On the local super antimagic total face chromatic number of plane graphs

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Abstract. Local antimagic total face labeling is a bijection $f : V(G) \cup E(G) \cup F(G) \rightarrow \{1, 2, 3, \ldots, |V(G)| + |E(G)| + |F(G)|\}$ and for every two adjacent $F_1$ and $F_2$, $w_t(F_1) \neq w_t(F_2)$, where $F \in G$, $w_t(F) = \sum_{v \in V(A)} f(v) + \sum_{e \in E(A)} f(e) + f(F)$. We assigned color $w_t(F)$ of local face antimagic total labeling that induces by proper edge coloring of $G$ for each face. It is considered to be a local super antimagic total face coloring, if we give vertex labeling first it call local super antimagic total face labeling. The local face super antimagic total chromatic number, denoted by $\gamma_{lfat}(G)$, is the minimum number of colors taken over all colorings induced by local super antimagic total face labelings of $G$. In this paper we study of local super antimagic total face chromatic number of graphs. Furthermore, we have determined exact value local super antimagic total face chromatic number of $Shack(C_m,v,n)$, friendship graph $F_n$, fan graph $F_n$, and triangular ladder graph $TL_n$.

1. Introduction

All graphs considered in this paper are finite, simple and connected. A graph $G(V, E, F)$ is obtained by of vertex $V(G)$, edge $E(G)$, and face $F(G)$. For detailed definition of graph refer to [9, 13]. The basic elements of a plane map are its vertices, edges, and faces. An edge is a closed jordan curve, its end-points are vertices. A plane graph is a plane map with neither loops nor multiple edges. For detail definition of plane graph, it refer to [4].

A plane graph divides the plane into regions which are called faces. For detail definition of plane graph can be seen [5]. A graph is said to be planar if there exists a planar drawing of it. This definition taken from gross et. al[8]. The labeling called antimagic if all the edge weights has the different value. The concept of antimagic
labeling is introduced by Hartsfield and Ringel [9]. An $n$ vertex and $m$ edge of $G$, the local labeling whose vertices and edges labeled the integer $1, 2, 3, \ldots, m+n$ so that the sum of the labels on any given face is different from the sum of the labels at any other face, none of two have same sum. The result of antimagic labeling studied by Daifik et. al [7, 6]. They conducted the super edge-antimagic total labelings and super edge-antimagicness.

We consider all weights associated with each face and examine the relation between antimagic labeling and coloring of graph, namely a local super antimagic total face coloring. Let $G$ be a plane graph with vertex set, edge set and face set are $V, E$ and $F$. Local face antimagic total labeling is a bijection $f : V(G) \cup E(G) \cup F(G) \rightarrow \{1, 2, 3, \ldots, |V(G)| + |E(G)| + |F(G)| \}$ if for any two adjacent $F_1$ and $F_2$. Obtain $w_l(F_1) \neq w_l(F_2)$ where for $F \in G$, $w_l(F) = \sum_{v \in V(A)} f(v) + \sum_{e \in E(A)} f(e) + f(F)$. Thus, any local antimagic total face labeling induces a proper face coloring $w_l(F)$ of $G$.

Local vertex antimagic coloring of a graph $G$ is introduced by Arumugam et al. [3]. Agustin et. al. [1] studied local antimagic edge coloring of graphs. Their result, the local antimagic edge chromatic number of some graphs and determined it is lower bound $\gamma_{leal}(G) \geq \Delta(G)$. Agustin et. al. [2] Super local edge antimagic total coloring of any graph was using EAVL technique and has found the lower bound denoted by $\gamma_{leal}(G) \geq \Delta(G)$. Kurniawati et. al.[11, 10] also studied local edge antimagic total coloring of comb product and amalgamation of graphs.

**Definition 1** Let $G(V, E)$ be a connected graph of order $n$ and size $m$. A bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, \ldots, n+m \}$ is called a local super antimagic face coloring if there is a bijective function $f : V(G) \rightarrow \{1, 2, \ldots, n \}$ and bijective function $f^* : E(G) \rightarrow \{n+1, n+2, \ldots, n+m \}$ such that for any two adjacent faces $A_1$ and $A_2$, $w(A_1) \neq w(A_2)$, where $w(A) = \sum_{v \in V(A)} f(v) + \sum_{e \in E(A)} f(e)$.

