Constraints on Neutrino Mass and Light Degrees of Freedom in Extended Cosmological Parameter Spaces

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From a combination of probes including the cosmic microwave background (WMAP7+SPT), Hubble constant (HST), baryon acoustic oscillations (SDSS+2dFGRS), and supernova distances (Union2), we have explored the extent to which the constraints on the effective number of neutrinos and sum of neutrino masses are affected by our ignorance of other cosmological parameters, including the curvature of the universe, running of the spectral index, primordial helium abundance, evolving late-time dark energy, and early dark energy. In a combined analysis of the effective number of neutrinos and sum of neutrino masses, we find mild (2.2σ) preference for additional light degrees of freedom. However, the effective number of neutrinos is consistent with the canonical expectation of 3 massive neutrinos and no extra relativistic species to within 1σ when allowing for evolving dark energy and relaxing the strong inflation prior on the curvature and running. The agreement improves with the possibility of an early dark energy component, itself constrained to be less than 5% of the critical density (95% CL) in our expanded parameter space. In extensions of the standard cosmological model, the derived amplitude of linear matter fluctuations $\sigma_8$ is found to closely agree with low-redshift cluster abundance measurements. The sum of neutrino masses is robust to assumptions of the effective number of neutrinos, late-time dark energy, curvature, and running at the level of 1.2 eV (95% CL). The upper bound degrades to 2.0 eV (95% CL) when further including the early dark energy density and primordial helium abundance as additional free parameters. Even in extended cosmological parameter spaces, Planck alone could determine the possible existence of extra relativistic species at 4σ confidence and constrain the sum of neutrino masses to 0.2 eV (68% CL).

I. INTRODUCTION

Observations of the cosmic microwave background (CMB) [1-4], large-scale structure [5-7], and type Ia supernovae (SNe) [8, 9] have established a flat ΛCDM model, with nearly scale-invariant, adiabatic, Gaussian primordial fluctuations as providing a consistent description of the global properties of our universe. At the same time, we do not yet understand the microscopic identities of the dark energy (Λ), cold dark matter (CDM), and inflaton (primordial fluctuations) that enter our standard cosmological model.

The neutrino sector is another area that the standard model is yet unable to fully describe, with open questions related to the effective number of neutrinos $N_{\text{eff}}$ and their masses $m_\nu$. The effective number of neutrinos is sensitive to both the number of neutrinos along with additional particle species that were relativistic at the photon decoupling epoch (e.g. [2, 10]). A joint analysis of CMB data from WMAP7 with baryon acoustic oscillation (BAO) distances from SDSS+2dF and Hubble constant from HST reveals a weak preference for extra relativistic species ($N_{\text{eff}} = 4.34 \pm 0.87$) [4]. When further combined with small-scale CMB data from ACT or SPT, this preference mildly increases and reaches the 2σ level ($N_{\text{eff}} = 4.56 \pm 0.75$ with addition of ACT [11] and $N_{\text{eff}} = 3.86 \pm 0.42$ with addition of SPT [12]; further see [13, 20]).

A primary objective of this manuscript is to clarify how robust these recent indications of additional light degrees of freedom are to assumptions of the under-
of the spectral index, primordial helium abundance, and to the sum of neutrino masses, which we know is nonzero from neutrino oscillation experiments [30, 32].

Constraining $N_{\text{eff}}$ with cosmology is mainly achieved through tight CMB measurements of the redshift at matter-radiation equality $z_{\text{eq}}$, the baryon density $\Omega_{b} h^{2}$, the angular size of the sound horizon $\theta_{s}$, and the angular scale of photon diffusion $\theta_{d}$ [15]. Keeping $z_{\text{eq}}$ and $\Omega_{b} h^{2}$ fixed as $N_{\text{eff}}$ increases can be achieved by increasing the dark matter density $\Omega_{c} h^{2}$ (assuming massless neutrinos), which manifests in a large correlation with $N_{\text{eff}}$ (shown in Fig. 2). Meanwhile, an increase in $N_{\text{eff}}$ and $Y_{p}$ both yield an enhanced Silk damping effect [13, 34, 36], and by fixing $\theta_{d}$, it can be shown that $\theta_{d} \propto (1 + f_{\nu})^{0.22}/\sqrt{1 + Y_{p}}$ [15], where $f_{\nu} \equiv \rho_{\nu}/\rho_{c}$ is proportional to $N_{\text{eff}}$. As a consequence, the suppression of the CMB damping tail can be picked out as a signature of extra relativistic species when $Y_{p}$ is known, while the constraints on $N_{\text{eff}}$ are relaxed when allowing for $Y_{p}$ as a free parameter.

An increase in $N_{\text{eff}}$ further shifts the acoustic peak locations [36], but this has been shown to be a small effect [13]. Instead, the constraint on $N_{\text{eff}}$ can be improved by the inclusion of low-redshift distances and a prior on the Hubble constant, $H_{0}$, as these are useful in constraining $\Omega_{c} h^{2}$ and by extension $N_{\text{eff}}$. However, when allowing for evolving dark energy, the ability to improve constraints on $N_{\text{eff}}$ from observations of the expansion history becomes diminished, as illustrated by the error ellipses for $\{N_{\text{eff}}, \Omega_{c} h^{2}, w\}$ in Fig. 2. Therefore, the inclusion of SN data becomes critical to a precise determination of the effective number of neutrinos.

The dark energy equation of state (EOS) is moreover anti-correlated with the sum of neutrino masses [8, 44, 57, 38]. In the CMB temperature power spectrum, the sum of neutrino masses shifts the first peak position to lower multipoles by changing the fraction of matter to radiation at decoupling, which can be compensated by a reduction in the Hubble constant (similar to the case for positive universal curvature) [3, 35, 39]. BAO distances and an $H_{0}$ prior can therefore be used to reduce correlations between the sum of neutrino masses and the dark energy EOS, but also the curvature density.

The strongest limits on the sum of neutrino masses from the CMB combined with probes of the expansion history and matter power spectrum place it at sub-eV level [1–4, 39–56]. We take the conservative approach in only combining CMB data with low-redshift measurements of the expansion history. While SN observations play an important role in constraining the dark energy EOS and thereby reduce the correlation between $\sum m_{\nu}$ and $w$, these observations are not powerful in constraining the curvature of the universe and therefore less helpful in reducing the correlation between $\sum m_{\nu}$ and $\Omega_{k}$ (e.g. [3, 8, 9]).

