Dark Energy and the mass of galaxy clusters

Cosimo Bambi
Istituto Nazionale di Fisica Nucleare, Sezione di Ferrara, I-44100 Ferrara, Italy
Dipartimento di Fisica, Università degli Studi di Ferrara, I-44100 Ferrara, Italy
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Up to now, Dark Energy evidences are based on the dynamics of the universe on very large scales, above 1 Gpc. Assuming it continues to behave like a cosmological constant Λ on much smaller scales, I discuss its effects on the motion of non-relativistic test-particles in a weak gravitational field and I propose a way to detect evidences of Λ ≠ 0 at the scale of about 1 Mpc: the main ingredient is the measurement of galaxy cluster masses.

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1. INTRODUCTION

Present observational data suggest that about 70% of the energy in the universe is made of a mysterious substance, the so-called Dark Energy, which would be unable to form structures and whose energy density would be constant in space and time. However, these conclusions are essentially based on the dynamics of the universe on very large scales, above 1 Gpc.

In the simplest case, Dark Energy would be the cosmological constant, maybe somehow related to the vacuum energy, and hence uniform on macroscopic distances. According to other proposals, it could instead be the energy of some new weakly interacting field, so it may not have an exactly homogeneous and isotropic distribution and it may be capable of clustering, even if not like standard matter. It would be therefore very important to observe Dark Energy effects on smaller scales, in order to distinguish different pictures and reject unsuccessful Dark Energy candidates. This becomes even more relevant by the light of the possibility that Dark Energy does not exist and that the accelerated expansion rate of the universe can be explained with standard physics or with modifications of General Relativity.

Effects of a non-null cosmological constant Λ on the Solar System have been considered and upper bounds on its local value have been deduced, but they are far from the value we get from cosmological observations.

In this paper I discuss the effects of a non zero cosmological constant in the “Newtonian limit”, i.e. under the approximations of slow motion and weak gravitational field. In particular, I focus the attention on the dynamics of systems such as galaxy clusters, which could be used to probe distances of about 1 Mpc, and I show the possibility of observing Dark Energy evidences by precise measurements of their gravitational masses. The existence of a cosmological constant at this relatively small scale could also be checked observing the “local expansion rate” of the universe (as opposed to the “global expansion rate”), as suggested in Ref.

The content of the paper is as follows. In the next section I derive the effective gravitational force acting on a non-relativistic test-particle in the case of a non zero cosmological constant and in Section 2 I consider general features and implications. In Section 3 I discuss cosmological constant effects on the measurements of galaxy cluster masses and I show the possibility of getting evidences of Λ at the scale of about 1 Mpc. In Section 4 there are final remarks and conclusion.

2. NEWTONIAN LIMIT

The Kottler spacetime, also known as Schwarzschild-de Sitter spacetime, is the unique spherically symmetric solution of Einstein’s vacuum field equation with a cosmological constant Λ. In static and spherically symmetric coordinates, the line element is

\[ ds^2 = A(r)dt^2 - \frac{dr^2}{A(r)} - r^2(d\theta^2 + \sin^2 \theta d\phi^2) \]

where

\[ A(r) = 1 - \frac{2G_NM}{r} - \frac{\Lambda}{3}r^2 \]

and \( M \) is the mass of the source. Eq. (1) reduces to the standard Schwarzschild metric for \( \Lambda = 0 \) and to the de Sitter one for \( M = 0 \); in the latter case the coordinates are not the ones often used in cosmology, where the expansion of the spacetime is explicit.

