Optical transparency modes in anisotropic media

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Abstract

The modes of nonlinear propagation of the two-component electromagnetic pulses through optically uniaxial media containing resonant particles are studied. The features of their manifestation in the "dense" media and in the media with expressed positive and negative birefringences are discussed. It is shown that exponentially and rationally decreasing solutions of the system of material and wave equations allow us also to describe the propagation of the self-induced transparency pulses in isotropic media in the case, when the direct electric dipole-dipole interaction between the resonant particles is taken into account.

Keywords: self-induced transparency, femtosecond optical pulses, nonlinear dynamics, optical anisotropy, resonance

1 INTRODUCTION

The development of the technologies of producing the low-dimensional quantum structures (wells, wires), growing the semiconductor crystals strengthens an interest to theoretical investigation of the nonlinear coherent phenomena in the anisotropic media. The quantum states of the particles being contained in such the media possess no certain parity, owing to what the diagonal elements of the matrix of the dipole moment operator and their differences, which are called the permanent dipole moments of the transitions, are distinct from zero.

The propagation in optically uniaxial media of the two-component laser pulses consisting of ordinary and extraordinary components was studied in. It was suggested that the linear velocities of both the components are equal, and the concentration of the resonant particles is small sufficiently for the approximation of the unidirectional propagation to be valid. Various modes of the optical transparency were classified with regard to the pulse propagation velocity and the degree of the resonant particles excitation. The mode of self-induced transparency (SIT) is characterized by strong excitation of the medium and substantial deceleration in the velocity of the pulse propagation relative to linear one. The self-induced supertransparency (SIST) mode differs from SIT in that the decrease of velocity of the pulses is small, but the medium is strongly excited as well. In this mode, the carrier frequency of ordinary component of the pulses, which cause full inversion of the quantum level population, is lower than the resonance frequency. The width of such the pulses is determined by the amount of the ordinary component detuning. (The full inversion of initial state of the resonant particles in the isotropic case happens under condition of the exact resonance only.) The SIST pulses have larger amplitudes and smaller duration as compared to the SIT ones, and local frequency of their ordinary component is strongly modulated. There exist also the modes in that the trapping of the quantum level population takes place. The pulses propagating in the extraordinary transparency (EOT) mode are characterized by small detuning of the ordinary component from the resonance and by dominant role of the extraordinary component. Their group...
velocity substantially changes and may become comparable to one of the pulses causing strong excitation. In the modes of positive nonresonant transparency (PNT) and negative nonresonant transparency (NNT), the pulse velocity changes insignificantly, and the absolute value of detuning is large. The extraordinary component of the pulse dominates in the former mode, whereas the ordinary-to-extraordinary amplitude ratio is arbitrary in the latter. The most substantial difference between these modes concerns the behavior of effective detuning. In the NNT mode, it remains virtually constant. If a pulse propagates in the PNT mode, then the effective detuning of the frequency of the ordinary component changes sign due to an influence of the extraordinary one. Since the local frequency passes through the resonance, a slightly stronger excitation of the medium occurs here, and the pulses are sharply peaked, as in the SIST mode.

In present report, we consider the propagation of the two-component electromagnetic pulses through the optically uniaxial medium containing two-level particles. Unlike the case studied in the approximation of unidirectional propagation is not applied. Also, linear velocities of ordinary and extraordinary components are not assumed to be equal. In Section 2, we write down the system of material and wave equations for resonant propagation of the electromagnetic pulses in a direction perpendicular to the optical axis of the anisotropic medium. Exponentially and rationally decreasing solutions of this system are given here as well. Using these solutions, we establish in Sections 3 and 4 the distinctive features of the manifestation of the transparency modes under discussion in ”dense” media and in the media with expressed positive and negative birefringences. The pulse solutions studied in the previous sections are modified in Section 5 on the case of isotropic, optically dense media, in which the direct electric dipole-dipole interaction of the resonant particles should be taken into account.

