EMC effect, few-nucleon systems and Poincaré covariance

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Received 23 February 2020, revised 6 April 2020
Accepted for publication 15 April 2020
Published 18 May 2020

Abstract

An approach for a Poincaré covariant description of nuclear structure and of lepton scattering off nuclei is proposed within the relativistic Hamiltonian dynamics in the light-front form. Indeed a high level of accuracy is needed for a comparison with the increasingly precise present and future experimental data at high momentum transfer. Therefore, to distinguish genuine QCD effects or effects of medium modified nucleon structure functions from conventional nuclear structure effects, the commutation rules between the Poincaré generators should be satisfied. For the first time in this paper a proper hadronic tensor for inclusive deep inelastic scattering of electrons off nuclei is derived in the impulse approximation in terms of the single nucleon hadronic tensor. Our approach is based: i) on a light-front spectral function for nuclei, obtained taking advantage of the successful non-relativistic knowledge of nuclear interaction, and ii) on the free current operator that, if defined in the Breit reference frame with the momentum transfer, q, parallel to the z axis, fulfills Poincaré covariance and current conservation. Our results can be generalized: i) to exclusive processes or to semi-inclusive deep inelastic scattering processes; ii) to the case where the final state interaction is considered through a Glauber approximation; iii) to finite momentum transfer kinematics. As a first test, the hadronic tensor is applied to obtain the nuclear structure function F_A² and to evaluate the EMC effect for ³He in the Bjorken limit. Encouraging results including only the two-body part of the light-front spectral function are presented.

Keywords: deep inelastic lepton scattering off nuclei, Poincaré covariance, light-front Hamiltonian dynamics

(Some figures may appear in colour only in the online journal)

1. Introduction

The picture of nuclei as bound systems of nucleons and pions within a non-relativistic framework has reached a wide success in describing the properties of nuclei, although the fundamental theory for strong interacting systems is QCD with quarks and gluons as elementary degrees of freedom.

One of the most challenging issues is to understand the dynamical mechanism that is able to confine quarks and gluons in the effective degrees of freedom investigated by nuclear physics, i.e. meson and nucleon. This issue is nowadays one of the main motivations of experimental efforts of an increasing worldwide community, like the initiatives for the development of the Electron Ion Collider (EIC) (see, e.g., [1–4]). In view of this, also theoretical efforts aiming at improving the description of nuclei are highly desirable, since after all nuclei represent the best QCD laboratory that Nature might yield. It is very important to take into account as many general principles as we can, particularly the Poincaré covariance, since it appears to be a prerequisite for avoiding possible ambiguities in the assessment of different effects.

A phenomenon that could indicate an explicit manifestation of QCD degrees of freedom in nuclei is the EMC effect
[5], which points to modifications of the nucleons bound in the nuclear medium [6]. However, to clearly isolate genuine QCD effects in nuclei the general relativistic constraints, as the proper commutation rules between the Poincaré generators, should be satisfied when processes involving nucleons with high 3-momentum are considered and a high precision is needed. For instance this is the case for the JLab program at 12 GeV, both for the inclusive and the semi-inclusive deep inelastic processes (see, e.g., [7] and [8–10], respectively) or for the future experiments to be performed at the EIC [1–3].

Traditional nuclear physics has achieved a deep knowledge of the nuclear interaction and has developed sophisticated methods for evaluating the bound state wave functions of nuclei in a non-relativistic framework. Our work points to merge this knowledge in a Poincaré covariant framework through the light-front (LF) form of the relativistic Hamiltonian dynamics of an interacting system [11, 12], where the LF components \([v^+, \not \! p] = [v^+ \not \! v, p_y, p_z, 0]\) of a four vector \(v\) are used, with \(v^+ = v_0 \pm \not \! v\). Indeed, within the relativistic Hamiltonian dynamics plus the Bakamjian-Thomas construction of the Poincaré generators [13], covariance of the Poincaré generators imposes to the interaction the same commutation rules as occurs for the non-relativistic interaction and then the phenomenological interactions obtained from the analysis of the phase shifts can be used in a Poincaré covariant environment (see, e.g., [14]). Furthermore LF dynamics allows for a meaningful Fock state expansion of wavefunctions [15].

The tool we use is the LF spectral function [14], which allows one to take advantage of the whole successful non-relativistic phenomenology for the nuclear interaction and to take care of the macroscopic locality (for a definition of macroscopic locality see, e.g., [12]). As shown in [14], from the LF spectral function one obtains a momentum distribution which fulfills at the same time the baryon number sum rule and the momentum sum rule.

