The effect of the demagnetizing field in cylindrical samples in high transverse field $\mu^+\text{SR}$ experiments

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Abstract. We investigate the spatial variation of the demagnetizing field for uniformly magnetized, cylindrical samples using a recently developed Fourier space approach. We show that the demagnetizing response of the sample leads to a position dependence of the magnetic field, which varies most strongly near the radial boundary of the cylinder. Furthermore, we demonstrate that the demagnetizing field leads to a subsequent broadening of the field distribution, as experienced by the muons implanted in the sample, with a characteristic shape including a low field tail and a sharp high-field cutoff. We present a detailed study of this field broadening as a function of the aspect ratio of the cylinder and find that it is significant and largest for aspect ratios typical for cylindrical samples grown in mirror furnaces. We identify two strategies to minimize this broadening: adding a degrader so that muons implant closer to the surface of the sample and using a circular mask to stop muons from implanting near the radial edge. This could help identify whether an experimentally observed broadening is caused by the demagnetization response or the intrinsic properties of the sample.

1. Introduction
The development of new high transverse field (TF) muon spectrometers such as Hifi, HiTime, GPD or HAL-9500 is opening up new possibilities for experimental investigations of samples with exotic magnetic phases. The high magnetic fields involved may magnetize the sample and subsequently induce a demagnetizing field. It is important to understand to what extent the measured variations in the internal magnetic field distribution might be caused by variations of the demagnetizing field rather than by intrinsic effects due to exotic sample physics. Many single crystals grown using mirror furnaces are cylindrical, with experiments carried out on thin cylindrical slices. Therefore, an understanding of the demagnetizing field inside such cylindrical samples is desirable for high transverse-field muon-spin rotation (TF-$\mu^+\text{SR}$) studies.

Demagnetizing fields inside magnetized objects are difficult to calculate, but if the object is ellipsoidal in shape then the demagnetizing field is uniform [1]. This simple case includes the sphere, the flat plate, and the infinitely long cylinder as subsets. In other geometries an analytical solution is not so forthcoming. Though a rather unwieldy analytic solution was outlined some time ago [2], a recently developed Fourier space approach [3] has extended the range of analytically solvable geometries in a much more general and elegant fashion. This approach has been used to calculate the demagnetizing tensor of a uniformly magnetized cylinder [4] analytically, but the published solution contains errors (see [5]).
2. Magnetic Fields
The total magnetic field acting on a muon implanted into a sample can be written as

\[ B_{\text{total}} = B_{\text{ext}} + B_{\text{hf}} + B_{\text{dip}} + B_{\text{L}} + B_{\text{D}}, \]

where \( B_{\text{ext}} \) represents the externally applied field, \( B_{\text{hf}} \) represents the contact and transferred hyperfine interactions, \( B_{\text{dip}} \) and \( B_{\text{L}} \) account for the dipolar fields inside and outside the Lorentz sphere respectively, and \( B_{\text{D}} \) corresponds to the demagnetizing field. We will restrict our attention to the last two terms, since we are interested in the general behaviour of magnetized samples regardless of the sample’s chemical structure, which is required to evaluate \( B_{\text{dip}} \). We assume that the sample acquires a uniform magnetization \( M \) along the direction of the external field \( B_{\text{ext}} \), taken to be along \( \hat{z} \), in which case the Lorentz field becomes \( B_{\text{L}} = \mu_0 M / 3 \). We define the shape function \( D(r) \) which takes the value 1 when \( r \) is inside the sample and 0 outside the sample. The demagnetizing tensor is then given by \( N_{ij}(r) = D(k) k_i k_j / |k|^2 \) and hence \[ N_{ij}(r) = \frac{1}{(2\pi)^2} \int d^3k \frac{D(k)}{k^2} k_i k_j e^{i k \cdot r}, \]

with the additional relation \( \text{Tr}[N_{ij}(r)] = D(r) \). Thus, the demagnetizing field becomes \( B_{D,i}(r) = -\mu_0 N_{ij}(r) M_j \) and we use our analytically computed expression for \( N_{ij}(r) \) for a cylinder \([5]\). Note that even if the sample is not ferromagnetic, it will acquire a magnetization \( M = \chi B_{\text{ext}} / \mu_0 \) in a strong applied field \( B_{\text{ext}} \). The purpose of this paper is to provide a guide in assessing whether an observed broadening is due to intrinsic sample physics affecting \( B_{\text{dip}} \), or a demagnetization effect due to \( B_{\text{D}} \).

