A consistent test of the distance–duality relation with galaxy clusters and Type Ia Supernovae

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ABSTRACT

We propose a new consistent method to test the distance–duality relation which related the angular diameter distances ($D_A$) to the luminosity distances ($D_L$) in a cosmology-independent way. In order to avoid any bias brought by redshift incoincidence between galaxy clusters and Type Ia Supernovae (SNe Ia), as well as to ensure the integrity of the galaxy clusters samples, we obtain the luminosity distance of a certain SN Ia point at the same redshift of the corresponding galaxy cluster by interpolating from the nearby SNe Ia. With the observational data pairs at the same redshifts of the angular diameter distances from the complete 38 galaxy clusters for the spherical model and the corresponding luminosity distances interpolated from the Union 2 set, we find that $\eta \equiv D_L(1 + z)^{-2}/D_A = 1$ is satisfied within 2σ confidence level for parametrizations of $\eta(z)$, which are more stringent than previous results without considering redshift bias.

Key words: supernovae: general – galaxies: clusters: general – distance scale.

1 INTRODUCTION

The distance–duality (DD) relation is known as the Etherington reciprocity relation (Etherington 1933), which is related the angular diameter distance (ADD, $D_A$) to the luminosity distance (DL) by means of a single parameter,

$$\eta \equiv \frac{D_A}{D_L}(1 + z)^{-2} = 1.$$  (1)

This equation is completely valid for all cosmological models based on Riemannian geometry (Ellis 2007). Therefore, the DD relation plays an essential role in observational astrophysics and modern cosmology, such as galaxy clusters observations (Lima, Cunha & Alcaniz 2003; Cunha, Marassi & Lima 2007), the anisotropies of cosmic microwave background (Komatsu et al. 2011), as well as gravitational lensing studies (Schneider, Ehlers & Falco 1999; Fu et al. 2008). In principle, if both $D_A$ and $D_L$ of cosmological sources at the common redshifts are known, the DD relation ($\eta = 1$) could be directly tested by means of astronomical observations. From Sunyaev–Zeldovich effect (SZE) (Sunyaev & Zel’dovich 1972) and X-ray surface brightness of galaxy clusters, the observational ADDs of galaxy clusters can be obtained (Silk & White 1978). By using an isothermal spherical model for which the hydrostatic equilibrium model and spherical symmetry assumed, Reese et al. (2002) selected 18 galaxy cluster sample and Mason et al. (2001) obtained 7 clusters from the X-ray-limited flux sample. The measurements of the two samples above have been corrected by using an isothermal elliptical model to get 25 ADDs of galaxy clusters (De Filippis et al. 2005). Recently, Bonamente et al. (2006) obtained 38 galaxy clusters sample by assuming the spherical model.

Uzan, Aghanim & Mellier (2004) considered ADDs of 18 galaxy cluster sample (Reese et al. 2002) to test the DD relation by assuming the Λ cold dark matter (CDM) model via the technique. $D_A^{\text{cluster}}(z) = D_L^{\Lambda\text{CDM}}(z)\eta(z)$, and showed that no violation of the DD relation is only marginally consistent. De Bernardis, Giusarma & Melchiorri (2006) considered 38 galaxy cluster for spherical model (Bonamente et al. 2006) to test the DD relation by assuming the CDM model. In order to test the DD relation in a model-independent way, one should use measurements of $D_L$ such as Type Ia Supernovae (SNe Ia) directly. By binning $D_L$ of SN Ia data and ADDs from FR IIb radio galaxies and ultracompact radio sources, Basset & Kunz (2004) found that the brightening excess of SNe Ia at $z > 0.5$ could cause a moderate violation at 2σ confidence level (CL). De Bernardis et al. (2006) binned ADD data of galaxy clusters (Bonamente et al. 2006) and SNe Ia data to find that the validity of $\eta = 1$ is consistent at 1σ CL.

