Making light of gravitational-waves

Justine Tarrant, Geoff Beck, and Sergio Colafrancesco

School of Physics, University of the Witwatersrand, Private Bag 3, 2050, Johannesburg, South Africa
E-mail: justine.tarrant@wits.ac.za, geoffrey.beck@wits.ac.za

Abstract. Coupling and mixing between photons and low-mass bosons is well considered in the literature, including the coupling of photons with hypothetical gravitons. In particular, we are concerned with the direct conversion of gravitons into photons in the presence of an external magnetic field. We examine whether such a process could produce direct low-frequency radio counterparts to gravitational-wave events that share the gravitational-wave spectrum. Our work differs from previous work in the literature in that we demonstrate that such a counterpart to a binary neutron star merger would be visible out to a distance of 50 Mpc with a hypothetical lunar array telescope (which is no longer science fiction). A very small probability exists of observing a counterpart with SKA-LOW, but only for distances less than 10 Mpc. Furthermore, we show that, for the case when no detection is made by either SKA-LOW or a lunar array, a lower bound, competitive with those from Lorentz-invariance violation, may be placed on the energy-scale of quantum gravitational effects. This direct conversion mechanism is valuable since it provides a means of studying the graviton-photon coupling, thereby providing a low-energy window onto high-energy physics in the domain of quantum gravity.

Keywords: lunar telescope, photon-graviton conversion, SKA-LOW, quantum gravity

ArXiv ePrint: 1904.12678

1This paper is dedicated to the memory of Prof. Sergio Colafrancesco.
1 Introduction

Electromagnetic counterparts to gravitational-waves were well considered in the literature [1–6] prior to the first detection of compact mergers by the Laser Interferometer Gravitational-wave Observatory (LIGO), made in September 2014, and produced by the merger of two black holes. Furthermore, this correlation was considered by [7, 8] following these events. It wasn’t until the discovery of counterparts associated with the fifth LIGO event [9] detected in August 2017, that the field was revived as scientists poured over the new data obtained. The successful detection was only possible due to an immense, well coordinated, global collaboration. This GW170817 event was the first detection of a binary neutron star merger, and was long hoped for since such events were expected to form a kilonova and collimated outflows, called jets, which result in electromagnetic counterparts.

Emissions across the entire electromagnetic spectrum were detected [10], thus heralding the dawn of multi-messenger astronomy. This includes the observation of radio emissions in the broad band afterglow occurring when the gamma-ray burst interacts with the interstellar medium [11]. However, there remains the mystery of what physical processes underpin the production of the electromagnetic counterparts which we observe. In this work, we approach this problem from a different angle using the idea that gravitational-waves can themselves directly generate plausibly detectable low-frequency radio counterparts while propagating from their source to Earth.

The idea that gravitational degrees of freedom may be converted into electromagnetic degrees of freedom is not a new one, see for example: [12–14] and [15]. Interest in this subject resulted in the need to find an indirect means of measuring gravitational-waves and also arose out of studies considering axion-photon mixing [16, 17]. For example, early studies involved the scattering of electromagnetic fields off time-dependant gravitational fields, showing that
there was a possible coherent interaction between linear gravitational-waves and electromagnetic waves in which energy could be transformed from one degree of freedom to the other [18]. Such conversions taking place inside an external magnetic field could be extremely bright, even if a small percentage of the gravitational-wave pulse is transformed [15, 19]. Modern calculations include considerations about plasma frequency and QED corrections [7], see also [8, 12]. As mentioned in [7], these articles dealt with the graviton-photon conversion at high frequencies $\omega/(2\pi)$ which exceeded the plasma frequency, $\omega_p/(2\pi)$, of the surrounding and intervening medium. This is problematic since typically the plasma frequency is approximately equal to $10 \, \text{kHz} \sqrt{n_e}$, where $n_e$ is the electron number density, furthermore, the frequency of gravitational-waves is usually less than $10 \, \text{kHz} \sqrt{n_e}$. It makes sense then, that higher frequencies were considered since low-frequency waves do not propagate through plasma. For the case of LIGO events concerning merging black holes, being roughly 20 - 30 $M_\odot$, the realistic gravitational-wave frequencies were approximately 100 - 200 Hz.

In [7], they consider the possibility that the energy transition from gravitational-waves into electromagnetic waves may still be possible despite their low frequency. In particular, they show that gravitational-waves travelling through a high-frequency plasma and non-zero magnetic field continue to transform some of their energy into non-propagating plasma waves which heat up the surrounding plasma, thus leading to a noticeable release of electromagnetic radiation. In this work, they use an asymmetric conversion regime for which conversion from photons into gravitons is less probable given that the wave vector for this solution is purely imaginary, corresponding to the damping of the electromagnetic wave travelling in the plasma with frequency higher than that of itself. The authors conclude that the graviton-photon conversion mechanism studied can hardly account for plasma heating in LIGO black hole merger events.

