Protons in High Density Neutron Matter

Misak M. Sargsian
Department of Physics, Florida International University, Miami, FL 33199, USA
E-mail: sargsian@fiu.edu

Abstract. We discuss the possible implication of the recent predictions of two new properties of high momentum distribution of nucleons in asymmetric nuclei for neutron star dynamics. The first property is about the approximate scaling relation between proton and neutron high momentum distributions weighted by their relative fractions \( x_p \) and \( x_n \) in the nucleus. The second is the existence of inverse proportionality of the high momentum distribution strength of protons and neutrons to \( x_p/n \). Based on these predictions we model the high momentum distribution functions for asymmetric nuclei and demonstrate that it describes reasonably well the high momentum characteristics of light nuclei. We also extrapolate our results to heavy nuclei as well as infinite nuclear matter and calculate the relative fractions of protons and neutrons with momenta above \( k_F \). Our results indicate that for neutron stars starting at three nuclear saturation densities the protons with \( x_p = \frac{1}{2} \) will populate mostly the high momentum tail of the momentum distribution while only 2% of the neutrons will do so. Such a situation may have many implications for different observations of neutron stars which we discuss.

1. Introduction
One of the exciting recent results in the studies of short-range correlations (SRCs) in nuclei is the observation of the strong (by factor of 20) dominance of the \( pn \) SRCs, relative to the \( pp \) and \( nn \) correlations, for nuclear internal momenta of \( \sim 300 - 600 \text{ MeV}/c \)[1, 2].

This observation is understood[1, 3, 4] based on the dominance of the tensor forces in NN interaction at the above mentioned momenta corresponding to average nucleon separations of \( \sim 1 \text{ Fm} \). At these distances the dominating NN central potential crosses the zero due to transition from attractive to repulsive interaction allowing tensor forces to dominate in this transition range. The tensor interaction projects the NN SRC part of the wave function into the isosinglet - relative angular momentum, \( L = 2 \) state, almost identical to the \( D \)-wave component of the deuteron wave function. As a results \( pp \) and \( nn \) components of the NN SRC will be strongly suppressed since they are dominated by the central NN potential with relative \( L = 0 \). The resulting picture for the nuclear matter consisting of protons and neutrons at densities in which inter-nucleon distances are \( \sim 1 \text{ Fm} \) is rather unique: it represents a system with suppressed \( pp \) and \( nn \) but enhanced \( pn \) interactions.

The goal of our study is to understand the implication of the above described conditions on the the momentum distribution of protons and neutrons in high density nuclei matter.

2. New Relation between High Momentum \( p \)- and \( n \)-distributions in Nuclei:
Due to short range nature of \( NN \) interaction the nuclear momentum distribution, \( n^A(p) \), for momenta, \( p \), exceeding the characteristic nuclear Fermi momentum \( k_F \) is predominantly defined...
by the momentum distribution in the SRCs. There is a rather large experimental body of information indicating that for the range of $k_F < p ≤ 600$ MeV/c the SRCs are dominated by 2N correlations, which consist of mainly the $pn$ pairs (for recent reviews see [5, 6]).

In recent work[7] based on the dominance of the $pn$ SRCs we predicted two new properties for the nuclear momentum distributions at $\sim k_F < p < 600$:

(i) There is an approximate equality of $p$- and $n$- momentum distributions weighted by their relative fractions in the nucleus $x_p = \frac{Z}{A}$ and $x_n = \frac{A-Z}{A}$:

$$x_p n_p^A(p) \approx x_n n_n^A(p),$$

(1)

with $\int n_{p/n}^A(p) d^3p = 1$.

(ii) The probability of proton or neutron being in high momentum NN SRC is inverse proportional to their relative fractions and can be related to the momentum distribution in the deuteron $n_d(p)$ as:

$$n_{p/n}^A(p) \approx \frac{1}{2x_{p/n}} a_2(A,y) \cdot n_d(p),$$

(2)

where $a_2(A,y)$ is interpreted as a per nucleon probability of finding 2N SRC in the given $A$ nucleus[8, 9, 10] and the nuclear asymmetry parameter is defined as $y = |1 - 2x_p| = |x_n - x_p|$.

