Superconformal field theories from IIB spectroscopy on $AdS_5 \times T^{11}$

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Abstract.

We report on tests of the AdS/CFT correspondence that are made possible by complete knowledge of the Kaluza–Klein mass spectrum of type IIB supergravity on $AdS_5 \times T^{11}$ with $T^{11} = SU(2)^2/U(1)$. After briefly discussing general multiplet shortening conditions in $SU(2,2|1)$ and $PSU(2,2|4)$, we compare various types of short $SU(2,2|1)$ supermultiplets on $AdS_5$ and different families of boundary operators with protected dimensions. The supergravity analysis predicts the occurrence in the SCFT at leading order in $N$ and $g_s N$, of extra towers of long multiplets whose dimensions are rational but not protected by supersymmetry.

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1. Introduction

One of the most stringent checks on the AdS/CFT correspondence \[1, 2, 3\] is the matching between the mass spectrum of the Kaluza–Klein (KK) supergravity models and the conformal dimensions of superconformal primary operators of the boundary superconformal field theory. This probes the correspondence at least in the regime where \( g_s N \) (\( g_s \) being the string coupling) and/or \( N \) are large. After the strong support provided by tests made for maximal supersymmetry, where the dynamics of \( N \) coincident D3 branes (for large \( N \)) is related to type IIB supergravity compactified on \( AdS_5 \times S_5 \) \[4\], it is natural to consider lower supersymmetry, where a far richer structure of matter multiplets leads to additional symmetries beside the original \( R \)-symmetry. Alternatively to orbifolding the sphere \( S_5 \), a very interesting way to reduce supersymmetry is to consider coset models such as \( T_{pq} = SU(2) \times SU(2) U(1) \) (\( p \) and \( q \) define the embedding of the \( H = U(1) \) group into the two \( SU(2) \) groups), which yield for \( p = q = 1 \) an \( N = 2 \) supergravity theory with a matter gauge group \( G = SU(2) \times SU(2) \) \[5\]. The corresponding \( CFT_4 \) description was constructed in \[6, 7\] as an \( N = 1 \) Yang–Mills theory with a flavour symmetry \( G \). One finds a conformal field theory with “singleton” degrees of freedom \( A \) and \( B \), each a doublet of the factor groups \( SU(2) \times SU(2) \) and with conformal anomalous dimension \( \Delta_{A,B} = 3/4 \). The gauge group \( G \) is \( SU(N) \times SU(N) \) and the two singleton (chiral) multiplets are respectively in the \((N,\overline{N})\) and \((\overline{N},N)\) of \( G \). The gauge potentials lying in the adjoint of one of the two \( SU(N) \) groups, whose field–strength in superfield notation is given by \( W_{\alpha} \), are singlet of the matter groups, carry unit \( U_R(1) \) charge and have \( \Delta = 3/2 \).

There is also a superpotential \[6\] \( V = \lambda \epsilon^{ij} \epsilon^{kl} Tr(A_i B_k A_j B_l) \) with \( \Delta = 3 \), \( r = 2 \) playing an important role in the discussion, since it determines to some extent both the chiral spectrum and the marginal deformations of the SCFT.

Chiral operators which are the analogue of the KK excitations of the \( SU(N) \) \( \mathcal{N} = 4 \) Yang–Mills theory is given by \( Tr(AB)^k \) with \( R \)-charge \( k \) and in the \( (k,k) \) representation of \( SU(2) \times SU(2) \) \[6\].

More generally \[8\], there exist a complete correspondence between all the CFT operators and the KK modes for the conformal operators of preserved scaling dimension. Even more intriguingly, there exist other operators related to long multiplets but having nonetheless non–renormalised conformal dimension in the large \( N \), \( g_s N \) limit. These seem to be the lowest dimensional ones for a given structure appearing in the supersymmetric Born–Infeld action of the \( D3 \)-brane on \( AdS_5 \times T^{11} \) \[9\].

It is now well known that states that are associated to shortened multiplets of the superconformal algebra \( SU(2,2|N) \) for the \( AdS_5/CFT_4 \) duality, in virtue of supersymmetry have protected conformal dimensions. They are BPS states from the point of view of the bulk theory and shortened superfields from the boundary perspective.

