COBE vs Cosmic Strings: An Analytical Model

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Abstract

We construct a simple analytical model to study the effects of cosmic strings on the microwave background radiation. Our model is based on counting random multiple impulses inflicted on photon trajectories by the string network between the time of recombination and today. We construct the temperature auto-correlation function and use it to obtain the effective power spectrum index $n$, the rms-quadrupole-normalized amplitude $Q_{rms-PS}$ and the rms temperature variation smoothed on small angular scales. For the values of the scaling solution parameters obtained in Refs. [10], [3] we obtain $n = 1.14 \pm 0.5$, $Q_{rms-PS} = (4.5 \pm 1.5)G\mu$ and $(\frac{\Delta T}{T})_{rms} = 5.5G\mu$. Demanding consistency of these results with the COBE data leads to $G\mu = (1.7 \pm 0.7) \times 10^{-6}$ (where $\mu$ is the string mass per unit length), in good agreement with direct normalizations of $\mu$ from observations.

1 Introduction

The recent detection of anisotropy in the Microwave Background Radiation (MBR) by the COBE (COnsomic Background Explorer) collaboration has provided a new powerful experimental probe for testing theoretical cosmological models. The DMR (Differential Microwave Radiometer) instrument of COBE has provided temperature sky maps leading to the rms sky variation $\sqrt{\langle (\frac{\Delta T}{T})^2 \rangle}$ (where $\Delta T \equiv T(\theta_1) - T(\theta_2)$, and $\theta_1 - \theta_2 = 60^\circ$ is the beam separation in the COBE experiment) and the rms quadrupole amplitude. A
fit of the data to spherical harmonic expansions has also provided the angular temperature auto-correlation function \( C(\Delta \theta) \equiv \langle \frac{\delta T}{T}(\theta) \frac{\delta T}{T}(\theta') \rangle \) where \( \langle \rangle \) denotes averaging over all directions in the sky, \( \delta T(\theta) \equiv T(\theta) - \langle T \rangle \) and \( \Delta \theta = \theta - \theta' \). This result was then used to obtain the rms-quadrupole-normalized amplitude \( Q_{\text{rms-PS}} \) and the index \( n \) of the power law fluctuation spectrum assumed to be of the form \( P(k) \sim k^n \). The published results are:

\[
\left( \frac{\Delta T}{T} \right)_{\text{rms}} = (1.1 \pm 0.2) \times 10^{-5} \quad (1)
\]
\[
Q_{\text{rms-PS}} = (5.96 \pm 0.75) \times 10^{-6} \quad (2)
\]
\[
n = 1.1 \pm 0.5 \quad (3)
\]

These results have imposed severe constraints on several cosmological models for large scale structure formation. Even the standard CDM model with bias \( 1.5 \leq b_8 \leq 2.5 \) is inconsistent with the COBE results for \( H_0 > 50 \text{km/}(\text{sec} \cdot \text{Mpc}) \) and is barely consistent for \( H_0 \approx 50 \text{km/}(\text{sec} \cdot \text{Mpc}) \) \( \text{[5][15]} \). Tensor mode perturbations have recently been shown however, to make standard CDM with specific inflationary models consistent with COBE for a wider cosmological parameter region \( \text{[23]} \).

It is therefore interesting to investigate the consistency of alternative models with respect to the COBE measurement. The natural alternative to models based on adiabatic Gaussian perturbations generated during inflation are models where the primordial perturbations are created by topological defects like cosmic strings, global monopoles or textures.

Cosmological models based on topological defects have a single free parameter \( v \), the scale of symmetry breaking leading to defect formation. Consistency with large scale structure observations, constrains this parameter to \( Gv^2 \approx 10^{-6} \) \( \text{[18][19]} \) where \( G \) is Newton’s constant. In what follows we will concentrate on the case of cosmic strings.

Previous analytical studies of MBR anisotropies due to cosmic strings \( \text{[14]} \) were based on the old picture of the cosmic string network evolution \( \text{[16]} \) and therefore focused on the effects of cosmic string loops. Loops however were later shown by more detailed simulations \( \text{[3][10]} \) to be unimportant compared to long strings.

