On Validating Attack Trees with Attack Effects: An Approach from Barwise-Seligman’s Channel Theory*

Hideaki Nishihara
SEI-AIST CyberSecurity Cooperative Research Lab.
National Institute of Advanced Industrial Science and Technology (AIST), Osaka, Japan
h.nishihara@aist.go.jp

Yasuyuki Kawanishi†
Cyber-Security R&D Office
Sumitomo Electric Industries, Ltd., Osaka, Japan
kawanishi-yasuyuki@sei.co.jp

Daisuke Souma†
Cyber-Security R&D Office
Sumitomo Electric Industries, Ltd., Osaka, Japan
souma-daisuke@sei.co.jp

Hirotaka Yoshida
SEI-AIST CyberSecurity Cooperative Research Lab.
National Institute of Advanced Industrial Science and Technology (AIST), Osaka, Japan
hirotaka.yoshida@aist.go.jp

Abstract. In security analysis, attack trees are a major tool for showing the structural decomposition of attacks and for supporting the evaluation of the quantitative properties (called attributes) of the attacks. However, the validities of decompositions are not established by attack trees themselves, and fallacious decisions about security may be made when the attack trees are inaccurate. This paper enriches attack trees with effects of attacks, with a formal system focusing on refinement scenarios. Relationships among effects indicate relationships among attacks and it allows for a systematic evaluation of attack decompositions. To describe effects this paper applies Barwise-Seligman’s channel theory. Infomorphisms, in particular, play a significant role to connect effects with distinct granularities. As a result, the consistency of a decomposition is formally defined and a condition for it is stated. This framework is applied to a case study of a vehicular network system. As an application of the idea of consistency, possible degrees of mitigation for attacks in attack trees are discussed.

Keywords: system security, threat modeling, refinement, mitigation, automotive security
1. Introduction

1.1. Background

Progress in information technology has led to the evolution of various systems worldwide. In particular, cyber-physical systems now have more flexible and finer functionalities, and cooperate with other systems via networks. However, security threats to these systems have also increased, and protecting against them has become an important issue recently.

Attack trees are a major tool in analyzing the security of a system [2–5], as they represent the decomposition of threats in the form of \textit{AND}/\textit{OR}-trees, alongside fault trees that represent the structures of faults in a system in the safety domain. As shown in the following example and Section 2, attack trees match formal approaches in that they enable us to understand a threat's logical structures and estimate its qualitative/quantitative properties.

Fig. 1 is an example of an attack tree representing the threat “authentication information in Infotainment module ([Aui] for short) is stolen” to a vehicle systems (see also Section 4.2). If the threat is realized, then a malicious user can connect their smartphone to Infotainment and control it or access critical subsystems in the vehicle. In the diagram, the threat to be analyzed is placed at the top node (the root) of the tree, and is decomposed into sub-attacks recursively. The branch just below the root node is an \textit{OR} branch. When at least one child of an \textit{OR} branch is done successfully, the threat or attack of the parent node is considered to be done. The branch below node A1 is a \textit{SAND} branch. In order for the attack of the parent node of the branch to be considered successful, all of the child nodes must be done in order from left to right. We draw a small arrow around a \textit{SAND} branch. Further, we can use \textit{AND} branches in attack trees. As with \textit{SAND} branches, all of the child nodes of an \textit{AND} branch are required for the attack of the parent node to be successful, but the order of their execution does not matter.

Address for correspondence: SEI-AIST CyberSecurity Cooperative Research Lab., National Institute of Advanced Industrial Science and Technology (AIST), Osaka, Japan.

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\† Also affiliated at SEI-AIST CyberSecurity Cooperative Research Lab. in AIST.
Fig. 1 shows three attack scenarios to achieve the threat on the top: reverse engineering of Infotainment module (A1), brute-force attack (A2), and eavesdropping (A3). Scenario A1 is concretized to the sequence of three actions: preparing the device that contains Infotainment module (A1.1), analysis of the module (A1.2), and identifying [AuI] (A1.3). Scenario A3 is concretized to the action sequence as well. When some attributes are assigned to each node at the bottom (the leaf), we can inductively estimate the attributes of the upper nodes, especially the root node. For example, let us consider the possibility of an attack. If the actions labeled A1.1, A1.2, A1.3, A3.1, and A3.3 are possible, then the intermediate action A1 is possible naturally, while actions A2 and A3 are not. This implies the action of the root A1 is possible as the result.

Simple and intuitive descriptions of attack trees allow various extensions of the concepts (see also Section 2.1). Examples include adding other types of nodes, connecting trees expressing defenses, and specifying the maximum number of children of a node.

Fortunately, attack trees are well formulated in research [6–9]; formal syntax and semantics are provided, the quantitative attributes are related to attacks formally, or attacks are linked to state transitions of the target system.

1.2. Problem and Result

Not all aspects of attack trees are supported by previous formulations. In particular, the relations among attacks around a branch have not been discussed thoroughly. In the literature on attack trees, one branch represents a concretization of the parent, another branch shows the preceding events for the parent, and these ideas are mingled together at the other branch. The resulting attack trees tend to be diverse, which poses a problem from a practical viewpoint. Although several studies [5, 9, 10] have addressed to this issue, the frameworks they proposed are rather indirect. Therefore, methodologies need to be developed for considering the consistency of attack trees simply.

This paper tackles this problem by introducing the effects of attacks. An effect, considered a post-condition of an event, is tightly related to an attack, and therefore we can use the concept of effects to discuss the relationships among attacks around a branch. Furthermore, when one attack depends on another one, they are related with effects; that is, the effect of the preceding attack functions as a pre-condition of the subsequent attack. Accordingly, the decomposition of an attack must correctly depict the relationships among effects, and equivalently relationship violations among effects reveal an inadequate decomposition of the attack. Establishing this idea is the primary goal of this paper.

We apply the theory of information flow and channel, developed by Barwise and Seligman [11], and define the consistency of a branch in terms of effects. The theory is originally used to describe whole-part relations of concepts appearing in distributed systems. A classification is assigned to each component in the system and they are linked with specific mappings called infomorphisms to the classification for the entire system. It means the components and their properties of the system with distinct granularities are dealt with in a united way. By the theory, the effects of nodes in an attack tree, that inhabit in some classifications, are “lifted” into the classification of their parent node. If the lifted effects imply the parent’s effect, then the latter is regarded as the abstraction of the child’s effect. On one hand, integrations of plural effects, that can reside in distinct classifications, are defined. Therefore, around a branch in an attack tree we can compare the integration of child nodes’ effects and the parent’s effect to check their abstraction-refinement relation. It leads to the definition of the consistency of the branch and this approach allows
us assess the validity of attack decompositions in formal way. Moreover, it supports rigorous analyses of attack trees, including evaluations of the completeness of the decompositions.

We define a novel semantics of attack trees in order to consider intermediate nodes. Horne et al. [7] provided a semantics of attack trees with sequential conjunctions, which takes values in a set of directed graphs whose nodes are labeled with primitive attacks. Here, a primitive attack corresponds to a leaf node in an attack tree. It means that attack trees are interpreted as combinations of only primitive attacks. However, this paper focuses on the relationships among an attack and its sub-attacks, especially at intermediate nodes. We consider that an attack tree expresses a collection of inseparable refinement scenarios. The semantics proposed in this paper takes value in the powerset of sub-trees without OR branches. These sub-trees can derive directed graphs labeled by the leaf nodes in the attack tree, and therefore the semantics can be related to the semantics proposed by Horne et al. (summarized in Appendix A).

As potential applications of attack trees with effects, we evaluate countermeasures and possible mitigation for attacks. A countermeasure or mitigation eliminates some of the consequences (i.e., effects) of attacks. Hence, the mitigated effects and the residuals can be described as fragments of effects. This enables us to link the mitigation to the consistency of the attack decomposition. We formalize the idea, and check that the residual effects of the sub-attacks tend to be stronger than the residual effect of the parent. Specifically, by looking at a case study of a vehicular network system and potential threats to it, we analyzes possible countermeasures in detail from this viewpoint.

Generally, an attack tree process for decomposing a threat consists of three steps: identification of the target, tree construction, and analysis. The first step is commonly carried out in system engineering, such as system development or risk management. To conduct this step in a systematic way, we can follow established methodologies such as with SysML [12]. On the one hand, for the third step, formal analyses with attack trees have been developed. With the use of attributes, logical or quantitative properties of a threat are integrated according to the tree structure, enabling a rigorous assessment of the threat (see also Section 2.1). However, the second step of the process, namely, a systematic approach to tree construction, seems to be overlooked, as we pointed out ambiguous relationships among attacks. Our results support tree constructions by observing consistency in a more direct, simpler and formal way. As a result, all steps in the attack tree process can be approached systematically, helping to improve the attack tree analysis.

This paper is an extension of [1]. Theoretical supports for the ideas and results constitute the main addition (Section 3). Relationships between attacks and effects are described based on the concept model and are formalized with Information flow theory. These justify the discussion on the refinement of attacks such as with abstract-refinement relations of effects, integrations of effects, and mitigation. Explicitly, the major differences are as follows:

- An introductory example is added (Section 1.1);
- Symbols that express the types of branches are simplified in diagrams;
- The formal definition is compared with [7] in detail (Section 2.2 and Appendix A);
- A concept model of effects is built, and the discussion treating effects is formalized with Information flow theory (Section 3.2);
- The definition of consistency (Definition 7) is generalized, with special consideration of the residual effects around SAND branches;
- The validity of decompositions is mentioned explicitly (Corollary 1);
2. Attack Trees with Sequential Conjunctions

2.1. Overview of Attack Trees

In this section, we review attack trees, particularly the formal descriptions as well as their practical applications. Comparisons with fault trees used in the safety domain are also discussed.

The concept of attack trees was firstly introduced by Schneier [2], who expressed the decomposition of an attack as an AND/OR-tree and demonstrated several examples of evaluating of attacks using the tree. That is, he evaluated an attack by integrating the evaluations of sub-attacks along with the tree structure. Subsequently, this idea was formalized by Mauw and Oostdijk [6], who specified a formal syntax of attack trees and defined the corresponding semantics as a set of multisets consisting of primitive attacks. Moreover, they discussed the equivalence and transformations of attack trees compatible with the semantics and called the evaluations of attacks as attributes. An attribute was defined as a function from the nodes of an attack tree to a set, where the function values did not conflict with AND and OR decomposition. Examples of attributes are possibility of attacks and the costs for attacks [2], attack time and the minimum number of experts for attacks [7], and attack probabilities [3]. The attribute of an attack in an attack tree was calculated with the attribute values of the children.

Attack trees are applied in various domains for the purpose of attack modeling, although in many cases, they are not defined formally. Recently, security analyses with attack trees were conducted for cyber-physical systems. One study [4] showed an attack tree for an implantable medical device and investigated whether communication protocols for the device had vulnerabilities or not. Another study [13] analyzed the security for a railway system. Attack trees were applied to identify detailed attack scenarios, while effect identification and risk evaluation was done with Failure Modes, Vulnerabilities, and Effects Analysis (FMVEA). In EVITA [3] for the automotive domain, attack trees were considered as major tools to identify attack scenarios and to estimate...
attack potentials. However, approaches to building trees were not discussed apart from abstract tree structures. JASO TP 15002 [14] for automobiles suggested tree decompositions of selected threats to analyze how the threats could be realized. In DO-356 [15] for the aviation domain, tree diagrams were introduced to analyze security aspects. They were called threat trees, as they were focused on threat condition events and vulnerability events as well as attacks.