After this introduction, we present a brief review of the multiplet shortening
conditions for the $SU(2,2|1)$ and $PSU(2,2|4)$ superalgebras, while in section 3 we show the results of the comparison between the dual AdS and CFT theories for the $T^{11}$ example.

2. Group theory lore: UIR’s of $SU(2,2|1)$ and $PSU(2,2|4)$

We first consider the unitarity bounds for the highest-weight representations of the $SU(2,2|\mathcal{N})$ superalgebra in the $\mathcal{N} = 1$ and $\mathcal{N} = 4$ cases, that are those relevant for the analysis of the $T^{11}$ and the $S_5$ IIB compactifications respectively.

For the $SU(2,2)$ algebra itself, a given UIR is denoted, following Flato and Frønsdal [10], as $D(E_0, J_1, J_2)$ where $E_0, J_1, J_2$ are the quantum numbers of the highest-weight state, given by a finite UIR of the maximal compact subgroup $SU(2) \times SU(2) \times U(1)$. The UIR’s fall in three series [11],

$$
\begin{align*}
\text{a) } J_1J_2 &\neq 0 & E_0 &\geq 2 + J_1 + J_2 \\
\text{b) } J_2J_1 &\neq 0 & E_0 &\geq 1 + J \\
\text{c) } J_1 &\neq J_2 & E_0 &\geq 0 .
\end{align*}
$$

In the bulk interpretation, the inequalities in a) and b) yield massive $AdS_5$ representations. Their saturation gives rise to massless particles of spin $J_1 + J_2$ in the case a) and to singlets of spin $J$ for the b) threshold.

Note that in the AdS/CFT map the bulk-boundary quantum numbers $(E_0, J_1, J_2)$ refer to the compact basis for the AdS states, while they refer to the non-compact basis $SL(2, C) \times O(1,1)$ for the boundary conformal operators [3, 12]. The highest weight state in AdS is related to a conformal operator $O(x)$ at $x = 0$, and thus the AdS energy $E_0$ becomes the conformal dimension $\Delta_0$ while the $(J_1, J_2)$ labels give the Lorentz spin of $O(x)$.

From the CFT perspective, the threshold value for the bound a) represents a conformal conserved currents of spin $J = J_1 + J_2$,

$$
E_0 = 2 + J_1 + J_2 \quad (J_1J_2 \neq 0) \quad \rightarrow \quad \partial^{a_1\dot{a}_1} J_{\alpha_1\dot{\alpha}_1\alpha_2\dot{\alpha}_2\ldots\alpha_{2J_2}}(x) = 0 ,
$$
while for the bound b) one gets massless spin $J$ conformal fields on the boundary,

$$
\begin{align*}
E_0 & = 1 + J \quad (J \neq 0) & \quad \rightarrow \quad & \partial^{a_1\dot{a}_1} O_{\alpha_1\ldots\alpha_{2J}} = 0 \\
& = 0 \quad (J = 0) & \quad \rightarrow \quad & \partial^2 O(x) = 0 .
\end{align*}
$$

The case c) gives rise to the identity representation.

Generalising to the $SU(2,2|\mathcal{N})$ superalgebras [13, 14], the highest weight state is denoted by $D(E_0, J_1, J_2; r, a_1, \ldots, a_{\mathcal{N}-1})$, where the quantum numbers in brackets indicates an UIR of $SU(2,2) \times U(1) \times SU(\mathcal{N})$, $r$ labelling the $U(1)$ R-symmetry and $a_1, \ldots, a_{\mathcal{N}-1}$ the Dynkin labels of a UIR of the non-abelian symmetry $SU(\mathcal{N})$. We will denote by $R$ the $U(1)$ generator inside $U(\mathcal{N})$.

Note that for $\mathcal{N} \neq 4$, the $SU(2,2|\mathcal{N})$ algebra is both a subalgebra and a quotient algebra of $U(2,2|\mathcal{N})$, since the supertrace generator (which is a central charge) can be
eliminated by a redefinition of the R generator. However, this redefinition is not possible for $\mathcal{N} = 4$ since in that case $R$ drops from the supersymmetry anti-commutators and becomes an outer automorphism of the algebra $\mathfrak{psu}(4)$. Therefore there are two inequivalent algebras (which do not include the R generator), $PSU(2,2|4)$ and $PU(2,2|4)$, depending on whether $r = 0$ or $r \neq 0$ (for $\mathcal{N} = 4$, $r$ denotes the central charge).

