Discrete Model for the Collapse Behavior of Unreinforced Random Masonry Walls

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Abstract. A discrete model with rigid blocks and elastic-plastic interfaces is adopted for studying the collapse behavior of in-plane loaded masonry panels with random texture. An existing random discrete model, originally developed in the elastic field, is here extended to the field of material nonlinearity by adopting a Mohr-Coulomb yield criterion for restraining actions at joint level. The resulting model turns out to be simple and effective in determining collapse loads and mechanisms of rectangular masonry panels, also accounting for a further perturbation parameter able to vary the height of each course of blocks into the masonry panel. The collapse loads turn out to be slightly smaller than those typical of regular assemblages, whereas mechanisms turn out to be influenced by local arrangement and size of blocks.

Introduction

The assessment of masonry structural behavior is of particular interest in the community of civil engineers and architects due to a large amount of historical masonry constructions in Europe and in Italy in particular. Refined models for investigating the nonlinear behavior of masonry represent an active field of research. As well known, masonry is a heterogeneous material obtained by composition of blocks connected by dry or mortar joints. Focusing on historical masonry, discrete models represent a class of models that may be successfully adopted for performing static and dynamic analyses of unreinforced masonry specimens in- and out-of-plane loaded \cite{1}. Such models are able to represent the mechanical properties of historical masonry, that is characterized by weak and small joints with respect to strong and well-sized blocks, allowing to assume that damage occurs more frequently along joints. Furthermore, historical masonry is characterized by blocks often arranged irregularly. Discrete and homogenized models for masonry are able to account for such randomness \cite{2-6}.

In this contribution, a simple and effective discrete model, with fixed contact topology and small displacement hypothesis, originally introduced for studying regular masonry in the elastic field \cite{7} and already extended to the field of material nonlinearity \cite{8}, is here extended to the case of random masonry by introducing a perturbation parameter able to vary the width of each block and block courses height, following in the first case the procedure already adopted by one of the authors in the elastic field \cite{5}. The proposed model is then able to better reproduce the microstructural behavior of historical masonry, characterized by blocks often arranged irregularly, but also to account for material nonlinearity, by assuming a Mohr-Coulomb yield criterion for restraining interface actions. The proposed model is a simple discrete model, given that it is based on rigid blocks and small displacements, leading to a fixed contact topology. Such simplifications allow adopting an efficient solution method based on the determination of the stiffness matrix of the masonry assemblage, that is extended to the case of random masonry.

A set of numerical tests of masonry specimens subject to self-weight and increasing horizontal loads are performed by assuming several different random arrangements, in order to evaluate the influence of randomness on masonry specimen behavior with respect to the regular case. Moreover,
the heterogeneity parameter represented by block standard size is increased and, as expected, when
the number of heterogeneities in the structure is large enough, the average response of the random
discrete model converges to an asymptotic response.

Discrete random model

In this paragraph, geometrical and mechanical characteristics of the model are described in order
to clearly state the hypotheses adopted. For instance, the random discrete model introduced by
Cecchi and Sab [5] is adopted and extended to the field of material nonlinearity and by defining a
further random parameter. Analysis is carried on in a linearized two-dimensional (2D) framework,
with 2D plane stress hypothesis, blocks are considered as rigid bodies, whereas the elastic and
inelastic behavior of the system are lumped at interface or joint level. Dry or mortar joints are
modeled as elastic-plastic interfaces. The random model still follows the hypothesis of fixed contact
topology and displacements are assumed to be small if compared with model size.

Geometric model. Blocks arranged in a 'running bond' pattern are first considered, hence a
generic block \( B^{i,j} \), having width \( b \), height \( a \), and thickness \( s \), is in contact with six surrounding
blocks by means of four horizontal and two vertical interfaces or contact surfaces, with horizontal
surface length equal to block half width \( b/2 \) and vertical surface length equal to block height \( a \) (Fig.
1a). Block center coordinates are \( y^{i,j} = i \, (b/2) \, e_1 + j \, a \, e_2 \), where \( j \) can assume any integer value from
1 (first row of blocks) to \( n_2 \) (last row of blocks in a masonry panel) and \( i \) identifies horizontal block
position, with \( i + j \) always even. Then the coordinates of the center of a vertical interface between
two adjacent blocks \( B^{i-2,j} \) and \( B^{i,j} \) are: \( z^{i,j} = (i - 1) \, (b/2) \, e_1 + j \, a \, e_2 \).

