Jets in Micro-Quasar SS 433: Analysis involving Acceleration

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Abstract

We analyze multi-wavelength observations of the jets of the micro-quasar SS 433 reported by Marshall, vis-à-vis the acceleration dependence of the Doppler Effect as reported by Wagh. Specifically, we are only interpreting the spectral shifts as arising due to the acceleration-dependent Doppler Effect and find the speed of blue-shifted jet to be ~ 0.022c and that of red-shifted jet to be ~ 0.29c. Our results have consequences for the energetics of the prime mover and jets in SS 433.

Keywords: Doppler shift; Velocity and acceleration; Jets in SS 433

Introduction

Micro-quasar SS 433 shows [1-3] emission lines of jets from compact object. Ha Doppler shifts led Margon and Anderson [4] to the kinematic model of SS 433 as a pair of oppositely directed, processing, jets with speed $V_J$ ~ 0.26. It has, since then, been used in analyzing data on SS 433, although its parameters as well as the model got updated to include nutation with a period of about half the orbital period and small variations in the jet velocity [5]. The distance to SS 433 is argued [5] to be either 5.5 ± 0.2 kpc or somewhat smaller [6] as 4.61 ± 0.35 kpc.

Margon and Anderson [4] determined jet orientation as processing with a 162.5 day period in a cone with half angle of 19.85° about an axis that is at 78.83° to the line of sight to SS 433. The jet lines are Doppler shifted with this period with the extreme red-shift being ~ 0.15 and the extreme blue-shift being ~ -0.08 as per this analysis.

Marshall et al. [7] report multi-wavelength study of jets of SS 433 using Chandra High Energy Transmission Grating Spectrometer with contemporaneous optical and VLBI observations. It is usual [4] to assume that Doppler shifts arise because of the source velocity, only. Then, assuming furthermore two perfectly oppositely directed jets at an angle $\theta$ to the line of sight, the line Doppler shifts of SS 433 are used to estimate [1,4,7,8] the relativistic $\gamma$ factor and the angular factor $\mu = \cos \theta$, which are then used to determine the jet velocity and the angle to the line of sight.

To the best of our knowledge, no study has been known to have focused on the role of acceleration for the jets in SS 433. It is therefore our aim here to explore it.

In what follows, we then analyze Doppler-shifted lines of SS 433 from Marshall et al. [7] and Marshall et al. [8] vis-à-vis the acceleration-dependence of the Doppler Effect [9]. We are only interpreting line shifts using the acceleration-dependence of the Doppler Effect; and are not modeling the jets in SS 433.

Acceleration and Doppler Shift

Recently, Wagh [9] showed that the Doppler shift of a source must include contribution from its acceleration, apart from that due to its velocity, and discussed [10,11] some of its direct implications.

Wagh [12] has also discussed how measurements of velocity and acceleration of the source can be effected from the Doppler shifts of its spectral lines. Cases of constant and temporally (sinusoidal) variable acceleration have, then, been analyzed in Wagh [12].

Jets from a prime mover impinge on external clouds that emit spectral lines. Then, the jet material emitted by the prime mover at a later instant should push material emitted by it at an earlier instant, against the obstruction by cloud. This leads to approximately sinusoidal temporal variability of acceleration, in the manner of a railway engine pushing a train of carriages. Then, jets should show time variable acceleration, in general.

In either case, pushing accelerates and friction decelerates. Sinusoidal variability of acceleration should then be first approximation for the above two situations.

Now, if we were to analyze the situation of jets assuming constant acceleration, we can expect to have overestimated acceleration. It will turn out that this is indeed the case in the analysis of jets in the system of SS433.

When the source $S$ is moving (towards observer $O$ in Figure 1 with

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velocity \( v \) making an angle \( \theta \) with the line \( SO \) and with acceleration \( a \), we can write:

\[
(MO)^2 = c^2T^2 = 2c^2t + 2c^2(\text{SP}) \cos \theta \tag{1}
\]

Where \( (SP) \) is the distance covered by the source in time \( T \), \( c \) is the speed of light (in vacuum), \( T \) is the period of the wave emitted by the source, and \( T \) is the measured period, all in the frame of observer.

