A Possible Universal Treatment of the Field Strength Correlator in the Abelian-Projected SU(2)-Theory

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Abstract

An integral relation between two functions parametrizing the bilocal field strength correlator within the Stochastic Vacuum Model is obtained in the effective Abelian-projected SU(2)-theory. This relation is independent of the concrete properties of the ensemble of vortex loops, which are present in the theory under study. By virtue of the lattice result stating that the infrared asymptotic behaviours of these functions should have the same functional form, the obtained relation enables one to find these behaviours, as well as the infrared asymptotics of the bilocal correlator of densities of the vortex loops. Those turn out to be exponentials, decreasing at the inverse mass of the dual vector boson, times certain polynomials in the inverse integer powers of the distance. This result agrees with the general predictions and the existing lattice data better than the results of previous calculations, where these powers were found to be half-integer ones.

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Stochastic Vacuum Model (SVM) \[1\] (see \[2\] for reviews) is nowadays commonly argued to be one of the most successful nonperturbative approaches to QCD. However, an important problem still requiring its clarification is the derivation of the bilocal field strength correlator, which is the main quantity in SVM, from the QCD Lagrangian. To simplify this problem it is natural to consider such a correlator in Abelian-type models, which allow for the analytical description of confinement. In this way, in the recent paper \[3\], the bilocal correlator has been calculated in the effective Abelian-projected \[4\] SU(2)- and SU(3)-theories. Those are just the 4D dual Ginzburg-Landau–type theories, in which confinement is realized as the dual Meissner effect according to the ’t Hooft-Mandelstam scenario \[5\]. The main difference of the calculation performed in Ref. \[3\] w.r.t. the pure classical calculations performed before \[3\] was the account for quantum vortex loops present in the above-mentioned theories. Being virtual (and therefore small-sized) objects, these loops have been treated within the natural, from the point of view of the standard superconductivity \[7\], model in which they form a dilute gas \[8\].

The aim of the present Letter is to improve on the calculation performed in Ref. \[3\] and to find the bilocal field strength correlator without any assumptions imposed on the ensemble of vortex loops. Such a model-independent study may become possible by virtue of the lattice data on the SVM correlator \[1, 10, 11\] (see \[12, 13\] for recent reviews). The present approach is based on the above-mentioned commonly accepted belief that confinement can really be viewed as the dual superconductivity. Clearly, it is this statement which enables one to employ the lattice results on the SVM correlator in QCD for the evaluation of an analogous quantity in the dual Ginzburg-Landau–type theories. In this respect, it is worth noting that recently a new support was given to the ’t Hooft-Mandelstam scenario of confinement (and consequently to the dual Abelian-type models) by the evidence of monopole condensation on the lattice \[14\] (see Ref. \[12\] for a review).

In what follows, we shall carry out our analysis for the simplest SU(2)-case, although the generalization to the SU(3)-case along with the lines of Ref. \[3\] is straightforward. Within the so-called Abelian dominance hypothesis \[15\], stating that the off-diagonal degrees of freedom are irrelevant to confinement, the effective IR theory under study can be shown (see e.g. \[16\]) to be nothing else, but the dual Abelian Higgs model with external electrically charged particles (quarks). In the London limit, \[i.e.\] the limit when the mass of the dual Higgs field is much larger than the mass of the dual vector boson, the respective action reads \[\[1\]\]

\[
S = \int d^4x \left[ \frac{1}{4} \left( F_{\mu\nu} + F^e_{\mu\nu} \right)^2 + \frac{\eta^2}{2} \left( \partial_\mu \theta - 2g_mB_\mu \right)^2 \right].
\]

(1)

Here, $\theta$ is the phase of the dual Higgs field describing the condensate of monopole Cooper pairs, $\eta$ is the v.e.v. of this field, and $2g_m$ is its magnetic charge with $g_m$ being the magnetic coupling constant related to the electric one, $g$, as $g_m g = 4\pi$. Next, in Eq. (1), $B_\mu$ stands for the gauge field dual to the diagonal gluonic field $A^3_\mu$, and $F^e_{\mu\nu}$ is a field strength tensor of an external quark obeying the equation $\partial_\mu \hat{F}^e_{\mu\nu}(x) = g \oint_C dx_\nu(\tau) \delta(x - x(\tau))$, where $\hat{O}_{\mu\nu} \equiv \frac{1}{2} \varepsilon_{\mu\nu\lambda\rho} O_{\lambda\rho}$. Note that the field $\theta$ contains the multivalued part which describes dual Abrikosov-Nielsen-Olesen strings \[17\]. The latter ones can either be open (those end up at the contour $C$ and provide the confinement of a quark moving along this contour) or closed. Such closed strings with minimal opposite winding numbers couple to each other and form virtual bound states, called vortex loops \[1, 8\].

