Abstract
These lectures provide a phenomenological introduction to supersymmetry, concentrating on the minimal supersymmetric extension of the Standard Model (MSSM). In the first lecture, motivations are provided for thinking that supersymmetry might appear at the TeV scale, including the naturalness of the mass hierarchy, gauge unification and the probable mass of the Higgs boson. In the second lecture, simple globally supersymmetric field theories are introduced, with the emphasis on features important for model-building. Supersymmetry breaking and local supersymmetry (supergravity) are introduced in the third lecture, and the structure of sparticle mass matrices and mixing are reviewed. Finally, the available experimental and cosmological constraints on MSSM parameters are discussed and combined in the fourth lecture, and the prospects for discovering supersymmetry in future experiments are previewed.

1. GETTING MOTIVATED

1.1 Defects of the Standard Model
The Standard Model agrees with all confirmed experimental data from accelerators, but is theoretically very unsatisfactory. It does not explain the particle quantum numbers, such as the electric charge $Q$, weak isospin $I$, hypercharge $Y$ and colour, and contains at least 19 arbitrary parameters. These include three independent gauge couplings and a possible CP-violating strong-interaction parameter, six quark and three charged-lepton masses, three generalized Cabibbo weak mixing angles and the CP-violating Kobayashi-Maskawa phase, as well as two independent masses for weak bosons.

As if 19 parameters were insufficient to appall you, at least nine more parameters must be introduced to accommodate neutrino oscillations: three neutrino masses, three real mixing angles, and three CP-violating phases, of which one is in principle observable in neutrino-oscillation experiments and the other two in neutrinoless double-beta decay experiments. Even more parameters would be needed to generate neutrino masses in a credible way, associated with a heavy-neutrino sector and/or additional Higgs particles.

Eventually, one would like to include gravity in a unified theory along with the other particle interactions, which involves introducing at least two more parameters, Newton’s constant $G_N = 1/m^2_P$ : $m_P \sim 10^{19}$ GeV that characterizes the strength of gravitational interactions, and the cosmological constant $\Lambda$ or some time-varying form of vacuum energy as seems to be required by recent cosmological data. A complete theory of cosmology will presumably also need parameters to characterize the early inflation of the Universe and to generate its baryon asymmetry, which cannot be explained within the Standard Model.

The Big Issues in physics beyond the Standard Model are conveniently grouped into three categories [1]. These include the problem of Unification: is there a simple group framework for unifying all the particle interactions, a so-called Grand Unified Theory (GUT), Flavour: why are there so many different types of quarks and leptons and why do their weak interactions mix in the peculiar way observed, and Mass: what is the origin of particle masses, are they due to a Higgs boson, why are the masses
so small? Solutions to all these problems should eventually be incorporated in a Theory of Everything (TOE) that also includes gravity, reconciles it with quantum mechanics, explains the origin of space-time and why it has four dimensions, etc. String theory, perhaps in its current incarnation of M theory, is the best (only?) candidate we have for such a TOE [2], but we do not yet understand it well enough to make clear experimental predictions.

Supersymmetry is thought to play a rôle in solving many of these problems beyond the Standard Model. As discussed later, GUT predictions for the unification of gauge couplings work best if the effects of relatively light supersymmetric particles are included [3]. Also, the hierarchy of mass scales in physics, and particularly the fact that \( m_W \ll m_P \), appears to require relatively light supersymmetric particles: \( M \lesssim 1 \) TeV for its stabilization [4]. Finally, supersymmetry seems to be essential for the consistency of string theory [5], although this argument does not really restrict the mass scale at which supersymmetric particles should appear.

Thus there are plenty of good reasons to study supersymmetry, and we return later to examine in more detail the motivations provided by unification and the mass hierarchy problem.

1.2 The Electroweak Vacuum

Generating particle masses within the Standard Model requires breaking its gauge symmetry, and the only consistent way to do this is by breaking the symmetry of the electroweak vacuum:

\[
m_{W,Z} \neq 0 \iff <0|X_{I,I_3}|0> \neq 0
\]

where the symbols \( I, I_3 \) denote the weak isospin quantum numbers of whatever object \( X \) has a non-zero vacuum expectation value. There are a couple of good reasons to think that \( X \) must have (predominantly) isospin \( I = 1/2 \). One is the ratio of the \( W \) and \( Z \) boson masses [6]:

\[
\rho \equiv \frac{m^2_W}{m^2_Z \cos^2 \theta_W} \simeq 1,
\]

and the other reason is to provide non-zero fermion masses. Since left-handed fermions \( f_L \) have \( I = 1/2 \), right-handed fermions \( f_R \) have \( I = 0 \) and fermion mass terms couple them together: \( m_f \bar{f}_L f_R \), we must break isospin symmetry by 1/2 a unit:

\[
m_f \neq 0 \iff <0|X_{1/2,\pm 1/2}|0> \neq 0.
\]

The next question is, what is the nature of \( X \)? Is it elementary or composite? In the initial formulation of the Standard Model, it was assumed that \( X \) should be an elementary Higgs-Brout-Englert [7, 8] field \( H \): \( <0|H^0|0> \neq 0 \), which would have a physical excitation that manifested itself as a neutral scalar Higgs boson [7]. However, as discussed in more detail later, an elementary Higgs field has problems with quantum (loop) corrections. Those due to Standard Model particles are quadratically divergent, resulting in a large cutoff-dependent contribution to the physical masses of the Higgs boson, \( W, Z \) bosons and other particles:

\[
\delta m^2_H \simeq \mathcal{O}\left(\frac{\alpha}{\pi}\right)\Lambda^2,
\]

where \( \Lambda \) represents the scale at which new physics appears.

The sensitivity [4] disturbs theorists, and one of the suggestions to avoid it was to postulate replacing an elementary Higgs-Brout-Englert field \( H \) by a composite field such as a condensate of fermions: \( <0|\bar{F}F|0> \neq 0 \). This possibility was made more appealing by the fact that fermion condensates are well known in solid-state physics, where Cooper pairs of electrons are responsible for conventional superconductivity, and in strong-interaction physics, where quarks condense in the vacuum: \( <0|\bar{q}q|0> \neq 0 \).

In order to break the electroweak symmetry at a large enough scale, fermions with new interactions that become strong at a higher mass scale would be required. One suggestion was that the Yukawa
interactions of the top quark might be strong enough to condense them: \( < 0|\bar{t}t|0 \neq 0 > \) \cite{2}, but this would have required the top quark to weigh more than 200 GeV, in simple models. Alternatively, theorists proposed the existence of new fermions held together by completely new interactions that became strong at a scale \( \sim 1 \) TeV, commonly called Technicolour models \cite{12}.

Specifically, the technicolour concept was to clone the QCD quark-antiquark condensate
\[
\langle 0| \bar{q}q |0 \rangle \sim \Lambda_{QCD}^3 \sim 1 \text{GeV}, \tag{5}
\]
on a much larger scale, postulating a condensate of new massive fermions \( \langle 0| \bar{Q}Q |0 \rangle \sim \Lambda_{TC}^3 \approx 1 \text{TeV} \) where \( \Lambda_{TC} \sim 1 \) TeV. Assigning the techniquarks to the same weak representations as conventional quarks, \( I_L = \frac{1}{2}, I_R = 0 \), the technicondensate breaks electroweak symmetry in just the way required to reproduce the relation (2). Just as QCD with two massless flavours would have three massless pions and a massive scalar meson, this simplest version of technicolour would predict three massless technipions that are eaten by the \( W^\pm \) and \( Z^0 \) to provide their masses via the Higgs-Brout-Englert mechanism, leaving over a single physical scalar state weighing about 1 TeV, that would behave in some ways like a heavy Higgs boson.

Unfortunately, this simple technicolour picture must be complicated, in order to cancel triangle anomalies and to give masses to fermions \cite{12}, so the minimal default option has become a model with a single technigeneration:
\[
\left( \begin{array}{c} \nu \\ \ell \\ u \\ d \end{array} \right)_{1,2,3} \longrightarrow \left( \begin{array}{c} N \\ L \\ U \\ D \end{array} \right)_{1,\ldots,N_{TC};1,2,3} \tag{6}
\]
One may then study models with different numbers \( N_{TC} \) of technicolours, and also different numbers of techniflavours \( N_{TF} \) if one wishes. The single-technigeneration model (6) above has \( N_{TF} = 4 \), corresponding to the technilepton doublet \( (N, L) \) and the three coloured techniquark doublets \( (U, D)_{1,2,3} \).

The absence of any light technipions is already a problem for this scenario \cite{12}, as is the observed suppression of flavour-changing neutral interactions \cite{13}. Another constraint is provided by precision electroweak data, which limit the possible magnitudes of one-loop radiative corrections due to virtual techniparticles. These may conveniently be parameterized in terms of three combinations of vacuum polarizations: for example \cite{14}
\[
T \equiv \frac{\epsilon_1}{\alpha} \equiv \frac{\Delta \rho}{\alpha}, \tag{7}
\]
where
\[
\Delta \rho = \frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{\Pi_{WW}(0)}{m_W^2} - 2 \tan \theta_W \frac{\Pi_{\gamma Z}(0)}{m_Z^2}, \tag{8}
\]
leading to the following approximate expression:
\[
T = \frac{3}{16 \pi \sin^2 \theta_W \cos^2 \theta_W} \left( \frac{m_Z^2}{m_W^2} \right) - \frac{3}{16 \pi \cos^2 \theta_W} \ln \left( \frac{m_H^2}{m_W^2} \right) + \ldots \tag{9}
\]
There are analogous expressions for the other two combinations of vacuum polarizations:
\[
S \equiv \frac{4 \sin^2 \theta_W}{\alpha} \epsilon_3 = \frac{1}{12 \pi} \ln \left( \frac{m_H^2}{m_Z^2} \right) + \ldots \tag{10}
\]
\[
U \equiv -\frac{4 \sin^2 \theta_W}{\alpha} \epsilon_2 \tag{11}
\]
The electroweak data may then be used to constrain \( \epsilon_{1,2,3} \) (or, equivalently, \( S, T, U \)), and thereby extensions of the Standard Model with the same \( SU(2) \times U(1) \) gauge group and extra matter particles that do not have important other interactions with ordinary matter. This approach does not include vertex corrections, so the most important one, that for \( Z^0 \rightarrow b\bar{b} \), is treated by introducing another parameter \( \epsilon_b \).
This simple parameterization is perfectly sufficient to provide big headaches for the simple technicolour models described above. Fig. 1 compares the values of the parameters $\epsilon_i$ extracted from the final LEP data with the values calculated in the Standard Model for $m_t$ within the range measured by CDF and D0, and for $113 \text{ GeV} < m_H < 1 \text{ TeV}$. We see that the agreement is quite good, which does not leave much room for new physics beyond the Standard Model to contribute to the $\epsilon_i$. Fig. 2 compares these measured values also with the predictions of the simplest one-generation technicolour model, with $N_{TC} = 2$ and other assumptions described in [15].

We see that the data seem to disagree quite strongly with these technicolour predictions. Does this mean that technicolour is dead? Not quite [17], but it has motivated technicolour enthusiasts to pursue epicyclic variations on the original idea, such as walking technicolour [18], in which the technicolour dynamics is not scaled up from QCD in such a naive way.

1.3 It Quacks like Supersymmetry

Electroweak radiative corrections may be bad news for technicolour models, but they do seem to provide hints for supersymmetry, as we now discuss.

Fig. 3 summarizes the indirect information about the possible mass of the Standard Model Higgs boson provided by fits to the precision electroweak data, including the best available estimates of leading multi-loop effects, etc. Depending on the estimate of the hadronic contributions to $\alpha_{em}(m_Z)$ that one uses, the preferred value of $m_H$ is around 100 GeV [19]. Including all the available electroweak data except the most recent NuTeV result on deep-inelastic $\nu$ scattering, and taking the value $\delta \alpha_{had} = 0.02747 \pm 0.00012$ for the hadronic contribution to the effective value of $\alpha_{em}(m_Z)$, one finds [19]

$$m_H = 98^{+53}_{-36} \text{ GeV}. \quad (12)$$

Also shown in Fig. 3 is the lower limit $m_H > 114.1 \text{ GeV}$ provided by direct searches at LEP [20]. We see that the most likely value of $m_H$ is 115 GeV, a point made graphically in Fig. 4, where precision electroweak data are combined with the lower limit coming from direct experimental searches [21]. Values of the Higgs mass up to 199 GeV are allowed at the 99 % confidence level, so any assertion that LEP has excluded the majority of the range allowed by the precision electroweak fit is grotesquely
Fig. 2: Two-dimensional projections comparing the allowed ranges of the $\epsilon_i$ shown in Fig. 1 with the predictions of the Standard Model (hatched regions) and a minimal one-generation technicolor model (chicken-pox regions) [15].

premature. On the other hand, any resemblance between the most likely value and the mass $m_H \sim 115$ GeV hinted by direct searches during the dying days of LEP is surely coincidental (?)

If $m_H$ is indeed as low as about 115 GeV, this would be prima facie evidence for physics beyond the Standard Model at a relatively low energy scale [22], as seen in Fig. 5. If $m_H$ is larger than the central range marked in Fig. 5 [23], the large Higgs self-coupling in the renormalization-group running of the effective Higgs potential causes it to blow up at some scale below the corresponding scale of $\Lambda$ marked on the horizontal axis. Conversely, if $m_H$ is below the central band, the larger Higgs-top Yukawa coupling overwhelms the relatively small Higgs self-coupling, driving the effective Higgs potential negative at some scale below the corresponding value of $\Lambda$. As a result, our present electroweak vacuum would be unstable, or at least metastable with a lifetime that might be longer than the age of the Universe [24]. In the special case $m_H \sim 115$ GeV, this potential disaster could be averted only by postulating new physics at some scale $\Lambda \lesssim 10^6$ GeV.

This new physics should be bosonic [22], in order to counteract the negative effect of the fermionic top quark. Let us consider introducing $N_I$ isomultiplets of bosons $\phi$ with isospin $I$, coupled to the conventional Higgs boson by

$$\lambda_{22} |H|^2 |\phi|^2. \quad (13)$$

It turns out [23] that the coupled renormalization-group equations for the $H, \phi$ system are very sensitive to the chosen value of $\lambda_{22}$ in (13). As seen in Fig. 6, if the coupling

$$M_0^2 \equiv \lambda_{22} < 0 |H|0|^2 \quad (14)$$

is too large, the effective Higgs potential blows up, but it collapses if $M_0^2$ is too small, and the typical amount of fine-tuning required is 1 in $10^3$! Radiative corrections may easily upset this fine-tuning, as seen in Fig. 7. The fine-tuning is maintained naturally in a supersymmetric theory, but is destroyed if one has top quarks and their supersymmetric partners $\tilde{t}$, but not the supersymmetric partners $\tilde{H}$ of the Higgs bosons.

If the new physics below $10^6$ GeV is not supersymmetry, it must quack very much like it!
Fig. 3: The $\chi^2$ function for the mass of the Higgs boson in the Standard Model provided by precision electroweak data, for two different estimates of the hadronic contribution to the effective value of $\alpha_{em}(m_Z)$. The shaded (blue) band covers other theoretical uncertainties [19].

Fig. 4: The probability distribution for the mass of the Higgs boson in the Standard Model obtained by combining precision electroweak data with the lower limit coming from direct experimental searches [21].
Fig. 5: Range of $m_H$ allowed in the Standard Model if it is to remain valid up to a scale $\Lambda$ \[23\]. When $m_H$ is too large, renormalization of the Higgs self-coupling causes it to blow up at some scale below $\Lambda$. When $m_H$ is too small, renormalization of the effective Higgs potential by the $t$-quark Yukawa coupling drives it negative, rendering the present electroweak vacuum unstable.

Fig. 6: Renormalization of the effective Higgs self-coupling for different values of the coupling $M_0$ to new bosons $\phi$. It is seen that the coupled system must be tuned very finely in order for the potential not to collapse or blow up \[23\].
1.4 Why Supersymmetry?

The main theoretical reason to expect supersymmetry at an accessible energy scale is provided by the hierarchy problem [4]: why is $m_W \ll m_P$, or equivalently why is $G_F \sim 1/m_W^2 \gg G_N = 1/m_P^2$? Another equivalent question is why the Coulomb potential in an atom is so much greater than the Newton potential: $e^2 \gg G_N m^2 = m^2/m_P$, where $m$ is a typical particle mass?

Your first thought might simply be to set $m_P \gg m_W$ by hand, and forget about the problem. Life is not so simple, because, as already mentioned, quantum corrections to $m_H$ and hence $m_W$ are quadratically divergent in the Standard Model:

$$\delta m_{H,W}^2 \sim O(\frac{\alpha}{\pi}) \Lambda^2,$$

which is $\gg m_W^2$ if the cutoff $\Lambda$, which represents the scale where new physics beyond the Standard Model appears, is comparable to the GUT or Planck scale. For example, if the Standard Model were to hold unscathed all the way up the Planck mass $m_P \sim 10^{19}$ GeV, the radiative correction (15) would be 36 orders of magnitude greater than the physical values of $m_{H,W}^2$!

