We consider the electroproduction of two vector mesons with a large rapidity gap between them on a nucleon target in the process $\gamma^* N \rightarrow \rho_1 \rho_2 N'$. We calculate the Born term for this process within the collinear factorization framework. The resulting scattering amplitude may be represented as a convolution of an impact factor describing the $\gamma^* \rightarrow \rho_1$ transition and an amplitude describing the $N \rightarrow \rho_2 N'$ transition. The latter amplitude is analogous to deeply virtual electroproduction of a meson, the virtual photon being replaced by two gluon (Pomeron) exchange. The long distance part of this amplitude is described by Generalized Parton Distributions (GPD) and meson light-cone distributions. The selection of a transversely polarized vector meson $\rho_2$ provides the first feasible selective access to chiral-odd GPD.
1. Generalized Parton Distributions (GPDs) are the non-perturbative objects encoding the information about the quark and gluon proton structure in the most complete way [1]. While the chiral even GPD may be probed in various hard exclusive processes, no single one has yet be proven to be sensitive to chiral-odd GPDs [2], [3].

In the massless quark limit, the chiral-odd functions may appear only in pairs in a non-vanishing scattering amplitude, so that chirality flip encoded in one of them is compensated by another. The natural probe for the forward chiral-odd distributions is the Drell-Yan process, containing the convolution of chiral-odd distributions of quark and antiquark [4]. Its nonforward analog is provided by the hard exclusive production of a transversely polarized vector meson, where the quark transversity distribution in the nucleon is substituted by the chiral-odd GPD and the antiquark one by the meson distribution amplitudes (DA). However, the simplest realization of this idea [2], namely the hard exclusive electroproduction of a transversely polarized vector meson, results in a zero contribution [5]. Formally, this comes from the vanishing value of the expression $\gamma^\mu \sigma_{\alpha\beta} \gamma_\mu$, appearing after the summation over the polarizations of the virtual gluon, which is required to transfer the hard momentum transfer in the case of collinear GPD and DA. This formal argument is supported by the consideration of angular momentum conservation in the collinear kinematics.

In the present paper we suggest another process which allows to avoid these effects by ”substituting” to the virtual photon a hard two gluon exchange, i.e. a perturbative Pomeron ($P$) in the lowest order, coming from a photon/meson transition. Let us consider a generic process

$$AN \rightarrow BN'$$

shown in Fig. 1 of scattering of a particle $A$, e.g. being a virtual or real photon, on a nucleon $N$, which leads via two gluon exchange to the production of particle $B$ (e.g. vector meson or photon) separated by a large rapidity gap from another produced meson $M$ and the scattered nucleon $N'$. We consider the kinematical region where the rapidity gap between $M$ and $N'$ is much smaller than the one between $B$ and $M$, that is the energy of the system ($M - N'$) is smaller than the energy of the system ($B - M$) but still large enough to justify our approach (in particular much larger than baryonic resonance masses).

We show that in such kinematical circumstances the Born term for this process is calculable consistently within the collinear factorization method. The final result is represented as an integral (over the longitudinal momentum fractions of the quarks) of the product of two amplitudes: the first one describing the transition $A \rightarrow B$ via two gluon exchange and the second one describing the subprocess $PN \rightarrow MN'$ which is closely related to the electroproduction process $\gamma^* N \rightarrow MN'$ where collinear factorization theorems allow to separate the long distance dynamics expressed through the GPDs from a perturbatively calculable coefficient function. The hard scale appearing in the process in Fig. 1 is supplied by the relatively large momentum transfer $p^2$ in the two gluon channel, i.e. by the virtuality of the Pomeron.

Such a process is a representative of a new class of hard reactions whose QCD description within the collinear factorization scheme involves in the described above kinematics.
Figure 1: Factorization of the process $A N \rightarrow B M N'$ in the asymmetric kinematics discussed in the text. $P$ is the hard Pomeron.

the impact factor $J$ appearing naturally in Regge-type perturbative description based on the BFKL evolution and it involves the collinear distributions (e.g. $F$), whose evolutions are governed by DGLAP-ERBL equations.

In this place we would like to mention Ref. [6] in which the factorization theorem was proven for electroproduction of multiple mesons of small invariant mass. Consequently, this bulk of mesons is described in [6] by one generalized distribution amplitude. This should be contrasted with the case which we consider below where the two produced mesons have a large invariant mass, which leads to a different, yet unproven to all orders, factorization theorem.

