Application of wavelet analysis for processing tomograms of narrow cracks

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Abstract. The previously proposed method of increasing the visibility of cracks on tomograms obtained from X-ray sensing of sections of metal elements of building structures has been developed. A multiscale decomposition of tomogram lines is performed using the Haar wavelet basis. The detailed decomposition coefficients corresponding to the level of the assumed crack width are multiplied by a constant, the value of which is determined based on the noise level and the difference between the amplitude of the crack image and the surrounding background. In contrast to the previous work, where these parameters were assumed to be set, here we suggest the methods for its evaluating from the measurements. In particular, a simple formula is obtained that relates the variance of uncorrelated noise in the projection data to that of noise in the tomogram. The method is implemented as a computational algorithm based on which a computer program is developed. Numerical modeling was performed. The errors of the first and second kind were used to evaluate the effectiveness of the method determining whether a pixel belongs to a crack image. Binary classification was used for their calculation. Dependences of errors on the number of projections and the noise level on them were obtained. The results of the simulations showed that the use of the proposed method can reduce errors by 1.7-2.5 times.

1. Introduction

The size and shape of internal cracks in the elements of building structures can be determined with high accuracy by X-ray tomography [1]. Its mathematical basis is the two-dimensional Radon transform [2], which converts a function of two variables \( g(x, y) \) to a set of its integrals along straight lines:

\[
f(p, \varphi) = \int_{-\infty}^{\infty} g\left( \sqrt{p^2 + r^2 \cdot \tan(\varphi)} \right) dp.
\]  

Here the function \( g \) is represented in polar coordinates; \( f(p, \varphi) \) denotes the value of the integral of \( g(x, y) \) along the line, which is defined by the distance to the origin \( p \) and by the angle of the normal to the axis of the abscissa \( \varphi \), see Figure 1. The function \( f(p, \varphi) \) is called projection data. In the case of X-ray tomography, it is usually assumed that \( f(p, \varphi) = \ln(I_0/I(p, \varphi)) \), where \( I(p, \varphi) \) is the radiation intensity measured by the detector; \( I_0 \) is the intensity of the X-ray source [3]. In this case, the function \( g(x, y) \)
describes the distribution of the X-ray attenuation coefficient in the studied cross-section of the object. The schematic diagram of registration of projection data is shown in Figure 1. Source $S$ and detector $D$ move synchronously in the direction indicated by the arrows, scanning the object. After scanning, the system rotates at an angle of $0.5^\circ - 1^\circ$ around the $Z$ axis, directed perpendicular to the drawing plane, and the next angle is taken, and so on.

The inversion formula for Radon transform is known. It is the basis for algorithms of image reconstruction from projections, [4, 5]:

$$g(x,y)=\frac{1}{8\pi}\int_0^{2\pi}\int_{-\infty}^{\infty} |\tilde{f}(\omega,\varphi)| \exp\left(i\omega(x \cos \varphi + y \sin \varphi)\right) d\omega d\varphi$$

(2)

where the "tilde" sign at the top means the Fourier transform over the first variable. Thus, having the data obtained as a result of X-ray sensing, it is possible, using the formula (2), to reconstruct the image of the distribution of the attenuation coefficient in the selected section. This image called a tomogram.

The specificity of the problem of X-ray diagnostics of cracks is that they usually have a low contrast against the surrounding background [6–8]. Therefore, the presence of even relatively weak noise in the tomogram may cause the crack to be visually indistinguishable. Today, this problem has not been fully resolved.

The existing methods of improving the quality of tomograms from the point of view of the problem under consideration have significant disadvantage. Low-frequency filtering of projection data, widely used for noise attenuation [9–11], leads to blurring of the crack boundaries, which makes it impossible to reliably describe them. The suppression of high-frequency components in the spectrum of the tomogram itself has a similar effect [12]. Sometimes a binary classification is used to estimate the size and shape of a crack [13, 14]. However, success here depends on how well the value of the dividing threshold was chosen. In particular, satisfactory results were obtained when the latter was determined from the minimum condition of the weighted sum of errors of the first and second kind, [14]. In this paper, we develop a method for selectively changing the amplitude of image fragments based on multiscale analysis. In this paper, a method for selectively changing the amplitude of image fragments based on multiple-scale analysis is developed to increase the contrast of the crack image.

