Complex Chern-Simons and Gribov

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Abstract

We explore a contact point between two distinct approaches to the confinement problem. We show that BLG-ABJM like theories generate gauge propagators with just the complex pole structure prescribed by the Gribov scenario for confinement. This structure, known as i-particles in Gribov-Zwanziger theories, effectively allows the definition of composite operators with a positive Källén-Lehmann spectral representation for their two-point functions. Then, these operators satisfy the criteria to describe glue-ball condensates. We calculate the (first order) contribution to the two-point function of the gauge invariant condensate in an ABJM environment, showing its interpretation as a physical particle along Källén-Lehmann. In the meantime, we argue for the necessity of absorbing Witten’s work on holomorphic complex theories in order to settle the physical interpretation of this non-perturbative scenario.

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1 Introduction

Three-dimensional gauge theory is an interesting lab for many studies in nonperturbative aspects of gauge field theories such as color confinement or topological properties such as obtained from Chern-Simons. Also, the Yang-Mills theory has local degrees of freedom and the coupling constant is dimensionful. This properties indicates that this theory can be seen as an approximation for the high temperature phase of QCD with the mass gap in the role of the magnetic mass. In particular one of the mechanisms to study color confinement comes from the analysis of copies of Gribov [1], known generally as Gribov problem, with special emphasis on the Gribov-Zwanziger model (GZ) [2, 3, 4, 5] and its refined version (RGZ) [6]. One of the Gribov mechanism properties is that it generates propagators for gauge fields with complex poles, known as “i particles” [7, 8], being impossible their identification with the propagation of simple massive particles but with the possibility to obtain condensates that behave like massive particles, which is interpreted as confinement and known generally as Gribov-Zwanziger scenario. The Gribov problem is a general characteristic of the quantization procedure of Yang-Mills theories that are a general property of all the local covariant renormalizable gauge fixing [9]. It is important to emphasize here that the Gribov problem is intrinsically linked to the Morse theory and this discussion was done by van Baal [10]. His work begins by interpreting the question of Gribov copies in this variational form as a problem in Morse theory (see also [11] for a previous use of Morse theory on topological quantum field theories and its relation to the Gribov ambiguity). Into simple terms, Morse theory searches for a characterization of topological invariants of any given manifold by the study of the critical points of functions defined on it [12, 13], like the Hilbert norm $I_S = \text{Tr} \int_M \tilde{A}_i^2$, that is the Gribov case. Morse theory is also important in the analytic continuation of three-dimensional Chern-Simons gauge theory away from integer values of the usual coupling parameter $k$. This analytic continuation can be carried out by generalizing the usual integration cycle of the Feynman path integral [14, 15]. Morse theory gives a natural framework for describing the appropriate integration cycles. Also is important to note that Bagger-Lambert-Gustavsson (BLG) theory [16, 17] or at least Aharony- Bergman-Jafferis-Maldacena (ABJM) theory [18] in the case of $U(1)_k \times U(1)_{-k}$ can be cast in the form of a complex holomorphic theory and this structure appears to be fundamental in every theory in which the infrared sector is under study. The Main objective of that paper is to present a connection between the complex theory and Gribov-Zwanziger scenario. With particular emphasis on the relation between the “i particles” pole structure, complex theory and observables that admits Källén-Lehmann spectral representation.

So in this paper we investigate the SCS ($N = 1, D = 3$) with superfields formalism taking into account the idea that a complex gauge structure can be related to the Gribov scenario, i.e. the “i particles” can be obtained from complex CS due to a symmetry breaking mechanism. Of course the symmetry breaking mechanism can not be understand in the context of a minimum of a scalar potential due to the non positivity of the action when written in terms of real gauge fields. The symmetry breaking mechanism must be understood as a critical point that permits the access to infrared properties due to the introduction of a scale and the fact that the theory is not topological any more.

The paper is organized as follows: in Section 2, the Gribov problem is presented for the YM-CS theory and the “i particles” pole structure is introduced and the relation to complex fields is discussed. In section 3 the Complex generalization of CS is presented and a scalar complex field is introduced, also the symmetry breaking mechanism is presented and the “i particles” structure is obtained and the construction of gauge invariant composite operators is obtained.

2 Superfield, $N = 1, D = 3$, SYM-CS theory Gauge fixing and Gribov

In three-dimensional Minkowski space-time the Lorentz group is $SL(2, R)$ and the corresponding fundamental representation acts on a two components real spinor (Majorana). In the case of Euclidean $D = 3$, the two components spinor shall be transformed under $SO(3)$ and as is well known [19, 20, 21] one can not have the usual Majorana condition. It’s the same question we are in $D = 4$ [22]. So we take the approach of generalizing the concept of complex conjugation of Grassmann algebra [23]. The notations and conventions are in Appendix A. Let us take the Euclidean version of this superspace action of SYM-CS [24]:
\[ S_{\text{SYMCS}} = S_{\text{SYM}} + S_{\text{SCS}}, \]  
with,
\[ S_{\text{SYM}} = \frac{1}{2} \int d^3x \theta W^a \theta W^a, \]  
and
\[ S_{\text{SCS}} = \text{im} \int d^3x d^2\theta \left[(D^a \Gamma^\alpha)(D_\beta \Gamma^a) + \frac{2}{3}gf^{abc}\Gamma^a \Gamma^b \Gamma^c - \frac{1}{6}g^2 f^{abc} f^{cde} \Gamma^a \Gamma^b \Gamma^c \right]. \]  
The field strength is given by:
\[ W^a = D^\beta D_\alpha \Gamma^a + igf^{abc}\Gamma^b \Gamma^c - \frac{1}{3}g^2 f^{abc} f^{cde} \Gamma^a \Gamma^b \Gamma^c, \]  
and superspace derivative:
\[ D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i\sigma^\mu \epsilon \epsilon^\gamma \gamma^\beta \theta^\beta \partial^\mu. \]  

2.1 Gauge-fixing

In order to quantize the theory correctly we have to fix the gauge and we can do covariantly using the usual procedure of Faddeev-Popov (FP) and the supersymmetric Landau gauge. We implement the conditions \( D_\alpha \Gamma^a = 0 \). 

