Cutoff-Free Propagation of Torsional Alfvén Waves Along Thin Magnetic Flux Tubes

Z.E. Musielak$^{1,2}$, S. Routh$^1$ and R. Hammer$^2$

ABSTRACT

Propagation of torsional Alfvén waves along magnetic flux tubes has been extensively studied for many years but no conclusive results regarding the existence of a cutoff frequency for these waves have been obtained. The main purpose of this paper is to derive new wave equations that describe the propagation of linear torsional Alfvén waves along thin and isothermal magnetic flux tubes, and use these wave equations to demonstrate that the torsional wave propagation is not affected by any cutoff frequency. It is also shown that this cutoff-free propagation is independent of different choices of the coordinate systems and wave variables adopted in the previous studies. A brief discussion of implications of this cutoff-free propagation of torsional tube waves on theories of wave heating of the solar and stellar atmospheres is also given.

Subject headings: stars: atmospheres – MHD – stars: late-type – wave motions

1. Introduction

Direct measurements show that solar magnetic fields outside sunspots are concentrated into flux-tube structures located primarily at the boundaries of supergranules (e.g., Solanki 1993). Similar magnetic structures are likely to exist in late-type stars (e.g., Saar 1996, 1998). The fundamental modes supported by solar and stellar magnetic flux tubes can be classified as sausage, kink, torsional Alfvén and fluting modes (for details, see excellent reviews by Hollweg 1990, Roberts 1991, and Roberts & Ulmschneider 1997, and references therein). The role played by these waves in the heating of different parts of the solar and stellar atmospheres was discussed by Narain & Ulmschneider (1996) and Ulmschneider &

---

$^1$Department of Physics, University of Texas at Arlington, Arlington, TX 76019, USA; zmusielak@uta.edu; sxr9573@uta.edu

$^2$Kiepenheuer-Institut für Sonnenphysik, Schöneckstr. 6, 79104 Freiburg, Germany; hammer@kis.uni-freiburg.de
Musielak (2003). The energy carried by sausage and kink tube waves was used as the input to the theoretical models of stellar chromospheres constructed by Cuntz et al. (1999) and Fawzy et al. (2002a, b).

In many theoretical studies of tube waves, it is assumed that these waves are linear and they propagate along vertically oriented and thin flux tubes. Under these conditions, the waves do not interact with each other, so they can be investigated independently. Important studies of sausage tube waves were performed by Defouw (1976), who derived the wave equation for these waves and demonstrated that there is a cutoff frequency, which restricts the wave propagation to only those frequencies that are higher than the cutoff (see also Webb & Roberts 1979; Rae & Roberts 1982; Edwin & Roberts 1983; Musielak, Rosner, & Ulmschneider 1987). Similar studies of kink tube waves were performed by Spruit (1981, 1982), who derived both the wave equation and the cutoff frequency for these waves (see also Musielak & Ulmschneider 2001).

Propagation of torsional Alfvén waves along solar and stellar magnetic flux tubes was extensively studied in the literature (e.g., Parker 1979; Priest 1982, 1990; Edwin & Roberts 1983; Hollweg 1985; 1990; Poedts, Hermans, & Goossens 1985; Ferriz-Mas, Schüssler, & Anton, 1989; Roberts 1991; Ferriz-Mas & Schüssler 1994; Roberts & Ulmschneider 1997; Hasan et al. 2003; Noble, Musielak, & Ulmschneider 2003). Two different approaches were considered and different sets of wave variables were used. In the first approach, the propagation of the waves was described in a global coordinate system (e.g., Ferriz-Mas et al. 1989; Noble et al. 2003), while in the second approach a local coordinate system was used (Hollweg 1978, 1981, 1992). The momentum and induction equations derived by Ferriz-Mas et al. (1989) were adopted by Ploner & Solanki (1999) in their studies of the influence of torsional tube waves on spectral lines formed in the solar atmosphere. In numerical studies of torsional tube waves performed by Kudoh & Shibata (1999) and Saito, Kudoh, & Shibata (2001), the basic equations originally derived by Hollweg were extended to more than one dimension and nonlinear terms were included.