The idea of attack trees is rather simple, and allows various extensions. Wang et al. [16] classified many variants of attack trees, and Fovino and Masera [17] enriched attack tree nodes with related information such as assertions, vulnerabilities, and operations. With such enrichment, attacks or threats can be analyzed from several viewpoints. The simplicity of attack trees also allows wider interpretations, which means practitioners may experience difficulties in building them. To the best of our knowledge, most related studies have explicitly discussed neither guidance for attack decompositions nor the validity of decompositions in detail. Although the research [5, 9, 10] discussed this issue, their frameworks dealt with attacks only indirectly.

One of the major extensions of attack trees was the addition of a new branch type, namely, sequential conjunction. In several cases, sub-attacks of an attack have causal dependency, and therefore it is natural to consider an attack tree together with the order of attack executions. Attack trees with sequential conjunction were discussed by Jhawar et al. [8] and Horne et al. [7]. Their studies extended Mauw and Oostdijk’s formalization [6], in particular, the semantics was extended from multisets to sets of graphs representing possible sequences of primitive attacks. Audinot et al. [9] also focused on attack trees with sequential conjunctions. They used pre- and post-conditions of attacks for the labels of nodes instead of the attacks themselves. The semantics was given as the sets of the target’s behaviors that satisfied the corresponding pre- and post-conditions. The framework clearly shows the relationships among the parent node and its children around a branch, but actual events by attacks are not presented explicitly. Furthermore, a transition model expressing possible behaviors in the target system should be prepared in advance. Audinot et al. discussed the consistency of decomposition by comparing the semantics of nodes around a branch. André et al. [18] formulated attack trees as representations of event sequences about attacks. A timed automaton is assigned to each leaf in an attack tree and the automata of lower nodes are composed at the upper nodes. Around a branch, the parent node activates the child nodes depending on the branch type, executes the automata assigned to the activated children in parallel or in series, and finally integrates their results.

In the safety domain, fault trees have been used for the reliability analysis of systems since the 1960s. Fault trees were presented as AND/OR-trees the same as attack trees, but they showed causal decompositions [19,20]. The literature often treated attack trees and fault trees similarly. For example, interpretations were given as sets of labels on the leaf nodes in the tree, or properties of the uppermost event of the tree were quantitatively calculated with the properties of leaf nodes [6,20]. Moreover, with a recent observation that security threats caused harms in safety-critical systems, the integration of attack trees and fault trees was proposed [21] to connect security analysis and safety analysis.

2.2. Formulation

We provide a formal definition of attack trees. The syntax is defined inductively, and the semantics represents possible scenarios of attack refinements. In the sequel, we denote attack trees, including sequential conjunctions.

1It is called correctness properties in their paper.
**Definition 1.** An attack tree is a labeled tree with three types of branches:

\[
\begin{align*}
t & ::= \text{\(Lf(n)\mid Nd(n, \text{op}, \langle t, t, \ldots, t \rangle)\)}, \\
\text{op} & ::= \text{\(\text{AND}\mid \text{OR}\mid \text{SAND}\)}
\end{align*}
\]

where \(\langle - \rangle\) means a non-empty finite sequence of its arguments and the symbol \(n\) is a label for the node, which expresses an action or event. The set of attack trees is denoted by \(\mathcal{AT}\).

Intuitively, \(Lf(n)\) corresponds to a primitive attack \(n\), which is no longer decomposed (a leaf node in a tree), and \(Nd(n', \text{op}, \langle t_1, \ldots, t_k \rangle)\) corresponds to an attack \(n'\), which has sub-trees \(t_1, \ldots, t_k\) with type \(\text{op}\) as its decomposition (an intermediate node in a tree). Attack trees can be diagrammatically represented, as illustrated in Fig. 1. The branch type is expressed around it: an arc for \(\text{AND}\) branch, a small arrow for \(\text{SAND}\) branch, and no symbols for \(\text{OR}\) branch. The child nodes around a \(\text{SAND}\) branch are executed from left to right. The uppermost node of an attack tree is called the root node.

We do not consider the order of children for \(\text{AND}\) or \(\text{OR}\) branches, and we do not care the branch type when it has only one child. Hence the following equalities are assumed for arbitrary subtrees \(\{t_j\}_{1 \leq j \leq k}\) and a subtree \(t\):

\[
\begin{align*}
Nd(n, \text{op}, \langle t_1, \ldots, t_i, t_{i+1}, \ldots, t_k \rangle) \\
&= Nd(n, \text{op}, \langle t_1, \ldots, t_{i+1}, t_i, \ldots, t_k \rangle) \quad (\text{\(\text{op} \in \{\text{AND}, \text{OR}\}\)} \\
Nd(n, \text{op}, \langle t \rangle) &= Nd(n, \text{op'}, \langle t \rangle) \quad (\text{\(\text{op}, \text{op'} \in \{\text{AND}, \text{SAND}, \text{OR}\}\}}
\end{align*}
\]

We denote an attack tree without \(\text{OR}\) branches as an \(R\)-tree\(^2\). Intuitively, an \(R\)-tree expresses an individual refinement scenario concerning the attack of the root node. The set of \(R\)-trees is denoted by \(\mathcal{AT}_R\).

A semantics \([\cdot]\) of attack trees is the function that maps an attack tree to a multiset of \(R\)-trees.

**Definition 2.** The function \([\cdot]\) on \(\mathcal{AT}\) is defined by the following rules, where \(\bar{\ell} = \langle t_1, \ldots, t_m \rangle\) and \(\bar{\ell} = ([t_1], \ldots, [t_m])\):

\[
\begin{align*}
[Lf(n)] &= \{Lf(n)\}, \\
[Nd(n, \text{AND}, \bar{\ell})] &= \{Nd(n, \text{AND}, \langle \tau_1, \ldots, \tau_m \rangle) \mid (\tau_1, \ldots, \tau_m) \in \bar{\ell}\}, \\
[Nd(n, \text{SAND}, \bar{\ell})] &= \{Nd(n, \text{SAND}, \langle \tau_1, \ldots, \tau_m \rangle) \mid (\tau_1, \ldots, \tau_m) \in \bar{\ell}\}, \\
[Nd(n, \text{OR}, \bar{\ell})] &= \bigcup_{1 \leq i \leq m} \{Nd(n, \text{AND}, \langle \tau \rangle) \mid \tau \in [t_i]\}.
\end{align*}
\]

The semantics is based on the idea that decompositions in attack trees are logical refinement. An \(\text{OR}\) branch is interpreted as a multiset union, indicating the branch corresponds to a case division. An aspect of the attack is refined, and possible detailed attacks are listed as sub-attacks. On the other hand an \(\text{AND/SAND}\) branch is interpreted as a factorization of an attack to sub-attacks. The collection of sub-attacks around the branch is inseparable, as a single sub-attack in it does not invoke the original attack. The causal dependency of attacks exists only between the children of each \(\text{SAND}\) branch and does not exist elsewhere, especially between an attack and its sub-attacks.

\(^2\)It corresponds to a codot term in [7].
Remark. A comparison of our formulation of attack trees (Definition 1, 2) with those of causal attack trees in [7] (reviewed in Appendix A.1) shows the syntaxes are very similar – the difference is whether or not a branch is limited to having two children. On one hand, our semantics keeps non-leaf nodes, and analyses of them are available, whereas the “intermediate semantics” for causal attack trees only considers leaf nodes. Therefore, we can project attack trees to causal ones, as stated in Proposition 6 in Appendix A.2. As a result, the attack trees in AT can be analyzed by structural methods for causal attack trees, after their consistencies are confirmed as discussed in Section 3.

![Attack tree and its interpretation](image)

Figure 2. Attack tree and its interpretation

2.3. Attributes

An attribute of an attack tree is defined as a function $f$ from the set of nodes. The codomain of $f$ depends on the node in general, but most of the known attributes have fixed common codomains like numbers or boolean values. Each codomain of $f$ is a set with the three operations $\mu_O$, $\mu_A$, and $\mu_S$, corresponding to OR, AND, and SAND, respectively. Around a branch in an attack tree, attribute values of the child nodes are summarized with these operations. Therefore, the following equalities are required to hold:

\[
\mu_O(\langle f(x_1), \ldots, f(x_i), f(x_{i+1}), \ldots, f(x_k) \rangle) = \mu_O(\langle f(x_1), \ldots, f(x_{i+1}), f(x_i), \ldots, f(x_k) \rangle),
\]

\[
\mu_A(\langle f(x_1), \ldots, f(x_i), f(x_{i+1}), \ldots, f(x_k) \rangle) = \mu_A(\langle f(x_1), \ldots, f(x_{i+1}), f(x_i), \ldots, f(x_k) \rangle),
\]

\[
\mu_A(\langle f(x) \rangle) = \mu_S(\langle f(x) \rangle) = \mu_O(\langle f(x) \rangle).
\]

One example of attributes is the minimum number of experts required to perform an attack, as discussed in [7]. When we denote the defining function of the attribute by $\nu$, its codomain is defined as the set of natural numbers $\mathbb{N}$ and $(\mu_O, \mu_A, \mu_S) = (\min, \text{Sum}, \max)$. Note that this
attribute is assumed to be determined by the values of lower nodes. Namely, \( \nu(Nd(n, \text{OR}, \vec{t})) = \min\{\nu(t_1), \ldots, \nu(t_k)\} \) where \( \vec{t} = \langle t_1, \ldots, t_k \rangle \), and similar equations hold for \text{AND/\mathcal{S}AND} branches. When there is another attribute for which we cannot make this assumption, the equalities may not be expected and we must consider contributions for the attribute by the intermediate nodes themselves. Such an attribute will be compatible with the semantics in Definition 2 but not with that in [7] using only leaf nodes. To distinguish the latter type of attribute, we often call them quasi-attributes. We see an example of a quasi-attribute in Section 3.3.

3. Validating Decompositions with Effects

3.1. Effects of Attacks

An effect is one of the major properties of an attack. It is a situation or property of a specific entity related to the target system and is caused by a specific action. For example, consider the attack “Message receive function is interfered.” After the attack, messages may be lost, the function may be unavailable, or other irregular behaviors may occur. As these situations did not occur before the attack, it can be considered that the attack caused them. As a result, we can identify the summarized situation “Messages are not processed correctly” as the effect of the attack.

Effects are also significant concepts in the areas adjacent to security. In ISO/IEC Guide 51 [22] safety standard, the primary issues to avoid are negative effects on people, property, or the environment caused by some events. In ISO 31000 [23] standard focused on risk management, a risk is defined as an effect of uncertainty on objectives. Owing to these observations, it seems reasonable that an effect of an attack meets the following conditions:

- The effect must be directly caused by the corresponding attack. It shows a change of the entity that the attack affects;
- Properties that hold before the attack must not be selected as effects;
- The effect occurs immediately after the corresponding attack, and no other properties invoked by the attack occur before the effect occurs.

Fig. 3 is a model of the relationship between an attack and an effect. An attack consists of the target object and action for the target, while an effect is a change of the target. The object is related to the target system or its environment, and the target of an attack coincides to the object of an effect.