We evaluate \( (SP) \) as follows. Let the temporal rate of acceleration be given by

\[
\frac{da}{dt} = a_0 \sin(\omega t), \hspace{1cm} a_0 = \text{constant} \tag{2}
\]

Then, on integration and assuming that the frequency \( \omega \) is appropriately small, we obtain

\[
(SP) = \frac{a_0}{\omega^2} \left( k_1 - \frac{a_0}{\omega} \right) \frac{t^2}{2} + k_f t + k_i \tag{3}
\]

Where \( k_1, k_2, k_3 \) are integration constants.

Substituting this in eq. (1), we then obtain after suitable manipulations:

\[
v_e \approx f(\beta, \theta) v_c = \left( \frac{a_0}{c \omega^2} \right) v_c^2 \cos \theta \left( 1 - \frac{a_0}{2c\omega} \right) \cos \theta \tag{4}
\]

Where \( v_e = 1/T \) is the observed frequency, \( v_c = 1/T \) is the emitted frequency, \( f(\beta, \theta) \) is as defined below with \( \beta = \beta_0 c \), and we have retained only first order terms in acceleration. We set \( k_i = 0 \) and \( k_1 = 0 \). (We recover the case of constant acceleration when \( k_i \neq 0 \) and \( a_0 = 0 \).)

The function

\[
f(\beta, \theta) = 1 - \frac{\beta^2}{2} \sin^2 \theta - \beta \cos \theta \tag{5}
\]

has the following characteristics. See Figure 2

For angular ranges \(-10, 110^\circ\) and \(-240^\circ, 360^\circ\), we have \( f(\beta, \theta) < 1 \); and within the angular range \(-110^\circ, 240^\circ\), we have \( 2 \geq f(\beta, \theta) > 1 \). (That \( f > 1 \) does not necessarily mean relativistic velocity. These ranges also overlap: for \([90^\circ, 110^\circ]\) and \([250^\circ, 270^\circ]\), \( f > 1 \) and \( f < 1 \) both.) When \( f(\beta, \theta) = 1 \), there exists a non-zero lower bound, \( \beta_{min} \), for velocity \( \beta \). For, \( \beta_{min} = 0 \), as \( \beta = 0 \) for all values of angle \( \theta \). (Note that for \( f < 1 \), \( \beta_{min} = 1 - f \) and for \( f > 1 \), \( \beta_{min} = f - 1 \).) Importantly, for \( f(\beta, \theta) = 1 \), velocity can be non-relativistic over quite large angular range, we may also note here.

Let source emit two spectral lines of rest frequencies \( v_{e(1)} \) and \( v_{e(2)} \), with corresponding observed frequencies being \( v_{e(1)} \) and \( v_{e(2)} \), respectively.

Now, let \( \Delta v_e = v_{e(1)} - v_{e(2)} \)

\[
\Delta v_e = v_{e(1)} - v_{e(2)}
\]

Then, from eq. (4), we have

\[
\frac{\Delta v_e}{\Delta v_e} = f(\beta, \theta) + h(\alpha, \omega, \theta) \frac{\Delta(v_c^2)}{\Delta v_e} \tag{6}
\]

where we have set \( h(\alpha, \omega, \theta) = -\frac{a_0}{c \omega} \cos \cos \theta \).

Equation (6) is, evidently, linear in \( \frac{\Delta v_e}{\Delta v_e} \) and \( \frac{\Delta(v_c^2)}{\Delta v_e} \). (We could select average in \( \Delta v_e \) or, equivalently, \( \lambda \) and \( \lambda \) where \( \lambda \) are the corresponding wavelengths, we then obtain eq. (4) \( k(\beta, \theta) \) by linear regression. The acceleration \( \frac{da}{dt} \) of the emitter can then be estimated from the observational data. \( \alpha \)

Velocity \( \beta_{min} \) corresponding to above \( f(\beta, \theta) \), gives us the minimum speed at which matter emitting frequency \( v_{e(1)} \) could be moving with. For the mean jet speed, \( \beta \), we may select average \( \beta_{min} \) or maximum of \( \beta_{max} \). But, angle to the line of sight \( s = \frac{1}{\gamma} - 1 \) will be zero for any \( \beta_{min} \) as the velocity of the line emitting material.