A further interesting paper is that of [8] where they considered the effects generating dispersion and coherence braking of the electromagnetic waves. Therein they obtained the energy power and energy power fluxes for quasiperpendicular external magnetic fields in the gravitational-wave propagation direction. The authors found that the energy power was large, but that the fluxes remained faint, as would be seen on earth. They considered waves with $\omega > \omega_p$, making the graviton-photon mixing for gravitational-waves of less than a few hundred Hertz less appealing. They noted, however, that the calculated plasma frequency through which the waves travel depends on the line of sight, producing varying frequency cutoff’s. Finally, they conclude that the detection of this graviton to photon mechanism is unlikely to be made on Earth or in interplanetary space due to the large cutoff frequencies. They suggest that detection is only probable outside of the solar system. We show that this is not necessarily true.

Our proposal uses the idea that gravitational-waves may be converted into electromagnetic waves following the mechanism laid out in [12]. Extending this mechanism, beyond the treatment of axions, to gravitons implies that the gravitational field is quantized. Therefore, this model also provides a potential way to probe quantum gravity and its associated energy scales. Using the characteristics of relatively well studied environments, such as the known external magnetic field, the environments characteristic size and a phenomenological energy scale $M$, we can compute the transition probability that gravitons are converted to photons in the weak mixing limit. It is not our aim to repeat the works reviewed in this section, we are not dealing directly with the technicalities faced therein. We simply suggest a model, that assumes that the graviton to photon mechanism is plausible, we then perform some back of the envelope calculations to determine whether a detection might be possible in the
neighbourhood of Earth, i.e. on the Moon.

Supposing that it may be possible to find the parameter $M$ through observation of the electromagnetic spectrum corresponding to the gravitational-wave event, we may then compare its value to the phenomenological $M_{\text{Pl}} = \sqrt{\hbar c/G} = 1.22 \times 10^{19}$ GeV/c$^2$, the Planck mass, as is widely used in the literature as an expected energy scale for the appearance of quantum gravitational effects. In this way, a bound may be placed on any deviation of the energy scale of quantum gravitational effects $M$ from that of the Planck mass. Furthermore, such an observation could determine the existence of the graviton and therefore that the gravitational field is quantized. Additionally, since no electromagnetic counterparts are expected to accompany binary black hole mergers, only kinematic properties may be inferred from such events [20]. Herein lies a further avenue which may be probed using the mechanism for converting gravitons into photons. Providing us with a unique source of information regarding black holes.

Our means of analysis is to make a phenomenological extrapolation of a numerically determined gravitational-wave spectrum produced by a binary neutron star merger. We then determine how much the detectability of the resulting counterpart photon spectrum depends upon the parameters of the extrapolation, which is justified in that it accounts for both scenarios encountered in the literature: mode decay and the contribution of higher order modes extending the spectrum to higher frequencies. If the dependence is strong we conclude that detection is improbable without significant fine-tuning of the extrapolated spectrum. On the other hand, if the counterpart’s detectability is largely independent of the choice of parameters, we take this to indicate that detection would be a strong possibility: in that, the extrapolation to detectable frequencies can be made far more naturally. To make the observations mentioned above, we consider a radio array on the far side of the moon and consider the cases: 100 and 1000 antennas. The far side of the moon is shielded from the interference from Earth and is thus preferable for conducting very low-frequency radio experiments. Considerations for building telescopes on the moon are already under way and have been considered in the literature\textsuperscript{1,2} [21–23]. Furthermore, we consider the transition probabilities required for a detection with the low-frequency Square Kilometre Array (SKA-LOW). Although the first attempts to search for these counterpart signals can be performed with the SKA-LOW, we find that a detection would require a considerably fine tuned behaviour with regards to the extrapolation of the gravitational-wave spectrum from neutron stars merger, and may be completely impossible in the most pessimistic scenario. This is largely due to the fact that the transition probabilities required for high detection probabilities with the SKA-LOW are too high to be supported by any well known environments. Very large magnetic fields, in addition to large intervening structures would be required. However, reasonable transition probabilities result in an achievable measurement using a lunar array. We consider these realistic values of the transition probability for two environments: the intra-cluster medium (ICM) of the Coma cluster and the intergalactic medium (IGM). Even if a very small fraction of gravitons are converted into photons, a radio telescope on the moon would be likely to detect the emissions since they possess a sufficiently high flux, [15] discuss this point as well.

In this work, we demonstrate that, for neutron star mergers, graviton-photon conversion emissions in the radio band are potentially detectable for various realistic transition probabilities. These radio emissions will have the same spectral shape as the original gravitational-

\textsuperscript{1}http://sci.esa.int/science-e/www/object/doc.cfm?fobjectid=53829
\textsuperscript{2}http://www.moonexpress.com/news/moon-express-announces-lunar-south-pole-mission-technology-development-contract-international-lunar-observatory-association
wave spectrum due to the relationship between the graviton and the photon, i.e. both particles are massless with the transition between the states being one-to-one. That is, real gravitons are converted into real photons in a one-to-one manner. Mediating the conversion is a virtual photon which is supplied by the external magnetic field, whose presence is necessary for spin conservation. Furthermore, the flux of the resulting photons will differ, in amplitude, to the source flux of gravitons by the factor $p$, the transition probability, which is the fraction of gravitons converted into photons through the aforementioned mechanism.

