Spectrum of Background X-rays from Moduli Dark Matter

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*(May 25, 2022)*

Abstract

We examine the X-ray spectrum from the decay of the dark-matter moduli with mass $\sim \mathcal{O}(100)$keV, in particular, paying attention to the line spectrum from the moduli trapped in the halo of our galaxy. It is found that with the energy resolution of the current experiments ($\sim 10\%$) the line intensity is about twice stronger than that of the continuum spectrum from the moduli that spread in the whole universe. Therefore, in the future experiments with higher energy resolutions it may be possible to detect such line photons. We also investigate the $\gamma$-ray spectrum emitted from the decay of the multi-GeV moduli. It is shown that the emitted photons may form MeV-bump in the $\gamma$-ray spectrum. We also find that if the modulus mass is of the order of 10 GeV, the emitted photons at the peak of the continuum spectrum loses their energy by the scattering and the shape of the spectrum is significantly changed, which makes the constraint weaker than that obtained in the previous works.
I. INTRODUCTION

Superstring theories [1], which may be the most attractive candidates to unify all known interactions including gravity, have a number of flat directions, called moduli fields, in a large class of classical ground states [1]. These moduli fields $\phi$ continuously connect infinitely degenerate supersymmetric vacua and they are generally expected to acquire their masses $m_\phi$ of the order of the gravitino mass $m_{3/2}$ once supersymmetry breaking effects are included [2].

These moduli fields cause different kinds of cosmological problems [3,4] depending on values of their masses. At present the thermal inflation proposed by Lyth and Stewart [5] seems to be the most plausible solution to the problems. In recent articles [8,9], we have shown by postulating the thermal inflation that only two regions of the moduli masses, $m_\phi \lesssim 500$ keV and $m_\phi \gtrsim \mathcal{O}(100)$ GeV, are cosmologically viable. In particular, the lighter mass region is more interesting since the original Affleck-Dine baryogenesis [10] does work here as shown first by de Gauvèa, Moroi and Murayama [11]. On the contrary, for $m_\phi \gtrsim \mathcal{O}(100)$ GeV we must invoke a variant type of Affleck-Dine baryogenesis [12] which has not been, however, well investigated yet.

If the moduli masses lie indeed in the region $m_\phi \simeq 10^{-2}$ keV–200 keV there is an intriguing possibility [9] that the moduli fields are the dark matter in our universe. Since the thermal inflation produces a tremendous amount of entropy at the late epoch of the universe’s evolution to dilute the moduli density substantially, there seems to be no candidate left for the dark matter beside the moduli themselves [1]. This would encourage us to consider the hypothesis of moduli being the dark matter in the universe.

In this paper we calculate spectrum of background X-rays emitted from the moduli dark matter and find that the spectrum is constituted of two distinct parts: one comes from the cosmic moduli filling homogeneously the whole universe and the other from the moduli condensed on the dark halo in our galaxy. The former has a relatively broad spectrum due to the redshift effect and the latter has a peak in the energy spectrum. We show that the peak in the X-ray spectrum can be detectable in future experiments if the moduli masses $m_\phi$ are around 100 keV. We also briefly comment on $\gamma$-ray spectrum emitted from more massive moduli of $m_\phi \simeq 1$ – 10 GeV, since this multi-GeV mass region is marginally allowed [8,9] if one assumes somewhat smaller values of the initial amplitudes of moduli fields, $\phi_0 \simeq (0.01 – 0.1) M_G$, where $M_G$ is the gravitational scale $M_G \simeq 2.4 \times 10^{18}$ GeV. We find that the $\gamma$-rays emitted from such moduli make a large bump in multi-MeV region and the shape of the spectrum depends heavily on the masses of moduli.

II. COSMOLOGICAL MODULI PROBLEM AND THE THERMAL INFLATION

In this section we briefly review the previous works [8,9] and show that only two mass regions such as $m_\phi \lesssim 500$ keV and $m_\phi \gtrsim \mathcal{O}(100)$ GeV survive various cosmological constraints.

1 The axion with high values of decay constant $F_a \simeq 10^{15}$–$10^{16}$ GeV could be another candidate for the dark matter [13].
We assume $m_\phi \simeq m^{3/2}$ throughout this paper.

