Measurements of $H(z)$ and $D_A(z)$ from the two-dimensional two-point correlation function of Sloan Digital Sky Survey luminous red galaxies

Chia-Hsun Chuang* and Yun Wang

Homer L. Dodge Department of Physics & Astronomy, University of Oklahoma, 440 W Brooks St, Norman, OK 73019, USA

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ABSTRACT

We present a method for measuring the Hubble parameter, $H(z)$, and angular diameter distance, $D_A(z)$, from the two-dimensional two-point correlation function and validate it using LasDamas mock galaxy catalogues. Applying our method to the sample of luminous red galaxies from the Sloan Digital Sky Survey Data Release 7, we measure $H(z = 0.35) = 82.1_{-4.9}^{+4.8}\,\text{km s}^{-1}\text{Mpc}^{-1}$ and $D_A(z = 0.35) = 1048_{-58}^{+60}\,\text{Mpc}$ without assuming a dark energy model or a flat universe. We find that the derived measurements of $H(0.35)r_s(z_d)/c$ and $D_A(0.35)/r_s(z_d)$ [where $r_s(z_d)$ is the sound horizon at the drag epoch] are nearly uncorrelated, have tighter constraints and are more robust with respect to possible systematic effects. Our galaxy clustering measurements of $H(0.35)r_s(z_d)/c$, $D_A(0.35)/r_s(z_d)$ = $0.0434 \pm 0.0018, 6.60 \pm 0.26$ (with the correlation coefficient $r = 0.0604$) can be used to combine with cosmic microwave background and any other cosmological data sets to constrain dark energy. Our results represent the first measurements of $H(z)$ and $D_A(z)$ (or $H(z)r_s(z_d)/c$ and $D_A(0.35)/r_s(z_d)$) from galaxy clustering data. Our work has significant implications for future surveys in establishing the feasibility of measuring both $H(z)$ and $D_A(z)$ from galaxy clustering data.

Key words: cosmology: observations – distance scale – large-scale structure of Universe.

1 INTRODUCTION

The cosmic large-scale structure from galaxy redshift surveys provides a powerful probe for dark energy and the cosmological model that is highly complementary to the cosmic microwave background (CMB; Bennett et al. 2003), supernovae (Riess et al. 1998; Perlmutter et al. 1999) and weak lensing (Bacon, Refregier & Ellis 2000; Kaiser, Wilson & Luppino 2000; van Waerbeke et al. 2000; Wittman et al. 2000). The scope of galaxy redshift surveys has dramatically increased in the last decade. The Point Source Catalog Redshift Survey (PSCz) surveyed ~15,000 galaxies using the Infrared Astronomical Satellite (IRAS; Saunders et al. 2000), the 2dF Galaxy Redshift Survey obtained 221,414 galaxy redshifts (Colless et al. 2001, 2003) and the Sloan Digital Sky Survey (SDSS) has collected 930,000 galaxy spectra in the Data Release 7 (DR7; Abazajian et al. 2009). The ongoing galaxy surveys will probe the Universe at higher redshifts; WiggleZ is surveying 240,000 emission-line galaxies at 0.5 < $z$ < 1 over 1000 square degrees (Blake et al. 2009), and the SDSS-III’s Baryon Oscillation Spectroscopic Survey (BOSS) is surveying 1.5 million luminous red galaxies (LRGs) at 0.1 < $z$ < 0.7 over 10,000 square degrees (Eisenstein et al. 2011). The planned space mission Euclid will survey over 60 million emission-line galaxies at 0.5 < $z$ < 2 over 20,000 square degrees (Cimatti et al. 2009; Wang et al. 2010).

Large-scale structure data from galaxy surveys can be analysed using either the power spectrum or the correlation function. Although these two methods are simple Fourier transforms of one another, the analysis processes are quite different and the results cannot be converted using the Fourier transform directly because of the finite size of the survey volume. The SDSS data have been analysed using both the power spectrum method (see, e.g., Tegmark et al. 2004; Hutsi 2005; Blake et al. 2007; Padmanabhan et al. 2007; Percival et al. 2007, 2010; Reid et al. 2010) and the correlation function method (see, e.g., Eisenstein et al. 2005; Okumura et al. 2008; Cabre & Gaztanaga 2009; Martinez et al. 2009; Sanchez et al. 2009; Kazin et al. 2010a; Chuang, Wang & Hemantha 2012). While previous work has focused on the spherically averaged two-point correlation function (2PCF), or the radial projection of the two-dimensional two-point correlation function (2D 2PCF), we measure and analyse the full 2D 2PCF of SDSS LRGs in this study.

The power of galaxy clustering as a dark energy probe lies in the fact that the Hubble parameter, $H(z)$, and the angular diameter distance, $D_A(z)$, can in principle be extracted simultaneously from data through the measurement of the baryon acoustic oscillation (BAO) scale in the radial and transverse directions (Blake & Glazebrook 2003; Seo & Eisenstein 2003; Wang 2006). This has not been achieved in the previous work in the analysis of real data. Okumura...
We select our LRG sample from the NYU-V AGC with the flag less affected by the collision corrections.

In our previous paper, Chuang et al. (2012), we presented the method to obtain dark energy and cosmological model constraints from the spherically averaged 2PCF, without assuming a dark energy model or a flat universe. We demonstrated the feasibility of extracting $H(z)$ and $D_A(z)$ by scaling the spherically averaged 2PCF (which leads to highly correlated measurements). In this paper, we obtain robust measurements of $H(z)$ and $D_A(z)$ through scaling, using the 2D correlation function measured from a sample of SDSS DR7 LRGs (Eisenstein et al. 2001). This sample is homogeneous and has the largest effective survey volume to date for studying the quasi-linear regime (Eisenstein et al. 2005). In Section 2, we introduce the galaxy sample used in our study. In Section 3, we describe the details of our method. In Section 4, we present our results. In Section 5, we apply some systematic tests to our measurements. We summarize and conclude in Section 6.