For the case where an event is detected by LIGO, has a suitable conversion environment on the line of sight, but is not detected by SKA-LOW or the hypothetical lunar array, we may also place a lower limit on the size of the scale $M$ by considering the largest $M$ attainable when detection does not require significant fine-tuning. We also compare our results to Lorentz invariance violation constraints and find that we produce competitive limits with respect to the results in [24].

Having established that detectable radio waves can result directly from gravitons produced in the associated gravitational-wave event a localization of the gravitational-wave event naturally follows. This is because the radio waves are coming from the same region, and electromagnetic signals are simpler to localize compared to gravitational-waves. It must be noted that, through this direct conversion, there may be some quantifiable delay between the radio emissions and the gravitational-waves arriving at the observer as, despite both gravitons and photons being massless [9], the plasma present in astrophysical media creates a refractive index different from the one for the propagating gravitons [25, 26].

This paper is structured in the following manner: After the introduction, section 2 discusses the model we use to formulate our study. Section 3 considers the construction of a lunar array in an optimal setting. Section 4 presents our results and discussion with the conclusion presented in section 5.

2 Model

Our model uses the spectrum for gravitational-waves resulting from a merging binary neutron star system found in [27, 28]. In this model the stars have equal masses of $1.5 \, M_\odot$ at a separation of 42.6 km. The resulting remnant is a low-mass black hole of about $2.8 \, M_\odot$. The total energy radiated away as gravitational-waves, in less than 3 ms, is $\Delta E_{GW} \sim 3 \times 10^{51}$ erg.

Now let us suppose that part of the radiation emitted as gravitational-waves, is converted into electromagnetic radiation via the conversion of gravitons into photons, as described below in Section 2.1. We denote this fraction by $p = E_{EM}/E_{GW}$, called the conversion probability, or conversion fraction. Then $E_{\text{total}} = E_{GW} + E_{EM} = (1 + p)E_{GW}$.

2.1 The mechanism

Here we summarize some of the details of the conversion mechanism described in [12], which illustrates the possibility for a low-mass (or zero-mass) particle to be created from a photon (spin-1) passing through an external magnetic field, and vice versa. This formalism is applicable to the case of gravitons, which have spin-2. Conversion requires the presence of an external magnetic field supplying one virtual photon in order to satisfy symmetry constraints, thus conversion of real gravitons to real photons is one-to-one, as the second photon in this interaction is virtual.

Furthermore, the strength of the mixing $S_M$ is
\[ S_M(\theta) = \frac{1}{2} \tan 2\theta, \]

where \( \theta \) is the mixing angle. We consider here only the well known weak conversion(or mixing) limit, in which the characteristic oscillation length, from graviton to photon and back, is much longer than the length scale considered, implying that \( \theta \ll 1 \) and that a very small fraction of photons will be converted back into gravitons. This may be illustrated by considering the critical energy \( E_c \) [25]. Energies much larger than \( E_c \) indicate the strong mixing regime. The critical energy is calculated as follows:

\[ E_c \approx \frac{50\omega_p^2}{(10^{-8} eV)^2} \left( \frac{10^{-6} G}{B_e} \right) \left( \frac{5 \times 10^{-11} GeV^{-1}}{g} \right), \]

where \( \omega_p \) is the plasma frequency, \( B_e \) is the external transverse magnetic field and \( g \) is the interaction strength. The plasma frequency is calculated as follows [25]

\[ \omega_p = 2\pi \times 10^4 \sqrt{\frac{n_e}{1 \text{ cm}^{-3}}} \text{ Hz} \]

where \( n_e \) is the electron density. For the IGM, \( \omega_p \sim 10^{-2} \) MHz, \( B_e \sim 10^{-9} \) G [29, 30]. This yields a critical energy of \( E_c \sim 10^5 \) GeV. The energies we are working with are given by \( E = h\nu \), where \( \nu \sim 10^5 \) Hz, i.e. the lower end of the radio band. Then for this case of the IGM, \( E \sim 10^{-19} \) GeV. Therefore we are well below the critical energy and the strong mixing regime, so that applying the weak mixing limit is not a problem. Similarly, for the Coma ICM with \( \omega_p \sim 10^{-3} \) MHz, \( B_e \sim 10^{-6} \) G [31], we find \( E_c \sim 1 \) GeV. So we are still below the strong mixing regime. For both cases we have used \( g \sim 10^{-19} \text{ GeV}^{-1} \).

