Electroweak double logarithms in inclusive observables for a generic initial state

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High energy observables are characterized by large electroweak radiative corrections of infrared origin; double logarithms are present even for inclusive cross sections, thus violating the Bloch-Nordsieck theorem. This effect, related to the initial states carrying nonabelian isospin charges, is here investigated for any inclusive cross section in the SU(2)⊗U(1) symmetric limit, that is appropriate for energies much higher than the weak scale. We develop a general formalism allowing to calculate the all order double log resummed cross sections once the hard (tree level) ones are known. The relevant cases of fermion-fermion, fermion-boson and boson-boson scattering are discussed.

I. INTRODUCTION

The behaviour of electroweak (EW) cross sections for energies \( \sqrt{s} \) much larger than the electroweak scale \( M \sim 100 \text{ GeV} \) is a theoretically interesting (and difficult) problem and a phenomenologically relevant one. The discovery of the physical origin of the leading asymptotic behaviour, related to the infrared (IR) structure of the electroweak theory, has opened the way to various attempts of all orders resummation, leading sometimes to controversial results, and to surprising features in some other cases. On one hand, electroweak symmetry breaking makes the calculation of exclusive quantities (i.e., observables including only photon emission) particularly hard, so that different results are present in the literature. On the other hand, precisely because of the unique features of weak interactions with respect to strong ones for instance, unsuppressed double logs (DL) of infrared origin are present also in inclusive quantities (where also W, Z emission is included), leading to violation of the Bloch-Nordsieck theorem.

In this paper we develop a general formalism that allows to obtain the expression for the leading EW radiative corrections to inclusive cross sections for any given hard process. Through this formalism, once the hard (Born level) cross section is known, resummation of EW effects at all orders at the leading (double log) level is easily obtained.

We work in the SU(2)⊗U(1) symmetric limit, with all invariants much bigger than the Higgs and heavy gauge bosons masses \( M_W \sim M_Z \sim M_H \equiv M \). To be specific, we consider processes with two partons (e.g., leptons, quarks, or bosons) in the initial state, characterized by a single hard scale, typically the c.m energy \( \sqrt{s} \), much greater than the EW symmetry breaking scale \( M \), and including emission of soft weak bosons \( \gamma, Z, W \) of energy \( \omega \ll \sqrt{s} \). All other hard scales are of the same order, namely \( |s| \sim |t| \sim |u| \gg M \). As has been noticed in the literature, despite being inclusive, this kind of process is characterized by large double logs \( \sim \log^2 \frac{\sqrt{s}}{M} \) of infrared origin, due to the fact that initial state particles carry nonabelian charges (weak isospin) which are fixed by the accelerator; this in contrast to QCD for instance where confinement forces averaging over initial colour.

Unlike the exclusive case, the radiative corrections to inclusive observables considered here are sensitive to the weak scale \( M \) only, because this scale represents the threshold for both the nonabelian double logs and gauge symmetry

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1 these (double) logarithms \( \sim \log^2 \frac{\sqrt{s}}{M} \) occur because at energies much larger than the EW scale \( M_Z \approx M_W \equiv M \), the latter acts as a cutoff for the collinear and IR divergences that would be present in the vanishing \( M \) limit.
restoration at the same time. For this reason, exponentiation to all orders was easily proved in [3] along the same lines as QCD, with a single IR cutoff $M$; this point is rediscussed in next Section. While only fermions were considered in the initial state in [3], we extend here the analysis to any initial particles belonging to a weak isospin multiplet, including gauge bosons. This generalization is essential in order to treat boson fusion processes, which are important for hadronic accelerators, like LHC.

II. RESUMMED OVERLAP MATRIX

In isospin space, the hard cross section structure is defined by the so called hard overlap matrix, describing the squared matrix element: $\langle \beta_1 \beta_2 | S_H^\dagger S_H | \alpha_1 \alpha_2 \rangle = O_H^{\beta_1 \beta_2, \alpha_1 \alpha_2}$ (see Fig. 2). While for cross sections we always have $\alpha_i = \beta_i$ ($\sigma_{\alpha \beta} \equiv O_H^{\alpha \beta, \alpha \beta}$), we leave open the possibility that $\alpha_i \neq \beta_i$ and see $O_H$ as an operator in isospin space with four indices.