2 DATA

The SDSS has observed one quarter of the entire sky and performed a redshift survey of galaxies, quasars and stars in five passbands u, g, r, i and z with a 2.5 m telescope (Fukugita et al. 1996; Gunn et al. 1998, 2006). We use the public catalogue, the NYU Value-Added Galaxy Catalog (VAGC; Blanton et al. 2005), derived from the SDSS II final public data release, DR7 (Abazajian et al. 2009). We select our LRG sample from the NYU-VAGC with the flag primTarger bit mask set to 32. K-corrections have been applied to the galaxies with a fiducial model [ΛCDM with $\Omega_m = 0.3$ and $h = 1$], and the selected galaxies are required to have rest-frame $g$-band absolute magnitudes $-23.2 < M_g < -21.2$ (Blanton & Roweis 2007). The same selection criteria were used in previous papers (Eisenstein et al. 2005; Zehavi et al. 2005; Okumura et al. 2008; Kazin et al. 2010a). The sample we use is referred to as ‘DR7full’ in Kazin et al. (2010a). Our sample includes 87,000 LRGs in the redshift range $0.16-0.44$. Spectra cannot be obtained for objects closer than 55 arcsec within a single spectroscopic tile due to the finite size of the fibres. To correct for these ‘collisions’, the redshift of an object that failed to be measured would be assigned to be the same as the nearest successfully observed one. Both fibre collision corrections and K-corrections have been made in NYU-VAGC (Blanton et al. 2005). The collision corrections applied here are different from what have been suggested in Zehavi et al. (2005). However, the effect should be small since we are using relatively large-scale data which are less affected by the collision corrections.

We construct the radial selection function as a cubic spline fit to the observed number density histogram with the width $\Delta z = 0.01$. The NYU-VAGC provides the description of the geometry and completeness of the survey in terms of spherical polygons. We adopt it as the angular selection function of our sample. We drop the regions with completeness below 60 per cent to avoid unobserved plates (Zehavi et al. 2005). The Southern Galactic Cap region is also dropped.

3 METHODOLOGY

In this section, we describe the measurement of the correlation function from the observational data, the construction of the theoretical prediction and the likelihood analysis that leads to constraints on dark energy and cosmological parameters.

3.1 Measuring the 2D 2PCF

We convert the measured redshifts of galaxies to comoving distances by assuming a fiducial model, ΛCDM with $\Omega_m = 0.25$. We use the 2PCF estimator given by Landy & Szalay (1993):

$$\xi(\sigma, \pi) = \frac{DD(\sigma, \pi) - 2DR(\sigma, \pi) + RR(\sigma, \pi)}{RR(\sigma, \pi)},$$

where $\pi$ is the separation along the line of sight (LOS), $\sigma$ is the separation in the plane of the sky, and $DD$, $DR$ and $RR$ represent the normalized data–data, data–random and random–random pair counts, respectively, in a distance range. The LOS is defined as the direction from the observer to the centre of a pair. The bin size we use in this study is $10 h^{-1}$ Mpc $\times 10 h^{-1}$ Mpc. The Landy and Szalay estimator has minimal variance for a Poisson process. Random data are generated with the same radial and angular selection functions as the real data. One can reduce the shot noise due to random data by increasing the number of random data. The number of random data we use is 10 times that of the real data. While calculating the pair counts, we assign to each data point a radial weight of $1/[1+n(z).P_{w}]$, where $n(z)$ is the radial selection function and $P_w = 4 \times 10^4 h^{-1}$ Mpc$^{-3}$ (Eisenstein et al. 2005). We use the same $P_w$ as in Eisenstein et al. (2005) in which they used the sample of the SDSS DR3. Although the data release versions are different, the properties of the galaxy sample should basically be the same. We find that the error bars estimated from LasDamas mock catalogues could be improved by 10 per cent while using the weighting (compared to the error bars obtained without using the weighting). We expect that the results should not be sensitive to the $P_w$ used.

3.2 Theoretical 2D 2PCF

We compute the linear power spectra at $z = 0.35$ by using CAMB (Lewis, Challinor & Lasenby 2000). To include the effect of non-linear structure formation on the BAOs, we first calculate the dewigged power spectrum

$$P_{dw}(k) = P_{lm}(k) \exp \left( -\frac{k^2}{2k_s^2} \right) + P_{nw}(k) \left[ 1 - \exp \left( -\frac{k^2}{2k_s^2} \right) \right],$$

where $P_{lm}(k)$ is the linear matter power spectrum, $P_{nw}(k)$ is the no-wiggle or pure CDM power spectrum calculated using equation (29) from Eisenstein & Hu (1998) and $k_s$ is marginalized over with a flat prior over the range of 0.09–0.13 $h$ Mpc$^{-1}$.

1 We have identified a bug while computing the weighting of each galaxy in the first draft of this paper. The bug was that we computed the weights of random data with the number density of random data instead of observed data. This would introduce a bias when the number density is not homogeneous.

2 Although $k_s$ can be computed by renormalization perturbation theory (Crocce & Scoccimarro 2006; Matsubara 2008), doing so requires knowing the amplitude of the power spectrum, which is also marginalized over in this study.

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We then use the software package \textsc{halofit} (Smith et al. 2003) to compute the non-linear matter power spectrum:

\begin{equation}
\rho_{\text{halofit}}(k) = \frac{P_{\text{halofit,ns}}(k)}{P_{\text{ns}}(k)},
\end{equation}

where \(P_{\text{halofit,ns}}(k)\) is the power spectrum obtained by applying \textsc{halofit} to the no-wiggle power spectrum and \(P_{\text{ns}}(k)\) is the non-linear power spectrum. We compute the theoretical real space 2PCF, \(\xi(r)\), by Fourier transforming the non-linear power spectrum \(P_{\text{nl}}(k)\).

In the linear regime (i.e. large scales) and adopting the small-angle approximation (which is valid on scales of interest), the 2D correlation function in the redshift space can be written as (Kaiser 1987; Hamilton 1992)

\begin{equation}
\xi^*(\sigma, \pi) = \xi(s) P_{\sigma}(\mu) + \xi(s) P_{\pi}(\mu) + \xi(s) P_1(\mu),
\end{equation}

where \(s = \sqrt{\sigma^2 + \pi^2}\), \(\mu\) is the cosine of the angle between \(s = (\sigma, \pi)\) and the LOS and \(P_1\) are Legendre polynomials. The multipole of \(\xi\) are defined as

\begin{equation}
\xi_0(r) = \left(1 + \frac{2\beta}{3} + \frac{\beta^2}{5}\right) \xi(r),
\end{equation}

\begin{equation}
\xi_2(r) = \frac{4\beta}{3} + \frac{2\beta^2}{7} \left[\xi(r) - \bar{\xi}(r)\right],
\end{equation}

\begin{equation}
\xi_4(r) = \frac{8\beta^2}{35} \left[\xi(r) + \frac{5}{2} \bar{\xi}(r) - \frac{7}{2} \pi \bar{\xi}(r)\right],
\end{equation}

where \(\beta\) is the redshift space distortion parameter and

\begin{equation}
\bar{\xi}(r) = \frac{3}{r} \int_0^r \xi(r') r' dr',
\end{equation}

\begin{equation}
\bar{\pi}(r) = \frac{5}{r^2} \int_0^r \xi(r') r^2 dr'.
\end{equation}

