Abstract. Among the multiple 5D thick braneworld models that have been proposed in the last years, in order to address several open problems in modern physics, there is a specific one involving a tachyonic bulk scalar field. Delving into this framework, a thick braneworld with a cosmological background induced on the brane is here investigated. The respective field equations — derived from the model with a warped 5D geometry — are highly non-linear equations, admitting a non-trivial solution for the warp factor and the tachyon scalar field as well, in a de Sitter 4D cosmological background. Moreover, the non-linear tachyonic scalar field, that generates the brane in complicity with warped gravity, has the form of a kink-like configuration. Notwithstanding, the non-linear field equations restricting character does not allow one to easily find thick brane solutions with a decaying warp factor which leads to the localization of 4D gravity and other matter fields. We derive such a thick brane configuration altogether in this tachyon-gravity setup. When analyzing the spectrum of gravity fluctuations in the transverse traceless sector, the 4D gravity is shown to be localized due to the presence of a single zero mode bound state, separated by a continuum of massive Kaluza-Klein (KK) modes by a mass gap. It contrasts with previous results, where there is a KK massive bound excitation providing no clear physical interpretation. The mass gap is determined by the scale of the metric parameter $H$. Finally, the corrections to Newton’s law in this model are computed and shown to decay exponentially. It is in full compliance to corrections reported in previous results (up to a constant factor) within similar braneworlds with induced 4D de Sitter metric, despite the fact that the warp factor and the massive modes have a different form.
1 Introduction

Within the framework of the braneworld models embedded in a spacetime with extra dimensions and after the success of the thin brane models — where singularities are present at the position of the branes — in solving the mass hierarchy and 4D gravity localization problems [1, 2], to find smooth braneworld solutions has cogently become a matter of interest (for an interesting review see, e. g., [3] and references therein). In some models, such solutions are obtained by introducing one or several scalar fields in the bulk. The large variety of scalar fields that can be used to generate these models elicit different scenarios [4–13]. By following this direction, and by using the freedom to choose a scalar field, several authors have been evoking a tachyonic scalar field in the bulk [12–14]. They are furthermore concerned to address issues like the mass hierarchy problem, and localization of gravity and matter fields, in both the thin and the thick branes models as well. In the original Randall-Sundrum (RS) model, a Standard Model or TeV brane is introduced at a certain fixed distance, say \( r_c \), from the gravitational or Planck brane, in order to achieve the desired warping, and hence, solve the hierarchy problem in a completely 5D geometrical way. However, this resolution mechanism impels to a new fine-tuning on the probe brane position, and therefore, to the need of stabilizing this brane separation. The stabilization of this brane separation is achieved through the Goldberger-Wise mechanism, by associating to it a radion scalar field that models the radius of the fifth dimension, when one ignores the brane back-reaction [15]. This mechanism was further generalized to the case when one takes into account the brane back-reaction in [4]. Moreover, in [16] the authors proposed a tachyonic scalar field action for modeling and stabilizing the brane separation for the full back-reacted system. They also obtained the desired warping from the Planck scale to the TeV one, resolving the fine tuning problem of the Higgs mass in a stable braneworld scenario and generalizing in a relevant way the RS model as well (see also [14]). This fact physically motivates the use of a tachyonic scalar field within the braneworld paradigm, since the back-reaction of the radion field must be taken into account in a self–consistent system.

On the another hand, the thick brane configuration constructed in [17] possesses an increasing warp factor, since most of the attempts to solve the highly non–linear field equations leads to imaginary tachyon field configurations. This fact translates into delocalization of gravity and other kinds of matter, like scalar and vector fields, while giving rise to localization of fermions. Alternatively, the localization of fermionic fields on thick branes was investigated in [18] with an auxiliary scalar field that couples to the fermionic mass term.

In this paper we propose a new thick brane tachyonic solution, with a decaying warp factor that enables localization of 4D gravity as well as other matter fields. By analyzing
the dynamics of the metric perturbations, we realize that their spectrum contains a single bound state corresponding to the 4D massless graviton of the model. Furthermore, it presents a continuum of KK excitations separated by a mass gap. We compute in addition the corresponding correction to Newton’s law, by analyzing the influence of these massive modes on the gravitational potential acting between two point massive particles, located along the center of the thick brane.

2 The thick brane model and its solution

The complete action for the tachyonic braneworld model is expressed as

\[ S = \int d^5x \sqrt{-g} \left( \frac{1}{2\kappa_5^2} R - \Lambda_5 \right) - \int d^5x \sqrt{-g} V(T) \sqrt{1 + g^{AB}\partial_A T \partial_B T} \]  

(2.1)

where the first term describes 5D gravity with a bulk cosmological constant \( \Lambda_5 \), the second is the action of the matter in the bulk, \( \kappa_5 \) is the 5D gravitational coupling constant, and \( A, B = 0, 1, 2, 3, 5 \). Hence, the tachyon field \( T \) represents the matter in the 5D bulk and \( V(T) \) denotes its self–interaction potential \([19, 20]\).

The tachyon action part in Eq. (2.1) above was proposed in \([19]\) within the context of a tachyon field living on the world volume of a non–BPS brane and found applications in braneworld cosmology \([21]\) and in string cosmology \([22]\). It was argued in \([23]\) that this form of the action can be used for any relativistic scalar field. Moreover, using this action as a scalar tensor theory, solar system constraints were analyzed in \([24]\).

The Einstein equations with a cosmological constant in five dimensions are given by

\[ G_{AB} = -\kappa_5^2 \Lambda_5 g_{AB} + \kappa_5^2 T_{AB}^{\text{bulk}}. \]  

(2.2)

For the background metric, the ansatz of a warped 5D line element, with an induced 3–brane in a spatially flat cosmological background, is used

\[ ds^2 = e^{2f(\sigma)} \left[ -dt^2 + a^2(t) \left( dx^2 + dy^2 + dz^2 \right) \right] + d\sigma^2, \]  

(2.3)

where \( f(\sigma) \) is the warp factor and \( a(t) \) is the scale factor of the brane.

The matter field equation is obtained by variation of the action with respect to the tachyon. It is expressed in the following form:

\[ \Box T - \nabla_C \nabla_D T \nabla^C T \nabla^D T \frac{1}{1 + (\nabla T)^2} = \frac{1}{V} \frac{\partial V(T)}{\partial T}. \]  

(2.4)

By using the ansatz (2.3), the Einstein tensor components read

\[ G_{00} = 3 \frac{\dot{a}^2}{a^2} - 3 e^{2f} \left( f'' + 2f' \right), \]

\[ G_{\alpha\alpha} = -2\ddot{a}a - \dot{a}^2 + 3a^2 e^{2f} \left( f'' + 2f' \right), \]

\[ G_{\sigma\sigma} = -3 e^{-2f} \left( \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) + 6f'', \]  

