Einstein’s general theory of relativity poses many problems to the quantum theory of point particle fields. Among them is the fate of a massive point particle. Since its rest mass exists entirely within its Schwarzschild radius, in the classical solutions of Einstein’s theory, the respective system should be a black hole. We address this issue using exact results in a new approach to quantum gravity based upon well-tested resummation methods in point particle quantum field theory. We show that the classical conclusion is obviated by quantum loop effects. We show that our new approach already passes two theoretical checks with the published literature; for, it reproduces known results on the one-loop correction to the graviton self-energy in scalar matter coupled to Einstein’s gravity as analyzed by ’t Hooft and Veltman and it is consistent with the asymptotic safety results of Bonnanno and Reuter on the behavior of Newton’s constant in the deep Euclidean regime. Indeed, our approach is consistent with the black hole phenomenology of the latter authors, including their results on the final state of the Hawking radiation for an originally massive black hole. Further black hole related phenomenological implications are also discussed.
1 Introduction

Einstein’s general theory of relativity has had many successful tests [1,2]. One of its most
direct classical predictions is that a massive elementary point particle should be a black
hole solution; for, in such a system, the rest mass lies entirely within the Schwarzschild
radius. For the Standard Model [3, 4] such a conclusion would be at best problematic, as
many of the elementary particles in the SM have non-zero rest mass and yet they are able
to communicate themselves entirely in the well-tested SM point-particle field theoretic
interactions – they do not only interact with the world beyond their Schwarzschild radii
by Hawking radiation [5]. A key point then is whether or not quantum loop effects obviate
the classical general relativistic conclusion that a massive point particle is a black hole.
It is this question that we answer in the following.

We base our analysis on pioneering work by Feynman [6,7] on the quantum theory of
general relativity in which he argued that Einstein’s theory is just another point particle
field theory in which the metric field of space-time undergoes quantum fluctuations as do
all the other point particle fields in the SM. Feynman worked-out the Feynman rules for
the simplest case of a massive scalar field coupled to Einstein’s gravity and we will use his
results in what follows. His formulation of quantum general relativity however was badly
behaved in the deep Euclidean (ultra-violet(UV)) regime and it can be characterized
as being non-renormalizable [8, 9]. What we do pedagogically beyond what is done in
Refs. [6, 7] is to re-arrange exactly the Feynman series for quantum general relativity
derived therein using our recent extension of YFS [10, 11] resummation methods to non-
Abelian gauge theories [12]. The resummed theory which we derive [13], which will be seen
to have improved UV behavior, is therefore not an approximation. We call it resummed
quantum gravity (RQG). Since we do not modify the theory of Einstein or the quantum
mechanics, we arrive at a minimal union of the ideas of Bohr and Einstein, in contrast to
the popular superstring theory [14, 15] or the recently developing loop quantum gravity
theory [16](LQG), where the existence of the smallest length parameter, the Planck length,
means that at least Einstein’s theory is modified in these two approaches to finite quantum
loop effects in quantum general relativity\(^1\).

The basic physical idea behind our resummation efforts is the following. In the over-all
space-time point of view of Feynman, a point particle of mass \(m\) at a point \(x\) and another
such particle at \(y\) experience an attractive force \(\propto m^2\) due to Newton’s law. When this
is considered in the deep Euclidean regime, wherein the effective value of \(m^2\) is large
and negative, it translates into a large repulsive interaction against the propagation of
the particle between the respective \(x\) and \(y\). One concludes that the propagation of the
particle should be severely damped in the deep Euclidean regime in the exact solutions of
quantum general relativity. This suggests that we resum the theory to get better behavior
of the Feynman series in the deep Euclidean regime.

We point-out that, in Ref. [18], Weinberg has noted the four approaches to the bad

\(^1\)In the superstring theory, it appears that quantum mechanics is also modified [17].
UV behavior of quantum general relativity:

• extended theories of gravitation: supersymmetric theories - superstrings; loop quantum gravity, etc.
• resummation
• composite gravitons
• asymptotic safety: fixed point theory (see Ref. [19, 20])

Here, we are developing a new version of the resummation approach. We will make contact with the more phenomenological asymptotic safety approach as realized in Refs. [19, 20] especially as it relates to black hole physics. Some speculations about the possible relationship between our new RQG theory and the superstring theory can be found in Ref. [13, 21], wherein we also point-out the complementarity between our analysis and the large distance analyses in Ref. [22].

