Indoor Positioning Based on Kernel Function and Analysis of Measurement Uncertainty

Yunling Wang¹, Yu Sang¹* and Jie Xu¹
¹Division of Electricity and Optics Measurement Technology, Shanghai Institute of Measurement and Testing Technology, China

*Corresponding author e-mail: sangy@simt.com.cn

Abstract. This paper proposes an indoor positioning algorithm based on kernel function, aiming at the problem that error of indoor positioning is large. Gaussian kernel function can capture the nonlinear correlation of received signal strength (RSS) between reference point fingerprint and the test point, and has a better matching effect than the KNN algorithm. The algorithm was verified by MATLAB simulation and real scene testing. At the end of the article, measurement uncertainty is analyzed to describe the reliability of the measurement results.

1. Introduction
With the development of science and technology all over the world, the positioning of mobile terminals has also developed. Outdoors, traditional satellite positioning technology perform excellently, such as GPS, GLONASS and Beidou system [1]. However, indoors, due to the small satellite signal strength, the positioning error is large. Considering that WLAN technology has been widely used indoors, people began to use WLAN [2] signals to achieve indoor positioning.

The study of indoor positioning began in the late 1990s. Reference [3] proposed the k-nearest neighbor (KNN) algorithm for positioning, and the weight KNN is a subsequent algorithm. Reference [4] proposed clustering and decision-tree-based method, which is limited when the number of access points (AP) is small. Reference [5,6] proposed an artificial neural network (ANN), and reference [7] proposed the support vector regression (SVR) method to improve accuracy.

This paper proposes a positioning method based on Gaussian kernel function [8], which is verified by MATLAB simulation and actual scene test. The measurement uncertainty is analyzed to assess the reliability of the measurement results.

2. Kernel function
X is defined as a non-empty set in the n-dimensional space \( \mathbb{R}_n \), and \( \Phi \) is a nonlinear mapping of \( X \) to space \( H \),

\[
\Phi : x \mapsto \Phi(x) \in H
\]  

(1)

If function \( K \) defined in space \( X \times X \), and \( \forall x, z \in X \)

\[
K(x, z) = < \Phi(x), \Phi(z) >
\]

(2)

then \( K(x, z) \) is a kernel function. \(<, > \) means inner product.

The kernel function can map the low-dimensional space to high-dimensional space, so as to analyze the nonlinear relationship between data in the high-dimensional space.
2.1. Gaussian kernel

The Gaussian kernel, also known as the radial basis function (RBF), is a highly used kernel function. It can map finite dimensional data to high dimensional space. Gaussian kernel function is as eq. 3

\[ K(x, z) = e^{-\frac{\|x-z\|^2}{2\sigma^2}} \]  

where \( \sigma \) is the bandwidth controlling the radial range.

2.2. Indoor positioning based on Gaussian kernel function

Distribute M APs in the indoor area. Choose N reference points, and mark location of each point as \( l_i(x_i, y_i) \). At each reference point, the RSS values from M APs can be acquired for several times. The average RSS value and other information are recorded as fingerprint information. This fingerprint information is an M-dimensional vector

\[ F_i = (r_{ss1}, r_{ss2}, ..., r_{ssM})^T, i \in (1, N) \]  

where \( r_{ss_i} \) is the average RSS value from the i-th AP. The fingerprint information of all reference points constitutes a fingerprint database

\[ F = (F_1, F_2, ..., F_N)^T \]

In test mode, RSS value of test points can be obtained. Traditional algorithm, such as KNN algorithm, is utilized to observe linear relation between RSS values of test points and ones of fingerprint database.

Kernel function algorithm can be used to observe nonlinear relation between RSS values of test points and fingerprint database, to calculate the similarity value between the test point and each reference point.

The Gaussian kernel function has good smoothing performance and the ability to approximate any nonlinear function, so it is utilized for indoor positioning. The weight coefficient formula of similarity is

\[ w(r, F_i) = \frac{1}{\sigma^2\sqrt{2\pi}} e^{-\frac{\|r-F_i\|^2}{2\sigma^2}} \]  

where \( F_i \) is fingerprint of the i-th reference point. M-dimensional vector \( r \) records RSS values of test point from M APs.

