Recently, it has been shown [1-4] that the electroweak symmetry of the Standard Model may be broken dynamically by a $t\bar{t}$ condensate. This is referred to in the literature [5] as “top-mode Standard Model”. The top quark, being much heavier than the other known fermions (and lying close in the mass spectrum to the electroweak scale $v = 246$ GeV), may, in this picture, be responsible for the breaking of the $SU(3)_c \times SU(2)_L \times U(1)_Y$ to $SU(3)_c \times U(1)_{em}$. It has been shown [2] that in this model, where the presence of a four-fermion interaction of the form $G(\overline{\psi}_L t_R)(\bar{t}_R \psi_L)$ induces the symmetry breaking, the bound-state spectrum consists of three massless Nambu-Goldstone bosons, which give masses to the massless gauge bosons, and one massive neutral scalar, which may be identified as the Higgs.

This model has several attractive features. First, the naturalness problem arising in the elementary scalar sector of the Standard Model can be isolated in the coupling constant $G$ once for all. Second, no elementary scalar is necessary for the theory, and there is no problem regarding the violation of the unitarity bound in $WW \to WW$ scattering. Third, a definite relationship can be established between the top mass and the Higgs masses reducing thereby (to some extent) the embarrassingly great laxity in the choice of parameters of the otherwise so successful Standard Model. Finally, there are physical examples of dynamical symmetry breaking at the eV scale (BCS theory of superconductivity) and the MeV scale (the breaking of chiral symmetry for nucleons) and it would be aesthetically satisfying if the mechanism should recur again at this higher scale of energy.

Although the above model is elegant and economical in the sense that it does not predict any new particle (even the Higgs scalar is a composite object), unfortunately the top-quark mass $m_t$ in this model, as determined from the renormalization group flow of the coupling constants, appears to be untenable with the present experimental upper bound of 190 GeV. To resolve this difficulty within the same framework, it was proposed [6-8] that one can include an additional $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant term in the Lagrangian, which is of the form

$$G'(\overline{\psi}_L^I A_P^P)(\overline{t}_R^J A_Q^Q)(\chi_{KM}^M \chi_{I}^I).$$

(1)

Here $G'$ is the coupling constant (of mass dimension $-2$) for the new interaction, $i$ is the $SU(2)_L$ index and $I, J, P, Q$ are the $SU(3)_c$ indices running from 1 to 3. The $A$ matrices are the real generators of $SU(3)_c$ à la Okubo which we find more convenient for our problem than the usual Gell-Mann matrices [9]. The four-fermionic interaction being nonrenormalizable in $3 + 1$ dimensions, a high energy cutoff $\Lambda$ is needed for the regularization of this theory. Effectively, this means that the theory ceases to be valid beyond $\Lambda$. For simplicity, we will use the same cutoff for all four-fermionic operators.

In a theory with strong coupling, one can use the perturbative analysis in the low-energy limit by introducing auxiliary fields [10] in the action. Alternatively, one can write down a low-energy Lagrangian, which, at a high-energy scale, when all auxiliary fields are integrated out, gives back the Lagrangian with four-fermionic interaction. For this, one has to suitably define the different renormalization constants, taking account of compositeness [2]. Following the latter approach, we will define the effective potential of our theory to be

$$V = -\mu^2 \phi^i \phi_i + m^2 \chi_{IJ}^i \chi_{IJ}^i + a_1 (\phi^i \phi_i)^2 + a_2 (\chi_{IJ}^i \chi_{IJ}^i)^2$$

$$+ a_3 (\chi_{IJ}^i \chi_{IK}^i \chi_{JK}^i \chi_{IK}^i) + a_4 (\chi_{IJ}^i \chi_{IJ}^i \chi_{KM}^M \chi_{KM}^M).$$

1
than (m), SU fields in an
note that though the field is an auxiliary one arising as a composite of two spinor
m gives so that succeeding analysis, we take some phenomenologically plausible values for
evident from eq. (4) that m
was done for the “Higgs scalar” in ref. 2, because here the strong interaction plays
a nontrivial part and the 1/N approximation is not valid. Not being determinable from the renormalization group equations, m^2 remains a free parameter of the
theory. When the symmetry is broken, two more terms of the form 1/2 m^{ij} v^2 and
1/2 a_6 v^2 contribute to m_{\chi}^2, the second term contributing only for the neutral field, so that
m_{\chi}^2 - m_{\chi^+}^2 = 1/2 a_6 v^2.

