Gravitational Excitons as Dark Matter *

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01.11.2000

Abstract

In earlier work it was pointed out that for warped product spacetimes the conformal (geometrical moduli) excitations of the internal compactified factor spaces should be observable as massive scalar fields in the external spacetime. Here we show that these scalar fields (gravitational excitons) describe weakly interacting particles and can be considered as dark matter component. Masses of the gravexcitons are defined by the form of the effective potential of the theory and the stabilization scales of the internal space. This implies that different stabilization scales result in different types of DM. An essential role is played by the effective potential. On the one hand, its minima fix possible stabilization scales of the internal spaces; on the other hand, they provide possible values for the effective cosmological constant.

PACS number(s): 04.50.+h, 98.80.Hw

1 Introduction

Modern observations lead to the conclusion that the cosmological constant \( \Lambda \) contributes about 0.7 to the total \( \Omega \), i.e., the cosmological constant is non-zero and positive. Matter contributes another 0.3 to \( \Omega \) and most of it consists of Dark Matter (DM). This implies that the Universe is spatially flat to a good approximation. The claim that the Universe is spatially flat is extremely strong: it results from the position of the first Doppler/Sakharov peak in the CMB anisotropy. This position is very well measured, notably, by the Boomerang experiment. One of the most important problems in modern cosmology is to construct a viable theoretical model which naturally explains the observed value of the cosmological constant and the DM contribution to \( \Omega \).

In the present paper we show with the help of a simple toy model that extra dimensions could give a good background for the resolution of these problems. More concrete, conformal excitations of internal spaces (geometrical moduli excitations) will propagate in our Universe as massive scalar fields (gravexcitons) and play the role of DM. Minima of their potential energy are treated as the \( \Lambda \) term.

*Report given at the Conference on Cosmology and Particle Physics, CAPP 2000, Verbier, Switzerland, July 17-28, 2000.
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2 Noninteracting Gravitational Excitons

We consider a cosmological toy model with metric

\[ g = g^{(0)} + \sum_{i=1}^{n} e^{2\beta^i(x)} g^{(i)}, \]  

(2.1)

which is defined on a manifold with warped product topology

\[ M = M_0 \times M_1 \times \ldots \times M_n. \]  

(2.2)

Here \( x \) denotes some coordinates of the \( D_0 = (d_0 + 1) \)-dimensional manifold \( M_0 \) and

\[ g^{(0)} = g^{(0)}_{\mu \nu}(x) dx^\mu \otimes dx^\nu. \]  

(2.3)

Let manifolds \( M_i \) be \( d_i \)-dimensional Einstein spaces with metric \( g^{(i)} \), i.e.

\[ R_{mn}[g^{(i)}] = \lambda^i g^{(i)}_{mn}, \quad m, n = 1, \ldots, d_i \quad \text{and} \quad R[g^{(i)}] = \lambda^i d_i = R_i. \]  

(2.4)

For constant curvature spaces the parameters \( \lambda^i \) are normalized as \( \lambda^i = k_i(d_i - 1) \) with \( k_i = \pm 1, 0 \). Later on we shall not specify the structure of the spaces \( M_i \). We require only \( M_i \) to be compact spaces with arbitrary sign of curvature.

With total dimension \( D = D_0 + \sum_{i=1}^{n} d_i \), \( \kappa^2_D \) a \( D \)-dimensional gravitational constant, and \( \Lambda \) - a \( D \)-dimensional cosmological constant, we consider an action of the form

\[ S = \frac{1}{2\kappa^2_D} \int_M d^D x \sqrt{|g|} \{ R[g] - 2\Lambda \} + S_m. \]  

(2.5)

\( S_m \) is a not specified action-term of matter fields. To illustrate the origin of gravexcitons it is sufficient to consider a pure geometrical model with \( S_m \equiv 0 \). Generalizations to models with included matter are obvious, and for different types of matter they can be found in our papers [1, 2].

Let \( \beta^0_0 \) be the scale of compactification of the internal spaces at present time. Instead of \( \beta^i \) it is convenient to introduce the shifted quantity: \( \tilde{\beta}^i = \beta^i - \beta^0_0 \).

