Near-Optimal Reviewer Splitting in Two-Phase Paper Reviewing and Conference Experiment Design

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Abstract

Many scientific conferences employ a two-phase paper review process, where some papers are assigned additional reviewers after the initial reviews are submitted. Many conferences also design and run experiments on their paper review process, where some papers are assigned reviewers who provide reviews under an experimental condition. In this paper, we consider the question: how should reviewers be divided between phases or conditions in order to maximize total assignment similarity? We make several contributions towards answering this question. First, we prove that when the set of papers requiring additional review is unknown, a simplified variant of this problem is NP-hard. Second, we empirically show that across several datasets pertaining to real conference data, dividing reviewers between phases/conditions uniformly at random allows an assignment that is nearly as good as the oracle optimal assignment. This uniformly random choice is practical for both the two-phase and conference experiment design settings. Third, we provide explanations of this phenomenon by providing theoretical bounds on the suboptimality of this random strategy under certain natural conditions. From these easily-interpretable conditions, we provide actionable insights to conference program chairs about whether a random reviewer split is suitable for their conference.

1 Introduction

Peer review is a widely-adopted method for evaluating scientific research. Careful assignment of reviewers to papers is critically important in order to ensure that reviewers have the requisite expertise and that the resulting reviews are of high quality. At large scientific conferences, the paper assignment is usually chosen by solving an optimization problem. Given a set of papers, a set of reviewers, and a similarity matrix consisting of scores representing the level of expertise each reviewer has for each paper, the standard paper assignment problem is to find an assignment of reviewers to papers that maximizes total similarity, subject to constraints on the reviewer and paper loads. This standard paper assignment problem is a simple matching problem and so can be efficiently solved (for example, through linear programming). Our work is motivated by two scenarios that arise in the context of paper assignment in conference peer review.

**Motivation 1: Two-phase paper assignment.** Many conferences (e.g., AAAI 2021-2022, IJCAI 2022) have adopted a two-phase review process. After the initial reviews are submitted, a subset of papers proceed to a second phase of reviews with additional reviewers assigned. There are a variety of reasons that a two-phase reviewing process can be helpful. For example, the process can be used to triage papers based on reviews in the first phase. This can allow the conference to solicit additional reviews only on papers that obtained sufficiently high ratings in the first phase and have any chance of getting accepted (as done at AAAI 2021). The second phase can also help focus on evaluation of the papers in the “messy middle”—the papers at the borderline between acceptance and rejection. This messy middle model [1], which hypothesizes...
that the acceptance decisions for some percentage of submitted papers are effectively random, was proposed
after the NeurIPS 2014 experiment [2] in order to explain the observed inconsistency in acceptance decisions.
Additional reviewers could improve the evaluation of these papers to more accurately discern which should
be accepted. Later analysis of the NeurIPS 2015 and 2016 review process found that the size of the messy
middle in these conferences was 45% and 30% of submissions respectively [3]. A second phase of reviews can
also be used to help compensate for reviewers who were unresponsive or minimally responsive in the first
phase, who can no longer review due to problems in their personal lives, who discovered conflicts they had
with a paper assigned to them and recused themselves from it, etc.

In all of these cases, the set of papers that will require additional review is unknown beforehand. While
some venues choose to recruit new reviewers after knowing which papers proceed to phase two, the tight
timeline of many conferences makes it hard to recruit new reviewers after phase one. In SIGMOD 2019 [4]:
“Additional reviews were solicited manually by the chairs and this was a huge time sink, especially when
some reviewers refused to take on the additional assignment. The additional review solicitation needs to be
automated and reviewer expectations need to be set appropriately beforehand.” For this reason, it is best if all
the reviewers are recruited at the beginning, and a key question is then how to assign reviewers to papers in
the first phase such that enough review capacity is saved for the second phase.

Motivation 2: Conference experiment design. Reviewers also need to be split into two groups when
conferences run controlled experiments on the paper review process. Conferences often run such experiments
to test changes to the review process. For example, the WSDM 2017 conference conducted an experiment to
test the effects of single-blind versus double-blind reviewing [5]. As another example, the aforementioned
NeurIPS 2014 experiment tested the consistency of acceptance decisions by providing some papers with a
second set of reviews from a separate group of reviewers. In these experiments, all papers receive reviews
conducted in the usual manner (the control condition), but a random subset of papers are additionally
assigned reviewers who provide reviews under an experimental condition. In the NeurIPS 2014 experiment,
a random 10% of papers were put in the experimental condition and received a second set of reviews. In
the WSDM 2017 experiment, the subset of papers was the full paper set; that is, all papers were reviewed
under both single-blind and double-blind conditions. The key question is then how to divide the reviewers
between the control and experimental conditions. As in the NeurIPS 2014 and WSDM 2017 experiments, this
is often done randomly for statistical purposes. However, conferences still want to ensure that the resulting
assignment of papers to reviewers is of high similarity.

As our results apply to both the two-phase and experiment design settings, we will use the generic
terminology of “stages” to refer to both phases and conditions simultaneously across the two settings.

Problem outline. In this paper, we formally analyze the two-stage paper assignment problem, which
encompasses both above motivations. As stated earlier, the standard paper assignment problem is to maximize
the total similarity of the assignment subject to load constraints and is efficiently solvable. However, in the
two-stage paper assignment problem, we must additionally decide how much of each reviewer’s capacity
should be saved to review papers in the second stage. We assume that the fraction of papers that will need
additional reviews is known and that the set of second-stage papers is chosen uniformly at random.

Because of constraints present in each setting, the maximum-similarity paper assignment across the two
stages cannot be achieved. In the two-phase setting, the set of papers that will need to be reviewed in the
second stage is unobserved when the first-stage assignment is made, making the problem one of stochastic
optimization. In the experiment design setting, reviewers are often randomized between stages for statistical
purposes. We show that a simple strategy for choosing reviewers to save for the second stage performs
near-optimally in terms of assignment similarity and can be used in either setting.

Contributions. Our contributions are as follows.

First, we identify and formulate the two-stage paper assignment problem, an issue of practical importance
to modern conferences, with applications to two-phase paper assignment and conference experiment design
(Section 3).

Second, we prove that a simplified version of the problem is NP-hard, suggesting that the problem may
not be efficiently solvable (Section 4).

Third, we empirically show that a very simple “random split” strategy, which chooses a subset of reviewers
uniformly at random to save for the second stage, gives near-optimal assignments on real conference similarity
scores (Section 5.1). This result is summarized in Figure 1, which shows the assignment similarity achieved
using random split as compared to the oracle optimal assignment (which views the set of second-stage papers
Figure 1: Range of assignment similarities found over 10 random reviewer splits on real conference data, as a fraction of the oracle optimal assignment’s similarity (computed after observing the second-stage papers). \( \beta \) indicates the fraction of papers in the second stage. The ICLR similarities \cite{6} (911 papers, 2435 reviewers) are constructed from text-matching between papers and reviewers’ past work, PrefLib3 \cite{7} (176 papers, 146 reviewers) and Bid1 \cite{8} (600 papers, 400 reviewers) similarities are constructed from bidding data, and SIGIR \cite{9} similarities (73 papers, 189 reviewers) are constructed from reviewer and paper subject areas.

before optimally assigning reviewers across both stages) for several datasets. We find that all random reviewer splits achieve at least 90% of the oracle optimal solution’s similarity on all datasets and at least 94% on all but two experiments. These results hold across similarities constructed via a variety of methods used in practice (including text-matching, bidding, and subject areas), indicating that random split is robust across methods of similarity construction. They also hold both when the second-stage papers are drawn uniformly at random (as in Figure 1a) and when they are selected based on the review scores of the papers (as in Figure 1b). In practice, this means that program chairs planning a two-phase review process or a conference experiment can simply split reviewers across the two phases/conditions at random without concerning themselves with the potential reduction in assignment quality.

We also show that this good performance is not achieved in general: there exist similarity matrices on which random split performs very poorly (Section 5.2).

Fourth, we theoretically explain why random split performs well on our real conference similarity matrices by deriving theoretical bounds on the suboptimality of this random strategy under certain natural conditions (Sections 6 and 7). We consider two such sufficient conditions here, which are met by our datasets: if the reviewer-paper similarity matrix is low-rank, and if the similarity matrix allows for a high-value assignment (in terms of total similarity) with a large number of reviewers assigned to each paper. From these results, we give key actionable insights to conference program chairs to help them decide—well before the reviewers and/or papers are known—if random split is likely to perform well in their conference.

All of the code for our empirical results is freely available online\cite{1}.

2 Related Work

Our work assumes that the “similarities” between reviewers and authors are given. In practice, there are several ways in which these similarities are computed, and different program chairs often make different decisions on how this computation is done. The similarities are generally computed using one or more of the following three sources of data:

- **Text-matching of papers**: Natural language processing techniques \cite{10,14} are used to match the text of the submitted paper with the text of the reviewers’ past papers.

\cite{1}https://github.com/sjecmen/multistage_reviewing_bounds
Subject areas: The program chairs create a list of subject areas relevant to the conference. Each reviewer selects a subset of these subject areas that are representative of their expertise, and each submitted paper is accompanied by the authors selecting the subject areas relevant to the paper.

Bids: A number of conferences adopt a bidding system, where reviewers are shown a list of (some of) the papers that are submitted to the conference (and which do not conflict with them) and asked to indicate the papers which they are willing to review. These computed similarities are then used to assign reviewers to papers. If more than one such source of data is used by the conference, they are combined in a manner deemed suitable by the program chairs. These computed similarities are then used to assign reviewers to papers. By far the most popular method of doing this assignment is to solve an optimization problem that maximizes the sum of the similarities of the assigned reviewer-paper pairs, subject to constraints on the reviewer and paper loads. Given its widespread popularity, we analyze this sum-similarity objective in our paper.

That being said, there are other objectives that are also proposed for automated assignment using the similarities, such as the max-min fairness objective. A recent work proposes assigning reviewers via optimizing the sum similarity achieving an approximation ratio of no greater than $0.5$. However, this guarantee is very weak in the paper assignment setting since it can be trivially achieved by maximizing similarity in the first stage alone.

One motivation for our work is that of running controlled experiments in peer review. Controlled experiments pertaining to peer review are conducted in many different scientific fields, including several controlled experiments recently conducted in computer science. These experiments have also led to a relatively nascent line of work on careful design of experimental methods for peer review, and our work sheds some light in this direction in terms of trading off assignment quality with randomization in the assignment. Some other experiments in conferences do not operate under controlled settings, but exploit certain changes in the conference policy such as a switch from single blind to double blind reviewing. Overall, experiments offer important insights into the peer review process; see for more discussion on challenges in peer review and some solutions.

3 Problem Formulation

In this section, we formally define the two-stage paper assignment problem. Given a set of $n$ papers $P = [n]$ and a set of $m$ reviewers $R = [m]$, define $S \in [0,1]^{m \times n}$ as the similarity scores between each reviewer and paper. An assignment of papers to reviewers is represented as a matrix $A \in \{0,1\}^{m \times n}$, where $A_{rp} = 1$ if reviewer $r$ is assigned to paper $p$ and $A_{rp} = 0$ otherwise. In the standard paper assignment problem, the objective is to find an assignment $A$ of reviewers to papers such that the total similarity $\sum_{r \in R, p \in P} A_{rp} S_{rp}$ is maximized, subject to constraints that each paper is assigned exactly a certain load of reviewers, each reviewer is assigned to at most a certain load of papers, and any reviewer-paper pairs with a conflict of interest are not assigned. In this work, we accommodate conflicts of interest by assuming
the corresponding similarities are set to 0. This problem can be formulated as a min-cost flow problem or as a linear program, and can be efficiently solved.

In a two-stage assignment, all papers \( P \) require a certain number of reviewers in the first stage and a subset of papers \( P_2 \subseteq P \) require additional review in the second stage. We assume that \( P_2 \) consists of a fixed fraction \( \beta \) of papers and is drawn uniformly at random from \( P \). Specifically, for some \( \beta \in \{ \frac{1}{n}, \ldots, \frac{n}{n} \} \), we assume that \( P_2 \sim \mathcal{U}_{\beta n}(P) \), the uniform distribution over all subsets of size \( \beta n \) of \( P \). In the two-phase setting, the fraction \( \beta \) itself can be viewed as a parameter that the program chairs set based on available reviewer resources, or it can be estimated from past editions of the conference. Our empirical results detailed in Section 5.1 also cover the case where papers are chosen for the second phase based on their first-phase review scores. In the conference experiment design setting, the value of \( \beta \) and the uniform distribution of \( P_2 \) are both experiment design choices. The choice of a uniform distribution for \( P_2 \) is common, as in the NeurIPS 2014 and WSDM 2017 experiments. The question we analyze is: how should reviewers be assigned to papers across the two stages?

Before continuing further, we introduce some notation. For subsets \( R' \subseteq R \) and \( P' \subseteq P \), desired paper load \( \ell_{pap} \in \mathbb{Z}_+ \), and maximum reviewer load \( \ell_{rev} \in \mathbb{Z}_+ \), define \( \mathcal{M}(R', P'; \ell_{rev}, \ell_{pap}) \subseteq \{0,1\}^{m \times n} \) as the set of valid assignment matrices on \( R' \) and \( P' \). Formally, \( A \in \mathcal{M}(R', P'; \ell_{rev}, \ell_{pap}) \) if and only if \( \sum_{r \in R'} A_{rp} = \ell_{pap} \) for all \( p \in P' \), \( \sum_{p \in P'} A_{rp} \leq \ell_{rev} \) for all \( r \in R' \), and \( A_{rp} = 0 \) for all \( (r, p) \notin R' \times P' \).

The two-stage paper assignment problem is to maximize the total similarity of the paper assignment across both stages. Without loss of generality, we instead consider the mean similarity so that later results will be easier to interpret. Fix a stage one paper load \( \ell_{pap}^{(1)} \), a stage two paper load \( \ell_{pap}^{(2)} \), and an overall reviewer load \( \ell_{rev} \) such that \( \ell_{pap}^{(1)} + \beta \ell_{pap}^{(2)} \leq \ell_{rev} \) (i.e., the number of reviews required by papers is no greater than the number of reviews that can be supplied by reviewers). Given \( P_2 \), the oracle optimal assignment has mean similarity

\[
Q^*(P_2) = \max_{A \in \mathcal{M}(R', P'; \ell_{rev}, \ell_{pap})} \frac{1}{\ell_{pap}^{(1)} + \ell_{pap}^{(2)} \beta n} \left[ \sum_{r \in R', p \in P} A_{rp} S_{rp} + \sum_{r \in R', p \in P_2} B_{rp} S_{rp} \right]
\]

subject to \( \sum_{p \in P} A_{rp} + B_{rp} \leq \ell_{rev} \quad \forall r \in R \).

