An Improved Particle Swarm Algorithm with Heuristic Factors and Empirical Operators

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Abstract. An improved particle swarm algorithm (PSA) is proposed based on the universal framework of the intelligence optimization algorithms (IOA). Regarded as increment to the standard PSA, the improved PSA takes the essence of the standard PSA, and additionally utilizes various heuristic factors and different empirical operators introduced by multiple other IOA. The capability of global and local optimization can be strengthened by the free search, the differential evolution and the quantum rotation comprehensively. The genetic variation is adopted to avoid premature convergence, while the greed strategy is adopted to guarantee monotonous convergence. The numerical simulation indicates that the improved PSA achieves better performance than the standard PSA.

1. Introduction

It is the field of IOA that has achieved rapid growth in the past few decades. IOA can deal with the problems which is difficult to be solved by those classical optimization methods such as gradient descent. The most attractive advantages of IOA include concise calculation, parallel pipeline, universal robustness and so on [1].

Given the following optimization problem:

\[
\begin{align*}
\arg \min_{X} & \quad f_{opt}(X) \\
\text{s.t.} & \quad X \in \mathbb{S} = [B_{1}^{L}, B_{1}^{U}] \times \cdots \times [B_{d}^{L}, B_{d}^{U}]
\end{align*}
\]

The symbol \( f_{opt} \) in equation (1) is the objective function, \( \mathbb{S} \) is the feasible region, and \( D_{S} \) is the dimension of \( \mathbb{S} \). \( B_{i}^{L} \) and \( B_{i}^{U} \) are the lower and upper bounds of the \( d \) -dimension \( (1 \leq d \leq D_{S}) \), respectively.

When adopting IOA to solve problems, neither the analytical properties such as continuity, differentiability, and integrability was essential to \( f_{opt} \), nor the topological properties such as connectivity, completeness and convexity was requisite to \( \mathbb{S} \). For the convenience of mathematics, let \( G \) be the maximum iterations. For \( g \) -iteration \( (1 \leq g \leq G) \), \( P_{g} = \{ f_{p,g} \mid p = 1, \cdots, N_{p} \} \) is the \( g \) -
population, where \( I_{p,g} \) is the \( p \)-individuality of \( P_g \), and \( N_g \) is the number of individuality. \( P_{g+1} \) is generated by \( P_g \). The spacial field \( X_{p,g} = (X_{1,p,g}, \ldots, X_{D_{p,g},g}) \) of \( I_{p,g} \) corresponds to the coordinate of \( I_{p,g} \) in \( \mathbb{S} \). \( \epsilon \) is the tolerate error.

The universal framework of IOA can be summarized as following procedures: [2]

1) Generate \( P_1 \) randomly in the range of \( \mathbb{S} \);
2) During \( g \)-iteration, for each \( I_{p,g} \), compute the increment \( \Delta_{p,g} \) by predetermined rules;
3) If \( \min_{p \in [1, \ldots, N_g]} f_{opt}(X_{p,g}) \leq f_{opt} + \epsilon \) or \( g \geq G \), stop; else, go to step 2.

\( X_s \) is the ultimate solution satisfying:

\[
f_{opt}(X_s) = \min_{P_g \in \mathbb{N}} f_{opt}(X_{p,g}).
\] (2)

The availability of \( X_s \) can be judged by the condition:

\[
f_{opt}(X_s) < f_{opt} + \epsilon.
\] (3)

The necessary and sufficient conditions about the convergence almost everywhere of \( \{P_g \mid g \in \mathbb{N}\} \) under the above framework has been investigated.

Regarded as increment to the previous literature, this paper mainly focuses on PSA, which is one typical IOA [3]. The rest of the paper is organized as follows. Section 2 presents the theoretical expressions of our improved PSA. In section 3, numerical simulation is performed, followed by conclusions in section 4.

2. Improved PSA

In this section, an improved PSA is proposed, which not only inherits the mechanism of the standard PSA, but also adopts multiple concepts introduced by other famous IOA. The consciousness proposed in the free search is adopted to balance the global and local optimization strategy [4]. The differential evolution and quantum rotation are adopted to improve the performance of global and local optimization, respectively [5, 6]. The genetic variation is adopted to avoid premature convergence [7]. The greed strategy is adopted to guarantee the monotonicity of convergence.

