Magnetohydrodynamic Simulation of DC Arc Plasma under AC Magnetic Field

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Direct current (DC) thermal plasma, shape controlled by use of alternative current (AC) magnetic field enables the wide area of thermal treatment and has potentials of applications other than welding. In this paper, the behavior of DC plasma under AC magnetic field imposed perpendicular to the plasma current is discussed experimentally and theoretically by changing various parameters such as plasma electric current, nozzle diameter, argon flow rate and magnetic flux density including its wave form. As the theoretical study, DC plasma was mathematically modeled by use of three dimensional magnetohydrodynamics (MHD) theory and discussed through numerical simulations performed by full finite volume method approach. Then the DC arc plasma motion under sinusoidal AC magnetic field is discussed through MHD analysis. By these experimental and theoretical investigations, the controlling parameters of DC plasma by AC magnetic field have been made clear quantitatively.

KEY WORDS: plasma; electromagnetism; magnetohydrodynamics; numerical simulation.

1. Introduction

DC thermal plasma is widely used for the heating and welding in metals processing because of high thermal efficiency and stability compared to other plasmas such as inductive plasma. In steel industry, many applications other than welding are utilized such as tundish plasma heating in steelmaking process. Among the applications, DC plasma shape-controlled by AC magnetic field is one way to get a wide area thermal treatment of materials, which opens the application of DC plasmas to processing other than welding.

On the numerical modeling of DC plasma, many previous researches have been performed mainly on the free burning arc plasma in two-dimensional axisymmetric case. In the thermofluid analyses described in the above literatures, the electric field is solved in the total region including an arc flow zone and both cathode and anode regions by giving heat flux condition between arc and electrodes without assuming the distributions of temperature and electric current on the electrode surfaces. The effects of arc current, inlet flow rate, electrode gap and cathode vertex angle on the thermofluid characteristics are clarified by two-dimensional computational simulation. In addition, the numerical model is adopted in the evaluation of tungsten electrode lifetime, considering the thorium diffusion inside the cathode and also thorium adsorption and desorption at the cathode tip. The time evolution of cathode surface temperature with the cathode vertex angle and applied arc current is evaluated by coupling with the cathode temperature distribution and electric current distribution previously obtained in the arc analysis.

In this paper, DC arc plasma is modeled by three-dimensional MHD theory and discussed through numerical simulations performed by full finite volume method approach. In which three-dimensional simulation is indispensable because DC plasma is driven by AC magnetic field imposed perpendicular to the plasma axis and consequently the MHD phenomena becomes three-dimensional.

2. Experiment

2.1. Experimental Apparatus and Conditions

Figure 1 shows the schematic view of experimental apparatus for the observation of DC plasma under AC magnetic field. The apparatus is composed of DC plasma torch, water-cooled copper anode, two turns AC coil and argon atmosphere chamber equipped with video camera for the observation of plasma behavior. The pressure in the chamber is controlled for standard pressure. The arc is generated by the imposition of high frequency voltage between W–2 wt%ThO2 cathode and nozzle made of water-cooled copper under the setting of anode-cathode distance as 20 mm. After the ignition, the torch is moved to the distance of 80 mm by fixing the DC plasma current to the arbitrary value. Simultaneously, AC electric current is imposed to the plasma drive coil, which generates magnetic field perpendicular to the DC plasma current and gives reciprocal motion to the plasma. The arc behavior was observed by using video camera. The evaluated parameters in the experiments are the effect of nozzle diameter, flow rate of argon, magnetic flux density of plasma drive coil and plasma current. The relation between these parameters and the stroke of oscillated plasma (fan-shape plasma width) is evaluated in order to
get the information of the controlling parameters giving larger width of reciprocal motion and uniform heat flux to the cathode surface. The experimental conditions are summarized in the Table 1.

### Table 1. Experimental conditions.

| Parameter                      | Value                        |
|--------------------------------|------------------------------|
| Cathode diameter              | 6 mm                         |
| Cathode tip angle             | 60 degree                    |
| Argon                         | 10 ~ 20 litre/min            |
| Cathode nozzle diameter       | 5 ~ 8 mm                     |
| Anode-cathode distance        | 80 mm                        |
| Cathode material              | W-2wt%ThO₂                   |
| Anode material                | Water-cooled copper          |
| Atmosphere                    | Argon (1.01 x 10^5 Pa)       |
| Magnetic flux density         | 0 ~ 28 Gauss                 |
| Plasma current                | 100 ~ 200 A                  |

2.2. Experimental Results and Discussions

**Figure 2** is the relation between anode–cathode distance and plasma DC voltage including estimated value of voltage drop at anode and cathode. The experimental conditions are summarized in the Table 1.

