Note on Artificial Deformation in Object Shapes Due to Pixelization

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Abstract

We qualitatively examine the properties of artificial deformation in the shapes of objects (galaxies and stars) induced by pixelization effects (also called aliasing effects) using toy mock simulation images. Two causes of these effects have been recognized. One is a consequence of observing the continuous sky with discrete pixels, which is called “first pixelization”. The other, called “second pixelization”, is a consequence of resampling a pixelized image onto another pixel grid whose coordinates are not perfectly adjusted to the input grid. We pay special attention to the latter, because it might be a potential source of a systematic noise in weak-lensing analysis. In particular, it is found that resampling with rotation induces artificial ellipticities in object shapes having a periodic concentric-circle-shaped pattern. Our major findings are as follows: (1) The root-mean-square (RMS) of artificial ellipticities in object shapes induced by the first pixelization effect can be as large as RMS $\geq 10^{-2}$ if the characteristic size of the objects (e.g., the FWHM) is smaller than twice of the pixel size. However, for larger objects, it quickly becomes very small (RMS $\lesssim 10^{-5}$). (2) The amplitude of shape deformation induced by the second pixelization effect depends on the object size. It also depends strongly on the interpolation scheme adopted to carry out resampling and on the grid size of the output pixels. The RMS of ellipticities in object shapes induced by the second pixelization effect can be suppressed to well below $10^{-2}$ if one adopts a proper interpolation scheme (implemented in popular image processing softwares). We also discuss the impact of pixelization effects on a weak-lensing analysis.

Key words: techniques: image processing

1. Introduction

A precise shape measurement is of fundamental importance in astronomical research, not only because the morphology of celestial bodies provides physical information, but also because any tiny deformation in shapes of distant galaxies caused by the gravitational-lensing effect allows us to explore the foreground mass distribution (see Fort & Mellier 1994; Mellier 1999; Bartelmann & Schneider 2001 for reviews). Analyses of a weak gravitational-lensing effect (e.g., the cosmic shear correlation) especially require very precise shape measurement, since the amplitude is at a few percents level (Refregier 2003 and references therein). Therefore, any artificial shape deformation that may arise during an observation as well as data reduction must be properly understood and must be controlled down to a sufficiently small level.

One known artificial shape deformation is caused by the pixelization of images, known as pixelization effects (also called aliasing effect). As pointed out by Rhodes et al. (2007), there are two causes of the pixelization effect (see figure 6 of Rhodes et al. 2007 for an illustration). One is a consequence of observing the continuous sky with discrete pixels, called first pixelization, which is an unavoidable effect. The other, called second pixelization, occurs when resampling a pixelized image onto another pixel grid whose coordinate is not perfectly adjusted to the input grid. During resampling, one input pixel may be resampled onto several output pixels, obviously resulting in deformations of the object shapes (see figure 2 for an illustration). The second pixelization occurs at several stages of the image processing processes involved in resampling, for example, the correction for a geometric distortion, mosaicking images from multiple CCD chips to generate a combined image, and stacking multiple dithered images.

In this paper, we concentrate on the second pixelization effect, while taking two examples of resampling: rotation and correction for an axially symmetric optical distortion. We especially pay an attention to rotation, which must be involved in the mosaic-stacking process of a mosaic CCD camera if multiple CCDs are not installed perfectly parallel to each other. In the case of the Subaru Prime Focus Camera (Suprime-Cam), rotations of $0'.025$–$0'.17$ are necessary for generating a properly mosaicked image. An important point to notice is that resampling with rotation induces artificial ellipticities in object shapes having a concentric-circle-shaped pattern (explained in detail in the following sections; see figures 2 and 4 for demonstrations). Therefore, it gives rise to artificial shear correlations that can potentially act as systematic noise in the measurement of cosmic shear correlation functions.