Following these procedure we ended with the action of gauge fixing
\[ S_{gf} = \frac{1}{4} s \left\{ \int d^3x d^2\theta (c^a D^a \Gamma^a) \right\}, \]  
where the Faddeev-Popov ghost fields will be scalar superfield \( c^a \) and \( \epsilon^a \) are the antighost and the ghost respectively. And \( s \) is the BRST nilpotent operator \( (s^2 = 0) \).

The total action \( S = S_{\text{SYMCS}} + S_{gf} \) is invariant under the BRST transformations \[ 24]:
\[ s\Gamma^a = (\nabla_\alpha c)^a \]
\[ sc^a = -\frac{1}{2}g f^{abc} c^b c^c \]
\[ sc^a = b^a \]
\[ sb^a = 0. \]  

Using this gauge fixing the massive gauge propagator for SYM-CS is:
\[ <\Gamma^a_\alpha(1)\Gamma^b_\beta(2)> = \frac{\delta^{ab}}{\delta^2(-\delta^2 + m^2)}(D^2 - im)D_\beta D_\alpha \delta^2(\theta_1 - \theta_2)\delta^3(x_1 - x_2). \]
2.1.1 SYM-SC and Gribov problem

The problem of Gribov copies is a general property of all the local covariant renormalizable gauge fixing \cite{9}.

It is straightforward to note that the Landau-gauge gauge condition is not ideal. If we consider two equivalents superfield, $\Gamma^\alpha_a$ and $\Gamma'^{\alpha'}_a$, connected by a gauge transformation \cite{14}, if both satisfy the same condition of the Landau gauge, $D^a(\Gamma^\alpha_a) = 0$ and $D^a(\Gamma'^{\alpha'}_a) = 0$, we have

$$D^a(\nabla_\alpha \Lambda)^a = 0. \quad (12)$$

Therefore, the existence of infinitesimal copies, even after FP quantization is related to the presence of the zero modes of the operator above. This suggests that we should restrict the functional integration to a region free of zero modes, and free of gauge superfields copies. To do this we would like to study the operator (12) in terms of modes of the operator above. This suggests that we should restrict the functional integration to a region free of zero modes, and thus define the Gribov problem is:

$$\mathcal{O}^{ab} = D^2 D^\alpha \nabla_{\alpha}^{ab}. \quad (13)$$

This operator is the correct generalization of the FP operator. So to see the zero mode problem we take the eigenvalues equation

$$D^2 D^\alpha \nabla_{\alpha}^{ab} \lambda = \lambda \Lambda \quad (14)$$

and the restriction of functional integration for the region free of zero modes, is given by the generalization of the Gribov region

$$\Omega := \{ \Gamma^\alpha_a \mid D^a(\Gamma^\alpha_a) = 0, \mathcal{O}^{ab}(\Gamma_a) > 0 \}. \quad (15)$$

In order to implement this restriction then we consider the GZ approach \cite{2, 3, 4, 5} where is included in the localization of the Gribov restriction was an extensive task and is done in $N = 1$, $D = 4$ super Yang-Mills directly in superspace in \cite{22} and in $D = 3$ SYMCS in \cite{23}. At this point we are just interested into the gauge propagator $\Gamma_\alpha$ and the “$i$-particle” structure. In order to calculate the gauge propagator we need only the bilinear of $S_{SGZ}$. Thus, for $S_{SGZ}$, we have:

$$S_{SGZ} = tr \int d^3 x d^2 \theta \Gamma_\alpha \frac{2 \gamma^4}{\gamma^4} \varepsilon^{\alpha \beta} \Gamma_{\beta}. \quad (17)$$

Similar to SYM-CS, the gauge propagator for SYM-CS-GZ:

$$< \Gamma^\alpha_a(1) \Gamma^\beta_b(2) > = \frac{1}{2} \mathcal{S}^{ab} \left[ \frac{(\partial^4 + \gamma^4) + im \partial^2 D^2}{-(\partial^4 + \gamma^4)^2 + m^2(\partial^2)^2} \right] D^2 D_\beta D_\alpha \delta^2(\theta_1 - \theta_2) \delta^3(x_1 - x_2). \quad (18)$$

To see how the introduction of $S_{SGZ}$ brings light on confinement of both bosons as fermions and to compare with literature, we shall observe the propagators in field components.

