Sensitivity Analysis in Ball Bar Measurement of Three-Dimensional Circular Movement Equivalent to Cone-Frustum Cutting in Five-Axis Machining Centers*

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Abstract
The present paper describes the sensitivity coefficient of measurement in the three-dimensional circular interpolation movement that is equivalent to cone-frustum cutting in five-axis machining centers with a tilting rotary table. The sensitive direction of a ball bar having a one-dimensional displacement sensor is parallel to its telescopic bar. In the present paper, the ratio of the measurement value to the actual error is defined as the sensitivity coefficient of measurement. The sensitivity coefficient of each axis is calculated by changing the apex angle and location of the cone-frustum. Different trajectories are obtained according to the attitude of the ball bar. This is due to the resulting variation in the sensitivity coefficient of the ball bar. If the ball bar is set parallel to the base circle of the cone-frustum, and if the center of the cone-frustum is positioned away from the centerline of the rotary table (in the positive direction of the linear axis that is perpendicular to the tilting axis of the table), the trajectory can be obtained appropriately.

Key words: Five-Axis Machining Center, Cone-Frustum, Sensitivity Coefficient, Ball Bar Measurement, Half Apex Angle

1. Introduction

One of the measurement methods for simultaneous five-axis control movement using the ball bar instrument prescribed in ISO/DIS 10791-6(1) is a three-dimensional circular interpolation movement that is equivalent to the machining of a cone-frustum. The movement conforms to National Aerospace Standard (NAS) 979(2) (enacted in 1969), which specifies the method of testing the machine accuracy by machining a cone-frustum. Five-axis control machining centers (MCs) in the era when NAS 979 was enacted were special machine tools used for manufacturing aircraft and rocket parts. The machining test requires the preparation of a dedicated jig to mount the workpiece. Moreover, after finishing...
the workpiece, it is necessary to measure the circularity on a coordinate measuring machine or a circularity measuring machine. Furthermore, it is necessary to change the jig in order to change the test condition of the truncated cone. For the machining test, it takes time for the setup of the workpiece, the machining, and the measurement, and changing the test conditions is not easy.

Thus, the development of a test methodology that does not require cutting is needed. A special measuring instrument is needed to measure the three-dimensional circular interpolation movement. The setup time can be reduced as compared with the machining test, the testing conditions can be easily changed, and the results of the measurement can be observed in place. Therefore, it is expected that the ball bar measurement will be widely used as an alternative to the cone-frustum machining test in the future.

Ihara et al. (3), (4) has reported that a motion equivalent to the cutter path for milling the cone-frustum can be conducted using a ball bar. Matsushita et al. (5) pointed out that the accuracy of the machined cone-frustum varies based on the workpiece location, and Matsushita and Matsubara (6) proposed a method to identify the geometric errors of the linear and rotational axes in five-axis machining centers using the ball bar. Uddin et al. (7) conducted the machining test using a cone-frustum as a case study to verify the prediction and compensation of machining geometric errors in five-axis machining. Hong et al. (8) presented an experimental investigation of the major causes of the observed contour error profile of the cone-frustum trajectory measured using the ball bar.

In addition, the authors have analyzed in detail the numerically controlled (NC) data that create a tool path for the two cone-frustums with half apex angles of 15° and 45° (9), which are the test conditions for checking the accuracy in ISO/DIS 10791-6 (1). This NC data can actually be used for five-axis MCs having a tilting rotary table. In the present paper, the displacement and feed speed in the five axes of motion were analyzed when the center of the cone-frustum was positioned away from the centerline of the rotary table. The maximum values of the command feed speed in the axes of motion depend on the center location of the cone-frustum on the XY plane, and the reversal positions on the axes of motion significantly depends on the center location.

The authors also reported that the trajectory of the three-dimensional circular interpolation movement measured using a ball bar can be reproduced by simulation (10). If the friction torque of the linear axes, the tracking delay of the servo system, and the backlash and pitch errors of the axes of rotation are introduced to the motion simulator as error sources in the five-axis MCs, the simulation results are in good agreement with the real trajectories. In addition, the influence of an individual error source on the trajectory was clarified by the motion simulator.

