Why size matters - the transport of expelled saliva droplets carrying SARS-CoV-2

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Abstract

There is evidence that one of the most likely routes of transmission of the corona virus disease (COVID-19) is through the saliva droplets, that are produced while speaking, coughing or sneezing by infected people. The expelled droplets can measure between 0.4 and 450 μm. Once the droplets are in the air, they are subject to the gravitational, and air frictional forces. Assuming a calm environment, i.e, no air currents, we solve numerically, to a very high accuracy, the coupled differential equations of motion, in two dimensions. We calculated, the residence time and the horizontal range, among other quantities, as a function of drop size and initial velocity, for particles expelled by a person at a height of 1.6 m. In the aerosol regime, i.e. sizes between 0.4 and 5 μm, the drag forces quickly stop the droplets in their horizontal movement and they fall extremely slowly, almost vertically, pulled down by the gravitational force. The residence time is, in that size, 3.83 days to 33.3 minutes. The droplets with a diameter of 100 μm, can take 3.28 s, to reach the ground at a horizontal distance of 57 cm. Finally, the most massive droplets, with sizes of 450 μm, may take only 0.67 s and fall 3.67 m away from the source. By modeling the production of the number of saliva droplets with log-log Gaussian distributions in the size range 0.4 – 450 μm, the amount of virions expelled into the environment is estimated. Since those droplets remain suspended for a very long time, they can be carried very easily by air currents. Since recent studies show that the use of face masks reduce drastically the amount of droplets emitted to the air by an infected person and inhaled by a healthy one, we emphasize the great importance of using adequate face protection to minimize COVID-19, transmission.
I. INTRODUCTION

It is alarming how rapidly the SARS-CoV-2, a spherical enveloped virus with the positive sense RNA genome and a diameter of about 0.12 microns, is spreading around the world. As of November 4th, there have been 47.66 million infected people reported, and more than 1.22 million deaths. The first reported case by the World Health Organization (WHO) in China [1] was on the last day of 2019. In a few months the virus has been propagated by travelers and became a global pandemic, named COVID-19. Despite an unprecedented world effort, to date, there is no vaccine and special measures have to be taken to reduce the virus transmission.

To prevent the virus spread, several countries have implemented emergency procedures reducing, as much as possible, the transit of people and goods across regions and countries. The WHO recommended[2], people to lock down, hand sanitizing, and social distancing as measures to prevent contracting COVID-19. More recently, while still a matter of debate, the infection by inhalation of small saliva droplets produced while talking, sneezing or coughing by infected people, has been shown as a highly probable route of infection[3–5]. This was supported by a recent study, in which high viral loads of SARS-CoV-2 were found in the oral fluids of patients suffering the coronavirus disease[6]. Furthermore, by analyzing the trends and mitigation measures implemented in China, Italy, and the USA, Zhang et al [5] came to the conclusion that airborne transmission represents the dominant route for viral spread. This path of virus propagation was discussed more than ten years ago, during the H1N1 pandemic. However, despite several investigations published over the last decade[7, 8] that showed risk of transmission through airborne droplets, health authorities only recently recognized the importance of using face protection, and now recommends their use.

The purpose of this communication is to present the results of aerodynamic calculations of saliva droplets that are expelled horizontally by a person at a height of 1.6 m. We assume that the air environment is calm (there are no air currents) and that the forces that determine the particle dynamics are gravity and those produced by drag. To our knowledge, there is no report on such a calculation, determining the time of residence and distance traveled as a function of droplet size. Furthermore, by modelling the expiratory saliva droplets by a log-log Gaussian distribution function, in the range 0.4 – 450 µm, we estimate the number of virions expelled to the air by infected persons.
II. THE PHYSICAL SCENARIO

