A FEEDBACK COMPRESSION STAR FORMATION MODEL
AND THE BLACK HOLE–BULGE RELATIONS

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ABSTRACT

We present a feedback compression model to describe galactic spheroid formation and its relation with the central nuclear activity. We suggest that the star formation itself can serve as the positive feedback in some extremely dense region to trigger the starburst. The star formation rate, as well as the related stellar feedback–induced turbulence, will be maximized under the regulation of the background dark halo’s gravity. There is also stellar feedback acting inward to confine and obscure the central black hole (BH) until the BH grows sufficiently large to satisfy a balance condition between the accretion disk wind and the inward stellar feedback. The extremely vigorous star formation activity, the BH-bulge relations, the maximum velocity dispersion, and the maximum BH mass are investigated on the basis of such a scenario and are found to be consistent with observations.

Subject headings: black hole physics — galaxies: formation — galaxies: nuclei — galaxies: starburst — galaxies: structure

1. INTRODUCTION

Observations have shown clear evidence that the mass of a supermassive black hole (SMBH) in the center of every galaxy is tightly correlated with the velocity dispersion of the bulge stars (Ferrarese & Merritt 2000; Gebhardt et al. 2000; Tremaine et al. 2002) and the mass of the whole bulge (Magorrian et al. 1998; McLure & Dunlop 2002; Marconi & Hunt 2003), which reflects the interplay between the nuclear activity and the spheroidal star formation activity (the potential well depth of bulge stars). Recent consensus emphasizes that the nuclear black hole (BH) feedback is the sticking point when explaining these correlations: the central BH can interact with the surrounding environment through BH feedback in a self-regulated way. However, the tightness of the correlations indicated by both the observations and numerical simulations poses a big challenge to many conventional feedback models, whose results are sensitive to the variable parameters, such as the gas fraction (the mass fraction of gas to dark matter; Di Matteo et al. 2005). Recently, some modified feedback models have been proposed to alleviate the inconsistency. Begelman & Nath (2005) suggest that the BH accretion physics on a local scale (relative to previous “global” models) should be considered. They propose that the “maximized” accreted gas mass will make the result insensitive to the gas fraction. Xu et al. (2007, hereafter XWZ07) provide another scenario with which to explain both the starburst activity in the gas fractions. The cooling by collisionally excited atomic and molecular emission processes can be very efficient. Hence, we hypothesize that the expanding gas shell is thin, its dynamics can be described by a momentum-driven way and trigger the new star formation events. The result can be understood by a simple example: if we assume that the expanding gas shell is thin, its dynamics can be described by

\[
\frac{d}{dt} \left( R^2 \frac{dR}{dt} \right) = \frac{3 M_w v_w}{4 \pi \rho_0},
\]

where \( M_w \) is the mass-loss rate, \( v_w \) is the velocity of the wind, and \( \rho_0 \) is the density of the homogeneous gas around the stars. We can easily find the following solution at large \( R \):

\[
R(t) = \left( \frac{3}{2} \frac{M_w v_w}{\pi \rho_0} \right)^{1/4} t^{1/2}.
\]
The shell will finally stall due to the ambient pressure, at the stalling radius \( R_s = (M_{\text{env}}/4\pi n_0 T)^{1/2} \). Using equation (2), the total compression timescale can be estimated as

\[
t_c = \frac{\mu m_H}{4kT} \left( \frac{2M_{\text{env}}v_w}{3\pi \rho_0} \right)^{1/2}.
\]

Comparing \( t_c \) with the dynamical timescale \( t_{\text{dyn}} = (3\pi/32G\rho)^{1/2} \), we have

\[
t_c \approx 5 \left( \frac{\rho_0}{\rho_s} \right)^{1/2} M_6^{1/2} v_{500}^{1/2} n_3^{1/2} T_10^{-1/2},
\]

where \( \rho_0 \) is the gas density of the compressed gas shell, \( M_6 = M_*/10^6 \) M_\odot yr\(^{-1} \), \( v_{500} = v_w/500 \) km s\(^{-1} \), and \( T_{10} = T/10 \) K. Note that the shell gas density \( \rho_0 \) is larger than \( \rho_s \), so \( t_c \) is at least 1 order of magnitude larger than \( t_{\text{dyn}} \). This demonstrates that in a very dense environment, the density inhomogeneities are amplified by the feedback compression, and the gas has enough time to collapse into the cloud.