In order to make our discussion as clear as possible we shall consider first a reference process with all longitudinally polarized vector particles

\[ \gamma^*_L(q) \ N(p_2) \rightarrow \rho^0_L(q_\rho) \ \rho^+_L(p_\rho) N' (p_2') , \tag{2} \]

which involves the emission of two gluons in the $\gamma^*_L \rightarrow \rho_L$ transition and which is more founded theoretically from the point of view of the collinear factorization. We choose a charged vector meson $\rho^+$ to select quark antiquark exchange with the nucleon line. We shall show below that the collinear factorization holds at least in the Born approximation of (2) and we shall derive the resulting scattering amplitude. At the same time it will be clear that all steps of derivation can be immediately applied to the description of a whole family of processes, in particular those involving the chiral-odd GPD, e.g. for

\[ \gamma^*_L(q) \ N(p_2) \rightarrow \rho^0_L(q_\rho) \ \rho^+_T(p_\rho) N' (p_2') , \tag{3} \]

which has been the main motivation for the present studies. We finish our paper by discussing some other processes which could be useful for studies of the transversity and present final conclusions.

2. Let us first summarize the details of the kinematics of the process (2). We introduce two light-like Sudakov vectors $p_{1/2}$. The momenta are parametrized as follows:

\[ q^\mu = p^\mu_1 - \frac{Q^2}{s} p^\mu_2 , \quad q^2 = -Q^2 , \quad s = 2(p_1 p_2) \]
where $\zeta$ is the skewedness parameter which can be written in terms of the two meson invariant mass

$$s_1 = (q_\rho + p_\rho)^2 = \frac{\vec{p}^2}{\alpha \bar{\alpha}}$$

and the photon virtuality $Q^2$ as

$$\zeta = \frac{1}{s} (Q^2 + s_1) .$$

The $\rho^+(p_\rho)$-meson - target invariant mass equals

$$s_2 = (p_\rho + p_\rho')^2 = s \bar{\alpha} \ (1 - \zeta) .$$

The kinematical limit with a large rapidity gap between the two mesons in the final state is obtained by demanding that $s_1$ is very large, being of the order of $s$

$$s_1 = s \zeta , \quad s_1 \gg Q^2, \vec{p}^2 ,$$

whereas $s_2$ is kept constant but large enough to justify the use of perturbation theory in the collinear subprocess $P N \rightarrow \rho^+_L N'$ and the application of the GPD framework

$$s_2 \rightarrow \frac{\vec{p}^2}{\zeta} (1 - \zeta) = constant .$$

In terms of the longitudinal fraction $\alpha$ the limit with a large rapidity gap corresponds to taking the limits

$$\alpha \rightarrow 1 , \quad \bar{\alpha} s_1 \rightarrow \vec{p}^2 , \quad \zeta \sim 1 .$$

We have chosen the kinematics so that the nucleon gets no transverse momentum in the process. Let us however note that in principle one may allow a finite momentum transfer, small with respect to $|\vec{p}|$. This case will involve additional GPDs in the expressions to follow.

Let us repeat in this place that the role of the main hard scale in the processes under discussion below is played by the virtuality $p^2 = -\vec{p}^2$, or by the large momentum transfer in the two-gluon exchange channel. If additionally the incoming photon has non zero, sufficiently large virtuality $Q^2$, then the theoretical description of the processes simplifies even more, as we can neglect within our approximation contribution of the hadronic component of the photon. The case with $Q^2 = 0$, i.e. the photoproduction at large momentum transfer is more complicated and may require to take into account both the perturbative and the hadronic (non-perturbative) contributions (see e.g. [4]).
3. We calculate the scattering amplitude $\mathcal{M}$ of the process (2) using the standard collinear factorization method, i.e. we write it in a form suggested by Fig. (1), as:

$$\mathcal{M} = \sum_{p=q,q'} \int_{0}^{1} dz \int_{0}^{1} du \int_{0}^{1} dx_1 T^p_H(x_1, u, z) F^p_\zeta(x_1) \phi_\rho^+(u) \phi_\rho(z). \quad (11)$$

Here $F^p_\zeta(x_1)$ is the generalized (skewed) parton $p$ distribution in the target at zero momentum transfer; $x_1$ and $x_2 = x_1 - \zeta$ are (see Fig. (2)) the momentum fractions of the emitted and absorbed partons (quarks) of the target, respectively (as usual the case $x_2 < 0$ is interpreted as an emitted antiquark). $\phi_\rho^+(u)$ and $\phi_\rho(z)$ are the distribution amplitudes of the $\rho^+$-meson and $\rho^0$-meson, respectively. $T^p_H(x_1, u, z)$ is the hard scattering amplitude (the coefficient function). For clarity of notation we omit in Eq. (11) the factorization scale dependence of $T^p_H$, $F^p_\zeta$, $\phi_\rho$ and $\phi_\rho^+$.

Eq. (11) describes the amplitude in the leading twist approximation. In other words all terms suppressed by powers of a hard scale parameter $1/|\vec{p}|$ are omitted. Within this approximation one neglects (in the physical gauge) the contributions of the higher Fock states in the meson wave functions and the many parton correlations (higher twist GPD’s) in the proton. Moreover, one can neglect in the hard scattering amplitude the relative (with respect to a meson momentum) transverse momenta of constituent quarks (the collinear approximation). This results in the appearance in the factorization formula (11) of the distribution amplitudes, i.e. the usual wave functions depending on the relative transverse momenta of constituents integrated over these momenta up to the collinear factorization scale.

