Measurement of the $e^+e^- \rightarrow K_SK_L\pi^0$ cross section in the energy range $\sqrt{s} = 1.3 - 2.0$ GeV

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The $e^+e^- \rightarrow K_SK_L\pi^0$ cross section is measured in the center-of-mass energy range $\sqrt{s} = 1.3 - 2.0$ GeV. The analysis is based on the data sample with an integrated luminosity of 33.5 pb$^{-1}$ collected with the SND detector at the VEPP-2000 $e^+e^-$ collider.

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I. INTRODUCTION

This paper is dedicated to the study of the reaction $e^+e^- \rightarrow K_SK_L\pi^0$. This reaction is one of three charge modes of the process $e^+e^- \rightarrow K\bar{K}\pi$, which gives a sizable contribution to the total cross section of $e^+e^-$ annihilation into hadrons in the center-of-mass energy range $\sqrt{s} = 1.5 - 1.8$ GeV. The process $e^+e^- \rightarrow K\bar{K}\pi$ is also important for spectroscopy of $s\bar{s}$ vector states. From these states, only the lowest $\phi(1020)$ is well studied. In particular, its branching fractions are measured up to $10^{-5}$ level. Spectroscopy of the first excited state $\phi(1680)$ is far from completion. The main decay mode of $\phi' \equiv \phi(1680)$ is $K^*(892)\bar{K}$ with the $K^*(892)$ decay to $K\pi$.

Processes of $e^+e^-$ annihilation to the $K\bar{K}\pi$ final state were studied in the DM1, DM2 and BABAR [1] experiments. Until recently, only the two subprocesses $e^+e^- \rightarrow K_SK_L\pi^0, K^+K^-\pi^0$ [8] were measured. The third, neutral subprocess $e^+e^- \rightarrow K_SK_L\pi^0$, is hard to study due to complexity of $K_L$-meson detection and identification. Recently, it was measured in the BABAR experiment [4]. The measurement uses the initial state radiation method, in which the $e^+e^- \rightarrow X$ cross section is determined from the mass spectrum of the hadron system $X$ in the reaction $e^+e^- \rightarrow X\gamma$. Detection of all final particles in the reaction $e^+e^- \rightarrow K_SK_L\pi^0\gamma$ was required, and the $K_L$ meson was identified as a single photon. The efficiency of $K_L$-meson detection was measured using $e^+e^- \rightarrow \phi(1020)\gamma \rightarrow K_SK_L\gamma$ events selected without any conditions on $K_L$ parameters. It should be noted that the $K_L$-meson energy was not measured. Therefore, good background suppression was not reached in Ref. [4]. A relatively large systematic uncertainty of the $e^+e^- \rightarrow K_SK_L\pi^0$ cross-section measurement ($\sim 10\%$ at the cross-section maximum) is due to an uncertainty in background subtraction.

In this paper, the process $e^+e^- \rightarrow K_SK_L\pi^0$ is studied using a data sample collected in the energy range $\sqrt{s} = 1.3 - 2.0$ GeV with the SND detector [5] at the VEPP-2000 $e^+e^-$ collider [6].

II. DETECTOR AND EXPERIMENT

SND is a general purpose nonmagnetic detector. Its main part is a three-layer electromagnetic calorimeter based on NaI(Tl) crystals. The calorimeter covers a solid angle of $95\%$ of $4\pi$. Its energy resolution is $\sigma_{E_c}/E_c = 4.2\%/\sqrt{E_c}$ (GeV), while the angular resolution is $\sigma_{\phi,\theta} = 0.82^\circ/\sqrt{E_c}$ (GeV), where $E_c$ is the photon energy. The tracking system is located inside the calorimeter, around the collider beam pipe. It consists of a nine-layer cylindrical drift chamber and a proportional chamber with cathode-strip readout. A solid angle covered by the

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1 Throughout this paper the use of charge conjugate modes is implied.
tracking is 94% of 4π. For charged kaon identification, the system of threshold aerogel Cherenkov counters is used. The calorimeter is surrounded by a muon system consisting of proportional tubes and scintillation counters.

The analysis uses a data sample with an integrated luminosity of 33.5 pb⁻¹ recorded in 2010–2012 in the energy region 1.3–2.0 GeV. Due to relatively small statistics of selected events of the process under study, data collected in 36 energy points are combined into 15 energy intervals listed in Table 1.

