A Two-Phase Maximum-Likelihood Sequence Estimation for Receivers with Partial CSI

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Abstract—The optimality of the conventional maximum-likelihood sequence estimation (MLSE), also known as the Viterbi Algorithm (VA), relies on the assumption that the receiver has perfect knowledge of the channel coefficients or channel state information (CSI). However, in practical situations that fail the assumption, the MLSE method becomes suboptimal and then exhaustive checking is the only way to obtain the ML sequence. At this background, considering directly the ML criterion for partial CSI, we propose a two-phase low-complexity MLSE algorithm, in which the first phase performs the conventional MLSE algorithm in order to retain necessary information for the backward VA performed in the second phase. Simulations show that when the training sequence is moderately long in comparison with the entire data block such as 1/3 of the block, the proposed two-phase MLSE can approach the performance of the optimal exhaustive checking. In a normal case, where the training sequence consumes only 0.14 of the bandwidth, our proposed method still outperforms evidently the conventional MLSE.

I. INTRODUCTION

In order to combat the signal distortion due to inter-symbol interference in frequency-selective fading channels, a receiver generally needs a channel estimator and an equalizer, where the former estimates the channel state information (CSI) based on a training sequence, while the latter performs the detection of data using the CSI obtained by the former. In the literature, a commonly used equalization method is the Euclidean-distance-based maximum-likelihood sequence estimation (MLSE) [1]. This MLSE is optimal if the estimator can perform perfect channel estimation; however, when the channel estimator cannot pass perfect CSI to the equalizer, the system performance degrades, thereby inducing the research about receivers with only partial CSI.

The detection criterion for a receiver with only partial CSI, usually referred to as partially coherent receiver, has been investigated in [2–6]. Specifically, they found that the ML criterion for a partial coherent receiver can actually be written as a weighted sum of the ML criterion assuming perfect CSI in the receiver and the ML criterion that assumes no CSI available in the receiver. Since exhaustive checking is the unique optimal method for performing ML sequence estimation for a receiver without CSI, their finding makes the usual Viterbi algorithm (VA) unsuitable for optimal sequence detection when only partial CSI is available [6].

For this reason, we propose in this paper a two-phase method to perform the sequence estimation for a partially coherent receiver. In short, the forward VA will be executed in the first phase, generating necessary information required by the backward VA that uses the partial-CSI ML criterion in the second phase. Simulation results confirm that the proposed two-phase method can considerably outperform the conventional MLSE over channels with only partial CSI available.

Throughout this paper, the following notations will be used: For a matrix $X$, $\det[X]$ is its determinant; $X^T$ and $X^H$ denote its transpose and Hermitian transpose, respectively. Also, I will be used to denote the identity matrix of a proper size.

II. SYSTEM MODEL

In this paper, we consider a signal $b = [b_1, \ldots, b_N]^T$ transmitted over a frequency-selective block fading (equivalently, quasi-static fading) channel of memory order $P - 1$. For $1 \leq i \leq N$ and $M > 0$, we restrict that $b_i$ is the output of constant-amplitude ($2^M$)-PSK modulation, and hence $|b_i|^2 = 1$. Among the $N$ components in signal $b$, the first $T$ components are the training sequence and are assumed known to the receiver, while the latter $(N - T)$ symbols are the data to be transmitted. The received vector $y$ can thus be

$$y = Bh + n,$$

where

$$B = \begin{bmatrix} B_P \\ B_D \end{bmatrix}$$

is formed by a $(T \times P)$ submatrix $B_P$ and a $((L-T) \times P)$ submatrix $B_D$, which are respectively defined as

$$B_P \triangleq \begin{bmatrix} b_1 & \cdots & 0 \\ b_2 & \ddots & \vdots \\ \vdots & & \ddots \\ b_T & \cdots & b_{T-P+1} \end{bmatrix},$$

$$B_D \triangleq \begin{bmatrix} b_{T+1} & \cdots & b_N \\ b_{T+2} & \vdots & \vdots \\ \vdots & \ddots & \vdots \\ b_{2T} & \cdots & b_{N+1} \end{bmatrix}.$$
and
\[
\mathbb{B}_D \triangleq \begin{bmatrix}
    b_{T+1} & \cdots & b_{T-P+2} \\
    \vdots & \ddots & \vdots \\
    b_N & \cdots & b_T \\
    0 & \ddots & \vdots \\
    0 & \cdots & b_N
\end{bmatrix}.
\]

