Quasideuteron states with deformed core

A. F. Lisetskiy\textsuperscript{1}, A. Gelberg\textsuperscript{1}, R. V. Jolos\textsuperscript{1,2}, N. Pietralla\textsuperscript{1,3}, P. von Brentano\textsuperscript{1}

\textsuperscript{1} Institut f"{u}r Kernphysik, Universit"{a}t zu K"{o}ln, 50937 K"{o}ln, Germany
\textsuperscript{2} Bogoliubov Theoretical Laboratory, Joint Institute for Nuclear Research, 141980 Dubna, Russia
\textsuperscript{3} Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut 06520-8124, USA

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Abstract

The M1 transitions between low-lying $T=1$ and $T=0$ states in deformed odd-odd $N=Z$ nuclei are analyzed in the frames of the rotor-plus-particle model. Using the representation of an explicit coupling of angular momenta we show that strong coupling of the quasideuteron configurations to the axially deformed core results in a distribution of the total $0^+ \rightarrow 1^+$ strength among a few low-lying $1^+$ states. Simple analytical formulae for $B(M1)$ values are derived. The realization of the M1 sum rule for the low-lying $1^+, T=0$ states is indicated. The calculated $B(M1)$ values are found to be in good agreement with experimental data and reveal specific features of collectivity in odd-odd $N=Z$ nuclei.
Due to the charge independence of the nuclear force, the isovector (T=1) neutron-proton (n-p) and like-nucleon (n-n and p-p) interactions have to be indistinguishable, as proved by the existence of isospin multiplets of nuclei. However a p-n pair can also exist in a T=0 state for which there is no need for equality with the T=1 n-p, p-p and n-n forces.

To understand the nuclear interaction in the T=0 channel considerable effort has recently been made but there are still substantial difficulties in understanding the T=0 n-p correlations \[1–5\]. However many features of the nuclear structure in the T=0 channel can be studied without reference to the peculiarity of the T=0 interaction. From this perspective the odd-odd N=Z nuclei, which are unique systems with both T=1 and T=0 modes coexisting at low energies, are the best laboratory to study the isospin antisymmetric (T=0) states and the transitions between T=1 and T=0 modes.

The remarkable fact that the magnetic dipole moments of the free proton (μ_π = +2.79μ_N) and the free neutron (μ_ν = −1.91μ_N) are comparable in magnitude and of opposite sign implies that the magnetic dipole (M1) transition operator is the most appropriate tool for the investigation of the transitions from T=0 to T=1 states. Moreover the M1 operator is very sensitive to the relative orientation of single nucleon spin (s) and orbital angular momentum (l) as is well illustrated by the familiar Schmidt values \[6\] for magnetic dipole moments of odd-mass nuclei. The observables which are most sensitive to the relative orientation of spin and orbital angular momentum are B(M1) values for isovector transitions between quasideuteron states in odd-odd N=Z nuclei \[7\]. The quasideuteron states were defined in \[7\] as one-proton-one-neutron [πj × νj]_{M}^{J,T} configurations in a single j orbital coupled to an inert even-even N=Z core nucleus in its ground state J=0^+, T=0. The [πj × νj]_{M}^{J,T} multiplet of states splits into two sequences with different isospin symmetry: T=0, (J - odd) and T=1,( J - even). The states with isospin quantum number T=0 and T=1 and spins J and J+1 are connected by M1 transitions. The B(M1) values for these transitions are given by simple analytical formulae \[7\]:

\[ B(M1; J \rightarrow J + 1) = \frac{3}{4\pi} \frac{J + 1}{2J + 1} \binom{j + 1 + J/2}{j - J/2} (g_{IV}^j)^2 \mu_N^2, \] (1)

where \[ g_{IV}^j = g_p^j - g_n^j = \frac{l + \alpha_q 4.706}{j} \] for \[ j = l + 1/2 \] (2)

and \[ g_{IV}^j = \frac{l + 1 - \alpha_q 4.706}{j + 1} \] for \[ j = l - 1/2, \] (3)