(2.5)

where “\( f' \)” and “\( \ddot{a} \)” are the derivative with respect to the extra dimension and time, respectively, while \( \alpha \) labels the spatial dimensions \( x, y \) and \( z \). The stress energy tensor components
do not depend explicitly on the time component, and read
\[
T_{AB}^{\text{bulk}} = \left[ -g_{AB} V(T) \sqrt{1 + (\nabla T)^2} + \frac{V(T)}{\sqrt{1 + (\nabla T)^2}} \partial_A T \partial_B T \right]. \tag{2.6}
\]
Since the non–diagonal components of the Einstein tensor vanish, consistency of Einstein equations demands that non–diagonal components of the stress energy tensors should vanish identically. This allows two possibilities: (a) the field \(T\) should depend merely on time and not on any of the spatial coordinates — which is the case for a scalar field in an homogeneous and isotropic background as in cosmology; (b) the field \(T\) depends only on the coordinate corresponding to the extra dimension. This simply amounts to a consistent time independence of the tachyon field, even if the background is time dependent, and shall be considered here, since we are not regarding cosmology. Hence Eq.(2.4) reads
\[
T'' + 4f' T' (1 + T'^2) = (1 + T'^2) \frac{\partial_r V(T)}{V(T)}, \tag{2.7}
\]
while the Einstein equations (2.2) can be rewritten in a straightforward way:
\[
f'' = -\kappa^2_5 V(T) \frac{T'^2}{3 \sqrt{1 + T'^2}} - e^{-2f} \ddot{a} \frac{\dot{a}}{a}, \tag{2.8}
\]
\[
f' = -\kappa^2_5 \frac{V(T)}{6 \sqrt{1 + T'^2}} - \frac{\kappa^2_5 \Lambda_5}{6} + e^{-2f} \left( \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right). \tag{2.9}
\]
Thereupon the case of a single brane configuration, in a 4D spatially flat cosmological background given by equations (2.7), (2.8), and (2.9), is evinced. The consistency between the component \(\alpha\alpha\) of the Einstein equation with Eqs. (2.8) and (2.9) demands that the scale factor has to be \(a(t) = k e^{H t}\), where \(H\) and \(k\) are integration constants. Therefore, it corresponds to a de Sitter 4D cosmological background defined by
\[
a(t) = e^{H t}, \tag{2.10}
\]
since the constant \(k\) can be absorbed into a coordinate redefinition. This result is dictated simply by the symmetry of the background. Thus, the action for the tachyonic scalar field is a non–trivial 5D configuration that leads to a braneworld in which the induced metric on the brane is described by \(dS_4\) geometry.

Here it is convenient to go to conformal coordinates through \(dw = e^{-f(y)} d\sigma\), leading to the following metric
\[
ds^2 = e^{2f(w)} \left[ -dt^2 + a^2(t) (dx^2 + dy^2 + dz^2) + dw^2 \right]. \tag{2.11}
\]
In the language of these coordinates the field equations (2.7), (2.8) and (2.9) can be written as follows
\[
T'' - f' T' (1 + e^{-2f} T'^2) = (e^{2f} + T'^2) \frac{\partial_r V(T)}{V(T)}, \tag{2.12}
\]
\[
f'' - f'^2 + H^2 = -\kappa^2_5 \frac{V(T) T'^2}{3 \sqrt{1 + e^{-2f} T'^2}}, \tag{2.13}
\]
\[
f'^2 + \frac{\kappa^2_5 \Lambda_5}{6} e^{2f} - H^2 = -\kappa^2_5 \frac{e^{2f} V(T)}{6 \sqrt{1 + e^{-2f} T'^2}}. \tag{2.14}
\]
where now the symbol \( {}' {}' \) stands for derivatives with respect to \( w \), and the form of the scale factor corresponding to (2.10) was taken into account.

According to the authors of [16], it is straightforward to obtain separate equations for the scalar field \( T \) and for the potential \( V(T) \) from Eqs.(2.13) and (2.14) by the following procedure: divide (2.13) into (2.14), isolate \( T' {}^2 \) and compute the square root of both sides of the equation; in order to get an explicit expression for \( T(w) \), one must be able to integrate the resulting expression. The self–interaction potential \( V(T(w)) \) can be obtained in a parametric form by isolating this quantity from (2.14) after replacing the corresponding expression for \( T' {}^2 \) in the radicand. If one is able to invert \( T(w) \), it is possible hence to express the potential \( V \) as a function of \( T \).

Thus, it turns out that despite the high non–linearity of these field equations, the derivative of the tachyonic scalar field \( T' {}^{} \), as well as the arbitrary potential \( V(T) \), can be expressed in terms of the warp and scale factors of the metric (and their respective derivatives), after some simple manipulations, leading to:

\[
T' = \pm e^f \sqrt{\frac{f'' - f' {}^2 + H^2}{2 (f' {}^2 + \frac{\kappa_5^2 \Lambda_5}{b} e^{2f} - H^2)}}, \tag{2.15}
\]

\[
V(T) = -\frac{3}{\kappa_5^2} e^{-2f} \sqrt{\frac{2 \left( f'' + f' {}^2 + \frac{\kappa_5^2 \Lambda_5}{3} e^{2f} - H^2 \right)}{\left( f'' + \frac{\kappa_5^2 \Lambda_5}{6} e^{2f} - H^2 \right)}} \left( f' {}^2 + \frac{\kappa_5^2 \Lambda_5}{6} e^{2f} - H^2 \right). \tag{2.16}
\]

Therefore, by determining a desired behavior for the geometry, the dynamics of the tachyon field is completely fixed, and vice–versa. However, the resulting solution must be real and have physical sense. This restriction is nonetheless cogently demanding, since several warp factors with “convenient” behavior lead to a complex tachyonic field \( T \) and/or self–interaction potential \( V(T) \).

It is straightforward to realize that the following warp factor

\[
f(w) = -\frac{1}{2} \ln \left[ \frac{\cosh \left[ H (2w + c) \right]}{s} \right], \tag{2.17}
\]

where \( H, c \) and \( s > 0 \) are constants, is a solution of the Einstein and field equations, if the tachyon scalar field adopts the form

\[
T(w) = \pm \sqrt{-\frac{3}{2 \kappa_5^2 \Lambda_5}} \arctanh \left[ \frac{\sinh \left[ H (2w + c) \right]}{\sqrt{\cosh \left[ H (2w + c) \right]}} \right], \tag{2.18}
\]

and the tachyon potential given by the following expression

\[
V(T) = -\Lambda_5 \sech \left( \sqrt{-\frac{2}{3} \kappa_5^2 \Lambda_5} T \right) \sqrt{6 \sech^2 \left( \sqrt{-\frac{2}{3} \kappa_5^2 \Lambda_5} T \right) - 1}
\]

\[= -\frac{\Lambda_5}{\sqrt{2}} \sqrt{1 + \sech \left[ H (2w + c) \right]} \sqrt{2 + 3 \sech \left[ H (2w + c) \right]}. \tag{2.19}
\]

In the last two equations we set

\[
s = -\frac{6H^2}{\kappa_5^2 \Lambda_5}. \tag{2.20}
\]
with a negative bulk cosmological constant $\Lambda_5 < 0$ for consistency.

