Minireview

Mathematical models of cell factories: moving towards the core of industrial biotechnology

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Summary
Industrial biotechnology involves the utilization of cell factories for the production of fuels and chemicals. Traditionally, the development of highly productive microbial strains has relied on random mutagenesis and screening. The development of predictive mathematical models provides a new paradigm for the rational design of cell factories. Instead of selecting among a set of strains resulting from random mutagenesis, mathematical models allow the researchers to predict in silico the outcomes of different genetic manipulations and engineer new strains by performing gene deletions or additions leading to a higher productivity of the desired chemicals. In this review we aim to summarize the main modelling approaches of biological processes and illustrate the particular applications that they have found in the field of industrial microbiology.

Introduction
The rapid progress in molecular biology and the development of tools for directed genetic modifications, high-throughput measurements and genome sequencing have made available important quantities of data that can be used to validate mathematical models and fit their parameters as well as the means to test in vivo the validity of the predictions of the models (Price et al., 2003). Combining knowledge of various disciplines (biology, mathematics, physics, biochemistry, molecular biotechnology and computer science) enhances the possibility to elucidate the complexity of biological systems and predict their behaviour.

In the field of industrial biotechnology there is currently much focus on how systems biology can improve the efficiency of cell factories and in particularly speed up the development process (Nielsen and Jewett, 2008), and hereby ensure that new products can be brought to the market faster or there can be a faster improvement of existing bioprocesses. The use of metabolic engineering for improvement of cell factories is not a novel concept (Nielsen, 2001; Stephanopoulos, 2002), but recent synergies with tools developed in systems biology enabled the production of variety of products through biotechnology with significantly reduced time and resources required for commercialization (Otero and Nielsen, 2010; Tyo et al., 2010).

Developing computational tools for data integration and generating in silico genome-scale metabolic models (GSMM) enables analysis of the effects of different media and specific mutations on growth and metabolic network adjustments. Numerous valuable predictions have been obtained from GSMM, with the relatively high success rate of 70–90%, depending of the organism and the predictions (Price et al., 2003). *Escherichia coli* and *Saccharomyces cerevisiae* are some of the most exploited organisms in industrial biotechnology. *Escherichia coli* has been used for production of many different recombinant proteins (like human growth hormone) and the yeast *S. cerevisiae* is used for bioethanol production, production of a range of pharmaceutical proteins, fine and bulk chemicals, and nutraceuticals (Table S1).

Biological systems are complex and generally not entirely understood. Mathematical models provide means to better understand processes and unravel some of the complexities. The aim is to construct the model in the simplest possible way, but still retain the most important features of the system. A good model will be able to agree as closely as possible with the real world observations of the phenomenon we are trying to model and at the same time be interrogative.

Depending on the process we want to model, the available data and the goal, biological processes can be modelled using either kinetic or stoichiometric methods.
Dynamic models

Dynamic modelling (Fig. 1) requires knowledge of the kinetics including the parameters of kinetic expressions. Kinetics of the different reactions is used to describe dynamic changes in the state variables, which are typically the concentrations of key compounds. These dynamic models are typically represented as difference equations (discrete dynamical systems) or differential equations (continuous dynamical systems).

The most common technique for dynamic modelling of biological systems is the ordinary differential equation (ODE) approach. The main characteristic of ODEs is that we can obtain deterministic time series for the variables under investigation. Linear ODE can be solved analytically, while non-linear ODEs are much harder, and in some cases it is impossible to find the solution analytically. In this case the approximate solution is derived using numerical algorithms for solving differential equations.

The rapid development in the field of systems biology led to enormous expansion of computational tools that can be used for system analysis. Most of the tools are freely available for the scientific community and their use will greatly depend on the user’s preferences and expertise (Klipp et al., 2007).

Dynamic modelling approaches are used in modelling regulatory processes, like central carbon metabolism (Savageau, 1969a,b; Savageau et al., 1970; Curto et al., 1995; Sorribas et al., 1998; 2000; Chassagnole et al., 2002), cell cycle regulation (Novak and Tyson, 1997; Rizzi et al., 1997; Novak et al., 1998; 2001; Chen et al., 2000; 2004; Barberis et al., 2007) and different signalling pathways (Klipp et al., 2005; Papin et al., 2005; Kholodenko, 2006). These models help in understanding complicated dynamic features, such as glycolytic oscillations, regulatory feedback effects or cell cycle oscillations.

