Automation calculation of the stability of drawing process of quartz optical fibers using different modifications of furnace shape

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Abstract. A one-dimensional model of quartz optical fibers extraction was considered and the problem of drawing parameters stability was solved. More over two numerical experiments demonstrating the states of stability or instability depending on the values of small harmonic oscillations introduced into the system, as well as on the furnace parameters were performed.

1. Introduction

Fiber drawing is one of the important stages in the production of quartz optical fibers. However, like any real phenomenon, this process is subject to fluctuations, which may have a negative impact on the quality of the finished product. In point of fact, the flow of a liquid jet injected through a nozzle is known as unstable due to capillary forces acting on the boundary and seeking to minimize the surface of the cylindrical jet in order to turn it into drops.

The instability of the process begins with the small axisymmetric disturbance spread of the jet radius, which amplitude increases with time. This can lead to inconsistency of the characteristics of the fiber along its length in practice at best; otherwise, it leads to the destruction of the jet being pulled out at worst. Despite the fact that the jet is mainly controlled by surface tension, other factors may also play an important role. Such factors, for example, are the ratio of the input / output velocities, inertial forces; the temperature of the heating element, etc. [1]. That is why it is very important to know how the system will react to these impacts, especially at the modeling stage.

The purpose of this paper is to evaluate the dependence of the fiber radius values on the values of the introduced oscillations of the radius, temperature and velocity at the boundaries for the non-stationary non-isothermal mode of extracting quartz optical fibers in a one-dimensional axisymmetric formulation with various features of the heating element.

2. Task statement

The authors consider a non-isothermal process of extracting quartz fiber, which is described by the following system of dimensionless partial differential equations [2-5]:

\[
\text{Equations...}
\]
We have \( \frac{\partial R}{\partial t} + V \frac{\partial R}{\partial x} + \frac{R}{2} \frac{\partial V}{\partial x} = 0, \)
\[
R^2 \left( \frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} \right) = \frac{3}{\text{Re}} \left( \frac{\partial}{\partial x} \left( \mu R^2 \frac{\partial V}{\partial x} \right) \right) + \frac{R^2}{\text{Fr}} + \frac{1}{\text{We}} \frac{\partial R}{\partial x},
\]
\[
R^2 \left( \frac{\partial T}{\partial t} + V \frac{\partial T}{\partial x} \right) = \frac{1}{\text{Pe}} \left( \frac{\partial}{\partial x} \left( \frac{R^3}{2} \frac{\partial T}{\partial x} \right) \right) - 2R \sqrt{1 + R^2} \cdot \text{St} \cdot (T - 1) + \]
\[+ 4 \chi R \cdot R_p \cdot \left( R_p - R \right) \left( \frac{\beta e_p}{V_p} T_p^4 - \epsilon T^4 \right) \left( (\eta - x)^2 + (R_p - R)^2 \right) d\eta, \]

The system of equations (1) is solved with initial and boundary conditions
\[
R(t,0) = \frac{R_{\text{pref}}}{L} = R_0, R(0,x) = R_{sf}(x), V(t,0) = \frac{V_0}{V_L} = \frac{1}{E}, V(t, L) = \frac{V_L}{V_0} = 1, E = \frac{V_L}{V_0}, V(0,x) = V_{sf}(x),
\]
\[T(t,0) = \frac{T_{p1}}{T_0}, T(0,x) = T_{sf}(x) \]

where \( \text{Re} = \frac{\rho V_L L}{\mu} \) - Reynolds number, \( \text{Fr} = \frac{V_L^2}{\gamma} \) - Froude number, \( \text{We} = \frac{\rho V_L^2 L}{\gamma} \) - Weber number,
\[\text{Pe} = \frac{\rho V_L C_p L}{\lambda T} \] - Peclet number, \( \chi = \frac{\mu^2 \sigma T_0^3}{\rho V_L C_p} \) - dimensionless complex, \( \text{St} = \frac{\alpha}{\rho V_L C_p} \) - Stanton number, \( C_p \) -specific heat, \( \alpha \) - heat transfer coefficient; \( \rho, \mu(T) \) - quartz melt density and viscosity, respectively, \( T_0 \) - air temperature, \( V_0 \) - preform feed rate, \( V_L \) - drawing speed, \( L \) - quartz melt sample length, \( R_{\text{pref}} \) - radius of the preform, \( E \) - speed coefficient, constant in the case when the preform feed rate \( V_0 \) and drawing speed \( V_L \) are constant both and equal to the ratio of the fiber drawing speed to the workpiece feed speed (the so-called drawing ratio), \( T_p(t,x) \) - temperature along the oven surface, \( R_p \) - furnace radius.