The simulation of the process $e^+e^- \rightarrow K_SK_L\pi^0$ is performed using a Monte-Carlo (MC) event generator based on formulas from Ref. [7]. It is assumed that the process proceeds via the $K^*(892)^0\bar{K}^0$ intermediate state.

Interaction of particles produced in $e^+e^-$ collision with the detector material is simulated using GEANT4 v.9.5 package [8]. Analyses of processes with $K_L$ meson in the final state critically depend on correct simulation of $K_L$ nuclear interaction. Unfortunately, both the total and inelastic low-energy cross section of the $K_L$ nuclear interaction are strongly overestimated in GEANT4 v.9.5 [9]. Therefore, we have modified the GEANT4 module responsible for $K_L$ cross-section calculation using the model from Ref. [10]. This model describes reasonably well the experimental data both on the total $K_L$ cross section in different materials (H, Be, C, Al, Fe, Cu, Pb) in the $K_L$ energy range 525–600 MeV [11] and on the inelastic cross section in the range 510–700 MeV [9]. Accuracy of the model is estimated by comparison of its prediction with the precisely measured value of the $K_L$ inelastic cross section at 510 MeV [9] and is found to be about 12%.

The simulation takes into account variation of experimental conditions during data taking, in particular dead detector channels, size and position of the collider interaction region, beam-induced background etc. The beam background leads to appearance of spurious photons and/or charged particles in data events. To take this effect into account in simulation, special background events are recorded during data taking with a random trigger. These events are then superimposed on simulated events.

In this paper the reaction $e^+e^- \rightarrow K_SK_L\pi^0$ is studied in the decay mode $K_S \rightarrow 2\pi^0$, with no charged particles in the final state. Therefore, the process $e^+e^- \rightarrow \gamma\gamma$ is used for normalization. As a result of the normalization a part of systematic uncertainties associated with event selection criteria for the process under study is canceled out. The accuracy of the luminosity measurement using $e^+e^- \rightarrow \gamma\gamma$ was studied in Ref. [12] and is estimated to be 1.4%.

III. EVENT SELECTION

The reaction $e^+e^- \rightarrow K_SK_L\pi^0$ is studied in the $K_S \rightarrow \pi^0\pi^0$ decay mode. The $K_L$ decay length is much larger than the radius of the SND calorimeter, and the length of its inelastic nuclear interaction is comparable with the calorimeter thickness [9]. In a significant fraction of $e^+e^- \rightarrow K_SK_L\pi^0$ events (25−30%) $K_L$ meson does not interact with the calorimeter, and only six photons from decays of three $\pi^0$ are detected. The $K_L$ meson undergoing a nuclear interaction inside the detector produces one or several clusters in the calorimeter, which are reconstructed as photons.

The selection of $e^+e^- \rightarrow K_SK_L\pi^0$ events is based on finding three pairs of photons forming three $\pi^0$ candidates. Two of these $\pi^0$’s having the invariant mass close to the $K^0$ mass form a $K_S$ candidate. The events of the process under study are selected in two stages. The primary selection is based on the following criteria:

- No charged tracks are reconstructed in the drift chamber. The number of hits in the drift chamber is less than four.
- The fired calorimeter crystals do not lie along a straight line. This requirement rejects cosmic-ray background.
- An event contains at least six “good” photons ($N_{\gamma} \geq 6$) and no charged particles. A “good” photon is a cluster in the calorimeter with the energy deposition larger than 20 MeV, which has a transverse energy profile consistent with expectations for a photon [13]. The latter condition rejects spurious photons originating from $K_L$ nuclear interaction or decay.
- There are three $\pi^0$ candidates in an event. The $\pi^0$ candidate is a pair of photons with the invariant mass in the range from 110 to 160 MeV/c².
- The invariant mass of two $\pi^0$ candidates lies in the range 450–550 MeV/c².

The fraction of signal events rejected by the condition on the number of hits in the drift chamber varies between 6% and 17%, depending on machine background conditions. It should be noted that the same condition is used for selection of $e^+e^- \rightarrow \gamma\gamma$ events. Therefore, the possible systematic uncertainty due to this condition cancels as a result of the luminosity normalization.

