Holographic dark energy in Brans–Dicke theory with logarithmic correction

A. Sheykhi · K. Karami · M. Jamil · E. Kazemi · M. Haddad

Received: 10 May 2011 / Accepted: 31 October 2011 / Published online: 18 December 2011
© Springer Science+Business Media, LLC 2011

Abstract In the derivation of holographic dark energy density, the area law of the black hole entropy plays a crucial role. However, the entropy-area relation can be modified from the inclusion of quantum effects, motivated from the loop quantum gravity, string theory and black hole physics. In this paper, we study cosmological implication of the interacting entropy-corrected holographic dark energy model in the framework of Brans–Dicke cosmology. We obtain the equation of state and the deceleration parameters of the entropy-corrected holographic dark energy in a non-flat Universe. As system’s IR cutoff we choose the radius of the event horizon measured on the sphere of the horizon, defined as $L = ar(t)$. We find out that when the entropy-corrected holographic dark energy is combined with the Brans–Dicke field, the transition from normal state where $w_D > -1$ to the phantom regime where $w_D < -1$ for the equation of state of interacting dark energy can be more easily achieved for than when resort to the Einstein field equations is made.
Keywords Holographic dark energy • Brans–Dicke cosmology • Corrected entropy-area relation

1 Introduction

Astrophysical data from type Ia supernovae (SNeIa), cosmic microwave background radiation (CMB) and large scale structure (LSS) have provided convincing evidence for the present observable Universe to be spatially flat and in the phase of accelerated expansion [1–5]. Also most of the portion of cosmic energy density is contained in the dark sectors i.e. dark energy (DE) and dark matter (DM) which are 73 and 23% respectively while ordinary baryonic matter (BM) is just 4%. In the framework of relativistic cosmology, the cosmic acceleration is described by any perfect fluid whose pressure \( p \) and energy density \( \rho \) satisfy \( \rho + 3p < 0 \), and such fluid is termed “DE” with negative pressure. In other words, the equation of state (EoS) parameter \( w = p/\rho < -1/3 \) theoretically while observationally it is a daunting task to constrain it. In theory, there are numerous candidates to explain DE including cosmological constant, quintessence, phantom energy, K-essence, quintom, Chaplygin gas, tachyon and modified gravity, to name a few (see [6–8] for comprehensive reviews on DE).

In literature, we have another candidate of DE namely holographic DE (HDE) which is motivated from the “holographic principle” [9–17]. It was shown in [18] that in quantum field theory, the UV cutoff \( \Lambda \) should be related to the IR cutoff \( L \) due to limit set by forming a black hole. If \( \rho_D = \Lambda^4 \) is the vacuum energy density caused by UV cutoff, the total energy of size \( L \) should not exceed the mass of the system-size black hole:

\[
E_D \leq E_{BH} \rightarrow L^3 \rho_D \leq M_p^2 L,
\]

where \( M_p \) is the reduced Planck mass \( M_p^{-2} = 8\pi G \). If the largest cutoff \( L \) is taken for saturating this inequality, we get the energy density of HDE as

\[
\rho_D = 3c^2 M_p^2 L^{-2},
\]

where \( c^2 \) is a dimensionless constant. Following Guberina et al. [19], there is an alternative derivation of HDE based on the entropy bound. In the thermodynamics of the black hole [20,21], there is a maximum entropy in a box of size \( L \), namely, the Bekenstein-Hawking entropy bound \( S_{BH} \sim M_p^2 L^2 \), which scales as the area of the box \( A \sim L^2 \), rather than the volume \( V \sim L^3 \). Also for a macroscopic system in which self-gravitation effects can be disregarded, the Bekenstein entropy bound \( S_B \) is given by the product of the energy \( E \sim \rho_D L^3 \) and the linear size \( L \) of the system. Requiring \( S_B \leq S_{BH}, \) namely \( EL \lesssim M_p^2 L^2 \), one has the same result \( \rho_D \lesssim M_p^2 L^{-2} \) obtained from energy bound argument.

The HDE is thoroughly investigated in the literature in various ways. In [22], the HDE is used to drive inflation in the early Universe. In [23], the EoS of HDE is studied with varying Newton’s gravitational constant and is shown that the EoS parameter can be modified significantly in the low redshift limit. In other papers [24–36], the
Holographic dark energy (HDE) is investigated with different IR cutoffs like the particle horizon, Hubble horizon, future event horizon and the Granda-Oliveros cutoff. Similarly, correspondences are established between HDE and other scalar field models of dark energy (DE) [37–39] while in other studies, HDE is studied in alternative gravity theories like Braneworld, $f(R)$, scalar-tensor gravity, Brans–Dicke (BD) and DGP model etc [40–50]. The HDE also best fits with the observational data of cosmic microwave background (CMB) and supernova of type Ia [51–56].

We emphasize that the black hole entropy $S$ plays a central role in the derivation of HDE density. Indeed, the definition and derivation of HDE density depends on the entropy-area relationship $S \sim A \sim L^2$ (or in general $S(A)$) of black holes in Einstein’s gravity, where $A \sim L^2$ denotes the area of the black hole horizon. However, this definition can be modified from the inclusion of quantum effects, motivated from the loop quantum gravity (LQG). These quantum corrections provided to the entropy-area relationship leads to the curvature correction in the Einstein–Hilbert action and vice versa [57–59]. The corrected entropy takes the form [60–65]

$$S = \frac{A}{4G} + \tilde{\alpha} \ln \frac{A}{4G} + \tilde{\beta}, \quad (1)$$