However, Holanda, Lima & Ribeiro (2010) argued that the above tests may have been influenced by the particular choice of redshift bin, and they tested the DD relation with two ADD samples (De Filippis et al. 2005; Bonamente et al. 2006) and the Constitution set of SNe Ia data (Hicken et al. 2009). For the biggest redshift...
difference between clusters and SNe Ia is $\Delta z = |z_{\text{clusters}} - z_{\text{SNe}}| \simeq 0.01$ for three clusters, a selection criteria ($\Delta z \leq 0.005$) for a given pair of data set are used to avoid the corresponding bias of redshift differences. With the incomplete spherical model sample (Bonamente et al. 2006) in which three ADD data have been discarded, they found a strong violation (>3σ) of the DD relation by using two parametrizations of $\eta$ parameter [$\eta(z) = 1 + \eta_1z$, and $\eta(z) = 1 + \eta_2z/(1+z)$]. More recently, Li, Wu & Yu (2011) used the same selection criteria for given pairs of observational data to remove more data points of the galaxy clusters corresponding to the Constitution set and found that the DD relation could be marginally accommodated at 3σ CL for the spherical model if the effect of the errors of SNe Ia considered. Additionally, they also examined the DD relation for two more general parametrizations of $\eta$ parameter to show that $\eta(z) = 1$ is compatible with the spherical model sample and the Union 2 set (Amanullah et al. 2010) at 2σ CL. Some authors have been proposed other astrophysical sources in context of testing the DD relation, such as the baryon acoustic oscillation observation (More, Bovy & Hogg 2009; Cardone et al. 2012), the observational Hubble parameter data (Avgoustidis et al. 2010), as well as the X-ray gas fraction ($f_{\text{gas}}$) data (Gonçalves, Holanda & Alcaniz 2011; Holanda, Gonçalves & Alcaniz 2012a). For recent works of DD relation on astrophysical research, see e.g. Cao & Zhu (2011), Nair, Jhingan & Jain (2011), Holanda, Lima & Ribeiro (2011, 2012b), Lima, Cunha & Zanchin (2011) and Holanda (2012).

It is obvious that testing results of the DD relation may be influenced by the particular choice of the selection criteria for a given pair of data set. The difference of redshifts between pairs of galaxy clusters and SNe Ia may cause obvious deviation in testing the DD relation. In principle, the only strict criterion to form a given pair is that galaxy clusters and SNe Ia locate at exactly the same redshift. At other hand, the more stringent selection criteria are used, the more data points should be removed. In order to avoid any bias of redshift differences between SNe Ia and galaxy clusters and ensure the integrity of observational data pairs, we can use the nearby SNe Ia points to obtain the luminosity distance of SN Ia point at the same redshift of the corresponding galaxy cluster; this situation is similar with the cosmology-independent calibration of gamma-ray burst relations directly from SNe Ia (Liang et al. 2008; Liang & Zhang 2008; Liang, Wu & Zhang 2010; Liang, Xu & Zhu 2011). In this work, we test the DD relation with the SNe Ia points in which a sub-sample are corrected by interpolating from the nearby points to the same redshifts of the corresponding galaxy clusters sample for a given pair of data set. We focus on the current observational data pairs of galaxy cluster sample under an assumption of spherical model and the Union 2 set in this work. When considering redshift bias of observational data pairs between the complete 38 galaxy clusters for the spherical model and the corresponding Union 2 set, we find that $\eta(z) = 1$ is satisfied within 2σ CL with current observations.

2 DATA ANALYSIS

In this work, we test the DD relation with the 38 ADD sample from galaxy clusters for the spherical model and the Union 2 set which consists of 557 SNe Ia. It is easy to find that differences of redshifts between the 38 galaxy clusters to the Union 2 set are more centred around $\Delta z = 0$ and the biggest value at $\Delta z = 0.005$ for a given pair of data set; this situation can provide the accuracy in the interpolating procedure. Therefore, we can obtain the luminosity distance of SN Ia at the same redshift of the corresponding galaxy cluster by interpolating from the nearby SNe Ia points with the biggest difference of redshifts $\Delta z_{\text{max}} = 0.005$ for a given pair of data set. Obviously, our method can successfully avoid the systematic errors brought by redshift incoincidence of the observational data pairs and ensure the integrity of observational data pairs.