Let us start with the cosmological moduli problems. As mentioned in the introduction the moduli fields of masses in the range of keV-TeV cause different kinds of cosmological problems depending on their masses. On one hand, the moduli with masses $\mathcal{O}(100)\text{ GeV} \lesssim m_\phi \lesssim \mathcal{O}(1)\text{ TeV}$ decay soon after nucleosynthesis and in consequence spoil the success of big-bang nucleosynthesis. On the other hand, the moduli with masses $m_\phi \lesssim 1\text{ GeV}$ are entangled with the problem that the abundance of moduli themselves overcloses the present universe, or the radiation expected from their decay may exceed the observed cosmic X(\gamma)-ray backgrounds. In any case, one finds it difficult to solve these problems as far as one is concerned with the standard cosmology.

We give now a brief account of the thermal inflation which is thought to be the only mechanism to overcome the above problems. In their original paper, Lyth and Stewart\footnote{The detailed analysis has been carried out in \textcolor{blue}{[5]}. For a more complete version, see Ref. \textcolor{blue}{[9]}.} proposed a solution for the case of moduli masses $\mathcal{O}(100)\text{ GeV} \lesssim m_\phi \lesssim \mathcal{O}(1)\text{ TeV}$, that corresponds to hidden sector supersymmetry breaking models\footnote{The imaginary part of the complex scalar field $X$ can be interpreted as a massless NG boson originating from the U(1) symmetry possessed by the potential (1). The NG bosons produced by the flaton decay diminish drastically the dilution effect of the thermal inflation. We have given in \textcolor{blue}{[8]} such a modification of the original thermal inflation model that this unfavorable decay mode is suppressed. It has been found, however, that the modification did not affect the original dynamics in Ref. \textcolor{blue}{[8]}.}. Thus, let us restrict ourselves in the following discussion to the mass region $10^{-2}\text{ keV} \lesssim m_\phi \lesssim 10\text{ GeV}$, that is relevant to gauge-mediated supersymmetry breaking models.

When we discuss the abundance of moduli energy density $\rho_\phi$, it is convenient to introduce a ratio $\rho_\phi/s$, where $s$ is the entropy density, since this quantity is invariant under the universe’s evolution as long as no entropy is produced. Then the problems stated above are reexpressed as that the present value of the ratio $\rho_\phi/s$ is predicted to be greater than the ratio $\rho_c/s$, where $\rho_c$ is the critical density of the present universe, by typical factors of $10^{11} - 10^{16}$. If the thermal inflation takes place, however, a significant increase in the entropy density leads to an extreme reduction of the quantity $\rho_\phi/s$.

The existence of flaton fields is required to provide vacuum energy, which is responsible for the thermal inflation to occur, and to produce entropy by their decay into radiation. The potential of the flaton $X$ is given by

$$V = V_0 - m_0^2 |X|^2 + \frac{1}{M_*^{2n}} |X|^{2n+4}, \quad (1)$$

where $-m_0^2$ is a negative mass squared induced by SUSY breaking effect and $M_*$ a cut-off scale of this effective theory. the vacuum energy density $V_0$ is determined so that the cosmological constant vanishes at the true vacuum.

If the flaton $X$ couples to some fields which are in thermal bath, the flaton potential (1) with finite temperature effects taken into account reads
\[ V_{\text{eff}} = V_0 + (cT^2 - m_0^2)|X|^2 + \frac{1}{M_\ast^{2n}}|X|^{2n+4}. \] (2)

Here, \( T \) is the cosmic temperature and \( c \) a constant of \( \mathcal{O}(1) \). As be easily seen from the form of the effective potential (2), the flaton sits near the origin at the temperature \( T > T_c \simeq m_0 \), and gives rise to the vacuum energy density \( V_0 \). This vacuum energy becomes greater than the radiation energy at the temperature \( T < T_* \simeq V_0^{1/4} \), since the radiation energy density is given by \( \rho_{\text{rad}} = \frac{\pi^2}{45}g_\ast T^4 \), where \( g_\ast \) is the effective number of degrees of freedom. Therefore, for the temperature \( T_c < T < T_* \) the flaton vacuum energy density dominates the cosmic energy density, and the thermal inflation takes place.

