Topological states of quantum matter have inspired both fascinating physics findings and exciting opportunities for applications. Due to the over-complicated structure of, as well as interactions between, real materials, a faithful quantum simulation of topological matter is very important in deepening our understanding of these states. This requirement puts the quantum superconducting circuits system as a good option for mimicking topological materials, owing to their flexible tunability and fine controllability. As a typical example herein, we realize a $\mathbb{Z}_2$-type topological insulator featuring the quantum spin Hall effect state, using a coupled system of transmission-line resonators and transmons. The single-excitation eigenstates of each unit cell are used as a pseudo-spin 1/2 system. Time reversal symmetry of the system is proved, and the boundary of the topological phase transition is fixed in the phase diagram. Topological edge states are shown, which can be experimentally verified by detecting the population at the boundary of the plane. Compared to the previous simulations, this compositional system is fairly controllable, stable and less limited. Therefore, our scheme provides a reliable platform for faithful quantum simulations of topological matter.

Recently, superconducting circuits [25–28], a scalable quantum computation platform, has been applied to simulate quantum many-body systems [29–38]. In the spin concern system [39–41], the hopping between each quasi-lattice could be adjusted separately and there is no limitation on nearly all parameters that are used to fix the physical properties of the system, such as the hopping strength, on site potential, hopping phase, etc. Here, we propose a coupled transmission-line resonators (TLRs) and transmons system to simulate a 2D topological insulator. We vary the coupling coefficients and hopping phases of the system, fix the parameters where the system is the topological insulator. Phase diagram is fixed to show where the QSHE can be realized. We also give out the edge states’ distribution for special cases, which can be detected by resonance absorption experimentally.

As shown in Fig. 1(a), we consider a 2D lattice with the following model Hamiltonian [14]

$$\mathcal{H} = -t_0 \sum_{m,n} c_{m+1,n}^\dagger e^{i\theta_x} c_{m,n} + c_{m,n+1}^\dagger e^{i\theta_y} c_{m,n} + \text{H.c.}$$

$$+ \sum_{m,n} \lambda_{m,n} c_{m,n}^\dagger c_{m,n},$$

where $t_0$ is the nearest-neighbor hopping strength; $c_{m,n} = (c_{m,n,\uparrow}, c_{m,n,\downarrow})^T$ is a 2-component operator defined on a lattice site $(x = ma, y = nb)$ with $a$ and $b$ being the lattice spacings and $m$ and $n$ being integers; $\theta_x = 2\pi \alpha y \sigma_z$ and $\theta_y = 2\pi \beta x \sigma_z$ with $(\sigma_x, \sigma_z)$ being the Pauli matrices and $(\alpha, \beta)$ being parameters governed by the magnetic flux and spin mixing; $\lambda_{m,n}$ is the on site potential which is here set to be staggered in $y$-direction, i.e., $\lambda_{m,n} = (-1)^n \lambda$. Define the time reversal operator as $T = i\sigma_y K$, where $\sigma_y$ is also the Pauli matrix and $K$ denotes the complex conjugation. We can verify the Hamiltonian in Eq. (1) has the time reversal symmetry, so
Each site consists of one transmon coupled to a TLR, the hopping between neighboring sites is realized by the SQUID. Different Peierls phases of the hoppings in x and y directions can be realized by setting appropriate parameters in the corresponding lattice and the SQUID. (c) Taking out one column in (a) for explanation. For the hopping between the lattices in y-direction 1-3-1, set the detuning in the No. 3 of each two lattices in y-direction, after unitary transformation we can get the staggered on-site potential as shown in (d). (d) Interacted lattices with staggered on-site potential along y-direction.

This system belongs to topological class with topological index $Z_2$ [42] and can be used to realize the QSHE.

Here we first take one rectangle block including four sites to introduce how to achieve the Hamiltonian in Eq. (1). As shown in Fig. 1(a), each circle presents a Jaynes-Cummings (JC) model made of one TLR and transmon, and the lattices are connected by a superconducting quantum interference device (SQUID) and a inductor $L$ [28]. The hoppings between nearest-neighbor lattices with Peierls phases $\theta_{x,y}$ can be adjusted by tuning the magnetic field through the connected SQUID. The hoppings in x-direction and y-direction are independent from each other and can be realized in the similar way. The Hamiltonian of that JC lattice is

