Analysis of Behaviour of Prefabricated Staircases with One-Sided Suspended Stairs

Jan Pěnčík¹, Miloš Lavický¹, Pavel Král², Zdeňka Havířová²

1 INTRODUCTION

When connecting two height levels during building of residential houses, the current trend is to use light and airy staircases with attractive and modern design. Staircases are often perceived by architects and end users, i.e. investors, as architecture elements that help to create a visual style and well-being of a modern home (Jiricna, 2001). The right choice of a staircase contributes to elegance, originality, and a unique style of a building (Karre, 2005).
The choice of the construction system of staircases is also related to the choice of material (Habermann, 2002). Nowadays, various material alternatives are combined; for example it is possible to see frequent use of wood with other material, e.g. stainless steel, glass, stone and fibreglass. In addition, there are many examples of use of just a single material, most commonly wood.

Some examples of staircases that meet the mentioned qualities, i.e. elegance, originality, airiness, and unique and modern style, include staircase bolts with inserted treads, or with central staircase bolt, or spiral staircases. The mentioned types of staircases are produced by a wide range of companies in the Czech Republic and in EU countries. The extensive list of companies includes e.g. SWN Moravia, s.r.o., TREPP-ART s.r.o., Bucher GmbH, Kenngott Treppen GmbH, and others.

Within a project of MPO ČR IMPULS, registration number FI-IM2/053 titled “Research and Development of a New Generation of Staircases to Residential and Civil Buildings”, the issue was the construction of wooden prefabricated staircases with one-sided suspended stairs. In accordance with the objectives of the project, a new generation of prefabricated staircases with one-sided suspended stairs was developed in the form of a prototype of a staircase in two versions. The modernized generation of prefabricated staircases improved the universality and variability of the construction system, brought lower costs on production thanks to material saving and simpler production and assembly.

The development of a new generation of staircases and the design of its prototype took advantage of a method of numerical modelling in combination with experimental testing. The combination was also used by other authors (Pousette, 2003; Pousette, 2006; Labans and Kalniņš, 2011; Fleischmann et al., 2005). The method of numerical modelling is used for the issues of construction mechanics, or dynamics, i.e. static analysis, dynamic analysis of analyzed structure or a detail with the use of a finite element method (Tankut et al., 2014). Using the outputs of numerical modelling, an evaluation of an analysed structure can be performed according to standard regulations, and critical construction points identified. These points can be modified and re-analysed thanks to the method of numerical modelling. Subsequently, the first phase of verifying details behaviour with the use of experimental tests of partial testing models will be performed. After verifying the correct design of details, an experimental analysis of the construction should be performed in the second phase, in order to find whether the designed structure complies with the existing standard criteria.

2 MATERIALS AND METHODS
2. MATERIJALI I METODE

2.1 Structure of prefabricated staircases with one-sided suspended stairs
2.1. Konstrukcija montažnega konzolnog stubišta

A prefabricated staircase with one-sided suspended stairs (Fig. 1) consists of stairs without risers, which leads to a lighter construction. The stairs at the side of the wall are usually anchored in the bearing wall with the use of 2 steel bars and partly anchored in the staircase bolt. In order to eliminate footfall sound spreading into bearing walls, the bars are put in rubber cases in the wall. A part from this type of mounting, the mounting used in the design of the staircase prototype can be used as well. At the outer side, the stairs are suspended with the system of bars anchored in a massive handrail. The height position of stairs is delineated with the use of distance elements, which are placed in between stairs on a stair edge. The details of the wooden prefabricated staircase are shown in Fig. 2.

The staircase is predominantly made of glued wooden profiles of European beech (Fagus sylvatica), European white oak (Quercus petrea), Scotch pine (Pinus sylvestris) and spruce (Picea abies), which improve
the shape durability and eliminate the effect of torsion of profiles, in the versions of connected profiles and non-connected profiles. The thickness of profiles ranges between 40 and 65 mm. The non-wooden parts are designed from stainless steel, or surface treated steel.

