Meson Electro-Magnetic Form Factors in an Extended Nambu–Jona-Lasinio model including Heavy Quark Flavors

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Abstract

Based on an extended NJL model including heavy quark flavors, we calculate the form factors of pseudo-scalar and vector mesons. After take into account of the vector-meson-dominance effect which introduce a form factor correction to the quark vector coupling vertices, the form factors and electric radii of $\pi^+$ and $K^+$ of pseudo-scalar meson in light flavor sector fit the experimental data well. Also the magnetic moments of light vector meson $\rho^+$ and $K^{*+}$ are comparable with other theoretical calculation. The form factors in light-heavy flavor sector are presented to confront with future experiments or theoretical calculations.

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I. INTRODUCTION

The Nambu–Jona-Lasinio (NJL) model \cite{1, 2} has been widely used in hadron physics as an effective model to study the chiral symmetry in the degree of quark freedom. Usually, this model dealt with light hadrons composed of only light quark flavors \( u, d, s \) with \( SU_f(3) \) symmetry \cite{3–6}.

In the hadron systems including heavy flavors such as a light-heavy meson, although the chiral symmetry is broken due to the mass of heavy quark, a complementary heavy flavor symmetry emerges and the so-called heavy quark effective theory (HQET) was formulated with the technique of \( 1/m_Q \) expansion \cite{7–14}. In Ref. \cite{15}, the NJL model was extended to include heavy quark flavors to investigate such light-heavy mesons like \( D^{(*)} \) and \( B^{(*)} \) mesons.

In our previous work \cite{16}, we also tried to extend NJL model to comprises heavy flavors by expanding the NJL interaction strengths in the inverse power of constituent quark masses according to HQET. Based on this extension, we obtained the meson masses and meson-quark coupling constants of all light and light-heavy mesons in a unified way. Furthermore, the decay widths of the mesons were calculated from those effective meson quark couplings \cite{17}.

In this work, we will further calculate the electro-magnetic form factors of mesons within this extended model. Electro-magnetic form factors play an important role in our understanding of hadronic structure. The form factors of pseudo-scalar meson \( \pi \) and \( K \) were measured in several experiments \cite{18–20}. In some previous theoretical works, the form factor of \( \pi \) and \( K \) mesons were studied in NJL model \cite{21, 22}. After consider the effect of vector-meson-dominance of the vector mesons, such as the \( \rho \) meson, in the calculation, typically the form factors of \( \pi \) were well fitted to the experimental data. Furthermore, the form factor of \( \pi \) was also studied in case of finite temperature with NJL model \cite{23}.

Certainly, the form factor of \( \pi \) was studied in many other theoretical approaches, such as Dyson-Schwinger equation using a confining quark propagator \cite{24}, light-cone or covariant quark wave functions \cite{25, 26}, lattice QCD method \cite{27, 28}.

The form factors of vector mesons have rather more complicate structure. Consequently, it can provide us more information about vector mesons such as magnetic moments and quadrupole moments. Presently there are only theoretical results about the form factors of vector mesons. Some works used constituent quark model and the light front dynamics.
or Dyson-Schwinger equations. Lattice QCD calculation were performed with the three-point functions method, or the background field method using only two-point functions. The magnetic moments of vector mesons were also calculated by dynamics with the external magnetic field, or QCD sum rules.

There are a few papers studying the form factor of light-heavy mesons. They were focus on the electroweak form factors. From the heavy flavor symmetry, those form factor should be unify described by the Isgur-Weise function when the masses of heavy flavor turn to infinity.

Here, we will make a systematic calculation to the meson form factors, including pseudo-scalar mesons and vector mesons, of both the light flavor sector and the light-heavy flavor sector, within the extended NJL model. In the next section, we will introduce our model and formalism. The numerical result and discussion will be presented in sec. III.

