Tunneling and Fluctuating Electron-Hole Cooper Pairs in Double Bilayer Graphene

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A strong low-temperature enhancement of the tunneling conductance between graphene bilayers has been reported recently, and interpreted as a signature of electron-hole pairing. The pairing in electron-hole double layers was first predicted more than forty years ago but has since avoided observation. Here we provide a detailed theory of conductance enhanced by electron-hole Cooper pair fluctuations, which are a precursor to equilibrium pairing, that reflects specific details of double graphene bilayer systems. Above the equilibrium condensation temperature, the pairs have finite temporal coherence and do not support dissipationless tunneling. Instead they strongly boost the tunneling conductivity via a fluctuational internal Josephson effect. We find dependences of the zero-bias peak in the differential tunneling conductance on temperature and electron-hole density imbalance that are in an enough good agreement with experiment.

I. INTRODUCTION

The possibility of the Cooper pairing in a system with spatially separated electrons and holes in semiconductor quantum wells was first anticipated more than forty years ago [1, 2]. According to theory, strong Coulomb interactions allow pairing at elevated temperatures, which would provide a physical realization of dipolar superfluidity that is potentially relevant for applications. The paired state is fragile however, and can be suppressed by disorder [3, 4] or by Fermi-line mismatches due to the differences between electron and hole anisotropies [5, 6] that are always present in conventional semiconductors [7]. In fact pairing has until recently been observed only in the presence of strong magnetic fields that quench the kinetic energies of electrons and holes and drive the system to the regime of strong correlations [8, 9].

Recent progress in fabricating a new class single-atomic-layer two-dimensional materials has renewed interest in electron-hole pairing [10–21]. Graphene-based two-dimensional electron systems not only have high mobility and almost perfect electron-hole symmetry but make it possible to fabricate closely-spaced, and therefore strongly interacting, independently gated and contacted double layer structures. Very recently low-temperature enhancement of the tunneling conductance between graphene bilayers has been observed at matched concentrations of electrons and holes [22]. A typical conductance trace is presented in Fig. 1, where we see a tunneling conductance that appears to diverge at $T_0 \approx 1.5$ K, signalling equilibrium pair condensation. This observation provides the first clear experimental signature of equilibrium electron-hole pair condensation [23].

Enhanced tunneling conductance has been observed previously in semiconductors bilayers in the strong field quantum Hall regime, [24] and has been interpreted as an internal Josephson effect [8]. The differential conductance, does not diverge however, and instead has a sharp peak at zero bias. The property that the conductance peak width is smaller than temperature, and smaller than the single-electron scattering rate (i.e. the Landau level width) nevertheless points to a collective origin of the peak. Bilayers in the quantum Hall regime are predicted to support dissipationless Josephson-like tunneling currents in the presence of the long-range electron-hole coherence [25, 26]. The development of a quantitative theory of enhanced tunneling in quantum Hall systems [27–30] that fully explains the peak width has been challenged by the importance of inhomogeneity and disorder, and by strong interactions in the presence of dispersionless Landau levels. Phase fluctuations that are inevitably present due to the two-dimensional Berezinskii-Kosterlitz-Thouless nature of the phase transition [31–34].

FIG. 1. Temperature dependence of the tunneling conductance $G_T$ between graphene bilayers at zero voltage bias. The calculated conductance (red) accurately fits the experimental data (blue dots) [22]. The purple dashed curve correspond to the theoretical conductance of a model that neglects interactions. Concentrations of electrons and holes do coincide and are equal to $7.4 \times 10^{11} \text{ cm}^{-2}$. Cooper pair fluctuations above the critical temperature $T_0 = 1.5$ K strongly enhance the tunneling conductance and are responsible for critical behavior in which $G_T \sim (T - T_0)^{-2}$. 

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also play a role. The theory of enhanced tunneling is simpler at zero magnetic field, at least in the weak coupling regime where the electron-hole pairing energy is small compared to the Fermi energy, allowing experiments to be explained more fully as we demonstrate below.

Since the pairing energy is small compared to the Fermi energy in the recent experiment with double bilayer graphene [22], as we explain more fully below, pairing is in the weak to moderate coupling regime. True internal Josephson behavior occurs only below a critical temperature $T_0$ and is preceded by enhancement of the tunneling conductance that diverges as $T_0$ is approached as illustrated in Fig. 1. This critical behavior has been predicted by one of the authors [35] and has been interpreted as a fluctuational internal Josephson effect. It originates from partly coherent fluctuating electron-hole Cooper pairs [35–37] that are a precursor of equilibrium pairing and reminiscent of Aslamazov-Larkin and related effects in superconductors [38–40]. Above $T_0$ Cooper pair fluctuations have a finite coherence time and cannot support a dissipationless tunneling current. While the recent observations do qualitatively agree with earlier theory, full agreement can be obtained by taking into account peculiarities of the electronic structure of bilayer graphene related to the system’s well known $2\pi$ momentum space Berry phases.

In the present work we have developed the theory of fluctuational internal Josephson effect in a system with closely spaced graphene bilayers. As we show below, the presence of valley and sublattice degrees of freedom provides three competing electron-hole channels for both intravalley and intervalley Cooper pairs. We show that the three channels are nearly independent and have different critical temperatures of the condensation and sublattice structures. The experimental fluctuation enhancement of the tunneling conductance can be explained only by the presence of competing channels that dominate in different temperature ranges. In the vicinity of $T_0$, the tunneling conductance at zero bias is predicted to have a critical divergence $G_T \sim (T - T_0)^{-2}$, that matches the experimental data well as we see in Fig. 1. The calculated dependence of the tunneling conductivity on interlayer voltage bias and carrier-density imbalance also match the experimental data [22] reasonably. We conclude that the observed enhancement of the tunneling conductance in double bilayer graphene is well explained by our fluctuation internal Josephson effect theory.

The rest of the paper is organized as follows. In Sec. II we introduce a model that describes the low-energy physics of two closely spaced graphene bilayers. Sec. III is devoted to a description of Cooper pair fluctuations above the critical temperature $T_0$. In Sec. IV we use these as the starting point for a theory of the tunneling conductance. In Sec. V we compare our calculations with the recent experimental data. Finally in Sec. VI we discuss limitations of our theory and aspects of the experimental data that are still not well understood, and present our conclusions.

II. MODEL

The system of interest contains two graphene bilayers separated by an insulator as sketched in Fig. 2-a. An external electric field perpendicular to the bilayers induces an excess of electrons in the top bilayer (t) and their deficit in the bottom bilayer (b). The electronic spectrum in the case of their matched concentrations, the most favorable regime for the Cooper pairing, is sketched in Fig. 2-b which illustrates the gaps at bilayer neutrality in the isolated bilayer spectra. Equilibrium electron-hole pairing in this system has been considered in a number of recent papers [16–19] that aim to provide realistic predictions of the critical temperature $T_0$ based on the microscopic model of the ideal system without disorder. Here we follow a different route and consider a minimal phenomenological model that describes the instability of the system towards electron-hole pairing in the weak coupling regime while accounting for disorder.