In principle, this is not a problem from the mathematical point of view of renormalization theory. All one has to do is postulate a tree-level value of $m_H^2$ that is (very nearly) equal and opposite to the ‘correction’ (15), and the correct physical value may be obtained. However, this strikes many physicists as rather unnatural: they would prefer a mechanism that keeps the ‘correction’ (15) comparable at most to the physical value.

This is possible in a supersymmetric theory, in which there are equal numbers of bosons and fermions with identical couplings. Since bosonic and fermionic loops have opposite signs, the residual one-loop correction is of the form

$$\delta m_{H,W}^2 \sim O(\frac{\alpha}{\pi})(m_B^2 - m_F^2),$$

which is $\lesssim m_{H,W}^2$ and hence naturally small if the supersymmetric partner bosons $B$ and fermions $F$ have similar masses:

$$|m_B^2 - m_F^2| \lesssim 1 \text{ TeV}^2.$$
This is the best motivation we have for finding supersymmetry at relatively low energies \([4]\).

In addition to this first supersymmetric miracle of removing the quadratic divergence, many logarithmic divergences are also absent in a supersymmetric theory. This is the underlying reason why supersymmetry solves the fine-tuning problem of the effective Higgs potential when \(m_H \sim 115\) GeV, as advertised in the previous section. Note that this argument is logically distinct from the absence of quadratic divergences in a supersymmetric theory.

Many other arguments for supersymmetry were proposed before this hierarchy/naturalness argument. Some of them remain valid, but none of them fixed the scale at which supersymmetry should appear.

Back in the 1960’s, there were many attempts to combine internal symmetries such as flavour SU(2) or SU(3) with external Lorentz symmetries, in groups such as SU(6) and \(\tilde{U}(12)\). However, it was shown in 1967 by Coleman and Mandula \([25]\) that no non-trivial combination of internal and external symmetries could be achieved using just bosonic charges. The first non-trivial extension of the Poincaré algebra with fermionic charges was made by Golfand and Likhtman in 1971 \([26]\), and in the same year Neveu and Schwarz \([27]\), and Ramond \([28]\), proposed two-dimensional supersymmetric models in attempts to make fermionic string theories that could accommodate baryons. Two years later, the first interesting four-dimensional supersymmetric field theories were written down. Volkov and Akulov \([29]\) wrote down a non-linear realization of supersymmetry with a massless fermion, that they hoped to identify with the neutrino, but this identification was soon found not to work, because the low-energy interactions of neutrinos differed from those in the non-linear supersymmetric model.

Also in 1973, Wess and Zumino \([30, 31]\) started writing down renormalizable four-dimensional supersymmetric field theories, with the objective of describing mesons and baryons. Soon afterwards, together with Iliopoulos and Ferrara, they were able to show that supersymmetric field theories lacked many of the quadratic and other divergences found in conventional field theories \([32]\), and some thought this was an attractive feature, but the physical application remained obscure for several years. Instead, for some time, phenomenological interest in supersymmetry was focused on the possibility of unifying fermions and bosons, for example matter particles (with spin 1/2) and force particles (with spin 1), or alternatively matter and Higgs particles, in the same supermultiplets \([33]\). With the discovery of local supersymmetry, or supergravity, in 1976 \([34]\), this hope was extended to the unification of the graviton with lower-spin particles. Indeed, for a short while, the largest supergravity theory was touted as the TOE: in the words of Hawking \([35]\), ‘Is the end in sight for theoretical physics?’.

These are all attractive ideas, and many play rôles in current theories, but I reiterate that the only real motivation for expecting supersymmetry at accessible energies \(\lesssim 1\) TeV is the naturalness of the mass hierarchy \([4]\).

### 1.5 What is Supersymmetry?

The basic idea of supersymmetry is the existence of fermionic charges \(Q_\alpha\) that relate bosons to fermions. Recall that all previous symmetries, such as flavour SU(3) or electromagnetic U(1), have involved scalar charges \(Q\) that link particles with the same spin into multiplets:

\[
Q \left| \text{Spin} J \right> = \left| \text{Spin} J \right>.
\]

Indeed, as mentioned above, Coleman and Mandula \([25]\) proved that it was ‘impossible’ to mix internal and Lorentz symmetries: \(J_1 \leftrightarrow J_2\). However, their ‘no-go’ theorem assumed implicitly that the prospective charges should have integer spins.

The basic element in their ‘proof’ was the observation that the only possible conserved tensor charges were those with no Lorentz indices, i.e., scalar charges, and the energy-momentum vector \(P_\mu\). To see how their ‘proof’ worked, consider two-to-two elastic scattering, \(1 + 2 \rightarrow 3 + 4\), and imagine that there exists a conserved two-index tensor charge, \(\Sigma_{\mu\nu}\). By Lorentz invariance, its diagonal matrix
elements between single-particle states \( |a\rangle \) must take the general form:

\[
< a | \Sigma_{\mu\nu} | a \rangle = \alpha P^{(a)}_\mu P^{(a)}_\nu + \beta g_{\mu\nu},
\]

where \( \alpha, \beta \) are arbitrary reduced matrix elements, and \( g_{\mu\nu} \) is the metric tensor. For \( \Sigma_{\mu\nu} \) to be conserved in a two-to-two scattering process, one must have

\[
P^{(1)}_\mu P^{(1)}_\nu + P^{(2)}_\mu P^{(2)}_\nu = P^{(3)}_\mu P^{(3)}_\nu + P^{(4)}_\mu P^{(4)}_\nu,
\]

where we assume that the symmetry is local, so that two-particle matrix elements of \( \Sigma_{\mu\nu} \) play no rôle. Since Lorentz invariance also requires \( P^{(1)}_\mu + P^{(2)}_\mu = P^{(3)}_\mu + P^{(4)}_\mu \), the only possible outcomes are \( P^{(1)}_\mu = P^{(3)}_\mu \) or \( P^{(4)}_\mu \). Thus the only possibilities are completely forward scattering or completely backward scattering. This disagrees with observation, and in fact theoretically impossible in any local field theory.

This rules out any non-trivial two-index tensor charge, and the argument can clearly be extended to any higher-rank tensor with more Lorentz indices. But what about a spinorial charge \( Q_\alpha \)? This can have no diagonal matrix element:

\[
< a | Q_\alpha | a \rangle \neq 0,
\]

and hence the Coleman-Mandula argument fails.

So what is the possible form of a ‘supersymmetry’ algebra that includes such spinorial charges \( Q^i_\alpha \)? Since the different \( Q^i \) are supposed to generate symmetries, they must commute with the Hamiltonian:

\[
\{Q^i, H\} = 0 : i = 1, 2, \ldots, N.
\]

So also must the anticommutator of two spinorial charges:

\[
\{\{Q^i, Q^j\}, H\} = 0 : i, j = 1, 2, \ldots, N.
\]

However, the part of the anticommutator \( \{Q^i, Q^j\} \) that is symmetric in the internal indices \( i, j \) cannot have spin 0. Instead, as we discussed just above, the only possible non-zero spin choice is \( J = 1 \), so that

\[
\{Q^i, Q^j\} \propto \delta^{ij} P_\mu + \ldots : i, j = 1, 2, \ldots, N.
\]

In fact, as was proved by Haag, Lopuszanski and Sohnius \[36\], the only allowed possibility is

\[
\{Q^i, Q^j\} = 2\delta^{ij} \gamma^\mu P_\mu C + \ldots : i, j = 1, 2, \ldots, N,
\]

where \( C \) is the charge-conjugation matrix discussed in more detail in Lecture 2, and the dots denote a possible ‘central charge’ that is antisymmetric in the indices \( i, j \), and hence can only appear when \( N > 1 \).

According to a basic principle of Swiss law, anything not illegal is compulsory, so there MUST exist physical realizations of the supersymmetry algebra \[25\]. Indeed, non-trivial realizations of the non-relativistic analogue of \[25\] are known from nuclear physics \[37\], atomic physics and condensed-matter physics. However, none of these is thought to be fundamental.

In the relativistic limit, supermultiplets consist of massless particles with spins differing by half a unit. In the case of simple \( N = 1 \) supersymmetry, the basic building blocks are chiral supermultiplets:

\[
\left( \begin{array}{c} \frac{1}{2} \\ 0 \end{array} \right) e.g., \left( \begin{array}{c} \ell (\text{lepton}) \\ \tilde{\ell} (\text{slepton}) \end{array} \right) \text{ or } \left( \begin{array}{c} q (\text{quark}) \\ \tilde{q} (\text{squark}) \end{array} \right)
\]

\(\text{gauge supermultiplets:}\)

\[1\] In what follows, I shall suppress the spinorial subscript \( \alpha \) whenever it is not essential. The superscripts \( i, j, \ldots, N \) denote different supersymmetry charges.
and the graviton supermultiplet consisting of the spin-2 graviton and the spin-3/2 gravitino.

Could any of the known particles in the Standard Model be linked together in supermultiplets? Unfortunately, none of the known fermions $q, \ell$ can be paired with any of the known bosons $\gamma, W^\pm Z^0, g, H$, because their internal quantum numbers do not match \[33\]. For example, quarks $q$ sit in triplet representations of colour, whereas the known bosons are either singlets or octets of colour. Then again, leptons $\ell$ have non-zero lepton number $L = 1$, whereas the known bosons have $L = 0$. Thus, the only possibility seems to be to introduce new supersymmetric partners (sparticles) for all the known particles: quark $\rightarrow$ squark, lepton $\rightarrow$ slepton, photon $\rightarrow$ photino, $Z \rightarrow$ Zino, $W \rightarrow$ Wino, gluon $\rightarrow$ gluino, Higgs $\rightarrow$ Higgsino, as suggested in (26, 27) above.

The best that one can say for supersymmetry is that it economizes on principle, not on particles!

1.6 (S)Experimental Hints

By now, you may be wondering whether it makes sense to introduce so many new particles just to deal with a paltry little hierarchy or naturalness problem. But, as they used to say during the First World War, ‘if you know a better hole, go to it.’ As we learnt above, technicolour no longer seems to be a viable hole, and I am not convinced that theories with large extra dimensions really solve the hierarchy problem, rather than just rewrite it. Fortunately, there are two hints from the high-precision electroweak data that supersymmetry may not be such a bad hole, after all.

One is the fact, already advertised, that there probably exists a Higgs boson weighing less than about 200 GeV \[19\]. This is perfectly consistent with calculations in the minimal supersymmetric extension of the Standard Model (MSSM), in which the lightest Higgs boson weighs less than about 130 GeV \[38\], as we discuss in more detail in Lecture 3.

The other hint is provided by the strengths of the different gauge interactions, as measured at LEP \[3\]. These may be run up to high energy scales using the renormalization-group equations, to see whether they unify as predicted in a GUT. The answer is no, if supersymmetry is not included in the calculations. In that case, GUTs would require

$$\sin^2 \theta_W = 0.214 \pm 0.004,$$

whereas the experimental value of the effective neutral weak mixing parameter at the $Z^0$ peak is $\sin^2 \theta = 0.23149 \pm 0.00017$ \[19\]. On the other hand, minimal supersymmetric GUTs predict

$$\sin^2 \theta_W \sim 0.232,$$

where the error depends on the assumed sparticle masses, the preferred value being around 1 TeV, as suggested completely independently by the naturalness of the electroweak mass hierarchy. This argument is also discussed in more detail in Lecture 3.

2. SIMPLE MODELS

2.1 Deconstructing Dirac

In this Section, we tackle some unavoidable spinorology. The most familiar spinors used in four-dimensional field theories are four-component Dirac spinors $\psi$. You may recall that it is possible to introduce projection operators

$$P_{L,R} \equiv \frac{1}{2}(1 \pm \gamma_5),$$

\[30\]
where \( \gamma_5 \equiv i \gamma^0 \gamma^1 \gamma^2 \gamma^3 \), and the \( \gamma_{\mu} \) can be written in the forms

\[
\gamma_{\mu} = \begin{pmatrix} 0 & \sigma_{\mu} \\ \bar{\sigma}_{\mu} & 0 \end{pmatrix},
\]

(31)

where \( \sigma_{\mu} \equiv (1, \sigma_i) \), \( \bar{\sigma}_{\mu} \equiv (1, -\sigma_i) \). Then \( \gamma_5 \) can be written in the form \( \text{diag}(-1, 1) \), where \(-1, 1\) denote \( 2 \times 2 \) matrices. Next, we introduce the corresponding left- and right-handed spinors

\[
\psi_{L,R} \equiv P_{L,R} \psi,
\]

(32)

in terms of which one may decompose the four-component spinor into a pair of two-component spinors:

\[
\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}.
\]

(33)

These will serve as our basic fermionic building blocks.

Antifermions can be represented by adjoint spinors

\[
\tilde{\psi} \equiv \psi^\dagger \gamma^0 = (\bar{\psi}_R, \bar{\psi}_L)
\]

(34)

where the \( \gamma^0 \) factor has interchanged the left- and right-handed components \( \psi_{L,R} \). We can now decompose in terms of these the conventional fermion kinetic term

\[
\bar{\psi} \gamma_{\mu} \partial^\mu \psi = \bar{\psi}_L \gamma_{\mu} \partial^\mu \psi_L + \bar{\psi}_R \gamma_{\mu} \partial^\mu \psi_R
\]

(35)

and the conventional mass term

\[
\bar{\psi} \psi = \bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R.
\]

(36)

We see that the kinetic term keeps separate the left- and right-handed spinors, whereas the mass term mixes them.

The next step is to introduce the charge-conjugation operator \( C \), which changes the overall sign of the vector current \( \bar{\psi} \gamma_{\mu} \psi \). It transforms spinors into their conjugates:

\[
\psi^c \equiv C \bar{\psi}^T = C (\psi^\dagger \gamma^0)^T = \begin{pmatrix} \bar{\psi}_R \\ \bar{\psi}_L \end{pmatrix},
\]

(37)

and operates as follows on the \( \gamma \) matrices:

\[
C^{-1} \gamma_{\mu} C = -\gamma_{\mu}^T.
\]

(38)

A convenient representation of \( C \) is:

\[
C = i \gamma^0 \gamma^2.
\]

(39)

It is apparent from the above that the conjugate of a left-handed spinor is right-handed:

\[
(\psi_L)^c = \begin{pmatrix} 0 \\ \bar{\psi}_L \end{pmatrix},
\]

(40)

so that the combination

\[
\bar{\psi}^c_L \psi_L = \psi_L \sigma_2 \psi_L
\]

(41)

mixes left- and right-handed spinors, and has the same form as a mass term (36).

It is apparent from (40) that we can construct four-component Dirac spinors entirely out of two-component left-handed spinors and their conjugates:

\[
\psi = \begin{pmatrix} \psi_i \\ \psi_i^c \end{pmatrix},
\]

(42)

a trick that will be useful later in our supersymmetric model-building. As examples, instead of working with left- and right-handed quark fields \( q_L \) and \( q_R \), or left- and right-handed lepton fields \( \ell_L \) and \( \ell_R \), we can write the theory in terms of left-handed antiquarks and antileptons: \( q_R \rightarrow q^c_L \) and \( \ell_R \rightarrow \ell^c_L \).
2.2 Simplest Supersymmetric Field Theories

Let us now consider a field theory containing just a single left-handed fermion $\psi_L$ and a complex boson $\phi$, without any interactions, as described by the Lagrangian

$$L_0 = i\bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + |\partial \phi|^2.$$  \hspace{1cm} (43)

We consider the simplest possible non-trivial transformation law for the free theory (43):

$$\phi \rightarrow \phi + \delta \phi, \text{ where } \delta \phi = \sqrt{2} E \bar{\psi}_L,$$  \hspace{1cm} (44)

where $E$ is some constant right-handed spinor. In parallel with (44), we also consider the most general possible transformation law for the fermion $\psi$:

$$\psi_L \rightarrow \psi_L + \delta \psi_L, \text{ where } \delta \psi_L = -a i \sqrt{2}(\gamma^\mu \partial_\mu \phi)E - FE^c,$$  \hspace{1cm} (45)

where $a$ and $F$ are constants to be fixed later, and we recall that $E^c$ is a left-handed spinor. We can now consider the resulting transformation of the full Lagrangian (43), which can easily be checked to take the form

$$\delta L_0 = \sqrt{2} \partial_\mu [\psi E \partial^\mu \phi + E \gamma^\mu \phi^* \gamma_\nu \partial_\nu \psi],$$  \hspace{1cm} (46)

if and only if we choose

$$a = 1 \text{ and } F = 0$$  \hspace{1cm} (47)

in this free-field model. With these choices, and the resulting total-derivative transformation law (46) for the free Lagrangian, the free action $A_0$ is invariant under the transformations (44,45), since

$$\delta A_0 = \delta \int d^4x L_0 = 0.$$  \hspace{1cm} (48)

Fine, you may say, but is this symmetry actually supersymmetry? To convince yourself that it is, consider the sequences of pairs (44,45) of transformations starting from either the boson $\phi$ or the fermion $\psi$:

$$\phi \rightarrow \psi \rightarrow \partial \phi, \hspace{0.5cm} \psi \rightarrow \partial \phi \rightarrow \partial \psi.$$  \hspace{1cm} (49)

In both cases, the action of two symmetry transformations is equivalent to a derivative, i.e., the momentum operator, corresponding exactly to the supersymmetry algebra. A free boson and a free fermion together realize supersymmetry: like the character in Molière, we have been talking prose all our lives without realizing it!

