Dynamics of the thermally radiative and chemically reactive flow of Sisko fluid in a tapered channel

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Abstract
This paper addresses the analysis of chemical reaction for peristaltic movement of electrically conducting Sisko fluid in an asymmetric tapered channel with velocity slip condition. An incompressible Sisko fluid saturates the porous medium. Modified Darcy's law has been employed for the porous medium effect. The impacts of thermal radiation and viscous dissipation are also taken into account. The resultant non-linear expressions are solved based on the approximation of lubrication theory. Such consideration is significant to predict human physiological characteristics especially in blood flow problems. The analytical outcomes for velocity, streamlines, pressure gradient, and temperature equations are found by utilizing the regular perturbation technique. The graphical illustrations are provided to explain the impressions of numerous emerging parameters on flow fields. The significant results of the current studies are that fluid velocity enhances for Darcy number and Sisko fluid parameter.

Keywords
Peristaltic motion, Sisko fluid, thermal radiation, chemical reaction, tapered asymmetric channel

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Introduction
The peristaltic movement is one of the most important phenomena as a result of its broad applications in different fields like biomedical, chemical industries, physiology, etc. This phenomenon is generally noticed in the human body such as urine bladder, bile carrying tube, cilia movement, and movement of blood through arteries, capillaries, and veins. Various biomedical appliances act on the peristaltic mechanism, for example, heart-lung gadgets, finger and roller pumps, blood pumps, and dialysis. Due to this significant quantity of applications, several investigators evaluated the peristaltic flow with distinct types of physical fluids in various geometries. Numerous inquiries have been done on the peristaltic activity of liquids with different physical implementations and numerous flow geometries. At first, the sinusoidal wall activity of viscous liquid was reported by Latham. Then Shapiro et al., in their article reported a succinct investigation of viscous fluid in the peristaltic movement through a channel and tube under lubrication approximation theory. Abdelsalam and Vafai discussed the blood flow with sinusoidal wall activity by applying lubrication theory through a relatively small flexible artery and found the impact of some parameters in the peristaltic region. After that,
Kavitha et al. explored the conveyance of Jeffrey liquid with Newtonian fluid following the peristaltic movement by assuming inertial forces as negligible and small wave number. Hussain et al. scrutinized the thermal radiative aspects of the peristaltic activity of electrically conducting viscous liquid passing through a porous medium. Bhatti et al. examined the theoretical modeling of magnetized Prandtl fluid through an annulus consisting of a porous medium. Rabinowitsch model was adopted with varying liquid properties in an article by Vaidya et al. to investigate the sinusoidal wall analysis under lubrication theory. Rashid et al. discussed the transportation of Williamson liquid through curved annulus having contracting and expanding boundaries. Rajashekar et al. presented the Rabinowitsch model between two flexible walls obeying peristalsis with the impact of varying liquid properties. Akram et al. scrutinized the heat source/sink aspects of the peristaltic transportation of Prandtl-Eyring nanomaterial through complaint channel walls. They acquired that liquid velocity upsurges by enhancing wall mass and wall tension parameters however velocity shows opposite behavior for wall damping parameter. The significance of activation energy exploration on the radiative peristaltic of Casson liquid in non-uniform tube utilizing lubrication approximation theory was delivered by Abbas et al. Some more significant articles about this topic can be consulted through.

Theoretical modeling of non-Newtonian fluids has key importance for predicting and understanding the behavior of several arising natural processes. Many fluids observed in nature like blood, petroleum, greases, oils, mud, and polymer solutions, show important rheological properties which do not obey the common linear relationship between strain and stress in Newton’s law viscosity. In modern industries and technologies, the performance of non-Newtonian fluids may cause a huge interest in the subject (see Abbas and Rafiq,). However, the constitutive expressions involved in modeling such fluids are usually complex and their exact solutions are difficult. One such generalized model predicting the non-Newtonian properties is Sisko fluid model exhibiting the shear thickening and thinning characteristics. The first attempt on Sisko fluid model was done by Sisko in 1958. The concept of Lorentz force on the peristaltic movement of Sisko liquid under the approximation of lubrication theory was analyzed by Wang et al. The homotopy analysis method (HAM) was adopted by Nadeem and Akbar to scrutinize the peristaltic propulsion of the Sisko fluid model inside the tube geometry. Tanveer et al. investigated the curved channel flow of Sisko liquid by claiming the velocity as a declining factor of the index parameter. Hydromagnetic blood flow of Sisko fluid in a non-uniform channel induced by the peristaltic wave was analyzed by Zeeshan et al. Again, Tanveer et al. focused on the modified Darcy’s law in the peristaltic analysis of Sisko liquid in a curved configuration using a long-wavelength approximation.