Taking components from \cite{16} we can project the propagator for the gauge field $A_\mu$:

$$< A^a_\mu(x_1) A^{b}_\mu(x_2) > = \delta^{ab} \left[ \frac{(\partial^4 + \gamma^4)(- \partial^2)}{(\partial^4 + \gamma^4)^2 - m^2(\partial^2)^2} \right] \left( \delta_{\mu \nu} - \frac{\partial_{\mu} \partial_{\nu}}{\partial^2} - \frac{im \partial^2 \varepsilon_{\mu \nu \sigma} \partial_{\sigma}}{(\partial^4 + \gamma^4)} \right) \delta^3(x_1 - x_2), \quad (19)$$

and gaugino $\lambda^\alpha$:

$$< \lambda^a_\alpha(x_1) \lambda^b_\beta(x_2) > = \frac{1}{4} \delta^{ab} \left[ \frac{(\partial^4 + \gamma^4)}{(\partial^4 + \gamma^4)^2 - m^2(\partial^2)^2} \right] \left( \partial^2 \partial_{\beta \alpha} - \frac{im(\partial^2)^3 \varepsilon_{\beta \alpha}}{(\partial^4 + \gamma^4)} \right) \delta^3(x_1 - x_2). \quad (20)$$

Despite being presented here the result of SYM-CS-GZ $D = 3$ it is also interesting to point the case of SYM-GZ $N = 1$, $D = 4$ in order to emphasize the complex pole structure. According \cite{22} the propagator for the gauge field $a_\mu$ and gaugino $\lambda^\alpha$ are:
\[
\Delta_{\alpha\mu,\nu}^c(1,2) = -\frac{2\Theta^2}{\partial^4 + \gamma^4} (\delta_{\mu\nu} - \frac{2\partial_{\mu}\partial_{\nu}}{\partial^2}) \delta^4(x_1 - x_2)
\]

(21)

\[
\Delta_{\lambda\lambda}^c(1,2) = \frac{5}{2} \frac{i\Theta^2}{\partial^4 + \gamma^4} \sigma^\mu \partial_\mu \delta^4(x_1 - x_2).
\]

(22)

It is fundamental here to stress the structure of the poles given by the introduction of the Gribov restriction. Looking for the SYM-CS-GZ $D = 3$ with $m = 0$ and SYM-GZ $D = 4$ is easy to observe the principal characteristic of the Gribov propagator ie: The complex pole structure.

\[
-\frac{2\Theta^2}{\partial^4 + \gamma^4} = \frac{1}{-\partial^2 + i\gamma^2} + \frac{1}{-\partial^2 - i\gamma^2}.
\]

(23)

These structure is fundamental in order to exclude the gluons from the physical spectrum of the theory and also turn possible the existence of local composite operators whose correlation functions exhibit the Källén-Lehmann spectral representation. At this point it is worth mentioning that this property may be related to the fact that the complex pole structure break the Osterwalder-Schrader reflection positivity condition turning impossible to obtain a Källén-Lehmann spectral representation for a single particle, or in simple terms a Källén-Lehmann representation of the two point correlation function is not positive. It is also important to note that not only the Gribov propagator breaks positivity but many works in lattice indicates that positivity is broken in the infrared limit of yang-mills theory Nevertheless it should be emphasized that proved that certain combination of composite operators could admit a positive Källén-Lehmann spectral representation. The construction of a Hilbert space for composite operators in a Gribov type model is a very difficult task and is still in initial studies.

3 Complex N= 1, BLG-ABJM like model and Gribov

The gauge symmetry in the BLG theory is generated by a Lie 3-algebra rather than a Lie algebra and $SO(4)$ is the only known example of a Lie 3-algebra. It is possible to decompose the gauge symmetry generated by $SO(4)$ into $SU(2) \times SU(2)$. In these way it is possible to write the gauge symmetry of BLG as generated by ordinary Lie algebras and the gauge sector of the theory is now given by two Chern-Simons cocycles with levels $\pm k$ and the matter fields exist in the bi-fundamental representation. The BLG theory represents two M2-branes due to the fact that its gauge symmetry is generated by the gauge group $SU(2)_k \times SU(2)_{-k}$. However, it has been possible to extend the gauge group to $U(N)_k \times U(N)_{-k}$, and the resultant theory is called ABJM theory. Let us present a typical gauge sector of two $SU(2)$ fields as it appears in a BLG-ABJM model:

\[
S_{SCS} = i \int d^3x d^2\theta \left[ (D^a \Gamma^{a\beta})(D_{\beta} \tilde{\Gamma}^a) + \frac{2}{3} ig f^{abc} \Gamma^{a\alpha} \Gamma^{b\beta}(D_{\beta} \tilde{\Gamma}^c) - \frac{1}{6} g^2 f^{abc} f^{cde} \Gamma^{a\alpha} \Gamma^{b\beta} \Gamma^{d\epsilon} \right].
\]

(24)

As we are interested in $N = 1$ supersymmetric gauge field theory with the gauge group $G \times G$, we write a second action for another gauge superfield $\tilde{\Gamma}^a$

\[
\tilde{S}_{SCS} = i \int d^3x d^2\theta \left[ (D^a \tilde{\Gamma}^{\alpha\beta})(D_{\beta} \tilde{\Gamma}^a) + \frac{2}{3} ig f^{abc} \tilde{\Gamma}^{a\alpha} \tilde{\Gamma}^{b\beta}(D_{\beta} \tilde{\Gamma}^c) - \frac{1}{6} g^2 f^{abc} f^{cde} \tilde{\Gamma}^{a\alpha} \tilde{\Gamma}^{b\beta} \Gamma^{d\epsilon} \right].
\]

(25)

And:

\[
S_{BLG}(\Gamma, \tilde{\Gamma}) = S_{SCS}(\Gamma) - \tilde{S}_{SCS}(\tilde{\Gamma}).
\]

(26)

It is interesting to note that the bilinear sector of these action

\[
S^0(\Gamma, \tilde{\Gamma}) = i \int d^3x d^2\theta \left[ (D^a \Gamma^{a\beta})(D_{\beta} \tilde{\Gamma}^a) - (D^a \tilde{\Gamma}^{a\beta})(D_{\beta} \tilde{\Gamma}^a) \right],
\]