The process we intend to study is the EMC effect, which gives evidence that the structure of nucleons may be different when bound together in nuclei and has been and still is the subject of extensive experimental programs in many laboratories (see, e.g., [6, 7] and references quoted therein). To this end we first obtain the proper LF expressions for the hadronic tensor and for the \(F_2\) structure nuclear functions in the impulse approximation. After that, the obtained expression for \(F_2\) is applied to the EMC effect for \(^3\)He (for some preliminary results see [16]).

In this paper, after a short review of the EMC effect in section 2, the definition of the LF spectral function is recalled in section 3. Then in sections 4 and 5 it is shown for the first time for a generic \(A\)-nucleon nucleus how the hadronic tensor and the nuclear structure function \(F_2\) can be obtained within LF Hamiltonian dynamics through the LF spectral function in the Bjorken limit. In section 6 preliminary results for the EMC effect in \(^3\)He with the LF spectral function are presented and our conclusions are drawn in section 7.

2. A short review of the EMC effect

The EMC effect [5] was observed in deep inelastic lepton scattering (DIS) experiments and is described by the ratio of the DIS cross section on an \(A\)-nucleus and on the deuteron

\[
R(x) = \frac{\sigma_A}{\sigma_D} = 1 \quad \text{with} \quad x = \frac{Q^2}{2m \nu} \quad (1)
\]

in the range \(0 \leq x \leq 1\), where \(m\) is the nucleon mass, \(Q^2 = -q^2\), \(q\) the four-momentum transfer and \(\nu\) the energy transfer from the lepton to the nucleus in the laboratory frame. It suggests that a picture of lepton scattering off nuclei as an incoherent sum of lepton scattering off the constituent protons and neutrons is incomplete and provides an indication for explicit QCD effects in nuclei. Since the original discovery, a large program of measurements at CERN, Fermilab, SLAC, DESY, and JLab aimed at understanding the EMC effect.

Following [6], we report here the main experimental conclusions: (i) the shape of the effect is universal and observed in all nuclei; (ii) it is relatively \(Q^2\) independent and slowly increases with \(A\); (iii) the \(A\) dependence is consistent with a dependence upon the local nuclear density.

A linear relation was found [17] between the size of the EMC effect measured as the slope of \(R(x)\) for 0.35 \(\leq x \leq 0.7\) and the short range correlations (SRCs) taken as the ratio

\[
a_2 = \frac{\sigma_A}{\sigma_D} \quad \text{at} \quad x > 1. \quad (2)
\]

However, it is unclear whether the SRCs cause the EMC effect or if there is some common underlying source of the two phenomena.

There are two categories for the theoretical models that have been proposed: (i) the first one takes care only of traditional nuclear physics, including binding and nucleon momentum distributions effects through spectral functions corresponding to realistic nuclear interactions; (ii) the second one invokes more exotic explanations, such as contributions of six or nine quark bags, or medium modification of the internal structure of nucleons bounded in a nucleus.

Our final aim is to provide a LF Poincaré covariant calculation of the nuclear part of the EMC effect for \(^3\)He and \(^3\)H as a solid starting point which allows one to safely investigate genuinely QCD-based effects.

3. The LF spectral function

The spectral function is the probability distribution to find inside a bound system a particle with a given 3-momentum when the rest of the system has a given energy \(\epsilon\).

The definition of the spin dependent LF spectral function of a nucleon in a nucleus of total 4-momentum \(P_A\) in the laboratory frame, with LF momentum components \([P_{A_1}, P_{A_2}, P_{A_3}, P_{A_4}]=\) based on the overlaps

\[
\langle \Lambda | T; \alpha; \epsilon; J'; \tau\sigma, \vec{K}| j, j_\ell = m; \epsilon; \Pi; T_A ; T_{A_2} \rangle. \quad (3)
\]

In (3) the state \(| j, j_\ell = m; \epsilon; \Pi; T_A ; T_{A_2} \rangle\) is the intrinsic ground eigenstate of the \(A\)-nucleon system, with angular
momentum \((j, j_z)\), energy \(E^A\), parity \(\Pi\), isospin \((T_A T_A)\), and polarized along \(\hat{z}\), with LF total energy \(P_A^z\). The quantum numbers \((J, J_z; \epsilon; T)\) describe the angular momentum, intrinsic energy and isospin, respectively, of the \((A-1)\)-nucleon system, while \(\alpha\) indicates the other quantum numbers needed to completely identify this system. The intrinsic state \([\mathbf{K}, \sigma; J_L j_L j; \epsilon, T]\) is a closed system composed by a fully-interacting intrinsic state of \((A-1)\) nucleons and a plane wave, describing a nucleon that freely moves in the intrinsic reference frame of the whole cluster \([1, (A-1)]\) with intrinsic momentum

\[
\mathbf{k}^+ = \mathcal{M}_0[1, (A-1)] \xi \\
\mathbf{k}_\perp = \mathbf{p}_\perp - \xi \mathbf{P}_A^\perp.
\]

