Is a generalized NJL model the effective action of massless QCD?

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A local and gauge invariant alternative version of QCD for massive fermions introduced in previous works, is considered here to just propose a theory which includes Nambu-Jona-Lasinio (NJL) terms in its defining action in a renormalizable form. The Lagrangian includes a special kind of new vertices which at first sight, look as breaking power counting renormalizability. However, these terms also modify the quark propagators, to become more decreasing that the Dirac propagator at large momenta, indicating that the theory is renormalizable. Therefore, it follows the surprising conclusion that the added NJL four fermions terms does not break renormalizability. The approach, can also be interpreted as a slightly generalized renormalization procedure for massless QCD, which seems able to incorporate the mass generating properties for the quarks of the NJL model, in a renormalizable way. The structure of the free propagator, given by the subtraction between a massive and a massless Dirac one in the Lee-Wick form, also suggests that the theory retains unitarity, if the radiative corrections make the massless quarks become non propagating. The appearance of finite masses in the theory is justified by the fact that the new action terms break chiral invariance. The scheme looks as being able to implement the Fritzsch Democratic Symmetry breaking approach to quark mass hierarchy. It seems also possible to further link the theory with the SM after employing the Zimmermann’s coupling constant reduction scheme in a similar way as the Top quark condensation model had been approximately reformulated as a Higgs field one.

Determining the origin of the wide range of values spanned by the quark masses, and more generally, the structure of the lepton and quark mass spectrum, is one of the central problems of High Energy Physics. We have considered a previous research activity associated to this question. It was motivated by the suspicion about that the large degeneration of the non-interacting massless QCD vacuum (the state which is employed for the construction of the standard Feynman rules of PQCD) in combination with the strong forces carried by the QCD fields, could be able to generate a large dimensional transmutation effect. This effect in turns, could then imply the generation of quark and gluon condensates and masses. The investigation of quark and gluon condensation effects had been widely considered in the literature [1–8]. Our previous works on the theme appeared in references [9–17]. Assumed that the idea in them is correct, the following picture could arise. A sort of Top condensate model might be the effective action for massless QCD. In this case, a Top quark condensate, arising within the same inner context of the SM, could play the role of the Higgs field. Thus, the SM could be “closed” by generating all the masses within its proper framework. We could imagine this effect to occurs as follows (see the figure 1 for illustrating the argue): In a first step, the six quarks could get their masses thanks to a flavor symmetry breaking determined by the quark and gluon condensates. Afterwards, the electron, muon and tau leptons, would receive their intermediate masses thanks to radiative corrections mediated by the mid strength electromagnetic interactions with quarks. Finally, the only weak interacting character of the three neutrinos with all the particles, could determine their even smaller mass values.

Is it is clear, that such a picture will need to satisfy the many experimental constraints imposed by the strong experimental evidence about the validity of SM model which had came from the LHC. However, we have no apriori sufficiently appealing reasons to discard the already obtained indications about the possibility for the idea to be correct.

Before starting to present the proposal of the model, let us briefly review the previous results which motivated it. In Ref. [10], with the use of a BCS squeezed state like vacuum state (formed with nearly zero momenta gluons and ghost particles) modified Feynman rules for massless QCD were derived. The case of gluon condensation in the absence of quark pair condensation was initially considered. Then, a proper selection of the parameters allowed to derive an addition to the gluon free propagator: a Dirac’s Delta function centered at zero momenta multiplied by the metric tensor. Such a term were before discussed by Munczek and Nemirovsky in [2]. Before, in reference [9] it was simply proposed this modification and used to argue that it predicts a non vanishing value of the gluon condensate in the first corrections. The physical state and zero ghost number conditions were also imposed to fix the parameters of the squeezed vacuum. Then, the results obtained for gluons motivated the idea of also considering the quarks as massless and to search for the possibility to generate their masses dynamically, thanks to the condensation of quark pairs. For this purpose the BCS like initial state was generalized in reference [11] to include the quark pair condensates in massless QCD. In this case, in a similar way as for gluons, the quark propagators simply were modified again by the addition of a term being the product of a Dirac’s Delta function at zero momentum and the spinor identity matrix. Next, in Refs. [11, 12] the main conclusion obtained from this starting approach followed from a simple discussion of the Dyson equation for quarks. It was considered by taking the quark self-energy in its lowest order in the power
expansion in the condensate parameters. The coefficient of the zero momentum Delta function was fixed to reproduce the estimate of the gluonic Lagrangian mean value, following from the sum rules approaches. After that, the solution of the Dyson equation was able to predict the “constituent” values of 1/3 of the nucleon mass for the light quarks. The initial approach was also further studied in [13, 14] in order to define a regularization scheme for eliminating the singularities which could appear in the Feynman diagram expansion due to the Delta functions at zero momentum entering in the modified propagators.

