Low-Mass X-Ray Binary as the Progenitor of PSR J1713+0747

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Abstract

We have calculated the evolution of low-mass X-ray binaries that lead to the formation of the binary radio pulsars, like PSR J1713+0747. We showed that the mass transfer is most likely to be nonconservative, due to unstable disk accretion, to account for the mass of PSR J1713+0747, which is close to its initial value. We assumed that part of the lost material from the binary may form a circumbinary disk, and found that it can significantly influence the mass-transfer processes. We briefly discuss the implications of the circumbinary disks on the evolution of low-mass X-ray binaries and the formation of low-mass binary pulsars.

Key words: stars: evolution — stars: mass-loss — stars: neutron — X-ray:binaries

1. Introduction

Since the first radio pulsar was discovered by Hewish et al. (1968), the number of pulsars has already increased to be more than 1500. In this population only $\sim 3\%$ are the members of binary systems. Most of the binary radio pulsars are millisecond pulsars (Backer et al. 1982) with a He or CO white-dwarf companion. Such binary and millisecond pulsars are thought to have been “recycled” from accreting low-mass or intermediate-mass X-ray binaries (L/IMXBs, see Bhattacharya, van den Heuvel 1991; Tauris, van den Heuvel 2006 for reviews). Mass accretion onto a neutron star induces magnetic field decay, and spins the star up to a short period. When mass transfer ceases, the end-point of the evolution is a circular binary containing a neutron star visible as a low-field, millisecond radio pulsar, and a white dwarf, the remaining He or CO core of the companion. The recent discovery of millisecond accreting pulsars has lent strong support to this scenario (van der Klis 2006).

The mechanisms that drive mass transfer in LMXBs depend on the initial separations of the binary components (Bhattacharya, van den Heuvel 1991). In narrow systems with
initial orbital periods of $P_{\text{orb}} < 1 - 2$ d, mass transfer is driven by the loss of orbital angular momentum via gravitational radiation and/or magnetic braking. Mass transfer in relatively wide ($P_{\text{orb}} > 1 - 2$ d) LMXBs is driven by the nuclear expansion of the secondary. Systems of this kind form a quite homogeneous group whose evolutionary history seems to be well understood (Webbink et al. 1983; Taam 1983). Ritter (1999) derived a simple analytical solution for the evolution of a close binary with nuclear time-scale driven mass transfer from a giant, based on the well-known fact that the luminosity and the radius of a giant scale to a good approximation as simple power laws of the mass of the degenerate helium core. It has been shown that the orbital binary periods gradually increase during evolution. Once the initial orbital period, $P_{\text{orb}}$, is long enough, the mass transfer rate, $\dot{M}$, will exceed the Eddington accretion rate, $\dot{M}_E \simeq 1.5 \times 10^{-8} M_{\odot} \text{yr}^{-1}$, of a neutron star, probably leading to mass loss, and the mass transfer becomes nonconservative.

The 4.57 ms radio pulsar PSR J1713+0747 is in a 67.8 day circular orbit with a low-mass white-dwarf companion (Foster et al. 1993). These characteristics indicate that this pulsar probably had evolved from a LMXB consisting of a neutron star and a low-mass red giant. Recent observations (Splaver et al. 2005) constrain the masses of the pulsar and secondary star to be $(1.3 \pm 0.2) M_{\odot}$ and $(0.28 \pm 0.03) M_{\odot}$, respectively (68% confidence), implying extensive mass loss during the previous LMXB evolution.

In this paper we explore nonconservative mass transfer in the LMXBs evolution, leading to systems like PSR J1713+0747, while taking into account mass loss resulting from unstable mass transfer, as well as its feedback to the binary evolution. In section 2 we describe our model for the evolution of LMXBs. In section 3 we present the calculated results, and compare them with the observational data of PSR J1713+0747 in section 4. We present a brief discussion and conclude in section 5.

2. Model

In a binary system consisting of a neutron star (of mass $M_X$) and a low-mass red giant (of mass $M$), the evolutionary expansion of the Roche lobe-filling giant will transfer mass to the neutron star through the inner Lagrangian point on the nuclear time scale. The mass transfer rate can be described by the following equation (e.g., Ritter 1999):

$$\dot{M} = -\left(\frac{\dot{R}}{R}\right) \frac{M}{\xi_e - \xi_R},$$  \hspace{1cm} (1)

where $\xi_e = (\partial \ln R / \partial \ln M)_{\text{eq}}$ is the thermal equilibrium mass radius exponent and $\xi_R = \partial \ln R_L / \partial \ln M$ is the mass radius exponent of the Roche lobe radius of the giant (Soberman et al. 1997); $R$ and $R_L$ are the radii of the giant and of the Roche lobe, respectively.
2.1. Nuclear Evolution of the Giant