When the cosmic temperature becomes lower than the critical temperature, i.e. \( T < T_c \), the flaton begins rolling down towards the true minimum of the potential (1), and oscillates around it. The flaton coherent oscillation energy is eventually transferred to the radiation energy through the flaton decay and reheats the universe, increasing the entropy density by a factor of

\[ \Delta \simeq \frac{4V_0}{3T_R} \frac{m_\phi^4}{g_\ast T_c^3} \simeq \frac{V_0}{70T_RT_c^3}, \] (3)

where \( T_R \) is the reheating temperature.

We are now at the point to evaluate the moduli energy density, with the notable effects of the thermal inflation considered. We assume only one modulus \( \phi \) to exist for simplicity. The generalization to the case of many moduli is straightforward, however.

When the Hubble parameter \( H \) becomes comparable to the modulus mass \( m_\phi \), the coherent oscillation of the modulus, which we refer to as ‘big-bang modulus’, starts with the initial amplitude \( \phi_0 \) which is likely to be of the order \( M_G \). Then, the abundance of ‘big-bang modulus’ after the thermal inflation is calculated as

\[ \left( \frac{\rho_\phi}{s} \right)_{\text{BB}} \simeq \frac{m_\phi^2\phi_0^2/2}{8.6m_\phi^3/2M_G^{3/2}} \frac{1}{\Delta}, \]
\[ \simeq 4 \left( \frac{T_c}{m_0} \right)^3 \left( \frac{\phi_0}{M_G} \right)^2 \frac{M_G^{1/2}m_\phi^{1/2}m_0^3T_R}{V_0}. \] (4)

We should not forget ‘thermal inflation modulus’ that is a secondary oscillation, which begins after the thermal inflation, stimulated by the shift \( \delta\phi \sim (V_0/m_\phi^2M_G^2)\phi_0 \) from the true minimum of the modulus potential. The abundance of this ‘thermal inflation modulus’ is

\[ \left( \frac{\rho_\phi}{s} \right)_{\text{TI}} \simeq \frac{m_\phi^2(\delta\phi)^2/2}{(2\pi^2/45)g_\ast T_c^3} \frac{1}{\Delta}, \]
\[ \simeq \frac{3}{8} \left( \frac{\phi_0}{M_G} \right)^2 \frac{V_0T_R}{m_\phi^2M_G^2}. \] (5)

The total energy density of the modulus \( \phi \) is then given by

\[ \frac{\rho_\phi}{s} \simeq \max \left[ \left( \frac{\rho_\phi}{s} \right)_{\text{BB}}, \left( \frac{\rho_\phi}{s} \right)_{\text{TI}} \right]. \] (6)
In Ref. [8], we regarded $m_0$ and $M_*$ as free parameters and obtained the theoretically predicted lower bound of (3), under the condition $T_R \gtrsim 10$ MeV that is required in order for the radiation created by the flaton decay not to upset the nucleosynthesis. As a result, we have shown that for all the region $10^{-2}$ keV $\lesssim m_\phi \lesssim 10$ GeV the lower bound of $\Omega_\phi h^2 \equiv \rho_\phi h^2/\rho_c$ ($h$ is the present Hubble parameter $H_0$ in units of 100 km/sec/Mpc) could be taken below the critical density $\Omega h^2 \simeq 0.25$.

A constraint from the observed X($\gamma$)-ray backgrounds, however, can be more stringent than that from the critical density in a certain modulus mass region. The modulus decays into two photons dominantly. Thus, we can derive another constraint on $\Omega_\phi h^2$ by requiring that the maximum value of the predicted photon flux should be less than the observed X($\gamma$)-ray backgrounds. It has been shown in [8] that this constraint excludes an interesting mass region $500$ keV $\lesssim m_\phi \lesssim 10$ GeV.

Let us summarize the conclusions that were obtained in [8,9]. First, we have found that only the theories with modulus mass $10^{-2}$ keV $\lesssim m_\phi \lesssim 500$ keV could survive the cosmological constraints. Second, we have pointed out that the modulus with mass $m_\phi \simeq 1 - 10$ GeV also had a chance to be allowed cosmologically if we could take $\phi_0 \simeq (0.01 - 0.1)M_G$. The final one, which has motivated us to work on this paper, is that in the modulus mass region $10^{-2}$ keV $\lesssim m_\phi \lesssim 200$ keV the equality $\Omega_\phi h^2 \simeq O(1)$ could be fulfilled because in this region the constraint from X($\gamma$)-ray backgrounds is weaker than that from the critical density. This observation is none other than the reason why we have stressed in the introduction that the moduli could be the dark matter in our universe.

