A Unified Hidden-Sector-Electroweak Model, Paraphotons and the X-Boson

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Abstract

Our contribution sets out to investigate a gauge model based on an $SU_L(2) \times U_R(1)_J \times U(1)_K$-symmetry group whose main goal is to accommodate, in the distinct phases the Higgs sector sets up according to different symmetry-breaking patterns, the para-photon of the dark matter sector, a heavy scalar whose mass is upper-bounded by 830 GeV and the recently discussed 17 MeV X-boson. As a result, there also emerge in the spectrum an extra massive charged fermion along with an additional neutral Higgs; their masses are fixed according to the particular way the symmetry breakings take place. In all situations contemplated here, we are committed with the 246 GeV electroweak breaking scale.

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I. INTRODUCTION

The search for new particles and interactions beyond the Standard Model (SM) has been challenging High-Energy Physics, both Theoretical and Experimental, over the recent decades. The results from the LHC’s ATLAS- and CMS-Collaborations may point to the existence of a (new) fifth interaction [1, 2]. Analysing data of pp-collisions at a center-of-mass energy scale of $\sqrt{s} = 8$ TeV has revealed the masses for the heavy $W'$- and $Z'$-bosons [3], which may be evidence for new Physics communicated from very high energies to the TeV-scale. We should keep in mind that the introduction of an extra Higgs may be needed to explain the heavy mass of the new bosons at the TeV-scale. These masses are associated to a new range of vacuum expectation values (VEVs) of supplementary Higgs scalars.

A well-known model in this direction is based on an $SU_L(2) \times SU_R(2) \times U(1)_{B-L}$-gauge symmetry [4–9]. The extra $SU_R(2)$-subgroup is introduced to account for the new $W'$- and $Z'$-bosons, besides the already known $W^\pm$- and $Z^0$-weak mediators of the Glashow-Salam-Weinberg (GSW) model. The right-handed sector also introduces fermion doublets with a charged lepton and its associated neutrino of right chirality. Two scalars doublets constitute the Higgs content so that the gauge bosons acquire their masses. The first one is introduced to break the $SU_R(2) \times U(1)_{B-L}$-gauge symmetry at the VEV scale $u > v = 246$ GeV. As a consequence, the hypercharge, $Y$, appears as a combination of generators of $SU_R(2) \times U(1)_{B-L}$. Next, the Standard Model Higgs breaks the remaining gauge symmetry to yield the electromagnetic interaction, $U(1)_{em}$. The scheme of these two spontaneous symmetry breaking is represented by \[ SU_L(2) \times SU_R(2) \times U(1)_{B-L} \overset{u}{\rightarrow} SU_L(2) \times U(1)_{em} \]

where $u$ is a VEV bringing the TeV-scale of the LHC experiments.

More recently, in a particular Nuclear Physics experiment, anomalies in the decay of the excited state of $^{8}\text{Be}^*$ to its ground state have suggested the existence of a new neutral boson, $X$, through the decay mode $^{8}\text{Be}^* \rightarrow 8\text{Be} + X$ [10]. The $X$-boson immediately decays into an electron-positron pair $X \rightarrow e^+ + e^-$. It is a vector-type spin-1 particle, like the photon, with mass around $m_X = 17$ MeV. Its origin could in principle be traced back to an extra gauge symmetry, $U(1)_X$, in addition to the symmetries of the SM. The discovery of the $X$-boson may be pointing to the existence of a fifth fundamental interaction in the Nature.

The effective Lagrangian proposed to describe this extra boson is [11]

\[
\mathcal{L}_{eff} = -\frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\chi}{2} X_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_X^2 X_{\mu} X^{\mu} + J_{\mu} X_{\mu},
\]

where $J^\mu$ is the current coupled to $X^\mu$

\[
J^\mu = \sum_\Psi e_{\chi,\Psi} \bar{\Psi} \gamma^\mu \Psi
\]

where $\Psi$ could be any fermion of the SM. The $X^\mu$-boson can also have a chiral interaction with the SM leptons via an axial current [12]

\[
e_{\chi,\Psi} \bar{\Psi} \gamma^\mu \gamma_5 \Psi.
\]

Here, the parameter $\chi$ mixes the $X^\mu$-boson with the usual electromagnetic (EM) photon, $A^\mu$, where $X^{\mu\nu} = \partial^\mu X^\nu - \partial^\nu X^\mu$ is the field-strength tensor for $X^\mu$, and $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, $\partial^\mu = \partial_\mu - i e X_\mu$, and $\gamma_5 \Psi = \bar{\Psi} \gamma_5$.
the corresponding tensor field of the photon. It is clear that the massive term spoils the $U(1)_X$-symmetry, and the Lagrangian exhibits only EM gauge symmetry, $U_{em}(1)$.

In the present paper, we take a different path and we propose a description based on an $SU_L(2) \times U_R(1)_J \times U(1)_K$-gauge symmetry. Endowing the model with a spontaneous symmetry breaking (SSB) mechanism spoils one of Abelian factors present in $U_R(1)_J \times U(1)_K$, so that a non-zero mass, $m_X$, like in (1) can be generated. Consequently, the experimental value of 17 MeV defines the scale of a vacuum expectation value (VEV), and we can estimate a range of mass for the extra Higgs field that came in. Furthermore, the $U(1)_K$-group allows to include a new charged fermion, in addition to the leptons and quarks of the SM. The same VEV responsible for a non-trivial $m_X$ also gives mass to the new charged fermion.

There is also in the literature a great deal of interest on the activity related to the phenomenology of hidden sector para-photons [13–15]. The para-photon is a neutral vector boson with a sub-eV mass and with the property of electromagnetic interactions with coupling constants referred to as millicharges. The para-photons are characterized by a mixed mass term with the photon and they appear with an extra gauge factor $U(1)$ in the full symmetry group.

Based on the model built up to describe the hidden sector para-photon, our attempt in this contribution consists in setting up a gauge model with an $SU_L(2) \times U_R(1)_J \times U(1)_K$-symmetry group that is twofold, according to the symmetry-breaking pattern. There are two Higgs scalars and, as we shall see, there can occur two situations: the extra Higgs breaks the extra symmetry $U(1)$ above or below the 246 GeV electroweak scale. These two possibilities open up new scenarios that may accommodate the extra Higgs an its associated extra gauge Boson in different scales: MeV, GeV and TeV-scales, according to the choice of the three parameters.

The organization of this paper follows the outline below: in Section II we present the model based on an $SU_L(2) \times U_R(1)_J \times U(1)_K$-symmetry. Sectors of fermions and scalar bosons are introduce with quantum numbers consistent with gauge invariance. Section III is divided in two Subsections. The first one is devoted to introduce the scalars fields as the Higgs sector to identify the hypercharge. In the second Subsection, we obtain the masses of $W^\pm$, $Z^0$ and the new particles as well; we also identify the basis of physical fields. Section IV is also divided in two Subsections, where we work out the renormalized propagators and the interactions of $W^\pm$, $Z^0$, $\gamma$- and $X$-bosons with the fermions. We also present results for the mass of the $X$-boson and we estimate the mass of the extra Higgs in a scenario of TeV-scale. Section VI exhibits the interpretation of the Higgs model at a lower scale, the MeV-scale, which also is useful to contemplate para-photons. Finally, our Concluding Comments are cast in Section VII.
II. SETTING UP THE MODEL

In this Section, we introduce the sector of gauge fields and fermions of the model \( SU_L(2) \times U_R(1)_J \times U(1)_K \)-model we shall discuss in the sequel. The fermion Lagrangian reads as follows below:

\[
\mathcal{L}_{\text{leptons}} = \bar{\Psi}_L i \not{D} \Psi_L + \bar{\ell}_R i \not{D} \ell_R + \bar{\zeta} i \not{D} \zeta ,
\]

where we have introduced the new \( \zeta \)-fermion associated to the \( U(1)_K \)-group; the notation \( \ell \) indicates the usual leptons of the SM, i.e., \( \ell = (e, \mu, \tau) \). The slashed notation corresponds to the contraction of the covariant derivatives with the usual Dirac’s matrices \( \not{D} = \gamma^\mu D^\mu \).

The \( \zeta \)-fermion can be interpreted as a hidden (dark matter) fermion. The covariant derivatives acting on the fermions of the model are cast below:

\[
D_\mu \Psi_L = \left( \partial_\mu + i g A_\mu^a \frac{\sigma^a}{2} + i J_L g' B_\mu + i K_L g'' C_\mu \right) \Psi_L ,
\]

\[
D_\mu \ell_R = \left( \partial_\mu + i J_R g' B_\mu + i K_R g'' C_\mu \right) \ell_R ,
\]

\[
D_\mu \zeta = \left( \partial_\mu + i J_\zeta g' B_\mu + i K_\zeta g'' C_\mu \right) \zeta ,
\]

where \( A^a = \{ A_\mu^1, A_\mu^2, A_\mu^3 \} \) are the gauge fields of \( SU_L(2) \), \( B_\mu \) is the Abelian gauge field of \( U_R(1)_J \), and \( C_\mu \) the similar one to \( U(1)_K \). Here, we have chosen the symbol \( J \) to represent the generator of \( U_R(1)_J \), \( K \) is the generator of \( U(1)_K \), and the Pauli matrices \( \frac{\sigma^a}{2} (a = 1, 2, 3) \) satisfy the Lie algebra

\[
\left[ \frac{\sigma^a}{2} , \frac{\sigma^b}{2} \right] = i \varepsilon^{abc} \frac{\sigma^c}{2} .
\]

In (5), \( g, g' \) and \( g'' \) are dimensionless gauge couplings. As usual, \( \Psi_L \) describes a doublet with a charged lepton and its neutrino partner (both left-handed). It transforms under the fundamental representation of \( SU_L(2) \) as

\[
\Psi_L(x) = \left( \begin{array}{c} \nu_{\ell L} \\ \ell_L \end{array} \right) \mapsto \Psi_L'(x) = e^{ig \frac{a^a}{2} \omega^a(x)} \Psi_L(x) .
\]

