Variation of fundamental constants in space and time: theory and observations

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Abstract

Review of recent works devoted to the temporal and spatial variation of the fundamental constants and dependence of the fundamental constants on the gravitational potential (violation of local position invariance) is presented. We discuss the variation of the fine structure constant $\alpha = e^2/\hbar c$, strong interaction and fundamental masses (Higgs vacuum), e.g. the electron-to-proton mass ratio $\mu = m_e/M_p$ or $X_e = m_e/\Lambda_{QCD}$ and $X_q = m_q/\Lambda_{QCD}$. We also present new results from Big Bang nucleosynthesis and Oklo natural nuclear reactor data and propose new measurements of enhanced effects in atoms, nuclei and molecules, both in quasar and laboratory spectra.

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I. INTRODUCTION

Theories unifying gravity with other interactions suggest temporal and spatial variation of the fundamental “constants” in expanding Universe (see e.g. review [1]). The spatial variation can explain fine tuning of the fundamental constants which allows humans (and any life) to appear. We appeared in the area of the Universe where the values of the fundamental constants are consistent with our existence. The fundamental constants may be slightly different near massive bodies (see e.g. review [2]). There are some hints for the variation of different fundamental constants in quasar absorption spectra [3, 4, 5, 6, 7, 8] and Big Bang nucleosynthesis [9, 10] data. However, a majority of publications report limits on the variations (see e.g. recent reviews [11, 12]).

The hypothetical unification of all interactions implies that variations of different fundamental constants may be related [13, 14, 15, 16, 17]. We can only detect variation of dimensionless fundamental constants. We will discuss variation of the fine structure constant $\alpha$ and dimensionless ratios $X_e = m_e/\Lambda_{QCD}$ and $X_q = m_q/\Lambda_{QCD}$ where $m_e$ and $m_q$ are the electron and quark masses, and $\Lambda_{QCD}$ is the quantum chromodynamics (QCD) scale (defined as the position of the Landau pole in the logarithm for the running strong coupling constant, $\alpha_s(r) \sim 1/\ln(\Lambda_{QCD}r/\bar{\hbar}c)$). The proton mass $m_p$ is proportional to $\Lambda_{QCD}$, therefore the relative variation of $\mu = m_e/M_p$ is equal to the relative variation of $X_e = m_e/\Lambda_{QCD}$ (if we neglect a small contribution of quark masses ($m_q \sim 5\text{ MeV}$) to the proton mass, $m_p = 938\text{ MeV}$). In the Standard model electron and quark masses are proportional to the vacuum expectation value of the Higgs field.

A simple estimate of the relations between the variations of different fundamental constants may be obtained using the idea of Grand Unification. The strong ($i=3$), and electroweak ($i=1,2$) inverse coupling constants have the following dependence on the scale $\nu$ and normalization point $\nu_0$:

$$\alpha_i^{-1}(\nu) = \alpha_i^{-1}(\nu_0) + b_i\ln(\nu/\nu_0)$$

(1)

In the Standard Model, $2\pi b_i = 41/10, -19/6, -7$; the electromagnetic $\alpha^{-1} = (5/3)\alpha_1^{-1} + \alpha_2^{-1}$ and the strong $\alpha_s = \alpha_3$. In the Grand Unification Theories (GUT) all coupling constants are equal at the unification scale, $\alpha_i(\nu_0) \equiv \alpha_{GUT}$. If we assume that $\alpha_{GUT}$ varies, then Eq. (1) gives us the same shifts for all inverse couplings:

$$\delta\alpha_1^{-1} = \delta\alpha_2^{-1} = \delta\alpha_3^{-1} = \delta\alpha_{GUT}^{-1}.$$  

(2)
We see that the variation of the strong interaction constant \( \alpha_3(\nu) \) at low energy \( \nu \) is much larger than the variation of the electromagnetic constant \( \alpha \), since \( \delta \alpha_3/\alpha_3 = (\alpha_3/\alpha_1,2)\delta \alpha_1,2/\alpha_1,2 \) and \( \alpha_3 \gg \alpha_1,2 \).

The variation of \( m/\Lambda_{QCD} \) can be estimated from the definition of \( \Lambda_{QCD} \). The running of \( \alpha_s \) near the electroweak scale is given by

\[
\alpha_s(\nu)^{-1} \approx b_s \ln(\nu/\Lambda_{QCD})
\]

Let us take \( \nu = m_z \) where \( m_z \) is the Z-boson mass. The variation of eq. (3) and relations above give

\[
\frac{\delta(m_z/\Lambda_{QCD})}{(m_z/\Lambda_{QCD})} = \frac{1}{b_s \alpha_s(m_z) \alpha_s(m_z)} \sim C \frac{\delta \alpha(m_z)}{\alpha(m_z) \alpha(m_z)}
\]

The value of the constant \( C \) here depends on the model used. However, the enhancement \( 1/\alpha \sim 100 \) should make the factor \( C/\alpha \) large. Note that \( m_z \) (as well as \( m_e \) and \( m_q \)) is proportional to the Higgs vacuum expectation value.

