1 Introduction

The large electron positron collider LEP was build to study properties of the $Z$ with very high precision. This was achieved with the LEPI program in the years 1989 to 1995 at centre-of-mass energies around the $Z$ mass. During the years 1996 to 2000, referred to as LEPII period, the energy was increased up to a maximum of 209 GeV yielding high sensitivity both for direct and indirect searches for physics beyond the Standard Model. A total luminosity of almost 700 pb$^{-1}$ was recorded by four experiments at energies above the W-pair threshold.

High precision measurements of cross-sections for fermion- and photon-pair final states are used to search for signals of new physics. Since the systematic errors are only partially correlated a combination of the statistically independent results of the four experiments leads to a significant improvement of the sensitivity. In combination with the high accuracy of the Standard Model prediction this leads to a search range well beyond the LEP energy region.

2 Fermion-pair final states

Fermion-pair production at LEPI was clearly dominated by the production and decay of an on-shell $Z$ boson. At higher energies the process becomes more complicated due to strong initial
state radiation, ISR, final state radiation, FSR, and a larger contribution of photon exchange leading to stronger interference effects. Initial state radiation results in a reduction of the effective centre-of-mass energy, $\sqrt{s'}$. For $\sqrt{s'} \sim M_Z$ the cross-section increases, leading to a radiative-return peak. Since the coupling of quarks to the Z is much stronger than to the photon, compared to charged leptons, the radiative return is more pronounced for hadronic final states than for $\mu$- and $\tau$- pairs. Only about 22% of the hadronic events have high $s'/s > 0.85$; for leptons this fraction is about 41%. The Bhabha cross-section is dominated by $t$-channel exchange and hence doesn’t show a pronounced radiative return peak. To enhance the sensitivity to new physics only high $s'$ events are discussed in the following report. Events from neutrino-pair production with ISR are mainly studied in the context of direct searches for invisible particles and are not covered here. An additional effect is initial-state pair-production, ISPP, leading to an effective four-fermion final state. However, ISPP should be treated similar to ISR as long as the propagator is a photon, not a Z boson. Careful treatment of these effects, both in the theoretical calculations and in the experimental measurement, lead to an improvement of the errors on the cross-section predictions. The expected experimental precision of the final results of the LEP combination are compared to the current theoretical errors in Table 1. For

Table 1: Expected final experimental precision and theoretical uncertainty for LEP combined results of different channels of fermion-pair production. The numbers for electron-pair production are for the region $|\cos \theta| < 0.7$.

|                | $\sigma(q)$ | $\sigma(e)$ | $\sigma(\mu)$ | $\sigma(\tau)$ | $A_{FB}(\mu)$ | $A_{FB}(\tau)$ |
|----------------|-------------|-------------|---------------|---------------|---------------|---------------|
| **experiment** | 1%          | 0.9%        | 1.6%          | 2.2%          | 0.012         | 0.015         |
| **theory**     | 0.26%       | 2.0%        | 0.4%          | 0.4%          | 0.004         | 0.004         |

Figure 1: LEP combined total cross-sections and forward-backward asymmetries for high $s'$ hadronic, $\mu$- and $\tau$-pair final states at energies between 130 and 209 GeV.
Table 2: Limits on contact interaction for different final states: Bhabhas, $\mu$ and $\tau$ pairs (labeled as $\ell^+\ell^-$), hadrons, $c$ and $b$ quarks. Models are specified by the combination of $(\eta_{LL}, \eta_{RR}, \eta_{LR}, \eta_{RL})$: $LL = (\pm 1,0,0,0)$, $RR = (0, \pm 1,0,0)$, $LR = (0,0, \pm 1,0)$, $RL = (0,0,0, \pm 1)$, $VV = (\pm 1, \pm 1,0,0)$, $AA = (\pm 1, \pm 1,0,0)$, $V0 = (\pm 1, \pm 1,0,0)$ and $A0 = (0,0,0,0)$.

| Model | $e^+e^- \rightarrow e^+e^-$ | $e^+e^- \rightarrow \ell^+\ell^-$ | $e^+e^- \rightarrow q\bar{q}$ | $e^+e^- \rightarrow c\bar{c}$ | $e^+e^- \rightarrow b\bar{b}$ |
|-------|-------------------------------|---------------------------------|-------------------------------|-------------------------------|-------------------------------|
| LL    | $\Lambda^-$ $\Lambda^+$     | $\Lambda^-$ $\Lambda^+$       | $\Lambda^-$ $\Lambda^+$     | $\Lambda^-$ $\Lambda^+$     | $\Lambda^-$ $\Lambda^+$     |
| RR    | $9.0$ $7.1$ $9.8$ $13.3$     | $3.7$ $6.0$ $5.7$ $6.6$       | $9.1$ $12.3$                 |                                |                                |
| VV    | $18.0$ $15.9$ $16.0$ $21.7$  | $8.1$ $5.3$ $8.2$ $10.3$      | $9.4$ $14.1$                 |                                |                                |
| AA    | $11.5$ $11.3$ $15.1$ $17.2$  | $5.1$ $8.8$ $6.9$ $7.6$       | $11.5$ $15.3$                |                                |                                |
| LR    | $10.0$ $9.1$ $8.6$ $10.2$    | $5.1$ $4.3$ $3.9$ $2.1$       | $3.1$ $5.5$                  |                                |                                |
| RL    | $10.0$ $9.1$ $8.6$ $10.2$    | $7.2$ $9.3$ $3.1$ $2.8$       | $7.0$ $2.4$                  |                                |                                |
| V0    | $12.5$ $10.2$ $13.5$ $18.4$  | $5.1$ $6.0$ $7.4$ $9.2$       | $10.8$ $14.5$                |                                |                                |
| A0    | $14.0$ $13.0$ $12.4$ $14.3$  | $8.0$ $3.9$ $4.5$ $2.7$       | $6.3$ $3.9$                  |                                |                                |

