Coherent scattering processes in the solar atmosphere cause the emitted radiation to be linearly polarized. The spectrum so obtained is referred to as the Second Solar Spectrum (SSS). This linearly polarized spectrum is modified at the line core in the presence of weak magnetic fields due to the Hanle effect. One of the most challenging aspects of such CLV modeling is to find a single model atmosphere which closely reproduces the observed Q/I at all µ but fails to reproduce the observed rest intensity at different µ. Hence we find that no single 1D model atmosphere succeeds in providing a good representation of the real Sun. This failure of 1D models does not, however, cause an impediment to the magnetic field diagnostic potential of the Ca I 4227 Å line. To demonstrate this we deduce the field strength at various µ positions without invoking the use of radiative transfer.

**Key words:** line: formation – methods: numerical – polarization – radiative transfer – scattering – Sun: atmosphere
They finally construct an anisotropy-modified single 1D atmospheric model to simultaneously fit $I$ and $Q/I$ at all $\mu$.

The CLV of the Ca\textsc{i} 4227 Å line away from the active regions was first observed by Stenflo et al. (1980) and analyzed by Auer et al. (1980). The CLV of the line center polarization observed by Stenflo et al. (1980) was later used by Faurobert-Scholl (1994) to study the Hanle effect due to the magnetic field canopies in the chromosphere. The CLV observations of this line were also done by Bianda et al. (1998, 1999). In this paper we attempt a detailed simultaneous modeling of the observed CLV of both the $I$ and $Q/I$ profiles of the Ca\textsc{i} 4227 Å line. For this purpose we solve the polarized RT equation by taking account of PRD effects in the nonmagnetic regime. Standard 1D atmospheric Fontenla–Avrett–Loeser (FAL) models (Fontenla et al. 1993; Avrett 1995) are used to obtain a fit to the $(I,Q/I)$ spectra. We find that it is not possible to achieve a simultaneous $(I,Q/I)$ fit to the CLV observations with a single 1D model atmosphere. If we consider the CLV of $Q/I$ alone, then we find it necessary to modify the original temperature structure of the standard FAL atmospheric models to obtain a fit. Such modifications of the original temperature structure were also used in previous works by Holzreuter & Stenflo (2007) and Smitha et al. (2012, 2013). In the present paper, the original temperature structure of the standard FAL-A atmosphere is modified. Later, the modified FAL-A (FALA) is combined with the FAL-X atmospheric model to construct a single-component model. It turns out that this newly constructed combined model can closely reproduce the observed $Q/I$ at different limb distances.

In Section 2 we give the details of the CLV observations. Section 3 is devoted to the modeling procedure and results. Section 4 describes the observational analysis to determine the magnetic fields. Concluding remarks are given in Section 5.

2. OBSERVATIONAL DETAILS

The CLV observations of the Ca\textsc{i} 4227 Å line presented in this paper were obtained with the Zurich Imaging Polarimeter-3 (Ramelli et al. 2010) at Istituto Ricerche Solari Locarno (IRSOL) in Switzerland on 2012 October 16. The observations were taken at 14 different $\mu$ positions (0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.60, 0.70, 0.80, 0.90, and 1.0), starting from the heliographic north pole at $\mu = 0.1$ up to the disk center at $\mu = 1$. Figure 1 shows the CCD images of the Ca\textsc{i} 4227 Å line at five selected positions on the solar disk. The polarization modulation was done with a piezoelectric modulator. The spectrograph slit was 60 μm wide, corresponding to a spatial extent of 0.′′5 on the solar disk. The CCD covered 190′′ along the slit. The CCD images have 140 effective pixel resolution elements in the spatial direction, with each element corresponding to 1′′.38, and 1240 pixels in the wavelength direction, with one pixel corresponding to 5.30 mÅ.

To keep the solar image position stable, the primary imaging system is used (Küveler et al. 2011). In addition, below $\mu = 0.35$ a rotating glass tilt plate is used to keep the distance between the spectrograph slit and the solar limb image under control. The slit jaw image is digitized by a dedicated CCD camera. An algorithm recognizes the solar limb and the spectrograph slit position on the image. This allows the calculated desired distance between the limb and the slit to be set by automatic control of the tilt plate. Note that this plate is set after the polarization analyzer and hence does not introduce spurious polarization signatures. At each $\mu$ position a measurement is obtained by adding 300 single frames, each of which is obtained with an exposure time of 1 s. Therefore the effective integration time is 8 minutes. The precision of the
pointing at a chosen \( \mu \) position over 8 minutes using the tilt plate is limited to about 1\(^\circ\), which is less than the size of 1 pixel.

An improvement of these measurements is related to the absolute precision which we could reach in measuring \( Q/I \). Previously, the zero polarization value needed to be defined manually was based on indirect considerations (e.g., the CLV of the continuum polarization). For the data set described in this paper we could reach an absolute precision of about 5 \times 10^{-5}.

This is mainly due to (1) the precise control of the limb distance, allowed by the tilt plate system described above; (2) the improved control of the rotation of the optical devices, including the polarization analyzer in front of the slit (to compensate for the image rotation originated by the Gregory Coudé Telescope); and (3) the optical compensation of the instrumental linear polarization, which is a source of variable offset effects, with an oriented glass plate set in front of the polarization analyzer.

