Secure Beamforming for Distributed Intelligent Reflecting Surfaces Aided mmWave Systems

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Abstract—In this letter, we investigate the beamforming in a distributed intelligent reflecting surfaces (IRSs)-aided millimeter-wave (mmWave) system, where a multi-antenna base station (BS) tries to send a secure message to a single antenna user. Assuming the user’s channel and eavesdropper’s channel is perfectly known at the BS. This problem is posed as a joint optimization problem of transmit beamforming and IRS control. Our objective is to maximize the secrecy rate under the total transmission power and unit-modulus constraints. The problem is difficult to solve optimally due to their nonconvexity and coupled variables, and we propose an alternating optimization (AO) scheme based on successive convex approximation (SCA) and manifold optimization (MO) techniques. Numerical simulations show that the proposed AO scheme can effectively improve the secrecy rate and outperforms conventional schemes.

Index Terms—Distributed intelligent reflecting surfaces, millimeter-wave, secrecy rate.

I. INTRODUCTION

Millimeter-wave (mmWave) technologies have been considered as a promising technology for 5G communications, due to its abundant spectrum and high data rates [1], [2]. However, due to the high propagation loss, the mmWave signals are easily blocked by obstacles. Intelligent reflecting surface (IRS) has emerged as a promising technology to solve these problems [3]–[5]. Specifically, by adjusting the phases shift of the IRS, the reflected signals are able to be strengthened. Due to the significant beamforming gain, the IRS can extend the coverage of mmWave communication systems as well.

On the other hand, the secrecy rate has been intensively investigated in recent years. Because the wireless channel can be configured by using phase shifts of the IRS, it can greatly improve the secrecy rate [6], [7]. Specifically, an IRS-aided secure single-user multiple-input single-output (MISO) system was investigated in [6]. The IRS’s phase shifts are adaptively adjusted to strengthen the received signal at the user but suppressed the eavesdropper. An alternating optimization scheme was proposed for maximizing the secrecy rate in the IRS-aided MISO communication multi-user systems [7].

Note that all aforementioned works about IRS-aided systems are based on a single IRS. In fact, distributed IRSs can cooperatively enhance the communication quality of the systems in future systems. Specifically, since multiple IRSs are deployed part from each other, distributed IRSs have higher robust in transmitting the information. Meanwhile, multiple IRSs can provide more propagation paths for signals, which enhances the signal strength. This motivates us to investigate a distributed IRSs-aided mmWave system with switches. In particular, assuming that the CSI of user and eavesdropper is perfectly known at BS. The secrecy rate of the system is maximized by jointly optimizing the phase shifts of all IRSs, the transmit beamforming of the transmitter, and the IRS on-off status vector. Because the formulated problems are non-convex, we propose an alternating optimization (AO) algorithm and first optimize the transmit beamforming by applying sequential convex approximate (SCA), then use a dual method to solve IRS on-off optimization subproblem. Finally, the manifold optimization (MO)-based algorithm is proposed to obtain the phase shift of the IRS. Numerical simulations show that the AO scheme can improve the secrecy rate compared to the benchmark schemes.

II. SYSTEM MODEL

As shown in Fig. 1, we consider a mmWave downlink with distributed IRSs, which consists of one BS, a set \( \mathcal{L} \) of \( L \) IRSs, one user, one eavesdropper. The BS is equipped with \( N_t \) antennas. Each, \( l \in \mathcal{L} \), has \( N_r \) reflecting elements. The user and eavesdropper are equipped with a single antenna, respectively. In this paper, it is assumed that the direct links between the BS and the user, and between the BS and the eavesdropper are blocked by obstacles, which usually occurs when the direct link is blocked due to long-distance path loss or obstacles [\textsuperscript{1}].

