Remarkable suppression of dc Josephson current on \(d\)-wave superconductor junction

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Josephson current in superconductor/insulator/superconductor junction is studied theoretically. It is well known that when the zero-energy resonance state exists both side of superconducting interface, the behavior of the temperature dependence of the critical Josephson current is striking enhancement at low temperature. On the other hand it is reported that if \(d\) + is-wave exists at the interface, Josephson current is suppressed at low temperature. In this paper, we discuss the existence of the imaginary part of the pair potential at the interface and remarkable suppresses of dc Josephson current on \(d\)-wave superconductor 110-junction.

I. INTRODUCTION

In two decade, transport property of the unconventional superconducting junctions is studied both theoretically and experimentally. In these junctions, zero-energy resonance state (ZES) plays an important role. It is well known that in the tunneling spectroscopy of the high-\(T_C\) superconductor the zero-bias conductance peak appears.

On the other hand Josephson current in superconductor/insulator/superconductor junction is one of the characteristic phenomena. Anomalous behavior is obtained on high-\(T_C\) superconducting junction, \(i.e.\) the critical Josephson current enhances at low temperature when the lattice orientation is \(\alpha = \pi/4\) (110-junction) as the Fig. 1. This is caused by the existence of the ZES formed at the interface. In previous papers we have known a general formula for Furusaki-Tskakada formulation for dc Josephson current, which include both macroscopic phase and ZES. This theory is based on a microscopic calculation of the current represented in terms of the coefficients of Andreev reflection. In this paper, we calculate and discuss dc Josephson current at 110-junction in the \(d\)-component superconductor/insulator/\(d\)-wave superconductor junction considering existence of is-wave state and imaginary part of the \(d\)-wave state. In these, we calculate spatial dependence of the pair potential self-consistently.

II. FORMULATION

In order to calculate Josephson current, we well know the Green’s function method like this:

\[
I = \frac{e\hbar}{2im} \left( \frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right) \text{Tr} G_{\omega_m}(x, x').
\]  

(1)

And now, we use the quasi-classical method in this paper. First of all, Nanbu-Gol’kov Green’s function is written as,

\[
G(x, x') = G_{++}(x, x') e^{ik_F(x-x')} + G_{--}(x, x') e^{-ik_F(x-x')} + G_{+-}(x, x') e^{ik_F(x+x')} + G_{-+}(x, x') e^{-ik_F(x+x')}.
\]  

(2)

Taking the differential into the equation, we obtain the following equation,

\[
I = \frac{e\hbar k_F}{m} \text{Tr} \left( G_{++}(x, x') e^{ik_F(x-x')} - G_{--}(x, x') e^{-ik_F(x-x')} \right) + O(1).
\]  

(3)

The differential for \(G_{\alpha\beta}(x, x')\) is order \(1 (\ll k_F)\), so it’s ignored. The quantity \(\alpha, \beta\) mean \(D\). The Green’s function \(G_{\pm\mp}(x, x')\) terms vanish in the differential.

We define the quasi-classical Green’s function

\[
\hat{g}_\alpha = f_{1\alpha} \hat{r}_1 + f_{2\alpha} \hat{r}_2 + g_\alpha \hat{r}_3, \quad (\hat{g}_\alpha)^2 = \mathbb{1}
\]  

(4)

Here \(\hat{r}_j (j = 1, 2, 3)\) are Pauli matrices and \(\mathbb{1}\) is a unit matrix. The quantities \(f_{1\alpha}, f_{2\alpha}, g_\alpha\) obey the following relations,

\[
f_{1\alpha} = \alpha \left[ i F_\alpha(x) + D_\alpha(x) \right]/\left[1 - D_\alpha(x) F_\alpha(x)\right],
\]  

\[
f_{2\alpha} = -\left[ F_\alpha(x) - D_\alpha(x) \right]/\left[1 - D_\alpha(x) F_\alpha(x)\right],
\]  

\[
g_\alpha = \alpha \left[ 1 + F_\alpha(x) D_\alpha(x) \right]/\left[1 - D_\alpha(x) F_\alpha(x)\right].
\]  

(5)

(6)

(7)

In these quasi-classical Green’s function, the quantity \(D_\alpha(x)\) and \(F_\alpha(x)\) obey the Ricatti equations

\[
h|v_F|D_\alpha(x) = \alpha \left[ 2\omega_m D_\alpha(x) + \Delta(x, \theta) D_\alpha^2(x) - \Delta^*(x, \theta) \right],
\]  

\[
h|v_F|F_\alpha(x) = \alpha \left[ -2\omega_m F_\alpha(x) + \Delta^*(x, \theta) D_\alpha^2(x) - \Delta(x, \theta) \right].
\]  