II. MODEL AND FORMALISM

A. Extended NJL model

To deal with both light and heavy mesons in the Nambu-Jona-Lasinio (NJL) mode, in Ref. the four-fermion point interactions are modified to

\[
\mathcal{L}_4^F = G_V (\bar{q} \lambda^a \gamma^\mu q) (\bar{q'} \lambda^a \gamma^\mu q') + \frac{\hbar}{m_q m_{q'}} [(\bar{q'} \lambda^a \gamma^\mu q') (\bar{q} \lambda^a \gamma^\mu q) + (\bar{q} \gamma^\mu \gamma^5 \lambda^a q)(\bar{q'} \gamma^\mu \gamma^5 \lambda^a q')]
\]

(1)

where \( \lambda^a \) are the generator of SU(3) in color space and \( q, q' = u, d, s, c, b \) including both the light and the heavy flavors. Here the second part of the interaction is required to improve spectra of light vector mesons and the factor of \( 1/(m_q m_{q'}) \) guarantees that the symmetry of heavy flavors will still be hold in the heavy quark limit according to HQET.

By solving Bethe-Salpeter equation (BSE), we have obtain meson masses and their coupling constants with quarks. We will use the effective Lagrangian to describe the quark interaction in mesons. In the case of \( \pi \) and \( \rho \), the effective Lagrangian reads

\[
\mathcal{L}_{\pi qq} = - g_{\pi q} \bar{q} i \gamma_5 \tau q \cdot \pi - \frac{\hat{g}_{\pi q}}{2 m_q} \bar{q} \gamma_\mu \gamma_5 \tau \partial^\mu \pi,
\]

(2)

\[
\mathcal{L}_{\rho qq} = - g_{\rho q} \bar{q} \gamma_\mu \tau q \cdot \rho.
\]

(3)

In ref. [17], we have calculate the strong and radiative decays of vector mesons. In this
work, we will use the above effective meson Lagrangian to further calculate form factors of mesons.

B. Form factor of pseudo-scalar meson

The definition of the form factor of a pseudo-meson is given by

$$\langle \pi^+(p_2)|\bar{\psi}\gamma_\mu\psi|\pi^+(p_1)\rangle = (p_1 + p_2)_\mu F(q^2),$$

where $q = p_1 - p_2$ is the transfer momentum. Its Feynmann diagrams are shown in Fig. 1 where $m_1$ and $m_2$ are the masses of the constitute quarks in the pseudo-scalar meson. Using the Feynman rules, the amplitude reads

$$ (p_1 + p_2)_\mu F^{(1)}(q^2) = (p_1 + p_2)_\mu [Q_1 F^{(1)}(q^2) + Q_2 F^{(2)}(q^2)] $$

$$ (p_1 + p_2)_\mu F^{(1)}(q^2) = - \text{Tr} \int \frac{d^4k}{(2\pi)^4} \gamma_\mu S_1(k + p_1)i(g - \tilde{g}_{\frac{\not{p}_1}{m_1 + m_2}})i\gamma_5 S_2(k) $$

$$ \times \frac{i}{g - \tilde{g}_{\frac{\not{p}_2}{m_1 + m_2}}}i\gamma_5 S_1(k + p_2), $$

$$ (p_1 + p_2)_\mu F^{(2)}(q^2) = - \text{Tr} \int \frac{d^4k}{(2\pi)^4} \gamma_\mu S_2(k - p_2)i(g + \tilde{g}_{\frac{\not{p}_2}{m_1 + m_2}})i\gamma_5 S_1(k) $$

$$ \times \frac{i}{g - \tilde{g}_{\frac{\not{p}_1}{m_1 + m_2}}}i\gamma_5 S_2(k - p_1), $$

where $F^{(1)}$ and $F^{(2)}$ are the form factors of quark and anti-quark respectively, $Q_i$ is the electron charge of $i$-th quark,

$$ S_i(p) = \frac{i}{p - m_i + i\epsilon} $$

is the propagator of $i$-th quark, $g$ and $\tilde{g}$ are the coupling constants of pseudo-scalar meson obtained in our previous work [16].