Beyond the vanilla parameters and the three additional parameters $\{N_{\text{eff}}, \sum m_{\nu}, w\}$, we relax the commonly employed strong inflation prior on the universal curvature $\Omega_{k}$ and running of the spectral index $d n_{s}/d \ln k$. Given that most popular models of inflation predict $d n_{s} / d \ln k \approx 10^{-3}$ [57, 58] and $|\Omega_{k}| \approx 10^{-4}$ (e.g. [37, 59, 60]), at the level of precision of present CMB data it is generally justified to fix these two parameters to their fiducial values of zero. However, given the mild preference for $N_{\text{eff}} > 3$ [1–4, 11, 12], we allow for the possible existence of inflationary models with large curvature or running. In particular, $|\Omega_{k}| \approx 10^{-2}$ may be generated in models of open inflation in the context of string cosmology [57, 61], while a large negative running may be produced by multiple fields, temporary breakdown of slow-roll, or several distinct stages of inflation [57, 62, 64].

| TABLE II. Constraints on Cosmological Parameters using SPT+WMAP+$H_{0}$+BAO. |
|-----------------------------------------------|
| $\Omega_{c}$ | $\Omega_{b}$ | $\theta_{s}$ | $\theta_{d}$ | $\Omega_{k}$ | $w$ |
|-----------------------------------------------|
| $h^{2}$ | $10^{3}h^{2}$ | $10^{4}h^{2}$ | $\tau$ | $n_{s}$ | $\ln(10^{10}A_{s})$ |
|-----------------------------------------------|
| $10^{2}h^{2}$ | $2.237 \pm 0.038$ | $2.261 \pm 0.042$ | $2.238 \pm 0.039$ | $2.272 \pm 0.043$ | $2.271 \pm 0.043$ | $2.273 \pm 0.044$ |
| $10^{3}h^{2}$ | $11.22 \pm 0.28$ | $12.80 \pm 0.92$ | $11.11 \pm 0.29$ | $13.05 \pm 0.94$ | $12.5 \pm 1.1$ | $13.0 \pm 1.2$ |
| $10^{4}h^{2}$ | $104.12 \pm 0.15$ | $103.95 \pm 0.17$ | $104.15 \pm 0.15$ | $103.95 \pm 0.18$ | $104.07 \pm 0.29$ | $103.96 \pm 0.29$ |
| $\tau$ | $0.086 \pm 0.014$ | $0.086 \pm 0.014$ | $0.088 \pm 0.014$ | $0.090 \pm 0.015$ | $0.098 \pm 0.014$ | $0.090 \pm 0.015$ |
| $n_{s}$ | $0.9648 \pm 0.0092$ | $0.981 \pm 0.013$ | $0.9661 \pm 0.0096$ | $0.987 \pm 0.013$ | $0.983 \pm 0.013$ | $0.987 \pm 0.013$ |
| $\ln(10^{10}A_{s})$ | $3.195 \pm 0.034$ | $3.186 \pm 0.035$ | $3.184 \pm 0.035$ | $3.170 \pm 0.037$ | $3.180 \pm 0.036$ | $3.169 \pm 0.037$ |

Mean of the posterior distribution of cosmological parameters along with the symmetric 68% confidence interval about the mean. We report the 95% upper limit on the sum of neutrino masses $\sum m_{\nu}$. The primordial helium mass fraction $Y_{p}$ is enforced consistent with standard BBN unless we allow it to vary as a free parameter.


**TABLE III.** Constraints on Cosmological Parameters using SPT+WMAP+H$_0$+BAO.

| Case | wCDM | ΛCDM | wCDM | ΛCDM | wCDM | wCDM |
|------|------|------|------|------|------|------|
|      | +N$_{\text{eff}}$ +∑$m_{\nu}$ +N$_{\text{eff}}$ +∑$m_{\nu}$ +N$_{\text{eff}}$ +∑$m_{\nu}$ +N$_{\text{eff}}$ +∑$m_{\nu}$ +N$_{\text{eff}}$ +∑$m_{\nu}$ +Y$_{p}$ |      |      |      |      |      |
| Primary | 1000h$^2$ | 2.219 ± 0.042 | 2.272 ± 0.043 | 2.224 ± 0.061 | 2.244 ± 0.054 | 2.192 ± 0.068 | 2.168 ± 0.079 |
| | 1000h$^2$ | 11.44 ± 0.45 | 13.05 ± 0.94 | 12.96 ± 0.96 | 13.2 ± 1.0 | 12.4 ± 1.2 | 13.1 ± 1.7 |
| | $10^3$θ$_s$ | 104.09 ± 0.16 | 103.95 ± 0.18 | 103.97 ± 0.18 | 103.97 ± 0.19 | 104.06 ± 0.21 | 103.83 ± 0.44 |
| | τ | 0.083 ± 0.014 | 0.090 ± 0.015 | 0.086 ± 0.015 | 0.090 ± 0.015 | 0.088 ± 0.015 | 0.091 ± 0.016 |
| | n$_s$ | 0.958 ± 0.011 | 0.987 ± 0.013 | 0.968 ± 0.022 | 0.978 ± 0.015 | 0.955 ± 0.025 | 0.949 ± 0.027 |
| | ln(10$^{10}$A$_S$) | 3.216 ± 0.042 | 3.170 ± 0.037 | 3.211 ± 0.052 | 3.179 ± 0.045 | 3.210 ± 0.052 | 3.209 ± 0.057 |
| Extended | $w$ | $-1.10 ± 0.11$ | $-1.31 ± 0.30$ | $-1.46 ± 0.39$ | $-1.35 ± 0.41$ |
| | N$_{\text{eff}}$ | $4.00 ± 0.43$ | $3.59 ± 0.57$ | $3.74 ± 0.58$ | $3.10 ± 0.74$ | $3.38 ± 0.86$ |
| | ∑$m_{\nu}$ [eV] | $< 0.67$ | $< 1.2$ | $< 1.2$ | $< 1.2$ | $< 1.4$ |
| | $dn_{l}/dln(k)$ | $-0.011 ± 0.019$ | $-0.018 ± 0.019$ | $-0.033 ± 0.031$ |
| | 1000h$^2$ | $-0.971 ± 0.019$ | $0.75 ± 0.93$ | $0.13 ± 0.99$ | $0.76 ± 1.5$ |
| | Y$_{p}$ | $0.00$ | $0.00$ | $0.00$ | $0.00$ | $0.00$ |
| Derived | $\sigma_8$ | $0.848 ± 0.049$ | $0.798 ± 0.053$ | $0.775 ± 0.063$ | $0.768 ± 0.070$ | $0.803 ± 0.085$ | $0.779 ± 0.091$ |

Same as Table III but with the addition of $\{w, dn_{l}/dln(k), \Omega_m\}$. Due to the large correlation between $n_s$ and $dn_{l}/dln(k)$ at our pivot scale $k_0 = 0.002$/Mpc, we quote values for $n_s$ at a less correlated scale $k_0 = 0.015$/Mpc. For the "wCDM+Λ_{CDM}+∑$m_{\nu}$ + $dn_{l}/dln(k)$ + Ω$_m$" case where $N_{\text{eff}}$ is closest to to the boundary at 3, we also considered a run where we impose a hard prior of $N_{\text{eff}} > 3$. Here, we find $N_{\text{eff}} = 3.65 ± 0.82, 4.62 ± 3.00$ where the two sets of upper and lower boundaries denote 68% and 95% C.Ls, respectively. The changes to the sum of neutrino masses and other parameters that weakly correlate with $N_{\text{eff}}$ are small (< 10%). While all within $1\sigma$, the largest changes are seen in 1000h$^2$ = 13.17±0.97 (compared to 1000h$^2$ = 12.4±1.2), $w = -1.25±0.30$ (compared to $w = -1.46±0.39$), $n_s = 0.971±0.019$ (compared to $n_s = 0.955±0.025$), $dn_{l}/dln(k) = -0.018±0.019$, and 1000h$^2$ = 0.2 ± 1.1 (compared to 1000h$^2$ = 0.13 ± 0.99). This particular configuration of parameter space and datasets shows the largest extent to which parameters may change with an $N_{\text{eff}} > 3$ prior as compared to our other runs. The changes to the parameters are more modest when including SNs because of the preference for larger values of $N_{\text{eff}}$, as seen in Table IV.