### III. X-RAY SPECTRUM FROM MODULI DARK MATTER

As shown in previous section, the modulus field with mass $m_\phi \simeq 10^{-2}$ keV – 200 keV is a candidate for the dark matter of our universe. This upper limit of the modulus mass originates from the constraint of the observed cosmic photon backgrounds. The modulus field decays most likely to two photons through non-renormalizable interaction suppressed by the gravity scale. \[5\] The lifetime of the modulus is estimated as \[4\]

$$\tau_\phi \simeq \frac{64\pi M_G^2}{b^2 m_\phi^3} \simeq 7.6 \times 10^{23} \text{ sec} \frac{1}{b^2} \left(\frac{1 \text{ MeV}}{m_\phi}\right)^3,$$

where $b$ denotes a parameter of order one which depends on the models of the superstring. In the following we take $b = 1$. From eq.(4) the modulus with mass $m_\phi \lesssim 100$ MeV has a lifetime longer than the age of the present universe. However, such modulus is continuously decaying at the rate of $1/\tau_\phi$ and produce photons which contribute to the diffuse photon backgrounds. This excludes the region 500 keV $\lesssim m_\phi \lesssim 1$ GeV. Furthermore, when we

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4 If we take the cut off scale $M_*$ in Eq.(1) as $M_* \gtrsim M_G$, which is a natural choice, only the modulus with $m_\phi \sim 100$ keV is cosmologically allowed for $m_\phi \lesssim 500$ keV and becomes the dark matter of our universe [1,8].

5 The modulus decay into two neutrinos is suppressed due to the chirality flip.
assume the cosmic modulus field as the dark matter of our universe ($\Omega_\phi \simeq \mathcal{O}(1)$), a region $10^{-2}\text{keV} \lesssim m_\phi \lesssim 200\text{ keV}$ survives from the cosmological constraints.

If the modulus field is indeed the dark matter, it would be the other contribution to the photon backgrounds. Some of the dark-matter moduli should be trapped in the halo of our galaxy. Since the Doppler spread due to the velocities of the moduli is negligible in this case, the narrow line spectrum is expected by the decay of the dark-matter moduli in our halo.

In this section we consider the dark-matter moduli with mass $m_\phi \sim 100\text{ keV}$ and investigate the X-ray spectrum of the produced photons in the decay, since such X-ray may be observable in the future experiments as we will describe below.

First we discuss the X-ray spectrum by the decay of the cosmic moduli distributed uniformly over the whole universe. Through the modulus decay two monochromatic photons with energy $E_\gamma = m_\phi/2$ are produced. When we integrate such line spectrum from the past to the present, the continuum spectrum below the energy $m_\phi/2$ is observed today. In Refs. [4,8,9] the photon flux from the decay of the moduli in the whole universe is estimated as

$$F_U(E_\gamma) \simeq \frac{1}{4\pi} \frac{2\Omega_\phi \rho_c}{m_\phi H_0} \left(\frac{2E_\gamma}{m_\phi}\right)^{3/2} f(m_\phi/2E_\gamma) \exp(-t(z = m_\phi/2E_\gamma - 1)/\tau_\phi), \quad (8)$$

with

$$f(x) = \left[\Omega_0 + (1 - \Omega_0 - \Omega_\Lambda)/x + \Omega_\Lambda/x^3\right]^{-1/2}, \quad (9)$$

where $\Omega_\Lambda$ is the density parameter of the cosmological constant, $z$ the redshift and $t(z)$ the cosmic time at $z$. This flux takes its maximal value at the photon energy

$$E_{\text{max}} \simeq \begin{cases} \frac{m_\phi}{2} & \text{for } \tau_\phi > t(z = 0) \\ \frac{m_\phi}{2} \left(\frac{3\tau_\phi H_0 \rho_c}{2}\right)^{2/3} & \text{for } \tau_\phi < t(z = 0) \end{cases} \quad (10)$$

In particular, when the modulus field is the dark matter, $\tau_\phi \gg t(z = 0)$ and $F_U(E_{\text{max}})$ is given by

$$F_U(E_{\text{max}}) = \frac{1}{4\pi} \frac{2\Omega_\phi \rho_c}{m_\phi H_0}. \quad (11)$$

It should be notice that this equation is independent of $\Omega_0$ and $\Omega_\Lambda$.