We notice that the artificial shape deformation induced by the second pixelization effect cannot in general be corrected by an anisotropic point spread function (PSF) correction (which is one of the important procedures in weak-lensing analyses [Kaiser et al. 1995]), simply because they originate from different causes. Shape deformation by anisotropic PSF arises during an observation with a real (thus, non-perfect) instrument, and is thus an unavoidable effect, and one has to develop a reliable correction scheme (e.g., Heymans et al. 2006; Massaey et al. 2007). However, the second pixelization effect occurs during image processing, and can be minimized by adopting an optimal resampling scheme. In fact, Rhodes et al. (2007) developed such a resampling scheme for the HST.
ACS data in an empirical manner by searching for optimal parameters (the interpolation kernel and output pixel size) of the image processing software MultiDrizzle.\(^1\)

The purpose of this paper is two-fold. The first is to quantitatively examine the effect of the second pixelization effect in order to understand its properties. The second is to explore an optimum way to minimizing it. To do these, we use a simple image simulation, which is described in section 2. Then, in section 3 we give some illustrative examples for a visual impression and for demonstrating the origin of the concentric-circle-shaped pattern induced by resampling with rotation. Results are presented in section 4. Finally, section 5 is devoted to a summary and discussion.

2. Simple Image Simulation

Since a very realistic mock simulation is not necessary for our purpose, we use a toy image simulation, described below. We adopt a two-dimensional Gaussian as the shape of an “object”. The full-width-half-maximums (FWHM) denoted by \(\theta_G\) of the Gaussian object that we consider are \(\theta_G = 0.4\), \(0.6\), \(0.8\), \(1.2\), and \(2.0\). These object sizes are chosen because: (i) the median seeing size (FWHM) of the Subaru telescope is about \(0.6\), and the best seeing is \(\sim 0.35\) (Miyazaki et al. 2002), and (ii) most of the objects used for weak-lensing analyses are galaxies (and reference stars) with the FWHMs being smaller than \(2.0\) (Hamana et al. 2003).

We take the same pixel size as the Suprime-Cam, namely \(l_{\text{pixel}} = 0.2\), since our primary science target is weak lensing, especially using the Suprime-Cam, or similar instruments. We create mock CCD images having \(N_x \times N_y\) pixels on which Gaussian objects (having an equivalent FWHM and intensity) are located on a regular interval of \(3\sigma\) arcsec. Note that the separations between objects are more than 10-times the \(\sigma\) of the Gaussian (\(\theta_G \sim 2.35 \times \sigma\)); thus, the overlapping of isophotes of neighbour objects does not cause any problem in the shape measurement. Note that the results given in this paper can be applied to any camera that uses a pixel array imaging device by properly translating the scaling ratio between \(\theta_G\) and \(l_{\text{pixel}}\).

Following the, so-called, KSB formalism (Kaiser et al. 1995), we quantify the image shapes by the ellipticity parameter, defined by

\[
e = \left(\frac{I_{11} - I_{22}}{I_{11} + I_{22}}\right),
\]

\[
I_{ij} = \int d^2\theta \ W_G(\theta) \delta_i \theta_j f(\theta),
\]

where \(W_G(\theta)\) is the Gaussian window function. Notice that the \(e_1\) (\(e_2\)) component represents the elongation in directions parallel (\(45^\circ\) rotated) to the coordinate system. The object detection and shape measurement are done with \texttt{hfindpeaks} and \texttt{getshapes} of the IMCAT software suite developed by Nick Kaiser, respectively.

Before investigating the second pixelization effect, here we examine the first pixelization effect. The Gaussian has, of course, no ellipticity, but its pixelized image may have a finite ellipticity if the center of an object does not fall onto special positions, such as the center of a pixel or an intersection of grids. We compute the root-mean-square (RMS) of the ellipticities among Gaussian objects in the simulation data of 2048 \(\times\) 2048 pixels. The RMS is defined by

\[
\left\langle e^2 \right\rangle = \frac{\sum_{i=1}^{N_{\text{obj}}} (e_{i1}^2 + e_{i2}^2) / N_{\text{obj}}}{\rangle/\rangle}.
\]