(27)

could be written as a complex CS into a holomorphic form using two complex fields

\[
\hat{\Gamma}^a_\beta = \frac{1}{\sqrt{2}} (\Gamma^a_\beta + i\tilde{\Gamma}^a_\beta)
\]

\[
\hat{\Gamma}^{a\dagger}_\beta = \frac{1}{\sqrt{2}} (\Gamma^a_\beta - i\tilde{\Gamma}^a_\beta),
\]

(28)
in a way that:

\[
S^0(I, I^\dagger) = i \int d^3 xd^2 \theta \left[ (D^\alpha I^\alpha I^\alpha I^\alpha I^\alpha)(D_\beta I^\alpha I^\alpha I^\alpha I^\alpha I^\alpha) + (D^\alpha I^\alpha I^\alpha I^\alpha I^\alpha I^\alpha)(D_\beta I^\alpha I^\alpha I^\alpha I^\alpha I^\alpha) \right],
\]

\[
S^0(\Gamma, \tilde{\Gamma}) = S^0_{ABJM}(I, I^\dagger).
\] (29)

In fact the construction of a complex CS action is not new and Witten [14, 15] uses Morse theory in order to give consistency to a complex CS theory and define suitable integration contours in a Feynmann functional integral of a CS action with complex field or into other terms for two complex CS into a holomorphic construction.

The \( S^0(I, I^\dagger) \) corresponds to a \( U(1)_k \times U(1)_{-k} \) and is clearly a holomorphic construction or in more simple terms a real abelian action that can be constructed with two complex fields that has complex gauge transformations ie:

\[
\delta I_\alpha^a = (D_\alpha(I)\Lambda)^a,
\]

\[
\delta I_\alpha^a = (D_\alpha(I)\Lambda)^a\dagger.
\] (30)

Before proceeding to build a complex version of the \( SU(2)_k \times SU(2)_{-k} \) it is interesting to remember that the Gribov type correlator forbids a Källén-Lehmann spectral representation for a single particle but simultaneously due to the complex conjugate structure of the pole it is possible to obtain composite operators that admits a positive Källén-Lehmann spectral representation. This is the fundamental point that connects the complex holomorphic action to the confinement Gribov scenario. Now the generalization from the complex \( U(1)_k \times U(1)_{-k} \) to the complex \( SU(2)_k \times SU(2)_{-k} \) is straightforward:

\[
\delta I_\alpha^a = (\nabla_\alpha(I)\Lambda)^a
\]

\[
\nabla^{ab} = \delta^{ab} D_\alpha + gf^{abc} I^c_{\alpha}
\] (31)

and the complex conjugate

\[
\delta I_\alpha^a = (\nabla_\alpha(I)\Lambda)^a\dagger
\]

\[
\nabla^{ab}_\dagger = \delta^{ab} D_\alpha + gf^{abc} I^c_{\alpha}\dagger.
\] (32)

The combination

\[
S_{ABJM} = i \int d^3 xd^2 \theta \left[ (D^\alpha I^\alpha I^\alpha I^\alpha I^\alpha I^\alpha)(D_\beta I^\alpha I^\alpha I^\alpha I^\alpha I^\alpha) + (D^\alpha I^\alpha I^\alpha I^\alpha I^\alpha I^\alpha)(D_\beta I^\alpha I^\alpha I^\alpha I^\alpha I^\alpha) \right]
\]

\[
S_{ABJM} = S_{SCS}(I) + S_{SCS}(I^\dagger),
\] (33)

is gauge invariant, real and the bilinear sector is exactly the same as presented in [26], also is holomorphic.

It is clear that we can not be able to generate the complex pole structure without the introduction of a scale, also is important to remember that in a BLG model there are matter fields in the bifundamental representation. In these way and taking into account that it is interesting to construct the matter sector in terms of complex fields and use a symmetry breaking mechanism in order to obtain a mass for the complex gauge fields. So let us suppose two scalar complex fields, in the adjoint representation, \( \varphi^a \) and \( \varphi^{a\dagger} \) that transforms according:

\[
\delta \varphi^a = gf^{abc} I^{c\alpha} \varphi^b
\]

\[
\delta \varphi^{a\dagger} = gf^{abc} I^{c\alpha\dagger} \varphi^{b\dagger},
\] (34)

and the two complex covariant derivatives

\[
(\nabla_\alpha \varphi)^a = \nabla^{ab} \varphi^b
\]

\[
(\nabla_\alpha \varphi)^{a\dagger} = \nabla^{ab\dagger} \varphi^{b\dagger},
\] (35)
that transforms as
\[
\delta (\nabla \varphi)^a = g f^{abc} \Lambda^e (\nabla \varphi)^b \\
\delta (\nabla \varphi)^a \dagger = g f^{abc} \Lambda^e (\nabla \varphi)^b \dagger.
\]

(36)

It is possible to introduce two different gauge invariant combinations that are real in the action. First the usual combination
\[
(\nabla^\alpha \varphi)^a (\nabla \varphi)^a + (\nabla^\alpha \varphi)^a \dagger (\nabla \varphi)^a \dagger,
\]

(37)
corresponding to a cocycle in the action very similar to the case of real fields and
\[
i (\nabla^\alpha \varphi)^a (\nabla \varphi)^a - i (\nabla^\alpha \varphi)^a \dagger (\nabla \varphi)^a \dagger,
\]

(38)
which corresponds to the second possibility for these complex gauge fields that are real, with the complex factor i, and gauge invariant. It is important to emphasize here that these are the only possible combinations in the action that are real, gauge invariant and both of them are holomorphic. Clearly these two combinations can be set in the form
\[
a (\nabla^\alpha \varphi)^a (\nabla \varphi)^a + a^\dagger (\nabla^\alpha \varphi)^a \dagger (\nabla \varphi)^a \dagger,
\]

