Phenomenology of High Energy Neutrinos in Low-Scale Quantum Gravity Models

J. Alvarez-Muñiz¹, F. Halzen², T. Han², D. Hooper²

¹Bartol Research Institute, University of Delaware, Newark, DE 19716
²Department of Physics, University of Wisconsin, 1150 University Avenue, Madison, WI 53706

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We show that neutrino telescopes, optimized for detecting neutrinos of TeV to PeV energy, can reveal threshold effects associated with TeV-scale gravity. The signature is an increase with energy of the cross section beyond what is predicted by the Standard Model. The advantage of the method is that the neutrino cross section is measured in an energy region where i) the models are characteristically distinguishable and ii) the Standard Model neutrino cross section can be reliably calculated so that any deviation can be conclusively identified.

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Motivated by the absence of a self-consistent theory of quantum gravity and the unresolved hierarchy problem between the electroweak scale (10² GeV) and the Planck scale (10¹⁹ GeV), a great deal of attention has been given to theories of low-scale quantum gravity which envision significant quantum gravity effects at an energy scale of the order of \( M_s \sim 1 \) TeV \[\mathrm{[1]}\]. In these scenarios, potentially large effects on high energy processes may occur due to the contributions from, \( \text{e.g.} \), Kaluza-Klein excitations of gravitons (KK) or other stringy states near \( M_s \). An interesting motivation for models in which cross sections at TeV scale become enhanced is the ultra-high energy cosmic ray problem. Protons above the GZK cutoff (\( \sim 10^{19} \) eV) interact with the cosmic microwave background cataclysmically by the \( \Delta \)-resonance \[\mathrm{[2]}, \mathrm{[3]}\]. Thus, the cosmic ray events observed above this energy must be produced by local sources, or involve new physics. Local sources of particles of such energy being unlikely, many exotic solutions have been proposed \[\mathrm{[4]}\]. A solution which has received a great deal of attention in recent literature proposes that neutrinos with enhanced cross sections at GZK energies constitute the highest energy cosmic rays \[\mathrm{[5]}\]. This solution requires neutrino-nucleon cross sections on the scale of 10's of mbarnes. Unfortunately, most scenarios of low-scale quantum gravity as low-energy effective theories are valid only up to the order of \( \lesssim M_s \). Above this scale, the naïve calculations typically violate unitarity \[\mathrm{[6]}\]. One has to introduce some ad hoc unitarization scheme, since the fundamental theory, such as a realistic string theory, is yet unavailable. It is also very difficult to reliably predict the parton distribution functions needed at GZK energies in neutrino-nucleon interactions. For these reasons, studies of ultra-high energy (\( \sim 10^{20} \) eV) quantum gravity enhancements to neutrino-nucleon interactions are extremely speculative.

These problems are far more manageable at energies below or near \( M_s \). Unitarity may not be violated at this scale, calculations are generally perturbative and the relevant parton distributions are known at these energies \[\mathrm{[7]}\]. The characteristics for different theoretical models can be also qualitatively distinguishable near and slightly above the threshold. Therefore, the TeV regime provides a natural scale for probing the features of low-scale quantum gravity models. These tests include direct searches in colliders such as the Fermilab Tevatron and the CERN Large Hadron Collider (LHC) \[\mathrm{[8]}, \mathrm{[9]}\]. This letter discusses another class of experiments capable of testing features of low-scale quantum gravity: multi-TeV to PeV neutrino astrophysics.

We have recently witnessed first light of neutrino telescopes optimized to detect neutrinos in the TeV to PeV energy range \[\mathrm{[10]}\]. This is the range of laboratory energies where the onset of TeV-scale gravity effects on the neutrino cross section will first manifest itself. For a neutrino flux \( \Phi \), the number of neutral currents events \( N_\nu \) observed as hadronic showers in a neutrino detector of effective area \( A \) is given by the convolution over energy of the quantity \( A^{3/2} \times \Phi \times n \times \sigma_\nu \). Here \( n \) is the density of the target that interacts with a neutrino with cross section \( \sigma_\nu \) to produce a hadronic shower. In our discussion the detected neutrino flux plays a secondary role. It may represent the atmospheric neutrino flux or the flux of neutrinos of hundreds of TeV anticipated from gamma ray bursts.

