THE SPECTRUM OF THE MILLISECOND PULSAR J0218+4232 – THEORETICAL INTERPRETATIONS

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We interpret the unique high-energy spectrum of the millisecond pulsar PSR J0218+4232 within polar cap scenarios. We show that the spectral data from BeppoSAX and EGRET impose very restrictive limitations on possible radiation mechanisms, energy spectrum of radiating charges as well as viewing geometry. Theoretical spectra are able to reproduce the data, however, this can be achieved provided very special – unusual within the conventional polar cap picture – conditions are satisfied. Those include off-beam viewing geometry along with one of the following alternatives: 1) strong acceleration of secondary pairs; 2) broad energy distribution of primary electrons extending down to $10^5$ MeV; 3) high-altitude synchrotron emission.

1 Introduction

PSR J0218+4232, with the spin period $P = 2.3$ ms and the inferred dipolar magnetic field at polar cap $B_{pc} \approx 8.6 \times 10^8$ G, is the only millisecond pulsar which has been marginally detected above 100 MeV. A broad-band high-energy spectrum of this object exhibits unusual features, completely different from those observed among young gamma-ray pulsars: Above 100 MeV the photon index $\alpha_{ph} \sim -2.6$, and the spectrum resembles the very soft spectra of middle latitude unidentified EGRET sources. Within the BeppoSAX range the spectrum is extremely hard: $\alpha_{ph} \sim -0.61 \pm 0.32$.

Kuiper et al. noticed that neither the polar cap nor the outer gap models could naturally explain the spectrum. The aim of this work is to carefully examine an ability of polar cap models to reproduce the spectrum of J0218+4232.

2 Directional characteristics of pulsar spectra predicted by the polar cap model

As we show in the accompanying paper (Woźna et al., these proceedings) the viewing geometry effects have crucial significance for the appearance of a pulsar spectrum for a given observer. For purely dipolar magnetic field the geometry is determined by two angles: $\alpha$ – the angle of dipole inclination relative to the rotation axis, and $\zeta$ – the angle between an observer’s line of sight and the rotation axis. Various combinations of $\alpha$ and $\zeta$ result in a large variety of spectral shapes and pulse profiles in high-energy domain. However, a particular energetic history of electrons in pulsar magnetosphere enables to extract two main cases: the on-beam and the off-beam geometry.

Fig. 1a presents the Lorentz factor $\gamma$ of electrons injected at the surface of neutron star along the "last open" magnetic field lines as a function of angle $\theta_m$ between magnetic dipole axis and a
local tangent to magnetic field line at the electrons’ position. The value of \( \theta_m = 26.5^\circ \pm 1.5 \theta_{pc} \) corresponds to the direction of \( \vec{B} \) at the polar cap rim (\( \theta_{pc} \) is the angular radius of the polar cap, measured from the center of neutron star). The rotation period \( P = 2.3 \text{ ms} \) and the initial Lorentz factor \( \gamma_0 = 2 \times 10^7 \) were assumed in these calculations. The electron energy losses noticeable in Fig. 1a are purely due to the emission of curvature radiation\(^a\). The CR energy loss rate of an electron is initially huge, but because of its strong dependence on electron energy \( \dot{\gamma}_{cr} \propto \gamma^4/\rho_{cr}^2 \), \( \rho_{cr} \) is the local radius of curvature of magnetic field lines) it becomes negligible after the electron traverses a length which is small in comparison with the light cylinder radius \( R_{lc} \). Thus, after the initial rapid drop, the electron’s energy starts to decrease very slowly (Fig. 1a).

The characteristic energy \( \epsilon_{cr} \) of curvature photons emitted by the electron in different directions \( \theta_m \) is shown in Fig. 1b. Because \( \epsilon_{cr} \) is very sensitive to \( \gamma \) (\( \epsilon_{cr} \propto \gamma^7/\rho_{cr} \)), the high-energy cutoff in the CR spectrum decreases rapidly for increasing angles \( \theta_m \) but then it starts to approach "asymptotically" the value of \( \sim 30 \text{ MeV} \). This is in part due to the slower decrease in \( \gamma \) at higher altitudes and in part due to an increase in \( \rho_{cr} \) which starts to take place for \( \theta_m > 68^\circ \). In fact, Rudak & Dyks\(^b\) (RD99) estimated an "absolute minimum" of \( \gamma_{break} \approx 3.8 \times 10^6 (\rho_{pc}/(10^7 \text{ cm}))^{1/3} \) for the Lorentz factor of CR-cooled electrons (eq. (7) in RD99) and the absolute lower limit of \( \sim 150 \text{ MeV} \) for \( \epsilon_{cr} \) (eq. (8) in RD99). These values are in reasonable agreement with the exact results presented in Fig. 1, given the crude method used in RD99. We recall here that the lower limit for \( \epsilon_{cr} \) practically does not depend on any pulsar parameters (eg. exact calculations for rotation period \( P = 0.1 \text{ s} \) give \( \epsilon_{cr} \sim 20 \text{ MeV} \) at \( R_{lc} \)).

