Understanding the physical properties of disordered elastic systems is a challenging question relevant to a host of experimental situations. Indeed such a situation is realized in many different systems, ranging from periodic ones, such as vortex lattices [1, 2, 3], charge density waves [4], Wigner crystals [5], to interfaces, such as magnetic [6, 7, 8] or ferroelectric [9] domain walls, fluid invasion in porous media [10] and domain growth [11]. The competition between disorder and elasticity in these systems leads to unique physical properties and in particular to glassy behavior. One particularly important question is the response of the system to an external force (magnetic or electric field for domain walls, current for vortices, etc.). At zero temperature, due to disorder the system is pinned and the velocity of the elastic structure remains zero up to a critical force \( F_c \). At finite temperature however, the barriers to motion due to pinning can always be passed by thermal activation an one expects thus a finite response at finite force. Although it was initially believed that the response was linear [12], it was subsequently proposed [13, 14] that due to the glassy nature of the disordered system, no linear response would exists. The slow dynamics of the disordered system, no linear response would exist.

Going beyond the simple scaling arguments or checking for such a law has proved to be very challenging. Although sub-linear response was clearly seen in various systems [1], with good agreement with [10], a precise determination of the exponent was clearly more difficult. Relation [2] has been confirmed experimentally only for magnetic domain walls (see also [17] for vortices). On the theoretical side the phenomenological predictions of [12] have been derived by a functional renormalization group calculation [18, 19], starting directly from the equation of motion, and valid in an \( \epsilon = 4 - D \) expansion. This calculation confirmed the phenomenological hypothesis made in the scaling derivation and the validity of [2] up to the lowest order in \( \epsilon \). Although the velocity found in the FRG calculation was identical to the one of the scaling derivation, important differences were also found, notably on the characteristic sizes involved in the motion.

In spite of these results, the physical picture of creep motion is still very phenomenological and many important questions remain open. A systematic study of the temperature (or disorder strength) dependence of the creep response is particularly lacking, both from experiments and theory. Moreover, from the theoretical point of view, the experimentally very relevant case of low dimensional interfaces (where thermal effects are expected to be very important), like elastic lines describing domain walls in thin films, poses a difficult problem to tackle analytically, since the FRG [18, 19] can hardly be used in \( D = 1 \). Such studies are quite crucial given the recent experimental results on creep in magnetic [6, 7, 8] and ferroelectric [9] systems. Numerical simulations are a valuable alternative theoretical tool to address this open issue. In this respect, creep simulations of elastic strings (\( D = 1 \)) have been done in the past [21, 22], but given the limited range of velocities available, they were neither systematic nor conclusive about the validity of [12].

In this work we used a Langevin dynamics method to study the Velocity-Force (V-F) characteristics and the dynamic roughness \( \zeta \) of an elastic string in a random potential. The range of velocities we can explore allows a precise check of the creep law. Although we find that

**Creep motion of an elastic string in a random potential**

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We study the creep motion of an elastic string in a two dimensional pinning landscape by Langevin dynamics simulations. We find that the Velocity-Force characteristics are well described by the creep formula predicted from phenomenological scaling arguments. We analyze the creep exponent \( \mu \), and the roughness exponent \( \zeta \). Two regimes are identified: when the temperature is larger than the strength of the disorder we find \( \mu \approx 1/4 \) and \( \zeta \approx 2/3 \), in agreement with the quasi-equilibrium-nucleation picture of creep motion; on the contrary, lowering enough the temperature, the values of \( \mu \) and \( \zeta \) increase showing a strong violation of the latter picture.

The slow dynamics of the system for \( \mu \ll F \ll F_c \) is an energy scale, and 

\[ V(F) \sim \exp \left[ - \frac{U_c}{T} \left( \frac{F}{F_c} \right)^\mu \right] \]  

(1)

where \( U_c \) is an energy scale, and \( \mu \) a characteristic exponent that can be obtained from the relation,

\[ \mu = \frac{D - 2 + 2\zeta}{2 - \zeta} \]  

(2)

where \( D \) is the dimensionality of the elastic system, and the exponent \( \zeta \) is the *equilibrium* roughness exponent of the static system. The above formulas are indeed derived under the assumption that the movement is so slow that static properties can be used. Relation [2] is remarkable since it links the statics with the nonlinear transport of a disordered elastic system.
the creep law does describe well the data, we also find that the equilibrium hypothesis for \( \zeta \) is not verified. This leads at low temperatures or strong disorder to creep and roughness exponents that become larger than the predicted values \( \mu = 1/4, \zeta = 2/3 \) respectively.

