Development of mathematical model of oil and gas wells drilling based on use of energy transmitted to bottom hole

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Abstract. A mathematical model of the well deepening process based on use of energy supplied to bottom-hole and used for rock destruction is considered. On the basis of this model, it is possible to obtain partial models for automated control of drilling, as well as a graphical model of technical and technological characteristics of the hydraulic bottom-hole motor (HBM) intended for effective bit drive selection.

1. Introduction
In oil and gas wells construction, the main expenses are associated with the well deepening [1,2,3]. Therefore, it is advisable to find solutions for optimal control of the well deepening process. This problem was studied by many domestic and foreign experts: V. S. Fedorov [1], A. I. Spivak [2], E. M. Galle and H. B. Woods [3], R. M. Ageles [4], G. A. Kyliabin [5] et al. However, a general method has not been proposed yet. Optimal control requires models of the controlled object which can be used for operational control of the well deepening process and can allow preliminary selection of effective bit drive for the drilling process using hydraulic bottom-hole motors.

2. Results
The goal of the work is to develop a mathematical model of drilling process which applies the hydraulic bottom-hole motor. Mechanical penetration speed $V$ was chosen as an optimization criterion.

It is known that penetration speed $V$ is defined by: $\delta_1$ – deepening into the rock of rock-breaking top of a bit tooth; $P$ – the bit load; $a_P$ – hardness of the rock; $K_Z$ - number of teeth in the bit which simultaneously affect the bottom-hole; $\tau_k$ – time of bit teeth contact with the rock. Thus, [1]:

$$\delta_1 = f(P; a_P; K_Z).$$

Analysis of many studies [2,3,4,5] shows that during the well deepening it is very difficult to define quickly and with sufficient accuracy values included in the formula describing $\delta_1$, and consequently, magnitude $\delta_1$ of the deepening itself. Automated operational control requires other methods based on defining indirect dependences of controlled values on controlling factors.

It is known that well deepening speed depends on the amount of energy used for rock destruction ($N_R$). Therefore, it is possible to use formula for $N_R$ as the basis of a deepening process mathematical model (DPM), which in general, according to analysis of [1,5,6], can be represented as:
\[ N_R = \frac{P_Z \delta_1 \kappa_Z}{\tau_{VD}} \]  

(2)

where: \( \tau_{VD} \) - time of penetration into the rock.

In the process of HBM operation, it is rather problematic to measure and control only \( N_R \) value, that is just a part of the drilling pump power \( N_{GN} \) and hydraulic power of the bottom-hole motor \( N_{TG} \). The turbo-drill power \( N_{TG} \) is also used for other operations: lifting the part of the drilling tool "due to dented surface of the rolling cutters"; friction of peripheral teeth of the rolling cutters against the well bottom; friction of the bit body against the well wall, etc. Components of \( N_i \) power and hydraulic power of the turbo-drill (except for consumption by the motor itself) in each half-cycle of axial bit tooth vibrations are used in a different way and not simultaneously (Figure 1).

![Figure 1. Scheme of power use by mud pumps during the drilling process](image)

In the first half-period, the main part of the power – \( N_R \) – under the action of the axial bit load \( P \) consisting of a static part \( P_C \) and a dynamic one \( P_D \) is used directly for the rock destruction, whereas during the second half-period, power \( N_{DP} \) supplied to the surface of the bottom-hole is transformed under action of the force \( P_C \) mainly to friction of the bit working surface against the rock. Here, \( N_{UP} \) and \( N_{TR} \) are the power used directly for the rock destruction, for elastic interaction of the bit and the drill string with the bottom-hole and friction of the bit against the well wall, etc. \( N_{DP} \) – power supplied from the motor to the bottom-hole (not taking into account \( N_{f1} \)); \( N_{f1} \) – power required for restoration of moment of inertia of the HBM rotor. \( N_G \) – a part of \( N_{TN} \) which is used for rock pieces refinement at the well bottom and the well bottom cleaning, for energy dissipation in the flow of liquid at \( N_{TN} \) vibrations, and due to hydraulic resistance \( N_{TS} \).

The presented cost scheme \( N_i \) allows obtaining DPM of various output variability for resulting parameters: deepening (elastic deformation) during a single tooth contact with the bottom-hole – \( \delta_1 \), penetration per a bit revolution, mechanical penetration rate \( V \) or penetration per one bit run.

According to Figure 1, deepening process model can be presented in different variants and with different detalization levels \( So \):

\[ N_R = (N_{DP} + N_D) - N_{RS} \]  

(3)

\[ N_{RR} = N_R + N_{UP} = N_{DP} + N_D \]  

(4)

where \( N_{DP} \) – power supplied to the bottom-hole during time \( T_1 \) and used for interaction of the bit cutters with bottom-hole without intense destruction of the latter;

\( N_D \) – dynamic component of \( N_R \);

\( N_{RS} \) – power dissipated in the rock.
Thus, \( N_{DP} \cong 2\pi M_U \cdot P_S \cdot n \) and \( N_D \cong 2\pi \cdot M_U \cdot P_D \cdot n \), where \( M_U \) – specific momentum at the bit operation in the bottom-hole associated with the friction coefficient (resistance) of its cutters with rock – \( \mu_{GP} \):

\[
N_{RR} = 2\pi n \cdot P_Z \cdot M_U.
\]  

(5)

According to recommendations [1,7] and our improvement, we use formula (2) for creation of DPM, and then:

\[
N_{UP} \cong P_Z \cdot K_Z \cdot n \cdot K_D \cdot \delta_1 \cdot \tau_v \cdot \mu_{GP}.
\]

