THE INITIAL HELIUM CONTENT OF GALACTIC GLOBULAR CLUSTER STARS FROM THE R-PARAMETER: COMPARISON WITH THE COSMIC MICROWAVE BACKGROUND CONSTRAINT

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ABSTRACT

Recent precise determinations of the primordial He abundance (\(Y_p\)) from cosmic microwave background (CMB) analyses and cosmological nucleosynthesis computations provide \(Y_p = 0.248 \pm 0.001\). On the other hand, recent works on the initial He abundance of Galactic globular cluster (GGC) stars, making use of the \(R\)-parameter as an He indicator, have consistently obtained \(Y_{GGC} \approx 0.20\). In light of this serious discrepancy, which casts doubt on the adequacy of low-mass He-burning stellar models, we have rederived the initial He abundance for stars in two large samples of GGCs by employing theoretical models computed using new and more accurate determinations of the equation of state for the stellar matter and of the uncertain \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) reaction rate. Our models include semiconvection during the central convective He-burning phase, while the breathing pulses are suppressed, in agreement with the observational constraints coming from the measurements of the \(R\)-parameter in a sample of clusters. By taking into account the observational errors on the individual \(R\)-parameter values, as well as uncertainties in the GGC [Fe/H] scale, treatment of convection, and \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) reaction rate, we have obtained a mean \(Y_{GGC} = 0.243 \pm 0.006\) and \(Y_{GGC} = 0.244 \pm 0.006\) for the two studied GGC samples. These estimates are now fully consistent with \(Y_p\) obtained from CMB studies. Moreover, the trend of the individual He abundances with respect to [Fe/H] is consistent with no appreciable He enrichment along the GGC metallicity range.

Subject headings: cosmology: miscellaneous — globular clusters: general — stars: abundances — stars: evolution — stars: horizontal-branch

1. INTRODUCTION

Galactic globular cluster (GGC) stars are the oldest objects in the Galaxy, and their initial He abundance \((Y_{GGC}\), where \(Y\) denotes the mass fraction of He) is supposed to be approximately equal to the primordial He abundance \((Y_p)\) produced during big bang nucleosynthesis (BBN).

The value of \(Y_p\) derived from spectroscopy of low-metallicity, extragalactic H\ II regions appears to be still subject to systematic uncertainties (see, e.g., the discussion in Bono et al. 2002 and references therein); as an example, Olive, Steigman, & Skillman (1997) determined \(Y_p = 0.234 \pm 0.002\), while Izotov & Thuan (1998) obtained \(Y_p = 0.244 \pm 0.002\), consistent with earlier findings by Kunth & Sargent (1983).

On the other hand, recent determinations of the cosmological baryonic matter density \((\Omega_b)\) from the cosmic microwave background (CMB) power spectrum obtained by the BOOMERANG, DASI, and MAXIMA experiments provide consistently (e.g., Pryke et al. 2002; Odman et al. 2002; Sievers et al. 2003) a value of \(\Omega_b h^2 = 0.022 \pm 0.003\) \((h\) is the Hubble constant in units of 100 km Mpc\(^{-1}\) s\(^{-1}\)). This baryon density coupled with standard BBN calculations (Burles, Nollett, & Turner 2001) provides \(Y_p = 0.248 \pm 0.001\); such an independent determination of \(Y_p\) is close to the spectroscopic determination by Izotov & Thuan (1998).

As for the value of \(Y_{GGC}\), empirical estimates are necessarily indirect, since He lines are not detectable in GGC star spectra, apart from the case of hot horizontal-branch (HB) objects, whose atmospheres are, however, affected by gravitational settling and radiative levitation, which strongly alter the initial chemical stratification (see, e.g., Michaud, Vauclair, & Vauclair 1983; Moehler et al. 1999). The \(Y_{GGC}\) estimates make use of results from stellar evolution, taking advantage of the fact that the evolution of low-mass Population II stars is affected by their initial He content. In more detail, the so-called \(R\)-parameter (Iben 1968), defined as the number ratio of HB stars to red giant-branch (RGB) stars brighter than the HB level \((R = N_{HB}/N_{RGB})\), can be employed in order to determine \(Y_{GGC}\) and therefore \(Y_p\). The basic idea behind the use of this parameter as an He indicator is that, at a given metallicity, a higher initial He content implies a brighter HB and, in turn, a lower value of \(N_{RGB}\) (\(N_{HB}\) is only slightly affected), with a consequent increase of \(R\). Other parameters derived from stellar evolution can also be employed (see, e.g., the
discussion in Sandquist 2000, hereafter S00; Zoccali et al. 2000, hereafter Z00), but they are better suited to determine relative He abundances than absolute ones.

Following earlier analyses by Buzzoni et al. (1983) and Caputo, Martinez Roger, & Paez (1987), S00 and Z00 have recently estimated $Y_{\text{GGC}}$ by measuring the value of $R$ in large samples of GGCs (43 objects taken from various sources, in the case of the S00 paper; 26 objects with homogeneous Hubble Space Telescope [HST] photometry, for the Z00 paper) and compared the observational values with results from stellar evolution models. In both cases, a value of $Y_{\text{GGC}} \approx 0.20$ was found, in severe disagreement with the CMB result and spectroscopic determinations for H ii regions. This large discrepancy between the CMB and $R$-parameter results casts doubt on the ability of stellar models to accurately predict the evolutionary times of these crucial phases of stellar evolution. Reasons for this low He content inferred from the $R$-parameter have been ascribed to the uncertainty of the $^{12}\text{C} (\alpha, \gamma)^{16}\text{O}$ reaction rate, which is relevant during the late stages of central He burning, but may in principle also be due, for example, to an improper treatment of the mixing in the central convective region of HB stars, which affects the HB evolutionary timescale and hence $N_{\text{HB}}$ (see, e.g., Z00).