An important implication of Fig. 1b is that pulsar radiation pattern can be considered as consisting of two components: a hollow cone of very high-energy emission extending up to GeV range (with opening half-angle \( \theta_m \approx 1.5 \theta_{pc} \)) and a much less anisotropic emission peaking close to 30 MeV. When the line of sight crosses the hollow cone beam (on-beam geometry) the recorded spectrum consists of two components: a curvature component and a synchrotron component due to synchrotron radiation (SR) from secondary electron-positron pairs. The shape of this spectrum is similar to the total spectrum emitted in all directions, shown in fig. 1 of RD99. Its qualitative features can also be assessed from Fig. 1b: The spectrum extends

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\(^a\)Resonant inverse Compton scatterings cannot influence the electron’s energy due to small relative energy losses per scattering in a weak magnetic field (eg. Dyks, Rudak, Bulik\(^c\)) whereas non-resonant scatterings (they occur in the Klein-Nishina regime) are too rare for surface temperature values relevant for neutron stars.\(^b\)Resonant inverse Compton scatterings cannot influence the electron’s energy due to small relative energy losses per scattering in a weak magnetic field (eg. Dyks, Rudak, Bulik\(^c\)) whereas non-resonant scatterings (they occur in the Klein-Nishina regime) are too rare for surface temperature values relevant for neutron stars.\(^c\)Resonant inverse Compton scatterings cannot influence the electron’s energy due to small relative energy losses per scattering in a weak magnetic field (eg. Dyks, Rudak, Bulik\(^c\)) whereas non-resonant scatterings (they occur in the Klein-Nishina regime) are too rare for surface temperature values relevant for neutron stars.
up to GeV range and is relatively soft within the EGRET range because it is composed of instantaneous CR spectra with different values of $\epsilon_{cr}$ (Fig. 1b). The value of photon index $\alpha_{ph}$ in this spectral range depends on the viewing angle $\zeta$ as well as on the rotation period $P$. For millisecond periods it is close to $\alpha_{ph} \sim -5/3$. Within the entire X-ray range (0.1 keV – a few MeV) the on-beam spectrum is dominated by the synchrotron component with the well-known photon index $\alpha = 0.61 \pm 0.32$). Any SR emitted close to the surface would have a broken power-law shape with $\alpha_{ph} = -1.5$ within the X-ray range, in clear disagreement with the BeppoSAX data. The slope $\alpha_{ph} = -2/3$ of the CR component does not depend on the viewing angle $\zeta$ as long as the line of sight misses the SR component, (which is concentrated around $1.5 \theta_{pc}$), i.e. as long as $\zeta > \alpha + 1.5 \theta_{pc} + 1/\gamma_{\parallel} \approx 40^\circ$ (see Woźna et al., this volume).

Furthermore, owing to the off-beam geometry the high energy cutoff in the total spectrum (which now consists of just a CR component) occurs at a relatively low photon energy of $\sim 100$ MeV. The cutoff is not caused by magnetic absorption – it corresponds to a maximum energy of $\epsilon_{cr}$ at high altitudes – see Fig. 1b.

Interestingly, the shape of the spectrum would not change much even if the line of sight sampled much larger range of $\theta_{in}$ within the off-beam region (say between $40^\circ$ and $140^\circ$). This is because of the remarkable stability of $\epsilon_{cr}$ at high altitudes – see Fig. 1b.
those electrons which emit observable CR. This is why the cutoff’s shape can easily mimic the very soft spectral shape as suggested by the two EGRET points (Fig. 2). Harding & Zhang\textsuperscript{7} used similar viewing geometry arguments to explain the very soft spectra of unidentified EGRET sources.