2 BASIC EQUATIONS AND THEIR SOLUTIONS

Consider a birefringent medium containing the resonant, two-level particles. Suppose that the plane, two-component pulse consisting of high-frequency ordinary component \( E_o \) and video-pulse extraordinary one \( E_e \), propagates through the medium in a direction perpendicular to its optical axes. The system of material and wave equations describing the dynamics of the quantum systems and the evolution of the components of the electric field in the slowly varying envelope approximation has the next form:

\[
\frac{\partial W}{\partial t} = \frac{i}{2} (\Omega_o R - \Omega_e R^*), \tag{1}
\]

\[
\frac{\partial R}{\partial t} = i (\Delta + \Omega_e) R + i\Omega_o W, \tag{2}
\]

\[
\frac{\partial \Omega_o}{\partial x} + \frac{n_o}{c} \frac{\partial \Omega_o}{\partial t} = -i\beta_o R, \tag{3}
\]

\[
\frac{\partial^2 \Omega_e}{\partial x^2} - \frac{n^2_e}{c^2} \frac{\partial^2 \Omega_e}{\partial t^2} = -2\beta_e \frac{n_e}{c} \frac{\partial^2 W}{\partial t^2}. \tag{4}
\]

Here the inversion population \( W (-1/2 \leq W \leq 1/2) \) and coherency \( R \) are expressed through the elements of the density matrix of the resonant particles; \( \Delta = \omega_0 - \omega \) is detuning of the carrier frequency \( \omega \) of the ordinary component from resonant frequency \( \omega_0 \) of the quantum transition \( (|\Delta| \ll \omega_0) \); \( \Omega_o = 2dE_o/h \); \( E_o \) is an envelope of the pulse ordinary component \( (E_o = E_o \exp(i\omega(t - x/c) + c.c.)) \); \( \Omega_e = DE_e/h \); and \( D \) are the dipole moment and the permanent dipole moment of the transition, respectively; \( h \) is the Plank’s constant; \( \beta_o = 4\pi N \bar{d}^2 \omega/(\hbar c n_o) \); \( \beta_e = 2\pi N \bar{d}^2 \omega/(\hbar c n_e) \); \( N \) is the concentration of the particles; \( c \) is the velocity of the light in vacuum; \( n_o \) and \( n_e \) are ordinary and extraordinary refractive indices; \( x \) and \( t \) are the physical distance and time.

The nonlinear equations arising from system (1)-(4) under condition \( n_e = n_o \) in the approximation of unidirectional propagation and their pulse solutions were investigated in work. The classification of the modes of nonlinear propagation of the two-component electromagnetic pulses, which are distinguished by the behavior of the quantum particles and by the characteristics of the pulses, was given. It turns out that the pulse solutions of the system of material and wave equations studied there can be modified properly for the case considered. Indeed, the following expressions are obtained

\[
\Omega_o = \sqrt{M} \exp(i\Phi), \tag{5}
\]
\[ \Omega_e = \frac{-\dot{D}}{4d^2\omega} M, \] (6)

\[ W = \left(1 - \frac{\tau_p^2}{2(1 + \alpha^2)}M\right)W_\infty, \] (7)

where

\[ M = \frac{8g}{\tau_p^2 (g - \alpha + \text{sgn}(g)\sqrt{1 + (g - \alpha)^2}\cosh(2\zeta))}; \] (8)

\[ \Phi = W_\infty \frac{\beta_o \alpha \tau_p}{1 + \alpha^2} x - \arctan\frac{\tanh \zeta}{s} + \text{const}; \] (9)

\[ \tilde{D} = \frac{A_e}{\left(1 + \frac{n_o - n_e}{n_o + n_e} \frac{1 + \alpha^2}{A\tau_p^2}\right)\left(1 + \frac{A\tau_p^2}{1 + \alpha^2}\right)}; \] (10)

\[ \alpha = \Delta \tau_p; \quad g = \frac{2\alpha_0 \tau_p \omega^2}{\tilde{D}}, \quad \zeta = \frac{t}{\tau_p} - \frac{x}{v_g \tau_p}; \]

\[ A_e = \frac{2n_o D^2}{n_o + n_e}, \quad A = -\frac{c\beta_o W_\infty}{n_o + n_e}; \]

\[ s = g - \alpha + \text{sgn}(g)\sqrt{1 + (g - \alpha)^2}; \quad v_g = \frac{c}{n_o + n_e + n_e + \alpha \frac{A\tau_p^2}{1 + \alpha^2}}; \]

