$\epsilon'/\epsilon$ Anomaly and Neutron EDM in $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model with Charge Symmetry

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Abstract

The Standard Model prediction for $\epsilon'/\epsilon$ based on recent lattice QCD results exhibits a tension with the experimental data. We solve this tension through $W_R^+$ gauge boson exchange in the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model with ‘charge symmetry’, whose theoretical motivation is to attribute the chiral structure of the Standard Model to the spontaneous breaking of $SU(2)_R \times U(1)_{B-L}$ gauge group and charge symmetry. We show that $M_{W_R} < 58$ TeV is required to account for the $\epsilon'/\epsilon$ anomaly in this model. Next, we make a prediction for the neutron EDM in the same model and study a correlation between $\epsilon'/\epsilon$ and the neutron EDM. We confirm that the model can solve the $\epsilon'/\epsilon$ anomaly without conflicting the current bound on the neutron EDM, and further reveal that almost all parameter regions in which the $\epsilon'/\epsilon$ anomaly is explained will be covered by future neutron EDM searches, which leads us to anticipate the discovery of the neutron EDM.
1 Introduction

The direct CP violation in $K \rightarrow \pi\pi$ decay parametrized by the $\epsilon'$ parameter is sensitive to physics beyond the Standard Model (SM) due to the suppressed SM contribution. Recent calculation of the hadronic matrix elements with lattice QCD [1, 2, 3] enables us to evaluate the $K \rightarrow \pi\pi$ decay amplitude without relying on any hadron model. On the basis of the above calculation, the same collaboration has reported that the SM prediction is separated from the experimental value [4, 5, 6] by 2.1$\sigma$, and other groups [8, 9] have also obtained predictions for $\epsilon'/\epsilon$ that show a discrepancy of 2.9$\sigma$ and 2.8$\sigma$, respectively. More importantly, the lattice result corroborates the calculation with dual QCD approach [10, 11], which has derived a theoretical upper bound on $\epsilon'/\epsilon$ that is violated by the experimental data and has thus claimed anomaly in this observable. (However, Ref. [12] presents a different calculation that claims the absence of the anomaly.) Some authors have tackled this $\epsilon'/\epsilon$ anomaly in new physics scenarios, such as a general right-handed current [13], the Littlest Higgs model with T-parity [14], supersymmetry [15, 16, 17], non-standard interaction with $Z'$ and/or $Z$ [18, 19], vector-like quarks [20] and $SU(3)_c \times SU(3)_L \times U(1)_X$ gauge group [21].

The $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge extension of the SM is a well-motivated framework for addressing the $\epsilon'/\epsilon$ puzzle, because the flavor mixing matrix for right-handed quarks automatically introduces new CP-violating phases, and $W^+_R$ gauge boson exchange contributes to $\Delta F = 1$ processes at tree level while it contributes to $\Delta F = 2$ processes at loop levels so that other experimental constraints, in particular the constraint from Re($\epsilon$), are readily evaded. Previously, Ref. [13] has shown that a general $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model with an arbitrary right-handed quark mixing can solve the $\epsilon'/\epsilon$ discrepancy. However, a major theoretical motivation for the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model lies in its capability of explaining the origin of the chiral nature of the SM, which is achieved by adding either the left-right parity [22] or the ‘charge symmetry’ [23]. The left-right parity requires invariance of the theory under the Lorentzian parity transformation plus the exchange of $SU(2)_L$ and $SU(2)_R$ gauge groups, while the charge symmetry requires invariance under the charge conjugation plus the exchange of $SU(2)_L$ and $SU(2)_R$, both of which endow the model with a symmetric structure for the left and right-handed fermions at high energies.

In this paper, we study $\epsilon'/\epsilon$ in the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model with charge symmetry. As a consequence of the charge symmetry, the Yukawa matrices are complex symmetric matrices, which restricts the quark mixing matrix associated with $W^+_R$ to be the complex con-

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1 The ‘charge symmetry’ is inspired by D-parity [24] in the SO(10) grand unification theory. However, the model we consider cannot be embedded in the SO(10) theory, since we assume the charge symmetry breaking scale to be below $O(100)$ TeV.
jugate of the SM Cabibbo-Kobayashi-Maskawa (CKM) matrix multiplied by a new CP phase factor for each quark flavor. Given the above restriction, one can evaluate $\epsilon'/\epsilon$ only in terms of two new CP phases, the mass of $W^+_R$ and the ratio of two vacuum expectation values (VEVs) of the bifundamental scalar, which leads to a specific prediction for the model parameters.

Our analysis on $\epsilon'/\epsilon$ proceeds as follows. By integrating out $W_R$, $W_L$ and the top quark, we obtain the Wilson coefficients for $\Delta S = 1$ operators that contribute to $K \rightarrow \pi\pi$ decay. The anomalous dimension matrix is divided into the same two 18 × 18 pieces for 36 operators, for which leading order expressions are obtainable from Refs. [25, 26]. The hadronic matrix elements for current-current operators are seized from the lattice results [2, 3]. We find that among new physics operators, $(\bar{s}u)_L(\bar{u}d)_R$ and $(\bar{s}u)_R(\bar{u}d)_R$ (each with two ways of color contraction) both dominantly contribute to $\epsilon'/\epsilon$. Their contributions are of the same order because the Wilson coefficients of the $(\bar{s}u)_L(\bar{u}d)_R$ operators are suppressed by the hierarchy of two bifundamental scalar VEVs $v_1/v_2 = \tan\beta$, which is about $m_b/m_t$ if there is no fine-tuning in accommodating the top and bottom quark Yukawa couplings, whereas this suppression does not enter into the Wilson coefficients of the $(\bar{s}u)_R(\bar{u}d)_R$ operators. On the other hand, the lattice computation has confirmed that the hadronic matrix elements for the former operators are enhanced compared to the latter. Thus, these operators possibly equally contribute to $\epsilon'/\epsilon$. This result is in contrast to the study of Ref. [13], which has concentrated solely on the $(\bar{s}u)_L(\bar{u}d)_R$ operators.

Once the $\epsilon'/\epsilon$ anomaly is explained in the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model with charge symmetry, correlated predictions for other CP violating observables are of interest. In particular, the neutron electric dipole moment (EDM), an observable sensitive to CP violation in the presence of CPT invariance, receives significant contributions from four-quark operators in $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ models [28, 29, 30, 31, 32, 13] allowing us to discuss future detectability of the neutron EDM in relation to the $\epsilon'/\epsilon$ anomaly.

Our analysis on the neutron EDM starts by integrating out $W_R$, $W_L$ and top quark to obtain the Wilson coefficients for CP-violating operators. The leading order expression for the anomalous dimension matrix is found in Refs. [35, 36, 37, 38]. Regarding the hadronic matrix elements of CP-violating operators, we reveal that the pion VEV $\langle \pi^0 \rangle$ induced by four-quark operators [27] gives the leading contribution to the neutron EDM, which is enhanced by the quark mass ratio $m_s/(m_u + m_d)$ in comparison to the rest. This enhancement is understood as follows: Since $W^+_R-W^+_L$ mixing gives rise to CP-odd and isospin-odd interactions, the pion VEV $\langle \pi^0 \rangle$, which is isospin-odd, can arise without the factor of $m_d - m_u$, and thus can be directly proportional to $1/(m_u + m_d)$. The pion VEV induces a CP-violating coupling for

\[ \text{See also Refs. [33, 34].} \]
neutron $n$, $\Sigma^-$ baryon, and kaon $K^+$ without the factor of $m_d - m_u$ because the $\bar{n}\Sigma^-K^+$ vertex is not isospin-even. Consequently, the CP-violating coupling for $n$, $\Sigma^-$, $K^+$ can appear with the factor of $m_d/(m_u + m_d)$. This coupling contributes to the neutron EDM at the leading chiral order through charged baryon-meson loops. Considering the above-mentioned importance of the pion VEV, we in this paper investigate meson condensation, the resultant CP-violating baryon-meson couplings, and their contributions to the neutron EDM through baryon-meson loops, using chiral perturbation theory.

This paper is organized as follows: In Sec. 2, we review the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model with charge symmetry, with emphasis on new sources of CP violation. In Sec. 3, we present the Wilson coefficients for $\Delta S = 1$ operators in the model, their RG evolutions and the hadronic matrix elements for these operators. The numerical result for $\epsilon'/\epsilon$ is shown at the end of the section. In Sec. 4, we give the Wilson coefficients for CP-violating operators contributing to the neutron EDM. Special care is taken in evaluating meson condensates and their impact on the neutron EDM. The final result is a prediction for the neutron EDM in light of the $\epsilon'/\epsilon$ anomaly. Section 5 is devoted to summary and discussions.

2 \textit{SU(2)}_L\times\textit{SU(2)}_R\times\textit{U(1)}_{B-L} \text{ Model with Charge Symmetry}

We consider $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge theory with charge symmetry. The field content is in Table 1. Hereafter, the fields are expressed in a way that they transform under a $SU(2)_L \times SU(2)_R$ gauge transformation as

$$
\Phi \rightarrow e^{i\theta^{\alpha}_{\tau a}} \Phi e^{-i\theta^{\alpha}_{R^b}}, \quad \Delta_L \rightarrow e^{i\theta^{\alpha}_{\tau a}} \Delta_L e^{-i\theta^{\alpha}_{\tau a}}, \quad \Delta_R \rightarrow (e^{i\theta^{k}_{R^b}})^* \Delta_R (e^{-i\theta^{k}_{R^b}})^*,
$$

$$
Q_L^i \rightarrow e^{i\theta^{\alpha}_{\tau a}} Q_L^i, \quad Q_R^{ci} \rightarrow (e^{i\theta^{k}_{R^b}})^* Q_R^{ci}, \quad L_L^i \rightarrow e^{i\theta^{\alpha}_{\tau a}} L_L^i, \quad L_R^{ci} \rightarrow (e^{i\theta^{k}_{R^b}})^* L_R^{ci}, \quad (1)
$$
with $\theta_L^a$ and $\theta_R^a$ being gauge parameters for $SU(2)_L$ and $SU(2)_R$, respectively. We demand the theory to be invariant under the following ‘charge symmetry’ transformation:

\[
\text{charge conjugation of all gauge fields,}
\]

\[
\text{and } SU(2)_L \leftrightarrow SU(2)_R, \quad Q_L^i \leftrightarrow Q_R^i, \quad L_L^i \leftrightarrow L_R^i, \quad \Phi \leftrightarrow \Phi^T, \quad \Delta_L \leftrightarrow \Delta_R. \tag{2}
\]

The part of the Lagrangian describing $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and Yukawa interactions of quarks is given by

\[
-L \supset Q_L^i \tilde{\sigma}_\mu \left( \frac{1}{2} g_L a^{a \mu} W_L^a \mu + \frac{1}{3} g_X X^\mu \right) Q_L^i + Q_R^i \tilde{\sigma}_\mu \left( -\frac{1}{2} g_R (\sigma^a)^T W_R^a \mu - \frac{1}{3} g_X X^\mu \right) Q_R^i
\]

\[
+ (Y_q)_{ij} Q_L^i \Phi \epsilon_s (Q_R^j)^* + (\bar{Y}_q)_{ij} Q_L^i (\epsilon^T g \epsilon_g) \epsilon_s (Q_R^j)^* + \text{H.c.,} \tag{3}
\]

where $g_L$, $g_R$ and $g_X$ are the gauge coupling constants for $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$ gauge groups, respectively, and $Y_q$ and $\bar{Y}_q$ are the quark Yukawa couplings. $\epsilon_s$ denotes the antisymmetric tensor for Lorentz spinors and $\epsilon_g$ denotes that for the fundamental representation of $SU(2)_L$ or $SU(2)_R$. Invariance under the charge symmetry transformation Eq. (2) leads to the following tree-level relations:

\[
g_L = g_R, \quad (Y_q)_{ij} = (Y_q)_{ji}, \quad (\bar{Y}_q)_{ij} = (\bar{Y}_q)_{ji}. \tag{4}
\]

The $SU(2)_R$ triplet scalar $\Delta_R$ develops a VEV, $v_R$, to break $SU_R(2) \times U(1)_{B-L} \rightarrow U(1)_Y$, and the bi-fundamental scalar $\Phi$ takes a VEV configuration, 

\[
\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v \sin \beta & 0 \\ 0 & v \cos \beta e^{i \alpha} \end{pmatrix}, \tag{5}
\]

to break $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$, where $\alpha$ is the spontaneous CP phase. The VEV of $\Delta_L$ is hereafter neglected, as it is severely constrained from $\rho$-parameter. The resultant mass matrices for $W_L^a$, $W_R^a$ and $X$ gauge bosons read

\[
-L \supset (W_L^-) \begin{pmatrix}
W_L^+ \\
W_R^+
\end{pmatrix} \begin{pmatrix}
g_L^2 v^2/4 & -g_L g_R \sin(2\beta) e^{-i \alpha} v^2/4 \\
-g_L g_R \sin(2\beta) e^{i \alpha} v^2/4 & g_R^2 (v_R^2 + v^2/4)
\end{pmatrix} \begin{pmatrix}
W_L^+ \\
W_R^+
\end{pmatrix} + \frac{1}{2} (W_L^3 \ W_R^3 \ X) \begin{pmatrix}
g_L^2 v^2/4 & -g_L g_R v^2/2 \\
-g_L g_R v^2/2 & g_R^2 (2v_R^2 + v^2/4)
\end{pmatrix} \begin{pmatrix}
W_L^3 \\
W_R^3
\end{pmatrix}. \tag{6}
\]

The mass matrix for the charged gauge bosons is diagonalized as

\[
-L \supset (W^- \ W'^-) \begin{pmatrix}
M_W^2 & 0 \\
0 & M_W^2
\end{pmatrix} \begin{pmatrix}
W^+ \\
W'^+
\end{pmatrix}, \quad \begin{pmatrix}
W^+_L \\
W^+_R
\end{pmatrix} = \begin{pmatrix}
\cos \zeta & -e^{-i \alpha} \sin \zeta \\
e^{i \alpha} \sin \zeta & \cos \zeta
\end{pmatrix} \begin{pmatrix}
W^+_L \\
W^+_R
\end{pmatrix}, \quad \sin(2\zeta) = \frac{2g_L g_R \sin(2\beta) v^2}{(g_L^2 + g_R^2) v^2 + 4g_R^2 v_R^2 - 8M_W^2}. \tag{7}
\]
For $v_R \gg v$ and $g_L = g_R$, we have an important relation for $\zeta$,

$$\sin \zeta \simeq \sin(2\beta) \frac{M_{W'}^2}{M_W^2},$$

which indicates that when we assume $\tan\beta \simeq m_b/m_t$ so that the top and bottom Yukawa couplings are naturally derived, the $W_L$-$W_R$ mixing angle $\zeta$ is smaller than $M_{W'}^2/M_W^2$ by the factor $2m_b/m_t \sim 0.05$.