However, due to the unusual characteristics of the approach, we decided in reference [15, 16] to also investigate the possibility of re-expressing the condensation effects in the modified propagators as equivalent vertices in the Lagrangian. The result of this work was the central step in leading to the model here presented. It was obtained that the condensate effects introduced in massless QCD by employing a squeezed state as modified vacuum, were equivalent to the addition of a new four legs vertex term in the Lagrangian, including two gluon and two quark lines. But, the vertex was a non local one including a zero momentum delta function. The obtained vertex structure clearly was not a fully gauge invariant one, but this drawback can be understood as due to the simple non gauge invariant form employed for the squeezed state vacuum. However, the curious structure directly led to the idea of constructing a local and gauge invariant form of the theory including a similar kind of two gluon and two quark vertices, presumably incorporating the gluon and quark condensates in a gauge invariant and local form. This modification was presented in reference [17], where it was also argued that the new terms of the action do not break the power counting renormalizability of massless QCD. This observation, then led to a surprising conclusion also exposed in reference [17], that the Nambu-Jona-Lasinio (NJL) four fermion vertices turn out to become renormalizable counterterms of the considered Lagrangian. The resulting theory included an additional set of six fermion fields showing a negative metric. However, the modified quark propagator also showed a rapidly decaying momentum dependence thanks to its Lee-Wick structure [19] which suggests that the negative metric states could result to be non propagating thanks to the radiative corrections [20]. The mentioned properties opened the opportunity that the proposed models can show the mass generation properties which the NJL models exhibit. The study of this possibility will be a main objective of the extensions of the this work.

Here, the previous results will be employed to motivate and propose a local and gauge invariant form of QCD including the special two gluons and two quark vertices and NJL four fermion terms and being power counting renormalizable. In section II, firstly we present the argument which allowed to represent the quark and gluon condensation effects in the previous discussion as a two-gluon-two-quark vertices in the massless QCD action. This section is a review of the results in [16]. Section III continues by resuming the construction done in [17] of a local and gauge invariant Lagrangian for massless QCD, by promoting the non local and gauge non-invariant action derived in the previous section to be local and gauge invariant. The given action expression included both gluon and quark condensate effects. However, in the next Section IV, as it was done in reference [17], the gluon condensation effects, which are assumed to contribute more importantly to low energy effects and confinement, are initially disregarded for to

FIG. 1: A qualitative illustration (the mass ratios are not correct ones) of the particle mass spectrum. Note the similar mass increasing behavior with the Family number for quarks and leptons. In addition, the reduction of the mass of the electron, muon and tau leptons with respect to the quarks, could be suspected to be due to the only electromagnetic strongest interaction of these particles. Further, the even smaller mass scales of the three neutrinos could be originated in the only weakly interacting character of them.
be considered separately. In this case the action following was simply the massless QCD one plus six additional terms, one for each flavor, of similar vertices formed by products of two quark and two covariant derivatives. Further, Section V defines a final form for the here proposed model which in addition includes the NJL four quark vertices, which are now allowed by the power counting renormalizability. There is also defined the specific notations to be employed in the extensions of this work. Section VI simply presents a qualitative argue suggesting that the proposed model can result to be unitary thanks to its similarities with the Lee-Wick theories. Next, Section VII also briefly exposes the possibilities that the discussed perturbative expansion can generate a quark mass hierarchy though a flavor symmetry breaking effect. Finally the Summary resume the content of the work and pose some remarks on its possible implications.

I. FROM MODIFIED PROPAGATORS TO A MODIFIED LAGRANGIAN

In this section we will describe how in reference [10], the gluon and quark condensate parameters appearing in the propagators, were "shifted" to appear as new couplings in a modified action for massless QCD. The Feynman diagram expansion of the modified theory was defined by the generating functional of Green functions having the form:

\[ Z[j, \eta, \xi, \bar{\eta}, \bar{\xi}] = \frac{I[j, \eta, \xi, \bar{\eta}, \bar{\xi}]}{I[0, 0, 0, 0]}, \]

\[ I[j, \eta, \xi, \bar{\eta}, \bar{\xi}] = \exp(V^{int} \frac{\delta}{\delta j} \frac{\delta}{\delta \eta} \frac{\delta}{\delta \xi} \frac{\delta}{\delta \bar{\eta}} \frac{\delta}{\delta \bar{\xi}}) \times \]

\[ \exp(\int \frac{dk}{(2\pi)^4} j(-k) \frac{1}{2} D(k) j(k)) \times \]

\[ \exp(\int \frac{dk}{(2\pi)^4} \eta(-k) G_j(k) \eta(k)) \times \]

\[ \exp(\int \frac{dk}{(2\pi)^4} \xi(-k) G_{gh}(k) \xi(k)), \]

\[ f = 1, 2, \ldots, 6. \] (2)