2.2 Static analysis of behaviour of prefabricated staircases with one-sided suspended stairs

In order to study the behaviour of prefabricated staircases with one-sided suspended stairs, a straight staircase was selected, which represents the most unfavourable arrangement in terms of statics.

Regarding the use of prefabricated staircases for building residential houses, a staircase made from Scotch pine (*Pinus sylvestris*) with the construction height of 3.0 m, aligned span of 4.862 m and ground distance of 3.98 m was considered (Fig. 3a). Dimensions of stairs without risers of 900 mm comply with the requirements of a Czech design standard ČSN 73 4130 for residential houses. The width of stairs at the walking line of 314 mm was designed with the stairs overlap of 10 mm. The thickness of stairs of 50 mm was designed taking into account the existing way of production. The dimensions of the handrail and newels were designed to be made of glued wooden profile 50 × 140 mm.

At the outer side, the stairs were suspended with the use of a system of steel bars (24 pieces) of profile of ø12/2 mm to the bearing massive handrail, which is taken along the outer side of the whole staircase. Each stair was suspended on three bars (Fig. 1) and (Fig. 2) and was connected with the previous and the following stair with the use of wood distance elements (Fig. 1a). At the wall side, the stairs were placed with the use of 2 steel bars ø16 mm that were embedded in the bearing wall through rubber cases (Fig. 2a).

2.3 FE modelling

A static analysis by FEM software systems was performed for loading in compliance with Czech design standard ČSN EN 1991-1-1 (73 0035) Eurocode 1: Action on Structures – Part 1-1: General actions - Densities, self-weight, imposed loads for buildings. Within the analyses dead load was considered, i.e. self-weight and live load, which was considered as uniform load, concentrated load of stairs, and concentrated load acting in vertical and horizontal direction of the handrail. The load was considered in characteristic and design values, which were set in compliance with article 6.10 of Czech design standard ČSN EN 1990 (73 0002) Eurocode: Basis of structural design.

The initial static analysis of behaviour of the selected prefabricated staircase with one-sided suspended stairs and the performance of a standard evaluation of this structure, according to Czech design standard ČSN EN 1995-1-1 (73 1701) Design of timber structures, Part 1-1: General - Common rules and rules for buildings, was performed with the use of a 3D beam analysis model and software IDA NEXIS 32 (2002).

In order to perform a detailed analysis of behaviour of the prefabricated staircase with one-sided suspended stairs, a 3D analysis model and partial analysis models of details (connection of the top and bottom newel with the handrail, a detail of the mounting of a stair on steel bars, detail of the connection of stairs through distance elements) were developed in the software ANSYS (2012a).

A 3D analysis model, where a fixed connection of all construction parts was assumed, was developed with the use of finite elements type of SOLID45, SOLID92, SOLID95 and SURF154 (ANSYS, 2012b). 3D models (Fig. 3) were developed for stairs, rubber cases, connecting screws, screw washers, bars, distance elements, handrail, and top and bottom newels.

Partial analysis models of massive handrail and bottom newel connections (Fig. 3b) using submodelling methods (ANSYS, 2012c) were developed with the use of finite elements of the type of SOLID92 (ANSYS, 2012b). The real glued connection between the handrail and newels was considered for these models. The connections were modelled with contact elements TARGE170 and CONTA174 (ANSYS, 2012b). In this case, the contact elements allowed the contact to
were simple properties in different anatomic directions of wood anisotropic material properties of wood caused by different orientation. This is the reason why the similar properties were considered for the tangential direction $T$ and radial direction $R$ (Požgaj et al., 1997; Kretschmann, 2010; Mascia and Lahr, 2006; Bucur, 2006). The possibility of using an orthotropic material model is related to the method of producing laminated wooden profiles. When producing the laminated wooden profiles, it is possible to clearly define just the longitudinal direction $L$, which is identical to the direction of wood grains. The other anatomic directions of wood cannot be clearly determined due to the different orientation. This is the reason why the similar properties were considered for the tangential $T$ and radial $R$ directions. Regarding the dimensions of the wooden elements, the material characteristics determined for the cylindrical system $LTR$ were used for the Cartesian system $XYZ$ (Danielsson and Gustafsson, 2013), where the material is considered to have similar properties in the direction $Y$ and $Z$.