(4)

Let us stress that the intrinsic momentum \(\mathbf{k} \equiv (k^+, \mathbf{k}_\perp)\) of a nucleon in the reference frame of the cluster \([1, (A-1)]\) is different from the momentum \(\mathbf{p} \equiv (p^+, \mathbf{p}_\perp)\) in the laboratory frame. As noticed in [12, 14], the cluster \([1, (A-1)]\) fulfills the macrocausality [12, 14]. In (4) one has \(\xi = p^+/P_A^z\) and \(\mathcal{M}_0[1, (A-1)]\) is the intrinsic energy of the cluster

\[
\mathcal{M}_0[1, (A-1)] = \frac{(k^+)^2 + (m^2 + k^2_\perp)^2}{2 k^+} + \left[\frac{(k^+)^2 + (m^2 + k^2_\perp)^2}{2 k^+} - M^2 - m^2\right]^{1/2}.
\]

(5)

where \(M^2\) is the mass of the interacting spectator \((A-1)\)-nucleon system.

For an \(A\)-nucleon system of total 4-momentum \(P_A\) and polarized along the polarization vector \(S\), the spin dependent LF spectral function of a nucleon with 4-momentum \(p \equiv (p^+, \mathbf{p}_\perp)\) in the point laboratory frame can be defined as follows (see (69) of [14])

\[
P^\tau_{\sigma \sigma'}(\mathbf{K}, \epsilon, S, M) = \sum_m \sum_{m'} D^{\tau m'}_{m\sigma \sigma'}(\alpha, \beta, \gamma) D^{\tau m \sigma'}_{m' \sigma \sigma'}(\alpha, \beta, \gamma) P^\tau_{\sigma \sigma'}(\mathbf{K}, \epsilon, \hat{z}),
\]

(6)

where \(D^{\tau m \sigma \sigma'}_{m' \sigma \sigma'}(\alpha, \beta, \gamma)\) is the rotation matrix, with \(\alpha, \beta, \gamma\) the Euler angles describing the proper rotation from the \(z\) axis to the polarization vector \(S\), and

\[
P^\tau_{\sigma \sigma'}(\mathbf{K}, \epsilon, \hat{z}) = \rho(\epsilon) \sum_{j_{L}, j} \sum_{T_L} \mathcal{W}_L(\mathbf{K}; \epsilon, j_{L}, j, j_z = m; \mathbf{K}, \sigma; J_L \mathbf{J}_L ; \epsilon, \alpha; T_L).
\]

(7)

with \(\rho(\epsilon)\) the energy density of the \((A-1)\)-nucleon states.

In conclusion, the LF spectral function is obtained through the overlaps between the cluster \([1, (A-1)]\) and the bound state of the \(A\)-particle system. In [14] it is shown for a three-particle system how these overlaps can be evaluated from the system ground state wave function and the two-particle system wave functions for the bound and the scattering states in momentum space.

Let us consider the unpolarized spectral function, independent of \(S\),

\[
P^\tau_{\sigma \sigma'}(\mathbf{K}, \epsilon, \hat{z}) = \frac{1}{2} \sum_{J, j_{L}} \sum_{T_L} \mathcal{W}_L(\mathbf{K}; \epsilon, j_{L}, j, j_z = m; \mathbf{K}, \sigma; J_L \mathbf{J}_L ; \epsilon, \alpha; T_L) P^\tau_{\sigma \sigma'}(\mathbf{K}, \epsilon, \hat{z}).
\]

(8)

Because of time-reversal and parity symmetries one has

\[
\sum_m P^\tau_{\sigma \sigma'}(\mathbf{K}, \epsilon, \hat{z}) = - \sum_m P^\tau_{\sigma' \sigma}(\mathbf{K}, \epsilon, \hat{z}).
\]

(9)

It is clear that the average on the spin directions gives a vanishing result for the non-diagonal terms of the spin-dependent spectral function. If \(\sigma = \sigma'\), then

\[
\sum_m P^\tau_{\sigma \sigma'}(\mathbf{K}, \epsilon, \hat{z}) = \sum_m P^\tau_{\sigma \sigma'}(\mathbf{K}, \epsilon, \hat{z}).
\]