Now we look at interactions in a supersymmetric field theory. The most general interactions between spin-0 fields $\phi^i$ and spin-1/2 fields $\psi^i$ that are at most bilinear in the latter, and hence have a chance of being renormalizable in four dimensions, can be written in the form

$$L = L_0 - V(\phi^i, \phi^*_j) - \frac{1}{2} M_{ij}(\phi, \phi^*) \bar{\psi}^i \psi^j$$  \hspace{1cm} (50)

where $V$ is a general effective potential, and $M_{ij}$ includes both mass terms and Yukawa interactions for the fermions. Supersymmetry imposes strong constraints on the allowed forms of $V$ and $M$, as we now see. Suppose that $M$ depended non-trivially on the conjugate fields $\phi^*$: then the supersymmetric variation $\delta(M \bar{\psi}^c \psi)$ would contain a term

$$\frac{\partial M}{\partial \phi^*} \bar{\psi}^c \psi$$  \hspace{1cm} (51)

that could not be compensated by the variation of any other term. We conclude that $M$ must be independent of $\phi^*$, and hence $M = M(\phi)$ alone.
Another term in the variation of the last term in (50) is
\[ \frac{\partial M_{ij}}{\partial \phi^k} \bar{E} \psi^k \bar{\psi}^c \psi^j. \] (52)

This term cannot be cancelled by the variation of any other term, but can vanish by itself if \( \frac{\partial M_{ij}}{\partial \phi^k} \) is completely symmetric in the indices \( i, j, k \). This is possible only if
\[ M_{ij} = \frac{\partial W}{\partial \phi^i \partial \phi^j} \] (53)
for some function \( W(\phi) \) called the superpotential. If the theory is to be renormalizable, \( W \) can only be cubic. The trilinear term of \( W \) determines the Yukawa couplings, and the bilinear part the mass terms.

We now re-examine the form of the supersymmetric transformation law (45) itself. Yet another term in the variation of the second term in (50) has the form
\[ i M_{jk} \bar{\psi}^c \gamma^\mu \partial^\mu \phi^k E + (\text{Herm.Conj.}) \] (54)
This can cancel against an \( F \)-dependent term in the variation of the fermion kinetic term
\[ -i \bar{\psi}_i \gamma_\mu \partial^\mu F_i E^c + (\text{Herm.Conj.}) \] (55)
if the following relation between \( F \) and \( M \) holds:
\[ \frac{\partial F_i^*}{\partial \phi^j} = M_{ij}, \] (56)
Thus the form of \( W \) also determines the required form of the supersymmetry transformation law.

The form of \( W \) also determines the effective potential \( V \), as we now see. One of the terms in the variation of \( V \) is
\[ \frac{\partial V}{\partial \phi^j} \bar{E} \psi^j + (\text{Herm.Conj.}) \] (57)
which can only be cancelled by a term in the variation of \( M_{ij} \bar{\psi}^c \psi^j \), which can take the form \( M_{ij} \bar{\psi}^c F_j^* E^c \) if
\[ \frac{\partial V}{\partial \phi^j} = M_{ij} F^i, \] (58)
which is in turn possible only if
\[ V = |\frac{\partial W}{\partial \phi^i}|^2 = |F^i|^2. \] (59)

We now have the complete supersymmetric field theory for interacting chiral (matter) supermultiplets [30]:
\[ L = i \bar{\psi}_i \gamma_\mu \partial^\mu \psi^i + |\partial_\mu \phi|^2 - |\frac{\partial W}{\partial \phi^j}|^2 - \frac{1}{2} \partial^2 W \bar{\psi}^c \psi^j \partial \phi^i + (\text{Herm.Conj.}) \] (60)
This Lagrangian is invariant (up to a total derivative) under the supersymmetry transformations
\[ \delta \phi^i = \sqrt{2} \bar{E} \psi^i, \quad \delta \psi^i = -i \sqrt{2} \gamma_\mu \partial^\mu \phi^3 E - F^i E^c : F^i = \left( \frac{\partial W}{\partial \phi^i} \right)^*, \] (61)
The simplest non-trivial superpotential involving a single superfield \( \phi \) is
\[ W = \frac{\lambda}{3} \phi^3 + \frac{m}{2} \phi^2. \] (62)
It is a simple exercise for you to verify using the rules given above that the corresponding Lagrangian is

$$L = i\bar{\psi}\gamma_{\mu}\partial^{\mu}\psi + |\partial_{\mu}\phi|^{2} - |m\phi + \lambda\phi^{2}|^{2} - m\bar{\psi}\psi - \lambda\phi\bar{\psi}\psi.$$  \hspace{1cm} (63)

We see explicitly that the bosonic component $\phi$ of the supermultiplet has the same mass as the fermionic component $\psi$, and that the Yukawa coupling $\lambda$ fixes the effective potential.

We now turn to the possible form of a supersymmetric gauge theory \[31\]. Clearly, it must contain vector fields $A_{\mu}^{a}$ and fermions $\chi^{a}$ in the same adjoint representation of the gauge group. Once one knows the gauge group and the fermionic matter content, the form of the Lagrangian is completely determined by gauge invariance:

$$L = i\bar{\chi}^{a}\gamma^{\mu}D_{ab}^{\mu}\chi^{b} - \frac{1}{4}F^{a}_{\mu\nu}F^{a,\mu\nu} - \frac{1}{2}(D^{a})^{2}. \hspace{1cm} (64)$$

Here, the gauge-covariant derivative

$$D_{ab}^{\mu} \equiv \delta_{ab}\partial^{\mu} - gf_{abc}A_{c}^{\mu}, \hspace{1cm} (65)$$

and the gauge field strength is

$$F^{a}_{\mu\nu} \equiv \partial_{\mu}A_{\nu}^{a} - \partial_{\nu}A_{\mu}^{a} + gf^{abc}A_{\mu}^{b}A_{\nu}^{c}, \hspace{1cm} (66)$$
as usual. We return later to the $D$ term at the end of \[64\]. Yet another of the miracles of supersymmetry is that the Lagrangian \[64\] is automatically supersymmetric, without any further monkeying around. The corresponding supersymmetry transformations may be written as

$$\delta A_{\mu}^{a} = -\bar{E}\gamma_{\mu}\chi^{a}, \hspace{1cm} (67)$$

$$\delta \chi^{a} = -\frac{i}{2}F^{a}_{\mu\nu}\gamma^{\mu}\gamma^{\nu}E + D^{a}E, \hspace{1cm} (68)$$

$$\delta D^{a} = -i\bar{E}\gamma_{\mu}\gamma^{\mu}D_{ab}^{\mu}\chi^{b}. \hspace{1cm} (69)$$

What about the $D$ term in \[64\]? It is a trivial consequence of equations of motion derived from \[64\] that $D^{a} = 0$. However, this is no longer the case if matter is included. Then, it turns out, one must add to \[64\] the following:

$$\Delta L = -\sqrt{2}g\chi^{a}\phi_{i}^{a}(T^{a})_{ij}\psi^{j} + \text{(Herm.Conj.)} + g(\phi_{i}^{a}(T^{a})_{ij}\phi^{j})D^{a}, \hspace{1cm} (70)$$

where $T^{a}$ is the group representation matrix for the matter fields $\phi^{i}$. With this addition, the equation of motion for $D^{a}$ tells us that

$$D^{a} = g\phi_{i}^{a}(T^{a})_{ij}\phi^{j}, \hspace{1cm} (71)$$

and we find a $D$ term in the full effective potential:

$$V = \Sigma_{i}|F_{i}|^{2} + \Sigma_{a}\frac{1}{2}(D^{a})^{2}, \hspace{1cm} (72)$$

where the form of $D^{a}$ is given in \[70\].

### 2.3 Further Aspects of Supersymmetric Field Theories

So far, we have taken a relatively unsophisticated approach to supersymmetry. However, one of the reasons why theorists are so enthusiastic about supersymmetry is because it is not just a new type of symmetry, but extends the concept of space-time itself. Recall the basic form of the supersymmetry algebra:

$$2\delta^{ij}\gamma_{\mu}P^{\mu}C = \{Q^{i}, Q^{j}\}. \hspace{1cm} (73)$$
The reason this is written backwards here is to emphasize that one can regard supersymmetric charges $Q^i$ as square roots of the translation operator. Recall how the translation operator acts on a bosonic field:

$$\phi(x + a) = e^{i a \cdot P} \phi(x) e^{-i a \cdot P}, \quad (74)$$

where the momentum operator $P$ is the generator of infinitesimal translations:

$$i [P_\mu, \phi(x)] = \partial_\mu \phi(x). \quad (75)$$

Expanding the formula (74), we find the following expression for a small finite translation:

$$\delta a \phi(x) \equiv \phi(x + a) - \phi(x) \simeq a_\mu \partial^\mu \phi(x) = ia_\mu [P^\mu, \phi(x)]. \quad (76)$$

Following this deconstruction of translations, we now can see better how the supersymmetric charge can be regarded, in some sense, as the square root: $Q \sim \sqrt{P}$, just as the Dirac equation can be regarded as the square root of the Klein-Gordon equation. There is an exact supersymmetric analogue of (76):

$$\delta E \phi(x) = \sqrt{2} \bar{\psi} \psi(x) = i \sqrt{2} \bar{E} [Q, \phi(x)]. \quad (77)$$

By analogy with (74, 76), one may consider the spinor $E$ as a sort of ‘superspace’ coordinate, and one can combine the bosonic field $\phi(x)$ and its fermionic partner $\psi(x)$ into a superfield:

$$\delta E \phi(x) = \Phi(x, \bar{E}) - \Phi(x) : \Phi(x, \bar{E}) \equiv \phi(x) + \sqrt{2} \bar{E} \psi(x). \quad (78)$$

At this level, the introduction of superspace and superfields may appear superfluous, but it gives deeper insights into the theory and facilitates the derivation of many important results, such as the non-renormalization theorems of supersymmetry, that we discuss next. In some sense, the next generation of accelerators such as the LHC is ‘guaranteed’ to discover extra dimensions, either bosonic ones, as discussed here by Antoniadis [40], or fermionic.

Many remarkable no-renormalization theorems can be proved in supersymmetric field theories [32]. First and foremost, they have no quadratic divergences. One way to understand this is to compare the renormalizations of bosonic and fermionic mass terms:

$$m_B^2 |\phi|^2 \leftrightarrow m_F \bar{\psi} \psi. \quad (79)$$

We know well that fermion masses $m_F$ can only be renormalized logarithmically. Since supersymmetry guarantees that $m_B = m_F$, it follows that there can be no quadratic divergence in $m_B$. Going further, chiral symmetry guarantees that the one-loop renormalization of a fermion mass has the general multiplicative form:

$$\delta m_F = \mathcal{O}(\frac{\alpha}{\pi}) m_F \ln(\frac{\mu_1}{\mu_2}), \quad (80)$$

where $\mu_{1,2}$ are different renormalization scales. This means that if $m_F$ (and hence also $m_B$) vanish at the tree level in a supersymmetric theory, then both $m_F$ and $m_B$ remain zero after renormalization. This is one example of the reduction in the number of logarithmic divergences in a supersymmetric theory.

In general, there is no intrinsic renormalization of any superpotential parameters, including the Yukawa couplings $\lambda$, apart from overall multiplicative factors due to wave-function renormalizations:

$$\Phi \to Z \Phi, \quad (81)$$

which are universal for both the bosonic and fermionic components $\phi, \psi$ in a given superfield $\Phi$. However, gauge couplings are renormalized, though the $\beta$-function is changed:

$$\beta(g) \neq 0: \quad -11 N_c \to -9 N_c \quad (82)$$
at one-loop order in an $SU(N_c)$ supersymmetric gauge theory with no matter, as a result of the extra gaugino contributions.

There are even fewer divergences in theories with more supersymmetries. For example, there is only a finite number of divergent diagrams in a theory with $N = 2$ supersymmetries, which may be cancelled by imposing a few simple relations on the spectrum of supermultiplets. Finally, there are no divergences at all in theories with $N = 4$ supersymmetries, which obey automatically the necessary finiteness conditions.

Many theorists from Dirac onwards have found the idea of a completely finite theory attractive, so it is natural to ask whether theories with $N \geq 2$ supersymmetries could be interesting as realistic field theories. Unfortunately, the answer is ‘not immediately’, because they do not allow the violation of parity. To see why, consider the simplest possible extended supersymmetric theory containing an $N = 2$ matter multiplet, which contains both left- and right-handed fermions with helicities $\pm 1/2$. Suppose that the left-handed fermion with helicity $+1/2$ sits in a representation $R$ of the gauge group. Now act on it with either of the two supersymmetry charges $Q_{1,2}$: they each yield bosons, that each sit in the same representation $R$. Now act on either of these with the other supercharge, to obtain a right-handed fermion with helicity $-1/2$: this must also sit in the same representation $R$ of the gauge group. Hence, left- and right-handed fermions have the same interactions, and parity is conserved. There is no way out using gauginos, because they are forced to sit in adjoint representations of the gauge group, and hence also cannot distinguish between right and left.

Thus, if we want to make a supersymmetric extension of the Standard Model, it had better be with just $N = 1$ supersymmetry, and this is what we do in the next Section.

### 2.4 Building Supersymmetric Models

Any supersymmetric model is based on a Lagrangian that contains a supersymmetric part and a supersymmetry-breaking part:

$$\mathcal{L} = \mathcal{L}_{\text{susy}} + \mathcal{L}_{\text{susyX}}.$$  \hfill (83)

We discuss the supersymmetry-breaking part $\mathcal{L}_{\text{susyX}}$ in the next Lecture: here we concentrate on the supersymmetric part $\mathcal{L}_{\text{susy}}$. The minimal supersymmetric extension of the Standard Model (MSSM) has the same gauge interactions as the Standard Model, and Yukawa interactions that are closely related. They are based on a superpotential $W$ that is a cubic function of complex superfields corresponding to left-handed fermion fields. Conventional left-handed lepton and quark doublets are denoted $L,Q$ and right-handed fermions are introduced via their conjugate fields, which are left-handed, $e_R \rightarrow E^c, u_R \rightarrow U^c, d_R \rightarrow D^c$. In terms of these,

$$W = \Sigma_{L,E^c} \lambda_L LE^c H_1 + \Sigma_{Q,U^c} \lambda_U QU^c H_2 + \Sigma_{Q,D^c} \lambda_D QD^c H_1 + \mu H_1 H_2.$$  \hfill (84)

A few words of explanation are warranted. The first three terms in (84) yield masses for the charged leptons, charge-$(+2/3)$ quarks and charge-$(-1/3)$ quarks respectively. All of the Yukawa couplings $\lambda_{L,U,D}$ are $3 \times 3$ matrices in flavour space, whose diagonalizations yield the mass eigenstates and Cabibbo-Kobayashi-Maskawa mixing angles.

Note that two distinct Higgs doublets $H_{1,2}$ have been introduced, for two important reasons. One reason is that the superpotential must be an analytic polynomial: as we saw in (84), it cannot contain both $H$ and $H^*$, whereas the Standard Model uses both of these to give masses to all the quarks and leptons with just a single Higgs doublet. The other reason is to cancel the triangle anomalies that destroy the renormalizability of a gauge theory. Ordinary Higgs boson doublets do not contribute to these anomalies, but the fermions in Higgs supermultiplets do, and two doublets are required to cancel each others’ contributions. Once two Higgs supermultiplets have been introduced, there is the possibility, even the necessity, of a bilinear term $\mu H_1 H_2$ coupling them together.
Once the MSSM superpotential (84) has been specified, the effective potential is also fixed:
\[
V = \sum_i |F_i|^2 + \frac{1}{2} \sum_a (D^a)^2 : \quad F_i^* = \frac{\partial W}{\partial \phi_i}, \quad D^a = g_a \phi_i^*(T^a)_j^i \phi^j,
\]
according to the rules explained earlier in this Lecture, where the sums run over the different chiral fields \(i\) and the \(SU(3), SU(2)\) and \(U(1)\) gauge-group factors \(a\).