Since the $\mathcal{N} = 4$ Yang–Mills multiplet has $r = 0$, we will only consider $PSU(2,2|4)$. In the boundary CFT language, where UIR’s can be realized as conformal superfields, the superhighest weight state corresponds to a superfield $\phi(x,\theta)$ at $x = \theta = 0$ 
\cite{14,13,25,17}.

The unitarity bounds for $SU(2,2|1)$ were given in \cite{10,14,18}. They generalize the cases a), b) and c) of eq. (1) and read

\begin{equation}
A) \quad E_0 \geq 2 + 2J_2 + \frac{3}{2}r \geq 2 + 2J_1 - \frac{3}{2}r \quad (\text{or } J_1 \rightarrow J_2, \quad r \rightarrow -r) \quad J_1, J_2 \geq 0
\end{equation}

which implies

\begin{equation}
E_0 \geq 2 + J_1 + J_2, \quad \frac{3}{2}r \geq J_1 - J_2, \quad 2 + 2J_1 - E_0 \leq \frac{3}{2}r \leq E_0 - 2 - 2J_2.
\end{equation}

\begin{equation}
B) \quad E_0 = \frac{3}{2}r \geq 2 + 2J - \frac{3}{2}r \quad (J_2 = 0, \quad J_1 = J, \quad \text{or } J_1 = 0, \quad J_2 = J, \quad r \rightarrow -r)
\end{equation}

and thus $E_0 \geq 1 + J$. Finally

\begin{equation}
C) \quad E_0 = J_1 = J_2 = r = 0
\end{equation}

which is the identity representation.

Shortening in the case A) takes place when

\begin{equation}
E_0 = 2 + 2J_2 + \frac{3}{2}r, \quad \left(\frac{3}{2}r \geq J_1 - J_2\right) \quad (\text{or } J_1 \rightarrow J_2, \quad r \rightarrow -r).
\end{equation}

This is a semi-long AdS$_5$ multiplet or, in conformal language, a semiconserved superfield 
\cite{8,19},

\begin{equation}
\bar{D}^{\alpha_1} L_{\alpha_1 \ldots \alpha_{2J_1}, \phi_{\alpha_{2J_2}}} (x, \theta, \bar{\theta}) = 0, \quad (\bar{D}^2 L_{\alpha_1 \ldots \alpha_{2J_1}} = 0 \text{ for } J_2 = 0)
\end{equation}

(in our conventions $\theta$ carries $\Delta = -1/2, r = 1, \bar{\theta}$ has $\Delta = -1/2, r = -1$).

Maximal shortening for the bound A) happens for $E_0 = 2 + J_1 + J_2, \quad r = J_1 - J_2$. This is a conserved superfield which satisfies both left and right constraints:

\begin{equation}
\bar{D}^{\alpha_1} J_{\alpha_1 \ldots \alpha_{2J_1}, \phi_{\alpha_{2J_2}}} = D^{\alpha_1} J_{\alpha_1 \ldots \alpha_{2J_1}, \phi_{\alpha_{2J_2}}} = 0
\end{equation}

Further, shortening in B) corresponds to chiral superfields $r = 2/3E_0$, while maximal shortening to massless chiral superfields, i.e. chiral singleton representations: $E_0 = \frac{3}{2}r = 1 + J$. The superfield, for $E_0 = \frac{3}{2}r$ satisfies

\begin{equation}
\bar{D}^{\alpha_1} S_{\alpha_1 \ldots \alpha_{2J}} = 0
\end{equation}

and, for $E_0 = 1 + J$, it also satisfies

\begin{equation}
D^{\alpha_1} S_{\alpha_1 \ldots \alpha_{2J}} = 0 \quad (D^2 S = 0, \text{ for } J = 0)
\end{equation}
These equations are the supersymmetric version of (4) and (5).