(6)

\[
N_{RR} = \frac{1.15P_Z K_Z \cdot \delta_1 \cdot K_D}{\tau_v}.
\]

(7)

Using these formulas, expanded form of DPM can be written:

\[
\frac{P_Z K_D \delta_1 K_Z}{\tau_v} = 2\pi \cdot P_Z \cdot n \cdot M_U - \frac{P_C \delta_1 K_Z}{1.8 \tau_v}.
\]

(8)

\( K_Z \) – number of bits teeth engaged in a simultaneous contact with the bottom-hole, according to recommendation of [8], \( K_Z = 3 \);

\( K_D \) – coefficient \( P \) of impact on the bottom; in general case, \( K_D = 1.0 - 1.5 \), but average value of \( K_D \leq 1.25 \);

\( \tau_V \), \( \tau_D \) – time periods during which the bit tooth penetrates to the rock and is withdrawal from the rock, given that:

\[
\tau_V + \tau_D = T_1.
\]

(9)

Let us reduce the formula (8) to a simpler form, taking into account that \( \tau_D \cong 1.1 \tau_V \cdot K_Z = 3 \), instantaneous radius of the bit rotation at contact of the rolling cutter with bottom-hole \( R_M = (0.55 - 0.99)R \). We will use the known method for \( M_U \) calculation:

\[
M_U = R_M \cdot \mu_{GP}.
\]

(10)

Then, the average value of \( \delta_1 \) will be equal to:

\[
\delta_1 = \frac{0.877 R \cdot \tau_V \cdot \mu_{GP}}{K_D}.
\]

(11)

where \( \mu_{GP} = 0.6 - 0.05 \) – for very soft rocks and 0.05 – for rocks of \( T, TK \) types;

\( R \) – the bit radius.

Time \( \tau_V \) can be defined in accordance with the known formula for calculating a period of axial bit tooth vibrations for the case of non-deformed bottom-hole:

\[
\tau_V \cong \frac{t_z}{2\pi R n}.
\]

(12)

where \( t_z \) – pitch of rolling cutter teeth on the peripheral row.

It can be obtained from formulas (11) and (12) that:

\[
\delta_1 \cong (7.6 \cdot 10^{2} \cdot t_z \cdot \mu_{GP})/K_D.
\]

(13)

The implementation of the derived DPM requires a much smaller number of hard-to-determine coefficients. The model includes all physically understandable parameters and is simpler than those proposed, for example, in [1].

The model obtained by means of component analysis of the power supplied to the bottom-hole is intended for creation of algorithms for control of well deepening performed with efficient bit drive, for which pre-selection can be performed using shown graphical model of HBM for turbo-drills – turbo-drill technical and technological characteristics proposed by the authors in [9].

The figure shows:

1 – non-working area for mechanical systems (motors), conditional line \( M_i = f_i(n) \) of \( M_i \) changes;
2 – area between curves corresponding to cyclic part of \( M_C = f_C(n) \) curve and point correspondent to work of rational momentum \( M_{RC} \) on the turbo-drill shaft;

3 – area in which the energy is supplied to the system and is transformed, area of braking with \( M_R \) at increase in the axial bit load;

4 – area of all resistances to the turbo-drill shaft, excluding working area 5 of effective working momentum \( M_R \) with maintenance of upper rational momentum \( M_{RC} \); \( M_{opi} \) – which is not reasonably used by some researchers, since it cannot be provided with an increase in the optimal \( M_{op} \) value in the operating mode of the turbo-drill;

![Figure 2. A graphical model of characteristics of HBM for turbo-drills - turbo-drill technical and technological characteristics](image)

\( M_{max} \) – maximum value \( M \) according to cyclic curve \( M_C = f_C(n) \), \( M_j \) – moment of inertia of the turbo-drill rotating parts, \( n_{MC} \) – value of \( n \) when \( M_i = M_{max} \) on the curve for one cycle \( M_C = f_C(n) \);

\( N_{Rmin} \) – minimum work \( n \) on the curve for \( M_{RC} \); \( n_{op} \) – value of \( n \), which is almost constant for all \( M_i \) curves;

\( n_{rac} \) – rational value of \( n \) for work \( M_R \) providing \( M_{RC} \);

\( n_x \) – idle \( n \) at \( M_C = f_C(n) \) which can be changed within a certain range;

\( \alpha \) – line \( M_i \) between areas 4 and 5 when \( n \) is changed from \( n_{Rmin} \) to \( n_{on} \), i.e. to the lower limit of \( M_R \);

\( \delta \) – the turbo-drill shaft braking curve in the case of excess of the turbo-drill thrust force and braking torques acting on the motor shaft; given that, to simplify the drawing, a curve part from \( n_{Rmin} \) to \( n_{mc} \) is not shown.

We presented in this paper the model of energy transfer in mechanical systems is much more accurate than models previously proposed in the sources.

3. Conclusion
The estimation with sufficient accuracy of penetration speed defined by deepening of rock-cutting tip of bit tooth into the rock is very difficult. An indirect method is proposed and a mathematical model of well deepening is derived based on the analysis of the energy components supplied to the bottom-hole.

A graphical model of a hydraulic bottom-hole motor for turbo-drills – turbo-drill technical and technological characteristics, intended for pre-selection of the effective drive bits is presented.
The problems of drilling operations, taking into account dynamics of the drilling tool, balance of hydraulic power used by bit drive and models of energy transfer in the bottom-hole motor, can be effectively solved using the proposed graphical and mathematical models of the well deepening process.

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