Leapfrog ADI FDTD Method for Lumped Elements

Wenbing Wang*, Yinhui Chen*, Liang Ma, Hui Zhou and Yifei Liu

State Key Laboratory of Intense Pulsed Radiation Simulation and Effect (Northwest Institute of Nuclear Technology), Xi’an, China

*Corresponding author e-mail: 18810628175@163.com, achenyinhui@nint.ac.cn,
maliang@nint.ac.cn, zhouchen@nint.ac.cn, dliuyifei@nint.ac.cn

Abstract. Leapfrog alternating direction implicit finite-difference time-domain (Leapfrog ADI-FDTD), a method with high computational efficiency, as well as it was proved to overcome CFL stability condition, the unconditional stability was given numerically, thus having a widely application in simulation. Lumped elements are introduced into Leapfrog ADI-FDTD method in this paper to further improvement of computational efficiency, single-grid updating equations of typical lumped elements are presented such as resistance, capacitor and inductor, what’s more, multi-grid model are provided to simulate special lumped elements which owned more than one grid, and situation of application is analyzed through iterated equations. Its high efficiency and stability were verified by numerical experiments, thus providing a foundation for Leapfrog ADI-FDTD applied to integrated circuits.

1. Introduction

The structure of electronic equipment tends to be complicated and miniaturized with the development of technology, it’s necessary to analyse electromagnetic characteristics of the electronic equipment in order to save the cost of design to optimize the circuit, numerical methods are the best choice at the moment [1-4, 8]. Leapfrog ADI FDTD method is one of the most efficient differential methods [5], it also could be used to solve microwave integrated circuits (IC) [6], development of IC makes linear components such as resistance, inductor, capacitor and nonlinear components like semi-conductor devices miniaturized increasingly, at the same time, the increase of signal frequency and the appearance of high speed pulses make the size of the components relative to the signal, thus we could treat the components as zero-dimensional unit, in other words, the method of lumped elements can be used to deal with them in this case [7], and the most frequently used are single-grid model and multi-grid model. Single-grid model means the lumped element occupies only one grid in space [8], while multi-grid model means the lumped element occupies several grids at least [9, 10].

2. Leapfrog ADI-FDTD method

Leapfrog ADI-FDTD method was obtained by eliminating split-steps of alternating direction implicit finite-difference time-domain (ADI-FDTD), it had reduced the computational requirement while keeping the same attractive numerical properties [11]. The ADI-FDTD formulation also use the traditional Yee lattice, the major difference was that ADI-FDTD method applied a split-step in time,
field components were iterated explicit and implicit in the split-steps, respectively [12]. The first half time step \( \{ n \to (n + 1/2) \} \) of the alternating-direction implicit (ADI) scheme was

\[
\begin{align*}
E^{n+1/2} &= E^n + \frac{\Delta t}{2\varepsilon} (AH^{n+1/2} - BH^n) \\
H^{n+1/2} &= H^n + \frac{\Delta t}{2\mu} (BE^{n+1/2} - AE^n)
\end{align*}
\]

For the second half time step \( \{ (n+1/2) \to (n+1) \} \), the term are split in the opposite sense, the corresponding equations are

\[
\begin{align*}
E^{n+1} &= E^{n+1/2} + \frac{\Delta t}{2\varepsilon} (AH^{n+1/2} - BH^{n+1}) \\
H^{n+1} &= H^{n+1/2} + \frac{\Delta t}{2\mu} (BE^{n+1/2} - AE^{n+1})
\end{align*}
\]

Where \( \bar{E}^{n+1/2} = [\bar{E}_x^{n+1/2}, \bar{E}_y^{n+1/2}, \bar{E}_z^{n+1/2}]^T \), \( \bar{H}^{n+1/2} = [\bar{H}_x^{n+1/2}, \bar{H}_y^{n+1/2}, \bar{H}_z^{n+1/2}]^T \), the expression of operators matrix A, B were:

\[
A = \begin{bmatrix}
0 & 0 & \frac{\partial}{\partial y} \\
0 & 0 & \frac{\partial}{\partial z} \\
0 & \frac{\partial}{\partial z} & 0
\end{bmatrix}, \quad B = \begin{bmatrix}
0 & \frac{\partial}{\partial z} & 0 \\
0 & 0 & \frac{\partial}{\partial x} \\
\frac{\partial}{\partial y} & 0 & 0
\end{bmatrix}
\]

