Coherent Polarization Shift Keying Modulated Free Space Optical Links Over a Gamma-Gamma Turbulence Channel

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Abstract: Problem statement: The optical signal propagating through the Free Space Optical (FSO) channel suffers from irradiance and phase fluctuations caused by the atmospheric turbulence, which results in Bit Error Rate (BER) performance degradation. Approach: In this study the performance of the Multilevel Coherent Polarization Shift Keying (M-POLSK) based FSO communication system operating over the gamma-gamma turbulence channel is investigated. To mitigate the turbulence induced fading, the convolutional coding and spatial diversity techniques are employed. The upper BER bounds are derived using the transfer function technique. Results: For example, with a SNR of 30 dB, the BERs for uncoded and coded M-POLSK are 0.047 and $1.4 \times 10^{-4}$, respectively in the weak turbulence regime. When the Maximum Ration Combining (MRC) technique employing four detectors are used in the receiver, the power gains of $\sim 31.4$, $\sim 29.5$ and $\sim 57.9$ dB are achieved for weak, moderate and strong turbulence regimes, respectively. Conclusion: We have also shown that the spatial diversity offers increased link margin as the scintillation level rises. Compared to the angular modulation, the proposed M-POLSK scheme offers high immunity to the phase noise, thus reducing the power penalties.

Key words: FSO, Maximum Ration Combining (MRC), Spatial Diversity (SD), atmospheric turbulence channel, Field of View (FOV), Polarization Controller (PC), Polarization Beam Splitter (PBS), Phase Modulator (PM)

INTRODUCTION

Unlike Radio Frequency (RF) based wireless systems which suffer from bandwidth constraints, FSO systems offers full-duplex gigabit rate throughput. This makes it a suitable technology for delivering broadband wireless services for certain applications including the metropolitan area network, enterprise/local area networks, optical fiber backup, enterprise connectivity and the last mile access networks (Uysal et al., 2006). FSO offers a number of advantages over the RF technology, including higher data rate, an unregulated spectrum, high immunity to the electromagnetic interference, high security, a small size transceiver, low cost and a lower power consumption (Popoola and Ghassemlooy, 2009; Popoola et al., 2008). In indoor applications, optical radiation is confined within rooms (assuming no windows and transparent barriers), thus making eavesdropping a difficult task. For outdoor applications, a laser transmitter with a highly directional and a cone-shaped beam profile normally installed high above the street level thus makes interception difficult. Therefore, anyone trying to tap into the communication link can be easily detected and any equipment placed within the narrow optical footprint could easily be identified.

However, the FSO link performance suffers from a number of phenomena such as the misalignment due to the building sway caused by the wind, thermal expansion and weak earthquakes (Uysal et al., 2006; Arnon, 2003; Tang et al., 2011). Another important issue with the outdoor FSO system is the susceptibility of the optical link to the atmospheric conditions. The laser beam propagating through the channel suffers...
from the high attenuation due to the atmospheric 
scatters such as haze, fog and rain, which limit the link 
range and the system reliability. Fog compared with 
rain and haze is the biggest contributor to the path loss. 
The attenuation due to thick fog and haze can reach up to 
300 dB km\(^{-1}\) thus limiting the link range to 100 m 
(Ramirez-iniguez et al., 2008). Smoke also has a similar 
effect as fog on the propagating optical signal (Ramirez-
iniguez et al., 2008).

Even in clear sky conditions, the optical signal 
suffers from the atmospheric turbulence, which is also 
known as the scintillation (Popoola and Ghassemlooy, 
2009; Kamalakis et al., 2006). Scintillation originates 
from the inhomogeneities in the refraction index of the 
atmosphere caused by the variation in the temperature 
and the pressure. Scintillation leads to significant 
fluctuations on the amplitude and phase of the optical 
field (i.e., channel fading) (Popoola and Ghassemlooy, 
2009). The knowledge of statistical distribution of the 
atmospheric turbulence is necessary to fully study and 
predict FSO performance operating over a clear 
atmospheric condition.

The performance impairments due to the 
scintillation can be mitigated by adopting several 
approaches including the aperture averaging and the 
diversity techniques (Uysal et al., 2006; Khalighi et al., 
2009; Zhu and Kahn, 2003), the adaptive optics 
(Weyrauch and Vorontsov, 2004), the saturated optical 
amplifiers (Abtahi et al., 2006), the modulation 
techniques and the error control coding (Uysal et al., 
2006; Zhu and Kahn, 2001; 2003). In (Uysal et al., 2006; Zhu 
and Kahn, 2001; 2003) the performances of coded FSO 
links for the log-normal and gamma-gamma channel 
models under atmospheric turbulence have been 
investigated. Study also presents an approximate upper 
bound for the Pairwise Error Probability (PEP) and the 
upper bounds on the BER using the transfer function 
technique for the coded FSO links with Intensity 
Modulation/Direct Detection (IM/DD). The diversity 
techniques comprising space, time, or frequency 
(wavelength) have been adapted to improve 
performance impairments due to the scintillation. With 
the spatial diversity technique, where a single receiver 
with a large Field of View (FOV) is replaced by a 
group of detectors with a narrow FOV, the possibility 
of all the detectors suffering from the deep fade 
simultaneously is much reduced. Moreover, the spatial 
diversity scheme limits the amount of background light 
from unwanted sources that impinges on the specific 
detector, which otherwise could be received by the 
single receiver with a wide FOV (Khalighi et al., 2009).

The type of modulation schemes in FSO systems is 
crucial to ensure the maximum power efficiency. The 
coherent system based on the angular modulation; such 
as Binary Phase Shift Keying (BPSK) and Differential 
Phase Shift Keying (DPSK) are highly sensitive to the 
phase noise effects (Betti et al., 1995). Alternatively, it 
has been shown that POLSK offers higher immunity to 
the phase noise and the atmospheric turbulence. The 
polarization states of the laser beam propagating 
through the FSO channel can be maintained over a long 
link range (Sugianto and Davis, 2006).

