1 Introduction

Impressive experimental results have been reported at this conference on the $Z$ boson parameters, the $W$ mass, and the top quark mass determination with $m_t = 175 \pm 6$ GeV. Also a sizeable amount of theoretical work has contributed over the last few years to a steadily rising improvement of the standard model predictions (for a review see ref. [5]). The availability of both highly accurate measurements and theoretical predictions, at the level of nearly 0.1% precision, provides tests of the quantum structure of the standard model thereby probing its empirically yet untested sector, and simultaneously accesses alternative scenarios like the minimal supersymmetric extension of the standard model (MSSM).

2 Status of precision calculations

2.1 Radiative corrections in the standard model

The possibility of performing precision tests is based on the formulation of the standard model as a renormalizable quantum field theory preserving its predictive power beyond tree level calculations. With the experimental accuracy being sensitive to the loop induced quantum effects, also the Higgs sector of the standard model is probed. The higher order terms induce the sensitivity of electroweak observables to the top and Higgs mass $m_t, M_H$ and to the strong coupling constant $\alpha_s$.

Before one can make predictions from the theory, a set of independent parameters has to be taken from experiment. For practical calculations the physical input quantities $\alpha, G_\mu, M_Z, m_f, M_H$: $\alpha_s$ are commonly used for fixing the free parameters of the standard model. Differences between various schemes are formally of higher order than the one under consideration. The study of the scheme dependence of the perturbative results, after improvement by resumming the leading terms, allows us to estimate the missing higher order contributions.

Two sizeable effects in the electroweak loops deserve a special discussion:

- The light fermionic content of the subtracted photon vacuum polarization corresponds to a QED induced shift in the electromagnetic fine structure constant. The evaluation of the light quark content yields the result

  $$(\Delta \alpha)_{had} = 0.0280 \pm 0.0007. \quad (1)$$

- The electroweak mixing angle is related to the vector boson masses by

  $$\sin^2 \theta = 1 - \frac{M_W^2}{M_Z^2} \frac{M_Z^2}{M_W^2} \Delta \rho + \cdots \quad (3)$$

  where the main contribution to the $\rho$-parameter is from the $(t, b)$ doublet, at the present level calculated to

  $$\Delta \rho = 3x_t \cdot [1 + x_t \rho^{(2)} + \Delta \rho_{QCD}] \quad (4)$$

  with

  $$x_t = \frac{G_\mu m_t^2}{8\pi^2 \sqrt{2}}. \quad (5)$$

  The electroweak 2-loop part is described by the function $\rho^{(2)}(M_H/m_t)$. $\Delta \rho_{QCD}$ is the QCD correction to the leading $G_\mu m_t^2$ term

  $$\Delta \rho_{QCD} = -2.86 a_s - 14.6 a_s^2, \quad a_s = \frac{\alpha_s(m_t)}{\pi}. \quad (6)$$
2.2 The vector boson masses

The correlation between the masses $M_W, M_Z$ of the vector bosons, in terms of the Fermi constant $G_\mu$, is in 1-loop order given by:

$$G_\mu = \frac{\pi\alpha}{2s_W^2M_W^2}[1 + \Delta r(\alpha, M_W, M_Z, M_H, m_t)].$$

The appearance of large terms in $\Delta r$ requires the consideration of higher than 1-loop effects. At present, the following higher order contributions are available:

- The leading log resummation\[3\] of $\Delta\alpha$:
  $$1 + \Delta\alpha \to (1 - \Delta\alpha)^{-1}$$
- The incorporation of non-leading higher order terms containing mass singularities of the type $\alpha^2 \log(M_Z/m_f)$ from the light fermions\[8\]
- The resummation of the leading $m_f^2$ contribution\[9\] in terms of $\Delta\rho$ in Eq. (4). Moreover, the complete $O(\alpha\alpha_s)$ corrections to the self energies are available\[10,11\] and part of the $O(\alpha\alpha_s^2)$ terms\[12\]
- The $G_\mu^2 m_f^2 M_Z^2$ contribution of the electroweak 2-loop order\[23\]

2.3 Z boson observables

With $M_Z$ as a precise input parameter, the predictions for the partial widths as well as for the asymmetries can conveniently be calculated in terms of effective neutral current coupling constants for the various fermions:

$$J_{\nu}^{\nu,NC} g_V^f \gamma_\nu - g_A^f \gamma_\nu \gamma_5 = (\rho_f)^{1/2} \left( (I_A^f - 2Q_f s_f^2) \gamma_\nu - I_V^f \gamma_\nu \gamma_5 \right).$$

with form factors $\rho_f$ and $s_f^2$ for the overall normalization and the effective mixing angle.

