Analytical model for calculation the radon-protective characteristics of underground walling

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Abstract. Radon and its progeny in the indoor air form the biggest part of the human annual individual dose obtained from all sources of ionizing radiation. The greatest amount of radon enters into the building from the soil base, so the building degree of radon safety is determined by the underground walling shape. Mathematical modeling is the most effective tool used for the description of a multifactor process of radon entry from soil air into the buildings. The dominant mechanism of radon transportation into the soil and walling materials is the basis of any model of the radon situation formation in the building. Diffusive and advective (convective) transport mechanisms can dominate or make a significant joint contribution to the radon transport in certain conditions of the "ground-atmosphere-building" media system. Insufficient information about the magnitude and action direction of the large number of radon transport factors make it possible to simulate the radon entry under the limited conditions of the one transport mechanism dominance. The article deals with the approach to the assessment of the required radon-protective properties of underground enclosing structures under the assumption of stationary diffusion transfer of radon in porous media.

1. Introduction
Throughout life a human being is exposed to the effects of ionizing radiation of a different nature. The greatest danger to health is caused by the exposure to radon and its progeny in buildings [1], which forms 55 to 90% of the annual individual dose. Radon exposure significantly increases the risk of lung cancer, the children under the age of ten and smokers are most likely to be adversely affected. For this reason considerable resources are aimed at identifying areas with high radon concentrations in the indoor air and minimizing radon entry from soils in Scandinavia, Western and Central Europe and America [2-4].

Large-scale monitoring studies of radon hazard in residential and office buildings have been carried out in most regions of the Russian Federation in the last two decades [5-7]. In order to limit exposure of the population to radon in buildings Radiation Safety Standards set control levels for the equivalent equilibrium radon concentration (EERC) in the indoor air: 200 Bq·m⁻³ for existing buildings and 100 Bq·m⁻³ for the new ones.

The following processes determine the radon entry into the indoor air of the lower floor apartments: radon generation in the soil; radon transport to the underground walling; radon transport into the building through the walling and driving forces presence for this transport of [8-9]. Radon entry is a complex problem due to the multifactorial side of these processes. Different orientation and simultaneity of a significant number of different nature factors actions do not allow us to limit
ourselves to “in-situ” studies, since their results can be misinterpreted or hidden under a large amount of experimental data. In such conditions mathematical modeling remains the only effective means of obtaining reliable information about radon entry into the buildings [10]. The current level of knowledge is still insufficient to create a universal model of the radon situation formation in buildings with different floor construction on the different soils types. As a result, now it is much more reasonable to develop and construct a model adequate to the limited states of the "soil-atmosphere-building" media system. The main problem is to select and justify the dominant mechanism of radon transport in a porous media (soil base and underground walling materials) [11].

The soil under the building provides up to 90% of the radon entry [12], so the decisive influence on the building radon safety level has the underground walling state. In the vast majority of cases, compliance with the established control levels of EERC can be achieved by rational underground walling design. It should be borne in mind that the vertical underground walling practically do not prevent the discharge of soil radon into the atmosphere, therefore radon comes through horizontal underground walling only [13].

2. Materials and methods
The driving force of the convective radon entry into the buildings is the indoor/outdoor air pressure difference (~ 2 ... 4 Pa), caused by the temperature differences in the indoor air and in the soil one [6]. In this case, the soil gas flow can be found from Darcy’s law:

\[ q = -(k / \mu) \nabla P, \]

where \( k \) is the porous media air permeability, \( m^2 \); \( \mu \) is the soil air dynamic viscosity \( Pa \cdot s^{-1} \); \( P \) is the pressure, \( Pa \).

The diffusive transportation is caused by the radon concentration differences in the soil air and in the indoor air of the ground floor, amounts to tens-hundreds of thousands of Bq \( m^{-3} \). This transportation mechanism is constantly in operation, as radioactive decay and radon removal into the building is compensated by its generation in the soil. Fick's law is used to describe the diffusive radon transport

\[ \partial A / \partial t = \nabla D \nabla A - \lambda A + W, \]

where \( A \) – radon concentration in soil air, Bq \( m^{-3} \); \( D \) – binary diffusion coefficient, \( m^2 \cdot s^{-1} \); \( \lambda \) – radon decay constant, \( s^{-1} \); \( W \) – radon generation rate in the media, Bq \( m^{-3} \cdot s^{-1} \).

Modern buildings have the low air permeability of the floor construction (below \( 10^{-14} m^2 \)) therefore we consider it advisable to adopt a hypothesis on the dominance of the diffusive radon transport mechanism in the soil and walling materials [14]. To describe the radon situation in the "soil-atmosphere-building" media system one can use the system of differential equations for the two-dimensional stationary diffusive radon transport (Fig. 1)

\[ D_1 \left( \partial^2 A_1 / \partial x^2 + \partial^2 A_1 / \partial y^2 \right) - \lambda \cdot A_1 (x,y) + W_i = 0, \quad i = 0, 1, 2 \]

where \( D_0, D_1 \) and \( D_2 \) are the radon diffusion coefficients in air, construction material of and the ground, respectively, \( m^2 \cdot s^{-1} \); \( A_0, A_1 \) and \( A_2 \) are the radon concentrations in air, construction material and soil air, respectively, Bq \( m^{-3} \); \( W_0 \), \( W_1 \) and \( W_2 \) are the radon generation rate in air, construction material and soil air, respectively, Bq \( m^{-3} \cdot s^{-1} \).

