Feasibility of underwater free space quantum key distribution

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Abstract: We investigate the optical absorption and scattering properties of underwater media pertinent to our underwater quantum key distribution (QKD) channel model. With the vector radiative transfer theory and Monte Carlo method, we calculate the bit rate, the fidelity, and the quantum bit error rate of photons when transmitting underwater. It can be observed from our simulations that maximally secure single photon underwater BB84 QKD is feasible with a distance of about 125 m in the clearest ocean water.

OCIS codes: (270.5568) Quantum cryptography; (010.4450) Oceanic optics.

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1. Introduction

Driven by the communication requirements of underwater sensor networks, submarines and all kinds of underwater vehicles, underwater wireless optical communication has been developing rapidly in recent years\(^\text{[1, 2, 3, 4, 5]}\). The new research shows that by using the current technology the light can transmit 350m in the clearest sea water with the bit rate of 10Mbps\(^\text{[6]}\). At the same time, a strong research effort has been devoted to study how quantum effects may be employed to manipulate and transmit information, that is called quantum information processing\(^\text{[7]}\). Quantum key distribution (QKD) is one important branch of quantum information processing, in particular, is on its way from research laboratories into the real world. Photons are very suitable for carrying informations and can be used as flying quantum bits (qubits) for QKD. Recent years, QKD based on photons makes great progress both in theoretical and experimental researches\(^\text{[8]}\). QKD has been proved absolutely safe in theory, and the maximum distance of QKD using single photon in free space has been reached 144Km\(^\text{[9]}\). It provides an important basis for the research of optical underwater free space quantum communication. The same as optical communication in free atmosphere, underwater QKD is also faced with two unavoidable problems, one is the attenuation of photons in sea water channel, and another is the error of information in communication. The complex structure of sea water and special optical properties become the biggest challenges for underwater QKD research. The absorption of water is stronger than that in atmosphere, however, the light attenuation is weaker than others in blue-green wavelength region, thus, underwater optical communication usually based on this wavelength region. On the other hand, during the transmission, part of the photons will be scattered and some of them could be received by the detector. According to Mie theory, the states of the scattering photons are different from the ones directly shoot at the detector. Fortunately, the recent developments of optical wireless communication technologies could be used for the realization of underwater QKD.

Some analyses of traditional underwater optical wireless communication have been proposed recently\(^\text{[10, 11, 12]}\). In this paper, we investigate the optical property of sea channel and present a analysis of the feasibility of underwater free space QKD using single photon (polarization coding based on BB84 QKD protocol). We mainly study the absorption and scattering, and we employ the vector radiative transfer (VRT) theory which can capture both the attenuation and multiple scattering effects. The VRT theory also takes into account the polarization, thus, the polarized optical signal communication can be modeled and evaluated. We investigate the characteristics of underwater media pertinent to the construction of our simulating channel model: the average distance between particles is approximately more than ten times of the wavelength of visible light, and the distribution and direction of the particles are random. To deal with such a random question, we use Monte Carlo simulation method. At last, we calculate the bit rate of underwater optical communication, the fidelity of the detected photons, and the quantum bit error rate (QBER) of photons when transmitting underwater in order to discuss the feasibility of underwater free space QKD.

2. Free space quantum key distribution

Secure communication is an important subfield of cryptography which is a field of applications that provides privacy, authentication, and confidentiality to users. Nowadays, most cryp-
tographic applications are based on our experience that some problems are hard to solve, and these schemes can be broken with a substantial amount of computational power [13]. In other words, security of these traditional cryptographic schemes cannot be proved in principle. The laws of quantum mechanics allow for the generation of a secret string which can subsequently be used as a key for symmetric encryption techniques, and these techniques are often called QKD. Quantum cryptography (QC) which is often identified with QKD could well be the first application of quantum mechanics at the single-quantum level, and is to become the most promising tasks of quantum information processing. The origin of the security of QKD can be traced back to some fundamental principles of quantum physics [14, 15, 16]. The fact that security can be based on general principles of physics suggests the possibility of unconditional security. The task of QKD makes sense only if two distant parties commonly referred to as Alice and Bob are separated by a macroscopic distance. As is well known, quantum states of light can be transmitted to distant locations basically without decoherence, therefore photons are very suitable for carrying informations. QKD is always implemented with light and there is no reason to believe that things will change in the future.

Fig. 1. Typical system for quantum key distribution using polarization coding based on BB84 protocol: LD, laser diode; F, neutral density filter; BS, beamsplitter; PBS, polarizing beamsplitter; $\lambda/2$, half waveplate; SPD, single photon detector.

There are several QKD protocols which describe how to establish a secret key, and the BB84 protocol is the first one which is described intuitively with polarization encoded photons as qubits [14]: Alice and Bob randomly use four quantum states that constitute two bases (the horizontal or vertical basis $|\pm\rangle$, and the complementary basis of linear polarizations $|\times\rangle$), for example, the states $|H\rangle$, $|V\rangle$, $|P\rangle$, and $|M\rangle$ are identified as the linear polarized photons “horizontal”, “vertical”, “45°”, and “135°” respectively. Conventionally, one attributes the binary value 0 to states $|H\rangle$, $|P\rangle$ and the value 1 to the other states $|V\rangle$, $|M\rangle$. Fig. 1 shows one of the typical systems for QKD using polarization coding based on BB84 protocol. BB84 protocol includes several steps: sending, measurement, sifting, and classical postprocessing, indeed, it is the conjunction of both Alice and Bob’s random choices that produces the secret key, neither Alice nor Bob can decide which key results before the protocol ending.

In practical communication, Alice and Bob may use the same transmission channel to implement both the quantum and the classical channels. The channel is any medium that propagates light with reasonable losses: typically, either an optical fiber or just free space provided Alice and Bob have a line of sight. Generally, the transmission channels of free space QKD are the atmosphere and the water. Low loss transmitting channel and high efficiency detector are two ways to extend the secure distance and improve the secret key generation rate. Free space QKD
in the atmosphere has attracted lots of attention for its low transmission losses, allowing almost unperturbed propagation of polarized photons. In the next section, we present a analysis of the optical propagation for underwater free space QKD. In particular, we investigate two crucial aspects which affect the propagation of polarized photons underwater: both absorption and scattering.

3. Propagation of polarized photons in the underwater environment

The optical properties of water can be divided into two classes, the inherent and the apparent optical properties of the medium [10, 17, 18]. Inherent optical properties (IOP) directly specify the true absorption and scattering characteristics of the medium and are dependent on the dissolved and suspended material in the water and the electromagnetic properties of the medium. In this paper, we mainly discuss the inherent optical properties of water. As indicated previously, an optical signal is attenuated by absorption and scattering in water [11, 12]. Absorption is an irreversible thermal process whereby photon energy is lost due to interaction with water molecules or other particulates. It will reduce the number of photons, and then influences the secret key generation rate of QKD. The effects of the light scattering in water are two sides: Firstly it attenuates the transmitted signal and reduces the signal to noise ratio (SNR), Secondly it creates the inter-symbol-interference (ISI) effect corrupting the signal waveform. In addition, the scattering will change the polarized state of photons which constitute the qubits, and increase the bit error rate. Therefore, the study of light propagation through an water channel is crucial to analyze the underwater free space QKD.

Fig. 2. Pure water absorption as a function of wavelength. (The blue-green wavelength region is suitable for underwater QKD.)

Fig. 3. (Color online) Optical attenuation for $\lambda = 480\text{nm}$ in clear, intermediate, and murky ocean waters.

Indeed, sea water is a mixture with extremely complex components. Depending on whether these components are correlated among themselves, water is classified as case 1 and case 2. Oceans are typical case 1 waters whereas coastal waters are often referred to as case 2 water. The optical properties of sea water tend to change with depth, seasonal changes, and weather effects. The characteristics of water and particulate matters in an underwater environment have been a subject of several studies for decades. In general [12], the materials only creating absorption are sea water and colored dissolved organic matter (CDOM), while the materials creating both absorption and scattering are (1) planktonic components, (2) detrital components, and (3) mineral components. For convenience, the oceanic waters have been divided in Jerlov water types that approximately share the same optical properties [6].

The scale of absorption and scattering in water is given in inverse meters, much shorter than the attenuation in the atmosphere which is in units of inverse kilometers. The cumulative
Effects of absorption and scattering are described by the beam extinction coefficient which is wavelength dependent, \( \mu_e(\lambda) = \mu_a(\lambda) + \mu_s(\lambda) \), where \( \mu_a(\lambda) \) is the absorption coefficient and \( \mu_s(\lambda) \) is the scattering coefficient, all in units of inverse meters. The photons that reach a certain location without being scattered or absorbed can be calculated by:

\[
N(l) = N_0 e^{-\mu_e(\lambda) l},
\]  

(1)

where \( N_0 \) is the initial number of photons, and \( N(l) \) is the number of photons after propagating a distance \( l \) in water.

Fig. 2 shows the curve of water absorption as a function of wavelength \( \lambda \). The data suggests that the absorption is much smaller for the blue-green wavelength region \( (400 \text{ nm} < \lambda < 550 \text{ nm}) \). The attenuation of photon numbers as a function of the distance is shown in Fig. 3. It can be observed that, for example, for the clearest oceanic waters, only 2.4% of photons reach a distance of 125 m. Therefore, the proposed communication system used in this situation may be required to operate with an optical attenuation in excess of 97.6% \[6\].

We employ the VRT theory which explains the behavior of wave propagation and scattering in a discrete random medium, and then we simulate the optical propagation in the underwater environment with the Monte Carlo method \[19\]. Fig. 4 is the sketch map of the polarized photons propagating in the sea water channel. The polarized state of a photon can be described by Stokes vector as follows \[20\]:

\[
S = \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \langle E_\parallel E_\parallel^* + E_\perp E_\perp^* \rangle \\ \langle E_\parallel E_\parallel^* - E_\perp E_\perp^* \rangle \\ \langle E_\perp E_\parallel^* + E_\parallel E_\perp^* \rangle \\ i(\langle E_\perp E_\parallel^* - E_\parallel E_\perp^* \rangle) \end{pmatrix}, \]

(2)

where \( E_\parallel \) and \( E_\perp \) are horizontally and vertically polarized electric fields, the symbol * and \(<\) denote complex conjugate and time average. In BB84 protocol, the Stokes vectors of four quantum states can be calculated as \( S_H = (1, 1, 0, 0)^T \), \( S_V = (1, -1, 0, 0)^T \), \( S_P = (1, 0, 1, 0)^T \), and \( S_M = (1, 0, -1, 0)^T \), where \( T \) is the transpose operator of a matrix.

In the beginning, we assume the initial position of the photon is the origin of the coordinate \( O \), and initial direction is \( z \)-axis \((0, 0, 1)\). The process of single scattering for the photon can be
calculated by a scattering Mueller matrix \(M(\theta)\), with \(4 \times 4\) elements. After the single scattering, the new Stokes vector changes to \(S' = M(\theta)S_0\), where \(\theta\) is the scattering angle. For a simple case, the scattering particles are considered as homogeneous and spherical particles, the Mueller matrix will have only four independent elements:

\[
M(\theta) = \begin{pmatrix}
m_1(\theta) & m_2(\theta) & 0 & 0 \\
m_2(\theta) & m_1(\theta) & 0 & 0 \\
0 & 0 & m_3(\theta) & m_4(\theta) \\
0 & 0 & -m_4(\theta) & m_3(\theta)
\end{pmatrix}, \tag{3}
\]

where the terms \(m_1(\theta), m_2(\theta), m_3(\theta)\) and \(m_4(\theta)\) are related to the scattering amplitudes which can be calculated by Mie theory.

To calculate the Mueller matrix, the index of refraction about mediums and the particle-size distributions (PSD) of scattering particles are required. The index of refraction about water and particulate matters in an underwater environment has been proposed in [12] and will not be described here. Usually a single distribution function \(N(D)\) can be used to describe the PSD whose radius ranges from 0.1\(\mu m\) to 100\(\mu m\). \(N(D)\) complies with the Jungle (also known as hyperbolic) cumulative size distribution [21]:

\[
N(D) = K\left(\frac{D}{D_0}\right)^{-\epsilon}, \tag{4}
\]

where \(D_0\) is a reference diameter for which the number concentration is \(K\), \(\epsilon\) is different for different types of particles, and usually ranges from 3 to 5 typically. In order to determine the diameters of scattering particles, we use Monte Carlo method to choose a random diameter \(D\) according to the distribution \(N(D)\).

The distance between two adjacent scattering of a photon is usually called free path. The photon will move a free path \(\Delta L\) which is calculated by a random number \(\eta\) generated in the interval \((0,1)\):

\[
\Delta L = -\frac{\ln(\eta)}{\mu_e}. \tag{5}
\]

For multiple scattering situation, the reference plane is different between two scatterings. For convenience, we can take scattering plane as the reference plane. The Stokes vectors of photons are rotated by an azimuth \(\phi\) and scattering angle \(\theta\) at each scattering step. In the case of \(n\) times scatterings, we get the last Stokes vector \(S'_n\),

\[
S'_n = M(\theta_n)R(\phi_n)\cdots M(\theta_1)R(\phi_1)S_0, \tag{6}
\]

where \(S_0\) is the initial Stokes vector \((l = 0)\), \(R(\phi)\) is a rotation matrix calculated by

\[
R(\phi) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(2\phi) & \sin(2\phi) & 0 \\
0 & -\sin(2\phi) & \cos(2\phi) & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}. \tag{7}
\]

According to the above equations [2][3][6][7], we can get the scattering phase function:

\[
P(\theta, \varphi) = m_1(\theta) I + m_2(\theta) [\cos(2\varphi) Q + \sin(2\varphi) U], \tag{8}
\]

and \(P(\theta, \varphi)\) can be used to sample the scattering angle \(\theta\) and the azimuth angle \(\varphi\) by the rejection method [19]. Fig.5 shows the distribution of the scattering angle \(\theta\) and the azimuth angle \(\varphi\) for four polarized states of photons, where red dots denote the statistics results simulated with
The scattering angle (range from 0° to 180°).

(b) The azimuth angle range from 0° to 360°.

Fig. 5. (Color online) The distribution of the scattering angle and the azimuth angle.

Monte Carlo method, and black lines are the fitting curve related to the statistics results. The total number of photons we use is $10^6$.

Other two important steps in our simulation are the determination of photons lifetime and the boundary problems. In order to simplify the calculation, we separate the scattering particles with both scattering and absorption (such as phytoplankton pigments and suspended sediments) from seawater and other components (mainly CDOM) which mainly take absorption effect. We define the weight of photons lifetime as $W$,

$$W(\Delta L) = \frac{\mu'_s}{\mu'_s + \mu'_a} e^{-\mu_m \Delta L}$$

where $\mu'_s$ and $\mu'_a$ are the scattering coefficient and the absorption coefficient of the scattering particles, $\mu_m$ is the absorption coefficient of the seawater and dissolved substances in it. $W$ can be considered as the probability of the surviving photons after moving a propagation distance $\Delta L$. In order to determine the lifetime of photons, a uniform random number $\xi$ between 0 and 1 should be generated firstly. The photon is regarded as being absorbed if $\xi > W$, then the next photon will be launched. Otherwise, the photon is regarded as being scattered if $\xi < W$, then it will continue to transmit. Another situation is when and where the photon arrives at the boundary (receiving plane, $l = L$). In our simulation, the position of photons should be recorded constantly, and the photon arrives at the boundary when $l \geq L$. Photons can be detected or not by the detector depending on the aperture size and angle of field of view (FOV) of the detector. In the next section, we will discuss some details about our simulation and analyze the feasibility of underwater QKD.

4. Discussion and conclusion

The bit rate and the bit error rate are two crucial points for any communication process. They depend on both the efficiency of the communication system, such as laser pulse frequency and detector efficiency, and the characteristics of communication channel. The efficiency of the system is not the focus of this article. We mainly discuss the influence of the channel characteristic. In underwater free space QKD, the attenuation of the water channel will reduce the bit rate, while scattering photons and the noise introduced in the system will increase the bit error rate. One cannot increase the signal power in order to have a good enough SNR since the signal
transmitted by Alice in QKD is ideally one photon (or a weak coherent pulse with very low mean photon number in many realistic implementations). Therefore the available methods to implement underwater free space QKD are reducing the water channel attenuation, scattering photons, and the background noise.