where the values of the orbital g-factors are taken to be bare \((g_p^{l,bare} = 1, g_n^{l,bare} = 0)\) and \(\alpha_q\) is a quenching factor for the spin bare g-factors \((g_p^{s,bare} = 5.58 \text{ and } g_n^{s,bare} = -3.82)\). The positive interference of large spin and orbital parts of the isovector ∆T=1 M1 reduced matrix elements in the \(j = l + 1/2\) case was shown to cause a strong enhancement of ∆T=1 M1 transitions between quasideuteron states with \(j = l + 1/2\) in odd-odd N=Z nuclei. Actually the strongest known M1 \(0^+ \rightarrow 1^+\) transitions between low-lying nuclear states are observed in odd-odd N=Z nuclei in the lower part of p-, sd- and pf-shells where the \(j = l + 1/2\) orbitals play a dominant role. On the contrary, for the odd-odd N=Z nuclei in the upper part of the p- and sd-shells, the M1 \(0^+ \rightarrow 1^+\) transitions are strongly suppressed due to the destructive

\[\]
interference of spin and orbital parts of the M1 matrix elements for one-proton-one-neutron configurations in a single \( j = l - 1/2 \) orbital.

Analyzing the strong M1, \( 0^+_1 \rightarrow 1^+_1 \) transitions in odd-odd N=Z nuclei we noted that in \(^6\text{Li}, ^{19}\text{F}\) and \(^{42}\text{Sc}\) the total M1 transition strength from the yrast \( 0^+, T = 1 \) state to the \( 1^+, T = 0 \) states is concentrated in the \( 0^+_1 \rightarrow 1^+_1 \) transition. This suggests that the structure of the low-lying states in these nuclei is dominated by simple quasideuteron configurations with \( j = l + 1/2 \). In deformed nuclei, e.g., \(^{10}\text{B}, ^{14}\text{N}, ^{22}\text{Na}\) and \(^{26}\text{Al}\) the total M1 strength is fragmented among two or three low-lying states indicating a more complex structure.

The aim of the present paper is to extend the investigation of the spherical shell model quasideuteron configurations to an axially deformed mean field and to explore the fragmentation mechanism of the isovector M1 strength in odd-odd N=Z nuclei. We will show that the low-energy structure of the deformed odd-odd N=Z nuclei where strong M1 transitions are observed could be reasonably well understood in terms of a rotor-plus-quasideuteron model and that the electromagnetic properties of the low-lying states are strongly affected by the deformed mean field.

The basic assumption of the model \(^8,9\) which we apply in this paper to the deformed odd-odd N=Z nuclei is that one has one proton and one neutron outside of an even-even deformed rotating core. We consider the simplified version of this model neglecting the Coriolis interaction and the residual interaction between the odd proton and odd neutron. Then the rotational motion of the nucleus is specified by the quantum numbers \( JMK \) and the total wave function has the form appropriate to a rotationally invariant system with axial symmetry which also posses the signature symmetry \(^10\):

\[
|JMKT\rangle = \frac{\sqrt{2J+1}}{16\pi^2(1+\delta_{K,0})} \left[ D^J_{MK} \Phi_{K,T} + (-1)^{J+K} D^J_{M-K} \Phi_{-K,T} \right],
\]

where \( \Phi_{K,T} \) is the wave function in the intrinsic system:

\[
\Phi_{K,T} = \frac{1}{\sqrt{2(1+\delta_{1,2})}} \left[ u_\pi(1)u_\nu(2) + (-1)^T u_\pi(2)u_\nu(1) \right] \cdot \zeta^{T=0}_{T_z=0}(1,2),
\]

where \( u_\rho(i) \) are single particle eigenfunctions of the Nilsson Hamiltonian with \( \Omega_i \) the 3-projection of particle angular momentum, \( K = \Omega_1 + \Omega_2 \), \( \zeta^{T=0}_{T_z=0}(1,2) \) - isospin wave function with the isospin quantum number \( T \). The states belonging to a \( K = 0 \) band have only even (odd) spins for the signature quantum number \( r = +1 \) \((-1)\). It can be shown that states belonging to the \( T = 1 \) \((T = 0)\) band have \( r = +1 \) \((-1)\). The coupling of the angular momenta of the odd proton, the odd neutron and the rotor that is implicit in Eq.\((4)\) can be exhibited by transforming to the representation of explicit coupling of angular momenta \(^{10}\) appropriate for a strongly coupled system. This representation, which can be treated also as an algebraic representation \(^{11}\), allows one to work with spherical shell model configurations and to study the interplay between different possible degrees of freedom generating M1 transitions.

