Natural Topcolor–Assisted Technicolor

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Abstract
We construct a prototype of topcolor–assisted technicolor in which, although both top and bottom quarks acquire some mass from extended technicolor, strong $U(1)$ couplings of technifermions are isospin symmetric and all gauge anomalies vanish. There is a mechanism for mixing between the light and heavy generations and there need be no very light pseudo–Goldstone bosons.

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Technicolor was invented to provide a natural, dynamical explanation for electroweak symmetry breaking [1]. Here, $SU(2) \otimes U(1)$ is broken down to $U(1)_{EM}$ by technifermion condensates $\langle \bar{T} T \rangle$ generated by strong technicolor (TC) interactions. These interactions have the characteristic energy scale $\Lambda_{TC} \simeq \Lambda_{EW} \simeq 1$ TeV. To account for explicit breaking of quark and lepton flavor symmetries within the spirit of technicolor, new gauge interactions, encompassing technicolor and known as extended technicolor (ETC), also had to be invented [2], [3]. Unfortunately, there seems to be no natural way to account for the extremely large mass, $m_t \simeq 175$ GeV, of the top quark [4] within the ETC framework [5].

Topcolor was invented as a minimal dynamical scheme to reproduce the simplicity of the one–doublet Higgs model and explain a very large top–quark mass [6]. Here, a large top–quark condensate, $\langle \bar{t} t \rangle$, is formed by strong interactions at the energy scale, $\Lambda_t \gg m_t \sim \Lambda_{EW}$ [7]. In order that the resulting low–energy theory simulate the standard model, this scale must be very high—$\Lambda_t \sim 10^{15}$ GeV. Unfortunately, the topcolor scenario is unnatural, requiring a fine–tuning of couplings of order one part in $\Lambda_t^2/m_t^2 \simeq 10^{25}$.

Recently, Hill has proposed joining technicolor and topcolor [8]. His idea is that electroweak symmetry breaking is driven mainly by technicolor interactions strong near 1 TeV and that light quark and lepton masses are generated by ETC. In addition, topcolor interactions with a scale also near 1 TeV generate $\langle \bar{t} t \rangle$ and the very large top–quark mass. This neatly removes the objections that topcolor is unnatural and that technicolor cannot generate a large top mass. In this scenario, topcolor is an ordinary asymptotically free gauge theory, but it is still necessary that technicolor be a walking gauge theory [9] to escape large flavor–changing neutral currents [3].

In detail, Hill’s scheme depends on separate color and weak hypercharge interactions for the third and for the first two generations of quarks and leptons. For example, the (electroweak eigenstate) third generation $(t, b)_{L,R}$ may transform with the usual quantum numbers under the gauge group $SU(3)_1 \otimes U(1)_1$ while $(u, d), (c, s)$ transform under a separate group $SU(3)_2 \otimes U(1)_2$. Topcolor is $SU(3)_1$. Leptons of the third and the first two generations transform in the obvious way to cancel gauge anomalies. At a scale of order 1 TeV (which may or may not be the same as the electroweak scale), $SU(3)_1 \otimes SU(3)_2 \otimes U(1)_1 \otimes U(1)_2$ is dynamically broken to the diagonal subgroup of ordinary color and weak hypercharge, $SU(3)_C \otimes U(1)$. At this energy scale, the $SU(3)_1 \otimes U(1)_1$ couplings are strong while the $SU(3)_2 \otimes U(1)_2$ couplings are weak. Top, but not bottom, condensation is driven by the fact that the $SU(3)_1 \otimes U(1)_1$ interactions are supercritical for top quarks,
but subcritical for bottom. This difference is caused by the $U(1)_1$ couplings of $t$ and $b$. If this topcolor–assisted technicolor (TC2) scenario is to be natural, i.e., no fine–tuning of the $SU(3)_1$ is required, the $U(1)_1$ couplings cannot be weak.