The additional simplification appears in the kinematics given by Eqs. (8-10). In this limit one can consider only the diagrams which involve two gluon exchange between two mesons, see Fig. 2. The other contributions (the fermion exchange diagrams) to the coefficient function $T^p_H$ are known [3] to be suppressed by the power of energy, $\sim \vec{p}^2/s$, therefore we will not discuss them in what follows. We will show that at the same accuracy, i.e. neglecting terms $\sim \vec{p}^2/s$, the contribution of gluon exchange diagrams shown in Fig. 2 is purely imaginary and involves GPD’s in the ERBL region only, $x_1 < \zeta$. The first property is of a general nature, it is valid for any diffractive processes calculated in the Born approximation (two gluon exchange), see [3]. The second one is specific for our process.

In the Born approximation the scattering amplitude $T^q_H(x_1, u, z)$ for the quark $q$ target is described by six diagrams, see Fig. 2. They are calculated for the on-mass-shell quarks carrying the collinear momenta $x_1, 2p_2$. The on-mass-shell quark and antiquark entering the $\rho$-mesons distribution amplitudes $\phi_\rho^+(u)$ and $\phi_\rho(z)$ carry fractions $u$ and $z$ of the momentum of a corresponding outgoing mason, $q_\rho$ and $p_\rho$, respectively.

We shall show below that for the process (2) the integrals over $x_1, u, z$ are convergent which justifies the validity of the factorization formula (11).

4. We define the light-cone distribution amplitudes $\phi_\parallel$ of $\rho_{L}^{0,+}$-mesons appearing
in (2) by the matrix elements

\[ \langle 0|\bar{q}(0)\gamma^\mu q(y)|\rho^0_L(q_\rho)\rangle = q_\rho^\mu f_\rho \int_0^1 dz \, e^{-iz\rho(y)} \phi|| (z) \]  
(12)

\[ \langle 0|\bar{q}(0)\gamma^\mu q(y)|\rho^+_L(p_\rho)\rangle = p_\rho^\mu f_\rho \int_0^1 du \, e^{-iu\rho(y)} \phi|| (u) \]  
(13)

\( \phi|| (z) \) is the meson distribution amplitude whose asymptotic form is \( \phi|| (z) = 6z\bar{z} \), \( f_\rho \) is the \( \rho \)-meson coupling constant, \( f_\rho = 198 \pm 7 \text{ MeV} \). The generalized (skewed) quark distribution in an unpolarized nucleon target \( F_\zeta \) is defined by the formula

\[ \langle N(p_2')|\bar{q}(0)\gamma^\mu q(y)|N(p_2)\rangle = \]  
(14)

\[ = \bar{u}(p_2')\gamma^\mu u(p_2) \int_0^1 dx_1 \left[ e^{-ix_1(p_2'y)} F^q_\zeta(x_1) - e^{ix_2(p_2'y)} F^\bar{q}_\zeta(x_1) \right]. \]

The necessary condition for the validity of the collinear factorization is the absence of pinch singularities in the integration over the momentum fraction \( x_1 \). The pinch contributions can produce terms which are singular in the end-point region of the momentum fraction \( u \), e.g. terms behaving as \( \sim 1/u \), \( 1/\bar{u} \) for \( u, \bar{u} \to 0 \), so that an integral over \( u \) would diverge. For example, the result of an integral with a pinch contribution leads to the result

\[ \int \frac{dx_1}{[x_2 + i\epsilon][x_2 + au - i\epsilon]} \xrightarrow{u \to 0} \sim -\frac{2i\pi F_\zeta(\zeta)}{au} \sim \frac{1}{u} \]  
(15)

whereas the one without pinch contribution gives

\[ \int \frac{dx_1}{[x_2 + i\epsilon][x_2 + au + i\epsilon]} \xrightarrow{u \to 0} \text{const.} \]  
(16)

The above example shows that in order to avoid pinch singularities it is necessary to have the same \( i\epsilon \) prescriptions in the gluon poles and the quark ones of diagrams in Fig. (2) and that both poles in expression (16) contribute (see for details [8]).