![Figure 1. Projection data recording system: S is X-ray source; D is detector.](image)

2. Materials and Method

Wavelet bases are derived from the function $\psi(x)$, which has certain properties [15], by scaling and shifting:

$$\psi_{m,k}(x)=a^{m/2}\psi(a^mx-k),$$

(3)

where $a$ is a real number; $m$ and $k$ are integers. Two interconnected orthonormal self-similar bases $\varphi_{m,k}(x)$ and $\psi_{m,k}(x)$, constructed according to (3) with the constant $a=2$, are used for multiscale analysis. In this case, $\varphi(x)$ is called a scaling function, and $\psi(x)$ is called a wavelet. Let there be a continuous signal $s(x)$ and the result of its digitization with length $N=2^M$ samples. The following representation takes place, [15]:
\[ s(x) = \sum_{k=0}^{2^n-1} c_{n,k} \varphi_{n,k}(x) + \sum_{m=1}^{M-1} \left( \sum_{k=0}^{2^m-1} d_{m,k} \psi_{m,k}(x) \right). \] (4)

Here \( c_{n,k} = (s, \varphi_{n,k}) \) and \( d_{m,k} = (s, \psi_{m,k}) \) are the scaling and detailing coefficients, respectively (the angle brackets denote the scalar product); \( n \) can take values from zero to \( M \), but for \( n=M \) the second term in (4) is missing. The first of the indexes numbering \( c_{m,k} \) and \( d_{m,k} \) defines their level. The practical value of expression (4) is greatly facilitated by the fact that there are recurrent formulas linking coefficients of different levels. They depend on the type of basic functions. In particular, for the simplest pair of functions \( \varphi(x) \) is a rectangular pulse, \( \psi(x) \) is a Haar wavelet, (see Figure 2) it takes place, [16]:

\[
\begin{align*}
    c_{m,1,k} &= \left( \frac{1}{\sqrt{2}} \right) \left( c_{m,2k} + c_{m,2k+1} \right), & d_{m,1,k} &= \left( \frac{1}{\sqrt{2}} \right) \left( c_{m,2k} - c_{m,2k+1} \right); \\
    c_{m,2,k} &= \left( \frac{1}{\sqrt{2}} \right) \left( c_{m,1,k} + d_{m,1,k} \right), & c_{m,2,k+1} &= \left( \frac{1}{\sqrt{2}} \right) \left( c_{m,1,k} - d_{m,1,k} \right).
\end{align*}
\] (5a)

Thus, knowing the higher scaling coefficients \( c_{M,k} \), using the formulas (5a), we can find all \( c_{m,k} \) and \( d_{m,k} \) in the expansion (4) to any level \( n \), without using the explicit form of the functions \( \varphi(x) \) and \( \psi(x) \). When conducting research, usually the digitized values of the signal itself are used as \( c_{M,k} \), i.e. \( c_{M,k} = s(x_k) \) is assumed.

The essence of the developed method is as follows. Without reducing generality, let's assume that it is known a priori that the crack is oriented along the Y axis, i.e. the tomogram lines are perpendicular to it. Each line represents as (4), where the level \( n \) is determined by the assumed width of the crack. If there is no such information, then one can conduct research for several, most likely \( M \). We calculate all scaling and detailing coefficients from the \( n \)-th to the \( M \)-th order inclusive. Let the position of the crack section in the line correspond to the coefficient \( d_{n,k} \). \( d_{n,k} \) and \( \left| d_{n,k} - d_{n,k+1} \right| \approx |d_{n,k} - d_{n,k+1}| \sim \delta \), where the difference between the crack amplitude and the surrounding background is denoted by \( \delta \). Utilizing the formula (4), we will assemble the lines using instead of each \( d_{n,k} \) its product by a certain number \( \alpha_n \). In the image obtained in this way, the differences in considered detail coefficients will be of the order \( \alpha_n \delta \), therefore, the contrast of the crack will increase, which will help to improve the conditions for its detecting and evaluating its shape and size.

![Figure 2. Functions that generate the used bases.](image)

In [17], a method for determining the value of \( \alpha_n \) is proposed. There were introduced random variables \( \xi = \frac{2(n-M+1)}{d_{n,k}} \) and \( \xi_n = \alpha_n \xi \). It was shown that \( \xi \) is an unbiased estimate for \( \delta \) and therefore has a mathematical expectation \( \mathbb{E}(\xi) = \delta \), and its variance is equal to \( \mathbb{D}(\xi) = 2^{n-M+1} \sigma^2 \), \( \sigma^2 \) is the variance of noise in the image. The factor \( \alpha_n \) is determined from the condition \( \mathbb{M}(\xi) + \sqrt{\mathbb{D}(\xi)} = \mathbb{M}(\xi_n) - \sqrt{\mathbb{D}(\xi_n)} \mathbb{M} \), from which