A dynamic model describing the central carbon metabolism in E. coli (Chassagnole et al., 2002) includes the phosphotransferase system (PTS), glycolysis, pentose–phosphate pathway and storage material and represents the first step towards systematic analysis of metabolism in E. coli. The main feature of this model is its application in improvement of microbial production processes. The underlying framework for improvement of production capabilities of desired compounds is built on metabolic control analysis (MCA). Assuming that in a metabolic network, the kinetics of the individual enzymes is known, MCA allows estimating the individual flux control coefficients and hereby identifying targets for overexpression with the objective to increase the flux through the pathway.

Flux control coefficients of glucose uptake by the PTS were calculated from the general model. The highest flux control coefficient was expectedly observed for the PTS since glucose uptake is irreversible and in the model it is independent of any preceding reactions. The results (Fig. S1) further suggest that there are other enzymes that control glucose uptake apart from the PTS. Phosphofructokinase (PFK) has the second highest value, and equivalent flux control coefficients are observed for pyruvate dehydrogenase (PDH) and glucose-6-phosphate dehydrogenase (G6PDH). The PTS is feedback inhibited by glucose-6-phosphate (G6P) and by its co-product pyruvate, and this explains why G6PDH has a high control on glucose uptake due to its role in G6P-consuming reaction and PDH is exerting flux control on glucose uptake as it is a pyruvate-degrading reaction.

Through combining kinetic modelling and MCA Hoefnagel and colleagues (2002) demonstrated that it was possible to identify targets for improving the flux trough biotechnologically relevant pathways in Lactococcus lactis. Their approach allowed for simple estimation of flux control coefficients and a further advantage was that the effect of genetic manipulations can be tested directly using the kinetic model. The kinetic model of L. lactis comprises of a set of ODEs which describes the time dependence of the metabolite concentrations, while enzymes were modelled using reversible Michaelis–Menten equation.

Metabolic control analysis of the pyruvate branches in L. lactis (Fig. S2) indicated that the highest flux control coefficients of the acetolactate branch are not within this branch, as intuitively one would assume, but can be found in the enzymes outside this branch – lactate dehydrogenase (LDH) and NADH oxidase (NOX) (Table S2). Further analysis indicated that 92% of the pyruvate is converted via the acetolactate branch when LDH knockout is combined with NOX overexpression.

Another approach to model complex biological system is to provide detailed representation of smaller modules and then stitch these together to describe a larger system. Fine tuning and wiring of the components in small modules is more effective and controllable than in larger systems. A challenge with this approach is the linking of the different modules, but this can be achieved by defining appropriate input and output signals for each module. Extending this further gives the possibility to link different pathways (modelled as single independent modules) into a larger network.

The high osmolarity glycerol (HOG) pathway has been intensively studied in the literature (Albertyn et al., 1994; Van Wuytswinkel et al., 2000; de Nadal et al., 2002; Hohmann, 2002). The pathway represents the fundamental process by which cells regulate their water balance. It consists of two branches – Sho1 and Sln1, both being transmembrane proteins, being placed upstream of other players in HOG signalling pathway. The fact that HOG
Formulation of the set of ODEs based on the given problem and a literature review. The set of ODEs, with the general representation:

$$\frac{dS_i}{dt} = \sum_{j=1}^{r} n_{ij} v_j$$

gives the rate of change of concentration $S_i$ of the $i$th compound, while $r$ denotes the number of reactions involved, $v_j$ the rate of $j$th reaction and $n_{ij}$ represents stoichiometric coefficient of $S_i$ in reaction $j$. The stoichiometric coefficients give stoichiometric matrix $N = \{n_{ij}\}$, which denotes the network properties. Analysis of matrix $N$ gives information of admissible set of fluxes and conservation relations of the system.

When building a model, not all parameters are known or could not be measurable directly, or there is inconsistency between different labs and strains that are used. In those cases, we have to estimate ‘unknown’ parameters by fitting the model to experimental data. Parameter estimation minimizes the error function over the parameters under investigation and using the goodness-of-fit measure the discrepancy between the observed values and the expected values in the model is monitored. Performing sensitivity analysis we get more information about the model. Some of the questions usually asked are: which parameters have the highest influence on the system behaviour or on the other hand which ones do not have any effect on the system, so they do not have to be considered further and can be fixed to some semi-arbitrary value. The main goal of sensitivity analysis is to better understand the dynamic behaviour of the system. Once the model is able to reproduce the input data obtained from experimental studies it can be used for the prediction of untested scenarios.
pathway activity can easily and rapidly be controlled experimentally by extracellular stimuli (Hohmann et al., 2007) makes it a suitable candidate for system level studies via mathematical modelling.