![Fig. 1. Representative elementary volumes (REVs) for masonry: (a) regular REV; (b) REV with
perturbed block width; (c) REV with perturbed block width and height.](image)

The random model is introduced by defining a first random parameter in horizontal direction \( p_1 \) in
order to vary the horizontal position of vertical interfaces between two adjacent blocks (Fig. 1b),
with \( 0 < p_1 < 1 \). Then, a second vertical random parameter \( 0 < p_2 < 1 \) is introduced in order to vary
the height of each block course (Fig. 1c), without varying the overall height of the masonry
assemblage considered (see Fig. 2c of the following case studies with an entire masonry panel). In
both cases, random parameters are defined together with a uniform random variable \( X^{i,j} \) defined on
[0 1].
The perturbed position of the vertical interface in horizontal and vertical directions turns out to be equal to:

\[ z_{p}^{i,j} = z^{i,j} + p_1 \left( \frac{1}{2} - X^{i,j} \right) b e_1 + p_2 \left( \frac{1}{2} - X^{i,j} \right) a e_2. \]

(1)

Then, the new horizontal positions of two consecutive vertical interfaces, \( z_{p}^{i,j} \) and \( z_{p}^{i+2,j} \), allow to define the position of the new block center \( y_{p}^{i,j} \) and the new block width \( b^{i,j} \). It is worth noting that the random model chosen maintains the original block contact topology characterized by six neighboring blocks around \( B^{i,j} \), by simply varying the length of the four horizontal contact surfaces around the block. Similarly, the new vertical positions of vertical interfaces allow defining the new height \( a' \) of each course of blocks, together with the new block vertical positions. Furthermore, the sum of the new block width along a generic course \( j \) still gives the initial width of the masonry assemblage: \( \Sigma_i b^{i,j} = L' = L \) and the sum of the new block course height still gives the initial height of the masonry assemblage: \( \Sigma_j a'^j = H \) (see the following Fig. 2b-d with panels having random arrangement).

The displacement of a generic block with a random position is given by the block centre translation and by the block rigid rotation with respect to its centre as follows:

\[ u^{i,j}(y) = u^{i,j} + \Omega^{i,j}(y - y_{p}^{i,j}) \quad \forall y \in B^{i,j}, \]

(2)

where \( u^{i,j} = u_{i}^{i,j} e_1 + u_{i}^{i,j} e_2 \) is the translation vector and \( \Omega^{i,j} \) is its rotation skew tensor, characterized by one component \( \omega_{i,j} \). Vector \( q^{i,j} = \{u_{i}^{i,j} \ u_{i}^{i,j} \ \omega_{i,j} \}^T \) collects displacement components of \( B^{i,j} \).

**Mechanical model.** As previously stated, model deformability and damage are considered at interface level only. Interfaces are assumed to have an elastic-perfectly plastic behavior that is not subject to randomness for simplicity. Following the notation of previous authors contributions, each interface may be identified by integers \( k_1, k_2 \), connecting blocks \( B^{i,j} \) and \( B^{i+k_1, j+k_2} \) [7]. Elastic behavior is given by interface stiffness that allows to define interface actions as function of block relative displacements. For instance, relative displacements \( d^{k_1,k_2} \) are horizontal and vertical relative translations and relative rotation between two adjacent blocks, that may be defined as functions of global block displacements by means of a 'compatibility matrix' [8,9] collecting distances between block centers:

\[ d^{k_1,k_2} = H^{k_1,k_2} q^{k_1,k_2} = \begin{bmatrix} I + D^{k_1,k_2} & 0 \\ 0 & -I + D^{k_1,k_2} \end{bmatrix} \begin{bmatrix} q^{i+k_1,j+k_2} \\ q^{i,j} \end{bmatrix} \]

(3)

where \( I \) is the 3x3 identity matrix and \( D^{k_1,k_2} \) is a 3x3 matrix having two non-zero components containing distances between block centres (\( D_{1,3} = dy_2/2 \) and \( D_{2,3} = -dy_1/2 \)). Interface actions \( f^{k_1,k_2} \) are obtained by integrating interface normal and shear stresses over the contact area \( S^{k_1,k_2} \), leading to normal and shear forces \( f_n \) and \( f_s \), together with an interface bending moment \( m \). Interface elastic behavior is represented by interface normal, tangential and rotational stiffness that characterize the linear relationship between interface relative displacements and actions: \( f^{k_1,k_2} = K^{k_1,k_2} d^{k_1,k_2} \). Such stiffness values may be related to actual mortar elastic and shear moduli in case of mortar joints [7], whereas in case of dry joints, a fictitious mortar material may be defined. Furthermore, interface stiffness values account also for contact surface area and moment of inertia and also for interface orientation. Assembling interface matrices in terms of block displacements by means of
compatibility matrices, the stiffness matrix of the whole masonry assemblage is obtained and the problem of a masonry panel subject to external forces may be statically solved: \( K_{\text{panel}} q_{\text{panel}} = F_{\text{ext}} \).