where $\tilde{\alpha}$ and $\tilde{\beta}$ are dimensionless constants of order unity. The exact values of these constants are not yet determined and still an open issue in quantum gravity. These corrections arise in the black hole entropy in LQG due to thermal and quantum fluctuations [66–70]. Moreover in Wald’s approach to classical gravity and in string theory, one can find similar corrections to entropy [71]. Generally the entropy-area relation can be expanded in an infinite series expression, but the contribution of these extra terms to the black hole entropy is negligible due to smallness of $\hbar$ [57–59]. Hence the leading order term in the expansion is the logarithmic term to entropy-area relation as considered in Eq. (1). The logarithmic term also appears in a model of entropic cosmology which unifies the early-time inflation and late-time cosmic acceleration of the Universe [72]. Taking the corrected entropy-area relation (1) into account, and following the derivation of HDE (especially the one shown in [19]), the energy density of the HDE will be modified as well. On this basis, Wei [73] proposed the energy density of the so-called “entropy-corrected HDE” (ECHDE) in the form

$$\rho_D = 3c^2M_p^2L^{-2} + \alpha L^{-4} \ln \left( \frac{M_p^2L^2}{\bar{L}^2} \right) + \beta L^{-4}, \quad (2)$$

where $\alpha$ and $\beta$ are dimensionless constants of order unity. In the special case $\alpha = \beta = 0$, the above equation yields the well-known HDE density. Since the last two terms in Eq. (2) can be comparable to the first term only when $L$ is very small, the corrections make sense only at the early stage of the Universe. When the Universe becomes large, ECHDE reduces to the ordinary HDE.

In particular, the HDE has been widely analyzed in the framework of BD gravity [74–83]. Since the HDE density belongs to a dynamical cosmological constant, we need a dynamical frame to accommodate it instead of general relativity. Further, taking $L = H^{-1}$, it fails to determine the EoS $w_D$ in the general relativity framework. In addition to these, the BD scalar field speeds up the expansion rate of a dust matter
dominated era (reduces deceleration), while slows down the expansion rate of cosmological constant era (reduces acceleration). Since our paper deals with the ECHDE, we generalize the above studies.

In the light of all mentioned above, the investigation on the HDE models in the framework of BD theory is well motivated. In these studies [74–83], several dynamical features of HDE have been explored in the flat/non-flat FRW background e.g. the phantom crossing \((w = -1)\) at the present time; cosmic-coincidence problem; effective EoS; the deceleration parameter and the quintom behavior.

This paper is outlined as follows. In Sect. 2 we study ECHDE in the framework of BD theory in a non-flat Universe. We also discuss some of the features of this model including effective EoS, deceleration parameter and evolution of dimensionless energy density in the absence of interaction between DE and DM in Sect. 2. In Sect. 3, we extend our study to the case where there is an interaction between ECHDE and DM. The last section is devoted to conclusions.

## 2 ECHDE in BD theory

The BD action is given by

\[
I = \int d^4x \sqrt{g} \left( -\frac{\omega}{8\varphi^2} R + \frac{\omega}{\varphi^2} \phi^2 g_{\mu\nu} \partial_\mu \phi \partial_\nu \phi + L_M \right).
\]  

(3)

Using the following definition

\[
\varphi = \frac{\phi^2}{8\omega},
\]

(4)

the above action can be rewritten in the canonical form [84,85]

\[
I = \int d^4x \sqrt{g} \left( -\frac{1}{8\omega} \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + L_M \right).
\]

(5)

where \(g, \omega, \phi, R, \) and \(L_M\) are the determinant of the metric \(g^{\mu\nu}\) of spacetime, the BD parameter, the BD scalar field, the scalar curvature, and the lagrangian of the matter, respectively. The non-minimal coupling term \(\phi^2 R\) replaces with the Einstein–Hilbert term \(R / G\) in such a way that \(G_{\text{eff}}^{-1} = 2\pi \phi^2 / \omega\), where \(G_{\text{eff}}\) is the effective gravitational constant as long as the dynamical scalar field \(\phi\) varies slowly.

We consider the Friedmann–Robertson–Walker (FRW) metric for the non-flat Universe as

\[
ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega_1^2 \right),
\]

(6)

where \(k = 0, 1, -1\) represent a flat, closed and open FRW Universe, respectively. Observational evidences support the existence of a closed Universe with a small positive curvature \((\Omega_k \sim 0.02)\) [86–90].
Taking the variation of the action (5) with respect to the metric (6), one can obtain the field equations for the non-flat Universe containing DE and pressureless dust matter as

\[
\frac{3}{4\omega} \phi^2 \left( H^2 + \frac{k}{a^2} \right) - \frac{1}{2} \phi^2 + \frac{3}{2\omega} H \phi \dot{\phi} = \rho_D + \rho_M, \tag{7}
\]

\[
-\frac{1}{4\omega} \phi^2 \left( \frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} \right) - \frac{1}{\omega} H \phi \dot{\phi} - \frac{1}{2\omega} \phi \ddot{\phi} - \frac{1}{2} \left( 1 + \frac{1}{\omega} \right) \dot{\phi}^2 = p_D, \tag{8}
\]

\[
\ddot{\phi} + 3H \dot{\phi} - \frac{3}{2\omega} \left( \frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} \right) \phi = 0, \tag{9}
\]

where \( \rho_D \) and \( p_D \) are the DE density and pressure, respectively. Also \( \rho_M = \rho_{DM} + \rho_{BM} \) is the total energy density of pressureless DM and BM. We neglect the contribution of radiation.

