Thermal Recombination: Beyond the Valence Quark Approximation

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Quark counting rules derived from recombination models agree well with data on hadron production at intermediate transverse momenta in relativistic heavy-ion collisions. They convey a simple picture of hadrons consisting only of valence quarks. We discuss the inclusion of higher Fock states that add sea quarks and gluons to the hadron structure. We show that, when recombination occurs from a thermal medium, hadron spectra remain unaffected by the inclusion of higher Fock states. However, the quark number scaling for elliptic flow is somewhat affected. We discuss the implications for our understanding of data from the Relativistic Heavy Ion Collider.

The production of hadrons with transverse momenta of a few GeV/c in Au+Au collisions at the Relativistic Heavy Ion Collider (RHIC) has been found to exhibit certain unusual features. The emission of baryons, in comparison with mesons, is significantly enhanced. For example, the ratio of protons to pions is about three times larger in p+p collisions.

Furthermore, the nuclear suppression factor $R_{AA}$ below 4 GeV/c is close to unity for protons and Λ-hyperons, while it is about 0.3 for pions. The azimuthal anisotropy (called “elliptic flow” $v_2$) of baryons lags behind that of mesons at low transverse momenta but exceeds the anisotropy of meson emission above 2 GeV/c. The recombination of quasi-thermal, deconfined quarks has been proposed as an explanation for these peculiarities. These recombination models are based on the concept of constituent quark recombination, which assumes that the probability for the emission of a hadron from a deconfined medium is proportional to the probability for finding the valence quarks of the hadron in the density matrix describing the source. The baryon enhancement, as well as the different momentum dependence of meson and baryon anisotropies, rely essentially on the different number of valence quarks in mesons (two) and baryons (three). The simplicity of this concept has been criticized, because it does not do justice to the complexity of the internal structure of hadrons in quantum chromodynamics (QCD). It is the purpose of this article to address some of these objections by showing that important features of the emission of relativistic hadrons by sudden recombination from a thermal deconfined medium do not depend on specific assumptions about their internal structure and survive in a more complete description of the emitted hadrons.

The quark-gluon content of a hadron in QCD, as that of a bound state in any strongly interacting quantum field theory, is a function of the resolution scale $Q^2$. The larger the scale $Q^2$ is, the more quantum fluctuations can be seen, and the number of gluons and sea quarks in a hadron increases. Moreover, the structure of a hadron depends on the frame of reference. This is so because the boost operator $K$ and the Hamiltonian $H$ generally do not commute: $[K, H] \neq 0$. Furthermore the quark-gluon structure depends on the measurement process, i.e. it is not universal. Different processes probe the Fock states in a hadron with different weights.

In the following we will use a light-cone frame, where formally the hadron momentum $P \rightarrow \infty$ and the momentum fractions of the partons are the only dynamic degrees of freedom. A meson $M$ with valence quarks $q_\alpha$ and $q_\beta$ can then be written as an expansion in terms of increasingly complex Fock states:

$$|M\rangle = \int_0^1 dx_a dx_b \delta(x_a + x_b - 1)c_1(x_a, x_b) |q_\alpha(x_a) \bar{q}_\beta(x_b)\rangle + \int_0^1 dx_a dx_b dx_c \delta(x_a + x_b + x_c - 1)c_2(x_a, x_b, x_c) |q_\alpha(x_a) \bar{q}_\beta(x_b) q_\gamma(x_c)\rangle + \int_0^1 dx_a dx_b dx_c dx_d \delta(x_a + x_b + x_c + x_d - 1)c_3(x_a, x_b, x_c, x_d) |q_\alpha(x_a) \bar{q}_\beta(x_b) q_\gamma(x_c) q_\delta(x_d)\rangle + \ldots$$

Hence $x_i P$ is the momentum of parton $i$. We have suppressed the momentum $P$ in the notation for simplicity.
For convenience, we adopt a probabilistic normalization for partons: $(q_a(x_a))|q_3(x_h)\rangle = \delta_{a,3}\delta(x_a - x_h)$. This implies that the completeness relation for the Fock space expansion takes the simple form:

\begin{align*}
1 &= C_1 + C_2 + C_3 + \cdots \\
&= \int_0^1 dx_a dx_b \delta(x_a + x_b - 1) |c_1(x_a, x_b)|^2 \\
&+ \int_0^1 dx_a dx_b dx_c \delta \left( \sum_i x_i - 1 \right) |c_2(x_a, x_b, x_c)|^2 \\
&+ \cdots
\end{align*}

A pure valence quark description would only include the first term, without accounting for its $Q^2$-dependence, which is governed by renormalization group equations coupling $c_1$ to $c_2$, $c_2$ to $c_1$ and $c_3$, and so forth. Generally speaking, for low values of $Q^2$, i.e. $Q^2 < 1 \text{ GeV}^2$, and most hadrons, with the possible exception of pions, the valence quark state will be the largest term in the expansion, but the contributions from other terms may not be negligible.