For a lower giant-branch star, the stellar luminosity, $L$, and radius, $R$, are uniquely determined by the mass of its degenerate helium core, $M_c$, independent of the hydrogen envelope mass (Refsdal, Weigert 1970),

$$L = L_\odot \exp(\sum a_i y^i),$$  \hspace{0.5cm} (2)

$$R = R_\odot \exp(\sum c_i y^i),$$  \hspace{0.5cm} (3)

where $y = \ln(4M_c/M_\odot)$ and $i = 0, 1, 2, 3$. The parameters $a_i$ and $c_i$ in the above equations for the solar chemical composition can be found in Webbink (1975).

From equation (3), the change in the radius of the donor star is related to the growth rate, $\dot{M}_c$, of the helium core by

$$\left(\frac{\dot{R}}{R}\right)_M = (c_1 + 2c_2y + 3c_3y^2) \frac{\dot{M}_c}{M_c}.$$  \hspace{0.5cm} (4)

Because the stellar luminosity is directly related to the growth in the core mass due to shell hydrogen burning,

$$L = X\varepsilon_H \dot{M}_c,$$  \hspace{0.5cm} (5)

where $X$ is the initial hydrogen mass fraction in the shell, and $\varepsilon_H \simeq 6 \times 10^{18} \text{erg g}^{-1}$ is the released energy rate by the CNO cycle (Webbink et al. 1983).

Since the giant remains in thermal equilibrium during the mass transfer, and its radius is only determined by the core mass, independent of total mass, $M$, $\xi_e \approx 0$ (Soberman et al. 1997). We then have

$$\dot{M} = \frac{c_1 + 2c_2y + 3c_3y^2}{\xi R} (\frac{M}{M_c}) \dot{M}_c.$$  \hspace{0.5cm} (6)

2.2. Mass and Angular Momentum Loss

The orbital evolution is determined by the angular-momentum loss rate for a binary system possessing mass communion. The total angular momentum of a system with a circular orbit is

$$J = a^2 \frac{MM_X}{M + M_X} \frac{2\pi}{P_{\text{orb}}},$$  \hspace{0.5cm} (7)

where $a = [G(M + M_X)P_{\text{orb}}^2/(4\pi^2)]^{1/3}$ is the binary separation; $G$ and $P_{\text{orb}}$ are the gravitational constant and the orbital period of system, respectively. We neglect the spin angular momentum of the components because it is small compared the total orbital angular momentum of the system, and consider the following processes for the mass and orbital angular momentum loss from the binary system.

1. isotropic wind

We consider the mass and angular momentum loss during nonconservative mass transfer in the evolution of LMXBs. If the magnitude of the secular mass transfer rate, $-\dot{M}$, is greater
than the Eddington accretion rate, $\dot{M}_E$, we assume that the neutron star accretes a fraction $(1 - \alpha)$ of the transferred mass, and the remaining fraction, $\alpha$, is ejected in the vicinity of the neutron star ($\alpha = 0$ if $|\dot{M}| \leq \dot{M}_E$, else $\alpha = 1 + \dot{M}_E/\dot{M}$).

Mass loss can also occur when the mass transfer is sufficiently low. It is well known that disk accretion in LMXBs is thermally and viscously unstable if the mass-transfer rate is less than a critical value, $\dot{M}_{cr}$ (van Paradijs 1996; Dubus et al. 1999)

$$\dot{M}_{cr} \simeq 3.2 \times 10^{-9} M_\odot \text{yr}^{-1} \left( \frac{M_X}{1.4 M_\odot} \right)^{0.5} \left( \frac{M}{1 M_\odot} \right)^{-0.2} \left( \frac{P_{\text{orb}}}{1 \text{d}} \right)^{1.4}. \quad (8)$$

When an accretion-disk instability occurs, the accreting neutron star will become a transient X-ray source, experiencing outbursts separated by long quiescent intervals. For the accretion behavior during outbursts, we adopt the following prescription suggested by Portegies Zwart et al. (2004). The accretion rate, $\dot{M}_X$, first reaches a peak value of $2 \dot{M}_E$, and then decays with an exponential timescale of $t_d = 6 \text{d} \times \min(1, P_{\text{orb}}/10 \text{hr})$. The relation of the recurrence time, $t_r$, and the total energy, $E$, in an outburst satisfies $t_r = E/(0.1 \dot{M} c^2)$, where $\log(E/\text{erg}) = 45 + \log(P_{\text{orb}}/\text{d})$. Since the accretion rate of a neutron star is $\dot{M}_X = 2\dot{M}_E \exp(-t/t_d)$, we can calculate the accreted mass of the neutron star within the recurrence time, $t_r$, to be

$$\Delta M_X = 2\dot{M}_E t_d [1 - \exp(-t_r/t_d)]. \quad (9)$$

Accordingly, the fraction $\alpha = 1 + \Delta M_X/(\dot{M} t_r)$ of the transferred mass is assumed to be lost from the system.