In particular, the sensitive direction of the ball bar has a significant influence on the results of the measurement for a half apex angle of 45°. Thus, the ratio of the measured value to the actual error is defined in this paper as the sensitivity coefficient of the measurement. The influence of the half apex angle and center location of the cone-frustum on the sensitivity coefficients of the five axes is clarified. If the sensitivity coefficients are considered, the difference in the sensitive direction of the ball bar can be understood well. Finally, the present paper recommends that the three-dimensional circular interpolation motion equivalent to the cone-frustum cutting be measured using the ball bar under the following three test conditions: (1) the half apex angle of the cone-frustum is 15°, (2) its center is positioned away from the center line of the rotary axis in the positive Y direction, and (3) the sensitive direction of the ball bar is set parallel to the base circle of the cone-frustum.
2. Measurement method and sensitive direction

2.1. Measurement method and simulation method

The structural configuration of the five-axis control machining center used in this study can be described as “w-C’A’ Y’bXZ(C)-t” according to the coordinate system prescribed by ISO 841(11), where w is the workpiece, b is the bed, t is the tool, (C) is the spindle axis, X, Y’, and Z are the axes of linear motion, A’ is the tilting axis of the table, and C’ is the axis of rotation of the table. In order to avoid complexity, the prime symbol is not put on the symbols of the coordinate system in this paper.

The present paper assumes that the three-dimensional circular interpolation movement of a five-axis MC with a tilting rotary table is measured using the ball bar. Outlines of two measurement setups are depicted in Fig. 1, which shows the relative position of the ball bar and the main spindle of the machining center. The half apex angle of the cone-frustum is 45°, and its inclination angle is 30°. Figure 1(a) shows one measurement method, in which the sensitive direction of the ball bar is set perpendicular to the conical surface (“perpendicular measurement method”). Figure 1(b) shows the other measurement method,

![Diagram](image-url)

(a) Perpendicular measurement method
(b) Parallel measurement method.

A half apex angle : \( \theta = 45° \), ball bar length : \( L = 100 \text{ mm} \).

Fig. 1 Two sensitive directions of the ball bar for measuring the conical circular path.

![Diagram](image-url)

(a) Block diagram of the axes of linear motion (\( G: Z \text{ axis only} \))
(b) Block diagram of the axes of rotation.

\( K_{pp} \): positional loop proportion gain; \( K_{vp} \): velocity loop proportion gain;
\( T_i \): velocity loop integration time; \( J \): total moment of inertia of the mechanism;
\( C \): viscous damping factor; \( l \): lead of the ball screw; \( f \): friction force;
\( G \): gravity; and \( R \): reduction ratio of the worm gear used for the \( A \) and \( C \) axes.

Fig. 2 Motion simulator for a five-axis machining center with a tilting rotary table.
A half apex angle: \( \theta = 45° \), inclination angle: \( \beta = 30° \), feed speed: \( F = 1000 \text{ mm/min} \),
\( f \): central angle of the base circle of the cone-frustum.

Fig. 3 Comparison of the circular trajectories simulated by the two measurement methods using a ball bar.
in which the ball bar is set parallel to the base circle of the cone-frustum ("parallel measurement method").

The simulation was conducted using a motion simulator developed by one of the authors\(^{[10]}\). The motion simulator depicted in Fig. 2 can well express various errors of the axes of motion, such as the reversal errors of the linear axis of motion, the pitch error and backlash of the axis of rotation, and the tracking delay of the servo system. Moreover, the simulation results are in good agreement with the measurement results\(^{[10]}\). Therefore, all of the trajectories depicted in the present paper are from the motion simulator results.

2.2. Sensitive direction and trajectory

The trajectories obtained by the two measurement methods shown in Fig. 1 were simulated. Figure 3 shows the trajectories simulated at three center locations of the cone-frustum corresponding to the two measurement methods; i.e., the first location is \( O_T = (0, 0, 150) \), the second location is \( O_T = (50, 0, 150) \), and the third location is \( O_T = (0, 50, 150) \). As shown in Fig. 3, when the center location is \( O_T = (0, 0, 150) \), the influence of the sensitive direction of the ball bar appears on the trajectory, and a considerable difference can be observed in the right half of the trajectory. For instance, there are large differences in the step heights at 0° and in the trajectory variation at approximately 45°.

When the center location is \( O_T = (50, 0, 150) \), the trajectories of both measurement methods are slightly affected by the sensitive direction of the ball bar. The periodic changes in the trajectory at approximately 90° and 270° are slightly larger in the parallel measurement method.

When the center location is \( O_T = (0, 50, 150) \), the trajectories of both measurement methods are slightly affected by the sensitive direction of the ball bar. The periodic changes in the trajectory at approximately 90° and 270° are slightly larger in the parallel measurement method.