**MATERIALS AND METHODS**

The aim of this study is to evaluate the 
performance of the Multilevel POLSK (M-POLSK) 
modulated FSO system with the coherent detection 
operating over the gamma-gamma atmospheric 
turbulence channel. For this purpose, the performance 
improvement by the error control coding and the spatial 
diversity will be also considered. However, the code 
should be short and simple in order to keep the complexity 
of this approach reasonably low. The system performance 
will be compared with the coherent BPSK and DPSK in 
terms of the average BER. The noise, comprising of both 
the background radiation and the thermal noise, is 
modeled as the additive white Gaussian process. We also 
assume that the transmitter and the receiver have the 
perfect link alignment.

**Gamma-gamma turbulence model:** The analysis for 
the atmospheric turbulence has been carried out by a 
number of researchers and several theoretical models 
have been developed to characterize its behaviour. The 
simplest and most widely reported model is the log 
normal turbulence, which is mathematically convenient 
and tractable. The log normal model is based on the 
Rytov approximation, which requires the unperturbed 
phase gradient to be large compared to the magnitude 
of the scattering field wave (Zhu and Kahn, 2003; 
Popoola and Ghassemlooy, 2009). However, the log 
normal model only covers the weak turbulence regime 
with a single scattering event. For the turbulence in the 
saturation regime with multiple scatterings, the log 
normal model becomes invalid (Uysal et al., 2006; 
Osche, 2002; Karp, 1988). The strength of turbulence 
can be described by the log intensity variance $\sigma_1^2$ and 
the log normal model is only valid for $\sigma_1^2$. Another 
important parameter for describing the turbulence 
strength is the scintillation index, which is the log 
intensity variance normalized by the square of the mean 
irradiance. The experimental results have indicated that 
the scintillation index does not only saturate, but also 
decreases after it researches the maximum value while 
the strength of turbulence continues rising (Karp, 
1988). In the saturation regime and beyond with the 
link length of several kilometres the intensity
fluctuation is experimentally verified to obey the Rayleigh distribution (Karp, 1988; Popoola and Ghassemlooy, 2009). To express the turbulence effects across all regimes, the gamma-gamma turbulence model is considered in study.

The gamma-gamma model first proposed by Andrews et al. (2001) is valid for all turbulence regimes from weak to strong regimes. It is based on the assumption that the fluctuation of the laser beam propagating through the turbulence medium consists of the refraction ($I_1$) and scattering ($I_2$) effects. Subsequently, the normalized irradiance is expressed as the normalized product of two independently statistical random processes $I_1$ and $I_2$ given by Eq. 1:

$$ I = I_1 I_2 $$

(1)

where, $I_1$ and $I_2$ represent the refraction and scattering effects, respectively and both are governed by the gamma distribution (Andrews et al., 2001; Al-Habash et al., 2001). Thus, the Probability Density Function (PDF) of the gamma-gamma model for the received irradiance fluctuation is derived as Eq. 2:

$$ p(l) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} l^{\alpha+\beta-1} \cdot K_{\alpha+\beta}(2\sqrt{\alpha\beta}l), l > 0 $$

(2)

where, $\Gamma(\cdot)$ represents the gamma function and $K_n(\cdot)$ is the modified Bessel function of the 2nd kind of order $n$. The parameters $\alpha$ and $\beta$ characterize the optical power fluctuation PDF which are related to the atmospheric conditions. Assuming that the optical radiation at the receiver is a plane wave, $\alpha$ and $\beta$ can be expressed as Eq. 3a-b (Popoola and Ghassemlooy, 2009; Andrews et al., 2001):

$$ \alpha = \left\{ \exp \left[ \frac{0.49\sigma^2}{(1 + 0.18d^2 + 0.56\sigma_0^2)^{1/6}} \right] - 1 \right\}^{-1} $$

(3a)

$$ \beta = \left\{ \exp \left[ \frac{0.51\sigma^2}{(1 + 0.9d^2 + 0.62d^2\sigma_0^2)^{1/6}} \right] - 1 \right\}^{-1} $$

(3b)

Where

- $\sigma^2 = 0.5C_n^2k^{1/2}L^{1/6}$ = Log irradiance variance
- $d = (kD^2/4L)^{1/2}$, $k = 2\pi/\lambda$ = Optical wave number
- $\lambda$ = Wavelength
- $D$ = Diameter of the receiver collecting lens aperture
- $L$ = Link length in meters
- $C_n^2 = 10^{-15} m^{-2/3}$ = The typical average value of refractive index structure parameter

For FSO links near the ground, $C_n^2$ can be taken approximately as $1.7 \times 10^{-14} m^{-2/3}$ and $8.4 \times 10^{-15} m^{-2/3}$ during daytime and night (Goodman, 1985; Uysal et al., 2006).

**M-Polsk modulation principles:**

**A. transmitter:** Figure 1 illustrates the block diagram of the M-POLSK transmitter. The state of the polarization of the laser beam is controlled by a Polarization Controller (PC) before being fed into the Polarization Beam Splitter (PBS). The Transmitting Laser (TL) beam is linearly polarized and has a $\pi/4$ polarization with respect to the principle axe of the external Phase Modulator (PM).

The emitted optical field of the carrier is expressed by a complex vector $\hat{E}_c(t)$ in the transverse plane, which is given as Eq. 4 (Cusani et al., 1992):

$$ \hat{E}_c(t) = \frac{P}{2\sqrt{\pi}} e^{i(\omega t + \phi)} \left\{ \hat{x} + \hat{y} \right\} $$

(4)

where, $P$, $\omega$ and $\phi(t)$ are the power, the angular frequency and the phase noise of the emitted optical carrier, respectively. The two vectors $\hat{x}$ and $\hat{y}$ denote the direction along which the field is polarized. Note that the square root of the field power directly provides the amplitude.

