MACROSCOPIC STRING-LIKE SOLUTIONS IN MASSIVE SUPERGRAVITY

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ABSTRACT

In this report we obtain explicit string-like solutions of equations of motion of massive heterotic supergravity recently obtained by Bergshoeff, Roo and Eyras. We also find consistent string source which can be embedded in these backgrounds when space-time dimension is greater than or equal to six.

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Recently, there has been regenerated interest in the study of massive supergravity theories \[1, 2, 3, 4\] and the generalised Scherk and Schwarz dimensional reduction scheme \[5, 6\]. In generalised Scherk-Schwarz (GSS) toroidal reduction some of the fields are given linear dependence along the coordinates of the torus. Thus the resultant compactified theory in the lower dimensions possesses mass like parameters. However, when these parameters are set to zero the massive theory reduces to the ‘standard’ (massless) supergravity. Another point to be emphasized here is that the massive supergravities obtained through GSS reduction, in general, possess smaller duality symmetry groups than their massless counterparts \[1, 2\].

Fundamental string solution \[6\] was obtained in the spacetimes which are asymptotically Minkowskian. Later on other fundamental solutions (with source terms) as well as solitonic solutions (without source terms) were obtained for any p-brane in D spacetime dimensions \[7\]. For these solutions masses and their respective charges saturate the Bogomol’nyi-Prasad-Sommerfeld bound and therefore the supersymmetry in the theory dictates that these classical solutions are the quantum mechanically exact solutions of the theory. It is now natural to ask what does happen if the spacetime around a p-brane is not asymptotically flat or if there is nontrivial dilatonic potential in the theory. Such examples are provided when we consider massive supergravities, \textit{e.g.}, massive IIA supergravity in \(D = 10\) \[8\] and its subsequent dimensional reductions\[1, 4\], various GSS reductions of massless type II \[1, 2\] and recently of heterotic strings in ten dimensions \[3\]. All these massive theories have some kind of dilatonic potentials either in the NS-NS sector as is the case with massive heterotic of Bergshoeff, Roo and Eyras or in the R-R sector as was the case with massive type IIA of Romans \[8, 2\] or in both the sectors as was anticipated in \[4\] in order to obtain maximally symmetric black hole solutions analogous to \[3\].

In the present work, first, we shall obtain explicit string like solutions of source free string equations of motion in arbitrary spacetime dimensions with nontrivial dilatonic potential of the form suggested in the massive heterotic string \[3\]. In these solutions spacetime has explicit \(O(1, 1) \times O(D - 2)\) symmetry and does not have asymptotic flatness. Secondly, we shall show that a string like source could be consistently embedded in these spacetimes when \(D \geq 6\).
We consider the following effective action in D-dimensional target space,

\[
S = \int d^D x \sqrt{-g} e^{-2\Phi} \left[ R_g + 4 \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2 \cdot 3!} H_{\mu\nu\lambda} H^{\mu\nu\lambda} - 2m^2 \right]
\]  

(1)

where \( m^2 \) is the mass term (or cosmological constant) as in [3] or an analog of central charge deficit term, \( \Phi \) is the dilaton field and \( g_{\mu\nu} \) is the \( \sigma \)-model metric. We have taken \( 2\kappa^2 = 1 = 2\pi\alpha' \). The antisymmetric field strength is expressed as,

\[
H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + \text{cyclic permutations.}
\]  

(2)

Above action for \( D = 4 \) can be obtained from the appropriate truncation of the massive theory described in [3]. Since it would be convenient to work in the Einstein (canonical) frame let us write down the action in the canonical metric, \( g_{\mu\nu} = e^{2\phi} G_{\mu\nu} \)

\[
S = \int d^D x \sqrt{-G} \left[ R_G - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2 \cdot 3!} e^{-a\phi} H_{\mu\nu\lambda} H^{\mu\nu\lambda} - 2m^2 e^{\frac{4}{g} \phi} - 2m^2 \right]
\]  

(3)

where \( a = \sqrt{\frac{8}{D-2}} \) and rescaled dilaton is \( \Phi = \frac{\phi}{a} \). One can derive equations of motion from the action (3). We obtain the solutions to these equations in two cases; when \( B_{\mu\nu} \neq 0 \) and when \( B_{\mu\nu} = 0 \). The latter solutions we describe as the vacuum solution (pure dilaton gravity).