This means that the diagonal, unpolarized spectral function is independent of \(\sigma\). In conclusion the properly normalized, unpolarized spectral function is [14]

\[
P^\tau(\mathbf{K}, \epsilon, \hat{z}) = \frac{1}{2} \sum_{J, j_{L}} \sum_{T_L} \mathcal{W}_L(\mathbf{K}; \epsilon, j_{L}, j, j_z = m; \mathbf{K}, \sigma; J_L \mathbf{J}_L ; \epsilon, \alpha; T_L) P^\tau(\mathbf{K}, \epsilon, \hat{z}).
\]

(10)

4. Hadronic tensor in light-front Hamiltonian dynamics

Within light-front Hamiltonian dynamics (LFHD), the hadronic tensor for inclusive lepton-nucleus scattering off an \(A\)-nucleon nucleus of LF state \([\psi_0]; S, M, T; P_A]\), which is polarized along \(S\) with spin projection \(M\), has isospin third component \(T_L\) and total 4-momentum \(P_A\) in the laboratory frame, is

\[
W^\mu_{\sigma \sigma'}(P_A, S, M, T_L, q) = \frac{1}{4\pi} \sum_{\mathcal{X}} \mathcal{W}_{\mathcal{X}}(\psi_0; S, M, T_L; P_A)[U^\mu(0)|X, P_X]\langle X, P_X|F_{\sigma \sigma'}(0)|\psi_0; S, M, T_L; P_A}\rangle
\]

\[
\times \langle X, P_X|F_{\sigma' \sigma}(0)|\psi_0; S, M, T_L; P_A\rangle \langle X, P_X]\langle X, P_X|\delta^4(q + P_A - P_X),
\]

(11)

where \([X, P_X]\langle X, P_X]\) is any LF final state.

Let us assume that the virtual photon scatters inherently from each one of the nucleons in the nucleus, so that the final state is composed of an \((A-1)\)-nucleon spectator system and the debris produced by the virtual photon impinging on a single nucleon. Then, instead of

\[
\sum_{\mathcal{X}} \langle X, P_X|F_{\sigma \sigma'}(0)|\psi_0; S, M, T_L; P_A\rangle
\]

\[
= I,
\]

(12)
let us introduce in (11) the quantity
\[ \sum_{J_L, \varepsilon} \sum_\tau \sum_{\gamma, \gamma'} \epsilon \int \frac{d\tilde{P}_S}{(2\pi)^2P_S} \sum_f \sum_{\gamma, \gamma'} \int \frac{d\tilde{p}_f}{(2\pi)^2p_f} \]
\[ \times \langle \tilde{p}_f, \sigma_f \gamma, \gamma' \rangle_{LF} \langle \tilde{P}_S; J_L, \varepsilon, \alpha, T_i \rangle_{LF} \]
\[ \times LF \langle T, \alpha, \epsilon, J_f; \tilde{P}_S \rangle \langle \tilde{p}_f, \sigma_f \gamma \rangle_{LF}, \]
(13)

where \( \tilde{P}_S \) is the total LF momentum of a fully interacting \((A-1)\)-nucleon spectator system in the laboratory.

The state \( \{\tilde{P}_S; J_L, \varepsilon, \alpha, T_i \}_{LF} \) is an eigenstate of the operator \( \tilde{p}_f \) with eigenvalue \( P_S \):
\[ \tilde{p}_f \langle \tilde{P}_S; J_L, \varepsilon, \alpha, T_i \rangle_{LF} = \frac{1}{P_S^2} \left[ \begin{array}{c} M_f^2 + (\mathbf{p}_f)^2 \end{array} \right] \langle \tilde{P}_S; J_L, \varepsilon, \alpha, T_i \rangle_{LF}. \]
(14)

For \((A-1) = 2\), the mass of the two-body interacting spectator system is \( M_S = [4(m^2 + m_e)^2]^{1/2} \). [14]

In (13) \( \tilde{p}_f, \sigma_f \gamma, \gamma' \) is the LF final state of the debris produced by the scattering of the virtual photon off a nucleon, and has isospin third component \( \gamma \), spin along the \( z \)-axis equal to \( \sigma_f \), LF momentum \( \tilde{p}_f \) in the laboratory frame and LF energy \( p_f = (m_f^2 + p_f^2 - p_f^2)\). The quantity \( p_f \) is an independent variable with respect to the components of the LF momentum \( \tilde{p}_f \), since the unknown mass \( m_f \) of the debris can assume any positive value compatible with conservation laws.