**Figure 3** shows the distribution of calculated magnetic flux density by the plasma drive coil, which is obtained by the following method and confirmed its correspondence with measured value. Magnetic flux density generated by an linear electric current $I_b$ at the position of the distance $r$ from the coil is described as $B=\mu_0 I_b/(2 \pi r)$. So that the $x$ component of the magnetic flux density on the $z$-axis in Fig. 4 is obtained as;

$$B_x = \mu_0 I_b \cdot \frac{(z - z_1)/r_1^2 + (z - z_2)/r_2^2}{r_1^2}$$

where, $r_1 = \sqrt{z_1^2 + (z_1 - z)^2}$ and $r_2 = \sqrt{z_2^2 + (z_2 - z)^2}$.

This relation is obtained by the superposition of the magnetic flux density from four linear current in Fig. 4. The $x$ component of the magnetic flux density takes its maximum value at the coil center and decreases along the $z$-directions as in Fig. 3.

**Figure 5** shows the typical snapshot of the arc without and with sinusoidal AC magnetic field. Stable fan-shaped arc is obtained as this time averaged figure by the reciprocal motion of the arc driven by sinusoidal AC magnetic field. In order to investigate the equilibrium state of the driven plasma by AC magnetic field, DC magnetic field, which has the value of maximum value of AC magnetic field, was imposed and the deviation from the center was measured. **Figure 6** represents the change in plasma deviation distance from the center with magnetic flux density of DC field in case of various nozzle diameters. **Figure 7** shows the observed plasmas under various value of DC magnetic field. As an equilibrium state, the plasma is well driven by the imposed magnetic field. The smaller the amount of argon flow rate and the higher the magnetic flux density of imposed DC magnetic field, the larger the width of the plasmas.
3. Numerical Simulation

3.1. Fundamental Equations and Numerical Methods

The hypothesis of Local Thermodynamic Equilibrium (LTE) state is applicable in this report’s conditions after the previous paper. The plasma can be treated as electrically conductive fluid. Supposing that the fluid is incompressible ideal gas and the displacement electric current is neglected, the fundamental equations become as follows.

Maxwell’s equations are written as follows.

\[ \nabla \cdot \mathbf{B} = 0 \] 
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \] 
\[ \nabla \cdot \mathbf{D} = 0 \] 
\[ \nabla \times \mathbf{H} = \mathbf{j} \]

Ohm’s law is written as follows.

\[ \mathbf{j} = \sigma(\nabla \phi + \mathbf{u} \times \mathbf{B}) \]

By the introduction of scalar potential \( \phi \) and vector potential \( \mathbf{A} \), the following relations are obtained.

\[ \mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t} \] 
\[ \mathbf{B} = \nabla \times \mathbf{A} \] 
\[ \mathbf{B} = \mu_0 \mathbf{H} \]

where, \( \mathbf{B}, \mathbf{E}, \mathbf{D}, \mathbf{H}, \mathbf{j}, \mathbf{u}, t, \sigma \) and \( \mu_0 \) represent magnetic flux density, electric field, electric flux density, magnetic field, electric current, flow velocity, time, electrical conductivity and magnetic permeability of the vacuum, respectively.

Poisson equation of scalar potential \( \phi \) and vector potential \( \mathbf{A} \) can obtained by taking the divergence of Eq. (6) as follows.

\[ \nabla \cdot \mathbf{E} = -\nabla^2 \phi - \frac{\partial(\nabla \cdot \mathbf{A})}{\partial t} \]

As the divergence of vector potential equals to zero, the second term of the right hand side of this relation becomes to zero.

This relation becomes as follows by using electric current conservation in the media.

\[ \nabla \cdot \mathbf{E} = \nabla \left( \frac{j}{\sigma} - \mathbf{u} \times \mathbf{B} \right) = -\nabla \cdot (\mathbf{u} \times \mathbf{B}) \]

From the formulas of vector operation, the following relation is obtained.

\[ \nabla \cdot (\mathbf{u} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{u}) - \mathbf{u} \cdot (\nabla \times \mathbf{B}) \]

Finally the following Poisson equation is obtained.
\[ \nabla^2 \phi = B \cdot (\nabla \times u) - u \cdot (\nabla \times B) \quad \cdots \cdots \quad (9) \]

By taking the rotation of Eq. (8), the following relation is obtained.

\[ \nabla \times B = \nabla \times (\mu H) = \mu \nabla \times H \]

Here, Eqs. (4) and (7) are given to Eq. (9).

\[ \nabla \times (\nabla \times A) = \mu j \]

Following relations are always true.

\[ \nabla \cdot (\nabla \times A) = \nabla \cdot \nabla \times A = 0 \]

By the combination of the above relations, the following Poisson equation of vector potential \( A \) is obtained.

\[ \nabla^2 A = -\mu j \quad \cdots \cdots \cdots \cdots \cdots \quad (10) \]

Once the distribution of scalar potential \( \phi \) is obtained by solving the Eq. (9) using suitable boundary conditions, the distribution of electric current density is obtained by the following relation.

\[ j = \sigma \left( -\nabla \phi - \frac{\partial A}{\partial t} + u \times B \right) \quad \cdots \cdots \cdots \cdots \cdots \quad (11) \]

The distribution of magnetic flux density can be obtained by the Eq. (7).