The spectrum sketched in Fig. 2-b has spin and valley degeneracy. The spin degrees of freedom simply added a factor of 2 in the tunneling conductance when the condensed state preserves spin-invariance and do not need to be treated explicitly. In graphene multilayers low energy states are always concentrated around two inequivalent valleys $K$ ($v = 1$) and $K'$ ($v = -1$) situated at the corners of the first Brillouin zone and are described by the following Hamiltonian

$$H_0 = \sum_p \left[ \hat{\psi}_p^+ (\hbar \hat{h}_p - \mu) \hat{\psi}_p + \hat{\psi}_p^+ (\hbar \hat{h}_{tp} + \mu_b) \hat{\psi}_{tp} \right].$$

where $\hat{\psi}_p = \{ \psi_{1p}, \psi_{2p} \}$ is a spinor of annihilation operators for electrons in both layers with sublattice indexes.
σ = 1, 2, that are numered according to the sketch in Fig. 2-a. We assume that there is a deficit of electrons in the bottom layer, but it is instructive not to perform the transformation to field operators of holes. μt = ϵ_F + h and μb = ϵ_F - h characterize the electric potentials in the top and bottom bilayers. Here ϵ_F is the average of the electron and hole Fermi energies at neutrality, while 2h ≪ ϵ_F is their difference. The matrices $h_{tp}$ and $h_{bp}$ are

$$
\hat{h}_{tp} = \left( \begin{array}{c} u \frac{p_{n}^{2}}{2m} \\ \frac{u}{2m} \end{array} \right), \quad \hat{h}_{bp} = \left( \begin{array}{c} -u \frac{p_{n}^{2}}{2m} \\ \frac{u}{2m} \end{array} \right). \quad (2)
$$

Here v = ±1 and $\bar{v} = \mp 1$ are valley indexes, m is the electron mass, and $p_{n} = p_{x} + iv_{y}$. The electric field perpendicular to bilayers opens a gap 2|u| separating conduction $\epsilon_{tp} = \epsilon_{p}$ and valence $\epsilon_{bp} = -\epsilon_{p}$ bands in each bilayer with $\epsilon_{p} = \sqrt{u^{2} + (p^{2}/2m)^{2}}$. Since the thickness of each bilayer is considerably smaller than the distance between them $d$, only the weak and moderate coupling BCS regime of electron-hole Cooper pairing can be realized in the experimental setup [22], and that limit is therefore considered here. In this regime only the conduction band of the top bilayer and the valence band of the bottom bilayer are relevant, and the corresponding spinor wave functions are

$$
|tp⟩ = \left( e^{-i\varphi_{p}} \right) \left( \begin{array}{c} c_{p} \\ s_{p} \end{array} \right), \quad |bp⟩ = \left( e^{-i\varphi_{p}} \right) \left( \begin{array}{c} c_{p} \\ -s_{p} \end{array} \right). \quad (3)
$$

Here $\varphi_{p}$ is the polar angle; $\epsilon_{p} = \cos(\varphi_{p}/2)$ and $s_{p} = \sin(\varphi_{p}/2)$ with $\cos(\varphi_{p}) = u/\epsilon_{p}$. The spinors have valley dependent chirality $\pm t_{t(b)} φ_{p}$ that defines a sublattice structure of fluctuating electron-hole Cooper pairs as will be shown below. We introduce disorder with the help of phenomenological decays rates $γ_{t(b)}$. It is important in this theory to observe that because the electron and hole components of the Cooper pair are spatially separated and have opposite charges, both short-range and long-range Coulomb disorder lead to pair-breaking [3, 4].

In experiment [22] the relative angle $\theta$ between graphene bilayers can be adjusted. Since valleys $K$ and $K'$ reside at the corners of the first Brillouin zone, the valley momenta in the two layers do not match in the presence of a twist. For momentum conserving tunneling, the current is maximized when the layers are aligned ($θ_{0} = 0$) or twisted by $θ_{0} = nπ/3$. For even $n$, valley $K(K')$ in one layer is aligned with valley $K(K')$ in the other layer whereas for $n$ odd valley $K(K')$ in one layer is aligned with valley $K'(K)$ in the other layer. When states are labelled by their momenta relative to the Brillouin-zone corners, the tunneling Hamiltonian for $θ$ close to $nπ/3$ is

$$
H_{t} = T^{+} + T = \sum_{p} \left[ \hat{ψ}^{+}_{tp} \hat{ψ}^{*}_{tp} + T^{+} \hat{ψ}^{+}_{tp} + T^{-} \hat{ψ}_{tp} \right]. \quad (4)
$$

Here $Q = Q_{B} + Q_{B}$ is the momentum splitting between valleys in the different layers. The twist contribution $Q_{B} = 2q_{K} \sin(θ/2)$ where $q_{K} = 4π\hbar/3a_{0}$ is the magnitude of the Brillouin-zone corner vector for bilayer graphene, $a_{0}$ is the corresponding Bravais lattice period and $θ$ is the twist angle. A relative momentum contribution, $Q_{B} = h/2a_{0}$, can also be induced by an in-plane magnetic field $B_{||}$. Here $t_{||} = \sqrt{\hbar c/eB_{||}}$ is a magnetic length. Because each bilayer is represented by a two-band model, the matrix $t$ has four matrix elements which we treat phenomenologically below, with the expectation that since $t_{22}$ corresponds to the tunneling between adjacent sublayers while $t_{11}$ to the tunneling between remote sublayers, $|t_{11}| \ll |t_{12}| \approx |t_{21}| \ll |t_{22}|$.

### III. COOPER PAIR FLUCTUATIONS

Due to Coulomb interactions between electrons and holes, the double-bilayer system is unstable towards Cooper pairing. Here we omit the repulsive interaction within each graphene bilayer since its main effect is a simple renormalization of the quasiparticle spectra. The interbilayer attraction

$$
H_{int} = \sum_{pp'} \sum_{\sigma \tau} U_{q} \hat{ψ}^{+}_{tp+q_{,}p\sigma} \hat{ψ}^{+}_{tp'-q_{,}σ} \hat{ψ}_{tp'σ} \hat{ψ}^{+}_{tpσ}. \quad (5)
$$

Here $U_{q}$ is the screened Coulomb potential estimated in our previous work [13–15]. Below we employ a multipole decomposition of the interaction and set the momenta

![Diagram](image-url)
magnitudes to the Fermi momentum so that $U$ reduces to a constant ($U_l$) for each orbital angular momentum channel $l$. The set of $U_l$ parameters are also treated as phenomenological parameters with the expectation that the $s$-wave moment $U_s \equiv U_0$ is largest. Since valleys are well separated in momentum space we neglect inter-valley scattering for electrons and holes.

The instability of double layer system towards electron-hole Cooper pairing is signaled by divergence of the electron-hole channel scattering vertex $\hat{\Gamma}_{\sigma_i \sigma_h}^{\sigma_i \sigma_h}(\omega, \mathbf{p}', \mathbf{p}, \mathbf{q})$ at zero frequency $\omega$. Note that in our approximation scattering conserves valley indices for electrons ($v_i$) and holes ($v_h$). The $\Gamma$-vertex satisfies the Bethe-Saltpeter equation presented in Fig. 3-a, and is algebraic within the multipole approximation. It is instructive to combine $\sigma_i$ and $\sigma_h$ into a single index $|\sigma \rangle$ that varies between 1 and 4 as $|1\rangle = |11\rangle$, $|2\rangle = |12\rangle$, $|3\rangle = |21\rangle$ and $|4\rangle = |22\rangle$. With this definition the Bethe-Saltpeter equation can be written in a compact matrix form:

$$\hat{\Gamma}_{l'1} = U_l \delta_{l'l} + \sum_{l''} U_{l'l''} \hat{M}_{l''l} \hat{\Pi}_{l'l}. \quad (6)$$

Here momentum and frequency dependences are suppressed, $l$ ($l'$) is the orbital momentum for the relative motion of two particles before (after) scattering, and $\hat{\Gamma}_{l'l}$ is the corresponding scattering matrix. We have separated a factor of $\Pi(\omega, \mathbf{q})$ which also appears as the single-step pair propagator in the Cooper ladder sum of a bilayer system without sub lattice degrees of freedom:

$$\Pi(\omega, \mathbf{q}) = \mathcal{N}_F \left\{ \ln \left[ \frac{\epsilon_c}{2\pi T} \right] - \frac{1}{2} \sum_{\zeta = \pm} \left\langle \hat{\Psi} \left( \frac{1}{2} + \frac{i[\omega + \zeta(\hbar + v_\mathbf{q} \cos \phi_p) + \gamma]}{4\pi T} \right) \right\rangle_{\phi_p} \right\}. \quad (7)$$