Then, after dimensional reduction and conformal transformation

\[ g^{(0)}_{\mu \nu} = \Omega^2 \tilde{g}^{(0)}_{\mu \nu} := \left( \prod_{i=1}^{n} e^{4d_i \tilde{\beta}^i} \right) \tilde{g}^{(0)}_{\mu \nu} \]  

(2.6)

action (2.5) reads

\[ S = \frac{1}{2\kappa^2_0} \int_{M_0} d^{D_0} x \sqrt{|\tilde{g}^{(0)}|} \left\{ \tilde{R}[\tilde{g}^{(0)}] - \tilde{G}_{ij} \tilde{g}^{(0)\mu \nu} \partial_\mu \tilde{\beta}^i \partial_\nu \tilde{\beta}^j - 2U_{eff} \right\}, \]  

(2.7)

where \( \tilde{R}_i := R_i e^{-2\tilde{\beta}^i} \), \( \tilde{G}_{ij} = d_i \delta_{ij} + \frac{\kappa^2_D}{V_D} d_i d_j \) is the midisuperspace metric and \( \kappa^2_0 := \kappa^2_D/V_{D'} \) denotes the \( D_0 \)-dimensional gravitational constant. \( (V_{D'}) \) is the total volume of the internal space.) If we take the TeV scale \( M_{TeV} \sim 1 \) TeV and the Planck scale \( M_{Pl} \sim 1.22 \times 10^{19} \) GeV as fundamental scales for the \( D \)-dimensional total spacetime and the 4-dimensional large scale spacetime, respectively: \( \kappa^2_0 = 8\pi/M_{Pl}^{2+2D'} \), \( \kappa^2_D = 8\pi/M_{Pl}^2 \) then we reproduce the well known relation [3]: \( M_{Pl} = V_{D'} M_{TeV}^{2+2D'}/V_{D'} \). Thus, the compactification scale of the internal space is fixed and of order

\[ a \sim V_{D'}^{1/D'} \sim 10^{32/D'-17} \, \text{cm}. \]  

(2.8)

The effective potential in (2.7) reads

\[ U_{eff}[\tilde{\beta}] = \left( \prod_{i=1}^{n} e^{4d_i \tilde{\beta}^i} \right) \left\{ -\frac{1}{2} \sum_{i=1}^{n} \tilde{R}_i e^{-2\tilde{\beta}^i} + \Lambda \right\}. \]  

(2.9)
With the help of a regular coordinate transformation \( \varphi = Q \tilde{\beta} \), \( \tilde{\beta} = Q^{-1} \beta \) midisuperspace metric (target space metric) \( G \) can be transformed to a pure Euclidean form: \( G_{ij} d\tilde{\beta}^i \otimes d\tilde{\beta}^j = \sigma_{ij} d\varphi^i \otimes d\varphi^j = \sum_{i=1}^n d\varphi^i \otimes d\varphi^i \), \( \sigma = \text{diag}(+1,+1,\ldots,+1) \) (see e.g. [1]).

Clearly, a stabilization of the internal spaces can be achieved if the effective potential \( U_{eff} \) has a minimum with respect to fields \( \tilde{\beta}^i \) (or fields \( \varphi^i \)). In general it is possible for the potential \( U_{eff} \) to have more than one extremum. For the pure geometrical toy model under consideration we can get only one extremum, which with respect to fields \( \tilde{\beta}^i \) plays the role of an effective cosmological constant in the external spacetime. These equations show that for our specific model a global minimum can only exist in the case of compact internal spaces with negative curvature \( R_k < 0 \) (\( k = 1, \ldots, n \)). The effective cosmological constant is then also negative: \( \Lambda_{eff} < 0 \). Models which include matter can have minima for internal spaces of positive curvature, and the effective cosmological constant in this case is usually positive.

For small fluctuations of the normal modes in the vicinity of the minima of the effective potential, action \( S \) reads

\[
S = \frac{1}{2\kappa_0} \int \frac{d^Dx}{M_0} \sqrt{|\tilde{g}^{(0)}|} \left\{ \tilde{R} \tilde{g}^{(0)} - 2\Lambda_{eff} \right\} - \frac{1}{2} \int \frac{d^Dx}{M_0} \sqrt{|\tilde{g}^{(0)}|} \left\{ \sum_{i=1}^n \left( \tilde{g}^{(0)}_{\mu\nu} \psi^i_{\mu} \psi^i_{\nu} + m_i^2 \psi^i \psi^i \right) \right\}.
\]

(For convenience we use here the normalizations: \( \kappa_0^{-1} \tilde{\beta} \rightarrow \tilde{\beta} \).) Thus, conformal excitations of the metric of the internal spaces behave as massive scalar fields developing on the background of the external spacetime. In analogy with excitons in solid state physics where they are excitations of the electronic subsystem of a crystal, we called the excitations of the subsystem of internal spaces gravitational excitons \([1]\). Later, since \([3]\) these particles are also known as radions.

