Optimisation of the lateral buckling strength of corrugated composite material plate by neural networks method

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Abstract. Enhancing the buckling strength of laminated composite materials can be achieved in numerous ways. One method involves corrugating the laminated composite material in one direction. Corrugation provides good buckling strength in the direction perpendicular to the corrugation but a low buckling strength in the same direction as the corrugation. This investigation used composite materials strips implanted in the direction of the laminate’s corrugation to modify the ability to buckle without excessive weight on the laminate. Finite elements were applied to analyse the problem. In addition, to overcome the extensive computational requirements, a neural network (NN) system was utilised to model the study case and then optimise the structure. The NN was trained by the results of the finite elements. The parameters examined and their effects on buckling strength include the number of strips, number of layers of strips and dimension of strips. Results confirmed that the technique of strengthening the laminate using strips in the direction of corrugation waves is beneficial for increasing the critical buckling load. Specifically, the optimisation result presented an increase of 52 times in the buckling load strength versus approximately twice the increase in the mass of the plate. Using the NN to simulate and optimise the structure is a powerful approach that consumes less time than employing the finite element method.

Keywords. Laminate corrugated composite materials; Buckling; Optimization; Neural Networks and Finite elements.

1. Introduction
Composite material corrugated laminates (CMCLs) have higher buckling strength than flat plate without any external reinforcement. This advantage explains the wide application of CMCLs, such as in aerospace and marine sectors, buildings and bridges. Such extensive application is possible because of the property of the buckling strength of CMCLs with regard to the directivity in its properties; that is, they have high strength in the direction perpendicular to the corrugation wave direction (span wise) and weakness in the direction of its corrugation wave [1]. This directivity (anisotropic) feature restricts designers from selecting this type of laminate for structures which include loads in multi-directional paths. Alternatively, this anisotropic property are employed in other applications, such as a flexible or morphing wing[1], [2] and [3].

Several researchers expressed interest in corrugated panels, including those made of metal or composite materials. Some scholars examined the methods for modelling corrugated plates with concepts like thin shell theory, finite elements, finite strips and a planer finite elements method. Other scholars attempted to extend the isotropic theory to an orthotropic one with an equivalent stiffness state.
for the model in [4], [5], [6], [7], [8] and [9]. In addition, extant literature involves diverse types of loads, such as compression (buckling), bending, tension and vibration. For instance, Nahas [10] investigated the buckling behaviour of corrugated plate made from different composite materials and inferred that the corrugated plate of unidirectional composite materials has lower buckling strength than those made of isotropic materials.

Yuning and Zhiping [11] posited a solution for the critical buckling loads for trapezoidal and sinusoidal corrugated laminates under a parabolic distribution of compression loads in a perpendicular direction to the corrugation. This solution was in line with the replacement of the components of equivalent stiffness in an anisotropic flat plate. The proposed solution demonstrated more precise results in estimating the critical buckling loads than the finite elements method. Alshabatat [12] examined the effect of corrugating the plate by alternating its natural frequency. Trapezoidal and sinusoidal waves of corrugation were studied using the finite elements method and revealed that corrugation increases the moment of inertia of the plate and changes the mode shape of the vibrated plate. Thurnherr et al. [13] developed an analytical model to investigate the geometrical nonlinearity of corrugated laminate when tensioned in the corrugation’s direction. The model showed a good agreement when verified experimentally and with the use of finite elements. To study the mechanics that drive the nonlinear stiffness response, Thurnherr et al. [14] also tested six load cases: two tensile load cases, two bending load cases, a shear load case and a torsion load case.

Many methods for optimising the structure are available. Some are represented by the classical method and others implemented modern techniques, such as the neural network (NN), genetic, and fuzzy methods in [15], [16], [17] and [18]. This work investigates the modification of the buckling strength for CMCLs in the direction of the corrugation by integrating composite material strips (CMS) with the same direction of the wave. The finite element method was utilised for modelling CMCLs. In addition, a NN was applied for simulating and accelerating the optimisation. This network was trained using the results of the finite element method. Optimisation was performed by ascertaining the optimal ratio of buckling load to weight.

2. Theory

2.1. Buckling and finite elements

The buckling phenomenon is caused by having a load large enough to destabilize structure. The best method for acquiring information about the critical buckling load for the laminate of a structure is according to the linear eigenvalue, as found in [19], [20] and [21]:

\[ [C][X] = \lambda[E][X] \]  \hspace{1cm} (1)

where \([C]\) and \([E]\) respectively represent stiffness and stress stiffness matrices, \([X]\) is the eigenvector and \(\lambda\) is the eigenvalue (load factor). The lowest value of \(\lambda\) indicates the predicted critical load factor. This value, when multiplied by an applied force, reveals the critical buckling load.

