The calculation of lean mixture burning in model combustors with flow swirling

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Abstract. The testing of various turbulence models was conducted when calculating separated swirling flows. For burning simulation, a simple phenomenological model is presented. The model takes into account the gas swirling effect on the velocity of flame propagation. The developed technique is applied to the calculation of the burn-out of a lean mixture in model combustion chambers with three-tier swirlers. It is shown that the reversed organization of combustion with outside ignition is more efficient than the scheme with inside ignition. In the calculations, the mutual swirling of the flows behind the swirlers was varied and its effect on the burn-out rate of the mixture was considered.

1. Introduction
In modern combustion chambers of gas turbine engines and stationary gas turbine plants, lean air-fuel mixtures are widely used to reduce toxic species emission. The stabilization of the combustion of a lean mixture, as a rule, occurs in the separation zone due to the interaction with a swirling flow of a small amount of combustion products of the rich mixture [1, 2]. Therefore, it is necessary to take into account the anisotropy of turbulent stresses and flow rotation effect on the burning rate, when modeling swirling flows of reacting gases. In the present work, the approbation of various turbulence models was conducted when calculating separated swirling flows. For burning simulation, a simple phenomenological model is presented. The model includes increasing of the lean region of ignition with temperature rise and gas swirling effect on the burning rate. The developed technique is applied to the calculation of the burn-out of a lean mixture in model combustion chambers with three-tier swirlers.

2. Calculation technique

2.1. The method of calculation of turbulent separated flows

To discretize elliptic differential equations describing the turbulent motion of a gas mixture, all transport equations were written in the following conservative form:

$$\text{div}(\rho \mathbf{U} \Phi - \Gamma_\Phi \text{grad} \Phi) = S_\Phi$$

(1)

Here $\mathbf{U}$ is the average velocity vector, $\Phi$ is the dependent variable, $\Gamma_\Phi$ is the diffusion transfer coefficient, $\rho$ is the density, $S_\Phi$ is the source term.

The two-dimensional system of stationary Reynolds equations and the preservation of total enthalpy $h$ written as (1) is solved by the finite-difference iterative method according to the author’s
program. The density of a gas mixture is calculated from the constituted equation of an ideal gas and Dalton's law. The system is complemented by equations of turbulence model and combustion model.

2.2. The turbulence model selection

The swirling flow of incompressible fluid in a round tube with sudden expansion was chosen as a test problem to approbate turbulence models. In the author's program, a two-parameter model of turbulence k-ε was used (k is the energy of turbulence per unit mass, ε is the rate of its dissipation). To calculate the mixing of swirling flows, the model was upgraded to take into account the influence of rotation on the turbulence structure. In the first approximation, anisotropy of the coefficients of eddy viscosity in turbulent flows can be modeled by replacing the constant in the dissipative term of the equation for \( \varepsilon \) with a function of the Richardson number, which can be considered as the dimensionless derivative of the angular momentum along the radius of curvature [3]:

\[
S_\varepsilon = S_g - \rho 1.8(1 - C_{ri} Ri) \frac{\varepsilon^2}{k}; \quad C_{ri} = 0.2;
\]

\[
Ri = \frac{2 \frac{U_\theta}{r} \frac{\partial}{\partial r} (r U_\theta)}{\left( \frac{\partial U}{\partial r} \right)^2 + \left( r \frac{\partial}{\partial r} \frac{U_\theta}{r} \right)^2}
\]

Here \( r \) is the distance to the axis of rotation, \( U, U_\theta \) are the axial and rotational components of the averaged velocity, respectively, and \( S_g \) is the generation of \( \varepsilon \). The calculated dependences of the length of the separation zone behind the step in the pipe on the input flow swirl parameter were compared with the known experimental data [4] shown in figure 1. In experiments, the air was supplied to a stationary pipe of diameter \( D \) from a pipe of diameter 0.5 \( D \) rotating with angular velocity \( \omega \). The swirl parameter \( \sigma = \omega D/U_0 \), where \( U_0 \) is the average velocity in a stationary pipe. This figure also shows numerical calculations using commercial programs with incorporated turbulence models. Testing has shown that the best match with the experiment is given by a simple model with an augmented dissipation equation, as well as models using Reynolds stress transfer equations. However, the latter requires 1.5-2.5 times more the computer time than k-ε model. Therefore, further calculations were carried out using the author's program with a modified k-ε model.

