Critical Behavior of $J/\psi$ across the Phase Transition from QCD Sum Rules

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We study behavior of $J/\psi$ in hot gluonic matter using QCD sum rules. Taking into account temperature dependences of the gluon condensates extracted from lattice thermodynamics for the pure SU(3) system, we find that the mass and width of $J/\psi$ exhibit rapid change across the critical temperature.

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I. INTRODUCTION

How $J/\psi$ changes its properties in the hot and dense medium is one of the main interests in high energy heavy ion physics. Pioneering works such as a mass shift caused by decreasing string tension [1] and suppression of the yield by Debye screening [2] are based on the fact that heavy quarkonium is a suitable tool for studying the confinement-deconfinement transition in QCD. Although experiments reveal things are much more complicated than expected, there are plenty of theoretically interesting issues. Especially recent lattice QCD, based on the maximum entropy method, has shown the spectral peak can survive even above the critical temperature [3, 4], but resolution of the peak is not sufficient and physics behind the result is still controversial [5].

Motivated by these lattice results, in this work we study the mass and width of $J/\psi$ in hot gluonic medium using QCD sum rules. QCD sum rules serve as a reliable non-perturbative theoretical tool for studying hadrons [6]. For charmonium, we can express the current-current correlation function with the perturbative contribution and the gluon condensate at the leading order of $\alpha_s$ because of much heavier mass of charm quark than $\Lambda_{QCD}$. If we set the energy scale of medium as the same order with $\Lambda_{QCD}$, effect of temperature can be entirely described by change of condensates [7]. Hence, once temperature dependence of relevant condensates are known, we can calculate properties of the heavy quarkonium in a quite similar manner to the vacuum case. Here we give a systematic study of $J/\psi$ using QCD moment sum rules by following the method described in Ref. [8] with relevant extensions to the hot gluonic matter case.

II. GLUON CONDENSATE AT FINITE TEMPERATURE

Since in-medium correlation function has no longer Lorentz invariance, we have to take into account higher twist operators in the operator product expansion (OPE). In the present case, we consider twist-2 gluon condensate as well as usual scalar gluon condensate which is related to

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the trace anomaly. By decomposing the twist-2 gluon operator, we define the scalar condensate term $G_0$ and the traceless and symmetric twist-2 term as

$$\left< \frac{\alpha_s}{\pi} G_{\alpha\mu} G^{\alpha\mu}_{\beta} \right>_T = \left( u_\alpha u_\beta - \frac{g_{\alpha\beta}}{4} \right) G_2(T) + G_0(T)$$

with $u^\mu$ being the four velocity of the medium. Comparing the energy-momentum tensor $T_{\alpha\beta} = (\varepsilon + p)u_\alpha u_\beta - pg_{\alpha\beta}$, we obtain

$$G_0(T) = G_{0,\text{vac}} - \frac{8}{11}(\varepsilon - 3p), \quad G_2(T) = -\frac{\alpha_s(T)}{\pi}(\varepsilon + p)$$

for pure SU(3) system. Here we put the value of the vacuum gluon condensate to $(0.35\text{GeV})^4$.

The three quantities of the righthand side are extracted from lattice SU(3) calculations [9, 10]. The resultant condensates are displayed in the left panel of Fig. 1.

**III. QCD MOMENT SUM RULES**

We analyzed the spectral property of $J/\psi$ using the moment sum rules. Definition of the moment of the correlation function is given by

$$M_n(Q^2) = \frac{1}{n!} \left( \frac{d}{dq^2} \right)^n T\langle q^2 \rangle \bigg|_{q^2=-Q^2}$$

where $T\langle q^2 \rangle$ is the correlation function for the vector current

$$(q^\mu q^\nu - g^{\mu\nu}q^2)T\langle q^2 \rangle = i \int d^4xe^{iqx} \langle T[\bar{c}(x)\gamma^\mu c(x)\bar{c}(0)\gamma^\nu c(0)] \rangle.$$  

The in-medium correlation function satisfies the dispersion relation

$$\Pi(q^2) = \int_0^\infty ds \frac{\rho(s)}{s-q^2},$$

FIG. 1: Left: temperature dependence of the gluon condensates. Right: ratio of the moment for the OPE side.
in which \( q = (\omega, 0) \) since we set the quarkonium at rest with respect to the medium. Taking \( q^2 < 0 \), the above dispersion relation gives the spectral density \( \rho(s) \) in the righthand side. In the QCD sum rules, we first calculate the correlation function using OPE which includes the condensates, and then relate it to the hadronic spectral density, which is modeled in a simple way. Generally the hadronic spectral density contains not only the desired pole term but also high-energy continuum part. In the momentum sum rule prescription, we can reduce the contribution from the continuum so that the pole term dominates the dispersion integral by optimizing the external parameters \( Q^2 \) and \( n \). Note that since \( Q^2 = -q^2 \) is an external parameter, we can take \( q^2 \) in the deep spacelike region such that \( \alpha_s(q^2) \) is small enough for the leading order perturbation. Hereafter we denote the energy scale in the unit of \( 4m_c^2 \), by \( \xi = Q^2/4m_c^2 \). Following the optimization criterion used in Ref. [8], we take the ratio of the \((n-1)\)-th moment to \( n \)-th one and picking up \( n \) at which the ratio of the OPE side takes its minimum. An example of this ratio for \( \xi = 1 \) is displayed in the right panel of Fig. 1. We can see that the ratio clearly decreases as temperature goes up. However, there is no minimum at \( T/T_c = 1.05 \), that indeed shows breakdown of the theory. This limitation in the maximally available temperature can be slightly improved by increasing the scale parameter \( \xi \), but current framework cannot be extended beyond \( T/T_c = 1.06 \). See Ref. [11, 12] for discussion. From the minima of the ratio, we can extract the mass and width of \( J/\psi \) by applying the relativistic Breit-Wigner form to the phenomenological hadronic spectral function and equating the ratio, i.e.,

\[
\frac{M_{n-1}}{M_n}\bigg|_{\text{OPE}} = \frac{M_{n-1}}{M_n}\bigg|_{\text{phen}}.
\]

The moment Eq. (6) enables us to extract to the pole term at \( \xi \) and \( n \) determined from the OPE side moment ratio.

IV. RESULTS AND DISCUSSION

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\(^1\) \( \alpha_s(T) \) in Eq. (2) should be taken separately because it expresses the effective coupling between heavy quark and thermalized gluonic medium.
Figure 2 shows the constraint between mass shift and width obtained from the sum rule. Reflecting the decreasing minimum of the moment ratio, the change of the mass and width becomes larger as temperature increases. A notable feature is the linear relation between the mass and width. Although a deviation from the linear and parallel behavior among different temperature cases is seen at $T = 1.04T_c$ in $\xi = 1$ case, this can be improved by increasing $\xi$ as shown in the right panel. In fact the minimum of the ratio obtained at $T = 1.04T_c$ is unstable as seen in Fig. 1. In such a case, coefficients of OPE become so large that applicability of the expansion might be suspicious. Therefore we can conclude that the linear relation holds as far as the sum rules do. For simultaneous determination of both mass and width, we need an additional constraint. See Ref. [13] for a recent investigation. Here we show the possible maximum change of the mass and width, in which the other quantity is assumed to remain with the vacuum value.

From Fig. 3 one finds both mass shift and width show order parameter-like behavior. Since temperature effect is incorporated into the gluon condensates, which clearly exhibit abrupt change due to the QCD phase transition, we can conclude the mass and width can be regarded as “order parameter” of the phase transition. Further discussion and details have been given in Refs. [11, 12].

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