Within the SVM, the (irreducible) bilocal field strength correlator can be parametrized by the two coefficient functions, $D(x^2)$ and $D_1(x^2)$, as follows:

\[\text{Throughout the present Letter, all the investigations will be performed in the Euclidean space-time.}\]
\[ \langle \langle f_{\mu \nu}(x)f_{\lambda \rho}(0) \rangle \rangle_{A_{\mu}^{2},j_{\mu}^{w}} = \left( \delta_{\mu \lambda} \delta_{\nu \rho} - \delta_{\mu \rho} \delta_{\nu \lambda} \right) D \left( x^2 \right) + \]
\[ + \frac{1}{2} \left[ \partial_{\mu} \left( x_{\lambda} \delta_{\nu \rho} - x_{\nu} \delta_{\lambda \rho} \right) + \partial_{\nu} \left( x_{\mu} \delta_{\lambda \rho} - x_{\lambda} \delta_{\mu \rho} \right) \right] D_{1} \left( x^2 \right). \]

(2)

Here, \( \langle \langle \mathcal{O} \mathcal{O}' \rangle \rangle \equiv \langle \mathcal{O} \mathcal{O}' \rangle - \langle \mathcal{O} \rangle \langle \mathcal{O}' \rangle \), and \( f_{\mu \nu} = \partial_{\mu} A_{\nu}^{2} - \partial_{\nu} A_{\mu}^{2} \). Next, the average \( \langle \ldots \rangle_{A_{\mu}} \) in Eq. (2) is just the standard Gaussian average over free diagonal gluons, whereas \( \langle \ldots \rangle_{j_{\mu}^{w}} \) is a certain average over trajectories of monopole Cooper pairs \( \langle \rangle \), which provides pair condensation. Note that it is the coupling of the dual field \( B_{\mu} \) to the currents of Cooper pairs \( j_{\mu}^{w} \)'s, which yields nonperturbative contents of the functions \( D \) and \( D_{1} \) in the model under study.

In Ref. [3], the following system of equations for the functions \( D \) and \( D_{1} \) has been obtained:

\[ D \left( x^2 \right) = m^2 D_{m}(x) + \left( 4 \pi g_{m} \eta^2 \right)^2 \int d^4 y \int d^4 z D_{m}(x-y) D_{m}(z) \partial^2 g(y-z), \]

(3)

\[ G \left( x^2 \right) = 4 D_{m}(x) + \left( 8 \pi g_{m} \eta^2 \right)^2 \int d^4 y \int d^4 z D_{m}(x-y) D_{m}(z) g(y-z). \]

(4)

Here, \( D_{m}(x) \equiv \frac{m}{4 \pi |x|} K_{1}(m|x|) \) is the propagator of the dual vector boson of the mass \( m \), \( m = 2 g_{m} \eta \), \( K_{1} \)'s henceforth stand for the modified Bessel functions, and \( G \left( x^2 \right) \equiv \int_{x^2}^{+\infty} dt D_{1}(t) \). Finally, in Eqs. (3) and (4), \( g(x) \) denotes a scalar function parametrizing the bilocal correlator of densities of the vortex loops, \( \langle \Sigma_{\mu \nu}(x) \Sigma_{\lambda \rho}(y) \rangle \) (with the average taken over the ensemble of these loops), as follows:

\[ \langle \Sigma_{\mu \nu}(x) \Sigma_{\lambda \rho}(y) \rangle = \varepsilon_{\mu \rho \alpha \beta} \varepsilon_{\lambda \nu \gamma \delta} \partial_{\alpha} \partial_{\beta} g(x-y). \]