The trajectory between 225° and 45° in the counter-clockwise direction is quite different from the other trajectories shown in Fig. 3. The influence of the mechanism errors measured by the perpendicular measurement method becomes considerably small as compared with the trajectories from the parallel measurement method, as shown Fig. 3(c); i.e., when the center location is \( O_T = (0, 50, 150) \), we can observe small periodic errors measured at approximately \( \phi = 180° \) by the perpendicular measurement method, but periodic errors and backlash of the axes of rotation do not appear. In the perpendicular measurement method, when the center location is \( O_T = (0, 50, 150) \),
(0, 50, 150), not only do the errors near $\phi = 0^\circ$ appear small but also all errors appear considerably small in a fairly wide range. When the half apex angle is 15°, as previously reported(10), the difference due to the sensitive direction of the ball bar is small.

As mentioned above, the shape of the trajectory is not only dependent on the center location but is also dependent on the sensitive direction of the ball bar. Therefore, it can be said that the errors obtained by the measurement method specified in ISO/DIS 10791-6 will be reduced because the sensitive direction is perpendicular to the conical surface. Thus, the causes of the difference in the trajectories between the two apex angles shown in Fig. 3 are investigated in detail based on the sensitivity analysis.

3. Sensitive direction of ball bar and sensitivity coefficients of the axes

3.1. Definition of sensitivity coefficient

The displacement detected by the perpendicular measurement method is an error component of the motion necessary to shape the cone-frustum with the end mill. In the axis of linear motion, when the sensitive direction of the ball bar is parallel to the movement direction of each axis, the error detected is equivalent to the real error of the axis, and when the sensitive direction is not parallel to the linear axis, the cosine component is detected. In the axis of rotation, when the sensitive direction of the ball bar is parallel to the tangential direction of the axis of rotation, the pitch error and backlash of the axis of rotation are directly detected by the ball bar(12). When it is in the other directions, the cosine component of the gyration error is detected.

The relationship between the real error vector of the axis and the measurement error vector can be depicted as in Fig. 4. The sensitivity coefficient of measurement can be defined using $v$ and $v'$ as follows:

$$k = \frac{v'}{|v|} = \cos \theta_b$$  \hspace{1cm} (1)$$

and the inner product of the error vector $v$ and the measurement direction vector can be expressed as follows:

$$v \cdot w = |v| \cdot |w| \cos \theta_b$$  \hspace{1cm} (2)$$

By substituting Eq. (1) into Eq. (2) and solving for $k$, the sensitivity coefficient $k$ can be written as the dot product of the unit error vector and the unit measurement direction vector:

$$k = \frac{v'}{|v|} = \frac{v \cdot w}{|v| \cdot |w|} = \frac{v}{|w|}$$  \hspace{1cm} (3)$$

3.2. Calculation of unit measurement direction vector

When the center position of the spindle side ball of the ball bar is expressed by $O_a(X, Y, Z)$ for the $A$ axis coordinate frame, whose origin is $O_a(0, 0, 0)$, and the center position of the table side ball is expressed by $O_t(X_t, Y_t, Z_t)$, the center coordinates of the table side ball $O'_t(X'_t, Y'_t, Z'_t)$ accompanying the rotation $\gamma$ of the $C$ axis can be expressed as follows:

![Fig. 4 Relationship between the real error vector and the measurement direction vector of the ball bar.](image-url)
\[ O'_T' = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} O_T = \begin{bmatrix} X'_T \\ Y'_T \\ Z'_T \end{bmatrix} \]  

(4)

In addition, the center coordinates \( O'_T'(X'_T, Y'_T, Z'_T) \) of the table side ball when the \( A \) axis rotates by \( \alpha \) can be expressed as follows:

\[
O'_T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix} O'_T = \begin{bmatrix} X'_T \\ Y'_T \\ Z' \end{bmatrix}
\]

(5)

Therefore, the unit measurement direction vector \( e_b \) can be expressed as follows:

\[
e_b = \frac{O'_T - O'_T'}{L} = \frac{1}{L} \begin{bmatrix} X - X'_T \cos \gamma + Y'_T \sin \gamma \\ Y - (X'_T \sin \gamma + Y'_T \cos \gamma) \cos \alpha + Z'_T \sin \alpha \\ Z - (X'_T \sin \gamma + Y'_T \cos \gamma) \sin \alpha - Z'_T \cos \alpha \end{bmatrix}
\]

(6)

where \( L \) is the reference length of the ball bar.

