Hawking Radiation of Topological Massive Warped-AdS$_3$ Black Holes via Particles Tunnelling

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Abstract. We investigate the Dirac and scalar particles tunnelling as a radiation of Warped AdS$_3$ black holes in Topological Massive Gravity. Using Hamilton-Jacobi method, we discuss tunnelling probability and Hawking temperature of the spin-1/2 and spin-0 particles for the black hole. We observe the tunnelling probability and Hawking temperature to be same for the spin-1/2 and spin-0. We also examined the same procedure for the extremal case of the Warped AdS$_3$ black holes, and thus, we show that the tunnelling process may occur, for both Dirac and scalar particles. Furthermore, in the extremal case, we find that the extremal case of the black hole has the Hawking Temperature in the Planck scale and thus it has a surface gravity although it has no surface gravity according to the classical method.
A self-consistent quantum gravity theory hasn’t been constructed yet. Therefore, the quantum mechanical properties of a classical gravitational field is studied by the quantum mechanical behaviour of a physical system effected from it. In particular, thanks to the extension of standard Quantum theory to curved spacetime, some events, such as particle creation and thermal radiation of a black hole, can be predicted. Moreover, the black holes as the most popular concepts of the classical gravity are just understood by the quantum mechanical concepts. From this point of view, the solutions of the relativistic quantum mechanical wave equations in a gravitational background became an important tool for getting information about its nature [1–3]. For this reason, the relativistic quantum mechanical wave equations in a curved spacetime background have been extensively studied [4–7].

The nature of black holes have been started to be understood by thermodynamical and quantum mechanical concepts since 1970 [8–13]. Among these concepts, especially, thermal radiation, known as Hawking radiation in the literature, has been investigated as a quantum tunnelling effect of the relativistic particles from a black hole [14–20]. Thanks to the studies, a black hole temperature, which is called hawking temperature in the literature, is related to the black hole surface gravity. Therefore, the Hawking temperature becomes an important concept to investigate a black hole physics. Since then, in the framework of standard Einstein general relativity, the Hawking radiation as a tunnelling process of the particles from various black holes has been studied, extensively, in the literature in both 3+1 and 2+1 dimensional [21–24, 29–31]. On the other hand, Kerner and Mann extended the tunnelling process to include the Dirac particle emission from a 3+1 dimensional black hole [19, 20]. Also, Ren and Li considered the Dirac particles’ tunnelling process to investigate the Hawking radiation for the 2+1-dimensional BTZ black hole using the tunneling method [24]. The particle tunnelling process in all these studies give useful information about the mathematical and physical properties of the black holes. In the similar way, the Hawking radiation is used to discuss the properties of a black hole in the context of modified gravity theories [25–28]. As an example, Gecim and Sucu discussed Hawking radiations for both Dirac and scalar particles from the New-type black hole in the framework of 2+1 dimensional New Massive Gravity theory [30]. However, according to the method, both particles probe the black hole in same way. Also, in the context of modified gravity theories, Qi investigates the fermion tunnelling radiation from the static Lifshitz black hole in 2+1 dimensional New Massive Gravity theory, and from New Class Black Holes in 3+1 Einstein-Gauss-Bonnet Gravity [31].
The (2+1) dimensional gravitational models provide a suitable area to investigate the quantum effects of the gravity [32–35]. Among these, Topologically Massive Gravity as an interesting modified three-dimensional gravitation theory is formed by adding a Chern-Simons term to the standard Einstein-Hilbert action [36]. With this term, the gravity theory has gained both physical and mathematical interesting properties. However, in contrast to other gravitational theories, the graviton becomes a massive particle.

The warped AdS$_3$ black holes for the solution of the Topological massive gravity is given by the following metric

$$ds^2 = N(r)^2 dt^2 - \frac{1}{N(r)^2 F(r)^2} dr^2 - F(r)^2 \left[ d\phi + N^\phi(r) dt \right]^2 \quad (1.1)$$

The abbreviations used in here are as follows;

$$F(r)^2 = r^2 + 4\omega r + 3\omega^2 + \frac{r_0^2}{3}$$

$$N(r)^2 = \frac{r^2 - r_0^2}{F(r)^2}, N^\phi(r) = -\frac{2r + 3\omega}{F(r)^2}$$

