In this study, nuclear ground-state properties of spherical nuclei, such as the total energy, nucleon local density, and nucleon local potential of Pb isotopes (especially 204-214Pb) are investigated by using Hartree - Fock method. The calculations have been performed by using Skyrme set parameters, especially SLy4, SkM*, Zn, and SIII set parameters. The calculation results have been compared to the related experiment results and the calculation results of the other researchers. All parameters used in this study are in good agreement with the results of the related experiments and the other researchers. In Pb nucleus, it is also obtained from this study that the total energy, mass radius, neutron radius, neutron skin thickness, neutron density, neutron density width, proton potential depth, and proton potential width increase accordingly with the increase of neutron number. In other hand, proton density and neutron potential decrease accordingly with the increase of neutron number. The increase of neutron number has minimum effect to the widths of proton density and neutron potential.

**Keywords:** Hartree - Fock, local density, nuclear binding energy, Pb isotopes, Skyrme interaction.

**Introduction**

It is commonly known that lead (Pb) has four stable isotopes, i.e. 204Pb, 206Pb, 207Pb, and 208Pb. 204Pb is entirely a primordial nuclide and is not a radiogenic nuclide. The next three isotopes, 206Pb, 207Pb, and 208Pb, represent the ends of three natural decay chains: the uranium series (or radium series), the actinium series, and the thorium series, respectively. In burn-up process, 212Pb and 208Pb are the products of 232Th series. The 214Pb, 210Pb, and 206Pb are the products of 238U series. 208Pb has an unusually low neutron capture cross-section (even lower than that of deuterium in the thermal spectrum) that makes it interesting to be applied in lead-cooled fast reactors [1]. The unique features of 208Pb lead to the economy of neutrons, hardening the neutron spectra, and other profitable factors [2]. It implies that Pb isotopes become very interesting to be investigated further, especially its ground-state properties.

The far-from-stability nuclei have attracted many scientists in recent years, especially observation about the new nuclear structure phenomena, such as the neutron halo and the neutron skin in light nuclei. Theoretically, microscopic mean-field approaches have been very successful in describing ground-state properties of nuclei to explain the experiment results. One of the methods that can be used is Skyrme - Hartree - Fock (SHF) method, where this model has been proven very successful for microscopic description of many nuclear properties near the stable line, such as the nuclear ground-state, collective motion, fission barrier, giant resonance, and heavy-ion collision. With a few adjusting parameters in effective interactions, one can quantitatively reproduce experiment data of nuclei near the stable line by the SHF model [3 - 5].

Nuclear structure investigations of some isotopes had been conducted by some researchers. The nuclear structure of the Be, Cr, and Cu isotopes have been investigated by Tel et al. using SHF method [6]. Radii and Density of 7-19Pb Isotopes have been also calculated by Tel et al. by using effective Skyrme force [7]. SHF method has been used by Alzubadi et al. to study the microscopic approach of nuclear structure for some Zr isotopes [8]. The aims of this study are to calculate numerically the ground-state energies of Pb isotopes (especially 204-214Pb) by using the SHF method with Skyrme set parameters (especially SLy4, SkM*, Zn, and SIII) and to simulate the local density as well as the local potential for both proton and neutron. The density distribution and nucleus radii are very important information to understand about nuclear structure. It can be used in scattering research to calculate the microscopic cross-section. Such data is needed in nuclear reactor analysis. Another purpose of this study is to investigate the effect of the increase of neutron number in Pb nucleus.