2.2. Neural network and optimization

A function approximation NN was used to formulate the objective functions from the finite element analysis.

A NN is characterised by expressing individual neurons, the weights associated with the different interconnections between neurons, the connectivity of the network and threshold function. The NN provides satisfactory mapping between the input and output spaces. To initiate the mapping, the network should be examined by presenting the given inputs with their coinciding outputs. A good learning process reveals the appropriate relation between inputs and their associated outputs [22]. After training the network, a quick mapping between the inputs and outputs is achieved. This process, in turn, can be employed to increase the effectiveness of the design procedure. Therefore, a NN technique was utilised to represent the objective functions used in the optimal design. Training a NN facilitates the identification of the relation between the strip’s variables (inputs) and the critical buckling load and mass of the corrugated laminates (outputs).
2.3. Optimisation

The first goal of this study is to create the optimal design for the buckling strength problem of CMCLs by specifying the maximum buckling strength and the minimum laminate mass. This goal is expressed as the objective functions indicated below:

Maximize: \( L_c (\bar{A}, \bar{B}, \bar{C}) \) \hspace{1cm} (2)

Minimize: \( W (\bar{A}, \bar{B}, \bar{C}) \) \hspace{1cm} (3)

Where: \( L_c \): Is the critical load, \( W \): Is the weight of corrugated composite materials with strips, \((\bar{A})\): Are the numbers of strip, \((\bar{B})\): Is the number of layers, \((\bar{C})\): Are the wide of strip \((\bar{A}, \bar{B}, \text{and} \bar{C}) \in R^3\)

The optimisation objective was subjected to the following constraints:

\[
0.6 \ A_T - \left| \bar{A}(i) * \bar{C}(j) \right| * l \geq 0 \quad \text{for} \ i = 1,2,3 \quad \text{and} \quad j = 1,2,3 \quad (4)
\]

\[
0.5 \ \sigma_{yield} - \sigma_{buckling} \geq 0 \quad (5)
\]

Where: \( A_T \): The total area of CMCL’s surface, \( l \): The length of strips, \( \sigma_{yield} \): The yield stress of composite materials; \( \sigma_{buckling} \): The stress in the CMCL due to the critical buckling load.

The second goal is to identify the optimal ratio of the critical buckling load to the weight of the CMCL \( \frac{L_c}{W} \). This multiobjective purpose was subjected to the same constraints. It is represented by the following formula:

Best ratio:

\[
\frac{L_c}{W} (\bar{A}, \bar{B}, \bar{C}) \quad (6)
\]

3. Modelling

3.1. CMCL modeling

The structure consisted of a composite material corrugated plate. The corrugation was modelled as a periodic sine wave in one direction with an amplitude of 5 mm and length of 42 mm for one cell (figure. 1). The coordinates x and z represent the wave direction and perpendicular direction of corrugation, respectively. The total length of the laminate is 500 mm in the x-direction and 100 mm in the z-direction (figure. 2).

The material used in this structure is a unidirectional (UD) fibre composite material E-glass/epoxy. To overcome the anisotropic characteristics of this kind of material, the unidirectional laminate was stacked from two layers of cross-ply laminate (0/90). This cross-ply laminate with thickness 1.2 mm was considered one layer to exclude the effect of the fibre direction when an odd number of layers is employed (Figure 3). This bundle of two layers of cross-ply laminate should follow the three assumptions listed below:

- The two cross-ply laminates do not slip (skilfully bonded).
- The laminate has the properties of thin sheets.
- The bond between the two cross-ply laminates is infinitely thin.
Table 1 lists the properties of the unidirectional E-glass/Epoxy fibre composite materials. The base material (without strips) consists of one layer (two cross-ply laminates), whereas various numbers of layers were used for strips. Three values each for the number and dimensions of strips were utilised for analysis in table 2. These variables of the strips are the factors that affect the analysis of the finite elements and the inputs of the NNs. Such values were selected to be congruent with constraint equation (4), that is, the values do not cover all areas of the laminate by strips.
Table 1. Properties of E-glass/Epoxy material for one ply.