![Figure 1](image.png)

**Figure 1.** The dependence of the length of the separation zone behind the step in the pipe on the swirl parameter.
2.3. Combustion simulation

To simulate combustion, a modification of a simple phenomenological model of combustion is used, based on the concept of burning a combustible mixture with moles of a burnt gas under the assumption of a simple one-step reaction [5]. For kerosene vapors, the reaction equation has the form (3).

\[ \text{C}_12\text{H}_24+18\text{O}_2 = 12 \text{CO}_2 + 12\text{H}_2\text{O} \] (3).

The model uses the equation of the form (1) for the average relative concentration of combustion products \( c \), in which the volumetric average product formation rate \( R_c \) is related to the kinematic turbulent viscosity coefficient \( \nu_t \) and the turbulent flame propagation velocity \( u_\tau \).

\[ R_c = \frac{\sigma_c u_t^2}{4} \nu_t \rho_c (1 - c) \] (4).

Here \( \sigma_c \) is the turbulent Schmidt number.

When considering the burning process of a homogeneous mixture, \( c = m_{\text{prod}} \) (combustion products fraction of total mass). Here we consider the mixing of flows with different chemical compositions. Therefore the relative concentration was determined as

\[ c = \frac{(m_{\text{prod}} - m_{\text{mix}})}{(m'_{\text{prod}} - m_{\text{prod}})} \] (5).

Here and further indexes \( \text{mix} \) and \( e \) denotes pure mixing and equilibrium value (diffusion combustion). So it’s necessary to include the transfer equation for conservative scalar – Shwab-Zeldovich function into the model:

\[ f = m_{\text{fu}} + m_{\text{prod}} / (1 + L_{\nu}) \] (6),

where \( L_0 \) is the stochiometric coefficient by oxygen, \( m_{\text{fu}} \) is the fuel fraction of total mass. It’s transfer is described by equation (1) with zero source term. Since this paper studies the mixing of three fluxes of different chemical composition, the model includes transport equations for two more conservative scalars describing the pure mixing of the oxidizer and combustion products. These equations differ from each other only by boundary conditions. After finding the extend of reaction development \( c \), concentrations of the mixture components are determined from (6), (5), and (3).

When determining \( u_\tau \), it was assumed that in a swirling flow, the flame propagates according to the fastest of two mechanisms: turbulent transfer and floating of burnt gas bubbles in a centrifugal force field (Lewis effect [6]):

\[ u_\tau = \max(u_{\tau}^0, u_b) \] (7).

Here \( u_b \) is the buoyancy rate of burnt gas moles.

We now associate \( u_b \) with the rotational component of the averaged velocity \( U_\phi \). The volume of combustion products moving in a swirling flow is affected by two forces directed in opposite directions: buoyancy force \( F_b \) and resistance force \( F_R \). The buoyancy force is the resulting force of buoyant \( F_A \) and centrifugal \( F_C \) forces, and depends, as in a gravitational field, on the density difference between hot combustion products and the surrounding cold mixture.

From the definition of centrifugal force we have

\[ F_c = \rho e S L g_\phi \frac{T_{\text{mix}}}{T_e} \] (8).

Here \( S \) is the effective cross-sectional area of the burnt gas bubble, \( L \) is its length, \( g_\phi = U_\phi^2 / r \) is the centripetal acceleration, \( T \) is the temperature. The buoyant force is determined similarly to the Archimedean force in the field of gravity:

\[ F_A = \rho_{\text{mix}} S L g_\phi \] (9).

Hence

\[ F_B = F_A - F_C = \rho_{\text{mix}} S L g_\phi (1 - T_{\text{mix}} / T_e) \] (10).
The resistance force is evaluated from the classic formula:

\[ F_R = 0.5C_R Su_b^2 \]  \hspace{1cm} (11). 

Here \( C_R \) is the dimensionless resistance coefficient.

We’ll find the buoyancy rate of a burnt gas bubble from the equilibrium condition of buoyancy and resistance forces:

\[ F_R = F_B \]  \hspace{1cm} (12).

Experiments show that the value of \( C_R \) is close to constant. Using the definitions of \( F_B, F_R \), we obtain from (12)

\[ u_b = \left[ C_1 L g \phi (1 - \frac{T_{mix}}{T_c}) \right]^{1/2} \]  \hspace{1cm} (13).

The length of the bubble of combustion products can be considered proportional to the distance to the axis of rotation: \( L = C_2 \). Then, substituting the definition of \( g \phi \) in (13), we get a formula for calculating \( u_b \) based on the characteristics of averaged motion:

\[ u_b = C_B V \phi (1 - \frac{T_{mix}}{T_c})^{1/2} \]  \hspace{1cm} (14).

Here \( C_1, C_2, C_R, C_B \) are dimensionless constants.