3.3. Calculation of unit error vector of each axis

The unit error vectors \( e_X, e_Y, \) and \( e_Z \) of the \( X \) axis, \( Y \) axis, and \( Z \) axis, respectively, are expressed as follows:

\[
e_X = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad e_Y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad e_Z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
\]

(7)

When the real error vector \( v \) and the symbols of the rotary axes are defined, as shown in Fig. 5(a), the error vector \( v_A \) of the \( A \) axis can be expressed as follows:

\[
v_A = \frac{\partial}{\partial \alpha} (O'_T - O_T) = \begin{bmatrix} 0 \\ (X'_T \sin \gamma + Y'_T \cos \gamma) \sin \alpha + Z'_T \cos \alpha \\ -(X'_T \sin \gamma + Y'_T \cos \gamma) \cos \alpha + Z'_T \sin \alpha \end{bmatrix}
\]

(8)

Therefore, the unit error vector \( e_A \) of the \( A \) axis is as follows:

\[
e_A = \frac{1}{|v_A|} \begin{bmatrix} 0 \\ (X'_T \sin \gamma + Y'_T \cos \gamma) \sin \alpha + Z'_T \cos \alpha \\ -(X'_T \sin \gamma + Y'_T \cos \gamma) \cos \alpha + Z'_T \sin \alpha \end{bmatrix}
\]

(9)

where \(|v_A| = \sqrt{(X'_T \sin \gamma + Y'_T \cos \gamma)^2 + Z'_T^2}\)

Fig. 5  Error vectors of rotary axes and definition of symbols.
As shown in Fig. 5(b), the error vector $v'_C$ of the C axis when the A axis is at the horizontal position can be expressed as follows using the rotational angle $\gamma$ of the C axis as well as the A axis:

$$v'_C = \frac{d}{dy} (O'_x - O_y) = \begin{bmatrix} \sin \gamma & \cos \gamma & 0 \\ -\cos \gamma & \sin \gamma & 0 \\ 0 & 0 & 0 \end{bmatrix} O_x$$

(10)

The error vector $v_C$ of the C axis from the rotation of the A axis is

$$v_C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix} v'_C = \begin{bmatrix} X_T \sin \gamma + Y_T \cos \gamma \\ -X_T \cos \alpha \cos \gamma + Y_T \cos \alpha \sin \gamma \\ -X_T \sin \alpha \cos \gamma + Y_T \sin \alpha \sin \gamma \end{bmatrix}$$

(11)

Therefore, the unit error vector $e_C$ of the C axis can be expressed as follows:

$$e_C = \frac{1}{|v_C|} \begin{bmatrix} X_T \sin \gamma + Y_T \cos \gamma \\ -X_T \cos \alpha \cos \gamma + Y_T \cos \alpha \sin \gamma \\ -X_T \sin \alpha \cos \gamma + Y_T \sin \alpha \sin \gamma \end{bmatrix}$$

(12)

where $|v_C| = \sqrt{X_T^2 + Y_T^2}$.

The unit error vectors of all axes were able to be calculated as described above. Then, the change in the sensitivity coefficient of each axis was investigated by changing the test conditions using NC data. The sensitivity coefficient takes a value of +1 to −1, as shown in Eq. (1). In the present paper, the sensitivity coefficient $k$ is presented as an absolute value. $k = 1$ means that the error of the axis is detected at 100%, and $k = 0$ means that the error of the axis is not detected at all.

4. Test conditions and sensitivity coefficients

4.1. Effect of half apex angle on the sensitivity coefficients of the axes

In this section, the simulation is conducted under the condition that the sensitive direction of the ball bar axis is set perpendicular to the conical surface, as shown in Fig. 1(a). Figure 6 shows the change in the sensitivity coefficient when the center location of two kinds of cone-frustums with half apex angles of 15° and 45° is $O_T(0, 0, 155)$. In the Figure, the solid lines present the results for a half apex angle of 15°, and the dashed lines present the results for a half apex angle of 45°. When the half apex angle is 15°, the X axis takes a value of $k = 0$ at 0° and 180°, and it takes a maximum value at 135° and 225°. The Y axis takes a value of $k = 1$ at 0° and 180°, and it takes a minimum value at 135° and 225°. The A axis takes a maximum value at 180° and takes a minimum value at 135° and 225°.

