Quark splitting in non-trivial $\theta$-vacuum

Hongxi Xing$^{a,b}$, Xin-Nian Wang$^b$, Feng Yuan$^{b,c}$

$^a$Institute of Particle Physics and Key Laboratory of Quark & Lepton Physics, Huazhong Normal University, Wuhan 430079, China
$^b$Nuclear Science Division, Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley, California 94720, USA
$^c$RIKEN BNL Research Center, Building 510A, BNL, Upton, New York 11973, USA

Abstract

Quark splitting in non-trivial $\theta$-vacuum with a given helicity is investigated in pQCD with a modified quark propagator. We found that the quark splitting functions were modified by the presence of a topologically non-trivial QCD background field, though there is no explicit helicity flip associated with the radiative processes. The interaction with the topological non-trivial field leads to the degeneracy of the quark splitting functions for left- and right-handed quarks. Such degeneracy can lead to imbalance of left- and right-handed quarks in quark jet showers. We also discuss phenomenological consequences of such imbalance if there exists non-trivial topological gluon field configuration in heavy-ion collisions.

1. Introduction

Quantum Chromodynamics (QCD) contains topologically non-trivial configurations of gauge fields that can be represented by degenerate vacuum states and the physical vacuum, so-called $\theta$-vacuum, could be a superposition of these degenerate states. The presence of the $\theta$-vacuum state can be characterized by a $\theta$-term in the QCD Lagrangian which would lead to CP violation in the strong interaction. Search for the violation of global CP-invariance in strong interaction has only lead to an upper bound on the value of $\theta$ from the neutron dipole moment, $\theta < 3 \times 10^{-10}$ [2], indicating the absence of global $P$ and $CP$ violation in QCD. Recently, the STAR Collaboration at RHIC observed charge asymmetry in the azimuthal angle of dihadron correlation with respect to the reaction plane in non-central heavy-ion collisions [3]. Such charge asymmetry is speculated to originate from the chiral magnetic effect [4] due to the presence of local $P$ and $CP$ violation in QCD at high temperature. However, the asymmetry is found to exist both in and out of the reaction plane which is so far not understood [5].

In this talk, we present a study of parity-odd effect in processes of parton shower in jet fragmentation by considering the interaction of a quark with the non-trivial gauge field in the $\theta$-vacuum. Assuming quarks propagate in the presence of a topological non-trivial gluon field, we find a modified quark splitting probability that is different for left- and right-handed quarks. QCD evolution equations with such modified splitting functions lead to a sizable imbalance of shower quark distributions for left- and right-handed quarks.

2. Quark splitting in normal vacuum ($\theta = 0$)

In general, the left-handed quark fragmentation function is the same as the right-handed because of the parity invariance of the QCD Lagrangian when $\theta = 0$. In terms of parton matrix elements, it can be defined as [6]:

$$
D_{q_L \rightarrow h}(z_h) = D_{q_R \rightarrow h}(z_h) = \frac{z_h}{2} \int \frac{dy^-}{2\pi} e^{-ip_y^+z_h} \sum_S \text{Tr} \left[ \frac{y^+}{2} \langle 0 | \phi(0) | p_y, S \rangle \langle S, p_y | \phi(y^-) | 0 \rangle \right],
$$

(1)
where \( D_{q\rightarrow h}(z_h) \) and \( D_{q\rightarrow h}(z_h) \) are left- and right-handed quark fragmentation functions, respectively, with \( z_h = p_h^\|/k^\| \) the momentum fraction of the quark carried by the hadron. Here we use light-cone notation \( k^\| = (k_0 \pm k^\perp) \sqrt{2} / \sqrt{2} \).

In normal vacuum, gluon bremsstrahlung or quark splitting, illustrated by the Feynman diagrams in Fig. 1, leads to the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi(DGLAP) \([7]\) QCD evolution equations for quark fragmentation functions with given helicity,

\[
\frac{\partial}{\partial \ln \mu^2} \begin{pmatrix} D_{q\rightarrow h}(z_h, \mu^2) \\ D_{\bar{q}\rightarrow h}(z_h, \mu^2) \end{pmatrix} = \alpha_s \int \frac{dz}{z} \begin{pmatrix} P_{q\bar{q},q}(z) & P_{q\bar{q},\bar{q}}(z) \\ P_{\bar{q}q,q}(z) & P_{\bar{q}q,\bar{q}}(z) \end{pmatrix} \begin{pmatrix} D_{q\rightarrow h}(z_h, \mu^2) \\ D_{\bar{q}\rightarrow h}(z_h, \mu^2) \end{pmatrix},
\]

where we only consider the quark branching via gluon bremsstrahlung to analyze the helicity distribution and \( P(z) \) are splitting functions:

\[
P_{q\bar{q},q}(z) = P_{\bar{q}q,q}(z) = C_F \left[ 1 + \frac{z^2}{(1-z)_+} + \frac{3}{2} \delta(z-1) \right],
\]

\[
P_{q\bar{q},\bar{q}}(z) = P_{\bar{q}q,\bar{q}}(z) = 0.
\]

