucleus scattering data.

The main difficulty involved in the generalisation of the approaches successfully employed to analyse electron scattering to the case of neutrino interactions stems from the fact that, while in electron scattering the beam energy is fixed, in neutrino scattering the measured cross section is obtained by averaging over different beam energies, distributed according to the neutrino flux. As a consequence, a measurement of the energy of the outgoing charged lepton does not specify the energy transfer to the nuclear target, which determines the dominant reaction mechanism. As shown in Ref. [9], the MiniBooNE double differential cross section corresponding to a specific muon energy bin turns out to receive comparable contributions from different mechanisms, which must be all taken into account.

Many authors have suggested that the excess CCQE cross section observed by the MiniBooNE collaboration is to be ascribed to the occurrence of events with two particle-two hole (2p2h) final states, not included in Monte Carlo simulations. A consistent description of these processes within a realistic model of nuclear dynamics requires that all mechanisms leading to their appearance – Initial State Correlations (ISC) among nucleons in the target ground state, Final State Correlations (FSC) between the struck nucleon and the spectator particles, and interactions involving two-nucleon meson-exchange currents (MEC) – be included. In existing calculations carried out in the kinematical region relevant to MiniBooNE analysis, however, the initial and final nuclear states are described within the Independent Particle Model (IPM), the deficiencies of which have been most clearly highlighted over fifty years ago by Blatt and Weisskopf, in their classic Nuclear Physics book.

In this Letter, we analyse the mechanisms leading to the appearance of 2p2h final states, and argue that interference between the amplitudes involving one- and two-nucleon currents, not consistently accounted for in Refs. [10] [11], may play a critical role. We also outline a novel approach, based on a generalisation of the factorisation ansatz implied in the impulse approximation (IA) scheme, allowing one to treat one- and two-nucleon current contributions on the same footing.

The nuclear electroweak current, determining the nuclear response to electron and neutrino interactions, can be written as a sum of one- and two-nucleon contribu-
tions according to (see, e.g., Ref. [13])

\[ J_A^\mu = \sum_i j_i^\mu + \sum_{j>i} j_{ij}^\mu. \]  

The one-body operator \( j_i^\mu \) describes interactions involving a single nucleon, and can be expressed in terms of the vector and axial-vector form factors. The two-body current \( j_{ij}^\mu \), on the other hand, accounts for processes in which the beam particle couples to the currents arising from meson exchange between two interacting nucleons.

It is very important to realise that, in scattering processes involving interacting many-body systems, 2p2h final states can be produced through the action of both one- and two-nucleon currents. Within the IPM, however, in which interaction effects are described in terms of a mean field, 2p2h states can only be excited by two-body operators, such as those describing MEC. In order for the the matrix element of a one-body operator between the target ground state and a 2p2h final state to be non vanishing, the effects of dynamical nucleon-nucleon (NN) correlations, ignored altogether in the IPM picture, must be included in the description of the nuclear wave functions.

Correlations give rise to virtual scattering between target nucleons, leading to the excitation of the participating particles to continuum states. The ISC contribution to the 2p2h amplitude arises from processes in which the beam particle couples to one of these high-momentum nucleons. The FSC contribution, on the other hand, originates from scattering processes involving the struck nucleon and one of the spectator particles, that also result in the appearance of 2p2h final states.

In the kinematical region corresponding to moderate momentum transfer, typically \(|q| < 400\) MeV, in which non relativistic approximations are expected to be applicable, ISC, FSC and MEC can be consistently described within advanced many-body approaches based on realistic models of nuclear dynamics, strongly constrained by the properties of the exactly solvable two- and three-nucleon systems [13]. The results of non relativistic calculations, whilst not being directly comparable to experimental data at large momentum transfer, can provide valuable insight on the interplay of the different mechanisms leading to the excitation of 2p2h final states.

The authors of Ref. [14] have recently reported the results of an accurate calculation of the sum rules of the electromagnetic response of carbon in the longitudinal and transverse channels, carried out within the Green’s Function Monte Carlo (GFMC) computational scheme. Exploiting the completeness of the set of final states entering the definition of the nuclear inclusive cross section, these sum rules can be easily related to the energy-loss integrals of the longitudinal and transverse components of the tensor describing the target response to electromagnetic interactions [8].