Although the tensor structure of this expression is unambiguously fixed by the condition of closeness of the vortex loops, \( \partial_{\mu} \Sigma_{\mu \nu} = 0 \), the form of the function \( g(x) \) depends on the properties of the ensemble of these objects. \( \langle \rangle \) (In particular, in Ref. [3], this function has been found in the framework of a dilute gas model for the ensemble of vortex loops.) As we shall see below, the IR asymptotics of the function \( g(x) \) can be found without any model assumptions, but rather on the basis of the statement that the respective asymptotics of the \( D \)- and \( D_{1} \)-functions have the same fall off, as it is suggested by the lattice data \[ [3, 10, 11, 12, 13]. \]

To proceed with the study of the system (3)-(4), notice that the integrals over \( z \) standing in these equations can be carried out \[ [\text{in Eq. (3)}, \text{by doing firstly the double partial integration}] \] by virtue of the formula \( \int d^4 u D_{m}(x-u) D_{m}(y-u) = \frac{1}{8 \pi^2} K_{0}(m|x-y|) \). \( 1 \) This yields

\[ D \left( x^2 \right) = m^2 D_{m}(x) + \left( 4 \pi g_{m} \eta^2 \right)^2 \int d^4 y \left[ \frac{m^2}{8 \pi^2} K_{0}(m|x-y|) - D_{m}(x-y) \right] g(y), \]

(5)

\[ G \left( x^2 \right) = 4 D_{m}(x) + 8 g_{m}^2 \eta^4 \int d^4 y K_{0}(m|x-y|) g(y). \]

(6)

Differentiating Eq. (3) \( w.r.t. \ x^2 \), one can completely eliminate the \( g(x) \)-dependence from the resulting system of equations without solving them \( w.r.t. \) this function. This yields the following relation between \( D \) and \( D_{1} \), which is thus independent of the properties of the ensemble of vortex loops:

\( ^{\text{Note once more that the classical, i.e. } g(x)\text{-independent, parts of Eqs. (3) and (4) have been found in Ref. [3].}} \)

\( ^{\text{The details of a derivation of a more general formula, where the masses on the L.H.S. are different, can be found in the Appendix to Ref. [3].}} \)
\begin{equation}
\mathcal{D}(x^2) + \mathcal{D}_1(x^2) + \frac{m^2}{4} \int_{x^2}^{\infty} dt D_1(t) = \frac{2}{x^2} D_m(x) + \left( \frac{m}{2\pi|x|} \right)^2 (K_0(m|x|) + K_2(m|x|)).
\end{equation}

By imposing some relation on the IR asymptotics $\mathcal{D}^{as}$ and $\mathcal{D}_1^{as}$ of the functions $\mathcal{D}$ and $\mathcal{D}_1$, one can employ Eq. (7) in order to find these asymptotics explicitly. The simplest relation of this kind can be imposed by disregarding the perturbative-type contributions to $\mathcal{D}^{as}$ and $\mathcal{D}^{as}_1$, which yields $\mathcal{D}^{as}_1 = \alpha \mathcal{D}^{as}$ with $\alpha \simeq 0.3$ \[\text{(11)}\]. Implementing this relation into Eq. (7) and differentiating the IR asymptotics of that equation w.r.t. $x^2$ we get

\begin{equation}
\frac{d\mathcal{D}^{as}(\xi^2)}{d\xi^2} - a \mathcal{D}^{as}(\xi^2) = \frac{1}{\alpha + 1} F(\xi),
\end{equation}

where $F(\xi) \equiv -\frac{m^4}{4\sqrt{2\pi^3/2}|\xi|^7/2} \left(1 + \frac{3}{\xi^2} + \frac{3}{\xi^3} \right) e^{-\xi}$ with $\xi \equiv m|x| \gg 1$ and $a \equiv \frac{\alpha}{4(\alpha+1)}$. The solution to this equation, vanishing at infinity, obviously reads

\begin{equation}
\mathcal{D}^{as}(\xi^2) = \frac{m^4}{4\sqrt{2\pi^3/2}(\alpha + 1)} e^{\alpha \xi^2} \int_{\xi^2}^{\infty} dt \frac{dt}{t^{7/4}} \left(1 + \frac{3}{\sqrt{t}} + \frac{3}{t} \right) e^{-at -\sqrt{t}}.
\end{equation}

The details of calculation of the last integral up to the terms of the order of $1/\xi$ are outlined in the Appendix, and the result has the form

\begin{equation}
\mathcal{D}^{as}(\xi^2) = \frac{2\sqrt{2} a^{3/4} m^4}{\pi^{3/2}(\alpha + 1)} \left[ C + \frac{1}{8a\xi} (3C_1 + 5C_2 + 7C_3) + O \left( \frac{1}{\xi^2} \right) \right] e^{-\xi}.
\end{equation}

