Quantum Depinning Transition of Quantum Hall Stripes

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(Dated: March 22, 2022)

PACS numbers: 73.43.Nq, 73.43.Lp, 73.43.Qt

We examine the effect of disorder on the electromagnetic response of quantum Hall stripes using an effective elastic theory to describe their low-energy dynamics, and replicas and the Gaussian variational method to handle disorder effects. Within our model we demonstrate the existence of a depinning transition at a critical partial Landau level filling factor $\Delta \nu_c$. For $\Delta \nu < \Delta \nu_c$, the pinned state is realized in a replica symmetry breaking (RSB) solution, and the frequency-dependent conductivities in both perpendicular and parallel to the stripes show resonant peaks. These peaks shift to zero frequency as $\Delta \nu \to \Delta \nu_c$. For $\Delta \nu \geq \Delta \nu_c$, we find a partial RSB (PRSB) solution in which there is free sliding only along the stripe direction. The transition is analogous to the Kosterlitz-Thouless phase transition.

There is strong evidence from DC transport experiments that states with stripe ordering form at certain fillings of higher Landau levels ($N \geq 2$). These states exhibit a highly anisotropic, and apparently metallic, conductivity. At low temperatures, as the partial filling of the highest occupied Landau level, $\Delta \nu$, moves away from 1/2, the electrons in this level cease to contribute to the transport properties, and the system behaves in a way typical of the integer quantum Hall effect. One likely interpretation of this change in behavior is that the electrons in the partially filled Landau level become pinned by disorder when $\Delta \nu < \Delta \nu_c$. The nature of this transition is the subject of this work.

Microwave absorption measurements provide additional information about these systems. These experiments probe the dynamical conductivity of the system, $\sigma_{\alpha\beta}(\omega)$, which, in pinned systems, exhibits a peak at a frequency determined by the effective restoring force due to the disorder. Existing data are suggestive of such a peak moving to zero frequency as the transition is approached from below, consistent with qualitative expectations for a quantum depinning transition. Interesting filling factor dependences of this peak have also been observed in Landau levels where a Wigner crystal is presumably pinned.

To examine the possibility of a depinning transition in the stripe state, we calculate the frequency-dependent conductivity using the replica trick and the Gaussian variational method (GVM), first introduced by Mézard and Parisi and further developed by Giamarchi, Le Doussal, and their coworkers. In the replica approach, a pinned state is represented by one replica, and the frequency-dependent conductivities in both perpendicular and parallel to the stripes show resonant peaks. These peaks shift to zero frequency as $\Delta \nu \to \Delta \nu_c$. For frequencies $\omega \geq \omega_c$, we find a partial RSB (PRSB) solution in which there is free sliding only along the stripe direction. The transition is analogous to the Kosterlitz-Thouless phase transition.

An example of our results is illustrated in Fig. 1. A prominent feature of the result is that the peak positions move to zero frequency as $\Delta \nu \to \Delta \nu_c$ from below.

For $\Delta \nu > \Delta \nu_c$, we find a different type of state in which the system is pinned for motion perpendicular to the stripes, but is free to slide along them. We call this a partial replica symmetry breaking (PRSB) state. The PRSB state has a number of striking properties, including a power law dependence of $\Re \sigma_{xx}(\omega) \sim \omega^{-\gamma}$ as a function of frequency with $\gamma$ continuously increasing with $\Delta \nu - \Delta \nu_c$; and a superconducting response at zero frequency in $\Re \sigma_{yy}(\omega)$, followed by an incoherent metallic response at finite frequency. The transition is in the Kosterlitz-Thouless universality class. It exhibits a jump in the low-frequency exponent $\gamma$ at the transition, analogous of the universal jump in the stiffness of a thin-film superfluid and in the critical exponent of correlation functions. The possibility of observing these properties in quantum Hall stripes and other analogous systems is discussed below.