The effects of attacks can be described by logical formulas. For example, the effect “The password becomes public or is overwritten” has the target password, and its changes are being public and being overwritten. Hence, the effect is intuitively decomposed to the pair of symbols \text{passwd} and \text{Disclosed} \lor \text{Modified}.

3.2. Refinement and Information Flow

In order to discuss effects and their refinement according to the model in Fig. 3, we apply Information flow and Channel theory [11] and describe several concepts and relationships.

\(^{3}\)Those effects are referred to as harms.
Intuitively, a system consists of several subsystems and they progress in cooperation with each other. Each subsystem has its individual state at an arbitrary time and some properties of it hold depending on the state. The states and properties of the entire system are represented as integrations of states and properties of subsystems. This idea is formalized by the theory of information flows and channel. Classifications defined in Channel theory describe relations between properties and situations on a system and its subsystems, while infomorphisms in the theory interlink elements in the classifications. With an infomorphism, a property of a subsystem is “transferred” into another subsystem and analyzed together with properties in the second subsystem.

Channel theory is used in development of cyber-physical systems. Gebreyohannes [24] applied Channel theory to model communications among a satellite and ground stations. The communication possibility to the satellite, which changes by time and distance, were caught by classifications and infomorphisms. Peters [25] presented a hierarchical model of a swarmbot. Components in the model with different granularities were linked with infomorphisms.

We start with reviewing basic definitions.

**Definition 3.** A classification $C$ is a triple $(\text{Tok}(C), \text{Typ}(C), \models)$ where $\text{Tok}(C)$ (‘tokens’ of $C$) and $\text{Typ}(C)$ (‘types’ of $C$) are sets, and $\models$ is a binary relation between them.

**Definition 4.** Let $C_1$ and $C_2$ be classifications. An infomorphism from $C_1$ to $C_2$ is a pair of mappings $(f^\land, f^\lor)$, where $f^\land : \text{Typ}(C_1) \to \text{Typ}(C_2)$, $f^\lor : \text{Tok}(C_2) \to \text{Tok}(C_1)$, and the following condition is satisfied:

$$f^\lor (a) \models_1 \gamma \iff a \models_2 f^\land (\gamma)$$  \hspace{1cm} (IM)

for all $a \in \text{Tok}(C_2)$ and $\gamma \in \text{Typ}(C_1)$. We write this infomorphism as $(f^\land, f^\lor) : C_1 \cong C_2$.

**Example.** Consider a classification $W_1$ and suppose

$\text{Tok}(W_1) = \{\text{passwd, pTimeout}\}$,

$\text{Typ}(W_1) = \{\text{Disclosed, Modified, Hidden}\}$, and

$\text{passwd} \not\models \text{Disclosed,}$  $\text{pTimeout} \not\models \text{Modified}$
that formalizes the status of the password and the timeout parameter in a software module. In addition, consider another classification \( W_2 \) and suppose

\[
\begin{align*}
\text{Tok}(W_2) &= \{ \text{auth} \}, \\
\text{pass-through} &\in \text{Typ}(W_2), \\
\text{auth} &\vdash \text{pass-through}, \text{and} \\
\text{auth} &\nvdash \alpha (\text{if } \alpha \in \text{Typ}(W_2) \text{ is not pass-through})
\end{align*}
\]

that formalizes the status of the authentication function (the lowest relation says “Authentication is not functioning”). Now take mappings \( f^\wedge : \text{Typ}(W_1) \to \text{Typ}(W_2) \) and \( f^\vee : \text{Tok}(W_2) \to \text{Tok}(W_1) \) with

\[
\begin{align*}
f^\wedge(\text{Disclosed}) &= \text{pass-through}, \\
f^\wedge(\text{Modified}) &\neq \text{pass-through} \neq f^\wedge(\text{Hidden}), \text{ and} \\
f^\vee(\text{auth}) &= \text{passwd}.
\end{align*}
\]

Then the pair \((f^\wedge, f^\vee)\) is an infomorphism. \(\Box\)

Infomorphisms represent whole-part relationships. Now let us see the classification \( W_2 \) modeling a function of a software system, and \( W_1 \) modeling its components. The relation \( \text{passwd} \vdash_1 \text{Disclosed} \) in \( W_1 \), whose token comes from the concepts of the entire function (i.e., it has a preimage by \( f^\wedge \)), is lifted to the relation \( \text{auth} \vdash_2 \text{pass-through} \) in \( W_2 \) with the infomorphism \((f^\wedge, f^\vee)\). We later apply this idea to representing abstract-refinement relations of effects.

Compositions of effects are required for our purpose; however, the original framework of classifications is not suitable for that, especially since related pairs of tokens and types cannot be composed when tokens are distinct. Now we develop some compound classifications. To begin, we suppose a set of indices \( \Omega \), where any index set appearing in this paper is its subset.

**Definition 5.** Let \( C, C_1, \) and \( C_2 \) be classifications that have finite tokens and types. They also have ‘un-connected tokens’ \( \varepsilon, \varepsilon_1, \varepsilon_2 \) respectively. Namely, \( \varepsilon \nvdash \alpha \) for \( \forall \alpha \in \text{Typ}(C) \) and similar conditions hold for \( \varepsilon_1 \) and \( \varepsilon_2 \).

1. The classification \( C_1 \oplus C_2 \) is defined as

   - \( \text{Tok}(C_1 \oplus C_2) = \text{Tok}(C_1) \sqcup \text{Tok}(C_2) \) (disjoint union)
   - \( \text{Typ}(C_1 \oplus C_2) = \text{Typ}(C_1) \sqcup \text{Typ}(C_2) \),
   - \( a \vdash_{C_1 \oplus C_2} \alpha \) holds if and only if either of the following statements holds
     - \( a' \vdash_1 \alpha' \) for some \( a' \in \text{Tok}(C_1) \) and \( \alpha' \in \text{Typ}(C_1) \) such that \( a = \text{in}_1(a') \) and \( \alpha = \text{in}_1(\alpha') \), where \( \text{in}_1 \) is the embedding to the disjoint union, or
     - \( a'' \vdash_2 \alpha'' \) for some \( a'' \in \text{Tok}(C_2) \) and \( \alpha'' \in \text{Typ}(C_2) \) such that \( a = \text{in}_2(a'') \) and \( \alpha = \text{in}_2(\alpha'') \), where \( \text{in}_2 \) is the embedding to the disjoint union.

2. The classification \( (C_1, C_2) \) is defined as

   - \( \text{Tok}((C_1, C_2)) = \text{Tok}(C_1) \times \text{Tok}(C_2) \),
   - \( \text{Typ}((C_1, C_2)) = \text{Typ}(C_1) \times \text{Typ}(C_2) \),
   - \( \langle a_1, a_2 \rangle \vdash_1 (\alpha, \beta) \iff a_1 \vdash_1 \alpha \text{ and } a_2 \vdash_2 \beta. \)

\(^4\)The assumption of \( \varepsilon_i \) can be ignored to construct \((C_1, C_2)\).
3. The classification $\mathcal{FD}(C)$ is about `family tokens' and `distributive lattice types' generated by $C$. It is defined as follows.

- A token of $\mathcal{FD}(C)$ is a family $\{a_\lambda\}_{\lambda \in \Lambda}$ where $a_\lambda \in \text{Tok}(C)$ and $\Lambda \subset \Omega$ is finite.
  In the sequel, we often write tokens like as $\{a_\lambda\}_\Lambda$ when it does not invoke some confusion.
- A primitive type of $\mathcal{FD}(C)$ is presented as $\alpha_\lambda$ where $\alpha \in \text{Typ}(C)$ and $\lambda \in \Omega$. A generic type is constructed as
  \[
  \tau := \chi | \top | \bot | \tau \wedge \tau | \tau \vee \tau \quad (\chi \text{ is a primitive type}).
  \]

$\text{Typ}(\mathcal{FD}(C))$ is subject to relations\(^5\) of distributive lattices and semantical relations among primitive types, that is the resulting lattice is not always a free-generated\(^6\)

- For a primitive type $\alpha_\mu$, $\{a_\lambda\}_{\lambda \in \Lambda} \models \mathcal{FD}(C) \alpha_\mu \iff \mu \in \Lambda$ and $a_\mu \models C \alpha$. Moreover $\{a_\lambda\}_\Lambda \models \top$ and $\{a_\lambda\}_\Lambda \not\models \bot$ for arbitrary token $\{a_\lambda\}_\Lambda$.
- For compound types,
  \[
  \{a_\lambda\}_\Lambda \models \mathcal{FD}(C) \Gamma \wedge \Delta \iff \{a_\lambda\}_\Lambda \models \mathcal{FD}(C) \Gamma \text{ and } \{a_\lambda\}_\Lambda \models \mathcal{FD}(C) \Delta.
  \]
  Similarly, $\{a_\lambda\}_\Lambda \models \mathcal{FD}(C) \Gamma \vee \Delta$ is defined.

Two types of relations are introduced in $\mathcal{FD}(C)$-type classifications. First, there are several structural deductions of relations concerning tokens:

- $\{a_\lambda\}_\Lambda \models \gamma_\mu \iff \{a_\lambda\}_\Lambda \models \gamma_{\mu'}$ if $a_\mu = a_{\mu'}$ where $\mu, \mu' \in \Lambda$.
- $\{a_\lambda\}_\Lambda \models \Gamma \iff \{a_\lambda\}_\Lambda \models \Gamma$ if $\rho$ has no occurrence in any primitive type in $\Gamma$.
- $\{a_\lambda\}_\Lambda \models \Gamma \iff \{a_\lambda\}_\Lambda \models \Gamma$ if $a_\rho = \varepsilon$.

Second, a distributive lattice structure defines a partial orders on $\text{Typ}(\mathcal{FD}(C))$, i.e., $\Gamma \leq \Delta \iff \Gamma \vee \Delta = \Delta \iff \Gamma \wedge \Delta = \Gamma$. We interpret this order as a derivation such that “the greater type can be derived from the smaller type.” As an example, consider that $\Gamma \wedge \Delta \leq \Gamma$ means $\Gamma$ can be derived from $\Gamma \wedge \Delta$. Hence, we often write $\Theta \Rightarrow Z$ instead of $\Theta \leq Z$. Moreover, phenomena in the modeled world are reflected as relations in the classification. For example, $(\text{passwd} \models \text{Disclosed}) \Rightarrow (\text{passwd} \models \text{Accessible})$ is assumed since the disclosure of the password includes the situation that the password is accessible. Introducing relations like this leads us to take a quotient of the distributive lattice $\text{Typ}(\mathcal{FD}(C))$.