![Feynmann diagrams of meson form factor.](image-url)
In the Breit frame, \( p_0^2 - p_1^0 = 0 \) and \( \mathbf{p}_1 = -\mathbf{p}_2 \). We introduce

\[
\begin{align*}
p_1 &= p + q/2, \\
p_2 &= p - q/2,
\end{align*}
\]  

where \( p \equiv \frac{1}{2}(p_1 + p_2) = (p_1^0, 0) \), \( q = (0, \mathbf{q}) \). Taking the direction of \( z \)-axis along momentum \( \mathbf{p}_1 \), we find

\[
F^{(1)}(q^2) = \text{in}_c n_f \int \frac{d^4k}{(2\pi)^4} \left[ g_2 S_1 - \frac{g\tilde{g}}{m_1 + m_2} S_2 - \frac{g\tilde{g}}{m_1 + m_2} S_3 + \frac{\tilde{g}^2}{(m_1 + m_2)^2} S_4 \right] 
\]  

\[
F^{(2)}(q^2) = F^{(1)} \left( m_1 \leftrightarrow m_2, q \rightarrow -q, p \rightarrow -p \right),
\]

where

\[
\begin{align*}
S_1 &= S_0(m_1, m_1, m_2), \\
S_2 &= m_1 S_0(m_1, (k + p - q/2)^2/m_1, m_2) + m_2 S_0(m_1, m_1, k^2/m_2), \\
S_3 &= m_1 S_0((k + p + q/2)^2/m_1, m_1, m_2) + m_2 S_0(m_1, m_1, k^2/m_2), \\
S_4 &= m_1^2 S_0((k + p + q/2)^2/m_1, (k + p - q/2)^2/m_1, m_2) + k^2 S_0(m_1, m_1, m_2) \\
&\quad + m_1 m_2 S_0((k + p + q/2)^2/m_1, m_1, k^2/m_2) \\
&\quad + m_1 m_2 S_0(m_1, (k + p - q/2)^2/m_1, k^2/m_2) \\
D &= [(k + p + q/2)^2 - m_1^2 + i\epsilon][(k + p - q/2)^2 - m_1^2 + i\epsilon](k^2 - m_2^2 + i\epsilon),
\end{align*}
\]

and

\[
S_0(m_1, m_2, m_3) = 2[m_3(m_1 + m_2) - 2k \cdot (k + p)] + 2\frac{k \cdot p}{p^2} [m_3(m_1 + m_2) - m_1 m_2 + p^2 - k^2 - q^2/4].
\]

Note that the denominator \( D \) of integrand is invariant under transform \( \mathbf{k} \rightarrow -\mathbf{k} \).

The electro-magnetic radius will be further obtained from the derivative of the form factor via

\[
r = \left[ \frac{6}{dF}{dq^2} \right]^{1/2}_{q^2 = 0}.
\]

We have

\[
r = \sqrt{Q_1 r_1^2 + Q_2 r_2^2},
\]

where

\[
r_i = \left[ \frac{6}{dF^{(i)}}{dq^2} \right]^{1/2}_{q^2 = 0},
\]

is the radius of \( i \)-th quark.
C. Form factor of vector meson

The definition of form factor of vector meson reads \[ 39, 40 \]

\[
\langle \rho^+(p_2, \lambda_2) | \bar{\psi} \gamma_\mu \psi | \rho^+(p_1, \lambda_1) \rangle = - \epsilon^*(p_2, \lambda_2) \cdot \epsilon(p_1, \lambda_1)(p_1 + p_2)_\mu F_1(q^2) \\
+ \left[ \epsilon_\mu(p_1, \lambda_1) q \cdot \epsilon(p_1, \lambda_1) - \epsilon_\mu^*(p_2, \lambda_2) q \cdot \epsilon(p_1, \lambda_1) \right] F_2(q^2) \\
+ \frac{q \cdot \epsilon^*(p_2, \lambda_2) q \cdot \epsilon(p_1, \lambda_1)}{2m^2}(p_1 + p_2)_\mu F_3(q^2),
\]

