The elliptic flow of proton as a function of mid-rapidity in heavy ion collision at intermediate energy by using a quantum molecular dynamics model

P Chaimongkon¹, S Junsen¹, W Wangwon¹ and P Srisawad²

¹Department of Physics, School of Science, University of Phayao, Amphoe Muang, Phayao 56000, Thailand
²Department of Physics, Faculty of Science, Naresuan University Amphoe Muang, Phitsanulok 65000, Thailand

E-mail: phacharatouch.ch@up.ac.th

Abstract. This work, concentrated on the effect of nuclear equation of state on the elliptic flow ($v_2$) in heavy ion collision by using quantum molecular dynamics model (QMD). The elliptic flow of proton in $^{197}$Au+$^{197}$Au collision at incident energy $0.25A$ GeV and impact parameter from 0 to 0.25 and from 0.25 to 0.45 within the quantum molecular dynamics model was studied. The theoretical calculations were performed with the nuclear equation of state (soft and hard EoS). The elliptic flow of proton as a function of the mid-rapidity ($y_0$) was computed and compared with those from experiments. The calculated results show that the behaviour of matter at high density and high temperature are described by a soft EoS. With the theoretical result, the elliptic flow of proton as a function of ($y_0$) is consistent with the experimental FOPI data.

1. Introduction
The experiment of the elliptic flow in heavy-ion collision at intermediate energy has been studied in the research field for the past years. For example, the direct and elliptic flow was measured from heavy ion collision at intermediate energy by the large acceptance apparatus FOPI (FOPI or $4\pi$ as synonyms, which means the entire solid angle, and as the name of a particle accelerator in GSI.) [1]. The experimental overview of collective flow phenomena in heavy-ion collisions in the incident energy regime from 100 $A$ MeV to 160 $A$ GeV was studied [2]. The elliptic flow as a function of pseudorapidity ($\eta$) in $^{197}$Au+$^{197}$Au collision at 200 $A$ GeV was computed and compared with the PHOBOS (PHOBOS is one of the four particle detectors at Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory.) experiments [3]. The elliptic flow in heavy ion collisions at the Relativistic Heavy Ion Collider was studied in a multiphase transport model [4]. The viscous potential flow was considered by the conditions for the development of a Kelvin-Helmholtz instability (KHI) for the quark-gluon plasma (QGP) flow [5] and the energy dependence of elliptic flow in $^{197}$Au+$^{197}$Au collisions was measured using the PHOBOS detector at Relativistic Heavy Ion Collider [6]. The nucleus-nucleus collisions at the energy lower than $A$ GeV in difference energy is the main motivation to study the isotopic flow and the nuclear equation of state. However, the effect of the nuclear equation of state in heavy-ion collision is not only to study the behavior of nuclear matter at high temperature and high density but also to study...
other fields in physics such as the evolution of the constellation, the evolution of the early universe and the mechanism of neutron star formation [7].

Elliptic flow at the mid-rapidity (previously called Squeeze-out) is a feature of the flow group that has been observed and was very interesting in the past. Based on previous research, YuMing Zheng et al. [8] examined the elliptic flow of proton in $^{48}$Ca+$^{48}$Ca reaction at incident energy 30-100 A GeV by used the iso-spin dependent transport model. It was found that the result of elliptic flow represented a change from positive to negative when the beam energy increased. The size of the elliptic flow depends on the nuclear equation of state and the nucleon-nucleon collision cross section. According to Andonic et al. [9] the elliptic flow dependence of transverse momentum was presented for the first time in this energy range and they indicated that, the elliptic flow could change rapidly at incident energy below 0.4 A GeV. The direct flow and elliptic flow from collision $^{197}$Au+$^{197}$Au collision at incident energy 40-150 A MeV has been observed by using INDRA (INDRA 4 ¶ multi detector) [10]. The results show that, the elliptic flow of atomic number, which is less than or equal has changed the flow from in plane to out of plane. When the energy is increased to 100 A MeV, the result is consistent with the FOPI experimental data [10]. From the above research, the elliptic flow has been generally recognized and observed in relativistic heavy-ion collisions at bombarding energies.

Consequently, this work investigates the elliptic flow of proton as a function of the mid-rapidity ($y_0$) in $^{197}$Au+$^{197}$Au at incident energy 0.25 A GeV and impact parameter from 0 to 0.25 ($b_0 < 0.25$) and from 0.25 to 0.45 (0.25<$b_0<0.45$) by using a quantum molecular dynamics model (QMD). Subsequently, the result are compared with the soft and hard nuclear equation of state. By the theoretical results, the elliptic flow of proton as a function of mid-rapidity are computed and compared with the FOPI experimental [1].