(39)
with \(a = 1 + i\) and \(a^\dagger = 1 - i\). The study of Morse theory and steepest descent for these action limits us to holomorphic real actions that can be associated to a suitable integration contour. It is also important that the same type of holomorphic structure can be applied to the potential in the scalar sector of the action. In order to stay as close as possible to the BLG-ABJM case we will limit our potential to a quartic one that admits a non zero expectation value for the scalar field. Again due to the gauge symmetry the most general candidate is:
\[
-\frac{\lambda}{4} (\varphi^a \varphi^a - \varphi^a \dagger \varphi^a \dagger)^2.
\]

(40)

It is important to note that there is another possibility for the potential, like in the kinetic term.
\[
\frac{\lambda}{4} (\varphi^a \varphi^a + \varphi^a \dagger \varphi^a \dagger)^2,
\]

(41)
this combination does not admit a non zero expectation value, so we disconsidered that. Also this cocycle is not bounded in any sense and is not acceptable as potential. With these set of values for \(a\) and \(a^\dagger\) it is important to note that the case \(a = 1\) and \(a^\dagger = 1\) which corresponds certainly to case of two real mass poles and the first possible cocycle for the kinetic term of the matter action. The cases \(a = 1 + i\) and \(a^\dagger = 1 - i\) certainly do not corresponds to real mass poles but surely to complex mass poles as it appear in Gribov correlator. This fact is related to the breaking of the Osterwal Schrader positivity condition. The breaking of positivity is fundamental in order to ensure that there is no single particle state. It is also clear that these combinations are the only possible in order to construct an action that can be real and holomorphic. So taking the holomorphic matter action as
\[
S_{\text{mix}} = \int d^3 x d^2 \theta (a (\nabla^\beta \varphi)^a (\nabla \varphi)^a + a^\dagger (\nabla^\beta \varphi)^a \dagger (\nabla \varphi)^a \dagger - \frac{\lambda}{4} (\varphi^a \varphi^a - \varphi^a \dagger \varphi^a \dagger)^2)
\]

(42)
the invariant action is given by
\[
S_{\text{inv}} = S_{\text{ABJM}} + S_{\text{mix}}.
\]

(43)
Now it is important to remember that due to the gauge symmetry it is necessary to perform a gauge fixing and following the Faddeev-Popov procedure and a Landau gauge for the two complex gauge fields we end with the gauge fixing action:
\[
S_{gf} = \frac{1}{4} s \left( \int d^3 x d^2 \theta (c^a D^a (I^a - i I^a) + c^\dagger a D^a (I^a \dagger + i I^a \dagger)) \right),
\]

(44)
where the Faddeev-Popov fields \(c^a\) and \(c^a \dagger\) are the antighost and the ghost respectively. And \(s\) is the BRST nilpotent operator \((s^2 = 0)\). It is Clear that this structure is constructed not only to write a BRST symmetry but also to define a real Gribov operator and permits the definition of the first Gribov region. It is clear that this gauge fixing,
It is easy to note that the propagator in terms of the real field $s$ must be understood in the context of consistent integration contours in a Feynman functional integral and can be taken just to access when written in terms of real fields offers the same property of the usual Landau gauge used in SYM-CS. The total action in momentum space are:

$$\Gamma = \delta_{mn} \int d^3 x d^2 \theta \left[ -\frac{1}{2} \left( \partial_{\alpha} I^\alpha_1 (D_\beta I^\beta_1) + (D_\beta I^\beta_1) (D_\beta I^\beta_1) + \gamma (a I^\alpha I^\alpha_1 + a I^\alpha I^\alpha_1) \right) \right]$$

where $j$ represents the sum over the nondiagonal directions. Remembering that $a$ and $a^\dagger$ are complex and $\gamma$ is the expectation value for two scalar fields, $\gamma = \langle \varphi^a \varphi^a \rangle = g^2$. Of course it is clear that we are taking the expectation value for the scalar field in the diagonal direction and are resuming the calculation for the $SU(2)$ group. The definition of the covariant derivatives in (31, 32) turns this calculation straightforward. It is easy to observe that the case with $a = 1$ and $a^\dagger = 1$ corresponds to two real mass poles. The most general case of a real holomorphic action is given by $a = 1 - i$ and $a^\dagger = 1 + i$ and the obtained correlator corresponds to a Gribov Zwanziger type one. The correlators in momentum space are:

$$\langle I^\alpha_1 (1) I^\alpha_1 (2) \rangle = \frac{1}{4} \delta_{\alpha\beta} \frac{2 i D^2 + (1 - i) \gamma}{p^2 - i^2 \frac{\gamma}{2}} \frac{D^2}{p^2} D_\beta D_\delta \delta^2 (\theta_1 - \theta_2) \delta^3 (x_1 - x_2),$$

$$\langle I^\alpha_1 (1) I^\alpha_1 (2) \rangle = \frac{1}{4} \delta_{\alpha\beta} \frac{2 i D^2 + (1 + i) \gamma}{p^2 + i^2 \frac{\gamma}{2}} \frac{D^2}{p^2} D_\beta D_\delta \delta^2 (\theta_1 - \theta_2) \delta^3 (x_1 - x_2).$$