We review general properties of the classifications defined above. First we see the construction of $\mathcal{FD}(C)$ is so-called functorial. Let $(f^\wedge, f^\vee) : C_1 \Rightarrow C_2$ be an infomorphism. We can define

\[
\begin{align*}
\mathcal{FD}f^\wedge : \text{Typ}(\mathcal{FD}(C_1)) & \rightarrow \text{Typ}(\mathcal{FD}(C_2)) : \\
\alpha_\lambda & \mapsto f^\wedge(\alpha)_\lambda, \\
\Gamma \vec{\wedge} \Delta & \mapsto \mathcal{FD}f^\wedge(\Gamma) \vec{\wedge} \mathcal{FD}f^\wedge(\Delta)
\end{align*}
\]

\(^5\)Relations are required to be compatible with token-type relation: $\forall \{a_\lambda\}_\Lambda, \{a_\lambda\}_\Lambda \models \rho \iff \{a_\lambda\}_\Lambda \models \sigma$ if $\rho$ and $\sigma$ are related.
\(^6\)Precisely, the structure of $\text{Typ}(\mathcal{FD}(C))$ depends on the relations and is not unique. However we fix one structure for every $C$ in this paper, and thus we denote the lattice by the abused name $\text{Typ}(\mathcal{FD}(C))$. 

where \( \not\in \{\land, \lor\} \). We assume that this map is compatible with relations among primitive types and is well-defined. On one hand, we can define
\[
\mathcal{F}D f^\lor : \text{Tok}(\mathcal{F}D(C_2)) \to \text{Tok}(\mathcal{F}D(C_1)) : \{a_\lambda\}_\Lambda \mapsto \{a'_\lambda\}_\Lambda
\]
where \( a'_\lambda = f^\lor(a_\lambda) \).

**Lemma 1.** Let \((f^\land, f^\lor) : C_1 \sqsupseteq C_2\) and \((g^\land, g^\lor) : C_2 \sqsupseteq C_3\) be infomorphisms between classifications \(C_1, C_2,\) and \(C_3\).

1. The construction above defines an infomorphism \((\mathcal{F}D f^\land, \mathcal{F}D f^\lor) : \mathcal{F}D(C_1) \sqsupseteq \mathcal{F}D(C_2)\).
2. \( \mathcal{F}D(g^\land \circ f^\land) = \mathcal{F}Dg^\land \circ \mathcal{F}D f^\land \) and \( \mathcal{F}D(g^\lor \circ f^\lor) = \mathcal{F}Dg^\lor \circ \mathcal{F}D f^\lor \) hold. As well, \( \mathcal{F}D(id_\Lambda^\land) = id_{\mathcal{F}D(C)}^\land \) and \( \mathcal{F}D(id_\Lambda^\lor) = id_{\mathcal{F}D(C)}^\lor \) hold.
3. \( \mathcal{F}D(f^\land) \) and \( \mathcal{F}D(g^\land) \) are order-preserving.

Statements 2 and 3 are obvious, and Statement 1 is derived from the following equivalence:
\[
\mathcal{F}D f^\lor({a_\lambda})_\Lambda \models \gamma_\mu \quad \text{(in} \ \mathcal{F}D(C_1))
\]
\[
\iff \mu \in \Lambda \text{ and } f^\lor(a_\mu) \models \gamma \text{ in } C_1
\]
\[
\iff \mu \in \Lambda \text{ and } a_\mu \models f^\land(\gamma) \text{ in } C_2
\]
\[
\iff \{a_\lambda\}_\Lambda \models \mathcal{F}D f^\land(\gamma_\mu) \quad \text{(in} \ \mathcal{F}D(C_2)).
\]

Here remark that, for \( \mathcal{F}D(C)\)-type classifications, it is enough to check the condition of infomorphisms (IM) for generators of \( \text{Typ}(C) \) only.

As stated in the next lemma, \( \mathcal{F}D(C)\) and \( C_1 \oplus C_2 \) are related to the original classifications \( C, C_1,\) and \( C_2,\) with infomorphism \( \iff \) We write \( in_i(x) \) as \( x^{(i)} \) for short.

**Lemma 2.**
1. Fix an index \( \mu \in \Omega \). The \( \mu \)-th embedding and the \( \mu \)-th projection
\[
\text{lift}^\land_\mu : \text{Typ}(C) \to \text{Typ}(\mathcal{F}D(C)) : \alpha \mapsto \alpha_\mu, \text{ and}
\]
\[
\text{lift}^\lor_\mu : \text{Tok}(\mathcal{F}D(C)) \to \text{Tok}(C) : \{a_\lambda\}_\Lambda \mapsto \begin{cases}
    a_\mu & (\mu \in \Lambda) \\
    \varepsilon & (\text{otherwise})
\end{cases}
\]
constitute an infomorphism \( C \sqsupseteq \mathcal{F}D(C). \)
2. For \( i \in \{1, 2\},\) the pair of the functions
\[
\text{inc}^{i^\land}(\zeta) = in_i(\zeta) = \zeta^{(i)}, \text{ and}
\]
\[
\text{inc}^{i^\lor}(x) = \begin{cases}
    x' & (\text{if there exists } x' \text{ s.t. } x = in_i(x')) \\
    \varepsilon & (\text{otherwise})
\end{cases}
\]
is an infomorphism \( C_i \sqsupseteq C_1 \oplus C_2. \)

\( \iff \) On one hand, relating \( C_i \) and \( (C_1, C_2) \) requires additional conditions. However, since the relationship between them does not appear in this paper, we do not consider the matter any further.
It is not difficult to check whether functions in the lemma satisfy the condition (IM).
As a corollary of Lemma 1, the next proposition describes \( F(D(C_1 \oplus C_2)) \). Notice that a token of \( F(D(C_i \oplus C_2)) \) is expressed as a family \( \{a^{(k_\lambda)}_{i}\}_{\Lambda} \). The parameter \( k_\lambda \) indicates the component where the token \( a^{(k_\lambda)}_{i} \) belongs in the original; that is, \( a_\lambda \in Tok(C_i) \) if \( k_\lambda = i \).

**Proposition 1.** The pair \((\text{inc}^\wedge_i, \text{inc}^\vee_i) = (F(D\text{inc}^\wedge_i), F(D\text{inc}^\vee_i))\) is an infomorphism \( F(D(C_1) \implies F(D(C_1 \oplus C_2)) \) for \( i \in \{1, 2\} \),

\[
\text{inc}^\wedge_i (\{a^{(k_\lambda)}_{i}\}_\Lambda) = \{a^{(k_\lambda)}_{i}\}_\Lambda, \quad \text{and}
\]

\[
\text{inc}^\vee_i (\{a^{(k_\lambda)}_{i}\}_\Lambda) = \{a^{(k_\lambda)}_{i} \Lambda_i \},
\]

for \( \Lambda_i = \{\lambda \in \Lambda | k_\lambda = i \} \). Especially, \( \text{inc}^\vee_i (\{a^{(k_\lambda)}_{i}\}_\Lambda) \) is the empty family if \( \Lambda_i = \emptyset \).

The next proposition says that the classification \((F(D(C_1)), F(D(C_2)))\) is embedded into \( F(D(C_1 \oplus C_2)) \).

**Proposition 2.** The mappings on \((\text{Typ}(F(D(C_1))), \text{Typ}(F(D(C_2))))\) and \(\text{Tok}(F(D(C_1)) \oplus F(D(C_2)))\) defined as

\[
\text{conj}^\wedge (\{\Gamma, \Delta\}) = \Gamma^{(1)} \wedge \Delta, \quad \text{and}
\]

\[
\text{conj}^\vee (\{a^{(k_\lambda)}_{i}\}_\Lambda) = \{a^{(k_\lambda)}_{i} \Lambda_1, \{a^{(k_\lambda)}_{i} \Lambda_2 \}
\]

constitute an infomorphism \((F(D(C_1)), F(D(C_2))) \implies F(D(C_1 \oplus C_2)), \) where \( \Gamma^{(1)} \) [re. \( \Delta^{(2)} \)] is obtained by replacing all primitive type like \( \alpha \) with \( \alpha^{(1)} \) [re. \( \alpha^{(2)} \)], and where \( \Lambda_i = \{\lambda \in \Lambda | k_\lambda = i \} \). Moreover this infomorphism is mono. i.e. if \( \text{conj} \circ g = \text{conj} \circ h \) then \( g = h \) holds where \( \text{conj} = (\text{conj}^\wedge, \text{conj}^\vee) \).

**Example.** Remember the classifications \( W_1 \) and \( W_2 \) in the last example. Additionally we introduce a new classification \( W_3 \):

\[
\text{dest} \in \text{Tok}(W_3), \quad \text{Modified} \in \text{Typ}(W_3), \quad \text{dest} \models \text{Modified}
\]

In \( W_1 \), (or \( W_2 \) as well) we can only describe relationships between single tokens and single types. Now relationships for plural tokens and types, especially transversal ones, can be described in single relations in \( F(D(\oplus W_1)) \). Concretely, under \( a_1 = \text{passwd} \), and \( a_2 = \text{pTimeout} \),

\[
\{a_1, a_2\} \models \text{Disclosed}_1 \wedge \text{Modified}_2.
\]

is a relation held in \( F(D(W_1)) \). Moreover, this relation is lifted to (with \( a_3 = \text{dest} \in \text{Tok}(W_3) \))

\[
\{a_1^{(1)}, a_2^{(1)}, a_3^{(3)}\} \models \text{Disclosed}_1^{(1)} \wedge \text{Modified}_2^{(1)}
\]

by \((\text{inc}^\wedge, \text{inc}^\vee)\). Like that, the relation \( \text{dest} \models_3 \text{Modified} \) is lifted to

\[
\{a_1^{(1)}, a_2^{(1)}, a_3^{(3)}\} \models \text{Modified}_3^{(3)},
\]
and finally they are composed to the relation in $\mathcal{F} \mathcal{D}(W_1 \oplus W_3)$:

$$\{a_1^{(1)}, a_2^{(1)}, a_3^{(3)}\} \models (\text{Disclosed}_1^{(1)} \wedge \text{Modified}_3^{(1)}) \wedge \text{Modified}_2^{(3)}.$$

Now we formalize the refinement of relations in classifications. Let $\mathcal{F} \mathcal{D}(C_C)$ and $\mathcal{F} \mathcal{D}(C_A)$ be classifications and $D$ be another classification embedded in $\mathcal{F} \mathcal{D}(C_C)$. Consider an infomorphism $(f^\wedge, f^\vee) : D \rightarrow \mathcal{F} \mathcal{D}(C_A)$. When a relation $(\vec{a} \models \Gamma)$ in $\mathcal{F} \mathcal{D}(C_C)$ belongs to the embedded image of $D$ and the token $\vec{a}$ has a preimage by $f^\vee$, it can be lifted to $(\vec{a}^\prime \models_A f^\wedge(\Gamma))$ via the embedding, and we can compare it with other relations in $\mathcal{F} \mathcal{D}(C_A)$.

**Definition 6.** In this situation, we say $(\vec{a}^\prime \models_A \Delta)$ is an abstraction of $(\vec{a} \models_C \Gamma)$ (or $(\vec{a} \models \Gamma)$ is a refinement of $(\vec{a}^\prime \models \Delta)$) by $(f^\wedge, f^\vee)$, if the following implication holds:

$$(\vec{a}^\prime \models_A f^\wedge(\Gamma)) \Rightarrow (\vec{a}^\prime \models_A \Delta)$$

i.e., $f^\wedge(\Gamma) \leq \Delta$ on the token $\vec{a}^\prime$.

### 3.3. Consistent Branches

As shown in Fig. 3, we consider objects in the target system and its environment, and a specific attack affects some of the objects. Remark that these objects depend on the granularity of the attack; tampering with a Telematics module does not target the authentication subfunction directly but rather the entire communication services.