More recently, numerical simulations have been used to investigate the MBR predictions of cosmic string models \( \text{[4]} \) and comparison of these predictions has been made with the COBE data \( \text{[7]} \). It was found that for \( Gv^2 \approx 10^{-6} \), \( (v^2 = \mu \) where \( \mu \) is the mass per unit length of the string)
cosmic strings are consistent with the COBE data for a wider range of cosmological parameters than the standard CDM model. The numerical analysis that has led to this result however, is rather complicated and currently there is still some controversy among the different groups about the details of the simulations involved in the analysis. In addition, studies based on string simulations have necessarily fixed scaling solution parameters and therefore, the dependence of the results on these parameters can not be revealed. These arguments make the construction of an analytical model for the study of the effects of topological defects on the MBR, a particularly interesting prospect. It is such an analytical model that we are constructing in this paper.

In particular, we use a multiple impulse approximation to obtain the temperature auto-correlation function $C(\Delta \theta)$ predicted by the string model. From $C(\Delta \theta)$ we obtain the mass per unit length $\mu$ of strings consistent with COBE and the effective power spectrum index $n$ predicted by the string model.

Our basic assumptions and approximations are the following:

1. We approximate the photon path from $t_{\text{rec}}$ to $t_0$ by a discrete set of $N$ Hubble time-steps $t_i$ such that $t_{i+1} = 2t_i$. For $z_{\text{rec}} \simeq 1400$ we have $N \simeq \log_2(1400)^{3/2} \simeq 16$.

2. At each Hubble time-step the photon beam is affected only by the long strings within a horizon distance from the beam. The effects of further strings are cancelled due to the compensating scalar field radiation.

3. The combined effects of all strings present within a horizon distance of the photon beam is a linear superposition of the individual effects.

4. Each string that affects the photon beam induces a temperature variation of the form:

$$\frac{\delta T}{T} = 4\pi G \mu \beta$$

with

$$\beta = \hat{k} \cdot (\vec{v}_s \gamma_s \times \hat{s})$$

where $\hat{k}$, $\hat{s}$ and $\vec{v}_s$ are the unit wave-vector, the unit vector along the string and the string velocity vector respectively (see Fig. 1).

5. The long strings within each horizon volume have random velocities positions and orientations.
6. The effects of loops are unimportant compared to the effects of long strings\cite{10}.

7. Initial temperature inhomogeneities at the last scattering surface are assumed negligible compared to those induced by the string network at later times.

We will also use the result that for any function $\theta_1(\theta)$ that takes random values as the independent variable varies we have:

$$< f(\theta_1(\theta)) >_{\alpha} = < f(\theta_1) >_{\theta_1}$$

where $<>_{\alpha}$ implies averaging with respect to $\alpha$. We will call this for obvious reasons the ‘ergodic hypothesis’.

Some of these assumptions are similar to those made in corresponding numerical studies. Others (like assumption 2 which is an attempt to take into account compensation) are improvements over those of the numerical analyses. We comment on the possibility of further improving some of these assumptions in section 4.

2 The Temperature Correlation Function

We begin with a description of our model. Consider a photon beam emerging from the last scattering surface at $t_1 = t_{rec}$. This beam of fixed temperature will initially suffer $M$ ‘kicks’ from the $M$ long strings within the horizon at $t_1$. At the Hubble time $t_2 \sim 2t_{rec}$, $M$ strings, uncorrelated with the previous ones will be within the horizon giving the photon $M$ further ‘kicks’ and the process will continue until the $N \simeq 16$ Hubble time-step corresponding to the present time $t_0$ ($t_0 \simeq 2^{16}t_{rec}$). Therefore the total temperature shift in the direction $\theta$ (in what follows we consider fixed $\varphi$ and omit it when defining direction unless otherwise needed) may be written as:

$$\frac{\delta T}{T}(\theta) = 4\pi G\mu \sum_{n=1}^{N} \sum_{m=1}^{M} \beta_{mn}(\theta)$$

(4)

where $\beta_{mn}(\theta)$ corresponds to the $m$th string at the $n$th Hubble time-step. Taking $\mathbf{v}_s$ and $\mathbf{s}$ to be unit vectors along the string velocity and string length directions we may write $\beta_{mn}(\theta) = v_s(\mathbf{k}_n) \cdot \mathbf{R}_n(\theta)$ where $\mathbf{R}_n(\theta) = \mathbf{\dot{v}}_{mn} \times \mathbf{s}_{mn}$ is a unit vector that varies randomly with $m$ and $n$. 
It is instructive for what follows to obtain $\frac{\delta T}{T}(\theta)$ averaged over all directions. Defining $\xi \equiv 4\pi G\mu \nu_s \gamma_s$ we have:

$$<\frac{\delta T}{T}(\theta)> = \xi \sum_{n,m} <\hat{k}(\theta) \cdot \hat{R}^{mn}_1(\theta)> = \frac{\xi}{4\pi} \sum_{m,n} \int d\cos\theta d\varphi \cos\theta_1(\theta, \varphi)$$

where $\theta_1$ is the angle between $\hat{R}^{mn}_1(\theta)$ and $\hat{k}(\theta)$ (see Fig. 3) and since $\hat{R}^{mn}_1(\theta)$ varies randomly with $m, n$ and with $\theta, \varphi$ on angular scales larger than $\theta_{\text{rec}}$, we may use the ergodic hypothesis to obtain:

$$<\frac{\delta T}{T}(\theta)> = \frac{\xi}{4\pi} \sum_{m,n} \int d\cos\theta d\varphi_1 \cos\theta_1 = 0$$

which is also the naively expected result. The same result could have been obtained without using the ergodic hypothesis by simply taking the large $M \times N$ limit and performing the sum over $m, n$ before the $\theta, \varphi$ integration.

Our goal is to investigate correlations of fluctuations $\delta T T(\theta) \delta T T(\theta')$. Using (4) we have:

$$\frac{\delta T}{T}(\theta) \frac{\delta T}{T}(\theta') = (4\pi G\mu)^2 \sum_{n,n'} M \sum_{m,m'} \beta^{mn}(\theta) \beta^{m'n'}(\theta')$$

Define now $\Delta\theta \equiv \theta - \theta'$ and focus on the case when $\Delta\theta = \theta_p$ where $\theta_p$ is the angular size of the horizon at the Hubble time-step $t_p$ ($1 \leq p \leq 16$). Long strings present at time $t$ will inflict ‘kicks’ on two photon beams separated by $\Delta\theta = \theta_p$ that are uncorrelated for $t \leq t_p$ but are equal for $t > t_p$ when $\Delta\theta$ is within the horizon scale $t$ (see Fig. 2). Therefore

$$\hat{R}^{jk}_1(\theta) = \hat{R}^{j'k'}_1(\theta') \quad \text{iff} \quad k > p, \ j = j', \ k = k'$$

$$\hat{R}^{jk}_1(\theta) \neq \hat{R}^{j'k'}_1(\theta') \quad \text{otherwise}$$

where $\neq$ here means ‘not equal and also uncorrelated’. We may therefore split the sum (5) in two parts consisting of correlated and uncorrelated products respectively.

$$\frac{\delta T}{T}(\theta) \frac{\delta T}{T}(\theta') = \xi^2 \sum_{n=n'=p \ m=m'=1}^M (\hat{k}(\theta) \cdot \hat{R}^{mn}_1(\theta))(\hat{k}(\theta') \cdot \hat{R}^{m'n'}_1(\theta')) +$$

$$+ \sum_{n,n',m,m'} (\hat{k}(\theta) \cdot \hat{R}^{mn}_1(\theta))(\hat{k}(\theta') \cdot \hat{R}^{m'n'}_1(\theta')) \equiv \xi^2 [\Sigma_1 + \Sigma_2]$$
where $\Sigma_1$ refers to the terms of the sum that are correlated on a scale $\Delta \theta$ while $\Sigma_2$ refers to the uncorrelated terms. Averaging $\Sigma_1$ over all directions we obtain

$$< \Sigma_1(\Delta \theta) > = \frac{1}{4\pi} \sum_{\theta_1^{mn}} \int d\cos \theta d\varphi \cos \theta_1^{mn}(\theta, \varphi) \cos(\theta_1^{mn}(\theta, \varphi) + \Delta \theta)$$

where $\theta_1^{mn}$ is the angle between $\hat{k}(\theta)$ and $\hat{R}_1^{mn}(\theta)$ (see Fig.3). Since by the construction of the model $\theta_1$ varies randomly from one correlated patch to another we may use the ergodic hypothesis and replace the average over all directions with an average over $\theta_1$. It is then easy to see that

$$< \Sigma_1(\Delta \theta) > = \cos(\Delta \theta) \frac{3}{N_{\text{cor}}(\Delta \theta)}$$

where $N_{\text{cor}}(\Delta \theta) = M(N - p(\Delta \theta))$ is the number of terms in $\Sigma_1$. For $\Sigma_2$ we have:

$$< \Sigma_2(\Delta \theta) > = \frac{1}{4\pi} \int d\cos \theta d\varphi \sum_{\theta_1^{mn}} \sum_{\theta_2^{m'n'}} \cos(\theta_2^{m'n'} + \Delta \theta) = 0$$