It was thus possible to subtract the polarization level measured at the disk center in a quiet region from every measurement done at a defined \( \mu \) position. For symmetry reasons the linear polarization in the continuum is expected to be zero at the disk center. In this way all residual instrumental linear polarization signatures are taken into account.

2.1. Stray Light Correction

The observed profiles contain a contribution from the spectrograph stray light which is about 2\% of the continuum intensity. Here we correct both the intensity and polarization profiles for stray light. The effect of stray light, including both its intensity and polarization, was treated in Stenflo (1974). Below are the details of the procedure we have followed.

In the absence of stray light but with instrumental polarization (crosstalk from \( I(\lambda) \)), the \( I(\lambda) \) and \( Q(\lambda) \) parameters after polarization calibration are \( I'(\lambda) = I(\lambda) \) and \( Q'(\lambda) = Q(\lambda) + M_{21}I(\lambda) \), if we assume that the Mueller matrix has been normalized \( (M_{11} = 1) \), there is no telescope depolarization or it has been calibrated away \( (M_{22} = 1) \), and that polarization crosstalk from \( U \) and \( V \) can be disregarded. \( M_{21} \) is the spectrally flat instrumental polarization, which for convenience will be renamed \( p_s \), since it represents a flat offset of the zero point of the polarization scale.

In the presence of stray light with an intensity that is a fraction \( s \) of the continuum intensity \( I_c \) and has a polarization \( p_s \), the apparent or observed Stokes parameters are

\[
\begin{align*}
I_{\text{obs}}(\lambda) &= I(\lambda) + sI_c, \\
Q_{\text{obs}}(\lambda) &= Q(\lambda) + p_s(I(\lambda) + sI_c) + p_sI_c.
\end{align*}
\]

We now introduce the notation \( r(\lambda) = I(\lambda)/I_c \) and \( \rho_{\text{obs}}(\lambda) = I_{\text{obs}}(\lambda)/I_{\text{obs,c}} \) for the rest intensities. For clarity we attach a \( \lambda \) to the quantities that are spectrally structured, in contrast to the three free parameters of our problem, namely, \( s \), \( p_s \), and \( p_c \), which are constant and spectrally flat. Then

\[
\rho_{\text{obs}}(\lambda) = (1 + s)r_{\text{obs}}(\lambda) - s.
\]

Similarly, we define the intrinsic polarization \( p(\lambda) = Q(\lambda)/I(\lambda) \) and the apparent polarization \( p_{\text{obs}}(\lambda) = Q_{\text{obs}}(\lambda)/I_{\text{obs}}(\lambda) \).

One can easily show that

\[
p(\lambda) = \left(1 + \frac{s}{r(\lambda)}\right)[p_{\text{obs}}(\lambda) - p_c] - \frac{s}{r(\lambda)}p_s.
\]

To calculate \( p(\lambda) \) from the observations we need to insert the expression for \( r(\lambda) \) from Equation (2) into Equation (3).

The intrinsic polarization that contributes to the stray light, as averaged over the wide spectral range, is represented by \( p_s \). Grating ghosts that sample discrete wavelengths spread over a large wavelength range are a major source of spectrograph stray light. In the absence of other information, the best estimate of \( p_s \) is probably \( p_s \approx p_c \), i.e., to set it equal to the continuum polarization.

The determination of \( p_s \) is best made for a disk center recording that is done immediately before or after the measurement at the given \( \mu \) position (so that one can assume that the instrumental polarization has not changed). At the disk center the solar-scattering polarization is zero, so the apparent polarization that we see is simply \( p_c \) (in contrast to measurements of the disk center, where \( p_c \) is mixed with the intrinsic solar polarization).

It is important to realize that the problem of correcting for the zero point of the polarization scale is entirely decoupled from the stray light issue. It is the first step to be done, and it gives us the spectrum

\[
p'(\lambda) = p_{\text{obs}}(\lambda) - p_c,
\]

which would equal to \( p(\lambda) \) in the absence of stray light. To correct \( p'(\lambda) \) for stray light we do not need to refer to \( p_s \) or disk center observations. The way in which the stray light correction enters can be seen by rewriting Equation (3) as

\[
p(\lambda) = p'(\lambda) + \frac{s}{r(\lambda)}(p'(\lambda) - p_c).
\]

If the stray light were unpolarized, then the stray light scaling factor \( s/r(\lambda) \), which is large where the rest intensity \( r(\lambda) \) is low, acts to amplify the polarization amplitudes \( p'(\lambda) \). In the presence of stray light polarization, however, the scaling factor only acts on the amplitude with respect to the \( p_s \) level rather than with respect to the zero level. The stray light polarization therefore reduces the effect of the stray light correction. For polarization amplitudes that are equal to \( p_s \) (which represents a broadband polarization background that may be approximated with the continuum polarization level \( p_c \)), the stray light correction does not have any effect at all. For the Ca\textsc{i} 4227 Å line, however, the core polarization is usually larger than \( p_c \). In this particular case the stray light polarization becomes a second-order effect (since \( sp_c \) is a product of two small quantities). The parameter \( s \) is determined exclusively by fitting the Fourier transform spectrum of Kurucz et al. (1984), in the same context as the spectral broadening is determined. The above considerations give us a rather well-defined procedure to determine (within the framework of our idealized model) unique estimates of the parameters \( s \), \( p_s \), and \( p_c \). Using these estimates we can correct both the \( I(\lambda) \) and \( Q(\lambda)/I(\lambda) \) spectra for stray light. In other sections we have dropped the \( \lambda \) dependence of \( I \) and \( Q \) for notational simplicity.

