$B_d - B_d$ mass difference in Little Higgs model

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Abstract

An alternate solution of hierarchy problem in the Standard Model namely, the Little Higgs model, has been proposed lately. In this work $B_d^0 - B_d^0$ mass difference in the framework of the Little Higgs model is evaluated. The experimental limits on the mass difference is shown to provide meaningful constraints on the parameter space of the model.
Our understanding of the Standard Model (SM) is plagued by a major issue, the “hierarchy problem”, arising out of the enormous difference between the electroweak and the Plank scale. For quite some time, Supersymmetry had provided an elegant framework for solving this problem although to-date there is no compelling experimental evidence in its support. During the last two years, an alternative possibility has been introduced in the literature where the Higgs mass remains small by virtue of it being a Goldstone boson of a global symmetry which is broken at a scale above the electroweak scale. These models are generically called the “Little Higgs” models and the simplest of these, the “Littlest Higgs” (LH) model [2], has the least number of additional particles involved.

In the gauge sector, the LH model contains weakly coupled gauge bosons with masses in the TeV scale in addition to the SM $W$ and $Z$ [3, 4]. These mix amongst themselves causing modification of SM gauge couplings of $W$; $Z$ with fermions and among themselves. In the quark sector, a vector-like heavy top quark comes into play with mass in TeV range, which has trilinear coupling with SM gauge bosons. Once again the heavy top quark has mixing possibility with the SM top quark, resulting in modification of coupling structure of quarks with $W$ and $Z$. In addition, the model has charged Higgs bosons which introduce scalar couplings with quark. Also, a heavier photon with mass in the TeV range emerges, which couples both to leptons and quarks.

The presence of these new particles as well as changes in the SM interaction vertices, can cause changes in a variety of measurable parameters. Some of them have already been calculated in the literature [3, 5–8]. These results provide good constraints on the parameters entering the LH model. Direct experimental confirmation of several aspects of LH, e.g., the masses of the heavy t-quark and the doubly charged Higgs, would require sharper estimates of the parameters of the theory. It is desirable therefore, to work out the consequences of the LH-model for as many observable quantities as possible in order to sharpen the constraints on the parameter space of such a model. In this note, we report on a calculation of $B_0 \rightarrow B_0$ and $K_0 \rightarrow K_0$ mixing in the context of LH model.

In SM, there is one basic box diagram responsible for generating the effective Hamiltonian for the mixing of $B_0 \rightarrow B_0$ and $K_0 \rightarrow K_0$. In LH, there are many more box diagrams (as shown in Figure 1) to be evaluated. The couplings and propagators required for calculating these diagrams are listed in [3].

The effective Hamiltonian resulting for the graphs in Fig.1 has the structure:

$$H_{\text{eff}} = \frac{G_F^2}{16 \pi^2} M_{\tilde{w}}^2 S_q (q_d \ell)_V \lambda (q_d \ell)_V \lambda$$

with $q = b, s$ for $(B_0 \rightarrow B_0)$ and $(K_0 \rightarrow K_0)$ mixing respectively. The invariant function $S_q$ has the following form:

$$S_q = S_{q \text{SM}} + S_{q \text{LH}}$$
where in both $S_b$ and $S_s$, the first term represents the SM contribution along with QCD corrections which are given in detail in [9]. The second term gives the LH contribution to the mass difference. As these are the corrections to the SM contribution, we do not consider QCD corrections to them which would arise from gluonic loops added to the diagrams of Fig 1. The effective Lagrangian in the LH model to order $\frac{v^2}{\xi^2}$ is well approximated by:

$$L_{\text{eff}}^4 = \frac{G_f^2}{16} M_W^2 S_{ij}^{\text{LH}} Q \quad (4 \ J = 2)$$

where $J = B; S$ and $j = b; s$ for $B_d, B^0$ and $K^0, K^0$ respectively. They are given as:

$$Q \ (4 \ B = 2) = (\bar{b} \ d)_V A (\bar{b} \ d)_V A$$
$$Q \ (4 \ S = 2) = (\bar{s} \ d)_V A (\bar{s} \ d)_V A$$

and

$$S_{ij}^{\text{LH}} = \frac{v^2}{\xi^2} \left( \sum_{i=1}^{8} \chi_i \chi_i^{\prime} \right) + \frac{2v^2}{\xi^2} \left( \sum_{i,j=1}^{8} \chi_i \chi_j \right)$$

Figure 1: Box diagrams in LH.