Results are plotted in figure 1. As expected, the RMS decreases with the object size. It quickly becomes large for smaller objects of \(\theta_G < 0.6\), and is \(\sim 0.01\) for the case of \(\theta_G = 0.4\) (\(\theta_G / l_{\text{pixel}} = 2\)). This should be compared with the RMS ellipticities of stars (before an anisotropic PSF correction), which is typically a few percent. Thus, if the seeing FWHM is less than twice the pixel size, the first pixelization effect can be one of the major sources of artificial shape deformation in small objects. An important finding here is that for objects with a FWHM larger than three-times the pixel size, the first pixelization effect is very small, but for smaller objects the first pixelization effect can be a non-negligible source of artificial ellipticities. Another point to be noticed here is that the pixelization effect does not necessarily generate the RMSs of \(e_1\) and that of \(e_2\) equally, because the pixels are square shaped, and so the pixelization effect is, generally not axially symmetric, but has some special directions. This is the reason why the RMSs of two components are generally, not equal as shown in figure 1.

\[\text{Fig. 1. RMSs of the ellipticities in object shapes caused by the first pixelization effect plotted as a function of the FWHM of the Gaussian objects. Note that the RMS ellipticities of stars before a PSF correction is typically a few percent, and the RMS of intrinsic galaxy ellipticities is about 40% (e.g., Hamana et al. 2003).}\]
3. Visual Impressions

Before moving on to a thorough examination of the second pixelization effect, it would be helpful to present some illustrative examples. Notice that in this section, for illustrative purposes, we use simulation data that are different from ones used in section 4 in the separation between objects. Figure 2 shows a demonstrative example for the origin of the periodic pattern caused by a resampling with rotation. Here, we create simulation data of Gaussian objects with $\theta_G/I_{\text{pixel}} = 2$ located on a regular interval of 10 pixels. Note that the objects are placed exactly at the center of pixels, so as to minimize the ellipticity induced by the first pixelization effect. The top-left panel shows the simulation image where the ellipticities of object shapes are over-plotted by ellipses (circles in this case). The top-right panel shows a zoom-in on the section enclosed by the dashed line in the top-left panel. In this plot, a new grid (rotated by $15^\circ$ relative to the original grid) on which the image is resampled is over-plotted. The bottom two panels show the image after resampling onto the new grid. The resampling is done with the 1st-order bilinear polynomial interpolation scheme. The over-plotted ellipses show the ellipticity of the objects. Note that the ellipticities are enlarged 10 times for clarify.
by the dashed line in the top-left panel. In this plot, a new grid (rotated 1°15 relative to the original grid) on which the image is resampled is over-plotted. Note that the rotation angle of 1°15 is much larger than a usual rotation angle involved in the mosaicking of multiple CCDs, but is chosen for demonstrative purposes. The bottom two panels show the images after resampling onto the new pixels. It is evident from these plots that a periodic pattern of artificial ellipticities in object shapes is induced by the rotation. The characteristic scale of the pattern is written in terms of the pixel size and the rotation angle, \( \phi \), as \( L_{\text{pattern}} = l_{\text{pixel}}/\tan \phi (\sim 50 \times l_{\text{pixel}} \text{ for } \phi = 1°15) \). The reason for this is as follows (see figure 3 for an illustration). The deformation is induced by the difference in the grid positions between the input and output grids. Thus, if the difference in the grid positions is the same at separate positions, the same deformation is induced at those positions. In the \( x \)- and \( y \)-directions, the same difference in the grid positions occurs at an interval of \( L_{\text{pattern}} \), because this is the length that the output grid diagonally crosses (with the angle of \( \phi \)) the input grid by one pixel length. Thus, \( L_{\text{pattern}} \) is the separation between positions (in \( x \)- and \( y \)-direction) where the same deformation is induced.

Next, in order to demonstrate the periodic patterns of ellipticities in object shapes appearing in realistic data, we create simulation data having the same dimensions as CCDs of Suprime-Cam (namely, 2048 \times 4096 pixels with \( l_{\text{pixel}} = 0.2\)) on which Gaussian images of \( \theta_G = 0.6 \) are placed on a regular interval of 20°. The data are rotated by 0°025, 0°075, or 0°15, and are resampled onto new pixels by adopting the 1st-order polynomial interpolation scheme (see section 4 for details). These rotation angles are chosen because the actual rotation involved in mosaicking of the Suprime-Cam’s CCDs ranges from 0°025 to 0°17. Ellipticity maps of resampled images are shown in figure 4, where the characteristic periodic patterns

Fig. 3. Sketch explaining the relation between the scale of the pattern \( L_{\text{pattern}} \), the pixel size \( l_{\text{pixel}} \) and the rotation angle \( \phi \). Two grids show the input and output grids that cross at an angle of \( \phi \). The ellipses show the ellipticites (which are enlarged arbitrarily for clarify) of originally circular objects resampled onto the output grid. Notice that the same deformation pattern appears at an interval of \( L_{\text{pattern}} \).

