Exchange Coulomb interaction in nanotubes: Dispersion of Langmuir waves

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Microscopic derivation of the Coulomb exchange interaction for electrons located on the nanotubes is presented. Our derivation is based on the many-particle quantum hydrodynamic method. We demonstrate the role of the curvature of the nanocylinders on the force of the exchange interaction. We calculate corresponding dispersion dependencies for electron oscillations on the nanotubes.

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I. INTRODUCTION

Studying of the Coulomb exchange interaction in electron gas has a long history [1]-[6]. These fundamental papers are dedicated to the three dimensional electron gas. In 1983 the Coulomb exchange interaction was considered in the two dimensional electron gas by Datta [7]. It was performed on the background of research of the two dimensional electron gas in the self-consistent field approximation [8]-[17].

Generalization of the Coulomb exchange interaction force field in the three-dimensional electron gas (quantum plasmas) includes the dependence on the spin polarization [18], [19], [20]. Same problem for the two dimensional quantum plasmas was solved in Ref. [21]. This topic is reviewed in Ref. [22] as a part of a detailed description of the many-particle quantum hydrodynamic method. Application of the generalized Coulomb exchange interaction force field was applied to the three-dimensional magnetized quantum plasmas in Ref. [23]. Kinetic model of the low frequency plasma oscillations with the Coulomb exchange interaction in the three-dimensional medium can be found in Ref. [24]. A brief review of development and application of the exchange interaction. For the three-dimensional electron gas can be found in Ref. [25]. Reviews of some recent results in quantum plasmas, including relativistic effects in quantum plasmas, can be found in Refs. [24], [25] and introduction of Ref. [26].

The spin-1/2 quantum plasmas, for the first time, were considered by Kuz’menkov et al in 2001 in Ref. [27], where corresponding quantum hydrodynamic equations were derived with the explicit consideration of many-particle interaction. The spin-spin interaction between electrons was explicitly included in the derivation. The self-consistent field approximation of the spin-1/2 quantum hydrodynamic equations was considered there. The following paper [18] presented generalization of the spin-1/2 quantum hydrodynamic equations including both the Coulomb exchange interaction and the exchange spin interaction. This model was a generalization of the many-particle quantum hydrodynamics with no explicit spin evolution [28], where the general forms of the exchange correlations were presented for both the systems of bosons and the systems of fermions. These general methods were applied to the quantum hydrodynamic derivation of the Gross-Pitaevskii equation, its generalization and similar equations for ultracold fermions for neutral particles with the short-range interaction [29]. Following Ref. [18] the contribution of the Coulomb and spin-spin exchange interactions in spectrums of magnetized quantum plasmas was studied in Ref. [19]. In Ref. [30] methods of many-particle quantum hydrodynamics [18], [27] were applied to derive the quantum kinetic equations for spinning particles in the three-dimensional and low-dimensional systems. Models for the separate description of the spin-up and spin-down electrons have been developed as well [31], [32], [33], [34].

Due to steady interest to waves on nanotubes we derive the force field of the exchange Coulomb interaction between electrons on small radius cylinders: nanotubes, where curvature of the surface is essential. To this end we consider electron gas with the concentrations \(10^{9} \div 10^{9}\) cm\(^{-2}\) on cylindrical surfaces with radius of 10 ÷ 30 nm.

One can consider spherical and cylindrical waves in the three-dimensional classical [35] or quantum [36]-[47] plasmas. However there are a lot of papers dedicated to plasmas located on the two-dimensional spherical and cylindrical surfaces. These surfaces exist in the three-dimensional physical space and they can be surrounded by other mediums. As a model problem one can consider spherical or cylindrical surface surrounded by a medium with unitary dielectric permittivity. Hence the surrounding medium does not affect the properties of the plasmas on these surfaces. One can study different properties of plasmas located on these surfaces [48]-[50]. Speaking of the quantum effects we mean the contribution of the quantum Bohm potential. In spite of the quantum nature of the Fermi pressure existing due to the Pauli exclusion principle it can be included in the classic model by an appropriate choice of the equation of state. Our paper is dedicated to another quantum effect in plasmas. This is the contribution of the exchange part of the Coulomb interaction. For the two-dimensional electron gas it was derived in Ref. [7] and generalized in Ref. [20]. Here we

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consider features of the Pauli pressure and the Coulomb exchange interaction for nanotubes including the curvature of the cylindrical surface. Effects of the Coulomb exchange interaction on properties of waves in nanotubes were considered in Refs. [65], [66], for the highly dense electron gas, when the curvature of the cylindrical surface is not essential. However, in Ref. [61], studying a two dimensional system authors applied the exchange interaction potential derived for three dimensional systems. Let us mention that the Fermi pressure for the electron gas on the spherical surface was recently derived in Ref. [21] (see formula (256)). We should also mention that application of the three-dimensional equations of state or the exchange interaction potentials contain the three dimensional concentrations [32], [33], [34].