\(\tau_p\) is a positive parameter; \(W_\infty\) is an initial population of the quantum levels. We will assume in the subsequent sections that the initial state of the quantum particles is stable: \(-1/2 \leq W_\infty < 0\).

It is seen that the ordinary component phase modulation \(\frac{\partial \Phi}{\partial t}\) is equal to \(\Omega_e\). Defining the pulse duration \(T_p\) as the double deviation from the zero point of \(t - x/v_g\), at which \(|\Omega_e|\) is half its maximum value, we obtain from formula (5):

\[ T_p = \tau_p \text{arccosh} \left(4 + 3 \text{sgn}(g)\frac{g - \alpha}{\sqrt{1 + (g - \alpha)^2}}\right). \]

Since parameter \(\tilde{D}\) depends not only on the characteristics of the medium, as it was\(^8\), but also from parameters \(\Delta\) and \(\tau_p\) of the pulse, the anisotropy caused by the permanent dipole moment becomes effective. Thereof, the domains of the pulse parameters values, in which exist the optical transparency modes described in\(^8\), will change.

It is remarkable that the condition of strong excitation of the medium is written in the same manner:

\[ g = \frac{1}{2} \left(\alpha + \frac{1}{\alpha}\right). \] (11)

The pulses, whose parameters satisfy this condition, will cause the full inversion of initial state of the quantum particles. The parameter \(|\tilde{D}|\), which does not vanish only for a medium having a permanent dipole moment, will be called the effective anisotropy in the sequel.

Taking the limit \(\tau_p \to \infty\) in formulas (6–10), we obtain rationally decreasing solution of system (1–4):

\[ \Omega_o = \frac{8i\omega_0^2 \kappa}{\tilde{D}(1 + i\kappa^2b)} \exp(iax), \] (12)

\[ W = \left(1 - \frac{8\kappa^2}{(1 + \kappa^2)^2(1 + \kappa^2b^2)}\right)W_\infty, \] (13)
where

\[ a = \frac{\beta_o W_\infty}{\Delta}; \quad b = 4\omega_0 \frac{d^2}{D} \left( t - \frac{x}{v_g} \right); \]
\[ \kappa = \sqrt{\frac{\Delta \tilde{D}}{2\omega_0 d^2} - 1}; \quad v_g = \left( \frac{n_o}{c} - \frac{\beta_o W_\infty}{\Delta^2} \right)^{-1}; \]
\[ \tilde{D} = \frac{A_e}{1 + \frac{n_o - n_e \Delta^2}{n_o + n_e A}} \left( 1 + \frac{A}{\Delta^2} \right). \]

This solution, as the algebraic solution of the derivative nonlinear Schrödinger equation (DNSE), has one parameter (without taking account of the shifts on variables \( x \) and \( t \)). This is not surprising, because the system of the equations considered in8 and DNSE belong to the same hierarchy of the nonlinear equations integrable in the frameworks of the inverse scattering transformation method. Also, rationally decreasing solution having no arbitrary parameters was found for the case of the one-component pulses propagating through the anisotropic media6.

Parameter \( \kappa \) in formulas (12) and (13) is supposed to be real. This imposes a constraint on detuning \( \Delta \) of the rationally decreasing pulses. It is seen from equation (13) that the full inversion of the medium takes place if detuning of the pulse satisfy condition \( \kappa = 1 \).

