Maieutic Prompting: Logically Consistent Reasoning with Recursive Explanations

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Abstract

Pre-trained language models (LMs) struggle with consistent reasoning; recently, prompting LMs to generate explanations that self-guide the inference has emerged as a promising direction to amend this. However, these approaches are fundamentally bounded by the correctness of explanations, which themselves are often noisy and inconsistent. In this work, we develop MAIEUTIC PROMPTING, which aims to infer a correct answer to a question even from the unreliable generations of LM. MAIEUTIC PROMPTING induces a tree of explanations abductively (e.g. \( X \) is true, because \( \ldots \)) and recursively, then frames the inference as a satisfiability problem over these explanations and their logical relations. We test MAIEUTIC PROMPTING for true/false QA on three challenging benchmarks that require complex commonsense reasoning. MAIEUTIC PROMPTING achieves up to 20% better accuracy than state-of-the-art prompting methods, and as a fully unsupervised approach, performs competitively with supervised models. We also show that MAIEUTIC PROMPTING improves robustness in inference while providing interpretable rationales.1

1 We share our code at https://github.com/jaehunjung1/Maieutic-Prompting.

1 Introduction

Following the remarkable success of few-shot prompting over large language models (e.g. Brown et al., 2020), recent studies on prompting methods suggest that LMs’ reasoning capability can be further promoted by generating a sequence of explanation for a given problem, prior to inferring the answer (Wei et al., 2022; Wang et al., 2022; Liu et al., 2021). The so-called explanation-based prompting helps an LM better elicit its knowledge and reason by leveraging its own generated explanations - whether it be commonsense knowledge (Liu et al., 2021), a solution for a math word problem (Wei et al., 2022; Wang et al., 2022; Liu et al., 2021), or the intermediate steps of program execution (Nye et al., 2021a).

Explanation-based prompting is intuitively motivated by the reasoning steps humans typically employ to solve a problem (Hausmann and Van-Lehn, 2007). However, we find that this intuition is faulty in practice, as model-generated explanations are often logically inconsistent and unreliable. For example, we manually inspected 100 samples from a QA task (Figure 1) and found that for a considerable number of cases, (1) the explanation does not logically lead to the answer, (2) model is invariant to negation, and (3) falsifies its own explanation. We prompt 175B GPT-3 with 100 questions sampled from Talmor et al. (2021).

Figure 1: Logical errors in explanation-based prompting: (1) explanation does not logically lead to the answer, (2) model is invariant to negation, and (3) falsifies its own explanation. We prompt 175B GPT-3 with 100 questions sampled from Talmor et al. (2021).

To this end, we propose MAIEUTIC PROMPT-
Our experiments show that the performance of MAIEUTIC PROMPTING exceeds that of all the few-shot prompting baselines (e.g., Chain of Thought; Wei et al., 2022) in three commonsense reasoning and fact verification benchmarks. MAIEUTIC PROMPTING performs up to 20% better than other prompting methods, and performs on par or even better than supervised models. Further analyses show that MAIEUTIC PROMPTING is robust to perturbations in both the questions and prompts, and offers an interpretable interface to understand the rationale behind the model’s inference.

2 Problem Setup and Background

Our goal is to infer whether a given statement $Q$ makes sense, i.e. inferring the truth value $A$ of $Q$. Conventionally, this can be done through prompting an LM with the following two methods:

**Standard Prompting** Let $Q$ be a statement we want to infer the truth value of (i.e., either True or False). In standard few-shot prompting, the model-inferred answer $\hat{A}$ is defined as:

$$\hat{A} = \arg\max_{A \in \{T,F\}} p_{LM}(A|Q,C),$$

where $C = \{(q_1, a_1), \ldots, (q_k, a_k)\}$ denotes the $k$ examples for in-context learning.

**Explanation-based Prompting** In explanation-based prompting, the inference process is factorized into two steps:

$$\hat{A} = \arg\max_{A \in \{T,F\}} \int_C p_{LM}(A|Q,E,C) p_{LM}(E|Q,C)$$

Here, $E$ denotes the explanation generated prior to inferring the answer label, and $C = \{(q_1, e_1, a_1), \ldots, (q_k, e_k, a_k)\}$ includes $k$ examples of questions, explanations and answers. Since marginalizing over all $E$ is intractable, prior works
resort to a sampling based approximation:
\[ \hat{A} = \argmax_{A \in \{T, F\}} p_{LM}(A|Q, E, C), \]
where \( E \sim p_{LM}(E|Q, C) \)

\[ (3) \]

3 Maieutic Prompting

In this section, we introduce MAIEUTIC PROMPTING, which performs inference over a maieutic tree of generated explanations. First, we introduce logical integrity, a key concept that is used to determine the reliability of propositions.