The analysis of magnetohydrodynamic (MHD) peristaltic motion has acquired tremendous attention from several investigators because of its trending uses in blood pumping, casting process, drug targeting, magnetic resonance imaging, magneto-therapy, hyperthermia, etc. The sinusoidal promulgation of hydromagnetic Williamson liquid in a curved configuration was developed by Rashid et al. Akram et al. scrutinized the transmission of hydromagnetic Prandtl nanomaterials across the non-uniform channel having sinusoidal walls peristaltic utilizing lubrication theory. The impacts of Lorentz force on the peristaltic activity of Jeffrey fluid in a channel under the approximation of lubrication theory were reported by Abbas et al. They found in this exploration that the thermal profile rises by enhancing the Brinkman number. Recently, Abbas et al. investigated the significance of entropy optimization for peristaltic transportation of hydromagnetic viscous liquid in a diverging tube utilizing the approximation of lubrication theory. Furthermore, heat transfer impacts subject to thermal radiation play a substantial role in the industry mainly in industrial and manufacturing equipment that comprise nuclear plants, missiles, space vehicles satellites, gas turbines, etc. Thermal radiation is found efficient in several high-temperature procedures. The radiative features of the peristaltic activity of Eyring-Powell nanomaterial in flexible complaint channel walls were deliberated by Nisar et al. The chief consequence of this research is that the radiative profile diminutions by amplying the thermal Biot number while it grows for Brownian motion and thermophoresis parameters. The radiative aspects of the peristaltic movement of Rabinowitsch liquid in a channel were reviewed by Imran et al. In addition, the impacts of hall current on the sinusoidal wall transportation of viscous nanomaterials through a channel were developed by Alsaedi et al. Recently, Abbas and Rafiq investigated the thermally radiative peristaltic flow of micropolar-Casson fluid in a symmetric channel under lubrication approximation theory.

Chemical reactions and mass transference interact in various ways, which may be seen in the consumption and synthesis of reactant species at various rates both inside the liquid and during mass transference. In the fields of chemical engineering and metallurgy, such as polymer manufacture and food processing, the study of heat and mass transport with chemical reactions on various fluids has become critical. The first investigations of chemical reactions on interface layer streams were done by Bestman. Several researchers have recently investigated the impact of chemical reactions on different fluid flow patterns across various surfaces.

The novelty of the present investigation is to inspect the significance of the chemically reactive peristaltic
flow of electrically conducting Sisko fluid through a porous medium in a two-dimensional tapered asymmetric channel which may be suitable to mimic the movement of intra-uterine liquid through a sagittal cross-section of the uterus. The problem of intrauterine liquid transportation uterus produced by myometrial contractions in a non-pregnant is a peristaltic-type liquid transport and this myometrial contraction may arise in both asymmetric and symmetric directions. In the human body, the transport of various biological fluids can easily be noticed actively to handle various types of diagnostic problems. The peristaltic pumping mechanism is very helpful in transporting different kinds of fluids, such as sanitary fluids, sensitive fluids, noxious fluids, and corrosive fluids. This phenomenon can also be found in a living body such as the gastro-intestinal tract. Furthermore, it is scrutinized that the intrauterine transport of liquid in a uterus sagittal cross-section divulges a narrow channel bounded by two moderately similar walls with wave trains taking dissimilar phase differences and amplitudes.\textsuperscript{36–39} The problem is first modeled and then analyzed by a lubrication approximation theory. The resulting equations are elucidated employing the perturbation technique and the physical attributes of pertinent variables are scrutinized and elaborated through graphs. Simpson’s rule was utilized to estimate the pumping feature such as pressure rise utilizing Mathematica software. To ensure the accuracy of the developed code, obtained results are compared with the results available in the literature and found in excellent agreement.

Problem statement

In this inspection, the radiative peristaltic movement of hydromagnetic Sisko liquid in a tapered channel with systematically contracting as well as relaxing sinusoidal walls has been considered. The geometry is visualized in Figure 1(a) with Cartesian coordinates implemented with the $X$-axis adapted along the center line and the $Y$-axis transverse to it. Further, a property geometrical display of tapered channel can be seen in Figure 1(b). Also, the fluid is electrically conducting under the radially imposed magnetic field $\mathbf{B}$ to linearize the flow stream is

$$\mathbf{B} = (B_0, 0, 0).$$

Here due to the low magnetic Reynolds number, induced magnetic impacts are negligible.

Ohm’s law enables attaining the required specific term that aids the MHD effect in curved flow dynamics

$$\mathbf{J} \times \mathbf{B} = (0, -\sigma B_0^2 w, 0).$$

The position of the left and right channel walls is specified by $\bar{Y} = H_1$ and $\bar{Y} = H_2$ (see Figure 1). The right and left walls are maintained at temperatures $T_0$ and $T_1$ and concentrations $C_0$ and $C_1$ respectively. The wave geometry is characterized by the following expressions\textsuperscript{38}:

$$H_1(\bar{X}, \bar{t}) = -d - m^* \bar{X} - a_1 \cos \left[ \frac{2\pi}{\lambda} (-ct + \bar{X}) + \phi \right],$$