The effective mixing angles are of particular interest since they determine the on-resonance asymmetries via the combinations

$$A_f = \frac{2g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2}. \quad (7)$$

Measurements of the asymmetries hence are measurements of the ratios

$$g_V^f/g_A^f = 1 - 2Q_f s_f^2 \quad (8)$$
or the effective mixing angles, respectively.

The total $Z$ width $\Gamma_Z$ can be calculated essentially as the sum over the fermionic partial decay widths. Expressed in terms of the effective coupling constants they read up to 2nd order in the fermion masses:

$$\Gamma_f = \Gamma_0 \left( (g_V^f)^2 + (g_A^f)^2(1 - \frac{6m_f^2}{M_Z^2}) \right)$$

$$(1 + Q_f^2 \frac{3\alpha}{4\pi}) + \Delta\Gamma_{QCD}^f$$

with $\left[ N_f^l = 1 \text{ (leptons), } N_f^q = 3 \text{ (quarks)} \right]$

$$\Gamma_0 = N_f^l \sqrt{2}G_\mu M_Z^2 \frac{12\pi}{13},$$

and the QCD corrections $\Delta\Gamma_{QCD}^f$ for quark final states\[24\]

2.4 Accuracy of the standard model predictions

For a discussion of the theoretical reliability of the standard model predictions one has to consider the various sources contributing to their uncertainties:

The experimental error of the hadronic contribution to $\alpha(M_Z^2)$, Eq. (2), leads to $\delta M_W = 13$ MeV in the $W$ mass prediction, and $\delta \sin^2 \theta = 0.00023$ common to all of the mixing angles, which matches with the experimental precision.

The uncertainties from the QCD contributions can essentially be traced back to those in the top quark loops for the $\rho$-parameter. They can be combined into the following errors:\[25\]

$$\delta(\Delta\rho) \simeq 1.5 \cdot 10^{-4}, \quad \delta s_f^2 \simeq 0.0001.$$ 

The size of unknown higher order contributions can be estimated by different treatments of non-leading terms of higher order in the implementation of radiative corrections in electroweak observables (‘options’) and by investigations of the scheme dependence. Explicit comparisons between the results of 5 different computer codes based on on-shell and $\overline{MS}$ calculations for the $Z$ resonance observables are documented in the “Electroweak Working Group Report”\[26\] in ref. \[3\].

Table 1 shows the uncertainty in a selected set of precision observables. Quite recently (not included in table 1) the non-leading 2-loop corrections $\sim G_\mu^2 m_t^2 M_Z^2$ have been calculated\[27\] for $\Delta r$ and $s_f^2$. They reduce the uncertainty in $M_W$ and $s_f^2$ considerably, by about a factor 0.2.
Table 1: Largest half-differences among central values ($\Delta_c$) and among maximal and minimal predictions ($\Delta_s$) for $m_t = 175$ GeV, $60$ GeV $< M_H < 1$ TeV, $\alpha_s(M_Z^2) = 0.125$ (from ref.\cite{23})