Denote the BS-to-\( l \)-th IRS mmWave channel, the \( l \)-th IRS-to-user mmWave channel, and the \( l \)-th IRS-to-eavesdropper mmWave channel as \( G_l \in \mathbb{C}^{N_r \times N_t} \), \( h_l \in \mathbb{C}^{N_t \times 1} \), \( g_l \in \mathbb{C}^{N_r \times 1} \), respectively. The signal at the \( l \)-th IRS is expressed as

\[
r_l = G_l w_s,
\]

where \( r_l \in \mathbb{C}^{N_r \times 1} \), \( s \in \mathbb{C}^{1 \times 1} \), and \( w \in \mathbb{C}^{N_t \times 1} \) denote the transmit data and the corresponding beamforming matrix at the BS with \( E[s_\mathrm{s}^H] = 1 \); Then the \( l \)-th IRS reflects it with a phase shift matrix \( \Theta_l = \text{diag}(\phi_l) \in \mathbb{C}^{N_r \times N_r} \), where \( \phi_l = \phi_{1,j} = e^{j\phi_j}, j = 1, \cdots, N_r \) with \( \phi_j \) being the reflection phase shift; IRS reflects \( s \) to the user while the eavesdropper eavesdrops. The received signals at the user and the eavesdropper are denoted as

\[
y = \sum_{l=1}^{L} x_l h_l^H \Theta_l G_l w_s + n,
\]

\[
y_e = \sum_{l=1}^{L} x_l g_l^H \Theta_l G_l w_s + n_e,
\]
where $x_l$ is a binary variable $x_l \in \{0, 1\}$, where $x_l = 1$ denotes that the $l$th IRS is active. When $x_l = 0$, the $l$th IRS does not work and consume any power. $n \sim \mathcal{CN}(0, \sigma^2)$ and $n_e \sim \mathcal{CN}(0, \sigma_e^2)$ denote additive Gaussian noises of the user and the eavesdropper, respectively.

From (2) and (3), the achievable rate between the BS and the user is

$$I = \log_2(1 + \frac{1}{\sigma^2} \| \sum_{l=1}^{L} x_l h_l^H \Theta_l G_l w \|^2),$$  

(4)

The achievable rate between the BS and the eavesdropper is

$$I_e = \log_2(1 + \frac{1}{\sigma_e^2} \| \sum_{l=1}^{L} x_l g_l^H \Theta_l G_l w \|^2).$$  

(5)

Therefore, the secrecy rate $I_s$ denotes as,

$$I_s = [I - I_e]^+, \quad \text{where} \quad [x]^+ = \max(0, x).$$

(6)

The transmit power constraint is

$$\text{tr}(ww^H) \leq P,$$

(7)

where $P$ is the maximum transmit power of the BS. Based on (4)-(8), the security beamforming optimization problem for distributed IRSs-aided mmWave system with power constraint is formulated as

$$\max_{w, \theta, x} I_s, \quad \text{s.t.} \quad \text{tr}(ww^H) \leq P, \quad \| \theta_{lj} \| = 1, \forall l \in L, j = 1, \cdots, N_r, \quad x_l \in \{0, 1\}, \forall l \in L,$$

(9a)-(9d)

where $x = [x_1, \cdots, x_L]^T$. The problem (9a) is highly non-convex because of the non-convexity of the objective and constraints. In the following, we propose one iterative algorithm to obtain suboptimal solutions of problem (9).

### III. ALTERNATING OPTIMIZATION ALGORITHM FOR PROBLEM

#### A. Transmit Beamforming Optimization

Since there is no general method to solve the non-convex problem (9) efficiently and optimally, we develop an alternating optimization algorithm to solve (9) in this paper. In this subsection, we fix the variables $\theta$ and $x$, problem (9) can be rewritten as

$$\max_w I_s, \quad \text{s.t.} \quad \text{tr}(ww^H) \leq P.$$  

(10a)-(10b)

To deal with the problem, we introduce a new matrix $W = ww^H$, $a^H = \sum_{l=1}^{L} x_l h_l^H \Theta_l G_l$, and $b^H = \sum_{l=1}^{L} x_l g_l^H \Theta_l G_l$. Thus, we have $\| \sum_{l=1}^{L} x_l h_l^H \Theta_l G_l \|^2 = \text{Tr}(WA)$ and $\| \sum_{l=1}^{L} x_l g_l^H \Theta_l G_l w \|^2 = \text{Tr}(WB)$, where $A = aa^H$ and $B = bb^H$. It is not difficult to find that if $W$ is regarded as the optimization variable, $W$ needs to satisfy the rank one constraint, i.e., rank($W$) = 1. Next, semi-definite relaxation (SDR) technique is used to omit the constraint rank($W$) = 1. (10a) can be transformed into

$$\max_w \log_2 \left( \frac{1}{\sigma^2} \text{Tr}(WA) + 1 \right) / \left( \frac{1}{\sigma_e^2} \text{Tr}(WB) + 1 \right),$$  

