Colorless States in Perturbative QCD: Charmonium and Rapidity Gaps

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Abstract

We point out that an unorthodox way to describe the production of rapidity gaps in deep inelastic scattering, recently proposed by Buchmüller and Hebecker, suggests a description of the production of heavy quark bound states which is in agreement with data. The approach questions the conventional treatment of the color quantum number in perturbative QCD.

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Buchmüller and Hebecker recently proposed a rather unconventional description of the formation of rapidity gaps in deep inelastic scattering [1]. The mechanism is illustrated in Fig. 1. The diagram shown represents the production of final state hadrons which are ordered in rapidity. From top to bottom we find the fragments of the intermediate partonic quark-antiquark state and those of the target. Buchmüller and Hebecker proposed that the origin of a rapidity gap corresponds to the absence of color between photon and proton, i.e. the $3 \times \bar{3}$ ($=1+8$) intermediate quark-antiquark state is in a color singlet state. Because color is the source of hadrons, only the color octet states yield hadronic asymptotic states. This leads to the simple, and experimentally verified, prediction that

$$F_2^{\text{gap}} = \frac{1}{1+8} F_2$$

(1)

This relation embodies the inevitable conclusion that events with and without gaps are described by the same short-distance dynamics. Essentially non-perturbative final-state interactions dictate the appearance of gaps whose frequency is determined by simple counting.

The orthodox description of rapidity gaps is sketched in Fig. 2. The $t$-channel exchange of a pair of gluons in a color singlet state is the origin of the gap. The color string which connects photon and proton in diagrams such as the one in Fig. 1 is absent and no hadrons are produced in the rapidity region separating them. The same mechanism predicts rapidity gaps between a pair of jets produced in hadronic collisions; see Fig. 3. These have been observed and occur with a frequency of order of one percent [2]. These gaps can, however, be accommodated as a mere final state color bleaching phenomenon à la Buchmüller and Hebecker. This can be visualized using the diagram shown in Fig. 4. At short distances it represents a conventional perturbative diagram for the production of a pair of jets. Also shown is the string picture for the formation of the final state hadrons. Color in the final state is bleached by strings connecting the $3$ jet at the top with the $\bar{3}$ spectator di-quark at the bottom and vice-versa. The probability to form a gap can be counted à la Buchmüller and Hebecker to be $1/(1+8)^2$ because it requires the formation of singlets in 2 strings. This is consistent with observation and predicts that, as was the case for electroproduction, the same short distance dynamics governs events with and without rapidity gaps. The data [2] is consistent with the prediction of this simple picture...
and, in fact, hard to understand with the alternative mechanism of Fig. 2.
The detailed dynamics may, however, be more complex and, for instance,
also involve initial state radiation.

It is clear from Figs. 2–4 that we have formulated alternative s- and
t-channel pictures to view the same physics. Although they seem at first
radically different, this may not be the case. Computation of the exchange
of a pair of colorless gluons in the t-channel is not straightforward and em-
body all the unsolved mysteries of constructing the “Pomeron” in QCD.
In a class of models where the Pomeron is constructed out of gluons with a
dynamically generated mass [3, 4], the diagram of Fig. 3 is, not surpris-
ingly, dominated by the configuration where one gluon is hard and the other soft.
The diagram is identical to the standard perturbative diagram except for
the presence of a soft, long-wavelength gluon whose only role is to bleach
color. Its dynamical role is minimal, events with gaps are not really different
from events without them. Soft gluons readjust the color at large distances
and long times. Their description is outside the realm of perturbative QCD.
Clearly this view of the t-channel exchange is completely compatible with
the s-channel picture. At the perturbative level the s- and t-channel views
are identical and the color structure of the event is dictated by large distance
interactions: string fragmentation in the s-channel and long-wavelength soft
gluons in the t-channel picture. In this class of models the genuinely hard
Pomeron is expected to be no more than an order $\alpha_s^2$ correction, a view which
can be defended on more solid theoretical ground [3]. Note that our discus-
sion is at best indirectly relevant to completely non-perturbative phenomena
like elastic scattering. There is no short distance limit defined by a large
scale. The Pomeron exists.

