Subleading power corrections in radiative leptonic $B$ decay

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Abstract

I discuss the two-particle subleading power corrections in radiative leptonic $B \rightarrow \gamma \ell \nu$ decay at next-to-leading order in $\alpha_s$ with the dispersion approach. Employing the method of regions, factorization of the $B \rightarrow \gamma^*$ form factors is demonstrated explicitly, at one loop, for a space-like hard-collinear photon. The two-particle soft (endpoint) contribution is shown to be suppressed by one power of $\Lambda/m_b$, in the heavy quark limit, compared with the leading power contribution computed from QCD factorization. I further report the recent calculation on the three-particle subleading power contribution to the on-shell $B \rightarrow \gamma$ form factors at tree level and demonstrate that the “soft” and the “hard” three-particle corrections are of the same power, in contrast to the two-particle counterparts, as already speculated from the rapidity divergence in the corresponding factorization formulae. Phenomenological implications of the subleading power contributions to the $B \rightarrow \gamma \ell \nu$ amplitude are also addressed in detail, focusing on the determination of the inverse moment of the leading-twist $B$-meson distribution amplitude.

Keywords:
Heavy Quark Physics, Perturbative QCD, Resummation

1. Introduction

Understanding subleading power corrections in heavy quark decays is of interest to explore the general properties of heavy-quark expansion and to perform a stringent test of the CKM mechanism of the Standard Model. The radiative leptonic $B \rightarrow \gamma \ell \nu$ decay process involving only a single hadron is considered to be one of benchmark channels to investigate the power suppressed contributions in exclusive $B$-meson decays. At leading power in $\Lambda/m_b$, soft-collinear factorization properties of $B \rightarrow \gamma \ell \nu$ have been explored in both QCD [1,2] and soft-collinear effective theory (SCET) [3,4]. Subleading power contributions in the $B \rightarrow \gamma \ell \nu$ amplitude including both the local and non-local hadronic effects have been discussed in QCD factorization at tree level [5], where the non-local power correction from the hard-collinear quark propagator was found to preserve the symmetry relations for the $B \rightarrow \gamma$ form factors due to the helicity conservation in the heavy quark limit. Subsequently, two-particle subleading power corrections to the $B \rightarrow \gamma$ form factors were computed from the dispersion approach at tree level [6], following the technique developed in the context of $\gamma^* \gamma \rightarrow \pi$ form factor [7]. I will discuss the soft two-particle contribution to $B \rightarrow \gamma \ell \nu$ at $O(\alpha_s)$ and the three-particle subleading power contribution at tree level, as computed in [8], with the dispersion approach.

The presentation is organized as follows. I will first outline the general strategy of applying the dispersion approach in the radiative leptonic $B \rightarrow \gamma \ell \nu$ decay and then demonstrate QCD factorization for the two-particle contribution to the generalized $B \rightarrow \gamma^*$ form factors at one loop. Afterwards the three-particle contribution to the $B \rightarrow \gamma \ell \nu$ amplitude will be discussed at tree level. Numerical impact of the newly computed power suppressed contributions on the $B \rightarrow \gamma$ form factors and on the extraction of the inverse moment $\lambda_B$ will be further presented with two different models for the two-particle $B$-meson distribution amplitudes (DA).
2. Dispersion relations for the radiative leptonic $B \to \gamma \ell \nu$ decay

We will start with some general aspects of the $B \to \gamma \ell \nu$ decay amplitude following the theory overview presented in [5, 8]. To the first order in the electromagnetic correction the transition amplitude for the $B \to \gamma \ell \nu$ decay can be expressed as

$$\mathcal{A}(B^- \to \gamma \ell \nu) = \frac{G_F}{\sqrt{2}} (i g_{\text{em}} \epsilon^\nu_\ell) \left[ T^{\mu\nu}(p, q) \bar{\ell} \gamma_\mu (1 - \gamma_5) \nu + Q_\ell \bar{f}_B T^\nu (1 - \gamma_5) \nu \right],$$

where the two terms in the bracket describe the photon radiation from the hadron constitute and the lepton, respectively, and the hadronic tensor $T^{\mu\nu}$ is defined as follows

$$T_{\mu\nu}(p, q) \equiv \int d^4x e^{ip \cdot x} \langle 0 | T \left\{ j_{\mu \nu}^c(x), \bar{\ell} \gamma_\mu (1 - \gamma_5) \nu | 0 \right\} | B^- (p + q) \rangle.$$ (2)

