Computational multicore on two-layer 1D shallow water equations for erodible dambreak

To cite this article: C A Simanjuntak et al 2018 J. Phys.: Conf. Ser. 971 012034

View the article online for updates and enhancements.

Related content
- Multicore runup simulation by underwater avalanche using two-layer 1D shallow water equations
  B A R H Bagustara, C A Simanjuntak and P H Gunawan
- Model reduction of unstable systems using balanced truncation method and its application to shallow water equations
  Kiki Mustaqim, Didik Khusnul Arif, Erna Apriliani et al.
- An averaging method for the weakly unstable shallow water equations in a flat inclined channel
  Richard Spindler
Computational multicore on two-layer 1D shallow water equations for erodible dambreak

C A Simanjuntak\textsuperscript{1,*}, B A R H Bagustara\textsuperscript{2} and P H Gunawan\textsuperscript{3}

School of Computing, Telkom University, Jalan Telekomunikasi No. 1 Terusan Buah Batu, Bandung 40257, Indonesia.

E-mail: \textsuperscript{1}casrio@student.telkomuniversity.ac.id; \textsuperscript{2}bajoerajahusein@student.telkomuniversity.ac.id; \textsuperscript{3}phgunawan@telkomuniversity.ac.id

*Corresponding author

Abstract. The simulation of erodible dambreak using two-layer shallow water equations and SCHR scheme are elaborated in this paper. The results show that the two-layer SWE model in a good agreement with the data experiment which is performed by Louvain-la-Neuve Universit\`e Catholique de Louvain. Moreover, the parallel algorithm with multicore architecture are given in the results. The results show that Computer I with processor Intel(R) Core(TM) i5-2500 CPU Quad-Core has the best performance to accelerate the computational time. Moreover, Computer III with processor AMD A6-5200 APU Quad-Core is observed has higher speedup and efficiency. The speedup and efficiency of Computer III with number of grids 3200 are 3.716050530 times and 92.9% respectively.

1. Introduction

Dambreak problem is widely used as a benchmark in fluid flow problems. Recently, this problem can be categorized in two types which are dambreak in hard [1] and sediment bed [2]. In sediment bed, the sediment can be moved by the interaction between the water and sediment itself. Thus, the mathematical model of this interaction problem will be needed.

Several mathematical models for bed morphological change due to water flow are available in some references [2, 3, 4]. For instance, the coupled model of shallow water-Exner equations is given [2, 5]. In [2, 5], the one-layer shallow water equations is used for describing the water flow. Meanwhile for the sediment, the Exner equation is elaborated.

In this paper, the two-layer shallow water equations (SWE) will be used to describe the fluid-sediment interaction phenomena. This model is chosen since it consists of various density for each layers, such that can be used to describe the sediment.

The two-layer SWE model is given as follows:

\[
\begin{align*}
\frac{\partial h_1}{\partial t} + \frac{\partial (h_1 u_1)}{\partial x} &= 0, \\
\frac{\partial (h_1 u_1)}{\partial t} + \frac{\partial \left(h_1 u_1^2 + \frac{g}{2} h_1^2\right)}{\partial x} &= -gh_1 \frac{\partial (h_2 + Z)}{\partial x},
\end{align*}
\]
\[
\frac{\partial h_2}{\partial t} + \frac{\partial (h_2 u_2)}{\partial x} = 0, \tag{3}
\]

\[
\frac{\partial (h_2 u_2)}{\partial t} + \frac{\partial \left( h_2 u_2^2 + \frac{g}{2} h_2^2 \right)}{\partial x} = -gh_2 \left( \frac{\rho_1}{\rho_2} \frac{\partial h_1}{\partial x} + \frac{\partial Z}{\partial x} \right), \tag{4}
\]

where the water height is denoted by \(h(x,t)\), the lateral velocity by \(u(x,t)\), the gravitational acceleration by \(g\), and the bottom elevation by \(Z(x)\). The systems (1-2) and (3-4) describe the mass and momentum equations of first and second layers respectively. Moreover, the space and time are denoted by \(x\) and \(t\) respectively. The detail of primitive variables from (1-4) can be seen in Figure 1. In Figure 1, the water level of second layer with density \(\rho_2\) is denoted by \(\eta_2(x,t) = h_2(x,t) + Z(x)\). Meanwhile, \(\eta_1(x,t) = h_1(x,t) + h_2(x,t) + Z(x)\) denotes the water level of first layer with density \(\rho_1\).