Language models often generate logically inconsistent propositions; for instance, in Figure 1, the model infers True when prompted with either “One is a number that comes before zero.” or “One is a number that comes after zero.”. In this sense, \( p(\text{True}|Q) \) does not provide a reliable value to determine whether \( Q \) is true or not. We formalize this idea as logical integrity: a proposition \( Q \) is logically integral when the LM consistently infers the truth value of \( Q \) and \( \neg Q \) (i.e., \( Q \) as True and \( \neg Q \) as False, or vice versa). Formally, we define a boolean function \( \text{integral}(E) \) as follows:

\[ 1. \ \argmax_{A \in \{T, F\}} p_{LM}(A|E, C) = T \text{ and } \argmax_{A \in \{T, F\}} p_{LM}(A|\neg E, C) = F \]

\[ 2. \ \argmax_{A \in \{T, F\}} p_{LM}(A|E, C) = F \text{ and } \argmax_{A \in \{T, F\}} p_{LM}(A|\neg E, C) = T \]

\[ \text{integral}(E) = \mathbb{1}\{1 \text{ or } 2 \text{ is satisfied}\}. \]

A statement is considered to be logically integral / True when condition 1 is met, and logically integral / False when condition 2 is met. Intuitively, the truth values of logically integral propositions are more credible than non-integral ones, to which LMs are inconsistent given a simple negation. For example, “One is a number that comes before zero.” in Figure 1 would not be logically integral, as the model assigns same truth value to both \( Q \) and \( \neg Q \).

For the rest of section, we first search for logically integral propositions by constructing the maieutic tree (Section 3.1), then quantify the relations between the propositions (Section 3.2), based on which we infer the final answer (Section 3.3).

3.1 Maieutic Tree Generation

3.1.1 Abductive Explanation Generation

Given a question, we require the LM to post-hoc rationalize both True and False labels. This abductive explanation generation has several advantages over an ad-hoc approach that first generates an explanation, then predicts the label. First, in the ad-hoc setting, the model is required to generate a discriminative explanation that helps in choosing one label over the other. Abductive generation (Bhagavatula et al., 2019), on the contrary, exposes the model to consider different possible answers rather than discriminating one, which often reveals an explanation that otherwise would not have been generated. Second, the label information would intuitively help LM elicit more specific explanations, mitigating the issue of a bland and generic generation which does not help the inference, a well-known weakness of LMs (Adiwardana et al., 2020).

Concretely, we define a function abduction which gets the statement \( Q \) as the input and outputs a tuple of two abductive explanations with True, False given as the answer, respectively:

\[ \text{abduction}(Q) = (E_T, E_F) \]

where \( E_{A \in \{T, F\}} \sim p_{LM}(E|Q, A, C) \).

Figure 2 shows a concrete example of generating \( E_T \) given \( Q \). With \( Q \), we prompt the model to rationalize True as the answer: “War cannot have a tie? True, because...”, which then is completed by an explanation by LM “In a context of war, there’s always a victor and a loser.”

3.1.2 Depth-wise Knowledge Spanning

As shown in Figure 1, LM-generated explanations are noisy and inaccurate by nature. Prior works indirectly compensate for the untrustworthy generations by independently sampling multiple generations then aggregating them at the answer level (e.g. through majority voting; Wang et al., 2022). Despite better performance, such an aggregation could still be brittle, as the inference fundamentally depends on the correctness of 1-hop explanations.

To enhance the robustness of reasoning, we hypothesize that the inference process should entail not only the breadth of reasoning, but also the depth of reasoning - whether the reasoning paths themselves are credible and consistent with each other. To do this, we require the LM itself to validate its own generations - by recursively prompting the
LM with the generated explanations. As Figure 2 shows, this corresponds to a depth-wise spanning of knowledge that induces a maieutic tree, a multi-depth structure of generated propositions and relations between them.

Let \( S_i \) denote the set of nodes at depth \( i \) in the maieutic tree \( T \). Each node in \( S_i \) is an explanation for an answer label (True or False), recursively generated given its parent node as the question:

\[
S_i \subseteq \bigcup_{t \in (T,F)^{i-1}} \{ E_{iT}, E_{iF} \},
\]

\[
(E_{iT}, E_{iF}) = \text{abduction}(E_i).
\]

Note that \( T \) is a full tree when the equality holds for all depths. For instance, in Figure 2, \( E_{TF} \) is generated by prompting the LM with its parent node \( E_T \) and False, i.e., \( E_{TF} \sim p_{LM}(\cdot|E_T, F, C) \).

In practice, we sample multiple explanations with the same \( Q \) and \( A \) through nucleus sampling (Holtzman et al., 2019). This corresponds to the width-wise spanning of knowledge, enhancing the diversity and coverage of generated explanations.

### 3.1.3 When to Stop Generating

Generating a full tree could be computationally expensive, as the number of generations grows exponentially with the maximum tree depth. Therefore, in each branch, we stop generating further once we reach a logically integral proposition; intuitively, this aligns with our goal to identify propositions that can be validated by the LM with confidence.

Figure 3 illustrates an example of maieutic tree generation where the maximum depth of the tree is set to 2. For visual clarity, we generate only 1 \( E_T \) and 1 \( E_F \) per question and omit the width-wise spanning of knowledge.

| Q: If you travel west far enough from the west coast, you will reach the east coast? | True, because \( E_T \). The Earth is round and if you travel in any direction long enough, you will eventually return to where you started. \[ \int \text{integral}(E_T) = 1 \] | False, because \( E_F \). You cannot reach the east coast by going west. \[ \int \text{integral}(E_F) = 0 \] |
| Q: You cannot reach the east coast by going west? | True, because \( E_T \). If you travel in a specific straight line, you will eventually reach the other side. \[ \int \text{integral}(E_T) = 1 \] | False, because \( E_F \). You can reach the east coast by going west by traveling around the world. \[ \int \text{integral}(E_F) = 0 \] |

Prune non-integral Branch

Figure 3: Illustrative example of maieutic tree generation, with the max tree depth set to 2. For visual clarity, we generate only 1 \( E_T \) and 1 \( E_F \) per question and omit the width-wise spanning of knowledge.

#### 3.2 Defining the Relations

Now that we have generated the maieutic tree, we seek to define the relations between propositions and quantify their strength into scalar weights. For illustration, assume that an LM has generated the following \( E_F \) for the given \( Q \):

\[
Q: \text{Captain Kirk is part of Star Wars?}
\]

\[
A: \text{False, because Captain Kirk is a character in Star Trek.}
\]

The generation can be logically interpreted as follows: (1) the LM believes that Captain Kirk is a character in Star Trek, (2) the LM believes that the proposition Captain Kirk is a character in Star Trek can be a reason to deny that Captain Kirk is part of Star Wars. Accordingly, we define belief and consistency to represent the two dimensions of the logical relationship.

**Belief** \( w_E \) corresponds to the LM’s belief that the proposition \( E \) is true (and therefore, \( \neg E \) is false). To quantify belief, we prompt the LM with \( E \) and \( \neg E \) respectively as a question, then comparing the probability assigned to True:

\[
w_E = \frac{p_{LM}(T|E, C) - p_{LM}(T|\neg E, C)}{p_{LM}(T|E, C) + p_{LM}(T|\neg E, C)}
\]

Note that calculating this does not require any additional prompting, as we already gained access to these values while checking for the logical integrity of each proposition.

**Consistency** \( w_{E,Q,A} \) corresponds to the consistency of the generated \( E \) with the given \( Q \) and \( A \). Intuitively, if the LM is logically consistent, the
likelihood of $E$ being generated given an answer (e.g., $E_F$ being generated given False) should be larger than its likelihood given the opposite answer (e.g., $E_F$ being generated given True). Following this intuition, we compute the consistency as:

$$w_{E,Q,A} := \frac{p_{LM}(E|Q,A,C)}{p_{LM}(E|Q,A,C) + p_{LM}(E|Q,\neg A,C)}.$$  

(8)

### 3.3 Inference

The two types of relations formulate a set of unary and binary logical constraints, based on which we assign the truth values to all nodes in the maieutic tree $T$, and in consequence, infer the answer to the original question. First, we represent $C_{blf}$ as the set of unary constraints. For each leaf node $E$ in $T$,

$$c_{blf} = \begin{cases} E & \text{if } E \text{ is logically integral / True} \\ \neg E & \text{if } E \text{ is logically integral / False}. \end{cases}$$

(9)

Note that all the leaf nodes in $T$ are logically integral, hence we can count on the credibility of belief for these nodes. We now define the set of all belief constraints $C_{blf}$ as:

$$C_{blf} = \{c_{blf} \text{ for } \forall E \in \text{leaf}(T)\}.$$  

(10)

For example, the nodes $E_F$ and $E_T$ in Figure 2 would have a belief constraint in $C_{blf}$.

Likewise, for consistency, we define $C_{con}$ as the set of binary constraints using logical implication. For each edge $(E_i, E_{i+1})$ in $T$,

$$c_{con} = \begin{cases} E_{i,A} \rightarrow E_i & \text{if } A = \text{True} \\ E_{i,A} \rightarrow \neg E_i & \text{if } A = \text{False} \end{cases}$$

(11)

$$C_{con} = \{c_{con} \text{ for } \forall (E_i, E_{i+1}) \in \text{edge}(T)\}.$$  

Our objective is to assign the truth values for all $E$s and the root node $Q$ in $T$, such that we maximize

$$\sum_{c \in C_{blf} \cup C_{con}} w_c \cdot \mathbb{1}\{c = \text{True}\},$$

(12)

which sums up the weights of satisfied constraints.

This problem is naturally formulated as weighted MAX-SAT, which is a problem of determining truth values of variables that maximize the weight of satisfied clauses. The problem can be algorithmically solved using an off-the-shelf solver.

### 3.4 Verifier Model

One limitation of the consistency definition in Section 3.2 is that it only considers the relationship between a parent node and a child node. Since the definition builds upon the likelihood of each generation from an LM, we cannot take into account the relationships across branches, e.g. $E_T$ and $E_F$ in Figure 3. This motivates us to introduce a small NLI model as a verifier, which can infer the relationship between an arbitrary pair of nodes in $T$.

Following previous works (Minervini and Riedel, 2018; Wang et al., 2019), we convert the NLI labels into logical relations as following:

$$\text{Entail}(E_1, E_2) : E_1 \rightarrow E_2$$

$$\text{Contradict}(E_1, E_2) : E_1 \rightarrow \neg E_2.$$  

(13)

For all pairs of nodes $(E_1, E_2) \in \text{node}(T)^2$, $E_1 \neq E_2$, we obtain either $E_1 \rightarrow E_2$ or $E_1 \rightarrow \neg E_2$ if $E_1$ entails or contradicts $E_2$. For NLI-based clauses, we fix the weights to 1. While the objective function (Eq. 12) stays the same, $c_{con}$ is now replaced with $C_{NLI}$, a set of clauses induced by the verifier model.

### 4 Experiments

#### Datasets

We evaluate MAIEUTIC PROMPTING on three commonsense reasoning and fact verification benchmarks in binary QA format: Com2Sense (Singh et al., 2021), CSQA 2.0 (Talmor et al., 2021), CREAK (Onoe et al., 2021). Despite the simple format, these datasets require a substantial amount of knowledge and robust reasoning, making them challenging even for the billion-scale fine-tuned LMs (Table 1).

#### Baselines

We compare our method with both the few-shot prompting methods and supervised models. Along with the standard prompting, we include Chain of Thought (Wei et al., 2022), Self-Consistency (Wang et al., 2022) and Generated Knowledge Prompting (GKP) (Liu et al., 2021). For supervised models, we consider the strong baselines used for the respective dataset, such as T5 (Raffel et al., 2020), UnifiedQA (Khashabi et al., 2020) and Unicorn (Lourie et al., 2021).

#### Configuration Details

For all prompting methods, we use the same set of 6 demonstration examples and the same version of GPT-3 (text-davinci-001) as the LM. We determine the hyperparameters of MAIEUTIC PROMPTING and baselines based on the dev set performance on the benchmarks. In maieutic tree generation, we set the maximum depth to 2. For depth 1, we use nucleus sampling ($p = 1.0$) (Holtzman et al., 2019) to generate $3 E_T$s.

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We also tried using the label probability assigned by NLI model as weight, but fixing it to 1 yielded better results.
and $3E_F$s from $Q$. For depth 2, we use greedy decoding to generate $1E_T$ and $1E_F$ from each parent node. This constrains the generated tree to have at most 18 nodes excluding the original $Q$. In Section 4.3, we conduct an ablation study on this depth-adaptive decoding scheme and analyze the effect of the tree size. For the main experiments, we use RoBERTa-large (Liu et al., 2019) fine-tuned on MNLI (Williams et al., 2018) as a verifier with 90.2% accuracy on MNLI dev set, and RC2 (Morgado et al., 2014) as a MAX-SAT solver.