When the linear interpolation is used, the interpolated distance modulus of a source at redshift $z$ can be calculated by $\mu(z) = \mu_i + ((z - z_i)/(z_{i+1} - z_i))(\mu_{i+1} - \mu_i)$, where $\mu_i$, $\mu_{i+1}$ are the distance moduli of the SNe at nearby redshifts $z_i$, $z_{i+1}$; and the uncertainty is $\sigma_{\mu_i} = ((z_{i+1} - z)/z_i)^2 \sigma_{\mu_i}^2 + ((z - z_i)/(z_{i+1} - z_i))^2 \sigma_{\mu_{i+1}}^2)^{1/2}$. It is noted that some SNe data locate at the same redshifts; therefore, we weighted the SNe data at the same redshifts each other in the interpolating procedure, $\bar{\mu}(z) = \sum(\mu_i/\sigma_{\mu_i}^2)/\sum 1/(\sigma_{\mu_i}^2)$, where $\bar{\mu}(z)$ stands for the weighted mean distance modulus at the same redshift $z$ with its uncertainty $\sigma_{\bar{\mu}} = (\sum 1/\sigma_{\mu_i}^2)^{-1/2}$.

In Fig. 1, we plot $D_A(z)$ data from the galaxy cluster and the corresponding corrected $D_L$ data from Union 2 sub-sample at the same redshifts of galaxy clusters. For galaxy cluster samples, the typical statistical and systematic uncertainties of galaxy clusters are around ±20 and ±12.4 per cent (Bonamente et al. 2006; Holanda et al. 2010). Following Holanda et al. (2010) and Li et al. (2011), we combine the statistical and systematic uncertainties of galaxy clusters in quadrature and treat the asymmetry uncertainties of galaxy clusters by a statistical approach (D’Agostini 2004).

The technique for determining the DD with the SZE+X-ray observations of galaxy clusters (Sunyaev & Zel’dovich 1972; Cavaliere & Fusco-Femrino 1978) is strongly dependent on the validity of the DD relation. It gives $D_A^{\text{cluster}}(z) = D_A(z)\eta^2$ when the DD relation does not hold. Therefore, $D_A(z)$ must be replaced with $D_A^{\text{cluster}}(z)\eta^{-2}$ when testing the DD relation (Holanda et al. 2010). The observed $\eta_{\text{obs}}$ in a redshift-dependent form can be determined by

$$\eta_{\text{obs}}(z) = (1 + z)^2 \frac{D_A^{\text{cluster}}(z)}{D_A^{\text{corrected}}(z)}$$

where $D_A^{\text{cluster}}$ is ADD from galaxy cluster at redshift $z$ inside the samples, and $D_A^{\text{corrected}}$ is the corrected luminosity distance interpolated from the nearby SNe Ia points $D_L^{\text{SNe}}$. We note that the data
points of SNe Ia are given in terms of the distance modulus, which could reduce to the luminosity distance by \(D_{L}(z) = 10^{\mu(z)/5} \). Accordingly, the uncertainty of the luminosity distance could be expressed as \(\sigma_{D_{L}} = (\ln 10/5) D_{L} \sigma_{\mu} \).

The DD relation can be tested with the combined observational data pairs of galaxy clusters and SNe Ia at the same redshift by the minimum \(\chi^{2}\) method and the total \(\chi^{2}\) can be given by

\[
\chi^{2}(p) = \sum_{i} \frac{[\eta(z_i; p) - \eta_{\text{obs}}(z_i)(1 + \alpha)]^2}{\sigma_{\text{obs}}^2},
\]

where \(\eta(p)\) represents the theoretical value with the parameter set \(p\) and \(\eta_{\text{obs}}\) associated with the observational technique with its error \(\sigma_{\text{obs}} = \eta_{\text{obs}}^2[(\sigma_{D_{L}}/D_{L})^{2} + (\sigma_{\text{corrected}}/D_{L})^{2}]\), the additional term \((1 + \alpha)\) is introduced to take into account a systematic uncertainty of the SN distance modulus.\(^1\)

It should be noted that we do not know explicitly the difference of redshifts between pairs of galaxy clusters and SNe Ia cause deviation in testing the DD relation by how much, and whether the redshift interpolation method indeed removes the bias. Therefore, it is important to check the interpolation scheme by using simulated data in testing the DD relation. We list the testing results by using the simulated data in the Appendix A.\(^2\)

### 3 RESULTS

We show testing results of the DD relation with ADDs and the Union 2 set by considering one-parameter parametrizations \([\eta(z) = 1 + \eta z\text{ and } \eta(z) = 1 + \eta_{\text{corr}} z / (1 + z)\] in Fig. 2. For comparison, the case with the corrected luminosity distance \(D_{L}^{\text{corrected}}\).

