Measurements of Branching Fractions for $B \to K\pi$ and $B \to \pi\pi$ Decays with 449 million $B\bar{B}$ Pairs

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Tests of the Standard Model (SM) can be performed in $B$-meson decays to $K\pi$ and $\pi\pi$ final states, which involve various interplays between dominant $b \to u$ tree diagram, $b \to s$, $d$ penguin diagrams and other sub-dominant contributions. In general, direct comparisons of the measured branching fractions with the SM predictions suffer from large hadronic uncertainties within the current theoretical framework. However, many of the uncertainties cancel out in ratios of branching fractions. Previous experimental results for the ratios $R_c \equiv 2\Gamma(B^+ \to K^+\pi^0)/\Gamma(B^+ \to K^0\pi^+) = 1.00 \pm 0.08$ and $R_n \equiv \Gamma(B^0 \to K^+\pi^-)/2\Gamma(B^0 \to K^0\pi^0) = 0.82 \pm 0.08$ deviate from the SM expectations within several approaches. For example, Ref. predicts the values $R_c = 1.15 \pm 0.05$ and $R_n = 1.12 \pm 0.05$, which are calculated assuming $SU(3)$ flavor symmetry. If the differences between these SM expectations and the measured values of $R_c$ and $R_n$ persist with more data, this would imply a large electroweak penguin contribution in $B \to K\pi$ decays.

In this letter, we report new measurements of the branching fractions for $B \to K^+\pi^-, K^+\pi^0, K^0\pi^+, \pi^+\pi^-$ and $\pi^0\pi^0$ decays with a data sample five times larger than that used in our previous study. Recent Belle results for $B \to K\bar{K}, B^+ \to K^0\pi^+$ and $B^0 \to \pi^0\pi^0$ decays have been reported elsewhere. The results are based on a sample of $(449.3 \pm 5.7) \times 10^6$ $B\bar{B}$ pairs collected with the Belle detector at the KEKB $e^+e^-$ asymmetric-energy (3.5 on 8 GeV) collider. The production rates of $B^+B^-$ and $B^0\bar{B}^0$ pairs are assumed to be equal. The inclusion of the charge-conjugate decay is implied, unless explicitly stated otherwise.

The Belle detector is a large-solid-angle magnetic spectrometer that consists of a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters, a barrel-like array of a large CsI(Tl) electromagnetic calorimeter, and an iron flux-return yoke comprised of a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return located inside the coil is instrumented to detect $K_L^0$ mesons and to identify muons. The detector is described in detail elsewhere. Two different inner detector configurations were used. For the first sample of 152 million $B\bar{B}$ pairs (set I), a 2.0 cm radius beam pipe and a three-layer silicon vertex detector were used; for the latter 297 million $B\bar{B}$ pairs (set II), a 1.5 cm radius beam pipe, a four-layer silicon detector and a small-cell inner drift chamber were used.

Primary charged tracks are required to have a distance of closest approach to the interaction point (IP) of less than 4 cm in the beam direction ($z$-axis) and less than 0.1 cm in the transverse plane. Charged kaons and pions are identified using $dE/dx$ information from the CDC and Cherenkov light yields in the ACC, which are combined to form a $K-\pi$ likelihood ratio $R(K/\pi) = \mathcal{L}_K/(\mathcal{L}_K+\mathcal{L}_\pi)$.
where $L_K$ ($L_z$) is the likelihood that the track is a kaon (pion). Charged tracks with $R(K/\pi) > 0.6$ ($<0.4$) are classified as kaons (pions). Typically, the kaon (pion) identification efficiency is 83% (90%), and 6% (12%) of selected kaons (pions) are misidentified as pions (kaons). Furthermore, we reject charged tracks that are consistent with an electron hypothesis. Candidate $K^0$ mesons are reconstructed as $K^0_s \rightarrow \pi^+ \pi^-$ decays with the branching fraction taken from Ref. [14]. We pair oppositely-charged tracks assuming the pion hypothesis and require the invariant mass of the pair to be within $\pm 18$ MeV/c$^2$ of the nominal $K^0_s$ mass. The intersection point of the $\pi^+ \pi^-$ pair must be displaced from the IP [13]. Pairs of photons with invariant masses in the range of 115 MeV/c$^2$ < $M_{\gamma\gamma}$ < 152 MeV/c$^2$ ($\pm 3\sigma$) are considered as $\pi^0$ candidates. The photon energy is required to be greater than 50 MeV in the barrel region, defined as $32^\circ < \theta_\gamma < 128^\circ$, and greater than 100 MeV in the end-cap regions, defined as $17^\circ < \theta_\gamma < 32^\circ$ or $128^\circ < \theta_\gamma < 150^\circ$, where $\theta_\gamma$ denotes the photon polar angle with respect to the direction anti-parallel to the $e^+$ beam.