### 2.2 The spectrum

Using the spectrum found in [27, 28], which we label as \( S_{gw}(\nu) \), we perform a power law extrapolation for frequencies \( \nu > \nu_f \) where \( \nu_f = 50 \) kHz is the largest frequency reached by [27, 28], simulating up to 7 modes contributing to the gravitational-wave spectrum

\[ S(\nu) = A\nu^\alpha \exp \left( -\frac{\nu}{\nu_c} \right), \]

so that the complete extrapolated spectrum is given by

\[ S_{gw,ex}(\nu) = \begin{cases} S_{gw}(\nu) & \nu \leq \nu_f \\ S(\nu) & \nu > \nu_f \end{cases}, \]

where \( \alpha \) is the power law index and \( \nu_c \) is the cut-off frequency. The choice of using a power law with a cutoff may be explained as follows: a cutoff is applied because we know that the quasi-normal modes decay exponentially according to [27, 28]. Furthermore, we require a power law to account for the fact that higher order modes may extend the spectrum [18], also seen in [27, 28]. Therefore, this extrapolation provides a parameter space of \( \alpha \) and \( \nu_c \) that effectively considers a large variety of scenarios for the extension of the gravitational wave spectrum to higher frequencies. The magnitude of the multi-order extrapolation is justified in that it deals effectively with the cases of quasi-normal mode decay (i.e. where \( \nu_c \simeq \nu_f \)) as well as the potential contribution of high order modes where \( \nu_c > \nu_f \). We normalise \( S_{gw,ex}(\nu) \) as
follows: we require that $E_{GW}$ is the energy emitted from the source at a luminosity distance $d_L$ over a time $\Delta t = 3$ ms and use this to determine the flux at Earth.

This power law is applied to the tail of the spectrum using the matching condition: $S_{gw}(\nu_f) = S(\nu_f)$. Furthermore, $pS(\nu_f)$ will then represent the spectrum obtained from the conversion of gravitational-waves into photons. The values $\alpha$ and $\nu_c$ provide us with the parameter space which allows us to determine the detectability of the electromagnetic counterparts with a radio telescope on the moon or with SKA-LOW. Note that this is a phenomenological extrapolation because it is difficult and beyond the scope of this work to calculate these spectra directly at frequencies high enough for radio-band detection. In order that the exact details of the extrapolation don’t bias the results, we will draw conclusions based on how insensitive the detection potential is to the choice of extrapolation parameters.

### 2.3 Conversion probabilities

The fraction of gravitational radiation converted into photons depends, in addition to the external magnetic field, on the distance travelled by the gravitons and the interaction strength $M$. As already mentioned above, we consider the weak mixing limit which leaves us with the following form of the transition probability [12, 25]

$$p(g \rightarrow \gamma) \approx B_e^2 d^2 g^2 = \frac{B_e^2 d^2}{M^2}$$

where $B_e$ is the external magnetic field component transverse to the propagation direction, $d$ is the propagation distance and $g$ is the interaction strength. The limit necessary to use this approximation is ensured by the weak magnetic fields and the small interaction strength $g$, that we will make use of in this work. These field magnitudes also negate the effects of quantum electrodynamics vacuum birefringence considered in [12].

### 2.4 The IGM and the Coma ICM

The magnetic field in the large scale IGM is very small, around 1 nG, but with a very long coherence length of $\sim 1$ Mpc [29, 30]. The distance, however, that the graviton has to travel through the IGM is therefore very large, order of $\sim 100$ Mpc.

The conversion probability is then given by

$$p_{IGM}(g \rightarrow \gamma) = \left(7.6 \times 10^{-2} \frac{g}{5 \times 10^{-11} \text{ GeV}^{-1}} \left(\frac{B_{IGM}}{10^{-6} \text{ G}}\right) \left(\frac{d}{1 \text{ kpc}}\right)\right)^2$$

where $B_{IGM} \sim 1$ nG and the interaction strength is taken as $1/M_{Pl} = 0.82 \times 10^{-19}$ GeV$^{-1}$. Then, we find that $p_{IGM}(g \rightarrow \gamma) \sim 1.55 \times 10^{-16}$.

For the Coma ICM, the magnetic field is larger by about three orders of magnitude, compared to that found in the IGM. However, the distance through which the gravitons may travel is much smaller, $\sim 4$ Mpc [32]. The conversion probability for the Coma cluster is, using equation (2.7) with an average $B_{coma} \sim 1 \mu$G [31], $p_{coma}(g \rightarrow \gamma) \sim 2 \times 10^{-14}$. So the larger magnetic field compensates for the smaller travel distance.