3 THE CASE OF "DENSE" MEDIA

In this section, we discuss the case of "dense" media, when the concentration of the resonant particles is large enough so that condition \( A\tau_p^2 \ll 1 \) is not satisfied. We use quotes to stress its difference from the case of optically dense media, in which the effects of the local field are considered. Since the refractive indices of typical anisotropic medium are such that inequality \( |n_e - n_o| \ll 1 \) holds, we can assume that the first multiplicand in the denominator in the right-hand side of (10) is close to unity. Then, formula (10) is rewritten as

\[ \tilde{D} = \frac{A_e}{1 + \frac{\tau_p^2}{1 + \alpha^2}}. \] (14)

It can be shown that the transparency modes described in8 exist for the "dense" media as well. If \( n_e < n_o \), then \( \tilde{D} \) can exceed \( D^2 \) in the general case. One can see that this takes place if

\[ \frac{1}{\tau_p^2} + \Delta^2 > \frac{n_o + n_e}{n_o - n_e} A. \]

However, expression (10) cannot be replaced by (14) under this condition. Therefore, the effective anisotropy cannot exceed \( D^2 \) in the "dense" medium case, and the modes of strong excitation (with \( g > 1 \)) must be more expressed.

Using (11) and (14), we find that parameters \( \tau_p \) and \( \Delta \) of the pulse causing the full inversion of the medium are related as given:

\[ \frac{1}{\tau_p} = \sqrt{F_{1,2}}, \] (15)

where

\[ F_{1,2} = -\Delta^2 + \left( \Delta \pm \sqrt{\Delta \left( \Delta + AA_e/(\omega_0 d^2) \right)} \right) \frac{2\omega_0 d^2}{A_e}. \] (16)

It is obvious that

\[ F_1 F_2 = \left( A_e \Delta^3/(4d^2) - \omega_0 \Delta^2 - A\omega_0 \right) \frac{4d^2 \Delta}{A_e}. \] (17)
It follows from (15) that detuning of the pulses, whose propagation is accompanied by the largest change of the level population, can only be such that at least one of the quantities $F_1$ and $F_2$ is positive. It is easy to see that this is possible only if $\Delta > 0$. Since the coefficients at $\Delta^3$ and $\Delta$ in the right-hand side of (17) are positive and the coefficient at $\Delta^2$ is zero, the product $F_1 F_2$ changes sign only once at $\Delta > 0$. Therefore, $F_2$ is negative in this domain, and $F_1$ is positive if $0 < \Delta < \Delta_d$, where $\Delta_d$ is the positive root of equation $F_1 F_2 = 0$. Thus, for a given detuning satisfying condition $0 < \Delta < \Delta_d$, only the pulses with a certain unique duration can strongly excite the medium. A similar result was obtained in the case, when the unidirectional propagation approximation was used, and the refractive indices were assumed to be equal\(^8\). The carrier frequency of the ordinary component of a pulse that strongly excites the medium was also lower than the resonance frequency. Note that parameter $\tau_p$ cannot be smaller than a certain minimum for such the pulses. Furthermore, in accordance with the classification\(^8\), both modes of strong excitation can be identified in this case: the SIT ($\alpha \to 0$) and SIST ($\alpha \to \infty$) modes correspond to $\Delta \to 0$ and $\Delta \to \Delta_d$ respectively (see Fig. 1).

![Curve of the strong excitation for “dense” medium](image)

**Fig. 1.** Curve of the strong excitation for “dense” medium. Domains of the pulse parameters are given in the case $\Delta_d \ll \omega_0$: (I) SIT; (II) SIST; (III) EOT; (IV) NNT; (V) PNT. Thin curve: $A = 0$.  

Let us note that the formulas, which appear in the frameworks of the unidirectional propagation approximation under condition $n_e = n_o$, are obtained from equations (15) and (16) if $A = 0$. The curve of the strong excitation for this case is depicted on Fig. 1 by thin line. The corresponding value of the maximum detuning is $\Delta_m = 4 \omega_0 d^2 / D^2$. One can show that the interval of admissible detuning is wider for the ”dense” medium ($\Delta_d > \Delta_m$). This can be explained as follows. The effective anisotropy is more expressed in a low-density resonant medium, and a pulse can be resonant with the medium on average of its duration and thus cause strong excitation of the medium within a narrower interval of detuning. Note also that the medium effectively becomes more isotropic with increasing $A$, and also the slope of the curve described by (14) increases with decreasing $\Delta$. This is a consequence of the fact that the largest change of the quantum-level population in an isotropic medium can occur under the exact resonance condition only.