While the observed spectra are corrected for stray light, they have not been corrected for spectral broadening, because the instrumental profile is not known with the precision that is needed to allow a deconvolution. The theoretical spectra, on the other hand, which are used for comparison with the observed spectra, need to be spectrally broadened to emulate the observations. However, one should not apply stray light to the theoretical spectra, since one can easily do the correction to the observed spectra itself. In this way we keep the presentation of the theoretical results independent of the particular properties of the instrument used for the observations, with the single exception of spectral broadening.
3. MODELING OF THE CLV OBSERVATIONS

In this section we describe the modeling procedure we have followed to model the CLV of the Ca I 4227 Å line. We started with the aim of finding a single 1D atmosphere model that can simultaneously fit the \((I, Q/I)\) CLV of the Ca I 4227 Å line. As a first step toward this, we begin the modeling of the observed profiles by solving the polarized RT equation for a two-level atom. The 1D polarized RT equation along with the other necessary equations in the nonmagnetic regime used in this paper are described in detail elsewhere (see Anusha et al. 2010). The elastic collision rates used in this paper are computed following the theory presented in Barklem & O’Mara (1997). Additionally, the modeling is done using a two-stage process wherein the intensities are computed using the PRD-capable MALI (Multi-level Approximate Lambda Iteration) code of Uitenbroek (2001) in the first stage. In the second stage, the polarization profiles are computed perturbatively by solving the polarized transfer equation. The details of such a two-stage modeling procedure are described in Holzreuter et al. (2005, see also Anusha et al. 2010). The atom model of the Ca I used in the present paper is the same as the one discussed in Anusha et al. (2010); hence we do not repeat the details here.

In our studies we use four standard 1D model atmospheres of the Sun, namely, FAL-A, FAL-C, FAL-F (Fontenla et al. 1993), and FAL-X (Avrett 1995). The temperature structures of these models are shown in Figure 2. Along with the standard models, the temperature structure of our newly constructed model FALA + FALX is also shown in the figure, details of which will be discussed in Section 3.2.

3.1. CLV Behavior at Three Wavelength Positions

The observed \(Q/I\) profiles of the Ca I 4227 Å line show three prominent features: the line center at 4226.7 Å, the blue wing PRD peak at 4226.2 Å, and the red wing PRD peak at 4227.1 Å. While the line center of the Ca I 4227 Å line is formed within a height range of 700–1000 km (covering \(0.9 \leq \mu \leq 0.1\)), the blue and red wing PRD peaks are formed at a height of 150–250 km above the level where the vertical continuum optical depth at 5000 Å is unity. To get an idea of the behavior of the polarized spectra as a function of \(\mu\), we plot the angular dependence of intensity (only at the line center) and linear polarization at these chosen wavelength positions in Figure 3, computed using the standard 1D model atmospheres. As expected, it can be seen from Figure 3 that the degree of linear polarization decreases to zero toward the disk center due to symmetry in the scattering geometry.

Panels (a) and (b) of Figure 3 show a comparison of the observed and theoretical CLV of \(I\) and \(Q/I\), respectively, at the line center wavelength. We see that the hottest model FAL-F (dot-dashed line) is more suited for modeling the CLV of the line center intensity. However, the same model is not at all good for \(Q/I\). Instead, it is the coolest model, FAL-X, which provides the closest fit to the observed \(Q/I\). This contrasting behavior seems to point to the fact that we need two different temperatures to simultaneously fit \(I\) and \(Q/I\) at the line center. From Figure 3(b) we see that the theoretical profile computed using FAL-A also falls close to the observed CLV profile of \(Q/I\). We consider the FAL-X atmosphere rather than FAL-A to provide a better fit to the observed \(Q/I\) at the line center for the following reasons: at the line center and when \(\mu\) is small, we always expect the nonmagnetic \(Q/I\) amplitude to be larger compared with the observed \(Q/I\), because the observed \(Q/I\) includes depolarization by magnetic fields. As we go to larger \(\mu\), the magnetic fields may enhance the core polarization amplitude. Such behavior was noted by Faurobert-Scholl (1994), who points out that there are enhancement effects due to magnetic fields when \(\mu > 0.4\) (this will be discussed in detail in Section 3.2.1). We found the FAL-X model to better satisfy this behavior. Thus we see from Figure 3(b) that the theoretical values of \(Q/I\) at the line center when computed with FAL-X are larger than the observed \(Q/I\) for \(\mu < 0.4\), while FAL-A shows this behavior only for \(\mu < 0.25\). Besides this, the theoretical \(Q/I\) computed with FAL-A falls much below the observed \(Q/I\) as we move toward larger values of \(\mu\). For these reasons we consider FAL-X to give a consistent overall fit to the observed CLV of \(Q/I\) at the line center, while FAL-A does not.

