TESTING PROPERTIES OF GRAPHS AND FUNCTIONS

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ABSTRACT

We define an analytic version of the graph property testing problem, which can be formulated as studying an unknown 2-variable symmetric function through sampling from its domain and studying the random graph obtained when using the function values as edge probabilities. We give a characterization of properties testable this way, and extend a number of results about “large graphs” to this setting.

These results can be applied to the original graph-theoretic property testing. In particular, we give a new combinatorial characterization of the testable graph properties. Furthermore, we define a class of graph properties (flexible properties) which contains all the hereditary properties, and generalize various results of Alon, Shapira, Fischer, Newman and Stav from hereditary to flexible properties.

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Graph property testing is a very active area in computer science. In its most restricted form (and this will be our concern in this paper), it studies properties of (very large) graphs that can be tested by studying a randomly chosen induced subgraph of bounded size.

To be more precise, we have to describe what kind of error is allowed. In this paper, by graph we always mean a finite simple graph. A graph property is a class of graphs invariant under isomorphism. The edit distance of two graphs $G_1, G_2$ on the same node set is $|E(G_1) \triangle E(G_2)|$. The edit distance of a graph $G$ from a graph property is the minimum number of edges we have to change (add or delete) to obtain a graph with the property. If no graph with the same number of nodes has the property, then this distance is infinite.

**Definition 1.1:** A graph property $\mathcal{P}$ is **testable**, if there exists another property $\mathcal{P}'$ (called a test property) satisfying the following conditions:

(a) if a graph $G$ has property $\mathcal{P}$, then for all $1 \leq k \leq |V(G)|$ at least a fraction of $2/3$ of its $k$-node induced subgraphs have property $\mathcal{P}'$, and

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