The above two properties are obtained assuming no contributions from $pp$, $nn$ as well as higher order SRCs. They follow from the assumption that the whole strength of nuclear high momentum distribution as well as per nucleon probability of proton and neutron to be in the SRC is defined by the same $pn$ correlation.

These properties can be checked for light nuclei by direct calculations of momentum distribution of nucleons in asymmetric nuclei (see below).

Predictions can be made also for heavy nuclei and infinite nuclear matter if one estimates the $a_2$ parameters entering in Eq.(2) for large $A$ and extrapolate for infinite nuclear matter. Since SRCs are defined by local properties of nuclei, one expects that the $A$ dependence of $a_2$ is related to the nuclear density, i.e. $a_2(A,y) = a_2(\rho,y)$. This could allow us to evaluate the high momentum part of the nucleon momentum distribution not only for finite[7] but also for infinite nuclear matter.

3. High Momentum Features of Light Nuclei

One can check the validity of the above two (Eqs.(1) and (2)) observations for light nuclei for which it is possible to perform realistic calculations based on the Faddeev equations for $A=3$ systems[11], Correlated Gaussian Basis(CGB) approach[12] as well as Variational Monte Carlo method(VMC) for light nuclei $A$ (recently being available for up to $A = 12$ [13]).

The validity of Eq.(1) is checked in Fig.1 for $^3He$ nucleus, based on the solution of Faddeev equation[11], and for $^{10}Be$ based on VMC calculations[13]. The solid lines with and without squares in Fig.1(a) represent neutron and proton momentum distributions for both nuclei weighted by their respective $x_n$ and $x_p$ factors.

As one can see for $^3He$ the proton momentum distribution dominates the neutron momentum distribution at small momenta reflecting the fact that in the mean field the probability of finding proton is larger than neutron just because there are twice as much protons in $^3He$ than neutrons. The same is true for $^{10}Be$ for which now the neutron momentum distribution dominates at small momenta. However at $\geq 300$ MeV/c for both nuclei, the proton and neutron momentum distributions become close to each other up to the internal momenta of 600 MeV/c. This is the region dominated by tensor interaction. As Fig.1(a) shows the high momentum distribution modeled according to Eq.(2) (dotted lines) agrees reasonably well with the realistic calculations.
Figure 1. (a) The momentum distributions of proton and neutron weighted by $x_p$ and $x_n$ respectively. The dotted lines represent the prediction for the momentum distribution according to Eq.(2). (b) The $x_{p/n}$ weighted ratio of neutron to proton momentum distributions. See the text for details.

Table 1. Kinetic energies (in MeV) of proton and neutron

| A   | y   | $E_{kin}^P$ | $E_{kin}^n$ | $E_{kin}^P - E_{kin}^n$ |
|-----|-----|-------------|-------------|-------------------------|
| $^3$He | 0.50 | 30.13       | 18.60       | 11.53                   |
| $^6$He | 0.33 | 27.66       | 19.06       | 8.60                    |
| $^9$Li | 0.33 | 31.39       | 24.91       | 6.48                    |
| $^3$He | 0.33 | 14.71       | 19.35       | -4.64                   |
| $^3$He$^{[11]}$ | 0.33 | 13.70       | 18.40       | -4.7                    |
| $^3$He$^{[12]}$ | 0.33 | 13.97       | 18.74       | -4.8                    |
| $^3$H | 0.33 | 19.61       | 14.96       | 4.65                    |
| $^8$Li | 0.25 | 28.95       | 23.98       | 4.97                    |
| $^{10}$Be | 0.2  | 30.20       | 25.95       | 4.25                    |
| $^7$Li | 0.14 | 26.88       | 24.54       | 2.34                    |
| $^9$Be | 0.11 | 29.82       | 27.09       | 2.73                    |
| $^{11}$B | 0.09 | 33.40       | 31.75       | 1.65                    |

This effect is more visible for the ratios of weighted n- to p- momentum distributions in Fig.1(b), demonstrating that the approximation of Eq.(1) in the range of 300 – 600 MeV/c is good on the level of 15%. Note that the similar features present for all other asymmetric nuclei calculated within the VMC method in Ref.[13].