On the other hand, Figures 3(c) and (d) show the CLV profiles of \(Q/I\) at the blue and red wing PRD peak wavelength positions, respectively. We notice that both the FAL-F and FAL-X model atmospheres fail to provide a fit to the PRD peaks. It is the theoretical CLV profiles from the FAL-A model that fall closest to the observed CLV of \(Q/I\). Thus we do not find a single 1D atmospheric model which can provide a fit to the entire Stokes \((I, Q/I)\) profiles simultaneously. As a next step we explore the possibility of obtaining a fit to the CLV of the Stokes profiles through a small modification of the temperature structure of the original FAL models at the appropriate heights.

3.2. Theoretical Fit to the CLV of the \(Q/I\) Profiles

From Figures 3(c) and (d) we see that although the theoretical profiles from the FAL-A model fall closest to the observed CLV of \(Q/I\), this model fails to provide a satisfactory fit to the \(Q/I\) observations. In order to obtain a better fit to the observed CLV of the \(Q/I\) profiles, we adopt a modification of the temperature structure at the heights where the PRD peaks are formed. We focus our attention only on the fit to the \(Q/I\) profiles. Since the
Figure 3. Observed (solid line) and calculated intensity and polarization signals as a function of $\mu$ (observed at 14 points) at three different wavelength positions in the line profile. The spectra are calculated for the standard models, FAL-X, FAL-C, FAL-A, and FAL-F.

FAL-A model atmosphere provides the closest fit to the observed $Q/I$ profiles, we choose this model for further modifications. Accordingly, the temperature of the FAL-A standard model at these heights is reduced by about 200 K. This newly constructed model is denoted as FALA. This new model provides a better fit to the wing PRD peaks at all the limb distances.

After achieving a fit to the PRD peaks, we concentrate on obtaining a fit to the linear polarization at the line center. From Figure 3(b) we see that it is the FAL-X model atmosphere which fits the observed profiles the closest. Thus, to obtain a satisfactory fit to the entire $Q/I$ profile we need to combine these two model atmospheres (FALA and FAL-X) at the appropriate heights. The two models are combined such that the new model atmosphere has the temperature structure of FALA up to a height of 400 km and the temperature structure of FAL-X at heights above 400 km. The temperature structure of the new combined model atmosphere (FALA + FALX) is shown as the solid line in Figure 2. The results obtained using this combined model atmosphere are discussed below.

3.2.1. Results from the New Combined Model Atmosphere

The theoretical profiles obtained using the FALA + FALX model atmosphere are shown in Figure 4 (dotted line). In addition, a comparison between the observed and theoretical $Q/I$ CLV curves at the blue and red wing PRD peaks and at the line center wavelength using the combined model is shown in Figure 5 (dotted line). These theoretical profiles show that we obtain an overall satisfactory fit to the $Q/I$ profiles at all the $\mu$ positions using the combined model atmosphere. The theoretical profiles computed using the combined model atmosphere in Figures 4 and 5 include suitable spectral smearing. This is done by convolving the theoretical spectra with a Gaussian profile having a FWHM of 50 mÅ. The smearing accounts for both the instrumental broadening (40 mÅ) and the broadening by macroturbulent velocity fields (30 mÅ). The macroturbulent smearing corresponds to a velocity of 1.28 km s$^{-1}$. For deep lines such as the Ca$^+$ 4227 Å, the effects of the stray light corrections (the stray light correction procedure is described in Section 2.1) are much more important than the smearing. The intensity ($I$) profiles from the combined model seem to fit the observed data at all the $\mu$ positions, with the exception of the line core, where we fail to get a satisfactory fit. At the formation heights of the line center, a rest intensity fit requires a hotter atmospheric model such as the FAL-F, which is not suitable for achieving a good fit to the $Q/I$ profile—which indeed requires cooler models such as the FAL-X. In spite of the carefully determined stray light correction (by $s = 2\%$) to the observed Stokes I profiles, the central line depth still does not come close to reproducing the very deep theoretical $I$ profiles. We have also carried out tests with the use of different microturbulent and macroturbulent velocities, and we found that the choice
Figure 4. Comparison between the observed (solid line) and the theoretical (dotted line) Stokes profiles ($I, Q/I$) at different limb distances. The combined model atmosphere FALA + FALX is used to compute the theoretical profiles.
of turbulent velocity does not significantly affect the rest intensity of the Stokes $I$ profiles.

From Figures 4 and 5 we notice that for $\mu \leq 0.35$ the observed line center $Q/I$ is less than the theoretical value, and for $\mu > 0.35$ it is greater than the $Q/I$ predicted theoretically. Such a discrepancy was also encountered by Faurobert-Scholl (1994) while modeling the CLV of the line center $Q/I$ of the Ca I 4227 Å line. The author found that the ratio of $(Q/I)_{\text{obs}}$ and $(Q/I)_{\text{theory}}$ at the line center was close to unity for smaller $\mu$ values ($\mu < 0.4$) and much greater than one for larger $\mu$ values. However, her treatment did not include the stray light corrections. We recall that the observed profiles in Figure 5 are corrected for the stray light. In Faurobert-Scholl (1994), although an explanation of the physical mechanism behind this enhancement in polarization for larger $\mu$ values was anticipated based on accelerated motions in the chromosphere, it was not completely justified.

