Helical phase inflation and its observational constraints

Mudassar Sabir,\textsuperscript{a} Waqas Ahmed,\textsuperscript{b} Yungui Gong,\textsuperscript{a,1} Tianjun Li\textsuperscript{c,d} and Jiong Lin\textsuperscript{a}

\textsuperscript{a}School of Physics, Huazhong University of Science and Technology, Wuhan, Hubei 430074, China
\textsuperscript{b}School of Physics, Nankai University, Nankai District, Tianjin, China
\textsuperscript{c}School of Physical Sciences, University of Chinese Academy of Sciences, Beijing, China
\textsuperscript{d}CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

E-mail: msabir@hust.edu.cn, waqasmit@nankai.edu.cn, yggong@mail.hust.edu.cn, tli@itp.ac.cn, jionglin@hust.edu.cn

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Abstract. We consider a class of helical phase inflation models from the $\mathcal{N} = 1$ supergravity where the phase component of a complex field acts as an inflaton. This class of models avoids the eta problem in supergravity inflation due to the phase monodromy of the superpotential. We study the inflationary predictions of this class of models in the context of both standard and large extra dimensional brane cosmology, and find that they can easily accommodate the Planck 2018 and BICEP2 constraints. We find that the helical phase inflation has $\alpha$-attractors and the attractors depend on one model parameter only.

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\textsuperscript{1}Corresponding author.
1 Introduction

Our observable universe has a finite age of 14 billion years, while it has already expanded to about 46 billion light years. And it appears spectacularly homogeneous and isotropic with cosmic microwave background temperature anisotropies only at the order of $10^{-5}$ or less. The current far off regions of the universe were in causal contact, and the universe may have gone through an era of exponential expansion which provides a solution to these puzzles in standard cosmology [1–6]. The inflationary models have specific predictions that can be tested experimentally. A plethora of inflation models are available in the literature, see ref. [7] for a detailed list. For slow-roll inflation, field excursions are related to the primordial gravitational wave that has not been detected so far. However, the future satellite experiments will have the required sensitivity to measure the tensor-to-scalar ratio up to $\sim 0.001$. For example, the future LiteBIRD, an experiment designed for the detection of B-mode polarization pattern embedded in the Cosmic Microwave Background anisotropies, is sensitive enough to detect primordial gravitational waves up to $r \sim 10^{-3}$ [8].

Helical phase inflation from $\mathcal{N} = 1$ supergravity was proposed where the phase of a complex field acts as an inflaton while the radial component is strongly stabilized [9–11]. The phase field rolls down along the deformed helicoid shaped potential. It is a generically difficult problem to generate a sufficiently flat scalar potential in supergravity due to the exponential Kähler potential factor in the scalar potential. To circumvent this eta problem usually additional symmetries are imposed. In helical phase inflation, we have the U(1) phase monodromy of superpotential to circumvent this eta problem automatically. Moreover, the helical model can easily interpolate between the natural inflation [12] and the Starobinsky-like inflation [2] in a single potential.

The four-dimensional single field chaotic inflationary models are in tension with the distance conjecture $\Delta \theta < 1$ [13] because of the large value of the tensor-to-scalar ratio. In the context of distance conjecture [13–15], although geodesic trajectories are quite well understood, the generalization to non-geodesic trajectories is still not clear at a quantitative level [16, 17]. In view of the quantum gravity constraint, it seems a good idea to explore the non-geodesic trajectories as they might be less constrained than geodesic ones, but it is still an open question whether the large field excursions for monodromic axions will eventually be compatible with quantum gravity.

In braneworld scenario the tension between the distance conjecture and the large value of tensor-to-scalar ratio is significantly reduced due to the modified Friedmann equation with a $\rho^2$ correction [18–26]. This makes brane inflation in large extra dimensional scenario an
interesting possibility to explore further. Braneworld scenario is also a prediction of brane gas cosmology [27] where our 4-dimensional spacetime is embedded in a higher dimensional bulk. Therefore, it is interesting to study the helical phase inflation in braneworld scenario.

