Coherent Photoproduction of \( \eta \)-mesons on Three-Nucleon Systems

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Abstract

A microscopic few-body description of near-threshold coherent photoproduction of the \( \eta \) meson on tritium and \(^3\)He targets is given. The photoproduction cross-section is calculated using the Finite Rank Approximation (FRA) of the nuclear Hamiltonian. The results indicate a strong final state interaction of the \( \eta \) meson with the residual nucleus. Sensitivity of the results to the choice of the \( \eta N T \)-matrix is investigated. The importance of obeying the two-body unitarity condition in the \( \eta N \) system is demonstrated.

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I. INTRODUCTION

Investigations of the \( \eta \)-nucleus interaction are motivated by various reasons. Some of them, such as the possibility of forming quasi-bound states or resonances \(^1\) in the \( \eta \)-nucleus system, are purely of nuclear nature. The others are related to the study of the properties and structure of the \( S_{11}(1535) \) resonance which is strongly coupled to the \( \eta N \) channel.

For example, it is interesting to investigate the behavior of the \( \eta \)-meson in nuclear media where, after colliding with the nucleons, it readily forms the \( S_{11} \) resonance. The interaction of this resonance with the surrounding nucleons can be described in different ways \(^2\), depending on whether the structure of this resonance is defined in terms of some quark configurations or by the coupling of meson-baryon channels, as suggested in Ref. \(^3\). The estimation by Tiwari et al. \(^4\) shows, that in case of pseudoscalar \( \eta NN \) coupling there is an essential density dependent reduction of the \( \eta \)-meson mass and of the \( \eta - \eta' \) mixing angle.

The importance of the influence of the nuclear medium on the mesons passing through it, was recently emphasized by Drechsel et al. \(^5\). If this influence is described in terms of self-energies and effective masses, then in the process of \( \pi \)-meson passing through the nucleus, ”saturation” of the isobar propagator (or self-energy) takes place. This phenomenon manifests itself even in light nuclei \(^6\). Similar ideas were discussed also in Ref. \(^7\). In other words, the propagation of \( \eta \)-mesons inside the nucleus is a new challenge for theorists.

Another interesting issue related to the \( \eta \)-nucleus interaction, is the study of charge symmetry breaking, which may partly be attributed to the \( \eta - \pi^0 \) mixing (see, for example, Refs. \(^8\) [\( \eta \)]). In principle, one can extract the value of the mixing angle from experiments involving \( \eta \)-nucleus interaction and compare the results with the predictions of quark models.
However, to do such an extraction, one has to make an extrapolation of the $\eta$-nucleus scattering amplitude into the area of unphysical energies below the $\eta$-nucleus threshold. This is a highly model dependent procedure requiring reliable treatment of the $\eta$-nucleus dynamics.

In this respect, few-body systems such as $\eta d$, $\eta^3$He, and $\eta^4$He, have obvious advantages since they can be treated using rigorous Faddeev-type equations. To the best of our knowledge, so far only the simplest of these systems, namely the $\eta(2N)$ system, was considered within the exact AGS theory [16].

A solution of the few-body equations presupposes the knowledge of the corresponding two-body $T$-matrices $t_{\eta N}$ and $t_{NN}$ off the energy shell. Due to the fact that at low energies the $\eta$ meson interacts with a nucleon mainly via the formation of the $S_{11}$-resonance, the inclusion of the higher partial waves ($\ell > 0$) is unnecessary. Furthermore, since the $\eta N$ interaction is poorly known, the effect of the fine tuned details of the “realistic” $NN$ potentials would be far beyond the level of the overall accuracy of the $\eta A$ theory. Indeed, in contrast to the well-established $NN$ forces, the $\eta N$ interaction is constructed using very limited information available, namely, the $\eta N$ scattering length and the parameters of the $S_{11}$-resonance. Furthermore, only the resonance parameters are known more or less accurately while the scattering length (which is complex) is determined with large uncertainties. Moreover, practically nothing is known about the off-shell behavior of the $\eta N$ amplitude. It is simply assumed that all mesons should have somewhat similar properties and therefore the off-shell behavior of this amplitude could be approximated (like in the case of $\pi$ mesons) by appropriate Yamaguchi form-factors (see, for example, Refs. [12–15]). However, if the available data are used to construct a potential via, for example, Fiedeldey’s inverse scattering procedure [19], the resulting form factor of the separable potential is not that simple. The problem becomes even more complicated due to the multichannel character of the $\eta N$ interaction with the additional off-shell uncertainties stemming from the $\pi$-meson channel.