Here $\epsilon_c$ is an energy cutoff that is required for momentum-independent interactions. The average $\langle \cdots \rangle_{\phi_p}$ is calculated respect to a polar angle $\phi_p$. $\gamma = \gamma_1 + \gamma_2$ is the pair breaking rate, which is the sum of the scattering rates for electrons $\gamma_1$ and holes $\gamma_2$. The expression for $\Pi(\omega, \mathbf{q})$ in Eq. (7) is well known [38, 39] from previous work on systems without layer degrees of freedom. The chiral nature of the bilayer graphene charge carriers is captured by the nontrivial matrix form-factor $\hat{M}_l$ in the Bethe-Salpeter Eq. (6). The matrix form-factor $\hat{M}_l$ is defined as the multipole moment of the two-particle matrix element

$$\hat{M}_{\sigma_i, \sigma_h}^\sigma(\phi_p) = \langle \sigma_i | t_{\mathbf{cp}} + \frac{\mathbf{q}}{2} \rangle \langle t_{\mathbf{cp}} + \frac{\mathbf{q}}{2} | \sigma_h \rangle \langle b_{\mathbf{p}} | b_{\mathbf{p}}^\dagger \rangle \langle b_{\mathbf{p}} - \frac{\mathbf{q}}{2} | \sigma_i \rangle. \quad (8)$$

Corrections to the form-factor $\hat{M}_l$ due to finite Cooper pair momentum $q$, $|\Delta M| = q^2 |u| / 4p_F^2 \epsilon_F$, are negligible in the weak coupling regime since $q \ll p_F$. As a result, the matrix (8) can be approximated as follows

$$\hat{M}(\phi_p) = \begin{pmatrix} c^4 & -c^3 s e^{-2iv_\mathbf{q}} \phi_p & c^3 s e^{-2iv_\mathbf{q}} \phi_p & -c^2 s^2 e^{-2iv_\mathbf{q} + iv_\mathbf{t}} \phi_p \\ -c^3 s e^{2iv_\mathbf{q}} \phi_p & c^2 s^2 & c^3 s e^{-2iv_\mathbf{q}} \phi_p & -c^2 s^2 e^{-2iv_\mathbf{q} + iv_\mathbf{t}} \phi_p \\ c^3 s e^{2iv_\mathbf{q} - iv_\mathbf{t}} \phi_p & -c^2 s^2 e^{2iv_\mathbf{q} - iv_\mathbf{t}} \phi_p & c^2 s^2 & c^3 s e^{2iv_\mathbf{q} - iv_\mathbf{t}} \phi_p \\ -c^2 s^2 e^{2iv_\mathbf{q} - iv_\mathbf{t}} \phi_p & -c^3 s e^{2iv_\mathbf{q} - iv_\mathbf{t}} \phi_p & -c^2 s^2 e^{-2iv_\mathbf{q} + iv_\mathbf{t}} \phi_p & c^3 s e^{-2iv_\mathbf{q} + iv_\mathbf{t}} \phi_p \end{pmatrix}. \quad (9)$$

Here the coefficient $c$ is $s$ correspond to the coherence factors $e_p$ and $s_p$ in (3) evaluated at the average Fermi energy $\epsilon_F$ for electrons and holes and are given by

$$c^2 = \frac{1}{2} \left( 1 + \frac{u}{\epsilon_F} \right), \quad s^2 = \frac{1}{2} \left( 1 - \frac{u}{\epsilon_F} \right). \quad (10)$$

The matrix form-factor (9) shapes the sublattice structure of fluctuating electron-hole Cooper pairs in double-bilayer graphene. Note that it couples scattering channels $\hat{\Gamma}_{l'l}$ with different orbital momenta. Importantly, $\hat{M}_l$ has only even harmonics $l = 0, \pm 2, \pm 4$ that forbid scattering between states with even and odd orbital momenta. For isotropic Coulomb interactions, the $s$-wave moment $U_s$ is expected to be largest. It is instructive
to start by neglecting all other moments. In that case only $\tilde{\Gamma}_{00}$ is nonzero. The latter depends on the $s$-wave moment of the form-factor $\tilde{M}$ that has a different form for intravalley and intervalley Cooper pairs. We discuss these two case separately below.

**Intravalley channel**

For intra-valley electron-hole Cooper pairs the matrix form-factor $\tilde{M}$ is given by

$$\tilde{M}_0 = \begin{pmatrix} c^4 & 0 & 0 & 0 \\ 0 & c^2 s^2 & -c^2 s^2 & 0 \\ 0 & -c^2 s^2 & c^2 s^2 & 0 \\ 0 & 0 & 0 & s^4 \end{pmatrix}. \quad (11)$$

The scattering problem decouples into the three channels identified in Ref. [18] and the corresponding scattering vertex is given by

$$\tilde{\Gamma}_{00} = \begin{pmatrix} s^4 U & 0 & 0 & 0 \\ 0 & c^2 s^2 U & -c^2 s^2 U & 0 \\ 0 & -c^2 s^2 U & c^2 s^2 U & 0 \\ 0 & 0 & 0 & s^4 U \end{pmatrix}. \quad (12)$$

Here $L_\alpha = 1/\lambda_\alpha^2 - \Pi/N_F$ is a dimensionless inverse Cooper propagator with label $\alpha$ and the corresponding coupling constant $\lambda_\alpha^2$ is specified in Tab. (I). $L_\alpha$ vanishes at the critical temperature $T_\alpha$ for the electron-hole pair instability in channel $\alpha$.

In the absence of disorder and electron-hole density imbalances, the critical temperatures are given by $T_\alpha = 2eC/\pi \epsilon_c [-1/\lambda_\alpha^2]$, where $C = 0.577$ is the Euler constant. Although the coupling constant values $\lambda_\alpha$ can be fine-tuned by the displacement field, as it is presented in Fig. 4, their hierarchy is universal for the case $|u| \ll \epsilon_F$. The coupling constant $\lambda_{12-21}^2 \approx 1/2$ is almost twice as large as the constants $\lambda_{11(22)}^2 \approx 1/4$, ensuring domination of the mixed $12-21$ channel. Physically the presence of two sub lattice combinations ([12] and [21]) doubles the number of states that take part in the Cooper pairing.

**Intervalley channel**

For intra-valley electron-hole Cooper pairs the matrix form-factor $\tilde{M}$ is given by

$$\tilde{M}_0 = \begin{pmatrix} c^4 & 0 & 0 & -c^2 s^2 \\ 0 & c^2 s^2 & 0 & 0 \\ 0 & 0 & c^2 s^2 & 0 \\ -c^2 s^2 & 0 & 0 & s^4 \end{pmatrix}. \quad (13)$$

This case also decouples into three independent channels, with scattering vertex

$$\tilde{\Gamma}_{00} = \begin{pmatrix} s^4 U & 0 & 0 & -c^2 s^2 U \\ 0 & c^2 s^2 U & 0 & 0 \\ 0 & 0 & c^2 s^2 U & 0 \\ -c^2 s^2 U & 0 & 0 & s^4 U \end{pmatrix}. \quad (14)$$

Interestingly, the sublattice structure of the Cooper pairs and the corresponding coupling constants are different in intravalley and intervalley cases. The latter are presented in the Tab. (II) and their dependence on displacement field parameter $u$ is shown in Fig. 4. In the regime $|u| \ll \epsilon_F$ the coupling constants are $\lambda_{12-21}^2 \approx 1/2$, and $\lambda_{11(22)}^2 \approx 1/4$, so that the mixed channel 11-22 is expected to have the highest critical temperature.