More precisely, after reviewing the formulation of Einstein’s theory by Feynman in the next Section, we use YFS resummation methods in Section 3 to arrive at a theory of quantum general relativity in which the loop corrections are finite. In Section 4, we use these finite loop corrections to address the fate of massive elementary particles from the standpoint of their being black holes. We also show in this Section how our results for the UV behavior of our quantum loop effects relate to the corresponding results of the asymptotic safety approach in Refs. [19, 20]. We argue that this relationship leads us to the same black hole physics phenomenology as that found in Refs. [19, 20]. Section 5 contains some summary remarks.

2 Point Particle Field Theoretic Formulation of Einstein’s Theory

In Feynman’s point particle formulation [6, 7] of Einstein’s theory we start with the Lagrangian density of the currently observed world

\[ \mathcal{L}(x) = -\frac{1}{2\kappa^2} \sqrt{-g} R + \sqrt{-g} L_{SM}^g(x) \]  

(1)

where \( R \) is the curvature scalar, \(-g\) is the negative of the determinant of the metric of space-time \( g_{\mu\nu} \), \( \kappa = \sqrt{8\pi G_N} \equiv \sqrt{8\pi/M_P^2} \), where \( G_N \) is Newton’s constant, and the SM Lagrangian density, which is well-known (see for example, Ref. [3, 4, 23]) when invariance under local Poincare symmetry is not required, is here represented by \( L_{SM}^g(x) \) which is readily obtained from the familiar SM Lagrangian density as described in Refs. [13, 21] as follows: since \( \partial_\mu \phi(x) \) is already generally covariant for any scalar field \( \phi \) and since
the only derivatives of the vector fields in the SM Lagrangian density occur in their curls, 
\( \partial_\mu A^J_\nu(x) - \partial_\nu A^J_\mu(x) \), which are also already generally covariant, we only need to give a rule for making the fermionic terms in usual SM Lagrangian density generally covariant. For this, we introduce a differentiable structure with \( \{ \xi^a(x) \} \) as locally inertial coordinates and an attendant vierbein field \( e^a_\mu \equiv \partial \xi^a / \partial x^\mu \) with indices that carry the vector representation for the flat locally inertial space, \( a \), and for the manifold of space-time, \( \mu \), with the identification of the space-time base manifold metric as \( g_\mu\nu = e^a_\mu e_\nu^a \) where the flat locally inertial space indices are to be raised and lowered with Minkowski’s metric \( \eta_{\mu\nu} \) as usual. Associating the usual Dirac gamma matrices \( \{ \gamma_a \} \) with the flat locally inertial space at \( x \), we define base manifold Dirac gamma matrices by \( \Gamma_\mu(x) = e^a_\mu \gamma_a \). Then the spin connection, \( \omega^a_{\mu\nu} = -\frac{1}{2} e^a_\rho \left( \partial_\mu e^b_\nu - \partial_\nu e^b_\mu + \partial_\rho e^b_\mu e^b_\nu e^b_\sigma \right) + \frac{1}{2} e^a_\mu e^b_\sigma \left( \partial_\rho e^b_\sigma - \partial_\sigma e^b_\rho \right) e^b_\nu \) when there is no torsion, allows us to associate the generally covariant Dirac operator for the SM fields by the substitution \( i \partial \rightarrow i \Gamma(x)^\mu \left( \partial_\mu + \frac{1}{2} \omega^a_{\mu\nu} \Sigma^b_{\nu a} \right) \), where we have \( \Sigma^b_{\nu a} = \frac{1}{4} \left[ \gamma^b, \gamma^a \right] \) everywhere in the SM Lagrangian density. This will generate \( \mathcal{L}_{SM}^G(x) \) from the usual SM Lagrangian density \( \mathcal{L}_{SM}(x) \) as it is given in Refs. [3, 4, 23], for example.