Candidate $B$ mesons are identified by the “beam-energy-constrained” mass, $M_{bc} = \sqrt{E_{\text{beam}}^2/c^4 - p_B^2/c^2}$, and the energy difference, $\Delta E = E_B - E_{\text{beam}}$, where $E_{\text{beam}}$ is the run-dependent beam energy, and $E_B$ and $p_B$ are the reconstructed energy and momentum of the $B$ candidates in the center-of-mass (CM) frame, respectively. Events with $M_{bc} > 5.20$ GeV/c$^2$ and $|\Delta E| < 0.3$ GeV are selected for the analysis.

The dominant background is from $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) continuum events. We use event topology to distinguish the $B\overline{B}$ events from the jet-like continuum background. We combine a set of modified Fox-Wolfram moments [16] into a Fisher discriminant. A signal/background likelihood is formed, based on a GEANT-based [17] Monte Carlo (MC) simulation, from the product of the probability density functions (PDFs) for the Fisher discriminant and that for the cosine of the polar angle of the $B$-meson flight direction. Suppression of the continuum is achieved by applying a requirement on the ratio $R = L_{\text{sig}}/(L_{\text{sig}} + L_{\text{qg}})$, where $L_{\text{sig}}$ ($L_{\text{qg}}$) is the signal (continuum) likelihood. Continuum background is further suppressed through use of the $B$-flavor tagging algorithm [18], which provides a discrete variable indicating the flavor of the tagging $B$ meson and a continuous quality parameter $r$ ranging from 0 (for no flavor-tagging information) to 1 (for unambiguous flavor assignment). Events with a high value of $r$ are considered well-tagged and hence are unlikely to have originated from continuum processes. We classify events separately as poorly-tagged ($r \leq 0.5$) and well-tagged ($r > 0.5$) in data set I and data set II and for each category we determine a continuum suppression requirement for $R$ that maximizes the value of $N_{\text{sig}}^{\text{exp}}/\sqrt{N_{\text{sig}}^{\text{exp}} + N_{\text{qg}}^{\text{exp}}}$. Here, $N_{\text{sig}}^{\text{exp}}$ denotes the expected signal yields based on MC simulation and the average branching fractions of the previous measurements [11, 12, 13], and $N_{\text{qg}}^{\text{exp}}$ denotes the expected continuum yields as estimated from sideband data ($M_{bc} < 5.26$ GeV/c$^2$ and $|\Delta E| < 0.3$ GeV).

Background contributions from $\Upsilon(4S) \rightarrow B\overline{B}$ events are investigated using a large MC sample that includes events from $b \rightarrow c$ transitions and charmless $B$ decays. After all the selection requirements, no $b \rightarrow c$ background is found, while a small contribution from charmless $B$ decays is present at low $\Delta E$ values for all studied modes. Due to $K - \pi$ misidentification, large $B^0 \rightarrow K^+\pi^-$ and $B^+ \rightarrow K^+\pi^0$ feed-down backgrounds appear in the $B^0 \rightarrow \pi^+\pi^-$ and $B^+ \rightarrow \pi^+\pi^0$ modes, respectively.

