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INPUT-OUTPUT MODELLING BASED ON TOTAL-USE RECTANGULAR TABLES: IS THIS A BETTER WAY?

FINANCIAL CONSTRAINTS AND EXPORTS: AN ANALYSIS OF PORTUGUESE FIRMS DURING THE EUROPEAN MONETARY INTEGRATION

1848: A PRIMEIRA CRISE DA TEORIA ECONÓMICA
Input-output tables can be presented in different formats, according to three main criteria: 1) symmetric or rectangular format; 2) total or domestic-use flows and 3) valuation prices (basic prices – bp or purchasers’ prices – pp). Official National Accounts (at least in EU) produce in a regular base a total use rectangular table at pp – also known as the Make and Use (M&U) format – that is different from the lay-out upon which traditional input-output models were developed (domestic use, symmetric, bp). The problem with this latter one is of course that it is only available at times in many countries. The objective of this paper is to prove (under common hypotheses) the equivalence between two alternative procedures, from the point of view of the results of an input-output model: 1) to convert the M&U input-output table into the traditional format – a domestic-use symmetric table at bp – and then implement the model; 2) to perform the direct modelling of the original table (the total-use rectangular table at pp). That equivalence is illustrated with Portuguese data for the year 2002.

Os quadros de Input-output podem obedecer a diferentes formatos, consoante três critérios principais: 1) formato simétrico ou rectangular; 2) inclusão ou não de produtos importados nos fluxos de uso; 3) sistema de valorização de preços (preços de base ou preços de aquisição). Pelo menos na UE, os quadros produzidos numa base regular por parte das Contas Nacionais oficiais são quadros de fluxos totais (incluindo importações), rectangulares e a preços de aquisição. Este é um formato diferente daquele em que os modelos tradicionais de input-output foram desenvolvidos (fluxos domésticos, simétricos, a preços de base). Obviamente, o problema é que, em muitos países, os quadros de input-output com essas características são disponibilizados apenas não regularmente. O objectivo deste artigo é provar a equivalência, sob hipóteses comuns, entre dois procedimentos alternativos: 1) converter a matriz de input-output rectangular no formato tradicional – matriz simétrica, de fluxos domésticos e a preços de base – e só depois implementar o modelo; 2) desenvolver o modelo directamente a partir do quadro original (rectangular, com fluxos totais e preços de aquisição). Esta equivalência é demonstrada usando dados das matrizes portuguesas, para o ano 2002.

**JEL Classification:** C67, E01.
Input-output tables can be classified according to three main criteria: 1) symmetric or rectangular format; 2) total use or domestic use flows and 3) valuation of goods and services. As a rule, the classical literature on input-output is based on symmetric matrices, with domestic flows, at basic prices. By a symmetric format we mean that the inner part of the input-output table has the same products or the same industries in its rows and columns. As a hypothesis, the classic Leontief tables assumed that each industry produced one and only one product. In input-output jargon, those tables depict product-by-product or industry-by-industry relationships. Remark, however, that in fact each industry may produce several secondary products beyond its main product that is referred in its denomination. Yet, since the end of the 1960’s, when the United Nations introduced the 1968 System of National Accounts, countries are recommended (at the national level) to compile and publish the input-output tables on a rectangular, or Make and Use format as it is known as well. In these tables the above-mentioned classical restrictive hypothesis is avoided. The idea is to combine two tables to depict Supply (or Make) and Use product-by-industry relationships. The Use matrix gives information on product consumption made by industries and final users. As to the Make matrix, its columns depict how the various industries contribute to the products’ output, while reading along the rows it gives us the distribution of each industry’s output over the several products: the primary product of that industry and its various secondary products. Since the number of products included in the model may be higher than the number of industries, this format is called rectangular.

As for the total or domestic-use criterion that refers to the type of flows represented in the intermediate transactions that are part of the Use table and also in the several components of the final demand. Intermediate consumption of products (made by industries) and final uses (made by households, government, firms and foreign countries) involve the use of products which are not only domestically produced, but are also imported. A total-use table records the whole amount of inputs used, whether these have been produced within the country (or the region, depending on whether we are dealing with a national or a regional model) or imported. Conversely, if intermediate and final use flows are expurgated from the value of imported products, then we are facing a domestic (or intra-regional) use table.

Finally, the third criterion is related to the different prices at which goods and services may be evaluated. Current input-output tables may involve two different price systems: basic prices (bp), the closest to the value of production factor costs, or purchasers’ prices (pp), which include taxes on the products (deducted from subsidies) and trade and transport margins.

Combining these criteria in several manners, many different types of input-output tables can be constructed. However, in practice, the starting point to the construction of these tables is usually the total flow Make and Use (M&U) rectangular table at purchasers’ prices, since this is the standard format in which statistical information is gathered and published by official statistical institutes, that follows the recommendations of international National Accounts manuals.

The main issue that this paper deals with is whether there is any benefit, for modelling purposes, in relying upon a domestic use symmetric table, or it is equivalent to implement the model directly from the total use rectangular table. That means that we aim to compare two different procedures for input-output modelling, when the original data is produced and available on a total use rectangular format: 1) firstly convert the table into a domestic use symmetric table at basic prices, and then implement the model or 2) perform the direct modelling of the total use rectangular table at purchasers’ prices, i.e., implementing the model on the basis of the table in its original format.

Many authors have thought the first procedure as the most adequate for input-output model applications. For example, in what respects the symmetric feature of the table, the EUROSTAT itself advocates in its Input-output manual that «For analytical purposes a relationship is needed
between the inputs and the outputs irrespective of whether the products have been produced by
the primary industry or by other industries as their secondary output» (EUROSTAT, 2002, p. 23);
as a consequence, symmetric input-output tables «are compiled mainly to be used in input-output
analysis» (p. 230). Concerning the content of the intermediate and final use flows, the same
manual states that «the separation of domestically-produced and imported goods and services is of
great importance for analytical purposes» (p. 145), leading to the option for domestic flow tables.

However, other authors, such as Madsen and Jensen-Butler (1999), Kauppila (1999) and
Piispala (1998), suggest that the direct use of the M&U format has considerable advantages at
different levels, namely:

• In the assembling process of the tables, since M&U tables are exempt of additional hypotheses
(conversely to product-by-product or industry-by-industry tables), being more directly connected
to the data collected by official statistical agencies.

• Make and Use tables are more easily intelligible for potential users of the model, since they
resemble reality in a closer way.

• M&U format is more suitable for application in certain fields of research which deal specifically
with spatial interaction flows of commodities such as: environmental modelling (for example,
when flows of products to be used in different industries are attached with flows of polluting
elements, such as CO2) and trade modelling (given that it is easier to incorporate trade
statistics, which report trade taking place with products and not with the output of industries, in
broad terms).

• Finally, as it will be demonstrated as well in this paper, the direct modelling of the rectangular
table is a more timesaving procedure, which can be considered as an advantage of this
alternative over the first one (involving the previous transformation into a symmetric table).

This paper is divided into five Sections, including this Introduction. In the next Section, the input-
output model based on the M&U framework will be presented. The three main criteria used to
classify input-output tables are the scope of Section 3. We proceed there to a detailed discussion
of the assumed hypotheses used in the transformation of the M&U format into the classic
symmetric domestic-use frame, that may be the same (and must be the same for comparison
purposes) that are implicit in the rectangular approach. A practical test will be carried out in
Section 4, aiming to compare with Portuguese data the results obtained from both above
mentioned procedures of building an input-output model. The last Section presents a summary of
the main conclusions.

2. Input-output modelling based on a M&U matrix, with total use flows, at purchasers’ prices

In this Section we deal with the rectangular or M&U model, with total-use flows, at purchasers’
prices (pp) – as it is a less well known procedure of implementing an input-output model --, in
order to demonstrate how it can de be directly modelled, avoiding its previous transformation in a
symmetric matrix of domestic flows at basic prices (bp).