There are important possible variations on the MSSM superpotential (84), which are impossible in the Standard Model, but are allowed by the gauge symmetries of the MSSM supermultiplets. These are additional superpotential terms that violate the quantity known as \(R\) parity:
\[
R \equiv (-1)^{3B+L+2S}, \tag{86}
\]
where \(B\) is baryon number, \(L\) is lepton number, and \(S\) is spin. It is easy to check that \(R = +1\) for all the particles in the Standard Model, and \(R = -1\) for all their sparticles, which have identical values of \(B\) and \(L\), but differ in spin by half a unit. Clearly, \(R\) would be conserved if both \(B\) and \(L\) were conserved, but this is not automatic. Consider the following superpotential terms:
\[
\lambda_{ijk} L_i L_j E_k^c + \lambda_{ijk} L_i Q_j D_k^c + \lambda_{ijk} U_i^c D_j^c D_k^c + \epsilon_i H L_i, \tag{87}
\]
which are visibly \(SU(3) \times SU(2) \times U(1)\) symmetric. The first term in (87) would violate \(L\), causing for example \(\tilde{\ell} \rightarrow \ell + \ell\), the second would violate both \(B\) and \(L\), causing for example \(\tilde{q} \rightarrow q + \ell\), the third would violate \(B\), causing for example \(\tilde{\tilde{q}} \rightarrow \tilde{q} + \tilde{q}\), and the last would violate \(L\) by causing \(H \leftrightarrow L_i\) mixing. These interactions would provide many exciting signatures for supersymmetry, such as dilepton events, jets plus leptons and multijet events. Such interactions are constrained by direct searches, by the experimental limits on flavour-changing interactions and other rare processes, and by cosmology: they would tend to wipe out the baryon asymmetry of the Universe if they are too strong [41]. They would also cause the lightest supersymmetric particle to be unstable, not necessarily a disaster in itself, but it would remove an excellent candidate for the cold dark matter that apparently abounds throughout the Universe. For simplicity, the conservation of \(R\) parity will be assumed in the rest of these Lectures.

We now look briefly at the construction of supersymmetric GUTs, of which the minimal version is based on the group \(SU(5)\) [42]. As in the transition from the Standard Model to the MSSM, one simply extends the conventional GUT multiplets to supermultiplets, so that matter particles are assigned to 5 representations \(\bar{F}\) and 10 representations \(T\), one doubles the electroweak Higgs fields to include both \(H, \bar{H}\) in 5, \(\bar{5}\) representations, and one postulates a 24 representation \(\Phi\) to break the \(SU(5)\) GUT symmetry down to \(SU(3) \times SU(2) \times U(1)\). The superpotential for the Higgs sector takes the general form
\[
W_5 = (\mu + \frac{3}{2} \lambda M) H \bar{H} + \lambda H \Phi \bar{H} + f(\Phi). \tag{88}
\]
Here, \(f(\Phi)\) is chosen so that the vacuum expectation value of \(\Phi\) has the form
\[
<0|\Phi|0> = M \times \text{diag}(1,1,1,-\frac{3}{2},-\frac{3}{2}). \tag{89}
\]
The coefficient of the \(H \bar{H}\) term has been chosen so that it almost cancels with the term \(\propto H <0|\Phi|0> \bar{H}\) coming from the second term in (88), for the last two components. In this way, the triplet components of \(H, \bar{H}\) acquire large masses \(\propto M\), whilst the last two may acquire a vacuum expectation value: \(<0|H|0> = \text{column}(0,0,0,0,0), <0|\bar{H}|0> = \text{column}(0,0,0,0,\bar{v})\), once supersymmetry breaking and radiative corrections are taken into account, as in the next Lecture.

In order that \(v, \bar{v} \sim 100\text{ GeV}\), it is necessary that the residual \(H \bar{H}\) mixing term \(\mu \lesssim 1\text{ TeV}\). Since, as we recall shortly, \(M \sim 10^{16}\text{ GeV}\), this means that the parameters of \(W_5\) (88) must be tuned finely to one part in \(10^{13}\). This fine-tuning may appear very unreasonable, but it is technically natural, in the sense
that there are no big radiative corrections. Thanks to the supersymmetric no-renormalization theorem for superpotential parameters, we know that $\delta \lambda, \delta \mu = 0$, apart from wave-function renormalization factors. Thus, if we adjust the input parameters of (88) so that $\mu$ is small, it will stay small. However, this begs the more profound question: how did $\mu$ get to be so small in the first place?

As already mentioned, a striking piece of circumstantial evidence in favour of the idea of supersymmetric grand unification is provided by the measurements of low-energy gauge couplings at LEP and elsewhere [3]. The three gauge couplings of the Standard Model are renormalized as follows:

$$\frac{dg_2^2}{dt} = b_a \frac{g^4}{16\pi^2} + \ldots, \quad (90)$$

at one-loop order, and the corresponding value of the electroweak mixing angle $\sin^2 \theta_W(m_Z)$ is given at the one-loop level by:

$$\sin^2 \theta_W(m_Z) = \frac{g^2}{g_2^2 + g^2} = \frac{3}{5} \frac{g_2^2(m_Z)}{g_2^2(m_Z) + \frac{4}{3} g_1^2(m_Z)} = \frac{1}{1 + 8x} \left[ 3x + \frac{\alpha_{em}(m_Z)}{\alpha_3(m_Z)} \right], \quad (91)$$

where

$$x = \frac{1}{5} \left( b_2 - b_3 \right). \quad (92)$$

One can distinguish the predictions of different GUTs by their different values of the renormalization coefficients $b_i$, which are in turn determined by the spectra of light particles around the electroweak scale. In the cases of the Standard Model and the MSSM, these are:

$$\frac{4}{3} N_G - 11 \leftarrow b_3 \rightarrow 2N_G - 9 = -3 \quad (93)$$
$$\frac{1}{6} N_H + \frac{4}{3} N_G - \frac{22}{3} \leftarrow b_2 \rightarrow \frac{1}{2} N_H + 2N_G - 6 = +1 \quad (94)$$
$$\frac{1}{10} N_H + \frac{4}{3} N_G \leftarrow b_1 \rightarrow \frac{3}{10} N_H + 2N_G = \frac{33}{5} \quad (95)$$
$$\frac{23}{218} = 0.1055 \leftarrow x \rightarrow \frac{1}{7}. \quad (96)$$

If we insert the best available values of the gauge couplings:

$$\alpha_{em} = \frac{1}{128}; \quad \alpha_3(m_Z) = 0.119 \pm 0.003, \quad \sin^2 \theta_W(m_Z) = 0.2315, \quad (97)$$

we find the following value:

$$x = \frac{1}{6.92 \pm 0.07}. \quad (98)$$

We see that experiment strongly favours the inclusion of supersymmetric particles in the renormalization-group equations, as required if the effective low-energy theory is the MSSM [26], as in a simple supersymmetric GUT such as the minimal $SU(5)$ model introduced above.

3. TOWARDS REALISTIC MODELS

3.1 Supersymmetry Breaking

This is clearly necessary: $m_e \neq m_\tau, m_\tau \neq m_\tau$, etc. The Big Issue is whether the breaking of supersymmetry is explicit, i.e., present already in the underlying Lagrangian of the theory, or whether it is spontaneous, i.e., induced by a non-supersymmetric vacuum state. There are in fact several reasons to disfavour explicit supersymmetry breaking. It is ugly, it would be unlike the way in which gauge symmetry is broken, and it would lead to inconsistencies in supergravity theory. For these reasons, theorists have focused on spontaneous supersymmetry breaking.
If the vacuum is not to be supersymmetric, there must be some fermionic state $\chi$ that is coupled to the vacuum by the supersymmetry charge $Q$:

$$<0|Q|\chi> \equiv f_{\chi}^2 \neq 0.$$  \hspace{1cm} (99)

The fermion $\chi$ corresponds to a Goldstone boson in a spontaneously broken bosonic symmetry, and therefore is often termed a Goldstone fermion or a Goldstino.

There is just one small problem in globally supersymmetric models, i.e., those without gravity: spontaneous supersymmetry breaking necessarily entails a positive vacuum energy $E_0$. To see this, consider the vacuum expectation value of the basic supersymmetry anticommutator:

$$\{Q, Q\} \propto \gamma_\mu P^\mu.$$  \hspace{1cm} (100)

According to (99), there is an intermediate state $\chi$, so that

$$<0|\{Q, Q\}|0> = |<0|Q|\chi>|^2 = f_{\chi}^4 \propto <0|P_0|0> = E_0,$$  \hspace{1cm} (101)

where we have used Lorentz invariance to set the spatial components $<0|P_i|0> = 0$. Spontaneous breaking of global supersymmetry (99) requires

$$E_0 = f_{\chi}^4 \neq 0.$$  \hspace{1cm} (102)

The next question is how to generate non-zero vacuum energy. Hints are provided by the effective potential in a globally supersymmetric theory:

$$V = \Sigma_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 + \frac{1}{2} \Sigma_\alpha g_\alpha^2 |\phi^* T^\alpha \phi|^2.$$  \hspace{1cm} (103)

It is apparent from this expression that either the first ‘$F$ term’ or the second ‘$D$ term’ must be positive definite.

The option $D > 0$ requires constructing a model with a $U(1)$ gauge symmetry \cite{13}. The simplest example contains just one chiral (matter) supermultiplet with unit charge, for which the effective potential is:

$$V_D = \frac{1}{2} \left( \xi + g\phi^* \phi \right)^2.$$  \hspace{1cm} (104)

the extra constant term $\xi$ is not allowed in a non-Abelian theory, which is why one must use a $U(1)$ theory. We see immediately that the minimum of the effective potential (104) is reached when $<0|\phi|0> = 0$, in which case $V_F = 1/2\xi^2 > 0$ and supersymmetry is broken spontaneously. Indeed, it is easy to check that, in this vacuum:

$$m_\phi = g\xi, \ m_\psi = 0, \ m_\nu = m_\tilde{\nu} = 0,$$  \hspace{1cm} (105)

exhibiting explicitly the boson-fermion mass splitting in the $(\phi, \psi)$ supermultiplet. Unfortunately, this example cannot be implemented with the $U(1)$ of electromagnetism in the Standard Model, because there are fields with both signs of the hypercharge $Y$, enabling $V_D$ to vanish. So, one needs a new $U(1)$ gauge group factor, and many new fields in order to cancel triangle anomalies. For these reasons, $D$-breaking models did not attract much attention for quite some time, though they have had a revival in the context of string theory \cite{14}.

The option $F > 0$ also requires additional chiral (matter) fields with somewhat ‘artificial’ couplings \cite{15}: again, those of the Standard Model do not suffice. The simplest example uses three chiral supermultiplets $A, B, C$ with the superpotential

$$W = \alpha A \tilde{B}^2 + \beta C (B^2 - m^2).$$  \hspace{1cm} (106)
using the rules given in the previous Lecture, it is easy to calculate the corresponding $F$ terms:

$$F_A = \alpha B^2, \quad F_B = 2B(\alpha A + \beta C), \quad F_C = \beta (B^2 - m^2),$$

(107)

and hence the effective potential

$$V_F = \sum_i |F_i|^2 = 4|B(\alpha A + \beta C)|^2 + |\alpha B^2|^2 + |\beta (B^2 - m^2)|^2.$$  

(108)

Likewise, it is not difficult to check that the three different positive-semidefinite terms in (108) cannot all vanish simultaneously. Hence, necessarily $V_F > 0$, and hence supersymmetry must be broken.

The principal outcome of this brief discussion is that there are no satisfactory models of global supersymmetry breaking, so we will now look at the options in local supersymmetry, i.e., supergravity theory.

### 3.2 Supergravity and Local Supersymmetry Breaking

So far, we have considered global supersymmetry transformations, in which the infinitesimal transformation spinor $E$ is constant throughout space. Now we consider the possibility of a space-time-dependent field $E(x)$. Why?

This step of making symmetries local has become familiar with bosonic symmetries, where it leads to gauge theories, so it is natural to try the analogous step with fermionic symmetries. Moreover, as we see shortly, it leads to an elegant mechanism for spontaneous supersymmetry breaking, again by analogy with gauge theories, the super-Higgs mechanism. Further, as we also see shortly, making supersymmetry local necessarily involves gravity, and even opens the prospect of unifying all the particle interactions and matter fields with extended supersymmetry transformations:

$$G(J = 2) \rightarrow \tilde{G}(J = 3/2) \rightarrow V(J = 1) \rightarrow q, \ell(J = 1/2) \rightarrow H(J = 0$$

(109)

in supergravity with $N > 1$ supercharges. In (109), $G$ denotes the graviton, and $\tilde{G}$ the spin-3/2 gravitino, which accompanies it in the graviton supermultiplet:

$$\begin{pmatrix} G \\ \tilde{G} \end{pmatrix} = \begin{pmatrix} 2 \\ 3/2 \end{pmatrix}$$

(110)

Supergravity is in any case an essential ingredient in the discussion of gravitational interactions of supersymmetric particles, needed, for example, for any meaningful discussion of the cosmological constant.

To gain insight into why making supersymmetry local necessarily involves gravity, consider what happens if one applies both of the following pair of supersymmetry transformations:

$$\delta_i \phi = \sqrt{2} \bar{E}_i \psi + \ldots,$$

$$\delta_j \psi = -i \sqrt{2} \gamma_\mu \partial^\mu \phi E_j + \ldots$$

(111)

(112)

on either of the fields $\phi, \psi$. One finds in each case:

$$[\delta_i, \delta_j](\phi, \psi) = -2(\bar{E}_j \gamma_\mu E_i) i \partial^\mu (\phi, \psi).$$

(113)

In either case, the effect is equivalent to a space-time translation, since $i \partial^\mu \leftrightarrow P_\mu$. Clearly, if the infinitesimal spinorial transformations $E_{i,j}$ are independent of $x$, this translation is global. However, if the $E_{i,j}$ depend on $x$, the net effect is equivalent to a local coordinate transformation, and we know that a theory invariant under these necessarily includes gravity.

To see further why gravity must be taken into account when making supersymmetry local, let us develop further the analogy with gauge theories. We consider the variation of a typical fermion kinetic term: $\delta(i \bar{\psi} \gamma^\mu \partial_\mu \psi)$. In a gauge theory, one makes a space-time-dependent phase transformation $\epsilon(x)$:

$$\psi(x) \rightarrow e^{i\epsilon(x)} \psi(x),$$

(114)
which leads to a term in the variation of the fermion kinetic term of the form:

$$- \overline{\psi} \gamma_{\mu} \psi \partial_{\mu} \epsilon(x).$$  \hfill (115)

This is cancelled in a gauge theory by the variation in the gauge interaction:

$$\overline{\psi}(x) \gamma_{\mu} \psi(x) A_{\mu}(x) : \delta A_{\mu}(x) = \partial_{\mu} \epsilon(x).$$  \hfill (116)

In the supersymmetric case, when the supersymmetric variation becomes local:

$$\delta \psi(x) = -i \gamma_{\mu} \partial_{\mu} (\phi(x) E(x)) + \ldots,$$  \hfill (117)

the variation in the fermionic kinetic term includes a piece

$$\propto \overline{\psi} \gamma_{\mu} \gamma_{\nu} \partial_{\nu} \phi \partial_{\mu} E(x),$$ \hfill (118)

which is cancelled by introducing a new field $\psi_{\mu}$ with coupling

$$\kappa \overline{\psi} \gamma_{\mu} \gamma_{\nu} \partial_{\nu} \phi \psi_{\mu}(x) : \delta \psi_{\mu}(x) = -\frac{2}{\kappa} \partial_{\mu} E(x).$$ \hfill (119)

The new field $\psi_{\mu}(x)$ may be regarded as a ‘gauge fermion’: it represents the gravitino.

OK, so now we are convinced that making supersymmetry local necessarily involves gravity, and
the analogy with gauge theory suggests the introduction of a gravitino field. Consider now the simplest
possible Lagrangian for a gravitino and graviton \[34\], which consists just of the Einstein Lagrangian for
general relativity and a Rarita-Schwinger Lagrangian for a spin-3/2 field, made suitably invariant under
general coordinate transformations by minimal substitution:

$$L = -\frac{1}{2\kappa^2} \sqrt{-g} R - \frac{1}{2} \epsilon^{\mu \nu \rho \sigma} \overline{\psi} \gamma_5 \gamma_{\mu} D_{\rho} \psi_{\sigma},$$ \hfill (120)

where $g \equiv \text{det}(g_{\mu \nu})$ with $g_{\mu \nu}$ the metric tensor:

$$g_{\mu \nu} \equiv \epsilon_{\mu}^{m} \epsilon_{\nu}^{m} \eta_{mn}$$ \hfill (121)

where $\epsilon_{\mu}^{m}$ is the vierbein, and

$$D_{\rho} \equiv \partial_{\rho} + \frac{1}{4} \omega_{\rho}^{mn} [\gamma_{m}, \gamma_{n}],$$ \hfill (122)

with $\omega_{\rho}^{mn}$ the spin connection, is the generally-covariant derivative. It is a remarkable fact that the simple
Lagrangian (120) is locally supersymmetric \[34\]. To check this invariance, you need the following local
supersymmetry transformation laws:

$$\delta \epsilon_{\mu}^{m} = \bar{E}(x) \gamma_{m} \psi_{\mu}(x),$$ \hfill (123)

$$\delta \omega_{\mu}^{mn} = 0,$$ \hfill (124)

$$\delta \psi_{\mu} = \frac{1}{\kappa} D_{\mu} E(x).$$ \hfill (125)

Once again, we have been speaking prose all our lives without realizing!