At this point our system of Eqs. (7)–(9) is not closed and we still have freedom to choose one. We shall assume that BD field can be described as a power law of the scale factor, \( \phi \propto a^n \). In principle there is no compelling reason for this choice. However, it has been shown that for small \( n \) it leads to consistent results [91,92]. A case of particular interest is that when \( n \) is small whereas \( \omega \) is high so that the product \( n\omega \) results of order unity [91]. This is interesting because local astronomical experiments set a very high lower bound on \( \omega \); in particular, the Cassini experiment implies that \( \omega > 10^4 \) [93,94]. Taking the derivative with respect to time of relation \( \phi \propto a^n \), we get

\[
\dot{\phi} = nH \phi, \tag{10}
\]

\[
\ddot{\phi} = n^2 H^2 \phi + n \phi \dot{H}. \tag{11}
\]

In the framework of BD cosmology, we write down the energy density of the ECH-DE model in the Universe as

\[
\rho_D = \frac{3c^2 \phi^2}{4\omega L^2} + \frac{\alpha}{L^4} \ln \left( \frac{\phi^2 L^2}{4\omega} \right) + \frac{\beta}{L^4}, \tag{12}
\]

which can be rewritten as

\[
\rho_D = \frac{3c^2 \phi^2}{4\omega L^2} \gamma_c, \tag{13}
\]

where

\[
\gamma_c = 1 + \frac{4\omega \alpha}{3c^2 \phi^2 L^2} \ln \left( \frac{\phi^2 L^2}{4\omega} \right) + \frac{4\omega \beta}{3c^2 \phi^2 L^2}. \tag{14}
\]

For \( \alpha = \beta = 0 \) we have \( \gamma_c = 1 \) and thus

\[
\rho_D = \frac{3c^2 \phi^2}{4\omega L^2}, \tag{15}
\]
which is the well-known HDE density in the BD cosmology [83]. In the limit of Einstein gravity where $G_{\text{eff}} \to G$, then the BD scalar field becomes trivial, i.e. $\phi^2 = \omega / 2\pi G = 4\omega M_p^2$, and Eq. (12) reduces to the ECHDE density (2) in Einstein gravity [73].

Following [12], the IR cut-off $L$ is defined as

$$ L = a(t) \frac{\sin n(\sqrt{|k|}y)}{\sqrt{|k|}}, $$

(16)

where

$$ \frac{\sin n(\sqrt{|k|}y)}{\sqrt{|k|}} = \begin{cases} 
\sin y, & k = 1, \\
y, & k = 0, \\
\sinh y, & k = -1,
\end{cases} $$

(17)

and

$$ y = \frac{R_h}{a(t)} = \int_0^r \frac{dr}{a(t)} = \int_0^r \frac{dr}{\sqrt{1 - kr^2}} = \begin{cases} 
\sin^{-1} r, & k = 1, \\
r, & k = 0, \\
\sinh^{-1} r, & k = -1.
\end{cases} $$

(18)

Here $R_h$ is the radial size of the event horizon measured in the $r$ direction and $L$ is the radius of the event horizon measured on the sphere of the horizon [12]. For a flat Universe, $L = R_h$.

The critical energy density, $\rho_{\text{cr}}$, and the energy density of the curvature, $\rho_k$, are defined as

$$ \rho_{\text{cr}} = \frac{3\phi^2 H^2}{4\omega}, \quad \rho_k = \frac{3k\phi^2}{4\omega a^2}. $$

(19)

The fractional energy densities are also defined as usual

$$ \Omega_M = \frac{\rho_M}{\rho_{\text{cr}}} = \frac{4\omega \rho_M}{3\phi^2 H^2}, $$

(20)

$$ \Omega_k = \frac{\rho_k}{\rho_{\text{cr}}} = \frac{k}{H^2 a^2}, $$

(21)

$$ \Omega_D = \frac{\rho_D}{\rho_{\text{cr}}} = \frac{c^2 \gamma_c}{L^2 H^2}. $$

(22)

For latter convenience we rewrite Eq. (22) in the form

$$ HL = \left( \frac{c^2 \gamma_c}{\Omega_D} \frac{1}{\Omega_D} \right)^{1/2}. $$

(23)
Taking time derivative of Eq. (16) and using (23) yields

\[ \dot{L} = \left( \frac{c^2 \gamma_c}{\Omega_D} \right)^{1/2} - \cos n(\sqrt{|k|}y), \tag{24} \]

where

\[ \cos n(\sqrt{|k|}y) = \begin{cases} 
\cos y, & k = 1, \\
1, & k = 0, \\
\cosh y, & k = -1. 
\end{cases} \tag{25} \]

Using Eqs. (16), (17), (21) and (23), one can rewrite Eq. (25) as

\[ \cos n(\sqrt{|k|}y) = \left[ 1 - \Omega_k \left( \frac{c^2 \gamma_c}{\Omega_D} \right) \right]^{1/2}. \tag{26} \]

Hence, Eq. (24) yields

\[ \dot{L} = \left( \frac{c^2 \gamma_c}{\Omega_D} \right)^{1/2} \left[ 1 - \left( \frac{\Omega_D}{c^2 \gamma_c} - \Omega_k \right)^{1/2} \right]. \tag{27} \]

For the FRW Universe containing the ECHDE and pressureless matter, the continuity equations are

\[ \dot{\rho}_D + 3H\rho_D(1 + w_D) = 0, \tag{28} \]
\[ \dot{\rho}_M + 3H\rho_M = 0, \tag{29} \]

where \( w_D = p_D/\rho_D \) is the EoS parameter of the ECHDE.