To examine this discrepancy further, we plot the variation exhibited by $Q/I$ along the spectrograph slit at the line center (solid line) and compare it with the $Q/I$ in the blue wings (dotted line) in Figure 6. The observed line center $Q/I$ values are smoothed over a rectangular box corresponding to 5″ to reduce contribution from noise. These smoothed values of the observed $Q/I$ at the line center are used for all further computations. From Figure 6 we see that the $Q/I$ at the line center shows more variation along the slit than at the blue wing peak. This indicates the presence of varying horizontal magnetic fields and their possible role in modifying the line center $Q/I$. These varying magnetic fields can in turn be used to understand the observed line center $Q/I$ which are greater than the theoretically predicted values. One possible explanation for this discrepancy could be that the observed line center $Q/I$ for $\mu > 0.35$ is enhanced due to the Hanle effect by these varying fields (see also Faurobert-Scholl 1994). This enhancement is very prominent in the case of the near-disk-center observations (see Anusha et al. 2011). However, in our case, it sets in for $\mu > 0.35$ and increases as $\mu \to 1$. This is clear evidence for highly structured, resolved, oriented magnetic fields (predominantly horizontal) in the solar atmosphere. However, the dotted line in Figure 6 (for the blue wing) does not exhibit the type of spatial fluctuation that is seen for the line center (solid line). This is because the Hanle effect is absent in the wings. We also note that the spatial variation close to the limb in the blue wing is not really spectrally flat. The details regarding this will be discussed in Section 4.

3.2.2. Impact of Temperature Structure Modifications on the Standard Model Atmospheres

In the previous section we described the necessity of constructing a new model in order to obtain a fit to the CLV of the Stokes profiles. To this end, a new model was constructed by combining two standard models after modifying their temperature structures at the desired heights. The new combined model thus constructed will provide a fit to the CLV of the observed $Q/I$. The physical consistency of the newly constructed atmospheric model with the modified temperature structure has been checked by verifying that it satisfies the hydrostatic equilibrium at all heights.

Next we examine the fit to the CLV of the continuum intensity over a wavelength range spanning from the visible to the infrared. The theoretical continuum intensity obtained using the new model should fit the observed data at all the limb distances and for a range of wavelengths. Figure 7 shows the limb-darkening function computed using the standard models and our new model atmosphere $\text{FAL} + \text{FALX}$ for a range of wavelengths and $\mu$ values. The theoretical values from different models are compared with the observed data from Neckel & Labs (1994). The dash-triple-dotted line represents the theoretical values from the new model $\text{FAL} + \text{FALX}$. We see that the best fit to the observations is provided by the FAL-C model. Although the combined model is successful in providing a CLV fit to the observed $Q/I$ and satisfies the equilibrium conditions, it does not provide the best fit to the observed CLV of the limb-darkening function and to the observed CLV intensity.

This leads us to the conclusion that it is indeed not possible to obtain a simultaneous fit to all the various types of data with a single 1D atmosphere model; a different atmosphere is needed for each observable. In search of a single model which satisfies all the observational constraints, the next obvious step would be to use the two-component modeling approach with appropriate mixing ratios, as was done in Holzreuter & Stenflo (2007). In the section below we discuss why we cannot adopt such a procedure in modeling the CLV of the Ca I 4227 Å line.
3.2.3. Two-component Modeling Approach

In modeling the CLV observations of the Ca\textsc{ii} K line, Holzreuter & Stenflo (2007) explored the possibility of constructing a two-component model atmosphere. This was constructed by mixing the results obtained from two standard model atmospheres in appropriate ratios. Such a method was adopted by making CLV plots of the Ca\textsc{ii} K line as shown in their Figure 1. As seen from their figure, the original models FAL-X and FAL-C produce theoretical CLV curves which fall above and below the observations.
below the observed CLV curve of $Q/I$, respectively. Hence the authors combine results from these two standard models with appropriate mixing ratios to achieve the required fit. In Figure 3 of the present paper we make similar plots of the CLV for the Ca i 4227 Å line. As seen from Figure 3, none of the standard model atmospheres produce a theoretical CLV curve which falls above the observed CLV curve. This does not allow us to apply the same kind of two-component modeling procedure as described by Holzreuter & Stenflo (2007). This suggests that we need to go beyond 1D modeling in the direction of two-dimensional or three-dimensional modeling to obtain a simultaneous fit to the $(I, Q/I)$ at all the limb distances. Such efforts are beyond the scope of this paper. However, 1D models with modified temperature structures serve as a good initial step to such elaborate computations. The failure of the 1D modeling approach does not preclude the use of a given line profile for purposes such as magnetic field determination. To demonstrate this fact, in the next section we perform observational analysis of the Ca i 4227 Å line to determine the field strengths for smaller $\mu$.