Models of quantum plasmas considering contribution of the quantum Bohm potential at huge densities do not include finite size of ions and dust. A hydrodynamic model explicitly including the effects of particle finite size was developed in Ref. [64].

One of most important examples of nanotubes is the carbon nanotube, which is rather different from the objects under consideration. The carbon nanotubes have different regime of the carriers concentration and conductivity. $\sigma$ and $\pi$ electrons in carbon nanotubes reveals different behavior since three $\sigma$ electrons of each atom are strongly bound within the layer of carbon atoms, while one $\pi$ electron of each atom is bound rather weakly. These properties are captured by the two-fluid hydrodynamic model of electrons for carbon nanotubes [63], [66]. Conductivity of the bounded electrons involves electrons from each atom, that reveals in the large concentration of carriers $n_0 \approx 10^{15}$ cm$^{-2}$. The Fermi pressure for $\sigma$ and $\pi$ electron fluids was introduced in Ref. [57]. Separate spin evolution of spin-up and spin-down electrons in three-dimensional and two-dimensional electron gas has recently been developed in literature [32], [33], [34].

This paper is organized as follows. In Sec. II the hydrodynamic equations for electron gas on the nanotube are presented and the equation of state for pressure of these electrons is derived. In Sec. III we derive the explicit form of the exchange force field. In Sec. IV we present the spectrum of collective excitations. In Sec. V a brief summary of obtained results is presented.

II. METHOD

A. Basic principles of the many-particle quantum hydrodynamics

We consider quantum mechanical system of N particles with various masses and charges, interacting by the Coulomb interaction, placed into external electromagnetic field. Derivation of quantum hydrodynamical equations is carried out by the method described in [28]. Microscopic number density is defined by formula

$$n(r, t) = \int dR \sum_{i=1}^{N} \delta(r - r_i) \psi^*(R, t) \psi(R, t),$$

where $R = (r_1, ..., r_N)$, $r_i$ are the coordinates of $i$th particle, $dR = \prod_{j=1}^{N} d r_j$ is the element of volume in 3N-dimensional configurational space, $d r_j$ is the element of volume in 3D space of radius-vector $r_j$. Definition (1) corresponds the fundamental quantum mechanical definition of a quantum observable value [68].

The Hamiltonian of systems under consideration has form

$$\dot{H} = \sum_{i=1}^{N} \left( \frac{D_i^2}{2m_i} + e_i \varphi_i \right) + \frac{1}{2} \sum_{i,j=1,i \neq j}^{N} \frac{e_i e_j}{|r_i - r_j|},$$

where $D_i = -i\hbar \nabla_i - e_i A_i/c$ is the long momentum including action of the external magnetic field on the charge of the electrons, $\nabla_i$ is the derivative over coordinates of $i$-th particle, functions $\varphi_i = \varphi_i(r_i, t)$, and $A_i = A_i(r_i, t)$ are the potentials of external electromagnetic field. The first group of terms in the Hamiltonian describes charged particles in the external electromagnetic field. The last term in formula (2) presents the interparticle Coulomb interaction. The Coulomb interaction gives two contributions in the Euler equation governing the collective motion of electrons: the self-consistent field part and the exchange part. Hamiltonian (2) shows that we consider systems of charged particles in the three dimensional space. Below we specify that particles are bound to the cylindrical surface of nanotubes. Our main goal is to present the microscopic derivation of the Coulomb exchange interaction force field of the electrons on the nanotubes.

