TESTING EXTENDED TECHNICOLOR WITH $R_b$ AND SINGLE TOP-QUARK PRODUCTION

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ABSTRACT

We review the connection between $m_t$ and the $Zb\bar{b}$ vertex in ETC models and discuss how data on $R_b$ constrains ETC models. Theories in which the ETC and weak gauge groups do not commute are consistent with electroweak data and predict effects on single top production that will be visible at Fermilab.

1. Introduction

Two outstanding questions in particle theory are the cause of electroweak symmetry breaking and the origin of the masses and mixings of the fermions. Because theories that use light, weakly-coupled scalar bosons to answer these questions suffer from the hierarchy and triviality problems, it is interesting to consider the possibility that electroweak symmetry breaking arises from strong dynamics at scales of order 1 TeV. This talk focuses on extended technicolor (ETC) models, in which both the masses of the weak gauge bosons and those of the fermions arise from gauge dynamics.

In extended technicolor models, the large mass of the top quark generally arises from ETC dynamics at relatively low energy scales. Since the magnitude of the CKM matrix element $|V_{tb}|$ is nearly unity, $SU(2)_W$ gauge invariance insures that ETC bosons coupling to the left-handed top quark couple with equal strength to the left-handed bottom quark. In particular, the ETC dynamics which generate the top quark’s mass also couple to the left-handed bottom quark thereby affecting the $Zb\bar{b}$ and $Wtb$ vertices.

This talk discusses how measurements of $R_b$ constrain ETC model building, shows that models in which $SU(2)_W$ is embedded in the ETC group are consistent with experimental data, and explains how measurements of single top quark production at the Fermilab Tevatron’s Run 3 will further test ETC.

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2. From $m_t$ To A Signal of ETC Dynamics

Consider an ETC model in which $m_t$ is generated by the exchange of a weak-singlet ETC gauge boson of mass $M_{ETC}$ coupling with strength $g_{ETC}$ to the current

$$\xi \bar{\psi}_i^L \gamma^\mu T^k_L + \frac{1}{\xi} \bar{t}_R \gamma^\mu U^k_R,$$

where $\psi_L \equiv \begin{pmatrix} t \\ b \end{pmatrix}_L, T_L \equiv \begin{pmatrix} U \\ D \end{pmatrix}_L$ (1)

where $U$ and $D$ are technifermions, $i$ and $k$ are weak and technicolor indices, and $\xi$ is an ETC Clebsch expected to be of order one. At energies below $M_{ETC}$, ETC gauge boson exchange may be approximated by local four-fermion operators. For example, $m_t$ arises from an operator coupling the left- and right-handed currents in Eq. (1)

$$-\frac{g_{ETC}^2}{M_{ETC}^2} \left( \bar{\psi}_i^L \gamma^\mu T^k_L \right) \left( \bar{U}^w_R \gamma^\mu t_R \right) + h.c.$$ (2)

Assuming, for simplicity, that there is only one weak doublet of technifermions and that technicolor respects an $SU(2)_L \times SU(2)_R$ chiral symmetry (so that the technipion decay constant, $F$, is $v = 246$ GeV) the rules of naive dimensional analysis give an estimate of

$$m_t = \frac{g_{ETC}^2}{M_{ETC}^2} \langle \bar{U} U \rangle \approx \frac{g_{ETC}^2}{M_{ETC}^2} (4\pi v^3).$$ (3)

for the top quark mass when the technifermions’ chiral symmetries break.

The ETC boson responsible for producing $m_t$ also affects the $Zb\bar{b}$ vertex when exchanged between the two left-handed fermion currents of Eq. (1) as in Fig. 1 (with $T \equiv D_L$ since the ETC boson is a weak singlet). This diagram alters the $Z$-boson’s tree-level coupling to left-handed bottom quarks $g_L = \frac{e}{\sin \theta \cos \theta} (-\frac{1}{2} + \frac{1}{2} \sin \theta^2)$ by

$$\delta g_L^{ETC} = -\frac{\xi^2 g_{ETC}^2 v^2}{2 M_{ETC}^2} \frac{e}{\sin \theta \cos \theta} (I_3) = \frac{1}{4} \xi^2 \frac{m_t}{4\pi v} \cdot \frac{e}{\sin \theta \cos \theta}.$$ (4)
where the right-most expression follows from applying eq. (3).

