Higgs sector CP violation in the Minimal Supersymmetric Model

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Abstract

We study the possibility that CP is spontaneously broken in the Minimal Supersymmetric Model when radiative corrections to the Higgs potential are included. We show that this can only occur if a light Higgs boson exists. Considering the recent ALEPH Higgs search, we exclude most of the parameter space of the model. The possibility of explicit CP violation in the model is also briefly discussed.
It has been known for a long time that when supersymmetry (SUSY) is imposed on the two Higgs doublet model (THDM), tree-level flavor changing neutral currents and CP violation are simultaneously avoided in the Higgs sector\(^\text{[1]}\). Nevertheless, since SUSY must be softly broken, new terms in the Higgs potential can be induced by radiative corrections and CP non-conserving effects could show up in the Higgs sector. CP may be broken in two different ways: explicitly and spontaneously. In the first case, CP violation derives from complex scalar self-couplings induced by radiative corrections by sectors of the theory which violate CP. In the second case, a relative phase between the vacuum expectation values (VEVs) of the two Higgs doublets arises which spontaneously breaks the CP symmetry\(^\text{[2]}\).

The purpose of this paper is to study the possibility that CP violation appears in one of these ways in the Higgs sector of the minimal supersymmetric model (MSSM). In such a case the CP-even and CP-odd neutral scalars would mix with each other giving rise to important phenomenological consequences\(^\text{[3]}\). It was recently pointed out\(^\text{[4]}\) that spontaneous CP violation (SCPV) in the MSSM can occur. However, it is known that in order that radiative corrections can cause a spontaneous broken vacuum, a light scalar is required\(^{[5,6]}\). Therefore, an analysis of the physical spectrum, not carried out in ref. [4], is necessary in order to determine the viability of this model.

Let $\Phi_1$ and $\Phi_2$ denote two Higgs doublets with hypercharges $Y = 1$. The most general renormalizable $SU(2)_L \times U(1)_Y$ gauge invariant two Higgs doublet potential is given by

$$V(\Phi_1, \Phi_2) = m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 - (m_{12}^2 \Phi_1^\dagger \Phi_2 + h.c.)$$

$$+ \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$

$$+ \frac{1}{2} \left[ \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + h.c. \right] + \frac{1}{2} \left[ \Phi_1^\dagger \Phi_2 \{ \lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2) \} + h.c. \right],$$

(1)

where by hermiticity only $m_{12}^2$, $\lambda_5$, $\lambda_6$ and $\lambda_7$ can be complex. Let us first consider the case where these parameters are real, i.e. CP is not explicitly violated. After spontaneous symmetry breaking, the VEVs of the neutral components of the Higgs
doublets are given by

\[ < \phi_1^0 >= v_1 , \quad < \phi_2^0 >= v_2 e^{i \xi} . \]

In order to have SCPV, ie. \( \xi \neq \frac{n \pi}{2} \) (\( n \in \mathbb{Z} \)), we need

\[ \lambda_5 > 0 , \quad (2) \]

\[ \left| \frac{2m_{12}^2 - \lambda_6 v_1^2 - \lambda_7 v_2^2}{4\lambda_5 v_1 v_2} \right| < 1 . \quad (3) \]

In this case, at the minimum of the potential,

\[ \cos \xi = \frac{2m_{12}^2 - \lambda_6 v_1^2 - \lambda_7 v_2^2}{4\lambda_5 v_1 v_2} . \]

When SUSY is imposed on the two Higgs doublet potential we have\(^{(1)}\),

\[ \lambda_1 = \lambda_2 = \frac{1}{8}(g^2 + g'^2) , \quad \lambda_3 = \frac{1}{4}(g^2 - g'^2) , \quad \lambda_4 = -\frac{1}{2} g^2 , \quad \lambda_5 = \lambda_6 = \lambda_7 = 0 . \]

Thus, eq. (3) does not hold and \( \xi \) must be 0 or \( \pi \). When radiative corrections are considered, new terms in the Higgs potential are induced. In the limit where the SUSY scale is large, \( M_{SUSY} \gg m_W \), only terms of dimension less than or equal to 4 are not suppressed by inverse powers of \( M_{SUSY} \). In this limit, the effective low-energy Higgs potential of the MSSM is given by eq. (1).