The physical picture is clear: when somebody sneezes, coughs or talks, hundreds of thousands of saliva droplets are expelled at high velocities [9, 10]. Various experimental techniques have been used in order to determine the droplet size distributions while sneezing, coughing or talking; however, there are discrepancies in the reported data[11–15]. For example, in the case of influenza, a very closely related human coronavirus, the reported[16] size range is 0.15 to 0.19 µm. Other studies on the cough, sneeze, and speech droplets expelled by two subjects infected with unknown bacteria[17], reported that the droplet size ranged from 50 to 860 µm and 76% were between 80 and 180 µm. Since the exact size distribution of saliva droplets ejected by a COVID-19 patient is not known, we model the number of saliva drops expelled while talking (5 minutes), coughing or sneezing, by a log-log gaussian-adjusted curve in the 0.4 to 450 µm range, shown in Figure 1. These curves are plotted in a log-log scale with the maximum number around 10 µm, reported by Deguid[11]. We are aware that more than one mode has been observed [13], but we expect that these single mode curves represent the most important contributions in the aerosol regime.

Another important question is: what is the viral load in the droplets? Virological testing using Reverse Transcription Polymerase Chain Reaction (RT-PCR) found an average viral load of $7 \times 10^6$ copies per milliliter[18]. Also important is that once the droplets are airborne, their size is reduced due to dehydration. COVID-19 transmission is a complex phenomenon and many studies remain to be performed by different specialists, but the answer to the questions, how far can the droplets travel?, how long can they linger in the air?, and how many virions are contained in the droplets expelled to the air?, during respiratory events, can shed some light into the importance of some of the proposed routes of infection.

Health authorities believe that dehydrated virus-laden droplets can travel as far as 1.5 meters in a relative short time, and suggest that people should avoid coming closer than that distance. They recognize the following two paths for infection. First, the droplets fall within an area described by a circle with a radius of 1.5 m, and contaminate the surfaces within that space. Thus, if the contaminated objects are touched by our hands and in turn, we touch our mouth, nose or eyes, we will contaminate ourselves. Another possibility is that the droplets are inhaled, by someone in that neighborhood. This is the reason for the recommendation to wash our hands as often as possible and to keep at least 1.5 m between...
individuals. Hence, physical distancing is a key recommendation. As we will show below, all the particles with diameters smaller than 202 $\mu m$ when sneezed, than 250 $\mu m$ when coughed, or than 325 $\mu m$ when spoken, fall within 1.5 m. Small droplets with sizes less than 100 $\mu m$ can take from a few seconds to several hours to settle on the ground. Within those, as shown very recently[5], are the aerosol droplets with diameters of less than 5 $\mu m$. These droplets are stopped almost immediately in their horizontal movement due to the air resistance and, in the absence of air disturbances, can take days to reach the ground. Since they can stay suspended for a very long time in the environment, any air current or difference in air pressure, can move them very easily and make them travel much farther.

III. CALCULATION

We assume that the droplets are spherical, with a mass that depends on the volume $m = \rho V_p$; we take a constant saliva density $\rho$ for all droplet sizes. The motion occurs in the two-dimensional plane $xz$, perpendicular to the ground and defined by the horizontal initial velocity $\vec{v}_0 = v_0 \hat{i}$. The gravitational force, $\vec{F}_g$, pulls the particle in the $-\hat{j}$ direction. On the other hand, the frictional force, $\vec{F}_f$, between the particle and the surrounding air, acts opposite to the particle velocity; thus its direction changes continuously as the particle moves down.

The formulation and the calculation is straightforward, we note that for the drag force we take into account both, linear and quadratic velocity terms. To solve the equation of motion, we separate the set of equations in the $x$ and $z$ components. Thus, one obtains a set of coupled differential equations that have to be solved numerically. To achieve that, we use the Runge-Kutta method to the fourth order with time intervals of $1.0 \times 10^{-6}s$.

