Zero-field precession and hysteretic threshold currents in a spin torque nano device with tilted polarizer

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Abstract. Using nonlinear system theory and numerical simulations, we map out the static and dynamic phase diagrams in the zero applied field of a spin torque nano device with a tilted polarizer (TP). We find that for sufficiently large currents, even very small tilt angles (\(\beta > 1^\circ\)) will lead to steady free layer precession in zero field. Within a rather large range of tilt angles, \(1^\circ < \beta < 19^\circ\), we find coexisting static states and hysteretic switching between these using only current. In a more narrow window (\(1^\circ < \beta < 5^\circ\)) one of the static states turns into a limit cycle (precession). The coexistence of \textit{current-driven} static and dynamic states in the \textit{zero magnetic field} is unique to the TP device and leads to large hysteresis in the upper and lower threshold currents for its operation. The nano device with TP can facilitate the generation of large amplitude mode of spin torque signals without the need for cumbersome magnetic field sources and thus should be very important for future telecommunication applications based on spin transfer torque effects.

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Spin torque, or the transfer of angular momentum from spin-polarized electrons to magnetic moments [1, 2], is currently receiving increasing interest due to potential use in magnetoresistive random access memory (MRAM) [3]–[5] and in microwave signal generators, so-called spin torque oscillators (STOs) [6]–[13]. While the first spin torque devices were based on (pseudo-)spin valves with in-plane magnetizations, recent devices utilize perpendicularly magnetized layers to achieve both higher stability in MRAM and zero-field operation in STOs. The resulting static and dynamic phase diagrams have been studied in detail [14]–[18]. The use of a fixed layer with tilted magnetoanisotropy has been demonstrated to improve the switching speed and reduce the switching field (current) in magnetic recording (spin transfer torque random access memory), while retaining thermal stability [19]–[23]. Recently, it has been shown that the tilted polarizer (TP) device has the significant advantage of zero-field operation while maintaining a high microwave output signal [24]–[28]. However, a systematic study of such a spin torque device based on a TP at zero applied field is still lacking.

In this paper, we study, both analytically and numerically, the static and dynamic phase diagrams of a novel spin torque device where the magnetization of the fixed layer is tilted at an arbitrary angle out of the film plane. We show, using nonlinear system analysis, that the new degree of freedom of the polarizer creates a surprisingly rich phase diagram of the free layer, in zero applied field, with coexistence of different static and dynamic states (including two different static states) within a certain range of the polarizer tilt angle $\beta$. Whereas the analytical treatment cannot determine the stability of the precessional states of the device, we carry out numerical simulations in order to completely reveal all the eigenstates of the system. The simulations also reveal hysteretic switching between different static states and between the static and dynamic states. The coexistence of static and dynamic states in zero field, and the associated hysteresis in current, is unique to the TP device and disappears for polarizer angles outside of $1^\circ < \beta < 19^\circ$. As a consequence, the TP device can exhibit unexpected large current-driven hysteresis in both the upper and lower threshold currents for precession. From a fundamental spin torque physics point of view, the polarizer angle introduces an entirely new degree of freedom to any spin torque device and opens up a wide range of additional phenomena.

The paper is organized as follows. In section 2, we describe how we apply standard nonlinear system theory to the TP device and carry out a stability analysis. In section 3, we determine the static phase diagram of a TP device as a function of electric current.
Figure 1. (a) The TP device structure: \( \mathbf{m} \) and \( \mathbf{M} \) are the magnetization vectors of free and fixed layer, respectively. \( \mathbf{M} \) lies in the \( x-z \) plane with angle \( \beta \) w.r.t. the \( x \)-axis. \( \theta \) and \( \phi \) are the polar and azimuthal angles of the free layer magnetization \( \mathbf{m} \). (b) Phase diagram of the high-\( J \) region. (c) Phase diagram of static and dynamic states of a TP device with nodes (N), spirals (S) and limit cycles (L). Parentheses denote states only found through dynamical simulation.

and out-of-plane angle, \( \beta \), of the polarizer, while always keeping the applied field equal to zero. In section 4, we use numerical simulations to carry out a more detailed analysis of the dynamic precession of the device, and the unique hysteretic switching between different static and dynamic states as a function of current. We also compare the numerical results with the analytical predictions. In section 5, we suggest a number of ways to fabricate a TP device from known materials. Finally, we give a brief summary of the main results in section 6.