In this paper we present a new determination of $Y_{\text{GGC}}$ from the $R$-parameter, employing in the model computation the most recent determination of the $^{12}\text{C} (\alpha, \gamma)^{16}\text{O}$ reaction rate (Kunz et al. 2002), together with an improved equation of state (EOS) for the stellar matter (A. W. Irwin, F. J. Swenson, D. A. VandenBerg, & F. J. Rogers 2003, in preparation). We show that with these two improved physical inputs the discrepancy between the CMB and $R$-parameter results almost completely disappears. This gives strong support to the accuracy of present HB stellar models and the adequacy of our convective core mixing treatment during the HB phase. In § 2 we briefly discuss the observational data and the theoretical stellar models, while in §§ 3 and 4 the results are presented and discussed. Our conclusions follow in § 5.

2. OBSERVATIONAL DATA AND THEORETICAL MODELS

We have determined $Y_{\text{GGC}}$ making use of our theoretical models and the observational databases presented by Z00 and S00. Z00 provide empirical $R$-values for 26 GGCs spanning all of the relevant metallicity range. The number of RGB stars is computed starting from the level of the observed $V$ magnitude of the zero-age HB (ZAHB; the lower envelope of the observed HB star distribution). This means that to compare the models with Z00 data one has to first transform theoretical bolometric luminosities to $V$ magnitudes and then determine the theoretical $R$-values following the definition of $R$ used by Z00.

S00 provides $R$-values for 43 GGCs but employs a slightly different definition of $R$, the same as Buzzoni et al. (1983): the reference level for the RGB counts is the bolometric luminosity corresponding to the average $V$ magnitude of HB stars. In order to determine the level of the RGB corresponding to this bolometric luminosity, S00 has applied to the observational data a relationship for the difference in bolometric corrections between HB and RGB stars (which is different from the one we applied to our models when using the Z00 definition for the $R$-parameter).

We have compared our theoretical results with these two databases taking into account these two different definitions of $R$. The derived He abundances thus reflect the theoretical uncertainties related to the different bolometric corrections employed in the two methods and the different observational samples.

In order to take into account current empirical uncertainties in the GGC metallicity scale, we have used for the individual clusters [Fe/H] values given by Rutledge, Hesser, & Stetson (1997) on both the Carretta & Gratton (1997, hereafter CG97) and Zinn & West (1984, hereafter ZW84) scales (the internal accuracy of these [Fe/H] values is of the order of 0.10 dex). For clusters not listed by Rutledge et al. (1997), we have used the original ZW84 values transformed to the CG97 scale using the conversion formula given by CG97.

We have determined the existence of possible He abundance versus [Fe/H] correlations by computing the corresponding correlation coefficient and evaluating its significance. In the case of no correlation, we have determined $Y_{\text{GGC}}$ by means of a weighted mean of the individual values, with weights inversely proportional to the square of the individual errors. We note that in Z00 the value of $Y_{\text{GGC}}$ was determined by simply considering the constant value of the He abundance best fitting the individual data points, without taking into account individual errors. The existence of a significant spread in the individual cluster He abundance has been studied by means of the statistical F-test.

We computed new theoretical models and isochrones for $Y = 0.23, 0.245$, and 0.26 using—as in the Z00 analysis—the same code, input physics, and bolometric corrections as in Cassisi & Salaris (1997), with the following modifications:

1. We have updated the energy-loss rates from plasmaneutroino processes using the most recent and accurate results provided by Haft, Raffelt, & Weiss (1994).

2. We have updated the nuclear reaction rates using the NACRE database (Angulo et al. 1999), with the exception of the $^{12}\text{C} (\alpha, \gamma)^{16}\text{O}$ reaction. For this reaction we employ the more accurate recent determination by Kunz et al. (2002), based on $\gamma$ angular distribution measurements of $^{12}\text{C} (\alpha, \gamma)^{16}\text{O}$ and a consistent $R$-matrix analysis of the process. The claimed relative uncertainty of this new rate is half the uncertainty quoted in previous determinations.

3. An improved EOS has been used.1 A full description of this EOS is in preparation (A. W. Irwin, F. J. Swenson, D. A. VandenBerg, & F. J. Rogers 2003, in preparation), so we will only summarize its principal characteristics here. The EOS is calculated using an equilibrium-constant approach to minimize the Helmholtz free energy. For realistic abundance mixtures, this approach greatly reduces the number of linear equations that must be solved per iteration so that the solution can be rapidly obtained. This speed makes it practical to call the EOS directly from the stellar-interior code without introducing the errors associated with interpolating EOS tables (S. Cassisi & A. W. Irwin 2003, in preparation; see also Dorman, Irwin, & Pedersen 1991). The equilibrium-constant approach gives numerical solutions of high quality with thermodynamic consistency that is typically better than 1 part in $10^{17}$. The “EOS1” mode of the free-energy model that is used for the present calcula-

