On the Periods of Negative Superhumps
and the Nature of Superhumps

J. S m a k

N. Copernicus Astronomical Center, Polish Academy of Sciences,
Bartycka 18, 00-716 Warsaw, Poland
e-mail: jis@camk.edu.pl

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ABSTRACT

Osaki and Kato (2012) interpreted variations of the negative superhump periods, discovered by them in dwarf nova V1504 Cyg, as evidence in favor of the thermal-tidal instability model for superoutbursts. It is shown that their interpretation was incorrect. The observational evidence is recalled showing that superoutbursts are due to the enhanced mass transfer rate.

Key words: accretion, binaries: cataclysmic variables, stars: dwarf novae

1. Introduction

In their recent paper Osaki and Kato (2012) presented results of the analysis of the Kepler light curve of the dwarf nova V 1504 Cygni. In particular they detected negative superhumps (nSH) and variations of their period. Interpreting those variations as being exclusively due to the variable radius of the disk they concluded that they provide support for the thermal-tidal model of superoutbursts.

The purpose of the present paper is to challenge those conclusions. In Section 2 it will be shown that their interpretation of the nSH period variations and – consequently – their main conclusion about the nature of superoutbursts were incorrect. In Section 3 the observational evidence will be recalled showing that superoutbursts are due to the enhanced mass transfer rate.

2. The Negative Superhump Periods and their Variations

2.1. Negative superhumps in V1504 Cyg

Osaki and Kato (2012) discovered negative superhumps (nSH) in the Kepler light curve of V1504 Cyg and studied them in detail during supercycle No.5. Results, presented in their Fig.5, show that the nSH frequency $\nu_{nSH}$ increased during
outbursts and superoutbursts and decreased during quiescence. In addition, the mean frequency, averaged over normal outburst cycles, increased during the supercycle.

According to the commonly adopted interpretation, negative superhumps are produced by the stream impact, as it transits across the face of the tilted, precessing disk. Adopting this interpretation Osaki and Kato assumed that the observed variations of the nSH frequency (or period) are exclusively due to the variations of the disk radius. In particular, they translated variations in $\nu_{nSH}$ into variations in $r_d$ using the following relation (from Larwood 1998)

$$\frac{\nu_{nSH}}{\nu_{orb}} = \frac{P_{nSH}}{P_{orb}} = 1 + \frac{3}{7} \frac{q}{(1+q)^{1/2}} \cos \theta \left(\frac{r_d}{R_d}\right)^{3/2},$$  

where $\theta$ is the disk tilt angle and $r_d = R_d/A$.

Variations of the nSH frequency in V1504 Cyg, when interpreted as being due to the variations of disk radius, are strikingly similar to the disk radius variations predicted by the thermal-tidal instability (TTI) model (Osaki 1989,2005). It is therefore not surprising that Osaki and Kato interpreted this similarity in favor of the TTI model.

Upon closer examination, however, it turns out that their interpretation of the nSH period variations leads to several discrepancies and inconsistencies and therefore cannot be correct. Consequently their main conclusion about the nature of superoutbursts is also incorrect.

2.2. Amplitudes of negative superhumps

Under the assumption that the mass transfer rate is constant, the nSH amplitude depends only on the amount of the kinetic energy of the stream dissipated during its collision with the disk, which is proportional to the (dimensionless) impact parameter $\Delta v^2$. From calculations of the stream particle trajectories one finds that the value of this parameter increases with decreasing radial distance. This implies that the nSH amplitude must be smaller when the disk is larger and vice versa.

Osaki and Kato (2012, Figs.7 and 8) determined the average nSH light curves and amplitudes in two cases:

(1) During the superoutburst No.4 the amplitude was $A = 0.04$ mag, or – corrected for the $\sim 3$ mag difference in brightness between superoutburst maximum and quiescence – $A_{corr} \approx 0.6$ mag. The nSH period, used by Osaki and Kato to calculate the average light curve, was $P_{nSH} = 0.067764$ d, which corresponds – via Eq.(1) – to $r_d = 0.48$.

(2) During the 8-day interval at quiescence (BJD 2455440-448) the amplitude was $A = 0.35$ mag. The nSH period used by Osaki and Kato was in this case $P_{nSH} = 0.068076$ d implying $r_d = 0.42$.

