$^3\text{He}$ Transport and the Question of Nonstandard Solar Models

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I consider phenomenological changes in the standard solar model with the goal of testing recent claims that the solar neutrino puzzle requires new particle physics. The assumption of a steady-state sun producing the correct luminosity and governed by standard microphysics appears to leave only one nonstandard solar model possibility open, slow mixing of the solar core on timescales characteristic of $^3\text{He}$ equilibration. The conjecture of such mixing raises striking physics issues connected with the standard model $^3\text{He}$ instability and the possibility that the $^3\text{He}$ abundance gradient might allow the sun’s early convective core to persist. While helioseismology might eventually rule out such a “model”, contrary to one recent claim I will argue that the helioseismology of a mixed two-fluid sun is as yet far from clear. Finally, I conclude by stressing that $^3\text{He}$-driven slow mixing is not being proposed as a solution of the solar neutrino problem, but as an example of a possibility that has not been quantitatively modeled and yet could produce neutrino fluxes far closer to experiment than the standard solar model. Thus, despite the attractiveness of neutrino oscillation solutions, astrophysical explanations of the solar neutrino problem are not yet, in my view, ruled out definitively.

In this talk I would like to summarize some recent work, done in collaboration with Andrew Cumming, on the possibility of an astrophysical solution to the solar neutrino problem [1]. It is widely appreciated that the results of the $^{37}\text{Cl}$, SAGE/GALLEX, and Kamioka II/III experiments are consistent with an unexpected pattern of neutrino fluxes,

$$
\phi(\text{pp}) \sim \phi^{SSM}(\text{pp})
$$

$$
\phi(\text{\ensuremath{^7}\text{Be}}) \sim 0
$$

$$
\phi(\text{\ensuremath{^8}\text{B}}) \sim 0.4\phi^{SSM}(\text{\ensuremath{^8}\text{B}})
$$

(1)

where $\phi^{SSM}$ denotes the standard solar model [2] (SSM) value. As $\phi(\text{\ensuremath{^8}\text{B}}) \sim T_c^{22}$ [3], where $T_c$ is the solar core temperature, the required reduction in this flux can be achieved by lowering $T_c$ to about 0.96 of the SSM value. However, as $\phi(\text{\ensuremath{^7}\text{Be}})/\phi(\text{\ensuremath{^8}\text{B}}) \sim T_c^{-10}$, this flux ratio then increases, contradicting Eq. (1). The difficulty of simultaneously reducing $\phi(\text{\ensuremath{^8}\text{B}})$ and $\phi(\text{\ensuremath{^7}\text{Be}})/\phi(\text{\ensuremath{^8}\text{B}})$ has been established for broad classes of solar models [3-5], leading many to favor nonastrophysical solutions to the solar neutrino problem.

It is clear that no solar model will give a perfect fit to the results of existing experiments: the measurements are inconsistent with any combination of undistorted $\text{\ensuremath{^8}\text{B}}$, $\text{\ensuremath{^7}\text{Be}}$, and pp neutrino fluxes at a confidence level of about
Figure 1: A comparison of SSM (ellipse in upper right hand corner) and various nonstandard model neutrino flux predictions to experiment. Notice that the theoretical results cluster along a trajectory corresponding to the naive $T_c$ dependence discussed in the text. (From Ref. [5])

2σ [6]. Yet a compelling argument for a resolution in terms of new particle physics must rest on the more dramatic $\sim 5\sigma$ discrepancy, illustrated in Fig. 1, that exists between experiment and the flux predictions of standard and nonstandard models. Thus it is important to determine whether a nonstandard model might exist in which the naive $T_c$ dependence described above is circumvented. The philosophy behind the calculations presented in Ref. [1] stemmed from a worry that, if a viable nonstandard model exists which is approximately compatible with the results in Eq. (1), its underlying physics might be subtle and thus difficult to anticipate. This seemed to argue for a simple minded approach — changing the SSM phenomenologically — putting aside for the moment the more difficult issue of the underlying physical mechanism, in the hope that Eq. (1) might then lead us to the proper solution. The procedures we followed are discussed in Ref. [1] and will not be repeated here. But the basic approach was to search for solutions more consistent with Eq. (1) constrained by three conditions. First, we retained all of standard nuclear and atomic microphysics, e.g., nuclear cross sections and opacities, be-
cause we felt current SSM “best values” for these parameters were sensibly chosen, given the existing body of measurements. Second, we required, to the extent possible in our phenomenological approach, that the modified “models” reproduce the known solar luminosity. Finally, we required the “models” to be steady state, thus demanding where appropriate an equilibrium in the production and consumption of pp-chain “catalysts” like D, $^3\text{He}$, and $^7\text{Be}$. Note that the SSM requires such equilibrium locally, while we allowed more freedom in the “model” by only enforcing this condition in the integral core abundances.

