Kerr-Newman Solution and Energy in Teleparallel Equivalent of Einstein Theory

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An exact charged axially symmetric solution of the coupled gravitational and electromagnetic fields in the teleparallel equivalent of Einstein theory is derived. It is characterized by three parameters “the gravitational mass $M$, the charge parameter $Q$ and the rotation parameter $a$” and its associated metric gives Kerr-Newman spacetime. The parallel vector field and the electromagnetic vector potential are axially symmetric. We then, calculate the total energy using the gravitational energy-momentum. The energy is found to be shared by its interior as well as exterior. Switching off the charge parameter we find that no energy is shared by the exterior of the Kerr-Newman black hole.
1. Introduction

The search for a consistence expression for the gravitating energy and angular momentum of a self-gravitating distribution of matter is undoubtedly a long-standing problem in general relativity. The gravitational field does not possess the proper definition of an energy momentum tensor and one usually defines some energy-momentum and angular momentum as Bergmann [1] or Landau-Lifschitz [2] which are pseudo-tensors. The Einstein’s general relativity can also be reformulated in the context of teleparallel geometry. In this geometrical setting the dynamical field quantities correspond to orthonormal tetrad field $e^i_{\mu}$ (i, $\mu$ are $SO(3,1)$ and spacetime indices, respectively). The teleparallel geometry is a suitable framework to address the notions of energy, momentum and angular momentum of any spacetime that admits a $3+1$ foliation [3]. Therefore, we consider the tetrad theory of gravitation.

The tetrad theory of gravitation based on the geometry of absolute parallelism [4] can be considered as the closest alternative to general relativity, and it has a number of attractive features both from the geometrical and physical viewpoints. Absolute parallelism is naturally formulated by gauging spacetime translations and underlain by the Weitzenböck geometry, which is characterized by the metricity condition and by the vanishing of the curvature tensor (constructed from the connection of the Weitzenböck geometry). Translations are closely related to the group of general coordinate transformations which underlies general relativity. Therefore, the energy-momentum tensor represents the matter source in the field equation for the gravitational field just like in general relativity.

The tetrad formulation of gravitation was considered by Møller in connection with attempts to define the energy of gravitational field [14, 15]. For a satisfactory description of the total energy of an isolated system it is necessary that the energy-density of the gravitational field is given in terms of first- and/or second-order derivatives of the gravitational field variables. It is well-known that there exists no covariant, nontrivial expression constructed out of the metric tensor. However, covariant expressions that contain a quadratic form of first-order derivatives of the tetrad field are feasible. Thus it is legitimate to conjecture that the difficulties regarding the problem of defining the gravitational energy-momentum are related to the geometrical description of the gravitational field rather than are an intrinsic drawback of the theory [16, 17].

It is well known that teleparallel equivalent of general relativity (TEGR) [18] provides an alternative description of Einstein’s general relativity. In this theory the gravitational field is described by the tetrad field $e^a_{\mu}$. In fact the first attempt to construct a theory of the gravitational field in terms of a set of four linearly independent vector fields in the Weitzenböck geometry is due to Einstein [30, 31, 32].

A well posed and mathematically consistence expression for the gravitational energy has been developed [17]. It arises in the realm of the Hamiltonian formulation of the TEGR [3] and satisfies several crucial requirements for any acceptable definition of gravitational energy. The gravitational energy-momentum $P^a$ [17, 33] obtained in the framework of the TEGR has been investigated in the context of several distinct configuration of the gravitational filed. For asymptotically flat spacetimes $P^0$ yields the ADM energy [34].
Kawai et. al. [10] assuming the tetrad field to have the form
\[ e^i_\mu = \delta^i_\mu + \frac{a}{2} l_k l_\mu - \frac{Q^2}{2} m_k m_\mu, \]
and
\[ \eta^{\mu\nu} l_\mu l_\nu = 0, \quad \eta^{\mu\nu} m_\mu m_\nu = 0, \quad \eta^{\mu\nu} l_\mu m_\nu = 0, \]
were able to obtain a charged Kerr metric solution in new general relativity. In extended new general relativity also Kawai et. al. [35] have examined axi-symmetric solutions of the gravitational and electromagnetic field equations in vacuum from the point of view of the equivalence principle.