2. Stability analysis

Figure 1(a) shows the schematic structure of the TP device and the coordinate system used in this work [24]. While all films are deposited in the \( x-y \) plane, the fixed layer magnetization, \( \mathbf{M} \), lies in the \( x-z \) plane, with an angle \( \beta \) w.r.t. the \( x \)-axis. The time evolution of the unit vector of the free layer magnetization \( \dot{\mathbf{m}} \) is found from the Landau–Lifshitz–Gilbert–Slonczewski (LLGS)
equation [1, 2],

\[
\frac{d\hat{m}}{dr} = -|\gamma|\hat{m} \times \mathbf{H}_{\text{eff}} + \alpha \hat{m} \times \frac{d\hat{m}}{dr} + |\gamma|\alpha_J \hat{m} \times (\hat{m} \times \hat{M})
\]  

(1)

and the last term is Slonczewski spin torque with the magnitude

\[
\alpha_J = \frac{\gamma \hbar \xi (1 + \chi) J}{ed\mu_0 M_s [2 + \chi (1 + \cos \varphi)]},
\]

(2)

where \(\gamma\) is the gyromagnetic ratio, \(\alpha\) is the Gilbert damping parameter, \(\mu_0\) is the magnetic vacuum permeability, \(M_s\) is the free layer saturation magnetization, \(\hbar\) is the reduced Planck constant, \(d\) is the free layer thickness, \(e\) is the electron charge, \(J\) is the electric current density, \(\xi\) is the spin polarization efficiency constant, \(\chi\) is the giant magnetoresistance (GMR) asymmetry parameter describing the deviation from sinusoidal angular dependence and \(\varphi\) is the angle between \(\hat{m}\) and \(\hat{M}\). The electric current is defined as positive when it flows from the fixed to the free layer and normalized by \(J_0 = 10^8\) A cm\(^{-2}\). Setting the applied field to zero and separating the effect of the demagnetizing tensor into a positive shape anisotropy field along \(x\) (easy-axis) and a negative out-of-plane demagnetizing field (easy-plane anisotropy), we get \(\mathbf{H}_{\text{eff}} = (H_k \hat{e}_z m_x - H_d \hat{e}_x m_x)/|\mathbf{m}|\). For the results presented here for the NiFe/Cu/FePt system, \(|\gamma| = 1.9 \times 10^{-11}\) Hz/T, \(\alpha = 10^{-2}\), \(\mu_0 H_d = 4\pi M_s = 1\) T, \(\mu_0 H_k = 10^{-2}\) T and \(\xi = 0.35\) [24, 29]. The lateral dimension of the NiFe thin film-free layer is assumed to be an elliptical shape of 130 nm \(\times\) 70 nm, with a thickness of 3 nm. The thickness of the fixed layer FePt is 20 nm. We use a symmetric torque term and a sinusoidal angular GMR dependence, i.e. \(\chi = 0\), so as not to introduce further complications which would obscure the main results. The material parameters are carefully chosen to conform to the experiments of TP-STO. However, it is found that both our analytical model and simulations will give qualitatively similar results if we choose different sets of material parameters such as Co/Cu/FePt. We used the Ito calculus and a numerical method described by the Milshtein integration scheme to solve the LLGS equation. The time average over the instantaneous magnetoresistance (MR) values at the steady state gives the resistance shown in the following figures for the precessional state. The details of the calculation and computation procedures for the MR can be found elsewhere [24, 30].

Equation (1) can be transformed into the following set of differential equations in a spherical coordinate system:

\[
\begin{aligned}
\dot{\theta} &= \frac{\gamma}{\alpha^2 + 1} (\alpha U + W), \\
\dot{\phi} &= \frac{\gamma}{\alpha^2 + 1} \left( \frac{\alpha U - W}{\sin \theta} \right)
\end{aligned}
\]

(3)

with \( U = [H_k \cos \theta \sin \theta \cos^2 \phi + H_d \cos \theta \sin \theta - \alpha J \cos \beta \sin \phi] \), and \( W = [-H_k \cos \phi \sin \theta \sin \phi - \alpha J \cos \beta \sin \phi \sin \theta \sin \beta] \).