The quark mass matrices are given by

$$M_u = \frac{v}{\sqrt{2}} \left( \sin \beta Y_q + \cos \beta e^{-i\alpha} \tilde{Y}_q \right), \quad M_d = \frac{v}{\sqrt{2}} \left( \cos \beta e^{i\alpha} Y_q + \sin \beta \tilde{Y}_q \right),$$

which we diagonalize as $M_u = V_u L, \text{diag}(m_u, m_c, m_t) V_u R$ and $M_d = V_d L, \text{diag}(m_d, m_s, m_b) V_d R$, with $V_u L, V_u R, V_d L, V_d R$ being unitary matrices. However, since $Y_q$ and $\tilde{Y}_q$ are complex symmetric matrices, so are $M_u$ and $M_d$, and one can most generally write

$$V_{uR} = \begin{pmatrix} e^{i\phi_u} & 0 & 0 \\ 0 & e^{i\phi_c} & 0 \\ 0 & 0 & e^{i\phi_t} \end{pmatrix} V_{uL}^*, \quad V_{dR} = \begin{pmatrix} e^{i\psi_d} & 0 & 0 \\ 0 & e^{i\psi_s} & 0 \\ 0 & 0 & e^{i\psi_b} \end{pmatrix} V_{dL}^*.$$ (10)

Hence, the SM CKM matrix, $V_L = V_{uL} V_{dL}^T$, and the corresponding flavor mixing matrix for right-handed quarks, $V_R = V_{uR} V_{dR}^T$, are related as

$$V_R = V_{uR} V_{dR}^T = \begin{pmatrix} e^{i\phi_u} & 0 & 0 \\ 0 & e^{i\phi_c} & 0 \\ 0 & 0 & e^{i\phi_t} \end{pmatrix} V_{uL}^* V_{dL}^T \begin{pmatrix} e^{-i\psi_d} & 0 & 0 \\ 0 & e^{-i\psi_s} & 0 \\ 0 & 0 & e^{-i\psi_b} \end{pmatrix} = \begin{pmatrix} e^{i\phi_u} & 0 & 0 \\ 0 & e^{i\phi_c} & 0 \\ 0 & 0 & e^{i\phi_t} \end{pmatrix} V_L^* \begin{pmatrix} e^{-i\psi_d} & 0 & 0 \\ 0 & e^{-i\psi_s} & 0 \\ 0 & 0 & e^{-i\psi_b} \end{pmatrix}.$$ (11)

Eventually, the part of the Lagrangian Eq. (3) describing flavor-changing $W, W'$ interactions is recast, in the unitary gauge, into the form,

$$-\mathcal{L} \supset \frac{g_L}{\sqrt{2}} (V_L)_{ij} U_R^{ij} W^L_{\mu} \bar{\sigma}_\mu D_L^j + \frac{g_R}{\sqrt{2}} (V_L^*)_{ij} e^{i(\phi_i - \psi_j)} U_R^{ij} W^R_{\mu} \sigma_\mu D_R^j + \text{H.c.}$$

$$= \frac{1}{\sqrt{2}} \tilde{U}^{ij} W^L_{\mu} \gamma_5 \left\{ g_L (V_L)_{ij} \cos \zeta P_L + g_R (V_L^*)_{ij} e^{i(\phi_i - \psi_j) + \alpha} \sin \zeta P_R \right\} D_R^j$$

$$+ \frac{1}{\sqrt{2}} \tilde{U}^{ij} W^R_{\mu} \gamma_5 \left\{ -g_L (V_L)_{ij} e^{-i\alpha} \sin \zeta P_L + g_R (V_L^*)_{ij} e^{i(\phi_i - \psi_j)} \cos \zeta P_R \right\} D_R^j + \text{H.c.},$$ (12)

where $U^i$ and $D^i$ denote the Dirac fields of the up and down-type quarks, respectively.

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$^3 U_R = \epsilon_s (U_R^*)^T$, $D_R = \epsilon_s (D_R^*)^T$. 

6
In this paper, we adopt the following convention for the quark phases and $\phi_u, \phi_c, \phi_t, \psi_d, \psi_s, \psi_b$:

First, we redefine the phases of five quarks to render the CKM matrix in the standard form,

$$V_L = \begin{pmatrix} 
  c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
  -s_{12}c_{23} - c_{12}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}e^{i\delta} & s_{23}s_{13}e^{-i\delta} \\
  s_{12}s_{23} - c_{12}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}e^{i\delta} & c_{23}^2c_{13}
\end{pmatrix}. \quad (13)$$

Next, we redefine $\phi_c, \phi_t, \psi_d, \psi_s, \psi_b$ to set $\phi_u = 0$. \quad (14)

Phase convention fixed in this way, all sources of CP violation are parametrized by $\text{Im}[(V_L)_{cd}]$, $\text{Im}[(V_L)_{cs}]$, $\text{Im}[(V_L)_{td}]$, $\text{Im}[(V_L)_{ts}]$, $\text{Im}[(V_L)_{cd}]$, the newly-defined $\phi_c, \phi_t, \psi_d, \psi_s, \psi_b$, and $\alpha$.

3 $\epsilon'/\epsilon$

3.1 Wilson Coefficients for $\Delta S = 1$ Operators

We match the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge theory with charge symmetry to the effective QCD×QED theory in which $W, W'$ bosons and the top quark are integrated out. In the effective theory, the $\Delta S = 1$ Hamiltonian is parametrized as

$$\mathcal{H}_{\Delta S=1} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{i=1,2,...,10,1c,2c} (C_iO_i + C'_iO'_i) + \sum_{j=1,2,1c,2c} (C_j^{RL}O_j^{RL} + C_j^{LR}O_j^{LR}) + \sum_{k=g,\gamma} (C_kO_k + C'_kO'_k) \right\} + \text{H.c.}, \quad (15)$$

where operators $O$'s are defined in Appendix A. We determine the Wilson coefficients as follows:

We approximate $g_R = g_L$ by ignoring difference in RG evolutions of $g_L$ and $g_R$ at scales below $M_{W'}$. Also, for each Wilson coefficient, if multiple terms have an identical phase, we only consider the one in the leading order of $M_W^2/M_{W'}^2$ or $\sin \zeta$. By integrating out $W'$, one obtains the following leading-order matching conditions at a scale $\mu \sim M_{W'}$ (note our convention with
\( \phi_u = 0 \):

\[
C'_2 = \frac{M_W^2}{M_W'^2} (V_L)_{us} (V_L^*)_{ud} e^{i(\psi_s - \psi_d)} \cos^2 \zeta,
\]

\( C'_{2c} = \frac{M_W^2}{M_W'^2} (V_L)_{cs} (V_L^*)_{cd} e^{i(\psi_s - \psi_d)} \cos^2 \zeta, \)

\( C'_4 = C'_6 = \frac{M_W^2}{M_W'^2} \frac{\alpha_s}{4\pi} \cos^2 \zeta \sum_{i=u,c,t} (V_L)_{is} (V_L^*)_{id} e^{i(\psi_s - \psi_d)} \frac{1}{2} F_1(y_i), \quad C'_3 = C'_5 = -\frac{1}{3} C'_4, \)

\( C'_7 = C'_9 = \frac{M_W^2}{M_W'^2} \frac{\alpha}{4\pi} \cos^2 \zeta \sum_{i=u,c,t} (V_L)_{is} (V_L^*)_{id} e^{i(\psi_s - \psi_d)} \frac{2}{3} E_{4d}(y_i), \)

\( \delta C_g = \frac{M_W^2}{M_W'^2} \cos^2 \zeta \sum_{i=u,c,t} m_d m_s (V_L)_{is} (V_L^*)_{id} F_2(y_i), \)

\( \delta C_\gamma = \frac{M_W^2}{M_W'^2} \cos^2 \zeta \sum_{i=u,c,t} (V_L)_{is} (V_L^*)_{id} E_{2d}(y_i), \)

\( \delta C'_g = \frac{M_W^2}{M_W'^2} \cos^2 \zeta \sum_{i=u,c,t} (V_L)_{is} (V_L^*)_{id} F_2(y_i), \)

\( \delta C'_\gamma = \frac{M_W^2}{M_W'^2} \cos^2 \zeta \sum_{i=u,c,t} (V_L)_{is} (V_L^*)_{id} E_{2d}(y_i), \)

with \( y_i \equiv m_i^2 / M_W'^2 \).
By further integrating out $W$ and the top quark, one gains the following leading-order matching conditions at a scale $\mu \sim M_W$ (note our convention with $\phi_u = 0$):

\begin{align}
C_2 &= (V_L^*)_{us}(V_L)_{ud} \cos^2 \zeta, \\
C_{2c} &= (V_L^*)_{cs}(V_L)_{cd} \cos^2 \zeta, \\
C_{2R}^L &= (V_L)_{us}(V_L)_{ud} e^{i(\psi_s - \alpha)} \sin \zeta \cos \zeta, \\
C_{2c}^L &= (V_L)_{us}(V_L)_{ud} e^{i(-\psi_s + \alpha)} \sin \zeta \cos \zeta, \\
C_{2R}^R &= (V_L)_{cs}(V_L)_{cd} e^{i(\phi_c + \psi_s - \alpha)} \sin \zeta \cos \zeta, \\
C_{2c}^R &= (V_L)_{cs}(V_L)_{cd} e^{i(\phi_c - \psi_s - \alpha)} \sin \zeta \cos \zeta, \\
C_4 &= C_6 = \frac{\alpha_s}{4\pi} \cos^2 \zeta \sum_{i=u,c,t} (V_L^*)_{is}(V_L)_{id} \frac{1}{2} F_1(x_i), \\
C_3 &= C_5 = -\frac{1}{3} C_4, \\
C_7 &= C_9 = \frac{\alpha}{4\pi} \cos^2 \zeta \sum_{i=u,c,t} (V_L^*)_{is}(V_L)_{id} \frac{2}{3} E_{1d}(x_i),
\end{align}

\begin{align}
\Delta C_g &= \sum_{i=u,c,t} \left\{ \cos^2 \zeta (V_L^*)_{is}(V_L)_{id} F_2(x_i) + \sin \zeta \cos \zeta \frac{m_i}{m_s} (V_L^*)_{is}(V_L)_{id} e^{i(-\phi_i + \psi_s - \alpha)} F_3(x_i) \right\}, \\
\Delta C_\gamma &= \sum_{i=u,c,t} \left\{ \cos^2 \zeta (V_L^*)_{is}(V_L)_{id} E_{2d}(x_i) + \sin \zeta \cos \zeta \frac{m_i}{m_s} (V_L^*)_{is}(V_L)_{id} e^{i(-\phi_i + \psi_s - \alpha)} E_{3d}(x_i) \right\}, \\
\Delta C_g' &= \sum_{i=u,c,t} \left\{ \cos^2 \zeta \frac{m_d}{m_s} (V_L^*)_{is}(V_L)_{id} F_2(x_i) + \sin \zeta \cos \zeta \frac{m_i}{m_s} (V_L^*)_{is}(V_L)_{id} e^{i(\phi_i - \psi_s + \alpha)} F_3(x_i) \right\}, \\
\Delta C_\gamma' &= \sum_{i=u,c,t} \left\{ \cos^2 \zeta \frac{m_d}{m_s} (V_L^*)_{is}(V_L)_{id} E_{2d}(x_i) + \sin \zeta \cos \zeta \frac{m_i}{m_s} (V_L^*)_{is}(V_L)_{id} e^{i(\phi_i - \psi_s + \alpha)} E_{3d}(x_i) \right\},
\end{align}

with $x_i \equiv m_i^2/M_W^2$,

where loop functions $F_1, F_2, F_3$ and $E_{1d}, E_{2d}, E_{3d}$ are defined in Appendix B. We are aware that the dipole operators receive two contributions with different phases when $W'$ is integrated out and when $W$ is. The two are expressed as $\delta C_g, \delta C_\gamma, \delta C_g', \delta C_\gamma'$ and $\Delta C_g, \Delta C_\gamma, \Delta C_g', \Delta C_\gamma'$, respectively.