According to the above discussion, it is clear that there is a problem in using the definitions of the energy-momentum complexes resulting from general relativity theory of gravitation. It is the aim of the present work to derive an exact charged axially symmetric solution in TEGR for the coupled gravitational and electromagnetic fields. Using this solution we calculate the energy using the gravitational energy-momentum. In \( \S 2 \) we derive the field equations of the coupled gravitational and electromagnetic fields. In \( \S 3 \) we first apply the tetrad field with sixteen unknown functions of \( \rho \) and \( \phi \) to the derived field equations. Solving the resulting partial differential equations, we obtain an exact analytic solution. In \( \S 4 \) we calculate the energy. The final section is devoted to discussion and conclusion.

2. The TEGR for gravitation and electromagnetism

In a spacetime with absolute parallelism the parallel vector field \( e_a^\mu \) define the nonsymmetric affine connection
\[ \Gamma^{\lambda}_{\mu\nu} \overset{\text{def.}}{=} e_a^\lambda e^a_{\mu\nu}, \] (1)
where \( e_a_{\mu\nu} = \partial_\nu e_a^\mu \). The curvature tensor defined by \( \Gamma^{\lambda}_{\mu\nu} \) is identically vanishing, however. The metric tensor \( g_{\mu\nu} \) is given by
\[ g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\nu. \] (2)

The Lagrangian density for the gravitational field in the TEGR, in the presence of matter fields, is given by\(^\dagger\) [17, 36]
\[ \mathcal{L}_G = e L_G = -\frac{e}{16\pi} \left( \frac{T^{abc} T_{abc}}{4} + \frac{T^{abc} T_{bac}}{2} - T^a T_a \right) - L_m = -\frac{e}{16\pi} \sum_{a} T_{abc} - L_m, \] (3)
where \( e = \text{det}(e^a_\mu) \). The tensor \( \sum_{a} \) is defined by
\[ \sum_{a} = \frac{1}{4} \left( T^{abc} + T^{bac} - T^{cab} \right) + \frac{1}{2} \left( \eta^{ac} T^{b} - \eta^{ab} T^{c} \right). \] (4)

\(^*\)spacetime indices \( \mu, \nu \) and SO(3,1) indices \( a, b \) run from 0 to 3. Time and space indices are indicated to \( \mu = 0 \), \( i \), and \( a = (0), (i) \).
\(^\dagger\)Throughout this paper we use the relativistic units, \( c = G = 1 \) and \( \kappa = 8\pi \).
$T^{abc}$ and $T^a$ are the torsion tensor and the basic vector field defined by

$$T^a_{\mu\nu} \equiv e^a_\lambda T^\lambda_{\mu\nu} = \partial_\mu e^a_\nu - \partial_\nu e^a_\mu, \quad T^a \equiv T^b_{\ b}.$$  (5)

The quadratic combination $\sum_{abc} T^{abc}$ is proportional to the scalar curvature $R(e)$, except for a total divergence term [16]. $L_m$ represents the Lagrangian density for matter fields. The electromagnetic Lagrangian density $L_{e.m.}$ is given by [10]

$$L_{e.m.} \equiv -\frac{e}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma},$$  (6)

where $F_{\mu\nu}$ being given by $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ and $A_\mu$ being the electromagnetic potential.

The gravitational and electromagnetic field equations for the system described by $L_G + L_{e.m.}$ are the following

$$\epsilon_a \epsilon_{b\mu} \partial_\nu \left( e \sum_{c}^{b\lambda} \right) - e \left( \sum_{c}^{b\nu} T_{b\nu\mu} - \frac{1}{4} e_{a\mu} T_{bcd} \sum_{c}^{b\nu} \right) = \frac{1}{2} \kappa e T_{a\mu},
\partial_\nu (e F^{\mu\nu}) = 0,$$