To show the absence of pinch singularities in the \( x_1 \)-integration it is then enough to examine the denominators appearing in all diagrams in Fig. (2). They involve two gluonic propagators which in a general kinematics, i.e. for arbitrary value of the parameter \( \alpha \), lead to denominators

\[ k_1^2 + i\epsilon = -s\bar{u}\alpha(x_1 - i\epsilon) \]  
(17)

\[ k_2^2 + i\epsilon = s\bar{u}\alpha(x_2 + i\epsilon) \]  
(18)

and result in the factor of the integrand

\[ \frac{1}{u\bar{u}(x_1 - i\epsilon)(x_2 + i\epsilon)}. \]  
(19)
As for the quark propagators which involve $x_{1,2}$ dependence, it is enough to consider, for example, only those which appear in diagrams $a$ and $a'$:

diagram $a$:  
$$
(zq_{\rho} - q - k_{1})^{2} + i\varepsilon = s(\bar{\alpha}u + \bar{z}\alpha) \left[ x_{2} + \bar{u} \frac{s_{1}\bar{z}}{s(\bar{\alpha}u + \bar{z}\alpha)} + i\varepsilon \right] \tag{20}
$$
diagram $a'$:  
$$
(zq_{\rho} - q - k_{2})^{2} + i\varepsilon = -s(\bar{\alpha}u + \bar{z}\alpha) \left[ x_{1} - u \frac{s_{1}\bar{z}}{s(\bar{\alpha}u + \bar{z}\alpha)} - i\varepsilon \right] \tag{21}
$$

All remaining quark propagators with $x_{1,2}$ dependence either coincide with (20) and (21) or they are obtained from (20) and (21) by the substitution $z \leftrightarrow \bar{z}$, which is unimportant for the present discussion. A comparison of Eqs. (17), (18), (20), (21) leads to the conclusion that in each separate diagram in Fig. (2) the $i\varepsilon$ prescriptions in quark and gluon propagators coincide and consequently there are no pinch singularities.

5. Now we pass to the calculation of the process (2) in the kinematics with large rapidity gap between $\rho_{L}$ and $\rho_{\rho}^{+}$, i.e. characterized by conditions

$$
s_{1} \sim s \gg Q^{2}, \vec{p}_{2}^{2}, \zeta \sim 1. \tag{22}
$$

They mean that $\rho_{L}^{0}$-meson is produced in the fragmentation region of the photon and consequently we should consider the limit $\alpha \to 1$. Thus we shall neglect terms suppressed by $\bar{\alpha} \to 0$ but keeping the condition (10).

In such kinematics we can substitute the numerator of the gluon propagators in the Feynman gauge by

$$
g^{\mu\nu} \to \frac{2}{s} \bar{p}_{1}^{\mu}p_{2}^{\nu}, \tag{23}
$$

where the Sudakov vector $p_{2}$ acts on the upper part of diagrams in Fig. (2).

The gauge invariance of these diagrams allows us to omit the component proportional to $q^{\mu}$ in the polarization vector $\varepsilon_{L}^{\mu}$ of the longitudinally polarized photon, i.e. to take it in the form

$$
\varepsilon_{L}^{\mu} \to \frac{2Q}{s} p_{2}^{\mu}. \tag{24}
$$

Calculations of diagrams in Fig. (4) for quark $u$ contribution and analogous set of diagrams for antiquark $\bar{d}$ contribution lead to the result

$$
\mathcal{M}_{\gamma^{\rho}p^{\mu}e^{\rho}_{L}p_{L}^{n}} = -\frac{e \alpha_{s}^{2} \pi^{2} 2^{2} C_{F} \sqrt{1 - \zeta} f_{\rho}^{2}}{4 \sqrt{2} s^{2} N^{2}} \left( F_{\zeta}^{u}(x_{1}) - F_{\zeta}^{d}(x_{1}) \right) \sum_{i=a,b,c,a',b',c'} I_{i}, \tag{25}
$$

with $C_{F} = \frac{N^{2} - 1}{2N}$, and

$$
I_{a} = Tr \left( \hat{q}_{\rho} \hat{\varepsilon}_{L} \frac{1}{z\hat{q}_{\rho} - \hat{q}} \hat{p}_{2} \frac{1}{z\hat{q}_{\rho} - \hat{q} - k_{1}} \hat{p}_{2} \right),
$$
where we have as usual denoted $\hat{v} = \gamma^\mu v_\mu$ and we have neglected all quark masses.

After calculation of the traces in Eq. (25) we can simplify the numerators of resulting expressions using conditions (10) and keeping the denominators exact. In this way we obtain that the scattering amplitude under study takes the form

$$I_b = \text{Tr} \left( \hat{q}_\rho \hat{q}_2 \frac{1}{z \hat{q}_\rho - k_2} \hat{p}_2 \frac{1}{\hat{q} - \hat{z} \hat{q}_\rho} \hat{\varepsilon}_L \right),$$

$$I_c = \text{Tr} \left( \hat{q}_\rho \hat{q}_2 \frac{1}{z \hat{q}_\rho - k_2} \hat{\varepsilon}_L \frac{1}{-\hat{z} \hat{q}_\rho + \hat{k}_1} \hat{p}_2 \right),$$

$$I_{a'} = I_a(k_1 \leftrightarrow k_2), \quad I_{b'} = I_b(k_1 \leftrightarrow k_2), \quad I_{c'} = I_c(k_1 \leftrightarrow k_2),$$

(26)