Under the Abelian factors, \( U_R(1)_J \) and \( U(1)_K \), these fermions transform as follows:

\[
\ell_R \mapsto \ell_R'(x) = e^{ig' J_R f(x)} \ell_R(x) ,
\]

\[
\zeta \mapsto \zeta'(x) = e^{ig'' K_\zeta h(x)} \zeta(x) ,
\]

where \( \omega^a = \{ \omega^1, \omega^2, \omega^3 \} \), \( f \) and \( h \) are arbitrary functions of the space-time. Defining the following gauge transformations

\[
A_\mu \mapsto A_\mu' = U A_\mu U^{-1} + \frac{i}{g} \partial_\mu U U^{-1} ,
\]

\[
B_\mu \mapsto B_\mu' = B_\mu - \partial_\mu f(x) ,
\]

\[
C_\mu \mapsto C_\mu' = C_\mu - \partial_\mu h(x) ,
\]
where $U(x) = e^{ig\frac{x^a}{2}\omega^a(x)}$; the covariant derivatives undergo the following transformations:

$$D_\mu \Psi_L \longrightarrow (D_\mu \Psi_L)'(x) = e^{ig\frac{x^a}{2}\omega^a(x)} D_\mu \Psi_L(x),$$

$$D_\mu \ell_R \longrightarrow (D_\mu \ell_R)'(x) = e^{ig'J_R f(x)} D_\mu \ell_R(x),$$

$$D_\mu \zeta \longrightarrow (D_\mu \zeta)'(x) = e^{ig''K_h(x)} D_\mu \zeta(x).$$

(10)

In the sector of gauge fields, the corresponding field-strength tensors are defined by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig \, [A_\mu, A_\nu],$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu,$$

$$C_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu,$$

and writing $F^{\mu\nu}$ in the basis $\frac{x^a}{2} = \{\frac{x^1}{2}, \frac{x^2}{2}, \frac{x^3}{2}\}$, where its components are given by

$$F^{a}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g \varepsilon^{abc} A^b_\mu A^c_\nu.$$

(12)

The Lagrangian for the gauge bosons is then given by

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{2} \text{tr} \left( F_{\mu\nu}^2 \right) - \frac{1}{4} B_{\mu\nu}^2 - \frac{1}{4} C_{\mu\nu}^2 - \frac{\chi}{2} B_{\mu\nu} C^{\mu\nu},$$

(13)

where $\chi$ is a real parameter that mixes the Abelian gauge fields of the $U_R(1)_J \times U(1)_K$-subgroup. Its currently estimated value is $10^{-3} < \chi < 10^{-6}$ for models that discuss hidden photons as dark matter [15]. With the field transformations shown above, the action is clearly $SU_L(2) \times U_R(1)_J \times U(1)_K$-invariant.

The interactions that emerge from (4) reveal how the leptons/neutrinos of the SM and the fermion-$\zeta$ of the dark sector interact with the all vector gauge fields of the model. We can initially write these interactions as

$$\mathcal{L}^{\text{int}}_{\text{leptons-gauge}} = -\frac{g}{\sqrt{2}} \bar{\nu}_L \left( A^1 - i A^2 \right) \ell_L - \frac{g}{\sqrt{2}} \bar{\ell}_L \left( A^1 + i A^2 \right) \nu_L$$

$$-\Psi \left( g A^3 I^3 + J g' \beta + K g'' \gamma \right) \Psi,$$

(14)

where we have defined $I^3 = \sigma^3/2$, for simplicity. The $\Psi$-field stands for any fermion of the model: $\Psi = \{ \Psi_L, \ell_R, \zeta \}$ in which $I^3 = 0$ for $\Psi = \ell_R$ or $\Psi = \zeta$. The previous expression motivates us to define the charged bosons $W^\pm$ in the usual manner, i.e., $\sqrt{2} W^\pm_\mu = A^1_\mu \mp i A^2_\mu$, which defines the interaction between leptons/neutrinos and bosons $W^\pm$. In terms of the interaction involving leptons, neutrinos, fermion-$\zeta$ and neutral bosons we can observe the need to define which one is the hypercharge generator of the model. This task will be done with help of a Higgs sector. In the next Section, we shall work out the Higgs sector in association with the hypercharge generator; the Higgses are so arranged to break the $U_R(1)_J \times U(1)_K$-symmetry in such a way that the Abelian boson and the fermion of the hidden sector both acquire mass. Here, we anticipate that this breaking shall be re-assessed in a different scenario, characterized by an energy scale much lower to the usual electroweak breaking scale.
III. THE HIGGS SECTOR AND THE HYPERCHARGE GENERATOR

A. The Higgs fields and their couplings

We have, up to now, proposed the general features of an Abelian gauge model to describe a possible sector of new particles beyond SM. This model exhibits an $SU_L(2) \times U_R(1)_J \times U(1)_K$-gauge symmetry. In this Section, we work out the Higgs sector, where two independent scalar fields are introduced; one of them will break the Abelian subgroup to give mass to the X-boson. Furthermore, these scalars justify a fundamental origin of the hypercharge generator. After the first spontaneous symmetry breaking (SSB) takes place, the residual (unbroken) symmetry is given by

$$SU_L(2) \times U_R(1)_J \times U(1)_K \overset{\xi}{\rightarrow} SU_L(2) \times U_Y(1),$$

where the $U_Y(1)$-group comes out as the mixing of the $U_R(1)_J$- and $U(1)_K$-subgroups. We denote this Higgs field by $\xi$. We next introduce a second Higgs field, $\phi$, to break the residual electroweak $SU_L(2) \times U_Y(1)$-symmetry and, consequently, it yields the masses for $W^\pm$ and $Z^0$. Finally, we end up with the exact electromagnetic symmetry.

$$SU_L(2) \times U_R(1)_J \times U(1)_K \overset{\xi}{\rightarrow} SU_L(2) \times U_Y(1) \overset{\phi}{\rightarrow} U_{em}(1).$$

To do that, we start off from the Higgs Lagrangian below:

$$L_{Higgs} = (D_\mu \xi)^\dagger D^\mu \xi - \mu_\xi^2 (\xi^\dagger \xi) - \lambda_\xi (\xi^\dagger \xi)^2 + (D_\mu \phi)^\dagger D^\mu \phi - \mu_\phi^2 (\phi^\dagger \phi) - \lambda_\phi (\phi^\dagger \phi)^2 - \lambda_\xi\phi (\xi^\dagger \xi) (\phi^\dagger \phi) - x_\ell \bar{\zeta}_L \ell_R - x_\ell^* \bar{\ell}_R \xi^\dagger \zeta_L
- y_\ell \bar{\Psi}_L \phi \ell_R - y_\ell^* \bar{\ell}_R \phi^\dagger \Psi_L
- z_\ell \bar{\Psi}_L \phi \zeta_R - z_\ell^* \bar{\zeta}_R \phi^\dagger \Psi_L,$$

where $\mu_\xi, \mu_\phi, \lambda_\xi, \lambda_\phi$ and $\lambda_\xi\phi$ are real parameters, and $\{x_\ell, y_\ell, z_\ell\}$ are Yukawa (complex) coupling parameters needed for the fermions to acquire non-zero masses. The covariant derivative of (17) acts on the $\xi$-Higgs as follows:

$$D_\mu \xi(x) = \left( \partial_\mu + i J_\xi g'B_\mu + i K_\xi g''C_\mu \right) \xi(x).$$

The $\xi$-field is a scalar singlet with the transformations under $U_R(1)_J \times U(1)_K$ given below:

$$\xi \mapsto \xi'(x) = e^{iJ_\xi g'f(x)} \xi(x),$$

$$\xi \mapsto \xi'(x) = e^{iK_\xi g''h(x)} \xi(x).$$

Using the previous gauge transformations, it can be readily checked that $D_\mu \xi$ has the same transformation as (19). The Yukawa interactions are gauge invariant under $U_{JR}(1) \times U_K(1)$, once we impose that $-J_{\xi L} + J_\xi + J_R = 0$, and $-K_{\xi L} + K_\xi + K_R = 0$; then, the Lagrangian (17) is invariant under gauge symmetry (19).
The X-Boson and a Unified Hidden-Sector-Electroweak Model

The second Higgs field, $\Phi$, couples to the gauge fields through the covariant derivatives which read as:

$$D_\mu \Phi(x) = \left( \partial_\mu + ig A^a_\mu \frac{\sigma^a}{2} + ig' J_\Phi B_\mu \right) \Phi(x) ,$$

(20)

where the $J_\Phi$ is the generator of $\Phi$-Higgs corresponding to the $U_R(1)_J$-subgroup. The $\Phi$-field is a complex scalar doublet that has the gauge transformation under $SU_L(2)$ given by

$$\Phi \mapsto -i \omega^a(x) \Phi(x) ,$$

(21)

and under $U_R(1)_J$ subgroup, it has the transformation

$$\Phi \mapsto -i g' J_\Phi f(x) \Phi(x) .$$

(22)

The Yukawa interactions of $SU_L(2)$ also guarantee the invariance under $U_R(1)_J$, whenever the generators satisfy the relations $-J_L + J_\Phi + J_R = 0$ and $-J_L + J_\Phi + J_{\zeta_R} = 0$. Then, the Higgs potential that has been adopted in (17) is reproduced below:

$$V_H (|\Xi|, |\Phi|) = \mu^2 |\Xi|^2 + \lambda_\Xi |\Xi|^4 + \mu_\Phi^2 |\Phi|^2 + \lambda_\Phi |\Phi|^4 + \lambda_{\Xi\Phi} |\Xi|^2 |\Phi|^2 ,$$

(23)

and all the Lagrangian is invariant under the symmetry $SU_L(2) \times U_R(1)_J \times U(1)_K$.