Applying the electromagnetic Ward identity $p_{\nu} T^{\mu\nu}(p, q) = -(Q_{B} - Q_{\ell}) f_B p_{\nu}^B$ and redefining the axial-vector $B \to \gamma$ form factor to absorb the second term in the bracket of [1] lead to the replacement rule

$$T_{\mu\nu}(p, q) \to -i v \cdot p \epsilon_{\mu\nu\rho\sigma} n^\rho v^\sigma F_V(n \cdot p) + \left[ g_{\nu\rho} v \cdot p - v_{\rho} p_{\nu} \right] \left[ \tilde{F}_A(n \cdot p) + \frac{Q_{\ell}}{v \cdot p} \right] \equiv \tilde{F}_A(n \cdot p).$$ (3)

From which the differential decay rate of $B \to \gamma \nu$ in the rest frame of the $B$-meson can be computed as

$$\frac{d \Gamma}{d E_\gamma} (B \to \gamma \nu) = \frac{\alpha^2_{\text{em}} G_F^2 |V_{ub}|^2}{6 \pi^2} m_B E_\gamma \left( 1 - \frac{2 E_\gamma}{m_B} \right) \left[ F_V^2 (n \cdot p) + F_A^2 (n \cdot p) \right].$$ (4)

To explain the essential technique of the dispersion approach for the evaluation of the subleading power contributions, we start with the correlation function describing the off-shell $B \to \gamma^*$ transition with a space-like hard-collinear (transverse polarized) photon following the discussion in [8, 9, 10, 11].

$$\tilde{T}_{\mu\nu}(p, q) \equiv \int d^4x e^{ip \cdot x} \langle 0 | T \left\{ j_{\mu \nu}^c(x), \bar{\ell} \gamma_\mu (1 - \gamma_5) \nu | 0 \right\} | B^- (p + q) \rangle \bigg|_{p^0 < 0},$$

$$= v \cdot p \left[ -i \epsilon_{\mu\nu\rho\sigma} n^\rho v^\sigma F_V^{B \to \gamma^*} (n \cdot p, \bar{n} \cdot p) + g_{\nu\rho} \tilde{F}_A^{B \to \gamma^*} (n \cdot p, \bar{n} \cdot p) \right],$$ (5)

with the power counting rule for the external momentum $n \cdot p \sim O(m_B), |\bar{n} \cdot p| \sim O(\Lambda)$. Taking advantage of the analytical property of the generalized $B \to \gamma^*$ form factors yields the hadronic dispersion relations

$$F_V^{B \to \gamma^*} (n \cdot p, \bar{n} \cdot p) = \frac{2}{3} \frac{f_0 m_p}{m_B^2 - m^2_{B^*} - i0} \left( 2 \frac{m_B}{m_B + m_p} \frac{V(q^2) + \frac{1}{\pi} \int_{\omega_-}^{\omega_+} d\omega' \text{Im} \omega' F_V^{B \to \gamma^*, \text{had}} (n \cdot p, \omega')}{\omega' - \bar{n} \cdot p - i0}. \right.$$ (6)

$$F_A^{B \to \gamma^*} (n \cdot p, \bar{n} \cdot p) = \frac{2}{3} \frac{f_0 m_p}{m_B^2 - m^2_{B^*} - i0} \left( 2 \frac{m_B + m_p}{n \cdot p} A_1(q^2) + \frac{1}{\pi} \int_{\omega_-}^{\omega_+} d\omega' \text{Im} \omega' \tilde{F}_A^{B \to \gamma^*, \text{had}} (n \cdot p, \omega') \right).$$ (7)
Applying the light-cone operator-product-expansion (OPE) technique and working out the dispersion representations for the resulting factorization formulae of the nonperturbative modification of the spectral density.

