Temporal control of light is a long-standing dream, which has recently demonstrated its potential to revolutionize optical and microwave technology, as well as our understanding of electromagnetic theory, overcoming the stringent constraint of energy conservation \[1\]. Along with the ability of time-dependent systems to violate electromagnetic reciprocity \[2–4\], realizing photonic isolators and circulators \[5–8\], amplify signals \[9\], perform harmonic generation \[10–12\] and phase modulation \[13\], new concepts from topological \[14–16\] and non-Hermitian \[17, 18\] physics are steadily permeating this field. However, current limitations to the possibility of significantly fast modulation in optics has constrained the concept of time-dependent electromagnetic to the radio frequency domain, where varactors can be used to modulate capacitance \[19\], and traveling-wave tubes are commonly used as (bulky) microwave amplifiers \[20\]. In the visible and near IR, optical nonlinearities have often been exploited to generate harmonics, and realize certain nonreciprocal effects \[21\]. However, nonlinearity is an inherently weak effect, and high field intensities are typically required.

In this Letter, we challenge the very need for high modulation frequencies, demonstrating that strong and broadband nonreciprocal response can be obtained by complementing the temporal periodic modulation of an electromagnetic medium with a spatial one, in such a way that the resulting traveling-wave modulation profile appears to drift uniformly at the speed of the wave, i.e., a luminal modulation. We show that unidirectional amplification and compression can be accomplished in luminal metamaterials, which thus constitute a broadband generalization of the narrowband concept of the parametric oscillator, enabling harmonic generation with exponential efficiency. We present a realistic implementation based on acoustic plasmons in double-layer graphene (DLG), that constitute a broadband nonreciprocal response can be obtained by introducing a traveling-wave modulation of the material parameters of the modulation is of a travelling-wave type, whereby Bragg scattering couples Fourier modes which differ by a discrete amount of both energy and momentum \[2, 7, 22–25\]. As shown in Fig. 1 for a 1D system, these space-time reciprocal lattice vectors can be defined to take any angle in phase space, depending on whether a generic traveling-wave modulation of the material parameters of the form \(\delta \epsilon (gx - \Omega t)\) is spatial (panel a: \(\Omega = 0\)), temporal (d: \(g = 0\)), or spatiotemporal (b,c: \(g \neq 0, \Omega \neq 0\)). Given the slope \(c\) of the bands in a Brillouin diagram, which denotes the velocity of waves in a dispersionless
medium, the speed of the traveling-wave modulation defines a subluminal regime \( \frac{\Omega}{g} < c \), whereby conventional vertical band gaps open \([22]\), and a superluminal one \( \frac{\Omega}{g} > c \), characterized by horizontal, unstable \( k \)-gaps \([23, 26]\). A common example of the latter is the parametric amplifier \((g = 0; \text{Fig. 1})\): when the parameters governing an oscillatory system are periodically driven at twice its natural frequency, exponential amplification occurs, as a result of the unstable \( k \)-gap at frequency \( \omega = \Omega/2 \). However, achieving such fast modulation at infrared frequencies remains a key challenge for dynamical metamaterials.

The transition between the regimes in Figs. 1b and 1c, i.e. \( \Omega/g = c \), is an exotic degenerate state that we name luminal metamaterial, whereby all forward-propagating modes are uniformly coupled. Due to its broadband spectral degeneracy in the absence of dispersion, this system is highly unstable, thus preventing a meaningful definition of its band structure. Nevertheless, if we consider transmission through a spatially (temporally) finite system with well-defined boundary conditions, causality can be imposed in the unmodulated regions of space (time), so that an expansion into eigenfunctions can be performed, as detailed in the S.M. \([27]\). In luminal metamaterials, the photonic transitions induced by the modulation of the refractive index are no longer interband \([28]\), but intraband, and can therefore be driven by means of any refractive index modulation, regardless of how adiabatic, whose reciprocal lattice vector \((g, \Omega)\) satisfies the speed-matching condition \( \Omega/g = c \). Hence, any limitation in modulation frequency \( \Omega \) can be compensated, in principle, by a longer spatial period \( L = 2\pi/\Omega \). Notably, these can be locally induced by modulating the properties of the medium, and can thus synthetically move at any speed, including and exceeding the speed of light, in analogy with the touching point of a water wave front propagating almost perpendicularly to a beach, or the junction between the blades of a pair of scissors.