Going to the limit (10) also in the denominators leads to

$$\tilde{I}_a = \frac{z}{z} \left[ Q^2 (1 - z \alpha) + \bar{\alpha} z s_1 \right] \left[ x_2 s (\bar{u} \bar{\alpha} + \bar{z} \alpha) + s_1 \bar{u} \bar{z} + i\epsilon \right],$$

$$\tilde{I}_{a'} = \frac{z}{z} \left[ Q^2 (1 - z \alpha) + \bar{\alpha} z s_1 \right] \left[ -x_1 s (\bar{\alpha} u + \bar{z} \alpha) + s_1 u \bar{z} + i\epsilon \right],$$

$$\tilde{I}_c = \left[ -x_1 s (\bar{z} + \bar{\alpha} (u + z - 1)) + s_1 u \bar{z} + i\epsilon \right] \left[ s x_2(z + \bar{\alpha} (1 - u - z)) + z \bar{u} s_1 + i\epsilon \right],$$

$$\tilde{I}_b = \tilde{I}_a(z \leftrightarrow \bar{z}), \quad \tilde{I}_{b'} = \tilde{I}_{a'}(z \leftrightarrow \bar{z}), \quad \tilde{I}_{c'} = \tilde{I}_c(z \leftrightarrow \bar{z}).$$

(27)

From Eqs. (28), (29), (30) and (31) we see that in our kinematics with a large rapidity gap, the sum of all $\tilde{I}$'s contributions leads to a result which is purely imaginary. Below
we identify it with the discontinuity across the sum of upper parts of diagrams in Fig. 2. Let us also observe that the delta function in the above expressions leads to the value 
\[ x_1 = u s_1 / s = u \zeta, \]
i.e. we probe the GPD’s in the ERBL region. Substituting (28), (29), (30), (31) into (27) we obtain the result
\[ \mathcal{M}^{\gamma^* p \rightarrow \rho^0 L^+ n} = - \frac{i 16 s \zeta e \alpha_s^2 \pi^3 C_F}{\sqrt{2} N^2 (\bar{p}^2)^2} \int_0^1 \frac{du \phi_{\parallel}(u)}{u^2 \bar{u}^2} \left[ F^\rho_{\zeta}(u \zeta) - F^0_{\zeta}(u \zeta) \right] \]
\[ \int_0^1 dz \phi_{\parallel}(z) \frac{1}{z \bar{z}} \left( \frac{1}{Q^2 z \bar{z} + z^2 \bar{p}^2} + \frac{1}{Q^2 z \bar{z} + \bar{z}^2 \bar{p}^2} - \frac{1}{Q^2 z \bar{z} + (1 - u - z)^2 \bar{p}^2} - \frac{1}{Q^2 z \bar{z} + (u - z)^2 \bar{p}^2} \right). \] (32)

In the result (32) all integrals are well defined and converge, which shows that the collinear factorization holds for our process. For its validity it is important that the last integral over \( z \) in (32) vanishes when \( u \) or \( \bar{u} \rightarrow 0 \). This fact is not surprising since this integral is well known in BFKL approach [9] as the impact factor describing in the Born approximation the transition \( \gamma^*_L \rightarrow \rho^0_L \) by two gluon exchange (Fig. 3).

The impact factor \( J^{\gamma^*_L \rightarrow \rho^0_L}(\bar{k}_1, \bar{k}_2) \) depends on the transverse components of the momenta of the \( t \)-channel gluons \( \bar{k}_1, \bar{k}_2, \bar{k}_1 + \bar{k}_2 = \bar{p} \). It is defined as the discontinuity in the channel \( (\gamma^*_L(q) g(k_1)) \), i.e. in the Mandelstam variable \( s^\beta = (q + k_1)^2 \)
\[ i \delta^{ab} J^{\gamma^*_L \rightarrow \rho^0_L}(\bar{k}_1, \bar{k}_2) = \frac{i d(s^\beta)}{2\pi i} D_{\text{disc}} \frac{S_{\mu^\nu}^{\gamma^*_L \rightarrow \rho^0_L, ab}}{s^2} p_{2u}^\mu p_{2b}^\nu. \] (33)

Here \( S_{\mu^\nu}^{\gamma^*_L \rightarrow \rho^0_L, ab} \) is the S-matrix element for the above transition. It contributes to the impact factor through its contraction with the light cone components of the Sudakov vector \( p_2 \). \( a, b \) are the colour indices of the gluons. In the Born approximation it is described by a quark loop with two \( t \)-channel gluons coupled to it in all possible ways, i.e. by the upper parts of Fig. 4. The calculation of \( J^{\gamma^*_L \rightarrow \rho^0_L}(\bar{k}_1, \bar{k}_2) \) is standard [10], leading to the result (for nonvanishing quark mass \( m_q \) case):
\[ J^{\gamma^*_L \rightarrow \rho^0_L}(\bar{k}_1, \bar{k}_2) = - f_p \frac{e \alpha_s 2\pi Q}{N_c \sqrt{2}} \int_0^1 dz \frac{z \bar{z} \phi_{\parallel}(z)}{P(k_1, k_2)}. \] (34)