This goes hand in hand with a change of the effective quark mass. While the assumption of quasi free partons with current quark masses less than 10 MeV for light flavors is valid for $Q^2 > 1 \ldots 2 \text{ GeV}^2$, we know from lattice calculations and other considerations that the dynamical quark mass $m(Q^2)$ increases rapidly for $Q^2 < 1 \text{ GeV}^2$ and approaches a constituent mass of order 300 MeV [12, 13, 14].

For the sudden recombination of medium constituents into a fast hadron, the relevant scale is determined by the average thermal momenta in the medium. The precise value of $Q^2$ depends on the process that is being considered, but generally the scale falls into the range $(\pi T)^2 \leq Q^2 \leq (2\pi T)^2$, where $T$ is the temperature of the medium [12]. For a quark-gluon plasma near the point of hadronization ($T_\text{c} \approx 170 \text{ MeV}$), this implies that the relevant scale $Q$ is in the range $0.5 \ldots 1 \text{ GeV}$. It is therefore natural to assume that the correct degrees of freedom for recombination act like constituent quarks and that the recombination probability is dominated by the lowest Fock states.

Nevertheless, even for very massive constituents, higher Fock states should be present, even though we can not calculate their contribution from first principles. We want to discuss in the following, how an admixture of higher Fock states alters the recombination formalism.

In the Boltzmann approximation, the probability for finding a quark (or gluon) with momentum $k = xP$ and energy $E_k$ in the medium is given by:

\begin{equation}
W_q(x) = \langle q(x)|\hat{\rho}|q(x)\rangle = e^{-E_k/T} = e^{-x P/T}
\end{equation}

where $\hat{\rho}$ denotes the thermal density matrix and masses are assumed to be much smaller than $Pc$. Hence for a state with $n$ quarks or other partons we have

\begin{equation}
\langle q(x_a)q(x_b)\ldots|\hat{\rho}|q(x_a)q(x_b)\ldots\rangle \\
= W_q(x_a)W_q(x_b)\ldots = e^{-(x_a+\ldots+x_n+\ldots)P/T}
\end{equation}

The emission probability for the single Fock space component of a meson then is

\begin{align*}
W_{q\bar{q}} &= \int_0^1 dx_a dx_b \delta(x_a + x_b - 1)|c_1(x_a, x_b)|^2 \langle q(x_a)\bar{q}(x_b)|\hat{\rho}|q(x_a)\bar{q}(x_b)\rangle = C_1 e^{-P/T}, \\
W_{q\bar{q}g} &= \int_0^1 dx_a dx_b dx_c \delta(x_a + x_b + x_c - 1)|c_2(x_a, x_b, x_c)|^2 \langle q(x_a)\bar{q}(x_b)g(x_c)|\hat{\rho}|q(x_a)\bar{q}(x_b)g(x_c)\rangle = C_2 e^{-P/T},
\end{align*}

Combining the contributions from all Fock space components, the probability for emission of a hadron with momentum $P$ is given by:

\begin{equation}
W(P) = W_{q\bar{q}} + W_{q\bar{q}g} + W_{q\bar{q}gg} + \cdots = e^{-P/T}
\end{equation}

where we applied the normalization condition [8]. Our result shows that the probability of relativistic emission of a complex state by recombination from a thermal ensemble does not depend on the degree of complexity of the state. Per spin-flavor degree of freedom the emission of a baryon with momentum $P$ is as likely as the emission of a meson with the same momentum, as long as the particle masses are negligible compared with $P = |P|$. 

We call this property the “egalitarian” nature of recombination from a thermal ensemble.

Let us next explore how the elliptic flow of hadrons is affected by the presence of higher Fock states in their wavefunction. Here we assume that there are no space-momentum correlations affecting the calculation and we limit our discussion to sufficiently small values of the elliptic flow parameter \( v_2 \), so that nonlinear corrections to the additivity rule of the flow of the constituents can be safely neglected.