1 The code is made publicly available at ftp://astroftp.phys.uvic.ca/pub/irwin eos/code eos demo fortran.tar.gz under the GNU General Public License.
tions includes the following: arbitrarily relativistic and degenerate free electrons (Eggleton, Faulkner, & Flannery 1973); a pressure-ionization occupation probability similar to that of Mihalas, Däppen, & Hummer (1988); a Planck-Larkin occupation probability (Rogers 1986); the exchange effect for arbitrarily relativistic and degenerate electrons (Kovetz, Lamb, & Van Horn 1972); and the Coulomb effect. The Coulomb effect is treated with the Debye-Hückel approximation in the weak coupling limit and an approximation (Pols et al. 1995) of the multicomponent combination of the one-component plasma result (DeWitt, Slattery, & Chabrier 1996) in the strong-coupling limit. A spline fit is used to interpolate between the weak and strong coupling limits. The size of the intermediate-coupling region and the size of the interaction radii that characterize the pressure-ionization occupation probability are adjusted to fit OPAL EOS tables.2

4. We have explicitly taken into account the α-enhanced chemical composition typical of Population II stars, using the same initial metal mixture employed by Salaris & Weiss (1998) and the same opacity tables; the heavy-element distribution has an average α-enhancement equal to 0.4 dex. This is potentially important for the upper metallicity end of the GGCs, since in that regime the well-known equivalence between low-mass scaled-solar and α-enhanced models with the same total metallicity (Salaris, Chieffi, & Straniero 1993) is no longer valid (Salaris & Weiss 1998; VandenBerg et al. 2000). We only note in passing that an average enhancement of 0.4 dex is in full agreement with abundance data from halo field stars (see, e.g., the discussion in Salaris & Weiss 1998), while the GGC data compiled by Carney (1996) seem to point to an α-enhancement slightly lower, about 0.3 dex. Such a small difference, if real, does not introduce any serious bias in our final Y_{GGC} estimates, because such a small difference in the α-enhancement between clusters and models can be fully compensated (also at high metallicities) by a small rescaling of the Z-[Fe/H] relationship of the theoretical models, which introduces a systematic effect of less than 0.001 on the individual cluster He-abundance estimates.

5. We have accounted for the different evolutionary times characterizing the red and blue parts of the HB. Ordinarily, the theoretical values of R are computed, as in Z00, by considering the HB evolutionary time of a star populating the middle of the RR Lyrae instability strip [log(T_{eff}) = 3.85]. This is strictly adequate only for those clusters with an HB populated at the RR Lyrae instability strip and redward (increasing total stellar mass), since the HB evolutionary timescales are basically unchanged when moving from the instability strip toward the red (see the discussion in Z00). However, stars populating the HB blueward of the instability strip do show different evolutionary times, which increase for decreasing total stellar mass. At the bluest end of a typical blue HB, the increase of the HB evolutionary time with respect to the RR Lyrae strip counterpart can amount to about 20% (Z00). We discuss in § 3.3 the implications for the derived He abundance in GGCs with a blue HB.

3. THE VALUE OF Y_{GGC} FROM THE R-PARAMETER

In this section we present separately our determination of Y_{GGC} using the Z00 and S00 samples.

2 These are distributed at ftp://www-phys.llnl.gov/pub/opacity/eos.

3.1. The Z00 Sample

Figure 1 displays the run of the empirical data by Z00 together with theoretical predictions for ages of 11 and 13 Gyr and Y = 0.245 (solid lines), as a function of [Fe/H]. At fixed age and Y, the theoretical value of R is very slowly decreasing up to [Fe/H] $\sim$ -1.15. Between [Fe/H] $\sim$ -1.15 and [Fe/H] $\sim$ -0.85, R increases steeply; this increase is due to the fact that the RGB bump, previously located at brightnesses larger than the ZAHB, moves below the ZAHB level with increasing metallicity, thus causing an abrupt decrease in the number of RGB stars brighter than the ZAHB (see, e.g., the discussion in Z00). At higher [Fe/H] values, R is again only very mildly decreasing with increasing [Fe/H]. It is also interesting to note how the dependence of R on age is restricted to the interval ranging from [Fe/H] $\sim$ -1.15 to [Fe/H] $\sim$ -0.85, which is exactly the metallicity range where the RGB bump crosses the ZAHB level. This is easily explained by the fact that the RGB bump luminosity does depend on the stellar age (e.g., Cassisi & Salaris 1997), while the ZAHB level is basically unaffected for ages typical of GGCs; in general, higher ages shift the RGB bump location toward lower luminosities.

In the same Figure 1, we also display the theoretical R-values for an age of 13 Gyr and $Y_{GGC} = 0.23$ to show the sensitivity of R to the model initial He content. The average value of the derivative $\delta R/\delta Y$ is $\sim$10.