Taking the time derivative of Eq. (12) and using (10) and (27), we obtain

\[ \dot{\rho}_D = \left( \frac{2H\rho_D}{\gamma_c} \right) \left[ 2n\gamma_c + \left[ \frac{1}{2} - 2\gamma_c + \frac{4\omega \alpha H^2}{\phi^2} \left( \frac{\Omega_D}{c^2 \gamma_c} \right) \right] \left[ 1 + n - \left( \frac{\Omega_D}{c^2 \gamma_c} - \Omega_k \right)^{1/2} \right] \right]. \tag{30} \]

For \( \alpha = 0 = \beta \) (\( \gamma_c = 1 \)) we recover

\[ \dot{\rho}_D = 2H\rho_D \left[ n - 1 + \left( \frac{\Omega_D}{c^2 \gamma_c} - \Omega_k \right)^{1/2} \right], \tag{31} \]

which is the same as the result obtained for the HDE in BD gravity [83]. While for \( n = 0 \) and \( \phi^2 = 4\omega M_p^2 \), we have

\[ \dot{\rho}_D = \left( \frac{2H\rho_D}{\gamma_c} \right) \left[ 1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M_p^2} \left( \frac{\Omega_D}{c^2 \gamma_c} \right) \right] \left[ 1 - \left( \frac{\Omega_D}{c^2 \gamma_c} - \Omega_k \right)^{1/2} \right]. \tag{32} \]
Using Eqs. (2), (22) and (23), one can rewrite Eq. (32) as

\[ \dot{\rho}_D = \left( \frac{c^2 \gamma_c}{\Omega_D} \right)^{1/2} \left[ \frac{2\alpha - 4\beta}{L^5} - \frac{4\alpha}{L^5} \ln(M_p^2 L^2) - \frac{6c^2 M_p^2}{L^3} \right] \left[ 1 - \left( \frac{\Omega_D}{c^2 \gamma_c} - \Omega_k \right)^{1/2} \right], \]

(33)

which is same as the result derived for the ECHDE in Einstein gravity [95]. Substituting Eq. (30) in (28) yields the EoS parameter of the ECHDE in BD gravity as

\[ w_D = -1 - \frac{4n}{3} - \frac{2}{3\gamma_c} \left[ 1 - 2\gamma_c + \frac{4\omega \alpha H^2}{\phi^2} \left( \frac{\Omega_D}{c^2 \gamma_c} \right) \right] \left[ 1 + n - \left( \frac{\Omega_D}{c^2 \gamma_c} - \Omega_k \right)^{1/2} \right]. \]

(34)

Note that as we already mentioned, at the very early stage when the Universe undergoes an inflation phase, the correction terms in the ECHDE density (12) become important. After the end of the inflationary phase, the Universe subsequently enters in the radiation and then matter dominated eras. In these two epochs, since the Universe is much larger, the entropy-corrected terms to ECHDE, namely the last two terms in Eq. (12), can be safely ignored. During the early inflation era the Hubble parameter \( H \) is constant and \( a = \exp(Ht) \). Hence, the Hubble horizon \( H^{-1} \) and the future event horizon \( R_h = a \int_0^\infty \frac{dt}{a} \) will coincide i.e. \( R_h = H^{-1} = \text{const} \). On the other hand, since an early inflation era leads to a flat Universe, i.e. \( \Omega_k = 0 \), we have \( L = R_h = H^{-1} = \text{const} \). Also from Eq. (23) we have \( \frac{\Omega_D}{c^2 \gamma_c} = 1 \). Therefore during the early inflation era, Eq. (34) reduces to

\[ w_D = -1 - \frac{2nc^2}{3\Omega_D} \left[ 1 + \frac{4\alpha \omega}{3c^2} H^2 e^{-2nHt} \right]. \]

(35)

Using \( \phi = a^n \), Eq. (35) yields

\[ w_D = -1 - \frac{2nc^2}{3\Omega_D} \left[ 1 + \frac{4\alpha \omega}{3c^2} H^2 e^{-2nHt} \right]. \]

(36)

It is worth while to mention that in BD gravity, besides the standard de Sitter inflation \( a = \exp(Ht) \), two other inflationary solutions namely the intermediate \( a = \exp(At^f) \), \( A \) and \( f \) are constants) and the power-law \( a = t^p, p > 1 \) inflation can also be realized. In the intermediate case, the expansion of the Universe is slower than de Sitter but faster than power-law inflation [96–99].

In the absence of correction terms \( (\alpha = \beta = 0) \) we have \( (\gamma_c = 1) \) and Eq. (34) recovers the EoS parameter of the HDE in BD theory [83]

\[ w_D = -1 - \frac{1}{3} - \frac{2n}{3} - \frac{2}{3} \left( \frac{\Omega_D}{c^2} - \Omega_k \right)^{1/2}. \]

(37)

Comparing Eq. (35) with (37) we see that in the presence of correction terms the scalar field \( \phi \) enters the EoS parameter explicitly. From Eq. (34) we see that when the
ECHDE is combined with BD field the transition from normal state where \( w_D > -1 \) to the phantom regime where \( w_D < -1 \) for the EoS of DE can be easily achieved. This is in contrast to Einstein gravity where the EoS of noninteracting HDE cannot cross the phantom divide \( w_D = -1 \) \([9–11]\). To illustrate this result in ample detail, we investigate it for the late-time Universe where \( \Omega_D = 1 \) and \( \Omega_k = 0 \). In this case we have \( L = R_h \neq H^{-1} \) and \( H \neq \text{const} \). Now from Eq. (34) we find \( w_D = -\frac{1}{3} - \frac{2}{3c} - \frac{2n}{3} \). If we take \( c = 1 \) \([83]\) then \( w_D = -1 - \frac{2n}{3} \). On the other hand for ECHDE in Einstein gravity (\( n \to 0 \)) with \( c = 1 \) we obtain \( w_D = -1 \). Thus in the late-time Universe, although the EoS parameter of ECHDE does not feel the presence of the last two correction terms in Eq. (12) for \( n \neq 0 \) it will necessary cross the phantom divide, i.e. \( w_D < -1 \) in BD theory. This is in contrast to Einstein gravity (\( n \to 0 \)) where \( w_D \) of ECHDE mimics a cosmological constant in the late-time.