Since we work in the $\text{SU}(2) \otimes \text{U}(1)$ symmetric limit, we can use isospin and hypercharge conservation in the form (notice the minus sign, due to the choice of momentum flow in Fig. 2)

$$\left( y_1 - y_1' \right) = \left( y_2 - y_2' \right) = \left( t_i^a - t_i'^a \right) = \left( t_2^a - t_2'^a \right) \quad a = 1, 2, 3$$

(1)

The isospin operator $t_1(t'_1)$ acts on the $\alpha_1$ ($\beta_1$) index, and so on. Here comes the important step: we change basis and couple the indices $\alpha_1, \beta_1$ that refer to leg 1 to form $\text{SU}(2)$ multiplets; the same is done for leg 2. Then, eqn. (1) can also be seen as the statement that the operator $O_H$ commutes with the total isospin in the $t$-channel $T_t \equiv t_1 - t_1'$.

From this, we can derive the form of $O_H$:

$$[T_t, O_H] = 0 \Rightarrow \langle t, t_3 | O_H | t', t'_3 \rangle = C_t \delta_{tt'} \delta_{t_3 t'_3} \Rightarrow O_H = \sum_t C_t P_t \quad ; \quad P_t = \sum_{t_3} |t, t_3 \rangle \langle t, t_3|$$

(2)

so that $O_H$ has the form of a sum of isospin multiplet projectors $P_t$, with coefficients $C_t$. The sum runs only over integers $t = 0, 1, 2, \ldots$ since we are coupling a particle and its own antiparticle in the $t$-channel. The coefficients $C_t$ depend on the specific process considered, and contain all the dynamics; for some relevant examples, see [3].

We now describe the dressing of the hard overlap matrix $O_H$ to give the dressed one $O$ through soft weak bosons emission and absorption with the external (initial) line insertions of the eikonal currents

$$J^u_a = g \frac{p^u_{k_1}}{k_{p_1}} (t^a_1 - t'^a_1) + \frac{p^u_{k_2}}{k_{p_2}} (t^a_2 - t'^a_2) \quad J^0_a = g' \left( \frac{p^u_{k_1}}{k_{p_1}} (y_1 - y_1') + \frac{p^u_{k_2}}{k_{p_2}} (y_2 - y_2') \right)$$

(3)

Here $a=1, 2, 3$ is the SU(2) index, we work in the unbroken basis $A_0 = cW^+ \gamma + sW^+ Z, A_3 = -sW^+ \gamma + cW^+ Z, g$ and $g'$ being the usual electroweak couplings with $\frac{sW}{cW} = \frac{\sqrt{2}}{2}$. Notice that, as is discussed thoroughly in [3], only initial state radiation needs to be included, since final state radiation automatically cancels between real and virtual corrections.

Before proceeding, we need to discuss an important and subtle point. Eqn. (3) includes, in the neutral sector, both the contributions of the massive Z boson and the massless photon. However, $\gamma, Z$ insertions are automatically cancelled out when summing real and virtual corrections; this is true already at the eikonal current insertion operator level, eqn. (3). This happens because, when considering cross sections, the diagonal operators $y_1 - y_1'$ and $t_1^a - t_1'^a$ give 0 contributions, since the quantum numbers of leg 1 and leg 1’ are obviously the same (see Fig. 2). For the sake of convenience, one can then reinset the vanishing $A_3$ contribution, ending up effectively with a SU(2) theory with infrared cutoff $M$. In other words, one can summarize like this: when considering completely inclusive observables in the SU(2)⊗U(1) electroweak sector of the Standard Model at the DL level in the recovered symmetry limit, one effectively needs to compute only initial state radiation in the corresponding SU(2) theory with gauge bosons $A_1^\pm, A_2^\pm, A_3^\pm$ and IR cutoff (effective mass) $M \sim M_Z \sim M_W$. Then, one ends up with the insertion operator:

$$I(k) \equiv g^2 \frac{2p_1 p_2}{(k_{p_1})(k_{p_2})} (t_1 - t_1') \cdot (t_2 - t_2') \quad (t_1 \cdot t_2 \equiv \sum_a t_1^a t_2^a)$$

(4)

where the charge factor can be replaced, because of the conservation (3), by

$$(t_1 - t_1') \cdot (t_2 - t_2') = -(t_1 - t_1')^2 = 2t_1 \cdot t_1' - t_1^2 - t_1'^2$$

(5)

The latter expression provides the charge computation in the axial gauge, because in this gauge the $W$ emission and absorption takes place on the same leg, for both virtual ($-2t_1'$) and real emission ($2t_1 \cdot t_1'$) contributions. We can see
from Fig. 1 the reason for noncancellation between virtual and real one loop corrections: the crucial point is that while $\gamma, Z$ emission does not change the initial state, $W$ emission does. Then, in the $W$ case virtual corrections and real corrections are of opposite sign but do not cancel completely.