The model developed by Klipp and colleagues (2005) represents a good example of an integrated approach towards a quantitative understanding of the osmotic shock response in yeast, and it also is a good example of linking regulation to a (small-scale) metabolic model. The model has four modules: Phosphorelay, MAP kinase cascade, Gene expression and Metabolism module. Each module was modelled and analysed individually.

This model comprises the HOG signalling pathway, gene expression, cellular metabolism of glycerol production and control of cellular volume and osmotic pressure. The entire reaction network consists of 32 ordinary equations and 70 parameters, which were estimated on the basis of steady state and time-course experiments. The model was validated with physiological and genetic perturbations.

Several new aspects of yeast osmoregulation have been revealed using in silico approach: (i) the contribution of osmotic and turgor pressure changes to the regulation of biochemical processes, (ii) the role of aquaglyceroporin Fps1p in controlling glycerol accumulation and signaling through the HOG pathway, and (iii) the function of the induced changes of gene expression as long-term contributions to the upregulation of glycerol (Klipp, 2007).

**Genome-scale metabolic models (GSMM)**

Kinetic models have their limitation in terms of describing large metabolic networks. Here simple stoichiometric models are more appropriate (Fig. 2), and with the appearance of genome sequences it became possible to reconstruct metabolic networks at genome scale. Four years after the first sequences were revealed, the first metabolic model was reconstructed (Haemophilus influenzae, Schilling and Palsson, 2000), and today more than 80 reconstructed models exist (Feist et al., 2009; Milne et al., 2009).

In GSMM the metabolic network is represented as a stoichiometric matrix containing the stoichiometric coefficients for all the metabolites in all the cellular reactions. Relevant data on the stoichiometry and occurrence of metabolic reactions are often extracted from annotated genome sequences, pathway databases (KEGG, ExPASy, ERGO), biochemical textbooks and research papers. Based on the stoichiometric coefficients for each metabolite in all reactions it is possible to set up mass balance equations for each metabolite. By assuming that the level of the metabolites is in steady state it is possible to constrain the set of fluxes (or reaction rates) of each of the cellular reactions, resulting in an underdetermined system of linear equations. The assumption of steady state implies that for each internal metabolite in the network, the sum of the rates of the reactions producing it is equal to the sum of the rates of the reaction consuming it. This assumption has proved to be realistic due to the fact that the relaxation time that the internal metabolites take to reach a steady state after a perturbation is several magnitude orders lower than the doubling time of the cells.

To identify a particular solution to this set of linear equations requires determination of constraints under which the system functions. These constraints can be based on thermodynamics, enzyme activity and ‘omics’ data (genomics, proteomics, transcriptomics, metabolomics) (Beard et al., 2002; Palsson, 2002; Price et al., 2002), and the integration of high-throughput biological data generally increases the predictive capabilities of genome-scale models.

For exploiting the capabilities of metabolic networks a number of tools are available. Constraint base modelling can be performed using the COBRA TOOLBOX (Becker et al., 2007) and web-based BioOpt application available in BioMet Toolbox (Cvijovic et al., 2010). Structural properties of metabolic networks, like elementary flux modes (EFMs) and null space matrix, can be analysed in METATOOL (von Kamp and Schuster, 2006) or FluxAnalyzer (Klamt et al., 2003). FluxAnalyzer is also able to detect dead-end metabolites in the network, that is, metabolites present only in one reaction, which can therefore not be used. BioMet Toolbox also features tools for transcriptome/proteome/metabolome data analysis (Reporter Features and Reporter Subnetworks applications).

Metabolic networks can be analysed using two different approaches: (i) identification of a unique solution and (ii) pathway analyses (Patil et al., 2004).

**Identification of a unique solution**

**Metabolic flux analysis (MFA)** is the solution of the system of linear equations obtained from the mass balances around each internal metabolite. In order to find a unique solution for the fluxes, as many fluxes as degrees of freedom should be determined experimentally. The number of degrees of freedom of the system is equal to the number of reactions minus the rank of the stoichiometric matrix. The number of degrees of freedom is typically big (several hundreds for genome-scale models) and exact solutions using MFA can only be found for simplified metabolic networks.