\text{s.t.} \quad \text{tr}(W) \leq P, \quad W \succeq 0, \quad i = 1, \cdots, K.$$  

(11a)-(11b)

However, problem (11) is still non-convex because the (11a) is non-convex. To cope with the non-convex parts, we continue to introduce the following auxiliary variables,

$$e^p = 1 + \frac{1}{\sigma^2} \text{Tr}(WA),$$

(12)

and

$$e^q = 1 + \frac{1}{\sigma_e^2} \text{Tr}(WB).$$

(13)

By substituting (12) and (13) into (11), one can reformulate problem (11) as the following

$$\max_{W, P, q} \log_2 \left( e^{p-q} \right),$$

\text{s.t.} \quad 1 + \frac{1}{\sigma^2} \text{Tr}(WA) \geq e^p,$$

(14a)-(14b)

$$1 + \frac{1}{\sigma_e^2} \text{Tr}(WB) \leq e^q,$$

(14c)

$$\text{tr}(W) \leq P, \quad W \succeq 0,$$

(14d)

$$\text{Tr}(WA) \geq 0, \quad \text{Tr}(WB) \geq 0.$$  

(14e)

According to the properties of the logarithmic function, the objective function in (14a) can be expressed as

$$\log_2 \left( e^{p-q} \right) = (p - q) \log_2(e).$$

(15)

Thus, (15) is linear and convex. We replace the equalities (12) and (13) with inequalities in (14b) and (14c). It is not difficult to find that because of the monotonicity of the objective function, the inequalities (14b) to (14c) would hold with equalities at the optimal points. To maximize (14a), we maximize $e^x$ which is the lower bound of $1 + \frac{1}{\sigma^2} \text{Tr}(WA)$, while minimizing $e^y$ which is the upper bound $1 + \frac{1}{\sigma_e^2} \text{Tr}(WB)$, as presented in (14b) and (14c). Thus, while solving the problem (14), the lower bound of the numerator of the (11a) is maximized and the upper bound of the denominator of the (11a) is minimized, which makes the (11a) maximize. In conclusion, the problem (14) is an alternative formulation of problem (11).

It can be observed that constraint in (14c) is non-convex, to deal with the non-convex constraint, we consider the successive convex approximation (SCA) algorithm to deal with the non-convex constraint. According to the first-order lower bounds of Taylor expansion of $e^q$ at $\bar{q}$ is given by

$$e^q + e^\bar{q}(q - \bar{q}).$$

(16)

Thus, a sufficient condition for the constraint in (14c) is

$$1 + \frac{1}{\sigma_e^2} \text{Tr}(WB) \leq e^\bar{q} + e^\bar{q}(q - \bar{q}).$$

(17)

By replacing (14a) with (17), the following problem can be obtained

$$\max_{W, P, q} \log_2 \left( e^{p-q} \right),$$

\text{s.t.} \quad 1 + \frac{1}{\sigma^2} \text{Tr}(WA) \leq e^\bar{q} + e^\bar{q}(q - \bar{q}),$$

(18a)-(18b)

\text{(14d), (14e).}

(18c)

Since problem (18) is a convex, CVX (8) can efficiently solve this problem. In the $t$th iteration, the convex approximate
problem is expressed as
\[
\begin{align*}
\max_{W, p, q} & \quad \log_2 \left( e^{p - q} \right), \\
\text{s.t.} & \quad 1 + \frac{1}{\sigma^2} \text{Tr}(WB) \leq e^{q' + e^{q'} (q - q')}, \\
& \quad (14d), (14e), (14f).
\end{align*}
\]
When update the \( \tilde{q} \), let \( \tilde{q}^{t+1} = q' \). It should be noticed that to get the initial values \( \tilde{q}' \), we first generate \( w^0 \) randomly, and compute \( W^0 = w^0 (w^0)^H \). The iterative process algorithm is summarized in Algorithm 1.