The specific problem of the existence or non-existence of a cutoff frequency for torsional Alfvén waves propagating along thin and isothermal magnetic flux tubes has not been discussed in the literature. An exception is the paper by Noble et al. (2003), who studied the generation rate of torsional tube waves in the solar convection zone and introduced the cutoff frequency, defined as the ratio of the Alfvén velocity to four times the pressure (or density) scale height, for these waves. In this paper, we revisit the problem by deriving new wave equations that describe the propagation of torsional tube waves and demonstrating that this propagation is cutoff-free. We also show that the cutoff-free propagation is independent of different choices of wave variables and coordinate systems used by Ferriz-Mas et al. (1989)
The paper is organized as follows. The momentum and induction equations derived in a global coordinate system are presented in §2. In §3, new wave equations for torsional Alfvén waves propagating along thin and isothermal magnetic flux tubes are obtained and it is shown that the wave propagation is not affected by any cutoff frequency. The fact that this cutoff-free propagation is independent of different choices of wave variables and coordinate systems is demonstrated in §4. A brief discussion of our results is given in §5, and conclusions are presented in §6.

2. Basic equations

We consider an isolated and vertically oriented magnetic flux tube that is embedded in a magnetic field-free, compressible and isothermal medium. The tube has a circular cross-section and is in temperature equilibrium with the external medium. Let us introduce a global cylindrical coordinate system \((r, \phi, z)\), with \(z\) being the tube axis, and describe the background medium inside the tube by the gas density \(\rho_0 = \rho_0(r, z)\), the gas pressure \(p_0 = p_0(r, z)\) and the magnetic field \(\vec{B}_0 = B_{0r}(r, z)\hat{r} + B_{0z}(r, z)\hat{z}\). The physical properties of the external medium are determined by \(\rho_e = \rho_e(r, z)\), \(p_e = p_e(r, z)\) and \(\vec{B}_e = 0\). Moreover, we also have \(T_0 = T_e = \text{const.}\)

To describe torsional Alfvén waves, we introduce \(\vec{v} = v_{\phi}(r, z, t)\hat{\phi}\) and \(\vec{b} = b_{\phi}(r, z, t)\hat{\phi}\), and assume that the waves are linear and purely incompressible, which means that both the perturbed density \(\rho\) and pressure \(p\) can be neglected. As a result of these assumptions, the propagation of the waves is fully described by the momentum and induction equations. The \(\phi\)-component of the momentum equation can be written in the following form:

\[
\frac{\partial}{\partial t} \left( \frac{v_{\phi}}{r} \right) - \frac{1}{4\pi \rho_0 r^2} \left[ B_{0r} \frac{\partial}{\partial r} + B_{0z} \frac{\partial}{\partial z} \right] (rb_{\phi}) = 0 ,
\]

and the \(\phi\)-component of the induction equation becomes

\[
\frac{\partial}{\partial t} (rb_{\phi}) - r^2 \left[ B_{0r} \frac{\partial}{\partial r} + B_{0z} \frac{\partial}{\partial z} \right] \left( \frac{v_{\phi}}{r} \right) = 0 .
\]

The derived momentum and induction equations are our basic equations for all the results derived and discussed in this paper.
3. Propagation along thin magnetic flux tubes

3.1. The thin flux tube approximation

Solar magnetic flux tubes are considered to be thin if their magnetic field is horizontally uniform, which means that at a given height all magnetic field lines have the same physical properties (e.g., Priest 1982). The essence of the so-called thin flux tube approximation (Roberts & Webb 1978, 1979; Spruit 1981, 1982; Priest 1982; Hollweg 1985; Ferriz-Mas, Schüssler, & Anton, 1989; Ferriz-Mas & Schüssler 1994; Musielak et al. 1995; Roberts & Ulmschneider 1997; Hasan et al. 2003) is that radial expansions around the axis of symmetry can be truncated at a low order. For the radial component of the magnetic field the leading term is of first order,

\[
B_0(r, z) = B_0(r, z) \big|_{r=0} + r \left[ \frac{\partial B_0(r, z)}{\partial r} \right]_{r=0} + ... ,
\]

(3)
since \(B_0(r = 0, z) = 0\) at the symmetry axis (cf. Ferriz Mas & Schüssler 1989). Away from the tube axis \(B_0(r, z)\) can be expressed in terms of \(r\) and \(B_0(z)\). Using the solenoidal condition \(\nabla \cdot \vec{B}_0 = 0\), we obtain

\[
B_0(r, z) = -\frac{r}{2} B'_0(z) ,
\]

(4)
where \(B'_0 = dB_0/dz\).

