The heat exchange of different atomicity gases at high thermal loads

Y Y Pechenegov

Engels Technological Institute (branch of Yuri Gagarin State Technical University of Saratov)
17, Svoboda square Engels Saratov Region 413100 Russia
E-mail: y.pechenegov@mail.ru

Abstract. The equation describing the heat exchange of a stream in a pipe is derived from the dependences for the turbulent boundary layer and the conservativeness of its characteristics. The equation includes the parameter considering a variability of physical properties in a flow area. The variability of properties affect the heat exchange for gases of more and less atomicity with different ways and it depends on the Reynolds number of the flow. The results of obtained formula evaluation are compared with the theoretical and experimental data of other authors.

1. Introduction

The operation of the modern heat power equipment at high thermal loads often is followed by the significant variability of physical properties of working medium and heat carrier. A considerable number of theoretical and experimental researches are devoted to the study of heat transfer under such conditions [1-9]. The available experimental data are obtained mainly for one- and diatomic gases and often will not be coordinated among themselves, and are sometimes directly opposite. In [5, 6] it is reported the effect of the variability of physical properties on heat exchange can be different for individual gases. Increasing requirements for the accuracy of heat transfer valuation make it necessary to further study this issue.

2. Calculated associations and correlation

The thermal boundary layer of a stabilized turbulent flow is represented as consisting of two zones. The first one is wall-adjacent with a predominantly molecular heat transfer mechanism. The second one is an external with a turbulent transport mechanism. Processes in the wall zone exert a determining influence on the heat exchange of the flow with the pipe wall. The thickness of wall-adjacent zone is calculated using the formula (1)

$$
\delta_n = CPr_c^{-n} \nu_c \sqrt{\sigma_c/\rho_c},
$$

where the factor C and the exponent n are determined from the experimental data; Pr_c is a Prandtl number; \( \nu_c \) is a kinematic-viscosity coefficient of a heat carrier; \( \rho_c \) is a density of a heat carrier; \( \sigma_c \) is shear stress. The subscript "c" which is used in this formula and further others shows that it is wall parameter or is determined at wall temperature T_c, K (or t_c, °C).

Experimental data show that if the wall-adjacent stream zone Reynolds number is close to \( 10^6 \) at a distance from the wall less as 0.1 mm then the temperature profile in the flow section is linear. So we write the following formula (2) for the heat flow density on the wall.

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\[ q_c = \frac{\lambda_c}{\delta}(t_e - t_n), \]  

where \( \lambda_c \) – heat carrier molecular thermal conductivity coefficient; \( t_n \) – temperature in stream section on external border of a wall zone (at a distance \( \delta_n \) from the wall).

The result of computing of the temperature distribution experimental measurements (there were used Reynolds numbers \( Re = \frac{w_{cp}d}{v_{cp}} \) in the range from \( 1 \cdot 10^4 \) to \( 3 \cdot 10^5 \) and a temperature factor \( T_e/T_{cp} = 1 \pm 2.1 \) ) [11] in the cross-section of the heated stabilized air flow in the pipe is the following approximating dependence for the excess temperature in the external zone of the boundary layer

\[ \frac{t_e - t}{t_e - t_0} = \left( \frac{y}{\delta} \right)^{1.2\xi}, \]  

where \( w_{cp} \) – a mean velocity in section; \( \delta \) – a boundary layer thickness, which is a half of a pipe diameter (\( \delta = 0.5d \)); \( t \) and \( t_0 \) – flow temperature at a distance \( y \) from the wall and at \( y = \delta \) respectively; \( \xi \) – resistance coefficient of flow friction, which is calculated on Filonenko's formula (4)

\[ \xi = (1.82 \ln Re - 1.64)^2 \left( \frac{T_e}{T_{cp}} \right)^k, \]  

where \( k = -0.57 \) for \( T_e/T_{cp} > 1 \) [6] and \( k= 0 \) for \( T_e/T_{cp} \leq 1 \) [1]. The subscript "cp" which is used in this formula and others shows that the parameter is determined at flow mean temperature \( T_{cp} \). K (or \( t_{cp}, \text{C} \)).

Using the hydrothermal analogy the formula (5) for the ratio between excess \( t_0 \) and average temperature \( t_{cp} \) in the flow section is determined

\[ \frac{t_e - t_0}{t_e - t_{cp}} = 1 + 1.3\sqrt{\xi}. \]  

The formula (5) corresponds to the expression for the ratio between maximum and average velocities in the cross section of the flow [12] \( w_{max}/w_{cp} = 1 + 1.3\sqrt{\xi} \).