The minimal value of the Higgs potential is attained by the non-trivial vacuum expectation value (VEV) of the Higgs field that keeps the full invariance of the model. It must be constant over space-time, $\langle \Xi \rangle_0 = u/\sqrt{2}$, where $u$ is the non-trivial VEV of the scalar field $\Xi$, defined by $u := \sqrt{-\mu_\Xi^2}$, with $\mu_\Xi^2 < 0$. We choose the parametrization of the $\Xi$-complex field as

$$\Xi(x) = \left( \frac{u + F(x)}{\sqrt{2}} \right) e^{i \eta(x) \over u} ,$$

(24)

where $F$ and $\eta$ are real functions. For the $\Phi$ Higgs, the minimal value of the potential is represented by the non-trivial VEV that we call $v$, where $v \gg u$ which breaks the electroweak symmetry of (17). Both scalar fields respect translational space-time symmetry. Thus, we choose the VEV of $\Phi$ as the constant doublet:

$$\langle \Phi \rangle_0 = \left( \begin{array}{c} 0 \\ v \sqrt{2} \end{array} \right) ,$$

(25)

where $\langle \phi^{(+)} \rangle_0 = 0$, $\langle \phi^{(0)} \rangle_0 = v$, and $v := \sqrt{-\mu_\Phi^2}$, when $\mu_\Phi^2 < 0$. We can choose the parametrization of the $\Phi$-complex field as given by

$$\Phi(x) = \left( \frac{v + H(x)}{\sqrt{2}} \right) e^{i \frac{\chi^a(x)}{\sqrt{2}}} \left( \begin{array}{c} 0 \\ \chi^a(x) \end{array} \right) ,$$

(26)

where $H$ and $\chi^a = \{ \chi^1, \chi^2, \chi^3 \}$ are real functions. Under these conditions, the Higgs potential is plotted below. We observe the four peaks that represent the vacuum states of the scalars fields.
FIG. 1: The Higgs potential as function of the variables $|\Xi|$ and $|\Phi|$ for $\mu_\Xi < 0$, $\mu_\Phi < 0$ and when $\lambda_\Xi > 0$, $\lambda_\Phi > 0$ and $\lambda_\Xi \Phi > 0$. The degenerated vacuum of the Higgs fields are illustrated by the four down peaks.

We shall initially analyze the sector $U_R(1)_J \times U(1)_K$. The $u$-VEV defines a scale for the breaking of the residual Abelian symmetry, in which one of the Abelian gauge fields acquires a mass term. The Abelian sector is given by

$$L_{\text{gauge}}^{B-C} = -\frac{1}{4} B_{\mu\nu}^2 - \frac{1}{4} C_{\mu\nu}^2 - \frac{\chi}{2} B_{\mu\nu} C^{\mu\nu} + \frac{u^2}{2} \left( g' B^\mu - g'' C^\mu \right)^2 + \frac{1}{2} \left( \partial_\mu \eta \right)^2 + u \partial_\mu \eta \left( g' B^\mu - g'' C^\mu \right),$$

(27)

where, for convenience, we have chosen $J_\Xi = -K_\Xi = +1$. The $B^\mu - C^\mu$-sector suggests us to introduce the orthogonal $SO(2)$-transformation given in (27) to eliminate the mixing terms:

$$C_\mu = \frac{1}{\sqrt{2}} X_\mu + \frac{1}{\sqrt{2}} Y_\mu,$$

$$B_\mu = -\frac{1}{\sqrt{2}} X_\mu + \frac{1}{\sqrt{2}} Y_\mu,$$

(28)

where the constant couplings satisfy the relation $g' = g''$. Thus, if we redefine $X^\mu = \sqrt{1-\chi} X^\mu$ and $Y^\mu = \sqrt{1-\chi} Y^\mu$, the Lagrangian (27) can be brought to the form

$$L_{\text{gauge}}^{X-Y} = -\frac{1}{4} X_{\mu\nu}^2 - \frac{1}{4} Y_{\mu\nu}^2 + \frac{1}{2} m_X^2 X^2 + \frac{1}{2} \left( \partial_\mu \eta \right)^2 - \frac{1}{2} m_X \partial_\mu \eta X^\mu,$$

(29)

where the mass of $X^\mu$ is identified in terms of the $u$-VEV as

$$m_X = \frac{\sqrt{2} g' u}{\sqrt{1-\chi}}.$$

(30)
The mass term for $X^\mu$ reveals a massive neutral boson associated to scale of the $u$-VEV. We identify the $X^\mu$-gauge boson as the new heavy particle beyond the observed spectrum of the SM, or, we can interpret it as a hidden photon candidate of the dark matter sector.

The initial gauge symmetry is spontaneously broken down to an Abelian symmetry which we shall refer to as $U_Y(1)$, i.e.,

$$SU_L(2) \times U_R(1)_J \times U(1)_K \xrightarrow{\langle \Xi \rangle} SU_L(2) \times U_Y(1) .$$

This can be readily understood by checking the symmetry that remains unbroken. After this SSB takes place, the transformation of the $X^\mu$- and $Y^\mu$-gauge bosons are given by

\begin{align}
X_\mu &\longrightarrow X'_\mu = X_\mu \\
Y_\mu &\longrightarrow Y'_\mu = Y_\mu - \partial_\mu \Lambda(x) ,
\end{align}

where the function $\Lambda$ is defined by $\Lambda = \sqrt{2} f = \sqrt{2} h$.

To define the weak hypercharge generator, we go back to the interactions between leptons/neutrinos and the gauge bosons, as highlighted in (14). We check that it can be written in terms of the $X^\mu$- and $Y^\mu$-fields so that the hypercharge turns out to be

\begin{align}
\mathcal{L}^{\text{int}}_{\text{leptons} - XY} = -\bar{\Psi} \left[ g A^3 I^3 + \frac{g'}{\sqrt{2 - 2 \chi}} ( J + K ) Y \right] \Psi \\
&- \bar{\Psi} \frac{g'}{\sqrt{2 - 2 \chi}} ( -J + K ) \chi \Psi .
\end{align}

Here, $Y^\mu$ is the gauge field associated with the hypercharge generator, which is taken as

\begin{equation}
g_Y Y := \frac{g'}{\sqrt{2 - 2 \chi}} ( J + K ) ,
\end{equation}

where $g_Y$ is the coupling constant that measure the interaction between $Y$ and the left- or right-components, after this first SSB occurs. We write

\begin{equation}
g_Y = \frac{g'}{\sqrt{2 - 2 \chi}} = \frac{g''}{\sqrt{2 - 2 \chi}} ,
\end{equation}

and this agrees with the coupling constant of the $U_Y(1)$-symmetry. So, we obtain that

\begin{equation}
Y = J + K .
\end{equation}

As an example, in the case of Higgs-$\Xi$, for $g_Y \neq 0$, we have

\begin{equation}
Y_\Xi = J_\Xi + K_\Xi = +1 - 1 = 0 .
\end{equation}

Therefore, the fermion transformation under $U_Y(1)$ reads as

\begin{align}
\ell_R &\longrightarrow \ell'_R(x) = e^{i g_Y J_R \Lambda(x)} \ell_R(x) , \\
\zeta &\longrightarrow \zeta'(x) = e^{i g_Y K_\zeta \Lambda(x)} \zeta(x) .
\end{align}
In view of these transformations, we establish that the $\zeta$-hidden fermion is neutral under $U_R(1)_J$, so that we impose $J_\zeta = 0$, and analogously, the charge of lepton right with respect to $U(1)_K$ is $K_R = 0$. The hypercharge generator yields the interactions

\[ \mathcal{L}_{\text{leptons-}XY}^{\text{int}} = -\bar{\Psi} \left( g A^3 + g_Y Y \right) \Psi - \bar{\Psi} g_Y (-Y + 2 K) X \Psi . \]  

We are now ready to obtain how the $A^3 - Y_\mu$-mixing defines the physical particles $Z^0$ and the (massless) photon, and after that, the electric charges of the particles. To accomplish this task, we need to break the residual electroweak symmetry $SU_L(2) \times U_Y(1)$.

### B. The physical fields and the masses of $W^\pm$ and $Z^0$

For this goal, we return to the Higgs sector using the transformation (28) in the covariant derivative (20)

\[ D_\mu \Phi(x) = \left( \partial_\mu + i g A^a_\mu \frac{\sigma^a}{2} + i \frac{g'}{\sqrt{2-2\chi}} J_\phi Y_\mu - i \frac{g'}{\sqrt{2-2\chi}} J_\phi X_\mu \right) \Phi(x) , \]  

and the hypercharge generator $Y_\Phi$ is identified as

\[ g_Y Y_\Phi = \frac{g'}{\sqrt{2-2\chi}} J_\phi . \]  

The relation (34) ensures that $J_\phi = Y_\phi$. Thus the covariant derivative is

\[ D_\mu \Phi(x) = \left( \partial_\mu + i g A^a_\mu \frac{\sigma^a}{2} + i g_Y Y_\Phi Y_\mu - i g_Y Y_\Phi X_\mu \right) \Phi(x) . \]  

As we know from the electroweak model, the value of the hypercharge of the Higgs represented by the doublet above is $Y_\Phi = +1/2$; then, the relation (35) for this Higgs gives us $K_\Phi = 0$. Thus, the parametrization (26) yields

\[ \mathcal{L}_{\text{gauge-}0} = -\frac{1}{2} W^\pm W^{\mu\nu} + \frac{g^2 v^2}{4} W^\mu W^\mu - \frac{1}{4} \left( \partial_\mu A^3_\nu - \partial_\nu A^3_\mu \right)^2 + \frac{1}{2} \frac{v^2}{4} \left( -g_Y X_\mu + g_Y Y_\mu - g A^3_\mu \right)^2 + \partial_\mu \chi^+ \partial^\mu \chi^- + \frac{g v}{2} \left( \partial_\mu \chi^+ W^{\mu-} + \partial_\mu \chi^- W^{\mu+} \right) + \frac{1}{2} \left( \partial_\mu \chi^3 \right)^2 + \frac{v}{2} \partial_\mu \chi^3 \left( g A^{\mu 3} - g_Y Y^\mu + g_Y X_\mu \right) , \]  

where we have defined $\sqrt{2} \chi^\pm := \chi^1 \mp i \chi^2$ and $W^\pm_{\mu \nu} := \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu$. As in the usual case, $W^\pm$ mass is

\[ m_{W^\pm} = \frac{1}{2} g v . \]
The mass term for the neutral bosons in (42) motivates us to introduce the orthogonal transformation

\begin{align*}
A^3_\mu &= \cos \theta_W Z_\mu + \sin \theta_W A_\mu, \\
Y_\mu &= -\sin \theta_W Z_\mu + \cos \theta_W A_\mu,
\end{align*}