energy, $w$, close to $-1$ [55, 72]. Like a cosmological constant, these models are fine-tuned to have dark energy dominate today. However, the requirement $w > -1$ currently, does not imply that dark energy was negligible at earlier times, specifically redshift $z > 2$, where we have no direct constraints. Given the degeneracy between dark energy and the sum of neutrino masses, we further consider a model that describes dark energy as non-negligible in the early universe in Sec. III.D.

We describe our analysis method in Section II. In Section III, we provide constraints on a ΛCDM model with three massive neutrinos and additional light degrees of freedom, then follow up with successive additions of a constant dark energy equation of state, universal curvature, running of the spectral index, and primordial helium abundance (all parameters defined in Table II). We also explore the constraints for a time-varying dark energy equation of state, including an early dark energy model. Lastly, we compare the constraints from present data to the constraints expected from the Planck experiment. Section IV concludes with a discussion of our findings.

II. METHODOLOGY

We employed a modified version of CosmoMC [73, 74] in performing Markov Chain Monte Carlo (MCMC) analyses of extended parameter spaces with CMB data from WMAP7 [4] and SPT [12], BAO distance measurements from SDSS+2dFGRS [75], the Hubble constant from HST [13], and SN distances from the SCP Union2 compilation [76]. In determining the convegence of our chains, we used the Gelman and Rubin $R$ statistic [77], where $R$ is defined as the variance of chain means divided by the mean of chain variances. To stop the runs, we generally required the conservative limit $(R - 1) < 10^{-2}$, and checked that further exploration of the tails does not change our results.

The CMB temperature and E-mode polarization power spectra were obtained from a modified version of the Boltzmann code CAMB [78, 79]. We approximated the effect of a dark energy component with a time-varying EOS by incorporating the PPF module by Fang, Hu, & Lewis (2008) [80] into CosmoMC. Given that the small scale CMB measurements of SPT come with much smaller error bars than ACT [11, 12], the further inclusion of the ACT dataset would not lead to significant improvements in our constraints, as we explicitly checked.

All parameters are defined in Table II. In our analyses, we always include the "vanilla" parameters, given by the full set $\{\Omega_b h^2, \Omega_c h^2, \theta_s, \tau, n_s, \ln(10^{10}A_S)\}$. Our constraints correspond to the mean of the posterior distribution of cosmological parameters along...
TABLE IV. Constraints on Cosmological Parameters using SPT+WMAP+$H_0$+BAO+SNe.

| WCDM | wCDM | wCDM | wCDM |
|------|------|------|------|
| +$N_{\text{eff}} + \sum m_{\nu}$ | +$N_{\text{eff}} + \sum m_{\nu}$ | +$N_{\text{eff}} + \sum m_{\nu} + Y_p$ | +$N_{\text{eff}} + \sum m_{\nu} + Y_p$ |
| Primary | 100$\Omega_k h^2$ | 2.223 ± 0.041 | 2.257 ± 0.048 | 2.226 ± 0.059 | 2.171 ± 0.080 |
| | 100$\Omega_k h^2$ | 11.36 ± 0.41 | 13.14 ± 0.94 | 13.1 ± 1.0 | 14.0 ± 1.4 |
| | $10^4 \theta_0$ | 104.10 ± 0.16 | 103.95 ± 0.17 | 103.99 ± 0.18 | 103.66 ± 0.36 |
| | $\tau$ | 0.082 ± 0.014 | 0.088 ± 0.015 | 0.089 ± 0.016 | 0.090 ± 0.015 |
| | $n_s$ | 0.960 ± 0.010 | 0.981 ± 0.015 | 0.970 ± 0.019 | 0.953 ± 0.026 |
| | $\ln (10^{10} A_s)$ | 3.209 ± 0.039 | 3.185 ± 0.041 | 3.194 ± 0.049 | 3.197 ± 0.053 |
| Extended | $w$ | −1.049 ± 0.072 | −1.09 ± 0.11 | −1.10 ± 0.11 | −1.13 ± 0.12 |
| | $N_{\text{eff}}$ | — | 3.88 ± 0.44 | 3.58 ± 0.60 | 3.78 ± 0.61 |
| | $\sum m_{\nu}$ [eV] | — | < 0.92 | < 1.2 | < 1.7 |
| | $dn/k$ [d ln k] | — | — | −0.013 ± 0.019 | −0.035 ± 0.030 |
| | 100$\Omega_k$ | — | — | 0.64 ± 0.95 | 1.2 ± 1.1 |
| | $Y_p$ | — | — | — | 0.176 ± 0.079 |

Same as Table III but with the addition of supernova distance measurements from the Union2 compilation. For the case “wCDM+$N_{\text{eff}} + \sum m_{\nu} + dn/s/d\ln k + \Omega_b$” we also considered a run where we impose a hard prior of $N_{\text{eff}} \geq 3$. Here, we find $N_{\text{eff}} = 3.74 \pm 0.92, 4.68$ where the two sets of upper and lower boundaries denote 68% and 95% CLs, respectively. The largest changes this prior induces in other parameters are in $n_s = 0.974 \pm 0.017$ (compared to $n_s = 0.970 \pm 0.019$), $dn/s/d\ln k = −0.010 \pm 0.017$ (compared to $dn/s/d\ln k = −0.013 \pm 0.019$), and $100\Omega_b = 0.52 \pm 0.92$ (compared to $100\Omega_b = 0.64 \pm 0.95$). All other parameters are modestly affected by our choice of prior ($< 10\%$).

with the symmetric 68% confidence interval about the mean. We impose uniform priors on the cosmological parameters, and let the prior ranges to be significantly larger than the posterior, such that the parameter estimates are unaffected by the priors. When including the sum of neutrino masses and early dark energy density, we report the 95% upper limit on these parameters.

When allowing for nonzero neutrino rest mass, we distribute the sum of neutrino masses ($\sum m_{\nu} = 94$ eV $\Omega_\nu h^2$) equally among 3 active neutrinos. We treat additional contributions to $N_{\text{eff}}$ as massless, such that $N_{\text{eff}} = (3 + N_{\text{ml}})$, where $N_{\text{ml}}$ denotes the massless degrees of freedom. In principle, these additional states could be massive, for example see recent treatments in Refs. [12, 21].