These new bosons can have profound consequences through various one-loop
effects which are experimentally observable, and we can place lower bounds on
their masses. One notes that the interaction in the minimal condensate scheme is
confined only to the third generation of quarks. The other quark generations take
part in the one-loop effects through the mixing between the mass eigenstates and the weak eigenstates of the quark wavefunctions [11]. This means that the physics of $K^0 - \bar{K}^0$, $B^0_d - \bar{B}^0_d$ and $B^0_s - \bar{B}^0_s$ mixing will be affected by $\chi$, and the same is true for the $CP$-violating $\epsilon$ parameter. In an earlier paper [8] we have discussed these effects in detail and showed that we can obtain a lower bound on the mass of the charged scalar $\chi^+$, which is of the order of a few hundreds of GeV. Another bound can be extracted from the observed rate of the radiative $B$-decays [12], which is of the same order of magnitude, and which is free from a number of undetermined or poorly determined parameters which entered in ref. 8. We have also shown that the maximum mass splitting in the doublet cannot be greater than 47 GeV [13].

The chief obstacle to putting phenomenological constraints on the model from low-energy data such as $B^0_d - \bar{B}^0_d$ mixing arises, as usual, from uncertainties in the hadronic parameters such as the decay constants $f_K$, $f_B$ and the bag parameters $B_K$, $B_B$. In this note we investigate a different observable, viz, the ratio $R_b$, defined as

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{hadrons})}.$$  \hfill (6)

$R_b$ is relatively free from uncertainties in hadronic parameters which tend to cancel out of numerator and denominator. It is also relatively insensitive to $m_t$ and QCD corrections. For this reason, the effects of new physics can show up in $R_b$ without being masked by uncertainties in $m_t$ etc., as in the case with a number of other phenomenologically interesting parameters. An analysis of the model using $R_b$ is also facilitated by the fact that the experimental error in its determination has come down drastically with the LEP measurements and the advent of microvertex detectors, and now stands at [15]

$$R_b \ (\text{expt.}) = 0.2201 \pm 0.0031$$  \hfill (7)

at 95% C.L., which is remarkably precise.

To fix ideas and notations, let us briefly discuss the features of $\Gamma(Z \to b\bar{b})$ and $R_b$ in the Standard Model [16-17]. The tree-level contribution to $\Gamma(Z \to b\bar{b})$ is

$$\Gamma^0(Z \to b\bar{b}) = \frac{G_\mu m_Z^3}{8\pi\sqrt{2}} \sqrt{1 - 4\mu_b} \left[1 - 4\mu_b + (1 - \frac{4}{3}\sin^2 \theta_W)^2(1 + 2\mu_b)\right]$$  \hfill (8)

where $\mu_b = m_b^2/m_Z^2$, $G_\mu$ is the Fermi coupling constant as obtained from muon decay and $\theta_W$ is the weak mixing angle.

The electroweak radiative corrections appear in the form of two form factors $\kappa_b$ and $\rho_b$, respectively for effective mixing angle and the overall renormalization. Thus, the decay width, calculated to one-loop, is given by

$$\Gamma^1(Z \to b\bar{b}) = \frac{G_\mu m_Z^3}{8\pi\sqrt{2}} \rho_b \sqrt{1 - 4\mu_b} \left[1 - 4\mu_b + (1 - \frac{4}{3}\sin^2 \theta_W \kappa_b)^2(1 + 2\mu_b)\right]$$  \hfill (9)

where $\sin^2 \theta_W \cos^2 \theta_W = \frac{\pi\alpha}{\sqrt{2}G_\mu m_Z^2(1 - \Delta r)}$,  \hfill (10)
\( \Delta r \) being the electroweak correction to \( \mu^\pm \) decay.

The one-loop correction is dominated by the top quark contribution. The vacuum polarization effect, which is common to all fermionic final states, is denoted by \( \Delta \rho_t \), which is given by

\[
\Delta \rho_t = \frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{\Pi_{WW}(0)}{m_W^2} = \frac{3G_{\mu}m_t^2}{8\pi^2\sqrt{2}} \approx \frac{\alpha m_t^2}{\pi m_Z^2},
\]

(11)

the \( b \)-mass having been neglected. The \( \Pi \) functions are the standard ones used to denote the vacuum polarization of the gauge bosons. For \( Z \rightarrow b\bar{b} \), the vertex corrections give

\[
\Delta \rho_b = -\frac{4}{3} \Delta \rho_t
\]