The simplified structure of an M&U matrix, with total-use flows, at purchasers’ prices can be
illustrated as in Figure 1, in which: U_{pp} and V_{bp} represent the Use and the Make matrix. The Use
matrix refers to the product intermediate consumption by industries. It is a product-by-industry
matrix: its rows refer to products and its columns to industries. It is also of the total-use kind and
it is a pp matrix. The Make matrix V_{bp} depicts the industries that produce each product, as
primary or secondary production. In Figure 1, industries are along the rows and products in the
columns. Although the M&U model works with pp flows, this specific matrix is bp. g_{bp} denotes the
vector of industry production, at bp; p_{pp} identifies the vector of product output, at pp; y_{pp} is the
vector of products’ final use (both domestically produced and imported); m, d and l, stand for the
vectors of product imports, margins and net taxes on products that proceeds either to the
transformation of domestic to total supply and from bp valuation to pp. Finally, w represents the
vector of the industries’ value added.
The relationships involved in the M&U setting can be written in algebraic terms. Using matrix and vector notation, the industry balance may be expressed by:

$$g_{bp} = V_{bp}i = \left(U_{pp}^{T}\right)i + w'$$  \hspace{1cm} (1)

At product level, the balance can be expressed as:

$$p_{pp} = \left(V_{bp}^{T}\right)i + m' + d' + I' = U_{pp}^{T}i + y_{pp}$$  \hspace{1cm} (2)

The nuclear part of the M&U table is represented by the shadowed quadrants in Figure 1. Dividing all the elements of $U_{pp}$ and $V_{bp}$ by the correspondent column totals $g_{bp}$ and $p_{pp}$, we obtain the following partitioned matrix, composed by the matrices $Q$ and $S$ and two zero-filled matrices:

$$D = \begin{bmatrix} 0 & Q \\ S & 0 \end{bmatrix}.$$  \hspace{1cm} (3)

Using matrix $D$, we can write the matrix system:

$$\begin{bmatrix} 0 & Q \\ S & 0 \end{bmatrix} \begin{bmatrix} p_{pp} \\ g_{bp} \end{bmatrix} + \begin{bmatrix} y_{pp} \\ 0 \end{bmatrix} = \begin{bmatrix} p_{pp} \\ g_{bp} \end{bmatrix}$$  \hspace{1cm} (3)

This system may be manipulated in order to the outputs vector:

$$\begin{bmatrix} p_{pp} \\ g_{bp} \end{bmatrix} = \begin{bmatrix} y_{pp} \\ 0 \end{bmatrix} \iff \begin{bmatrix} p_{pp} \\ g_{bp} \end{bmatrix} = (I-D)^{-1}\begin{bmatrix} y_{pp} \\ 0 \end{bmatrix} \iff \begin{bmatrix} p_{pp} \\ g_{bp} \end{bmatrix} = \begin{bmatrix} I & -Q \end{bmatrix}^{-1} \begin{bmatrix} I \\ S \end{bmatrix} = \begin{bmatrix} I \\ -S \end{bmatrix} = \begin{bmatrix} I \\ S \end{bmatrix}$$  \hspace{1cm} (4)

1. We will use the vector $i$, consisting of a column-vector filled by 1s, to compute the column sum of the correspondent matrix and the sign ‘$^T$’ to indicate a transpose of a matrix or a column-vector.
2. It should be noted that even if the matrices $U_{pp}$ and $V_{bp}$ are not square, the partitioned matrix composed of these two (and of zero matrices of the appropriate dimension) will be square. Consider, for example, that there are 30 industries and 50 products. In this case, the matrix $U_{pp}$ will have a dimension of 50*30 and $V_{bp}$ will be a 30*50 matrix. Consequently, the partitioned matrix $D$ will have a dimension of 80*80 and $I-D$ can be inverted.
Applying the general formulas for computing the inverse of a partitioned matrix\(^3\), we obtain:

\[
\begin{bmatrix}
I & -Q \\
-S & I
\end{bmatrix}^{-1} =
\begin{bmatrix}
I + Q(I - SQ)^{-1}S & Q(I - SQ)^{-1} \\
(I - SQ)^{-1}S & (I - SQ)^{-1}
\end{bmatrix}
\]  

(5)

or

\[
\begin{bmatrix}
I & -Q \\
-S & I
\end{bmatrix}^{-1} =
\begin{bmatrix}
(I - QS)^{-1} & (I - QS)^{-1}Q \\
S(I - QS)^{-1} & I + S(I - QS)^{-1}Q
\end{bmatrix}
\]  

(6)

Inserting equation (6) into (4), and multiplying these partitioned matrices, we get:

\[
p^{pp} = (I - QS)^{-1}y^{pp}
\]  

(7)

and

\[
g^{bp} = S(I - QS)^{-1}y^{pp}
\]  

(8)

The first equation allows us to compute the impact on total product supply originated by changes in final demand for products \(\frac{\partial p^{pp}}{\partial y^{pp}}\). Therefore, this is a product-by-product relationship. The second equation is an industry-by-product relationship; it shows the impact on industry’s supply caused by changes in final demand for products \(\frac{\partial g^{bp}}{\partial y^{pp}}\). As for the right hand blocks in (5), the lower right hand, \((I - SQ)^{-1}\), depicts an industry-by-industry relationship: it gives us \(\frac{\partial y^{pp}}{\partial (Sy^{pp})}\), where \(Sy^{pp}\) is the final demand by industries, transformed from \(y^{pp}\). The upper right hand corner, \(Q(I - SQ)^{-1}\), accounts for the impact on product demand, including imports, margins and taxes, created by changes in the demand directed at domestic industries \(\frac{\partial p^{pp}}{\partial (Sy^{pp})}\). Hence, it is a product-by-industry relationship.

We may then conclude that by performing a rectangular or M&U model (at \(pp\), with total flows) we get within one single model product-by-product, industry-by-industry, product-by-industry and even industry-by-product relationships. This may be seen as an advantage over symmetric models. In these latter, each model provides only one type of relationship: the product-by-product symmetric model generates only a product-by-product impact equation; if we want to quantify an industry-by-industry impact, we will need to build an industry-by-industry symmetric table and develop the corresponding model.

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3 These formulas can be found, for example, in Barnett (1990), pp. 71-72.
It is now time to look at the other way, namely at the previous transformation of the rectangular, total-use, \( pp \) table (the M&U format) into the classic Leontief-type symmetric, domestic-use, \( bp \) structure, that in this alternative procedure is the base of the input-output modelling. As a rule the rooted-survey information to deal with this transformation is very scarce, so we mainly have to resort to reasonable assumptions. These assumptions must be the same, for comparisons purposes, of those that are implicit in the M&U direct modelling. This section is devoted to the discussion of these assumptions.

### 3.1. Symmetric and rectangular input-output tables revisited.

The simplifying hypothesis adopted by the traditional symmetric input-output table is that each product is produced by one single industry and each industry produces one single product. However, in reality, the most common situation is that each industry produces a growing diversity of products, one of these being the primary product and the others the secondary ones. These secondary products can be divided into two categories: subsidiary products and by-products (EUROSTAT, 2002); subsidiary products are those secondary products which are technologically dissociated from the primary product; by-products are outputs that unavoidably result from the primary product production process, therefore being technologically related to it. As a rule, national Make and Use tables, following the SNA (System of National Accounts) recommendations, involve some partial refining in the Industry classification. This is due to the fact that industries are grouped according to the concept of kind-of-activity unit, and not according to the concept of enterprise. The term kind-of-activity unit (KAU) is used to denote a part of an institutional unit in which only one particular type of economic activity is carried out (Jackson, 2000). Thus, as a rule, enterprises «must be partitioned into smaller and more homogeneous units, with regard to the kind of production» (ESA, 1995, paragraph 2.105). So, in the National Accounts’ industry classification, each industry consists of a group of KAs which are «engaged in the same or a similar kind of activity» (ESA, 1995, paragraph 2.108). This means that most of the subsidiary products produced in each enterprise is classified under a different industry heading, the one that produces those products as its main activity. Exceptions to this procedure occur whenever it is not possible to separate the secondary from the primary activity, either because secondary production is of by-product nature, or because the available information obtained from enterprises does not allow for separation (this being the case with most small firms, which have no accounting documents which allow for their partitioning into different KAs). As a result, the values of production recorded outside the main diagonal in the Make matrix are mostly by-products, along with some residual subsidiary products that could not be separated from the main activity in the firms in which they were produced. The presence of these flows outside the main diagonal of the Make matrix — that represent the production by industries of products that do not fall in their core business — is the reason why the M&U model is not of symmetric type. As a consequence in the Use matrix each column refers to one industry that may produce more than one product; but their inputs still consist of single products. The Use matrix is then of product-by-industry kind.