From eq. \((2.10)\) follows that

\[
|\Lambda_{eff}| \sim m_i^2 \sim a_{(0)i}^{-2},
\]

where \( a_{(0)i} = \exp \beta_0^i \) are the scale factors of stabilized internal spaces.

The calculations above were performed in a model with the TeV scale \( M_{TeV} \) as fundamental scale of the \( D \)-dimensional theory (see eq. \((2.8)\)). Clearly, it is also possible to choose the Planck scale as the fundamental scale.

For this purpose, we will not fix the compactification scale of the internal spaces at their present time values. We consider them as free parameters of the model and demand only that \( L_{Pl} < a_{(0)i} = e^{\beta_0^i} < L_F \sim 10^{-17} \text{cm} \). So, we do not transform \( \beta^i \) to \( \tilde{\beta}^i \). In this case, \( \kappa_D^2 \sim M_{Pl}^{(2+D)} \) so that the Planck scale becomes the fundamental scale of the \( D \)-dimensional theory. In this approach eqs. \((2.3)\), \((2.7)\) and \((2.9)\) preserve their form, with only substitutions \( \tilde{\beta} \rightarrow \beta \) and \( \tilde{R}_t \rightarrow R_t \). The Einstein frame metrics of the external spacetime in both approaches are equivalent to each other up to a numerical prefactor:

\[
\tilde{g}_{\mu\nu}^{(0)}|_{TeV} = v_0^{-2/(D_0-2)} \tilde{g}_{\mu\nu}^{(0)}|_{Pl},
\]

where \( v_0 = \prod_{i=1}^n \exp(d_i \beta_0^i) \). Obviously, the same rescaling takes place for the masses squared of the gravitational excitons, and the effective cosmological constant:

\[
m_i^2 \rightarrow (v_0)^{-2/(D_0-2)} m_i^2 \quad \text{and} \quad \Lambda_{eff} \rightarrow (v_0)^{-2/(D_0-2)} \Lambda_{eff}.
\]

Thus, in the latter approach we get instead of \((2.13)\) the relation:

\[
|\Lambda_{eff}| \sim m_i^2 \sim (a_{(0)i})^{-(D-2)},
\]

\[\text{(2.15)}\]
where we set $D_0 = 4$. This expression shows that due to the power $(2-D)$ the effective cosmological constant and the masses of the gravitational excitons can be very far from planckian values, even for scales of compactification of the internal spaces close to the Planck length.

Let us return to the comparison of the TeV scale and the Planck scale approaches. E.g., within the TeV scale approach for $6 \leq D < \infty$ the internal space scale factors, gravexciton masses and effective cosmological constant run, correspondingly, as: $10^{-11} \text{cm} \leq a_{(0)1} < 10^{-17} \text{cm}$, $10^{-4} \text{eV} \leq m_i < 1 \text{TeV}$ and $10^{-64} \Lambda_{Pl} \leq |\Lambda_{eff}| \leq 10^{-32} \Lambda_{Pl}$. For this approach the scale factors of the internal spaces are defined by eq. (2.3), due to the requirement that the $D$-dimensional gravitational constant is of order of the TeV scale. In the Planck scale approach this condition is absent, and $a_{(0)i}$ are free parameters. Let us take, e.g., $a_{(0)1} \sim 10^{-18} \text{cm}$. Then, within the Planck scale approach for $6 \leq D \leq 10$ the gravexciton masses and the effective cosmological constant run correspondingly as: $10^{-7} \text{eV} \leq m_i \leq 10^{-32} \text{eV}$ and $10^{-60} \Lambda_{Pl} \leq |\Lambda_{eff}| \leq 10^{-120} \Lambda_{Pl}$.

These estimates show that for the TeV scale approach the effective cosmological constant is much greater than the present day observable limit $\Lambda \leq 10^{-122} \Lambda_{Pl} \sim 10^{-57} \text{cm}^{-2}$ (for our model $|\Lambda_{eff}| |_{\text{TeV}} \geq 10^2 \text{cm}^{-2}$), whereas in the Planck scale approach we can satisfy this limit even for very small compactification scales. For example, if we demand in accordance with observations $|\Lambda_{eff}| \sim 10^{-122} \Lambda_{Pl}$ then eq. (2.13) gives a compactification scale $a_{(0)1} \sim 10^{22/(D-2)} L_{Pl}$. Thus, $a_{(0)1} \sim 10^{15} L_{Pl} \sim 10^{-18} \text{cm}$ for $D = 10$ and $a_{(0)1} \sim 10^5 L_{Pl} \sim 10^{-28} \text{cm}$ for $D = 26$, which is not in contradiction to observations because for this approach the compactification scales should be $a_{(0)1} \leq 10^{-17} \text{cm}$. Assuming an estimate $\Lambda_{eff} \sim 10^{-122} L_{Pl}$, we automatically get from eq. (2.13) the value of the gravitational exciton mass: $m_1 \sim 10^{-61} M_{Pl} \sim 10^{-33} \text{eV} \sim 10^{-66} \text{eV}$, which is extremely light. Nevertheless such light particles are not in contradiction with observations, because these particles do not overclose the Universe $\[2\].