In the saturation regime, the relationship is more complicated and the scaling law is apparently violated by the contribution from a thermal ensemble. Here it is important to be in a light cone frame where the masses of the particles can be neglected.

In the hydrodynamic regime, where \( v_2(k) = ak \), one finds again trivially \( v_2^{(H)}(P) = aP \) using \( \alpha \). The elliptic flow of all hadrons then follows the same universal line. In the saturation regime, the relationship is more complicated and one may suspect that higher Fock states alter the \( v_2 \) of the hadron.

For the derivation of the familiar scaling law for elliptic flow it is assumed that the wave function of the hadron is narrow: all partons in a Fock state carry roughly equal momentum \( x_i \approx 1/n_\nu \), where \( n_\nu \) is the number of partons. The scaling law follows when only the lowest Fock state with \( n_1 \) partons is taken into account.

\[
v_2^{(H)}(P) = n_1 v_2(P/n_1)
\]

The experimental data is described very well by this equation \( \alpha \). We can easily generalize \( \beta \) to higher Fock states in the limit of a very narrow wave function \( \delta(x_1 - 1/\nu) = C_\nu^{(M)}(M)x_1 - 1/\nu) = \delta(x_1 - 1/\nu). \) Then

\[
v_2^{(H)}(P) \approx \sum_\nu C_\nu n_\nu v_2(P/n_\nu)
\]

and the scaling law is apparently violated by the contributions from higher Fock states. In principle, this violation should be visible in a scaling analysis. The data are usually plotted with scaled axes \( P_T/n_1 \) and \( v_2/n_1 \), where \( n_1 = 2.3 \) is the valence quark number for the hadron. Equation \( \gamma \) implies that the scaled elliptic flow for mesons and baryons, respectively, is

\[
v_2^{(M)}(p) = \sum_\nu C_\nu^{(M)} n_\nu^{(M)} v_2 \left( \frac{2p}{n_\nu^{(M)}} \right)
\]

\[
v_2^{(B)}(p) = \sum_\nu C_\nu^{(B)} n_\nu^{(B)} v_2 \left( \frac{3p}{n_\nu^{(B)}} \right).
\]

Clearly, if all \( C_\nu = 0 \) except the lowest Fock states, for which \( C_1^{(M)} = 1 \) and \( C_1^{(B)} = 1 \), the scaled elliptic flow curve is the same for mesons and baryons. This is what has been found in the data and has been interpreted as evidence that the scaled curve reflects the partonic elliptic flow before hadronization: \( \tilde{v}_2^{(M)}(p) = \tilde{v}_2^{(B)}(p) = v_2(p) \).

Can higher Fock states be present despite of the scaling law holding? We notice that for states with the same number \( s \) of sea quarks or gluons the ratio of prefactors \( n_\nu/n_1 \) for mesons and baryons in \( \delta \) is

\[
\frac{3s + 2}{2s + 3}
\]

and the ratio of prefactors inside the argument, multiplying the momentum \( p \) is the inverse of that. This ratio is always between 1 for the pure valence state and 3/2 for states with an infinite number of sea quarks and gluons. This is not a large variation, and there are two other effects that are also important. First, we note that the prefactors in front of \( v_2 \) and the momentum \( p \) in each line of \( \epsilon \) tend to cancel if \( v_2 \) is a rising function. Second, with increasing number of sea quarks and gluons the average momentum of each parton decreases. For large Fock states we expect the partons to be in the hydrodynamic regime where \( v_2 \) is a linear function and scaling holds for arbitrary parton numbers.

How large can the scaling violation be expected to be in practice? In order to present a numerical estimate, we consider the lowest Fock state and the next higher one, with an additional gluon, for mesons and baryons respectively. In focusing on the one-gluon admixture, we assume that the weights of higher Fock states, including two or more additional gluons or one or more quark-antiquark pairs, die out rapidly with growing complexity. For this analysis we abandon the simplistic \( \delta \)-function shaped wave functions. We use the following more realistic light-cone distribution amplitudes for mesons as guidance:

\[
C_1^{(M)}(x_a, x_b) = \frac{\sqrt{C_1^{(M)}} x_a x_b}{2\sqrt{35}},
\]

\[
C_2^{(M)}(x_a, x_b, x_g) = \frac{\sqrt{C_2^{(M)}} x_a x_b x_g^2}{12\sqrt{10010}};
\]

and for baryons:

\[
C_1^{(B)}(x_a, x_b, x_c) = \frac{\sqrt{C_1^{(B)}} x_a x_b x_c}{4\sqrt{330}},
\]

\[
C_2^{(B)}(x_a, x_b, x_c, x_g) = \frac{\sqrt{C_2^{(B)}} x_a x_b x_c x_g^2}{1680\sqrt{4862}}.
\]

