High precision displacement-measuring interferometer based on phase modulation technique and modulation index effect elimination

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Abstract. A high precision displacement-measuring interferometer based on a phase modulation technique was developed. A PZT actuator was utilized to drive a mirror of a Michelson interferometer by applying a sinusoidal voltage to the PZT controller. The path difference between two arms of the interferometer was modulated leading to modulation in the phase of the interference signal with a frequency of 3 kHz. The first and second harmonics of the interference signal were detected at the modulation index of 2.63 rad, a special value when the values of the first and second orders of Bessel function are equal. The displacement was determined by the ratio of the first and second harmonic in which the effects of modulation index instability and intensity fluctuation were neglected. The phase modulation interferometer using a PZT actuator has no amplitude modulation effect compared with other modulation interferometers using EOM (electro-optics modulator) or modulation the injection current of a laser diode. Moreover, the direction of the displacement that was ambiguous of the traditional interferometers was clarified in a real time. A measurement precision of 60 nm was obtained using the phase modulation interferometer.

1. Introduction

Laser interferometers are widely utilized for displacement measurements with nanometer-order uncertainty because of their inherent accuracy and their traceability to the metric standard through the frequency of the laser source. Various signal processing techniques have been developed for displacement-measuring interferometers such as homodyne [1, 2], heterodyne [3, 4] and phase or frequency modulation techniques [5-7].

The homodyne interferometer technique is widely utilized in small-displacement measurements with very high measurement resolution. In particular, a measurement accuracy of 10 pm [8] and a resolution of sub-picometer [9] order have been reported. The interference signal of a homodyne interferometer is time independent, and therefore it enables an ultrafast response because interference converts instantaneously phase variations into intensity variations. The upper bandwidth limit is determined by the response time of the photodetector and the bandwidth of the signal-processing electronics. Therefore, homodyne interferometers have the potential to be used for high-speed applications. However, homodyne interferometers require highly stable laser intensity during each measurement. This means that the misalignment of the optics, disturbance of the environment or
shifting of a measured point will strongly affect the measurement uncertainty [10]. A heterodyne interferometer is less sensitive to temperature and pressure variations but it is slower because of the delay introduced by electronic signal processing for phase acquisition. The maximum measurable speed of a heterodyne interferometer is limited by the heterodyne frequency [4]. A high-cost, voluminous, voluminous, and complicated system are also disadvantages of heterodyne interferometers.

Among these techniques, the sinusoidal phase modulated (SPM) and sinusoidal frequency modulated (SFM) techniques have many advantages. The signal of SPM or SFM interference, which is a continuous function of time, is a series of harmonics of the modulation frequency. The phase shift, which is induced by the displacement of the target mirror in the interferometer, can be accurately extracted from the interference signal using an lock-in amplifier (LIA) [5-7]. Moreover, the measurement speed of an SPM or SFM interferometer is only limited by the modulation frequency, for which a very high frequency can be obtained by using an electro-optic modulator (EOM) or by modulating the injection of laser diodes. However, the disadvantaged feature of the SFM technique is the modulation index change when the unbalanced between two arms of the interferometer changes. The modulation index of the SPM interferometer is unchanged and the modulation index measurement is unnecessary during the operating time. However, the main problem remains of both the SPM and SFM is the residual amplitude modulation (RAM). The RAM reduces the accuracy of the measured modulation index value when the values is calculated from the amplitude of harmonics.

In this paper, a high precision displacement measuring interferometer using a phase modulation technique was proposed. The effect of the Bessel function values was neglected by using a suitable modulation index. The RAM effect is removed from the system because the phase of the interferometer is modulated using a PZT. Consequently, compared with other techniques, SPM is the most competitive for achieving high precision as well as a much wider measurement range.

2. Measurement principle

Figure 1 illustrates a sinusoidal phase modulation (SPM) Michelson interferometer. A laser beam goes through an isolator which protects the source from the reflection light. On beam splitter, the beam is divided into two paths, one goes to the reference mirror which is attached to a piezoelectric transducer (PZT). The movement of the reference mirror is modulated by sinusoidally modulating the applied voltage of PZT. Consequently, the phase of the interference signal is modulated. Another beam comes to the measurement mirror and returns the beam splitter. Two beams recombine and interfere on the beam splitter. The interference signal is detected using a photodetector.

The electric field in the reference arm is modulated sinusoidally and it can be expressed as:

\[ E_r(r,t) = E_{0r} \times e^{i(\omega_0 t + \omega_r r)}, \]  

(1)
where $E_0$ and $\omega_0$ represent electric amplitude and carrier frequency of the laser source, $\omega_m$ and $m$ are modulation angular frequency and a modulation index, respectively. The beam returning from the measurement mirror is represented by:

$$E_m(r, t) = E_{0m} \times e^{i\left(\omega_0 t + \frac{4\pi m}{\lambda_0} \Delta L\right)},$$  \hspace{1cm} (2)

where $\Delta L$ is measured displacement and $\lambda_0$ is the wavelength of the light source.