**Figure 1.** The configuration of primitive variables from two-layer shallow water equations.

Here, the source-centered hydrostatic reconstruction (SCHR) scheme will be used to approximate the (1-4). This numerical method is satisfied the mathematical properties for SWE model \([6, 7, 8, 9]\). The properties are such as, preserving the non-negative of water height \((h > 0)\), satisfying well-balanced, discrete entropy balance, and the dry or vacuum condition \((h = 0)\).

The goal of this paper is to perform the two-layer SWE for erodible dambreak problem using SCHR scheme and to analyze the performance of its parallel computing. Here, the numerical simulation will be compared with the data experiment given in \([2, 10]\). Originally, the experiment were performed in Louvain-la-Neuve Université Catholique de Louvain, which is reported by Fraccarollo and Capart 2002 \([11]\). Moreover, in this paper, the parallel computing in multicore architecture with OpenMP platform will be elaborated \([12]\).

2. **Source-centered Hydrostatic Reconstruction Scheme**

In this paper, the source-centered hydrostatic reconstruction (SCHR) scheme will be used to discretize (1-4). This scheme is a modified original hydrostatic reconstruction (HR) scheme \([7, 8, 13]\) from one-layer SWE. The modification is made due to the momentum conservation problem occurs when the original HR is used to discretized two-layer SWE. In \([14]\), two numerical schemes which are called splitting and sum scheme use original HR for approximating two-layer SWE. However, both of them produce wrong solutions because of the total momentum did not conserve. Therefore in \([6]\), the SCHR is introduced to approximate the two-layer SWE which is a robust scheme.
Here, the brief explanations about SCHR will be given, the more detail about this scheme can be found in [6]. Given the discrete domain of 1D spatial space \( \Omega = [-L : L] \), \( L \in \mathbb{R} \) and time \( T = [0, \infty) \) as follows

\[
\Delta x = \frac{2L}{N_x}, \quad N_x \in \mathbb{Z}^+, \quad x_j = j \times \Delta x - L, \quad j \in \mathcal{M} = \{0, 1, \ldots, N_x\},
\]

and

\[
t^n = n \times \Delta t, \quad n \in \mathcal{T} = \{0, 1, \ldots\}.
\]

Here, the discrete space \( \Delta x \) is considered as the uniform step and the time step \( \Delta t \) is depend on the stability condition (16). Let the primitive variables in (1-4) in a vector form,

\[
\mathbf{U} = \left( \begin{array}{c} h_i \\ h_i u_i \end{array} \right), \quad i \in \{1, 2\}.
\]

(5)

Therefore, the finite volume method for approximating (1-4) is given as

\[
\mathbf{U}_{i,j}^{n+1} = \mathbf{U}_{i,j}^n - \frac{\Delta t}{\Delta x} \left( \mathcal{F}_l(\mathbf{U}_{i,j}, \mathbf{U}_{i,j+1}, b_{i,j}, b_{i,j+1}) - \mathcal{F}_r(\mathbf{U}_{i,j-1}, \mathbf{U}_{i,j}, b_{i,j-1}, b_{i,j}) \right),
\]

(6)

where

\[
b_i = \sum_{k<i}^{2} \rho_k h_k + \sum_{k>i}^{2} h_k + Z_i, \quad i \in \{1, 2\}.
\]

(7)

Here, the variable \( \mathbf{U}_{i,j}^n \) denotes the the value of variable \( \mathbf{U} \) at point \( x_j \) and current time \( t^n \). The important part of SCHR is to define the numerical fluxes such that satisfy the total momentum conservation. The numerical scheme SCHR defines the numerical fluxes \( \mathcal{F}_{l,r}(u_l, u_r, b_l, b_r) \) as follows

\[
\mathcal{F}_l = \mathcal{F}^{HR}_l + \mathcal{P}_l, \quad \mathcal{F}_r = \mathcal{F}^{HR}_r + \mathcal{P}_r,
\]

(8)

where \( \mathcal{F}^{HR}_{l/r} \) is the numerical fluxes of the original hydrostatic reconstruction scheme for one-layer SWE which can be found in [8] for more detail and the additional terms \( \mathcal{P}_{l/r} = (\mathcal{P}^0_{l/r}, \mathcal{P}^1_{l/r}) \) for the balancing in momentum conservation.

