Vibrating kinetics of the Luders front

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Abstract. The regularities of the inhomogeneous plastic deformation such as Lüders bands are considered at the yield plateau. It has been established that the Luder front has stick-slip motion under these conditions. Such a character of motion at the deformation front is associated with the inhomogeneous distributions of the components of the plastic distortion tensor. A plausible estimate of the oscillation frequency was obtained during the development of the Lüders front. The estimation of the spatial period (the length of the autowave of localized deformation) also corresponds to reality.

1. Introduction

According to Seger and Frank \cite{1}, the development of plastic flow is self-organization of defect structure of a deformed medium. As it was possible to establish that such self-organization of plastic deformation of solids is equivalent to localization of plastic flow and can be described as the nucleation and evolution of localized plasticity autowaves. Macroscopic patterns of localized plasticity (plastic deformation pattern), that are experimentally observed during deformation, qualitatively and quantitatively characterize plastic flow at various stages of the process. At the same time, each of the localized plasticity autowaves corresponds to one of a small number of patterns that can be observed. The autowave approach to plasticity based on these ideas allowed us to understand many regularities of plastic flow \cite{2–5} that were not convincingly explained within the framework of the existing dislocation mechanisms of plasticity.

2. Experimental part

One of the most well-known and thoroughly studied patterns of localized plasticity is the Lüders band, which develops during deformation at the yield plateau \cite{6–8}. In this work, an experimental analysis of the deformation of single-crystal Hadfield’s high-manganese steel (Fe-12% Mn-0.9% C), that has axis of elongation oriented along the [3 1 7] direction, was carried out. In this alloy, the length of yield plateau was 0.25. Using the speckle photograph technique \cite{5}, we were able to obtain data on the space-time behavior of the components of the plastic distortion tensor (elongation $\varepsilon_{xx}$, shear $\varepsilon_{xy} = \varepsilon_{yx}$, and rotation $\omega_z$) when the sample is stretched along the $x$ axis. The $X$-$t$ (Lüders front position and time) diagram, which is shown in Fig. 1 for the components of the shear $\varepsilon_{xy}$ and rotation $\omega_z$, was used to determine the average velocity of motion of the Lüders front. It was $\langle V_{aw} \rangle \approx 9 \cdot 10^{-6}$ m/s.

3. Results and discussion

The data presented in Fig. 1 can be considered as an indication of a rather complex jumping nature of the kinetic of the Lüders fronts development. In particular, the duration of the front stops reaches $\sim 200$ s, and the time jump is $\sim 10$ s. Interestingly, arrest of development and jumps of different components
of the plastic distortion tensor are not synchronized with each other (Figure 1). In this article, the causes of such behavior of Lüders fronts were discussed.

Figure 1. Dependence of the position of the maxima of the local shift $\varepsilon_{xy}$ and rotation $\omega_z$ on the time of deformation

To understand the mechanism of development of the Lüders localized plasticity fronts, the behavior of components of the plastic distortion tensor was analysed when total deformation of the sample increased from 0.1 to 0.13 (Figure 2). As shown in Figure 2, at deformation $\varepsilon_{tot}=0.1$ the maxima of the spatial distributions of $\varepsilon_{xx}(x)$, $\varepsilon_{xy}(x)$ and $\omega_z(x)$ coincide (Figure 2 a) when $x$ is approximately equal to 10 mm. Then the total deformation increases to 0.13, and the $\varepsilon_{xx}$ component reaches a peak at $x \approx 12$ mm, while distributions of $\varepsilon_{xy}(x)$ and $\omega_z(x)$ are divided into two components with different signs which are shifted at about 3 mm with respect to the maximum of the distribution $\varepsilon_{xx}(x)$ to the left and right (Fig. 2 b).

In the localization zone, the Lüders front motion begins after rearrangement of the component distributions ($\varepsilon_{xx}$, $\varepsilon_{xy}$ and $\omega_z$) in the order shown in figure 2. This conclusion is drawn from a comparison of the figure 1 and figure 2. The convergence of the extrema of the distributions of all components of the plastic distortion tensor (Fig. 2 a) corresponds to the constancy of the X value for them. In contrast, the discontinuous changes of X, noted in figure 1, correspond to the separation point of extremum of spatial distributions $\varepsilon_{xx}(x)$, $\varepsilon_{xy}(x)$ and $\omega_z(x)$ shown in Fig. 2 b. It is obvious from these data that the spatial distributions of shear $\varepsilon_{xy}(x)$ and rotation $\omega_z(x)$ are asymmetric. They differ in an amplitude in addition to the different sign of deformations. Lüders front is moving towards higher peaks of distributions on the boundary of the elastically deformed part of the sample.

It can be assumed that redistribution of the components of $\varepsilon_{xx}(x)$, $\varepsilon_{xy}$ and $\omega_z$ is the cause of jumping development of plastic flow at all stages, which corresponds to the broken lines in Figure 1. It can be seen that the rearrangements of distributions of the components of the distortion tensor occur repeatedly when the deformation zone moves along the sample. In the deformation localization zone of the materials studied the matching of the peak of the elongation with peak of the shear and rotation is noted not only at yield plateau but at the stages of easy sliding, linear and parabolic deformation hardening.
Figure 2. Evolution of the distributions of local elongations $\varepsilon_{xx}$, shifts $\varepsilon_{xy}$ and rotations $\omega_z$ on the yield plateau of a single crystal of Hadfield steel $^{[377]}$ during movement of Lüders front with a total deformation of $\varepsilon_{\text{tot}} = 0.1$ (a) and 0.13 (b). The Lüders front moves from left to right.