Thus, symmetric input-output tables (SIOT) cannot be built directly with the statistical data collected by regular firm surveys. As a consequence this kind of tables can only be achieved in a derivative way, departing from the M&U tables, and assuming some hypotheses in order to calculate the product-by-product (or industry-by-industry) intermediate consumption flows\(^4\). Two alternative hypotheses, connecting the products’ output and the industries’ output may be used in the transformation of product-by-industry matrixes in symmetric ones, either of product-by-product or of industry-by-industry type: the industry technology assumption (ITA) and the commodity technology assumption (CTA).

\(^4\) As well as to compute the value added by products, or, in industry-by-industry tables, the final demand by industries.
In the ITA case each industry has its own technology, which is common to all the commodities it produces. Thus, the technology assigned to each product depends on the industry where it is produced (ten Raa and Rueda-Cantuche, 2007). This kind of assumption is usually pointed out as preferable when the majority of secondary production is of by-product nature (Miller and Blair, 2009). On its turn, CTA assumes that each product is always produced by the same technology, regardless of the industry in which it is produced. For this reason, it is best suited to treat subsidiary production (Miller and Blair, 2009). In this paper we will deal with both the hypotheses, namely when in Section 4 we proceed with real data and compare the actual values of the input-output multipliers.

3.2. Total use flows versus domestic use flows.

Another major issue concerning input-output tables is the treatment of imported products. In a total Use table, as the one that is comprised in the M&U format, all the use flows (intermediate and final) also include imported products, beyond national produced flows. In fact, this means that the intermediate Use matrix reflects true technical relationships: each of its elements indicates the total amount of a certain input used to produce a certain output. Data collected by means of surveys to firms can be directly used to produce these types of tables. The same does not apply to domestic flow tables. In this case, a Use matrix of imported products is needed in order to subtract its value from the total Use table. Direct information to construct such an Imports matrix is very rare. It is in fact very difficult for many firms to know the origin (imported or domestically produced) of several of their inputs. In many cases, firms buy inputs from wholesale traders, hence ignoring their origin. For similar reasons, the computation of final demand domestic flows, based on direct information, is also very hard (or even more complex, since the number of intermediate traders between the importing firm and the final user is usually greater). Being so, Import matrices are very often built merely by resorting to plausible assumptions, seldom complemented by direct information on some particular products.

The most common assumption – and the one that we adopt in this paper – is the imports proportionality hypothesis which asserts that, for each product, the share of imports in any type of use (intermediate or final) of that product is the same and is given by the proportion of imports on total supply of the same product. For example, if 40% of steel’s total supply is imported, it is assumed that, in every industry which uses steel, 40% is imported and the same applies to any type of final use. This means that imports are differentiated by type of product but not by type of use.

Although controversial, this hypothesis is adopted in many cases, alone or combined with the incorporation of direct information, even when the domestic-use symmetric table is assembled by the official entities. In what concerns to the estimation of the imports matrix, for example, even OECD recognizes that this happens, stating that «Techniques used to construct the import matrix data vary between countries, but every country in the OECD database made, to some extent, use of the import proportionality assumption in the construction of their import matrices» (OECD, 2000, p.12). Moreover, the Input-output database provided by OECD (consisting of symmetric industry-by-industry tables) is compiled using this kind of assumptions, whenever supplementary information is not available (Yamano and Ahmad, 2006).

One crucial point on this assumption is the product disaggregation level that is applied (EUROSTAT, 2002). If the import coefficients are calculated at a much aggregated level, the imports proportionality hypothesis may not be acceptable. Thus, the most detailed level of disaggregation available on import data should be used. This does not usually originate a great deal of trouble on national tables since international imports data by products is available at a very detailed product level5. On the other hand, several authors note that some final uses, like

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5 The magnitude of the errors coming from such an assumption, however, can only be accounted for when there is a benchmark survey-based imports matrix against which the estimated one can be compared. This is done in Oosterhaven and Stelder (2007), in their comparison between four alternative non-survey inter-country input-output table construction methods, for nine Asian countries and the USA. In one of the non-survey input-
exports, for example, have less incorporation of imported products than others, like investment. In order to take this differentiation into account, they have proposed to exclude exports from the import proportionality assumption, assuming that there are no re-exports. This is done, for example, in Miller and Blair (2009), and Jackson (1998). As emphasized by Lahr (2001), this approach should be preferred only in those cases in which the researcher knows that the export vector has no (or almost no) re-exports. In the present work, however, the import proportionality assumption will be taken uniformly throughout the various types of intermediate and final uses.

3.3. Basic versus purchasers’ prices.

Different concepts can be used in the valuation of input-output flows of goods and services, ranging from the factor cost to the purchaser’s price. The valuation at factor costs represents the production price and reflects better the production function of each product (Martins, 2004). At the opposite, the purchasers’ prices represent the amount paid to obtain «a unit of a good or service at the time and place required by the purchaser» (EUROSTAT, 2002, p. 121). In spite of this multiplicity of concepts, however, in practice SNA input-output tables use only two price concepts: basic price and purchaser’s price. Basic prices are similar to factor costs, except for the fact that basic prices include other taxes and subsidies on production, which are not possible to allocate to specific products. Basic prices \(bp\) can be obtained from purchasers’ prices \(pp\), subtracting the taxes on products less subsidies on products and the trade and transport margins.

The published M&U tables usually employ \(pp\) concept to balance supply and use. It is however, sometimes argued, that this valuation is not sufficiently homogeneous to be used for input-output analytical purposes; for example, the ESA’s Input-Output Manual states that «a valuation at purchasers’ prices is a less homogeneous option as the shares of trade and transport margins differ from industry to industry and also from and between the final uses; the same is true for the shares of product taxes less subsidies» (EUROSTAT, 2002, p.124). It is also true that basic prices are closer to the concept of production costs involved in the technical relationships used in input-output analysis. These relationships assume that a certain amount of an input represents the same physical unit irrespective of the production process in which it is used (EUROSTAT, 2002).

Hence, it would be desirable that prices were cleared from margins and taxes which differently affect the diverse uses of the products. The problem lies in the compilation of the valuation matrices required to transform \(pp\) into \(bp\), since direct information on the value of margins and taxes comprised in each use flow is very scarce. In fact, when someone buys a certain item, he/she doesn’t know very often the amount of margins and sometimes taxes comprised in the price that has to be paid. In the absence of direct information to construct valuation matrices and obtain a basic price valued table, the proposal is to assume the same kind of proportionality hypothesis than for imports: the margin (net taxes) rate comprised in any type of use (intermediate or final) of that product is assumed to be the same for each product, and is given by the proportion of margins (net taxes) on total supply of the same product.

What is the plausibility of such an assumption? In this case, it is useful to look at each of the following items separately: Value Added Tax (VAT), margins, other taxes on products and subsidies on products. In what concerns non-deductible VAT, the problem is quite complex. Ideally, direct information should be available in order to: 1) identify the type of users who support non-deductible VAT. Non-deductible VAT is, in fact, supported mainly by households and, in output tables, they assume that there is no imports matrix and use the imports proportionality assumption to indirectly estimate it. The comparison between this table and the benchmark (which is a semi-survey based inter-country table) allow the authors to conclude that in general, «The tests show that the impact of using self-sufficiency ratios to estimate the domestic flows is small (…)» (Oosterhaven and Stelder, 2007, p. 258).