In addition, the other definition worth considering is as follow is:

**Definition 2** The local super antimagic face coloring chromatic number $\gamma_{laf}(G)$ defined to be the minimum number of colors taken over all colorings of $G$ induced by local super antimagic face coloring of $G$.

Local super antimagic total face can be applied to the spread of exam questions during the exam. Vertex on the plane graph is the code exam question. The edge is the relationship between the exam question where for each edge of the adjacent must use different exam question code. So that during the exam there is no code of questions that are next to each other and there is a small possibility that the test participants cheat. This method is very effective to do examinations, to avoid cheating.

2. Main Results

In this paper we present our result local super antimagic total face chromatic number of plane graph. Furthermore, we have determined chromatic number of local super
antimagic total face coloring of $Shack(C_m, v, n)$ graph, friendship graph $F_n$, fan graph $F_n$, and triangular ladder graph $TL_n$.

**Theorem 1.** Let $Shack(C_m, v, n)$ be a graph with $n \geq 2$, the local super antimagic total face chromatic number of $Shack(C_m, v, n)$ is $\gamma_{fat}(Shack(C_m, v, n)) = 1$

**Proof.** The graph $Shack(C_m, v, n)$ is a graph with vertex set $V(Shack(C_m, v, n)) = \{x_i; 1 \leq i \leq n + 1\} \cup \{x_i; 1 \leq j \leq m - 2, 1 \leq i \leq n\}$, edge set $E(Shack(C_m, v, n)) = \{x_i x_{i+1}; 1 \leq i \leq n\} \cup \{x_i x_j; 1 \leq i \leq n, j = 1\} \cup \{x_i x_{ij+1}; 1 \leq i \leq n, 1 \leq j \leq m - 3\} \cup \{x_i x_{i+1}; 1 \leq i \leq n, j = m - 2\}$, face set $F(Shack(C_m, v, n)) = \{F_i, 1 \leq i \leq n\}$. Hence $|V(Shack(C_m, v, n))| = mn - n + 1$, $|E(Shack(C_m, v, n))| = nm$, and $|F(Shack(C_m, v, n))| = n$. If there are two adjacent sides, then the two faces should be colored with different colors. We will show $\gamma_{fat}(Shack(C_m, v, n)) \geq 1$. The function of labeling is $\gamma_{fat}(Shack(C_m, v, n)) \leq 1$. To prove the above theorem, we will describe the proof into four cases:

**Case 1:** For $n, m$ is odd, we will show that function of the local super antimagic total face coloring of $Shack(C_m, v, n)$ is $\gamma_{fat}(Shack(C_m, v, n)) \leq 1$. We define bijection of vertex labeling $f : V(Shack(C_m, v, n)) \rightarrow \{1, 2, 3, \ldots, nm - n + 1\}$ as follows:

$$f(x_i) = \begin{cases} 
\frac{i+1}{2}, & \text{i is odd; } 1 \leq i \leq n \\
\frac{n+1+i}{2}, & \text{i is even; } 1 \leq i \leq n 
\end{cases}$$

we define the bijection of edge labeling $f : E(Shack(C_m, v, n)) \rightarrow \{nm - n + 2, \ldots, 2nm - n + 1\}$ as follows:

$$f(x_ixj) = nm - n + i + 1; \quad 1 \leq i \leq n, j = 1$$

$$f(x_ixj+1) = \begin{cases} 
\frac{nm + j - i + 2}{2}, & \text{if } 1 \leq i \leq n, j = m - 2, j \text{ is odd; } \\
\frac{nm + j - n + i + 1}{2}, & \text{if } 1 \leq i \leq n, j = m - 2, j \text{ is even;} 
\end{cases}$$

$$f(x_ixj+2) = 2nm - 2n - i + 2; \quad 1 \leq i \leq n$$

$$f(x_ixj+11) = 2nm - 2n + i + 1; \quad 1 \leq i \leq n, j = m - 2$$

we define the bijection of face labeling $f : F(Shack(C_m, v, n)) \rightarrow \{2nm - n + 2, \ldots, 2nm + 1\}$ as follows:

$$f(F_i) = 2nm - n + 2; 1 \leq i \leq n$$

It is easy to see that $F$ is a local super antimagic total face coloring of $Shack(C_m, v, n)$ with face weights are as follows:

$$w_t(F_i) = \frac{9nm^2 - 6nm + 11n + 12m + 2}{4}; 1 \leq i \leq n$$

...
**Case 2:** For $n$ is odd and $m$ is even, we will show that function of the local edge antimagic total coloring of $\text{Shack}(C_m,v,n)$ is $\gamma_{fat}(\text{Shack}(C_m,v,n)) \leq 1$. We define bijection of vertex label $f : V(\text{Shack}(C_m,v,n)) \rightarrow \{1, 2, 3, \ldots, nm - n + 1\}$ as follows:

$$f(x_i) = \begin{cases} 
    \frac{i + 1}{2}; & i \text{ is odd, } 1 \leq i \leq n \\
    \frac{n + i + 1}{2}; & i \text{ is even, } 1 \leq i \leq n 
\end{cases}$$

we define the bijection of edge labeling $f : E(\text{Shack}(C_m,v,n)) \rightarrow \{nm - n + 2, \ldots, 2nm - n + 1\}$ as follows:

$$f(x_{ij}) = \begin{cases} 
    n_j + n - i + 2; & \text{if } 1 \leq i \leq n, j \text{ is odd, } 1 \leq j \leq m - 2 \\
    n_j + i + 1; & \text{if } 1 \leq i \leq n, j \text{ is even, } 1 \leq j \leq m - 2 
\end{cases}$$

we define the bijection of face labeling $f : F(\text{Shack}(C_m,v,n)) \rightarrow \{2nm - n + 2, \ldots, 2nm + 1\}$ as follows:

$$f(F_i) = 2nm - n + 2; 1 \leq i \leq n$$

it is easy to see that $F$ is a local super antimagic total face coloring of $\text{Shack}(C_m,v,n)$ with face weights are as follows:

$$w_i(F_i) = \frac{4nm^2 + n + 6m + 1}{2}; 1 \leq i \leq n$$

**Case 3:** For $n$ is even and $m$ is odd, we will show that function of the local super antimagic total face coloring of $\text{Shack}(C_m,v,n)$ is $\gamma_{fat}(\text{Shack}(C_m,v,n)) \leq 1$. We define bijection of vertex label $f : V(\text{Shack}(C_m,v,n)) \rightarrow \{1, 2, 3, \ldots, nm - n + 1\}$ as follows:

$$f(x_i) = \begin{cases} 
    \frac{i + 1}{2}; & i \text{ is odd, } 1 \leq i \leq n \\
    \frac{n + i + 1}{2}; & i \text{ is even, } 1 \leq i \leq n 
\end{cases}$$

we define the bijection of edge labeling $f : E(\text{Shack}(C_m,v,n)) \rightarrow \{nm - n + 2, \ldots, 2nm - n + 1\}$ as follows:

$$f(x_{ij}) = \begin{cases} 
    n_j + n - i + 2; & \text{if } 1 \leq i \leq n, j \text{ is odd, } 1 \leq j \leq m - 2 \\
    n_j + i + 1; & \text{if } 1 \leq i \leq n, j \text{ is even, } 1 \leq j \leq m - 2 
\end{cases}$$
with face weights are as follows: 

\begin{align*}
  f(x_i) &= 2nm - 2n - i + 2; \quad \text{if } 1 \leq i \leq n \\
  f(x_{i+1}) &= 2nm - 2n + i + 1; \quad \text{if } 1 \leq i \leq n, j = m - 2
\end{align*}

we define the bijection of face labeling \( f : F(\text{Shack}(C_m, v, n)) \rightarrow \{2nm - n + 2, \ldots, 2nm + 1\} \) as follows:

\[ f(F_i) = 2nm - i + 2; 1 \leq i \leq n \]

it is easy to see that \( F \) is a local super antimagic total face coloring of \( \text{Shack}(C_m, v, n) \) with face weights are as follows:

\[ w_i(F_i) = \frac{9nm^2 - 4nm + 5n + 12n + 4}{4}; 1 \leq i \leq n \]