We study the creep motion of an elastic string in two dimensions driven through a random potential. The string is described by a single valued function \( u(z, t) \), which measures its transverse displacement \( u \) from the \( z \) axis at a given time \( t \). We therefore exclude overhangs and pinched-off loops that eventually could be produced in the motion of domain walls. Assuming a linear short-range elasticity and a purely relaxational dynamics the phenomenological Langevin equation describing the motion (per unit length) is given by,

\[
\gamma \partial_t u(z, t) = c \partial^2_z u(z, t) + F_p(u, z) + F + \eta(z, t) \tag{3}
\]

where \( \gamma \) is the friction coefficient, \( c \) is the elastic constant, \( F \) is the driving force, and \( F_p(u, z) = -\partial_z U(u, z) \) is the pinning force derived from the disordered pinning potential \( U(u, z) \). The stochastic force \( \eta(z, t) \) ensures a proper thermal equilibrium and satisfies \( \langle \eta(z, t) \rangle = 0 \), \( \langle \eta(z, t)\eta(z', t') \rangle = 2\gamma T \delta(z - z')\delta(t - t') \), with \( \langle \ldots \rangle \) denoting thermal average. The sample to sample fluctuations of the random potential are given by,

\[
|U(u, z) - U(u', z)|^2 = -2\delta(z - z') R(u - u') \tag{4}
\]

where the over-line denotes average over disorder realizations. In this work we consider a random-bond type of disorder, characterized by a short-ranged correlator \( R(u) \), of range \( r_f \) and strength \( R(0) \). A physical realization of this kind of disorder is for instance the random anisotropy for magnetic domain walls [6].

To perform numerical simulations of equation (3) we discretize the string along the \( z \) direction, \( z \to j = 0, \ldots, L - 1 \), keeping \( u_j(t) \) as a continuous variable. A second order stochastic Runge-Kutta method 22 [23, 24, 25] is used to integrate the resulting equations. To model a continuous random potential satisfying (1), we generate, for each \( j \), a cubic spline \( U(u_j, j) \) passing through \( M \) regularly spaced uncorrelated Gaussian random points, with zero mean and variance \( R(0)^2 \) 26. Moreover the random potential satisfies periodic boundary conditions,

\[
U(u_j + M, j) = U(u_j, i + L) = U(u_j, j) \tag{5}
\]

this defines a finite sample of size \( (L, M) \).

We are interested in the V-F characteristics and the dynamic roughness exponent \( \zeta \) in the creep regime, for different values of \( T \) and disorder strength \( R(0) \). The average center of mass velocity \( V \) is defined as,

\[
V(F) = \left\langle \frac{1}{L} \sum_{j=0}^{L-1} \frac{d}{dt} u_j(t) \right\rangle. \tag{6}
\]

The roughness exponent \( \zeta \) is obtained from the average structure factor, defined as,

\[
S(q) = \left\langle \left| \frac{1}{L} \sum_{j=0}^{L-1} u_j(t) e^{-iq_0 j} \right|^2 \right\rangle \tag{7}
\]

where \( q = 2\pi n/L \), with \( n = 1, \ldots, L - 1 \). From dimensional analysis we know that for small \( q \), \( S(q) \sim q^{-(1+2\zeta)} \) 27. Fitting our numerical data with this function we extract \( \zeta \).

We simulate systems of sizes \( L = 64, 128, \ldots, 1024 \) and \( M = 2L \), with \( c = r_f = \gamma = 1 \). In this paper we show the results for \( L = 512 \), where finite size effects are negligible. We take \( R(0) = 0.12, 0.30 \) and temperatures ranging from \( T = 0.8R(0) \) to \( T = 3.5R(0) \). We start each simulation with a flat initial configuration \( u = 0 \) at the force \( F = F_c/10 \), and then decrease slowly \( F \) in steps of \( \Delta F = 0.01F_c \) up to \( F/F_c \approx 0.01 \). \( F_c \) is calculated to high precision for each disorder realization using a fast-convergent algorithm 26. The properties of interest have to be calculated when the stationary state is achieved. In practice we let the string complete two turns around the system for the initial force and one turn for the following forces. After this equilibration we estimate numerically 6 and 17 approximating the average over disorder and thermal realizations by a single time average over one turn.

Typical V-F characteristics obtained in the simulations are shown in Fig. 4. In the whole range of temperature and pinning strength analyzed we find that the V-F curve can be well fitted by the creep formula 11 with \( U_c \) and \( \mu \) as fitting parameters. We thus confirm the predicted
stretched exponential behavior, being the range of velocities in our simulations sufficient to rule out other proposed forms \[12, 28\]. However, contrarily to the naive creep relation \(2\) we find that both fitting parameters, and not only \(U_c\) as predicted in Refs. \(20, 26\), can depend on temperature.