6 Taxes (subsidies) on products are those that «are payable per unit for some goods or services produced or transacted» (EUROSTAT, 2002, p. 200); examples: Value added taxes, import duties or tobacco product tax. Taxes (or subsidies) on production are those paid (or received) by firms as a direct result of their production activity, «independently of the quantity or value of the goods and services produced or sold» (idem, p. 200).

7 Deductible VAT is not included in the \(pp\) valuation.
some exceptional cases, by firms, either falling upon intermediate consumption or Gross Fixed Capital Formation (e.g. firms exempt from VAT and sometimes not allowed to deduct it from their purchases) and 2) Perform the linkage between the different VAT taxes and the product classification in the Use matrix; if the level of aggregation is high, some problems can arise because groups of products may well involve different VAT taxes (EUROSTAT, 2002).

Treating margins on a proportional assumption basis is also not completely realistic. In fact, it has to be recognized that different users of a product pay different margins on it. For example, a manufacture will certainly pay a smaller amount of margins on stationery materials than the final consumer. Finally, the use of the proportional assumption in the case of other taxes and subsidies is less controversial. These taxes and subsidies fall upon specific products and as a rule all the users have to support them. For example, taxes on gasoline have to be paid equally by any type of user of this product. As for imports, in any of the items mentioned in this Section, the proportionality assumption must be applied at the most disaggregated level of product classification. This is important in order to avoid situations in which groups of products are heterogeneous in respect to margins or tax rates.

In this paper, however, as our purpose is confined to the theoretical argument of the equivalence of different approaches, the proportionality assumption is allowed by simplification to all these flows, concerning the transformation of \( pp \) on \( bp \).

4. A test with Portuguese data

In this Section, it will be shown that the direct modelling of the rectangular M&U matrices, with total use flows and at \( pp \), that adopts a framework that is equal or very close from the official statistics, is exactly equivalent to the modelling of a domestic flow symmetric table (at \( bp \)), when it is derived from the former one, using similar assumptions. To do so, we will begin by computing the input-output multipliers obtained both through the direct modelling of the rectangular table, and through the product-by-product and industry-by-industry symmetric tables that can be obtained from the same rectangular frame. Then we focus on the analysis of the multipliers and conclude that insofar of the method we use for achieving them, we get exactly the same results.

Although this paper focuses in real data from the Portuguese economy, and makes the option of showing the results obtained by both the methods, to conclude that actually they are the same, a mathematical proof of our argument is also provided in an Appendix.

4.1. Deriving the input-output multipliers

With the purpose of comparing the multipliers produced by both methods, we begin by performing a rectangular model, including the computation of the associated inverse matrices. The data in which we based this experience is the Portuguese Make and Use tables for the year 2002, at current prices, provided by the Portuguese Statistics National Institute (INE)\(^8\). Every year, since 1995, INE provides a set of National Accounts tables, which includes a M&U table. Products and industries are usually presented in a 60 by 60 disaggregate level (ESA95 – A60 classification). The level of aggregation used in this paper, however, corresponds to a less disaggregated classification also provided by INE containing only 31 products and 31 industries. The Portuguese Make matrices are heavily diagonal, meaning that most of the production has been affected by its primary producing industry, in the process of partial refining of Industries’ classification, as it has been previously explained. Intermediate and final uses of goods and services are composed of both domestically produced and imported products, but no import matrices are regularly compiled. Additionally, these Use flows are valuated at \( pp \). Thus our first step was to implement an input-output rectangular model, as the one described in section 2, based in the M&U table provided by the INE.

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\(^8\) We are thankful to INE, for its kindness in providing us with the Make table, for the working year, which is not currently published. For the remaining information we downloaded it from the INE’s official website: www.ine.pt.
It is important to emphasize that the model developed in section 2 implicitly assumes the ITA hypothesis. Although we did not develop that model in that section, it is possible as well to settle a CTA-based rectangular model. In this model the sub-matrix $S$ of (3) – $S$ represents the relative contributions of each industry to the supply of each product – is replaced by $H^{-1}(I - \hat{c})(I - \hat{f} - \hat{n})$. $H$ is derived as well from $V_{bp}$, but it displays the product’s structure of each industry output. That means that we have now calculated fixed coefficients along the rows of $V_{bp}$, and not anymore along its columns as we had done in $S$. $c(\hat{c})$, $f(\hat{f})$ and $n(\hat{n})$ mean the import, margin and taxes (less subsidies) coefficient vectors (diagonal matrices), that result from dividing vectors $m$, $d$ and $l$ (inserted in Figure 1) by the total supply of products $p_{bp}$. In fact, pre-multiplying by $(I - \hat{c})(I - \hat{f} - \hat{n})$ transforms one vector of total supplies at purchasers prices in its equivalent with domestic supplies at basic prices.

Therefore under CTA, instead of equation (3), we have:

$$
\begin{bmatrix}
0 \\
H^{-1}(I - \hat{c})(I - \hat{f} - \hat{n}) \\
\end{bmatrix}
\begin{bmatrix}
Q \\
p_{bp} \\
\end{bmatrix}
\begin{bmatrix}
0 \\
g_{bp} \\
\end{bmatrix}
+ 
\begin{bmatrix}
y_{bp} \\
0 \\
\end{bmatrix}
= 
\begin{bmatrix}
p_{bp} \\
g_{bp} \\
\end{bmatrix}
\Rightarrow

(9)
$$

$$
\begin{bmatrix}
p_{bp} \\
g_{bp} \\
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
H^{-1}(I - \hat{c})(I - \hat{f} - \hat{n}) \\
\end{bmatrix}
\begin{bmatrix}
I \\
- Q^{-1}y_{bp} \\
\end{bmatrix}
$$

The multipliers produced by that version of the rectangular model are the cells of the inverted block matrix defined as follows:

$$
\begin{bmatrix}
I + Q[I - H^{-1}(I - \hat{c})(I - \hat{f} - \hat{n})Q]^{-1}H^{-1}(I - \hat{c})(I - \hat{f} - \hat{n}) \\
I - H^{-1}(I - \hat{c})(I - \hat{f} - \hat{n}) \\
\end{bmatrix}
\begin{bmatrix}
I - H^{-1}(I - \hat{c})(I - \hat{f} - \hat{n})Q \\
I - H^{-1}(I - \hat{c})(I - \hat{f} - \hat{n})Q \\
\end{bmatrix}
\Rightarrow

(10)
$$

After computing the two sets (ITA and CTA-based) of M&U multipliers, we have to derive as well the product-by-product and the industry-by-industry domestic-flow symmetric tables valuated at $bp$, in order to allow for the comparison of the two kinds of the multipliers. Remember that there is no regular production and publication of any symmetric tables (product-by-product or industry-by-industry) in Portugal and in several other EU countries. Thus, whenever the researcher wants to make use of symmetric domestic flow tables he/she may have to assemble the import matrix and to symmetrize the table, relying on a set of different hypotheses. The methodology used to build the SIOTs must follow exactly the same hypotheses than when we were dealing with the direct modelling of the M&U tables. The method involved three stages:

1. Computing Use matrices for margins and for taxes (less subsidies), in order to subtract them from the purchasers’ prices Use table and obtain the basic prices Use table.
2. Computing the Use matrix of imported products, in order to subtract it from the basic prices Use table and thus obtain the domestic flow basic prices Use table. To do this, the proportionality hypotheses were used. In practice, most of the countries that construct an official import matrix also support their work in this kind of hypothesis (OECD, 2000).
3. Obtaining the product-by-product and industry-by-industry symmetric tables, resorting either to the ITA hypothesis or accepting the CTA instead.