with
\[ P(k_1, k_2 = \bar{p} - \bar{k}_1) = \frac{1}{z \bar{z} \bar{p}^2 + m_q^2 + Q^2 z \bar{z}} + \frac{1}{z \bar{z} \bar{p}^2 + m_q^2 + Q^2 z \bar{z}} - \frac{1}{(k_1 - z \bar{p})^2 + m_q^2 + Q^2 z \bar{z}} - \frac{1}{(k_1 - z \bar{p})^2 + m_q^2 + Q^2 z \bar{z}}. \] (35)

Let us note also that the impact factor \( J^{\gamma^*_L \rightarrow \rho^0_L}(\bar{k}_1, \bar{k}_2) \) vanishes linearly when the transverse momentum of one of the \( t \)-channel gluons, \( \bar{k}_1 \) or \( \bar{k}_2 \), vanishes. This fact is
related to the gauge invariance of QCD. Its physical interpretation in this case is that a soft gluon cannot probe the structure of a colourless scattered object (photon dissociating into a small quark dipole).

The last integral over \( z \) in Eq. (32) is proportional to \( J_{\gamma^* L \to \rho^0 L} (k_1 = u \vec{p}, k_2 = \bar{u} \vec{p}) \) for \( m_q = 0 \). The values appearing as arguments of this impact factor follow in an obvious way from the conservation of transverse momentum in the diagrams in Fig. (3) and Eq. (4): the quark and antiquark momenta \( x_1 p_2 \) and \( x_2 p_2 \), respectively, are longitudinal ones, consequently \( \vec{k}_1 = -u \vec{p} = u \vec{p} \) and \( \vec{k}_2 = -\bar{u} \vec{p} = \bar{u} \vec{p} \).

Expressing the result (32) in terms of the impact factor \( J_{\gamma^* L \to \rho^0 L} \) we obtain

\[
\mathcal{M}_{\gamma^* p \to \rho^0 L}^{\rho} = \frac{i8\pi^2 \zeta s_{\rho} f_{\rho} \sqrt{1 - \zeta}}{N (p^2)^2} \int_0^1 \frac{du \phi_{\parallel}(u)}{u^2 \bar{u}^2} J_{\gamma^* L \to \rho^0 L}(u \vec{p}, \bar{u} \vec{p}) \left[ F^u_\zeta(u \zeta) - F^d_\zeta(u \zeta) \right] ,
\]

Eq. (36) exhibits an interesting property which we would like to emphasize. Our calculations were done consistently within the framework of the collinear factorization. Nevertheless the result (36) shows that the end-point singularities related to the singular behaviour of the hard t-channel two gluon propagators, \( \sim \frac{1}{u^2 \bar{u}^2} \) for \( u, \bar{u} \to 0 \), are regularized by both the \( \rho \)-meson distributions amplitudes ( \( \phi_{\parallel}(u) \) ) characteristic of the collinear factorization and by the impact factor ( \( J_{\gamma^* L \to \rho^0 L}(u \vec{p}, \bar{u} \vec{p}) \) ) appearing naturally in the BFKL approach. It would be very desirable to investigate whether such compensation of the end-point singularities persists also after taking into account of radiative corrections.

6. All steps of the derivation which led us to Eq. (36) can be now repeated for the process (9) which involves the chiral-odd transversity distribution whose investigation was the main motivation of our studies.

To proceed with this program we need to define the corresponding distribution amplitudes. We describe the production of a transversely polarized \( \rho \)-meson by means of its chiral-odd light-cone distribution amplitude [11] defined by the matrix element

\[
\langle \rho_T(p_\rho, T) | \bar{q}(x) \sigma^{\mu \nu} q(-x) | 0 \rangle = i f_T \left( p_\rho^\mu \xi_T^{\nu} - p_\rho^\nu \xi_T^{\mu} \right) \int_0^1 du e^{-i(2u-1)(p_\rho x)} \phi_\perp(u) ,
\]

where the \( \phi_\perp(u) = 6u \bar{u} \) and \( f_T(\mu) = 160 \pm 10 \text{ MeV} \) at the scale \( \mu = 1 \text{ GeV} \).

