THE CHALLENGE OF SMALL $x$

R D Ball$^*$
Department of Physics and Astronomy
University of Edinburgh, EH9 3JZ, Scotland†

P V Landshoff
DAMTP, Centre for Mathematical Sciences
Cambridge, CB3 0AW, England†

Abstract

We review the current understanding of the behaviour of inclusive cross sections at small $x$ and large $Q^2$ in terms of Altarelli-Parisi evolution, the BFKL equation, and Regge theory, asking in particular to what extent they are mutually consistent.

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* Royal Society University Research Fellow
† email addresses: rdb@th.ph.ed.ac.uk  pvl@damtp.cam.ac.uk
Introduction

A striking discovery at HERA has been the rapid rise with $1/x$ of the proton structure $F_2$ at small $x$. If one fits this rise to an effective power $x^{-\lambda(Q^2)}$ then, even at quite small values of $Q^2$, $\lambda(Q^2)$ is found to be significantly greater than the value just less than 0.1 associated with soft pomeron exchange that is familiar in purely hadronic collisions [1]. Moreover, $\lambda(Q^2)$ increases rapidly with $Q^2$. Similarly, and perhaps equally importantly, the size of the scaling violations is seen to increase dramatically as we go to smaller $x$ (see figure 1).

At first it was believed that $\lambda(Q^2)$ could be calculated from the BFKL equation [3]. However it was soon realised that this approach could not explain the observed rise of $\lambda$ with $Q^2$, nor the large scaling violations. Instead, the experimental data are in good agreement [4] with the double-logarithmic rise

$$F_2(x,Q^2) \sim \exp(\sqrt{(48/\beta_0) \ln 1/x \ln \ln Q^2}),$$

predicted long ago [5] from the lowest-order Altarelli-Parisi equations [6]. The data can also be fitted in Regge theory [7], by adding the exchange of a ‘hard pomeron’ to that of the soft pomeron; this achieves an effective power $\lambda(Q^2)$ as the result of combining fixed-power terms whose relative weights vary with $Q^2$.

In this note we review the present difficulties with the BFKL equation, the uncertainties related to the resummation of small $x$ logarithms in Altarelli-Parisi equations, and discuss whether either of these approaches is consistent with Regge theory and in particular the assumption that the dominant singularities are Regge poles. The central question concerns the extent to which the behaviour of cross-sections in the small $x$ limit may be calculated from perturbative QCD.

These are important issues, as the accuracy of any extractions of parton distribution functions from HERA data and thus of many of the predictions for the LHC relies crucially on our understanding of them. Most of these analyses are currently based on conventional fixed order perturbation theory.

The Regge Approach

The ZEUS collaboration has recently published [9] new data on events in which a $D^*$ particle is produced, which they use to extract the contribution $F_2^c(x,Q^2)$ to the complete structure function $F_2(x,Q^2)$ from events where the $\gamma^*$ is absorbed by a charmed quark. Their data for $F_2^c(x,Q^2)$ have the property [8] that, over a wide range of $Q^2$ they can be described by a fixed power of $x$:

$$F_2^c(x,Q^2) = f_c(Q^2)x^{-\epsilon_0},$$

with $\epsilon_0 \approx 0.4$ and $f_c(Q^2)$ fitted to the data: see figure 2.

If the behaviour (2) were literally true, it would imply that the Mellin transform $F_2^c(j,Q^2)$ would have a pole at $j = 1 + \epsilon_0$. Such poles in the complex angular momentum plane are called Regge poles, and the theory of Regge poles has a long history [10]. It has been used very successfully to correlate together a huge amount of data from soft hadronic reactions: total cross-sections such as $pp$ and $\bar{p}p$, partial cross-sections such as $\gamma p \rightarrow pp$, differential cross-sections such as $pp \rightarrow pp$, and diffraction dissociation (events where the final state has a very fast hadron). It is well established [1] that $j$-plane amplitudes have a pole near to $j = 1/2$, resulting from vector and tensor meson exchange, and another singularity, called the soft-pomeron singularity, near to $j = 1$. It is possible to obtain a good description of the soft hadronic data by assuming that this singularity too is a
Figure 1: a) Measurements of $F_2$ by ZEUS [2]. The curves show a NLO perturbative fit, with scaling violations as predicted by perturbative QCD. b) $\lambda(Q^2)$ extracted from ZEUS and E665 data on $F_2(x, Q^2)$ [2]. The solid line above 1 GeV$^2$ is from a NLO Altarelli-Parisi fit, while the lines below 1 GeV$^2$ are from Regge fits.
pole, at \( j = 1.08 \). Its dynamical origin is poorly understood \[11\]; it is presumably the result of some kind of nonperturbative gluonic exchange, or perhaps glueball exchange.