We note that despite the occurrence of spinless Higgs couplings to quarks, the ultimate structure of the effective Hamiltonian in LH retains the same $(V \ A)$ form as in SM to order $\frac{v^2}{\xi^2}$. Given the form $\xi$ is the scale at which the global SU(5) symmetry is spontaneously broken via a vacuum expectation value which is expected to be in the TeV range and roughly of the order of masses of heavy bosons and $v$ is the vev of standard model Higgs.
of the effective Hamiltonian, we can proceed exactly as in SM and calculate its matrix element between $K_0$ or $B_0$ states using the vacuum saturation approximation. There are no divergences in the SM amplitude because of the unitarity of the CKM matrix; this statement holds even in LH model\(^2\) where once again the unitarity of CKM ensures that all divergences vanish to order $(v=f)^2$. Neglecting QCD corrections and long distance contributions we can get the mass difference to be:

$$4 \frac{M(B_0 \to B_0)^{LH}}{M(B_0 \to K_0)^{LH}} = \frac{G^2_F}{2} M_B \frac{M_2^2}{f_B^2} s_B^{LH}$$

and

$$4 \frac{M(K_0 \to K_0)^{LH}}{M(B_0 \to K_0)^{LH}} = \frac{G^2_F}{2} M_K \frac{M_2^2}{f_K^2} s_S^{LH}$$

where $M_B$ and $f_B$ are the masses and decay constants of B mesons respectively.

It should be mentioned that the renormalization group evolution of the matrix elements has been the subject of much work and has been summarized in [10] and is far from trivial since the matrix elements are controlled by long distance dynamics and are generally parameterized by a “Bag factor” $B_{K, B}$. However for the neutral B meson case, the long range interactions arising from the intermediate virtual states are negligible because of the large B mass, being far from the region of hadronic resonances.

The LH involves not only heavy vector bosons and quarks but also a large number of parameters over and above those in the SM. The global symmetry in the theory is broken at TeV range scale $s$ ($s = 4 f$); the scalar bosons, doublets and triplets, acquire vacuum expectation values $v$ and $v_0$ respectively at the EW-scale, providing the convenient small parameters $v=f$ and $v_0=f$. The mixing of the charged and neutral vector bosons results in two mixing angle parameters $\theta$ and $\theta_0$ (with $\cos \theta = c_s \cos \theta_0$; $\sin \theta = s_\theta \cos \theta_0$). Finally the Yukawa coupling of the fermions involves two parameters $\lambda_1$ and $\lambda_2$ with the combination $x_L = \frac{\lambda_1}{\lambda_1 + \lambda_2}$ occurring frequently. To the leading order in $(v=f)$, the masses of all the heavy particles in LH can be expressed in terms of SM masses $m_W$ and $m_Z$ as:

$$\begin{align*}
\frac{m_{W_{\pm}}}{m_W} &= \frac{1}{2} \frac{f}{v} \\
\frac{m_t}{m_W} &= \frac{\cos \theta}{\sqrt{v}} \\
\frac{m_{\tau}}{m_{W^+}} &= \frac{1}{2} \frac{2 + \frac{f}{v}}{\sqrt{v}} \\
\frac{m}{m_H} &= \frac{\sqrt{2}}{2} \frac{f}{\sqrt{v}}
\end{align*}$$

The coupling of all heavy particles to SM particles as well among themselves are expressible in terms of these parameters with the SM ones.

The parameter space is obviously too large. Requiring that the heavy particles have masses in TeV range results in the condition $\frac{1}{2} f < 10$. There is another restriction arising out of the requirement that the mass of the triplet scalars be positive definite [3]:

$$\frac{v^2}{v^2} < \frac{v^2}{16f^2}$$

\(^2\)the CKM matrix is unitary in LH up to order $v^2 = f^2$

\(^3\)The QCD corrections to these have been worked out in literature
Figure 2: $4 M (B_d \rightarrow B_d)$ in ps$^{-1}$ with $v=f$. For these plots we have used $s^0 = s$. Shaded area indicates the experimental bounds.