For energies above 1.9 GeV an additional selection criterion is applied to suppress the background from the process $e^+e^- \rightarrow K_SK_L\pi^0\pi^0$, $K_S \rightarrow \pi^0\pi^0$. Events containing more than three $\pi^0$ candidates selected with the mass window 100–170 MeV/c² are rejected.
FIG. 1: The distribution of the invariant $K_L \pi^0$ and $K_S \pi^0$ masses (two entries per event) for data events from the energy region $\sqrt{s} = 1.600 - 1.750$ GeV (points with error bars). The histogram represents the simulated distributions obtained in the model with the $K^*(892)^0 K^0$ intermediate state.

FIG. 2: The distribution of the $K_S K_L$ invariant mass for data events from the energy region $\sqrt{s} = 1.600 - 1.750$ GeV (points with error bars). The histogram represents the simulated distributions obtained in the model with the $K^*(892)^0 K^0$ intermediate state. The shaded histogram represents the expected contribution of the $e^+ e^- \rightarrow \phi \pi^0$ process estimated using MC simulation.

The selected events are then kinematically fitted with three $\pi^0$ mass constraints and a $K_S$ mass constraint. The $\chi^2$ of the kinematic fit is required to be less than 15. The refined photon parameters are used to calculate the mass recoiling against the $K_S \pi^0$ system ($M_{\text{rec}}$).

IV. ANALYSIS OF INTERMEDIATE STATES IN THE REACTION $e^+ e^- \rightarrow K_S K_L \pi^0$

In Ref. [4] it is shown that the dominant mechanism of the $e^+ e^- \rightarrow K_S K_L \pi^0$ reaction is the transition via the $K^*(892)^0 K^0$ intermediate state. The fraction of $e^+ e^- \rightarrow \phi \pi^0 \rightarrow K_S K_L \pi^0$ events near the maximum of the $e^+ e^- \rightarrow K_S K_L \pi^0$ cross section (1.7 GeV) is about 1% [3]. Also a small contribution of the $K_2^*(1430)^0 K^0$ state was observed in Ref. [4]. The $e^+ e^- \rightarrow K_2^*(1430) K$ cross section was measured in Ref. [3] in the charge modes $K^+ K^- \pi^0$ and $K_S K^{\pm} \pi^\mp$. It proceeds in $D$ wave and is expected to be negligibly small in the VEPP-2000 energy region, below 2 GeV.

Figures 1 and 2 represent the distributions of the $K_S (L) \pi^0$ invariant mass (two entries per event) and the $K_S K_L$ invariant mass, respectively, for six-photon data events from the energy region $\sqrt{s} = 1.600 - 1.750$ GeV selected with
the extra condition $400 < M_{\text{rec}} < 600$ MeV/c$^2$. Six-photon events are used to maximize the signal-to-background ratio. The fraction of background events in these distributions is estimated on the tails of the $M_{\text{rec}}$ distribution (see Sec. [V]) and does not exceed 3%. It is seen, that the data spectra in Figs. 1 and 2 are in good agreement with the simulated spectra obtained in the model with the $K^* (892)^0 K^0$ intermediate state. The shaded histogram in Fig. 2 represents the expected contribution of the $e^+ e^- \rightarrow \phi \pi^0$ process estimated using MC simulation. With current statistics, we cannot observe the signal of the $\phi \pi^0$ intermediate state.

V. FIT TO THE $M_{\text{rec}}$ SPECTRUM

The number of signal events is determined from the fit to the $M_{\text{rec}}$ spectrum (Fig. 3) by a sum of distributions for signal and background events. The signal distribution is described by a sum of three Gaussian functions with parameters determined from the fit to the simulated signal $M_{\text{rec}}$ spectrum. To account for a possible inaccuracy of the signal simulation, two parameters are introduced: mass shift $\Delta M$ and smearing parameter $\Delta \sigma^2$. The latter is added to all Gaussian sigmas squared ($\sigma^2 \rightarrow \sigma^2 + \Delta \sigma^2$). The parameters $\Delta M$ and $\Delta \sigma^2$ are determined from the fit to the $M_{\text{rec}}$ spectrum for events from the energy range $\sqrt{s} = 1.6 - 1.75$ GeV shown in Fig. 3. They are found to be $\Delta M = (5 \pm 5)$ MeV/c$^2$ and $\Delta \sigma^2 = 1800 \pm 770$ MeV$^2$/c$^4$.

