Proton-air collisions in a model of soft interactions at high energies

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We show that Pomeron interactions generate important corrections to the Gribov-Glauber formula, which is used to extract proton-proton cross sections from proton-air collisions at high energy. We show that these corrections are larger than the errors for proton-air cross sections measured at ultra high energies in cosmic ray experiments. We present a description of these data in our model for soft interactions at high energies, which describes all available accelerator data including that from the LHC.

I. INTRODUCTION

In this letter we examine the problem of hadron-nucleus interaction at high energies. It is well known that in the Gribov-Glauber approach \cite{1,2}, where the total cross section of the hadron-nucleus interaction is expressed through the inelastic cross section of hadron-proton scattering, can only be justified at rather low energies, where the corrections due to Pomeron interactions may be neglected. A more general approach has been developed\cite{3–6} in which the Pomeron interaction has been taken into account in the high energy range for:

\begin{equation}
g_i S_A(b) G_{3\mathcal{P}} e^{A_{\mathcal{P}} Y} \propto g G_{3\mathcal{P}} A^{1/3} e^{A_{\mathcal{P}} Y} \approx 1; \quad G_{3\mathcal{P}}^2 e^{A_{\mathcal{P}} Y} \ll 1. \tag{I.1} \end{equation}

$G_{3\mathcal{P}}$ is the triple Pomeron coupling, $g$ is the vertex of Pomeron nucleon interaction, and $1 + \Delta_{\mathcal{P}}$ denotes the Pomeron intercept. For the nuclear profile $S_A(b)$ we use the general expression

\begin{equation}S_A(b) = \int_{-\infty}^{+\infty} dz \rho(z,b) = \int_{-\infty}^{+\infty} dz \, \frac{\rho_0}{1 + e^{\frac{\sqrt{z^2 + b^2} - R_A}{R}}}; \quad \int d^2 b S_A(b) = A; \tag{1.2} \end{equation}

In the Wood-Saxon parametrization (see the last equation) $\rho_0 = 0.171 (1/fm^3)$ \cite{7}.

In this approach, in order to calculate the hadron-nucleus cross sections, one needs to know the values of $\Delta_{\mathcal{P}}, g$ and $G_{3\mathcal{P}}$.

In this letter we will discuss the non-Glauber-type corrections to hadron-nucleus interaction. Our re-analysis of the problem is based on two recent achievements.

First, the Auger Collaboration has published the measurement of the proton-air total cross section at extremely high energies ($W = 57$ TeV) with sufficiently small errors ($\sigma_{\text{tot}}(p - \text{Air}) = 505 \pm 22(\text{stat}) \pm 28(\text{syst}) \text{ mb}$ \cite{8}).

Second, a model for high energy hadron-hadron scattering has been proposed which successfully described LHC data, and which has included all theoretical ingredients that have been found in QCD and N=4 SYM\cite{9}. The latter allows us to calculate the non-Glauber corrections, and to estimate the influence of these corrections on the value of the proton-air cross section at very high energies.

In the next section we briefly review the main theoretical formulae for hadron-nucleus interactions. In section III we calculate the proton-air cross section using the model, all parameters of which have been fitted from the proton-proton data. In this section we estimate the difference between the Glauber-Gribov approach, and the alternative approach that includes the Pomeron interactions. In the conclusions we summarize our results.
II. HADRON-NUCLEUS COLLISIONS

A. General approach

In the kinematic region of Eq. (I.1), the hadron-nucleus scattering amplitude can be written in an eikonal form in which the opacity \( \Omega \) is given by sum of the 'fan' diagrams (see Fig. 1b).

\[
A_{hA}(Y, b) = i \left( 1 - \exp\left( - \frac{\Omega_{hA}(Y; b)}{2} \right) \right),
\]

with

\[
\Omega_{hA}(Y; b) = \frac{g_h g G_{enh}(Y) S_A(\vec{b})}{1 + g G_{3P} G_{enh}(Y) S_A(\vec{b})}.
\]

For diagrams of Fig. 1, \( G_{enh}(Y) \) is the Green’s function of the Pomeron exchange which is equal to

\[
G_{enh}(Y) = e^{\Delta P Y}.
\]

From Eq. (II.3) and Eq. (II.4), we obtain that

\[
\sigma_{tot}^{hA} = 2 \int d^2b \left( 1 - \exp\left( - \frac{\Omega_{hA}(Y; b)}{2} \right) \right); \\
\sigma_{el}^{hA} = \int d^2b \left( 1 - \exp\left( - \frac{\Omega_{hA}(Y; b)}{2} \right) \right)^2; \\
\sigma_{in}^{hA} = \int d^2b \left( 1 - \exp\left( -\Omega_{hA}(Y; b) \right) \right).
\]

The processes of diffractive production have been discussed in Refs.[5, 10].