In real space, amplification in this system can be modelled as follows: consider a non-dispersive, lossless medium where \( \varepsilon(x,t) = 1 + 2\alpha \cos(gx - \Omega t) \), with \( \Omega/g = \alpha_0 \). Following the derivation of Poynting’s theorem, we can write:

\[
\nabla \cdot (\mathbf{E} \times \mathbf{H}) = -\frac{\mu_0}{2} \frac{\partial H^2}{\partial t} - \frac{\varepsilon_0 \varepsilon}{2} \frac{\partial E^2}{\partial t} - \varepsilon_0 \frac{\partial \varepsilon}{\partial t} E^2,
\]

so that the total time-derivative of the local energy density is:

\[
\frac{dU}{dt} = -\frac{1}{\varepsilon} \frac{\partial \varepsilon}{\partial t} U - \frac{\partial P}{\partial x} + \alpha_0 \frac{\partial U}{\partial x} = -\frac{1}{\varepsilon} \frac{\partial \varepsilon}{\partial t} U - \frac{\partial P'}{\partial x},
\]

where the compensated Poynting vector, \( P' \), consists of a local and an advective part (due to the moving frame) \([27]\). The first term in Eq. 2 is responsible for gain, whereas the second describes the Poynting flux, which drives the compression of the pulse. Ignoring the Poynting contribution to zero-order yields \( U(X,t) = e^{-2\alpha t \sin(gX)} \), where \( X = x - \Omega t/g \). Feeding the zero-order solution into the resulting compensated Poynting vector \( P' = \alpha_0 (\varepsilon(X,t)^{-\frac{3}{2}} - 1)U \) in Eq. 2 we obtain a

[FIG. 2. (a) An incident plane wave is concentrated and exponentially amplified as it propagates through a luminal metamaterial \((g = \Omega = 1, \alpha = 0.04)\) of length \( d \) (inset), at whose exit the field is calculated. Continuous lines correspond to Floquet-Bloch theory, whereas dashed lines and circles were obtained from our analytic model to zeroth and first \((\text{Eq. } 3)\) order respectively. (b) Waves preceding (following) the gain-point \((\Omega t = \pi/2)\) experience a lower (higher) permittivity, hence a higher (lower) phase velocity, thus being attracted towards the gain-point, at which amplification occurs. Conversely, waves preceding (following) the loss-point \((\Omega t = 3\pi/2)\) are drawn away from it, depleting it of energy. (c) An incident monochromatic wave with input frequency \( \omega_0 = \Omega \) is efficiently coupled to higher harmonics at an exponential rate. Beating arises from the different \( \Omega \) and \( \omega_0 \) (d) The frequency content (log-scale) of a DC input applied to a luminal metamaterial spreads out exponentially in Fourier space, generating a supercontinuum.]
corrected expression for the energy density:

\[ U(X, t) = \exp[-2\alpha\Omega \sin(gX) - \alpha^2\Omega^2 t^2 \cos^2(gX)]. \]  

(3)

Alternatively, the system can also be modelled with a semi-analytic Floquet-Bloch expansion of the fields, and the transmission coefficient can be calculated for a finite slab, validating our analytical expressions [27]. Assuming a slab of length \( d \), and substituting \( \Omega = g d \) in Eq. 3 we calculate the temporal profile of the electric field intensity at the output \( x = d \) (Fig. 2a). The modulation is able to exponentially amplify and concentrate the signal at the point with phase \( \Omega = \pi/2 \), and exponentially suppress it at \( \Omega = 3\pi/2 \). The reason is apparent from Fig. 2b: those field amplitudes which sit at \( -\pi/2 < \Omega t < \pi/2 \) experience a lower permittivity, and hence a higher phase velocity, whereas those sitting at \( \pi/2 < \Omega t < 3\pi/2 \) lag, so that the point corresponding to a phase \( \Omega = \pi/2 \) acts as an attractor, or gain-point, where the modulation imparts energy into the wave. Conversely, \( \Omega = 3\pi/2 \) is a repellor, or loss-point, where energy is absorbed by the modulation drive (further numerical simulations are provided in the S.M. [27]).