The gravitational force acting on a test-particle in the Newtonian limit can be deduced as follows (see e.g. Ref. 11). First, we write the geodesic equation, which describes the motion of a test-particle in a background gravitational field

\[ \frac{d^2x^\sigma}{d\lambda^2} + \Gamma^\sigma_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0. \]

Here and in the following, Greek letters \( \mu, \nu, \ldots \) (\( \mu = 0, 1, 2, 3 \)) denote spacetime indices, Latin letters \( i, j, \ldots \) (\( i = 1, 2, 3 \)) denote space indices and \( \lambda \) is an affine parameter. In the “Newtonian limit” we assume that the
motion of the test-particle is slow and that the gravitational field is weak. The first hypothesis means
\[ \frac{dt}{d\lambda} \gg \frac{dx^i}{d\lambda}, \]
while the assumption of weak gravitational field lets us write the metric tensor as \( g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \), where \( \eta_{\mu\nu} \) is the Minkowski metric and \( h_{\mu\nu} \) a small perturbation. To first order in \( dx^i/d\lambda \) and \( h_{\mu\nu} \) and for time independent metrics, like the one in Eq. (1), the geodesic equation can be written as
\[ \frac{d^2 \mathbf{r}}{dt^2} = -\frac{1}{2} \nabla h_{tt}, \]
where \( \mathbf{r} \) is the flat position 3-vector of the test-particle and \( \nabla \) is the flat nabla operator. In the case of the Kottler metric, Eq. (5) becomes
\[ \frac{d^2 \mathbf{r}}{dt^2} = \left( \frac{G_N M}{r^2} + \frac{\Lambda}{3} \right) \frac{\mathbf{r}}{r}. \]

Eq. (6) is the non-relativistic acceleration acting on the test-particle and consists of the standard Newtonian term, which goes like \( 1/r^2 \), plus a correction proportional to \( r \), due to the cosmological constant \( \Lambda \). This equation can also be written as
\[ \frac{d^2 \mathbf{r}}{dt^2} = -\frac{G_N M_{\text{eff}}}{r^2} \frac{\mathbf{r}}{r}, \]
where \( M_{\text{eff}} \) is the effective Newtonian mass enclosed within the radius \( r \)
\[ M_{\text{eff}}(r) = M - \frac{8}{3} \pi r^3 \rho_\Lambda. \]
and \( \rho_\Lambda \) is the energy density associated with the cosmological constant: \( \Lambda = 8\pi G_N \rho_\Lambda \). Since \( M_{\text{eff}} \) depends on the distance from the source, for \( \Lambda \neq 0 \) the test-particle feels an effective violation of standard gravitational inverse square law. As the matter of the fact, the new force is an inertial effect, related to the choice of the coordinate system. This becomes more evident in the limit \( M = 0 \), where the test-particle continues to feel the force proportional to its distance from the origin of the coordinate system, but where the latter is a point like all the others; the phenomenon disappears in a comoving reference frame.

3. GENERAL FEATURES

As we have seen in the previous section, in a static coordinate reference frame, cosmological constant effects can be interpreted by a non-relativistic test-particle as an effective violation of the gravitational inverse square law. However, one should not confuse this assertion with present attempts of many authors focusing on deviations from standard gravity. There, deviations at small distances (\( \lesssim 1 \) mm) are usually referred to in order to solve the huge gap between the observed value of the Dark Energy and the one we could naively predict for the cosmological constant from particle physics considerations. On the other hand, deviations at larger scales (\( \gtrsim 1 \) Gpc) would aim at explaining the present accelerated expansion rate of the universe without invoking Dark Energy, but just modifying General Relativity [13]. Here, the picture is more conservative: the framework is the one of the theory of General Relativity with a small cosmological constant in the Newtonian limit and the target is to consider Dark Energy effects on small distances.

Let us now discuss the main features emerging from this picture. From Eq. (7) we can see that a test-particle is attracted by a body of mass \( M \) with a weaker force (we take \( \Lambda > 0 \)) than the case with null cosmological constant. Nevertheless, for \( \rho_\Lambda \approx 6 \cdot 10^{-30} \) g/cm\(^3\) (the value we deduce from present cosmological data [1]) the effect is so tiny that it is essentially impossible to detect.