The last constraint ensures that each reviewer’s assignment across both stages does not exceed the maximum reviewer load. Just like the standard paper assignment problem, the oracle optimal assignment for a given \( P_2 \) can be found efficiently. However, in both the two-phase and experiment design settings, this oracle optimal assignment is either unachievable or undesirable. In the two-phase setting, the set of papers \( P_2 \) requiring additional review is unknown until after the stage one assignment is chosen. Thus, the oracle optimal assignment cannot be computed beforehand. In the experiment design setting, the assignment of reviewers to conditions is commonly randomized in order to gain statistical power, as was done in the WSDM 2017 and NeurIPS 2014 experiments. Thus, a deterministic choice of assignment may not be desirable. Additionally, depending on the experiment setup, it may not be possible for a reviewer to review papers in both conditions. In what follows, we use this oracle optimal assignment value as an unachievable baseline for comparison.

We instead consider simple strategies for the two-stage assignment problem that choose a subset \( R_2 \subseteq R \) of reviewers to save for the second stage without observing \( P_2 \), leaving reviewers \( R_1 = R \setminus R_2 \) to be assigned to papers in the first stage. Unlike the oracle optimal assignment, such strategies are feasible to implement in both settings since they do not require knowledge of \( P_2 \), do not split reviewer loads across conditions, and allow for a random choice of \( R_2 \). The mean similarity of the optimal assignment when reviewers \( R_2 \) and papers \( P_2 \) are in the second stage is

\[
Q(R_2, P_2) = \frac{1}{\ell_{pap}^{(1)} + \ell_{pap}^{(2)} \beta n} \left[ \max_{A \in \mathcal{M}(R_1, R_2, P_2; \ell_{rev}, \ell_{pap}^{(1)})} \sum_{r \in R \setminus R_2, p \in P} A_{rp} S_{rp} + \max_{B \in \mathcal{M}(R_1, R_2, P_2; \ell_{rev}, \ell_{pap}^{(2)})} \sum_{r \in R_2, p \in P_2} B_{rp} S_{rp} \right]
\]

We require that \( \ell_{rev}|R_2| \geq \ell_{pap}^{(2)} \beta n \) and \( \ell_{rev}(m - |R_2|) \geq \ell_{pap}^{(1)} \) for feasibility in both stages. Given \( R_1, R_2 \), and \( P_2 \), the optimal paper assignment in each stage can be efficiently computed using standard methods. Thus, the difficulty of the problem lies entirely in choosing \( R_2 \).
The expected mean similarity of the optimal assignment when saving reviewers \( R_2 \) for the second stage is
\[
f(R_2) = \mathbb{E}_{\mathcal{P}_2 \sim \mathcal{U}_{\beta n}(\mathcal{P})} \left[ Q(R_2, \mathcal{P}_2) \right].
\]
We can also evaluate the suboptimality of \( R_2 \) as compared to the oracle optimal assignment as
\[
Q^*(\mathcal{P}_2) - Q(R_2, \mathcal{P}_2), \quad \text{where} \quad \mathcal{P}_2 \sim \mathcal{U}_{\beta n}(\mathcal{P}).
\]
Note that \( Q^* \) and \( Q \) are bounded in \([0, 1]\), so that both \( f \) and the suboptimality are also bounded in \([0, 1]\).

In our theoretical analysis, for simplicity, we assume that \( \ell_{rev} = \ell_{pap}^{(1)} = \ell_{pap}^{(2)} = 1 \), leaving this implicit in \( f, Q \), and \( Q^* \) throughout the paper. We also assume \( m = (1 + \beta)n \) in our analysis unless specified otherwise, so that \( |R_2| = \beta n \). The intuition behind our results carries over to the cases of general loads and excess reviewers, which are covered by our empirical results in Section 5.1. All asymptotic bounds are given as \( n \) grows.

4 Hardness

In the two-phase setting, the oracle optimal assignment is unachievable because \( R_2 \) must be chosen before observing \( \mathcal{P}_2 \). Therefore, conferences must choose \( R_2 \) to maximize \( f \), the expected mean similarity of the assignment across both stages. In this section, we demonstrate that maximizing a variant of \( f \) is NP-hard, indicating that it is unlikely that \( f \) can be optimized efficiently.

First, note that evaluating \( f(R_2) \) requires computing an expectation over the draw of \( \mathcal{P}_2 \), which naively requires evaluating a sum over the optimal assignment value for \( \binom{n}{\beta n} \) possible choices of \( \mathcal{P}_2 \). This number is exponential in the input size, so an efficient algorithm for this problem would have to either optimize \( f \) without evaluating it or compute this expectation without computing the optimal assignment for each possible \( \mathcal{P}_2 \).

Instead of attempting to optimize \( f \) exactly, a standard approach from two-stage stochastic optimization is to simplify the problem by sampling as follows [53, 54]. First, take some fixed number of samples \( \mathcal{P}_2^{(1)}, \ldots, \mathcal{P}_2^{(K)} \) from \( \mathcal{U}_{\beta n}(\mathcal{P}) \). Then, rather than optimizing an average over all \( \mathcal{P}_2 \) in the support of \( \mathcal{U}_{\beta n}(\mathcal{P}) \), choose \( R_2 \) to optimize an average over only all sampled sets:
\[
\bar{f}(R_2) = \frac{1}{K} \sum_{i=1}^{K} Q(R_2, \mathcal{P}_2^{(i)}).
\]
This is a natural simplification of the two-stage paper assignment problem, because the sum in the objective is now taken over only a constant \( K \) subsets rather than an exponential number. However, this problem is still not efficiently solvable, as the following theorem shows.

**Theorem 1.** It is NP-hard to find \( R_2 \subseteq \mathcal{R} \) such that \( \bar{f}(R_2) \) is maximized, even when \( K = 3 \).

**Proof sketch.** We reduce from 3-Dimensional Matching [55], which asks if there exists a way to select \( k \) tuples from a set \( T \subseteq X \times Y \times Z \) where \( |X| = |Y| = |Z| = k \) such that all elements of \( X, Y, \) and \( Z \) are selected exactly once. We construct 3 samples of second-stage papers corresponding to \( X, Y, \) and \( Z \) respectively, and construct reviewers corresponding to elements of \( T \). These reviewers have 1 similarity with the papers in their tuple, and 0 similarity with all other papers. Thus, checking if there exists a choice of \( R_2 \) which gives full expected similarity in the second stage would require solving 3-Dimensional Matching. We add additional reviewers and papers to ensure that this choice of \( R_2 \) is optimal over both stages.

The full proof is presented in Appendix D.1.

Since it is NP-hard to find the optimal \( R_2 \) even when estimating the objective by sampling three random choices of \( \mathcal{P}_2 \), this suggests that the original objective \( f \) may be hard to optimize efficiently. Therefore, in the two-phase assignment setting, we instead look for efficient approximation algorithms.
5 Our Approach: Random Split

Our proposed approach for finding a two-stage assignment is extremely simple: choose \( R_2 \) uniformly at random (i.e., \( R_2 \sim \mathcal{U}_{\beta n}(R) \)). We refer to this as a “random split” of reviewers into the two review stages.

In the two-phase setting, random split is an efficient approximation algorithm for the problem of optimizing \( f \), which is likely difficult (as shown in Section 4). Because random split does not execute \( f \), it produces a two-stage paper assignment without needing to estimate \( f \) by sampling.

In the conference experiment design setting, our proposed random-split strategy corresponds to a uniform random choice of reviewers for the experimental condition. Recall that in this setting, assigning reviewers to conditions uniformly at random is already a common experimental setup. The performance of random split on \( f \) therefore indicates how well this common setup performs in terms of the expected assignment similarity.

In our theoretical results, we often refer to the suboptimality of random split, defined as the suboptimality of \( R_2 \) chosen via random split when \( P_2 \) is chosen uniformly at random:

\[
Q^*(P_2) - Q(R_2, P_2), \quad \text{where} \quad P_2 \sim \mathcal{U}_{\beta n}(P), R_2 \sim \mathcal{U}_{\beta n}(R).
\]

Recall from Section 3 that \( Q^*(P_2) \) is the mean similarity of the oracle optimal assignment given second-stage papers \( P_2 \) and that \( Q(R_2, P_2) \) is the mean similarity of the optimal assignment given second-stage reviewers and papers \( R_2, P_2 \). Additionally, many of our results evaluate the expected mean similarity under random split:

\[
\mathbb{E}_{R_2 \sim \mathcal{U}_{\beta n}(R) \mid f(R_2)]} = \mathbb{E}_{R_2 \sim \mathcal{U}_{\beta n}(R), P_2 \sim \mathcal{U}_{\beta n}(P)} [Q(R_2, P_2)] .
\]

In the following subsections, we first elaborate on the good performance random split displays empirically before showing that there exist cases where random split performs very poorly.

5.1 Empirical Performance

As introduced earlier in Figure 1, random split performs very well in practice on four real conference similarity matrices. The first is a similarity matrix recreated using text-matching on ICLR 2018 data \([6]\). The second is constructed using reviewer bid data for an AI conference (conference 3) from PrefLib dataset MD-00002 \([7]\). The third (denoted Bid1) is a sample of the bidding data from a major computer science conference \([8]\). In both of these bidding datasets, we transformed “yes,” “maybe,” and “no response” bids into similarities of 1, 0.5, and 0.25 respectively, as is often done in practice \([3]\). The fourth similarity matrix is constructed from the subject areas of ACM SIGIR 2007 papers and the subject areas of the past work of their authors (assumed to be the reviewers) \([9]\); we set the similarity between each reviewer and paper to be equal to the number of matching subject areas out of the 25 total, normalized so that each entry of the matrix is in \([0, 1]\).

In Appendix A we present additional empirical results particularly relevant to the conference experiment design setting.

We run several experiments, each corresponding to a choice of dataset and \( \beta \). Each experiment consists of 10 trials, where in each trial we sample a random reviewer split and a set of second-stage papers. We then present the range of assignment values achieved across the trials as percentages of the oracle optimal assignments for each trial. The oracle optimal assignment for a trial is found by choosing the optimal assignment of reviewers across both stages after observing \( P_2 \). We set paper loads of 2 in each stage (as done in AAAI 2021), and limit reviewer loads to be at most 6 (a realistic reviewer load \([3]\)). Since these datasets have excess reviewers, we choose \( R_2 \) to have size \( \frac{2}{1+\beta}m \) so that the proportions of reviewers and papers in the second stage are equal.

In Figure 1a \( P_2 \) is drawn uniformly at random in each trial (as in the problem formulation). We see that all trials of random split achieve at least 90\% of the oracle optimal solution’s similarity on all datasets, with all trials on all but two experiments achieving at least 94\%. We see additionally that the randomness of the reviewer choice does not cause much variance in the value of the assignment, as there is at most a 5\% difference between the minimum and maximum similarity (as a percentage of oracle optimal) for each experiment. Note that this is true despite the fact that the similarity matrices of the different datasets are constructed in several different ways, indicating that random split is robust across methods of similarity construction.
In Figure 1b, \( \mathcal{P}_2 \) is chosen as a fixed set for all trials based on the actual review scores received by the papers at ICLR 2018 \cite{56} (as review scores were not available for other datasets). We run trials where either the top-scoring papers or the messy-middle papers are given additional reviews. Since about 37% of papers were accepted, we define the messy middle as the range of \( \frac{\beta}{1+\beta} m \) papers centered on the 63rd-percentile paper when ordered by score. These are sets of papers that a conference may potentially want to assign additional reviewers to. In all cases, random split shows consistently good performance, similar to when \( \mathcal{P}_2 \) was drawn uniformly at random. All trials achieve at least 95% of the oracle optimal similarity, with at most a 2% difference between the minimum and maximum for each experiment. This suggests that the good performance of random reviewer split naturally holds in these practical cases.

5.2 A Counterexample

The good results random split shows in practice may be somewhat surprising because random split does not perform well in all settings. The following theorem shows that for any \( \beta \), there exist instances of the two-stage paper assignment problem where the suboptimality of random split (1) is \( \Omega(1) \) in expectation.

Theorem 2. For any constant \( \beta \in [0,1] \), there exists \( n_0 \) such that for all \( n \geq n_0 \) where \( \beta n \in \mathbb{Z}_+ \), there exist instances of the two-stage paper assignment problem where the suboptimality of random split is at least \( \frac{\beta^4}{(1+\beta)^3} \) in expectation.

Proof sketch. Consider \( \beta = 1 \). We construct a similarity matrix where every reviewer has similarity 1 with only 1 paper, and all papers have similarity 1 with only 2 reviewers. The optimal reviewer split puts the two good reviewers for each paper in separate stages and always achieves a mean similarity of 1. Random split puts both good reviewers in the same stage with at least constant probability for each paper, giving a constant mean similarity \(<1\).

The full proof is presented in Appendix D.2.

Note that the above lower bound on the objective value of random split holds even in the easy case of \( \beta = 1 \), where the problem could be solved simply through standard paper assignment methods. This case is particularly relevant in the conference experiment setting, where all papers are commonly reviewed under both conditions (as in the WSDM 2017 experiment).

Although the above lower bound demonstrates that random split cannot hope to do well in general, the constructed example is unrealistic for real conferences. However, program chairs may understandably want some guarantee that a random reviewer split will work well for their conference before deciding to use it. Ideally, this guarantee should be given before the precise similarity matrix for the conference is known, since the similarities may not be known in full until late in the planning process.

In the following sections, we provide such guarantees, thereby showing that the good performance of random split is not just an artifact of our specific datasets. We focus our attention on two sufficient conditions on the similarity matrix under which we show random split performs well. These conditions are natural for real similarity matrices, implying that random split will perform well for many real conferences, whether in the context of a two-phase review process or a conference experiment. Using these conditions, we provide actionable insights to program chairs based on simple properties of their conference’s similarities that they may have intuition about. These insights are designed to be useful well before the full paper and reviewer sets are known.

6 Condition 1: Low-Rank Similarity Matrix

The first condition we consider is that the similarity matrix \( S \) has low rank \( k \). This condition naturally arises in practice when reviewer-paper similarities are calculated from the number of subject area agreements between reviewers and papers; in such cases, the rank is no greater than the number of subject areas. For example, the SIGIR similarity matrix used in Figure 1 is constructed in this way and thus has rank no greater than 25 (the number of subject areas). In this section, we provide asymptotic upper and lower bounds on the suboptimality of random split for constant-rank similarity matrices.
6.1 Theoretical Bounds

We first provide an upper bound on the suboptimality of random split. This shows that random reviewer splits perform well on constant-rank similarity matrices, including the SIGIR similarity matrix examined earlier. More precisely, the following theorem shows that if the similarity matrix $S$ has constant rank $k$, the suboptimality of random split is at most $O(n^{-\frac{1}{2}})$ when $k = 1$, $O(n^{-\frac{1}{2} + o(1)})$ when $k = 2$, and $O(n^{-\frac{1}{2} + o(1)})$ when $k \geq 3$ with high probability.