In (1), noise \( n \) is zero-mean circular symmetric complex Gaussian distributed with correlation matrix \( \sigma_n^2 I \), and \( h = [h_1, \ldots, h_P]^T \) denotes the channel taps that remain constant during an \( L \)-symbol transmission block, where \( L = N + P - 1 \).

The underlying assumptions in the system we consider are given below. It is assumed that perfect frame synchronization can be achieved, and adequate guard periods are added between consecutive transmission blocks so that there is no inter-block interference. In addition, both the transmitter and the receiver know nothing about the channel coefficients \( h \) except the multipath parameter \( P \). Notably, the training sequence does not have to be placed at the beginning of \( b \), but can be distributed over the entire transmission block. It however has been shown that placing the training sequence at the beginning of \( b \), together with \( \mathbb{B}_P \mathbb{B}_P = T \mathbb{I} \), can minimize the variance of estimation error [2]. This justifies the model in (1), where \( \mathbb{B}_P \) is placed ahead of \( \mathbb{B}_D \). The condition \( \mathbb{B}_P \mathbb{B}_P = T \mathbb{I} \) is accordingly assumed following [2].

### III. Criterion and Algorithm of the Proposed Two-Phase Method

Based on the system model in (1), we can divide the received signal \( y \) into two parts:
\[
\begin{align*}
    y_P &= \mathbb{B}_P h + n_P \\
    y_D &= \mathbb{B}_D h + n_D
\end{align*}
\]
where \( y_P \) and \( y_D \) are defined via \( y = [y_P, y_D] \), and \( n_P \) and \( n_D \) are similarly defined. Under the reasonable premise that \( T \geq P \), the least square estimate of \( h \), given \( \mathbb{B}_P \) and \( y_P \), is equal to
\[
\hat{h} = (\mathbb{B}_P \mathbb{B}_P)^{-1} \mathbb{B}_P y_P.
\]
Then the ML decoding criterion for a receiver with only partial CSI is given by [2]:
\[
\hat{b}_{ML} = \arg \max_{\hat{b}_D} \Pr(y_D | \mathbb{B}_D, h = \hat{h}) = \arg \min_{\hat{b}_D} \left\{ \| y_D - \mathbb{B}_D \hat{h} \|^2 \right\}
\]
\[
- (y_D - \mathbb{B}_D \hat{h})^H \mathbb{Q}_B (y_D - \mathbb{B}_D \hat{h}) + \sigma_n^2 \log \det \left( \mathbb{I} + (\mathbb{B}_P \mathbb{B}_P)^{-1} \mathbb{B}_B \mathbb{B}_B \right),
\]
where
\[
\mathbb{Q}_B = \mathbb{B}_D (\mathbb{B}_D \mathbb{B}_D + \mathbb{B}_P \mathbb{B}_P)^{-1} \mathbb{B}_D.
\]

At medium to high SNRs, the last term in (2) becomes negligible when it is compared with the first two terms; hence, a near-ML decoding criterion can be yielded as follows:
\[
\hat{b}_{near-ML} = \arg \min_{\hat{b}_D} \left\{ \| y_D - \mathbb{B}_D \hat{h} \|^2 - (y_D - \mathbb{B}_D \hat{h})^H \mathbb{Q}_B (y_D - \mathbb{B}_D \hat{h}) \right\}. \quad (3)
\]

It is noted that the criteria for both \( \hat{b}_{ML} \) and \( \hat{b}_{near-ML} \) contain the Euclidean distance \( \| y_D - \mathbb{B}_D \hat{h} \|^2 \) as their first term, which can be easily decomposed into finite-state recursive expression that readily suits the need of the VA. However, the remaining terms in (2) and (3) do not have finite-state recursive expressions, so the VA cannot be applied to obtain either \( \hat{b}_{ML} \) or \( \hat{b}_{near-ML} \).