Now we turn to discuss the computation of the one-loop hard and jet functions in the factorization formulae for the on-shell $B \rightarrow \gamma^*$ form factors lead to the light-cone sum rules for the form factors $V(q^2)$ and $A_1(q^2)$

\[
\frac{2}{3} \frac{f_{\pi}}{n \cdot p} \text{Exp} \left[ \frac{m_{B}^{2} - m_{p}^{2}}{n \cdot p \omega_{M}} \right] \frac{2 m_{B}}{m_{B} + m_{p}} V(q^2) = \frac{1}{\pi} \int_{0}^{\omega_{M}} d\omega' \ e^{-\omega'|\omega_{M}} \text{Im}_{\omega'} F_{V}^{B \rightarrow \gamma^*}(n \cdot p, \omega'),
\]

\[
\frac{2}{3} \frac{f_{\pi}}{n \cdot p} \text{Exp} \left[ - \frac{m_{p}^{2}}{n \cdot p \omega_{M}} \right] \frac{2 (m_{B} + m_{p})}{n \cdot p} A_{1}(q^2) = \frac{1}{\pi} \int_{0}^{\omega_{M}} d\omega' \ e^{-\omega'|\omega_{M}} \text{Im}_{\omega'} F_{A}^{B \rightarrow \gamma^*}(n \cdot p, \omega').
\]

Substituting the above sum rules into (6) and (7) and setting $n \cdot p \rightarrow 0$ give rise to improved dispersion relations for the on-shell $B \rightarrow \gamma$ form factors

\[
F_{V}(n \cdot p) = \frac{1}{\pi} \int_{0}^{\infty} d\omega \ \frac{n \cdot p}{m_{B}^{2}} \text{Exp} \left[ \frac{m_{B}^{2} - \omega n \cdot p}{n \cdot p \omega_{M}} \right] \text{Im}_{\omega} F_{V}^{B \rightarrow \gamma^*}(n \cdot p, \omega'),
\]

\[
\hat{F}_{A}(n \cdot p) = \frac{1}{\pi} \int_{0}^{\infty} d\omega \ \frac{n \cdot p}{m_{B}^{2}} \text{Exp} \left[ \frac{m_{B}^{2} - \omega n \cdot p}{n \cdot p \omega_{M}} \right] \text{Im}_{\omega} F_{A}^{B \rightarrow \gamma^*}(n \cdot p, \omega'),
\]

where the second term on the right-hand side of (10) and (11) corresponds to the soft (end-point) contribution due to the nonperturbative modification of the spectral density.

At tree level, the generalized $B \rightarrow \gamma^*$ form factors can be readily computed as

\[
F_{V,\gamma^*}(n \cdot p, \bar{n} \cdot p) = \hat{F}_{A,\gamma^*}(n \cdot p, \bar{n} \cdot p) = \frac{Q_{s} f_{B}(\mu) m_{B}}{n \cdot p} \int_{0}^{\infty} d\omega \ \frac{\phi_{B}^{3}(\omega, \mu)}{\omega - \bar{n} \cdot p - i0} + O(\alpha_{s}, \Lambda/m_{B}).
\]

Now we turn to discuss the computation of the one-loop hard and jet functions in the factorization formulae

\[
F_{V}^{B \rightarrow \gamma^*}(n \cdot p, \bar{n} \cdot p) = \hat{F}_{A}^{B \rightarrow \gamma^*}(n \cdot p, \bar{n} \cdot p)
\]

\[
= \frac{Q_{s} f_{B}(\mu) m_{B}}{n \cdot p} \int_{0}^{\infty} d\omega \ \frac{\phi_{B}^{3}(\omega, \mu)}{\omega - \bar{n} \cdot p - i0} J_{\perp}(n \cdot p, \bar{n} \cdot p, \omega, \mu) + \ldots,
\]

at leading power in $\Lambda/m_{B}$, employing the method of regions [12] which has been extensively used for evaluating the multi-scale amplitudes (see, for instance [13, 14, 15, 16, 17]).
Taking the weak vertex diagram displayed in figure 1(a) as an example, the corresponding QCD amplitude defined by the convolution integral of the partonic distribution amplitude at next-to-leading-order (NLO) in $\alpha_s$ and the leading order (LO) hard kernel. The resulting contribution to the hard coefficient function $C_\perp$ can be extracted from Eq. (3.7) of [8] with the integrand of the loop-momentum integral expanded in the hard region [8].

\[
\tilde{T}^{(1),h}_{\nu\mu,\text{weak}}(p,q) = -i g_2^2 C_F \int \frac{d^D l}{(2\pi)^D} \frac{\tilde{T}_{\nu\mu}^{(0)}(p,q)}{[l^2 + n \cdot p n \cdot l + i0][l^2 + 2 l \cdot q + i0][l^2 + i0]} \\
\times [n \cdot l][(D - 2) \tilde{n} \cdot l + 2 m_b] + (D - 4) \tilde{l}_2 \tilde{l}_2 + 2 n \cdot p (\tilde{n} \cdot l + m_b)] \\
\equiv C_{\perp,\text{weak}}(n \cdot p) \tilde{T}_{\nu\mu}^{(0)}(p,q),
\]

(15)

where $T_{\nu\mu}^{(0)}$ is the tree-level contribution to the correlation function [5] and $C_{\perp,\text{weak}}$ can be found in Eq. (3.7) of [8].