The generalized (skewed) transversity distribution in nucleon target described by the polarization vector \( n^\mu \) is defined by the formula

\[
\langle N(p_2, n) | \bar{q}(0) \sigma^{\mu \nu} q(y) | N(p_2, n) \rangle = \int_0^1 dx_1 \left[ e^{-ix_1(p_2 y)} F^T_\zeta q(x_1) - e^{ix_2(p_2 y)} F^T_\bar{\zeta} q(x_1) \right] ,
\]

9
where \( \sigma^{\mu \nu} = i/2[\gamma^\mu, \gamma^\nu] \). Using the formulas (37) and (38) we obtain for the sum of the contributions of diagrams (2) the expression

\[
\mathcal{M}_{\gamma^L \rho^0_{T} \rho_{T}^+ n}^{\gamma_{l} p \to \rho_{T}^0 \rho_{T}^+ n} = -\frac{i e \alpha_s^2 \pi^2 22 C_F}{64 \sqrt{2} s^2 N^2} \sqrt{1 - \zeta} f_{T} f_{T} \]

\[
\int_0^1 dz \phi_\parallel(z) du \phi_{\perp}(u) dx_1 \frac{[F_{T}^T u(x_1) - F_{T}^T d(x_1)]}{[k_1^2 + i\epsilon][k_2^2 + i\epsilon]} \sum_{i=a,b,c,a',b',c'} I_i
\]

\[
Tr (\sigma^{\alpha \beta} \hat{p}_1 \sigma_{\mu \nu} \hat{p}_1) Tr (\sigma^{\alpha \beta} \hat{p}_2 \gamma^5 \hat{n}) \left( p_{T}^{\mu} \epsilon_{T}^{\nu} - p_{T}^{\nu} \epsilon_{T}^{\mu} \right)
\]

where \( I_i \)’s are given by Eq. (26) and \( \gamma^5 = \gamma_5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3 \). This formula is the analog of Eq. (23). The product of traces in Eq. (38) comes from the definitions of distributions in Eqs. (37) and (38). This is the main difference between this expression and Eq. (25). The calculation of the above traces leads to the formula

\[
\mathcal{M}_{\gamma^L \rho^0_{T} \rho_{T}^+ n}^{\gamma_{l} p \to \rho_{T}^0 \rho_{T}^+ n} = -i \sin \theta \frac{e \alpha_s^2 \pi^2 22 C_F}{\sqrt{2} s N^2} \sqrt{1 - \zeta} f_{T} f_{T} \]

\[
\int_0^1 dz \phi_\parallel(z) du \phi_{\perp}(u) dx_1 \frac{[F_{T}^T u(x_1) - F_{T}^T d(x_1)]}{[k_1^2 + i\epsilon][k_2^2 + i\epsilon]} \sum_{i=a,b,c,a',b',c'} I_i
\]

where \( \theta \) is the angle between the transverse polarization vector of the target \( \vec{n} \) and the polarization vector \( \vec{c}_T \) of the produced \( \rho_{T}^+ \)-meson. The comparison of Eq. (40) for transversity with the analogous expression for \( \rho_{T}^+ \)-meson production (23) leads to the conclusion that apart from obvious changes of coupling constants and distribution amplitudes the only difference between these two expressions is the presence in (40) of the additional factor \( i \sin \theta \). Thus the final result for the scattering amplitude for the process \( \gamma_{l} p \to \rho_{L}^0 \rho_{T}^+ n \) with transversity distribution takes the form

\[
\mathcal{M}_{\gamma^L \rho^0_{T} \rho_{T}^+ n}^{\gamma_{l} p \to \rho_{L}^0 \rho_{T}^+ n} = -\sin \theta 8\pi^2 \frac{\zeta s \alpha_s f_{T}}{N(\vec{p}^2)^2} \sqrt{1 - \zeta} \frac{C_F}{N(\vec{p}^2)^2} \]

\[
\int_0^1 \frac{du \phi_{\perp}(u)}{u^2 \vec{n}^2} J_{\gamma^L \rho^0_{L} \rho_{T}^+ n}(u \vec{p}, \vec{n}) \left[ F_{T}^T u(u \zeta) - F_{T}^T d(u \zeta) \right], \quad (41)
\]

where \( J_{\gamma^L \rho^0_{L}} \) is the same impact factor as in (30) given by Eqs. (34) and (35).

7. The simple form of the scattering amplitude (41) (and of (30)) involving the convolution of the impact factor, the vector meson distribution amplitude and the GPD suggests a search for other processes in which the transversity can be described in a similar framework and which are may be easier for experimental studies. If for such a new process the impact factor of corresponding transition is known it is enough to replace in Eq. (41) the impact factor \( J_{\gamma^L \rho^0_{L}} \) by another one to obtain the appropriate scattering amplitude.
The most obvious candidate is the process (3) with the transversely polarized photon \( \gamma_T^* \, N \rightarrow \rho_\perp^0 \, \rho_T^+ \, N' \). The impact factor \( J_{\gamma_T^* \rightarrow \rho_\perp^0} \) has then the form:

\[
J_{\gamma_T^* \rightarrow \rho_\perp^0}(k_1, k_2 = \vec{p} - \vec{k}_1) = -\frac{e \alpha_s \pi f_{\rho}}{\sqrt{2} N} \int_0^1 dz \,(2z - 1) \phi_{\parallel}(z) \left( \vec{e} \vec{Q}_p \right).
\]