$$
\mathcal{H}_{\text{JC}} = \sum_{\mathbf{r}} h_{\mathbf{r}} + \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} J_{\mathbf{r}\mathbf{r}'}(t) (\hat{a}_{\mathbf{r}}^\dagger \hat{a}_{\mathbf{r}'} + \text{H.c.}),
$$

where $\mathbf{r}$ is the label of the unit cell at $(x, y)$; $h_{\mathbf{r}} = \hbar \omega_{\mathbf{r}} (\sigma_+^r \sigma_-^r + \delta^r_{\mathbf{r}} \hat{a}^\dagger_{\mathbf{r}} \hat{a}^r_{\mathbf{r}})$ is the JC Hamiltonian in unit cell $\mathbf{r}$ with $\sigma_+^r = |e\rangle_{\mathbf{r}} \langle g|$ and $\sigma_-^r = |g\rangle_{\mathbf{r}} \langle e|$ the raising and lowering operators of the transmon qubit at $\mathbf{r}$; $\hat{a}^r_{\mathbf{r}}$ and $\hat{a}^\dagger_{\mathbf{r}}$ are the annihilation and creation operators of the photon in the TLR at $\mathbf{r}$; $J_{\mathbf{r}\mathbf{r}'}(t)$ is the inter-cell hopping strength between the unit cell in $\mathbf{r}$ and its neighbor cells. In the following of the paper, for each $\mathbf{r}' = (m, n)b$ we set $\mathbf{r} = (ma, (n + 1)b)$ or $\mathbf{r} = ((m + 1)a, nb)$. The lowest three eigenstates of the JC Hamiltonian $h_{\mathbf{r}}$ are denoted as $|0g\rangle_{\mathbf{r}}, |\uparrow \rangle_{\mathbf{r}} = (|0e\rangle_{\mathbf{r}} + |1g\rangle_{\mathbf{r}})/\sqrt{2}$ and $|\downarrow \rangle_{\mathbf{r}} = (|0e\rangle_{\mathbf{r}} - |1g\rangle_{\mathbf{r}})/\sqrt{2}$, where $|ng\rangle_{\mathbf{r}}$ and $|ne\rangle_{\mathbf{r}} (n = 0, 1, 2, \ldots)$ are the states containing $n$ photons while the transmon is at the ground and excited states. The corresponding eigen-energies are $E_{r,0g} = 0$ and $E_{r,\uparrow \downarrow} = \omega_{r} \pm g_{r}$. We choose the two single-excitation eigenstates $|\uparrow \rangle_{\mathbf{r}}$ and $|\downarrow \rangle_{\mathbf{r}}$ to simulate the effective spin-up and spin-down states in the lattice at $\mathbf{r}$. We can control each hopping separately by adjusting the pulse shape of $J_{\mathbf{r}\mathbf{r}'}(t)$. Based on the current experiments $[26]$, setting $t_0/2\pi = 3 \text{ MHz}$, $\omega_1/t_0 = 2700$, $\omega_2/t_0 = 3000$, $\omega_3/t_0 = 2650$, and $\omega_4/t_0 = 2900$. And $g_1/t_0 = 250$, $g_2/t_0 = 150$, $g_3/t_0 = 150$, $g_4/t_0 = 200$. With those, the energy interval $|E_{\mathbf{r},\uparrow} - E_{\mathbf{r},\downarrow}|$ of the 16 hopping between each two of them is much larger than (or equal to) 20 times of the effective hopping strength $t_0$, such distance is enough for selective frequency addressing. And the hopping strength $J_{\mathbf{r}\mathbf{r}'}(t)$ contains 16 tunes,

$$
J_{\mathbf{r}\mathbf{r}'}(t) = \sum_{\eta, \eta'} 4t_{0,rr'\eta\eta'} \cos \left( \omega_{rr'\eta\eta'} t + s_{rr'\eta\eta'} \varphi_{rr'\eta\eta'} \right),
$$

where $s_{rr'\eta\eta'} = \text{sgn}(E_{\mathbf{r},\eta} - E_{\mathbf{r}',\eta'})$ is the sign of the hopping phase, and $\omega_{rr'\eta\eta'} = |E_{\mathbf{r},\eta} - E_{\mathbf{r}',\eta'}|$ is the energy difference between the nearest lattice. $4t_{0,rr'\eta\eta'}$ and $s_{rr'\eta\eta'} \varphi_{rr'\eta\eta'}$ are the amplitudes and phases corresponding to the hopping $|\eta\rangle_{\mathbf{r}} \rightarrow |\eta'\rangle_{\mathbf{r}'}$, respectively. In experiments, this time-dependent coupling strength $J_{\mathbf{r}\mathbf{r}'}(t)$ can be realized by adding external magnetic fluxes with dc and ac components through the SQUIDs $[41]$. Both the hopping strengths and phases can be controlled by inducing controllable spin transition process in a certain rotating frame via Eq. (3).