With an abuse of language, we may call off-shell singletons chiral superfields since in an interacting conformal field theory singletons may acquire anomalous dimension, and thus fall in (13).

It is also evident, from superfield multiplication, that by taking suitable products of several free supersingletons one may get any other superfield of type (11), (12) or (13).

One can remark that, since the shortening condition just implies a relation between $E_0$ and $r$ without fixing their value, superfields obeying (11), (13) may have anomalous dimensions.

The basic singleton multiplets for $\mathcal{N} = 1$ gauge theories arise for $J = 0, 1/2$ in (13), i.e. chiral scalar superfields $S$ (Wess-Zumino multiplets) and Yang-Mills field strength multiplets $W_\alpha$. Any other conformal operator is obtained by suitable multiplication of these two sets of basic superfields.

In type IIB supergravity on $T^{11}$ long, semi-long and chiral multiplets do indeed occur [3, 21, 8]. Chiral WZ singleton multiplets have in this case an anomalous dimension $\gamma = -1/4 (\Delta = 1 + \gamma)$ and R-symmetry $r = 3/4$.

The $\mathcal{N} = 4$ superalgebra is of great interest because it corresponds to $\mathcal{N} = 4$ superconformal Yang-Mills theory and lives, in the dual description, at the boundary of $\text{AdS}_5$ [1, 2, 3]. The supergravity theory emerges as the low energy limit of type IIB string theory compactified on $\text{AdS}_5 \times S_5$.

The highest weight UIR’s of the $\text{PSU}(2,2|4)$ superalgebra are denoted by $D(E_0, J_1, J_2; p, k, q)$, where $(p, k, q)$ are the $SU(4)$ Dynkin labels.

There exist three classes of UIR’s

\begin{equation}
A' \quad E_0 \geq 2 + J_1 + J_2 + p + k + q, \quad J_2 - J_1 \geq \frac{1}{2}(p - q),
\end{equation}

with maximal shortening occurring when,

\begin{equation}
E_0 = 2 + J_1 + J_2 + p + k + q, \quad J_2 - J_1 = \frac{1}{2}(p - q).
\end{equation}

Massless bulk multiplets arise for $p = k = q = 0$ and $J_1 = J_2$.

\begin{equation}
B' \quad E_0 = \frac{1}{2}(p + 2k + 3q) \geq 2 + 2J + \frac{1}{2}(3p + 2k + q)
\end{equation}

$(J_2 = 0, J_1 = J$ or $J_1 \rightarrow J_2, (p, k, q) \rightarrow (q, k, p))$

with

\begin{equation}
E_0 \geq 1 + J + p + k + q \quad 1 + J \leq \frac{1}{2}(q - p).
\end{equation}

Maximal shortening occurs when $1 + J = \frac{1}{2}(q - p)$, with highest weight $D(3 + 3J + 2p + k, J, 0; p, k, p + 2 + 2J)$. No supersingletons appear in this series. Finally,

\begin{equation}
C' \quad E_0 = 2p + k, \quad p = q, \quad J_1 = J_2 = 0.
\end{equation}

The highest weight states are $D(2p + k, 0, 0; p, k, p)$. The $p = 0, k \geq 2$ UIR’s correspond to the KK states of type IIB on $\text{AdS}_5 \times S_5$, the $k = 2$ case being associated with the
bulk graviton multiplet. The \( p = 0, k = 1 \) UIR yields the only supersingleton of the \( PSU(2, 2|4) \) algebra \[21, 22\]. The infinite sequence of UIR’s with \( p = 0 \), multiplets with \( J_{MAX} = 2 \) have been obtained in \[23\] with the oscillator construction. They are associated with the harmonic holomorphic superfields of \[24\]. The case \( p \neq 0 \), which may be relevant for multiparticles supergravity states, has been discussed in \[16\].