\[
\alpha_n = \left( \mathbb{M}(\xi) + \sqrt{\mathbb{D}(\xi)} \right) / \left( \mathbb{M}(\xi_n) - \sqrt{\mathbb{D}(\xi_n)} \right) = \left( \delta + 2^{n-M+1/2} \sigma \right) / \left( \delta - 2^{n-M+1/2} \sigma \right) \] (6)
Formula (6) includes the parameters $\delta$ and $\sigma$, which were assumed to be set in [14]. Here, based on the specific conditions of the problem under consideration, their estimates are obtained. In tomography problems, it is often possible to approximately determine the variance of the noise in the recorded data $\sigma_p^2$. In [18], we investigated the relationship between noise in projections and in the tomogram reconstructed from them. According to the inverse formula (2), data registered at a single angle, called a parallel projection, is processed by a high-pass filter with the characteristic $|\omega|$ (ramp-filter). After that, the backprojection (integration by angle) is performed. In [15], an expression for noise dispersion after filtering was obtained:

$$\sigma^2 = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \Omega^2(\omega) W(\omega) d\omega. \tag{7}$$

Here $W(\omega)$ is the spectral power density of the noise, and $\Omega(\omega)$ is an approximation of the frequency response of the ramp-filter. Assume that the noise is white, and $\Omega(\omega)=|\omega|$ if $|\omega|<\omega_N$ and $\Omega(\omega)=0$ otherwise ($\omega_N$ is the Nyquist cyclic frequency). Then it is easy to show that $\sigma^2 = \sigma_p^2 \omega_N/2$. To get to the noise variance in the tomogram, note that in the numerical implementation of (2), integration by angle is replaced by summation by scanning angles, and the variance of the sample average falls in direct proportion to the sample size. As a result, we get $(\sigma_p^2 \omega_N)/(2K)$, where $K$ is the number of one-dimensional projections (scanning angles).

Let us now proceed to estimate the value of $\delta$. Denote by $\hat{g}(x_i,y_j)$ the values of the reconstructed function in the grid nodes. Let $\hat{n}_c$ be the estimated number of pixels occupied by the crack on the tomogram. In the area of interest $D_i$, the selection of which is usually not difficult, the $\hat{n}_c$ of the smallest values of $\hat{g}(x_i,y_j)$ are found, and the average $\bar{g}_{\min}$ is calculated for them. It is obvious that its variance is equal to $\sigma^2 / \hat{n}_c$. For the other nodes lying in $D_i$, we also calculate the average value $\bar{g}$. The difference in the tomogram of the crack and material amplitudes is assumed to be the value $\delta = \bar{g} - \bar{g}_{\min} + \sqrt{\sigma^2 / \hat{n}_c}$.

### 3. Numerical simulation results

The method described in the previous section was investigated by a computational experiment. A numerical algorithm was developed and then implemented as a computer program. The mathematical phantom developed for testing is shown in Figure 3.a. It simulates the cross section of a steel rebar with a crack. A set of 360 projections in the form of a synogram is shown in Figure 3.b. Here the horizontal axis corresponds to the coordinate $p$, and the vertical axis corresponds to the angle $\phi$ (see formula (1) and its explanations). The tomogram reconstructed from these data by an algorithm based on the formula (4) is shown in Figure 3.c. The image resolution is 512 x 512 pixels.

![Figure 3. Mathematical phantom (a); projection data (b); tomogram (c).](image-url)
To evaluate the accuracy of the method we use normalized errors of the first and second kinds:

\[ \Delta_1 = \frac{1}{n_c} \sum_{(x_i, y_m) \in D_c} \hat{x}(x_i, y_m), \quad \Delta_2 = \frac{1}{n_c} \sum_{(x_i, y_m) \in D_i} \left(1 - \hat{x}(x_i, y_m)\right) \]  

(8)

Here \( D_c \) is the area occupied by the crack in the image of the mathematical phantom; \( D_i \) is the area of interest, in Figure 3,a its borders are schematically marked with a dotted line; \( n_c \) denotes the number of pixels that belong to the crack. The function \( \hat{x}(x_i, y_m) \) is 0 in pixels assigned by the classifier to the crack, and 1 otherwise. To calculate errors (8), a binary classification was applied to the image in the area \( D_i \). The value of the threshold \( g_{th} \) was calculated according to [12]:

\[ g_{th} = \frac{\sigma_y^2}{\bar{g} - \bar{g}_c} \ln \left(\frac{1 - P_c}{P_c}\right) + \frac{\bar{g} + \bar{g}_c}{2}, \]  

(9)

where \( \bar{g}_c \) and \( \bar{g} \) are the average values for the \( D_c \) and \( D_i \setminus D_c \) regions, respectively; \( P_c \) is the probability that a randomly taken pixel from \( D_i \) belongs to the crack image. In this work, it was estimated as \( n_c / n_i \), \( n_i \) is the number of pixels in the \( D_i \) region. In particular, for Figure 3, with \( \Delta_1 = 0.093 \) and \( \Delta_2 = 0.051 \). In particular, for Figure 3,c \( \Delta_1 = 0.093 \) and \( \Delta_2 = 0.051 \).