**Flux balance analysis (FBA)** is an approach that relies on imposing an objective function to be maximized (or minimized) on the system and then find among the many possible solutions for the system, one that maximizes (or
Fig. 2. Genome-scale models – flowchart. This figure illustrates the work-flow from the reconstruction of genome-scale metabolic models to their different applications as predictive tools for metabolic engineering. More detailed information about each step is included in the text.
minimizes) the selected objective function. The objective function used for microorganisms is normally the specific growth rate (Edwards et al., 2002), which is consistent with the evolutionary advantage of fastest-growing species. FBA problems are solved using linear programming and the obtained solution is in a corner of the feasible region in the solution space. In order to have growth rate as an objective function, it is crucial to define a biomass stoichiometric equation which should be obtained from the biomass composition of the corresponding microorganism. The biomass equation determines the drainage of the different biomass precursors per unit of biomass produced and therefore the metabolic demands for growth. The processes involved in biomass production, such as protein synthesis or DNA replication, have also an cost in terms of ATP, which is difficult to estimate a priori. The right estimation of the energetic costs for biomass production is extremely relevant for the predictive power of the metabolic models.

**Pathway analysis**

The set of constrains will determine all possible functions of the reconstructed network or all feasible phenotypes. We refer to this set of possible solutions as a solution space. The solution space can be characterized by a set of solutions that span the whole spectrum of steady-state solutions attainable by a metabolic network. The elements of this set are EFMs and extreme pathways (EPs). A recent review on pathway analysis and its applications has been published by Trinh and colleagues (2009).

**Elementary flux modes (EFMs)** are minimal sets of reactions that can operate in steady state. This means that if any reaction used by an EFM is removed, the remaining reactions will not be able to operate together in steady state (Schuster et al., 1999). The EFMs of a network provide an easy way to understand the effects of gene deletions due to the fact that the deletion of a particular reaction simply makes disappear all the EFMs in which it was involved. This allows the identification of essential reactions and the evaluation of the degree of coupling between different fluxes. However, the number of EFMs grows exponentially with the size of a metabolic network and makes its calculation impossible for genome-scale metabolic networks. Despite the mentioned limitations, the pathway analysis of simplified metabolic models has led to several successful metabolic engineering applications involving the identification of both deletion and overexpression targets as well as the introduction of heterologous genes.

The first successful practical application of pathway analysis for a metabolic engineering application was the optimization of 3-deoxy-D-arabino-heptulosonate-7-phosphate (DAHP) production in *E. coli* (Liao et al., 1996). A recent application was the development of high-ethanol-producing *E. coli* strains (Trinh et al., 2008). These approaches consist essentially in the deletion of reactions involved in pathways with low yields of the product of interest and the overexpression of reactions involved in the pathways with high yields. However, as Liao and co-workers highlighted in their paper (Liao et al., 1996), there are also examples of unsuccessful overexpression strategies deduced from the EFM approach.

**Extreme pathways (EPs)** are the elements of the convex basis that characterizes the solution space (Schilling et al., 2000). They are a subset of the EFMs and none of them can be expressed as a positive linear combination of the others. Schilling and colleagues (2000) showed how the projection of the different EPs on a plane defined by the input fluxes of oxygen and the carbon source defines several differentiated regions in the phenotypic phase plane.

**Algorithms for in silico strain optimization**

Traditionally, the improvement of the industrial strains producing valuable compounds was carried out by inducing random mutations and selecting the strains that showed improvements in the production yield. This iterative blind method allowed obtaining high-yield microorganisms; however, the availability of reconstructed metabolic networks allows much more rational and efficient approaches to the problem.

The first rational manipulations of metabolic pathways were focused on genes directly connected with the product-synthesizing pathway (Mingot et al., 1999; Stafford et al., 2002; Koffas et al., 2003; Padilla et al., 2004). All these manipulations part from the identification of a key branch point in a precursor of the desired product. The flux through the path leading to the target product is then increased by deleting genes in the competing pathways or by overexpressing enzymes involved in the desired pathway. In some cases the path leading to the product of interest was not present in the wild-type strain and was introduced by the heterologous expression of enzymes from a different organism.