The behaviour of connecting elements and rubber cases was ideally modelled with the use of an isotropic material model. The steel elements were included in the analysis through material characteristics for steel S235. The isotropic material model of rubber cases was described by modulus of elasticity 10 MPa (2012), density 50 kg/m$^3$ and Poisson’s ratio 0.475.

Boundary conditions concerning the 3D analysis model originated from the real support. The simple support was considered at the contact of the bottom newel to the bearing floor structure. The fixation of the staircase to the bearing ceiling structure was considered with the use of a board under the last stair anchored in the ceiling with three screws. Regarding the rubber cases, boundary conditions were defined to their cylindrical surface in the cylindrical coordinate system while preventing the case face movement out of the wall.

The analyses made with the use of the 3D analysis model and detailed analysis models were materially linear and geometrically nonlinear. The 3D analysis model was loaded in compliance with a Czech design standard ČSN EN 1991-1-1. Due to the use of the method of sub-modelling, the partial analysis models were only loaded by deformation load, which was determined with the use of an analysis of the 3D analysis model, i.e. the load of the partial analysis models was taken over from the output of the 3D analysis model.

After the solution of the 3D analysis model, as well as detailed analysis models, the evaluation of results was performed. The evaluation determined the field of displacement ($U_x$, $U_y$, $U_z$) and field of stress ($S_x$, $S_y$, $S_z$, $S_{xy}$). The vertical displacement $U_z$ and normal stress in the direction of grains for uniformly loaded

---

**Table 1** Material properties of Scotch pine (*Pinus sylvestris*) in notation of ANSYS (ANSYS, 2012a)

| Property | Value       |
|----------|-------------|
| $E_X$, MPa | 14300       |
| $E_Y$, MPa | 545         |
| $E_Z$, MPa | 700         |
| DENS, kg/m$^3$ | 505         |
| $G_{XY}$, MPa | 800         |
| $G_{YZ}$, MPa | 500         |
| $G_{ZX}$, MPa | 1230        |
| $NU_{XY}$ | 0.04        |
| $NU_{YZ}$ | 0.38        |
| $NU_{XZ}$ | 0.03        |
staircase in intensity of uniform serviceability load $V = 3.0 \text{kN/m}^2$ and self-weight is shown in Fig. 4. The average values of normal stress and principal stress in individual wooden construction parts of the staircase were in the range of the interval of the wood strength. The stress was distributed uniformly in the majority of construction parts of the staircase. Using interactive failure criteria Hoffman’s criterion (Hoffman, 1967; Berthelot, 1998; Galicky and Czech, 2013) and Tsai-Wu criterion (Tsai and Wu, 1971; Danielsson and Gustafsson, 2013; Galicky and Czech, 2013) “critical” places of the staircase structure were identified (places of the contact of distance elements with stairs, at places of laying stairs on steel bars, at places of the suspension of stairs with the use of steel bars, and at places of the handrail contact with the top, or bottom, newel).

The “critical” places of the staircase structure were subsequently changed in order to reduce the concentration of stress at places of these details. The designed changes were numerically reanalysed. After their numerical verification, they were integrated in the prototype design of the prefabricated staircase with one-sided suspended stairs (Fig. 5).

3 RESULTS AND DISCUSSION

Based on the results of numerical analyses from the 3D model, a prototype of a wooden straight prefabricated staircase was made from Scotch pine (*Pinus sylvestris*) with one-sided suspended stairs in two versions. The prototype design applied the proposed changes based on the numerical analyses. The changes included the reduction of the number of bars from 24 pieces for the whole staircase (Fig. 5) to 4 pcs, the reduction of the thickness of steps from 50 mm to 40 mm, the change of the anchoring of stairs in the bearing
wall, the modification of solutions of distance elements through improved universality by height rectification, and the change of the contact of the top and bottom newel with the handrail was designed.