**Theorem 3.** Consider any constants $\beta \in [0, 1]$ and $k \in \mathbb{Z}_+$. There exists $n_0$ and constants $C, \eta$ such that, for any $n \geq n_0$ where $\beta n \in \mathbb{Z}_+$ and for any similarity matrix $S \in [0, 1]^{(1+\beta)n \times n}$ of rank $k$, the suboptimality of random split is at most:

- $C(\log n)^n n^{-\frac{1}{2}}$ if $k = 1$
- $C(\log n)^n n^{-\frac{1}{2} + \frac{1}{\log \log n}}$ if $k = 2$
- $C(\log n)^n n^{-\frac{1}{2} + \frac{1}{\log \log n}}$ if $k \geq 3$

with probability at least $1 - \frac{1}{n}$ (where $\log$ indicates the base-2 logarithm).

**Proof sketch.** By Lemma 4 of [57], a rank $k$ similarity matrix $S \in [0, 1]^{m \times n}$ can be factored into vectors $u_r \in \mathbb{R}^k$ for each $r \in \mathcal{R}$ and $v_p \in \mathbb{R}^k$ for each $p \in \mathcal{P}$ such that $S_{rp} = (u_r, v_p)$, $||u_r||_2 \leq k^{1/4}$, and $||v_p||_2 \leq k^{1/4}$. We cover the $k$-dimensional ball containing all paper vectors with smaller cells, and consider a reviewer to be in one of these cells if the oracle optimal assignment (given $\mathcal{P}_2$) assigns it to a paper in that cell. Using a concentration inequality on the number of reviewers and papers in each cell in each stage, we can upper bound the number of reviewers that we cannot match to papers within the same cell. We then increase the size of the cells and attempt to match the remaining reviewers in this way, continuing until all reviewers are matched. We upper bound the suboptimality of the resulting assignment by the L2 distance between a reviewer’s assigned paper and the paper they are assigned by the oracle optimal assignment.

The constants $C$ and $\eta$ may depend on $k$, which is itself assumed to be constant. The full proof is presented in Appendix D.3.

For constant-rank similarity matrices, the suboptimality diminishes as $n$ grows, unlike when the rank of the similarity matrix is unrestricted. Conceptually, our proof technique of finding a minimum-distance matching between two samples of points resembles the optimal transport problem solved when finding the Wasserstein distance between a probability distribution and its empirical measure. Thus, our upper bounds nearly match those found in the literature on the expected empirical 1-Wasserstein distance for continuous measures (see [55] and references therein).

We now complement the above upper bound with lower bounds on the suboptimality of random split for constant rank similarity matrices. The following theorem shows that, for similarity matrices of constant rank $k$, the suboptimality of random split is $\Omega(n^{-1/2})$ in expectation and $\Omega(n^{-2/k})$ with high probability.

**Theorem 4.** Suppose $\beta = 1$. For any constant $k \in \mathbb{Z}_+$, there exists $n_0$ and constants $C, \zeta$ such that for all $n \geq n_0$:

(a) There exist instances of the two-stage paper assignment problem with similarity matrices $S \in [0, 1]^{2n \times n}$ of rank $k$ such that the suboptimality of random split is at least $Cn^{-1/2}$ in expectation.

(b) There exist instances of the two-stage paper assignment problem with similarity matrices $S \in [0, 1]^{2n \times n}$ of rank $k$ such that the suboptimality of random split is at least $Cn^{-2/k}$ with probability $1 - \zeta e^{-n/10}$.

**Proof sketch.** (a) We construct $k$ groups of reviewers and papers, where all reviewers and papers in the same group have similarity 1 with each other and similarity 0 with all other reviewers/papers. The first group contains $\frac{n}{2}$ papers and $n$ reviewers. The optimal reviewer split puts half of each group’s reviewers in each stage and assigns all reviewers to papers with similarity 1. By an anti-concentration inequality, with constant probability, at least $\Omega(\sqrt{n})$ reviewers in the first group cannot be assigned to a paper in their group under random split.

(b) We construct a vector in $\mathbb{R}^k$ for each reviewer and each paper and set the similarity between that reviewer and that paper to be the inner product of their corresponding vectors. We place one paper vector
and two reviewer vectors at each point in an evenly-spaced grid throughout the cube $[0, 1/\sqrt{k}]^k$. The resulting similarity matrix has rank $k$. The optimal assignment assigns the two reviewers at each point to the paper at that point. With high probability, random split places $\Omega(n)$ pairs of reviewer vectors into the same stage. One of each of these reviewer pairs must be assigned to a paper at a different point, which is at least $\Omega(n^{-1/k})$ away in L2 distance. The suboptimality of the resulting assignment can be written in terms of the total squared L2 distance between each reviewer and their assigned paper, giving the stated bound. 

The constants $C$ and $\zeta$ may depend on $k$, which is itself assumed to be constant. The full proof is presented in Appendix D.4.

6.2 Interpretation of Results

As discussed earlier in this section, certain methods of constructing similarities (such as counting subject area agreements) may inherently lead to low-rank similarity matrices. If a conference is using such a method, the results in this section provide guarantees to the program chairs that random split will perform well, particularly if the rank of the matrix is low compared to the number of papers and reviewers. Alternatively, program chairs may be able to estimate that their reviewers and papers can be grouped into a small number of communities with little variation within them, in which case the similarity matrix may also be low rank.

7 Condition 2: High-Value, Large-Load Assignment

A natural condition on the similarity matrix to consider is that each paper has a large number $\mu$ of reviewers with high similarity for that paper. It turns out that this condition is insufficient for guaranteeing good performance of random split, since the same group of $\mu$ reviewers could have high similarity with all papers, thus satisfying this condition without changing the assignment value by much (since we can only assign these reviewers to a few papers). In this section, we consider a condition on the similarity matrix that is similar in spirit: the existence of a high-value assignment (in terms of total similarity) on the full reviewer and paper sets where each paper is assigned a large number $(1 + \beta)\mu$ of reviewers. Our proposed condition handles the issue with the naive “large number of reviewers” condition by requiring that the high-value reviewers for each paper can all be simultaneously assigned.

In the following subsections, we first provide theoretical guarantees about the performance of random split under this condition. We then demonstrate that this condition helps to explain the good performance of random split on the real similarity matrices presented earlier.

7.1 Theoretical Bounds

The first result of this section gives a lower bound on the expected value of random split in terms of the value of a single, large-load assignment. All results in this section still hold if there are excess reviewers (i.e., if $m \geq (1 + \beta)n$ and $R_2 \sim \mathcal{U}_{\frac{m}{1+\beta}}(\mathcal{R})$).

**Theorem 5.** Consider any $\mu \in [10, 000]$ and $\beta \in \{\frac{1}{100}, \ldots, \frac{100}{100}\}$ such that $\beta \mu \in \mathbb{Z}_+$. If there exists an assignment $A^{(\mu)} \in \mathcal{M}(\mathcal{R}, \mathcal{P}; \mu, (1 + \beta)\mu)$ with mean similarity $s^{(\mu)}$, choosing $R_2$ via random split gives that

$$\mathbb{E}_{R_2} [f(R_2)] \geq s^{(\mu)} \left[ 1 - \sqrt{\frac{\beta}{2\pi(1 + \beta)^2 \mu}} \left( 2\sqrt{\frac{1}{1 + \beta} + \sqrt{1 - \beta}} \right) \right].$$

A similar bound holds when $\beta \mu$ is not integral, with some additional small terms due to rounding.

**Proof sketch.** We construct assignments with paper and reviewer loads of at most $\mu$ in stage one and at most $\beta \mu$ in stage two using the reviewer-paper pairs assigned by $A^{(\mu)}$. We drop any extra assignments at random so that no reviewers and papers are overloaded, and assume any pairs that must be assigned from outside of $A^{(\mu)}$ have similarity 0. From within each of these larger assignments, we can find an assignment with paper and reviewer loads of 1 with at least the same mean similarity. The expected mean similarity of these assignments can be written as the expectation of a function of binomial random variables. Approximating
Then, choosing an assignment $A$ similar bound holds when $m < n$. Suppose there exists an assignment $A^{(1)} \in \mathcal{M}(R, P; 1, 2)$ with mean similarity $s^{(1)}$. Suppose there also exists an assignment $A^{(\mu)} \in \mathcal{M}(R, P; \mu, 2\mu)$ with mean similarity $s^{(\mu)}$ that does not contain any of the pairs assigned in $A^{(1)}$. Then, choosing $R_2$ via random split gives that
\[
\mathbb{E}_{R_2} [f(R_2)] \geq \frac{3}{4} s^{(1)} + \left(1 - \frac{1.44}{\sqrt{\mu}}\right) \frac{1}{4} s^{(\mu)}.
\]
A similar bound holds when $\frac{m}{n}$ is not integral, with some additional small terms due to rounding.

**Proof sketch.** We first attempt to assign as many pairs as possible from within $A^{(1)}$; in expectation we can assign $\frac{3}{4}$ of them. Among the remaining reviewers and papers, we attempt to construct assignments with paper and reviewer loads of $\frac{m}{n}$ in both stages from within the reviewer-paper pairs assigned by $A^{(\mu)}$. This is done in a similar way as in Theorem 5.

The more general version of the bound and the full proof are stated in Appendix D.6.

If we consider $A^{(1)}$ as the optimal assignment and assume that $\mu$ is divisible by 4, we get an approximation ratio (between the random split assignment and oracle optimal assignment’s similarities) of $\frac{3}{4} + \frac{2\mu}{4} \left(1 - \frac{1.44}{\sqrt{\mu}}\right)$ where $\gamma_{\mu} = \frac{s^{(\mu)}}{s^{(1)}}$. With $\mu = 8$, we achieve an approximation ratio of at least $\frac{3}{4} + \frac{2\times 8}{4} \left(1 - \frac{1.44}{\sqrt{8}}\right)$. Additionally, if $\gamma_{\mu} \to 1$ as $n$ grows for any $\mu = \omega(1)$, the suboptimality of random split approaches 0. For example, this means that the suboptimality of random split approaches 0 as $n$ grows if the mean similarity of an assignment with paper loads of $\log n$ improves faster than the mean similarity of the optimal assignment.

**7.2 Empirical Evaluation**

We now show the performance of these bounds on our real conference datasets in order to evaluate the extent to which they explain the good performance of random split. We use three of the conference datasets introduced earlier with $\beta = 1$. In Appendix A, we evaluate the bounds on additional datasets (including the SIGIR dataset). On PrefLib3 and Bid1, the problem is infeasible with paper and reviewer load constraints of 1 since $m < 2n$, so we modify the datasets by splitting each reviewer into 3 copies as follows. For each paper, we arbitrarily give one of the copies the same similarity as the original reviewer and give the other copies similarity 0. In this way, the similarity of the optimal assignment on this modified dataset is no greater than the similarity of the optimal assignment on the original dataset.

In Figure 2, we vary the value of the parameter $\mu$ (indicating the loads of the assignment $A^{(\mu)}$) and show the bounds of Theorem 5 and Theorem 6 as compared to the estimated expected value of random split. The estimated expected value is averaged over 10 trials with the standard error of the mean shaded, although it is sometimes not visible because it is small. We see that on these datasets, the bound of Theorem 5 performs best for low values of $\mu$ and not very well for higher values, likely due to the presence of a few “star” reviewers for each paper which hold a lot of the value. By making use of extra information about the values of these reviewers, the bound of Theorem 6 achieves a high fraction of the actual random split value. Although this bound is maximized at large values of $\mu$ on these datasets, it is close to its maximum even with reasonably low values of $\mu$. For example, on ICLR, the lower bound achieves 86% of the estimated expected value of random split with $\mu = 8$. This indicates the good performance of random split is explained well by the presence of just a few good reviewers per paper that can be simultaneously assigned.
7.3 Interpretation of Results

Although our results in this section are stated in terms of the precise values of high-load assignments, they can be interpreted by program chairs in a simple and practical way. Roughly, our results indicate that if several good reviewers can be simultaneously assigned to each paper (as was the case for the three conference similarity matrices in Figure 2), random split will perform well. When considering the potential performance of randomly splitting reviewers, program chairs should consider the reviewer and paper pools they expect to have at their conference and make a judgement about how many good-quality reviewers they think could be assigned to each paper (if the reviewer loads are scaled up proportionately). For example, the program chairs of a large AI conference might be confident that the top several reviewers for each paper are about equally valuable (due to the depth of the reviewer pool) and could be assigned to each paper with only a modest loss in average review quality; this would imply that random split would perform very well for this conference.

8 Conclusion

We showed that randomly splitting reviewers between two reviewing phases or two reviewing conditions produces near-optimal assignments on realistic conference similarity matrices. Our analysis of this phenomenon can help future program chairs make decisions about whether random split will work well for their conference’s two-phase review process, based on their assessment of whether a few simple conditions are applicable to their case. In the setting of conference experiment design, our analysis allows program chairs to understand if running an experiment on their review process will significantly impact their assignment quality.

In addition, our results can potentially be further generalized to related reviewing models such as those of academic journals (which accept submissions on a rolling basis), or to other multi-stage resource allocation problems that involve matching resources based on similarities. For example, datacenters receiving a large batch of jobs may have to select some to run on various servers immediately and some to run later when additional servers have been freed, or hospitals may want to assign nurses to shifts based on expertise but without knowledge of which expertise will be most applicable in later shifts.

One limitation of our work is that while our empirical results demonstrate the effectiveness of the random-split strategy with real conference data, our theoretical results make the simplifying assumption that paper and reviewer loads are 1, which is unrealistic for real conferences. However, we believe that incorporating this detail would not change our explanations for the good performance of random split. Another limitation is that we assume the set of papers requiring reviews in the second stage is drawn uniformly at random. Although this is a reasonable belief without further information in the two-phase setting, one direction for future work is to consider non-uniform distributions of second-stage papers and analyze if a form of random split still performs well there.

Our work could potentially produce negative outcomes in the form of worse paper assignments if program...
chairs decide to use random split on an incorrect belief that their conference will fit our conditions. However, program chairs are required to make such decisions about how to perform the paper assignment anyway, so this is not a significant increase in risk. The use of random reviewer splits, as opposed to some alternate strategy where reviewers can self-select their stage, could also negatively impact reviewers with strong preferences over which stage they review in (e.g., due to schedule constraints). These preferences should ideally be taken into account along with the similarity of the resulting assignment when choosing the reviewer split; we leave this as an interesting direction for future work.

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Appendices
## A Additional Empirical Results

We first present empirical evaluations showing the performance of random split on additional values of $\beta$ for the similarity matrices used in Section 5.1 (Figure 3a), as well as on additional similarity matrices constructed from bidding data (Figure 3b). Two of these additional similarity matrices were constructed using bid data for the two other AI conferences (conferences 1 and 2) from PrefLib dataset MD-00002 with sizes $(n = 54, m = 31)$ and $(n = 52, m = 24)$ respectively. Another additional similarity matrix (marked Bid2) was constructed from another sample of the bidding data from a major computer science conference with size $(n = 1200, m = 300)$. As in the bidding datasets shown earlier, we transformed “yes,” “maybe,” and “no response” bids into similarities of 1, 0.5, and 0.25 respectively.