Fig. 4. Ellipticity maps showing the periodic patterns of object shape deformations arising on realistic data. Simulation data having the same dimensions as CCDs of Suprime-Cam (2048 \times 4096 pixels with \( l_{\text{pixel}} = 0.2\)) on which Gaussian images of \( \theta_G = 0.6 \) are placed on a regular interval of 20°, are rotated by 0°025, 0°075, and 0°15 (from left to right). Note that the ellipticities are enlarged 20 times for clarify. For comparison, an ellipse with \( |e| = 2\% \) (enlarged 20 times) is displayed in the small panel at the top-left corner.
are clearly observed. The scales of the pattern are \( L_{\text{pattern}} = 0'2 / \tan \phi \sim 7.6, 2'5 \) and \( 1'3 \) for \( \phi = 0'025, 0'075, \) and \( 0'15, \) respectively. This explains, at least qualitatively, the origin of the concentric-circle-shaped pattern observed in the real data displayed in figure 2 of Miyazaki et al. (2007). As evidently shown in figure 4, the artificial ellipticities induced by image rotation mostly lead to E-mode shear. It is thus very important to note that in the presence of such systematic ellipticities, the smallness of the B-mode shear does not guarantee a successful correction of this systematic noise, and it may be difficult to distinguish this from signals arising from gravitational lensing. Thus it is necessary to develop a resampling procedure that suppresses the systematic ellipticities to a sufficiently small level. This is exactly the purpose of this paper, and we explore the best way to minimize the systematic ellipticities in an empirical manner in the next section. Notice that actual mosaick-stacking involves the rotation, displacement and enlargement/reduction of images; also, a high-order warping is operated to remove optical distortion (e.g., see Miyazaki et al. 2007). Thus, actual data may have a more complex ellipticity pattern than that found in the simple simulation discussed in this section. In the next section, we qualitatively examine the second pixelization effect while taking two realistic examples of resampling: namely, rotation operated in mosaicking and correction for the optical distortion in the case of Suprime-Cam.

4. Results

The magnitude of the second pixelization effect depends on the interpolation scheme used to resample an image. We examine the following interpolation schemes that are implemented in some popular image-processing software: (i) the polynomial interpolation of 1st, 3rd, and 5th order, for which we utilize transformimage of IMCAT. (ii) the sinc kernel \( \text{sinc}(x) = \sin(\pi x)/\pi x \) (truncated at 31 by 31 pixels), for which we utilize rotate of IRAF.2 (iii) the Lanczos kernel \( \text{sinc}(x) \text{sinc}(x/a) \) of \( a = 2, 3, \) and \( 4 \) (called Lanczos2, Lanczos3, and Lanczos4, respectively; implemented e.g., Swarp developed by Emmanuel Bertin), for which we utilize a resampling program developed by ourself. Also, we examine the performance of adopting a finer grid for output pixels, which we call grid refinement. Actually, it has been recognized that the grid refinement can reduce the object shape deformation by pixelization effects (Rhodes et al. 2007; Miyazaki et al. 2007) at the cost of computational overheads. Grid refinement was tested in combination with the 1st and 3rd-order polynomial interpolation schemes for which we utilized transformimage of IMCAT.

4.1. Rotation

The \( 2048 \times 2048 \) pixels simulation data described in section 2 are rotated by \( 0'16, \) and are resampled onto a new grid by applying one of the interpolation schemes mentioned above. The rotation angle of \( 0'16 \) is chosen so that it is within the range of Suprime-Cam’s actual rotation angles in the mosaic-stacking procedure (\( 0'025 - 0'17 \)). Note that the RMS of the ellipticities after resampling does not depend on the rotation angle, though the size of the concentric-circle-shaped pattern does.