The signal $\hat{E}_c(t)$ is decomposed by the PBS into two orthogonally polarized components with the equal amplitudes. The amplitude and phase of the optical component polarized along the $\hat{x}$ axis are modulated externally by the data while the $\hat{y}$ component is transmitted as the reference carrier. The applied voltage to the LiNbO$_3$ based external phase modulator is equal to either zero or $V_{pi}$. The applied voltage $V_s$ induces $\pi$ phase shift in the $\hat{x}$-component and zero phase shift in the $\hat{y}$-component, thus leading to a $\pi/2$ rotation of the polarization of the optical carrier. The amplitude modulation combined with the phase modulation in $\hat{x}$ component is described as an eight-level modulation where the time axis is divided into symbol intervals.
No, components which are then mixed and transmitted are supposed to be instantaneous. Under these conditions, the transmitted optical field is constant. The transitions between subsequent symbol intervals are supposed to be linear. The vector \( m(t) \) is expressed as Eq. 6:

\[
m(t) = \sum b_k \text{rect}_T (t-kT)
\]

where:

- \( b_k = (0,1) \) = Transmitted bit
- \( T \) = Symbol period and the rectangular pulse shaping function
- \( \text{rect}_T(t) \) = The rectangular pulse shaping function

Receiver: Figure 2 illustrates the block diagram of the proposed optical coherent heterodyne receiver. The ideal Optical Band-Pass Filter (OBPF) with a narrow bandwidth typically 1 nm is used to limit the background ambient light. The optical field of the local oscillator \( \tilde{E}_{lo}(t) \) is linearly polarized at \( \pi/4 \) with respect to the receiver reference axes. An Automatic Frequency Control (AFC) is used to compensate for the slow-frequency fluctuations occurring in the Local Oscillator (LO), whose control signal is derived from the intermediate frequency electric signal. Generally, the AFC is a closed-loop circuit acting on the bias current (Betti et al., 1995).

The received optical field and are uncorrelated and can be expressed as Eq. 7a-b:

\[
\tilde{E}_s(t) = \sqrt{P_s} e^{j(m_1 t + \varphi_1(t))} e^{j\xi + j\gamma} \\
\tilde{E}_r(t) = \sqrt{P_r} e^{j(m_2 t + \varphi_2(t))} e^{j\xi + j\gamma}
\]

where, \( P_s \) and \( \varphi_1(t) \) are the power and the phase noise of the received optical signal, respectively. Both \( P_r \) and \( \varphi_2(t) \) are time-variant due to the atmospheric turbulence. The parameters \( P_{lo}, \omega_0 \) and \( \varphi_0(t) \) represent the power, angular frequency and the phase noise of the local oscillator, respectively and \( \omega \neq \omega_0 \).

The received optical signal \( \tilde{E}_s(t) \) is split by the PBS into \( \hat{x} \) and \( \hat{y} \) components which are then mixed with the corresponding optical fields emitted from the local oscillator. Therefore, the decomposed orthogonally polarized components \( \tilde{E}_{s\hat{x}}(t) \) and \( \tilde{E}_{s\hat{y}}(t) \) with equal amplitudes are given as Eq. 8a-b:

\[
\tilde{E}_{s\hat{x}}(t) = \sqrt{P_s} e^{j(m_1 t + \varphi_1(t))} \hat{x} \\
\tilde{E}_{s\hat{y}}(t) = \sqrt{P_s} e^{j(m_1 t + \varphi_1(t))} \hat{y}
\]

Following optical-to-electrical conversion, the signals \( c_{s\hat{x}}(t) \) and \( c_{s\hat{y}}(t) \) at the output of two identical Photo Diodes (PD) are expressed as Eq. 9a-b (Betti et al., 1995):

\[
c_{s\hat{x}}(t) = R \left[ \sqrt{P_s/2} e^{j\omega_0 t + j\gamma + j\phi_0(t)} + n_s(t) \right] \\
c_{s\hat{y}}(t) = R \left[ \sqrt{P_s/2} e^{j\omega_0 t + j\gamma + j\phi_0(t)} + n_s(t) \right]
\]

where, \( R \) represents the photodiode responsively, \( \omega_0=\omega-\omega_0 \) and \( \phi_0(t)=\phi_0(t)-\phi_0(t) \) are the intermediate angular frequency and the phase noise, respectively. The noise terms \( n_s(t) \) and \( n_c(t) \) represent the background radiation and the thermal noise which are assumed to be statistically independent, stationary Gaussian processes with zero-mean and a variance of \( \sigma_n^2 = \frac{1}{2}N_0 \), \( N_0 \) being the double-sided noise power spectral density consisting of the background radiation noise and the thermal noise. The electrical signals \( c_{s\hat{x}}(t) \) and \( c_{s\hat{y}}(t) \) are passed through the ideal Band-Pass Filters (BPF) to reject the constant term and to limit the additive noise. The bandwidth of the BPF is expressed as \( B_{BPF} = 2(R_\gamma + k_\gamma B_k) \) with the center frequency at \( \omega_0 \), where \( R_\gamma \) is the symbol rate and \( B_k \) is the sum of the transmitting and local oscillator lasers’ line width.
The parameter $k_F$ must be chosen to transmit the signals undistorted through the filters (Cusani et al., 1992). The BPF in the lower branch is assumed to have a very narrow bandwidth in order to only pass the carrier signal with negligible distortion. Therefore, the electrical currents at the output of the BPFs are expressed as Eq. 10a-b:

$$c_{x}(t) = \sqrt{R^2P \rho _{0}}/2 \cos \left[ \omega _{B}t + \gamma + \phi _{p}(t) \right] + n_{x}(t)$$  \hspace{1cm} (10a) \\
$$c_{y}(t) = \sqrt{R^2P \rho _{0}}/2 \cos \left[ \omega _{B}t + \phi _{p}(t) \right]$$  \hspace{1cm} (10b)

where, $n_{x}(t)\sim (0, \sigma _{n}^2)$ is additive Gaussian noise at the output of the BPF. The additive narrowband noise components can be expressed by using the simple trigonometric identities given as Eq. 11 (Proakis, 2001):

$$n_{x}(t) = n_{x}^{1}(t) \cos [ \omega _{B}t + \phi _{p}(t) ]$$
$$- n_{x}^{Q}(t) \sin [ \omega _{B}t + \phi _{p}(t) ]$$  \hspace{1cm} (11)

where, $n^{1}_{x}(t)$ and $n^{Q}_{x}(t)$ are the phase and the quadrature components, respectively, which are band-limited Gaussian with zero mean and a variance of $\sigma _{n}^2$.

After passing through a Low Pass Filter (LPF), the electric signal $c(t)$ is given as Eq. 12:

$$c(t) = c_{x}(t) + h_{p}(t)$$

where, $* \text{ indicates the convolution operation, } h_{p}(t)\text{ is the impulse response of the low-pass filter with a bandwidth of } \Phi _{p} = R + k_{b} B_{p}$. The electric signal $c(t)$ is integrated over the symbol period $T$, sampled at time $t = \tilde{T}$. Hence, the output of the correlation-type demodulator (Proakis, 2001) is Eq. 13:

$$V_i = \frac{1}{T} \int \left[ \sqrt{\frac{R^2P \rho _{0}}{2}} \left[ \sqrt{\frac{R^2P \rho _{0}}{2}} \cos \gamma + n_{x}^{1}(t) \right] \right] dt$$

$$= \frac{1}{2} \sqrt{\frac{R^2P \rho _{0}}{2T}} \int \left[ \sqrt{\frac{R^2P \rho _{0}}{2}} \cos \gamma dt + \frac{1}{2} n_{x}^{1}(t) dt \right]$$

$$= \frac{R^2P \rho _{0}}{8} \sqrt{\frac{R^2P \rho _{0}}{2}} \cos \gamma + n_{x}^{1}(t)$$  \hspace{1cm} (13)

The decision rule which maximizes the correlation metrics is applied to determine the average probability of error. It follows that the detector compares the demodulator output $V_i$ with seven Threshold (TH) levels: $0, \pm R^2P \rho _{0}/2, \pm R^2P \rho _{0}, \pm 3R^2P \rho _{0}/2$. Therefore, a decision is made in favor of the amplitude level closest to $V_i$.

**Error rate analysis:** It has been assumed that the data transmission is independent and identically distributed (i.i.d.). For equally probable signals, the decision rule which maximizes the correlation metrics is applied to determine the average probability of error. Compared with the encoded BPOLSK (Tang et al., 2010) the average power for equiprobable coded M-POLSK symbol is increased by a factor of 11, so that the average power of each symbol in the coded M-POLSK system must be reduced by a factor of 11. Therefore, the average power per bit is defined by $R^2P \rho _{0} = P_{av}/22$.

Since an error can only occur in one direction, the BER conditioned on the received optical power before the decoder is expressed as Eq. 14:

$$P_{e} = 1 - \frac{1}{4\sqrt{2\Pi \sigma _{n}}} \left[ \int_{-\infty }^{-\frac{1}{4\sqrt{2\Pi \sigma _{n}}}} e^{-\frac{1}{2}(\xi - \frac{1}{4\sqrt{2\Pi \sigma _{n}}})^2} d\xi \right] + \frac{1}{4\sqrt{2\Pi \sigma _{n}}} \left[ \int_{\frac{1}{4\sqrt{2\Pi \sigma _{n}}}}^{\infty } e^{-\frac{1}{2}(\xi - \frac{1}{4\sqrt{2\Pi \sigma _{n}}})^2} d\xi \right]$$

$$+ \frac{1}{4\sqrt{2\Pi \sigma _{n}}} \left[ \int_{\frac{1}{4\sqrt{2\Pi \sigma _{n}}}}^{\frac{1}{4\sqrt{2\Pi \sigma _{n}}}} e^{-\frac{1}{2}(\xi - \frac{1}{4\sqrt{2\Pi \sigma _{n}}})^2} d\xi \right]$$

$$+ \frac{1}{4\sqrt{2\Pi \sigma _{n}}} \left[ \int_{-\infty }^{-\frac{1}{4\sqrt{2\Pi \sigma _{n}}}} e^{-\frac{1}{2}(\xi - \frac{1}{4\sqrt{2\Pi \sigma _{n}}})^2} d\xi \right]$$

$$+ \frac{1}{4\sqrt{2\Pi \sigma _{n}}} \left[ \int_{\frac{1}{4\sqrt{2\Pi \sigma _{n}}}}^{\infty } e^{-\frac{1}{2}(\xi - \frac{1}{4\sqrt{2\Pi \sigma _{n}}})^2} d\xi \right]$$