$$H_2(\bar{X}, \bar{t}) = d + m^* \bar{X} + a_2 \cos \left[ \frac{2\pi}{\lambda} (-ct + \bar{X}) \right],$$

where $a_1$ and $a_2$ are the amplitudes of lower and upper walls respectively, $\lambda$ is the wavelength, $m^*(m^* \ll 1)$ is the non-uniform parameter of the tapered asymmetric channel, $c$ is the velocity of propagation, $t$ is the time, the phase difference $\phi$ varies in the range $0 \leq \phi \leq \pi$, $\phi = 0$ corresponds to symmetric channel with waves out of phase, that is, both walls move outward or inward simultaneously. Additionally, $d$, $a_1$, $a_2$, and $\phi$ satisfy the condition $a_1^2 + a_2^2 + 2a_1a_2 \cos \phi \approx (2d)^2$. 

Figure 1. (a) Geometry of the problem and (b) property geometrical display of tapered channel.
The suitable equations that govern the flow are given by\textsuperscript{16}:

\[
div V = 0, \tag{5}
\]

\[
\rho \frac{dV}{dt} = -PI + divS + J \times B - u_{eff}^2 \frac{V}{K}, \tag{6}
\]

\[
\rho C_p \frac{dT}{dt} = L \cdot S + k\nabla^2 T - \nabla \cdot q_r, \tag{7}
\]

\[
\frac{dC}{dt} = D_B \nabla^2 C + \frac{D_B K_T}{T_m} (\nabla^2 T) - k_c (C - C_0). \tag{8}
\]

where \(d/dt\) denotes the material derivative, \(S\) embodies the extra stress tensor, \(-PI\) is the spherical part of the stress because of incompressibility constraint, \(V\) characterizes velocity vector, \(C_p\) is the specific heat at constant pressure, \(K\) is the permeability of the porous medium, \(u_{eff}\) represents the effective viscosity of the porous medium, \(k\) denotes the thermal conductivity, \(D_B\) is the coefficient of mass diffusivity, \(K_T\) denotes the thermal diffusivity, \(T_m\) mean temperature, dissipation function, \(\rho\) is the fluid density, and \(k_c\) is the chemical reaction parameter.

The extra stress tensors for the Sisko fluid model are given by\textsuperscript{18}:

\[
\tau = -pI + S, \tag{9}
\]

\[
\tilde{S} = \left[ a_s + b_s \left| \sqrt{\Pi} \right| \right] A_1, \tag{10}
\]

where \(A_1 = \text{grad} V + (\text{grad} V)^t\) and \(\Pi = \frac{1}{2} \text{tr}(A_1^2)\).

In Sisko liquid model, \(a_s, b_s,\) and \(n (\geq 0)\) signify the infinite shear rate viscosity, consistency index, and power-law index respectively. Newtonian model deduces by adjusting \((n = 1, a_s = 0, b_s = \mu\) or \(b_s = 0, a_s = \mu\)). Furthermore, this model predicts shear-thinning and shear-thickening liquid characteristics for \(n < 1\) and \(n > 1\) respectively. Also,

\[
\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.
\]

At this point, we will define the porous medium for the Darcy resistance termed as \(R = (R_x, R_y, 0)\), hence by operating the modified Darcy law, we get the filtered relation between velocity and pressure indicated below\textsuperscript{23}:

\[
\nabla \bar{p} = -\frac{\xi}{K} \left[ a_s + b_s \sqrt{\Pi^{n-1}} \right] V, \tag{11}
\]

here \(-\xi/K\) and \(\xi\) implies the permeability and the porosity of porous medium respectively. An extra benefit of the generalized form stated above relates to its potential of regaining the outcomes of Darcy law by taking \(n = 1\) or \(b_s = 0\). Keeping in mind the case of flow with porous space, resistance is noted by the pressure gradient, hence equation (11) is interpreted in the form:

\[
R = -\frac{\xi}{K} \left[ a_s + b_s \sqrt{\Pi^{n-1}} \right] V. \tag{12}
\]

The velocity vector for the current liquid flow analysis is

\[
V = [W(X, Y, T), V(X, Y, T), 0], \tag{13}
\]

Utilizing the velocity field given in equation (13), we obtain

\[
\frac{\partial W}{\partial x} + \frac{\partial V}{\partial y} = 0, \tag{14}
\]

\[
\rho \left( \frac{\partial W}{\partial t} + \bar{W} \frac{\partial W}{\partial x} + \bar{V} \frac{\partial W}{\partial y} \right) = -\frac{\partial P}{\partial x} + \frac{\partial S_{XX}}{\partial x} + \frac{\partial S_{XY}}{\partial y} + \frac{\partial S_{YY}}{\partial y} + R_x, \tag{15}
\]

\[
\rho \left( \frac{\partial V}{\partial t} + \bar{W} \frac{\partial V}{\partial x} + \bar{V} \frac{\partial V}{\partial y} \right) = -\frac{\partial P}{\partial y} + \frac{\partial S_{XY}}{\partial x} + \frac{\partial S_{YY}}{\partial y} + R_y, \tag{16}
\]