The two versions of the prototype of a wood straight prefabricated staircase differed in the way the stairs were fixed into the bearing wall. In the first version marked A, the stairs were supported at the entry edge by a steel profile L 80 × 60 × 8 mm and at the exit edge by a distance element made of stainless steel (Fig. 6a) without being fixed to the bearing wall. In the other version marked B, the stairs were supported at the entry and exit edge by a steel profile L 80 × 60 × 8 mm (Fig. 6b).

In accordance with the selected numerically analysed prefabricated staircase with one-sided suspended stairs, two prototypes of the staircase in versions A and B in the scale 1:1 were made in the testing laboratory of the Institute of Building Testing, Faculty of Civil Engineering, Brno University of Technology. The staircase prototypes consisted of 15 stairs of the length of 900 mm, width of 314 mm and thickness of 40 mm and an atypical exit stair (Fig. 3). The height of both staircases was 3.0 m, the handrail and the entry and exit newels were of a rectangular cross-section 50 × 140 mm. The position of the handrail was secured with the entry and exit newels. The handrail was connected to four stairs No. 4, 7, 10 and 13 with steel bars ⌀12/2 mm, which ran through a stainless steel newel and a height-rectifiable distance element ⌀32/1.85 mm made of stainless steel. The steel profiles L 80 × 60 × 8 mm were fixed to the wall with fixings Fischer FUR 10 × 115 T and screws ⌀7 mm.

The prototypes of staircases were experimentally tested in accordance with ETAG 008 – Guideline for European technical approval of prefabricated stair kits, edition January 2002. The results of tests were used for the verification of the precision and function of the designed modifications. The load tests monitored the response of the structures to the effect of a static load.

Within the experimental tests, the measured time data, i.e. the size of vertical displacement, were continuously recorded. The loading scheme selected at the effect of the static load was chosen so as to model the effects of the uniform serviceability load (V = 3.0 kN/m²) determined on the basis of Czech design standard ČSN EN 1991-1-1. The load was applied on the staircase with the use of loading boxes (Fig. 7). The boxes were placed on the staircase in such order, that the course of the bending moment drew as close to the course of the homogeneous distributed load, i.e. 2nd degree parabola. The reverse action was applied for the unloading.

During the static loading tests, the values of vertical displacements at the stair faces at the selected 12 stairs (Fig. 5a) were continuously recorded with the MS04 with the accuracy of 0.05 mm and the measurement units HBM SPIDER 8 (2006). The measuring points were located in the middle of the width and thickness of the stairs. Two stairs No. 6 and 11 were also equipped with potentiometric trajectory sensors (Fig. 5a) in order to monitor vertical displacements at the bearing wall. The position of the measuring points was selected at the lower side of the stairs in the middle of the stairs 30 mm of the edge.

3.1 Staircase A

3.1. Stubište A

The staircase A was loaded in compliance with Czech design standard ČSN 73 2030 (1994) in two steps. In the first step, the staircase was loaded with uniform serviceability load (V = 3.0 kN/m²), which was increased in the second step by the 0.3 multiple of the uniform serviceability load (Fig. 7). Under the effects of the increased load (1.3 multiple of uniform serviceability load), an extreme value of vertical displacement of 20.66 mm occurred at the stair No. 8, which is lower than the limit value according to (ETAG 008/2002, 2002; ČSN EN 1995-1-1, 2006) amounting to 29.797 mm ((1/200)×L/\cos α; L₅ = 4.862 m (Fig. 5b); α = 35.3281°) (Table 2). The course of vertical displacements measured under gradual loading allows to clearly identify the course of loading and the moments, when the glued contact of the top (or bottom)
When relieving the load, there is a structure response and after the complete unloading, permanent irreversible deformations (notation PD in Table 3) appear amounting to 1.3 multiple of the uniform serviceability load of approx. 13.7%. Under the loading, breaking occurred as well as partial opening of the glued contact of the top (or bottom) newel with the handrail, while the screw contact showed no faults and the joint between the top and bottom newel and stairs was slightly opened and a distance element was partially displaced from the bottom washer. Some anchoring screws from the bearing wall were slightly pulled out at some stairs, which was manifested by the turning of the steel L 80 × 60 × 8 profile. After the load relief of the staircase A, the partially opened glued contact of the bottom and top newel, respectively, with the handrail was closed up.

