Geometrical diagnostic for purely kinetic k-essence dark energy

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Geometrical diagnostic, involving the statefinder \( \{r, s\} \) and \( Om(x) \), is widely used to discriminate different dark energy models. We apply the statefinder \( \{r, s\} \) and \( Om(x) \) to purely kinetic k-essence dark energy model with Dirac-Born-Infeld-like Lagrangian which can be considered as scalar field realizations of Chaplygin gas. We plot the evolution trajectories of this model in the statefinder parameter-planes and \( Om(x) \) parameter-plane. We find that the statefinder \( \{r, s\} \) and \( Om(x) \) fail to distinguish purely kinetic k-essence model from \( \Lambda \)CDM model at 68.3% confidence level for \( z \ll 1 \).

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I. INTRODUCTION

In the last decade a convergence of independent cosmological observations suggested that the Universe is experiencing accelerated expansion. An unknown energy component, dubbed as dark energy, is proposed to explain this acceleration. Dark energy almost equally distributes in the Universe, and its pressure is negative. The simplest and most theoretically appealing candidate of dark energy is the vacuum energy (or the cosmological constant \( \Lambda \)) with a constant equation of state (EoS) parameter \( w = -1 \). This scenario is in general agreement with the current astronomical observations, but has difficulties to reconcile the small observational value of dark energy density with estimates from quantum field theories; this is the cosmological constant problem. It is thus natural to pursue alternative possibilities to explain the mystery of dark energy. Over the past decade numerous dark energy models have been proposed, such as quintessence, phantom, k-essence, tachyon, (Generalized) Chaplygin Gas, DGP, etc. k-essence, a simple approach toward constructing a model for an accelerated expansion of the Universe, is to work with the idea that the unknown dark energy component is due exclusively to a minimally coupled scalar field \( \phi \) with non-canonical kinetic energy which results in the negative pressure [1]. This scenario has received much attention, considerable efforts have been made in understanding the role of k-essence on the dynamics of the Universe. A feature of k-essence models is that the negative pressure results from the non-linear kinetic energy of the scalar field. Secondly, because of the dynamical attractor behavior, cosmic evolution is insensitive to initial conditions in k-essence theories. Thirdly, k-essence changes its speed of evolution in dynamic response to changes in the background equation-of-state. Here we only concentrate on a special class of k-essence with Dirac-Born-Infeld-like Lagrangian \( p(X) = -V_0 \sqrt{1-2X} \). This class of k-essence can be considered as scalar field realizations of Chaplygin gas [2, 5] and have been studied intensively (see e. g. [6–11]).

Since more and more dark energy models have been constructed, the problem of discriminating between various dark energy models is important. To solve this problem, Sahni et al. [12] and Alam et al. [13] introduced a geometrical diagnostic, called statefinder. The statefinder probes the expansion dynamics of the Universe through higher derivatives of the expansion factor \( a \) and is a natural companion to the deceleration parameter \( q \) which depend on \( a \). The statefinder pair \( \{r, s\} \) is defined as

\[
\begin{align*}
    r &= \frac{\ddot{a}}{aH^2} \\
    s &= \frac{r - 1}{3(q - 1/2)},
\end{align*}
\]

where \( a \) is the scale factor, \( H \equiv \dot{a}/a \) is the Hubble parameter and \( q \equiv \dot{q}/(aH^2) \) is the deceleration parameter.

Trajectories in the \( r - s \) plane corresponding to different cosmological models exhibit qualitatively different behaviors. The spatially flat \( \Lambda \)CDM scenario corresponds to a fixed point \( \{r, s\} = \{1, 0\} \) in the diagram. Departure of a given dark energy model from this fixed point provides a good way of establishing the distance of the model from \( \Lambda \)CDM. If this distant can be measured, models can be distinguished. It has been demonstrated that the statefinder can successfully differentiate between a wide variety of dark energy models, such as \( \Lambda \)CDM, quintessence [12, 14, 15], Ricci Dark Energy model [16], DGP [19], Generalized Chaplygin Gas Model [20, 22], Agegraphic Dark Energy Models [23], quintom dark energy model [24], etc.

Another diagnostic \( Om(x) \) was introduced to differentiate \( \Lambda \)CDM from other dark energy models except of \( \{r, s\} \) [25]. \( Om(x) \) is a combination of the Hubble parameter and the cosmological redshift and provides a null test of dark energy being a cosmological constant \( \Lambda \). Namely, if the value of \( Om(x) \) is the same at different redshift, then dark energy is \( \Lambda \) exactly. The slope of \( Om(x) \) can distinguish dynamical dark energy from the cosmological constant in a robust manner both with and without reference to the value of the matter density, which can be a significant source of the uncertainty for cosmological reconstruction. It has been shown that the \( Om(x) \) can successfully differentiate between a wide
variety of dark energy models, such as quintessence [22], phantom [23], Ricci Dark Energy model [18], holographic dark energy [16, 17], etc. In this Letter we apply the statefinder \{r,s\} and \( Om(x) \) to purely kinetic k-essence dark energy models with Dirac-Born-Infeld-like Lagrangian \( p(X) = -V_0\sqrt{1-2X} \) which can be considered as scalar field realizations of Chaplygin gas. We plot the evolution trajectories of this model in the statefinder parameter-planes and \( Om(x) \) parameter-plane. We find that the statefinder \{r,s\} and \( Om(x) \) fail to distinguish purely kinetic k-essence model from ΛCDM model.