Next, we convolve the 2D correlation function with the distribution function of random pairwise velocities, \(f(v)\), to obtain the final model \(\xi(\sigma, \pi)\) (Peebles 1980),

\begin{equation}
\xi(\sigma, \pi) = \int_{-\infty}^{\infty} \xi^2(\sigma, \pi - \frac{v}{H(z)a(c)}) f(v) dv,
\end{equation}

where the random motions are represented by an exponential form (Ratcliffe et al. 1998; Landy 2002)

\begin{equation}
f(v) = \frac{1}{\sigma_v \sqrt{2\pi}} \exp \left(-\frac{v^2}{2\sigma_v^2}\right),
\end{equation}

where \(\sigma_v\) is the pairwise peculiar velocity dispersion.

The parameter set we use to compute the theoretical correlation function is \([H(z), D_L(z), \beta, \Omega_m h^2, \Omega_b h^2, n_s, \sigma_v, k_s]\), where \(\Omega_m\) and \(\Omega_b\) are the density fractions of matter and baryons, \(n_s\) is the power-law index of the primordial matter power spectrum and \(h\) is the dimensionless Hubble constant \((H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1})\). We set \(h = 0.7\) while calculating the non-linear power spectra. On the scales we use for comparison with data, the theoretical correlation function only depends on cosmic curvature and dark energy through parameters \(H(z)\) and \(D_L(z)\), assuming that dark energy perturbations are unimportant (valid in simplest dark energy models). Thus, we are able to extract constraints from data that are independent of a dark energy model and cosmic curvature.

Fig. 1(a) shows the 2D 2PCF measured from SDSS LRGs compared with a theoretical model. The measured 2D 2PCF of the SDSS LRGs has been smoothed by a Gaussian filter with an rms variance of 2 \(h^{-1}\) Mpc to illustrate the comparison of data with model in this figure as the unsmoothed data are very noisy. Smoothing is not used in our likelihood analysis to avoid possibly the introduction of systematic biases. Fig. 1(b) shows the 2D 2PCF measured from a single LasDamas SDSS LRG mock catalogue for comparison. The similarity between the data and the mock in the range of scales we used (indicated by the shaded disc) is apparent.

Fig. 2 shows the averaged 2D 2PCF measured from the LasDamas mock catalogues compared with a theoretical model. The contour levels are apparent in the measured 2D 2PCF even though no smoothing is used; this is due to the reduction of shot noise achieved by averaging over 160 mock catalogues. Clearly, our 2D theoretical model provides a reasonable fit to data on intermediate (and quasi-linear) scales. The deviations on smaller scales may be due to the simplicity of the peculiar velocity model we have used. We do not use the smaller scales (\(s < 40 h^{-1}\) Mpc), where the scale dependence of redshift distortion and galaxy bias are not negligible and cannot be accurately determined at present. According to fig. 5 in Eisenstein et al. (2005) and fig. 4 in Blake et al. (2011), these effects are negligible at \(s > 40 h^{-1}\) Mpc. On large scales, data become very noisy as sample variance dominates. For these reasons, we will only use the scale range of \(s = 40-120 h^{-1}\) Mpc in our analysis. We do not consider wide-angle effects since they have been shown to be small on the length scales of interest here (Samushia, Percival & Raccanelli 2011). Samushia et al. (2011) showed that the corrections (i.e. non-linear effect and wide-angle effect) to the Kaiser formula are small compared to the statistical errors on the measurement of the correlation function from SDSS DR7 LRG for the scale range interested (\(s = 40-120 Mpc h^{-1}\)). In this study, we include the largest correction, dwigelling (non-linear BAO) and the non-linear effects at small scales. Since including a larger range of scales gives more stringent constraints, our choice of \(s = 40-120 h^{-1}\) Mpc represents a conservative cut in data to reduce contamination by systematic uncertainties.

### 3.3 Covariance matrix

We use the mock catalogues from the LasDamas simulations (McBride et al., in preparation) to estimate the covariance matrix of the observed correlation function. LasDamas provides mock catalogues matching SDSS main galaxy and LRG samples. We use the LRG mock catalogues from the LasDamas gamma release with the same cuts as the SDSS LRG DR7-full sample, \(-23.2 < M_r < -21.2\) and \(0.16 < z < 0.44\). We have diluted the mock catalogues to match the radial selection function of the observed data by randomly selecting the mock galaxies according to the number density of the data sample. We calculate the 2D correlation functions of the mock catalogues and construct the covariance matrix as

\begin{equation}
C_{ij} = \frac{1}{N-1} \sum_{k=1}^{N} \left(\bar{\xi}_i - \bar{\xi}_j\right) \left(\bar{\xi}_j - \bar{\xi}_j\right),
\end{equation}

where \(N\) is the number of the mock catalogues, \(\bar{\xi}_i\) is the mean of the \(n\)th bin of the mock catalogue correlation functions and \(\bar{\xi}_i\) is the value in the \(n\)th bin of the \(k\)th mock catalogue correlation function. Note that the covariance matrix constructed from the LasDamas mock catalogues is noisy because only 160 mock catalogues are available. The covariance matrix contains noisy information.

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3 http://lss.phy.vanderbilt.edu/lasdamas/
The correlation function with parameters close to the best-fitting values in the likelihood analysis (dashed red contours). The theoretical model has $H(z = 0.35) = 81.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $D_A(z = 0.35) = 1042 \text{ Mpc}$, $\beta = 0.35$, $\Omega_m h^2 = 0.117$, $\Omega_b h^2 = 0.022$, $n_s = 0.96$, $\sigma_8 = 300 \text{ km s}^{-1}$ and $k_s = 0.11 \text{ h Mpc}^{-1}$. (b) The 2D 2PCF measured from a single mock catalogue, compared to a theoretical model with the input parameters of the LasDamas simulations, and $\{\beta, \sigma_8, k_s\}$ are set to $\{0.35, 300 \text{ km s}^{-1}, 0.11 \text{ h Mpc}^{-1}\}$ (dashed red contours). In both figures, the shaded disc indicates the scale range considered ($s = 40–120 \text{ h}^{-1} \text{ Mpc}$) in this study. The thick dashed blue circle denotes the BAO scale. The observed 2D 2PCF has been smoothed by a Gaussian filter with the rms variance of $2h^{-1}$ Mpc for illustration in these figures only; smoothing is not used in our likelihood analysis. The contour levels are $\xi = 0.5, 0.1, 0.025, 0.01, 0.005, 0$. The $\xi = 0$ contours are denoted with dotted lines for clarity.