In our Monte Carlo simulations, we mainly study on the transmission of the polarized photons for underwater QKD with the limitation of aperture (in unit of cm) and FOV (in unit of mrad). We choose the wavelength of photons as 480nm, and choose the radius of Mie scattering particles from 0.1µm to 3µm. To simplify the calculation we take the mean complex refractive index of particles as 1.41 $- 0.00672i$ (in the case of plankton).

4.1. Signal attenuation

In order to reduce the water channel attenuation, free space QKD should work in the blue-green light wavelengths because it suffers less attenuation in water compared to other colors. In our Monte Carlo simulations, we investigate the effects of the total extinction coefficient on the detected photons. As discussed in section 3, Fig.3 shows the number of photons change at different propagation distance in three types of ocean waters, where the solid lines are the theoretical values while the scatter dots are results of our simulation in small aperture (1cm) and small FOV (87mrad), they are almost coincide with another in the same water type. As expected, the number of photons decrease as the distance increases, therefore, the secret key generation rate (related to the bit rate) of QKD decreases. In practical QKD system, take BB84 protocol for example, after attenuation and sifting, the raw key generation rate is calculated as

$$\kappa = f \cdot \langle N \rangle \cdot (1-a) \cdot (1-BER) \cdot DE/2,$$

where $f$ is laser pulse frequency, $\langle N \rangle$ is the mean photon number per pulse, $a$ is attenuation rate of channel, $BER$ is bit error rate, and $DE$ is detection efficiency. Assume that $f = 1GHz$, $\langle N \rangle = 0.1$, $BER = 10\%$, and $DE = 20\%$, we can estimate the raw key generation rate of underwater free space QKD, i.e., $\kappa \approx 215kbps$ at the distance of 125m in the clearest ocean water (Jerlov Type I). It could be used for encrypting most audio information and some low bit rate video information in underwater communications.

Fig. 6. The fidelity of the detected photons as a function of distance in Jerlov Type I ocean water, where $A = 1cm$, $FOV = 87mrad$, $\mu_e = 0.03/m$.
4.2. Fidelity of the detected photons

In quantum communication, fidelity is usually used to describe how similar two quantum states are. The fidelity of states $\omega$ and $\sigma$ is defined as [7]:

$$F(\omega, \sigma) = \text{tr} \sqrt{\omega^{1/2} \sigma \omega^{1/2}}, \quad (11)$$

where $\text{tr}$ is the trace operator.

In particular, suppose an arbitrary quantum system is in one of a number of states $|\psi\rangle_i$, where $i$ is an index, with respective probabilities $p_i$. The density operator for the system is defined as $\rho = \sum p_i |\psi\rangle_i \langle \psi|$. The fidelity between a pure state $|\psi\rangle$ and the arbitrary state is

$$F(|\psi\rangle, \rho) = \sqrt{\langle \psi | \rho | \psi \rangle}. \quad (12)$$

We calculate the fidelity of the detected polarized photons in the clearest ocean waters (shown in Fig.6). If we ignore the influence of the background noise in water channel, the fidelities of the four situations are almost close to 1 as the increase of the distance. The result shows that the polarized states of the detected photons keep almost invariant as the increase of distance.

In our simulations, the reason why the fidelity is close to 1 is that, with the limitation of the aperture and FOV, most of the detected photons are without scattering. Even if the scattering photons is detected that the majority of those detected photons are scattered less than two times (generally scattered only once according to our statistics). In this case, we are convenient to define the probability of the scattered photons which are detected by the detector as:

$$P_s(l) = W(l) \times \int_0^{\tan^{-1}\left(\frac{A}{2l} \right)} P(\theta, \varphi) \, d\theta, \quad (13)$$

where $L$ is the detector position, and $l$ is the scattering position with $l < L$. $A$ is the aperture of the detector.