Therefore we may write

$$< \frac{\delta T}{T}(\theta) \frac{\delta T}{T}(\theta') > = \frac{\xi^2}{3} N_{\text{cor}}(\Delta \theta) \cos(\Delta \theta)$$

(6)

Using the relations $t_p = 2^pt_{\text{rec}}$ and $\Delta \theta = \theta_{t_p} = z_p^{-1/2}$ (for $\Omega_o = 1$) where $z_p$ is the redshift at $t_p$, it is straightforward to show that

$$p(\Delta \theta) = 3 \log_2 \left( \frac{\Delta \theta}{\theta_{\text{rec}}} \right)$$

(7)

Using (6) and (7) we obtain

$$C(\Delta \theta) \equiv < \frac{\delta T}{T}(\theta) \frac{\delta T}{T}(\theta') > = \frac{\xi^2}{3} M(N - 3 \log_2 \left( \frac{\Delta \theta}{\theta_{\text{rec}}} \right)) \cos(\Delta \theta)$$

Our assumptions for the construction of $C(\Delta \theta)$ clearly break down for $\Delta \theta < \theta_{\text{rec}} \simeq 1^\circ$ (since we must have $t_p \geq t_{\text{rec}}$) and for $\Delta \theta > \frac{2\pi}{9}$ (since we can not have $N_{\text{cor}} < 0$). However, the region of validity of (8) may be easily extrapolated by slightly shifting $\Delta \theta$ by $\theta_{\text{rec}} \simeq 1^\circ$ to $\Delta \theta + \theta_{\text{rec}}$ in the log (thus extrapolating $C(\Delta \theta)$ to $\Delta \theta = 0$) and keeping $C(\Delta \theta) = 0$ for $\Delta \theta > \frac{2\pi}{9}$ as is physically expected since $N_{\text{cor}}$ goes to 0 at large angular separations. We may now write the extrapolated auto-correlation function as:
\[ C(\Delta \theta) = \frac{\xi^2}{3} M (N - 3 \log_2(1 + \frac{\Delta \theta}{\theta_{rec}})) \cos(\Delta \theta) \quad 0 \leq \Delta \theta \leq \frac{2\pi}{9} \quad (8) \]

\[ C(\Delta \theta) = 0 \quad \frac{2\pi}{9} \leq \Delta \theta \leq \pi \]

The validity of this result for \( C(\Delta \theta) \) has also been verified\[21\] by using the model introduced here to calculate directly the rms sky variation and showing that the result is identical to the one obtained below from (8) and (14) (see section 3). In what follows we will use (8) to compare COBE’s data with the string model predictions. A typical value for \( M \), the number of long strings per horizon volume, is \( M \approx 10 \) while for \( z_{rec} = 1400 \) we have \( N \approx 16 \).

### 3 Predictions of the Model

Consider an expansion of the temperature pattern on the celestial sphere in spherical harmonics

\[ \frac{\delta T}{T} = \sum_{lm} a_l^m Y_l^m(\theta, \varphi) \]

Defining \( C_l \equiv |a_l^m|^2 \) it may be shown using the addition theorem that

\[ C(\Delta \theta) \equiv \langle \frac{\delta T}{T}(\theta) \frac{\delta T}{T}(\theta') \rangle = \frac{1}{4\pi} \sum_l (2l + 1) C_l P_l(\cos(\Delta \theta)) \]

Assuming now a power spectrum of perturbations of the form \( P(k) \sim k^n \) it may be shown\[15\] that

\[ C_l = C_2 \frac{\Gamma(l + \frac{n+1}{2})\Gamma(\frac{9-n}{2})}{\Gamma(l + \frac{5-n}{2})\Gamma(\frac{3+n}{2})} \quad (9) \]

where \( C_2 = \frac{4\pi}{5} (Q_{rms-PS})^2 \), \( Q_{rms-PS} \) being the rms-quadrupole-normalized amplitude\[4\].