By setting \((U, W) = 0\), we get a series of possible equilibrium solutions \(\tilde{\phi}_i(\beta, J)\), where \(i \leq i_i\) is the \(i\)th solution of total \(i_i\) solutions. However, these \(i_i\) equilibrium states are not all stable. We linearize equation (3) in the vicinity of \((\tilde{\phi}_i, \tilde{\phi}_i)\) and get:

\[
\begin{pmatrix}
\dot{\theta} \\
\dot{\phi}
\end{pmatrix} =
\begin{pmatrix}
A(\beta, J, \tilde{\phi}_i) & B(\beta, J, \tilde{\phi}_i) \\
C(\beta, J, \tilde{\phi}_i) & D(\beta, J, \tilde{\phi}_i)
\end{pmatrix}
\begin{pmatrix}
\dot{\theta} \\
\dot{\phi}
\end{pmatrix},
\]

(4)
where $A$, $B$, $C$ and $D$ are the explicit functions of variables $\beta$, $J$ and other material parameters. Following [31], the eigenvalues of the corresponding Jacobian, which determine the stability of the system, can therefore be solved and can always be expressed as: $\mu_{1,2} = E(\beta, J) \pm \sqrt{F(\beta, J)}$. For a solution to be stable, it must satisfy $\Re\{\mu_{1,2}\} < 0$. For real eigenvalues and $F > 0$, the eigenvalue with larger magnitude dominates and defines the only eigenvector governing the approach towards the final state, in this case a node (N). For $F = 0$, the two eigenvectors are identical and again define a node. For $F < 0$, the complex conjugate eigenvalues define two complex eigenvectors generating an oscillatory trajectory towards equilibrium, characteristic of a spiral-like (S) solution [32].

As an illustrating example, we show the procedure for determining the stability and type of solution for the case $\beta = 0^\circ$, i.e. for a conventional in-plane spin torque MRAM cell in zero field. The well-known solution for negative current is $(\theta, \phi) = (\pi/2, 0)$, i.e. parallel alignment of the free and fixed layer magnetizations. Expanding equation (1) around this point yields:

$$
\begin{bmatrix}
\dot{\theta} \\
\dot{\phi}
\end{bmatrix} = 
\begin{bmatrix}
\alpha J - \alpha (H_d + H_k) & -H_k - \alpha \alpha J \\
H_d + H_k + \alpha \alpha J & \alpha H_k + \alpha J
\end{bmatrix}
\begin{bmatrix}
\hat{\theta} \\
\hat{\phi}
\end{bmatrix},
$$

(5)

with eigenvalues:

$$
\mu_{1,2} = -\frac{\alpha H_d}{2} + \alpha J \pm \sqrt{\left(\frac{\alpha H_d}{2}\right)^2 + H_k (H_k + H_d) (\alpha^2 - 1) + f(\alpha J)},
$$

(6)

where $f(\alpha J) = \alpha \alpha J (H_d - 2H_k - \alpha \alpha J)$. By entering the parameters into equation (6), we see that the type of solution and its stability depend upon the value of $\alpha J$ (i.e. $J$), if all other parameters are fixed. For $|J| < 1.5 \times 10^8 \text{ A cm}^{-2}$, the solution is of spiral-type (S), while outside this region, where the torque is larger, the solution is a node (N). Point Q in figure 1 denotes the $S \rightarrow N$ transition at negative currents. This result is well known in conventional spin torque switching where switching between S states proceeds by slow spiraling out of the unstable state and into the stable S state, while large currents will switch the magnetization without much precession into a stable N state [6].