It is well known from the practice that the temperature along the surface of the furnace changes and in the central part of the furnace it is possible to distinguish a zone (core) of width \( H \), where the temperature is constant and much nearer than near the edges. Taking this into account, further the authors will define the temperature distribution as following:
\[
T_p(x) = \begin{cases} 
\frac{T_{p1}}{T_0}, x \in [0; \frac{(1-h) \frac{L}{L}}{2}] \\
\frac{T_{p2}}{T_0}, x \in [\frac{(1-h) \frac{L}{L}}{2}; \frac{(1+h) \frac{L}{L}}{2}] \\
\frac{T_{p1}}{T_0}, x \in [\frac{(1+h) \frac{L}{L}}{2}; \frac{L}{L}] 
\end{cases}
\]

where \( h = \frac{H}{L} \) - the relative width of the heating element core. \( R_{sf}(x), V_{sf}(x), T_{sf}(x) \) - fiber radius, drawing speed and temperature at the initial time, state of the system \( R(t,x), V(t,x), T(t,x) \) - fiber radius, drawing speed and temperature functions. The longitudinal coordinate \( x \) is directed in the direction of the fiber,
in dimensional form \( x \in [0, L] \), \( t \) - time, \( t \in [0, \tau] \). Further, the temperature of the core \( T_{p2} \) will be fixed and will be chosen as 2100 °C, which in the dimensionless form corresponds to the temperature \( T_{p2} = 7 \) i.e. that the control parameters \( T_{p1}, h \) were chosen in the calculations.

Based on the model, the furnace design influence on the stability of the drawing process was studied, and the stability conditions were obtained in the form of a range of values for the furnace parameters. The stability assessment was carried out according to the classical scheme of testing stability by the first approximation (i.e. Lyapunov stability).

The study of the original nonlinear system was reduced to the consideration of the linearized system (i.e. the system of the first approximation equations). Particularly, all eigenvalues of the Jacobian of a differential operator \( \omega \) were calculated, and the signs of their imaginary parts were evaluated. Since the system goes into an unstable state, if at least one contains a non-negative imaginary part, then the stability was estimated only by one maximum value of the imaginary part \( \omega^{(1)}_i \) (by the value of the attenuation coefficient of the 1st mode) [4, 6-7]. The problem was solved with the following parameters:

\[
\begin{align*}
Re & \approx 3 \cdot 10^{-5}, \quad Fr \approx 5 \cdot 10^{-5}, \quad We \approx 0.147, \quad Pe \approx 634.9, \quad St = 9.4 \cdot 10^{-4}, \quad \chi \approx 4.8 \cdot 10^{-5}, \quad \mu(T) = e^{-4.22 T + 29.01} \\
R_p & = 0.03, \quad T_0 = 300, \quad T_{p2} = 7, \quad L = 0.2, \quad \frac{R_{pref}}{L} = 8 \cdot 10^{-4}, \quad E = \frac{10^{-2}}{2.5 \cdot 10^{-5}}
\end{align*}
\]

Thereby, a parameter space which guarantees continuous stable fiber formation, was obtained (Table 1).

**Table 1** Dependence \( \omega^{(1)}_i \) on \( h; T_{p1} \).

| \( h \) | 4    | 4.5  | 5    | 5.5  | 6    |
|-------|------|------|------|------|------|
| 0.1   | -0.00446 | -0.00963 | -0.01345 | -0.01008 | -0.00044 |
| 0.3   | 0.00356  | -0.02144 | -0.03513 | -0.02929 | -0.01840 |
| 0.6   | -0.00017 | -0.00411 | -0.01927 | -0.01930 | -0.01148 |
| 0.8   | -0.00396 | -0.00172 | -0.00217 | -0.00023 | 0.00359 |
| 1     | 0.03088  | 0.03113 | 0.03106 | 0.03098 | 0.03089 |

The obtained results show that the drawing stability strongly depends on the furnace shape. There are also optimal parameters of the heating element, which increase the stability of the drawing process significantly (For the studied case parameters were selected as follow: \( T_{p1} = 5, T_{p2} = 7 \) for dimensional form the temperature coresponds 1500°C and 2100°C, \( h \in [0.2; 0.5] \)).

The direct nonlinear problem (1) was solved for the obtained values of the pairs (\( h; T_{p1} \)) to get more detailed results and for a complete analysis of the system states. The values of parameters pairs were chosen both from the obtained stability space and from the instability space of the drawing process. In this case, small perturbations acting on the boundaries for the quartz billet feed rate \( V(t, 0) \), as well as for the functions of drawing speed \( V(t, 1) \) and perform radius \( R(t, 0) \), were artificially introduced into the system (1). The effect of these fluctuations on the finished fiber characteristics and on its radius, particularly, was evaluated.

3. **Implementation of stability calculations and discussion of simulation results**

For carrying out numerical experiments in the system of multiphysical modeling Comsol Multiphysics a software package was developed.

