Effects of $c$-axis Hopping in the Interlayer Tunneling Model of High-$T_c$ Layered Cuprates

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Abstract

We consider the interlayer pair-tunneling model for layered cuprates, including an effective single particle hopping along the $c$-axis. A phenomenological suppression of the $c$-axis hopping matrix element, by the pseudogap in cuprate superconductors, is incorporated. At optimal doping, quantities characteristic to the superconducting state, such as the transition temperature and the superconducting gap are calculated. Results from our calculations are consistent with the experimental observations with the noteworthy point that, the superconducting gap as a function of temperature shows excellent match to the experimental data. Predictions within the model, regarding $T_c$ variation with interlayer coupling, are natural outcomes which could be tested further.

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1. Introduction

A salient common characteristic feature of high temperature layered superconductors (HTLS) is the existence of well separated CuO$_2$ planes and the presence of strong electronic correlations in these planes. This characteristic, together with the observed interlayer contact, though very weak, often forms the basis for theoretical modelling of HTLS. Many of the to-date models of HTLS are intraplanar, focussing mainly on the electronic interaction in a single CuO$_2$ layer and ignoring, in the first approximation, the weak interlayer contact. But unlike these intraplanar models, the interlayer pair tunneling (ILPT) model due to Anderson [1] is the one where interlayer contact has duly been incorporated and interlayer coupling plays an important role in this model to make it more compatible with the realistic HTLS materials than other existing models.

The ILPT model of interacting electrons on coupled superconducting layers could naturally yield high transition temperatures ($T_c$) observed in the layered superconducting materials. In the ILPT model, single particle hopping along the $c$-direction was argued to be blocked owing to strong electronic correlations [2] and the tunneling of only pairs of carriers, caused by interlayer coupling, was considered. The idea behind pair tunneling is to amplify the pairing mechanism within a given layer, and as a consequence $T_c$ gets enhanced [3].

It is important to note that, the interlayer hypothesis was based on certain experimental observations on underdoped cuprates, that is, the $c$-axis infrared conductivity is small, and the $c$-axis resistivity shows semiconducting behaviour in contrast to the $ab$-plane resistivity which is metallic [4]. These observations, which are signatures of the marginal presence of $c$-axis transport, led Anderson and co-workers to enforce complete suppression of interlayer single particle hopping (ISPH) in their ILPT model [5]. However, later development in sample preparation and the emergence of more accurate experimental techniques, made it possible to study the $c$-axis transport for wide range of dopings and it is observed that for sufficiently overdoped cuprates, both $c$-axis and $ab$-plane resistivities show similar temperature dependence [6] implying a three dimensional metallic behaviour of the material. In other words, for overdoped HTLS materials,
c-axis transport is metallic-like. This view is further supported by the c-axis optical conductivity measurements on overdoped cuprates [3, 7], where the presence of Drude peak in the spectrum indicates the existence of interlayer single particle hopping. Also, band structure calculations yield substantial value for the interlayer single particle hopping matrix element for bilayer cuprates [8].

Thus, guided by the to-date experimental results regarding the c-axis transport in variedly doped HTLS, we consider an extended or more generalized version of the ILPT model including an effective ISPH between the layers. Consideration of an effective ISPH is essential to remain content with the experimentally observed doping dependence of the c-axis transport that, the effective ISPH is strongest in the overdoped regime, decreases gradually through optimal doping and becomes negligible in the underdoped regime. In case of HTLS, the interrelation between the effects of electronic correlations and the variation of dopant concentration, also supports this doping dependence of effective ISPH. Effects of electronic correlations [9], which inhibits ISPH within the Anderson’s ILPT model, is strongest in highly underdoped cuprates leading to non-Fermi liquid characteristics [10], but becomes weaker in the overdoped regime where a Fermi-liquid behavior is expected. Thus, it is suggestive theoretically that the restriction on ISPH be relaxed in the overdoped regime and ISPH be given due consideration. To remain consistent with these observations and arguments, we introduce a probability factor P to describe the effective ISPH, such that, P becomes very small or negligible for highly underdoped systems and increases with doping leading to significant values of the effective ISPH in the overdoped region.