To obtain the background distribution we analyze simulation for the processes $e^+ e^- \rightarrow K_S K_L$, $e^+ e^- \rightarrow K_S K_L \pi^0 \pi^0$, $e^+ e^- \rightarrow \phi \eta$, $e^+ e^- \rightarrow \omega \pi^0$, $e^+ e^- \rightarrow \omega \pi^0 \pi^0$, $e^+ e^- \rightarrow \omega \pi^0 \eta$ with decays $\phi \rightarrow K_S K_L$ and $\omega \rightarrow \pi^0 \eta$. For all these processes the existing experimental data on Born cross sections are approximated and then used in event generators for calculation radiative corrections, and generation of extra photons emitted from the initial state. The obtained simulated background distribution is fitted with a smooth function. The largest contributions into expected background come from the processes $e^+ e^- \rightarrow K_S K_L$ and $e^+ e^- \rightarrow K_S K_L \pi^0 \pi^0$.

In the fit to the data $M_{\text{rec}}$ spectra the background distribution obtained from simulation is multiplied by a free scale factor. For all energy regions the fitted value of the scale factor was found to be consistent with unity. The example of the fit for the energy region $\sqrt{s} = 1.60 - 1.75$ GeV is presented in Fig. 3.

Some fraction of selected signal events contains a hard photon emitted from the initial state. These initial-state-radiation (ISR) events have $M_{\text{rec}}$ larger than the $K^0$ mass and distort the shape of the signal $M_{\text{rec}}$ distribution. The distortion is most significant at energies $\sqrt{s} > 1.92$ GeV, where the fraction of ISR events becomes larger than 50%. For these energies, the fitting procedure is modified. The signal spectrum is represented as a sum of two spectra, for events with the photon energy $E_\gamma$ smaller and larger than 120 MeV. The number of events in the latter spectrum and its error are calculated using the Born $e^+ e^- \rightarrow K_S K_L \pi^0$ cross section measured in this work in the lower-energy interval. The number of signal events with $E_\gamma < 120$ MeV is determined from the fit to the data $M_{\text{rec}}$ spectrum.

The fitted numbers of signal events obtained for 15 energy intervals are listed in Table I.
VI. DETECTION EFFICIENCY

The detection efficiency for $e^+e^- \rightarrow K_SK_L\pi^0$ events is determined using MC simulation. The simulation takes into account radiative corrections [14], in particular, emission of extra photon from the initial state [15]. The Born cross section for the process $e^+e^- \rightarrow K_SK_L\pi^0$ is taken from Ref. [4].

The detection efficiency obtained from MC simulation is corrected to take into account difference between data and simulation in photon conversion in detector material before the tracking system. This difference is measured using $e^+e^- \rightarrow \gamma\gamma$ events. The conversion probability for two photons is canceled when normalizing to luminosity. The remaining data-MC simulation difference for 4 photons is $- (1.35 \pm 0.05)\%$ [10].

We also study the data-simulation difference in the photon transverse energy-deposition profile in the calorimeter. To do this, $e^+e^- \rightarrow \omega\pi^0 \rightarrow \pi^0\pi^0\gamma \rightarrow 5\gamma$ events are used, which can be selected without background [12]. The dependence of the photon loss due to the “good” photon requirement on the photon energy is measured in data and simulation. The obtained data-simulation difference is used to determine the efficiency correction for simulated $e^+e^- \rightarrow K_SK_L\pi^0$ events. The correction is found to be practically independent of $\sqrt{s}$ and is equal to $- (3.4 \pm 1.3)\%$.

The high-statistics study of the systematic uncertainty associated with selection of multiphoton events based on the kinematic fit was performed in Refs. [12, 17] using $e^+e^- \rightarrow \omega\pi^0 \rightarrow \pi^0\pi^0\gamma$ events. We estimate that the systematic uncertainty due to conditions on invariant masses and $\chi^2$ of the kinematic fit does not exceed 5%.