In order to make some estimates, it is convenient to introduce the quantity
\[ \beta(r) = \frac{8}{3} \pi r^3 \rho_\Lambda / M, \]
which is the ratio of the repulsive cosmological constant force to the attractive standard term. From Eq. (9), we can see that in laboratory experiments, where for example \( M \sim 100 \) kg and \( r \sim 100 \) cm, \( \beta \) is about \( 10^{-28} \). In the Solar System, with \( M \sim M_\odot \) the Solar mass and \( r \sim 10^{13} \) cm the mean Earth-Sun distance, \( \beta \) is at the level of \( 10^{-23} \). The correction is so small that relativistic effects may be much more important.

Of course, if Dark Energy was not the cosmological constant \( \Lambda \), its value could vary from one point of the spacetime to another. In this case, we could put phenomenological upper bound on its local magnitude, even if probably this approach is not theoretically well motivated and we should expect other more relevant phenomena (for example violation of the universality of free fall or spacetime variation of fundamental constants), depending on the unknown origin of Dark Energy.

As the distance between test-particle and massive body increases, \( \Lambda \) repulsion term becomes more and more relevant and \( M_{\text{eff}} \) decreases. Gravitational attraction dominates until when \( M_{\text{eff}} > 0 \). The distance \( R \) for which \( M_{\text{eff}}(R) = 0 \) is an unstable equilibrium point and for \( r > R \) the real spacetime expansion overcomes the attractive gravitational force of the body of mass \( M \). For example, a test-particle feels an effective attraction towards the Sun up to a distance \( R \approx 100 \) pc. As for the Milky Way, whose mass is about \( 10^{12} M_\odot \), the distance is \( R \approx 1 \) Mpc: this means that the Local Group is a gravitationally bound system. On the other hand, from this simple picture follows that the Virgo Cluster is not exerting an effective attractive force on us: its mass is \( M \sim 10^{15} M_\odot \), implying \( R \sim 10 \) Mpc, whereas it is at a distance of about 20 Mpc from us. Here, however, the situation is more subtle, since the Virgo Cluster and the Local Group are not two objects in an empty space,
but between them there are other galaxies and clusters, whose effect may be to form a sort of “chain” or “gravitationally bound filament” (see the end of Section 5), so that they may be part of the same bound system.

In this connection, we can work out the following simple description for $N$ point-like massive particles which interact gravitationally. First, we choose a static system of coordinates, whose origin appears as the source of an effective radial repulsive force for all the particles. Second, we consider the standard Newtonian gravitational force acting on each particle and equal to the sum of all the gravitational forces exerted on the particle by the other ones. The result is that the acceleration of the $i$th particle is

$$\frac{d^2 \mathbf{r}_i}{dt^2} = -G_N \sum_{i \neq j} \frac{M_j \mathbf{r}_{ij}}{r_{ij}^3} + \frac{\Lambda}{3} \frac{\mathbf{r}_i}{r_i},$$

(10)

where $M_j$ is the (real) mass of the $j$th particle, $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$ and $r_{ij} = |\mathbf{r}_{ij}|$. This example shows clearly that the $\Lambda$ force has to be an effective (or apparent) force, due to the choice of the reference frame. In addition to this, considering the limit $\sum \to f$, we can easily generalize Eq. (8) in the case of spherically symmetric mass distribution

$$M_{eff}(r) = M(r) - \frac{8}{3} \pi r^3 \rho_{\Lambda},$$

(11)

where

$$M(r) = \int_0^r \rho(x) 4\pi x^2 dx,$$

(12)

is the matter mass within the radius $r$ and $\rho(x)$ the matter mass density at the distance $x$ from the origin.

$N$-body simulations are often used to study cluster structure. Particular interest is devoted to mass density profile and substructure, because it is believed that they retain informations on the evolutionary history. They can also provide a measurement of $\Omega_{mat} = \rho_{mat}/\rho_c$, the matter energy density in the universe to the critical energy density ratio, but are (at least usually) essentially insensitive to $\Lambda$: for a typical cluster of mass $M \approx 10^{15} M_\odot$ and size 1 Mpc, $\beta$ is no more than $10^{-3}$.