Differentiating number density (1) with respect to time and applying the Schrödinger equation with the Hamiltonian (2), we obtain the continuity equation

$$\partial_t n + \nabla (nv) = 0,$$

where $j = nv$ is the particle current. Its explicit definition via the many-particle wave function can be found in Refs. [26], [29], [69], [70]. This definition allows us to derive the Euler equation. The exchange interaction appears beyond the self-consistent field approximation. Hence we need to calculate the quantum two-particle correlations as it was done in Ref. [20] for three-dimensional and the plane-like two-dimensional electron gas.
The derivation method of the quantum hydrodynamic (QHD) equations for systems of many charged particles was suggested by Kuz’menkov and Maksimov in 1999 in Ref. [28]. The explicit derivation of the QHD equations in the cylindric coordinates \((\rho = \sqrt{x^2 + y^2}, \varphi = \arctan(y/x)\) and \(z = z\) has been recently performed in Ref. [21]. Nevertheless this form of the QHD equations had been applied earlier (see Refs. [22]-[27]). Substituting \(\rho = R\) and assuming \(v_\rho = 0\), and dropping the derivatives over \(\rho\) we find QHD equations for electrons on the nanotube

\[
\partial_t n + \frac{1}{R} \partial_\varphi (n \nu_\varphi) + \partial_z (n \nu_z) = 0, \quad (4)
\]

\[
 mn \partial_t \nu_\varphi + mn \left( \frac{1}{R} \nu_\varphi \partial_\varphi + \nu_z \partial_z \right) \nu_\varphi + \frac{1}{R} \partial_\varphi P
\]

\[
- \frac{\hbar^2}{2m} n \frac{1}{R} \partial_\varphi \left( \frac{\Delta \sqrt{n}}{\sqrt{n}} \right) = q_e n E_\varphi + F_{E\varphi}^{E_x}, \quad (5)
\]

\[
 mn \partial_t \nu_z + mn \left( \frac{1}{R} \nu_\varphi \partial_\varphi + \nu_z \partial_z \right) \nu_z + \partial_z P
\]

\[
- \frac{\hbar^2}{2m} n \partial_z \left( \frac{\Delta \sqrt{n}}{\sqrt{n}} \right) = q_e n E_z + F_z^{E_x}, \quad (6)
\]

where \(F_{E\varphi}^{E_x}\) and \(F_z^{E_x}\) are the force fields associated with the Coulomb exchange interaction, \(P\) is the pressure related to the distribution of particles on different quantum states, \(q_e = -e\) is the charge of electron. Equation (4) is the continuity equation, and equations (5), (6) are the Euler equations for two projections of the velocity field \(\nu_\varphi\) and \(\nu_z\).

Initially the interaction force field, presented in the Euler equations (5) and (6) as \(F_{\alpha \text{int}} = q_e n E_\alpha + F_{\alpha \text{ex}}^{E_x}\), appears as follows

\[
 F_{\alpha \text{int}} = -q_e \int dr' (\nabla G(|r - r'|)) \times
\]

\[
 [q_e n_{2(\alpha)}(r, r', t) + q_i n_{2(\alpha)}(r, r', t)], \quad (7)
\]

where \(n_{2(\alpha)}(r, r', t)\) is the two-particle concentration, with \(\alpha = e, i\). Function \(n_{2(\alpha)}(r, r', t)\) does not contain the exchange part (or in other words the Fock part of the Hartree-Fock approximation), since electrons and ions are not identical particles. Consequently we have

\[
n_{2(\alpha)}(r, r', t) = n_e(x, r) n_i(r', t). \quad (8)
\]

In the limit of weakly interacting electrons we can write

\[
n_{2(\alpha)}(r, r', t) = n_e(r, t)n_e(r', t) - \rho(r, r', t) \quad (8)
\]

for system of electrons being in the same spin state. Result (8) appears due to symmetry of the spin part of the many-particle wave function and antisymmetry of the coordinate part of the wave function. What corresponds to the antisymmetry of the full wave function relatively to permutation of identical particles.

The ratio of spin polarization affects the second term in formula (8). If we want to consider the system of partially polarized fermions we need to include that exchange part of the Coulomb interaction gives equal shifts of the energy, but in opposite directions, relatively to the relative direction of spins of these particles. As the first step we consider system of fully spin polarized electrons to present the generalization for partially spin polarized electrons below.

In the case of fermions \(F_{E\varphi}^{E_x}\) has next form:

\[
 F_{E\varphi}^{E_x} = e^2 \int d r' \nabla G(r - r') |\rho(r, r', t)|^2. \quad (9)
\]

We direct the external magnetic field parallel to the cylinder axes \(B = B_0 e_z\). In this regime the Lorentz force equals to zero. However, the magnetic field gives contribution via the spin polarisation of electrons, which affects the equation of state and the force of the Coulomb exchange interaction, as we show it below.

