Subwavelength beam focusing using a ball-shaped lens for ultrasound imaging

Victor H. S. Santos\textsuperscript{a}, Everton B. Lima\textsuperscript{a}, André L. Baggio\textsuperscript{a}, José H. Lopes\textsuperscript{b}, Glauber T. Silva\textsuperscript{a},

\textsuperscript{a}Physical Acoustics Group, Instituto de Física, Universidade Federal de Alagoas, Maceió, AL 57072-970, Brazil
\textsuperscript{b}Grupo de Física da Matéria Condensada, Núcleo de Ciências Exatas, Universidade Federal de Alagoas, Arapiraca, Alagoas 57309-005, Brazil

\textbf{Abstract}

Ultrasonic superresolution images can be generated by means of (super) focusing acoustic beams to subwavelength dimensions or using algorithm-based methods. Here, we demonstrate that ultrasonic pulses which are superfocused by a ball-shaped lens can be used to produce superresolution images. The imaging system is comprised of a circular flat transducer with an operation frequency of 1 MHz, and a ball lens centered in the beam axis of symmetry. The corresponding wavelength in water is $\lambda_0 = 1.53$ mm. The system resolution is $0.6\lambda_0$ in the focal plane at one wavelength away from the lens. The superresolution method is compared with a conventional ultrasonic system based on a spherically focused transducer. Our method presents twice more resolution with a shorter depth-of-field of $2\lambda_0$. Possible applications that take advantage of these features are discussed, as well as some limitations of the proposed technique.

\textbf{Keywords:} Ultrasound superresolution, Image formation, Beamforming

\section{1. Introduction}

The spatial resolution of ultrasonic imaging systems is primarily restricted by the diffraction limit (i.e., beam focusing in a disk roughly with one wavelength diameter) \cite{1}. This effect thwarts subwavelength focusing at a given frequency. A finer resolution can be achieved by increasing the frequency at the expense of less ultrasound penetration due to absorption and also a high-cost electronics.

Different methods have overcome the ultrasonic diffraction limit. Notably, harmonic generation was used to make images at a doubled frequency, which means an improvement of 100\% is the spatial resolution \cite{2,3}. Nonlinear ultrasonic wave mixing forms images at the difference-frequency with the fundamental high-frequency resolution \cite{4,5,6}. Also, the nonlinear wave interaction gives rise to a sum-frequency component (i.e., the sum of the fundamental frequencies) that can be used to form superresolution images \cite{7}. Another approach uses the time-reversal wave phenomenon to focus an ultrasound beam \cite{8}. Lenses made
2. Methods and materials

2.1. Superresolution image formation

Consider ultrasonic waves propagating in a homogeneous non-viscous fluid of density \(\rho_0\) and speed of sound \(c_0\). The acoustic pressure is described at position \(r\), concerning the coordinate system set in the right-hand pole of the ball lens (see Fig. 1), and time \(t\).

We assume that the active surface of the transducer vibrates uniformly with normal velocity denoted by \(v_n\). For a linear and time invariant system, the transmitted pressure reads\[17\]

\[
p_{\text{tr}}(r, t) = \rho_0 v_n(t) \ast \partial_t h(r, t),
\]

where \(\partial_t\) is time derivative, \(h\) is the spatial impulse response of the transducer and \(\ast\) means convolution in time—see Eq. (A.4). The vibration velocity is expressed by

\[
v_n(t) = v_{\text{exc}}(t) \ast e_{\text{tr}}(t),
\]

where \(v_{\text{exc}}(t)\) is the excitation voltage and \(e_{\text{tr}}(t)\) is the electromechanical impulse response of the transducer in the transmitting mode.
Figure 1: (a) Schematics of the superresolution ultrasonic (SU) imaging system. The incident (blue) pulse hits a target object at the focal plane $z = z_0$. The system receives the backscattered pressure (red pulse). The acquired signal is post-processed and displayed on a computer. (b) Photography of the SU system.