| Observable $O$ | $\Delta_c O$ | $\Delta_s O$ |
|----------------|---------------|---------------|
| $M_W$ (GeV)   | $4.5 \times 10^{-3}$ | $1.6 \times 10^{-2}$ |
| $\Gamma_Z$ (MeV) | $1.3 \times 10^{-2}$ | $3.1 \times 10^{-2}$ |
| $s_\beta^2$   | $5.5 \times 10^{-4}$ | $1.4 \times 10^{-4}$ |
| $s_\beta^2$   | $5.0 \times 10^{-5}$ | $1.5 \times 10^{-4}$ |
| $R_{b\text{had}}$ | $4.0 \times 10^{-3}$ | $9.0 \times 10^{-3}$ |
| $R_b$         | $6.5 \times 10^{-5}$ | $1.7 \times 10^{-4}$ |
| $R_c$         | $2.0 \times 10^{-5}$ | $4.5 \times 10^{-5}$ |
| $\sigma_{b\text{had}}^0$ (nb) | $7.0 \times 10^{-3}$ | $8.5 \times 10^{-3}$ |
| $A_{FB}^L$    | $9.3 \times 10^{-5}$ | $2.2 \times 10^{-4}$ |
| $A_{FB}^C$    | $3.0 \times 10^{-4}$ | $7.4 \times 10^{-4}$ |
| $A_{FB}^B$    | $2.3 \times 10^{-4}$ | $5.7 \times 10^{-4}$ |
| $A_{LR}$      | $4.2 \times 10^{-4}$ | $8.7 \times 10^{-4}$ |

3 Standard model and precision data

In table 2 the standard model predictions for $Z$ pole observables and the $W$ mass are put together for a light and a heavy Higgs particle with $m_t = 175$ GeV. The last column is the variation of the prediction according to $\Delta m_t = \pm 6$ GeV. The input value $\alpha_s = 0.123$ is the one from QCD observables at the $Z$ peak\cite{24}. Not included are the uncertainties from $\delta \alpha_s = 0.006$, which amount to 3 MeV for the hadronic $Z$ width. The experimental results on the $Z$ observables are from combined LEP and SLD data. $\rho_t$ and $s_\beta^2$ are the leptonic neutral current couplings in eq. (6), obtained from partial widths and asymmetries under the assumption of lepton universality. Compared to the previous year, the deviation in $R_c$ has disappeared and $R_b$ is on its way towards the standard model value. On the other hand, the deviation in $A_b$ has become somewhat stronger.

Table 2 also illustrates the sensitivity of the various quantities to the Higgs mass. The effective mixing angle turns out to be the most sensitive observable, where both the experimental error and the uncertainty from $m_t$ are small compared to the variation with $M_H$. Since a light Higgs boson corresponds to a low value of $s_\beta^2$, the strong upper bound on $M_H$ is from $A_{LR}$ at the SLC\cite{25}, whereas LEP data alone allow to accommodate also a relatively heavy Higgs.

4 Fermion pair production above the $Z$ resonance

Also above the $Z$ peak, the production of fermion pairs is an important class of processes since they are the dominant ones at LEP 2. The cross sections are measurable with high accuracy: 1.2% for $e^+e^- \rightarrow \mu^+\mu^-$, and 0.7% for $e^+e^- \rightarrow \text{hadrons}$\cite{26}. From the theoretical side they are of special interest because box diagrams with two heavy boson exchange are no longer negligible, contributing several percent to the integrated cross section. Large QED corrections from the radiative tail of the $Z$ resonance to both integrated cross sections and forward-backward asymmetries occur and require a careful theoretical treatment to obtain a theoretical precision of about 0.5% (see\cite{26} for more information).
Table 2: Precision observables: experimental results and standard model predictions.

| observable | exp. (1996) | $M_H = 65$ GeV | $M_H = 1$ TeV | $\Delta m_t$ |
|------------|-------------|----------------|---------------|--------------|
| $M_Z$ (GeV) | 91.1863 ± 0.0020 | input | input | |
| $\Gamma_Z$ (GeV) | 2.4946 ± 0.0027 | 2.5015 | 2.4923 | ±0.0015 |
| $\sigma_{\text{had}}$ (nb) | 41.508 ± 0.056 | 41.441 | 41.448 | ±0.003 |
| $\Gamma_{\text{had}}/\Gamma_e$ | 20.773 ± 0.029 | 20.798 | 20.770 | ±0.002 |
| $\Gamma_b/\Gamma_{\text{had}} = R_b$ | 0.2178 ± 0.0011 | 0.2156 | 0.2157 | ±0.0002 |
| $\Gamma_c/\Gamma_{\text{had}} = R_c$ | 0.1715 ± 0.0056 | 0.1724 | 0.1723 | ±0.0001 |
| $\rho_\ell$ | 0.867 ± 0.022 | 0.9350 | 0.9340 | ±0.0001 |
| $s_\ell^2$ | 0.23165 ± 0.00024 | 0.23115 | 0.23265 | ±0.0002 |
| $M_W$ (GeV) | 80.356 ± 0.125 | 80.414 | 80.216 | ±0.038 |