At this background, we propose a two-phase method to perform sequence estimation for a partially coherent receiver. The first phase is exactly the MLSE using the Euclidean distance \( \phi_{L-T} \equiv \| y_D - \mathbb{B}_D \hat{h} \|^2 \) in recursive form, i.e.,
\[
\phi_t = \phi_{t-1} + |y_{t+T} - u^t \hat{h}|^2,
\]
where \( \equiv \) denotes that the two sides are equivalent metrics in decoding, and \( y_{t+T} \) and \( u^t \) are respectively the \((t + T)\)th component of \( y \) and the \( t \)th row of \( \mathbb{B}_D \). In order to apply the recursive metric in (4) on a VA trellis, we reformatulate the accumulated metric \( \phi_t \) as a function of the trellis state \( i \) as follows:
\[
\phi_t(i) = \min_{1 \leq j \leq 2^M(P-1)} \{ \phi_{t-1}(j) + |c_t(j, i)|^2 \},
\]
where \( t \) and \( i \) are respectively ranged from 1 to \( L - T \) and from 1 to \( 2^M(P-1) \),
\[
c_t(j, i) = y_{t+T} - u_{t}(j, i)^H \hat{h},
\]
and \( u_{t}(j, i) = [b_{t+T-P+1}(j, i), \ldots, b_{t+T}(j, i)] \) denotes the signals corresponding to the trellis branch from state \( j \) at time \( t - 1 \) to state \( i \) at time \( t \). Meanwhile, two variables will be calculated during the execution of the first phase so that they can be used in the second phase, which are:
\[
\begin{align*}
    \eta_{t}^f(i) &= \eta_{t-1}^f(j) + c_t(j, i) \cdot b_{t+T-\ell}(j, i) \\
    \rho_{t}^f(i) &= \rho_{t-1}^f(j) + (b_{t+T}(j, i))^* \cdot b_{t+T-\ell}(j, i)
\end{align*}
\]
where in the above two formulas, \( j \) is the minimizer of (5), and \( t \) and \( \ell \) are ranged from 1 to \( L - T \) and from 0 to \( P - 1 \), respectively.

In the second phase, a backward VA is performed. Since the simulations in [2] show that \( \hat{b}_{ML} \) and \( \hat{b}_{near-ML} \) yield almost the same performance, we adopt the criterion in (3) to save the computational complexity. We then reexpress the criterion in (3) into an indirect backward recursive form:
\[
\Sigma_t(j, i) = \varphi_{t+1}(j) - \lambda_t(j, i) + \phi_t(i) + |c_{t+1}(i, j)|^2,
\]
where \( j \) and \( i \) are respectively the previous and current states that define the concerned branch in the backward trellis, \( c_{t+1}(i, j) \) is defined in (6), and except that \( \phi_t(i) \) is from the
The First Phase (Forward VA):

**Step 1-1. Initialization:**
For $1 \leq \ell \leq P - 1$ and for $1 \leq i \leq 2^{M(P-1)}$, initialize $\eta_0^{(i)}(i) = 0$ and $\rho_0^{(i)}(i) = 0$. Let $\phi_0(1) = 0$ and $\phi_0(i) = \infty$ for $2 \leq i \leq 2^{M(P-1)}$.

**Step 1-2. Recursion (From $t = 1$ to $t = L - T$):**
For $1 \leq i \leq 2^{M(P-1)}$ and for $1 \leq \ell < P$, compute
\[
\phi_t(i) = \min_{1 \leq j \leq 2^{M(P-1)}} (\phi_{t-1}(j) + |c_t(j, i)|^2),
\]
\[
\xi_t(i) = \arg\min_{1 \leq j \leq 2^{M(P-1)}} (\phi_{t-1}(j) + |c_t(j, i)|^2).
\]

**Update**
\[
\eta_t^{(i)}(i) = \eta_{t-1}^{(i)}(\xi_t(i)) + c_t(\xi_t(i), i) \cdot b_{t+T-\ell}(\xi_t(i), i),
\]
\[
\rho_t^{(i)}(i) = \rho_{t-1}^{(i)}(\xi_t(i)) + (b_{t+T}(\xi_t(i), i)) \cdot b_{t+T-\ell}(\xi_t(i), i),
\]
where $c_t(j, i) = y_{t+T} - u(j, i)^T h$ and $u(j, i) = [b_{t+T-P-1}(j, i), \cdots, b_{t+T}(j, i)]$ consists of $P$ symbols corresponding to the trellis branch between state $j$ at time $t-1$ and state $i$ at time $t$.