We take into account RG evolutions of the Wilson coefficients at order $O(\alpha_s)$. The fact that four sets of operators, $\{O_i\}, \{O_i'\}$ ($i = 1, 2, ..., 10, 1c, 2c$), $\{O_j^{RL}\}, \{O_j^{LR}\}$ ($j = 1, 2, 1c, 2c$), do not mix with each other facilitates the computation. For $\{C_i\}, \{C_j^{RL}\}$ and $\{C_j^{LR}\}$, we assume that their initial conditions at scale $\mu = M_W$ are given by Eqs. \[24-29\] and solve the RG equations from $\mu = M_W$ to the scale for which the lattice results are reported. For $\{C_i'\}$, we assume that their initial conditions at $\mu = M_W'$ are provided by Eqs. \[16-23\] and solve the RG equations from $\mu = M_W'$ to the scale of lattice results. Finally, we compute RG evolutions of
the coefficients of the dipole operators \((C_g, C_γ)\) and \((C'_g, C'_γ)\), which receive contributions from \(\{C_i, C'_i\}\) and \(\{C'_i, C''_i\}\), respectively. The \(O(α_s)\) RG equations for \(\{C_i\}\) and \(\{C'_i\}\) are found in Ref. [26], and those for \(\{C''_i\}\) and \(C_g, C_γ\) are in Ref. [25].

### 3.2 Hadronic Matrix Elements

We employ the lattice calculations of hadronic matrix elements \(⟨(ππ)I|O_i|K^0⟩\) for \(i = 1, 2, ..., 10\) for \(I = 0, 2\) reported by RBC/UKQCD in Refs. [2, 3].

Since lattice calculations for the matrix elements of \(O_{LR}^1\) and \(O_{LR}^2\) are missing, we estimate them from the RBC/UKQCD results using isospin symmetry. In the limit of exact isospin symmetry, we find, for \(ΔI = 3/2\) amplitudes,

\[
⟨(ππ)_{I=2}|O_7|K^0⟩ = \frac{3}{4}⟨(ππ)_{I=2}|(s\bar{d})_L(\bar{u}u - \frac{1}{2}\bar{d}d - \frac{1}{2}\bar{s}s)R|K^0⟩ \tag{34}
\]

where we have discarded \(ΔI = 1/2\) part when obtaining the second line, and when deriving the third line, we have inserted Clebsch-Gordan coefficients for constructing the \(ΔI = 3/2\) operator from a \(ΔI = 1/2\) one and a \(ΔI = 1\) one. For \(ΔI = 1/2\) amplitudes, we find

\[
⟨(ππ)_{I=0}|(\frac{4}{3}O_7 + \frac{2}{3}O_5)|K^0⟩ = ⟨(ππ)_{I=0}|\{(s\bar{d})_L(\bar{u}u + \bar{d}d)R + (s\bar{d})_L(\bar{u}u - \bar{d}d)R\}|K^0⟩ \tag{37}
\]

where in the first line, we have separated \((\bar{u}u)R\) into \(ΔI = 0\) and \(ΔI = 1\) parts, and in the second line, we have inserted Clebsch-Gordan coefficients for constructing the \(ΔI = 1/2\) operator from a \(ΔI = 1/2\) one and a \(ΔI = 0\) one or a \(ΔI = 1\) one. The same relations hold between the matrix elements for \(O_{LR}^1\) and \(O_8, O_6\).

The hadronic matrix elements for the chromo-dipole operators \(O_g, O'_g\) are extracted from the calculation based on dual QCD approach [39]. Note that the above calculation is corroborated by the fact that it is consistent with a lattice calculation of the \(K-π\) hadronic matrix element [40], which is related to the \(K-π\) one by chiral perturbation theory.
3.3 Numerical Analysis of $\epsilon'/\epsilon$

The definition for the decay amplitudes of $K^0 \to \pi\pi$ is

$$A_0 e^{i\delta_0} = \langle (\pi\pi)_{I=0} | H_{\Delta S=1} | K^0 \rangle, \quad A_2 e^{i\delta_2} = \langle (\pi\pi)_{I=2} | H_{\Delta S=1} | K^0 \rangle,$$

where $\delta_{0,2}$ represent the strong phases. In terms of the above amplitudes, one writes the direct CP violation parameter divided by the indirect one as

$$\text{Re} \left( \frac{\epsilon'}{\epsilon} \right) = \text{Re} \left( \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}} \left( \frac{\text{Im} A_2}{\text{Re} A_2} - \frac{\text{Im} A_0}{\text{Re} A_0} \right) \right),$$

where $\omega = \text{Re} A_2/\text{Re} A_0$ is a suppression factor due to the $\Delta I = 1/2$ rule. For the strong phases, we use the values of Refs. [3, 2], $\delta_2 = 23.8 \pm 5.0$ degree and $\delta_0 = -11.6 \pm 2.8$ degree. For the real parts of the decay amplitudes, we employ the experimental data [7], $\text{Re} A_2 = 1.479 \times 10^{-8}$ GeV and $\text{Re} A_0 = 33.20 \times 10^{-8}$ GeV, which leads to $\omega = 4.454 \times 10^{-2}$. In our analysis, we separate the SM and new physics contributions as

$$\text{Re} \left( \frac{\epsilon'}{\epsilon} \right) = \text{Re} \left( \frac{\epsilon'}{\epsilon} \right)_{\text{SM}} + \text{Re} \left( \frac{\epsilon'}{\epsilon} \right)_{\text{NP}}.$$

For the SM part, we quote the calculation in the literature $\text{Re}(\epsilon'/\epsilon)_{\text{SM}} = (1.38 \pm 6.90) \times 10^{-4}$ [3]. It is the new physics part,

$$\text{Re} \left( \frac{\epsilon'}{\epsilon} \right)_{\text{NP}} = \text{Re} \left( \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}} \left( \frac{\text{Im} A_2^{\text{NP}}}{\text{Re} A_2} - \frac{\text{Im} A_0^{\text{NP}}}{\text{Re} A_0} \right) \right),$$

that we compute in this paper. In doing so, we approximate $\cos^2 \zeta = 1$ in the Wilson coefficients Eqs. [24–33], so that the SM contribution is separated from the new physics one at the operator level.

In the analysis, we fix the ratio of the bifundamental scalar VEVs at its natural value as $\tan \beta = m_b/m_t$. We have found numerically that for $M_{W'} > 1$ TeV, the chromo-dipole contribution to $\text{Re}(\epsilon'/\epsilon)$ does not exceed $O(10^{-4})$ and is hence safely neglected. Consequently, only two combinations of new CP phases, $\alpha - \psi_d$ and $\alpha - \psi_s$, and the $W'$ mass determine the new physics contribution.

First, we choose specific values for the new CP phases in the calculation of $\epsilon'/\epsilon$ to illustrate the model prediction. In Fig. [1] the prediction for $\text{Re}(\epsilon'/\epsilon)$ is presented with specific choices of $\alpha - \psi_d$ and $\alpha - \psi_s$.\footnote{When we use a calculation based on the chiral quark model in Ref. [41] to evaluate the hadronic matrix elements of the chromo-dipole operators, we are again lead to the result that the chromo-dipole contribution to $\text{Re}(\epsilon'/\epsilon)$ is below $O(10^{-4})$ for $M_{W'} > 1$ TeV.}
\[ \alpha - \psi_d = \pi/3, \alpha - \psi_s = \pi/4 \]
\[ \alpha - \psi_d = \pi/2, \alpha - \psi_s = \pi/3 \]
\[ \alpha - \psi_d = \pi, \alpha - \psi_s = -\pi/4 \]

Figure 1: Numerical result of \( \text{Re}(\epsilon'/\epsilon) \). The blue band represents the 1\( \sigma \) range of experimental data given by PDG \(^7\), while model predictions with specific choice of phases are shown by the lines.

Next, we randomly vary \( \alpha - \psi_d \) and \( \alpha - \psi_s \) in the range \([0, 2\pi]\), since they are free parameters. In Fig. 2 we show the region of \( \text{Re}(\epsilon'/\epsilon) \) obtained by varying \( \alpha - \psi_d \) and \( \alpha - \psi_s \). One observes that \( M_{W'} < 58 \text{ TeV} \) is necessary for 1\( \sigma \) explanation of the anomaly.

Figure 2: Numerical result of \( \text{Re}(\epsilon'/\epsilon) \). The blue band represents the experimental data given by PDG \(^7\), while each red dot corresponds to the model prediction with a randomly generated set of \((\alpha - \psi_d, \alpha - \psi_s)\).
We have confirmed that among the terms of $\Delta S = 1$ Hamiltonian Eq. (15), $\sum_{i=1,2} C'_i O'_i$ and $\sum_{j=1,2} (C'^R_j O'^R_j + C'^L_j O'^L_j)$ are the leading sources of the new physics contribution.

4 Neutron Electric Dipole Moment

4.1 Wilson Coefficients for Operators contributing to the neutron EDM

In the effective QCD×QED theory in which $W, W'$ and the top quark are integrated out, the part of the CP-violating Hamiltonian that contributes to the neutron EDM is parametrized as

$$\mathcal{H}_{nEDM} = \frac{G_F}{\sqrt{2}} \left\{ \sum_q \sum_{i=1,2,4,5} C_{iq} O_{iq} + C_3 O_3 + \sum_{q \neq q'} \sum_{i=1}^4 C_{iq'q} O_{iq'q} \right\},$$

where operators $O$’s are defined in Appendix C.

We determine the Wilson coefficients $C$’s as follows: Again, for each coefficient, if multiple terms have an identical phase, we exclusively consider the one in the leading order of $M_W^2/M_{W'}^2$, or $\sin \zeta$. By integrating out $W$ and the top quark, one obtains the following leading-order matching conditions at $\mu \sim M_W$ (note our convention with $\phi_u = 0$):

$$C_{2du} = -C_{2ad} = 4 \sin \zeta \cos \zeta \ \text{Im} \left[ (V_L)_{ud} (V_L)_{ud} e^{i(\psi_d - \alpha)} \right], \quad (d \to s),$$

$$C_{2dc} = -C_{2cd} = 4 \sin \zeta \cos \zeta \ \text{Im} \left[ (V_L)_{cd} (V_L)_{cd} e^{i(-\phi_c + \psi_d - \alpha)} \right], \quad (d \to s),$$

$$C_{1u} = \frac{e}{4\pi^2} \sin \zeta \cos \zeta \ \sum_{i=d,a,b} \frac{m_i}{m_u} \ \text{Im} \left[ (V_L)_{ui} (V_L)_{ui} e^{i(\psi_i - \alpha)} \right] E_{3u}(x_i),$$

$$C_{1c} = \frac{e}{4\pi^2} \sin \zeta \cos \zeta \ \sum_{i=d,a,b} \frac{m_i}{m_u} \ \text{Im} \left[ (V_L)_{ui} (V_L)_{ui} e^{i(-\phi_c + \psi_i - \alpha)} \right] E_{3u}(x_i),$$

$$C_{1d} = \frac{e}{4\pi^2} \sin \zeta \cos \zeta \ \sum_{i=u,c,t} \frac{m_i}{m_d} \ \text{Im} \left[ (V_L)_{id}(V_L)_{id} e^{i(-\phi_i + \psi_d - \alpha)} \right] E_{3d}(x_i), \quad (d \to s),$$

$$C_{2u} = -\frac{g_s}{4\pi^2} \sin \zeta \cos \zeta \ \sum_{i=d,a,b} \frac{m_i}{m_u} \ \text{Im} \left[ (V_L)_{ui}(V_L)_{ui} e^{i(\psi_i - \alpha)} \right] F_3(x_i),$$

$$C_{2c} = -\frac{g_s}{4\pi^2} \sin \zeta \cos \zeta \ \sum_{i=d,a,b} \frac{m_i}{m_u} \ \text{Im} \left[ (V_L)_{ui}(V_L)_{ui} e^{i(-\phi_c + \psi_i - \alpha)} \right] F_3(x_i),$$

$$C_{2d} = \frac{g_s}{4\pi^2} \sin \zeta \cos \zeta \ \sum_{i=u,c,t} \frac{m_i}{m_d} \ \text{Im} \left[ (V_L)_{id}(V_L)_{id} e^{i(-\phi_i + \psi_d - \alpha)} \right] F_3(x_i), \quad (d \to s),$$

$$C_3 \simeq \frac{4g_s^3}{(16\pi^2)^2} \sin \zeta \cos \zeta \ \frac{m_t}{m_b} F_3(x_t) \ \text{Im} \left[ (V_L)_{tb}(V_L)_{tb} e^{i(-\phi_t + \psi_b - \alpha)} \right],$$

with $x_i \equiv m_i^2 / M_W^2$,

where loop functions $F_1, F_2, F_3$ and $E_{1d}, E_{2d}, E_{3d}, E_{3u}$ are defined in Appendix B. In Eq. (53) (which corresponds to the Weinberg operator $[42]$), we present the dominant part proportional
to $m_l$. Terms obtained by integrating out $W'$ possess the same phases as Eqs. (45–53) and are simply suppressed by $M_W^2/M_{W'}^2$ compared to Eqs. (45–53). They are therefore neglected in our analysis.