**Algorithm 1: Proposed SCA-based Algorithm for Problem (11).**

1. **Initialization:** \( t = 0 \), given \( w^0 \) that is satisfy conditions, calculate \( q^0 \) based on (13) and let \( q^1 = q' \).
2. **Repeat:**
3. Solve problem in (19) to obtain the optimal solution \( W^t \) and \( q^t \).
4. Update \( \tilde{q}^{t+1} = q' \).
5. Set \( t = t + 1 \).
6. **Until:** The stopping criterion is met.
7. **Output:** Obtain \( w \) by decomposition of \( W \) when the rank(\( W \)) = 1; otherwise the Gaussian Randomization would be utilized to get a rank-one approximation.

### B. Phase Shift and IRS On-Off Optimization

Substituting the transmit beamforming variables \( W \) obtained in the previous section and given variables \{\( \Theta_l \)\}, problem (9) becomes
\[
\begin{align*}
\max_{x} & \quad 1 + \frac{1}{\sigma^2} \left\| \sum_{l=1}^{L} x_l h_l^H \Theta_l G_l w \right\|^2 \\
\text{s.t.} & \quad 1 + \frac{1}{\sigma^2} \left\| \sum_{l=1}^{L} x_l g_l^H \Theta_l G_l w \right\|^2 = 0.
\end{align*}
\]
There are two difficulties in solving problem (20). The first difficulty is that objective function (20a) is non-convex. The second difficulty is that constraint (20b) is non-convex. To deal with the first difficult, we optimize \( \{x_l\} \), and rewrite \( \left\| \sum_{l=1}^{L} x_l h_l^H \Theta_l G_l w \right\|^2 \) and \( \left\| \sum_{l=1}^{L} x_l g_l^H \Theta_l G_l w \right\|^2 \) as
\[
\begin{align*}
\left\| \sum_{l=1}^{L} x_l h_l^H \Theta_l G_l w \right\|^2 &= \sum_{l=1}^{L} C_l x_l + \sum_{l=2}^{L} L \sum_{l=1}^{L} \sum_{m=1}^{L-1} C_{lm} x_l x_m, \\
\left\| \sum_{l=1}^{L} x_l g_l^H \Theta_l G_l w \right\|^2 &= \sum_{l=1}^{L} D_l x_l + \sum_{l=2}^{L} D_{lm} x_l x_m,
\end{align*}
\]
where \( C_{lm} = h_l^H \Theta_l G_l \omega_m G_m h_l \) and \( D_{lm} = g_l^H \Theta_l G_l \omega_m G_m h_l \). We use the parametric approach in (20) and consider the following problem
\[
G(\lambda) = \max_{x} A(1 + \frac{1}{\sigma^2} \sum_{l=1}^{L} C_l x_l + \sum_{l=2}^{L} \sum_{m=1}^{L-1} C_{lm} x_l x_m)) \\
- \lambda(1 + \frac{1}{\sigma^2} \sum_{l=1}^{L} D_l x_l + \sum_{l=2}^{L} \sum_{m=1}^{L-1} D_{lm} x_l x_m),
\]
where \( C \) denotes the feasible set of \( x \) satisfying constraint (20b). According to [9], solving (23) is equivalent to obtaining the root of \( G(\lambda) \), and Dinkelbach method can obtain the root. After introduced the parameter \( \lambda \), (20a) can be transformed as formula in (23).