We run several experiments, each corresponding to a choice of dataset and $\beta$. Each experiment consists of 10 trials, where in each trial we sample a random reviewer split and a set of second-stage papers, and report the range of assignment values found as percentages of the oracle optimal assignments for each trial. We set paper loads of 2 in each stage, and limit reviewer loads to be at most 6 for all datasets except PrefLib2 and Bid2, which limit reviewer loads to be at most 12 (for feasibility). As in Section 5.1, we draw $R_2$ uniformly at random with size $\frac{\beta}{1+\beta}m$ and draw $P_2$ uniformly at random with size $\beta n$. In general, we see that random split performs very well on these datasets as well. We see that all trials of random split achieve at least 88% of the oracle optimal solution’s similarity on all datasets, with all trials on all but three experiments achieving at least 94%. The range of values on each experiment is generally small (at most 7%), with the largest ranges occurring on the small PrefLib datasets.

We additionally test the bounds of Section 7 on these datasets as well as the SIGIR dataset to evaluate...
how well they explain the performance of random split. On PrefLib1, Preflib2, and Bid2, we scale up the number of reviewers by 4, 5, and 8 respectively for feasibility, as described in Section 7.2. On the x-axis we vary the parameter $\mu$, which determines the loads of the assignment $A(\mu)$ used in the bound.

In Figure 4, we see similar results to those shown earlier. The Theorem 5 bound performs best at low values of $\mu$. The Theorem 6 bound performs better at higher values of $\mu$, although for some datasets a more moderate value of $\mu$ is better since the assignment value $s(\mu)$ drops too quickly at higher $\mu$. From the Theorem 6 bound, we see that the good performance of random split on these datasets is generally explained fairly well by the large-load, high-similarity assignment.

All empirical evaluations in this paper were run on a computer with 8 cores and 16 GB of RAM, running Ubuntu 18.04 and solving the LPs with Gurobi 9.0.2 [59].

B Empirical Results for Paper-Split Variant

In this section, we provide some additional empirical results that are particularly relevant to the conference experiment design setting. Sometimes, conferences may not have the reviewing resources to provide a significant number of papers with two sets of reviews as part of an experiment. Instead, they may want to provide reviews to each paper under only one of the conditions. If the papers and reviewers are both split between conditions uniformly at random, this can be seen as a variant of our standard two-stage paper
Figure 5: Range of values found over 10 random reviewer splits when papers split between stages.

assignment problem where only papers in $P_1 = P \setminus P_2$ are assigned reviewers in stage one.

To test whether such experiments will still give high-similarity assignments in practice, we conduct additional empirical evaluations. The results of these experiments are shown in Figure 5. For each dataset of those introduced in Section 5.1, we take 10 samples of random reviewer and paper splits where $|R_2| = m/2$ and $|P_2| = n/2$ so that half of the reviewers and papers are in each stage (i.e., each condition). We then find assignments in each stage with paper loads of 3 and reviewer loads of at most 6 (standard conference loads), and display the range of assignment values found as a fraction of the oracle optimal assignment’s value. On all datasets, all trials of random reviewer split achieve over 75% of the oracle optimal assignment’s total similarity with low variation (at most 4%). On ICLR and SIGIR, all trials achieve over 90% of the oracle optimal similarity. Overall, the average assignment quality is slightly worse than in the standard model (where all papers are in stage one). This is likely because it is more difficult for reviewers to be assigned to their optimal papers when each paper is in only one of the two stages.

C Submodularity of $f$

In this section, we show that the problem of optimizing $f$ (or $\mathcal{F}$) is actually an instance of submodular optimization. For simplicity, we consider the case where $m = (1+\beta)n$ and $\ell_{rev} = \ell_{pap}^{(1)} = \ell_{pap}^{(2)} = 1$.

For some set $N$, a function $g : 2^N \to \mathbb{R}$ is submodular if $g(A \cup \{u\}) - g(A) \geq g(B \cup \{u\}) - g(B)$ for all $A \subseteq B \subseteq N$ and all $u \in N \setminus B$. Since $f$ and $\mathcal{F}$ are defined only for $R_2$ where $|R_2| = \beta n$, we modify them to be defined over $2^B$.

Recall that for subsets $R' \subseteq R$ and $P' \subseteq P$, desired paper load $\ell_{pap}$, and maximum reviewer load $\ell_{rev}$, $\mathcal{M}(R', P'; \ell_{rev}, \ell_{pap})$ is the set of assignment matrices that assign a load of exactly $\ell_{pap}$ to all papers in $P'$ (and a load of at most $\ell_{rev}$ to all reviewers in $R'$). Define $\mathcal{M}'(R', P'; \ell_{rev}, \ell_{pap})$ as the set of assignment matrices that assign a load of at most $\ell_{pap}$ to all papers in $P'$ (and a load of at most $\ell_{rev}$ to all reviewers in $R'$). Formally, $A \in \mathcal{M}'(R', P'; \ell_{rev}, \ell_{pap})$ if and only if $\sum_{r \in R'} A_{rp} \leq \ell_{pap}$ for all $p \in P'$, $\sum_{p \in P'} A_{rp} \leq \ell_{rev}$ for all $r \in R'$, and $A_{rp} = 0$ for all $(r, p) \notin R' \times P'$.

Consider the modified version of $Q$

$$Q'(R_2, P_2) = \frac{1}{(1+\beta)n} \left[ \max_{A \in \mathcal{M}'(R, P_2; 1, 1)} \sum_{r \in R \setminus R_2, p \in P} A_{rp} S_{rp} + \max_{B \in \mathcal{M}'(R_2, P_2; 1, 1)} \sum_{r \in R_2, p \in P_2} B_{rp} S_{rp} \right]$$

which allows papers to be underloaded and so is defined for all $R_2$ and $P_2$. Define $f_{sub}(R_2) = E_{P_2}[Q'(R_2, P_2)]$ and $\mathcal{F}_{sub}(R_2) = \frac{1}{K} \sum_{k=1}^{K} Q'(R_2, P_2^{(k)})$ as modifications of $f$ and $\mathcal{F}$. Since $S \geq 0$, there exists a maximum-similarity assignment from within $\mathcal{M}'(R', P'; 1, 1)$ that meets all paper load constraints with equality when $|R'| \geq |P'|$ and thus is contained in $\mathcal{M}(R', P'; 1, 1)$. Also, $\mathcal{M}(R', P'; 1, 1) \subseteq \mathcal{M}'(R', P'; 1, 1)$. Thus, when $|R_2| = \beta n$, $Q(R_2, P_2) = Q'(R_2, P_2)$. Therefore, subject to the constraint $|R_2| = \beta n$, maximizing $f_{sub}$ (or $\mathcal{F}_{sub}$) is equivalent to maximizing $f$ (or $\mathcal{F}$).
Proposition 1. $f_{\text{sub}}$ and $\overline{f}_{\text{sub}}$ are submodular in $R_2$.

Proof. Note that $\max_{A \in M'(R', P_2) \cap \beta} \sum_{r' \in R'} \sum_{p \in P_2} A_{r'p} S_{rp}$ is a submodular function of the reviewer set $R'$ when the paper set $P'$ is held fixed [60]. Submodularity in $R_2$ is equivalent to submodularity in $R_1 = R \setminus R_2$, so $Q'(R_2, P_2)$ is submodular in $R_2$. As sums over terms submodular in $R_2$, $f_{\text{sub}}$ and $\overline{f}_{\text{sub}}$ are submodular in $R_2$.

Therefore, the two-stage paper assignment problem is an instance of maximizing a non-monotone submodular function subject to a cardinality constraint $|R_2| = \beta n$. However, no value oracle for the function $f$ is available due to the expectation over $P_2$. Since a polynomial-time value oracle is available for $\overline{f}$, the paper [38] gives an approximation algorithm achieving an approximation ratio of no greater than 0.5 (depending on $\beta$). This guarantee does not imply much about the assignment quality, since it can be trivially achieved by maximizing assignment similarity in the first stage only. Furthermore, it is known that achieving an approximation ratio of greater than 0.5 requires an exponential number of queries to the value oracle; this holds true even without the cardinality constraint [61][62]. Thus, generic algorithms for submodular maximization are not helpful for our problem.

D Proofs

D.1 Proof of Theorem 1

We will show that it is NP-hard to determine if there exists a choice of $R_2$ with value $\overline{f}(R_2) = 1$ when $K = 3$, for some instance of $P^{(1)}_2, P^{(2)}_2, P^{(3)}_2$. If such a choice exists, it would be the optimal solution. Therefore, any algorithm to optimize $\overline{f}$ would be able to determine if there exists a solution with value 1, solving an NP-hard problem. This implies the NP-hardness of optimizing $\overline{f}$.

We reduce from 3-Dimensional Matching, an NP-complete problem [55]. An instance of 3-Dimensional Matching consists of three sets $X, Y, Z$ of size $s$, and a collection of tuples $T \subseteq X \times Y \times Z$. It asks whether there exists a selection of $s$ tuples from $T$ that includes each element of $X, Y$, and $Z$ exactly once.

Given such an instance of 3-Dimensional Matching, we construct an instance of the two-stage paper assignment problem with $n = |T| + 2s, m = |T| + 3s$, and $\beta = \frac{m}{s} - 1$ ($\ell_{\text{rev}} = \ell_{\text{pap}} = \ell_{\text{pap}} = 1$). $\beta n = s$ papers and reviewers will be in stage two. The first $s$ papers correspond to elements of $X$, the next $s$ to elements of $Y$, and the next $s$ to elements of $Z$; the remaining $|T| - s$ papers are “dummy papers” that all reviewers can review. The first $3s$ reviewers are “specialty reviewers” corresponding to each of the first $3s$ papers, and the remaining $|T|$ reviewers correspond to each of the elements of $T$. We construct the $K = 3$ sampled subsets $P^{(1)}_2, P^{(2)}_2, P^{(3)}_2$ where the elements of these sets correspond to the elements of $X, Y$, and $Z$ respectively. We then construct $S$ as follows. For $i \in [3s]$, set $S_i = 1$ and $S_{ij} = 0$ for all $j \in [3s], j \neq i$. For the remaining reviewers $i \in \{3s + 1, \ldots, 3s + |T|\}$ and for papers $j \in [3s]$, set $S_{ij} = 1$ if the element corresponding to $j$ in $X \cup Y \cup Z$ is included in the tuple corresponding to $i$ in $T$. Finally, for the remaining papers $j \in \{3s + 1, \ldots, |T| + 2s\}$, set $S_{ij} = 1$ for all reviewers $i$.

Suppose we have a “yes” instance of 3-Dimensional Matching, so there exists a choice of $s$ tuples from $T$ that cover each element of $X, Y$, and $Z$. Choose the corresponding $s$ reviewers as $R_2$ and the remaining reviewers as $R_1$. In stage one, we can assign each specialty reviewer to each of their corresponding papers and each of the remaining $|T| - s$ reviewers in $R_1$ to dummy papers. In stage two, for each of the three possible samples, there exists one reviewer that has similarity 1 with each paper since the corresponding choice of tuples from $T$ cover $X, Y$, and $Z$. Therefore, this partition achieves $\overline{f}(R_2) = 1$.

Suppose we have a “no” instance of 3-Dimensional Matching, so no choice of $s$ tuples from $T$ covers each element of $X, Y$, and $Z$. We claim that no choice of $R_2$ will achieve $s$ total similarity in the second stage. First, suppose we include a specialty reviewer in $R_2$. This reviewer has similarity 1 with only one paper, so there exists a sample of stage two papers $P^{(i)}_2$ such that this reviewer must be assigned to a paper with which it has similarity 0. Therefore, $\overline{f}(R_2)$ cannot be 1 when a specialty reviewer is in $R_2$ and so $R_2$ must be chosen from the reviewers corresponding to elements of $T$. However, no choice of $s$ tuples covers each element of $X, Y$, and $Z$. Therefore, for every choice of $R_2$, some reviewer must be assigned to a paper with which they have similarity 0 for at least one of the sampled sets of stage two papers. This means that $\overline{f}(R_2) = 1$ is unachievable.
D.2 Proof of Theorem 2

For any $\beta \in [0, 1]$, choose any $n$ such that $\beta n \in \mathbb{Z}_+$. We construct the following similarity matrix. Paper $i$ has similarity 1 with reviewer $i$, and also with reviewer $n + i$ if $i \leq \beta n$. All other similarities are 0.

On this example, the oracle optimal assignment for any $\mathcal{P}_2$ is to assign reviewers $\{1, \ldots, n\}$ to papers in the first stage, since this maximizes the similarity across both stages. This choice gives a total similarity of $n$ in stage one and an expected similarity of $\beta^2 n$ in stage two (since each reviewer’s matching paper is in stage two with probability $\beta$), for a total similarity of $n(1 + \beta^2)$. Since there are $(1 + \beta)n$ total assignments, the expected mean similarity is $\frac{1 + \beta^2}{1 + \beta}$.

Now consider the assignment after randomly splitting reviewers. Any paper $p \leq \beta n$ has two reviewers $a, b$ with similarity 1. For sufficiently large $n \geq \frac{1 + \beta^2}{1 + \beta}$, the expected value of this paper’s assignment is

$$(P[a \in \mathcal{R}_1 \land b \in \mathcal{R}_2] + P[b \in \mathcal{R}_1 \land a \in \mathcal{R}_2])(1 + P[p \in \mathcal{P}_2]$$

$$+ P[a, b \in \mathcal{R}_1] + P[a, b \in \mathcal{R}_2]P[p \in \mathcal{P}_2]

= \left(2 \frac{n}{(1 + \beta)n (1 + \beta)n - 1} \right) (1 + \beta) + \frac{n - 1}{n - 1} + \frac{\beta n}{(1 + \beta)n (1 + \beta)n - 1}

\leq 2 \frac{\beta n}{(1 + \beta)n (1 + \beta)n - 1}

\leq \frac{1 + 4\beta}{2(1 + \beta)} + \frac{1}{(1 + \beta)^2} + \frac{\beta^3}{(1 + \beta)^2}.$$

There are $\beta n$ of these papers.

Any of the remaining papers $p > \beta n$ has only one reviewer $a$ with similarity 1. The expected value of this paper’s assignment is

$$P[a \in \mathcal{R}_1] + P[a \in \mathcal{R}_2]P[p \in \mathcal{P}_2]

= \frac{1 + \beta^2}{1 + \beta}.$$

There are $(1 - \beta)n$ of these papers.