(44)

where \( \theta_W \) is known as the Weinberg’s angle, that satisfies the relation

\[ \tan \theta_W = \frac{g_Y}{g} . \]

(45)

Using this diagonalization, the mass term for the \( Z^0 \)- and \( X^\mu \)-neutral fields can be written, in matrix form, as follows below:

\[
L_{\text{mass} - Z - X} = \frac{1}{2} \left( m_Z Z^\mu + \frac{m_X}{4x} X^\mu \right)^2 + \frac{1}{2} m_X^2 X^\mu = \frac{1}{2} (V^\mu)^t \eta_{\mu\nu} M_{Z-X}^2 V^\nu ,
\]

(46)

in which \( (V^\mu)^t = (Z^\mu \ X^\mu) \) and \( M_{Z-X}^2 \) is the mass matrix to the square

\[
M_{Z-X}^2 = \left( \frac{m_Z}{4x} \frac{m_X}{16x^2} m_X^2 + m_X^2 \right) .
\]

(47)

Here, the dimensionless parameter \( x \) has been defined by \( x := u/v \), and \( m_Z \) is the mass of \( Z^0 \) at the tree level approximation of GSW model

\[
m_Z = \frac{g v}{2 \cos \theta_W} .
\]

(48)

The eigenvalues of the matrix \( M_{Z-X}^2 \) are found:

\[
M_{\pm}^2 = \frac{1}{2} \left[ m_Z^2 + m_X^2 + \frac{m_X^2}{16x^2} \pm \sqrt{(m_Z^2 - m_X^2)^2 + (m_Z^2 + m_X^2) \frac{m_X^2}{8x^2} + \frac{m_X^2}{256x^4}} \right] .
\]

(49)

We are interested, at this stage, in VEV scales that satisfy the approximation \( u \gg v \). Therefore, we define a VEV scale-\( u \) at a higher energy scale for which the TEV scale could be the candidate to describe a new particle phenomenology. This condition implies that \( m_X \gg m_Z \), and the eigenvalues correspond to the mass of \( Z^0 \) and \( X^\mu \) with the corrections of the VEVs ratio, \( u/v \):

\[
m_{Z^0} \simeq \frac{g v}{2 \cos \theta_W} \left( 1 - \frac{v^2}{32u^2} \right) ,
\]

\[
m_X \simeq 2 g_Y u \left( 1 + \frac{v^2}{32u^2} \right) .
\]

(50)

To eliminate the mixed terms \( A^{\mu 3} - \chi^3 \) and \( W^\pm - \chi^\pm \) in (42) and \( X^\mu - \eta \) in (29), we introduce the gauge fixing Lagrangian

\[
L_{gf} = -\frac{1}{2a} (\partial_\mu A^\mu)^2 - \frac{1}{2g} \left( \partial_\mu Z^\mu - \gamma m_{Z^0} \chi^3 \right)^2
\]

\[ -\frac{1}{2\beta} \left( \partial_\mu X^\mu + \frac{\beta}{2} m_X \eta - \beta m_{Z^0} \sin \theta_W \chi^3 \right)^2
\]

\[ -\frac{1}{\delta} \left( \partial_\mu W^{\mu +} - \delta m_{W^+} \chi^+ \right) \left( \partial_\nu W^{\nu -} - \delta m_{W^-} \chi^- \right) ,
\]

(51)
where \(\{\alpha, \beta, \gamma, \delta\}\) are real parameters. The surface terms are eliminated as usually done in dealing with the topologically-trivial sector; so, we get all the terms for the gauge fields:

\[
\mathcal{L}_{\text{gauge}} = -\frac{1}{4} (\partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu})^2 - \frac{1}{2\alpha} (\partial_{\mu} A^\mu)^2 - \frac{1}{2} W_{\mu\nu} W^{\mu\nu} - \frac{1}{\delta} \partial_{\mu} W_{\mu} \partial_{\nu} W^{\nu} + m_{W \pm}^2 W_{\mu} W^\mu - \frac{1}{4} (\partial_{\mu} Z_{\nu} - \partial_{\nu} Z_{\mu})^2 - \frac{1}{2\gamma} (\partial_{\mu} Z^\mu)^2 - \frac{1}{2} m_{Z0}^2 Z_{\mu} Z^\mu - \frac{1}{4} (\partial_{\mu} X_{\nu} - \partial_{\nu} X_{\mu})^2 - \frac{1}{2\beta} (\partial_{\mu} X^\mu)^2 + \frac{1}{2} m_X^2 X_{\mu} X^\mu ,
\]

(52)

and the Lagrangian of the four would-be-Goldstone bosons of the model

\[
\mathcal{L}_{\text{Goldstone bosons}} = \frac{1}{2} (\partial_{\mu} \eta)^2 - \frac{1}{2} \beta \left( m_X^2 \eta + m_{Z0} \sin \theta_W \chi^3 \right)^2 + \frac{1}{2} (\partial_{\mu} \chi^3)^2 - \frac{1}{2} m_{Z0}^2 (\chi^3)^2 + \partial_{\mu} \chi^+ \partial^\mu \chi^- - m_{\chi^\pm}^2 \chi^+ \chi^- ,
\]

(53)

where the masses of would-be-Goldstone bosons \(\chi^\pm\) are given by

\[
m_{\chi^\pm} = \sqrt{\delta} m_{W^\pm} .
\]

(54)

To obtain the masses of \(\chi^3\) and \(\eta\), we perform a diagonalization in the mixed terms of (53)

\[
\mathcal{L}_{\text{mass-\eta-\chi^3}} = \left( \eta \quad \chi^3 \right) \left( \begin{array}{cc}
\frac{1}{4} m_X^2 & \frac{1}{2} m_X m_{Z0} \sin \theta_W \\
\frac{1}{2} m_X m_{Z0} \sin \theta_W & \frac{1}{2} m_{Z0}^2 + m_{Z0}^2 \sin^2 \theta_W
\end{array} \right) \left( \begin{array}{c}
\eta \\
\chi^3
\end{array} \right) ,
\]

(55)

where the eigenvalues of the matrix correspond to the following masses:

\[
m_\eta \simeq \frac{\sqrt{\beta}}{2} m_X \\
m_{\chi^3} \simeq m_{Z0} \left( \gamma + \beta \sin^2 \theta_W \right)^{1/2} .
\]

(56)

It is clear that the would-be-Goldstone bosons can be eliminated upon the choice of the unitary gauge, i.e., \(\alpha = \beta = \gamma = \delta = 0\).

We observe that only the field \(A^\mu\) does not have quadratic term \(A_{\mu} A^\mu\) in the Lagrangian (52). Thus, when we inspect the interactions with leptons, we are ready to identify it as the photon. After this second SSB, we have the final symmetry

\[
SU_L(2) \times U_R(1)J \times U(1)_K \overset{\Xi}{\longrightarrow} SU_L(2) \times U_Y(1) \overset{\Phi}{\longrightarrow} U_{em}(1) .
\]

This can be checked by using the transformations of the gauge fields.

\[
Z_{\mu} \mapsto Z'_{\mu} = Z_{\mu} \\
A_{\mu} \mapsto A'_{\mu} = A_{\mu} - \partial_{\mu} \omega(x) \\
W_{\mu}^{\pm} \mapsto W'_{\mu}^{\pm} = e^{\mp i \omega(x)} W_{\mu}^{\pm} .
\]

(57)
The Higgs sector, after the SSBs take place and the unitary gauge is adopted, is reduced
to the scalar fields $F$ and $H$, described by the Lagrangian

$$\mathcal{L}_{\text{Higgs-0}}^{F-H} = \frac{1}{2} \left( \partial_\mu F \right)^2 - \frac{1}{2} \left( 2 \lambda_\Xi u^2 + \frac{1}{2} \lambda_\Xi \phi v^2 \right) F^2$$

$$+ \frac{1}{2} \left( \partial_\mu H \right)^2 - \frac{1}{2} \left( 2 \lambda_\phi v^2 + \frac{1}{2} \lambda_\Xi \phi u^2 \right) H^2 + \lambda_\Xi \phi u v F H \, . \quad (58)$$

We cast the mass terms in matrix form $\mathcal{L}_{\text{mass}}^{F-H} = \frac{1}{2} V^t M_{F-H}^2 V$, where $V^t = (F H)$, and $M_{F-H}^2$ is the mass matrix.

$$M_{F-H}^2 = \begin{pmatrix} 2 \lambda_\Xi u^2 + \frac{1}{2} \lambda_\Xi \phi v^2 & 2 \lambda_\Xi \phi u v \\ 2 \lambda_\Xi \phi u v & 2 \lambda_\phi v^2 + \frac{1}{2} \lambda_\Xi \phi u^2 \end{pmatrix} \, , \quad (59)$$

where the eigenvalues are related to the masses of $F$ and $H$

$$(M_{F-H}^\pm)^2 = \left( \lambda_\phi v^2 + \lambda_\Xi u^2 + \frac{1}{2} \lambda_\Xi \phi u^2 + \frac{1}{4} \lambda_\Xi \phi v^2 \right) \times$$

$$\times \left[ 1 \pm \sqrt{1 - \frac{4 \lambda_\Xi \phi \lambda_\phi v^2 + 8 \lambda_\Xi \lambda_\phi u^2 v^2 + \lambda_\phi \lambda_\Xi \phi v^4}{(4 \lambda_\Xi u^2 + 2 \lambda_\Xi \phi u^2 + 4 \lambda_\phi v^2 + \lambda_\Xi \phi v^2)^2}} \right] \, . \quad (60)$$