The material probably leaves the binary system in the form of winds, outflows, or jets. We assume that a fraction of $(1 - \delta)$ of the lost matter is ejected in the vicinity of the neutron star, carrying away a specific orbital angular momentum of the neutron star,

$$j_w = \frac{M}{M_T M_X} J, \quad (10)$$

where $M_T$ is the total mass of the system.

2. CB disk

Spruit and Taam (2001) suggested that a Keplerian, circumbinary (CB) disk could be formed as the result of mass outflow from an accreting star or accretion disk. Tidal torques are put on the CB disk, and carry away angular momentum from the binary orbiting inside it. We assume that the other part (with a fraction of $\delta$) of mass loss forms a CB disk, extracting the orbital angular momentum from the binary system.

At the inner edge, $r_i$, of the disk, the viscous torque exerted by the CB disk on the binary can be shown to be (Spruit, Taam 2001; Taam, Spruit 2001)

$$T = \gamma \left( \frac{2\pi a^2}{P_{\text{orb}}} \right) \delta \dot{M} \left( \frac{t}{t_{vi}} \right)^{1/2}, \quad (11)$$

where, $\gamma^2 = r_i/a$, $a$ is the binary separation, and $t_{vi}$ the viscous timescale at the inner edge of the CB disk,
where $\nu_i$ is the viscosity at the inner edge of the CB disk. We estimate $\nu_i$ using the standard $\alpha$ prescription (Shakura, Sunyaev 1973),

$$\nu_i = \alpha_{SS} c_s H_i,$$

where $c_s$ and $H_i$ are the sound speed and the height of the disk at the inner edge of disk, respectively. In the following calculations we set the viscosity parameter, $\alpha_{SS} = 0.001$, and assume that the disk was hydrostatically supported and geometrically thin with $H_i/r_i \sim 0.03$ (Belle et al. 2004). The sound speed can be obtained from the equation of vertical hydrostatic equilibrium,

$$c_s \simeq \Omega_K H_i,$$

where $\Omega_K = (GM_T/r_i^3)^{1/2}$ is the Keplerian angular velocity at $r_i$.

3. The gravitational and baryonic mass of the neutron star

For neutron stars, one should consider the discrepancy between its baryonic mass, $M_b$, and the gravitational mass, $M_g$. By differentiating the binding energy equation (Lattimer, Yahil 1989),

$$M_b = M_g + 0.084 M_\odot (M_g/M_\odot)^2,$$

with respect to $t$, the relation between the growing rate of the gravitational and of the baryonic masses is

$$\dot{M}_X = \dot{M}_g = (1 - \beta) \dot{M}_b,$$

where $\beta = 1 - (1 + 0.168 M_X/M_\odot)^{-1}$. We assume that the fraction $\beta$ of the accreted baryonic mass disappears along with the specific angular momentum of the neutron star.

2.3. Computing $\xi_R$

Considering various kinds of mass loss discussed in the above subsection, we can write the rate of change of the orbital angular momentum of the binary to be

$$\frac{\dot{J}}{J} = [(1 - \delta) \frac{M}{M_T M_X} + \eta \frac{\gamma}{\mu}] \alpha \dot{M} + \frac{M}{M_T M_X} (1 - \alpha) \beta \dot{M},$$

where $\eta = \delta (t/t_{vi})^{1/2}$ is the efficiency factor of transfer mass removing angular momentum from the system through the CB disk; $\mu = M M_X/M_T$ is the reduced mass of the binary. The first, second, and third terms on the right-hand side of equation (17) represent the change in the orbital angular momentum due to mass loss through ejected outflows, the CB disk and the baryonic mass loss, respectively.