(8)

(9)

The quantity \(\theta\) is the angle between quasi-particle going through the interface and \(x\) direction, here \(x\)-axis is the...
vertical to the interface. The boundary conditions at the interface are given by

\[
F_{+L} = \frac{D_{-R} - RD_{+R} - (1 - R)D_{-R}}{D_{-L}(RD_{-R} - D_{+R}) + (1 - R)D_{+R}D_{-R}}, \quad (10)
\]

\[
F_{-L} = \frac{RD_{-R} - D_{+R} + (1 - R)D_{+L}}{D_{+L}(D_{-R} - RD_{+R}) - (1 - R)D_{+R}D_{-R}}, \quad (11)
\]

\[
F_{+R} = \frac{RD_{+L} - D_{-R} - (1 - R)D_{+R}}{D_{+R}(RD_{-L} - D_{+R}) - (1 - R)D_{+R}D_{-R}}, \quad (12)
\]

\[
F_{-R} = \frac{D_{+R} - RD_{-L} - (1 - R)D_{+R}}{D_{-R}(RD_{-L} - D_{+R}) + (1 - R)D_{+R}D_{-R}}, \quad (13)
\]

where we omit the index \((x = 0)\). The quantity \(R = Z^2/(4 + Z^2)\) with \(Z = 2mH/h^2k_F\). Here the quantity \(H\) is the height of the barrier potential. Then we treat the insulator as the \(\delta\)-functional barrier potential. The boundary condition for \(D_\alpha(x)\) at \(x = \pm\infty\) is

\[
D_\alpha(\pm\infty) = \frac{\Delta^*(\pm\infty, \theta)}{\omega_m + \alpha\Omega_\alpha}, \quad (14)
\]

In these relation, we can write down Josephson current as following,

\[
I(\theta) = \frac{2e\hbar k_F}{m} i\left([g_+(x, \theta)] - [g_-(x, \theta)]\right). \quad (15)
\]

Josephson current in this formula is obtained by \(x \to 0\).

The spatial dependent pair potential is calculated as following

\[
\Delta(x, \theta) = \frac{2T}{\ln T/T_C + \sum_{0 \leq m} \frac{1}{m+1/2}} \times \sum_{0 \leq m} \int_{-\pi/2}^{\pi/2} d\theta' V(\theta, \theta') f_{2+} \quad (16)
\]

where \(V(\theta, \theta') = 2\sin 2\theta \sin 2\theta'\) for 110-junction and \(V(\theta, \theta') = 2\cos 2\theta \cos 2\theta'\) for 100-junction, respectively for \(d\)-wave component, and \(V(\theta, \theta') = 1\) for \(s\)-wave component for both 110- and 100-junction case. In this equation, we can calculate spatial dependent of the pair potential self-consistently (SCF).

Josephson current \(I\) in these formula is obtained numerically solving Eq. 8, 13 under the boundary conditions Eq. 10, 11, 12, 13.

Calculated result of Josephson current is normalized by normal conductance \(\sigma_N\),

\[
I(\eta) = \int_{-\pi/2}^{\pi/2} I(\theta) \cos \theta d\theta / \sigma_N, \quad (17)
\]

\[
\sigma_N = \int_{-\pi/2}^{\pi/2} \frac{4 \cos \theta^2}{4 \cos \theta^2 + Z^2} \cos \theta d\theta. \quad (18)
\]

Here, we define \(\eta = \eta_L - \eta_R\), where \(\eta_L, \eta_R\) is the macroscopic phase of left and right side of the superconductors. In every thing, we chose the temperature \(T = 0.05T_C\), where \(T_C\) is the transition temperature of superconductor. And the cutoff frequency \(\omega_D\) is set to be \(\omega_D/2\pi T_C = 1\) for summation of Matsubara frequency \(m\).

### III. Results

In this section, we show the calculated results of the superconducting macroscopic phase \(\eta\) dependence of the pair potential and Josephson current. In all case we chose \(T_C = 0.2T_{Cu}\), where \(T_C\), and \(T_{Cu} (= T_C)\) are the transition temperature of \(s\)-wave component and \(d\)-wave component, respectively.

![FIG. 2: The x-dependence of the pair potential of the right and left side of the superconductors at \(\eta = 0\). The a, b, c mean \(Z = 0, 5, 10\), respectively. \(\xi\) is the coherent length of the superconductor.](image)

First of all, we show the \(\eta\)-dependence of the pair potential. The system is 110-junction. In the Fig. 2, 3, 4, the solid line is real number, and dotted line is imaginary number, respectively. The \(x\) axis is normalized by coherent length \(\xi\). For \(\eta = 0\), \(Z\)-dependences don’t appear in the pair potential as Fig. 2. The reducing of the pair potential for \(Z = 0\) near the interface receives spatial changing. In these, \(is\) state exists near the interface. This state doesn’t appear in the 100-junction’s case. When the quasi-particle goes through the inter-
face, it feels the opposite sign of the pair potential. So the reducing occurs in spite of $Z = 0$.