The deceleration parameter is given by

\[
q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2}.
\]

(38)

Dividing Eq. (8) by \( H^2 \), and using Eqs. (12), (23), (10) and (11), we obtain

\[
q = \frac{1}{2n+2} \left[ (2n+1)^2 + 2n(n\omega - 1) + \Omega_k + 3\Omega_D w_D \right].
\]

(39)

Replacing \( w_D \) from Eq. (34), we obtain the deceleration parameter for the ECHDE in BD theory as

\[
q = \frac{1}{2n+2} \left[ (2n+1)^2 + 2n(n\omega - 1) + \Omega_k + (2n+1)\Omega_D - 2\Omega_D \left( \frac{\Omega_D}{c^2\gamma_c} - \Omega_k \right)^{1/2} \right. \\
- \left. \frac{2\Omega_D}{\gamma_c} \left[ \frac{4\omega \alpha H^2}{\phi^2} \frac{\Omega_D}{c^2\gamma_c} + 1 \right] - \left[ 1 + n - \left( \frac{\Omega_D}{c^2\gamma_c} - \Omega_k \right)^{1/2} \right] \right].
\]

(40)

For \( \alpha = \beta = 0 \) (\( \gamma_c = 1 \)) Eq. (40) reduces to the case of HDE in BD gravity \([83]\)

\[
q = \frac{1}{2n+2} \left[ (2n+1)^2 + 2n(n\omega - 1) + \Omega_k - (2n+1)\Omega_D - 2\Omega_D \left( \frac{\Omega_D}{c^2} - \Omega_k \right)^{1/2} \right].
\]

(41)

If we take \( \Omega_D = 0.73 \) and \( \Omega_k \approx 0.01 \) for the present time and choosing \( c = 1 \), \( n\omega \approx 1 \) and \( \omega = 10^4 \), we obtain \( q \approx -0.48 \) for the present value of the deceleration parameter which is in good agreement with recent observational results \([100]\).

### 3 Interacting ECHDE in BD theory

Here our aim is to extend our study for the case that there is an interaction between ECHDE and DM. The recent observational evidence provided by the galaxy cluster Abell A586 supports the interaction between DE and DM \([101–103]\). This kind of
Interaction can be detected in the formation of large scale structures. It was suggested that the dynamical equilibrium of collapsed structures such as galaxy clusters would get modification due to the coupling between DE and DM [101–103]. The idea is that the virial theorem is modified by the energy exchange between the dark sectors leading to a bias in the estimation of the virial masses of clusters when the usual virial conditions are employed. This provides a near Universe probe of the dark coupling. The other observational signatures on the dark sectors’ mutual interaction can be found in the probes of the cosmic expansion history by using the SNeIa, baryonic acoustic oscillation (BAO), and CMB shift data, etc. [104–109].

The total energy density satisfies the following conservation law

\[ \dot{\rho} + 3H(\rho + p) = 0, \]  

(42)

where \( \rho = \rho_M + \rho_D \) and \( p = p_D \). Interaction causes the ECHDE and DM do not conserve separately and they must rather enter the energy balances [110]

\[ \dot{\rho}_D + 3H\rho_D(1 + w_D) = -Q, \]  

(43)

\[ \dot{\rho}_{DM} + 3H\rho_{DM} = Q, \]  

(44)

\[ \dot{\rho}_{BM} + 3H\rho_{BM} = 0, \]  

(45)

where we have assumed the BM dose not interact with DE. Here \( Q \) represents the interaction term. It is important to note that the continuity equations imply that the interaction term should be a function of a quantity with units of inverse of time (a first and natural choice can be the Hubble factor \( H \)) multiplied with the energy density. Therefore, the interaction term could be in any of the following forms: (i) \( Q \propto H\rho_D \), (ii) \( Q \propto H\rho_M \), or (iii) \( Q \propto H(\rho_M + \rho_D) \). The more general choice is \( Q = 3b^2H(\rho_M + \rho_D) \) with \( b^2 \) is a coupling constant [111–114]. The freedom of choosing the specific form of the interaction term \( Q \) stems from our incognizance of the origin and nature of DE as well as DM. Moreover, a microphysical model describing the interaction between the dark components of the Universe is not available nowadays. Thus, in the absence of such a theory, we rely on pure dimensional basis for choosing an interaction \( Q \).

Combining Eqs. (19) and (10) with the first Friedmann equation (7), we can rewrite this equation as

\[ \rho_{cr} + \rho_k = \rho_{BM} + \rho_{DM} + \rho_D + \rho_\phi, \]  

(46)

with the definition

\[ \rho_\phi \equiv \frac{1}{2}nH^2\phi^2\left(n - \frac{3}{\omega}\right). \]  

(47)

Dividing Eq. (46) by \( \rho_{cr} \), it can be rewritten as

\[ \Omega_{BM} + \Omega_{DM} + \Omega_D + \Omega_\phi = 1 + \Omega_k, \]  

(48)
where

$$\Omega_\phi = \frac{\rho_\phi}{\rho_{cr}} = -2n \left(1 - \frac{n\omega}{3}\right). \quad (49)$$