**Case-I** \( B_{\mu\nu} \neq 0 \)

We find that the equations of motion derived from (3) are satisfied for the following choice of the background fields

\[
\begin{align*}
ds^2 &= U^{\frac{D-3}{D-2}} (-dt^2 + dx^2) + U^{-\frac{2}{D-2}} (dy_1^2 + \cdots + dy_{D-2}^2), \\
\phi &= -\frac{a}{2} \ln U, \\
B &= \frac{1}{2} B_{\mu\nu} dx^\mu \wedge dx^n = -\frac{1}{U} dx^0 dx^1,
\end{align*}
\]

(4)

provided the potential has the following form

\[
U = 1 + \frac{m^2}{2} (t^2 - x^2) + \frac{Q_2}{|y - y_0|^{D-4}}, \quad \text{for } D > 4
\]

\[
= 1 + \frac{m^2}{2} (t^2 - x^2) - Q_2 \ln |y - y_0|, \quad D = 4
\]

(5)

where \( Q_2 \) is the charge associated with the 2-from gauge field \( B_{\mu\nu} \) and is given by

\[
Q_2 = \int_{S^{D-3}} e^{-a\phi} \ast H.
\]

(6)
The symbol \( \ast \) stands for Hodge dual operation and \( y_0 \) is some point in the transverse \( y \)-plane.

Obviously, the background solution obtained in (4) is similar to the spacetime of a fundamental string solution in [6]. Only difference is of an explicit \( m \)-dependent term in (5). In the limit \( m \to 0 \) background in (4) reduces to asymptotically flat spacetime around a fundamental string. There is a curvature singularity at \( y = y_0 \) which can be smoothened by introducing some string-like source at \( y = y_0 \).

**Case-II** \( B_{\mu\nu} = 0 \)

In this case we have a pure dilatonic gravity with the dilatonic potential. The equations of motion are still satisfied by the same ansatz as in (4) for the remaining fields while the potential in (5) becomes

\[
U = 1 + \frac{m^2}{2}(t^2 - x^2), \quad \text{for } D > 2.
\]

Thus we also have solution independent of the antisymmetric field strength. When this background is substituted into the action it becomes

\[
S = \int d^D x \left[ \frac{4m^2}{D - 2} \right],
\]

which shows there is finite lagrangian density \( \frac{4m^2}{D-2} \) per unit \( D \)-dimensional Minkowski volume.

We now introduce string-like source in the form of \( \sigma \)-model world-sheet action

\[
S_\sigma = -\frac{1}{2} \int d\tau d\sigma \left[ \sqrt{-\gamma} \gamma^{ij} \partial_i X^M \partial_j X^N G_{MN} e^{2\phi} + \epsilon^{ij} \partial_i X^M \partial_j X^N B_{MN} \right]
\]

where \( \gamma_{ij} \) is the induced metric on the string world-sheet. The \( \phi \)-dependence is chosen in accordance with [7] so that, when \( m \to 0 \), under the rescaling

\[
\begin{align*}
G_{MN} &\to \lambda^{\frac{4}{D-2}} G_{MN}, \quad B_{MN} \to \lambda^2 B_{MN} \\
e^\phi &\to \lambda^{\frac{4(D-4)}{(D-2)^2}} e^\phi, \quad \gamma_{ij} \to \lambda^2 \gamma_{ij}
\end{align*}
\]

both actions \( S \) and \( S_\sigma \) scale in the same way

\[
S \to \lambda^2 S, \quad S_\sigma \to \lambda^2 S_\sigma.
\]
When \( m \neq 0 \) such rescaling of the action does not hold, see [10]. Now, the above string like source can be embedded in the spacetime (4) if we make the following choice of static gauge for the world-sheet coordinates,

\[
X^0 = \tau, \quad X^1 = \sigma, \quad X^r = \text{constant}, \quad (r = 1, \ldots, D - 2),
\]

\[
\gamma_{ij} = \partial_i X^M \partial_j X^N G_{MN} e^{\frac{2\phi}{2}}.
\]

(12)

That is to say, eqs. (4) and (12) together satisfy all the field equations derived from actions \( S \) and \( S_\sigma \) if considered together at least for \( D \geq 6 \). The essential requirement for the embedding of the source (9) is that the string coupling \( e^\phi \) vanishes at \( y = y_0 \), i.e.