In addition, simplifying (14) and substituting the average power per bit, the BER expression for the coded M-POLSK before the decoder is expressed as Eq. 15:

$$P_{e} = \frac{7}{4\sqrt{2\Pi \sigma _{n}}} \left[ \frac{1}{4\sqrt{2\Pi \sigma _{n}}} \int_{-\frac{1}{4\sqrt{2\Pi \sigma _{n}}}}^{\frac{1}{4\sqrt{2\Pi \sigma _{n}}}} e^{-\frac{1}{2}(\xi - \frac{1}{4\sqrt{2\Pi \sigma _{n}}})^2} d\xi \right]$$

$$+ \frac{7}{4\sqrt{2\Pi \sigma _{n}}} \left[ \frac{1}{4\sqrt{2\Pi \sigma _{n}}} \int_{\frac{1}{4\sqrt{2\Pi \sigma _{n}}}}^{\infty } e^{-\frac{1}{2}(\xi - \frac{1}{4\sqrt{2\Pi \sigma _{n}}})^2} d\xi \right]$$

where, $Q(.)$ is the Gaussian-Q function and $r = P_{av}/N_0$ is the electrical Signal to Noise Ratio (SNR) at the input of the demodulator.
The PEP is the basic method for the union bounds calculation, which is used as the main criterion for code design (Sandalidis, 2011; Simon and Alouini, 2005). Under the assumption of the maximum likelihood soft decoding with perfect Channel State Information (CSI), the conditional PEP subject to the fading coefficients $I=I_1I_2...I_k$ is expressed as Eq. 16 (Zhu and Kahn, 2003; Uysal et al., 2006):

$$P(X|\hat{X}|l) = \frac{1}{4} \pi \int_0^{\pi} \exp \left( -\frac{x^2}{2\sin^2 \theta} \right) d\theta$$

(16)

where, $\tilde{x} = (\tilde{x}_1,...,\tilde{x}_n)$ and $x = (x_1,...,x_n)$ stand for the chosen coded sequence and the transmission sequence, respectively and $\Omega$ is the set of bit intervals’ locations where $X$ and $\tilde{X}$ differ from each other. The alternative function form for the Gaussian-Q function is Eq. 17 (Sandalidis, 2011; Simon and Alouini, 2005):

$$Q(x) = \frac{1}{\pi} \int_0^{\pi} \exp \left( -\frac{x^2}{2\sin^2 \theta} \right) d\theta$$

(17)

Substituting Eq. 16 into Eq. 17 and considering the received signal light subjected to fading, Eq. 16 becomes Eq. 18:

$$P(X|\hat{X}|l) = \frac{1}{4} \pi \int_0^{\pi} \Pi_{n=0}^{n-1} \exp \left( -\frac{r^2 I^2}{4\sin^2 \theta} \right) d\theta$$

(18)

We need to take Eq. 18 as regards $I_k$ to obtain the unconditional PEP. Assuming perfect interleaving, the independency among fading coefficients $I_k$ can be exploited and the unconditional PEP is written as Eq. 19:

$$P(X|\hat{X}|l) = \frac{7}{4} \pi \int_0^{\pi} \Pi_{n=0}^{n-1} \exp \left( -\frac{r^2 I^2}{4\sin^2 \theta} \right) d\theta$$

(19)

$$= \frac{7}{4} \pi \int_0^{\pi} \exp \left( -\frac{r^2 I^2}{4\sin^2 \theta} \right) p(l) d\theta$$

Where:

$E(.) = \text{The expectation operation}$

$|\Omega| = \text{The cardinality of } \Omega \text{ corresponding to the length of error event}$

Note that Eq. 19 has no closed form solution. The unconditional PEP Eq. 19 is derived for uncoded M-POLSK modulated FSO communication systems operating over gamma-gamma turbulence channel. The unconditional PEP expression is the tool for the derivation of upper bounds on the error probability of the coded M-POLSK communication system.

**Maximum likelihood sequence detection:** In order to reduce the effects of the turbulence induced fading on the received optical signal, a convolution coded M-POLSK-FSO communication system has been considered. The union upper bound on the average BER with uniform error probability codes can be found as Eq. 20 (Sandalidis, 2011; Simon and Alouini, 2005):

$$P_s \leq \frac{7}{4} \pi \int_0^{\pi} \left[ \exp \left( -\frac{r^2 I^2}{4\sin^2 \theta} \right) T(D(\theta), N) \right] d\theta$$

(20)

where, $N$ is an indicator taking into account the number of bits in error, $n$ is the number of information bits per transmission, the transfer function $T(D(\theta), N)$ is in conjunction with the particular state diagram of a coded modulation and $D(\theta)$ depends on the derived PEP. Here we have applied a convolutional code with the rate of $1/3$ and the constraint length of $3$, as illustrated in Fig. 8.2-2 of (Proakis, 2001). The function generators of the convolutional encoder are given as and $g_1 = (100)$, $g_2 = (101)$ and $g_3 = (111)$. The transfer function is expressed as Eq. 21:

$$T(D(\theta), N) = \frac{D(\theta)N}{1-B_{2}(D(\theta))}$$

(21)

The BER is thus obtained as Eq. 22:

$$P_e \leq \frac{7}{4} \pi \int_0^{\pi} \frac{D(\theta)}{1-B_{2}(D(\theta))} d\theta$$

(22)

$D(\theta)$ is defined based on the underlying PEP expression. In this study, using the integrand of PEP expression given by Eq. 20, the approximation $D(\theta)$ formula for the channels under consideration is Eq. 23:

$$D(\theta) = \int \exp \left( -\frac{r^2 I^2}{4\sin^2 \theta} \right) p(l) d\theta$$

(23)

**Spatial diversity techniques:** Employing multiple photo-detectors can mitigate the turbulence induced fading in the received signal, thus leading to further improvement in the error performance. To avoid any correlation in the received irradiance the detectors are sufficiently spaced as shown in Fig. 3.