Choosing the \( z \)-axis along the direction of the momentum transfer, \( q \), the transverse sum rule can be written in the form

\[ S_T(q) = \int d\omega S_T(q, \omega), \]  

where

\[ S_T(q, \omega) = S^{xx}(q, \omega) + S^{yy}(q, \omega), \]  

with \((\alpha, \beta = 1, 2, \text{ and } 3)\) label the \( x\)-, \( y\)- and \( z\)-component of the current, respectively

\[ S^{\alpha\beta} = \sum_N \langle 0 | J_A^\alpha | N \rangle \langle N | J_A^\beta | 0 \rangle \delta(E_0 + \omega - E_N) . \]  

In the above equation, \(|0\) and \(|N\) denote the initial and final nuclear states, the energies of which are \( E_0 \) and \( E_N \). The generalisation of Eqs. (2)-(4) to the case of charged current weak interactions is discussed in Ref. [15].

We have employed the approach of Ref. [14] to pin down the contribution of the terms arising from interference between correlations and MEC to the transverse sum rule, which is long known to be strongly affected by processes involving two-nucleon currents.

![FIG. 1: Sum rule of the electromagnetic response of carbon in the transverse channel. The dashed line shows the results obtained including the one-nucleon current only, while the solid line corresponds to the full calculation. The dot-dash line represents the sum rule computed neglecting interference terms, the contribution of which is displayed by the dotted line. The results are normalised so that the dashed line approaches unity as \(|q| \to \infty\). Monte Carlo errors bars are not visible on the scale of the figure.](image)
interference terms are generated by adding ad hoc contributions to the two-body current \[16\]. However, this procedure does not properly account for correlations arising from the strong repulsive core of the NN interaction. Furthermore, it disregards correlations among the spectator particles altogether.

The results of Fig. 1 clearly point to the need for a consistent treatment of correlations and MEC within a formalism suitable for application in the kinematical regime in which non relativistic approximations are known to fail. The relativity issue is of paramount relevance to the analysis of neutrino data, because the mean momentum transfer of CCQE events obtained by averaging over the MiniBooNE \[4\] and Minerva \[17\] neutrino fluxes turn out to be \(\sim 640\) and \(\sim 880\) MeV, respectively. Comparison between the solid and dashed lines of Fig. 2 showing the nuclear matter response to a scalar probe delivering momentum \(|q| = 800\) MeV, demonstrates that the non relativistic approximation fails to predict both position and width of the quasi elastic bump. The calculations have been carried out using the formalism described in Ref. \[8\].

The effects of ISC on the nuclear cross section at large momentum transfer can be taken into account within the formalism based on the IA using realistic spectral functions \[18, 19\]. The IA scheme rests on the assumptions that at momentum transfer such that \(|q|^{-1} \ll d, d\) being the average separation distance between nucleons in the target nucleus, the contribution of the two-nucleon current can be disregarded and the final state \(|N\rangle\) of Eq. (4) can be written in the factorized form

\[ |N\rangle = |pp\rangle \otimes |n_{A-1}, p_n\rangle, \]

where the state \(|pp\rangle\) describes a non interacting nucleon carrying momentum \(p\), while \(|n_{A-1}, p_n\rangle\) describes the \((A-1)\)-particle spectator system in the state \(n\), with momentum \(p_n\). Note that, owing to NN correlations, \(|n_{A-1}, p_n\rangle\) is not restricted to be a bound state.

Within the IA, the contribution to the nuclear cross section arising from interactions involving the one-nucleon current is written in terms of the cross section of the elementary scattering process involving an individual nucleon and the nuclear spectral function \(P(k, E)\), dictating its energy and momentum distribution, according to \[8\]

\[
d\sigma_{IA} = \int d^3k \, dE \, P(k, E) \, d\sigma_{elem}. \tag{6}\]

![FIG. 2: Nuclear matter response to a scalar probe delivering momentum \(|q| = 800\) MeV. The solid and dashed lines have been obtained using relativistic and non relativistic kinematics, respectively.](image)

Figure 3 illustrates the \(2p2h\) contribution to the electron-carbon cross section, at beam energy \(E_e = 730\) MeV and electron scattering angle \(\theta_e = 37\) deg, plotted as a function of the energy loss. The solid and dashed lines represent the result of the full calculation and the contribution arising from amplitudes involving \(2p2h\) final states.