All processes, apart from Bhabha scattering, the theory error is well below the experimental precision. For endcap Bhabhas the theory error is smaller, around 0.5%, and the experimental precision is limited by systematic errors inspite of high statistics.

LEP combined results for the differential cross-sections of Bhabhas, $\mu$- and $\tau$-pairs at energies between 183 and 209 GeV show good agreement with the Monte Carlo expectation². Measured total cross-sections and forward-backward asymmetries are shown in Figure and compared to the prediction from ZFITTER³. Similar results are available for heavy flavour final states, $R_c$, $R_b$, $A_b^c$ and $A_b^b$, though not all inputs are provided by all collaborations².

Results of cross-sections and asymmetries are used to search for indirect signals of physics beyond the Standard Model which would in most cases arise from a new particle as a propagator. Examples are leptoquarks being exchanged in the $t$- or $u$-channel in the production of hadronic events, Kaluza-Klein gravitons in models with large scale extra dimensions or an additional heavy vector boson $Z'$. A general approach to test such models is the study of four-fermion contact interactions with the effective Lagrangian:

$$L_{\text{eff}} = \frac{g^2}{(1 + \delta)\Lambda^2} \sum_{i,j=L,R} \eta_{ij} \bar{e}_i \gamma_\mu e_j \bar{f}_j \gamma^\mu f_j ,$$

where the coupling is fixed to $g = 4\pi$, $\delta = 1$ for Bhabhas and 0 else, $\eta_{ij} = 0, \pm 1$ describe the chiral structure and $\Lambda$ is the scale of the interaction. The limits summarized in Table range from about 2 to 20 TeV.

Limits are also derived within the specific models. The couplings of leptoquarks can be restricted depending on the leptoquark mass. Bhabha scattering gives the best sensitivity for models with large extra dimensions, restricting the energy scale to 1.2 TeV and 1.09 TeV depending on the model. Cross-sections and asymmetries measured at LEP-II are sensitive to the interference of a $Z'$ with the ordinary $Z$ leading to limits on the $Z'$ mass between 342 and 1787 GeV, depending on the model. In general a $Z'$ can mix with the Standard Model $Z$, thus changing the lineshape and couplings observed at LEP-I. A lineshape fit to LEP-I data within the $Z'$ model provides information about this mixing angle $\theta_M$. Since no combined cross-sections and asymmetries for LEP-II data are available this study is performed by separate experiments only. Limits are in the range $|\theta_M| < 4$ mrad.

The most precise determination of the Mass of the $Z$ is derived from the LEP-I lineshape fit to hadronic and leptonic cross-sections and leptonic asymmetries. Most of the LEP-I data...
were recorded in three centre-of-mass energy regions: at the peak and about 1.8 GeV above and below. Since the coupling of the Z is stronger to quarks than to charged leptons about 88% of all data are hadronic events at these three energies, resulting in effectively three measurements dominating the determination of the lineshape parameters which describe the mass of the Z, $M_Z$, its width $\Gamma_Z$, its coupling to quarks via the total hadronic cross-section $\sigma^0_{\text{had}}$ and leptonic parameters. The mentioned three parameters can be accurately determined from the three measurements if all other aspects are assumed to be known. This includes the couplings of the photon to quarks and the interference between photon and Z contributing to the hadronic cross-section.

From the results of the standard lineshape fit it is difficult to determine which effect a change in the interference or additional contributions from physics beyond the Standard Model might have. Therefore a more model independent ansatz, the S-matrix formalism\(^7\) is applied to determine the Z mass. It assumes two neutral bosons as propagators, one of them massless, but leaves the contribution of the Z exchange, $J^\text{tot}_{\text{had}}$, and the interference, $J^\text{tot}_{\text{had}}$, to the hadronic cross-section explicitly free. However, four parameters cannot be accurately determined from three effective measurements, leading to a large correlation between $M_Z$ and $J^\text{tot}_{\text{had}}$ shown in Figure\(^2\). Including more measurements from LEP\(II\) solves this problem, reducing the correlation. The final result of $M_Z = 91.186.9 \pm 2.3$ MeV\(^8\) is in very good agreement with the result of the standard lineshape fit $M_Z = 91.187.6 \pm 2.1$ MeV\(^9\) with only slightly increased error.