In such a situation, it is desirable to narrow as much as possible the uncertainty intervals for the parameters of $\eta N$ interaction. This could be done by demanding consistency of theoretical predictions based on these parameters, with existing experimental data for two-, three-, and four-body $\eta$-nucleus processes. This is one of the objectives of the present work. To do this, we calculate the cross sections of coherent $\eta$-photoproduction on $^3$He and $^3$H nuclei and study their sensitivity to the parameters of $\eta N$ amplitude.

II. FORMALISM

We start by assuming that the Compton scattering on a nucleon,

$$\gamma + N \rightarrow N + \gamma,$$

as well as the processes of multiple re-appearing of the photon in the intermediate states,

$$\gamma + N \rightarrow N + \eta \rightarrow \gamma + N \rightarrow N + \eta \rightarrow \ldots,$$

give a negligible contribution to the coherent $\eta$-photoproduction on a nucleus $A$. Then the process
\[
\gamma + A \rightarrow A + \eta ,
\]
(1)
can be formally described in two steps: at the first step, the photon produces the \(\eta\) meson on one of the nucleons,
\[
\gamma + N \rightarrow N + \eta ,
\]
(2)
and at the second step (final state interaction), the \(\eta\) meson is elastically scattered off the nucleus,
\[
\eta + A \rightarrow A + \eta .
\]
(3)
An adequate treatment of the scattering step is, of course, the most difficult and crucial part of the theory. Among the few-body systems \(\eta d\), \(\eta^3H\), \(\eta^3He\), and \(\eta^4He\), so far only the simplest three-body one \((\eta d)\) was considered in the framework of exact Faddeev-type AGS equations. The first microscopic calculations concerning the low-energy scattering of \(\eta\)-meson from \(^3H\), \(^3He\), and \(^4He\) nuclei were done in Refs. [20–26] where the few-body dynamics of these systems were treated by employing the Finite-Rank Approximation (FRA) [27] of the nuclear Hamiltonian. This approximation consists in neglecting the continuous spectrum in the spectral expansion
\[
H_A = \sum_n \mathcal{E}_n |\psi_n\rangle \langle \psi_n| + \text{continuum}
\]
of the Hamiltonian \(H_A\) describing the nucleus. Since the three- and four-body nuclei have only one bound state, FRA reduces to
\[
H_A \approx \mathcal{E}_0 |\psi_0\rangle \langle \psi_0| .
\]
(4)
Physically, this means that we exclude the virtual excitations of the nucleus during its interaction with the \(\eta\) meson. It is clear that the stronger the nucleus is bound, the smaller is the contribution from such processes to the elastic \(\eta A\) scattering. By comparing with the results of the exact AGS calculations, it was shown [28] that even for \(\eta d\) scattering, having the weakest nuclear binding, the FRA method works reasonably well, which implies that we obtain sufficiently accurate results by applying this method to the \(\eta^3H\), \(\eta^3He\), and even more so to the \(\eta^4He\) scattering.

In essence, the FRA method can be described as follows (for details see Ref. [27]). Let
\[
H = h_0 + V + H_A
\]
be the total \(\eta A\) Hamiltonian, where \(h_0\) describes free \(\eta\)-nucleus motion and
\[
V = \sum_{i=1}^A V_i
\]
the sum of the two-body \(\eta\)-nucleon potentials. The Lippmann-Schwinger equation
\[
T(z) = \sum_{i=1}^A V_i + \sum_{i=1}^A V_i(z - h_0 - H_A)^{-1}T(z)
\]
(5)
for the \( \eta \)-nucleus \( T \)-matrix can be rewritten as

\[
T(z) = W(z) + W(z)M(z)T(z)
\]

where

\[
M(z) = G_0(z)H_AG_A(z),
\]

\[
G_0(z) = (z - h_0)^{-1},
\]

\[
G_A(z) = (z - h_0 - H_A)^{-1},
\]

and the auxiliary operator \( W(z) \) is split into \( A \) components of Faddeev-type,

\[
W(z) = \sum_{i=1}^{A} W_i(z),
\]

satisfying the following system of equations

\[
W_i(z) = t_i(z) + t_i(z)G_0(z) \sum_{j \neq i}^{A} W_j(z)
\]

with \( t_i \) being the two-body \( T \)-matrix describing the interaction of the \( \eta \)-meson with the \( i \)-th nucleon, i.e.