An important recent development in HTLS is the observation of a normal state pseudogap. This gap has been inferred from the NMR [11, 12], optical conductivity [13, 14], heat capacity [15] and transport data [16] in different cuprate materials. In addition, the angle resolved photoemission spectroscopy (ARPES) experiments have shown that the symmetry of the normal state pseudogap is d-wave like [17], similar to that of the superconducting gap, and its magnitude ($E_g$) is large for underdoping, but falls off rapidly with the increase of doping [12, 13, 17]. C. C. Homes et al., by optical conductivity measurements on Y123 material [13], found that for oxygen-reduced sample (underdoped),
where $E_g$ is appreciable, the $c$-axis dc (zero frequency) conductivity decreases with lowering of temperature, whereas at frequencies well above the pseudogap the conductivity is temperature independent. Similar evidences are also reported in experiments on other cuprates \cite{13, 14}. These findings reveal a clear correspondence between the pseudogap magnitude and the suppression of $c$-axis conductivity. Motivated by these facts we express the probability factor $P$ in terms of $E_g$ \cite{18, 19} in such a way, that the doping dependence of $E_g$ makes $P$ to follow the observed doping dependence of $c$-axis transport, i.e. the effective ISPH becomes small for underdoping and significant for overdoping. Initially we choose an exponential form for the probability factor $P = e^{-E_g/T}$, which is very small for underdoping (because of large $E_g$) ensuring a negligibly small value of the effective ISPH. With the increase of doping, $E_g$ falls off and $P$ increases, yielding significant values of effective ISPH for the overdoped systems. Notice that, when ISPH probability is finite, a fraction of the available charge carriers will take part in ISPH, and only those particles, which are not participating in ISPH, will be available for pair tunneling. Since pair tunneling is a two particle process the probabilistic weight for such a process is taken as $(1 - P)^2$. Here, in the generalised pair tunneling model, we assume that the interlayer tunneling of particles could occur via two channels: (i) single particle hopping and (ii) pair tunneling, and these two processes are complementary.

Having introduced the extended ILPT model, we do a mean-field analysis within the BCS approximation and obtain self consistent equations for the gap parameter and the chemical potential. Focussing mainly on the optimal doping situation (at which $T_c$ becomes maximum), we calculate $T_c$ and the superconducting gap $\Delta_k^{\text{max}}(T)$. Main results from our calculations are as follows. Phase diagram of the model ($T_c$ versus $\delta$) shows that the transition temperature $T_c$ at optimal doping increases with the increase of $E_g$. $T_c$ as a function of bare interlayer hopping ($t_{\perp}$) shows upward or downward trend depending on the values of $E_g$. High values of the ratio of the superconducting gap to $T_c$ ($\sim 6 - 9$), as observed in HTLS \cite{20}, are recovered for realistic range of parameters. Gap variation $\Delta_k^{\text{max}}(T)/\Delta_k^{\text{max}}(0)$ as a function of scaled temperature $T/T_c$ shows qualitatively correct behaviour as in high-$T_c$ cuprates. A rigorous fit to the experimental gap-variation data \cite{21} is obtained with a $T$-linear choice of the probability factor $P$. 
The paper is organised as follows. In section-2 we present a detailed account regarding the formulation and justification of the extended ILPT model. Section-3 includes a brief presentation of the steps involved in the mean-field calculations leading to the gap equation. Results from our calculations are discussed in section-4 and section-5 contains a summary of results and some comments.

2. Extended interlayer pair tunneling model: formulation and justification

The model Hamiltonian for the coupled bilayer complex [19] is given by

\[
H = \sum_{i,k,\sigma} (\epsilon_k - \mu)c_{ik\sigma}^\dagger c_{ik\sigma} - \sum_{i,k,k',\sigma} \left[ V_{k,k'} c_{ik\uparrow}^\dagger c_{-k'k\downarrow}^\dagger c_{-k'k\downarrow} c_{k\uparrow} + h.c \right] \\
- \sum_{i\neq j,k} \left[ T_p^{\text{eff}}(k) c_{ik\uparrow}^\dagger c_{-k'j\downarrow}^\dagger c_{-k'j\downarrow} c_{k'j\uparrow} + h.c \right] + \sum_{i\neq j,k,\sigma} \left[ t_\perp^{\text{eff}}(k) c_{ik\sigma}^\dagger c_{kj\sigma} + h.c \right]
\]  