Model and method. We start with an action for an elastic system in a magnetic field to describe the low-energy degrees of freedom of the quantum Hall stripes and their nonlinear coupling to the disorder,

$$S = S_0 + S_{\text{imp}}$$

$$S_0 = \frac{1}{2T} \sum_{q,\omega_n} \sum_{\alpha,\beta=x,y} u_{\alpha}(q, \omega_n) G_{\alpha\beta}^{(0)}(q, i\omega_n) \times u_{\beta}(-q, -\omega_n)$$

$$S_{\text{imp}} = \int dr \int_0^{1/T} d\tau V(r) n(r, \tau),$$

where $u_{\alpha}$ is a displacement degree of freedom, $G_{\alpha\beta}^{(0)}(q, i\omega_n) = D_{\alpha\beta}(q) - \epsilon_{\alpha\beta} \omega_n / l_B^2$ is the inverse Green’s function of $u$ in the pure limit ($\epsilon_{xy} = -\epsilon_{yx} = 1, \epsilon_{xx} = \epsilon_{yy} = 0$), $l_B$ is the magnetic length, and $D_{\alpha\beta}(q)$ is the dynamical matrix. This last quantity is determined.
by matching the electron density-density correlation function obtained from the elastic model with that computed from microscopic time-dependent Hartree-Fock (HF) calculations \[17\]. In the low-energy sector, \(D_{\alpha\beta}(q)\) has a smeared form, \(D_{\alpha x}(q) \approx \frac{\alpha(q) + n_q a_\alpha}{a_\alpha} \), \(D_{\alpha y}(q) = D_{\beta x}(q) \approx \frac{\beta(q) + n_q a_\beta}{a_\beta} \), and \(D_{\beta y}(q) \approx \frac{\beta(q) + n_q a_\beta}{a_\beta} \). We note that alternative estimates of \(D\) were made by using an edge state model for the stripe system [18], which leads to different results than ours. We will comment on this difference below. The disorder potential \(V(r)\) in Eq. (4) is assumed to be Gaussian distributed with zero average, \(\langle V(r)V(r') \rangle = V_0^2 a_x a_y \delta (r - r')\), where \(a_x\) and \(a_y\) are the lattice constants of the stripe crystal. In Eq. (6) \(n(r, \tau)\) is the electron density operator whose Fourier transform we approximate by \[10\] \(n(q, \tau) \approx n_0 \{1 - q \cdot u(q) + \sum_{K \neq 0} K^2 f dt \, e^{-iK \cdot [r - u(q, \tau)] - iq \cdot \tau}\} \), where \(n_0\) is the average electron density and \(K\) is a stripe crystal reciprocal lattice vector. For simplicity, we drop the \(i q \cdot u\) term in \(n(q, \tau)\) since this cannot pin the electron system \[10\], and we keep only the lowest harmonics of the reciprocal lattice vectors, \(K_x = 0, \pm 2\pi/a_x\) and \(K_y = 0, \pm 2\pi/a_y\). These approximations should not qualitatively change our results.

We use replicas and the GVM to handle the disorder effects. This method has been employed with great success in a number of systems \[10, 11\], although some controversy has arisen over previous applications to quantum Hall systems, for reasons that do not apply to our present study \[13, 19\]. We introduce \(n\) replicas of the system where eventually we will take \(n \rightarrow 0\). This involves setting \(S_0 \rightarrow S_{\text{eff}}^{(0)} = \sum_{a=1}^{n} S_0^{(a)}\) with \(a\) the replica index; averaging over disorder introduces coupling among the replicas in the form \(S_{\text{imp}}^{(0)} \approx -t_{\text{imp}} \sum_{a=1}^{n} \int_0^{1/T} d\tau_1 d\tau_2 \int dt \sum_{K \neq 0} \cos \{K \cdot [u^a(r, \tau_1) - u^a(r, \tau_2)]\}\), with \(t_{\text{imp}} = \frac{\hbar^2}{2m a_x a_y^2}\). The GVM \[3\] consists of replacing \(S_0^{(a)} + S_{\text{imp}}^{(a)}\) with a variational Gaussian action, characterized by variational Green’s function \(G_{\alpha\beta}^{ab}(q, \omega_n)\) which is chosen to minimize the free energy. This leads to a set of saddle point equations (SPE’s). It is convenient to parameterize the matrix \(G\) in terms of a self-energy matrix \(\Sigma\), such that \((G^{-1})_{\alpha\beta}^{ab}(q, \omega_n) = G_{\alpha\beta}^{(0)}(q, \omega_n) - \Sigma_{\alpha\beta}^{ab}(\omega_n)\). Note that we can safely assume \(\Sigma\) has no \(q\) dependence because none emerges in the SPE’s. We also set \(\Sigma_{xy} = \Sigma_{yx} = 0\) since it is a valid solution of the SPE’s, and preserves reflection symmetry.