| Properties          | Symbol | Value | Unit  |
|---------------------|--------|-------|-------|
| Young’s modulus-1   | $E_1$  | 45    | GPa   |
| Young’s modulus-2   | $E_2$  | 10    | GPa   |
| Young’s modulus-3   | $E_2$  | 10    | GPa   |
| Shear modulus-12    | $G_{12}$ | 5   | GPa   |
| Shear modulus-23    | $G_{23}$ | 3.85 | GPa   |
| Shear modulus-13    | $G_{13}$ | 5   | GPa   |
| Poisson’s ratio-12  | $\nu_{12}$ | 0.3 |       |
| Poisson’s ratio-23  | $\nu_{23}$ | 0.4 |       |
| Poisson’s ratio-13  | $\nu_{13}$ | 0.3 |       |
| Tensile - 1         | $\sigma_{\text{yield-1}}$ | 1.1 | GPa   |
| Tensile - 2         | $\sigma_{\text{yield-2}}$ | 1.1 | GPa   |
| Tensile - 3         | $\sigma_{\text{yield-3}}$ | 35  | MPa   |
| Compression -1      | $\tau_1$ | -675 | MPa   |
| Compression -2      | $\tau_2$ | -120 | MPa   |
| Compression -3      | $\tau_3$ | -120 | MPa   |
| Thickness of ply    | $t_{\text{ply}}$ | 0.6 | mm    |

Table 2. Strip’s variables.

| Variable            | Symbols | Values |
|---------------------|---------|--------|
| Number of strips    | A       | 1 2 3  |
| Number of layers    | B       | 2 3 4  |
| Dimension of strips (mm) | C      | 10 15 20 |

Figure 4 shows an example of the distribution of the strips in the y-direction of the laminate. This distribution differs according to the number and dimension of strips.

3.2. Finite element modelling

Workbench ANSYS was used for the section on finite elements. A shell element compatible with this kind of structure was applied for meshing the model. The element size was selected according to the mesh density buckling load convergence and time consumption for processing. Figure 5 shows the convergence in the buckling load for element sizes smaller than 2.5 mm. The test was conducted on the CLMC without strips. To balance the processing time with the convergence of the buckling load, a size of 2 mm was chosen as the element size. For element sizes smaller than 2 mm, processing time increases considerably without a significant difference in load. The convergence test showed that the difference between the buckling loads at 2 mm and 0.5 mm was 0.00029 N, with an increase in time of more than seven hours. The two side edges of laminate were fixed in the z-direction but were free in the x- and y-directions. As for the other two ends, one was fixed and the other had a load applied to it (figure 6).
**Figure 4.** Distribution of strips.

**Figure 5.** Convergence mesh density test.
3.3. Finite element modelling
In this work, the NN function approximation method was used to predict a relationship between the strip’s variables and the buckling load and mass of the laminate. This relation was employed to simulate and optimise the analyses of the laminate. MATLAB software was utilised to design the NN architectures and algorithm. The hyperbolic tangent function is the transfer function ($\tanh$) for hidden layers and is a purely linear function for the output layer [22]:

$$\tau_{sec}(x) = \tanh(x) = \frac{e^{\beta x} - e^{-\beta x}}{e^{\beta x} + e^{-\beta x}}$$  (7)

A back-propagation training algorithm was utilised for NN training. The training data were extracted from the finite element results. The training input data include the number of strips, number of layers for strips and the width of the strips, with three different values for each variable. Conversely, the data on target outputs include the critical buckling load and mass. The structure of the NN with the best fit (figure 7) involves the following elements:

I. One input layer
II. Two hidden layers:
   a. First hidden layer with 25 neurons
   b. Second hidden layer with 10 neurons
III. Output layer with two neurons
The resulting NN had 0.9984 and 0.9938 correlation coefficients for buckling load (figure 8) and mass (figure 9), respectively. However, for the performance (figure 10) and regression curves (figure 11), the samples were divided to 70% for training, 15% for testing and 15% for validating.

**Figure 7.** The architecture of the best fit neural network.

**Figure 8.** Buckling load comparison between FEM and NN results.

**Figure 9.** Mass comparison between FEM and NN results.
3.4. Methodology
A program built using MATLAB was employed along with the designed NN tool to achieve optimisation. Figure 12 presents the optimisation flowchart.
4. Results

4.1. Finite element analysis

Finite element analysis was performed on 27 trials to cover all the possible combinations for each strip’s parameters. The tests generated 27 critical buckling loads with conformable mass table 3. For each parameter’s combination, six modes for buckling were specified when running the simulation. The mode with the smallest load was chosen, whereas the others indicated the deformation behaviour at larger critical buckling loads. These six modes presented a sample of the modes for one combination of the strip’s variables (three strips, two layers and 20 mm width per strip). Moreover, the six modes showed similar deformations for all parameter combinations (figure 13).
Table 3. Finite elements result.