**Proposition 1.** In order to satisfy the semi-discrete entropy condition, thus \( \mathcal{P}^0 \) and \( \mathcal{P}^1_{l/r} \) are defined as follows

\[
\mathcal{P}^0 = \frac{\kappa ((1 + \theta)u_l + (1 - \theta)u_r)}{2},
\]

(9)

\[
\mathcal{P}^1_l = \frac{\kappa (1 + \theta)(h_r - h_i + \Delta b)g}{2} + u_l \max(\mathcal{P}^0, 0) + u_r \min(\mathcal{P}^0, 0),
\]

(10)

\[
\mathcal{P}^1_r = -\frac{\kappa (1 - \theta)(h_r - h_l + \Delta b)g}{2} + u_l \max(\mathcal{P}^0, 0) + u_r \min(\mathcal{P}^0, 0).
\]

(11)
with $\theta$ and $\kappa$ are the parameters for hydrostatic solver which are given as follows

$$
\theta = \min \left( 1, \frac{\max(u_l, 0)}{\sqrt{gh_l}} \right) - \max \left( 1, \frac{\max(-u_r, 0)}{\sqrt{gh_r}} \right),
$$

(12)

$$
\kappa = \min(\kappa, 2.5 \times \min(h_l, h_r)),
$$

(13)

$$
\bar{\kappa} = \frac{\Delta b}{2} \begin{cases} 
(\Delta b - h_l)h_r & \text{if } \Delta b > h_l, \\
0 & \text{if } -h_l \leq \Delta b \leq h_l, \\
-(h_r + \Delta b)h_l & \text{if } \Delta b < -h_r,
\end{cases}
$$

(14)

$$
\Delta b = \begin{cases} 
\min(\Delta b, h_l) & \text{if } \Delta b \geq 0, \\
\max(\Delta b, -h_r) & \text{otherwise.}
\end{cases}
$$

(15)

Proof. The proof of this proposition can be found in the paper of Bouchut [6] in detail.

The numerical scheme (6) with the numerical fluxes of SCHR (8 - 15) has the stability condition in order to keep the positivity of water height for each layer. The stability condition of the SCHR scheme is given as follows

$$
\Delta t \times \lambda(U^n_{i,j}, U^n_{i,j+1}, b^n_{i,j+1} - b^n_{i,j}) \leq 1,
$$

(16)

where the function $\lambda(U^n_{i,j}, U^n_{i,j+1}, b^n_{i,j+1} - b^n_{i,j})$ is the wave speed which can found in [6] in detail.

3. Parallel Algorithm for SCHR Numerical Scheme

OpenMP platform is one of shared memory architecture in parallel computing which are the threads allowed to access main memory for executing algorithm at the same time. In this paper shared parallel by using OpenMP platform will be used for computing the numerical scheme of two-layer because this platform is simple to implement and straightforward [12, 15, 16, 17, 18, 19]. Moreover, the performance of parallel implementation can be evaluated by measuring speedup and efficiency [20]. Figure 2 shows the parallel implementation for computing the numerical scheme.

The parallel algorithm of erodible dambreak simulation is described as follow :

- **Initializing all parameter**: here the parallel computing will be started. Initialize the values of water elevation, density, and velocity for each layer and spatial grid.
- **Computing the mass conservation**: the unknown values $h_{1,j}^{n+1}$ and $h_{2,j}^{n+1}$ will be computed using (6).
- **Computing the momentum conservation**: the momentum equation (6) will be used for computing the unknown variables of velocity $u$.
- **Update the previous values**: all unknowns variables should be updated for the next time iteration.
- **Output of Simulation**: the data of unknown variables and CPU time are collected.

In this algorithm, each processors will execute the same process on parallel part. The number of tasks in loop process is depend on the number of processors. For instance, using number of grids $N_x$ and processor $p$, the maximum number of tasks for a processor is defined as $\lceil N_x/p \rceil$. 