3. Confronting with experiment

\( T^{pq} = SU(2)^2/U(1) \) cosets are Einstein spaces having \( \mathcal{N} = 2 \) supersymmetry only when the subgroup \( U(1) \) generator \( T_H = p\sigma_3 + q\hat{\sigma}_3 \) is defined with \( p = q = 1 \), where \( \sigma_3, \hat{\sigma}_3 \) are Pauli matrices generating the two \( SU(2) \) groups in the numerator. The \( U(1) \) \( R \)-symmetry generator is \( T_R = \sigma_3 - \hat{\sigma}_3 \). Since \( T^{11} \) is topologically the product \( S_2 \times S_3 \), it has non–trivial Betti numbers \( b_2 = b_3 = 1 \), and therefore the full isometry group is \( SU(2, 2|1) \times SU(2) \times SU(2) \) with an extra \( U_B(1) \) gauge symmetry related to the existence of non–trivial three cycles \[23\].

Knowing the fundamental degrees of freedom of the conformal field theory, one could try to write the conformal operators by simply combining the above fields while respecting the symmetries of the theory. Next to the already mentioned \( Tr(AB)^k \) chiral primaries, one could also have an operator given by \( Tr[W_\alpha(AB)^k] \) or \( Tr[W^2(AB)^k] \), and so on. The important point is that the correspondence with the KK states is true only for the protected operators, and thus one needs to know these latters to make the comparison.

3.1. CFT \( \rightarrow \) AdS

The operators with protected conformal dimension correspond to the short representation of the \( SU(2, 2|1) \) supergroup described in the previous section. In our case we have only three types of such operators, namely the chiral \[13\], conserved \[12\] and semi–conserved \[11\] superfields. Since these fields satisfy certain specific constraints effecting their quantum numbers, their anomalous dimension is also fixed in terms of their spin and \( R \)–symmetry charge.

It is easy to relate operators of different type by superfield multiplication. The product of a chiral \((J_1, 0)\) and an anti–chiral \((0, J_2)\) primary gives a generic superfield with \((J_1, J_2), \Delta = \Delta^c + \Delta^a\) and \(r = \frac{2}{3}(\Delta^c - \Delta^a)\). By multiplying a conserved current superfield \( J_{\alpha_1...\alpha_2J_1, \dot{\alpha}_1...\dot{\alpha}_2J_2} \) by a chiral scalar superfield one gets a semi–conserved superfield with \(\Delta = \Delta^c + 2 + J_1 + J_2\) and \(r = \frac{2}{3}(\Delta - 2 - 2J_2)\).

These are the basic rules to construct operators with protected dimensions beside the chiral ones, and they also apply in superconformal field theories of lower or higher dimensions. For instance, beyond the chiral operators with \(\Delta = r\) (hypermultiplets), in \(d = 3\) \(OSp(2|4)\) superconformal field theories one replaces the shortening condition \[11\] by the simpler constraint

\[ D^{-\alpha_1}L_{\alpha_1...\alpha_2s}(x, \theta, \bar{\theta}) = 0 \quad s \neq 0 \]  \hspace{1cm} (20)
defining semiconserved tensor operators \((s = 0, \frac{1}{2}, 1\) in the KK context) with protected dimensions \(\Delta = 1 + s + r\) (or \(r \to -r\) if \(- \to +\)). As before, \(L\) is obtained by multiplying a conserved spin \(s\) superfield (for which both \(D^+\) and \(D^-\) constraints are satisfied) and a chiral superfield\(\S\).

Since the anomalous dimensions of these operators is fixed in terms of their spin and \(R\)-symmetry, it must be given by rational number. This yields a very restrictive condition when searching for the corresponding supergravity states, as it imposes strong constraints on the allowed masses and matter group quantum numbers.

The AdS/CFT correspondence provides a fixed relation between the anomalous dimension of the various fields at the boundary and the masses of the bulk states. A result of our computations is that the requirement for the anomalous dimensions to be rational implies that one must look for dual KK states having also rational masses.

The virtue of KK harmonic analysis on a coset space \([28]\) hinges on the possibility of reducing the computation of the mass eigenvalues of the various kinetic differential operators to a completely algebraic problem. Harmonics are identified by \(G\) quantum numbers, and they are acted upon by derivatives that are reduced to algebraic operators. Such elegant technique can be quite cumbersome for complicated cosets, but it is quite straightforward for the simple \(T^{11}\) manifold. Indeed, it allows to go beyond the computation of the scalar laplacian eigenvalues \([20]\), or of specific sectors of the mass spectrum \([29]\).