The prediction of Eq.(2) can be checked by comparing the kinetic energies of proton and neutron, in which case we expect that per nucleon kinetic energy of the lesser component to be larger. As it can be seen from Table 1 this prediction is confirmed too by realistic calculations.
4. High Momentum Properties of Heavy Nuclei

Presently, no realistic calculations exist for asymmetric heavy nuclei for the predictions of Eqs.(1) and (2) to be checked. The main predictions of Eq.(2) is that high momentum protons and neutrons became increasingly unbalanced with an increase of the nuclear asymmetry, \( y \). To quantify this, using Eq.(2) one can calculate the fraction of the nucleons having momenta \( \geq k_F \) as:

\[
P_{p/n}(A,y) \approx \frac{1}{2x_{p/n}} a_2(A,y) \int_{k_F}^{\infty} n_d(p) d^3p,
\]

where the parameter \( a_2(A,y) \) can be taken from the experimental analysis of inclusive \( A(e,e')X \) data at kinematics dominated by SRC\[10, 14, 15, 16\]. The results of the estimates of \( P_{p/n}(A,y) \) for medium to heavy nuclei are presented in Table 2.

Table 2. Fractions of high momentum (\( \geq k_F \)) protons and neutrons in nuclei A

| A   | \( P_p(\%) \) | \( P_n(\%) \) | A   | \( P_p(\%) \) | \( P_n(\%) \) |
|-----|--------------|--------------|-----|--------------|--------------|
| 12  | 20           | 20           | 56  | 27           | 23           |
| 27  | 24           | 22           | 197 | 31           | 20           |

As it follows from the table, as the asymmetry increases the imbalance between the high momentum fractions of proton and neutron grows. For example, in the Gold, the relative fraction of high momentum (\( \geq k_F \)) protons is 50\% more than that of the neutrons.

Such a dominance of the proton high momentum component in heavy neutron rich nuclei can be checked in high momentum transfer single proton knock-out processes in which long-range nuclear effects are well controlled\[17, 18\]. The first such experimental verification for heavy nuclei is currently underway in quasi elastic \( A(e,e,p)X \) measurement at Jefferson Lab, where the ratio of high momentum fractions of nucleons in \( ^{56}\text{Fe} \) and \( ^{208}\text{Pb} \) to that of \( ^{12}\text{C} \) is extracted. The results\[19\] are in reasonably good agreement with the prediction of Eq.(3) (Table 2) and they are being prepared for publication.

5. High Momentum Properties of Asymmetric Nuclear Matter

To estimate the fractions of energetic protons and neutrons in asymmetric nuclear matter using Eq.(3) one needs to extrapolate \( a_2(A,y) \) for infinite nuclear matter. This was achieved in Ref.[20] where based on the local property of SRCs the \( a_2(A,y) \) parameters were represented as a function of local nuclear density \( \rho \) and asymmetry \( y \): \( a_2(\rho,y) \). Then all the available data on inclusive \( A(e,e')X \) scattering\[21\] at SRC kinematics were used to fit and extrapolate \( a_2 \) magnitudes for saturation and above saturation densities of nuclear matter at given asymmetry \( y \). The accuracy of the fit was checked for symmetric nuclear matter at the saturation density, for which the obtained value of \( a_2(\rho_0,0) \approx 7.03 \pm 0.41 \) is in reasonable agreement with other estimates of \( a_2 \) for symmetric nuclear matter\[22\].