To determine the empirical constant \( C_B \), we turn to experiments [7]. In this paper, methane flame propagation in vortex rings (PIV measurements) was studied. The observed ratio between the flame propagation velocity to the vortex center and the maximum rotational velocity component in the vortex was close to linear. Considering that for lean mixtures, the buoyancy rate of combustion product moles (\( u_c^0 < u_b \)) was measured in practice, from the dependence obtained in [7] for a lean mixture (heating ratio = 6) we found \( C_B \approx 0.82 \).

In the absence of a swirl, the quantity \( u_c = u_c^0 \) is related to the normal propagation velocity of the laminar flame \( u_n \) by the relation following from the theory of turbulent flame propagation [8].

\[ \frac{u_c^0}{\sqrt{k}} = \theta u_n + 2.4 \sqrt{\ln \left(1 + \frac{\sqrt{k} 2/\pi \arctg(l_1 / l_2)}{u_n} \right)} \]  \hspace{1cm} (15).

According to [8] the ratio \( u_c / \sqrt{k} \) depends on two values \( u_n / \sqrt{k} \) and \( l_1 / l_2 \), where \( k, l_1 \) are the energy and scale of turbulence, \( u_n \) is the normal velocity of laminar flame, \( l_2 \) is its width. Here \( \theta \) is the heating ratio by the flame in the case of complete mixture burning. When calculating the mixing of flows with different initial temperatures, we set \( \theta = T_c / T_{mix} \).

To calculate \( u_n \), formulas approximating the experimental data were proposed in [5], which take into account the expansion of the “lean” ignition region with temperature increase:

\[
 u_n = \begin{cases} 
 u_{lim} + A_1 \left( \frac{1}{\alpha} - \frac{1}{\alpha_L} \right)(\frac{1}{\alpha_L} - \frac{1}{\alpha}) & \text{при } \alpha_M \leq \alpha \leq \alpha_L \\
 u_{lim} + A_2 \left( \frac{1}{\alpha} - \frac{1}{\alpha_L} \right)(\frac{1}{\alpha} - \frac{1}{\alpha_R}) & \text{при } \alpha_R \leq \alpha \leq \alpha_M \\
 0 & \text{при } \alpha > \alpha_L, \alpha < \alpha_R 
\end{cases} 
\]  \hspace{1cm} (16).

\[ 100/(1 + \alpha_L L_{M}) = c_{TL} = c_{TL}^0 (T_L - T)/(T_L - 293) \]

\[ u_{max} = u_{max}^0 (T/T^0)^{2}(p/p^0)^{0.25} \]

Here \( \alpha \) is the local air excess coefficient, \( \alpha_L, \alpha_R \) are low and upper concentration limits of flame propagation, \( A_1, A_2 \) are constants determined by maximum velocity of flame propagation (for kerosene at normal condition \( \alpha_L^0 = 1.4, \alpha_R^0 = 0.65 \), \( u_{lim} = 0.06 \text{m/s}, u_{max}^0 = 0.4 \text{ m/s at } \alpha_M = 0.9 \); index 0 corresponds to \( T^0 = 293 \text{K}, p^0 = 100 \text{ kPa} \), \( 1/\alpha_1 = 1/\alpha_M + (1/\alpha_M - 1/\alpha_L) \), \( 1/\alpha_2 = 1/\alpha_M + (1/\alpha_M - 1/\alpha_R) \), \( L_M \) is the molar stochiometric coefficient, \( T_L \) is the combustion products temperature at low
concentration limit of flame propagation $C_{fl}^0$. The limit velocity $u_{lim}$ was considered as constant dependent only on the kind of fuel.

3. Results of calculations of flows in model combustors

Diagrams of annular model combustion chambers are shown in figure 2. Mixing and burning of two swirling flows occur in a pipe with an internal diameter of $2R_o= 100 \text{ mm}$. It was assumed that a mixture of kerosene vapors with air at $\alpha = 1.6$ is fed to the chamber upstream of the calculated area. The mixture was divided into stoichiometric one, which was burned in the inner or outer tube, while the remaining lean mixture entered the second channel. The pressure drop for the fresh mixture and combustion products supply channels was considered to be the same. It was assumed that axial swirlers were installed at the channels inlet. Additionally, 66% of the total air flow was supplied through a 20 mm high ring tangential swirler with 45° blade angles.

The main parameters of the problem are the relative area of the channel through which the hot combustion products are supplied – $S_{rel}$, and the blades installation angles of the internal and external swirls $\beta_I, \beta_O$ (minus means that the swirl is opposite in direction to the tangential swirler). It was previously [9] proved, that $S_{rel}$ 30 -50% is the optimum value. Now we will vary the mutual swirl of the flows behind the axial swirlers and analyze its effect on the burn-out rate of the lean mixture.