\[
\frac{\partial S_{XX}}{\partial x} + \frac{\partial S_{XY}}{\partial y} + \frac{\partial S_{YY}}{\partial y} = \frac{a_s}{\frac{\partial W}{\partial x}} + \frac{a_s}{\frac{\partial V}{\partial y}} + \frac{a_s}{\frac{\partial W}{\partial x}} - \frac{\partial q_r}{\partial y}, \tag{17}
\]

\[
\frac{\partial C}{\partial t} + W \frac{\partial C}{\partial x} + V \frac{\partial C}{\partial y} = D_B \left( \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial x^2} \right) \tag{18}
\]

where,

\[
S_{XX} = 2 \left[ a_s + b_s \left( \frac{\partial W}{\partial x} \right)^2 + \left( \frac{\partial W}{\partial y} + \frac{\partial V}{\partial x} \right)^2 + 2 \left( \frac{\partial V}{\partial y} \right)^2 \right] \frac{\partial W}{\partial x}, \tag{19}
\]

\[
S_{XY} = \left[ a_s + b_s \left( \frac{\partial W}{\partial x} \right)^2 + \left( \frac{\partial W}{\partial y} + \frac{\partial V}{\partial x} \right)^2 + 2 \left( \frac{\partial V}{\partial y} \right)^2 \right] \frac{\partial V}{\partial y}, \tag{20}
\]

\[
S_{YY} = 2 \left[ a_s + b_s \left( \frac{\partial W}{\partial x} \right)^2 + \left( \frac{\partial W}{\partial y} + \frac{\partial V}{\partial x} \right)^2 + 2 \left( \frac{\partial V}{\partial y} \right)^2 \right] \frac{\partial V}{\partial y}. \tag{21}
\]
Here $\bar{W}$ and $\bar{V}$ are the velocity components in the axial and transverse directions respectively, $\bar{P}$ is the pressure, $\bar{T}$ and $\bar{C}$ the fluid temperature and concentration.

The suitable conditions are $^{23}$:

$$
\begin{align*}
\psi &= -\frac{q}{2}, \quad \frac{\partial \psi}{\partial \bar{y}} - \beta \bar{S}X_{\bar{Y}} = 0, \quad T - T_0 = 1, \\
\bar{C} - C_0 &= 1 \quad \text{at} \quad \bar{Y} = \bar{H}_1, \\
\psi &= \frac{q}{2}, \quad \frac{\partial \psi}{\partial \bar{y}} + \beta \bar{S}X_{\bar{Y}} = 0, \quad \bar{T} - T_0 = 0, \\
\bar{C} - C_0 &= 0 \quad \text{at} \quad \bar{Y} = \bar{H}_2.
\end{align*}
$$

(19)

where, $\beta$ is the slip parameter.

The radiative thermal heat flux $q_r$ is $^{28}$

$$
q_r = -\frac{16 T_0^2 \sigma a^* T}{3 \cdot k' \partial \bar{Y}}.
$$

(20)

The dimensionless quantities are

$$
x = \frac{X}{\lambda}, \quad y = \frac{Y}{d}, \quad Re = \frac{pcd}{\mu}, \quad w = \frac{W}{c}, \quad \lambda_2 = \frac{\alpha_2 c}{\nu d}, \quad \beta = \frac{B_r}{d}, \\
\theta = \frac{T - T_0}{T_1 - T_0}, \quad Ec = \frac{c^2}{C_p(T_1 - T_0)}, \quad Pr = \frac{\mu C_p}{k}, \quad h_1 = \frac{\bar{H}_1}{d}, \quad Da = \frac{K}{d^2}, \\
\delta = \frac{2 \pi d}{\lambda}, \quad M = \sqrt{\sigma / \mu B_d}, \quad t = \frac{c_i}{c}, \quad v = \frac{V}{c_i}, \quad B_r = EcPr, \\
h_2 = \frac{h_2}{d}, \quad R = \frac{16 T_0^2 \sigma^*}{3 k' k}, \quad p = \frac{pd}{c \mu \lambda}, \quad b = \frac{a_2}{d}, \quad \psi = \frac{\psi}{c d}, \\
a = \frac{a_1}{d}, \quad S_y = \frac{d}{\mu} S_y(\bar{X}), \quad \gamma = \frac{k d^2}{v}, \quad \lambda^2 = M^2 + \frac{1}{Da}, \quad S_r = \frac{v}{D_0}, \\
\phi = \frac{\bar{C} - C_0}{C_1 - C_0}, \quad S = \frac{\mu D_0 K_r(T_1 - T_0)}{\mu T_m(C_1 - C_0)}, \quad A = b_3 / a_3(d/c)^{n-1}.
$$

(21)

where $\delta$ is the wave number, $S_r$ is the Schmidt number, $\gamma$ signifies the chemical reaction parameter, $Da$ is the Darcy number, $Pr$ is the Prandtl number, $Ec$ is the Eckert number, $B_r$ is the Brinkman number, $M$ is the magnetic parameter, $S_t$ denotes the Soret number, $R$ is the radiation parameter, and $A$ is the fluid parameter.

