Finite element modelling reinforced concrete beam in transverse bending zone

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Abstract. The article presents methods and results of studying the stress-strain state of a reinforced concrete beam in its area of transverse bending, using the finite element models. The advantage of the finite element modelling method is relative simplicity of mechanical and physical interpretation of the finite element models. Herein, we present a description of finite element model and the method for numerical investigation during its loading and evaluation of the stress-strain state. Testing of the finite element models were performed using different lengths of shear span, taking into account the inelastic properties of concrete. During the models’ tests, the process of cracks formation and development was studied while taking into account the bond of reinforcement to concrete, which changes in line with the adopted law. We also evaluated the parameters of stress-strain state of the beam blocks separated by cracks and the stressed state of concrete at the tip of an inclined crack, depending on the loading level. Based on the results of numerical experiment, we obtained distribution patterns of principal stresses for the concrete and those of shear stresses along the reinforcement and concrete contact surface, depending on the level of loading, the degree of bond failure and the length of shear span.

1. Introduction
As a rule, reinforced concrete structures of buildings operate in conditions of complex stress state. The vast majority of structures are subjected to cumulative effect of bending moments, of longitudinal and transverse forces. Therefore, the issue of the nature of stress-strain state of transverse bending area, and the factors affecting it, is of great practical importance [1-4].

The resistance of a transverse bending area depends on a large number of factors, which - in an explicit or implicit form - the authors of numerous experimental and theoretical studies are trying to estimate. This area comprises significant studies by various authors; however, it is commonly agreed that, the stress-strain state of reinforced concrete elements in the area of transverse bending is a very complicated thing, and in some cases, has not been sufficiently studied [5-7]. The analysis of the stress-strain state becomes yet more complicated due to inconsistency of the available experimental data concerning the weight of this or that factor. For the count and analysis of influencing factors, there exists a fairly reliable and universal method that allows us to evaluate the stress-strain state of a reinforced concrete structure at different stages of its operation – mathematical modelling based on the finite element method. One of this method’s advantages is a relative simplicity of mechanical and physical interpretation of finite element models. An essential point in a model development is verification of the model to achieve an adequate reproduction of the real physical processes occurring.
in a loaded reinforced concrete structure. A model’s adequate behaviour can be assessed by comparing it with the results of experimental studies using test samples [8-10]. To verify the model adequacy, we used results of experimental studies of stress-strain state of reinforced concrete beams in the area of their transverse bending, performed by the authors [11-12]. In addition, the results of experimental samples testing obtained by foreign authors were analyzed [13-18].

2. Methodology

The following issues were addressed when modeling the stress-strain state of beam transverse bending area:

- studying the process of crack formation and development, taking into account the reinforcement and concrete bond, changing as per the applied law;
- estimating the stress-strain state parameters of interacting beam blocks and the stress state of concrete at the top of inclined crack, depending on the load level.

These issues were handled by testing finite element models with different shear spans, taking into account inelastic properties of concrete. The model for studying the stress-strain state of transverse bending area was developed in accordance with the provisions described in [19]. The validity of the model was evaluated by comparing it with the results of experimental studies [11,12] conducted with three groups of test samples with different lengths of their transverse bending area. The bond degree between longitudinal reinforcement and concrete in the test samples varied from a full bonding to a complete failure. The model consists of plane finite elements whose dimensions are selected so that model’s cross-section matched the cross-section of a test sample. In addition, the finite elements dimensions make it possible to analyze the stress-strain state of the concrete in its contact area with the reinforcement. The longitudinal reinforcement was also modeled with plane finite elements, whose dimensions matched 20 mm diameter reinforcement bar. There was no transverse reinforcement in the models. Deformation characteristics for the concrete and reinforcement were taken based on the experiment results.

The model (figure 1) consists of rectangular finite elements of a plane 10×10 mm problem, which allow simulating a plane stress state.

Figure 1. The finite element model of the first group test samples ($a/h_0 = 1$).

The contact area of the reinforcement with concrete was modelled by three thin layers of plane finite elements. The width of layers was assumed to be $\pi d_s/2$ - a half perimeter of reinforcement bar, to obtain shear stresses matching the actual contact surface. Existence or lack of bond was modelled accordingly by the presence or absence of elements directly adjacent to reinforcing bar. When the...
shear stresses in the reinforcement to concrete contact area reached the limit values, which were assumed as $2R_{bt}$ according to experiment results, the interaction of reinforcement with concrete was modelled as follows. The element adjacent to reinforcing bar, whose shear stresses reached the limit values, was deleted, and contra-directional forces $N_t$ were applied to the reinforcement and concrete, whose force value was

$$N_t = R_{bt} \times \pi d_t \times l_v$$  \hspace{1cm} (1)$$

To transmit compressive stresses normal to contact surface of reinforcement with concrete, a system of rigid links was arranged, which having sufficient length and minor relative displacements of the elements modelling concrete and reinforcement, ensured a reliable transfer of vertical forces to opposite nodes without interfering with their mutual horizontal displacements. Lower ends of the rigid links were attached to a low stiffness link, which took up almost no forces.