Let us first look into the dependence of the second pixelization effect on the object size for various interpolation schemes. Figure 5 compares the RMSs of ellipticities in object shapes as a function of the object size. The left panel of figure 5 compares the three polynomial interpolation schemes, while revealing that the higher is the order of the polynomials, the better is the performance one obtains. It is also found that the higher is the order of polynomials, the steeper the slope becomes. To be specific, the RMSs depend on roughly the object size, \( (e^2)^{1/2} \propto \theta_G^2 \) for a linear polynomial, and \( (e^2)^{1/2} \propto \theta_G^4 \) for the 3rd order, and a further steeper slope for the 5th order. The crosses in the same plot show the RMS of the \( e_2 \) component only for the case of the 3rd-order polynomial, from which it is found that the \( e_2 \) component is much smaller than the total RMS. In fact, we found that the second pixelization effect preferentially induces the \( e_1 \) component, irrespective of the interpolation schemes. This is due to the fact that the pixels are square shaped, and so the pixelization effect has some special directions.

The middle panel of figure 5 shows the results for the sinc and Lanczos kernels. It is found that Lanczos2 works as well as does the 3rd-order polynomial. Lanczos3 and Lanczos4 are better than 3rd and 5th-order polynomials for small objects (\( \theta_G < 0'6 \)), but for larger objects (\( \theta_G > 1' \)) they work only a little better than does Lanczos2. The sinc kernel shows the best performance among the interpolation schemes (without the grid-refinement) that we consider in this paper. We note that the sinc kernel is computationally expensive, since it extends to a very large area (e.g., 31 by 31 pixels for the default setting of IRAF).

The right panel of figure 5 shows that the grid refinement nicely suppresses the second pixelization effect. This is also observed in figure 6, where the improvement gained by the grid refinement is plotted as a function of the ratio between the input and output pixel sizes. If combined with the linear polynomial interpolation scheme, taking a twice finer output grid reduces the RMSs by about one third while keeping the slope of \( (e^2)^{1/2} \propto \theta_G^2 \) mostly unchanged. The use of a 4-times finer grid reduces the RMSs by about one order of magnitude for objects with \( \theta_G < 1'2 \), and by a lesser extent for larger objects. If combined with the 3rd-order polynomial, improvements gained by the grid refinement behave irregularly, as observed in figure 6. Interestingly, in the case of an input/output pixel ratio of 3, adopting the 3rd-order polynomial makes only a slight improvement over the 1st order case. An important message of this is that certain combinations may not give a good improvement for the computational overhead, and thus care must be paid when one combines grid refinement with a higher order interpolation scheme. Our experiment suggests that a reasonably good improvement is stably obtained when one adopts a twice finer grid with the 3rd-order polynomial interpolation.

4.2. Optical Distortion

We now examine the second pixelization effect induced during the correction for optical distortion. To do so, we take
Fig. 5. RMSs of ellipticity in object shapes as a function of the object size are shown for comparison among various interpolation schemes. **Left panel (a):** The 1st, 3rd and 5th order polynomial for the filled circles, filled triangles and open circles respectively. The crosses show the contribution from \( e_2 \) component (\((e_2^2)^{1/2}\)) of the 3rd order polynomial, which demonstrates that the second pixelization effect mostly induces \( e_1 \) component. **Middle panel (b):** The Lanczos2, Lanczos3, Lanczos4 and sinc resampling schemes for the filled circles, filled triangles, open circles and crosses, respectively. **Right panel (c):** The 1st order polynomial with twice and 4 times finer grid for the filled circles and filled triangles respectively, and the 3rd order polynomial with twice finer grid for the open triangles.

Fig. 6. RMSs of ellipticity in object shapes as a function of the ratio between the input and output pixel size. Open circles are for the linear polynomial interpolation scheme, while filled triangles are for the 3rd-order polynomial. Different line styles are for different object size: \( \theta_G = 0.6, 1.2, \) and 2.0 for the solid, dotted and dashed line, respectively.

The optical distortion of the Suprime-Cam is axially symmetric with respect to the optical axis, and is well approximated by a forth-order polynomial function of the distance from the optical axis [see equation (8) of Miyazaki et al. 2002]. As shown in figure 21 of Miyazaki et al. (2002), the distortion rapidly increases with the distance from the optical axis.