It follows from the consideration carried out above that the decrease in the effective anisotropy in the ”dense” medium does reinforce characteristic properties of pulses in the SIT and SIST modes. Conversely, the NNT, PNT, and EOT modes (with $g \ll 1$) become less expressed. The difference between the refractive indices of anisotropic ”dense” medium is not essential. The domains of the pulse parameters values in different modes of optical transparency are marked on Fig. 1.

The expression for parameter $\tilde{D}$ of the rationally decreasing pulse (12), (13) is obtained from formula (14) in limit $\tau_p \to \infty$. The interval of possible values of detuning of this pulse is determined by the condition of reality of parameter $\kappa$. It is easy to show that detuning of rationally decreasing pulse should be positive and can be smaller than $\Delta_d$. 

5
Suppose that the concentration of the resonant particles is so small that inequality $A\tau_p^2 \ll 1$ is valid. Then we have from equation (10):

$$\tilde{D} = \frac{A_e}{1 + \frac{n_o - n_e}{n_o + n_e} A\tau_p^2 1 + \alpha^2}.$$  \hspace{1cm} (18)

In contrast to the case of "dense" medium considered in the previous section, the difference of refractive indices plays significant role here.

Let us assume at first that the medium is positively birefringent ($n_e > n_o$). In this case, $\tilde{D}$ is negative if the pulse parameters $\tau_p$ and $\Delta$ satisfy condition

$$\frac{1}{\tau_p^2} + \Delta^2 > \frac{n_e + n_o}{n_e - n_o} A.$$  

The polarity of the extraordinary component changes the sign here (see formula (6)). This leads to important consequences concerning the modes of the strong excitation of the medium.

As $\tau_p \to \tilde{\tau}_p$ and $\Delta \to \tilde{\Delta}$, where $\tilde{\tau}_p$ and $\tilde{\Delta}$ are connected by relation

$$\frac{1}{\tilde{\tau}_p^2} + \tilde{\Delta}^2 = \frac{n_e + n_o}{n_e - n_o} A,$$  

the effective anisotropy increases indefinitely ($|\tilde{D}| \to \infty$), and the group velocity of the pulse tends to the extraordinary-wave velocity ($v_g = c/n_e$). A pulse with $\tau_p$ and $\Delta$ close to $\tilde{\tau}_p$ and $\tilde{\Delta}$ propagates in the EOT mode (see Fig. 2), as its extraordinary component is stronger than the ordinary one. Since the effective anisotropy is larger in the parameter domain in question, the EOT regime is more expressed, especially when $\Delta$ is small.

If

$$\frac{1}{\tau_p^2} + \Delta^2 > \frac{n_e + 3n_o}{n_e - n_o} A,$$  \hspace{1cm} (19)

then the effective anisotropy cannot exceed $D^2$. Therefore, the NNT and PNT modes, for which $g \ll 1$ and $|\alpha| \gg 1$, are less expressed in this parameter domain. Since $\tilde{D} < 0$ here, the detuning corresponding to these modes changes the sign; i.e., the former regime exists at $\alpha < 0$; the latter, at $\alpha > 0$. 