From (2) we see that the eikonal insertion is described by the Casimir operator $T^2_i$. The resummed expression for the overlap matrix is given by the following expression, involving the energy ordered $(w_1 < w_2 < ... w_n$ where $w_i$ are the soft bosons energies) product $P_w$:

$$O(s) = P_w \left\{ \int dk I(k) \right\} O^H = \exp[-L_W T^2_i] O^H = \sum_t P_t C_t e^{-L_W (t+1)}$$

where we have defined the eikonal radiation factor for $W$ exchange:

$$L_W = \frac{g^2}{2} \int_{E_M}^{E} \frac{d^3k}{2w_k(2\pi)^3} \frac{2p_1p_2}{(kp_1)(kp_2)} = \frac{\alpha_W}{4\pi} \log^2 \frac{s}{M^2} \quad (\alpha_W = \frac{g^2}{4\pi})$$

and we have taken into account the fact that the SU(2) Casimir $T^2_i$ is diagonal on the projection operators $T^2_i P_t = t(t + 1) P_t$, turning the $w$-ordered exponential into a regular one.

Equation (4) is our main result, since it allows to know the all order DL resummed overlap matrix (and consequently the physical cross sections) at any energy $\sqrt{s}$, once the Born cross sections (or, equivalently, the coefficients $C_t$) for a given hard process are known.

It is clear from (3) that the only component that survives at very high energies is the singlet one $(t = 0)$, that doesn’t evolve with energy. This component corresponds to the averaged cross section $\bar{\sigma} = \sum_{i,j} \sigma_{ij}$ that is left unchanged by soft boson emission and gives the asymptotic value of all cross sections. On the other hand, all other components get bigger and bigger effects due to the presence of a isospin 1 state on leg 2:

These results agree with the ones already obtained in (4).

III. GENERALIZATION TO INITIAL TRANSVERSE BOSONS

Initial partons in practice are either gauge bosons or fermions; thus the phenomenologically relevant cases are fermion-fermion scattering, already analyzed in the previous section, and the boson-fermion and boson-boson cases that we consider here. The case of initial bosons deserves a special attention due to the presence of longitudinally polarized gauge bosons, which are peculiar since they are sensitive to the symmetry breaking Higgs sector even in the limit $\frac{M}{\sqrt{s}} \rightarrow 0$. Leaving the case of longitudinal bosons to a more detailed forthcoming analysis, we consider here only transverse gauge bosons, that we label with indices $+,3,-$ for the triplet and 0 for the singlet. Note that since we rely only on SU(2) invariance, the index 0 can represent not only the $A_0$ gauge boson, but also any SU(2) singlet like a gluon or a righthanded fermion.

Let us consider the case of fermion-boson scattering first. Again, the composition of two isospin $\frac{1}{2}$ states on leg 1 produces only two projection operators; this time however the explicit form of these operators is different from (6) due to the presence of an isospin 1 state on leg 2:

$$O_{\beta_1\beta_2,\alpha_1\alpha_2} = C_0 \delta_{\beta_1\alpha_1} \delta_{\beta_2\alpha_2} + C_1 T_{\beta_2\beta_1}^\alpha e^{-2L_W} \quad \sigma_{\alpha_1\alpha_2} = O_{\alpha_1\alpha_2,\alpha_1\alpha_2} = C_0 + C_1 T_{3\alpha_2}^3 \tau_{\alpha_1\alpha_1}^3 e^{-2L_W}$$

These results agree with the ones already obtained in (3).
where the generators in the adjoint representation are chosen to be:

\[
T^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad T^2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad T^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}
\]

and from which it is straightforward to obtain:

\[
\sigma_{1+} = \frac{\sigma^2_{++} + \sigma^2_{--}}{2} + \frac{\sigma^1_{++} - \sigma^1_{--}}{2} e^{-2L_W} \quad \sigma_{1-} = \frac{\sigma^2_{++} + \sigma^2_{--}}{2} - \frac{\sigma^1_{++} - \sigma^1_{--}}{2} e^{-2L_W}
\]

\[
\sigma_{13} = \sigma_{23} = \frac{\sigma_{1+} + \sigma_{1-}}{2} = \frac{\sigma^2_{++} + \sigma^2_{--}}{2} \quad \sigma_{2-} = \sigma_{1+} \quad \sigma_{2+} = \sigma_{1-}
\]