The coefficients here read: $C_1 = \frac{1}{21(3/4)}$, $C_2 = \frac{4\sqrt{2}}{15\Gamma(1/4)}$, $C_3 = \frac{4a}{7\Gamma(3/4)}$, $C = C_1 + C_2 + C_3$ with “Γ” denoting the gamma function. Thus we see that the lattice-inspired suggestion that the IR asymptotic behaviours of both functions $\mathcal{D}$ and $\mathcal{D}_1$ are proportional to each other yields for those the exponential fall off at the inverse mass of the dual vector boson times some polynomial in the inverse integer powers of the distance. Such a preexponential behaviour differs from that of the classical calculation \[\text{(8)}\] and the one which was obtained in the dilute gas model of the ensemble of vortex loops \[\text{(9)}\]. Indeed, in that cases the preexponentials were half-integer inverse powers of the distance, which is less favourable from the point of view of the lattice data \[\text{(10)}\], \[\text{(11)}\], \[\text{(12)}\], \[\text{(13)}\], \[\text{(14)}\].

Let us now investigate the stability of the obtained solution \[\text{(11)}\] w.r.t. some possible power-like corrections. Namely, let us insert into Eq. \[\text{(8)}\] the following modified \textit{Ansatz}:

\begin{equation}
\mathcal{D}^{as}_1 \left( \xi^2 \right) = \alpha \left( \xi^2 \right)^{-\lambda} \mathcal{D}^{as} \left( \xi^2 \right)
\end{equation}

with $\lambda \to 0$. In the limit $\xi \gg 1$ under study this leads to the equation

\begin{equation}
\left[ \frac{1}{\alpha} \left( \xi^2 \right)^{-\lambda} + 1 \right] \frac{d\mathcal{D}^{as}_1}{d\xi^2} - \frac{1}{4} \mathcal{D}^{as}_1 = F(\xi),
\end{equation}

or equivalently

\begin{equation}
\frac{d\mathcal{D}^{as}_1}{d\xi^2} - a(1 + \varepsilon) \mathcal{D}^{as}_1 = 4a(1 + \varepsilon) F(\xi),
\end{equation}

with $\varepsilon \to 0$.
where $\varepsilon \equiv \frac{1-(\xi^2)^{\lambda}}{\alpha+1}$. In what follows we assume that $\varepsilon \ll 1$, which is obviously true for sufficiently small $\lambda$, and seek for the solution to Eq. (12) in the form $D_1^{as} = D_1^{as(0)} + \varepsilon D_1^{as(1)}$. Thus, to check the stability of the obtained solution (10), one should find $D_1^{as(1)}$ and prove that it is not dramatically large w.r.t. $D_1^{as(0)}$. The leading term of the latter one reads $B e^{-\xi}$, where $B \equiv \frac{8\sqrt{2}\pi^{7/4}Cm^4}{\pi^{3/2}}$, and for the desired function $D_1^{as(1)}$ we get the following equation:

$$\frac{dD_1^{as(1)}}{d\xi^2} - aD_1^{as(1)} = 4a \left[ F + \frac{1}{4}D_1^{as(0)} \right] \approx aB e^{-\xi}.$$  

This equation is straightforward to be integrated, and the resulting integral can be evaluated in the large-$\xi$ limit by virtue of the known asymptotics for the probability integral [19]: $\frac{2}{\sqrt{\pi}} \int_0^x dt e^{-t^2} \to 1 - \frac{1}{\sqrt{\pi}} e^{-x^2}$ at $x \gg 1$. In this way, we obtain $D_1^{as(1)} = -D_1^{as(0)}$, and thus finally $D_1^{as} = B(1 - \varepsilon)e^{-\xi}$. We conclude that while $\varepsilon \ll 1$, i.e. for small enough $\lambda$, namely $\lambda \ll (\ln \xi^2)^{-1}$, contributions to the obtained solution (11) stemming from the power-like correction (1) are small, and thus this solution is stable.