However, with the off-beam CR spectrum normalized to reproduce the level of the EGRET emission, the BeppoSAX data points are located about 3 orders of magnitude above the extrapolated level of the CR component (thick line in Fig. 2). The intrinsically hard shape of the off-beam spectrum results in too low ratio of X-ray level to gamma-ray level of emission. In our opinion it is not possible to solve this problem within the standard polar cap model.

3 Can the secondary $e^\pm$ pairs emit CR photons within the keV range?

In order to explain the BeppoSAX data we propose a contribution of CR from secondary $e^\pm$ pairs accelerated within the polar gap. Upon the acceleration energy $\gamma_\parallel$ of the $e^\pm$ pairs should reach values in the range between $1.5 \times 10^5$ and $6 \times 10^5$. For $\gamma_\parallel < \gamma_{\text{break}} \sim 3 \times 10^6$ the CR energy loss length scale considerably exceeds the size of magnetosphere ($R_\text{c}$). The pairs do not suffer thus considerable energy losses and at all altitudes emit roughly the same spectrum of CR. In Fig. 2 it is shown as a thin solid line (we took $\gamma_\parallel = 3 \times 10^5$).

The level of CR spectrum at energies below the characteristic energy $\epsilon_\text{cr} \propto \gamma_\parallel^3/\rho_\text{cr}$ does not depend on $\gamma_\parallel$, but solely on the radius of curvature of magnetic field lines $\rho_\text{cr}$. Since the pairs follow nearly the same field lines as the primary electrons, the ratio between the level of BeppoSAX data points (diamonds in Fig. 2) and the level of X-ray emission from primary electrons (thick solid line) is a direct measure of the number of pairs $n_\pm$ per single primary electron required to reproduce the level of the X-ray data. The level of the primary CR spectrum (thick solid line) within the BeppoSAX range can be determined by fitting the high-energy cutoff of the spectrum to the EGRET data points. Apart from $n_\pm$, the same fit could also determine the value of $\zeta$, because the shape of the cutoff depends on the viewing angle\textsuperscript{1}, however, the EGRET data do not allow to determine $n_\pm$ and $\zeta$ with high accuracy. "By eye" fits give $n_\pm$ of at least a few hundred; for $\zeta = 48^\circ$ (Fig. 2), $n_\pm \sim 10^3$.

Thus, to reproduce the broad-band high-energy spectrum of J0218+4232, at least a few hundred of pairs should acquire an energy roughly equal to 1% of primary electron energy. According to most works on the physics of the pair formation front (eg. Harding & Muslimov\textsuperscript{8}) acceleration of such a large number of pairs within the polar gap is difficult, because redistribution of the pairs screens out the electric field of the polar gap. However, existing works on the physics of the polar gap neglect the fact that the $e^\pm$ pairs are created at different magnetic field lines than those along which parent primary electrons propagate. Because of this 1D-approach as well as plenty of other approximations our present knowledge of processes at the pair formation front is far from being well established.

Because of the bimodal energy distribution of radiating charges (primary electrons, secondary pairs), the CR spectral components which correspond to them are separated by a dip. A precise shape of this dip would depend on the shape of high-energy cutoff in the energy distribution of $e^\pm$ pairs and the dip may extend in principle between $\sim 10$ keV and 100 MeV (dashed line in Fig. 2 suggests a possible situation).

4 What is the energy spread among primary electrons?

One of popular assumptions present in polar cap models is that all primary electrons accelerated within the gap acquire roughly the same energy. This is not realistic because the potential drop

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\textsuperscript{1} Because of the near alignment of the magnetic dipole, the value of $\zeta$ cannot be well determined from radio polarisation data on J0218+4232 (Stairs et al\textsuperscript{13})
across the polar cap depends on magnetic colatitude: it decreases towards the rim of the polar cap (eg. [19]). Moreover, stochastic energy losses caused by resonant Compton scatterings may significantly soften the energy spectrum of primary electrons provided the surface temperature $T$ and the magnetic field $B$ are sufficiently high (see fig.4 in paper[1]). In the case of weak-$B$ millisecond pulsars the latter mechanism is not efficient because the energy loss rate due to the resonant ICS ($\propto B$) is low. However, no calculations of primary electron energy distribution have been performed so far for very high temperatures in excess of $10^7$ K – a value considered for J0218+4232[10].