This functionals is associated to the action \( S_g \) depending on the gauge interaction coupling \( g \), and \( S_0 \), which is \( S_g \) for \( g = 0 \), defines the free action. The action \( S_g \) and the vertex part Lagrangian \( V^{int} \) are then defined in terms of the six quark \( \Psi_f \) (\( f = 1, \ldots, 6 \)), gluons \( A \) and ghost \( \chi \) fields in the usual massless QCD form

\[ S_g = \int dx \left( -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a - \frac{1}{2\alpha} \partial_\mu A_\mu^a \partial_\nu A_\nu^a + \bar{\psi}_f \gamma^\mu D_\mu \psi_f - \sum_f \bar{\psi}_f i\gamma_\mu D^{ij}_\mu \psi_f \right), \] (3)

\[ V^{int} = S_g - S_0, \] (4)

where the field intensity and covariant derivatives follow the Euclidean conventions

\[ F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - gf^{abc} A_\mu^b A_\nu^c, \]

\[ D^{ij}_\mu = \partial_\mu \delta^{ij} + ig A_\mu \gamma^{ij}, \quad D^{ab}_\mu = \partial_\mu \delta^{ab} + gf^{abc} A_\mu^c, \] (5)

\[ \{\gamma_\mu, \gamma_\nu\} = -2\delta_{\mu\nu}, \quad [T_a T_b] = i f^{abc} T_c. \]

The \( D, G_f \) and \( G_{gh} \) are the mentioned in the Introduction modified gluon, quark and ghost propagators including condensate effects:

\[ D^{ab}_{\mu\nu}(k) = \delta^{ab} \left( \frac{1}{k^2} (\delta_{\mu\nu} - (1 - \alpha) \frac{k_\mu k_\nu}{k^2}) \theta_N(k) + C_g \delta^D(k) \delta_{\mu\nu} \right), \]

\[ G^{ij}_f(k) = \delta^{ij} \left( \frac{\theta_N(k)}{m + \gamma_\mu k_\mu} + C_f \delta^D(k) I \right), \]

\[ G^{ab}_{gh}(k) = \delta^{ab} \frac{\theta_N(k)}{k^2}. \] (6)

Note, that the propagators are basically the same Feynman’s ones of PQCD, after adding Dirac delta functions at zero momentum \( \langle 2 \rangle \) that represent the gluon and quark condensates. They also include the Heaviside functions,
introduced by Nakanishi to solve difficulties in the quantization of the free gauge theory \[21\]. These functions make the Feynman contribution to the propagator vanish in a small neighborhood of the zero value of the momentum. As mentioned before, their consideration allowed in reference \[13\] to get rid of various singular contributions to the Feynman expansion that could have been appeared, due to the distributional character of the Dirac delta function terms in the propagators.

Then, in reference \[14\], for the purpose of “shifting” the gluon and quark condensate parameters to appear in vertex terms within a modified action for massless QCD, it was employed the property that the “condensates” corrections to the propagators define quadratic forms in the zero momentum component of the spatial Fourier transforms of the sources. Then, the exponential of those terms in the generating functional \(Z\) were expressed as Gaussian integrals over auxiliary vector and fermion parameters, not functions. This change transformed in linear ones the dependence of the Feynman integrands defining \(Z\) on the sources which are contracted with the condensates dependent part of the propagators. Then, after evaluating the commutator of an exponential having an argument being linear in the sources, the generating functional \(Z\) was written in the form

\[
Z[j, \eta, \bar{\eta}, \xi, \bar{\xi}] = \frac{1}{N} \int \int d\bar{\chi} d\chi \exp[-\sum_{f} \chi_{j}, r \chi_{j}, r - \frac{\alpha_{u} \alpha_{a}}{2}] \\
\exp[S_{g}^{\ast} \delta \eta \delta \bar{\eta} - \delta \bar{\eta} \delta \xi - \delta \xi \delta \alpha, \chi, \bar{\chi}] \times \\
\int \mathcal{D}[A, \bar{\Psi}, \Psi, \bar{\tau}, \tau, c] \exp[\int dx \ (j(x)A(x) + \sum_{f} (\bar{\eta}_{f}(x)\Psi_{f}(x) + \bar{\Psi}_{f}(x)\eta_{f}(x)))] \\
= \frac{1}{N} \int \int d\bar{\chi} d\chi \exp[-\sum_{f} \chi_{j}, r \chi_{j}, r - \frac{\alpha_{u} \alpha_{a}}{2}] \times \\
\int \mathcal{D}[A, \bar{\Psi}, \Psi, \bar{\tau}, \tau, c] \exp[\int dx \ (S_{g}^{\ast}[A, \Psi, \bar{\Psi}, c, \bar{\tau}, \tau, \alpha, \chi, \bar{\chi}] + \\
 j(x)A(x) + \sum_{f} (\bar{\eta}_{f}(x)\Psi_{f}(x) + \bar{\Psi}_{f}(x)\eta_{f}(x)) + \bar{\xi}(x)c(x) + \bar{\tau}(x)\xi(x))] ,
\]

