Bianchi-V string cosmology with power law expansion in $f(R,T)$ Gravity

Anil Kumar Yadav

Department of Physics, Anand Engineering College, Keetham, Agra 282 007, India

Abstract: In this paper, we search the existence of Bianchi-V string cosmological model in $f(R,T)$ gravity with power law expansion. Einstein’s field equations have been solved by taking into account the law of variation of Hubble’s parameter that yields the constant value of deceleration parameter (DP). We observe that the massive strings dominate the early universe but they do not survive for long time and finally disappear from the universe. We examine the nature of classical potential and also discuss the physical properties of universe.

Keywords: early universe, $f(R,T)$ gravity and cosmological parameters.

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I. INTRODUCTION

The spontaneous breakdown of symmetries in the early universe that produce linear discontinuities in the field is called cosmic string (Kibble 1976). The cosmic strings are also common in modern string cosmologies. In 2003, general interest in cosmic strings was heightened by the discovery of what seemed at first to be a plausible candidate for lensing by a cosmic string. A pair of images of elliptical galaxies separated by 1.8 arc seconds was found to have the same redshift, $z = 0.46$, and the same spectra. Sazhim et al (2003) confirmed that these images were not distorted in the way that would be expected for lensing of a single galaxy but are consistent with lensing by cosmic string. In the literature, the line like structure of cosmic strings with particle attached to them are considered as possible seeds for galaxy formation at the early stages of the evolution of universe. In the past time, Stachel (1980), Letelier (1983) and Vilenkin et al (1987) have studied different aspects of string cosmological models in general relativity. Reddy (2003), Reddy and Naidu (2007) and recently Yadav (2013) have investigated anisotropic string cosmological models in scalar-tensor theory of gravitation.

Harko et al (2011) proposed $f(R,T)$ gravity theory by taking into account the gravitational Lagrangian as the function of Ricci scalar $R$ and of the trace of energy-stress tensor $T$. They have obtained the equation of motion of test particle and the gravitational field equation in metric formalism both. Myrzakulov (2011) presented point like Lagrangian’s for $f(R,T)$ gravity. The $f(R,T)$ gravity model that satisfy the local tests and transition of matter from dominated era to accelerated phase was considered by Houndjo (2012). Recently Chaubey and Shukla (2013), Naidu et al (2013) and Ahmad and Pradhan (2013) have investigated anisotropic cosmological model in $f(R,T)$ gravity. In this paper, our aim is to study the role of strings in $f(R,T)$ gravity and Bianchi-V space time. Since Bianchi-V models are natural generalization of FRW models hence the Bianchi-V cosmological models permit one to obtain more general cosmological model, in comparison to FRW model. In the recent years several authors (Yadav 2011, Kumar and Yadav 2011, Yadav 2012, Pradhan et al 2005, Singh et al 2008) have studied Bianchi-V cosmological models in different physical context.

In this paper, we establish the existence of Bianchi-V string cosmological model in $f(R,T)$ gravity and examine a cosmological scenario by proposing power law expansion. We observe that strings do not survive for long time and eventually disappear from universe. The organization of the paper is as follows: The model and basic theory of $f(R,T)$ gravity are presented in section 2. The field equations are established in section 3 and their solution is presented in section 4. Section 5 deals with the cosmological parameters and classical potential of derived model. Finally conclusions are presented in section 6.
II. THE METRIC AND $f(R, T)$ GRAVITY

We consider the Bianchi-V metric in following form

$$ds^2 = -dt^2 + A^2dx^2 + e^{2\alpha z}(B^2dy^2 + C^2dz^2) \quad (1)$$

Here, $A(t)$, $B(t)$ & $C(t)$ are scale factors in the x, y & z direction respectively and $\alpha$ is constant.

The action of $f(R, T)$ gravity is given by

$$S = \frac{1}{16\pi} \int f(R, T)\sqrt{-g}d^4x + \int L_m\sqrt{-g}d^4x \quad (2)$$

Where $R$, $T$ and $L_m$ are the Ricci scalar, the trace of the stress-energy tensor of matter and the matter Lagrangian density respectively.

The stress-energy tensor of matter is given by

$$T_{ij} = \frac{-2}{\sqrt{-g}} \delta\sqrt{-gL_m}\delta g_{ij} \quad (3)$$

The gravitational field equation of $f(R, T)$ gravity is given by

$$f_R(R, T)R_{ij} - \frac{1}{2} f(R, T)g_{ij} + (g_{ij}\nabla^i \nabla - \nabla_i \nabla_j) f(R, T) = 8\pi T_{ij} - f_T(R, T)T_{ij} - f_T(R, T)\Theta_{ij} \quad (4)$$

where $f_R(R, T) = \frac{\partial f(R, T)}{\partial R}$, $f_T = \frac{\partial f(R, T)}{\partial T}$, $\Theta_{ij} = -2T_{ij} - pg_{ij}$ and $\nabla_i$ denotes the covariant derivative.