Therefore, the selected value \( \beta_{min} \), or max \( \beta_{max} \), is added to each \( \beta_{min} \) to obtain line-speeds \( \beta_i \). Then, angle \( \theta \) can be obtained from the \( f \)-value of each line using:

\[
\cos \cos \theta = \frac{1}{2 \beta_i^2} \left( 1 - \beta_i^2 + 2(f - 1) \right) \tag{7}
\]

The jet speed is then the average of these line speeds \( \beta_i \). We therefore obtain observational values of kinematical parameters of the material of the jet.

We then note that, for \( a_0 = 0 \), the Left Hand Side of eq. (6) directly yields \( f(\beta, \theta) \), as \( h(\alpha_0, \omega, \theta) \). The value of acceleration \( k_1 \), which is constant, is then to be obtained from eq. (4) \( k_1 = 0 \) replacing \( \Delta a_0 \) in its last term. This value is unreasonable for the jet in SS 433 as it implies that jet material halts instantaneously. See later.

Nevertheless, we note the following. The order of the term \( \frac{\Delta v_e}{\Delta v_e} \) is unity and so is that of \( f(\beta, \theta) \). Then, as the order of \( \frac{\Delta(v_c^2)}{\Delta v_e} \) is \( 10^{-8} \), the order of \( h(\alpha, \omega, \theta) = 10^{-10} \), Thus, the acceleration \( \frac{a_0}{} \) is of order \( 10^{-8} \), which yields reasonable value. (The last term of eq. (4) will then become negligible. The time-scale of change in velocity is now, of the order of \( \frac{\beta c}{\omega} \), \( \frac{a_0}{\omega} \) of \( \beta c \times 10^{-8} \).

Temporally (sinusoidal) variable acceleration thence allows, in general, reasonable value(s) for the magnitude of acceleration in the jet system.

The aforementioned summarizes the role of acceleration in Doppler shift(s) of spectral lines from a source. As will be seen in the next section, the function \( f(\beta, \theta) \) clearly identifies (by way of \( f \) being

![Figure 2: \( \beta \) as a function of \( \theta \) for values of \( f(\beta, \theta) \).](image_url)
greater than 1 for such lines) certain blue-shifted lines of SS 433 as being emitted by the material beyond angle of 90° to the line of sight.

The role of acceleration-dependence of Doppler Effect is therefore an important one for the analysis not only of astronomical jet situations but in general, also.

In what follows, we adopt the above strategy to analyze data on Doppler-shifted spectral lines of jets in SS 433 from Marshall et al. [7] and Marshall et al. [8].

**Jets in SS433**

Consider therefore Table 4 of Marshall et al. [7] reporting various blue-shifted spectral lines from the micro-quasar SS 433 with their observed and rest wavelengths. (The rest wavelengths have been computed for blends by applying weights equal to the fractional flux contribution to the blend, according to the multi-temperature plasma model). We use these data in the following analysis performed as per the details outlined in Section 2.

Firstly, of significance is the value of acceleration that we get, namely, a ∼ -10^2 cm/s^2, when assuming constant acceleration and the details outlined in Section 2.

We then find the blue-shifted jet of SS 433 to be angularly limited with velocity vJ ∼ -10^9 cm/s and encountering this deceleration, then it would be brought to rest almost instantaneously in time ∼ 10^{-11} s! This is a certain indication that the material of the jet is also being pushed through the obstructing matter as it propagates through. That is to say, the material of the jet emitting blue-shifted lines cannot be free-streaming, and the mechanism of its acceleration is operating within the emission regions of these lines.

For these above reasons, we question the assumption of the constancy of acceleration, also. This issue was discussed in Section 2. We therefore also obtain justification for the procedure outlined in Section 2, then.

Results of our analysis (following Section 2, now) of the data on SS 433 in Table 4 of Marshall et al. [7] are given in Table 1 for jet speed β_J ∼ 0.022 and in Table 2 for jet speed β_J ∼ 0.076. Notice that every spectral line with f > 1 has θ ≥ 90°.

Figure 3 then compares angular plots of line-velocities β for β_J ∼ 0.022 and β_J ∼ 0.076. Data points are fitted with cubic spline curves.

