A. $O(w)$ bitvector merging algorithm

The DP extension algorithm requires merging bitvectors at nodes with an in-degree of at least two. In our previous work Rautiainen et al. (2019) we described an $O(\log w)$ algorithm for merging two bitvectors. This was based on finding difference masks between the bitvectors, describing indices where one bitvector was greater than the other and vice versa.

We reuse the notation from our previous work. A column $A$ consists of $w$ cells (where $w = 64$ in practice), each of which has an integer value. We notate the value at the $i$'th cell as $S^A_i$. The score difference between two adjacent cells is $S^A_i - S^A_{i-1} \in \{-1, 0, 1\}$. This enables the column to be encoded by two bitvectors $V_P^A$ for indices where $S^A_i = S^A_{i-1} + 1$ and $V_N^A$ for indices where $S^A_i = S^A_{i-1} - 1$ and a score at end $S^A_{end} = S^A_{w-1}$, which can be used to calculate the score before the first cell $S^A_b = S^A_{end} + \text{popcount}(V_N^A) - \text{popcount}(V_P^A)$.

Given two columns $A$ and $B$, we want to find two difference masks $M_{A>B}$ and $M_{B>A}$, where $M_{A>B}$ is set in indices where $S^A > S^B$ and $M_{B>A}$ is set in indices where $S^B > S^A$. The idea is to first create a difference bitvector of the difference between the two bitvectors, which describes how the score difference $S^A - S^B$ changes at each index.

The difference bitvector consists of four bitvectors $\text{twosmaller}$, $\text{onesmaller}$, $\text{onebigger}$ and $\text{twobigger}$, which correspond to the cases where $(S^A_i - S^B_i) - (S^A_{i-1} - S^B_{i-1}) \in \{-2, -1, 1, 2\}$ respectively. Given the difference bitvector, we find the first index where the score difference $S^A - S^B$ increases, and the first index where the $S^A - B^A$ decreases. Assume without loss of generality that we have two bitvectors with the same score before the first cell $S^A_b = S^B_b$, the first (least significant) index $i$ where the difference $S^A - S^B$ increases and the first index $j$ where the difference $S^A - S^B$ decreases, with $i < j$. Then, regardless of the scores elsewhere in the bitvector, $S^A_x > S^B_x$ in indices $i \leq x < j$. Using this idea, we can create an algorithm which calculates at least one index of the difference masks.
in each iteration. The algorithm repeatedly takes the smallest indices i and j where
the difference increases and decreases, sets the difference masks in (i, j), then unsets
the difference bitvectors at the indices i and j. Once the smaller bitvector has no more
set bits, the remaining masks are set according to whether the bigger bitvector has
bits left, and similarly vice versa. This results in an algorithm with a runtime of \(O(w)\).