Next, the fluid and energy equations are shown as follows.

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \quad \cdots \cdots \cdots \cdots \cdots \quad (12) \]

\[ \frac{\partial (\rho u)}{\partial t} + (u \cdot \nabla)(\rho u) = -\nabla p + \nabla (\mu \nabla u) + j \times B \quad \cdots \cdots \cdots \cdots \cdots \quad (13) \]

\[ \frac{\partial (\rho c_p T)}{\partial t} + (u \cdot \nabla)(\rho c_p T) = \nabla (k \nabla T) + \frac{j^2}{\sigma} + \frac{k_b}{2e} (j \cdot \nabla (\rho T)) - R_e \quad \cdots \cdots \cdots \cdots \cdots \quad (14) \]

where, \( u, p, \mu, t, \rho, C_p, T, k, k_b, e \) and \( R_e \) represent fluid velocity, pressure, viscosity which is molecular viscosity at the laminar part and effective viscosity for turbulent part, time, density of fluid, specific heat under constant pressure, temperature, thermal conductivity, Boltzmann constant, electron charge and radiation loss, respectively. The radiation loss \( R_e \) from plasma is given by the multiple terms approximation as a function of plasma temperature from the experimental data for argon. For the turbulence model, RNG \( k-e \) model is used, because almost all the part near plasma is in laminar state and mixed setting of laminar and turbulent zone should be set.

The equation of state of the ideal gas is given as follows.

\[ p = \rho RT \quad \cdots \cdots \cdots \cdots \cdots \quad (15) \]

where, \( R \) represents the gas constant.

The physical properties of carrier gas argon, which depend on the temperature, is given by the ones in the reference, which are simplified as listed in Table 2. In the three dimensional simulation of this report, the treatment of anode and cathode is simplified by supposing some boundary conditions which yields not so inexact result by solving only the gas region after the previous paper.

Commercial code FLUENT 6, which enables the use of unstructured scheme under finite volume method, was used to solve the fluid flow field by adding MHD solver developed by the authors based on the aforementioned logic. Pressure equation is solved by SIMPLE algorithm. The other variables were solved by QUICK scheme.

Figure 8 shows the calculated region. As a mesh, hexagonal unstructured cell is used. Figure 9 presents the boundary conditions for fluid flow and thermal field. The physical properties of carrier gas argon, which depend on the temperature, is given by the ones in the reference, which are simplified as listed in Table 2.
anode and cathode, suitable voltage for arbitrary plasma electric current is given at the lower part of conical surface of the electrode and 0 V is given on the surface of the anode and Neumann condition is given for the other parts. As for the vector potential, it should be zero at the infinity, however, here, the following approximation is taken for finite calculation domain. That is, Neumann condition at the periphery of calculation domain. For the boundary condition of fluid flow, Neumann condition at the periphery of gas domain is given and constant velocity, which corresponds to argon flow rate, at the inlet condition for the nozzle inlet. Standard wall function is used for the walls. On the temperature field, constant temperature of 300 K is given at the anode and nozzle surfaces. Mixed boundary condition of heat transfer, which is composed of heat flux coefficient of $15 \text{ W/m}^2\text{K}$ and radiation coefficient of 0.3 was used. Cathode surface was treated as adiabatic condition. The Stefan–Boltzmann constant is given by \(5.669 \times 10^{-8} \text{ W/m}^2\text{K}^4\).