The “light” outflow accelerating against the dense cold gas shell will trigger the Rayleigh-Taylor instability (RTI). Moreover, the Kelvin-Helmholtz instability (KHI) may also occur as the outflow punches into the gas shell and moves through the cold gas. The combination of these two instabilities will prevent the gas collapse and destruct the star-forming cloud. If the density contrast is large, the dispersion relation for the RTI is \( w \approx (ka)^{1/2} \), where \( a \) is the acceleration (Chandrasekhar 1961).

The characteristic growth timescale for the RTI responsible for the cloud destruction is \( \tau_{\text{RT}} \approx (2\pi\alpha/\alpha)^{1/2} \), where \( \alpha \) is the Jeans length. The acceleration can be derived from equation (2):

\[
a(t) = \frac{1}{4} \left( \frac{3M_{\text{env}}v_w}{2\pi \rho_0} \right)^{1/4} t^{-3/2}.
\]

Initially, the growth timescale \( \tau_{\text{RT}} \) may be smaller than the dynamical timescale \( t_{\text{dyn}} \). The RTI will make the dense cold shell highly porous to the hot wind and will entrain cold gas into the wind. The efficiency of the driving wind is enhanced (Silk 2001). Noting that \( \tau_{\text{RT}} \propto t^{3/4} \), we define the duration in which the RTI dominates by setting \( \tau_{\text{RT}} = t_{\text{dyn}} \). Using equation (5), the duration is

\[
t_d = 2.5 \times 10^9 M_6^{1/6} v_{500}^{1/2} n_3^{-1/2} T_10^{-1/3} \left( \frac{\rho_0}{\rho_s} \right)^{1/3} \text{yr},
\]

where \( n_3 = n_0/10^3 \text{ cm}^{-3} \). The duration is smaller than the dynamical timescale \( t_{\text{dyn}} \), which indicates that the cloud destruction by the RTI and the KHI (its growth timescale is of the same order as the RTI’s; Agertz et al. 2006) may not be important in the momentum-driven phase, at least at a large radius \( R \). So it is reasonable to assume that the feedback-induced compression can trigger and expedite the cloud collapse and star formation.

3. HALO-REGULATED STAR FORMATION AND BLACK HOLE GROWTH

Star formation in local disk galaxies is usually inefficient, and the typical star formation efficiency (SFE) is \(~2\%\) (Kennicutt 1998). This is because the star formation is regulated by negative feedback (heating, winds), which keeps the star formation rate (SFR) from becoming too large. However, as mentioned in the previous section, this may not be the case in the starburst region. Without the heating effects, the feedback-induced compression can serve as positive feedback, which would trigger the intense star formation activities, unless the total mechanical energy generated by the stellar feedback is larger than the binding energy of these regions. In other words, once the total feedback from those massive stars is unable to disrupt or unbind the whole star-forming region, the star formation process may continue for a relatively long time, analogous to the formation of bound clusters (Elmegreen & Efremov 1997).

Following XWZ07, we adopt the NFW density profile for the background dark matter and assume the standard cosmological parameters, with \( \Omega_m = 0.3, \Omega_\Lambda = 0.7, \) and \( h = 0.7 \) (\( z = 0 \)). The inner NFW profile is given by (Navarro et al. 1997; Komatsu & Seljak 2001)

\[
\rho_{\text{NFW}}(r) \approx \frac{1}{2} \left( M_{*12} / 5.5 \times 10^{12} M_\odot \right)^{1/3} \rho_c, \quad \rho_c = 1.3 \times 10^{-27} \text{ g cm}^{-3},
\]

where \( M_{*12} = M_*/10^{12} M_\odot \) and \( M_* \) is the virial mass of the halo. The concentration parameter, \( \xi(z) = \delta_{\text{c}}(z)/\Omega_m(z) \), where \( \xi(z) = (1 + z)^{-1/2} \), and \( \Delta_c = 18.3 \pi^2 + 82d - 39d^2 \), where \( d = \Omega_m(z) - 1 \) (Bryan & Norman 1998; Barkana & Loeb 2001). The dimensionless parameter \( \xi_{c,0.58} = \ln(1 + c) - c/(1 + c) \) is the concentration parameter (Bullock et al. 2001). Such a \( r^{-1} \) profile in the inner region is almost a universal profile: it does not change during galaxy mergers or interactions, and it is also insensitive to the redshift and the halo mass. Another interesting thing to note is that such an inner profile gives a nearly constant gravitational acceleration:

\[
g_{\text{DM}}(r) = \frac{GM_{\text{DM}}(r)}{r^2} = 2\pi G \Pi \sim 1.2 \times 10^{-8} \text{ cm s}^{-2}.
\]