We shall discuss later the coupling of supergravity to matter. First, however, now is a good time to
mention the remarkable phenomenon of spontaneous breaking of local supersymmetry: the super-Higgs
effect \[46, 47\]. You recall that, in the conventional Higgs effect in spontaneously broken gauge theories,
a massless Goldstone boson is ‘eaten’ by a gauge boson to provide it with the third polarization state it
needs to become massive:

$$\left(2 \times V_{m=0}\right) + \left(1 \times GB\right) = \left(3 \times V_{m\neq 0}\right).$$ \hfill (126)
In a locally supersymmetric theory, the two polarization states of the massless Goldstone fermion (Goldstino) are ‘eaten’ by a massless gravitino, giving it the total of four polarization states it needs to become massive:

\[
(2 \times \psi_{m=0}^{\mu}) + (2 \times GF) = (4 \times \psi_{m \neq 0}^{\mu}).
\]  

(127)

This process clearly involves the breakdown of local supersymmetry, since the end result is to give the gravitino a different mass from the graviton: \(m_G = 0 \neq m_{\tilde{G}} \neq 0\). It is indeed the only known consistent way of breaking local supersymmetry, just as the Higgs mechanism is the only consistent way of breaking gauge symmetry. We shall not go here through all the details of the super-Higgs effect, but there is one noteworthy feature: this local breaking of supersymmetry can be achieved with zero vacuum energy:

\[
<0|V|0> = 0 \leftrightarrow \Lambda = 0.
\]  

(128)

As we discuss shortly in more detail, there is no inconsistency between local supersymmetry breaking and a vanishing cosmological constant \(\Lambda\), unlike the case of global supersymmetry breaking that we discussed earlier.

### 3.3 Effective Low-Energy Theory

The coupling of matter particles to supergravity is more complicated than the globally supersymmetric case discussed in the previous lecture. Therefore, it is not developed here in detail. Instead, a few key results are presented without proof, and then we study the general form of the effective low-energy theory [48] derivable from a supergravity theory.

The superpotential of global supersymmetry is upgraded in \(N = 1\) supergravity to a ‘Kähler potential’, which describes the geometry of the internal space parameterized by the scalar fields in the chiral supermultiplets. This Kähler potential is a Hermitean function \(G(\phi, \phi^*)\) of the chiral fields and their complex conjugates, and plays several rôles. It is an order parameter for supersymmetry breaking:

\[
m_{\tilde{G}} \equiv m_{3/2} = e^G.
\]  

(129)

It also determines the kinetic terms for the chiral fields:

\[
L_K = G_i^j \partial^\mu \phi^*_j \partial_\mu \phi^i
\]  

via the Kähler metric

\[
G_i^j = \frac{\partial^2 G}{\partial \phi^i \partial \phi^j},
\]  

(131)

as well as the effective potential:

\[
V = e^G [G_i(G^\mu)_{j}^{-1} G^j - 3] : G_i \equiv \frac{\partial G}{\partial \phi^i}.
\]  

(132)

The first term in (132) corresponds to the effective potential in a globally supersymmetric theory.

The second term in (132) is novel: it permits the reconciliation of supersymmetry breaking: \(m_{\tilde{G}}^2 = e^G \neq 0\) with a vanishing effective potential: \(V = 0\), as a result of a cancellation between the first and second terms in (132). This is certainly good news, but there is also bad news. For general forms of \(G\), there are certain values of the fields where \(V\) is negative, with values \(-O(m^{4}_P)\). This would be a catastrophe for cosmology, since our Universe would surely fall down one of these holes. Fortunately, there is a particular class of Kähler potentials, known as no-scale supergravities [15], where the effective potential is positive semidefinite. Fortunately but not fortuitously, this is the class of supergravity that emerges from string theory [50].

Just as the Kähler potential determines the geometry and kinetic terms for chiral fields, there is a corresponding function that describes the geometry and determines the kinetic terms for gauge fields.
For generic choices of this function, there are also non-vanishing supersymmetry-breaking masses for the gauginos:

\[ m_{1/2} \propto m_{\tilde{G}} \equiv m_{3/2}. \]  

(133)

It is not inevitable that the masses of the \( SU(3), SU(2) \) and \( U(1) \) gauginos be universal, but this emerges naturally if the geometry is not too complicated.

Expanding the effective potential (133), one also finds in general terms \( \propto |\phi|^2 \), that are interpreted as supersymmetry-breaking scalar masses:

\[ m_0 \propto m_{\tilde{G}} \equiv m_{3/2}. \]  

(134)

In this case, there is no particularly good theoretical motivation for universality: indeed, this is broken in many string models. There are, however, phenomenological reasons to think that the supersymmetry-breaking scalar masses for sparticles with the same charge, e.g., all the sleptons, should be universal, in order to suppress flavour-changing neutral interactions mediated by virtual sparticles [51].

Generic forms of the effective potential (132) also yield trilinear supersymmetry-breaking interactions among the scalar particles:

\[ A_\lambda \lambda \phi^3 : \ A_\lambda \propto m_{\tilde{G}} \equiv m_{3/2}. \]  

(135)

where here \( \phi \) denotes a scalar component of a generic chiral supermultiplet. If the supersymmetric theory also includes bilinear interactions \( \mu \phi^2 \), as is the case in the minimal supersymmetric extension of the Standard Model (MSSM), one also expects an analogous bilinear supersymmetry-breaking term \( B_\mu \mu \phi^2 \) among the scalar components.

Thus, the final form of the effective low-energy theory suggested by spontaneous supersymmetry breaking in supergravity is:

\[
- \frac{1}{2} \sum_{a} m_{1/2a} \bar{V}_a \tilde{V}_a - \sum_i m_{0i}^2 |\phi_i|^2 - (\Sigma_{\lambda} A_{\lambda} \lambda \phi^3 + \Sigma_{\mu} B_{\mu} \mu \phi^2 + \text{Herm.Conj.}),
\]  

(136)

which contains many free parameters and phases. The breaking of supersymmetry in the effective low-energy theory (136) is explicit but ‘soft’, in the sense that the renormalization of the parameters \( m_{1/2a}, m_0, A_\lambda \) and \( B_\mu \) is logarithmic. Of course, these parameters are not considered to be fundamental, and the underlying mechanism of supersymmetry breaking is thought to be spontaneous, for the reasons described at the beginning of this lecture.

The logarithmic renormalization of the parameters means that one can calculate their low-energy values in terms of high-energy inputs from a supergravity or superstring theory, using standard renormalization-group equations [52]. In the case of the low-energy gaugino masses \( M_a \), the renormalization is multiplicative and identical with that of the corresponding gauge coupling \( \alpha_a \) at the one-loop level:

\[
\frac{M_a}{m_{1/2a}} = \frac{\alpha_a}{\alpha_{GUT}}
\]  

(137)

where we assume GUT unification of the gauge couplings at the input supergravity scale. In the case of the scalar masses, there is both multiplicative renormalization and renormalization related to the gaugino masses:

\[
\frac{\partial m_0^2}{\partial t} = \frac{1}{16\pi^2} [\lambda^2 (m_0^2 + A_\lambda^2) - g_a^2 M_a^2]
\]  

at the one-loop level, where \( t \equiv \ln(Q^2/m_{GUT}^2) \), and the \( O(1) \) group-theoretical coefficients have been omitted. In the case of the first two generations, the first terms in (138) are negligible, and one may integrate (138) trivially to obtain effective low-energy parameters

\[
m_0^2 = m_0^2 + C_i m_{1/2}^2,
\]  

(139)
where universal inputs are assumed, and the coefficients $C_i$ are calculable in any given model. The first terms in (138) are, however, important for the third generation and for the Higgs bosons of the MSSM, as we now see.

Notice that the signs of the first terms in (138) are positive, and that of the last term negative. This means that the last term tends to increase $m_{0_i}^2$ as the renormalization scale $Q$ decreases, an effect seen in Fig. 8. The positive signs of the first terms mean that they tend to decrease $m_{0_i}^2$ as $Q$ decreases, an effect seen for a Higgs squared-mass in Fig. 8. Specifically, the negative effect on $H_u$ seen in Fig. 8 is due to its large Yukawa coupling to the $t$ quark: $\lambda_t \sim g_2,3$. The exciting aspect of this observation is that spontaneous electroweak symmetry breaking is possible [52] when $m_{H_u}^2(Q) < 0$, as occurs in Fig. 8. Thus the spontaneous breaking of supersymmetry, which normally provides $m_{0}^2 > 0$, and renormalization, which then drive $m_{H_u}^2(Q) < 0$, conspire to make spontaneous electroweak symmetry breaking possible. Typically, this occurs at a renormalization scale that is exponentially smaller than the input supergravity scale:

$$\frac{m_W}{m_P} = \exp(-O(1) \alpha_t) : \quad \alpha_t \equiv \frac{\lambda_t^2}{4\pi}. \quad (140)$$

Typical dynamical calculations find that $m_W \sim 100$ GeV emerges naturally if $m_t \sim 60$ to 200 GeV, and this was in fact one of the first suggestions that $m_t$ might be as high as was subsequently observed.

To conclude this section, let us briefly review the reasons why soft supersymmetry breaking might be universal, at least in some respects. There are important constraints on the mass differences of squarks and sleptons with the same internal quantum numbers, coming from flavour-changing neutral interactions [51]. These are suppressed in the Standard Model by the Glashow-Iliopoulos-Maiani mechanism [53], which limits them to magnitudes $\propto \Delta m^2_q/m^2_W$ for small squared-mass differences $\Delta m^2_q$. Depending on the process considered, it is either necessary or desirable that sparticle exchange contributions, which would have expected magnitudes $\propto \Delta m^2_q/m^2_W$, be suppressed by a comparable factor. In particular, one would like

$$m_{0}^2(\text{first generation}) - m_{0}^2(\text{second generation}) \sim \delta m^2_q \times \frac{m^2_q}{m^2_W}. \quad (141)$$
The limits on third-generation sparticle masses from flavour-changing neutral interactions are less severe, and the first/second-generation degeneracy could be relaxed if $m_{\tilde{q}}^2 \gg m_W^2$, but models with physical values of $m_{\tilde{0}}^2$ degenerate to $\mathcal{O}(m_q^2)$ are certainly preferred. This is possible in models with a universal Kähler geometry for the scalar fields $\phi^i$. For example:

$$G = |\phi^i|^2 \rightarrow \frac{\partial^2 G}{\partial \phi^i \partial \phi^*_j} = \delta^i_j,$$

resulting in universal $m_{\tilde{0}}^2$, and there are other examples such as certain no-scale models. However, this restriction is not respected in many low-energy effective theories derived from string models.

The desirability of degeneracy between sparticles of different generations help encourage some people to study models in which this property would emerge naturally, such as models of gauge-mediated supersymmetry breaking or extra dimensions [40]. However, for the rest of these lectures we shall mainly stick to familiar old supergravity.

### 3.4 Sparticle Masses and Mixing

We now progress to a more complete discussion of sparticle masses and mixing.

**Sfermions**: Each flavour of charged lepton or quark has both left- and right-handed components $f_{L,R}$, and these have separate spin-0 boson superpartners $\tilde{f}_{L,R}$. These have different isospins $I = \frac{1}{2}, 0$, but may mix as soon as the electroweak gauge symmetry is broken. Thus, for each flavour we should consider a $2 \times 2$ mixing matrix for the $\tilde{f}_{L,R}$, which takes the following general form:

$$M^2_{\tilde{f}} = \begin{pmatrix} m^2_{\tilde{f}_{LL}} & m^2_{\tilde{f}_{LR}} \\ m^2_{\tilde{f}_{LR}} & m^2_{\tilde{f}_{RR}} \end{pmatrix}$$

The diagonal terms may be written in the form

$$m^2_{\tilde{f}_{LL,RR}} = m^2_{\tilde{f}_{LR}} + m^2_{D^2_{\tilde{f}_{LR}}} + m^2_{\tilde{f}}$$

where $m_f$ is the mass of the corresponding fermion, $m^2_{\tilde{f}_{LR}}$ is the soft supersymmetry-breaking mass discussed in the previous section, and $m^2_{D^2_{\tilde{f}_{LR}}}$ is a contribution due to the quartic $D$ terms in the effective potential:

$$m^2_{D^2_{\tilde{f}_{LR}}} = m_Z^2 \cos 2\beta \left( I_3 + \sin^2 \theta_W Q_{em} \right)$$

where the term $\propto I_3$ is non-zero only for the $\tilde{f}_L$. Finally, the off-diagonal mixing term takes the general form

$$m^2_{\tilde{f}_{LR}} = m_f \left( A_f + \mu^\tan \beta \right) \text{ for } f = e, \mu, \tau, d, s, b, u, c, t$$

It is clear that $\tilde{f}_{L,R}$ mixing is likely to be important for the $\tilde{t}$, and it may also be important for the $\tilde{b}_{L,R}$ and $\tilde{t}_{L,R}$ if $\tan \beta$ is large.

We also see from (144) that the diagonal entries for the $\tilde{t}_{L,R}$ would be different from those of the $\tilde{u}_{L,R}$ and $\tilde{c}_{L,R}$, even if their soft supersymmetry-breaking masses were universal, because of the $m_{\tilde{t}}^2$ contribution. In fact, we also expect non-universal renormalization of $m^2_{\tilde{t}_{LL,RR}}$ (and also $m^2_{\tilde{b}_{LL,RR}}$ and $m^2_{\tilde{c}_{LL,RR}}$ if $\tan \beta$ is large), because of Yukawa effects analogous to those discussed in the previous section for the renormalization of the soft Higgs masses. For these reasons, the $\tilde{t}_{L,R}$ are not usually assumed to be degenerate with the other squark flavours. Indeed, one of the $\tilde{t}$ could well be the lightest squark, perhaps even lighter than the $t$ quark itself [54].
Charginos: These are the supersymmetric partners of the $W^\pm$ and $H^\pm$, which mix through a $2 \times 2$ matrix

$$-rac{1}{2} (\tilde{W}^- , \tilde{H}^-) \cdot M_C \left( \begin{array}{c} \tilde{W}^+ \\ \tilde{H}^+ \end{array} \right) + \text{herm.conj.}$$

(147)

where

$$M_C \equiv \left( \begin{array}{c c c c} M_2 & \sqrt{2}M_W \cos \beta \\ \sqrt{2}M_W \sin \beta & \mu \end{array} \right)$$

(148)

Here $M_2$ is the unmixed $SU(2)$ gaugino mass and $\mu$ is the Higgs mixing parameter introduced in (84).

Fig. 9 displays (among other lines to be discussed later) the contour $m_{\chi^\pm} = 91 \text{ GeV}$ for the lighter of the two chargino mass eigenstates [55].

Neutralinos: These are characterized by a $4 \times 4$ mass mixing matrix [56], which takes the following form in the ($\tilde{W}^3, \tilde{B}, \tilde{H}^0_2, \tilde{H}^0_1$) basis:

$$m_N = \begin{pmatrix} M_2 & 0 & \frac{g_{21}^W}{\sqrt{2}} & \frac{g_{21}^H}{\sqrt{2}} \\ 0 & M_1 & \frac{g_{13}^W}{\sqrt{2}} & -\frac{g_{13}^H}{\sqrt{2}} \\ -\frac{g_{21}^W}{\sqrt{2}} & \frac{g_{13}^W}{\sqrt{2}} & 0 & \mu \\ \frac{g_{21}^H}{\sqrt{2}} & -\frac{g_{13}^H}{\sqrt{2}} & \mu & 0 \end{pmatrix}$$

(149)

Note that this has a structure similar to $M_C$ (148), but with its entries replaced by $2 \times 2$ submatrices. As has already been mentioned, one conventionally assumes that the $SU(2)$ and $U(1)$ gaugino masses $M_{1,2}$ are universal at the GUT or supergravity scale, so that

$$M_1 \simeq M_2 \frac{\alpha_1}{\alpha_2}$$

(150)
so the relevant parameters of (149) are generally taken to be \( M_2 = (\alpha_2/\alpha_{GUT})m_{1/2} \), \( \mu \) and \( \tan \beta \).

Figure 20 also displays contours of the mass of the lightest neutralino \( \chi \), as well as contours of its gaugino and Higgsino contents [55]. In the limit \( M_2 \to 0 \), \( \chi \) would be approximately a photino and it would be approximately a Higgsino in the limit \( \mu \to 0 \). Unfortunately, these idealized limits are excluded by unsuccessful LEP and other searches for neutralinos and charginos, as discussed in more detail in the next Lecture.