The signal yields are extracted by performing extended unbinned maximum likelihood fits to the $(M_{bc}, \Delta E)$ distributions of the selected candidate events. The likelihood function for each mode is defined as

$$L = \frac{\exp \left( -\sum_{i,k,j} N_{i,k,j} \right)}{N!} \prod_{i,k,j} \left( \sum_{l,k,j} N_{l,k,j} P_{l,k,j} \right),$$

where $N$ is the total number of events, $i$ is the event identifier, $l$ indicates set I or set II, $k$ distinguishes the two $r$ regions and $j$ runs over all components included in the fitting function: signal, continuum background, feed-down, and charmless $B$ background. The variable $N_{i,k,j}$ denotes the number of events, and $P_{i,k,j} = P_{i,k,j}(M_{bc}^{\text{sig}}, \Delta E^{\text{r}})$ are two-dimensional PDFs, which are the same in the two $r$ regions for all fit components except for the continuum background.

All the signal PDFs ($P_{i,k,j}^{\text{signal}}(M_{bc}, \Delta E)$) are parameterized by smoothed two-dimensional histograms obtained from correctly reconstructed signal MC based on the set I and set II detector configurations. Signal MC events are generated with the PHOTOS [19] simulation package to take into account final-state radiation. Since the $M_{bc}$ signal distribution is dominated by the beam-energy spread, we use the signal-peak positions and resolutions obtained from $B^+ \rightarrow D^0 \pi^+$ data to refine our signal MC (the $D^0 \rightarrow K^+\pi^-\pi^-$ sub-decay is used for modes with a $\pi^0$ in the final state, while $D^0 \rightarrow K^+\pi^-$ is used for the other modes). The resolution for the $\Delta E$ distribution is calibrated using the invariant mass distribution of high momentum ($p_{\text{lab}} > 3$ GeV/c) $D$ mesons. The size of the final-state radiation effects can be assessed if we take signal PDFs from MC without PHOTOS and use these PDFs to extract the signal yields from the signal MC with PHOTOS. The extracted yields decrease by 5.8% for $B^0 \rightarrow K^+\pi^-$, 9.4% for $B^0 \rightarrow \pi^+\pi^-$ and 3.6% for $B^+ \rightarrow K^+\pi^0$ and $B^+ \rightarrow \pi^+\pi^0$, respectively.

The continuum background PDF is described by a product of a linear function for $\Delta E$ and an ARGUS function, $f(x) = x \times (1 - x^2) \times \exp \left[ -\xi \times (1 - x^2) \right]$, where $x = M_{bc}c^2/E_{\text{beam}}$ [20]. The overall normalization, $\Delta E$ slope and ARGUS parameter $\xi$ are free parameters in the fit. The background PDFs for charmless $B$ decays are mod-
eled by a smoothed two-dimensional histogram, obtained from a large MC sample. We also use a smoothed two-dimensional histogram to describe the feed-background, since the background events have \((M_{bc}, \Delta E)\) shapes similar to the signal, except for a \(\Delta E\) peak position shift of \(\simeq 45\) MeV. We perform a simultaneous fit for \(B^0 \to K^+\pi^-\) and \(B^0 \to \pi^+\pi^-\), since these two decay modes feed across each other. The feed-across fractions are constrained according to the identification efficiencies and fake rates of kaons and pions. A simultaneous fit is also used for the \(B^+ \to K^+\pi^0\) and \(B^+ \to \pi^+\pi^0\) decay modes.

When likelihood fits are performed, the yields are allowed to float independently for each \(l\) (set I or set II) and \(k\) bin (low or high \(r\) region). The \(M_{bc}\) and \(\Delta E\) projections of the fits are shown in Fig. 1, while Table II summarizes the fit results for each mode. The branching fraction of each mode is calculated by dividing the total signal yield by the number of \(B\bar{B}\) pairs and by the average reconstruction efficiency. The calculation of this average efficiency takes into account the differences between various \(l\) and \(k\) bins, and sub-decay branching fractions.