Since we impose $1.047 < N_{\text{eff}} < 10$, the number of relativistic species is always positive at early times. At late times, our prior on $N_{\text{eff}}$ implies that the number of relativistic species can be negative when the three active neutrinos are massive ($−1.953 < N_{\text{ml}} < 7$). However, the total radiation energy density is always positive (at late times $\propto 1 + 0.227N_{\text{ml}}$ when the three active neutrinos are massive). We choose this particular prior on $N_{\text{eff}}$ in order for the data set to rule out a given part of parameter space. In Figs 241 we find that the marginalized contours on $N_{\text{eff}}$ close before the lower end of our prior, such that the data itself is constraining the radiation content from below. For completeness, we also considered several conventional runs with the prior $N_{\text{eff}} \geq 3$, such that $N_{\text{ml}} \geq 0$, and we find no qualitative changes in our results. For complete details, see the captions of Tables III, IV, VI.

As part of our analysis of extended parameter spaces, we consider cases with the primordial fraction of baryonic mass in helium $Y_p$ as an unknown parameter to be determined by the data. However, when we do not allow $Y_p$ to vary freely, it is determined in a BBN-consistent manner within CAMB via the PArthENoPE code [61], which enforces

$$Y_p = 0.2485 + 0.0016 \left[ (273.99h^2 - 6) + 100(S - 1) \right].$$

Here $S = \sqrt{1 + (7/4\Delta N_\nu)}$ encapsulates deviations from standard BBN [32, 64], and we let $\Delta N_\nu = (N_{\text{eff}} - 3.046)$ in agreement with the SPT analysis. Aside from the derived limits on $Y_p$, we explicitly checked that our results do not significantly change ($< 10\%$) when passing $\Delta N_\nu = 0$ to PArthENoPE instead.

Furthermore, in our analysis we either consider “enforcing the strong inflation prior” on the curvature and running, by which $\{\Omega_k \equiv 0, dn/s/d\ln k \equiv 0\}$, or “relaxing the strong inflation prior” such that $\{\Omega_k, dn/s/d\ln k\}$ are allowed to vary as free parameters to be constrained by the data. We define the running of the spectral index via the dimensionless power spectrum of primordial curvature perturbations:

$$\Delta^2 R(k) = \Delta^2 R(k_0) \left( \frac{k}{k_0} \right)^{n_s - 1 + \frac{1}{2} \ln(k/k_0)dn/s/d\ln k},$$

where the pivot scale $k_0 = 0.002$ Mpc. Due to the large correlation between $n_s$ and $dn/s/d\ln k$ at this scale, we always quote our values for $n_s$ at a scale $k_0 = 0.015$ Mpc, where the tilt and running are
III. RESULTS

We now explore the constraints on extended parameter spaces with the CMB (WMAP7+SPT), BAO distances (SDSS+2dFGRS), and an HST prior on the Hubble constant. Beginning with Sec. III C, we always also consider SN distance measurements from the Union2 compilation. In Sec. III F, we discuss the expected constraints from Planck [111, 112].

A. ΛCDM with Massive Neutrinos

1. Enforcing the inflation prior on \( \{ \Omega_k, d\sigma_s/d\ln k \} \)

In Table IV, we begin by allowing the effective number of neutrinos, sum of neutrino masses, and primordial helium abundance to vary as free parameters, both separately and jointly, in a ΛCDM universe.

First for ΛCDM alone, then with \( N_{\text{eff}} \) and \( Y_p \) added separately, we reproduce the results in Ref. [12]. In particular, with \( N_{\text{eff}} = 3.87 \pm 0.42 \) in the parameter space given by “vanilla+\( N_{\text{eff}} \),” we recover the reported 2σ deviation from canonical \( N_{\text{eff}} = 3.046 \) [11, 12].

Given the well-known degeneracy between \( N_{\text{eff}} \) and \( Y_p \) [11, 12, 13] (also see discussion in Sec. I), we find \( N_{\text{eff}} = 3.70 \pm 0.54 \) when further allowing \( Y_p \) to vary as a free parameter irrespectively of the BBN expectation. Allowing for three active neutrinos to have mass, and treating additional contributions to \( N_{\text{eff}} \) as massless, we find an even larger deviation with the standard value as \( N_{\text{eff}} = 4.00 \pm 0.43 \) for the parameter combination “vanilla+\( N_{\text{eff}}+\sum m_\nu \)” (consistent with [12]).

Here, the upper bound on the sum of neutrino masses is 0.67 eV (95% CL) and we find the spectral index to be consistent with unity within 1σ (\( n_s = 0.987 \pm 0.013 \)). The neutrino mass constraint is to be compared with 0.45 eV at 95% CL in “vanilla+\( \sum m_\nu \)” (consistent with [12]). This 0.45 eV constraint is competitive with the robust upper bound of 0.36 eV when including CMASS [11], the conservative upper bound of 0.34 eV from the MegaZ photometric redshift catalog of luminous red galaxies [12], and the conservative upper bound of 0.41 eV from the CFHTLS galaxy angular power spectrum [13].

When we consider “vanilla+\( N_{\text{eff}}+\sum m_\nu+Y_p \),” the statistical significance of the \( N_{\text{eff}} \) deviation is reduced from 2.2σ to 1.6σ, and the upper bound on the sum of neutrino masses moderately weakens to 0.73 eV (95% CL). While the primordial helium abundance from the CMB+BAO+\( H_0 \) has been found mildly in tension (\( \sim 2\sigma \)) [11, 12] with that from observations of metal-poor extragalactic H II regions [80–84], we find constraints on \( Y_p \) consistent to within 1σ with these observations. This is mainly due to the strong negative correlation between \( Y_p \) and \( N_{\text{eff}} \) (as reported in [11, 12, 15]) and detailed in Sec. I. For instance, Aver, Olive, & Skillman (2011) [86] determine \( Y_p = 0.2534 \pm 0.0083 \) via an MCMC analysis that accounts for both statistical and systematic uncertainties, which agrees with \( Y_p = 0.277 \pm 0.037 \) in “vanilla+\( N_{\text{eff}}+Y_p \)” and \( Y_p = 0.261 \pm 0.039 \) in “vanilla+\( N_{\text{eff}}+\sum m_\nu+Y_p \)”.

Moreover, when \( N_{\text{eff}} \) and \( \sum m_\nu \) are analyzed in a joint setting, we find that the data is both consistent with higher values of \( \Omega_k h^2 = 0.13 \pm 0.01 \) and lower values of \( \sigma_8 = 0.80 \pm 0.05 \), which perfectly agrees with low-redshift measurements of \( \sigma_8 \) from the abundance of clusters [92, 96] (as also noted in Ref. [12]). This is because the amount of suppression in mat-
ter clustering by the free-streaming of light neutrinos increases with mass [2, 97–99], which gives a large anti-correlation between $\sum m_\nu$ and $\sigma_8$, an example of which can be seen in Fig. [1].

2. Relaxing the inflation prior on $\{\Omega_k, dn_s/d\ln k\}$

Let us now relax the strong inflation prior on the curvature of the universe and running of the spectral index by considering the parameter combination “vanilla+$N_{\text{eff}}+\sum m_\nu+\sigma_8/d\ln k+\Omega_k” in Table III.