Interface plastic behavior is activated when actions reach their corresponding elastic limits, that are based on a tension strength and Mohr-Coulomb yield criterion. Such limits are defined by the tensile strength \( f_t \) for the normal force, the shear strength following a Mohr-Coulomb yield criterion for the shear force and the maximum eccentricity for the moment:

\[
\begin{align*}
    f_n &\leq f_t = \bar{\sigma}_t S, \\
    |f_s| &\leq \mu(f_t - f_n), \\
    |m| &\leq l_c(f_t - f_n). \\
\end{align*}
\]

where apex \( k_1,k_2 \) is omitted and \( \bar{\sigma}_t \) is the tensile strength of the mortar interface, that is related for simplicity to interface cohesion \( c \) by means of interface friction ratio \( \mu: \bar{\sigma}_t = c / \mu = c / \tan \phi \); moreover, \( l_c \) is the interface characteristic length, that represents the maximum eccentricity of normal force that can be stand by the interface (for instance \( l_c \) is equal to one half of interface length). When actions reach the corresponding elastic limits, the corresponding stiffness values are set equal to zero. Then, interface stiffness matrix and the stiffness matrix of the whole masonry assemblage are updated together with interface actions and an incremental analysis accounting for decreasing structural stiffness can be performed, following the common procedures adopted for nonlinear analysis with finite elements accounting for material nonlinearity.

It is worth noting that interface mechanical properties are frequently subject to uncertainties, in particular if a random arrangement is considered. For this reason, a sensitivity analysis of interface mechanical properties should be performed in order to investigate the effect of uncertainties on the overall behavior in further developments of this work.

**Numerical tests**

A set of numerical tests is performed in order to evaluate the effectiveness of the random discrete model in representing the behavior of masonry panels with random texture subject to self-weigh and in-plane increasing lateral loads, denoted by the load multiplier \( \lambda \). Several case studies characterized by masonry with regular texture and dry joints, already studied by authors [8] are taken as reference. Then, the effect of randomness is evaluated for a set of rectangular masonry panels with varying height-to-width ratio and considering randomness in horizontal direction only and both in horizontal and vertical directions. For instance, the following blocks arrangement cases are considered:

- a) regular 'running bond' (Fig. 2a);
- b) horizontal randomness (Fig. 2b);
- c) horizontal and vertical randomness (Fig. 2c);
- c) 'stack bond' (horizontal randomness with \( p_1 = 0.99 \), Fig. 2d).

Random block arrangement is considered by applying for first the randomness in vertical interface positions, then by applying randomness both for vertical interfaces and block height. For instance, block dimensions in case or regular arrangement are \( b = 240 \text{ mm}, a = 60 \text{ mm}, s = 120 \text{ mm} \), with joint thickness assumed equal to 1 mm. Interfaces are assumed to have negligible tensile strength and cohesion, whereas friction ratio \( \mu \) is assumed equal to 0.6. Masonry panels having \( n_1 = 5 \) blocks along horizontal direction and \( n_2 = 11, 21, 27 \) blocks along vertical direction are considered. Then, panel width \( L \) is equal to 1.25 m, whereas panel height \( H \) turns out to be equal to 0.77 m, 1.47 m, 1.89 m, leading to \( H/L \) ratios equal to, respectively, 0.62, 1.18 and 1.51. Fig. 2 shows the case with \( n_2 = 23 \) block courses, leading to a panel having an almost square shape. In particular, Fig. 2a shows the case having regular texture (i.e. 'running bond'), whereas Fig. 2b shows the same panel with
randomness in horizontal direction and Fig. 2c shows the case with randomness in both plane directions. Fig. 2d shows a particular case of variation of vertical interfaces horizontal position, for instance with $p_1 = 0.99$, leading to a panel with almost vertically aligned interfaces (i.e. 'stack bond').

Fig. 2. Square-shaped masonry panel having (a) regular texture ('running bond'), (b) texture with random position of vertical interfaces, (c) texture with random position of vertical interface and random block height, (d) texture with almost vertically aligned interfaces ('stack bond').