**Case 4:** For \( n, m \) is even, we will show that function of the local super antimagic total face coloring of \( \text{Shack}(C_m, v, n) \) is \( \gamma_{\text{fat}}(\text{Shack}(C_m, v, n)) \leq 1 \). we define bijection of vertex label \( f : V(\text{Shack}(C_m, v, n)) \rightarrow \{1, 2, 3, \ldots, nm - n + 1\} \) as follows:

\[ f(x_i) = \begin{cases} 
  \frac{i + 1}{2}; & \text{i is odd, } 1 \leq i \leq n \\
  \frac{n + i + 2}{2}; & \text{i is even, } 1 \leq i \leq n
\end{cases} \]

\[ f(x_{ij}) = \begin{cases} 
  nj + n - i + 2; & \text{if } 1 \leq i \leq n, j \text{ is odd, } 1 \leq j \leq m - 2 \\
  nj + i + 1; & \text{if } 1 \leq i \leq n, j \text{ is even, } 1 \leq j \leq m - 2
\end{cases} \]

we define the bijection of edge labeling \( f : E(\text{Shack}(C_m, v, n)) \rightarrow \{nm - n + 2, \ldots, 2nm - n + 1\} \) as follows:

\[ f(x_{ij}) = nm - i + 2; \quad \text{if } 1 \leq i \leq n, j = 1 \]

\[ f(x_{ij}) = \begin{cases} 
  nm + nj - n + i + 1; & \text{if } 1 \leq i \leq n, j = m - 2, j \text{ is odd} \\
  nm + nj - i + 2; & \text{if } 1 \leq i \leq n, j = m - 2, j \text{ is even}
\end{cases} \]

\[ f(x_{i+1}) = 2nm - 2n - i + 2; \quad \text{if } 1 \leq i \leq n \]

\[ f(x_{i+1}) = 2nm - 2n + i + 1; \quad \text{if } 1 \leq i \leq n, j = m - 2 \]

we define the bijection of face labeling \( f : F(\text{Shack}(C_m, v, n)) \rightarrow \{2nm - n + 2, \ldots, 2nm + 1\} \) as follows:

\[ f(F_i) = 2nm - i + 2; 1 \leq i \leq n \]

it is easy to see that \( F \) is a local super antimagic total face coloring of \( \text{Shack}(C_m, v, n) \) with face weights are as follows:

\[ w_i(F_i) = \frac{8nm^2 + 2n + 12n + 4}{4}; 1 \leq i \leq n \]

Hence, from the above the in case, it gives \( \gamma_{\text{fat}}(\text{Shack}(C_m, v, n)) \leq 1 \). Also it follows from Theorem 2.1 that \( \gamma_{\text{fat}}(\text{Shack}(C_m, v, n)) \geq 1 \) and hence \( \gamma_{\text{fat}}(\text{Shack}(C_m, v, n)) = 1 \).

For the illustration, we give the follows graph.

Based on the figure 1(a), the graph \( \text{Shack}(C_6, v, 4) \) that has is \( \gamma_{\text{fat}}(\text{Shack}(C_6, v, 4)) = 1 \), because \( \text{Shack}(C_6, v, 4) \) not have two adjacent sides, than can have the same color. Labeling of local super antimagic total face can be shown on the figure 1(a), it is clear that of \( \text{Shack}(C_6, v, 4) \) have 1 the weight of face= 309. That weight induce a proper coloring thus, \( \gamma_{\text{fat}}(\text{Shack}(C_6, v, 4)) = 1 \).
Figure 1. (a) Local super antimagic total face coloring of $\text{Shack}(C_6, v, 4)$ and (b) Local super antimagic total face coloring of $\text{Shack}(C_7, v, 4)$

**Theorem 2.** Let $\mathcal{F}_n$ be a friendship graph where $n \geq 3$, the local super antimagic total face chromatic number of $\mathcal{F}_n$ is $\gamma_{\text{fat}}(\mathcal{F}_n) = 1$.