If this estimate is correct, the variation in \( X_{e,q} = m_{e,q}/\Lambda_{QCD} \) or \( \mu = m_e/M_p \) may be easier to detect than the variation in \( \alpha \). The cosmological variation of \( m_q/\Lambda_{QCD} \) can be extracted from the big bang nucleosynthesis (BBN), quasar absorption spectra and Oklo natural nuclear reactor data [18, 19, 20]. For example, the factor of three disagreement between the calculations and measurements of the BBN abundance of \(^7\text{Li} \) may, in principle, be explained by the variation of \( m_q/\Lambda_{QCD} \) at the level of \( \sim 10^{-3} - 10^{-2} \) [9, 10] (see also recent work [21]). The claim of the variation of the fundamental constants based on the Oklo data in Ref. [22] is not confirmed by recent studies [23, 24, 25] which give a stringent limit on the possible variation of the resonance in \(^{150}\text{Sm} \) during the last two billion years. The search for the variation of \( m_e/\Lambda_{QCD} \) using the quasar absorption spectra gave a non-zero result in Ref. [8] but zero results in Refs. [26, 27, 28]. The present time variation of \( m_{e,q}/\Lambda_{QCD} \) can be extracted from comparison of different atomic [29], molecular or nuclear [31, 32] clocks. New enhanced effects have been proposed.

II. BIG BANG NUCLEOSYNTHESIS

The result of our work Ref. [9] suggested that a reduced deuteron binding energy of \( \Delta Q/Q = -0.019 \pm 0.005 \) would yield a better fit to observational data (the WMAP value
of barion-to-photon ratio $\eta$ and measured $^{2}$H, $^{4}$He, and $^{7}$Li abundances) for Big Bang Nucleosynthesis. Using our calculations we obtained in Ref. [9] an estimate of the strange quark mass variation.

Recently Dent, Stern, and Wetterich [21] calculated the sensitivity of BBN abundances for $^{2}$H, $^{4}$He and $^{7}$Li to the variation of binding energies of $^{2,3}$H, $^{3,4}$He, $^{6,7}$Li and $^{7}$Be in a linear approximation. In the works [10, 33, 34, 35] we calculated dependence of these binding energies on the light quark mass variation and estimated the sensitivity of BBN yields to variation of the quark mass. Then we used the observational data to obtain the following equations for $^{2}$H, $^{4}$He and $^{7}$Li [10]:

$$1 + 7.7x = \frac{2.8 \pm 0.4}{2.61 \pm 0.04} = 1.07 \pm 0.15 , \quad (5)$$

$$1 - 0.95x = \frac{0.249 \pm 0.009}{0.2478 \pm 0.0002} = 1.005 \pm 0.036 , \quad (6)$$

$$1 - 50x = \frac{1.5 \pm 0.5}{4.5 \pm 0.4} = 0.33 \pm 0.11 , \quad (7)$$

where $x = \delta X_q/X_q$. These equations yield 3 consistent values of $x$: $0.009 \pm 0.019$, $-0.005 \pm 0.038$ and $0.013 \pm 0.002$. The statistically weighted average of $\delta X_q/X_q = 0.013 \pm 0.002$ is dominated by the $^{7}$Li data. A more accurate calculation should take into account the effect of the $^{8}$Be binding energy variation (which is not calculated in Ref. [21]), the variation of the virtual $^{1}$S$_0$(np) level, and non-linear corrections in $x$ which are important for $^{7}$Li. Allowing for the theoretical uncertainties we should understand this BBN result as $\delta X_q/X_q = K \cdot (0.013 \pm 0.002)$ where $K \sim 1$, where the expected accuracy in $K$ is about a factor of 2. Note that here we neglected effects of the strange quark mass variation. A rough estimate of these effects on BBN due to the deuteron binding energy variation was made in Refs. [9, 20].

### III. OKLO NATURAL NUCLEAR REACTOR

The results from Oklo natural nuclear reactor are based on the measurement of the position of very low energy resonance ($E_r = 0.1$ eV) in neutron capture by $^{149}$Sm nucleus. The estimate of the shift of this resonance induced by the variation of $\alpha$ have been done long time ago in works [36, 37]. Recently we performed a rough estimate of the effect of the variation of $m_q/\Lambda_{QCD}$ [9, 18, 20]. The final result is

$$\delta E_r \approx 10^6 eV(\frac{\delta \alpha}{\alpha} - 10 \frac{\delta X_q}{X_q} + 100 \frac{\delta X_s}{X_s}) \quad (8)$$
where \( X_q = m_q/\Lambda_{QCD} \), \( X_s = m_s/\Lambda_{QCD}, m_q = (m_u + m_d)/2 \) and \( m_s \) is the strange quark mass. Refs. [23, 24, 25] found that \( |\delta E_r| < 0.1 \) eV. This gives us a limit

\[
|0.01 \frac{\delta \alpha}{\alpha} - 0.1 \frac{\delta X_q}{X_q} + \frac{\delta X_s}{X_s}| < 10^{-9}
\]

The contribution of the \( \alpha \) variation in this equation is very small and should be neglected since the accuracy of the calculation of the main term is low. Thus, the Oklo data cannot give any limit on the variation of \( \alpha \). Assuming linear time dependence during last 2 billion years we obtain an estimate \( |\dot{X}_s/X_s| < 10^{-18} \) yr\(^{-1}\).

IV. OPTICAL ATOMIC SPECTRA

A. Comparison of quasar absorption spectra with laboratory spectra

To perform measurements of \( \alpha \) variation by comparison of cosmic and laboratory optical spectra we developed a new approach [38, 39] which improves the sensitivity to a variation of \( \alpha \) by more than an order of magnitude. The relative value of any relativistic corrections to atomic transition frequencies is proportional to \( \alpha^2 \). These corrections can exceed the fine structure interval between the excited levels by an order of magnitude (for example, an s-wave electron does not have the spin-orbit splitting but it has the maximal relativistic correction to energy). The relativistic corrections vary very strongly from atom to atom and can have opposite signs in different transitions (for example, in s-p and d-p transitions). Thus, any variation of \( \alpha \) could be revealed by comparing different transitions in different atoms in cosmic and laboratory spectra.