3. Sample Details
We will assume a cylindrical geometry with a uniform magnetization of magnitude \( |M| = M \) along the cylindrical axis, taken to be the \( z \)-axis, and the muons being implanted along the same axis. Furthermore, we will assume a typical experimental sample with a radius \( R \) of 10mm, a height \( h \) of 2mm, and a muon implantation depth of 0.1mm, in line with the stopping range of surface muons of about 110 mg cm\(^{-2} \), unless explicitly stated otherwise. Figure 1 shows a schematic of the experimental setup we are simulating.

**Figure 1.** Schematic of high-field TF-\( \mu^+ \)SR experiment, showing the demagnetization field variation within a cylindrical sample (shown in cross section) at the muon implantation depth.
4. Demagnetizing Field Distribution

We can use the analytic solution for the demagnetization tensor to compute the demagnetizing field at the stopping depth of the muons as a function of the radial position. The resulting fields are plotted in Figure 2 for the demagnetizing field and the sum of Lorentz and demagnetizing field and their respective z-components. The reason for plotting the z-components is that the large external field is applied along $\hat{z}$ and therefore $(B_D + B_L) \cdot \hat{z}$ will add to the total field in first order, whereas the components perpendicular to $\hat{z}$ add in second order. Thus, the former will have the largest effect in a TF-$\mu^+$SR experiment.

![Figure 2. Magnetic field strengths and components along the cylinder axis at the muon stopping depth. The inset shows the z dependence of the fields at the centre and edge of the cylinder. $R$ and $h$ are the cylinder radius and height respectively; the cylinder centre is at $z = r = 0$. Note that the Lorentz field $B_L = \mu_0 M / 3$ is homogenous and that the demagnetizing field $B_D$ is not.](image.png)

Figure 2 shows that the demagnetizing field is position dependent and more importantly that its z-component almost vanishes near the radial boundary, while the overall field strength does not. Qualitatively, this is due to a realigning of the field to satisfy the boundary condition at the radial edge, as indicated in Figure 1, but we note that the region near the radial edge actually constitutes a significant portion of the total volume. Thus, even though the field realignment only happens close to the boundary, it can be a significant effect due to the volume weighting.

The impact of the demagnetizing field becomes apparent in the probability distributions, which are plotted in Figure 3 and show a clear and sizeable broadening of the magnetic field. The discrepancy between the two distributions for the field strength and its z-component can be understood in the light of the realigning of the field near the radial boundary and the demagnetizing field being stronger near the flat z-boundary, which is often modelled to contain “magnetic charges” induced by the external field.

The calculations described above assume all muons to stop precisely at a depth of 0.1mm and in a radially uniform way, which is a good approximation if the muon beam covers the entire sample. However, when one repeats the above calculations and includes a Gaussian stopping profile centred around this implantation depth, the outcomes are effectively unchanged. In fact, the field distribution results were largely insensitive to all stopping profiles we tested. Thus, we restrict the subsequent discussion to a simple delta-function stopping profile.
5. Effect of Aspect Ratio

We now examine the impact of the edge effects on the field broadening as a function of the cylinder’s aspect ratio, defined as the ratio of height $h$ to diameter $d = 2R$ (the calculations above assumed $h/d = 0.1$). Computing the field distributions at the muon stopping depth for different values of $h/d$ yields the results shown in Figure 4. Plotted in the figure are the mean and standard deviation of both the field strength and its $z$-component. Figure 4 shows that the correct results are recovered in the limits of a flat cylinder (disc: $h/d \to 0$, $B_D = -\mu_0 M$, $B_D + B_L = -2\mu_0 M/3$) or long cylinder (wire: $h/d = \infty$, $B_D = 0$, $B_D + B_L = \mu_0 M/3$). Furthermore, it illustrates that the field broadening is strongest when the aspect ratio is of order 1, which applies to most experimental samples, and that it can exceed 20% of the average demagnetizing field in such cases. Therefore, we conclude that the field broadening due to the demagnetizing response and edge effects can in general be significant for the typical samples used for high-field TF-$\mu^+$SR experiments.