We have estimated $Y_{GGC}$ and the associated 1σ dispersion of the individual abundances (the latter is thoroughly discussed at the end of this section) by first assuming an average age of 13 Gyr for the clusters (see, e.g., the analyses by VandenBerg 2000; Salaris & Weiss 1998, 2002); the error on the individual cluster $Y$-values has been obtained from the quoted errors on the value of R. When considering all 26 clusters together with the CG97 [Fe/H] scale, we obtained a...
weighted mean $Y_{\text{GGC}} = 0.240 \pm 0.003$ (1σ error). However, we found a clear correlation between $Y_{\text{GGC}}$ and [Fe/H] in the sense that the mean $Y_{\text{GGC}}$ obtained for clusters with [Fe/H] between $-1.15$ and $-0.85$ (the metallicity range influenced by the assumed cluster age) is $Y_{\text{GGC}} = 0.231 \pm 0.005$, while for [Fe/H] $< -1.15$ and [Fe/H] $> -0.85$ we found, respectively, $Y_{\text{GGC}} = 0.247 \pm 0.005$ and $Y_{\text{GGC}} = 0.244 \pm 0.003$ (no correlation of the individual $Y$ estimates with [Fe/H] has been found in these latter two metallicity ranges). It is evident that the mean values of $Y$ determined for [Fe/H] $< -1.15$ and [Fe/H] $> -0.85$ are in good agreement, while a substantially lower value is obtained for the clusters whose $R$-parameter is affected by the precise value of the age. We have therefore rederived the He content with a different assumption about the cluster ages. Rosenberg et al. (1999) and Salaris & Weiss (2002) have shown how clusters with [Fe/H] larger than $\sim -1.2$ (on the CG97 metallicity scale) display a large age spread and are on average younger by $\approx 2$ Gyr than the more metal-poor clusters. We have therefore recomputed the values of $Y$ assuming an age of 13 Gyr for the clusters with [Fe/H] $< -1.2$ and 11 Gyr for more metal-rich ones. As expected, the mean $Y$-values for [Fe/H] $< -1.15$ and [Fe/H] $> -0.85$ are unchanged, but this time, in the [Fe/H] range between $-1.15$ and $-0.85$, we obtain a mean $Y_{\text{GGC}} = 0.239 \pm 0.004$, which, within the 1σ error bar, is in better agreement with the results at higher and lower metallicities. It is therefore important to notice that the precise individual cluster ages do matter when determining an accurate $Y_{\text{GGC}}$ value for clusters in this [Fe/H] range.

We have repeated the previous analysis by employing the ZW84 [Fe/H] scale. Adopting an age of 13 Gyr for all clusters, we derive a mean $Y_{\text{GGC}} = 0.242 \pm 0.003$ for the whole cluster sample, and we do not find any correlation between $Y$ and [Fe/H]. However, Salaris & Weiss (2002) have shown that, when considering the ZW84 metallicity scale, clusters with [Fe/H] $> -1.6$ show a large age spread and are on average younger than the more metal-poor ones (see also VandenBerg 2000). We therefore repeated the previous calculation considering an age of 13 Gyr when [Fe/H] $< -1.6$ and 11 Gyr at higher [Fe/H]; we obtain a mean $Y_{\text{GGC}} = 0.243 \pm 0.003$, consistent with the value determined for a constant age of 13 Gyr. This result comes from the fact that, when using the ZW84 metallicities, there are no clusters populating the [Fe/H] range, which is strongly affected by age.

Another important question to be addressed is the significance of the dispersion of the individual cluster values around the mean $Y_{\text{GGC}}$. In particular, it is important to know if the observed 1σ dispersion, of the order of 0.02, is entirely due to the error on the individual cluster estimates. To address this point we have applied the statistical $F$-test (see, e.g., an application to the case of GGC ages by Chaboyer et al. 1996; Salaris & Weiss 1997, 2002) to our sample of He determinations. In the case of the CG97 [Fe/H] scale, we have restricted the analysis to the clusters within the metallicity range unaffected by age, so that an age spread will not affect the observed He-abundance dispersion. For each individual cluster we have calculated a set of synthetic He abundances by randomly generating—using a Monte Carlo procedure—10,000 abundance values, according to a Gaussian distribution with mean value equal to the observed mean $Y_{\text{GGC}}$ and $\sigma$ equal to the individual He-abundance error. This is repeated for all clusters in the selected sample, and the 10,000 values for each individual cluster are joined to produce an “expected” $Y_{\text{GGC}}$ distribution for the entire cluster sample, on the assumption that the detected He-abundance spread is not intrinsic but merely due to the individual error bars. The $F$-test has been then applied in order to determine if this “expected” distribution has the same variance as the observed one. We state that a $Y_{\text{GGC}}$ range does exist if the probability that the two distributions have different variance is larger than 95%. In the case in which this condition is verified, the size of the true $Y_{\text{GGC}}$ range ($\sigma_Y$) can be estimated according to $\sigma_Y = (\sigma_{\text{obs}}^2 + \sigma_{\text{exp}}^2)^{0.5}$, where $\sigma_{\text{obs}}$ and $\sigma_{\text{exp}}$ are, respectively, the 1σ dispersion of the actual data and of the expected distribution.

The result of this test applied to the Z00 sample with our two choices of the [Fe/H] scale indicates that the observed dispersion around the mean $Y_{\text{GGC}}$ is entirely due to the formal errors (the probability that the observed and the synthetic distributions have different variance is below 70% in both cases) on the individual determinations; therefore, no statistically significant spread in the individual He abundances is found.

### 3.2. The S00 Sample

Figure 2 displays the run of the empirical data by S00 together with theoretical predictions for ages of 11 and 13 Gyr and $Y = 0.245$ (solid lines), as a function of [Fe/H]. At fixed age and $Y$, the theoretical value of $R$ is very slowly decreasing up to [Fe/H] $\sim -0.85$. At higher metallicities the value of $R$ increases, due again to the fact that the bolometric luminosity of the RGB bump crosses the reference HB bolometric luminosity. The shift to higher metallicities of this crossing region with respect to the $R$-definition previously used arises from the fact that the bolometric luminosity of the RGB reference level corresponds to $V$ magnitudes fainter than the $V$ magnitude level of the HB. This implies that RGB bump stars are included in the determination of $R$ up to higher metallicities than in the case of the Z00 sample.
definition. This high-metallicity region is also the only one affected by age (see discussion in § 3.1); therefore, the precise choice of the GGC ages does not affect at all the results when using the S00 definition of the $R$-parameter, since only very few clusters show these high values of $[\text{Fe/H}]$ (see Fig. 2), and only on the ZW84 scale.