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\(^1\) We thank the anonymous referee for pointing this out. For the absolute magnitude of SNe is known up to \(\pm 0.05\) mag, the luminosity distance can be shifted by a factor 0.023 percent. We marginalize over this term as a nuisance parameter with a Gaussian prior centred on \(< \alpha > = 0\) and with standard deviation \(\sigma_{\alpha} = 0.023\) (Cardone et al. 2012).

\(^2\) We thank the anonymous referee for pointing this out. By using simulated data, we find that the difference of redshifts between pairs of galaxy clusters and SNe Ia could cause deviation in testing the DD relation explicitly and the redshift interpolation method indeed removes the bias of the difference of redshifts between data pairs.

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\(^3\) In order to find the impact of the extra factor \((1 + \alpha)\) in equation (3), we also obtain results without the additional term. For comparing to results with and without the factor, we could find that the factor does NOT affect the testing results significantly.

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**Figure 2.** Likelihood contours with the 38 ADDs of galaxy clusters for spherical model and the corrected luminosity distances of the Union 2 set in the \(\eta_{1} - \Delta \chi^{2}\) plane (left: for \(\eta(z) = 1 + \eta z\), and in the \(\eta_{a} - \Delta \chi^{2}\) plane (right: for \(\eta(z) = 1 + \eta_{\text{corr}} z / (1 + z)\)). The blue real lines represent the case with the corrected luminosity distance interpolated from the nearby SNe Ia (Union 2*), the black dashed lines represent the case with the SNe Ia set (Union 2) directly, and the red vertical lines represent \(\eta = 1\).

**Table 1.** Fitting results with the 38 ADDs of galaxy clusters for spherical model and the Union 2 set and Constitution set, and \(\chi_{\text{min}}^2\) (the minimum \(\chi^{2}\), \(\chi_{\text{min}}^2/(\text{dof})\), for \(\eta(z) = 1 + \eta z\) and \(\eta(z) = 1 + \eta_{\text{corr}} z / (1 + z)\), respectively. The asterisk represents the case with the corrected luminosity distance interpolated from the nearby SNe Ia.

| Parametrization (SN Ia*) | \(\eta_{1}/\eta_{0}\) | \(\chi_{\text{min}}^2\) | \(\chi_{\text{min}}^2/(\text{dof})\) |
|------------------------|---------------------|----------------------|---------------------|
| 1 + \(\eta_{1} z\) (Union 2*) | \(\eta_{1} = -0.232 \pm 0.232(2\sigma)\) | 28.78 | 0.778 |
| 1 + \(\eta_{1} z\) (Union 2) | \(\eta_{1} = -0.257 \pm 0.225(2\sigma)\) | 28.99 | 0.783 |
| 1 + \(\eta_{0} z\) (Union 2*) | \(\eta_{0} = -0.351 \pm 0.368(2\sigma)\) | 29.13 | 0.783 |
| 1 + \(\eta_{0} z\) (Union 2) | \(\eta_{0} = -0.387 \pm 0.353(2\sigma)\) | 29.39 | 0.794 |
| 1 + \(\eta_{1} z\) (Constitution*) | \(\eta_{1} = -0.431 \pm 0.303(3\sigma)\) | 33.10 | 0.895 |
| 1 + \(\eta_{1} z\) (Constitution) | \(\eta_{1} = -0.517 \pm 0.286(3\sigma)\) | 40.97 | 1.107 |
| 1 + \(\eta_{0} z\) (Constitution*) | \(\eta_{0} = -0.664 \pm 0.457(3\sigma)\) | 32.33 | 0.874 |
| 1 + \(\eta_{0} z\) (Constitution) | \(\eta_{0} = -0.793 \pm 0.436(3\sigma)\) | 40.46 | 1.094 |