Chivukula, Dobrescu and Terning (CDT) have argued that the TC2 proposal cannot be both natural and consistent with experimental measurements of the parameter $\rho = \frac{M_W^2}{M_Z^2} \cos^2 \theta_W$ \[10\]. Their strongest criticism is that even degenerate up and down technifermions are likely to have custodial–isospin violating couplings to the strong $U(1)_1$ and that this leads to large contributions to $\rho$. To prevent this, CDT showed that the $U(1)_1$ coupling must be so small that it is necessary to tune the $SU(3)_1$ coupling to within 1% of the critical value for top condensation and to increase the topcolor boson mass above 4.5 TeV.

The argument presented by CDT that the technifermions’ $U(1)_1$ couplings violate custodial isospin proceeds as follows: The $(t, b)$ chiral symmetries must be broken explicitly by ETC interactions to avoid unwanted massless bosons. In particular, part of $m_t$ must arise from ETC (see Ref. \[8\]). Thus, technifermions must couple to $U(1)_1$. If ETC commutes with electroweak $SU(2)$ and if $m_b$ also arises in part from ETC, then the right–handed technifermions $U_R$ and $D_R$ to which $t$ and $b$ couple must have different $U(1)_1$ couplings.

CDT further state that custodially–invariant $U(1)_1$ couplings to technifermions may be difficult to arrange because of the need to cancel all gauge anomalies. The difficulty here is that, in cancelling the anomalies with extra fermions, one must not introduce extra unbroken chiral symmetries \[3\]. Finally, they stress that there must be mixing between the third and first two generations and this further constrains hypercharge assignments.

In this Letter, we construct a prototype of TC2 that can overcome these difficulties. In particular, provided that technifermion condensates align properly:

1.) Both $t$ and $b$ get some mass from ETC interactions.

2.) $U(1)_1$ couplings of technifermions preserve custodial $SU(2)$.

3.) All gauge anomalies vanish.

4.) There is mixing between the third and first two generations.

5.) The only spontaneously broken technifermion chiral symmetries that are not also explicitly broken by ETC are the electroweak $SU(2) \otimes U(1)$.

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1. A large bottom condensate is not generated because the topcolor $SU(3)$ symmetry is broken and the interaction does not grow stronger as one descends to lower energies.

2. This may not be necessary if $SU(3)_1$ instanton effects can produce all of $m_b$; see \[3\].
Thus, the $U(1)_1$ interaction can be moderately strong and the TC2 interactions natural.

Our prototype is incomplete in several ways: First, we do not specify the ETC gauge group, $G_{ETC}$; the existence of the desired ETC four–fermion interactions is assumed. They are invariant under the $SU(3)$ and $U(1)$ groups. For simplicity, we assume that $G_{ETC}$ commutes with electroweak $SU(2)$. Second, we do not specify how $SU(3)_1 \otimes SU(3)_2 \otimes U(1)_1 \otimes U(1)_2$ is broken. An example of this was given in Ref. [8]. We might have to modify our $U(1)_1 \otimes U(1)_2$ charge assignments and anomaly cancellations to accommodate this symmetry breaking. In our model, there is $U(1)_1 \otimes U(1)_2$ breaking due to technifermion condensation. We do not know yet whether this is sufficient. Third, we do not discuss leptons other than to assume that they are paired with quarks to cancel gauge anomalies in the usual way. Their masses may arise from ETC interactions similar to those in Eq. (1) below.

Our model has three doublets of technifermions, all of which are assumed to transform according to the same complex irreducible representation of the technicolor gauge group, $G_{TC}$. They are also assumed to be singlets under $SU(3)_1 \otimes SU(3)_2$\footnote{Thus, ETC bosons connecting these fermions to quarks must be triplets under the appropriate $SU(3)$ group.}. The technifermion doublets are denoted by $T^l_{L,R} = (U^l, D^l)_{L,R}$, coupling to the first two (“light”) generation quarks via ETC; $T^t_{L,R} = (U^t, D^t)_{L,R}$, giving the top quark its ETC–mass; and $T^b_{L,R} = (U^b, D^b)_{L,R}$ giving the bottom quark its ETC–mass. To simplify our presentation, we assume for now that condensates are flavor–diagonal: $\langle \bar{U}^l_L U^j_R \rangle = \langle \bar{D}^l_L D^j_R \rangle \propto \delta_{ij}$ in the correct chiral–perturbative vacuum [11]. Below, when we discuss vacuum alignment, we shall see that these condensates are matrices in flavor space. We shall find that, generically, they still induce nonzero quark masses, but the precise outcome depends on the details of ETC symmetry and its breaking.