In order to predict the effects of genetic manipulations on a broader scale, two main methods have been used: FBA and minimization of the metabolic adjustment (MOMA). The FBA method is based on the assumption that the metabolic fluxes in a microorganism are such to maximize the cell growth yield (Edwards and Palsson, 2000). The MOMA approach (Segre et al., 2002) minimizes the distance between the flux distributions of the wild-type strain and the deletion mutant. Another algorithm developed with the same purpose as MOMA is the regulatory On/Off minimization of metabolic flux changes (ROOM). It aims to minimize the number of fluxes that...
pass zero to a non-zero flux (or the opposite) between the two metabolic states (Shlomi et al., 2005).

The mentioned methods provided prediction tools for the effects of gene deletions; however, the combinatorial number of possible gene deletions to be tested required the introduction of computer algorithms to find the optimal combinations of deletions necessary to optimize the desired target.

The first algorithm introduced for microbial strain optimization was OptKnock (Burgard et al., 2003). The function to be optimized by this algorithm is the production rate of the target product and the variables are the genes to be deleted. The solution involves two nested optimizations, which makes it difficult to solve. In order to overcome this problem, the dual optimization problem for growth is also formulated (Fig. 3). In this way the nested optimization can be transformed into a simple optimization of the production rate of the desired chemical with the constraints corresponding to the primal and dual problems plus the condition of equality between the two solutions.

The OptKnock algorithm was used to construct lactic acid-producing strains of E. coli with very positive results (Fong et al., 2005).

The OptKnock algorithm found three optimal solutions for the optimization of lactic acid production. The first one involved the deletion of the adaHE and pta genes (Fig. S3). This solution seems trivial, as those genes are involved in the production of ethanol and acetate, which are the two main products secreted under anaerobic conditions. The second solution involved pta and pfk deletions. The pfk deletion is a less trivial solution. It deviates flux through the Entner–Doudoroff pathway and increases the production of NADH and pyruvate, the two necessary metabolites for lactic acid production. The third solution involves the already three mentioned targets plus the glk gene. The deletion of the glucokinase couples the phosphorylation of glucose to the transformation of PEP into pyruvate.

Three new strains were built with the adaHE-pta, pta-pfk and the adaHE-pta-pfk-glk deletions. The designed deletion strains did not show optimal growth rates and lactic acid yields in the first generation; however, after a process of adaptive evolution, the growth rate and the lactic acid production (which is coupled to the growth rate in these strains) showed up to threefold increases. This finding clearly shows that the growth rate is a suitable objective function for microorganisms.

OptKnock has found a number of industrial applications, which is reflected in an increasing number of patents. Some examples are: ‘Methods and organisms for the growth-coupled production of succinate’ (Burgard and Van Dien, 2007); ‘Methods and Organisms for Growth-Coupled Production of 3-Hydroxypropionic Acid’ (Burgard and Van Dien, 2008a,b); ‘Methods and organisms for the growth-coupled production of 1,4-butanediol’ (Burgard et al., 2009).

The OptKnock algorithm, when it is applied to genome-scale models, is computationally demanding, and normally only a relatively small set of deletion candidates can be tested, such as the reactions in the central carbon metabolism (Burgard et al., 2003) or amino acid metabolism (Pharkya et al., 2003).

Other authors (Alper et al., 2005) have used an iterative approach that consists in simulating the effects of single
deletions, selecting the deletion that results in the highest increase in the product yield and performing new deletions on the selected mutant. This approach was successfully applied to the production of lycopene in E. coli.

The parallelism existing with the process of biological evolution and their lower computational cost make genetic algorithms an attractive tool for the design of new strains. OptGene (Fig. S4) (Patil et al., 2005) is an algorithm that relates the production rate of the desired compound to the evolutionary fitness of the microorganism.

The OptGene algorithm is more computationally efficient than the OptKnock algorithm and allows using non-linear objective functions (such as the amount of product per unit of time). However OptGene does not guarantee finding the absolute optimal solution. The optimal convergence rates for OptGene were found for a population of 125 individuals and a mutation probability of 1 per genome size and generation. The method was used to optimize the production of vanillin, glycerol and succinate in S. cerevisiae and it converged to the final solution in less than 1000 generations.

OptGene has been used to improve the production of sesquiterpenes in S. cerevisiae (Asadollahi et al., 2009). It has also inspired the apparition of new genetic algorithms such as CiED (Fowler et al., 2009), which differs in the mutant selection method and has been used to increase the production of flavanone in E. coli.