For $M \leq 0.8 M_X$, the Roche lobe radius of the donor can be approximately denoted by (Paczyński 1971)
\[ R_L = 0.462a \left( \frac{q}{1 + q} \right)^{1/3}, \]  

where \( q = M/M_X \). We then obtain \( \xi_R \) combining equations (7), (17), and (18):

\[ \xi_R = -\frac{5}{3} + 2(1 - \alpha)(1 - \beta)q + \frac{2}{3}(\alpha + \beta - \alpha\beta) \frac{q}{1 + q} + 2\alpha(1 - \delta) + \beta(1 - \alpha) \frac{q^2}{1 + q} + 2\gamma\eta(1 + q). \]  

(19)

When \( \alpha = \beta = 0 \), equation (19) recovers to the standard conservative mass-transfer case,

\[ \xi_R = -\frac{5}{3} + 2q. \]  

(20)

When \( \beta = \delta = 0 \), equation (19) becomes

\[ \xi_R = -\frac{5}{3} + 2(1 - \alpha)q + \left( \frac{2}{3} + 2q \right) \frac{\alpha q}{1 + q} \]  

(21)

for nonconservative mass transfer with only outflows included (Li, Wang 1998).

3. Numerical Results

We adopted the semi-analytical method by Webbink et al. (1983) to calculate the evolutionary sequences of LMXBs. We set the initial mass of the donor \( M = 1.0M_\odot \) (with solar chemical composition \( X=0.7, Y=0.28 \)) and the core mass in the range of \( 0.15M_\odot \leq M_c \leq 0.35M_\odot \), the initial mass of the accreting neutron star being \( M_X = 1.4M_\odot \).

For stable mass transfer, we let \( \dot{M}_X = -(1 - \alpha)(1 - \beta)\dot{M} \) when \( -\dot{M} > \dot{M}_E \). When the accretion disk becomes unstable, we calculate \( \Delta M_X \) according to equation (8). For the radius \( r_i \) of the inner edge and the mass feeding rate \( \delta \) of CB disk, we took \( r_i/a = \gamma^2 = 1.7 \) (Artymowicz, Lubow 1994) and \( \delta = 0, 0.01 \) and 0.02. We stopped the calculations when the mass of the hydrogen envelope of the giant was reduced to be \( \sim 2\% \) of the core mass.

Figure 1 shows the calculated \( \xi_R \) against the mass ratio, \( q = M/M_X \), for the initial \( M_c = 0.25M_\odot \) and \( \delta = 0, 0.01, 0.02 \). One can see that the larger is \( \delta \), the smaller is \( -\xi_R \), and hence the higher the mass transfer rates. This can also be seen in Figure 2, which compares the detailed evolutions of the the giant mass, the orbital period, the neutron star mass, and the mass-transfer rate for different values of \( \delta \). Obviously, the existence of the CB disk can accelerate the evolutionary processes quite efficiently. If \( \delta \) is further increased, the orbital period will decrease, finally leading to runaway mass transfer. The variation of \( \dot{M} \) is also much larger if the CB disk effect is included. Because of unstable mass transfer induced by the accretion-disk instability, the efficiency of neutron-star accretion is quite low. The resulting averaged accretion rates of the neutron star, \( \dot{M}_X = \Delta M_X/t_r \), are shown in Figure 3.
4. Application to PSR J1713+0747

We have performed many calculations for LMXBs evolution with different initial parameters in order to fit the observed data ($P_{\text{orb}}$, $M_X$, and $M$) of PSR J1713+0747. We found reasonable results when the initial core mass was $0.258 M_\odot$, $0.246 M_\odot$, $0.205 M_\odot$ for $\delta = 0.02$, $0.01$, $0$, respectively, while conservative evolution failed to produce results compatible with all of the measured data simultaneously. Table 1 summarizes the results of our calculations. It seems that both models with and without a CB disk can well fit the observed data for this source, if unstable mass transfer is taken into account. However, there exist considerable differences in the initial binary parameters and the evolutionary timescales, suggesting that the CB disks may have important implications for the LMXB evolution (see below).

Previous investigations suggested a simple relation between the orbital period, $P_{\text{orb}}$, and the mass, $M$, of the white dwarf for low-mass binary radio pulsars. Assuming the mass-feeding rate to be $\delta = 0.002$ (a larger $\delta$ possibly cause infinite mass transfer rate when initial core mass $M_c = 0.15 M_\odot$, see Figure 1) of the CB disk, we calculated 21 LMXBs evolution sequences, where the initial core mass of giant ranged from 0.15 to $0.35 M_\odot$. The relation between $P_{\text{orb}}$ and $M$ when evolution ceases for $0.15 M_\odot \leq M_{c1} \leq 0.35 M_\odot$ was plotted with the solid curve in Figure 4; the dot and dashed curves correspond to the relations obtained by Rappaport et al. (1995) and Tauris and Savonije (1999). It can be seen that there is a power-law relation between $P_{\text{orb}}$ and $M$ with $P_{\text{orb}}(d) \propto (M/M_\odot)^{6.55}$.