Suppose there are no differences between the number of left- and right-handed quarks, or zero chirality, before the start of branching processes, the QCD evolution in normal vacuum according to Eq. \(2\) does not induce any non-zero chirality. This means that hadron distributions from left and right-handed quarks should be identical. Therefore, alignment of left and right-handed quarks in the presence of large magnetic field in heavy-ion collisions, does not lead to any asymmetry in the final hadron spectra. However, in the presence of a topological non-trivial gluon field, the above evolution equation will be different for left and right-handed quarks that might lead to hadron spectra asymmetry under a strong magnetic field. This is essentially a dynamic mechanism for the proposed chiral magnetic effect \([4]\).

### 3. Quark splitting in non-trivial \( \theta \)-vacuum \( (\bar{\theta} \neq 0) \)

In QCD, the axial anomaly in quantum theory might lead to a non-trivial \( \theta \)-vacuum which violates the parity conservation in strong interaction. In this case, the vacuum wave function in QCD is a linear combination of wave functions with different winding numbers. This vacuum state can be reproduced by adding to the QCD Lagrangian a new term,

\[
\mathcal{L}_\theta = \theta/(32\pi^2) \bar{q}^\gamma \sigma^\mu \epsilon_{\mu\nu} \sigma^\nu q.
\]

The observable effect of the above parity-violating interaction can be mimicked by an effective space-time dependent dynamical ”spurion” field, i.e., \( \theta = \theta(x, t) \) \([3]\). By performing an axial U(1) rotation and omit a full derivative term, the extra \( \theta \)-term in QCD Lagrangian can be transformed into the fermionic contribution \( \mathcal{L}_\theta \rightarrow 1/(2N_f) \partial_\mu \bar{\psi} \gamma^\mu \gamma^5 \psi \) \([3]\). Here \( \partial_\mu \bar{\psi} \theta \) represents the fluctuation of the \( \theta \)-vacuum, whose zero component is the chiral chemical potential \( \mu_5 \). The existence of this new term will yield a modified quark propagator \([9]\), in massless case,

\[
i\bar{\psi}(k, \bar{\theta}) = i \left[ \mathcal{P}_R S(k + \bar{\theta}) + \mathcal{P}_L S(k - \bar{\theta}) \right],
\]
where \( P_{RL} \) are the right (left) projection operator \( P_{RL} = (1 \pm \gamma^5)/2 \), \( iS(k) = i/k \) is the conventional quark propagator in normal vacuum.

With this modified quark propagator in non-trivial \( \theta \)-vacuum, the parity symmetry will be broken in the process of quark propagation and branching during quark fragmentation. This parity-odd effect can be described by the modified DGLAP evolution equation in \( \theta \)-vacuum,

\[
\frac{\partial}{\partial \ln Q^2} \left( \frac{d\hat{D}_{RL}(z, \mu^2)}{d\hat{D}_{L/R}(z, \mu^2)} \right) = \frac{\alpha_s}{2\pi} \int_z^1 \frac{dz}{z} \left( \hat{P}_{q RL}(z) \right) \left( \hat{D}_{RL-\to RL}(z, \mu^2) \right).
\]

with the modified splitting functions,

\[
\hat{P}_{q RL}(z, t) = C_F \left[ \frac{1 + z^2}{(1 - z)^{+}} - t(1 + t) \left( \frac{1 + z}{z + t} \right)^2 + \left( \frac{3}{2} + a(t) \right) \delta(z - 1) \right];
\]

\[
\hat{P}_{q RL}(z, t) = C_F \left[ \frac{1 + z^2}{(1 - z)^{+}} + t(1 - t) \left( \frac{1 + z}{z - t} \right)^2 + \left( \frac{3}{2} + a(-t) \right) \delta(z - 1) \right];
\]

\[
\hat{P}_{q RL}(z, t) = \hat{P}_{q RL}(z, t) = 0,
\]

where \( t = \hat{\theta}^* / k^* \), and

\[
a(t) = t \left( 1 + t \right) \log \frac{1 + t}{z_0 + t} + \left( 1 - t \right) \frac{1 - z_0}{z_0 + t}.
\]

\( z_0 \) is the minimum value of the momentum fraction bounded by the virtuality of the initial quark. There are no helicity flip because of the pseudovector quark-gluon coupling. However, the parity is not conserved because of the degeneracy in the right-handed splitting function \( \hat{P}_{RL}(z, t) \) and left-handed splitting function \( \hat{P}_{QL}(z, t) \).