6. Effect of Muon Stopping Depth

Another possible way of influencing the effect of the demagnetizing field experienced by the muons, besides changing the aspect ratio of the sample, is to reduce the implantation depth of the muons by placing a degrader on top of the sample. This has the effect that more muons stop in regions closer to the flat $z$-boundary than the radial edge, and Figure 5 shows the two main resulting consequences for the case $h/d = 0.1$. Firstly, the average demagnetizing field strength increases as the stopping distance is reduced, which shows that the field is stronger near the cylinder’s flat surfaces where $\nabla \cdot M \neq 0$. Secondly, the broadening of the field distribution decreases as the implantation depth is shortened. Therefore, by adding a degrader it can be possible to reduce the field broadening at the expense of raising the average demagnetizing field strength.
Figure 4. Field distribution versus cylinder aspect ratio. The standard deviations are scaled up by a factor of 5. The inset graph shows the ratio of standard deviations and mean values.

Figure 5. Field distribution dependence on the muon implantation depth for the case $h/d = 0.1$. Solid lines and the left hand axis correspond to mean values, dashed lines and the right hand axis to standard deviations.
7. Discussion
Our investigations show that the demagnetizing response of typical cylindrical samples can be significant due to edge effects. If we assume the external field $\mathbf{B}_{\text{ext}} \parallel \hat{z}$ to be very large then to a good approximation the demagnetizing and Lorentz field components parallel and perpendicular to $\hat{z}$ add to the total field in first and second order respectively. In this case, a possible signature that the observed broadening is caused by the demagnetizing field could be a field distribution resembling that of the $z$-component of $\mathbf{b} = \mathbf{B}_D + \mathbf{B}_L$ shown in Figure 3, which has a long low-field tail due to the radial edge behaviour and a sharp high-field cutoff (this behaviour is reversed if $\chi < 0$). If an experimentally observed field distribution resembles this characteristic behaviour it could be useful to consider the demagnetization response more closely.

Additionally, we can think of two potential ways of influencing the demagnetization effects experimentally. One possibility is to place a degrader on top of the sample in order to shorten the muon implantation depth and subsequently reduce the demagnetizing field broadening as discussed above. This has the disadvantage that we might then get additional field contributions due to muons stopping in the degrader and the demagnetizing response of the degrader itself. Another possibility to reduce the demagnetizing field broadening is to place a thick circular mask with radius $< R$ in front of the sample, such that the muons that would stop in the regions near the radial boundary are completely stopped in the mask. In Figure 2 this would correspond to only considering regions away from the radial edge, thereby shortening the low-field tail of the $b_z$ distribution shown in Figure 3. However, the presence of the mask will lower the data rate and potentially shift the background signal experienced by muons. An alternative to the mask could be to reduce the muon beamspot radius below $R$, with the sample well centred in the beam.

8. Conclusions
We have shown that the demagnetizing field inside a uniformly magnetized cylinder is position dependent, which results in a field broadening caused by edge effects. Furthermore, we demonstrated that this field broadening has a characteristic shape and is non-negligible for sample geometries close to typical experimental ones, with standard deviations in excess of 10% of the mean demagnetizing field. We have identified two strategies to ameliorate this effect. A degrader can be used to shorten the muon implantation depth, at the expense of increasing the mean demagnetizing field experienced by the muons, or a circular mask could be deployed to ensure no muons implant near the radial edge of the sample. These strategies can be used to minimize the demagnetization broadening, or diagnostically, to ascertain how significant the effects are in a particular geometry. It is hoped that our calculations may serve as a useful guide for future high-field TF-$\mu^+$SR experiments and allow an assessment to be made of whether an observed broadening is intrinsic due to exotic sample physics or simply caused by a field-induced artifact.

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