The Warped-AdS$_3$ Black holes have two horizon at $r \mp r_0$. The parameters $\omega$ and $r_0$ are related to the physical parameters of the black hole, mass and angular momentum [41]. For the metric, the surface gravity is calculated by classical (standard) method as,

$$\kappa = \frac{1}{2} \left[ F(r) \frac{\partial}{\partial r} \left[ N^2(r) \right] \right]_{r=r_0}$$

and thus,

$$\kappa = \sqrt{3} \left( \frac{r_0}{2r_0 + 3\omega} \right)$$

The Hawking temperature, $T_H$, is defined in terms of the surface gravity as $T_H = \frac{\hbar \kappa}{2\pi}$ and, for the black hole, it is given as follows

$$T_H = \frac{\hbar \sqrt{3}}{2\pi} \left( \frac{r_0}{2r_0 + 3\omega} \right).$$

The Warped-AdS$_3$ Black hole becomes extremal at $r_0 = 0$. According to (1.2) the surface gravity becomes zero in the extremal case, hence the Hawking temperature of the extremal black hole is zero. Additionally, in the extremal case, the black hole has a double horizon at $r = 0$. Moreover, this result does not depend on parameter $\omega$. In an even more special case ($\omega = r_0 = 0$), the metric (1.1) is reduced to the horizonless metric that is characterized as the ground state or ‘vacuum’ of the black-hole [41].

To understand the quantum mechanical properties of the black hole, we find the probability of tunnelling and Hawking temperature by using the solutions of the relativistic quantum mechanical wave equation for the scalar and Dirac particles. We also show the extremal case of this black hole to have a Hawking temperature, and therefore, quantum mechanically, a surface gravity.

The organization of this work are follows. In the Section 2, we write the Dirac equation in Warped-AdS$_3$ Black holes background, and calculate the tunnelling possibility of the Dirac
particle by using the semi-classical method. Also, we find Hawking temperature. In the Section 3, Klein-Gordon equation is rewritten in Warped-AdS$_3$ Black hole spacetime. The tunnelling probability of scalar particles from the black hole and their Hawking temperature is also calculated. In the Section 4, we carry out the same calculation for the extremal case, $r_0 = 0$, and for the ground state ($\omega = r_0 = 0$) of the black hole. Finally, we evaluate and summarize the results.

2 Tunnelling of Dirac particles

To investigate tunnelling the Dirac particles from Warped-AdS$_3$ Black hole, we write Dirac equation in (2+1) dimensional spacetime in the following representation [42],

$$\{i \vec{\sigma}^\mu (x) [\partial_\mu - \Gamma_\mu (x)]\} \Psi (x) = \frac{m_0}{\hbar} \Psi (x). \quad (2.1)$$

In this representation; Dirac spinor, $\Psi (x)$, has only two components corresponding positive and negative energy eigenstates which has only one spin polarization. $\vec{\sigma}^\mu (x)$ are the spacetime depended Dirac matrices and they are written in terms of constant Dirac matrices, $\vec{\sigma}^i$, by using triads, $e^\mu_{(i)} (x)$, as follows

$$\vec{\sigma}^\mu (x) = e^\mu_{(i)} (x) \vec{\sigma}^i, \quad (2.2)$$

where $\vec{\sigma}^i$ are Dirac matrices in a flat spacetime and given as

$$\vec{\sigma}^i = (\sigma^0, \sigma^1, \sigma^2) \quad (2.3)$$

with

$$\sigma^0 = \sigma^3, \quad \sigma^1 = i \sigma^1, \quad \sigma^2 = i \sigma^2, \quad (2.4)$$

where $\sigma^1$, $\sigma^2$ and $\sigma^3$ Pauli matrices, and $\Gamma_\mu (x)$ are the spin affine connection by the following definition,

$$\Gamma_\mu (x) = \frac{1}{4} g_{\lambda \alpha} (e^\nu_{(i)} e^\alpha_{(i)} - \Gamma^\alpha_{\nu \mu}) s^{\lambda \nu} (x). \quad (2.5)$$

Here, $\Gamma^\alpha_{\nu \mu}$ is Christoffell symbol, and $g_{\mu \nu} (x)$ is spacetime depended metric tensor and it is given in term of triads as follows,

$$g_{\mu \nu} (x) = \epsilon^{(i)}_{\nu} (x) \epsilon^{(j)}_{\mu} (x) \eta_{(i)(j)}, \quad (2.6)$$

where $\mu$ and $\nu$ are curved spacetime indices running from 0 to 2. $i$ and $j$ are flat spacetime indices running from 0 to 2 and $\eta_{(i)(j)}$ is the metric of (2+1) dimensional Minkowski spacetime, with signature (1,-1,-1), and $s^{\lambda \nu} (x)$ is a spin operator given by

$$s^{\lambda \nu} (x) = \frac{1}{2} [\vec{\sigma}^\lambda (x), \vec{\sigma}^\nu (x)]. \quad (2.7)$$