In Fig.1 we show the spectrum of the photon flux [8] for various moduli masses. Then the X-ray intensity from the whole universe is given by

$$I_U(E_\gamma) \simeq \frac{1}{E_\gamma} F_U(E_\gamma). \quad (12)$$

As shown in Fig.1, the flux from the moduli decay is comparable to the observed X-ray backgrounds if the mass of the modulus is $\sim 100\text{keV}$.

Next we consider the X-ray spectrum coming from the dark-matter moduli trapped in the our galactic halo. Since the cosmological redshift is negligible in this case, the photons produced by the decay have a monochromatic energy $m_\phi/2$. Here we estimate the intensity
of this line spectrum. The mass density of the halo of our galaxy at the distance \( r \) from the center of the galaxy is expressed as

\[
\rho_H(r) \simeq \frac{\rho_0}{1 + \frac{r}{r_c}},
\]

(13)

where \( r_c \simeq 2 \) kpc and the halo density in the solar neighborhood \( (r \simeq R_0 \simeq 8.5 \) kpc) is \( \rho_H(R_0) \simeq 0.38 \) GeV/cm\(^3\). Then the density of the modulus component in the halo is given by \( \rho_H(r) \times (\Omega_\phi/\Omega_0) \). Using the halo density (13) the line flux is estimated as

\[
F_H \simeq \frac{1}{4\pi \tau_\phi m_\phi} \frac{2}{\Omega_0} \int dx \frac{\Omega_\phi}{\Omega_0} \frac{\rho_0 r_c^2}{(x - R_0 \cos b \cos l)^2 + R_0^2 (1 - \cos^2 b \cos^2 l) + r_c^2},
\]

(14)

where \( x \) is the distance to the modulus particle from the sun and \( l (b) \) is the galactic longitude (latitude). After the \( x \) integration, we obtain the following expression

\[
F_H \simeq \frac{1}{4\pi \tau_\phi m_\phi} \frac{2}{\Omega_0} \frac{\Omega_\phi}{\Omega_0} \frac{R_0^2 + r_c^2 \rho_H(R_0)}{R_{eff}} \left[ \frac{\pi}{2} \tan^{-1} \left( \frac{R_0 \cos b \cos l}{R_{eff}} \right) \right],
\]

(15)

with \( R_{eff} = R_0^2 (1 - \cos^2 b \cos^2 l) + r_c^2 \). Then the diffuse line intensity from the galactic halo is given by

\[
I_H \simeq \frac{F_H}{\Delta E},
\]

(16)

where \( \Delta E \) denotes the energy resolution at \( E_\gamma \simeq m_\phi/2 \) of the experiment. Here it should be noted that the line flux depends on the direction of the incoming photon, i.e. \( b \)- and \( l \)-dependence, and that the intensity of the line X-ray spectrum from the galactic halo becomes more significant in the experiments with higher energy resolution, which contrasts to the continuous spectrum produced by the decay of the moduli in the whole universe.

Then we compare the line intensity from the moduli in our galactic halo to the maximum value of the X-ray intensity from the moduli that spread over the whole universe. For this end it is convenient to introduce the ratio \( R_I \) defined as

\[
R_I(b, l) = I_H/I_U(E_{max}).
\]

(17)

For the dark-matter moduli this ratio \( R_I \) is almost independent on the modulus mass \( m_\phi \) since we can neglect the exponential factor in eq.\((8)\). If we see the direction of the north or south galactic poles, this ratio becomes

\[
R_I(b = \pm\pi/2, l) \simeq 0.16 \frac{1}{\Omega_0} \left( \frac{E_{max}}{\Delta E} \right).
\]

(18)

In Fig.2 we show the contour of the ratio \( R_I \) in the \( b-l \) plane for the case \( m_\phi \simeq 200 \) keV (i.e. \( E_\gamma = 100 \) keV) and \( \Delta E/E = 10 \% \).\(^6\) We find that the line intensity is about

