Optimal dimensions of connecting tubes for dynamic measurements of pressure

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Abstract. In many industrial applications a connecting tube between the measured object and the pressure meter is a component part of the pressure measurement system, which may significantly influence the accuracy of the dynamic measurements of pressure. The aim of the paper is to determine the optimal dimensions of the connecting tube from the viewpoint of magnitudes of amplitude dynamic errors. The limit frequencies of the pneumatic pressure measurement system with connecting tubes of different dimensions were obtained with Bergh and Tijdeman mathematical model and experiments.

1. Introduction
Accurate measurements of time-varying pressure are important in many industrial and scientific applications [1]. Due to spatial, temperature and other constraints, pressure sensors often have to be placed at a certain distance from the measured object and a tube that connects the measured object and the pressure sensor is then a component part of the pressure measurement system. The dynamic characteristics of the resulting fluid oscillator can significantly influence the accuracy of dynamic measurements of the pressure, where the magnitudes of dynamic errors in the transmission of the measured pressure quantities depend on the thermodynamic and transport properties of the transmission medium, and on the material and geometric characteristics of the connecting tube and the internal volume of the pressure transducer [2–5].

The useful frequency range of the measurement system can be defined with the limit frequency $f_{\text{lim}}$, which is the largest frequency up to which the amplitude response of the dynamic system does not exceed the defined maximum permissible dynamic error. As an example, the limit frequency $f_{\text{lim}}$ can be explained for the case of the linear second-order dynamic system. The amplitude-frequency characteristic of such a system can be written as [6]:

$$
\beta(\omega) = \frac{1}{\sqrt{1 - \left(\frac{\omega}{\omega_0}\right)^2 + \left(2\xi\frac{\omega}{\omega_0}\right)^2}},
$$

where $\beta$ is the amplitude ratio of the output and input, $\omega = 2\pi f$ is the angular frequency, $\omega_0$ is the natural angular frequency of the system and $\xi$ is its damping ratio. By assuming a constant natural frequency of the system, $f_{\text{lim}}$ depends only on the damping ratio. The optimal damping ratio $\xi_{\text{opt}}$ of such dynamic system from perspective of maximum value of $f_{\text{lim}}$ can be obtained by equalizing the
maximum of the amplitude ratio with \(1 + e_r\), where \(e_r\) is a relative maximum permissible dynamic error. The optimal damping ratio \(\xi_{\text{opt}}\) can be therefore expressed as:

\[
\xi_{\text{opt}} = \sqrt{-\frac{1}{2} \left(1 - \frac{e_r(2 + e_r)}{1 + e_r}\right)}
\]  

(2)

Figure 1 presents the amplitude-frequency characteristics for the underdamped system (\(\xi = 0.1\)), critically damped system (\(\xi = 1\)) and the system with \(\xi_{\text{opt}}\) for \(e_r = 0.05\) (\(\xi = 0.59\)). From the figure it is seen that \(f_{\text{lim}}\) strongly depends on the damping of the system and reaches the maximum value for the system with \(\xi_{\text{opt}}\).

**Figure 1.** Amplitude-frequency characteristics of the linear second-order dynamic system.

The aim of this paper is to study the optimal dimensions of the connecting tube for dynamic measurements of pressure. The limit frequencies \(f_{\text{lim}}\) of the pneumatic pressure measurement system with connecting tubes of different dimensions were obtained with Bergh and Tijdeman mathematical model, which is briefly presented in section 2, and experimentally. In section 3 the theoretical and experimental results are discussed and compared.

2. **Bergh and Tijdeman mathematical model**

Bergh and Tijdeman (B&T) mathematical model [2] is currently considered the state of the art for calculating the frequency response of pneumatic pressure measurement system. The model is derived from the fundamental flow equations, i.e., the Navier-Stokes equations (radial and axial), the continuity equation, the equation of state and the energy equation. The main assumptions of the model are the following: the sinusoidal oscillations in the pressure, density, temperature and velocity are small in comparison to the mean values, the length to internal diameter ratio of the tube \(L/D\) is large so that end effects are negligible, the flow conditions in the tube are laminar, the thermal conductivity of the wall of the tube is assumed to be large compared to that of the fluid, so that the temperature oscillations at the wall can be neglected, the pressure and the density of the fluid in the internal volume of the pressure transducer \(V\) are not locally dependent, the natural frequency of the transducer diaphragm is large in comparison with the frequency of the pressure oscillations. The analytical
solution (for details of its derivation see the original paper [2]) for the complex frequency response of the pneumatic pressure measurement system derived by B&T gives:

\[
P_d(i\omega) = \frac{1}{\cosh(\phi(i\omega)L) + \frac{n(i\omega)V}{V_i}(\sigma + \frac{1}{k})\phi(i\omega)L\sinh(\phi(i\omega)L)},
\]