To generate light and heavy quark masses, we assume that ETC interactions produce couplings of the following form:

\[
\begin{align*}
\mathcal{H}_{\bar{u},u_j} &= \frac{g^2_{ETC}}{M^2_{ETC}} \bar{q}_L^l \gamma^\mu T^l_L \bar{U}^j_R \gamma_\mu u_j R + \text{h.c.} \\
\mathcal{H}_{\bar{d},d_j} &= \frac{g^2_{ETC}}{M^2_{ETC}} \bar{q}_L^l \gamma^\mu T^l_L \bar{D}^j_R \gamma_\mu d_j R + \text{h.c.} \\
\mathcal{H}_{\bar{t},t} &= \frac{g^2_{ETC}}{M^2_{ETC}} \bar{q}_L^l \gamma^\mu T^l_L \bar{U}^j_R \gamma_\mu t R + \text{h.c.} \\
\mathcal{H}_{\bar{b},b} &= \frac{g^2_{ETC}}{M^2_{ETC}} \bar{q}_L^l \gamma^\mu T^l_L \bar{D}^j_R \gamma_\mu b R + \text{h.c.}
\end{align*}
\]

(1)
Here, \( g_{ETC} \) and \( M_{ETC} \) stand for generic ETC couplings and gauge boson mass matrices; also, \( q^i_L = (u_i, d_i)_L \) for \( i = 1, 2 \) and \( q^b_L = (t, b)_L \). The assignments of the \( U(1)_1 \) and \( U(1)_2 \) hypercharges, \( Y_1 \) and \( Y_2 \), for the quarks and those for technifermions consistent with these interactions and with electric charge, \( Q = T_3 + Y_1 + Y_2 \), are listed in Table 1 in terms of six parameters \( (x_{1,2}, y_{1,2}, z_{1,2}) \) to be determined. The strong \( U(1)_1 \) couplings of the right and left–handed technifermions are isospin symmetric. This is possible because different technifermions give mass to \( t \) and \( b \).

The conditions that gauge anomalies vanish are:

\[
U(1)_1[SU(2)_{EW}]^2 : \quad x_1 + y_1 + z_1 = 0
\]
\[
U(1)_2[SU(2)_{EW}]^2 : \quad x_2 + y_2 + z_2 = 0
\]
\[
[U(1)_1]^3 : \quad x_1(y_1 - z_1 + \frac{1}{2}) = 0
\]
\[
[U(1)_2]^3 : \quad x_2(y_2 - z_2 - \frac{1}{2}) = 0
\]
\[
[U(1)_1]^2U(1)_2 : \quad x_1 - x_2 + 4(y_1y_2 - z_1z_2) = 0
\]

The \( U(1)_1[G_{TC}]^2 \) anomaly conditions are automatically satisfied by the hypercharge assignments in Table 1. Earlier anomaly conditions in Eqs. (3) were imposed on the later ones. Thus, the \([U(1)_2]^2U(1)_1\) condition is the same as for \([U(1)_1]^2U(1)_2\).

To choose among the solutions to Eqs. (3), we insist that there mixing between the third and first two generations. Specifically, we require that there exist ETC–generated four–technifermion (4T) interactions which connect \( T^l \) to \( T^t \) or to \( T^b \) and which are consistent with \( SU(2) \otimes U(1)_1 \otimes U(1)_2 \). Then, once color and hypercharge symmetries break to \( SU(3)_C \otimes U(1) \), these operators can mix the heavy and light generations. Still assuming \( \langle \mathcal{T}^i_{L,R} \rangle \propto \delta_{ij} \), there are four possible 4T operators: \( \mathcal{T}^i_{L,R} \gamma^\mu \gamma^\nu T^l_{L,R} \tilde{D}^i_{R} \gamma^\nu D^i_{R} \), \( \mathcal{U}^i_{L,R} \gamma^\mu T^l_{L,R} \tilde{D}^i_{R} \gamma^\nu D^i_{R} \), \( \mathcal{U}^i_{L,R} \gamma^\mu \gamma^\nu T^l_{L,R} \tilde{D}^i_{R} \gamma^\nu \), and \( \mathcal{U}^i_{L,R} \gamma^\mu \gamma^\nu T^l_{L,R} \tilde{D}^i_{R} \gamma^\nu \). These have the potential to induce the mixings \( b_R-s_L, d_L; b_L-s_R, d_R; t_R-c_L, u_L; \) and \( t_L-u_R, c_R \), respectively.