The thin flux tube approximation also requires that \(\rho_0 = \rho_0(z)\), \(p_0 = p_0(z)\), \(B_{0z} = B_{0z}(z)\), \(\rho_e = \rho_e(z)\), \(p_e = p_e(z)\) and \(T_0 = T_e = \text{const}\). In addition, the horizontal pressure balance must be satisfied, \(p_0 + B_{0z}^2/8\pi = p_e\) at \(r = R_t\), where \(R_t\) is the tube radius. The increase of \(R_t\) with height is determined by the conservation of the magnetic flux \(\pi R_t^2 B_{0z} = \text{const}\). As a result of the above assumptions, the Alfvén velocity \(c_A = B_{0z}/\sqrt{4\pi\rho_0}\) remains constant along the entire length of a thin and isothermal magnetic flux tube, and \(B'_0 = -B_{0z}/2H\), where the pressure (density) scale height \(H\) is also constant. For reasons explained in the next subsection, the wave variables \(v_\phi\) and \(b_\phi\) are considered here to be functions of time and both spatial coordinates \(r\) and \(z\).

3.2. Wave equations

Using Eq. (4) and taking into account the fact that the variables \(r\) and \(z\) are independent in the global coordinate system, we write Eqs. (1) and (2) as

\[
\frac{\partial v_\phi}{\partial t} + \frac{B'_0}{8\pi\rho_0} \left( r \frac{\partial b_\phi}{\partial r} + b_\phi \right) - \frac{B_{0z}}{4\pi\rho_0} \frac{\partial b_\phi}{\partial z} = 0 ,
\]

(5)
\[
\frac{\partial b_\phi}{\partial t} + \frac{B_{0z}}{2} \left( r \frac{\partial v_\phi}{\partial r} - v_\phi \right) - B_{0z} \frac{\partial v_\phi}{\partial z} = 0 .
\] (6)

We combine the above equations and derive the wave equations for the wave variables \(v_\phi(r, z, t)\) and \(b_\phi(r, z, t)\)

\[
\frac{\partial^2 v_\phi}{\partial t^2} - c_A^2 \frac{\partial^2 v_\phi}{\partial z^2} + c_A^2 \frac{\partial v_\phi}{2H \partial z} - \frac{c_A^2}{16H^2} v_\phi
\]

\[- c_A^2 \left( \frac{r}{4H} \right) \left[ \left( \frac{r}{4H} \right) \frac{\partial^2 v_\phi}{\partial r^2} + 2 \frac{\partial^2 v_\phi}{\partial r \partial z} - \frac{1}{4H} \frac{\partial v_\phi}{\partial r} \right] = 0 ,
\] (7)

and

\[
\frac{\partial^2 b_\phi}{\partial t^2} - c_A^2 \frac{\partial^2 b_\phi}{\partial z^2} - c_A^2 \frac{\partial b_\phi}{2H \partial z} - \frac{c_A^2}{16H^2} b_\phi
\]

\[- c_A^2 \left( \frac{r}{4H} \right) \left[ \left( \frac{r}{4H} \right) \frac{\partial^2 b_\phi}{\partial r^2} + 2 \frac{\partial^2 b_\phi}{\partial r \partial z} + \frac{3}{4H} \frac{\partial b_\phi}{\partial r} \right] = 0 .
\] (8)

The derived wave equations show that the value of the parameter \((r/4H)\) determines the contributions of the \(r\)-dependence of the wave variables \(v_\phi\) and \(b_\phi\) to the propagation of torsional tube waves. For wide flux tubes, the contributions are important, however, for very thin flux tubes with \((r/4H) \ll 1\), the contributions become negligible. It must be noted that the limit \((r/4H) \to 0\) is not allowed because \(v_\phi(r, z, t)|_{r=0} = 0\) and \(b_\phi(r, z, t)|_{r=0} = 0\). From a physical point of view, this means that a flux tube reduced to a single magnetic field line cannot support torsional waves.