In this paper, we discuss the helical phase inflation in the braneworld scenario. In section 2, we briefly review the helical phase inflation from the $\mathcal{N} = 1$ supergravity. By varying the parameters, we can interpolate from natural inflation to Starobinsky-like inflation. We study the observational constraints on the model parameters, and present the viable parameter space which is consistent with the Planck 2018 data and BICEP2 results [28, 29]. It is found that the natural inflation is marginally consistent with the observations at the 2σ level. Next, we consider the helical phase inflation in the setup of the large extra dimensional scenario where our four-dimensional world is embedded in a five-dimensional space-time [30]. we discuss the modified Friedmann equation, and then study the observational constraints. Similarly, we present the viable parameter space which is consistent with the Planck 2018 data and BICEP2 results as well. In particular, the natural inflation is excluded by the observations. While the Starobinsky-like inflation provides the favorable central value for spectral index $n_s$, and the range of values of tensor-to-scalar ratio $r$ spans the full experimental and theoretical estimate. The constraints on the temperature of reheating are also discussed.

## 2 Helical phase inflation

In helical phase inflation, the inflaton $\theta$, i.e., the phase component of a complex field, is a pseudo Nambu-Goldstone boson (PNGB) [9–11]. The potential of a complex field admits helicoid structure and the inflaton evolves along a local valley, tracing a helical trajectory. The Kähler potential $K$ and the holomorphic superpotential $W$ are

$$
\begin{align*}
K &= \Phi \bar{\Phi} + X \bar{X} - g(XX)^2, \\
W &= a \frac{X}{\Phi} (\Phi^\chi - 1), \quad \chi = b + ic.
\end{align*}
$$

(2.1)

(2.2)

It was thoroughly shown in ref. [9] that the exponent $\chi$ has a geometrical origin associated with non-geometric flux compactification of Type IIB string theory.

The eta problem of supergravity theory is solved due to a global $U(1)$ symmetry of the Kähler potential that introduces phase monodromy in the superpotential.

$$
\Phi \rightarrow e^{2\pi i} \Phi, \quad K \rightarrow K, \quad W \rightarrow W + a \frac{X}{\Phi} \Phi^\chi(e^{2\pi i \chi} - 1).
$$

(2.3)

The scalar potential is determined by the Kähler potential $K$ and superpotential $W$

$$
V = e^K (K_{ij} D_i W D_j \bar{W} - 3W \bar{W}),
$$

(2.4)

where $K_{ij} = \partial_i \partial_j K$ and $D_i W = \partial_i W + K_i W$. During inflation the field $X$ is strongly fixed at its vacuum expectation value $\langle X \rangle = 0$. Hence the scalar potential can be written as follows

$$
V(r, \theta) = a^2 e^{r^2} \left( 1 + r^{2b} e^{-2c\theta} - 2r^b e^{-c\theta} \cos(b\theta + c \log r) \right).
$$

(2.5)

in reduced Planck units ($M_P \equiv 1/(8\pi G) = 1$), with $\Phi \equiv re^{i\theta}$.

The norm of $\Phi$ needs to be stabilized otherwise it will generate notable iso-curvature perturbation that contradicts with observations. However, it is a non trivial task to stabilize
the norm of $\Phi$ while keep the phase light as the norm and phase couple to each other. Curvature along the radial direction is determined by the coefficient $e^{r^2/r^2}$, which admits a global minimum at $r = 1$ and gives a large mass above the Hubble scale. Extra couplings between the field norm $r$ and the phase $\theta$ in $V(r, \theta)$ can partially affect the stabilization of $|\Phi|$, although $b \ll 1$ and $e^{-c\theta} \ll 1$ during inflation, such corrections to observables are of order $O(b^2)$ and can be ignored. With stabilized field norm $|\Phi| = 1$ and $b \ll 1$, but no constraint on $c$, the scalar potential $V(r, \theta)$ becomes

$$V(\theta) = a^2 \left[ 1 + e^{-2c\theta} - 2e^{-c\theta}\cos(b\theta) \right].$$

(2.6)

By varying $b$ and $c$ we can interpolate from natural inflation ($c = 0$, $b \ll 1$) to Starobinsky-like inflation ($b = 0$, $c > 0$).