where the changed action \(S_{g}^{\ast}\) had the form

\[
S_{g}^{\ast} = S_{g}^{\ast}[A, \Psi, \bar{\Psi}, c, \bar{\tau}, \tau, \alpha, \chi, \bar{\chi}] \\
= \int dx [ - \frac{1}{4} F_{\mu\nu}^{a}(A) (\frac{2C_{g}}{(2\pi)^{2}})^{\frac{1}{2}} \alpha_{\mu}^{a} ) F_{\mu\nu}^{a}(A) (\frac{2C_{g}}{(2\pi)^{2}})^{\frac{1}{2}} \alpha_{\mu}^{a}) \\
- \frac{1}{2C_{g}} \partial_{\mu} A_{\nu}^{a} \partial_{\nu} A_{\mu}^{a} - c^{\alpha} \partial_{\mu} D_{\mu}^{ab}(A) (\frac{2C_{g}}{(2\pi)^{2}})^{\frac{1}{2}} \alpha_{\mu}^{a} ) c^{b} ) \\
- \sum_{f} (\bar{\Psi}_{f} \gamma_{\mu} D_{\mu}^{ij}(A) \chi_{f} (\frac{C_{f}}{(2\pi)^{2}})^{\frac{1}{2}} + (\frac{C_{f}}{(2\pi)^{2}})^{\frac{1}{2}} \gamma_{\mu} D_{\mu}^{ij}(A) \Psi_{f} \\
+ (\frac{C_{f}}{(2\pi)^{2}})^{\frac{1}{2}} \gamma_{\mu} i g_{\mu} T_{a}^{ij} \chi_{f} )] .
\]

This was the expression which directly suggested the local form for QCD proposed in reference \[17\]. Thus, let us resume below the elements from the above relations leading to the proposal. Firstly, note that the auxiliary boson and fermion parameters \((\alpha_{\mu}^{a} \) and \(\chi_{f}^{j}\) appearing thanks to the Gaussian integral representations of the condensate dependent sources terms, are constants independent of the space-time coordinates. As already pointed out before, this property was a direct consequence of the simple perturbative modifications of the free QCD vacuum employed to connect the interaction in the construction of the Wick expansion. Therefore, all the space time derivatives of these parameters in the above formula, in fact vanish, which makes that the expression can not be explicitly seen as corresponding to a quantized gauge invariant theory. This represents a conceptual limitation of the version of the modified massless QCD before considered. However, the above written form of the action suggests a direct possibility to overcome this problem. It consists in simply modifying this action to become gauge invariant and local as suggested by the various gauge invariant constitutive elements entering its structure. The reasonable character of this idea to modify the action, comes from the fact that the obtained generating functional was derived, by considering very particular forms of the initial modified free vacuum states incorporating gluon and quark condensates. Therefore, it can be conceived that the gradual connection of the interaction (or a better constructed initial state) could eventually
lead to the below simply to be proposed gauge invariant form of the action. In the next section, this construction will be reviewed.

II. THE LOCAL FORM OF THE MODEL

Then, in this section we will resume the proposal done in reference [17] of a local and gauge invariant QCD Lagrangian including gluon and quark condensate effects in a way resembling the former study. The construction started by “promoting” the constant space-time independent condensate parameters to be full space-time dependent functions. Further, the new action (without considering the Fadeev-Popov gauge fixing terms) was taken as the same as in the previous modified QCD, but in which now the new gauge and fermion fields \( \alpha, \chi \) fields transform in a homogeneous way under the same gauge transformation leaving invariant the massless QCD action. The generating functional expression resulted in the form

\[
Z[j, \eta, \chi, \xi] = \frac{1}{\mathcal{N}} \int \mathcal{D}[\alpha, \chi, \chi] \exp \left[ - \sum_{\Psi} \mathcal{L}_{\text{matter}}(x, r(x)) + \frac{\alpha^2(x, \alpha^2(x))}{2} \right] \times \int \mathcal{D}[A, \Psi, \chi, \chi] \exp \left[ \int dx \left( S_0^*[A, \Psi, \chi, \chi, \chi] \right) \right] \]

where now, the local \( S_0^* \) action is defined as

\[
S_0^* = S_0^*[A, \Psi, \chi, \chi] = \int dx \left[ -\frac{1}{4} F_{\mu
u}^a(A + \frac{2C_g}{(2\pi)^4})^2 \alpha^a_{\mu
u} + \frac{2C_g}{(2\pi)^4} \alpha^a_{\mu
u} \right]
\]

Note that now, the covariant derivatives over the new auxiliary fields \( \chi, \chi \) include the spatial derivatives which were absent in the former action. The bosonic spatially dependent field \( \alpha \), entered in similar way that a gauge background field perturbation: that is, transforming by the homogeneous part of the gauge transformation. It should be also noticed that the action terms corresponding to the Fadeev-Popov gauge fixation had been adopted in the usual Lorentz gauge.