5. Discussion and Summary

The mass measured for PSR J1713+0747 indicates that the neutron star had accreted $< 0.1 M_\odot$ mass during its previous LMXB evolution, if formed with an initial mass of $1.4 M_\odot$. We suggest that unstable mass transfer in the accretion disk may lead to extensive mass loss in the evolution of relatively wide LMXBs. The mass accreted by the neutron star is considerably smaller than in the stable accretion case (see also Li, Wang 1998).

The evolutionary path of the binary also depends on the form of mass and angular momentum loss when mass transfer is nonconservative. Traditionally the excess material is assumed to leave the binary with a specific orbital angular momentum of either the neutron star or the binary. However, based on arguments proposed by Spruit and Taam (2001) and Taam and Spruit (2001), we considered the case with a CB disk originating from part of the mass outflow. Actually, radio observations of super-Eddington accretion systems SS 433 (Blundell et al. 2001) and Cygnux X-3 (Miller-Jones et al. 2004) have shown evidence of equatorial, disk-like outflows.

Our calculations show that the CB disk can remove the orbital angular momentum of the binary at a rate significantly higher than simple mass outflows. This may have important implications on the LMXBs evolution and its relation with low-mass binary pulsars. Recent
binary population synthesis studies (Podsiadlowski et al. 2002; Pfahl et al. 2003) fail to produce enough luminous LMXBs, as observed. This problem seems to be related to the discrepancy between the birthrates of LMXBs and low-mass binary pulsars (e.g. Kulkarni, Narayan 1988), the solution of which may lie in an unknown mechanism for angular-momentum loss to speed up the mass transfer. Moreover, it has been pointed out by Sills et al. (2000) that the traditional magnetic braking, which has served as the basis for the evolution of LMXBs and cataclysmic variables (CVs), is significantly reduced. The CB disk may provide an efficient mechanism to drain the orbital angular momentum, accelerating the binary evolution and enhancing the mass-transfer rates. Dubus et al. (2002) found that the spectral energy distributions expected of CB disks in CVs can dominate the emission from the donor star and the accretion disk of the white dwarf at wavelengths $\gtrsim 3$ $\mu$m. At longer wavelengths the relative contribution from the CB disk to the total emission from the system increases. Since LMXBs and CVs have similar binary parameters, searching for (mid- and far-) infrared emission from the CB disks will be of great importance for the evolution of both LMXBs and CVs.

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### Table 1. Calculated results for different evolutionary models. *

| Model | $M_c(t_0)$ | $t_f - t_0$ | $\log|\dot{M}(t_f)|$ | $R(t_f)$ | $P_{\text{orb}}(t_f)$ | $M_c(t_f)$ | $M_X(t_f)$ |
|-------|------------|-------------|----------------------|----------|----------------------|------------|------------|
| $\delta = 0.02$ | 0.258 | 23.9 | $-7.95$ | 21.38 | 68.36 | 0.284 | 1.402 |
| $\delta = 0.01$ | 0.246 | 47.1 | $-8.15$ | 21.20 | 67.71 | 0.283 | 1.403 |
| $\delta = 0$ | 0.205 | 267.5 | $-8.406$ | 21.67 | 69.78 | 0.284 | 1.408 |
| $\alpha = \beta = 0$ | 0.215 | 179.00 | $-8.400$ | 20.99 | 66.70 | 0.283 | 2.117 |

* $t_0$ and $t_f$ denote the ages at the beginning and end of the mass transfer, respectively.

**Fig. 1.** Plot of $\xi_R$ vs. $q$ for LMXBs with an initial core mass of $M_c = 0.25M_\odot$. The dot, dashed, and solid curves correspond to $\delta = 0$, 0.01, and 0.02, respectively.
Fig. 2. Evolutions of an LMXB with an initial donor mass of \( M = 1.0 M_\odot \) and a core mass of \( M_c = 0.25 M_\odot \). The dot, dashed, and solid curves correspond to \( \delta = 0, 0.01, \) and 0.02, respectively.
Fig. 3. Evolution of the accretion rate of the neutron star (mean accretion rate in a recurrence time $t_r$). The dot, dashed, and solid curves correspond to $\delta = 0, 0.01, \text{ and } 0.02$, respectively.
Fig. 4. Predicted relation between $P_{\text{orb}}$ and $M$ for low-mass binary radio pulsars. The dashed, dot, and solid curves correspond to the relations obtained by Tauris and Savonije (1999), Rappaport et al. (1995), and this work, respectively.