4. Numerical Simulations
In this section, the numerical simulation of erodible dambreak using SCHR will be given. The comparison numerical result of two-layer SWE using SCHR with data experiment obtained in [2, 10, 11] also will be elaborated in Subsection 4.1. There are two experiments of erodible dambreak flow can be found in the paper of Capart and Young 1998 [21] and [11] which are performed in Taipei University of Taiwan and Louvain-la-Neuve Université Catholique de Louvain respectively. In this paper, the numerical results will be compared only with the data experiment in Louvain. Moreover, the parallel performance with OpenMP platform to accelerate the computational time will be given in Subsection 4.2.

4.1. Simulation of erodible dambreak
In this simulation, the comparison of results two-layer SWE using SCHR scheme and data experiment in Louvain will be elaborated. The initial condition of erodible dambreak is given...
Figure 3. The initial condition of erodible dambreak simulation with spatial domain of this simulation is $\Omega = [-1 : 1]$. Here, first and second layer describe the water surface and erodible bottom or sediment respectively. The density of each layers will be defined in a ratio in the simulation.

in Figure 3 and described as the following equations,

$$
\eta_1(x, 0) = \begin{cases} 
0.1 & \text{if } x < 0, \\
0 & \text{otherwise}, 
\end{cases}
$$

(17)

$$
\eta_2(x, 0) = 0,
$$

(18)

$$
u_1(x, 0) = u_2(x, 0) = 0.
$$

(19)

First layer acts as the water surface and second layer is used to describe the sediment or erodible bottom. At the initial condition, velocity of water and sediment is set to be zero and the sediment is given in a flat profile.

The results using the ratio of $\rho_1/\rho_2 = 10^{-3}, 6 \times 10^{-2}, 10^{-1}, 4 \times 10^{-1}, 6 \times 10^{-1}$ and $8 \times 10^{-1}$ are given in Figure 4. It can be seen clearly that, the numerical results are depend on the ratio of density of each layer. If the ratio of density $\rho_1/\rho_2$ small enough, thus the influence of first layer to the second layer is small and less erodible of sediment can be obtained. Meanwhile, when the ratio is high, the relation between water and sediment is more strong. Therefore the water flow causes the erodible on bottom of simulation.

From the results (Figure 4), the lines in the upper and bottom are shown the numerical solutions by SCHR of first and second layer respectively. Meanwhile, the dots ‘+’ and ‘∗’ are described the data experiment of water surface and sediment respectively. Figures 4(a), (b) and (c) show the numerical solutions of the water surface and sediment are slightly far from the experiment data. These results are using the ratio of density $\leq 0.1$. In other side using the ratio of density $> 0.1$, Figures 4(d), (e) and (f) show the numerical solutions is in a good agreement with the data experiment for both water surface and sediment profile.

The experiment of erodible dambreak in Louvain uses the sediment porosity $\phi = 0.3$, diameter of sediment particle 3.2 mm, and sediment density $1,540$ kg/m$^3$. If the water density is given $1,000$ kg/m$^3$, thus the ratio of water and sediment is obtain around 0.64. Therefore, without consider the porosity and diameter of sediment particle, the numerical result with density ratio $\rho_1/\rho_2 = 0.6$ can be seen close enough to the data experiment (see Figure 4(e)).

4.2 Parallel performances using OpenMP platform

In this paper, three different computers are used for obtaining the performance of parallel algorithm. The specifications of each computer are shown on Table 1. In order to obtain the
Figure 4. The comparison of two-layer SWE and SWE-Exner model at final time of simulation $t = 0.36$ seconds. (a) The comparison using ratio $\rho_1/\rho_2 = 10^{-3}$. (b) The comparison using ratio $\rho_1/\rho_2 = 6 \times 10^{-2}$. (c) The comparison using ratio $\rho_1/\rho_2 = 10^{-1}$. (d) The comparison using ratio $\rho_1/\rho_2 = 4 \times 10^{-1}$. (e) The comparison using ratio $\rho_1/\rho_2 = 6 \times 10^{-1}$. (f) The comparison using ratio $\rho_1/\rho_2 = 8 \times 10^{-1}$.

performance of each computers, here, the parallel computing use 4 (four) cores/threads for each computers.
Table 1. The specifications of three computers for measuring the performance of parallel computing in the simulation.