The lens focuses the transmitted pressure through a scattering process \cite{14}. In turn, ultrasound scattering can be model as a linear and shift-invariant system. The incident pressure $p_{\text{in}}$ to a target object can be expressed as the spatial convolution—see Eq. (A.5)—between the transmitted pressure by the transducer and the lens spatial response function $h_{\text{lens}}$,

$$p_{\text{in}}(r, t) = p_{\text{tr}}(r, t) \ast h_{\text{lens}}(r).$$  (3)

The backscattered pressure by the target is given by \cite{18}

$$p_{\text{sc}}(r, t) = p_{\text{in}}(r, t) \ast f(r),$$  (4)

with $f$ being the object’s function. The scattered wave is also focused by the lens yielding the pressure to be received by the transducer,

$$p_{\text{rec}}(r, t) = p_{\text{sc}}(r, t) \ast h_{\text{lens}}(r).$$  (5)

The signal in the transducer terminals is described by \cite{17}

$$s(t) = e_{\text{rec}}(t) \ast p_{\text{rec}}(r, t) \ast h(r, t),$$  (6)

where $e_{\text{rec}}(t)$ is the electromechanical impulse response of the transducer in the receiving mode. Substituting Eqs. (2)–(5) into this equation yields

$$s(t) = v_{\text{pe}}(t) \ast \partial_t g(r, t) \ast g(r, t) \ast f(r),$$  (7a)  
$$g(r, t) = h(r, t) \ast h_{\text{lens}}(r),$$  (7b)  
$$v_{\text{pe}}(t) = \rho_0 v_{\text{exc}}(t) \ast e_{\text{tr}}(t) \ast e_{\text{rec}}(t).$$  (7c)
Equation (7a) has a similar structure of Eq. (45) in Ref. [18], which describes a conventional ultrasonic pulse-echo system. The spatial impulse function of the SU system $g(r, t)$ accounts for diffraction effects of the transducer and lens. The pulse-echo wavelet $v_{pe}(t)$ includes the voltage excitation and the transducer electromechanical impulse response.

According to Eqs. (A.2), (A.3) and (A.8), the detected signal can be expressed in the frequency-domain as

$$S(r, \omega) = i\rho_0 \omega V_{exc}(\omega) E_{tr}(\omega) E_{rec}(\omega) G^2(r, \omega) \ast f(r).$$  (8)

Hereafter, an uppercase function denotes the Fourier transform of its corresponding time-domain counterpart. From Eq. (3), we see that the incident pressure in the frequency-domain is

$$P_{in}(r, \omega) = -i\omega \rho_0 V_{exc}(\omega) E_{tr}(\omega) G(r, \omega).$$  (9)

Combining Eqs. (8) and (9) results in

$$S(r, \omega) = A(\omega) P_{in}^2(r, \omega) \ast f(r),$$  (10)

where $A(\omega) = [i\rho_0 \omega V_{exc}(\omega) E_{tr}(\omega)]^{-1}$.

The SU imaging system is considered as a spatially invariant system. The image formation is then described by the point spread function (PSF). This function describes how what should be a point target is spread out by diffraction. We determine the PSF by assuming that the object function is a Dirac delta distribution

$$f(r) = \delta(r' - r).$$  (11)

Substituting this function into Eq. (10) and following Eq. (A.7), we find the detected signal of a point target as

$$S_\delta(r, \omega) = A(\omega) P_{in}^2(r, \omega).$$  (12)

The SU PSF can be defined as the detected signal at the focal plane $(z = z_0)$ divided by the same signal in the focus point both at the center-frequency $\omega = \omega_0$,

$$h_{PSF}(x, y) = \left| \frac{S_\delta(x, y, z_0, \omega_0)}{S_\delta(0, 0, z_0, \omega_0)} \right|^2 = \left| \frac{P_{in}(x, y, z_0, \omega_0)}{P_{in}(0, 0, z_0, \omega_0)} \right|^2.$$  (13)