As a starting point we use particle plus-rotor-model basis states written in terms of spherical single-particle wave functions in a strong coupling approximation \(^{10,11}\):

\[
|JMK\rangle = \sum_{R,j} \sqrt{(1+\delta_{KR,0})(2R+1)} \frac{C_{JK}^{KK} \Omega \chi^j_{j}}{2J+1} |R\rangle \otimes |j\rangle^J_M,
\]
where $\chi_{j}^{\Omega}$ are projection coefficients of single particle Nilsson $[Nn_{z}\Lambda]\Omega$ orbitals on the spherical single particle $|nlj\Omega\rangle$ basis [12]:

$$|Nn_{z}\Lambda;\Omega\rangle = \sum_{j=\Omega}^{N+1/2} \chi_{j}^{\Omega}|nlj\Omega\rangle,$$

(7)

$C_{RJKl_{p}\Omega}^{J_{K}K_{l_{p}}}^{J_{K}K_{l_{p}}}$ are Clebsch-Gordan coefficients, $R$ is the core angular momentum quantum number and $[|R\rangle \otimes |j\rangle]_{M}^{J} = \sum_{M_{R},m} C_{R_{M}R_{j}m}^{JM} |RM_{R}\rangle \cdot |nm_{j}\rangle$ are weakly coupled (SU(2) coupling) rotor-plus-particle states. The wave functions for two particle states coupled to the $K_{R} = 0,T=0$ rotational core can be easily constructed applying Eq.(6). After some simple transformations one obtains:

$$|JMKT\rangle = \sum_{R,J} \sqrt{\frac{2(2R+1)}{2J+1}} C_{RJKl_{p}T}^{J_{K}K_{l_{p}}}^{J_{K}K_{l_{p}}} \left[|R\rangle \otimes |j\rangle\right]_{M_{T_{z}}=0}^{JT},$$

(8)

where the $|J_{q}\rangle$ is a one-proton-one-neutron state in the deformed field:

$$|J_{q}\rangle \equiv |J_{q}M_{q}KT\rangle = \sum_{j_{1},j_{2}} \chi_{j_{1}}^{\Omega_{1}} \chi_{j_{2}}^{\Omega_{2}} C_{j_{1}j_{2}l_{p}T_{l_{p}}}^{J_{q}L_{p}K_{l_{p}}} \left[|j_{1}\rangle \otimes |j_{2}\rangle\right]_{M_{q}T_{z}=0}^{JT},$$

(9)

and $\Omega_{i}$ is the Nilsson quantum number of angular momentum projection on the symmetry axis for the odd proton and the odd neutron.

To calculate M1 matrix elements we start with a nuclear magnetic dipole operator, which is the sum of proton and neutron one-body terms for spin and orbital contributions:

$$T_{R}(M1) = \sqrt{3} \sum_{i=1}^{Z} \left[ g_{l_{i}}^{P} l_{i}^{P} + g_{s_{i}}^{P} s_{i}^{P} \right] + \sum_{i=1}^{N} \left[ g_{l_{i}}^{N} l_{i}^{N} + g_{s_{i}}^{N} s_{i}^{N} \right] \mu_{N},$$

(10)

where $g_{l_{i}}^{P}$ and $g_{s_{i}}^{P}$ are the orbital and spin $g$-factors and $l_{i}^{P}$, $s_{i}^{P}$ are the single particle orbital angular momentum operators and spin operators. For the purposes of our paper it is convenient to rewrite the expression for the M1 transition operator in another form:

$$T_{R}(M1) = T_{R}(M1) + T_{q}(M1),$$

(11)

where the $T_{R}(M1)$ is an M1 operator for the even-even $N=Z$ core and $T_{q}(M1)$ is the M1 operator for the odd proton and odd neutron subsystem. Since only the $T = 0, K = 0$ states of the even-even $N=Z$ core nucleus are assumed to be taken into account, and the angular momentum quantum number $R$ can be only even, the core does not contribute to the $\Delta T=1$ M1 matrix elements. Taking this into account we can calculate directly the matrix elements of the $T_{q}(M1)$ operator defined in the laboratory coordinate frame using the wave functions given by Eq.(8). Using the general reduction formula for the reduced matrix elements [13] and performing summation over all possible values of $R$ we get:

$$\langle JKT||T(M1)||J'K'T'\rangle = \sqrt{2J+1} C_{J_{K_{1}l_{p}}K'_{1}l_{p}}^{J'_{K_{1}l_{p}}K'_{K_{1}l_{p}}} \sum_{J_{q},J'_{q}} (-1)^{J_{q}} C_{J_{q}l_{p}K'_{l_{p}}}^{J_{q}l_{p}K'_{l_{p}}} (J_{q}T||T_{q}(M1)||J'_{q}T'\rangle \sqrt{2J_{q}+1},$$