| Computer | Operating System | Processor | Memory |
|----------|------------------|-----------|--------|
| I        | Ubuntu 12.04.3 LTS | Intel(R) Core(TM) i5-2500 CPU Quad-Core | 8 GB   |
| II       | Ubuntu 14.04.3 LTS | AMD Opteron 63xx class CPU Quad-Core | 8 GB   |
| III      | Ubuntu 14.04.3 LTS | AMD A6-5200 APU Quad-Core | 6 GB   |

The serial and parallel CPU time using Computer I is given in Table 2. Using processor Intel(R) Core(TM) i5-2500 CPU Quad-Core, the CPU time of serial and parallel is obtained 2.406260 s and 0.763619 s respectively using grid sizes 3200. The speedup is shown increasing along the increasing of grid sizes and reached 3.15 times when the number of grid sizes is 3200.

Table 2. The CPU time and speedup performance of parallel implementation of Computer I.

| $N_x$ | Serial time | Parallel time | Speedup |
|-------|-------------|---------------|---------|
| 200   | 0.168599    | 0.056266      | 2.996463228 |
| 400   | 0.328057    | 0.108305      | 3.029010664 |
| 800   | 0.622869    | 0.202799      | 3.071361299 |
| 1600  | 1.178440    | 0.379376      | 3.071361299 |
| 3200  | 2.406260    | 0.763619      | 3.151126413 |

The parallel performance using Computer II with processor AMD Opteron 63xx class CPU Quad-Core can be seen in Table 3. By various numbers of grid sizes, the CPU time of serial and parallel is shown increasing along the $N_x$ is doubled. When the $N_x = 200$, the CPU time in serial code is 0.400121 s, which is approximately 2.9 times the parallel time. In other side, by $N_x = 3200$ the CPU time in parallel code is obtained 2.008820 s which is almost 1/3.15 times of serial time.

Table 3. The CPU time and speedup performance of parallel implementation of Computer II.

| $N_x$ | Serial time | Parallel time | Speedup |
|-------|-------------|---------------|---------|
| 200   | 0.400121    | 0.136395      | 2.933545951 |
| 400   | 0.777113    | 0.258098      | 3.010922208 |
| 800   | 1.525450    | 0.500182      | 3.049789876 |
| 1600  | 3.063830    | 0.988666      | 3.098953539 |
| 3200  | 6.298810    | 2.008820      | 3.135577105 |

Computer III with processor AMD A6-5200 APU Quad-Core gives the parallel performance as shown in Table 4. Using this computer, the CPU time of serial and parallel using $N_x = 3200$ is 8.798530 s and 2.367710 s respectively. Thus the speedup of parallel computing is reached 3.71 times than the serial time.

Table 4. The CPU time and speedup performance of parallel implementation of Computer III.

| $N_x$ | Serial time | Parallel time | Speedup |
|-------|-------------|---------------|---------|
| 200   | 8.798530    | 2.367710      | 3.71    |
Table 4. The CPU time and speedup performance of parallel implementation of computer III.

| \(N_x\) | Serial time | Parallel time | Speedup  |
|---------|-------------|--------------|----------|
| 200     | 0.589111   | 0.173978     | 3.386123533 |
| 400     | 1.161250   | 0.330499     | 3.513626365 |
| 800     | 2.334270   | 0.648217     | 3.601062607 |
| 1600    | 4.605950   | 1.264240     | 3.643256027 |
| 3200    | 8.798530   | 2.367710     | 3.716050530 |

From Tables 2, 3 and 4, it can be seen that Computer I with processor Intel(R) Core(TM) i5-2500 CPU Quad-Core has the best performance of CPU time for both serial and parallel computing. However, according to the speedup measurement, Computer III with processor AMD A6-5200 APU Quad-Core has the best performance for speedup. The speedup is obtained more than 3.3 times along the increasing of the grid size \(N_x\). Comparing with the Computer I and II, their speedup (around 3.15 and 3.13 times respectively) is observed less than the speedup of Computer III which is achieved 3.71 times using \(N_x = 3200\).