![Fig. 3: Spatial diversity receiver with H-PDs Photo-Detector (PD); Semiconductor Optical Amplifier (SOA)](image-url)
Since the transverse correlation size \( \rho_0 \) of the laser radiation operating over the atmospheric turbulence channel is nearly a few centimeters, the parameter \( \rho_0 \) can be assumed to be greater than the spacing between the detectors (Popoola and Ghassemlooy, 2009). Since the spacing between the PDs is much shorter than the wireless link range, the difference in the propagation delay across the receiver array would be negligible (Popoola and Ghassemlooy, 2009). Note that the received optical power is assumed to be constant and time invariant during \( T < \tau_0 \), where the coherence time \( \tau_0 \) of the atmospheric fluctuation is in the order of milliseconds (Shin and Chan, 2004).

The received signal from each branch is scaled by a gain factor \( \{G_i\}_{i=1}^H \). The output of the combiner is the sum of the weighted and co-phased signals as illustrated in Fig. 3. Each receiver aperture size of the H-photo detector is \( 1/H \) th of the aperture area of the single receiver. Accordingly, the background noise variance on each branch is proportional to the receiver aperture area, which is reduced by a factor of H. Whereas the thermal noise on each branch is not affected. It follows that the total noise variance at the combiner output is \( \sigma^2_{\text{th}} = H\sigma^2_{\text{thermal}} + \sigma^2_{\text{bg}} \). The background noise is not considered here because the ideal OBPF with a narrow FOV is applied at the receiver. Hence, \( \sigma^2_{\text{bg}} < H\sigma^2_{\text{thermal}} \) and \( \sigma^2_{\text{th}} = \sigma^2_{\text{thermal}} \) the variance of the overall noise with a zero mean becomes as Eq. 24:

\[
N_T = \sum_{i=1}^H G_i^2 No, \quad i = 1,2,...,H
\]  
(24)

The total received optical power during the symbol duration at the output of the combiner is given as Eq. 25:

\[
E_T(t) = \frac{1}{H} \sum_{i=1}^H \frac{G_i^2 P_i}{2} e^{-\left(\gamma T_i + \phi_i\right)} \{\gamma T_i + \phi_i\}.
\]  
(25)

The optimum post detection electrical SNR \( r_T \) at the M-POLSK demodulator input becomes as Eq. 26:

\[
r_T = \frac{\sum_{i=1}^H G_i^2 P_i}{\sum_{i=1}^H G_i^2 No}
\]  
(26)

where, \( P = (\sum_{i=1}^H I_i) \) and \( p(\Gamma) = \prod_{i=1}^H p(I_i) \) represents the joint PDF of the uncorrelated irradiance. Hypotheses are taken that all the received optical power is independent and all obey gamma-gamma distribution. Therefore, \( p(\Gamma) \), being the joint PDF for H-photodetector receiving uncorrelated signals, is expressed as Eq. 27:

\[
p(\Gamma) = \int \cdots \int p(I_1) \cdots p(I_H) dI_1 \cdots dI_H = \prod_{i=1}^H p(I_i)
\]  
(27)

The system performance of the coded M-POLSK-based FSO communication system operating over gamma-gamma turbulence channel can thus be evaluated from Eq. 2, 22 and 27 using numerical integration since the resulting expression has no closed form. In determining \( r_T \), only MRC spatial diversity techniques are considered in this study. As the MRC linear combiner results in a maximum-likelihood receiver structure (Simon and Alouini, 2005), it is optimal regardless of the fading statistics.

The received power level on every branch has to be estimated prior for the coherent summation. The gain factor \( \{G_i\}_{i=1}^H \) is proportional to the received optical power. Applying the Cauchy inequality (Gradshtein et al., 2007), \( \sum_{i=1}^H G_i^2 P_i \leq \left( \sum_{i=1}^H G_i^2 \right) \left( \sum_{i=1}^H P_i / No \right) \), the

\[
r_{\text{MRC}} \leq \frac{\sum_{i=1}^H G_i^2 No (\sum_{i=1}^H P_i / No)}{H^2 \sum_{i=1}^H G_i^2 No} = \frac{1}{H^2} \left( \sum_{i=1}^H P_i / No \right)
\]  
(28)

Therefore, the unconditional BER is derived as Eq. 29:

\[
P_{\text{unc}} = \frac{2}{\pi d} \sqrt{\frac{2}{\pi} \int_{0}^{\infty} \left\{ \frac{1}{\sqrt{2\pi\sigma_{\text{th}}^2}} e^{-\frac{(r-\mu)^2}{2\sigma_{\text{th}}^2}} \right\}^2} \int_{r}^{\infty} p(\Gamma) d\Gamma
\]  
(29)

RESULTS

To evaluate the system performances of the proposed coded M-POLSK based FSO communication link, the error probability of the system employing the coherent BPOLSK, BPSK and DPSK schemes are illustrated. For the purpose of like-to-like comparison, the average optical signal power \( E[RP_{\text{no}}] \) is normalized to unity and the electrical SNRs are made equal for different modulation schemes. Furthermore, the channel turbulence under consideration is from weak to strong regimes. The values of \( \alpha \) and \( \beta \) at any given regimes can be calculated with the corresponding value of \( \sigma_1^2 \) using Eq. 3. The values of all the parameters used in calculations are illustrated in Table 1.
Table 1: Simulation parameters with respect to weak, moderate and strong turbulence regimes

| Parameter                  | Value                |
|----------------------------|----------------------|
| Wavelength \( \lambda \)   | 1550 nm              |
| Refractive index \( \varepsilon_n^2 \) | 1.7 x 10^{-14} |
| Diameter of the receiver \( D \) | < \( L \) and \( d = 0 \) |
| Link length \( L \)         | 3 km, 4 km, 6 km     |
| \( \sigma_l^2 \)           | 1.03, 1.75, 3.67     |
| \( \alpha \)               | 2.902, 2.296, 2.064  |
| \( \beta \)                | 2.51, 1.822, 1.342   |