Since our model predicts \( C(\Delta \theta) \), \( C_l \) may be obtained for any \( l \) using the orthogonality relations for the Legendre functions. We may then find \( \bar{n} \) and \( Q_{rms-PS} \) that give the best fit to (9). Using the symbol-manipulating package Mathematica\[22\] we calculated \( C_l \) for \( l = 3...30 \). We excluded the
quadrupole $C_2$ from our fit (as was done with the COBE data) and considered harmonics up to $l_{max} = 30$ to account for the small angle cutoff of COBE due to the finite antenna beam size. Our results are rather insensitive to the specific cutoff $l_{max}$ for $l_{max} > 15$.

Minimizing the sum

$$
\sum_{l=3}^{30} (C_l - C_2 \frac{\Gamma(l + \frac{n-1}{2})\Gamma\left(\frac{n-2}{2}\right)}{\Gamma(l + \frac{n+5}{2})\Gamma\left(\frac{3+n}{2}\right)})^2
$$

with respect to $C_2$ and $n$ we obtain:

$$\bar{n} = 1.14 \pm 0.24 \quad (11)$$

$$Q_{rms-PS} \equiv \sqrt{\frac{5}{4\pi} \bar{C}_2} = (4.5 \pm 1.5)(G\mu)_6(v_s\gamma_s)_{0.15} \sqrt{M_{10}} \times 10^{-6} \quad (12)$$

as the parameters giving the best fit, where $(G\mu)_6$, $(v_s\gamma_s)_{0.15}$ and $M_{10}$ are the corresponding quantities in units of $10^{-6}$, 0.15 and 10 respectively. The error bars were $\sqrt{<\bar{n}_l - n_l^2>}$; and similarly for $C_2$, where $n_l$ (or $C_2^l$) is obtained by equating to 0 the $l$th term of the sum (10) while fixing $C_2$ (or $n$) to its best fit value.

In Fig. 4 we show a plot of $C_l$ vs $l$ obtained from $C(\Delta \theta)$ of (8) (continuous thin line), superimposed with $C_l$ obtained from (9) for the best fit values $n = \bar{n}$ and $Q_{rms-PS} = Q_{rms-PS}$ (dashed thick line). The corresponding results coming from the COBE data are given by (3) and (2). The agreement between (11) and (3) is not too surprising since it is well known that topological defects with a scaling solution naturally lead to a slightly tilted scale invariant Harrison-Zeldovich spectrum on large angular scales. Clearly however, the confirmed prediction of (11) provides a good test showing that our simple analytic model has fairly robust and realistic features.

Comparison of the observational result (2) with the prediction (12) leads to an estimate of $G\mu$. According to the recent simulations of Ref. [3], preferred values of the scaling solution parameters on the horizon scale are $v_s\gamma_s \approx 0.15$ and $M \approx 10$. Using these values we obtain

$$G\mu = (1.3 \pm 0.5) \times 10^{-6} \quad (13)$$

This result is in good agreement with the result obtained numerically in Ref. [4].

Another way to obtain an estimate for $G\mu$ is to compare the measured

$$(\Delta T^2)_{rms} \equiv \sqrt{<\left(\frac{T(\theta_1) - T(\theta_2)}{T}\right)^2>}$$

with the corresponding value predicted by our model.
The rms temperature variation may be expressed in terms of the temperature auto-correlation function for a two beam experiment by the relation:

\[
\left( \frac{\Delta T}{T} \right)_{\text{rms}}^2 = 2(C(0) - C(\alpha))
\]

(14)

where \(\alpha\) is the beam separation angle \((60^\circ\) for COBE). Since however \(\left( \frac{\Delta T}{T} \right)_{\text{rms}}^2\) for COBE was obtained with a 3.2\(^\circ\) Gaussian beam smoothing we must introduce a similar smoothing before we compare with the COBE result. Thus, we first expand \(C(\theta)\) in spherical harmonics and then reconstruct it using the window function \(W(l) = e^{-\frac{(l+1)^2}{2(l^2+1)}}\) as was done with the COBE data for the construction of \(C(\Delta \theta)\). This leads to

\[
C(\Delta \theta) = \frac{1}{4\pi} \sum_{l=3}^{30} (2l + 1)C_l P_l(\cos(\Delta \theta))W(l)^2
\]

We now use the reconstructed \(C(\theta)\) in (14) to obtain

\[
\left( \frac{\Delta T}{T} \right)_{\text{rms}} = \sqrt{2.4\xi^2 M/3} = 0.55(G\mu)_{6}(v_s\gamma_s)_{0.15}\sqrt{M_{10}} \times 10^{-5}
\]