Intense star formation activity and related stellar feedback will make the protospheroid environment highly turbulent. Because the bulge star density distribution implies that the protospheroid density profile may follow \( \rho \sim r^{-3} \) (Tremaine et al. 1994), here we assume an isothermal density profile for the protospheroid baryons or mixture of gas and stars:

\[
\rho_b(r) = \frac{c_s^2}{2\pi G r^2}, \quad M_b = \frac{2c_s^2 R}{G},
\]

where \( \rho_b(r) \) is the total baryon density or the density of the mixture of gas and stars, \( c_s \) is the isothermal turbulent sound speed, and \( M_b \) is the enclosed mass within the radius \( r \). The momentum transport during the compression requires that

\[
\frac{\dot{P}_s}{4\pi r^2} = \rho_b(r)c_s^2,
\]

where \( \dot{P}_s \) is the net outward momentum deposition rate. The star formation activity and the feedback-induced turbulence will be maximized until the whole system evolves to a virial equilibrium state. Once the whole system becomes a little more turbulent, the deposited momentum flux will make the whole system deviate from the equilibrium state, and the SFR is hence prohibited from increasing. Such a self-regulating mechanism will make the whole system maintain the equilibrium state.

We can write the equation of the virial equilibrium for the protospheroid as

\[
3M_{*12} c_s^2 = \frac{GM_{\odot}^2}{r} + \pi G M_b r,
\]
where $c_{s,m}$ is the maximum turbulent sound speed. The first term on the right-hand side of equation (11) denotes the total baryonic self-gravity, while the second term denotes the gravity from the background dark matter. Substituting equation (9) into equations (10) and (11), we obtain

$$\dot{P}_s = \frac{2c_{s,m}^4}{G} = \pi G M_b. \tag{12}$$

Equation (12) shows that the maximum turbulent sound speed or the depth of the potential well of the protospheroid is directly related to the background dark matter. In other words, both the star formation activity and the feedback compression are regulated by the dark halo’s gravity, as XWZ07 proposed. It only requires the homogeneous turbulent environment and the virial equilibrium, without involving detailed gas assembly physics, such as monolithic collapse or merger-driven inflow. So the feedback compression combined with the halo regulation scenario provides a more general description to the star formation process during the spheroid formation.

Following XWZ07, during the formation of the protospheroid, the stellar feedback can act in both inward and outward directions. At small scales (e.g., the galactic nuclear region), the inward stellar feedback (required to conserve the local momentum) obscures and regulates the BH growth, while the outward stellar feedback resists the gravity at large scales. In particular, the inward stellar feedback regulates the BH growth by interacting with the Compton-thick wind launched from the accretion disk if super-Eddington accretion is assumed (King & Pounds 2003). The final balance between the inward stellar feedback and the disk wind is achieved when

$$\dot{P}_s = \frac{8\pi G M_{BH}}{\kappa}, \tag{13}$$

where $\dot{P}_s$ is the momentum flux transported by inward stellar feedback and the right-hand side of equation (13) is the momentum deposition of the disk wind. Then, if the BH’s feedback is large enough to halt the further supply of gas, it will end its main growth phase after equation (13) is satisfied.

At a late epoch of the coeval evolution, star formation consumes most of the gas and gradually ceases. Without the continuous ejections of energy and momentum from stellar feedback, the feedback-induced turbulence will decay on a crossing time of the system (Stone et al. 1998). Then the remaining stellar system will be virialized through violent relaxation under the combined gravity from itself and the background dark matter. We say that the system is at its initial state after turbulence decay and before virialization, and at its final state after virialization. If we assume that the total energy of the initial state is $E$, the kinetic energy of the final virialized state is $K = -E$, according to the virial theorem. So we have

$$3\sigma_f^2 = 2\left(\frac{G M_b}{r} + \pi G \rho r\right) = 6c_{s,m}^2. \tag{14}$$

Using equation (12), we obtain

$$\dot{P}_s = \frac{\sigma_f^4}{2G}, \quad \rho_b = \frac{\sigma_f^2}{2\pi G}. \tag{15}$$

where $\rho_b$ is the boundary radius of the initial state. We note that the total momentum deposition rate from stars is only related to the velocity dispersion of the final stellar system, independent of any parameters of the detailed star formation physics.