The BER expressions for BPOLSK with and without turbulence are given as Eq. 30a-b (Tang et al., 2010):

\[
P_{\text{BPOLSK}} = Q\left(\sqrt{\sum_{k=0}^{l} I_k^2}\right) \tag{30a}
\]

\[
P_{\text{BPOLSK}} = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{e^{-\frac{t^2}{2\sigma^2}}}{\sigma} dt \tag{30b}
\]

Following the approach adopted in (Zhu and Kahn, 2002), the error probabilities of the DPSK modulated coherent system in the absence and presence of the turbulence channel is given as Eq. 31a-b:

\[
P_{\text{DPSK}} = \text{erfc}\left(\sqrt{\frac{2\sigma^2}{\lambda^2}}\right) \tag{31a}
\]

\[
P_{\text{DPSK}} = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{e^{-\frac{t^2}{2\sigma^2}}}{\sigma} \left(1 - \frac{1}{2}\text{erfc}\left(\sqrt{\frac{2\sigma^2}{\lambda^2}}\right)\right) dt \tag{31b}
\]

The conditional BER expressions for the coherent BPSK modulation technique without and with the phase tracking errors are given as Eq. 32a-b (Betti et al., 1995):

\[
P_{\text{BPSK}} = Q\left(\sqrt{2\sum_{k=0}^{l} I_k^2}\right) \tag{32a}
\]

\[
P_{\text{BPSK}} = \frac{1}{\sqrt{2\pi}} \int_0^\infty \frac{e^{-\frac{t^2}{2\sigma^2}}}{\sigma} \left(1 - \frac{1}{2}\text{erfc}\left(\sqrt{\frac{2\sigma^2}{\lambda^2}}\right)\right) dt \tag{32b}
\]

where, \( \Delta \phi \) denotes the phase tracking error due to the PLL circuit and \( \sigma_{\Delta \phi} = 1/\sigma^2 \). Therefore, the theoretical unconditional BER for the BPSK scheme in the gamma-gamma turbulence channel is derived as Eq. 32c:

\[
P_{\text{BPSK}}(P_e) = \int_0^\infty Q\left(\sqrt{2\sum_{k=0}^{l} I_k^2} \cos(\Delta \phi)\right) e^{-\frac{\lambda^2}{2\sigma^2}} dt \tag{32c}
\]

**DISCUSSION**

The error performances of coherent BPOLSK, DPSK and BPSK schemes can be predicted for any given value of SNR across all turbulence regimes using Eq. 30, 31 and 32 as shown in Fig. 4.
However, as revealed out by the results under all turbulence assumptions, increasing SNR results in a relatively smaller change in the slope of the BER curves for uncoded M-POLSK. The reduction in the SNR for coded M-POLSK is achieved by the use of convolutional code in mitigating the effect of turbulence induced irradiance fluctuation. For example, with the SNR of 30 dB, the BERs for uncoded and coded M-POLSK are 0.047 and $1.4 \times 10^{-4}$ respectively in weak turbulence regime. With the same SNR in the moderate regime, the BER drops from 0.08 to $5.13 \times 10^{-4}$ for the uncoded and coded M-POLSK, respectively.

The BER based on the derived PEP yields a very good performance, which is achieved without the need for increasing the SNR. For example, with a SNR of 30 dB, the BERs for uncoded and coded M-POLSK schemes are 0.047 and $1.4 \times 10^{-4}$ respectively, in the weak turbulence regime. Around 69 dB power gain is achievable when ten detectors are used in strong turbulence regime. The spatial diversity with MRC technique ($H = 4$) outperforms the uncoded M-POLSK employing the single receiver by $\sim 31.4$, $\sim 29.5$ and $\sim 57.9$ dB respectively in weak, moderate and strong turbulence regimes. We also have shown that the spatial diversity offers increased link margin as the scintillation level rises. The performances of BPOLSK, BPSK and DPSK based FSO systems have been evaluated across all turbulence regimes. The BPSK without phase tracking error outperforms BPOLSK in terms of the SNR to achieve the same BER performance for a range of turbulence regimes. However, the BPSK scheme suffers the penalties due to the phase tracking errors. The performance degradation increases with the phase error variances.

**CONCLUSION**

This study has outlined the theoretical analysis of a coherent M-POLSK modulated FSO communication system operating over the gamma-gamma turbulence channel. To mitigate the turbulence induced fading the convolution coding and the spatial diversity with the MRC technique have been applied. The upper BER bound based on the derived PEP has been obtained using the transfer function. The BER yields a very good performance, which is achieved without increasing the SNR. For example, with a SNR of 30 dB, the BERs for uncoded and coded M-POLSK schemes are 0.047 and $1.4 \times 10^{-4}$ respectively, in the weak turbulence regime. Around 69 dB power gain is achievable when ten detectors are used in strong turbulence regime. The spatial diversity with MRC technique ($H = 4$) outperforms the uncoded M-POLSK employing the single receiver by $\sim 31.4$, $\sim 29.5$ and $\sim 57.9$ dB respectively in weak, moderate and strong turbulence regimes. We also have shown that the spatial diversity offers increased link margin as the scintillation level rises. The performances of BPOLSK, BPSK and DPSK based FSO systems have been evaluated across all turbulence regimes. The BPSK without phase tracking error outperforms BPOLSK in terms of the SNR to achieve the same BER performance for a range of turbulence regimes. However, the BPSK scheme suffers the penalties due to the phase tracking errors. The performance degradation increases with the phase error variances.

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**REFERENCES**

Abtahi, M., P. Lemieux, W. Mathlouthi and L.A. Rusch, 2006. Suppression of turbulence-induced scintillation in free-space optical communication systems using saturated optical amplifiers. J. Lightwave Technol., 24: 4966-4973. DOI: 10.1109/JLT.2006.884561
Al-Habash, M.A., L.C. Andrews and R.L. Phillips, 2001. Mathematical model for the irradiance probability density function of a laser beam propagating through turbulent media. Opt. Eng., 40: 1554-1562.