(12)
where $\nu = K' - K$. This expression clearly shows that configurations with various possible $J_q$ values contribute to the total $M1$ 0$^+ \rightarrow 1^+$ m.e. with the weight given by the familiar Clebsch-Gordan coefficient $C_{j_0 K}^{j' K'}$. We consider further a particular case assuming that the initial state is characterized by $K=0$ and that $T=1$ ($J$ even). Then, using Eq. (11) and well known properties of Clebsch-Gordan coefficients and $6-j$ symbols, we get the following formula for the $B(M1)$ values:

$$B(M1; J, K = 0 \rightarrow J', K' = \Omega'_1 + \Omega'_2) = \frac{\mu_N^2}{4\pi} \left[ C_{j_0 K}^{j' K'} \sum_{j_1 j'_1} \chi_{j_1}^{\Omega_1} \chi_{j'_1}^{\Omega'_1} C_{j_1 j'_1 \Omega_1 \Omega'_1}^{1 K' \Omega'} M_{j j'_1} \right]^2,$$  

(13)

where $M_{j j'_1} = \sqrt{j(j+1)(2j+1)} g_{IV}^{j j'}$ for $j' = j$ i.e., for quasideuteron configurations and

$$M_{j j'_1} = \sqrt{l(l+1)(2l+1)} \frac{(g_{IV}^j - g_{IV}^{j'})}{2}$$

for spin-orbit partner orbitals with $j = l \pm 1/2, j' = l \mp 1/2$. The difference of proton and neutron $g$-factors ($g_{IV}^j = g_p^j - g_n^j$) is given by Eqs. (12) and (13). $\Omega$ and $\Omega'$ are 3-projections of particle angular momentum for the Nilsson $[Nn\Lambda]\Omega$ and $[Nn'\Lambda']\Omega'$ orbitals used for the construction of the final and initial states involved in the transition. The terms with $j = j'$ in Eq. (13) represent the individual contributions of two nucleon configurations in the single-$j$-orbital while the terms with $j \neq j'$ are related to the single particle isovector spin-flip ($j = l \pm 1/2 \rightarrow j' = l \mp 1/2$) mechanism. It is interesting to note that the isovector spin-flip part is proportional to the difference of the isovector $g$-factors for spin-orbital partner orbitals, i.e. it is itself decomposed into two different parts produced by the quasideuteron configurations with $j = l + 1/2$ and $j' = l - 1/2$ orbitals.

When the Fermi surface coincides with the Nilsson level with $\Omega \neq 1/2$ then it can be stated for a certainty that the lowest 1$^+$ state in the odd-odd N=Z nucleus is a bandhead of a $K = 0, T = 0$ ($r = -1$) band [3]. This means that both initial $J$ and final $J' = J + 1$ states are characterized by $K' = K = 0$ quantum number (i.e. $\Omega' = \Omega$ in Eq. (13)) and the expression (13) for the $B(M1)$ values reduces to the following analytical form:

$$B(M1; J \rightarrow J + 1) = \frac{3}{4\pi} \Omega^2 \mu_N^2 \frac{J + 1}{2J + 1} \left\{ \sum_j \chi_j^{\Omega} \left[ \chi_j^{\Omega} \mp \frac{\chi_j^{\Omega+1}}{\sqrt{2}} \left( \frac{l + 1/2}{\Omega} \right)^2 \right] g_{IV}^j \right\}^2,$$  

(15)

where the upper sign is to be used for the $j = l+1/2$ and the lower sign for the $j = l-1/2$ case. The property of the “spin-flip” m.e. given by Eq. (14) allows to represent the total $M1$ m.e. as a sum of partial contributions of a single-$j$-orbitals. These individual contributions are proportional to the isovector $g_{IV}^j$ factor as in the familiar case of two nucleon configurations in a single-$j$-orbital [see Eq. (14)]. The $\chi_j^{\Omega}$ coefficients were calculated using the deformation parameter $\beta_{eff}$ deduced (see [10]) from known $B(E2; 2^+ \rightarrow 0^+)$ values (see Table I). For

1If $\Omega = 1/2$ then the Coriolis interaction mixes $\Omega = 1/2$ and $\Omega = -1/2$ single particle states and subsequently $K=1$ and $K=0$ bands with $T=0$. 