3. Static phase diagram of a TP device

Following this procedure, we now construct the static part of the phase diagram in figure 1(c) by finding all eigenvalues in the parameter space $0^\circ < \beta < 90^\circ$ and $|J/J_0| < 1.6 \times 10^3$. While the entire parameter range was studied, figure 1 focuses on $0^\circ < \beta < 30^\circ$ and $|J/J_0| < 10$, where coexistence of several different stable solutions are observed. It should be noted that only the static solutions (S and N) can be found from the eigenvalue analysis. To find the precessional states (L) we have to resort to numerical simulations below (see section 4). However, according to the Poincaré–Bendixson theorem [33, 34], the only possible final states of the system are either static states (fixed points) or limit cycles (self-oscillation) and chaos is precluded since the free layer evolves on the unit sphere surface ($\hat{m} = 1$) [18, 35]. In regions where there are neither S nor N solutions we can hence infer that steady precession (L) must take place.

For small enough $J$, there is a single S state at all tilt angles $\beta$, corresponding to the usual P/AP orientation of the free layer with respect to the in-plane projection of the fixed layer magnetization. For $\beta > 2^\circ$ this static state disappears with increasing current, and as discussed
above, we can infer a precessional L state in this region. The crossover between the S and L regions defines the critical current for the onset of precession ($J_{c1}$), where the negative damping from the spin-polarized current destabilizes the S state and sustains continuous precession. It is hence possible to have zero-field operation down to very small tilt angles if only large enough current densities can be realized. At yet smaller tilt angles (as in the in-plane case above) the eigenvalue analysis indicates that the S state changes into a node (N) with increasing current. It will however become apparent in the magnetodynamic simulations below that for $\beta > 1^\circ$ this region also contains a precessional state (hence the additional label (L)).

If the current is increased further, precession stops at an upper threshold current ($J_{c2}$) where the L state turns into a single spiral state located close to the north/south pole of the unit sphere. If we increase the current in the N/(L) region the same L $\rightarrow$ S transition occurs but in addition, the node also remains stable. We hence observe a region S/N where two different stable static states coexist. The two states are located far from each other at two different points on the unit sphere: the north/south pole (S), and at P/AP alignment (N), respectively. As will become clear in the magnetodynamic simulation below, this separation allows both states to be realized by only sweeping the current.

Finally, at large currents (figure 1(b)) there is a node state at P/AP alignment for $\beta < 40^\circ$ and a spiral state close to the north/south poles for $\beta > 40^\circ$. At extremely high currents, the S state gradually turns away from the poles, approaches P/AP orientation, and finally replaces the N state at about $J/J_0 > 10^3$ as the free layer magnetization aligns completely with the fixed layer. At these current densities ($J \sim 10^{11}$ A cm$^{-2}$) any real sample would break down; we include this region for completeness.

4. Numerical simulations: precession in zero applied field and hysteretic switching

To determine the dynamic states and also study the hysteretic switching between the coexisting static and dynamic states, we now solve equation (1) using numerical simulation within the macro-spin approximation. To simulate actual hysteresis loops as a function of current, we start out at very large negative current, let the simulation reach a steady state, determine the type of state and its dynamic or static properties, and then let this state be the initial condition for the next simulation at the next current step. At very large negative (positive) currents, the free layer always aligns (anti-aligns) with the fixed layer, as was confirmed by a large set of random initial conditions. There is hence no dependence on the initial high-current state in our simulation.

The oscillation regions are shown in figure 2, for (a) increasing and (b) decreasing currents and for the angular region of interest. The precession frequency varies from 0 to about 31 GHz. The critical current for the onset of magnetization precession depends strongly on the tilt angle $\beta$ and is reduced by almost two orders of magnitude when $\beta$ increases from almost in-plane ($\beta = 2^\circ$) to perpendicular ($\beta = 90^\circ$). Typical experimentally measured critical current density for STO with in-plane anisotropy lies in the range of $10^7$–$10^8$ A cm$^{-2}$, while it is much reduced to the range of $10^6$ A cm$^{-2}$ for perpendicular STO. Our calculated current density for these two mostly investigated cases agrees well with the reported values [14, 30, 36, 37].