Let us compare our model with the usual $\alpha$-attractor models. From no-scale like Kähler potential, one obtains two kinds of $\alpha$-attractor models: the T-Model and E-Model [31]. The potential for the T-Model is

$$V(\phi) = \alpha\mu^2 \tanh^2 \frac{\phi}{\sqrt{6}\alpha}.$$  

(2.7)

In general, our model is different from the usual $\alpha$-attractor T-models derived from no-scale like Kähler potential [31]. If $e^{2\theta \sqrt{6\alpha}}$ is much larger than 1, we can obtain the T-model from our model (2.6) by choosing

$$a^2 = \alpha\mu^2, \quad b = 0, \quad c = \sqrt{\frac{2}{3\alpha}}, \quad \theta = \phi - \ln 2/c.$$  

(2.8)

The inflaton potential for the E-Model is

$$V(\phi) = \alpha\mu^2 \left( 1 - e^{-\sqrt{\frac{2}{3\alpha}}\phi} \right)^2,$$  

(2.9)

and we can obtain the above inflaton potential for E-Model from our model (2.6) by choosing

$$a^2 = \alpha\mu^2, \quad b = 0, \quad c = \sqrt{\frac{2}{3\alpha}}, \quad \theta = \phi.$$  

(2.10)

Therefore, our model is more general than the usual $\alpha$-attractor T-models and E-models from no-scale like Kähler potential [31].

In figure 1, we show the numerical results of the scalar spectral index $n_s$ and the tensor-to-scalar ratio $r$ for the helical phase inflation in the framework of general relativity (GR). By comparing with the Planck 2018 and BICEP2 observations [28], we get the constraints on the parameters of the model. Note that $b$ and $c$ control the period and amplitude of potential, respectively. As $b$ increases, the potential becomes steep and large $c$ is needed to flatten the potential and realize inflation. In the limit of $b\theta \ll 1$ and $c\theta \ll 1$, we get the polynomial potential

$$V \simeq \frac{1}{2}a^2 b^2 \theta^2 (1 - c\theta).$$  

(2.11)

This potential is excluded by Planck 2018 and BICEP2 observations as shown in figure 1.
For the Starobinsky-like inflation ($b = 0$) the number of e-folds, scalar spectral index and tensor-to-scalar ratio are
\begin{align}
N_* &= \frac{e^{\theta_*} - e^{\theta_e} - c(\theta_* - \theta_e)}{2c^2}, \\
n_s &= 1 - \frac{4c^2 \left( e^{\theta_*} + 1 \right)}{(e^{\theta_*} - 1)^2}, \\
r &= \frac{32c^2}{(e^{\theta_*} - 1)^2},
\end{align}
where $\theta_e$ is the field value evaluated when inflation ends and can be solved from $\epsilon(\theta_e) = 1,$
\begin{equation}
c_{\theta_e} = \ln(1 + \sqrt{2}c).
\end{equation}
In the limit $c \gg 1$, $N_* \simeq (e^{\theta_*} - e^{\theta_e})/(2c^2)$, we get the $\alpha$-attractor result [32]
\begin{align}
n_s &\simeq 1 - \frac{4c^2}{e^{\theta_*}} \simeq 1 - \frac{2}{N_*}, \\
r &\simeq \frac{32c^2}{e^{2\theta_*}} \simeq \frac{8}{c^2 N_*^2}.
\end{align}
As discussed in eqs. (2.8) and (2.10), we can obtain T-models and E-models if we identify $c = \sqrt{2/(3\alpha)}$, thus it is no surprise that we obtain the $\alpha$-attractors and large $c$ plays the role of suppressing the tensor-to-scalar ratio. The results are shown in figure 1 with yellow and green lines.