Considering

\[ \sigma_c = \frac{\xi p_{cp} w_{cp}^2}{8}, \]  

for \( y = \delta_n \) and \( t = t_n \) and using formula (1), (2), (3) and (5), the expression for the density of heat flow on the pipe wall is

\[ q_c = \frac{\lambda_c}{\delta} \left( 1 + 1.3\sqrt{\xi} \right) \left( t_e - t_{cp} \right) \left( \frac{\Re_c Pr_c v_{cp}}{C_v} \sqrt{\frac{\xi p_{cp}}{8\rho_c}} \right)^p, \]  

where \( p = 1 - 1.2\sqrt{\xi}; \ Re_c = \frac{w_{cp}\delta}{v_{cp}} \).

Considering formula (7) and \( \delta = d/2 \), Nusselt number is

\[ Nu = \frac{q_c d}{(t_e - t_{cp})\lambda_{cp}} = \left( 2 + 2.6\sqrt{\xi} \right) \left( 0.177 RePr_c Pr_{cp}^{-1} \sqrt{\xi}/C \right)^p \varepsilon_\epsilon, \]  

where \( \varepsilon_\epsilon \) – the parameter considering a variability of physical properties in a flow area,

\[ \varepsilon_\epsilon = \left[ \frac{\lambda_c}{\lambda_{cp}} \right]^{p-1} \left( \frac{\rho_c}{\rho_{cp}} \right)^{0.4} \left( \frac{c_c}{c_{cp}} \right); \]

\( c \) – specific heat of heat carrier.
The results of calculations using the formula (8) at εₜ = 1 are tallied with the experimental data for heat transfer in pipes with a small temperature difference |t_c – tₚt|, if C = 12.7 and n = 0.35 + 0.058 lg Re.

The value of C can be interpreted as a dimensionless coordinate yₜ = δ_n(σ_c/ρ_c)₀.₃₅ Pr_c/ν_c = C in the cross-section of the boundary layer, where the linear temperature profile in the wall zone intersects the power profile (3), so they have a shared point.

Calculations show that temperature drop at wall zone thickness δₚt constitutes a significant part of a thermal gradient from wall temperature t_c to value it in the center of a flow t. So 0.6 of differential temperature t_c-t δₚt is triggered at Re = 5000 and Pr = 0.7 in a wall zone. While Re increases, the quantity (t-t₀)/(t_c-tδ) decreases.

3. The heat exchange at constant physical properties

The results of calculations using the formula (8) at εₜ = 1 are were compared with the calculation by the formula of Petukhov and Kirillov

\[ \text{Nu}_m = \frac{(\zeta/8)\text{RePr}}{1 + 900/\text{Re} + 12.7\sqrt{\zeta/8}\left(\text{Pr}^{2/3} - 1\right)}, \]

which is considered the one of the most universal and reliable [6], and also by the Dittus-Belter formula

\[ \text{Nu} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4}. \]

The equation (8) is triggered with a formula (9) in the wide range of Prandtl numbers up to Pr = 50 at Re ≤ 10⁶. For Re > 10⁶ the fit is limited to Pr = 15. Equation (10) shows a lower value of Nu in relation to the formula (9) at raised numbers of Pr and Re. The formula (10) has a larger error at high numbers of Re and in relation to a formula (9) at the raised numbers of Pr and Re > 2 [9].

4. Heat exchange at variable physical properties of gases of various atomicity

The formula of dependence of physical properties of gases on temperature is

\[ A = A_0 \left(\frac{T}{T_0}\right)^{m_A}, \]

where A is property at temperature T, K; A₀ is property at temperature T₀ = 273 K; m_A is exponent for property A, the parameter εₜ is calculated using expression (12)

\[ \varepsilon_t = \left(\frac{T_c}{T_{cp}}\right)^m, \]

where m = m_z - p (m_z - m_c + 0.5); m_z and m_c are exponent of formula (11) for a thermal conductivity and a thermal capacity respectively. The exponent in (11) is equal to unit for density of gases.

So if C = 12.7 and Pr_c = Pr_{cp} (for gases),

\[ \text{Nu} = \left(2 + 2.6\sqrt{\zeta}\right)\left(0.014 \text{RePr}_c^{0.14}\sqrt{\zeta}\right)^p \left(T_c/T_{cp}\right)^m \]

where ξ is known from expression (4).