For an attack $A$, let us consider a classification $C_A$. $\text{Tok}(C_A)$ is the set of objects in the target system and its environment such that their granularities are in keeping with the attack $A$. $\text{Typ}(C_A)$ is the set of properties about some elements in $\text{Tok}(C_A)$. For $a \in \text{Tok}(C_A)$ and $\gamma \in \text{Typ}(C_A)$, $a \models \gamma$ holds if and only if $\gamma$ is an effect on $a$, i.e., $\gamma$ is a property of $a$ satisfying the three conditions addressed in Section 3.1. With $C_A$ and the semantical relations between its types, we can consider the classification $\mathcal{F} \mathcal{D}(C_A)$ and compound properties of the attack $A$. The order of $\text{Typ}(\mathcal{F} \mathcal{D}(C_A))$ expresses the strength of effects. In our context, effects mean some negative impacts, and thus, holding plural properties indicates stronger effects. For example, $(\vec{a} \models \Gamma_1)$, $(\vec{a} \models \Gamma_2)$, and $\Gamma_1 \leq \Gamma_2$ mean $\Gamma_1 = \Gamma_1 \wedge \Gamma_2$ and thus $\Gamma_1$ is stronger than $\Gamma_2$.

Now we can assign an effect to each node of an attack tree. For a node $N$ in the tree, choose a pair $(\vec{a}, \Gamma) \in \text{Tok}(\mathcal{F} \mathcal{D}(C_N)) \times \text{Typ}(\mathcal{F} \mathcal{D}(C_N))$ such that $\vec{a} \models \Gamma$ holds, and regard it as the effect of $N$. In diagrams, we put the round node labeled by the effect around $N$ and connect them with a blue edge (see Fig. 4).

Since a branch in an adequately constructed attack tree represents a refinement of the attack on the parent node, a similar structure can be expected for effects due to the concept model (Fig. 3). For instance, the effect of the parent node will be derived from the conjunction of all effects of the child nodes for an AND branch, because all of the attacks corresponding to the child nodes are executed. By contrast, when there are several conflicts among the effects around a branch, the
decomposition of the attack will have inconsistencies, such as a misunderstanding of the situation or inadequate refinement.

To describe refinement around SAND branches, we introduce cut sequences. For a sequence \( (\vec{a}^1 \models \Gamma_1, \vec{a}^2 \models \Gamma_2, \ldots, \vec{a}^m \models \Gamma_m) \), we pick up the rightmost elements with respect to each token and form a new sub-sequence \( (\vec{a}^{i_1} \models \Gamma_{i_1}, \vec{a}^{i_2} \models \Gamma_{i_2}, \ldots, \vec{a}^{i_k} \models \Gamma_{i_k}) \) \( (i_1 < i_2 < \cdots < i_k) \) as the cut sequence. Namely, if tokens \( \vec{a}^i \) and \( \vec{a}^j \) \( (i < j) \) in the original sequence are equivalent, then the element \( \vec{a}^i \) is removed from the sequence. By definition, all tokens in a cut sequence are distinct.

The consistencies of branches in attack trees are defined by the use of infomorphisms. Consider a branch in an attack tree and denote by \( p \) [re. \( i \)] an index for the parent node [re. the \( i \)-th child node]. For instance, \( C_p \) and \( (\vec{a}^p \models \Gamma_p) \) express the classification and the effect assigned to the parent node, respectively.

**Definition 7.** A branch with \( n \) child nodes in an attack tree is called consistent, if the condition below holds regarding its type:

- **OR** branch: For each index \( i \), there exists an infomorphism \( f_i : D_i \to \mathcal{F}D(C_p) \) such that the \( i \)-th effect is a refinement of the parent node’s effect by \( f_i \), where \( D_i \) is an embedded classification in \( \mathcal{F}D(C_i) \) to which the \( i \)-th effect belongs.
- **AND** branch: There exists an infomorphism \( f : D \to \mathcal{F}D(C_p) \) such that the tuple of child’s effects is a refinement of the parent node’s effect by \( f \), where \( D \) is an embedded classification in \( (\mathcal{F}D(C_i))_{1 \leq i \leq n} \) to which the tuple of child’s effects belongs.
- **SAND** branch: The two conditions below hold.
   - Every \( i \)-th effect \( (\{a^{i}_\lambda\}_{\lambda \in \Lambda_i} \models \Gamma_i) \) obtained by the \( i \)-th attack with assuming the effects in the cut sequence of the preceding effects \( (\{a^{i-1}_\lambda\}_{\lambda \in \Lambda_{i-1}} \models \Gamma_{i-1}) \).
   - For the cut sequence of the tuple of child’s effects, there exists an infomorphism \( f \) from an embedded classification \( D \) in \( (\mathcal{F}D(C_i))_{r \in I} \) to \( \mathcal{F}D(C_p) \) such that the same conditions for AND branches hold, where \( I \) is the index set of the cut sequence.

These conditions are depicted in Fig. [4]

If all branches in an attack tree are consistent, then the entire attack tree is called consistent.

Examples of attack trees with effects are provided in Section [4]

Let us denote the mapping from nodes in an attack tree to their effects by \( \varphi \). The codomain of \( \varphi \) with respect to the node \( N \) is \( \text{Tok}(\mathcal{F}D(C_N)) \times \text{Typ}(\mathcal{F}D(C_N)) \). Assume that \( \text{Tok}(C_i) \) is finite for each \( i \)-th child node around a branch and write \( \varphi(N_i) = (\{a^{i}_\lambda\}_{\lambda \in \Lambda_i} \models \Gamma_i) \). The integration of \( \varphi(N_i) \)s around the branch is defined with values in \( \mathcal{F}D(\oplus C_i) \) as follows:

\[
\mu_O((\varphi(N_i))_{1 \leq i \leq n}) = \left( \{ a^{(k)}_\lambda \}_{\lambda \in \Lambda_O} \models \bigvee_{1 \leq i \leq n} \text{inc}^{\forall i}(\Gamma_i) \right),
\]
\[
\mu_A((\varphi(N_i'))_{1 \leq i' \leq n'}) = \left( \{ a^{(k)}_\lambda \}_{\lambda \in \Lambda_A} \models \text{conj}^{\forall i'}(\{\Gamma_{i'}\}_{1 \leq i' \leq n'}) \right), \text{ and}
\]
\[
\mu_S((\varphi(N_i))_{1 \leq i' \leq n'}) = \left( \{ a^{(k)}_\lambda \}_{\lambda \in \Lambda_S} \models \text{conj}^{\forall i''}(\{\Gamma_{i''}\}_{1 \leq i'' \leq I}) \right),
\]
where \( \forall i, \Lambda_i \subset \Lambda_O, \forall i', \Lambda_{i'} \subset \Lambda_A, \) and where \( \forall i'' \in I, \Lambda_{i''} \subset \Lambda_S \) for the index set of the cut sequence of \( \varphi(N_i) \)s and \( \text{conj}^{\forall} \) is a ‘cut sequence-version’ of \( \text{conj} \).
Proposition 3. The function $\varphi$ is a quasi-attribute. Moreover, the integrated effects reflect child nodes’ effects.

(Proof) Remark that the index set $\Lambda_i$ of the $i$-th token $\{a_{\lambda}\}_{\lambda \in \Lambda_i}$ in $\mathcal{FD}(C_i)$ can be reduced due to the structural deductions mentioned in Section 3.2. Hence we can assume $\Lambda_i$ is isomorphic to a subset of $\text{Tok}(C_i)$. In particular, $\Lambda_i \cap \Lambda_j = \emptyset$ if $i \neq j$.

Around the branch, the values of $\varphi$ for child nodes are integrated with the use of Proposition 1 and Proposition 2 as follows.

- **OR branch:** For the $i$-th child’s effect $\langle \{a_{\lambda}\}_{\lambda \in \Lambda_i} \mid \Gamma_i \rangle$, we observe its target $\{a_{\lambda}\}_{\lambda \in \Lambda_i}$ has a preimage of $\text{inc}^\top$; that is, $\{a_{\lambda}^{(k)}\}_{\lambda \in \Lambda}$ where $\Lambda_i \subset \Lambda_0$ and both $a_{\lambda}$ and $a_{\lambda}^{(k)}$ point to the same token if $\lambda \in \Lambda_i$. In particular, we can take $\{a_{\lambda}^{(k)}\}_{\lambda \in \Lambda_0}$ as the common preimage of $\text{inc}^\top$ for all $j$. Therefore, the $i$-th effect $\langle \{a_{\lambda}\}_{\lambda \in \Lambda_i} \mid \Gamma_i \rangle$ is lifted to $\langle \{a_{\lambda}^{(k)}\}_{\lambda \in \Lambda_0} \mid \text{inc}^\top(\Gamma_i) \rangle$. Finally, we have $\langle \{a_{\lambda}^{(k)}\}_{\lambda \in \Lambda_0} \mid \bigvee_{1 \leq i \leq n} \text{inc}^\top(\Gamma_i) \rangle$ in $\mathcal{FD}(\oplus C_i)$ as the integrated effect. It is $\mu_0(\langle \varphi(N_i) \rangle_{1 \leq i \leq n})$, and obviously, it holds if and only if $\langle \{a_{\lambda}\}_{\lambda \in \Lambda_i} \mid \Gamma_i \rangle$ hold for some $i$.

- **AND branch:** Notice that the tuple of child’s effects $\langle \{a_{\lambda}^i\}_{\lambda \in \Lambda_i} \mid \Gamma_i \rangle_{1 \leq i \leq n}$ is regarded as the relation $\langle \{a_{\lambda}^i\}_{\lambda \in \Lambda_i, 1 \leq i \leq n} \mid \langle \Gamma_i \rangle_{1 \leq i \leq n} \rangle$ in $\langle \mathcal{FD}(C_i) \rangle_{1 \leq i \leq n}$. We can see that the target $\{a_{\lambda}^i\}_{\lambda \in \Lambda_i, 1 \leq i \leq n}$ has a preimage by $\text{conj}^\top$, presented as $\{a_{\lambda}^{(k)}\}_{\lambda \in \Lambda_A}$ where $\Lambda_i \subset \Lambda_A$ for all $i$, and where both $a_{\lambda}^i$ and $a_{\lambda}^{(k)}$ point to the same token if $\lambda \in \Lambda_i$. Therefore, the tuple $\{a_{\lambda}^i\}_{\lambda \in \Lambda_i, 1 \leq i \leq n}$ is lifted to $\{a_{\lambda}^{(i)}\}_{\lambda \in \Lambda_A} \vdash \text{conj}^\top(\langle \Gamma_i \rangle_{1 \leq i \leq n})$ in $\mathcal{FD}(\oplus C_i)$, and it is the integration $\mu_A(\langle \varphi(N_i) \rangle_{1 \leq i \leq n})$. As in the previous case, it holds if and only if $\langle \{a_{\lambda}^i\}_{\lambda \in \Lambda_i, 1 \leq i \leq n} \mid \Gamma_i \rangle$ hold for all $i$.

- **SAND branch:** Consider the cut sequence of child’s effects. Similar to the AND branch, the sequence $\langle \{a_{\lambda}^i\}_{\lambda \in \Lambda_i} \mid \Gamma_i \rangle_{1 \leq i \leq n}$ is lifted to a relation in $\mathcal{FD}(\oplus C_i)$ and the integration $\mu_S(\langle \varphi(N_i) \rangle_{1 \leq i \leq n})$ is defined.