\[
t_i(z) = V_i + V_iG_0(z)t_i(z).
\]

It should be emphasized that up to this point no approximation has been used yet and therefore the set of equations (6-12) is equivalent to the initial equation (5). However, to solve Eq. (6), we have to resort to the approximation (4) which simplifies its kernel (7) to

\[
M(z) \approx \frac{E_0|\psi_0\rangle\langle\psi_0|}{(z - h_0)(z - E_0 - h_0)}.
\]

With this approximation, the sandwiching of Eq. (6) between \( \langle\psi_0| \) and \( |\psi_0\rangle \) and the partial wave decomposition give a one-dimensional integral equation for the amplitude of the process (3).

The question then arises on how can a photon be included into this formalism in order to describe the photoproduction process (1). This can be achieved by following the same procedure as in Ref. [29] where the reaction (1) with \( A = 2 \) was treated within the framework of the exact AGS equations, and the photon was introduced by considering the \( \eta N \) and \( \gamma N \) states as two different channels of the same system. This implies that the operators \( t_i \) should be replaced by \( 2 \times 2 \) matrices. It is clear, that such replacements of the kernels of the integral equations (11) and subsequently of the integral equation (6) lead to solutions having a similar matrix form

\[
t_i \rightarrow \begin{pmatrix} t_i^{\gamma\gamma} & t_i^{\gamma\eta} \\ t_i^{\eta\gamma} & t_i^{\eta\eta} \end{pmatrix} \quad \Rightarrow \quad W_i \rightarrow \begin{pmatrix} W_i^{\gamma\gamma} & W_i^{\gamma\eta} \\ W_i^{\eta\gamma} & W_i^{\eta\eta} \end{pmatrix} \quad \Rightarrow \quad T \rightarrow \begin{pmatrix} T^{\gamma\gamma} & T^{\gamma\eta} \\ T^{\eta\gamma} & T^{\eta\eta} \end{pmatrix}.
\]

Here \( t_i^{\gamma\gamma} \) describes the Compton scattering, \( t_i^{\gamma\eta} \) the photoproduction process, and \( t_i^{\eta\eta} \) the elastic \( \eta \) scattering on the \( i \)-th nucleon. What is finally needed is the cross section
\[
\frac{d\sigma}{d\Omega} = \frac{2}{9(2\pi)^2} \frac{k_\gamma}{k_\eta} \frac{E_\gamma m_A}{E_\gamma + m_A} \mu_{\eta A} \left| \langle \vec{k}_\eta, \psi_0 | T^{\eta\gamma} (E_0 + E_\gamma) | \psi_0, \vec{k}_\gamma \rangle \right|^2
\]  

(15)
of the reaction (1), where \( \vec{k}_\gamma \) and \( \vec{k}_\eta \) are the momenta of the photon and \( \eta \) meson, \( E_\gamma \) is the energy of the photon, \( m_A \) the mass of the nucleus, and \( \mu_{\eta A} \) the reduced mass of the meson and the nucleus.

However, it is technically more convenient to consider the \( \eta \)-photoabsorption, i.e. the inverse reaction. Then the photoproduction cross section can be obtained by applying the detailed balance principle. The reason for this is that all the processes in which the photon appears more than once, i.e. the terms of the integral equations of type \( W^{\gamma\gamma} MT^{\eta\gamma} \) or \( W^{\eta\gamma} MT^{\gamma\eta} \) involving more than one electromagnetic vertex, can be neglected. Omission of these terms in (6) results in decoupling the elastic scattering equation

\[
T^{\eta\eta} = W^{\eta\eta} + W^{\eta\eta} MT^{\eta\eta}
\]  

(16)
from the equation for the photoabsorption

\[
T^{\gamma\eta} = W^{\gamma\eta} + W^{\gamma\eta} MT^{\gamma\eta}.
\]  

(17)
Once the \( T^{\eta\eta} \) is calculated, the photoabsorption \( T \)-matrix (17) can be obtained by integration.