9 We are using notation $^\wedge$ to represent a diagonal matrix with the non-null entries being the elements of the correspondent column or row-vector.
10 It must be noted, however, that semi-official domestic flow symmetric input-output tables at basic prices has been provided every five years, since 1995. The compiling work was not directly done by the INE, but by a partnership between it and a governmental body: the Planning and Prospective Department. The description of the methodology of assembling these tables, and the matrices themselves, are available, for instance, at Dias (2008).
4.2 Multipliers’ comparison

The results of the partitioned matrix inversion, based on the M&U table, are displayed in Annex A.1 and A.2, corresponding to ITA and CTA hypotheses, respectively. We may find the product-by-product multipliers in the upper left-hand blocks of these partitioned matrices. For example, when we assume ITA in the Annex A.1, this upper left-hand block corresponds to \((I - QS)^{-1}\) in (6) and it shows the impact of changes in \(\gamma_{pp}\) over \(p_{pp}\). Let’s take value 0.0217, located at [EE, DJ] in that matrix: this cell means that when final demand for «DJ – Basic metals and fabricated metal products» valued at \(pp\) is exposed to an unitary increase, the direct and indirect extra demand (at \(pp\)) for product «EE – Electricity, gas and water supply» increases 0.0217 units. This increase also includes the increase for imported «EE» products, since the effect evaluated here is on \(p_{pp}\) as a whole. The correspondent product-by-product multiplier in the CTA-based partitioned matrix in Annex A.2 is 0.0229, which illustrates the fact that a different technological assumption does not originate extremely diverse values.

However, these multipliers comprised in the upper left-hand blocks of the matrices of Annexes A.1 and A.2 cannot be directly compared with the results obtained through domestic flows \(bp\) product-by-product symmetric tables, displayed in the Annexes A.3 and A.4. The reason is that in those blocks of those two annexes we have the impacts of the total demands – addressed to the domestic economy but also to imports, at \(pp\) – on total transactions, also at \(pp\), imports included; that is of \(\gamma_{pp}\) on \(p_{pp}\). On the other hand, in symmetric models the results we should reach concern only shocks on domestic perceived demand, at \(bp\), and their effect on domestic production valued at \(bp\) as well. That means that for comparison purposes the upper left-hand blocks of the matrices of Annexes A.1 and A.2 must be previously transformed by pre-multiplying those blocks by the diagonal matrixes \((I - \hat{c})\) and \((I - \hat{f} - \hat{n})\), where \(c\), \(f\) and \(n\) mean the import, margin and taxes (less subsidies) coefficients, in a first step, and then in a second stage post-multiplying by the inverses of those matrixes\(^{11}\). When we do that with our [EE, DJ] entry of 0.0217 pulled apart of ITA-based Annex A.1 matrix, we divide it by 0.6086 and 0.8874 and multiply it by 0.9886 and 0.9815, getting 0.0390. This is exactly the same value that is displayed in the [EE, DJ] cell of the domestic flow product-by-product inverse matrix \((bp)\) of the Annex A.3. As for the CTA-technology, we proceed in the same way with 0.0229 extracted from the [EE, DJ] upper left-hand block of the matrix of Annex A.2, and we obtain 0.0412 that is the cell [EE, DJ] of the domestic flow, \(bp\), product-by-product matrix derived by CTA, depicted in Annex A.4. In fact, the matrices included in Annexes A.3 and A.4 as a whole may be obtained starting from the upper left-hand blocks of the matrices of Annexes A.1 and A.2 and applying the recommended transformations.

The lower right-hand blocks of the partitioned inverse matrices (Annexes A.1 and A.2) tell us about the industry-by-industry relationships. They correspond to the inverse matrices implicit in equations (5) and (6) for ITA and (10) for CTA. From these matrices one can assess the effects in each industry and in the total economy-wide caused by changes in the demand addressed to each industry. Looking again at the ITA case (Annex A.1), if the demand addressed to the output of industry «DJ» increases by 1, the «EE» industry will have to increase 0.0395 (through direct and indirect effects). As referred to before, the values of these lower right-hand block matrices should be equal to the values of the inverse matrices derived from a domestic flow industry-by-industry symmetric table (valuated at \(bp\)), constructed taking as original data the same rectangular table, and using similar hypotheses. Such matrices are presented in Annex A.5 for ITA and in Annex A.6 for CTA. In this case direct comparison is allowed, so then the same value 0.0395 may be found in the corresponding entry of the matrix of the Annex A.5. The same conclusion may be drawn to CTA-based matrices: as can be easily checked the lower right-hand block of the table in A.2 is the same matrix that is depicted in A.6.

\(^{11}\) Because the final impacts on \(p_{pp}\) and the initial shocks on \(\gamma_{pp}\) must be both transformed multiplying by \((I - \hat{c})(I - \hat{f} - \hat{n})\), then, for counterbalancing, each multiplier is multiplied by the transformation coefficient corresponding to its row and divided by the one corresponding to its column.
5. Conclusions

The main issue of the present essay fell upon input-output modelling when the starting available matrix produced by official statistics is a total-flow rectangular table at purchasers’ prices. Two alternative procedures have been analyzed: 1) to perform the direct modelling of the total-flow rectangular table at purchasers’ prices; 2) to convert the initial matrix into a domestic-flow symmetric table at basic prices and then implement the traditional Leontief-type input-output model. It has then been proved that, when the hypotheses used to make the table symmetric and to operate the conversion from total use to domestic use flows (and from purchasers’ prices to basic prices) are also used in the direct modelling of the starting rectangular matrix, the results we obtain are exactly the same. Thus, there is not a clear advantage, in most cases, in performing a previous transformation of the original tables, as some authors advise, into the symmetric domestic flow format, before implementing the model. Of course, in specific context – for instance, if one wishes to infer only the direct and indirect impact on domestic production resulting from an increase of final demand towards domestic products, it may be more appropriate to build the adequate symmetric input-output table (domestic-use and basic prices), instead of going into the process of solving the whole rectangular system previously described.

The equivalence between the results of both alternative procedures has been attested through a numerical example. In fact, an algebraic proof may be produced as well, as we have done in the Mathematical Appendix ahead. The numerical example consisted in using the Portuguese M&U table as a starting point (which is a total-flow rectangular table at purchasers’ prices) and implementing the input-output model, applying both the previously referred procedures. As we expected, the input-output multipliers when referring to the same impact and the same effect are exactly the same, either by one or by the other procedure. We may even say, following that equivalence, that the direct use of the rectangular format has an important advantage over the use of symmetric tables: in the rectangular framework, the simple inversion of a partitioned matrix generates a set of four different inverse matrices (product-by-product, industry-by-industry, product-by-industry and industry-by-product ones); conversely, the symmetric tables originate only one type of inverse matrix (product-by-product or industry-by-industry).

In this paper, the development of the input-output model directly from the total-flow rectangular table at purchasers’ prices, involved the use of proportionality hypotheses concerning imports, margins and taxes comprised in the intermediate and final use flows. Additionally, the model was developed in two versions – one using ITA and another using CTA. The proportionality and the technology hypotheses adopted are of course controversial. This doesn’t however jeopardize the validity of the conclusions, as the important is that the same hypotheses have been used either in the direct modelling of the starting matrix, or in the conversion of this matrix into a domestic-flow symmetric table at basic prices. Besides, in many cases, even the official organisms of statistics use these kinds of simplifying hypotheses (or similar procedures) when assembling symmetric tables. In other cases, of course, these hypotheses are sometimes complemented or substituted by the inclusion of direct information, which however and as a rule can be incorporated in the rectangular model as well. For example, if a true import matrix is available, it is obviously better to use such information than to use the proportionality hypothesis (even though the gathering of direct information involves high costs and, in many cases, originates only a marginal improvement in the results). That however does not refute our point that equivalent hypotheses generate equivalent results.
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Annex A.1 – Partitioned matrix inverse; ITA (results from the rectangular M&U model with total flows at pp)

|   | AA  | BB  | CC  | DD  | EE  | FF  | GG  | HH  | II  | JJ  | KK  | LL  | MM  | NN  | OO  | PP  |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| AA | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| BB | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| CC | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| DD | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| EE | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| FF | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| GG | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| HH | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| II | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| JJ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| KK | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| LL | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| MM | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 |
| NN | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 |
| OO | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 |
| PP | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 |