4.1 Benchmark Performance

Table 1 presents overall evaluation results of MAIEUTIC PROMPTING along with the prompting and supervised baselines. MAIEUTIC PROMPTING significantly outperforms all prompting methods across all benchmarks. Notably, GKP and Self Consistency ensemble more 1-hop explanations than the maximal size of the maieutic tree; our superior performance compared to these methods confirms the sample efficiency of depth-wise knowledge spanning. Moreover, MAIEUTIC PROMPTING is the only prompting method that performs better than even the smallest supervised baseline (RoBERTa-large) in Com2Sense and CREAK. In fact, MAIEUTIC PROMPTING allows us to use an off-the-shelf LM to achieve comparable performance to a large fine-tuned LM by simply plugging in our inference algorithm. In Appendix C we also provide experiments on StrategyQA (Geva et al., 2021), to evaluate the generalizability of MAIEUTIC PROMPTING in multi-hop setting.

4.2 Robustness Analysis

We perform additional analyses to understand the working of our method under semantic perturbations and different prompt formats.

**Robustness to semantic perturbations** In addition to the standard accuracy, we report two additional metrics called pairwise accuracy and contrast set accuracy in Table 1. In Com2Sense test set and CREAK contrast set, each question is paired with its complimentary counterpart, of which the surface form is similar but the answer should be the opposite (e.g. “Barack Obama has no daughter.” vs “Barack Obama has daughters.”), testing the models’ robustness to semantic perturbations. In these metrics, the gap between MAIEUTIC PROMPTING and baselines widens substantially, indicating the robustness of our method against semantic perturbations.
Table 2: Ablation study on Com2Sense Dev set. The best configuration is with abductive generation, depth-adaptive decoding and verifier-based consistency.

| Model                                      | Accuracy |
|--------------------------------------------|----------|
| Non-abductive generation                   | 68.4     |
| All greedy decoding (no depth-adaptive)    | 67.2     |
| All nucleus sampling (no depth-adaptive)   | 72.0     |
| Likelihood-based consistency               | 65.6     |
| Maieutic Prompting                         | 72.5     |

Table 3: Performance of Maieutic Prompting on Com2Sense with different maieutic tree sizes.

| Dimension | 1 | 2 | 3 | 5 | 10 |
|-----------|---|---|---|---|----|
| Depth     | 61.3 | 72.5 | 72.4 | -  | -  |
| Width     | 62.4 | 66.5 | 72.5 | 71.5 | 72.1 |

Robustness to different prompts Prior works revealed that prompting performance could be sensitive to few-shot examples and their order (Lu et al., 2021b; Zhao et al., 2021). We investigate whether this holds true for Maieutic Prompting, as shown in Figure 4. We compare different prompting methods run with 3 different sets of few-shot examples (left), and 5 different permutations of the few-shot examples (right). In both settings, while Self Consistency and Maieutic Prompting are much more stable then the other two, our method has slightly less variance.

4.3 Ablation Study

We ablate different components of Maieutic Prompting to investigate their respective contributions as shown in Table 2.

Generation First, we consider Maieutic Prompting without abductive generation — we generate each explanation without providing an answer label, i.e. in an identical fashion to Chain of Thought. In this setting, the performance of Maieutic Prompting degrades by 4%, alluding to the importance of abductive generation in eliciting the latent knowledge from LM. Next, we ablate the depth-adaptive decoding mechanism (Section 4), by applying either greedy decoding or nucleus sampling for all depths of the maieutic tree. All greedy decoding restraints width-wise spanning of knowledge, hence leads to large degradation of performance. All nucleus sampling performs much more comparably with our best configuration, although the stochastic decoding produces slightly more errors in the explanations.

Consistency We ablate the NLI-based clauses and replace them with the original $C_{con}$ discussed in Section 3.2. With the likelihood-based $C_{con}$, the accuracy reduces by about 7%, but still prevails over the prompting baselines in Table 1. The verifier model indeed benefits the inference process by providing more accurate relations between generated explanations, although our method performs competently even without the access to the verifier.