To show that $\delta g_L$ provides a test of ETC dynamics, we must relate it to a shift in the value of an experimental observable. Because ETC gives a direct correction to the $Zb\bar{b}$ vertex, we need an observable that is particularly sensitive to direct, rather than oblique, effects. A natural choice is the ratio of $Z$ decay widths

$$R_b \equiv \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{hadrons})}$$

(5)

since both the oblique and QCD corrections largely cancel in this ratio. One finds

$$\frac{\delta R_b}{R_b} \approx -5.1\% \xi^2 \left( \frac{m_t}{175\text{GeV}} \right).$$

(6)

Such a large shift in $R_b$ would be readily detectable in current electroweak data. In fact, the experimental value of $R_b = 0.2179 \pm 0.0012$ lies close enough to the standard model prediction (.2158) that a 5% reduction in $R_b$ is excluded at better than the $10\sigma$ level. ETC models in which the ETC and weak gauge groups commute are therefore ruled out.

3. Non-commuting ETC Models

The next logical step is to examine models in which the weak and ETC gauge groups do not commute. We begin by describing the symmetry-breaking pattern that enables “non-commuting” ETC models to include both a heavy top quark and approximate Cabibbo universality. A heavy top quark must receive its mass from ETC dynamics at low energy scales; if the ETC bosons responsible for $m_t$ are weak-charged, the weak group $SU(2)_{\text{heavy}}$ under which $(t, b)_L$ is a doublet must be embedded in the low-scale ETC group. Conversely, the light quarks and leptons cannot be charged under the low-scale ETC group lest they also receive large contributions to their masses; hence the weak $SU(2)_{\text{light}}$ group for the light quarks and leptons must be distinct from $SU(2)_{\text{heavy}}$. To approximately preserve low-energy Cabibbo universality the two weak $SU(2)$’s must break to their diagonal subgroup before technicolor dynamically breaks the remaining electroweak symmetry. The resulting symmetry-breaking pattern is:

$$ETC \times SU(2)_{\text{light}} \times U(1)_Y$$

$$\downarrow f$$

$$TC \times SU(2)_{\text{heavy}} \times SU(2)_{\text{light}} \times U(1)_Y$$

$$\downarrow u$$

$$TC \times SU(2)_W \times U(1)_Y$$

$$\downarrow v$$

$$TC \times U(1)_{EM},$$

(7)
where ETC and TC stand, respectively, for the extended technicolor and technicolor gauge groups, while \( f, u, \) and \( v = 246 \text{ GeV} \) are the expectation values of the order parameters for the three different symmetry breakings. Note that, since we are interested in the physics associated with top-quark mass generation, only \( t_L, b_L, \) and \( t_R \) must transform non-trivially under ETC. However, to ensure anomaly cancelation we take both \((t, b)_L\) and \((\nu_\tau, \tau)_L\) to be doublets under \(SU(2)_{\text{heavy}}\) but singlets under \(SU(2)_{\text{light}}\), while all other left-handed ordinary fermions have the opposite \(SU(2)\) assignment.

Once again, the dynamics responsible for generating the top quark’s mass contributes to \( R_b \). This time the ETC gauge boson involved transforms as a weak doublet coupling to

\[
\xi \bar{\psi}_L \gamma^\mu U_L + \frac{1}{\xi} \bar{f}_R \gamma^\mu T_R
\]

where \( \psi_L \equiv (t, b)_L \) and \( T_R \equiv (U, D)_R \), are doublets under \(SU(2)_{\text{heavy}}\) while \( U_L \) is an \(SU(2)_{\text{heavy}}\) singlet. The one-loop diagram involving exchange of this boson (Figure 1 with \( T \equiv U_L \)) shifts the coupling of \( b_L \) to the \( Z \) boson by

\[
\delta g_L = -\frac{e}{\sin \theta \cos \theta} \frac{\xi^2 v^2}{2 f^2} \approx -\frac{\xi^2}{4} \frac{e}{\sin \theta \cos \theta} \frac{m_t}{4 \pi v}.
\]

Since the tree-level \( Z b_L \bar{b}_L \) coupling is also negative, the ETC-induced change tends to increase the coupling. Hence \( R_b \) increases by

\[
\frac{\delta R_b}{R_b} \approx +5.1% \xi^2 \left( \frac{m_t}{175 \text{GeV}} \right).
\]

The change is similar in size to what was obtained in the commuting ETC models (Eq. (6)), but is opposite in sign.