In general, the field equations depend through the tensor $\Theta_{ij}$, on the physical nature of the matter field. Hence we obtain several theoretical models for different choice of $f(R, T)$ depending on the nature of the matter source. In the literature, Chaubey & Shukla (2013), Reddy et al. (2012a, 2012b) and Naidu et al (2013) have been studied the cosmological models, assuming $f(R, T) = R + 2f(T)$. Recently Ahmad & Pradhan studied consequence of Bianchi-V cosmological model in $f(R, T)$ gravity by considering $f(R, T) = f_1(R) + f_2(T)$. They have assumed perfect fluid as source of matter while in this paper, we assumed the string fluid as source of matter to describe the physical consequences of early universe. Thus our paper is all together different from the paper of Ahmad and Pradhan (2013). Assuming $f_1(R) = \mu R$ and $f_2(T) = \mu T$ where $\mu$ is arbitrary parameter.

Now equation (4) can be rewritten as

$$R_{ij} - \frac{1}{2}g_{ij}R = \left(\frac{8\pi + \mu}{\mu}\right) T_{ij} + \left(p + \frac{1}{2}T\right) g_{ij} \quad (6)$$

Throughout the paper, we use units $c = G = 1$.

The expansion scalar ($\theta$) and shear scalar ($\sigma$) have the form

$$\theta = 3H = \dot{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \quad (7)$$

$$2\sigma^2 = \left[\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2}\right] - \frac{\theta^2}{3} \quad (8)$$

III. FIELD EQUATIONS IN $F(R, T)$ GRAVITY

The Einstein’s field equation (6) for the line-element (1) leads to the following system of equations

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\left(\frac{8\pi + \mu}{\mu}\right) \left(p + \lambda\right) \quad (9)$$
\[ \frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}C}{AC} - \frac{\alpha^2}{A^2} = -\left(\frac{8\pi + \mu}{\mu}\right)p \tag{10} \]

\[ \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}B}{AB} - \frac{\alpha^2}{A^2} = -\left(\frac{8\pi + \mu}{\mu}\right)p \tag{11} \]

\[ \frac{\dot{A}B}{AB} + \frac{\dot{A}C}{AC} + \frac{\dot{B}C}{BC} - \frac{3\alpha^2}{A^2} = \left(\frac{8\pi + \mu}{\mu}\right)p \tag{12} \]

\[ \frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{13} \]

**IV. SOLUTION OF FIELD EQUATION**

We have system of five equation (10)–(13) involving six unknown variables, namely A, B, C, ρ, p & λ. Thus, in order to completely solve the fields equations, we need at least one physical assumption among the unknown parameters.

In the literature, it is common to use the law of variation of Hubble’s parameter which yields the constant value of deceleration parameter. This law deals with two type of cosmology (i) power law cosmology (ii) exponential law cosmology. It is well known that the exponential law projects the dynamics of future universe and such type of model does not have consistency with present day observations. Since we are looking for a model, describing the late time acceleration of universe therefore we choose the average scale factor in following form

\[ a = (nDt)^{1/n} \tag{14} \]

where \( n \neq 0 \), is positive constant.

Eq. (14) can be easily obtain by law of variation of Hubble’s parameter.

We define average scale factor (a) and mean Hubble’s parameter (\( \dot{H} \)) of the Bianchi-V model as

\[ a = (ABC)^{1/3} \tag{15} \]

\[ \dot{H} = \frac{\dot{a}}{a} = \frac{1}{3}(H_1 + H_2 + H_3) \tag{16} \]

where \( H_1 = \frac{\dot{A}}{A} \), \( H_2 = \frac{\dot{B}}{B} \) and \( H_3 = \frac{\dot{C}}{C} \) are directional Hubble’s parameters in the direction of x, y, and z- direction respectively.

From equation (10) and (11), we obtain the following relation

\[ \frac{B}{C} = b_1\exp\left( x_1 \int \frac{dt}{ABC} \right) \tag{17} \]

where \( b_1 \) and \( x_1 \) are constant of integrations.