After diagonalising different operators for fields of various spin, we have found that all the masses have a fixed dependence on the scalar laplacian eigenvalue

\[
H_0(j, l, r) = 6[j(j + 1) + l(l + 1) - 1/8r^2] \tag{22}
\]

where \((j, l, r)\) refer to the \(SU(2)^2\) and \(R\)-symmetry quantum numbers. This is due to the fact that on a rank one coset we have only one functionally independent Laplace-Beltrami operator.

The full analysis \([30]\) reveals that the supergravity theory has one graviton multiplet with conformal dimensions

\[
\Delta = 1 + \sqrt{H_0(j, l, r) + 4}, \tag{23}
\]

four gravitino multiplets with

\[
\Delta = -1/2 + \sqrt{H_0(j, l, r \pm 1) + 4}, \quad \Delta = 5/2 + \sqrt{H_0(j, l, r \pm 1) + 4}, \tag{24}
\]

and four vector multiplets, with

\[
\Delta = -2 + \sqrt{H_0(j, l, r) + 4}, \\
\Delta = 4 + \sqrt{H_0(j, l, r) + 4}, \\
\Delta = 1 + \sqrt{H_0(j, l, r \pm 2) + 4}. \tag{25}
\]

\(\S\) The above basic rules have been recently used in the AdS/CFT correspondence of some \(M\)-theory models compactified on \(AdS_4 \times X_7\) \([27]\).
The above formulae clearly show that rational values of the conformal dimensions occur when the square roots assume rational values

\[ H_0 + 4 \in Q^2. \]  

This equation is found to admit some special solutions for

\[ j = l = |r/2|, \]  
\[ j = l - 1 = |r/2| \text{ or } l = j - 1 = |r/2|. \]

At this point we have some strong constraints on the possible \( SU(2,2|1) \) quantum numbers as well as on the \( SU(2) \times SU(2) \) ones. It is therefore an easy task to build the conformal operators satisfying such constraints and find the corresponding bulk supermultiplets.

While referring to [8] for all details, we now show some interesting examples.

The chiral operators of the conformal field theory are given by

\[ S^k = Tr(AB)^k \]  
\[ \Phi^k = Tr \left[ W^2(AB)^k \right] \]  
\[ T^k = Tr \left[ W_\alpha(AB)^k \right] \]

and are shown to correspond to hyper–multiplets containing massive recursions of the dilaton or the internal metric \((29 \text{ and } 30)\) or to tensor multiplets \((31)\).

More interesting are the towers of operators associated to the semi–conserved currents. Some of them are given by the following operators

\[ J^{\alpha \dot{\alpha}}_k = Tr \left( W_\alpha e^V \bar{W}_{\dot{\alpha}} e^{-V} (AB)^k \right), \]  
\[ J^k = Tr \left( A e^V \bar{A} e^{-V} (AB)^k \right), \]

which lead to short multiplets whose highest state is a spin 2 and spin 1 field respectively, with masses given by

\[ M_{J^{\alpha \dot{\alpha}}_k} = \sqrt{\frac{3}{2} k \left( \frac{3}{2} k + 4 \right)}, \quad \text{and} \quad M_{J^k} = \sqrt{\frac{3}{2} k \left( \frac{3}{2} k + 2 \right)}. \]

These bulk states correspond to massive recursion of the graviton and of the gauge bosons of the matter groups.

It has been explained that under certain conditions the semi–conserved superfields can become conserved, and this is indeed the case. If we set \( k = 0 \) we retrieve the conserved currents related to the stress–energy tensor and the matter isometries. In fact \( M_{J^{\alpha \dot{\alpha}}_0} = M_{J^0} = 0 \) are the massless graviton and gauge bosons of the supergravity theory.