This method provides an order of magnitude precision gain compared to measurements of the fine structure interval. Relativistic many-body calculations are used to reveal the dependence of atomic frequencies on \( \alpha \) for a range of atomic species observed in quasar absorption spectra [38, 39, 40, 42, 43, 44, 45] (a 2004 summary may be found in Ref. [46]).

It is convenient to present results for the transition frequencies as functions of \( \alpha^2 \) in the form

\[
\omega = \omega_0 + qx,
\]

where \( x = (\frac{\alpha}{\alpha_0})^2 - 1 \approx \frac{2\delta \alpha}{\alpha} \) and \( \omega_0 \) is a laboratory frequency of a particular transition. We stress that the second term contributes only if \( \alpha \) deviates from the laboratory value \( \alpha_0 \). We performed accurate many-body calculations of the coefficients \( q \) for all transitions of
astrophysical interest (strong E1 transitions from the ground state) in Mg, Mg II, Fe II, Fe I, Cr II, Ni II, Al II, Al III, Si II, Zn II, Mn II (and many other atoms and ions which have not been used in the quasar measurements yet because of the absence of accurate laboratory wavelenths - see [46]). It is very important that this set of transitions contains three large classes: positive shifters (large positive coefficients $q > 1000 \text{ cm}^{-1}$), negative shifters (large negative coefficients $q < -1000 \text{ cm}^{-1}$) and anchor lines with small values of $q$. This gives us an excellent control of systematic errors since systematic effects do not “know” about sign and magnitude of $q$. Comparison of cosmic frequencies $\omega$ and laboratory frequencies $\omega_0$ allows us to measure $\frac{\delta \alpha}{\alpha}$.

Three independent samples of data containing 143 absorption systems spread over red shift range $0.2 < z < 4.2$. The fit of the data gives $\frac{\delta \alpha}{\alpha} = (-0.543 \pm 0.116) \times 10^{-5}$. If one assumes the linear dependence of $\alpha$ on time, the fit of the data gives $d \ln \alpha/dt = (6.40 \pm 1.35) \times 10^{-16}$ per year (over time interval about 12 billion years). A very extensive search for possible systematic errors has shown that known systematic effects can not explain the result (It is still not completely excluded that the effect may be imitated by a large change of abundances of isotopes during last 10 billion years. We have checked that different isotopic abundances for any single element can not imitate the observed effect. It may be an improbable “conspiracy” of several elements).

Recently our method and calculations [38, 39, 40, 42, 43, 44] were used by two other groups [47, 48, 49]. However, they have not detected any variation of $\alpha$. Recently the results of [47] were questioned in Refs. [6, 7]. Re-analysis of Ref. [47] data revealed flawed parameter estimation methods. The authors of [6, 7] claim that the same spectral data fitted more accurately give $\frac{\delta \alpha}{\alpha} = (-0.64 \pm 0.36) \times 10^{-5}$ (instead of $\frac{\delta \alpha}{\alpha} = (-0.06 \pm 0.06) \times 10^{-5}$ in Ref. [47]). However, even this revised result may require further revision.

Note that the results of [3, 4, 5] are based on the data from the Keck telescope which is located in the Northern hemisphere (Hawaii). The results of [6, 7, 47, 48, 49] are based on the data from the different telescope (VLT) located in the Southern hemisphere (Chile). Therefore, some difference in the results may appear due to the spatial variation of $\alpha$.

Using opportunity I would like to ask for new, more accurate laboratory measurements of UV transition frequencies which have been observed in the quasar absorption spectra. The “shopping list” is presented in [46]. We also need the laboratory measurements of isotopic shifts - see [46]. We have performed very complicated calculations of these isotopic shifts.
However, the accuracy of these calculations in atoms and ions with open d-shell (like Fe II, Ni II, Cr II, Mn II, Ti II) may be very low. The measurements for at least few lines are needed to test these calculations. These measurements would be very important for a study of evolution of isotope abundances in the Universe, to exclude the systematic effects in the search for $\alpha$ variation and to test models of nuclear reactions in stars and supernovae.

A comparison of the hyperfine transition in atomic hydrogen with optical transitions in ions, was done in Refs. [26, 27]. This method allows one to study time-variation of the parameter $F = \alpha^2 g_p \mu$, where $g_p$ is proton $g$-factor. Analysis of 9 quasar spectra with redshifts $0.23 \leq z \leq 2.35$ gave

$$\frac{\delta F}{F} = (6.3 \pm 9.9) \times 10^{-6},$$

$$\frac{\dot{F}}{F} = (-6 \pm 12) \times 10^{-16} \text{ yr}^{-1}.$$  

B. Optical atomic clocks

Optical clocks also include transitions which have positive, negative or small contributions of the relativistic corrections to frequencies. We used the same methods of the relativistic many-body calculations to calculate the dependence on $\alpha$ [39, 40, 41, 55, 56, 57]. A 2004 summary of the results for the coefficients $q$ is presented in [58]. The coefficients $q$ for optical clock transitions may be substantially larger than in cosmic transitions since the clock transitions are often in heavy atoms (Hg II, Yb II, Yb III, etc.) while cosmic spectra contain mostly light atoms lines ($Z < 33$). The relativistic effects are proportional to $Z^2 \alpha^2$.