The increment of standard PSA: [3]

\[
\Delta_{p,g} = \omega \Delta_{p,g-1} + c_1 u_1 (X_{p,\text{pbest}} - X_{p,g}) + c_2 u_2 (X_{g,\text{best}} - X_{p,g}).
\] (4)

\( \omega \geq 0 \) is the inertia factor. \( c_1 \in [0, 4] \) and \( c_2 \in [0, 4] \) are emulate factor. \( u_1 \) and \( u_2 \) follow the standard uniform distribution, respectively. \( X_{p,\text{pbest}} \) is equal to \( I_{X_{p,g}} \) which corresponds to \( \min_{g \in [1, \ldots, G]} f_{opt}(X_{p,g}) \), and \( X_{g,\text{best}} \) is the optimal solution after \( g \)-iteration.

In our improved PSA, \( I_{p,g} = (X_{p,g}, Y_{p,g})^T \) consists of \( X_{p,g} \) and the extensional field:

\[
Y_{p,g} = (y_{1,g}, y_{2,g}, y_{3,g}, y_{4,g}, y_{5,g}, y_{6,g}, y_{7,g}).
\] (5)

The extensional field includes multiple heuristic factors. \( y_{1,g} \) is the adaptability factor:
\[ y_{1,g} = \frac{f_{opt}(X_{p,g}) - \min_{p \in [1, \ldots, N_p]} f_{opt}(X_{p,g})}{\max_{p \in [1, \ldots, N_p]} f_{opt}(X_{p,g}) - \min_{p \in [1, \ldots, N_p]} f_{opt}(X_{p,g})}. \]  

\[ y_{2,g} \] is the consciousness factor of free search, and follows the standard uniform distribution. \[ y_{3,g} \] is the weight factor:
\[
y_{3,g} = \begin{cases} 
\frac{\pi}{4} & g = 1, \\
y_{3,g-1} + y_{4,g}, & g > 1.
\end{cases}
\]

\[ y_{4,g} = \sum_{d=1}^{D_g} \arcsin \left( \frac{x_{d,g}^p - B_d^L}{B_d^U - B_d^L} \right) \] is the update factor. \[ y_{5,g} \sim \text{Beta}(\mu_{y_{5,g}}, \mu_{y_{5,g}}) \] is the scale factor, and follows the Beta distribution [8]. \[ y_{6,g} \sim \text{Tikhonov}(\mu_{y_{6,g}}, 1) \] is the threshold factor, and follows the Tikhonov distribution [8]. The undetermined parameter of the Beta distribution is:
\[
\mu_{y_{5,g}} = \begin{cases} 
0.5, & g = 1, \\
(1-c)\times\mu_{y_{5,g-1}} + c\times\frac{\sum_{p \in P_{y_{5,g-1}}}(y_{5,g-1})^2}{y_{5,g-1}}, & g > 1.
\end{cases}
\]

And the undetermined parameter of the Tikhonov distribution is:
\[
\mu_{y_{6,g}} = \begin{cases} 
0, & g = 1, \\
(1-c)\times\mu_{y_{6,g-1}} + c\times\frac{\sum_{p \in P_{y_{6,g-1}}}(y_{6,g-1})^2}{y_{6,g-1}}, & g > 1.
\end{cases}
\]

c and \( \delta \) are the shrink coefficient and the proportion coefficient, respectively. \( P_{y_{g-1}} \) stands for the set of these top \( \delta \% \) \( I_{p,g} \) of \( P_k \). The typical values of \( c \) and \( \delta \) usually range from 0.05 to 0.2 and from 5 to 20, respectively. Their values are insensitive to the performance of the optimization algorithm when solving different optimization problems [9]. \[ y_{7,g} = \left[ 2^{D_h} \times y_{2,g} \right] \] is the intersect factor, where \( \left[ \cdot \right] \) stands for rounding to the nearest integer less than or equal to the input element. \( \overline{b_{D_1} \cdots b_1}_2 = \sum_{d=1}^{D_g} 2^{d-1} \times b_d = y_{7,g} \) is the binary number of \( y_{7,g} \), where \( \left[ \cdot \right]_2 \) stands for the binary system. The \( d \)-bit of \( \overline{b_{D_1} \cdots b_1}_2 \) is \( b_d \in \{0,1\} \).