**Column Sum:**

|   | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

**Row Sum:**

|   | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
Annex A.1 – Partitioned matrix inverse; ITA (results from the rectangular M&U model with total flows at $pp$) (cont.)

|       | AL | EM | EN | ET | FP | GG | HH | IJ | JK | LI | MM | N | NO | PP |
|-------|----|----|----|----|----|----|----|----|----|----|----|---|---|----|
| AL    | 0.022 | 0.015 | 0.024 | 0.020 | 0.012 | 0.085 | 0.084 | 0.025 | 0.031 | 0.027 | 0.029 | 0.020 | 0.016 | 0.000 |
| EM    | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| EN    | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| ET    | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| FP    | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| GG    | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| HH    | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| IJ    | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| JK    | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LI    | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| MM    | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| N     | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| NO    | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| PP    | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

**COLUMN SUM**

| AL | EM | EN | ET | FP | GG | HH | IJ | JK | LI | MM | N | NO | PP |
|----|----|----|----|----|----|----|----|----|----|----|---|---|----|
| 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

**COLUMN SUM**

| AL | EM | EN | ET | FP | GG | HH | IJ | JK | LI | MM | N | NO | PP |
|----|----|----|----|----|----|----|----|----|----|----|---|---|----|
| 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
Annex A.1 – Partitioned matrix inverse; ITA (results from the rectangular M&U model with total flows at pp) (cont.)

|   | MA | SB | CA | CB | JB | JC | JG | JH | JI | JJ |
|---|----|----|----|----|----|----|----|----|----|----|
| AA | 0.1659 | 0.0251 | 0.0030 | 0.0240 | 0.420 | 0.0069 | 0.0135 | 0.0107 | 0.0070 | 0.0058 |
| JS | 0.0031 | 0.0048 | 0.0019 | 0.0033 | 0.0024 | 0.0010 | 0.0016 | 0.0014 | 0.0014 | 0.0011 |
| JA | 0.0621 | 0.0001 | 0.0002 | 0.0019 | 0.0019 | 0.0017 | 0.0013 | 0.0012 | 0.0011 | 0.0009 |
| EA | 0.0157 | 0.0141 | 0.0013 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| BB | 0.0029 | 0.0024 | 0.0010 | 0.0009 | 0.0007 | 0.0004 | 0.0002 | 0.0002 | 0.0001 | 0.0001 |
| KC | 0.0023 | 0.0016 | 0.0006 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| RD | 0.0021 | 0.0019 | 0.0004 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| DE | 0.0133 | 0.0181 | 0.0151 | 0.0144 | 0.0148 | 0.0149 | 0.0147 | 0.0146 | 0.0147 | 0.0147 |
| KG | 0.0090 | 0.0090 | 0.0090 | 0.0090 | 0.0090 | 0.0090 | 0.0090 | 0.0090 | 0.0090 | 0.0090 |
| KH | 0.0082 | 0.0082 | 0.0082 | 0.0082 | 0.0082 | 0.0082 | 0.0082 | 0.0082 | 0.0082 | 0.0082 |
| JL | 0.0509 | 0.0509 | 0.0509 | 0.0509 | 0.0509 | 0.0509 | 0.0509 | 0.0509 | 0.0509 | 0.0509 |
| EN | 0.0176 | 0.0176 | 0.0176 | 0.0176 | 0.0176 | 0.0176 | 0.0176 | 0.0176 | 0.0176 | 0.0176 |
| FF | 0.0236 | 0.0236 | 0.0236 | 0.0236 | 0.0236 | 0.0236 | 0.0236 | 0.0236 | 0.0236 | 0.0236 |
| UH | 0.0030 | 0.0030 | 0.0030 | 0.0030 | 0.0030 | 0.0030 | 0.0030 | 0.0030 | 0.0030 | 0.0030 |
| JJ | 0.0125 | 0.0125 | 0.0125 | 0.0125 | 0.0125 | 0.0125 | 0.0125 | 0.0125 | 0.0125 | 0.0125 |
| JP | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 |
| COLUMN SUM | 0.8292 | 0.8292 | 0.8292 | 0.8292 | 0.8292 | 0.8292 | 0.8292 | 0.8292 | 0.8292 | 0.8292 |
Annex A.1 – Partitioned matrix inverse; ITA (results from the rectangular M&U model with total flows at pp) (cont.)

|    | AL | JM | LN | EE | FF | HH | IL | MM | NN | PP |
|----|----|----|----|----|----|----|----|----|----|----|
| AL | 0.049 | 0.045 | 0.043 | 0.035 | 0.015 | 0.010 | 0.013 | 0.009 | 0.056 | 0.055 |
| JM | 0.009 | 0.005 | 0.007 | 0.003 | 0.006 | 0.002 | 0.002 | 0.002 | 0.001 | 0.001 |
| LN | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| EE | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| FF | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| HH | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| IL | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| MM | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| NN | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| PP | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |

COLUMN SUM

|    | 1.006 | 1.009 | 1.191 | 1.144 | 1.385 | 1.029 | 1.503 | 1.543 | 1.429 | 1.415 | 0.324 | 0.370 | 1.039 | 0.350 |
|----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| AL | 0.009 | 0.006 | 0.028 | 0.015 | 0.006 | 0.002 | 0.002 | 0.002 | 0.001 | 0.001 | 0.004 | 0.005 | 0.002 | 0.001 |
| JM | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.004 | 0.005 | 0.002 | 0.001 |
| LN | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.004 | 0.005 | 0.002 | 0.001 |
| EE | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.004 | 0.005 | 0.002 | 0.001 |
| FF | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.004 | 0.005 | 0.002 | 0.001 |
| HH | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.004 | 0.005 | 0.002 | 0.001 |
| IL | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.004 | 0.005 | 0.002 | 0.001 |
| MM | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.004 | 0.005 | 0.002 | 0.001 |
| NN | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.004 | 0.005 | 0.002 | 0.001 |
| PP | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.004 | 0.005 | 0.002 | 0.001 |

COLUMN SUM

|    | 1.006 | 1.009 | 1.191 | 1.144 | 1.385 | 1.029 | 1.503 | 1.543 | 1.429 | 1.415 | 0.324 | 0.370 | 1.039 | 0.350 |
Annex A.2 – Partitioned matrix inverse: CTA (results from the rectangular M&U model with total flows at pp)

|     | AA   | BB    | CA    | DB    | OC    | DD    | DE    | EF    | DG    | DG    | DJ    | DJ    | DK    |
|-----|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| AA  | 1.239| 0.000 | 0.000 | 0.217 | 0.025 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| BB  | 0.000 | 1.015 | 0.000 | 0.022 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| CA  | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| DB  | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| OC  | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| DD  | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| DE  | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| EF  | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| DG  | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| DJ  | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 |
| DJ  | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 |
| DK  | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 |
Annex A.2 – Partitioned matrix inverse: CTA (results from the rectangular M&U model with total flows at pp) (cont.)