While the assumption that the soft-pomeron singularity is a pole describes a large amount of data well, Regge theory admits other types of singularity. For example, powers of logarithms of \( W^2 \) have been used to obtain equally good fits to total-cross-section data \[12\]. These fits have the advantage that they automatically satisfy standard unitarity bounds when extrapolated to arbitrarily high \( W^2 \), but they have the disadvantage that Regge factorization and quark counting rules become rather harder to understand. Nor can they readily be extended to other applications, such as \[13\] \( pp \) and \( \bar{p}p \) elastic scattering, and diffraction dissociation \[14\].

Regge theory should be applicable whenever \( W^2 \) is much greater than all the other variables, in particular when \( W^2 \gg Q^2 \) (and thus \( x \ll 1 \)), even if \( Q^2 \) is large. However, the tensor-meson and soft-pomeron poles are insufficient to fit all the HERA \( F_2 \) data. An excellent fit can be obtained \[7\] by including a further fixed pole at \( j = 1 + \epsilon_0 \), so that

\[
F_2(x, Q^2) = \sum_{i=0,1,2} f_i(Q^2) x^{-\epsilon_i}
\]

This ansatz fits the data all the way from photoproduction at \( Q^2 = 0 \) to \( Q^2 = 2000 \text{ GeV}^2 \), the highest value available at small \( x \). The soft-pomeron power is \( \epsilon_1 = 0.08 \), the tensor-meson power is \( \epsilon_2 \approx -0.5 \), while the new power is \( \epsilon_0 \approx 0.4 \), which we have already seen is what is needed to fit the data for \( F_2^C \) shown in figure 2. The new leading singularity at \( j = 1 + \epsilon_0 \) is sometimes referred to as the ‘hard pomeron’ singularity. This does not explain what causes it; it has often been conjectured that its origin is perturbative QCD, and we will see below the extent to which it is consistent with our current understanding based on the summation and resummation of small \( x \) logarithms.

Although there is no sign of any contribution from the hard pomeron in data for purely hadronic processes, it does seem to be present in \( F_2(x, Q^2) \) even at extremely small \( Q^2 \): measurements \[15\] indicate that even for \( Q^2 \) as low as \( 0.045 \text{ GeV}^2 \), \( F_2 \) is rising quite steeply in \( x \). Even at \( Q^2 = 0 \) the effective power \( \lambda \) may well be greater than that associated with soft purely-hadronic collisions.
Similarly, the data for $\gamma p \rightarrow J/\psi p$ are described well by the sum of two powers in the amplitude, $(W^2)^{\epsilon_0}$ and $(W^2)^{\epsilon_1}$ at $t = 0$. One does not expect a contribution from tensor meson exchange, because of Zweig's rule. The Regge picture also successfully describes the differential cross-section away from $t = 0$.

The striking feature of these fits is that such a wide variety of different data may be described using a simple parameterization: this suggests a universal underlying mechanism, and raises the hope that the hard component at least might be derivable from perturbative QCD. However, the $j$-plane singularities need not be poles, so the $x$ dependence need not be simple powers of $x$: powers of $\ln 1/x$ could do as well. Furthermore, Regge theory does not determine the coefficient functions $f_i(Q^2)$ in (3). Nor is it clear that three terms in (3) will always be enough: as the range in $x$ and $Q^2$ increases still further, it may be that yet more terms are required.

Thus although the $x$ and $Q^2$ of the existing data can be fitted using a Regge pole ansatz, the uncertainties in any extrapolation outside the existing kinematic range (such as from HERA to the LHC) are difficult to quantify. Moreover, it is not possible using Regge theory alone to predict jet cross sections, or indeed vector boson or top or Higgs production cross sections: we need more dynamics. Our only candidate for a complete theory of strong interactions at high energies is perturbative QCD, and it is to the understanding of perturbative QCD at small $x$ that we now turn.

**QCD: Resummation of Logs of $x$**

At first it was hoped that the BFKL equation provided a purely perturbative calculation of the value of $\lambda(Q^2)$. This hope was based on the leading contribution to the BFKL kernel $K(Q^2, k^2)$ with fixed coupling. Its Mellin transform $\chi(M)$ has a minimum at $M = \frac{1}{2}$, which gives rise to a power rise of the form $x^{-\lambda}$, with $\lambda = \lambda_0 \equiv \chi(\frac{1}{2}) = 12 \ln 2 \alpha_s/\pi$, in qualitative agreement with the first data sets. However this agreement was superficial, essentially because the $Q^2$ dependence was incorrect (see figure 1): $\lambda$ did not rise with $Q^2$, but remained fixed. There were suggestions that this was because the BFKL equation did not take sufficient account of energy conservation and of nonperturbative effects [16]: it is difficult to avoid important contributions from soft gluons, which cannot be estimated using perturbation theory. For this reason attempts to improve the kernel by making the coupling run were never entirely successful [17]: running couplings make the equation unstable, leading to unphysical effects.