It can be remarked, that in the case of the vanishing quark condensate parameters, we expect that the expansion can lead to interesting physical consequences in the low energy region around 1 GeV, were a similar discussion in the previous non local expansion gave predictions for the constituent quark masses of the light fermions and reproduced the Savvidy chromomagnetic effective potential form, as a function of the gluon condensate parameter [11]. Further, an interesting circumstance is the fact that the appearance of Gaussian integration over the new auxiliary fields directly suggests a possible link with the so called "stochastic vacuum" vacuum approach initiated by Dosch [22]. However, in the rest of this article we will only study the case of the vanishing gluon condensation parameter \( C_g \). It will be considered first by the mentioned suspicion about the possibilities for quark mass generation in the proposed model.
III. THE VANISHING GLUON CONDENSATE LIMIT

As remarked before, in reference [17], seeking to simplify the discussion in a first stage, the limit of vanishing gluon condensate parameter was initially assumed in [9]. Then, the $Z$ functional reduced to

$$Z[j, \eta, \bar{\eta}, \xi, \bar{\xi}] = \frac{1}{N} \int \int \mathcal{D}[\alpha, \chi, \bar{\chi}] \exp \left[ - \sum_f \int_a \chi^f (x) \chi^f (x) - \frac{\alpha^a_\mu(x) \alpha^a_\mu(x)}{2} \right] \times$$

$$\int \mathcal{D}[A, \bar{\Psi}, \Psi, c, \bar{c}, \chi, \bar{\chi}] \exp \left[ \int dx \left( S^g[A, \bar{\Psi}, \psi, c, \bar{c}, \chi, \bar{\chi}] + j(x)A(x) + \sum_f \left( \bar{\psi}_f (x) \psi_f (x) + \bar{\psi}_f (x) \psi_f (x) \right) + \bar{\chi}(x) c(x) + \bar{\chi}(x) \xi(x) \right) \right],$$

where the action adopts the simpler form

$$S^g = S^g[A, \bar{\Psi}, \psi, c, \bar{c}, \chi, \bar{\chi}]$$

$$= \int dx \left[ -\frac{1}{4} F^a_{\mu \nu} F^a_{\mu \nu} - \frac{1}{2\alpha} \partial_\mu A^a_\mu \partial_\nu A^a_\nu - \bar{c} \partial_\mu D^a_{\mu \nu} c^b \right]$$

$$- \sum_f \left[ \bar{\psi}_f j^i \chi^i + \bar{\psi}_f j^i \chi^i \left( \frac{C_f}{(2\pi)^2} x^i + \left( \frac{C_f}{(2\pi)^2} x^k \right) \bar{\chi}_f j^i \chi^i \right) \right],$$

which is basically the massless QCD action plus two linear terms in the new fermion fields $\chi$ and $\bar{\chi}$. Now, the Gaussian integral over the auxiliary functions were evaluated by solving the Lagrange equations, that after substituting the solutions for the fields in the expression for $Z$ led to

$$Z = \frac{1}{N} \int \mathcal{D}[A, \bar{\Psi}, \psi, c, \bar{c}, \chi, \bar{\chi}] \exp[S[A, \bar{\Psi}, \psi, c, \bar{c}, \chi, \bar{\chi}]],$$

$$S[A, \bar{\Psi}, \psi, c, \bar{c}, \chi, \bar{\chi}] = S_{mqcd}[A, \bar{\Psi}, \psi, c, \bar{c}, \chi, \bar{\chi}] + S^q[A, \bar{\Psi}, \psi, c, \bar{c}, \chi, \bar{\chi}]$$

$$S_{mqcd}[A, \bar{\Psi}, \psi, c, \bar{c}, \chi, \bar{\chi}] = \int dx \left[ -\frac{1}{4} F^a_{\mu \nu} F^a_{\mu \nu} - \frac{1}{2\alpha} \partial_\mu A^a_\mu \partial_\nu A^a_\nu - \sum_f \bar{\psi}_f j^i \chi^i \left( \frac{C_f}{(2\pi)^2} x^i + \left( \frac{C_f}{(2\pi)^2} x^k \right) \bar{\chi}_f j^i \chi^i \right) \right],$$

where the action $S_{mqcd}$ is the usual one for massless QCD and as in Ref. [17], new six action terms appeared, one for each quark flavor $f$. They have the expressions

$$S^q[A, \bar{\Psi}, \psi, c, \bar{c}, \chi, \bar{\chi}] = - \sum_f \frac{C_f}{(2\pi)^2} \int dx \bar{\psi}_f j^i \chi^i \left( \frac{C_f}{(2\pi)^2} x^i + \left( \frac{C_f}{(2\pi)^2} x^k \right) \bar{\chi}_f j^i \chi^i \right).$$

These new action terms are local and gauge invariant and most relevantly, also they do not disturb power counting renormalizability, because they also make the quark propagator to decrease with the square of the momentum.