Therefore, the procedure of calculating the photoproduction cross section (15) consists of the following steps:

- Solving the system of equations

\[
W_i^{\eta\eta} = t_i^{\eta\eta} + t_i^{\eta\eta} G_0 \sum_{j \neq i}^A W_j^{\eta\eta}
\]  

(18)
for the auxiliary elastic-scattering operators \( W_i^{\eta\eta} \).

- Calculating (by integration) the auxiliary matrices \( W_i^{\gamma\eta} \) from

\[
W_i^{\gamma\eta} = t_i^{\gamma\eta} + t_i^{\gamma\eta} G_0 \sum_{j \neq i}^A W_j^{\eta\eta}.
\]  

(19)

- Solving the integral equation

\[
T^{\eta\eta} = \sum_{i=1}^A W_i^{\eta\eta} + \sum_{i=1}^A W_i^{\eta\eta} MT^{\eta\eta}
\]  

(20)
for the elastic-scattering \( T \)-matrix.

- Calculating (by integration) the photoabsorption \( T \)-matrix

\[
T^{\gamma\eta} = \sum_{i=1}^A W_i^{\gamma\eta} + \sum_{i=1}^A W_i^{\gamma\eta} MT^{\gamma\eta}.
\]  

(21)

- Substituting this \( T \)-matrix into Eq. (15) to obtain the differential cross section for the photoproduction. This is possible because the absolute values of the photoproduction and photoabsorption \( T \)-matrices coincide.
III. TWO-BODY INTERACTIONS

To implement the calculation steps described in the previous section, we need the two-body $T$-matrices $t^{\eta\eta}$ and $t^{\gamma\eta}$ for the elastic $\eta N$ scattering and the photoabsorption $N(\eta, \gamma)N$ on a single nucleon respectively. Furthermore, all equations (18-21) have to be sandwiched between $\langle \psi_0 |$ and $| \psi_0 \rangle$ (ground state wave function of the nucleus). Since at low energies both the elastic scattering and photoproduction of $\eta$ meson on a nucleon proceed mainly via formation of the $S_{11}$ resonance, we may retain only the $S$-waves in the partial wave expansions of the corresponding two-body $T$-matrices.

A. Elastic $\eta N$ scattering

The problem of constructing an $\eta N$ potential or directly the corresponding $T$-matrix $t^{\eta\eta}$ has no unique solution since the only experimental information available consists of the $S_{11}$-resonance pole position $E_0 - i\Gamma/2$ and the $\eta N$ scattering length $a_{\eta N}$. In the present work, we use three different versions of $t^{\eta\eta}$.

1. Version I

With the absence of any scattering data it is practically impossible to construct a reliable $\eta N$ potential. However, we can make use of the fact that the $S_{11}$-resonance is the dominant feature of the $\eta N$ interaction in the low-energy region, where the elastic scattering can be viewed as the process of formation and subsequent decay of this resonance, i.e.

$$\eta + N \longrightarrow S_{11} \longrightarrow N + \eta . \quad (22)$$

This implies that in this region the corresponding Breit-Wigner formula could be a good approximation for the $\eta N$ cross section. Therefore, we may adopt the following ansatz

$$t^{\eta\eta}(k', k; z) = g(k') \tau(z) g(k) \quad (23)$$

where the propagator $\tau(z)$ describing the intermediate state of the process (22), is assumed to have a simple Breit-Wigner form

$$\tau(z) = \frac{\lambda}{z - E_0 + i\Gamma/2} , \quad (24)$$

which guarantees that the $T$-matrix (23) has a pole at the proper place. The vertex function $g(k)$ for the processes $\eta N \leftrightarrow S_{11}$ is chosen to be

$$g(k) = (k^2 + \alpha^2)^{-1} \quad (25)$$

which in configuration space is of Yukawa-type. The range parameter $\alpha = 3.316 \text{ fm}^{-1}$ was determined in Ref. [30] while the parameters of the $S_{11}$-resonance

$$E_0 = 1535 \text{ MeV} - (m_N + m_{\eta}) , \quad \Gamma = 150 \text{ MeV}$$
are taken from Ref. [31]. The strength parameter $\lambda$ is chosen to reproduce the $\eta$-nucleon scattering length $a_{\eta N}$,

$$
\lambda = 2\pi \frac{\alpha^4 (E_0 - i\Gamma/2)}{\mu_{\eta N}} a_{\eta N} .
$$