Here the variable \( x_1 \) denotes the light-cone momentum fraction of the additional gluon in the hadron wave function. The model wave functions on the right-hand side of \( \delta \) and \( \epsilon \) are standard forms of higher twist distributions for hadrons \( \gamma \). The additional kinematic factors
on the left-hand side arise because of our non-standard probabilistic normalization of the states.

The constants $C_i$ give the relative normalization of the two Fock states. In the following, we shall consider two cases: (i) lowest Fock state only ($C_1 = 1, C_2 = 0$) and (ii) moderate $|qar{q}g\rangle$ or $|qqqg\rangle$ contribution ($C_1 = 0.7, C_2 = 0.3$). It is important to recognize that even in the first case, although valence quark scaling is well realized, the scaled elliptic flow function $\tilde{v}_2^{(H)}$ of the hadrons differs from the input function $v_2$ describing the quark flow. This is illustrated in Fig. 1 which shows the scaled elliptic flow of mesons and baryons (dotted lines) together with the quark flow (solid line). The difference between the parton and scaled hadron flow is due to the smearing by the internal hadron wave functions. Figure 2 shows the scaled hadron elliptic flow in comparison with the scaled STAR data [4] for neutral kaons and $\Lambda$ hyperons. The quark flow function was chosen so that the theoretical values obtained by use of Eq. (8) fit the data. The figure demonstrates that a single input curve for quarks can describe the elliptic flow of both, mesons and baryons.

Figures 3 and 4 show the same quantities for the case (ii), i.e. for hadron wavefunctions with a 30 percent component containing an additional gluon.
component containing an additional gluon. We assume that the elliptic flow for the gluons is the same as that for light quarks. Note that the parton elliptic flow has been slightly adjusted to yield a good fit to the data again for this case. As anticipated, the scaling is not as good as for the lowest Fock state alone but, as Fig. 3 demonstrates, the scaling is only modestly violated. However, we also expect deviations to start in this region because fragmentation starts to replace recombination as the dominant hadronization mechanism. The present precision of the data, as seen in Fig. 4, is insufficient to observe the deviations from the scaling law and a gluon contribution of 30% is compatible with the data.

Since the parton elliptic flow can be adjusted after including higher Fock states, the real test is the systematic deviation between baryons and mesons. In Figure 5 we show the relative difference \((\tilde{v}_2^{(B)} - \tilde{v}_2^{(M)}) / (\tilde{v}_2^{(B)} + \tilde{v}_2^{(M)})\) between the scaled meson and baryon elliptic flow for three different sizes of the higher Fock state component (0%, 30%, 50%). In all cases, baryons have a slightly larger scaled \(\tilde{v}_2\) than mesons at small momenta. This effect is likely to be overwhelmed by the influence of mass differences, which have been neglected in the sudden recombination model. At larger momenta, the scaled meson \(\tilde{v}_2\) is slightly larger. As stated, these differences are well within present experimental errors.

In summary, we have investigated the effects of higher Fock states on the recombination of hadrons from thermalized quark distributions. We showed that the yield of relativistic parton clusters is independent of the number of partons in the cluster. Therefore, hadron spectra remain unaffected by the inclusion of higher Fock states.

One important implication is that gluon degrees of freedom could be accomodated during hadronization. They simply become part of the quark-gluon wave functions of the produced hadrons, but remain hidden constituents because the commonly produced hadrons do not contain valence gluons.

On the other hand higher Fock states introduce deviations from the scaling law for elliptic flow. We showed that an additional 30% contribution from gluons is compatible with the existing data on elliptic flow from RHIC. We emphasize that the interpretation that elliptic flow data from RHIC proves the existence of quark degrees of freedom in the bulk matter produced in the heavy ion collision is still valid. However, the connection of the measured elliptic flow to the quark elliptic flow might be less straightforward than anticipated.

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