|     | AA | BB | CC | DD | EE | FF | GG | HH | II | JJ | KK | LL | MM | NN | OO | PP |
|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| AA  | 0.017 | 0.013 | 0.627 | 0.010 | 0.015 | 0.043 | 0.052 | 0.023 | 0.054 | 0.034 | 0.025 | 0.045 | 0.039 | 0.062 | 0.037 | 0.035 |
| BB  | 0.010 | 0.012 | 0.629 | 0.010 | 0.015 | 0.043 | 0.052 | 0.023 | 0.054 | 0.034 | 0.025 | 0.045 | 0.039 | 0.062 | 0.037 | 0.035 |
| CC  | 0.013 | 0.012 | 0.629 | 0.010 | 0.015 | 0.043 | 0.052 | 0.023 | 0.054 | 0.034 | 0.025 | 0.045 | 0.039 | 0.062 | 0.037 | 0.035 |
| DD  | 0.015 | 0.010 | 0.629 | 0.010 | 0.015 | 0.043 | 0.052 | 0.023 | 0.054 | 0.034 | 0.025 | 0.045 | 0.039 | 0.062 | 0.037 | 0.035 |
| EE  | 0.045 | 0.023 | 0.629 | 0.010 | 0.015 | 0.043 | 0.052 | 0.023 | 0.054 | 0.034 | 0.025 | 0.045 | 0.039 | 0.062 | 0.037 | 0.035 |
| FF  | 0.023 | 0.015 | 0.629 | 0.010 | 0.015 | 0.043 | 0.052 | 0.023 | 0.054 | 0.034 | 0.025 | 0.045 | 0.039 | 0.062 | 0.037 | 0.035 |
| GG  | 0.015 | 0.010 | 0.629 | 0.010 | 0.015 | 0.043 | 0.052 | 0.023 | 0.054 | 0.034 | 0.025 | 0.045 | 0.039 | 0.062 | 0.037 | 0.035 |
| HH  | 0.023 | 0.015 | 0.629 | 0.010 | 0.015 | 0.043 | 0.052 | 0.023 | 0.054 | 0.034 | 0.025 | 0.045 | 0.039 | 0.062 | 0.037 | 0.035 |
| II  | 0.015 | 0.010 | 0.629 | 0.010 | 0.015 | 0.043 | 0.052 | 0.023 | 0.054 | 0.034 | 0.025 | 0.045 | 0.039 | 0.062 | 0.037 | 0.035 |
| JJ  | 0.054 | 0.034 | 0.629 | 0.010 | 0.015 | 0.043 | 0.052 | 0.023 | 0.054 | 0.034 | 0.025 | 0.045 | 0.039 | 0.062 | 0.037 | 0.035 |
| KK  | 0.034 | 0.025 | 0.629 | 0.010 | 0.015 | 0.043 | 0.052 | 0.023 | 0.054 | 0.034 | 0.025 | 0.045 | 0.039 | 0.062 | 0.037 | 0.035 |
| LL  | 0.054 | 0.034 | 0.629 | 0.010 | 0.015 | 0.043 | 0.052 | 0.023 | 0.054 | 0.034 | 0.025 | 0.045 | 0.039 | 0.062 | 0.037 | 0.035 |
| MM  | 0.034 | 0.025 | 0.629 | 0.010 | 0.015 | 0.043 | 0.052 | 0.023 | 0.054 | 0.034 | 0.025 | 0.045 | 0.039 | 0.062 | 0.037 | 0.035 |
| NN  | 0.054 | 0.034 | 0.629 | 0.010 | 0.015 | 0.043 | 0.052 | 0.023 | 0.054 | 0.034 | 0.025 | 0.045 | 0.039 | 0.062 | 0.037 | 0.035 |
| OO  | 0.034 | 0.025 | 0.629 | 0.010 | 0.015 | 0.043 | 0.052 | 0.023 | 0.054 | 0.034 | 0.025 | 0.045 | 0.039 | 0.062 | 0.037 | 0.035 |
| PP  | 0.054 | 0.034 | 0.629 | 0.010 | 0.015 | 0.043 | 0.052 | 0.023 | 0.054 | 0.034 | 0.025 | 0.045 | 0.039 | 0.062 | 0.037 | 0.035 |

Continued...
### Annex A.2 – Partitioned matrix inverse: CTA (results from the rectangular M&U model with total flows at pp) (cont.)

|    | AA | BB | CA | CS | DA | DB | DC | DD | DE | DF | DG | DH | DI | DJ | DK |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| AA | 0.1960 | 0.0400 | 0.0363 | 0.0433 | 0.0363 | 0.0313 | 0.0363 | 0.0276 | 0.0198 | 0.0102 | 0.0102 | 0.0102 | 0.0102 | 0.0102 | 0.0102 |
| BB | 0.0000 | 0.0472 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| CA | 0.0614 | 0.0381 | 0.1145 | 0.0116 | 0.0363 | 0.0433 | 0.0363 | 0.0276 | 0.0198 | 0.0102 | 0.0102 | 0.0102 | 0.0102 | 0.0102 | 0.0102 |
| CS | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| DA | 0.0707 | 0.0283 | 0.0287 | 0.0287 | 0.0287 | 0.0287 | 0.0287 | 0.0287 | 0.0287 | 0.0287 | 0.0287 | 0.0287 | 0.0287 | 0.0287 | 0.0287 |
| DB | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| DC | 0.0600 | 0.0150 | 0.0333 | 0.0333 | 0.0333 | 0.0333 | 0.0333 | 0.0333 | 0.0333 | 0.0333 | 0.0333 | 0.0333 | 0.0333 | 0.0333 | 0.0333 |
| DD | 0.0774 | 0.0317 | 0.0317 | 0.0317 | 0.0317 | 0.0317 | 0.0317 | 0.0317 | 0.0317 | 0.0317 | 0.0317 | 0.0317 | 0.0317 | 0.0317 | 0.0317 |
| DE | 0.1686 | 0.0843 | 0.1765 | 0.1765 | 0.1765 | 0.1765 | 0.1765 | 0.1765 | 0.1765 | 0.1765 | 0.1765 | 0.1765 | 0.1765 | 0.1765 | 0.1765 |
| DF | 0.0367 | 0.0625 | 0.3175 | 0.3175 | 0.3175 | 0.3175 | 0.3175 | 0.3175 | 0.3175 | 0.3175 | 0.3175 | 0.3175 | 0.3175 | 0.3175 | 0.3175 |
| DG | 0.0336 | 0.0352 | 0.0352 | 0.0352 | 0.0352 | 0.0352 | 0.0352 | 0.0352 | 0.0352 | 0.0352 | 0.0352 | 0.0352 | 0.0352 | 0.0352 | 0.0352 |
| DH | 0.0926 | 0.0356 | 0.0356 | 0.0356 | 0.0356 | 0.0356 | 0.0356 | 0.0356 | 0.0356 | 0.0356 | 0.0356 | 0.0356 | 0.0356 | 0.0356 | 0.0356 |
| DI | 0.1771 | 0.0732 | 0.0732 | 0.0732 | 0.0732 | 0.0732 | 0.0732 | 0.0732 | 0.0732 | 0.0732 | 0.0732 | 0.0732 | 0.0732 | 0.0732 | 0.0732 |
| DJ | 0.1044 | 0.0448 | 0.0448 | 0.0448 | 0.0448 | 0.0448 | 0.0448 | 0.0448 | 0.0448 | 0.0448 | 0.0448 | 0.0448 | 0.0448 | 0.0448 | 0.0448 |
| DK | 0.0503 | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0086 | 0.0086 |
| DL | 0.0494 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 | 0.0079 |
| DM | 0.0041 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 |
| DN | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| DP | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| PP | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
Annex A.2 – Partitioned matrix inverse: CTA (results from the rectangular M&U model with total flows at pp) (cont.)