Since the derived wave equations have different forms for \(v_\phi\) and \(b_\phi\), the wave variables behave differently. In the following, we transform these wave equations to new variables that obey the same wave equations.

### 3.3. Transformed wave equations

Using the transformations \(v_\phi(r, z, t) = v(r, z, t)\rho^{-1/4}\) and \(b_\phi(r, z, t) = b(r, z, t)\rho^{1/4}\) (see Musielak et al. 1995; Musielak & Ulmschneider 2001; Noble et al. 2003), we obtain

\[
\frac{\partial^2 v}{\partial t^2} - c_A^2 \frac{\partial^2 v}{\partial z^2} - c_A^2 \left( \frac{r}{4H} \right) \left[ \left( \frac{r}{4H} \right) \frac{\partial^2 v}{\partial r^2} + 2 \frac{\partial^2 v}{\partial r \partial z} + \frac{1}{4H} \frac{\partial v}{\partial r} \right] = 0 ,
\] (9)

and

\[
\frac{\partial^2 b}{\partial t^2} - c_A^2 \frac{\partial^2 b}{\partial z^2} - c_A^2 \left( \frac{r}{4H} \right) \left[ \left( \frac{r}{4H} \right) \frac{\partial^2 b}{\partial r^2} + 2 \frac{\partial^2 b}{\partial r \partial z} + \frac{3}{4H} \frac{\partial b}{\partial r} \right] = 0 .
\] (10)
Clearly, the behavior of the transformed wave variables \( v \) and \( b \) is identical.

To remove the first-order derivatives from the above equations, we use the transformation \( d\zeta = (4H/r)dr \), which gives

\[
\left( \frac{\partial^2}{\partial t^2} - c_A^2 \frac{\partial^2}{\partial z^2} - c_A^2 \frac{\partial^2}{\partial \zeta^2} - 2c_A^2 \frac{\partial^2}{\partial z \partial \zeta} \right) [v(\zeta, z, t); b(\zeta, z, t)] = 0 ,
\]

where \( \zeta = 4H \ln |r| \). This is the most general equation that describes the propagation of torsional waves along thin and isothermal magnetic flux tubes. The equation shows that there is no cutoff frequency for torsional tube waves (see Sec. 3.4).

### 3.4. Dispersion relation

Since all the coefficients in Eq. (11) are constant, we make Fourier transforms in time and space, and derive the following dispersion relation

\[
\omega^2 = (k_z^2 + 2k_z k_\zeta + k_\zeta^2)c_A^2 ,
\]

where \( \omega \) is the wave frequency and \( k_z \) and \( k_\zeta \) are the \( z \) and \( \zeta \) components of the wave vector \( \vec{k} \), respectively. Note that the same dispersion relation is obtained for each wave variable.

Let us define \( \kappa = k_z + k_\zeta \) and write

\[
\omega^2 = \kappa^2 c_A^2 ,
\]

which shows that the propagation of linear torsional Alfvén waves along thin and isothermal magnetic tube waves is not affected by any cutoff frequency.

### 4. Other approaches

To demonstrate that the propagation of torsional tube waves is cutoff-free, we used the global coordinate system and the original wave variables \( v_\phi \) and \( b_\phi \), which were transformed to the new variables \( v \) and \( b \). Two different approaches were developed by Ferriz-Mas et al. (1989), who adopted the same coordinate system but used different wave variables, and by Hollweg (1978, 1981, 1992), who chose a local coordinate system and introduced different wave variables (see also Edwin & Roberts 1983 and Poedts et al. 1985). In addition, Noble et al. (2003) considered the global coordinate system and used the wave variables \( v_\phi \) and \( b_\phi \).
However, their assumption that $B_0 r = 0$ was inconsistent with the solenoidal condition, which makes their claim of the existence of the cutoff frequency for torsional Alfvén waves invalid. In the following, we demonstrate that the momentum and induction equations derived by Ferriz-Mas et al. and Hollweg lead to the same results as those found in Sec. 3 of this paper.