In our improved PSA, \( I_{p,g} \) can dynamically adjust to the global and local optimization strategy by \( y_{1,g} \) and \( y_{2,g} \). On one hand, a lower (higher) value of \( y_{1,g} \) will lead to a higher (lower) probability of local (global) optimization strategy. On the other hand, let the adjust function be:
\[ f_a(g) = \sqrt{\frac{g}{G}}. \] (10)

It is helpful to increase the probability of local (global) optimization strategy in the later (early) stage of the iterations. If \( y_{1,g}^p \geq y_{2,g}^p \) or \( y_{2,g}^p \geq f_a(g) \), \( I_{p,g} \) adjust to the global optimization strategy, while if \( y_{1,g}^p < y_{2,g}^p \) and \( y_{2,g}^p < f_a(g) \), \( I_{p,g} \) adjust to the local optimization strategy. This adjustment can ensure the ergodicity of \( P_g \) for \( S \).

For global optimization strategy, \( X_{p',g} \) can be generated by the global displacement operator based on the quantum rotation:

\[ X_{p',g} = X_{p,g} + \Delta_{p,g} + \text{max}(\sin^2 y_{3,g}^p, \cos^2 y_{3,g}^p) \times \hat{X}_{p,g}. \] (11)

And for local optimization strategy, \( X_{p',g} \) can be generated by the local displacement operator based on the differential evolution:

\[ X_{p',g} = X_{p,g} + \Delta_{p,g} + y_{3,g}^p \times (X_{pS,g} - X_{p,g}). \] (12)

\( I_{pS,g} \in P_{g} \) and \( \hat{X}_{p,g} \in S \) are both selected randomly. If certain component \( x_{d',g}^p \) of \( X_{p',g} \) exceeds \([B_d^L, B_d^U] \), the truncation operator is activated:

\[ x_{d',g}^p = \begin{cases} B_d^L, & x_{d',g}^p < B_d^L, \\ x_{d',g}^p, & B_d^L \leq x_{d',g}^p \leq B_d^U, \\ B_d^U, & x_{d',g}^p > B_d^U. \end{cases} \] (13)

Based on equation (11) and equation (12), the step size of the local displacement operator is smaller, because it takes the neighbourhood of \( X_{p,g} \) into account. Each component of \( X_{p',g} \) can be generated by the intersection operator based on the genetic variation:

\[ x_{d',g}^p = \begin{cases} x_{d',g}^p, & y_{6,g}^p < 0 \land b_d = 1, \\ x_{d',g}^p, & y_{6,g}^p > 0 \land b_d = 0. \end{cases} \] (14)

And \( X_{p,g+1} \) can be generated by the selection operator based on the greed strategy:

\[ X_{p,g+1} = \begin{cases} X_{p',g}, & f_{opt}(X_{p',g}) \leq f_{opt}(X_{p,g}), \\ X_{p,g}, & f_{opt}(X_{p',g}) > f_{opt}(X_{p,g}). \end{cases} \] (15)

After \( g \)-iteration, update \( Y_{p,g+1} \) for next iteration.

3. Simulation

In this section, \( \omega \), \( c_1 \), and \( c_2 \) in equation (4) adopt the default values of Matlab R2016a. Other parameters relevant to the simulation include: \( c = 0.1 \), \( \delta = 20 \), \( \epsilon = 10^{-6} \), and \( G = 1000 \). When \( D_S = 5 \), \( N_p = 50 \). When \( D_S = 10 \), \( N_p = 100 \).
To evaluate the performance of our improved PSA, four classical test functions are adopted, which are listed in table 1. $f_1$ is the Ridge function with inseparable independent variables. $f_2$ is the Step function with discontinuous dependent variable. $f_3$ is the Rastrigin function with multimodal points. $f_4$ is the Quartic function with stochastic noise [1].

**Table 1. Test functions**

| $f_{opt}$ | $f_{min}$ | Optimum |
|-----------|-----------|----------|
| $f_1(x) = \sum_{i=1}^{D_x} \left( \sum_{j=1}^{D_x} x_j \right)^2$ | $[-100,100]^{D_x}$ | 0 ($0, \ldots, 0$) |
| $f_2(x) = \sum_{i=1}^{D_x} \left| x_i + 0.5 \right|^2$ | $[-100,100]^{D_x}$ | 0 ($0, \ldots, 0$) |
| $f_3(x) = \sum_{i=1}^{D_x} \left[ x_i^2 - 10 \cos (2\pi x_i) + 10 \right]$ | $[-5.12, 5.12]^{D_x}$ | 0 ($0, \ldots, 0$) |
| $f_4(x) = \sum_{i=1}^{D_x} i x_i^4 + \text{rand}(0, 1)$ | $[-1.28, 1.28]^{D_x}$ | 0 ($0, \ldots, 0$) |