|   | AA | AL | AM | AN | AO | AP | AR | AS | AT | AU | AV | AW | AX | AY | AZ |
|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| AA | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| AL | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| AM | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| AN | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| AO | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| AP | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| AR | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| AS | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| AT | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| AU | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| AV | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| AW | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| AX | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| AY | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
| AZ | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 |
Annex A.3 – Domestic flow $bp$ product-by-product inverse matrix: ITA (symmetric model)

| A   | B   | C   | D   | E   | F   | G   | H   | I   | J   | K   | L   | M   | N   | O   | P   |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| A   | 1.131 | 0.0033 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| B   | 0.0033 | 1.131 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| C   | 0.0000 | 0.0000 | 1.131 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| D   | 0.0000 | 0.0000 | 0.0000 | 1.131 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| E   | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.131 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| F   | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.131 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| G   | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.131 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| H   | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.131 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| I   | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.131 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| J   | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.131 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| K   | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.131 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| L   | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.131 | 0.0000 | 0.0000 | 0.0000 |
| M   | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.131 | 0.0000 | 0.0000 |
| N   | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.131 | 0.0000 |
| O   | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.131 |
| P   | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.131 |
Annex A.4 – Domestic flow *bp* product-by-product inverse matrix: CTA (symmetric model)

|     | A  | B  | C  | D  | E  | F  | G  | H  | I  | J  | K  |
|-----|----|----|----|----|----|----|----|----|----|----|----|
| A   | 1.23 |     |     |     |     |     |     |     |     |     |     |
| B   | 0.02 | 0.01 |     |     |     |     |     |     |     |     |     |
| C   | 0.02 | 0.01 | 0.01 |     |     |     |     |     |     |     |     |
| D   | 0.02 | 0.01 | 0.01 | 0.01 |     |     |     |     |     |     |     |
| E   | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 |     |     |     |     |     |     |
| F   | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |     |     |     |     |     |
| G   | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |     |     |     |     |
| H   | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |     |     |     |
| I   | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |     |     |
| J   | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |     |
| K   | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
Annex A.5 – Domestic flow $bp$ industry-by-industry inverse matrix: ITA (symmetric model)

|   | A1 | A2 | A3 | A4 | A5 | A6 | B1 | B2 | B3 | B4 | B5 | B6 | C1 | C2 | C3 | C4 | C5 | C6 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A1 | 1.257 | 0.020 | 0.003 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| A2 | 0.001 | 1.000 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| A3 | 0.001 | 0.001 | 1.000 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| A4 | 0.001 | 0.001 | 0.001 | 1.000 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| A5 | 0.001 | 0.001 | 0.001 | 0.001 | 1.000 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| A6 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 1.000 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |

... (Table continued)
Annex A.6 – Domestic flow bp industry-by-industry inverse matrix: CTA (symmetric model)

|   | AA | BB | CC | DD | EE | FF | GG | HH | II | JJ |
|---|----|----|----|----|----|----|----|----|----|----|
| AA | 1.149 | 0.327 | 0.267 | 0.383 | 0.287 | 0.308 | 0.334 | 0.399 | 0.319 | 0.324 |
| BB | 0.000 | 0.310 | 0.104 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| CC | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| DD | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| EE | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| FF | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| GG | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| HH | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| II | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| JJ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

|   | AA | BB | CC | DD | EE | FF | GG | HH | II | JJ |
|---|----|----|----|----|----|----|----|----|----|----|
| AA | 1.265 | 1.321 | 1.318 | 1.309 | 1.324 | 1.332 | 1.351 | 1.399 | 1.375 | 1.382 |
| BB | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| CC | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| DD | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| EE | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| FF | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| GG | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| HH | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| II | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| JJ | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
This appendix makes the mathematical proof of the full equivalence of the two methods considered in this paper, besides the practical example that was provided above. ITA is the reference technological assumption in this appendix, although an equivalent proof can be made for CTA.

Let us start with the product-by-product case. Our aim is to show that the multipliers inserted in the inverse matrix of the symmetric, domestic-flow, bp table (like the ones of Annex A.3), are the same that may be inferred of the upper left-hand part of the M&U’s inverse (as in Annex A.1).

The multipliers of this upper left-hand block correspond to equation (7). The first step is to re-write (7) for domestic flows at bp. Defining \( \hat{t} = (I - \hat{c}) (I - \hat{f} - \hat{n}) \) as the diagonal matrix that proceeds (through pre-multiplication) to the transformation of a column-vector (or of each column of one matrix) of total-flows at pp into its equivalent with domestic-flows at bp, then:

\[
(p^N)^{\text{bp}} = \hat{t} (y^{\text{pp}}) \quad \text{and} \quad (y^N)^{\text{bp}} = \hat{t} (y^{\text{pp}}) \quad \text{(A.1)}
\]

(in this appendix the superscript \( N \) means domestic flows and \( bp/pp \) basic or purchasers prices)

On the other hand:

\[
(Q^N)^{\text{bp}} = \hat{t} Q \quad \text{(A.2)}
\]

(because \( Q = U^{\text{pp}} (\hat{g}^{\text{bp}})^{-1} \) and \( (Q^N)^{\text{bp}} = (U^N)^{\text{bp}} (\hat{g}^{\text{bp}})^{-1} \) by the «technical» coefficient definition. So \( (U^{\text{bp}})^{\text{pp}} = \hat{t} U^{\text{pp}} \Rightarrow (Q^N)^{\text{bp}} = \hat{t} Q \).

As for the matrix \( S \) (in (7) as well), this one was already calculated with domestic production only, and it was evaluated at \( bp \). However, the denominator in these coefficients were the cells of \( p^{\text{pp}} \): the total product supplies, with imports included, at \( pp \). This means: \( S = v^{\text{bp}} (\hat{p}^{\text{pp}})^{-1} \).

Being \( S^N = v^{\text{bp}} (\hat{p}^{\text{bp}})^{-1} \) instead, then:

\[
S^N = v^{\text{bp}} (\hat{p}^{\text{pp}})^{-1} = v^{\text{bp}} (\hat{p}^{\text{pp}})^{-1} \hat{t}^{-1} = S \hat{t}^{-1} \quad \text{(A.3)}
\]

Making use of these results, and returning to (7), we may then conclude that:

\[
p^{\text{pp}} = (I - QS)^{-1} y^{\text{pp}}
\]

\[
\hat{t} p^{\text{pp}} = \hat{t} (I - QS)^{-1} \hat{t}^{-1} t y^{\text{pp}}
\]

\[
(p^N)^{\text{bp}} = [\hat{t} (I - QS) \hat{t}^{-1}]^{-1} (y^N)^{\text{bp}} \quad \text{by (A.1)}
\]

\[
(p^N)^{\text{bp}} = (I - \hat{t} QS \hat{t}^{-1})^{-1} (y^N)^{\text{bp}}
\]

\[
(p^N)^{\text{bp}} = \left[ I - (Q^N)^{\text{bp}} S^N \right]^{-1} (y^N)^{\text{bp}} \quad \text{by (A.1) and (A.3)}
\]
On the other hand, remark that the symmetric model is not based on \((U^N)^{bp}\), as this is a product-by-industry matrix. Let \((Z^N)^{bp}\) denote instead the intermediate consumption matrix of the symmetric product-by-product domestic-flow table (at basic prices). This matrix includes the products needed for the production of each product. Making use of the ITA assumption, \((Z^N)^{bp}\) can be computed by the equation:

\[
(Z^N)^{bp} = (Q^N)^{bp} V^{bp}
\tag{A.5}
\]

The corresponding «technical» coefficients matrix is:

\[
A^N = (Z^N)^{bp} \left(\hat{\rho}^N\right)^{bp}^{-1} = (Q^N)^{bp} V^{bp} \left[\hat{\rho}^N\right]^N_{bp}^{-1} = (Q^N)^{bp} S^N
\tag{A.6}
\]

which is the same than the outcome of (A.4).

Concerning now the industry-by-industry case, the symmetric domestic-flow ITA-based table (at basic prices) comprises an intermediate consumption matrix \((Z_1^N)^{bp}\) derived as follows:

\[
(Z_1^N)^{bp} = S^N (U^N)^{bp}, \quad \text{so}
\]

\[
(Z_1^N)^{bp} (g^{bp})^{-1} = S^N (U^N)^{bp} (g^{bp})^{-1}
\]  

\[
A_1^N = S^N (Q^N)^{bp} \Rightarrow (I - A_1^N)^{-1} = (I - S^N (Q^N)^{bp})^{-1}
\tag{A.7}
\]

It is straightforward that this is the same than the lower right-hand block in equation (6), displayed as well in our example in Annex A.1, as:

\[
S^N (Q^N)^{bp} = \hat{S} \hat{i}^{-1} \hat{Q} = SQ
\]

by (A.2) and (A.3)  

\tag{A.8}