**Proof.** Friendship graph $\mathcal{F}_n$ is a graph with vertex set $V(\mathcal{F}_n) = \{x_i; 1 \leq i \leq n\} \cup \{y_i; 1 \leq i \leq n\} \cup \{z_i; 1 \leq i \leq n\}$, and edge set $E(\mathcal{F}_n) = \{xy_i; 1 \leq i \leq n\} \cup \{xz_i; 1 \leq i \leq n\} \cup \{yi z_i; 1 \leq i \leq n\}$, face set $F(\mathcal{F}_n) = \{F_i; 1 \leq i \leq n\}$. Hence $|V(\mathcal{F}_n)| = 2n - 1, |E(\mathcal{F}_n)| = 3n, |f(\mathcal{F}_n)| = n$.

If there are two adjacent sides, then the two faces should be colored with different colors. We will show $\mathcal{F}_n$ is $\gamma_{\text{fat}}(\mathcal{F}_n) \geq 1$. The function of labeling is $\gamma_{\text{fat}}(\mathcal{F}_n) \leq 1$.

To prove the above theorem, we define bijection of vertex label $f : V(\mathcal{F}_n) \rightarrow 1, 2, 3, \ldots, 2n + 1$ as follows:

\[
f(x) = 1 \\
f(y_i) = i + 1; 1 \leq i \leq n \\
f(z_i) = 2n - i + 2; 1 \leq i \leq n
\]

we define the bijection of labeling $f : E(\mathcal{F}_n) \rightarrow \{2n + 2, 2n + 3, \ldots, 5n + 1\}$ as follows:

\[
f(xy_i) = 2n + i + 1; \text{ for } 1 \leq i \leq n \\
f(xz_i) = 4n - i + 2; \text{ for } 1 \leq i \leq n \\
f(yi z_i) = 4n + i + 1; \text{ for } 1 \leq i \leq n
\]

we define the bijection of face labeling $f : F(\mathcal{F}_n) \rightarrow \{5n + 2, 5n + 3, \ldots, 6n + 1\}$ as follows:

\[
f(F_i) = 6n - i + 2; \text{ for } 1 \leq i \leq n
\]

it is easy to see that $F$ is a local super antimagic total face coloring of $\mathcal{F}_n$ with a face weights are as follows:

\[
w_t(F_i) = 18n + 10; \text{ for } 1 \leq i \leq n
\]

Hence, from the above the face weight, it gives $\gamma_{\text{fat}}(\mathcal{F}_n) \leq 1$. Also it follows from Theorem 2 that $\gamma_{\text{fat}}(\mathcal{F}_n) \geq 1$ and hence $\gamma_{\text{fat}}(\mathcal{F}_n) = 1$.

For the illustration, we give the follows graph.

Based on the figure 2, the graph $(\mathcal{F}_5)$ that has is $\gamma_{\text{fat}}(\mathcal{F}_5) = 1$, because $(\mathcal{F}_5)$ not have two adjacent sides, than can have the same color. Labeling of local super antimagic total face can be shown on the figure 2, it is clear that of $(\mathcal{F}_5)$ have 1 the weight of face= 100. That weight induce a proper coloring thus, $\gamma_{\text{fat}}(\mathcal{F}_5) = 1$. 
Figure 2. Local super antimagic total face coloring of $F_5$

**Theorem 3.** Let $F_n$ be a fan graph with $n \geq 3$, the local super antimagic total face chromatic number of $F_n$ is $\gamma_{lfat}(F_n) = 2$

**Proof.** $F_n$ is a graph with vertex set $V(F_n) = \{x_i; 1 \leq i \leq n+1\} \cup \{y; i = 1\}$, edge set $E(F_n) = \{x_ix_{i+1}; 1 \leq i \leq n\} \cup \{xy, 1 \leq i \leq n+1\}$, face set $F(F_n) = \{F_i, 1 \leq i \leq n\}$. Hence $|V(F_n)| = n+2$, $|E(F_n)| = 2n+1$, and $|F(F_n)| = n$. If two adjacent sides, then the two faces should be colored with different colors. We will show the local super antimagic total face coloring of $F_n$ is $\gamma_{lfat}(F_n) \geq 2$. The function of labeling is $\gamma_{lfat}(F_n) \leq 2$. To prove the above theorem, the proof will describe into two cases:

**Case 1:** For $n$ is odd, we will show that function of the local super antimagic total face coloring of $F_n$ is $\gamma_{lfat}F_n \leq 2$. we define bijection of vertex label $f : V(F_n) \rightarrow \{1, 2, 3, \ldots, n+2\}$ as follows:

$$f(x_i) = \begin{cases} \frac{2n-i+5}{n-1}; & i \text{ for } 1 \leq i \leq n+1, i \text{ is odd} \\ \frac{n-i+5}{2}; & i \text{ for } 1 \leq i \leq n+1, i \text{ is even} \end{cases}$$

$$f(y) = 1$$

we define the bijection of edge labeling $f : E(F_n) \rightarrow \{n+3, n+4, \ldots, 3n+3\}$ as follows:

$$f(x_ix_{i+1}) = \begin{cases} 3n-i+6; & i \text{ for } 1 \leq i \leq n, i \text{ is odd} \\ \frac{4n-i+6}{2}; & i \text{ for } 1 \leq i \leq n, i \text{ is even} \end{cases}$$

we define the bijection of face labeling $f; F(F_n) \rightarrow \{3n+4, 3n+5, \ldots, 4n+3\}$ as follows:

$$f(F_i) = \begin{cases} \frac{7n-i+8}{2}; & i \text{ for } 1 \leq i \leq n, i \text{ is odd} \\ \frac{8n-i+8}{2}; & i \text{ for } 1 \leq i \leq n, i \text{ is even} \end{cases}$$

it is easy to see that $F$ is a local super antimagic total face coloring of $F_n$ with a face weights which are as follows:

$$w_t(F_i) = \begin{cases} \frac{21n+35}{2}; & 1 \leq i \leq n, i \text{ is odd} \\ \frac{23n+35}{2}; & 1 \leq i \leq n, i \text{ is even} \end{cases}$$
Figure 3. (a) Local super antimagic total face coloring of $F_5$ and (b) Local super antimagic total face coloring of $F_6$

Case 2: For $n$ is even, we will show that function of the local super antimagic total face coloring of $F_n$ is $\gamma_{lfat}(F_n) \leq 2$. We define bijection of vertex label $f : V(F_n) \rightarrow \{1, 2, 3, \ldots, n+2\}$ as follows:

$$f(x_i) = \begin{cases} \frac{n-i+5}{2} & \text{if } 1 \leq i \leq n+1, i \text{ is odd} \\ \frac{2n-i+6}{2} & \text{if } 1 \leq i \leq n+1, i \text{ is even} \end{cases}$$

$$f(y) = 1$$

we define the bijection of edge labeling $f : E(F_n) \rightarrow \{n+3, n+4, \ldots, 3n+3\}$ as follows:

$$f(x_ix_{i+1}) = \begin{cases} \frac{4n-i+5}{2} & \text{if } 1 \leq i \leq n, i \text{ is odd} \\ \frac{3n-i+6}{2} & \text{if } 1 \leq i \leq n, i \text{ is even} \end{cases}$$

$$f(x_iy) = 2n + i + 2; \text{ if } 1 \leq i \leq n+1$$

we define the bijection of face labeling $f : F(F_n) \rightarrow \{3n+4, 3n+5, \ldots, 4n+3\}$ as follows:

$$f(F_i) = \begin{cases} \frac{8n-i+7}{2} & \text{if } 1 \leq i \leq n, i \text{ is odd} \\ \frac{7n-i+8}{2} & \text{if } 1 \leq i \leq n, i \text{ is even} \end{cases}$$

it is easy to see that $F$ is a local super antimagic total face coloring of $F_n$ with a face weights are as follows:

$$w_t(F_i) = \begin{cases} \frac{23n+34}{2} & 1 \leq i \leq n, i \text{ is odd} \\ \frac{21n+36}{2} & 1 \leq i \leq n, i \text{ is even} \end{cases}$$

Hence, from the above case, it gives $\gamma_{lfat}(F_n) \leq 2$. Also it follows from Theorem 3 that $\gamma_{lfat}(F_n) \geq 2$ and hence $\gamma_{lfat}(F_n) = 2$.

For the illustration, we give the follows graph.

Based on the figure 3a, the graph $(F_5)$ that has is $\gamma_{lfat}(F_5) = 2$, because $(F_5)$ have two adjacent sides, than can have the different color. Labeling of local super antimagic total face can be shown on the figure 3a, it is clear that of $F_5$ have 2 the weight of face= 70, and face = 75 . That weight induce a proper coloring thus, $\gamma_{lfat}(F_5) = 2$.