Then, assuming no interaction in the final state between the \((A-1)\)-nucleon system and the state \( \tilde{p}_f, \sigma_f \gamma \), one has \( P_X = P_f + p_f \) and the hadronic tensor becomes
\[ W_A^{\mu\nu} = \frac{1}{4\pi} \sum_{J_L, \varepsilon} \sum_{\gamma, \gamma'} \epsilon \int \frac{d\tilde{P}_S}{(2\pi)^2P_S} \sum_f \sum_{\gamma, \gamma'} \int \frac{d\tilde{p}_f}{(2\pi)^2p_f} \]
\[ \times \delta^4(q + P_A - p_f - p_f) \frac{1}{M_0[1, (A-1)]}. \]
(18)

where the completeness for the one-nucleon momentum eigenstates \( \tilde{p}_f, \sigma_f \gamma \) has been inserted, with \( \tilde{p}_f \) the nucleon LF momentum in the laboratory frame.

By considering that the LF-momentum is conserved (the interaction is contained only in the minus component of the momenta) and the kinematical nature of the LF-booster, one has (see appendix)
\[ \langle \tilde{P}_S; J_L, \varepsilon, \alpha, T_i \rangle_{LF} = \langle \tilde{P}_S; J_L, \varepsilon, \alpha, T_i \rangle_{LF} \]
\[ \times LF \langle T, \alpha, \epsilon, J_f; \tilde{P}_S \rangle \langle \tilde{p}_f, \sigma_f \gamma \rangle_{LF} \]
(18)

In impulse approximation the current of the \( A \)-nucleon system is
\[ J_L^\mu(0) = \sum_{J_L, \varepsilon} J_L^\mu(0). \]
(16)

As shown in [18], this current satisfies Poincaré covariance and current conservation in the Breit frame with the momentum transfer, \( \mathbf{q} \), along the \( z \) axis and in any frame that can be obtained through a LF boost parallel to the \( z \) axis. To be more definite let us take \( \mathbf{q} \) opposite to the \( z \) axis: \( \mathbf{q} \equiv (0, 0, q_z = -|\mathbf{q}|) \).

Let us assume that the interference terms between the contributions of different constituent nucleons yield a negligible contribution to the hadronic tensor. Then the hadronic tensor can be approximated as follows
\[ W_A^{\mu\nu} = \frac{1}{4\pi} \sum_{J_L, \varepsilon} \sum_{\gamma, \gamma'} \epsilon \int \frac{d\tilde{P}_S}{(2\pi)^2P_S} \sum_f \sum_{\gamma, \gamma'} \int \frac{d\tilde{p}_f}{(2\pi)^2p_f} \]
\[ \times \delta^4(q + P_A - p_f - p_f) \frac{1}{M_0[1, (A-1)]}. \]
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\[ \langle \tilde{P}_S; J_L, \varepsilon, \alpha, T_i \rangle_{LF} = \langle \tilde{P}_S; J_L, \varepsilon, \alpha, T_i \rangle_{LF} \]
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\[ \times \delta^4(q + P_A - p_f - p_f) \frac{1}{M_0[1, (A-1)]}. \]
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\[ \langle \tilde{P}_S; J_L, \varepsilon, \alpha, T_i \rangle_{LF} = \langle \tilde{P}_S; J_L, \varepsilon, \alpha, T_i \rangle_{LF} \]
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where \(|\psi_0; S, M, T_{A,S})|_{LF}\) is the intrinsic state of the nucleus with LF energy \(P_A\). In (20) the equalities \(\bar{p} = \bar{p}' = P_A = P_S\) and \(\tau = \tau' = T_{A,S} = \bar{t}\) have been used and the sum over \(\tau\) is implicit in the sum over \(N\).

Let us insert in (20) the definition of the LF spin-dependent spectral function (see (69) and (66) of [14] and section 3) and the definition of the hadronic tensor for a single constituent

\[
W_{\mu\nu}^{\mu\nu}(p, q) = \frac{1}{2\pi^3} \int_{4\pi} d\xi \frac{d^3p_f}{(2\pi)^3} \left[ \frac{1}{2} \mathbf{p}_f \cdot (\mathbf{p}_f - p_f) \right] \frac{M_{A,S}}{M_{A}(1 - \xi)} \left( \begin{array}{c} E_s \\ p_f^\perp \end{array} \right)
\]

(21)

where \(p^+ = P_A - P_S\) (i.e., the nucleon with four momentum \(p\) is an off-shell nucleon) and \(\bar{p} = \bar{p}' = P_A = P_S\). Then the hadronic tensor can be written as follows

\[
\frac{1}{2} \sum_{\sigma, \sigma'} \int d\xi \int_{4\pi} d\xi' \int_{4\pi} d\xi'' \left[ \frac{1}{2} \mathbf{p}_f \cdot (\mathbf{p}_f - p_f) \right] \frac{M_{A,S}}{M_{A}(1 - \xi)} \left( \begin{array}{c} E_s \\ p_f^\perp \end{array} \right)
\]

(22)

where the isospin \(\tau\) is defined by \(N\).