By assuming $t = 13$ Gyr for all GGC, we find again a $Y_{\text{GGC}}$ distribution uncorrelated with $[\text{Fe/H}]$. A mean value of $Y_{\text{GGC}} = 0.246 \pm 0.005$ is obtained when considering the CG97 [Fe/H] scale, while $Y_{\text{GGC}} = 0.241 \pm 0.004$ is derived when the ZW84 metallicities are employed. As for the dispersion of the $Y_{\text{GGC}}$ values around these means, we obtain in both cases $\sigma_Y = 0.04$; we have applied the $F$-test also in this case and derived that the dispersion cannot be completely explained by the formal errors on the individual determinations (the probability that the variances of the He-abundance distributions in the observed sample and in the synthetic one are different is larger than 99%) and has an intrinsic component equal to $\sigma_Y = 0.03$ (analogous conclusions were reached by S00).

### 3.3. Clusters with a Blue HB

All the $Y_{\text{GGC}}$ values given before have been obtained by considering the evolutionary time of HB stars in the instability strip when computing the theoretical value of $R$; this is also what has been done by Z00 and S00.

While this assumption is well founded in the case of HBs populated at the strip and redward, it is less adequate in the case of very blue HBs (see the discussion in Caputo et al. 1987 and Z00); this is particularly true when the location of the average mass populating the observed HB corresponds to a $V$ about 0.5–1.0 mag fainter than the instability strip level. This is due to the fact that, as discussed in § 2, the HB evolutionary times increase when one greatly reduces the total stellar mass with respect to the values attained at the instability strip. To correct for this possible systematic uncertainty caused by our assumption, we have applied the following procedure.

For both the Z00 and S00 samples we have identified those clusters whose HB is mainly populated at the blue side of the instability strip; among these clusters, through comparison with our HB models, we have identified the objects whose average HB mass is located more than 0.7 mag below the RR Lyrae level. For these clusters, we have recomputed the theoretical $R$-values by taking as representative of the HB evolutionary lifetime the corresponding value for the average mass. There are only six clusters in the Z00 sample and eight clusters in the S00 sample that satisfy this condition.

When applying these corrected evolutionary times to the blue HB clusters, we find that the $Y_{\text{GGC}}$ values obtained in the previous analysis are reduced by only 0.001–0.002. The size and significance of the $Y_{\text{GGC}}$ spread and the behavior with the respect to $[\text{Fe/H}]$ are unchanged with respect to the previous results. In Table 1 we summarize the $Y_{\text{GGC}}$ results, with and without the correction for the blue HB clusters. In the case of the Z00 sample and the CG97 [Fe/H] scale, we display the results for the metallicity range that is insensitive to the choice of the cluster ages. Figure 3 displays the distribution of the individual GGC He abundance for both samples and both choices of the [Fe/H] scale, taking into account the correction for the blue HB GGCs. The Z00 sample clearly has a significantly narrower abundance range than the S00 sample.

| Sample  | [Fe/H]  | Blue HB Correction | GGC Selection | $Y_{\text{GGC}}$ | $\sigma_Y$ |
|---------|---------|--------------------|---------------|------------------|-----------|
| Z00     | CG97    | No                 | [Fe/H] $< -1.15$ or $[\text{Fe/H}] > -0.85$ | 0.245 ± 0.003 | 0.0       |
| Z00     | ZW84    | No                 | All           | 0.242 ± 0.003 | 0.0       |
| S00     | CG97    | No                 | All           | 0.246 ± 0.005 | 0.03      |
| S00     | ZW84    | No                 | All           | 0.241 ± 0.004 | 0.03      |
| Z00     | CG97    | Yes                | [Fe/H] $< -1.15$ or $[\text{Fe/H}] > -0.85$ | 0.243 ± 0.003 | 0.0       |
| Z00     | ZW84    | Yes                | All           | 0.240 ± 0.003 | 0.0       |
| S00     | CG97    | Yes                | All           | 0.244 ± 0.004 | 0.03      |
| S00     | ZW84    | Yes                | All           | 0.240 ± 0.004 | 0.03      |

![Fig. 3.—Histograms representing the distribution of the individual cluster He abundances for the Z00 (top) and S00 (bottom) samples. Shaded histograms display the abundance distribution when the CG97 [Fe/H] scale is adopted; dashed lines represent the corresponding histograms for the ZW84 scale. In the case of the Z00 data and the CG97 [Fe/H] scale, we have included only clusters with [Fe/H] $< -1.15$ or $[\text{Fe/H}] > -0.85$, that is, the ranges unaffected by the precise choice of the GGC ages.](image)
4. DISCUSSION

In the previous section we found that the $R$-parameter provides values of $Y_{GGC}$ between $-0.240$ and $-0.245$, independent of [Fe/H]; the exact values are summarized in Table 1, together with the size of the intrinsic spread $\sigma_Y$ of the individual cluster He abundances.