Figure 1. (a) The 2D 2PCF measured from SDSS DR7 LRGs in a redshift range $0.16 < z < 0.44$ (solid black contours), compared to a theoretical correlation function with parameters close to the best-fitting values in the likelihood analysis (dashed red contours). The theoretical model has $H(z = 0.35) = 81.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $D_A(z = 0.35) = 1042 \text{ Mpc}$, $\beta = 0.35$, $\Omega_m h^2 = 0.117$, $\Omega_b h^2 = 0.022$, $n_s = 0.96$, $\sigma_8 = 300 \text{ km s}^{-1}$ and $k_s = 0.11 \text{ h Mpc}^{-1}$. (b) The 2D 2PCF measured from a single mock catalogue, compared to a theoretical model with the input parameters of the LasDamas simulations, and $\{\beta, \sigma_8, k_s\}$ are set to $\{0.35, 300 \text{ km s}^{-1}, 0.11 \text{ h Mpc}^{-1}\}$ (dashed red contours). In both figures, the shaded disc indicates the scale range considered ($s = 40–120 \text{ h}^{-1} \text{ Mpc}$) in this study. The thick dashed blue circle denotes the BAO scale. The observed 2D 2PCF has been smoothed by a Gaussian filter with the rms variance of $2h^{-1}$ Mpc for illustration in these figures only; smoothing is not used in our likelihood analysis. The contour levels are $\xi = 0.5, 0.1, 0.025, 0.01, 0.005, 0$. The $\xi = 0$ contours are denoted with dotted lines for clarity.

Figure 2. The average 2D 2PCF measured from 160 LasDamas SDSS LRG-full mock catalogues (solid black contours), compared to a theoretical model with the input parameters of the LasDamas simulations, and $\{\beta, \sigma_8, k_s\}$ are set to $\{0.35, 300 \text{ km s}^{-1}, 0.11 \text{ h Mpc}^{-1}\}$ (dashed red contours). The grey area is the scale range considered ($s = 40–120 \text{ h}^{-1} \text{ Mpc}$) in this study. The thick dashed blue circle denotes the BAO scale. The contour levels are apparent in the 2D 2PCF measured from mock catalogues, even though no smoothing is used. The contour levels are $\xi = 0.5, 0.1, 0.025, 0.01, 0.005, 0$. The $\xi = 0$ contours are denoted with dotted lines for clarity.
where \( \xi_{\text{fid}}(s) \) is the observed correlation function assuming the fiducial model. This allows us to rewrite \( \chi^2 \) as

\[
\chi^2 = \sum_{i,j=1}^{N_{\text{obs}}} \left\{ T^{-1} \{ \xi_{\text{obs}}(s_i) \} - \xi_{\text{fid}}(s_i) \right\} C_{\text{fid},ij}^{-1} T^{-1} \left\{ \xi_{\text{obs}}(s_j) \right\} - \xi_{\text{fid}}(s_j),
\]

where we have used equations (13) and (15).

To find the operator \( T \), note that the fiducial model is only used in converting redshifts into distances for the galaxies in our data sample. In the analysis of galaxy clustering, we only need the separation of a galaxy pair and not the absolute distances to the galaxies. For a thin redshift shell, the separation of a galaxy pair in the transverse direction is proportional to \( D_s(z) \Delta \theta (\Delta \theta \) is the angular separation of the galaxy pair), and the separation along the direction of the line of sight is proportional to \( \Delta z/H(z) \) (\( \Delta z \) is the redshift difference between the galaxy pair). Thus, we can convert the separation of one pair of galaxies from the fiducial model to another model by performing the scaling (see, e.g., Seo & Eisenstein 2003)

\[
(\sigma', \pi') = \left( \frac{D_s(z)}{D_A(z)} \frac{H(z)}{H(z)}, \pi \right).
\]

Therefore, we can convert the measured 2D correlation function from some model to the fiducial model as follows:

\[
\xi_{\text{fid}}(s, \pi) = T^{-1} \{ \xi_{\text{obs}}(s, \pi) \} = \xi_{\text{obs}} \left( \frac{D_s(z)}{D_A(z)} \frac{H(z)}{H(z)} \frac{H(z)}{\pi} \right).
\]

This mapping defines the operator \( T \).

We now apply the inverse operation to the theoretical correlation function:

\[
T^{-1} \{ \xi_{\text{obs}}(s, \pi) \} = \xi_{\text{obs}} \left( \frac{D_s(z)}{D_A(z)} \frac{H(z)}{H(z)} \frac{H(z)}{\pi} \right).
\]

\( \chi^2 \) can be calculated by substituting equation (19) into equation (16).

### 3.5 Markov chain Monte Carlo likelihood analysis

We use cosmomc in a Markov chain Monte Carlo (MCMC) likelihood analysis (Lewis & Bridle 2002). The parameter space that we explore spans the parameter set of \( \{H(0.35), D_A(0.35), \Omega_m h^2, \Omega_b h^2, n_s, \sigma_8, k_1\} \). Only \( H(0.35), D_A(0.35), \Omega_m h^2 \) are well constrained using SDSS LRGs alone. We marginalize over the other parameters, \( \{\Omega_b h^2, r_s, \sigma_8, k_1\} \), with the flat priors, \{0.1, 0.6\}, (0.01859, 0.02657), (0.865, 1.059), (0, 500) km s\(^{-1}\), (0.09, 0.13) Mpc\(^{-1}\), where the flat priors of \( \Omega_b h^2 \) and \( n_s \) are centred on the measurements from Wilkinson Microwave Anisotropy Probe 7 (WMAP7) and has a width of \( \pm 1\sigma_{\text{WMAP}} \) (with \( \sigma_{\text{WMAP}} \) from Komatsu et al. 2010). These priors are wide enough to ensure that CMB constraints are not double counted when our results are combined with CMB data (Chuang et al. 2012). We also marginalize over the amplitude of the galaxy correlation function, effectively marginalizing over a linear galaxy bias.