V. ENHANCED EFFECTS OF $\alpha$ VARIATION IN ATOMS

An enhancement of the relative effect of $\alpha$ variation can be obtained in transition between the almost degenerate levels in Dy atom [39, 40, 57]. These levels move in opposite directions if $\alpha$ varies. The relative variation may be presented as $\delta \omega / \omega = K \delta \alpha / \alpha$ where the coefficient $K$ exceeds $10^8$. Specific values of $K = 2q/\omega$ are different for different hyperfine components and isotopes which have different $\omega$; $q = 30,000$ cm$^{-1}$, $\omega \sim 10^{-4}$ cm$^{-1}$. An experiment is currently underway to place limits on $\alpha$ variation using this transition [59, 60]. The current
limit is $\dot{\alpha}/\alpha = (-2.7 \pm 2.6) \times 10^{-15} \text{ yr}^{-1}$. Unfortunately, one of the levels has quite a large linewidth and this limits the accuracy.

Several enhanced effects of $\alpha$ variation in atoms have been calculated in [61, 62].

VI. ENHANCED EFFECT OF VARIATION OF $\alpha$ AND STRONG INTERACTION IN UV TRANSITION OF $^{229}$Th NUCLEUS (NUCLEAR CLOCK)

A very narrow level $(7.6 \pm 0.5)$ eV above the ground state exists in $^{229}$Th nucleus [63]. The position of this level was determined from the energy differences of many high-energy $\gamma$-transitions to the ground and excited states. The subtraction produces the large uncertainty in the position of the 7.6 eV excited state. The width of this level is estimated to be about $10^{-4}$ Hz [64]. This would explain why it is so hard to find the direct radiation in this very weak transition. However, the search for the direct radiation continues [65].

The $^{229}$Th transition is very narrow and can be investigated with laser spectroscopy. This makes $^{229}$Th a possible reference for an optical clock of very high accuracy, and opens a new possibility for a laboratory search for the variation of the fundamental constants [32].

As it is shown in Ref. [31] there is an additional very important advantage. According to Ref. [31] the relative effects of variation of $\alpha$ and $m_q/\Lambda_{QCD}$ are enhanced by 5 orders of magnitude. This estimate has been confirmed by the recent calculation [66] and preliminary results of our new calculations. The accurate results of the calculations will be published soon. A rough estimate for the relative variation of the $^{229}$Th transition frequency is

$$\frac{\delta \omega}{\omega} \approx 10^5 (0.1 \frac{\delta \alpha}{\alpha} + \frac{\delta X_q}{X_q})$$

(13)

where $X_q = m_q/\Lambda_{QCD}$. Therefore, the Th experiment would have the potential of improving the sensitivity to temporal variation of the fundamental constants by many orders of magnitude. Indeed, we obtain the following energy shift in 7.6 eV $^{229}$Th transition:

$$\delta \omega \approx \frac{\delta X_q}{X_q} MeV$$

(14)

This corresponds to the frequency shift $\delta \nu \approx 3 \cdot 10^{20} \delta X_q/X_q$ Hz. The width of this transition is $10^{-4}$ Hz so one may hope to get the sensitivity to the variation of $X_q$ about $10^{-24}$ per year. This is $10^{10}$ times better than the current atomic clock limit on the variation of $X_q$, $\sim 10^{-14}$ per year.
Note that there are other narrow low-energy levels in nuclei, e.g. 76 eV level in $^{235}U$ with the 26.6 minutes lifetime (see e.g. [32]). One may expect a similar enhancement there. Unfortunately, this level can not be reached with usual lasers. In principle, it may be investigated using a free-electron laser or synchrotron radiation. However, the accuracy of the frequency measurements is much lower in this case.

VII. ATOMIC MICROWAVE CLOCKS

Hyperfine microwave transitions may be used to search for $\alpha$-variation [67]. Karshenboim [68] has pointed out that measurements of ratios of hyperfine structure intervals in different atoms are also sensitive to variations in nuclear magnetic moments. However, the magnetic moments are not the fundamental parameters and can not be directly compared with any theory of the variations. Atomic and nuclear calculations are needed for the interpretation of the measurements. We have performed both atomic calculations of $\alpha$ dependence [39, 40, 41, 55, 56, 57, 58] and nuclear calculations of $X_q = m_q/\Lambda_{QCD}$ dependence [29] (see also [35]) for all microwave transitions of current experimental interest including hyperfine transitions in $^{133}\text{Cs}$, $^{87}\text{Rb}$, $^{171}\text{Yb}^+$, $^{199}\text{Hg}^+$, $^{111}\text{Cd}$, $^{129}\text{Xe}$, $^{139}\text{La}$, $^{1}\text{H}$, $^{2}\text{H}$ and $^{3}\text{He}$. The results for the dependence of the transition frequencies on variation of $\alpha$, $X_e = m_e/\Lambda_{QCD}$ and $X_q = m_q/\Lambda_{QCD}$ are presented in Ref. [29] (see the final results in the Table IV of Ref. [29]). Also, one can find there experimental limits on these variations which follow from the recent measurements. The accuracy is approaching $10^{-15}$ per year. This may be compared to the sensitivity $\sim 10^{-5} - 10^{-6}$ per $10^{10}$ years obtained using the quasar absorption spectra.