An increase with energy of the cross section for neutrinos to interact with matter beyond a level calculated in the Standard Model will signal the onset of new physics including the increase anticipated as a consequence of TeV-scale gravity effects. It is important to recognize that the Standard Model cross section is computed from nucleon structure functions probed by HERA experiments in this energy range. The Standard Model baseline against which to measure new physics is known. The important observation here is that due to the geometry of the Earth and detector, the angular distributions of events with new physics can be significantly different from the Standard Model prediction. These measurements, though challenging for existing instruments like AMANDA and the detectors in the Mediterranean, should be feasible for second-generation detectors such as IceCube. Unlike first generation neutrino telescopes, IceCube can separate interesting high energy events from...
the large background of lower energy atmospheric neutrinos by energy measurement. The instrument can identify high energy neutrinos over 4π solid angle, and not just in the lower hemisphere where they are identified by their penetration of the Earth, as is the case with AMANDA. Furthermore, due to the enhanced neutrino cross section from new physics, the Earth eventually becomes opaque to those high energy neutrinos. This leads to the striking phenomena that only down-going events from new physics at high energies are enhanced.

For the sake of illustration, we will examine three classes of low-scale quantum gravity/stringy models:

1) ADD scenario [1]: Large extra dimensions (R < 0.1 mm) with a flat Minkowski metric. The large effects are due to the high degeneracy of the light KK gravitons of mass \( m_{KK} \sim 1/R \). The theory saturates perturbative unitarity at \( \sqrt{s} \gtrsim M_s \) and some unitarization scheme has to be introduced.

2) RS scenario [2]: One anti-de Sitter dimension with a non-factorizable “warped” geometry. The physical consequences relevant to our interests are the contributions from the KK gravitons of a TeV mass [1].

3) Veneziano amplitude: Due to the lack of a fundamental theory of quantum gravity, a reasonable parameterization to the new physics at the scale \( M_s \) perhaps is to include a sum of possible “stringy” states [2]. The Veneziano amplitude serves for this purpose in describing the stringy physics, before non-perturbative quantum gravity effects become overall dominant. It also manifestly preserves unitarity [3].

There are several free parameters in these models. The choices for these parameters were selected to illustrate a variety of phenomenological features and are not inclusive. First of all, we take the quantum gravity or string scale \( M_s \) as 1 TeV throughout our studies. In addition, the ADD scenario is subject to the number of extra dimensions, although the effect under consideration is not sensitive to it [10]. The RS model varies with the scale of the theory (\( \Lambda = e^{-kR} M_{pl} \)) and the mass of graviton resonances (\( m_{\sigma} \)). In our analysis, we will only keep the first graviton resonance and ignore the heavier states. Models calculated with Veneziano amplitudes are characterized by two constants \( a \) and \( b \), which parameterize the Chan-Paton traces for string models [8], in addition to \( M_s \).

Perturbative calculations for the ADD scenario lead to a parton-level cross section \( \sigma(\nu g \rightarrow \nu g) \sim s^3/M_s^8 \), which violates partial wave unitarity \( |Re(a_\ell)| < 1/2 \) at energies above \( 1.5 M_s \). To have a sensible estimate, partial wave amplitudes are taken to saturate unitarity when the unitarity bound is reached. Calculations are then made with only \( S \)-wave, with 5 leading partial waves, as well as with all partial waves that need to be unitarized. This variety of choices reflects our ignorance of how nature chooses to restore unitarity at a higher energy scale. However, we consider the \( S \)-wave unitarization a conservative estimation for the ADD scenario. The RS model is less likely to violate unitary as the rapid growth of amplitudes only occurs way above graviton mass scale. The Veneziano amplitudes automatically respect unitarity bounds.

![Figure 1](image.png)

**FIG. 1.** Neutrino-nucleon cross sections in a variety of models compared to the Standard Model neutral current prediction. ADD (large extra dimension) models are for all, 5 and 1 partial waves, up to unitarity saturation (top to bottom). RS models are shown for \( \Lambda = 1 \) TeV, \( m_{\sigma} = 500 \) GeV; \( \Lambda = 1 \) TeV, \( m_{\sigma} = 1 \) TeV; \( \Lambda = 3 \) TeV, \( m_{\sigma} = 500 \) GeV (top to bottom). Models using Veneziano amplitudes are for \( a = b = 5 \) and \( a = b = 0 \) (top to bottom). \( M_s = 1 \) TeV for all models.