(26)

the imaginary part of which accounts for the flux losses into the $\pi N$ channel. Here $\mu_{\eta N}$ is the $\eta N$ reduced mass.

The two-body scattering length $a_{\eta N}$ is not accurately known. Different analyses [32] provided values for $a_{\eta N}$ in the range

$$
0.27 \text{ fm} \leq \text{Re } a_{\eta N} \leq 0.98 \text{ fm} , \quad 0.19 \text{ fm} \leq \text{Im } a_{\eta N} \leq 0.37 \text{ fm} .
$$

(27)

In most recent publications, the value used for Im $a_{\eta N}$ is around 0.3 fm. However, for Re $a_{\eta N}$ the estimates are still very different (compare, for example, Refs. [33] and [34]). In the present work we assume that

$$
a_{\eta N} = (0.55 + i0.30) \text{ fm} .
$$

(28)

The $T$-matrix $t^m$ constructed in this way, reproduces the scattering length (28) and the $S_{11}$ pole, but apparently violates the two-body unitarity since it does not obey the two-body Lippmann-Schwinger equation.

2. Version II

An alternative way of constructing the two-body $T$-matrix $t^m$ is to solve the corresponding Lippmann-Schwinger equation with an appropriate separable potential having the same form-factors (25). However, a one-term separable $T$-matrix obtained in this way, does not have a pole at $z = E_0 - i\Gamma/2$. To recover the resonance behavior in this case, we use the trick suggested in Ref. [18], namely, we use an energy-dependent strength of the potential

$$
V(k, k'; z) = g(k) \left[ \Lambda + C \frac{\zeta - z}{\zeta} \right] g(k')
$$

where $\Lambda$ is complex while $C$ and $\zeta$ are real constants. With this ansatz for the potential, the Lippmann-Schwinger equation gives the $T$-matrix in the form (23) with

$$
\tau(z) = - \left( \frac{4\pi\alpha^3}{\mu_{\eta N}} \right) \frac{\Lambda(\zeta - z) + C\zeta}{\zeta - z - [\Lambda(\zeta - z) + C\zeta]/(1 - i\sqrt{2z\mu_{\eta N}/\alpha})^2} .
$$

(29)

The constants $\Lambda$, $C$, and $\zeta$ can be chosen in such a way that the corresponding scattering amplitude reproduces the scattering length $a_{\eta N}$ and has a pole at $z = E_0 - i\Gamma/2$.

This version of $t^m$ also reproduces the scattering length (28) and the $S_{11}$ pole. Moreover, it is consistent with the condition of the two-body unitarity.
3. Version III

We can also construct the $t^{\eta\eta}$ having the same form as version I, namely (23), with the same $\tau(z)$ as in (24) but obeying the unitarity condition

$$(1 - 2\pi i t^{\eta\eta})(1 - 2\pi i t^{\eta\eta})^\dagger = 1. \quad (30)$$

Of course, with the simple form (23), we cannot satisfy the condition (30) at all energies. To simplify the derivations, we impose this condition on $t^{\eta\eta}$ at $z = E_0$. Since Eq. (30) is real, it can fix only one parameter and we need one more condition to fix both the real and imaginary parts of the complex $\lambda$. As the second equation, we used the imaginary part of Eq. (26) with $a_{\eta N}$ given by (28).

This procedure guarantees the two-body unitarity and gives the correct position of the resonance pole, but the resulting $t^{\eta\eta}$ gives $a_{\eta N}$ which, of course, is different from the value (28), namely, it gives

$$a_{\eta N} = (0.76 + i0.61) \text{ fm}. \quad (31)$$

In what follows we use the three versions of the matrix $t^{\eta\eta}$ described above. All of them have the same separable form (23) but different $\tau(z)$. A comparison of the results obtained with these three $T$-matrices can give us an indication of the importance of the two-body unitarity in the photoproduction processes.