There are some concerns [71], [72], [73] about derivation of the hydrodynamic equations from the single particles models and disregards of the area of applicability of hydrodynamic equations. Formulae presented above demonstrate that we have derived our equations from a many-particle model. Therefore appearance of the Fermi pressure and the Coulomb exchange interaction is justified. Area of applicability of our equation is discovered in the text through the derivation of the explicit forms for the Fermi pressure and the exchange interaction. Corresponding limits are discussed below in the paper.

The QHD equations in the cylindrical coordinates were explicitly derived in Ref. [21], where was discussed the contribution of the quantum part of the inertial forces, which, together with the quantum part of the momentum flux, forms the familiar form of the quantum Bohm potential presented in equations (3) and (6).

Function \(\rho(r, r', t)\) has the following definition:

\[
\rho(r, r', t) = \sum_f n_f \varphi_f^*(r, t) \varphi_f(r', t), \quad (10)
\]

where \(n_f\) is number of particles in the quantum state described by the set of quantum numbers \(f, \varphi_f(r, t)\) are the wave functions of these states [28], [29]. For our approximate calculation of the exchange Coulomb interaction we apply the wave functions of free particles on the cylindrical surface

\[
\varphi_{p,t}(r, t) = \frac{1}{\sqrt{2\pi i RL}} \exp \left(-\frac{it}{\hbar}\right) \exp(i l \varphi + ipz/\hbar), \quad (11)
\]

where \(L\) and \(R\) are the length and radius of cylinder (nanotube) correspondingly, \(l\) is the quantum number.
electrons have the quasi-one-dimensional inspite the two-dimensional distribution of particles in the physical space. Where $d$ is the dimensionless momentum space with the momentum over the coordinate $z$.

\[ \rho = \rho_c \]

\[ \bar{\rho} = \bar{\rho}_c \]

\[ \frac{\pi}{\bar{\rho}} \]

\[ \frac{\pi R}{\bar{\rho}} \]

\[ G(k,l) \]

\[ H(k,l) \]

\[ E = -q_e \nabla \int \frac{n_e - n_{0i}}{|r - r'|} dr', \quad (12) \]

where $dr'$ is the differential of two dimensional surface: $dr' = dx'dy'$ for planes, $\nabla = \{(1/R)\partial_{\varphi}, \partial_z\}$ and $dr' = R d\varphi' dz'$ for cylinders. We assume that equilibrium concentrations of electrons and ions equal to each other. Hence the difference $n_e - n_{0i}$ gives us the perturbation of electron concentration $\delta n$. Equations (11)-(13) together with formula (12) correspond to the two-dimensional hydrodynamics developed by Fetter in Refs. [9], [10].

Since we consider the electrons on the cylindrical surface we use the equation (12) in a more explicit form including the symmetry of our problem

\[ E = -q_e \nabla \int_{l=-\infty}^{+\infty} \int_{l=-\infty}^{+\infty} \frac{dk}{(2\pi)^2} G(k,l) \times \]

\[ \times \int R d\varphi' dz'[n_e - n_{0i}] \exp[i(\varphi - \varphi') + ik(z - z')] , \quad (13) \]

where $G(k,l) = 4\pi I_n(kR)K_n(kR)$, where we have applied the following factorization for the Green function of the Coulomb interaction [24] (see page 104 formula 3.148):

\[ \frac{1}{|r - r'|} = \frac{1}{\pi} \int_{-\infty}^{+\infty} dk \sum_{l=-\infty}^{+\infty} e^{il(\varphi - \varphi')} \times \]

\[ e^{ik(z - z')} I_l(k\rho_<) K_l(k\rho_>) , \quad (14) \]

where $\rho_<$, $\rho_>$ are the smaller and the larger variable from $\rho$ and $\rho'$. $I_l(k\rho_<)$ and $K_l(k\rho_>)$ are modified Bessel functions of the first and second kind. For our case of the electrons located on the cylindrical surface we use the following limit $\rho = \rho' = R$.

Fig. (1) shows the distribution of the quantum states in the momentum space for the ideal electron gas located on the cylindrical surface. Fig. (2) shows the position of the Fermi surface $|p| \leq p_F$ relatively to the quantum states of electrons at rather low concentration of electrons.