From Eq.(1.1) and (2.6), the triads of $e^\alpha_{(i)}$ are written as;

$$e^\mu_{(0)} = \left( \frac{1}{N}, 0, -\frac{N^0}{N} \right)$$

$$e^\mu_{(1)} = (0, FN, 0)$$

$$e^\mu_{(0)} = \left( 0, 0, \frac{1}{F} \right)$$
The tunnelling probability for the classically forbidden trajectory from inside to outside of the black hole horizon is given by

$$\Gamma = e^{-\frac{\pi}{2} \text{Im} S} \quad (2.8)$$

where $S$ is the classical action function of a particle trajectory [24, 43, 45]. Therefore, in order to discuss tunneling probability, one needs to calculate the imaginary part of a classical action function, $S$, in regards to the tunnelling probability. To investigate the tunnelling probability of a Dirac particle from the black hole, we use the following ansatz for the wave function in the Eq.(2.1):

$$\Psi(x) = \exp \left( \frac{i}{\hbar} S(t, r, \phi) \right) \begin{pmatrix} A(t, r, \phi) \\ B(t, r, \phi) \end{pmatrix} \quad (2.9)$$

where $A(t, r, \phi)$ and $B(t, r, \phi)$ are functions of space-time [24, 45]. To apply the Hamilton-Jacobi method, we insert the Eq.(2.9) in the Dirac equation given by Eq.(2.1). Dividing by the exponential term and neglecting the terms with $\hbar$, we derive the following two coupled differential equations.

$$A \left[ m_0 N(r) + \frac{\partial S}{\partial t} - N^\phi(r) \frac{\partial S}{\partial \phi} \right] + B \left[ iF(r) N(r)^2 \frac{\partial S}{\partial r} + \frac{N(r) \partial S}{F(r) \partial \phi} \right] = 0$$

$$A \left[ iF(r) N(r)^2 \frac{\partial S}{\partial r} - \frac{N(r) \partial S}{F(r) \partial \phi} \right] + B \left[ m_0 N(r) - \frac{\partial S}{\partial t} + N^\phi(r) \frac{\partial S}{\partial \phi} \right] = 0. \quad (2.10)$$

These two equations have nontrivial solutions for $A(t, r, \phi)$ and $B(t, r, \phi)$ when the determinant of the coefficient matrix is vanished. Accordingly,

$$F(r) \left( \frac{\partial S}{\partial t} \right)^2 - 2F(r)^2 N^\phi(r) \left( \frac{\partial S}{\partial t} \right) \left( \frac{\partial S}{\partial \phi} \right) + \left( F(r)^2 N^\phi(r)^2 - N(r)^2 \right) \left( \frac{\partial S}{\partial \phi} \right)^2$$

$$- N(r)^4 F(r)^4 \left( \frac{\partial S}{\partial r} \right)^2 - N(r)^4 F(r)^2 m_0^2 = 0. \quad (2.11)$$

As $(\partial_t)$ and $(\partial_\phi)$ are two killing vectors we can separate $S(t, r, \phi)$ to the variables as follows

$$S(t, r, \phi) = -Et + j\phi + K(r) + C, \quad (2.12)$$

where $E$ and $j$ are the energy and angular momentum of a Dirac particle, respectively, and $C$ is a complex constant. Inserting Eq.(2.12) in Eq.(2.11) and solving for the radial function, $K(r)$, for fixed $\phi = \phi_0$ we get

$$K_\pm(r) = \pm \int \sqrt{\frac{F(r)^2 E^2 - F(r)^2 N(r)^2 m_0^2}{F(r)^2 N(r)^2}} dr$$

$$= \pm i\pi \sqrt{3E} (2r_0 + 3\omega) \quad (2.13)$$

where $K_+(r)$ is outgoing and $K_-(r)$ is incoming solutions of radial part and the $K_\pm(r)$ values stem from the first order poles of the complex integral. The total imaginary part of the action is $\text{Im} S(t, r, \phi) = \text{Im} K_\pm(r) = \text{Im} K_+(r) - \text{Im} K_-(r) [30, 47]$. Hence, the two kind
probabilities of crossing the outer horizon, from outside to inside or from inside to outside, are given by

\[
P_{\text{out}} = \exp \left[ -\frac{2}{\hbar} ImK_{+} (r) \right],
\]
\[
P_{\text{in}} = \exp \left[ -\frac{2}{\hbar} ImK_{-} (r) \right].
\]

(2.14)