### 3 Lunar telescopes: Are they just science fiction?

A perhaps not so well known fact is that proposals for lunar development are under way as plans to return to the moon have recently been announced [23]. Driven by the need to
understand early universe physics such as the cosmic dawn, the International Lunar Observatory Association, in collaboration with Moon Express have announced plans for the first delivery of an international lunar observatory on the south pole of the moon by 2019\(^3\). The far side of the moon is the best place in the inner solar system to monitor low-frequency radio waves, to which the Earth is opaque [22]. Additionally, the far side of the moon shields the observatory from interference coming from the Earth. Therefore, whilst currently there is no telescope on the moon, we consider here some optimal specifications for a lunar array, that may in principle, detect electromagnetic counterparts to gravitational-waves.

3.1 Building a telescope on the moon

In this section we provide some details for designing an optimal lunar radio telescope. As a test case, we consider two set-ups: firstly, a configuration using \(N = 10^3\) log-periodic dual-polarized dipole antennas with bandwidth \(b = 300\) MHz, and secondly, the case with \(N = 100\) antennas and the same bandwidth \(b\). We calculate the minimum observable flux for the array as follows\(^4\)

\[
S_{\text{min}} = \frac{2k_B T_{\text{sky}}}{N\sqrt{b\tau A_e}} \quad (3.1)
\]

where \(A_e = \lambda^2/4\pi\) is the effective collecting area and \(\lambda\) is the incoming wavelength. Here \(k_B\) is the Boltzmann constant, \(\tau\) is the integration time. The sky temperature is given by [21]

\[
T_{\text{sky}} = \begin{cases} 
16.3 \times 10^6 K \left(\frac{\nu}{2 \text{MHz}}\right)^{-2.53}, & \nu > 2\text{MHz} \\
16.3 \times 10^6 K \left(\frac{\nu}{2 \text{MHz}}\right)^{-0.3}, & \nu < 2\text{MHz}
\end{cases} 
\quad (3.2)
\]

We can see, in figure 1, that a lunar array would be more sensitive than the SKA-LOW with a minimum detectable flux of roughly \(S_{\text{min}} \sim 0.5 \times 10^{-16}\) erg cm\(^{-2}\) s\(^{-1}\). The curve, for a lunar array, has an expected break or kink in its uniformity due to the behaviour of equation (3.2). In the aforementioned figure, we have computed both telescope sensitivities for a total integration time of \(\tau = 1.3 \times 10^3\) s in accordance with the emission time of the gravitational-wave signal [27, 28]. For the lunar telescope we consider two cases, one for \(N = 100\) dipoles and the other for \(N = 1000\) dipoles. The SKA-LOW itself will possess \(N \sim 10^5\) dipoles. The orange dashed line considers the conservative case of only \(N = 100\) dipole antennas, and as can seen we achieve a lower sensitivity than the case of \(N = 1000\) antennas given by the solid blue line. However, the setup is still more sensitive than SKA-LOW (green dashed-dotted line) with its larger array, by almost one order of magnitude at \(\nu = 50\) MHz, the lower end of the SKA-LOW bandwidth (i.e. \(50 - 350\) MHz).

3.2 Detectability

In order for these counterparts to be detected by an observer on Earth or on the moon, the frequency of these photons must be larger than the plasma frequency, \(\omega_p\), of the environments through which they travel. This is to ensure that the photons are not absorbed by the intervening medium. For the Coma cluster (**n\(_e\)** = \(3.43 \times 10^{-3}\) cm\(^{-3}\)) we have \(\omega_p \sim 3.67 \times 10^{-3}\) MHz, whilst the IGM (**n\(_e\)** = \(3 \times 10^{-2}\) cm\(^{-3}\)) has \(\omega_p \sim 1.2 \times 10^{-2}\) MHz. For the same reasons,

\(^3\)See footnote 2.

\(^4\)See footnote 1.
Figure 1: Minimum flux of photons that may be observed by a telescope on the moon when there are $N = 1 \times 10^2$ antennas (orange/dashed), and for $N = 1 \times 10^3$ antennas (solid/blue) and finally we plot the sensitivity of the SKA-LOW (green/dashed-dotted) which has $N \sim 1 \times 10^4$ antennas.

The photons must have frequencies higher than the plasma frequency of the atmosphere through which they travel to arrive at the detector. Whilst the moon has no atmosphere it does possess an ionosphere with $n_e \sim 10^2 \text{ cm}^{-3}$ \cite{33} and therefore has $\omega_p \sim 0.62$ MHz and the Earth ($n_e \sim 10^5 \text{ cm}^{-3}$) \cite{34} has $\omega_p \sim 19.9$ MHz.

The sensitivity profile for SKA-LOW may be found in \cite{35}, whilst for the moon radio telescope we use the setup established in this section. The plasma frequency for the moon, as calculated above, is roughly 0.62 MHz. Therefore, the telescope operating frequency should start just above this value, and we chose to start at 1 MHz. Hence, the frequency range is $1 - 300$ MHz.

Note that the photons we are considering are those that do strike the antennas after having passed through the moon’s ionosphere. Therefore, they have also survived travel through the intervening space between the moon and gravitational-wave event.