Additionally, Computer III also has a good performance of efficiency using 4 cores/threads which is shown in Figure 5. The gap between the efficiency of Computer III and the rest of computers is shown slightly far. For instance, using \(N_x = 200\) the efficiency for Computer I, II and III are observed 74.9%, 73.3% and 84.6% respectively. Moreover, by \(N_x = 3200\), the efficiency using Computer I, II and III are obtained 78.7%, 78.3% and 92.9% respectively.

Remark 1. Computers I and II are the server computers, whereas computer III is the personal computer (PC). The performance of CPU time for server computers is observed better than PC. In terms of computational time, the server computer will obviously be superior to the PC due to its high specifications such as memory and processor type. However, the speedup and efficiency by using computer server are obtained lower than PC when using number of grids \(N_x = 3200\). This can happen because the high specification computer is not efficient enough to compute small number of grids. Nevertheless, it would be different if the large number of grids is used.

5. Conclusion
The comparison of numerical solution of two-layer SWE with the data experiment in erodible dambreak simulation is elaborated. Without considering the characteristics of sediment, the
numerical solutions for ratio of density greater than 0.1 are shown in a good agreement with the data experiment in Louvain case. In parallel performance during erodible dambreak simulation, Computer I with processor Intel(R) Core(TM) i5-2500 CPU Quad-Core is shown the best computer in order to accelerate the computational time. However, the best measurement in speedup and efficiency is observed for Computer III with processor AMD A6-5200 APU Quad-Core. Using the maximum grid size $N_x = 3200$, the speedup is obtained 3.716050530 times and the efficiency is calculated 92.9%.

References

[1] Song L, Zhou J, Li Q, Yang X and Zhang Y 2011 International Journal for Numerical Methods in Fluids 67 960–980
[2] Gunawan P H and Lh´ebrard X 2015 Computers & Fluids 121 44–50
[3] Simpson G and Castelltort S 2006 Computers & Geosciences 32 1600–1614
[4] Cao Z, Pender G, Wallis S and Carling P 2004 Journal of hydraulic engineering 130 689–703
[5] Gunawan P, Eymard R and Pudjaprasetya S 2015 Computational Geosciences 19 1197–1206
[6] Bouchut F and Zeitlin V 2010 Discrete and Continuous Dynamical Systems-Series B 13 739–758
[7] Bouchut F 2004 Nonlinear stability of finite Volume Methods for hyperbolic conservation laws: And Well-Balanced schemes for sources (Springer Science & Business Media)
[8] Audusse E, Bouchut F, Bristeau M O, Klein R and Perthame B t 2004 SIAM Journal on Scientific Computing 25 2050–2065
[9] Doyen D and Gunawan P H 2014 Finite Volumes for Complex Applications VII-Methods and Theoretical Aspects (Springer) pp 227–235
[10] Wu W and Wang S S 2007 Journal of hydraulic engineering 133 48–58
[11] Fraccarollo L and Capart H 2002 Journal of Fluid Mechanics 461 183–228
[12] Gunawan P H 2016 Information and Communication Technology (ICoICT), 2016 4th International Conference on (IEEE) pp 1–5
[13] Bouchut F 2007 Edited Series on Advances in Nonlinear Science and Complexity 2 189–256
[14] Bouchut F and de Luna T M 2008 ESAIM: Mathematical Modelling and Numerical Analysis 42 683–698
[15] Iryanto and Gunawan P H 2017 Information and Communication Technology (ICoICT), 2017 5th International Conference on (IEEE) pp 1–5
[16] Alamsyah M N A, Simanjuntak C A, Bagustara B A R H, Pradana W A and Gunawan P H 2017 Information and Communication Technology (ICoICT), 2017 5th International Conference on (IEEE) pp 1–6
[17] Pahlevi M R and Gunawan P H 2017 Information and Communication Technology (ICoICT), 2017 5th International Conference on (IEEE) pp 1–4
[18] Juliati S and Gunawan P H 2017 Information and Communication Technology (ICoICT), 2017 5th International Conference on (IEEE) pp 1–5
[19] Mulyani, Putri N D and Gunawan P H 2017 Information and Communication Technology (ICoICT), 2017 5th International Conference on (IEEE) pp 1–5
[20] Rauber T and Rünger G 2013 Parallel programming: For multicore and cluster systems (Springer Science & Business Media)
[21] Capart H and Young D 1998 Journal of Fluid Mechanics 372 165–187