For the initial conditions $\vec{r}_0$ and $\vec{v}_0$, the solution gives the trajectory of any droplet with mass $m$. In principle, for virus-laden droplets, we should take into account that the droplet contains SARS-CoV-2 viruses; however, it been reported that his amount is small[18], so we assume that in saliva droplets water is the main contributor, and that the density and shape do not change much with the inclusion of virions.

The velocity at which the saliva droplets are ejected is still an area of debate. We assume initial velocities of 120 km/h (33.33 m/s), 60 km/h (16.66 m/s), and 30 km/h (8.33 m/s), when sneezing, coughing or talking, respectively, and consider the droplet sizes from 0.4 to
450 \mu m. For the calculation we assume that the droplets are expelled by a person at a height of 1.6 m and that the initial velocity vector is parallel to the ground.

IV. RESULTS

We present in Figure 2, the time that the droplets would take to hit the ground (\( \tau \) in s, continuous lines), and the distance that they travel along the x-axis (\( \lambda \) in m, discontinuous lines), as a function of the droplet size. It is worth noting that the plots are in a log-log, and semi-log scale. Although we plot the results for the three initial velocities, in the residence time there are no noticeable difference. On the other hand, in the case of \( \lambda \) we start noticing differences around 50 \( \mu m \).

From the results, it is important to note that for a wide range of small droplets (0.4 \( \mu m \leq D \leq 200 \mu m \)), the time of residence differs by less than 0.01 s, for the three ejecting velocities. For droplets with 0.4 \( \mu m \) diameter, the time of flight is 3.83 days for the three cases and the horizontal distances traveled are and 7.75 \times 10^{-6} m, 1.54 \times 10^{-5} m, and 3.07 \times 10^{-5} m, when talking, coughing and sneezing, respectively. These droplets would remain suspended in the air for several days in the absence of air currents, and the distance traveled horizontally would be negligible. For larger droplets measuring 1 \( \mu m \) in diameter, still in the aerosol regime, the time needed to fall to the ground decreases to 14.7 hours and the distance traveled increases to 4.43 \times 10^{-5} m, 8.8 \times 10^{-5} m, and 1.74 \times 10^{-4} m, for speaking, coughing and sneezing, respectively. This is still a very long residence time and the distance traveled is now near the millimeter regime. The 10 \( \mu m \) droplets stay in the air 8.82 min and travel 4.17 mm, 7.83 mm and 1.41 cm. Although the particles remain suspended only minutes, there is enough time for events characteristic of daily life activities to impact their travel distance. These droplets move some millimeters away from the source. Much larger droplets with diameter \( D = 100 \mu m \), would be in the air 3.28 seconds and will land 26.8 cm, away when speaking, 40.6 cm away, when coughing, and half a meter away, when sneezing from the emitting person. Finally, the 450 \( \mu m \) droplets ejected at the talking, coughing, and sneezing velocities take only 0.66 s, 0.67 s and 0.68 s, and they travel horizontally 1.82 m, 2.89 m and 3.75 m.

Thus, while speaking, coughing, or sneezing, the droplets that will fall within the excluded area described by the WHO recommendations, of 1.5 m are those with \( D \) smaller than
TABLE I: The total amount of virions expelled \( N_v \), contained in the droplets with sizes in the four ranges: aerosol regime \((0.4 - 5 \, \mu m)\), the droplet intervals \((5.1 - 10 \, \mu m)\), \((10.1 - 100 \, \mu m)\), and \((100.1 - 450 \, \mu m)\). Columns three and four show the time range that they remain in the air \( \tau(s) \), and the distance interval that they travel \( \lambda(m) \) from the source, while talking, coughing or sneezing, respectively.