4. DETERMINATION OF THE FIELD STRENGTH

In the present section we use an approach similar to that of Bianda et al. (1998, 1999) to determine the field strength at different limb distances. Since the observed $Q/I$ is influenced by so many factors besides the magnetic field, it is imperative to apply differential techniques to isolate the Hanle effect from the multitude of other effects. This can be done by using the wing polarization as a reference, since it has been well established that the Hanle effect only operates in the line core but is absent in the wings.

If in Figure 6 we compare $Q/I$ in the line core (solid lines) and in the blue wing (dotted lines), we notice that the line core exhibits large spatial variations along the slit, in contrast to the blue wing. Nevertheless, the blue wing polarization exhibits large-scale slow drifts along the slit, which increase significantly as we approach the limb. Much of this can be explained in terms of a geometric effect due to the limb curvature. Since the solar limb is curved, while the slit is straight, the limb distance (or $\mu$) will vary along the slit. This effect will increase in significance as we get closer to the limb. It is an effect that is nearly identical for the line core and wing (since the core and wing have nearly the same relative CLVs) and therefore can be eliminated when forming the core-to-wing ratio. Similarly, any other unidentified instrumental effect would ratio out. In principle, there may also be nonmagnetic effects of solar origin, such as spatial variations of the radiation-field anisotropy, which may be different between core and wings and therefore would not fully ratio out (although they should be suppressed when forming the ratio, since the nonmagnetic fluctuations in the core and wings are not uncorrelated). However, with our rather low spatial resolution and long integration times, these solar effects are expected to be miniscule.

We therefore have strong reasons to believe that practically all the spatial fluctuations that we see in the $Q/I$ core-to-wing ratio are due exclusively to magnetic fields via the Hanle effect. Instead of directly using this ratio as our differential measure, we can scale it with the slit average of the wing polarization to express it in polarization units. This scaling is equivalent to the assumption that the wing polarization should be spatially flat after all effects of limb curvature, unidentified instrumental effects, and solar nonmagnetic effects have been corrected for. We thus correct the line core polarization amplitude with the following relation:

\[
(Q/I)_{\text{line center}} \text{corrected} = \frac{(Q/I)_{\text{line center}} \text{uncorrected}}{P_b} \langle P_b \rangle,
\]

where $P_b = (Q/I)_{\text{blue wing peak}}$ for each pixel and $\langle P_b \rangle$ is the spatial average of $P_b$ along the slit. With this correction we plot in Figure 8 the CLV of the spatially averaged $Q/I$ at the blue wing and the corrected $Q/I$ at the line center. Each “plus” symbol in the bottom panel of Figure 8 represents the value of $Q/I$ at each pixel corresponding to the line center. We notice large spatial variations along the slit in the corrected $Q/I$ line center data. This effect is due exclusively to the magnetic fields via the Hanle effect. In order to find the field strengths that contribute to such spatial variation, we follow the method used in Bianda et al. (1998, 1999). We would like to note that in both of these papers, the authors use observations taken at different periods for the data analysis. However, in our analysis...
we consider only one single set of observations and the variation of \(Q/I\) along the slit in these observations. To this end we construct the envelopes (continuous lines in Figure 8) to our data set, using the analytical relation

\[
\frac{Q}{I} = \frac{a(1 - \mu^2)}{\mu + b}. \tag{7}
\]

This relation was first introduced by Stenflo et al. (1997) where \(a\) and \(b\) are the best-fit free parameters. For our studies we have chosen the same set of free parameters as given in Bianda et al. (1998, 1999). From the top panel of Figure 8 we see that the dashed line \((a = 0.33\%, \ b = 0.02)\) gives a good fit to the spatially averaged observed CLV profile in the blue wing. In the bottom panel of Figure 8 we use three different sets of free parameters \(a\) and \(b\) to construct envelopes for the line center data. The envelopes constructed using the analytical relation given in Equation (7) represent the “nonmagnetic value,” and all the values lying below this envelope are considered as the depolarized \(Q/I\) values due to the Hanle effect. From our modeling efforts we know that the magnetic fields cause an enhancement in the polarization value for \(\mu > 0.35\) (see also Faurobert-Scholl 1994). Hence this envelope-fitting method is good for \(\mu \leq 0.35\) and becomes questionable for \(\mu\) larger than about 0.35. However, the transition between the large-angle scattering and small-angle scattering is gradual and smooth. For large \(\mu\) we gradually enter into the regime of the forward scattering Hanle effect, for which the kind of techniques developed by Anusha et al. (2011) have to be adopted to derive the field strengths. Full RT modeling is naturally needed for intermediate \(\mu\) values. Only for smaller \(\mu\) values is it possible to use a method that avoids the need for RT. In this approach, we first extract an observed depolarization factor via the envelope method and then convert this depolarization into field strength.

Thus we first determine the ratio between the line center \(Q/I\) and the corresponding envelope value. This ratio represents the depolarization factor caused by the Hanle effect for each pixel. The conversion of this factor into field strength is dependent on the choice of the envelope, since it represents a single observable, while the magnetic field vector is characterized by three parameters (its spatial components). The magnetic field is therefore underdetermined, so a conversion cannot be unique, but it is still meaningful in a statistical sense, as explained in the following.