4 Results and discussion

We provide the parameter space which indicates values of $\alpha$ and $\nu_c$ for which detections may be possible within the bandwidths of SKA-LOW or a lunar array as described in section 3. That is, we study how the flux varies compared to the sensitivity of the detector. When the flux of the incoming photon is higher than the sensitivity of the detector, we have made a detection, the shaded regions of the parameter space are those in which detections are possible. SKA-LOW is represented by blue/dark shading and the lunar array by the green/light shading. Plots showing parameter space detection coverage were computed at a 5$\sigma$ confidence level and will assume the energy scale of quantum gravitational effects is that of the Planck mass, making these results potentially conservative.

\footnote{http://solar-center.stanford.edu/SID/science/Ionosphere.pdf}
With regards to our choice of the parameter space, we require $\nu_c$ to be larger than the final point in the spectrum in [27, 28], but smaller than 100 MHz, above which $\nu_c$ becomes irrelevant for SKA-LOW. Ultra-steep radio-band power-law indexes have been shown [36] to extend up until -2. In keeping with this we chose our parameter space to lie within $\alpha \in [-2, 0]$.

As we make use of a phenomenological extrapolation of the gravitational-wave spectrum from [27, 28], we will discuss the implications of the results qualitatively. We are interested in the breadth of the detectable parameter space, rather than the specific model values for which signals will be detectable. This is to ensure that our conclusions are not strongly dependent on the choice of extrapolation. So we will examine how strongly detection of a signal depends on the choice of model parameters. A weak dependence will be taken to indicate that the signal is highly detectable, as it can be naturally extrapolated to detectable frequencies with little or no fine-tuning. Whereas, a requirement of very particular values for $\alpha$ and $\nu_c$ will imply that detection is improbable, as significant fine tuning of the spectral extrapolation is needed to reach detectable frequencies. We make a reasonable apriori assumption that a detection requiring fine-tuning is less plausible the more fine-tuning that is required.

Figure 2 shows the detectability parameter space for the conversion of gravitons into photons using $N = 1000$ antennas in the lunar array. The left three plots are for the case of $p = 10^{-13}$ and the right three are for $p = 10^{-16}$. As one moves vertically down, the distance to the gravitational-wave source increases, the 3 values used are: 10 Mpc, 50 Mpc, and 100 Mpc. Note that the conversion probabilities we consider here are similar in magnitude to the conversion probabilities found for the real cases: the IGM and Coma ICM. Figure 3 follows the same formatting but has $N = 100$ antennas in the lunar array.

Consider the parameter space for SKA-LOW in figure 2 or 3, what is immediately noticeable is that a very small amount (18%) of the parameter space is covered even for the case where the source distance is relatively small and the conversion fraction is relatively large. This indicates that the spectral extrapolation must be almost flat. The specificity of the required extrapolation suggests detection is a somewhat marginal possibility.

For the lunar array with both $N = 100$ and $N = 1000$ antennas, when considering $p = 10^{-13}$, it is clear that the parameter space is largely covered (roughly 99% for the case of 10 Mpc and the stronger conversion fraction and $N = 100$), implying that the possibility of detection is largely independent of the parameters. However, when the conversion becomes weak, i.e. $p = 10^{-16}$, we start needing very specific values to make a detection probable. In the case of the smaller number of antennas, at 100 Mpc, detections are completely impossible. Although, the detectable parameter space for the weaker conversion at 10 Mpc has broad coverage, allowing for detections without being strongly reliant on the parameter values.

Figure 4 shows the lower limit that can be placed on the scale $M$ via non-observation with either radio array. We fixed $p = 2 \times 10^{-14}$, in agreement with an environment like the Coma cluster when $M = M_{pl}$, and searched for the largest $M$ such that 95% of the parameter space was covered at a confidence interval of $2\sigma$. As the distance from the array increases, this coverage will drop as the parameter space shrinks with decreasing array sensitivity. Therefore, we see a decline in the mass fraction and the lower bound on $M$, producing poorer limits. This limit allows us to place constraints on the energy-scale of quantum gravity effects supposing a detection is not made with SKA-LOW or a lunar array. This also places limits on the interaction strength as it is inversely proportional to $M$. An interesting comparison may be made with the Lorentz invariance violation limits for gamma-ray bursts. The Lorentz invariance violations can be parameterized by $E^n$, where $E$ is the gamma-ray photon energy and $n$ is an unknown parameter. When $n = 1$, it is shown that a maximum energy-scale of
$M_1 = 9.23 \times 10^{19} \text{ GeV} \ [24]$ can be probed. This is nearly one order of magnitude bigger than the Planck mass $M_{Pl}$. We show here that SKA-LOW, although with weaker limits, may have $M$ up to $10^{-3}$ of the Planck mass, whilst, the lunar array provides competitive results when compared to the $n = 1$ case. For $n = 2$, the limits from Lorentz invariance violation are much weaker: that is $M_2 = 1.3 \times 10^{11} \text{ GeV}$ versus $M = 1.998 \times 10^{13} \text{ GeV}$ with SKA-LOW. Therefore, in both cases we are able to produce competitive non-observational limits on the energy-scale of quantum gravitational effects. We also note that Lorentz invariance violation limits strongly depend on the model-dependent parameter $n$, whereas our results don’t have any unknown such dependence.