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the resulting Nilsson wave functions we use the following short notation $|lj,\Omega\rangle$, where $lj$ indicates the dominant spherical orbital in the Eq. (7). Finally, $B(M1; 0_{1+} \rightarrow 1_{1+}^\pm)$ values for the deformed odd-odd $N=Z$ nuclei with the $1_{1+}^\pm$ state characterized by the $K=0$ are collected in the Table I. The experimental $B(M1; 0_{1+}^+ \rightarrow 1_{1+}^\pm)$ values for $^{46}$V [14,15], $^{50}$Mn [16] and $^{54}$Co [17] were taken as $B(M1; 0_{1+}^+ \rightarrow 1_{1+}^\pm) = D^{th}B(M1; 3_{1+}^+ \rightarrow 2_{1+}^+)$ were $D^{th}$ is the ratio of the calculated $B(M1)$ values: $D^{th} = B^{th}(M1; 0_{1+}^+ \rightarrow 1_{1+}^\pm)/B^{th}(M1; 3_{1+}^+ \rightarrow 2_{1+}^+)$. These data are plotted in Figure I giving also the predictions for the cases where the $g_{9/2}$ orbital (N=4) is expected to be dominant.

First, we conclude from Table I and Figure I a surprisingly good agreement of the experimental data with the theoretical results. It indicates that to a large extent, the structure of the low-lying states in the odd-odd $N=Z$ nuclei considered, is determined by the deformed mean field.

Second, we note from Eq.(13) the proportionality of the $B(M1)$ values to $\Omega^2$. This theoretical result is clearly supported by the known experimental data as well as by full pf-shell model calculations with the KB3 residual interaction for $^{46}$V [14] and $^{50}$Mn [18]. Following the predictions of the rotor-plus-quasideuteron model (see Table I) one finds that the ratio of $B(M1; 0_{1+}^+ \rightarrow 1_{1+}^\pm)$ values in $^{46}$V and $^{50}$Mn is 0.45 that is very close to the shell model value of 0.44 [14,18]. It shows that $B(M1)$ values can provide us with additional information on collective states in odd-odd $N=Z$ nuclei which can not be obtained from the $B(E2)$ values.

When one of the two nucleons occupying a Nilsson orbital with $\Omega \neq 1/2$ lying on the Fermi surface is moved to the higher orbital with $\Omega' = \Omega \pm 1$ or when one of the nucleons from the lower lying Nilsson orbital is moved to the orbital on the Fermi surface, then one can construct other low-lying $1^+$ states which are the bandheads of $K=1, T=0$ bands. Using Eq.(13) for the $M1$ m.e. we have calculated $B(M1; 0_{1+}, K = 0 \rightarrow 1_{1+}, K = 1)$ values for some of the odd-odd $N=Z$ nuclei. We collect our predictions in Table I to compare with the known experimental data. While the quality of the agreement with experiment is good, it is worse than for the $K=0$ $1_{1+}^\pm$ states discussed above. This indicates larger admixtures of other configurations than in the $K=0$ $1_{1+}^\pm$ case. From Table I one can see that if the Nilsson orbital with $\Omega' = \Omega \pm 1$ contains also a large component with $j = l + 1/2$ (similarly to the orbital with $\Omega$ quantum number) then one gets a large strength of the $\Delta K=1, 0^+ \rightarrow 1^+_1$ transition. This strength is comparable with the strength of the $\Delta K=0, 0^+ \rightarrow 1^+_1$ transition (see, for example, results for $^{22}$Na) discussed above. Moreover if one sums the strengths of $0^+ \rightarrow 1^+_1$ transitions with $\Delta K=1$ and $\Delta K=0$, one gets the value which is approaching the one given by the Eq. (1) for the quasideuteron spherical configurations. The simplest way to see the realization of a kind of quasideuteron sum rule mentioned above is to consider the deformed single j orbital approximation. In this case the $\chi_j^{\Omega}$ coefficients with $j \neq N+1/2$ vanish while $\chi_j^{\Omega}_{j=N+1/2} = 1$. Then using Eq. (13) we get