Theorem 4. Let $TL_n$ be a triangular ladder graph with $n \geq 3$, the local super antimagic total face chromatic number of $TL_n$ is $\gamma_{lfat}(TL_n) = 2$
Theorem 4 that

follows:

Figure 4. Local super antimagic total face coloring of $TL_5$

Proof. Triangular ladder graph $TL_n$ is a graph with vertex set $V(TL_n) = \{x_i; 1 \leq i \leq n\} \cup \{y_i; 1 \leq i \leq n\}$, edge set $E(TL_n) = \{y_i y_{i+1}; 1 \leq i \leq n-1\} \cup \{x_i x_{i+1}; 1 \leq i \leq n-1\} \cup \{x_i y_i; 1 \leq i \leq n\} \cup \{x_i y_{i+1}; 1 \leq i \leq n-1\}$, and face set $F(TL_n) = \{f_i; 1 \leq i \leq n+1\}$. Hence $|V(TL_n)| = 2n$, $|E(TL_n)| = 4n - 3$, and $|F(TL_n)| = 2n - 2$. If there are two adjacent sides, then the two faces should be colored with different colors. We will show the local super antimagic total face coloring of $TL_n$ is $\gamma_{tfat}(TL_n) \geq 2$. The function of labeling is $\gamma_{tfat}(TL_n) \leq 2$. To prove the above theorem, we define bijection of vertex label $f : V(TL_n) \to \{1, 2, 3, \ldots, 2n\}$ as follows:

$$f(x_i) = i; \text{ for } 1 \leq i \leq n$$
$$f(y_i) = n + i; \text{ for } 1 \leq i \leq n$$

We define the bijection of edge labeling $f_1 : E(TL_n) \to \{2n + 1, 2n + 2, \ldots, 6n - 3\}$ as follows:

$$f(y_i y_{i+1}) = 2n + 2i - 1; \text{ for } 1 \leq i \leq n - 1$$
$$f(x_i x_{i+1}) = 2n + 2i; \text{ for } 1 \leq i \leq n - 1$$
$$f(x_i y_i) = 6n - 2i - 1; \text{ for } 1 \leq i \leq n$$
$$f(x_i y_{i+1}) = 6n - 2i - 2; \text{ for } 1 \leq i \leq n - 1$$

We define the bijection of face labeling $f : F(TL_n) \to \{6n - 2, 6n - 1, \ldots, 8n - 5\}$ as follows:

$$f(F_i) = \begin{cases} 8n - \left(\frac{i+1}{2}\right) - 4; & \text{for } 1 \leq i \leq n - 1, \text{ i is odd} \\ 7n - \frac{i+1}{2} - 3; & \text{for } 1 \leq i \leq n - 1, \text{ i is even} \end{cases}$$

It is easy to see that $F$ is a local super antimagic total face coloring of $TL_n$ with a face weights are as follows:

$$w_t(F_i) = \begin{cases} 24n - 7; & \text{for } 1 \leq i \leq n - 1, \text{ i is odd} \\ 22n - 6; & \text{for } 1 \leq i \leq n - 1, \text{ i is even} \end{cases}$$

Hence, from the above the face weight, it gives $\gamma_{tfat}(TL_n) \leq 2$. Also it follows from Theorem 4 that $\gamma_{tfat}(TL_n) \geq 2$ and hence $\gamma_{tfat}(TL_n) = 2$.

For the illustration, we give the follows graph.

Based on the figure 4, the graph $(TL_5)$ that has is $\gamma_{tfat}(TL_5) = 2$, because $(TL_5)$ have two adjacent sides, than can have the different color. Labeling of local super antimagic total face can be shown on the figure 4, it is clear that of $TL_5$ have 2 the weight of face= 113, and face = 104. That weight induce a proper coloring thus, $\gamma_{tfat}(TL_5) = 2$. 


3. Concluding Remarks

In this paper we have given result on local face super antimagic total coloring of plane graphs. We also determine the chromatic number of local super antimagic total face coloring of $Shack(C_m,v,n), F_n, F_n$ and $TL_n$.

**Open Problem 1.** Determine the chromatic number of local super antimagic total face coloring of another plane graph.

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