In the limit $c\theta \gg 1$ with $b \neq 0$, the potential becomes
\begin{equation}
V \simeq a^2 \left( 1 - 2\cos(b\theta)e^{-\epsilon\theta} \right),
\end{equation}
Note that as $c$ gets larger, the field value decreases and $b\theta$ becomes small. Thus when $c \gg 1$, the number of e-folds before the end of inflation can be expanded in terms of $b\theta$ as
\begin{equation}
N_* \simeq -\int_{\theta_0}^{\theta_*} \frac{e^{\epsilon\theta}}{2\cos(b\theta)}d\theta \simeq -\int_{\theta_0}^{\theta_*} \frac{2c(1 + b^2\theta^2/2)d\theta}{e^{\epsilon\theta_0}(1 + b^2\theta^2/2)^2/2c^2},
\end{equation}
the scalar spectral index and tensor-to-scalar ratio are
\begin{align}
n_s &\simeq 1 - \frac{4c^2 \cos(b\theta_0 - \theta_0)}{e^{\epsilon\theta_0}} \simeq 1 - \frac{4c^2}{e^{\epsilon\theta_0}} (1 - b^2\theta^2/2) \simeq 1 - \frac{2}{N_*}, \\
r &\simeq \frac{32c^2 \cos^2(b\theta_0 - \theta_1)}{e^{2\epsilon\theta_*}} \simeq \frac{32c^2}{e^{2\epsilon\theta_*}} (1 - b^2\theta^2/2)^2 \simeq \frac{8}{c^2 N_*^2},
\end{align}
where $\theta_0 = \arctan(2b/c) \approx 0$ and $\theta_1 = \arctan(b/c) \approx 0$. The attractors are the same as $\alpha$-attractors (2.16) in Starobinsky-like case and are independent of the value of $b$ in the large $c$ limit if we identify $c = \sqrt{2/(3\alpha)}$. The $\alpha$-attractors are shown in figure 1.

For natural inflation ($c = 0$), the corresponding analytic expressions for the number of e-folds, spectral index and tensor-to-scalar ratio are
\begin{align}
N_* &= \frac{2}{b^2} \ln \left[ \frac{\cos \left( b \theta_0/2 \right)}{\cos \left( b \theta_*/2 \right)} \right], \\
n_s &= -2b^2 \csc^2 \left( \frac{b \theta_0}{2} \right) + b^2 + 1, \\
r &= 8b^2 \cot^2 \left( \frac{b \theta_0}{2} \right),
\end{align}
and the end of inflation condition $\epsilon(\theta_e) = 1$ determines the field $\theta_e$ as

$$b\theta_e = \arccos \left( \frac{2 - b^2}{2 + b^2} \right).$$  \hfill (2.23)

The results are marginally consistent with the observations at the 2σ confidence level as shown with cyan line in figure 1. The field excursions in four-dimensional case are typically of the order of $\sim O(10)$ in reduced Planck units as elaborated in ref. [9].

3 Brane inflation

In the braneworld cosmology, our four-dimensional world is a 3-brane embedded in a higher-dimensional bulk. The Friedmann equation is modified due to high energy corrections to Einstein equations on the brane [33–37]

$$H^2 = \frac{\rho}{3M_P^2} \left( 1 + \frac{\rho}{2\lambda} \right),$$  \hfill (3.1)

where $\rho$ is the energy density of the scalar field, $M_P = M_4/\sqrt{8\pi}$ is the reduced Planck mass, and $\lambda$ is the brane tension that relates the four-dimensional Planck scale $M_4$ with the five-dimensional Planck scale $M_5$ as below