The RG equations at order $O(\alpha_s)$ for the Wilson coefficients are obtainable in Refs. [35, 36, 37, 38]. We assume that the initial conditions at $\mu = M_W$ for the RG equations are given by Eqs. (45–53), and solve the equations from $\mu = M_W$ to $\mu = 1$ GeV. At the 1 GeV scale, we evaluate the hadronic matrix elements.

### 4.2 Hadronic Matrix Elements

#### 4.2.1 Four-quark operators $O_{1q'q}$, $O_{2q'q}$

In the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model with charge symmetry, the Wilson coefficients for the four-quark operators $O_{1q'q} = \langle \bar{q} q' (\bar{q} i\gamma_5 q') \rangle$ and $O_{2q'q} = \langle \bar{q}_\alpha q'_\beta (\bar{q}_\beta \gamma_5 q_\alpha) \rangle$ ($q \neq q'$; $q, q' = u, d, s$) are particularly large. Therefore, we scrutinize how these operators contribute to the neutron EDM. Operators $O_{1q'q}$ contribute in the following three ways:

- The first one is through meson condensation [27]: $O_{1q'q}$ operators give rise to tadpole terms for pseudoscalar mesons and induce their VEVs. These VEVs generate CP-violating interactions for baryons and mesons, which contribute to the neutron EDM through baryon-meson loop diagrams.

- The second one is through hadronic matrix elements of $O_{1q'q}$ with baryons and mesons, $\langle BM | O_{1q'q} | B \rangle$ ($B$ denotes a baryon and $M$ a meson), which contribute to the neutron EDM through baryon-meson loop diagrams.

- The third one is directly through the hadronic matrix element of $O_{1q'q}$ with neutrons and photon.

On the other hand, $O_{2q'q}$ operators do not yield meson condensation, but do contribute to the neutron EDM in the latter two ways. Later, it will be shown that the contribution from the pion VEV $\langle \pi^0 \rangle$, which belongs to the first category, is enhanced by the factor $m_s/(m_u + m_d)$ compared to the latter two. We therefore investigate how $O_{1q'q}$ operators bring about meson condensation, thereby contributing to the neutron EDM.

We are aware that if Peccei-Quinn mechanism [43] exists, it affects the meson condensation and also induces an effective non-zero $\bar{\theta}$ term due to incomplete cancellation between the genuine $\bar{\theta}$ term and the axion VEV. Alternatively, it is logically possible to assume $\bar{\theta} = 0$ without Peccei-Quinn mechanism, by considering an unknown mechanism or through fine-tuning, in
which case we do not need to take into account the effect of Peccei-Quinn mechanism or that of non-zero $\bar{\theta}$. In this paper, we consider both cases where (i) one has $\bar{\theta} = 0$ without Peccei-Quinn mechanism, and (ii) Peccei-Quinn mechanism is at work.

We start from the case with $\bar{\theta} = 0$ without Peccei-Quinn mechanism. The meson condensation contribution is evaluated by the following steps:

(1) First, we implement $\sum C_{1q'q}O_{1q'q}$ part of the Hamiltonian Eq. (44) into the meson chiral Lagrangian. To this end, we rewrite

$$\sum_{q \neq q', q, q' = u, d, s} C_{1q'q} O_{1q'q} = \sum_{i, j, k, l = u, d, s} i C_{ijkl} LRLR^{ijkl} (\bar{q}_i L q_j R) (\bar{q}_k L q_l R) + i C_{ijkl} RLLR^{ijkl} (\bar{q}_i R q_j L) (\bar{q}_k L q_l R) \} - (L \leftrightarrow R),$$

with $C_{ijkl} LRLR^{ijkl} = C_{ijkl} RLLR^{ijkl} \equiv \sum_{q \neq q'} C_{1q'q} \delta_{iq} \delta_{jq} \delta_{kq} \delta_{lq}$.

It then becomes clear that the theory would be invariant (except for $U(1)_A$ anomaly) if coefficients $C_{ijkl} LRLR^{ijkl}$ and $C_{ijkl} RLLR^{ijkl}$ transformed under $U(3)_L \times U(3)_R$ rotations, $L \times R$,

$$C_{ijkl} LRLR^{ijkl} \rightarrow \sum_{m, n, o, p} (L)_{im} (L)_{ko} C_{mnop}^{LRLR} (R^\dagger)_{nj} (R^\dagger)_{pl},$$

$$C_{ijkl} RLLR^{ijkl} \rightarrow \sum_{m, n, o, p} (R)_{im} (L)_{ko} C_{mnop}^{RLLR} (L^\dagger)_{nj} (R^\dagger)_{pl}.$$

From the above transformation property and the parity invariance of QCD, the meson chiral Lagrangian at order $O(p^2)$ plus the leading CP-violating terms is found to be (remind that $\bar{\theta} = 0$ has been assumed)

$$\mathcal{L}_{\text{mesons}} = \frac{F_\pi^2}{4} \text{tr} [(D_\mu U)^\dagger D^\mu U + \chi(U + U^\dagger)] + \frac{F_0^2 - F_\pi^2}{12} \text{tr} [UD_\mu U^\dagger] \text{tr} [U^\dagger D^\mu U]$$

$$+ a_0 \text{tr} [\log U - \log U^\dagger]^2$$

$$+ \frac{G_F}{\sqrt{2}} \sum_{i, j, k, l = u, d, s} \left\{ i C_{ijkl} LRLR^{ijkl} (c_1[U]_{ji}[U]_{lk} - c_1[U^\dagger]_{ji}[U^\dagger]_{lk} + c_2[U]_{jl}[U]_{jk} - c_2[U^\dagger]_{jl}[U^\dagger]_{jk})

+ i C_{ijkl} RLLR^{ijkl} (c_3[U^\dagger]_{ji}[U]_{lk} - c_3[U]_{ji}[U^\dagger]_{lk}) \right\},$$

(56)

where $C_{ijkl} LRLR^{ijkl}$, $C_{ijkl} RLLR^{ijkl}$ have been defined in Eq. (55). Here, $U$ is a nonlinear representation of the nine Nambu-Goldstone bosons that transforms under $U(3)_L \times U(3)_R$ rotations $L \times R$ as
\( U \to RUL \), and \( \chi \) includes the quark mass term, which are given by

\[
U = \exp \left[ \frac{2i}{\sqrt{6}F_0} \eta_0 I_3 + \frac{2i}{F_\pi} \Pi \right], \quad \Pi \equiv \begin{pmatrix}
\frac{1}{2} \pi^0 + \frac{1}{2\sqrt{3}} \eta_8 & \frac{1}{2\sqrt{2}} \pi^+ & \frac{1}{\sqrt{2}} K^+
\frac{1}{\sqrt{2}} \pi^- & \frac{1}{2\sqrt{3}} \eta_8 & \frac{1}{2\sqrt{2}} K^0
\frac{1}{\sqrt{2}} \pi^- & \frac{1}{2\sqrt{2}} K^0 & -\frac{1}{\sqrt{2}} \eta_8
\end{pmatrix}, \quad (57)
\]

\( I_3 \equiv \text{diag}(1,1,1) \),

\[
\chi = 2B_0 \text{ diag}(m_u, m_d, m_s). \quad (58)
\]

\([U]_{ij}\) denotes the \((i,j)\) component of matrix \(U\). \(F_\pi\) is the pion decay constant in the chiral limit and \(F_0\) is the decay constant for \(\eta_0\), which we approximate as \(F_0 \simeq F_\pi\). \(B_0\) satisfies \(B_0 \simeq m_0^2/(m_u + m_d)\). The term with \(\log U\) represents instanton effects, whose expression is exact in the large \(N_c\) limit [14], and \(a_0\) satisfies \(48a_0/F_0^2 \simeq m_\eta^2 + m_{\eta'}^2 - 2m_K^2\). \(c_1\), \(c_2\) and \(c_3\) are unknown low energy constants (LECs), which can be estimated by naive dimensional analysis [15] as

\[
c_1 \sim c_2 \sim c_3 \sim \frac{(4\pi F_\pi)^6}{(4\pi)^4}. \quad (59)
\]

(2) The CP-violating part of the Lagrangian Eq. [56] contains tadpole terms for mesons, which lead to non-zero meson VEVs. Assuming that electric charge and strangeness are not broken spontaneously, we obtain the following potential for neutral mesons \(\pi^0\), \(\eta_8\) and \(\eta_0\):

\[
V(\pi^0, \eta_8, \eta_0) = F_\pi^2 B_0 \left\{ m_u \cos \left( \frac{\pi^0}{F_\pi} + \frac{\eta_8}{\sqrt{3}F_\pi} + \frac{2\eta_0}{\sqrt{6}F_0} \right) + m_d \cos \left( -\frac{\pi^0}{F_\pi} + \frac{\eta_8}{\sqrt{3}F_\pi} + \frac{2\eta_0}{\sqrt{6}F_0} \right) + m_s \cos \left( -\frac{2\eta_8}{\sqrt{3}F_\pi} + \frac{2\eta_0}{\sqrt{6}F_0} \right) \right\} - 24 \frac{a_0}{F_0^2} (\eta_0)^2
\]

\[- G_F \sqrt{2} \left\{ (C_{1ud} + C_{1du}) \sin \left( \frac{2\eta_8}{\sqrt{3}F_\pi} + \frac{4\eta_0}{\sqrt{6}F_0} \right) + (C_{1us} + C_{1su}) \sin \left( \frac{\pi^0}{F_\pi} - \frac{\eta_8}{\sqrt{3}F_\pi} + \frac{4\eta_0}{\sqrt{6}F_0} \right) \right\} \right\}
\]

\[- G_F \sqrt{2} \left\{ (C_{1ud} - C_{1du}) \sin \left( \frac{-\pi^0}{F_\pi} \right) + (C_{1us} - C_{1su}) \sin \left( \frac{-\pi^0}{F_\pi} - \frac{3\eta_8}{F_\pi} \right) \right\} \right\}, \quad (60)
\]

The above potential is minimized with non-zero meson VEVs, \(\langle \pi^0 \rangle\), \(\langle \eta_8 \rangle\) and \(\langle \eta_0 \rangle\). Insofar as we are concerned with vertices with one meson, the physical modes of \(\pi^0\), \(\eta_8\) and \(\eta_0\) fields can
be approximated as
\[ \pi^0_{\text{phys}} \simeq \pi^0 - \langle \pi^0 \rangle, \quad \eta_{\text{phys}} \simeq \eta - \langle \eta \rangle, \quad \eta_{0\text{phys}} \simeq \eta_0 - \langle \eta_0 \rangle. \] (61)

In the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model with charge symmetry, there hold relations $C_{1ud} \simeq -C_{1du}$ and $|C_{1ud}| \gg |C_{1sq}|, |C_{1qs}|$ ($q = u, d$). When $(C_{1ud} + C_{1da})$ and $C_{1sq}, C_{1qs}$ are neglected accordingly, one finds, for small VEVs,

\[ \frac{\langle \pi^0 \rangle}{F_\pi} \simeq \frac{G_F}{\sqrt{2}} (C_{1ud} - C_{1du}) \frac{c_3}{B_0 F_\pi^2} B_0 F_\pi^2 (m_u + m_d) m_s + 8a_0 (m_u + m_d + 4m_s), \]
\[ \frac{\langle \eta \rangle}{F_\pi} \simeq \frac{G_F}{\sqrt{2}} (C_{1ud} - C_{1du}) \frac{c_3}{\sqrt{3} B_0 F_\pi^2} (m_d - m_u) B_0 F_\pi^2 m_u m_d m_s + 8a_0 (m_u m_d + m_d m_s + m_u m_s) + 24a_0, \]
\[ \frac{\langle \eta_0 \rangle}{F_0} \simeq \frac{G_F}{\sqrt{2}} (C_{1ud} - C_{1du}) \frac{\sqrt{2} c_3}{\sqrt{3} B_0 F_\pi^2} (m_d - m_u) B_0 F_\pi^2 m_u m_d m_s + 8a_0 (m_u m_d + m_d m_s + m_u m_s). \] (62)

Note that $\langle \eta \rangle$ and $\langle \eta_0 \rangle$ are proportional to $m_d - m_u$. This is because these VEVs are isospin singlets and hence must be constructed from the product of isospin-odd coefficient $C_{1ud} - C_{1du}$ and isospin-odd mass term $m_d - m_u$. In contrast, $\langle \pi^0 \rangle$ does not contain $m_d - m_u$ because this VEV is isospin-violating. Since $20a_0 \sim B_0 F_\pi^2 m_s$ holds empirically, we find from Eq. (62) that $\langle \pi^0 \rangle$ is much larger than $\langle \eta \rangle$ and $\langle \eta_0 \rangle$ by the factor $m_s/(m_d - m_u)$.