**Theory**

**Skyrme - Hartree - Fock**

The most convenient force used in the description of the ground-state properties of nucleus is the phenomenological Skyrme force first proposed by Skyrme [9]. According to this effective interaction, the force of a zero-range, density, and momentum-dependent can be approximated as

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\[ V_{\text{Skyrme}} = t_0 \left( 1 + x_0 P_0 \right) \delta(\vec{r}) + \frac{1}{2} t_1 \left( 1 + x_1 P_1 \right) \left\{ \delta(\vec{r}) \vec{k}^2 + \vec{k}^2 \delta(\vec{r}) \right\} + t_2 \left( 1 + x_2 P_2 \right) \vec{k} \cdot \delta(\vec{r}) \vec{k} + \frac{1}{6} t_3 \left( 1 + x_3 P_3 \right) \rho^a \left( \vec{r}_f^a + \vec{r}_l^a \right) \]

\[ E_{\text{Skyrme}} = 4\pi \int_0^\infty dr \, r^2 \left\{ \frac{\hbar^2}{2m} \gamma + \frac{1}{2} t_0 \left( 1 + \frac{1}{2} x_0 \right) \rho^2 - \frac{1}{2} \gamma t_0 \left( 1 + \frac{1}{2} x_0 \right) \sum_q \rho_{qq} + \frac{1}{12} t_0 \left( 1 + \frac{1}{2} x_0 \right) \rho_{a^2} \right\} \]

\[ - \frac{1}{12} t_1 \left( 1 + x_1 \right) \rho^a \sum_q \rho_{qq}^2 + \frac{1}{4} \left[ t_1 \left( 1 + \frac{1}{2} x_1 \right) \rho - t_2 \left( 1 + \frac{1}{2} x_2 \right) \right] \rho \gamma \]

\[ - \frac{1}{4} \left[ t_1 \left( x_1 + \frac{1}{2} \right) - t_2 \left( x_2 + \frac{1}{2} \right) \right] \sum_q \rho_{qq}^2 \rho_{a^2} - \frac{1}{16} t_2 \left( 1 + \frac{1}{2} x_2 \right) \rho \nabla^2 \rho \]

\[ + \frac{1}{16} \left[ 3 t_1 \left( 1 + \frac{1}{2} x_1 \right) + t_2 \left( 1 + \frac{1}{2} x_2 \right) \right] \sum_q \rho_q \nabla^2 \rho_q - \frac{1}{2} t_4 \left[ \rho \nabla J + \sum_q \rho_q \nabla J_q \right] \]

where \( \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \), \( \rho_q \) is the density of nucleon, \( \gamma_q \) is the kinetic energy density of nucleon, and \( J_q \) is the spin-orbit density of nucleon. The isospin label \( q \) runs over \( q \in \{ p, n \} \), where the densities, without an isospin label in all equations, represent total densities, summed over both species, such as \( \rho = \rho_p + \rho_n \), \( \gamma = \gamma_p + \gamma_n \), and \( \nabla J = \nabla J_p + \nabla J_n \). The total energy follows the form of:

\[ E = E_{\text{Skyrme}} + E_{\text{Coul}} + E_{\text{Pair}} - E_{\text{CM}} \tag{3} \]

where \( E_{\text{Skyrme}} \) is the Skyrme interaction energy, \( E_{\text{Coul}} \) is the Coulomb interaction energy, \( E_{\text{Pair}} \) is the nucleon interaction pairing energy, and \( E_{\text{CM}} \) is the correction for the spurious center-of-mass motion of the mean field. In other hand, the densities in spherical representation are expressed as

\[ \rho_q(r) = \sum_{\eta \eta' \eta''} w_\eta \left[ \frac{2 j_\eta + 1}{4\pi} \left( \frac{R_\eta}{r} \right)^2 \right] \]

\[ \tau_q(r) = \sum_{\eta \eta' \eta''} w_\eta \left[ \frac{2 j_\eta + 1}{4\pi} \left( \frac{\partial R_\eta}{\partial r} \right)^2 \right] + \frac{l_\eta l_\eta' + 1}{r^2} \left( \frac{R_\eta}{r} \right)^2 \]

where \( t_0, t_1, t_2, t_3, t_4, x_0, x_1, x_2, x_3, \) and \( a \) are Skyrme set parameters, \( \vec{k} \) and \( \vec{k}' \) are the relative momentum acting on the right and the left, \( P_q \) is the space exchange operator, \( \rho \) represents the density, \( \delta(\vec{r}) \) is the delta function, and \( \hat{\sigma} \) is the vector of Pauli spin matrices \([7, 10, 11]\). For spherical representation, Skyrme energy can be expressed as \([3, 10]\)