The formula (13) is used for following graphs (Figure 1 and 2). The Figure 1 shows functions \( \text{Nu}/\text{Nu}_0 = f(T_c/T_{cp}) \) for different gases. There also are data of other authors for comparison. \( \text{Nu}_0 \) is determined from (13), ξ is determined from (4) at \( T_c/T_{cp} = 1 \) (quasi-isothermal flow condition). As is clear from Figure 1, the function \( \text{Nu}/\text{Nu}_0 = f(T_c/T_{cp}) \) is specific for different gases, it can be explained by distinctions in the various characters of physical properties of all kinds of gases at height of temperature. For mono- and biatomic gas (helium, air) evaluation by formula (13) is in good
agreement with the theoretical calculation for hydrogen and air streams [2] and with experimental data [3, 4] at $T_e/T_{cp} > 1$, but this consistency with authors [2-3] is worse at $T_e/T_{cp} < 1$.

**Figure 1.** The plot of the ratio of Nusselt numbers $\frac{Nu}{Nu_0}$ against the temperature factor $T_e/T_{cp}$ at $Re = 10^4$: 1 - cyclohexane C$_6$H$_{12}$; 2-ethane C$_2$H$_6$; 3 - methane CH$_4$; 4 - ammonia NH$_3$; 5 - carbon dioxide CO$_2$; 6 - line generalizing air experiments [3]; 7 - line generalizing ammonia NH$_3$ experiments [5]; 8-helium He; 9 - air; 10 - combustion fuel products (average composition); 11 - water vapor H$_2$O; 12 - theoretical calculation [2]; circles are experimental points, $x/d = 52$, air [14].

Calculations using equation (13) show (Figure 1) character of the dependence $\frac{Nu}{Nu_0} = f \left( \frac{T_e}{T_{cp}} \right)$ changes significantly with the growth of the atomicity of gases. For three- and four-atomic gases (CO$_2$, H$_2$O, NH$_3$) the influence of the temperature factor on $\frac{Nu}{Nu_0}$ is minimal and the curves of the dependence $\frac{Nu}{Nu_0} = f \left( \frac{T_e}{T_{cp}} \right)$ are flat. For hydrocarbon gases, while the degree of their atomicity increases, the slope of the curves $\frac{Nu}{Nu_0} = f \left( \frac{T_e}{T_{cp}} \right)$ also increases. It is important that $\frac{Nu}{Nu_0} > 1$ for polyatomic gases, unlike mono- and diatomic gases, at $T_e/T_{cp} > 1$ and $\frac{Nu}{Nu_0} < 1$ at $T_e/T_{cp} < 1$.

The functional connection between $\frac{Nu}{Nu_0}$ and the number of atoms $N$ in the gas molecule (Figure 2) shows that an increase in the gases atomicity up to approximately $N = 8$ leads to an increase $\frac{Nu}{Nu_0}$ at the heating conditions and to a decrease $\frac{Nu}{Nu_0}$ at cooling of the stream. For $N \geq 8$ the function $\frac{Nu}{Nu_0} = f \left( N \right)$ is self-similar to $N$. For two- and triatomic gases under flow cooling conditions (at $T_e/T_{cp} < 1$) the ratio $\frac{Nu}{Nu_0}$ is close to unity, which is tallied with the experimental data [1, 13]. If $T_e/T_{cp} > 1$ and $N = 4 \div 6$, then $\frac{Nu}{Nu_0} \approx 1$. 

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The Reynolds number \( Re \) affects the exponent \( m \) in equation (13). For if \( \frac{T_c}{T_{cp}} = 4 \), then value of \( \text{Nu}/\text{Nu}_0 \) differ by 10 per cent at \( Re = 5 \cdot 10^3 \) and \( Re = 10^6 \).

Figure 2. The plot of the ratio of Nusselt numbers \( \text{Nu}/\text{Nu}_0 \) against the number of atoms (N) in the gas molecule: \( Re = 10^5 \); \( Pr = 0.6 \div 0.9 \) (avg 0.7); 1 \( T_c/T_{cp} = 0.2 \); 2 \( T_c/T_{cp} = 0.4 \); 3 \( T_c/T_{cp} = 3 \); 4 \( T_c/T_{cp} = 5 \); points are obtained by calculation according to equation (13) for following gases: He, Ne, air, N₂, CO, H₂O, SO₂, NH₃, CH₄, C₂H₆, C₃H₆O, C₄H₁₀, C₅H₁₂, C₆H₁₄, C₇H₁₆, C₈H₁₈.

5. Conclusion
The function \( \frac{\text{Nu}}{\text{Nu}_0} = f\left(\frac{T_c}{T_{cp}}\right) \) can be either decreasing or increasing, when turbulent gas flows are heated or cooled, it depends on gas atomicity. The heat transfer result equation (13) allows effectively to consider the effect of the variability of the physical properties of gases of different atomic properties at high thermal loads and can be recommended for engineering applications.

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