The hadronic tensor can be expressed as an integral on the intrinsic momentum \(k\). Since [14]

\[
P_{A, S} = P_{A, S} - P_{A, S} \rho = P_{A, S} - \kappa_\perp - \xi P_{A, S}
\]

(23)

\[
P_S^S = P_S^A - p^+ = P_S^A - \xi P_A
\]

(24)

one has

\[
\frac{\partial (P_{A, S}^A)}{\partial (\kappa_\perp, \xi)} = P_A^A.
\]

Then using the equality [14]

\[
\frac{\partial E_s}{\partial \xi} = \frac{E_s}{(1 - \xi)}
\]

(25)

the hadronic tensor becomes

\[
W_A^{\mu\nu} = \sum_{\kappa, \sigma} \int d\xi \int \frac{d^3k}{(2\pi)^3} \frac{d\kappa^+}{2\pi^3} \frac{1}{\kappa^+} \left| P_A^A \right| \left| \psi_0; \kappa, \xi\right|_{LF}^{\mu\nu}(p, q).
\]

(27)

In the rest reference frame for the nucleus, one has \(P_{A, S} = 0\) and \(p^+ = P_A = M_A\). Therefore one obtains

\[
\bar{p} = \kappa_\perp, \quad p^+ = \xi M_A, \quad p^+ = M_A - [M^2_A + (\kappa_\perp)^2] M_A (1 - \xi).
\]

(28)

Let us notice that the hadronic tensor for a single constituent depends on the energy \(\epsilon\) both through the plus component of the momentum \(p\) and through the minus component. Indeed, since \(M_{S}\) depends on \(\epsilon\), the same occurs for \(M_{A,S}(1 - \xi)\) (see (5)) and in turn for \(\xi\) (see (4)).

If the hadronic tensor for the nucleus is averaged on the spin projections of the nucleus along \(S\),

\[
W_{A,S}^{\mu\nu}(p_A, T_{A,S}, q) = \frac{1}{2} \sum_{\mu} \sum_{\nu} W_{A,S}^{\mu\nu}(p_A, S, M, T_{A,S}, q).
\]

(29)

then one obtains the hadronic tensor for an unpolarized nucleus, which is expressed through the unpolarized spectral function, independent of \(S\).

Eventually the hadronic tensor for an unpolarized nucleus reads

\[
W_A^{\mu\nu}(p_A, T_{A,S}, q) = \frac{1}{2} \sum_{\mu} \sum_{\nu} W_{A,S}^{\mu\nu}(p_A, S, M, T_{A,S}, q) \times \mathcal{P}^N(\kappa, \epsilon) \left. W_{A,s}^{\mu\nu}(p, q) \right|_{\epsilon}
\]

(30)

5. Nuclear structure function \(F_2\)

In the Bjorken limit the nuclear structure function \(F_2\) can be obtained from the hadronic tensor for an unpolarized nucleus as follows

\[
F_2^A(x) = -\frac{1}{2} \sum_{\mu, \nu} g_{\mu\nu} W_{A}^{\mu\nu}(p_A, T_{A,S}) = \frac{1}{2} \sum_{\kappa, \sigma} \sum_{\mu, \nu} \int d\xi \frac{d\kappa^+}{2\pi^3} \frac{d\kappa^+}{\kappa^+} \frac{1}{\xi}
\]

\[
\times \mathcal{P}^N(\kappa, \epsilon) \left. (-x_A) \frac{1}{2} g_{\mu\nu} W_{A,S}^{\mu\nu}(p, q) \right|_{\epsilon}, \quad (31)
\]

(31)

where

\[
x_A = \frac{Q^2}{2P_A \cdot q}.
\]

(32)

In (31) the integration limits on \(\kappa^+\) are made explicit. The minimum value for \(\kappa^+\) (see (4) and (A.3)) is the value corresponding to \(\xi_{\text{min}},\) i.e.,

\[
\kappa^+_{\text{min}} = \frac{m^2 + (\vec{k}_\perp)^2}{\xi_{\text{min}}} + \frac{1}{(1 - \xi_{\text{min}})^{3/2}}
\]