From the Eq.(2.13), we find that \( ImK_{+} (r) = -ImK_{-} (r) \). And, the tunneling probability of the Dirac particle from the outer event horizon is given by \([43, 45, 46]\),

\[
\Gamma = \frac{P_{\text{out}}}{P_{\text{in}}} = \exp \left[ -\frac{2\pi E (2r_{0} + 3\omega)}{\hbar\sqrt{3}r_{0}} \right]
\]

(2.15)

If one expands the classical action in terms of the particle energy, the Hawking temperature is obtained at the lowest order (linear order). So, we can write

\[
\Gamma = e^{-\frac{\hbar}{2} ImS} = e^{-\beta E}
\]

(2.16)

where \( \beta \) is the inverse temperature of the outer horizon. Where, the Hawking temperature is given as follows

\[
T_{H} = \frac{\hbar\sqrt{3}}{2\pi} \left( \frac{r_{0}}{2r_{0} + 3\omega} \right).
\]

This result consistent with the previous results [41].

3 Tunnelling of Scalar Particles

The scalar field \( \Psi (t, r, \phi) \) is represented by the Klein-Gordon equation. In the curved spacetime, the Klein-Gordon equation is given as follows,

\[
\frac{1}{\sqrt{-g}} \partial_{\mu} \left[ \sqrt{-g} g^{\mu\nu} \partial_{\nu} \right] \Psi (t, r, \phi) = \frac{m_{0}^{2}}{\hbar^{2}} \Psi (t, r, \phi),
\]

(3.1)

where \( m_{0} \) is mass of a scalar particle, \( \hbar \) is Planck’s constant, and \( g \) is the determinant of the metric tensor given in Eq.(1.1). To study the quantum tunnelling of scalar particles from the Warped-AdS_3 Black hole, we assume an ansatz for the solution in a form that is similar to Eqs.(2.9) as,

\[
\Psi (t, r, \phi) = A \exp \left( \frac{i}{\hbar} S (t, r, \phi) \right),
\]

(3.2)

where \( A \) is a constant and \( S (t, r, \phi) \) is the classical action term for the outgoing trajectory. Now Substituting equation Eq.(3.2) into Eq.(3.1) and ignoring the small terms of \( \hbar \) as multiplicator via semi-classical approximation, we obtain the Hamilton-Jacobi equation in the following way

\[
F (r) \left( \frac{\partial S}{\partial t} \right)^{2} - 2F (r)^{2} N^{\phi} (r) \left( \frac{\partial S}{\partial t} \right) \left( \frac{\partial S}{\partial \phi} \right) + \left( F (r)^{2} N^{\phi} (r)^{2} - N (r)^{2} \right) \left( \frac{\partial S}{\partial \phi} \right)^{2} - N (r)^{4} F (r)^{4} \left( \frac{\partial S}{\partial r} \right)^{2} + N (r)^{2} F (r)^{2} m_{0}^{2} = 0.
\]

(3.3)
Because of \( \partial_t \) and \( \partial_\phi \) are Killing vectors of the Warped-AdS\(_3\) black hole, we can assume the following separation of variables for the classical action as a solution of the Eq.\((3.3)\),

\[
S(t, r, \phi) = -Et + j\phi + W(r) + C.
\]

Here \( E \) and \( j \) are the energy and angular momentum of the scalar particle, respectively, and \( C \) is a complex constant. We are only considering radial trajectories for fixed \( \phi = \phi_0 \). Using this assumption in Eq.\((3.3)\), after some simplification, we get

\[
W_{\pm}(r) = \pm \int \frac{\sqrt{F(r)^2 E^2 - F(r)^2 N(r)^2 m_0^2}}{F(r)^2 N(r)^2} dr = \pm i\pi \sqrt{3E(2r_0 + 3\omega)}
\]

Here \( '+' \) and \( '-' \) are representing the outgoing and incoming trajectories of the tunnelling scalar particles, respectively, and the \( W_{\pm}(r) \) values stem from the first order poles, as are the complex integral of the \( K_{\pm}(r) \). The tunnelling probabilities of crossing the horizon from inside to outside and outside to inside given by the Eq.\((2.15)\). This means that the probability of the scalar particle tunnelling from inside to outside the horizon is

\[
\Gamma = \exp \left[ -\frac{4}{\hbar} ImW_+(r) \right] = \exp \left[ -\frac{2\pi E(2r_0 + 3\omega)}{\hbar \sqrt{3r_0}} \right],
\]

which is the same result for both Dirac particles: particle and anti-particle. Accordingly, the hawking temperature is also the same,

\[
T = \frac{\hbar \sqrt{3}}{2\pi} \left( \frac{r_0}{2r_0 + 3\omega} \right).
\]

4 Tunnelling of the Particles in the Extremal Case

Extremal black hole solutions have an important role in the black-hole thermodynamics. The common idea in the literature is that the Hawking temperature of an extremal black hole vanishes because the surface gravity is zero according to the classical method. On the other hand, according to analogies between the thermodynamics and the Black hole dynamics laws, the surface gravity of a black hole can not be reach to zero (The third law of black-hole dynamics) \[48\]. In this section, we use the Hamilton-Jacobi method to show that the tunnelling event may occur in the extremal case. Therefore, quantum mechanically, the extremal black hole has a temperature and hence a surface gravity.