It is easy to note that the propagator in terms of the real fields $\Gamma, \tilde{\Gamma}$ is real and is in the first Gribov Region. In fact in terms of real fields is easy to obtain the propagators as:

$$\langle \Gamma^\alpha_1 (1) \Gamma^\alpha_1 (2) \rangle = \frac{1}{4} \delta_{\alpha\beta} \frac{2 i D^2 p^2 + \gamma (p^2 + \frac{\gamma}{2})}{p^4 + \frac{\gamma^2}{4}} \frac{D^2}{p^2} D_\beta D_\delta \delta^2 (\theta_1 - \theta_2) \delta^3 (x_1 - x_2),$$

$$\langle \Gamma^\alpha_1 (1) \tilde{\Gamma}^\alpha_1 (2) \rangle = - \frac{1}{4} \delta_{\alpha\beta} \frac{i D^2 \gamma^2 + \gamma (p^2 - \frac{\gamma^2}{2})}{p^4 + \frac{\gamma^2}{4}} \frac{D^2}{p^2} D_\beta D_\delta \delta^2 (\theta_1 - \theta_2) \delta^3 (x_1 - x_2).$$

It is important to emphasize here that these propagators are used to calculate the two point ghost function and the Gribov pole ensure that we are at the first Gribov region due to the fact that the two point ghost function goes to infinity at the Gribov frontier. This calculation in $N = 1$ is done in [25] for real gauge field and is straightforward that those propagators imply into the first Gribov Region.

\footnote{It is always important to remember that in a complex theory like the one that we are discussing the concept of a symmetry breaking must be understood in the context of consistent integration contours in a Feynman functional integral and can be taken just to access consistently the infrared sector of the model.}
It now remains to obtain a candidate to composite operator that exhibit the Källén-Lehmann spectral representation. That operator must obey the definition in 3 dimensions:

\[
\langle O(k)O(-k) \rangle \rightarrow I(k^2, m_1, m_2) = \int \frac{d^3p}{(2\pi)^3} \frac{F(p, k - p)}{((k - p)^2 + m_1^2)(p^2 + m_2^2)}
\]

and \( I(k^2, m_1, m_2) \) can be written as

\[
I(k^2, m_1, m_2) = \int_{\mu^2}^{\infty} d\tau \rho(\tau) \left( \frac{1}{\tau + k^2} \right),
\]

with a positive spectral density.

It is well known that the Gribov correlator only admits the Källén-Lehmann spectral representation if a product of two conjugate “i-particle” appears simultaneously. Using the results obtained in [31], it is clear that a direct product of two “i-particle” correlators \( I(k^2) \) is of the form:

\[
I(k^2) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{((k - p)^2 + i\sqrt{2}g^2)(p^2 - i\sqrt{2}g^2)},
\]

which can be written as

\[
I(k^2) = \int_{\mu^2}^{\infty} d\tau \rho(\tau) \left( \frac{1}{\tau + k^2} \right),
\]

with the spectral density

\[
\rho(\tau) = \frac{1}{8\pi} \frac{1}{\sqrt{\tau}},
\]

which is positive throughout the integration range and ensure that a two "i-particles" composite operator admits a Källén-Lehmann spectral representation and this operator can be seen as a massive particle. It is also clear that due to the holomorphic structure of the action there is no mix between the two complex correlators and even taking high order loop corrections the spectral representation is always sustained. So the natural candidate for a composite operator is of the form:

\[
O = \hat{d}^2 \theta \{ iD^\gamma D^\beta I^\alpha (I^\dagger) D^\alpha D^\beta I^{\dagger \alpha} \}
\]

which, its two point correlation function, can be cast in the form of a spectral representation with positive spectral function.

\[
\langle O(k)O(-k) \rangle = \int_{\mu^2}^{\infty} d\tau \rho(\tau) \frac{\rho(\tau)}{\tau + k^2}.
\]

It is clear that this operator is not gauge invariant but the composite operator constructed from this is a candidate that offers a gauge invariant composite particle and is given by:

\[
O = \int d^3x d^2\theta \{ W^{a\alpha}(I)W^{a\alpha}_\alpha(I^\dagger) \}.
\]

It is important to stress here that due to the complex holomorphic structure of the action this properties could be extended beyond the one loop level and take into account loop corrections. It is also important to note that not all operators constructed with the complex fields are acceptable physical operators. For example the gauge sector of the action does not offer the same type of structure if we take into account a wick prescription as defined in [31]. It is clear that only a mixing with equal numbers of \( I \) and \( I^\dagger \) has the possibility to be associated to an observable. The study of all kind of observables with these type of structure will be done in a future work.

4 Connection with ABJM and SO(3,1)

A 3-algebra is a vector space with basis \( T^a \), \( a = 1, 2, ..., N \) with a triple product

\[
[T^a, T^b, T^c] = f^{abc}_{\quad d} T^d,
\]
In that case one finds equations of motion that are invariant under 16 supersymmetries and SO(8) R-symmetry. In order to construct a Lagrangean it is also necessary an inner product

\[ h^{ab} = Tr(\tilde{T}^a, T^b). \]  

(58)

In the particular case of real \( h^{ab} \) and \( f^{abc}_d \) it is clear that \( f^{abcd} = f^{abc} f^{de} \) is totally antisymmetric and it is clear that \( f^{abcd} \propto \epsilon^{abcd} \) is the only possible choice. If we assume a Lorentz type signature it is possible to define a not totally antisymmetric \( f^{abcd} \) only observing that the metric signature does not define a positive norm. In general these property reflects into states with non positive norm. This is solved by the maximally supersymmetric gauge theory. In our case it is important to note that we are already interested into an action that does not has single particle states and condensed states has positive defined norm. Taking complex generators and reducing the number of supersymmetries it is possible to define a \( f^{abcd} \) in order that \( f^{abcd} = - f^{bacd} \), \( f^{abcd} = - f^{abcd} \) and \( f^{abcd} = f^{cdab} \).