Analysis of various values of disorder and temperature show essentially two different regimes of creep motion. To investigate further these regimes we show in Fig. 2(a) the V-F characteristics for two values of disorder and temperature representative of each regime. For the small disorder case we get the exponent \(\mu = 0.26 \pm 0.01\) which is compatible with the predicted theoretical value \(\mu = 1/4\), obtained from \(2\) using the equilibrium roughening exponent \(\zeta_e = 2/3\) \(30, 31\). The situation is quite different for the strong disorder case, where although the fit with the creep formula \(14\) is still excellent, the value of the exponent \(\mu \approx 0.36\) is now clearly in excess with respect to the predicted theoretical value.

To understand in more detail the nature of the two regimes we calculate the roughness exponent \(\zeta\) using the structure factor \(7\). Quite generally, one can predict that the short distance behavior of an elastic string is dominated by thermal fluctuations \((\zeta_T = 1/2)\). On the other hand, because of the finite velocity, the quenched disorder acts effectively as a thermal noise at the largest length scale. Thus, in this case, the expected exponent is also \(\zeta_V = 1/2\) \(32\). Finally, at intermediate length scales, the physics is determined by the competition between disorder and elasticity. In particular, in our simulations we verified that the Larkin length \(32\) is negligible. Therefore, a random manifold scaling, characterized by a non trivial roughness exponent, takes place. In Fig. 2(b) and (c) we show the structure factor for the two cases analyzed in Fig. 2(a). As predicted, we get \(\zeta \sim \zeta_T = 1/2\) for large \(q\). At a certain scale we observe a crossover between the thermal and the random manifold scaling. The location of this crossover decreases as \(T (R(0))\) is increased (decreased). We can also observe that the second velocity-controlled crossover is not achieved in our finite-size simulation due to the very slow dynamics. Interestingly, for the small disorder case, the random manifold scaling gives \(\zeta \approx 0.67\), in excellent agreement with the equilibrium value \(\zeta_e = 2/3\), while a much higher roughness exponent \(\zeta \approx 0.89\) is found for the strong disorder case. These results are consistent with the previous ones for the creep exponent \(\mu\). This conclusion holds for the whole range of temperature and disorder strength analyzed, as we can see in Fig. 3(a). We find that the relevant parameter to define the two regimes is \(T/R(0)\). It would be interesting to determine whether such a type of scaling has a theoretical justification. Moreover, we notice that although the values of \(\zeta\) and \(\mu\) depart from the equilibrium values, the relation \(2\) seems still to hold, within the error bars for the two exponents. This is highly non-trivial since equation \(2\) is derived from a calculation of the barriers in an equilibrium situation.

We discuss finally the temperature dependence of the barriers \(U_c(T)\), shown in Fig. 3(b). We remark that the observed linear temperature dependence is of course peculiar to the one dimensional wall, where thermal fluctuations lead to unbounded displacements, contrarily to what happens in higher dimensions \(21, 26\).

In conclusion we have find two regimes of creep motion. The first one occurs when the temperature is larger than the strength of the disorder, giving \(\mu \sim 1/4\) and \(\zeta \sim 2/3\) as predicted by assuming a quasi-equilibrium nucleation picture of the creep motion. This implies that the domain wall has time to re-equilibrate between hops, being the underlying assumption behind \(2\) essentially satisfied. The second regime occurs for temperatures smaller than the strength of the disorder, and is characterized by anomalously large values of both exponents. This clearly shows that in this regime the domain wall stays out of
FIG. 3: (a) Roughness exponent, $\zeta(T)$, and creep exponent, $\mu(T)$, vs $T$. $\zeta(T)$ (+ symbols), $\mu(T)$ (□ symbols) correspond to $R(0) = 0.30$ and $\zeta(T)$ (△ symbols), $\mu(T)$ (○ symbols) correspond to $R(0) = 0.12$. The dashed line gives the equilibrium roughness exponent $\zeta_{eq} = 2/3$, and the dotted line the purely thermal roughness $\zeta_T = 1/2$. The dashed-dotted line corresponds to the creep exponent predicted from phenomenological scaling arguments. (b) Effective energy barriers $U_c(T)$ vs $T$, for $R(0) = 0.30$ (○ symbols) and $R(0) = 0.12$ (□ symbols).

equilibrium, and that the naive creep hypothesis does not apply. Note that the measured roughness exponent is intermediate between the equilibrium value and the depinning value $\zeta_{dep} \approx 1.2$. The fact that the thermal nucleation which is the limiting process in the creep velocity, is in fact followed by depinning like avalanches was noted in the FRG study of the creep [11]. Whether such avalanches and the time it would take them to relax to equilibrium is at the root of the observed increase of the exponent, is clearly an interesting but quite complicated open question.

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