We proceed to show directly how the selective control of individual hopping in the JC lattice can be achieved by adjusting $J_{\mathbf{r}\mathbf{r}'}(t)$ in Eq. (3) via the ac flux. First mapping the Hamiltonian in Eq. (2) into the single excitation subspace by using $|\eta\rangle_{\mathbf{r}}$ to denote one excitation state with ‘spin’ $\eta = \uparrow, \downarrow$ and get the Hamiltonian in the dressed states,

$$
\mathcal{H}_{\text{JC}}^S = \sum_{\eta, \eta'} E_{\mathbf{r},\eta} |\eta\rangle_{\mathbf{r}} \langle \eta| + \frac{1}{2} \sum_{\mathbf{r}, \mathbf{r}', \eta \eta'} J_{\mathbf{r}\mathbf{r}'}(t) |\eta\rangle_{\mathbf{r}} \langle \eta|_{\mathbf{r}'} |\eta'|_{\mathbf{r}'} + \text{H.c.}.
$$

Then for each $J_{\mathbf{r}\mathbf{r}'}(t)$, we add four tunes, each in resonant to one of the 16 inter-cell hoppings $[41]$, and contains its independent tunable amplitude, frequency and phase as shown in Eq. (3). The form of $J_{\mathbf{r}\mathbf{r}'}(t)$ will be determined by $t_{0,rr'\eta\eta'}$ and $s_{rr'\eta\eta'} \varphi_{rr'\eta\eta'}$ depending on the topological phase in simulated scheme. The target Hamiltonian we need to simulate topological insulator could be got in the rotating frame transformed by $U = \exp(-i \sum_{\mathbf{r}} h_{\mathbf{r}} - (\hbar \omega_{\mathbf{r}} / 2) \lambda(|\uparrow\rangle_{\mathbf{r}} \langle \uparrow| + |\downarrow\rangle_{\mathbf{r}} \langle \downarrow|) t)$, where $n_{\mathbf{r}}$ is the same number as $n$ in the $r = (x, y) = (ma, nb)$. After the unitary transformation $\mathcal{H}_{\text{JC}}^S = U \mathcal{H}_{\text{JC}} U^\dagger + iU \partial_t U$, and if the conditions $\{t_{0,rr'\eta\eta'}\}_{\eta, \eta' = \uparrow, \downarrow} \leq \{\omega_{rr'\eta\eta'}, \omega_{rr'\eta\eta'}, \omega_{rr'\eta\eta'}, \omega_{rr'\eta\eta'}\}_{\eta, \eta' = \uparrow, \downarrow} \neq 0$, all the other terms are fast rotating term that can be dropped with the rotating wave approximation. As a result, we derive the 2D tight-binding model with tunable hopping.
coefficients as
\[ \mathcal{H}_{TB} = \sum_{\langle rr' \rangle, n, n'} t_{0, rr', nn'} e^{i \varphi_{rr', nn'}} \hat{c}_{r, n} \hat{c}^\dagger_{r', n'} + \text{H.c.} \]
\[ + \sum_r (-1)^n \lambda \sigma^0_r, \]
where \( t_{0, rr', nn'} \) is just the effective coupling strength and \( \hat{c}_{r, n}^\dagger = |\eta \rangle \langle G | \) is the creation operator of quasi-electron with ‘spin’ \( \eta \) in the lattice at \( r \). Comparing the Hamiltonian in Eqs. (1) and (5), we should choose appropriate \( t_{0, rr', nn'} \) and \( \varphi_{rr', nn'} \) to get the target model in Eq. (1). In \( x \)-direction, choose
\[ t_{0, ((m+1),n),(m,n),\uparrow \uparrow} = t_{0, ((m+1),n),(m,n),\downarrow \downarrow} = 0, \]
\[ \varphi_{((m+1),n),(m,n),\uparrow \downarrow} = 2n \pi \alpha, \]
\[ \varphi_{((m+1),n),(m,n),\downarrow \uparrow} = -2n \pi \alpha. \]
And in \( y \)-direction, set
\[ t_{0, ((m,n+1),(m,n),\uparrow \uparrow} = t_{0, ((m,n+1),(m,n),\downarrow \downarrow} = 0, \]
\[ \varphi_{((m,n+1),(m,n),\uparrow \downarrow} = \varphi_{((m,n+1),(m,n),\downarrow \uparrow} = \varphi_{0, ((m,n+1),(m,n),\downarrow \uparrow} = 2\pi \beta. \]
When all other coefficients not mentioned above are set to be zero, we can realize the tight binding model in Eq. (1). Equations (6) and (7) show that \( t_0 \) could be adjusted by the corresponding hopping strength and \( \alpha, \beta \) could be varied through changing hopping phases.