Here, $N_{\text{eff}}$ becomes increasingly consistent with the canonical value at 1.2$\sigma$ (down from 2.2$\sigma$), mainly as a result of the positive correlation with $dn_s/d\ln k$, which also brings the tilt down to $n_s = 0.978 \pm 0.015$ (from $n_s = 0.987 \pm 0.013$). Further, we find that the correlation between $\sum m_\nu$ and $\Omega_k$ degrades the upper bound on the sum of neutrino masses by close to a factor of 2 to $\sum m_\nu < 1.2$ eV (95\% CL). As a consequence of the well known anti-correlation with the sum of neutrino masses, which increases when relaxing the strong inflation prior, the amplitude of matter fluctuations is seen to prefer smaller values at $\sigma_8 = 0.768 \pm 0.070$ (as compared to $\sigma_8 = 0.798 \pm 0.053$ when $dn_s/d\ln k$ and $\Omega_k$ are held fixed).

However, as the strong inflation prior is relaxed, both the running and curvature are consistent with zero to 1$\sigma$. It is therefore far from certain that shifts in parameters other than $\{\Omega_k, dn_s/d\ln k\}$ that come about from relaxing the strong inflation prior are true manifestations that will hold with improved data.

B. $w$CDM with Massive Neutrinos

1. Enforcing the inflation prior on $\{\Omega_k, dn_s/d\ln k\}$

In the previous section, we considered cases with neutrinos as massless and cases with neutrinos as massive. However, as it is well established that neutrinos are indeed massive [30–33], we account for the sum of neutrino masses as a free parameter in all further treatments of neutrinos. In Table III we explore possible degeneracies between $\{N_{\text{eff}}, \sum m_\nu\}$ and a constant EOS of the dark energy ($w \neq -1$).

Beginning with “vanilla+$w$,” we constrain a con-
stant dark energy EOS: $w = -1.10 \pm 0.11$ (as compared to $w = -1.10 \pm 0.14$ without SPT). Considering $w$ in conjunction with “vanilla+$N_{\text{eff}}+\sum m_\nu$” we find a reduction in $N_{\text{eff}} = 3.59 \pm 0.57$ (down from $N_{\text{eff}} = 4.00 \pm 0.43$), rendering it consistent with the canonical value to within 1σ. This is caused by the $w - N_{\text{eff}}$ correlation discussed in Sec. I and shown in Fig. 2. Expectedly, we also find a correlation between the dark energy EOS and the sum of neutrino masses (discussed in Sec. II), the latter of which degrades by close to a factor of 2 to $\sum m_\nu < 1.2$ eV (95% CL).

The joint impact of $\{N_{\text{eff}}, \sum m_\nu\}$ on the dark energy EOS is to weaken the constraint on it by roughly a factor of 3 to $w = -1.31 \pm 0.30$. Moreover, with the introduction of $w$, the amplitude of linear matter fluctuations is mildly shifted to smaller values at $\sigma_8 = 0.775 \pm 0.063$ (compared to $\sigma_8 = 0.798 \pm 0.053$) because of the anti-correlation between $w$ and $\sigma_8$ that mainly enters through the growth function (e.g. see [3]). The spectral index shifts further away from unity to $n_s = 0.968 \pm 0.022$ (down from $n_s = 0.987 \pm 0.013$).

2. Relaxing the inflation prior on $\{\Omega_k, dn_s/d\ln k\}$

In Table III we now consider the parameter combination “vanilla+$N_{\text{eff}}+\sum m_\nu+w$” in conjunction with the running of the spectral index and curvature of the universe. We also consider a case with the primordial helium abundance as a free parameter.

Given that we already identified separate degeneracies between $N_{\text{eff}} - dn_s/d\ln k$ and $N_{\text{eff}} - w$ in past sections, it is not surprising that we obtain $N_{\text{eff}} = 3.10 \pm 0.74$ to be in even closer agreement with the canonical value for our extended parameter space. This is a result of the even more negative values preferred by $dn_s/d\ln k = -0.018 \pm 0.019$ and $w = -1.46 \pm 0.39$, shown in Figs. 2 and 3. However, the upper bound on $\sum m_\nu < 1.2$ eV is robust to the further expansion of the parameter space, such that this bound holds for all three cases: “vanilla+$N_{\text{eff}}+\sum m_\nu+w$”, “vanilla+$N_{\text{eff}}+\sum m_\nu+\Omega_k+dn_s/d\ln k””, as well as “vanilla+$N_{\text{eff}}+\sum m_\nu+w+\Omega_k+dn_s/d\ln k””.

In addition, when allowing for $Y_p$ as an independent parameter (i.e. considering the case “vanilla+$N_{\text{eff}}+\sum m_\nu+w+\Omega_k+dn_s/d\ln k+Y_p$”), the upper bound on the sum of neutrino masses is only mildly weakened, while the error bars are large enough that the effective number of neutrinos is consistent with values of both 3 and 4 (to within 68% CL). In all of the above cases we continue to find $\sigma_8$ consistent with that from cluster abundance measurements to within 68% CL, while the spectral index is consistent with unity to within 95% CL.

While our results are based on the construction of 3 massive neutrinos, and $(N_{\text{eff}}-3)$ massless degrees of freedom (as discussed in Sec. I), we also considered imposing a hard prior of $N_{\text{eff}} \geq 3$ in a new run with
TABLE V. Constraints on Cosmological Parameters using SPT+WMAP+$H_0+BAO+SNe$.

| Parameter | $w(a)CDM$ | $w(a)CDM$ | $w(a)CDM$ | $w(a)CDM$ |
|-----------|-----------|-----------|-----------|-----------|
| $\Omega_k h^2$ | 2.226 ± 0.042 | 2.249 ± 0.047 | 2.224 ± 0.057 | 2.163 ± 0.078 |
| $\Omega_b h^2$ | 11.37 ± 0.46 | 13.3 ± 1.1 | 13.3 ± 1.1 | 14.1 ± 1.4 |
| $10^3 \theta_s$ | 104.11 ± 0.15 | 103.96 ± 0.18 | 103.99 ± 0.18 | 103.66 ± 0.36 |
| $\tau$ | 0.083 ± 0.014 | 0.088 ± 0.015 | 0.089 ± 0.016 | 0.090 ± 0.016 |
| $n_s$ | 0.963 ± 0.011 | 0.978 ± 0.015 | 0.979 ± 0.019 | 0.950 ± 0.026 |
| $\ln(10^{10} A_s)$ | 3.203 ± 0.041 | 3.194 ± 0.044 | 3.194 ± 0.054 | 3.200 ± 0.055 |