In section II, we will briefly review purely kinetic k-essence model. In section III, we plot the evolutionary trajectories of this model in the statefinder parameter planes. In the last section we will give same conclusions.

II. BRIEFLY REVIEW ON K-ESSENCE

As a candidate of dark energy, k-essence [1] is defined as a scalar field \( \phi \) with non-linear kinetic terms which appear generically in the effective action in string and supergravity theories, and its action minimally coupled with gravity generically may be expressed as

\[
S_\phi = \int d^4x \sqrt{-g} \left[ -\frac{R}{2} + p(\phi, X) \right],
\]

where \( X = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi \), and we take \( 8\pi G = 1 \) throughout this Letter. The Lagrangian \( p \) and the energy density of k-essence take the forms: \( p_k = V(\phi)F(X) \) and \( \rho_k = V(\phi)[2XF_X - F] \). Here \( F(X) \) is a function of the kinetic energy \( X \) and \( F_X \equiv dF/dX \). The corresponding equation of state (EoS) parameter and the effective sound speed are given by

\[
w_k = \frac{F}{2XF_X - F},
\]

\[
c_s^2 = \frac{\partial \rho_k / \partial X}{\partial \rho_k / \partial X} = \frac{F_X}{F_X + 2XF_XX'},
\]

with \( FXX' \equiv d^2F/dX^2 \). The definition of the sound speed comes from the equation describing the evolution of linear perturbations in a k-essence dominated Universe [20]. In this Letter, we consider a class of k-essence with constant potential \[2, 10]: p_k(X) = -V_0\sqrt{1-2X}, \] where \( V_0 \) is a constant. Such models can be considered as scalar field realizations of Chaplygin gas [2, 4]. For this Lagrangian, the EoS parameter and the sound speed take the form respectively [3],

\[
w_k = -c_s^2 = -\frac{1}{1 + 2k_0^2 a^{-6}} ,
\]

where \( k_0 = \sqrt{2}F_0/2V_0 \) is a constant \((-\infty < k_0 < +\infty\), but because of the exponent 2, the case \( k_0 \geq 0 \) and the case \( k_0 \leq 0 \) are equivalent). For \( k_0 = 0 \), the above EoS reduces to \(-1\); meaning the ΛCDM model is contained in k-essence model as one special case. The behavior of the EoS [3], being \( \approx -0 \) in the early Universe, runs closely to \(-1\) in the future for \( k_0 \neq 0 \). Such behavior can, to a certain degree, solve the fine-turning problem [1, 27].

III. GEOMETRICAL DIAGNOSTIC FOR KINETIC K-ESSENCE DARK ENERGY

Statefinder \{r,s\} introduced in Refs. [12, 13] is a useful method to differentiate kinds of dark energy models. Researches have shown that it can differentiate ΛCDM from many dark energy models including ΛCDM, quintessence [12, 14, 15], holographic dark energy model [16, 17], Ricci Dark Energy model [18], DGP [19], Generalized Chaplygin Gas [20, 22], Agegraphic Dark Energy Models [23], quintom dark energy model [24], etc. For model of dark energy with equation of state \( w_\rho \) and density parameter \( \Omega_D \), the statefinder parameters \{r,s\} can be expressed as follows [12]

\[
r = 1 + \frac{9}{2} \Omega_D w_D (1 + w_D) - \frac{3}{2} \Omega_D \frac{\dot{w}_D}{H} ,
\]

\[
s = 1 + w_D - \frac{1}{3} \frac{\dot{w}_D}{H} \frac{\dot{H}}{H} .
\]

For the purely kinetic k-essence dark energy, the statefinder parameters and the deceleration parameter can be expressed as

\[
r = 1 + 9\Omega_k \frac{k_0^2 (1 + z)^6}{[1 + 2k_0^2 (1 + z)^6]^2} ,
\]

\[
s = \frac{2k_0^2 (1 + z)^6}{1 + 2k_0^2 (1 + z)^6} ,
\]

and

\[
q = \frac{\Omega_{m_0} (1 + z)^3 + 2k_0^2 (1 + z)^6 - 2}{2\Omega_{m_0} (1 + z)^3 + 2(1 - \Omega_{m_0}) f(z)} ,
\]

where \( f(z) = \exp\left[3 \int_0^z \frac{1 + w_\rho(z')}{1 + z'} dz' \right] \) and \( \Omega_k(z) = \Omega_{m_0}(f(z)) + \Omega_{m_0}(1 + z)^3 \).

Constrained form 307 SNIa data [25], the shift parameter \( R \) [29], and the acoustic scale \( l_a \) [29], the best-fit values of the parameters at 68% confidence level were found to be: \( \Omega_{m_0} = 0.36 \pm 0.01 \) and \( k_0 = 0.067 \pm 0.011 \) [3]. With those parameters, we plot the evolution trajectories of purely kinetic k-essence model in the statefinder parameter-planes.

