Do Quark Mass Effects Survive in the High-$Q^2$ Limit of DIS?

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Abstract
Quark mass effects are analyzed at high $Q^2$ in the current fragmentation region of DIS. It is found that the linear combination $F_2 - 2.75F^c_2$ scales at large $Q^2$ and small $x$. We obtained a lower bound for the ratio $F^c_2/F_2$ which lies very close to the data from HERA.

Keywords
Heavy Quark Production – Coefficient Functions – Specific Scaling
1 Introduction

The data on open-charm production in Deep Inelastic Scattering (DIS) from HERA collider \[1, 2\] show that its contribution, $F_{c^2}$, to the structure function runs up to 40% at measured values of $x$ and $Q^2$ and increases faster than $F_2$ when $x$ becomes smaller. The contribution of b-quarks, $F_{b^2}$, to the total structure function is of the order 2-3%, as one can see from recent measurements of open-beauty production \[3\].

One often believes that, with increasing energy of colliding particles, $W$, and momentum transfer squared $Q^2$, mass effects become insignificant. However, arguments were given in \[4, 5\] that the difference between the DIS structure functions with open heavy quark production in the current fragmentation region and structure functions of the process without heavy flavours scales for large $Q^2$, i.e. it depends on the Bjorken variable $x$ and heavy quark mass $m_Q$ only. This result gives a possibility to obtain model independent (i.e. independent on the concrete choice of the gluon distribution inside the nucleon) lower bound for $F_{c^2}$, which is in an agreement with the experimental data on $F_{c^2}$ \[4, 5\].

In the present work we analyse the problem of mass scale influence on the behaviour of physical quantities in the framework of the operator product expansion \[6\].

It is convenient to introduce the quantity

$$F_{Q^2} = F_{Q^2}/e_Q^2$$

instead of $F_{Q^2}^Q$ ($Q = c, b$). Analogously for the light quarks $q = u, d, s$ (that we treat massless) we define

$$F_{q^2} = F_{q^2}/e_q^2,$$

where $e_{Q(q)}$ are quark electric charges.

The operator product expansion gives the following expression for the $F_{Q^2}^Q$:

$$\frac{1}{x} F_{Q^2}^Q(x, Q^2, m_Q^2) = C_g \left( \frac{Q^2}{\mu^2}, \frac{m_Q^2}{\mu^2}, \alpha_s(\mu^2) \right) \otimes f_g(\mu^2)[x] + C_Q \left( \frac{Q^2}{\mu^2}, \frac{m_Q^2}{\mu^2}, \alpha_s(\mu^2) \right) \otimes f_Q(\mu^2)[x] + C_q \left( \frac{Q^2}{\mu^2}, \frac{m_Q^2}{\mu^2}, \alpha_s(\mu^2) \right) \otimes f_q(\mu^2)[x].$$

In Eq. (3) the quantities $C_i$ are coefficient functions, $f_i$ are matrix elements of corresponding composite operators, which can be identified with distributions of quarks and gluons inside the nucleon, and $\mu$ is a renormalization scale of the composite operators. Symbol $\otimes$ means convolution in the variable $x$,

$$a \otimes b[x] = \int_x^1 \frac{dy}{y} a(y) b \left( \frac{x}{y} \right).$$

In Eq. (4) we have neglected higher twists contributions. To simplify notations, we do not show below $\alpha_s$-dependence of the coefficient functions.
It is well known that $C_i$ and $f_i$ separately depend on a renormalization scheme. Usually, the MS-scheme is more preferable than the MOM-scheme, because of more complicated calculations in the framework of the latter.

Nevertheless, the MOM-scheme has some advantages. One of them is the universality of the calculational algorithm for coefficient functions in any order of perturbation theory. Advantages of use of the scheme with momentum subtraction in the case of heavy quarks were noted in [7], where authors proposed a mixed, so called, CWZ renormalization scheme.

The purpose of the present work is to show that effects related to the heavy quark mass $m_Q$, "survive" in the large $Q^2$ limit. There is a linear combination of the structure functions $F_2$ and $F_2^c$ that has scaling properties. It appears that this scaling takes place in different renormalization schemes. The lower bound for the ratio $F_2^c/F_2$ as the function of $x$ at fixed values of $Q^2$ is also calculated. The results are compared with the experimental data.

2 Asymptotic relations between structure functions

We are interested in the behaviour of $F_2^c$ at large $Q^2$ and small $x$. We suppose that the production of heavy quarks in this region results from gluons, and heavy quarks are not included in the evolution of light quarks and gluons.