Whenever $u \gg v$, we obtain

$$M_{F-H}^+ \simeq \sqrt{2 \lambda_\Xi u^2} \left( 1 + \frac{\lambda_\Xi \phi v^2}{8 \lambda_\Xi u^2} \right) \, ,$$

$$M_{F-H}^- \simeq 2 u^2 \left( \frac{\lambda_\Xi \phi}{4} + \lambda_\phi v^2/u^2 \right) \, . \quad (61)$$

The interactions between $F$ and $H$ are given by

$$\mathcal{L}_{\text{int}}^{\text{Higgs}} = -\frac{\lambda_\Xi}{4} F^4 - \frac{\lambda_\Xi}{4} H^4 - \lambda_\Xi u F^3 - \lambda_\phi v H^3$$

$$- \frac{\lambda_\Xi}{2} u F H^2 - \frac{\lambda_\Xi \phi}{2} v H F^2 - \frac{\lambda_\Xi \phi}{4} F^2 H^2 \, . \quad (62)$$

The leptons and the $-\zeta$-fermion acquire mass terms that can be read below:

$$\mathcal{L}_{\text{leptons-} \zeta \text{-0}} = -\tilde{\ell} i \bar{\phi} \ell + \zeta i \bar{\phi} \bar{\zeta} \zeta - \frac{u}{\sqrt{2}} \left( y_\ell \ell L \bar{\ell} R + y_\ell^* \ell R \bar{\ell} L \right)$$

$$- \frac{u}{\sqrt{2}} \left( z_\ell \ell L \zeta R + z_\ell^* \zeta R \ell L \right) - \frac{u}{\sqrt{2}} \left( x_\ell \ell L \bar{\ell} R + x_\ell^* \ell R \zeta L \right) \, , \quad (63)$$

since we write the coupling constants in terms of global phases $x_\ell = |x_\ell| e^{i \delta_\ell}$, $y_\ell = |y_\ell| e^{i \delta_\ell}$ and $z_\ell = |z_\ell| e^{i \delta_\zeta}$, and it can absorb in the fields

$$e^{-i \delta_\zeta} \zeta L \rightarrow \zeta L \, , \quad e^{i \delta_\ell} \ell R \rightarrow \ell R \, , \quad e^{i \delta_\ell} \zeta R \rightarrow \zeta R \, . \quad (64)$$
Thus, the kinetic part remains unchanged, and the mass terms are written in terms of \((\ell, \zeta)\) as
\[
L_{\text{leptons-}\zeta-0} = \bar{\ell} i \not{\partial} \ell + \bar{\zeta} i \not{\partial} \zeta - \frac{|y_\ell| v}{\sqrt{2}} \bar{\ell} \ell - \frac{|z_\ell| v}{\sqrt{2}} (\bar{\zeta} L \ell + \bar{\ell} R \zeta) - \frac{|x_\ell| u}{\sqrt{2}} (\bar{\zeta} R \ell + \bar{\ell} L \zeta),
\]
where \(R\) and \(L\) are the Right and Left projectors that satisfy the relations \(R^2 = R, L^2 = L, RL = LR = 0\) and \(R + L = \mathbb{1}\). It is convenient to write it in matrix form
\[
L_{\text{leptons-}\zeta-0} = \xi (i \not{\partial} - M_{\ell-\zeta} \mathbb{1}) \xi,
\]
in which \(\xi^t = (\ell, \zeta)^t\) and \(M_{\ell-\zeta}\) is the square matrix
\[
M_{\ell-\zeta} = \begin{pmatrix}
\frac{|y_\ell| v \mathbb{1}}{\sqrt{2}} & \frac{|x_\ell| u + |z_\ell| v R}{\sqrt{2}} \\
\frac{|x_\ell| u + |z_\ell| v L}{\sqrt{2}} & 0
\end{pmatrix}.
\]
This mass matrix can be diagonalized by the unitary transformation
\[
\xi \mapsto \xi' = S \xi,
\]
where \(S\) is a unitary matrix \(S^t S = \mathbb{1}\); so, we obtain
\[
\bar{\xi} M_{\ell-\zeta} \xi = \bar{\xi}' (S M_{\ell-\zeta} S^t) \xi' = (\bar{\ell}' \bar{\zeta}') \begin{pmatrix} m_\ell \mathbb{1} & 0 \\ 0 & -m_\zeta \mathbb{1} \end{pmatrix} (\ell' \zeta') = m_\ell \ell' \ell' - m_\zeta \bar{\zeta}' \bar{\zeta}'.
\]
Here \(m_\ell\) is the lepton mass, and \(m_\zeta\) is the hidden fermion-\(\zeta\) mass given by the eigenvalues
\[
m_\ell = \frac{|y_\ell| v}{2\sqrt{2}} \left[ 1 + \sqrt{1 + 4 \frac{|x_\ell||z_\ell|}{|y_\ell|^2} \frac{u}{v}} \right], \quad m_\zeta = -\frac{|y_\ell| v}{2\sqrt{2}} \left[ 1 - \sqrt{1 + 4 \frac{|x_\ell||z_\ell|}{|y_\ell|^2} \frac{u}{v}} \right].
\]
It is convenient to make use of the approximation \(v \gg u\), so that the eigenvalues can be re-written as
\[
m_\ell \approx \frac{|y_\ell| v}{2\sqrt{2}} \left( 1 + \frac{|x_\ell||z_\ell|}{|y_\ell|^2} \frac{u}{v} \right), \quad m_\zeta \approx \frac{|g_\zeta| u}{2\sqrt{2}},
\]
where \(|g_\zeta|\) defined above is a constant coupling much smaller than \(|y_\ell|\)
\[
|g_\zeta| := \sum_{\ell = e, \mu, \tau} \frac{|x_\ell||z_\ell|}{|y_\ell|}.
\]
Therefore, upon the SSB, the fermionic sector is given by
\[
L_{\text{leptons-}\zeta-0} = \bar{\ell}' (i \not{\partial} - m_\ell) \ell' + \bar{\zeta}' (i \not{\partial} + m_\zeta) \zeta'.
\]
We have obtained all mass terms coming from the two SSBs, depending on the VEVs \(u\) and \(v\). We have two massive charged bosons \(W^\pm\), two massive neutral bosons \(Z^0\) and \(X^\mu\), the latter being identified as a new gauge boson. The \(A^\mu\)-gauge field is identified as the photon of the model, and it mass is exclusively due to the topological term. In the next Section, we shall derive the propagators of all the fields of the model to explicitly read the masses in terms of the \(u, v\)-VEVs, the mixing angle, \(\theta_W\) and the coupling constants.
IV. THE PROPAGATORS, INTERACTIONS AND THE MASSES OF HIDDEN PARTICLES

In this Section, we shall work to identify the masses of the gauge fields in the Lagrangian (52) by calculating the poles of the propagators at the tree approximation.

A. The propagators and interactions

The $W^\pm$ terms are not related to the neutral gauge bosons terms in (52); then, the same propagators as in the EW model come out:

$$\langle W_\mu^+ W_\nu^- \rangle = -\frac{i}{k^2 - m_{W^\pm}^2} \left[ \eta_{\mu \nu} + (\delta - 1) \frac{k_\mu k_\nu}{k^2 - \delta m_{W^\pm}^2} \right], \tag{74}$$

where the pole is $k^2 = m_{W^\pm}^2$ in the momentum space. The would-be-Goldstone bosons of Lagrangian (53) have the propagators

$$\langle \eta \eta \rangle = i \frac{k^2}{k^2 - m_\eta^2},$$

$$\langle \chi^3 \chi^3 \rangle = i \frac{k^2}{k^2 - m_{\chi^3}^2},$$

$$\langle \chi^+ \chi^- \rangle = i \frac{k^2}{k^2 - m_{\chi^\pm}^2}. \tag{75}$$

From sector of fermions in (73), the following propagators can be derived:

$$\langle \bar{\ell} \ell \rangle = i \frac{\hat{p} + m_\ell}{\hat{p} - m_\ell} = i \frac{\hat{p} + m_\ell}{p^2 - m_\ell^2}, \tag{76}$$

where the propagator has a pole for $p^2 = m_\ell^2$, and $m_\ell$ is the leptonic mass at the tree-level approximation. The propagator of the hidden $\zeta$-fermion is

$$\langle \bar{\zeta} \zeta \rangle = i \frac{\hat{p} - m_\zeta}{\hat{p} + m_\zeta} = i \frac{\hat{p} - m_\zeta}{p^2 - m_\zeta^2}, \tag{77}$$

where $m_\zeta$ is the mass associated to the pole of this propagator.

The masses of the neutral fields in (52) are not so easy to identify as in the previous cases. The Lagrangian of the neutral gauge fields can be better written in terms of the spin operators:

$$\mathcal{L}_{\text{gauge-0}} + \mathcal{L}_{gf} = \frac{1}{2} A^\mu \Box \left( \theta_{\mu \nu} + \frac{1}{\alpha} \omega_{\mu \nu} \right) A^\nu + \frac{1}{2} X^\mu \left( \Box + m_X^2 \right) \theta_{\mu \nu} + \left( \frac{\Box}{\beta} + m_X^2 \right) \omega_{\mu \nu} \right] X^\nu + \frac{1}{2} Z^\mu \left( \Box + m_Z^2 \right) \theta_{\mu \nu} + \left( \frac{\Box}{\gamma} + m_Z^2 \right) \omega_{\mu \nu} \right] Z^\nu, \tag{78}$$
where we have defined
\[ \theta_{\mu\nu} = \eta_{\mu\nu} - \frac{\partial_{\mu} \partial_{\nu}}{\Box} \quad \text{and} \quad \omega_{\mu\nu} = \frac{\partial_{\mu} \partial_{\nu}}{\Box}, \]
and they satisfy the algebra \( \theta^2 = \theta, \omega^2 = \omega, \theta \cdot \omega = \omega \cdot \theta = 0, \) and \( \theta + \omega = 1 \). The elements of the inverse matrix in (78) correspond to the following propagators:
\[ \langle A_\mu A_\nu \rangle = -\frac{i}{k^2} \left[ \eta_{\mu\nu} + (\alpha - 1) \frac{k_\mu k_\nu}{k^2} \right], \]
(80)
\[ \langle X_\mu X_\nu \rangle = -\frac{i}{k^2 - m_X^2} \left[ \eta_{\mu\nu} + (\beta - 1) \frac{k_\mu k_\nu}{k^2 - \beta m_X^2} \right], \]
(81)
\[ \langle Z_\mu Z_\nu \rangle = -\frac{i}{k^2 - m_{Z^0}^2} \left[ \eta_{\mu\nu} + (\gamma - 1) \frac{k_\mu k_\nu}{k^2 - \gamma m_{Z^0}^2} \right]. \]
(82)
All propagators are well-behaved in the ultraviolet regime, i.e., \( k^\mu \rightarrow \infty \), which is compatible with the requirement of renormalizability.