We could obtain via simplification:

\[
\begin{align*}
\left( I - \frac{\Delta t^2}{4\mu\varepsilon} AB \right) \bar{E}^{n+1/2} &= \left( I - \frac{\Delta t^2}{4\mu\varepsilon} AB \right) \bar{E}^{n-1/2} + \frac{\Delta t}{\varepsilon} (AH^n - BH^n) \\
\left( I - \frac{\Delta t^2}{4\mu\varepsilon} AB \right) \bar{H}^{n+1} &= \left( I - \frac{\Delta t^2}{4\mu\varepsilon} AB \right) \bar{H}^{n} + \frac{\Delta t}{\mu} (BE^{n+1/2} - AE^{n+1/2})
\end{align*}
\]

Where I is unit matrix. Equation (4) described the formulation of Leapfrog ADI-FDTD method, this form is similar compared with traditional FDTD method, its unconditional stability was proven analytically.

3. The formulation of Leapfrog ADI-FDTD incorporated with lumped elements

3.1. Signal-grid model

When receiving the paper, we assume that the corresponding authors grant us the copyright to use the paper for the book or journal in question. Should authors use tables or figures from other Publications, they must ask the corresponding publishers to grant them the right to publish this material in their paper.

Assuming that the size of lumped elements is less than the length of the grid, the single-grid model should be used in this case as shown in Fig. 1[13, 14]. Without loss of generality, the Maxwell curl equation should be rewritten after considering the influence of lumped elements as
$$\nabla \times H = \frac{\partial D}{\partial t} + J_c + J_L$$  \hspace{1cm} (5)

Where $J_c = \sigma E$ is the conduct current density, $J_L$ is current density of lumped element. We suppose the lumped element is placed along the z-direction and located in a lossless region with permittivity $\varepsilon$ and permeability $\mu$, $I_L$ is used to symbol current, then there is a relationship between $J_L$ and $I_L$

$$J_L = I_L / (\Delta x \Delta y)$$  \hspace{1cm} (6)

However, it's obvious $J_c = 0$ in vacuum. The iterative equations of Leapfrog ADI-FDTD combined with $J_c$ are obtained after algebraic transformation

$$\frac{\partial E_n^n}{\partial t} = \frac{1}{4 \mu \varepsilon} \left( \frac{\partial}{\partial x} \left( E_{x}^{n+1/2} - E_{x}^{n-1/2} \right) + \frac{1}{\varepsilon} \left( \frac{\partial}{\partial x} H_{x}^{n} - \frac{\partial}{\partial y} H_{y}^{n} - (J_{z}^{n-1/4} + J_{z}^{n+1/4}) \right) \right)$$  \hspace{1cm} (7)

According to the principle of Taylor expansion, there are

$$\begin{cases}
J_{z}^{n-1/4} = J_{z}^{n} - \frac{1}{4} \frac{\partial J}{\partial t} y_{n \Delta y} \Delta t + O((\Delta t)^2) \\
J_{z}^{n+1/4} = J_{z}^{n} + \frac{1}{4} \frac{\partial J}{\partial t} y_{n \Delta y} \Delta t + O((\Delta t)^2)
\end{cases}$$  \hspace{1cm} (8)

Substituting (8) into (7) and simplify, we could obtain

$$J_{z}^{n-1/4} + J_{z}^{n+1/4} = 2J_{z}^{n} + O((\Delta t)^2)$$  \hspace{1cm} (9)

$$\left(1 - \frac{\Delta t^2}{4 \mu \varepsilon} \frac{\partial}{\partial x} \right) E_{x}^{n+1/2} \left|_{i,j,k} \right. = \left(1 - \frac{\Delta t^2}{4 \mu \varepsilon} \frac{\partial}{\partial x} \right) E_{x}^{n-1/2} \left|_{i,j,k} \right. + \Delta t \left( \frac{\partial}{\partial x} H_{x}^{n} - \frac{\partial}{\partial y} H_{y}^{n} - J_{z}^{n-1/4} - J_{z}^{n+1/4} \right)$$  \hspace{1cm} (10)

Where $i, j, k$ are spatial indices in the $x$, $y$ and $z$-directions and cyclic permutation as usual. Finally, the recursive equations of Leapfrog ADI-FDTD method with lumped elements can be rewritten as (10).