Integrating eq. (13), we get

\[ A^2 = BC \tag{18} \]

From equations (14), (15), (17), and (18), the metric functions can be explicitly written as

\[ A = (nDt)^{\frac{2}{3}} \tag{19} \]

\[ B = \sqrt{b_1}(nDt)^{\frac{7}{3}}\exp\left[ -\frac{x_1}{2D(n-3)}(nDt)^{\frac{n-2}{n-3}} \right] \tag{20} \]

\[ C = \frac{1}{\sqrt{b_1}}(nDt)^{\frac{7}{3}}\exp\left[ -\frac{x_1}{2D(n-3)}(nDt)^{\frac{n-2}{n-3}} \right] \tag{21} \]

provided \( n \neq 3 \).
V. THE COSMOLOGICAL PARAMETERS AND CLASSICAL POTENTIAL

The average Hubble’s parameter \( H \), expansion scalar \( \theta \), spatial volume \( V \), deceleration parameter (DP) and shear scalar \( \sigma \) are given by

\[
H = \frac{1}{nt} \tag{22}
\]

\[
\theta = 3H = \frac{3}{nt} \tag{23}
\]

\[
V = (nDt)^\frac{3}{n} \tag{24}
\]

\[
q = -\frac{a\ddot{a}}{a^2} = n - 1 \tag{25}
\]

\[
\sigma = \frac{nx_1}{6}(nDt)^\frac{n-6}{n} \tag{26}
\]

For accelerating universe, we impose the restriction on the value of \( n \) \((0 < n < 1)\). If we take \( n = 0.27 \) then the value of DP is \(-0.73\) which exactly matches with the observed value of DP at present epoch (Cunha et al. 2009). Hence we constrain \( n = 0.27 \) in graphical representations and discussion of physical parameters of derived model.

The isotropic pressure \( (p) \), proper energy density \( (\rho) \), string tension density \( (\lambda) \) and particle energy density \( (\rho_p) \) are found to be

\[
p = \frac{\mu}{(8\pi + \mu)} \left[ \frac{\alpha^2}{(nDt)^\frac{2}{n}} + \frac{2n - 3}{n^2t^2} - \frac{7x_1}{6t}(nDt)^\frac{n-6}{n} - \frac{n^2x_1^2}{36}(nDt)^{\frac{2(n-6)}{3}} - \frac{n^2(n-6)Dx_1}{18}(nDt)^{\frac{n-9}{n}} \right] \tag{27}
\]

\[
\rho = \frac{\mu}{(8\pi + \mu)} \left[ \frac{3}{n^2t^2} - \frac{n^2x_1^2}{36}(nDt)^{\frac{2(n-6)}{3}} - 3\alpha^2(nDt)^{-\frac{2}{n}} \right] \tag{28}
\]

\[
\lambda = \frac{\mu}{(8\pi + \mu)} \left[ \frac{7x_1}{6t}(nDt)^\frac{n-6}{n} + \frac{n^2(n-6)Dx_1}{18}(nDt)^{\frac{n-9}{n}} - \frac{2(2n-3)}{n^2t^2} \right] \tag{29}
\]

\[
\rho_p = \frac{\mu}{(8\pi + \mu)} \left[ \frac{4n - 3}{n^2t^2} - \frac{n^2x_1^2}{36}(nDt)^{\frac{2(n-6)}{3}} - \frac{7x_1}{6t}(nDt)^\frac{n-6}{n} - \frac{n^2(n-6)Dx_1}{18}(nDt)^{\frac{n-9}{n}} - 3\alpha^2(nDt)^{-\frac{2}{n}} \right] \tag{30}
\]

From eq. (17), we obtain

\[
\dot{V} = 3D(nDt)^{\frac{3}{n}-1} \tag{31}
\]

Following Saha and Boyadjiev (2004) and Yadav(2013), we have the following equation of motion of a single particle with unit mass under force \( F(V) \)

\[
\dot{V} = \sqrt{2(\epsilon - U(V))} \tag{32}
\]

Where \( U(V) \) and \( \epsilon \) are the classical potential and amount of energy respectively.

Eqs. (31) and (32) lead to

\[
U(V) = 2\epsilon - 9D^2(nDt)\frac{2(3-n)}{3} \tag{33}
\]

The classical potential in terms of Hubble’s parameter is given by

\[
U(V) = 2\epsilon - 9D^2 H\frac{2n-6}{n} \tag{34}
\]
The age of universe in connection with DP is given by

\[ T = \frac{1}{q + 1} H^{-1} \]  

(35)

From eq. (35), it is clear that the value of \( q \) in the range \(-1 < q < 0\) increase the age of universe.

Also we know that the speed of sound \( v_s \) is less than the speed of light \( (c) \). In gravitational unit we take \( c = 1 \).

Therefore for a physical viable model, \( v_s \) lies between 0 and 1.