Totalling over all papers and dividing by the total number of assignments, the mean expected similarity of random split is at most

$$\frac{1 + 4\beta}{2(1 + \beta)} + \frac{1}{(1 + \beta)^2} + \frac{\beta^3}{(1 + \beta)^2} \frac{\beta}{1 + \beta} + \frac{(1 + \beta^2)(1 - \beta)}{(1 + \beta)^2}.$$

The suboptimality is therefore at least

$$\frac{1 + \beta^2}{1 + \beta} - \left(\frac{1 + 4\beta}{2(1 + \beta)} + \frac{1}{(1 + \beta)^2} + \frac{\beta^3}{(1 + \beta)^2}\right) \frac{\beta}{1 + \beta} - \frac{(1 + \beta^2)(1 - \beta)}{(1 + \beta)^2}

= \frac{(1 + \beta^2)2\beta}{(1 + \beta)^2} - \left(\frac{1 + 4\beta}{2(1 + \beta)} + \frac{1}{(1 + \beta)^2} + \frac{\beta^3}{(1 + \beta)^2}\right) \frac{\beta}{1 + \beta}

= \left(2(1 + \beta^2)(1 + \beta) - \frac{1}{2}(1 + 4\beta)(1 + \beta) - 1 - \beta^3\right) \frac{\beta}{(1 + \beta)^3}

= \left(\frac{1}{2} - \frac{1}{2\beta + \beta^3}\right) \frac{\beta}{(1 + \beta)^3}

\geq \frac{\beta^4}{(1 + \beta)^3}.$$

D.3 Proof of Theorem 3

By Lemma 4 of [57], a rank $k$ similarity matrix $S \in [0, 1]^{(1+\beta)n \times n}$ can be factored into vectors $u_r \in \mathbb{R}^k$ for each reviewer $r$ and $v_p \in \mathbb{R}^k$ for each paper $p$ such that $S_{rp} = \langle u_r, v_p \rangle$, $\|u_r\|_2 \leq k^{1/4}$, and $\|v_p\|_2 \leq k^{1/4}$. 

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Consider the ball of radius \(k^{1/4}\) in \(\mathbb{R}^k\) in which the paper vectors \(v_p\) lie. We cover this ball with smaller “cells” by dividing the containing \(k\)-dimensional hypercube with side length \(2k^{1/4}\) along each dimension to create some number of smaller hypercubes. If we divide the containing hypercube into \(t\) equal-sized segments along each dimension, there are \(t^k\) cells in total and the maximum \(L_2\) distance between two points in a cell is \(\frac{2k^{3/4}}{t}\).

We construct \(L\) layers of cells in this way, where the cells increase in size between layers. Denote by \(t_i\) the number of divisions along each dimension at layer \(i\). We choose \(t_{i} = 2^{2i}\) for some integer \(Z_i\) for all layers \(i\) so that each cell at layer \(i\) is fully contained within a single cell at each higher layer. Denote by \(s_i\) the desired maximum within-cell distance at layer \(i\). This distance is achieved if \(t_{i}\) is at least \(\frac{2k^{3/4}}{s_i}\), so the minimum such \(t_{i}\) that is also a power of two is at most \(\frac{4k^{3/4}}{s_i}\). This gives that there are at most \(z_i = \left(\frac{4k^{3/4}}{s_i}\right)^k\) cells in layer \(i\).

In what follows, we say that a paper \(p\) is in some cell if its vector \(v_p\) is in the cell. (Papers on the border of multiple cells at layer 1 are considered to be in an arbitrary one of the bordering cells so that each paper is in exactly one cell. At higher layers, such papers are considered to be in the cell containing their layer 1 cell.) We say that a reviewer is in a cell if it is assigned to a paper in that cell by the oracle optimal paper assignment (given \(P_2\)).

Given \(P_2\) and \(R_2\) produced by random split, we proceed through layers from 1 to \(L\) in order to match reviewers to papers in the same stage. We match as many reviewers as possible to papers that are within the same cell at each layer \(i\), and then continue to layer \(i+1\). Define \(n_i\) as an upper bound on the number of reviewers unmatched before matching within layer \(i\); \(n_1 = (1 + \beta)n\). The difference in value between the assignment \(A\) produced in this way and the oracle optimal assignment \(A^*\) (which we call the “value gap”) is

\[
\sum_{r \in R_p, p \in P} (A^*_{rp} - A_{rp}) \langle u_r, v_p \rangle = \sum_{r \in R_p, p \in P} A^*_{rp} A^*_{rp} \langle u_r, v_p^* - v_p \rangle \\
\leq \sum_{r \in R_p, p \in P} A^*_{rp} A^*_{rp} ||u_r||_2 ||v_p^* - v_p||_2 \\
\leq k^{1/4} \sum_{r \in R_p, p \in P} A^*_{rp} ||v_p^* - v_p||_2.
\]

Consider some cell containing \(x\) papers. All \(x\) of these papers are in stage one. Define \(Hyp(N, K, M)\) as the hypergeometric distribution where \(N\) is the population size, \(M\) is the number of draws, and \(K\) is the number of successes in the population; by symmetry \(Hyp(N, K, M)\) is equivalent to \(Hyp(N, M, K)\). The number of stage two papers has distribution \(Hyp(n, x, \beta n)\). With probability \(1 - 2\delta\), by Hoeffding’s inequality \(63\) and using the symmetry property, within \(\beta x \pm \sqrt{\frac{x}{2} \ln(1/\delta)}\) of the papers in this cell are also in stage two. (In this section, \(ln\) indicates the logarithm with base \(e\) and \(\log\) indicates the logarithm with base \(2\).) Call \(y\) the total number of reviewers in the cell. There are exactly the same number of reviewers as total stage one and two papers in this cell, so \(y\) is within \((1 + \beta)x \pm \sqrt{\frac{x}{2} \ln(1/\delta)}\) and is at most \(2x\). Since \(R_2\) is produced by random split, the number of reviewers in this cell in stage one has distribution \(Hyp((1 + \beta)n, y, n)\) and the number of reviewers in this cell in stage two has distribution \(Hyp((1 + \beta)n, y, \beta n)\). By Hoeffding’s inequality and again using symmetry, the number of reviewers in the cell in stage one is at most \(\frac{y}{\beta} + \sqrt{\frac{x}{2} \ln(1/\delta)} \leq x + \sqrt{\frac{x}{2} \ln(1/\delta)} + \sqrt{x \ln(1/\delta)} \leq x + \sqrt{3x \ln(1/\delta)}\) with probability \(1 - 2\delta\) (conditioned on the earlier event concerning the number of stage-two papers). By this argument, with probability \(1 - 4\delta\) (again conditioned on the earlier event), there are within \(x + \sqrt{3x \ln(1/\delta)}\) reviewers in stage one in the cell and within \(\beta x \pm \sqrt{3x \ln(1/\delta)}\) reviewers in stage two in the cell. In total, there are at most \(nL\) cells with a non-zero number of papers across all layers and so the total probability of error in any of the bounds is at most \(6\delta LN\).

Assume that this high probability event occurs. In layer \(i\), in any cell \(j\) with \(x_{ij}\) papers (all of which are in stage one), the number of stage one reviewers is within \(x_{ij} \pm \sqrt{3x_{ij} \ln(1/\delta)}\). Any reviewers in this cell matched at earlier layers must have been matched to papers also in this cell. Therefore, the number of unmatched stage one reviewers after matching within this cell is at most \(\sqrt{3x_{ij} \ln(1/\delta)}\). The number of stage two reviewers is within \(\beta x_{ij} \pm \sqrt{3x_{ij} \ln(1/\delta)}\) and the number of stage two papers is within \(\beta x_{ij} \pm \sqrt{3x_{ij} \ln(1/\delta)}\). Therefore, the number of unmatched stage two reviewers after matching within this cell is at most \(\sqrt{6x_{ij} \ln(1/\delta)}\). In total over both stages, the total number of unmatched reviewers after matching in layer \(i\) is at most
\[ n_{i+1} = \sum_{j=1}^{z_{i+1}} \sqrt{18 z_{j} \ln(1/\delta)} \leq \sqrt{18 z_{i} n \ln(1/\delta)}. \] All of the reviewers matched at layer \( i \) are matched to papers at most \( s_{i} \) away from their optimal paper assignment. Across all layers, the value gap is therefore bounded by \( k^{1/4} \left( \sum_{i=1}^{L-1} n_{i} s_{i} + 2 n_{L} k^{1/4} \right) \), since everything at layer \( L \) is matched to whatever remains regardless of \( s_{L} \).

We now determine how to set \( s_{i} \) for all layers \( i \). We choose \( s_{1} = s \) and set other \( s_{i} \) such that \( n_{i} s_{i} = (1+\beta) n s \) for all \( i \). This leads to the recursively-defined values of \( s_{i} = (1+\beta) n s \), \( z_{i} = (4 k^{3/4})^{k} s^{-k} \), \( n_{i+1} = \sqrt{z_{i} n 18 \ln(1/\delta)} \) with initial values \( n_{1} = (1 + \beta) n \) and \( s_{1} = s \). Unrolling the iteration, we see that

\[ s_{i} = \frac{1}{2} s_{i-1} \sqrt{(z_{i})^{1/2} (18 \ln(1/\delta))^{-1/2} (1 + \beta)} \]
\[ = \left( 1 + \frac{1}{2} \beta n \right) \frac{1}{2} s_{i-1}^{-1} \frac{1}{2} \sum_{j=0}^{i-1} \left( \frac{4}{2} \right)^{j} \left( 4 k^{3/4} \right)^{-1/2} \left( 18 \ln(1/\delta) \right)^{-1/2} \sum_{j=0}^{i-1} \left( \frac{4}{2} \right)^{j} \left( 1 + \beta \right)^{i-1} \sum_{j=0}^{i-1} \left( \frac{4}{2} \right)^{j} \]

for \( i \geq 2 \). Defining \( \epsilon \) such that \( s = \left( \frac{(1+\beta)^{2} n}{18 \ln(1/\delta)} \right)^{\epsilon} \),

\[ s_{i} = \left( \frac{(1 + \beta)^{2} n}{18 \ln(1/\delta)} \right)^{1-\frac{1}{2^{i}}} \left( 4 k^{3/4} \right)^{-1+\frac{1}{2^{i}}} \]

for \( i \geq 2 \). This gives a value gap of at most \( k^{1/4}(1 + \beta)^{1+2^{i}} n^{1+1/2} (18 \ln(1/\delta))^{-1} ((L - 1) + 2 k^{1/4} s_{L}^{-1}) \). We now continue in cases on the value of \( k \).

**Case k = 1.** Note that \( \sum_{j=0}^{i-1} \left( \frac{4}{2} \right)^{j} = 2 \left( 1 - \frac{1}{2} \right) \), so

\[ s_{i} = \left( \frac{(1 + \beta)^{2} n}{18 \ln(1/\delta)} \right)^{1-\frac{1}{2^{i}}} \left( 4 k^{3/4} \right)^{-1+\frac{1}{2^{i}}} \]

Choose \( \epsilon = -\frac{1}{2} + \frac{1}{2^{i}} \) so that \( s_{L} = (4 k^{3/4})^{-1+\frac{1}{2^{i}}} \). Setting \( \delta = (2 n)^{-3} \) and \( L = \log \log n \), for sufficiently large \( n \), the value gap is bounded by

\[ \left( 1 + \beta \right)^{2} n \frac{1}{2^{i}} \frac{1}{2^{i}} \left( 2^{i} \right)^{1} \left( 4 k^{3/4} \right)^{1} \left( 2^{i} \right)^{1} \]

for some constants \( C, \eta \) with probability \( 1 - \frac{6 \log \log n}{8 n^{\eta}} \geq 1 - \frac{1}{n} \).

**Case k = 2.** Note that \( \sum_{j=0}^{i-1} \left( \frac{4}{2} \right)^{j} = i \), so

\[ s_{i} = \left( \frac{(1 + \beta)^{2} n}{18 \ln(1/\delta)} \right)^{\frac{1}{2^{i}} \frac{1}{2^{i}}} \left( 4 k^{3/4} \right)^{-i+1} \]

Choose \( \epsilon = -\frac{1}{2} + \frac{1}{2^{i}} \) so that \( s_{L} = (4 k^{3/4})^{-L+1} \). Setting \( \delta = (2 n)^{-3} \) and \( L = \log \log n \), for sufficiently large \( n \), the value gap is bounded by

\[ k^{1/4}(1 + \beta)^{2} n \frac{1}{2^{i}} \left( 2^{i} \right)^{1} \left( 4 k^{3/4} \right)^{1} \left( 2^{i} \right)^{1} \]

for some constants \( C, \eta \) with probability \( 1 - \frac{6 \log \log n}{8 n^{\eta}} \geq 1 - \frac{1}{n} \).

**Case k \geq 3.** Note that \( \sum_{j=0}^{i-1} \left( \frac{4}{2} \right)^{j} = \frac{i}{2} + \frac{i}{2} - 1 \), so

\[ s_{i} = \left( \frac{(1 + \beta)^{2} n}{18 \ln(1/\delta)} \right)^{\frac{1}{2^{i}} \frac{1}{2^{i}}} \left( 4 k^{3/4} \right)^{-i+1} \left( 4 k^{3/4} \right)^{-i+1} \left( 2^{i} \right)^{1} \left( 2^{i} \right)^{1} \]

for some constants \( C, \eta \) with probability \( 1 - \frac{6 \log \log n}{8 n^{\eta}} \geq 1 - \frac{1}{n} \).
Choose $\epsilon = -\frac{1}{k} + \frac{(4 - k)}{(k/2) - 1}$ so that $s_L = (4k^{3/4})^{\frac{1}{2}} \left(\frac{(n/2)L^{-1}}{\log(k/2)}\right)^{\frac{1}{2}} + \left(\frac{4}{k}\right)^{\frac{1}{2}}$. Setting $\delta = (2n)^{-3}$ and $L = \log \log \log n$, for sufficiently large $n$, the value gap is bounded by

$$k^{1/4}(1 + \beta)^{1 - \frac{1}{k}} \frac{4^{1/2}}{(k/2) - 1} n \frac{1}{\log \log n - \log(k/2)} \frac{1}{n} \left(18 \ln(1/\delta) \right)^{k/2} \left(\frac{4}{k}\right)^{k/2 - 1} \left(\frac{(n/2)L^{-1}}{\log(k/2)}\right)^{\frac{1}{2}} \left(\frac{4}{k}\right)^{\frac{1}{2}} \left(\frac{(n/2)L^{-1}}{\log(k/2)}\right)^{\frac{1}{2}}\left((L - 1) + 2k^{1/4}(4k^{3/4})^{\frac{1}{2}} \right)^{\frac{1}{2}}$$

$$\leq k^{1/4}(1 + \beta)n^{-\frac{1}{k} + \frac{1}{\log \log n - \log(k/2)}}\left(54 \ln(2n)\right)^{k/2} \frac{\log \log n}{\log(k/2)} - 1 + 2k^{1/4}(4k^{3/4})^{\frac{1}{2}}\log \log n$$

$$\leq k^{1/4}(1 + \beta)n^{-\frac{1}{k} + \frac{1}{\log \log n - \log(k/2)}}\left(54 \ln(2n)\right)^{k/2} \frac{\log \log n}{\log(k/2)} - 1 + 2k^{1/4}(\log(n)^2)\log \log(n^{4k^{3/4}})$$

$$\leq C(\log n)n^{-\frac{1}{k} + \frac{1}{\log \log n}}$$

for some constants $C, \eta$ with probability $1 - \frac{6\log \log n}{\log(k/2)n^2} \geq 1 - \frac{1}{n}$. To get the suboptimality, divide the value gap by $(1 + \beta)n \leq 2n$.