\[
U_{y \to y_0} \to \infty.
\]

(13)

For \( D < 6 \) above condition (13) is violated near the end points of the string, i.e., when \( x \to \pm \infty \). Therefore we have difficulty to find appropriate source when \( D < 6 \). However, if the end points are excluded, which is possible for temporal evolution (\( t > x \)), then the source action in (9) is a good choice even for \( D < 6 \). Thus in (9) we have got a background configuration in which a charged macroscopic string is embedded in a cosmological spacetime. These solutions have explicit \( O(1, 1) \times O(D - 2) \) symmetry. The symmetry gets automatically enhanced to \( P_2 \times O(D - 2) \) when \( m \to 0 \). Here \( P_2 \) stands for Poincaré symmetry in two dimensions. This constitutes our main result.

Next we shall try to find multi-string solution in asymptotically non-flat spacetime (4). First we calculate the net transverse force a test string is subjected to when another string is brought close to it. We still have

\[
\frac{d^2}{d\tau^2} X^m = 2\Gamma^m_{00} + H^m_{00} = 0,
\]

(14)

where \( \Gamma^\nu_{\mu\sigma} \) are the Christoffel connections in the canonical metric \( G_{\mu\nu} \). The result in eq. (14) suggests that net transverse force between two such strings vanishes. Thus it can be argued that a multi-string solutions could also be obtained in this configuration. For multi-string case the potential \( U \) in (5) modifies to

\[
U = 1 + \frac{m^2}{2} (t^2 - x^2) + \sum_n \frac{Q_2}{|y - y_n|^{D-4}}, \quad \text{for } D > 4
\]

\[
= 1 + \frac{m^2}{2} (t^2 - x^2) - Q_2 \sum_n \ln |y - y_n|, \quad D = 4
\]

(15)
where \( n \) is the number of strings fixed at the positions \( y_n \) in the \( y \)-plane. It is not easy to calculate ADM energy for these solutions. We do not know whether their masses and respective charges will saturate the BPS bound as they do in the case when \( m = 0 \). However, since we can control the value of the mass parameter \( m \) it appears to us that for \( m \sim 0 \) the solutions obtained above will only be slightly off extremal.

To analyse briefly the evolution and spacetime properties of the metric in (14) we consider specific case of \( D = 6 \) and of single string source positioned at \( y = 0 \). The metric is

\[
\begin{align*}
    ds^2 = U^{-\frac{1}{2}} \left[ \left( -dt^2 + dx^2 \right) + U \left( dy_1^2 + \cdots + dy_4^2 \right) \right],
\end{align*}
\]

(16)

with \( U = 1 + \frac{m^2}{2} \left( t^2 - x^2 \right) + \frac{Q^2}{2y^2} \). The spacetime is non-static and the evolution is dragged by a Lorentz invariant quantity involving the directions tangential to the string world-sheet. The evolution could be light-like if the quantity \( (t^2 - x^2) \) is greater than zero and it could be also space-like if \( (t^2 - x^2) \) is less than zero. The spacetime can be viewed as a solenoid whose axis is identified with the \( x \)-direction. At any given time the metric in (16) is not well defined in whole space since \( U \) becomes negative whenever \( 1 + \frac{m^2}{2} \left( t^2 - x^2 \right) + \frac{Q^2}{2y^2} < \frac{m^2}{2} x^2 / 2 \). However, a light signal travelling in \( x \)-direction will not see any discontinuity and only will detect the geometry around a fundamental string straightened at \( y = 0 \). For a possible \( m^2 < 0 \) (anti-de-Sitter) case, situation is more interesting. There is a critical time \( t_c = \sqrt{\frac{x}{|m^2|}} \) such that for \( t < t_c \) the metric is well defined in the whole space except at \( y = 0 \) which is the position of the string source. As \( |m^2| \rightarrow 0 \) we can see that \( t_c \rightarrow \infty \). This implies that for infinitesimally small values of the cosmological constant, or for a slow rate of evolution, it will take infinitely longer time for any irregularities to set in in the space. But \( m^2 \) cannot become negative if it is the mass term coming from GSS reduction \( \mathbb{R} \). Certainly there must be some other source for negative cosmological constant in the NS-NS sector if that has to happen.

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