4. GALAXY CLUSTER MASSES

Galaxy clusters are the largest gravitationally bound systems in the universe, containing usually some few hundreds galaxies spread over a region of roughly 1 Mpc. At present there exist three independent methods to measure their masses, based respectively on galaxy kinematics [11], X-ray profile [12] and gravitational lensing [13].

The first approach focuses on galaxy motion within the cluster. Basically, we assume that the cluster is in hydrostatic equilibrium and is spherically symmetric, so that the acceleration of a galaxy (here considered as a test-particle) at the distance $r$ from the cluster center is

$$\frac{v^2}{r} = \frac{G_N M(r)}{r^2},$$

(13)

where $v$ is the galaxy velocity and $M(r)$ the total cluster mass within the radius $r$ (for more details, see e.g. Ref. [11]). For $\Lambda \neq 0$, the effective gravitational force is not provided by $M(r)$ but by $M_{eff}(r)$, so that we really measure the latter quantity.

As for the second approach, the key point is that the hot low-density intracluster gas is expected to have a distribution similar to the one of the galaxies in the cluster and to be able to trace the cluster gravitational potential of all the matter. Assuming that the gas is in hydrostatic equilibrium, we can write

$$\nabla P = -\rho \nabla \phi,$$

(14)

where $P$ is the gas pressure, $\rho$ the gas density and $\phi$ the gravitational potential of the cluster. If the latter is spherically symmetric

$$\phi = -\frac{G_N M(r)}{r},$$

(15)

However, for a non zero cosmological constant the Newtonian gravitational potential is not exactly that given in Eq. (15), but we have to perform the substitution $M(r) \to M_{eff}(r)$: even in this case we do not measure the real cluster mass but also the $\Lambda$ contribution.

The last method is based on gravitational lensing and can measure cluster masses from the produced distortion of background galaxies. In this case the slow motion approximation considered in this paper is clearly inadequate and a relativistic treatment is necessary; this can be found in the literature. The important feature is that light deflection is not affected by a non-zero cosmological constant [3,13], implying that the method measures the “real” cluster mass $M$.

Since the three independent techniques provide consistent cluster masses, typically within radii of about 1 Mpc, it is common belief that we can reliably determine them with an accuracy at the level of 30%, the observed scatter of the data. As for Dark Energy effects on these measurements, they are indeed usually negligible: for $\rho_{\Lambda} \approx 6 \cdot 10^{-30}$ g/cm$^3$, the theoretical ratio of galaxy kinematics or X-ray mass $M_{eff}$ to the gravitational lensing one $M$ within the radius $r$ is

$$\frac{M_{eff}}{M} = 1 - 0.007 \left(\frac{10^{14} M_\odot}{M}\right) \left(\frac{r}{1 \text{ Mpc}}\right)^3.$$

(16)

Since standard value are $M \approx 10^{12} - 10^{15} M_\odot$ and $r \approx 0.2 - 1$ Mpc, the discrepancy is irrelevant for present accuracy.

However, if we are interested in the observation of Dark Energy effects on these gravitationally bound systems, we could select suitable galaxy clusters with features favorable for our purpose. What we would need are light and non-compact galaxy clusters: for example, for $M \approx 10^{13} M_\odot$ and $r \approx 2$ Mpc the mass measured by the first two methods with respect to the gravitational lensing one should differ at the level of 60%. Of course
this kind of measurements are challenging, but they are not impossible to reach.

An alternative approach is to measure cluster masses through galaxy kinematics and to study the behavior of the gravitational force [19]; for $\Lambda \neq 0$ the gravitational force can not decrease faster than $1/r^2$, whereas for $\Lambda > 0$ it can. Here we would need a very compact cluster with few small satellites at larger distances which can be used to determines $M_{\text{eff}}$ as a function of $r$.