Here we have also used Eq. (12). Finally, the image of an object is given by

$$i(x, y) = h_{PSF}(x, y) \ast f(x, y) + n(x, y),$$  (14)

where $f(x, y)$ is the 2D object function and $n(x, y)$ is the system noise. Although after determining the PSF the SU images can be further enhanced by deconvolution algorithms [19], the proposed method is not an algorithm-based technique [20].
Figure 2: Description of the domains used in the finite element simulation of the ultrasonic pulse superfocusing.

We shall compare the superresolution method with conventional ultrasonic (CU) technique that employs a spherically focused transducer. The spatial resolution of the CU system is defined as the beam intensity at full width at half maximum (FWHM) $w_{\text{CU}}$,

$$w_{\text{CU}} = 1.02\lambda_0 N = 1.13\lambda_0,$$

where $N$ is the transducer f-number. The CU depth-of-field is described as the full depth at half maximum (FDHM) of the axial beam intensity $d_{\text{CU}}$,

$$d_{\text{CU}} = 7.1\lambda_0 N^2 = 8.74\lambda_0.$$

2.2. Finite element simulations

We now explain how to obtain the system PSF with numerical simulations. The incident pressure $p_{\text{in}}$ is computed in time-domain using the finite element method in Comsol Software (Comsol Inc., USA). We also assume that the transducer is a circular rigid piston with the normal vibration velocity given by a Gaussian vibration packet,

$$v_n(t) = v_0 e^{-((t-t_c)^2)/\Delta t^2} \sin \omega_0 t,$$

where $v_0$ is the oscillation amplitude, $\Delta t$ and $t_c$ are the pulse width and time delay, respectively. The pressure $P_{\text{in}}$ is calculated via the fast Fourier transform algorithm. The PSF is then computed through Eq. (13). The simulation domain and mesh are described in Fig. 2. The propagation medium is water, which is assumed to be an inviscid fluid. The ball lens is made of reolite polymer. In the simulations, we used the Transient Pressure Acoustics and Solid...
Table 1: Parameters of the numerical simulation at room temperature and pressure, and computational information.

| Parameter                                      | Value                                      |
|------------------------------------------------|--------------------------------------------|
| **Medium (water)**                             |                                            |
| Density \( (\rho_0) \)                        | 1000 kg m\(^{-3}\)                        |
| Speed of sound \( (c_0) \)                    | 1480 m s\(^{-1}\)                        |
| Wavelength \( (\lambda_0) \)                  | 1.53 mm                                    |
| Mesh element size \( \lambda_0/11 \)          | 89.6 \(\mu\) m                           |
| Dimensions (free triangular mesh)             | 12.24 mm (W) × 150.8 mm (L)               |
| PML (mapped mesh)                             | 3.06 mm (W) × 3.06 mm (L)                 |
| **Transducer**                                |                                            |
| Diameter                                       | 12 mm                                     |
| Pressure at the active surface \( (p_0) \)    | 1.5 MPa                                    |
| Normal velocity \( (v_0) \)                   | 1 m s\(^{-1}\)                            |
| Pulse width \( (\Delta t) \)                  | 2.06 \(\mu\) s                            |
| Time delay \( (t_0) \)                        | 6.18 \(\mu\) s                            |
| Center frequency \( (f_0) \)                  | 1 MHz                                     |
| Sampling frequency \( (f_s) \)                | 5.25 MHz                                   |
| **Ball lens (rexolite 1422)**                 |                                            |
| Diameter                                       | 12 mm                                     |
| Distance to the transducer \( (d) \)          | 120 mm                                    |
| Density                                        | 1049 kg m\(^{-3}\)                       |
| Longitudinal speed of sound                   | 2337 m s\(^{-1}\)                        |
| Shear speed of sound                          | 1157 m s\(^{-1}\)                        |
| Mesh element size                             | 70.1 \(\mu\) m                           |
| **Additional information**                    |                                            |
| Computation time                               | 53 h                                      |
| CPU                                           | E5-2690 3.00GHz, 20 cores                 |
| Operating system                              | Linux                                     |

Mechanics modules. The numerical boundary conditions are the Perfect Match Layer (PML) combined with the plane wave radiation condition. The simulation parameters are summarized in Table 1.