It can be shown that for sufficiently narrow tube all particles will be in states with $l = 0$. This case is considered further. Let us to obtain expressions for Fermi momentum and pressure. Number of particles $N$ in system equals to volume of our system in the momentum space $V = 2p_F h/R$ divided on the volume of the single electron quantum state on nanotubes $V_s = \frac{(2\pi h)^2}{2\pi LR}$ where the volume of the quantum state in the momentum space relatively to the motion in the $\varphi$ direction $V_{\varphi} = 2\pi h/(2\pi R) = h/R$. In the limit of full spin polarization, when each quantum state occupied by one electron, we obtain

\[ N = \frac{2p_F h}{2\pi LR} , \quad (15) \]

Factor $h/R$ in the numerator arrives from the fact that momentum over coordinate $\varphi$ equals to $p_\varphi = lh/R$, from which we see that width of one state is $h/R$ as it depicted in Fig. (4). We obtain that the maximal momentum of occupied states $p_\varphi$ of fully polarized electrons on the cylindrical surface equals to

\[ p_\varphi = 2\pi^3 h R n , \quad (16) \]

where $n = N/V_{2D}$, $V_{2D} = 2\pi LR$.
The Fermi momentum corresponds to double occupation of the low lying quantum states \( p_{Fe} = \pi^2 \hbar R n \), which is two times smaller than \( p_{\uparrow\uparrow} \). Corresponding Fermi energy has the traditional definition \( E_{Fe} = p_{Fe}^2/(2m) \).

The average energy of fully spin polarized electrons \( E \) is obtained by formula

\[
E_{\uparrow\uparrow} = \frac{\int_{-p_{\uparrow\uparrow}}^{p_{\uparrow\uparrow}} \frac{1}{3m} p_z^2 dp_z \int_{-h/2R}^{h/2R} dp_{\varphi}}{\int_{-p_{\uparrow\uparrow}}^{p_{\uparrow\uparrow}} dp_z \int_{-h/2R}^{h/2R} dp_{\varphi}}
\]

\[
E_{\uparrow\uparrow} = \frac{2\pi^4}{3m} \hbar^2 R^2 n^2 = \frac{4}{3} E_{Fe}.
\] (17)

Curvature of the cylindrical surface with radius of order of 20 nm gives contribution in the Fermi pressure and the Coulomb exchange interaction if the concentration of electrons is of order of \( 10^8 \div 10^{10} \text{ cm}^{-2} \). It corresponds to the Fermi temperature of order of \( T_{Fe} = \frac{E_{Fe}}{k_B} = 1 \text{ K} \), where we apply the reduced Planck constant \( \hbar = 1.05 \times 10^{-27} \text{ erg}\cdot\text{s} \), the Boltzmann constant \( k_B = 1.38 \times 10^{-16} \text{ erg}\cdot\text{K}^{-1} \), and the electron mass \( m_e = 9 \times 10^{-28} \text{ g} \). Therefore our results corresponds to low temperatures.

The average energy allows us to find the equation of state. Pressure of the ideal gas of particles having two degrees of freedom arises as

\[
P_{\uparrow\uparrow} = E_{\uparrow\uparrow} n.
\] (18)

In 3D case in formula (18) coefficient 2/3 is placed before \( E \), in 2D case coefficient equals to 2/2 = 1. As a result next expression is obtained for the pressure of the spin polarized electrons:

\[
P_{\uparrow\uparrow} = \frac{2\pi^4}{3m} \hbar^2 R^2 n^3.
\] (19)

Similar calculations for unpolarized electrons gives us the Fermi pressure \( P_{Fe} \):

\[
P_{Fe} = \frac{\pi^4}{6m} \hbar^2 R^2 n^3.
\] (20)

III. FORCE FIELD OF EXCHANGE INTERACTION

In this section we present the calculation of force field of the Coulomb exchange interaction \( F_{ex} \) for electrons located on the cylinder surface. Calculations is presented for fully spin polarized gas: \( n_f = 1 \).