3.2 Staircase B

The staircase B was loaded with the uniform serviceability load in three steps in compliance with standard ČSN 73 2030 (1994). In the first step, the staircase was loaded with uniform serviceability load ($V = 3.0 \, \text{kN/m}^2$), which was increased in the second step by the 0.3 multiple $V = 3.0 \, \text{kN/m}^2$ increased by the 0.3 multiple.

The values of vertical displacements in individual load steps are shown in Table 3. The graphical time record of vertical displacement of the static load test of the staircase A is shown in Fig. 8.

### Table 2: Maximum measured vertical displacement $U_y$ (mm) for staircase A and B for uniform serviceability load with self-weight compared with theoretical values (ETAG 008/2002, 2002; ČSN EN 1995-1-1, 2006)

| Staircase A / Stubište A | Staircase B / Stubište B |
|--------------------------|--------------------------|
| $U_y \times (1/200) \times L_y / \cos \alpha$ | $U_y \times (1/200) \times L_y / \cos \alpha$ |
| 20.540 < 29.797 | 13.320 < 29.797 |

Condition is satisfied. / Uvjet je zadovoljen.
multiple and in the third step by 0.2 multiple of the uniform serviceability load. Under the effects of the increased load (1.5 multiple of uniform serviceability load), an extreme value of vertical displacement of 13.48 mm occurred at the stair No. 8, which is lower than the limit value according to (ETAG 008/2002, 2002; ČSN EN 1995-1-1, 2006) amounting to 29.797 mm (Table 2).

The second part of the graph (Fig. 9) shows a noticeably sharp rise of vertical displacements caused by the broken contact of the bottom newel with the handrail. The values of vertical displacements in individual load steps are shown in Table 3.

When relieving the load, there is again a noticeable structure response and after the complete load relief, permanent deformations (PD in Table 3) appear amounting to 1.5 multiple of the uniform serviceability load of approx. 12.8 %. Similarly to the situation with staircase A, breaking and partial opening of the glued contact of the top newel with the handrail occurred under this load. In addition, a partial split of the handrail...
appeared. The distance of the handrail and the newel reached approx. 3 mm. Some anchoring screws from the bearing wall were slightly pulled out at some stairs, which was manifested by the turning of the steel L 80 × 60 × 8 profile. After the load relief of the staircase A, the opened glued contact of the bottom newel with the handrail was not closed up.

3.3 Evaluation of static loading tests of staircase A and B

The evaluation of static loading tests of prefabricated staircases with one-sided suspended stairs in versions A and B was performed according to (ETAG 008/2002, 2002; ČSN EN 1995-1-1, 2006). This condition was met by both staircases. Staircase A was loaded by 1.3 multiple of the uniform serviceability load. After applying the same load, the staircase B continued to be loaded up to 1.5 multiple of the uniform serviceability load. The values of the ratio between the permanent and total deformation for staircases A and B are lower than the coefficient λ1, which, according to Section D. 8 (ČSN 73 2030, 1994), amounts to 0.25 and 25 %, respectively, for glued structures. Both staircases met these criteria of reliability in terms of ultimate limit state.

The comparison of experimentally measured values of vertical displacements for comparable load shown in Table 3 for 1.0 multiple and 1.3 multiple of uniform serviceability load shows that staircase B is stiffer than staircase A.

4 CONCLUSION

4. ZAKLJUČAK

The comparison of the measured data showed that the prototype of staircase B is stiffer and more resistant to the applied load than the prototype of staircase A. Regarding statics, this finding indicates that supporting stairs with two steel profiles L 80 × 60 × 8 mm is more advantageous than the combination of a steel profile and a rectifiable stainless steel distance element.