We have performed the mesh convergence analysis for the simulated super-resolution system. The two parameters in this analysis are the full width at half maximum (FWHM) and full depth at half maximum (FDHM). We computed the relative error of these parameters by varying the mesh element size \( \lambda_0/n \), for which \( n = 5, 7, 9, 11 \). The relative error is defined as

\[
\epsilon = \left| 1 - \frac{x_n}{x_{11}} \right|, \tag{18}
\]

where \( x_n = \text{FWHM, FDHM} \) with the corresponding number of points per sampling wavelength. The correct value in the error computation is assumed to be \( x_{11} \). For \( n = 9 \) the errors are below 0.5\%.
Figure 3: Error analysis of the mesh convergence (a) FWHM and (b) FDHM.
2.3. Superresolution ultrasonic system

The SU imaging system is depicted in Fig. 1. It is composed of a circular flat transducer (ISG014SM, NdtXducer LCC, USA), with a diameter of 12 mm and center frequency of \( \omega_0/2\pi = 1 \text{ MHz} \). A ball-shaped lens made of rexolite with a 12.2 mm-diameter is suspended at \( d = 120 \text{ mm} \) away from the transducer active element along its central axis. At this distance, the incident beam intensity from the transducer reaches its maximum. Rexolite material was chosen due to its low attenuation and acoustic impedance close to water. These parameters are chosen to attain a spatial resolution close to half-wavelength and a depth-of-field of few wavelengths \[14\]. The experimental apparatus is immersed in a water tank with dimensions of 109 cm (L) \( \times \) 54 cm (W) \( \times \) 57 cm (H). The characteristic wavelength of a pulse is \( \lambda_0 = 1.53 \text{ mm} \). The transducer is driven by a pulse/receiver (DPR300, JSR Ultrasonics, USA) with negative spike pulses (excitation voltage of 200 V, damping of 333 Ω, the energy of 16 mJ, and repetition frequency of 100 Hz). The emitted pulse is focused in the shadow region of the lens at \( z = z_0 = 1.04\lambda_0 = 1.6 \text{ mm} \). This defines the imaging plane of the system. The peak pressure in the system focus of 7.6 kPa was measured by a 0.2 mm needle hydrophone (NH0200, Precision Acoustics, UK). The backscattered pressure by a target object is re-focused by the ball lens and acquired by the transducer. The corresponding signal is pre-processed with a low-pass filter (3 MHz-cutoff frequency and 43 dB-gain), digitized by a 12-bit A/D converter with 60 MSample/s (PCI-5105, National Instruments, USA), and gated in a 4 μs-time window. The image pixel corresponds to the magnitude of the acquired signal in the frequency-domain at 1 MHz. The SU beam raster-scans the object in steps of 150 μm that is much smaller than the wavelength.

A typical echo signal measured as the response to a point target (e.g., the tip of a needle with a 125 μm-diameter) placed at the system focus (0, 0, 1.6 mm) is shown in Fig. 4, panel (a). The Fourier transform of the gated echo at a 4 μs-window is depicted in panel (b).