(12a)

\[
\sin^2 \theta_W \Delta \kappa_b = \frac{2}{3} \sin^2 \theta_W \Delta \rho_t.
\]

(12b)

Taking both these factors into account, we can write

\[
\rho_b = 1 + \Delta \rho_t + \Delta \rho_b + \cdots
\]

(13a)

\[
\kappa_b \sin^2 \theta_W = \sin^2 \theta_W + \cos^2 \theta_W \Delta \rho_t + \frac{2}{3} \sin^2 \theta_W \Delta \rho_t + \cdots
\]

(13b)

After some straightforward algebra, it can be shown that

\[
R_b = \frac{13}{59} \left( 1 + \frac{46}{59} \delta^{(t)}(m_t) + \frac{24}{767} \frac{g_{VL}}{g_A l} + \frac{0.1\alpha_s(m_Z^2)}{\pi} \right)
\]

(14)

with \( \sin^2 \theta_W = 0.2324 \), and \( \delta^{(t)}(m_t) \) and \( g_{VL}/g_A l \) are given by

\[
\delta^{(t)}(m_t) \simeq -\frac{20\alpha}{13\pi} \frac{m_t^2}{m_Z^2} - \frac{10\alpha}{3\pi} \ln \frac{m_t^2}{m_Z^2}
\]

(15)

\[
\frac{g_{VL}}{g_A l} = 1 - 4 \sin^2 \theta_W.
\]

(16)

The top-dependent contribution is of the order of \( 10^{-3} \), the same as the order of experimental errors, and thus the variation of \( R_b \) with \( m_t \) over the allowed range of the latter is rather flat. As stated above, this is one of the reasons for which \( R_b \) is phenomenologically interesting. In our model, another term of the form

\[
\delta^X(R_b) = \frac{13}{59} \frac{46}{59} (\delta^X(m_t) - \delta^X(0))
\]

(17)

gets added to the above contribution. We take only the non-oblique part as it is known [13] that the oblique part has negligible contribution. It is noteworthy that the effective Lagrangian only favors the production of left-handed \( b \) quarks, but since the same is true for the tree-level case, it will not cause any significant change in the electroweak asymmetries.
In the limit \( m_b \to 0 \), we can introduce the effects of the new physics through a change in the vertex factors for the \( Z \to b \bar{b} \) coupling:

\[
v_L' = v_L + \frac{8}{3} \frac{\alpha}{4\pi \sin^2 \theta_W} F_L(P^2, m_t) \tag{18a}
\]

\[
v_R' = v_R \tag{18b}
\]

where \( P \) is the four-momentum of the external \( Z \), and the color factor of \( 8/3 \) comes from the octet nature of \( \chi \) under \( SU(3)_c \). The right-handed coupling is not changed as no term of the form \( \bar{t}_L b_R \chi \) is allowed in the Lagrangian. The function \( F_L \) represents the total of all one-loop correction effects, depicted in Fig. 1. It can be written as the sum of three terms,

\[
F_L = F_L^a + F_L^b + F_L^c, \tag{19}
\]

where \( F_L^a, F_L^b \) and \( F_L^c \) denote the contributions from the figures 1a, 1b and 1c respectively. The correction works out to be

\[
\delta_b^X(m_t) - \delta_b^X(0) = \frac{8}{3} \frac{\alpha}{4\pi \sin^2 \theta_W} \frac{2v_L}{v_L^2 + v_R^2} F_L(m_Z^2, m_t) \tag{20}
\]

with

\[
v_L = -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W, \quad v_R = \frac{1}{3} \sin^2 \theta_W. \tag{21}
\]

The \( F_L \) functions are

\[
F_L^a = b_1(m_\chi, m_t, m_b^2) v_L \lambda_L^2, \tag{22a}
\]

\[
F_L^b = \left[ \frac{P^2}{\mu_R^2} c_6(m_\chi, m_t, m_t) - \frac{1}{2} - c_0(m_\chi, m_t, m_t) \right] v_R^{(t)}
+ \frac{m_t^2}{\mu_R^2} c_2(m_\chi, m_t, m_t) v_L^{(t)} \lambda_L^2, \tag{22b}
\]

\[
F_L^c = c_0(m_t, m_\chi, m_\chi)(\frac{1}{2} - s^2) \lambda_L^2 \tag{22c}
\]

where \( \lambda_L = g_t^L/g, g \) being the usual \( SU(2)_L \) coupling constant, \( m_\chi \) is the mass of the charged \( \chi \), \( \mu_R \) is the mass scale arising in dimensional regularization, and

\[
v_R^{(t)} = -\frac{2}{3} \sin^2 \theta_W, \quad v_L^{(t)} = \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W. \tag{23}
\]

