EMBEDDABILITY OF JOINS AND PRODUCTS OF POLYHEDRA

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Abstract. We present a short proof of S. Parsa’s theorem that there exists a compact $n$-polyhedron $P$, $n \geq 2$, non-embeddable in $\mathbb{R}^{2n}$, such that $P \ast P$ embeds in $\mathbb{R}^{4n+2}$. This proof can serve as a showcase for the use of geometric cohomology. We also show that a compact $n$-polyhedron $X$ embeds in $\mathbb{R}^m$, $m \geq 3(n+1)/2$, if either

- $X \ast K$ embeds in $\mathbb{R}^{m+2k}$, where $K$ is the $(k-1)$-skeleton of the $2k$-simplex; or
- $X \ast L$ embeds in $\mathbb{R}^{m+2k}$, where $L$ is the join of $k$ copies of the 3-point set; or
- $X$ is acyclic and $X \times (\text{triod})^k$ embeds in $\mathbb{R}^{m+2k}$.

1. Introduction

It was shown by Flores, van Kampen and Grünbaum [9] that every $n$-dimensional join of $k_i$-skeleta of $(2k_i + 2)$-simplexes does not embed into $\mathbb{R}^{2n}$ (see also [11, Examples 3.3, 3.5], [12], [20]). Some other $k_i$-polyhedra with this property are constructed in [12].

As noted by S. Parsa [15], it is implicit in a paper by Bestvina, Kapovich and Kleiner [5] that if compact polyhedra $P^m$ and $Q^m$ both have non-zero mod 2 van Kampen obstruction, then $P \ast Q$ does not embed in $\mathbb{R}^{2(n+m+1)}$. An $n$-dimensional polyhedron, non-embeddable in $\mathbb{R}^{2n}$ but with vanishing mod 2 van Kampen obstruction was constructed by the author for each $n \geq 2$ [11], settling...