The mean values of $Y_{GGC}$ deduced from the Z00 and S00 samples are in excellent agreement within the associated 1σ error, in spite of the (in principle) different bolometric corrections applied to the data analysis and the different photometric samples employed. It is, however, important to mention the fact that the Z00 data do not provide any indication of a statistically significant spread of $Y_{GGC}$, while the opposite is true for the S00 data. One possible reason for this occurrence may be the inhomogeneity of the S00 sample, which is made of photometries taken with very different instruments and detectors (photographic, photoelectric, and CCD photometries), reduced with different procedures in the course of the last 25 years and with possibly different methods to correct for incompleteness, as opposed to the homogeneously observed, reduced, and analyzed $HST$ sample by Z00.

Another possibility to explain this He-abundance spread is related to the existence of population gradients within the observed clusters, coupled with the fact that the $HST$ data employed by Z00 mainly sample regions of the clusters’ cores, whereas the ground-based photometries adopted by S00 sample more external regions located at various distances from the cores.

We have also performed another test by comparing the individual He abundance for 13 clusters in common between the Z00 and S00 data. In Figure 4 we display the individual He abundance for the 13 clusters derived from the Z00 and S00 data (we have chosen to use in this figure the ZW84 [Fe/H] scale, but this choice does not affect the result of the comparison), considering the corrections for the blue HB clusters. The Z00 data provide a mean $Y_{GGC} = 0.237 \pm 0.004$, in very good agreement with the result from the whole sample displayed in Table 1 ($Y_{GGC} = 0.240 \pm 0.003$); the dispersion around the mean is again due (as for the whole sample) only to the error on the individual determinations. In the case of the S00 data for the same 13 clusters, the mean $Y_{GGC} = 0.224 \pm 0.006$ is smaller than for the Z00 data and also significantly smaller than the mean value for the whole sample ($Y_{GGC} = 0.240 \pm 0.004$); the dispersion around the mean $Y_{GGC}$ is larger than in the Z00 case. Therefore, whereas a somewhat randomly selected sizable cluster subsample (the 13 common clusters span all the relevant [Fe/H] range as well as show both red and blue HBs) shows the same properties as the whole sample in the case of the Z00 data, the opposite is true for the S00 data. This may lend some support to the idea that the significant dispersion of $Y_{GGC}$ for the whole S00 sample is due to some inhomogeneity intrinsic to the data used for determining the observed $R$-values. On the other hand, when the four most metal-rich clusters ([Fe/H] > $-1$) are excluded from the comparison shown in Figure 4, the dispersion of the S00 data becomes comparable to the Z00 one, while the mean He abundance is similar to the value for the whole S00 sample.

This seems to point to some metallicity-related effect, which, however, does not explain the dispersion for the whole S00 sample. In fact, if we apply the F-test discussed in §§ 3.1 and 3.2 to the S00 sample without the clusters with [Fe/H] > $-1$, we still obtain a statistically significant dispersion of the individual He abundances.

In spite of this difference regarding the spread in the cluster He abundance for the Z00 and S00 samples, our results clearly indicate a mean value of $Y_{GGC}$ that is not in significant contradiction with the CMB constraint. This is very different from the conclusions reached by the Z00 and S00 analyses, which derived an unrealistically low He abundance, namely, $Y_{GGC} \sim 0.20$, completely inconsistent with the CMB constraint. When we redetermine $Y_{GGC}$ by using the same observational data and theoretical scenario adopted by Z00, but using the same weighted average method employed in our analysis and considering the metallicity range unaffected by the selected cluster age, we obtain $Y_{GGC} = 0.21$, still largely incompatible with the CMB constraint.

The new $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate and the new EOS are the two physical ingredients that have strongly modified the theoretical $R$-values with respect to the results by Z00, whose employed stellar models we have updated for this work. In particular, the recent estimate of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate (Kunz et al. 2002) has reduced the HB evolutionary times (at a fixed core mass and envelope composition) by $\sim 7\%$–$8\%$, and the new EOS has further reduced the HB evolutionary times by $\sim 10\%$. On the other hand, the new EOS also slightly reduces the value of the He-core mass at the He flash for a given age, which has the effect of increasing by $\sim 2\%$ the HB evolutionary time; there is also a further increase by $\sim 4\%$ for the value of $N_{\text{RGB}}$ because a larger portion of the RGB is considered in the evaluation of the $R$-parameter. These effects cause a total reduction of $R$ by $\sim 20\%$, which, for a typical average observed value of $R$ (i.e., with the Z00 definition of $R$) equal to $\sim 1.4$–1.5, corresponds to an increase of the estimated $Y_{GGC}$ by about 0.03.

The $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate by Kunz et al. (2002) has a relative uncertainty of about $\pm 30\%$, which translates into a systematic uncertainty of about $\pm 0.008$ around the values...
obtained in our analysis. It is also very interesting to note at this point that Metcalfe & Handler (2003) find, from asteroseismology data for two local white dwarfs, central oxygen abundances consistent with the value obtained by using the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ rate by Kunz et al. (2002) during the progenitor He-burning phase.

We believe the EOS calculations for the current set of stellar models do not contribute significant errors to the final results. The new EOS has been adjusted to fit the tabulated OPAL results. The quality of the fit is quite good. For example, the residuals for solar conditions are less than 0.06% in the pressure, and this good agreement should also extend to all but the highest density portions of evolved models, where there are some EOS uncertainties in the treatment of the Coulomb and electron exchange effects. However, the large variety of effects on the models caused by these nonideal effects largely cancel each other, so the calculated $R$-values are insensitive to these uncertainties.