Effect of tree size We also investigate how the size of the maieutic tree influences the performance. In Table 3, we present the performance of Maieutic Prompting on Com2Sense dev set with various values of maximal depth and width. In both dimensions, the accuracy saturates after a certain threshold. We attribute this to (1) the topic drift in generation which intensifies as the depth grows, (2) larger overlaps in generated knowledge as we sample more explanations width-wise.

4.4 Human Evaluation

We qualitatively analyze actual inference results of Maieutic Prompting through human evaluation. For each sample, we first retrieve true Es (the set of generated Es that are inferred to be True by Maieutic Prompting), then evaluate them over the four criteria from Liu et al. (2021): (1) Grammaticality of the explanations, (2) Relevance of the explanations to the question, (3) Factuality: whether the explanations states facts, and (4) Helpfulness: whether the explanation explicitly leads to the correct answer. Six NLP experts scored 100 examples sampled from CSQA 2.0 dev set, of which...
50 were answered correctly (Set 1) and 50 were answered wrongly by the model (Set 2).

Figure 5 presents the evaluation results. For both sets, over 99% of the true Es are grammatically perfect, and most of them provide relevant evidence to the question. Surprisingly, the LM often generates both factual and helpful explanations even when its answer is different from the ground truth: 42% of the true Es for incorrectly answered examples are perfectly factual, and 23% of them are completely helpful in correctly answering the question. We find that in many of these cases, the questions did not have a clear-cut answer; as exemplified in Figure 6, the explanations generated and validated by MAIEUTIC PROMPTING are compelling enough as an alternative to the ground-truth answer.

5 Related Work

Prior works have leveraged natural language explanations (NLEs) to promote model reasoning, either by training a model to explain (Rajani et al., 2019; Camburu et al., 2018; Chen et al., 2022; Wiegrefe and Marasović, 2021), or generating answers to templated queries and distantly supervised rationales (Schwartz et al., 2020; Brahman et al., 2021). Incorporated with in-context learning (Brown et al., 2020; inter alia), these efforts have led to explanation-based prompting (Wei et al., 2022; Wang et al., 2022; Liu et al., 2021; Lampinen et al., 2022). Other works aim to improve model interpretability with NLEs, training a model that explains its inference post-hoc or in parallel with the answer (Camburu et al., 2018; Narang et al., 2020; Jacovi et al., 2021). Unlike these works, the explanations in our work are designed to be intrinsic (Du et al., 2019); the explanations themselves explicitly participate in the inference.

Meanwhile, recent observations reveal that LM explanations are unreliable, as they often lack logical consistency and are not factually grounded (Ye and Durrett, 2022; Kassner and Schütze, 2020). This is in part due to the broader limitations of generative LMs, which assign high probability to unlikely sentences (Welleck et al., 2020; Holtzman et al., 2021) and are sensitive to semantic perturbations (Elazar et al., 2021). MAIEUTIC PROMPTING overcomes these limitations by avoiding the use of explanations “as-is”, and modeling the relationships between explanations.

Another line of works apply symbolic methods on top of LMs to improve their consistency, spanning from a lexical constraint on sequence decoding (Lu et al., 2021a) to a symbolic world model (Nye et al., 2021b) and discrete operations (Chen et al., 2019; Cobbe et al., 2021). Other works explore how to train a model that simulates the symbolic reasoning process, such as logical transformation (Bostrom et al., 2021) and consistent generation of beliefs (Kassner et al., 2021; Dalvi et al., 2022). However, these models require a curated set of human annotations that limits their application to specific domains. MAIEUTIC PROMPTING generalizes these neuro-symbolic approaches in an unsupervised setup, employing MAX-SAT algorithm to symbolically determine the true subset.
from a noisy pool of neural generations.

6 Conclusion

In this work, we propose MAIEUTIC PROMPTING, a novel few-shot inference method inspired by the Socratic way of conversation. We systematically generate a tree of explanations that bear logical relations between each other, then find the truth values that max-satisfy these relations. Empirical results show that MAIEUTIC PROMPTING is both competitive and robust compared to diverse baselines, while providing intrinsic interpretations over its inference.

Limitations

Extension to different task formats In this work, we limit our experiments to validating a given statement. In future works, we aim to extend our method over a broader range of tasks, e.g. multiple-choice QA. A potential strategy could be binarizing multiple-choice options to respective statements and scoring them with MAIEUTIC PROMPTING, e.g. using the sum of weight of satisfied clauses from MAX-SAT.