\[ \nabla J_q(\vec{r}) = \left( \frac{\partial}{\partial r} + \frac{2}{r} \right) J_q(r) \tag{6} \]

\[ J_q(r) = \sum_{\eta \eta' \eta''} w_\eta \left[ \frac{2 j_\eta + 1}{4\pi} \left( j_\eta + 1 \right) \right] - l_\eta \left( l_\eta + 1 \right) - \frac{3}{4} \left( \frac{R_\eta}{r} \right)^2 \tag{7} \]
using the mean of the least-squares fitting technique, which optimizes the force parameters such that the Hartree-Fock calculations reproduce the experimentally determined nuclear ground-state properties, e.g. binding energy, radii, and surface width [4].

Table 1. The SLy4 [12], SkM* [13], Zα [4], and SIII [14] set parameters for Skyrme interaction

| Parameter | SLy4   | SkM*  | Zα     | SIII   |
|-----------|--------|-------|--------|--------|
| t₀        | -2488  | -2645 | -1983.76 | 1128.75 |
| t₁        | 486.82 | 410   | 362.252 | 395    |
| t₂        | -546.3 | -135  | -104.27 | -95    |
| t₃        | 13777  | 15595 | 11861.4 | 14000  |
| x₀        | 0.834  | 0.09  | 1.1717  | 0.45   |
| x₁        | -0.544 | 0     | 0       | 0      |
| x₂        | -1     | 0     | 0       | 0      |
| x₃        | 1.354  | 0     | 1.762   | 1      |
| t₄        | 123    | 130   | 123.69  | 120    |
| α         | 0.167  | 0.167 | 0.25    | 1      |

Hartree-Fock Method

The single-particle Hartree-Fock equation for the radial wave function \( R_\beta \) can be expressed as [10]:

\[
\hbar^2 \frac{\partial^2}{\partial r^2} R_\beta = \epsilon_\beta R_\beta \tag{8}
\]

with the mean-field Hamiltonian

\[
U_q = t_0 \left( 1 + \frac{1}{2} x_0 \right) \rho - t_0 \left( x_0 + \frac{1}{2} \right) \rho_q + \frac{1}{12} t s \rho^3 \left[ 2 + \alpha \left( 1 + \frac{1}{2} x_3 \right) \rho 
\right.
- 2 \left( x_3 + \frac{1}{2} \right) \rho_q - \alpha \left( x_3 + \frac{1}{2} \right) \left( \frac{\rho_p^2 + \rho_n^2}{\rho} \right) + \frac{1}{4} t_1 \left( 1 + \frac{1}{2} x_1 \right) + t_2 \left( 1 + \frac{1}{2} x_2 \right) \tau
\]

\[
- \frac{1}{8} t_1 \left( x_1 + \frac{1}{2} - t_2 \left( x_2 + \frac{1}{2} \right) \right) \tau_q - \frac{1}{8} t_1 \left( 1 + \frac{1}{2} x_1 \right) - t_2 \left( 1 + \frac{1}{2} x_2 \right) \Delta \rho
\]

\[
+ \frac{1}{8} t_1 \left( x_1 + \frac{1}{2} + t_2 \left( x_2 + \frac{1}{2} \right) \right) \Delta \rho_q - \frac{1}{2} t_4 \nabla \vec{J} + \vec{\psi} \nabla \vec{J} + U_{\text{Coul}} \tag{11}
\]

\[
U_{\mu,q} = \frac{1}{4} t_4 \rho + \rho_q + \frac{1}{8} t_1 - t_2 J_q - \frac{1}{8} x f_1 + x f_2 J, \tag{12}
\]

where the \( U_{\mu,q} \) is the form factor of the one-body spin-orbit potential, \( U_{\text{Coul}} \) is the Coulomb potential, and \( U_q \) is the potential of nucleon. Coulomb interaction is well-known part of the nucleus Hamiltonian. The Coulomb energy density can be expressed as [3, 17]