From eq. (27) and (28), the speed of sound is given by

\[ v_s^2 = \frac{dp}{d\rho} = \frac{2D\alpha^2(nDt)^{-\frac{n+2}{n}} + \frac{4n-6}{n+2} + \zeta_1 t^{\frac{n-12}{3}} + \zeta_2 (nDt)^{\frac{2n-15}{3}} + \zeta_3 (nDt)^{\frac{n-12}{n}}}{\frac{6}{n+1} + \frac{n^3 D x^1}{36} (nDt)^{\frac{2n-15}{3}} - 6D\alpha^2 (nDt)^{\frac{10n+2}{n}}} \]  

(36)

where

\[ \zeta_1 = \frac{7n(n-9)Dx^1}{18} (nD)^{\frac{n-6}{n}} \]
\[ \zeta_2 = \frac{n^3 (2n-12) d^2x^2}{108} \]
\[ \zeta_3 = \frac{n^6 (n-6)(n-9) D}{54} \]

In figure panel 1, we graphed the parameters of derived model against \( t \) for \( n = 0.27 \). The behaviour is quite evident; the scale factors along axial direction satisfies the anisotropic nature of universe; \( \rho > 1 \) shows that the particles dominate over the strings with the evolution of universe hence the strings are not observed today; the classical potential \( (U(V)) \) is positive and speed of sound \( (v_s) \) is less the speed of light throughout the expansion of universe from big bang to present epoch.
VI. CONCLUDING REMARKS

In the present paper, we have considered \( f(R, T) \) gravity model with an arbitrary coupling between matter and geometry in Bianchi-V space-time. We have derived the gravitational field equations for string fluid corresponding to \( f(R, T) \) gravity model. We observed that string tension density \( (\lambda) \) decreases with time and it approaches to zero at present epoch. Therefore strings could not survive with the evolution process of universe. That is why the strings are not observed today but it play significant role in early universe. The classical potential \( (U(V)) \) is positive and it decreases with time. The scale factors vanish at \( t = 0 \). Thus the model has point type singularity at \( t = 0 \). As \( t \to \infty \) the scale factors diverge and the physical parameters such as expansion, scalar \( (\theta) \), energy density \( (\rho) \) and Hubble’s parameters \( (H) \) tend to zero. Therefore in the derived model, all matter and radiation are concentrated in the big bang. It is important to note that \( q = -1 \) and \( \frac{dH}{dt} = 0 \) for \( t \to \infty \) in the derived model which implies the fastest rate of expansion of universe. So, the derived model can be utilized to describe the dynamics of universe at present epoch. Since \( \lim_{t \to \infty} \frac{\theta}{2} = 0 \), thus the model approaches isotropy at late times.

[1] Ahmad, N., Pradhan, A. 2013 Int. J. Theor. Phys. DOI: 10.1007/s10773-013-1809-7
[2] Chaubey, R, Shukla, A. K. 2013 Ap & SS 343, 415
[3] Cunha C. E. et al. 2009 Mon. Not. Roy. Astron. Soc. 396, 2379
[4] Harko, T., Lobo, F. S. N., Nojiri, S., Odintsov, S. D. 2011 Phys. Rev. D 84, 024020
[5] Houndjo, M. J. S. 2012 Int. J. Mod. Phys. D 21, 1250003
[6] Kibble W. B. 1976 J. Phys. A 9, 1387
[7] Kumar, S., Yadav, A. K. 2011 Mod. Phys. Lett. A 26, 647
[8] Letelier, P. S. 1983 Phys. Rev. D 28, 2414
[9] Myrzakulov, R 2011 Phys. Rev. D 84, 024020
[10] Naidu, R. L., Reddy, D. R. K., Ramprasad, T., Ramana, K. V. 2013 Ap & SS, DOI: 10.1007/s10509-013-1540-0
[11] Pradhan, A., Yadav, A. K., Yadav, L. 2005 Czech J. Phys. 55, 487
[12] Reddy, D. R. K. 2003 Astroph. Space Sc. 286, 356
[13] Reddy, D. R. K., Naidu, R. L. 2007 Ap & SS, 307, 395
[14] Reddy, D. R. K. et al. 2012a Ap & SS 342, 249
[15] Reddy, D. R. K. et al. 2012b Int. J. Theor. Phys. 51, 3222
[16] Saha, B., Boyadjiev, T. 2004 Phys. Rev. D 69, 124010
[17] Sazhim, M. V. et al. 2003 Mon. Not. Roy. Astron. Soc. 343, 353
[18] Singh, C. P., Ram, S., Zeyauddin, M. 2008 Ap & SS 315, 181
[19] Stachel, J. 1980 Phys. Rev. D 21, 2171
[20] Vilenkin, A. et al. 1987 Three Hundred Years of Gravitation. Cambridge Press, Cambridge
[21] Yadav, A. K. 2013 Research in Astronomy and Astrophysics 13, 772
[22] Yadav, A. K. 2011 Ap & SS 335, 565
[23] Yadav, A. K. 2012 Research in Astronomy and Astrophysics 12, 1467