The boundary conditions for the corresponding regions have the form:

- for \( G_{01} \) \[ \partial A_{01} / \partial x = 0, \quad \text{at } x = 0, x = d, h_2 \leq y \leq h_1; \]
- for \( G_{11} \) \[ \partial A_{11} / \partial x = 0, \quad \text{at } x = 0, x = d, h \leq y \leq h_2; \]
- for \( G_{12} \) \[ \partial A_{12} / \partial x = 0, \quad \text{at } x = d, x = H_o, h \leq y \leq 0; \]
- for \( G_{22} \) \[ \partial A_{22} / \partial x = 0, \quad \text{at } x = 0, x = H_o, h_2 \leq y \leq h; \]
- for \( G_{01} \) \[ \partial A_{01} / \partial y = 0, \quad \text{at } y = h_1, 0 \leq x \leq d; \]
- for \( G_{22} \) \[ \partial A_{22} / \partial y = 0, \quad \text{at } y = H_o, 0 \leq x \leq H_1; \]
- for \( G_{12} \) \[ \partial A_{12} / \partial y = \alpha A_{12}(x,y), \quad \text{at } y = 0, d \leq x \leq H_1; \]
The internal boundary conditions (at the boundaries of the adjacent regions of the system) for equations (3) are the following:

\[ A_{01}(x, h_2) = A_{11}(x, h_2), \quad D_0(\partial A_{01}(x, h_2)/\partial y) = D_1(\partial A_{11}(x, h_2)/\partial y), \quad \text{at } 0 \leq x \leq d, \quad (11) \]

\[ A_{11}(x, h) = A_{22}(x, h), \quad D_1(\partial A_{11}(x, h)/\partial y) = D_2(\partial A_{22}(x, h)/\partial y), \quad \text{at } 0 \leq x \leq d, \quad (12) \]

\[ A_{12}(x, h) = A_{22}(x, h), \quad D_2(\partial A_{12}(x, h)/\partial y) = D_2(\partial A_{22}(x, h)/\partial y), \quad \text{at } d \leq x \leq H_x. \quad (13) \]

The method of variables separation is applied for each equation of the system (3) with the corresponding boundary conditions. The solutions \( A(x, y) \) are sought in the form

\[ A(x, y) = \frac{G_s}{1} \left( 1 - N_0 e^{-\sqrt{\frac{\pi}{\mu x}}} \right) \sum_{n=1}^{\infty} \left( p_n(s) \cos u_n(s) (x - a_1(s)) R_n e^{-\nu_n(s)(y - b_1(s))} + e^{-\nu_n(s)(y - b_1(s))} \right) + q_n(s) \cos u_n(s) (y - a_2(s)) \cosh(\nu_n(s)(x - b_2(s))), \quad (14) \]

where \( p_n(s), q_n(s), n = 0, 1, 2, \ldots; s = 0, 1, 2 \) are the undetermined coefficients.

These coefficients are found by means of replacing the series in (14) by finite sums and solving the linear equations system (3), which are obtained after equating the values of \( A(x, y) \) and its derivative on the common boundaries of adjacent regions. The \( A(x, y) \) and other quantities values are calculated using a software package MAPLE.

3. Results

The proposed model allows:

1. To evaluate the radon load \( N \) on the underground horizontal walling of the building and depending on the soil base physical characteristics (Fig. 2).
The soil radon potential is the equilibrium radon concentration per unit of the soil block volume without activity gradients at its boundaries, Bq·m\(^{-3}\). Its value is reached at a depth of 5-10 m and is determined by the formula

\[ P_{Rn} = C_{Ra} \cdot \rho \cdot k, \]  

(15)

where \( C_{Ra} \) is the radium-226 activity, Bq·kg\(^{-1}\); \( \rho \) is soil density, kg·m\(^{-3}\); \( k \) is radon emanation coefficient.

2. To evaluate the radon load \( N \) on the underground horizontal walling depending on the building geometric characteristics. Figure 3 (a) shows the radon load on the underground walling increases with the building width, which can be explained by the radon outflow decrease from the building towards the more permeable open ground. The radon load increases nonlinearly with the building depth increases and approaching to the soil radon potential (Figure 3 (b)).

3. To estimate the radon concentration distribution in the soil under the building. Figure 4 shows the radon concentration isolines in the ground, which is determined at a diffusion coefficient \( D_2 = 5 \cdot 10^{-8} \) m\(^2\)·s\(^{-1}\) for the building depths 3 and 6 m, respectively. The lower line represents the soil radon potential which is equal to 45,000 Bq·m\(^{-3}\) in this case. The next isolines values decrease toward the soil in increments of 1,000 Bq·m\(^{-3}\).

Figure 3. Dependence of radon load on the building walling from the building width (a) and from the building depth (b)

Figure 4 shows that the building significantly changes the natural radon distribution in the soil (the contours are parallel to the ground surface). Because the building prevents the radon discharge into the atmosphere, the higher radon concentration values are in the contact plane of the construction with the soil than a few meters outside the building where the undisturbed concentration field. Moreover, the distortion of the concentration field increases with the embedding of the building.

Figure 4. Radon concentration isolines in the soil at the building depth 3 m (a) and 6 m (b)
4. Conclusions
The prospects of our further researches are in expanding the adequacy area of the proposed model with a view to describe the radon entry into the different types of the buildings. It is also important to investigate in detail the behavior of all the main factors of the indoor radon situation on a basis of a comparison of model calculations and the “in situ” results of radon transport process studies in the special laboratory experiment.

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