Our SPE’s are a natural generalization of those found using the replica and GVM on isotropic systems \[10, 11\]. Here, we will present only the final equations, deferring details to a future publication \[20\]. After taking the limit \(n \rightarrow 0\), the self-energy is characterized by a replica diagonal component, \(\Sigma_\alpha^{\alpha}(\omega_n) \rightarrow \bar{\Sigma}_\alpha^{\alpha}(\omega_n)\) and an off-diagonal function, \(\Sigma_\alpha^{\beta}(\omega_n) \rightarrow \bar{\Sigma}_\alpha^{\beta}(\omega_n, 0)\), where \(0 < u < 1\). It is the first of these that directly enters the frequency-dependent conductivity, while the second of these determines whether the system is in a replica symmetric state (constant \(\bar{\Sigma}(u)\)), a RSB state (\(\bar{\Sigma}(u)\) varying with \(u\)) or some other state. The dynamical conductivity is obtained \[10\] via

\[
\sigma_{\alpha\beta}(\omega) = (e^2/a_x a_y) i \omega \bar{G}_{\alpha\beta}^{\text{ret}}(q = 0, \omega),
\]

with \(\bar{G}_{\alpha\beta}^{\text{ret}}(q, \omega) = \bar{G}_{\alpha\beta}(q, \omega_n \rightarrow \omega + i\delta)\), where \(\bar{G}_{\alpha\beta}\) is the \(n \rightarrow 0\) limit of \(G_{\alpha\beta}^{(0)}\). The self energy \(\bar{G}_{\alpha\beta}^{\text{ret}}(\omega)\) is the analytic continuation of \(\bar{\Sigma}_\alpha^{\alpha}(\omega_n)\), which satisfies the following SPE’s \[20\]:

\[
\bar{\Sigma}_\alpha^{\text{ret}}(\omega) = e_\alpha - \left[ F_\alpha(\omega) - F_\alpha(0^+) \right],
\]

\[
F_\alpha(\omega) = 4t_{\text{imp}} \sum_{K \neq 0} K^2 \int_0^\infty dt \, e^{iK \cdot y - e^{i(\omega - |q|\omega_n)\tau}} - \int_0^\infty d\tau \langle u^\dagger u \rangle,
\]

where \(e_\alpha = \int_0^\infty du \, \bar{\Sigma}_\alpha^{\alpha}(u) - F_\alpha(0^+)\) and \(\bar{\Sigma}_\alpha^{\beta}(\omega_n)\) is chosen to minimize the free energy. This constraint also can be justified by requiring marginal stability of the replica model \[10\]. The second constraint can be found from the SPE’s for \(\bar{\Sigma}_\alpha^{\alpha}(u)\) and \(\bar{\Sigma}_\alpha^{\beta}(\omega_n)\) with the assumption of a one-step RSB, which is a common solution in low-dimensional systems \[10, 11, 12\]. This leads after some work \[20\] to the condition

\[
e_{g}/e_x = \sum_{K \neq 0} K^2 e^{-W(K)}/[ \sum_{K \neq 0} K^2 e^{-W(K)}],
\]

where \(W(K) = \frac{1}{\pi} \sum_{\mu} K^2 f_{\mu}^\infty df A_\mu(f)\) is Debye-Waller factors. These play a prominent role in the depinning transition: as we shall see, \(W(K)\) diverges whenever \(K\) has a component along the \(\bar{y}\) axis as \(\Delta \tau_c\) is approached from below, leading to a suppression of \(e_y\). This behavior cannot be captured by the semiclassical approximation.