By the help of the above definitions, one can rewrite the interaction term as

$$Q = 3b^2 H (\rho_{DM} + \rho_D) = 3b^2 H \rho_D (1 + r), \quad (50)$$

where

$$r = \frac{\Omega_{DM}}{\Omega_D} = -1 + \frac{1}{\Omega_D} \left[1 + \Omega_k - \Omega_{BM} + 2n \left(1 - \frac{n\omega}{3}\right)\right]. \quad (51)$$

shows the ratio of the energy densities of two dark components. Using the continuity equation (45), one can easily obtain

$$\rho_{BM} = \rho_{BM_0} a^{-3} = \rho_{BM_0} (1 + z)^3.$$

Dividing the above relation by $\rho_{cr} = 3\phi^2 H^2 / (4\omega)$ gives

$$\Omega_{BM} = \left(\frac{\rho_{cr_0}}{\rho_{cr}}\right) \Omega_{BM_0} a^{-3} = \left(\frac{\rho_{cr_0}}{\rho_{cr}}\right) \Omega_{BM_0} (1 + z)^3,$$

where $\rho_{cr_0} = 3\phi^2 H_0^2 / (4\omega)$ and $\Omega_{BM_0} \sim 0.04$ is the fractional energy density of baryonic matter at the present time. Here $z = a^{-1} - 1$ is the cosmological redshift.

Inserting Eqs. (30), (50) and (51) in (43) we obtain the EoS parameter of the interacting ECHDE in BD theory as

$$w_D = -1 - \frac{4n}{3} - \frac{2}{3\gamma_c} \left[1 - 2\gamma_c + \frac{4\omega \alpha H^2}{\phi^2 c^2} \left(\frac{\Omega_D}{c^2 \gamma_c}\right)\right] \left[1 + n \left(1 - \frac{n\omega}{3}\right)\right]$$

$$- \frac{b^2}{\Omega_D} \left[1 + \Omega_k - \Omega_{BM} + 2n \left(1 - \frac{n\omega}{3}\right)\right]. \quad (52)$$

Comparing Eq. (52) with (34) shows that in the presence of interaction since the last expression in Eq. (52) has a negative contribution, hence crossing the phantom divide, i.e. $w_D < -1$, can be more easily achieved for than when the interaction between ECHDE and DM is not considered.

During the early inflation era ($L = R_b = H^{-1} = \text{const.}$) when the correction terms make sense in the ECHDE density (12), Eq. (52) yields

$$w_D = -1 - \frac{2nc^2}{3\Omega_D} \left[1 + \frac{4\omega \alpha H^2 e^{-2Ht}}{3} \right] - \frac{b^2}{\Omega_D} \left[1 - \Omega_{BM} + 2n \left(1 - \frac{n\omega}{3}\right)\right]. \quad (53)$$
In the absence of correction terms ($\alpha = \beta = 0$), Eq. (52) reduces to

$$w_D = -\frac{1}{3} - \frac{2n}{3} - \frac{2}{3} \left( \frac{\Omega_D}{c^2} - \Omega_k \right)^{1/2} - \frac{b^2}{\Omega_D} \left[ 1 + \Omega_k - \Omega_{BM} + 2n \left( 1 - \frac{n\omega}{3} \right) \right].$$

(54)

which is exactly the result obtained for the HDE in BD gravity [83]. On the other hand, when $n = 0 (\omega \rightarrow \infty)$ the BD scalar field becomes trivial, i.e. $\phi^2 = \omega/2\pi G = 4\omega M_P^2$, and Eq. (52) yields

$$w_D = -1 - \frac{2}{3\gamma_c} \left[ 1 - 2\gamma_c + \frac{\alpha H^2}{3c^2 M_P^2 \left( \frac{\Omega_D}{c^2} \right)} \right] \left[ 1 - \left( \frac{\Omega_D}{c^2 \gamma_c} - \Omega_k \right)^{1/2} \right]$$

$$- \frac{b^2}{\Omega_D} \left[ 1 + \Omega_k - \Omega_{BM} \right].$$

(55)

Using Eqs. (2), (22) and (23), one can rewrite Eq. (55) as

$$w_D = -1 - \frac{2}{3\gamma_c} \left[ \frac{(2\alpha - 4\beta) L^{-2} - 4\alpha L^{-2} \ln(M_P^2 L^2) - 6c^2 M_P^2}{3(3c^2 M_P^2 + \alpha L^{-2} \ln(M_P^2 L^2) + \beta L^{-2})} \right] \left[ 1 - \left( \frac{\Omega_D}{c^2 \gamma_c} - \Omega_k \right)^{1/2} \right]$$

$$- \frac{b^2}{\Omega_D} \left[ 1 + \Omega_k - \Omega_{BM} \right].$$

(56)

which recovers its respective expression in ECHDE model in Einstein gravity [95]. If we compare Eq. (52) with (55) we find out that when ECHDE is combined with BD field the transition from normal state where $w_D > -1$ to the phantom regime where $w_D < -1$ for the EoS of interacting DE can be more easily achieved for than when resort to the Einstein field equations is made.

Following [115], if we define the effective EoS parameter

$$w_D^\text{eff} = w_D + \frac{\Gamma}{3H},$$

(57)

where $\Gamma = 3b^2 (1 + r) H$. Then, the continuity equation (43) can be rewritten in the standard form

$$\dot{\rho}_D + 3H\rho_D (1 + w_D^\text{eff}) = 0.$$

(58)

Substituting Eq. (52) into (57) yields

$$w_D^\text{eff} = -1 - \frac{4n}{3} - \frac{2}{3\gamma_c} \left[ 1 - 2\gamma_c + \frac{4\omega \alpha H^2}{\phi^2 3c^2 \left( \frac{\Omega_D}{c^2 \gamma_c} \right)} \right] \left[ 1 + n - \left( \frac{\Omega_D}{c^2 \gamma_c} - \Omega_k \right)^{1/2} \right].$$