where \( \epsilon(p_1) \) and \( \epsilon(p_2) \) are the polarization vector of the initial and the final vector meson respectively. Based on the Feynmann diagrams, LHS of eq. (21) can be written as

\[
\epsilon_\nu(p_1, \lambda_1) \epsilon_\lambda^*(p_2, \lambda_2) G_{\mu\nu}^{\lambda\lambda},
\]

where

\[
G_{\mu\nu}^{\lambda\lambda} = Q_1 G_{\mu}^{(1)\lambda\lambda} + Q_2 G_{\mu}^{(2)\nu\lambda},
\]

\[
G_{\mu}^{(1)\lambda\lambda} = - \text{Tr} \int \frac{d^4k}{(2\pi)^4} \gamma_\mu S_1(k + p_1)i\gamma^\nu S_2(k)ig_\nu \gamma^\lambda S_1(k + p_2),
\]

\[
G_{\mu}^{(2)\nu\lambda} = - \text{Tr} \int \frac{d^4k}{(2\pi)^4} \gamma_\mu S_2(k - p_2)i\gamma^\nu \gamma^\lambda S_1(k)ig_\nu \gamma^\nu S_2(k - p_1).
\]

Still we will work in the Breit frame and take the z-axis along the momentum \( p_1 \). The polarization vectors are chosen to be

\[
\epsilon(p_1, \pm) = \frac{1}{\sqrt{2}}(0,1,\pm i,0), \quad \epsilon(p_1, 0) = \frac{1}{m}(p_{1z}, 0, 0, p_{10}),
\]

\[
\epsilon(p_2, \pm) = \frac{1}{\sqrt{2}}(0,1,\mp i,0), \quad \epsilon(p_2, 0) = \frac{1}{m}(p_{2z}, 0, 0, p_{20}).
\]

To retrieve \( F_1 \), we take the time component in eq. (21) and find that

\[
\epsilon_\nu(p_1, \lambda_1) \epsilon_\lambda^*(p_2, \lambda_2) G_{0}^{\lambda\lambda} = - \epsilon^*(p_2, \lambda_2) \cdot \epsilon(p_1, \lambda_1)(p_1 + p_2)_0 F_1(q^2) \\
+ \frac{q \cdot \epsilon^*(p_2, \lambda_2) q \cdot \epsilon(p_1, \lambda_1)}{2m^2}(p_1 + p_2)_0 F_3(q^2).
\]

Then \( F_1 \) can be obtained via the transversely polarization

\[
\epsilon_\nu(p_1, \pm) \epsilon_\lambda^*(p_2, \pm) G_{0}^{\lambda\lambda} = - \epsilon^*(p_2, \pm) \cdot \epsilon(p_1, \pm)(p_1 + p_2)_0 F_1(q^2).
\]

To retrieve \( F_2 \), we take the spatial components in eq. (21) and find that

\[
\epsilon_\mu(p_1, \lambda_1) \epsilon_\nu^*(p_2, \lambda_2) G_{\mu\nu}^{\lambda\lambda} = - \left[ \epsilon_\nu(p_1, \lambda_1) q \cdot \epsilon(p_2, \lambda_2) - \epsilon_\nu^*(p_2, \lambda_2) q \cdot \epsilon(p_1, \lambda_1) \right] F_2(q^2) \\
= \left\{ \left[ \epsilon(p_1, \lambda_1) \times \epsilon^*(p_2, \lambda_2) \right] \times q \right\} F_2(q^2).
\]
Still each form factor $F_{j}$ is a charge weight average of form factors of quark and anti-quark in the vector meson,

$$F_{j}(q^{2}) = Q_{1}F_{j}^{(1)}(q^{2}) + Q_{2}F_{j}^{(2)}(q^{2}),$$

and

$$F_{j}^{(2)}(q^{2}) = F_{j}^{(1)}\left(m_{1} \leftrightarrow m_{2}, q \rightarrow -q, p \rightarrow -p\right).$$