$$\lambda = 3 \frac{M_5^6}{4\pi M_4^2}. \hfill (3.2)$$

The nucleosynthesis limit implies that $\lambda \gtrsim (1 \text{ MeV})^4 \sim (10^{-21})^4$ in the reduced Planck unit. A more stringent constraint can be obtained by requiring the theory to be reduced to Newtonian gravity on scales larger than 1 mm corresponding to $\lambda \gtrsim 5 \times 10^{-53}$, i.e., $M_5 \gtrsim 10^5 \text{ TeV}$ [35]. Notice that in the limit $\lambda \to \infty$, we recover the standard Friedman equation in four dimensions.
The $\rho^2$ correction term in the modified Friedmann equation makes the slow-roll parameters smaller for a given potential such that \cite{30, 35}

$$
\epsilon_H = \epsilon_V \frac{1 + V/\lambda}{[1 + V/(2\lambda)]^2},
$$

$$
\eta_H = \eta_V \frac{1}{1 + V/(2\lambda)},
$$

(3.3)

where $\epsilon_V = (V'/V)^2/2$ and $\eta_V = V''/V$. Slow-roll brane inflation can be realized when $\epsilon_H \ll 1$ and $\eta_H \ll 1$. The number of $e$-folds during inflation becomes

$$
N_\ast = - \int_{\theta_*}^{\theta} \frac{V}{V'} \left(1 + \frac{V}{2\lambda}\right) d\theta.
$$

(3.4)

Note that in the high energy limit $V \gg \lambda$, to get the same number of $e$-folds, brane inflation requires smaller field excursion than that in GR. For the Randall-Sundrum model II \cite{38}, the amplitudes for the tensor and scalar power spectrum are \cite{30, 35}

$$
A^2_t = \frac{2}{3\pi^2} V \left(1 + \frac{V}{2\lambda}\right) F^2,
$$

(3.5)

$$
A^2_s = \frac{1}{12 \pi^2} \frac{V^3}{V''} \left(1 + \frac{V}{2\lambda}\right)^3,
$$

(3.6)

where

$$
F^2 = \left[\frac{1}{\sqrt{1 + x^2} - x \sinh^{-1}\left(\frac{1}{x}\right)}\right]^{-1},
$$

(3.7)

with

$$
x = \left(\frac{3 H^2}{4\pi \lambda}\right)^{1/2} = \left[\frac{2 V}{\lambda} \left(1 + \frac{V}{2\lambda}\right)\right]^{1/2}.
$$

(3.8)

Note that the right-handed sides of eqs. (3.5) and (3.6) should be evaluated at the horizon crossing. In the low-energy limit $V/\lambda \ll 1$, $F^2 \approx 1$, we recover the results in standard cosmology.\footnote{The results (3.5) and (3.6) differ from those in refs. \cite{30, 35} by factors of $(1/16) \times (4/25)$ and $4/25$ respectively because they use matter density perturbations when modes re-enter the Hubble scale during the matter dominated era.}

The scalar spectral tilt and the tensor-to-scalar ratio under slow-roll approximation can be written as \cite{35, 39}

$$
n_s - 1 \simeq -6\epsilon_H + 2\eta_H,
$$

$$
r = \frac{A^2_t}{A^2_s} \simeq 8 \left(\frac{V'}{V}\right)^2 \frac{F^2}{[1 + V/(2\lambda)]^2}.
$$

(3.9)

In the low energy limit $V/\lambda \ll 1$, $F^2 \approx 1$, we recover the standard result $r \simeq 16\epsilon_V$. In the high energy limit where $V/\lambda \gg 1$ and $F^2 \approx 3V/2\lambda$, the tensor-to-scalar ratio and the scalar power spectrum amplitude become

$$
r \simeq 24\epsilon_H,
$$

$$
A^2_s \simeq \frac{1}{12 \pi^2} \frac{V}{\epsilon_H} \left(\frac{V}{2\lambda}\right)^2.
$$

(3.10)
Hence in brane inflation, the inflation energy scale and the brane tension cannot be completely fixed by the observational values of $A_s^2$ and $r$.