Obviously then in case (2), when the disk was smaller, the nSH amplitude should be larger than in case (1). Instead it was much smaller.
2.3. Negative superhumps during normal outburst cycles

Fig. 5 in Osaki and Kato (2012) shows that during all normal outburst cycles the minima of $\nu_{nSH}$ occurred $\sim 3$ days before the initial rise to outburst maximum and the following increase of $\nu_{nSH}$ till its maximum lasted for $\sim 3$ days. Within their interpretation this would imply that the disk begins to expand $\sim 3$ days before the initial rise and continues to expand for about 3 days.

Model calculations for dwarf nova outbursts show, however, that the expansion of the disk occurs nearly simultaneously with rising light and – in the case of short period systems – lasts for only $\sim 0.5$ day (see, for example, Fig. 3 in Hameury et al. 1998, or Fig. 1 in Osaki and Kato 2012). The interpretation is simple: Accretion of the material from the outer parts of the disk causes its luminosity to increase, while the excess angular momentum transported outward causes the disk to expand; this occurs on a viscous time scale which – in this case – is shorter than 1 day. Obviously then the expansion of the disk cannot begin 3 days earlier and cannot last 3 days.

2.4. Negative superhumps during superoutbursts

Fig. 5 in Osaki and Kato (2012) shows that during the main part of both superoutbursts, lasting for about 8 days, the nSH frequency decreased from $\nu_{nSH} = 14.78$ to $\nu_{nSH} = 14.70$. Using Eq. (1) we get for the corresponding disk radii: $r_d = 0.51$ and $r_d = 0.44$ and, consequently, $\frac{dr_d}{dt} = -0.007 \, \text{d}^{-1}$.

For comparison we have the radii of the disk in dwarf nova Z Cha during its superoutbursts determined from eclipses of the hot spot (Appendix). The least squares fit to those data gives

$$r_{d,0} = 0.481 \pm 0.016, \quad \text{and} \quad \frac{dr_d}{dt} = 0.0003 \pm 0.0043 \, \text{d}^{-1}, \quad (2)$$

with no difference between $r_d(\Phi_i)$ and $r_d(\Phi_e)$. This shows that the radius of the disk remains constant throughout superoutburst. Worth noting is also that $r_d \approx r_{\text{tidal}}$, as expected in the case of steady-state accretion with high mass transfer/accretion rate.

2.5. Comparison with other systems

Negative superhumps and their variable periods have been observed in several other dwarf novae at various stages of their outburst and superoutburst cycles. The
best documented examples are:

**V503 Cyg.** Harvey et al. (1995) detected negative superhumps during quiescence as well as during outbursts and superoutbursts. According to them the nSH period was longer during quiescence and shorter during outbursts, which is similar to the situation observed in V1504 Cyg. However, unlike in the case of V1504 Lyr, the nSH period, averaged over normal outburst cycles, did not show any significant change between the two successive superoutbursts.

**BK Lyn.** The light curve and the (O-C) diagram in Kato et al. (2012, Fig.26) show that between the two successive superoutbursts the nSH period decreased – like in V1504 Cyg. On the other hand, however, unlike in the case of V1504 Cyg, no variations were present during normal outburst cycles.

**V344 Lyr.** Wood et al. (2011) detected negative superhumps in the Kepler light curve of this star, present occasionally during some of the quiescence intervals and during some of normal outbursts. As can be seen from their Figs.23 and 26, the behavior of the nSH period was similar to that of V1504 Cyg: it was increasing during quiescence, decreasing during normal outbursts, and decreasing during supercycles.

**ER UMa.** The light curve and the (O-C) diagram in Ohshima et al. (2012, Fig.2) show that the nSH period variations in this system are similar to those in BK Lyn.

Those examples show that (1) the decreasing nSH period during supercycles is a common phenomenon among dwarf novae with superoutbursts, while (2) the nSH period variations during their normal outbursts cycles occur only in some of them. In particular, the nSH period variations observed in V1504 Cyg cannot be considered as representative for all such systems.