(1)

which is similar to that of pair tunneling model [1], except the last term that accounts for the interlayer single particle hopping. Here, \( c_{ik\sigma}^\dagger \) (\( c_{ik\sigma} \)) is the fermion creation (annihilation) operator with momentum \( k \) and spin sigma, \( i (= 1, 2) \) is the layer index, \( V_{k,k'} \) is the pairing potential forming Cooper pairs in the \( ab \)-plane and \( \mu \) is the chemical potential. The \( ab \)-plane band dispersion \( \epsilon_k \) is taken to be that of Bi2212, obtained from a six parameter tight binding fit \([t_0, t_1, t_2, t_3, t_4, t_5] = [0.131, -0.149, 0.041, -0.013, -0.014, 0.013] \) eV to the ARPES data [22]. This six parameter band dispersion, used elsewhere [23], shows flat bands in the Brillouin zone and the corresponding density of states (DOS) has a power law singularity known as extended van Hove singularity. This is characteristic to the high-\( T_c \) cuprates [24]. \( T_p^{\text{eff}}(k) \) and \( t_\perp^{\text{eff}}(k) \) represent effective matrix elements for pair tunneling and single particle hopping respectively between the layers which involve the probability factor \( P \). Effective ISPH is taken to be \( t_\perp^{\text{eff}}(k) = t_\perp^b(k)P \), where \( t_\perp^b(k) = t_\perp((\cos k_x a - \cos k_y a)/2)^2 \) is the \( k \)-dependent ISPH as predicted by the band structure calculations [8] with \( t_\perp \) being the bare ISPH matrix element (\( a \) is the lattice constant). The bare pair tunneling, following the original ILPT model [3], is taken as \( T_p(k) = (t_\perp^b(k))^2/|t_1| \) where \( t_1 \) is the nearest neighbour hopping matrix element of the \( ab \)-plane band dispersion [22], and the effective pair tunneling, as mentioned earlier, is represented by \( T_p^{\text{eff}}(k) = T_p(k)(1 - P)^2 \).

Regarding the origin of the pseudogap in cuprates no consensus has been reached.
so far. Some authors believe that it is precursor to the superconducting gap \cite{25} and it represents the pairing energy of preformed pairs without phase coherence. Absence of phase coherence among the preformed pairs, is believed to be due to the strong phase fluctuations in the quasi two dimensional systems, and superconductivity sets in at the phase locking transition temperature of these pairs. However, such a scenario is contradicted by recent experiments \cite{26}, which observe absence of any isotope effect in $E_g$, even though there remains an isotope effect in $T_c$, suggesting that the interactions responsible for superconductivity and the pseudogap are independent and the pseudo gap cannot be attributed to short-range superconducting pairing correlations. Similar views are reflected by M. Suzuki et al. by tunneling spectroscopy measurements on Bi2212 material \cite{27}. These experiments also found that the normal state pseudogap coexists with the superconducting gap below $T_c$.

A second school of thought pertains to the strong correlation viewpoint for HTLS \cite{28}, where spin and charge degrees of freedom of electrons are decomposed into spinons and holons. For underdoped systems, below a temperature $T_{RVB}$, the spinons are paired as Resonating Valence bond (RVB) \cite{29} in the singlet state, and $T_{RVB}$ is much higher than the superconducting transition temperature \cite{30}. The pseudogap may be interpreted as the spin excitation gap over the RVB singlet state. Within this picture, a holon has to combine with a spinon to form a real hole which can hop between the layers. Consequently, for underdoped systems where spinons are paired below $T_{RVB}$, the ISPH is heavily suppressed. Such a picture, which forms the basis of Anderson’s pair tunneling model, also motivates us to assume that charge carriers, which are available for ISPH, decreases with $E_g$. This phenomenon (for the case of short-range RVB) could be represented by an exponential probability factor $P = \exp(-E_g/T)$. Exponential form of $P$ signifies that the particles have to be activated above the gap $E_g$ to be available for ISPH. This activation is favoured as $T$ goes up or $E_g$ gets lowered, whereas an increase in $E_g$ would degrade the activation and lead to the suppression of ISPH. Notice that the form of $P$, as it stands, could mimic the observed doping dependence of $c$-axis transport by virtue of the parameter $E_g$, that is, $c$-axis transport is small for underdoping ($E_g$ large), gradually increases through optimal doping ($E_g$ gets lowered) and becomes
significant for overdoping ($E_g$ very small).