### 4.1. Global coordinate system and different wave variables

In their work on propagation of waves along thin magnetic flux tubes, Ferriz-Mas et al. (1989) used the global coordinate system and derived the first and second-order equations that describe the propagation of sausage, kink and torsional Alfvén tube waves. In their approach, each wave variable is expanded in a Taylor series and, specifically for torsional tube waves, the new variables $v_{\phi 1}$ and $b_{\phi 1}$, which represent the first order expansion in the series, are introduced. These variables are given by

$$v_{\phi 1}(z, t) = \frac{\partial v_\phi}{\partial r}|_{r=0} \quad \text{and} \quad b_{\phi 1}(z, t) = \frac{\partial b_\phi}{\partial r}|_{r=0}.$$  \hspace{1cm} (14)

Since $v_\phi(r, z, t)|_{r=0} = 0$ and $b_\phi(r, z, t)|_{r=0} = 0$ (see Sec. 3.2), we may use Eq. (14) to write $v_\phi(r, z, t) = rv_{\phi 1}(z, t)$ and $b_\phi(r, z, t) = rb_{\phi 1}(z, t)$ in first order. Substituting these new variables into Eqs. (5) and (6), we obtain

$$\frac{\partial v_{\phi 1}}{\partial t} + \frac{1}{4\pi \rho_0} \left( B_{0z} b_{\phi 1} - B_{0r} \frac{\partial b_{\phi 1}}{\partial z} \right) = 0,$$  \hspace{1cm} (15)

and

$$\frac{\partial b_{\phi 1}}{\partial t} - B_{0z} \frac{\partial v_{\phi 1}}{\partial z} = 0,$$  \hspace{1cm} (16)

which are the same equations as those obtained by Ferriz-Mas et al. (1989, see their Eqs. 14 and 16; note that our $B_{r0}$ corresponds to their $r B_{r1}$, and $v_z$ and $v_r$ vanish in our case). Despite the fact that $v_{\phi 1}$ and $b_{\phi 1}$ depend solely on $z$ and $t$, the above equations are not valid at the tube axis (see discussion above).

The wave equations resulting from the above momentum and induction equations become

$$\frac{\partial^2 v_{\phi 1}}{\partial t^2} - c_A^2 \frac{\partial^2 v_{\phi 1}}{\partial z^2} = 0,$$  \hspace{1cm} (17)

and

$$\frac{\partial^2 b_{\phi 1}}{\partial t^2} - c_A^2 \frac{\partial^2 b_{\phi 1}}{\partial z^2} - \frac{c_A^2}{H} \frac{\partial b_{\phi 1}}{\partial z} - \frac{c_A^2}{4H^2} b_{\phi 1} = 0.$$  \hspace{1cm} (18)
Clearly, the derived wave equations have different forms, which implies that the wave variables $v_{\phi 1}$ and $b_{\phi 1}$ behave differently. To remove the first-order derivative from Eq. (18), we use the transformation $b_{\phi 1}(z, t) = \tilde{b}_{\phi 1}(z, t)\rho^{1/2}$, and obtain

$$\frac{\partial^2 \tilde{b}_{\phi 1}}{\partial t^2} - c_A^2 \frac{\partial^2 \tilde{b}_{\phi 1}}{\partial z^2} = 0.$$ (19)

Hence, the behavior of the wave variables $v_{\phi}$ and $\tilde{b}_{\phi 1}$ is the same and there is no cutoff frequency that affects the wave propagation. This is an important result as it shows that the non-existence of a cutoff frequency for torsional tube waves is independent of the choice of the wave variables.

### 4.2. Local coordinate system and different wave variables

Propagation of torsional Alfvén waves along magnetic flux tubes can also be described by using a local orthogonal curvilinear coordinate system $(\xi, \theta, s)$, with $s$ being the length measured along a magnetic field line, $\theta$ the azimuthal angle about the axis of symmetry, and $\xi$ a coordinate in the direction $\hat{\xi} = \hat{\theta} \times \hat{s}$. In this case, $\vec{B}_0 = B_{0s}(s)\hat{s}$, $B_{0\xi} = 0$ and $B_{0\theta} = 0$. We also have $\vec{v} = v_{\theta}(s, t)\hat{\theta}$, $\vec{b} = b_{\theta}(s, t)\hat{\theta}$ and $R = R(s)$, where $R$ represents the distance from the magnetic field line to the tube axis. This approach was first considered by Hollweg (1978), who also applied it to solar magnetic flux tubes (see Hollweg 1981, 1992).