As evidenced by the absence of any frequency dependence in Eq. 3 and in contrast to conventional time-modulated systems, parametric amplification in a luminal medium is a fully broadband phenomenon, enabling exponentially efficient generation of frequency-wavevector harmonics, as shown in Fig. 2. Remarkably, even a DC input can be transformed into a broadband pulse train at an exponential rate, as revealed by Floquet-Bloch calculations (see Fig. 2a). Our closed-form analytic solution enables us to exactly quantify the power amplification rate as \( 2\alpha \Omega \), which needs to overcome the loss, for amplification to occur. However, the reactive behaviour responsible for the compression performance is unaffected by losses, which only reduce the overall output power efficiency. Furthermore, these systems are transparent to counterpropagating waves, thus entailing the additional advantage of nonreciprocity. Moreover, while nonreciprocal response is typically observed only near band gaps in conventional systems [2], it is achieved at virtually any frequency in a luminal metamaterial.

Due to their ease of manipulation, metasurfaces offer the most promising playground to realize dynamical effects [1 29 30], also due to the rise of tunable two-dimensional materials [31 32]. Recently, graphene has emerged as a platform to enhance light-matter interactions [33 34], realizing atomically thin metasurfaces [13 35 40]. Its doping level, which can be tuned with ion-gel techniques to be as high as 2 eV [41, 42], has emerged as a platform to enhance light-matter interactions [33–36], realizing atomically thin metasurfaces [31, 32]. Recently, graphene has also been theoretically proposed, such as drift currents [49–51], periodic doping modulation [52], adiabatic doping suppression [53], and plasmonic Čerenkov emission by hot carriers [54].

We assume a semiclassical (Drude) conductivity model, which is accurate as long as \( \hbar \omega \ll \epsilon_F \) and \( k \ll k_F \). Our setup consists of two graphene layers, whose Fermi levels are modulated as \( \epsilon_F(x, t) = \epsilon_{F,0}[1+2a \cos(gx-\Omega t)] \) (Fig. 3). Dispersion is accounted for, by expressing the constitutive relation for the current \( J(x, t) \) in Fourier space, where the conductivity modulation couples neighbouring frequency harmonics:

\[ J_n = \frac{e^2}{\pi \hbar^2} \frac{E_n}{\gamma - i(\omega + n\Omega)}, \]  

(5)

where \( \gamma \) is the loss rate and \( E_n \) is the \( n^{th} \) Fourier amplitude of the in-plane electric field, which is continuous.
at the layer positions \( z = 0 \) and \( z = \delta_0 \), as detailed in the S.M. [27]. The magnetic field of the p-polarized wave \( H_p(x, z, t) \) is discontinuous at the layers by the surface current [45]. This system can be accurately described within an adiabatic regime, since the modulation frequency \( \Omega \ll \omega \). Furthermore, since acoustic plasmons carry much larger momentum than photons, the modes are strongly quasistatic, so that the out of plane decay constant \( \kappa_n \approx k + ng \), and coupling to radiation is negligible, given that both spatial and temporal frequencies of the doping modulation are much smaller than the plasmon wavevector and frequency. Taking advantage of the adiabatic assumption, we can conveniently solve the scattering problem in the time-domain, as detailed in the S.M. [27].

In our calculations we assume a Fermi energy \( \epsilon_F = 1.5 \) eV \( \approx 2\pi \hbar \times 362 \) THz, and a loss rate \( \gamma = \frac{\nu_F^2 e}{mc \epsilon_F \hbar} \approx 60 \) GHz, where \( m = 10^6 \) cm\(^2\)/(V-s) is the electron mobility, and the Fermi velocity \( v_F \approx 9.5 \times 10^8 \) m/s. Fig. 3 demonstrates plasmon amplification and compression for different modulation times \( T_f \). Here, we use a modulation amplitude \( \alpha = 0.05 \), interlayer gap \( \delta_0 = 1 \) nm, for an input frequency \( \omega/2\pi = 1 \) THz and a modulation frequency \( \Omega/2\pi = 120 \) GHz, which corresponds to a modulation period \( \tau = 2\pi/\Omega \approx 8 \) ps and length \( L \approx 26 \) μm, such that the long-wavelength phase velocity of the plasmon is matched by the modulation speed \( c_p = \Omega/g \). Since the DLG plasmon bands are approximately linear, we can set \( c_0 = c_p \) in our closed-form solution (Eq. 3), and verify the analogous amplification mechanism, showing excellent agreement with Floquet-Bloch theory (Fig. 3c). Finally, Fig. 3d demonstrates the total power amplification achieved by our luminal graphene metasurface: initially the unit input power of the wave is predominantly dissipated by the uniform losses, except at the gain-point, so that this first propagation moment is dominated by damping. Once sufficient power is accumulated at the gain-point, the energy fed by the modulation into the plasmon ensures that its propagation is effectively loss-compensated, as in the case of \( \alpha = 0.08 \), extending its lifetime by orders of magnitude, or even amplifying it, as in the \( \alpha = 0.1 \) case.