The full extent of the difficulties was reinforced by the calculation of the next-to-leading order correction to the kernel [18]: the correction turned out to be very large and negative, inverting the minimum of the BFKL function $\chi(M)$, which was responsible for the power behaviour at leading order (see figure 4a). Since the saddle points of the inverse Mellin transform were now off the real axis, the NLLx equation gave rise to negative cross-sections in the Regge region [19]. This destroyed any faith that might have remained in the leading-order prediction.

Various proposals to fix up the BFKL equation have been put forward: for example a particular choice of the renormalization scale [20], or a different identification of the large logs which are resummed [21]. However the root of the problem [22] is that the perturbative contributions to $\chi(M)$ become progressively more and more singular at integer values of $M$, due to unresummed logarithms of $Q^2$ and $k^2$ in the kernel $K$. In particular, near $M = 0$ the expansion oscillates wildly. It follows that a perturbative expansion which sums logarithms of $x$ must also resum the large logarithms of $Q^2$ to all orders in perturbation theory if it is to be useful.
QCD: Resummation of Logs of $Q^2$

The usual way to resum logarithms of $Q^2$ is to use Altarelli-Parisi evolution equations, with the splitting functions calculated at a given fixed order in perturbation theory. If one starts at some initial scale $Q_0^2$ with parton distributions that rise less steeply than a power in $1/x$, then fixed order evolution to higher $Q^2$ leads to distributions that become progressively steeper in $1/x$ as $Q^2$ increases, in agreement with the $F_2$ data from HERA. More significantly the prediction[5] of the specific form (1) of the rise is in good agreement [4] with the data over a wide region of $x$ and $Q^2$. This is widely seen as a major triumph for perturbative QCD, as direct evidence for asymptotic freedom [23]: the coefficient $\beta_0$ in (1) which determines the slope of the rise is the first coefficient of the QCD $\beta$-function.

The success of fixed-order perturbative QCD in describing the increasingly precise HERA $F_2$ data when $Q^2 \gtrsim 1$ GeV$^2$ has been confirmed many times by successful NLO fits [24]. From these a gluon distribution may be extracted, (see figure 3a), and predictions for $F_2^c$ (figure 3b), dijet production, and $F_L$, all of which have now been supported by direct measurements [25]. Clearly fixed order perturbative QCD works well at HERA; none of these predictions is trivial, and all are successful. Of course once $Q_0^2$ is as small as 1 GeV$^2$ or less a perturbative treatment is no longer appropriate, and indeed an instability develops in the NLO gluon distribution at around such a scale (see figure 3a).

It is perhaps useful to compare figure 2 with figure 3b: the data are the same on each figure, but the curves on the former are the result of a power fit that assumes a flavour-blind hard pomeron, while those on the latter are from a straightforward parameter-free prediction made using NLO perturbative QCD. Interestingly the conclusions are also different: the slope of the rise in $x$ manifestly increases with $Q^2$ in figure 3b (corresponding to the rise of the slopes in figure 1a and figure 3a), while in figure 2 it is fixed.

It is important to realise that the success of the NLO perturbative QCD predictions is crucially dependent on the nonperturbative input at the initial scale $Q_0^2 \sim 1$ GeV$^2$ being ‘soft’ — not rising too quickly with $x$ — so that the rise in $x$ can be generated dynamically. If instead the rise were input in the form (3), growing as $x^{-\epsilon_0}$ with $\epsilon_0$ as large as 0.4, this would when evolved perturbatively with the NLO anomalous dimension lead to a $Q^2$ dependence which was independent of $x$ and thus inconsistent with the data [4] (see figure 1). If one were to insist on such a hard pomeron singularity, one would thus to be consistent also have to argue that NLO perturbative QCD could not be applied in this region. The many quantitative successes of NLO perturbative QCD at HERA [4,24,25] would then have to be considered merely fortuitous. Conversely, if one instead accepts that the success of the perturbative predictions is significant, one would then have to conclude that the simple assumption (3) that the rightmost singularity in the $j$-plane is a simple pole is incorrect, since the perturbative results rely for their success on a soft input.