The phenomenological behaviour of $P$, and consequently that of the effective ISPH, is also supported by certain existing ideas and results in HTLS. Kumar and Jayannavar [31], while trying to explain the power law dependence of the semiconducting-like $c$-axis resistivity ($\rho_c$) of HTLS, showed that the low temperature up-turn of $\rho_c$ could come from the $T$-dependence of the interlayer coupling $t_\perp$ which gets renormalized as a power of temperature $t_\perp^{\text{eff}} = t_\perp T^\alpha$ (Eq.(7) of Ref.[31]) where $\alpha$ is an exponent of the order of unity. This renormalization of $t_\perp$, which is an adiabatic modification, was originally derived by A. J. Leggett et al. considering a coupling of the slow interplanar electron tunneling to some bosonic degrees of freedom [32]. It has also been established that the strong electronic correlations, present in the CuO$_2$ planes of HTLS in the form of strong intraplane electron-electron scattering, could lead to the complete blocking of single particle tunneling between CuO$_2$ layers at zero temperature ($T = 0$), but the tunneling of electron pairs remain uninhibited [9]. These views, if translated in the language of our model, would mean that at $T = 0$, $P = 0$ and $t_\perp^{\text{eff}} = 0$ but $T_p^{\text{eff}} \neq 0$, whereas at any finite temperature ($T \neq 0$) $P \neq 0$ and $t_\perp^{\text{eff}}$ is nonzero. Precisely, the effective ISPH and the effective pair tunneling matrix elements within our model show similar temperature dependence.

Thus, our proposed model for HTLS is consistent with the experimental observations and theoretical ideas/results regarding the $c$-axis electrical transport. Effects of intraplanar correlations, which is strongest in the too-underdoped region ($E_g \rightarrow \infty$), can be thought of as coming to picture in our model as blocking the ISPH at any temperature [9] in the limit of very small doping. As the doping is increased, intraplanar correlations become weaker ($E_g$ decreases) and ISPH becomes more and more favourable at nonzero temperatures, however at $T = 0$, ISPH remains completely blocked at any doping. Thus, in the limit $E_g \rightarrow \infty$ or at $T = 0$, the probability factor $P$ vanishes leading to $t_\perp^{\text{eff}} = 0$ and our model becomes equivalent to the original ILPT model, and in this sense the proposed model is more general.

It is important to note, that the presence of ISPH gives rise to bilayer splitting, which
however has not been observed in Bi2212 system [33], particularly at low temperatures.
In this light, we need to check the validity of our proposed model. A study of the
electronic density of states (DOS) of our model [18] reveals that for optimally doped
or underdoped systems, when \( E_g \geq T_c \), bilayer splitting is absent at low temperatures
\( T \leq 20 \text{ K} \) and within an energy resolution of 1 \text{ meV}. This is because of strong suppression
of ISPH by \( E_g \). At higher temperatures, broadening of the quasiparticle states would be
important to obscure such splitting in real systems. A detailed calculation of the ARPES
intensity curves within our proposed model clearly establishes that bilayer splitting would
not be observable at low temperatures \( \sim 13 \text{ K} \) as in actual experiment of Ref.[33]. Even at
a higher temperature \( \sim 40 \text{ K} \) and with a resolution much better than that in experiment,
splitting remains absent [18]. These outcomes further embolden the consistency of our
proposed model with experimental results on HTLS.

3. Superconducting gap equation within the BCS approximation

Mean field decoupling of the four fermion terms in the Hamiltonian of Eq.(1) yields

\[
H = \sum_{i,k,\sigma} (\epsilon_k - \mu) c_{i,k\sigma}^{(i)} c_{i,k\sigma}^{(i)} - \sum_{i,k} [\Delta_k c_{k\uparrow}^{(i)} c_{-k\downarrow}^{(i)} + h.c] \\
+ \sum_{i \neq j,k,\sigma} \left[ t_{\perp}^{\text{eff}}(k) c_{k\sigma}^{(i)} c_{j\sigma}^{(j)} + h.c \right] 
\]

(2)

with the gap parameter being defined as

\[
\Delta_k = \Delta_{i,k} = \sum_{k'} V_{k,k'} \langle c_{i,k'}^{(*)} c_{j,k'}^{(*)} \rangle + T_p^{\text{eff}}(k) \langle c_{i,k}^{(*)} c_{j,k}^{(*)} \rangle 
\]

(3)

Layer index \((i, j)\) are equivalent, since by symmetry, in-plane pairing is identical in both
the layers. Presence of interlayer single particle hopping produces two quasiparticle
bands \( E_k^- = \sqrt{\{\epsilon_k - \mu - t_{\perp}^{\text{eff}}(k)\}^2 + \Delta_k^2} \) and \( E_k^+ = \sqrt{\{\epsilon_k - \mu + t_{\perp}^{\text{eff}}(k)\}^2 + \Delta_k^2} \).