To handle the constraint (24), we introduce new variable \( z_{lm} = x_l x_m \). Owing to \( x_l \in \{0, 1\} \), constraint \( z_{lm} = x_l x_m \) is equivalent to
\[
\begin{align*}
\sum_{l=1}^{L} x_l &+ x_m - 1, 0 \leq z_{lm} \leq 1, z_{lm} \leq x_l, z_{lm} \leq x_m.
\end{align*}
\]
According to (23) and (24), problem in (20) is rewritten as
\[
\begin{align*}
\max_{x,z} & \quad A(1 + \frac{1}{\sigma^2} \sum_{l=1}^{L} C_l x_l + \sum_{l=2}^{L} \sum_{m=1}^{L-1} C_{lm} z_{lm})) \\
- \lambda(1 + \frac{1}{\sigma^2} \sum_{l=1}^{L} D_l x_l + \sum_{l=2}^{L} \sum_{m=1}^{L-1} D_{lm} z_{lm})),
\end{align*}
\]
where \( z = [z_{21}, z_{31}, \cdots, z_{L(L-1)}]^T \). Due to constraint (24), handling problem (25) is difficult. By We relax the (24) with \( x_l \in [0, 1] \), all constraints in (25) are convex. The problem in (25) can be rewritten as
\[
\begin{align*}
\max_{x,z} & \quad A(1 + \frac{1}{\sigma^2} \sum_{l=1}^{L} C_l x_l + \sum_{l=2}^{L} \sum_{m=1}^{L-1} C_{lm} z_{lm})) \\
- \lambda(1 + \frac{1}{\sigma^2} \sum_{l=1}^{L} D_l x_l + \sum_{l=2}^{L} \sum_{m=1}^{L-1} D_{lm} z_{lm})),
\end{align*}
\]
For problem (26) with relaxed constraints, the optimal solution can be obtained through the dual method [8]. The integer solution is obtained by the dual method and it guarantees both optimality and feasibility of the original problem. To obtain the optimal solution of the problem (26), we give the following theorem.

**Theorem 1.** For problem (26) these variables \( x \) and \( y_l, z_{lm} \) are respectively denoted as
\[
\begin{align*}
x_l &= \begin{cases} 
1 & S_l > 0 \\
0 & S_l \leq 0 
\end{cases}, \\
y_l &= \begin{cases} 
1 & S_{lm} < 0 \\
0 & S_{lm} \geq 0 
\end{cases},
\end{align*}
\]
where \( S_l \) is given at the top of next page and
\[
S_{lm} = (\lambda_{lm}^1 + \lambda_{lm}^2 + \lambda_{lm}^3) + \frac{1}{\sigma^2} A - \frac{1}{\sigma^2} \lambda D_{lm}.
\]
where \( \{\lambda_{lm}^1, \lambda_{lm}^2, \lambda_{lm}^3\} \) are the Lagrange multipliers associated with corresponding constraints of problem (26).

**Proof:** please refer to see Appendix A.
introducing additional interference from eavesdropper, which means that the secrecy rate can be improved when the IRS is on. The values of \( \{ \lambda^1_{im}, \lambda^2_{im}, \lambda^3_{im} \} \) are updated by the subgradient method \(8\), they are denoted as

\[
\begin{align*}
\lambda^1_{im} &= [\lambda^1_{im} - \beta(\hat{z}_{im} - x_i - x_m + 1)]^+, \\
\lambda^2_{im} &= [\lambda^2_{im} - \beta(\hat{z}_{im} - x_i)]^+, \\
\lambda^3_{im} &= [\lambda^3_{im} - \beta(z_{im} - x_m)]^+, 
\end{align*}
\]

where \( \beta > 0 \) is a step-size sequence. By iteratively optimizing \((x, z)\) and \( \{ \lambda^1_{im}, \lambda^2_{im}, \lambda^3_{im} \} \), the optimal \( x \) is obtained. The dual method for solving the problem \((28)\) and the Dinkelbach method to update parameter \( \lambda \) are given in Algorithm 2. It is not difficult to find that the optimal \( x_l \) is either 0 or 1 according to \((29)\), even though \( x_l \) is relaxed as \((26)\). Using the Dinkelbach method, we can obtain the root of \( G(\lambda) = 0 \), which indicates that the optimal solution of the secrecy rate optimization problem \((20)\) is obtained.