Commutativities for quasi-attributes are derived easily. □

Although we do not consider it in detail in this paper, the completeness of the decompositions of attacks is defined as follows: an integrated effect is derived from the parent’s effect, that is $\Gamma^p$ is less than the type part of $\mu_\sharp(\langle \varphi(N_i) \rangle_{1 \leq i \leq n})$ where $\sharp \in \{O, A, S\}$. 
The following proposition links between the consistency of a branch and the quasi-attribute of effects. Owing to Proposition 3, it is sufficient to consider the relationship between \( \varphi(N_p) \) and the integration of child nodes’ effects. As a corollary, we can judge the inconsistency of a branch.

**Proposition 4.** Let us consider a consistent branch in an attack tree. Suppose the infomorphism realizing the consistency satisfies the following properties: the token-part of the infomorphism is determined by \( \text{Tok}(C_p) \); that is, it preserves the disjoint unions of families. Then, there is an infomorphism from an embedded classification with the integration of \( \langle \varphi(N_i) \rangle_{1 \leq i \leq n} \) to \( FD(C_p) \) such that it realizes the original abstraction.

**Corollary 1.** For a branch in an attack tree, if the effect \( \varphi(N_p) \) and the integration \( \mu_p(\langle \varphi(N_i) \rangle_{1 \leq i \leq n}) \) (\( \forall \in \{O, A, S\} \)) cannot be linked with an infomorphism, then the branch is inconsistent.

Rigorously speaking, the corollary negates only infomorphisms with the properties stated in Proposition 4. However, it is sufficient from the viewpoint of refinement. In our context, the token-part of the infomorphism refines individual objects in \( C_p \) and it is unrealistic that a specific combination \( \{a_1, a_2\} \in \text{Tok}(FD(C_p)) \) is refined to some objects independently on the refinement of \( \{a_1\} \) and \( \{a_2\} \). That is, \( f^\vee(\{a_1, a_2\}) = f^\vee(\{a_1\}) \cup f^\vee(\{a_2\}) \) holds.

(Proof of Proposition) The required infomorphism is constructed regarding types of branches.

First consider the case of OR branch. Suppose the \( i \)-th effect \( \{a^i_\lambda\}_{\Lambda_i} \models \Gamma_i \) is a refinement of \( \{a^p_\lambda\}_{\Lambda_p} \models \Gamma_p \) by an infomorphism \( (f^i_\lambda, f^\vee_\lambda) : D_i \Rightarrow FD(C_p) \) for each \( i \) and the embedded classification \( D_i \). We define an infomorphism \( (g^\wedge, g^\vee) : FD(\oplus C_i) \Rightarrow FD(C_p) \) which derives the refinement between \( \varphi(N_p) = \{a^p_\lambda\}_{\Lambda_p} \models \Gamma_p \) and the integration \( \mu_O(\langle \varphi(N_i) \rangle_{1 \leq i \leq n}) \).

For types, it is enough that \( g^\wedge \) is defined for each generator \( \alpha(i)_\lambda \in FD(\oplus C_j) \). This generator is the embedded type of \( \alpha(i)_\lambda \in \text{Typ}(FD(C_1)) \) by \( \text{inc}^\wedge \) and we can define \( g^\wedge(\alpha(i)_\lambda) = f^i_\lambda(\alpha(i)_\lambda) \).

For tokens, the image \( f^\vee_\lambda(\{c^p_\lambda\}_M) \) for an element \( \{c^p_\lambda\}_M \in \text{Tok}(FD(C_p)) \) equals to \( \bigcup_{\lambda \in M} f^\vee_\lambda(\{c^p_\lambda\}) \) by assumption, and it can be written as the form \( \{b^k_{\lambda_i}\}_{\Lambda_i} \in \text{Tok}(FD(C_1)) \) with a fresh index set \( \Lambda_i \). We can aggregate the images for all \( i \) and define \( g^\vee(\{c^p_\lambda\}_M) = \{b^k_{\lambda_i}\}_{\bigcup_i \Lambda_i} \in \text{Tok}(FD(\oplus C_j)) \) where \( k = i \) and \( b^k_{\lambda_i} = b^k_{\lambda} \) if \( \lambda \in \Lambda_i \).

Assume that \( g^\vee(\{a^p_\lambda\}_{\Lambda_p}) \models (\alpha(i))_\mu \) in \( FD(\oplus C_j) \). It is rewritten to \( \{b^k_{\lambda_i}\}_{\bigcup_i \Lambda_i} \models (\alpha(i))_\mu \), and is reduced to \( \{b^k_{\lambda}\}_{\Lambda_i} \models (\alpha(i))_\mu \). The following equivalence

\[
\{b^k_{\lambda}\}_{\Lambda_i} \models (\alpha(i))_\mu \quad (\text{in } FD(\oplus C_j))
\iff \{b^k_{\lambda}\}_{\Lambda_i} \models (\alpha(i))_\mu \quad (\text{in } FD(C_i))
\iff f^\vee_\lambda(\{a^p_\lambda\}_{\Lambda_p}) \models (\alpha(i))_\mu \quad (\text{in } FD(C_i))
\iff \{a^p_\lambda\}_{\Lambda_p} \models f^\vee_\lambda(\alpha(i))_\mu \quad (\text{in } FD(C_p))
\iff \{a^p_\lambda\}_{\Lambda_p} \models g^\wedge(\alpha(i)_\mu) \quad (\text{in } FD(C_p))
\]

shows \( (g^\wedge, g^\vee) \) is an infomorphism.

The token of the integrated effect \( \mu_O(\langle \varphi(N_i) \rangle_{1 \leq i \leq n}) \) is of the form \( \{a^{(k\lambda)}_\lambda\}_\Lambda \) where \( \Lambda_i \subset \Lambda \) and \( a^{(k\lambda)}_\lambda \) equals to \( a^\lambda_\lambda \) in the token part of \( \varphi(N_i) \) if \( \lambda \in \Lambda_i \). Namely, this token is \( \bigcup_{1 \leq i \leq n} f^\vee_\lambda(\{a^p_\lambda\}_\Lambda) \) which equals to \( g^\vee(\{a^p_\lambda\}_\Lambda) \).
Finally,
\[
\mu_0(\langle \varphi(N_i) \rangle_{1 \leq i \leq n}) = \bigvee_{1 \leq i \leq n} \text{inc}^\wedge(\Gamma_i)
\]
\[\iff\]
\[
g^\wedge(\{a^p_\lambda\}_\Lambda) = \bigvee_{1 \leq i \leq n} \text{inc}^\wedge(\Gamma_i)
\]
\[\iff\]
\[
\{a^p_\lambda\}_\Lambda \models g^\wedge(\bigvee_{1 \leq i \leq n} \text{inc}^\wedge(\Gamma_i))
\]
\[\iff\]
\[
\{a^p_\lambda\}_\Lambda \models \bigvee_{1 \leq i \leq n} g^\wedge(\text{inc}^\wedge(\Gamma_i))
\]
\[\iff\]
\[
\{a^p_\lambda\}_\Lambda \models \bigvee_{1 \leq i \leq n} f^\wedge_i(\Gamma_i)
\]
and the type \(\bigvee_{1 \leq i \leq n} f^\wedge_i(\Gamma_i)\) in the last is less than \(\varphi(N_p)\).

Next, consider the case of AND branch (SAND branch case is proved in the same way). Suppose the tuple \(\langle \{a^i_\lambda\}_\Lambda \models \Gamma_i \rangle_{1 \leq i \leq k}\) is a refinement of \(\{a^i_\lambda\}_\Lambda \models \Gamma_p\) by an infomorphism \((f^\wedge, f^\vee) : (\mathcal{F}D(C_i))_{1 \leq i \leq k} \Rightarrow \mathcal{F}D(C_p)\).

By Proposition \(2\), the classification \((\mathcal{F}D(C_i))_{1 \leq i \leq n}\) is embedded into \(\mathcal{F}D(\oplus C_i)\), and we denote the embedded image by \(D\). Here,
\[
\text{Tok}(D) = \{ \{c^{(k_\lambda)}_M \mid 1 \leq i \leq n, \exists \lambda \in M, k_\lambda = i \}, \text{ and}
\]
\[
\text{Typ}(D) = \{ \bigwedge_{1 \leq i \leq n} \Delta_i^{(i)} \mid \Delta_i \in \text{Typ}(\mathcal{F}D(C_i)) \}.
\]
We can define the infomorphism \((g^\wedge, g^\vee) : D \Rightarrow \mathcal{F}D(C_p)\) by
\[
g^\wedge(\bigwedge_{1 \leq i \leq n} \Delta_i^{(i)}) = f^\wedge((\bigwedge_{1 \leq i \leq n} \Delta_i)_{1 \leq i \leq n}), \text{ and}
\]
\[
g^\vee(\{c^p_\lambda\}_M) = \bigcup_{1 \leq i \leq n} f^\vee(\{c^p_\lambda\}_M)^{(i)}.
\]
It indeed satisfies the relation (IM) due to the assumption.

Since the family \(\{a^i_\lambda\}_\Lambda\), is the \(i\)-th component of \(f^\vee((\{a^p_\lambda\}_\Lambda)_i)\), its aggregation \(\bigcup_{1 \leq i \leq n} f^\vee(\{a^p_\lambda\}_\Lambda)_i^{(i)}\), the token part of \(\mu_A(\langle \varphi(N_i) \rangle_{1 \leq i \leq n})\), has the preimage by \(f^\vee\). Therefore, \(\mu_A(\langle \varphi(N_i) \rangle_{1 \leq i \leq n})\) is lifted to \(\{a^p_\lambda\}_\Lambda \models f^\wedge((\bigwedge_{1 \leq i \leq n} \Gamma_i)_{1 \leq i \leq n})\) by (IM) and we can see its type part is less than \(\Gamma_p\).

\[\Box\]

### 3.4. Mitigation and Effects

Attacks can be treated by countermeasures. A countermeasures prevents an attack or modifies its results, and therefore it mitigates the effect of the attack. Let us consider the classification \(\mathcal{F}D(C)\) for an attack. When the original effect is \((\bar{a} \models \Gamma)\) and \(\Gamma = \Gamma' \land \Gamma''\), we can reduce the effect to \(\bar{a} \models \Gamma'\) by canceling the part \(\Gamma''\). This reduction is also presented as \(\Gamma \leq \Gamma'\) with respect to the order of \(\text{Typ}(\mathcal{F}D(C))\). For example, if a countermeasure prevents the effect completely,
then the whole of $\Gamma$ is canceled and the residual part $\Gamma'$ is the top element $\top$. It can be interpreted as a valid situation that any primitive type in $\text{Typ}(\mathcal{F}D(C))$ does not hold for any token.

Next we consider abstract-refinement relations and the reduction of effects. Although we can assign a countermeasure to each effect independently, these countermeasures may break the refinement relation in general. In order to keep refinement relations, mitigations must be also related.

The next proposition states the maximal mitigation for effects in the classification of refinement. Assume $(\bar{a} \models_C \Gamma)$ in $\mathcal{F}D(C_C)$ is a refinement of $(\bar{b} \models_A \Delta)$ in $\mathcal{F}D(C_A)$ by $(f^\wedge, f^\vee)$.

**Proposition 5.** Consider type $\Delta'$, which is a reduction of $\Delta$. If the reduction $\Gamma'$ of $\Gamma$ preserves the abstract-refinement relation with $\Delta'$, then they are restricted by the inequality $f^\wedge(\Gamma') \vee \Delta \leq \Delta'$.