5 Muon anomalous magnetic moment

The anomalous magnetic moment of the muon,

$$a_\mu = g_\mu - \frac{2}{2}$$

provides a precision test of the standard model at low energies. Within the present experimental accuracy of $\Delta a_\mu = 840 \cdot 10^{-11}$, theory and experiment are in best agreement, but the electroweak loop corrections are still hidden in the noise. A new experiment, E 821 at Brookhaven National Laboratory, is being prepared for 1997 to reduce the experimental error down to $40 \pm 10^{-11}$ and hence will become sensitive to the electroweak loop contribution.

For this reason the standard model prediction has to be known with comparable precision. Recent theoretical work has contributed to reduce the theoretical uncertainty by calculating the electroweak 2-loop terms and updating the contribution from the hadronic photonic vacuum polarization (first reference of $a_\mu$).

$$a_{\mu,\text{had}}^{\text{vacuum pol.}} = (7024 \pm 153) \cdot 10^{-11}$$

which agrees within the error with the result of $a_\mu$. The main sources for the theoretical error at present are the hadronic vacuum polarization and the light-by-light scattering mediated by quarks, as part of the 3-loop hadronic contribution.

Table 3 shows the breakdown of $a_\mu$. The hadronic part is supplemented by the higher order $\alpha^3$ vacuum polarization effects but is without the light-by-light contribution.

Table 3: Contributions $\Delta a_\mu$ to the muonic anomalous magnetic moment and their theoretical uncertainties, in units of $10^{-11}$.

| source | $\Delta a_\mu$ | error |
|--------|----------------|-------|
| QED $^3$ | 116584706 | 2 |
| hadronic $^8$ | 6916 | 153 |
| EW, 1-loop $^2$ | 195 | |
| EW, 2-loop $^4$ | -44 | 4 |
| light-by-light $^3$ | -52 | 18 |
| light-by-light $^3$ | -92 | 32 |
| future experiment | | 40 |

The 2-loop electroweak contribution is as big in size as the expected experimental error. The dominating theoretical uncertainty at present is the error in the hadronic vacuum polarization. But also the contribution involving light-by-light scattering needs improvement in order to reduce the theoretical error.

6 The MSSM and precision data

The MSSM deserves a special discussion as the most predictive framework beyond the minimal model. Its structure allows a similarly complete calculation of the electroweak precision observables as in the standard model in terms of one Higgs mass (usually taken as $M_A$) and $\tan\beta = v_2/v_1$, together with the set of SUSY soft breaking parameters fixing the chargino/neutralino and scalar fermion sectors. It has been known since
quite some time\cite{1} that light non-standard Higgs bosons as well as light stop and charginos predict larger values for the ratio $R_b$\cite{2}. Complete 1-loop calculations are available for $\Delta r$\cite{3} and for the $Z$ boson observables\cite{3}. For obtaining the optimized SUSY parameter set a global fit to all the electroweak precision data, including the top mass measurement, has been performed with the new data\cite{4}. Figure 1 displays the experimental data normalized to the best fit results in the SM and MSSM, with the data from this conference. The difference between the experimental and the SM value of $R_b$ can now be fully explained by the MSSM. Other quantities are practically unchanged. In total, the $\chi^2$ of the fit is slightly better than in the standard model, but due to the larger numbers of parameters, the probability for the standard model is higher. A similar situations occurs for large $\tan \beta$ with light Higgs bosons $h^0, A^0$ around 50 GeV.

In conclusion, the theoretical predictions for electroweak precision observables are reliably calculated, with the main uncertainty from $\Delta \alpha$ in (2). In view of the new data, the standard model is in a very good shape. The MSSM is competitive to the standard model, but it is no longer prefered by the data.

Acknowledgements: I want to thank W. de Boer, P. Gambino, M. Grünewald, G. Passarino, U. Schwickerath and G. Weiglein for helpful discussions and valuable informations.

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