The known mixing between the third and the first two generations is in the Kobayashi–Maskawa matrix for left–handed quarks. It is \( |V_{cb}| \approx |V_{ts}| \approx 0.03–0.05 \sim m_s/m_b \) and \( |V_{ub}| \approx |V_{td}| \approx 0.002–0.015 \sim \sin \theta_C m_s/m_b \). A nonzero term \( \delta m \sim m_s \) in the \( \tilde{s}_L b_R \) element of the quark mass matrix is needed to produce mixing of this magnitude. Thus, only the first of the 4T operators above has the correct flavor and chiral structure. Requiring this operator leads to two solutions to the anomaly conditions, which we call cases A and B. The case A solution is:

\[
\text{Case A : } \quad x_1 = -\frac{1}{2}, \quad y_1 = 0, \quad z_1 = \frac{1}{2}; \quad x_2 = \frac{1}{2}, \quad y_2 = 0, \quad z_2 = -\frac{1}{2}. \tag{3}
\]
The 4T operators allowed by these hypercharges (and that influence the vacuum’s alignment) are \( H_{\bar{l}t}, H_{\bar{b}t}, H_{\bar{lt}}, H_{\bar{ib}} \) and \( H_{\text{diag}} \), where, for example,

\[
H_{\bar{l}t} = \frac{g_{ETC}^2}{M_{ETC}^2} T_L^i \gamma^\mu T^i_L \left( a_u \bar{U}^i_R \gamma_\mu U^b_R + a_D \bar{D}^i_R \gamma_\mu D^b_R \right) + \text{h.c.}
\]

\[
H_{\text{diag}} = \frac{g_{ETC}^2}{M_{ETC}^2} T_L^i \gamma^\mu T^i_L \left( b_u \bar{U}^i_R \gamma_\mu U^j_R + b_D \bar{D}^i_R \gamma_\mu D^j_R \right).
\]

The constants \( a_{U,D}, \ldots \) stand for unknown ETC–model–dependent factors and, in the diagonal interaction, \( i, j = l, t, b \). These flavor–diagonal interactions may arise, for example, from broken \( U(1) \) subgroups of \( G_{ETC} \). The case B solution to the anomaly conditions are

\[
\text{CaseB : } \quad x_1 = 0, \ y_1 = -1, \ z_1 = 1; \quad x_2 = 0, \ y_2 = 1, \ z_2 = -1.
\]

The allowed operators are \( H_{\bar{l}t}, H_{\bar{b}t} \) and \( H_{\text{diag}} \).

We obtain a constraint on \( b–s \) mixing as follows: As noted above, the transition \( b_R \to D^b_R \to D^l_L \to s_L \) requires both the interaction \( H_{\bar{l}t} \) and breaking of the separate color and hypercharge groups to \( SU(3)_C \otimes U(1) \). Since the technifermions in our prototype are \( SU(3)_1 \otimes SU(3)_2 \) singlets, there must be operator effecting this breaking in the ETC boson mass matrix. This operator transforms as \( (3, 3; \frac{5}{6}, -\frac{5}{6}) \) in cases A,B. Let us denote the corresponding mass–mixing term by \( \delta M^2_{ETC} \). Also, denote by \( M_s \) and \( M_b \) the ETC boson masses that generate \( m_s \) and the ETC contribution to the \( b–quark \) mass, \( m_{ETC}^b \). We expect \( M_s \gtrsim 100 \text{TeV} \) and \( M_s/M_b \approx m_{ETC}^b/m_s \) in a walking technicolor model. Then, we estimate that

\[
\frac{\delta m}{m_{ETC}^b} \sim \frac{\delta M^2_{ETC}}{M^2_s}.
\]