The Second Phase (Backward VA):

**Step 2-1. Initialization:**
For $1 \leq \ell \leq P - 1$ and for $1 \leq i \leq 2^{M(P-1)}$, initialize $\zeta_{L-T+1}^{(i)}(i) = 0$ and $\sigma_{L-T+1}^{(i)}(i) = 0$. Let $\varphi_{L-T+1}(1) = 0$ and $\varphi_{L-T+1}(i) = \infty$ for $2 \leq i \leq 2^{M(P-1)}$.

**Step 2-2. Recursion (From $t = L - T$ down to $t = 1$):**
For $1 \leq i \leq 2^{M(P-1)}$, compute
\[
\xi_t(i) = \arg\min_{1 \leq j \leq 2^{M(P-1)}} (\varphi_{t+1}(j) + |c_t(i, j)|^2) + \phi_t(i) - \lambda_t(i, j),
\]
where the terms involved in the above computations have been introduced previously.

**Update**
\[
\varphi_t(i) = \varphi_{t+1}(\xi_t(i)) + |c_t(i, \xi_t(i))|^2,
\]
\[
\zeta_t^{(i)}(i) = \zeta_{t+1}^{(i)}(\xi_t(i)) + c_t(i, \xi_t(i)) \cdot b_{t+1+T-\ell}(i, \xi_t(i)),
\]
\[
\sigma_t^{(i)}(i) = \sigma_{t+1}^{(i)}(\xi_t(i)) + (b_{t+1+T}(i, \xi_t(i))) \cdot b_{t+1+T-\ell}(i, \xi_t(i)).
\]

**Step 2-3. Trace Back:**
Output the best state sequence $[1, s_1, \cdots, s_{L-T}, 1]$, where $s_t = \xi_t(s_{t-1})$, and its corresponding decision symbol sequence.

IV. Complexity Analysis

The computational complexity of the proposed algorithm consists of the forward VA complexity $C_F$ and backward VA complexity $C_B$. Since both the forward VA and backward VA operate on a trellis having $2^{M(P-1)}(N - T)$ states and there are $2^M$ branch metric calculations for each state, these two complexities can be expressed as
\[
C_F = N_F \cdot 2^M \cdot 2^{M(P-1)}(N - T)
\]
and
\[
C_B = N_B \cdot 2^M \cdot 2^{M(P-1)}(N - T),
\]
where $N_F$ and $N_B$ are the branch metric computational complexities in forward VA and backward VA, respectively.

By convention, the branch metrics dominate the branch metric computational complexity; therefore, $N_F$ and $N_B$ can be approximated by the number of complex multiplications required in forward VA and backward VA, respectively. As a result, in forward VA, there is a $P$-tag filter and two additional complex multiplications for each branch; so, we set $N_F = P + 2$. In backward VA, each branch metric calculation needs a $P$-tag filter for the calculation of $\varphi_t(\cdot), 2P^2$ complex multiplications for $\lambda_t(\cdot, \cdot)$, $P^3$ complex multiplications for $\delta_u(\cdot, \cdot)$ and $4P$ complex multiplications for the remaining variables. We then obtain $N_B = 5P + 2P^2 + P^3$. The total computational complexity is accordingly given by
\[
C_F + C_B = (N_F + N_B) \cdot 2^{MP}(N - T) = (2 + 6P + 2P^2 + P^3) \cdot 2^{MP}(N - T).
\]

The complexity is considerably more than the complexity of conventional MLSE, which is $P \cdot 2^{MP}(N - T)$. However, it
is much smaller than the complexity of the optimal exhaustive checking decoder, which is
\[(N_F + N_B) \cdot 2^{MN+1}\] (10)

We remark at the end that because the complexity of on-line computations of \(\delta_{n,r}(\cdot,\cdot)\) is high for a large \(P\), the proposed two-phase method may be more suitable for channels with small \(P\), or for a system with pre-filters at the receiver to reduce the tap number of channels [7], [8].