Interpolated from the nearby SNe Ia and the case with the Union 2 set directly \(D_{L}^{\text{corrected}}\) are given simultaneously. For the case with \(D_{L}^{\text{corrected}}\), the best-fitting values are \(\eta_{1} = -0.232 \pm 0.232\) at 2σ CL with \(\chi_{\text{min}}^2 = 28.78\), and \(\eta_{0} = -0.351 \pm 0.368\) at 2σ with \(\chi_{\text{min}}^2 = 28.98\). For the case with the Union 2 set directly, the best-fitting values are \(\eta_{1} = -0.257 \pm 0.225\) (2σ), and \(\eta_{0} = -0.387 \pm 0.353\) (2σ), which are consistent with those obtained by Li et al. (2011) \((\eta_{1} = -0.22 \pm 0.21\) and \(\eta_{0} = -0.33 \pm 0.33(2\sigma)\)). We summarize the testing results with the 38 ADDs of galaxy clusters and the corrected luminosity distances of the Union 2 set in Table 1.\(^3\)
$\eta_a = -0.793 \pm 0.436 \ (3\sigma)$. Results with the Constitution set are summarized in Table 1.\footnote{For simplicity, we do not consider the additional term $(1 + \alpha)$ in the $\chi^2$ function for the case with the Constitution set.}

From Figs 2 and 3 and Table 1, we have obtained some new results and insights, which summarized as follows. (1) From comparing to results of the case with the corrected luminosity distance and the case with SN Ia set (the Union 2 set and the Constitution set) directly, we can see a shift (significantly with the Constitution set) between the best-fitting values and the likelihood contours towards the standard DD relation ($\eta = 1$) with lower $\chi^2_{\text{min}}$ for using of the interpolating method to obtain $D_L^{\text{corrected}}$. This situation shows that the using of the interpolating method tend to avoid the corresponding systematic bias of redshift differences and make testing results be more compatible with the DD relation. (2) Compared to results of the case with the Union 2 set and the case with Constitution set, it is shown that the DD relation of the one-parameter parametrizations with the Union 2 set for the interpolating method is satisfied within $2\sigma$ CL; while the DD relation is inconsistent with the Constitution set for both cases at $3\sigma$ CL, which shows that the redshift incoincidence of the observational data pairs can bring the systematic errors and the systematic errors of the observational data pairs exist significantly within the Constitution set. For Comparison with previous results from the incomplete ADD sample and the Constitution set (Holanda et al. 2010; Li et al. 2011), our analyses with the complete spherical model sample (38 ADDs) and the Constitution set directly are consistent with previous results obtained by Holanda et al. (2010) with the incomplete spherical model sample (35 ADDs, three points removed by selection criteria) and the Constitution set, where $\eta_1 = -0.42 \pm 0.34$, and $\eta_{\text{a}} = -0.66 \pm 0.50$ at $3\sigma$ CL; and inconsistent with those obtained by Li et al. (2011) with the incomplete spherical model sample (26 ADDs, 12 points removed by selection criteria) and the Constitution set, where $\eta_1 = -0.30 \pm 0.34$ and $\eta_{\text{a}} = -0.46 \pm 0.51\ (3\sigma)$. It indicates that the choice of selection criteria to remove ADD points with large bias of redshift differences may play an important role in testing of the DD relation (Cao & Liang 2011), which also means that the systematic errors brought by redshift incoincidence exist significantly within the Constitution set, and the using of the interpolating method tend to alleviate the corresponding systematic bias of redshift differences significantly.