We have now checked the correspondence as far as what the conformal field theory predicts on the bulk states, but what can we learn on the CFT from the analysis of the supergravity states?
3.2. \( AdS \rightarrow CFT \)

There are essentially two aspects of the supergravity theory which can give us new insight in the dual CFT. The first is the existence of the so–called Betti multiplets \[31\], which give rise to additional symmetries of the boundary theory, and the other is the presence of long multiplets with rational scaling dimensions, which could provide us with new non–renormalization theorems at least in the large \( N, g_sN \) limit. Let us now turn to the first aspect. The non–trivial \( b_2 \) and \( b_3 \) numbers of the \( T^{11} \) manifold imply the existence of closed non–exact 2–form \( Y_{ab} \) and 3–form \( Y_{abc} \). These forms must be singlets under the full isometry group, and thus they signal the presence of new additional massless states in the theory than those connected to the \( SU(2) \times SU(2) \times U_R(1) \) isometry.

From the KK expansion of the complex rank 2 \( A_{MN} \) and real rank 4 \( A_{MNPQ} \) tensors of type IIB supergravity we learn that we should find in the spectrum a massless vector (from \( A_{\mu abc} \)), a massless tensor (from \( A_{\mu \nu ab} \)) and two massless scalars (from \( A_{ab} \)). This implies the existence of the so called Betti vector, tensor and hyper–multiplets, the last two being a peculiar feature of the \( AdS_5 \) compactification. The additional massless vector can be seen to be the massless gauge boson of an additional \( U_{B}(1) \) symmetry of the theory.

From the boundary point of view we need now to find an operator counterpart for such a vector multiplet and seek an interpretation of the additional symmetry. The task of finding the conformal operator is very easy, once we take into account that it must be a singlet of the full isometry group and must have \( \Delta = 3 \). The only operator we can write is \[3,32\]

\[
U = \text{Tr} \ Ae^V \bar{A}e^{-V} - \text{Tr} \ Be^V \bar{B}e^{-V} \quad (D^2U = \bar{D}^2U = 0),
\]

which represents the conserved current of a baryon symmetry of the boundary theory under which the \( A \) and \( B \) field transform with opposite phase. We have shown that the occurrence of such Betti multiplets is indeed due to the existence of non–trivial two and three–cycles on the \( T^{11} \) manifold. This implies that, from the stringy point of view, we can wrap the \( D3 \)–branes of type IIB superstring theory around such 3–cycles and the wrapping number coincides with the baryon number of the low–energy CFT \[32\].

We finish by commenting on the second AdS prediction on the CFT. We have shown that the conformal operators with protected dimension are given by chiral ones or by their products with the conserved currents. The surprising output of the supergravity analysis is that there exist some long operators (not protected by supersymmetry) which have rational conformal dimension.

If we take for example the chiral operator \( \text{Tr}(W^2(AB)^k) \), we can make it non–chiral by simply inserting into the trace an antichiral combination of the gauge field–strength \( \text{Tr}(W^2e^V \bar{W}^2e^{-V}(AB)^k) \). This operator then corresponds to a long multiplet in the bulk theory and one should expect its scaling dimension to be renormalized to an irrational number. If we search for the corresponding vector multiplet in the supergravity theory, we see that its anomalous dimension is instead rational and matches exactly the naive sum of the dimensions of the operators inside the trace. From our analysis this appears
to be the case for all the lowest non–chiral operators of general towers with irrational scaling dimension. For instance, the towers of operators

$$\begin{align*}
Tr \left[ W_\alpha (Ae^V \bar{A}e^{-V})^n (AB)^k \right] \\
Tr \left[ e^V \bar{W}_\alpha e^{-V} (Ae^V \bar{A}e^{-V})^n (AB)^k \right]
\end{align*}$$

have an irrational value of $\Delta$ for generic $n$, but when $n = 1$ we have found that they do have rational anomalous dimension $\Delta = 5/2 + 3/2k$. When $n = 0$ we retrieve the chiral, or semi–conserved operators with protected $\Delta$.