In the case of boson-boson scattering, the composition of two isospin 1 multiplets in the t-channel produces also a spin 2 projector, whose component is highly suppressed because of the factor 2(2+1) = 6. The general decomposition in this case is therefore:

\[
\mathcal{O}_{b_1 b_2 a_1 a_2} = C_0 \delta_{b_1 a_1} \delta_{b_2 a_2} + C_1 T^a_{b_2 a_2} T^a_{b_1 a_1} e^{-2L_W} + C_2 \left( T^a T^b \right)_{b_1 a_1} \left( T^a T^b \right)_{b_2 a_2} - \frac{16}{3} \delta_{b_1 a_1} \delta_{b_2 a_2} e^{-6L_W}
\]

\[
\sigma_{a_1 a_2} = \mathcal{O}_{a_1 a_2 a_1 a_2} = C_0 + C_1 T^3_{a_2 a_2} T^3_{a_1 a_1} e^{-2L_W} + C_2 \left( \delta_{a_2 -} - 2 \delta_{a_3 +} + \delta_{a_2 -} \right) (T^3_{a_1 a_1} - 2) e^{-6L_W}
\]

from which we derive the relations:

\[
\sigma_{--} = \sigma_{++} \quad \sigma_{3-} = \sigma_{3+} \quad \sigma_{33} = \sigma_{++} + \sigma_{--} - \sigma_{33}
\]

and, with \( \sigma_a \equiv \sigma_{a+} \):

\[
\sigma_+ = \sigma^H_+ \left( \frac{1}{3} + \frac{e^{-2L_W}}{2} + \frac{e^{-6L_W}}{6} \right) + \sigma^H_- \left( \frac{1}{3} + \frac{e^{-2L_W}}{2} + \frac{e^{-6L_W}}{6} \right) + \sigma_3^H \left( \frac{1}{3} - \frac{e^{-6L_W}}{3} \right)
\]

\[
\sigma_- = \sigma^H_+ \left( \frac{1}{3} - \frac{e^{-2L_W}}{2} + \frac{e^{-6L_W}}{6} \right) + \sigma^H_- \left( \frac{1}{3} + \frac{e^{-2L_W}}{2} + \frac{e^{-6L_W}}{6} \right) + \sigma_3^H \left( \frac{1}{3} - \frac{e^{-6L_W}}{3} \right)
\]

\[
\sigma_3 = \sigma^H_+ \left( \frac{1 - e^{-6L_W}}{3} \right) + \sigma^H_- \left( \frac{1 - e^{-6L_W}}{3} \right) + \sigma_3^H \left( \frac{1 + 2e^{-6L_W}}{3} \right)
\]

We also give for completeness the expression for \( \sigma_{33} \) that can be derived from the above equations:

\[
\sigma_{33} = (\sigma^- + \sigma^H^+ \frac{1 + 2e^{-6L_W}}{3} + \sigma_3^H \frac{1 - 4e^{-6L_W}}{3})
\]

We now briefly discuss mixing in the weak bosons sector. The physical states in the neutral sector are linear combinations \((\gamma, Z) = M(A_0, A_3)\) where \(M\) is the 2x2 matrix \(M_{11} = M_{22} = c_W, M_{12} = M_{21} = s_W\). It is straightforward to obtain physical cross sections involving neutral gauge bosons on the external legs. For instance, if \(A, B\) are \(\gamma, Z\) indices we have:

\[
\sigma_{AB} = \mathcal{O}_{AB,AB} = \sum_{a_1, a_2, b_1, b_2} M_{Aa_1} M_{Ab_1} M_{Ba_2} M_{Bb_2} \mathcal{O}_{b_1 b_2, a_1 a_2}
\]

In this equation also overlap matrix elements that do not correspond to physical cross sections appear; this is the case for \(\mathcal{O}_{00,33}\) for instance. Therefore we need to know also all overlap matrix elements involving the \(A_0\) boson, that we label with an index 0, on the external legs. There are obviously 4 possible cases, with one, two or three or four \(A_0\) bosons on the external legs. Because of isospin invariance we can write:

\[
\mathcal{O}_{ij,00} = \mathcal{O}_{00,ij} = C_1 T^k_{ij} e^{-2L_W} \quad \mathcal{O}_{ij,0k} = \mathcal{O}_{0k,ij} = C_1 T^k_{ij} e^{-2L_W}
\]