The fundamental issue we address here is the fate of the massive point particles in the SM. We do not expect the respective spin representation to be crucial to the conclusions we reach so we follow Feynman and replace (1) with the simplest example of our problem, the Lagrangian of a massive scalar field coupled to Einstein’s gravity, where we have in mind the physical Higgs field of the SM, whose rest mass is known to be greater than 114 GeV with a 95% CL [24]:

\[
\mathcal{L}(x) = -\frac{1}{2\kappa^2} R \sqrt{-g} + \frac{1}{2} \left( g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - m_\phi^2 \phi^2 \right) \sqrt{-g}
\]

\[
= \frac{1}{2} \left\{ \eta^{\mu\nu,\lambda} \bar{h}_{\mu\nu,\lambda} - 2 \eta^{\mu\nu,\rho} \phi^\rho \bar{h}_{\mu\nu,\lambda} \right\}
\]

\[
+ \frac{1}{2} \left\{ \phi^\mu \phi_\mu - m_\phi^2 \phi^2 \right\} - \kappa h^\mu_{\mu} \left[ \bar{\phi}^\mu \phi_\mu + \frac{1}{2} m_\phi^2 \phi^2 \eta_{\mu\mu} \right]
\]

\[
- \kappa^2 \left[ \frac{1}{2} h_{\lambda\rho} \bar{\phi}^\lambda \left( \phi^\mu \phi_\mu - m_\phi^2 \phi^2 \right) - 2 \eta_{\mu\nu} h^\mu_{\rho\nu} \bar{\phi}^\rho \phi_\mu \phi_\nu \right] + \cdots
\]

Here, \( \phi(x)_{\mu} \equiv \partial_\mu \phi(x) \), and \( g_{\mu\nu}(x) = \eta_{\mu\nu} + 2 \kappa h_{\mu\nu}(x) \) where we follow Feynman and expand about Minkowski space so that \( \eta_{\mu\nu} = \text{diag}\{1, -1, -1, -1\} \). Following Feynman, we have introduced the notation \( \bar{y}_{\mu\nu} \equiv \frac{1}{2} \left( y_{\mu\nu} + y_{\nu\mu} - \eta_{\mu\nu} y_\rho^\rho \right) \) for any tensor \( y_{\mu\nu} \). Thus, \( m_\phi \) is the bare mass of our free Higgs field and we set the small tentatively observed [25] value of the cosmological constant to zero so that our quantum graviton has zero rest mass. The Feynman rules for (2) have been essentially worked out by Feynman [6, 7], including the rule for the famous Feynman-Faddeev-Popov [6, 26] ghost contribution that must be added to it to achieve a unitary theory with the fixing of the gauge (we use the gauge of Feynman in Ref. [6], \( \partial^\mu \bar{h}_{\mu\nu} = 0 \)), so we do not repeat this material here.

\footnote{Our conventions for raising and lowering indices in the second line of (2) are the same as those in Ref. [7].}
We wish to use the quantum loop corrections predicted by [2] to study the black hole character of our massive elementary particle $\varphi$. The original Feynman series derived in Refs. [6, 7] is badly UV divergent. For example, the graphs in Fig. 1 are superficially quarticly divergent and, since they give us the scalar one-loop contribution to the graviton propagator, they will lead to a logarithmic UV divergence in the coefficient of $q^4$ in the respective 1PI 2-point function, a divergence which can not be removed by any amount of field and mass renormalization, i.e., these graphs already exhibit the non-renormalizability of the original Feynman series derived in Ref. [6, 7]. In the next Section, with an eye toward our black hole physics analysis, we improve on it here by using the extension of the methods of Yennie, Frautschi and Suura [10] to non-Abelian gauge theories as we have developed this extension in Ref. [12].

### 3 Resummed Quantum Gravity

In this section, we will YFS resum the propagators in the theory in [2]: from the YFS formula (see eq.(5.18) in Ref. [10])

$$iS_F'(p) = \frac{ie^{-\alpha B''}}{S_F^{-1}(p) - \Sigma_F'(p)},$$

where $\Sigma_F'(p)$ is the sum of the YFS loop residuals, we need to find for quantum gravity the analogue of

$$\alpha B'' = \int d^4 \ell \frac{S''(k, k, \ell)}{\ell^2 - \lambda^2 + i\epsilon}$$