For randomness cases b and c, ten pushover analyses are performed in order to consider ten different random cases, whereas only one analysis is done for the running and stack bond cases, then results are collected in Fig. 3, that shows collapse load multipliers varying $H/L$. The continuous line represents the collapse load multiplier obtained analytically by means of a homogenized nonlinear model [10-12], dots represent the numerical solutions obtained in case of regular block arrangement (Fig. 2a), crosses represent ultimate load multipliers of panels having horizontal randomness only
(Fig. 2b) and plus symbols represent ultimate load multipliers of panels having both horizontal and vertical randomness (Fig. 2c). Furthermore, the ultimate load multipliers obtained with almost aligned vertical interfaces (Fig. 2d) are denoted by square symbols. In general, ultimate load multipliers decrease for increasing $H/L$ both with regular and with random texture. As expected, random texture lead to smaller load multipliers with respect to the regular case.

| $H/L$ | 0.62 | 1.18 |
|-------|------|------|
| a (regular, 'running bond') | ![image](a.png) | ![image](a_1.18.png) |
| b (random, $p_1$) | ![image](b.png) | ![image](b_1.18.png) |
| c (random, $p_1$, $p_2$) | ![image](c.png) | ![image](c_1.18.png) |
| d (stack bond) | ![image](d.png) | ![image](d_1.18.png) |

Fig. 4. Collapse mechanisms obtained with regular and random masonry panels, varying $H/L$ ratio and with several block arrangements.
Considering the average values of the load multipliers obtained with each random model type, the strength reduction is close to 10-11% in case of both randomness cases for $H/L = 0.6$, whereas it is close to 8-10% for $H/L$ equal to 1.18 and 1.51. The limit case characterized by almost aligned vertical interfaces is characterized by collapse load multipliers with a strength reduction close to 17% for the three $H/L$ cases considered.

Fig. 4 shows several collapse mechanism obtained with the first two $H/L$ ratios considered and with regular texture (a, first row), texture with random vertical interface positions (b, second row), texture having random vertical interface positions and random block height (c, third row), and texture with almost aligned vertical interfaces (d, fourth row). Focusing on cases a, b and c, all mechanisms are characterized by joint shear and detachment failures, generating a triangular or trapezoidal portion of the panel close to the upper-right corner that moves far from the rest of the panel. The shape and the size of the panel portion involved in the mechanism is strictly related to the local random block size. Considering the limit case characterized by a 'stack bond' texture, the collapse mechanism is always characterized by the almost rigid rotation of each column of blocks, with the detachment or shear failure of the almost aligned vertical interfaces.

**Concluding remarks**

In this contribution, a discrete model with rigid blocks and elastic-plastic interfaces has been adopted for studying the collapse behavior of in-plane loaded masonry panels with random texture. An existing random discrete model, originally developed in the elastic field, has been extended to the field of material nonlinearity by adopting a Mohr-Coulomb yield criterion for restraining actions at interface level. The resulting model turned out to be simple and effective in determining collapse load and mechanisms of rectangular masonry panels, accounting for both a horizontal and a vertical random parameter. The former parameter has been responsible of perturbations in the horizontal position of vertical interfaces, leading to blocks having different width along the panel; the latter parameter has been responsible of perturbations of the height of each row of blocks. The static solution method already adopted by authors in case of regular masonry turned out to be effective and equally fast for studying the random cases. The collapse load multipliers turned out to be generally smaller than those typical of regular assemblages, with a strength reduction almost close to 10% for all the cases considered. However, such a difference turned out to be slightly decreasing for increasing panel dimensions and blocks number, in agreement with numerical results obtained in the linear elastic case; moreover the additional random parameter in vertical direction did not reduce significantly the collapse load multipliers. Focusing on collapse mechanisms, instead, perturbation parameters turned out to be more important, given that mechanisms turned out to be influenced by local arrangement and size of blocks.

Further developments of this work will focus on a more detailed statistical study of limit load multipliers, together with further case studies with different panel height-to-width ratios. A more accurate evaluation of the small influence of random parameters on collapse loads for increasing panel size and blocks number (i.e. number of heterogeneities) will be assessed. Moreover, a comparison of the proposed discrete random model with existing homogenized solutions for random masonry will be taken in consideration. Finally, as already stated in the section dedicated to mechanical parameters, further developments of this work will also evaluate the influence of mechanical uncertainties of joint stiffness and strength in addition to the geometrical uncertainties considered here.

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