(3) Meson condensation breaks CP symmetry (and $U(3)_L \times U(3)_R$ symmetry) and induces CP-violating interactions for baryons and mesons. To study these interactions, we write the baryon chiral Lagrangian at order $O(p^2)$ as (terms irrelevant in the current discussion are omitted)

\[ \mathcal{L}_{\text{baryons}} = \text{tr} \left[ \bar{B} i \gamma^\mu (\partial_\mu B + [\Gamma_\mu, B]) - M_B \bar{B} B \right] \\
- \frac{D}{2} \text{tr} \left[ \bar{B} \gamma^\mu \gamma_5 \xi_\mu, B \right] - \frac{F}{2} \text{tr} \left[ \bar{B} \gamma^\mu \gamma_5 [\xi_\mu, B] \right] - \frac{\lambda}{2} \text{tr} \left[ \xi_\mu \right] \text{tr} \left[ B \gamma^\mu \gamma_5 B \right] \\
+ b_D \text{tr} \left[ \bar{B} \{\chi_+, B \} \right] + b_F \text{tr} \left[ B \{\chi_+, B \} \right] + b_0 \text{tr} \left[ \chi_+ \right] \text{tr} \left[ \bar{B} B \right] + \ldots, \] (63)

where $B$ represents baryons and $\xi_L, \xi_R$ include mesons as

\[ B = \begin{pmatrix} \Sigma^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda^0 & n \\ \Xi^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda^0 & -\frac{2}{\sqrt{3}} \Lambda^0 \end{pmatrix}, \] (64)

\[ U = \xi_R^\dagger \xi_L, \] (65)

\[ \xi_R = \xi_L^\dagger. \] (66)
\[ \Gamma_\mu \equiv \frac{1}{2} \xi_\mu^R (\partial_\mu - i r_\mu) \xi_R + \frac{1}{2} \xi_\mu^L (\partial_\mu - i l_\mu) \xi_L, \]

\[ \xi_\mu \equiv i \xi_\mu^R (\partial_\mu - i r_\mu) \xi_R - i \xi_\mu^L (\partial_\mu - i l_\mu) \xi_L, \]

\[ \chi_+ \equiv 2 B_0 \xi^\dagger_L \text{diag}(m_u, m_d, m_s) \xi_R + 2 B_0 \xi^\dagger_R \text{diag}(m_u, m_d, m_s) \xi_L. \]

\[ M_B \] is the baryon mass in the chiral limit. We insert meson VEVs \( \langle \pi^0 \rangle, \langle \eta_8 \rangle, \langle \eta_0 \rangle \) into the baryon chiral Lagrangian Eq. (63) and extract CP-violating interaction terms involving neutron \( n \). We thus obtain

\[ L_{\text{baryons}} \supset \bar{g}_{n\pi} \bar{n}n \pi^0_{\text{phys}} + \bar{g}_{nn8} \bar{n}n \eta_{\text{phys}} + \bar{g}_{nn0} \bar{n}n \eta_{0\text{phys}} + \bar{g}_{nn\pi}(\bar{n}n\pi^+ + \bar{n}n\pi^-) + \bar{g}_{n\Sigma K^0}(\bar{n}n\Sigma^0 K^0 + \bar{n}n\Sigma^0 K^0) + \bar{g}_{n\Sigma^- K^+}(\bar{n}n\Sigma^- K^+ + \bar{n}n\Sigma^- K^+) + \bar{g}_{n\Lambda K}(\bar{n}n\Lambda K^0 + \bar{n}n\Lambda K^0), \]
where the coupling constants are given by

\[
\bar{g}_{n\pi} = \frac{B_0}{F_\pi} \left[ \frac{4}{\sqrt{3}} \left\{ b_0(m_d - m_u) + (b_D + b_F)m_d \right\} \langle \pi^0 \rangle \right] F_\pi \left( \frac{\langle \eta_8 \rangle}{F_\pi} + \sqrt{2} \frac{\langle \eta_0 \rangle}{F_0} \right), \tag{71}
\]

\[
\bar{g}_{n\eta} = \frac{B_0}{F_\pi} \left[ \frac{4}{\sqrt{3}} \left\{ b_0(m_d - m_u) + (b_D + b_F)m_d \right\} \langle \pi^0 \rangle \right] F_\pi \left( \frac{\langle \eta_8 \rangle}{F_\pi} + \frac{1}{\sqrt{3}} \langle \eta_0 \rangle \right), \tag{72}
\]

\[
\bar{g}_{n\eta} = \frac{B_0}{F_\pi} \left[ \frac{4\sqrt{2}}{\sqrt{3}} \left\{ b_0(m_d - m_u) + (b_D + b_F)m_d \right\} \langle \pi^0 \rangle \right] F_\pi \left( \frac{\langle \eta_8 \rangle}{F_\pi} + \frac{1}{\sqrt{3}} \langle \eta_0 \rangle \right), \tag{73}
\]

\[
\bar{g}_{n\pi} = \frac{B_0}{F_\pi} \left[ \frac{4\sqrt{2}}{\sqrt{3}} \left\{ b_0(m_d - m_u) + (b_D + b_F)m_d \right\} \langle \pi^0 \rangle \right] F_\pi \left( \frac{\langle \eta_8 \rangle}{F_\pi} + \frac{1}{\sqrt{3}} \langle \eta_0 \rangle \right), \tag{74}
\]

\[
\bar{g}_{n\eta} = \frac{B_0}{F_\pi} \left[ \frac{4\sqrt{2}}{\sqrt{3}} \left\{ b_0(m_d - m_u) + (b_D + b_F)m_d \right\} \langle \pi^0 \rangle \right] F_\pi \left( \frac{\langle \eta_8 \rangle}{F_\pi} + \frac{1}{\sqrt{3}} \langle \eta_0 \rangle \right), \tag{75}
\]

\[
\bar{g}_{n\pi} = \frac{B_0}{F_\pi} \left[ \frac{4\sqrt{2}}{\sqrt{3}} \left\{ b_0(m_d - m_u) + (b_D + b_F)m_d \right\} \langle \pi^0 \rangle \right] F_\pi \left( \frac{\langle \eta_8 \rangle}{F_\pi} + \frac{1}{\sqrt{3}} \langle \eta_0 \rangle \right), \tag{76}
\]

\[
\bar{g}_{n\lambda} = \frac{B_0}{F_\pi} \left[ \frac{4\sqrt{2}}{\sqrt{3}} \left\{ b_0(m_d - m_u) + (b_D + b_F)m_d \right\} \langle \pi^0 \rangle \right] F_\pi \left( \frac{\langle \eta_8 \rangle}{F_\pi} + \frac{1}{\sqrt{3}} \langle \eta_0 \rangle \right), \tag{77}
\]

Note in particular that \( \langle \pi^0 \rangle \) enters into the expression for \( \bar{g}_{n\pi} \) without the factor of \( m_d - m_u \), which is allowed because the coupling \( \bar{g}_{n\pi} \) violates isospin. It follows that \( \bar{g}_{n\pi} \) is enhanced by the factor \( m_\pi/(m_u + m_d) \), as it contains a term \( m_\pi \langle \pi^0 \rangle \).

We compare the above meson-VEV-induced CP-violating couplings with those arising from direct hadronic matrix elements of \( O_{1q'q} \) and \( O_{2q'q} \). The latter are estimated by naïve dimen-
sional analysis [45] as \(\bar{g}_{BBM}|_{\text{direct}} \sim \frac{G_F}{\sqrt{2}} \sum_{i=1,2} \sum_{q,q'} |C_{iq'q}| \frac{1}{F_\pi} \frac{(4\pi F_\pi)^3}{2^i (4\pi)^2}. \) (78)

On the other hand, \(\bar{g}_{n\Sigma^-K^+}\) Eq. (76), for example, is estimated to be

\[
\bar{g}_{n\Sigma^-K^+} \simeq -\frac{B_0}{F_\pi} \frac{m_s}{\sqrt{2}} \bar{g}_{np\pi} \frac{\langle p^0 \rangle}{F_\pi} \sim -\frac{G_F}{\sqrt{2}} \frac{m_s}{\sqrt{2}} \frac{(4\pi F_\pi)^6}{(4\pi)^4 F_\pi^3}, \tag{79}
\]

where Eq. (62) and the naïve dimensional analysis on \(c_3\) Eq. (59) are in use. Noting that \((b_D - b_F)(4\pi F_\pi) \sim 1\) holds numerically, we observe that the meson VEV contribution Eq. (79) dominates over the direct hadronic matrix element one Eq. (78) by the factor \(m_s/(m_u + m_d)\).

This fact allows us to neglect the latter contribution in the rest of the analysis.

(4) The neutron EDM receives contributions from baryon-meson loop diagrams involving a CP-violating coupling of Eqs. (70), a CP-conserving baryon-meson axial-vector coupling and a photon coupling. We refer to the loop calculation of Ref. [50] performed with infrared regularization [51, 52], from which the neutron EDM, \(d_n\), is obtained as

\[
d_n|_{\text{loop}} = \frac{e}{8\pi^2 F_\pi} \left\{ \bar{g}_{np\pi} \frac{\langle p^0 \rangle}{\sqrt{2}} (D + F) \left( \frac{1}{\bar{\epsilon}} - 1 - \log \frac{m_\pi^2}{\mu^2} + \frac{\pi m_\pi}{2m_N} \right) \right. \\
- \frac{\bar{g}_{n\Sigma^-K^+}}{\sqrt{2}} (D - F) \left( \frac{1}{\bar{\epsilon}} - 1 - \log \frac{m_K^2}{\mu^2} + \frac{\pi m_K}{2m_N} - \frac{\pi (m_\Sigma - m_N)}{m_K} \right) \right\}. \tag{80}
\]

Here, the divergent part \(1/\bar{\epsilon} \equiv 1/\epsilon - \gamma_E + \log(4\pi)\) and the scale \(\mu\) stem from dimensional regularization in \(4 - 2\epsilon\) dimension with mass parameter \(\mu\). In fact, the baryon chiral Lagrangian contains a LEC which cancels the above divergence and whose finite part contributes to the neutron EDM. The impact of the finite part of the LEC is assessed by naïve dimensional analysis [45] as

\[
d_n|_{\text{LEC}} \sim \frac{G_F}{\sqrt{2}} \sum_{i=1,2} \sum_{q,q'} |C_{iq'q}| e^\frac{4\pi F_\pi}{(4\pi)^2}. \tag{81}
\]

On the other hand, from Eqs. (62) and (76) and the estimate on \(c_3\) Eq. (59), the finite part of the loop contribution Eq. (80) is estimated to be

\[
d_n|_{\text{loop}} \sim \frac{e}{8\pi^2 F_\pi} \frac{\bar{g}_{n\Sigma^-K^+}}{\sqrt{2}} (D - F) \\
- \frac{e}{8\pi^2 F_\pi} \frac{G_F}{\sqrt{2}} (C_{1ud} - C_{1du})(b_D - b_F) \frac{2m_s}{m_u + m_d} (4\pi F_\pi)^6 (D - F). \tag{82}
\]

There are also studies in which the direct hadronic matrix elements are estimated with vacuum saturation approximation [46, 47, 48, 49] and with hadron models [29, 50].
Since \((b_D - b_F)(4\pi F_\pi) \sim 1\) and \(D - F \sim 1\), we find that the loop contribution Eq. \((82)\) dominates over the LEC one Eq. \((81)\) by the factor \(m_s/(m_u + m_d)\). It is thus justifiable to estimate \(d_n\) by simply extracting the finite part of the loop contribution. We further set \(\mu = m_N\), since \(m_N\) is a natural cutoff scale, and arrive at

\[
d_n \sim \frac{e}{8\pi^2 F_\pi} \left\{ \frac{\bar{g}_{n\pi}}{\sqrt{2}} (D + F) \left( -1 - \log \frac{m_\pi^2}{m_N^2} + \frac{\pi m_\pi}{2m_N} \right) - \frac{\bar{g}_{nK^-K^+}}{\sqrt{2}} (D - F) \left( -1 - \log \frac{m_K^2}{m_N^2} + \frac{\pi m_K}{2m_N} - \frac{\pi (m_\Sigma - m_N)}{m_K} \right) \right\}. \tag{83}\]