\[
\epsilon_{\text{Coul}} = \frac{e^2}{2} \int \frac{\rho_p \: \vec{p} \: \vec{r}}{\sqrt{\vec{p}^2 - \vec{r}^2}} d^3 r = \frac{3}{4} e^2 \left( \frac{3}{\pi} \right)^{1/3} \rho_p^{4/3} \vec{r}' \tag{13}
\]

\[
E_{\text{pair}} = - \sum_q G_q \left( \sum_{W_p} W_p \right)^2, \tag{14}
\]

where the pairing matrix elements \( G_q \) are
constant for each type of nucleon \((G_p = \frac{22\text{ MeV}}{A})\) and \(G_n = \frac{29\text{ MeV}}{A}\), where \(A\) is the total nucleon number of a nucleus) and \(w_\beta\) is the pairing weights of the proton and neutron, which can be calculated by using

\[
w_\beta = \frac{1}{2} \left( 1 - \frac{\epsilon_\beta - \epsilon_{F,q}}{\epsilon_\beta - \epsilon_{F,q} + \Delta_q^2} \right),
\]

where

\[
\frac{\Delta_q}{G_q} = \sum_{\beta=q} \sqrt{w_\beta} \left( 1 - w_\beta \right).
\]

The \(\Delta_q\) is the pairing gap, \(\epsilon_\beta\) is the single-particle energy of state \(\beta\), and \(\epsilon_{F,q}\) is the Fermi energy. The detailed explanation of pairing energy can be found in the following references [10, 16]. The root-mean-square (rms) radii of charge, mass, neutron and proton can be defined as the following formula [6, 18]

\[
r_q = \left( \frac{1}{\rho_q} \right)^{\frac{1}{2}} = \left[ \int \rho_q^2 \, r^2 \, d^3r \right] \left[ \int \rho_q \, r^3 \, d^3r \right]^{\frac{1}{2}}
\]

and the neutron skin thickness can be defined as the difference between the neutron rms radius and the proton rms radius as [6, 19]

\[
t = r_n - r_p.
\]

**Result and Discussion**

**Total Energy and Radii Calculations**

Calculation results of total energy for each nucleus used in this study are presented in Table 2. It can be seen in Table 2 that the total energies calculated by \(Z_\sigma\) are in very good agreement with the related experiment results where the discrepancies are in the range of 0.008 - 0.243\%, followed by SkM* results as the second best fit with 0.618 - 0.934\% of discrepancies. Next, SLy4 results, as the third best fit, have the discrepancies of about 1.127 - 1.526\%, followed by SIII results, as the fourth best fit, with 1.261 - 1.541\% of discrepancies.

In this study, the charge, mass, proton, and neutron radii for each nucleus have been also calculated. From Table 2 to Table 7, it can be seen that all calculation results are in good agreement with the experiment results as well as Tel et al. results. For charge radius calculations (Table 3), it can be seen that SkM* results are in very good agreement with the experiment results, followed by the results of SLy4, SIII, and \(Z_\sigma\), respectively. For proton radius of \(^{208}\text{Pb}\) (Table 5), the SIII result is the best fit to the experiment result, followed by the results of SLy4, SkM*, and \(Z_\sigma\), respectively. For neutron radius of \(^{208}\text{Pb}\) (Table 6), the SLy4 result is the best fit to the experiment result, followed by the results of SkM*, SIII, and \(Z_\sigma\), respectively.