5 Conclusion

In the presented work, we have demonstrated that it is plausible to detect low-frequency radio counterparts to gravitational-waves, produced by a binary neutron star merger within 50 Mpc, using a lunar dipole antenna array. Radio telescopes are being planned and are expected to land on the moon by 2019. Experiments such as the Dark Ages Radio Explorer (DARE) and International Lunar Observatory Association (ILOA)’s science objectives are to study early universe physics. The results presented here demonstrate that such a lunar instrument will have the potential to probe the realm of quantum gravity at radio frequencies, using this mechanism of mixing between photons and low-mass bosons. It may even be possible to rule out or confirm the existence of the graviton. In particular, a non-observation of such a direct radio counterpart will place lower limits on the energy scale of putative quantum gravitational effects. These lower limits being competitive with Lorentz invariance violation experiments. With a sufficiently well characterized conversion environment and gravitational-wave event, the observation of these signals will allow one to infer the conversion probability and thus the characteristic energy scale of the interaction.

We have assumed that the photons being produced are made directly by conversion of gravitons into photons as the gravitational-waves pass through the Coma cluster ICM or through the IGM, for example. The advantage of this route is that we may even consider low-frequency radio counterparts coming from binary black hole mergers, which are currently believed to lack any associated electromagnetic counterparts. This is mainly due to lack of accretion processes surrounding the merger event. With the mechanism we consider here, accretion is not required and we may therefore gain knowledge about the elusive compact objects also.

6 Acknowledgements

JT and GB honour the memory of Prof. Sergio Colafrancesco, who contributed to this paper, and helped shape our academic lives, before his passing in September 2018. We thank Dr Dmitry Prokhorov for his invaluable comments. This work is supported by the South African Research Chairs Initiative of the Department of Science and Technology and National Research Foundation of South Africa (Grant No 77948). J.T. and G.B. acknowledges support from the DST/NRF SKA post-graduate bursary initiative. G.B also acknowledges support from a National Research Foundation of South Africa Thuthuka grant no. 117969.
Figure 2: Detectability parameter space for the conversion of gravitons into photons using $N = 1000$ antennas. The left three plots are for the case of $p = 10^{-13}$ and the right three are for $p = 10^{-16}$. As one moves vertically down, the fiducial distance increases: 10Mpc, 50Mpc, 100Mpc. Notice that the parameter space decreases in size as the distance decreases, as one might expect. SKA-LOW is represented by the blue/dark region and the lunar array by the green/light region.

References
[1] J. D. Schnittman, *Coordinated Observations with Pulsar Timing Arrays and ISS-Lobster*, 1411.3994.
Figure 3: Detectability parameter space for the conversion of gravitons into photons using $N = 100$ antennas. The left three plots are for the case of $p = 10^{-13}$ and the right three are for $p = 10^{-16}$. As one moves vertically down, the fiducial distance increases: 10Mpc, 50Mpc, 100Mpc. Notice that the parameter space decreases in size as the distance decreases, as one might expect. SKA-LOW is represented by the blue/dark region and the lunar array by the green/light region.

[2] R. Essick et al., *Localization of short duration gravitational-wave transients with the early advanced LIGO and Virgo detectors*, ApJ 800 (2015) 81 [1409.2435].
Figure 4: Maximum detectable energy-scale $M$, as a fraction of the Planck scale $M_{\text{pl}}$, versus distance. This plot illustrates the lower bounds on $M$ at distances up to 100 Mpc, using SKA-LOW and a lunar array with 1000 dipoles. Here $p = 2 \times 10^{-14}$.

[3] F. Pannarale and F. Ohme, Prospects for joint gravitational-wave and electromagnetic observations of neutron-star–black-hole coalescing binaries, ApJ Lett. 791 (2014) 7 [1406.6057].

[4] A. R. Williamson et al., Improved methods for detecting gravitational waves associated with short gamma-ray bursts, Phys. Rev. D. 90 (2014) 122004 [1410.6042].

[5] D. Kasen, R. Fernandez and B. Metzger, Kilonova light curves from the disc wind outflows of compact object mergers, MNRAS 450 (2015) 1777 [1411.3726].

[6] A. Kamble, A. Soderberg, E. Berger, A. Zauderer, S. Chakraborti and P. Williams, Radio Supernovae in the Local Universe, 1401.1221.

[7] A. Dolgov and K. Postnov, Electromagnetic radiation accompanying gravitational waves from black hole binaries, JCAP 2017 (2017) 018 [1706.05519].

[8] D. Ejlli and V. R. Thandlam, Graviton-photon mixing, Phys. Rev. D. 99 (2019) 044022.