The fitting systematic errors are due to signal PDF modeling, charmless \(B\) background modeling, and feed-across constraints. The first and last of these errors are estimated from the fit deviations after varying each parameter of the signal PDFs or the yields of the feed-across backgrounds by one standard deviation. The effects due to fake-rate uncertainties are also included in the systematic error of the feed-across backgrounds. The systematic error due to the charmless \(B\) background modeling is evaluated by requiring that \(\Delta E > -0.12\) GeV, since the \(\Delta E\) values of the charmless \(B\) events are typically smaller than \(-0.12\) GeV. The above deviations in the signal yield are added in quadrature to obtain the overall systematic error due to fitting.

The MC-data efficiency difference due to the requirement on the likelihood ratio \(R\) is investigated with \(B^+ \to D^0\pi^+\) samples. The systematic error due to the charged-track reconstruction efficiency is estimated to be 1% per track using partially reconstructed \(D^*\) events. The systematic error due to the \(R(K/\pi)\) selection is 1.3% for pions and 1.5% for kaons, respectively. The \(K^0_S\) reconstruction and the systematic error is verified by comparing the ratio of \(D^+ \to K^0_S\pi^+\) and \(D^+ \to K^-\pi^+\pi^+\) yields with the MC expectations. The \(\pi^0\) reconstruction efficiency and the systematic error is verified by comparing the ratio of \(D^0 \to K^+\pi^-\) and \(D^0 \to K^+\pi^-\pi^0\) yields with the MC expectations. Possible systematic uncertainties due to the description of final-state radiation have been studied by comparing the latest theoretical calculations with the PHOTOS MC [21]. These uncertainties were found to be negligible and thus no systematic error is assigned due to PHOTOS. The systematic error due to the uncertainty of the total number of \(B\bar{B}\) pairs is 1.3% and the error due to signal MC statistics is between 0.4% and 0.7%. The final systematic uncertainty is obtained by quadratically summing all the contributions, as shown in Table III.

The ratios of partial widths can be used to extract the angle \(\phi_3\) and to search for new physics [3, 4, 7]. These ratios (listed in Table III) are obtained from the five measurements in Table I and the new measurement of \(B(B^+ \to K^0\pi^+) = (22.8^{+0.8}_{-0.7} \pm 1.3) \times 10^{-6}\) described in Ref. [3]. The ratio of charged to neutral \(B\) meson lifetime, \(\tau_{B^+}/\tau_{B^0} = 1.076 \pm 0.008\) [4], is used to convert the branching-fraction ratios into the ratios of partial widths. The total errors are reduced because of the cancellation.
TABLE I: Extracted signal yields, product of efficiencies and sub-decay branching ratios ($B_i$), and calculated branching fractions for individual modes. The branching fraction errors are statistical and systematic, respectively.

| Mode       | Yield | Eff.$\times B_i$ | $B(10^{-6})$ |
|------------|-------|-----------------|--------------|
| $K^+\pi^-$ | 3585$^{+69}_{-68}$ | 40.16 | 19.9 $\pm$ 0.4 $\pm$ 0.8 |
| $\pi^+\pi^-$ | 872$^{+41}_{-40}$ | 37.98 | 5.1 $\pm$ 0.2 $\pm$ 0.2 |
| $K^+\pi^0$ | 1493$^{+57}_{-55}$ | 26.86 | 12.4 $\pm$ 0.5 $\pm$ 0.6 |
| $\pi^+\pi^0$ | 693$^{+46}_{-55}$ | 23.63 | 6.5 $\pm$ 0.4 $\pm$ 0.4 |
| $K^0\pi^0$ | 379$^{+28}_{-27}$ | 9.17 | 9.2 $\pm$ 0.7 $\pm$ 0.7 |

TABLE II: Summary of systematic errors, given in percent.