![Figure 7](image)

Fig. 7. (Color online) The probability distributions of the scattering photons which are detected by the detector with different aperture and field of view in Jerlov Type I ocean water.

Fig.7 shows the probability distributions of the scattering photons which are detected by the detector with different aperture and field of view: (a) with small aperture and small FOV of
the detector, while (b) with large aperture and large FOV. The simulation results show that: a) the majority of the scattering photons locate at a distance closer to the detector, and the scope of scattering angle whose maximum depends on the aperture and FOV of the detector increases as the increase of distance $l$; b) the changes of the polarized states for the received scattering photons are extremely small due to the small changes of the scattering angle; c) changing the size of aperture or FOV can affect the total number of received photons, apparently, the small aperture and small FOV can prevent large amount of scattering photons to be detected.

4.3. Noise and quantum bit error rate

The quantum bit error rate (QBER) is an important quantity used to quantify the security of QKD system [8,13]. It describes the probability of false detection in total probability of detection per pulse. For underwater quantum communication, the false detection is connected with dark current noise, thermal noise, background noise, and shot noise. From above discussion, we know that the shot noise produced by scattering photons is negligible. The mainly noise source is from QKD systems and the background light. The dark current and thermal noise are the typical system noises, they are restricted by the processing technic of the single photon detector. It is worth mentioning that, another kind of system noise is usually produced by the imperfect optical devices, i.e., polarizing film and non-polarizing splitting prisms (NPBS). In fact, background light is far more serious than the system noise. There are two kinds of background light underwater. One is the radiation or reflection from the sun, moon and stars. These lights when travelling through the sea could be scattered by molecules and particles in water, then collected by the detector as background noises. So the detector should be placed back to the sea surface. Fortunately, light is hard to through over the depths of 200 meters, and what is more, marine environment is completely dark in the depths of more than 600 meters. Another kind of background light is created by the illuminant in the water, such as marine luminous organisms. It should be kept away from these luminous bodies in underwater free space QKD.

![QBER as functions of the detector aperture and FOV in Jerlov Type I ocean water. Where black triangle denotes QBER without background noise, red square denotes QBER with background noise, the background noise is calculated according to the real data of QKD system which measured in our dark laboratory.](a) $L = 60m$, $FOV = 87mrad$, $\mu_e = 0.03/m$. (b) $L = 60m$, $A = 1cm$, $\mu_e = 0.03/m$.]

When background noise is considered, the influence of background noise will far exceed the scattering photons, background noise becomes the most important factor which affects the
We investigate the QBER as functions of the detector aperture and FOV at the same transmitting distance (shown in Fig. 8). With the increase of the aperture and FOV, both the scattering photons and background noises will increase, however, the effects of scattering photons are far less than that of the background noises. Fig. 8 shows that without considering the background noise QBER is almost always close to 0, and with the background noise QBER increases as the increase of the detector aperture and FOV. Hence, small aperture and FOV are suitable for underwater quantum communication. On the other hand, the numbers of both scattering photons and without scattering photons reduce as the increase of transmitting distance, that is to say the signal photons received by detector reduce. Thus the SNR of quantum communication decreases due to the increasing proportion of the background light. Its influence is especially obvious with the increasing distance. Fig. 9 shows the results of QBER with the increase of transmitting distance.

In particular, for the case of BB84, the system secure against a sophisticated quantum attack if $QBER \leq 10\%$ [6]. From above discussions, it can be observed from our simulation that maximally secure single photon underwater BB84 QKD is feasible with a single photon detection up to about 125$m$ in the clearest ocean water.

In practical underwater QKD, reducing the background noise is one of the crucial technologies. The recent researches show that it can avoid a very strong background noise by implementing a strong filtering in the spatial, spectral and temporal domains emitted photons [22, 23]. In principle, the detector aperture and FOV could suppress the background noise effectively, so we can control the size of aperture and FOV to implement a spatial filter. Usually the background light is of continuous wavelength so a narrow band filter must be used before the detector to prevent it. Another effective method to decrease the number of background photons is using a time-gate filter. Only when the signal photons are expected to arrive is a narrow time gate opened to allow them to enter the detector, thus noise photons arriving outside the time window are blocked.

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