Extended

| Parameter | $w_0$ | $w_a$ | $N_{\text{eff}}$ | $\sum m_\nu$ [eV] | $d_{\text{ln} k}$ |
|-----------|--------|--------|----------------|-------------------|----------------|
| $w_0$     | −1.10 ± 0.17 | −1.05 ± 0.19 | −1.08 ± 0.21 | −1.12 ± 0.21 | $< 1.2$ |
| $w_a$     | 0.20 ± 0.64 | −0.4 ± 1.0 | −0.3 ± 1.2 | −0.3 ± 1.2 | $< 1.4$ |
| $N_{\text{eff}}$ | — | 3.84 ± 0.45 | 3.57 ± 0.59 | 3.75 ± 0.68 | $< 1.8$ |
| $\sum m_\nu$ [eV] | — | $< 1.2$ | $< 1.4$ | $< 1.8$ | $< 1.8$ |
| $d_{\text{ln} k}$ | — | — | −0.012 ± 0.020 | −0.038 ± 0.030 | — |
| $100 \Omega_k$ | — | — | 0.7 ± 1.1 | 1.3 ± 1.2 | $0.168 ± 0.079$ |
| $Y_\nu$ | — | — | — | — | — |

Derived

| Parameter | $\sigma_8$ | $\sigma_8$ | $\sigma_8$ | $\sigma_8$ |
|-----------|------------|------------|------------|------------|
| $\sigma_8$ | 0.827 ± 0.046 | 0.779 ± 0.062 | 0.763 ± 0.076 | 0.736 ± 0.084 |

Same as Table IV but for a time-dependent parameterization of the dark energy equation of state, of the form $w(a) = w_0 + (1 - a)w_a$ (as opposed to time-independent $w$).

“vanilla+$N_{\text{eff}}$+$\sum m_\nu$+$\Omega_k$+$d_{\text{ln} k}$" , as this is the case where $N_{\text{eff}}$ would be the most affected by the prior. Given the weak correlation between $\sum m_\nu$ and $N_{\text{eff}}$, the upper bound on the sum of neutrino masses doesn’t change, while the data is still consistent with no extra relativistic species as $N_{\text{eff}} = 3.65 \pm 3.00, 3.60$, where the two sets of upper and lower boundaries denote 68% and 95% CLs, respectively. It is clear that our findings are qualitatively unchanged with this alternative choice of prior on the effective number of neutrinos (also see captions of Tables III and IV).  

C. $wCDM$ with Massive Neutrinos, Running, and Curvature: Including Supernovae

Since much of the work in bringing $N_{\text{eff}}$ in agreement with the canonical value is done by the possibility of evolving dark energy, for which the constraints from the CMB, $H_0$, and BAO measurements that we have considered are relatively weak, we further include SN data from the Union2 compilation in order to more effectively constrain a constant dark energy EOS and parameters with which it strongly correlates.

In Table IV we find that the addition of SN observations help constrain the dark energy EOS to $w = −1.05 ± 0.07$ when analyzed along with the vanilla parameters (35% reduction in uncertainty compared to no SNe). This constraint degrades to $−1.10 ± 0.11$ when expanding the parameter space to further include $\{N_{\text{eff}}, \sum m_\nu, \Omega_k, d_{\text{ln} k}\}$, but is still a factor of 4 stronger than the equivalent case where SNe are not included in the analysis. Improving the constraint on $w$ is helpful in breaking much of the degeneracy between dark energy and the effective number of neutrinos, resulting in $N_{\text{eff}} = 3.88 ± 0.44$ for the case of “vanilla+$N_{\text{eff}}$+$\sum m_\nu$" (as compared to $N_{\text{eff}} = 3.59 ± 0.57$ without SNe, and as compared to $N_{\text{eff}} = 4.00 ± 0.43$ with a prior $w = −1$). However, as before, when relaxing the strong inflation prior on $\{\Omega_k, d_{\text{ln} k}\}$ we find the effective number of neutrinos becomes consistent with the canonical value to 68% CL (as $N_{\text{eff}} = 3.58 ± 0.60$).

Low-redshift SN measurements are useful in reducing the correlation between $\{\sum m_\nu, w, H_0\}$, which drives the 1.2 eV (95% CL) upper bound on the sum on neutrino masses for the case “vanilla+$N_{\text{eff}}$+$\sum m_\nu$" down to 0.9 eV (Tables II and III). However, since SN observations do not much improve the constraint on the curvature when added to CMB+$H_0+BAO$ (shown in Fig. 2), the SNe are unable to lower the upper bound on the sum of neutrino masses from 1.2 eV (95% CL) when relaxing the strong inflation prior (i.e. for the case “vanilla+$N_{\text{eff}}$+$\sum m_\nu$+$\Omega_k$+$d_{\text{ln} k}$”). The parameter that most strongly increases the upper bound on the sum of neutrino masses when singularly added to “vanilla+$\sum m_\nu$" is the curvature, which renders $\sum m_\nu < 1.0$ eV (95% CL) in “vanilla+$\sum m_\nu$+$\Omega_k$.”

Expanding the parameter space to allow $Y_\nu$ to vary as an independent parameter (i.e. considering “vanilla+$N_{\text{eff}}$+$\sum m_\nu$+$\Omega_k$+$d_{\text{ln} k}$+$Y_\nu$"), we find a mild shift in $N_{\text{eff}} = 3.78 ± 0.61$ (as compared to $N_{\text{eff}} = 3.58 ± 0.60$), and a stronger shift in $\sum m_\nu < 1.7$ eV (as compared to $\sum m_\nu < 1.2$ eV at 95% CL). Meanwhile, $Y_\nu = 0.176 ± 0.079$ shows a preference for lower values but is still consistent with measurements of $Y_\nu$ from low-metallicity H II regions [89-91]. For all of the non-minimal cases considered in Table IV $n_s$ is consistent with unity to at least 95%
TABLE VI. Constraints on Cosmological Parameters using SPT+WMAP+$H_0+BAO+SN$.

|            | eCDM | eCDM | eCDM | eCDM |
|------------|------|------|------|------|
|            | $+N_{\text{eff}}+\sum m_\nu$ | $+N_{\text{eff}}+\sum m_\nu$ | $+N_{\text{eff}}+\sum m_\nu+Y_p$ | $+N_{\text{eff}}+\sum m_\nu+Y_p$ |
| Primary    | $100\Omega_0h^2$ | 2.223 ± 0.041 | 2.256 ± 0.047 | 2.196 ± 0.058 | 2.173 ± 0.081 |
|           | $100\Omega_0h^2$ | 11.56 ± 0.42 | 13.23 ± 0.96 | 13.4 ± 1.1 | 13.9 ± 1.5 |
|           | $10^4\theta$ | 104.00 ± 0.17 | 103.89 ± 0.18 | 103.89 ± 0.19 | 103.72 ± 0.39 |
|           | $\tau$ | 0.085 ± 0.014 | 0.090 ± 0.015 | 0.091 ± 0.015 | 0.094 ± 0.016 |
|           | $n_s$ | 0.966 ± 0.011 | 0.984 ± 0.015 | 0.966 ± 0.019 | 0.957 ± 0.027 |
|           | $\ln(10^{10}A_s)$ | 3.194 ± 0.041 | 3.174 ± 0.042 | 3.176 ± 0.051 | 3.182 ± 0.055 |