Adopting the forth-order polynomial model given in Miyazaki et al. [2002; their equation (8)], we generate mock Suprime-Cam images of \(10456 \times 8282\) pixels, on which Gaussian objects with \( \theta_G = 0.6^\prime \) with the optical distortion artificially operated, are distributed in the manner described in section 2. In the left panel of figure 7, an ellipticity map of the distorted Gaussian objects is shown. As shown there, the Suprime-Cam's optical distortion induces a radial elongation in the object shapes because the distortion becomes larger as the distance from the optical axis increases. The RMS of the ellipticities as a function of the distance from the optical axis is shown in figure 8. At the central region where the distortion is smallest, the RMS ellipticity is as small as that induced by the first pixelization effect, as expected. However at the largest distance it becomes one percent. Note that in this case the first pixelization effect induces the RMSs of \( e_1 \) and \( e_2 \) almost equally, and the turnover in the \( e_1 \) component seen at \( \theta \sim 17^\prime \) is an artifact due to anisotropic sampling (objects in the largest distance are located only at the four corners, which preferentially have the \( e_2 \) component, as observed in figure 8).
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5. Summary and Discussions

We have qualitatively examined the ellipticities in object shapes induced by pixelization effects while paying a special attention to the periodic concentric-circle-shaped pattern induced by the resampling of pixels with rotation. Our major findings are summarized as follows:

- Artificial ellipticities induced by the first pixelization effect can be as large as $\langle e_2^2 \rangle^{1/2} \approx 10^{-2}$ if the characteristic size of objects (e.g., the FWHM) is smaller than twice of the pixel size. However, for objects
with the characteristic size being larger than three-times the pixel size, the RMS becomes negligibly small \((\langle e^2 \rangle^{1/2} \lesssim 10^{-5})\).

- The second pixelization effect preferentially induces the \(e_1\) component (parallel to the grids). The reason for this is that pixels are square shaped, and so the pixelization effect is, in general, not axially symmetric, but has some special directions.

- The size (e.g., RMS of \(e\)) of the shape deformation caused by the second pixelization effect depends on the object size. It also strongly depends on the interpolation scheme for resampling and on the grid size of the output pixels. If we set an upper limit of the RMS ellipticities by \(\langle e^2 \rangle^{1/2} < 5 \times 10^{-5}\) for objects with FWHM \(> 2.5 \times l_{\text{pixel}}\) (corresponding to FWHM \(> 0.5\) for the case of Suprime-Cam), the interpolation schemes passing the above condition are (see figure 5) the 5th-order polynomial, Lanczos3, Lanczos4 and sinc kernel (as far as among ones considered in this paper). Adopting grid refinement results in a great improvement. Actually, if one adopts a twice-finer grid for the output pixels, even a linear polynomial can pass the above condition.

- Resampling of a pixelized image with rotation induces a periodic concentric-circle-shaped pattern of artificial ellipticities in object shapes. The scale of the pattern is related to the pixel size and the rotation angle, \(\phi\), by \(L_{\text{pattern}} = l_{\text{pixel}}/\tan \phi\).

Before closing this paper, we would like to comment on an impact of the second pixelization effect on the actual weak-lensing analysis using Suprime-cam data presented in Miyazaki et al. (2007). Miyazaki et al. (2007) carried out resampling while adopting the 3rd-order polynomial interpolation scheme, and typically combined 4 dithered images.\(^3\) Thus, for images with FWHM \(\gtrsim 0.6\) (the typical PSF size), the RMS of the ellipticities induced by the second pixelization effect should be well below \(10^{-2}\). However, the RMSs measured from stellar images are about a few \(\times 10^{-2}\); therefore, we may safely conclude that the second pixelization effect is sufficiently suppressed, and is not a major source of artificial ellipticities in object shapes.

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\(^3\) Combining dithered images reduces the RMS ellipticities roughly as \(\propto N^{-1/2}\) for \(N\) dithered images (Rhodes et al. 2007), because an object falls onto a different sub-pixel position in different exposures as a consequence of dithered exposures which results in different ellipticities with basically random orientations. Combining those images can mitigate the pixelization effects.