Fig. 2. Curve of the strong excitation for positively birefringent medium. Parameter domains are shown for $|\Delta_2| \ll \omega_0$ and labelled as on Fig. 1. Thin curve: $v_g = c/n_e$. 
Consider the case of strong excitation of the medium with positive birefringence. It follows from (11) and (18) that the largest change of the population of the quantum levels occurs if the pulse parameters \( \tau_p \) and \( \Delta \) are such that

\[
\frac{1}{\tau_p} = \sqrt{F}, \tag{20}
\]

where

\[
F = \frac{\omega_0 \Delta}{A_1 + A_2 \Delta} - \Delta^2; \tag{21}
\]

\[
A_1 = \frac{A_e}{4d^2}, \quad A_2 = \frac{n_e - n_o}{n_e + n_o} \frac{\omega_0}{A}. \tag{22}
\]

Condition \( F > 0 \) determines the range of admissible detuning. Using (20), we rewrite (18) as

\[
\tilde{D} = 4d^2 (A_1 + A_2 \Delta). \tag{23}
\]

We denote by

\[
\Delta_0 = -\frac{A_1}{A_2},
\]

and

\[
\Delta_{1,2} = \left( -A_1 \pm \sqrt{A_1^2 + 4A_2\omega_0} \right) \frac{1}{2A_2}, \tag{23}
\]

the root of equation \( \tilde{D} = 0 \) and the nonzero roots of equation \( F = 0 \). Since \( n_e > n_o \) in the case considered, it holds that \( \Delta_2 < \Delta_0 < \Delta_1 \) (see Fig. 2).

One can see that the largest change in the quantum level population can be achieved for both positive and negative values of detuning \( 0 < \Delta < \Delta_1 \) and \( \Delta_2 < \Delta < \Delta_0 \). The existence of the latter interval of admissible detuning is due to the possibility of change in the sign of \( \tilde{D} \) for positively birefringent media. Indeed, if \( \Delta_2 < \Delta < \Delta_0 \), then \( \tilde{D} < 0 \) and therefore \( \Omega_e > 0 \). This means that a negatively detuned pulse can strongly excite the medium, being resonant with it on average of the pulse duration because of the dynamic shift induced by its extraordinary component.

The transparency modes accompanied by the strong excitation of the medium in the case under consideration are similar to those described in Ref. 8, but they can exist in two parameter domains. If \( \Delta \to 0 \) or \( \Delta \to \Delta_0 \), then \( \alpha \to 0 \), and the pulse propagates in the SIT mode. In the latter limit case, both components have larger amplitudes and, therefore, higher velocities. If \( \Delta \to \Delta_1 \) or \( \Delta \to \Delta_2 \), then \( |\alpha| \to \infty \), and the pulse propagates in the SIST mode, in which the pulse shape is sharper and velocity is higher as compared to the case when \( \Delta \to 0 \). The corresponding transparency modes domains and the curve of strong excitation are shown on Fig. 2.

If \( n_e \to n_o \), then \( \Delta_1 \to \omega_0/A_1 \), and both \( \Delta_0 \) and \( \Delta_2 \) increase indefinitely. The curve of strong excitation for this case is represented in Fig. 1 by a thin line.

It should be noted that positively birefringent media, as well as isotropic media, are strongly excited by pulses with arbitrary \( \tau_p \). Since the present analysis relies on condition \( A\tau_p^2 \ll 1 \), the effective anisotropy exceeds \( D^2 \) in the parameter domain where inequality (19) is not valid. Therefore, both SIT and SIST modes are less expressed in the case of positive detuning, whereas the SIT mode corresponding to \( \Delta \to \Delta_0 \) must be more expressed because the effective anisotropy tends to zero.

Taking limit \( \tau_p \to \infty \) in formula (18), we obtain expression for parameter \( \tilde{D} \) of rationally decreasing pulse (12), (13) in the media with expressed birefringences. In positively birefringent media, detuning of this pulse can be both positive: \( \tilde{\Delta}_1 < \Delta < \Delta_+ \), and negative: \( \tilde{\Delta}_2 < \Delta < \Delta_- \). Here we use notations

\[
\tilde{\Delta}_{1,2} = \left( -A_1 \pm \sqrt{A_1^2 + 4A_2\omega_0} \right) \frac{1}{A_2};
\]

\[
\Delta_{\pm} = \pm \sqrt{\frac{\omega_0}{A_2}}
\]
Detunings $\Delta_1$ and $\Delta_2$ belong to the ranges of possible values of detuning for such the pulses (see Fig. 2).