If topcolor instanton and ETC contributions to \( m_b \) add, then \( \delta m/m_b \lesssim \delta M^2_{ETC}/M^2_s \). The quark masses in Eq. (3) are renormalized at the ETC scale \( M_b \), which is above the scale at which \( SU(3)_1 \otimes SU(3)_2 \otimes U(1)_1 \otimes U(1)_2 \) is broken. If the the effect of renormalization down to about 1 TeV of the operators involved in this ratio is small, then \( |V_{cb}| \approx |V_{ts}| \lesssim \delta M^2_{ETC}/M^2_s \). This requires \( \delta M_{ETC} \sim 10 \text{TeV} \), too large to be compatible with a topcolor breaking scale near 1 TeV. Obviously, the issue of renormalization of the mixing parameters

\[4\] The need for custodial isospin violation in these operators was discussed in Ref. [3]. Since the operators generate technifermion “hard” masses of at most a few GeV, they are not expected to contribute excessively to \( \rho - 1 \); see Ref. [13].
down to the QCD scale must be addressed in a more complete model. Obtaining mixing
of the right magnitude will be a challenge.

Turn now to the question of vacuum alignment. If broken ETC and \( U(1) \) interactions
of technifermions may be treated as a perturbation, the flavor symmetry group of the
technicolor sector is \( G_x = SU(6)_L \otimes SU(6)_R \). When TC interactions become strong, \( G_x \)
breaks spontaneously to an \( SU(6) \) subgroup. For simplicity, we have assumed that this
is the diagonal subgroup, \( S_x = SU(6)_V \). This pattern of symmetry breaking assumes
that the ground state minimizing the expectation value of the chiral symmetry breaking
interactions is characterized by the flavor–diagonal condensates:

\[
\langle \bar{U}_L U_R^j \rangle = \langle \bar{D}_L D_R^j \rangle = -\frac{1}{2} \Delta_T \delta_{ij}, \quad (i, j = l, t, b).
\] (7)

Whether this or another pattern is preferred depends on the relative strengths and
signs of the explicit \( G_x \)-breaking interactions in Eqs. (6) and the 4T operators and strong
\( U(1)_1 \) interactions in cases A or B. For example, the \( U(1)_1 \) interactions of case A prefer
the condensates to align as \( \langle \bar{T}^i_L T^i_R \rangle = \langle \bar{T}^t_L T^t_R \rangle = \langle \bar{T}^b_L T^b_R \rangle = -\frac{1}{2} \Delta_T \), while those in case B
prefer the diagonal alignment in Eq. (7). If the former alignment occurred, it would not
be possible to generate proper ETC masses for the \( t \) and \( b \) quarks. In a TC theory whose
coupling evolves very little below \( M_{ETC} \), the three types of interaction make contributions
to the vacuum energy which are nominally of order \( m_{ETC}^2 \langle \bar{t}t \rangle, g_{ETC}^2 \Delta_T^2 / \Lambda_T^2 \) (where \( \Lambda_T \simeq \) few \( \times F_T \)), and \( g_{U(1)_1}^2 F_T^4 \). The \( U(1)_1 \) coupling is strong, but so is \( g_{ETC}^2 \) in a walking gauge
theory \[15\]. Therefore, it seems likely that the ETC interactions will be the decisive ones.
In any case, it is easy to see that the ETC and \( U(1)_1 \) interactions in either case explicitly
violate all spontaneously broken chiral symmetries except for the electroweak ones. Thus,
there are no light Goldstone bosons left over.

A full discussion of vacuum alignment is not possible in this Letter. We need to
construct definite ETC models and determine the allowed chiral symmetry breaking in-
teractions and their strengths before we can state what vacuum alignment patterns occur
and whether they produce the desired quark and lepton masses. In lieu of that, we briefly
summarize the results of a study for our case B choice of hypercharges.