### 2.6. Negative superhumps periods and their variations

In the simplest case, when both – the precession period and the nSH period are constant, we have

$$\frac{1}{P_{nSH,o}} = \frac{1}{P_{prec}} + \frac{1}{P_{orb}}. \quad (3)$$

The precession period depends not only on the effective radius of the disk but also on the distribution of its surface density $\Sigma(r)$ (cf. Larwood 1998, Montgomery 2009 and references therein)

$$P_{prec} = f[r_d, \Sigma(r)], \quad (4)$$
where both \( r_d \) and \( \Sigma(r) \) are known to change during the dwarf nova cycle.

Turning to the observed nSH period we must note that it is defined as the interval of time between two successive maxima resulting from the collision of the stream with the surface of the tilted disk. The effective location of the stream impact depends on the disk tilt and on its geometrical thickness, described by \( z/r = f(r) \), which also changes during the dwarf nova cycle. Taking this into account we can write

\[
P_{nSH,obs} = P_{nSH,o} \left(1 + \frac{d\Delta t}{dt}\right),
\]

where

\[
\Delta t = f[\theta, z/r(r)]
\]

is the "flight" time of the stream elements from L\(_1\) to the effective point of collision.

Combining Eqs.(3-6) we obtain the following general formula describing the nSH period variations

\[
\frac{dP_{nSH,obs}}{dt} = \varepsilon_{nSH} \frac{\partial P_{prec}}{\partial r_d} \frac{dr_d}{dt} + \varepsilon_{nSH} \frac{\partial P_{prec}}{\partial \Sigma(r)} \frac{d\Sigma(r)}{dt}
\]

\[
+ \varepsilon_{nSH} \frac{\partial P_{prec}}{\partial \Delta t} \frac{d\Delta t}{dt} + P_{nSH,o} \frac{d^2\Delta t}{dt^2},
\]

where \( \varepsilon_{nSH} = 1 - P_{nSH}/P_{orb} \).

This shows that the nSH period variations can be due to many different causes. Osaki and Kato limited their discussion only to the first term on the right-hand side of Eq.(7) thereby making their conclusions unreliable.

3. Evidence for the Enhanced Mass Transfer Rate

The presence and luminosity of the hot spot in Z Cha (Smak 2007,2008a) and in other dwarf novae (e.g. OY Car; Smak 2008b) during their superoutbursts provide direct observational evidence showing that superoutbursts are due to strongly enhanced mass transfer (EMT) rate.
Osaki and Kato ignore this evidence. Instead they write: "In the EMT model mass-transfer rate is thought to be greatly increased during a superoutburst". Furthermore, in their Table 1, the "enhanced hot spot" is listed as a "consequence" of the model, which – according to them – is "not in agreement with observation"!

Appendix

The radii of the disk in dwarf nova Z Cha during its superoutbursts were determined (Smak 2007) from phases of ingress and egress of the hot spot eclipses. The most reliable values of \( r_d(\phi_i) \) and \( r_d(\phi_e) \), obtained from eclipses observed at beat phases \( \phi_b = 0.4 - 0.6 \) (which have not been published before) are listed in Table 1. The parameter \( \delta t \) is the time (in days) since the beginning of superoutburst.

Table 1

| Eclipse | \( \delta t \) | \( \phi_b \) | \( r_d(\phi_i) \) | \( r_d(\phi_e) \) |
|---------|--------------|--------------|----------------|----------------|
| E54023  | 2.5          | 0.56         | 0.48           | 0.39           |
| E54024  | 2.5          | 0.56         | 0.46           | 0.43           |
| E54025  | 2.5          | 0.56         | 0.46           | 0.54           |
| E54076  | 6.5          | 0.41         | 0.47           | 0.50           |
| E54077  | 6.5          | 0.41         | 0.46           | 0.51           |
| E59875  | 4.5          | 0.46         | 0.50           | 0.47           |
| E67837  | 1.5          | 0.55         | 0.52           | 0.51           |
| E67838  | 1.5          | 0.55         | 0.46           | 0.49           |
| E67864  | 3.5          | 0.45         | 0.51           | 0.48           |
| E77865  | 1.5          | 0.55         | 0.51           | 0.47           |

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