This said, to obtain reliable predictions for processes at the LHC it is not sufficient to confirm NLO QCD within experimental errors at HERA: we must also be able to understand theoretical errors. In particular, at small $x$ the approximation to the splitting functions given by retaining only the first few terms in an expansion in powers of $\alpha_s$ is not necessarily very good: as soon as $\xi = \log 1/x$ is sufficiently large that $\alpha_s\xi \sim 1$, all terms of order $\alpha_s(\alpha_s\xi)^n$ (LLx) and $\alpha_s^2(\alpha_s\xi)^n$ (NLLx) must also be considered in order to achieve a result which is reliable up to terms of order $\alpha_s^3$. In fact $\alpha_s\xi \gtrsim 1$ throughout most of the HERA kinematic region, so one might expect these effects to be significant. The fact that empirically they seem to be small is thus a mystery requiring some explanation.

This argument may be sharpened by consideration of the $j$-plane singularities of the Mellin trans-
Figure 3: a) The gluon distribution extracted from a NLO fit to ZEUS data for $F_2$ [2]. b) The ZEUS data for $F_2^c$ [9], compared to the QCD prediction obtained from the gluon a).
form $F_2(j, Q^2)$. At the $n$-th order in fixed order perturbation theory the iteration of small $x$ logarithms in the evolution gives rise to essential singularities of the form

$$(j - 1)^{-1} \exp(\alpha_s^n/(j - 1)^n)$$

(4)

The $j = 1$ singularity thus becomes more severe order by order in perturbation theory. This is not necessarily a problem phenomenologically, since (4) corresponds to a sequence of predictions for measurable quantities such as $F_2(x, Q^2)$ that are strictly convergent [26] provided only that $x > 0$. It follows that although (4) may not be correct actually at the point $j = 1$ it may be a good numerical approximation to the correct behaviour away from $j = 1$.

Furthermore there is good reason to believe that a resummation over all orders $n$ might remove the singularity [27]. The argument is that, if there is a singularity at a fixed point in the complex $j$-plane for large values of $Q^2$, such as a naive application of (4) might seem to imply, then considerations of analyticity in $Q^2$ suggest that it might also be present at small $Q^2$. While this is not completely excluded, the Mellin transform variable $j$ is essentially a complex angular momentum and studies made more than a quarter of a century ago [28] never found any need for a worse singularity than a fixed pole at $j = 1$ in Compton-scattering amplitudes, with no singularity at all at that point in $F_2$.

The problem with this argument is that although it suggests that the singularity structure (4) is incorrect, it still doesn’t tell us precisely what or where the rightmost singularities are in the $j$-plane. Furthermore it is clearly not possible to deduce precisely what it is from the data: to do this we would need to do experiments of arbitrarily high precision at arbitrarily high energies. It is thus interesting to ask whether we can instead deduce it from perturbative QCD. To do this, we would at least need a sensible resummation of small $x$ logarithms. We now discuss the difficult problem of constructing such a resummation.

**QCD: Resummation of Logs of $x$ and Logs of $Q^2$**

Using the BFKL kernel it is possible [30] to deduce the coefficients of the LLx singularities of the splitting function to all orders in perturbation theory, ie of all terms in the anomalous dimension $\gamma(N)$ of the form $\alpha_s^n/N^n$, where $N = j - 1$. Summing up these singularities converts the sum of poles into a cut starting from $N = \lambda_0$, apparently confirming the Regge expectation about the behaviour at $j = 1$: it is this cut which at fixed coupling gives the power rise of the BFKL pomeron. This procedure may be extended beyond LLx [26,31,32]: the anomalous dimension $\gamma(\alpha_s, N)$ in a particular factorization scheme (such as MS) is related to a BFKL function $\chi(\alpha_s, M)$ through the ‘duality’ relation

$$\chi(\alpha_s, \gamma(\alpha_s, N)) = 1.$$  

(5)

Expanding this relation to NLLx, and using calculations of the coefficient function and gluon normalization [33] and of the NLLx kernel [18], we can compute the coefficients of all terms of the form $\alpha_s\alpha_s^n/N^n$ in the anomalous dimension. Such an approach has several advantages over the direct solution of the BFKL equation: there is a clean factorization of hard and soft processes, running coupling effects are properly taken care of by well formulated renormalization group arguments, and it is easy to arrange for a smooth matching to the large $x$ region.