Therefore, the set of equations (2.10), (2.17), (2.18), and (2.19) are the necessary and sufficient ingredients that provide a tachyonic thick brane solution. The warp factor has a decaying and vanishing asymptotic behavior, whereas the tachyon scalar is real and possesses a kink or antikink-like profile as shown in Fig. 1. In contrast with solutions found in [17], here we manage to express the tachyon potential $V(T)$ in terms of the tachyonic scalar field $T$. This potential has a maximum/minimum at the position of the brane, but it is positive/negative definite as it can be seen from (2.19). In fact, since the tachyon field is bounded, the potential remains real as well as bounded (see Fig. 2).

It is worth noticing that due to the relation (2.20), we can compute the limit of this field configuration when the bulk cosmological constant vanishes. In order to do this, we consider the limit when $\Lambda_5 \to 0$ with the same rapidity as $H^2 \to 0$ does in such a way that $s$ remains finite. In this limit, the tachyon field becomes linear $T = \pm \sqrt{s}(2w + c)$, the self-interaction potential vanishes $V(T) = 0$, while the scale and warp factors become constant that can be absorbed into the metric coordinates, leading to a flat 5D spacetime. This situation is similar to that which arises in the RS model [2], among others [4–6, 25], when taking such a limit. For instance, in the RS solution the warp factor becomes equal to unity, while the brane tensions vanish, leading to a flat spacetime as well.
By computing the 5D curvature scalar for our solution (2.17)

$$R = - \frac{14}{3} \kappa_5^2 \Lambda_5 \sech[H(2w+c)]$$

we see that this 5D invariant is positive definite and asymptotically vanishes, yielding an asymptotically 5D Minkowski spacetime [26]. In the absence of matter, usually the presence of a negative cosmological constant $\Lambda_5$ leads to an asymptotically AdS$_5$ spacetime. However, if we look at the equation (2.1) and/or (2.2), we see that both the cosmological constant $\Lambda_5$ and the self–interaction potential $V(T)$ contribute to the overall effective cosmological constant of the 5D spacetime of our scalar tensor setup.

3 Gravity localization and corrections to Newton’s law

In order to study the metric and field fluctuations of our system, both must be perturbed. Since the relevant geometry of the four–dimensional background is of de Sitter type, the fluctuations of the metric may be classified into tensorial, vector, and scalar modes, with respect to the transformations associated to the symmetry group $dS_4$ [27]. It is of prominent importance, since at first (linear) order these modes evolve independently. Hence, their dynamical equations decouple, even when the perturbed Einstein and tachyon field equations had been highly coupled non–linear equations. According to it, the dynamics of the tensorial metric fluctuations is now studied, which have the physical interpretation of the 5D braneworld graviton. These tensorial metric fluctuations are gauge invariant and enable us forthwith to determine whether the localization of 4D gravity, and hence the physics of our 4D world, is feasible or not within this model.

To begin with, the Einstein equations are written in the form:

$$R_{AB} = \frac{2}{3} \kappa_5^2 \Lambda_5 g_{AB} + \kappa_5^2 \hat{T}_{AB},$$

(3.1)

where the reduced energy–momentum tensor

$$\hat{T}_{AB} = \frac{2}{3} V \sqrt{1 + (\nabla T)^2} g_{AB} - \frac{V}{\sqrt{1 + (\nabla T)^2}} \left[ \frac{1}{3} g_{AB} (\nabla T)^2 - \nabla_A \nabla_B T \right]$$

(3.2)

is employed. In the language of the conformal metric coordinates (2.11) the 00–component of (3.1) reads

$$f'' + 3f'^2 = 3H^2 - \frac{2}{3} \kappa_5^2 \Lambda_5 \epsilon^{2f} - \frac{\kappa_5^2 \epsilon^{2f}}{3} \frac{2 + (\nabla T)^2}{\sqrt{1 + (\nabla T)^2}} V(T),$$

(3.3)

since

$$R_{00} = -3H^2 + f'' + 3f'^2$$

(3.4)

and

$$\hat{T}_{00} = -\frac{V}{3} \epsilon^{2f} \frac{2 + (\nabla T)^2}{\sqrt{1 + (\nabla T)^2}}.$$

(3.5)

Now let the metric be perturbed as follows

$$ds_p^2 = (g_{AB} + h_{AB}) dx^A dx^B = (g_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu + \epsilon^{2f(w)} dw^2,$$

(3.6)

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where \( g_{AB} = (g_{\mu\nu}, e^{2f(w)}) = e^{2f(w)g_{AB}} = e^{2f(w)}(\bar{g}_{\mu\nu}, 1) \), \( \bar{g}_{\mu\nu} = \text{diag}([-1, \delta_{ij}a^2]) \), \( \mu, \nu = 0, \ldots, 3 \) and \( i, j = 1, 2, 3 \). Since we are studying the tensorial sector of the metric fluctuations, we can ignore the \( h_{M5} \) perturbations and set them to zero. From them, the \( h_{m5} \) components belong to the vector sector, while the \( h_{55} \) mode has scalar nature and couples to the fluctuations of the scalar tachyon field \( T \).

In this approach we shall impose the transverse traceless condition on these metric fluctuations: \( \nabla^\mu h_{\mu\nu} = h''_{\mu\nu} = 0 \), where \( \nabla \) is the covariant derivative operator with respect to the metric \( g_{AB} \).

After some algebraic work, the linearized Einstein equations for the transverse traceless perturbations and set them to zero. From them, the metric fluctuations one has (see [28])

\[
\delta R_{AB} = \frac{2}{3} \kappa^2 \Lambda_5 h_{AB} + \frac{\kappa^2}{3} \nabla^2 \frac{2 + (\nabla T)^2}{1 + (\nabla T)^2} h_{AB}. \tag{3.7}
\]

At this stage, it is extremely useful to define a new fluctuation variable \( h_{\mu\nu} = e^{2f} \bar{h}_{\mu\nu} \). By making use of the conformal transformation for the Ricci tensor, in terms of the barred fluctuations we have

\[
\delta R_{\mu\nu} = \delta \bar{R}_{\mu\nu} - \frac{3}{2} f' \bar{h}'_{\mu\nu} - \bar{h}_{\mu\nu}(f'' + 3f'^2) \tag{3.8}
\]

where all the barred quantities are computed with respect to the metric \( \bar{g}_{AB} \). On the other hand, this quantity can also be computed as follows (see, for instance, [28])

\[
\delta R_{\mu\nu} = \frac{1}{2} \left( -\square h_{\mu\nu} + \nabla_A \nabla_\mu \bar{h}_A^\nu + \nabla_A \nabla_\nu \bar{h}_A^\mu - \nabla_\mu \nabla_\nu \bar{h}_A^A \right), \tag{3.9}
\]

where \( \square = \bar{g}^{\alpha\beta} \nabla_\alpha \nabla_\beta \bar{h}_{\mu\nu} + \bar{g}^{5\alpha} \nabla_5 \nabla_\alpha \bar{h}_{\mu\nu} \). In order to compute the involved quantities in this expression, recall that the non–null Christoffel symbols are

\[ \Gamma^i_{ij} = a^2 H \delta_{ij}, \quad \Gamma^j_{i0} = H \delta^j_i. \]