The four legs vertex is the local counterpart of the non local one appearing in the previous expansion, which exactly represent massless QCD on a modified free vacuum. The pure gluon and ghost one loop diagrams are the same as in massless QCD because the gluon propagators is the same as in usual massless QCD. The evaluated in [17] additional contributions to the one loop polarization operator became transversal, thus satisfied the Ward identity associated to the gauge invariance.

In ending this section, it should be remarked, that the action terms written in [18] are central elements in the present work. There is one of such terms for each quark flavor and they introduce six dimensional parameters which are their coupling constants. Also, they are precisely corresponding to the quark condensate parameters of the former approach discussed in [16], when the auxiliary fields are assumed as space-time independent constants. Their relevance comes from the fact that they do not disturb the power counting renormalizability of massless QCD, thanks to the fact that they modify the original Dirac’s propagator to show a more rapidly decreasing behavior at large momenta. This special property, after taking into account the need of renormalizing the theory, with its central procedure of determining the appropriate counterterms, opens the surprising opportunity of consider the standard Nambu-Jona-Lasinio action terms as appropriate counterterms. This inclusion in the normal case is completely excluded due to slowly decreasing behavior of the Dirac’s propagator. The described outcomes are employed in next section to propose a definite model for a local and gauge invariant QCD including NJL terms in a renormalizable way.
IV. A QCD MODEL INCLUDING NJL TERMS IN A RENORMALIZABLE FORM

The action of the here proposed model is written in concrete the form:

\[ S = \int dx \left( -\frac{1}{4} F^a_{\mu\nu}(x) F^{a\mu\nu}(x) - \frac{1}{2\alpha_x} \partial_\mu A^a_\mu(x) \partial^{\mu} A^a_\mu(x) + c^a(x) \partial_\mu D^{abh} c^b(x) - \sum_f \bar{\Psi}_f(x) \gamma^\mu D^f_\mu \Psi^f(x) - \sum_f \kappa_f \bar{\Psi}_f(x) \gamma_\mu \tilde{D}^f_\mu \gamma_\nu D^{ik\nu} \Psi^f(x) + \sum_{f_1, f_2, f_3, f_4} \Lambda_{f_1 f_2 f_3 f_4} \bar{\Psi}_f^{f_1}(x) \gamma_\mu \tilde{D}^f_\mu \gamma_\nu D^{ik\nu} \Psi^f(x) \Psi^{f_2}_f(x) \Psi^{f_3}_f(x) \Psi^{f_4}_f(x) \right), \]

(17)

where the index \( k \) is the color one \( k = 1, 2, 3 \) and the spinor indices are hidden to simplify notation, \( f \) indicates the flavor of the quarks. It should be underlined that the main different elements in this action with respect to the massless QCD, are the presence of the two last terms and the change in the sign of the Dirac Lagrangian. The last term is the added Nambu-Jona-Lasinio like four quarks action. The coefficients \( \Lambda_{f_1 f_2 f_3 f_4} \) are assumed to be consistent with all the symmetries of QCD. For bookkeeping purposes, the conventions for the various quantities are defined now as follows

\[ F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu, \]

\[ \Psi^f_\nu(x) \equiv \begin{pmatrix} \Psi_{f,1}^k(x) \\ \Psi_{f,2}^k(x) \\ \Psi_{f,3}^k(x) \\ \Psi_{f,4}^k(x) \end{pmatrix}, \]

(18)

\[ \Psi^{fk}_f(x) = (\Psi^f_k(x))^T s, \]

\[ = \left( (\Psi^f_k(x))^* (\Psi^f_k(x))^* (\Psi^f_k(x))^* (\Psi^f_k(x))^* \right), \]

(19)

where \( f = 1, ..., 6 \) indicates the flavor index. The expressions for the Dirac conjugate spinors and covariant derivatives are

\[ \bar{\Psi}_f(x) = \Psi^{fk}_f(x) \gamma^0, \]

(21)

\[ D^{ij}_\mu = \partial_\mu \delta^{ij} - i g A^a_\mu T^{ij}_a, \quad \tilde{D}^{ij}_\mu = -\partial_\mu \delta^{ij} - i g A^a_\mu T^{ij}_a, \]

(22)

\[ D^{ab}_\mu = \partial_\mu \delta^{ab} - g f^{abc} A^c_\mu, \]

(23)

in which the Dirac’s matrices, \( SU(3) \) generators and the metric tensor are defined in this section in the conventions of reference [23], as

\[ \{ \gamma^\mu, \gamma^\nu \} = 2g^{\mu\nu}, \quad [T_a, T_b] = i f^{abc} T_c, \quad \gamma^0 = \beta, \quad \gamma^j = \beta \alpha^j, \quad j = 1, 2, 3, \]

\[ g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \alpha^j = \begin{pmatrix} 0 & \sigma^j \\ \sigma^j & 0 \end{pmatrix}, \quad j = 1, 2, 3, \]

\[ \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \]