Extended

$w_0$ | −1.082 ± 0.079 | −1.11 ± 0.10 | −1.16 ± 0.12 | −1.17 ± 0.14 |
$\Omega_c$ | < 0.030 | < 0.025 | < 0.049 | < 0.049 |
$N_{\text{eff}}$ | — | 3.85 ± 0.43 | 3.24 ± 0.63 | 3.37 ± 0.68 |
$\sum m_\nu$ [eV] | — | < 0.96 | < 1.6 | < 2.0 |
$\sum \nu s$ | — | — | −0.023 ± 0.021 | −0.034 ± 0.032 |
$100\Omega_b$ | — | — | 1.5 ± 1.2 | 1.7 ± 1.3 |
$Y_p$ | — | — | — | 0.206 ± 0.086 |

Derived

$\sigma_8$ | 0.805 ± 0.045 | 0.773 ± 0.063 | 0.703 ± 0.095 | 0.692 ± 0.097 |

Same as Table [IV] but for an early dark energy model with present EOS $w_0$ and density at high redshift $\Omega_c$ (as opposed to time-independent $w$). We report the 95% upper limit on $\Omega_c$ (and $\sum m_\nu$ as before). For the “$eCDM+N_{\text{eff}}+\sum m_\nu+dn_s/d\ln k+\Omega_k$” case, we also considered a run where we impose a hard prior of $N_{\text{eff}} \geq 3$. Here, we find $N_{\text{eff}} = 3.60^{+4.47}_{-3.09} \pm 3.00$, where the two sets of upper and lower boundaries denote 68% and 95% CLs, respectively. The largest changes this prior induces in other parameters are seen in $n_s = 0.975 \pm 0.015$ (compared to $n_s = 0.966 \pm 0.009$), $dn_s/d\ln k = -0.014 \pm 0.017$ (compared to $dn_s/d\ln k = -0.023 \pm 0.021$), $100\Omega_b = 1.0 \pm 1.1$ (compared to $100\Omega_b = 1.5 \pm 1.2$), and $\Omega_c < 0.042$ (as compared to $\Omega_c < 0.049$). All of the other parameters are modestly affected (< 10%) by our choice of prior on $N_{\text{eff}}$. For the vanilla+$w_0+\Omega_k$ case, we also considered a run with a hard prior $w > -1$, for which we find $\Omega_c < 0.023$ at 95% CL (as compared to $\Omega_c < 0.019$ in Ref. [108]).

For the particular parameter combination that shifts $N_{\text{eff}}$ the closest to a value of 3 from above (i.e., “vanilla+$N_{\text{eff}}+\sum m_\nu+w+\Omega_k+dn_s/d\ln k$”), we also considered a run with the prior $N_{\text{eff}} \geq 3$ imposed. Here, we continue to find the effective number of neutrino species to be consistent with the standard value, as $N_{\text{eff}} = 3.74^{+3.92}_{-3.00} \pm 3.60^{+4.68}_{-3.00}$, where the two sets of upper and lower boundaries denote 68% and 95% CLs, respectively. The constraints on other parameters such as $w$ and $\sum m_\nu$ change by less than 10% with this alternative choice of prior.

As compared to the case with a constant dark energy EOS, we find that the new parameterization for late-time dark energy doesn’t significantly change our constraints on other cosmological parameters. Expectedly, the two most sensitive parameters are $\Omega_k$.
FIG. 4. Joint two-dimensional marginalized constraints on the early dark energy density $\Omega_e$ against $\{N_{\text{eff}}, \sum m_{\nu}\}$ for the extended parameter combination “vanilla+$N_{\text{eff}}+\sum m_{\nu}+w_0+w_a$.” When we instead use SNe from the “Constitution” compilation [106], we find $w_0 = -0.93 \pm 0.13$ and $w_a = -0.36 \pm 0.65$, which are consistent with the constraints on these parameters in Ref. [4] (perfect agreement when excluding SPT). The difference in constraints may be traced to the larger number of SNe in the Union2 compilation (557 SNe) as compared to the Constitution compilation (397 SNe), along with the use of the SALT2 light curve fitter for the Union2 compilation as compared to the SALT fitter for the Constitution compilation. Clearly, precise SN measurements are critical to understanding the true values of these EOS parameters. In further extensions of our parameter space, the constraint on $w_0$ degrades by up to 20%, while the constraint on $w_a$ degrades by up to a factor of 2.

2. Early Dark Energy

Late-time dark energy models suffer from the well known coincidence problem. The value of the dark energy density has to be fine-tuned so that it only affects the dynamics of the universe at present. This coincidence problem motivates the exploration of models in which the evolution of the dark energy density is such that it is large enough to affect the universal dynamics even at $z > 2$.

A realization of early dark energy (EDE) is given by the “tracker” parameterization of Doran & Robbers (2006) [107], where the dark energy tracks the dominant component in the universe. In a sense, it is simpler to parameterize the dark energy density evolution directly, rather than express it in terms of an evolving equation of state. We use a modified form of the original parameterization that tracks the equation of state of the dominant energy [107, 108],

$$\Omega_d(z) = \frac{\Omega_{d0}}{h^2_e(z)} \left(1 + \frac{(1 + z)^{3+3w_0}}{h_w(z)}\right),$$

where $\Omega_{d0}$ is the dark energy density at large redshift and $\Omega_{d}(0) = \Omega_{d0}$. We use $v(z) = 1 - (1 + z)^{3w_0}$ [107], but any other parameterization such that $d\ln(v)/d\ln(z) = O(1)$ will give similar results. Note that the first term proportional to $\Omega_{d0}$ is the dark energy density as a function of redshift for a model with present density of dark energy $\Omega_{d0}$ and constant EOS $w_0$. Thus, in this parameterization with early dark energy, the effect on the dynamics of the universe at low redshift is the same as a model with constant EOS.

By approximating the effect of a dark energy component with time-varying EOS using the PPF module of Ref. [80], we allow $w_0$ to freely vary above and below the $w = -1$ boundary, unlike the treatments in Refs. [26, 109]. We compute the equation of state using the expression $w(z) = -1 + \frac{(1+z) d\ln(\Omega_d(z)) H^2(z)}{3 d\ln z}$, where $H(z)$ is the Hubble parameter in a universe with radiation, matter, curvature, and dark energy
out a run with an explicit $N_{\text{eff}} \geq 3$ prior, finding $N_{\text{eff}} = 3.60^{+3.74}_{-3.00} \times 3.47^{+4.47}_{-3.00}$, where the two sets of upper and lower boundaries denote 68% and 95% CLs, respectively. This prior lowers the upper bound on the EDE density to $\Omega_c < 0.042$ at 95% CL (as compared to $\Omega_c < 0.049$), while the constraints on other parameters such as $w$ and $\sum m_\nu$ change by less than 10%. When further including $\Gamma_0$ as a free parameter, we find qualitatively modest changes in our constraints, similar in nature to those discussed in sections IIIA and IIIc.