In the SSM the core $^3\text{He}$ abundance profile is very distinctive, rising steeply as a function of the distance $r$ from the center of the sun: the equilibrium abundance $X_3$ varies as $T^{-6}$, where $T$ is the local temperature. But the characteristic result of our phenomenological explorations was a somewhat cooler sun with a remarkably different $^3\text{He}$ profile, one elevated by an order-of-magnitude, relative to the equilibrium value, at small $r$ and depleted at large $r$. Such an altered profile is compared to the SSM result in Fig. 2. It is readily seen why such a change moves the neutrino flux predictions towards the results of Eq.
First, a large fraction of the produced $^3$He is burned out of equilibrium at small $r$. The ppI terminations are governed by the reaction $^3$He+$^3$He, which is quadratic in the $^3$He abundance, while the competing reaction $^3$He+$^4$He leading to higher energy neutrinos is linear. Thus the rate of ppII+ppIII terminations relative to ppI terminations is reduced in direct proportion to the $^3$He excess, suppressing both the $^7$Be and $^8$B neutrino fluxes. However, when the reaction $^3$He+$^4$He does occur, short-lived $^7$Be is produced at small $r$, where the ambient temperature is high. This favors ppIII terminations over ppII terminations, leading to a suppressed $\phi(\overline{7}\text{Be})/\phi(\overline{8}\text{B})$ flux ratio. The combined effects of the reduced (ppII+ppIII)/ppI and enhanced ppIII/ppII branching ratios yield a somewhat reduced $^8$B neutrino flux and a significantly reduced $^7$Be flux.

Now how should Fig. 2 be interpreted? Clearly the nonstandard $^3$He profile represents something quite different from the static SSM profile with which it is compared in the figure: such an out-of-equilibrium profile can only result from core mixing of $^3$He on a timescale characteristic of $^3$He equilibration ($\sim 10^7$ years in the outer core). One soon concludes that the nonstandard profile depicts where $^3$He is burned in a postulated “model”, which may be quite different from the abundance distribution itself. Such localized burning of $^3$He at small $r$ can arise from a rather specific pattern of core mixing. First, there must be a relatively rapid downward flow of $^3$He-rich material from large $r$; the speed must be sufficient to take a mass element well past the usual equilibrium point, into a region where the rapidly decreasing local lifetime of $^3$He finally results in sudden $^3$He ignition. This mass element, now depleted in $^3$He and buoyant because of the energy release, must return to large $r$ sufficiently slowly to allow the p+p reaction to replenish the $^3$He. This flow is depicted in Fig. 3. As we are assuming a steady-state process in which any mass element is roughly equivalent to any other, each mass element must, on average, remain within a radial shell bounded by $r$ and $r+dr$ for a time proportional to the mass $dM(r)$ contained within that shell. This condition would be satisfied if the slow upward flow is broad with a local velocity inversely proportional to $dM(r)$ - the kind of flow that would result from displacement from below. Such upward flow will produce a positive $^3$He gradient, as in the SSM; but the upward flow must be sufficiently fast to keep the $^3$He below its local equilibrium value to prevent burning at large $r$. To keep the circulation steady, the rapid downward flow clearly must be localized, e.g., perhaps in narrow plumes. This flow was simulated numerically in Ref. [1]. The derived $^3$He burning pattern and neutrino fluxes ($\phi(8\text{B}) \sim 0.4 \phi^{SSM}(8\text{B})$, $\phi(7\text{Be})/\phi(8\text{B}) \lesssim \phi^{SSM}(7\text{Be})/\phi^{SSM}(8\text{B})$) emerged for a variety of downward ($\tau_L \sim \text{few} \cdot 10^6$ y) and upward ($\tau_U \sim \text{few} \cdot 10^7$ y) flow time scales, provided the mixing encompasses most of the energy-
producing core. It is notable that a simple flow pattern like that of Fig. 3 can produce the $^3$He burning pattern of Fig. 2: the latter was deduced phenomenologically, so it is not obvious that it will necessarily arise from any mixing pattern. Furthermore, there exist some speculative but rather intriguing corrections to well known idiosyncracies of the SSM:

1) In their work on the “solar spoon”, Dilke and Gough [7] showed that the SSM $^3$He gradient implied an instability to large amplitude perturbations: the energy released by enhanced $^3$He burning can exceed the work against gravity required to push a volume element at large $r$ through the denser material below. They speculated that this overstability could trigger periodic mixing of the core. Our phenomenology suggests an interesting variation on the solar spoon mixing mechanism. In the case of the continuous flow depicted in Fig. 3, the core would remain homogeneous in H and $^4$He while still permitting a $^3$He gradient, an amusing change in the solar spoon as the flow would be nearly adiabatic. Large-scale adiabatic flow that would allow the sun to produce the required luminosity more efficiently (i.e., in a cooler sun that derives more energy per $4p \rightarrow ^4\text{He}$ conversion) has a certain attractiveness.