We have varied $v=f$ in the range 0 to 0.1. $s; s^0$ in range 0.2 to 0.8 and $x_L$ in range 0.2 to 0.8 in our numerical analysis. Other parameters used are given in the Appendix B.

Our results for the $B_d^0 \rightarrow B_d^0$ case are shown in Figure 2. Varying $s^0$ doesn’t significantly change our conclusions. The corresponding $K^0 \rightarrow K^0$ results too have similar trend. However, since there are large error bars in them because of QCD corrections involved, it makes it difficult to draw any definitive conclusions. Hence we haven’t shown them. In the plots shown in Fig.2 the shaded area corresponds to the mass difference $4 M (B_d \rightarrow B_d) = 0.5 \pm 0.01$ ps$^{-1}$ which is consistent with current experimental bounds [11].

From Fig 2, it is easy to note that for low values of $x_L$, there is a region of parameter space (in terms of the parameters $s$ and $v=f$) that is consistent with the experimental limits. Very specifically for $x_L = 0.2$, the bound on the scale $f$ can be very close to 1 TeV for almost all the $s$ values. However, as $x_L$ is increased, the LH contribution starts deviating significantly from the SM results. Therefore, in principle the experimentally allowed band for the $B_d^0 \rightarrow B_d^0$ mass difference provides significant constraints for the parameter space of generic Little Higgs models, and in particular the Littlest Higgs model.

It will be fruitful to compare the limits on the parameters coming from precision electroweak data [12]. To this end we recall that the Littlest Higgs model does not have the custodial symmetry inherently
built into it and can therefore, in principle, lead to large corrections, arising both from heavy gauge 
bozon exchange diagrams and the triplet VEV. A naive way out would be to have the extra gauge boson 
masses raised by some means. However, this would spoil the motivation of circumventing the hierarchy 
problem and would also bring in the issue of fine tuning. It has been found that global fits to precision 
electroweak data imply the following bound (at 95% C.L.) on the scale $f$ for any generic coupling 
(specifically varying $c$ and $c^0$ between 0.1 and 0.995):

$$f > 4 \text{ TeV}$$  \hspace{1cm} (9)

It is worth noting that this very stringent bound is almost (practically) independent of any variation of 
Higgs mass upto 200 GeV. Further, the bound still holds for any order unity value of the parameters 
encapsulating the physics due to proper UV-completion of the theory. To be noted is the fact that these 
are very strong limits and the origin can simply be traced back to the absence of custodial symmetry. 
In the second reference of [12] it was noted that considering only precision electroweak data allows 
for a small region in parameter space where the bound on the scale $f$ can be lowered to about 1 TeV. 
However, electroweak data combined with Drell-Yan production excludes this region and a combined 
analysis implies a bound very similar to the one quoted above. Constraints from low energy precision 
data like $(g - 2)$ and atomic weak charge of Cesium also indicate similar bounds, though it should be 
remembered that the rather small $(g - 2)$ corrections may not serve to put any meaningful bounds.

Interestingly enough, in almost all the variations of the Littlest Higgs model [13], the constraints 
remain quite strong and generically very similar to the minimal version, though for some very specific 
choices of the parameters the constraints on $f$ are relaxed to 1 – 2 TeV. This can be understood as 
arising due to small mixing between the two sets of gauge bosons and also small coupling between the 
fermions and heavy $U(1)$ gauge boson. Nevertheless, these arise only in very specific models and for 
very special choices of the parameters and are not a generic feature of LH models. It may also be useful 
to keep in mind that the positivity of triplet mass squared imposes severe constraints on the triplet VEV 
and therefore the parameters or combination of parameters entering the mass squared relation. This 
strong constraint considerably reduces the allowed parameter space. Therefore, it is natural to expect 
that in the variation of the minimal model where there is no triplet Higgs, the bounds are partially 
relaxed as is the case in models which have custodial symmetry built into them.