There exist also the more favorable possibility that Dark Energy is not uniformly distributed and that in some galaxy cluster $\rho_\Lambda$ is larger than its mean value. For instance, it would be relatively easy to observe Dark Energy effects if $\rho_\Lambda > 10^{-27}$ g/cm$^3$, only 2 – 3 order of magnitude larger than its mean value. At the moment we can only say that the general agreement between the three techniques rejects a frequently incluster cosmological constant of this magnitude: even if from systems of size of about 1 Mpc, it represents a constraint much stronger than the ones coming from the Solar System [20].

5. CONCLUSION

Often it is assumed, without particular cure, that cosmological constant effects enter into the dynamics of the universe on large scales but that they are completely negligible for the dynamics of gravitationally bound systems. This is indeed true in general and in this paper I have discussed in some detail the topic. Moreover, I have shown that measurements of galaxy cluster masses can provide evidences of Dark Energy in the dynamics of gravitationally bound systems with typical size of 1 Mpc. This kind of measurements would be very important for a future solution of the Dark Energy puzzle and of the mysterious accelerated expansion rate of the universe. The key point is to find light and non-compact galaxy clusters and then to be able to perform precise mass measurements which are sensitive and insensitive to $\Lambda$. An alternative possibility is to look for very compact clusters with few distant satellites and to measure the gravitational force behavior as a function of the distance from the cluster center.

Here I have considered cosmological constant effects in the Newtonian limit, i.e. under the assumptions of slow motion and weak gravitational field. First order relativistic corrections of Eq. (16) are suppressed by

$$\frac{GMr}{r^2} \sim 10^{-7} - 10^{-4},$$

where the estimates are for $M \sim 10^{12} - 10^{15} M_\odot$, $r \sim 0.2 - 1$ Mpc and $v \sim 300$ km/s. Corrections of the same order of magnitude have to be expected in all the results coming from Eq. (16), so that the Newtonian limit is a good approximation for our purpose and there are no reasons to go beyond the non-relativistic picture. If it was not so, exact General Relativity equations should be used: in the special case of spherical symmetry, they reduce to ordinary differential equations (such as Eq. (20b) of Ref. [17]) that can be solved numerically.

Finally, the Newtonian framework suggests us the following simple picture of the universe. Each galaxy can be thought of as a point-like particle of mass $M$ at the center of a bubble of radius $R$, with $M_{\text{eff}}(R) = 0$. Inside the bubble, gravitational attraction towards the galaxy overcomes the effective repulsive force due to the cosmological constant $\Lambda$. Gravitationally bound systems such as galaxy clusters are essentially overlapping bubbles. On larger scale, the universe can be seen as a space filled with larger bubbles, representing galaxy clusters; if two bubbles are not in direct contact, they exert an effective repulsive force each other. On the other hand, if the distance between the center of two bubbles is smaller than the radius of the larger bubble, the two bubbles are surely exerting attractive force each other (even if at a given time their distance is increasing because of the expansion of the universe, the acceleration of the relative separation is negative). As the universe expands, $\Omega_\Lambda = \rho_\Lambda/\rho_c$ increases and islands of gravitationally bound systems will be more and more diluted. Structure formation goes on inside bubbles. If there exist "chains" of overlapping bubbles, they may be gravitationally bound and collapse, generating super-bubbles, but disjointed chains are destined to go away and not to interact.

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There exist also attempts to modify gravity at the galactic scale, i.e. at distances of order 10 – 100 kpc, to explain galaxy rotation curves without Dark Matter.

For this purpose, X-ray profile is probably not competitive, since we would need large volumes with very low matter density, so that X-ray measurements are difficult.

The statement is strictly valid only if Dark Energy can be described by a cosmological constant on scales around 1 Mpc, even if not necessary with the value we deduce from cosmological data. For observational signatures of other Dark Energy candidates, see e.g. Ref. [10].