To model the deformations of the contact layer between reinforcement and concrete, the bond law proposed in [19] was used

$$\tau_{\text{bond}} (x) = F_g\left[ g(x) \right]$$  \hspace{1cm} (2)$$

The law of bond was taken in the form of elastic-plastic deformations diagram:

$$\tau_{\text{bond}} (x) = \left( \frac{\tau_0}{g_\star} \right) \times g \hspace{1cm} \text{when} \hspace{0.5cm} g \leq g_\star$$  \hspace{1cm} (3)$$

$$\tau_{\text{bond}} (x) = \tau_0 \hspace{1cm} \text{when} \hspace{0.5cm} g > g_\star$$  \hspace{1cm} (4)$$

In equations (3) and (4), it is assumed:

$\tau_0 = 2R_{bt}$;

$\tau_{\text{bond}} (x)$ – conditional bond stresses along the contact area of concrete and reinforcement;

$g(x)$ – conditional mutual displacements on the contact area of concrete and reinforcement;

$x$ – a cross-section coordinate along the reinforcement bar considered.

When testing models in a numerical experiment, parameters of stress-strain state of the model were evaluated depending on the load level for the full and failed bonds of reinforcement with concrete. Besides, the process of inclined crack formation and development was studied for the case of full bond changing as per the applied law (2).

The law of concrete deformation was applied as power dependence determined by initial modulus of elasticity and the values of limit stresses and deformations during compression and tension. The effect of the shear span length and bonding conditions on formation of normal and inclined cracks was studied. The bent element model without transverse reinforcement items was tested. Criterion for cracks formation on the tension side of a sample was achieving a value of $1.75R_{bt}$ by main tensile stresses, which corresponds to the maximum tensile strength of concrete during bending. Criterion for inclined cracks formation in a plane stress state (in the middle part of cross-section vertically) was adopted as per the Pisarenko-Lebedev theory for heterogeneous materials. Formation of cracks was modeled by splitting nodes of the neighboring elements. The direction of crack development was taken in accordance with the position of main tensile stress planes. The crack development was considered complete when the values of main tensile stresses in the elements adjacent to the crack tip did not exceed $R_{bt}$. At the same time, in each stage of applying load to a model, bond stresses values along the contact surface of reinforcement and concrete were monitored.

3. Results and Discussion

Based on the results of numerical experiment, we obtained distribution patterns of main tensile (compressive) stresses and distribution of shear stresses along the reinforcement and concrete contact surface, depending on the level of loading, degree of bond failure and the length of shear span.
The results of studies performed on finite element models show that there are two patterns for cracks to form in the transverse bending area of a reinforced concrete beam. For small shear spans \((a/h_0 = 1)\), cracking begins in the middle part height-wise of a model cross section (figure 2). The reason for inclined cracks forming is the main tensile stresses of a plane stress state.

![Figure 2. Distribution of the main tensile stresses.](image)

The nature of crack formation changes with increase of a shear span length \((a/h_0 = 2.3)\). Initially, normal cracks form on the tension side of the model, which develop into inclined cracks as the loading level increases. The opening of cracks and the mutual turn of the blocks separated by a crack lead to the appearance of dowel effect in the longitudinal reinforcement items. Based on the results of this study stress fields \(\sigma_x, \sigma_y\) were obtained in the concrete (figures 3, 4) as well as diagrams of tensile stresses \(\sigma_s\) along the length of reinforcement rods (figure 5). Analysis of the stress fields and diagrams revealed satisfactory agreement with the results of experimental studies carried out earlier [11, 12, 20].

![Figure 3. Stress distribution \(\sigma_x\) in concrete.](image)
The finite element models in figures 2 – 4 are shown in a deformed state at an operational load level ($\sigma_s = 270$ MPa). Longitudinal reinforcement is not shown for clarity.

Cracking in reinforced concrete beams occurs in most cases following such sequence: initially, normal cracks form in area of pure bending and then normal cracks occur in the area of transverse forces, becoming slightly inclined as the load increases, towards increasing bending moment.

Two types of inclined cracks were observed during the models testing. One type of cracks formed as a normal crack in area of transverse bending from the beam’s lower face and then turned into an inclined crack. The second type of cracks was formed in area of transverse bending at small shear spans above or below the center of gravity in the model’s cross-section. Then the cracks developed quickly in two directions to the upper and lower faces of beam.

4. Conclusion

The results obtained during the models testing allow us recognize the validity of accepted design assumptions, and the qualitative correspondence of the operational nature of the near supporting area of a reinforced concrete element being bent, to the results of previously performed experimental and theoretical studies [11,12,14,16,20]. The proposed model permits analyzing the stress-strain state of transverse bending area of a reinforced concrete structure and evaluate the qualitative effect of external and internal factors on its operation.
Based on analysis of the results obtained during the models testing, we can conclude that the developed model and its testing methods adequately reflect the nature and operational features of the transverse bending area of reinforced concrete elements. The results obtained during the models testing show the validity of accepted design assumptions, qualitative and quantitative compliance with the results of previously performed experimental studies on test samples [11,12].

The inelastic properties of concrete in the area of adhesion of longitudinal reinforcement to concrete were taken into account when carrying out a numerical experiment using the proposed model. The implementation of a nonlinear calculation will allow taking into account the inelastic properties of a concrete element as a whole and moving from qualitative to quantitative estimates, comparing those with the results of the previous experimental studies. Experimental studies using the finite element models have shown the possibility of implementing this modeling method to assess the crack resistance and strength of reinforced concrete beams during their transverse bending.

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