**Algorithm 2: Proposed Langrange Dual Algorithm for Problem \((20)\).**

1. **Initialization:** \( t = 0 \), \( \lambda^0 \) and set the accuracy \( \epsilon \).
2. **Repeat:**
3. **Initialization:** \( \{ \lambda^1_{im}, \lambda^2_{im}, \lambda^3_{im} \}^0 \).
4. **Repeat:**
5. Update the IRS on-off vector \( x \) according to \((29)\).
6. Update dual variables \( \{ \lambda^1_{im}, \lambda^2_{im}, \lambda^3_{im} \} \) based on \((30)-(32)\).
7. Set \( t = t + 1 \).
8. **Until:** The objective value converges.
9. Denote the objective value \((26a)\) by \( G(\lambda) \).
10. Update \( \lambda \) as

\[
\lambda = A\left(1 + \frac{1}{\sigma^2} \sum_{i=1}^L C_i x_i \sum_{m=1}^L \frac{C_i z_{im}}{z_{im} + 2 \sigma^2}ight).
\]

11. Set \( t_1 = t + 1 \).
12. **Until:** \( G(\lambda) < \epsilon \).
13. **Output:** The solution \( x^* \).

Then given \( x \) and \( \{ w \} \), problem \((9)\) can be simplified as

\[
\begin{align*}
\max_{\theta} \log_2 (1 + \frac{1}{\sigma^2} \sum_{l=1}^L x_l H_l \Theta_l G_l(w) \|w\|^2) \\
- \log_2 (1 + \frac{1}{\sigma^2} \sum_{l=1}^L x_l g_l H_l \Theta_l G_l w \|w\|^2),
\end{align*}
\]

s.t. \((9d)\).

The problem \((33)\) can be solved efficiently by the manifold optimization (MO) algorithm as \(10\). Details are omitted for simplicity.

**IV. NUMERICAL RESULTS**

In this section, numerical results are provided to demonstrate the effectiveness of the proposed alternating optimization schemes. As shown in Fig. 1 BS’s coordinate is \((0, 0, 0)\) and three IRSs are respectively located at \((0, 20, 20)\), \((0, 40, 20)\) and \((0, 60, 20)\) in meters. While user and eavesdropper are located at \((0, 40, 0)\) and \((0, 60, 0)\) in meters, respectively. The mmWave channel is modeled based on the channel model in \(1\). We set \( N_t = 16 \), \( N_r = 16 \) and \( \sigma^2 = \sigma_r^2 = -110 \) dBm. We compare the proposed scheme with the conventional scheme: the conventional scheme with a single IRS located at \((0, 60, 20)\) in meters and the number of reflecting elements of the IRS is set as the total number of reflecting elements for all IRSs in distributed IRSs-aided system.

Fig. 2 depicts the secrecy rate versus the power for different beamforming algorithms. It is found that the secrecy rate of all schemes linearly increases with the maximum transmit power of the BS. We can see that the proposed algorithm achieves the best performance. From Fig. 2 distributed IRS
can increase up to 20% secrecy rate compared to the mmWave system with a single IRS. This is due to the benefits of distributed deployment. Multiple IRSs are spatially distributed, which can provide more than one path of the received signal compared to the mmWave system with only one central IRS. Meanwhile, the average secrecy rate of the proposed AO scheme significantly increases, but those of maximum ratio transmission (MRT)-based and random beamforming (RB)-based schemes increase slowly. This is due to the fact that the MRT-based and RB-based scheme aims to maximize the achievable rate of the user while ignoring the eavesdropper, which results in significant information leakage, while the proposed AO scheme can effectively prevent eavesdropping.

Fig. 3 plots the secrecy rate versus the number of reflecting elements for each IRS and the number of IRSs, respectively. From Fig. 3 we can see that the secrecy rate of IRSs monotonically increases with the number of reflecting elements and the number of IRSs. This is because large numbers of reflecting elements and IRSs can lead to high signal gain and can suppress the eavesdropper, which results in high secrecy rate of the system. In addition, it is also found that the mmWave system with distributed IRSs increases faster with the number of IRSs than the mmWave system with one IRS, which shows that distributed IRSs is more efficient in improving secrecy rate of mmWave system.