**Table 2. Simulation results (D_S = 5)**

| $f_{opt}$ | PSA | $f_{opt}(X_{opt})$ | $\mu f_{opt}(X_{opt})$ | $\sigma f_{opt}(X_{opt})$ | Availability Rate (%) | Average Iterations |
|-----------|-----|--------------------|------------------------|--------------------------|-----------------------|-------------------|
| $f_1$     | Standard | 7.74e-08 | 5.55e-07 | 2.68e-07 | 98 | 93 |
| $f_1$     | Improved | 2.41e-09 | 9.87e-08 | 1.53e-07 | 100 | 173 |
| $f_2$     | Standard | 0.00e-00 | 2.00e-02 | 1.41e-01 | 98 | 43 |
| $f_2$     | Improved | 0.00e-00 | 0.00e-00 | 0.00e-00 | 100 | 28 |
| $f_3$     | Standard | 3.10e-11 | 7.50e-01 | 7.47e-01 | 42 | 127 |
| $f_3$     | Improved | 1.28e-13 | 3.63e-07 | 3.21e-07 | 100 | 182 |
| $f_4$     | Standard | 2.71e-17 | 5.01e-11 | 1.99e-10 | 100 | 38 |
| $f_4$     | Improved | 2.22e-12 | 1.82e-07 | 2.60e-07 | 100 | 20 |

**Table 3. Simulation results (D_S = 10)**

| $f_{opt}$ | PSA | $f_{opt}(X_{opt})$ | $\mu f_{opt}(X_{opt})$ | $\sigma f_{opt}(X_{opt})$ | Availability Rate (%) | Average Iterations |
|-----------|-----|--------------------|------------------------|--------------------------|-----------------------|-------------------|
| $f_1$     | Standard | 3.19e-07 | 8.61e-07 | 3.30e-07 | 64 | 244 |
| $f_1$     | Improved | 2.76e-08 | 1.15e-06 | 1.12e-06 | 100 | 355 |
| $f_2$     | Standard | 0.00e-00 | 0.00e-00 | 0.00e-00 | 100 | 65 |
| $f_2$     | Improved | 0.00e-00 | 0.00e-00 | 0.00e-00 | 100 | 43 |
| $f_3$     | Standard | 9.95e-08 | 7.40e-00 | 3.75e-00 | 20 | 172 |
| $f_3$     | Improved | 3.35e-10 | 3.49e-07 | 2.76e-07 | 100 | 288 |
| $f_4$     | Standard | 7.53e-12 | 2.35e-09 | 1.16e-08 | 100 | 61 |
| $f_4$     | Improved | 5.85e-17 | 1.66e-07 | 2.01e-07 | 100 | 32 |
To initialize our improved PSA and the standard PSA, $I_{p,1}$ are randomly distributed in $S = [B_{L}^{L}, B_{R}^{L}] \times \cdots \times [B_{D}^{L}, B_{D}^{R}]$. Besides, assign values to those coefficients such as $c$, $\delta$ and so on. Thus, $Y_{p,1}$ corresponding to $X_{p,1}$ can be computed by relevant equations, and iterations begin.

The optimization conducted to these four test functions repeats 100 times independently for each configuration of simulation. Table 2 and table 3 list the statistical information of the numerical simulation for $D_{L} = 5$ and $D_{S} = 10$, respectively. In table 2 and table 3, $\min f_{\text{opt}}(X)$, $\mu f_{\text{opt}}(X)$ and $\sigma f_{\text{opt}}(X)$ are the minimum, the average and the standard deviation of 100 simulation results, respectively. The average iterations is the average of $g$ when iterations stops. The numerical simulation indicates that better performance of our improved PSA is achieved compared with the standard PSA. Our improved PSA is capable of rapidly converging to the global optimum with higher availability rate.

4. Conclusion
This paper investigates an improved PSA under the universal procedures of the IOA. Our improved PSA retains the essential rules of the standard PSA to generate the increment during the iterations. Simultaneously, various heuristic factors and different empirical operators introduced by multiple IOA are adopted in our improved PSA. The consciousness proposed in the free search is adopted to balance the global and local optimization strategy. The differential evolution and quantum rotation are adopted to improve the performance of global and local optimization, respectively. The premature convergence can be avoided by the genetic variation, whereas the monotonous convergence can be guaranteed by the greed strategy. Four classical test functions are chosen to evaluate the feasibility and applicability of our improved PSA. The numerical simulation indicates that our improved PSA is capable of rapidly converging to the global optimum with higher availability rate than the standard PSA.

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