Reply to the comment on “Frame-dragging: meaning, myths, and misconceptions” by A. Deriglazov

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Abstract

It has been claimed in [arXiv:2110.09522] that the expression for the Sagnac coordinate time delay given in [arXiv:2109.14641] “differs from the standard interpretation described in the book by Landau-Lifshitz (LL)”. We note that: 1) the Sagnac effect is not even discussed in LL; 2) the expression in [arXiv:2109.14641] is standard, given in countless papers and even textbooks; 3) the expression by LL quoted by the author consists of the (infinitesimal) two-way trip travel time for a light signal, which the author confuses with the Sagnac time delay (when they are actually very different things); 4) such confusion would negate the existence, both in special and general relativity, of the well-known and experimentally tested Sagnac effect; 5) the claims that it sheds doubt in any of the assertions made in [arXiv:2109.14641] are completely unfounded.

1 Two-way trip of light signals, and Landau-Lifshitz space metric

The line element $ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$ of a stationary spacetime can generically be written as

$$ds^2 = -e^{2\Phi}(dt - A_i dx^i)^2 + h_{ij} dx^i dx^j,$$

(1)

where $e^{2\Phi} = -g_{00}$, $\Phi \equiv \Phi(x^i)$, $A_i \equiv A_i(x^j) = -g_{0i}/g_{00}$, and $h_{ij} \equiv h_{ij}(x^k) = g_{ij} + e^{2\Phi} A_i A_j$.

Consider two infinitesimally close observers at rest in the coordinates of [1]: observer $E$, carrying a flashlight, and observer $R$, carrying a mirror. Observer $E$ emits a light flash at position $x^i_E$, which is reflected by observer $R$’s mirror at $x^i_R = x^i_E + dx^i$, returning then to $E$; see Fig. 18 in Sec. §84 of the Landau-Lifshitz (LL) textbook [1]. Along the photon’s worldline, $ds^2 = 0$; by [1], this yields two solutions for $dt$, of which the one corresponding to a future-oriented null worldline is

$$dt = A_i dx^i + e^{-\Phi} dl ; \quad dl = \sqrt{h_{ij} dx^i dx^j} \quad \text{(for a photon)}.$$

(2)

For the trips $E \rightarrow R$, and $R \rightarrow E$ we have, respectively,

$$dt_{ER} = A_i dx^i + e^{-\Phi} dl ; \quad dt_{RE} = -A_i dx^i + e^{-\Phi} dl .$$

Observe that the term $A_i dx^i$, but not $dl$, changes sign with an inversion of direction. Hence, for the photon’s two-way trip $E \rightarrow R \rightarrow E$,

$$dt_{ER} + dt_{RE} = 2e^{-\Phi} dl ,$$

(3)

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Figure 1: (a) Light beams propagating in opposite directions along an optical fiber loop $C$ around a spinning body (Fig. 1(a) of [10]). (b) Spacetime diagram for this setup (inspired on Fig. 2 of [19]): the difference in arrival times $\Delta t_S$ in Eq. (5) (Sagnac coordinate time delay) is an interval along a $t$-coordinate line (which translates, in the observer’s proper time, to $\Delta \tau_S = e^\Phi \Delta t_S$). The quantity $t_p = \int_C dl$, dubbed “true time” in [2], yields in fact the length of the spatial curve $C$. Confusing it with the beams’ travel time would lead to the erroneous conclusion that they arrive at the same time, in contradiction with measurement (by e.g. a Sagnac interferometer).

which corresponds to the unnumbered equation below Eq. (84.5) in [1] quoted by the author of [2].

In terms of observer $E$’s proper time, $d\tau = e^\Phi dt$, this time interval equals twice $dl$, which is thus the measured spatial distance between $E$ and $R$. This standardly defines $h_{ij}$ as the spatial metric, introduced in [1], and subsequently widely used in the literature on 1+3 spacetime splittings (e.g. [3–9], including [10]).

From the above discussion it follows that integrating the quantity

$$dl = e^\Phi (dt - A_i dx^i) \equiv dt_p$$  \hspace{1cm} (4)

along the worldline of a photon propagating in an optical fiber, as suggested by the author of [2] (but not by LL), will simply result in the length of the optical fiber as measured by the observers at rest in the coordinates of [1] (which of course is the same in both directions), see Fig. 1(b). Notice that such integral does not even represent the time measured by some observer $E$ for a finite photon two-way trip along the optical fiber (unless $\Phi$ is constant along it), since such interval would, by Eq. (2), be $\Delta \tau_{ERE} = e^\Phi (\Delta t_{ER} + \Delta t_{RE}) = 2 e^\Phi \int_{E}^{R} e^{-\Phi} dl$. More importantly: this has nothing to do with the Sagnac effect.

2 Sagnac effect

The Sagnac effect (e.g. [9,[11],[31]) consists of the difference in arrival times of light beams propagating in opposite directions around a spatially closed loop $C$. Consider an optical fiber loop, where observer $E$ at rest in the coordinates of [1] injects light beams in opposite directions, as depicted in Fig. 1 (a) (similar to Fig. 1 (a) of [10]). Using the $+$ ($-$) sign to denote the anti-clockwise (clockwise) directions, the coordinate time it takes for a full loop is, from Eq. (2), respectively $	ext{LL write this as the difference between the coordinate time intervals calculated with respect to the reflection event (thus one being typically positive and the other typically negative); this is perhaps the source of the confusion by the author of [2].}$
therefore, the Sagnac time delay is, in coordinate time and in observer $E$’s proper time, respectively (cf. Eq. (4) of [10]),

$$\Delta t_S \equiv t_+ - t_- = 2 \int_C A_i dx^i = 2 \int_C A,$$

$$\Delta \tau_S = e^\Phi \Delta t_S$$ (5)

(above and $[10]$). Equation (5) is a well known, standard result, given, precisely in (one, or both of) these two forms, in e.g. [8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25]. It applies both to rotating frames in flat spacetime as well as to arbitrary stationary gravitational fields, has been thoroughly experimentally tested, plays a key role in the relativistic corrections for the Global Position System [22,23], and is the basis of optical gyroscopes [14,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34]. Based on this equation, proposals for experimental detection of frame-dragging have also been put forth [16,17,27,28,29].

Finally, we remark that although this effect is not explicitly discussed in LL [1], a closely related quantity — the synchronization gap along $C$, which is well known (e.g. [9,20]) to be one half the corresponding Sagnac time delay — is computed therein, see Eq. (88.5) in Sec. §88; the result is, as would be expected, exactly $\Delta t_S/2$, as given by Eq. (5) above (Eq. (4) in [10]).

### 3 Conclusions

To conclude, we have shown that the claims in [2] regarding the results presented in [10] are completely unfounded: the Sagnac effect is not even discussed in [1] (just the closely related synchronization gap, and with results entirely consistent with those in [10]); the integral (5) in [2], that the author dubs “true time”, does not actually correspond to the travel time as measured by any observer, but instead to the usual definition of spatial length; and the expression for the Sagnac coordinate time delay in [10] is completely standard.

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