Table 1: Parameters of blue-shifted lines of the jet of SS 433 for β_J ∼ 0.022. Average angle is 86°.

| Identity | λ_a(Å) | λ_c(Å) | h_w | f | β | θ(deg) | a_w/λ |
|----------|-------|-------|-----|---|----|--------|------|
| Ni-XXVIII | 1.532 | 1.526 ± 0.005 | 0.8890315 | 0.93532 | 0.075639 | 33 | -0.10583205 |
| Ni-XXVII | 1.592 | 1.590 ± 0.002 | 0.88267751 | 0.939096 | 0.0702075 | 34 | -0.09912994 |
| Fe-XXVI | 1.780 | 1.786 ± 0.001 | 0.8510851 | 0.960209 | 0.050963 | 40 | -0.06300088 |
| Fe-XXV | 1.855 | 1.860 ± 0.001 | 0.8313152 | 0.979514 | 0.035657 | 47 | -0.0452725 |
| Fe-XXIII | 8.815 | 8.851 ± 0.010 | 0.82612262 | 0.988381 | 0.022791 | 60 | -0.05211100 |
| Ni-XX | 12.435 | 12.428 ± 0.010 | 0.1795082 | 0.989686 | 0.021485 | 62 | -0.03805716 |
| Si-XXII | 6.648 | 6.652 ± 0.002 | 0.0838549 | 0.991826 | 0.019346 | 66 | -0.02022888 |
| Ar-XVII | 3.962 | 3.970 ± 0.002 | 0.0671313 | 0.993540 | 0.018253 | 68 | -0.02469117 |
| Ne-XX | 12.134 | 12.146 ± 0.006 | 0.1102429 | 0.994347 | 0.016825 | 71 | -0.03355997 |
| Si-XXIV | 6.182 | 6.186 ± 0.001 | 0.02612262 | 0.996812 | 0.014359 | 78 | -0.01379550 |
| Ca-XX | 3.020 | 3.025 ± 0.005 | 0.00643031 | 0.994555 | 0.016616 | 71 | -0.02088114 |
| Si-XXIV | 5.217 | 5.224 ± 0.006 | 0.00661529 | 0.997036 | 0.014135 | 78 | -0.03260272 |
| Fe-XXIV | 10.634 | 10.633 ± 0.005 | 0.00016857 | 1.000772 | 0.011944 | 106 | 0.00238848 |
| Ni-XXVI | 9.759 | 9.730 ± 0.007 | 0.00369856 | 0.998642 | 0.012530 | 121 | -0.03560158 |
| Fe-XXIV | 8.316 | 8.309 ± 0.004 | 0.00671313 | 0.999703 | 0.014135 | 78 | -0.03260272 |
| Mg-XII | 7.101 | 7.083 ± 0.003 | 0.03774527 | 1.022585 | 0.033756 | 133 | -0.05565784 |

Table 1: Parameters of blue-shifted lines of the jet of SS 433 for β_J ∼ 0.022. Average angle is 86°.
Variation of line-velocity $\beta$ with parameter $\frac{a_3}{w^3}$ is then depicted in Figure 6 for $\beta_j \sim 0.022$ and in Figure 7 for $\beta_j \sim 0.076$. Data are also fitted with cubic spline curves to indicate these variations.

Of interest to modeling of jets and considerations of their prime mover is angular distribution of elements emitting observed spectral lines. We therefore provide the angular distribution of elements, following Tables 2 and 4.
0.29 (Table 2) for \( \beta_J \sim 0.32 \) (Table 4). Notice here that the jet velocity is substantially larger for the red-shifted jet than that for the blue-shifted jet of SS 433.

Figure 11 now shows the angular variation of the line-acceleration parameter \( \frac{\alpha_J}{w} \) for \( \beta_J \sim 0.29 \) (Table 2) and for \( \beta_J \sim 0.32 \) (Table 4).

**Discussion**

Acceleration dependence of Doppler Effect has an important consequence: the spectral shift is not dependent on only the velocity of the source. Acceleration also contributes to the spectral shift, as in eq. (4), in a significant manner.

In our analysis of the micro-quasar SS 433, we then find that acceleration plays one important role. Lines of the blue-shifted jet with
If \( f > 1 \) then correspond to angles larger than 90° to the line of sight. (For the red-shifted jet, \( f > 1 \) would imply angles greater than 270°, we note. However, we do not find such lines in the data on SS 433 for the red-shifted jet. However, see later, also.) This kind of identification of angle is not possible without the acceleration dependence of the Doppler shift.