For photospheric spectral lines an interpretational model with a spatially averaged microturbulent field distribution could be used to convert Hanle depolarization into field strength (Stenflo 1982, 1994), because the absence of \(U/I\) polarization in combination with insignificant spatial variations (at resolved scales) in \(Q/I\) made such a microturbulent interpretational model unavoidable (see also Stenflo 2013). The situation is, however, entirely different for strong chromospheric lines, such as the \(\text{Ca} \ iv\) 4227 Å line, which always exhibit large spatial line core variations in both \(Q/I\) and \(U/I\), such as those in Figure 6. As we have spatially resolved these variations, they should ideally be interpreted in terms of resolved, oriented fields rather than angular distributions. However, since the field vector is underdetermined by the single depolarization factor, we need to eliminate the ambiguity by using the statistical approach, dealing with each depolarization factor as if it were obtained through averaging over an ensemble of field elements. This approach will give field strength values that are meaningful as averages in a statistical sense.

Vertical fields are immune to the Hanle effect; depolarization can only occur if the field has a substantial inclination with respect to the vertical direction. Horizontal fields give the largest depolarization. If we assume the fields to be horizontal, but with orientations that are random in azimuth angle, and let the Hanle depolarization be determined by an ensemble average over such a field distribution, then the field strengths that we extract from this model can be considered to represent lower limits to the true average field strength (since there may exist less inclined fields that are less “visible” to the Hanle effect).

Chromospheric fields are expected to be largely horizontal, forming a “canopy” over the underlying photosphere. The Hanle depolarization factor \(k_H\) for a horizontal field distribution with random azimuths can be written as (Stenflo 1982, 1994)

\[
k_H = 1 - 0.75 \sin^2 \alpha_2, \tag{8}
\]

where the Hanle mixing angle \(\alpha_K\) is given by

\[
\tan \alpha_K = \frac{KB}{B_0/k_{(K)}^{(K)}}. \tag{9}
\]

\(B\) is the field strength to be determined, \(K = 1\) or 2, \(k_{(K)}^{(K)}\) is the collisional branching ratio for the 2\(K\)-multipole, and \(B_0\) is the characteristic field strength for the Hanle effect.
Figure 9. Histogram of the field strength at different \( \mu \) positions. Field strengths are computed for each depolarization value in the spatial direction. Different panels along the row for each \( \mu \) correspond to field strengths obtained using different envelopes. The solid, dotted, and dashed lines correspond to the solid, dotted, and dashed envelopes in Figure 8, respectively.

Figure 9 shows the histograms of the field strengths obtained when using Equations (8) and (9) for all pixels along the spatial direction. By definition, a depolarization factor should be less than or equal to unity; otherwise, it is unphysical. However, some points give an unphysical depolarization factor, both because there is scatter of the \( Q/I \) values due to measurement noise and because the chosen envelope may be too low. In such cases the field strength used for the histograms in Figure 9 is set to zero. The number of such zero field points depends on the choice of envelope and increases as we move away from the limb because of the increasing contribution from the forward scattering Hanle effect.

Figure 9 shows how the field strength fluctuations along the slit vary with differing limb distance. For the derivation of the CLV of the average field strength, which is shown in Figure 10, we do not average the field strength histograms of Figure 9, because they are affected by measurement noise in a nonlinear way (including the truncation used for the unphysical values); instead, we average the measured \( Q/I \) along the slit (causing the Gaussian instrumental noise to be greatly suppressed) and then convert the average \( Q/I \) to field strength. Since the height of the line formation increases with decreasing \( \mu \), the \( \mu \) variation displayed by Figure 10 may be interpreted in terms of a height variation of the field. In view of the limited statistical sample and the crudeness of the interpretational model, the minor variations with \( \mu \) in Figure 10, which are similar to the ones obtained by Bianda et al. (1998, 1999), are not significant but are compatible with the approximate constancy of the average field strength over the height range covered by our \( \mu \) range.

Note that we have limited the \( \mu \) range in Figure 10 to 0.1–0.35, because as previously mentioned the envelope method is not applicable for larger \( \mu \) values. Note also how the derived mean
field strength depends on the choice of envelope. In spite of these uncertainties, the values are generally limited to the range 6−10 G. We cannot choose envelopes significantly lower than the one represented by the solid line (in the bottom panel of Figure 6), because one would then get an excessive number of unphysical depolarization factors. Therefore the 6 G value can be seen as representing a kind of lower limit for the average field strength.