![Figure 1. Space location of lumped elements in FDTD grid](image)

Expressions for calculating the current density of resistances, capacitors and inductors were given by Xianghua Wang [6]. Based on that, the iterative equations of some lumped elements like resistance were given in detail. What’s more, the equations of lumped diode also derived.
3.1.1. Resistance. The lumped element supposed to be a resistance, as it shown in Figure 2 (a). The current density could be obtained by (6)

\[ J^r_{i,j,k+\frac{1}{2}} = \frac{\Delta z}{2R\Delta x\Delta y} \left[ E^r_{i,j,k+\frac{1}{2}} + E^r_{i,j,k-\frac{1}{2}} \right] \]  

(11)

Substituting (11) into (10), and the iterative equation of field components \( E_z \) could be written as

\[ -\frac{\Delta t^2}{4\mu_0\epsilon_0} E_z^{n+1/2}_{i-1,j,k+1/2} + \left( 1 + \frac{\Delta t^2}{2\mu_0\epsilon_0} + \frac{\Delta t\Delta z}{2R\epsilon_0\Delta x\Delta y} \right) E_z^{n+1/2}_{i,j,k+1/2} - \frac{\Delta t^2}{4\mu_0\epsilon_0} E_z^{n+1/2}_{i+1,j,k+1/2} = \]

\[ -\frac{\Delta t^2}{4\mu_0\epsilon_0} E_z^{n-1/2}_{i-1,j,k+1/2} + \left( 1 + \frac{\Delta t^2}{2\mu_0\epsilon_0} - \frac{\Delta t\Delta z}{2R\epsilon_0\Delta x\Delta y} \right) E_z^{n-1/2}_{i,j,k+1/2} - \frac{\Delta t^2}{4\mu_0\epsilon_0} E_z^{n-1/2}_{i+1,j,k+1/2} \]

\[ + \frac{\Delta t}{\epsilon} \left[ H_x^{n+1}_{i+1/2,j,k+1/2} - H_x^{n+1}_{i-1/2,j,k+1/2} - H_y^{n+1}_{i,j+1/2,k+1/2} - H_y^{n+1}_{i,j-1/2,k+1/2} \right] \]

(12)

Other field components could be got by cyclic permutation of \( \{x,y,z\} \).

3.1.2. Voltage source. If the lumped element is a Voltage source \( sU \) with internal resistance \( sR \) in Figure 1. With the same principle, we obtained the time-advance scheme for the electric field \( E_z \)

\[ -\frac{\Delta t^2}{4\mu_0\epsilon_0} E_z^{n+1/2}_{i-1,j,k+1/2} + \left( 1 + \frac{\Delta t^2}{2\mu_0\epsilon_0} + \frac{\Delta t\Delta z}{2R\epsilon_0\Delta x\Delta y} \right) E_z^{n+1/2}_{i,j,k+1/2} - \frac{\Delta t^2}{4\mu_0\epsilon_0} E_z^{n+1/2}_{i+1,j,k+1/2} = \]

\[ -\frac{\Delta t^2}{4\mu_0\epsilon_0} E_z^{n-1/2}_{i-1,j,k+1/2} + \left( 1 + \frac{\Delta t^2}{2\mu_0\epsilon_0} - \frac{\Delta t\Delta z}{2R\epsilon_0\Delta x\Delta y} \right) E_z^{n-1/2}_{i,j,k+1/2} - \frac{\Delta t^2}{4\mu_0\epsilon_0} E_z^{n-1/2}_{i+1,j,k+1/2} \]

\[ + \frac{\Delta t}{\epsilon} \left[ H_x^{n+1}_{i+1/2,j,k+1/2} - H_x^{n+1}_{i-1/2,j,k+1/2} - H_y^{n+1}_{i,j+1/2,k+1/2} - H_y^{n+1}_{i,j-1/2,k+1/2} \right] \]

\[ \frac{\Delta tU^n_s}{R\epsilon_0\Delta x\Delta y} \]

(13)

And similarly with cyclic permutation of \( \{x,y,z\} \).