Remembering that \( SO(3, 1) \simeq [SU(2) \otimes SU(2)]/\mathbb{Z}_2 \) it is possible to define \( T^I \) and \( T^{Ii} \) in order to obtain

\[
[T^I, T^J] = i\epsilon^{ijk}T^k, \quad [T^{Ii}, T^{Jj}] = i\epsilon^{ijk}T^{ikj},
\]

\[
(f^{ij})^c_d = (1 - i)\epsilon^{ijk}(T^k)^c_d - (1 + i)\epsilon^{ijk}(T^k)^c_d,
\]

\[
(f^{0i})^c_d = (1 - i)(T^i)^c_d + (1 + i)(T^{iI})^c_d,
\]

(59)

and the gauge field \( (\tilde{A}_\mu)^c_d \) can be written as

\[
(\tilde{A}_\mu)^c_d = i(\hat{A}^I_\mu)^c_d + i(\hat{A}^{Ii}_\mu)^c_d,
\]

(60)

with the gauge symmetry given by

\[
\delta(\tilde{A}_\mu)^c_d = -i(\partial_\mu \lambda^i + \epsilon^{ijk} A^j_\mu \lambda^k)(T^i)^c_d - i(\partial_\mu \lambda^i + \epsilon^{ijk} A^j_\mu \lambda^k)(T^{iI})^c_d.
\]

(61)

This configuration applied to a Chern-Simons action

\[
\int d^3x \frac{1}{2} \epsilon^{\mu\nu\lambda} f^{abcd} \tilde{A}_{\mu\nu\lambda} \partial_\nu \tilde{A}_{\lambda cd} + \frac{2}{3} \int f^{a} g f^{gb} \tilde{A}_{\mu} \partial_\nu \tilde{A}^{\nu}_{abc} \partial_\lambda \tilde{A}^{\lambda}_{abc},
\]

(62)

gives rise to a combination of two complex Chern-Simons actions as appears in the gauge sector of \( N = 1 \) model described in the full text like

\[
\int d^3x \left[ \frac{1}{2} \epsilon^{\mu\nu\lambda} A^I_\mu \partial_\nu A^\lambda + \frac{2}{3} \epsilon^{ijk} A^I_\mu X^J_\nu X^K_\lambda + \frac{1}{2} \epsilon^{ijk} A^I_\mu \partial_\nu A^{Ii}_\lambda + \frac{2}{3} \epsilon^{ijk} A^I_\mu X^J_\nu X^K_\lambda \right],
\]

(63)

and the necessary \( 1 + i \) and \( 1 - i \) coefficients necessary for a holomorphic structure appears in directly in the gauge sector. It is clear that the very best solution is to obtain a \( N = 6 \) theory that can be splitted into a desired holomorphic action. Following the usual procedure of [16, 33] we introduce the scalar fields \( (X^I_a)_c^b \), the fermions \( (\tilde{\psi}_a)_c^b \), the gauge fields \( (\tilde{A}_\mu)_c^b \) and a pair of auxiliary scalar and fermionic fields \( (N^I_a)_c^b \), \( (F_a)_c^b \) that are defined into a \( D = 11 \) space where the indices \( I, J, K \) take values \((3, ..., 10)\) and they specify the transverse directions of the brane; \( \mu, \nu = 0, 1, 2 \) describing the longitudinal directions. The indices \( a, b, c \) take values according the Lie 3-algebra previously introduced. The candidate to supersymmetry transformations are:

\[
\delta(N^I_a)_c^b = \epsilon^{ijk} \Gamma^T \partial_\nu (F_a)_c^b,
\]

\[
\delta(F_a)_c^b = (N^I_a)_c^b \Gamma^I \epsilon,
\]

\[
\delta(X^I_a)_c^b = \epsilon^{ijk} \Gamma^T \partial_\nu (\tilde{\psi}_a)_c^b,
\]

\[
\delta(\tilde{\psi}_a)_c^b = (D_a X^I_a)_c^b \Gamma^T \Gamma^I \epsilon + ((A^I_a)_c^b \Gamma^I J + (N^K_a)_c^b \Gamma^K \epsilon,
\]

\[
\delta(\tilde{A}_\mu)_c^b = \epsilon^{ijk} \Gamma^T \partial_\nu (X^I_a)_c^b g f^{debc},
\]

(64)

with

\[
(D_a X^I_a)_c^b = (\partial_\mu X^I_a)_c^b + (\tilde{A}_\mu)_c^b (X^I_a)_c^b - (X^I_a)_c^b (\tilde{A}_\mu)_c^b,
\]

\[
(A^I_a)_c^b = (X^I_a)_c^b g f^{debc} (X^K_a)_c^b g f^{debc} (X^K_a)_c^b - (X^K_a)_c^b),
\]

(65)
The next step is to require that the above transformations close on shell. Of course at this level of the calculation we do not know the full action and the requirement is a way to obtain the for of the action. For the scalar, gauge field and the auxiliary fields we obtain:

\[
\begin{align*}
[\delta_1, \delta_2](X^I_a)_c^b &= \nu^\mu \partial_\mu (X^I_a)_c^b + (\nu^\mu (\bar{A}_\mu)_c^l + (\Lambda^I)_l^b)(X^I_a)_c^l - (\nu^\mu (\bar{A}_\mu)_c^l + (\Lambda^I)_l^b)(X^I_a)_c^l \\
[\delta_1, \delta_2](\bar{A}_\mu)_c^b &= \nu^\mu \partial_\mu (\bar{A}_\mu)_c^b + D_\mu (\nu^\mu \bar{A}_\mu + \Lambda)_c^b \\
[\delta_1, \delta_2](N^I_a)_c^b &= \nu^\mu \partial_\mu (N^I_a)_c^b \\
[\delta_1, \delta_2](F_a)_c^b &= \nu^\mu \partial_\mu (F_a)_c^b,
\end{align*}
\]