3.1 **Calculation of the model with furnace parameters** (\( h, T_{p1} = (0.2; 1650) \) - model steady state)
The solution of system (1) was carried out with the following initial and boundary conditions:

\[
R(t,0) = \frac{R_{\text{pref}}}{L} = 8 \cdot 10^{-4}, \quad R(0,x) = \frac{R_{\text{pref}}}{L} = 8 \cdot 10^{-4}, \quad V(t,0) = \frac{1}{E}, \quad V(t,1) = 1, \quad E = \frac{10^{-2}}{2.5 \cdot 10^{-5}}, \quad V(0,x) = \frac{1}{E},
\]

\[
T(t,0) = \frac{T_{\text{p1}}}{T_0} = 1650 = 5.5
\]

At the first stage of the study, the solution of the problem was obtained in the stationary mode. The simulation results are presented in Figure 1.

**Figure 1.** a – Function of radius; b – Function of temperature; c – Function of speed in the stationary mode of extraction

Then, in each boundary condition of the model (1), harmonic perturbations were introduced alternately for the values of the preform radius, the billet temperature and the inlet / outlet drawing speeds:

\[
R(t,0) = R(t,0) \cdot (1 + A \sin(2\pi \omega t)),
\]

\[
V(t,0) = V(t,0) \cdot (1 + A \sin(2\pi \omega t)),
\]

\[
V(t,0) = V(t,0) \cdot (1 + A \sin(2\pi \omega t)),
\]

\[
T(t,0) = T(t,0) \cdot (1 + A \sin(2\pi \omega t))
\]
The frequency of the introduced oscillations $\omega$ was taken equal to 25, the amplitude $A = 0.05$. The type of these disturbances is shown in Figure 2.

It should be noted that the absolute results of the calculation were not estimated, but the relative parameters of the model, depending simultaneously on the solutions of the problem in the original formulation and on the solutions of the problem with the influence of the introduced fluctuations were determined. Here, relative parameters were understood as calculated values of the following type

$$\Delta R(t, x) = \frac{R^*(t, x) - R(t, x)}{R(t, x)}$$

Where $R(t, x)$ is the solution to the original problem, $R^*(t, x)$ is the solution obtained after introducing perturbing influences. Figure 3 presents the calculated values of one of the relative values - $\Delta R(t, 1)$ depending on time for various disturbing influences.

The form of solutions presented in Figure 3 allows analyzing the time vibrations damping. It also shows that such vibrations do not have a significant negative effect on the characteristics of the drawn out fiber, in general, and on the jet geometry, in particular. Thus, the process reaches the steady state, despite of the various introduced disturbances with time, and the shapes of the jet radius profiles, as well as melt velocity, approach to the shapes of the corresponding stationary states. This confirms the stability of the drawing process with the parameters previously selected.
Figure 3. Values ΔR(t, l) with disturbing effects on: a - radius of the preform R(t, 0) ; b - temperature of the preform T(t, 0); c - the rate of extraction of the finished fiber V(t, L); d - preform speed V(t, 0)

3.2 Calculation of the model with the parameters (h, Tp1) = (1; 1650) - model unstable state

The solution of system (1) was carried out with the following initial and boundary conditions:

\[
R(t, 0) = \frac{R_{\text{pref}}}{L} = 8 \cdot 10^{-4}, R(0, x) = \frac{R_{\text{pref}}}{L} = 8 \cdot 10^{-4}, V(t, 0) = \frac{1}{E}, V(t, l) = 1, E = \frac{10^{-2}}{2.5 \cdot 10^{-5}}, V(0, x) = \frac{1}{E}.
\]

\[
T(t, 0) = \frac{T_{p1}}{T_0} = \frac{1650}{300} = 5.5
\]

Similarly to the previous, the authors introduce disturbing effects for the radius of the workpiece, the drawing speed and the feed rate of the preform.
Figure 4. Values $\Delta R(t, l)$ with disturbing effects on: a - radius of the preform $R(t, 0)$; b - temperature of the preform $T(t, 0)$; c - the rate of extraction of the finished fiber $V(t, L)$; d - preform speed $V(t, 0)$

Figure 4 demonstrates obtained calculated values of the one relative magnitude depending on time and for different disturbing influences. The solutions of the problem are most sensitive to the impacts on the radius and the temperature of the preform.

As it can be seen from the plot above, the fluctuations have not damped with time and it has a qualitative impact on the characteristics of the fiber. Thus, such results certainly indicate the instability of the process within the framework of this model with parameters previously selected in this section.

4. Conclusion
In this paper, a one-dimensional model of quartz optical fibers extraction was considered, the problem of drawing parameters stability was solved, two numerical experiments, demonstrating states of stability or instability depending on the values of small harmonic oscillations introduced into the system, as well as on the furnace forms were performed. In addition to this, two software packages were developed. The first one makes it possible to evaluate the linear stability of the fiber drawing process with different furnace shape. The second one allows estimating the influence of external disturbances on this process.

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