In order to know whether eqs. (2) and (3) hold, we must calculate the induced \( \lambda_5 \) parameter. The \( \lambda_6 \) and \( \lambda_7 \) parameters are in fact not relevant because \( m_{12}^2 \) is a free parameter. The only contribution that generates a positive \( \lambda_5 \) comes from diagrams involving loops of charginos and neutralinos (fig. 1). Squark and Higgs loops give a negative contribution to \( \lambda_5 \) but they can be neglected in the case of
small $\tilde{q}_R - \tilde{q}_L$ mixing and small $m_{12}^2$ respectively. Quarks and gauge bosons do not contribute. In the limit of equal mass charginos and neutralinos, we find

$$\lambda_5 = \frac{g^4}{32\pi^2} \sim 5 \cdot 10^{-4}.$$ 

Therefore, we see from eq. (3) that, in order that SCPV occur, the tree level parameter $m_{12}^2$ must be of $\mathcal{O}(\lambda_5 v_1 v_2) \sim (3 \text{ GeV})^2$. This seems to contradict the Georgi–Pais theorem \cite{6} which says that SCPV can only be generated by radiative corrections when a tree-level massless scalar field, other than the Goldstone boson, exists*. Notice, however, that this theorem is strictly true only for first order corrections to the effective potential. When two-loop corrections are considered, it is easy to see that the scalar can have a tree-level mass whose magnitude is of one-loop order \cite{6}. Of course, the theorem can only be applied when the true minimum is close to the tree-level minimum.

To analyze the physical spectrum, let us make the following rotation

$$\Phi'_1 = \cos \beta \Phi_1 + \sin \beta e^{-i\xi} \Phi_2 = \begin{pmatrix} G^+ \\ v + \frac{1}{\sqrt{2}} (h^0 + iG^0) \end{pmatrix},$$

$$\Phi'_2 = -\sin \beta \Phi_1 + \cos \beta e^{-i\xi} \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}} (H^0 + iA^0) \end{pmatrix},$$

where $\tan \beta = v_2/v_1$, $v = \sqrt{v_1^2 + v_2^2}$, $G^+$ and $G^0$ are the goldstone bosons, $h^0$ and $H^0$ are CP-even fields and $A^0$ is a CP-odd field \cite{7}. The three physical neutral Higgs boson mass eigenstates are mixtures of $h^0$, $H^0$ and $A^0$. The relevant elements of the neutral scalar mass matrix are given by

$$M^2_{h^0A^0} = -2m_{12}^2 \sin \xi,$$

$$M^2_{H^0A^0} = [\lambda_5(v_2^2 - v_1^2) \cos \xi + (\lambda_6 - \lambda_7)v_1 v_2] \sin \xi,$$

$$M^2_{A^0A^0} = 2\lambda_5(v_1^2 + v_2^2) \sin^2 \xi.$$ \hspace{1cm} (4)

It is clear from eq. (4) that there is a light Higgs boson for any value of $\xi$ and $\tan \beta$.

* We must have $m_{12}^2 = 0$ in order to have a massless Higgs boson ($A^0$) at tree-level.
Let us first consider the case where the other neutral Higgs bosons are much heavier. These will be predominantly CP-even states with a small admixture of $A^0$. In this case our model will be similar to the MSSM without CP violation and with a light $A^0$:

$$m_{A^0}^2 \simeq M_{A^0 A^0}^2 \lesssim (6 \text{ GeV})^2.$$  

Since the recent limit from ALEPH Collaboration\(^8\) implies a lower bound of 20 GeV for the CP-odd scalar mass, this possibility is ruled out\(^\star\).