B. Photoabsorption $N(\eta, \gamma)N$

In constructing the photoabsorption $T$-matrix $t^{\gamma\eta}$, the $S_{11}$ dominance in the near-threshold region also plays an important role. It was experimentally shown [35] that, at low energies, the reaction (2) proceeds mainly via formation of the $S_{11}$-resonance, i.e.

$$\gamma + N \rightarrow S_{11} \rightarrow N + \eta. \quad (32)$$

This implies that $t^{\gamma\eta}$ in this energy region can be written in a separable form similarly to (23). To construct such a separable $T$-matrix, we use the results of Ref. [36] where the $t^{\gamma\eta}$ was considered as an element of a multi-channel $T$-matrix which simultaneously describes experimental data for the processes

$$\pi + N \rightarrow \pi + N, \quad \pi + N \rightarrow \eta + N, \quad \gamma + N \rightarrow \pi + N, \quad \gamma + N \rightarrow \eta + N$$

on the energy shell in the $S_{11}$-channel. In the present work, we take the $T$-matrix $t^{\gamma\eta}_{on}(E)$ from Ref. [36] and extend it off the energy shell via

$$t^{\gamma\eta}_{off}(k', k; E) = \frac{\kappa^2 + E^2}{\kappa^2 + k'^2} t^{\gamma\eta}_{on}(E) \frac{\alpha^2 + 2\mu_{\eta N} E}{\alpha^2 + k^2}, \quad (33)$$

where $\kappa$ is a parameter. The Yamaguchi form-factors used in this ansatz, go to unity on the energy shell. Since $\kappa$ is not known, this parameter is varied in our calculations within a
reasonable interval \(1 \text{ fm}^{-1} < \kappa < 10 \text{ fm}^{-1}\) which is a typical range for meson-nucleon forces. It is known that \(t^{\gamma\eta}\) is different for neutron and proton. In this work we assume that they have the same functional form (33) and differ by a constant factor,

\[ t_n^{\gamma\eta} = A t_p^{\gamma\eta}. \]

Multipole analysis [37] gives for this factor the estimate \(A = -0.84 \pm 0.15\).

C. Nuclear subsystem

Since the \(T\)-matrices \(t^m\) and \(t^{\gamma\eta}\) are poorly known and their uncertainties significantly limit the overall accuracy of the theory, it is not necessary to use any sophisticated (“realistic”) potential to describe the \(NN\) interaction. Therefore we may safely assume that the nucleons interact with each other only in the \(S\)-wave state.

To obtain the necessary nuclear wave function \(\psi_0\), we solve the few-body equations of the Integro-Differential Equation Approach (IDEA) [38,39] with the Malfliet-Tjon potential [40]. This approach is based on the Hyperspherical Harmonic expansion method applied to Faddeev-type equations. In fact, in the case of \(S\)-wave potentials, the IDEA is fully equivalent to the exact Faddeev equations. Therefore, the bound states used in our calculations, are derived, to all practical purposes, via an exact formalism.

IV. RESULTS AND DISCUSSION

Figures 1-5 show the results of our calculations for the total cross section

\[ \sigma = \int \left( \frac{d\sigma}{d\Omega} \right) d\Omega \]

of the coherent process (1). The calculations were done for two nuclear targets, \(^3\text{H}\) and \(^3\text{He}\), using the three versions of \(t^m\) described in the previous section. The curves corresponding to these three \(T\)-matrices are denoted by (I), (II), and (III), respectively.

As can be seen in Fig. 1, the two versions of \(t^m\), (I) and (II), give significantly different results despite the fact that both of them reproduce the same \(a_{\eta N}\) and the \(S_{11}\)-resonance. This indicates that the scattering of the \(\eta\) meson on the nucleons (final state interaction) is very important in the description of the photoproduction process. This conclusion is further substantiated when our curves are compared to the corresponding points (triangles) calculated for the \(^3\text{He}\) target in Ref. [41] where the final state interaction was treated using an optical potential of the first order. It is well-known that the first-order optical theory is not adequate at the energies near resonances. This is the reason why the calculations of Ref. [41] underestimate \(\sigma\) near the threshold where with \(a_{\eta N} = (0.55 + i0.30) \text{ fm}\) the systems \(\eta \, ^3\text{H}\) and \(\eta \, ^3\text{He}\) have resonances [23].