### 3.5 The Lightest Supersymmetric Particle

This is expected to be stable in the MSSM, and hence should be present in the Universe today as a cosmological relic from the Big Bang [57, 56]. Its stability arises because there is a multiplicatively-conserved quantum number called \( R \) parity, that takes the values +1 for all conventional particles and -1 for all sparticles [33]. The conservation of \( R \) parity can be related to that of baryon number \( B \) and lepton number \( L \), since

\[
R = (-1)^{3B+L+2S}
\]

where \( S \) is the spin. Note that \( R \) parity could be violated either spontaneously if \( \langle \bar{\nu} \nu \rangle \neq 0 \) or explicitly if one of the supplementary couplings (87) is present. There could also be a coupling \( HL \), but this can be defined away by choosing a field basis such that \( \tilde{H} \) is defined as the superfield with a bilinear coupling to \( H \). Note that \( R \) parity is not violated by the simplest models for neutrino masses, which have \( \Delta L = 0, \pm 2 \), nor by simple GUTs, which violate combinations of \( B \) and \( L \) that leave \( R \) invariant. There are three important consequences of \( R \) conservation:

1. sparticles are always produced in pairs, e.g., \( \bar{p}p \to \bar{q}qX, e^+e^- \to \mu^+ + \bar{\mu}^- \),
2. heavier sparticles decay to lighter ones, e.g., \( \bar{q} \to q\tilde{g}, \tilde{\mu} \to \mu\tilde{\gamma} \), and
3. the lightest sparticles is stable, because it has no legal decay mode.

This feature constrains strongly the possible nature of the lightest supersymmetric sparticle. If it had either electric charge or strong interactions, it would surely have dissipated its energy and condensed into galactic disks along with conventional matter. There it would surely have bound electromagnetically or via the strong interactions to conventional nuclei, forming anomalous heavy isotopes that should have been detected. There are upper limits on the possible abundances of such bound relics, as compared to conventional nucleons:

\[
\frac{n(\text{relic})}{n(p)} \lesssim 10^{-15} \text{ to } 10^{-29}
\]

for \( 1 \text{ GeV} \lesssim m_{\text{relic}} \lesssim 1 \text{ TeV} \). These are far below the calculated abundances of such stable relics:

\[
\frac{n(\text{relic})}{n(p)} \gtrsim 10^{-6} \left( 10^{-10} \right)
\]

for relic particles with electromagnetic (strong) interactions. We may conclude [56] that any supersymmetric relic is probably electromagnetically neutral with only weak interactions, and could in particular not be a gluino. Whether the lightest hadron containing a gluino is charged or neutral, it would surely bind to some nuclei. Even if one pleads for some level of fractionation, it is difficult to see how such gluino nuclei could avoid the stringent bounds established for anomalous isotopes of many species.

Plausible candidates of different spins are the sneutrinos \( \tilde{\nu} \) of spin 0, the lightest neutralino \( \chi \) of spin 1/2, and the gravitino \( \tilde{G} \) of spin 3/2. The sneutrinos have been ruled out by the combination of LEP experiments and direct searches for cosmological relics. The gravitino cannot be ruled out, but we concentrate on the neutralino possibility for the rest of these Lectures.

A very attractive feature of the neutralino candidature for the lightest supersymmetric particle is that it has a relic density of interest to astrophysicists and cosmologists: \( \Omega_\chi h^2 = \mathcal{O}(0.1) \) over generic
domains of the MSSM parameter space \([58]\), as discussed in the next Lecture. In these domains, the lightest neutralino \(\chi\) could constitute the cold dark matter favoured by theories of cosmological structure formation.

### 3.6 Supersymmetric Higgs Bosons

As was discussed in Lecture 2, one expects two complex Higgs doublets \(H_2 \equiv (H_2^+, H_2^0)\), \(H_1 \equiv (H_1^+, H_1^0)\) in the MSSM, with a total of 8 real degrees of freedom. Of these, 3 are eaten via the Higgs mechanism to become the longitudinal polarization states of the \(W^\pm\) and \(Z^0\), leaving 5 physical Higgs bosons to be discovered by experiment. Three of these are neutral: the lighter CP-even neutral \(H\), the CP-odd neutral \(A\), and charged bosons \(H^\pm\). The quartic potential is completely determined by the \(D\) terms

\[
V_4 = \frac{g^2 + g'^2}{8} \left( |H_1^0|^2 - |H_2^0|^2 \right)
\]  

(154)

for the neutral components, whilst the quadratic terms may be parametrized at the tree level by

\[
m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 + (m_3^2 H_1 H_2 + \text{herm. conj.})
\]  

(155)

where \(m_3^2 = B_{\mu \mu}\). One combination of the three parameters \((m_{H_1}^2, m_{H_2}^2, m_3^2)\) is fixed by the Higgs vacuum expectation \(v = \sqrt{v_1^2 + v_2^2} = 246\) GeV, and the other two combinations may be rephrased as \((m_A, \tan \beta)\). These characterize all Higgs masses and couplings in the MSSM at the tree level. Looking back at (154), we see that the gauge coupling strength of the quartic interactions suggests a relatively low mass for at least the lightest MSSM Higgs boson \(h\), and this is indeed the case, with \(m_h \leq m_Z\) at the tree level:

\[
m_h^2 = m_Z^2 \cos^2 2\beta
\]  

(156)

This raised considerable hope that the lightest MSSM Higgs boson could be discovered at LEP, with its prospective reach to \(m_H \sim 100\) GeV.

However, radiative corrections to the Higgs masses are calculable in a supersymmetric model (this was, in some sense, the whole point of introducing supersymmetry!), and they turn out to be non-negligible for \(m_t \sim 175\) GeV \([58]\). Indeed, the leading one-loop corrections to \(m_h^2\) depend quartically on \(m_t\):

\[
\Delta m_h^2 = \frac{3 m_t^4}{4 \pi^2 v^2} \ln \left( \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right) + \frac{3 m_t^4 \tilde{A}_t^2}{8 \pi^2 v^2} \left[ 2 h(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + \tilde{A}_t^2 f(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) \right] + \ldots
\]  

(157)

where \(m_{\tilde{t}_1,2}\) are the physical masses of the two stop squarks \(\tilde{t}_{1,2}\) to be discussed in more detail shortly, \(\tilde{A}_t \equiv A_t - \mu \cot \beta\), and

\[
h(a, b) \equiv \frac{1}{a-b} \ln \left( \frac{a}{b} \right), \quad f(a, b) = \frac{1}{(a-b)^2} \left[ 2 - \frac{a+b}{a-b} \ln \left( \frac{a}{b} \right) \right]
\]  

(158)

Non-leading one-loop corrections to the MSSM Higgs masses are also known, as corrections to coupling vertices, two-loop corrections and renormalization-group resummations \([58]\). For \(m_{\tilde{t}_1,2} \lesssim 1\) TeV and a plausible range of \(A_t\), one finds

\[
m_h \lesssim 130\ \text{GeV}
\]  

(159)

as seen in Fig. 14. There we see the sensitivity of \(m_h\) to \((m_A, \tan \beta)\), and we also see how \(m_A, m_H\) and \(m_{H^\pm}\) approach each other for large \(m_A\).
4. PHENOMENOLOGY

4.1 Constraints on the MSSM

Important experimental constraints on the MSSM parameter space are provided by direct searches at LEP and the Tevatron collider, as compiled in Fig. 11. One of these is the limit \( m_{\chi^\pm} > \sim 103.5 \text{ GeV} \) provided by chargino searches at LEP [59], where the third significant figure depends on other CMSSM parameters. LEP has also provided lower limits on slepton masses, of which the strongest is \( m_{\tilde{e}} > \sim 99 \text{ GeV} \) [60], again depending only slightly on the other CMSSM parameters, as long as \( m_{\tilde{e}} - m_{\chi} > 10 \text{ GeV} \). The most important constraints on the \( u,d,s,c,b \) squarks and gluinos are provided by the Tevatron collider: for equal masses \( m_{\tilde{q}} = m_{\tilde{g}} > \sim 300 \text{ GeV} \). In the case of the \( \tilde{t} \), LEP provides the most stringent limit when \( m_{\tilde{t}} - m_{\chi} \) is small, and the Tevatron for larger \( m_{\tilde{t}} - m_{\chi} \) [59].

Another important constraint is provided by the LEP lower limit on the Higgs mass: \( m_H > 114.1 \text{ GeV} \) [20]. This holds in the Standard Model, for the lightest Higgs boson \( h \) in the general MSSM for \( \tan \beta < \sim 8 \), and almost always in the CMSSM for all \( \tan \beta \), at least as long as CP is conserved. Since \( m_h \) is sensitive to sparticle masses, particularly \( m_{\tilde{t}} \), via loop corrections:

\[
\delta m_h^2 \propto \frac{m_{\tilde{t}}^4}{m_W^2} \ln \left( \frac{m_{\tilde{t}}^2}{m_W^2} \right) + \ldots
\]

(160)

the Higgs limit also imposes important constraints on the CMSSM parameters, principally \( m_{1/2} \) [64] as seen in Fig. 11. The constraints are here evaluated using FeynHiggs [58], which is estimated to have a residual uncertainty of a couple of GeV in \( m_h \).

Also shown in Fig. 11 is the constraint imposed by measurements of \( b \to s \gamma \) [52]. These agree with the Standard Model, and therefore provide bounds on MSSM particles, such as the chargino and charged Higgs masses, in particular. Typically, the \( b \to s \gamma \) constraint is more important for \( \mu < 0 \), as seen in Fig. 11a and c, but it is also relevant for \( \mu > 0 \), particularly when \( \tan \beta \) is large as seen in Fig. 11d.

The final experimental constraint we consider is that due to the measurement of the anomalous magnetic moment of the muon. The BNL E821 experiment reported last year a new measurement of

\footnote{The lower bound on the lightest MSSM Higgs boson mass may be relaxed significantly if CP violation feeds into the MSSM Higgs sector [63].}
Fig. 11: Compilations of phenomenological constraints on the CMSSM for (a) $\tan \beta = 10, \mu < 0$, (b) $\tan \beta = 10, \mu > 0$, (c) $\tan \beta = 35, \mu < 0$ and (d) $\tan \beta = 50, \mu > 0$, assuming $A_0 = 0, m_t = 175$ GeV and $m_0(m_0)_{\overline{MS}} = 4.25$ GeV \cite{61}. The near-vertical lines are the LEP limits $m_{\chi^\pm} = 103.5$ GeV (dashed and black) \cite{59}, shown in (b) only, and $m_h = 114.1$ GeV (dotted and red) \cite{20}. Also, in the lower left corner of (b), we show the $m_{\tilde{e}} = 99$ GeV contour \cite{60}. In the dark (brick red) shaded regions, the LSP is the charged $\tilde{\tau}_1$, so this region is excluded. The light (turquoise) shaded areas are the cosmologically preferred regions with $0.1 \leq \Omega_\chi h^2 \leq 0.3$ \cite{61}. The medium (dark green) shaded regions that are most prominent in panels (a) and (c) are excluded by $b \to s\gamma$ \cite{62}. The shaded (pink) regions in the upper right regions delineate the $\pm 2 \sigma$ ranges of $g_\mu - 2$. For $\mu > 0$, the $\pm 1 \sigma$ contours are also shown as solid black lines.
\( a_\mu \equiv \frac{1}{2}(g_\mu - 2) \) which deviated by 2.6 standard deviations from the best Standard Model prediction available at that time \([65]\). The largest contribution to the errors in the comparison with theory was thought to be the statistical error of the experiment, which will soon be significantly reduced, as many more data have already been recorded. However, it has recently been realized that the sign of the most important pseudoscalar-meson pole part of the light-by-light scattering contribution \([66]\) to the Standard Model prediction should be reversed, which reduces the apparent experimental discrepancy to about 1.6 standard deviations. The next-largest error is thought to be that due to strong-interaction uncertainties in the Standard Model prediction, for which recent estimates converge to about \(7 \times 10^{-10} \) \([67]\).

As many authors have pointed out \([68]\), a discrepancy between theory and the BNL experiment could well be explained by supersymmetry. As seen in Fig. 11, this is particularly easy if \(\mu > 0\). With the change in sign of the meson-pole contributions to light-by-light scattering, good consistency is also possible for \(\mu < 0\) so long as either \(m_{1/2}\) or \(m_0\) are taken sufficiently large. We show in Fig. 11 as medium (pink) shaded the new \(2\sigma\) allowed region: \(-6 < \delta a_\mu \times 10^{10} < 58\).

The new regions preferred by the \(g - 2\) experimental data shown in Fig. 11 differ considerably from the older ones \([68]\) which were based on the range \(11 < \delta a_\mu \times 10^{10} < 75\). First of all, the older bound completely excluded \(\mu < 0\) at the \(2\sigma\) level. As one can see this is no longer true: \(\mu < 0\) is allowed so long as either (or both) \(m_{1/2}\) and \(m_0\) are large. Thus, for \(\mu < 0\), one is forced into either the \(\chi - \tilde{\tau}\) coannihilation region or the funnel region produced by the s-channel annihilation via the heavy Higgses \(H\) and \(A\), as described below. Second, whereas the older limits produced definite upper bounds on the sparticle masses (which were accepted with delight by future collider builders), the new bounds, which are consistent with \(a_\mu = 0\), allow arbitrarily high sparticle masses. Now only the very low mass corner of the \((m_{1/2}, m_0)\) plane is excluded.

Fig. 11 also displays the regions where the supersymmetric relic density \(\rho_\chi = \Omega_\chi\rho_{\text{critical}}\) falls within the preferred range

\[
0.1 < \Omega_\chi h^2 < 0.3
\]

The upper limit is rigorous, since astrophysics and cosmology tell us that the total matter density \(\Omega_m \lesssim 0.4\), and the Hubble expansion rate \(h \sim 1/\sqrt{2}\) to within about 10% (in units of 100 km/s/Mpc). On the other hand, the lower limit in (161) is optional, since there could be other important contributions to the overall matter density.

As is seen in Fig. 11, there are generic regions of the CMSSM parameter space where the relic density falls within the preferred range (161). What goes into the calculation of the relic density? It is controlled by the annihilation cross section \([54]\):

\[
\rho_\chi = m_\chi n_\chi, \quad n_\chi \sim \frac{1}{\sigma_{\text{ann}}(\chi\chi \rightarrow \ldots)},
\]

where the typical annihilation cross section \(\sigma_{\text{ann}} \sim 1/m_\chi^2\). For this reason, the relic density typically increases with the relic mass, and this combined with the upper bound in (161) then leads to the common expectation that \(m_\chi \lesssim O(200)\) GeV.

However, there are various ways in which the generic upper bound on \(m_\chi\) can be increased along filaments in the \((m_{1/2}, m_0)\) plane. For example, if the next-to-lightest particle (NLSP) is not much heavier than \(\chi\): \(\Delta m/m_\chi \lesssim 0.1\), the relic density may be suppressed by coannihilation: \(\sigma(\chi + \text{NLSP} \rightarrow \ldots)\) \([59]\). In this way, the allowed CMSSM region may acquire a ‘tail’ extending to larger sparticle masses. An example of this possibility is the case where the NLSP is the lighter stau: \(\tilde{\tau}_1\) and \(m_{\tilde{\tau}_1} \sim m_\chi\), as seen in Figs. 11(a) and (b) and extended to larger \(m_{1/2}\) in Fig. 2(a) \([70]\). Another example is coannihilation when the NLSP is the lighter stop \([71]\): \(\tilde{t}_1\) and \(m_{\tilde{t}_1} \sim m_\chi\), which may be important in the general MSSM or in the CMSSM when \(A\) is large, as seen in Fig. 2(b) \([72]\). In the cases studied, the upper limit on \(m_\chi\) is not affected by stop coannihilation. Another mechanism for extending the allowed CMSSM region to large \(m_\chi\) is rapid annihilation via a direct-channel pole when \(m_\chi \sim\)
\[ \tan \beta = 10, \ A_0 = 2000 \text{ GeV}, \ \mu > 0 \]

Fig. 12: (a) The large-\( m_{1/2} \) ‘tail’ of the \( \chi - \tilde{\tau}_1 \) coannihilation region for \( \tan \beta = 10, \ A = 0 \) and \( \mu < 0 \) \[70\], superimposed on the disallowed dark (brick red) shaded region where \( m_{\tilde{\tau}_1} < m_\chi \), and (b) the \( \chi - \tilde{\tau}_1 \) coannihilation region for \( \tan \beta = 10, \ A_0 = 2000 \text{ GeV} \) and \( \mu > 0 \) \[72\], exhibiting a large-\( m_0 \) ‘tail’.

This may yield a ‘funnel’ extending to large \( m_{1/2} \) and \( m_0 \) at large \( \tan \beta \), as seen in panels (c) and (d) of Fig. 11 \[61\]. Yet another allowed region at large \( m_{1/2} \) and \( m_0 \) is the ‘focus-point’ region \[74\], which is adjacent to the boundary of the region where electroweak symmetry breaking is possible, as seen in Fig. 13.