The factorisation ansatz of Eq. (7) can be readily extended to allow for a consistent treatment of the matrix elements of one- and two-nucleon currents. The resulting expression is

\[
|N\rangle = |pp'\rangle \otimes |m_{A-2}, p_m\rangle, \tag{7}\]

where the states \(|pp'\rangle\) and \(|m_{A-2}, p_m\rangle\) describe two non interacting nucleons of momenta \(p\) and \(p'\) and the \((A-2)\)-particle spectator system, respectively.

Using Eq. (7), the nuclear matrix element of the two-nucleon current can be written in terms of two-body matrix elements according to

\[
\langle N|j_{ij}^{\mu}|0\rangle = \int d^3kd^3k' \, M_m(k, k')|pp'\rangle|j_{ij}^{\mu}|kk'\rangle, \tag{8}\]
with the amplitude $M_m(k,k')$ given by

$$M_m(k,k') = \langle n_{(A-2)}, p_m | \otimes \langle kk' | 0 \rangle . \quad (9)$$

Within the scheme outlined in Eqs. [7]–[9], the nuclear amplitude $M_m(k,k')$ turns out to be independent of $q$, and can therefore be obtained within non-relativistic many-body theory without any problems. On the other hand, the two-nucleon matrix element can be evaluated using the fully relativistic expression of the current.

The connection with the spectral function formalism becomes apparent noting that the two-nucleon spectral function $P(k,k',E)$, yielding the probability of removing two nucleons from the nuclear ground state leaving the residual system with excitation energy $E$, is defined as [20]

$$P(k,k',E) = \sum_m |M_m(k,k')|^2 \delta(E + E_0 - E_m) , \quad (10)$$

with $M_m(k,k')$ given by Eq. [9].

The two-nucleon spectral function of uniform and isospin symmetric nuclear matter at equilibrium density has been calculated by the authors of Ref. [20] using a realistic hamiltonian. The resulting relative momentum distribution, defined as

$$n(Q) = 4\pi|Q|^2 \int d^3 K n \left( \frac{Q}{2} + K, \frac{Q}{2} - K \right) \quad (11)$$

where $K = k + k'$, $Q = (k - k')/2$, and

$$n(k,k') = \int dE \, P(k,k',E) , \quad (12)$$

is shown by the solid line of Fig. 4. Comparison with the prediction of the Fermi Gas (FG) model, represented by the dashed line, indicates that correlation effects are sizeable, and give rise to a quenching of the peak of the distribution, along with the appearance of a high momentum tail.

Note that the ansatz of Eq. [5] implies neglecting all Final State Interactions (FSI) between the nucleon interacting with the beam particles and the spectators, including FSC.

In inclusive processes, FSI lead to: i) a shift of the energy loss spectrum, arising from interactions between the knocked out nucleon and the mean field of the recoiling nucleus, and ii) a redistribution of the strength from the quasi free bump to the tails, resulting from FSC. Theoretical studies of electron-nucleus scattering suggest that in the kinematical region relevant to the MiniBooNE analysis the former mechanism, which does not involve the appearance of 2p2h final states, provides the dominant contribution [8]. The inclusion of FSI within the IA scheme has been recently discussed in Ref. [21].

The results presented in this Letter show that interference between the different reaction mechanisms leading to the excitation of 2p2h final states plays an important role, and must be taken into account using a description of nuclear structure that includes NN correlations. Models in which the processes involving MEC are treated within the framework of the IPM, such as those of Refs. [10, 11], appear to be conceptually inconsistent, although the impact of this issue on the numerical results needs to be carefully investigated. The treatment of MEC based on the extension of the $y$-scaling analysis, extensively employed in electron scattering studies, also fails to account for interference effects [22].

The extension of the factorisation scheme underlying the IA appears to be a viable option for the development of a unified treatment of processes involving one- and two-nucleon currents in the region of large momentum transfer. We believe that the implementation of the approach outlined in this work may in fact be regarded as a first step towards the new paradigm advocated by the authors of Ref. [9].

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