Find out K fingerprints with the highest similarity weight, then estimated test point location is

\[ (\hat{x}, \hat{y}) = \sum_{i=0}^{K} w(r, F_i) \cdot l_i(x_i, y_i) \]  

3. Measurement results

This paper compares results of KNN algorithm and kernel function algorithm, based on MATLAB simulation and real scene.

3.1. MATLAB Simulation

This paper use MATLAB for simulation. The scene is shown in figure. 45 reference points are at one-meter intervals, marked with red stars in figure.

Fig. 1. Simulated scene
Assume that the strength of every AP is known, the signal strength received from every AP at any point can be calculated based on attenuation model [9], as shown in Eq. 8

\[ p(d) = p(d_0) - 10\alpha \log\left(\frac{d}{d_0}\right) + n \]  

(8)

where \( d \) presents the distance between AP and test point. \( p(d) \) presents the received signal strength at distance \( d \). \( p(d_0) \) presents the received signal strength at distance \( d_0 \), and \( d_0 \) is generally 1 meter. \( \alpha \) is environment factor, related to materials and structures of architecture, generally between 2 and 4. \( n \) presents noise of environment, and we assume it Gauss noise with the mean \( \mu = 0 \) and variance \( \sigma = 1 \). Parameters are set as in Table 1.

| Table 1. Basic parameters |
|---------------------------|
| Parameter | \( p(d_0) \) | \( \alpha \) |
| value | 20dbm | 2 |

We test the same point for 10 times and calculate the mean and variance of the error, shown in Table 2.

| Table 2. Result |
|-----------------|
| Algorithm | \( \mu \) | \( \sigma \) |
| KNN | 0.21m | 0.07m |
| Kernel function | 0.08m | 0.05m |

The result shows that keener function algorithm is better than KNN algorithm.

3.2. Real scene

Our indoor experiment is conducted in SIMT, and floor plan is shown in Fig. 2.

![Fig. 2. Real scene](image)

The area is 50m long and 20m wide, with 13 APs distributed along the corridor. 110 reference points are at one-meter intervals along the corridor. Each reference point is measured for 180 times. We test the same point for 10 times and calculate the mean and variance of the error, shown in Table 3.

| Table 3. Result |
|-----------------|
| Algorithm | \( \mu \) | \( \sigma \) |
| KNN | 1.9m | 1.3m |
| Kernel function | 1.6m | 1.2m |

The result shows that keener function algorithm is better than KNN algorithm.
KNN algorithm only calculates the weighted average position of the nearest k reference points. Kernel function considers the RSS of every reference point, and the nonlinear relationship of RSS, which makes the results more accurate.

4. Evaluation of measurement uncertainty of distance error

We use measurement uncertainty to describe the reliability of the result. In our experiment, sources of uncertainty are made of surveying and mapping, size of mobile phones, instrument resolution and repeatability of measurement.

4.1. Standard uncertainty (SU) component $u_1$ caused by surveying and mapping

Before conducting experiments, we should survey the scene and draw the floor plan, which introduces measurement uncertainty. This component is transmitted by the length measurement department, $U = 0.2\text{m} \ (k = 2)$, so

$$u_1 = 0.1\text{m}$$

4.2. SU component $u_2$ caused by size of mobile phones

Mobile phones are parallel to the horizontal plane when working. Different from an abstract point, a mobile phone takes up space, which introduces uncertainty. Take the long size of a phone, about 15 cm.

$$a_2 = 0.075\text{m}$$

The distribution is assumed to be uniform, so the coverage factor (CF) is $\sqrt{3}$.

$$u_2 = 0.043\text{m}$$

4.3. SU component $u_3$ caused by resolution of instrument

The resolution of the system is 0.01 m,

$$a_3 = 0.005\text{m}$$

The distribution is assumed to be uniform, so the CF is $\sqrt{3}$.