For the Starobinsky-like case with $b = 0$, the analytic expressions for $N_s$, $n_s$ and $r$ in the high energy and large $c$ limits are

$$N_s \simeq \frac{a^2}{4\lambda c^2} e^{c\theta_*}, \quad \text{(3.11)}$$

$$n_s \simeq 1 - \frac{8\lambda c^2}{a^2} e^{c\theta_*} \simeq 1 - \frac{2}{N_s},$$

$$r \simeq \frac{192\lambda c^2}{a^2} e^{c\theta_*} \simeq \frac{12}{c^2 N_s^2}. \quad \text{(3.12)}$$

The attractors (3.12) are the same as $\alpha$-attractors (2.16) in GR except that in brane case $r$ is $3/2$ times larger. Comparing with E-model $\alpha$-attractors in braneworld [25] in the large $N$ limit, again we have the relation $c = \sqrt{2/(3\alpha)}$.

For the general case with $b \neq 0$, in the large $c$ and high energy limits, $c\theta \gg 1$ and $V \gg \lambda$, $b\theta$ is small and we can expand the observables in terms of $b\theta$. The number of $c$-folds is

$$N_s \simeq -\int_{\theta_*}^{\theta_e} \frac{a^2 e^{c\theta}}{4\lambda c \cos(b\theta)} d\theta \simeq -\frac{a^2}{2\lambda} \int_{\theta_*}^{\theta_e} \frac{e^{c\theta}}{2c} (1 + b^2 \theta^2/2) d\theta \simeq \frac{a^2 e^{c\theta_1} (1 + b^2 \theta^2_2/2)}{4\lambda c^2}. \quad \text{(3.13)}$$

The scalar spectral index and tensor-to-scalar ratio are

$$n_s \simeq 1 - \frac{8\lambda c^2 \cos(b\theta_*)}{a^2 e^{c\theta_*}} \simeq 1 - \frac{8\lambda c^2}{a^2 e^{c\theta_*}} (1 - b^2 \theta^2_*/2) \simeq 1 - \frac{2}{N_s},$$

$$r \simeq \frac{192\lambda c^2 \cos^2(b\theta_*)}{a^2 e^{2c\theta_*}} \simeq \frac{192\lambda c^2}{a^2 e^{2c\theta_*}} (1 - b^2 \theta^2_*/2)^2 \simeq \frac{12}{c^2 N_s^2}. \quad \text{(3.14)}$$

Therefore, the attractors for $b \neq 0$ are the same as those with $b = 0$ and are independent of the value of $b$ in the large $c$ limit.

Figure 2 shows the numerical results of the scalar spectral index $n_s$ and the tensor-to-scalar ratio $r$ in brane helical phase inflation for $N = 60$ $c$-folds and the ratio of scale of inflation to the brane tension $a^2/\lambda = 100$. Because we are interested in the effect of brane correction, so we choose the high energy limit $a^2/\lambda \gg 1$. The predictions of the $n_s - r$ plane shrink from a tree-like shape ($b \neq 0$) to a single branch ($b = 0$) as we increase the value of $a^2/\lambda$, here we take a relatively large value $a^2/\lambda = 100$. A larger value of $a^2/\lambda$ just gives the Starobinski-like branch as derived in eq. (3.14). By comparing with the Planck 2018 and BICEP2 observations, we get the constraints on the model parameters $b$ and $c$. The above attractors (3.12) and (3.14) are confirmed from the numerical results in figure 2. Note that due to the tensor-to-scalar ratio $r$ receives the correction $F^2/[1 + V/(2\lambda)]^2$, in the high energy limit the tensor-to-scalar ratio $r$ is larger in brane inflation than that in GR. Because of that, larger values of $c$ are needed. The same reason leads to the exclusion of natural inflation on a brane by the observations. Due to the high energy correction factor $V/\lambda$ in brane inflation, as shown in figure 2, attractors can be reached more easily and the observables $n_s$ and $r$ are almost independent of $b$. By using constrained parameters $b$ and $c$ in figure 2, we also calculate the field excursion $\Delta \theta$ and the results for $r$ and $\Delta \theta$ are shown in figure 3. Thus, the sub-Planckian field excursion $\Delta \theta < 1$ is obtained if $r < 0.03$. 