Figure 4. Tomograms reconstructed from noisy data: \( \xi = 0.01 \) (a); \( \xi = 0.025 \) (b); \( \xi = 0.05 \) (c).

The tomogram in Figure 3, c was obtained from data that does not contain random noise. To model it a normally distributed centered random variable with the variance \( \sigma_y^2 = \xi^2 \bar{f}^2 \) was added to the function \( f(p, \varphi) \) at each point, where \( \xi \) is a positive number and \( \bar{f} \) is the average value of the projection data at the absence of noise. Figure 4,a,b,c show tomograms obtained from 360 projections at \( \xi = 0.01, 0.025, \) and 0.05, respectively. The results of applying the developed method to them are shown in Figure 5,a,b,c. The detailed coefficients of the seventh and eighth levels were transformed. Thus, it was assumed that the crack has a width of 2–4 pixels.

Figure 5. Results of applying the developed method to tomograms in Figure 4.
Figure 6. Dependences of errors $\Delta_1$ (left) and $\Delta_2$ (right) on the noise level; 360 projections.

For Figure 4,a,b,c, errors of the first and second kind are equal to $\Delta_1 = 0.152$, $0.331$, $0.664$ and $\Delta_2 = 0.099$, $0.172$, $0.303$. For Figure 5,a,b,c, $\Delta_1 = 0.102$, $0.150$, $0.291$; $\Delta_2 = 0.051$, $0.089$, $0.214$. Figure 6 shows the dependencies $\Delta_1(\xi)$ (left) and $\Delta_2(\xi)$ (right), the number of projections is 360. Here, as in Figure 9, curve 1 is obtained for the original tomograms, and curve 2 is obtained for those processed by the proposed method. Similar results obtained for a different number of projections at a noise level corresponding to $\xi = 0.025$ are shown in Figure 7 – Figure 9.

![Figure 6](image_url)

Figure 7. Tomograms reconstructed from different numbers of projections: 60 (a); 180 (b); 540 (c).

![Figure 7](image_url)

Figure 8. Results of applying the developed method to tomograms in Figure 7.
Figure 9. Dependences of errors $\Delta_1$ (left) and $\Delta_2$ (right) on the number of projections; noise level $\xi = 0.025$.

For Figure 6, $a,b,c$, errors of the first and second kind are equal to $\Delta_1 = 0.822$, 0.587, 0.231 and $\Delta_2 = 0.781$, 0.375, 0.142. For Figure 7, $a,b,c$, $\Delta_1 = 0.567$, 0.326, 0.106; $\Delta_2 = 0.573$, 0.238, 0.065.

4. Discussion
The results of the computational experiment illustrate two problems typical for X-ray tomography of metal structures. First, due to the high absorption of X-ray radiation by metals, in particular steel, the effect of the presence of cracks is weak in the recorded data. In Figure 3, the crack projection is represented by two barely noticeable sloping lines, which are indicated by arrows. The second problem is inherent in the whole tomography: image reconstruction using the formula (2) is an ill-posed problem. In practice, this means that even a negligible distortion of the source data can cause significant distortion in the tomogram.

The usual method of noise reduction is low-frequency filtering. But it is unacceptable in this case, since it also destroys the weak distortion of projections caused by the presence of defects. In this regard, the developed method is very relevant. In general, the results of the simulation confirmed its effectiveness. Visual comparison of tomograms before and after processing in some cases allows us to confidently speak about improving their quality. For example, in Figure 8, in contrast to Figure 7, the details of the bifurcated tip of the crack are clearly distinguished. However, the best confirmation is a comparison of quantitative criteria. Errors of the first and second kind (8) characterize the accuracy of determining the shape of crack. It is this information that allows us to judge its potential danger. As it is seen from the graphs in Figure 6, Figure 9, as well as from the numerical values given, when using the method, the errors $\Delta_1$ and $\Delta_2$ are reduced by $1.7 – 2.5$ times.

5. Conclusion
The paper considers the actual problem of flaw detection. It is a diagnostics of internal cracks in metal products by X-ray tomography. To improve the image quality of defects, a method for processing of tomograms based on their line-by-line multiscale analysis is proposed. Numerical simulations have shown that errors that characterize the accuracy of determining the shape of crack are reduced by $1.7 – 2.5$ times.

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