The Coulomb effect arises because the attractive Coulomb force between ions and electrons tends to correlate the two kinds of particles. The exchange effect arises because the total eigenfunction of electrons, which is antisymmetric with respect to electron exchange, anticorrelates the electrons with each other. For fixed density and temperature, both the Coulomb correlation and the exchange anticorrelation reduce the amount of pressure required to confine the gas to its volume and also reduce the adiabatic gradient.

To determine how Coulomb and exchange effects alter the theoretical $R$-values, we did some test stellar-evolution calculations with and without the Coulomb or exchange effects for main-sequence, RGB, and HB phases. The calculated $R$-value is equal to $t_{\text{HB}}/t_{\text{RGB}}$, where $t_{\text{HB}}$ is the duration of the HB evolutionary phase and $t_{\text{RGB}}$ is the duration of that part of the RG phase whose luminosity is greater than the luminosity of the HB. By analyzing the various numerical experiments, we found that although the evolutionary rate along the RGB is slightly affected by Coulomb and exchange effects, the quantity $t_{\text{RGB}}$ is not significantly changed as a consequence of the variation of the HB luminosity level, which compensates the change in the RGB evolutionary rate. On the contrary, we found the following results for $t_{\text{HB}}$:

1. Coulomb and exchange effects for the precursor phase (i.e., the main sequence and the RGB) decrease the core mass by a small amount, but this reduction in fuel is more than compensated by the accompanying decrease in the helium-burning luminosity, so the total precursor effect for Coulomb and exchange is a 4% increase in $t_{\text{HB}}$. The exchange effect accounts for about one-sixth of this total.

2. The Coulomb effect (which for weak coupling and full ionization is proportional to the cube of the atomic number of the element) is considerably enhanced for later stages of the HB phase because He burning in the core substantially increases the abundance of C and O.

3. Coulomb and exchange effects for the HB phase increase the convective core mass by roughly 5%, but that is more than compensated by a helium-burning luminosity increase of roughly twice as much. Thus, the total HB effect for Coulomb and exchange is a 6% decrease in $t_{\text{HB}}$. The exchange effect accounts for about one-third of this total.

4. When one combines the opposite precursor and HB effects, a further cancellation occurs, so the total effect for Coulomb and exchange is only a 2% decrease in $t_{\text{HB}}$.

5. An alternative spline fit to the Coulomb effect (see the EOS description in §2), with a substantially enlarged range of intermediate coupling, changed the Coulomb results by about 10% of their size. This translates to a 0.2% uncertainty in $t_{\text{HB}}$ and calculated $R$ and a negligible uncertainty in the derived $Y_{\text{GGC}}$ value.

Models of stellar interiors are sensitive to EOS interpolation errors (Dorman, Irwin, & Pedersen 1991), so the most reliable calculational procedure is to eliminate EOS interpolation errors by calling the EOS code directly from the stellar interior code. The present EOS is fast enough so that such direct use is practical on workstation-type computers but, of course, still substantially slower than calculations done with interpolated EOS tables. Thus, in the interests of reducing the required computer time for the computations, we interpolated tables of EOS results that were tabulated with the present EOS for the required ranges of pressure, temperature, $Y$, $X_C$, $X_N$, and $X_0$ for a fixed non-CNO metal abundance mix. The adopted grid spacings are small enough in all coordinates that the resulting $t_{\text{HB}}$ values gave excellent agreement with one test calculation using direct EOS results.

As an additional test for the adequacy of our models and therefore of our inferred $Y_{\text{GGC}}$, we have also considered the so-called $R_2$-parameter, defined as the number ratio of asymptotic giant branch (AGB) to HB stars (Caputo et al. 1989). This parameter is strongly sensitive to the extension of the convective cores during the HB phase, while it is fairly insensitive to the initial metal and He abundance of the models and the precise value of the age. A test for our treatment of the convection in the HB stellar cores is of fundamental importance, since the extension of the convective core strongly affects the evolutionary time along the HB phase. An underestimate of the size of the HB convective cores would cause an underestimate of the HB evolutionary times, with a consequent spurious increase of $Y_{\text{GGC}}$. To compare theory with observations we have used the database by S00, which also provides the number of AGB stars for each cluster. These empirical data confirm the negligible effect of [Fe/H] on $R_2$; the mean value for the 43 clusters by S00 is $R_2 = 0.14 \pm 0.05$.

In our models we have treated the convective mixing during central He burning by including semiconvection, following the prescriptions by Castellani et al. (1985). We have suppressed the onset of the breathing pulses during the latest phases of central He burning by imposing that the allowed extension of the convective core not lead to an increase of the central He abundance from one model to the next (Caputo et al. 1989). All our models have reached the thermal pulse phase along the AGB; from this moment on the evolution is so fast that neglecting the computation of the thermal pulses does not affect the theoretical value of $R_2$. Our computations provide $R_2 = 0.12$, in good agreement with the empirical result; this confirms the adequacy of our mixing treatment in the HB stellar cores. If breathing pulses are not inhibited, HB evolutionary times are longer, because of the ingestion of fresh He into the central convective region following the onset of the pulses. We obtain in this case $R_2 \sim 0.08$, in disagreement with observations (a similar conclusion was reached by Caputo et al. 1989 by comparing their models with data about the GGC M5).