The detection efficiency calculated in the model of the $K^0(S(892))^0\bar{K}^0$ intermediate state ($\varepsilon_{K^\ast\cdot R}$) is modified to take into account a small contribution of the $\phi\pi^0$ intermediate state:

$$\varepsilon = \varepsilon_{K^\ast\cdot R}(1 + \frac{\varepsilon_{\phi\pi^0} - \varepsilon_{K^\ast\cdot R} f_{\phi\pi^0}}{\varepsilon_{K^\ast\cdot R}}),$$

(1)

where $\varepsilon_{\phi\pi^0}$ is the detection efficiency for the process $e^+e^- \rightarrow \phi\pi^0 \rightarrow K_SK_L\pi^0$, and $f_{\phi\pi^0}$ is the ratio of the $e^+e^- \rightarrow \phi\pi^0 \rightarrow K_SK_L\pi^0$ cross section [3] and the total $e^+e^- \rightarrow K_SK_L\pi^0$ cross section obtained in this work. The relative difference $(\varepsilon_{\phi\pi^0} - \varepsilon_{K^\ast\cdot R})/\varepsilon_{K^\ast\cdot R}$ varies in the range 10–30%. The efficiency correction is 0.1-0.7% in the range 1.40–1.85 GeV, and about 2% above and below this interval. The associated systematic uncertainty is determined by the accuracy of the $e^+e^- \rightarrow \phi\pi^0$ cross section [3] and does not exceed 0.6% in the range 1.4–1.9 GeV, and is 2% above and below. The maximum possible contribution of the $K^0_S(1430)^0\bar{K}^0$ mechanism can be estimated from the measurement of the isovector and isoscalar $e^+e^- \rightarrow K^0_S(1430)^0\bar{K}^0$ cross sections in Ref. [3] and isospin relations [18] assuming constructive interference of the isovector and isoscalar amplitudes. It does not exceed 10% of the total $e^+e^- \rightarrow K_SK_L\pi^0$ cross section at $\sqrt{s} > 1.9$ GeV and negligible below. The detection efficiency for $e^+e^- \rightarrow K^0_S(1430)^0\bar{K}^0 \rightarrow K_SK_L\pi^0$ events is about 40% larger than $\varepsilon_{K^\ast\cdot R}$. Therefore, we estimate that the systematic uncertainty on the detection efficiency due to the possible contribution of the $K^0_S(1430)^0\bar{K}^0$ mechanism does not exceed 4% at $\sqrt{s} > 1.9$ GeV.

The corrected detection efficiencies for the 15 energy regions are listed in Table I. For the two intervals with $\sqrt{s} > 1.92$ GeV, the efficiency is calculated with the additional requirement $E_\gamma < 120$ MeV. The systematic uncertainty on detection efficiency is 5.2% in the range $\sqrt{s} = 1.4 – 1.9$ GeV, 5.5% at $\sqrt{s} < 1.4$ GeV, and 6.8% at $\sqrt{s} > 1.9$ GeV.

VII. THE BORN CROSS SECTION

The visible cross section for the process $e^+e^- \rightarrow K_SK_L\pi^0$ is obtained from data as:

$$\sigma_{vis,i} = \frac{N_i}{\varepsilon_i L_i}$$

(2)

where $N_i$ is the number of $K_SK_L\pi^0$ events obtained from the fit to the $M_{rec}$ spectrum in Sec. IV, $\varepsilon_i$ is the detection efficiency, and $L_i$ is the integral luminosity for the $i$th energy region.

The Born cross section $\sigma$ relates to the visible cross section as:

$$\sigma_{vis}(\sqrt{s}) = \int_{x_{\min}}^{x_{\max}} W(s, x)\sigma(\sqrt{s(1-x)})dx,$$

(3)

where $W(s, x)$ is the so-called radiator function, which describes the probability of emission of photons with the energy $x\sqrt{s}/2$ by the initial electron and positron [14].