In Fig. 1, ΛCDM scenario corresponds to a fixed point: \{r,s\} = \{1,0\}. As \( s \) varies in the interval \([-1,0]\), \( r \) first increases from \( r = 1 \) to its maximum values and then decreases to the ΛCDM fixed point. We clearly see that the ‘distance’ from today’s values of purely kinetic k-essence to ΛCDM model is hardly identified in this diagram at 68% confidence level. Meanwhile today’s values get closer and closer to the ΛCDM model by decreasing \( k_0 \) and \( \Omega_{m_0} \). Hence, the statefinder diagnostic can’t
discriminate purely kinetic k-essence model and ΛCDM model at 68% confidence level for $z \ll 1$.

As a complementarity, we plot the evolution trajectory in $q - r$ plane. In Fig. 2, both purely kinetic k-essence model and ΛCDM model commence evolving from the same point in the past $q = 0.5, r = 1$ which corresponds to a matter dominated SCDM (standard cold dark matter) Universe, and end their evolution at the same point in the future. Meanwhile, we plot the evolution trajectory in $q - s$ plane (see Fig. 3). At the beginning, the difference between purely kinetic k-essence model and ΛCDM model is very obvious in $q - s$ plane. When evolving, purely kinetic k-essence model is getting closer and closer ΛCDM model.

According the discussions above, we see that statefinder $\{r, s\}$ is fail to distinguish purely kinetic k-essence model from ΛCDM model. Now, we apply another diagnosis method, $Om(x)$, to distinguish them.

$Om(x)$ diagnostics is defined as \cite{25}

$$Om(x) \equiv \frac{h^2(x) - 1}{x^3 - 1}, \quad (11)$$

where $x = 1 + z$ and $h(x) = H(x)/H_0$. The slope of $Om(x)$ can distinguish dynamical dark energy from the cosmological constant in a robust manner both with and without reference to the value of the matter density \cite{16-18, 25}. For ΛCDM, the $Om(x)$ is

$$Om(x) = \Omega_{m0}. \quad (12)$$

For purely kinetic k-essence model, the $Om(x)$ is

$$Om(x) = \frac{\Omega k_0 \left( \frac{x^2 + 1}{2k_0^2 + 1} \right)^{1/2} + \Omega_{m0} x^3 - 1}{x^3 - 1}. \quad (13)$$

Similarly, we plot the evolutions of $Om(x)$ in Fig. 4.

For purely kinetic k-essence model, we plot evolution trajectories of $Om(x)$ with the values \cite{9} constrained from 307 SNIa data \cite{28}, the shift parameter $R$ \cite{29}, and
the acoustic scale $l_a$. At present, $Om(x)$ of purely kinetic k-essence model is greater than that of ΛCDM model when $\Omega_m^0 = 0.36$ and $\Omega_m^0 = 0.37$, while less than that of ΛCDM model when $\Omega_m^0 = 0.35$. However, the difference between purely kinetic k-essence model and ΛCDM model isn’t obvious near present. It is obviously that the deviations of $Om(x)$ between purely kinetic k-essence and ΛCDM is less than 0.06 ($\Delta Om(x) < 0.06$) even at $z \leq 6$. Namely, the $Om(x)$ cannot discriminate those two models at 68.3% confidence level.

To understand the results above more well, we calculate analytically the lowest order of $h(x) = H(x)/H_0$ of purely kinetic k-essence model and ΛCDM model. For ΛCDM model, we find $h(x) \approx 1 + \frac{3}{2} \Omega_m^0$ at low redshifts. For purely kinetic k-essence model, we find $h(x) \approx 1 + \frac{1}{2} z \Omega_m^0 + \frac{6 k_0^2 (1 - \Omega_m^0)}{2 k_0 + 1}$ at low redshifts. Taking $\Omega_m^0 = 0.36$ and $k_0 = 0.067$, we find the deviation of $Om(x)$ between purely kinetic k-essence and ΛCDM is very small for $z = 0.01$: $\Delta Om(x) \approx 0.0001$. So the $Om(x)$ cannot discriminate these two models at low redshifts.

IV. CONCLUSIONS AND DISCUSSIONS

In this Letter, we applied two geometrical diagnostics of dark energy, involving the statefinder \{r, s\} and $Om(x)$, to distinguish purely kinetic k-essence with Dirac-Born-Infeld-like Lagrangian from ΛCDM model. We plotted the evolution trajectories in statefinder $s - r$ plane. We found that the current values of purely kinetic k-essence are close to the ΛCDM fixed point. The ‘distant’ between two models cannot be identified explicitly. Obviously, the distances between these cases can’t be easily measured. Therefore, the statefinder cannot differentiate purely kinetic k-essence model from ΛCDM model at 68.3% confidence level for $z \ll 1$. As another diagnostic method, $Om(x)$ is widely used to distinguish different dark energy models. We found, however, the $Om(x)$ also cannot discriminate purely kinetic k-essence model and ΛCDM model at 68.3% confidence level. In order to differentiate these two models, it is necessary to find a new method.

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