5. CONCLUSIONS

To understand the depth dependence of various physical quantities in the Sun, such as the magnetic fields, it is important to model the CLV observations of suitable atomic and molecular lines. In this paper we have attempted to model such CLV observations of the well-known Ca i 4227 Å line. In our approach we take into account the effects of PRD and RT. The observations of this line were carried out in quiet regions on the Sun at 14 positions starting from the limb up to the disk center on 2012 October 16 at IRSOL in Switzerland. This line has the largest degree of linear polarization in the visible region of the SSS and can be modeled by considering a simple two-level atom picture. When trying to model this CLV data we find that none of the standard atmospheric models we attempted, such as the FAL-F, FAL-A, FAL-C, and FAL-X models, could simultaneously fit the observed \( I, Q/I \) profiles at all the limb distances. To model the CLV of the line center intensity we need the FAL-X model, which is the hottest, and to model the CLV of the line center polarization at the line center we need the coolest model, FAL-X. In order to obtain a fit to the observed Stokes profiles, modifications in the temperature structure of the standard models become necessary. With suitable modifications in the desired height range, we constructed FAL-A and later combined it with FAL-X. While the FAL-A model gives a good fit to the PRD peaks, the FAL-X model gives a good fit to the line center. The combined model has the temperature structure of FAL-A up to 400 km and that of FAL-X in the upper layers. This new combined model atmosphere gives a good fit to the entire \( Q/I \) at all values of \( \mu \). In modeling efforts we also found that the Hanle effect not only depolarizes the line core of \( Q/I \) (which is true for smaller \( \mu \)) but also enhances the line core \( Q/I \) for larger \( \mu \) values. This might be due to the highly structured horizontal magnetic fields in the solar atmosphere.

Although the new combined model provides a fit to the CLV of the observed \( Q/I \), it fails to reproduce the observed CLV of the continuum limb-darkening function and the CLV of the observed line core intensity. This failure of the 1D models in order to simultaneously fit the observed \( I, Q/I \) CLV profiles does not restrain the use of the Ca i 4227 Å as a tool to map the magnetic fields. To support this claim, we carried out observational analysis to determine field strength using the Ca i 4227 Å for smaller \( \mu \) values.

To conclude, it appears that no single 1D atmosphere model can completely provide a good representation of the actual solar atmosphere. This shows that the solar atmosphere has a far more complex structure. To simultaneously satisfy the various observational constraints it is therefore unavoidable to go beyond such 1D models—a difficult problem that needs to be approached step by step. This conclusion is not at all a technical failure, meaning that our inability to obtain a simultaneous perfect fit to the CLV of the \( I, Q/I \) has nothing to do with the weakness of our approach or the method followed in using 1D solar atmosphere models. Instead it is a “profound failure,” indicating that the atmosphere of the Sun has such complexity that it is not possible to represent it in terms of a single 1D atmosphere model. It could mean that the use of 1D models for interpretations of the SSS may give results that are physically incorrect (since they do not represent solar conditions), although the results may formally be mathematically correct. However, 1D modeling efforts may still provide a guideline to the more systematic and sophisticated modeling efforts.

We are grateful to Dr. Han Uitenbroek for providing us with his realistic atmospheric modeling code. We acknowledge the use of the HYDRA cluster at the Indian Institute of Astrophysics for computations in this work. H.D.S. thanks COST ACTION MP1104 for the financial support provided to visit IRSOL in connection with this project. Research at IRSOL is financially supported by State Secretariat for Education, Research and Innovation (SERI), Canton Ticino, the city of Locarno, the local municipalities, the Foundation Aldo e Cele Daccò, and the Swiss National Science Foundation grant 200021-138016). R.R. acknowledges financial support by the Carlo e Albina Cavallargna foundation. We are grateful to the referee for very detailed and useful comments which helped improve the paper substantially.

REFERENCES

Anusha, L. S., Nagendra, K. N., Bianda, M., et al. 2011, ApJ, 737, 95
Anusha, L. S., Nagendra, K. N., Stenflo, J. O., et al. 2010, ApJ, 718, 988
Auer, L. H., Rees, D. E., & Stenflo, J. O. 1980, A&A, 88, 302
Avrett, E. H. 1995, in Infrared Tools for Solar Astrophysics: What’s Next?, ed. J. R. Kuhn & M. J. Penn (Singapore: World Scientific), 303
Barklem, P. S., & O’Marra, B. J. 1997, MNRAS, 290, 102
Bianda, M., Ramelli, R., Anusha, L. S., et al. 2011, A&A, 530, L13
Bianda, M., Solanki, S. K., & Stenflo, J. O. 1998, A&A, 331, 760
Bianda, M., Stenflo, J. O., Gandorfer, A., & Gisler, D. 2003, in ASP Conf. Ser. 286, Current Theoretical Models and Future High Resolution Solar Observations: Preparing for ATST, ed. A. A. Pevtsov & H. Uitenbroek (San Francisco, CA: ASP), 61
Bianda, M., Stenflo, J. O., & Solanki, S. K. 1999, A&A, 350, 1060
Faurobert-Scholl, M. 1992, A&A, 258, 521
Faurobert-Scholl, M. 1994, A&A, 285, 655
Fontenla, J. M., Avrett, E. H., & Loeser, R. 1993, ApJ, 406, 319
Gandorfer, A. 2002, The Second Solar Spectrum, Vol. 2, 3910 Å to 4630 Å Line (Zurich: VdF Hochschulverlag)
Holzreuter, R., Fluri, D. M., & Stenflo, J. O. 2005, A&A, 434, 713
Holzreuter, R., & Stenflo, J. O. 2007 A&A, 467, 695

Figure 10. Mean value of field strength (G) derived from the averaged \( Q/I \) value along the slit. The solid, dotted, and dashed lines correspond to the mean field strength derived from the corresponding envelopes indicated in Figure 8.