(7)

where

$$\frac{\delta L_m}{\delta e^{a\mu}} = e T_{a\mu}.$$

It is possible to prove by explicit calculations that the left hand side of the symmetric part of the field equations (7) is exactly given by [17]

$$\frac{e}{2} \left[ R_{a\mu}(e) - \frac{1}{2} e_{a\mu} R(e) \right].$$

3. Exact Analytic Solution

Let us begin with the tetrad field which can be written in the spherical polar coordinates as

$$\left( e^i_\mu \right) = \begin{pmatrix}
  A_1(\rho, \phi) & A_2(\rho, \phi) & A_3(\rho, \phi) & A_4(\rho, \phi) \\
  B_1(\rho, \phi) \sin \theta \cos \phi & B_2(\rho, \phi) \sin \theta \cos \phi & B_3(\rho, \phi) \cos \theta \cos \phi & B_4(\rho, \phi) \sin \phi \sin \theta \\
  C_1(\rho, \phi) \sin \theta \sin \phi & C_2(\rho, \phi) \sin \theta \sin \phi & C_3(\rho, \phi) \cos \theta \sin \phi & C_4(\rho, \phi) \cos \phi \sin \theta \\
  D_1(\rho, \phi) \cos \theta & D_2(\rho, \phi) \cos \theta & D_3(\rho, \phi) \sin \theta & D_4(\rho, \phi) \cos \theta
\end{pmatrix},$$  (8)

\(^1\text{Heaviside-Lorentz rationalized units will be used throughout this paper}\)
where $A_i(\rho, \phi), B_i(\rho, \phi), C_i(\rho, \phi)$ and $D_i(\rho, \phi), i = 1 \ldots 4$ are unknown functions of $\rho$ and $\phi$. Applying (8) to the field equations (7) we obtain a set of nonlinear partial differential equations. Due to the lengthy of writing these partial differential equations we will just write the solution that satisfy these differential equations.

The Exact Solution

If the arbitrary functions take the following values

$$ A_1 = 1 - \frac{2M\rho - Q^2}{2\Omega}, \quad A_2 = \frac{2M\rho - Q^2}{2\Upsilon}, \quad A_3 = 0, \quad A_4 = -\frac{(2M\rho - Q^2)a\sin^2 \theta}{2\Omega}, $$
$$ B_1 = \frac{(2M\rho - Q^2)}{2\Omega}, \quad B_2 = \frac{1}{2\Upsilon \cos \phi} \left( 2\rho \alpha - (2M\rho - Q^2) \cos \phi \right), $$
$$ B_3 = \frac{\alpha}{\cos \phi}, \quad B_4 = \frac{-2\beta + (2M\rho - Q^2)a\sin^2 \theta \cos \phi}{2\sin \phi}, $$
$$ C_1 = \frac{(2M\rho - Q^2)}{2\Omega}, \quad C_2 = \frac{1}{2\Upsilon \sin \phi} \left( 2\rho \beta - (2M\rho - Q^2) \sin \phi \right), $$
$$ C_3 = \frac{\beta}{\sin \phi}, \quad C_4 = \frac{2\alpha + (2M\rho - Q^2)a\sin^2 \theta \sin \phi}{2\cos \phi}, $$
$$ D_1 = \frac{(2M\rho - Q^2)}{2\Omega}, \quad D_2 = 1 + \frac{(2M\rho - Q^2)}{2\Upsilon}, $$
$$ D_3 = -\rho, \quad D_4 = \frac{(2M\rho - Q^2)a\sin^2 \theta}{2\Omega}, $$

(9)

where $\Omega$, $\Upsilon$, $\alpha$, and $\beta$ are defined by

$$ \Omega = \rho^2 + a^2 \cos^2 \theta, \quad \Upsilon = \rho^2 + a^2 - 2M\rho + Q^2, $$
$$ \alpha = \rho \cos \phi + a \sin \phi, \quad \beta = \rho \sin \phi - a \cos \phi, $$

(10)

$M$, $Q$ and $a$ are the gravitational mass, the charge parameter and the angular momentum of the rotating source [9, 10].