Following Li et al. (2011), we also treat the redshift-independent model parameter $\eta_0$ as a free parameter to examine the DD relation. Results with two-parameter parametrizations $[\eta(z) = \eta_0 + \eta_1 z$ and $\eta(z) = \eta_0 + \eta_1 z/(1 + z)]$ are shown in Fig. 4. With the corrected $D_L$ of the Union 2 set, the best-fitting values are $(\eta_0, \eta_1) = (1.007, -0.219)$ and $(\eta_0, \eta_1) = (1.027, -0.396)$, respectively. For the case with the Union 2 set directly, the best-fitting values are $(\eta_0, \eta_1) = (1.035, -0.302)$ and $(\eta_0, \eta_1) = (1.071, -0.575)$, respectively. It might be worth mentioning that marginalizing over $\eta_0$ with two-parameter parametrizations is similar to adding the $\alpha$ parameter with a very wide prior.\footnote{We thank the anonymous referee for pointing this out. The best-fitting value of $\eta_0 - 1$ for the data is comparable to the width of the prior on $\alpha$ from footnote 1.} From Fig. 4, we can see that a significant shift between the best fitting values and the likelihood contours towards the standard DD relation ($\eta = 1$) for the interpolating method compared to the testing results of the case with the Union 2 set directly. Our results suggest that the DD relation for two-parameter parametrizations are consistent with the observational data marginally at $1\sigma$ CL, which are more stringent than those obtained in Li et al. (2011), where the DD relation is marginally accommodated at $2\sigma$ CL for two-parameter parametrizations.

4 CONCLUSIONS AND DISCUSSION

In this work, we perform a new consistent test of the distance–duality relation $[\eta(z) \equiv D_L (1 + z)^{-2}/D_L = 1]$ in a cosmology-independent way. It is obvious that the redshift differences of observational samples may cause deviation of the DD relation. Testing results from given pairs of data set with the corresponding galaxy clusters and SNe Ia at nearby redshift may be influenced by the particular choice of the selection criteria; the more stringent selection criteria are used, the more data points should be removed. In order to avoid any bias of difference of redshift and ensure the integrity of the ADD samples, we correct the luminosity distance of an SN Ia to the same redshift of the corresponding galaxy cluster directly from the nearby SN Ia points.

With the 38 ADD sample from galaxy clusters under an assumption of spherical model and the corrected luminosity distances of
the Union 2 set by using the interpolating method to alleviate the corresponding systematic bias of redshift differences, fitting results of the DD relation are \( \eta_1 = -0.232 \pm 0.232 \) for parametrization \( \eta(z) = 1 + \eta_0 z \), and \( \eta_0 = -0.351 \pm 0.368 \) for parametrization \( \eta(z) = 1 + \eta_0 z/(1+z) \) at 2\( \sigma \) CL, respectively. We also find the DD relation are consistent with the spherical model and the corrected luminosity distances marginally at 1\( \sigma \) CL for the two-parameter forms of parametrization \( \eta(z) = \eta_0 + \eta_1 z \) and \( \eta(z) = \eta_0 + \eta_1 z/(1+z) \). Our results show that there exists no conceivable evidence for variations in the duality distance relation when the current SNe Ia (Union 2) and the complete 38 sample of galaxy clusters data are confronted, since various parametrizations of \( \eta(z) \) are satisfied within 2\( \sigma \) CL which are more stringent than those obtained in Li et al. (2011), where the DD relation is only marginally accommodated at 3\( \sigma \) CL for the spherical model sample without considering redshift bias. We also show that the systematic errors brought by redshift incoherence of the data pairs exist explicitly within the simulated data by considering a fiducial cosmological model as well as within the Constitution set, and the using of the interpolating method tend to alleviate the corresponding systematic bias significantly.

In this work, we focus on the 38 galaxy cluster sample under an assumption of spherical model (Bonamente et al. 2006). When considering the 25 galaxy clusters under isothermal elliptical model (De Filippis et al. 2005) and the Union 2 set, we also find that \( \eta(z) = 1 \) is well satisfied at 1\( \sigma \) CL for case with the corrected luminosity distance, which is consistent with previous works using SN Ia set directly (Holanda et al. 2010; Li et al. 2011). This situation shows that testing results of the DD relation still depend on the choice of assumptions on the cluster geometry model significantly.

Although the excursion of the DD relation is not significantly reduced with the corrected sub-sample of the Union 2 set and the complete 38 sample of galaxy clusters data, it should be noted that the testing results of the DD relation would be improved by considering the systematic bias of redshift.