Expanding [14] in the hard-collinear region

\[
\tilde{T}^{(1),hc}_{\nu\mu,\text{weak}}(p,q) = -i g_2^2 C_F \int \frac{d^D l}{(2\pi)^D} \frac{2 m_b n \cdot (p + l)}{[n \cdot (p + l) \tilde{n} \cdot (p + l) + \tilde{l}_2 + i0][m_b n \cdot l + i0][l^2 + i0]} \\
\times [n \cdot (p + l) \tilde{n} \cdot (p + l) + \tilde{l}_2 + i0][m_b n \cdot l + i0][l^2 + i0] \\
\equiv J_{\perp,\text{weak}}(n \cdot p, \tilde{n} \cdot p, \omega) \tilde{T}_{\nu\mu}^{(0)}(p,q),
\]

(16)

where the explicit expression of $J_{\perp,\text{weak}}$ can be found in Eq. (3.9) of [8].

Evaluating the leading power contributions to remaining diagrams with the same technique leads to

\[
C_{\perp} = 1 - \frac{\alpha_s C_F}{4\pi} \left[ 2 \ln^2 \frac{\mu}{m_b} + 5 \ln \frac{\mu}{m_b} - 2 \text{Li}_2 \left( 1 - \frac{1}{r} \right) - \ln^2 r + \frac{3r - 2}{1 - r} \ln r + \frac{\pi^2}{12} + 6 \right],
\]

(17)

Figure 1: Two-particle contribution to the correlation function [5] at one loop.

Similarly, the leading-power hard-collinear contribution to the weak vertex diagram can be obtained by expanding [14] in the hard-collinear region.
\[ J_\perp = 1 + \frac{\alpha_s C_F}{4\pi} \left\{ \ln^2 \frac{\mu^2}{n \cdot p (\omega - \tilde{n} \cdot p)} - \frac{\pi^2}{6} - 1 - \frac{\tilde{n} \cdot p - \omega}{\omega} \ln \frac{\tilde{n} \cdot p - \omega}{\omega} \left[ \ln \frac{\mu^2}{\tilde{n} \cdot p} + \ln \frac{\mu^2}{n \cdot p (\omega - \tilde{n} \cdot p)} + 3 \right] \right\}. \] (18)

Resummation of the parametrically large logarithms in the hard function \( C_\perp \) and in the \( B \)-meson decay constant \( \tilde{f}_B \) can be achieved by solving the following evolution equations

\[
\frac{dC_\perp(n \cdot p, \mu)}{d\ln \mu} = -\Gamma_{\text{caep}}(\alpha_s) \ln \frac{\mu}{n \cdot p} + \gamma(\alpha_s) \right] C_\perp(n \cdot p, \mu) \right), \quad \frac{d\tilde{f}_B(\mu)}{d\ln \mu} = \gamma(\alpha_s) \tilde{f}_B(\mu), \] (19)

It is then straightforward to write down the resummation improved factorization formulae for the \( B \to \gamma' \) form factors, from which one can derive the following dispersion relations for the two-particle contributions with the aid of (10) and (11).

\[
F_{V,2p}(n \cdot p) = \hat{F}_{A,2p}(n \cdot p) = -\frac{Q_u m_B}{(n \cdot p)^2} \left\{ U_2(n \cdot p, \mu_3, \mu) \tilde{f}_B(\mu_3) \right\} \left\{ U_1(n \cdot p, \mu_1, \mu) C_\perp(n \cdot p, \mu_1) \right\}
\times \left\{ \int_0^\infty d\omega \frac{\phi^p_{\text{s}}(\omega, \mu)}{\omega} J_\perp(n \cdot p, 0, \omega, \mu) + \int_0^\infty d\omega' \left\{ \frac{n \cdot p}{m_B^2} - \frac{\omega' n \cdot p}{n \cdot p \omega_M} \right\} \exp \left\{ \frac{m_B^2}{n \cdot p \omega_M} \right\} - \frac{1}{\omega'} \right\} \phi^p_{\text{eff}}(\omega', \mu) \right\}. \] (20)

where the explicit expression of \( \phi^p_{\text{eff}}(\omega', \mu) \) is displayed in Eq. (3.31) of [3].