### D.4 Proof of Theorem 4

(a) Choose $n$ large enough such that $k \leq \frac{n}{2}$. We define $k$ groups of reviewers and papers such that all papers have similarity 1 with all reviewers within the same group and similarity 0 with all other reviewers. Group 1 contains all papers $p \leq \lfloor \frac{n}{2} \rfloor$ and all reviewers $r \leq \lfloor \frac{k}{2} \rfloor$. Each other group $2, \ldots, k$ contains 2 reviewers and 1 paper. All papers and reviewers not in any group have all similarities 0. This similarity matrix has rank $k$.

The oracle optimal assignment for any $P_2$ will split the reviewers in each group evenly between stages, so all papers in any group can be assigned a similarity-1 reviewer in both stages. This gives a total similarity of at least $n + 2(k - 1)$.

Define $X$ as the random variable representing the number of reviewers from group 1 selected to be in $R_2$. $X \sim \text{Hyper}(2n, 2\lfloor n/2 \rfloor, n)$, the hypergeometric distribution corresponding to the number of successes when $n$ items are sampled without replacement from a population of $2n$ items where $2\lfloor n/2 \rfloor$ of them are successes. By Lemma 2.1 of [64], $P[X = t] \leq \frac{C}{\sigma}$ for any $t \geq 0$ where $\sigma^2 = \frac{\lfloor n/2 \rfloor}{n} \left(1 - \frac{\lfloor n/2 \rfloor}{n} \right)$ and $C$ is an absolute constant. Since $\sigma^2 \geq \frac{n}{2} \left(1 - \frac{1}{2} - \frac{1}{n} \right) \geq \frac{n}{16}$ for $n \geq 4$, $P[X = t] \leq \frac{4C}{\sqrt{n}}$ for sufficiently large $n$. Therefore, $P\left[\frac{n}{2} - \frac{\sqrt{n}}{16} \leq X \leq \frac{n}{2} + \frac{\sqrt{n}}{16}\right] \leq \frac{1}{2}$. With probability at least $\frac{1}{2}$, at least $\frac{\sqrt{n}}{16}$ of the reviewers in group 1 cannot be matched to an optimal paper in their stage. Therefore, the total expected similarity is no greater than $n - \frac{\sqrt{n}}{16} + 2(k - 1)$ and the expected difference in value from the oracle optimal assignment is at least $\frac{\sqrt{n}}{32C}$. Since there are $2n$ assignments, the suboptimality is at least $\frac{1}{64C\sqrt{n}}$.

(b) We construct a similarity matrix by creating a vector in $\mathbb{R}^k$ for each reviewer and each paper, and setting the similarity between that reviewer and that paper to be the inner product of their corresponding vectors. Consider the cube in $\mathbb{R}^k$ contained in $[0, 1/\sqrt{k}]^k$. We construct a grid of points within this cube by evenly spacing $z = \lfloor n^{1/k} \rfloor$ along each axis and filling in the remaining points so that there are $z^k \geq n$ grid points in total. Place the $n$ paper vectors at arbitrary (unique) points on this grid, so that each vector is at least $\frac{1}{\sqrt{k}n^{1/k}} \geq \frac{1}{2\sqrt{k}n^{1/k}}$ away from any other paper vector. Place the $2n$ reviewer vectors such that 2 are at each grid point with a paper vector. The inner product of any two vectors is in $[0, 1]$, so this is a valid similarity matrix. The $2n \times k$ and $n \times k$ matrices where the rows are the reviewer and paper vectors respectively have linearly independent columns and so have rank $k$; thus, the similarity matrix has rank $k$.

We claim that the oracle optimal matching across both stages chooses one reviewer from each grid point and matches it to the paper at the same point. Suppose we have a matching where this is not the case. There must exist a cycle of matched reviewer and paper pairs where the corresponding vectors are not paired with themselves and are instead paired $(x_1, x_2), (x_2, x_3), \ldots, (x_K, x_1)$. This cycle has a total similarity of (using
grid points have both reviewers in stage one after a random reviewer split is at most 

with probability at least 

would assign them to the paper with vector 

Therefore, the probability that less than 

There are at most 

Proof. There are at most \( n^3 - a \) ways to assign reviewers to stages such that \( a \) pairs of reviewers at the same grid point are in stage one. For all \( n \) and \( a \) such that \( n + 1 \geq 4a \), \( (n^3 - a) \leq (n^3 - a) \geq (n^3 - a) \). Setting \( a = n/100 \), \( (n^3 - a) \leq \exp(0.06n) \) and \( 3^{n - (n/100)} \leq \exp(1.09n) \). Therefore, the number of ways to assign reviewers to stages such that less than \( n/100 \) pairs are in stage one is at most 

the total number of ways to assign reviewers to stages is \( \frac{2\sqrt{\pi} a^{2n}}{\sqrt{n}} \geq \frac{2\sqrt{\pi} e^{1.35n - 0.5\ln(n)}}{\exp(0.1n) \exp(-0.1n + 0.5\ln(n))} \leq \frac{2\sqrt{\pi} e^{-0.1n}}{\frac{1}{100}} \) for sufficiently large \( n \).

Therefore, with high probability, at least \( n/100 \) reviewers must be assigned to a paper at a different grid point.

Consider the assignments produced in each stage after random split, and consider the reviewers not assigned to their optimal papers by these assignments. From the set of vectors corresponding to these suboptimally-assigned reviewers, we can construct some number \( K \) of disjoint cycles \( C_j = \{x_1^{(j)}, \ldots, x_{K_j}^{(j)}\} \), where a reviewer with vector \( x_i^{(j)} \) is assigned to the paper with vector \( x_{i+1}^{(j)} \) when the optimal assignment would assign them to the paper with vector \( x_i^{(j)} \). By Lemma 1, \( \sum_{j=1}^{K} \sum_{i=1}^{K_j} |C_j| \geq \frac{n}{100} \). The difference in value between the random-split assignments and the optimal assignment is 

\[
\sum_{j=1}^{K} \sum_{i=1}^{K_j} (x_i^{(j)}, x_{i+1}^{(j)}) - x_i^{(j)} = \frac{1}{2} \sum_{j=1}^{K} \sum_{i=1}^{K_j} ||x_i^{(j)} - x_{i+1}^{(j)}||^2 \\
\geq \frac{1}{8k} n^{-2/k} \sum_{j=1}^{K} |C_j| \\
\geq \frac{1}{8k} n^{-2/k} \left( \frac{n}{100} \right)
\]

with probability at least \( 1 - \zeta e^{-n/10} \) for sufficiently large \( n \). Dividing by \( 2n \), the suboptimality is at least \( \frac{1}{16000} n^{-2/k} \).
D.5 Proof of Theorem 5

In this section, we state and prove a more general version of the bound in Theorem 5 that does not require that \( \beta \mu \) be integral. This result immediately implies the result of Theorem 5.

In the proof, we use the following lemma. We prove this lemma following the proof of the main theorem. For some set \( N \) and some constant \( p \in [0, 1] \), define distribution \( I_p(N) \) as the distribution over all subsets \( A \subseteq N \) induced by choosing each item \( x \in N \) to be in \( A \) independently with probability \( p \). Recall from Appendix C the definition of \( Q' \), a modified version of \( Q \) that allows papers to be underloaded (i.e., assigned fewer reviewers than their load).

**Lemma 2.** Consider the modified version of \( f \): \( f'(R_2) = E_{P_2 \sim I_{\beta}(P)}[Q'(R_2, P_2)], f' \) draws \( P_2 \sim I_{\beta}(P) \) rather than \( P_2 \sim U_{\beta\mu}(P) \) and allows papers to be underloaded. Then,

\[
E_{R_2 \sim I_{\beta/(1+\beta)}(R)}[f'(R_2)] \leq E_{R_2 \sim U_{\beta\mu}(R)}[f(R_2)].
\]

This lemma shows that when attempting to lower bound the expected value of random split, we can analyze as if the second-stage reviewers and papers were drawn independently.

We now state and prove the main theorem. We abuse notation slightly by defining \( M(R, P; \ell_{rev}, \ell_{pap}) \) to include all assignments where papers are assigned either \( [\ell_{pap}] \) or \( [\ell_{pap}] \) reviewers when \( \ell_{pap} \) is not integral; i.e., \( A \in M(R', P'; \ell_{rev}, \ell_{pap}) \) if and only if \( [\ell_{pap}] \leq \sum_{r \in R'} A_{rp} \leq \ell_{pap} \) for all \( p \in P' \), \( \sum_{p \in P'} A_{rp} \leq \ell_{rev} \) for all \( r \in R' \), and \( A_{rp} = 0 \) for all \((r, p) \notin R' \times P' \).

**Theorem 5 (Generalized).** Consider any \( \mu \in [10, 000] \) and \( \beta \in \{1, \frac{1}{100}, \ldots, \frac{100}{100}\} \). Let \( \epsilon = [\beta \mu] - [\beta \mu] \). If there exists an assignment \( A^{(\mu)} \in M(R, P; \mu, (1 + \beta)\mu) \) with mean similarity \( s^{(\mu)} \), choosing \( R_2 \) via random split gives that

\[
E_{R_2}[f(R_2)] \geq s^{(\mu)} \left[ 1 - \frac{\beta}{\sqrt{2\pi(1+\beta)[(1+\beta)\mu]}} \left( 2^{\frac{1}{1+\beta}} + \sqrt{1-\beta} \right) - \frac{(1+\beta)\mu}{(1+\beta)\mu} \right] \right] \cdot [1 - \epsilon] \cdot \left[ \frac{1}{(1+\beta)\mu} \right].
\]

**Proof.** By Lemma 2 we can consider drawing \( P_2 \sim I_{\beta}(P) \) and \( R_2 \sim I_{\beta/(1+\beta)}(R) \) and allowing papers to be underloaded. For all reviewers \( r \in R \), define the random variables \( Z_r = \begin{cases} 1 \text{ w.p. } \frac{1}{1+\beta} \\ 2 \text{ w.p. } \frac{\beta}{1+\beta} \end{cases} \) representing the stage that reviewer \( r \) is randomly chosen to be in. Define the random variables \( Y_p = \begin{cases} 1 \text{ w.p. } \frac{\beta}{1+\beta} \\ 0 \text{ w.p. } \frac{1}{1+\beta} \end{cases} \) representing whether \( p \in P_2 \). All of these random variables are independent. Also, denote by \( u^{(\mu)} = s^{(\mu)}(1+\beta)\mu \) the total similarity value of assignment \( A^{(\mu)} \), and denote by \( v_r^{(\mu)} \) and \( v_r^{(\mu)} \) the total similarity value of the assignments for paper \( p \) and reviewer \( r \) respectively in assignment \( A^{(\mu)} \).

The proof works as follows. We form an assignment \( B^{(1)} \) in stage one with paper loads of at most \( \mu \) and reviewer loads of at most \( \mu \), and form an assignment \( B^{(2)} \) in stage two with paper loads of at most \( [\beta \mu] \) and reviewer loads of at most \( [\beta \mu] \). We do this by initially assigning all reviewer-paper pairs in \( A^{(\mu)} \) that are present in the same stage, and then randomly removing assignments from each paper or reviewer that is overloaded. We then find “final assignments” (i.e., assignments that are feasible solutions for the two-stage assignment problem) from within \( B^{(1)} \) and \( B^{(2)} \).

**Stage One:** First, consider stage one. Define \( Binom(N, p) \) as the binomial distribution with \( N \) trials and \( p \) probability of success; denote by \( f \) the Binomial pmf. The number of reviewers assigned by \( A^{(\mu)} \) to paper \( p \) and present in stage one is a \( Binom \left( \lambda_p, \frac{1}{1+\beta} \right) \) variable, where \( \lambda_p \in \{(1+\beta)\mu, [1+\beta]\mu\} \). Suppose that we observe the set of such reviewers, randomly remove reviewers from this set until its size is at most \( \mu \), and then assign these reviewers to \( p \) in our stage one assignment \( B^{(1)} \). Since each reviewer has at most \( \mu \) assigned papers in \( A^{(\mu)} \), \( B^{(1)} \) satisfies the desired load constraints on both sides. The expected total value of
the assigned reviewers after we drop reviewers from each paper at random is

\[
E \left[ \sum_{r \in R} B^{(1)}_{rp} S_{rp} \right] = \sum_{x=0}^{\mu} f \left( x; \lambda_p, \frac{1}{1 + \beta} \right) \frac{v_p(x) x}{\lambda_p} + \sum_{x=\mu+1}^{\lambda_p} f \left( x; \lambda_p, \frac{1}{1 + \beta} \right) \frac{v_p(x) \mu}{\lambda_p}
\]

\[
= \frac{v_p(\mu)}{\lambda_p} E_{X \sim \text{Binom} \left( \lambda_p, \frac{1}{1 + \beta} \right)} \left[ \min (X, \mu) \right]
\]

\[
\geq \frac{v_p(\mu)}{\left( 1 + \beta \right) \mu} E_{X \sim \text{Binom} \left( \left( 1 + \beta \right) \mu, \frac{1}{1 + \beta} \right)} \left[ \min (X, \mu) \right].
\]

Summing over all papers,

\[
E \left[ \sum_{p \in P} \sum_{r \in R} B^{(1)}_{rp} S_{rp} \right] \geq \frac{v(\mu)}{\left( 1 + \beta \right) \mu} E_{X \sim \text{Binom} \left( \left( 1 + \beta \right) \mu, \frac{1}{1 + \beta} \right)} \left[ \min (X, \mu) \right].
\]

Due to the loads, the matrix \( \frac{1}{\mu} B^{(1)} \) has row sums at most 1 and column sums at most 1. By a generalization of the Birkhoff-von Neumann theorem \[66\], this can be written as a convex combination of matrices with all entries in \( \{0, 1\} \), all row sums at most 1, and all column sums at most 1. Each of these matrices represents an assignment obeying the reviewer and paper load constraints for the final assignment (since we allow papers to be underloaded), so they are all valid final assignments in stage one. At least one of these assignments must have a total value at least \( \frac{1}{\mu} \) of the value of \( B^{(1)} \).