Compared to the DGLAP equation in normal vacuum, Eq. (4), the renormalization equation for the modified quark fragmentation function in non-trivial \( \theta \)-vacuum Eq. (6) is the same as the evolution equation in normal vacuum except for the modification of the splitting functions. As indicated in Eq. (7), the modified splitting functions have an extra term that is different for left- and right-handed quarks. It is these different extra terms that lead to the parity-odd effect in quark branching processes for non-vanishing \( \hat{\theta} \). One can see from Eq. (6), the DGLAP evolution equations in normal vacuum are recovered when \( \hat{\theta} = 0 \).

4. Shower parton distribution

To illustrate the imbalance of the left- and right-handed quark distributions, we solve the modified DGLAP evolution equations with an initial shower parton distribution,

\[
D_0^q(z, Q_0^2) = \delta(z - 1),
\]

Figure 2: Ratio defined in Eq. (10) for different values of \( \hat{\theta} \) and momentum scale \( Q^2 \) for left- and right-handed quarks.
for both right and left-handed quarks. The index $a, b$ denote the quark flavors. To quantify the effect of the $\theta$-vacuum, we define the ratio:

$$R \equiv \frac{D_a(z, Q^2)_{\theta}}{D_a(z, Q^2)_{\theta=0}}.$$  \hspace{1cm}(10)

Shown in Fig. 2 are the ratios for different values of $\tilde{\theta}$ (left panel) and momentum scale $Q^2$ (right panel) as a function of the momentum fraction $z$. One can see that the interaction with the topological non-trivial gauge field configuration lead to the degeneracy of left- and right-handed quark distributions. This degeneracy or imbalance between the left- and right-handed quark distributions is proportional to the value of $\tilde{\theta}$. Therefore, a non-zero chirality proportional to the $\theta$-vacuum fluctuation was generated by the modified DGLAP evolution equations.

In the right panel of Fig. 2, we also observe that the imbalance of right and left-handed quark distributions or chirality from the modified evolution equations is mostly in the region of small and moderate momentum fraction $z$ and the degeneracy increases with momentum scale $Q^2$. In principle, one can obtain the final state charged hadron distribution by convolution of this shower quark distribution function with the normal fragmentation function, assuming quark hadronization below scale $Q_h$ happens in normal vacuum.

We have illustrated that the parity-odd terms in the modified DGLAP evolution equations lead to a net helicity for quarks in the parity-odd domain, which is proportional to the fluctuation of the $\theta$-vacuum. Under a strong magnetic field, the left and right-handed quarks would orient their spin parallel (anti-parallel) to the magnetic field due to electromagnetic interaction through their magnetic moments. The above net chirality would then lead to hadron or charge asymmetry with respect to the direction of the magnetic field. Therefore, a combination of topological non-trivial gluon field and strong magnetic field would lead to hadron asymmetry in the final state of quark fragmentation. In high-energy heavy-ion collisions, strong magnetic field is induced by two high-energy nuclei with finite impact-parameter. Such magnetic field exists only for a very short period of time during the early stage of heavy-ion collisions, during which hard processes and jet parton branching happen. Therefore, existence of non-trivial gluon field would then lead to hadron asymmetry in the final hadrons from jet fragmentation.

5. Summary

In this paper, we have studied quark splitting in a non-trivial QCD vacuum with a given helicity. We found that the quark splitting functions were modified differently for left- and right-handed quarks for quark branching under such topologically non-trivial QCD background. This difference will induce non-zero chirality. In our calculation, this chirality is not induced by the flipping of the quark helicity, but rather by the branching asymmetry for left- and right-handed quarks in the presence of $\theta$-vacuum. The modifications of the splitting functions are parity-odd and depend on the size of the $\theta$ fluctuation. We have estimated this parity violation effect in the quark distribution in the parton shower of a quark jet. We found a sizable imbalance of quark distribution for left- and right-handed quarks. Such imbalance could provide an mechanism for final hadron asymmetry induced by the $\theta$-vacuum fluctuation.

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