3.1.3. Capacitor. For a lumped capacitor C as shown in Figure 2(b). We have

\[ -\frac{\Delta t^2}{4\mu_0\epsilon_0} E_z^{n+1/2}_{i-1,j,k+1/2} + \left( 1 + \frac{\Delta t^2}{2\mu_0\epsilon_0} + \frac{C\Delta z}{\epsilon_0\Delta x\Delta y} \right) E_z^{n+1/2}_{i,j,k+1/2} - \frac{\Delta t^2}{4\mu_0\epsilon_0} E_z^{n+1/2}_{i+1,j,k+1/2} = \]

\[ -\frac{\Delta t^2}{4\mu_0\epsilon_0} E_z^{n-1/2}_{i-1,j,k+1/2} + \left( 1 + \frac{\Delta t^2}{2\mu_0\epsilon_0} - \frac{C\Delta z}{\epsilon_0\Delta x\Delta y} \right) E_z^{n-1/2}_{i,j,k+1/2} - \frac{\Delta t^2}{4\mu_0\epsilon_0} E_z^{n-1/2}_{i+1,j,k+1/2} \]

\[ + \frac{\Delta t}{\epsilon} \left[ H_x^{n+1}_{i+1/2,j,k+1/2} - H_x^{n+1}_{i-1/2,j,k+1/2} - H_y^{n+1}_{i,j+1/2,k+1/2} - H_y^{n+1}_{i,j-1/2,k+1/2} \right] \]

(14)

3.1.4. Inductor. For a lumped inductor L as shown in Figure 2(c). We also have

\[ I^n_s = \frac{1}{L} \sum_{m=1}^{n-1} U^n_m \]

\[ = \frac{\Delta t\Delta z}{2L \sum_{m=1}^{n-1}} E^n_{i,j,k+\frac{1}{2}} + E^n_{i,j,k-\frac{1}{2}} \]

\[ \frac{\Delta tU^n_s}{R\epsilon_0\Delta x\Delta y} \]

(15)
3.2. Multi-grid model

However, the size of lumped elements is bigger than Leapfrog ADI-FDTD grids occasionally. In this case, the parasitic inductor would be formed without considering the size and location of the lumped element by using the single-grid model, which leads to the numerical dispersion and makes the calculation result inaccurate. Obviously, multi-grid model is more reasonable in such a situation [9].

Taking resistance as an example, if the resistance is crossed through several grids along the z-direction, as shown in Figure 3(a). The current density could be written as

\[
I_{i,j,k} = \frac{1}{\Delta t \Delta y} J_{i,j,k} = \frac{\Delta z}{R \Delta x \Delta y} \sum_{k=-N}^{k=N} E_{i,j,k} + E_{i,j,k+1}
\]

Substituting (17) into (10), we could obtained equation of field component \( E_z \) under multi-grid model,

\[
\left( -\frac{\Delta^2}{4 \mu \varepsilon \partial x \partial x} \right) E_z^{n+1/2} = \left( -\frac{\Delta^2}{4 \mu \varepsilon \partial y \partial y} \right) E_z^{n-1/2} + \frac{\Delta t \mu}{\varepsilon} \left( \frac{\partial}{\partial y} H_y^{n+1/2} + E_y^{n+1/2} \right)
\]

And cyclic permutation. It’s remarkable that original Leapfrog ADI-FDTD requires the solution of tri-diagonal equations at each step, therefore it’s quite difficult to solve equations like (18) with multi-grid current density.

According to the characteristics of equation (17), it will be ordinary to solve if we could take the problem in two-dimensional. The TE wave was used to illustrate, a rectangular coordinate was set up which take the direction of the lumped element as y-axis shown in Figure 3(b). The iterative equations of Leapfrog ADI-FDTD in two-dimensional are

\[
\left( -\frac{\Delta^2}{4 \mu \varepsilon \partial x \partial x} \right) E_y^{n+1/2} = \left( -\frac{\Delta^2}{4 \mu \varepsilon \partial y \partial y} \right) E_y^{n-1/2} + \frac{\Delta t \mu}{\varepsilon} \frac{\partial}{\partial y} H_y^{n+1/2}
\]

\[
\left( -\frac{\Delta^2}{4 \mu \varepsilon \partial x \partial x} \right) H_y^{n+1} = \left( -\frac{\Delta^2}{4 \mu \varepsilon \partial y \partial y} \right) H_y^{n} + \frac{\Delta t \mu}{\varepsilon} \frac{\partial}{\partial y} E_y^{n+1/2}
\]