The parallel vector field (8) using solution (9) is axially symmetric in the sense that it is form invariant under the transformation

$$ \vec{\phi} \rightarrow \phi + \delta \phi, \quad e^{(0)}_{\mu} \rightarrow e^{(0)}_{\mu}, \quad e^{(1)}_{\mu} \rightarrow e^{(1)}_{\mu} \cos \delta \phi - e^{(2)}_{\mu} \sin \delta \phi, $$
$$ e^{(2)}_{\mu} \rightarrow e^{(1)}_{\mu} \sin \delta \phi + e^{(2)}_{\mu} \cos \delta \phi, \quad e^{(3)}_{\mu} \rightarrow e^{(3)}_{\mu}. $$

(11)

The form of the antisymmetric electromagnetic tensor field $F_{\mu\nu}$ and the energy-momentum tensor are given by

$$ F = \frac{Q}{2\sqrt{\pi \Omega}^2} \left\{ \left( (a^2 \cos^2 \theta - \rho^2) d\rho + 2\rho a^2 \cos \theta \sin \theta d\theta \right) \wedge dt \right. $$
$$ + \left. \left( 2\rho a \cos \theta \sin \theta (\rho^2 + a^2) d\theta - a \sin^2 \theta (\rho^2 - a^2 \cos^2 \theta) d\rho \right) \wedge d\phi \right\}, $$

(12)
\[ T_1^1 = -T_2^2 = -\frac{Q^2}{8\pi \Omega^2}, \]
\[ T_3^3 = -T_0^0 = \frac{Q^2(\rho^2 + a^2 + a^2 \sin^2 \theta)}{8\pi \Omega^3}, \]
\[ T_0^3 = \frac{-T_3^0}{\sin^2 \theta(\rho^2 + a^2)} = \frac{Q^2 a}{4\pi \Omega^2}. \]  

Solution (9) satisfies the field equations (7) and the associated metric has the following form

\[
\begin{align*}
\begin{aligned}
ds^2 &= \left( \frac{\Upsilon - a^2 \sin^2 \theta}{\Omega} \right) dt^2 - \frac{\Omega}{\Upsilon} d\rho^2 - \Omega d\theta^2 - \left( \frac{\rho^2 + a^2}{\Upsilon} \right) \sin^2 \theta \sin^2 \theta d\phi^2 \\
&\quad - 2\frac{(2M\rho - Q^2)a \sin^2 \theta}{\Omega} dt d\phi,
\end{aligned}
\end{align*}
\]

which is the Kerr-Newman black hole in the Boyer-Lindquist coordinate.

The previously obtained solutions (Schwarzschild, Reissner Nordström and Kerr spacetimes) can be generated as special solutions of the tetrad (8) using (9) by an appropriate choice of the arbitrary functions. Now we are going to write the tetrad field (8) using (9) in the Cartesian coordinate.

Performing the following coordinate transformation [10]

\[
\begin{align*}
t &\rightarrow t - M \ln \Upsilon - 2M^2 \left( 1 - \frac{Q^2}{4M^2} \right) \int \frac{d\rho}{\Upsilon}, \\
x &\rightarrow (\rho \cos \phi + a \sin \phi) \sin \theta, \\
y &\rightarrow (\rho \sin \phi - a \cos \phi) \sin \theta, \\
z &\rightarrow \rho \cos \theta,
\end{align*}
\]

where

\[
r = \sqrt{x^2 + y^2 + z^2} = \frac{\sqrt{\rho^4 + a^2 \rho^2 - a^2 z^2}}{\rho},
\]

(16)
to the tetrad (8) we obtain*

\[
\begin{align*}
e^{(0)}_0 &= 1 - \frac{(2M\rho - Q^2)\rho^2}{2\rho_1}, \\
e^{(0)}_\alpha &= \left\{ -n_\alpha - \frac{a}{\rho} \epsilon_{\alpha j 3} n^j \right\} \left( \frac{2M\rho - Q^2}{\rho_1(\rho^2 + a^2)} \right)^{\frac{1}{2}} = -b^{(l)}_0, \\
e^{(l)}_\beta &= \delta^l_\beta + \left\{ x^k x_{k, \beta} + \frac{2a}{\rho} \epsilon_{k 3(\beta x^l)} x^l + \frac{a^2}{\rho^2} \left[ \epsilon_{k 3} \epsilon_{m 3} x^k x^m + 2z\{ \rho x^l - \frac{a}{\rho} \epsilon_{k 3} x^k \delta^l_\beta \} \right] \right\} \frac{(2M\rho - Q^2)\rho^4}{2\rho_1(\rho^2 + a^2)^2},
\end{align*}
\]