2. Theories

2.1 Quantum Molecular Dynamics (QMD) Model

The nuclear equation of state has explained the possibility of compacting nuclear matter. The quantum molecular dynamics [11] in which each nucleon is represented by a coherent state of the form

$$\psi(r,p,t) = \frac{1}{(2\pi L)^{3/4}} \exp \left[ -\frac{(r-r_0)^2}{4L} \right] \exp \left\{ i \mathbf{p} \cdot (r-r_0) \right\}, \quad (1)$$

where $r_0$ is the center of a Gaussian wave pocket and $L = 1.08$ fm$^2$ is the width of the wave pocket. Consequently, the density of the system with $N$ nucleons in a coordinate space is given below:

$$\rho(r,t) = \sum_{i} \frac{1}{(2\pi L)^{3/2}} \exp \left[ -\frac{(r-r_0)^2}{2L} \right]. \quad (2)$$

The time evolution of the $N$ - body distribution is determined by the motion of the centroid of Gaussian $\{r_{i0}, p_{i0}\}$, which obeys the Poisson brackets,

$$\dot{r}_{i0} = \{p_{i0}, H\}, \quad (3)$$

$$\dot{p}_{i0} = \{r_{i0}, H\}, \quad (4)$$

with $H$ being the nuclear Hamiltonian and $c = \hbar = 1$, where $c$ is the speed of light, $\hbar$ is the reduced Planck constant.

$$H = \sum_{i} \sqrt{p_{i0}^2 + m_i^2} + \sum_{i<j} \left( U_{ij}^{\text{inv}} + U_{ij}^{\text{coul}} \right). \quad (5)$$
In this work, the nuclear Hamiltonian is MeV unit. Here \( U_{ij}^{\text{Str}} \) is a nuclear mean field, and \( U_{ij}^{\text{Coul}} \) is the Coulomb interaction.

The strength of the nuclear compression is quoted normally in terms of the incompressibility by value of constant \( K \) (compressibility) defined as below [12]:

\[
K = 9 \rho \frac{\partial^2}{\partial \rho^2} \left( \frac{E}{A} \right),
\]

(6)

for the description of the energy per nucleon \( E/A \) as a function of density, usually Skyrme-parameterizations \( U \) are used in Equation (7). With the soft EoS is represented by a value of \( K = 200 \text{ MeV} \), while a hard EoS is represented by the value of \( K = 380 \text{ MeV} \).

\[
U = \alpha \left( \frac{\rho}{\rho_0} \right) + \beta \left( \frac{\rho}{\rho_0} \right) \gamma,
\]

(7)

where \( \rho \) is the nuclear density which is frequently measured in unit of the situation density \( \rho_0 \) of cold nuclear matter \( (\rho_0 \approx 0.16 \text{ fm}^{-3}) \).

| K (MeV) | \( \alpha \) | \( \beta \) | \( \gamma \) | EOS       |
|---------|-------------|-------------|-------------|-----------|
| 200     | -356        | 303         | 7/6         | Soft      |
| 380     | -124        | 70.5        | 2           | Hard      |

Table 1. Parameters in the equations (6) and (7) for the soft and hard a nuclear equation of state (EoS) [12].

2.2 Elliptic flow
The phenomenon of collective flow could be quantitatively described in terms of anisotropies of the azimuthal emission pattern, expressed by a Fourier series [13]

\[
\frac{dN}{d\phi} (\phi) \propto 1 + 2v_1 \cos (\phi) + 1 + 2v_2 \cos (2\phi),
\]

(8)

and

\[
-v_2 = \left( \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right),
\]

(9)

whereas \( \phi \) is the azimuthal angle of the outgoing particle with respect to the reaction plane. The second order Fourier coefficient, \( v_2 \) describes the elliptic flow [14] as shown in figure 1.
Figure 1. Isotropic flow as the elliptic flow: if \( v_2 \) is large, the circle will move on the \( y \) axis.

2.3. Rapidity

Rapidity, as its names implies, is related to the velocity. It is a dimensionless variable, \( y \), described the rate at which a particle is moving with respect to the chosen reference point situated on the line of motion. Thus, a particle is shifting by the momentum in the \( z \) axis (\( p_z \)) and \( E \) is also total energy. Mathematically, it is defined as [15]

\[
y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right). \tag{10}
\]

The mid-rapidity (\( y_0 \)) is the rapidity of the proton that moves out from the central collision (\( y \)) scaled by the rapidity of the beam energy (\( y_p \)) [1]:

\[
y_0 = \frac{y}{y_p}. \tag{11}\]

2.4. The centrality transverse component \( t \) of the 4-velocity

The magnitude of the velocity of the reference frame on the transverse coordinates is described by [16]

\[
u_{0t} = \frac{u_t}{u_p}, \tag{12}\]

where \( u_{0t} \) is the velocity of reference frame on the transverse coordinates, described to

\[
u_t = \beta_t \gamma_t, \tag{13}\]

taking \( \beta_t = p_t/E \) and \( \gamma_t = 1/\sqrt{1 - \beta_t^2} \), where \( \beta_t \) is the velocity magnitude of the transverse plane and \( u_p \) is the velocity of the reference frame in the reaction coordinates, related to

\[
u_p = \beta_p \gamma_p, \tag{14}\]
taking $\beta_p = p_z/E$ and, $\gamma_p = 1/\sqrt{1 - \beta^2}$ where $\beta_p$ is the velocity magnitude of the reaction plane, which help connect two experiments of a particle system in two different frames together.