(33)
with
\[ \xi_{\text{min}} = x \frac{m}{M_A}. \]

Then, defining the nucleon structure function \( F_2^N(z) \)
\[ F_2^N(z) = -\frac{1}{2} z g_{\mu\nu} \frac{1}{2} \sum_{\sigma} w_{\mu\nu,\sigma}(p, q) \]
where
\[ z = \frac{Q^2}{2p \cdot q}, \]
on one obtains
\[ F_2^A(x) = \sum_N \sum_{\ell} \tilde{\ell} \int \frac{d\kappa_n}{(2\pi)^3} \int_{\kappa_n^-}^{\infty} \frac{d\kappa^+}{2 \kappa^+} \times P_N(\kappa, \epsilon) \frac{P_A^+}{p^+} \frac{Q^2}{2p \cdot q} \frac{2p \cdot q}{Q^2} F_2^N(z) \]
\[ = \sum_N \sum_{\ell} \tilde{\ell} \int \frac{d\kappa_n}{(2\pi)^3} \int_{\kappa_n^-}^{\infty} \frac{d\kappa^+}{2 \kappa^+} \times P_N(\kappa, \epsilon) F_2^N(z), \]
since in our reference frame \( P_{A\perp} = 0 \), \( q_{\perp} = 0 \) and, in the Bjorken limit, \( q^+ \) is vanishing small with respect to \( q \).

Let us stress that one cannot integrate over \( \epsilon \) to obtain the momentum distribution because both \( p^+ \) and \( p^- \) depend on \( \epsilon \).

6. Preliminary results for the EMC effect in \( ^3\text{He} \)

We aim to compare the results of our novel approach for the spectral function and for the structure function \( F_2^A(x) \) in the DIS limit with the available experimental data for the EMC effect in \( ^3\text{He} \). To this end we use the Pisa group \( ^3\text{He} \) wave function [19] corresponding to the AV18 NN interaction [20] to obtain the LF spectral function from (7).

Then we have to evaluate \( F_2^A(x) \) from (37), the ratio
\[ R^A(x) = \frac{F_2^A(x)}{Z F_2^N(x) + (A - Z) F_2^D(x)}. \]
for \( A = 3 \) (i.e. for \( ^3\text{He} \)) and for \( A = 2 \) (i.e. for the deuteron) and eventually the EMC ratio
\[ r_{\text{EMC}}(x) = \frac{R^{\text{He}}_2(x)}{R^A_2(x)}. \]

In figure 1 the experimental data of the JLab E03-103 experiment [21] (red dots) and the outcomes of [22] (black squares), where a carefully reanalysis of the same experimental data was carried out, are compared with theoretical calculations. The authors of [22] in their analysis indicate that a normalization factor is required in the E03-103 data for \( ^3\text{He} \) to be consistent with the \( F_2^A/F_2^D \) ratio from the NMC experiment [23]. With this normalization the E03-103 data for \( ^3\text{He} \) are in good agreement with the HERMES data [24] in the overlap region at \( x \sim 0.35 \).

The solid line is our renormalized two-body contribution to the EMC ratio. In figure 1 the results for \( F_2^A(x) \) obtained through a convolution formula with the approach of [25] for the spectral function are also shown. The dotted line is the full result, while the dashed line is the contribution from the two-body channel, properly renormalized. The essential difference of our LF spectral function with respect to the approach of [25] is the use of the intrinsic momentum \( \vec{\kappa} \equiv (\kappa^0, \vec{\kappa}) \) of a free nucleon in the reference frame of the cluster \([1, (A - 1)]\), where the system of \((A - 1)\) nucleons is fully-interacting. Noteworthy, the intrinsic momentum \( \vec{\kappa} \equiv (\kappa^0, \vec{\kappa}) \) is different from the momentum \( \vec{p} \equiv (p^+, \vec{p}) \) in the laboratory frame, used for the spectral function in [25], and from the intrinsic momentum in a system of \( A \) free nucleons [12].

It is clear that the LF spectral function pulls down the two-body contribution to the EMC effect with respect to the approach of [25], and suggests that a complete calculation including the two- and three-body contributions can be near to the \( ^3\text{He} \) data, especially in view of the analysis of Kulagin and Pett[22]. Therefore our next step will be the full calculation of the EMC effect for \( ^3\text{He} \), including the exact three-body contribution. For an approximate evaluation of the three-body contribution see [16].
7. Conclusion and perspectives

A Poincaré covariant description of nuclei, based on the light-front relativistic Hamiltonian dynamics, has been proposed. The Bakamjian-Thomas construction of the Poincaré generators allows one to embed the successful phenomenology of interacting nucleons for few-nucleon systems in a Poincaré covariant framework.