Although we have now made a connection between both pictures, we have
also made a fundamental shift in the way color is viewed in perturbation
theory. It should, in fact, be ignored. Color structure is dictated by large-
distance fluctuations of quarks and gluons. It is complex enough that the
occupation of different states probably respects statistical counting, *e.g.*, in
determining the fraction of singlet and octet states in a given rapidity region.
In retrospect it does, in fact, seem illogical to worry about color at short
distances, given that soft partons have an infinite time to readjust any color
structure previous to the formation of asymptotic states. The main point
of this paper is to suggest that the issue can be settled by studying the
production of colorless states which are theoretically, and experimentally,
more well-defined than rapidity gaps: heavy quark bound states.

In Figs. 5 and 6 we show typical diagrams for the production of $\psi$-particles using the competing treatments of the color quantum number. In the diagram of Fig. 5 the $\psi$ is associatively produced with a final state gluon which is required to conserve color. The diagram is related by crossing to the hadronic decay $\psi \rightarrow 3$ gluons. In the alternative approach, however, the color singlet property of the $\psi$ is not enforced at the perturbative level. The $\psi$ can, for instance, be produced to leading order by gluon-gluon, as well as quark-antiquark, annihilation into $c\bar{c}$. The latter mechanism is the color-equivalent of the Drell-Yan process. Both mechanisms can be calculated perturbatively. Their dynamics are dictated by short-distance interactions of range $\Delta x \approx m_\psi^{-1}$. At large distances the exchange of soft gluons between the $c$ and $\bar{c}$ and the spectators partons will dictate the final color of the pair. As before, the probability that a singlet is formed and produces an onium-state is $1/(1+8)$; the colored states result in the production of $D\bar{D}$ pairs. We predict

$$\sigma_{onium} = \frac{1}{9} \int_{2m_c}^{2m_D} dm \frac{d\sigma_{c\bar{c}}}{dm}$$

$$\sigma_{open} = \frac{8}{9} \int_{2m_c}^{2m_D} dm \frac{d\sigma_{c\bar{c}}}{dm} + \int_{2m_D}^{2m_c} dm \frac{d\sigma_{c\bar{c}}}{dm}$$

where $\sigma_{c\bar{c}}$ is the cross section for producing heavy quark pairs. It is computed perturbatively. These relations can be generalized in an obvious way to differential cross sections, such as $d\sigma/dx_F$ and $d\sigma/dp_T$.

The approach embodied in Eqs. (2) and (3) seems reasonable: it is indeed hard to imagine that a color singlet state formed at a range $m_\psi^{-1}$, as in Fig. 4, automatically survives to form a $\psi$. This formalism was, in fact, proposed almost twenty years ago [6, 7, 8] and subsequently abandoned for no good reason. Maybe the time has come to contemplate the possibility that the phenomenological problems [9] of the current perturbative approach in describing data on the hadronic production of onium-states is connected to the inappropriate treatment of color. Other approaches of similar spirit can be found in Refs. [10] and [11].

This also raises the question whether the alternative approach is (still) consistent with data. In the limit that the masses $m_\eta$, $m_\psi$ and $2m_D$ are equal, Eqs. (2) and (3) state that the production of onium- and open charm states
is dictated by identical dynamics. Only a normalization factor connected to
color differentiates the two cases. This way of viewing the approach mirrors
Eq. (1) for deep inelastic scattering. One may be able to take the idea of
final state counting one step further by dividing the total color-singlet cross
section, which is rigorously predicted, into onium-states according to the
simple statistical counting

$$\sigma_X = \rho_X \sigma_{onium}$$  \hspace{1cm} (4)

with

$$\rho_X = \frac{2 J_X + 1}{\sum_i (2 J_i + 1)},$$  \hspace{1cm} (5)

where $J_X$ is the spin of any onium state $X$; the sum runs over all onium
states.

The above formalism can, at best, be approximate; we expect phase space
corrections favoring the lighter states. It is, in this context, interesting to
note that any approach must satisfy the sum rule that the sum of the cross
sections of all onium-states is given by Eq. (2), i.e.

$$\sum_i \sigma_i = \frac{1}{9} \int_{2m_c}^{2m_{\psi}} dm \frac{d\sigma_{e\bar{e}}}{dm}.$$  \hspace{1cm} (6)

This relation is, unfortunately, difficult to test experimentally since it requires
measuring cross sections for all of the charmonium bound states at a given
energy.