![Figure 2: Correlation between the mass of the Z and $J^\text{tot}_{\text{had}}$. Results are shown for LEP\(I\) data only and for a combined fit to LEP\(I\) and LEP\(II\) data. The yellow band indicates the 1σ error from the 9 parameter fit.](image)

3 Photon-pair final states

The process $e^+e^- \rightarrow \gamma\gamma$ is purely electromagnetic with weak contributions at the considered energies of about 0.2%. The theory is well known, the coupling constant $\alpha$ at zero momentum transfer\(^a\) is precisely measured. However, cross-section predictions are available only for Born level and next to leading order\(^10\). The theoretical error is proportional to the applied radiative corrections, which depend on the selected phase space, especially the acolinearity restriction.

\(^a\)Photons don’t appear in the propagator but as real particles in the final state.
Table 3: Selected phase space of the four experiments depending on the photon angle $\theta$, the acolinearity $\xi$ and the photon energies $E$. OPAL replaces the cut on the acolinearity by a restriction on the missing longitudinal momentum. The resulting radiative corrections in the barrel ($|\cos \theta| < 0.6$) range from an up correction of 5% (ALEPH) to a down correction of 28% (L3).

| Experiment | polar angle | acolinearity | energy | rad. correction |
|------------|-------------|--------------|--------|-----------------|
| ALEPH      | $|\cos \theta| < 0.95$ | $\xi < 20^\circ$ | $E > 0.25\sqrt{s}$ | -5% to -2%       |
| DELPHI     | $0.035 < |\cos \theta| < 0.731$ | $\xi < 50^\circ$ | $E > 0.15\sqrt{s}$ | 5% to 8%         |
| L3         | $|\cos \theta| < 0.961$ | $\xi < 175^\circ$ | $E > 5$ GeV, $\Sigma E > 0.5\sqrt{s}$ | 18% to 28%       |
| OPAL       | $|\cos \theta| < 0.93$ | $p_l < E_{1,2}$ | $E > 1$ GeV, $\Sigma E + p_l > 0.6\sqrt{s}$ | 4% to 7%         |

Table 3 summarizes the phase space selected by the four experiments. Though the corrections from the next to leading order prediction to Born level are different for the experiments, the preliminary combination assumes a common theory error of 1%.

The combination based on final results from ALEPH, L3 and OPAL as well as preliminary results from DELPHI yields an average of $0.982\pm0.010$ for the ratio of the measured over the expected total cross-section at energies between 183 and 209 GeV. The differential cross-sections determined by the four experiments are analyzed in a combined fit to set limits on scales of physics beyond the Standard Model: anomalous couplings $\Lambda_+ > 392$ GeV, $\Lambda_- > 364$ GeV, contact interactions $\Lambda' > 831$ GeV, low scale gravity $M_s > 933$ GeV($\lambda = +1$), $M_s > 1010$ GeV($\lambda = -1$), and the coupling $f/\Lambda < 3.9$ TeV$^{-1}$ of excited electrons with a mass $M_{e^*} = 200$ GeV. The best limits on $f/\Lambda$ are derived from LEP direct searches but are restricted to the kinematic region $M_{e^*} < 210$ GeV while the indirect limits from $e^+e^- \rightarrow \gamma\gamma$ depend only weakly on $M_{e^*}$.

All of these models affect only $d\sigma/d\cos \theta$. However, there are also models like non-commutative

![Graph showing the dependence of the $e^+e^- \rightarrow \gamma\gamma$ cross-section on $\phi$ in comparison to different expectations of non-commutative QED.](image_url)
QED, which lead to deviations of the azimuthal angle $\phi$. Non-commutative geometry is predicted by the quantization of strings in the presence of a background field [12]. This background field defines a unique direction which leads to a dependence of the cross-section not only on $\theta$ but also on $\phi$, on time and the orientation of the detector. A non-commutative Standard Model has not yet been formulated, but QED based on a non-commutative geometry (NCQED) exists, albeit with some limitations: only integer charges are allowed and the size of higher order corrections is unclear.

Figure 3 shows the cross-section as a function of $\phi$ integrated over $|\cos \theta| < 0.6$ and averaged over time [11]. It is compared to lowest order predictions of NCQED [13] for different angles $\eta$ between the unique direction and the rotation axis of the earth. For OPAL the $\cos \theta$ distribution provides a $\eta$ independent limit on the scale $\Lambda > 141$ GeV which can be improved by the $\phi$ distribution up to $\Lambda > 167$ GeV for $\eta = 0^\circ$.

4 Conclusion

LEP data taking was finalized in 2000 and the final LEP II results now start to come up. Besides the study of W-pair production and direct searches the precise measurements of Standard Model processes like fermion- and photon-pair production contributed to the large variety of processes studied. LEP II is a prime example of how four collaborations can act as in fact one experiment. The careful combination of results in the LEPEWWG working group exploits fully the power of electroweak measurements possible under the clean conditions of an $e^+e^-$ collider. As a result the high precision of the data forced the theorists to improve the Standard Model predictions to the sub % level. Fermion- and photon-pair cross-sections mainly provide indirect limits which complement the direct searches done at the Tevatron and HERA. No discovery was made at LEP, but we gained a much better understanding of the Standard Model.

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