In order to illustrate OptGene's applications it is interesting to mention its recent utilization for the design of high-succinic-acid-producing strains of E. coli. OptGene was used to increase the production of flavanone in E. coli.

The mutant sdh3ser3ser3Δ was constructed. The new strain was cultivated in a series of six flasks with decreasing concentrations of glycine in order to create a selective pressure favouring the production of glycine from isocitrate and therefore releasing succinate. An increase of almost eightfold in the succinate yield was observed in the evolved strain. Three more cultures in absence of glycine were performed in order to select the strain for faster growth.

The algorithms mentioned so far are used to improve microbial phenotypes by performing gene deletions. In many cases, metabolic engineering involves expressing heterologous genes in host organisms that lack reactions involved in the production of the desired compound. The creators of the OptKnock algorithm developed the OptStrain algorithm (Pharkya et al., 2004), which uses a database of known biological transformations to find the minimal set of non-native genes to be expressed in a host organism in order to obtain the desired product with an optimal yield.

OptStrain is limited by the reactions contained in the available database. A more general algorithm named BNICE (Hatzimanikatis et al., 2005) has been proposed to generate de novo pathways using generalized enzyme reactions. This method can involve metabolic intermediates that are not included in the available reaction databases.

As we have seen, the existing algorithms for strain optimization rely on stoichiometric considerations and do not include regulatory information. The obtained outputs are sets of reactions to be removed or added to a metabolic network in order to modify its topology and couple the growth rate with the production of the desired compounds. The effects of overexpression of metabolic genes are still poorly understood and a systematic algorithm to find overexpression targets with efficiency comparable to OptKnock or OptGene is still missing.

**Integration of thermodynamics in the analysis of metabolic networks**

According to the second law of thermodynamics, a chemical transformation at constant pressure and temperature occurs in the direction of negative Gibbs free energy.

The thermodynamic feasibility condition must be satisfied by all the reactions in the metabolic network simultaneously. This condition allows defining additional constraints to the mass balances. It has been used to determine the feasibility of biosynthetic pathways under physiological conditions (Mavrovouniotis, 1993; Pissarra and Nielsen, 1997) and is a useful tool for the design of de novo biosynthetic pathways (Hatzimanikatis et al., 2005).

The analysis of thermodynamic feasibility in metabolic networks has been denominated Network-embedded...
thermodynamic analysis (NET analysis, Fig. 4) and is based on the integration of metabolomics data (Kümmel et al., 2006). NET analysis consists in the application of an optimization procedure to find the interval of variation of reaction Gibbs free energy for each metabolic reaction. The concentrations of unmeasured metabolites are allowed to vary between 0.001 and 10 mM (Fraenkel, 1992).

Network-embedded thermodynamic analysis can be used to check the thermodynamic consistency of metabolomic data and the assumed flux directions. It also allows specifying concentration ranges for unmeasured metabolites and determining whether each reaction operates far or close to the equilibrium. The reactions operating far from the equilibrium are more likely to be flux controlling (Wang et al., 2004).

There are several important limitations for the use of NET analysis. The metabolomics data are often incomplete and lacking enough precision to predict accurately the in vivo chemical potentials. For example, from seven metabolomics data sets analysed (Kümmel et al., 2006), only four were thermodynamically consistent. The effects of ionic strength and pH have been often overlooked and in many cases change completely the results of the analysis (Maskow and von Stockar, 2005). The standard chemical potentials of most of the metabolites in genome-scale models are unknown and the errors in their estimations play also a role in decreasing the accuracy of the thermodynamic analysis. An improved group contribution method for the determination of standard chemical potentials in complex metabolic networks has been recently published (Jankowski et al., 2008).

The first effort to estimate the reaction Gibbs free energies for all the reactions in a genome-scale model (Henry et al., 2006) led to the iHJ873 model for E. coli. This model included standard Gibbs free energies for each of its reactions. It contains fewer reactions than the iJR904 model because those reactions involving metabolites with unknown chemical potentials were lumped together. The iHJ873 model was used to identify the less thermodynamically favourable reactions in the network and analyse the biological implications of removing them from the model.

The same authors moved a step forward (Henry et al., 2007) and included in their analysis the effects of ionic strength and pH. In the same paper, an improved analysis framework with respect to NET analysis was proposed. The new framework was named thermodynamics-based metabolic flux analysis (TMFA) and differs from NET analysis in the fact that the reaction rates are not obtained independently of the thermodynamic analysis. In TMFA, the thermodynamic directionality constraints are added to the mass balance constraints during the calculation of the flux distribution.