For the case of asymmetric nuclear matter we considered a neutron star matter, in which for asymmetry parameter \( y \), we used the threshold value of \( x_p = \frac{1}{9} (y = \frac{1}{9}) \) below of which the direct URCA processes:

\[
n \to p + e^- + \bar{\nu}_e, \quad p + e^- \to n + \nu_e
\]

will stop in the standard model of superdense nuclear matter consisting of degenerate protons and neutrons\[23\]. Estimating the Fermi momenta of protons and neutrons in Eq.(3) with \( k_{FN} = \left(3\pi^2x_N\rho\right)^{\frac{1}{3}} \), in Fig.2 we present the off-Fermi-shell fractions of protons and neutrons as a function of nuclear density. The most interesting result of these estimates is that in the equilibrium, \( pn \) SRCs move the large fraction of protons above the Fermi-shell: at \( 3\rho_0 \) densities.
half of the protons will be off-Fermi-shell while at \( \rho \geq 4.5\rho_0 \) all the protons will populate the high momentum tail of the momentum distribution. The situation however is not as dramatic for neutrons, with only about 2% of neutrons populating the high momentum part of the momentum distribution.

![Figure 2. Density dependence of the fraction of off-Fermi-shell nucleons in \( x_p = \frac{1}{5} \) matter.](image)

It is worth mentioning that the present result of strong modification of proton momentum distribution in high density asymmetric matter is in qualitative agreement with the nuclear matter calculation based on Green function method[25].

6. Possible Implications for Nuclei and Neutron Stars:

Our main observation is that with an increase of nuclear asymmetry the lesser component become more energetic. This is confirmed[7] for light nuclei by direct estimates of the average kinetic energies of proton and neutron using realistic wave function calculations (Table 1). In the case of neutron rich heavy nuclei (\( A \geq 40 \)), a larger fraction of protons will occupy the high momentum tail of the momentum distributions (Table 2). This may have several verifiable implications for large \( A \) nuclear phenomena[7]. Particularly our estimates show that it may explain[26] the large \( A \) part of the recently observed correlation between the strengths of the EMC (nuclear partonic distribution modification) and SRC effects[27].

The energetic protons in neutron rich nuclei will result also to the stronger nuclear modification of \( u \)-quarks as compared to \( d \)-quarks and the effect will grow with \( A \). This provides an explanation[28] of the NuTeV anomaly[29]. The predicted effect also can be checked in parity violating deep inelastic scattering off heavy nuclei.

However, our observation may have more dramatic implications for the dynamics of neutron stars. Some of them are:

- **Cooling of a Neutron Star:** Large concentration of protons above the Fermi momentum will allow the condition for Direct URCA processes \( p_p + p_e > p_n \) to be satisfied even if \( x_p < \frac{1}{5} \). This will allow a situation in which intensive cooling of the neutron stars continues well beyond the critical point \( x_p = \frac{1}{5} \) (see also Ref.[5]).

- **Superfluidity of Protons:** Transition of protons to the high momentum tail will smear out the energy gap which will remove the superfluidity condition for the protons.

- **Protons in the Neutron Star Cores:** The concentration of protons in the high momentum tail will result in proton densities \( \rho_p \sim p_p^0 \gg k_{F,p}^3 \). This will favor an equilibrium condition with ”neutron skin” effect in which large concentration of protons populate the core rather than the crust of the neutron star. This and the proton superfluidity condition violation may provide different dynamical picture for generation of magnetic fields in the stars.
- Isospin locking and the stiff equation of state of neutron stars: With an increase in density more and more protons move to the high momentum tail where they are in short range tensor correlations with neutrons. In this case one would expect that high density nuclear matter to be dominated by configurations with quantum numbers of tensor correlations ($S=1, I=0$). In such a scenario protons and neutrons at large densities will be locked in the NN iso-singlet state. This will double the threshold of inelastic excitation from $NN \rightarrow N\Delta$ to $NN \rightarrow \Delta\Delta (NN^*)$ transition thereby stiffening the equation of state which is favored by the recent large neutron star mass observation[24].

7. Possible Universality of the Obtained Results

Our observation is relevant to any asymmetric two-component Fermi system in which the interaction within each component is suppressed while the mutual interaction between two components is enhanced. It is interesting that the similar situation is realized for two-fermi-component ultra-cold atomic systems[30] but with the mutual s-state interaction. One of the most intriguing aspects of such systems is that in the asymmetric limit they exhibit very rich phase structure with indication of the strong modification of the small component of the mixture[31]. In this respect our case is similar to that of ultra-cold atomic systems with the difference that the interaction between components has a tensor nature.

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