Following Hollweg, Jackson, & Galloway (1982), the curvilinear scale factors are $h_\phi = R$ and $h_s = 1$, and we determine $h_\xi$ from the condition $h_\xi R B_{0s} = \text{const}$, which results from $\nabla \cdot \vec{B}_0 = 0$. To conserve the magnetic flux, we must choose $h_\xi = R$. Using these scale factors, the explicit form of the momentum and induction equations is

$$\frac{\partial}{\partial t} \left( v_\theta \frac{R}{R} \right) - \frac{B_{0s}}{4\pi \rho_0 R^2} \frac{\partial}{\partial s} (Rb_{\theta}) = 0,$$ (20)

and

$$\frac{\partial}{\partial t} (Rb_{\theta}) - R^2 B_{0s} \frac{\partial}{\partial s} \left( \frac{v_\theta}{R} \right) = 0.$$ (21)

It is easy to see that the magnetic field $\vec{B}_0(s)$ in the local coordinate system can be described in the global coordinate system (see Eqs. 1 and 2) by the $B_{0r}(r, z)$ and $B_{0z}(r, z)$ field components. This means that the following relation must hold between the spatial operators in these two coordinate systems

$$B_{0r} \frac{\partial}{\partial r} + B_{0z} \frac{\partial}{\partial z} = B_{0s} \frac{\partial}{\partial s}.$$ (22)
This relation is consistent with the fact that $\mathbf{B}_0 \cdot \nabla$ must be the same in the global (the LHS of Eq. 22) and local (the RHS of Eq. 22) coordinate systems. Moreover, $v_\phi$ and $v_\theta$ are also related, as the former can be treated as a projection of the latter on the $\phi$-axis of the global coordinate system. Obviously, the same is true for the wave variables $b_\phi$ and $b_\theta$.

We follow Hollweg (1978, 1981) and introduce the new variables $x = v_\theta/R$ and $y = Rb_\theta$. The wave equations for these variables are

$$\frac{\partial^2 x}{\partial t^2} - c_A^2 \frac{\partial^2 x}{\partial s^2} = 0, \tag{23}$$

and

$$\frac{\partial^2 y}{\partial t^2} - \frac{\partial}{\partial s} \left( c_A^2 \frac{\partial y}{\partial s} \right) = 0, \tag{24}$$

where in general $c_A = c_A(s)$. However, for thin magnetic flux tubes $c_A = \text{const}$ (see Sec. 3.1) and the wave equations become

$$\left( \frac{\partial^2}{\partial t^2} - c_A^2 \frac{\partial^2}{\partial s^2} \right) [x(s,t); y(s,t)] = 0. \tag{25}$$

Again, no cutoff frequency exists. It is a significant (but expected) result that this non-existence of a cutoff frequency for torsional tube waves is independent of the choice of the coordinate system and the wave variables.

5. Discussion

We considered the propagation of linear torsional Alfvén waves along thin and isothermal magnetic flux tubes using the global coordinate system, and derived new wave equations describing this propagation. The derived wave equations were then used to demonstrate that no cutoff frequency exists for these waves, which means that torsional waves of any frequency are freely propagating along the tubes. We also showed that this result is independent of different choices of the coordinate systems and wave variables adopted by Ferriz-Mas et al. (1989) and Hollweg (1978, 1981, 1992).

As first shown by Defouw (1976) for sausage tube waves and by Spruit (1981) for kink tube waves, the propagation of both modes is affected by their corresponding cutoff frequencies. With their cutoff-free propagation, torsional Alfvén waves seem to be exceptional among the tube modes. In general, the existence of cutoff frequencies is caused by either gravity or gradients of the characteristic wave velocities, which result from an inhomogeneity
of the background medium. In the cases discussed in this paper, the characteristic wave velocities are constant for all tube modes because of the thin flux tube approximation. Hence, it is gravity which leads to the origin of the cutoff through either stratification (sausage tube waves) or buoyancy force (kink tube waves).