Utilizing equation (21) with stream functions $(u = -\partial \psi / \partial y, v = -\partial \psi / \partial x)$ accompanied by long wavelength approximation $\delta \ll 1$ as well as negligible Reynolds number $Re$, the equations (14) to (18) becomes

$$
\left(1 + R Pr\right) \frac{\partial^2 \theta}{\partial y^2} + B_r \left(\frac{\partial^2 \psi}{\partial y^2}\right)^2 \left(\frac{\partial^2 \psi}{\partial y^2} + A \left(\frac{\partial^2 \psi}{\partial y^2}\right)^n\right) = 0,
$$

(24)

$$
\frac{\partial^2 \phi}{\partial y^2} + S_r S_c \left(\frac{\partial^2 \theta}{\partial y^2}\right) - S_c \gamma \phi = 0.
$$

(25)

Upon eliminating pressure from equations (22) and (23) we have

$$
\frac{\partial^2 \theta}{\partial y^2} \left(\frac{\partial^2 \psi}{\partial y^2} + A \left(\frac{\partial^2 \psi}{\partial y^2}\right)^n\right) - L^2 \frac{\partial^2 \psi}{\partial y^2} \frac{A \left(\frac{\partial^2 \psi}{\partial y^2}\right)^n}{D_a \left(\frac{\partial^2 \psi}{\partial y^2}\right)} = 0.
$$

(26)

The transformed surface conditions are

$$
\begin{align*}
\psi &= -\frac{F}{2}, \quad \frac{\partial \psi}{\partial y} - \beta S_{xy} = 0, \quad \theta = 0, \\
\phi &= 0, \quad \text{at} \quad y = h_1, \\
\psi &= \frac{F}{2}, \quad \frac{\partial \psi}{\partial y} + \beta S_{xy} = 0, \quad \theta = 1, \\
\phi &= 1, \quad \text{at} \quad y = h_2.
\end{align*}
$$

(27)

The mean flow rate expression is specified by

$$
\Theta = 1 + d + F.
$$

(28)

The instantaneous volumetric flow rate $F(x, t)$ is calculated by:

$$
F(x, t) = \Theta + b \sin 2\pi(x - t) + a \sin[b\phi + 2\pi(x - t)].
$$

With $F = \int_{h_1}^{h_2} wd\bar{y}$.

The computations of pressure per wavelength $\Delta p$ are performed by utilizing

$$
\Delta p = \int_0^1 \int_{-h_2}^{h_2} \left(\frac{dp}{dx}\right) dxdt.
$$

(29)

The skin friction and Nusselt number at the channel’s upper wall in dimensionless form can be written as

$$
S_f = \left. \left(\frac{\partial^2 \psi}{\partial y^2} + A \left(\frac{\partial^2 \psi}{\partial y^2}\right)^n\right) \right|_{h_2}, \quad Nu = \left. \frac{\partial \theta}{\partial y}\right|_{h_2}.
$$

(30)

**Method of solution**

The significance of the perturbation method is well established in literature for its ability to provide an approximate analytical solution to nonlinear problems. The outcomes reported for the other comparatively
sophisticated approximate analytical methods to non-linear problems have decent accuracy, but they are more difficult in applications and analysis than perturbation methods. Thus, for many years, the relative simplicity and high precision in the limitation of small parameters have made perturbation methods fascinating tools among the most frequently utilized approximate analytical procedures. Perturbation methods usually use convenient mathematical formulations to provide accurate outcomes for small perturbation parameters. Therefore, the method of perturbation has been implemented to acquire the expressions for velocity, pressure gradient, temperature, and concentration for small values of the Sutterby liquid parameter. For this intention, we expand the flow quantities as follows:

\[
\psi = \psi_0 + A\psi_1 + \ldots \quad \rho = \rho_0 + A\rho_1 + \ldots \quad \theta = \theta_0 + A\theta_1 + \ldots \quad \phi = \phi_0 + A\phi_1 + \ldots
\]  

Invoking equation (31) into equations (22) to (27), we acquire the following system