Next, we study the case with Peccei-Quinn mechanism. We incorporate the axion field, \(a\), into the meson Lagrangian Eq. \((56)\) by performing \(U(3)_A\) chiral rotations to remove the gluon theta term and transform the quark fields as

\[
\begin{align*}
    u_L &\to e^{-i\alpha_u/2} u_L, \quad u_R \to e^{i\alpha_u/2} u_R, \quad d_L \to e^{-i\alpha_d/2} d_L, \quad d_R \to e^{i\alpha_d/2} d_R, \\
    s_L &\to e^{-i\alpha_s/2} s_L, \quad s_R \to e^{i\alpha_s/2} s_R,
\end{align*} \tag{84}\]

where \(\alpha_u, \alpha_d, \alpha_s\) include the axion field \(a\) as

\[
\begin{align*}
    \alpha_u &= \frac{m_d m_s}{m_u m_d + m_d m_s + m_s m_u} \left( \frac{a}{f_a} + \tilde{\theta} \right), \\
    \alpha_d &= \frac{m_s m_u}{m_u m_d + m_d m_s + m_s m_u} \left( \frac{a}{f_a} + \tilde{\theta} \right), \\
    \alpha_s &= \frac{m_u m_d}{m_u m_d + m_d m_s + m_s m_u} \left( \frac{a}{f_a} + \tilde{\theta} \right),
\end{align*}
\]

with \(f_a\) denoting the axion decay constant and \(\tilde{\theta}\) being the genuine theta term. (With the above choice of \(\alpha_u, \alpha_d, \alpha_s\), the axion does not mix with \(\pi^0\) or \(\eta_8\).) As a result, the axion field is associated with the quark masses and the coefficients \(C_{1q'q}\), and can thus be implemented in the meson chiral Lagrangian through these terms. Accordingly, the meson potential Eq. \((60)\)
is modified to the potential of $\pi^0$, $\eta_8$, $\eta_0$ and axion $a$,

\[
V(\pi^0, \eta_8, \eta_0, a) = F^2_\pi B_0 \left\{ m_u \cos \left( \frac{\pi^0}{F_\pi} + \frac{\eta_8}{\sqrt{3} F_\pi} + \frac{2 \eta_0}{\sqrt{6} F_0} + \alpha_u \right) \\
+ m_d \cos \left( -\frac{\pi^0}{F_\pi} + \frac{\eta_8}{\sqrt{3} F_\pi} + \frac{2 \eta_0}{\sqrt{6} F_0} + \alpha_d \right) \\
+ m_s \cos \left( -\frac{2 \eta_8}{\sqrt{3} F_\pi} + \frac{2 \eta_0}{\sqrt{6} F_0} + \alpha_s \right) \right\} - 24 \frac{a_0}{F_0^2} (\eta_0)^2
\]

\[
= -2c_1 \left\{ (C_{1ud} + C_{1du}) \sin \left( \frac{2 \eta_8}{\sqrt{3} F_\pi} + \frac{4 \eta_0}{\sqrt{6} F_0} + \alpha_u + \alpha_d \right) \\
+ (C_{1us} + C_{1su}) \sin \left( \frac{\pi^0}{F_\pi} - \frac{\eta_8}{\sqrt{3} F_\pi} + \frac{4 \eta_0}{\sqrt{6} F_0} + \alpha_u + \alpha_s \right) \\
+ (C_{1ds} + C_{1sd}) \sin \left( \frac{\pi^0}{F_\pi} - \frac{\eta_8}{\sqrt{3} F_\pi} + \frac{4 \eta_0}{\sqrt{6} F_0} + \alpha_d + \alpha_s \right) \right\}
\]

\[
- 2c_3 \left\{ (C_{1ud} - C_{1du}) \sin \left( -\frac{2 \pi^0}{F_\pi} - \alpha_u + \alpha_d \right) + (C_{1us} - C_{1su}) \sin \left( -\frac{\pi^0}{F_\pi} - \sqrt{3} \eta_8 - \alpha_u + \alpha_s \right) \\
+ (C_{1ds} - C_{1sd}) \sin \left( \frac{\pi^0}{F_\pi} - \sqrt{3} \eta_8 - \alpha_d + \alpha_s \right) \right\}.
\]  

(85)

where it should be reminded that $\alpha_u, \alpha_d, \alpha_s$ are functions of $a$. The minimization condition for Eq. (85) yields meson VEVs $\langle \pi^0 \rangle$, $\langle \eta_8 \rangle$, $\langle \eta_0 \rangle$ and an axion VEV $\langle a \rangle$. When only the term $(C_{1ud} - C_{1du})$ is non-zero, these VEVs are given by

\[
\frac{\langle \pi^0 \rangle}{F_\pi} \approx \frac{G_F}{\sqrt{2}} (C_{1ud} - C_{1du}) \frac{c_3}{B_0 F^2_\pi} \frac{m_u + m_d + 4m_s}{m_u m_d + m_d m_s + m_s m_u},
\]

\[
\frac{\langle \eta_8 \rangle}{F_\pi} \approx \frac{G_F}{\sqrt{2}} (C_{1ud} - C_{1du}) \sqrt{3} \frac{c_3}{B_0 F^2_\pi} \frac{(m_d - m_u)}{m_u m_d + m_d m_s + m_s m_u},
\]

\[
\frac{\langle \eta_0 \rangle}{F_0} \approx 0,
\]

\[
\frac{\langle a \rangle}{f_a} + \bar{\theta} \approx \frac{G_F}{\sqrt{2}} (C_{1ud} - C_{1du}) \frac{2c_3}{B_0 F^2_\pi} \frac{(m_d - m_u)}{m_u m_d}.
\]  

(86)

The VEVs of $\pi^0$ and $\eta_8$ remain of the same order as the case without Peccei-Quinn mechanism, and hence they contribute to the neutron EDM in an analogous way. The axion VEV no longer cancels the genuine $\overline{\theta}$ term and the leftover induces an effective $\overline{\theta}$ term; we estimate its contribution by employing the result of Ref. [53] as

\[
d_n|_{\text{ind } \overline{\theta}} = -(2.7 \pm 1.2) \times 10^{-16} \left( \frac{\langle a \rangle}{f_a} + \bar{\theta} \right) \text{ e cm}.
\]  

(87)

The final result is the sum of the meson VEV contribution estimated analogously to Eq. (83), plus Eq. (87).
4.2.2 Other CP-violating operators $O_{1q}$, $O_{2q}$ and $O_3$

The contributions of the dipole operators in Eq. (102) and the Weinberg operator in Eq. (103) to the neutron EDM can be obtained with the QCD sum rule. The former is calculated in Ref. [54] while the latter is in Ref. [55], resulting in the following relations:

$$d_n|_{\text{quark}} = 0.47d_d - 0.12d_u + e(0.18d^c_d - 0.18d^c_u - 0.008d^c_s),$$  \hspace{1cm} (88)

$$d_n|_{\text{PQ}} = 0.47d_d - 0.12d_u + e(0.35d^c_d + 0.17d^c_u),$$  \hspace{1cm} (89)

$$d_n|_{\text{Weinberg}} = \frac{G_F}{\sqrt{2}}e g_s C_3 \times (10 - 30) \text{ MeV},$$  \hspace{1cm} (90)

where r.h.s. must be evaluated at 1 GeV. In Eqs. (88, 89), $d_q$ and $d^c_q (q = u,d,s)$, so-called quark EDM and quark chromo-EDM, are defined as,

$$d_q(\mu) = -\frac{G_F}{\sqrt{2}} e q_c C_{1q}(\mu) m_q(\mu),$$  \hspace{1cm} (91)

$$d^c_q(\mu) = -\frac{G_F}{\sqrt{2}} C_{2q}(\mu) m_q(\mu).$$  \hspace{1cm} (92)

Equations (88) and (89) represent the quark EDM contributions without and with Peccei-Quinn mechanism, respectively. For the case without Peccei-Quinn mechanism, we have taken $\bar{\theta} = 0$.

4.3 Numerical Analysis of Neutron EDM versus $\epsilon'/\epsilon$

For numerical analysis of $d_n$, we employ the following values: The chiral-limit pion decay constant $F_\pi$ is obtained from a lattice calculation as $F_\pi = 86.8$ MeV [56]. $D, F$ have been measured to be $D = 0.804$ and $F = 0.463$. For $b_D, b_F$, we quote the result of Ref. [57, 58] with a NLO calculation in Lorentz covariant baryon chiral perturbation theory with decuplet contributions, which reads $b_D = 0.161$ GeV$^{-1}$ and $b_F = -0.502$ GeV$^{-1}$. Since the same calculation formalism, combined with experimental data $\sigma_{\pi N} \simeq 59(7)$ MeV, predicts a small value of the strange quark contribution to the nucleon mass $\sigma_s$ [57], we infer that these values of $b_D, b_F$ are most robust. For the quark masses, we adopt lattice results in Ref. [59], $m_{ud}(2 \text{ GeV}) = 3.373$ MeV and $m_s(2 \text{ GeV}) = 92.0$ MeV, and further evaluate QCD five-loop RG evolutions to obtain the masses at 1 GeV in MS scheme, which are used in our analysis. Also, we exploit an estimate $m_u/m_d = 0.46$ [59].

The main source of uncertainty in our analysis is the unknown LEC $c_3$ in the meson chiral Lagrangian Eq. (56). The other unknown LEC $c_1$ is ineffective, because the Wilson coefficients satisfy $|C_{1ud} - C_{1u}| \gg |C_{1ud} + C_{1u}|, |C_{1sq}|, |C_{qs}|$. Our calculations of loop-induced $d_n$ Eq. (83) and
axion-induced $d_n$ Eq. (83) are hence proportional to $c_3$ and subject to $O(1)$ uncertainty originating from its naïve dimensional analysis Eq. (59). The fact that our results depend only on one LEC $c_3$ is good news, because it excludes the possibility of accidental cancellation between contributions with different LECs. Another source of uncertainty is the renormalization scale $\mu$ in the loop calculation Eq. (80), but this is subdominant compared to the uncertainty of $c_3$.

In the analysis, the ratio of the bifundamental scalar VEVs is again fixed as $\tan \beta = m_b / m_t$. The values of the new CP phases $\phi_c, \phi_t, \psi_d, \psi_s, \psi_b, \alpha$ are randomly generated. We find that the contribution of the Weinberg operator is suppressed by roughly $10^{-7} - 10^{-9}$ compared with that of the four-quark operators, and thus we neglect it in the analysis.

First, we show the numerical result for the neutron EDM without the constraint from $\epsilon'/\epsilon$ in Fig. (3). One observes that the contribution of the four-quark operators is dominant over that of the quark EDMs.

![Figure 3: Prediction for the neutron EDM in the case with $\bar{\theta} = 0$ without Peccei-Quinn mechanism. Only the contributions of four-quark operators and quark-level EDMs including both quark EDM and chormo-EDM are shown. A dashed line represents the current bound on the neutron EDM [60], while a dashed dotted line stands for the future bound [61].](image)

As stated previously, an effective $\bar{\theta}$ term is induced in the presence of Peccei-Quinn mechanism. In Fig. 4 we additionally show the numerical prediction based on Eq. (87). One finds that the induced $\bar{\theta}$ gives subleading contribution to the neutron EDM.
Figure 4: Comparison between the contribution of the induced $\bar{\theta}$ to the neutron EDM and others in the presence of Peccei-Quinn mechanism. A gray dashed line represents the current bound of EDM [60] while a black dashed dotted line stands for the future bound [61].

Next, the correlated prediction for $|d_n|$ and Re($\epsilon'/\epsilon$) is presented in Figs. 5 and 6 in the cases without and with Peccei-Quinn mechanism, respectively. Here, small contributions from the quark EDMs are neglected. The cases with and without Peccei-Quinn mechanism yield almost identical results because the induced $\bar{\theta}$ has a subdominant effect, as seen in Fig. 4. We observe that $M_{W'} = 20$ TeV and 50 TeV can be consistent with the data on Re($\epsilon'/\epsilon$) at 1$\sigma$ level, whereas the case with $M_{W'} = 70$ TeV cannot explain it. However, the case with $M_{W'} = 20$ TeV has already been excluded by the current bound on the neutron EDM, and only $M_{W'} = 50$ TeV can be compatible with the neutron EDM bound and the data on Re($\epsilon'/\epsilon$). Figs. 5 and 6 further inform us that almost all parameter points that account for the Re($\epsilon'/\epsilon$) data will be covered by future neutron EDM searches [61]. Therefore, unless the tree-level $\bar{\theta}$ in the case without Peccei-Quinn mechanism miraculously cancels the contribution of the model, we anticipate the discovery of the neutron EDM in the near future.
Figure 5: Correlation plot for the direct CP violation in $K \rightarrow \pi \pi$ decay and the neutron EDM in the case with $\bar{\theta} = 0$ without Peccei-Quinn mechanism. A gray dashed line and a black dashed dotted line represent the current [60] and the future [61] bounds on the neutron EDM, while a cyan band stands for the $1\sigma$ range of the direct CP violation in $K \rightarrow \pi \pi$ decay obtained from PDG [7].