According to Ref. [29] the frequency ratio $Y$ of the 282-nm $^{199}\text{Hg}^+$ optical clock transition to the ground state hyperfine transition in $^{133}\text{Cs}$ has the following dependence on the fundamental constants:

$$\dot{Y}/Y = -6\dot{\alpha}/\alpha - \dot{\mu}/\mu - 0.01\dot{X}_q/X_q$$  \hspace{1cm} (15)

In the work [30] this ratio has been measured: $\dot{Y}/Y = (0.37 \pm 0.39) \times 10^{-15}$ yr$^{-1}$. Assuming linear time dependence we obtained the quasar result [28] $\dot{\mu}/\mu = \dot{X}_e/X_e = (1 \pm 3) \times 10^{-16}$ yr$^{-1}$. A combination of this result and the atomic clock result [30] for $Y$ gives the best limit on the variation of $\alpha$: $\dot{\alpha}/\alpha = (-0.8 \pm 0.8) \times 10^{-16}$ yr$^{-1}$. Here we neglected the small ($\sim 1\%$) contribution of $X_q$. 

VIII. ENHANCEMENT OF VARIATION OF FUNDAMENTAL CONSTANTS IN ULTRACOLD ATOM AND MOLECULE SYSTEMS NEAR FESHBACH RESONANCES

Scattering length $A$, which can be measured in Bose-Einstein condensate and Feshbach molecule experiments, is extremely sensitive to the variation of the electron-to-proton mass ratio $\mu = m_e/m_p$ or $X_e = m_e/A_{QCD}$ [69].

$$\frac{\delta A}{A} = K \frac{\delta \mu}{\mu} = K \frac{\delta X_e}{X_e},$$

where $K$ is the enhancement factor. For example, for Cs-Cs collisions we obtained $K \sim 400$. With the Feshbach resonance, however, one is given the flexibility to adjust position of the resonance using external fields. Near a narrow magnetic or an optical Feshbach resonance the enhancement factor $K$ may be increased by many orders of magnitude.

IX. MOLECULAR SPECTRA

Recently we wrote a review about search for the variation of the fundamental constants in quasar and laboratory molecular spectra [70]. Below I present several examples related to our works.

A. Comparison of hydrogen hyperfine and molecular rotational quasar spectra

The frequency of the hydrogenic hyperfine line is proportional to $\alpha^2 \mu g_p$ atomic units, molecular rotational frequencies are proportional to $\mu$ atomic units. The comparison places limit on the variation of the parameter $F = \alpha^2 g_p$ [71]. Recently similar analysis was repeated by Murphy et al [72] using more accurate data for the same object at $z = 0.247$ and for a more distant object at $z = 0.6847$, and the following limits were obtained:

$$\frac{\delta F}{F} = (-2.0 \pm 4.4) \times 10^{-6}$$

(17)

$$\frac{\delta F}{F} = (-1.6 \pm 5.4) \times 10^{-6}$$

(18)

The object at $z = 0.6847$ is associated with the gravitational lens toward quasar B0218+357 and corresponds to the backward time $\sim 6.5$ Gyr.
B. Enhancement of variation of $\mu$ in inversion spectrum of ammonia and limit from quasar spectra

Few years ago van Veldhoven et al suggested to use decelerated molecular beam of ND$_3$ to search for the variation of $\mu$ in laboratory experiments [73]. Ammonia molecule has a pyramidal shape and the inversion frequency depends on the exponentially small tunneling of three hydrogens (or deuteriums) through the potential barrier. Because of that, it is very sensitive to any changes of the parameters of the system, particularly to the reduced mass for this vibrational mode. This fact was used in [28] to place the best limit on the cosmological variation of $\mu$.

The inversion vibrational mode of ammonia is described by a double well potential with first two vibrational levels lying below the barrier. Because of the tunneling, these two levels are split in inversion doublets. The lower doublet corresponds to the wavelength $\lambda \approx 1.25$ cm and is used in ammonia masers. Molecular rotation leads to the centrifugal distortion of the potential curve. Because of that, the inversion splitting depends on the rotational angular momentum $J$ and its projection on the molecular symmetry axis $K$:

$$\omega_{\text{inv}}(J, K) = \omega_{\text{inv}}^0 - c_1 \left[ J(J + 1) - K^2 \right] + c_2 K^2 + \cdots,$$

(19)

where we omitted terms with higher powers of $J$ and $K$. Numerically, $\omega_{\text{inv}}^0 \approx 23.787$ GHz, $c_1 \approx 151.3$ MHz, and $c_2 \approx 59.7$ MHz.

In addition to the rotational structure (19) the inversion spectrum includes much smaller hyperfine structure. For the main nitrogen isotope $^{14}$N, the hyperfine structure is dominated by the electric quadrupole interaction ($\sim 1$ MHz). Because of the dipole selection rule $\Delta K = 0$ the levels with $J = K$ are metastable. In astrophysics the lines with $J = K$ are also narrower and stronger than others, but the hyperfine structure for spectra with high redshifts is still not resolved. We obtained the following results for NH$_3$ [28] (in atomic units):

$$\frac{\delta \omega_{\text{inv}}^0}{\omega_{\text{inv}}^0} \approx 4.46 \frac{\delta \mu}{\mu}. \quad (20)$$

$$\frac{\delta c_{1,2}}{c_{1,2}} = 5.1 \frac{\delta \mu}{\mu}. \quad (21)$$

For ND$_3$ the inversion frequency is 15 times smaller and this leads to a higher relative
sensitivity of the inversion frequency to \( \mu \):

\[
\frac{\delta \omega_{\text{inv}}^0}{\omega_{\text{inv}}^0} \approx 5.7 \frac{\delta \mu}{\mu} .
\]  