3. Results and discussion

In Fig. 5, we show images of letters ‘PSF’ made of thin wires with a 0.5 mm-diameter: panel (a), (b), and (c) show, respectively, the SU, conventional ultrasonic (CU), and photography. The CU system is based on a spherically focused transducer with a diameter of 45 mm, 50 mm-focal distance (image plane), and f-number \( N = 1.11 \). The transducer operates in the pulse-echo mode at 1 MHz. The same electronic hardware and signal processing are used for both the SU and CU systems. The SU and CU images have 100 \( \times \) 100 pixels. These images differ in several important ways. The letters can be seen in the SU image, while they are not recognized in the CU image. The horizontal bar is shown in panel (c) is placed 0.5 mm in front of the vertical wires. The bar is visible in the CU image, whereas it is not seen in the SU image. Moreover, the SU and CU images have dynamic range of 30 dB and 16 dB, respectively.

In Fig. 6, we compare the numerical simulation and experimental results. The physical parameters used in the numerical simulation are summarized in
Figure 4: (a) Detected echo from a point target placed at the system focus (0, 0, 3 mm). (b) Fourier transform of the echo signal gated in a 4 μs-interval depicted by the gray region.
The target object is the tip of a needle with a diameter of 125 µm, which is much smaller than the characteristic wavelength \( \lambda_0 = 1.53 \text{ mm} \). Panel (a) shows the SU PSF. The FWHM of the experimental and numerical PSF are 0.6\( \lambda_0 = 0.91 \text{ mm} \) and 0.5\( \lambda_0 = 0.76 \text{ mm} \), respectively. From Eq. (15), we see that the FWHM of the CU system is 1.13\( \lambda_0 = 1.70 \text{ mm} \). Panel (b) illustrates the depth-of-field of the SU system. We see then that the experimental and numerical FDHM are 2\( \lambda_0 = 3.06 \text{ mm} \) and 2.13\( \lambda_0 = 3.27 \text{ mm} \), respectively. Referring to Eq. (16), the experimental FDHM is four times smaller than the CU FDHM. We have also numerically calculated the focusing gain of the lens (i.e., the ratio in dB of the pressure at focus to transmitted pressure at \( z = -12 \text{ mm} \)) to be 13 dB. For comparison, the theoretical gain of the CU system is 26 dB [1].

In Fig. 7, panel (a) displays the experimental depth-of-field image of the SU system. The PSF image is presented in panel (b). Moreover, the ultrasonic pulse propagation is illustrated by the video in the Supplementary material. The propagating pressure and axial component stress tensor are shown outside and inside the lens, respectively. It is worth noticing the pulse build-up inside the lens due to the focusing effect. Subsequently, the pulse is transmitted into the surrounding liquid. A rigid sphere of diameter \( \lambda_0 \) is placed at the system focus. The sphere scatters the incident pulse, and the echo propagates backwardly towards the transducer.

In respect to the features mentioned above, superfocused beams can be used in ultrasound biomicroscope (UBM) that is employed in ophthalmic imaging [21]. While this technology requires a frequency above 35 MHz, the superfocusing method would need half of that frequency to produce images with similar lateral resolution. For instance, consider a superresolution system with frequency of 25 MHz, wavelength of \( \lambda_0 = 60 \mu\text{m} \), and lens diameter of \( 20\lambda_0 = 1.2 \text{ mm} \). According to [14], this system achieves a lateral resolution of 0.6\( \lambda_0 = 36 \mu\text{m} \), depth-of-field of 6.5\( \lambda_0 = 390 \mu\text{m} \), and focal distance of 7\( \lambda_0 = 420 \mu\text{m} \).
Figure 6: Comparison between the experimental and numerical results. (a) SU PSF, (b) Normalized SU depth-of-field. The SU system operates in the pulse-echo mode with 1 MHz-center frequency.

These parameters are compatible with those of a UBM with a 50 MHz-center frequency [21]. The obtained depth-of-field makes the superfocused system more suitable for C-scan images.

The main limitations of the SU method using a ball lens is having a relatively low contrast (dynamic range of 30 dB) and a fixed focus. Post-processing amplification and improving focus gain by the lens may increase the dynamic range. The beam steering of the SU system has to be done mechanically. Combining electronic beam steering with the proposed method is yet to be developed.