So, we can write

$$\frac{1}{x}F_2^{q\bar{q}}(x, Q^2, m_Q^2) = C_g\left(\frac{Q^2}{\mu^2}, \frac{m_Q^2}{\mu^2}\right) \otimes f_g(\mu^2)[x] + C_Q\left(\frac{Q^2}{\mu^2}, \frac{m_Q^2}{\mu^2}\right) \otimes f_Q(\mu^2)[x].$$

Taking $\mu^2 = \mu_0^2$, where $\Lambda^2_{\text{QCD}} \ll \mu_0^2 \ll Q^2$, and neglecting "intrinsic charm" ("beauty") inside the nucleon, i.e. assuming, that there is a scale $\mu_0^2$, at which heavy quark distribution function is small as compared to the gluon one, we simplify Eq. (5) to the relation

$$\frac{1}{x}F_2^{q\bar{q}}(x, Q^2, m_Q^2) = C_g\left(\frac{Q^2}{\mu_0^2}, \frac{m_Q^2}{\mu_0^2}\right) \otimes f_g(\mu_0^2)[x].$$

Let us now define the quantity

$$\Delta F_2 = F_2^{q\bar{q}}(x, Q^2) - F_2^{q\bar{q}}(x, Q^2, m_Q^2).$$

Assuming that at small $x$ the main contribution to the DIS structure function without heavy quark production is determined by gluons, i.e. by the formula analogous to (5), we find

$$\frac{1}{x}\Delta F_2 = \Delta C_g \otimes f_g(\mu_0^2)[x],$$

where

$$\Delta C_g = C_g(y, \frac{Q^2}{\mu_0^2}, 0) - C_g(y, \frac{Q^2}{\mu_0^2}, \frac{m_Q^2}{\mu_0^2}).$$
Calculations of the gluon coefficient function in the order $O(\alpha_s)$ in the MOM-scheme were given in [8]. Using the expression obtained there, we get from (9):

$$\Delta C_g \simeq \Delta C^{(1)}_g(y, \frac{m_Q^2}{\mu_0^2}) = \frac{\alpha_s}{8\pi} \left\{ \left( y^2 + (1-y)^2 \right) \ln \left[ 1 + \frac{m_Q^2}{\mu_0^2 y (1-y)} \right] - \frac{m_Q^2 (1 + 2y(1-y))}{m_Q^2 + \mu_0^2 y (1-y)} \right\} .$$

Thus, the quantity $\Delta F_2$ tends to the finite limit $\Delta F_2(x, m_Q^2)$ at large $Q^2$. It was shown in [8], that this result is not the artifact of the MOM-scheme, although the very expression for the quantity $\Delta g$ depends, of course, on the renormalization scheme.

Starting from the expression for $\Delta C_g$, Eq. (10), it is easy to see, that for $y \leq 0.1$ (small $y$ region is most important in the analysis of the behavior of the structure function at $x \ll 1$)

$$\Delta C_g > 0 .$$

It follows from the explicit form of $C_g(y, \frac{Q^2}{\mu_0^2})$ in the MOM-scheme for the massless case in the order $O(\alpha_s)$ for any $Q^2$ and $\mu^2$ that [8]

$$C_g(y, \frac{Q^2}{\mu_0^2}) \bigg|_{Q^2=m_Q^2} > \Delta C_g(y, \frac{m_Q^2}{\mu_0^2}) .$$

Then from Eqs. (8), (11), (12) we conclude:

$$F_{2q}(x, Q^2) \bigg|_{Q^2=m_Q^2} > \Delta F_2(x, m_Q^2) > 0 .$$

Inequalities (13) are used below to obtain the lower bound for the ratio $F_2^c/F_2$.

### 3 Charm contribution to the structure function

It has been found in the previous section (see Eqs. (8), (14)), that the difference between contributions of light and heavy flavours to the DIS structure function scales for $Q^2 \to \infty$.

Taking this result as a basis, it is easy to see, that the following linear combination [4]

$$\Sigma_\alpha(x, Q^2) \equiv F_2(x, Q^2) + \alpha F_2^c(x, Q^2, m_c^2) - (4\alpha + 11)F_2^b(x, Q^2, m_b^2)$$

has scaling behaviour, with $\alpha$ being an arbitrary constant.

To cancel the contribution of b-quarks from (14), we have taken $\alpha = -2.75$. Then we obtain the prediction, that the linear combination

$$\Sigma = F_2 - 2.75 F_2^c$$

must tend to a function, that depends only on the Bjorken variable $x$ (and the heavy quark mass) for $Q^2 \to \infty$. 

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Using the expression for $\Delta C_g$ in the leading order in $\alpha_s$, we find, that the above difference (13) tends to the scaling limit in the region $m_Q^2 \ll Q^2$ in the following way

$$\frac{1}{x} \Sigma = \frac{1}{9} \left[ 7 \Delta C_g^{(1)}(\frac{m_c^2}{\mu_0^2}) - \Delta C_g^{(1)}(\frac{m_b^2}{\mu_0^2}) \right] \otimes f_g(\mu_0^2)[x] +$$

$$+ \frac{m_b^2 - 7m_c^2}{Q^2} \ln(\frac{Q^2}{\mu_0^2}) \cdot h \otimes f_g(\mu_0^2)[x],$$

where

$$h(y) = \frac{1}{9} y(1-y)[(2-3y)^2 + 3y^2].$$

(17)

Since $m_b^2 - 7m_c^2 > 0$, we conclude, that the correction in the expression for $\Sigma(x, Q^2)$, (16), is positive and decreasing on $Q^2$.