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**Figure 4.** Likelihood contours with the 38 ADDs of galaxy clusters and the corrected luminosity distances of the Union 2 set for two-parameter forms in the \( \eta_0 - \eta_1 \) plane (left: for \( \eta(z) = \eta_0 + \eta_1 z \)), and in the \( \eta_0 - \eta_0 \) plane (right: for \( \eta(z) = \eta_0 + \eta_0 z/(1+z) \)). The blue real lines represent the case with the corrected luminosity distance interpolated from the nearby SNe Ia, and the black dashed lines represent the case with the Union 2 set directly. The contours correspond to 1\( \sigma \) and 2\( \sigma \) confidence regions, and the blue crosses and black stars represent the best-fitting points and the red stars represent \( \eta(z) = 1 \).

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Note that our results with the 38 ADD sample and the corrected Union 2 set are also consistent with more recent works, e.g. Fu et al. (2011); Meng et al. (2012); Cardone et al. (2012), which considers redshift bias of observational data pairs to test the DD relation.
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APPENDIX A: THE SIMULATED DATA

In order to check whether the redshift interpolation indeed removes the bias of the difference of redshifts between pairs, we test \( \eta(z) \) of parametrizations \([\eta(z) = 1 + \eta_1 z] and [\eta(z) = 1 + \eta_2 z/(1 + z)]\) with the simulated data pairs. In the simulating procedure, we consider the concordance model \((\Omega_m = 0.27, \Omega_v = 0, w = -1)\), with Hubble constant \(H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}\) as a fiducial cosmological model to simulate cluster and SNe Ia data. For the simulated SNe Ia, we obtain the distance moduli at \(\Delta z_{\text{SN}} = 0.005 (0.14 < z < 0.9)\) from a Gaussian distribution centred on the theoretical value and with variance \(\sigma_{\Delta z} = \sigma_{\Delta z, \text{obs}} (\mu_{\text{sim}} / \mu_{\text{obs}})\) (Cardone et al. 2012). For the simulated cluster sample, we calculate 38 ADDs from the concordance model at \(z_{\text{cluster}} \in [0.142, 0.89]\) from a Gaussian distribution centred on the theoretical value and with variance \(\sigma_{\Delta z} = \sigma_{\Delta z, \text{obs}} (D_{\text{sim}} / D_{\text{obs}})\). In the \(\chi^2\) fit, we combine the systematic uncertainties of galaxy clusters and SNe to calculate \(\sigma_{\text{obs}}\) and produces a reduced \(\chi^2\) of unity.

We show testing results with the simulated data pairs in Fig. A1. For the case with the simulated data pairs directly, we choose the closest SN point to the redshift of cluster to build up the pair of galaxy clusters and SNe Ia. The best-fitting values are \(\eta_1 = -0.016 \pm 0.006(1\sigma) \pm 0.012(2\sigma) \pm 0.017(3\sigma)\) for \(\eta(z) = 1 + \eta_1 z\), and \(\eta_2 = -0.034 \pm 0.011(1\sigma) \pm 0.022 (2\sigma) \pm 0.033(3\sigma)\) for \(\eta(z) = 1 + \eta_2 z/(1 + z)\), which shows that \(\eta(z) = 1\) are excluded at \(2\sigma\) for both cases. It is indicated that the difference of redshifts between pairs of galaxy clusters and SNe Ia could cause deviation in testing the DD relation explicitly. For the case with the interpolated SNe Ia data, we choose the corrected SN point at the same redshift of cluster, which interpolated from the nearby points, to build up data pairs of galaxy clusters and SNe Ia. The best-fitting values are \(\eta_1 = (5.9 \pm 6.0) \times 10^{-5}\) and \(\eta_2 = (9.5 \pm 9.6) \times 10^{-5}\) with the \(1\sigma\) uncertainties, which shows that \(\eta(z) = 1\) is fully satisfied with the data pairs interpolated to the same redshifts. Therefore, we conclude that the redshift interpolation method indeed removes the bias of the difference of redshifts between data pairs.

![Figure A1. Likelihood contours with the simulated data pairs in the \(\eta_1 - \Delta \chi^2\) plane (left: for \(\eta(z) = 1 + \eta_1 z\), and in the \(\eta_2 - \Delta \chi^2\) plane (right: for \(\eta(z) = 1 + \eta_2 z/(1 + z)\)). The black dashed lines represent the case with the simulated SNe Ia directly, and the red vertical lines represent \(\eta(z) = 1\), which overlap the contours of the case with the corrected luminosity distance interpolated from the simulated SNe Ia.]