On Dissipation Mechanism in the Intrinsic Josephson Effect in Layered Superconductors with d-wave Pairing

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Abstract

Conductivity mechanism in the regime of the intrinsic Josephson effect in layered superconductors with singlet d-wave pairing is studied theoretically. The cases of coherent and incoherent interlayer tunneling of electrons are considered. The theory with coherent tunneling describes main qualitative features of the effect observed in highly anisotropic high-\(T_c\) superconductors at voltages smaller than the amplitude of the superconducting gap, mechanism of the resistivity being related to excitations of the quasiparticles via the d-wave gap. At voltages of the order of the gap amplitude and larger, the I-V curves are better described by the incoherent tunneling. Interaction of the Josephson junctions formed by the superconducting layers due to charging effects is shown to be small.

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I. INTRODUCTION

Theoretical understanding of the intrinsic Josephson effect (IJE) in layered high-$T_c$ superconductors is limited by insufficient knowledge of transport properties of both the superconducting and normal states. Studies of the energy structure show that electronic spectral density in directions $(0, \pi)$ is affected by strong interaction with spin fluctuations (see e. g. Ref. 1–3). In the same time at low energies, in directions $(\pi, \pi)$ corresponding to zeros of the order parameter $\Delta(\phi)$, electronic structure can be described in terms of the Fermi liquid. This gives chances to describe the IJE at low temperatures and small voltages in terms of an approach based on the Fermi liquid theory. The aim of this work is to study the resistive properties of layered high-$T_c$ superconductors and to understand to which extent they can be described in the frame of the BCS model with intralayer singlet d-wave pairing.

The IJE is expected to be qualitatively different in cases of coherent and incoherent interlayer transfer of electrons. If the interlayer tunneling is coherent, a layered superconductor is a strongly anisotropic 3D crystal the normal-state resistivity of which is induced by scattering. On the other hand, a superconductor with incoherent tunneling can be considered as a stack of Josephson tunnel junctions.

If the tunneling is incoherent the in-layer momentum is not conserved in the interlayer transitions. This results in a finite tunneling resistivity along the c-axis perpendicular to conducting layers. If the tunnel junctions are formed by conventional s-wave superconductors the product of critical current and normal state resistivity $V_c = I_c R_N$ is of the order of the gap value which corresponds to experimental data for high-$T_c$ superconductors. However, for the d-wave symmetry of the order parameter, independent averaging over directions of the electron momentum in neighboring layers, in the case of incoherent tunneling would result in zero critical current along c-axis. Then in order to explain experimentally observed large values of critical current along c-axis one must assume a special d-wave-like angle dependence for the probability of random interlayer hopping. Fraction of the s-component in the order parameter in BSCCO was found in the recent tunneling measurements to be below...
Without assumption of the special momentum dependence of the interlayer transfer integral for the incoherent interlayer tunneling such symmetry would result in negligibly small critical current $I_c$ and in $V_c$ about three orders of magnitude smaller, than experimentally observed values. Furthermore, with such small values of $I_c$ the regime of branching in which some junctions are in superconducting state while others are in a resistive state would be impossible in the range of voltages $V \sim \Delta_0/e$ per one resistive junction. Such branching is the most typical manifestation of the IJE. Thus it is difficult to understand main features of the IJE in the frame of incoherent interlayer tunneling and d-wave pairing.

In the case of a layered superconductor with coherent tunneling, which is an anisotropic 3D crystal, one must not assume a special angle dependence for the interlayer transfer integral in order to explain large experimental values of $I_c$. But the question whether a clean quasi two-dimensional crystal can exhibit the IJE is not still clear. The finite normal-state resistivity in crystals is induced by scattering, therefore, $\rho_c \propto 1/\tau$. In a clean superconductor the scattering rate is small, $\hbar/\tau \ll \Delta_0$. Typical voltages in experimental studies of IJE are by few times smaller, than the maximum energy gap $\Delta_0/e$, i. e., much larger than $\hbar/e\tau$. At such voltages (and frequencies of Josephson oscillations) resistivity is expected to decrease as voltage and frequency increase because of the Drude-like regime of scattering, however such decaying I-V curves are not observed. The region of finite resistivity may become more pronounced if the intralayer scattering by impurities is resonant. Then gapless states are formed by such scattering at relatively large energy region around Fermi energy, $\gamma \sim \sqrt{\Delta_0\hbar/e\tau}$, which results in finite conductivity at $V < \gamma$. Calculated value of conductivity at voltages $V \ll \gamma$ was found to be consistent with the experimental data. But in a clean superconductor the region where such a mechanism is expected to work is limited by voltages much smaller, than $\Delta_0$. Thus, typical voltage per one junction in the resistive state, $eV \sim \Delta_0$, can be explained only provided an additional mechanism for the resistivity is present at frequencies $\omega > \hbar/\tau, \gamma$. Such a mechanism was considered recently in the study of bandwidth of the Josephson plasma resonance at low temperatures, it is related to the dissipation induced by electron excitation via the d-wave gap. However,
this mechanism dies out at voltages larger than the amplitude of the d-wave gap, and to get finite resistivity at higher voltages one must take into account some other mechanisms of scattering. A finite resistivity at large voltages, may be induced by some contribution of incoherent interlayer tunneling or by inelastic scattering, e.g. scattering on spin wave excitations is expected to become important when large energies and ”hot spot” regions are involved. We do not consider these processes here limiting our study by low voltages.

In this paper we study, first, the IJE in a perfect (i.e. without scattering) highly anisotropic superconducting crystal with coherent interlayer charge transfer and d-wave symmetry of the order parameter. We derive equations for charge and current densities and calculate I-V curves at voltages and frequencies of the order of the gap, paying attention to interaction between oscillations in different layers due to charging effects, considered first by Koyama and Tachiki. Then for comparison we calculate I-V curves for the case of incoherent interlayer tunneling and found that the calculated curve looks rather similar to the experimental data at large voltages.

II. MAIN EQUATIONS

Having in mind to study large frequencies and voltages much larger, than the inverse scattering time we calculate expressions for current and charge densities using collisionless transport equations in Keldysh approach for Green’s functions at coinciding times \( \hat{G}(t,t) \) derived by Volkov and Kogan. We generalize these equations to the case of layered superconductors assuming the interlayer interaction described by the tight-binding approximation and the superconductivity described by a BCS type Hamiltonian leading to singlet d-wave pairing. This approach to interlayer transitions describes the case of layered single crystal, it is opposite to the case of random interlayer hopping considered in Ref. 11. Our results were also checked using quasiclassical transport equations for Green’s functions integrated over momenta generalized for layered superconductors with coherent interlayer tunneling. For the case when time dependence of the scalar potential can be neglected the results can be
calculated using a standard expression for the conductivity in terms of the spectral densities in neighboring layers. Using this approach we calculate I-V curves for the case of incoherent interlayer tunneling.

We consider homogeneous interlayer currents along the c-axis, this case is realized in narrow samples with the width in the ab-plane smaller, than the Josephson length. We solve equations for diagonal and off-diagonal with respect to spin indices components $g_{nm}(t)$ and $f_{nm}(t)$ of Keldysh propagator.

$$
\begin{align*}
  i\hbar \frac{\partial}{\partial t} g_{nm} + \Delta_n f_{nm}^* - f_{nm} \Delta_m + (\mu_n - \mu_m)g_{nm} &= t_\perp \sum_{i=\pm 1} (A_{n+i}A_{n+i} - g_{n+i}A_{m+i}) ; \\
  i\hbar \frac{\partial}{\partial t} f_{nm} - 2\xi f_{nm} - \Delta_n g_{nm}^* + g_{nm} \Delta_m + (\mu_n + \mu_m)f_{nm} &= t_\perp \sum_{i=\pm 1} (A_{n+i}f_{n+i} - f_{n+i}A_{m+i}^*) .
\end{align*}
$$

(1)

Functions $g_{nm}(\xi, \phi, t)$, $f_{nm}(\xi, \phi, t)$ are matrices in layer indices. Here $\xi = \epsilon(p) - \epsilon_F$, where $\epsilon(p)$ is the single electron energy in the normal state, $\epsilon_F$ is the Fermi energy, and $p$ is the electron momentum in the ab plane, $\phi$ is the angle of the in-plane electron momentum, $t_\perp$ is the interlayer transfer integral. Furthermore, $\Delta_n = \Delta(\phi)_n$ and $\chi_n$ are the anisotropic superconducting order parameter and its phase in layer $n$, $\mu_n = e\Phi_n + \frac{\hbar}{2} \frac{d\chi_n}{dt}$ is the gauge-invariant scalar potential, $\Phi_n$ is the electric potential, $A_{n+1} = \exp(i\varphi_n)$, where $\varphi_n = \chi_{n+1} - \chi_n - \frac{2\pi s}{\Phi_0}A_z$ is the gauge-invariant phase difference between the layers, and $A_z$ is the component of the vector potential in the direction perpendicular to the layers. Electric field between the layers is expressed via the gauge-invariant potential as

$$
e E_n s = \mu_n - \mu_{n+1} + \frac{\hbar}{2} \frac{d\varphi_n}{dt} ,
$$

(2)

The scalar potential $\mu$ is related to branch imbalance, it is responsible for charging effects in Josephson plasma oscillations and in IJE.

The current density between layers $n$ and $n+1$ and charge density in layer $n$ can be calculated as

$$
\begin{align*}
  j_{n,n+1} &= \frac{e t_\perp}{2s} \int \frac{d\mathbf{p}}{(2\pi\hbar)^2} (A_{n+1}g_{n+1} - g_{n+1}A_{n+1}) ,
\end{align*}
$$

(3)
\[ \rho_n = -\frac{e}{4is} \int \frac{dp}{(2\pi \hbar)^2} (g_{nn} + g_{nn}^*), \] (4)

where \( s \) is the crystalline period in c-direction.

**III. CHARGE AND CURRENT DENSITIES**

We solve equations (1) perturbatively with respect to \( t_\perp \) and consider the case of low temperatures \( T \ll \Delta_0 \). Contributions of interlayer transitions to charge density are quadratic in \( t_\perp \), therefore, in the main approximation we can take into account only intralayer contribution which in the Fourier representation reads

\[ g_{nn} + g_{nn}^* = \left[ \frac{2\xi}{\varepsilon} - \frac{8\Delta^2 \mu_\omega}{\varepsilon(4\varepsilon^2 - \omega^2)} \right] \tanh \frac{\varepsilon}{2T}, \] (5)

with \( \varepsilon = \sqrt{\xi^2 + \Delta^2} \). Inserting this expression to (4) we get in the time representation

\[ \rho_n = -\frac{\kappa^2}{8\pi} \int_0^\infty dt_1 F(t_1) \mu_n(t - t_1), \] (6)

where \( \kappa \) is the inverse Thomas-Fermi screening radius,

\[ F(t) = \left\langle \int_{-\infty}^{\infty} d\xi \frac{2\Delta^2 \sin 2\varepsilon t}{\varepsilon^2} \tanh \frac{\varepsilon}{2T} \right\rangle, \] (7)

\( \langle \cdots \rangle \) means averaging over \( \phi \).

Integral with function \( F \) describes non-exponential relaxation of \( \mu \) with relaxation time of the order of \( \hbar/\Delta_0 \). We will need equation (6) in the case of slow variations of the scalar potential when \( F(t) \to 2\delta(t) \) so that the integral in Eq.(6) reduces to \( 2\mu(t) \). Then inserting the expression for the charge density to the Poisson’s equation with electric field determined by Eq.(2) we express the difference of scalar potentials between neighboring layers, \( \delta \mu_n = \mu_{n+1} - \mu_n \), in terms of time derivative of phase differences.

\[ \delta \mu_n = -\frac{a}{16\sqrt{1 + a}} \sum_m (\dot{\varphi}_{n+m+1} + \dot{\varphi}_{n+m-1} - 2\dot{\varphi}_{n+m}) \left( \frac{\sqrt{1 + a - 1}}{\sqrt{1 + a + 1}} \right)^{|m|}, \] (8)

with \( a = 4\epsilon_\perp/(\kappa s)^2 \), where \( \epsilon_\perp \) is a high frequency dielectric constant in perpendicular direction. For \( s = 15 \) A, \( 1/\kappa = 2 \) A, \( \epsilon_\perp = 12 \) we get \( a \approx 0.8 \). Then the factor before the sum in
Eq. (8) is about 0.04 and the last factor describing contributions from non-nearest neighbors is about 0.15. Thus, $\delta \mu_n$ is small compared to time derivatives of the phase differences, therefore, the charging effects and contribution of $\delta \mu_n$ to the electric field between the layers must be small as well (cf. Eq. (2)).

To calculate the current density we need to solve equations (1) in the linear approximation in $t_{\perp}$. But equations are still difficult to solve for arbitrary $\mu_n(t)$ and $\varphi_n(t)$, therefore, we calculate expressions for current density in two limiting cases. First, we find the solutions in the linear approximation with respect to potential $\mu$ which describes the charging effects. The conditions for smallness of such effects will be discussed later.


g_{n,n+1} = \frac{4t_{\perp} \Delta^2}{\varepsilon} \tanh \frac{\varepsilon}{2T} \left\{ \frac{C_\omega}{\varepsilon(4\varepsilon^2 - \omega^2)} + \int_{-\infty}^{\infty} \frac{d\omega_1}{2\pi} \left[ \frac{(2\omega_1 - \omega)C_{\omega-\omega_1}}{4\varepsilon^2 - (\omega - \omega_1)^2} - \frac{i\omega S_{\omega-\omega_1}}{4\varepsilon^2 - \omega_1^2} \right] \frac{\mu_{\omega_1}}{4\varepsilon^2 - \omega_1^2} \right\}

(9)

In this limit the current density between layers $n$ and $n+1$ consists of a component determined by the phase difference $\varphi_n$ between the same layers only, and of a component which, in addition, depends on the difference of scalar potentials in these layers,

$$j_{n,n+1}(t) \equiv j^\varphi(t) + j^\mu(t).$$

Assuming that $t_{\perp}$ does not depend on momentum we get

$$j^\varphi(t) = j_c \int_0^\infty dt_1 F(t_1) \cos \frac{\varphi_n(t-t_1)}{2} \sin \frac{\varphi_n(t)}{2};$$

$$j^\mu(t) = j_c \int_0^\infty dt_1 \int_0^\infty dt_2 F(t_2) \left\{ \left[ \cos \frac{\varphi_n(t-t_1)}{2} \delta \mu_n(t-t_1-t_2) - \cos \frac{\varphi_n(t-t_1-t_2)}{2} \delta \mu_n(t-t_1) \right] \cos \frac{\varphi_n(t)}{2} + \sin \frac{\varphi_n(t-t_1)}{2} \delta \mu_n(t-t_1-t_2) \sin \frac{\varphi_n(t)}{2} \right\}. \tag{11}$$

Since according to Eq. (8) $\delta \mu_n(t)$ depends on phase differences between different layers, the component (11) of the current describes interaction between the "Josephson junctions" due to charging effects. Interaction due to charging effects was studied in the phenomenological model by Koyama and Tachiki, however, the results are not similar.