To compare our results with the recent experimental data, we parametrize $F_2^c$ according to the expression (14) for $\Sigma(x, Q^2, m_Q^2)$ and fit the data from HERA collider [2].

For $F_2$ we use the parametrization of H1 collaboration [1]:

$$F_2(x, Q^2) = \left[ ax^b + cx^d (1 + \sqrt{x}) \left( \ln Q^2 + f \ln^2 Q^2 + \frac{h}{Q^2} \right) \right] (1-x)^g,$$

(18)

with parameters

|  a  |   b  |   c   |   d   |
|-----|------|-------|-------|
| 3.1 | 0.76 | 0.124 | -0.188|
| e   | f    | g     | h     |
| -2.91 | -0.043 | 3.69  | 1.4 GeV^2 |

For $F_2^c$ we choose the expression qualitatively coherent to the asymptotic behaviour of the quantity $\Sigma = F_2 - 2.75F_2^c$ in the variable $Q^2$ (14)

$$F_2^c(x, Q^2) = \frac{1}{2.75} F_2(x, Q^2) - \bar{a}x^\bar{b}(1-x)^\bar{g} \left[ 1 + x^\bar{c} \frac{\bar{h}}{Q^2} \ln Q^2 \right],$$

(19)

where $F_2$ is defined above (18). Fitting the data from HERA [2] for $6.5 \text{ GeV}^2 \leq Q^2 \leq 130 \text{ GeV}^2$ gives the following values of parameters

| $\bar{a}$ | $\bar{b}$ | $\bar{c}$ | $\bar{g}$ | $\bar{h}$ |
|-----------|-----------|-----------|-----------|-----------|
| 0.28      | 0.15      | -0.08     | 5.00      | 1.86 GeV^2 |

and $\chi^2/n.d.f. = 34.6/36 = 0.96$.

In Fig. 1 we show the dependence of the quantity $\Sigma$ as a function of $Q^2$ for two values of $x$, for which we have the array of experimental data, obtained at different $Q^2$ and $x$, closest to the given values $x = 0.01$ and $x = 0.001$. As we see, experimental data are in a good agreement with our result on the approach to scaling behaviour from above but the very existence of the scaling may be checked at higher $Q^2$.

As has been found in [3], inequalities (13) allow us to obtain the following estimation for the ratio of structure functions:

$$\frac{F_2^c(x, Q^2)}{F_2(x, Q^2)} > 0.4 \left( 1 - \frac{F_2(x, m_c^2)}{F_2(x, Q^2)} \right).$$

(20)
It is important to stress, that we did not use any parametrization for $F^c_2$ to obtain the inequality, and also, that it does not depend on the behaviour of the gluon distribution.

Curves represented in Figs. 2, 3 are calculated by using the formula (20) for two values of $c$-quark mass and are compared with the data of ZEUS collaboration [2]. Despite the fact that these curves are only lower bounds for the ratio $F^c_2/F_2$, they lay very close to the experimental points.

Our estimations show that lower-bound curves are also in a good agreement with new preliminary data of ZEUS collaboration [3], including the data for maximal measured value $Q^2 = 565$ GeV$^2$.

4 Conclusions

In the present work we analyzed quark mass effects in DIS with the help of OPE. It is shown by calculating in the leading order that the new scaling takes place in DIS; certain linear combination of the DIS structure function and the DIS structure function with open charm production scales in the limit of large $Q^2$.

It is found that this specific scaling is in a good agreement with the $Q^2$-trend of recent experimental data on $F^c_2$ and $F_2$ obtained at HERA.

Certainly, higher orders could change the theoretical conclusion about such a scaling. At present the first order result can serve as an indication of the existence of an interesting physical phenomenon. Influence of higher orders is a subject of our further work.

We also calculated the lower bound for the ratio $F^c_2/F_2$ as a function of $x$ at fixed $Q^2$, independent on the shape of the gluon distribution in the nucleon, and compared it with the data from ZEUS collaboration. This lower bound appears to be quite close to the data.

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Figure captions

Fig. 1: $Q^2$-dependence of the difference of structure functions at two fixed values of $x$. Solid curves are obtained by fitting the data on $F_c^2$ from [2, 9]. The experimental points have $x$ closest to the given values $x = 0.01$ and $x = 0.001$.

Fig. 2: The ratio $F_c^2/F_2$ as the function of the variable $x$ at fixed values of $Q^2$. Dashed curves represent the lower bound for $F_c^2/F_2$ for the c-quark mass $m_c = 1.7$ GeV, dotted curves correspond to $m_c = 1.3$ GeV. Experimental points are taken from [2].

Fig. 3: The same as in Fig. 2, but for other values of $Q^2$. 

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Figure 1:
Figure 2:
Figure 3: