An accurate method for detecting moving circular face with chamfer in video

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Abstract. A Circle is a common shape in mechanical parts, which projects as ellipse in image or video. In industrial video, target ellipse is often chaotic due to chamfer interference and external interference. An algorithm is proposed to precisely detect ellipse in video on basis of arc analysis in this paper. Compared with the previous methods, the algorithm adds the step of re-extracting the omitted arcs in first detection, and distinguishes target from interference, so it is more accurate. Combined with a video about the fatigue experiment of self-lubricating joint bearing to analyze, the effectiveness and accuracy of this algorithm are verified intuitively.

Key words: Ellipse Detection, Chamfer, Arc Analysis, Secondary Detection for Refinement (SDR), Hough Transform

1. Introduction
Geometric shape recognition in digital images has always been a basic task in Machine vision [1]. For motion detection of mechanical parts, there are many advantages of machine-vision-based method [2]. Circle is a common artificial nonlinear geometric shape, which projects as ellipse in image or video, and ellipse detection in video becomes the key technology for motion detection.

Many researchers have proposed various methods for ellipse detection in the past [3, 4], ellipse detection is usually done in two steps, ellipse recognition and ellipse extraction. The typical methods for ellipse recognition can be roughly divided into two categories: Hough Transform [5, 6, 7] and Arc-based methods. In order to improve the efficiency of Hough transform, researchers have designed improved methods, such as Probabilistic Hough Transform (PHT) [8], Random Hough Transform (RHT) [9], Hough Transform with geometric constraints [10], and Hough Transform with convolution neural network [11]. The main drawback is that they are very sensitive to the choice of the scale of the parameter space. Furthermore, the performance of HT algorithms deteriorates when the number of ellipses in image increases. The main strategy of Arc-based methods is to detect arcs and then group them in order to detect ellipses, such as Hahn’s ellipse detection by arc [12], Dong H’s arc clustering based on geometric constraints [13], iteration for ellipse proposed by Kanatani [14] and arc combination for ellipse proposed by F.Mai [15], etc. Compared with point, arc contains relatively more geometric information, which makes Arc-based methods more efficient [16]. The methods for ellipse extraction is mainly represented by Least Square based methods. The basic idea of Least Square based methods is to fit a set of data in image with the target equation, which solves parameters of the equation by minimizing...
the error between the data and the target equation [17, 18]. The disadvantage of Least Square based methods is that it has poor anti-interference and is greatly affected by noise points, so noise should be eliminated as far as possible before ellipse extraction.

Because the traditional method of collecting motion data is not simple enough and is limited in actual conditions, data could be collected by detecting the circular face on a moving part in video. Ideally, ellipse detection is only removing the external interference in an image or video. However, two ellipses projected by chamfer are identified as one ellipse if chamfering radius is small, which also results in detection errors.

The ideal methods for ellipse detection can’t apply directly, which is unable to meet the requirement of precision in industrial detection, an algorithm is proposed to improve the accuracy on detect ellipse of the moving target in video.

2. Algorithm

The main process of this algorithm is divided into Edge detection, Determination of Region of Interest (ROI), Arc matching, and Secondary Detection for Refinement (SDR). The flowchart of detect target ellipse in frame is shown in Figure 1. Since this algorithm is based on arc analysis, arc without sufficient information should be filtered out to improve efficiency.

![Diagram of Algorithm](image)

**Figure 1.** The flowchart of algorithm

2.1. Determination of ROI

For target ellipse, most of features in the whole image are interferential, and the efficiency of sifting is obviously lower. Therefore, before extracting arcs, the ROI where target arcs are located should be defined according to the continuity of motion.

Considering that the motion of mechanical parts is continuous, the motion of a feature on two adjacent frames of a video is also limited. It is assumed that the target ellipse’s parameters of the first frame in video are known. The variant between the parameters of ellipses on the next frame are slight. Ellipse detection in the latter frame utilizes target ellipse’s parameters in the previous frame, which will greatly improve the efficiency of extracting arcs.

The ellipse equation of any position is formula (1).

\[
\frac{[(x-x_c)\cos\theta+(y-y_c)\sin\theta]^2}{a^2} + \frac{[-(x-x_c)\sin\theta+(y-y_c)\cos\theta]^2}{b^2} = 1
\]

(1)

Where, \(a\) is the length of major semi-axes and \(b\) is minor; \((x_c,y_c)\) is the coordinate of center; \(\theta\) is angle between longer axis of ellipse and X-axis, which is known as angle of deflection. Ellipse detection is to determine these five parameters.

Here, five parameters \(a, b, x_0, y_0, \theta\) are provided by target ellipse in the previous frame. Set a variable range \(\Delta_1\) in advance and traverse all pixels \(P_i = (x_i, y_i)\) in the latter frame after edge detection. When \(P_i = (x_i, y_i)\) meets the inequation (2), the pixel value \(P_i\) is retained.

\[
1 - \Delta_1 < \frac{[(x_i-x_0)\cos\theta+(y_i-y_0)\sin\theta]^2}{a^2} + \frac{[-(x_i-x_0)\sin\theta+(y_i-y_0)\cos\theta]^2}{b^2} < 1 + \Delta_1
\]

(2)

\(\Delta_1\) is a margin of amplitude which can roughly measure the amplitude of motion between targets in two adjacent frames.

2.2. Arc extraction and processing

The edge points detected in ROI are \(P_i = (x_i, y_i), i = 1, 2, 3 \ldots N\), \(\Omega_P\) is a set of all edge points in ROI. \(P_i \in \Omega_P\). After vectorization, the set of edge points is divided into a number of eight-connected point sets [19]. \(\Omega_P = \{\Omega_{n-1}, \Omega_{n-2}, \ldots\}\). A set of eight-connected pixels are neighbors to every pixel that touches one of their edges or corners. These pixels are connected horizontally, vertically, and
diagonally. \( \Omega_{b-t} = \{ (P^m_{i,1}, \ldots, P^m_{i,n}) \mid i \in \text{Connected}(P^m_{i-1}, P^m_i) \} \). Where \( n \) is the number of edge points in \( \Omega_{b-t} \). Connected\((P^m_{i-1}, P^m_i)\) said that \( P^m_{i-1}, P^m_i \) are subordinate to a relationship that they are adjacent points in eight directions (eight-connected).

\( \Omega_{b-t} \) is processed by gradient direction to obtain a non-branched arc [20]. Then, according to chain code in eight-connected, points in \( \Omega_{b-t} \) are sorted in arc direction to get arc \( C \). A set of non-branched arcs extracted in ROI is denoted as \( \Omega_{ROI} = \{ C^1, C^2, \ldots \} \).

In fact, ellipse arc is smooth without abruptness. However, due to the interference of interactive features in edge extraction, a non-branched arc \( C^1 \) may not belong to a single feature, and there may be cusps or inflection points in the arc. Therefore, identifying cusp or inflection point in an arc plays a key role in separating interference feature from target feature.

Arc is discrete in digital image. In order to measure bending degree of discrete arc, we adopt a method of L-curvature on precision digital arc [21] to assess the discrete curvature of each point in arc. Select \( P_i(x_i, y_i) \)'s neighborhood \( \Phi(P_i) = \{ P_{i-k}, \ldots, P_{i-1}, P_i, P_{i+1}, \ldots, P_{i+k} \} \) in arc \( C \), where \( k \) is radius of neighborhood, discrete curvature at \( P_i \) is calculated through the coordinate data of these points.

In order to improve the accuracy of discrete curvature, assuming that the cumulative chord length is \( 2L \), two rough points \( P_{i-L_x}(x_{i-L_x}, y_{i-L_y}), P_{i+L_x}(x_{i+L_x}, y_{i+L_y}) \) with the cumulative chord length of about \( L \) on both sides of \( P_i \) are determined, \( |P_{i-L_x}P_i| = |P_iP_{i+L_x}| = L \), \( x_{i-L_x}, y_{i-L_y}, x_{i+L_x}, y_{i+L_y} \) are positive integers, formula (3) represents the cumulative length between two points.

\[
|P_iP| = \sum_{k=1}^{i+1} \sqrt{(x_{i+k} - x_{i+k-1})^2 + (y_{i+k} - y_{i+k-1})^2} \tag{3}
\]

To be precise now, \( P_{i-L_x} \) meets the condition that \( |P_{i-L_x}P_i| < L \) and \( |P_{i-1}P_{i+1}| \geq L \). \( P_i(x_i, y_i) \) is determined by linear interpolation to satisfy \( |P_iP_0| = L \) accurately between \( P_{i-L_x} \) and \( R_{i-L_x+1} \), similarly, \( P_i(x_i', y_i') \) will be also determined to satisfy \( |P_iP'| = L \) accurately between \( P_{i+L_x} \) and \( R_{i+L_x+1} \).

Now, the neighborhood of \( P_i \) updates to \( [P_i^{L}, P_i^{R}] \), discrete curvature at \( P_i(x_i, y_i) \) is expressed as formula (4).

\[
\Delta_2 = \text{sign}\{(x_i - x_i')(y_i - y_i') - (x_i' - x_i)(y_i - y_i')\} \sqrt{1 - \left( \frac{D_i}{2L} \right)^2} \tag{4}
\]

\( D_i \) is Euclidean distance between \( P_i^{L} \) and \( P_i^{R} \), \( 2L \) is accumulative length of chord.

If there are cusps and inflection points in an arc, its position \( P_{\text{change}} \) can be determined by the sign and value of curvature. Originally, \( P_{\text{change}} \in C^m \), \( P_{\text{change}} \) is removed from \( C^m \), which makes arc \( C^m \) broken into \( C^{m_1} \) and \( C^{m_2} \).

2.3. Arc matching
As mentioned above, the target ellipse is very close to the reference ellipse, which is usually taken as the result of the previous frame. An arc is assessed whether to be remained in terms of the difference between the tangent slope of the arc and that of the reference ellipse.

Assume a point \( P(x_p, y_p) \) on an arc \( A \), and take the ellipse with equation (1) as the reference ellipse. \( P'(x_p', y_p') \) is the intersection of the ellipse and the line joining the center \( C(x_c, y_c) \) and \( P \), as shown in Figure 2.
Now describe these geometric items in uCv coordinate system (Fig.3). The coordinate of \( P \) is transformed as \((u_p, v_p)\), where
\[
\begin{align*}
    u_p &= |PC| \cos \phi = (x - x_C) \cos \theta + (y - y_C) \sin \theta \\
    v_p &= |PC| \sin \phi = -(x - x_C) \sin \theta + (y - y_C) \cos \theta
\end{align*}
\] (5)
and \( \phi \) stands for the angle of line PC to u-axis.

The coordinate of \( P' \) is \((|P'C| \cos \phi, |P'C| \sin \phi)\). The tangent vector of the ellipse at point \( C \) can be easily obtained,
\[
\tilde{t}_p = \left(-|P'C| \frac{a \sin \phi}{b}, |P'C| \frac{b \cos \phi}{a}\right)
\] (6)

The unit tangent vector of the arc A at point \( P \) also can be evaluated by linear fit, denoted as \( \hat{t}_p(t_u, t_v) \). Let \( \alpha \) the angle of vectors \( \hat{t}_p \) and \( \tilde{t}_p \), which can be got by inner product between the vectors:
\[
\cos \alpha = \frac{\hat{t}_p \cdot \tilde{t}_p}{\| \hat{t}_p \| \| \tilde{t}_p \|}
\] (7)

Using the equations (5) and (6), we have
\[
\cos \alpha = \frac{t \cdot b^2 \left((x - x_C) \cos \theta + (y - y_C) \sin \theta\right) - t \cdot a^2 \left(-(x - x_C) \sin \theta + (y - y_C) \cos \theta\right)}{\sqrt{a^4 \left((x - x_C) \cos \theta + (y - y_C) \sin \theta\right)^2 + b^4 \left(-(x - x_C) \sin \theta + (y - y_C) \cos \theta\right)^2}}
\] (8)

Hence, a criterion for arc matching is formed. Let
\[
\Delta_3 = 1 - \frac{t \cdot b^2 \left((x - x_C) \cos \theta + (y - y_C) \sin \theta\right) - t \cdot a^2 \left(-(x - x_C) \sin \theta + (y - y_C) \cos \theta\right)}{\sqrt{a^4 \left((x - x_C) \cos \theta + (y - y_C) \sin \theta\right)^2 + b^4 \left(-(x - x_C) \sin \theta + (y - y_C) \cos \theta\right)^2}}
\] (9)

For arc A, if \( \Delta_3 \) of any point on arc A is less than a threshold specified, it is considered that arc A matches successful. Accordingly, \( \Omega_{\text{smooth}} \) is divided into the successful-matching set \( \Omega_{\text{true}} \) and the failed-matching set \( \Omega_{\text{false}} \).
2.4. **Secondary Detection for Refinement (SDR)**

The arcs in $\Omega_{\text{false}}$ ought to discard. However, because of the existence of features formed by chamfer in image, arcs in $\Omega_{\text{true}}$ belongs to two ellipse. If arcs in $\Omega_{\text{true}}$ are involved in final process to fit a ellipse, the error will be too large. Additionally, if there are omitted arcs of target ellipse in edge detection, the error will be larger.

In view of above situation, we return to gray image before edge detection, focus on the neighborhood of target arcs in $\Omega_{\text{true}}$, and detect weak edges, so as to fully find target arc and reduce omission. At the same time, the number of arcs belong to two ellipses projected by chamfer will increase.

Firstly, we acquire the appropriate neighborhood of target arcs in $\Omega_{\text{true}}$ one by one from gray image and lower the threshold of edge detection to a value to ensure that more edge points can be obtained without excessive noise. Then, arcs in neighborhood are extracted, since there may be inferential arcs, after verifying the match of these newly emerged arcs, the arc which matches successfully is added to $\Omega_{\text{true}}$.