It’s easy to see that the equation of \( E_y \) is explicit. (18) Could be simplified as
After that, field components $E_y$ at the location of the lumped elements can be solved by linear equations.

\begin{equation}
E_y^{n+1/2}_{i,j,k} = E_y^n_{i,j,k} - \frac{\Delta t}{\varepsilon \Delta x} \left( H_z^{n+1/2}_{i+1/2,j,k} - H_z^n_{i-1/2,j,k} \right) - \frac{\partial}{\partial y} H_x^n_{i,j+1/2,k} - \frac{\Delta \Delta z}{2 \varepsilon \Delta x} \sum_{j'=-h}^{j+h} \left( E_y^{n+1/2}_{i,j',k} + E_y^{n-1/2}_{i,j',k} \right) \tag{19}
\end{equation}

Figure 3. Multi-grid model of lumped elements

3.3. Exponential lumped elements

Exponential lumped elements refer to special elements like diode which voltage-current characteristics appear exponential. Because equations of the Leapfrog ADI-FDTD are implicit and the field updating are modified to include the solution of sets of tri-diagonal equations at each time step, we could compute exponential lumped elements only by making the equation explicit. The lumped element supposed to be a diode in Figure 1. The expression of current density is

\begin{equation}
I_d = I_0 \left[ \exp \left( \frac{q U_d}{k T} \right) - 1 \right] \tag{20}
\end{equation}

Where $q$ is electron charge, $U_d$ is supply voltage, $k$ is Bolzmann’s constant, $T$ is absolute temperature. We could get the current density of the diode along the $y$-direction under TE wave after time discrete

\begin{equation}
J_y^{n+1/2}_{i,j,k} = \frac{1}{\Delta x} I_y^n_{i+1/2,j,k} - \Delta t \frac{q \Delta y}{k T} E_y^n_{i+1/2,j,k} - \frac{1}{\Delta x} I_y^n_{i-1/2,j,k} + \Delta t \frac{q \Delta y}{k T} E_y^n_{i-1/2,j,k} - \frac{1}{\Delta x} I_y^0_{i,j,k} + \Delta t \frac{q \Delta y}{2 k T} \left( E_y^{n+1/2}_{i+1/2,j,k} + E_y^{n-1/2}_{i-1/2,j,k} \right) - 1 \right] \tag{21}
\end{equation}

The iterative equation of the field component $E_y$ can also be obtained by substituting (22) into (6).

4. Numerical experiments

4.1. Verification in three-dimension

Based on the recursive equations of the Leapfrog ADI-FDTD method with lumped elements, a simple micro-strip circuit shown in Figure 4 was calculated based on MATLAB. The space is divided into cubes as $\Delta x = \Delta y = \Delta z = 1$ mm, the time step was selected as $\Delta t = \Delta x / (\sqrt{5} \cdot c)$, the length of the circuit is 8 mm along $x$-direction and 6 mm along $y$-direction and 1 mm along $z$-direction, and the width of the conduction band is 2 mm. The convolution perfectly matched layer (CPML) absorbing boundary condition was used and there are 10-layer in each direction [15], the micro-strip circuit was separated
from the CPML by a Yee grid. The lumped element is supposed to be a resistance $R$ valued 50 Ohm, and the voltage source was sine wave source with an internal resistance valued 50 Ohm. The expression of the voltage source could be written as $U_s = \sin(2\pi ft)$, where $f$ represented as frequency valued 10 MHz.

Figure 5 showed the voltage of load resistance $R$, where CFLN represented the ratio of the time-step between Leapfrog ADI-FDTD and the maximum time step named $\Delta t_0$ of the conventional FDTD method. It can be seen that the voltage computed by Leapfrog ADI-FDTD agreed well with that computed by FDTD method, which is generally in line with the theoretical results. Moreover, when the CFLN is 4, the results could still keep a high accuracy.