During a static loading test, staircases A and B were loaded by their own weight and then by uniform serviceability load of the intensity of \( V = 3.0 \text{ kN/m}^2 \). Subsequently, the load of staircases A and B was increased up to 1.3 multiple for staircase A and up to 1.5 multiple of the load for staircase B. Even under the higher load, the vertical displacements of selected measuring points at staircase B were lower than those at staircase A. Under the effects of increased loading, which models the ultimate limit state, the staircase structure showed no serious faults and deficiencies. It should be emphasised that the pressing of newel washers into stairs occurred as well as opening of the contact between the bottom newel and handrail for staircases A and B, and opening of the contact between the top newel and handrail for staircase B. Despite these slight faults, the structures of staircases were reliable, which was documented by subsequently performed loading tests of broken and opened contacts.

Staircases A and B were evaluated in accordance with (ETAG 008/2002, 2002; ČSN EN 1995-1-1, 2006; ČSN 73 2030, 1994) in terms of ultimate and serviceability limit state. Both staircases A and B met the required criteria of the mentioned regulations.

Based on the behaviour of staircases in the course of loading tests, it was recommended to increase ultimate limit state of staircases structure and their general stiffness by changes in the detail of the contact of the top and bottom newel with handrail, and the detail of the passage of the exit stair through the top newel.

Acknowledgements - Zahvala

The experimental testing of prefabricated staircases with one-sided suspended stairs has been financially supported by the project of MPO ČR IMPULS, registration number FI-IM2/053. The article is supported by a research project FAST-S-15-2757 from the Internal Grant Agency, Brno University of Brno, Brno and by the European Social Fund and the state budget of the Czech Republic, project “The Establishment of an International Research Team for the Development of New Wood-based Materials” reg. no. CZ.1.07/2.3.00/20.0269. The authors also acknowledge the contribution of participating laboratories.

5 REFERENCES

1. Berthelot, J. M., 1998: Composite Materials, Mechanical Behavior and Structural Analysis. Berlin: Springer.
2. Bucur, V., 2006: Acoustics of Wood. Berlin: Springer.
3. Danielsson, H.; Gustafsson, P. J., 2013: A three-dimensional plasticity model for perpendicular to grain cohesive fracture in wood. Engineering Fracture mechanics, 98: 137-152. http://dx.doi.org/10.1016/j.engfracmech.2012.12.008
4. Fleischmann, M.; Müllner, H. W.; Krenn, H., Eberhardsteiner, J., 2005: Experimental and Numerical Investigation of timber Structures for the Validation of an Orthotropic Plasticity Model. 22nd Danubia-Adria Symposium on Experimental Methods in Solid Mechanics (DAS-22), Monticelli Terme - Parma, Italy.
5. Franke, B.; Quenneville, P. J., 2011: Numerical Modeling of the Failure Behavior of Dowel Connections in Wood. Journal of Engineering Mechanics. 137(3): 186-195. http://dx.doi.org/10.1061/(ASCE)EM.1943-7889.0000217
6. Galicky, J.; Czech, M., 2013: A new approach to formulate the general strength theories for anisotropic discontinuous materials. Part A: The experimental base for a new approach to formulate the general strength theories for anisotropic materials on basis of wood. Applied Mathematical Modelling, 37(3): 815-827. http://dx.doi.org/10.1016/j.apm.2012.03.004
7. Habermann, K. J., 2002: Staircases: Design and Construction. Basel, Boston, Berlin: Birkhäuser.
8. Hoffman, O., 1967: The Brittle Strength of Orthotropic Materials. Journal of Composite Materials, 1: 200-206. http://dx.doi.org/10.1177/00219836700100210
9. Jirina, E., 2001: Staircases. New York, NY: Watson-Guptill Publications.
10. Karre, A., 2005: Stairscaping: A Guide to Buying, Remodeling, and Decorating Interior and Exterior Staircases. Massachusetts: Quarry Books.

11. Kretschmann, D. E., 2010: Wood Handbook, Chapter 05: Mechanical Properties of Wood. General Technical Report FPL-GTR-190. Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory: 5-1-5-46.