Now, arcs in $\Omega_{\text{true}}$ mostly belong to two ellipses projected by chamfer. We select Hough transform based on midpoint of chord to detect ellipse [22]. What get the highest number of votes represents the center of target ellipse, and then the remaining parameters of ellipse can be further solved.

Under current situation, points in arcs from $\Omega_{\text{true}}$ will vote in parameter space, and two ellipse centers with the highest and the second highest number of votes must belong to two ellipses projected by chamfer. We distinguish two ellipses by a prior condition. For example, both the long axis and the short axis of up-chamfer ellipse are shorter than those of down-chamfer ellipse. If we want to detect the up-chamfer ellipse, target ellipse can be distinguished by comparing two ellipse’s parameters.

3. **Experiment**

The self-lubricating bearing tester for fatigue is taken as an experimental equipment.

![Inner ring of bearing](image1)

![Three-ear oscillating sleeve](image2)

**Figure 3.** The image of the self-lubricating bearing tester for fatigue

The shape of pressure ring on end is circular and is projected as ellipse in image. After pre-processing such as graying, smoothing and edge detection for each frame, the goal is to distinguish two ellipses caused by chamfer and detect the up-chamfer ellipse in image. Take a frame to detect target ellipse as an example.
Canny operator [23] is selected for edge detection, and Figure 4 is obtained. ROI is determined based on Figure 4. Then the non-branched arcs are extracted in ROI. Figure 5 is a diagram of non-branched arcs in ROI. For the sake of clarity, we invert the original picture.

$P_{\text{change}}$ can be detected by discrete curvature. Red circle marks cusp or inflection point detected by above algorithm, and a cusp is amplified to display. Figure 6 is the curvature of arc where that point is located, and that point can be found easily.

**Figure 4.** A frame after Canny detection

**Figure 5.** Cusps or inflection points of non-branched arcs in ROI

**Figure 6.** The curvature of Arc 1 in Figure 6
According to Figure 5, most of cusps and inflection points are detected, which basically meet the requirement of application. $\Omega_{\text{smooth}}$ could be achieved.

After filtering, arcs in $\Omega_{\text{smooth}}$ are matched. The successful-matching arcs in $\Omega_{\text{true}}$ are shown in Figure 7. arcs are labeled for explanation.

**Figure 7.** The successful-matching arcs in $\Omega_{\text{true}}$

Each arc in $\Omega_{\text{true}}$ was processed with SDR, and weak edges could appear, as a result the corresponding arcs are extracted. Take part of arc 4 as an example, as shown in Figure 8. After the matching of new arc 4’ is verified, the arc is added to $\Omega_{\text{true}}$ (Figure 9).

**Figure 8.** Detection for arc 4’

**Figure 9.** The successful-matching arcs in $\Omega_{\text{true}}$ after secondary detection
The points in arcs from $Ω_{\text{true}}$ are voted by Hough transform based on midpoint of chord, two peaks are voted out in parameter space (Figure 10), and we solve other parameters of two ellipses further. The ellipses are drawn on Figure 9, as shown in Figure 11: Statistics of elliptic features are display in table 1.

![Figure 10. Voting in parameter space](image1)

![Figure 11. Two ellipses detected by Hough transform](image2)

| Feature | Enumeration of arc | Number of pixel |
|---------|--------------------|-----------------|
| Ellipse 1 | 1,2,3,4,4',5,6,10 | 159             |
| Ellipse 2 | 2,4,5',6          | 125             |

Since the goal of Algorithm is to detect the up-chamfer ellipse, we select the target ellipse according to the comparison of axes. In order to reflect the accuracy caused by the addition of SDR, an intuitive comparison was made according to ellipses extracted by two methods, as shown in Figure 12(a) and (b). By comparing results, we can see the effect of SDR.

![Figure 12. Compared](image3)

A frame can be processed by above algorithm to detect the corresponding accurate ellipse, which can be extended to video. As long as ellipse’s parameters in first frame are given, the detection of moving ellipse can be realized in video. The results of other four frames are shown in Figure 13.
4. Conclusion
This paper focuses on solving such a practical problem that the accurate detection of moving ellipse can be realized for each frame in video, even if target ellipse is incomplete and interfered by chamfer which often exists in industry. On basis of ellipse detection summarized by predecessors, we propose an algorithm for the specific situation of problem. The principle is described in detail, and video is captured from the self-lubricating joint bearing as material for analysis, which verifies the effectiveness and accuracy of the algorithm.

this paper only focuses on the principle and the process of case analysis, without considering for environment and hardware of video acquisition. Actually, video acquisition is the basis of all processing. Research on video acquisition is also a direction to improve the accuracy of detection.

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