Explicitly we obtain

$$F_{1}^{(1)}(q^{2}) = \epsilon_{\nu}(p_{1}, +)\epsilon^{*}_{\lambda}(p_{2}, -)G_{0}^{(1)\nu\lambda}/(2p_{0})$$

$$= in_{c}n_{f}g_{V}^{2} \int \frac{d^{4}k}{(2\pi)^{4}} G_{1},$$

where

$$G_{1} = 4[(k + p) \cdot k - m_{1}m_{2} + k_{x}^{2} + k_{y}^{2}]$$

$$- 2\frac{p \cdot k}{p^{2}}(p^{2} - k^{2} - q^{2}/4 - m_{1}^{2} + 2m_{1}m_{2} - 2(k_{x}^{2} + k_{y}^{2})],$$

and

$$F_{2}^{(1)}(q^{2}) = \frac{m_{V}}{p_{0}|q|} \left[ \frac{1 - i}{\sqrt{2}}\epsilon_{\nu}(p_{1}, +)\epsilon^{*}_{\nu}(p_{2}, 0)G_{1}^{(1)\mu\nu} + \frac{1 + i}{\sqrt{2}}\epsilon_{\nu}(p_{1}, -)\epsilon^{*}_{\nu}(p_{2}, 0)G_{2}^{(1)\mu\nu} \right]$$

$$= in_{c}n_{f}g_{V}^{2} \int \frac{d^{4}k}{(2\pi)^{4}} G_{2},$$

where

$$G_{2} = 4[k \cdot (k + p) - m_{1}m_{2} + k_{z}^{2}]$$

$$- 2\frac{k \cdot p}{p^{2}}[(k + p)^{2} - q^{2}/4 - m_{1}^{2} + 2(k_{x}^{2} + k_{y}^{2})].$$

We will not consider the form factor $F_{3}$ in this work.

D. Vector-Meson-Dominance and Quark Loop Correction

According to the vector-meson-dominance picture, the $\pi$ and $K$ form factor are dominated by the $\rho$, $\omega$ and $\phi$ intermediate vector meson states [21]. In NJL model, the vector-meson-dominance is represented by the correction to quark-photon vertex as shown in the Feynmann diagram Fig. 2. The correction will introduce a form factor to the constituent quark [5]. For
the $i$-th quark

\[ F^{(i)}_q(q^2) = \frac{1}{1 - K^V J^{(T)}_{VV}}, \tag{35} \]

where $K^V$ is the NJL vector coupling constant and $J^{(T)}_{VV}$ represents the transversely vector loop integral \[4, 16\]. The meson form factor will be modified to

\[ F(q^2) = Q_1 F^{(1)}_q(q^2) F^{(1)}_q(q^2) + Q_2 F^{(2)}_q(q^2) F^{(2)}_q(q^2). \tag{36} \]

### III. NUMERICAL RESULTS

The parameters of the extended NJL model were fixed by fit the meson mass spectra and decay constants in a previous work \[16\]. The input parameters were the current masses of light quarks and the constituent masses of heavy quarks, two coupling constants and the 3-dimensional cutoff:

\[
\begin{align*}
    m_0^u/d &= 2.79\text{MeV}, & m_0^s &= 72.0\text{MeV}, \\
    m_c &= 1.62\text{GeV}, & m_b &= 4.94\text{GeV}, \\
    \Lambda &= 0.8\text{GeV}, & G_V &= 2.41, \\
    h &= 0.65.
\end{align*}
\tag{37}
\]