\(^6\)The X-ray backgrounds at energy \( E_\gamma \simeq 100 \) keV were measured by HEAO-I experiment \(^1\) whose energy resolution \( \Delta E/E \) is about 10 \%.\(^2\)
two times stronger than the peak of the continuum spectrum in the wide region of the sky. In Fig. 3 we also show the intensity of the line photons in the direction $b = \pi/2$ together with the continuous spectrum \(^{(12)}\) for the energy resolution $\Delta E/E = 10\%$ and $5\%$ and for the modulus mass $100$keV and $200$keV. It is seen that the photon intensity from the moduli in the whole universe is below or marginal to the observed one for $m_\phi = 100$keV or $200$keV, but the line intensity from the halo moduli is above the observed one. Thus, future experiments with energy resolution $\Delta E/E \lesssim 10\%$ at energy around $E_\gamma = m_\phi/2$ can detect the line intensity from the dark-matter moduli in our halo for $m_\phi \gtrsim 100$keV. Furthermore, by observing the $b$- or $l$-dependence of the X-ray intensity of the peak, we may confirm the origin of the peak, i.e. it originates from the line spectrum produced by the decay of the dark-matter moduli in the halo of our galaxy.

\section*{IV. $\gamma$-RAY SPECTRUM FROM COSMIC MULTI-GEV MODULI}

The cosmic modulus field with multi-GeV mass decays into two photons until the present. Since the produced photons are redshifted by the cosmic expansion, they form the continuum spectrum which takes it maximal value at MeV region. Thus the decay of the multi-GeV modulus field may be observed as a MeV-bump in the spectrum of the background $\gamma$-rays if the produced photons reach us directly. However, such high energy photons may be scattered off the background photons and its spectrum may be deformed. For the case of the multi-GeV modulus, we can neglect the double photon pair creation process: $\gamma + \gamma_{BG} \to e^+ e^-$ because the energy of the produced photons is below the effective threshold $E_\ast \simeq m_e^2/(22T)(m_e$: electron mass, $T$: background temperature) \(^{[11]}\). Thus we take into account only the photon photon scattering process: $\gamma + \gamma_{BG} \to \gamma + \gamma$ by which the emitted photons from the moduli lose their energy. Since the total cross section of the photon photon scattering is proportional to $E_\gamma^3$, ($E_\gamma$ denotes an energy of the emitted photon.), this process becomes significant only for modulus with mass larger than $O(1)$GeV. In this section we estimate the photon spectrum emitted from the multi-GeV moduli including the effect of the scattering with the background’s photons.

In order to obtain the photon spectrum we solve the following Boltzmann equation for the distribution function $f_\gamma$ \(^{[17]}\):

\begin{align}
\frac{\partial f_\gamma(E_\gamma)}{\partial t} &= \frac{1112}{10125} \frac{\alpha^4}{\pi m_\phi^8} \int_{E_\gamma}^{\infty} d\epsilon \gamma \int_{\epsilon_\gamma}^{\infty} d\epsilon' \gamma \left( 1 - \frac{E_\gamma}{\epsilon_\gamma} \right)^2 \left( \frac{E_\gamma}{\epsilon_\gamma} \right)^2 \int_{0}^{\infty} d\epsilon \gamma \frac{\epsilon^2}{\pi^2} \frac{\epsilon}{\epsilon_\gamma} \bar{f}(\tau) \\
&\quad - \frac{1946}{50625} \frac{\alpha^4}{\pi m_\phi^8} E_\gamma^3 f_\gamma(E_\gamma) \int_{0}^{\infty} d\epsilon \gamma \frac{\epsilon^2}{\pi^2} \bar{f}(\tau) \\
&\quad - 2H f_\gamma(E_\gamma) \\
&\quad + \frac{1}{4\pi \tau_\phi m_\phi} e^{-t/\tau_\phi} \delta \left( E_\gamma - \frac{m_\phi}{2} \right), \tag{19}
\end{align}

where $\bar{f}$ denotes the distribution function of the background photon at temperature $T$:

\begin{equation}
\bar{f}(\epsilon) = \frac{\epsilon^2}{\pi^2} \frac{1}{\exp(\epsilon/T) - 1}. \tag{20}
\end{equation}
We solve the Boltzmann equation (19) numerically including the evolution of the universe. The continuum $\gamma$-ray spectrum of the modulus decay is obtained using the present distribution function as $I_{\gamma}(E_{\gamma}) = f_\gamma(E_{\gamma})|_{t=t_0}$.