In the neutron skin thickness calculation for \(^{208}\text{Pb}\) (Table 7), by using experiment result given in Tables 5 and 6, it can be obtained theoretically that the neutron skin thickness of \(^{208}\text{Pb}\) is 0.06 fm. By using this calculation result, it can be seen that the \(Z_\sigma\) result is the best fit, followed by the results of SIII, SLy4, and SkM*, respectively. It can be indicated that the \(Z_\sigma\) is good enough to explain surface phenomena. In other hand, the \(Z_\sigma\) results are in contradiction with the results of SIII, SLy4, and SkM* set parameters. The results of SIII, SLy4, and SkM* are similar to each other but the results of \(Z_\sigma\) are twice smaller than the results of SIII and three times smaller than those of SLy4 and SkM*. These contradictions need further investigation. From those results, it can be seen that, in Pb nucleus, the total energy, mass radius, neutron radius, and neutron skin thickness increase with the increase of neutron number. In contrast, the increase of neutron number has smaller effect to the proton radius compared to that of neutron.

**Table 2. Calculation results for the total energy of Pb isotopes (all units are in MeV)**

| Nucleus | Exp. [20] | This study |
|---------|-----------|------------|
|        |          | SLY4 | SkM* | Z_σ | SIII |
| ^{209}\text{Pb} | -1607.506 | -1586.060 | -1592.495 | -1605.806 | -1585.495 |
| ^{208}\text{Pb} | -1614.238 | -1594.414 | -1601.143 | -1614.083 | -1593.442 |
| ^{207}\text{Pb} | -1622.324 | -1602.639 | -1609.630 | -1622.719 | -1601.122 |
| ^{206}\text{Pb} | -1629.062 | -1610.687 | -1617.891 | -1630.062 | -1608.520 |
| ^{205}\text{Pb} | -1636.430 | -1617.492 | -1624.896 | -1636.816 | -1614.805 |
| ^{204}\text{Pb} | -1640.367 | -1621.865 | -1630.219 | -1641.434 | -1619.592 |
| ^{203}\text{Pb} | -1645.552 | -1625.307 | -1634.713 | -1645.233 | -1623.488 |
| ^{202}\text{Pb} | -1649.387 | -1628.557 | -1639.007 | -1648.830 | -1627.145 |
| ^{201}\text{Pb} | -1654.514 | -1631.721 | -1643.186 | -1652.358 | -1630.699 |
| ^{200}\text{Pb} | -1658.240 | -1634.846 | -1647.282 | -1655.817 | -1634.196 |
| ^{199}\text{Pb} | -1663.290 | -1637.909 | -1651.319 | -1659.241 | -1637.660 |
Table 3. Calculation results for charge radius of Pb isotopes (all units are in fm)

| Nucleus | SLy4 | SkM* | Z_σ | SIII | SIII | SkM* | Exp. [21] |
|---------|------|------|------|------|------|------|-----------|
| 209Pb   | 5.505| 5.495| 5.427| 5.559| 5.555| 5.494| 5.4803 ± 0.0014 |
| 210Pb   | 5.509| 5.500| 5.432| 5.564| -    | -    | 5.4828 ± 0.0015 |
| 211Pb   | 5.513| 5.505| 5.436| 5.570| 5.566| 5.503| 5.4902 ± 0.0014 |
| 212Pb   | 5.517| 5.509| 5.440| 5.575| -    | -    | 5.4943 ± 0.0014 |
| 213Pb   | 5.522| 5.514| 5.445| 5.581| 5.578| 5.513| 5.5012 ± 0.0013 |
| 214Pb   | 5.529| 5.520| 5.451| 5.588| -    | 5.51 | -         |
| 215Pb   | 5.535| 5.526| 5.455| 5.595| -    | -    | 5.5208 ± 0.0016 |
| 216Pb   | 5.541| 5.531| 5.460| 5.603| -    | -    | 5.529 ± 0.0017  |
| 217Pb   | 5.547| 5.536| 5.464| 5.610| -    | -    | 5.5396 ± 0.0019 |
| 218Pb   | 5.552| 5.541| 5.469| 5.617| -    | -    | -         |
| 219Pb   | 5.558| 5.546| 5.473| 5.625| -    | -    | 5.5577 ± 0.0023 |