Modeling relationships between trees MAIEUTIC PROMPTING models the relations between the nodes in each maieutic tree to infer a consistent answer. The scope of modeled relationships, however, could be further generalized beyond a single tree - a span of knowledge generated for one question could serve as the evidence for another question. Indeed, modeling the relationship between questions is an active area of research (Kossen et al., 2021). We envision that the knowledge elicited from MAIEUTIC PROMPTING could further be enriched through this type of generalization.

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A  Tree Generation Algorithm

Algorithm 1 Maieutic tree generation

Input: Question Q, Max tree depth D
Output: Maieutic tree T

\[ T \leftarrow \text{init}(Q) \] // initialize the tree with Q

for \( d \in \{1, \ldots, D\} \) do // generate nodes

\( S_i \leftarrow \emptyset \)

for \( E \in S_{i-1} \) do

if \( \text{integral}(E) = 1 \) then

\( S_i \leftarrow S_i \cup \text{abductive}(E) \)

end if

end for

end for

V \leftarrow \{E ; \text{integral}(E) = 0 \text{ for all } E \in \text{leaf}(T)\} // \text{set of non-integral leaf nodes}

while V \neq \emptyset do

T.remove(V) // prune the non-integral leaf nodes

V \leftarrow \{E ; \text{integral}(E) = 0 \text{ for all } E \in \text{leaf}(T)\}

end while

B  Dataset Details

| Dataset | Com2Sense | CSQA 2.0 | CREAK |
|---------|-----------|---------|-------|
| Train / Dev / Test split size | 804 / 402 / 2779 | 9282 / 2544 / 2517 | 10176 / 1371 / 1371 |
| Average # of tokens | 21 | 11.3 (words) | 10.8 |

Table 4: We evaluate MAIEUTIC PROMPTING in three commonsense reasoning and fact verification benchmarks - Com2Sense, CSQA 2.0 and CREAK. Com2Sense and CSQA 2.0 consist of adversarial commonsense questions generated to mislead a proxy model. CREAK tests for a combination of commonsense reasoning and accurate fact retrieval, consisting of long-tail questions such as “Harry Potter can teach how to fly on a broomstick?”. Table 4 presents key statistics of the three datasets.

C  Multi-hop Reasoning on StrategyQA

| Model | Standard | C-o-T | Maieutic | C-o-T (Multi-hop) | Maieutic (Multi-hop) |
|-------|----------|-------|----------|-------------------|---------------------|
| Accuracy | 56.3 | 58.2 | 60.7 | 57.9 | 61.4 |

Table 5: Results on StrategyQA

To further evaluate the generalizability of MAIEUTIC PROMPTING, we conduct additional experiments on multi-hop reasoning over StrategyQA (Geva et al., 2021) dev split. Note that the original evaluation setting for StrategyQA presupposes access to Wikipedia articles, from which the gold knowledge could be retrieved from; hence the benchmark as-is may not represent the best evaluation setting for few-shot prompting methods.

To better address the multi-hop nature of the dataset, we add a straightforward adjustment to both C-o-T and Maieutic Prompting, to first decompose the original question into 2-3 minor questions and then generate the explanation and answer. We denote this as Multi-hop in Table 5.

Consistent with the original experimental results, Maieutic Prompting yields promising improvement compared to both the standard / C-o-T prompting. The result attests to both the generalizability of Maieutic Prompting to multi-hop setting and the importance of reasoning algorithm in challenging scenarios.
D Inference Examples

Figure 7: Example of correct inference by MAIEUTIC PROMPTING. We show the generated maieutic tree along with the assigned truth-values to each propositions.

Figure 8: (continued) Example of correct inference by MAIEUTIC PROMPTING.
Every living being is capable of getting energy from lower in the food chain.

Humans are omnivores that can consume both plants and animals.

Some people are not capable of digesting complex carbs and proteins and thus, must eat food from the higher end of the food chain.

Some people are not able to digest complex proteins, and therefore, need to eat food that is lower in the food chain.

A city is a place where many people live.

A city is a place where people live and work.

A city will have residents who have permanent addresses and commuters who have temporal addresses.

A city will always have transient traffic.

People passing through a city will always be there.

A city is a place where many people live and work.

A city will have a mix of both transient and local traffic.

A city will have both transient and non-transient traffic.

Figure 9: Example of incorrect inference by MAIEUTIC PROMPTING.

Figure 10: (continued) Example of incorrect inference by MAIEUTIC PROMPTING.