(24)

Other precise definitions and relations for the coordinates are

\[ x \equiv x^\mu = (x^0, \vec{x}) = (x^0, x^1, x^2, x^3), \quad x_\mu = g_{\mu\nu} x^\nu, \quad x^0 = t. \]

As underlined before, a basic new element in the proposed action are the six vertices of the form

\[ -\sum_f \kappa_f \bar{\Psi}_f(x) \gamma_\mu \tilde{D}^f_\mu \gamma_\nu D^{ik\nu} \Psi^f(x), \]

where the six coefficients \( \kappa_f \) will be called “condensate parameters” since they enter in similar ‘positions’ of the parameters appearing in the “motivating” non local vertex derived in the previous work [10]. As before remarked,
these terms play the relevant role in the action of allowing the validity of the power counting renormalizability of the model, even with the inclusion of the four fermion terms. The $\kappa_f$ are the only dimensional parameters in the theory.

The last new element with respect to the massless QCD in the action is the changed sign of the Dirac Lagrangian. The need of this change was discussed in reference [18]. This modification leads to a free propagator which is expressed as usual positive metric Dirac propagator of massive fermions plus a negative metric massless propagators. The usual sign assignment determines that the massive propagator shows negative metric. Since experiments seem to indicate that the massive quarks in QCD should be the ones which should be physically relevant within the model, the negative sign of the Dirac action was imposed. However, it can be remarked that in reference [18] it was also argued that such a change in the sign can be introduced in the same physical action by a change of field and coordinates transformation.

After finding the inverse of the kernel associated with quadratic form in the quark fields, it follows that the quark propagator can be expressed as the difference between a usual massive Dirac propagator and one also usual but massless Dirac one, in the form

$$S_f(p) = \frac{1}{\gamma^{\nu} p^{\nu} - \kappa_f p^2}$$

$$= \frac{1}{\gamma^{\nu} p^{\nu}} - \left( \frac{1}{\gamma^{\nu} p^{\nu} - m_f} \right).$$

The gluon free propagator is the usual one. Then, both propagators behaves as $\frac{1}{p^2}$ and the maximal number of fields in a Lagrangian term is 4, therefore the model is power counting renormalizable. The masses of the massive quarks become just the inverses of the six condensate parameters $\kappa_f, \ f = 1, 2, \ldots, 5, 6$. As it was mentioned, the massive propagator has the appropriate sign corresponding to positive norm states. On another hand, the massless component has the sign related with negative norm states.

The new action terms also create two new vertices in the modified Feynman expansion as depicted in figure 2. The line at the bottom represents the full quark propagator defined by (25).

![FIG. 2: The two new types vertices, one pair for each flavor, determined by the six new vertices. The three legs one, at difference with the usual thee legs vertex in usual QCD, is proportional to a product of two gamma matrices. The two-gluon-two-quark four legs vertex is the local counterpart of the non local vertex representing quark condensate effects obtained in reference. The line represents the free quark propagator defined by (25).](image)

The expectation associated with the approach presented in this work, is that the proposed scheme can incorporate quark condensation effects in a way being able in describing a quark mass hierarchy.

For QCD, assumed to describe Nature, it is currently interpreted that nor gluons or quarks show asymptotic states. Thus, the negative metric of the massless free states seem not be a direct drawback of the model. However, the fact that in very high energy processes, a description based in massive quarks in short living asymptotic states, seems to describe the experiences, suggests that an approach in which the massive quarks have positive norms and massless do not appear thanks to radiative corrections would be convenient. As remarked before, this was the purpose of the change in the sign of the Dirac action. For renormalizing the model, the new two-gluons-two-quarks vertices should be included in the counterterm action of the theory. But, as noted above, additional ones are permitted by the new more decreasing behavior of the quark propagator at large momenta. Thus, it follows that many of the Lagrangian terms which define the NJL models are allowed to be included as counterterms. Henceforth, the mass generation properties embodied in such usually non-renormalizable phenomenological theories, seem that can dynamically work now in the proposed context.
The sum of fourth order terms in the quark fields should be a general expression being invariant under the symmetries of QCD, to allow the cancelation of the divergences. In the normal NJL theory, the usual Dirac propagator, with its “one over the momentum modulus” behavior at large momenta, makes the model non-renormalizable. Here, the one “over modulus of the momentum squared” behavior of the new quark propagators, makes the Feynman expansion power counting renormalizable. This property can be easily seen, by noting that the power counting rules for the proposed model are identical to the ones in the simpler $\lambda\phi^4$ scalar field theory with the usual scalar field propagator $\frac{1}{p^2}$.