![Fig. 6 Sensitivity coefficients of the five axes from the perpendicular measurement method.](image-url)
The sensitivity coefficients of the X, Y, and A axes significantly change depending on the angle position $\phi$, as shown in Fig. 6. In particular, the sensitivity coefficients of the Y and A axes are always larger than 0.7, and the sensitivity coefficients of the Z and C axes are zero. Thus, errors due to the movement of the two axes are not detected. The errors of the X, Y, and A axes are only detected by the ball bar, though the five axes are simultaneously moved. As reported previously by Bossoni\(^{(13)}\) and the present authors\(^{(9)}\), the travel ranges of these axes for a half apex angle of 45° are larger than those for a half apex angle of 15°. The sensitivity coefficients of the Y and A axes are small (except around 0° and 180°), though the sensitivity coefficient of the X axis is slightly larger than that for a half apex angle of 15°, as shown in Fig. 6. The sensitivity coefficient of the A axis is decreased to approximately 0.25 from 0.91; i.e., the detected error of the A axis is only approximately 1/4 of the real error around 0°. This decrease is remarkable.

As mentioned above, the errors of the Z and C axes cannot be detected, though the errors of the X, Y, and A axes are detected when the sensitive direction of the ball bar is set perpendicular to the conical surface and when the center location of the cone-frustum is $O_T(0, 0, 155)$. In addition, the sensitivity coefficients of the X and Y axes change only slightly, but that of the A axis changes remarkably.

### 4.2. Effect of the center location of the cone-frustum on the sensitivity coefficients of the axes

Figure 7 shows the relationship between the sensitivity coefficients of all five axes and the central angle $\phi$ of the base circle of the cone-frustum when the center location is moved by 50 mm in the X direction. In addition, the results for half apex angles of 15° and 45° are shown in the Figure. It is found that the error of the C axis as well as those of the X, Y, and A axes can be detected by moving the center location in the X direction, as shown in the Figure.

The sensitivity coefficient of the C axis is zero when the center location is $O_T(0, 0, 150)$ and is approximately $k = 1$ at 90° when moving the center position by 50 mm in the case of a half apex angle of 15°. The sensitivity coefficients of all of the linear axes do not change, although their results are not shown in the Figure when the center location $O_T$ is moved. In the case of a half apex angle of 45°, the change in the sensitivity coefficient of the A axis is also remarkable.

The sensitivity coefficient is nearly $k = 0.55$ at 0° (although its coefficient is $k = 0.25$) before moving the center location, as shown in Fig. 6. However, the sensitivity coefficient at $\phi = 180°$ is $k = 0.8$, and its value is lower than that for a half apex angle of 15°. The tendency of the sensitivity coefficient to decrease can be seen in the C axis. The maximum

![Fig. 7 Sensitivity coefficients of the five axes from the perpendicular measurement method when the center was located at 50 mm in the X direction.](image-url)
value is \( k = 0.9 \) when the half apex angle is 15°, but the maximum value is \( k = 0.7 \) when the half apex angle is 45°; i.e. the sensitivity coefficients of the axes of rotation are generally low compared with those for a half apex angle of 15°, although a high part appears in the sensitivity coefficient of the \( A \) axis when the center location moves in the \( X \) direction. The sensitivity coefficients of the linear axes do not change even if the center location moves in the \( X \) and \( Y \) directions.

Then, we investigated the sensitivity coefficients of the two axes of rotation by changing the center location of the cone-frustum. Figure 8 shows the results. Figure 8(a) shows the change in the sensitivity coefficients of the \( A \) and \( C \) axes when the center is located on the centerline of the \( C \) axis and when it moves by 50 mm in the \( X \) and \( Y \) directions in the case of a half apex angle of 15°. The change in the sensitivity coefficient of the \( A \) axis is small even if the center location moves in any direction, as shown in the Figure. Even if the center location moves from the centerline of the rotary table, the sensitivity coefficient of the \( C \) axis does not become zero; i.e., the sensitivity coefficient of the \( C \) axis changes like a sine wave when moving the center location in the \( X \) direction and also changes like a cosine wave when moving in the \( Y \) direction. Each maximum value is close to \( k = 1 \).

In other words, the sensitivity coefficient takes a maximum value at 90° and 270° when moving the center location in the \( X \) direction, and it takes a maximum value at 0° and 180° when moving it in the \( Y \) direction. However, the sensitivity coefficient at 180° is higher than that at 0°. The sensitivity coefficient of the \( A \) axis is only high around 180°, as shown in Fig. 8(b) in the case of a half apex angle of 45°. The change in the sensitivity coefficient is symmetric at 180° when the center location moves in the \( X \) direction, and its change is asymmetric when its position moves in the \( Y \) direction.