Self consistent equations, for chemical potential and the superconducting gap are
obtained as

\[
1 - \delta = 1 - \frac{1}{N} \sum_k (\epsilon_k - \mu - t_{\perp}^{\text{eff}}) \chi(E_k^-) - \frac{1}{N} \sum_k (\epsilon_k - \mu + t_{\perp}^{\text{eff}}) \chi(E_k^+) 
\]

(4)
and

\[ \Delta_k = \frac{\sum_{k'} \Delta_{k',k} V_{k',k'} \left( \chi(E_{k'}^-) + \chi(E_{k'}^+) \right)/2}{1 - T_{\text{eff}}(k') \left( \chi(E_{k}^-) + \chi(E_{k}^+) \right)/2} \]  

(5)

where \( \chi(E_{k}^\pm) = \frac{1}{2E_{k}^\pm} \tanh \left( \frac{\beta E_{k}^\pm}{2} \right) \), \( \beta = 1/T \) (in a scale of \( k_B = 1 \)), \( \delta = 1 - n \) with \( n \) being the number of electrons per site and \( N \) is the total number of lattice sites. The pairing interaction \( V_{k,k'} \) is separable as \( V_{k,k'} = V \eta_k \eta_{k'} \), which also makes the \( k \)-dependence of \( \Delta_k \) to be shoved in the symmetry factor \( \eta_k \). Finally, the expression for the superconducting gap (from Eq.(5)) becomes

\[ \frac{1}{4V} = \frac{1}{N} \sum_k \frac{\eta_k^2 \left( \chi(E_k^-) + \chi(E_k^+) \right)/2}{1 - T_{\text{eff}}(k) \left( \chi(E_k^-) + \chi(E_k^+) \right)/2} \]  

(6)

In our calculations, the \( d_{x^2-y^2} \) symmetry of the pairing state is considered because of growing evidence of the same in high-\( T_c \) cuprates \cite{34, 35} and this implies the symmetry factor to be \( \eta_k = (\cos k_x a - \cos k_y a)/2 \). The parameter \( V \), in our case, is the nearest neighbour attractive interaction. In experiments, momentum dependence of the normal state pseudogap is found to be of \( d_{x^2-y^2} \) symmetry \cite{17}, and we take \( E_g(k) = E_g | \cos k_x a - \cos k_y a | \). This introduces a \( k \) dependence in the probability factor \( P(k) = \exp(-E_g(k)/T) \).

4. Results and discussions

We numerically solve the self consistent equations (4) and (6) for the superconducting gap parameter and the chemical potential. Basically, we focus on the properties characterizing the superconducting state, since in this state the quasiparticle picture is valid at any doping. We calculate the superconducting gap \( \Delta_k^{\text{max}}(T) \) and the transition temperature \( T_c \) (at which the gap magnitude drops to zero) and also study their variations with model parameters.