Using equations (13) and (15), we obtain the final BH mass,

$$M_{BH}^{\text{final}} = \frac{\kappa \sigma_f^4}{16\pi G^2} = 1.5 \times 10^8 \sigma_{200}^4 M_\odot. \tag{16}$$

This result is remarkably consistent with low-redshift observations (Tremaine et al. 2002). The stellar bulge mass is approximately equal to $M_b$. Using equations (12) and (13), the ratio of the BH mass to the bulge mass can be expressed as

$$\frac{M_{BH}}{M_{\text{bulge}}} = \frac{\kappa \Pi}{8} \approx 1.4 \times 10^{-3} M_{\odot}^{-0.07} [\xi(\varepsilon)]^{2/3} \Psi_{0.58}^{-1}, \tag{17}$$

which matches the Magorrian relation found for local galaxies (Magorrian et al. 1998; McLure & Dunlop 2002; Marconi & Hunt 2003).

4. APPLICATION TO THE HIGH-REDSHIFT STAR-FORMING GALAXIES

The gas fraction of high-redshift galaxies is much larger than that of local galaxies. In an extreme case, we take the fraction to be $\sim 1$. In other words, a protospheroid with a mass of $M_b$ is almost totally in gaseous form. The large amount of gas accumulating in the central region will trigger the central vigorous starburst, accompanied by strong star-forming feedback. We mainly focus on two primary sources of star-forming feedback: radiation pressure and supernovae. The combined momentum flux deposited into these sources of star-forming feedback can be written as (Murray et al. 2005; XWZ07)

$$\dot{P}_s = \dot{P}_\text{tp} + \dot{P}_\text{sn} = \xi_m \dot{M}_* c, \tag{18}$$

where $\dot{M}_*$ is the star formation rate and $\xi_m = 1 + \dot{P}_\text{sn}/\dot{P}_\text{tp}$ is on the order of unity in our model. Combining equations (13), (16), and (18), we can easily obtain the star formation rate of high-redshift starburst galaxies:

$$\dot{M}_* = \frac{\sigma_f^4}{2G \xi_m c^2} \approx 600 \sigma_{200}^4 \epsilon_3^{-1} M_\odot \text{ yr}^{-1}. \tag{19}$$

Although the star formation law in the protospheroid is far from clear, we adopt an equivalent Schmidt-Kennicutt law in order to compare it with the local disk galaxies (Schmidt 1959; Kennicutt 1998):

$$\dot{M}_* = \eta \dot{M}_{bdyn}, \tag{20}$$

where $\eta$ is the equivalent star formation efficiency and $t_{dyn} = (3\pi/32G \rho_b)^{1/2}$ is the dynamical timescale. In our model we take $\rho_b = 3M_b/4\pi r_b^3$ as the average gas density, where $r_b$ is the outer boundary radius given in equation (15).

From equations (12) and (18), we have

$$\xi_m \dot{M}_* c = \pi G M_b. \tag{21}$$

Using equation (20) to eliminate $\dot{M}_*/\dot{M}_*$ in equation (21), we get the equivalent star formation efficiency as

$$\eta = \sqrt{2\sigma_f \pi} \xi_m = 0.4 \sigma_{200} \epsilon_3^{-1} \xi_3^{-1}, \tag{22}$$

where $\epsilon_3 = \epsilon/10^{-3}$. 

\[\]
We find that our derived equivalent SFE is much higher than that in normal disk galaxies as inferred from the Kennicutt law, but it is consistent with the high-redshift star formation observations and some small-scale star formation observations (e.g., SFE in the formation of the protocluster). We note that the derived SFE is independent of both the radius of the star-forming region and time, and the larger the velocity dispersion is, the higher the SFE is. Equation (22) also gives us another implication for the maximum stellar velocity dispersion. We can rewrite equation (22) as

$$\sigma_f = \frac{4\sqrt{G_m} \eta \zeta c}{\pi}.$$ (23)

The physical limit requires that $\eta \leq 1$, so the maximum stellar velocity dispersion is

$$\sigma_f^{\text{max}} = \frac{4\sqrt{G_m} \zeta c}{\pi} = 540 \zeta_{m,3} \text{ km s}^{-1}.$$ (24)