\[ \text{Figure 2} \]
Figure 2. The observational constraints on helical phase inflation on a brane taking $N = 60$ and $a^2/\lambda = 100$. The left panel shows the attractors. The 1$\sigma$ and 2$\sigma$ constraints on the parameters $b$ and $c$ are shown with light blue and pink colors respectively in the right panel.

Figure 3. The results for $r$ and $\Delta \theta$ by using the constrained parameters in figure 2.

3.1 Reheating

In order to calculate the reheating temperature we add an interaction term in superpotential as,

$$ W \supset \gamma \Phi H_u H_d $$

(3.15)

that induces an inflaton decay into Higgsinos via the decay width given by [40],

$$ \Gamma_{\Phi}(\Phi \to H_u H_d) = \frac{\gamma^2}{8\pi} m_{\Phi}, $$

(3.16)
where $\gamma$ is the decay coupling parameter and $m_\Phi = 2a^2 (b^2 + c^2)$ is the inflaton mass. The reheating temperature $T_r$ is estimated to be \[ T_r \approx \left( \frac{90}{\pi^2 g_*} \right) \sqrt{\Gamma_\Phi}, \] \[ = 3.025 \times 10^{17} \gamma \sqrt{a^2 (b^2 + c^2)} \text{GeV}. \]

where $g_*$ is taken to be 228.75 for MSSM. Figure 4 shows plots of reheating temperature versus parameter $c$ for the case of Starobinsky-like inflation with $b = 0$ and $N = 60$ e-folds. The solid lines correspond to $\gamma = 10^{-3}$ and the reheating temperature is $T_r \simeq 10^9$ GeV. The LHC bounds on gravitino mass constraints the reheating temperature to be $T_r \lesssim 10^9$ GeV [42]. The dashed lines correspond to $\gamma = 10^{-5}$ and the reheating temperature is $T_r \simeq 10^6$ GeV.

Due to the correction factor $(1 + V/2\lambda)^3$ to scalar perturbation amplitudes of brane inflation, the reheating temperature for brane inflation is always less than GR inflation in the high energy regime $V/\lambda \gg 1$.

### 4 Conclusions

We have considered the helical phase inflation models from the $\mathcal{N} = 1$ supergravity where the phase component of a complex field is inflaton. This class of models can solve the eta problem in supergravity inflation due to the phase monodromy of the superpotential.

We studied the observational constraints on the model parameters, and present the viable parameter space which is consistent with the Planck 2018 and BICEP2 results. In GR, the natural inflation is marginally consistent with the observations at the $2\sigma$ level. We also find that the helical phase inflation has $\alpha$-attractors in the large $c$ limit and the attractors are independent of the parameter $b$. In the brane case, the natural inflation lies well outside the $2\sigma$ region in $n_s$-$r$ plane and it is excluded by the observations. The $\alpha$-attractors are also found except that the value of $r$ is $3/2$ times larger than that in GR. The high energy correction makes the attractors easily reached and the attractors for the general case with
\( b \neq 0 \) is almost the same as the Starobinsky-like case with \( b = 0 \). The field excursions in the brane scenario can be subplanckian if \( r < 0.03 \).

From the reheating analysis in section 3.1, the brane inflation having an additional parameter can easily accommodate a wide range for the reheating temperature. The upper bound for the temperature can be lowered with either taking the coupling to Higgsinos to be small or taking a higher ratio of scale of inflation to the brane tension.

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