Figure 4. Diagrammatic sketch of a simple micro-strip circuit

Figure 5. Voltage wave of load resistance in micro-strip circuit

4.2. Experiments in two-dimension

In fact, the micro-strip circuit shown in Fig. 4 can be simplified as a model in two-dimensional showed in Figure 6(a) under certain conditions. We chose $l_x = 20$ mm, $l_y = 5$ mm, $R_s = R = 50$ Ohm, $\Delta t = \Delta x / (\sqrt{2} \cdot c)$, the size of grid retained $\Delta x = \Delta y = 1$ mm, and the voltage source is Gauss pulse, its expression was

$$U_s = \exp\left(\frac{4\pi(t - t_0)^3}{\tau}\right)$$  \hspace{1cm} (22)

Where $\tau = 1 \times 10^{-7}$ s, $t_0 = 0.8\tau$. The voltage of load resistance is computed by single-grid and multi-grid model respectively, when the multi-grid model was applied, we supposed resistance crossed three grids along the y-direction, and the resistor value was evenly distributed in the three grids. The results were shown in Figure 7(a). Similarly, the lumped element is assumed as a parallel connection of a resistance and a capacitor shown in Figure 6(b). The voltage source was still the Gauss pulse as (24), we chose $R = 150$ Ohm, $C = 15$ pF, the single-grid and multi-grid model were used to compute the voltage of load, when the multi-grid model was applied, the load also crossed three grids. Figure 7(b) showed the voltage of load.
Computing results show that the voltage of resistance calculated by Leapfrog ADI-FDTD is consistent with the conventional FDTD method under the same time step, which indicate that Leapfrog ADI-FDTD method had a high calculating precision. With the increasing of time step, computational error of Leapfrog ADI-FDTD is also rising, nevertheless, it was small totally. When CFLN is 8, the result of Leapfrog ADI-FDTD method also keep a high accuracy. The precision of the multi-grid model is the same as that of the conventional FDTD method basically, which proves that the multi-grid model is feasible in two-dimensional.

What’s more, the lumped element in Figure 6 supposed to be a diode, the characteristic parameters in (22) were \( I_0 = 0.1 \) pA, \( T = 300 \) K, the voltage source is still the Gauss pulse like (24), and the internal resistance \( R_s \) is 0. Figure 8 shows the voltage waveform on the diode, it can be seen that the voltage on the diode is consistent with the power supply voltage.

In order to verify the stability of the Leapfrog ADI-FDTD in two-dimensional, the CFLN is 8 and we keep running for a long time. The result is showed in Figure 9, which indicated that Leapfrog ADI-FDTD method possess a high stability when taking a large time step.
Figure 8. Voltage wave between two ends of diode

Figure 9. Stability of Leapfrog ADI-FDTD method with lumped elements in two-dimension

Computing time under several conditions for Figure 6(a) is given in Table 1, as can be seen, when Leapfrog ADI-FDTD takes the same time-step as the conventional FDTD method, there is longer computing time needed, because the Leapfrog ADI-FDTD must solving tri-diagonal equations in every step. However, when the value of CFLN is increased, the calculating time of Leapfrog ADI-FDTD is decreasing almost inversely so that the computing efficiency is improved rapidly, if the multi-grid model is used, the computing time was slightly higher than the single-grid under the same conditions because the multi-grid model required to solve several equations simultaneously.

Table 1. Computing time comparison under different computing conditions

| Model         | Method                        | Space  | Time step/s | Steps  | Compute time/s |
|---------------|-------------------------------|--------|-------------|--------|----------------|
| Signal-grid   | FDTD                          | 42*27  | 2.36e-12    | 90000  | 1625.46        |
|               | Leapfrog ADI-FDTD(CFLN=1)     |        | 2.36e-12    | 90000  | 2156.34        |
|               | Leapfrog ADI-FDTD(CFLN=2)     |        | 4.72e-12    | 45000  | 1103.85        |
|               | Leapfrog ADI-FDTD(CFLN=4)     |        | 9.43e-12    | 22500  | 536.9          |
|               | Leapfrog ADI-FDTD(CFLN=8)     |        | 1.89e-11    | 11250  | 265.74         |
| Multi-grid    | Leapfrog ADI-FDTD(CFLN=1)     |        | 2.36e-12    | 90000  | 2288.95        |