### 4 RESULTS

We now present the model-independent measurements of the parameters \( \{H(0.35), D_A(0.35), \Omega_m h^2\} \), obtained by using the method described in previous sections. We also present the derived parameters including \( H(0.35) r_s(z_0)/c, D_A(0.35)/r_s(z_0), D_V(0.35)/r_s(z_0) \) and \( A(0.35) \), where

\[
D_V(z) = \left( 1 + z \right) D_A^2(z) c^2 H(z)^2
\]

and

\[
A(z) = \frac{\Omega_m h^2}{c^2}.
\]

We recommend to use \( \{H(0.35) r_s(z_0)/c, D_A(0.35)/r_s(z_0)\} \) instead of \( \{H(0.35), D_A(0.35), \Omega_m h^2\} \) because they are more robust measurements from this study (see Section 5 for more detail). We apply our method to the 2D 2PCF of the LasDamas mock catalogues and find that our measurements are consistent with the input parameters of the simulations.

#### 4.1 Constraints on \( H(0.35) \) and \( D_A(0.35) \) independent of a dark energy model

Fig. 3 shows 1D and 2D marginalized contours of the parameters \( \{H(0.35), D_A(0.35), \Omega_m h^2, H(0.35) r_s(z_0)/c, D_A(0.35)/r_s(z_0), D_V(0.35)/r_s(z_0), A(0.35)\} \), derived in an MCMC likelihood analysis from the measured 2D 2PCF of the SDSS LRG sample. Table 1 lists the mean, r.m.s variance and 68 per cent confidence level limits of these parameters. Table 2 gives the normalized covariance matrix for this parameter set. These are independent of a dark energy model and obtained without assuming a flat universe.

The constraints on \( \{H(0.35), D_A(0.35), \Omega_m h^2, H(0.35) r_s(z_0)/c, D_A(0.35)/r_s(z_0), D_V(0.35)/r_s(z_0), A(0.35)\} \), summarized in Tables 1 and 2, can be used to combined with any other cosmological data set to constrain dark energy and the cosmological model. We recommend to use only \( \{H(0.35) r_s(z_0)/c, A(0.35)\} \) since they have tighter constraints than \( \{H(0.35), D_A(0.35)\} \) and are robust in the systematic tests we have carried out (see Section 5). In addition, \( H(0.35) r_s(z_0)/c \) and \( D_A(0.35)/r_s(z_0) \) are basically independent of \( \Omega_m h^2 \) which might not be a robust measurement in this study (see Section 5).

The best-fitting model from the MCMC likelihood analysis has \( \chi^2 = 112 \) for 99 bins of data used for a set of nine parameters (including the overall amplitude of the correlation function) and the \( \chi^2 \) per degree of freedom (\( \chi^2/\text{d.o.f.} \)) is 1.24. Note that a 10 h\(^{-1}\) Mpc \( \times 10 h^{-1}\) Mpc bin is used if the centre of the bin is in the scale range of \( 40 < s < 120 h^{-1}\) Mpc.

### 4.2 Validation using mock catalogues

In order to validate our method, we have applied it to 80 2D 2PCFs from 80 LasDamas mock catalogues (which are indexed with 01a–40a and 01b–40b). Again, we apply the flat and wide priors (\( \pm 1\sigma_{\text{WMAP}} \)) on \( \Omega_b h^2 \) and \( n_s \), centred on the input values of the simulation (\( \Omega_b h^2 = 0.0196 \) and \( n_s = 1 \)).

Table 3 shows the means and standard deviations of the distributions of our measurements of \( \{H(0.35), D_A(0.35), \Omega_m h^2, H(0.35) r_s(z_0)/c, D_A(0.35)/r_s(z_0), D_V(0.35)/r_s(z_0), A(0.35)\} \) from each of the LasDamas mock catalogues (80 total) of the SDSS LRG sample. These are consistent with the input parameters, establishing the validity of our method. We also show the measurements of \( H(0.35) r_s(z_0)/c \) and \( D_A(0.35)/r_s(z_0) \) of each mock catalogue in Figs 4 and 5. One can see that the measurements of \( H(0.35) r_s(z_0)/c \) and \( D_A(0.35)/r_s(z_0) \) are consistent with the input parameters of the simulations.
Figure 3. 2D marginalized contours (68 and 95 per cent CL) for \( \{ H(0.35), \Omega_m h^2, H(0.35) r_s(z_d)/c, D_A(0.35)/r_s(z_d), D_V(0.35)/r_s(z_d), A(0.35) \} \). The diagonal panels represent the marginalized probabilities. The unit of \( H \) is \( \text{km s}^{-1} \text{Mpc}^{-1} \). The unit of \( D_A, D_V \) and \( r_s(z_d) \) is Mpc.

4.3 Cross-check with measurements from multipoles of the correlation function

As a cross-check of our results, we now present the measurements from the monopole–quadrupole method of the correlation function for comparison. The detail of the method is described in Appendix B. Table 4 lists the mean, rms variance and 68 per cent confidence level limits of these parameters. The measurements are consistent with those from our main method (see Table 1). However, the constraints are much weaker (>8 per cent, which is twice as large as our main results). This is most likely due to the fact that the information used in the monopole–quadrupole method is much less than what we use in our main method (as presented in this paper). It is possible to obtain better measurements using the multipole method by including higher order multipoles of the correlation function; this is explored in Chuang & Wang (2012).

5 SYSTEMATIC TESTS

Table 5 shows the systematic tests that we have carried out varying key assumptions made in our analysis. These include the range
of scales used to calculate the correlation function, the non-linear damping factor, the bin size and an overall shift in the measured correlation function due to a systematic error.

First, we fix the non-linear damping factor, \( k_s = 0.11 \text{ hMpc}^{-1} \), and then find that the results are basically the same. To speed up the computation, we fix \( k_s \) for the rest of the tests.

In this study, we marginalize over \( \beta \) with a wide flat prior (0.1–0.6) since our method is not sensitive to \( \beta \). We test fixing the value of \( \beta \) to 0.35, which is close to the measurement from previous work with similar data but using different method (Cabre & Gaztanaga 2009), and find that our measurements of \( H(0.35) \) and \( D_A(0.35) \) change by less than 1 per cent compared to that of marginalizing over \( \beta \).

We vary the range of the scale and find that \( H(0.35) \) and \( D_A(0.35) \) are insensitive to it. However, \( \Omega_m h^2 \) is sensitive to the minimum scale chosen which could imply that the scale-dependent bias or redshift distortion is distorting larger scale than we have expected. Therefore, we do not recommend to use \( \Omega_m h^2 \) from this study. In the case of \( s = 40–130 \text{ h}^{-1} \text{ Mpc}, D_A(0.35) \) is different from the fiducial result with about 2\( \sigma \), which is likely due to systematic errors responsible for the anomalously high tail in the spherically averaged correlation function (see, e.g., Chuang et al. 2012) on large scales.