Table 4. Calculation results for mass radius of Pb isotopes (all units are in fm)

| Nucleus | SLy4 | SkM* | Z_σ | SIII | SIII | SkM* | Tel et al. [18] |
|---------|------|------|------|------|------|------|----------------|
| 209Pb   | 5.534| 5.529| 5.403| 5.573| 5.567| 5.527| -             |
| 210Pb   | 5.542| 5.537| 5.408| 5.580| -    | -    | -             |
| 211Pb   | 5.549| 5.546| 5.414| 5.588| 5.583| 5.544| -             |
| 212Pb   | 5.557| 5.554| 5.419| 5.597| -    | -    | -             |
| 213Pb   | 5.565| 5.563| 5.425| 5.605| 5.603| 5.562| -             |
| 214Pb   | 5.578| 5.575| 5.435| 5.617| -    | -    | -             |
| 215Pb   | 5.588| 5.585| 5.442| 5.627| -    | -    | -             |
| 216Pb   | 5.599| 5.595| 5.449| 5.637| -    | -    | -             |
| 217Pb   | 5.610| 5.605| 5.456| 5.648| -    | -    | -             |
| 218Pb   | 5.621| 5.616| 5.463| 5.658| -    | -    | -             |
| 219Pb   | 5.631| 5.626| 5.470| 5.668| -    | -    | -             |

Table 5. Calculation results for proton radius of Pb isotopes (all units are in fm)

| Nucleus | SLy4 | SkM* | Z_σ | SIII | SIII | SkM* | Exp. [18, 22] |
|---------|------|------|------|------|------|------|---------------|
| 204Pb   | 5.450| 5.441| 5.375| 5.508| 5.502| 5.439| -             |
| 205Pb   | 5.454| 5.446| 5.380| 5.514| -    | -    | -             |
| 206Pb   | 5.459| 5.451| 5.384| 5.519| 5.513| 5.448| -             |
| 207Pb   | 5.463| 5.455| 5.388| 5.524| -    | -    | -             |
| 208Pb   | 5.468| 5.461| 5.393| 5.531| 5.528| 5.460| 5.5          |
| 209Pb   | 5.476| 5.467| 5.400| 5.538| -    | -    | -             |
| 210Pb   | 5.482| 5.473| 5.404| 5.545| -    | -    | -             |
| 211Pb   | 5.488| 5.478| 5.410| 5.552| -    | -    | -             |
| 212Pb   | 5.494| 5.484| 5.414| 5.559| -    | -    | -             |
| 213Pb   | 5.500| 5.490| 5.420| 5.566| -    | -    | -             |
| 214Pb   | 5.507| 5.495| 5.425| 5.573| 5.592| 5.500| -             |

Table 6. Calculation results for neutron radius of Pb isotopes (all units are in fm)

| Nucleus | SLy4 | SkM* | Z_σ | SIII | SIII | SkM* | Exp. [18, 22] |
|---------|------|------|------|------|------|------|---------------|
| 204Pb   | 5.589| 5.587| 5.421| 5.615| 5.61 | 5.585| -             |
| 205Pb   | 5.599| 5.597| 5.427| 5.625| -    | -    | -             |
| 206Pb   | 5.608| 5.608| 5.434| 5.634| 5.628| 5.606| -             |
| 207Pb   | 5.618| 5.618| 5.439| 5.643| -    | -    | -             |
| 208Pb   | 5.627| 5.628| 5.446| 5.654| 5.65 | 5.627| 5.56         |
| 209Pb   | 5.642| 5.643| 5.457| 5.667| -    | -    | -             |
| 210Pb   | 5.656| 5.656| 5.466| 5.679| -    | -    | -             |
| 211Pb   | 5.669| 5.668| 5.474| 5.691| -    | -    | -             |
| 212Pb   | 5.682| 5.681| 5.482| 5.702| -    | -    | -             |
| 213Pb   | 5.695| 5.693| 5.490| 5.714| -    | -    | -             |
| 214Pb   | 5.707| 5.705| 5.498| 5.725| -    | -    | -             |
3.2. The Density and the Potential of Nucleons

In this paper, it is also investigated the effect of the increase of neutron number in Pb nucleus. First, it is investigated the density and the potential of nucleons calculated by SLy4, SkM*, Zσ, and SIII set parameters.