We have also experimented with an alternative procedure to inhibit the onset of the breathing pulses; following the
suggestions by Dorman & Rood (1993), we have set to zero the gravitational term in the energy generation equation for the central stellar regions during the later stage of core He burning. In this way, the breathing pulses are also effectively inhibited (see the detailed discussion by Dorman & Rood 1993), and we obtained a decrease of both AGB and HB evolutionary time by about 2% with respect to the procedure followed in our reference models; this leaves the value of $R_2$ unchanged ($R_2 \sim 0.12$) and causes a systematic increase of $Y_{GGC}$ by $\sim 0.003$.

The error on the individual $Y_{GGC}$ values displayed in Table 1 comes basically from the random error on the individual He-abundance determinations (i.e., from the random error on the individual $R$-parameter estimates). In order to give a best estimate for $Y_{GGC}$, including also the sources of systematic error described before (associated to uncertainties in the theoretical models) and the effect of the still uncertain [Fe/H] scale, we have resorted to a Monte Carlo technique briefly explained in the following. We have considered as reference values for $Y_{GGC}$ the ones determined by adopting the CG97 [Fe/H] scale, taking into account the corrections for the blue HB clusters (the fifth and seventh rows of Table 1 for the Z00 and S00 samples, respectively); we note that in the case of the Z00 data we consider the subsample unaffected by the precise choice of the clusters’ ages. Starting from each of these two reference $Y_{GGC}$ values, we have generated a set of 10,000 synthetic He-abundance values by applying (through a Monte Carlo simulation) to each generated abundance value a set of random and systematic errors, according to a given probability distribution. In particular, random errors have been modeled according to a Gaussian distribution with a mean value equal to the reference one and $\sigma$ equal to the corresponding random error on $Y_{GGC}$ provided in Table 1. The systematic uncertainties due to the choice of the [Fe/H] scale (which causes a decrease of $Y_{GGC}$ by 0.003 with respect to the reference value), $^{12}{C}(\alpha,\gamma)^{16}{O}$ reaction rate (variation by $\pm 0.008$), and breathing pulse suppression technique (increase by 0.003) have been modeled using a uniform distribution spanning the appropriate range.

The mean values for the two final synthetic distributions of He abundances are $Y_{GGC} = 0.243 \pm 0.006$ in the case of the Z00 sample and $Y_{GGC} = 0.244 \pm 0.006$ for the S00 sample. These values are, as expected, in very good reciprocal agreement and moreover compare well with the primordial He abundance $Y_p = 0.248 \pm 0.001$ inferred from the CMB in conjunction with primordial nucleosynthesis computations.

Another important result of our analysis is the fact that there is no statistically significant increase of $Y_{GGC}$ with [Fe/H], at least within the analyzed GGC samples. This bears considerable interest for studies of Galactic chemical evolution. As a test for the reliability of this result, we have performed the following numerical experiment. We have considered the clusters of the Z00 sample and the ZW84 metallicity scale (we obtain an analogous result when using the CG97 scale); for each cluster we have considered a reference $R$-value obtained from our theoretical models, using a primordial He mass fraction of 0.245 and assuming a value for the chemical enrichment ratio $\Delta Y/\Delta Z$. We have then generated, using a Monte Carlo procedure, 10,000 synthetic He abundances for each individual cluster and a given choice of $\Delta Y/\Delta Z$, using a Gaussian distribution with mean value equal to the reference theoretical $R$-value and $\sigma$ equal to the actual random error on the observed $R$-value.

For each of these synthetic samples we have then tried to recover the input $\Delta Y/\Delta Z$ value; we conclude from this analysis that ratios $\Delta Y/\Delta Z > 1$ should have been unambiguously detected even taking into account the actual observational errors on the determination of $R$.

5. SUMMARY

Following recent precise determinations of the primordial He abundance coming from CMB analyses and primordial nucleosynthesis computations, we have rederived the initial He abundance for stars in two samples of GGCs (Z00 and S00), using the $R$-parameter as an abundance indicator. We have employed theoretical models computed adopting new and more accurate determinations of the EOS for the stellar matter and of the crucial $^{12}{C}(\alpha,\gamma)^{16}{O}$ reaction rate. Our models include semiconvection, while the breathing pulses are suppressed, in agreement with the observational constraints coming from the measurements of the $R_2$-parameter in the S00 sample.

By taking into account the uncertainties in the observed individual $R$-value, as well as the uncertainties in the GGC metallicity scale, the $^{12}{C}(\alpha,\gamma)^{16}{O}$ reaction rate, and the method for the breathing pulse suppression, we obtain $Y_{GGC} = 0.243 \pm 0.006$ in case of the Z00 sample and $Y_{GGC} = 0.244 \pm 0.006$ in case of the S00 sample. These abundances are in good reciprocal agreement and fully consistent with the $Y_p = 0.248 \pm 0.001$ recently determined from CMB analyses and primordial nucleosynthesis computations. Within the S00 sample we find a statistically significant spread of the individual He abundances. This spread in the He abundances is not found in the Z00 sample, and we argue that it is due to the inhomogeneity of the observational database used by S00, as opposed to the homogeneously observed and reduced photometry employed by Z00.

It is important to remark that neither of the two samples shows any statistically significant increase of $Y_{GGC}$ with the cluster [Fe/H], a fact that is relevant in the context of the chemical evolution of the Galaxy.

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