Since \( \Gamma^5_{AB} = \Gamma^5_{5A} = 0 \) and \( \bar{h}_{5A} = 0 \) it follows that

\[
\nabla_5 \nabla_5 \bar{h}_{\mu\nu} = \bar{h}_{\mu\nu}'' \quad \bar{g}^{\alpha\beta} \nabla_\alpha \nabla_\beta \bar{h}_{\mu\nu} = \square \bar{h}_{\mu\nu}, \tag{3.10}
\]

where \( \square \) is the d’Alembert operator on \( ds_4 \). While it is evident that \( h = 0 \) implies \( \bar{h} = 0 \) from their conformal relationship, it is not straightforward to see that \( \nabla^\mu \bar{h}_{\mu\nu} = 0 \) follows from \( \nabla^\mu h_{\mu\nu} = 0 \). Hereon we show that this is indeed the case. The definition of the covariant derivative implies that

\[
\nabla^\mu h_{\mu\nu} = \bar{g}^{\mu\alpha} \nabla_\alpha \bar{h}_{\mu\nu} = \bar{g}^{\mu\alpha} \left( \partial_\alpha \bar{h}_{\mu\nu} - \Gamma^E_{\alpha\mu} \bar{h}_{E\nu} - \Gamma^E_{\alpha\nu} \bar{h}_{E\mu} \right). 
\]

Likewise, from the relationship between the Christoffel symbols of two conformal metrics, for the case here considered it follows that

\[
\Gamma^A_{BC} = \Gamma^A_{BC} + \left( \delta^A_B \nabla_C f + \delta^A_C \nabla_B f - \bar{g}_{BC} \nabla^A f \right). 
\]

From both of these relations one concludes that \( \Gamma^5_{\alpha\gamma} = \Gamma^5_{\beta\gamma} \) and, therefore, that \( \nabla^\mu h_{\mu\nu} = \nabla^\mu \bar{h}_{\mu\nu} = 0 \). Since in a curved spacetime the double covariant derivatives do not commute, for the the metric fluctuations one has (see [29], for instance)

\[
\nabla_C \nabla_\mu \bar{h}_{\nu} = \nabla_\mu \nabla_C \bar{h}_{\nu} + \nabla_{E\mu} \bar{h}^E_{\nu} - \nabla_{E\nu} \bar{h}^E_{\mu} \bar{h}_{C\mu} \bar{h}_{E}. \tag{3.11}
\]
By making use of the transverse conditions $\nabla^\mu \bar{h}_{\mu \nu} = 0$ and the fact that $\bar{\Gamma}^5_{AB} = \bar{\Gamma}^B_{5A} = 0$, it reads

$$\nabla_C \bar{h}^C_\nu = \nabla^5_5 \bar{h}^5_\nu + \nabla_\alpha \bar{h}^\alpha_\nu = 0, \quad \nabla_\mu \nabla_C \bar{h}^C_\nu = 0.$$  \hspace{1cm} (3.12)\

Since $h_{5A} = 0$ and $\bar{R}_{5ABC} = \bar{R}_{5A} = 0$, one gets

$$\nabla_C \nabla_\mu \bar{h}^C_\nu = \bar{\Gamma}^{\alpha}_\mu \bar{h}^\alpha_\nu - \bar{\Gamma}^{\alpha}_\nu \bar{h}^\alpha_\mu.$$  \hspace{1cm} (3.13)\

Moreover, for a dS$_4$ spacetime the following relations hold

$$R_{\mu \nu \alpha \beta} = H^2 (g_\mu \alpha g_\nu \beta - g_\mu \beta g_\nu \alpha), \quad R_{\mu \nu} = 3H^2 g_{\mu \nu}.$$  \hspace{1cm} (3.14)\

Hence, we get the following result for the linear variation of the Ricci tensor

$$\delta R_{\mu \nu} = \frac{1}{2} \left( -\Box \bar{h}_{\mu \nu} - \bar{h}''_{\mu \nu} + 8H^2 \bar{h}_{\mu \nu} \right).$$  \hspace{1cm} (3.15)\

Finally, by making use of the relations (3.3), (3.7), (3.8) and the last expression we have

$$\Box \bar{h}_{\mu \nu} + \bar{h}''_{\mu \nu} + 3f \bar{h}_{\mu \nu} - 2H^2 \bar{h}_{\mu \nu} = 0,$$  \hspace{1cm} (3.16)\

under the imposed transverse and traceless conditions $\nabla^\mu \bar{h}_{\mu \nu} = 0, \bar{h}^\alpha_\alpha = 0$.

Furthermore, by performing the following separation of variables for the metric fluctuations $\bar{h}_{\mu \nu} = e^{-\frac{3}{2}f(w)}\Psi(w)\phi_{\mu \nu}(x)$, it leads (3.16) into a Schrödinger–like equation along the extra dimension:

$$(-\partial_z^2 + V_{QM} - m^2) \Psi(w) = 0,$$  \hspace{1cm} (3.17)\

where the analogue quantum mechanical potential $V_{QM}$ reads

$$V_{QM} = \frac{9}{4} f'^2 + \frac{3}{2} f''.$$  \hspace{1cm} (3.18)\

Now, the 4D equation indited from (3.16) is

$$(-\partial_t^2 - 3H \partial_t + e^{-2Ht} \nabla^2 - 2H^2) \phi(x) = -m^2 \phi(x),$$  \hspace{1cm} (3.19)\

where $m^2$ represents the mass that a 4D observer perceives in a de Sitter spacetime [30, 31]. The indices of the function $\phi_{\mu \nu}(w)$ were omitted, for convenience.

By substituting the expression for the warp factor (2.17) into (3.18) the analog quantum mechanical potential adopts the form of a modified Pöschl–Teller potential that reads

$$V_{QM} = \frac{3H^2}{4} \left[ 3 - 7 \text{sech}^2(2Hw) \right].$$  \hspace{1cm} (3.20)\

The fact that this potential possesses a definite positive asymptotic value ensures the existence of a mass gap in the graviton spectrum of KK massive fluctuations determined by $\frac{3H^2}{4}$, or equivalently $m = \frac{3H}{2}$ [25, 31–35].

By performing the following rescaling of the fifth coordinate $v = 2Hw$ we recast (3.17) into

$$\left(-\partial_v^2 - \frac{21}{16} \text{sech}^2v \right) \Psi(v) = \left( \frac{m^2}{4H^2} - \frac{9}{16} \right) \Psi(v),$$  \hspace{1cm} (3.21)\