**Zeroth order system**

\[
\frac{\partial^4 \psi_0}{\partial y^4} - L^2 \frac{\partial^2 \psi_0}{\partial y^2} = 0, \quad \frac{\partial \rho_0}{\partial x} = \frac{\partial^3 \psi_0}{\partial y^3} - L^2 \frac{\partial \psi_0}{\partial y},
\]

\[
(1 + R \text{Pr}) \frac{\partial^2 \theta_0}{\partial y^2} + Br \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^2 = 0,
\]

\[
\frac{\partial^2 \phi_0}{\partial y^2} + \frac{S_x S_y}{C_0} \left( \frac{\partial^2 \theta_0}{\partial y^2} \right) - \gamma S_x \phi_0 = 0,
\]

with boundary conditions

\[
\psi_0 = -\frac{F_0}{2}, \quad \frac{\partial \psi_0}{\partial y} - \beta S_y \psi_0 = 0, \quad \theta_0 = 0, \quad \phi_0 = 0, \quad \text{at } y = h_1,
\]

\[
\psi_0 = \frac{F_0}{2}, \quad \frac{\partial \psi_0}{\partial y} + \beta S_y \psi_0 = 0, \quad \theta_0 = 1, \quad \phi_1 = 1, \quad \text{at } y = h_2.
\]

(36)

**First-order system**

\[
\frac{\partial^4 \psi_1}{\partial y^4} - L^2 \frac{\partial^2 \psi_1}{\partial y^2} = - \frac{\partial^2}{\partial y^2} \left[ \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^3 \right] - \frac{1}{Da} \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^3,
\]

\[
\frac{\partial \rho_1}{\partial x} = \frac{\partial}{\partial y} \left[ \frac{\partial^2 \psi_1}{\partial y^2} \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^3 \right] - L^2 \frac{\partial \psi_1}{\partial y},
\]

\[
(1 + R \text{Pr}) \left( \frac{\partial^2 \theta_1}{\partial y^2} \right) + Br \left( \frac{\partial^2 \psi_0}{\partial y^2} \right)^2 + \frac{\partial^2 \psi_0}{\partial y^2} = 0,
\]

\[
\frac{\partial^2 \phi_1}{\partial y^2} + \frac{S_x S_y}{C_0} \left( \frac{\partial^2 \theta_0}{\partial y^2} \right) - \gamma S_x \phi_1 = 0,
\]

(40)

with boundary conditions

\[
\psi_1 = -\frac{F_1}{2}, \quad \frac{\partial \psi_1}{\partial y} - \beta S_y \psi_0 = 0, \quad \theta_1 = 0, \quad \phi_1 = 0, \quad \text{at } y = h_1,
\]

\[
\psi_1 = \frac{F_1}{2}, \quad \frac{\partial \psi_1}{\partial y} + \beta S_y \psi_0 = 0, \quad \theta_1 = 1, \quad \phi_1 = 1, \quad \text{at } y = h_2.
\]

(41)

**Solution for the zeroth order system**

The outcomes at zeroth order are

\[
\psi_0 = \frac{B_1}{L^2} (\sinh Ly + \cosh Ly) + \frac{B_2}{L^2} (\cosh Ly - \sinh Ly) + B_3 y + B_4,
\]

(42)

\[
\frac{\partial \rho_0}{\partial x} = \frac{F(h_1 - h_2)L^3(\cosh h_1 L + \cosh h_2 L + \sinh h_1 L + \sinh h_2 L)}{(-2 + h_1 L + h_2 L)(\cosh L + \sinh L) + (2 + h_1 L - h_2 L)(\cosh L + \sinh L)}.
\]

(43)

\[
\theta_0 = B_5 y + B_6 - \frac{B_x B_1 B_2 L^2}{M(1 + \text{Pr} R)} - \frac{B_x B_2^2}{M(1 + \text{Pr} R)} (\sinh 2Ly + \cosh 2Ly)
\]

\[
+ \frac{B_x B_2^2}{M(1 + \text{Pr} R)} (\sinh 2Ly - \cosh 2Ly),
\]

(44)
\[
\phi_0 = \frac{1}{4L^2(1 + \Pr R)} \left( \frac{B_{12}B_{11}^{1/2}}{M(1 + \Pr R)} (\cos 2L y - \sinh 2L y) \
\right) \\
\left( B_1^2 + 4B_1B_2L^2y^2 \cos 2M y + B_{12}^2 \cosh 4L y \right) \\
+ 4B_1B_2L^2y^2 \sinh 2L y + B_1^2 \sinh 2L y \\
+ B_7y + B_8.
\]

(45)

**Solution for the first order system**

The outcomes at first order are

\[
\psi_1 = C_3y + C_4 + \frac{(\cos 3L y - \sin 3L y)}{8L^2} \times \\
\left( B_2^3 + B_1^2 \cos 6L y - 6B_1B_2^2(5 + 2L y)(\cos 2L y + \sin 2L y) + \right) \\
\left( 8(\cos 2L y + \sin 2L y)(C_2 + C_1 \cos 2L y + C_1 \sin 2L y) + \right) \\
\left( 6B_1^2B_2(-5 + 2L y)(\cos 4L y + \sin 4L y) + B_1^2 \sin 6L y \right)
\]

\[
\frac{\delta p_1}{\delta x} = -\frac{1}{8} \left( \cosh 3h_1L - \sin 3h_1L \right) \left( A_1 + 2A_3 \cosh 2L_1y + A_7 \sin 3h_1L + 2A_2 \sin 4h_1L \right) + \\
\left( \cosh 3h_2L - \sin 3h_2L \right) \left( A_4 + 2A_6 \cosh 2L_2y + 2A_5 \sin 4h_2L \right) + \\
\left( 2A_3 \sin 3h_1L + 2A_2 \sin 4h_1L \right)
\]