The equation (3) can be represented as:

$$\sigma_{vis}(\sqrt{s}) = \sigma(\sqrt{s})[1 + \delta(\sqrt{s})],$$

(4)
where $\delta(\sqrt{s})$ is the radiation correction, which is calculated as a result of the fit to the visible-cross-section data with Eq. (3) and a theoretical model for the Born cross section. The vector-meson dominance (VMD) model [19] is used to describe the energy dependence of the $e^+e^- \rightarrow K_SK_L\pi^0$ cross section. In principle, it should include contributions of all vector resonances of the $\rho$, $\omega$, and $\phi$ families. In Ref. [3] it is shown that the isoscalar contribution dominates only near the maximum of the $\phi(1680)$ resonance. Below 1.55 GeV and above 1.8 GeV the isoscalar and isovector amplitudes are the same order of magnitude. However, for the purpose of calculating the radiation correction, a simple model with the $\phi(1020)$ and $\phi(1680)$ resonances is sufficient. This model describes the experimental data well. However, its fitted parameters should not be considered when measuring the parameters of the $\phi(1020)$ and $\phi(1680)$ resonances. The Born cross section for the process $e^+e^- \rightarrow K_SK_L\pi^0$ is described by the following formula:

$$\sigma(\sqrt{s}) = |A_0(s) + e^{i\alpha}A_1(s)|^2 \frac{P(s)}{s^{3/2}}$$  \hspace{1cm} (5)

where $A_0$ and $A_1$ are the amplitudes of the $\phi(1020)$ and $\phi(1680)$ decays to $K_SK_L\pi^0$, and $\alpha$ is their relative phase. It was assumed that the decays proceed via $K^*(892)^0K^0$ intermediate state. So, the function $P(s)$ describes energy dependence of the $K^*(892)^0K^0$ phase space [19]:

$$P(s) = \frac{1}{\pi} \int \frac{d^2q}{(m_{\omega} + m_{K^*})^2} \frac{m_{K^*}\Gamma_{K^*}}{q^2 - m_{K^*}^2 + m_{K^*}^2\Gamma_{K^*}^2} p^3(q^2) dq^2$$  \hspace{1cm} (6)

$$p(q^2) = \frac{\sqrt{(s - m_{K^*}^2)^2 - 4m_{K^*}^2q^2}}{4s}$$

where $m_{K^*}$ and $\Gamma_{K^*}$ are the $K^*(892)^0$ mass and width [21], and $p(q^2)$ is the momentum of the $K^0\pi^0$ system. The $\phi(1020)$ amplitude is parametrized as

$$A_0(s) = A_{\phi} \frac{M_{\phi}\Gamma_{\phi}}{(M_{\phi}^2 - s) - i\sqrt{s}\Gamma_{\phi}}$$  \hspace{1cm} (7)

where $A_{\phi}$ is a real constant, $M_{\phi}$ and $\Gamma_{\phi}$ are the $\phi(1020)$ mass and width [21]. while the $\phi' \equiv \phi(1680)$ amplitude is given by

$$A_1(s) = \sqrt{\frac{\sigma_{\phi'}M_{\phi'}^3}{P(M_{\phi'})M_{\phi'}^2 - s - i\sqrt{s}\Gamma_{\phi'}}}$$  \hspace{1cm} (8)

where $\sigma_{\phi'}$ is the cross section of the process $e^+e^- \rightarrow \phi' \rightarrow K_SK_L\pi^0$ at $\sqrt{s} = M_{\phi'}$. $M_{\phi'}$ and $\Gamma_{\phi'}$ are the $\phi'$ mass and width. The free fit parameters are $A_{\phi}$, $\sigma_{\phi'}$, $\alpha$, $M_{\phi'}$, and $\Gamma_{\phi'}$. The model describes data well ($\chi^2/ndf = 7/15$, where ndf is the number of degrees of freedom). The fitted $\phi'$ mass ($1700 \pm 23$ MeV/$c^2$) and width ($300 \pm 50$ MeV) are close to the Particle Data Group values for $\phi(1680)$ [21].

The radiation corrections calculated with the fitted model parameters are listed in Table IV. The experimental values of the Born cross section are then obtained from the measured values of the visible cross sections using Eq. (4). They are listed in Table IV and shown in Fig. 4 together with the fitted curve.

VIII. SYSTEMATIC UNCERTAINTY

Several sources give contribution to the systematic uncertainty of the measured cross section. These are the uncertainties of luminosity measurement and detection efficiency, the systematic uncertainty in determination of the number of signal events from the fit to the $M_{\text{rec}}$ spectrum, the model uncertainty of the radiative correction. A possible source of the systematic uncertainty on the number of signal events is imperfect simulation of the shape of the signal and background $M_{\text{rec}}$ distributions.