One important remark following from the previous discussion should be added here. Let us assume that a renormalization procedure of pure massless QCD is being reconsidered. Then, the new two-gluons-two-quarks vertices, can be identified as possible counterterms for this purpose, since they do not destroy power counting renormalizability. Therefore, the idea comes to the mind that the proposed model could result to be physically equivalent to the quantized massless QCD.

In next section we simply expose an argue suggesting the possibility of the theory to be also unitary. A closer investigation of this property is expected to be considered in the extension of this work.

V. POSSIBLE UNITARITY OF THE S MATRIX IN THE LEE-WICK SENSE.

The scattering among the physical particles of the proposed model should not give rise to states in which unphysical particles exist as described by propagating waves (unitarity). This property is illustrated in the picture at the left: matrix elements of the scattering between a state defined by incident physical states with states in which at least one non-physical particle propagates should vanish. Therefore, it should be checked whether or not the first corrections to the quark propagator will allow the existence of non physical propagating negative metric quark states in the proposed model. That is, showing poles in the momentum squared at positive values (mass squared) for modes of negative metric which in the model are the massless quark fields. The possibility for unitarity to be valid in the theory is strongly suggested by the existence of the so called Lee-Wick theories \[19, 20\]. In them, precisely helped

FIG. 3: An illustration of the unitarity conditions for the S matrix in the Lee-Wick sense.

FIG. 4: The picture illustrates the structure of the propagator in the presented model. It is expressed as the sum of a positive metric massive propagator plus negative metric massless one. This form leads to the expectation that after evaluating the first radiative corrections the propagator will not show poles corresponding to negative metric massless states, at positive momentum squared values.
by the propagator being given as the substraction of two propagators one of negative metric and another of positive metric, as illustrated in figure 1 these theories do not show propagating negative metric states, as a consequence of the radiative corrections to the propagators. Therefore, assumed that the model shows a similar property, it can result to be unitary in the state space of the physical particles in high energy scattering. The validity of this property is expected to be studied in the extensions of the present work.

VI. A QUARK MASS HIERARCHY FROM MINIMIZING THE VACUUM ENERGY?

Finally, it can be noted that the vacuum energy of the model, that is, minus the effective action at vanishing mean fields, is a function of the six masses $m_f$, $f = 1, 2, 3, ..., 6$ (the reciprocal of the condensate parameters $\kappa_f$). This quantity can be expected to show all “the mass generating” properties of the NJL model, which are associated to the “four fermions” vertices entering the new diagram expansion. Therefore, if we consider the evaluation of the energy as a function of only two of those parameters, the fact that the fermion propagators are functions of these two masses, and the existence of two loops “interference between flavors” effective potential diagrams, like the one shown in the figure 5 leads to the possibility that those terms make the energy to rise when the two condensate takes similar values. Such an effect will lead to a flavor symmetry breaking in which one mass could take a larger value with respect to the other. This could be the first step in a hierarchy structure. In this connection it can be also remarked that the two-gluon-two-quark diagrams directly break chiral invariance, thus the appearance of masses is natural within the model. This is an important property which could allow the generation of a flavor symmetry breaking, which requires the existence of a chiral non-invariance as follows from current algebra results [24]. These possibilities will be explored in the extension of this work.

VII. SUMMARY

We reviewed a recently proposed improved version of the modified massless QCD discussed in previous works. Motivated by it, it was proposed an alternative to massless QCD including NJL action terms in a local and renormalizable way. It is also underlined that the analysis done for constructing the proposal suggests its equivalence with massless QCD. The new terms determine masses for all the six quarks which are given by the reciprocal of the new
six flavor condensate couplings linked with each quark type. The approach suggests a possibility to explain the quark mass hierarchy as a dynamical flavor symmetry breaking. In it, the contributions of diagrams showing two kinds of fermion lines might tend to rise the energy of the configurations with equal values of the quark masses, making them more energetic that ones in which a single quark mass parameter gets a finite value. It is interesting to remark that the occurrence of this flavor breaking, in spite of the known need of a chiral symmetry for the appearance of quark condensates which arose from the current algebra analysis, might be allowed by the fact that included two-gluon-two-quark vertices directly break chiral invariance. The considered framework seems appropriate to realize the so called Democratic Symmetry Breaking properties of the mass hierarchy remarked by H. Fritzsch [25]. Finally, it can be also imagined that the appearance of six different couplings in the theory, could be reduced to only one by employing the Zimmermann’s reduction of the couplings approach [26]. This possibility also suggests a way for linking the model with the SM assuming that the single coupling could be expected to play the role of the Higgs field. This property is also suggested by the known results which show that the Top condensate models can be re-formulated in a form being closer to the SM. It can be concluded that the discussion supports the starting idea of the study about that massless QCD could generate an intense dimensional transmutation effect. Its feasibility will be investigated in the extension of this work. In ending, it must be remarked that in the gluodynamic limit (which was not considered here) the appearance of Gaussian means over color fields suggests the possibility of a first principles derivation of the linear confining effects predicted by the stochastic vacuum models of QCD [22].

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