4.1. Transition temperature:

In Fig.1, we plot transition temperature \( T_c \) as a function of \( \delta \), for various values of the normal state pseudogap \( E_g \) as shown explicitly. For \( E_g = 0 \), where ISPH takes
its full value \((P = 1)\) and the pair tunneling is absent, \(T_c\) has two peaks. This is a consequence of the splitting of DOS due to the presence of unsuppressed ISPH between the layers \([18, 19]\). As \(E_g\) increases, the ISPH gets suppressed and the two peak form gradually merge into one peak, yielding the maximum transition temperature \((T_c^m)\) at optimal doping. For realistic values of \(E_g\), the one peak form persists which is the case in experiments. We choose the bare value of \(t_\perp = 40\, \text{meV}\), in accordance with the band structure calculations \([8]\) suggesting \(t_\perp/|t_1| \sim 0.2 - 0.3\). Interaction parameter is fixed at \(V = 70\, \text{meV}\) which makes \(T_c^m \sim 100\, \text{K}\) for \(E_g \sim T_c^m\). This value of \(V\) is kept fixed throughout in this communication. Clearly, for fixed values of \(V\) and \(t_\perp\), \(T_c^m\) increases with the increase of \(E_g\) and highest value of \(T_c^m\) is obtained in the Anderson limit \((E_g = \infty)\) where ISPH is completely suppressed giving vent to only pair tunneling between the layers. It should be noted that \(E_g\) is kept fixed for every curve in Fig.1, whereas, in reality it should change with doping. Here, we are interested in quantities at optimal doping (as in Fig.2 and Fig.3) and hence consider different fixed \(E_g\) values which might represent different cuprate materials. Increase of \(T_c^m\) with \(E_g\) is consistent with the scaling behaviour of the pseudogap \([12]\), according to which, at a fixed doping the ratio \(E_g/T_c^m\) should ideally fall on the same point for different cuprate materials.

Variation of \(T_c^m\) as a function of \(t_\perp\) is shown in Fig.2. Numerical values of \(E_g\) for different plots are \((A, B, C, D, E) = (2, 4, 6, 8, \infty)\) in \(\text{meV}\). For small \(E_g\), \(T_c^m\) decreases with increasing \(t_\perp\). Increase of \(E_g\) makes interlayer pair tunneling more probable. Consequently, \(T_c^m\) rises with \(t_\perp\) and in the Anderson limit \(T_c^m\) grows rapidly with \(t_\perp\). It may be noted that regarding the role of the interlayer coupling on \(T_c\), there is mixed experimental evidence (see Ref.\([3]\), page 141). Our results show that \(T_c^m\) may increase or decrease or may even remain unaffected with the increase of interlayer coupling \(t_\perp\), depending upon the values of \(E_g\). This is a prediction of our calculations and a systematic study of the variation of \(T_c\) with \(t_\perp\) for differently doped or different class of cuprate materials (with changing \(E_g\)) is called for. A possible mechanism to vary \(t_\perp\) could be the application of pressure along \(c\)-axis.
4.2. Superconducting gap parameter:

Two important issues relating the superconducting gap, which figures frequently in the HTLS literature, are the ratio of the superconducting gap to $T_c$ and the temperature variation of the gap. Superconducting gap-ratio (gap-width) $\Delta_{k}^{\text{max}}(0)/T_c$, as a function of $t_{\perp}$ for different $E_g$, is presented in Fig.3. Alphabetic labels correspond to different $E_g$ as written in the figure (similar to the values of $E_g$ as in Fig.2). In the Anderson limit, gap-ratio could reach the value $\sim 4.5$ for $t_{\perp}$ within realistic range ($\sim 30 - 45\text{ meV}$). But, for finite $E_g$, the gap-ratio within this range of $t_{\perp}$ increases as $E_g$ is lowered. For example, with $E_g = 4\text{ meV}$ one could get the gap-ratio as high a value as $\sim 6$ for $t_{\perp} \sim 45\text{ meV}$. To understand this, note that for any finite $E_g$ the probability factor $P \to 0$ as $T \to 0$. This situation corresponds to the Anderson limit where ISPH is completely suppressed. Hence at $T = 0$, $\Delta_{k}^{\text{max}}(0)$ is quantitatively same for any $E_g$ whatsoever for a fixed $t_{\perp}$. But for $T \neq 0$, as $E_g$ is lowered, the suppression of ISPH gets weaker and correspondingly $T_c$ falls off from its value in the Anderson limit which is evident from the plot in Fig.2. Consequently, the ratio $2\Delta_{k}^{\text{max}}(0)/T_c$ is small for $E_g = \infty$ and increases as $E_g$ is decreased.

Finally, We study the temperature variation of the superconducting gap. In Fig.4 we plot $\Delta_{k}^{\text{max}}(T)/\Delta_{k}^{\text{max}}(0)$ as a function of reduced temperature $T/T_c$ (dashed lines), for different $E_g$ values shown. Here $t_{\perp} = 40\text{ meV}$ and the doping is at the optimum level. The solid line represents results for BCS superconductors and solid square symbols are experimental data [21] for Bi-cuprate. Clearly, the locus of experimental data is well below the BCS-curve. Results from our calculations are all below the BCS-curve, but high values of $E_g$ makes them closer to the BCS-curve, with the closest one being that in the Anderson limit. For small $E_g$ our calculations are close to the experimental data. However, our results with $P = \exp(-E_g/T)$ could not match the experimental data for the whole range of $T/T_c$. Maximum mismatch is observed in the low temperature region where experimental data registers a near linear fall from its $T = 0$ saturation.