In experiments, adding detuning to the transition frequencies between the nearest-neighbor lattice in \( y \)-direction could also simulate staggered on-site potential. We take the column 1-3-1 to illustrate how the detuning is added as in Figs. 1(c) and 1(d). And the hopping strength could be controlled by varying the amplitudes of each unit cell. The hopping phase could be adjusted by the SQUIDs between each lattices, which could drive phase transition between topological and trivial phases. That Peierls phase is hard to implement in cold atom systems because of the limitation that \( \beta \) cannot be small values [14].

The validity of individual frequency addressing of the intercell transitions can be verified by numerical simulation. We find that in the present of the unmatched driving, all the initial non-target states remain almost unchanged, thus justify our individual frequency addressing method. So far, we have shown how to realize 2D tight-binding model by the combined TLRs and transmons system, next we will use this system simulate the 2D lattice in Fig. 1(a).

Taking periodic boundary condition in \( x \)-direction, we numerically calculate the topological invariant [43–45] and fix phase diagrams with corresponding Fermi energy, using the methods for analysing the \( Z_2 \) topological insulators [1, 46]. In the numerical work we set the hopping coefficient \( t_0 \) as energy unit, lattice spacings \( a, b = 1 \) and \( \alpha = 1/3 \).

When \( \beta = 0 \) there is no spin-mixing, and if \( \lambda = 0 \) the Hamiltonian in Eq. (1) are just two Hofstadter models with different magnetic fluxes \( \pm 2\alpha \pi \) for spin-up and spin-down branches. With the parameter \( \lambda = 0 \) and \( \beta = 0 \) the band structure of the system is plotted in Fig. 2(a). With the numerical results of topological invariants, we can get Fig. 2(b), which is the phase diagram with Fermi energy between \( t_0 \) to \( 2t_0 \). The plane is divided into two kinds: (I) topological insulator with edge states which is shown the so called QSHE; (II) metal state. (c), (d) Distributions of the edge states’ wave functions for (c) \( 42 \times 42 \) and (d) \( 6 \times 6 \) lattices.

![Figure 2](image-url)

**FIG. 2.** (a) Energy band for \( \lambda = 0 \) without spin-mixing \( \beta = 0 \). (b) Phase diagram in \( \beta - \lambda \) plane, based on the band structure in (a) with Fermi energy between \( t_0 \) to \( 2t_0 \). The plane is divided into two kinds: (I) topological insulator with edge states which is shown the so called QSHE; (II) metal state. (c), (d) Distributions of the edge states’ wave functions for (c) \( 42 \times 42 \) and (d) \( 6 \times 6 \) lattices.

We next show how to trigger phase transition between topological insulator and metal state by changing the coupling parameter \( \beta \) between spin-up and spin-down terms. In our scheme, adjusting the SQUIDs between lattices along \( y \)-direction could varying \( \beta \) in Eq. (1), different from cold atom case [14], there is no limitation of the value of \( \beta \), it is not necessary to set \( \beta \) to be a large value which means spin-up and spin-down states have to be mixed deeply. Look at Figs. 2(c) and 2(d), when \( \lambda = 0 \), no spin-mixing case \( \beta = 0 \), the system is topological insulator and if the spin-mixing efficient became larger the system will transform to metal state at last. We choose \( \lambda = t_0, \beta = 0 \) and 0.1 to show that phase transi-
Figs. 3(a) and 3(b) are the band structures for $\lambda = t_0$, and different spin-mixing terms $\beta = 0$ and $\beta = 0.1$. And in the same column are the wave functions of the corresponding spin up state for systems of $42 \times 42$ and $6 \times 6$ lattices. In Figs. 3(c) and 3(e), topological invariant $\nu = 1$ and edge states appear along the boundary of the system. While Figs. 3(d) and 3(f) show the wave functions for the metal states.