V. SIMULATIONS

For simplicity, only BPSK modulation is considered in simulations; thus \(M = 1\). The channel coefficients \(h\) are zero-mean complex-Gaussian distributed with \(E[hh^H] = (1/P)I\) and \(P = 2\). By the system model introduced in Section III, the signal-to-noise power ratio per information bit is given by

\[
\frac{E_b}{N_0} (\text{dB}) = 10 \log_{10} \frac{\text{tr} (E[hh^H])}{\sigma_n^2} = 10 \log_{10} \frac{E[\nu^2]}{\sigma_n^2} = 10 \log_{10} \frac{1}{\sigma_n^2}. 
\]

We first examine our proposed two-phase method using a data sequence of length \(N = 15\), in which 5 of them are training sequence and are equal to \([-1, -1, -1, 1, 1]\). Figure 1 then shows that the word error rate (WER) of our proposed two-phase method is almost the same as that of the exhaustive checking scheme using criterion 3. This figure also indicates that our proposed two-phase method outperforms the conventional MLSE by about 0.8 dB. All three schemes estimate channel coefficients via a least square (LS) estimator. This result confirms that our proposed two-phase MLSE (designed based on criterion 3 for complexity saving) can achieve the optimal performance of exhaustive checking when the length of the training sequence is moderately large (for example, 1/3) in comparison with the entire block size.

Next, we consider a longer block of length \(N = 70\), in which only 10 of them are training sequence and are equal to \([1, 1, 1, 1, 1, -1, -1, -1, 1, 1]\). Note that the training sequence consumes around 10/70 = 0.14 of the bandwidth. Figure 2 then shows that the proposed two-phase method still maintains a 0.7 dB advantage in comparison with the conventional MLSE with LS estimation. Because at this block length, the exhaustive checking method is no longer feasible, we provide the performance of the conventional MLSE with perfect CSI in this figure as a reference genie-aided performance lower bound.

In Figs. 3 and 4 we examine our proposed two-phase method over the Gauss-Markov fading channel [11], [12]. In this channel, the channel coefficients that are fixed within a data burst period are varying according to

\[
h_t = \alpha \cdot h_{t-1} + \sqrt{1 - \alpha^2} v_t \tag{11}
\]

This number is smaller than what is considered in a GSM data burst, where a 148-bit normal burst contains a 26-bit training sequence.

Fig. 1. Word error rates (WERs) of three MLSE schemes in block fading channels for a data burst of length 15, in which 5 of them are training sequence.

There are 10 of them are training sequence.

Fig. 2. An additional scheme is added in comparison with our proposed two-phase method, which is the MLSE with an adaptive least mean square (LMS) filter [9], [10]. This filter has been proved to be effective in tracking the time-varying nature of time-varying channels. Under the assumption that the receiver can perfectly estimate the value of \(\alpha\), the step size of the LMS filter used in our simulations is set to be \(\sqrt{1 - \alpha^2}/2\).

Figure 3 then shows that for \(\alpha = 0.9999\), our two-phase method outperforms the other two equalization schemes. The simulation result under \(\alpha = 0.9999\) also indicates similar performance gain of our two-phase method over the other two equalization schemes except that a performance floor appears at high SNR. We again provide the performances of the conventional MLSE with perfect CSI in these two figures as reference genie-aided performance lower bounds.

VI. CONCLUSION

After establishing the recursive expression of ML criterion for partially coherent receiver, we propose a two-phase MLSE algorithm in this paper. Simulation results show that our method outperforms the conventional MLSE in both quasi-static block fading channels and time-varying Gauss-Markov channels. A possible future work could be to modify our
algorithm to provide soft-outputs so that it can iteratively co-work with an outer coding scheme.

VII. ACKNOWLEDGMENT

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