The inequality is derived from $f^\wedge(\Gamma') \leq \Delta'$ and $\Delta \leq \Delta'$.

By this proposition, type $\Gamma$ cannot be mitigated completely if $\Delta$ is not as well, for example.

We can observe an application of Proposition 5 to a branch in an attack tree. With the same notations used in Section 3.3, the effects of child nodes are written as $\bar{a}^1 \models \Gamma_1$, $\bar{a}^2 \models \Gamma_2$, ..., $\bar{a}^n \models \Gamma_n$, and the effect of the parent node is written as $\bar{a}^p \models \Gamma_p$. For a consistent branch, if each $\Gamma_i$ is reduced to $\Gamma'_i$, then the reduced effects must satisfy the inequality

$$f^\wedge(\mathbb{I}_{i} \wedge \Gamma'_i) \vee \Gamma_p \leq \Gamma'_p$$

where $\mathbb{I}$ is either $\wedge$ or $\vee$, depending on the branch type. Moreover, the LHS is equal to $\mathbb{I}_i f^\wedge(\Gamma'_i)$, if $f^\wedge$ preserves $\wedge$ and $\vee$. Then, we can state a corollary as follows.

**Corollary 2.** For a consistent OR branch, if the residual effects preserve consistency, then none of $f^\wedge(\Gamma'_i)$'s is greater than $\Gamma'_p$. In other words, the mitigated effect of the parent node is weaker than that of each child node.

However, the same cannot be said for AND branches. There may exist $\Gamma_i$'s such that each $f^\wedge(\Gamma'_i)$ is not less than $\Gamma'_p$, while their conjunction $\wedge_i f^\wedge(\Gamma'_i)$ is less than $\Gamma'_p$. Furthermore, another factor prevents us from consideration for SAND branches. The effects of attacks in a SAND branch depend on the preceding effects and attacks, and thus mitigations for effects are constrained by the relationship with not only a parent’s effect but also the preceding ones.

### 4. Case Study

Here, we apply the ideas described in Section 3 to practical situations. The first case study is about improving an attack tree (Section 4.3). We make explicit invalid branches in an attack tree with effects and propose correct decompositions for them. The second case study concerns mitigating effects (Section 4.4). Consistency is given for an attack tree rigorously, and possible degrees of mitigation are considered. Beforehand, the entire security analysis and the target system are described as a context.
4.1. Process Overview

The security analysis process specified in JASO TP15002 [14] consists of five phases: ToE (Target of Evaluation) definition, Threat analysis, Risk assessment, Define security objectives, and Security requirement selection. We follow a concretization of it studied in [26]. It also reflects a refactoring of the process and data in the original security analysis process.

A deliverable of Risk assessment phase is a list of identified threats to ToE. Threats in this list are prioritized, and significant ones are analyzed in depth using tree diagrams. Then, countermeasure goals for the threats are discussed in Define security objectives phase.

4.2. Target of Evaluation (ToE)

Based on the model in [27], a vehicular network system can be specified as the ToE. While the entire network system is analyzed in [26], the focus of this paper is on specific parts of the network (Fig. 5). Concretely, in Section 4.3 we consider the identified threat “Authentication function in Powertrain is interfered via a Tire Pressure Monitoring System (TPMS).” Similarly, in Section 4.4 we consider “Authentication information in Infotainment is stolen via smartphone.” Both threats are estimated to be significant, since the target modules are close to their entry points and the potential damages to the vehicle system are severe.

4.3. Verifying Decompositions

In this subsection, we consider the threat “Authentication function in Powertrain is interfered via TPMS.” Fig. 5(a) shows an early version of the attack tree for the threat (only the upper part is outlined). It should be noted that attacks on node labels include the events that contribute to invoking the parent but are not performed by the attacker. Policies and methods used to construct the tree are slightly ambiguous, and the structures of modules (Fig 5) are not considered well. Therefore, sub-attacks of an attack are intuitively selected; some of the sub-attacks do not refine the parent but are expected to occur preceding the parent. Here, decomposition is interpreted as...
causal ones implicitly. Moreover only OR and AND branches are considered. As a result, the following two inconsistencies are found:

1. A temporal gap among attacks around a branch. Attack A1 has to occur before its parent A0, but attack A2 and its parent A0 occur simultaneously.

2. A violated refinement order. The Msg. identification function in A1 refines the Authentication function in A0, whereas Powertrain Software mentioned in A1.1 is a wider entity than Msg. identification function in A1.

These un-structural situations are made explicit by considering the effects of attacks. As mentioned in Section 3, effects are changes in ToE and its environment caused by specific actions. Here, we consider the entities affected by attacks for all nodes and identify the effects on them derived from the attacks. The results are illustrated in Fig. 6(b).

First, let us examine the relations between A0 and its children A1 and A2. The correspondences of tokens are appropriate; namely, TPMS msg identification function presented by \( \text{MsgIdF}_{\text{PT}} \) is a refined entity of Authentication function. On the other hand, we observe that an unintended behavior expresses a dynamic aspect of the function while invalidness is a static one. The invalidness cannot be related to the unintended behavior semantically. Therefore, we conclude that no abstract-refinement relation exists between E0 and E1 and the branch around A0 is inconsistent, if we consider infomorphisms reflecting ToE.

Next, let us move on to the branch around A1. Clearly Powertrain Software is a higher concept in Fig. 5 and we cannot define any mapping of tokens that maps \( \text{MsgIdF}_{\text{PT}} \) to \( \text{Software}_{\text{PT}} \). The branch is inconsistent as we cannot construct infomorphisms realizing the abstract-refinement relation.

The revised version of the attack tree is given in Fig. 6(c). The second inconsistency mentioned above is resolved by emphasizing the target (A1.1.1). We modify node A1 and its subtree to ensure consistency. The targets of nodes in the subtree are unified to TPMS msg identification function presented by \( \text{MsgIdF}_{\text{PT}} \), yielding the common classification for considering their effects; in particular, token parts of infomorphisms realizing abstract-refinement relations are identity maps. Moreover, the cut sequence of effects E1.1 and E1.2 consists of a single relation \( \text{MsgIdF}_{\text{PT}} \models \text{UI} \text{behavior} \). At last, all branches are consistent in the tree, and the entire attack tree is consistent as well.

### 4.4. Degrees of Possible Mitigation

In this subsection, we consider the threat “**Authentication information in Infotainment is stolen via BT/Wifi/IR (smartphone)**.” The attack tree for the threat is depicted in Fig. 7. In particular, we focus on branch A1, where reverse engineering is attempted on the mobile device that has connected to the Infotainment in the past.

Primitive tokens and types to describe effects are listed in Table 1. Just two kinds of classifications are shown, since plural nodes refer their tokens and types by the same names. We assume the order \( \text{Disc} < \text{Acc} \), meaning that Information disclosure includes the situation that the information is accessible (but not vice versa), while orders of other pairs of primitive elements are not cared.

First, we check that the branch at node A1 is consistent by defining an infomorphism \( (f^\wedge, f^\vee) \). Attack A1.2 (analyzing the device) can be done under effect E1.1 (data in the device are accessible), and attack A1.3 can be done under effect E1.2 as well. Since the cut sequence of the
Node labels:
([AuF.PT], [msgId.PT] mean ‘Authentication function in Powertrain,’ ‘TPMS message identification function in Powertrain’.)
A0: [AuF.PT] is interfered with via TPMS.
A1: [msgId.PT] is tampered with.
A1.1: Powertrain Software is tampered with.
A2: [msgId.PT] is interfered with by DoS.

Effects
E0: AuthF_{PT} \nvDash UI_{behavior} (Unintended behaviors in Authentication function of Powertrain)
E1: MsgIdF_{PT} \nvDash invalid (Installed TPMS msg identification function is invalid)
E1.1: Software_{PT} \nvDash invalid (Installed Powertrain Software is invalid)
E2: MsgIdF_{PT} \nvDash unavailable (TPMS msg identification function is not available)

(a) Early version
(b) Early version with effects

Revised node labels:
A1: Unauthorized [msgId.PT] is invoked.
A1.1: [msgId.PT] is tampered with.
A1.1.1: [msgId.PT] is tampered with by replacing Powertrain software.
A1.2: (Unauthorized) [msgId.PT] is invoked.

Revised effects:
E1: MsgIdF_{PT} \nvDash UI_{behavior} (Unintended behaviors in TPMS msg identification function)
E1.1: MsgIdF_{PT} \nvDash invalid (Installed Powertrain Software is invalid)
E1.1.1: MsgIdF_{PT} \nvDash invalid (Installed TPMS msg identification function is invalid)
E1.2: MsgIdF_{PT} \nvDash UI_{behavior} (Unintended behaviors in TPMS msg identification function).

(c) Improved attack tree

Figure 6. Improvement of an attack tree
\textbf{Triangular Nodes:}

The triangles connected to A2 and A3 indicates their decompositions are left undeveloped.

\textbf{Node Labels:}

({\[AuI.I\]} and {\[AuF.I\]} means Authentication Information and Authentication Function in Infotainment respectively.)

A0: [\[AuI.I\]] is stolen via BT/Wifi/IR (smartphone).
A1: [\[AuI.I\]] is stolen by reverse engineering.
A2: [\[AuI.I\]] is obtained by brute-force.
A3: [\[AuI.I\]] is obtained by eavesdropping BT/Wifi/IR.
A1.1: Procuring a device which had connected to the target.
A1.2: Analyzing the device.
A1.3: Identifying [\[AuI.I\]].

\textbf{Effects:}

E0, E1, E2, E3, E1.3: \[\text{Disc}(\text{[\[AuI.I\]} is disclosed)]
E1.1: \[\text{Acc}(\text{Data in the device is accessible})
E1.2: \[\text{Disc}(\text{Data in the device is disclosed})

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{attack_tree.png}
\caption{Attack tree for “Authentication information is stolen”}
\end{figure}

\begin{table}[h]
\centering
\caption{Table 1. Token and types to describe effects for the tree in Fig. 7}
\begin{tabular}{|c|c|}
\hline
\textbf{Token} & \textbf{Type} \\
\hline
\[\text{AuI.I}, \text{AuF.I}, (\varepsilon)\] & \text{Disc (Disclosed), Acc (Accessible), Mod (Modified), Inv (Invalid), Unav (Unavailable), Ubhv (Unintended behavior)} \\
\hline
\multicolumn{2}{|c|}{Token-Type relations} \\
\hline
\text{} & \text{x} \Leftrightarrow \text{Disc}, \text{x} \Leftrightarrow \text{Acc} (x \in \{\text{AuF, AuI}\}) \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{The classification $C_{0}$ [re. $C_{1}, C_{1.3}$] for Node A0 [re. A1, A1.3]}
\begin{tabular}{|c|c|}
\hline
\textbf{Token} & \textbf{Type} \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{The classification $C_{1.1}$ [re. $C_{1.2}$] for Node A1.1 [re. A1.2]}
\begin{tabular}{|c|c|}
\hline
\textbf{Token} & \textbf{Type} \\
\hline
\text{Mech (mechanical part), Data, Pgm, (\varepsilon)} & \text{Disc, Acc, Mod, Inv, Unav, Ubhv} \\
\hline
\multicolumn{2}{|c|}{Token-Type relations} \\
\hline
\text{} & \text{x} \Leftrightarrow \text{Disc}, \text{x} \Leftrightarrow \text{Acc, Mech} \Leftrightarrow \text{Acc} (x \in \{\text{Data, Pgm}\}) \\
\hline
\end{tabular}
\end{table}
effects of child nodes A1.1, A1.2 and A1.3 is \((\text{Data} \models \text{Disc}, \text{AuI.I} \models \text{Disc})\), the infomorphism is \((\mathcal{FD}(C_{1.2}), \mathcal{FD}(C_{1.3})) \models \mathcal{FD}(C_1)\), and we can define the token part and the type part as follows.