Turning to contribution to $4 M \left( B_d - B_d \right)$, we would indeed get a bound similar to eqn (9) above 
if we require that the LH contribution be no more than the experimental band for $4 M \left( B_d - B_d \right)$. 
However, as has been pointed out by Buras et.al. [1] that there is a hadronic uncertainty of about 10% 
in calculation of $4 M \left( B_d - B_d \right)$. In view of this a reasonable constraint on $(v = f)$ could be given if the 
variation in the mass difference could be more than 10%. For a contribution of about 10%, the LH 
model would require $v = f \approx 0 \text{.2}$, which is not very useful in view of the stronger constraint like eqn(9) 
above. However, should it become possible for hadronic uncertainties to be reduced by a factor of 2 
or 3, then the bound on $(v = f)$ values becomes much lower, leading to constraints on the value of $f$ 
comparable with the value in equation (9) above.
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A Loop Functions

\( W_L; W_H \) in eqn (5) refer to the light & heavy W-boson in LH; c,s are the mixing angles in LH. Various 's are

\[
\begin{align*}
W^2_1 &= 2c^2 (c^2 + s^2) \frac{x^2_1}{i}; \quad i = u; c \\
W^2_t &= 2c^2 (c^2 + s^2) + x^2_t \frac{x^2_1}{i}; t
\end{align*}
\]

(A.1)

(A.2)

(A.3)

(A.4)

(A.5)

(A.6)

(A.7)

(A.8)

the functions (E) used in eqn. (5) are:

\[
\begin{align*}
E(x_i; x_j; W_L) &= \frac{x_i x_j}{x_i x_j} \left( \frac{1}{4} + \frac{3}{2} \left( \frac{x_i}{x_j} \right) \log(x_i) \right) + \frac{3}{4} \left( \frac{x_i x_j}{x_i x_j} \right) \left( \frac{1}{4} + \frac{9}{4} \left( \frac{x_i}{x_j} \right) \log(x_i) \right) \left( \frac{1}{4} + \frac{3}{2} \left( \frac{x_i}{x_j} \right) \log(x_j) \right) \left( \frac{1}{4} + \frac{3}{2} \left( \frac{x_i}{x_j} \right) \log(x_j) \right) \\
E(x_i; W_L) &= \frac{3}{2} \left( \frac{x_i}{x_i} \right) \log(x_i) \left( \frac{1}{4} + \frac{9}{4} \left( \frac{x_i}{x_i} \right) \log(x_i) \right) \left( \frac{1}{4} + \frac{3}{2} \left( \frac{x_i}{x_i} \right) \log(x_i) \right) \left( \frac{1}{4} + \frac{3}{2} \left( \frac{x_i}{x_i} \right) \log(x_i) \right) \\
E(x_i; x_j; W_L; W_H) &= \frac{x_i x_j}{x_i x_j} \left( \frac{1}{4} + \frac{1}{x_H} \frac{x_i}{x_H} \left( \frac{1}{4} + \frac{1}{x_H} \frac{x_i}{x_H} \log(x_i) \right) \left( \frac{1}{4} + \frac{1}{x_H} \frac{x_i}{x_H} \log(x_i) \right) \left( \frac{1}{4} + \frac{1}{x_H} \frac{x_i}{x_H} \log(x_i) \right) \left( \frac{1}{4} + \frac{1}{x_H} \frac{x_i}{x_H} \log(x_i) \right) \right)
\end{align*}
\]

(A.9)

(A.10)

(A.11)
\[ E (x_i; W_L, W_R) = \frac{3}{4} \frac{x_1^3}{x_R^2 (1 - x_i)^2} \left( \frac{1}{x_R} \right) \log x_1 + \frac{3}{4} \frac{1}{x_R} \left( \frac{1}{x_R} \right) \left( \frac{1}{x_R} \right) \log (x_R) \] (A.12)

\[ E (x_i, x_j; W_L, x) = \frac{x_i x_j}{2} - \frac{x_1}{(x_i, x_j) (1 - x_i)} (x_i, x_j) \log (x_i) + \frac{x_1}{(x_i, x_j) (1 - x_i)} (x_i, x_j) \log (x_j) \] (A.13)

\[ E (x_i; W_L, x) = \frac{x_1^2}{2} \left( \frac{1}{x_i} \right) (x_i, x_j) \log (x_i) + \frac{x_1}{(x_i, x_j) (1 - x_i)} (x_i, x_j) \log (x_j) \] (A.14)

**B  Input parameters**

\[ G_F = 1 \times 10^{-5} \text{ GeV}^{-2} ; \epsilon_B = 0.21 ; m_B = 5.3 \text{ GeV} ; \]
\[ m_{W_L} = 80.4 \text{ GeV} ; m_{Z_L} = 91.2 \text{ GeV} \]

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