A second possibility is that the mass of one of the CP-even scalars is also small and mixes substantially with the $A^0$. In this case, the ALEPH data must be carefully examined to determine if this possibility is excluded. In particular, the lower scalar mass limits from ALEPH are not valid in a CP violating THDM. To see why this is so, let us denote by $h^0_1$ and $h^0_2$ the two lightest Higgs bosons, and by $\frac{g(p_1+p_2)}{2\cos\theta_W}\Theta_{h^0_1 h^0_2 Z}$ and $\frac{i g m_Z}{\cos\theta_W} g^{\mu\nu}\Theta_{h^0_i ZZ}$ the Feynman rules for the tree level $h^0_1 h^0_2 Z$ and $h^0_i ZZ (i = 1, 2)$ couplings respectively. For a CP conserving Higgs sector ($h^0_2 \equiv A^0$),

$$\Theta_{h^0_i ZZ}^2 + \Theta_{h^0_i h^0_2 Z}^2 = 1.$$  

This relation, which plays a crucial role in inferring lower mass limits for the Higgs bosons, need not be satisfied when CP is violated in the Higgs sector. Nevertheless, a sum rule similar to eq. (5) can be also derived for a CP violating THDM. It is given by\(^7\)

$$\Theta_{h^0_1 ZZ}^2 + \Theta_{h^0_2 ZZ}^2 + \Theta_{h^0_1 h^0_2 Z}^2 = 1.$$  

On the other hand, assuming that $m_{h^0_1} \simeq m_{h^0_2} \lesssim 20 \text{ GeV}$, the ALEPH Higgs

\(^\star\) The ALEPH limit is only valid in the region $\tan \beta > 1$. For $\tan \beta < 1$ there exists a region of the MSSM parameter space in which a light $A^0$ is not excluded by ALEPH. However, regions of parameter space where $\tan \beta < 1$ are strongly disfavored in low-energy supersymmetric models\(^9\).
search[^6] implies the following limits on the $\Theta$'s:

$$\Theta^2_{h^0_1 Z Z} \lesssim 0.1,$$

$$\Theta^2_{h^0_1 h^0_2 Z} \lesssim 0.7.$$  (7)

Combining eq. (6) and eq. (7) we can rule out the possibility of two light Higgs bosons of indefinite CP. If the $h^0_1$ and $h^0_2$ were light enough, they would decay outside the detector and the constraints of eq. (7) could not be deduced. Nevertheless, since any contribution to the $Z$ width from non-standard processes is limited to less than $0.26 \Gamma_{\nu \bar{\nu}}[^{10}]$, bounds on the $h^0_1 h^0_2 Z$ and $h^0_i Z Z$ couplings can also be inferred[^8,11], which turn out to be in contradiction with eq. (6).

Finally, let us briefly consider the case when other sectors of the theory violate CP. In that case, the induced couplings $\lambda_5$, $\lambda_6$ and $\lambda_7$ can be complex[^†] and we have a CP violating Higgs sector even for real VEVs. In supersymmetric theories there are a number of new sources of CP violation from the various supersymmetric sectors. Nevertheless, experimental limits on the neutron electric dipole moment require any such CP violating phases, $\varphi$, to be less than $10^{-2}[^{12}]$. As a result, explicit CP violation effects in the Higgs potential will be of order

$$\lambda_{5,6,7} \cdot \varphi \sim 10^{-5}.$$

These effects are too small to have any significant phenomenological implications.

Summarizing, we have seen that the MSSM with SCPV require the existence of a Higgs boson with a mass of the order of a few GeV. Based on the recent ALEPH Higgs search[^8], we have seen that this model is easily ruled out (except perhaps for a small disfavored region of parameter space where $\tan \beta < 1$). Although explicit CP violation in the Higgs sector is in principle possible, it turns out to be too small to be phenomenologically relevant.

[^†]: $m^2_{12}$ can be made real by a redefinition of the Higgs doublets.
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