(22)

\[
\frac{\delta c_{1,2}}{c_{1,2}} = 6.2 \frac{\delta \mu}{\mu} .
\]  

(23)

We see that the inversion frequency \( \omega_{\text{inv}}^0 \) and the rotational intervals \( \omega_{\text{inv}}(J_1, K_1) - \omega_{\text{inv}}(J_2, K_2) \) have different dependencies on the constant \( \mu \). In principle, this allows one to study time-variation of \( \mu \) by comparing different intervals in the inversion spectrum of ammonia. For example, if we compare the rotational interval to the inversion frequency, then Eqs. (20) and (21) give:

\[
\frac{\delta \left\{ \frac{\omega_{\text{inv}}(J_1, K_1) - \omega_{\text{inv}}(J_2, K_2)}{\omega_{\text{inv}}^0} \right\}}{\omega_{\text{inv}}(J_1, K_1) - \omega_{\text{inv}}(J_2, K_2)} \approx 0.6 \frac{\delta \mu}{\mu} .
\]  

(24)

The relative effects are substantially larger if we compare the inversion transitions with the transitions between the quadrupole and magnetic hyperfine components. However, in practice this method will not work because of the smallness of the hyperfine structure compared to typical line widths in astrophysics.

We compared the inversion spectrum of \( \text{NH}_3 \) with rotational spectra of other molecules, where

\[
\frac{\delta \omega_{\text{rot}}}{\omega_{\text{rot}}} = \frac{\delta \mu}{\mu} .
\]  

(25)

High precision data on the redshifts of \( \text{NH}_3 \) inversion lines exist for already mentioned object B0218+357 at \( z \approx 0.6847 \) [74]. Comparing them with the redshifts of rotational lines of CO, HCO\(^+\), and HCN molecules from Ref. [75] one can get the following limit:

\[
\frac{\delta \mu}{\mu} = \frac{\delta X_e}{X_e} = (-0.6 \pm 1.9) \times 10^{-6}.
\]  

(26)

Taking into account that the redshift \( z \approx 0.68 \) for the object B0218+357 corresponds to the backward time about 6.5 Gyr and assuming linear time dependence, this limit translates into the most stringent present limit for the variation rate \( \dot{\mu}/\mu \) and \( X_e \) [28]:

\[
\dot{\mu}/\mu = \dot{X}_e/X_e = (1 \pm 3) \times 10^{-16} \text{ yr}^{-1}.
\]  

(27)

A combination of this result and the atomic clock results [29, 30] gives the best limit on variation of \( \alpha \):

\[
\dot{\alpha}/\alpha = (-0.8 \pm 0.8) \times 10^{-16} \text{ yr}^{-1}.
\]  

(28)
X. PROPOSALS OF ENHANCED EFFECTS IN DIATOMIC MOLECULES

In transitions between very close narrow levels of different nature in diatomic molecules the relative effects of the variation may be enhanced by several orders of magnitude. Such levels may occur due to cancelation between either hyperfine and rotational structures \[76\], or between the fine and vibrational structures of the electronic ground state \[77\]. The intervals between the levels are conveniently located in microwave frequency range and the level widths are very small, typically \(\sim 10^{-2}\) Hz.

A. Molecules with cancelation between hyperfine structure and rotational intervals

Consider diatomic molecules with unpaired electron and ground state \(^2\Sigma\). It can be, for example, LaS, LaO, LuS, LuO, YbF, etc. \[78\]. Hyperfine interval \(\Delta_{\text{hfs}}\) is proportional to \(\alpha^2 Z F_{\text{rel}}(\alpha Z) \mu g_{\text{nuc}}\), where \(F_{\text{rel}}\) is additional relativistic (Casimir) factor. Rotational interval \(\Delta_{\text{rot}} \sim \mu\) is roughly independent on \(\alpha\). If we find molecule with \(\Delta_{\text{hfs}} \approx \Delta_{\text{rot}}\) the splitting \(\omega\) between hyperfine and rotational levels will depend on the following combination

\[
\omega \sim \left[ \alpha^2 F_{\text{rel}}(\alpha Z) g_{\text{nuc}} - \text{const} \right].
\]

Relative variation is then given by

\[
\frac{\delta \omega}{\omega} \approx \frac{\Delta_{\text{hfs}}}{\omega} \left[ (2 + K) \frac{\delta \alpha}{\alpha} + \delta g_{\text{nuc}} \frac{g_{\text{nuc}}}{\mu} \right],
\]

where factor \(K\) comes from variation of \(F_{\text{rel}}(\alpha Z)\), and for \(Z \sim 50\), \(K \approx 1\). Using data from \[78\] one can find that \(\omega = (0.002 \pm 0.01)\) cm\(^{-1}\) for \(^{\text{139}}\text{La}^{32}\text{S}\) \[76\]. Note that for \(\omega = 0.002\) cm\(^{-1}\) the relative frequency shift is:

\[
\frac{\delta \omega}{\omega} \approx 600 \frac{\delta \alpha}{\alpha}.
\]

B. Molecules with cancelation between fine structure and vibrational intervals

The fine structure interval \(\omega_f\) rapidly grows with nuclear charge \(Z\):

\[
\omega_f \sim Z^2 \alpha^2,
\]
The vibration energy quantum decreases with the atomic mass:

\[ \omega_{\text{vib}} \sim M_{r}^{-1/2} \mu_{\text{vib}}^{1/2}, \]  

where the reduced mass for the molecular vibration is \( M_{r}m_{p} \). Therefore, we obtain equation \( Z = Z(M_{r}, v) \) for the lines on the plane \( Z, M_{r} \), where we can expect approximate cancelation between the fine structure and vibrational intervals:

\[ \omega = \omega_{f} - v \omega_{\text{vib}} \approx 0, \quad v = 1, 2, ... \]