The separation of scattering problem into three channels is not an artifact of the $s$-wave truncation, but is maintained when higher multipole momenta of interac-

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**TABLE I. Coupling constants $\lambda_\alpha$ for intra-valley electron-hole Cooper pairs**

| Label, $\alpha$ | s-wave, $\lambda^s$ | d-wave, $\lambda^d$ |
|----------------|---------------------|---------------------|
| 11             | $c^3 N_F U_s$       | $2c^2 c^2 N_F U_d$ |
| 22             | $s^4 N_F U_s$       | $2c^2 c^2 N_F U_d$ |
| 12-21          | $2s^2 c^2 N_F U_s$  | $(c^4 + s^4) N_F U_d$ |
TABLE II. Coupling constants $\lambda_\alpha$ for Intra-valley electron-hole Cooper pairs

| Label, $\alpha$ | $s$-wave, $\lambda^s$ | $d$-wave, $\lambda^d$ |
|-----------------|----------------------|----------------------|
| 12              | $c^s s^2 N_F U_6$    | $(c^s + s^d) N_F U_6$ |
| 21              | $c^s s^2 N_F U_6$    | $(c^s + s^d) N_F U_6$ |
| 11-22           | $(c^s + s^d) N_F U_6$| $2\sqrt{2} s s^2 N_F U_6$ |


The dependence of the critical temperature $T_0$ on the pair-breaking rate $\gamma$ and the electron-hole imbalance expressed as a difference in Fermi energies $h$, are illustrated in Fig. 5-a. The electron-hole pair instability is suppressed when the pair-breaking rate exceeds a critical value $\gamma \approx 1.78 T_0$. If the rate does not exceed $\gamma \approx T_0$ the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state with finite Cooper pair momentum $Q_0$ is stabilized by finite imbalance. The dependence of the instability momentum $Q_0$ is presented in Fig. 5-b. Its magnitude can be approximated by $v_T Q_0 \approx h$, corresponding to the difference between the Fermi momenta for electrons and holes.

In the absence of electron-hole imbalance, the Cooper propagator $L_0^{-1}$ of the dominating channel at small frequencies and momenta simplifies to

$$L_0^{-1}(\omega, q) = \frac{1}{i\omega \tau - \epsilon_q}, \quad \epsilon_q = \epsilon + \frac{\xi^2 q^2}{\hbar^2}. \quad (16)$$

Here $\epsilon = \ln |T/T_0|$ specifies the energy scale that is required to create a uniform Cooper pair fluctuation and vanishes at critical temperature, and $\tau$ and $\xi$ are characteristic time and spatial scales for the Cooper pairs which are given by

$$\tau = \frac{\hbar \Psi' (\frac{1}{2} + \frac{\gamma}{4\sqrt{T}}) \Psi'' (\frac{1}{2} + \frac{\gamma}{4\sqrt{T}}) \frac{1}{2}}{4\pi T}, \quad \xi = \frac{\hbar v_F |\Psi'' (\frac{1}{2} + \frac{\gamma}{4\sqrt{T}}) \frac{1}{2}}{8\pi T}. \quad (17)$$

They are connected to the coherence time and length of Cooper pairs by $\tau^* = \tau/2\epsilon$ and $\xi^* = \xi/\sqrt{\epsilon}$ that diverge at the critical temperature for electron-hole condensation $T_0$. The Cooper propagator (16) has its only pole on the imaginary frequency axis at $\omega_q = -i/2\tau_q^*$ with $\tau_q^* = \tau/2\epsilon_q$, reflecting the dissipative nature of Cooper pairs dynamics. Due to the presence of a finite temporal renormalization of coupling constants $\lambda_\alpha = \lambda^{s}_\alpha + \lambda^{d}_\alpha$. The $d$-wave coupling constants $\lambda^{d}_\alpha$ are summarized in Tabs. I and II.

Disorder and Density Imbalances

Disorder and electron-hole density imbalances both reduce the critical temperature $T_0$. The latter can stabilize the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) pairing state [5, 6, 41, 42] with finite Cooper pair momentum $Q_0$. The critical temperature $T_0$ and the instability momentum $Q_0$ for a channel with highest critical temperature satisfy the equation $L_0(0, Q_0) = 0$, which can be recast as follows:

$$\ln \left(\frac{T_0}{T_0'}\right) + \frac{1}{2} \sum_{\xi = \pm} \left\langle \left| \frac{1}{2} + \frac{i\xi h v_F Q_0 \cos \phi_p}{4\pi T} \right\rangle \left\langle \frac{1}{2} \right\rangle \right\rangle_T = 0. \quad (15)$$

FIG. 5. Dependence of the critical temperature (a) and the instability momentum of Cooper pairs on the pair-breaking rate $\gamma$ and electron-hole imbalance, parametrized by the difference between electron and hole Fermi energies $h$. When disorder is weak $h$ can stabilize the Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) state with finite Cooper pair momentum.
IV. TUNNELING CONDUCTIVITY

When interlayer tunneling amplitudes are treated in leading order of perturbation theory, the interbilayer tunneling conductance at finite voltage bias $V$ is [43]

$$G_T(V) = \frac{8Ae^2}{\hbar} \text{Im}[\chi(eV, Q)] eV.$$  \hspace{1cm} (18)

Eq. (18) accounts for the fourfold degeneracy due to the presence of valley and spin degrees of freedom and $A$ is the sample area. Here $\chi(\Omega)$ is the retarded correlation function corresponding to the imaginary-time ordered correlation function constructed from tunneling operators

$$\chi(\tau, q) = -\langle T_M T(\tau, q) T^+(0, q) \rangle.$$  \hspace{1cm} (19)

$\chi(\omega, q)$ can be constructed from $\chi(\tau, q)$ by the usual Fourier transform and analytical continuation steps. Without Coulomb interactions between electrons and holes the response function $\chi(\omega, q)$ corresponds to the single electron-hole loop diagram depicted in Fig. 3-b and is given by

$$\chi_0(\omega, q) = i^+ \hat{M}_0 \Pi i^-,$$  \hspace{1cm} (20)

Here $i^\dag = \{t_{11}, t_{12}, t_{21}, t_{22}\}$ is the vector of tunneling matrix elements in the compact representation, and $\Pi \equiv \Pi(\omega, q)$ is a single step pair propagator in the Cooper ladder defined in Eq. (7). In the presence of Coulomb interactions, the single-particle Green functions and the tunneling vertex need to be renormalized. The renormalization of Green functions results in a dip of the density of states at the Fermi level [37], that does not produce any singularities in the tunneling conductivity and thus is unimportant and can be neglected. The renormalized tunneling vertex $i^\dag \rightarrow \tilde{i}^\dag \equiv \tilde{i}^\dag(\omega, q)$ diverges at the critical temperature $T_0$ and is responsible for the drastic enhancement of the tunneling conductivity in its vicinity. The renormalization of tunneling vertex is presented in Fig. 3-d and the corresponding equation for $\tilde{i}^\dag$ can be written as

$$\tilde{i}^\dag_t = t \delta_{t0} + \sum_{t'} U_{tt'} M_{t-t'} \Pi \tilde{i}^\dag_{t'}.$$  \hspace{1cm} (21)

The matrix form-factor $\hat{M}$ couples even orbital channels and we neglect all multipole moments except for $s$- and $d$. (The $p$-wave multipole moment is decoupled from the $s$-wave one and is irrelevant.) Since the form-factors $\hat{M}$ are different for intravalley and intervalley Cooper pairs, we again consider these two cases separately.

Intravalley tunneling

Without Coulomb interactions between electrons and holes the response function $\chi(\omega, q)$ is a sum of three non-interfering terms that correspond to three channels $\alpha$ introduced in Sec. III and identified in Ref. 18:

$$\chi^0 = (c^2|t_{11}|^2 + s^2|t_{22}|^2 + |t_{12} - t_{21}|^2 c^2 s^2) \Pi$$  \hspace{1cm} (22)

The three channels are not coupled by Coulomb interactions and are renormalized independently as follows:

$$\chi = \left(\frac{c^2|t_{11}|^2}{\lambda_{11} L_{11}} + \frac{s^2|t_{22}|^2}{\lambda_{22} L_{22}} + \frac{|t_{12} - t_{21}|^2 c^2 s^2}{\lambda_{12-21} L_{12-21}}\right) \Pi.$$  \hspace{1cm} (23)

Here $L_\alpha$ is the inverse Cooper propagator for each channel and $\lambda_\alpha = \lambda^s_\alpha + \lambda^d_\alpha$ is the corresponding coupling constant. Importantly the only role of the $d$-wave interaction moment is the renormalization the coupling constant $\lambda_\alpha$. For intravalley Cooper pairs the channel $\alpha = 12-21$ has the largest critical temperature $T_0 = T_{12-21}$. $G^I_{12-21}$ is therefore buried under $G^I_T$, since the latter channel corresponds to tunneling between adjacent sublayers, except in the vicinity of the critical temperature $T_0$. As we explain in the next section, this property is consistent with the experimental data [22], suggesting that it is intervalley tunneling that is probed in the experiment.