| Range (\( \mu m \)) | \( N_v \) | \( \tau(s) \) | \( \lambda(m) \) |
|---------------------|-----------|----------------|-----------------|
| **Speaking \((v_0 = 30 km/h)\)** |           |                |                 |
| 0.4 – 5.0           | 1.05 \times 10^3 | 3.31 \times 10^5 - 2.12 \times 10^3 | 7.75 \times 10^{-6} - 1.13 \times 10^{-3} |
| 5.1 – 10            | 3.41 \times 10^3 | 2.12 \times 10^3 - 5.29 \times 10^2 | 1.13 \times 10^{-3} - 4.17 \times 10^{-3} |
| 10.1 – 10^2         | 1.75 \times 10^4 | 5.29 \times 10^2 - 3.3 | 4.17 \times 10^{-3} - 2.68 \times 10^{-1} |
| 1.01 \times 10^2 – 4.5 \times 10^2 | 6.14 \times 10^2 | 3.3 - 0.7 | 2.68 \times 10^{-1} - 2.1 |
| **Coughing \((v_0 = 60 km/h)\)** |           |                |                 |
| 0.4 – 5.0           | 2.09 \times 10^4 | 3.31 \times 10^5 - 2.13 \times 10^3 | 1.54 \times 10^{-6} - 2.19 \times 10^{-3} |
| 5.1 – 10            | 7.03 \times 10^4 | 2.13 \times 10^3 - 5.29 \times 10^2 | 2.19 \times 10^{-3} - 7.8 \times 10^{-3} |
| 10.1 – 10^2         | 3.4 \times 10^5 | 5.29 \times 10^2 - 3.3 | 7.8 \times 10^{-3} - 4.06 \times 10^{-1} |
| 1.01 \times 10^2 – 4.5 \times 10^2 | 3.84 \times 10^3 | 3.3 - 0.7 | 4.06 \times 10^{-1} - 3.1 |
| **Sneezing \((v_0 = 120 km/hr)\)** |           |                |                 |
| 0.4 – 5.0           | 5.83 \times 10^6 | 3.31 \times 10^5 - 2.13 \times 10^3 | 3.07 \times 10^{-5} - 4.14 \times 10^{-3} |
| 5.1 – 10            | 1.64 \times 10^7 | 2.13 \times 10^3 - 5.29 \times 10^2 | 4.14 \times 10^{-3} - 1.41 \times 10^{-2} |
| 10.1 – 10^2         | 4.61 \times 10^7 | 5.29 \times 10^2 - 3.3 | 1.41 \times 10^{-2} - 5.7 \times 10^{-1} |
| 1.01 \times 10^2 – 4.5 \times 10^2 | 1.2 \times 10^5 | 3.4 - 0.7 | 5.7 \times 10^{-1} - 3.7 |

327 \( \mu m \), 250 \( \mu m \), and 202 \( \mu m \), respectively. From Figure 1, one notices that based in our results the number of droplets falling farther away from that area is very small and therefore it is a correct recommendation. However, the safe distance of 2 \( m \) adopted by some countries is even a better advice.

To estimate the number of virions expelled in the respiratory events, we calculate from Figure 1, the volume associated to the number of droplets for each size. Then, this volume is multiplied by the average viral density, \( 7 \times 10^6 \) virions per milliliter[18]. This amounts for the virial load of that particular set of droplets. To obtain the total number of virions expelled,
we sum all the contribution from the droplets with diameter in the range $0.4 \leq D \leq 450 \mu m$.

To better discern the importance of the various range sizes, we divide the whole range in four intervals: aerosols droplets from $0.4$ to $5 \mu m$, droplets with sizes between $5.1$ to $10 \mu m$, $10.1$ to $100 \mu m$, and $100.1$ to $450 \mu m$. In Table 1 we present the results for the total number of virions for each set of $D$ values, $N_v$, the range of residence time $\tau(s)$, and the range of horizontal distances traveled $\lambda(m)$.

Droplets with diameters of $10 - 100 \mu m$ convey the largest viral load due to the abundance of droplets of this size (calculated as $17,500$ when talking, $340$ thousand when coughing and $46.1$ million when sneezing). Those drops stay in the air from $529$ to $3.28$ seconds, a range of time very dangerous for an infection. However, their horizontal range is less than $1 \ m$. The safe distance of $1.5 \ m$ recommended by the OMS, worldwide, is correct according to the aerodynamic calculation presented in this work.