Our prediction of Eqs. (2) and (3) that the production of hidden and open
charm have similar dynamics is supported by Fig. 7, where we have plotted
the experimental data for the production cross section of $J/\psi$ [12] and $D\bar{D}$
[13] as a function of the center-of-mass energy. The $J/\psi$ data has been mul-
tiplied by a constant, which, in the aforementioned limit where $m_{\psi} = 2m_D$, is simply $8/\rho_\psi$. The agreement is remarkable. Obviously, higher statistics
data would exhibit deviations from our predictions since threshold effects
have not been taken into account. In Fig. 8 we directly compare our leading-
order predictions of Eqs. (2) and (3) with data on $J/\psi$ and $D\bar{D}$ production.
We here used the MRS A parameterization of the nucleon structure function
with the scale $Q^2 = \sqrt{s}$ and $m_c = m_\eta/2$ in our calculations. In order to
fit the data, we multiplied the leading order prediction of Eq. (3) by a “K”
factor of $K_{D\bar{D}} = 6.4$. We multiplied the prediction of Eq. (2) by a factor of
\( \rho_\psi K_{onium} = 3.0 \). It is interesting to notice that the energy dependencies of the cross sections are well explained by our simple model. Assuming that the next to leading order corrections (\( K \) factors) to both processes are equal, \( i.e. \ K_{DD} = K_{onium} \), we determine the fraction of \( J/\psi (\rho_\psi) : \rho_\psi = 3.0/6.4 = 0.47 \).

In the same spirit, starting from Eqs. (2) – (4), our model predicts that the normalized \( x_F \) distribution for \( J/\psi \) and \( DD \) pairs should be the same at a given center-of-mass energy. This is indeed the case [14, 15]; see Fig. 9.

In our model the production of charmonium states proceeds in two stages. First the \( c\bar{c} \) pair is produced perturbatively, subsequently it evolves non-perturbatively into the asymptotic states. Therefore, the study of the production of specific onium-states should shed light on the non-perturbative dynamics responsible for onium formation. The available experimental data [16] for the ratios \( \sigma(\chi)/\sigma(\psi) \) and \( \sigma(\psi')/\sigma(\psi) \) indicate that these quantities are not only independent of the center-of-mass energy but also independent of the target and incident beams [17]. We summarize the experimental results for fractions of individual charmonium states in Table 1. Simple counting predicts that the fraction of \( c\bar{c} \) pair forming a given state is given by (5). However, this does not take into account the feed-down from the decays of heavier states into the lighter ones. In Table 1 we also list the predictions corrected for this effect. This very simple model clearly captures the general features of the data. A quantitative comparison requires a model which incorporates phase space. It is nevertheless important to point out that simple counting predicts a substantial production of \( \psi' \) in agreement with observations. This should be contrasted with the conventional treatment.

A more detailed discussion of both theory and experiment, including applications to bottomonium, will be published elsewhere.

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Observable | Experimental results | Statistical model
---|---|---
Fraction of $\psi$ from $\chi$ decay | $0.36 \pm 0.02$ | $0.24$
Fraction of $\psi$ from $\psi'$ decay | $0.059 \pm 0.005$ | $0.28$
$\sigma(\psi')/\sigma(\psi)$ | $0.10 \pm 0.01$ | $0.48$
$\sigma_{direct}(\psi)$ | $0.57 \pm 0.02$ | $0.48$

Table 1: Components of the charmonium production. We have averaged the available experimental data.

Figure 1: Mechanism for the production of rapidity gaps in deep inelastic scattering.
Figure 2: Pomeron mechanism for the formation of rapidity gaps.

Figure 3: Pomeron mechanism for the formation of rapidity gaps in hadron collisions.
Figure 4: Color bleaching picture for the formation of rapidity gaps in hadron collisions.

Figure 5: Mechanism for the production of $J/\psi$ in the color singlet model.
Figure 6: Mechanism for the production of $J/\psi$ in the color bleaching model.
Figure 7: Total cross section per nucleon for the production of $D\bar{D}$ (circles) and $J/\psi$ (squares) in proton-nucleon collisions. The $J/\psi$ cross section has been multiplied by a constant factor of 50.
Figure 8: Total cross section per nucleon for the production of $D\bar{D}$ (circles) and $J/\psi$ (squares) in proton-nucleon collisions.
Figure 9: Normalized $x_F$ distribution for the production of $D\bar{D}$ (histogram) at $\sqrt{s} = 27.4$ GeV and $J/\psi$ (squares) in proton-nucleon collisions at $\sqrt{s} = 23.7$ GeV. The curves are the prediction of our model.