Equations (10) and (11) can be simplified in limiting cases. At $T \ll \hbar \omega, eV \ll \Delta_0$
\[
\begin{align*}
\dot{\varphi}(t) &= j_c \sin \varphi_n + j_c \frac{\pi}{2\Delta_0} \frac{d\varphi_n}{dt} (1 - \cos \varphi_n) ; \quad (12) \\
\dot{j}(t) &= 2j_c \int_0^\infty dt_1 \sin \frac{\varphi_n(t - t_1)}{2} \delta\mu_n(t - t_1) \sin \frac{\varphi_n(t)}{2} . \quad (13)
\end{align*}
\]

So the current can be considered as consisting of superconducting, normal and interference components, and of the quasiparticle contribution related to the difference of scalar potential. In the limit of linear response the dissipative term in Eq. (12) vanishes because in spatially uniform case excitations of the quasiparticles via the superconducting gap are forbidden. In the nonlinear regime this dissipation mechanism is effective because in the presence of current along the $c$-axis the phase is dependent on the layer index. This makes the system non-uniform and excitations via the gap become allowed. In the regime of the linear response this dissipation mechanism becomes possible due to scattering. Taking into account scattering as it was done in Ref. 8 we get

\[
\frac{j(t)}{j_c} = \varphi_n + \frac{2}{3\tau \Delta_0^2} \frac{d\varphi_n}{dt} - \frac{\pi}{2\Delta_0} \delta\mu_n . \quad (14)
\]

The expression for current density simplifies also at large frequencies

\[
\omega, V \gg \omega_p \quad (15)
\]

where $\omega_p$ is the Josephson plasma frequency. In the most anisotropic materials like Bi- and Tl-based cuprates this frequency is small enough, $\Delta_0 \gg \omega_p$. Under condition (15) the electronic AC current is shunted by the displacement current,

\[
V_{AC} \sim V_{DC} \left( \frac{\omega_p^2}{\omega^2} \right) \ll v_{DC} ,
\]

and time dependences of the phase differences become simple, $\varphi_n \approx 2\omega_n t + \varphi_\omega$, $\varphi_\omega \ll 1$.

When all layers are in the resistive state then $\delta\mu = 0$, and I-V curve has the form

\[
j = \begin{cases}
\theta (2\Delta_0 - V) \left[ \frac{2\Delta_0}{V} K \left( \frac{V}{2\Delta_0} \right) - E \left( \frac{V}{2\Delta_0} \right) \right] + \\
\theta (V - 2\Delta_0) \left[ K \left( \frac{2\Delta_0}{V} \right) - E \left( \frac{2\Delta_0}{V} \right) \right] \right] \tanh \frac{V}{4T} ,
\end{cases}
\]

where, again, $V$ is the voltage per one junction. This expression is also valid in the limit $a \to 0$ in which $\delta\mu = 0$. 

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At low voltages the I-V curve (16) is described by the linear quasiparticle conductivity
\[ \sigma_q = \pi esj_c/\Delta_0 = e^2/(8\pi\lambda^2\Delta_0) \]. This value agrees with the experimental data by Latyshev et al.\(^7\) and differs from the linear conductivity calculated for the case of the resonant scattering\(^7\) by factor \(8/\pi^2 \approx 1\). At larger voltages the shape of the I-V curve (16) is different from that observed experimentally. The calculated curve has a logarithmic peak at \(V = 2\Delta_0\), and current decreases with voltage at \(V > 2\Delta_0\), while the experimental curves\(^7\) exhibit a peak in the conductivity at \(V = 2\Delta_0\) and does not decay at \(V > 2\Delta_0\). This demonstrates that our approach is not valid at large \(V\) when contributions of electrons with energy of order \(\Delta_0\) are important. Such electrons are strongly scattered by spin fluctuations which we did not take into account. One may expect that any additional mechanism of scattering, in particular, inelastic processes or presence of the component with the incoherent charge transfer in the interlayer tunneling will smear the logarithmic anomaly at \(V = 2\Delta_0\) and add an Ohmic contribution to I-V curves at large voltages.