A significant differences between the corresponding curves (I) and (II) in Fig. 1 imply that two-body unitarity is important as well. To clarify this statement, we compare in Fig. 2 the results corresponding to the three choices of \(\tau(z)\) in (24). Surprisingly, the curves (II) and (III) almost coincide despite the fact that they correspond to different \(a_{\eta N}\) while both obey the two-body unitarity condition.
The last three figures, Figs. 3, 4 and 5, show the dependence of the results on the choices of the parameters $\kappa$ and $A = t^\gamma_\eta / t^\gamma_p$. Since nothing is known about $\kappa$, we assume $\kappa = \alpha$ as the basic value for it. This can be motivated by the fact that both the elastic scattering and photoabsorption (production) of the $\eta$ meson on the nucleon go via formation of the same $S_{11}$ resonance. This means that at least one vertex, namely, $\eta N \leftrightarrow S_{11}$ should be the same for both the elastic scattering and photoabsorption. To find out how crucial the choice of $\kappa$ is, we did two additional calculations with $\kappa = 1 \text{ fm}^{-1}$ and $\kappa = 10 \text{ fm}^{-1}$ (see Fig. 3). We see that even with this wide variation the curves are not far from each other especially in the immediate vicinity of the threshold energy. Therefore the dependence on $\kappa$ is not very strong and the choice $\kappa = \alpha$ can give us a reasonable estimate for the photoproduction cross section. Fig. 3 also shows an interesting tendency of increasing $\sigma$ when the range of the interaction becomes smaller (when $\kappa$ grows).

Figs. 4 and 5 for the $^3\text{H}$ and $^3\text{He}$ targets, respectively, show the dependence of $\sigma$ on the choice of the parameter $A$. An interesting observation here is that the cross section for $\eta$ photoproduction is more sensitive to this parameter with the tritium rather than the $^3\text{He}$ target. This means that between these two nuclei, the tritium is a preferable candidate for a possible experimental determination of the ratio $A$.

The cusp exhibited by all the curves at the threshold of total nuclear break-up, reflects losses of the flux into the non-coherent channel. In a sense, this is a reflection of the four-body unitarity which the FRA equations are consistent with.

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FIG. 1. Cross section of the coherent $\eta$-photoproduction on the $^3$H and $^3$He targets, calculated with the two versions of $t^{\eta\eta}$ which are denoted as (I) and (II) respectively. All curves correspond to $a_{\eta N} = (0.55 + i0.30)$ fm, $\kappa = \alpha = 3.316$ fm$^{-1}$, and $A = -0.84$. The triangles represent the points calculated in Ref. [41] for the $^3$He target within the optical model.
FIG. 2. Cross section of the coherent $\eta$-photoproduction on $^3$He, calculated with the three versions of $t^{\eta\gamma}$ which are denoted as (I), (II), and (III) respectively. All three curves correspond to $\kappa = \alpha = 3.316 \text{ fm}^{-1}$ and $A = -0.84$. For the curves (I) and (II) the $a_{\eta N}$ is given by Eq. (28) while for the third curve by Eq. (31).
FIG. 3. Cross section of the coherent $\eta$-photoproduction on $^3\text{He}$, calculated with the version (II) of $t^m$ with three values of the parameter $\kappa$. All three curves correspond to $A = -0.84$. 
FIG. 4. Cross section of the coherent $\eta$-photoproduction on $^3\text{He}$, calculated with the version (II) of $t^{\eta\eta}$ with three different values of the parameter $A$. All three curves correspond to $\kappa = \alpha = 3.316 \text{ fm}^{-1}$.
FIG. 5. Cross section of the coherent $\eta$-photoproduction on $^3$H, calculated with the version (II) of $t^m$ with three different values of the parameter $A$. All three curves correspond to $\kappa = \alpha = 3.316 \text{ fm}^{-1}$.