Our main tool is the LF spectral function introduced in [14], which allows one to satisfy the baryon number sum rule and the momentum sum rule at the same time. The definition of LF spectral function is based on the overlaps between the ground state of the $A$-nucleon system and the cluster [1, $(A-1)]$, which is the tensor product of a momentum eigenstate of a free nucleon of momentum $k$ in the intrinsic reference frame of the cluster $[1, (A-1)]$ times the state of a fully interacting $(A-1)$-nucleon spectator subsystem. The definition of the nucleon momentum and the use of the tensor product allows one to take care of macrocausality and to introduce a novel effect of binding in the spectral function.

The LF spectral function can be obtained from non-relativistic wave functions with a realistic nuclear interaction including two- and three-body forces. As shown in section 5, through the LF spectral function and a current which satisfies Poincaré covariance and current conservation one can write an hadronic tensor for inclusive lepton-nucleus scattering to evaluate DIS processes in the impulse approximation. Generalizations are straightforward: (i) to describe exclusive processes [26] or semi-inclusive deep inelastic scattering (SIDIS) processes [27, 28]; (ii) to include the final state interaction between the $(A-1)$-spectator system and the final debris with a Glauber approximation through a distorted LF spectral function as in [28]; (iii) to finite momentum transfer kinematics [29].

As a first application of a Poincaré covariant description of nuclei, we aim to provide a calculation of the nuclear part of the EMC effect for $^3$He and $^3$H to unambiguously investigate genuinely QCD-based phenomena. Indeed a Poincaré covariant evaluation of this effect could indicate the size of the difference with respect to the experimental data to be filled by manifestations of non-nucleonic degrees of freedom or by modifications of nucleon structure in nuclei.

Up to now only the two-body contribution to the LF nucleon spectral function has been used. Encouraging improvements clearly appear with respect to a convolution approach with a momentum distribution. Our next step will be a full calculation of the three-body contribution.

The laboratory system

\[
\begin{align*}
LF_\Lambda(\gamma S T S, \alpha, e' J J; \tilde{P}_5; \tilde{P}_5; JJ, \epsilon, \alpha, T S T S)_\Lambda &= 2p^+(2\pi)^3\delta(k' - k) \times \frac{E_s}{M_0[1, (A-1)]} \\
&\times LF_\Lambda(\gamma S T S, \alpha, e' J J; \tilde{P}_5; \tilde{P}_5; JJ, \epsilon, \alpha, T S T S)_\Lambda \\
&\quad \times LF_\Lambda(\gamma S T S, \alpha, e' J J; \tilde{P}_5; \tilde{P}_5; JJ, \epsilon, \alpha, T S T S)_\Lambda \\
&= 2(2\pi)^3\delta(k' - k) \times \frac{E_s}{M_0[1, (A-1)]} \\
&\times LF_\Lambda(\gamma S T S, \alpha, e' J J; \tilde{P}_5; \tilde{P}_5; JJ, \epsilon, \alpha, T S T S)_\Lambda.
\end{align*}
\]

As a consequence one has

\[
\begin{align*}
\langle \tilde{P}_5; JJ, \epsilon, \alpha, T S T S \rangle_{\Lambda} &= |\tilde{P}_5; JJ, \epsilon, \alpha, T S T S \rangle_{\Lambda} \\
&= \frac{\sqrt{E_s}}{M_0[1, (A-1)]}.
\end{align*}
\]

The factor $\sqrt{E_s}/M_0[1, (A-1)]$ takes care of the proper normalization of the momentum eigenstates $|\tilde{P}_5\rangle_{\Lambda}, |\tilde{P}_5\rangle_{\Lambda}$ and $|\tilde{P}_5\rangle_{\Lambda}$.

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Appendix. Orthogonality properties of the momentum eigenstates of the $[1, (A-1)]$ cluster states

Let us consider the orthogonality properties of the eigenstates of the $[1, (A-1)]$ cluster states $|\tilde{P}_5; JJ, \epsilon, \alpha, T S T S\rangle_{\Lambda}$ in the laboratory system.

\[
LF_\Lambda(\gamma S T S, \alpha, e' J J; \tilde{P}_5; \tilde{P}_5; JJ, \epsilon, \alpha, T S T S)_\Lambda = 2p^+(2\pi)^3\delta(k' - k) \times \frac{E_s}{M_0[1, (A-1)]}.
\]
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