As the luminal modulation couples the frequency content of the pulse to very high frequency-wavevector harmonics, these will experience the nonlinearity of the bands. In Fig. 4 we use a wider inter-layer gap \( \delta_0 = 15 \) nm and higher mobility \( m = 10^6 \) cm\(^2\)/(V-s), to highlight the effects of dispersion on the pulse profile (a) and its spectral content (b) for different modulation times \( T_f \). At a first stage, since higher frequency components experience a slightly lower phase velocity, the gain-point must shift back to \( gX < \frac{k_\perp}{\pi} \), where the increase in local phase velocity determined by the modulated Fermi energy compensates for the curvature of the band (Fig. 4 inset). In addition, Fourier components propagating with phase velocity \( c_p \) are amplified near the conventional gain-point, thus skewing the pulse \( (T_f = 5\tau) \). Finally, for even longer propagation times, the wave will cease to compress, and break into a train of pulses. This is due to the existence of a finite regime of phase velocities: \((1 + 2\alpha)^{-1/2} < v_p/c_p < (1 - 2\alpha)^{-1/2}\), within which the interaction between co-propagating bands is strongly coupled to higher harmonics, and its power spectrum is effectively reflected, resulting in beating. Thus, dispersion plays the important role of stabilizing these systems. Subsequently, the power spectrum oscillates within the extended luminal region, although beating between different space-time harmonics, no longer in-phase, induces fast oscillations, reminiscent of comb formation in nonlinear optics [55].

In this Letter, we have introduced the concept of luminal metamaterials, realized by inducing a traveling-wave modulation in the permittivity of a material, whose phase

\[ v_p(k_c)/c_p = \frac{(\omega(k_c) / k)}{\left(\frac{\Omega}{g}\right)} \approx \left(\frac{1 - e^{-\delta_0k}}{\delta_0k}\right)^{1/2}, \]

decreases approximately linearly with increasing wavevector [27]. Equating the latter to the lower threshold velocity ratio \( v_p(k_c)/c_p = 1/(1 + 2\alpha) \), where \( \alpha = 0.05 \) and expanding the exponential to second order, we get an analytical estimate for the critical wavevector \( k_c \approx 13 \) μm, beyond which the pulse is no longer strongly coupled to higher harmonics, and its power spectrum is effectively reflected, resulting in beating.

![Figure 4](image_url)
velocity matches that of the waves propagating in it, in the absence of modulation. We have shown that these dynamical structures generalize the concept of parametric amplification to cover a virtually unlimited bandwidth, thus being capable of reinforcing and compressing input waves of any frequency, including a DC field. We have demonstrated their robustness against moderate dispersion, and proposed a realistic implementation exploiting acoustic plasmons in double-layer graphene, thus paving a new viable route towards the amplification of graphene plasmons, and terahertz generation. Furthermore, luminal metamaterials exhibit an inherent, strongly nonreciprocal response at any frequency, due to the directional bias induced by the modulation, whose phase velocity can be made as high as needed by extending the spatial period of the modulation, the only limitation being the propagation length of the excitation, and hence the loss. Furthermore, thanks to its ability to couple incident electromagnetic waves to higher frequency-momentum harmonics at an exponential rate, the luminal metamaterial concept constitutes a fundamentally new path towards efficient harmonic generation, which can work even with a DC input, necessitating only of low modulation speeds, as opposed to conventional parametric systems. Finally, we remark that this concept can be translated to any wave system which exhibits a linear or weakly dispersive regime, such as acoustic, elastic and shallow-water waves, and the reach of this mechanism could be further extended by introducing chirping, in analogy with the tuning of the frequency of a driving field with the energy of electrons accelerated in a synchrocyclotron.

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