Figure 1 depicts the sum of the neutral current neutrino nucleon cross section within the Standard Model and the corresponding cross section due to new physics for the different models discussed above. For the reasons explained earlier, we wish to explore new physics not too far from the threshold, and thus do not address the energy range above \( E_\nu = 5 \times 10^3 \) TeV. The increase in the cross section beyond the Standard Model due to TeV-scale quantum gravity starts at a threshold energy \( E_\nu ^t \sim 10^3 \) TeV, corresponding to a neutrino-nucleon center-of-mass energy near 1 TeV. Indeed, the three models under consideration have distinctive characteristics near the threshold, as clearly seen in Fig. 1. For the large extra dimension scenario (ADD), the three (solid) curves correspond to those of unitarized amplitudes (\( S \)-wave only, 5 partial waves, and all partial waves that saturate unitarity). Up to an energy \( E_\nu = 5 \times 10^3 \) TeV, the three solid curves are identical, indicating the \( S \)-wave dominance. At higher energies, there will be more partial waves to reach the unitarity bound. We consider the \( S \)-wave unitarization scheme a conservative representation for this model. We see that above \( E_\nu ^t \) the effects of the new physics in ADD should be clearly visible for which the increase in the cross section beyond the Stan-
ard Model expectations could be orders of magnitude at $E_{\nu} = 5 \times 10^4 \text{ TeV}$. For the RS scenario, the influence with a single KK graviton exchange should be also visible if the graviton is not too heavy. The result with $\Lambda = 1 \text{ TeV}$ and a graviton mass $m_g = 500 \text{ GeV}$ could lead to a maximum increase in the cross section of a factor of 30 at $E_{\nu} = 5 \times 10^4 \text{ TeV}$. On the other hand, the effects would be unobservable if we take $m_g = 1 \text{ TeV}$ and $\Lambda = 3 \text{ TeV}$. The stringy model of Veneziano amplitude [8] predicts an increase about a factor of 6 with $E_{\nu} = 5 \times 10^4 \text{ TeV}$, while the enhancement is negligibly small if we take $a = b = 0$.

The upper panels of Fig. 2 show the up-going neutrino events and the lower panels the down-going events. Figures 2 and 3 illustrate the qualitative behavior explained above in the IceCube detector. We explore three different theoretical predictions of the neutrino flux. These have been chosen mostly for illustrative purposes. The panel labeled charm refers to maximal predictions of neutrinos from the decay of charmed particles produced by cosmic ray interactions in the atmosphere [3]. The W-B refers to the Waxman and Bahcall (W-B) limit on the neutrino flux from astrophysical sources that are optically thin to proton-photon and proton-proton interactions. This represents a result of these fluctuations [17]. Although these results can widely vary, those shown are typical.

The upper panels of Fig. 2 show the up-going neutrino induced shower events and the energy spectrum for down-going shower events $0 < \cos \theta_{\text{zenith}} < 1$ is shown in the lower panels. The event rate peaks at the neutrino energies at which the product $\Phi(E_{\nu}) \times \sigma_{\nu}(E_{\nu})$ maximizes. Figure 3 shows the zenith angle distribution of the neutrino induced shower events for the same three neutrino fluxes. Notice that the shower rate has a contribution from Standard Model charged current neutrino interactions which is present in all plots. The angular distribution of the up-going events is dominated by absorption in the Earth whereas for down-going events absorption is not important and the angular distribution reflects the detector’s effective volume to contained showers. Little sensitivity to new physics is anticipated when looking for up-going events, while increase in the event rate for down-going due to new physics may be visible if the neutrino fluxes are large enough at high energies. Within the large extra dimension model (ADD), the signal is observable even for the conservative scenario with $m_g \lesssim 500 \text{ GeV}$, and similar sensitivity is predicted for the Veneziano scenario if $a, b \sim 5$. The excess of events due to new physics in the charm case is very small ($< 1 \text{ per km}^2 \text{ and year}$) because the flux drops quickly in the energy range where the cross section increases, although there is large uncertainty for the atmospheric neutrino flux at this energy regime. The W-B flux gives a sizeable signal due to the larger neutrino flux rate near the threshold energies of new physics. The inte-

FIG. 2. Energy distribution of $\nu_{\mu} + \bar{\nu}_{\mu} + \nu_e + \bar{\nu}_e$ induced shower events in IceCube. The upper panels show the up-going neutrino events and the lower panels the down-going events. The solid line is the event rate for $\sigma_{\text{SM}} + \sigma_{\text{ADD}}(S\text{-wave})$; the dashed line for $\sigma_{\text{SM}} + \sigma_{\text{Veneziano}}$ with $a = b = 5$; the dashed line for $\sigma_{\text{SM}} + \sigma_{\text{RS}}$ with $m_g = 500 \text{ GeV}$ and $\Lambda = 1 \text{ TeV}$ and, the dotted for $\sigma_{\text{SM}}$ alone. $M_e = 1 \text{ TeV}$ for all models.

FIG. 3. $\cos \theta_{\text{zenith}}$ distribution of $\nu_{\mu} + \bar{\nu}_{\mu} + \nu_e + \bar{\nu}_e$ induced shower events in IceCube. $\cos \theta_{\text{zenith}} = -1$ corresponds to vertical up-going neutrinos, and $\cos \theta_{\text{zenith}} = 0$ to neutrinos coming from the horizon. The panels are labeled with the corresponding theoretically predicted neutrino flux that has been used to obtain the event rate (see text). The legend for different models is the same as in Fig. 2.
grated numbers of events are summarized in Table I for the different neutrino fluxes and new physics scenarios studied. The numbers in parenthesis are the event rates with muons. They include contributions from muons in charged current $\nu_\mu + \bar{\nu}_\mu$ interactions within the Standard Model and muons produced in hadronic showers induced by $\nu_\mu + \bar{\nu}_\mu + \nu_e + \bar{\nu}_e$. The IceCube detector is sensitive to down-going neutrino induced muons above 1 PeV since it can determine the energy of the events and hence separate them from the background produced by cosmic rays in the atmosphere, which is smaller than the neutrino induced muon rate for energies above $\sim 1 \sim 10$ PeV [19].