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References

[1] J. Maldacena, Adv. Theor. Math. Phys. 2, (1998) 231.
[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys. Lett. B428, (1998) 105.
[3] E. Witten, Adv. Theor. Math. Phys. 2, (1998) 253.
[4] O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, Large N field theories, string theory and gravity, preprint hep-th/9905111.
[5] L. J. Romans, Phys. Lett. 153 B, (1985) 392.
[6] I.R. Klebanov and E. Witten, Nucl. Phys. B536, (1998) 199.
[7] D.R. Morrison and M.R. Plesser, Nonspherical horizons. 1, hep-th/9810201.
[8] A. Ceresole, G. Dall’Agata, R. D’Auria and S. Ferrara, Spectrum of Type IIB Supergravity on $AdS_5 \times T^{11}$: Predictions on $N=1 SCFT$’s, hep-th/9905226.
[9] S. Ferrara, M.A. Lledo and A. Zaffaroni, Phys. Rev. D58, (1998) 105029.
[10] M. Flato and C.Fronsdal, Lett. Math. Phys. 2, (1978) 421; Phys. Lett. B97, (1980) 236; J. Math. Phys. 22, (1981) 1100; Phys. Lett. B172, (1986) 412; Lett. Math. Phys. 89 (1984) 159; Essays in Supersymmetries, Math. Phys. Stud. 8, D. Reidel, Dordecht (1986).
[11] B Binegar, C. Fronsdal and W.Heidenreich, Conformal QED, J. Math. Phys. 24 (1983) 2828; S. Ferrara, R. Gatto and A. F. Grillo, Phys. Rev. D9, (1974) 3564.
[12] M. Gunaydin, D. Minic and M. Zagermann, Nucl. Phys. B544, (1999) 737.
[13] I. Bars and M. Gunaydin, Commun. Math. Phys.91 (1983) 31; M. Gunaydin, J. Math. Phys. 29 (1988) 1275.
[14] V.K. Dobrev and V.B. Petkova, Phys. Lett. B162, 127 (1985); Lett. Math. Phys. 9 1985) 287; Fortschr. Phys. 35 (1987), 537.
[15] L. Andrianopoli and S. Ferrara, Phys. Lett. B430, (1998) 248; Lett. Math. Phys. 46, (1998) 265.
[16] S. Ferrara and A. Zaffaroni, Superconformal Field Theories, Multiplet Shortening, and the $AdS_5 \times SCFT_4$ Correspondence, preprint hep-th/9908163.
[17] J. Park, Int. J. Mod. Phys. A13 (1998) 1743, hep-th/9703191: Superconformal Symmetry and Correlation Functions, hep-th/9903230.
[18] D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, Renormalization group flows from holography supersymmetry and a c theorem, preprint hep-th/9904017.
[19] Osborne, Ann. Phys. 272 (1999) 243.
[20] S.S. Gubser, Phys. Rev. D59, (1999) 025006.
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[21] B. Binegar, Phys. Rev. D34 (1986) 525.
[22] M. Gunaydin, D. Minic and M. Zagermann, Nucl. Phys. B534 (1998) 96.
[23] M. Gunaydin and N. Marcus, Class. Quant. Grav. 2, (1985) L11.
[24] P. S. Howe and P. C. West, Phys. Lett. B389 (1996) 273, hep-th/9607061; Phys. Lett. B400 (1997) 307, hep-th/9611075.
[25] S. Ferrara and A. Zaffaroni, Phys. Lett. B431, (1998) 49.
[26] S.S. Gubser and I.R. Klebanov, Phys. Rev. D58, (1998) 499.
[27] D. Fabbri, P. Fre', L. Gualtieri, C. Reina, A. Tomasiello, A. Zaffaroni, A. Zampa, preprint hep-th/9907219.
[28] L. Castellani, R. D’Auria and P. Fre, Supergravity and superstrings: A geometric perspective. Vol. 1–2, Singapore, Singapore: World Scientific (1991); A. Salam and J. Strathdee, Annals Phys. 141, (1982) 316.
[29] D.P. Jatkar and S. Randjbar-Daemi, Type IIB string theory on $AdS_5 \times T^{11}$, preprint hep-th/9904187.
[30] A. Ceresole, G. Dall’Agata and R. D’Auria, KK Spectroscopy of Type IIB Supergravity on $AdS_5 \times T^{11}$, hep-th/9907210.
[31] R. D’Auria and P. Fre, Ann. Phys. 162, (1985) 372.
[32] I.R. Klebanov and E. Witten, AdS/CFT correspondence and symmetry breaking, preprint hep-th/9905104.