Now we consider the regime of branching. At current values \(I < I_c\) the regime is possible in which some ”junctions” are in the superconducting state while others are in the resistive state. In this case the DC current through the superconducting junctions is transported in the form of the superconducting current, and the AC current is transported mainly as the displacement current. The total voltage is formed by the sum of the voltages across resistive junctions, and the I-V curves consist of branches corresponding to different numbers of the junctions in the resistive state. The number of branches is equal to the total number of the layers in the sample. In the limit of small \(a\) the n-th branch is described by the first term in Eq. (16) with \(V\) substituted by \(V/n\). Under condition (15) the current density was calculated also for arbitrary relation between \(\delta\mu_n\) and \(\dot{\varphi}_n\). At \(\dot{\varphi}_n \gg T\)

\[ j_{n,n+1} = \sqrt{\frac{\varphi_n + \delta\mu_n}{(\dot{\varphi}_n - \delta\mu_n)^3}} \left[ K \left( \frac{\sqrt{\varphi_n^2 - \delta\mu_n^2}}{2\Delta_0} \right) - E \left( \frac{\sqrt{\varphi_n^2 - \delta\mu_n^2}}{2\Delta_0} \right) \right] \]

Note that the voltage across the resistive junction differs from \(\varphi_n\) by \(\delta\mu_n\) (cf. Eq.(8)). In the limit \(a \to 0\), when \(\delta\mu_n = 0\), the current as function of the voltage \(V\) per one resistive junction is identical for all branches. At finite \(a\), according to (8), the shape of branches depends on
the neighboring junctions whether they are in the resistive state or not, the difference being smaller at smaller values of the parameter $a$. The shape of branches presented as function of the mean voltage per one junction in the resistive state $V$, which is defined as the total voltage divided by the number of resistive junctions, is shown in Fig.1 and 2. One can see that already at $a = 0.2$ the branches almost coincide as it is observed experimentally.
FIG. 1. I-V curve branches normalized per one layer for the system of 10 layers. The curves from left to right correspond to 10, 9 junctions in the resistive state, 5 and 2 neighboring junctions in the resistive state, 5 junctions in the resistive state separated by junctions in the superconducting state, and 1 junction in the resistive state. Parameter $a = 0.8$. Current is measured in units of $J_c$, and voltage – in units of $2\Delta_0$.

FIG. 2. Similar branches for $a = 0.2$. 
IV. INCOHERENT TUNNELING

Since a finite resistivity may be caused also by some contribution of incoherent interlayer tunneling at large voltages, we shall consider now briefly the case of incoherent tunneling.

Note that expressions for the interlayer current can be derived via 2D Green’s functions. These Green’s functions are easy to calculate for the case when time dependence of $\mu$ can be neglected. To calculate the interlayer current we need the spectral density determined by the imaginary part of the Green’s function. It has the form

$$A(\varepsilon, \xi) = \frac{1}{\pi} (g^n_R - g^n_A) = \pi [u^2_{\xi_n} \delta(\varepsilon - \sqrt{\xi_n + \Delta^2}) + v^2_{\xi_n} \delta(\varepsilon + \sqrt{\xi_n + \Delta^2})],$$  \hfill (18)

where $\xi_n = \xi + \mu_n$, $u^2_{\xi_n}$ and $v^2_{\xi_n}$ are Bogolyubov amplitudes

$$u^2_{\xi_n} = \frac{1}{2} \left( 1 + \frac{\xi_n}{\sqrt{\xi_n^2 + \Delta^2}} \right), \quad v^2_{\xi_n} = \frac{1}{2} \left( 1 - \frac{\xi_n}{\sqrt{\xi_n^2 + \Delta^2}} \right).$$ \hfill (19)