| Events/(km$^2$ yr) in 2$\pi$ sr | W-B limit | GRB |
|---------------------------------|-----------|-----|
|                                 | Up        | Down | Up    | Down |
| Standard Model                  | 85 (399)  | 103 (315) | 5 (8)  | 5 (20) |
| Large extra-D (all waves)       | 85 (399)  | 219 (433) | 5 (8)  | 54 (71) |
| Large extra-D (5 waves)         | 85 (399)  | 130 (325) | 5 (8)  | 12 (24) |
| Large extra-D (1 wave)          | 85 (399)  | 112 (310) | 5 (8)  | 9 (22) |
| R-S. $m_N = 500$ GeV, $A = 1$ TeV | 85 (399)  | 106 (318) | 5 (8)  | 7 (21) |
| R-S. $m_N = 1$ TeV, $A = 1$ TeV | 85 (399)  | 103 (316) | 5 (8)  | 6 (20) |
| R-S. $m_N = 500$ GeV, $A = 3$ TeV | 85 (399)  | 103 (315) | 5 (8)  | 5 (20) |
| Veneziano a=b=0                 | 85 (399)  | 103 (315) | 5 (8)  | 5 (20) |
| Veneziano a=b=5                 | 85 (399)  | 106 (316) | 5 (8)  | 7 (21) |

| FIG. 4. | $\nu_\mu + \bar{\nu}_\mu + \nu_e + \bar{\nu}_e$ shower event rate and $\mu$ event rate (in parenthesis) per km$^2$ yr in IceCube for different theoretically predicted $\nu$ fluxes and $\nu$-nucleon cross sections. $\mu$ events include contributions from muons in charged current $\nu_\mu + \bar{\nu}_\mu$ interactions within the Standard Model and muons produced in hadronic showers induced by $\nu_\mu + \bar{\nu}_\mu + \nu_e + \bar{\nu}_e$. The muon energy threshold is $E_{\mu} = 100$ GeV, the shower energy threshold is 1 TeV and the maximum neutrino energy is $5 \times 10^4$ TeV. |

In summary, TeV scale quantum gravity or stringy models have distinctive characteristics near the new physics threshold and may be probed in large high energy neutrino telescopes. The ADD scenario with TeV scale quantum gravity has distinctive phenomenological features for high energy neutrino scattering. We illustrated this point by a conservative unitarized $S$-wave approximation. The RS model with a single graviton exchange can also provide observable signatures if the graviton mass is lighter than about 1 TeV. TeV Stringy models using Veneziano amplitudes can produce interesting features in the case with large Chan-Paton trace factors. Studies of these features may allow for discovery of such models, or for stronger constraints on the scale of new physics. Neutrino astrophysics can provide a means to complement searches for TeV scale quantum gravity in collider experiments.

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[1] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429, 263 (1998); Phys. Rev. D59, 086004 (1999); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B436, 257 (1998).
[2] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999).
[3] K. Greisen, Phys. Rev. Lett. 16 (1966) 748.
[4] G. T. Zatsepin and V. A. Kuzmin, Pis’ma Zh. Eksp. Teor. Fiz. 4 (1966) 114 [JETP. Lett. 4 (1966) 78].
[5] T. J. Weiler, hep-ph/0103023; R. Gandhi, Nucl. Phys. Proc. Suppl. 91, 453 (2000); H. Davoudiasl, J. L. Hewett and T. G. Rizzo, hep-ph/0010065; D. S. Gorbunov, G. G. Raffelt and D. V. Semikoz, hep-ph/0103175.
[6] S. Nussinov and R. Shrock, Phys. Rev. D59, 105002 (1999); ibid. D64, 047702 (2001); G. Domokos and S. Kovesis-Domokos, Phys. Rev. Lett. 82, 1366 (1999); G. Domokos, S. Kovesis-Domokos and P. T. Mikulski, hep-ph/0006322.
[7] P. Jain, D. W. McKay, S. Panda and J. P. Ralston, Phys. Lett. B484, 267 (2000); A. Jain, P. Jain, D. W. McKay and J. P. Ralston, hep-ph/0011310; C. Tyler, A. Olinto, and G. Sigl, Phys. Rev. D63, 053001 (2001).
[8] R. Cornet, J. L. Illana and M. Masip, Phys. Rev. Lett. 86, 4235 (2001).