Suppose that the primary electrons assume the steady-state energy distribution in a form of a power law $dN_\gamma/d\gamma \propto \gamma^\alpha$ extending down to $\gamma_{\text{min}} = a \text{ few } \times 10^5$. Then, the resulting CR spectrum could easily reproduce both the observed slopes and levels of X-ray and gamma-ray emission for appropriate values of $\alpha$ and $\gamma_{\text{min}}$. It would have a broken power law shape with $\alpha_\text{ph} = -2/3$ within the BeppoSAX range, and $\alpha_\text{ph} = (p - 1)/3$ above a break somewhere between $10$ keV and $100$ MeV (the exact value of the break energy $\epsilon_{\text{br}} = \epsilon_{\text{cr}}(\gamma_{\text{min}})$). Again, if primary electrons really achieved $\gamma$ in excess of $10^7$ in the polar gap of millisecond pulsars, the agreement of the model spectrum with EGRET data points could be easily achieved only by a proper choice of the viewing angle $\zeta$ for the off-beam geometry. There would be no dip in the spectrum and a measure of $\alpha_\text{ph} = \epsilon_{\text{br}} = \epsilon_{\text{cr}}(\gamma_{\text{min}})$ would give us direct information about $p$ and $\gamma_{\text{min}}$, respectively. BeppoSAX data points and COMPTEL upper limits constrain $\alpha_\text{ph}$ to the range between $-1.5$ and $-2.15$ which corresponds to rather soft electron energy distribution with the index $-3.5 > p > -5.5$ and with the low energy cutoff at $10^5 < \gamma_{\text{min}} < 2.5 \times 10^5$.

5 High-altitude synchrotron emission

In the case when the synchrotron radiation (SR) is emitted close to the neutron star surface, its spectrum extends down to a blue-shifted local cyclotron energy $\gamma\bar{h}\omega_B$ with $\alpha_\text{ph} = -1.5$ (RD99). For high-altitude emission, however, the cooling length scale due to SR can become longer than the length scale of decrease in $B$. Simple calculations show that this may happen for a local magnetic field $B < 2 \times 10^6$ G. For PSR J0218+4232 such a field is expected at $r > 0.5 R_\text{lc}$.

In the case of SR in such a low $B$, electrons do not lose their entire energy $\gamma \gamma \text{mc}^2$ corresponding to their motion across $B$. In consequence, a new break appears in the SR spectrum, at photon energy greater than $\gamma\bar{h}\omega_B$ (Chang et al.[1]). Below the break, the photon index assumes the value $\alpha_\text{ph} \simeq -2/3$, characteristic for instantaneous spectrum of SR (or CR) emission.

Fig. 3 shows that such a kind of spectrum is also able to reproduce the high-energy data on J0218+4232. Between the BeppoSAX and EGRET range the SR spectrum has a well known slope of $\alpha_\text{ph} = -1.5$, and its flux level is about 10 times below the upper limits from OSSE and COMPTEL. This SR spectrum was calculated within the following simplified model: The electrons were injected at a distance of $0.8R_\text{lc}$ from a neutron star, with initial energy $\gamma = 3 \times 10^7$ and were propagated radially up to $5R_\text{lc}$ with a constant value of $\gamma = 10^5$ (ie. constant pitch angle during the SR emission was assumed). Viewing geometry effects were not taken into account in these simple calculations but the lack of strong GeV emission from J0218+4232 again implies the off-beam case. The agreement between the data and the model spectrum of SR shown in Fig. 3 is remarkable, however, the emission of 100 MeV synchrotron photons close to the light cylinder requires very high energy of primary electrons near $R_\text{lc}$: ($\gamma_{\perp} > 10^2$, $\gamma_{\parallel} \sim 10^5$). Various mechanisms of acquiring $\gamma_{\perp} > 1$ in the outer parts of magnetosphere have been considered so far within the polar cap model (eg. Malov & Machabeli[1]) but they usually concerned the optical emission from pulsars.
6 Conclusions

The combined X-ray and gamma-ray data on the millisecond pulsar J0218+4232 impose severe limitations on possible radiation mechanisms, energy spectrum of radiating charges as well as viewing geometry. The standard polar cap model is not able to explain the spectrum of J0218+4232 even when viewing geometry effects are taken into account. We find three possible interpretations which require non-orthodox assumptions about the electron energy distribution or emission altitude. The spectra corresponding to these possibilities have unique features (the MeV dip or the characteristic slope of SR) which may enable to identify them with a high-sensitivity and high-angular resolution gamma-ray telescope (INTEGRAL?).

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