But that is not the full story of \( R_b \) in non-commuting ETC. Recall that there are two sets of weak gauge bosons which mix at the scale \( u \). Of the resulting mass eigenstates, one set is heavy and couples mainly to the third-generation fermions while the other set is nearly identical to the \( W \) and \( Z \) of the standard model. That ‘nearly’ is important: it leads to a shift in the light \( Z \)’s coupling to the \( b \) of order

\[
\delta g_L = \frac{e}{2 \sin \theta \cos \theta} \frac{g_{ETC}^2 v^2}{u^2} \sin^2 \alpha
\]

where \( \tan \alpha = g_{\text{light}}/g_{\text{heavy}} \) is the ratio of the \(SU(2)\) gauge couplings. The couplings of the light \( Z \) to other fermions are similarly affected. Mixing thus alters \( R_b \) by

\[
\frac{\delta R_b}{R_b} \approx -5.1% \sin^2 \alpha \frac{f^2}{u^2} \left( \frac{m_t}{175 \text{GeV}} \right).
\]

Because the two effects on \( R_b \) in non-commuting ETC models are of similar size and opposite sign, these theories can yield values of \( R_b \) that are consistent with experiment.
Since $R_b$ alone cannot confirm or exclude non-commuting ETC, we should apply a broader set of precision electroweak tests. This requires describing the $SU(2) \times SU(2)$ symmetry breaking sector in more detail. The two simplest possibilities for the $SU(2)_{\text{heavy}} \times SU(2)_{\text{light}}$ transformation properties of the order parameters that mix and break these gauge groups are:

\begin{align*}
\langle \varphi \rangle & \sim (2, 1)_{1/2}, \quad \langle \sigma \rangle \sim (2, 2)_0, \quad \text{“heavy case”} \\
\langle \varphi \rangle & \sim (1, 2)_{1/2}, \quad \langle \sigma \rangle \sim (2, 2)_0, \quad \text{“light case”}.
\end{align*}

Here the order parameter $\langle \varphi \rangle$ is responsible for breaking $SU(2)_L$ while $\langle \sigma \rangle$ mixes $SU(2)_{\text{heavy}}$ with $SU(2)_{\text{light}}$. We refer to these two possibilities as “heavy” and “light” according to whether $\langle \varphi \rangle$ transforms non-trivially under $SU(2)_{\text{heavy}}$ or $SU(2)_{\text{light}}$. In the heavy case, the technifermion condensation responsible for providing mass for the third generation of quarks and leptons is also responsible for the bulk of electroweak symmetry breaking. The light case corresponds to the opposite scenario in which different physics provides mass to the third generation fermions and the weak gauge bosons.

We have performed a global fit for the parameters of the non-commuting ETC model ($s^2$, $1/x \equiv v^2/u^2$, and the $\delta g$'s) to all precision electroweak data: the $Z$ line shape, forward backward asymmetries, $\tau$ polarization, and left-right asymmetry measured at LEP and SLC; the $W$ mass measured at FNAL and UA2; the electron and neutrino neutral current couplings determined by deep-inelastic scattering; the degree of atomic parity violation measured in Cesium; and the ratio of the decay widths of $\tau \rightarrow \mu \nu \bar{\nu}$ and $\mu \rightarrow e \nu \bar{\nu}$. We find that both the heavy and light cases provide a good fit to the data. Furthermore, the extra $W$ and $Z$ bosons can be relatively light\footnote{These mass limits are stronger than current limits from direct searches for heavy weak bosons at FNAL.} Figure 2 displays the 95% confidence level lower bound (heavy solid line) on the heavy $W$ mass ($M^H_W$) for different values of $s^2$ (with $\alpha_s(M_Z) = 0.115$); at large $s^2$, the extra $W$ can weigh as little as 400 GeV. In the heavy case, similar work shows that the lowest possible heavy $W$ mass at the 95% confidence level is $\approx 1.6$ TeV, for $0.7 < s^2 < 0.8$.

We conclude that non-commuting ETC is consistent with all electroweak tests proposed so far. Clearly a new test is needed! In the last section of the talk, we show that single top quark production may fit the bill.

4. Single Top Production

It has been suggested that a sensitive measurement of the $Wtb$ coupling can be made at the Tevatron collider by studying single top production through quark/anti-quark annihilation ($q\bar{q}' \rightarrow W \rightarrow tb$), and normalizing to the Drell-Yan process ($q\bar{q}' \rightarrow Wq \rightarrow \ell\nu$) to control theoretical systematic uncertainties (e.g. in the initial
parton distributions). This method should be more precise than alternative methods involving single top production via $W$-gluon fusion, because there is no similar way to eliminate the uncertainty associated with the gluon distribution function.