Anticipating that the linear fall of $\Delta_{k}^{\text{max}}(T)$ in the vicinity of $T = 0$ could come from
a linear increment of effective ISPH, we consider another possible choice of $P$ as

$$P = \frac{T}{E_g + T e^{-E_g/T}}$$

(7)

Note that the choice of $P$ is not unique. While making a choice, the conditions to be satisfied are, $P \propto T$ for $E_g \gg T$ and $P = 1$ for $E_g = 0$, which are ensured in Eq.(7). Results, with linear-$T$ dependent $P$ and $t_\perp = 40 \text{meV}$, $E_g = 8 \text{meV}$ [36], are presented in the inset of Fig.4 as a dashed line. The matching with experimental data (solid squares) is excellent. To our knowledge, this is the first ever calculations that correctly reproduces experimental data, affirming the merits of the model under consideration. Use of $P$ from Eq.(7) does not qualitatively change other results discussed earlier, rather yields higher values of the gap-ratio than those obtained with exponential $P$ for the same set of parameters. As for example, $t_\perp = 40 \text{meV}$ and $E_g = 4 \text{meV}$ yields a gap-ratio 7.1 for the present case.

The $T$-linear form of the probability factor could have phenomenological support from the work by Leggett et al., where the renormalization of $t_\perp$ involves a power law dependence of temperature as $T^\alpha$ with $\alpha$ being of the order of unity. Arguments could also be given within the spin-charge separation picture of Anderson and coworkers, where spinons and holons are the quasiparticles in a layer, and a holon has to combine with a spinon to form a real hole that can hop to another layer. Thus, within this picture, $c$-axis transport is proportional to the spinon density, and since the density of spin excitations grows as $T$ within the (long-range) RVB model [29], the $c$-axis transport is expected to increase linearly with $T$. These views, at least on the phenomenological level, justifies the form of $P$ as in Eq.(7).

So far, we have presented results within the extended ILPT model incorporating the pseudogap as a phenomenological parameter which affects only the out-of-plane charge transport (suppresses the ISPH), and our focus remained on the optimally doped situation (fixed $E_g$). But in actual case, the pseudogap should also affect the in-plane charge dynamics and should vary with doping as observed in experiments [12, 17, 17]. Regarding the in-plane effects of $E_g$, based on the analysis of experimental pseudogap data for several cuprates, it has been suggested that, the pseudogap suppresses the in-plane spectral
weight or single particle DOS \cite{12}, and to this effect, the total measurable spectroscopic
gap on the Fermi surface (FS) would be an additive combination of the superconducting
gap and the pseudogap. Following this idea, we modify the quasiparticle band energies as
\[ E_{\pm k} = \sqrt{\left(\epsilon_{k} - \mu \pm t_{\text{eff}}^{\perp}(k)\right)^{2} + \Delta_{k}^{2} + E_{g}(k)^{2}} \]
do preliminary calculations incorporating
the variation of \( E_{g} \) with doping as in Ref\cite{12}. Our results show that, \( T_{c} \) in this case, falls
off rapidly towards underdoping because of the rapid suppression of spectral weight by
high values of \( E_{g} \). We also find that, the total spectroscopic gap at zero temperature
(\( T = 0 \)) shows non-trivial doping dependence (different from \( T_{c} \)) as noted by tunneling
experiments on HTLS \cite{37}. Around the optimal doping, in-plane effects of \( E_{g} \) does not
change the qualitative features of the superconducting gap or the transition tempera-
ture \cite{38}, and the conclusions drawn in this paper remains unaffected. However, to have
better understanding about the effects of \( E_{g} \) on the \( ab \)-plane charge dynamics, and to
draw definite conclusions regarding the behaviour of \( T_{c} \) and gap parameter as a function
of doping within the DOS-suppression picture, detailed calculations would be necessary.
Results on these matters will be presented in future communications.