In Figs. 1 - 5, for proton density (part a), at r < 0.5 fm, the SLy4 obtained the largest results and the SIII obtained the smallest results. In radius 2 - 6 fm, the Zσ obtained the largest results and the SIII obtained the smallest results. From neutron density (part b), at r < 0.5 fm, the Zσ obtained the largest results and the SLy4 obtained the smallest results. In radius 2 - 6 fm, for both proton and neutron, the Zσ obtained the largest results and the SIII obtained the smallest results. The results of SLy4 and SkM* are similar in this area. Proton density has maximum value at r < 0.5 fm and neutron density has maximum value in radius 2 - 5 fm. In radius 6 - 8 fm, the densities of proton and neutron decreased drastically and it approached to zero smoothly at r > 8 fm.

![Fig. 1](image.png)

Table 7. Calculation results for neutron skin thickness of Pb isotopes (all units are in fm)

| Nucleus | This study | Tel et al. [18] |
|---------|------------|----------------|
|         | SLy4 | SkM* | Zσ | SIII | Zσ | SkM* |
| 204Pb   | 0.139 | 0.146 | 0.046 | 0.107 | 0.108 | 0.146 |
| 206Pb   | 0.145 | 0.151 | 0.047 | 0.111 | - | - |
| 208Pb   | 0.149 | 0.157 | 0.050 | 0.115 | 0.115 | 0.158 |
| 209Pb   | 0.155 | 0.163 | 0.051 | 0.119 | - | - |
| 209Pb   | 0.159 | 0.167 | 0.053 | 0.123 | 0.122 | 0.167 |
| 210Pb   | 0.166 | 0.176 | 0.057 | 0.129 | - | - |
| 212Pb   | 0.174 | 0.183 | 0.062 | 0.134 | - | - |
| 214Pb   | 0.181 | 0.190 | 0.064 | 0.139 | - | - |
| 216Pb   | 0.188 | 0.197 | 0.068 | 0.143 | - | - |
| 218Pb   | 0.195 | 0.203 | 0.070 | 0.148 | - | - |
| 220Pb   | 0.200 | 0.210 | 0.073 | 0.152 | - | - |

In this study, it has been taken the results of SkM* set parameters to investigate the effect of the increase of neutron number in Pb nucleus. In Fig. 6, a, in region 0 - 6 fm, the proton density of 176Pb is the largest and 220Pb is the smallest. The proton density widths of 178Pb, 208Pb, and 220Pb are almost similar. In Fig. 6, b, in region 2 - 4 fm, the neutron density of 176Pb is the smallest and 220Pb is the largest. The neutron density width of 178Pb is the smallest and 220Pb is the biggest. It is indicated that the increase of neutron number causes the decrease of proton density and has minimum effect to the proton density width. In contrast, the increase of neutron number causes substantial effect for neutron density, where the increase of neutron number causes the increase of neutron density and its width.

In Fig. 7, a, in region 0 - 4 fm, the proton potential depth of 178Pb is the shallowest and 220Pb is the deepest. The proton potential width of 178Pb is the smallest and 220Pb is the largest. In Fig. 7, b, in region 2 - 4 fm, the neutron potential of 178Pb is the deepest and 220Pb is the shallowest. The neutron potential widths of 178Pb, 208Pb, and 220Pb are almost similar to each other. It can be indicated that the
depth and width of proton potentials increase with
the increase of neutron number. In other hand, the
increase of neutron number causes the decrease of
neutron potential depth and has minimum effect to
the neutron potential width.

Fig. 2. Local densities of proton (a) and neutron (b) of $^{206}\text{Pb}$.

Fig. 3. Local densities of proton (a) and neutron (b) of $^{210}\text{Pb}$.

Fig. 4. Local densities of proton (a) and neutron (b) of $^{212}\text{Pb}$.
Proton density, fm$^3$

Neutron density, fm$^3$

Fig. 5. Local densities of proton (a) and neutron (b) of $^{214}$Pb.