The flux distributions obtained using TMFA do not include loops, which are by definition thermodynamically infeasible. The infeasibility of loops has been mentioned in the literature (Beard et al., 2004); however, a reliable method to remove loops from the flux distributions has not yet been implemented in the common calculus packages. TMFA would be the ideal framework.

Other important finding of Henry and co-workers was the fact that the reactions that operate far from the equilibrium are likely to be situated at the beginning of linear pathways or in branching points. This is consistent with their expected role as flux-controlling steps.

The range of concentration variability allowed for the metabolites was between 20 and 0.001 mM. Only one essential reaction appeared to be infeasible under the used concentration rank, it was the dihydroorotase. The reaction became feasible by allowing the estimated reference Gibbs free energy to vary in an interval of two standard deviations (obtained from the group contribution method) or increasing the concentration rank of its metabolites by 20%. The thermodynamic bottleneck nature of dihydroorotase is consistent with the fact that in mammals, this enzyme belongs to a multienzyme complex, which could be a way to use substrate channelling to overcome unfavourable Gibbs free energies.

The main weakness of the existing thermodynamic analysis approaches is the fact that they assume a concentration rank for the metabolites a priori. Once the reaction directions have been determined using TMFA, a new concentration rank can be defined for each metabolite; however, this rank is still very broad (Henry et al., 2008).
and is not really a useful predictor of the cellular metabolic pools.

Conclusions and perspectives

Mathematical models have shown to be promising tools for the design of new highly productive microbial strains. These models provide a predictive framework that allows testing in silico the outcomes of genetic manipulations. In this way some manipulations can be discarded a priori and others can be selected for further validation. This allows saving both time and resources in the strain development process.

Mechanistic models based on differential equations have been applied to study relatively small systems such as metabolic or signalling pathways but their scale-up at the genome-scale level has proved a complex task that would require collecting kinetic information from all the biochemical processes in the cell. Genome-scale stoichiometric metabolic models have together with various simulation algorithms shown to be very efficient for identification of complex metabolic engineering strategies. A major advantage of these models is that they do not rely on any information about the kinetics of the individual reactions within the metabolic network.

In this review some interesting examples of the applications of mathematical modelling to the design of efficient cell factories have been discussed. Biological systems are still too complex to be analysed in the deterministic way typically applied in other branches of science, but systems biology is providing an increasing number of success stories in the field of industrial biotechnology. These examples are a very encouraging trend that could be the beginning of a shift from the traditionally descriptive approach of biology to a predictive one.

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**Supporting information**

Additional Supporting Information may be found in the online version of this article:

**Fig. S1.** Flux control coefficients of the reactions in the system on the glucose uptake by phosphotransferase system.

**Fig. S2.** Schematic representation of pyruvate branch in *L. lactis*.

**Fig. S3.** The OptKnock algorithm found four deletions to improve the lactic acid production in *E. coli*.

**Fig. S4.** Flow chart of an evolutionary algorithm. OptGene defines a population with a fixed number of individuals. Each individual is characterized by a ‘chromosome’, which is a list of genes labelled with ones if they are present or zeros if they are absent. The algorithm can be initialized by assigning present status to all genes or by assigning present or absent status randomly. Each individual in the population is assigned a fitness score that determines whether it will be propagated to the next generation. The fitness score is calculated using FBA, MOMA or other optimization criteria. The individuals to be propagated to the next generation are selected with a probability determined from the fitness score. The ‘chromosomes’ of the selected individuals are crossed over to generate the next population and the next iteration starts. The iterations are repeated until an individual with the desired phenotype appears.

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Fig. S5. Deletions coupling succinate production to growth. After deleting the genes ser3 and ser33, the cell is forced to synthesize the serine necessary for growth from glyoxylate. Glyoxylate itself is obtained from isocitrate, generating succinate as a by-product. The deletion of sdh3 avoids the degradation of succinate, which has to be secreted from the cell. The first-generation mutant grows very slowly and needs external serine supply, but after evolution in a medium with decreasing serine concentration the cells become adapted to produce serine via glyoxilate-secreting succinate as a by-product.

Table S1. Overview of available genome-scale metabolic models of some of the industrially most exploited organisms.

Table S2. Flux control coefficients predicted by the model through acetolactate synthesis (ALS) in *Lactococcus lactis*.

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