Intervalley tunneling

When opposite valleys are aligned by a twist angle between bilayers close to $\theta = \pi/3$, the response function $\chi(\omega, q)$ for noninteracting electrons and holes is

$$\chi_0 = (c^2 s^2|t_{12}|^2 + c^2 s^2|t_{21}|^2 + c^2 t_{11} - s^2 t_{22}) \Pi.$$  \hspace{1cm} (24)

It again is a sum of three non-interfering terms that correspond to three channels $\alpha$. Coulomb interactions do not couple the channels but renormalize as follows

$$\chi = \left(\frac{c^2 s^2|t_{12}|^2}{\lambda_{12} L_{12}} + \frac{c^2 s^2|t_{21}|^2}{\lambda_{21} L_{21}} + \frac{|t_{11} - t_{22}|^2}{\lambda_{11-22} L_{11-22}}\right) \Pi.$$  \hspace{1cm} (25)

Each channel acquires its own Cooper propagator $L_\alpha$ with the coupling constant $\lambda_\alpha = \lambda^s_\alpha + \lambda^d_\alpha$.

Importantly, in this case the channel that has the highest critical temperature $\alpha = 11-22$ also dominates the conductance $G_T$ away from the critical temperature because it involves tunneling between adjacent sublayers, not within the bilayer. As the critical temperature $T_0 = T_{11-22}$ is approached $G^I_{11-22}$ diverges. The two other terms $G^I_{12-21}$ and $G^I_T$ are small, free of singularities and are unimportant. The intervalley alignment case has not been studied experimentally yet, but according to our theory is most favorable of observations of the fluctuational internal Josephson effect.

V. COMPARISON WITH EXPERIMENT

In Ref. [22] the twist angle between graphene layers can in principle be tuned to access the intravalley ($\theta_0 = 0$)
and intervalley \((\theta_0 = \pi/3)\) tunneling cases, although experimental results for tunneling conductance \(G_T\) have so far been reported only for one alignment. A preliminary analysis of the data suggests that a divergent zero bias peak appears on the top of a background with a weaker temperature dependence. This behavior can be explained by the competition between \(G^{12\rightarrow 21}_T\) and \(G^{22\rightarrow 21}_T\) expected in the case of intravalley tunneling \((\theta_0 = 0)\). The top and bottom graphene bilayers originate from the same flake, and are aligned with a \(\theta_0 = 0\) relative twist. Calculations for the intravalley tunneling case are presented below and compared with experimental data, while ones for the intervalley alignment are presented in Appendix A.

The experimental data has already been carefully analyzed [22] within a model of noninteracting charge carriers. The non-interacting model explains the tunneling data very well except in the case of opposite polarity charge carriers with nearly equal electron and hole densities. Based on the fits to experimental data away from matched electron and hole densities we confidently assign values for the adjacent layer tunneling amplitude, \(|t_{22}| = 30 \mu \text{eV}\), and the disorder-broadening energies \(\gamma_{(1b)} = 4 \text{ meV}\). The non-interacting electron model severely underestimates the tunneling conductance \(G_T\) in the case of nearly matched electron and hole concentrations, as it is clearly seen in Fig. 1, where interactions are crucial to explain enhanced tunneling conductance at low temperature and singular behavior in the vicinity of the critical temperature \(T_0\) for Cooper pair condensation.

In Figs. 1 and 6 we compare theory with experimental data at the doping level \(n = 7.4 \times 10^{11} \text{ cm}^{-2}\). At this density, the effective mass parameter of bilayer graphene can be approximated as \(m \approx 0.04 \, m_0\), while the electric potential difference between layers in each bilayer can be estimated as \(2|\epsilon| \approx 6.6 \, \text{meV}\) [44]. The latter is much smaller than the corresponding Fermi energy \(\epsilon_F \approx 20 \, \text{meV}\). It follows that \(|\epsilon|/\epsilon_F \approx 0.16\) implying that \(\epsilon \approx 0.58\) and \(s^2 \approx 0.42\); sublayer polarization within the bilayers is modest. The measured tunneling conductance at zero voltage bias was presented in Fig. 1, and diverges at temperatures below \(T_0 \approx 1.5 \, \text{K}\). The temperature should be the dominant channel \((\alpha = 12 \rightarrow 21)\) critical temperature \(T_{12 \rightarrow 21}\). For these parameters, the bare critical temperature in the absence of disorder \(T_0\) calculated from Eq. (15) is equal to \(T_0 \approx 50\, \text{K}\), suggesting that much higher critical temperatures could be achieved in less disordered samples. In our interpretation then, the system is in the regime of strong pair breaking \(T_0 \ll \bar{T}_0 \sim \gamma\) and the value \(\gamma/T_0 \approx 1.74\) is very close to the critical value \(1.78\). Experimentally, singular behavior of the tunneling conductance is observed only in cleaner samples. Both the characteristic time \(\tau \approx \hbar/\gamma \approx 82\, \text{fs}\) and the characteristic length \(\xi \approx \hbar\nu_F/2\gamma \approx 11\, \text{nm}\) defined in Eq. (17) are governed by the pair breaking rate \(\gamma\) and are quite small. Approximating the high energy cutoff as \(\epsilon_c \approx 2\epsilon_F\) the dimensionless coupling constant for the dominant channel \(\alpha = 12 \rightarrow 21\) can be estimated to be \(\lambda_0 \approx 0.44\). We use this value in numerical calculations below. The system is in the weak to moderate coupling regime \(\lambda_0 \approx 1\) that justifies the approximations used in our theory.

If we neglect \(G^{11}_T\), which requires tunneling between remote sublayers and is free of singularities, the tunneling conductance can be rewritten as follows

\[
G_T = \frac{8Ae^2N_F|t_{22}|^2 \text{Im}[L]}{\hbar \lambda_0^2}\frac{4e^4}{\Delta \lambda^{-1} + L_0^2} + 4e^2 s^2 r^2 \left(\left|\frac{L_0^2}{L_0}\right|\right),
\]

Here \(L_0 \equiv L_0(eV, Q)\) is the Cooper propagator in the dominating channel and \(\Delta \lambda^{-1} = \lambda_0^{-1} - \lambda_0^{-1}\) is the difference between inverse coupling constants that correspond to channels \(\alpha = 22\) and \(\alpha = 12-21\). The parameter \(r = |t_{12} - t_{21}|/4|t_{22}|^2\) scales the contribution of \(G^{12\rightarrow 21}_T\) to the tunneling conductance and should be small \(r \ll 1\) because of the hierarchy of tunneling matrix elements \(|t_{12}| \approx |t_{21}| \ll |t_{22}|\). As a result, the tunneling conductance in the wide temperature range \(T \ll T_0\) is dominated by \(G^{22\rightarrow 21}_T\), and only in the vicinity of critical temperature \(T_0\) does the singular term \(G^{12\rightarrow 21}_T\) prevail. We treat \(r\) and sample area \(A\) as fitting parameters for the model. The value of \(A\) can easily be obtained by matching the high temperature behavior of \(G_T\) with experimental data yielding \(A \approx 397\, \mu\text{m}^2\). The value of \(r \approx 1.9 \times 10^{-5}\) is obtained by fitting the singular behavior of the tunneling conductance in the vicinity of the critical temperature. The corresponding theoretical curve in the absence of imbalance and in-plane magnetic field is presented in Fig. 1 and matches with the experimental data reasonably well over a wide temperature range.

A comparison between theory and experiment for the voltage dependence of the tunneling conductance be-
between graphene bilayers $G_T$ is presented in Fig. 6. The zero bias peaks emerge with a decreasing temperature on a top of a smooth background that corresponds to $G_T^2$. The width of the background $eV_{HM}$ is governed by the single-particle energy scales $2\pi T$ and $\gamma$ and is approximately equal to the largest of them. The width of the zero bias peak in the vicinity of $T_0$ is much smaller than the single-particle disorder scale, that demonstrates its collective origin. Overall, the calculated curves agree reasonably with experimental data, although the latter exhibit a voltage asymmetry that becomes more prominent at low temperatures. Within our phenomenological model an asymmetrical voltage dependence of the tunneling conductance can be obtained if the scattering rates of electrons and holes $\gamma_{t(b)}$ that define Cooper pair scattering rate as $\gamma = \gamma_t + \gamma_h$ are energy-dependent. The simple linear dependence $\gamma = \gamma + \gamma'\omega$ with a phenomenological parameter $\gamma'$ does not capture the observed asymmetry however. A quantitative understanding of the asymmetry requires a microscopic understanding of disorder mechanisms that is outside the scope of the present work.