where \(P_d(i\omega)\) is the complex amplitude of pressure oscillations in \(V\), \(P_i(i\omega)\) is the complex amplitude of pressure oscillations at the inlet of the tube, \(V_i = \pi D^2 L / 4\) is the internal volume of the tube, \(\sigma\) is the relative transducer volume increase due to deflection of the transducer diaphragm, \(k\) is the polytropic index for the fluid in \(V\) and \(\phi(i\omega)\) is the complex wave propagation factor defined by:

\[
\phi(i\omega) = \frac{\omega}{a} \sqrt{\frac{J_0(\alpha(\omega))}{J_1(\alpha(\omega))}} \sqrt{\frac{\gamma}{n(i\omega)}},
\]

where \(J_0\) an \(J_2\) are Bessel functions of the first kind, of the zeroth and second order, respectively, \(\gamma\) is the adiabatic index, \(\alpha(i\omega)\) the complex shear wave number defined by:

\[
\alpha(i\omega) = i\sqrt{\frac{\rho \omega}{2\mu}},
\]

where \(i\) is the imaginary unit, \(\rho\) is the density of the fluid, \(\mu\) is the dynamic viscosity of the fluid and \(n(i\omega)\) can be interpreted as a complex polytropic index of the fluid in the tube:

\[
n(i\omega) = \left[1 + \frac{\gamma - 1}{\frac{\gamma}{J_0(\alpha(i\omega)\sqrt{\text{Pr}})}} \right]^{-1},
\]

where \(\text{Pr}\) is the Prandtl number of the fluid. The absolute value of the complex frequency response (3) gives the amplitude-frequency characteristic of the pneumatic pressure measurement system:

\[
\beta(\omega) = \left|\frac{P_d(i\omega)}{P_i(i\omega)}\right|.
\]

3. Results

The study of the limit frequencies of the pneumatic pressure measurement system with connecting tubes of different dimensions were carried out for \(V' = 1993.45\text{ mm}^3\) and \(e_v = 0.05\). Due to the fact that the thermodynamic process in the internal volume of the pressure transducer is approximately isothermal, \(k = 1\), for the lowest oscillation frequencies and it is almost adiabatic, \(k = \gamma \approx 1.4\), for higher oscillation frequencies, an average value of \(k = 1.2\) is considered in the mathematical model.

Figure 2 shows the theoretical results obtained for dry air properties at standard environmental conditions \((\rho = 101.3\text{ kPa}, T = 20\text{ °C})\). The results show that by increasing the tube length from 0.25 m up to 2 m, its optimal diameter increases from 0.96 mm \((f_{\text{lim}} = 43\text{ Hz})\) up to 1.47 mm \((f_{\text{lim}} = 20\text{ Hz})\), while for the shortest observed tube with a length of 0.1 m \(f_{\text{lim}}\) reaches the largest value at the largest observed tube diameter \((f_{\text{lim}} = 73\text{ Hz} \text{ for } D = 4\text{ mm})\).

Figure 3 shows the comparison of theoretically obtained \(f_{\text{lim}}\) (for dry air properties under the conditions of the measurements) with the results obtained from three repeated measurements for \(D = 1.078\text{ mm}\). In the experimental system one piezoelectric pressure transducers was flush mounted to the sinusoidal dynamic pressure generator based on loudspeaker and measured the tube inlet pressure, while the other was connected to the dynamic pressure generator with the connecting tube under test and measured the tube outlet pressure (the details of the measurement system are presented...
in [7]). Trends of the experimental results show relatively good agreement with those obtained with the mathematical modelling, where \( f_{\text{lim}} \) reaches the largest value for \( L = 0.42 \, \text{m} \). Deviations between the experimental and theoretical results can be explained by the fact that the B&T mathematical model considers only the major losses in the connecting tube, although there are also other sources of irreversible pressure losses in the actual measurement system, such as the minor losses at the inlet and outlet of the pressure connecting tube and in the internal volume of the pressure transducer. The amplitude-frequency characteristic resembles the overdamped response for all the tube lengths except for \( L = 0.25 \, \text{m} \), where it resembles the underdamped response. Consequently, experimentally obtained \( f_{\text{lim}} \) for the actual measurement system with higher damping are lower for all the tubes lengths except for \( L = 0.25 \, \text{m} \).

Figure 2. Theoretically obtained dependence of the limit frequency on the tube dimensions.

Figure 3. Theoretically and experimentally obtained limit frequencies for \( D = 1.078 \, \text{mm} \).

4. Conclusions
The results presented in this paper show that the limit frequency of the pneumatic pressure measurement system up to which its amplitude response does not exceed the defined maximum permissible error strongly depends on the dimensions of the pressure connecting tubes. The determined optimal dimensions of the connecting tubes in this paper can help to improve the accuracy of the dynamic measurements of pressure with the pneumatic pressure measurement system.

5. References
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