For simplicity, consider only the allowed ETC interactions. If only the interaction
\( \mathcal{H}_{\bar{t}t \bar{b}b} \) exists and has the sign given indicated in Eq. (6), the condensates align as \( \langle \bar{T}^i_L T^i_R \rangle =

\[5\] On dimensional grounds, we expect \( \Delta_T \simeq 4\pi F_T^3 \[14\], where \( F_T \simeq 246 \text{ GeV}/\sqrt{3} \), consistent
with the \( SU(6) \otimes SU(6) \) chiral symmetry of technifermions.
\[ \langle T_L^b T_R^b \rangle = \langle T_L^t T_R^t \rangle = -\frac{1}{2} \Delta_T. \]

If only \( \mathcal{H}_{tib} \) exists, interchange \( t \) and \( b \). If both interactions \( \mathcal{H}_{titb} \) and \( \mathcal{H}_{tibt} \) have positive signs, the one with the dominant coefficient determines the condensation pattern. If they have the same strength, either pattern minimizes the vacuum energy, so that the vacuum is doubly degenerate. If the diagonal ETC interactions \( \mathcal{H}_{tti\bar{t}} \) and \( \mathcal{H}_{b\bar{b}b} \) also appear with comparable strength (or if the \( U(1)_1 \) interactions are as strong), the condensates in the correct vacuum form fully mixed matrices in techniflavor space.

It is clear that there is a broad range of possibilities for vacuum alignment and that phenomenologically interesting patterns can arise quite naturally.

Much work remains to construct a satisfactory model of topcolor–assisted technicolor. One important issue is topcolor breaking. It is easy to break topcolor using spectator fermions that introduce no gauge anomalies nor unwanted Goldstone bosons. However, we believe it is preferable to incorporate the breaking of topcolor with that of electroweak symmetry. An even more ambitious program is to construct an ETC model, based on an assumed pattern of symmetry breaking of some \( G_{ETC} \), and to complete the vacuum alignment analysis. We are hopeful that progress can be made on these issues.

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| Particle   | \(Y_1\) | \(Y_2\) | \(Q = T_3 + Y_1 + Y_2\) |
|-----------|---------|---------|-------------------------|
| \(q_L^l\) | 0       | \(\frac{1}{6}\) | \(\frac{2}{3}, -\frac{1}{3}\) |
| \(c_R, u_R\) | 0       | \(\frac{2}{3}\) | \(\frac{2}{3}\) |
| \(d_R, s_R\) | 0       | \(-\frac{1}{3}\) | \(-\frac{1}{3}\) |
| \(q_L^b\) | \(\frac{1}{6}\) | 0 | \(\frac{2}{3}, -\frac{1}{3}\) |
| \(t_R\) | \(\frac{2}{3}\) | 0 | \(\frac{2}{3}\) |
| \(b_R\) | \(-\frac{1}{3}\) | 0 | \(-\frac{1}{3}\) |
| \(T_L^l\) | \(x_1\) | \(x_2\) | \(\pm \frac{1}{2} + x_1 + x_2\) |
| \(U_R^l\) | \(x_1\) | \(x_2 + \frac{1}{2}\) | \(\frac{1}{2} + x_1 + x_2\) |
| \(D_R^l\) | \(x_1\) | \(x_2 - \frac{1}{2}\) | \(-\frac{1}{2} + x_1 + x_2\) |
| \(T_L^b\) | \(y_1\) | \(y_2\) | \(\pm \frac{1}{2} + y_1 + y_2\) |
| \(U_R^b\) | \(y_1 + \frac{1}{2}\) | \(y_2\) | \(\frac{1}{2} + y_1 + y_2\) |
| \(D_R^b\) | \(y_1 + \frac{1}{2}\) | \(y_2 - 1\) | \(-\frac{1}{2} + y_1 + y_2\) |
| \(T_L^b\) | \(z_1\) | \(z_2\) | \(\pm \frac{1}{2} + z_1 + z_2\) |
| \(U_R^b\) | \(z_1 - \frac{1}{2}\) | \(z_2 + 1\) | \(\frac{1}{2} + z_1 + z_2\) |
| \(D_R^b\) | \(z_1 - \frac{1}{2}\) | \(z_2\) | \(-\frac{1}{2} + z_1 + z_2\) |

**TABLE 1:** Quark and technifermion hypercharges and electric charges. The parameters \(x_i, y_i, z_i\) are determined in the text.