In the fit to the $M_{\text{rec}}$ spectrum we use the simulated background distribution multiplied by a free scale factor. To estimate the systematic uncertainty due imperfect simulation of the background shape, another approach to background description is applied, by a sum of predicted background plus a linear function. The difference in the number of signal events obtained with the standard and new background descriptions does not exceed 5% in the energy region 1.60–1.75 GeV. This value is used as an estimate of the uncertainty.

The signal $M_{\text{rec}}$ distribution has the asymmetric line shape (see Fig. 3). The tail of the distribution at $M_{\text{rec}} > m_{K^0}$ originates from events with $N_{\gamma} > 6$, in which wrong combination of photons forming the $K_SK_L\pi^0$ system is chosen. For six-photon events the line shape is symmetric and close to Gaussian. To estimate the systematic uncertainty
TABLE I: The energy interval \((\sqrt{s})\), integrated luminosity \((L)\), number of selected \(e^+e^- \rightarrow K_SK_L\pi^0\) events \((N)\), detection efficiency \((\varepsilon)\), radiative correction factor \((1 + \delta)\), and the \(e^+e^- \rightarrow K_SK_L\pi^0\) Born cross section \((\sigma)\). The shown cross-section errors are statistical. The systematic error is 12%.

| \(\sqrt{s}\) (GeV) | \(L\) (nb\(^{-1}\)) | \(N\) | \(\varepsilon\) | \(1 + \delta\) | \(\sigma\) (nb) |
|---------------------|-------------------|------|----------------|----------------|---------------|
| 1.300 – 1.350       | 2546              | 18 ± 7 | 0.058          | 0.872          | 0.14 ± 0.05   |
| 1.360 – 1.375       | 1468              | 10 ± 5 | 0.057          | 0.867          | 0.14 ± 0.06   |
| 1.400 – 1.440       | 2783              | 70 ± 10 | 0.057       | 0.851          | 0.52 ± 0.08   |
| 1.450 – 1.475       | 1082              | 48 ± 9 | 0.057          | 0.860          | 0.90 ± 0.16   |
| 1.500               | 2081              | 135 ± 13 | 0.055        | 0.867          | 1.35 ± 0.13   |
| 1.520 – 1.525       | 1437              | 105 ± 12 | 0.056       | 0.874          | 1.49 ± 0.18   |
| 1.550 – 1.575       | 1100              | 135 ± 13 | 0.056       | 0.886          | 2.48 ± 0.24   |
| 1.600 – 1.650       | 2997              | 408 ± 28 | 0.054       | 0.899          | 2.80 ± 0.19   |
| 1.675 – 1.700       | 2257              | 314 ± 22 | 0.053       | 0.923          | 2.85 ± 0.20   |
| 1.720 – 1.750       | 1575              | 180 ± 17 | 0.051       | 0.968          | 2.30 ± 0.21   |
| 1.760 – 1.800       | 3362              | 311 ± 26 | 0.049       | 1.039          | 1.81 ± 0.15   |
| 1.825 – 1.850       | 1973              | 109 ± 22 | 0.049       | 1.135          | 1.00 ± 0.23   |
| 1.870 – 1.900       | 3659              | 123 ± 15 | 0.049       | 1.249          | 0.55 ± 0.09   |
| 1.920 – 1.950       | 2667              | 23 ± 7  | 0.022        | 0.992          | 0.39 ± 0.12   |
| 1.960 – 2.000       | 2458              | 20 ± 7  | 0.020        | 0.974          | 0.41 ± 0.15   |

FIG. 4: The Born cross section for the process \(e^+e^- \rightarrow K_SK_L\pi^0\) measured in this work (filled squares) in comparison with the BABAR data [4] (open squares). The curve represents the result of the fit to SND data with the VMD model. The band represents the prediction for the \(e^+e^- \rightarrow K_SK_L\pi^0\) cross section obtained using isospin relations from the BABAR measurements of the \(e^+e^- \rightarrow K_SK_L\pi^0\), \(e^+e^- \rightarrow K^+K^-\pi^0\), and \(e^+e^- \rightarrow \phi\pi^0\) cross sections [3].