Figure 1: The scalar one-loop contribution to the graviton propagator. $q$ is the 4-momentum of the graviton.
where $\lambda$ is the IR cut-off and

$$S''(k, k, \ell) = \frac{-i8\alpha}{(2\pi)^3} \left( \frac{kk'}{(\ell^2 - 2\ell k + \Delta + i\epsilon)} \right) - \frac{1}{(\ell^2 - 2\ell k' + \Delta' + i\epsilon)} \bigg|_{k=k'},$$

for $\Delta = k^2 - m^2$, $\Delta' = k'^2 - m^2$. To this end, note also

$$\alpha_{\gamma}'' \gamma = \int \frac{d^4 \ell}{(2\pi)^4} \left( \frac{-i\eta^\mu\nu (\ell^2 - \lambda^2 + i\epsilon)}{(\ell^2 - 2\ell k + \Delta + i\epsilon)} \right) - \frac{-i\eta^\mu(2k_\mu)}{(\ell^2 - 2\ell k' + \Delta' + i\epsilon)} \bigg|_{k=k'},$$

where $\Delta = k^2 - m^2$, $\Delta' = k'^2 - m^2$ and $\lambda$ is the IR cut-off. With the identifications [9] of the conserved graviton charges via $e \to \kappa k_{\mu}$ for soft emission from $k$ we get the analogue $-B''_g(k)$, of $\alpha_{\gamma}'' \gamma$ by replacing the $\gamma$ propagator in (6) by the graviton propagator,

$$\frac{i\frac{1}{2}(\eta^\mu\nu \eta^{\overline{\mu}\overline{\nu}} + \eta^\mu\overline{\nu} \eta^{\overline{\mu}\nu} - \eta^\mu\overline{\mu} \eta^{\nu\overline{\nu}})}{(\ell^2 - \lambda^2 + i\epsilon)},$$

and by replacing the QED charges by the corresponding gravity charges $\kappa k_{\mu}$, $\kappa k'_{\nu}$. This yields [13] for our scalar propagator

$$i\Delta_F(k)|_{\text{Resummed}} = \frac{-ie B''_g(k)}{(k^2 - m^2 - \Sigma'_{s} + i\epsilon)},$$

where

$$B''_g(k) = -2i\kappa^2 k^4 \frac{d^4 \ell}{16\pi^4} \left( \frac{1}{(\ell^2 - \lambda^2 + i\epsilon)} \right)$$

so that $B''_g(k) = \frac{\kappa^2 |k^2|}{8\pi^2} \ln \left( \frac{m^2}{m^2 + |k^2|} \right)$ in the deep Euclidean regime. If $m$ vanishes, using the usual $-\mu^2$ normalization point we get $B''_g(k) = \frac{\kappa^2 |k^2|}{8\pi^2} \ln \left( \frac{\mu^2}{|k^2|} \right)$. In both cases the resummed propagator falls faster than any power of $|k^2|$! This is the basic result. Note that $\Sigma'_{s}$ starts in $O(\kappa^2)$, so we may drop it in calculating the one-loop effects which are at the heart of our black hole physics analysis. Our result that the resummed propagator falls faster than any power of $|k^2|$ means that one-loop corrections are finite! Indeed, all quantum gravity loops are UV finite and the all orders proof, as well as the explicit finiteness of $\Sigma'_{s}$ at one-loop, is given in Refs. [13].

### 4 Massive Elementary Particles and Black Hole Physics in RQG

The one-loop corrections to Newton’s law implied by the diagrams in Fig. [1] are central to our discussion, as they directly impact our black hole physics issues. Using the YFS
resummed propagators in Fig. 1, as we have shown in Refs. [13,21] we get the Newtonian potential \( \Phi_N(r) = -\frac{G_{NM}M_1M_2}{r}(1 - e^{-ar}) \) where \([13, 21]\) \( a \approx 3.96M_{Pl} \) when for definiteness we set \( m \approx 120\text{GeV} \). Our gauge invariant analysis can be shown [13] to be consistent with the one-loop analysis of QG in Ref. [27].