In a vicinity of $T_0$ the tunneling conductance is dominated by the divergent contribution $G_T^{12-21}$. In the absence of the electron-hole imbalance it has an universal Lorentzian shape and is governed by the factor

$$F(eV, Q) = \frac{\text{Im}[L_0^{-1}(eV, Q)]}{eV} \frac{\tau}{(eV\tau)^2 + \epsilon_Q^2}. \quad (26)$$

At $Q = 0$ the peak height has a long high temperature tail $F(0, 0) = \tau/\ln^2[T/T_0]$ and diverges $F(0, 0) \approx \tau T_0^2/\Delta T^2$ with a critical index 2 at the critical temperature $T_0$. Its width at half maximum $eV_{HM} = 1/\tau^* = 2\Delta T/T_0\tau$ is equal to the inverse coherence time $\tau^*$ of fluctuating Cooper pairs and vanishes linearly at the critical temperature $T_0$. Their spatial coherence length $\xi^*$ can be extracted from the dependence of peak width on in-plane magnetic field $B$. The latter results in a relative shift of electronic dispersions between layers and makes it possible to probe fluctuating Cooper pairs with finite momentum $Q_B = \bar{h}d/2B$. In our theory the peak width

$$eV_{HM} = \frac{2\epsilon_Q}{\tau} = 1 \left(1 + \frac{\xi^2}{2^2\bar{h}^2} \right). \quad (27)$$

grows quadratically with in-plane magnetic field $B$, and has a universal dependence on the coherence time $\tau^*$ and length $\xi^*$.

The comparison between theory and experiment for the dependence of the tunneling conductance $G_T$ on the electron-hole imbalance $\Delta n$ and the voltage bias $V$ are summarized in Fig. 7 and a zero-bias-voltage cut of the comparison made in this color plot at zero voltage bias is presented in Fig. 8. The theoretical curves again agree reasonably well with the data. A density imbalance splits the Fermi lines of the electrons and holes and disfavors their Cooper pairing. As seen in Fig. 5, the FFLO state with a finite Cooper pair momentum can not be stabilized in the strong pair breaking regime realized in the experiment. The critical temperature $T_0$ is maximal for Cooper pairs with zero Cooper pair momentum and decreases monotonically in the presence of imbalance and vanishes if the latter exceeds the critical temperature. As a result, the dependence of tunneling conductance on imbalance $\Delta n$ and in-plane magnetic filed $B$ that is presented in
FIG. 9. Theoretical dependence of tunneling conductivity on in-plane magnetic field and electron-hole density imbalance \( \Delta n \) at temperature \( T = 3.5 \) K.

Fig. 9 is smooth and featureless. Fig. 9 presents theoretical curves since the experimental data for magnetic field and density-balance dependence at this temperature is not yet available. The theory qualitatively explains the decrease of peak width with magnetic field studied experimentally at lower temperatures \( T_0 \approx 1.5 \) K, but considerably overestimates its effect. In this case the system is in the paired state whose behavior lies outside the range of validity of the present theory. Other less fundamental limitations might also explain this discrepancy as we discuss in more details in the next section. We conclude that our theory of electron-hole pair fluctuation enhanced conductivity, combined with specific features of the multiple-channel structure of pairing in double bilayer graphene provides a good overall description of experiment.

VII. DISCUSSIONS

Our theory of the internal fluctuational Josephson effect does not account for interactions between Cooper pair fluctuations. The Gaussian nature of the theory we employ is more clearly seen within an alternate derivation of the tunneling conductance. The latter employs the auxiliary field approach and is presented in Appendix B. Interactions between fluctuations can be safely omitted in the wide range of temperatures \( \Delta T \sim T_0 \), and are important only in the critical region \( \Delta T_{Gi} \lesssim GiT_0 \) where fluctuations are large and strongly interfere with each other. Here \( Gi = T_0/E_F \) is the Ginzburg number calculated in Appendix C. The double bilayer graphene system studied experimentally [22] is close to the weak coupling regime, and the critical region \( \Delta T_{Gi} \approx T_0^2/E_F \approx 10 \) mK is much smaller than the temperature range \( \Delta T \approx 4 \) K where the tunneling conductance is strongly enhanced. The picture of noninteracting fluctuating Cooper pairs is therefore well justified to address the basic phenomenon identified in experiment.

Cooper pair fluctuations in conventional superconductors alter the thermodynamics of the normal state only in the critical region \( \Delta T_{Gi} \). The conductivity and the magnetic susceptibility [38] are however not only singular at the critical temperature, but have long high temperature tails [40]. The high temperature tail for the diamagnetic susceptibility \( \chi \sim \chi_L/\ln^2[T/T_0] \), where \( \chi_L \) is the Landau diamagnetic susceptibility in the normal state, was predicted [45–47] theoretically and observed in experiments [40]. The reason is the paired state is superconducting and mediates the perfect diamagnetism that makes even a small number of fluctuating Cooper pairs important. Similarly the equilibrium paired state of spatially separated electrons and holes provides in fluctuational internal Josephson effect that colossally enhances the interlayer tunneling. That is why even a small density of fluctuating Cooper pairs can make a strong impact on the tunneling conductance above the critical temperature \( T_0 \) which also has the high-temperature tail proportional to \( 1/\ln^2[T/T_0] \) that is clearly seen in Fig. 1.

In the vicinity of the critical temperature \( T_0 \) zero bias peak shape is universal and governed by the factor \( F(eV,Q) = \text{Im}[L^{-1}(eV,Q)]/eV \). Thus an imaginary part of the Cooper propagator \( L_0^{-1}(eV,Q) \), that can be interpreted as Cooper pair susceptibility [48], is directly probed in tunneling experiments [49]. It should be noted that Cooper pair-susceptibility of a superconductor can also be probed in tunneling Josephson junction in which one side is near its critical temperature while the other is well below its critical temperature [50, 51]. The junction does not support dissipationless Josephson tunneling current, but the tunneling current at finite voltage is strongly enhanced by Cooper pair fluctuations that grow in the vicinity of \( T_0 \). The latter has been observed experimentally [52, 53].

The enhancement of intravalley tunneling that corresponds to the alignment \( (\theta_0 = 0) \) has only been reported so far. In this case the instability happens in one channel, while the main contribution to the tunneling conductance comes form the different one. As a result the divergent contribution to the conductance due to Cooper pair fluctuations appears on the top of background with weak temperature dependence that dominates at higher temperatures. We predict an enhancement of intervalley tunneling \( (\theta_0 = \pi/3) \) to be much more profound because Cooper pairs in the dominating pairing channel involve electrons and holes settled at adjacent sublayers. We present calculations for this case in Appendix A.

The Larkin-Ovchinnikov-Fulde-Ferrell (FFLO) state with finite Cooper pair momentum has been predicted in conventional superconductors more than sixty years ago [41, 42]. It requires the splitting of Fermi surfaces/lines for pairing electrons with opposite spins. The splitting can be induced by a magnetic field provided that its paramagnetic effect is larger than its diamagnetic one (Chandrasekhar-Clogston limit [54, 55]). This condition is rarely satisfied even in layered conventional supercon-
ductors subjected to in-plane magnetic field. There are few observations in heavy-fermion and organic superconductors where FFLO state signatures have been claimed but are still debated (See Ref. [56] and Refs. [57, 58] for reviews of progress in solid state and cold atoms systems). So far the FFLO state has not been unambiguously identified. In double bilayer graphene the densities of electrons and holes can be controlled separately in a way that opens the FFLO state up for experimental study in a condensed matter system [5, 6]. The FFLO can be unambiguously identified if it appears from the dependence of the zero-bias peak on imbalance and in-plane magnetic field, since the latter makes it possible to probe Cooper pairs with finite momentum. In the vicinity of the instability to the uniform paired state, the tunneling conductance monotonically decreases with in-plane magnetic field (as presented in Fig. 9). In the vicinity of an instability to the FFLO state the tunneling conductance achieves a maximum at finite field-induced momentum shift $Q_B = Q_0$, where $Q_0$ is the corresponding momentum of Cooper pairs. We discuss how distinguish these states in more details in Appendix A, where calculations for the intervalley tunneling are presented.