Using Eqs. (32-34) it is easy to find dependence of the transition frequency on the fundamental constants:

\[ \frac{\delta \omega}{\omega} = \frac{1}{\omega} \left( 2 \omega_{f} \frac{\delta \alpha}{\alpha} + \frac{v}{2} \omega_{\text{vib}} \frac{\delta \mu}{\mu} \right) \approx K \left( 2 \frac{\delta \alpha}{\alpha} + \frac{1}{2} \frac{\delta \mu}{\mu} \right), \]  

where the enhancement factor \( K = \frac{\omega_{f}}{\omega} \) determines the relative frequency shift for the given change of fundamental constants. Large values of factor \( K \) hint at potentially favorable cases for making experiment, because it is usually preferable to have larger relative shifts. However, there is no strict rule that larger \( K \) is always better. In some cases, such as very close levels, this factor may become irrelevant. Thus, it is also important to consider the absolute values of the shifts and compare them to the linewidths of the corresponding transitions.

Because the number of molecules is finite we can not have \( \omega = 0 \) exactly. However, a large number of molecules have \( \omega/\omega_{f} \ll 1 \) and \( |K| \gg 1 \). Moreover, an additional “fine tuning” may be achieved by selection of isotopes and rotational, \( \Omega \)-doublet, and hyperfine components. Therefore, we have two large manifolds, the first one is build on the electron fine structure excited state and the second one is build on the vibrational excited state. If these manifolds overlap one may select two or more transitions with different signs of \( \omega \). In this case expected sign of the \( |\omega| \)-variation must be different (since the variation \( \delta \omega \) has the same sign) and one can eliminate some systematic effects. Such control of systematic effects was used in [59, 60] for transitions between close levels in two dysprosium isotopes. The sign of energy difference between two levels belonging to different electron configurations was different in \( ^{163}\text{Dy} \) and \( ^{162}\text{Dy} \).

Among the interesting molecules where the ground state is split in two fine structure levels and (34) is approximately fulfilled, there are \( \text{Cl}_{2}^{+} \) (enhancement \( K = 1600 \), SiBr
(\(K = 360\)), CuS (\(K = 24\)) and IrC (\(K = 160\)). The list of molecules is not complete because of the lack of data in [78]. The molecules Cl₂⁺ and SiBr are particularly interesting. For both of them the frequency \(\omega\) defined by (34) is of the order of 1 cm\(^{-1}\) and comparable to the rotational constant \(B\). That means that \(\omega\) can be reduced further by the proper choice of isotopes, rotational quantum number \(J\) and hyperfine components, so we can expect \(K \sim 10^3 - 10^5\). New dedicated measurements are needed to determined exact values of the transition frequencies and find the best transitions. However, it is easy to find necessary accuracy of the frequency shift measurements. According to (35) the expected frequency shift is

\[
\delta \omega = 2 \omega_f \left( \frac{\delta \alpha}{\alpha} + \frac{1}{4} \frac{\delta \mu}{\mu} \right) \tag{36}
\]

Assuming \(\delta \alpha/\alpha \sim 10^{-15}\) and \(\omega_f \sim 500\) cm\(^{-1}\), we obtain \(\delta \omega \sim 10^{-12}\) cm\(^{-1}\) \(\sim 3 \times 10^{-2}\) Hz (in order to obtain similar sensitivity comparing hyperfine transition frequencies for Cs and Rb one has to measure the shift \(\sim 10^{-5}\) Hz). This shift is larger than the natural width \(\sim 10^{-2}\) Hz.

**C. Molecular ion HfF⁺**

The ion HfF⁺ and other similar ions are considered by Cornell’s group in JILA for the experiment to search for the electric dipole moment (EDM) of the electron. Recent calculation by [79] suggests that the ground state of this ion is \(^1\Sigma^+\) and the first excited state \(^3\Delta_1\) lies only 1633 cm\(^{-1}\) higher. Calculated vibrational frequencies for these two states are 790 and 746 cm\(^{-1}\) respectively. For these parameters the vibrational level \(v = 3\) of the ground state is only 10 cm\(^{-1}\) apart from the \(v = 1\) level of the state \(^3\Delta_1\). Thus, instead of (34) we now have:

\[
\omega = \omega_{el} + \frac{3}{2} \omega_{vib}^{(1)} - \frac{7}{2} \omega_{vib}^{(0)} \approx 0 ,
\]

where superscripts 0 and 1 correspond to the ground and excited electronic states. Electronic transition \(\omega_{el}\) is not a fine structure transition and (32) is not applicable. Instead, we can write:

\[
\omega_{el} = \omega_{el,0} + qx , \quad x = \alpha^2/\alpha_0^2 - 1 .
\]

Our estimate is [77]

\[
\frac{\delta \omega}{\omega} \approx \left( \frac{2q \delta \alpha}{\omega \alpha} + \frac{\omega_{el} \delta \mu}{2 \omega \mu} \right) \approx \left( 2000 \frac{\delta \alpha}{\alpha} + 80 \frac{\delta \mu}{\mu} \right) ,
\]

\[15\]
\[ \delta \omega \approx 20000 \text{ cm}^{-1}(\delta \alpha/\alpha + 0.04\delta \mu/\mu). \] (40)

Assuming \( \delta \alpha/\alpha \sim 10^{-15} \) we obtain \( \delta \omega \sim 0.6 \) Hz. The natural width is about 2 Hz.