(17)

where

\[
\rho_1 = \rho^4 + a^2 z^2,
\]

(18)

*We will denote the symmetric part by ( ), for example, \( A_{(\mu\nu)} = (1/2)(A_{\mu\nu} + A_{\nu\mu}) \) and the antisymmetric part by the square bracket [ ], \( A_{[\mu\nu]} = (1/2)(A_{\mu\nu} - A_{\nu\mu}) \).
and the $\epsilon_{\alpha\beta\gamma}$ are the three dimensional totally antisymmetric tensor with $\epsilon_{123} = 1$. Therefore, we are interested in this solution to calculate its associated energy.

4. Energy associated with the axially symmetric solution

Now we are going to calculate the energy content of the tetrad field (8) using (9). Before this let us give a brief review of the derivation of the gravitational energy-momentum.

Multiplication of the symmetric part of Eq. (7) by the appropriate inverse tetrad field yields it to have the form [17, 36]

$$\partial_{\nu} \left( -e \sum^{a=\lambda\nu} \right) = -\frac{ee^{a\mu}}{4} \left( 4\sum^{b=\lambda\nu} T_{b\nu\mu} - \delta^{a}_{\mu} \sum^{bc\nu} T_{bcd} \right) - 4\pi e^{a\mu} T_{\lambda\mu}. \quad (19)$$

By restricting the spacetime index $\lambda$ to assume only spatial values then Eq. (19) takes the form [17]

$$\partial_{0} \left( e \sum^{a\lambda j} \right) + \partial_{k} \left( e \sum^{a\lambda j} \right) = -\frac{ee^{a\mu}}{4} \left( 4\sum^{bcj} T_{bc\mu} - \delta^{j}_{\mu} \sum^{bcd} T_{bcd} \right) - 4\pi e^{a\mu} T_{\lambda j}. \quad (20)$$

Note that the last two indices of $\sum^{abc}$ and $T^{abc}$ are anti-symmetric. Taking the divergence of Eq. (20) with respect to $j$ yields

$$-\partial_{0} \partial_{j} \left( \frac{1}{4\pi} e \sum^{a\lambda j} \right) = -\frac{1}{16\pi} \partial_{j} \left[ ee^{a\mu} \left( 4\sum^{bcj} T_{bc\mu} - \delta^{j}_{\mu} \sum^{bcd} T_{bcd} \right) - 16\pi (ee^{a\mu} T_{\lambda j}) \right]. \quad (21)$$

In the Hamiltonian formulation of the TEGR [3],[37]~[39] the momentum canonically conjugated to the tetrad components $e_{aj}$ is given by

$$\Pi^{aj} = -\frac{1}{4\pi} e \sum^{a0j},$$

and that the gravitational energy-momentum $P^{a}$ contained within a volume $V$ of the three-dimensional spacelike hypersurface is defined by [17]

$$P^{a} = -\int_{V} d^{3}x \partial_{j} \Pi^{aj}. \quad (22)$$

If no condition is imposed on the tetrad field, $P^{a}$ transforms as a vector under the global $SO(3,1)$ group. It describes the gravitational energy-momentum with respect to observers adapted to $e^{a}_{\mu}$. Let us assume that the spacetime is asymptotically flat. The total gravitational energy-momentum is given by

$$P^{a} = -\oint_{S \rightarrow \infty} dS_{k} \Pi^{ak}. \quad (23)$$
The field quantities are evaluated on a surface $S$ in the limit $r \to \infty$.