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which can be directly compared to the canonical form of the classical eigenvalue problem for the Schrödinger equation with a modified Pöschl–Teller potential

\[
\left[ -\partial_v^2 - n(n+1)\text{sech}^2 v \right] \Psi(v) = E \Psi(v) \tag{3.22}
\]

with \( n = 3/4 \) and \( E = \frac{m^2}{4H^2} - \frac{9}{16} \). Since \( n < 1 \) there is just one bound state in the mass spectrum of KK metric fluctuations: the zero mode massless state which accounts for the massless 4D graviton. To the best of our knowledge, this is the first model that presents just a single bound state in this kind of spectra within the framework of thick braneworlds. Usually one encounters a second bound state which represents a massive KK excitation with no clear physical interpretation (see [5], [25, 31–35], for instance). Eq. (3.21) can be integrated for an arbitrary mass and possess the following general solution:

\[
\Psi(w) = C_1 P_{\frac{3}{4}}^{\mu}(\text{tanh}(2Hw)) + C_2 Q_{\frac{3}{4}}^{\mu}(\text{tanh}(2Hw)) \tag{3.23}
\]

where \( C_1 \) and \( C_2 \) are constants, while \( P_{\frac{3}{4}}^{\mu} \) and \( Q_{\frac{3}{4}}^{\mu} \) are associated Legendre functions of first and second kind, respectively, with degree \( \nu = 3/4 \) and order \( \mu = \sqrt{\frac{9}{16} - \frac{m^2}{4H^2}} \). For the massless case the zero mode has \( \mu = \nu = 3/4 \) and the exact solution can be expressed as

\[
\Psi_0(w) = -k_1 \left[ P_{\frac{3}{4}}^{\mu}(\text{tanh}(2Hw)) + \frac{2}{\pi} Q_{\frac{3}{4}}^{3/4}(\text{tanh}(2Hw)) \right], \tag{3.24}
\]

where now \( k_1 = -C_1 > 0 \) and we have set \( C_2 = 2C_1/\pi \) in order to get a localized configuration. This bound state is physically interpreted as a stable graviton localized on the brane, since there are no states with negative squared masses due to the positive definite character of the zero mode (3.24) and the structure of the potential (3.22). The behavior of both the modified Pöschl–Teller potential and the graviton zero mode is displayed in Fig. 3.

**Figure 3.** The profile of the modified Pöschl–Teller potential (thin line) and the localized 4D graviton zero mode (thick line) along the fifth dimension. Here we have set \( c = 0, H = 1/2 \) and \( k_1 = 1 \) for simplicity.

Finally, there is also a continuum of KK massive modes in the spectrum that starts from \( m \geq 3H/2 \) and is described by eigenfunctions with imaginary order \( \pm \mu = \pm i\rho \) [33]:

\[
\Psi_m(w) = \sum_{\pm} C_{\pm} P_{\frac{3}{4}}^{\pm i\rho}(\text{tanh}(2Hw)) \tag{3.25}
\]
where \( C_\pm \) are arbitrary constants and \( \rho = \sqrt{\frac{m^2}{4H^2} - \frac{9}{16}} \). These KK massive modes must behave as plane waves asymptotically \([31, 32]\). This fact can be seen by considering masses \( 2m > 3H \) and taking into account that the constants \( C_\pm \) depend on \( \rho \) in general, i.e.

\[
\Psi_\pm^\mu(w) = C_\pm(\rho)P_{3/4}^{\pm i\rho}(\tanh(2Hw)).
\]  

(3.26)

We further make an expansion for large \( w \) of the argument of the associated Legendre functions of first kind:

\[
\tanh(2Hw) \approx 1 - 2e^{-4Hw}
\]  

(3.27)

and compute the asymptotic behavior of \( P_{3/4}^{\pm i\rho}(\tanh(2Hw)) \) according to Eq.(8) of Sec. (3.9.2) in \([36]\): 

\[
P_{3/4}^{\pm i\rho}(\tanh(2Hw)) \sim \frac{1}{\Gamma(1 \mp i\rho)}e^{\pm 2iH\rho w}.
\]  

(3.28)

The normalization condition of (3.26) in the plane wave sense leads to the following normalization constants

\[
C_+(\rho) = C_-(\rho) = \frac{|\Gamma(1 + i\rho)|}{\sqrt{2\pi}}
\]  

(3.29)

since \(|\Gamma(1 - i\rho)| = |\Gamma(1 + i\rho)|\). Thus, by substituting (3.28) and (3.29) into (3.26) we obtain the asymptotic behavior of these associated Legendre functions:

\[
\Psi_\pm^\mu(w) \sim \frac{1}{\sqrt{2\pi}}e^{\pm 2iH\rho w}
\]  

(3.30)

which corresponds to plane waves as expected.

Once an analytical expression for the KK massive modes was obtained, we should be able to compute the corresponding small corrections to Newton’s law due to these 5D massive modes. This is achieved by taking the thin brane limit \( H \to \infty \), locating a probe mass, \( M_1 \), in the center of the brane in the transverse direction. Subsequently, by computing the gravitational potential generated by this particle, felt by another massive particle with mass \( M_2 \). The corrections to the Newtonian potential generated by massive gravitons in the thin brane limit can be expressed as follows \([6]\)

\[
U(r) \sim \frac{M_1M_2}{r} \left( G_4 + M_5^{−3} \int_{m_0}^{\infty} dm e^{−mr} \left| \Psi^{\mu(m)}(w_0) \right|^2 \right) = \frac{M_1M_2}{r} (G_4 + \Delta G_4),
\]  

(3.31)

where \( w = w_0 \) sets the position where the brane is located, \( m_0 = 3H/2 \) for our case, \( G_4 \) is the gravitational 4D coupling constant and \( \Psi^{\mu}(w_0) \) denotes the continuum of KK massive modes that must be integrated over their masses in order to get the searched corrections.

Now we proceed to calculate \( |\Psi^{\mu}(0)|^2 \) at \( w_0 = 0 \) and get

\[
|\Psi^{\mu}(0)|^2 = \left| \frac{\Gamma\left(1 + i\rho\right)}{\Gamma\left(\frac{11}{8} + \frac{\nu}{2}\right)\Gamma\left(\frac{1}{8} + \frac{\nu}{2}\right)} \right|^2,
\]  

(3.32)

where we set \( \nu = 3/4 \) and made use of the following relation for the associated Legendre functions of first kind \([37]\):

\[
P_{\nu}^{\mu}(0) = \frac{2^\mu \sqrt{\pi}}{\Gamma\left(\frac{1-\nu-\mu}{2}\right)\Gamma\left(1 + \frac{\nu-\mu}{2}\right)}.
\]  

(3.33)
Substitution of (3.32) into the second term of (3.31) yields the following expression for $\Delta G_4$:

$$\Delta G_4 = M_s^{-3} \int_{m_0}^{\infty} dm \, e^{-m r} \left| \frac{\Gamma(1 + i \rho)}{\Gamma(\frac{11}{8} + \frac{i \rho}{2}) \Gamma(\frac{1}{8} + \frac{i \rho}{2})} \right|^2 .$$

(3.34)

In order to calculate this integral it is convenient to perform a change of variable from $m$ to $\rho$:

$$\Delta G_4 = 2M_s^{-3} H \int_{0}^{\infty} e^{-2 \sqrt{\rho^2 + \frac{9}{16} H r}} \left| \frac{\Gamma(1 + i \rho)}{\Gamma(\frac{11}{8} + \frac{i \rho}{2}) \Gamma(\frac{1}{8} + \frac{i \rho}{2})} \right|^2 d\rho .$$

(3.35)

This integral can be calculated in the thin brane limit $H \to \infty$, where it is dominated by the region of small $\rho$. Thus (3.35) can be well–approximated by expanding the factor that multiplies the exponential at $\rho = 0$ [33]. Thus, the contribution to Newton’s law made by the continuum of KK massive modes in the thin brane limit reads:

$$\Delta G_4 \sim \frac{M_s^{-3}}{\left| \Gamma(\frac{11}{8}) \Gamma(\frac{1}{8}) \right|^2} e^{-\frac{9}{16} H r} \left( 1 + O\left(\frac{1}{H r}\right) \right) .$$

(3.36)

These corrections are exponentially suppressed as in other braneworld models with an induced 4D Minkowski [38], (see also [33]–[34]) or de Sitter metric [25, 35]. This result shows that the form of the corrections to Newton’s law coming from the extra dimension is quite robust against the explicit form of the KK massive modes as well as the type of the field that couples to gravity to generate the braneworld model.