We also present the result for transition between close levels in \( \text{Cs}_2 \) molecule suggested in [80, 81]. Our estimate is [70]:

\[ \delta \omega \approx (-240 \frac{\delta \alpha}{\alpha} - 1600 \frac{\delta \mu}{\mu}) \text{cm}^{-1}, \] (41)

XI. CHANGING PHYSICS NEAR MASSIVE BODIES

In this section I follow Ref. [82].

The reason gravity is so important at large scales is that its effect is additive. The same should be true for massless (or very light) scalars: its effect near large body is proportional to the number of particles in it.

For not-too-relativistic objects, like the usual stars or planets, both their total mass \( M \) and the total scalar charge \( Q \) are simply proportional to the number of nucleons in them, and thus the scalar field is simply proportional to the gravitational potential

\[ \phi - \phi_0 = \kappa (GM/rc^2). \] (42)

Therefore, we expect that the fundamental constants would also depend on the position via the gravitational potential at the measurement point.

Gravitational potential on Earth is changing due to ellipticity of its orbit: the corresponding variation of the Sun gravitational potential is \( \delta(GM/rc^2) = 3.3 \cdot 10^{-10} \). The accuracy of atomic clocks in laboratory conditions is about \( 10^{-16} \). As an example we consider recent work [30] who obtained the following value for the half-year variation of the frequency ratio of two atomic clocks: (i) optical transitions in mercury ions \( ^{199}\text{Hg}^+ \) and (ii) hyperfine splitting in \( ^{133}\text{Cs} \) (the frequency standard). The limit obtained is

\[ \delta \ln \left( \frac{\omega_{\text{Hg}}}{\omega_{\text{Cs}}} \right) = (0.7 \pm 1.2) \cdot 10^{-15} \] (43)

For \( \text{Cs}/\text{Hg} \) frequency ratio of these clocks the dependence on the fundamental constants was evaluated in [29] with the result

\[ \delta \ln \left( \frac{\omega_{\text{Hg}}}{\omega_{\text{Cs}}} \right) = -6 \frac{\delta \alpha}{\alpha} - 0.01 \frac{\delta (m_q/\Lambda_{\text{QCD}})}{(m_q/\Lambda_{\text{QCD}})} - \frac{\delta (m_e/M_p)}{(m_e/M_p)} \] (44)

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Another work \cite{83} compare $H$ and $^{133}Cs$ hyperfine transitions. The amplitude of the half-year variation found were
\[
|\delta \ln(\omega_H/\omega_{Cs})| < 7 \cdot 10^{-15} \tag{45}
\]
The sensitivity \cite{29}
\[
\delta \ln(\frac{\omega_H}{\omega_{Cs}}) = -0.83 \frac{\delta \alpha}{\alpha} - 0.11 \frac{\delta (m_q/\Lambda_{QCD})}{(m_q/\Lambda_{QCD})} \tag{46}
\]
There is no sensitivity to $m_e/M_p$ because they are both hyperfine transitions.

As motivated above, we assume that scalar and gravitational potentials are proportional to each other, and thus introduce parameters $k_i$ as follows
\[
\frac{\delta \alpha}{\alpha} = k_\alpha \delta \left(\frac{GM}{rc^2}\right) \tag{47}
\]
\[
\frac{\delta (m_q/\Lambda_{QCD})}{(m_q/\Lambda_{QCD})} = k_q \delta \left(\frac{GM}{rc^2}\right) \tag{48}
\]
\[
\frac{\delta (m_e/\Lambda_{QCD})}{(m_e/\Lambda_{QCD})} = \frac{\delta (m_e/M_p)}{(m_e/M_p)} = k_e \delta \left(\frac{GM}{rc^2}\right) \tag{49}
\]
where in the r.h.s. stands half-year variation of Sun’s gravitational potential on Earth.

In such terms, the results of Cs/Hg frequency ratio measurement \cite{30} can be rewritten as
\[
k_\alpha + 0.17k_e = (-3.5 \pm 6) \cdot 10^{-7} \tag{50}
\]
The results of Cs/H frequency ratio measurement \cite{83} can be presented as
\[
|k_\alpha + 0.13k_q| < 2.5 \cdot 10^{-5} \tag{51}
\]
Finally, the result of recent measurement \cite{84} of Cs/H frequency ratio can be presented as
\[
k_\alpha + 0.13k_q = (-1 \pm 17) \cdot 10^{-7} \tag{52}
\]
The sensitivity coefficients for other clocks have been discussed above.

Two new results have been obtained recently. From transition between close levels in Dy we obtained \cite{85}
\[
k_\alpha = (-8.7 \pm 6.6) \cdot 10^{-6} \tag{53}
\]
From optical Sr/hyperfine Cs comparison we obtained \cite{86}
\[
k_\alpha + 0.36k_e = (1.8 \pm 3.2) \cdot 10^{-6} \tag{54}
\]
Combination of the data gives

\[ k_\alpha = (-2.3 \pm 3.1) \cdot 10^{-6} \] (55)

\[ k_e = (1.1 \pm 1.7) \cdot 10^{-5} \] (56)

\[ k_\alpha = (1.7 \pm 2.7) \cdot 10^{-5} \] (57)

XII. ACKNOWLEDGMENTS

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