| Source | $K^+\pi^-\pi^-\pi^-\pi^+\pi^+\pi^0\pi^0$ |
|--------|------------------------------------------|
| Signal PDF | $\pm0.2$ $\pm0.3$ $\pm0.4$ $\pm0.5$ $\pm0.4$ |
| Charmless $B$ background | $\pm0.0$ $\pm0.6$ $\pm0.0$ $\pm0.0$ $\pm0.0$ |
| Feed-across background | $\pm0.2$ $\pm0.0$ $\pm0.9$ $\pm2.0$ $\pm4.0$ |
| $\mathcal{R}$ requirement | $\pm1.0$ $\pm1.0$ $\pm1.3$ $\pm1.4$ $\pm1.5$ |
| Tracking | $\pm2.0$ $\pm2.0$ $\pm1.0$ $\pm1.0$ $\pm0.0$ |
| $\mathcal{R}(K/\pi)$ requirement | $\pm2.9$ $\pm2.8$ $\pm1.5$ $\pm1.3$ $\pm0.0$ |
| $K_S^0$ reconstruction | $0.0$ $0.0$ $0.0$ $0.0$ $\pm4.9$ |
| $\pi^0$ reconstruction | $0.0$ $0.0$ $\pm4.0$ $\pm4.0$ $\pm4.0$ |
| $\#$ of $B\bar{B}$ | $\pm1.3$ $\pm1.4$ $\pm1.3$ $\pm1.3$ $\pm1.3$ |
| Signal MC statistics | $\pm0.6$ $\pm0.4$ $\pm0.4$ $\pm0.5$ $\pm0.7$ |
| Total | $\pm4.0$ $\pm4.5$ $\pm3.8$ $\pm3.4$ $\pm4.7$ |

TABLE III: Partial width ratios of $B \to K\pi$ and $\pi\pi$ decays. The errors are quoted in the same manner as in Table II.

| Modes | Ratio |
|-------|-------|
| $2\Gamma(K^+\pi^0)/\Gamma(K^0\pi^+)$ | 1.08 $\pm$ 0.06 $\pm$ 0.08 |
| $\Gamma(K^+\pi^-)/\Gamma(K^0\pi^0)$ | 1.08 $\pm$ 0.08 $\pm0.09$ |
| $\Gamma(K^+\pi^-)/\Gamma(K^0\pi^+)$ | 0.94 $\pm$ 0.04 $\pm0.05$ |
| $\Gamma(\pi^+\pi^-)/\Gamma(K^0\pi^-)$ | 0.26 $\pm$ 0.01 $\pm$ 0.01 |
| $\Gamma(\pi^+\pi^-)/2\Gamma(\pi^+\pi^-)$ | 0.42 $\pm$ 0.03 $\pm0.03$ |
| $\Gamma(\pi^+\pi^-)/\Gamma(K^0\pi^0)$ | 0.66 $\pm$ 0.07 $\pm$ 0.05 |
| $2\Gamma(\pi^+\pi^-)/\Gamma(K^+\pi^-)$ | 0.57 $\pm$ 0.04 $\pm0.05$ |

In conclusion, we have measured the branching fractions for $B \to K\pi$ and $B \to \pi\pi$ decays with 449 million $B\bar{B}$ pairs collected at the $\Upsilon(4S)$ resonance with the Belle detector. We confirm the expected hierarchy of branching fractions: $B(K^0\pi^+)$ $\geq$ $B(K^+\pi^-)$ $>$ $B(K^+\pi^0)$ $\geq$ $B(K^0\pi^0)$ $>$ $B(\pi^+\pi^-)$ $\geq$ $B(\pi^-\pi^0)$ and find no significant deviation from SM expectations in the ratios of partial widths. We also find that the ratios $R_n$ and $R_c$ are both in good agreement with SM expectations, in contrast to early measurements [1,2,3].

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We use PHOTOS version 2.13 allowing the emission of up to two photons, with an energy cut-off at 1% of the energy available for photon emission (i.e. approximately 26 MeV for the first emitted photon). PHOTOS also takes into account interference between charged final-state particles.

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