(46)

\[
\theta_1 = C_3y + C_6 - \frac{1}{8M(1 + \Pr R)} B_r \times \\
\left( 64B_1B_2(B_1C_2 - B_2C_1)L^2y^3 + e^{-2M}(A_{12} + y(-A_{13} + 72B_1^2B_2^2L y^2)) \right) \\
\left( -48B_1^2B_2^2L^4y + e^{-4M}(A_0 + 27B_1^2B_4) + e^{4M}(A_2 + 27B_1^2B_2) \right) \\
\left( e^{2M}(A_{10} + y(-A_{11} + 72B_1^2B_2^2L y^2)) + 2A_7L y^2 + \text{terms} \right)
\]

(47)

\[
\phi_1 = \frac{1}{4L^2(1 + \Pr R)} \left( \frac{B_{12}B_{11}^{1/2}}{M(1 + \Pr R)} (\cos 2L y - \sinh 2L y) \
\right) \\
\left( B_1^2 + 4B_1B_2L^2y^2 \cos 2M y + B_{12}^2 \cosh 4L y \right) \\
+ 4B_1B_2L^2y^2 \sinh 2L y + B_1^2 \sinh 2L y \\
+ C_7y + C_8.
\]

(48)

(49)

**Results and discussion**

This portion characterizes the impacts of sundry variables on the velocity, pressure rise, temperature, concentration, and streamlines through graphical outcomes. Integral specified in equation (29) cannot be resolved analytically so a numerical solution based on a suitable algorithm is required. Therefore, this integral is calculated numerically by employing the composite Simpson’s rule with spatial discretization number for the numerical process taken at 200. The computed convergence criterion was 10\(^{-10}\). In this analysis, the following default parameter values are adopted for computations: \( Da = 0.2, M = 1, t = 0.2, a = 0.3, \Theta = 0.8, A = 0.1, B_r = 2, R = 0.2, S_r = 0.5, \gamma = 0.3, m = 0.2, x = 0.4, \phi = \pi/2, \) and \( b = 0.2 \). We have designated the parameter values according to the previous literature\(^5,22,27,37,38\) and these are beneficial for experimental purposes. Furthermore, the outcomes of the current study are in decent agreement with the outcomes accessible in the literature\(^36\) which suggests the validity of the present model.

Figure 2(a) to (c) is designed to scrutinize and deliberate the impacts of significant parameters on the momentum profile \( w \). The influence of the velocity slip parameter \( \beta \) on liquid velocity is offered in Figure 2(a). From this graph, it can be seen that velocity is increasing in the central part of the channel by increasing the values of the velocity slip parameter. This is be that...
cells. The profile drawn in Figure 2(c) depicts the deviations of Darcy number $Da$ on liquid velocity. The impact of higher values in the Darcy number cause enhancement in liquid velocity $Da$. It occurs because of a more permeable porous medium which enhances liquid velocity by diminishing its resistance to liquid movement, resulting in a rise in fluid velocity. This concept is beneficial in some medicine systems such as arteries in the human body to scrutinize the circulation of blood. Figure 2(d) is outlined to elucidate the performance of the momentum profile for numerous values of the Sisko liquid parameter $A$. It can be distinguished that the momentum profile enhances with enhancing values of Sisko fluid parameter. In fact, that for larger values of Sisko model parameter $A$, it decreases the effective conductivity of fluid, and consequently it reduced the magnetic damping effect. Therefore, velocity is increases.

In a human body, the transport of various biological fluids can easily be notice actively to handle various types of diagnostic problems. Peristaltic pumping mechanism is very helpful in transporting different kinds of fluids, such as sanitary fluids, sensitive fluids, noxious fluid, and corrosive fluids. This phenomenon can also be found in a living body such as gastrointestinal tract. Figure 3(a) to (c) is outlined to explore the impressions of significant parameters on average pressure rise $\Delta p$. These diagrams demonstrate a linear relationship of $\Theta$ and $\Delta p$. Generally, in accordance with the modification of $\Delta p$, the mechanism of pumping is distributed into three sections. The segment that corresponds to $\Delta p$ greater than zero is termed as the pumping region. Further, the pumping area is divided into sections of positive pumping $\Theta > 0$ and negative pumping $\Theta < 0$. At last, the region with $\Delta p = 0$ implies the free pumping region and a co-pumping region also exists where $\Delta p$ is less than zero. Figure 3(a) is exposed to distinguish the impacts of $M$ on $\Delta p$. It is detected from this graph that the pumping rate diminutions with amplifying $M$ in the peristaltic pumping region, whereas the reverse trend is monitored in the co-pumping region. The graphs illustrated in Figure 3(b) display the variation of rising in pressure with altered values of Darcy number. From this diagram, we observe that the pumping rate enhances with Darcy number in the peristaltic pumping portion and diminishes in the co-pumping region. The impression of the Sisko liquid parameter $A$ on $\Delta p$ is exhibited in Figure 3(c).
diagram exhibits that greater values of the Sutterby liquid parameter amplify the pressure rise in the retrograde pumping region.