One observes in figure 2 that the precession region is asymmetric and depends on the direction of the current sweep. In the low-angle region, both the lower and upper threshold currents for precession exhibit hysteresis. We define $|J_{c1, +}|$ and $|J_{c2, +}|$ as the lower and upper (absolute) threshold currents for increasing $|J|$, and similarly $|J_{c1, -}|$ and $|J_{c2, -}|$ as the corresponding currents for decreasing $|J|$. As seen in figure 2(a), $|J_{c2, +}|$ can be more than five
times greater than \( |J_{c2}^-| \) at small \( \beta \). The hysteresis in \( J_{c1} \) is less obvious in figure 2, but will be discussed in detail in figure 4 at the end of the paper.

Using figure 2, we can now add information about the dynamical steady states and their boundaries to the phase diagram in figure 1. It is noteworthy that the boundaries in the two figures agree. Figure 2 first confirms our assumption that the lack of a stable static state infers the existence of a limit cycle in the L region. Secondly, it adds a steady dynamic state to the region where our eigenvalue analysis only indicated N; hence we label this region \( N/(L) \). While figure 2 indicates that this state only exists for \( \beta > 2^\circ \), it does indeed extend all the way to the \( N/(L) - N \) boundary at about \( \beta = 1^\circ \), as was confirmed by choosing initial conditions closer to precession. For \( \beta > 2^\circ \) the S state at P/AP orientation must transform into L, but below \( \beta = 2^\circ \), it can simply turn into the equivalent N state at P/AP orientation. Once in the N state there is no energetically favorable path to the L state. Finally, one realizes that the asymmetry in figure 2 stems from the selective realization of either the N or L state in the \( N/(L) \) region. When approaching this region from above in a node state, the system stays in the node; if this region is approached from above in a spiral state, the system enters the dynamic precessional state L. To confirm this picture, we simulated minor loops where we limited the current sweep to remain in the S/N region before reversing the current direction. In this case, the high current N state is

**Figure 2.** Precession frequency \( f \) as a function of tilt angle and current. Current is swept from (a) negative to positive and (b) positive to negative. The hysteretic switching threshold current for a selected angle \( \beta = 3^\circ \) is also shown here for illustration purpose.
Figure 3. Current-driven hysteresis loops for different $\beta = 3^\circ$, $8^\circ$, $15^\circ$ and $20^\circ$, showing large hysteresis in both the upper threshold current $J_{c2}$ and the transition between the static S and N states.

never realized and the S state again nucleates a precessional state already at the high value of $|J_{c2,+}|$ and not at the much lower $|J_{c2,-}|$.

It should be noted that although the above results are obtained by macrospin simulation, which assumes that the magnetization of the free layer stays spatially uniform during its precession, we have also performed micromagnetics simulations at room temperature and found that the dynamic precessional modes also exist in a wide range of $[J, \beta]$ space at zero applied field. The study of the sensitivity of such precessional modes on various material parameters is underway and will be addressed in a future work [38].

Not only is the precessional state hysteretic, the two static states in the S/N region also exhibit hysteresis. In figure 3, we plot the reduced MR $r = (R - R_P)/(R_{AP} - R_P)$, where $R_P$ and $R_{AP}$ denote the resistance in the parallel and antiparallel configurations, respectively, at four different polarizer angles. Close to $J = 0$ we observe the usual spin torque switching region between P and AP states. As the current is increased we first observe a linear $r$ versus $J$ region at all angles characteristic of the average resistance within the precessional state L. As precession stops at $|J_{c2,+}|$, $r$ reaches a plateau characteristic of the S state located close to the north/south poles. At a certain current value ($J_{S\rightarrow N}$), this state becomes unstable and $m$ switches to its N state close to P/AP alignment. If the current is again decreased, $m$ stays within its N state well below $J_{S\rightarrow N}$ and only switches back at a much smaller current, either to an S state at $J_{N\rightarrow S}$ ($\beta > 5^\circ$) or to an L state at $|J_{c2,-}|$ ($\beta < 5^\circ$). At about $\beta = 20^\circ$, this hysteresis disappears and $m$ rotates continuously $S \leftrightarrow N$.