The sensitivity of Cooper pairing to a disorder opens a possibility of a granular electron-hole state in the presence of its strong long-range variations. In this state the pairing happens in disconnected or weakly coupled regions with minimal amount of disorder and does not support the spatial coherence. It makes the transport properties of the system including Coulomb drag effect to be different from ones in the uniform paired state. A tunneling conductance in the granular state is still colossally enhanced since the latter requires temporal coherence of Cooper pairs but not the spatial one.

The interpretation of experiment provided by our theory suggested that the pairing critical temperature would be substantial if samples with weaker disorder could be fabricated. This finding is perhaps a bit surprising since the experiments are for the most part conducted in the weak to moderate coupling weak coupling regime where some researchers have argued that critical temperatures should be strongly suppressed by screening, especially accounting for spin and valley degeneracy [59] (See also arguments that this approach considerably underestimates the critical temperature [14, 15, 60]). Our theory also suggests that high pairing temperatures should be achievable in double single-layer graphene systems, since there is nothing in its structure that puts single layers at a disadvantage relative to bilayer. Future experimental work which seeks to weaken pair-breaking by disorder and which explores double single-layer graphene systems as well, is therefore important. We note that double bilayer graphene and double single-layer graphene have the important distinction that the former system possesses a BEC strong coupling regime by virtue of its gate-voltage induced gaps, whereas the latter does not. Experiment may reveal whether or not this distinction remains important at weak coupling, and more generally what influence high energy physics has on the bare pair interaction amplitudes treated here as phenomenological parameters.

In summary, the theory of the fluctuational internal Josephson effect developed here explains the anomalies in the tunneling conductance between graphene bilayers observed experimentally at equal electron and hole densities, including their dependence on temperature, bias voltage bias and electron-hole imbalance. Some aspects of the observations are nevertheless not understood. First of all the observed dependence of the tunneling conductance on bias voltage has an asymmetry between positive and negative bias that becomes more prominent with decreasing temperature. At first glance the asymmetry is unexpected and surprising since the electronic spectrum of two graphene bilayers with matched concentrations of electrons and holes is symmetric, as it is clearly seen in Fig. 2. The symmetry can be broken by Coulomb impurities if most of them are of the same charge. For example positive charges (ionized donors) provide repulsive scattering for holes and attractive scattering for electrons. Our model takes the scattering rates for electrons and holes $\gamma_{t(h)}$ to be momentum and energy independent and ignores these common complications. Phenomenologically, an asymmetry can be introduced by making the assumption that the Cooper pairbreaking time is energy dependent $\gamma(\omega)$. Approximating it by a linear function does predict an asymmetry of the tunneling conductance that grows with decreasing temperature, but the shape of the experimental curves is not captured by this simple ansatz. To clarify whether or not the observed asymmetry can be explained by the presence of charge impurities, a more microscopic description of their scattering characteristics is needed and this is outside of the scope of the present work. Secondly it is not clear whether or not our theory can capture the dependence of tunneling conductance on magnetic field since more experimental data is needed. Comparison with data obtained at $T \approx 1.5$ K suggests that the theory considerably overestimates the effect of magnetic field. Nevertheless, at such temperatures the system is in the paired state or in the critical regime that is outside of the applicability range of the theory of Gaussian fluctuations. This discrepancy can be due to other reasons. Bilayer graphene as other two-dimensional systems have long-range density variations and if the corresponding length is smaller than $\hbar/Q_B$ the effect of the magnetic can not be reduced just to the relative shift of electronic dispersions form different layers. To better understand capabilities of the theory more experimental data is needed.

To conclude, we have developed a theory of the fluctuational internal Josephson effect in the system of two closely spaced graphene bilayers. The presence of valley and sublattice degrees of freedom provides three competing electron-hole channels for both intravalley and in-
tervalley Cooper pairs. We show that the three channels are nearly independent and have different temperature dependences and sublattice structures. The experimental fluctuation enhancement of the tunneling conductance can be explained only by the presence of competing channels that dominate in different temperature ranges. The theory enough well captures the dependence of the conductance on temperature, voltage bias between bilayers and electron-hole imbalance. We also argue that the enhancement is much stronger for intervalley tunneling than for the intravvalley one that has been reported recently. We also discuss how to distinguish the uniform state and the FFLO state with finite Cooper pair momentum that can be stabilized in the system by an electron-hole imbalance.

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FIG. 10. The temperature dependence of the intervalley tunneling conductance $G_T$ between graphene bilayers at zero voltage bias. Three curves correspond to pairbreaking rates $\gamma = 2$, 4, and 8 meV. Its main effect is to reduce the critical temperature of electron-hole condensation.

\[ G_T = \frac{8Ae^2}{\hbar} \frac{N_F s^4 |t_{22}|^2 \text{Im}[L]}{eV}. \] (28)

FIG. 11. The dependence of tunneling conductance $G_T$ on in-plane magnetic $B$ and the electron-hole density imbalance $\Delta n$. Three subplots correspond to $T = 45$ K (a), 25 K (b) and 5 K (c). The dashed line corresponds to the phase boundary of equilibrium electron-hole paired state. In (b) and (c) the conductance achieves maximum at finite value of magnetic field that demonstrates that the Cooper pair fluctuations with finite pair momentum $Q \approx Q_B$ are the most intensive and the system is in the vicinity of the instability to the FFLO state. Kink in the phase boundary in (c) clearly shows the presence of the equilibrium FFLO state.

APPENDIX A. INTERVALLEY TUNNELING AND IDENTIFICATION OF FFLO STATE

In the main text of the paper calculations for intravalley tunneling ($\theta_0 = 0$) are discussed, while ones for the intervalley alignment ($\theta_0 = \pi/3$) are presented here. The tunneling conductance is dominated by $G_T^{11-22}$ because of Cooper pairs in the dominating pairing channel involve electrons and holes settled at adjacent sublayers. The tunneling conductance can be written as

\[ G_T = \frac{8Ae^2}{\hbar} \frac{N_F s^4 |t_{22}|^2 \text{Im}[L]}{eV}. \] (28)
We use the same set of parameters that has been used above to fit the experimental data except for pairbreaking rate $\gamma$. For the latter we use $\gamma = 2, 4$, and 8 meV. The first two values correspond to cleaner samples compared to ones that have been studied experimentally [22]. The temperature dependence of tunneling conductance is presented in Fig. 10. Its enhancement of the conductance is considerable stronger than that for the intervalley tunneling. The pairbreaking rate $\gamma$ determines the critical temperature of pair condensation, but weakly influences the temperature dependence of tunneling conductance above it.

For the pairbreaking rate $\gamma = 2$ meV the critical temperature is $T_0 \approx 42$ K. According to the phase diagram Fig. 5 the ratio $\gamma/T_0 \approx 0.52$ is small enough to stabilize the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state by the electron-hole imbalance. In-plane magnetic field makes it possible to probe fluctuating Cooper pairs with finite momentum $Q_B = \hbar d/l_B^2$. The dependence of tunneling conductance at zero voltage bias on temperature is presented in Fig. 10. Its enhancement of the conductance has a monotonic dependence demonstrating that the system is far away from the instability to the uniform BCS one. Nevertheless, the tunneling conductance achieves maximum at finite value of magnetic field that shows that the systems is close to the FFLO state and fluctuations with finite Cooper momentum are the most intensive. The dependence at $T = 5$ K is presented in Fig. 11-c. The dashed line that denotes the phase boundary is non-monotonous and has a kink at finite value of magnetic field. It clearly demonstrates that the equilibrium FFLO state is stabilized by the electron-hole imbalance.