Figure 6: The same figure as Fig. 5 with Peccei-Quinn mechanism.
5 Summary and Discussions

We have addressed the $\epsilon'/\epsilon$ anomaly in the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge extension of the SM with charge symmetry. Since the charge symmetry gives strong restrictions on the mixing matrix for right-handed quarks, $\epsilon'/\epsilon$ can be evaluated only in terms of two new CP phases $\alpha - \psi_d$ and $\alpha - \psi_s$, the mass of $W'$ gauge boson (mostly composed of $W_R$), and the bifundamental scalar VEV ratio $\tan \beta$. By fixing $\tan \beta$ at its natural value $m_b/m_t$, and by randomly varying $\alpha - \psi_d$ and $\alpha - \psi_s$, we have shown that $M_{W'} < 58$ TeV must be satisfied to account for the experimental value of $\epsilon'/\epsilon$ at $1\sigma$ level.

Next, we have made a prediction for the neutron EDM $d_n$ when the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model with charge symmetry solves the $\epsilon'/\epsilon$ anomaly. We have investigated the contribution of meson condensates induced by four-quark operators, and revealed that the $\pi^0$ VEV dominantly contributes to the neutron EDM, whose impact is enhanced by $m_s/(m_u + m_d)$ compared to other contributions. This enhancement is attributable to the isospin violating coupling of $W'$ gauge boson, which allows the $\pi^0$ VEV to arise without the factor of $m_d - m_u$. Additionally, we have found that the induced $\tilde{\theta}$ term in the presence of Peccei-Quinn mechanism yields only a subleading effect on $d_n$. On the basis of the above observations, we have shown that the $\epsilon'/\epsilon$ anomaly can be explained without conflicting the current experimental bound on $d_n$, and that the parameter space where the $\epsilon'/\epsilon$ data are accounted for will be almost entirely covered by future experiments [61].

We comment on the constraint from Re($\epsilon$) on the model. Since $W'$ gauge boson contributes to $\Delta F = 2$ processes only at loop levels, for $M_{W'} > 20$ TeV, its contribution to Re($\epsilon$) is safely below the experimental bound [62]. However, the heavy neutral scalar particles coming from the bifundamental scalar induce $\Delta F = 2$ processes at tree level. Since their mass is of the same order as or below $M_{W'}$ if there is no fine-tuning in the scalar potential, these particles may lead to a tension with the data on Re($\epsilon$) [62] (constraint from Re($\epsilon$) on general left-right models is found in Ref. [63], and that on the model with left-right parity is in Ref. [64]) (for early studies on the Re($\epsilon$) constraint, see, e.g., Ref. [65]).

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Appendix A

\( O_1 = (\bar{s}_\alpha u_\beta)_L(\bar{u}_\beta d_\alpha)_L, \quad O_2 = (\bar{s}u)_L(\bar{u}d)_L, \)  
\( O_{1c} = (\bar{s}_\alpha c_\beta)_L(\bar{c}_\beta d_\alpha)_L, \quad O_{2c} = (\bar{s}c)_L(\bar{c}d)_L, \)  
\( O_{1L}^{RL} = (\bar{s}_\alpha u_\beta)_R(\bar{u}_\beta d_\alpha)_L, \quad O_{2L}^{RL} = (\bar{s}u)_R(\bar{u}d)_L, \)  
\( O_{1c}^{RL} = (\bar{s}_\alpha c_\beta)_R(\bar{c}_\beta d_\alpha)_L, \quad O_{2c}^{RL} = (\bar{s}c)_R(\bar{c}d)_L, \)  
\( O_3 = \sum_{q=u,d,s} (\bar{s}d)_L(\bar{q}q)_L, \quad O_4 = \sum_{q=u,d,s} (\bar{s}_\alpha d_\beta)_L(\bar{q}_\beta q_\alpha)_L, \)  
\( O_5 = \sum_{q=u,d,s} (\bar{s}d)_L(\bar{q}q)_R, \quad O_6 = \sum_{q=u,d,s} (\bar{s}_\alpha d_\beta)_L(\bar{q}_\beta q_\alpha)_R, \)  
\( O_7 = \frac{3}{2} \sum_{q=u,d,s} (\bar{s}d)_L\epsilon_q(\bar{q}q)_R, \quad O_8 = \frac{3}{2} \sum_{q=u,d,s} (\bar{s}_\alpha d_\beta)_L\epsilon_q(\bar{q}_\beta q_\alpha)_R, \)  
\( O_9 = \frac{3}{2} \sum_{q=u,d,s} (\bar{s}d)_L\epsilon_q(\bar{q}q)_L, \quad O_{10} = \frac{3}{2} \sum_{q=u,d,s} (\bar{s}_\alpha d_\beta)_L\epsilon_q(\bar{q}_\beta q_\alpha)_L, \)  
\( O_g = \frac{g_s}{8\pi^2}m_s\bar{s}\sigma_{\mu\nu}G^{\mu\nu}T^aP_Ld, \quad O_\gamma = \frac{e}{8\pi^2}m_s\bar{s}\sigma_{\mu\nu}F^{\mu\nu}P_Ld, \)  

where \((\bar{q}q')_L \equiv \bar{q}_\gamma(1 - \gamma_5)q'\) and \((\bar{q}q')_R \equiv \bar{q}_\gamma(1 + \gamma_5)q'\), \(\alpha, \beta\) are color indices, and color summation is taken in each quark bilinear unless \(\alpha, \beta\) are displayed. \(e_u = 2/3\) and \(e_d = e_s = -1/3\).

The operators \(O'_i, O_{LR}^j\) are obtained by interchanging \(L \leftrightarrow R\) in the corresponding operators.
Appendix B

The loop functions in the main text are defined as follows:

\[ F_1(x) = \frac{x(-18 + 11x + x^2)}{12(1-x)^3} + \frac{x^2(-15 + 16x - 4x^2)}{6(1-x)^4} \log x + \frac{2}{3} \log x + \frac{2}{3}, \]

\[ F_2(x) = \frac{x(2 + 5x - x^2)}{4(1-x)^3} + 3x^2 \log x \]

\[ F_3(x) = \frac{4 + x + x^2}{2(1-x)^2} + 3x \log x \]

\[ E_{1d}(x) = \frac{25x^2 - 19x^3}{36(1-x)^3} + \frac{x^2(6 + 2x - 5x^2)}{18(1-x)^4} \log x + \frac{4}{9} \log x + \frac{4}{9}, \]

\[ E_{2d}(x) = \frac{x(7 - 5x - 8x^2)}{12(1-x)^3} + \frac{x^2(2 - 3x)}{2(1-x)^4} \log x, \]

\[ E_{3d}(x) = \frac{20 - 31x + 5x^2}{6(1-x)^2} + \frac{x(2 - 3x)}{(1-x)^3} \log x, \]

\[ E_{3u}(x) = \frac{8 - 16x + 2x^2}{3(1-x)^2} + \frac{x(1 - 3x)}{(1-x)^3} \log x. \]

Appendix C

\[ O_{1q} = -\frac{1}{2} e_q m_q \bar{q} \sigma_{\mu\nu} q F_{\mu\nu}, \quad O_{2q} = -\frac{g_s}{2} m_q \bar{q} \sigma_{\mu\nu} i g_5 T^a q G^{a\mu\nu}, \]

\[ O_{3} = -\frac{1}{6} \epsilon^{abc} e^{\mu\nu\rho\sigma} \eta^{\tau\nu} G_{\tau\mu} G_{\rho\sigma}, \]

\[ O_{4q} = \bar{q} \gamma_5 q, \quad O_{5q} = \bar{q} \sigma_{\mu\nu} q \bar{q} \sigma_{\mu\nu} g_5 q, \]

\[ O_{1q'q} = \bar{q}' \gamma_5 q, \quad O_{2q'q} = \bar{q}' \bar{q} \gamma_5 q, \]

\[ O_{3q'q} = \bar{q}' \sigma^{\mu\nu} q \bar{q} \sigma_{\mu\nu} q, \quad O_{4q'q} = \bar{q}' \sigma^{\mu\nu} q \bar{q} \sigma_{\mu\nu} q \gamma_5 q, \]

where \( q, q' = u, d, s \) and \( q' \neq q \). \( \alpha, \beta \) are color indices, and color summation is taken in each quark bilinear unless \( \alpha, \beta \) are displayed.

References

[1] T. Blum et al., “The \( K \to (\pi\pi)_{I=2} \) Decay Amplitude from Lattice QCD,” Phys. Rev. Lett. 108, 141601 (2012) [arXiv:1111.1699 [hep-lat]]; “Lattice determination of the \( K \to (\pi\pi)_{I=2} \) Decay Amplitude \( A_2 \),” Phys. Rev. D 86, 074513 (2012) [arXiv:1206.5142 [hep-lat]].
[2] T. Blum et al., “$K \to \pi\pi$ $\Delta I = 3/2$ decay amplitude in the continuum limit,” Phys. Rev. D 91, no. 7, 074502 (2015) [arXiv:1502.00263 [hep-lat]].

[3] Z. Bai et al. [RBC and UKQCD Collaborations], “Standard Model Prediction for Direct CP Violation in K Decay,” Phys. Rev. Lett. 115, no. 21, 212001 (2015) [arXiv:1505.07863 [hep-lat]].

[4] J. R. Batley et al. [NA48 Collaboration], “A Precision measurement of direct CP violation in the decay of neutral kaons into two pions,” Phys. Lett. B 544, 97 (2002) [hep-ex/0208009].

[5] A. Alavi-Harati et al. [KTeV Collaboration], “Measurements of direct CP violation, CPT symmetry, and other parameters in the neutral kaon system,” Phys. Rev. D 67, 012005 (2003) Erratum: [Phys. Rev. D 70, 079904 (2004)] [hep-ex/0208007].

[6] E. Abouzaid et al. [KTeV Collaboration], “Precise Measurements of Direct CP Violation, CPT Symmetry, and Other Parameters in the Neutral Kaon System,” Phys. Rev. D 83, 092001 (2011) [arXiv:1011.0127 [hep-ex]].

[7] C. Patrignani et al. [Particle Data Group], “Review of Particle Physics,” Chin. Phys. C 40, no. 10, 100001 (2016).

[8] A. J. Buras, M. Gorbahn, S. Jäger and M. Jamin, “Improved anatomy of $\epsilon'/\epsilon$ in the Standard Model,” JHEP 1511, 202 (2015) [arXiv:1507.06345 [hep-ph]].

[9] T. Kitahara, U. Nierste and P. Tremper, “Singularity-free next-to-leading order $\Delta S = 1$ renormalization group evolution and $\epsilon'_K/\epsilon_K$ in the Standard Model and beyond,” JHEP 1612, 078 (2016) [arXiv:1607.06727 [hep-ph]].

[10] A. J. Buras and J. M. Gerard, “Upper bounds on $\epsilon'/\epsilon$ parameters $D_6^{(1/2)}$ and $D_8^{(3/2)}$ from large N QCD and other news,” JHEP 1512, 008 (2015) [arXiv:1507.06326 [hep-ph]].

[11] A. J. Buras and J. M. Gerard, “Final state interactions in $K \to \pi\pi$ decays: $\Delta I = 1/2$ rule vs. $\epsilon'/\epsilon$,” Eur. Phys. J. C 77, no. 1, 10 (2017) [arXiv:1603.05686 [hep-ph]].

[12] H. Gisbert and A. Pich, “Direct CP violation in $K^0 \to \pi\pi$: Standard Model Status,” arXiv:1712.06147 [hep-ph].

[13] V. Cirigliano, W. Dekens, J. de Vries and E. Mereghetti, “An $\epsilon'$ improvement from right-handed currents,” Phys. Lett. B 767, 1 (2017) [arXiv:1612.03914 [hep-ph]].
[14] M. Blanke, A. J. Buras and S. Recksiegel, “Quark flavour observables in the Littlest Higgs model with T-parity after LHC Run 1,” Eur. Phys. J. C 76, no. 4, 182 (2016) arXiv:1507.06316 [hep-ph].

[15] M. Tanimoto and K. Yamamoto, “Probing SUSY with 10 TeV stop mass in rare decays and CP violation of kaon,” PTEP 2016, no. 12, 123B02 (2016) arXiv:1603.07960 [hep-ph].

[16] T. Kitahara, U. Nierste and P. Tremper, “Supersymmetric Explanation of CP Violation in $K \to \pi\pi$ Decays,” Phys. Rev. Lett. 117, no. 9, 091802 (2016) arXiv:1604.07400 [hep-ph].

[17] M. Endo, S. Mishima, D. Ueda and K. Yamamoto, “Chargino contributions in light of recent $\epsilon'/\epsilon$,” Phys. Lett. B 762, 493 (2016) arXiv:1608.01444 [hep-ph].