In the standard model, the ratio of single top production and Drell-Yan cross-sections

$$\frac{\sigma(q\bar{q}' \rightarrow W \rightarrow tb)}{\sigma(q\bar{q}' \rightarrow W \rightarrow \ell\nu)} \equiv R_{\sigma}^{SM}$$

(15)

is proportional to the top quark decay width $\Gamma(t \rightarrow Wb)$ and, therefore, to $|V_{tb}|^2$. Recent work has shown that with a 30 fb$^{-1}$ data sample from Run 3 at the Tevatron with $\sqrt{s} = 2$ TeV it should be possible to use single top-quark production to measure $R_{\sigma}$, and hence $|V_{tb}|^2$ in the standard model, to an accuracy of at least $\pm 8\%$. By that time, the theoretical accuracy in the standard model calculation is projected to become at least this good.

The enlarged gauge group in non-commuting ETC models provides two potential sources of non-standard contributions to $R_{\sigma}$. Exchange of ETC gauge bosons can potentially make a large direct correction to the $Wtb$ vertex, similar to the direct effect on the $Zb\bar{b}$ vertex. Furthermore, these models include two sets of $W$ bosons; both sets contribute to the cross-sections, and mixing between the two sets alters the couplings of the lighter $W$ state to fermions. If the resulting fractional change in the cross-section ratio $\Delta R_{\sigma}/R_{\sigma}$ is at least $16\%$, it should be detectable in Run 3.

![Figure 2: Region (shaded) where light-case non-commuting ETC models predict a visible increase ($\Delta R_{\sigma}/R_{\sigma} \geq 16\%$) in single top quark production at TeV33. The dark line marks the lower bound (at 95% c.l.) on the mass of the heavy weak bosons $M_{W^H}$ (as a function of mixing parameter $\sin^2 \phi$) by electroweak data. Below the dashed line, the predicted value of $\Delta R_{\sigma}/R_{\sigma} \geq 24\%$.](image)

A priori, it appears that the $Wtb$ vertex should be affected by ETC gauge boson exchange through a diagram similar to Figure 1. However, a closer look at the operator that gives rise to the top quark mass (the product of the currents in Eq. (8))
demonstrates that there are no direct ETC contributions to the $Wtb$ vertex of order $m_t/4\pi v$ in non-commuting ETC models. Because the left-left piece of this operator includes $(t_l, b_l, U_L)$ but not $D_L$ and because its purely right-handed piece contains $(t_R, U_R, D_R)$ but not $b_R$, this operator does not contribute to the $Wtb$ vertex.

On the other hand, the presence of two sets of weak boson does alter $R_\sigma$. Diagonalizing the mass matrix of the $W$ bosons yields the masses, widths, and couplings to fermions of the $W$ mass eigenstates. Using this information, we have calculated the size of $\Delta R_\sigma/R_\sigma$ in both the heavy and light cases of non-commuting ETC. In the heavy case, the constraint $M_{WH} \approx 1.6$ TeV from electroweak data prevents $|\Delta R_\sigma/R_\sigma|$ from exceeding 9%. This effect is too small to be clearly visible at Tev33.

The light case of non-commuting ETC, where $M_{WH}$ can be as small as 400 GeV, yields more encouraging results. Since lighter extra $W$ bosons produce larger shifts in $R_\sigma$, there is a significant overlap between the experimentally allowed portion of parameter space and the region in which $|\Delta R_\sigma/R_\sigma| \geq 16\%$, as shown in Figure 2. In fact, the predicted fractional shift in $R_\sigma$ is greater than 24% for much of this overlap region. More precisely, the shift in $R_\sigma$ is towards values exceeding $R_\sigma^{SM}$, so that non-commuting ETC models with the “light” symmetry breaking pattern predict a visible increase in the rate of single top-quark production.

What allows the corrections to single top-quark production to be relatively large in non-commuting ETC models is the fact that there is no direct ETC effect on the $Wtb$ vertex to cancel the contributions from weak gauge boson mixing. This is in contrast to the calculation of $R_b$, where such a cancelation does occur. Hence within the context of these models it is possible for $R_b$ to have a value close to the standard model prediction while $R_\sigma$ is visibly altered.

Finally, we note that no model other than light non-commuting ETC has been found to predict a visible increase in $R_\sigma$. Thus, single top quark production can provide a clear signal of dynamical electroweak symmetry breaking.

5. Conclusions

Extended technicolor models predict distinctive alterations in the $Zb\bar{b}$ coupling and the rate of single top quark production. Measurements of $R_b$ have already excluded models in which the ETC and weak gauge groups commute, in favor of “non-commuting” models. Studies of single top quark production in Run 3 at the Tevatron will provide the next stringent test of non-commuting ETC.

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