Due to the charge conservation, the form factor should be normalized to $F(q^2 = 0) = 1$ for any hadron carrying +1 charge. We will make a self-consistent calculation by using the theoretical values of meson masses and the quark coupling constants together. This will guarantee the strict normalization of the form factor at $q^2 = 0$ \[23\]. The theoretical values of pseudo-scalar and vector meson are listed in Table. I and Table. II respectively.
### TABLE I. The masses and quark coupling constants of pseudo-scalar mesons

|     | π      | K      | D      | D_s    | B      |
|-----|--------|--------|--------|--------|--------|
| mass(MeV) | 139 | 496 | 1870 | 1940 | 5280 |
| g    | 4.25 | 4.32 | 4.71 | 5.03 | 5.92 |
| g̃   | 1.56 | 1.61 | 2.04 | 2.09 | 2.84 |

### TABLE II. The masses and quark coupling constants of vector mesons

|     | ρ      | K*     | D*     | D_s*   | B*     |
|-----|--------|--------|--------|--------|--------|
| mass(MeV) | 771 | 918 | 1990 | 2120 | 5310 |
| g    | 1.29 | 1.31 | 1.64 | 1.83 | 2.51 |

**A. Pseudo-scalar meson**

The form factor of π⁺ and K⁺ are compared with experimental data in Fig.3 and Fig.4. The theoretical results fit the experimental data well. In the theoretical calculation, the quark loop correction is included to account the important effect of vector-meson-dominance.

The heavy-light pseudo-scalar meson like D⁺, D_s⁺ and B⁺ have no experiment data of form factor yet. In Fig.5 we present the form factor of all positive pseudo-scalar mesons.

![FIG. 3. The form factor of π⁺ is compared to the experimental data from ref. 20.](image-url)
FIG. 4. The form factor of $K^+$ is compared to the experimental data from ref. [19].

FIG. 5. The form factor of all positive pseudo-scalar mesons.

The form factor at low momentum $q^2$ can be well illustrated by the electro-magnetic radius. The radii of all positive pseudo-scalar mesons are listed in Table. [III]

As had been noticed in ref. [21], it is the quark loop correction of vector-meson-dominance that make $\pi$ radius bigger than that of $K$. In eq. (19), the radius of a meson is a charge weight average of individual quark radii. From eq. (36), after consider the quark loop correction,
TABLE III. Comparison of electric radii $\langle r^2 \rangle^{1/2}$ of pseudo-scalar mesons vs. experimental data. $r_1$ and $r_2$ are constituent quark and anti-quark radii in the meson.

| Meson | $\langle r_1^2 \rangle$ (fm$^2$) | $\langle r_2^2 \rangle$ (fm$^2$) | $\langle r_{2\text{cal.}}^2 \rangle$ (fm) | $\langle r_{2\text{exp.}}^2 \rangle$ (fm) |
|-------|-----------------|-----------------|-----------------|-----------------|
| $\pi^+$ | 0.322 | 0.322 | 0.57 | 0.66 |
| $K^+$ | 0.341 | 0.206 | 0.54 | 0.56 |
| $D^+$ | 0.080 | 0.473 | 0.46 | |
| $D_S^+$ | 0.083 | 0.283 | 0.39 | |
| $B^+$ | 0.795 | 0.041 | 0.74 | |

we have

$$\langle r_i^2 \rangle = \langle r_i^2 \rangle_{\text{int}} + \langle r_i^2 \rangle_{\text{q}},$$

where

$$\langle r_i^2 \rangle_{\text{int}} = \left[ 6 \frac{dF^{(i)}}{dq^2} \right]_{q^2=0},$$

$$\langle r_i^2 \rangle_{\text{q}} = \left[ 6 \frac{dF^{(i)}}{dq^2} \right]_{q^2=0},$$

are the “intrinsic” charge radius and the quark loop correction respectively. The quark loop correction decreases as the quark mass increases. So the lighter quark has a larger radius than its heavier partner in any meson. We show the individual form factor of quark and anti-quark in $\pi$ and $K$ mesons in Fig. 6 and also list the individual quark radii in Table. III.