Droplets with sizes in the $5 - 10 \mu m$ range produced when sneezing create a spread of $16.4$ million virions. These droplets remain in the air from $33.33$ to $8.8$ minutes with barely noticeable horizontal travel ranges. These particles, in closed spaces and in the presence of air currents can be transported large distances and inhaled by other people in the neighborhood. Thus, there is great danger if a sick person sneezes without any protection; mouth and nose covering is essential to avoid virus transmission. In addition, face covering by healthy people around, reduce their infection probability.

The range of particle sizes that has lately deserved great interest (perhaps controversial) is aerosols, ranging from $0.4$ to $5 \mu m$. Although they remain very close to the issuing person, their persistence times are even more dangerous since they take values from $331$ to $2.1$ thousand seconds. They carry a load of about $5.8$ million virions when sneezed. Again, mouth and nose coverings are highly recommended.

It is important to note that this work does not take into account the evaporation of the water droplets. It has been estimated [19] that the droplet size may shrink between $20$ to $30\%$. That produces a larger amount of smaller droplets with a higher virus density and longer residence time than those originally expelled.
V. CONCLUSIONS

From our results, one may observe that if the highest quantity of droplets exhaled while talking, coughing or sneezing by infected people, are in the 100 $\mu m$ range, the advice by the World Health Organization, to keep a distance of 1.5 $m$ between individuals, is well founded. On the other hand, they also stress the danger presented by aerosol droplets ($< 5 \mu m$), whose residence time is extremely large in calm environments. Aerosol droplets, once in the air, can be transported by environmental currents to much longer distances, than expected in a calm environment. These virus-laden particles can be inhaled by healthy subjects and infect them. A more dangerous scenario is that of group gatherings in confined environments like hospitals, schools, airplanes, or city public transportation, where air ventilation must be maximized.

Our calculations supports the recent report which shows that airborne transmission represents one of the most dangerous routes for the transmission of the COVID-19 disease and that to mitigate viral spread it is important to enforce the use of masks or other face coverings. There is no doubt, that this recommendation may effectively reduce contaminations and protect the public, prior to the development of a vaccine.

Another important observation, is that it has been recently shown[20], that common materials, like cotton or silk, can be effectively used in the production of face masks. A mask made with a combination of cotton and silk provides high filtration efficiencies of droplets in the range of 10 $nm$ to 6 $\mu m$. Thus, there is no reason for shortage of masks and they should be available at very low cost. These low cost measures, can be implemented easily in developing countries to effectively reduce the virus propagation.

Our results are consistent with reported experimental data on the time of flight and travel distance for saliva droplets. However, experiments by epidemiologists and infectologists, to find the minimum number of virions needed for a person to acquire the infection, is a field of research. We anticipate that our results will stimulate further experiments and allow sanitary authorities to implement scientifically supported measures to reduce virus transmission while a vaccine is produced.

This work is dedicated to the memory of the Nobel Laureate, Mario Molina, who made seminal contributions to the field.
Competing interests

The authors declare no competing interests.

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FIG. 1: The saliva droplets distribution functions. Adjusted by log-log gaussian curves for the respiratory events of talking, coughing and sneezing. The parameter values were taken from the data published by Deguid[11].

FIG. 2: Droplet’s time of flight $\tau$ and distance traveled $\lambda$. The time of residence in the air (left scale in s, continuous lines), and the distance traveled away from the source (right scale in m) by a saliva droplets as a function of its diameter $D$. The initial velocity is assumed parallel to the ground with values 8.33, 16.66, and 33.33 m/s, (discontinuous lines) typical of saliva droplets ejected while speaking, coughing or sneezing.
Figure 1
Figure 2