The fact that stratification leads to a cutoff frequency for acoustic waves propagating in a stratified and isothermal medium was first demonstrated by Lamb (1908, 1911). Since sausage tube waves are essentially acoustic waves guided by the tube magnetic field, and since they propagate in a stratified and isothermal medium inside the tube, it is stratification of the background medium that is responsible for the existence of the cutoff frequency for these waves.

The nature of kink tube waves is significantly different than sausage tube waves and yet it is again gravity that is responsible for the existence of the cutoff frequency for these waves. The main reason is that magnetic tension and buoyancy are the restoring forces for kink tube waves, and that the buoyancy force through its dependence on gravity leads to the cutoff frequency (e.g., Spruit 1982; Hollweg 1985), which is lower than that for longitudinal tube waves.

Now, despite some similarities between kink and torsional Alfvén tube waves, the main difference is that magnetic tension is the only restoring force for the latter. Since linear torsional tube waves have only purely axisymmetric twists in the $\varphi$-direction and show no pressure fluctuations, the twists are neither coupled to the gravitational force nor affected by stratification. As a result, no cutoff frequency can exist for linear torsional Alfvén waves propagating along thin and isothermal magnetic flux tubes.

The cutoff-free propagation of torsional tube waves may have important implications on theories of wave heating of the solar and stellar atmospheres. The theoretical models of stellar chromospheres constructed by Fawzy et al. (2002a,b) are based on the amount of energy carried by acoustic waves and by sausage and kink tube waves; these waves are generated by turbulent motions in the solar and stellar convection zones. The models point to a missing amount of heating for stars with high levels of activity. It is likely that the energy carried by torsional Alfvén waves could be used, at least partially, to account for these “heating gaps”. Since there is no cutoff frequency for torsional tube waves, a broad spectrum of these waves is expected to be generated in the solar and stellar convection zones. The waves of different frequencies of this spectrum may transfer energy to different parts of the solar and stellar atmospheres. Hence, new studies are required to determine the efficiency of generation of torsional tube waves and their dissipation rates.
6. Conclusions

We derived new wave equations that describe the propagation of linear torsional Alfvén waves along thin and isothermal magnetic flux tubes, and used them to demonstrate that this propagation is cutoff-free. Our study also showed that the result is independent of different choices of the coordinate systems and wave variables used by Ferriz-Mas et al. (1989) and Hollweg (1978, 1981, 1992).

Since the existence of cutoff frequencies for sausage and kink tube waves is caused by stratification and buoyancy force, respectively, our results clearly show that neither stratification nor buoyancy force affects the torsional wave propagation. The physical reason is that magnetic tension is the only restoring force for these waves.

This lack of any cutoff frequency for torsional tube waves implies that a broad wave energy spectrum for these waves will be generated in the solar and stellar convection zones. The energy carried by the waves of different frequencies of this spectrum may be used to account for the “heating gaps” discovered by Fawzy et al. (2002a,b) and may also contribute to the wave heating of different parts of the solar and stellar atmospheres.

This work was supported by NSF under grant ATM-0538278 (Z.E.M. and S.R.) and NASA under grant NAG8-1889 (Z.E.M. and S.R.). Z.E.M. also acknowledges the support of this work by the Alexander von Humboldt Foundation.