Next, we consider negatively birefringent media ($n_o > n_e$). It is easy to show by using condition $A\tau_p^2 \ll 1$ that $\tilde{D} < D^2$; i.e., the effective anisotropy is weaker for such media. The reason is that the linear extraordinary-wave velocity is higher as compared to the ordinary one and, therefore, is always higher than the pulse velocity. In this case, the role of the extraordinary component in the pulse formation occurs to be less significant.

![Diagram](image)

**Fig. 3.** Curves of the strong excitation of negatively birefringent medium for $-A_2\omega_0 < A_1^2/4$, $-A_2\omega_0 = A_1^2/4$ (thin curve), and $-A_2\omega_0 > A_1^2/4$ (dotted curve).

Since negative birefringence reduces the effective anisotropy, the modes of transparency characterized by the strong excitation are more expressed. Equations (20–23) corresponding to these modes remain valid. Note that condition $A\tau_p^2 \ll 1$ imposes a constraint on parameter $\tau_p$ and, therefore, on detuning (see (20)). Accordingly, the detuning in (22) must be such that parameter $\tilde{D}$ is less than $D^2$ even though the first summand in the righthand side is greater than $D^2$.

It is seen from formula (21), that $F < 0$ for negative $\Delta$; i.e., the pulses that cause the largest change of the level population can be positively detuned only. This agrees with the fact that $\tilde{D}$ does not change sign in the case under study, and, therefore, the extraordinary component induces a dynamic red shift. However, as in the case of positive detuning, two intervals of admissible detuning corresponding to strong excitation of the medium can exist. This takes place if the medium is such that $0 < -A_2\omega_0 < A_1^2/4$. In this case, $\Delta_1$ and $\Delta_2$ are real and distinct, at that $\Delta_1 < \Delta_2 < \Delta_0$. The SIT mode takes place when either $\Delta \to 0$ or $\Delta \to \Delta_0$; the SIST mode, when either $\Delta \to \Delta_1$ or $\Delta \to \Delta_2$. If $-A_2\omega_0 = A_1^2/4$, then, obviously, $\Delta_1 = \Delta_2 = \Delta_0/2$. The parameter domain corresponding to SIST lies in the neighborhood of point $\Delta = \Delta_0/2$ in this case. If $-A_2\omega_0 > A_1^2/4$, then the roots $\Delta_1$, $\Delta_2$ are complex quantities, and the curve of strong excitation consists of a single branch. Under this condition, the SIST mode may not exist. As in the case of an isotropic medium and a positively birefringent medium, the parameter $\tau_p$ can have arbitrary values for pulses that cause the largest change of the population of the quantum levels. Figure 3 shows the curves of strong excitation corresponding to several values of parameters of the medium.

The NNT, PNT, and EOT modes must be less expressed in negatively birefringent media because of weaker effective anisotropy. This behavior resembles one of ”dense” media, except that the effective anisotropy decreases with increasing absolute value of detuning in the present case.

Rationally decreasing pulses (12), (13) exist in negatively birefringent media if $-A_2\omega_0 < A_1^2$. Detuning of these pulses can be positive only ($\Delta_1 < \Delta < \Delta_2$).
In recent years, there was a growing interest of investigating the coherent phenomena in the optically dense media$^{13-21}$. The difference between the macroscopic field and microscopic one acting on a separate particle has to be taken into account at the description of the interaction of radiation with the quantum systems in such the media. The effect of the local field leads, for example, to the internal optical bistability$^{15}$ and to the upconversion of the emission in dimeric structures$^{16}$. A consideration of near electric dipole-dipole interaction of the particles causes the dynamic shift of the frequency of resonant transition, which depends on the density population of levels$^{14}$. In that case, the propagation of the electromagnetic pulses in optically dense isotropic two-level media is described in the frameworks of the slowly varying envelope approximation by the equations (1), (3) and by the equation

$$\frac{\partial R}{\partial t} = i (\Delta + B(W - W_\infty)) R + i \Omega_o W. \quad (24)$$

Here $B$ is the phenomenological parameter taking account of the interaction between dipoles; variable $\Omega_o$ has a meaning of the Rabi frequency of the complex envelope of the electric field of the pulse.