We can rewrite formula (2) as follows

\[
F_{ex} = e^2 \int dr' \nabla G(r - r') \times \nabla \sum_{f,f'} \sum_{n_f,n_{f'}} \frac{n_f n_{f'} \varphi_f^*(r,t) \varphi_{f'}(r',t) \varphi_{f'}^*(r',t)}.
\] (21)

As mentioned above, the occupation numbers for states inside the sphere of radius \( p_{\uparrow\uparrow} \) in the momentum states equal to one: \( n_f = 1 \). Let us substitute wave functions \( \varphi_f \) in formula (21). Sum over states appears as

\[
\sum_{f} = \frac{1}{2\pi^2} \int_{-p_{\uparrow\uparrow}}^{p_{\uparrow\uparrow}} dp.
\] (22)

We do not need to perform the summation over all \( l \) since in our case \( l = 0 \) for all particles. As a result next expression for \( F_{ex} \) is obtained:

\[
F_{ex} = \frac{e^2}{4\pi^3 \hbar^2 R} \times
\]
FIG. 5: (Color online) The figure shows the comparison of the potential of the exchange forces for the plane-like electron gas and the electron gas on the nanotube with the occupation of one level \( l = 0 \). The potential corresponds to the force field as \( \mathbf{F}^{Ex} = \nabla \mathbf{U} \). The upper (blue, dashed) curve describes the exchange interaction in the plane-like electron gas. The lower (red) curve presents the exchange interaction in the cylindrical two-dimensional electron gas.

\[
\times \nabla \int_{-p_{\uparrow \uparrow}}^{p_{\uparrow \uparrow}} dp \int_{-p_{\uparrow \uparrow}}^{p_{\uparrow \uparrow}} dp' I_0((p - p')R/\hbar)K_0((p - p')R/\hbar).
\]

We consider the limit of small wave vectors \( kR \ll 1 \), therefore, the asymptotic expressions of the modified Bessel functions can be used for calculation of the exchange interaction. We apply

\[
I_0(x) \simeq 1,
\]

and

\[
K_0(x) \simeq -\ln \left| \frac{x}{2} \right| - \gamma,
\]

where \( \gamma \approx 0.577215... \) is the Euler-Mascheroni constant (see [74] page 92 formulae 3.102 and 3.103). As a result for the force field density next expression is obtained:

\[
\mathbf{F}^{Ex} = 2\pi e^2 R \nabla \left[ \left( 3 - 2\gamma - 2 \ln[2\pi^2 R^2 n]\right)n^2 \right],
\]

with \( \ln[2\pi^2 R^2 n] < 0 \).

It is interesting and important to compare result (26) with the two-dimensional analog \( \mathbf{F}^{Ex}_{pl} = 192 \cdot \text{arcsh} \cdot e^{\gamma} \sqrt{\pi}/(3\sqrt{\pi}) \) [1, 20]. To present the comparison it is rather useful to introduce the potential of these forces \( \mathbf{F}^{Ex} = \nabla \mathbf{U}_{Cyl} \), and \( \mathbf{F}^{Ex}_{pl} = \nabla \mathbf{U}_{pl} \). Dependence of the potentials on the concentration \( \mathbf{U}_{Cyl}(n) \), \( \mathbf{U}_{pl}(n) \) are presented in Fig. 5.

A. Partial spin polarisation of electrons

Above we have presented the equation of state (19) and the force field of the Coulomb exchange interaction (26) for electrons on nanotubes in the case of the full spin polarisation. This is a simple case, which allows to describe the derivation in a simple way. Now we are going to realistic regime of the partial spin polarisation.

The potential of the exchange interaction is significantly larger than the Fermi pressure in wide range of the spin polarizations \( \eta \in [10^{-5}, 1] \). We illustrate it by Fig. 6, where we compare the exchange interaction potential and the Fermi pressure for the electron gas in the regime of the full spin polarization \( \eta = 1 \). We see that the...
The exchange interaction potential is five orders larger than the Fermi pressure.

Fig. 7, similarly to Fig. 6, gives a comparison of the exchange interaction potential $U$ with the Fermi pressure $P$. In the regime of low spin polarization $\eta = 10^{-6}$ and $\eta = 10^{-8}$. In this regime $P$ and $U$ are comparable. The Fermi pressure becomes larger than the exchange interaction potential $U$ at small enough spin polarization.