[9] VIRGO, LIGO Scientific collaboration, GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral, Phys. Rev. Lett. 119 (2017) 161101.

[10] B. P. Abbott et al., Multi-messenger Observations of a Binary Neutron Star Merger, ApJ Lett. 848 (2017) 12 [1710.05833].

[11] K. D. Alexander et al., The Electromagnetic Counterpart of the Binary Neutron Star Merger LIGO/VIRGO GW170817. VI. Radio Constraints on a Relativistic Jet and Predictions for Late-Time Emission from the Kilonova Ejecta, ApJ 848 (2017) L21 [1710.05457].

[12] G. Raffelt and L. Stodolsky, Mixing of the Photon with Low Mass Particles, Phys. Rev. D. 37 (1988) 1237.

[13] V. I. Denisov, Interaction of a weak gravitational wave with the field of a rotating magnetic dipole, Sov. Phys. JETP 47 (1978) 209.

[14] U. H. Gerlach, Beat Frequency Oscillations near Charged Black Holes and Other Electrovacuum Geometries, Phys. Rev. Lett. 32 (1974) 1023.
[15] M. E. Gertsenshtein, *Wave resonance of light and gravitational waves*, Sov. Phys. JETP **14** (1962) 84.

[16] R. D. Peccei and H. R. Quinn, *Cp conservation in the presence of instantons*, Phys. Rev. Lett. **38** (1977) 1440.

[17] D. B. Kaplan, *Opening the Axion Window*, Nucl. Phys. B. **260** (1985) 215.

[18] M. Marklund, G. Brodin and P. K. S. Dunsby, *Radio wave emissions due to gravitational radiation*, ApJ **536** (2000) 875 [astro-ph/9907350].

[19] D. Fargion, *Radio Bangs at Kilohertz by SN 1987A: a Test for Graviton-Photon Conversion*, Gravitation and Cosmology **1** (1995) 301 [astro-ph/9604047].

[20] H. Wang et al., *GW170817/GRB 170817A/AT2017gfo association: some implications for physics and astrophysics*, 1710.05805.

[21] S. Jester and H. Falcke, *Science with a lunar low-frequency array: From the dark ages of the universe to nearby exoplanets*, New Astron. Rev. **53** (2009) 1 [0902.0493].

[22] I. A. Crawford and J. Zarnecki, *Astronomy from the moon*, Astron. Geophys. **49** (2008) 2.17.

[23] J. Silk, *Put telescopes on the far side of the moon*, Nature **553** (2018) 6.

[24] V. Vasileiou, A. Jacholkowska, F. Piron, J. Bolmont, C. Couturier, J. Granot et al., *Constraints on Lorentz Invariance Violation from Fermi-Large Area Telescope Observations of Gamma-Ray Bursts*, Phys. Rev. D. **87** (2013) 122001 [1305.3463].

[25] D. Horns et al., *Hardening of TeV gamma spectrum of AGNs in galaxy clusters by conversions of photons into axion-like particles*, Phys. Rev. D. **86** (2012) 075024 [1207.0776].

[26] J. A. R. Cembranos, M. C. Díaz and P. Martín-Moruno, *Graviton-photon oscillation in alternative theories of gravity*, Class. Quantum Gravity **35** (2018) 205008.

[27] M. Kawamura et al., *General relativistic numerical simulation on coalescing binary neutron stars and gauge-invariant gravitational wave extraction*, astro-ph/0306481.

[28] M. Kawamura and K. Oohara, *Gauge - invariant gravitational wave extraction from coalescing binary neutron stars*, Prog. Theor. Phys. **111** (2004) 589 [astro-ph/0404228].

[29] P. P. Kronberg, *Extragalactic magnetic fields*, Reports on Progress in Physics **57** (1994) 325.

[30] P. Blasi et al., *Cosmological magnetic field limits in an inhomogeneous universe*, ApJ Lett. **514** (1999) L79.

[31] A. Bonafede et al., *The coma cluster magnetic field from faraday rotation measures*, A&A **513** (2010) A30 [1002.0594].

[32] S. Colafrancesco, S. Profumo and P. Ullio, *Multi-frequency analysis of neutralino dark matter annihilations in the Coma cluster*, A&A **455** (2006) 21 [astro-ph/0507575].

[33] E. A. Lisin et al., *Lunar dusty plasma: A result of interaction of the solar wind flux and ultraviolet radiation with the lunar surface*, J. of Phys. Conference Series **653** (2015) 012139.

[34] D. Anderson and T. Fuller-Rowell, *The ionosphere*, Space Environment Topics (1999).

[35] P. Dewdney and others.*SKA baseline design document* (2012).

[36] C. De Breuck, W. van Breugel, H. J. A. Rottgering and G. Miley, *A Sample of 669 ultrsteep spectrum radio sources to find high redshift radio galaxies*, A&A **143** (2000) 303 [astro-ph/0002297].