The results obtained in$^8$ and in the previous sections are extended on the case of the optically dense media as follows. Let us assume $n_e = n_o$. Then the pulse solutions of the equations (1), (3), (24) are given by formulas (5–10) and (12), (13) if we put

$$\tilde{D} = 2BW_\infty \frac{\omega_0 \tau_p^2 d^2}{1 + \alpha^2},$$

and

$$\tilde{D} = 2BW_\infty \frac{\omega_0 d^2}{\Delta^2},$$

respectively. Hence, the electromagnetic pulses can propagate through the optically dense media in the transparency modes classified in$^8$. The solution (12), (13) describing the propagation of rationally decreasing pulse of the self-induced transparency seems to be new. It exists when detuning of the pulse satisfy condition $BW_\infty / \Delta > 0$. The rational form of the decay of the free polarization for such the media was revealed in$^{21}$.

![Fig. 4. Curve of the strong excitation for optically dense medium. Parameter domains are shown for $B > 0$ and labelled as on Fig. 1.](image)

It is interesting, that the largest excitation of the medium will be caused in accordance with formula (11) by the pulses having constant detuning: $\Delta = BW_\infty / 2$ (see Fig. 4). The pulses can propagate in this case in the SIT mode ($\tau_p \rightarrow 0$) or in SIST one ($\tau_p \rightarrow \infty$). If parameter

$$g = \frac{1 + \alpha^2}{BW_\infty \tau_p},$$

...
satisfy condition $|g| \ll 1$ (extremely dense media), then pulses can propagate in the EOT mode ($|\alpha| \ll 1$), NNT or PNT modes ($|\alpha| \gg 1$). The domains corresponding to different modes of the optical transparency are given on Fig. 4.

The term responsible for the dipole-dipole interaction can be entered into equations \(\mathcal{H}_1\) also. The solutions of the corresponding system follow from ones presented in Section 2 after suitable redefinition of coefficient $\tilde{D}$.

6 CONCLUSION

The analysis of the solutions of system of material and wave equations \(\mathcal{H}_1\) shows that the modes of the pulse propagation through the anisotropic media described in\(^7\),\(^8\) exist in more general physical situation, when the approximation of the unidirectional propagation is inapplicable, and linear velocities of ordinary and extraordinary components are different. The anisotropy of the medium becomes effective now, i.e. its degree depends on the parameters of the pulse also. Another important feature of the anisotropic media is that the rationally decreasing pulses of the self-induced transparency can propagate through it.

If the concentration of the particles is not small, or the medium is negatively birefringent, then the effective anisotropy decreases. For this reason, the modes of the transparency, which are accompanied by the strong excitation of the resonant particles, become expressed. As in the case studied previously\(^8\), the carrier frequency of the ordinary component of the pulses in these modes is less than the frequency of resonant transition. Under this condition only, the extraordinary component of the pulses can shift the energy levels and, simultaneously, generate the phase modulation in such a manner, that the ordinary component turns out to be in the resonance with the medium in an average and causes its strong excitation.

The more interesting case takes place when the concentration of the particles is small, and the medium possesses the positive birefringence. Its distinctive feature is that the effective anisotropy can change the sign. As a result, the modes of the strong excitation exist not only if the carrier frequency of ordinary component is less than resonant one, but if it exceeds the resonant frequency also. Besides, the degree of the effective anisotropy becomes unbounded, when the pulse group velocity tends to linear velocity of the extraordinary wave. Such the pulses propagate in the mode of the extraordinary transparency\(^7\).

The transparency modes introduced for the anisotropic media exist as well in isotropic, optically dense media, in which the direct electric dipole-dipole interaction between the resonant particles is considered\(^14\). The role of the dipole-dipole interaction coincides here with one of the extraordinary component in anisotropic case.

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