**Results - RSB solution.** We have solved numerically Eqs. \[5\] and \[6\] along with the two constraints using an
the conductivity appears to tend toward a non-vanishing value when $\Delta \nu$ is sufficiently below $\Delta \nu_c$. The quantity $e_y$ turns out to be rather small due to a large Debye-Waller factor, and in this frequency range the system displays a behavior similar to the incoherent metal response at non-vanishing frequency of the depinned (PRSB) phase which we discuss below. For $\omega \ll e_y$, $\text{Re} \sigma_{yy}(\omega)$ vanishes quadratically with $\omega$ (not visible on the scale of Fig. 1), as required for a pinned state. As $\Delta \nu \to \Delta \nu_c$, we eventually reach a situation in which $e_y$ and $\Omega_{py}$ are of similar order, in which case the pinning peak sharpens and grows quite large. This peak continuously evolves into a $\delta$-function at zero frequency as the system enters into the PRSB state, so that the transition from pinned to depinned behavior is very continuous.

**Results – PRSB solution.** For $\Delta \nu \geq \Delta \nu_c$ the state is characterized by $e_x \neq 0$ but $e_y = 0$. This corresponds to a RSB solution for $\zeta_y(u)$ but a replica symmetric solution for $\zeta_y(u)$. We call this the partial RSB (PRSB) state. In this situation, the system is pinned perpendicular to the stripe direction, but is free to slide along it. This is consistent with the results from a perturbative renormalization group study of the same model [4], where the coupling of the disorder to the motion of the system parallel to the stripes was shown to be irrelevant when $\Delta \nu$ is close enough to $1/2$. This irrelevance suggests that the PRSB phase should be in a superconducting state. This observation is born out by the presence of a $\delta$-function in $\text{Re} \sigma_{yy}$ at zero frequency. Remarkably, $\text{Re} \sigma_{xx}$ vanishes at zero frequency, so that we find the PRSB state is one with an infinite anisotropy in the DC conductivity. This is not observed in DC transport experiments [3], and we comment below on what is missing from our model that leads to this discrepancy. The possibility of such behavior for quantum Hall stripes was first suggested in Ref. [14].

The origin of the PRSB and its structure may be understood from the SPE [4]. The state is characterized by $e_x \neq 0$ and $\text{Im} \zeta_{xx}^\text{ret}(\omega) \sim \omega$ at small $\omega$, and $\zeta_{yy}^\text{ret}(\omega)$ vanishing faster than linearly in $\omega$. It is easy to show in this situation, $A_y(f) \sim 1/f$ at small $f$. After some algebra, we find [21] that when

$$\gamma = \frac{a_y l_B}{a_x} \int dq_x \frac{d_{xx} - c_{xx}}{\sqrt{|d_{xx} - c_{xx}| d_{yy} - d_{xy}^2}} \geq 1, \quad (8)$$

a self-consistent solution of Eq. (6) emerges, with $\text{Re} \zeta_{yy}^\text{ret}(\omega) \sim \omega^2$ and $\text{Im} \zeta_{yy}^\text{ret}(\omega) \sim \omega^{\gamma+1}$. The large value of this last exponent leads both to a power law dependence of $\text{Re} \sigma_{xx} \sim \omega^{-\gamma}$ and the $\delta$-function response at zero frequency in $\text{Re} \sigma_{yy}$. It is also easy to show that $\text{lim}_{\omega \to 0} \text{Re} \sigma_{yy} = \text{const.} > 0$, so that, at low but finite frequency the response is metallic. This combination of infinite and “incoherent” metallic response is typical of superconductors [21].

