A Gamma-Ray Bursts’ Fluence-Duration Correlation

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Summary. We present an analysis indicating that there is a correlation between the fluences and the durations of gamma-ray bursts, and provide arguments that this reflects a correlation between the total emitted energies and the intrinsic durations. For the short (long) bursts the total emitted energies are roughly proportional to the first (second) power of the intrinsic duration. This difference in the energy-duration relationship is statistically significant, and may provide an interesting constraint on models aiming to explain the short and long gamma-ray bursts.

1 Introduction

The gamma-ray bursts (GRBs) measured with the BATSE instrument on the Compton Gamma-Ray Observatory are usually characterized by 9 observational quantities (2 durations, 4 fluences, 3 peak fluxes) [12], [17], [13]. In a previous paper [1] we have shown that these 9 quantities can be reduced to only two significant independent variables (principal components). Here we present a new statistical analysis of the correlation between these variables and show that there is a significant difference between the power law exponents of long and short bursts. The details of this analysis will be presented elsewhere [2].

2 Distributions of Durations and Total Emitted Energies

We consider here those GRBs from the current BATSE Gamma-Ray Burst Catalog [13] which have measured $T_{90}$ durations and fluences ($F_1, F_2, F_3, F_4$). Therefore, we are left with $N = 1929$ GRBs, all of which have defined $T_{90}$ and $F_{tot}(= F_1 + F_2 + F_3 + F_4)$, as well as peak fluxes $P_{256}$.

The distribution of the log$T_{90}$ clearly displays two peaks reflecting the existence of two groups of GRBs [8]. This bimodal distribution can be fitted by
two log-normal distributions [6]. The fact that the distribution of \( T_{90} \) within a subclass is log-normal has important consequences. Let us denote the observed duration of a GRB with \( T_{90} \) (which may be subject to cosmological time dilatation) and with \( t_{90} \) those measured by a comoving observer (intrinsic duration). Then one has \( T_{90} = t_{90}f(z) \) where \( z \) is the redshift, and \( f(z) \) measures the time dilatation. For the concrete form of \( f(z) \) one can take \( f(z) = (1 + z)^k \), where \( k = 1 \) or \( k = 0.6 \), depending on whether energy stretching is included or not.

Taking the logarithms of both sides of this equality one obtains the logarithmic duration as a sum of two independent stochastic variables. According to a theorem of Cramér [3], if a variable \( \zeta \), which has a Gaussian distribution, is given by a sum of two independent variables, i.e. \( \zeta = \xi + \eta \), then both \( \xi \) and \( \eta \) have Gaussian distributions. Therefore, from this theorem it follows that the Gaussian distributions of \( \log T_{90} \), confirmed for the two subclasses separately [6], implies the same type of distribution for the variables of \( \log t_{90} \) and of \( \log f(z) \). However, unless the space-time geometry has a very particular structure, the distribution of \( \log f(z) \) cannot be Gaussian. This means that the Gaussian nature of the distribution of \( \log T_{90} \) must be dominated by the distribution of \( \log t_{90} \), and the latter must then necessarily have a Gaussian distribution. This holds for both duration subgroups separately. (Note here that several other authors, e.g. [20], [15], [16], have already suggested, that the distribution of \( T_{90} \) reflects predominantly the distribution of \( t_{90} \).)

One also has \( F_{\text{tot}} = (1 + z)E_{\text{tot}}/(4\pi d_l^2(z)) = c(z)E_{\text{tot}} \), where \( d_l \) is the luminosity distance, and \( E_{\text{tot}} \) is the total emitted energy. Once there is a log-normal distribution for \( F_{\text{tot}} \) (for the two subgroups separately), then the previous application of Cramér theorem is also possible here. The existence of this log-normal distribution is not obvious, but may be shown as follows.

Assume both the short and the long groups have distributions of the variables \( T_{90} \) and \( F_{\text{tot}} \) which are log-normal. In this case, it is possible to fit simultaneously the values of \( \log F_{\text{tot}} \) and \( \log T_{90} \) by a single two-dimensional ("bivariate") normal distribution. This distribution has five parameters (two means, two dispersions, and the angle \( \alpha \) between the axis \( \log T_{90} \) and the semi-major axis of the "dispersion ellipse"). Its standard form can be seen in [19] (Chapt. 1.25). When the \( r \)-correlation coefficient differs from zero, then the semi-major axis of the dispersion ellipse represents a linear relationship between \( \log T_{90} \) and \( \log F_{\text{tot}} \) with a slope of \( m = \tan \alpha \). This linear relationship between the logarithmic variables implies a power law relation of form \( F_{\text{tot}} = (T_{90})^m \) between the fluence and the duration, where \( m \) may be different for the two subgroups. Then a similar relation will exist between \( t_{90} \) and \( E_{\text{tot}} \).

We obtain the best fit through a maximum likelihood estimation (e.g., [7], Vol.2., p.57-58). From this estimation we obtain the dependence of the total emitted on the intrinsic duration in form

\[
E_{\text{tot}} \propto \begin{cases} 
(t_{90})^{1.1} & \text{(short bursts)}; \\
(t_{90})^{2.3} & \text{(long bursts)}.
\end{cases}
\] (1)

Several papers discuss the biases in the BATSE values of \( F_{\text{tot}} \) and \( T_{90} \) (cf. [4], [9], [10], [18], [11], [17], [5], [14]). All types of biases are particularly essential.
for faint GRBs. To discuss these effects we provide several different additional calculations (for more details see [2]), which give the same results.

3 Conclusion

The exponent in the power laws differ significantly for the two subclasses of short \( (T_{90} < 2 \text{ s}) \) and long \( (T_{90} > 2 \text{ s}) \) bursts. These new results may indicate that two different types of central engines are at work, or perhaps two different types of progenitor systems are involved. While the nature of the progenitors remains so far indeterminate, our results indicate strongly that the nature of the energy release process giving rise to the bursts is different between the two burst classes. In the short ones the total energy released is proportional to the duration, while in the long ones it is proportional roughly to the square of the duration. This result is completely model-independent, and provides an interesting constraint on the two types of bursts.

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