However it was known some time ago that reconciling the summed logarithms with the HERA data was actually rather difficult [35]. Once all the NLLx corrections were known it became clearer why: the expansion in summed anomalous dimensions at LLx, NLLx, . . . is unstable [32,34], the
ratio of NLLx/LLx contributions growing rapidly as $\xi = \log 1/x \to \infty$. It follows that the previous theoretical estimates of the size of the effects of the small $x$ logarithms based on the fixed order BFKL equation, either at LLx or NLLx, were all hopelessly unreliable. Indeed any calculation which resums LO and NLO logs of $Q^2$, but sums up only LO and NLO logarithms of $x$ is seen to be insufficient: some sort of all order resummation of the small $x$ logarithms is always necessary. Clearly there are many ways in which such a resummation might be attempted: what is needed are guiding principles to keep it under control.

One such principle is momentum conservation [29]: before using $\chi(M)$ to compute the corrections to $\gamma(N)$ through the duality eqn.(5), we should first resum all the LO and NLO singularities at $M = 0$. 

Figure 4: (a) the BFKL function $\chi(M)$ and (b) the corresponding anomalous dimension $\gamma(N)$ in various approximation schemes [29].
discussed above, and impose the momentum conservation condition \(\gamma(\alpha_s, 1) = 0\), whence (from eqn.(5)) \(\chi(\alpha_s, 0) = 1\). Since these are collinear singularities, their coefficients may be determined from the usual LO and NLO anomalous dimensions, again using the duality relation eqn.(5), but this time in the reverse direction. It turns out that when the \(M = 0\) singularities are resummed they account for almost all of \(\chi\) in the region of \(M = 0\) (see figure 4a): this explains already why the remaining small \(x\) corrections have not yet been seen at HERA. Small \(x\) logarithms are simply numerically much less important than collinear logarithms.

The second principle is perturbative stability. The instability found at NLLx can be shown to follow inevitably from the shift in the value \(\lambda\) of \(\chi\) at the minimum due to subleading corrections [32]. This shifts the position of the singularity from \(N = \lambda_0\) to \(N = \lambda_0 + \Delta \lambda\), and this shift must be accounted for exactly if a sensible resummed perturbative expansion is to be obtained. Since in practice the correction \(\Delta \lambda\) is of the same order as the leading term \(\lambda_0\), it seems probable that \(\lambda = \lambda_0 + \Delta \lambda\) is not calculable in perturbation theory: rather the value of \(\lambda\) may be used to parameterise the uncertainty in the value of \(\chi\) in the vicinity of \(M = \frac{1}{2}\).

This uncertainty is clearly due to the unresummed infrared logarithms at \(M = 1\). In [36] an attempt is made to resum these singularities through a symmetrization of \(\chi\) about \(M = \frac{1}{2}\): \(\chi\) is then supposedly determined for all \(0 \leq M \leq 1\), and \(\lambda\) is given by the height of its minimum. The main shortcoming of this approach is that it makes implicit assumptions about the validity of perturbation theory when \(Q^2\) is very small.

Putting together the two principles of momentum conservation and perturbative stability, we can compute fully resummed NLO anomalous dimensions (see figure 4b). The result depends on the unknown parameter \(\lambda\). Provided \(\lambda \lesssim 0\), the corrections to Altarelli-Parisi evolution in the HERA region are tiny: for larger values they may be significant at low \(x\) and low \(Q^2\), and it might then be possible to determine \(\lambda\) from the data. It can be seen from the plot that the singularity structure at \(N = 0\) (and thus \(j = 1\)) is still completely undetermined: this is a reflection of the uncertainty in the \(\chi\) plot at \(M = 1\), which makes it not only unclear as to the value of \(\chi\) at its minimum, but even whether there is a minimum at all. To determine the position and nature of the rightmost singularities in the \(j\)-plane would presumably require control of \(\chi(M)\) at \(M = 1, 2, \ldots\), which is clearly beyond current perturbative technology.

It seems that to make further progress we require either genuine nonperturbative input, or a substantial extension of the perturbative domain. A possible way in which this might be done through a new factorization procedure was explored in [37], from which the main conclusion was that at small \(x\) the coupling should run not with \(Q^2\), but with \(W^2\). Preliminary calculations [38] suggest that this is not phenomenologically unacceptable. However much more work remains to be done.

Summary

At low \(Q^2\) but high \(W^2\) Regge theory works well and gives nontrivial and successful predictions. At high \(Q^2\) and small \(x\) NLO perturbative QCD works well and gives nontrivial and successful predictions, with quantifiable uncertainties due to the need for a controlled resummation of small \(x\) logarithms. In the same region, Regge theory can also fit data successfully, but without the predictive power of perturbative QCD. Neither Regge theory, nor conventional perturbative QCD, nor even the data, seem to be able to predict the precise form of cross sections in the Regge limit \(W^2 \to \infty\) with \(Q^2\) large. To do this, new ideas will probably be needed.
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