It is also worth making the following comment. In previous calculations, the functions $D(x^2)$ and $G(x^2)$ were obviously proportional to some propagators. For example, in the classical approximation those were just $D_m$ [see the first terms on the R.H.S.'s of Eqs. (3)-(4)]. However, this fact is not valid any more in the present approach. While this statement is obvious for the $D$-function whose IR asymptotics is given by Eq. (11), some comment is in order for the function $G(x^2)$. Namely, we should check that the leading term in the IR asymptotics of this function differs from the respective asymptotics of any propagator. In this way, we have

$$G(x^2) \left| x \gg m^{-1} \right. \approx \frac{8\sqrt{2}Ca^{7/4}m^4}{\pi^{3/2}} \int_{x^2}^{+\infty} dt e^{-m\sqrt{t}} \approx \frac{16\sqrt{2}Ca^{7/4}m^2}{\pi^{3/2}} \xi e^{-\xi},$$  

which explicitly proves our statement. The fact that in the present approach the functions $D$ and $G$ are not proportional to any (massive) propagator makes it closer to QCD than the previous ones. Indeed, would such a proportionality takes place in QCD, it might be suspicious of the appearance of an asymptotic state carrying colour in some processes involving the bilocal field strength correlator.

Another comment which is in order concerns the comparison of the preexponent in Eq. (11) with the respective value known from the lattice measurements. In this way, we immediately face the following problem. Namely, in our calculation we employed the assumption that $D_\text{as} \sim D_1^{as}$ at $\xi \gg 1$. This relation has been checked on the lattice for the quenched $SU(3)$ QCD in Ref. [10] within the interval of distances between 0.4 fm and 1 fm, which corresponds to the values of $\xi$ in

\[ 4 \] Moreover, assuming that Eqs. (10) and (13) remain valid also for $|x| \leq m^{-1}$, one can explicitly see that the Fourier transforms of these functions possess only cuts rather than poles. Indeed, in that case we have for the leading terms of the $D$- and $G$-functions:

$$D(p^2) \approx \frac{2\sqrt{2}Ca^{3/4}m^4}{\pi^{3/2}(\alpha+1)} \int d^4xe^{-m|x|+ipx} = \frac{24\sqrt{2}\pi Cm^5}{\alpha+1} \frac{2m^2-3p^2}{(p^2+m^2)^{7/2}},$$

$$G(p^2) \approx \frac{16\sqrt{2}Ca^{7/4}m^3}{\pi^{3/2}} \int d^4xe^{-m|x|+ipx} = 192\sqrt{2}\pi C a^{7/4}m^3 \frac{3p^4-24m^2p^2+8m^4}{(p^2+m^2)^{9/2}}.$$
the range between 1.8 and 4.6. However, it is straightforward to see that the \( \frac{1}{x} \)-term on the R.H.S. of Eq. (10) becomes smaller than \( C \) only for \( \xi \geq 5.3 \). This means that the assumption on the proportionality of \( \mathcal{D}^{as} \) and \( \mathcal{D}_1^{as} \), which led to Eq. (11), should be extended to the distances exceeding those which were used up to now in the lattice measurements. Thus, since these measurements yield the preexponential factor at the distances where the next-to-constant terms on the R.H.S. of Eq. (11) are important, the only way to get this factor within our approach is to integrate numerically the exact (rather than the asymptotic) equation for the function \( \mathcal{D} \) down to that distances. Such an equation is given by the formula (8) with the asymptotic function \( F \) replaced by the exact one:

\[
- \frac{m^4}{4\pi^2\xi^3} \left[ \frac{3K_1(\xi)}{\xi^2} + \frac{3}{2\xi}(K_0(\xi) + K_2(\xi)) + \frac{1}{4}(3K_1(\xi) + K_3(\xi)) \right].
\]

By virtue of the MATHEMATICA program, one gets for the quantity \( \mathcal{D}(\xi^2) e^{\xi/m^4} \) at \( |x| = 0.4 \) fm the value 0.03, which differs in one order from the lattice result [11] equal to 0.32(3). Clarification of the origin of this numerical discrepancy requires more precise investigations, which will be performed in future publications.