3. Method
The quantum molecular dynamics approach, an n-body theory to describe heavy-ion reactions between 100 A MeV and 2 A GeV, is used in this work [11]. We start with a simulation of $^{197}$Au+$^{197}$Au collisions at the incident energy of 0.25 A GeV for impact parameters ranging from 0 and 0.25 ($b_0 < 0.25$) and 0.25 to 0.45 ($0.25 < b_0 < 0.45$). Then, we calculate the elliptic flow ($-v_2$) of the proton as a function of the mid-rapidity ($y_0$) using from the equation (8) and (9). Finally, we give a detailed comparison between our results and the mid-rapidity data from FOPI.

4. Results and Discussion

![Figure 2](image)

**Figure 2.** Elliptic flow of proton ($-v_2$) as a function of the mid-rapidity ($y_0$) in $^{197}$Au+$^{197}$Au collisions at incident energy 0.25 A GeV and impact parameter from 0.00 to 0.25 (a) and 0.25 to 0.45 (b) for collision for collision.

Figures 2 display the elliptic flow of proton ($-v_2$) as a function of the mid-rapidity ($y_0$) in heavy ion collisions of $^{197}$Au+$^{197}$Au at incident energy 0.25 A GeV for impact parameter from 0 to 0.25 and 0.25 to 0.45 by including with the nuclear equation of state (EoS; soft and hard). The result from the figure shows that, the proton discharge is observed to the elliptic flow of proton moving out of plane as represented by the negative $v_2$ values and the calculated results with soft EoS are consistent with the FOPI data. The result of the theoretical calculation is in a good agreement with the experiment data. The calculated results by using the soft EoS have the smallest RMSE. This indicates that the equation of state can describe behavior of matter in high pressure and high temperature that is soft EoS. For the centrality, they are reasonable with the quantum molecular dynamics calculations, this pattern similar to other work [16].
5. Conclusions
In summary, the QMD model has been used to study the elliptic flow of proton as a function of the mid-rapidity ($y_0$) in $^{197}$Au+$^{197}$Au collisions at incident energy 0.25A GeV for impact parameter from 0 to 0.25 and 0.25 to 0.45, for the soft and hard nuclear equation of state (EoS). Our simulated results show that the rapidity dependence of proton elliptic flow sensitivity to properties tightly is connected with the nuclear EoS. The quasi-parabolic shapes leading to the high elliptic flow pattern at hard EoS; this work suggests a preference for the soft EoS. One reason for the preference of the soft EoS to the hard EoS is due to a gradual tendency of the shape change, which tends to be more compact at the hard EoS.

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References
[1] Reisdorf W et al 2012 Nucl. Phys. A 876 1
[2] Herrmann N, Wessels J P and Wienold T 1999 Annu. Rev. Nucl. Part. Sci. 49 581
[3] Hirano T, Heinz U, Kharzeev D, Lacey R and Nara Y 2006 Phys. Lett. B 636 299
[4] Lin Z and Ko C H 2002 Phys. Rev. C 65 034904
[5] Uphoff J, Fochler O, Xu Z and Greiner C 2011 Phys. Rev. C 87 024908
[6] Back B B et al 2005 Phys. Rev. Lett. 94 122303
[7] Shen H, Toki H, Oyamatsu K and Sumiyoshi K 1998 Nucl. Phys. A 637 435
[8] Zheng Y, Ko C M, Li B A and Zhang B 1999 Phys. Rev. Lett. 83 2534
[9] Andronic A et al 2005 Phys. Lett. B 94 173
[10] Lukasik J et al 2005 Phys. Lett. B 608 223
[11] Aichelin J 1991 Phys. Rep. 202 233
[12] Hartnack C et al 1998 Eur. Phys. J. A 1 151
[13] Xing Y, Zhang Y, Srisawad P and Yan Y 2010 EPL 90 12002
[14] Yan T Z et al 2006 Phys. Lett. B 638 50
[15] Best D et al 1997 Nucl. Phys. A 612 173
[16] Chaimongkon P, Prommi Y, Roumsuk T, Srisawad P and Zheng Y 2018 J. Phys. Conf. Ser. 1144 012027
[17] Chai T and Draxler R R 2014 Computer Model Dev. 7 1250

Table 2. The root mean square errors [17] (RMSE) for calculated result of the elliptic flow of proton as a function mid-rapidity in $^{197}$Au+$^{197}$Au collision at incident energy 0.25 A GeV.

| Parameter Range | Soft EoS | Hard EoS |
|-----------------|----------|----------|
| $0 < b_0 < 0.25$| 0.01     | 0.05     |
| $0.25 < b_0 < 0.42$| 0.03     | 0.10     |