\[
f^\wedge((x, y)) = \begin{cases} 
\text{Disc} & (x, y) = (\text{Disc}, \text{Disc}) \\
\text{Acc} & (x, y) = (\text{Disc}, \text{Acc}), (\text{Acc}, \text{Disc}), \text{or } (\text{Acc}, \text{Acc}) \\
\top & \text{(otherwise)}
\end{cases}
\]

and the mapping is extended for compound types naturally. For tokens, all except one token are mapped to the trivial pair.

\[
f^\lor(x) = \begin{cases} 
\{\{\text{Data}\}, \{\text{AuI.I}\}\} & (x = \{\text{AuI.I}\}) \\
\{\{\varepsilon\}, \{\varepsilon\}\} & \text{(otherwise)}
\end{cases}
\]

By \((f^\wedge, f^\lor)\), the cut sequence mentioned above is equivalent to \((\text{AuI.I} \models \text{Disc})\) in \(\mathcal{FD}(C_1)\), which is the effect of node A1. Therefore, \((f^\wedge, f^\lor)\) realizes refinement and the branch is consistent.

Now let us consider reducing effect E1.2 to \((\text{AuI.I} \models \text{Acc})\). The definition of \(f^\wedge\) indicates that the type parts of E1.2 or E1.3 are reduced to \text{Acc} in order to keep the abstract-refinement relation. Moreover, the dependency between them restricts the situation that E1.2 remains unchanged. Hence, the possible mitigated effects are that the type of E1.3 is reduced to \text{Acc}. Finally, as a mitigation, we can encrypt authentication information in the device so that it is not identified.

\section{5. Concluding Remarks}

Attack trees are a major tool for security analysis, as they allow us to formally derive qualitative/quantitative properties of attacks. However, the attack decompositions in attack trees have been justified only by reviews. The chains of formalism in attack tree processes are unlinked at the construction of the trees.

At first in this paper, we defined a new formal system of attack trees (with sequential conjunctions). It is a generalization of the existing framework [7], and an attack tree is interpreted as a set of its refinement scenarios.

Next, we proposed a validation of decompositions in attack trees. An effect is considered for every attack, and based on those effects, the consistency of a branch is defined. The framework for describing effects applies the theory of the classifications with infomorphisms. We sought to verify the abstract-refinement relations around branches in attack trees rigorously and to analyze the structure of the attack. The chains of formalism are linked with each other, resulting in a complete application of the formal approach to security analysis.

One application of the framework relates to mitigation of attacks. A mitigation for an attack cancels some part of the original effect of the attack. Possible degrees of mitigation can be measured with effects.

Several issues remain for future research:

- From a practical viewpoint, the simplification of attack trees is important. Equivalence relations between trees can give us a solution. Indeed, the frameworks [6, 7] introduced the
equivalences and discussed detailed properties with transforming attack trees. Though the formal system of attack trees in this paper is relatively rigid, appropriate equivalence and transformation will improve security analysis with attack trees.

- The discussion on mitigation can be improved. In this paper, we considered the distributive lattice structures of $\text{Typ}(FD(C))$. The description is weak in the sense that the canceled part of the original effect is not always identified. Namely, not all $\Gamma \in \text{Typ}(FD(C))$ can be expressed in the form $\Gamma' \land \Gamma''$. Hence, we can operate only the residual part. When we focus on the specified effects occurring in a target system, we may identify the canceled part like the above and evaluate the possible degrees of mitigation more concretely.

- We can verify the consistency of a branch in an attack tree with effects, but how do we decompose an attack accurately? In the case study, we show a way to do it following the structure of the target system. It seems to work well in many cases but is not comprehensive. In particular, it is difficult to find sequential attacks (for SAND branches) by observing the static structure. Methodologies deriving attack decompositions systematically are expected.

- Tool support. Defining and managing classifications for every node in an attack tree with infomorphisms is a tedious work. Software for attack trees with effects will improve the quality of security analyses.

A. Projection on Causal Attack Trees

A.1. Intermediate Semantics of Attack Trees

Here, we review the formal definition of attack trees proposed in Section 4 of [7]. Attack trees are binary trees with three types of branches, and they allow several equivalent transformations. They are referred to as causal attack trees, and we also use this term in order to distinguish them from our definition. Causal attack trees are interpreted as sets of directed graphs such that their vertices are labeled with the attacks of leaf nodes in the original trees.

**Definition 8.** A causal attack tree is a binary tree constructed from the following rule:

$$t ::= a \mid t \land t \mid t \lor t \mid t \cdot t,$$

where $a \in \text{Atom}$, the set of symbols presenting atomic attacks.

The operator $\land$ [re. $\lor$, $\cdot$] corresponds to an AND [re. OR, SAND] branch. The associativity and commutativity for $\land$ and $\lor$ are assumed, and distributivity $(t \lor u) \land v = (t \land v) \lor (u \land v)$ and idempotency $u \land u = u$ as well. However, for $\cdot$, only associativity and distributivity with OR, i.e., $t \cdot (u \lor v) = (t \cdot u) \lor (t \cdot v)$ and $(u \lor v) \cdot t = (u \cdot t) \lor (v \cdot t)$, are assumed.

Although two kinds of semantics are introduced to deal with the specialization of the tree effectively, they are induced from the following generic semantics.

**Definition 9.** The intermediate semantics of causal attack trees $[t]_M$, taking values in the set of directed graphs whose vertices are labeled with Atom, is defined as follows:

- $[a]_M = \{G_a\}$, where $G_a$ is the graph consisting of a single vertex labeled with $a$ and no edge.
• \([t_1 \lor t_2]_M = \lfloor t_1 \rfloor_M \cup \lfloor t_2 \rfloor_M\).
• \([t_1 \triangle t_2]_M = \{\tau_1 \sqcup \tau_2 \mid \tau_1 \in \lfloor t_1 \rfloor_M, \tau_2 \in \lfloor t_2 \rfloor_M\}\), where the juxtaposition of graphs \(S\) and \(T\) is denoted by \(S \sqcup T\).
• \([t_1 \cdot t_2]_M\) is the set of pointwise sequential composition of \(\lfloor t_1 \rfloor_M\) and \(\lfloor t_2 \rfloor_M\). i.e. the set of graphs constructed with every pair of \(g_1 \in \lfloor t_1 \rfloor_M\) and \(g_2 \in \lfloor t_2 \rfloor_M\), where these two graphs are juxtaposed, and all of possible edges from \(g_1\)'s vertices to \(g_2\)'s vertices are added.

Fig. 8 illustrates an interpretation of a causal attack tree.

A.2. The Projection to Causal Attack Trees

Here, we see the relationship between attack trees in \(AT\) and causal attack trees defined in Section A.1.

First, R-trees, the atomic refinement scenarios of attack trees, are projected to directed graphs.

**Definition 10.** The projection \(\pi\) from \(AT_R\) to the set of directed graphs with labels is defined as follows:

- \(\pi(Lf(a)) = G_a\). i.e. the R-tree having only the root node is projected to the singleton graph.
- \(\pi(Nd(n, \text{AND}, (\tau_1, \ldots, \tau_m))) = \pi(\tau_1) \sqcup \ldots \sqcup \pi(\tau_m)\).
- \(\pi(Nd(n, \text{SAND}, (\tau_1, \ldots, \tau_m)))\) is the graph \(\pi(\tau_1) \sqcup \ldots \sqcup \pi(\tau_m)\) with additional edges \(\{(v^{(i)}, w^{(i+1)}) \mid v^{(i)} \in \pi(\tau_i), w^{(i+1)} \in \pi(\tau_{i+1}), 1 \leq i \leq m - 1\}\).

We abuse the symbol \(\pi\) as a function on the set of attack trees. Namely \(\pi(\{t_1, t_2, \ldots, t_m\}) = \{\pi(t_1), \pi(t_2), \ldots, \pi(t_m)\}\).
Attack trees in $\mathcal{AT}$ can be interpreted with intermediate semantics for causal attack trees. Notice that it is easy to transform an attack tree into a causal one. Indeed, a sequence of each node’s children can be expressed as nested binary terms (dummy labels are required for intermediate nodes). This transformation is denoted by $\beta$. By the following proposition, semantics are linked as well.

**Proposition 6.** The following diagram commutes:

$$
\begin{array}{ccc}
\mathcal{AT} & \xrightarrow{\beta} & \{\text{causal attack trees}\} \\
[\mathcal{I}] & \downarrow & \mathcal{I}_M \\
\text{Pow}(\mathcal{AT}_R) & \xrightarrow{\pi} & \text{Pow}(\{\text{directed graphs with labels}\})
\end{array}
$$

where $\text{Pow}(X)$ means the power set of $X$.

**Proof.** We can check the proposition for the type of the top branch. Denote a sequence $\langle t_1, \ldots, t_m \rangle$ by $\vec{t}$.

Let us take the element $Lf(a)$, the tree having only the root node. The equalities $\pi([Lf(n)]) = \{G_n\}$ and $[\beta(Lf(n))]_M = [Lf(n)]_M = \{G_n\}$ indicates the proposition.

For the compound attack tree, the proof is divided with respect to the type of the branch:

- **For an AND branch,**
  $$\pi([Nd(n, \text{AND}, \vec{t})]) = \pi(\{Nd(n, \text{AND}, \langle \tau_1, \ldots, \tau_m \rangle) | (\tau_1, \ldots, \tau_m) \in [\vec{t}]\}) = \{\tau_1 \sqcup \cdots \sqcup \tau_m | (\tau_1, \ldots, \tau_m) \in [\vec{t}]\}$$
  holds, and each element in RHS appears in $[\beta(Nd(n, \text{AND}, \vec{t}))]_M$.

- **For a SAND branch,** the proposition is proved in similar way. An element in $\pi([Nd(n, \text{SAND}, \vec{t})])$ is of the form $\tau_1 \sqcup \cdots \sqcup \tau_m$ with additional edges, and it also appears in $[\beta(Nd(n, \text{SAND}, \vec{t}))]_M$.

- **For an OR branch,**
  $$\pi([Nd(n, \text{OR}, \vec{t})]) = \pi(\bigsqcup_{1 \leq i \leq m} \{Nd(n, \text{AND}, \langle \tau \rangle) | \tau \in [t_i]\})$$
  $$= \bigsqcup_{1 \leq i \leq m} \pi(\{Nd(n, \text{AND}, \langle \tau \rangle) | \tau \in [t_i]\})$$
  $$= \bigsqcup_{1 \leq i \leq m} \pi([t_i])$$
  holds. It equals to $[\beta(Nd(n, \text{OR}, \vec{t}))]_M$. \hfill $\square$

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