The two- and three-point functions \( b_1, c_0, c_2 \) and \( c_6 \) in terms of the well-known Passarino-Veltman functions \([18]\) are \([16]\)

\[
b_1(m_1, m_2) = B_1(m_2, m_1) + \frac{1}{2}(\Delta - \ln \mu_R^2), \tag{24a}
\]
\[ c_0(m_1, m_2, m_3) = -2C_{24}(m_2, m_1, m_3) + \frac{1}{2}(\Delta - \ln \mu_R^2), \]  
(24b)

\[ c_2(m_1, m_2, m_3) = \mu_R^2 C_6(m_2, m_1, m_3), \]  
(24c)

\[ c_6(m_1, m_2, m_3) = -\mu_R^2[C_{23} + C_{11}](m_2, m_1, m_3) \]  
(24d)

where \( \Delta = 2/(4 - d) - \gamma - \ln \pi \) in \( d \) dimensions, and this divergence cancels out in the final formula for \( F_L \).

In Fig. 2 we show the plot of \( \delta \chi(R_b) \) with \( m_\chi \) for the top mass ranging from 110 GeV to 200 GeV. The corresponding \( g'_t \) values can be obtained from eq. (4). We have taken \( \sin^2 \theta_W = 0.2324 \), \( m_b = 4.7 \) GeV and \( \alpha_s(m_Z^2) = 0.117 \). We have checked that very little change in the final results occur if we take into account the errors in \( \alpha_s(m_Z^2) \) and \( \sin^2 \theta_W \).

In Fig. 3 we plot the lower bound on \( m_\chi \) for different \( m_t \), ranging from 100 to 180 GeV. It may be noted that this bound goes as \( m_t^2 \) and \( m_t = 190 \) GeV is the maximum allowed limit. For \( m_t = 150 \) GeV, we get \( m_\chi = 380 \) GeV as the lower limit. It is to be noted that the \( \chi \)-contribution, being negative, makes the bound very stringent in nature. The new physics decouples in the limit \( m_\chi \rightarrow \infty \); this result is in conformity with those obtained earlier [8, 12-13]. Of course, this is just a technical point since \( \chi \) is a composite object and it is meaningless to carry \( m_\chi \) beyond the compositeness scale. So it can be claimed that within the framework of this model, \( m_\chi \) has both an upper as well as a lower bound, and these come closer and finally coincide as the cutoff \( \Lambda \) is decreased. This behaviour can be easily explained; if we decrease \( \Lambda \), \( (m_t)_{BHL} \) will increase, so there will be a corresponding increase in \( g'_t \), and \( \delta \chi_b \) functions are proportional to \( g'_t^2 \). The coincidence occurs at about \( \Lambda = 1 \) TeV.

In this work, therefore, we have investigated the effects of isodoublet color-octet composite scalars arising in a realistic model with dynamical breaking of electroweak symmetry. The specific process focussed on is the decay \( Z \rightarrow b\bar{b} \), since the ratio \( R_b \) is precisely determined and well-known to be relatively free from uncertainties in the Standard Model parameters such as \( m_t \). We find that a stringent lower bound can be placed on the masses of these composite colored scalars, which is around 400 GeV for a top mass of 150 GeV, and increases quadratically with \( m_t \). At the present moment, one of the main issues confronting \( Z \) decay experiments is to determine the difference between the Standard Model prediction of \( R_b \) and the experimental number, because this is one more possible gateway to look into the new physics. The Minimal Supersymmetric Standard Model predicts a positive (but small) \( \delta R_b \), and nearly all extensions and modifications in the scalar sector (whether elementary or composite) predict \( \delta R_b \) to be slightly negative. None of the alternatives can be ruled out at the present moment, and further precision experiments could help discriminate between models.

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Figure Captions
1. The one-loop diagrams involving the colored scalars $\chi^{\pm}$ which contribute to $\Gamma(Z \to b\bar{b})$.
2. The contribution of the $\chi^{\pm}$ to the parameter $R_b$ as a function of $m_{\chi^+}$. The shaded region depicts the phenomenologically allowed region for the top mass.
3. The lower bound $\bar{m}_\chi$ on the mass of $\chi^{\pm}$ as a function of $m_t$ for $R_b = 0.2170$. 