Finally, by virtue of the obtained results it is also possible to derive the IR asymptotics of the bilocal correlator of densities of the vortex loops, \( i.e. \) the function \( g(x) \). To this end, let us apply the operator \( (\partial^2 - m^2) \) to both sides of Eq. (6) differentiated \( w.r.t. \) \( x^2 \), which yields

\[
g(x) \xrightarrow{|x| \gg m^{-1}} \frac{1}{(2\pi\eta)^2} \left( \frac{\partial^2}{\partial \xi_\mu^2} - 1 \right) \left[ \frac{2}{x^2} D_m(x) + \left( \frac{m}{2\pi|x|} \right)^2 (K_0(\xi) + K_2(\xi)) - \frac{8\sqrt{2}\alpha^{7/4}m^4}{\pi^{3/2}} \left( C + \frac{1}{8\xi}(3C_1 + 5C_2 + 7C_3) + O \left( \frac{1}{\xi^2} \right) \right) e^{-\xi} \right],
\]

where \( \xi_\mu \equiv m x_\mu \). Taking into account that \( \frac{\partial^2}{\partial \xi_\mu^2} f(\xi) = \frac{3}{\xi} f' + f'' \), we finally obtain

\[
g(\xi) \xrightarrow{\xi \gg 1} \frac{8\sqrt{2}\alpha^{7/4}(g m m)^2}{\pi^{1/2}} \left[ \frac{3C}{\xi} + O \left( \frac{1}{\xi^2} \right) \right] e^{-\xi}.
\]

This IR behaviour of the function \( g(x) \) differs from the one obtained in Ref. [3] within the dilute gas model for the ensemble of vortex loops. In particular, the obtained expression does not contain the asymptotics of the massless propagator, which in that case was the origin of a novel nonperturbative \( 1/|x|^4 \)-term in the \( \mathcal{D}_1 \)-function. In fact, this term in the function \( \mathcal{D}_1 \) is now absent, which could be anticipated beforehand owing to the equation \( \mathcal{D}_1^{as} = \alpha \mathcal{D}^{as} \). Clearly, due to this equation, if such a term was present in the function \( \mathcal{D}_1 \), it would unavoidably appear in the function \( \mathcal{D} \) as well. The latter fact would however contradict the general principles of SVM [1, 2], which state that the nonperturbative part of the function \( \mathcal{D} \) cannot contain any \( 1/|x|^4 \)-term.

In conclusion of the present Letter, on the basis of the 't Hooft-Mandelstam scenario of confinement [which suggests that the dual Abelian Higgs model is relevant to the description of confinement in the \( SU(2) \)-QCD] and the lattice result on the bilocal field strength correlator in QCD (which states that the IR asymptotic behaviours of the two coefficient functions parametrizing this correlator in SVM have the same exponential form), we have derived the IR asymptotics of this quantity in the London limit of the dual Abelian Higgs model. In this way, the demand of correspondence with the above-mentioned lattice result enabled not to employ any particular model of the ensemble of vortex loops, present in the theory under study. It rather occurred...
that the proposed approach yielded as a by-product the bilocal correlator of densities of the vortex loops itself. As far as the field strength correlator is concerned, it turned out to decrease exponentially at the inverse mass of the dual vector boson with the preexponential given by a certain polynomial in the inverse powers of the distance. These powers were found to be integer ones, which is in the better agreement with the existing lattice data than the half-integer powers found in the previous calculations. Besides that, the obtained leading terms in the IR asymptotic behaviours of the functions describing the surface and contour exchanges by means of the bilocal field strength correlator have been found to be different from that of the massive propagator. This result is more favourable from the point of view of QCD than the opposite one obtained in the previous approaches.

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Appendix. Calculation of the integral (9)

In this Appendix, we shall present some details of calculation of the integral standing on the R.H.S. of Eq. (9). Let us start with the contribution to this integral brought about by the addendum dominant in the large-$\xi$ limit under study,

\[
\int_{\xi^2}^{+\infty} \frac{dt}{t^{7/4}} e^{-at-\sqrt{t}} = \frac{1}{\Gamma(7/4)} \int_0^{+\infty} d\tau \tau^{3/4} \int_{\xi^2}^{+\infty} dt e^{-t(\tau+a)-\sqrt{\tau}} = \\
\frac{1}{\Gamma(7/4)} \int_0^{+\infty} d\tau \tau^{3/4} \int_0^{+\infty} d\lambda \frac{\lambda}{\sqrt{\pi \lambda}} e^{-\lambda} \int_{\xi^2}^{+\infty} dt e^{-t(\tau+A)} = \frac{1}{\Gamma(7/4)} \int_0^{+\infty} d\lambda \frac{\lambda}{\sqrt{\pi \lambda}} e^{-\lambda} \int_0^{+\infty} d\tau \tau^{3/4} \frac{e^{-(\tau+A)\xi^2}}{\tau+A}, \quad (A.1)
\]