[18] A. J. Buras, “New physics patterns in $\epsilon'/\epsilon$ and $\epsilon_K$ with implications for rare kaon decays and $\Delta M_K$,” JHEP 1604, 071 (2016) arXiv:1601.00005 [hep-ph].

[19] M. Endo, T. Kitahara, S. Mishima and K. Yamamoto, “Revisiting Kaon Physics in General $Z$ Scenario,” Phys. Lett. B 771, 37 (2017) arXiv:1612.08839 [hep-ph].

[20] C. Bobeth, A. J. Buras, A. Celis and M. Jung, “Patterns of Flavour Violation in Models with Vector-Like Quarks,” JHEP 1704, 079 (2017) arXiv:1609.04783 [hep-ph].

[21] A. J. Buras and F. De Fazio, “$\epsilon'/\epsilon$ in 331 Models,” JHEP 1603, 010 (2016) arXiv:1512.02869 [hep-ph]; “331 Models Facing the Tensions in $\Delta F = 2$ Processes with the Impact on $\epsilon'/\epsilon$, $B_s \to \mu^+\mu^-$ and $B \to K^*\mu^+\mu^-$,” JHEP 1608, 115 (2016) arXiv:1604.02344 [hep-ph].

[22] J. C. Pati and A. Salam, “Lepton Number as the Fourth Color,” Phys. Rev. D 10, 275 (1974) Erratum: [Phys. Rev. D 11, 703 (1975)]; R. N. Mohapatra and J. C. Pati, “A Natural Left-Right Symmetry,” Phys. Rev. D 11, 2558 (1975); G. Senjanovic and R. N. Mohapatra, “Exact Left-Right Symmetry and Spontaneous Violation of Parity,” Phys. Rev. D 12, 1502 (1975).

[23] A. Maiezza, M. Nemevsek, F. Nesti and G. Senjanovic, “Left-Right Symmetry at LHC,” Phys. Rev. D 82, 055022 (2010) arXiv:1005.5160 [hep-ph].

[24] D. Chang, R. N. Mohapatra and M. K. Parida, “Decoupling Parity and $SU(2)_R$ Breaking Scales: A New Approach to Left-Right Symmetric Models,” Phys. Rev. Lett. 52, 1072 (1984); D. Chang, R. N. Mohapatra and M. K. Parida, “A New Approach to Left-Right Symmetry Breaking in Unified Gauge Theories,” Phys. Rev. D 30, 1052 (1984).
[25] P. L. Cho and M. Misiak, “$b \to s \gamma$ decay in $SU(2)_L \times SU(2)_R \times U(1)$ extensions of the Standard Model,” Phys. Rev. D 49, 5894 (1994) [hep-ph/9310332].

[26] M. Ciuchini, E. Franco, G. Martinelli, L. Reina and L. Silvestrini, “Scheme independence of the effective Hamiltonian for $b \to s \gamma$ and $b \to sg$ decays,” Phys. Lett. B 316, 127 (1993) [hep-ph/9307364]; A. J. Buras, M. Misiak, M. Munz and S. Pokorski, “Theoretical uncertainties and phenomenological aspects of $B \to X_s \gamma$ decay,” Nucl. Phys. B 424, 374 (1994) [hep-ph/9311345].

[27] J. de Vries, E. Mereghetti, R. G. E. Timmermans and U. van Kolck, “The Effective Chiral Lagrangian From Dimension-Six Parity and Time-Reversal Violation,” Annals Phys. 338, 50 (2013) [arXiv:1212.0990 [hep-ph]].

[28] G. Beall and A. Soni, “Electric Dipole Moment of the Neutron in a Left-right Symmetric Theory of CP Violation,” Phys. Rev. Lett. 47, 552 (1981).

[29] H. An, X. Ji and F. Xu, “P-odd and CP-odd Four-Quark Contributions to Neutron EDM,” JHEP 1002, 043 (2010) [arXiv:0908.2420 [hep-ph]].

[30] F. Xu, H. An and X. Ji, “Neutron Electric Dipole Moment Constraint on Scale of Minimal Left-Right Symmetric Model,” JHEP 1003, 088 (2010) [arXiv:0910.2265 [hep-ph]].

[31] W. Dekens, J. de Vries, J. Bsaisou, W. Bernreuther, C. Hanhart, U. G. Meissner, A. Nogga and A. Wirzba, “Unraveling models of CP violation through electric dipole moments of light nuclei,” JHEP 1407, 069 (2014) [arXiv:1404.6082 [hep-ph]].

[32] A. Maiezza and M. Nemevsek, “Strong P invariance, neutron electric dipole moment, and minimal left-right parity at LHC,” Phys. Rev. D 90, no. 9, 095002 (2014) [arXiv:1407.3678 [hep-ph]].

[33] Y. Zhang, H. An, X. Ji and R. N. Mohapatra, “Right-handed quark mixings in minimal left-right symmetric model with general CP violation,” Phys. Rev. D 76, 091301 (2007) [arXiv:0704.1662 [hep-ph]].

[34] Y. Zhang, H. An, X. Ji and R. N. Mohapatra, “General CP Violation in Minimal Left-Right Symmetric Model and Constraints on the Right-Handed Scale,” Nucl. Phys. B 802, 247 (2008) [arXiv:0712.4218 [hep-ph]].

[35] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, “On the Weak Radiative Decays (Effects of Strong Interactions at Short Distances),” Phys. Rev. D 18, 2583 (1978) Erratum: [Phys. Rev. D 19, 2815 (1979)].
[36] J. Dai and H. Dykstra, “QCD Corrections to CP Violation in Higgs Exchange,” Phys. Lett. B 237, 256 (1990); E. Braaten, C. S. Li and T. C. Yuan, “The Gluon Color - Electric Dipole Moment and Its Anomalous Dimension,” Phys. Rev. D 42, 276 (1990).

[37] G. Boyd, A. K. Gupta, S. P. Trivedi and M. B. Wise, “Effective Hamiltonian for the Electric Dipole Moment of the Neutron,” Phys. Lett. B 241, 584 (1990).

[38] J. Hisano, K. Tsumura and M. J. S. Yang, “QCD Corrections to Neutron Electric Dipole Moment from Dimension-six Four-Quark Operators,” Phys. Lett. B 713, 473 (2012) [arXiv:1205.2212 [hep-ph]].

[39] A. Buras and J. M. Gerard, “$K \rightarrow \pi\pi$ and $K-\pi$ Matrix Elements of the Chromomagnetic Operators from Dual QCD,” [arXiv:1803.08052 [hep-ph]].

[40] M. Constantinou et al. [ETM Collaboration], “$K \rightarrow \pi$ matrix elements of the chromomagnetic operator on the lattice,” [arXiv:1712.09824 [hep-lat]].

[41] S. Bertolini, J. O. Eeg, A. Maiezza and F. Nesti, “New physics in $\epsilon'$ from gluomagnetic contributions and limits on Left-Right symmetry,” Phys. Rev. D 86, 095013 (2012) Erratum: [Phys. Rev. D 93, no. 7, 079903 (2016)] [arXiv:1206.0668 [hep-ph]].

[42] S. Weinberg, “Larger Higgs Exchange Terms in the Neutron Electric Dipole Moment,” Phys. Rev. Lett. 63, 2333 (1989).

[43] R. D. Peccei and H. R. Quinn, “CP Conservation in the Presence of Instantons,” Phys. Rev. Lett. 38, 1440 (1977).

[44] E. Witten, “Large N Chiral Dynamics,” Annals Phys. 128, 363 (1980); P. Di Vecchia and G. Veneziano, “Chiral Dynamics in the Large n Limit,” Nucl. Phys. B 171, 253 (1980).

[45] A. Manohar and H. Georgi, “Chiral Quarks and the Nonrelativistic Quark Model,” Nucl. Phys. B 234, 189 (1984).

[46] V. M. Khatsimovsky, I. B. Khriplovich and A. S. Yelkhovsky, “Neutron Electric Dipole Moment, T Odd Nuclear Forces and Nature of CP Violation,” Annals Phys. 186, 1 (1988).

[47] X. G. He and B. McKellar, “Large contribution to the neutron electric dipole moment from a dimension-six four quark operator,” Phys. Rev. D 47, 4055 (1993).

[48] X. G. He and B. McKellar, “Constraints on CP violating four fermion interactions,” Phys. Lett. B 390, 318 (1997) [hep-ph/9604394].

33
[49] C. Hamzaoui and M. Pospelov, “The Limits on CP odd four fermion operators containing strange quark field,” Phys. Rev. D 60, 036003 (1999) [hep-ph/9901363].

[50] K. Ottnad, B. Kubis, U.-G. Meissner and F.-K. Guo, “New insights into the neutron electric dipole moment,” Phys. Lett. B 687, 42 (2010) arXiv:0911.3981 [hep-ph]; F. K. Guo and U. G. Meissner, “Baryon electric dipole moments from strong CP violation,” JHEP 1212, 097 (2012) arXiv:1210.5887 [hep-ph].

[51] T. Becher and H. Leutwyler, “Baryon chiral perturbation theory in manifestly Lorentz invariant form,” Eur. Phys. J. C 9, 643 (1999) [hep-ph/9901384].

[52] P. J. Ellis and H. B. Tang, “Pion nucleon scattering in a new approach to chiral perturbation theory,” Phys. Rev. C 57, 3356 (1998) [hep-ph/9709354].

[53] N. Yamanaka, B. K. Sahoo, N. Yoshinaga, T. Sato, K. Asahi and B. P. Das, “Probing exotic phenomena at the interface of nuclear and particle physics with the electric dipole moments of diamagnetic atoms: A unique window to hadronic and semi-leptonic CP violation,” Eur. Phys. J. A 53, 54 (2017) arXiv:1703.01570 [hep-ph].

[54] J. Hisano, J. Y. Lee, N. Nagata and Y. Shimizu, “Reevaluation of Neutron Electric Dipole Moment with QCD Sum Rules,” Phys. Rev. D 85, 114044 (2012) arXiv:1204.2653 [hep-ph].

[55] D. A. Demir, M. Pospelov and A. Ritz, “Hadronic EDMs, the Weinberg operator, and light gluinos,” Phys. Rev. D 67, 015007 (2003) [hep-ph/0208257].

[56] S. Borsanyi, S. Durr, Z. Fodor, S. Krieg, A. Schafer, E. E. Scholz and K. K. Szabo, “SU(2) chiral perturbation theory low-energy constants from 2+1 flavor staggered lattice simulations,” Phys. Rev. D 88, 014513 (2013) arXiv:1205.0788 [hep-lat].

[57] J. M. Alarcon, L. S. Geng, J. Martin Camalich and J. A. Oller, “The strangeness content of the nucleon from effective field theory and phenomenology,” Phys. Lett. B 730, 342 (2014) arXiv:1209.2870 [hep-ph].

[58] J. Martin Camalich, L. S. Geng and M. J. Vicente Vacas, “The lowest-lying baryon masses in covariant SU(3)-flavor chiral perturbation theory,” Phys. Rev. D 82, 074504 (2010) arXiv:1003.1929 [hep-lat].

[59] S. Aoki et al., “Review of lattice results concerning low-energy particle physics,” Eur. Phys. J. C 77, no. 2, 112 (2017) arXiv:1607.00299 [hep-lat].
[60] C. A. Baker et al., “An Improved experimental limit on the electric dipole moment of the neutron,” Phys. Rev. Lett. 97, 131801 (2006) [hep-ex/0602020]; J. M. Pendlebury et al., “Revised experimental upper limit on the electric dipole moment of the neutron,” Phys. Rev. D 92, no. 9, 092003 (2015) [arXiv:1509.04411 [hep-ex]].

[61] K. Kumar, Z. T. Lu and M. J. Ramsey-Musolf, “Working Group Report: Nucleons, Nuclei, and Atoms,” arXiv:1312.5416 [hep-ph]; T. Chupp and M. Ramsey-Musolf, “Electric Dipole Moments: A Global Analysis,” Phys. Rev. C 91, no. 3, 035502 (2015) [arXiv:1407.1064 [hep-ph]].

[62] S. Bertolini, A. Maiezza and F. Nesti, “Present and Future K and B Meson Mixing Constraints on TeV Scale Left-Right Symmetry,” Phys. Rev. D 89, no. 9, 095028 (2014) [arXiv:1403.7112 [hep-ph]].

[63] M. Blanke, A. J. Buras, K. Gemmler and T. Heidsieck, “$\Delta F = 2$ observables and $B \to X_q\gamma$ decays in the Left-Right Model: Higgs particles striking back,” JHEP 1203, 024 (2012) [arXiv:1111.5014 [hep-ph]].

[64] N. Haba, H. Umeeda and T. Yamada, “Semialigned two Higgs doublet model,” Phys. Rev. D 97, no. 3, 035004 (2018) [arXiv:1711.06499 [hep-ph]].

[65] K. Kiers, J. Kolb, J. Lee, A. Soni and G. H. Wu, “Ubiquitous CP violation in a top inspired left-right model,” Phys. Rev. D 66, 095002 (2002) [hep-ph/0205082].