3.2. Calculated Results and Discussions

Figure 11 is the change in calculated temperature field with various values of DC magnetic field. The temperature measurement could not be performed in this report, however, it shows a reasonable temperature level written in the literature. \(^3\) Figure 12 presents the calculated velocity field without and with DC magnetic field. The velocity at the nozzle outlet reaches to 160 m/s, however, it becomes the value of 120 m/s at the middle part between anode and cathode, which corresponds to the described level in the above mentioned literatures. Figure 13 shows the calculated temperature field under DC magnetic field, which corresponds to the experimental results. Comparison between two results is demonstrated in Fig. 14, which shows a good agreement. Figure 15 is the temperature field change during one oscillation cycle of the plasma under AC magnetic field, which is obtained by the unsteady numerical simulation. By the dynamic effect, the plasma shows a deformed shape due to the velocity component in the horizontal direction. As the time-averaged heat flux depends on plasma sectional area and arc residence time at the arbitrary position along the arc trajectory on the cathode, it is important to evaluate the time averaged heat flux and to control it to obtain a possibly uniform heat flux along the plasma trajectory. Figure 16 presents the heat flux evaluated by the simulation, which shows a peak at the center of the trajectory. A trapezoidal wave form of imposed AC field might improve the heat flux. Figure 17 shows the calculated heat flux of DC plasma driven by AC magnetic field, which
shows the improvement of the heat flux uniformity, in which 50 Hz trapezoidal wave composed by 80% of constant region and 20% of linear change region was used. The uniformity is the function not only of the AC magnetic field but the characteristics of DC arc itself, so that it will be necessary to obtain suitable relation among all parameters of the system by the numerical approach.

4. Conclusions

Three dimensional MHD model for DC arc plasma is established and evaluated by the comparison with experimental data. The simulation results showed quantitative correspondence with experimental data and the following results have been obtained.

From the experiments, the following things are obtained.

(1) Plasma is widely driven by perpendicularly imposed AC magnetic field and shows a fan-shape when the time average is taken for the video figure.

(2) The width of the driven plasma becomes wider when the flow rate of argon decreases, the magnetic flux density of imposed field increases, the nozzle diameter increases and the DC current of plasma decreases. The width of plasma driven by perpendicularly imposed magnetic field is proportional to the square of nozzle diameter.

From the numerical calculations, the following things are obtained.

(3) Calculated width of plasma driven by DC and AC magnetic field shows a good agreement with experimental data and showed same dependence of plasma width on the control parameters.

(4) Trapezoidal AC magnetic field showed more uniform heat flux to the anode from the plasma rather than sinusoidal one. This approach to get uniform heat flux to the anode surface will give an effective controllability to the DC plasma application.

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Nomenclature

A : Vector potential (T · m)
**B**: Magnetic flux density (T)

**Bx**: x-component of magnetic flux density generated by the plasma drive coil (T)

**B_{xm}**: Maximum value of **Bx** (T)

**Cp**: Specific heat at constant pressure (J/kg · K)

**D**: Electric flux density (C/m²)

**d_n**: Nozzle inner diameter (m)

**E**: Electric field (V/m)

**e**: Electron charge (C)

**f**: Lorentz force (N/m³)

**H**: Magnetic flux (A/m)

**h**: Enthalpy (J/kg)

**I_a**: DC electric current for plasma (A)

**I_b**: Electric current in the plasma drive coil (A)

**j**: Electric current density (A/m²)

**j_e**: Emission current density (A/m²)

**k**: Thermal conductivity (W/m · K)

**k_b**: Boltzmann constant (J/K)

**L**: Half of the distance between plasma drive linear coils (m)

**p**: Pressure (Pa)

**R**: Gas constant

**R_a**: Radiation loss (J/m³)

**r, r_1, r_2**: Distance between arbitrary point in space and the plasma drive linear coil (m)

**z, z_1, z_2**: Vertical coordinate from the origin (m)

**t**: Time (s)

**T**: Temperature (K)

**T_A**: Temperature on the surface of anode (K)

**T_C**: Temperature on the surface of cathode other than its tip (K)

**T_N**: Temperature on the surface of nozzle (K)

**v**: Flow velocity (m/s)

**x**: Coordinate (m)

**μ**: Molecular or effective viscosity (Pa · s)

**θ**: Cathode vertex angle (deg)

**ϕ**: Scalar potential (V)

**μ**: Molecular or effective viscosity of fluid (Pa · s)

**μ_0**: Magnetic permeability of vacuum (H/m)

**ρ**: Density (kg/m³)

**σ**: Electrical conductivity (S/m)

**REFERENCES**

1) K. Takeda: Bull. Iron Steel Inst. Jpn., 5 (2000), 103.

2) K. Takeda: Kouon Gakkai-shi, 16 (1990), 357.

3) K. C. Hsu, K. Etemadi and E. Pfender: J. Appl. Phys., 54 (1983), 1293.

4) X. Chen and H.-P. Li: Int. J. Heat Mass Transfer, 44 (2001), 2541.

5) H. Nishiyama, T. Sato and K. Takamura: ISIJ Int., 43 (2003), 950.

6) T. Iwao, H. Miyazaki, T. Ishida, Y. Liu and T. Inaba: ISIJ Int., 40 (2000), 275.