We now turn to the hysteresis in the onset current of precession, $J_{c1}$. In figure 4, we plot $r$ versus $J$ in the low-$J$ region for $\beta = 2^\circ$. $J_{c1}$ is clearly hysteretic with $|J_{c1,+}| = 0.3J_0$ and $|J_{c1,-}| = 0.15J_0$. For $0.15 < J/J_0 < 0.3$, the S-type P/AP states hence coexist with a precessional L state with a very wide cone angle (inset 1 in figure 4). This hysteresis persists at all polarizer angles as shown in inset 2 in figure 4.
Figure 4. Reduced MR versus $J$ for $\beta = 2^\circ$. Inset 1 shows the wide-angle orbit of the L state within the $S/(L)$ region. Inset 2 shows the angular dependence of the hysteretic threshold current $J_{c1}$.

5. Tilted polarizer

We finally address the question of how a TP device may be realized and how one may freely vary the tilt angle $\beta$. Earlier work on perpendicular magnetic recording in the 1980s demonstrated strong perpendicular anisotropy in a wide range of Co-based alloys. In CoCr, there was evidence for a tilted anisotropy direction, which furthermore varied continuously with the film thickness [39, 40]. The variation was argued to arise from competition between the shape anisotropy of individual grains with a growth direction perpendicular to the film plane, and the dipolar and exchange coupling between neighboring grains. There are however no reports of any spin valves fabricated based on this material.

Amorphous TbFeCo presents an alternative route to a tunable tilted anisotropy. While TbFeCo films typically exhibit strong perpendicular anisotropy, one may freely induce an in-plane anisotropy of similar magnitude, by biasing the substrate (resputtering the film) while applying an in-plane magnetic field [41]. The in-plane pair-ordering anisotropy of the Co and Fe ions then adds to the perpendicular anisotropy of the Tb ions, and by varying the CoFe content and/or the resputtering rate, $\beta$ can be varied continuously from fully in-plane to fully perpendicular. While TbFeCo-based spin valves with perpendicular anisotropy have been fabricated with GMR of up to 4% [42, 43], there are no reports of TbFeCo-based spin valves with a TP.

A third approach is to use a polarizer material with strong magnetocrystalline anisotropy and grow it with a texture that directs the easy axis at a tilted angle. (111)-oriented $L1_0$ FePt with a large anisotropy (up to $K_u \sim 7 \times 10^7$ erg cm$^{-3}$) can either be grown on Si (001) substrates with suitable seed layers, or on MgO(111) substrates [44]. In this orientation, the tilt angle is fixed at about $\beta = 36^\circ$ [45]. TP spin valves based on (111) FePt thin films have indeed recently been demonstrated with GMR values approaching 5% [26, 27]. To change $\beta$ one can also grow $L1_0$ FePt films with a different orientation, such as the (101) orientation with $\beta = 45^\circ$ deposited.
on bcc CrW (110) seed layers [46]. However, no spin valves with this orientation have so far been reported. To overcome the limitation of a fixed $\beta$ one may add an in-plane CoFe layer on top of the FePt film. Due to strong exchange coupling, the CoFe magnetization will align with the FePt for thin CoFe. However, as the CoFe layer thickness increases, the CoFe top spins will gradually turn into the plane [47]. By varying the CoFe thickness, one may hence vary $\beta$ from $36^\circ$ to $0^\circ$, and consequently cover the region where the TP device phase diagram shows coexistence of a number of stable static and dynamic states.

6. Conclusions

In conclusion, we have shown by nonlinear system analysis and magnetodynamical simulations that a spin valve with a tilted fixed layer magnetization possesses a surprisingly rich phase diagram of static and dynamic states in the zero magnetic field. The coexistence of several of these states leads to a number of large hysteretic switching behaviors between both static and dynamic states and in particular to hysteresis in the threshold currents for magnetic precession. The analytical eigenvalue analysis and the prediction of large amplitude mode oscillation over a wide range of parameter spaces by our macrospin simulations will be useful to explore new device design and optimization of spin torque nano devices based on TP. We hope our study can stimulate further experimental effort toward this direction and can serve as a guideline for designing such devices.

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