**Stage Two:** Now, consider stage two. The number of reviewers assigned by \( A^{(\mu)} \) to paper \( p \) present in stage two is a \( \text{Binom} \left( \lambda_p, \frac{\beta}{1 + \beta} \right) \) variable, where \( \lambda_p \in \{ \left( 1 + \beta \right) \mu, \left( (1 + \beta) \mu \right) \} \). The number of papers assigned by \( A^{(\mu)} \) to a reviewer \( r \) present in stage two is a \( \text{Binom} \left( \mu, \beta \right) \) random variable. We first calculate the total expected value of all assignments in \( A^{(\mu)} \) and present in stage two (without dropping assignments from overloaded reviewers/papers):

\[
E \left[ \sum_{r \in R_{2, p \in P_2}} A^{(\mu)}_{rp} S_{rp} \right] = \frac{\beta^2}{1 + \beta} v(\mu).
\]

We then construct assignment \( B^{(2a)} \) from the pairs assigned in \( A^{(\mu)} \) and present in stage two by dropping reviewers from each paper at random until all papers have a load of at most \( \lceil \beta \mu \rceil \), with a value on paper \( p \) (if present in stage two) of

\[
E \left[ \sum_{r \in R} B^{(2a)}_{rp} S_{rp} \middle| p \in P_2 \right] = \sum_{x=0}^{\lceil \beta \mu \rceil} f \left( x; \lambda_p, \frac{\beta}{1 + \beta} \right) \frac{v_p(x) x}{\lambda_p} + \sum_{x=\lceil \beta \mu \rceil + 1}^{\lambda_p} f \left( x; \lambda_p, \frac{\beta}{1 + \beta} \right) \frac{v_p(x) \lceil \beta \mu \rceil}{\lambda_p}
\]

\[
= \frac{v_p(\mu)}{\lambda_p} E_{X \sim \text{Binom} \left( \lambda_p, \frac{\beta}{1 + \beta} \right)} \left[ \min (X, \lceil \beta \mu \rceil) \right]
\]

\[
\geq \frac{v_p(\mu)}{\left( 1 + \beta \right) \mu} E_{X \sim \text{Binom} \left( \left( 1 + \beta \right) \mu, \frac{\beta}{1 + \beta} \right)} \left[ \min (X, \lceil \beta \mu \rceil) \right].
\]

Each paper is present in stage two with probability \( \beta \), so the overall value is

\[
E \left[ \sum_{p \in P} \sum_{r \in R} B^{(2a)}_{rp} S_{rp} \right] \geq \frac{\beta v(\mu)}{\left( 1 + \beta \right) \mu} E_{X \sim \text{Binom} \left( \left( 1 + \beta \right) \mu, \frac{\beta}{1 + \beta} \right)} \left[ \min (X, \lceil \beta \mu \rceil) \right].
\]

We separately construct assignment \( B^{(2b)} \) from the pairs assigned in \( A^{(\mu)} \) and present in stage two by dropping papers from each reviewer at random until all reviewers have a load of at most \( \lceil \beta \mu \rceil \), with a value on reviewer
Totalling across all reviewers, since each reviewer is present in stage two with probability $\frac{\beta\mu}{1+\beta}$,
\[
\mathbb{E} \left[ \sum_{r \in \mathcal{R}} B_r(2) S_r \bigg| r \in \mathcal{R}_2 \right] = \sum_{x=0}^{[\beta\mu]} \mathbb{E}_{X \sim \text{Binom}(\mu, \beta)}[\min(X, [\beta\mu])]
\]
\[
\sum_{x=[\beta\mu]+1}^{\mu} f(x; \mu, \beta) v^{(\mu)}(x) \frac{\beta\mu}{\mu} + \sum_{x=[\beta\mu]+1}^{\mu} f(x; \mu, \beta) v^{(\mu)}(x) \frac{\beta\mu}{\mu}
\]
\[
= v^{(\mu)} \mathbb{E}_{X \sim \text{Binom}(\mu, \beta)}[\min(X, [\beta\mu])].
\]

Totalling across all reviewers, since each reviewer is present in stage two with probability $\frac{\beta\mu}{1+\beta}$,
\[
\mathbb{E} \left[ \sum_{r \in \mathcal{R}} \sum_{p \in \mathcal{P}} B_r(2) S_r \right] = \frac{\beta v^{(\mu)}}{(1+\beta)\mu} \mathbb{E}_{X \sim \text{Binom}((1+\beta)\mu, \frac{\beta}{1+\beta})}[\min(X, [\beta\mu])]
\]
\[
+ \frac{\beta v^{(\mu)}}{(1+\beta)\mu} \mathbb{E}_{X \sim \text{Binom}(\mu, \beta)}[\min(X, [\beta\mu])].
\]

Define $B^{(2)}$ as the intersection of the assigned pairs in $B^{(2a)}$ and $B^{(2b)}$; $B^{(2)}$ satisfies the desired load constraints on both sides. Its expected value is lower-bounded by the total expected value of $B^{(2a)}$ and $B^{(2b)}$ less the expected value of the pairs assigned in $A^{(\mu)}$ and present in stage two, since the pairs assigned in $B^{(2a)}$ and $B^{(2b)}$ are subsets of the stage two pairs assigned in $A^{(\mu)}$.
\[
\mathbb{E} \left[ \sum_{r \in \mathcal{R}} \sum_{p \in \mathcal{P}} B_r(2) S_r \right] \geq \frac{\beta v^{(\mu)}}{(1+\beta)\mu} \mathbb{E}_{X \sim \text{Binom}((1+\beta)\mu, \frac{\beta}{1+\beta})}[\min(X, [\beta\mu])]
\]
\[
+ \frac{\beta v^{(\mu)}}{(1+\beta)\mu} \mathbb{E}_{X \sim \text{Binom}(\mu, \beta)}[\min(X, [\beta\mu])]
\]
\[
+ \frac{\beta^2}{1+\beta} v^{(\mu)}.
\]

By the same Birkhoff-von Neumann argument as used in stage one, there exists a valid final assignment in stage two with paper loads of at most 1, reviewer loads of at most 1, and value at least $\frac{1}{[\beta\mu]}$ of the value of $B^{(2)}$.

**Total:** Sum the total value of the 1-load assignment in both stages and divide by $(1+\beta)n$ to get a lower bound on the expected mean similarity:
\[
\frac{\mu}{[(1+\beta)\mu]} \mathbb{E}_{X \sim \text{Binom}((1+\beta)\mu, \frac{\beta}{1+\beta})}[\min(X, \mu)]
\]
\[
+ \frac{\beta}{(1+\beta)} \mathbb{E}_{X \sim \text{Binom}(\mu, \beta)}[\min(X, [\beta\mu])]
\]
\[
+ \beta \left[ \mathbb{E}_{X \sim \text{Binom}((1+\beta)\mu, \frac{\beta}{1+\beta})}[\min(X, [\beta\mu])]ight]
\]
\[
+ \mathbb{E}_{X \sim \text{Binom}(\mu, \beta)}[\min(X, [\beta\mu])].
\]

Since the above bound is a function of the binomial pmf, we search for a simpler approximation. Say that $X \sim \text{Binom}(N, p)$ and $q = 1 - p$. The above bound is a function of $\mathbb{E} \left[ \min \left( \frac{X}{\mu}, 1 \right) \right]$ for three binomial random variables where $Np \leq \ell$. We approximate these binomials as if they were normals $Z \sim \mathcal{N}(Np, Npq)$,
since $X - N_{pq} f_Z(\ell)/\sqrt{Npq}$ converges in distribution to a standard normal. We use $f_Z$ as the pdf of $Z$, $F_Z$ as the cdf of $Z$, and $\Phi$ as the standard normal cdf.

\[
E \left[ \min \left( \frac{X}{\ell}, 1 \right) \right] \approx E \left[ \min \left( \frac{Z}{\ell}, 1 \right) \right] \\
= E \left[ \min \left( \frac{Z}{\ell}, 1 \right) \mid Z \leq \ell \right] P[Z \leq \ell] + E \left[ \min \left( \frac{Z}{\ell}, 1 \right) \mid Z > \ell \right] P[Z > \ell] \\
= \frac{1}{\ell} \left( Np - Npq \frac{f_Z(\ell)}{F_Z(\ell)} \right) F_Z(\ell) + 1 - F_Z(\ell) \\
= 1 - \frac{Npq}{\ell} f_Z(\ell) \left( 1 - \frac{Np}{\ell} \right) \\
\geq 1 - \sqrt{\frac{q}{2\pi Np} - \Phi \left( \frac{\ell - Np}{\sqrt{Npq}} \right) \left( 1 - \frac{Np}{\ell} \right)} \\
\geq 1 - \sqrt{\frac{q}{2\pi Np} - \left( 1 - \frac{Np}{\ell} \right)} .
\]

In total, defining $\epsilon^+ = \lceil \beta \mu \rceil - \beta \mu$, $\epsilon^- = \beta \mu - \lfloor \beta \mu \rfloor$, and $\epsilon = \epsilon^+ + \epsilon^-$, this approximation to the lower bound gives

\[
s^{(\mu)} \left( \frac{\mu}{(1 + \beta)\mu} \right) \left[ 1 - \sqrt{\frac{\beta}{2\pi [(1 + \beta)\mu]}} - \Phi \left( \frac{\mu - \frac{(1 + \beta)\mu}{\beta}}{\sqrt{(1 + \beta)\mu}} \right) \left( 1 - \frac{(1 + \beta)\mu}{\beta} \right) \right] \\
+ \beta \left( 1 - \sqrt{\frac{1}{2\pi [(1 + \beta)\mu]}} - \sqrt{1 - \frac{1}{2\pi \beta}} \right) - \Phi \left( \frac{\lceil \beta \mu \rceil - \frac{(1 + \beta)\mu}{\beta}}{\sqrt{(1 + \beta)\mu}} \right) \left( 1 - \frac{(1 + \beta)\mu}{\beta} \right) - \Phi \left( \frac{\beta - \beta \mu}{\sqrt{(1 + \beta)\mu}} \right) \left( 1 - \frac{\beta - \beta \mu}{\beta} \right) \right] \\
\geq s^{(\mu)} \left[ 1 - \frac{2}{(1 + \beta)} \sqrt{\frac{\beta}{2\pi [(1 + \beta)\mu]}} - \frac{1}{(1 + \beta)} \sqrt{\frac{\beta(1 - \beta)}{2\mu}} \right] \\
- \frac{1 + 2\beta}{1 + \beta} \left( 1 - \frac{(1 + \beta)\mu}{\beta \mu} \right) \left( \frac{(1 + \beta)\mu}{\beta \mu} \right) \\
\geq s^{(\mu)} \left[ 1 - \frac{\beta}{2\pi[(1 + \beta)\mu]} \left( \frac{1}{1 + \beta} + \sqrt{1 - \beta} \right) \right] \\
- \frac{(1 + 2\beta)}{(1 + \beta)\beta \mu} \left( \frac{\beta}{1 + \beta} \epsilon^+ + \epsilon^- \right) \left( 1 - \frac{\epsilon^+}{\beta \mu} \right) \\
\geq s^{(\mu)} \left[ 1 - \frac{\beta}{2\pi[(1 + \beta)\mu]} \left( \frac{1}{1 + \beta} + \sqrt{1 - \beta} \right) \right] \\
- \frac{(1 + 2\beta)}{(1 + \beta)\beta \mu} \epsilon^+ \left( 1 - \frac{\epsilon^+}{\beta \mu} \right) .
\]

Via simulation, we confirm that the approximation in (3) is in fact a lower bound on the expression in (2) for all $\mu \in [10^4]$ and $\beta \in \{\frac{1}{100}, \ldots, \frac{100}{100}\}$. In Figure 2 of Section 7 we plot the more precise bound of (3).

\[\square\]

**D.5.1 Proof of Lemma 2**

It remains to prove Lemma 2. We first prove a supplementary lemma, from which the main lemma follows.
Lemma 3. Consider a set $\mathcal{N}$ of $N$ items and a submodular function $g : 2^\mathcal{N} \to \mathbb{R}$. Then, $\mathbb{E}_{A \sim \mathcal{U}_p(\mathcal{N})}[g(A)] \leq E_{A \sim \mathcal{U}_{pN}(\mathcal{N})}[g(A)]$.

Proof. Consider the following randomized procedure $h$, which takes in a set $D \subseteq \mathcal{N}$ and constructs a set containing exactly $pN$ items. If $|D| = pN$, return $h(D) = D$. If $x = |D| - pN > 0$, then choose a subset $B \subseteq D$ uniformly at random such that $|B| = x$ and return $h(D) = D \setminus B$. If $x = pN - |D| > 0$, choose a subset $C \subseteq \mathcal{N} \setminus D$ uniformly at random such that $|C| = x$ and return $h(D) = D \cup C$.

If $D \sim \mathcal{I}_p(\mathcal{N})$ then $h(D) \sim \mathcal{U}_{pN}(\mathcal{N})$, since all subsets of size $pN$ have an equal chance to be created. We will show that $\mathbb{E}_{D \sim \mathcal{I}_p(\mathcal{N})}[g(h(D)) - g(D)] \geq 0$, proving that $\mathbb{E}_{D \sim \mathcal{I}_p(\mathcal{N})}[g(D)] \leq \mathbb{E}_{A \sim \mathcal{U}_{pN}(\mathcal{N})}[g(A)]$. More specifically, we show that for each $x > 0$,

$$
\mathbb{E}_{D \sim \mathcal{I}_p(\mathcal{N})}[g(h(D)) - g(D) \mid |D| = pN + x] + \mathbb{E}_{D \sim \mathcal{I}_p(\mathcal{N})}[g(h(D)) - g(D) \mid |D| = pN - x] 
$$

$$
= \frac{1}{\binom{N}{pN + x}} \sum_{D \subseteq \mathcal{N} : |D| = pN + x} g(D) - g(D \cup C) - g(D \setminus B) + g(C) - g(D) \cup C - g(D) \setminus B 
$$

$$
= \frac{1}{\binom{N}{pN}} \sum_{A \subseteq \mathcal{N} : |A| = pN} g(A) - g(A \cup B) + g(A \setminus C) 
$$

Since $g$ is submodular, for any subsets $A \subseteq \mathcal{N}$, $C \subseteq A$, $B \subseteq \mathcal{N} \setminus A$, we have that $g((A \setminus C) \cup B) - g(A \setminus C) \geq g(A \cup B) - g(A)$.

Proof. Consider the following randomized procedure $h$, which takes in a set $D \subseteq \mathcal{N}$ and constructs a set containing exactly $pN$ items. If $|D| = pN$, return $h(D) = D$. If $x = |D| - pN > 0$, then choose a subset $B \subseteq D$ uniformly at random such that $|B| = x$ and return $h(D) = D \setminus B$. If $x = pN - |D| > 0$, choose a subset $C \subseteq \mathcal{N} \setminus D$ uniformly at random such that $|C| = x$ and return $h(D) = D \cup C$.