$$u_3 = 0.0029\text{m}$$

4.4. SU component $u_4$ caused by measuring repeatability

We test one point 10 times. The experimental standard deviation of a single measurement [10] is

$$s(x) = \sqrt{\frac{\sum_{i=1}^{n}(x_i-x)^2}{n-1}}$$ (9)

$$u_4 = s = 1.2\text{m}$$

4.5. Combined SU

SU components are listed in Table 4.

| Number | Source                          | Type | Value | Distribution | CF   | SU   |
|--------|---------------------------------|------|-------|--------------|------|------|
| 1      | surveying and mapping           | B    | $a_1=0.2\text{m}$ | /       | 2    | $u_1=0.1\text{m}$ |
| 2      | size of mobile phones           | B    | $a_2=0.15\text{m}$ | Uniform | $\sqrt{3}$ | $u_2=43\text{mm}$ |
| 3      | resolution of instrument        | B    | $a_3=0.01\text{m}$ | Uniform | $\sqrt{3}$ | $u_3=2.9\text{mm}$ |
| 4      | measuring repeatability         | A    | /     | /           | /    | $u_4=1.2\text{m}$ |
4.6. The combination of SU

Uncertainty components of $u_1$, $u_2$ and $u_3$ are independent from each other. $u_3$ and $u_4$ are positively related, and we use the biggest one.

Combined SU is marked as $u_c$

$$u_c = \sqrt{u_1^2 + u_2^2 + u_3^2} = 1.2\text{m}$$

4.7. Expanded uncertainty

CF is set to be 2, so the expanded uncertainty is as follow.

$$U = 2.4\text{m} \ (k = 2)$$

5. Conclusion

This paper uses kernel function algorithm for indoor positioning system, and compares results of KNN algorithm and kernel function algorithm based on MATLAB simulation and real scene. KNN algorithm only calculate the weighted average position of nearest k reference points. Kernel function consider RSS of every reference point, and nonlinear relationship of RSS, which makes results more accurate. This paper also analyzes measurement uncertainty which describe reliability of measurement results.

Acknowledgments

This work was financially supported by program of Development of virtual instrument based on IOT intelligent terminal for location performance detection (F00RJ1605).

References

[1] China's ‘Beidou’ navigation test satellite launch success. Global Positioning System, vol.4, pp. 43, 2000.

[2] Ma L, Xu Y. Received signal strength recovery in green WLAN indoor positioning system using singular value thresholding. Sensors, 2015, 15(1): 1292-1311.

[3] Bahl P, Padmanabhan V N. RADAR: An in-building RF-based user location and tracking system. INFOCOM 2000. Nineteenth Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. IEEE. Ieee, 2000, 2: 775-784.

[4] Chen Y, Yang Q, Yin J, et al. Power-efficient access-point selection for indoor location estimation. IEEE Transactions on Knowledge and Data Engineering, 2006, 18(7): 877-888.

[5] Fang S H, Lin T N. Indoor location system based on discriminant-adaptive neural network in IEEE 802.11 environments. IEEE Transactions on Neural networks, 2008, 19(11): 1973-1978.

[6] Outemzabet S, Nerguizian C. Accuracy enhancement of an indoor ANN-based fingerprinting location system using Kalman filtering. Personal, Indoor and Mobile Radio Communications, 2008. PIMRC 2008. IEEE 19th International Symposium on. IEEE, 2008: 1-5.

[7] Wu Z, Li C, Ng J K Y, et al. Location estimation via support vector regression. IEEE Transactions on mobile computing, 2007, 6(3): 311-321.

[8] Keerthi S S, Lin C J. Asymptotic behaviors of support vector machines with Gaussian kernel. Neural computation, 2003, 15(7): 1667-1689.

[9] Xu X, Indoor positioning technology of IoT, Publishing House of Electronics Industry, 2017.

[10] Chinese Society for Measurement. First-level registered measurer, basic knowledge and professional practice. China Quality Press, 2013.