4.2 Fine Tuning

The filaments extending the preferred CMSSM parameter space are clearly exceptional, in some sense, so it is important to understand the sensitivity of the relic density to input parameters, unknown higher-order effects, etc. One proposal is the relic-density fine-tuning measure \[75\]:

\[ \Delta \Omega \equiv \sqrt{\sum_i \left( \frac{\partial \ln(\Omega h^2)}{\partial \ln a_i} \right)^2} \tag{163} \]

where the sum runs over the input parameters, which might include (relatively) poorly-known Standard Model quantities such as \( m_t \) and \( m_b \), as well as the CMSSM parameters \( m_0, m_{1/2}, \) etc. As seen in Fig. 14, the sensitivity \( \Delta \Omega \) \[163\] is relatively small in the ‘bulk’ region at low \( m_{1/2}, m_0, \) and \( \tan \beta \). However, it is somewhat higher in the \( \chi - \tilde{\tau}_1 \) coannihilation ‘tail’, and at large \( \tan \beta \) in general. The sensitivity measure \( \Delta \Omega \) \[163\] is particularly high in the rapid-annihilation ‘funnel’ and in the ‘focus-point’ region. This explains why published relic-density calculations may differ in these regions \[76\], whereas they agree well when \( \Delta \Omega \) is small: differences may arise because of small differences in the treatments of the inputs.

It is important to note that the relic-density fine-tuning measure \[163\] is distinct from the traditional measure of the fine-tuning of the electroweak scale \[77\]:

\[ \Delta = \sqrt{\sum_i \Delta_i^2}, \quad \Delta_i \equiv \frac{\partial \ln m_W}{\partial \ln a_i} \tag{164} \]

Sample contours of the electroweak fine-tuning measure are shown \[164\] are shown in Figs. 15. This electroweak fine tuning is logically different from the cosmological fine tuning, and values of \( \Delta \) are
m_h = 114 GeV

m_t = 171 GeV, \tan \beta = 10, \mu > 0

m_0 (GeV)
m_1/2 (GeV)

m_h = 114 GeV

m_t = 171 GeV, \tan \beta = 50, \mu > 0

m_0 (GeV)
m_1/2 (GeV)

Fig. 13: An expanded view of the m_{1/2} – m_0 parameter plane showing the focus-point regions \[4] at large m_0 for (a) \tan \beta = 10, and (b) \tan \beta = 50. In the shaded (mauve) region in the upper left corner, there are no solutions with proper electroweak symmetry breaking, so these are excluded in the CMSSM. Note that we have chosen m_t = 171 GeV, in which case the focus-point region is at lower m_0 than when m_t = 175 GeV, as assumed in the other figures. The position of this region is very sensitive to m_t. The black contours (both dashed and solid) are as in Fig. 11, we do not shade the preferred g – 2 region.

not necessarily related to values of \( \Delta \Omega \), as is apparent when comparing the contours in Figs. 14 and 15. Electroweak fine-tuning is sometimes used as a criterion for restricting the CMSSM parameters. However, the interpretation of \( \Delta \) \[64\] is unclear. How large a value of \( \Delta \) is tolerable? Different physicists may well have different pain thresholds. Moreover, correlations between input parameters may reduce its value in specific models, and the regions allowed by the different constraints can become very different when we relax some of the CMSSM assumptions, e.g. the universality between the input Higgs masses and those of the squarks and sleptons, a subject beyond the scope of these Lectures.

4.3 Prospects for Observing Supersymmetry at Accelerators

As an aid to the assessment of the prospects for detecting sparticles at different accelerators, benchmark sets of supersymmetric parameters have often been found useful \[78\], since they provide a focus for concentrated discussion. A set of proposed post-LEP benchmark scenarios in the CMSSM \[79\] are illustrated schematically in Fig. 16. They take into account the direct searches for sparticles and Higgs bosons, \( b \to s\gamma \) and the preferred cosmological density range (161). The proposed benchmark points are consistent with \( g_\mu - 2 \) at the 2 \( \sigma \) level, but this was not imposed as an absolute requirement.

The proposed points were chosen not to provide an ‘unbiased’ statistical sampling of the CMSSM parameter space, whatever that means in the absence of a plausible \textit{a priori} measure, but rather are intended to illustrate the different possibilities that are still allowed by the present constraints \[79\].\footnote{This study is restricted to \( A = 0 \), for which \( \tilde{t}_1 - \chi \) coannihilation is less important, so this effect has not influenced the selection of benchmark points.} Five of the chosen points are in the ‘bulk’ region at small \( m_{1/2} \) and \( m_0 \), four are spread along the coannihilation ‘tail’ at larger \( m_{1/2} \) for various values of \( \tan \beta \), two are in the ‘focus-point’ region at large \( m_0 \), and two are in rapid-annihilation ‘funnels’ at large \( m_{1/2} \) and \( m_0 \). The proposed points range over the allowed values of \( \tan \beta \) between 5 and 50. Most of them have \( \mu > 0 \), as favoured by \( g_\mu - 2 \),
Fig. 14: Contours of the total sensitivity $\Delta \Omega$ (163) of the relic density in the $(m_{1/2}, m_0)$ planes for (a) $\tan \beta = 10, \mu > 0, m_t = 175$ GeV, (b) $\tan \beta = 35, \mu < 0, m_t = 175$ GeV, (c) $\tan \beta = 50, \mu > 0, m_t = 175$ GeV, and (d) $\tan \beta = 10, \mu > 0, m_t = 171$ GeV, all for $A_0 = 0$. The light (turquoise) shaded areas are the cosmologically preferred regions with $0.1 \leq \Omega_c h^2 \leq 0.3$. In the dark (brick red) shaded regions, the LSP is the charged $\tilde{\tau}_1$, so these regions are excluded. In panel (d), the medium shaded (mauve) region is excluded by the electroweak vacuum conditions.
Fig. 15: Contours of the electroweak fine-tuning measure $\Delta$ in the $(m_{1/2}, m_0)$ planes for (a) $\tan \beta = 10, \mu > 0, m_t = 175$ GeV, (b) $\tan \beta = 35, \mu < 0, m_t = 175$ GeV, (c) $\tan \beta = 50, \mu > 0, m_t = 175$ GeV, and (d) $\tan \beta = 10, \mu > 0, m_t = 171$ GeV, all for $A_0 = 0$. The light (turquoise) shaded areas are the cosmologically preferred regions with $0.1 \leq \Omega_\chi h^2 \leq 0.3$. In the dark (brick red) shaded regions, the LSP is the charged $\tilde{\tau}_1$, so this region is excluded. In panel (d), the medium shaded (mauve) region is excluded by the electroweak vacuum conditions.
but there are two points with $\mu < 0$.

Various derived quantities in these supersymmetric benchmark scenarios, including the relic density, $g_{\mu} - 2$, $b \to s\gamma$, electroweak fine-tuning $\Delta$ and the relic-density sensitivity $\Delta \Omega$, are given in [79]. These enable the reader to see at a glance which models would be excluded by which refinement of the experimental value of $g_{\mu} - 2$. Likewise, if you find some amount of fine-tuning uncomfortably large, then you are free to discard the corresponding models.

The LHC collaborations have analyzed their reach for sparticle detection in both generic studies and specific benchmark scenarios proposed previously [80]. Based on these studies, Fig. 17 displays estimates of how many different sparticles may be seen at the LHC in each of the newly-proposed benchmark scenarios [79]. The lightest Higgs boson is always found, and squarks and gluinos are usually found, though there are some scenarios where no sparticles are found at the LHC. The LHC often misses heavier weakly-interacting sparticles such as charginos, neutralinos, sleptons and the other Higgs bosons.

It was initially thought that the discovery of supersymmetry at the LHC was ‘guaranteed’ if the BNL measurement $g_{\mu} - 2$ was within $2\sigma$ of the true value, but this is no longer the case with the new sign of the pole contributions to light-by-light scattering. This is the case, in particular, because arbitrarily large values of $m_{1/2}$ and $m_0$ are now compatible with the data at the $2\sigma$ level [81].

The physics capabilities of linear $e^+e^-$ colliders are amply documented in various design studies [82]. Not only is the lightest MSSM Higgs boson observed, but its major decay modes can be measured with high accuracy. Moreover, if sparticles are light enough to be produced, their masses and other properties can be measured very precisely, enabling models of supersymmetry breaking to be tested [85].

As seen in Fig. 17, the sparticles visible at an $e^+e^-$ collider largely complement those visible at the LHC [73, 81]. In most of benchmark scenarios proposed, a 1-TeV linear collider would be able to discover and measure precisely several weakly-interacting sparticles that are invisible or difficult to detect at the LHC. However, there are some benchmark scenarios where the linear collider (as well as
Fig. 17: Summary of the prospective sensitivities of the LHC and linear colliders at different $\sqrt{s}$ energies to CMSSM particle production in the proposed benchmark scenarios G, B, ..., which are ordered by their distance from the central value of $g_{\mu} - 2$, as indicated by the pale (yellow) line in the second panel. We see clearly the complementarity between an $e^+e^-$ collider [82, 83] (or $\mu^+\mu^-$ collider [84]) and the LHC in the TeV range of energies [79], with the former excelling for non-strongly-interacting particles, and the LHC for strongly-interacting sparticles and their cascade decays. CLIC [83] provides unparalleled physics reach for non-strongly-interacting sparticles, extending beyond the TeV scale. We recall that mass and coupling measurements at $e^+e^-$ colliders are usually much cleaner and more precise than at hadron-hadron colliders such as the LHC. Note, in particular, that it is not known how to distinguish the light squark flavours at the LHC.
the LHC) fails to discover supersymmetry. Only a linear collider with a higher centre-of-mass energy appears sure to cover all the allowed CMSSM parameter space, as seen in the lower panels of Fig. 17, which illustrate the physics reach of a higher-energy lepton collider, such as CLIC [83] or a multi-TeV muon collider [84].

4.4 Prospects for Other Experiments

4.4.1 Detection of Cold Dark Matter

Fig. 18 shows rates for the elastic spin-independent scattering of supersymmetric relics [86], including the projected sensitivities for CDMS II [87] and CRESST [88] (solid) and GENIUS [89] (dashed). Also shown are the cross sections calculated in the proposed benchmark scenarios discussed in the previous section, which are considerably below the DAMA [90] range ($10^{-5} - 10^{-6}$ pb), but may be within reach of future projects. The prospects for detecting elastic spin-independent scattering are less bright, as also shown in Fig. 18. Indirect searches for supersymmetric dark matter via the products of annihilations in the galactic halo or inside the Sun also have prospects in some of the benchmark scenarios [86], as seen in Fig. 19.

4.4.2 Proton Decay

This could be within reach, with $\tau(p \rightarrow e^+ \pi^0)$ via a dimension-six operator possibly $\sim 10^{35} y$ if $m_{GUT} \sim 10^{16}$ GeV as expected in a minimal supersymmetric GUT. Such a model also suggests that $\tau(p \rightarrow \bar{\nu}K^+)<10^{32} y$ via dimension-five operators [92], unless measures are taken to suppress them [93]. This provides motivation for a next-generation megaton experiment that could detect proton decay as well as explore new horizons in neutrino physics [94].

4.5 Conclusions

We have compiled in this Lecture the various experimental constraints on the MSSM, particularly in its constrained CMSSM version. These have been compared and combined with the cosmological constraint on the relic dark matter density. As we have shown, there is good overall compatibility between these
Fig. 19: Left panel: prospects for detecting photons with energies above 1 GeV from annihilations in the centre of the galaxy, assuming a moderate enhancement there of the overall halo density, and right panel: prospects for detecting muons from energetic solar neutrinos produced by relic annihilations in the Sun, as calculated [86] in the benchmark scenarios using Neutdriver[91].

Various constraints. To exemplify the possible types of supersymmetric phenomenology compatible with all these constraints, a set of benchmark scenarios have been proposed.

We have discussed the fine-tuning of parameters required for supersymmetry to have escaped detection so far. There are regions of parameter space where the neutralino relic density is rather sensitive to the exact values of the input parameters, and to the details of the calculations based on them. However, there are generic domains of parameter space where supersymmetric dark matter is quite natural. The fine-tuning price of the electroweak supersymmetry-breaking scale has been increased by the experimental constraints due to LEP, in particular, but its significance remains debatable.

As illustrated by these benchmark scenarios, future colliders such as the LHC and a TeV-scale linear $e^+e^-$ collider have good prospects of discovering supersymmetry and making detailed measurements. There are also significant prospects for discovering supersymmetry via searches for cold dark matter particles, and searches for proton decay also have interesting prospects in supersymmetric GUT models.

One may be disappointed that supersymmetry has not already been discovered, but one should not be disheartened. Most of the energy range where supersymmetry is expected to appear has yet to be explored. Future accelerators will be able to complete the search for supersymmetry, but they may be scooped by non-accelerator experiments. In a few years’ time, we expect to be writing about the discovery of supersymmetry, not just constraints on its existence.

References

[1] J. R. Ellis, Lectures at 1998 CERN Summer School, St. Andrews, Beyond the Standard Model for Hillwalkers, arXiv:hep-ph/9812235.

[2] J. R. Ellis, The Superstring: Theory Of Everything, Or Of Nothing?, Nature 323 (1986) 595.

[3] J. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. B 260 (1991) 131; U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. B 260 (1991) 447; C. Giunti, C. W. Kim and U. W. Lee, Mod. Phys. Lett. A 6 (1991) 1745.

[4] L. Maiani, Proceedings of the 1979 Gif-sur-Yvette Summer School On Particle Physics, 1; G. ’t Hooft, in Recent Developments in Gauge Theories, Proceedings of the Nato Advanced Study Insti-
[5] M. B. Green, J. H. Schwarz and E. Witten, *Superstring Theory*, (Cambridge Univ. Press, 1987).

[6] D. A. Ross and M. J. Veltman, Nucl. Phys. B *95* (1975) 135.

[7] P. W. Higgs, Phys. Lett. *12* (1964) 132; Phys. Rev. Lett. *13* (1964) 508.

[8] F. Englert and R. Brout, Phys. Rev. Lett. *13* (1964) 321.

[9] C. T. Hill, Phys. Lett. B *266* (1991) 419; for a recent review, see: C. T. Hill and E. H. Simmons, arXiv:hep-ph/0203079.

[10] For a historical reference, see: E. Farhi and L. Susskind, Phys. Rept. *74* (1981) 277.

[11] S. Dimopoulos and L. Susskind, Nucl. Phys. B *155* (1979) 237; E. Eichten and K. Lane, Phys. Lett. B *90* (1980) 125.

[12] J. R. Ellis, M. K. Gaillard, D. V. Nanopoulos and P. Sikivie, Nucl. Phys. B *182* (1981) 529.

[13] S. Dimopoulos and J. R. Ellis, Nucl. Phys. B *182* (1982) 505.

[14] G. Altarelli and R. Barbieri, Phys. Lett. B *253* (1991) 161; M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. *65* (1990) 964.

[15] J. R. Ellis, G. L. Fogli and E. Lisi, Phys. Lett. B *343* (1995) 282.

[16] G. Altarelli, F. Caravaglios, G. F. Giudice, P. Gambino and G. Ridolfi, JHEP *0106* (2001) 018 arXiv:hep-ph/0106029.

[17] For a recent reference, see: K. Lane, *Two lectures on technicolor*, arXiv:hep-ph/0202255.

[18] B. Holdom, Phys. Rev. D *24* (1981) 1441.

[19] LEP Electroweak Working Group, http://lepewwg.web.cern.ch/LEPEWWG/Welcome.html.

[20] LEP Higgs Working Group for Higgs boson searches, OPAL Collaboration, ALEPH Collaboration, DELPHI Collaboration and L3 Collaboration, *Search for the Standard Model Higgs Boson at LEP*, ALEPH-2001-066, DELPHI-2001-113, CERN-L3-NOTE-2699, OPAL-PN-479, LHWG-NOTE-2001-03, CERN-EP/2001-055, arXiv:hep-ex/0107029. *Searches for the neutral Higgs bosons of the MSSM: Preliminary combined results using LEP data collected at energies up to 209 GeV*, LHWG-NOTE-2001-04, ALEPH-2001-057, DELPHI-2001-114, L3-NOTE-2700, OPAL-TN-699, arXiv:hep-ex/0107030.

[21] J. Erler, Phys. Rev. D *63* (2001) 071301 [arXiv:hep-ph/0010153].

[22] J. R. Ellis and D. Ross, Phys. Lett. B *506* (2001) 331 [arXiv:hep-ph/0012067].

[23] For a review, see: T. Hambye and K. Riesselmann, arXiv:hep-ph/9708416.

[24] G. Isidori, G. Ridolfi and A. Strumia, Nucl. Phys. B *609* (2001) 387 [arXiv:hep-ph/0104016].

[25] S. R. Coleman and J. Mandula, Phys. Rev. *159* (1967) 1251.