5. Summary and comments

In this communication, we consider an extension of the interlayer pair tunneling model
including an effective interlayer single particle hopping and calculate the superconducting
gap as well as the mean-field \( T_{c} \). Here, we briefly state the results from our calculations.
Increase of \( T_{c}^{m} \) with the increase of \( E_{g} \) is in qualitative agreement with the scaling
behaviour of the pseudogap. Within the model, high values for the gap-ratio are obtained
and the experimental gap-variation data are reproduced with a \( T \)-linear probability factor
for the effective ISPH. An outcome of our calculations, that \( T_{c} \) may increase or decrease
as a function of interlayer coupling depending on \( E_{g} \), comes as a natural prediction which
could be put to test. Thus, several contentious issues of high-\( T_{c} \) cuprates are explained
within the extended interlayer pair tunneling model and new predictions are made.

Certain comments regarding the phase diagram and the extended ILPT model in
general, would be relevant at this point. In Fig.1 we find that for the no doping (\( \delta = 0 \))
situation a finite \( T_{c} \) is obtained, which is not the case in real layered cuprate materials.
This is due to the non-consideration of the effect of the strong on-site repulsions (strong correlations) on the kinetic energy term removing any double occupancy. The inconsistency can be removed by calculating $T_c$ within a $t-J-V$ model where hopping is restricted in a space with no double occupancy [39]. But the point is that, in both of these analyses (in the limit of weak and strong correlations), properties characterising the superconducting state would remain unaltered. Moreover, we would like to note that, in our extended ILPT model, strong intraplanar correlations do exist, which plays role only to suppress the single particle hopping between CuO$_2$ layers. This is similar in spirit to the views adapted by Anderson and collaborators relating to the original ILPT model [3].

In the context of ILPT model, Chakravarty and Anderson showed that, in the limit of very small temperature or ($T \rightarrow 0$), the phenomenon of the blocking of ISPH is exhibited by those HTLS materials characterized by non-Fermi liquid behaviour [10]. We would like to note, that non-Fermi liquid behaviour, as mentioned in the introduction, is the characteristic of too-underdoped HTLS ($E_g$ very large or $E_g \rightarrow \infty$) and the original ILPT model, which is a special case of our extended model, is valid for very little doped HTLS. For overdoping, on the other hand, HTLS show Fermi liquid like characteristics and the $c$-axis charge transport becomes more like that in the $ab$-plane. This implies that, for overdoped HTLS, ISPH gains significance and an extension of ILPT model, as considered in this communication, becomes essential. Thus one finds that the extended ILPT model is applicable to HTLS for a wide range of doping starting from too little to very high, whereas the original ILPT model is applicable only in the underdoped region. Furthermore, comparison of our results with those from experiments on HTLS show that, the extended ILPT model captures HTLS phenomenology better than the original ILPT model.

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Figure captions:

Fig.1. Mean-field superconducting transition temperature \( T_c \) as a function of dopant concentration \( \delta \) for different values of \( E_g \) (magnitude of the normal state pseudogap) as shown in the figure. Other parameters are chosen to be \( V = 70 \, meV \) and \( t_\perp = 40 \, meV \). Anderson limit corresponds to \( E_g = \infty \).

Fig.2. Plot of \( T_c^m \) as a function of bare interlayer single particle hopping \( t_\perp \), for different \( E_g \) values denoted by alphabetic labels \( (A, B, C, D, E) = (2, 4, 6, 8, \infty) \) in \( meV \). Varied behaviour of \( T_c^m \) flow with \( t_\perp \) is apparent from the curves.

Fig.3. Maximum value of the gap-ratio \( (2\Delta_{k}^{max}(0)/T_c) \) is plotted as a function of \( t_\perp \) at optimal doping. Alphabetic labels corresponding to different \( E_g \) values are the same as in Fig.2 and are listed in the inset.

Fig.4. Finite temperature gap scaled to its zero temperature value \( (\Delta_{k}^{max}(T)/\Delta_{k}^{max}(0)) \), as a function of reduced temperature \( (T/T_c) \). Solid line is the BCS-form, solid square symbols are experimental data from Ref.\[21\] and dashed lines are from our calculations for different \( E_g \) shown. [Inset: Dashed line is from our calculations with the renormalizing factor as in Eq.(7). Solid squares are experimental data and solid line is the BCS-form as in the main figure].