(59)
For $\alpha = \beta = 0$ then $\gamma_c = 1$ and we have

$$w_D^{\text{eff}} = -\frac{1}{3} - \frac{2n}{3} - \frac{2}{3} \left( \frac{\Omega_D^{c^2} - \Omega_k}{\Omega_D^{c^2}} \right)^{1/2},$$

which is same as the result obtained for HDE in BD theory [83]. It is important to note that in the literature, “the effective EoS” is also defined as the EoS for the total energy density $\rho_{\text{tot}}$ and pressure $P_{\text{tot}}$ of the Universe which in the flat FRW Universe, is given by $w_{\text{eff}} = -1 - 2\dot{H} / (3H^2) = P_{\text{tot}} / \rho_{\text{tot}}$ [116,117]. However, for the interacting HDE models the effective EoS parameter is defined as in Eq. (57) with adding the interaction term to $w_D$ [115]. So this definition differs from that of [116,117]. From Eqs. (59) and (60), one can easily see that $w_{\text{eff}}$ in BD theory can cross the phantom line provided the model parameters are chosen suitably.

Substituting Eq. (52) into (39) yields the deceleration parameter for the interacting ECHDE in BD gravity as

$$q = \frac{1}{2n+2} \left\{ (2n+1)^2+2n(n\omega - 1)+\Omega_k - (2n+1)\Omega_D - 2\Omega_D \left( \frac{\Omega^{c^2}D}{c^2} - \Omega_k \right)^{1/2} \right. - \left. \frac{2\Omega_D}{\gamma_c} \left[ \frac{4\omega H^2}{\phi^2 3c^2} \left( \frac{\Omega_D^{c^2}}{c^2} + 1 - \gamma_c \right) \right] \left[ 1 + n - \left( \frac{\Omega_D^{c^2}}{c^2} - \Omega_k \right)^{1/2} \right] + 3\Omega_D \zeta \right\},$$

where

$$\zeta = -\frac{b^2}{\Omega_D} \left[ 1 + \Omega_k - \Omega_{BM} + 2n \left( 1 - \frac{n\omega}{3} \right) \right].$$

In the absence of correction terms, i.e. $\alpha = \beta = 0$, Eq. (61) reduces to the deceleration parameter for the interacting HDE in BD gravity [83]

$$q = \frac{1}{2n+2} \left\{ (2n+1)^2+2n(n\omega - 1)+\Omega_k - (2n+1)\Omega_D - 2\Omega_D \left( \frac{\Omega^{c^2}D}{c^2} - \Omega_k \right)^{1/2} \right. - \left. 3b^2 \left[ 1 + \Omega_k - \Omega_{BM} + 2n \left( 1 - \frac{n\omega}{3} \right) \right] \right\},$$

We can also obtain the equation of motion for $\Omega_D$. Taking time derivative of Eq. (22) and using relation $\Omega_D = H\Omega_D'$, we obtain

$$\Omega_D' = 2\Omega_D \left[ q + \left( \frac{\Omega^{c^2}D}{c^2} - \Omega_k \right)^{1/2} \right] + 2\Omega_D \left[ \frac{4\omega H^2}{\phi^2 3c^2} \left( \frac{\Omega_D^{c^2}}{c^2} + 1 - \gamma_c \right) \right] \left[ 1 + n - \left( \frac{\Omega^{c^2}D}{c^2} - \Omega_k \right)^{1/2} \right].$$
where the prime denotes the derivative with respect to $x = \ln a$. Also $q$ is given by Eq. (61). For $\alpha = \beta = 0$, the above expression reduces to the case of interacting HDE in BD gravity [83]

$$
\Omega_D' = \Omega_D \left\{ (1 - \Omega_D) \left[ 1 + 2 \left( \frac{\Omega_D}{c^2} - \Omega_k \right)^{1/2} \right] - 3b^2(1 + \Omega_k - \Omega_{BM}) + \Omega_k \right\}.
$$

(65)

## 4 Conclusions

In this paper, we investigated the model of HDE with the logarithmic corrections. These corrections are motivated from the LQG which is one of the promising theories of quantum gravity. We started by taking a non-flat FRW background in the BD gravitational theory. This theory involves a scalar field which accounts for a dynamical gravitational constant. We assumed an ansatz by which the BD scalar field evolves with the expansion of the Universe. We then established a correspondence between the field and the ECHDE to study its dynamics. The dynamics are governed by few dynamical parameters like its EoS, deceleration and energy density parameters. For the sake of generality, we calculated them in the non-flat background with the interaction of ECHDE with the matter. The study favors the phantom crossing scenario due to the availability of abundant parameters. It is not our purpose to fix or fit these parameters and we left it till the availability of the observational data. We hope that the future high precision observations like the type Ia supernovae (SNeIa) surveys, the shift parameter of the cosmic microwave background (CMB) observed by the Wilkinson Microwave Anisotropy Probe (WMAP) and the Planck Mission, and the baryon acoustic oscillation (BAO) measurement from the Sloan Digital Sky Survey (SDSS) may be capable for determining the fine property of the interacting entropy-corrected holographic model of DE in BD gravity and consequently reveal some significant features of the underlying theory of DE.

### Acknowledgments

The works of A. Sheykhi and K. Karami have been supported financially by Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), Maragha, Iran. M. Jamil would like to thank the Abdus Salam International Center for Theoretical Physics (ICTP), Trieste, Italy for its kind hospitality during which part of this work was completed. Useful comments from the anonymous referee are also gratefully acknowledged.