Now we turn to discuss the three-particle contribution to the generalized \( B \to \gamma' \) form factors at tree level, which can be obtained by evaluating the partonic diagram presented in figure 3 of [3]. Applying the background field approach for the light-quark propagator we obtain

\[
F_{V,3p}^{B,\gamma'}(n \cdot p, \tilde{n} \cdot p) = \hat{F}_{A,3p}^{B,\gamma'}(n \cdot p, \tilde{n} \cdot p)
\]

\[
F_{V,3p}(n \cdot p) = \hat{F}_{A,3p}(n \cdot p) = -\frac{Q_u \tilde{f}_B(\mu)}{(n \cdot p)^2} \left\{ \frac{n \cdot p}{m_B^2} \exp \left\{ \frac{m_B^2}{n \cdot p \omega_M} \right\} \hat{f}_B^p(\omega_3, \omega_M) + \hat{f}_B^{II}(\omega_3, \omega_M) \right\}, \] (22)

where the first and second terms in the bracket correspond to the “soft” and “hard” three-particle contributions. One can conclude from the power counting rules for the external momenta and the sum-rule parameters that both the “soft” and “hard” three-particle contributions scale as \((\Lambda/m_B)^{3/2}\) in the heavy quark limit.
To evaluate the numerical impact of the subleading power two-particle correction at $O(\alpha_s)$ and the three-particle contribution at tree level, we adopt the two different models for the two-particle $B$-meson DA inspired from the QCD sum rule analysis at LO and at NLO (see [18] for an improvement including perturbative constraints) and employ the exponential model for the three-particle $B$-meson DA. The key quantity entering the parametrization of the above-mentioned non-perturbative functions is the inverse moment $\lambda_B$ of the leading-twist $B$-meson DA, which also serves as a fundamental hadronic input for the theoretical description of many other exclusive processes [19] 20 21 22 23. We will take the interval $\lambda_B(\mu_0) = 354^{+38}_{-30}$ MeV determined from the matching of the two different types of light-cone sum rules for the $B \to \pi$ form factors with the pion DA [24] and with the $B$-meson DA [15], respectively. With the default theory inputs, perturbative QCD corrections to the two-particle soft contribution is found to shift the tree-level prediction by an amount of $(10 \sim 20)\%$, and the LO three-particle correction to the $B \to \gamma$ form factors turns out to be negligible numerically [8]. However, the subleading power two-particle contribution can be enhanced significantly with the decrease of $\lambda_B$ and it is even comparable to the leading power contribution computed in QCD factorization at $\lambda_B \lesssim 100$ MeV as implied by the power counting analysis [8]. Moreover, the model dependence of the two-particle $B$-meson DA on the theoretical predictions of $F_V$ and $F_A$ also becomes more important at small $\lambda_B$ and at small $E_\gamma$. Finally, we are in a position to discuss the determination of the inverse moment $\lambda_B$ from the Belle measurement of the integrated branching ratio of $B \to \gamma\ell\nu$ [25]. Taking into account the newly computed subleading power corrections, no interesting constraint on $\lambda_B$ can be obtained for the Grozin-Neubert model [25] due to the rather weak experiment limit, while a meaningful bound $\lambda_B > 214$ MeV can be deduced for the Braun-Ivanov-Korchemsky model [27] of the leading-twist $B$-meson DA. This fact can be easily understood from the strong sensitivity of the $B \to \gamma$ form factors on the precise shape of the $B$-meson DA $\phi^B_\gamma(\omega)$ at small light-quark momentum $\omega$.

3. Conclusions

Applying the dispersion approach, the subleading power two-particle soft correction to the $B \to \gamma\ell\nu$ transition amplitude was shown to be sizeable in particular at small $\lambda_B$ and the inverse moment $\lambda_B$ is not sufficient to describe the strong interaction dynamics of the $B \to \gamma$ form factors in general. In contrast, the tree-level three-particle contribution can only lead to the negligible impact on the $B \to \gamma$ form factors. Further improvement including perturbative QCD corrections to the three-particle DA and the yet higher-twist corrections will be crucial to deepen our understanding of the factorizaton properties in the heavy quark system and to achieve precision determinations of the CKM matrix elements.

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