Since $I \propto E^2$, the interfering signal of two beams detected by the photodetector is written as

$$I = |E_r(r, t) + E_m(r, t)|^2 = \langle E_r(r, t) \rangle^2 + \langle E_m(r, t) \rangle^2 + 2 \langle E_r(r, t) \rangle \langle E_m(r, t) \rangle \cos(\omega_m t)$$

Using the Bessel function to expand Eq. (3) and it is give

$$I = I_0 \left[ 1 + \cos \left( \frac{4\pi m}{\lambda_0} \Delta L \right) \times \left[ J_0(m) + 2 \sum_{k=1}^{\infty} J_{2k}(m) \cos(2k\omega_m t) \right] \right] \left[ 1 - \sin \left( \frac{4\pi m}{\lambda_0} \Delta L \right) \times 2 \sum_{k=1}^{\infty} J_{2k-1}(m) \sin[(2k - 1) \times \omega_m t] \right]$$  \hspace{1cm} (4)

LIAs are used to obtain 1st and 2nd harmonic terms from Eq.(4)

$$I_1 = - I_0 J_1(m) \sin \left( \frac{4\pi m}{\lambda_0} \Delta L \right),$$  \hspace{1cm} (5)

$$I_2 = I_0 J_2(m) \cos \left( \frac{4\pi m}{\lambda_0} \Delta L \right).$$  \hspace{1cm} (6)

Equation (5) and (6) show that the 1st and 2nd harmonics of the interference signal are two quadrature phase signals. A Lissajous diagram obtained from the two signals can be used to clarify the direction of movement and to measure the phase shifting caused by displacement concurrently. The displacement $\Delta L$ is given by

$$\Delta L = \frac{\lambda}{4\pi m} \times \tan^{-1} \left( \frac{I_2}{I_1} \times J_1(m) \right).$$  \hspace{1cm} (7)

In Eq. (7), $\Delta L$ depends on the intensity of 1st and 2nd harmonics, and Bessel functions $J_1(m)$ and $J_2(m)$. Normally, the intensity fluctuation of laser source limits the measurement accuracy of homodyne interferometer. Using the ratio of 1st and 2nd harmonics ($I_1/I_2$) the effect of intensity fluctuation is neglected.

However, the Bessel functions, $J_1(m)$ and $J_2(m)$, which depend on the value of $m$ can reduce the signal to noise ratio of the 1st and 2nd harmonics. In this research, a method to neglect the effect of the modulation index is proposed.

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**Figure 2.** Bessel function.
Figure 2 shows the Bessel functions $J_1(m)$, $J_2(m)$, $J_3(m)$, and $J_4(m)$. There are some critical points where two consecutive Bessel functions are equal. $J_1(m) = J_2(m)$ when $m=2.63$ rad and $J_2(m) = J_3(m)$ when $m=3.77$ rad. In this research, the modulation index $m=2.63$ rad is used and Eq. (7) becomes

$$\Delta L = \frac{\lambda}{4m} \times \tan^{-1} \left( \frac{J_1}{J_2} \right).$$  \hspace{1cm} (8)

Equation (8) shows that the displacement $\Delta L$ is independent on the modulation index $m$. The Lissajous diagram is a circular and the normalized method for a nonstandard Lissajous diagram is unnecessary [5]. Therefore, the measurement uncertainties of modulation index measurement and approximation Bessel function value are removed from uncertainty sources of the proposed interferometer.

3. Measurement principle

The experimental system and the data processing module are shown in figure 3. A collimated laser diode (CPS532-C2, Thorlabs Inc.) was used as a light source for the interferometer. The movement of the reference mirror was sinusoidally modulated by a PZT actuator (PA4FKW, Thorlabs Inc.). The PZT actuator was driven by a voltage controller (PK4DMP1, Thorlabs Inc.) with the smallest increment of nanometer order. The interference signal was detected using a photodetector (PDA36A-EC, Thorlabs Inc.), figure (3a). A signal processing module was built by combining analog lock-in amplifiers and high-resolution data acquisition, figure (3b). The experimental condition is shown in table 1.

![Experimental system and signal processing module.]

**Figure 3.** Phase modulation interferometer system.

| Table 1. Experimental condition. |
|-----------------------------------|
| Wavelength of laser source       | 532 nm       |
| Maximum power                    | 1 mW         |
| Modulation frequency of PZT      | 3 kHz        |
| Modulation index                 | 2.63 rad     |
| Resonant frequency of PZT        | 270 kHz      |
| Spectral response range of detector | 350-1000 nm |
| Frequency bandwidth of detector  | DC-10 MHz    |
| Resolution of ADC                | 24 Bit       |
| Sample rates of ADC              | 512 kSPS     |

The proposed interferometer was used to measure a displacement which was generated by another PZT stage. The measuring result was compared with the reference displacement that was generated using another PZT (PK4DMP1, Thorlabs Inc.). The reference displacement can be determined directly by using a capacitive sensor (D100, Physik Instrumente). The resolution of the capacitive sensor is less than 0.5 nm, and the measurement accuracy was less than 2 nm with the measurement range of 10 µm. The interference signal and 1st and 2nd harmonics were shown in figure (4a) and figure (4b),
respectively. The Lissajous diagram of 1<sup>st</sup> and 2<sup>nd</sup> harmonic was used to track the movement direction and to calculate the phase change due to the displacement of the object, figure (4c). The measured displacement obtained by the interferometer and reference displacement were depicted in figure (5a).

![Figures](image.png)

**Figure 4.** Demodulated signals of the phase modulation interferometer.

![Figures](image.png)

**Figure 5.** The difference between the measuring result using the interferometer and the reference.

The experimental system was performed in an open space. However, 1<sup>st</sup> and 2<sup>nd</sup> harmonics were detected purely and then the displacement can be determined. It means that the phase modulation interferometer can work well even if there was the existence of the environment effect. To clarify the measurement accuracy, the difference of the displacement measurement results using the interferometer and the reference is shown in figure. (5b). The difference from peak to peak was about 60 nm. There were some uncertainty sources can be listed such as the refractive index fluctuation, vibration, and imperfectly optical polarization.

4. Conclusion
A phase modulation displacement measuring interferometer was successfully developed. The measuring system is compact, low-cost, and stable. The measurement accuracy was less than 60 nm. It
can be used for industrial applications. For future work, the proposed interferometer should be compared concurrently with heterodyne interferometer to clarify clearly the measurement accuracy and measurement resolution.

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