$$B(M1; 0_{1+},(T=1,K=0) \rightarrow 1_{1+},(T=0,K=0)) = \frac{3}{4\pi} \Omega^2 (g^j_{IV})^2 \mu^2_N, \quad \text{and} \quad (16)$$

$$B(M1; 0_{1+},(T=1,K=0) \rightarrow 1_{1+},(T=0,K=0)) = \frac{3}{8\pi} (j + \Omega_{2,3}) (j - \Omega_{2,3} + 1) (g^j_{IV})^2 \mu^2_N, \quad (17)$$

where $\Omega_{2,3} = 1 \pm |\Omega|$. Summing up the strengths for three $1^+$ states we obtain.
\[
\sum_{i,K} B(M1; 0^+_1, (T=1, K=0) \rightarrow 1^+_i, (T=0, K)) = \frac{3}{4\pi} j(j+1) (g^j_{IV})^2 \mu^2_N,
\] (18)

that is exactly the same as the expression yielded by the Eq. (11) for the J=0 case. This exercise shows that one of the consequences of deformation is a splitting of the quasideuteron states, i.e. the splitting of the single particle states and their coupling to the different spins of the deformed core result in the appearance of a few low-lying \(1^+\), \(T = 0\) states connected with the lowest \(0^+, T = 1\) state by comparably strong M1 transitions. Another effect caused by the deformation is the mixing of the different single \(j\) orbitals. This leads to the modification of the sum rule discussed above but it does not change substantially the whole picture – the main fragments of the quasideuteron strength given by Eq. (18) will be still concentrated in a few lowest \(1^+\) states. As an example we present in Table I the results of the exact calculations using Eq. (13) for M1 m.e. and calculations in the deformed single-\(j\)-orbital approximation using Eqs. (16) and (17).

Very interesting specific case is the one when the odd proton and the odd neutron occupy Nilsson orbital with \(\Omega = N + 1/2\). Then only one spherical component, namely with \(j = N+1/2\), contributes to the Nilsson single particle state (see Eq. (1)) and subsequently the \(B(M1; 0^+_1 \rightarrow 1^+_i)\) value is insensitive to the deformation and is given by Eq. (16). Moreover using Eq. (13) one can prove that in this case the sum of \(B(M1; 0^+_1 \rightarrow 1^+_i)\) values for all possible \(1^+, T = 0\) states within single N-oscillator shell does not depend on the deformation. This is the case of \(^{10}\)B, \(^{26}\)Al and \(^{54}\)Co nuclei. The existence of this sum rule for the above discussed specific case but for even-even nucleus \(^{12}\)C was noted recently by L. Zamick and N.Auerbach [19].

Our present consideration was focused on the M1 \(0^+_1 \rightarrow 1^+_i\) transitions. However we want to note that the transitions between the states with spin values different from \(J = 0\) and \(J = 1\) are also of great importance. Beside the yrast \(K = 0, T = 1\) and \(K = 0, T = 0\) bands the yrast \(K = 2\Omega, T = 0\) band is present in odd-odd N=Z nuclei at low energies. Then in the collective model the isovector M1 transitions between the states of the \(K = 0, T = 1\) and \(K = 2\Omega, T = 0\) (with \(\Omega \geq 3/2\)) bands are forbidden. Thus these forbidden isovector M1 transitions on the background of enhanced isovector M1 transitions can be used as effective indicators of the goodness of the \(K\) as a quantum number in odd-odd N=Z nuclei. For instance, it was illustrated recently [20] that this collective model selection rule, which arises also in large scale shell model calculations, works perfectly in \(^{46}\)V and \(^{50}\)Mn.

In the present paper we have avoided discussion of the E2 transition strengths which are traditional tools to investigate the collective properties of nuclear states. They were frequently used to explore the collectivity in odd-odd N=Z nuclei, too. It was well established (see, for example, [12][22]) that the behavior of experimental and shell model E2 m.e.’s in deformed odd-odd N=Z nuclei in the sd-shell and in the pf-shell is reproduced by the simple geometrical model rather well for low-spin states.