We vary the bin size to \( 8 \text{ h}^{-1} \text{ Mpc} \times 8 \text{ h}^{-1} \text{ Mpc} \) and find \( \chi^2/\text{d.o.f.} = 1.72 \), which can be explained by the increase in the noise level with the increased number of bins. The number of the mock catalogues used to construct the covariance matrix is 160, and the number of bins used with bin size = \( 8 \text{ h}^{-1} \text{ Mpc} \times 8 \text{ h}^{-1} \text{ Mpc} \) is 159. One can expect that the covariance matrix would be too noisy to give reasonable results.

We also show that the results are insensitive to the constant shift by lowering down the data points of the observed correlation function by 0.001 and 0.002.

### Table 1. Normalized covariance matrix of the measured and derived parameters, \{H(0.35), D_A(0.35), \Omega_m h^2, H(0.35) r_s(z_d)/c, D_A(0.35) r_s(z_d), D_V(0.35) r_s(z_d), A(0.35)\}.

| Parameter | H(0.35) | D_A(0.35) | \Omega_m h^2 | H(0.35) r_s(z_d)/c | D_A(0.35) r_s(z_d) | D_V(0.35) r_s(z_d) | A(0.35) |
|-----------|---------|-----------|-------------|-------------------|-------------------|-------------------|---------|
| H(0.35)   | 1       | -0.4809   | 0.7088      | 0.7297            | 0.0827            | -0.2631          | 0.2618  |
| D_A(0.35) | -0.4809 | 1         | -0.6339     | -0.0605           | 0.6730            | 0.6167           | 0.0379  |
| \Omega_m h^2 | 0.7088 | -0.6339 | 1           | 0.0867            | 0.0888            | 0.0427           | 0.7042  |
| H(0.35) r_s(z_d)/c | 0.7297 | -0.0605 | 0.0867      | 1                 | 0.0604            | -0.4104          | -0.1934 |
| D_A(0.35) r_s(z_d) | 0.0827 | 0.6730 | 0.0888      | 1                 | -0.4104           | 1                 | 0.6807  |
| D_V(0.35) r_s(z_d) | -0.2631 | 0.6167 | 0.0427      | -0.4104           | 1                 | 0.6807           | 1       |
| A(0.35) | 0.2618   | 0.0379   | 0.7042      | -0.1934           | 0.6447            | 0.6807           | 1       |

### Table 2. Measurements of the means of \( H(0.35) r_s(z_d)/c \) from 80 individual mock catalogues (indexed as 01a–40a and 01b–40b).

| Parameter | Mean | \( \sigma \) | Lower | Upper |
|-----------|------|--------------|-------|-------|
| \( H(0.35) \) | 82.1 | 5.0          | 77.2  | 86.9  |
| \( D_A(0.35) \) | 1048 | 58           | 990   | 1107  |
| \( \Omega_m h^2 \) | 0.118 | 0.017       | 0.101 | 0.133 |
| \( D_V(0.35) r_s(z_d) \) | 6.60 | 0.26         | 6.34  | 6.85  |
| \( A(0.35) \) | 8.62 | 0.25         | 8.38  | 8.86  |
| \( \Omega_m h^2 \) | 0.445 | 0.021       | 0.425 | 0.465 |

### Table 3. Measurements of the mean, standard deviation and the 68 per cent CL bounds of the distributions of the measured values of \( H(0.35), D_A(0.35), \Omega_m h^2, H(0.35) r_s(z_d)/c, D_A(0.35) r_s(z_d), D_V(0.35) r_s(z_d), A(0.35) \) from the 2D 2PCF of each of 80 LasDamas mock catalogues (which are indexed with 01a–40a and 01b–40b). Our measurements are consistent with the input values within 1\( \sigma \), where each \( \sigma \) is computed from the 80 means measured from the 80 mock catalogues. The unit of \( H \) is \( \text{ km s}^{-1} \text{ Mpc}^{-1} \). The unit of \( D_A, D_V \) and \( r_s(z_d) \) is Mpc.

| Parameter | Mean | \( \sigma \) | Input Value |
|-----------|------|--------------|-------------|
| \( H(0.35) \) | 81.1 | 4.1          | 81.79       |
| \( D_A(0.35) \) | 1009 | 56           | 1032.8      |
| \( \Omega_m h^2 \) | 0.121 | 0.013       | 0.1225      |
| \( D_V(0.35) r_s(z_d) \) | 6.26 | 0.30         | 6.48        |
| \( A(0.35) \) | 8.33 | 0.31         | 8.51        |

### Figure 4. Measurements of the means of \( H(0.35) r_s(z_d)/c \) from 80 individual mock catalogues (indexed as 01a–40a and 01b–40b). The black solid line shows the mean of these 80 measurements, and the blue dashed lines show the range of ±\( \sigma \). The red dotted line shows the theoretical value computed with the input parameters of the simulations.

### 6 CONCLUSION AND DISCUSSION

We have obtained the first measurements of \( H(z) \) and \( D_A(z) \) from galaxy clustering data in an MCMC likelihood analysis. Our constraints for the measured and derived parameters, \{\( H(0.35), D_A(0.35), \Omega_m h^2, H(0.35) r_s(z_d)/c, D_A(0.35) r_s(z_d), D_V(0.35) r_s(z_d), A(0.35) \}\), from the 2D 2PCF of the sample of SDSS DR7 LRGs are summarized in Tables 1 and 2. Our results are...
robust and independent of a dark energy model, obtained without assuming a flat universe, and represent the first measurements of $H(z)$ and $D_A(z)$ from galaxy clustering data.

Our galaxy clustering measurements of $H(0.35)r_z(z_0)/c$ and $D_A(0.35)/r_z(z_0)$ (see Tables 1 and 2) can be used to combine with CMB and other cosmological data sets to probe dark energy. In a companion paper (Wang, Chuang & Mukherjee 2012), we explore the implications of our results for dark energy constraints.