| No. of trail | \( \bar{A} \) | \( \bar{B} \) | \( \bar{C} \) | \( L_c \) | m  | \( L_c/W \) |
|--------------|--------------|--------------|--------------|---------|----|-----------|
| 1            | 1            | 2            | 10           | 5.691   | 0.138 | 4.203     |
| 2            | 2            | 2            | 10           | 8.612   | 0.151 | 5.815     |
| 3            | 3            | 2            | 10           | 11.267  | 0.163 | 7.046     |
| 4            | 1            | 3            | 10           | 13.16   | 0.151 | 8.886     |
| 5            | 2            | 3            | 10           | 23.557  | 0.176 | 13.64     |
| 6            | 3            | 3            | 10           | 33.659  | 0.201 | 17.068    |
| 7            | 1            | 4            | 10           | 27.732  | 0.163 | 17.343    |
| 8            | 2            | 4            | 10           | 52.708  | 0.201 | 26.728    |
| 9            | 3            | 4            | 10           | 77.363  | 0.239 | 32.991    |
| 10           | 1            | 2            | 15           | 6.064   | 0.144 | 4.292     |
| 11           | 2            | 2            | 15           | 10.099  | 0.163 | 6.316     |
| 12           | 3            | 2            | 15           | 14.197  | 0.182 | 7.954     |
| 13           | 1            | 3            | 15           | 17.234  | 0.163 | 10.778    |
| 14           | 2            | 3            | 15           | 32.612  | 0.201 | 16.538    |
| 15           | 3            | 3            | 15           | 47.618  | 0.239 | 20.306    |
| 16           | 1            | 4            | 15           | 39.06   | 0.182 | 21.882    |
| 17           | 2            | 4            | 15           | 76.282  | 0.239 | 32.53     |
| 18           | 3            | 4            | 15           | 112.98  | 0.295 | 39.039    |
| 19           | 1            | 2            | 20           | 8.401   | 0.151 | 5.673     |
| 20           | 2            | 2            | 20           | 12.952  | 0.176 | 7.5       |
| 21           | 3            | 2            | 20           | 18.349  | 0.201 | 9.305     |
| 22           | 1            | 3            | 20           | 23.291  | 0.176 | 13.486    |
| 23           | 2            | 3            | 20           | 42.744  | 0.226 | 19.28     |
| 24           | 3            | 3            | 20           | 63.018  | 0.276 | 23.271    |
| 25           | 1            | 4            | 20           | 48.388  | 0.201 | 24.566    |
| 26           | 2            | 4            | 20           | 100.97  | 0.276 | 37.286    |
| 27           | 3            | 4            | 20           | 150.13  | 0.352 | 43.539    |
4.2. Neural network results
The resulting NN has a good fit between the target and the NN outputs. Figure 14 and figure 15 depict the response surfaces for the outcome load and mass, respectively. Both surfaces were used in the optimisation of the laminate.

**Figure 13.** Six buckling modes for one set of parameters.

**Figure 14.** Critical Buckling load from NN.

**Figure 15.** Mass from NN.
Figure 16 to figure 18 show the samples of comparison between the finite element and NN results when only one strip was utilised.

![Graph](image)

**Figure 16.** Comparison between FE and NN results (wide of the strip -10 mm).

![Graph](image)

**Figure 17.** Comparison between FE and NN results (wide of the strip -15 mm).

![Graph](image)

**Figure 18.** Comparison between FE and NN results (wide of the strip -20 mm).

### 4.3 Optimisation
Optimisation was conducted according to the maximum ratio of the buckling load to the mass of the laminate and the corresponding combination of the strip’s parameters. The results of optimisation are:

- No. of strips: 3
- No. of layers: 4
- The wide of the strip: 19 mm
The critical Buckling load: 150.4932N
The mass: 0.3511 Kg
Lc/W: 43.693

To verify these results, a new simulation was performed using a finite elements model with the same values for the strip’s parameters (3, 4 and 19 mm). The outcome is as follows:

The critical buckling load: 149.12 N
The mass: 0.34024 Kg
Lc/W: 44.677

The finite elements’ simulation results for CMCL without strips are as follows:

The critical Buckling load: 2.8465 N
The mass: 0.12567 Kg
Lc / W: 2.31

Comparison of the buckling loads between the CLMC without strips and those from the optimisation indicate an increase of 2.79 times in mass and 52.87 times in the consolidation in the buckling load.

5. Conclusion
This study included two parts. The first part investigated the benefit of integrating the composite materials strips with the CMCL for strengthening the capabilities of buckling. For this analysis, the finite elements method was used. The results from this method were utilised to perform the second part of this research. The second part implemented the NN method to simulate the CMCL computations for optimisation and analysis. The outcomes from these two parts revealed that the addition of strips enhanced the stability and buckling strength of the laminate in the direction of corrugation. Using NNS also attained good feasibility in the simulation and optimisation of corrugated buckling phenomenon, acceptable time consumption and satisfactory function fitting between the strip parameters and buckling responses. In addition, utilising NNS generated a broader range of variables for strips relative to that obtained by finite element analysis. This outcome was demonstrated in the section on optimisation wherein the NN results presented values that were absent in the list of input parameters for finite element simulation and NN training parameters.

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