with \( \vec{e} \) being the polarization vector of the initial photon, and

\[
\vec{Q}_p(k_1, k_2 = \vec{p} - \vec{k}_1) = \frac{z \vec{p}}{z^2 \vec{p}^2 + Q^2 z \vec{z} + m_q^2} - \frac{\vec{z} \vec{p}}{z^2 \vec{p}^2 + Q^2 z \vec{z} + m_q^2} + \frac{\vec{k}_1 - z \vec{p}}{(\vec{k}_1 - z \vec{p})^2 + Q^2 z \vec{z} + m_q^2} - \frac{\vec{k}_1 - \vec{z} \vec{p}}{(\vec{k}_1 - \vec{z} \vec{p})^2 + Q^2 z \vec{z} + m_q^2}.
\]

Another example is the process (3) with \( \rho^0 \) replaced by heavy \( J/\Psi \)-meson. One could study either \( \gamma_L^* \, N \rightarrow J/\Psi_L \, \rho_T^+ \, N' \) involving the impact factor

\[
J_{\gamma_L^* \rightarrow J/\Psi_L}(k_1, k_2 = \vec{p} - \vec{k}_1) = \frac{-8 \pi e \alpha_s Q f_{J/\psi}}{3 N} \left( \frac{1}{\vec{p}^2 + Q^2 + 4 m_c^2} - \frac{1}{(2 \vec{k}_1 - \vec{p})^2 + Q^2 + 4 m_c^2} \right),
\]

or the process \( \gamma_T^* \, N \rightarrow J/\Psi_T \, \rho_T^+ \, N' \) for which the impact factor \( J_{\gamma_T^* \rightarrow J/\Psi_T} \) has the form

\[
J_{\gamma_T^* \rightarrow J/\Psi_T}(k_1, k_2 = \vec{p} - \vec{k}_1) = \frac{4 e \alpha_s \pi m_c f_{J/\psi}}{3 N} \left( \vec{e} \vec{e}^{*} \right) \left( \frac{1}{\vec{p}^2 + Q^2 + 4 m_c^2} - \frac{1}{(2 \vec{k}_1 - \vec{p})^2 + Q^2 + 4 m_c^2} \right).
\]

These impact factors are obtained from the corresponding ones for light quarks by applying a standard non-relativistic approximation for the \( J/\Psi \)-meson vertex, i.e. by approximating the distribution amplitude by \( \phi_{J/\psi}(z) = \delta(z - 1/2) \). The coupling constant

\[
f_{J/\psi}^2 = \frac{27 m_J \Gamma_{J/\psi \rightarrow \mu^+ \mu^-}}{16 \pi \alpha_{em}^2}
\]

is expressed in terms of the width \( \Gamma_{J/\psi \rightarrow \mu^+ \mu^-} \) and \( \alpha_{em} = e^2/4\pi \).

This last process (45) can be of course also studied both for virtual as well as real photon.

One can also study the transversity in the inelastic DVCS: \( \gamma_{L/T}^* \, N \rightarrow \gamma \, \rho_T^+ \, N' \). The expressions for the corresponding impact factors \( J_{\gamma_{L/T}^* \rightarrow \gamma \rho_T} \) and \( J_{\gamma_{L/T}^* \rightarrow \gamma \rho_T} \) are also known and can be found e.g. in (12) (Eq. (11) and Eq. (12), respectively).

Another very natural and interesting choice is the hadroproduction of a vector meson in such "asymmetric" kinematics. Hadron collider experiments should in principle give a powerful opportunity to test our factorization scheme and give access to transversity. Diffractive physics is indeed a topical subject of investigation of the Tevatron at FNAL, RHIC at BNL and LHC at CERN facilities and high quality data will be obtained in the near future at these places. Contrary to the \( \gamma^* \mathcal{P} \rho \)-coupling, the \( p \mathcal{P} p \)-coupling is not yet under theoretical control (diffractive factorization breaking in hadron hadron collisions[13] may forbid the extension of the electroproduction case; moreover, one should take into account the Landshoff mechanism[14]), and deserves much more study before we
can apply the ideas of the present paper to meson production in diffractive \( p \bar{p} \) scattering with asymmetric kinematics.

8. In conclusion, the electroproduction process of two mesons with large rapidity gap can be described consistently within the collinear factorization approach. Higher order studies are necessary to establish its validity beyond the Born order. On the other hand the chiral-odd GPD may now be accessed in a feasible reaction, namely \( \gamma^* p \rightarrow \rho^0 L, \rho^+ n \). An estimate of the cross-section of this reaction requires a knowledge of the chiral-odd GPD. No model has yet been proposed for this quantity. We believe that further improvement of the theoretical understanding of hadronic impact factors may help us to access this chiral-odd GPD also in hadronic diffractive reactions at RHIC and LHC.

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Figure 2: Diagrams contributing to the scattering amplitude in the Born approximation