Warped-AdS\(_3\) Black hole becomes extremal in the case, \( r_0 = 0 \). If we repeat the calculations for this case, the Dirac and scalar particles’ tunnelling probability and Hawking temperature are calculated as follows, respectively,

\[
\Gamma = \exp \left[ -\frac{4\pi E}{\hbar \sqrt{3}} \right]
\]

and

\[
T_H = \frac{\hbar \sqrt{3}}{8\pi}
\]
where all of the contribution to the tunnelling probability and the temperature stem from the second order pole of the complex integral in Hamilton-Jacobi method. And also, if we apply the Hamilton-Jacobi method to more special case ($\omega = r_0 = 0$) which is characterized the ground state of the black hole, we find the same Hawking temperature. These results show that the Warped-AdS$_3$ Black holes are radiated in both extremal and ground state. As we have shown above, according to Hamilton-Jacobi method, the particles could be tunnelled from the singularity. The Hawking temperature of this radiation becomes a constant in planck scale and, as the result of this, the block hole has a surface gravity of $\kappa = \sqrt{\frac{3}{8}}$. These results are in agreement in regards to the analogies between the thermodynamics laws and the Black hole dynamics laws [48].

5 Summary and Conclusion

In this study, we have studied Hawking radiation of fermion and scalar particles as a quantum tunnelling effect from the Warped-AdS$_3$ Black holes. By using Hamilton-Jacobi method, we have derived the tunneling probability of the relativistic particles (fermions and scalar) from the Warped-AdS$_3$ Black holes. Subsequently, using the obtained these particle tunnelling probabilities, we have calculated the Hawking temperature for the black hole. These results are consistent with surface gravity method based previous works.

We have also examined the particle tunneling whether it is possible for the extremal cases of the black hole or not. Our results show that the extremal Warped-AdS$_3$ black holes may radiate both Dirac fermion and scalar particles. Hawking temperature of these radiations are $T_H = \frac{\hbar}{4\pi M}$. We infer from the quantum mechanical result that the extremal black hole has a surface gravity which is, classically, not predicted. From the Hawking temperature and surface gravity relation, we get the surface gravity, $\kappa = \sqrt{\frac{3}{4}}$. The $\kappa$ expression can be interpreted as quantized ground state surface gravity of the extremal Warped-AdS$_3$ black hole.

Another interesting example for an extremal black hole is the Reissner-Nordström (RN) black hole where $M = Q$, $M$ and $Q$ being mass and charge of the black hole, respectively, [43]. For an extremal RN black hole despite the presence of a horizon at $r = M$, the surface gravity is seem to have vanished, hence, the Hawking temperature is zero. However, if we apply the quantum mechanical tunnelling process of the charged fermions and charged scalar particles to extremal RN black hole case, we obtain a non-zero temperature. From the Eq.(2.8), the tunnelling probabilities of the both charged fermions and charged scalar particles from the extremal RN black hole are same and given as $\Gamma = \exp\left[-\frac{4\pi M}{\hbar}\right]$ where $\omega_0$ is the energy of the particle. By using this result and Eq.(2.16), the temperature of the extremal RN black hole is obtained as, $T_H = \frac{\hbar}{4\pi M}$. This result is twice the result obtained by Kerner for $n = 0$ [43]. On the other hand, the tunnelling probabilities of the neutral particles (both fermion and scalar) vanish, hence, the temperature is zero and the extremal RN black hole emits only charged particles. We conclude that an incipient extremal black hole emits particles that are associated with the Hawking temperature.

All of these results show that the classical surface gravity is in accordance with Hawking temperature calculated from the imaginary part of the complex integral with the first order pole in the Hamilton-Jacobi method. The classical surface gravity becomes zero in case the complex integral pole is second order, but the particles keep tunneling from the extremal black holes, i.e. the extremal black hole has a Hawking temperature thus it has a surface gravity, quantum mechanically. Although the Hamilton-Jacobi method predicts a surface
gravity for an extremal black hole it, unfortunately, terminates the spin effect of the particles to the results. Therefore, each particle, no matter what their spins are, probe a black hole in same way [30, 44].

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