Second, let us show the $\eta = \pi/2$ case. For $Z = 0$, $s$ and $is$ state not exist. Since the pair potential contains the superconducting macroscopic phase at the right side of the superconductor, in the Fig. 3, right side of the pair potential of $d$-wave is imaginary number. Similarly $\eta = 0$ case, since the quasi-particle through the interface feels the different sign of the pair potential, real number of $d$-wave (since it contains the macroscopic phase of the superconductor, real number of $d$-wave appears as the imaginary number.) at the right side of the pair potential is connected to the imaginary $d$-wave at the left side of the pair potential.

For $Z = 5$, pair potential behaves as Fig. 3 b. The existence of the barrier potential affects the suppression of the right and left side of the pair potential for both real and imaginary numbers near the interface. At the region of the coherence length near the interface, imaginary parts of the $d$-wave is enhanced for the right and left side of the superconductors.

When $Z = 10$, $s$ and $is$ state appear for both side of the superconductors near the interface. The existence of the $is$-wave is same reason as the ordinary discussion for $is$-wave state at the edge of the $d$-wave superconductor on $\alpha = \pi/4$ (110-junction).

Next, we show the $\eta = \pi$ case. The phase factor is $\exp(i\eta) = -1$, so 110-junction is same as in the 100-junction ($\alpha = 0$). Therefore real part of the $d$-wave is not spacial dependence in the $Z = 0$ case, i.e. pair potential is constance for all region. $s$ and $is$-wave don’t appear and $id$-wave doesn’t appear too.

When $Z = 5$, since existence of the barrier potential makes the reflection of the quasi-particle at the interface, $d$-wave factors are reduced. $Z = 10$ case is same as $Z = 5$. The different point at the $Z = 10$ is the existence of the $is$ state at the interface.

Finally, we show the normalized dc Josephson current for 110-junction and 100-junction. For 110-junction, Josephson current is suppressed at the $\eta = \pi/2 \sim \pi$ region for $Z = 10$, and it suppressed at all area for $Z = 5$. For $Z = 15$, Josephson current behaves $\sin \eta$. On the other hand, for 100-junction, Josephson current is not suppressed. And it is consistent with the non-SCF calculation.

Comparing the 110-junction to 100-junction, the height of the Josephson current for 100-junction is higher than that for 110-junction. It is not consistent with non-SCF calculations. This is our new dissolve.

FIG. 3: The $x$-dependence of the pair potential of the right and left side of the superconductors at $\eta = \pi/2$. The a, b, c mean $Z = 0, 5, 10$, respectively. $\xi$ is the coherent length of the superconductor.

FIG. 4: The $x$-dependence of the pair potential of the right and left side of the superconductors at $\eta = \pi$. The a, b, c mean $Z = 0, 5, 10$, respectively. $\xi$ is the coherent length of the superconductor.
IV. SUMMARY

In this section, we summarize the obtained results. Now we have seen the imaginary part of the pair potential exists at the $\eta \neq 0, \pi$ for 110-junction. That occurs by the existence of the macroscopic phase of the superconductors. This results are different from the situation of the surface of superconductor or junction between normal metal and superconductor. Since the both $is$- and $id$-wave state exist near the interface, Josephson current is reduced on 110-junction. This results is not only by the $is$-wave state but also by the existence of the imaginary part of the pair potential of $d$-wave. This reducing is same as in the $s$-wave superconductor / $p$-wave-superconductor / $s$-wave superconductor junction.

In this paper’s case $id$-wave component plays the different symmetry for the $d$-wave component. On the other hand, Josephson current is not reduced on 100-interface. These results are unusual. These appear only in the SCF calculation. In the non-SCF calculation, these don’t appear.

These result for 110-junction is consistent with Ref. 10, where it’s a high barrier limit case.

And adding one more thing, Josephson current disappears on $Z = 0$ both for 110-junction and 100-junction. In the physical point of view, it is expected that Josephson current only exists when insulating barrier or something (normal metal or different type of superconductor) exist at the interface. Therefore these results are valid physically.

In this paper, we discuss dc Josephson current for the 110-junction. Pair potential has the imaginary part for $\eta \neq 0$, and Josephson current is suppressed. This result appears only in the SCF calculation of the pair potential.

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FIG. 5: Josephson current at the 110-junction (a) and 100-junction (b) for $Z = 5, 10$ and $Z = 15$. 