The impacts of sundry variables on thermal profile are accessible in Figure 4(a) to (c). In Figure 4(a) we portray the mutual impacts of $B_r$ on thermal profile. It is distinguished that thermal profile enhances with augmenting values of $B_r$. Physically, the Brinkman number enhances the resistance to the flow because of its shear strength, which in turn leads to an enhancement in heat due to the viscous dissipation impact. So, upsurges the liquid temperature. Figure 4(b) illustrates the deviation of Sisko liquid parameter and thermal radiation parameter $R$ on the liquid temperature. It can observe from this graph, liquid temperature improves for greater values of the Sisko liquid parameter however, the graph
represents that liquid temperature diminishes for augmenting values of the parameter $R$. The reason is that the radiative parameter is inversely related to the absorption coefficient $k^*$. This leads to better heat absorption of the liquid and means that excess heat is dissipated. Therefore, the liquid temperature is reduced. This concept is employed in the liver cancer and lungs treatment, tissue coagulation, and stomach acid reflux.

Figure 5(a) and (b) are sketched to depict the deviation of concentration profile. The impression of $Sc$ on the liquid concentration is depicted in Figure 5(a) and analyzed that fluid concentration is reduced with improved values of $Sc$. The physical implication of this observation is that $Sc$ embodies the ratio of diffusivities of momentum to the mass (species) and therefore there is a reduction in the concentration. Figure 5(b) is graphed to see the deviations of liquid concentration with enhancing values of $\gamma$. It is established from this graph that with an increment in the values of the chemical reaction parameter causes the reduction in concentration profile. Physically, raising the parameter $\gamma$ yields a reduction in the interfacial mass transfer rate which diminishes concentration.

The heat transfer rate $Nu$ against the Brinkman number $Br$ is analyzed in Figure 6 for the different values of Sisko fluid parameter $A$. It can be seen that Nusselt number enhances with increasing the values of $A$. Moreover, it continually increases with the increasing rate of Brinkman number.

The development of an internally circulating bolus that transports waves is a very relatable mechanism in fluid dynamics. This phenomenon has physical examples in blood clots and the transportation of food bolus in the gastrointestinal tract. This kind of event is characterized as trapping. The trapping mechanism of the streamlines $\psi$ has been accessible in Figures 7 to 10. Figure 7 exhibits contour plots for several values of $M$. It divulges that trapped bolus size diminishes with an enhancement in magnetic parameter $M$. This decrease in the size of the bolus is attributed to the Lorentz forces which act as a retarding force. Figure 8 portrays the trapping mechanism for multiple $m$. The number of boluses improves as the non-uniform parameter upsurgs. Figure 9 designates the streamline pattern for rising values of $B$. From this plot, we can observe that number of bolus improves with amplifying values of $B$. The deviations in streamlines for the Sisko liquid parameter are visualized in Figure 10. From this graph, it can be depicted that number of trapped bolus augmented with higher values of the Sisko liquid parameter.

**Validation**

The purpose of this section is to check the accuracy of our outcomes. To verify obtained results, a comparison of limiting case of present investigation for the velocity profile in the absence of Darcy number, velocity slip parameter, and fluid parameter with the results reported by Kothandapaniet al. is provided in the
absence of fluid parameter (see Figure 11). This graph indicates that both findings are in good agreement.

**Conclusions**

We have theoretically scrutinized the problem of peristaltic flow of electrically conducting Sisko liquid in a tapered with modified Darcy law. The outcomes are acquired for the flow quantities employing the regular perturbation method for small values of the Sisko liquid parameter. The substantial outcomes from the current model are:

- Momentum profile decreased by increasing the values of magnetic and velocity slip parameters while it rises by the Sisko liquid parameter.
- Pressure rise is enhanced in the retrograde pumping region by increasing the values of Sisko fluid parameter.
- The declining behavior of the thermal profile is viewed by enhancing the radiation parameter while it enhances with the Brinkman number and Sisko liquid parameter.
- The bolus size is perceived to reduce with the enhancement of the magnetic parameter.
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Figure 9. Streamlines for various values of velocity slip parameter $\beta$: (a) $\beta = 0.01$ and (b) $\beta = 0.03$.

Figure 10. Streamlines for various values of Sisko fluid parameter $A$ on $\psi$: (a) $A = 0.1$ and (b) $A = 0.2$.

Figure 11. Comparison of limiting case of the present study with the results of Kothandapani et al.$^{36}$
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