B. Main formulae

To describe the experimental data, we replace the Glauber-Gribov eikonal formula by Eq. (II.3) and Eq. (II.4). In addition, we need to adjust this formula to our description of hadron-hadron data given in Ref.[9]. In this model we introduce two additional ingredients that have not been included in Eq. (II.3) and Eq. (II.4).

1) A two channel Good-Walker model[13] which is exclusively responsible for low mass diffraction.
2) Enhanced Pomeron diagrams that lead to a different Pomeron Green’s function. This mechanism provides the main contribution for high mass diffraction.
The model we develop is based on a two channel model which takes into account the Good-Walker mechanism, in which the observed physical hadronic and diffractive states are written in the form

$$\psi_h = \alpha \Psi_1 + \beta \Psi_2; \quad \psi_D = -\beta \Psi_1 + \alpha \Psi_2,$$

where $\alpha^2 + \beta^2 = 1$. Note that Good-Walker diffraction is presented by a single wave function $\psi_D$. The vertex of the interaction of a Pomeron with a nucleon ($g$ in Eq. (II.3) and Eq. (II.4)) has in this model a more complex structure $\tilde{g} = \alpha^2 g^{(1)} + \beta^2 g^{(2)}$. $g^{(k)}$ denotes the vertex of the Pomeron interaction with the state $k$ that have been described by either the wave functions $\Psi_1$ or $\Psi_2$.

In our model [9] we sum all enhanced diagrams for proton-proton scattering (see Fig. 2 and calculated $G_{\text{enh}}(Y)$ in the MPSI approximation [11]. We refer our reader to Ref. [6] for the details of the calculations which lead to

$$G_{\text{enh}}(Y) = \frac{1}{\gamma} \left( 1 - \exp \left( \frac{1}{T(Y)} \right) \frac{1}{T(Y)} \Gamma \left( 0, \frac{1}{T(Y)} \right) \right)$$

where $k_{T,i}$ are the transverse momenta of three Pomerons, and $T(Y)$ is given by

$$T(Y) = \gamma e^{\Delta P Y}$$

The final formula that includes both the Good-Walker mechanism of low mass diffraction production, and the enhanced Pomeron diagrams, takes the following form:

$$\sigma_{\text{in}}(p + A; Y) = \int d^2b \left( 1 - \exp \left( - \left\{ \begin{array}{c} \sigma_{\text{tot}}^{pp} S_A(b) \\ \tilde{g} G_{3P} G_{\text{enh}}(Y) S_A(b) \end{array} \right\} \left( \begin{array}{c} \sigma_{el}^{pp} + \sigma_{dif}^{pp} \\ 1 + \tilde{g} G_{3P} G_{\text{enh}}(Y) S_A(b) \end{array} \right)^2 \right) \right).$$

One can see that for $G_{3P} \to 0$ Eq. (II.10) reduces to Gribov-Glauber formula with one difference: instead of $\sigma_{\text{in}} = \sigma_{\text{tot}} - \sigma_{\text{cl}}$ in the formula we have $\sigma_{\text{in}} = \sigma_{\text{tot}} - \sigma_{\text{cl}} - \sigma_{\text{dif}}$ where $\sigma_{\text{dif}} = 2\sigma_{\text{el}}^{GW} + \sigma_{\text{el}}^{GW}$ as it was advocated in Ref. [14]. Note, that the low mass of both single and double diffraction enters Eq. (II.10). We consider the simplest model for proton-proton interaction in which elastic and diffraction processes are taken into account (see Fig. 8): the total cross section is given as a sum of one and two Pomeron exchanges(see Fig. 8-a and Fig. 8-b). The Pomeron diagrams for this case, are shown in Fig. 8-c.