5. Conclusion

Leapfrog ADI-FDTD is a implicit method with high efficient and unconditionally stability, it’s also extended to the field of circuit computing. The iterative equations of Leapfrog ADI-FDTD with several typical lumped elements are given under the single-grid model, then, this method incorporated the multi-grid model is presented first time in this letter, the recursive equations considering the location of lumped elements are derived, and Leapfrog ADI-FDTD combined with multi-grid model is suitable for solving two-dimensional problem via analysis. For exponential lumped elements like diode, it needed to make the iteration equations explicit through setting the computing environment correctly, and the equations could be solved by Newton iteration method. Finally, computational efficiency and accuracy are verified by several numerical experiments, the results indicated that Leapfrog ADI-FDTD method is available to be used in complex electronic system.

Acknowledgments

This work was financially supported by the Northwest Institute of Nuclear Technology Foundation (13131805).
References

[1] Xin Wen, Wei Zhuang, and Sheng Liu, Analysis of microstrip circuits with lumped components by Laguerre-FDTD method, J. China Sciencepaper, 20 (2015) 2442-2446.
[2] Yingying Wu, The simulation of hybrid circuit by higher order FDTD method [D]. Anhui University, 2013.
[3] Xinghua Zhao, Application research of FDTD in microstrip circuits, D. Xi'an, Xidian University, 2008.
[4] Yu Er, Analysis of micro-strip circuits with lumped elements by CN-FDTD method, D. Nanjing, Nanjing University of Science and Technology, 2008.
[5] Shunchuan Yang, Zhizhang Chen and Yiqiang Yu, The unconditionally stable one-step leapfrog adi-fdtd method and its comparisons with other fdtd methods, J.. IEEE Microwave and Wireless Components Letters. 21 (2011) 640-642.
[6] Xianghua Wang, Wenyan Yin and Zhizhang Chen, One-Step Leapfrog ADI-FDTD Method Including Lumped Elements and Its Stability Analysis, J. IEEE Antennas and Wireless Propagation Letters. 11 (2012) 1406-1409.
[7] Debiao Ge, Yubo Yan. Finite-Difference Time-Domain Method for Electromagnetic Waves third ed., Xi'an, Xidian University Press, 2011.
[8] Haizhao Zhao, Yan Li, Simulation of micro-strip circuits with microwave devices by FDTD method, J. Modern Electronics Technique, 21 (2009) 19-24.
[9] Zhengyu Yuan, Minliu Zou and Zhengfan Li, FDTD models of lumped-elements across multiple cells and its application in hybrid circuits, J. Journal of Shanghai Jiaotong University. 5 (1999) 526-529.
[10] Jingjing Tang, Ronghong Jin and Junping Gen, Study on modeling of arbitrary lumped network spanning two-dimensional multiple FDTD cells, J. Journal of China Academy of Electronics and Information Technology, 5 (2012) 490-495.
[11] S J Cooke, M Botton, T M Antonsen Jr. and Levush B, A leapfrog formulation of the 3-D ADI-FDTD algorithm, J. International Journal of Numerical Modelling. 22 (2009) 187-200
[12] Takefumi N, A new FDTD algorithm based on alternating-direction implicit method, J. IEEE Transactions on Microwave Theory and Techniques. 47 (1999) 2003-2007.
[13] Lin Guan and Yonggang Zhou. Extending the Three-Dimensional LOD-FDTD Method to Lumped Load and Voltage Source with Impedance, C. 2009 International Conference on Microwave Technology and Computational Electromagnetics (ICMTCE 2009) 341-343.
[14] Zhihui Cheng and Qingxin Chu, Stability analysis of the extended ADI-FDTD technique including lumped models, J. Science in China. 51 (2008) 1607-1613.
[15] Xianghua Wang, Wenyan Yin and Yu Yiqiang, A convolutional Perfect matched layer (CPML) for one-step leapfrog ADI-FDTD method and its applications to EMC problems , J. IEEE Transactions on Electromagnetic Compatibility. 54 (2012) 1066-1076.