Now we are going to apply Eq. (23) to the tetrad field (8) with Eq. (9) to calculate the energy content. We perform the calculations in the Boyer-Lindquist coordinate. The only required component of $\sum_{\mu\nu\lambda}$

$$
\sum_{001} = \frac{-1}{4(\rho^2 + a^2 \cos^2 \theta)} \left[ 4M \rho^6 + 6Ma^2 \rho^4 + 2Ma^4 \rho^2 + 2MQ^2 \rho^4 + 4MQ^2 a^2 \rho^2 \\
-2\rho^8 Q^2 - 4a^2 \rho^6 Q^2 - 2a^4 \rho Q^2 + 2Ma^2 \cos^2 \theta (\rho^4 + \rho^2 a^2 \cos^2 \theta + a^2 Q^2 \cos^2 \theta + a^4 \sin^2 \theta) \right]
$$

Further substituting (24) in (23) we obtain

$$
P^{(0)} = E = -\oint_{S \to \infty} dS_k \Pi^{(0)k} = -\frac{1}{4\pi} \oint_{S \to \infty} dS_k e^{(0)}_0 \sum_{00j} \approx \left( M - \frac{Q^2}{4\rho} \left[ 1 + \left\{ \frac{3a^2 + \rho^2}{a \rho} \right\} \tan^{-1} \left( \frac{a}{\rho} \right) \right] \right).
$$

As is clear from (25) that the energy content is shared by both the interior and exterior of the Kerr-Newman black hole. The total energy when $\rho \to \infty$ gives the ADM (Arnowitt-Deser-Misner) mass.

5. Main results and discussion

In this paper we have studied the coupled equations of the gravitational and electromagnetic fields in the TEGR, applying the most general tetrad (8) with sixteen unknown function of $\rho$ and $\phi$ to the field equations (7). Exact analytic solution is obtained (9). This solution (9) is a general solution from which we can generate the other solutions (Schwarzschild, Reissner-Nordström and Kerr spacetimes) by an appropriate choice of the arbitrary functions of the tetrad (8). The tetrad field and the electromagnetic vector potential are axially symmetric as is clear from Eq. (11). We then transform tetrad (8) with (9) into the Cartesian coordinate using the transformation given by (13).

The geometrical framework determined by the tetrad field and torsion has proven to be suitable to investigate the problem of defining the gravitational energy-momentum [17]. A consistent expression developed in the realm of the TEGR shares many features with the expected definition. In the context of the tetrad theories of gravity, asymptotically flat spacetime may be characterized by the asymptotic boundary condition

$$
e_{\alpha \mu} \approx \eta_{\alpha \mu} + \frac{1}{2} h_{\alpha \mu}(1/r),
$$

and by the condition $\partial_{\mu} e^\mu_{\alpha \nu} = O(1/r^2)$ in the asymptotic limit $r \to \infty$. An important property of the tetrad field that satisfy the above condition is that in the flat spacetime limit we have $e^\mu_{\alpha}(t, x, y, z) = \delta^\mu_{\alpha}$ and therefore $T^\alpha_{\mu \nu} = 0$. Hence for the flat spacetime we normally consider a set of tetrad field such that $T^\alpha_{\mu \nu} = 0$ in any coordinate system. [17, 3, 36]
Using the definition of the torsion tensor given by Eq. (5) and apply it to the tetrad field (8) using (9) we can show that the torsion of flat spacetime is vanishing identically. Therefore, we use the gravitational energy-momentum given by (23). The use of (23) is not restricted to Cartesian coordinate. Therefore, we apply (23) to the tetrad field (8) with (9) and obtain the energy content (25). As is clear from (25) that the energy content is shared by both the interior and exterior of the Kerr-Newman black hole. The asymptotic value of (25) up to \( O(a^4) \) is given by

\[
E \approx M - \frac{Q^2}{\rho} \left( \frac{1}{2} + \frac{a^2}{3\rho^2} + \frac{a^4}{60\rho^4} \right). 
\]

(26)

This result is consistent with that obtained before [40, 41] up to \( O(a^2) \). Switching off the rotation parameter, (i.e., \( a = 0 \)) the energy associated will be the same as that of Reissner-Nordström metric [42, 43]. Setting the charge parameter \( Q \) to be equal zero, i.e., in the case of a Kerr black hole, it is clear from expression (26) that there is no energy contained by the exterior of the Kerr black hole and hence the entire energy is confined to its interior only.
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