4 Discussion

In this paper we have presented a thick braneworld model generated by a tachyon scalar field coupled to gravity with a bulk cosmological constant and a de Sitter metric induced on the brane. We were able to obtain an exact solution with a decaying warp factor that gives rise to localization of 4D gravity when studying the metric fluctuations. The structure of the corresponding graviton spectrum is novel in the sense that it contains just one bound state (the massless zero mode) which is physically interpreted as the 4D graviton, separated by a mass gap from a continuum of KK massive excitations. We provided an explicit expression for these KK massive modes, a fact that enables us to analytically compute the corrections to Newton’s law coming from the extra dimension. As we mentioned above, these corrections are exponentially suppressed and coincide (up to a coefficient factor) with previous results reported in the literature within the framework of similar braneworld models with the same 4D de Sitter induced metric. This result reflects the robustness of the form of the corrections to Newton’s law since they are computed for KK massive modes with different form compared to other braneworld models previously reported.

When analyzing the curvature scalar of our model we realize that it is positive definite on the whole patch and asymptotically vanishes even when we have a negative cosmological constant. Thus, our de Sitter tachyonic thick braneworld model interpolates between two 5D Minkowski spaces. On the other hand, we should remember that our analysis is carried out in a local basis. Thus, the study of the global structure of our 5D spacetime, which involves
the continuation of our coordinate system defined by (2.3), could lead to a more complex picture in which the curvature is positive in our chart, but is asymptotically negative, for instance. The study of this interesting properties is beyond the scope of the present paper and will be reported elsewhere.

Since here a braneworld with an induced 4D de Sitter metric is considered, in principle, we are able to reproduce the early inflation and accelerated expansion epochs of our universe within our model. However, a more general and realistic ansatz for our setup that attempts to describe the late time behavior of our 4D universe should involve both a time–depending tachyon field and a time–depending warp factor, since from the cosmological viewpoint one needs to obtain scale factors that reproduce in a better way (closer to the observations) the accelerated expansion of the universe (which takes into account its dark matter component) than the de Sitter metric does. This line of research is under current investigation and could lead to interesting new results in cosmology.

Another interesting issue that should be approached is related to the stability of this braneworld configuration. This study involves computing the field equations of the perturbed tachyonic scalar field coupled to the scalar modes of the metric fluctuations in the linear approximation. Recall that the scalar sector of the perturbed metric must be defined with respect to the transformations associated to the $dS_4$ symmetry group. This is a rather involved task that is already under performance and will be reported in the near future. In the lack of such a rigorous analysis we can instead study the stability of the brane in the limit of small gradient for the tachyonic scalar field. This is done by considering the dynamics of scalar perturbations when taking into account the back reaction of the brane itself.

Thus, we shall consider a sort of “slow–roll” approximation in the action (2.1). In this limit, the kinetic term of the tachyon field is considered small compared to the self–interaction potential. However, we shall instead assume a small gradient approximation $(\nabla T)^2 << 1$, so that the action for the tachyon field could be expanded up to quadratic first derivative terms

\[
S = - \int d^5x V(T) \left( 1 + \frac{1}{2} g^{AB} \partial_A T \partial_B T + \ldots \right),
\]

This action can be further written as an action for a standard scalar field:

\[
S = \int d^5x \left( -\frac{1}{2} g^{AB} \partial_A \varphi \partial_B \varphi - V(\varphi) + \ldots \right),
\]

where the new scalar field $\varphi = \varphi(T)$ and we have assumed that

\[
V = (\varphi_T)^2 = \left( \frac{\partial \varphi}{\partial T} \right)^2
\]

in order to identify $\varphi_T \partial_A T$ with $\partial_A \varphi$. Thus, in the small gradient approximation, the tachyon dynamics is described by the action for the standard scalar field $\varphi$. Moreover, from the latter relation (4.3) one can easily read the expression for its first derivative with respect to the extra coordinate in terms of the tachyonic field and its first derivative: $\varphi' = \sqrt{V} T'$.

In order to study the stability behaviour of our tachyonic brane in the small gradient approximation, and under the ansatz (2.11), we must consider the regime $(\nabla T)^2 << 1$.

The analysis of scalar perturbations $\varphi = \varphi_0 + \delta \varphi$ for fluctuated non–singular solutions of the Einstein equations with non–trivial standard scalar fields and self–interaction potentials was performed in [8]. These authors computed the effective potentials for the master
Schrödinger–like equation of scalar perturbations for a family of smooth solutions corresponding to different bulk curvatures and found that they are positive definite as well as the spectrum of massive modes, consequently, these systems were found to be stable under small perturbations.

We can perform a similar analysis in the small gradient approximation of our relevant tachyonic thick braneworld. Thus, by quoting the relevant Schrödinger–like equation obtained in [8] for scalar perturbations $F(w, x^\mu)$ (see this work for details):

$$- F''(w, x^\mu) + V_{eff}(w) F(w, x^\mu) = m^2 F(w, x^\mu),$$  \hspace{1cm} (4.4)

where we have taken into account that $\delta \varphi = \delta \varphi(F, F')$, $m$ is the mass of the scalar perturbation in a 4D de Sitter space [30] and the effective potential $V_{eff}$ reads

$$V_{eff} = -\frac{5}{2} f'' + \frac{9}{4} f^2 + f' \varphi_0'' - \frac{\varphi_0''}{\varphi_0} + 2 \left( \frac{\varphi_0''}{\varphi_0} \right)^2 - 4H^2,$$  \hspace{1cm} (4.5)

where $\varphi_0 = \sqrt{V T'}$.

The form of this effective potential corresponds to a potential barrier which asymptotically approaches the positive value $\frac{17H^2}{4}$ and possesses a maximum at $\frac{51H^2}{10}$ (see Fig. 4). Thus, the spectrum of massive scalar fluctuations is positive definite, a fact that indicates that we have a stable scalar field configuration around the origin, where the 3–brane is located, in the small gradient approximation for the scalar field.

However, it should be pointed out that since $(\nabla T)^2 = e^{-2f}(T')^2 = \frac{1}{4} \text{sech}^2(Hw)$ (here we have set $c = 0$), at the origin we have a 25% of a unity instead of a small quantity. This means that our result seems to have sense in the small gradient approximation unless the potential barrier configuration turns into a potential well sufficiently deep in order to get unstable modes in the massive spectrum of scalar perturbations when one computes the exact dynamical equation and, hence, is not so robust.

![Figure 4](image.png)

**Figure 4.** The shape of the approximated self–interaction potential for the tachyonic scalar field in the small gradient approximation. We set here $c = 0$, $H = 1/2$ and $2\kappa T = 1$ for simplicity.