Andrews, L.C., R.L. Philips and C.Y. Hopen, 2001. Laser Beam Scintillation with Applications. 1st Edn., SPIE Press, Bellingham, ISBN: 10: 0819441031, pp: 375.

Arnon, S., 2003. Effects of atmospheric turbulence and building sway on optical wireless-communication systems. Optics Lett., 28: 129-131. DOI: 10.1364/OL.28.000129

Betti, S., G.D. Marchis and E. Iannone, 1995. Coherent Optical Communications Systems. 1st Edn. John Wiley, New York, ISBN: 10: 0471576374, pp: 171.

Kamalakis, T., T. Sphicopoulos, S.S. Muhammad and E. Leitgeb, 2006. Estimation of the power scintillation probability density function in free-space optical links by use of multicanonical Monte Carlo sampling. Optics Lett., 31: 3077-3079. DOI: 10.1364/OL.31.003077

Goodman, J.W., 1985. Statistical optics. 1st Edn., Wiley-Interscience, New York, ISBN: 10: 0471015024, pp: 550.

Karp, S., 1988. Optical Channels: Fibers, Clouds, Water and the Atmosphere. 1st Edn., Plenum Press, New York, ISBN: 10: 0306426544, pp: 405.

Khalighi, M.A., N. Schwartz, N. Aitamer and S. Bourennane, 2009. Fading reduction by aperture averaging and spatial diversity in optical wireless systems. J. Opt. Commun. Netw., 1: 580-593. DOI: 10.1364/JOCN.1.000580

Osche, G.R., 2002. Optical Detection Theory For Laser Applications. 1st Edn., Wiley-Interscience, Hoboken, New Jersey, ISBN: 0471224111, pp: 412.

Popoola, W.O. and Z. Ghassemlooy, 2009. BPSK subcarrier intensity modulated free-space optical communications in atmospheric turbulence. J. Lightwave Technol., 27: 967-973. DOI: 10.1109/JLT.2008.2004950

Popoola, W.O., Z. Ghassemlooy, J.I.H. Allen, E. Leitgeb and S. Gao, 2008. Free-space optical communication employing subcarrier modulation and spatial diversity in atmospheric turbulence channel. IET Optoelect., 2: 16-23. DOI: 10.1049/i et-opt:20070030

Proakis, J.G., 2001. Digital Communications. 4th Edn., McGraw-Hill, Boston, ISBN: 10: 0072321113, pp: 1002.

Ramirez-Iniguez, R., S.M. Idrus and Z. Sun, 2008. Optical Wireless Communications: IR for Wireless Connectivity. 1st Edn., CRC Press, London, ISBN: 10: 0849372097, pp: 344.

Sandalidis, H.G., 2011. Coded free-space optical links over strong turbulence and misalignment fading channels. IEEE Trans. Commun., 59: 669-674. DOI: 10.1109/TCOMM.2011.121410.090318

Shin, E.J. and V.W.S. Chan, 2004. Part 1: Optical communication over the clear turbulent atmospheric channel using diversity. IEEE J. Select. Areas Commun., 22: 1896-1906. DOI: 10.1109/JSAC.2004.835751

Simon, M.K. and M.S. Alouini, 2005. Digital Communication over Fading Channels. 2nd Edn., John Wiley and Sons, Hoboken, New Jersey, ISBN: 9780471649533, pp: 900.

Sugianto, T. and C.C. Davis, 2006. Performance of free space optical communication systems using polarization shift keying modulation. Proc. SPIE Int. Soc. Opt. Eng., 6304: 63040V-63040V.

Tang, X., Z. Ghassemlooy, S. Rajbhandari, W.O. Popoola and C.G. Lee, 2011. Coherent optical binary polarisation shift keying heterodyne system in the free-space optical turbulence channel. IET Microw. Antennas Propag., 5: 1031-1038. DOI: 10.1049/iet-map.2010.0221

Tang, X., Z. Ghassemlooy, S. Rajbhandari, W.O. Popoola and C.G. Lee et al., 2010. Free-space optical communication employing polarization shift keying coherent modulation in atmospheric turbulence channel. Proceedings of the 7th International Symposium on Communication Systems, Networks and Digital Signal Processing, Jul. 21-23, IEEE Xplore Press, Newcastle Upon Tyne, UK, pp: 615-620.

Uysal, M., J. Li and M. Yu, 2006. Error rate performance analysis of coded free-space optical links over gamma-gamma atmospheric turbulence channels. IEEE Trans. Wireless Commun., 5: 1229-1233. DOI: 10.1109/TWC.2006.1638639

Weyrauch, T. and M.A. Vorontsov, 2004. Free-space laser communications with adaptive optics: Atmospheric compensation experiments. J. Opt. Fiber Commun. Res., 1: 355-379. DOI: 10.1007/s10297-005-0033-5
Zhu, X. and J.M. Kahn, 2002. Free-space optical communication through atmospheric turbulence channels. IEEE Trans. Commun., 50: 1293-1300. DOI: 10.1109/TCOMM.2002.800829

Zhu, X. and J.M. Kahn, 2003. Performance bounds for coded free-space optical communications through atmospheric turbulence channels. IEEE Trans. Commun., 51: 1233-1239. DOI: 10.1109/TCOMM.2003.815052

Zhu, X. and J.M. Kahn, 2001. Pairwise codeword error probability for coded free-space optical communication through atmospheric turbulence channels. Proceedings of the IEEE International Conference on Communications, Jun. 11-14, IEEE Xplore Press, Helsinki, Finland, pp: 161-164. DOI: 10.1109/ICC.2001.936294