associated with the signal line shape, we repeat the analysis selecting events with \(N_\gamma = 6\). The visible cross section near the maximum is found to be \((20 \pm 5)\)\% lower than the cross section obtained with the standard selection criteria. The observed difference is partly explained by incorrect simulation of \(K_L\) nuclear interaction. At the \(K_L\) energy 510 MeV the inelastic nuclear interaction length used in simulation [10] is larger than the measured one by \((12 \pm 5)\)\%. The six-photon selection in contrast to the standard selection is very sensitive to the value of the \(K_L\) nuclear interaction length: its decrease by 12\% is translated to the 11\% decrease of the detection efficiency [22]. The remaining difference
TABLE II: The systematic uncertainties (%) on the measured $e^+e^- \rightarrow K_SK_L\pi^0$ Born cross section in four c.m. energy intervals.

| Source                     | 1.3–1.4 GeV | 1.40–1.8 GeV | 1.8–1.9 GeV | 1.9–2.0 GeV |
|----------------------------|-------------|-------------|-------------|-------------|
| Luminosity                 | 1.4         | 1.4         | 1.4         | 1.4         |
| Detection efficiency       | 5.5         | 5.2         | 5.2         | 6.8         |
| Background subtraction     | 5.0         | 5.0         | 5.0         | 5.0         |
| Signal line shape          | 9.0         | 9.0         | 9.0         | 9.0         |
| Radiative corrections      | 0.5         | 0.5–1.2     | 1.9–2.4     | 2.6–3.5     |
| Total                      | 12          | 12          | 12          | 13          |

(9 ± 7)% is used as an estimate of the uncertainty associated with the signal line shape.

The model uncertainty of the radiation correction is estimated by varying the fitted parameters of the VMD model [Eq. (5)–(8)] within their errors. It is below 0.5% at $\sqrt{s} < 1.65$ GeV and increases up to 3.5% near 2 GeV. The systematic uncertainty on the detection efficiency is discussed in Sec. VI and is 5.2% in the range $\sqrt{s} = 1.4–1.9$ GeV. The systematic uncertainty of the luminosity measurement studied in Refs. [12, 17] is 1.4%.

The systematic uncertainties from different sources are summarized in Table II for four c.m. energy intervals. The total systematic uncertainty including all the contributions discussed above combined in quadrature is estimated to be 12% below 1.9 GeV and 13% above.

IX. DISCUSSION AND SUMMARY

The cross section for the process $e^+e^- \rightarrow K_SK_L\pi^0$ has been measured with the SND detector at the VEPP-2000 $e^+e^-$ collider in the energy range 1.3–2.0 GeV. The comparison of the SND data with the only previous measurement, done by the BABAR Collaboration [4], is presented in Fig. 4. Only statistical errors are shown. The systematic uncertainty of the SND data is 12–13%, while the BABAR systematic uncertainty increases from 10% at 1.7 GeV and below to about 20% at 2 GeV [4]. Near the maximum of the cross section (1.7 GeV) the SND points lie below the BABAR points, but agree within systematic errors. The same trend persists at higher energies, up to 2 GeV. The largest difference, about 2 standard deviations including systematic uncertainties, between the SND and BABAR data is observed in the energy points 1.875 and 1.925 GeV.

It is discussed in Sec. IV that the dominant mechanism of the $e^+e^- \rightarrow K_SK_L\pi^0$ reaction at $\sqrt{s} < 2$ GeV is $K^*(892)^0K^0$. Under this assumption the cross section of the process under study can be predicted using the isospin relation [18]

$$\sigma(e^+e^- \rightarrow K_SK_L\pi^0) = \sigma(e^+e^- \rightarrow K_SK^+\pi^-) - \sigma(e^+e^- \rightarrow K^+K^-\pi^0) + B(\phi \rightarrow K\bar{K})\sigma(e^+e^- \rightarrow \phi\pi^0)$$

and the BABAR measurements [3] of the $e^+e^- \rightarrow K_SK^+\pi^-, e^+e^- \rightarrow K^+K^-\pi^0$, and $e^+e^- \rightarrow \phi\pi^0$ cross sections. In Eq. (9) we take into account that both $\sigma(e^+e^- \rightarrow K_SK_L\pi^0)$ and $\sigma(e^+e^- \rightarrow K^+K^-\pi^0)$ contain contributions of the $\phi\pi^0$ intermediate state. The predicted cross section is shown in Fig. 4 by the green band and is found to be in good agreement with our measurement.

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