Comparing (c), (e) and (d), (f) in Fig. 3, predictably we see the size effects in these new results in Figs. 3(e) and 3(f). However, when $\lambda = 0$ and $\beta = 0$, see Figs. 2(c) and 2(d), the QSHE is not changed so much as $\lambda = t_0$ and $\beta = 0$ case. After numerical calculation, we find $m, n \geq 6$ is good enough to realize the QSHE, and of course the more the better. And mark $\alpha = 1/q$, where $q$ is an integer, set $n$ be an integral multiple of $q$ could effectively reduce the size effects. Considering the real status of experiments $n = 6$ could be a good choice for the present $\alpha = 1/3$.

With the parameters of the edge states shown in Figs. 2(c) and 2(d), prepare an original spin up state $| \uparrow \rangle$ with energy around $E_{\text{topo}}$ of the edge states. Theoretically, only the lattices at the boundary of the 2D plane that will have excited states and photons could be detected, that is the so called QSHE. Considering the progress of the experiments about the superconducting circuits, there is limitation of the lattices’ size in experiment for realizing the QSHE. As shown in Fig. 2(d), $6 \times 6$ lattice is enough to find the edge state, and can be achieved experimentally soon [47–50].

As an addition, we investigate the quantum decoherence effects in our proposal for detecting the edge states. We use the Lindblad master equation and take three main decoherence factors in numerical calculation: the losses of the photon, the decay and dephasing of the transmon into account. The Lindblad master equation can be written as:

$$\dot{\rho} = -i[H_{\text{SC}}, \rho] + \sum_{r} \sum_{i=1}^{3} \gamma \left( \Gamma_{r,i} \rho \Gamma_{r,i}^\dagger - \frac{1}{2} \{ \Gamma_{r,i}^\dagger \Gamma_{r,i}, \rho \} \right),$$

(8)

where $\rho$ is the density operator of the system, $\gamma$ is the decay rate or noise strength which are set to be the same here, $\Gamma_{r,1} = a_r$, $\Gamma_{r,2} = \sigma_+^r$ and $\Gamma_{r,3} = \sigma_-^r$ are the photon-loss, transmon-loss and the transmon-dephasing operators in the lattice at $r$, respectively. In Fig. 4, we plot the edge-site and the inner-site populations:

$$P_1(t) = \text{tr}[\rho(t) \sum_{r_{\text{edge}}} (a_r^\dagger a_r + \sigma_+^r \sigma_-^r)],$$

$$P_2(t) = \text{tr}[\rho(t) \sum_{r_{\text{inner}}} (a_r^\dagger a_r + \sigma_+^r \sigma_-^r)],$$

(9)

after 2$\mu$s for different decay rates. It shows that both the edge state population $P_1$ and the inner state population $P_2$ decrease smoothly when the decay rate increases. Fortunately, the detection in our scheme can tolerate the decay rate up to the order of $2\pi \times 10$ kHz, while the typical decay rate is $2\pi \times 5$ kHz. We use initial state $|\uparrow\rangle_{r=(1,1)}$ in the numerical calculation in the consideration that it is easier to preparing such excited state in one site than the eigenstate of the Hamiltonian in Eq. (1) which concerns all 36 sites on the 2D plane. That initial state caused a little leakage from edge state to inner state, however that not effects much in the detection.

In summary, we have proposed a circuit quantum electrodynamics system with TLRs and transmons connected by SQUIDS, the hopping coefficients between these quasi-lattices could be adjusted separately. With that a Z$_2$ topological insulator in 2D lattices is realized and the phase transitions between topological and trivial states are simulated. With different parameters the wave functions for metal and topological states are shown all over the 2D lattices separately and
edge states of topological states along the boundaries are illustrated. Considering the real status of superconducting circuits experiment, based on a 6 × 6 lattice, we have given out a probable proposal to detect the topological edge state which could tolerate the decoherence at present experiments. Our proposal is stable and well controllable, especially using SQUIDs to realize the hopping is smooth and steady. The state of each quasi-lattice could be prepared and detected separately and precisely. These new characteristics surpass the previous methods and will shed light not only on the realization of the topological systems but also topological quantum computation.

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