Table 5. The systematic tests with the damping factor, the scale range, the bin size and the assumed constant shift from a systematic error ($\xi_{obs}(s) = \xi_{true}(s)+\text{shift}$). The fiducial results are obtained by considering the scale range ($40 < s < 120 h^{-1}$ Mpc), the bin size $=10 h^{-1}$ Mpc $\times 10 h^{-1}$ Mpc and the damping factor, $k_s$, marginalized over with a flat prior (0.09 < $k_s$ < 0.13 h Mpc$^{-1}$). The other results are calculated with only specified quantities different from the fiducial one. The unit of $H$ is km s$^{-1}$ Mpc$^{-1}$. The unit of $D_A$, $D_V$ and $r_z(z_0)$ is Mpc. The unit of $k_s$ is h Mpc$^{-1}$.

| $H(0.35)$ | $D_A(0.35)$ | $\Omega_m h^2$ | $H(0.35)$ | $D_A(0.35)$ | $A(0.35)$ | $\chi^2$/d.o.f. |
|-----------|-------------|----------------|-----------|-------------|-----------|----------------|
| $k_s = 0.11$ | 82.1$^{+4.8}_{-4.9}$ | $1048^{+59}_{-55}$ | 0.118$^{+0.016}_{-0.017}$ | 0.0434$^{+0.0017}_{-0.0017}$ | 6.60$^{+0.25}_{-0.25}$ | 8.62$^{+0.24}_{-0.24}$ | 0.445$^{+0.020}_{-0.020}$ | 1.24 |
| $\beta = 0.35$ | 81.7$^{+4.5}_{-4.9}$ | $1051^{+59}_{-55}$ | 0.116$^{+0.017}_{-0.017}$ | 0.0434$^{+0.0017}_{-0.0017}$ | 6.60$^{+0.25}_{-0.25}$ | 8.60$^{+0.24}_{-0.24}$ | 0.445$^{+0.020}_{-0.020}$ | 1.24 |
| $30 < s < 120$, $k_s = 0.11$ | 83.8$^{+5.5}_{-5.4}$ | $1038^{+52}_{-51}$ | 0.120$^{+0.013}_{-0.014}$ | 0.0437$^{+0.0019}_{-0.0019}$ | 6.59$^{+0.22}_{-0.22}$ | 8.59$^{+0.24}_{-0.24}$ | 0.446$^{+0.016}_{-0.016}$ | 1.24 |
| $50 < s < 120$, $k_s = 0.11$ | 83.5$^{+4.5}_{-4.9}$ | $1015^{+52}_{-51}$ | 0.134$^{+0.024}_{-0.023}$ | 0.0428$^{+0.0019}_{-0.0019}$ | 6.59$^{+0.22}_{-0.22}$ | 8.65$^{+0.24}_{-0.24}$ | 0.460$^{+0.024}_{-0.024}$ | 1.06 |
| $40 < s < 110$, $k_s = 0.11$ | 80.6$^{+5.1}_{-5.6}$ | $1087^{+69}_{-61}$ | 0.115$^{+0.016}_{-0.016}$ | 0.0429$^{+0.0019}_{-0.0019}$ | 6.78$^{+0.27}_{-0.27}$ | 8.81$^{+0.27}_{-0.27}$ | 0.454$^{+0.022}_{-0.022}$ | 1.09 |
| $40 < s < 130$, $k_s = 0.11$ | 84.8$^{+6.4}_{-6.7}$ | $987^{+60}_{-61}$ | 0.115$^{+0.016}_{-0.016}$ | 0.0451$^{+0.0026}_{-0.0026}$ | 6.17$^{+0.27}_{-0.27}$ | 8.14$^{+0.28}_{-0.28}$ | 0.418$^{+0.019}_{-0.019}$ | 1.31 |
| Bin size = $8 h^{-1}$ Mpc $\times 8 h^{-1}$ Mpc | 87.5$^{+4.5}_{-4.9}$ | $1037^{+60}_{-63}$ | 0.139$^{+0.017}_{-0.018}$ | 0.0447$^{+0.0023}_{-0.0023}$ | 6.78$^{+0.27}_{-0.27}$ | 8.70$^{+0.27}_{-0.27}$ | 0.470$^{+0.021}_{-0.021}$ | 1.72 |
| Shift = 0.001, $k_s = 0.11$ | 84.3$^{+4.5}_{-4.9}$ | $1041^{+60}_{-63}$ | 0.124$^{+0.019}_{-0.020}$ | 0.0433$^{+0.0023}_{-0.0023}$ | 6.63$^{+0.26}_{-0.26}$ | 8.65$^{+0.26}_{-0.26}$ | 0.453$^{+0.021}_{-0.021}$ | 1.25 |
| Shift = 0.002, $k_s = 0.11$ | 85.2$^{+4.5}_{-4.9}$ | $1024^{+63}_{-65}$ | 0.135$^{+0.021}_{-0.021}$ | 0.0435$^{+0.0021}_{-0.0021}$ | 6.67$^{+0.28}_{-0.28}$ | 8.67$^{+0.28}_{-0.28}$ | 0.463$^{+0.022}_{-0.022}$ | 1.24 |
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APPENDIX A: ALGORITHM OF SMOOTHING THE COVARIANCE MATRIX

The original covariance matrix, $C_0$, is computed by equation (13). Since the correlation function is in two dimensions, we could relabel the covariance matrix as a function of the indices of two bins, $C(\sigma_i, \pi_i)$, $C(\sigma_j, \pi_j)$. The covariance matrix would be noisy since it is constructed with a small number of mock catalogues compared to the number of bins used. One might not obtain converging results when applying MCMC analysis on the observed data or an individual mock catalogue, especially considering larger scale range or larger number of bins. We find that smoothing the covariance matrix could solve the problem. We have also checked that the smoothing procedure would not introduce bias by comparing the results obtained from applying the smoothed and the original covariance matrix on the averaged correlation function from 160 mock catalogues. The concept of our smoothing procedure is that the new value of an element of the covariance matrix would be determined by its original value and the values of its neighbouring elements. For example, to smooth an array, $f[n]$, we assign the new value at the index $n'$ by $\bar{f}[n'] = (1 - p) \cdot f[n'] + p \cdot (f[n' - 1] + f[n' + 1])/2$, where $0 \leq p \leq 1$. The goal is to make the value to be closer to the mean of the neighbours. Note that while one of the neighbours is not available (i.e. $f[n']$ is the first or last element of the array), we let $f[n']$
be fixed since the mean of the neighbours is also not available. The algorithm can be expressed by

\[
\tilde{C}(\sigma_i, \pi_i, \sigma_j, \pi_j) = (1 - p) \cdot C(\sigma_i, \pi_i, \sigma_j, \pi_j) + \frac{p}{m} \times \left[ (C(\sigma_i + \Delta s, \pi_i, \sigma_j, \pi_j) + C(\sigma_i - \Delta s, \pi_i, \sigma_j, \pi_j)), \text{if both elements on the left are available} \right] \\
\times \left[ (C(\sigma_i, \pi_i + \Delta s, \sigma_j, \pi_j) + C(\sigma_i, \pi_i - \Delta s, \sigma_j, \pi_j)), \text{if both elements on the left are available} \right] \\
\times \left[ (C(\sigma_i, \pi_i, \sigma_j + \Delta s, \pi_j) + C(\sigma_i, \pi_i, \sigma_j - \Delta s, \pi_j)), \text{if both elements on the left are available} \right] \\
\times \left[ (C(\sigma_i, \pi_i, \sigma_j, \pi_j + \Delta s) + C(\sigma_i, \pi_i, \sigma_j, \pi_j - \Delta s)), \text{if both elements on the left are available} \right].
\]  