We show the spectra for $\Omega_\phi h^2 = 1$ and $m_\phi = 1, 10$ and 20 GeV in Fig.4. The effect of the photon photon scattering off the background photons is negligible for $m_\phi \sim 1$ GeV. On the other hand, for the modulus with mass $m_\phi \gtrsim 10$ GeV, we find that photons at the peak of the spectrum significantly lose their energy by the scattering and the peak of the spectrum moves to a lower energy region. Therefore, comparing with the observed background photon spectrum, it is found that the constraint becomes slightly weaker for the modulus with mass $m_\phi = \mathcal{O}(10)$ GeV than that obtained in Ref. [9].

V. CONCLUSION

In this paper we have examined the photon spectra from the decay of the cosmic modulus field. First we have considered the modulus mass region $m_\phi \simeq 10^{-2}$ keV–200 keV. This region is interesting because the modulus field can be the dark matter in our universe. We have calculated the X-ray continuum spectrum from the decay of the dark-matter moduli that spread homogeneously in the whole universe and the line spectrum from the dark-matter moduli trapped in the halo of our galaxy. It is found that with the energy resolution of the current experiments ($\sim 10\%$) the line intensity is about twice stronger than that of the continuum spectrum in the wide region of the sky. If the modulus mass is around 100 keV, both intensities are comparable with the present observed photon backgrounds. Therefore, in the future experiments with higher energy resolutions it may be possible to detect the line photons produced by the decay of dark-matter moduli in our halo. Moreover, by measuring the dependence of the line intensity on the galactic longitude and latitude, we will be able to confirm the origin, i.e. it comes from the halo of our galaxy rather than from the whole universe.

We have also investigated the $\gamma$-ray spectrum emitted from the decay of the multi-GeV modulus field. In this modulus mass region, the emitted photons are redshifted and have a peak in the MeV region of the spectrum. Thus we may observed those photons as a MeV-bump in the $\gamma$-ray backgrounds.

The produced high energy photon may be scattered off the background photons and lose their energy. It is found that the effect of the scattering is negligible for modulus with mass less than $\mathcal{O}(1)$ GeV. However, if the modulus mass is of the order of 10 GeV, the emitted photons at the peak of the continuum spectrum loses their energy by the scattering and the shape of the spectrum is significantly changed. This makes the constraint from the present observed $\gamma$-ray backgrounds weaker than the result in Ref. [9].

ACKNOWLEDGMENTS

We would like to thank T. Kamae for useful comments and encouragement.
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FIG. 1. The spectra of the photon flux from the decay of the cosmic moduli filling all the universe for various moduli masses. We take $\Omega_{\phi} h^2 = 1$. We also show the observed spectrum of the background photons by the thick solid line.
\[ m_\phi = 200 \text{ keV}, \Delta E/E = 10\% \]

FIG. 2. The contours of the ratio of the line intensity from our galactic halo moduli to the maximum value of the continuum spectrum from the whole universe moduli. We use the galactic coordinates \((b, l)\). We assume the energy resolution \(\Delta E/E = 10\%\) and \(m_\phi = 200\text{keV}\).
FIG. 3. The predicted intensity of the photons from the moduli in our halo (white region) and from the moduli in the whole universe (dark region) for the modulus mass $m_\phi = 100\text{ keV}$ and $200\text{ keV}$. We assume that the energy resolution $\Delta E/E = 10\%$ and $\Delta E/E = 5\%$. For the line spectrum of the our halo, we take the galactic latitude $b = \pi/2$. We also show the observed spectrum of the background photons by the thick solid line.
FIG. 4. The photon spectra from the decay of the modulus with mass $m_\phi = 1$ GeV (a), $m_\phi = 10$ GeV (b) and $m_\phi = 20$ GeV (c). The solid or dot-dashed line represents the case with or without the effect of the photon-photon scattering. We take $\Omega_\phi h^2 = 1$. 