[26] Y. A. Golfand and E. P. Likhtman, JETP Lett. *13* (1971) 323 [Pisma Zh. Eksp. Teor. Fiz. *13* (1971) 452].
[27] A. Neveu and J. H. Schwarz, Nucl. Phys. B 31 (1971) 86.
[28] P. Ramond, Phys. Rev. D 3 (1971) 2415.
[29] D. V. Volkov and V. P. Akulov, Phys. Lett. B 46 (1973) 109.
[30] J. Wess and B. Zumino, Phys. Lett. B 49 (1974) 52; Nucl. Phys. B 70 (1974) 39.
[31] J. Wess and B. Zumino, Nucl. Phys. B 78 (1974) 1.
[32] S. Ferrara, J. Wess and B. Zumino, Phys. Lett. B 51 (1974) 239; S. Ferrara, J. Iliopoulos and B. Zumino, Nucl. Phys. B 77 (1974) 413.
[33] P. Fayet, as reviewed in Supersymmetry, Particle Physics And Gravitation, CERN-TH-2864, published in Proc. of Europhysics Study Conf. on Unification of Fundamental Interactions, Erice, Italy, Mar 17-24, 1980, eds. S. Ferrara, J. Ellis, P. van Nieuwenhuizen (Plenum Press, 1980).
[34] D. Z. Freedman, P. van Nieuwenhuizen and S. Ferrara, Phys. Rev. D 13 (1976) 3214; S. Deser and B. Zumino, Phys. Lett. B 62 (1976) 335.
[35] S. W. Hawking, Is The End In Sight For Theoretical Physics? Phys. Bull. 32 (1981) 15.
[36] R. Haag, J. T. Lopuszanski and M. Sohnius, Nucl. Phys. B 88 (1975) 257.
[37] F. Iachello, Phys. Rev. Lett. 44 (1980) 772.
[38] Y. Okada, M. Yamaguchi and T. Yanagida, Prog. Theor. Phys. 85 (1991) 1; J. R. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B 257 (1991) 83; H. E. Haber and R. Hempfling, Phys. Rev. Lett. 66 (1991) 1815.
[39] For an early review, see: P. Fayet and S. Ferrara, Phys. Rept. 32 (1977) 249; see also: H. P. Nilles, Phys. Rept. 110 (1984) 1; H. E. Haber and G. L. Kane, Phys. Rept. 117 (1985) 75.
[40] I. Antoniadis, Lectures at this school; for a recent phenomenological review, see: I. Antoniadis and K. Benakli, Int. J. Mod. Phys. A 15 (2000) 4237 [arXiv:hep-ph/0007226].
[41] B. A. Campbell, S. Davidson, J. R. Ellis and K. A. Olive, Phys. Lett. B 256 (1991) 457; W. Fischler, G. F. Giudice, R. G. Leigh and S. Paban, Phys. Lett. B 258 (1991) 45.
[42] S. Dimopoulos and H. Georgi, Nucl. Phys. B 193 (1981) 150; N. Sakai, Z. Phys. C 11 (1981) 153.
[43] P. Fayet and J. Iliopoulos, Phys. Lett. B 51 (1974) 461.
[44] M. Dine, N. Seiberg and E. Witten, Nucl. Phys. B 289 (1987) 589.
[45] L. O'Raifeartaigh, Nucl. Phys. B 96 (1975) 331; P. Fayet, Phys. Lett. B 58 (1975) 67.
[46] J. Polonyi, Hungary Central Inst. Res. preprint KFKI-77-93 (1977).
[47] E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and P. van Nieuwenhuizen, Nucl. Phys. B 147 (1979) 105.
[48] R. Barbieri, S. Ferrara and C. A. Savoy, Phys. Lett. B 119 (1982) 343; A. H. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. 49 (1982) 970.
[49] E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, Phys. Lett. B 133 (1983) 61; J. R. Ellis, A. B. Lahanas, D. V. Nanopoulos and K. Tamvakis, Phys. Lett. B 134 (1984) 429; J. R. Ellis, C. Kounnas and D. V. Nanopoulos, Nucl. Phys. B 241 (1984) 406.
[50] E. Witten, Phys. Lett. B 155 (1985) 151.

[51] J. R. Ellis and D. V. Nanopoulos, Phys. Lett. B 110 (1982) 44; R. Barbieri and R. Gatto, Phys. Lett. B 110 (1982) 211.

[52] K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita, Prog. Theor. Phys. 68 (1982) 927 [Erratum-ibid. 70 (1982) 330]; L.E. Ibáñez and G.G. Ross, Phys. Lett. B 110 (1982) 215; L.E. Ibáñez, Phys. Lett. B 118 (1982) 73; J. Ellis, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. B 121 (1983) 123; J. Ellis, J. Hagelin, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. B 125 (1983) 275; L. Alvarez-Gaumé, J. Polchinski, and M. Wise, Nucl. Phys. B 221 (1983) 495.

[53] S. L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D 2 (1970) 1285.

[54] J. R. Ellis and S. Rudaz, Phys. Lett. B 128 (1983) 248.

[55] J. R. Ellis, T. Falk, G. Ganis, K. A. Olive and M. Schmitt, Phys. Rev. D 58 (1998) 095002 [arXiv:hep-ph/9801445].

[56] J. Ellis, J.S. Hagelin, D.V. Nanopoulos, K.A. Olive and M. Srednicki, Nucl. Phys. B 238 (1984) 453; see also H. Goldberg, Phys. Rev. Lett. 50 (1983) 1419.

[57] H. Goldberg, Phys. Rev. Lett. 50 (1983) 1419.

[58] S. Heinemeyer, W. Hollik and G. Weiglein, Comput. Phys. Commun. 124, 76 (2000) [arXiv:hep-ph/9812320]; S. Heinemeyer, W. Hollik and G. Weiglein, Eur. Phys. J. C 9 (1999) 343 [arXiv:hep-ph/9812472].

[59] Joint LEP 2 Supersymmetry Working Group, Combined LEP Chargino Results, up to 208 GeV, http://lepsusy.web.cern.ch/lepsusy/www/inos_moriond01/charginos_pub.html.

[60] Joint LEP 2 Supersymmetry Working Group, Combined LEP Selenon/Smuon/Stau Results, 183-208 GeV, http://alephwww.cern.ch/~ganis/SUSYWG/SLEP/sleptons2k01.html.

[61] J. R. Ellis, T. Falk, G. Ganis, K. A. Olive and M. Srednicki, Phys. Lett. B 510 (2001) 236 [arXiv:hep-ph/0102098].

[62] M.S. Alam et al., [CLEO Collaboration], Phys. Rev. Lett. 74 (1995) 2885 as updated in S. Ahmed et al., CLEO CONF 99-10; BELLE Collaboration, BELLE-CONF-0003, contribution to the 30th International conference on High-Energy Physics, Osaka, 2000. See also K. Abe et al., [Belle Collaboration], [arXiv:hep-ex/0107065]; L. Lista [BaBar Collaboration], [arXiv:hep-ex/0110010]; C. Degrandi, P. Gambino and G. F. Giudice, JHEP 0012 (2000) 009 [arXiv:hep-ph/0009337]; M. Carena, D. Garcia, U. Nierste and C. E. Wagner, Phys. Lett. B 499 (2001) 141 [arXiv:hep-ph/0010003].

[63] M. Carena, J. R. Ellis, A. Pilaftsis and C. E. Wagner, Nucl. Phys. B 586 (2000) 92 [arXiv:hep-ph/0003180], Phys. Lett. B 495 (2000) 155 [arXiv:hep-ph/0009212]; and references therein.

[64] J. R. Ellis, G. Ganis, D. V. Nanopoulos and K. A. Olive, Phys. Lett. B 502 (2001) 171 [arXiv:hep-ph/0009355].

[65] H. N. Brown et al. [Muon g-2 Collaboration], Phys. Rev. Lett. 86, 2227 (2001) [arXiv:hep-ex/0102017].
[66] M. Knecht and A. Nyffeler, arXiv:hep-ph/0111058; M. Knecht, A. Nyffeler, M. Perrottet and E. De Rafael, arXiv:hep-ph/0111059; M. Hayakawa and T. Kinoshita, arXiv:hep-ph/0112102; I. Blokland, A. Czarnecki and K. Melnikov, arXiv:hep-ph/0112117; J. Bijnens, E. Pallante and J. Prades, arXiv:hep-ph/0112255.

[67] R. Alemany, M. Davier and A. Hocker, Eur. Phys. J. C 2 (1998) 123 [arXiv:hep-ph/9703220]; M. Davier and A. Hocker, Phys. Lett. B 419 (1998) 419 [arXiv:hep-ph/9711108]; M. Davier and A. Hocker, Phys. Lett. B 435 (1998) 427 [arXiv:hep-ph/9805470]; S. Narison, Phys. Lett. B 513 (2001) 53 [arXiv:hep-ph/0103199]; J. F. De Troconiz and F. J. Yndurain, arXiv:hep-ph/0106025.

[68] L. L. Everett, G. L. Kane, S. Rigolin and L. Wang, Phys. Rev. Lett. 86, 3484 (2001) [arXiv:hep-ph/0102145]; J. L. Feng and K. T. Matchev, Phys. Rev. Lett. 86, 3480 (2001) [arXiv:hep-ph/0102146]; E. A. Baltz and P. Gondolo, Phys. Rev. Lett. 86, 5004 (2001) [arXiv:hep-ph/0102147]; U. Chattopadhyay and P. Nath, Phys. Rev. Lett. 86, 5854 (2001) [arXiv:hep-ph/0102157]; S. Komine, T. Moroi and M. Yamaguchi, Phys. Lett. B 506, 93 (2001) [arXiv:hep-ph/0102204]; J. Ellis, D. V. Nanopoulos and K. A. Olive, Phys. Lett. B 508 (2001) 65 [arXiv:hep-ph/0102331]; R. Arnowitt, B. Dutta, B. Hu and Y. Santos, Phys. Lett. B 505 (2001) 177 [arXiv:hep-ph/0102344]; S. P. Martin and J. D. Wells, Phys. Rev. D 64, 035003 (2001) [arXiv:hep-ph/0103067]; H. Baer, C. Balazs, J. Ferrandis and X. Tata, Phys. Rev. D 64, 035004 (2001) [arXiv:hep-ph/0103280].

[69] S. Mizuta and M. Yamaguchi, Phys. Lett. B 298 (1993) 120 [arXiv:hep-ph/9208251]; J. Edsjo and P. Gondolo, Phys. Rev. D 56 (1997) 1879 [arXiv:hep-ph/9704361].

[70] J. Ellis, T. Falk and K. A. Olive, Phys. Lett. B 444 (1998) 367 [arXiv:hep-ph/9810361]; J. Ellis, T. Falk, K. A. Olive and M. Srednicki, Astropart. Phys. 13 (2000) 181 [arXiv:hep-ph/9905481]; M. E. Gómez, G. Lazarides and C. Pallis, Phys. Rev. D 61 (2000) 123512 [arXiv:hep-ph/9907261] and Phys. Lett. B 487 (2000) 313 [arXiv:hep-ph/0004028]; R. Arnowitt, B. Dutta and Y. Santos, Nucl. Phys. B 606 (2001) 59 [arXiv:hep-ph/0102181].

[71] C. Boehm, A. Djouadi and M. Drees, Phys. Rev. D 62 (2000) 035012 [arXiv:hep-ph/9911496].

[72] J. Ellis, K.A. Olive and Y. Santos, arXiv:hep-ph/0111213.

[73] M. Drees and M. M. Nojiri, Phys. Rev. D 47 (1993) 376 [arXiv:hep-ph/9207234]; H. Baer and M. Brhlik, Phys. Rev. D 53 (1996) 597 [arXiv:hep-ph/9508321] and Phys. Rev. D 57 (1998) 567 [arXiv:hep-ph/9706509]; H. Baer, M. Brhlik, M. A. Diaz, J. Ferrandis, P. Mercadante, P. Quintana and X. Tata, Phys. Rev. D 63 (2001) 015007 [arXiv:hep-ph/0005027]; A. B. Lahanas, D. V. Nanopoulos and V. C. Spanos, Mod. Phys. Lett. A 16 (2001) 1229 [arXiv:hep-ph/0009065].

[74] J. L. Feng, K. T. Matchev and T. Moroi, Phys. Rev. Lett. 84, 2322 (2000) [arXiv:hep-ph/9908309]; J. L. Feng, K. T. Matchev and T. Moroi, Phys. Rev. D 61, 075005 (2000) [arXiv:hep-ph/9909334]; J. L. Feng, K. T. Matchev and F. Wilczek, Phys. Lett. B 482, 388 (2000) [arXiv:hep-ph/0004043].

[75] J. R. Ellis and K. A. Olive, Phys. Lett. B 514 (2001) 114 [arXiv:hep-ph/0105004].

[76] For other recent calculations, see, for example: A. B. Lahanas, D. V. Nanopoulos and V. C. Spanos, Phys. Lett. B 518 (2001) 94 [arXiv:hep-ph/0107151]; V. Barger and C. Kao, Phys. Lett. B 518, 117 (2001) [arXiv:hep-ph/0106189]; L. Roszkowski, R. Ruiz de Austri and T. Nihei, JHEP 0108, 024 (2001) [arXiv:hep-ph/0106334]; A. Djouadi, M. Drees and J. L. Kneur, JHEP 0108, 055 (2001) [arXiv:hep-ph/0107316].

[77] J. Ellis, K. Enqvist, D. V. Nanopoulos and F. Zwirner, Mod. Phys. Lett. A 1, 57 (1986); R. Barbieri and G. F. Giudice, Nucl. Phys. B 306 (1988) 63.
[78] See, for example: I. Hinchliffe, F. E. Paige, M. D. Shapiro, J. Soderqvist and W. Yao, Phys. Rev. D 55 (1997) 5520; TESLA Technical Design Report, DESY-01-011, Part III, Physics at an e+e− Linear Collider (March 2001).

[79] M. Battaglia et al., Eur. Phys. J. C 22 (2001) 535 [arXiv:hep-ph/0106204].

[80] ATLAS Collaboration, ATLAS detector and physics performance Technical Design Report, CERN/LHCC 99-14/15 (1999); S. Abdullin et al. [CMS Collaboration], arXiv:hep-ph/9806366; S. Abdullin and F. Charles, Nucl. Phys. B 547 (1999) 60 [arXiv:hep-ph/9811402]; CMS Collaboration, Technical Proposal, CERN/LHCC 94-38 (1994).

[81] M. Battaglia et al., in Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001), eds. R. Davidson and C. Quigg, arXiv:hep-ph/0112013.

[82] S. Matsumoto et al. [JLC Group], JLC-1, KEK Report 92-16 (1992); J. Bagger et al. [American Linear Collider Working Group], The Case for a 500-GeV e+e− Linear Collider, SLAC-PUB-8495, BNL-67545, FERMILAB-PUB-00-152, LBNL-46299, UCRL-ID-139524, LBL-46299, Jul 2000, arXiv:hep-ex/0007023; T. Abe et al. [American Linear Collider Working Group Collaboration], Linear Collider Physics Resource Book for Snowmass 2001, SLAC-570, arXiv:hep-ex/0106055, hep-ex/0106056, hep-ex/0106057; TESLA Technical Design Report, DESY-01-011, Part III, Physics at an e+e− Linear Collider (March 2001).

[83] R. W. Assmann et al. [CLIC Study Team], A 3-TeV e+e− Linear Collider Based on CLIC Technology, ed. G. Guignard, CERN 2000-08; CLIC Physics Study Group, http://clicphysics.web.cern.ch/CLICphysics/.

[84] Neutrino Factory and Muon Collider Collaboration, http://www.cap.bnl.gov/mumu/ mu_home_page.html; European Muon Working Groups, http://muonstoragerings.cern.ch/Welcome.html.

[85] G. A. Blair, W. Porod and P. M. Zerwas, Phys. Rev. D63 (2001) 017703 [arXiv:hep-ph/0007107].

[86] J. Ellis, J. L. Feng, A. Ferstl, K. T. Matchev and K. A. Olive, arXiv:astro-ph/0110225.

[87] CDMS Collaboration, R. W. Schnee et al., Phys. Rept. 307, 283 (1998).

[88] CRESST Collaboration, M. Bravin et al., Astropart. Phys. 12, 107 (1999) [arXiv:hep-ex/9904005].

[89] H. V. Klapdor-Kleingrothaus, arXiv:hep-ph/0104028.

[90] DAMA Collaboration, R. Bernabei et al., Phys. Lett. B 436 (1998) 379.

[91] G. Jungman, M. Kamionkowski and K. Griest, Phys. Rept. 267, 195 (1996) [arXiv:hep-ph/9506380]; http://t8web.lanl.gov/people/jungman/neut-package.html.

[92] H. Murayama and A. Pierce, arXiv:hep-ph/0108104.

[93] J. R. Ellis, J. S. Hagelin, S. Kelley and D. V. Nanopoulos, Nucl. Phys. B 311 (1988) 1.

[94] C. K. Jung, arXiv:hep-ex/0005046; Y. Suzuki et al. [TITAND Working Group Collaboration], arXiv:hep-ex/0110005.