3 Interacting Gravitational Excitons

By definition, Dark Matter (DM) consists of particles interacting with usual matter mainly via gravitational forces. Thus, to define gravexcitons as DM we should show that their interaction with usual matter is very weak. Here, we do this with respect to their interaction with electromagnetic (e.m.) fields. As we saw above, gravexcitons are neutral particles and for this reason cannot interact with e.m. fields via current. Their interaction with e.m. fields originates in their multidimensional nature, and can be easily understood if we consider a zero-mode approximation of the e.m. fields in the multidimensional spacetime. Due to the multidimensional determinant of the metric (which depends on the scale factors of the internal space), the dimensionally reduced action $S_m$ will contain nonlinear terms describing the interaction between matter and gravexcitons. In the simplest case of one internal space ($n = 1$), the gravexciton-photon interaction is in lowest order approximation described by the term $[1]:$

$$2 \sqrt{\frac{d_i}{(D_0 - 2)(D - 2)}} \kappa_0 \psi F_{\mu\nu} F^{\mu\nu}.$$  

A corresponding first order diagram of this interaction describes the decay of gravexcitons into photons: $\psi \rightarrow 2\gamma$. The probability of this decay is easily estimated as

$$\Gamma = \frac{2d_i}{d_i + 2 M_{Pl}^3} \frac{m^3}{d_i + 2} \frac{2d_i}{d_i + 2} \left( \frac{m}{M_{Pl}} \right)^3 \frac{1}{T_{Pl}},$$

which results in a life-time of the gravitational excitons with respect to this decay

$$\tau = \frac{1}{\Gamma} = \frac{d_i + 2}{2d_i} \left( \frac{M_{Pl}}{m} \right)^3 T_{Pl}.$$  

These relations show that gravexcitons with masses $m \leq 10^{-21} M_{Pl} \sim 10^{-2} \text{GeV} \sim 20 m_e$ (where $m_e$ is the electron mass) have a life-time $\tau \geq 10^{16} \text{sec} > t_{\text{min}} \sim 10^{18} \text{sec}$, which is greater than the age of the Universe. Thus, they are stable particles with respect to this process. In other words, such gravexcitons interact weakly (Planck scale suppressed) with e.m. fields and can be considered as DM. Similar estimates hold for the interaction

\[1\] In the case of matter located on a 4-dimensional brane (which is simulated by delta-function-fixing of the brane position in the multidimensional space) such interaction terms exist also due to the time dependence of the scale factors of the internal space.
of gravexcitons with other types of matter. The type of the DM depends on the DM particle masses. It is hot for $m_{DM} \leq 50 - 100\text{eV}$, warm for $100\text{eV} \leq m_{DM} \leq 10\text{KeV}$ and cold for $m_{DM} \geq 10 - 50\text{KeV}$. Gravexciton masses are closely related with the compactification scales of the internal space (see (2.13) and (2.15)). As shown in the previous section, gravexcitons may be hot DM as well as cold DM, depending on $a_{(0)i}$. However, gravexcitons with masses $m \geq 10^{-56}M_{Pl}\left(M_{Pl}/\varphi_{in}\right)^{4}$ (3.4) overclose the Universe, i.e. their energy density at the present time is greater than the critical energy density of the Universe. Usually, it is assumed that the amplitude of initial gravexciton oscillations $\varphi_{in} \sim O(M_{Pl})$ can be considerably less than $M_{Pl}$ (although it depends on the form of $U_{eff}$). If we assume $\varphi_{in} \sim O(M_{Pl})$ then excitons with masses $m \lesssim 10^{-28}\text{eV}$ will not overclose the Universe. Thus, gravexcitons are either very hot DM with masses $m \leq 10^{-28}\text{eV}$ (and with negligible contribution to the total amount of Dark Matter) or the amplitude of initial oscillations is $\varphi_{in} \ll M_{Pl}$ and gravexcitons may be cold DM.

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