From our analysis, presented in Table 1 and Table 3, we find that the speed of the blue-shifted jet is non-relativistic, that is, \( \beta_J \sim 0.022 \) or \( \beta_J \sim 0.076 \). But, the red-shifted jet is mildly relativistic \( \beta_J \sim 0.29 \) or \( \beta_J \sim 0.32 \).

As discussed in Section 2, values \( \beta_J \sim 0.022 \) and \( \beta_J \sim 0.29 \) have been obtained by adding average \( \langle \beta_{\text{min}} \rangle \) of the line-values of \( \beta_{\text{min}} \), while the values of \( \beta_J \sim 0.076 \) and \( \beta_J \sim 0.32 \) have been obtained by adding maximum of the line-values of \( \beta_{\text{min}} \).

We, of course, suggest the addition of the average \( \langle \beta_{\text{min}} \rangle \) of the line-values of \( \beta_{\text{min}} \) to each of them. Maximum of \( \beta_{\text{min}} \) corresponds, from the analogy of a train; to maximum effect of the push material receives. We have therefore used it only to compare the jet characteristics at \( \beta_J = \beta_{\text{min}} \) with those at any higher value for \( \beta_J \).

We therefore measure \( \beta_J \sim 0.022 \) for the blue-shifted jet and \( \beta_J \sim 0.029 \) for the red-shifted jet as our observed values. We also find variations in line velocities in both these jets of SS 433.

We have not provided errors of various quantities here, as we find that errors do not change the main conclusions of our analysis in any significant manner. That the speed of the blue-shifted jet is substantially non-relativistic, that the speed of the blue-shifted jet is different than that of the red-shifted jet, that there are interesting variations of line-velocity, etc. hold.

The bulk flow speed of the material \( 0.26c \) of the blue-shifted jet is, in particular, substantially smaller than obtained [7] from the standard analysis with no acceleration dependence of the Doppler shift. Nevertheless, the bulk flow speed, \( \beta_J \sim 0.29 \), of the red-shifted jet is close to this value, we then note.

We therefore find blue-shifted and red-shifted jets to be possessing different speeds in the system of SS 433. Then, any model assuming the same speed for the two oppositely directed jets appears to be in difficulty here.

But, we could be viewing an early (meaning “closer” to the prime mover) part of the receding jet (for which the speed is higher) and the later (meaning “away from” the prime mover) part of the approaching jet (for which the speed is lower as a result of its passage through matter), perhaps. This could then be a possible reason for difference in speeds of the (blue-shifted) approaching and (red-shifted) receding jets.
Angular dependence of the line-acceleration parameter $\alpha_w$ of Figure 4 and the dependence of line-velocity $\beta_L$ on the line-acceleration parameter of Figure 6 are of definite significance, now.

In this context, we note that variations of $\alpha_w$ correspond directly with those of $\beta_L$, the line-acceleration. We can, consequently, interpret these variations as providing us the angular distribution of the clouds causing deceleration of the jet material.

We emphasize then that the multi-epoch monitoring of these variations will provide us valuable information on the jet advance and the parameters of these jet-obstructing clouds. Furthermore, such monitoring will also provide us information about temporal character of the activity of the prime mover of these jets, we emphasize.

Now, just the line Doppler shifts do not provide us any information on the radial distance of the jet material from its prime mover. Nevertheless, the angular distribution of elements in Figure 8 is of definite significance for it shows us the shell encountered by the jet.

Lastly, Marshall et al. [7] have also observed aperiodic variability of Doppler shifts (of blue-shifted jet) over timescale much shorter than any of the known periodicities in the system, like that of precession, orbit, and nutation. We then only note here that aperiodic variability could come from the material of jet experiencing aperiodic changes of deceleration. Any aperiodic variability of acceleration could then be related to the mechanisms of the jet acceleration and deceleration, both.

In summary, we find that analysis of data on Doppler-shifted lines of SS 433, specifically X-ray emission lines seen using Chandra HETGS, implies non-relativistic speed of the material of the jet causing their emission. Our results have consequences for the energetics of the prime mover as well as for the model of the jets in SS 433.

Detailed model of the jets in SS 433 consistent with the aforementioned results of observational nature is a subject of our separate considerations.

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