4. Summary and conclusions

In summary, we have introduced a superresolution ultrasonic (SU) imaging system composed of a circular flat transducer and a ball-shaped lens. Excellent agreement is found between numerical simulations of the system PSF and experimental data. The SU system has nearly a half-wavelength (0.6\(\lambda_0\)) spatial resolution. The images are formed in the nearfield at 1.04\(\lambda_0\) away from the lens with a depth-of-field of 2\(\lambda_0\). These features add substantial improvements to SU images compared with conventional ultrasonic systems, i.e., higher resolution and sharper focusing.

The SU method may find fruitful applications for diagnostic imaging of skin and eye, which requires shallow image scanning, and detecting near-surface flaws in materials. Also, this method might be suitable for enhancing acoustic microscopy resolution while keeping the same operational frequency and electronic hardware.
Figure 7: (a) Normalized depth-of-field of the SU system. The vertical dotted line in (a) indicates the focal plane at $z = 1.6$ mm. (b) System PSF.

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Appendix A. Mathematical background

We summarize here some mathematical expressions used in the main text. For a time-dependent function denoted by $g(t)$, the Fourier transform is given by

$$G(\omega) = \mathcal{F}[g(t)] = \int_{-\infty}^{\infty} g(t)e^{-i\omega t} \, dt,$$
(A.1)
where $\omega$ is angular frequency. The inverse Fourier transform reads

$$g(t) = \mathcal{F}^{-1}[G(\omega)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega)e^{i\omega t} \, d\omega. \quad (A.2)$$

The Fourier transform of a time derivative of a function is given by

$$\mathcal{F} \left[ \frac{dg}{dt} \right] = i\omega \mathcal{F} [g(t)] = i\omega G(\omega). \quad (A.3)$$

The convolution in time between two functions $g_1$ and $g_2$ is defined as

$$g_1(t) * g_2(t) = \int_{-\infty}^{\infty} g_1(t')g_2(t - t') \, dt'. \quad (A.4)$$

While the spatial convolution between two functions of configuration space $g_1(r)$ and $g_2(r)$ is expressed by

$$g_1(r) * g_2(r) = \int_{\mathbb{R}^3} g_1(r)g_2(r - r') \, dV', \quad (A.5)$$

where $dV'$ is the volume element. In the $xy$-plane, the convolution is reduced to

$$g_1(x, y) * g_2(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_1(x, y)g_2(x - x', y - y') \, dx' \, dy'. \quad (A.6)$$

The convolution between a function $g(r)$ and the Dirac delta distribution $\delta(r)$ results

$$g(r) * \delta(r) = g(r). \quad (A.7)$$

The Fourier transform of a convolution in time of two functions is given by

$$\mathcal{F} [g_1(t) * g_2(t)] = \mathcal{F} [g_1(t)] \mathcal{F} [g_2(t)] = G_1(\omega)G_2(\omega). \quad (A.8)$$

References

[1] G. S. Kino, Acoustic Waves: Devices, Imaging and Analog Signal Processing, Prentice-Hall, Englewood Cliffs, NJ, 1987.

[2] D. Rugar, Resolution beyond the diffraction limit in the acoustic microscope: A nonlinear effect, J. Appl. Phys. 56 (1984) 1338–1346.

[3] B. Ward, A. C. Baker, V. F. Humphrey, Nonlinear propagation applied to the improvement of resolution in diagnostic medical ultrasound, J. Acoust. Soc. Am. 101 (1997) 143.

[4] M. Fatemi, J. F. Greenleaf, Ultrasound-stimulated vibro-acoustic spectrography, Science 280 (1998) 82–85.
[5] G. T. Silva, A. C. Frery, M. Fatemi, Image formation in vibro-acoustography with depth-of-field effects, Comput. Med. Imaging Graph. 30 (2006) 321–327.