Now, we can see how the electromagnetic, weak and a new possible interaction appear in the Lagrangian (38). By replacing (44) in (38) yields the interactions between fermions and neutral gauge bosons:
\[ \mathcal{L}_{int}^{\text{leptons-AZX}} = -e Q_{em} \bar{\Psi} A \Psi - e Q_Z \bar{\Psi} Z \Psi - e Q_X \bar{\Psi} X \Psi. \]
(83)
We identify the fundamental electromagnetic charge as follows:
\[ e = g \sin \theta_W = g_Y \cos \theta_W, \]
(84)
where the generator electric charge is given by the relation
\[ Q_{em} = I^3 + Y. \]
(85)
So, the coupling constants can be written in terms of the fundamental charge and the mixing angles:
\[ g = \frac{e}{\sin \theta_W} \quad \text{and} \quad g' = g'' = \frac{e}{\cos \theta_W} \sqrt{2 - 2 \chi}, \]
(86)
in such way that we define the fundamental electric charge with the correction due to the \( \chi \)-parameter
\[ e^{-1}_X = \sqrt{\frac{1}{g^2} + \frac{1}{g'^2} + \frac{1}{g''^2}} = e^{-1} \sqrt{1 + \chi \cos^2 \theta_W} \simeq e^{-1} \left( 1 + \frac{\chi}{2} \cos^2 \theta_W \right), \]
(87)
for \( \chi \ll 1 \). For convenience, we also define effective charge associated to the interaction of the fermions with \( Z^0 \)
\[ Q_Z := \frac{1}{\sin \theta_W \cos \theta_W} \left( I^3 - \sin^2 \theta_W \right) Q_{em}, \]
(88)
Both generators emerge in the usual electroweak model, but we have here another charge, $Q_X$, of the interaction between the fermions and the the hidden photon, $X^\mu$

$$Q_X := \frac{1}{\cos \theta_W} \left( -Y + 2K \right). \quad (89)$$

The values of $Q_{em}$, $I^3$, $Y$ and of the primitive charges $J$ and $K$ are summarized in the following table:

| Fields & Particles               | $Q_{em} = I^3 + Y$ | $I^3$ | $J = J + K$ | $J$ | $K$ |
|----------------------------------|--------------------|------|-------------|-----|-----|
| leptons-left                     | $-1$               | $-1/2$ | $-1/2$     | $-1/2$ | $0$ |
| leptons-right                    | $-1$               | $0$   | $-1$        | $-1$ | $0$ |
| neutrinos-left                   | $0$                | $+1/2$ | $-1/2$     | $-1/2$ | $0$ |
| neutrinos-right                  | $0$                | $0$   | $0$         | $0$  | $0$ |
| Hidden fermion left-ζ            | $-1$               | $0$   | $-1$        | $0$  | $-1$ |
| Hidden fermion right-ζ           | $-1$               | $0$   | $-1$        | $-1$ | $0$ |
| bosons $W^\pm$                   | $\pm1$             | $\pm1$ | $0$         | $0$  | $0$ |
| neutral bosons                   | $0$                | $0$   | $0$         | $0$  | $0$ |
| Higgs - Ξ                        | $0$                | $0$   | $0$         | $+1$ | $-1$ |
| Higgs - Φ                        | $0$                | $-1/2$ | $+1/2$     | $+1/2$ | $0$ |

Using these values, we list below all interactions from (83):

a) Interactions of leptons, neutrinos, hidden ζ-fermion with photons ($\gamma$)

$$\mathcal{L}_{\ell \ell - \gamma}^{int} = +e \bar{\ell} A \ell, \quad (90)$$

$$\mathcal{L}_{\zeta \zeta - \gamma}^{int} = +e \bar{\zeta} A \zeta. \quad (91)$$

b) Interactions of leptons, neutrinos, hidden ζ-fermion with the $Z^0$-boson

$$\mathcal{L}_{\ell \ell - Z}^{int} = \frac{g}{4 \cos \theta_W} \bar{\ell} Z \left( 1 - 4 \sin^2 \theta_W - \gamma_5 \right) \ell, \quad (92)$$

$$\mathcal{L}_{\bar{\nu}_\ell \nu_\ell - Z}^{int} = \frac{g}{4 \cos \theta_W} \bar{\nu}_\ell Z \left( 1 - \gamma_5 \right) \nu_\ell, \quad (93)$$

$$\mathcal{L}_{\zeta \zeta - Z}^{int} = -\frac{g}{\cos \theta_W} \bar{\zeta} Z \zeta. \quad (94)$$

c) Interactions of leptons, neutrinos, hidden ζ-fermion with hidden neutral $X^\mu$-boson

$$\mathcal{L}_{\ell \ell - X}^{int} = -\frac{e}{2 \cos \theta_W} \bar{\ell} X \ell, \quad (95)$$

$$\mathcal{L}_{\bar{\nu}_\ell \nu_\ell - X}^{int} = \frac{e}{4 \cos \theta_W} \bar{\nu}_\ell X \left( 1 - \gamma_5 \right) \nu_\ell, \quad (96)$$

$$\mathcal{L}_{\zeta \zeta - X}^{int} = -\frac{e}{\cos \theta_W} \bar{\zeta} X \gamma_5 \zeta. \quad (97)$$
B. Estimating the masses

The parametrization (84) gives us the masses of $W^\pm$, $Z^0$ and $X^\mu$ in terms of $e$, $v$, $u$, and the mixing angle, $\theta_W$. The $W^\pm$ mass is

$$m_{W^\pm} = \frac{e v}{2 \sin \theta_W}, \quad (98)$$

and the masses of $Z^0$ and $X^\mu$, when $u \gg v$, are given by

$$m_{Z^0} \simeq \frac{e v}{2 \sin 2 \theta_W} \left( 1 - \frac{v^2}{32 u^2} \right),$$

$$m_X \simeq \frac{2 e u}{\cos \theta_W} \left( 1 + \frac{v^2}{32 u^2} \right). \quad (99)$$

The relation between $g$ and the Fermi’s constant $G_F = 1.166 \times 10^{-5} (\text{GeV})^{-2}$, i.e., $g^2 = 4\sqrt{2} m_{W^\pm}^2 G_F$, leads us to $v = (\sqrt{2} G_F)^{-1/2} \simeq 246 \text{ GeV}$. We identify $\theta_W$ as the Weinberg angle, and considering that the experimental value is $\sin^2 \theta_W \simeq 0.23$, the mass of $W^\pm$ is unaltered

$$m_{W^\pm} = \frac{37 \text{ GeV}}{|\sin \theta_W|} \simeq 77 \text{ GeV}. \quad (100)$$

To estimate a possible mass for the hidden boson, $X^\mu$, we adopt the experimental value of the mass of $Z^0$

$$m_{Z^0} = 91.1876 \pm 0.0021 \text{ GeV}, \quad (101)$$

whose uncertainty we take to fix the VEV scale $u$. Thus, the $Z^0$-mass with the correction due to the ratio $v/u$ is

$$m_{Z^0} \simeq 89 \left( 1 - \frac{v^2}{32 u^2} \right) \text{ GeV}, \quad (102)$$

so that the uncertainty in (101) fixes the scale $u \simeq 8.9 \text{ TeV}$. Therefore, the $X^\mu$ mass is around the value

$$m_X \simeq 6 \text{ TeV}. \quad (103)$$

The presently-known experimental result for the $H$-Higgs mass is by $m_H = 125.7 \pm 0.4 \text{ GeV}$, so the $v$-VEV scale fixes the coupling constant $\lambda_\Phi \simeq 0.13$ in the EW Higgs sector. Using (61), the uncertainty in the Higgs mass estimates the mixed coupling constant in order of $\lambda_{\Xi \Phi} = 4 \times 10^{-6}$. Since we hope that the mass of the unknown Higgs-$F$ must have the order of VEV-$u$, we adopt the range of $0.01 < \lambda_{\Xi} < 0.09$ to estimate a range for the mass of $F$. Furthermore, the coupling constants of the original symmetry group have the values $g \simeq 0.62$ and $g' = g'' \simeq 0.48$. Thereby the mass of Higgs-$F$ for this scale-$u$, it is in the range

$$280 \text{ GeV} < M_F < 830 \text{ GeV}, \quad (104)$$

and the coupling constant that mixes the fields $H$ and $F$ is estimated as $\lambda_{\Xi \Phi} = 1.3 \times 10^{-6}$. 
The leptonic sector determines each Yukawa coupling constant $|y_\ell| (\ell = e, \mu, \tau)$, where we take the known masses $m_e = 0.5$ MeV, $m_\mu = 105$ MeV and $m_\tau = 1777$ MeV, these coupling constants are given by

$$
|y_e| \simeq 3 \times 10^{-6},
|y_\mu| \simeq 6 \times 10^{-4},
|y_\tau| \simeq 10^{-2}.
$$

(105)