where \(\nu^\mu = 2\tau_i \Gamma^\mu \varepsilon\) and \(A^b_c = 2\tau_i \Gamma^{MN} \varepsilon (X^M_d)_f ^g (X^N_c)_g ^f f^{debc}\) however only if the following equations of motion are observed

\[
\begin{align*}
(N^I_a)_c^b + 2(A^{IJ}_a)_c^b &= 0 \\
(\bar{F}_{\mu\nu})^b_c - \frac{i}{2} \epsilon_{\mu\nu\lambda}(X^I_d)_f ^g (X^N_c)_g ^f f^{debc}\ &= 0 \\
- \frac{1}{4} \epsilon_{\mu\nu\lambda}(\bar{\psi})_f ^g D\lambda(\psi_c)_g ^f f^{debc} - \eta_{\mu\nu}(X^I_d)_f ^g (N^I_a)_f ^g + 2(A^{IJ}_a)_f ^g f^{debc} \ &= 0 \\
\Gamma^\mu \partial_\mu (F_a)_c^b &= 0.
\end{align*}
\]

The first equation is for the lagrange multiplier \((N^I_a)_c^b\) and the main expectation is that when integrated we could recover \(N = 8\). The second equation is the gauge equation and the last one is for the auxiliar fermion.

Unfortunately the requirement of \(N = 6\) is not observed for the fermion sector \(\psi\) due to the algebra of \(\Gamma^I\) matrices and the requirement that \(f^{debc}\) is not totally antisymmetric. Of course if we relax the requirement of \(N = 6\) and \(N = 8\) it is possible to generate the desired holomorphic action in order to obtain a Gribov type propagator. It is important to stress now that the requirement of a maximally supersymmetric action is not necessary due to the non existence of single particle states into these model and the fact that the holomorphic structure generates condensed states with positive norm. relaxing the need for superconformal theory the holomorphic construction gave us the structured action presented directly into superfield \(N = 1\) formalism.

5 Conclusions

In this work we have studied the complex pole structure of the Gribov propagator and the possibility of obtaining this pole structure from a complex holomorphic action, also it is presented the most general complex scalar gauge invariant holomorphic action. Due to a symmetry breaking mechanism for this complex scalar action the characteristic Gribov pole structure is obtained in the case of the most general kinetic holomorphic cocycle of the scalar action. Also a good candidate to a composite operator in order to generate a Källén-Lehmann spectral representation that has positive spectral density, which characterize a composite particle state, is obtained. It is interesting to note that the holomorphic structure is fundamental in order to introduce interaction terms in the definition of the composite operator and perform loop corrections without a mix between correlators that destroy the spectral representation. This mechanism could be fundamental in the study of confinement in the Gribov scenario and will be matter for a future work.

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A Notation, conventions and some useful formulas

We work with Euclidean metric: diag(+++). So we choose the gamma matrices being the Pauli matrices $\sigma_i$ [32]:

$$\gamma^\mu \equiv (\sigma_\mu)_\alpha^\beta$$

which are OS self-conjugate and:

$$\{\sigma^\mu, \sigma^\nu\} = 2\delta^{\mu\nu}I,$$  \hspace{1cm} (69)

$$[\sigma^\mu, \sigma^\nu] = 2i\epsilon^{\mu\nu\sigma\tau}\sigma^\tau.$$  \hspace{1cm} (70)

The invariant anti-symmetric tensor is defined as

$$\varepsilon^{-+} = \varepsilon_{-+} = +1,$$  \hspace{1cm} (71)

$$\varepsilon^{\gamma\beta}\varepsilon_{\beta\alpha} = -\delta_\alpha^\gamma,$$  \hspace{1cm} (72)

and are used to raise and lower indices:

$$\psi^\alpha = \varepsilon^{\alpha\beta}\psi_\beta,$$  \hspace{1cm} (73)

$$\psi_\alpha = \psi^\beta\varepsilon_{\beta\alpha}.$$  \hspace{1cm} (74)

The representation of differential operator of the generators of super algebra in D=3, with the concept of graded Majorana [32]:

$$Q_\alpha = -\partial_\alpha + \partial_{\alpha\beta}\theta^\beta,$$  \hspace{1cm} (75)

with

$$\partial_{\alpha\beta} = i\sigma^{\alpha\gamma}\varepsilon_{\gamma\beta}\partial_\mu.$$  \hspace{1cm} (76)

As well as the superspace derivative:

$$D_\alpha = \partial_\alpha + \partial_{\alpha\beta}\theta^\beta,$$  \hspace{1cm} (77)

with the following relations:

$$\{D_\alpha, D_\beta\} = 2\partial_{\alpha\beta},$$  \hspace{1cm} (78)

$$[D_\alpha, D_\beta] = -2\varepsilon_{\alpha\beta}D^2,$$  \hspace{1cm} (79)

$$D^\beta D_\alpha = \partial_{\alpha\beta} - \varepsilon_{\alpha\beta}D^2,$$  \hspace{1cm} (80)

$$D^\beta D_\alpha D_\beta = 0.$$  \hspace{1cm} (81)

$$[Q_\alpha, D_\beta] = 0.$$  \hspace{1cm} (82)

Another useful relations:

$$\partial_{\alpha\beta}\partial^{\alpha\gamma} = \partial^2\delta^\gamma_\beta,$$  \hspace{1cm} (83)

$$(D^2)^2 = -\partial^2,$$  \hspace{1cm} (84)

$$\int d^2\theta = -\frac{1}{4}D^2.$$  \hspace{1cm} (85)