As we did for the rms quadrupole variation we may compare the predicted rms variation with the observational result (1) of COBE to obtain the value of \(G\mu\) that makes our model consistent with the COBE data. For \((v_s\gamma_s) = 0.15\) and \(M = 10\) the resulting value for \(G\mu\) is:

\[
G\mu = (2.0 \pm 0.5) \times 10^{-6}
\]

(15)

consistent with our corresponding result from \(Q_{\text{rms}} - PS\). An estimate of \(G\mu\) may be obtained by averaging (13) and (15) and extending the error bars to cover both (13) and (15)

\[
G\mu = (1.7 \pm 0.7) \times 10^{-6}
\]

(16)

The interesting consequences of this result will be discussed in the next section.

4 Discussion

The main result of this letter is that the COBE data constrain the single free parameter \(G\mu\) of the cosmic string model to be in the range given
by (16). This range is in good agreement with normalizations of $G\mu$ from galaxy [18] [19] and large scale structure formation [8] [11]. Therefore the cosmic string model for structure formation remains a viable model, consistent with the COBE data.

Recent studies have used large scale structure observations to derive the dependence of $G\mu$ on the bias factor $b_8$, defined such that the rms fluctuation in a sphere of radius $8h^{-1}\text{Mpc}$ is $1/b_8$ [1] [2]. In Fig. 5 we plot the allowed range of $b_8$ for four different cosmic string models, demanding that the value of $G\mu$ in [1] [2] is consistent with (16). For comparison we also plot the allowed range of $b_8$ for the standard CDM model, with cosmological parameters chosen to give the most reasonable value for it [15] [5]. Our estimate for the allowed range of the bias factor in the string model is slightly lower than that of Ref. [7] but is clearly consistent with it. Since a value of $b_8$ in the range 1 to 3 is generally well motivated physically, Fig. 5 suggests that both the string model and standard CDM (with $h = 0.5$) are consistent with the COBE data.

Even though our model is based on fairly simple assumptions it has provided results that are not only self-consistent but also in good agreement with much more complicated numerical analyses. This fact indicates that our assumptions even though simple are fairly realistic and economical. In spite of that, there is certainly room for improvement. One interesting improvement that could be implemented in a straightforward way is the consideration of the effects of loops. Even though we do not expect loops to affect the shape of the auto-correlation function on the angular scales considered here they may introduce an overall multiplicative factor slightly larger than one, thus reducing the predicted value of $G\mu$ by the same factor. Another improvement that would modify our results in a similar way is the consideration of string induced perturbations on the last scattering surface by strings present in the final stages of the radiation era. Finally, it would be interesting to include the effects of compensation in a more realistic way. For example, instead of introducing an abrupt cutoff of the deficit angle on the horizon scale we could smoothly reduce it to zero as the horizon scale is approached. The effect of this modification would increase the predicted value of $G\mu$ thus tending to cancel the effect of the previous two modifications.

It is straightforward to apply the analysis presented here to other seed based cosmological models including the global monopole and texture models. Work in that direction is currently in progress.
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Figure Captions

Figure 1:
The geometry of the vectors $\vec{v}$, $\hat{s}$ and $\hat{k}$.

Figure 2:
Two photon beam paths in directions $\theta$ and $\theta'$ from $t_{rec}$ to $t_0$. The horizon in three Hubble time-steps is also shown. The effects of strings during the first two Hubble time-steps are uncorrelated for the particular angular beam separation shown.

Figure 3:
The geometry of the vectors $\hat{k}(\theta)$ and $\hat{R}^{mn}(\theta)$ in the term $\Sigma_1(\Delta \theta)$.

Figure 4:
The multipole coefficients $C_l$ (continuous thin line) superimposed with the best fit of (9) (dashed thick line), obtained by using $n = 1.14$.

Figure 5:
The allowed range of the bias factor $b_8$ for four cosmic string based models obtained using our result for $G_{\mu}$ and the results of Ref. [1][2]. The following cases are shown:
a) Strings + CDM, $h=1$ and $h=0.5$
b) Strings + HDM, $h=1$ and $h=0.5$
The allowed range for standard CDM with $h = 0.5$ and $\Omega_b = 0.1$ (the most consistent case with COBE’s data) is also shown for comparison.
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