V. CONCLUSION

The secrecy rate maximization problem for mmWave communications with distributed IRSs was investigated in this paper. The IRS phase shifts, BS transmit beamforming, and IRS on-off status were jointly optimized to maximize the secrecy rate under transmit power constraints and unit-modulus constraints. To solve this problem, we have proposed AO algorithms. In particular, the transmit beamforming problem was solved by using the SCA method and IRS on-off vector was solved by using the dual method. Finally, we used the MO algorithm to solve the phase shift optimization problem. Numerical results have shown that the proposed AO algorithm outperforms conventional schemes in terms of secrecy rate.

APPENDIX A

THE PROOF OF THEOREM 1

The Lagrange function of (26) with relaxed constraints is expressed as

\[
L(x, z, \lambda_{lm}^1, \lambda_{lm}^2, \lambda_{lm}^3) = A \left(1 + \frac{1}{\sigma_1^2} \left( \sum_{l=1}^{L} C_l x_l + \sum_{l=2}^{L} \sum_{m=1}^{L-1} C_{lm} z_{lm} \right) \right) - \lambda \left( \frac{1}{\sigma_1^2} \left( \sum_{l=1}^{L} D_l x_l + \sum_{l=2}^{L} \sum_{m=1}^{L-1} D_{lm} z_{lm} \right) \right) + \sum_{l=2}^{L} \sum_{m=1}^{L-1} \left( \lambda_{lm}^1 (z_{lm} - x_l - x_m + 1) + \lambda_{lm}^2 (z_{lm} - x_l) + \lambda_{lm}^3 (z_{lm} - x_m) \right).
\]

(34)

To maximize the objective function in (26a), let \( \frac{\partial L(x, z, \lambda_{lm}^1, \lambda_{lm}^2, \lambda_{lm}^3)}{\partial x_l} = 0 \) and \( \frac{\partial L(x, z, \lambda_{lm}^1, \lambda_{lm}^2, \lambda_{lm}^3)}{\partial z_{lm}} = 0 \) for each \( l \) and \( m \) respectively, we have

\[
\frac{1}{\sigma_1^2} A C_l - \frac{1}{\sigma_2^2} \lambda D_l - \sum_{m=2}^{L} \left( \lambda_{lm}^1 + \lambda_{lm}^2 + \lambda_{lm}^3 \right) = 0.
\]

(35)

When \( 2 \leq l \leq N - 1 \), we have

\[
\frac{1}{\sigma_1^2} A C_l - \frac{1}{\sigma_2^2} \lambda D_l - \sum_{m=1}^{L-1} \left( \lambda_{lm}^1 + \lambda_{lm}^2 \right) - \sum_{m=l+1}^{L} \lambda_{ml}^3 = 0.
\]

(36)

When \( l = N \), we have

\[
\frac{1}{\sigma_1^2} A C_l - \frac{1}{\sigma_2^2} \lambda D_l - \sum_{m=1}^{L-1} \lambda_{lm}^1 = 0.
\]

(37)

The relationship between \( C_{lm} \) and \( D_{lm} \) is given by

\[
\frac{1}{\sigma_1^2} A C_{lm} - \frac{1}{\sigma_2^2} \lambda (\lambda_{lm}^1 + \lambda_{lm}^2 + \lambda_{lm}^3) = 0.
\]

(38)

To maximize the objective function in (21a), it is equivalent to maximize the following formulation

\[
1 + \frac{1}{\sigma_1^2} \left( \sum_{l=1}^{L} C_l x_l + \sum_{l=2}^{L} \sum_{m=1}^{L-1} C_{lm} z_{lm} \right) + \frac{1}{\sigma_2^2} \left( \sum_{l=1}^{L} D_l x_l + \sum_{l=2}^{L} \sum_{m=1}^{L-1} D_{lm} z_{lm} \right)
\]

(39)

If \( C_l \leq D_l \) and \( x_l = 1 \), we have

\[
C_l > D_l.
\]

(40)

According to (35), (38) and (41), we have following inequalities in (29) and (29), which set as \( x_l = 0 \), i.e., IRS is off. Therefore, when \( l \)th IRS is on, we hope the user gain which is generated by IRS is larger than the eavesdropper gain, i.e.,

\[
C_l > D_l.
\]

(41)

Similarly, when \( z_{lm} = 1 \), we have

\[
C_{lm} > D_{lm}.
\]

(42)

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