At partial spin polarization of degenerate electrons we have that the spin-down (the spin-up) electrons occupy the quantum states lying inside the circle with radius $p_{\downarrow \downarrow}$ ($p_{\uparrow \uparrow}$) in the momentum space, and $p_{\downarrow \downarrow} > p_{\uparrow \uparrow}$. Hence the area at $p < p_{\downarrow \downarrow}$ is occupied by pair of electrons with opposite spins, and the area $p_{\downarrow \downarrow} > p > p_{\uparrow \uparrow}$ is occupied by the spin-down electrons only. The full pressure of the system $P$ appears as the sum of the partial pressures of spin-up and spin-down electrons $P = P_{\uparrow \uparrow} + P_{\downarrow \downarrow}$, where the partial pressures $P_{\uparrow \uparrow}$, $P_{\downarrow \downarrow}$ are given by formula (19) with the concentration of the spin-up electrons $n_{\uparrow}$ and the spin-down electrons $n_{\downarrow}$ correspondingly. The partial concentrations $n_{\uparrow}$ and $n_{\downarrow}$ can be easily presented via the full concentration of electrons and the ratio of the spin polarization

\[
n_{\uparrow} = \frac{n}{2} - \frac{\Delta n}{2}, \quad n_{\downarrow} = \frac{n}{2} + \frac{\Delta n}{2}, \quad (27)
\]

where $n$ is the full concentration, and $\Delta n$ is the concentration of particles being in the partially occupied states at $p_{\downarrow \downarrow} > p > p_{\uparrow \uparrow}$. Applying the notion of spin polarization ratio $\eta = |n_{\uparrow} - n_{\downarrow}| / (n_{\uparrow} + n_{\downarrow})$. Therefore $n_{\uparrow} = n(1 - \eta)/2$, and $n_{\downarrow} = n(1 + \eta)/2$.

Finally, for the case of the intermediate spin polarization we have

\[
P_{\downarrow \downarrow} = (1 + 3\eta^2) \frac{\pi^4}{6m} \hbar^2 R^2 n^3. \quad (28)
\]

Fig. 8 shows the Fermi step (the distribution function) for the partially polarized and the non-polarized systems of electrons.

Next we need to consider the force field of the exchange Coulomb interaction for the partially spin polarized electrons.

### IV. DISPERSION EQUATION

Using equations obtained above, let us investigate dispersion for the linear plasma waves in the electron gas on
nanotubes. Let us present the concentration of particles and the velocity field as \( n = n_0 + \delta n \), \( v_x = 0 + \delta v_x \), and \( v_z = 0 + \delta v_z \), where \( n_0 \) is the equilibrium value of concentration, \( \delta n \) is the perturbation of the concentration due to wave propagation. We assume that there are no currents in the equilibrium. Now let us apply the Fourier transformation to linearized form of equations (30)-(32) in accordance with formula (11)

\[
- \omega \delta n + k \delta v_z = 0, \\
\omega \delta v_x = 0, \\
\text{and}
\]

\[
mn_0 \omega \delta v_z - (1 + 3\eta^2)\frac{2\pi^4}{m} \hbar^2 R^2 n_0^2 k \delta n - \frac{\hbar^2}{4m} k^3 \delta n = e^2 n_0 G(k) \delta n
\]

\[+ 4\pi e^2 R n_0 k \left[ 2(1 - \gamma)\eta + \frac{1}{2}(1 + \eta^2) \ln \left( \frac{1 - \eta}{1 + \eta} \right) \right] \delta n.
\]

At derivation of equations (30), (31), we have included that derivatives on the angle \( \varphi \) equal to zero since we have \( l = 0 \).

Equations (30)- (32) allows to obtain the dispersion dependence of the Langmuir waves on the cylindrical surface of the nanotubes

\[
\omega^2(k) = \frac{e^2 n_0 k}{m} G(k) + (1 + 3\eta^2)\frac{2\pi^4}{m^2} \hbar^2 R^2 n_0^2 k^2
\]

\[ - \frac{4\pi e^2}{m} R n_0 k^2 \left[ 2(1 - \gamma)\eta + \frac{1}{2}(1 + \eta^2) \ln \left( \frac{1 - \eta}{1 + \eta} \right) \right]
\]

\[- \eta \ln(1 - \eta^2) + 2 \ln(2\pi^2 n_0 R^2)].
\]

At zero spin polarization \( \eta = 0 \) from formula (33) we find

\[
\omega^2(k) = \frac{e^2 n_0 k}{m} G(k) + \frac{2\pi^4}{m^2} \hbar^2 R^2 n_0^2 k^2.
\]

In the opposite limit, the full spin polarization \( \eta = 1 \) formula (33) gives

\[
\omega^2(k) = \frac{e^2 n_0 k}{m} G(k) + \frac{8\pi^4}{m^2} \hbar^2 R^2 n_0^2 k^2
\]

\[- \frac{8\pi e^2}{m} R n_0 k^2 [1 - \gamma - \ln(2\pi^2 n_0 R^2)].
\]

with \( 1 - \gamma - \ln(2\pi^2 n_0 R^2) > 0 \), since \( \ln(2\pi^2 n_0 R^2) < 0 \).