In the vanishing disorder limit, the minimum value of $\gamma$ from the formula (8) for which we can obtain a PRSB solution agrees precisely with the condition found

![Image](https://via.placeholder.com/150.png?text=FIG. 1: Real parts of conductivities as functions of frequency in the pinned state (a) perpendicular to the stripes, (b) along the stripes. The $h = 1$ unit and $v_{imp} = 0.0005e^4/l_B^2$ are used. In (a) all curves start from $\text{Re} \sigma_{xx} = 0$ at $\omega = 0$, and curves except for $\Delta \nu = 0.36$ are lifted upward for clarity. Curves from right to left in (b) correspond to $\Delta \nu = 0.36, 0.38, 0.4, 0.42, 0.43, 0.44, 0.45, 0.452, 0.454$, respectively. Inset in (b): peak positions $\Omega_{px}, \Omega_{py}$, in units of $e^2/l_B$, as functions of $\Delta \nu$. In what follows we present some results for electrons in the $N = 3$ Landau level, with a disorder level $v_{imp} = 0.0005e^4/l_B^2$. This is likely to be somewhat larger than experimental values, but we choose it for numerical convenience and do not expect our results to qualitatively change with smaller disorder strengths. The dynamical conductivities for these parameters when the system is in a pinned (RSB) phase are presented in Fig. 1. For $\Delta \nu$ well below $\Delta \nu_c \approx 0.46$, $\text{Re} \sigma_{xx}$ has a pinning peak whose lineshape is qualitatively similar to what is found using the semiclassical approximation [12]. The prominent behavior visible in Fig. 1 (a) is the collapse of the peak frequency $\Omega_{px} \to 0$ as the depinning transition is approached. Experimental observations are so far consistent with this [4, 6]. $\text{Re} \sigma_{yy}$ also has a collapsing peak, but the observed lineshape is more interesting. Below the peak frequency $\Omega_{py}$, in the range $e_y < \omega < \Omega_{py}$...
in Ref.\textsuperscript{7} for pinning along the stripes to become irrelevant. One remarkable consequence of this limiting value is that $\text{Re}\sigma_{xx} \sim \omega^\gamma$ with $\gamma \to 1$ as the transition is approached from above, whereas just below the transition we expect, in the pinned state, $\text{Re}\sigma_{xx} \sim \omega^2$. Thus, the low frequency exponent \textit{jumps} at the transition, in a way that is analogous to the universal jump in the superfluid stiffness and the critical exponent of correlation functions of the Kosterlitz-Thouless transition \textsuperscript{10}. This behavior is similar to what happens in the roughening transition \textsuperscript{17,22}.

In real DC transport experiments, one observes a finite anisotropy rather than the infinite one found in the PRSB state. The missing ingredients from our model are processes allowing hopping of electrons between stripes. These processes are very difficult to incorporate into an elastic model. It is clear that, if relevant, such processes can broaden the $\delta$-function response to yield anisotropic metallic behavior. Our results should apply at frequency scales above this broadening. Indeed, microwave absorption experiments become quite challenging at low frequencies, and it is unclear whether existing measurements of the dynamical conductivity can access the low frequency conductivity in the unpinned state, whether or not it is broadened. In any case, it is interesting to speculate that a true $\delta$-function response might be accessible in structured environments where barriers between stripes may suppress electron hopping among stripes \textsuperscript{23}, or that there may be analogous states for layered 2+1 dimensional classical systems of long string-like objects, which has been shown \textsuperscript{15} to be closely related to the two-dimensional quantum stripe problem.

The quantum depinning transition we find is unlikely to occur in models which preserve particle-hole symmetry (PHS) at $\Delta \nu = 1/2$ \textsuperscript{18}. Our model overcomes this limitation because the HF state we use spontaneously breaks PHS at this filling to arrive at a lower energy state than the simpler “box-filled” state \textsuperscript{2} which has been used in the edge state description of the quantum Hall smectic \textsuperscript{15}. It is at present unclear if quantum fluctuations restore PHS to the quantum Hall smectic at $\Delta \nu = 1/2$. Our results offer a falsifiable experimental test that can settle this question.

\textbf{Acknowledgements.} The authors are especially grateful to R. Lewis, L. Engel, and Y. Chen for many stimulating discussions about this problem, and for showing us their experimental data prior to publication. We are also indebted to G. Murthy, E. Orignac, E. Poisson, and A.H. MacDonald for useful discussions and suggestions. This work was supported by a NSF Grant No. DMR-0108451, by a grant from the Fonds Qu\'{e}b\'{e}cois de la recherche sur la nature et les technologies and a grant from the Natural Sciences and Engineering Research Council of Canada, and by a grant from SKORE-A program.

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