If $D \sim \mathcal{I}_p(\mathcal{N})$ then $h(D) \sim \mathcal{U}_{pN}(\mathcal{N})$, since all subsets of size $pN$ have an equal chance to be created. We will show that $\mathbb{E}_{D \sim \mathcal{I}_p(\mathcal{N})}[g(h(D)) - g(D)] \geq 0$, proving that $\mathbb{E}_{D \sim \mathcal{I}_p(\mathcal{N})}[g(D)] \leq \mathbb{E}_{A \sim \mathcal{U}_{pN}(\mathcal{N})}[g(A)]$. More specifically, we show that for each $x > 0$,

$$
\mathbb{E}_{D \sim \mathcal{I}_p(\mathcal{N})}[g(h(D)) - g(D) \mid |D| = pN + x] + \mathbb{E}_{D \sim \mathcal{I}_p(\mathcal{N})}[g(h(D)) - g(D) \mid |D| = pN - x] 
$$

$$
= \frac{1}{\binom{N}{pN + x}} \sum_{D \subseteq \mathcal{N} : |D| = pN + x} g(D) - g(D \cup C) - g(D \setminus B) + g(C) - g(D) \cup C - g(D) \setminus B 
$$

$$
= \frac{1}{\binom{N}{pN}} \sum_{A \subseteq \mathcal{N} : |A| = pN} g(A) - g(A \cup B) + g(A \setminus C) 
$$

Since $g$ is submodular, for any subsets $A \subseteq \mathcal{N}$, $C \subseteq A$, $B \subseteq \mathcal{N} \setminus A$, we have that $g((A \setminus C) \cup B) - g(A \setminus C) \geq g(A \cup B) - g(A)$.

We also show that $f'$ and $Q'$ are submodular.

Proposition 2. $Q'(R_2, \mathcal{P}_2)$ is submodular in $R_2$ and $\mathcal{P}_2$. Further, $f'$ is submodular in $R_2$.

Proof. Note that $\max_{A \in \mathcal{M}(\overline{R}_2, \mathcal{P}_2)} \sum_{r \in \mathcal{R}' \subseteq \mathcal{R}_2' : \mathcal{P}' = \mathcal{P}_2} A_r S_{r,p}$ is a submodular function of the reviewer set $\mathcal{R}'$ when the paper set $\mathcal{P}'$ is held fixed and of the paper set $\mathcal{P}'$ when the reviewer set is held fixed [60]. Submodularity in $R_2$ is equivalent to submodularity in $R_1 = R_2 \setminus R_1$, so $Q'(R_2, \mathcal{P}_2)$ is submodular in $R_2$ and $\mathcal{P}_2$. As a sum over terms submodular in $R_2$, $f'$ is submodular in $R_2$. □
We now prove the main lemma. Since $S \geq 0$, there exists a maximum-similarity assignment from within $\mathcal{M}'(\mathcal{R}', \mathcal{P}'; 1, 1)$ that meets all paper load constraints with equality when $|\mathcal{R}'| \geq |\mathcal{P}'|$, and thus is contained in $\mathcal{M}(\mathcal{R}' \setminus \mathcal{R}, \mathcal{P}' \setminus \mathcal{P}, 1, 1)$. Also, $\mathcal{M}(\mathcal{R}' \setminus \mathcal{R}, \mathcal{P}' \setminus \mathcal{P}, 1, 1) \subseteq \mathcal{M}'(\mathcal{R}', \mathcal{P}'; 1, 1)$. Thus, when $|\mathcal{R}'| \geq \beta n$ and $m - |\mathcal{R}'| \geq n$, $Q(\mathcal{R}_2, \mathcal{P}_2) = Q'(\mathcal{R}_2, \mathcal{P}_2)$. Further, by Proposition 2 $Q'$ is submodular in $\mathcal{P}_2$. Therefore, by Lemma 3 $f(\mathcal{R}_2) \geq f'(\mathcal{R}_2)$ whenever $|\mathcal{R}_2| = \frac{\beta}{1 + \beta} m$ (since $m \geq (1 + \beta) n$). This shows that

$$\mathbb{E}_{\mathcal{R}_2 \sim \mathcal{U}(\beta/(1 + \beta))m(\mathcal{R})}[f(\mathcal{R}_2)] \geq \mathbb{E}_{\mathcal{R}_2 \sim \mathcal{U}(\beta/(1 + \beta))m(\mathcal{R})}[f'(\mathcal{R}_2)].$$

By Proposition 2 $f'$ is submodular in $\mathcal{R}_2$. Therefore, by Lemma 3

$$\mathbb{E}_{\mathcal{R}_2 \sim \mathcal{U}(\beta/(1 + \beta))m(\mathcal{R})}[f'(\mathcal{R}_2)] \geq \mathbb{E}_{\mathcal{R}_2 \sim \mathcal{U}(\beta/(1 + \beta))m(\mathcal{R})}[f'(\mathcal{R}_2)].$$

D.6 Proof of Theorem 6

In this section, we state and prove a more general version of the bound in Theorem 6 that does not require that $\frac{\beta}{4}$ be integral. This result immediately implies the result of Theorem 6.

**Theorem 6 (Generalized).** Suppose $\beta = 1$, and consider any $\mu \in [10, 000]$. Define $\epsilon = [\frac{\pi}{4}] - \frac{\beta}{4}$. Suppose there exists an assignment $A^{(1)} \in \mathcal{M}(\mathcal{R}, \mathcal{P}; 1, 2)$ with mean similarity $s^{(1)}$. Suppose there also exists an assignment $A^{(\mu)} \in \mathcal{M}(\mathcal{R}, \mathcal{P}; \mu, 2\mu)$ with mean similarity $s^{(\mu)}$ that does not contain any of the pairs assigned in $A^{(1)}$. Then, choosing $\mathcal{R}_2$ via random split gives that

$$\mathbb{E}_{\mathcal{R}_2}[f(\mathcal{R}_2)] \geq \frac{3}{4} s^{(1)} + \frac{s^{(\mu)}}{4} \left[ 1 - \frac{\sqrt{7} + \sqrt{6}}{2\sqrt{\pi} \mu} - \frac{3\epsilon}{\mu/4} \right].$$

**Proof.** We attempt to construct an assignment in each stage in two rounds. We first match all available pairs from $A^{(1)}$ (tiebreaking randomly between the two reviewers if both are available), and then attempt to construct a larger assignment from $A^{(\mu)}$.

By Lemma 2, we can consider drawing $\mathcal{P}_2 \sim \mathcal{I}_\beta(\mathcal{P})$ and $\mathcal{R}_2 \sim \mathcal{I}_\beta/(1 + \beta)(\mathcal{R})$ and allowing papers to be underloaded. For all reviewers $r \in \mathcal{R}$, define the random variables $Z_r = \begin{cases} 1 & \text{w.p. } 1/2 \\ 2 & \text{w.p. } 1/2 \end{cases}$ representing the stage that reviewer $r$ is randomly chosen to be in. For each pair of reviewers $(i, j)$ that are matched to the same paper in $A^{(1)}$, define the random variables $F_{ij} = \begin{cases} i & \text{w.p. } 1/2 \\ j & \text{w.p. } 1/2 \end{cases}$ representing the reviewer that will be assigned in round one if both are in the same stage. All of these random variables are independent. Define the total similarity value of the assignments as $v^{(1)} = 2ms^{(1)}$ and $v^{(\mu)} = 2n\mu s^{(\mu)}$. For $A^{(\mu)}$, define the total similarity value assigned to paper $p$ and reviewer $r$ respectively as $v_p^{(\mu)}$ and $v_r^{(\mu)}$.

**Round One:** We first match all available pairs from $A^{(1)}$. For any paper $p \in \mathcal{P}$, call $a, b$ the two reviewers assigned to $p$ by $A^{(1)}$. The value assigned to paper $p$ across both stages is represented by a random variable $V_p = 1[Z_a \neq Z_b](S_{ap} + S_{bp}) + 1[Z_a = Z_b](S_{ap} + S_{bp})[F_{ab} = a] + S_{ap} + S_{bp}[F_{ab} = b]$. $\mathbb{E}[V_p] = \frac{3}{4}(S_{ap} + S_{bp})$, so $\mathbb{E}[\sum_{p \in \mathcal{P}} V_p] = \frac{3}{4} v^{(1)}$ is the total expected value assigned in round 1.

**Round Two:** Fixing the round one assignments, we now attempt to find a matching for all remaining papers and reviewers by matching pairs from within $A^{(\mu)}$. We first attempt to find an assignment with paper and reviewer loads of at most $\theta = [\mu/4]$ among the remaining reviewers and papers in each stage. We start with the pairs from $A^{(\mu)}$ that both are present in this stage and were not matched in round one, and randomly drop entries from each reviewer and paper until they are no longer overloaded. This argument mirrors the one made in the proof of Theorem 4.

We consider stage one without loss of generality. We start by constructing an assignment $C$ to include all pairs assigned in $A^{(\mu)}$ where the reviewer and paper both were unmatched in round one and are in stage one. Each reviewer-paper pair in $A^{(\mu)}$ can be assigned in $C$ with probability $\frac{1}{32}$, so $\mathbb{E} \left[ \sum_{r \in \mathcal{R}, p \in \mathcal{P}} C_{rp} S_{rp} \right] = \frac{v^{(\mu)}}{32}$. 

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We then construct an assignment $B^{(1a)}$ from $C$ by removing assigned reviewers from each paper at random until each paper has load at most $\theta$. Fix some paper $p$, and define $W_p$ as the event that paper $p$ was not assigned in round one. The number of reviewers assigned to $p$ in $A^{(\mu)}$ that are in stage one and not assigned in round one is a $\text{Binom}(2\mu, 1/8)$ random variable. The expected value assigned to $p$ in this assignment is (using $f$ as the Binomial pmf),

\[
E \left[ \sum_{r \in R} B^{(1a)}_{rp} S_{rp} \mid W_p \right] = \sum_{x=0}^{\theta} f(x; 2\mu, 1/8) v^{(\mu)}_p \frac{x}{2\mu} + \sum_{x=0}^{2\mu} f(x; 2\mu, 1/8) v^{(\mu)}_p \frac{\theta}{2\mu} = \frac{v^{(\mu)}_p}{2\mu} E_{X \sim \text{Binom}(2\mu, 1/8)}[\min(X, \theta)].
\]

Summing over all papers, since each paper has a $1/4$ change of being unmatched in round one,

\[
E \left[ \sum_{p \in P} \sum_{r \in R} B^{(1a)}_{rp} S_{rp} \right] = \frac{v^{(\mu)}_P}{8\mu} E_{X \sim \text{Binom}(2\mu, 1/8)}[\min(X, \theta)].
\]

We separately construct an assignment $B^{(1b)}$ from $C$ by removing assigned papers from each reviewer at random until each reviewer has load at most $\theta$. Fix some reviewer $r$, and define $W_r$ as the event that reviewer $r$ was not assigned in round one. The number of papers assigned to $r$ in $A^{(\mu)}$ that are not assigned in round one is a $\text{Binom}(\mu, 1/4)$ random variable. The expected value assigned to $r$ in this assignment is,

\[
E \left[ \sum_{p \in P} B^{(1b)}_{rp} S_{rp} \mid W_r \right] = \sum_{x=0}^{\theta} f(x; \mu, 1/4) v^{(\mu)}_r \frac{x}{\mu} + \sum_{x=0}^{\mu} f(x; \mu, 1/4) v^{(\mu)}_r \frac{\theta}{\mu} = \frac{v^{(\mu)}_r}{\mu} E_{X \sim \text{Binom}(\mu, 1/4)}[\min(X, \theta)].
\]

Summing over all reviewers, since each reviewer has a $1/4$ change of being both unmatched in round one and present in stage one,

\[
E \left[ \sum_{r \in R} \sum_{p \in P} B^{(1b)}_{rp} S_{rp} \right] = \frac{v^{(\mu)}_R}{8\mu} E_{X \sim \text{Binom}(\mu, 1/4)}[\min(X, \theta)].
\]

Finally, we construct $B^{(1)}$ to include all pairs assigned in both $B^{(1a)}$ and $B^{(1b)}$. It has value at least equal to the total value of $B^{(1a)}$ and $B^{(1b)}$ less the value of $C$, since the assigned pairs in $B^{(1a)}$ and $B^{(1b)}$ are subsets of the assigned pairs in $C$.

\[
E \left[ \sum_{r \in R, p \in P} B^{(1)}_{rp} S_{rp} \right] \geq \frac{v^{(\mu)}}{8\mu} \left[ E_{X \sim \text{Binom}(2\mu, 1/8)}[\min(X, \theta)] + E_{X \sim \text{Binom}(\mu, 1/4)}[\min(X, \theta)] - \frac{\mu}{4} \right].
\]

By construction this assignment has paper loads of at most $\theta$ and reviewer loads of at most $\theta$ (among all reviewers and papers unmatched in round one and present in stage one).

By a generalization of the Birkhoff-von Neumann theorem \cite{66}, there exists an assignment with paper loads of at most 1 and reviewer loads of at most 1 among all reviewers and papers unmatched in round one and present in stage one, with value at least $\frac{1}{2}$ of the value of $B^{(1)}$. Tottaling over both stages and dividing by $2n$, the round two assignments contribute at least

\[
\frac{s^{(\mu)}}{4} \left[ E_{X \sim \text{Binom}(2\mu, 1/8)} \left[ \min \left( \frac{X}{\theta}, 1 \right) \right] + E_{X \sim \text{Binom}(\mu, 1/4)} \left[ \min \left( \frac{X}{\theta}, 1 \right) \right] - \frac{\mu}{4\theta} \right].
\]
to the mean assignment value.

If $X \sim \text{Binom}(N, p)$, the above bound is a function of $\mathbb{E} \left[ \min \left( \frac{X}{\ell}, 1 \right) \right]$ for two binomial random variables where $Np \leq \ell$. Using the normal approximation presented in the proof of Theorem 5, we get the following approximation to the above bound (defining $\epsilon = \lceil \mu/4 \rceil - (\mu/4)$):

$$\frac{s(\mu)}{4} \left[ 1 - \frac{\sqrt{7} + \sqrt{6}}{2\sqrt{\pi \mu}} - \left( 1 - \frac{\mu/4}{\lceil \mu/4 \rceil} \right) \left( \Phi \left( \frac{\epsilon}{\sqrt{\frac{7}{32} \mu}} \right) + \Phi \left( \frac{\epsilon}{\sqrt{\frac{3}{16} \mu}} \right) + 1 \right) \right] \quad (8)$$

$$\geq \frac{s(\mu)}{4} \left[ 1 - \frac{\sqrt{7} + \sqrt{6}}{2\sqrt{\pi \mu}} - 3 \left( 1 - \frac{\mu/4}{\lceil \mu/4 \rceil} \right) \right]$$

$$= \frac{s(\mu)}{4} \left[ 1 - \frac{\sqrt{7} + \sqrt{6}}{2\sqrt{\pi \mu}} - \frac{3 \epsilon}{\lceil \mu/4 \rceil} \right].$$

Via simulation, we confirm that the approximation in (8) is in fact a lower bound on the expression in (7) for all $\mu \in [10^4]$. In Figure 2 of Section 7, we plot the more precise bound of (8).