In formula (35) we have used

\[
\frac{1}{2} \left( 1 + \eta^2 \right) \ln \left( \frac{1 - \eta}{1 + \eta} \right) = \eta \ln(1 - \eta^2) - 2 \ln(2\pi^2 n_0 R^2)
\]

\[
= \ln(1 - \eta) - \ln(1 - \eta) - 2 \ln 2 - 2 \ln(2\pi^2 n_0 R^2)
\]

\[
= -2 \ln 2 - 2 \ln(2\pi^2 n_0 R^2) = -2 \ln(2\pi^2 n_0 R^2).
\]

The first term in the dispersion dependence (35) corresponds to the plasma frequency \( \omega_{\text{pl}}^2 = e^2 n_0 k G(k)/m \). The Fourier image of the Green function of the potential of Coulomb interaction on the cylindrical surface in the three-dimensional space is obtained as \( G(k) = 4\pi I_0(kR)K_0(kR) \). This function is the signature of geometrical properties of our system. In plane-like two-dimensional electron gas one can find \( \omega_{\text{pl}}^2 = 2\pi e^2 n_0 k/m \).

All of these Langmuir frequencies are analogs of well-known expression for three dimensional medium \( \omega_{\text{pl},3D}^2 = 4\pi e^2 n_{0,3D}/m \), where \( n_{0,3D} \) is the three dimensional concentration of particles \( [n_{0,3D}] = cm^{-3} \). The second term arises from the Fermi pressure (19), and the third term presents the quantum Bohm potential. The last term in the dispersion dependence arises from the exchange interaction. All terms, except the last one, are positive, while the exchange interaction gives a negative contribution decreasing the Langmuir frequency. In spite the quantum nature of the exchange interaction, the force field (20) and its contribution in the dispersion dependence (33) does not contain the Planck constant.

Numerical comparison between the Fermi pressure and the exchange interaction (see Fig. (9)) shows that we can drop contribution of the Fermi pressure in compare with the Coulomb exchange interaction in our regime.

Fig. (9) demonstrates difference of the Langmuir frequencies for the plane-like electron gas and the cylindrical electron gas at \( R = 20 \text{ nm} \).

V. CONCLUSION

Nanotubes with radiuses in interval from 10 nm to 30 nm containing free charge carriers at concentration \( 10^9 \text{ cm}^{-2} \) and smaller reveal large difference of properties from the charge carriers on the plane-like two dimensional structures. To obtain the quantum hydrodynamic description of these objects we have derived corresponding equation of state for the pressure of degenerate electrons. We have included dependence of the pressure on the spin polarization. Zero spin polarization gives the Fermi pressure for electrons on the nanotubes, which appears to be proportional to the third degree of the electron two-dimensional concentration. We have also performed the explicit derivation of the Coulomb exchange interaction force field. The force field is a potential field,
and its potential is proportional to the square of the concentration multiplied by the sum of a constant and the logarithm of the concentration. Dependence of the exchange interaction force field on the spin polarization is derived. It has been shown that the force field monotonically increases with the increase of the spin polarization. Numerical analysis of the difference of the pressures and the exchange interaction forces between plane-like electron gas and the electron gas on the nanotubes have been presented.

We have applied obtained model to study the dispersion of collective excitations of charge carriers on the nanotubes assuming that the external magnetic field is directed parallel to the tube axes. We have considered dispersion of the Langmuir waves obtaining contribution of the Fermi pressure, the exchange Coulomb interaction, and the ratio of the spin polarization.

Finally we should note that we have developed a new set of closed hydrodynamic equations for nanotubes with the electron concentration of order $10^{10}$ cm$^{-2}$ and smaller. Our model allows to study different plasma-wave phenomena in these objects.

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