Since $|x_\ell|$ and $|z_\ell|$ are coupling constants of Yukawa interactions involving the $\zeta$-fermion and leptons left or right components, we consider it to be the same order, that is, $|x_\ell| \simeq |z_\ell|$. Thus if we use a mass of order $m_\zeta = 1$ TeV for the $\zeta$-fermion, we obtain $g_\zeta \simeq 0.15$, and the new Yukawa constants coupling are estimated as

$$
|x_e| \simeq 1.6 \times 10^{-5},
|x_\mu| \simeq 3 \times 10^{-2},
|x_\tau| \simeq 0.55.
$$

(106)

To place our results within an experimental scenario with the results obtained in LHC [1], we can compare the mass of $Z'$ with the mass obtained for the boson $X^\mu$, for instance. A range for the mass of $Z'$, according to ATLAS and CMS experiments, is given by

$$3.5 \text{ TeV} < m_{Z'} < 4.5 \text{ TeV}.$$  

(107)

which means that the result in (103) is of the same order (TeV scale), but the X-boson has a mass above the expected $Z'$-mass. We rely on an Abelian factor to get our $X^\mu$-boson, whereas $Z'$ is associated with an $SU(2)$ subgroup. We are actually saying that we expect another neutral gauge boson, which is not to be identified as $Z'$. Consequently, the new Higgs that we have put in, with mass estimated in the range (104) can be heavier too. Since we know that the VEV associated with $W'$ and $Z'$ masses is of order $3 - 4 \text{ TeV}$ [4], the $\Xi$-Higgs must emerge near the 9 TeV. If we adopt now $0.1 < \lambda_\Xi < 0.9$, the mass of $F$-Higgs is estimated to be

$$2.8 \text{ TeV} < M_F < 8.3 \text{ TeV}.$$  

(108)

V. PARAPHOTONS AND THE BREAKING DRIVEN BY THE EXTRA HIGGS

In contrast with the previous Section, with the new Higgs and its associated SSB, we here analyze the case of a VEV-scale such that $u \ll v$. This corresponds to a SSB at a lower scale, $u$, with respect to $v = 246$ GeV of the EW model. To this end, we re-start from the original $SU_L(2) \times U_R(1)_J \times U(1)_K$-symmetry, but we first couple the Higgs of EW model as done in (17). After this SSB occurs, we obtain the remaining symmetry

$$SU_L(2) \times U_R(1)_J \times U(1)_K \xrightarrow{(\Phi)_{\theta}} U(1)_G \times U(1)_K,$$

(109)

where the $U(1)_G$-group is a result from the mixing of $SU_L(2) \times U_R(1)_J$. The generator $G$ of $U(1)_G$ is given by $G = I^3 + J$. Thus, the free Lagrangian for the neutral gauge fields, after this SSB takes place, reads as follows:

$$
\mathcal{L}_{\text{gauge}} = -\frac{1}{4} \left( \partial_\mu A_\nu^3 - \partial_\nu A_\mu^3 \right)^2 - \frac{1}{4} B_{\mu \nu}^2 - \frac{1}{4} C_{\mu \nu}^2 - \frac{\chi}{2} B_{\mu \nu} C_{\mu \nu} + \frac{1}{2} v^2 \left( g' B^\mu - g A_\mu^3 \right)^2.
$$

(110)
Now, to diagonalize the mixed $A^{\mu 3} - B^\mu$ term in \((110)\), we introduce the orthogonal transformation

\[
A^3_\mu = \cos \theta_W Z_\mu + \sin \theta_W G_\mu \\
B_\mu = -\sin \theta_W Z_\mu + \cos \theta_W G_\mu ,
\]

where $\theta_W$ satisfies the relation

\[
\tan \theta_W = \frac{g'}{g} .
\]

Here, $G^\mu$ is the vector field associated to the $U(1)_G$ subgroup. To reduce the residual $U_R(1)_J \times U(1)_K$-symmetry to the electromagnetism $U(1)_{em}$, we introduce the second Higgs-$\Xi$ sector coupled to the fields $X^\mu$ and $B^\mu$, like in Section III. The $\Xi$-(Higgs) field breaks now the symmetry

\[
SU_L(2) \times U_R(1)_J \times U(1)_K \xrightarrow{\Phi_0} U(1)_G \times U(1)_K \xrightarrow{\Xi_0} U(1)_{em} ,
\]

where we have the VEV scale-$u$ that defines a new massive gauge boson. This VEV gives the neutral sector

\[
\mathcal{L}_{gauge} = -\frac{1}{4} Z_{\mu \nu}^2 + \frac{1}{2} m_Z^2 Z_{\mu \nu} - \frac{1}{4} G_{\mu \nu}^2 - \frac{1}{4} C_{\mu \nu}^2 \\
+ \frac{\chi_W}{2} \sin \theta_W Z_{\mu \nu} C_{\mu \nu} - \frac{\chi_W}{2} \cos \theta_W G_{\mu \nu} C_{\mu \nu} + \frac{u^2}{2} \left( g' B^\mu - g'' C^\mu \right)^2 .
\]

Using the orthogonal transformation

\[
C_\mu = \frac{1}{\sqrt{2}} X_\mu + \frac{1}{\sqrt{2}} A_\mu \\
G_\mu = -\frac{1}{\sqrt{2}} X_\mu + \frac{1}{\sqrt{2}} A_\mu ,
\]

the mixing $G - B$ is eliminated in \((114)\), where we have used this second parametrization

\[
\sqrt{2} e = g'' = g \sin \theta_W = g' \cos \theta_W ,
\]

but the electric charge generator remains unchanged, that is, $Q_{em} = G + K = I^3 + J + K$. Consequently, all terms of neutral bosons are gathered together in the Lagrangian

\[
\mathcal{L}_{gauge} = -\frac{1}{4} Z_{\mu \nu}^2 + \frac{1}{4} X_{\mu \nu}^2 - \frac{1}{4} A_{\mu \nu}^2 + \frac{1}{2} m_Z^2 Z_{\mu \nu}^2 \\
+ \frac{\chi_W}{2} Z_{\mu \nu} X_{\mu \nu} + \frac{\chi_W}{2} Z_{\mu \nu} A_{\mu \nu} + \frac{1}{2} \left( m_X X_{\mu} + x m_Z Z_{\mu} \right)^2 ,
\]

where we have defined

\[
\chi_W = \frac{\chi}{\sqrt{2}} \sin \theta_W , \\
x = 2 \frac{u}{v} \sin^2 \theta_W .
\]
To simplify our task, we diagonalize the Lagrangian to eliminate the mixed terms $Z - X$ and $Z - A$. To achieve this goal, we shift the photon-$A^\mu$ and $Z^0$ Boson as

\[ Z^\mu \longrightarrow \tilde{Z}^\mu = (1 + \chi_W^2) \tilde{Z}^\mu \]
\[ A^\mu \longrightarrow \tilde{A}^\mu = \tilde{A}^\mu + \chi_W \tilde{Z}^\mu , \]

so the Lagrangian is reduced to $2 \times 2$ diagonalization of the mixing $\tilde{Z} - X$

\[
\mathcal{L}_{gauge} \tilde{Z} - X = -\frac{1}{4} \tilde{Z}^2_{\mu\nu} - \frac{1}{4} X^2_{\mu\nu} + \frac{\chi_W}{2} \tilde{Z}_{\mu\nu} X^{\mu\nu} + \frac{1}{2} \tilde{m}_Z^2 \tilde{Z}^2_{\mu} + \frac{1}{2} \left( m_X X^\mu + x \tilde{m}_Z \tilde{Z}^\mu \right)^2 ,
\]

where $\tilde{m}_Z$ is defined by

\[
\tilde{m}_Z \simeq m_Z \left( 1 + \frac{\chi_W^2}{2} \right) .
\]

The Lagrangian involving $\tilde{Z}^\mu$ and $X^\mu$ is so written into the matrix form

\[
\mathcal{L}_{gauge} \tilde{Z} - X = \frac{1}{2} (V^\mu)^t \Box \theta_{\mu\nu} K V^{\nu} + \frac{1}{2} (V^\mu)^t \eta_{\mu\nu} M^2 V^{\nu} ,
\]

where $(V^\mu)^t = \begin{pmatrix} \tilde{Z}^\mu & X^\mu \end{pmatrix}$, $K$ is the matrix

\[
K := \begin{pmatrix} 1 & -\chi_W \\ -\chi_W & 1 \end{pmatrix} .
\]

The mass matrix $M^2$ is not diagonal

\[
M^2 = \begin{pmatrix} \tilde{m}_Z^2 (1 + x^2) & \tilde{m}_Z m_X x \\ \tilde{m}_Z m_X x & m_X^2 \end{pmatrix} ,
\]

so to diagonalize the Lagrangian (122), we perform the orthogonal transformation $V \longrightarrow \tilde{V} = R V$, where $R^t R = I$. Thus, if we define the diagonal matrix as $K_D = R K R^t$, the eigenvalues of $K_D$ are given by $1 \pm \chi_W$, so we obtain the diagonal matrix

\[
K_D = \begin{pmatrix} 1 + \chi_W & 0 \\ 0 & 1 - \chi_W \end{pmatrix} .
\]

Finally, the Lagrangian in terms of $\tilde{V}^\mu$ takes over the form

\[
\mathcal{L}_{gauge} = \frac{1}{2} (\tilde{V}^\mu)^t \Box \theta_{\mu\nu} K_D \tilde{V}^{\nu} + \frac{1}{2} (\tilde{V}^\mu)^t \eta_{\mu\nu} \tilde{M}^2 \tilde{V}^{\nu} ,
\]

where $\tilde{M}^2 = R M^2 R^t$. It can be readily checked that the solution for $R$ is the following $SO(2)$-matrix

\[
R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} ,
\]
so, the matrix mass $\tilde{M}^2$ is given by
\[
\tilde{M}^2 = \frac{1}{2} \begin{pmatrix}
\tilde{m}_Z^2 (1 + x^2) + m_X^2 + 2 \tilde{m}_Z m_X x & m_X^2 - \tilde{m}_Z^2 (1 + x^2) \\
m_X^2 - \tilde{m}_Z^2 (1 + x^2) & \tilde{m}_Z^2 (1 + x^2) + m_X^2 - 2 \tilde{m}_Z m_X x
\end{pmatrix}.
\] (128)