where we have denoted for brevity $a + \frac{1}{4\lambda}$ by $A$. The two other addendums can be treated analogously:

\[
3 \int_{\xi^2}^{+\infty} \frac{dt}{t^{9/4}} e^{-at-\sqrt{t}} = \frac{3}{\Gamma(9/4)} \int_0^{+\infty} d\tau \tau^{5/4} \int_0^{+\infty} d\lambda \frac{\lambda}{\sqrt{\pi \lambda}} e^{-\lambda} \frac{e^{-(\tau+A)\xi^2}}{\tau+A}, \quad (A.2)
\]

\[
3 \int_{\xi^2}^{+\infty} \frac{dt}{t^{11/4}} e^{-at-\sqrt{t}} = \frac{3}{\Gamma(11/4)} \int_0^{+\infty} d\tau \tau^{7/4} \int_0^{+\infty} d\lambda \frac{\lambda}{\sqrt{\pi \lambda}} e^{-\lambda} \frac{e^{-(\tau+A)\xi^2}}{\tau+A}. \quad (A.3)
\]

One can further write down the following dominant contribution to the integral over $\tau$ in Eq. (A.1):
\[
\frac{e^{-A\xi^2}}{A} \int_0^A d\tau \tau^{3/4} + \int_A^\infty d\tau \frac{e^{-\tau\xi^2}}{\tau^{1/4}} = \left[ \frac{4}{7} A^{3/4} + O \left( \frac{1}{\xi^2} \right) \right] e^{-A\xi^2}.
\] (A.4)

Here, we have used the following asymptotics of the incomplete gamma function at large values of its second argument [19]: \[ \Gamma(c, z) = z^{c-1} e^{-z} \left[ 1 + O \left( \frac{1}{z} \right) \right], \quad z \gg 1. \] The integrals over \( \tau \) entering Eqs. (A.2) and (A.3) can be evaluated analogously and read

\[
\left[ \frac{4}{9} A^{5/4} + O \left( \frac{1}{\xi^2} \right) \right] e^{-A\xi^2}
\] (A.5)

and

\[
\left[ \frac{4}{11} A^{7/4} + O \left( \frac{1}{\xi^2} \right) \right] e^{-A\xi^2},
\] (A.6)

respectively. Inserting now Eqs. (A.4)-(A.6) into Eqs. (A.1)-(A.3) and using the formula \( \Gamma(z+1) = z\Gamma(z) \) we get from the original Eq. (9):

\[
D(\xi^2) = \frac{2\sqrt{2}m^4}{\pi^{3/2}(\alpha+1)} \int_0^\infty \frac{d\lambda}{\sqrt{\pi}\lambda} e^{-\lambda} \left[ \frac{1}{21\Gamma(3/4)} \left( a - \frac{\partial}{\partial\xi^2} \right)^{3/4} + \frac{4}{15\Gamma(1/4)} \left( a - \frac{\partial}{\partial\xi^2} \right)^{5/4} \right.
\]

\[
+ \left. \frac{4}{77\Gamma(3/4)} \left( a - \frac{\partial}{\partial\xi^2} \right)^{7/4} + O \left( \frac{1}{\xi^2} \right) \right] e^{-\frac{\xi^2}{4\lambda}} =
\]

\[
= \frac{2\sqrt{2}a^{3/4}m^4}{\pi^{3/2}(\alpha+1)} \left[ C_1 \left( 1 - \frac{1}{a} \frac{\partial}{\partial\xi^2} \right)^{3/4} + C_2 \left( 1 - \frac{1}{a} \frac{\partial}{\partial\xi^2} \right)^{5/4} + C_3 \left( 1 - \frac{1}{a} \frac{\partial}{\partial\xi^2} \right)^{7/4} + O \left( \frac{1}{\xi^2} \right) \right] e^{-\xi^2}. \]

The constants \( C_1, C_2, \) and \( C_3 \) entering this result, are introduced after Eq. (10). Finally, taking into account that \( \left( 1 - \frac{1}{a} \frac{\partial}{\partial\xi^2} \right)^q e^{-\xi^2} = \left[ 1 + \frac{q}{2a\xi} + O \left( \frac{1}{\xi^2} \right) \right] e^{-\xi^2} \), we arrive at Eq. (10) of the main text.
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