Just like $\pi^+$ and $K^+$, the radii of the light-heavy mesons $\langle r_{D^+}^2 \rangle^{1/2}$, $\langle r_{D^+_s}^2 \rangle^{1/2}$ decrease as the meson mass increase. However the radius of $B^+$ meson increases by roughly a factor 2. This mainly because that, in a light-heavy meson, the heavy quark’s contribution is much smaller than the light one. If we neglect the contribution of the heavy quark, in $B^+$ meson, $u$-quark has a 2/3 charge weight of contribution. On the contrary, in $D^+$, $D^+_s$, $d$-quark has only a 1/3 charge weight. The form factors of individual constituent quarks in $D$ and $B$ are shown in Fig. 7.
FIG. 6. The form factor of constituent quark and anti-quark in π and K.

FIG. 7. The form factor of constituent quark and anti-quark in B and D.

B. Vector meson

The electric form factors $F_1$ of vector mesons are shown in Fig. 8. The electric radii are listed in Table IV. Because all vector mesons’ masses are close to their thresholds, their bound energies are small and their radii are larger than their pseudo-scalar partners.
The magnetic form factors $F_2$ are presented in Fig. 9. They are connected to magnetic momentum through

$$\mu_V = \frac{F_2(0)e}{2m_V}.$$  \hfill (41)

The magnetic moments are also listed in Table. IV. The magnetic moments are given in the unit of nuclear magneton $\mu_n$. Generally the magnetic momentum decreases as the meson mass increases. In our results, the magnetic moments of $D^*$ and $D_s^*$ are smaller than that of light meson $\rho$ and $K$. However the magnetic moment of $B^*$ is larger than that of $D^*$ and $D_s^*$. The reason is still that the main contribution comes from the light quark but $u$’s charge is larger than $d$’s and $s$’s by a factor of 2. Up to now, no experiment data is available. We compare our results of $\rho^+$ and $K^{*+}$ mesons with other theoretical work \cite{29} and \cite{33}.

### IV. SUMMERY

With the extended NJL model including heavy flavors, we have make a systematic calculation of the form factor of mesons, including the pseudo-scalar mesons and their vector partners, of both the light flavor sector and the light-heavy flavor sector. The form factors of $\pi$ and $K$ mesons fit the experimental data. Other form factors of mesons, especially of the
### TABLE IV. the radius and magnetic moment of vector meson

| meson | $r_1^2 (fm^2)$ | $r_2^2 (fm^2)$ | $r_c (fm)$ | $\mu_1(\mu_n)$ | $\mu_2(\mu_n)$ | $\mu(\mu_n)$ | $\mu(\mu_n)[29]$ | $\mu(\mu_n)[33]$ |
|-------|----------------|----------------|------------|-----------------|----------------|---------------|-------------------|-------------------|
| $\rho^+$ | 1.267          | 1.267          | 1.12       | 1.69            | 0.85           | 2.54          | 2.56              | 3.25              |
| $K^{*+}$ | 1.304          | 0.697          | 1.05       | 1.63            | 0.63           | 2.26          |                   | 2.81              |
| $D^{*+}$ | 0.095          | 1.366          | 0.72       | 0.42            | 0.74           | 1.16          |                   |                   |
| $D_s^{*+}$ | 0.083         | 0.567          | 0.49       | 0.42            | 0.56           | 0.98          |                   |                   |
| $B^{*+}$ | 1.359          | 0.025          | 0.96       | 1.4             | 0.07           | 1.47          |                   |                   |

**FIG. 9.** $F_2$ of all positive vector meson with respect to $q^2$

light-heavy mesons are presented here to confront with future experiments and theoretical calculation such as the lattice calculation.

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