[6] A. L. Baggio, H. A. S. Kaminura, J. H. Lopes, A. A. O. Carneiro, G. T. Silva, Parametric array signal in confocal vibro-acoustography, Appl. Acoustics 126 (2017) 143–148.

[7] F. G. Mitri, G. T. Silva, J. F. Greenleaf, M. Fatemi, Simultaneous sum-frequency and vibro-acoustography imaging for nondestructive evaluation and testing applications, J. Appl. Phys. 102 (2007) 114911.

[8] M. Fink, D. Cassereau, A. Derode, C. Prada, P. Roux, M. Tanter, J.-L. Thomas, F. Wu, Time-reversed acoustics, Rep. Prog. Phys. 63 (2000) 1933–1995.

[9] A. Sukhovich, B. Merheb, K. Muralidharan, J. O. Vasseur, Y. Pennec, P. A. Deymier, J. H. Page, Experimental and Theoretical Evidence for Subwavelength Imaging in Phononic Crystals, Phys. Rev. Lett. 102 (2009) 154301.

[10] J. Zhu, J. Christensen, J. Jung, L. Martin-Moreno, X. Yin, L. Fok, X. Zhang, F. J. Garcia-Vidal, A holey-structured metamaterial for acoustic deep-subwavelength imaging, Nat. Comm. 7 (2011) 52–55.

[11] C. Errico, J. Pierre, S. Pezet, Y. Desailly, Z. Lenkei, O. Couture, M. Tanter, Ultrafast ultrasound localization microscopy for deep super-resolution vascular imaging, Nature 527 (2015) 499.

[12] C. Fan, M. Caleap, M. Pan, B. W. Drinkwater, A comparison between ultrasonic array beamforming and super resolution imaging algorithms for non-destructive evaluation, Ultrasonics 54 (2014) 1842–1850.

[13] S. S. George, M. C. Huang, Z. Ignjatovic, Portable ultrasound imaging system with super-resolution capabilities, Ultrasonics 94 (2019) 391–400.

[14] J. H. Lopes, M. A. B. Andrade, J. P. Leao-Neto, J. C. Adamowski, I. V. Minin, G. T. Silva, Focusing acoustic beams with a ball-shaped lens beyond the diffraction limit, Phys. Rev. Applied 8 (2017) 024013.

[15] O. V. Minin, I. V. Minin, Acoustojet: acoustic analogue of photonic jet phenomenon based on penetrable 3D particle, Opt. Quant. Electron. 49 (2017) 54.

[16] D. V. Canle, T. Kekkonen, J. Mkinen, T. Puranen, H. J. Nieminen, A. Kuronen, S. Franssila, T. Kotiaho, A. Salmi, E. Hggstrom, Practical realization of a sub-λ/2 acoustic jet, Sci. Rep. 9 (2019) 5189.

[17] P. R. Stepanishen, Pulsed transmit/receive response of ultrasonic piezoelectric transducers, J. Acoust. Soc. Am. 69 (1981) 1815–1827.
[18] J. A. Jensen, A model for the propagation and scattering of ultrasound in tissue, J. Acoust. Soc. Am. 89 (1991) 182–190.

[19] T. Perciano, M. W. Urban, N. D. A. Mascarenhas, M. Fatemi, A. C. Frery, G. T. Silva, Deconvolution of vibroacoustic images using a simulation model based on a three dimensional point spread function, Ultrasonics 53 (2013) 36–44.

[20] N. Zhao, Q. Wei, A. Basarab, D. Kouam, J.-Y. Tourneret, Single image super-resolution of medical ultrasound images using a fast algorithm, in: 2016 IEEE 13th International Symposium on Biomedical Imaging (ISBI), IEEE, Prague, Czech Republic, 2016, pp. 473–476.

[21] R. H. Silverman, High-resolution ultrasound imaging of the eye—a review, Clin. Experiment Ophthalmol. 37 (2009) 54–67.