1. Introduction

In the report the usage of the properties of the statistically dual distributions $^3$ (Poisson and Gamma) as well as the concept the “confidence density of parameter” $^4$ allows us to show the presence of a bias in reconstruction of the initial asymmetry, which produced the observed asymmetry. We use here the confidence density as a posteriori density with assumption that we have uniform prior.

2. The modeling of the asymmetry

Under the initial asymmetry we keep in mind the difference between the relative mean numbers of events $(\mu_1$ and $\mu_2)$ of two different flows of events:

$$A = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}. \quad (1)$$

Under the observed asymmetry we keep in mind the difference between the relative observed numbers of events $(\frac{n_1}{n_1 + n_2}$ and $\frac{n_2}{n_1 + n_2})$ from the same pair of flows of events:

$$\hat{A} = \frac{n_1 - n_2}{n_1 + n_2}. \quad (2)$$

In ref $^3$ (see, also, $^2, ^5$) it is shown that the confidence density of the Poisson distributed parameter $\mu$ in the case of a single observation $n$ is the Gamma distribution $\Gamma_{1,1+n}$ with mean, mode, and variance $n+1$, $n$, and $n+1$ respectively. This statement was checked by the Monte Carlo experiment $^6$.

The difference between the most probable and mean values of the parameter of the Poisson distribution suggests that there takes place (in the case of the measurement of asymmetry) a deviation which can be approximately estimated by the expression

$$\hat{A}_{\text{cor}} = \frac{(n_1 + 1) - (n_2 + 1)}{(n_1 + 1) + (n_2 + 1)} = \hat{A} \cdot \frac{n_1 + n_2}{n_1 + n_2 + 2}, \quad (3)$$

where values $n_1$ and $n_2$ are the observed numbers of events from two Poisson distributions with parameters $\mu_1$ and $\mu_2$ correspondingly.

We carried out the uniform scanning of parameter $A$, varying $A$ from value $-1$ to value $1$ using step size 0.01. By playing with the two Poisson distributions (with parameters $\mu_1$ and $\mu_2$) and using 30000 trials for each value of $A$ we used the RNPSSN function $^7$ to construct the conditional distribution of the probability of the production of the observed value of asymmetry $\hat{A}$ by the initial asymmetry $A$. We assume that an integral luminosity is a constant $\mu_1 + \mu_2 = \text{const}$. The parameters $\mu_1$ and $\mu_2$ are chosen in accordance with the given initial asymmetry $A$.

In Fig.1 (left) the distribution of $\hat{A}$ for given values of $\mu_1 + \mu_2 = 100$ and $A = 0.5$ is shown. The
distribution of the observed asymmetry $\hat{A}$ versus the initial asymmetry $A$ (Fig. 1, right) shows the result of the full scanning. The distribution of the probability of the initial asymmetries $A$ to produce the observed value of $\hat{A} = 1$ in case of $\mu_1 + \mu_2 = 10$ is presented in Fig. 2 (left). This figure clearly shows the difference between the most probable value of the initial asymmetry ($A=1$) and the mean value of the initial asymmetry ($A=0.76$). As seen in Fig. 2 (right), the r.m.s. (root-mean-square) of the distribution of the initial asymmetry $A$ is dependent on the observed value of asymmetry $\hat{A}$. This distribution characterizes the resolution of the determination of the initial asymmetry $A$ by the observed value $\hat{A}$. The dependence of the initial asymmetry $A$ on the observed asymmetry $\hat{A}$ for $\mu_1 + \mu_2 = 60$ can be seen in Fig. 3. The deviation from the straight line is essentially dependent on the integral luminosity.

3. Conclusions

The Monte Carlo experiment confirms the presence of the bias between the mean value of the initial asymmetry and the observed asymmetry. The conditional distribution of the probability of the initial asymmetry $A$ to give the observed value $\hat{A}$ has an asymmetric shape for large values of $\hat{A}$. The resolution of the determination of the initial asymmetry $A$ by the observed value $\hat{A}$ is dependent on the value of the observed asymmetry. We propose a simple formula Eq. (3) for correction of the observed asymmetry. The correct account for the uncertainty of the observed value must use the distribution of the initial asymmetry, i.e. the reconstructed confidence density of the parameter $A$ (see, Fig. 2, left).

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References

1. S.I. Bityukov, N.V. Krasnikov, V.A. Taperechkina, Preprint IFVE 2000-61, Protvino, 2000; hep-ex/0108020, 2001.
2. S.I.Bityukov, JHEP 09 (2002) 060; S.I.Bityukov and N.V.Krasnikov, Nucl.Instr.&Meth. A502, 795 (2003).
3. S.I. Bityukov and N.V. Krasnikov, “Statistically dual distributions and conjugate families”, in Proc. of 25th International Workshop on Bayesian Inference and Maximum Entropy Methods in Science and Engineering (MaxEnt’05), San Jose State University, San Jose CA USA, Aug 7-12,2005.
4. B. Efron, Stat.Sci. 13 95 (1998).
5. S.I. Bityukov and N.V. Krasnikov, AIP Conf. Proc., 707, 455 (2004).
6. S.I. Bityukov et al., Nucl.Instr.&Meth. **A534**, 228 (2004);  
7. CERNLIB, CERN PROGRAM LIBRARY, (CERN, Geneva, Switzerland, Edition - June 1996)