In conclusion, we have analyzed the properties of the rotor-plus-particle model wave functions with respect to the quasideuteron degree of freedom which is related to the very strong M1 transitions in odd-odd N=Z nuclei. We have found that strong coupling of the quasideuteron to the different states of the deformed core results in several comparably strong M1 \(0^+_1 \rightarrow 1^+_i\) transitions. This is in contrast to the simple quasideuteron scheme with a \(J = 0^+\) spherical core where the M1 strength is concentrated in one strong M1 transition between the lowest \(0^+\) and \(1^+\) states. The results of calculations are found to be in good
agreement with experimental data and demonstrate an $\Omega^2$-dependence of $B(M1)$ values for $\Delta T = 1, \Delta K = 0$ transitions. The existence of sum rule for the low-lying $1^+$ states in collective model was demonstrated. The predictions for heavier proton rich nuclei indicate that strong enhancement of M1 transitions in odd-odd $N=Z$ nuclei is to be expected in the exotic region up to $^{100}$Sn.

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FIG. 1. Calculated (Eq. [14] with $\alpha_q = 0.9$) and experimental $B(M1; 0^+_1, K = 0 \rightarrow 1^+_1, K = 0)$ values as a function of principal quantum number $N$ for different values of the $\Omega$ quantum number. The filled circles with error bars and filled squares connected with dashed lines are used for experimental and theoretical data, respectively. For the nuclei marked with an asterisk (*), the experimental data are deduced following the procedure discussed in the text.
The structure of the $1^+_f$, $K$ states is shown in the fourth column, where $|l_j|$ indicates the dominant spherical component in Eq.(7). The calculated B(M1) values are given for bare spin $g$-factors ($\alpha_q=1.0$) and quenched ones with $\alpha_q=0.9$. Experimental B(M1) values shown in the last column are taken from [14–17,23].

### TABLE I

| Nucleus | $\beta_{\text{eff}}$ | $J_f^+, K_f$ | $|[l_j]$, $\Omega\rangle \times |[l'_{j'}], \Omega'\rangle$ | B(M1; $0^+_1 \rightarrow 1^+_f$, $K_f$), ($\mu_B^2$) |
|---------|---------------------|----------------|---------------------------------|-----------------------------------------------|
| $^{10}$B | 0.8 | $1^+_f$, 0 | $|p_{3/2}, 3/2\rangle \times |p_{3/2}, 3/2\rangle$ | 7.8 | 6.5 | 7.5(32) |
| $^22$Na | 0.43 | $1^+_f$, 0 | $|d_{5/2}, 3/2\rangle \times |d_{5/2}, 3/2\rangle$ | 5.4 | 4.6 | 5.0(3) |
| $^{26}$Al | 0.38 | $1^+_f$, 0 | $|f_{7/2}, 3/2\rangle \times |f_{7/2}, 3/2\rangle$ | 3.7 | 3.2 | 5(2) |
| $^{46}$V | 0.23 | $1^+_f$, 0 | $|f_{7/2}, 3/2\rangle \times |f_{7/2}, 3/2\rangle$ | 7.6 | 6.7 | 8(2) |
| $^{50}$Mn | 0.25 | $1^+_f$, 0 | $|f_{7/2}, 3/2\rangle \times |f_{7/2}, 3/2\rangle$ | 8.2 | 7.2 | 6.7(14) |
| $^{54}$Co | 0.16 | $1^+_f$, 0 | $|f_{7/2}, 3/2\rangle \times |f_{7/2}, 3/2\rangle$ | 5.6 | 5.0 | 5(2) |

### TABLE II

Individual B(M1; $0^+_1 \rightarrow 1^+_f$, $K_f$) and summed $\sum = \sum_f B(M1; 0^+_1 \rightarrow 1^+_f, K_f)$ values for the three lowest $1^+ , T = 0$ states. Results of calculations using exact formula (13) and deformed single-$j$-orbital approximation formulae (16) and (17) are shown. The bare spin $g$-factors were used.

| State | $J_f^+, K_f$ | $^{46}$V | B(M1; $0^+_1 \rightarrow 1^+_f$, $K_f$), ($\mu_B^2$) |
|-------|--------------|----------|-----------------------------------------------|
|       | Eq. [13]     | Eqs. [16,17] | Eq. [13]     | Eqs. [16,17] |
| $1^+_f$, 0 | 3.7 | 2.6 | 8.2 | 7.2 |
| $1^+_f$, 1 | 7.6 | 8.7 | 5.6 | 6.9 |
| $1^+_f$, 1 | 5.7 | 6.9 | 3.2 | 4.1 |
| $\sum$ | 17.0 | 18.2 | 17.0 | 18.2 |

2The values represent an experimental estimations (see text).
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