\[
\mathcal{O}_{ij,00} = \mathcal{O}_{00,ij} = C_1 \delta_{ij} e^{-2L_W} \quad \mathcal{O}_{01,00} = \mathcal{O}_{01,00} = C_1 \delta_{ij} e^{-2L_W} \quad \mathcal{O}_{01,00} = \mathcal{O}_{01,00} = C_0 \delta_{ij}
\]
\[ O_{i,0,0} = O_{0,i,0} = O_{0,0,i} = 0 \]  
\[ O_{0,0,0} = C_0 \equiv \sigma_{00} \]  
where \( i, j, k \neq 0 \). From (13) we obtain the physical cross sections after accounting for mixing:

\[ \sigma_{\gamma\gamma} = s_W^4 \sigma_{33} + c_W^4 \sigma_{00} + 2s_W^2 c_W^2 (O_{00,33} + O_{03,03} + O_{03,30}) \]  
\[ \sigma_{ZZ} = c_W^4 \sigma_{33} + s_W^4 \sigma_{00} + 2s_W^2 c_W^2 (O_{00,33} + O_{03,03} + O_{03,30}) \]  
\[ \sigma_{Z\gamma} = s_W^2 c_W^2 (\sigma_{33} + \sigma_{00}) - 2s_W^2 c_W^2 O_{00,33} + (c_W^4 + s_W^4) O_{03,03} - 2s_W^2 c_W^2 O_{03,30} \]  
\[ \sigma_{Z\delta} = s_W^2 \sigma_{0\delta} + c_W^2 \sigma_{3\delta} + 2s_W c_W O_{3\delta,0\delta} \]  
\[ \sigma_{\gamma\delta} = c_W^2 \sigma_{0\delta} + s_W^2 \sigma_{3\delta} - 2s_W c_W O_{3\delta,0\delta} \]

(21a)

(21b)

(21c)

(21d)

(21e)

The formalism developed here is completely general. Given a specific process, all that one has to do is to find out the corresponding form of the \( C_i \) coefficients, given by the hard tree level cross sections. Besides the explicit examples in (1) that refer to the fermion-fermion scattering case, we give here an example with the purpose of seeing the general formalism at work and of having an idea of the order of magnitude of the effects that are expected. With this in mind, let us consider the cross section for initial transverse gauge bosons into two hadron jets, obtained by summing the fermion-fermion, t,u-channel fermion exchange and s-channel annihilation. In the limit \( g' \to 0 \) it is easy to obtain the cross sections

\[
\frac{d\sigma_H^H}{dc_0} = 0 \quad \frac{d\sigma_{3+}}{dc_0} = \frac{\pi \alpha_W^2 N_c N_f c_3^2 (1 + c_\beta^2)}{8s} \quad \frac{d\sigma_{33}}{dc_0} = \frac{\pi \alpha_W^2 N_c N_f (1 + c_\beta^2)}{8s} \quad \frac{d\sigma_{H+}^H}{dc_0} = \frac{\pi \alpha_W^2 N_c N_f (1 + c_\beta^2)^2}{8s} s_\theta^2
\]  

(22)

where \( \theta \) is the angle between the \( W^- \) (or \( A_3 \)) and the fermion. The energy dependence is now obtained by inserting the hard cross sections values (22) into the energy evolution equations (17). Notice that the isospin relations (16) are satisfied, as expected, for any energy value. The case of \( \sigma_{++} \) is particularly interesting, since this cross section is zero at the tree level. However, a \( W^+ \) can radiate a soft \( W^+ \) becoming an \( A_3 \) which has a sizeable tree level cross section. Asymptotically, \( \sigma_{++} \) tends to the singlet value which is the cross sections average. At 1 TeV for instance \( \frac{\sigma_{3+}}{\sigma_{++}} \approx 12\% \) so that \( \sigma_{++} \) despite being 0 at tree level, reaches at the TeV scale a value comparable to the other cross sections. Analogous interesting effects can be found in the angular dependence since, for instance, \( \sigma_{3+} \) is zero for \( \theta = \frac{\pi}{2} \). The relative effects one finds for \( \sigma_{33}^H, \sigma_{H+}^H \) are in the 12 - 20 % range at the TeV scale. In conclusion, pretty large effects are expected for boson fusion processes: a realistic calculation, though, has to include the luminosity weights for the various initial states, and the longitudinal contributions (14) as well.

FIG. 1. Unitarity diagrams for (b) virtual and (a) real emission contributions to lowest order initial state interactions. Sum over gauge bosons \( a= \gamma, Z, W \) is understood.
FIG. 2. Operator insertions for the overlap matrix. For physical cross sections, $\alpha_1 = \beta_1$, the contributions from the neutral sector, that are proportional to $y_1 - y'_1$ and $t_1^\alpha - t'_1^\alpha$, are identically zero.

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