12. Labans, E.; Kalniņš, K., 2012: Numerical Modelling and Experimental Validation of Dendrolight Cellular Wood Material. In: The 8th Meeting “Northern European Network for Wood Science and Engineering (WSE)”: Proceedings: The 8th Meeting of the Northern European Network for Wood Science and Engineering (WSE), Lithuania, Kaunas, 13-14 September, 2012. Kaunas: 2012, pp.177-184. (online: http://www.nordicforestresearch.org/wp-content/uploads/2011/03/26_Edgars_Labans.pdf)

13. Matovič, A., 1993: Fyzikální a mechanické vlastnosti dřeva a materiálů na bázi dřeva, Brno: Vysoká škola zemědělská.

14. Mascia, N. T.; Lahr, F. A. R., 2006: Remarks on orthotropic elastic models applied to wood. Materials Research, 9 (3): 301-310. http://dx.doi.org/10.1590/S1516-14392006000300010

15. Pencík, J.; Lavický, M., 2006: Možnosti modelování lepených dřevěných spojů pomocí prvků CONTA17x. (Modeling capabilities of glued wood joints with contact elements CONTA17x) 14. ANSYS Users Meeting pro Českou republiku a Slovensko 2006, Tábor. Tábor: SVS FEM s.r.o., pp. 1-9.

16. Pousette, A., 2003: Full-scale test and finite element analysis of a wooden spiral staircase. Holz als Roh- und Werkstoff, 61(1): 1-7. http://dx.doi.org/10.1007/s00107-002-0345-6

17. Pousette, A., 2006: Testing and modeling of the behavior of wooden stairs and stair joints. J Wood Sci, 52: 358-362. http://dx.doi.org/10.1007/s10086-005-0778-8

18. Požgaj, A.; Chovanec, D.; Kurjatko, S.; Babiak, M., 1997: Štruktura a vlastnosti dřeva, Priroda, Bratislava.

19. Tankut, N.; Tankut, A. N.; Zor, M., 2014: Finite Element Analysis of Wood Materials. Drvna industrija, 65 (2): 159-171. http://dx.doi.org/10.5552/drind.2014.1254

20. ***2004: “Fyzikální a mechanické vlastnosti dřeva” (online), Mendel University in Brno, wood.mendelu.cz/cz/sections/Props/?q-node/56. First published 2004 (Accessed Aug. 1, 2013).

21. ***2006: “Spider8 from HBM” (online), HBM Inc. http://www.hbm.com/fileadmin/mediapool/hbmdoc/technical/b0409.pdf. First published 2006 (Accessed May 20, 2013).

22. ***2012: “Elastic Properties and Young Modulus for some Materials” (online), The Engineering Toolbox, www.engineeringtoolbox.com/young-modulus-d.417.html. First published 2012 (Accessed May 15, 2013).

23. *** ANSYS® Academic Research, Release 14.5, ANSYS, Inc., 2012a.

24. *** ANSYS® Academic Research, Release 14.5, Help System, Elements Reference, ANSYS, Inc., 2012b.

25. *** ANSYS® Academic Research, Release 14.0, Help System, Advanced Analysis Techniques Guide, ANSYS, Inc., 2012c.

26. *** ČSN 49 1531-1, 1998: Structural timber - Part 1: Visual strength grading.

27. *** ČSN 73 2030, 1994: Loading tests of building structures Common regulations.

28. *** ČSN 73 4130, 2010: Stairways and sliding ramps – Basic requirements.

29. *** ČSN EN 1990 (73 0002) Eurocode, 2004: Basis of structural design.

30. *** ČSN EN 1991-1-1 (73 0035) Eurocode 1, 2004: Action on Structures – Part 1-1: General actions – Densities, self-weight, imposed loads for buildings.

31. *** ČSN EN 1995-1-1 (73 1701), 2006: Design of timber structures, Part 1-1: General - Common rules and rules for buildings.

32. *** ETAG 008/2002, 2002: Guideline for European technical approval of prefabricated stair kits. Prefabricated stair kits in general.

33. *** IDA NEXIS 32, 2002: SCIA Group.

34. *** JEMA Svitavy a.s., 2006: Wooden staircase. Product catalog.

Corresponding address:
Assist. Prof. Ing. JAN PĚNČÍK, Ph.D.
Institute of Building Structures
Faculty of Civil Engineering
Brno University of Technology
CZECH REPUBLIC
e-mail: pencik.j@fce.vutbr.cz