Let’s consider the flow pattern in a model combustor with a three-tier swirlers (figure 3).

Figure 3. Vector velocity field in model combustor with three-tier swirlers; $\beta_O = 60^\circ, \beta_I = -60^\circ; S_{rel} = 50\%$.

As calculations show, as a result of the mixing of hot combustion products with a fresh mixture, the mixture is ignited. In a non-swirling flow, the combustion zone is localized within the mixing layer. When swirling, the internal flow of the combustible mixture or combustion products is discarded to the outer wall. Near the axis of the pipe, a pre-separated flow region forms. Regardless of $S_{rel}$, for $\beta_I > 35^\circ$, a recirculation arises at the end of the inner tube and, with a further increase in swirling, a low-velocity return flow zone is formed, elongated along the axis. The same happens with countercurrent flows.

Figure 4. The dependence of combustion efficiency on the blade angle of outer swirler in model combustor with three-tier swirlers. $S_{rel}=30\%; x/R_o=5$; 1 – $\beta_I =60^\circ$, 2 – $\beta_I =-60^\circ$; solid lines – burning outside; dotted line – burning inside.
Analysis of the calculation data shows that swirling is an effective way to increase the completeness of combustion. This is due to four factors: 1 - the growth of the contact surface of two flows as the mixing layer is displaced outward by near-axial preseparated or separated flow; 2. - increase in the intensity of turbulence and, consequently, the mixing process due to an increase in the gradient of the axial velocity components at the boundary of the separation zone and due to the appearance of a tangential shear layer; 3. - the spread of flame near the axis upstream. This mechanism manifests itself at large swirl angles (with the formation of a long separation flow region near the pipe axis, the flow velocity decreases, which stabilizes the combustion process); 4. - an increasing the burning rate due to the transfer of emissions of combustion products to the center of rotation (“Lewis effect”). When supplying a stream of “cold” air through a tangential swirler, a near-wall separation zone and a mixing layer (axial and tangential) of air with a stream from the external supply channel are also formed. In the traditional scheme (with the supply of combustion products along the axis), this jet depletes the fresh mixture and at the same time prevents the penetration of hot combustion products into it. Combustion occurs mainly in the low-temperature, lean mixing region of the fresh mixture with air. In a reversed organization of combustion with ignition from outside, an air stream, on the contrary, enhances the drift of hot combustion products to the axis and their mixing with a fresh mixture. At large swirl angles, this process is enhanced by the “Lewis effect” (transfer of moles of combustion products to the center of rotation by centrifugal force). Therefore, the reversed organization of combustion with ignition from the outside is more effective than the scheme with ignition from the inside in almost the entire studied area (figure 4). In the calculations, for a fixed blade angle of the internal axial swirler $\beta_i$, the blade angle of the external axial swirler $\beta_o$ varied. The results of calculating the combustion efficiency are shown in figure 4.

As we can see from figure 4, when hot combustion products are fed into the external channel, the combustion efficiency $\eta$ depends quite strongly on the relative angle of the axial swirlers, since in this case combustion occurs mainly in the layer of mixing of combustion products with a fresh mixture. The highest value of $\eta$ is achieved when counter-swirling due to the higher value of tangential shear stresses comparing with swirling in one direction. When the combustion products are fed into the inner annular channel (igniting from the inside) $\eta$ depends weakly on the swirl in the outer channel, since combustion efficiency is determined mainly by the axial layer of mixing with a jet of "cold" air passing through the tangential swirler.

Conclusions
The testing of various turbulence models was conducted when calculating separated swirling flows. Testing has shown that the best match with the experiment is achieved using a modified $k$-$\varepsilon$ model with an augmented dissipation equation, as well as models using Reynolds stress transfer equations. However, the latter requires computer time 1.5 - 2.5 times more. Presented combustion model is based on the idea of igniting a combustible mixture by moles of a burnt gas. The model takes into account the expansion of the “lean” ignition region with increasing temperature and the influence of the swirl on the burning rate. The developed technique is applied to the calculation of the burn-out of a lean mixture in model combustion chambers with three-tier swirlers. It is shown that the reverse scheme of combustion with the ignition of the mixture from the outside is more effective than that with ignition from the inside. If hot combustion products are fed into the outer channel, the combustion efficiency depends strongly on the relative angle of the axial swirlers and reaches the greatest value when counter-swirling. When supplying combustion products along the chamber axis, the combustion efficiency depends weakly on the relative angle of the axial swirlers. It is determined mainly by a stream of “cold” air flowing through the tangential swirler.

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