The above assumed that the scores before the first cells are equal, that is, \(S^A_b = S^B_b\).
This is not always true. To solve this corner case, we run a preprocessing step where
we pretend that the difference bitvectors have extra bits set "before" the first cell,
and run the bit unsetting described above on those extra bits. The source code of the
merge operation in C++ is below.

```cpp
//Word is an unsigned integer of arbitrary width, in practice uint64_t
template <typename Word>
std::pair<Word, Word> differenceMasks(Word VP_A, Word VN_A, int Sb_A, Word VP_B, Word VN_B, int Sb_B) {
    Word ASmaller = 0;
    Word BSmaller = 0;
    // unset bits which are common in both bitvectors
    // that is, both bitvectors increase or decrease
    Word VPCommon = ~(VP_A & VP_B);
    Word VNCommon = ~(VN_A & VN_B);
    VP_A &= VPCommon;
    VN_A &= VNCommon;
    VP_B &= VPCommon;
    VN_B &= VNCommon;
    // difference bitvectors describing the score change at each index
    Word twosmaller = VN_A & VP_B; // A is two smaller
    Word onesmaller = (VP_B & ~VN_A) | (VN_A & ~VP_B);
    Word onebigger = (VP_A & ~VN_B) | (VN_B & ~VP_A);
    Word twobigger = VN_B & VP_A; // A is two bigger
    onebigger |= twobigger;
    onesmaller |= twosmaller;
    // preprocessing: handle the cases where Sb_B != Sb_A
    if (Sb_B > Sb_A) {
        for (int i = 1; i < Sb_B - Sb_A; i++) {
            Word leastSignificant = onebigger & ~(onebigger - 1);
            onebigger ^= (~twobigger & leastSignificant);
            twobigger &= ~leastSignificant;
            if (onebigger == 0) {
                return std::make_pair(-1, 0);
            }
        }
    }
    return std::make_pair(0, 0);
}
```
Word leastSignificant = onebigger & ~(onebigger - 1);
ASmaller |= leastSignificant - 1;
onebigger = (~twobigger & leastSignificant);
twobigger &= ~leastSignificant;
}
else if (Sb_A > Sb_B)
{
    for (int i = 1; i < (Sb_A - Sb_B); i++)
    {
        Word leastSignificant = onesmaller & ~(onesmaller - 1);
onesmaller = (~twosmaller & leastSignificant);
twosmaller &= ~leastSignificant;
        if (onesmaller == 0)
        {
            return std::make_pair(0, -1);
        }
    }

    Word leastSignificant = onesmaller & ~(onesmaller - 1);
    BSmaller |= leastSignificant - 1;
onesmaller = (~twosmaller & leastSignificant);
twosmaller &= ~leastSignificant;
}

// unset bits one at a time
for (int i = 0; i < sizeof(Word) * 8; i++)
{
    if (onesmaller == 0)
    {
        if (onebigger == 0) break;
        Word leastSignificant = onebigger & ~(onebigger - 1);
        BSmaller |= ~leastSignificant;
        break;
    }
    if (onebigger == 0)
    {
        assert(onesmaller != 0);
        Word leastSignificant = onesmaller & ~(onesmaller - 1);
        ASmaller |= ~leastSignificant;
        break;
    }

    Word leastSignificantBigger = onebigger & ~(onebigger - 1);
    Word leastSignificantSmaller = onesmaller & ~(onesmaller - 1);
    assert((onebigger & leastSignificantBigger) != 0);
    assert((onesmaller & leastSignificantSmaller) != 0);
    assert(leastSignificantSmaller != leastSignificantBigger);
Figure S1: A tangled subgraph of a whole human genome de Bruijn graph. Visualized with Bandage Wick et al. (2015).

```cpp
assert (leastSignificantSmaller != 0);
assert (leastSignificantBigger != 0);
if (leastSignificantBigger > leastSignificantSmaller)
{
    ASmall = leastSignificantBigger - leastSignificantSmaller;
}
else
{
    BSmall = leastSignificantSmaller - leastSignificantBigger;
}
onebig = (~twobig & leastSignificantBigger);
twobig &~ leastSignificantBigger;
onesm = (~twosm & leastSignificantSmaller);
twosm &~ leastSignificantSmaller;
assert ((ASmall & BSmall) == 0);
assert (onesm == 0 || onebig == 0);
return std::make_pair(ASmall, BSmall);
}```
Figure S2: An alignment produced by vg Garrison et al. (2018) which is inconsistent with graph topology. A read simulated by PBSIM Ono et al. (2013) was aligned with vg to a variant graph of the chromosome 22 created by vg. The alignment was then visualized with vg, and manually cropped to a position containing two consecutive SNPs. The solid boxes represent nodes in the graph, and the thick black lines between the corners are edges in the graph. There are two SNPs adjacent to each others, the first with alleles A and C, and the second with alleles T and C. The alignment is represented by the blue and yellow texts connected by the thin black lines. The alignment passes through both branches of both SNPs, in fact covering the allele A in the leftmost SNP three times, and covers the flanking regions multiple times as well. Since the graph is acyclic, this alignment cannot be consistent with any path in the graph.
References

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