2) If a flow similar to Fig. 3 were established, it is conceivable that it might persist, as it is both driven by and maintains the $^3$He gradient: if the mixing
is slowed through some perturbation, the $^3$He gradient powering the mixing will steepen. The subsequent more violent ignition of $^3$He in the descending plumes would then act to return the cycle to equilibrium.

However this begs the question of how the cycle itself gets started. As the solar spoon overstability is a large-amplitude one, the consequences for the static SSM are not obvious. While a trigger for large-scale flow was discussed in the solar spoon [7], this has been a point of contention [8].

But a speculation made by Roxburgh [9] in a different context opens up another possibility: could the difference between the SSM and a model like that sketched in Fig. 3 have to do with initial conditions? The core of the early sun is believed to be convectively unstable prior to the establishment of equilibrium in the pp and CNO cycles: $\eta = d\log \epsilon / d\log T$, where $\epsilon$ is the energy generation rate, is initially in excess of the critical value of about 5.0 due to the out-of-equilibrium burning of $^{12}$C to $^{14}$N. This condition persists for a time in excess of the few $\cdot 10^7$ y characterizing the flow of Fig. 3. The convective epoch is conventionally ignored in SSM simulations: if this phase is transient, one can avoid a great deal of work by starting from a static, one-dimensional primordial sun. Roxburgh raised the issue, with perhaps a hint of embarrassment: Could $^3$He transport by convective overshooting cause the early core mixing to persist, perhaps up to modern times?

This suggestion underscores a potential flaw in SSM logic. The SSM arises from a one-dimensional solution of the equations describing stellar evolution, beginning with the assumption of a static primordial sun. Although the resulting modern SSM sun is affected by the solar spoon overstability, modelers have taken comfort in the fact that the $^3$He profile is only unstable to large-amplitude perturbations. Thus it is possible – some might argue likely [8] – that the $^3$He overstability has no consequences for today’s SSM sun. Roxburgh’s concern is that a more detailed modeling of the early convective core might allow one to evolve onto a trajectory where the $^3$He gradient naturally becomes involved in driving persistent convection. Thus there could be a bifurcation, leading respectively to the SSM and to a modern convective sun that exploits the $^3$He gradient, associated entirely with how realistically initial conditions are treated. The physics concerns raised by Refs. [7] and [9] are substantial ones, and I find it disturbing that our entirely phenomenological exercise in solar model neutrino physics leads us back to those papers.

In closing, let me stress that Andrew and I presented our work as a highly speculative but quite amusing possibility. It is not being proposed as a solution to the solar neutrino problem, but as an existence proof: there do exist nonstandard model possibilities that have not been explored quantitatively yet could substantially reduce the problem summarized in Eq. (1). For this reason
I think it will remain unwise to rule out astrophysical solutions, at least until we have the results from Superkamiokande and SNO. It is also noteworthy that only a single class of steady-state nonstandard models, those with core mixing on timescales characteristic of $^3$He mixing, appears helpful in reducing the solar neutrino discrepancies.

For those who might be motivated to pursue an astrophysical solution, the present work presents two challenges:

1) Can the “model” be implemented dynamically? That is, in a realistic 2D or 3D model with an early convective core, would the establishment of the $^3$He gradient allow the star to remain convective and evolve into flow patterns similar to Fig. 3? This is quite a challenge requiring the modeling of two-fluid flows where the fluids are both chemically and thermally distinct.

2) Is such a “model” compatible with other astrophysical constraints? As mentioned in Ref. [1], core convection would likely alter galactic $^3$He synthesis, evolution along the color-magnitude diagram, and helioseismology. The first might be a welcome change in galactic chemical evolution, while the latter two are substantial tests that a viable nonstandard model must be able to pass. In fact, in a recent paper, John Bahcall et al. [10] have argued that such core mixing models are definitively ruled out by helioseismology. I agree with John that the SSM helioseismology successes are very significant. But I regard the conclusions about mixed models to be premature. Here and in Ref. [1] it is clear that we have not provided a model, but merely sketched a somewhat provocative idea. Thus the use of quotation marks on “model”. The work of Ref. [10] rules out a “model” analogous to the static cartoon of Fig. 2. Such a model is also ruled out by $\vec{F} = m(r)\vec{a}$, as no attempt has been made to enforce the standard rules of stellar hydrodynamics. Likewise, were the SSM described in a similarly crude fashion, it would almost certainly fail an analogous helioseismology test. My point is that if we succeed in item 1) above - if a detailed model can be constructed - we will then have the density and temperature profiles necessary for determining sound speeds $c(r)$ and helioseismology. Helioseismology will then be both an appropriate test of the model, and a major challenge to its viability. I would purpose such a statement as a more conservative summary of the work reported in Ref. [10].

Acknowledgments

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