According to equation (16), the maximum BH mass is $8 \times 10^8 M_\odot$. Using equations (12) and (15) and the expression of $f_{\text{dyn}}$, we can also obtain the characteristic star formation timescale,

$$t_s = \frac{t_{\text{dyn}}}{\eta} = \frac{2\pi \zeta_{m,3} \zeta c}{\sigma_f^{\text{max}}} = \frac{\zeta_{m,3} \zeta c}{\pi \Pi G} \approx 1.0 \times 10^8 \zeta_{m,3} M_{12}^{-0.07} [\zeta(z)]^{-2/3} \Psi_{c,0.58} \text{ yr.}$$ (25)

Through the feedback compression, the dark halo’s gravity regulates the SFR and SFE to a higher level during the spheroid formation. We note that for some luminous elliptical galaxies whose velocity dispersion is $\sigma \approx 300 \text{ km s}^{-1}$, the predicted star formation rate can be as high as $3000 M_\odot \text{ yr}^{-1}$, which is consistent with the observations of high-redshift starburst galaxies (Solomon & Vanden Bout 2005). It is also interesting to note that the star formation timescale is independent of the velocity dispersion $\sigma$. Furthermore, the characteristic star formation time scale is larger than the Salpeter timescale, which is usually taken to be the typical timescale for BH growth. This means that the BH will first grow relatively fast to reach the balance condition (eq. [13]) and then grow relatively slowly in response to the outside stellar feedback.

5. DISCUSSION

Unlike previous momentum feedback models (King 2003; Murray et al. 2005; Begelman & Nath 2005), which mainly consider the BH feedback dynamics, our scenario focuses more on the star formation activity in the protospheroid and its relation to the nuclear BH growth. Our model favors a gas-rich environment at high redshift because a large amount of gas is needed to form stars and obscure the BH. We argue that at an early epoch of BH growth (when the BH is relatively small), a large-scale outflow or jet may not be crucial in producing BH-bulge relations, although they become important after the main growth epoch of the BH (Churazov et al. 2002). In addition, the BH does not generate an outflow or a jet until the balance condition (eq. [13]) is satisfied. The derived $M_{\text{BH}} - \sigma$ relation based on our model is insensitive to the gas fraction and other variables, because the velocity dispersion is the measure of the maximized turbulent velocity of the total baryon component rather than the gas only, as equation (9) shows. The derived $M_{\text{BH}} - M_{\text{bulge}}$ relation has weak dependences on the redshift and the halo mass, which offer their intrinsic scatters to the relation.

Extremely high SFRs and SFEs are the results of certain “positive” feedback that is probably due to either “internal” or “external” effects. Silk (2005) suggests that the high SFRs and SFEs are triggered by the super-Eddington outflow driven by the SMBH. Such external positive feedback naturally leads to a top-heavy initial mass function (IMF), which is preferred by the early generation of star formation and predicts an antihierarchical trend of SMBH growth (Merloni et al. 2004). However, recent optical, infrared, and X-ray studies of submillimeter galaxies (SMGs) indicate that SMGs harbor relatively smaller SMBHs than those of typical quasars (Ivison et al. 1998; Vernet & Cimatti 2001; Smail et al. 2003; Alexander et al. 2005), and the main growth phase of the SMBH (the “prequasar” phase) is heavily obscured. Considering that the SMGs themselves are massive galaxies, which are reckoned as the progenitors of local elliptical galaxies (Greve et al. 2005), the vigorous star formation activity cannot be regulated by the small BH. Conversely, the star formation activity should have great impact on the small BH. So we argue that the internal positive feedback that is produced by the star formation itself may exist in some extremely dense starburst region at median redshifts, and the SMBH outflow-triggered star formation mode may only be available at very high redshifts (Walter et al. 2004). In addition, the maximized positive stellar feedback is actually related to the dark halo’s gravity in our model. Under the regulation of the dark halo’s gravity, the maximized velocity dispersion also has its maximum value, which is determined by the physical upper limit of the SFE. The physical upper limit of the BH mass ($\sim 10^{10} M_\odot$) can then be obtained by the $M_{\text{BH}} - \sigma$ relation, and the dynamical signature of such SMBHs should be detectable (Wyithe & Loeb 2003). We note that some observational evidence does support such a result (Netzer 2003; Vestergaard 2004), although all of the observations still contain many uncertainties. We expect that more accurate SMBH mass measurements in the future will confirm our result.

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