In the \( C \) axis, the sensitivity coefficient takes a maximum value at approximately 90° and 270° when the center moves in the \( X \) direction, and it takes a value of zero at 0° and 180°. However, the sensitivity coefficient is less than \( k = 0.7 \). The sensitivity coefficient is approximately \( k = 1 \) at about 180° when the center moves in the \( Y \) direction. However, the sensitivity coefficient is lower than \( k = 0.25 \) in the range of 0° to 90° and 270° to 360°. Therefore, the error of the \( C \) axis will be undervalued within these ranges

The sensitivity coefficient for a half apex angle of 15° is high overall compared with that of a half apex angle of 45°, and the sensitivity coefficients of the \( A \) and \( C \) axes have an angular position that is approximately equal to \( k = 1.0 \) regardless of the direction that the center location moves. On the other hand, the sensitivity coefficient of the \( C \) axis is low overall when the half apex angle is 45°, and the maximum sensitivity coefficient significantly changes depending on the direction in which the center moves. Therefore, it
can be said that the errors of the axes of rotation are small overall in the cone-frustum with a half apex angle of 45° when the sensitive direction of the ball bar is set perpendicular to the conical surface, as shown in Fig. 1(a). The relationship between the sensitive direction and the sensitivity coefficient of the ball bar in each axis is investigated in the following section.

4.3. Effect of sensitive direction of the ball bar on the sensitivity coefficients of the axes

4.3.1. Sensitivity coefficients of the axes of linear motion

As shown in Fig. 3, there was a large difference between the two measurement methods depicted in Fig. 1. When measuring the three-dimensional circular interpolation motion that is equivalent to the cutter path for the cone-frustum machining, the influence of the sensitive directions of the ball bar on the measured trajectories was investigated based on the sensitivity coefficient.

Figure 9 shows the change in the sensitivity coefficients of the three linear axes. Figure 9(a) shows the results for a half apex angle of 15°. The difference in the sensitivity coefficients of the X and Y axes due to the difference in the sensitive direction of the ball bar is small, and the sensitivity coefficient from the perpendicular measurement method is slightly higher than that from the parallel measurement method. However, the sensitivity coefficient of the Z axis is $k = 0$ from the perpendicular measurement method, and its coefficient is $k = 0.26$ from the parallel measurement method.

In contrast, in the case of a half apex angle of 45°, when the error is measured using the perpendicular measurement method, the sensitivity coefficients of the X and Y axes are increased by approximately 30%, as shown in Fig. 9(b), and the measurement sensitivity is high. However, movement of the Z axis is not detected because the sensitivity coefficient is zero.

When the error is measured using the parallel measurement method, the sensitivity coefficient of the Z axis is equal to $k = 0.7$. As mentioned above, high sensitivity coefficients of the X and Y axes can be obtained by the perpendicular measurement method rather than the parallel measurement method, and its tendency clearly appears in the sensitivity coefficient for a half apex angle of 45°. However, the sensitivity coefficient of the Z axis is also zero in this case.

![Fig. 9](image-url)
4.3.2. Sensitivity coefficients of the axes of rotation

As shown in Figs. 7 and 8, the sensitivity coefficient of the $A$ axis was strongly affected by the half apex angle and the center location of the cone-frustum. When the sensitive direction of the ball bar is set perpendicular to the conical surface, as shown in these Figures, it has been understood that the range of high sensitivity coefficients for the $A$ axis is narrow in the case of a half apex angle of 45°.

Then, the sensitivity coefficient was calculated based on the parallel measurement method. The results are shown in Fig. 10. As shown in the Figure, the sensitivity coefficient $k$ is over 0.7 when the center location moves in the $X$ direction. The sensitivity coefficient for a half apex angle of 45° is also increased by moving the center location in the positive $X$ direction.

Thus, the sensitivity coefficient changes as shown in Fig. 11 when the center location moves in the $Y$ direction. The range of low sensitivity coefficients is extended by moving the center location significantly in the positive and negative $Y$ directions. In particular, its tendency strongly appears in the case of a half apex angle of 45°.

It is understood from the viewpoint of the sensitivity coefficient that there is no difference whether the center location is moved in the positive or negative $Y$ direction. As shown in Figs. 7 and 8, when the center location moved in the $X$ direction, the same results were obtained for half apex angles of 15° and 45°. In addition, there was no substantial change in the sensitivity coefficient of the $C$ axis except taking a maximum value at 0° and 180° when moving it in the $Y$ direction. In order to clarify the relativity of these sensitivity coefficients and the trajectory, the next section investigates how the pitch errors of the rotary axes affect the circular trajectory using the motion simulator depicted in Fig. 2.
5. Sensitivity coefficient and trajectory shape

5.1. Pitch error of the A axis and trajectory shape

The pitch error of the axis of rotation was only introduced into the simulation model shown in Fig. 2, but the frictional torque of the linear axes, the tracking delay of the servo systems, and the backlash of the A axis were not considered for the simulation. The sensitivity coefficient influences the amplitude of the pitch error that appears in the circular trajectories. Therefore, if the amplitude of the pitch error with periodicity is observed, the influence of the sensitivity coefficient on the circular trajectories can be easily understood.