Now, we write the matrix $K_D$ as $K_D = \left( K_D^{1/2} \right)^t \left( K_D^{1/2} \right)$ to adsorb it in the kinetic term redefining $\tilde{V} \rightarrow K_D^{1/2} \tilde{V}$. The solution for the matrix $K_D^{1/2}$ is
\[
K_D^{1/2} = \begin{pmatrix}
\sqrt{1 + \chi W} & 0 \\
0 & \sqrt{1 - \chi W}
\end{pmatrix},
\] (129)
so that
\[
\mathcal{L}_{\tilde{Z}^c - \tilde{X}^c} = \frac{1}{2} \left( \tilde{V}^\mu \right)^t \Box \theta_{\mu\nu} \tilde{V}^\nu + \frac{1}{2} \left( \tilde{V}^\mu \right)^t \eta_{\mu\nu} M_D^2 \tilde{V}^\nu ,
\] (130)
where the mass matrix is now $M_D^2 = \left( K_D^{1/2} \right)^{-1} \tilde{M}^2 \left( K_D^{1/2} \right)^{-1}$, that is,
\[
M_D^2 = \frac{1}{2} \begin{pmatrix}
\tilde{m}_Z^2 (1 + x^2) + m_X^2 + 2 \tilde{m}_Z m_X x & m_X^2 - \tilde{m}_Z^2 (1 + x^2) \\
m_X^2 - \tilde{m}_Z^2 (1 + x^2) & \tilde{m}_Z^2 (1 + x^2) + m_X^2 - 2 \tilde{m}_Z m_X x \\
\sqrt{1 - \chi W} & \sqrt{1 - \chi W}
\end{pmatrix}.
\] (131)

Since $M_D^2$ is also symmetric, it can be diagonalized by an orthogonal matrix, $S$; if we define $\tilde{V}^\mu = S \tilde{V}^\mu$, we end up with a fully diagonal Lagrangian as given below:
\[
\mathcal{L}_{\tilde{Z}^c - \tilde{X}^c} = \frac{1}{2} \left( \tilde{V}^\mu \right)^t \Box \theta_{\mu\nu} \tilde{V}^\nu + \frac{1}{2} \left( \tilde{V}^\mu \right)^t \eta_{\mu\nu} M_{\text{diag}}^2 \tilde{V}^\nu ,
\] (132)
with $M_{\text{diag}}^2 = S M_D^2 S^t$ is given by
\[
M_{\text{diag}}^2 = \begin{pmatrix}
M_Z^2 & 0 \\
0 & M_X^2
\end{pmatrix}.
\] (133)

Here, the $Z^0$-mass acquires a correction coming from the mixing parameter, $\chi$. For $\chi \ll 1$, we can write
\[
M_Z \simeq m_Z \left[ 1 + \frac{4 u^2}{v^2} \sin^4 \theta_W \left( 1 - \frac{8 \chi}{\sqrt{2}} \right) \right],
\]
\[
M_X \simeq m_X \left[ 1 - \frac{4 u^2}{v^2} \sin^4 \theta_W \left( 1 - \frac{8 \chi}{\sqrt{2}} \right) \right].
\] (134)

The $S$-matrix also corresponds to an $SO(2)$-transformation, but with rotation angle of $3\pi/4$; so, we have
\[
S = \frac{1}{\sqrt{2}} \begin{pmatrix}
-1 & 1 \\
1 & -1
\end{pmatrix}.
\] (135)
To avoid confusion in notation, the doublet represented by $\tilde{V}^\mu$ is renamed as the original fields $Z^\mu, X^\mu$, and the photon is also renamed as $A^\mu$. Thus, the full diagonal Lagrangian is more explicitly written as

$$L^{AZX}_{\text{gauge}} = -\frac{1}{4} A^2_{\mu\nu} - \frac{1}{4} Z^2_{\mu\nu} + \frac{1}{2} M_Z^2 Z^2_{\mu\nu} - \frac{1}{4} X^2_{\mu\nu} + \frac{1}{2} M_X^2 X^2_{\mu\nu}. \tag{136}$$

The uncertainty on (101) can be taken to fix the VEV-scale as $u \simeq 2.5$ GeV, and therefore, the mass of the $X^\mu$-boson becomes

$$m_X \simeq 1.7 \text{ GeV}. \tag{137}$$

Whenever $u \ll v$, the eigenvalues of (60) are given by

$$M_H \simeq \sqrt{2 \lambda_\phi} v^2 \left( 1 + \frac{\lambda_\Xi}{8 \lambda_\phi} \frac{u^2}{v^2} \right),$$

$$M_F \simeq 2 v^2 \left( \frac{\lambda_\Xi}{4} + \lambda_\Xi \frac{u^2}{v^2} \right), \tag{138}$$

and mass of the corresponding $F$-Higgs is comes out in the range $0.35 \text{ GeV} \lesssim M_F \lesssim 1 \text{ GeV}$. Under these conditions, if we take $|g_\zeta| = 0.15$, the mass of new fermion turns out to be around $m_\zeta = 266 \text{ MeV}$. Another possibility, which seems more appealing, is to fix the scale $u$ by adopting the value $m_X = 17 \text{ MeV}$. In so doing, we obtain the VEV $u = 25 \text{ MeV}$, so that the mass of the $F$-Higgs is estimated in the range

$$3.5 \text{ MeV} \lesssim M_F \lesssim 10.6 \text{ MeV}. \tag{139}$$

In this range, the hidden $\zeta$-fermion has a mass around $m_\zeta = 2.6 \text{ MeV}$.

The interactions in the scenario $u \ll v$ are obtained by replacing the transformations (111) and (115) into the expression (14), and using the parametrization (116); this yields:

$$L^{\text{int}}_{\text{leptons-}AZX} = -e Q_{em} \bar{\Psi} A \Psi - \frac{e}{\sqrt{1 + \chi_W}} (Q_Z + \chi_W Q_{em}) \bar{\Psi} Z \Psi - \frac{e Q_X}{\sqrt{1 + \chi_W}} \bar{\Psi} X \Psi, \tag{140}$$

where here we have defined the generators $Q_Z$ and $Q_X$ as follows

$$Q_Z = \frac{\sqrt{2}}{\sin \theta_W \cos \theta_W} \left( I^3 - \sin^2 \theta_W Q_{em} + \sin^2 \theta_W K \right),$$

$$Q_X = -G + K. \tag{141}$$

We observe a resulting millicharge given by $e \chi_W Q_{em}$. Furthermore, we also obtain the correction induced by the $\chi_W$-parameter to each interaction.
VI. CONCLUDING COMMENTS

We have made efforts to set up a model with an extra $U(1)_K$-factor that may describe a scenario of a Particle Physics beyond the Standard Model. The model is based on the gauge group $SU_L(2) \times U_R(1)_J \times U(1)_K$, where the extra $U(1)_K$-factor introduces a (new) massive neutral boson. The mass of the latter is generated upon a spontaneous symmetry breaking mechanism that defines an extra VEV scale, beyond that VEV of 246 GeV associated to the Higgs field of the Standard Model. In our proposal, the Higgs sector displays two scalars fields and the gauge symmetry allows interactions between them. This mechanism can be introduced in two ways: in the first case, the SSB occurs through a VEV scale-$u$, where $u > 246$ GeV, and as consequence, we obtain a heavy neutral boson with a mass around the $m_X = 6$ TeV. Next, the Higgs of VEV 246 GeV breaks the electroweak symmetry to give the masses for $W^\pm$ and $Z^0$. The sequence of SSB mechanisms is as follows:

$$SU_L(2) \times U_R(1)_J \times U(1)_K \overset{(\Xi)_q}{\rightarrow} SU_L(2) \times U_Y(1) \overset{(\Phi)_q}{\rightarrow} U_{em}(1).$$

The result for the mass of $X$ is in agreement with the mass values for the boson $Z'$ estimated by the ATLAS and CMS experiment. The scale $u = 8.9$ TeV estimates the mass of new Higgs within the range $280$ GeV $< M_F < 830$ GeV, that is agreement with the diphoton resonance at 750 GeV. Furthermore, the subgroup $U(1)_K$ also introduces a new fermion in the TeV scale that would be a candidate to the Dark Matter.

In the second scenario, the SSB mechanism is introduced to break the composite subgroup $SU_L(2) \times U_R(1)_J$ in the VEV scale of 246 GeV to give mass for $Z^0$, and so, a further SBB defines the VEV scale-$u$, such that $u < 246$ GeV. If this is the situation, the X-boson acquires a lower mass, that may describe a phenomenology of elementary particle physics at the lower energy scale. The scheme for the SSBs follows now the pattern below:

$$SU_L(2) \times U_R(1)_J \times U(1)_K \overset{(\Xi)_q}{\rightarrow} U(1)_G \times U(1)_K \overset{(\Xi)_q}{\rightarrow} U(1)_{em}.$$

This proposal may be discussed in connection with the para-photon or hidden photon phenomenology, at Sub-eV scale, which can be related to the composition of dark matter.

Other context is in the recent proposal of a new boson needed to explain the excited 8-Beryllium decay. In this case, the mass of boson $X$ is fixed at the scale $m_X = 17$ MeV. Therefore, the results of SSB lead us to the VEV scale of $u = 25$ MeV that must be origin of a new Higgs within the range mass $3.5$ MeV $\lesssim M_F \lesssim 10.6$ MeV. The decay of new Higgs and the influence on the X-boson can be the motivation to a forthcoming paper. A unification scheme including $W'$- and $Z'$-bosons in the framework of an $SU_L(2) \times SU_R(2) \times U(1)_J \times U(1)_K$-gauge group may also be the subject of further investigation to be pursued.

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