**APPENDIX B. BOSONIC PICTURE OF FLUCTUATIONAL JOSEPHSON EFFECT**

Here we present a bosonic picture of the fluctuational internal Josephson effect that can be employed with a help of the field integral formalism. We consider the contact interactions $U$ between electrons and holes that correspond to the truncation of all multipole moments of interactions except the $s$-wave one. In the main text of the paper we have demonstrated that they are unimportant. The correlation function of tunneling operators $\chi(\omega, \mathbf{q})$ that defines the tunneling conductance $G_T$ according to Eq. (18) can be extracted from the corresponding imaginary time correlation function $\chi(\tau, \mathbf{r})$ given by

$$\chi(\tau, \mathbf{r}) = \frac{\delta^2 Z}{Z \delta \Lambda_{\tau, r} \delta \Lambda_{\tau, r}^\dagger}.$$  

Here $Z[\Lambda^\dagger_{\tau, r}, \Lambda_{\tau, r}]$ is the statistical sum with an auxiliary bosonic field $\Lambda_{\tau, r}$ introduced to the action $S$ of the system as follows

$$S = \int_0^\beta d\tau \int d\mathbf{r} \left[ \hat{\psi}^\dagger_\mathbf{t}(\partial_\tau + \hat{h}_{tr} - \mu_t) \hat{\psi}_\mathbf{t} + \hat{\psi}^\dagger_\mathbf{b}(\partial_\tau + \hat{h}_{br} + \mu_b) \hat{\psi}_\mathbf{b} + \hat{\psi}^\dagger_\mathbf{b} \Lambda^s \hat{\psi}_\mathbf{t} + \hat{\psi}^\dagger_\mathbf{t} \Lambda^s \hat{\psi}_\mathbf{b} + U \hat{\psi}^\dagger_\mathbf{t} \hat{\psi}^\dagger_\mathbf{b} \hat{\psi}_\mathbf{b} \hat{\psi}_\mathbf{t} + \cdots \right].$$

Here $\hat{\psi}_\mathbf{t} = \hat{\psi}_{\mathbf{t}, \mathbf{r}}$ and $\hat{\psi}_\mathbf{b} = \hat{\psi}_{\mathbf{b}, \mathbf{r}}$ are spinor fermionic fields for electrons from top (t) and bottom (b) layers with labeling described in Sec. II. If they are integrated out and the corresponding action is expanded in the lowest order in tunneling matrix elements $\mathbf{t}$, the Fourier transform $\chi^q$ do appears in the action as follows $S = -\sum_q \chi^q |q|^2$. Here $q = \{q_n, \mathbf{q}\}$ with bosonic Matsubara frequency $q_n = 2\pi T n$. For noninteracting electrons and holes calculations are straightforward and result in

$$\chi^q = \hat{i} + N \hat{M}_q \Pi_q \hat{i}.$$  

After analytical continuation we get the Eq. (20) from the main text. In the case of interacting electrons and holes it is instructive start with the Hubbard-Stratonovich transformation. It eliminates interactions but introduces the bosonic field $\Delta_{\tau, r}$ corresponding to electron-hole Cooper pairs. The action $S$ is modified as follows

$$S = \int_0^\beta d\tau \int d\mathbf{r} \left[ \hat{\psi}^\dagger_\mathbf{t}(\partial_\tau + \hat{h}_{tr} - \mu_t) \hat{\psi}_\mathbf{t} + \hat{\psi}^\dagger_\mathbf{b}(\partial_\tau + \hat{h}_{br} + \mu_b) \hat{\psi}_\mathbf{b} + \hat{\psi}^\dagger_\mathbf{b} \Delta^s \hat{\psi}_\mathbf{t} + \hat{\psi}^\dagger_\mathbf{t} \Delta^s \hat{\psi}_\mathbf{b} + \frac{1}{U} \text{tr} \left[ \Delta^+ \Delta \right] \right].$$

where $\Delta^s_{\tau, r} = \Delta_{\tau, r} + \hat{i} \Lambda_{\tau, r}$. Above the critical temperature $T_0$ of the electron-hole pairing the saddle point of
the action is trivial $\langle \hat{\Delta} \rangle = 0$ and the field $\hat{\Delta}$ corresponds to Cooper pair fluctuations [66]. In the wide temperature range $\Delta T_{Gi} \lesssim \Delta T \sim T_0$ outside the critical regime $\Delta T \lesssim \Delta T_{Gi}$ fluctuations can be approximated by the noninteracting Gaussian theory. Here $\Delta T_{Gi} = GiT_0$ with Ginzburg number $Gi = T_0/E_F$ calculated in Appendix C. Integrating out fermions and expanding the action up to the second order in the bosonic field $\hat{\Delta}'$ results in

$$S = \sum_q \left[ \frac{\hat{\Delta}_q^+ \hat{\Delta}_q}{U} - \hat{\Delta}_q^+ \hat{M}_0 \Pi_q \hat{\Delta}_q \right] = \sum_q \left[ \hat{\Delta}_q^+ \Gamma_q^{-1} \hat{\Delta}_q - \hat{\Delta}_q^+ \hat{M}_0 \Pi_q t \Lambda_q - \Lambda_q^+ i \hat{M}_0 \Pi_q \hat{\Delta}_q - \Lambda_q^+ \xi^0_q \Lambda_q \right]. \quad (32)$$

Here all matrices are in the compact representation, and $\Gamma_q$ is the scattering vertex calculated in the Sec. III of the paper. Some of its components vanish at the critical temperature $T_0$ of the electron-hole Cooper pairing. The last term corresponds to the response $\chi_q^0$ function for noninteracting electrons and holes that is given by Eq. (30). The action represents the bosonic picture of the Josephson effect and is valid outside the weak coupling regime. The action (32) is quadratic in the bosonic field $\hat{\Delta}_q$, and after its integration we get the tunneling response function

$$\chi_q = \chi_q^0 + i \hat{M}_0 \Pi_q \Gamma_q^{-1} \hat{M}_0 \Pi_q \hat{t}. \quad (33)$$

With a help of Eqs. (12) and (11) we get the tunneling response function for the intravalley tunneling (23). In the same way with a help of Refs. (14) and (13) we get the response function for the intravalley one (25).

**APPENDIX C. GINZBURG CRITERION**

The developed theory of the internal fluctuational Josephson effect implies that Cooper pair fluctuations are Gaussian and do not interact with each other. The interactions can be safely omitted in the wide range of temperatures $\Delta T \sim T_0$ except the critical region $\Delta T \lesssim \Delta T_{Gi}$ where fluctuations are overgrown and are strongly interfere with each other. The range $\Delta T_{Gi}$ can be estimated from the Ginzburg criterion [67] that compares the contribution of Gaussian fluctuations to the heat capacity $C_{FL} = \alpha T_0/4\pi c \Delta T$ with predictions of the mean-field theory below the critical temperature $C_{MF} = \alpha^2/bT_0$. Here $\alpha$, $b$ and $c$ are given by

$$\alpha = N_F \quad b = \frac{N_F |\Psi''(1/2 + \gamma/2\pi\pi)|}{16\pi^2 T^2}, \quad c = \frac{\hbar^2 v^2 b}{4}. \quad (34)$$

and define the Ginzburg-Landau functional for the contribution of Cooper pair fluctuations to free energy of the system as follows

$$F = \int dr \left[ a |\Delta_r|^2 + c |\partial_r \Delta_r|^2 + \frac{b}{2} |\Delta(r)|^4 \right], \quad (35)$$

with $a = \alpha \log[T/T_0] \approx \alpha \Delta T/T_0$. The contribution of fluctuations $C_{MF}$ grows with decreasing of temperatures and dominates in the temperature range $\Delta T_{Gi} = Gi T_0$ with Ginzburg number $Gi = b T_0/4 \pi a c = T_0/E_F$. It does not depend explicitly on the pairbreaking rate $\gamma$ but only on the critical temperature $T_0$. It should be noted that the Ginzburg criterion can be derived microscopically in a more strict way by explicit analysis of the role of interactions between fluctuations [68].