REFERENCES

Cuntz M., Rammacher W., Ulmschneider P., Musielak Z.E., & Saar S.H. 1999, ApJ, 522, 1053
Defouw, R.J. 1976, ApJ, 209, 226
Edwin, P.M., & Roberts, B. 1983, Sol. Phys., 88, 179
Fawzy, D.E., Rammacher, W., Ulmschneider, P., Musielak, Z.E., & Stępień, K. 2002a, A&A, 386, 971
Fawzy, D.E., Ulmschneider, P., Stępień, K., Musielak, Z.E., & Rammacher, W. 2002b, A&A, 386, 983
Ferriz-Mas, A., Schüssler, M., & Anton, V. 1989, A&A, 210, 425
Ferriz-Mas, A., & Schüssler, M. 1989, Geophys. Astrophys. Fluid Dynamics, 48, 217
Ferriz-Mas, A., & Schüssler, M. 1994, ApJ, 433, 852
Hasan, S. S., Kalkofen, W., van Ballegooijen, A. A., & Ulmschneider, P. 2003, ApJ, 585, 1138
Hollweg J.V. 1978, Solar Phys., 56, 305
Hollweg J.V. 1981, Solar Phys., 70, 25
Hollweg J.V. 1985, in Advances in Space Plasma Physics, ed. B. Buti, Singapore: World Scientific Publ., 77
Hollweg J.V. 1990, in Physics of Magnetic Flux Ropes, eds. C.T. Russell, E.R. Priest, & L.C. Lee, Geophys. Monograph 58, AGU, 23
Hollweg J.V. 1992, ApJ, 389, 731
Hollweg J.V., Jackson S., & Galloway D. 1982, Solar Phys., 75, 35
Kudoh, T., & Shibata, K. 1999, ApJ, 514, 493
Lamb, H. 1908, Proc. Lond. Math. Soc., 7, 122
Lamb, H. 1911, Proc. R. Soc. London, A, 84, 551
Musielak, Z.E., Rosner, R., & Ulmschneider, P. 1987, in Lecture Notes in Physics: Cool Stars, Stellar Systems and the Sun, eds. J.L. Linsky & R.E. Stencel, 291, 66
Musielak, Z.E., Rosner, R., Gail, H. P., & Ulmschneider, P. 1995, ApJ, 448, 865
Musielak, Z.E., & Ulmschneider, P. 2001, A&A, 370, 541
Musielak, Z.E., & Ulmschneider, P. 2003, A&A, 400, 1057
Narain U., & Ulmschneider P. 1996, Space Sci. Rev., 75, 453
Noble, M.W., Musielak, Z.E., & Ulmschneider, P. 2003, A&A, 409, 1085
Parker, E. N. 1979, Cosmic Magnetic Fields: Their Origin and Their Activity, Oxford and New York: Clarendon Press
Ploner, S.R.O., & Solanki, S.K. 1999, A&A, 345, 986
Poedts, S., Hermans, D., & Goossens, M. 1985, A&A, 151, 16
Priest, E. R. 1982, Solar Magnetohydrodynamics, Dordrecht: Reidel
Priest, E. R., 1990, in Physics of Magnetic Flux Ropes, eds. C.T. Russell, E.R. Priest, & L.C. Lee, Geophys. Monograph 58, AGU, 1

Rae, I.C., & Roberts, B. 1982, ApJ, 256, 761

Roberts B. 1991, Geophys. Astrophys. Fluid Dynamics, 62, 83

Roberts, B., & Ulmschneider, P. 1997, in: Solar and Heliospheric Plasma Physics, eds. G.M. Simett, C.E. Alissandrakis, & L. Vlahos, Berlin: Springer-Verlag, 75

Roberts, B., & Webb, A.R. 1978, Solar Phys., 56, 5

Roberts, B., & Webb, A.R. 1979, Solar Phys., 64, 77

Saar, S.H. 1996, in IAU Symp. 176, Stellar Surface Structure, eds. K. Strassmeier, & J.L. Linsky, Dordrecht: Kluwer, 237

Saar, S.H. 1998, in Cool Stars, Stellar Systems, and the Sun, eds. R.A. Donahue, & J.A. Bookbinder, ASP Conf. Ser. 154, 211

Saito, T., Kudoh, T., & Shibata, K. 2001, ApJ, 554, 1151

Solanki, S.K. 1993, Space Sci. Rev., 63, 1

Spruit H.C. 1981, A&A 98, 155

Spruit H.C. 1982, Solar Phys., 75, 3

Ulmschneider, P., & Musielak, Z.E. 2003, in 21st NSO/SP Workshop on Current Theoretical Models and Future High Resolution Solar Observations: Preparation for ATST, eds. A.A. Pevtsov, & H. Uitenbroek, ASP Conf. Ser., 286, 363

This preprint was prepared with the AAS \LaTeX{} macros v5.2.