To understand the nature of the considered processes, the data in fig. 1 describing the stick-slip temporary behavior of the $\varepsilon_{xx}(t)$, $\varepsilon_{xy}(t)$ and $\omega_z(t)$ components of the distortion tensor are analyzed at Luders deformation. To explain the stick-slip nature, we assume that the development of the Lüders band is a thermally activated process, the propagation rate of which $^{[9]}$ is defined as

$$V_m = V_0 \exp \left( -\frac{U - \gamma \sigma}{k_B T} \right) = V_0 \exp \left( -\frac{U - \gamma \sigma}{k_B T} \right)$$

(1)

On the yield plateau it's customary for the deformation to proceed at a stress which equals to the lower yield stress of $\sigma_y(\ell)$. In equation (1) $V_0$ is constant, $\gamma$ is the activation volume, $U$ is the height of the potential barrier, $T$ is the temperature, $k_B$ is the Boltzmann constant.

Since there is no strain hardening at the yield plateau, $\sigma_y^{(0)}$ can be close to the internal stress of $\sigma_{in} = Gb\sqrt{\Delta \rho}$ $^{[10]}$ but with the sign reversed. Where $G$ is shear modulus and $\Delta \rho$ is the dislocation density jump at the Lüders front, which depends on conditions realized. On condition of $\sigma_y^{(0)} = -\sigma_{in} = -Gb\sqrt{\Delta \rho}$ the equation (1) can be described in terms

$$V_m = V_0 \exp \left( -\frac{U + \gamma Gb\sqrt{\Delta \rho}}{k_B T} \right)$$

(2)
Considering the nonmonotonic nature of dislocation density changes whenever sharp yield point and the Lüders front [11] arise, it can be assumed that \( \Delta \rho = \Delta \rho(\epsilon) = \rho_m - \rho_0 \), where \( \rho_0 \) is dislocation density in unstrained medium before Lüders front and \( \rho_m \) is density of mobile dislocation born in Lüders front. Two cases may be taken into consideration:

i. When \( \rho_m - \rho_0 > 0 \) on the yield plateau the Lüders front moves at a constant speed \( V_{aw} \approx \exp \left( \frac{i \gamma G b}{k_B T} \right) \). This case corresponds to the possibility of average data shown in Fig. 1 (dashed line).

ii. When \( \rho_m - \rho_0 < 0 \) from equation (2) it follows

\[
V_m = V_e \exp \left( - \frac{U + iyGb\sqrt{\Delta \rho}}{k_B T} \right)
\]  

(3)

Where \( i = \sqrt{-1} \). Equation (3) points to the possibility of oscillations during the development of the front Lüders. The quantity of \( b/\sqrt{\Delta \rho} = b/\left( \sqrt{\Delta \rho} \right)^{-1} = \epsilon \) is deformation. In this case, the equation (4) describes the fluctuations of the velocity of the Lüders front in the developing deformation.

\[
V_m = V_e \exp \left( - \frac{U}{k_B T} \right) \exp \left( \frac{i \gamma G \epsilon}{k_B T} \right)
\]  

(4)

To determine the dependence of this speed on time, we formally introduce a deformation \( \epsilon = \delta l/l = (V_m/l)t = \dot{\epsilon}t \), where \( V_m \) is the rate of the flexible tension grip, and \( l \) is the sample length. Then equation (4) takes the form

\[
V_m \sim \exp \left( -i \frac{\gamma G \epsilon}{k_B T} t \right) \sim \exp \left( -i \omega_{osc} t \right)
\]  

(5)

Accordingly, during the motion of the Lüders front the frequency of oscillations is

\[
\omega_{osc} \approx \frac{\gamma G \epsilon}{k_B T} \approx \frac{\gamma G}{k_B T}
\]  

(6)

There are \( \gamma \approx 10^2 b^3 \approx 10^{-29} \text{ m}^3 \), \( G \approx 4 \cdot 10^{10} \text{ Pa} \), \( k_B T \approx 0.025 \text{ eV} = 4 \cdot 10^{-21} \text{ J} \), \( \dot{\epsilon} \approx 2 \cdot 10^{-5} \text{ s}^{-1} \) for the experimental conditions used. In this case, from equation (6) we have \( \omega_{osc} \approx 2 \cdot 10^{-3} \text{ Hz} \). This estimate corresponds to the observed data on the period of change in the local deformations shown in Fig. 1 (full lines), and points to the validity of the considered mechanism for the propagation of the Lüders front on the yield plateau. The estimation of the spatial period (the length of the autowave of localized deformation) of \( \lambda \approx \langle V_{aw} \rangle \cdot \omega_{osc}^{-1} \approx 5 \cdot 10^{-3} \text{ m} \) also corresponds to reality [1].

4. Conclusion

It can be assumed that, in such an approach, the nature of the phenomena, which is responsible for the development of the Lüders deformation and the Portevin-Le Chatelier hopping deformation [12], can be viewed from a unified position. It is likely that these effects differ only in the spatial and temporal scales of the corresponding stress and strain jumps.

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