With reasonable estimates and measurements [13, 28, 29] of the SM particle masses, including the various bosons, the corresponding results for the analogs of the diagrams in Fig. 1 imply [13] that in the SM \( a_{eff} \approx 0.349 \). To make direct contact with black hole physics, note that, if \( r_S \) is the Schwarzschild radius, for \( r \to r_S \), \( a_{eff}r \ll 1 \) so that \( |2\Phi_N(r)|_{m_1=m_2} \ll 1 \). This means that \( g_{00} \approx 1 + 2\phi_{newton}(r)|_{m_1=m_2} \) remains positive as we pass through the Schwarzschild radius. It can be shown [13] that this positivity holds to \( r = 0 \). Similarly, \( g_{rr} \) remains negative through \( r_S \) down to \( r = 0 \) [13].

In resummed QG, a massive SM point particle is not a black hole.

The value of \( a_{eff} \) given here is incomplete, as there may be as yet unknown massive particles beyond those already discovered – these would only decrease \( a_{eff} \). For example, in the minimal supersymmetric Standard Model we expect approximately that \( a_{eff} \to \frac{1}{\sqrt{2}}a_{eff} \).

One can also use the results for the complete one-loop UV divergent corrections of Ref. [27] to see that the remaining interactions at one-loop order not discussed here (vertex corrections, pure gravity self-energy corrections, etc.) also do not increase the value of \( a_{eff} - a_{eff} \) is a parameter which is bounded from above by the estimates we give here. In general, we expect that the precise value of \( a_{eff} \) should be determined from cosmological and/or other considerations. Such implications will be taken up elsewhere.

Our results imply the running Newton constant \( G_N(k) = G_N/(1 + \frac{k^2}{a_{eff}}) \) which is fixed point behavior for \( k^2 \to \infty \), in agreement with the phenomenological asymptotic safety approach of Ref. [20]. Our result that an elementary particle has no horizon also agrees with the result in Ref. [20] that a black hole with a mass less than \( M_{cr} \sim M_{Pl} \) has no horizon. The basic physics is the same: \( G_N(k) \) vanishes for \( k^2 \to \infty \).

Because our value of the coefficient of \( k^2 \) in the denominator of \( G_N(k) \) agrees with that found by Ref. [20], if we use their prescription for the relationship between \( k \) and \( r \) in the regime where the lapse function vanishes, we get the same Hawking radiation phenomenology as they do: a very massive black hole evaporates until it reaches a mass \( M_{cr} \sim M_{Pl} \) at which the Bekenstein-Hawking temperature vanishes, leaving a Planck scale

\(^3\text{Our deep Euclidean studies are complementary to the low energy studies of Ref. [22].}\)
We can carry this argument further as follows. In Ref. [20] there is some uncertainty about the precise value of $M_{cr}$; for, this value depends on a parameter $\gamma$ which varies between 0 and $9/2$. However, independent of the value of $\gamma$, the result that the value of $M_{cr}$ is of the order of $M_{Pl}$ means that the quantum corrections to the Newtonian potential found above must be taken into account in the analysis of Ref. [20] when the mass of the black hole approaches this critical value in the Hawking process. When we do this, we find that the horizon is obviated [31]. This would mean that eventually the entire mass of the originally massive black hole would become accessible to the rest of the universe, which is consistent with Hawking’s recent conclusions [32] on the very important subject of information loss in black hole physics.

5 Conclusions

YFS resummation renders quantum gravity finite so that quantum loop corrections are now cut off dynamically. We call the resultant new approach to quantum gravity resummed quantum gravity (RQG), which represents a minimal union of the ideas of Bohr and Einstein. Physics below the Planck scale is accessible to point particle quantum field theory. Early universe studies may be able to test some of our predictions \(^4\) and we point-out that a theoretical cross check with the analysis of Ref. [27] has been done. In the discussion presented here, we focused on some consequences of RQG for black hole physics.

We have shown that, contrary to classical expectations, a massive elementary SM particle is not a black hole in resummed quantum gravity. Our results are also consistent with the asymptotic safety analysis in Ref. [20] that a black hole of mass less than a critical mass $\sim M_{Pl}$ does not have a horizon in quantum gravity and that the final state of the Hawking radiation of a massive black hole is a Planck scale remnant, which our recent work now shows becomes accessible to the rest of the universe due to our finite quantum loop corrections as well – a result which agrees with Hawking’s recent results [32] on information loss in black hole physics. Further checks are under investigation.

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\(^4\)This is controversial [33] and is under investigation.
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