Proton density, fm$^3$

Neutron density, fm$^3$

Fig. 6. Local densities of proton (a) and neutron (b) with SkM* set parameters.

Proton potential, MeV

Neutron potential, MeV

Fig. 7. Local potentials of proton (a) and neutron (b) with SkM* set parameters.
Conclusion

It has been performed an investigation of nuclear ground-state properties of spherical nuclei, such as the total energy, nucleon local density, and nucleon local potential of Pb isotopes (especially 204,206Pb) by using SHF method, with Skyrme set parameters, especially SLy4, SkM*, Zn, and SIII set parameters. All used parameters are in good agreement with the results of the related experiments and the other researchers. From this study, it is also obtained that in Pb nucleus, the total energy, mass radius, neutron radius, neutron skin thickness, neutron density, neutron density width, proton potential depth, and proton potential width increase accordingly with the increase of neutron number. In other hand, proton density and neutron potential decrease accordingly with the increase of neutron number. The increase of neutron number has minimum effect to the widths of proton density and neutron potential.

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ЕНЕРГІЯ ЗВ’ЯЗКУ ЯДЕР ТА РОЗПОДІЛ ГУСТИНИ
В ІЗОТОПАХ СВИНЦЮ В МЕТОДІ СКІРМА - ХАРТРІ - ФОКА

У цьому дослідженні вивчено основні властивості сферичних ядер, такі як сумарна енергія, локальна густина нуклонів та локальний нуклонний потенціал ізотопів свинцю (особливо 204-214Pb) за допомогою методу Хартрі - Фока. Розрахунки були виконано з використанням параметрів Скірма, зокрема SLy4, SkM*, Zө та SIII. Результати розрахунків порівнюються з експериментальними результатами та результатами розрахунків інших дослідників. Усі використані параметри добре узгоджуються з результатами відповідних експериментів та інших розрахунків. З дослідження випливає, що в ізотопах свинцю сумарна енергія, масовий радіус, нейтронний радіус, товщина нейтронної шкіри, ширина нейтронного розподілу, глибина та шириня протонного потенціалу зростають зі збільшенням числа нейтронів. З іншої сторони, густина протонів та нейтронний потенціал зменшуються зі збільшенням числа нейтронів. Збільшення кількості нейтронів мінімально впливає на ширину густини протонів та нейтронний потенціал.

Ключові слова: Хартрі - Фок, локальна густина, енергія зв’язку ядер, ізотопи свинцю, взаємодія Скірма.

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ЭНЕРГИЯ СВЯЗИ ЯДЕР И РАСПРЕДЕЛЕНИЕ ПЛОТНОСТИ
В ИЗОТОПАХ СВИНЦА В МЕТОДЕ СКИРМА - ХАРТРИ - ФОКА

В этом исследовании изучены основные свойства сферических ядер, такие как суммарная энергия, локальная плотность нуклонов и локальный нуклонный потенциал изотопов свинца (особенно 204-214Pb) с помощью метода Хартрі - Фок. Расчеты были выполнены с использованием параметров Скірма, в частности SLy4, SkM*, Zө и SIII. Результаты расчетов сравниваются с экспериментальными результатами и результатами других расчетов. Все используемые параметры хорошо согласуются с результатами соответствующих экспериментов и других расчетов. Из исследования вытекает, что в изотопах свинца суммарная энергия, массовый радиус, нейтронный радиус, толщина нейтронной кожи, нейтронная плотность, ширина нейтронного распределения, глубина и ширина протонного потенциала увеличиваются при увеличении числа нейтронов. С другой стороны, плотность протонов и нейтронный потенциал уменьшаются при увеличении числа нейтронов. Увеличение числа нейтронов минимально влияет на ширину плотности протонов и нейтронный потенциал.

Ключевые слова: Хартри - Фок, локальная плотность, энергия связи ядер, изотопы свинца, взаимодействие Скірма.