(A1)

where \( m \) is the number of the neighbour elements used which should be 0, 2, 4, 6 or 8, and \( \Delta s \) is the size of the bin in one direction. We use \( p = 0.01 \) and \( \Delta s = 10 \text{ Mpc} h^{-1} \) in this study, but \( p = 0 \) if \( m = 0 \). And then, we iterate this procedure for 10 times.

While equation (A1) can be applied on most of the elements of the covariance matrix, there are some special cases as described below.

1. **Diagonal elements**: these elements are only determined by the nearby diagonal elements by equation (A2), since the diagonal elements should not be smoothed using the latter.

\[
\tilde{C}(\sigma_i, \pi_i, \sigma_i, \pi_i) = (1 - p) \cdot C(\sigma_i, \pi_i, \sigma_i, \pi_i) + \frac{p}{m} \times \left[ (C(\sigma_i + \Delta s, \pi_i, \sigma_i + \Delta s, \pi_i) + C(\sigma_i - \Delta s, \pi_i, \sigma_i - \Delta s, \pi_i)), \text{if both elements on the left are available} \right] \\
\times \left[ (C(\sigma_i, \pi_i + \Delta s, \sigma_i, \pi_i + \Delta s) + C(\sigma_i, \pi_i - \Delta s, \sigma_i, \pi_i - \Delta s)), \text{if both elements on the left are available} \right].
\]

(A2)

2. **First off-diagonal elements**: these elements are only determined by the first off-diagonal elements nearby, i.e. \( C(\sigma_i, \sigma_i, \sigma_j, \pi_j) \) would be assigned a new value by equation (A3), since a first off-diagonal element could be very different from its neighbouring diagonal elements and should not be smoothed using the latter.

\[
\tilde{C}(\sigma_i + \Delta s, \pi_i, \sigma_i, \pi_i) = (1 - p) \cdot C(\sigma_i + \Delta s, \pi_i, \sigma_i, \pi_i) + \frac{p}{m} \times \left[ (C(\sigma_i + 2\Delta s, \pi_i, \sigma_i + \Delta s, \pi_i) + C(\sigma_i, \pi_i - \Delta s, \pi_i)), \text{if both elements on the left are available} \right] \\
\times \left[ (C(\sigma_i + \Delta s, \pi_i + \Delta s, \sigma_i, \pi_i + \Delta s) + C(\sigma_i + \Delta s, \pi_i - \Delta s, \sigma_i, \pi_i - \Delta s)), \text{if both elements on the left are available} \right].
\]

(A3)

**APPENDIX B: MEASURING \( H \) AND \( D_A \) WITH MULTIPOLES OF THE CORRELATION FUNCTION**

Using monopole–quadrupole of the power spectrum to break the degeneracy of \( H(z) \) and \( D_A(z) \) is introduced by Padmanabhan & White (2008). Kazin, Sanchez & Blanton (2011) tested the method of monopole–quadrupole of the correlation function with mock catalogues. The method used in Kazin et al. (2011) is similar to but not exactly the same as our method. We describe our method below.

First, we compute the 2D correlation function with bin size 1 \( h^{-1} \text{ Mpc} \times 1 \text{ h}^{-1} \text{ Mpc} \). Then, we compute the monopole and quadrupole by

\[
\xi_0(s) = \sum_{s - \frac{\Delta s}{2} < \sqrt{\sigma^2 + \pi^2} < s + \frac{\Delta s}{2}} \xi(\sigma, \pi) \sqrt{1 - \mu^2}
\]

(Number of bins used in the numerator)

\[
\xi_2(s) = \sum_{s - \frac{\Delta s}{2} < \sqrt{\sigma^2 + \pi^2} < s + \frac{\Delta s}{2}} \frac{\xi(\sigma, \pi)(3\mu^2 - 1) \sqrt{1 - \mu^2}}{2}
\]

(Number of bins used in the numerator).

(B1)

(B2)

Figure B1. Measurement of monopole of the correlation function from SDSS DR7 LRGs in a redshift range 0.16 < z < 0.44. The error bars are the square roots of the diagonal elements of the covariance matrix we have derived from mock catalogues.
Figure B2. Measurement of quadrupole of the correlation function from SDSS DR7 LRGs in a redshift range $0.16 < z < 0.44$. The error bars are the square roots of the diagonal elements of the covariance matrix we have derived from mock catalogues.

where

$$\mu \equiv \frac{\pi}{\sqrt{\sigma^2 + \pi^2}} \quad (B3)$$

and $\Delta r = 5 h^{-1} \text{Mpc}$, and the scale range used is $40 < s < 120 h^{-1} \text{Mpc}$ in this study. Therefore, there are 32 data points for the monopole–quadrupole method, with 16 measurements each for monopole and quadrupole, respectively. Figs B1 and B2 show the measurements of the monopole and quadrupole of the correlation function from the observed galaxy sample.

Just like our main method, the covariance matrix (for the 32 monopole and quadrupole measurements) is constructed from the mock catalogues. The theoretical multipoles are computed using equations (B1) and (B2).

Now, by exploring the same parameters and ranges using MCMC (with $\chi^2$ given by equation 16), one could measure $H(z)$ and $D_A(z)$ following the same steps as our main method in this study. The results are shown in Section 4.3 (see Table 4). Like our main method, our monopole–quadrupole method has only one approximation that the observed 2D correlation function using different fiducial models can be converted from one to another with two stretching factors. However, the multipole method tested in Kazin et al. (2011) neglected some additional terms while measuring the stretching factors. Although the effect of the neglected terms might be small, it could be completely avoided by using the method described here.

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