Then, the measurement by the perpendicular measurement method was simulated for the cone-frustum with a half apex angle of 45°. In this half apex angle, the pitch error of the A axis is affected by the sensitivity coefficient. Figure 12 shows the sensitivity coefficient and circular trajectories. The simulation is conducted by moving the center location in the X direction. Figure 12(b) shows two trajectories; one is for \( X_T = 200 \) mm, and the other is for \( X_T = -50 \) mm. The sensitivity coefficient for \( X_T = 200 \) mm takes a maximum value of 0.9 at 0° (360°) and takes a minimum value of 0.4 at 180° according to Fig. 12(a). The amplitude of the pitch error that appears in the circular trajectory takes a maximum value at 0°, then reduces gradually along the path from 0° to 180°, and takes a minimum value at 180° for \( X_T = 200 \) mm, as shown in Fig. 12(b).

In contrast, in the case of \( X_T = -50 \) mm, the sensitivity coefficient takes a maximum value of 1.0 at 180°, but it is less than 0.3 between 0° and 90° and between 270° and 360°. Therefore, for \( X_T = -50 \) mm, it is difficult to identify the effect of the pitch error of the A axis on the trajectory between 0° and 90° and between 270° and 360°.

Fig. 12 Effect of the center of the cone-frustum on the sensitivity coefficients and the circular trajectories in the perpendicular measurement method (center \( O_T(X_T, 0, 150) \)).

Fig. 13 Effect of the center of the cone-frustum on the sensitivity coefficients and the circular trajectories in the perpendicular measurement method (center \( O_T(0, Y_T, 150) \)).
Next, the changes in the sensitivity coefficient and the circular trajectory when the center location of the cone-frustum moves in the $Y$ direction are shown in Fig. 13. When the center location moves in the positive and negative $Y$ direction, as shown in Fig. 13(a), the curves of the sensitivity coefficient become symmetric about the angle of 180°. The pitch error of the $A$ axis appears symmetric, as shown in Fig. 13(b), when the circular trajectories in the case of $Y_T = 200$ mm and $Y_T = -200$ mm are observed. The pitch error can be observed within the range from 40° to 200° when moving the center location of the cone-frustum in the positive $Y$ direction, and the pitch error can be observed within the range from 160° to 320° when moving it in the negative $Y$ direction. The sensitivity coefficient $k$ is over 0.6 within the range where this pitch error can be clearly observed.

5.2. Pitch error of the C axis and trajectory shape

The sensitivity coefficient of the $C$ axis is dependent on the offset direction, although its sensitivity coefficient is independent of the center location of the cone frustum, as shown in Figs. 7 and 8. In this section, the trajectory was simulated considering only the pitch error of the $C$ axis based on the perpendicular measurement method. The results are shown in Figs. 14 and 15.

In this simulation, the center of the cone-frustum was set at $X_T = 200$ mm. Figure 14(a) shows the effect of the half apex angle on the sensitivity coefficient of the $C$ axis. As shown in this Figure, both curves take a maximum value at 90° and 270° and take a value of zero at 0° and 180°. As shown in Figs. 14(b) and (c), the amplitude of the pitch error in the circular trajectory takes a maximum value at 90° and 270° and takes a minimum value at 0° and 180°. The maximum value of the sensitivity coefficient for a half apex angle of 45° is approximately 30% lower than that for a half apex angle of 15°, and the maximum amplitude at 90° and 270° for a half apex angle of 45° is smaller than that for a half apex angle of 15°.
Then, when the center of the cone-frustum is located at $Y_T = 200$ mm, the sensitivity coefficient and the trajectory were simulated. Figure 15(a) shows the effect of the half apex angles on the sensitivity coefficient of the $C$ axis. As shown in this Figure, the maximum sensitivity coefficient at 180° for a half apex angle of 45° is slightly lower than that for a half apex angle of 15°, but the sensitivity coefficient for a half apex angle of 45° is considerably low within the range from 0° to 90° and from 270° to 360° and is zero at 55° and 305°. Therefore, the amplitude of the pitch error of the $C$ axis will be detected as considerably small in these ranges. Its tendency can be observed in the circular trajectories shown in Fig. 15(b).