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References

[1] M. Gogberashvili, *Hierarchy problem in the shell universe model*, Int. J. Mod. Phys. D11 (2002) 1635 [hep-ph/0108296]; *Four dimensionality in noncompact Kaluza-Klein model*, Mod. Phys. Lett. A 14 (1999) 2025 [hep-ph/9904383].

[2] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, Phys. Rev. Lett. 83 (1999) 3370 [hep-ph/9905221]; *An alternative to compactification*, Phys. Rev. Lett. 83 (1999) 4690 [hep-th/9906064].

[3] V. Dzhunushaliev, V. Folomeev and M. Minamitsuji, *Thick brane solutions*, Rept. Prog. Phys. 73 (2010) 069901 [arXiv:0904.1775 [gr-qc]].

[4] O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, *Modeling the fifth dimension with scalars and gravity*, Phys. Rev. D62 (2000) 046008 [hep-th/9909134].

[5] M. Gremm, *Four-dimensional gravity on a thick domain wall*, Phys. Lett. B478 (2000) 434 [hep-th/9912060]; *Thick domain walls and singular spaces*, Phys. Rev. D62 (2000) 044017 [hep-th/0002040].

[6] C. Csaki, J. Erlich, T. Hollowood and Y. Shirman, *Universal Aspects of gravity localized on thick branes*, Nucl. Phys. B581 (2000) 309 [hep-th/0001033].

[7] M. Giovaninni, *Gauge-invariant fluctuations of scalar branes*, Phys. Rev. D64 (2001) 064023 [hep-th/0106041]; *Localization of metric fluctuations on scalar branes*, Phys. Rev. D65 (2002) 064008 [hep-th/0106131]; K. Farakos and P. Pasipoularides, *Gauss–Bonnet gravity, brane world models, and non–minimal coupling*, Phys. Rev. D75 (2007) 024018, [hep-th/0610010]; K. Farakos, G. Koutsoumbas and P. Pasipoularides, *Graviton localization and Newton’s law for brane models with a non-minimally coupled bulk scalar field*, Phys. Rev. D76 (2007) 064025 [arXiv:0705.2364 [hep-th]]; A. Herrera–Aguilar, D. Malagón–Morejón, R.R. Mora–Luna and I. Quiros, *Thick braneworlds generated by a non-minimally coupled scalar field and a Gauss-Bonnet term: conditions for localization of gravity*, Class. Quantum Grav. 29 (2012) 035012 [arXiv:1105.5479 [hep-th]]; H. Guo, Y.–X. Liu, Z.–H. Zhao and F.–W. Chen, *Thick branes with a non-minimally coupled bulk–scalar field*, Phys. Rev. D85 (2012) 124033 [arXiv:1106.5216 [hep-th]].

[8] S. Kobayashi, K. Koyama and J. Soda, *Thick brane worlds and their stability*, Phys. Rev. D65 (2002) 064014 [hep-th/0107025].

[9] V.I. Afonso, D. Bazeia, R. Menezes and A.Yu. Petrov, *f(R)-Brane*, Phys. Lett. B658 (2007) 71 [arXiv:0710.3790 [hep-th]]; D. Bazeia, A.R. Gomes, L. Losano, *Gravity localization on thick branes: a numerical approach*, Int. J. Mod. Phys. A24 (2009) 1135 [arXiv:0708.3530]; D. Bazeia, F. A. Brito, F. G. Costa, *First-order framework and domain-wall/brane-cosmology correspondence*, Phys. Lett. B661 (2008) 179 [arXiv:0707.0680]; D. Bazeia, F. A. Brito, L. Losano, *Scalar fields, bent branes, and RG flow*, JHEP 0611 (2006) 064 [arXiv:hep-th/0610233]; D. Bazeia, A.R. Gomes, *Bloch Brane*, JHEP 0405 (2004) 012 [arXiv:hep-th/0403141]; D. Bazeia, C. Furtado, A.R. Gomes, *Brane Structure from a Scalar Field in Warped Spacetime*, JCAP 0402 (2004) 002 [arXiv:hep-th/0308034]; Y. Zhong, Y.–X. Liu and K. Yang, *Tensor perturbations of f(R)-branes*, Phys. Lett. B699 (2011) 398 [arXiv:1010.3478 [hep-th]]; M. Gogberashvili and D. Singleton, *Brane in 6-D with increasing gravitational trapping potential*, Phys. Rev. D69 (2004) 026004 [hep-th/0305241]; V. Dzhunushaliev, V. Folomeev, D. Singleton and A. Aguilar-Rudametkin, *Thick branes from scalar fields*, Phys. Rev. D77 (2008) 044006 [hep-th/0703043]; M. Gogberashvili and D. Singleton, *Anti-de-Sitter Island-Universes from 5D Standing Waves*, Mod. Phys. Lett. A25 (2010)
[10] O. Arias, R. Cardenas and I. Quiros, Thick Brane Worlds Arising From Pure Geometry, Nucl. Phys. B 643 (2002) 187 [hep-th/0202130]; N. Barbosa-Cendejas and A. Herrera-Aguilar, 4D gravity localized in non $Z_2$-symmetric thick branes, JHEP 0510 (2005) 101 [hep-th/0510507]; N. Barbosa-Cendejas and A. Herrera-Aguilar, Localization of 4D gravity on pure geometrical thick branes, Phys. Rev. D73 (2006) 084022; Erratum–ibid. D 77 (2008) 049901 [hep-th/0603184].

[11] J.M. Hoff da Silva and R. da Rocha, Braneworld remarks in Riemann–Cartan manifolds, Class. Quantum Grav. 26 (2009) 055007, Corrigendum–ibid. 26 (2009) 179801 [arXiv:0804.4261 [gr-qc]]; J.M. Hoff da Silva and R. da Rocha, Reply to Comment on 'Braneworld remarks in Riemann-Cartan manifolds', Class. Quant. Grav. 26 (2009) 178002; (2010) 024021 [arXiv:0912.5186 [hep-th]]; J.M. Hoff da Silva and R. da Rocha, Gravitational constraints of $dS$ branes in $AdS$ Einstein–Brans–Dicke bulk, Class. Quantum Grav. 27 (2010) 250008 [arXiv:1006.5176 [gr-qc]]; J.M. Hoff da Silva and R. da Rocha, Effective Monopoles within Thick Branes, Europhys. Lett. 100 (2012) 11001 [arXiv:1209.0989 [hep-th]]; M. C. B. Abdalla, J. M. Hoff da Silva and R. da Rocha, Notes on the Two-brane Model with Variable Tension Phys. Rev. D 80 (2009) 046003 [arXiv:0907.1321 [hep-th]]; R. da Rocha and J. M. Hoff da Silva, Black string corrections in variable tension braneworld scenarios, Phys. Rev. D 85 (2012) 046009 [arXiv:1202.1256 [gr-qc]].

[12] D. Bazeia, F.A. Brito and J.R. Nascimento, Supergravity brane worlds and tachyon potentials, Phys. Rev. D 68 (2003) 085007 [hep-th/0306284].

