First results from a parametrized Fixed-Point QCD action

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We have constructed a new fermion action which is an approximation to the (chirally symmetric) Fixed-Point action, containing the full Clifford algebra with couplings inside a hypercube and paths built from renormalization group inspired fat links. We present an exploratory study of the light hadron spectrum and the energy-momentum dispersion relation.

1. Introduction

Fixed-Point (FP) fermions are a solution to the Ginsparg-Wilson equation \cite{1} and thus preserve chiral symmetry on the lattice \cite{2}. While there exist a range of lattice fermion formulations with exact or approximate chiral symmetry \cite{3–7}, we expect FP fermions to have additional improved properties like small scaling violations. For the FP gauge action we work with, good scaling behaviour has been found for measurements of $T_c$ and glueballs up to lattice spacings of $\sim 0.3$ fm \cite{8}. We report first results for light hadron spectroscopy with a hypercubic FP fermion action \cite{9} which includes all Clifford algebra elements and uses gauge paths constructed out of fat links. Results for the chiral condensate and other observables are given in \cite{10}.

2. Simulation parameters

At a gauge coupling of $\beta = 3.0$, the lattice spacing determined from $r_0$ is $a = 0.16$ fm. Lattice volumes of $6^3 \times 16$ and $9^3 \times 24$ are compared to identify finite volume effects. We use gaussian smeared sources and point sinks. Configurations are fixed to Landau gauge. Bare quark masses $m_q a$ range from 0.016 to 0.23. At the smallest mass we get $m_\pi/m_\rho = 0.33(3)$. In this exploratory study we introduced the mass by defining $D(m_q) = D^{FP} + m_q$. A multimass BiCGStab algorithm is used to invert the Dirac operator. We symmetrize the correlators, make correlated fits and calculate bootstrap errors. More details on the simulation and results can be found in \cite{11}.

Fig. 1 shows the masses of the $\pi$, $\rho$ and $N$ measured with the parametrized FP Dirac operator $D^{FP}$ on the $9^3 \times 24$ lattice. The inversion of the Dirac operator converged for all configurations, although for one configuration the number of iterations was about 2.5 times larger than the typical value. This indicates that the residual chiral symmetry breaking leads to fluctuations of the order of the smallest mass for the real eigenvalues of $D^{FP}$. The scale determined from the rho mass is $a = 0.17$ fm. Fig. 2 shows an Edinburgh plot of the data.

![Figure 1. $\pi$, $\rho$ and $N$ masses for $D^{FP}$ at $\beta = 3.0$ from 70 $9^3 \times 24$ gauge configurations.](image)
3. Spectroscopy with $D^{FP}$

We use three different correlators to extract the pion mass: The pseudoscalar $P$, the axial vector $A$ and the difference of pseudoscalar and scalar $P-S$. In the quenched theory, the $P$ correlator picks up a contribution proportional to $Q/m_q^2V$. This quenched enhanced finite size effect is cancelled in the difference $P-S$ if the action is exactly chiral. For the $A$ correlator, the divergence is expected to be partially suppressed [12].

Figs. 3 and 4 show a comparison of $m^2_\pi$ vs. $m_q$ at the two lattice sizes using the parametrized FP Dirac operator. On the smaller volume, the three different pion correlators give different values at small quark masses. The $P$ correlator lies highest, while the $P-S$ correlator gives considerably smaller pion masses. The $A$ correlator lies in between. This behaviour is in qualitative agreement with the results from domain-wall fermions [12]. For larger $m_q$, the $P-S$ correlator deviates from the pion due to the the closely lying scalar state. The discrepancy between the three correlators in the chiral limit is reduced at our larger lattice volume, where the topological finite-volume effects are on the order of the statistical error.

On the $9^3 \times 24$ lattice, we also measure the unrenormalized PCAC quark mass

$$2m^{\text{PCAC}}(t) = \sum_{\vec{x}} \langle \partial_4 A_4(\vec{x}, t) P(0) \rangle / \sum_{\vec{x}} \langle P(\vec{x}, t) P(0) \rangle,$$

which should go to zero as $m_q \to 0$ if the Dirac operator respects chiral symmetry. From a linear fit to all but the smallest two masses we get $m^{\text{PCAC}}_{\text{res}}a = 0.003(4)$. A linear fit to $m^2_\pi$ gives $m^{\text{res}}_\pi a = -0.007(3), -0.006(2)$ and $-0.002(3)$ for the $P$, $A$ and $P-S$ correlators, respectively. Hence we do not see additive mass renormalization due to residual chiral symmetry breaking within our statistical and systematical errors.

Figure 2. Edinburgh plot for $D^{FP}$.

Figure 3. $m^2_\pi$ for the parametrized FP Dirac operator $D^{FP}$ from 100 $6^3 \times 16$ gauge configurations.

In Fig. 5 the squared speed of light $c^2 = (E(p)^2 - m^2)/p^2$ measured on the $9^3 \times 24$ lattice is shown for the smallest momentum $|\vec{p}| = 2\pi/9$. While $c^2$ for the pion is consistent with 1 within errors, it is not for the rho meson. However, the error bars do not include systematic uncertainties from choosing the fit range. The dispersion relation is significantly improved compared to the Wilson or clover action.

4. Overlap improved FP Dirac operator

We define the massive overlap Dirac operator

$$D^{FP}_\text{ov}(m_q) = (1 - m_q/2)D^{FP}_\text{ov}(0) + m_q/2R,$$

with $R$ from the Ginsparg-Wilson relation and

$$D^{FP}_\text{ov}(0) = (2R)^{-1/2}(1 - A/\sqrt{A^\dagger A})(2R)^{-1/2}.$$
In the kernel $A = 1 - \sqrt{2}R^D\sqrt{2} R$ the parametrized FP Dirac operator enters. We use a 3rd order Legendre expansion of the inverse square root and project out the smallest 20 eigenvalues of $A^\dagger A$. At the smallest mass $m_q a = 0.012$ we get $m_\pi / m_\rho = 0.27(4)$. Fig. 5 shows $m_\pi^2$ for the overlap improved FP Dirac operator. A linear fit to the $P$-$S$-correlator at the six smallest masses is consistent with zero at $m_q = 0$. Similar behaviour has been observed at $\beta = 3.2$ ($a = 0.13$ fm).

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