The motion errors of four axes can be detected in the cone-frustum for a half apex angle of 45° when the sensitive direction of the ball bar is set perpendicular to the conical surface and the center is positioned away from the center of the rotary table in the $Y$ direction. However, the pitch errors of the $A$ and $C$ axes cannot be sufficiently detected. Consequently, there is a possibility of undervaluing the errors when the half apex angle is 45°.

6. Measurement condition of the movement equivalent to cone-frustum cutting

In the preceding sections, the center of the cone-frustum was located at a position away from the centerline of the rotary table, and the change in the sensitivity coefficient was investigated. As a result, it is found that the motion errors of only three axes (the $X$, $Y$, and $A$ axes) are detected, though the measurement is conducted by simultaneous five-axis control movement when the center location of the cone-frustum is set on the centerline of the rotary table and when the sensitive direction of the ball bar is set in the direction perpendicular to the conical surface (refer to Fig. 6).

If the center of the cone-frustum is located at a position away from the centerline of the rotary table in the $X$ direction or in the $Y$ direction under such a condition, the motion error of the $C$ axis can be detected (refer to Fig. 7). The motion error of the $Z$ axis of a tilting table type five-axis MC cannot be detected as long as the sensitive direction of the ball bar is set perpendicular to the conical surface (refer to Figs. 8 and 9).

It is necessary to set the sensitive direction of the ball bar to be parallel to the base circle of the cone-frustum in order to detect the motion error of the $Z$ axis. In this setting condition of the sensitive direction of the ball bar, even if the center of the cone-frustum moves in the $X$ and $Y$ directions, the motion errors of the five axes can be detected. The sensitivity coefficient of the $Z$ axis becomes large for a half apex angle of 45° when the center moves, but the sensitivity coefficients of the $C$ and $A$ axes become lower in a wide range, and the influence of the pitch error of the $A$ axis appears on one side (refer to Figs. 12 and 13).

For a half apex angle of 15°, the sensitivity coefficients of the $A$ axis (refer to Figs. 10 and 11) and the $C$ axis (refer to Figs. 14 and 15) are higher than those for a half apex angle of 45°, though the sensitivity coefficient of the $Z$ axis is lower than that of a half apex angle of 45°, and all of the motion errors inherent to the axes of rotation of the tilting rotary table type five-axis MCs can be detected evenly.

Therefore, as shown in Table 1, it can be concluded that the motion errors of five axes can be detected when the sensitive direction of the ball bar is set parallel to the base circle.

| Sensitive direction of ball bar | Center $O_T$ of the table side ball of the ballbar |
|-------------------------------|--------------------------------------------------|
| Perpendicular to conical surface | $(0, 0, 150)$ | $(X_T, 0, 150)$ | $(0, Y_T, 150)$ |
| 3 axes $(X, Y, A)$ | 4 axes $(X, Y, A, C)$ | 4 axes $(X, Y, A, C)$ |
| Parallel to base circle | 4 axes $(X, Y, Z, A)$ | 5 axes $(X, Y, Z, A, C)$ | 5 axes $(X, Y, Z, A, C)$ |

Table 1. The number of axes for which the movement errors are able to be acquire.
of the cone-frustum for a half apex angle of 15° and its center is located 50 mm away from the centerline of the rotary table in the Y direction. For this test condition, as previously reported\(^{(6)}\), the reversal positions of the axes can be detected separately on the circular trajectory, and interference with the ball bar itself does not occur.

7. Conclusion

In the present paper, the sensitivity coefficient of the measurement was newly defined for five-axis MCs with a tilting rotary table, and the trajectory error in the three-dimensional circular interpolation movement equivalent to cone-frustum cutting was simulated. In the simulation, a ball bar with a one-dimensional displacement sensor was used, and the correlation between the circular trajectory and the sensitivity coefficient of each axis was investigated. In addition, the influences of the position of the table side ball and the sensitive direction of the ball bar were investigated. As a result, the following conclusions were obtained:

1) Even if the measurements of the motion errors of five axes are conducted with the ball bar perpendicular to the conical surface, the motion errors of only four axes are detected (the exception being the Z axis).

2) It is possible to detect the motion errors of four axes when the sensitive direction of the ball bar axis is set perpendicular to the conical surface of the cone-frustum for a half apex angle of 45° and its center is located away from the centerline of the rotary table in the Y direction. However, the pitch errors of the A and C axes might be undervalued.

3) In the cone-frustum for a half apex angle of 15°, the motion errors of five axes can be detected with good sensitivity without interference with the ball bar itself when the center is positioned away from the table center in the Y direction and the sensitive direction of the ball bar is set parallel to the base circle of the cone-frustum.

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