### References

1. Riess, A.G., et al.: Astron. J. 116, 1009 (1998)
2. Perlmutter, S., et al.: Astrophys. J. 517, 565 (1999)
3. de Bernardis, P., et al.: Nature 404, 955 (2000)
4. Perlmutter, S., et al.: Astrophys. J. 598, 102 (2003)
5. Seljak, U., et al.: Phys. Rev. D 71, 103515 (2005)
6. Padmanabhan, T.: Phys. Rep. 380, 235 (2003)
7. Copeland, E.J., et al.: Int. J. Mod. Phys. D 15, 1753 (2006)
8. Cai, Y.F., et al.: Phys. Rep. 493, 1 (2010)
9. Li, M.: Phys. Lett. B 603, 1 (2004)
10. Myung, Y.S.: Phys. Lett. B 649, 247 (2007)
11. Myung, Y.S., Seo, M.G.: Phys. Lett. B 671, 435 (2009)
12. Huang, Q.G., Li, M.: JCAP 08, 013 (2004)
13. Susskind, L.: J. Math. Phys. 36, 6377 (1995)
14. 't Hooft, G.: hep-th/0003004
15. 't Hooft, G.: Int. J. Mod. Phys. D 15, 1587 (2006)
16. Bigatti, D., Susskind, L.: hep-th/0002044
17. Fischler, W., Susskind, L.: hep-th/9806039
18. Cohen, A., et al.: Phys. Rev. Lett. 82, 4971 (1999)
19. Guberina, B., Horvat, R., Nikolic, H.: JCAP 01, 012 (2007)
20. Bekenstein, J.D.: Phys. Rev. D 7, 2333 (1973)
21. Hawking, S.W.: Phys. Rev. D 13, 191 (1976)
22. Chen, B., et al.: Nucl. Phys. B 774, 256 (2007)
23. Jamil, M., et al.: Phys. Lett. B 679, 172 (2009)
24. Sadjadi, H.M.: JCAP 02, 026 (2007)
25. Xu, L.: JCAP 09, 016 (2009)
26. Jamil, M., et al.: Eur. Phys. J. C 61, 471 (2009)
27. Jamil, M., Farooq, M.U.: Int. J. Theor. Phys. 49, 42 (2010)
28. Karami, K., Fehri, J.: Int. J. Theor. Phys. 49, 1118 (2010)
29. Jamil, M., Sheykhi, A.: Int. J. Theor. Phys. 50, 625 (2011)
30. Jamil, M., Karami, K., Sheykhi, A.: Int. J. Theor. Phys. 50, 3069 (2011)
31. Sadjadi, H.M., Jamil, M.: Gen. Relativ. Gravit. 43, 1759 (2011)
32. Karami, K., Jamil, M., Moos, H., Ghaffari, S., Abdolmaleki, A.: arXiv:1101.1774
33. Wang, B., Gong, Y., Abdalla, E.: Phys. Lett. B 624, 141 (2005)
34. Wang, B., Lin, C.Y., Abdalla, E.: Phys. Lett. B 637, 357 (2005)
35. Wang, B., Lin, C.Y., Pavón, D., Abdalla, E.: Phys. Lett. B 662, 1 (2008)
36. Sheykhi, A.: Class. Quantum Grav. 27, 025007 (2010)
37. Chattopadhyay, S., Debnath, U.: Astrophys. Space Sci. 319, 183 (2009)
38. Karami, K., Fehri, J.: Phys. Lett. B 684, 61 (2010)
39. Karami, K., Abdolmaleki, A.: Phys. Scr. 81, 055901 (2010)
40. Feng, C.J., Li, X.Z.: Phys. Lett. B 679, 151 (2009)
41. Feng, C.J., Zhang, X.: Phys. Lett. B 680, 399 (2009)
42. Wei, H.: Nucl. Phys. B 819, 210 (2009)
43. Bisabr, Y.: Gen. Relativ. Gravit. 41, 305 (2009)
44. Nozari, K., Rashidi, N.: Int. J. Theor. Phys. 48, 2800 (2009)
45. Nozari, K., Rashidi, N.: Int. J. Mod. Phys. D 19, 219 (2010)
46. Jamil, M., Sheykhi, A., Farooq, M.U.: Int. J. Mod. Phys. D 19, 1831 (2010)
47. Karami, K., Khaledian, M.S.: JHEP 03, 086 (2011)
48. Setare, M.R.: Phys. Lett. B 644, 99 (2007)
49. Setare, M.R., Jamil, M.: Europhys. Lett. 92, 49003 (2010)
50. Setare, M.R., Jamil, M.: Phys. Lett. B 690, 1 (2010)
51. Feng, C., et al.: JCAP 09, 005 (2007)
52. Wang, B., et al.: Nucl. Phys. B 778, 69 (2007)
53. Wu, Q., et al.: Phys. Lett. B 659, 34 (2008)
54. Li, M., et al.: JCAP 12, 014 (2009)
55. Lu, J., et al.: JCAP 03, 031 (2010)
56. Zhang, X.: Phys. Lett. B 683, 81 (2010)
57. Banerjee, R., Modak, S.K.: JHEP 05, 063 (2009)
58. Modak, S.K.: Phys. Lett. B 671, 167 (2009)
59. Banerjee, R., Gangopadhyay, S., Modak, S.K.: Phys. Lett. B 686, 181 (2010)
60. Banerjee, R., Majhi, B.R.: Phys. Lett. B 662, 62 (2008)
61. Banerjee, R., Majhi, B.R.: JHEP 06, 095 (2008)
62. Zhang, J.: Phys. Lett. B 668, 353 (2008)
63. Majhi, B.R.: Phys. Rev. D 79, 044005 (2009)
64. Karami, K., et al.: Gen. Relativ. Gravit. 43, 27 (2011)
65. Karami, K., Khaledian, M.S., Jamil, M.: Phys. Scr. 83, 025901 (2011)
66. Rovelli, C.: Phys. Rev. Lett. 77, 3288 (1996)