[13] R. Koley and S. Kar, A Novel braneworld model with a bulk scalar field, Phys. Lett. B623 (2005) 244, Erratum–ibid. B631 (2005) 199 [hep-th/0507277].

[14] A. Das, S. Kar and S. SenGupta, Stable two–brane models with bulk tachyon matter, Int. J. Mod. Phys. A24 (2009) 4457 [arXiv:0804.1757 [hep-th]].

[15] W.D. Goldberger and M.B. Wise, Modulus stabilization with bulk fields, Phys. Rev. Lett. 83 (1999) 4922 [hep-ph/9907447].

[16] D. Maity, SenGupta, and S. Sur, Stability analysis of the Randall–Sundrum braneworld in presence of a bulk scalar, Phys. Lett. B643 (2006) 348 [hep-th/0604195].

[17] S. Pal and S. Kar, de Sitter branes with a bulk scalar, Gen. Rel. Grav. 41 (2009) 1165, [hep-th/0701266].

[18] A. Melfo, N. Pantoja and J.D. Tempo, Fermion localization on thick branes, Phys. Rev. D73 (2006) 044003 [hep-th/0601161]; X.-H. Zhang, Y.-X. Liu, and Y.-S. Duan, Localization of Fermionic Fields on Braneworlds with Bulk Tachyon Matter, Mod. Phys. Lett. A23 (2008) 2093, [arXiv:0709.1888 [hep-th]].

[19] A. Sen, Supersymmetric world volume action for nonBPS D-branes, JHEP 9910 (1999) 008 [hep-th/9909062]; E. A. Bergshoeff, M. de Roo, T. C. de Wit, E. Eyras, S. Panda, $T$ duality and actions for non-BPS D-branes, JHEP 0005 009 (2000) [arXiv:hep-th/0003221].

[20] A. Sen, Rolling tachyon, JHEP 0204 (2002) 048 [hep-th/0203211]; A. Sen, Tachyon matter, JHEP 0207 (2002) 065 [hep-th/0203265]; A. Sen, Field theory of tachyon matter, Mod. Phys. Letts. A17 (2002) 1797 [hep-th/0204143]; A. Sen, Time and tachyon, Int. J. Mod. Phys. A18 (2003) 4869 [hep-th/0209122].

[21] A. Mazumdar, S. Panda, A. Perez-Lorenzana, Assisted inflation via tachyon condensation, Nucl.Phys. B 614 (2001) 101 [arXiv: hep-ph/0107058].

[22] G. W. Gibbons, Cosmological evolution of the rolling tachyon, Phys. Lett. B 537 (2002) 1 [arXiv: hep-th/0204608]; D. Choudhury, D. Ghoshal, D. P. Jatkar, S. Panda, On the cosmological relevance of the tachyon, Phys. Lett. B 544 (2002) 231 [arXiv:hep-th/0204204]; D. Choudhury, D. Ghoshal, D. P. Jatkar, S. Panda, Hybrid inflation and brane - anti-brane system, JCAP 0307 (2003) 009 [arXiv:hep-th/0305104].

[23] T. Padmanabhan, Accelerated expansion of the universe driven by tachyonic matter, Phys. Rev. D 66 (2002) 021301 [arXiv:hep-th/0204150].
[24] N. Chandrachani Devi, S. Panda, A. A. Sen, *Solar System Constraints on Scalar Tensor Theories with Non-Standard Action*, Phys. Rev. D **84** (2011) 063521 [arXiv:1104.0152].

[25] A. Herrera–Aguilar, D. Malagón–Morejón and R.R. Mora–Luna, *Localization of gravity on a thick braneworld without scalar fields*, JHEP **1011** (2010) 015 [arXiv:1009.1684[hep-th]]; H. Guo, A. Herrera–Aguilar, Y. X. Liu, D. Malagón–Morejón and R.R. Mora–Luna, *Localization of bulk matter fields on a pure de Sitter thick braneworld*, [arXiv:1103.2430 [hep-th]].

[26] P. D. Mannheim, *Brane–Localized Gravity*, World Scientific, Singapore (2005).

[27] Ch. Charmousis, R. Gregory, N. Kaloper an A. Padilla, *DGP spectroscopy*, JHEP **0610** (2006) 066 [hep-th/0604086].

[28] G. ’t Hooft, *Introduction to General relativity*, Rinton Press, Princeton (2001).

[29] S. Carroll, *Spacetime and geometry: An introduction to general relativity*, Addison-Wesley, San Francisco (2004).

[30] C. Gabriel and P. Spindel, *Massive spin-2 propagators on de Sitter space*, J. Math. Phys. **38** (1997) 622 [hep-th/9912054]; J. Garriga and M. Sasaki, *Brane world creation and black holes*, Phys. Rev. D**62** (2000) 043523 [hep-th/9912118]; T. Garidi, J.P. Gazeau and M.V. Takook, 'Massive’ spin two field in de Sitter space, J. Math. Phys. **44** (2003) 3838 [hep-th/0302022]. J.P. Gazeau and M. Novello, *The question of Mass in (anti–) de Sitter Spacetimes*, J. Phys. A**41** (2008) 304008.

[31] A. Wang, *Thick de Sitter 3-Branes, Dynamic Black Holes and Localization of Gravity*, Phys. Rev. D**66** (2002) 024024 [hep-th/0201051].

[32] M.K. Parikh and S.N. Solodukhin, *De Sitter brane gravity: From close–up to panorama*, Phys. Lett. B**503** (2001) 384 [hep-th/0012231].

[33] N. Barbosa–Cendejas, A. Herrera–Aguilar, M. A. Reyes Santos and C. Schubert, *Mass gap for gravity localized on Weyl thick branes*, Phys. Rev. D**77** (2008) 126013 [arXiv:0709.3552 [hep-th]].

[34] N. Barbosa–Cendejas, A. Herrera–Aguilar, K. Kanakoglou, U. Nucamendi and I. Quiros, *Mass hierarchy and mass gap on thick branes with Poincaré symmetry*, [arXiv:0712.3098 [hep-th]]; A. Herrera–Aguilar, D. Malagón–Morejón, R.R. Mora–Luna, U. Nucamendi, *Aspects of thick brane worlds: 4D gravity localization, smoothness, and mass gap*, Mod. Phys. Lett. A**25** (2010) 2089 [arXiv:0910.0363 [hep-th]].

[35] H. Guo, Y. X. Liu, S. W. Wei and C.–E Fu, *Gravity Localization and Effective Newtonian Potential for Bent Thick Branes*, Europhys. Lett. **97** (2012) 60003 [arXiv:1008.3686 [hep-th]].

[36] A. Erdélyi (Ed.), *Higher transcendental functions*, Vol. I, McGraw–Hill, New York (1953), reprint edition R.E. Krieger Publishing Company, Malabar (1981).

[37] I.S. Gradsteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products*, Academic Press, San Diego (2007).

[38] A. Brandhuber and K. Sfetsos, *Nonstandard compactifications with mass gaps and Newton’s law*, JHEP **10** (1999) 013 [arXiv:hep-th/9908116].

[39] J.A. Minahan and B. Zwiebach, *Effective tachyon dynamics in superstring theory*, JHEP **03** (2001) 038 [arXiv:hep-th/0009246].