Then we can find the interlayer current for both coherent and incoherent tunneling. For the case of coherent tunneling at $T = 0$ the parallel component of the momentum conserves in the interlayer transitions, and we find

$$j \propto \int_{-U/2}^{U/2} d\varepsilon \int d^2 p A(\varepsilon + U/2, \xi_n) A(\varepsilon - U/2, \xi_{n+1}),$$ \hfill (20)

where $U = \dot{\varphi}_n$. We calculate this integral in the linear approximation with respect to $\mu$ when the result can be found easily

$$j \propto \int d\xi \left\langle \frac{(\delta \mu + \sqrt{\xi_n^2 + \Delta^2})\Delta^2}{(\xi_n + \Delta^2)^{3/2}} \delta(2\sqrt{\xi_n^2 + \Delta^2} - U) \right\rangle = \left\langle \frac{(2\delta \mu + U)\Delta^2}{U^3} \right\rangle.$$ \hfill (21)

This expression coincides with the version of Eq.(17) linearized with respect to $\mu$.

For the case of incoherent tunneling the momenta of the electrons in different layers are integrated independently, and we have

$$j \propto \int_{-U/2}^{U/2} d\varepsilon \int d^2 p_1 A(\varepsilon + U/2, \xi_n) \int d^2 p_2 A(\varepsilon - U/2, \xi_{n+1}).$$ \hfill (22)

In this case the current is determined by the density of states in the layers
\[ N(\varepsilon) = \int \frac{d\xi}{\pi} \langle A(\varepsilon, \xi) \rangle = \frac{2}{\pi} \left[ K \left( \frac{\Delta_0}{\varepsilon} \right) \theta(\varepsilon - \Delta_0) + \frac{\varepsilon}{\Delta_0} K \left( \frac{\varepsilon}{\Delta_0} \right) \theta(\Delta_0 - \varepsilon) \right] . \]  

Since the density of states does not depend on \( \mu \) (at least when the latter does not depend on time, otherwise equation (18) is not valid), the current does not depend on \( \delta \mu \) as well. Thus the interaction between the "junctions" due to charging effects here is absent. Inserting Eq. (23) into (22) we get the I-V curve presented in Fig. 3. This curve resembles experimental curve at large voltages, however, it does not contain the region of linear resistivity at small voltages.

**FIG. 3.** I-V curve for the case of incoherent tunneling and all layers in the resistive state. Current is measured in arbitrary units, and voltage – in units of 2\( \Delta_0 \).

**V. CONCLUSION**

Thus the model with BCS-type d-wave pairing and coherent interlayer transport describes qualitatively such features of the IJE like branching with typical voltages per one junction \( V < \Delta_0 \) at low temperatures. The charging effects are found to be small at reasonable values of parameters. Damping in the system and, hence, the resistivity at voltages larger than inverse scattering time can be attributed to the excitations of the quasiparticles via the d-wave gap. At small voltages such transitions take place near the nodes of the gap, and the region of the angles where such transitions are possible increases with the voltage
increasing. The conductivity value induced by this mechanism is sample independent, in contrast to the damping due to scattering.

However, our model based on the Fermi liquid approach does not describe the experimental curves at voltages of the order $\Delta_0$ and larger. Namely, in the regime when all layers are in the resistive state it results in the logarithmic singularity at $V = \Delta_0$ and decaying I-V curve at larger voltages. This discrepancy appears when contribution of the quasiparticles with larger energies to the conductivity becomes important, it may be related either to the inapplicability of the simple BSC-type model with d-wave pairing or to some other mechanisms of scattering becoming effective at larger energies, like inelastic scattering on spin wave excitations, or other effects which smear the spectral density. We do not address these processes here. The I-V curve calculated for the case of incoherent tunneling looks rather similar to the experimental data at large voltages but is inconsistent with the experimental data at small voltages.

Note that at elevate temperatures, especially near $T_c$, one may expect a different regime, in which the resistivity is related to quasiparticle scattering. Such mechanism is expected to die out at large frequencies and voltages $\omega, V \gg 1/\tau$.

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