Table 1 presents the parameters of our model [9] that are needed to calculated the modified Gribov-Glauber formula of Eq. (II.10). For the scattering with air we use $S_{\text{Air}}(b) = 0.87 S_{Ng}(b) + 0.22 S_{O}(b)$ where $Ng$ and $O$ stand for nitrogen and oxygen, respectively. For each of these nuclei we used the oscillatory parametrization, following Ref. [7]. Our estimates will be discussed below.
III. COMPARISON WITH THE EXPERIMENT

Before comparing with the experimental results, we would like to draw the reader’s attention to the fact that some of the experimental results shown might be overestimated, due to the possibility of the airshowers being created by helium nuclei, as well as protons. The importance of this phenomena has been investigated by Block[15] and the Auger collaboration[8]. We refer the reader to these references for further details. All experiments, except Auger, assume a pure proton composition of the projectile. Auger concludes that they have a contamination of about 25% of helium, which produces an uncertainty of about 30 mb (which is less than 10% of their final result), and is included in their systematic error.

The results of our calculations are shown in Fig. 4. One can see that our model describes the Auger and HiRes[16] quite well. In Fig. 4 the two different simplifications of the exact formula of Eq. (II.10) are plotted: Gribov-Glauber formula in which the interaction with the Pomerons are neglected ($G_{3P} = 0$) as well as the cross section of diffraction dissociation is considered to be small ($\sigma_{\text{dif}} = 0$); and our model with $G_{3P} = 0$. The first lesson that we learn is that these three approaches produce results whose difference are larger than the experimental errors of the Auger and HiRes experiments. The second lesson is that our model gives a good description of the data at very high energies ($W = 57\, \text{TeV}$ and $W = 77.5\, \text{TeV}$). Our total proton-proton cross section at $W = 57\, \text{TeV}$ is equal to 130 mb. It is instructive to note that all cosmic ray data lie between our two predictions: our model with $G_{3P} = 0$, and our model with $G_{3P} = 0$. Since the systematic errors are large we cannot exclude the case with $G_{3P} = 0$. However, this case has been eliminated by the LHC experiments where high mass diffraction has been measured.

It should be stressed that our results show that the Pomeron interaction leads to a considerable contribution and cannot be neglected. We would like to stress that our model[9] gives the smallest contribution for Pomeron interactions, in comparison with other attempts to describe the LHC data[21, 22].

Fig. 5 shows our predictions for the total and inelastic cross sections for proton-lead interaction at high energy. One can see that for heavy nuclei the difference between our approach and Gribov-Glauber formula is not large reaching about 11% for total and 5% for inelastic cross sections. It is instructive to note that the inelastic cross section for heavy nuclei is not sensitive to the Pomeron interactions and the major difference from Gribov-Glauber formula stems from Good-Walker mechanism for low mass diffraction in proton-proton collisions.

IV. CONCLUSIONS

In this paper we show that Pomeron interactions generate valuable corrections to Gribov-Glauber formula which is used for the extraction of proton-proton cross sections from proton-air collisions at high energies. Bearing this in mind, we can attempt to use the cosmic ray data at ultra high energies ($W > 1\, \text{TeV}$) to check our knowledge of very high energy phenomenology.

Although the cosmic ray experiments have large errors, we conclude that the Pomeron interaction and low mass diffraction production, have to be taken into account with all needed modifications of Gribov-Glauber formula for proton-air scattering.

Our model which describes all accelerator data at high energies also gives a good description of the cosmic ray data. Therefore, we conclude that our model[9] can be used for ultra high energies.
Our model with $G_{3P} = 0$

- AGASA
- HiRes
- Fly’s-eye
- Knurenko et al.

**FIG. 4**: Comparison the energy dependence of the total cross section for proton-Air interaction with the high energy experimental data. Data are taken from Refs. [16–20].

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Our model

Our model with $G_{3p} = 0$

G-G formula ($G_{3p} = 0; \sigma_{\text{dif}} = 0$)

FIG. 5: Predictions of our model for proton-lead cross sections. In this figure $\sigma$ in barns while $W$ in TeV.

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