### E. Parameter Forecasts for Planck

Having discussed the present status of constraints on expanded parameter spaces with CMB, $H_0$, BAO, and SN measurements, we next explore the constraints from CMB temperature, E-mode polarization, and lensing potential power spectrum measurements with Planck. To this end, we employed a Fisher matrix formalism, such that the parameter covariance matrix is given by the inverse of

$$ F_{\alpha\beta} = \sum_\ell \text{Tr} \left[ \tilde{C}_\ell^{-1} \partial^2 C_\ell \partial \tilde{C}_\ell^{-1} \partial^2 C_\ell \right], \quad (5) $$

where the CMB temperature ($T$), E-mode polarization ($E$), and lensing potential ($\phi$) power spectra enter the symmetric matrix

$$ C_\ell = \begin{pmatrix} C_\ell^{\phi\phi} & C_\ell^{\phi E} & 0 \\ C_\ell^{E\phi} & C_\ell^{ EE} & C_\ell^{ E\phi} \\ 0 & C_\ell^{ E\phi} & C_\ell^{ E E} \end{pmatrix}. \quad (6) $$

The noise power spectra contribute additively to $C_\ell^{ab}(\ell) = f_{\text{sky}}^{1/2} \left( 2/(2\ell+1) \right)^{1/2} \left( C_\ell^{ab}(\ell) + \delta_{ab}N_\ell^{ab}(\ell) \right)$, where $\{a, b\} \in \{\phi, E, T\}$. The derivatives of the Fisher matrix are taken with respect to cosmological parameters that are defined in Table III and we let the “vanilla” parameters to be given by the set $\{\Omega_b h^2, \Omega_c h^2, \theta_s, \tau, n_s, \ln(10^{10} A_s)\}$. The experimental specifications are listed in Table VII.

We take a flat $\Lambda$CDM model for the fiducial cosmology (where parameter values are based on WMAP), with $N_{\text{eff}} = 3.04$ and $\sum m_\nu = 0.17$ eV. When EDE is included, the fiducial cosmology includes $\Omega_c = 0.01$. We have checked that our results are not significantly affected by the choice of fiducial neutrino mass and EDE density, and expect that this holds true for the other parameters as well (e.g. 108). For the terms in Eqs. [3] we carried out two-sided numerical derivatives with steps of 2% in most parameter values. We have confirmed the robustness of our results to other choices of step size. For further details on our prescription, including how to obtain the noise power spectra, see Ref. 108. Therein, we also consider weak lensing tomography, galaxy tomography, supernovae, and
extensive set of cross-correlations for future wide and deep surveys.

As shown in Figs. 4, 5, 6 Planck will be extremely helpful in improving the constraints on extended parameter spaces. At the $1\sigma$ level, considering the combination “vanilla+$N_{\text{eff}}+\sum m_\nu+w+\Omega_k+dn_s/d\ln k$”, the effective number of neutrinos could be constrained to $\sigma(N_{\text{eff}}) = 0.23$, mainly from the CMB temperature power spectrum, and the sum of neutrino masses to $\sigma(\sum m_\nu) = 0.19$ eV, mainly from the CMB lensing potential power spectrum (consistent with Refs. 108, 115, 116). These constraints on $N_{\text{eff}}$ and $\sum m_\nu$ are a factor of 3 stronger for both parameters than the present constraints from a joint analysis of “WMAP7+SPT+H0+BAO+SN” data.

When further allowing for the possible existence of a non-negligible component of dark energy in the high-redshift universe (i.e. considering “vanilla+$N_{\text{eff}}+\sum m_\nu+w_0+\Omega_k+dn_s/d\ln k$”), Planck constraints on $\sum m_\nu$ and $N_{\text{eff}}$ only degrade by 10% and 20%, respectively. This is because the CMB temperature power spectrum achieves a factor of 6 improvement in the constraint on the EDE density at $\sigma(\Omega_k) = 0.0087$ (which only improves by 5% in the full analysis), removing much of the degeneracy with other parameters obtained from the CMB. Expectedly, the orientation of error ellipses for Planck and present data in Figs. 2, 3, 4 match for parameters that are mainly constrained by the CMB ($T, E$), while they differ for parameters, such as $w$, for which the CMB temperature and E-mode polarization provide inferior constraints.

Moreover, we allow for the primordial helium abundance to vary in “vanilla+$N_{\text{eff}}+\sum m_\nu+Y_p$”, where the $1\sigma$ constraints on $\{N_{\text{eff}}, \sum m_\nu, Y_p\}$ are at the level of $\{0.25, 0.14\text{eV}, 0.015\}$. In the extended space “vanilla+$N_{\text{eff}}+\sum m_\nu+w_0+\Omega_k+dn_s/d\ln k+\frac{d\ln k}{k}$”, these constraints degrade to $\{0.27, 0.20\text{eV}, 0.019\}$, respectively (such that these neutrino constraints are comparable with the $\Omega_k$ case). However, the expected parameter constraints worsen significantly when considering $\{N_{\text{eff}}, \sum m_\nu, Y_p, \Omega_k\}$ in conjunction. For the maximal parameter extension (“vanilla+$N_{\text{eff}}+\sum m_\nu+w_0+\Omega_k+\Omega_{\Lambda}+\Omega_\gamma+dn_s/d\ln k+Y_p$”), we find 1$\sigma$ constraints on these four correlated parameters at the level of $\{0.51, 0.29\text{eV}, 0.030, 0.013\}$, respectively. In this scenario, Planck will need to be combined with external datasets to break the parameter degeneracies, in particular low-redshift measurements of the expansion history.

Thus, assuming a strong BBN prior on $Y_p$ (alternatively, at less than 10% degradation, assuming the dark energy is a pure late-time phenomenon at the level of $\Omega_k \lesssim 3 \times 10^{-3}$), it is expected that Planck alone will be able to determine the possible existence of extra relativistic species to $4\sigma$ confidence and the sum of neutrino masses to 0.2 eV, regardless of the extent of the remaining parameter space. With the ability to strongly constrain $\sum m_\nu$, there is promise for Planck to find evidence for nonzero neutrino mass, in particular when combined with probes of the large-scale structure (also see 117, 126).

**IV. CONCLUSIONS**

With the latest cosmological data sets of the cosmic microwave background (WMAP7+SPT), baryon acoustic oscillations (SDSS+2dF), supernovae (Union2), and the Hubble constant (HST), we have explored in closer detail the dependence of constraints on the effective number of neutrino species and the sum of neutrino masses on our assumptions of other cosmological parameters, including the curvature of the universe, running of the spectral index, primordial helium abundance, evolving late-time dark energy, and early dark energy.

In a combined analysis of the effective number of neutrinos and sum of neutrino masses (with 6 other $\Lambda$CDM parameters), we find mild ($2.2\sigma$) preference for additional light degrees of freedom. However, the effective number of neutrinos is consistent with three massive neutrinos and no extra relativistic species to $1\sigma$ when further including $\{w, \Omega_k, dn_s/d\ln k\}$ as free parameters. The transformation of a constant EOS to one that varies with time ($w_0, w_a$) doesn’t significantly change the constraints on $N_{\text{eff}}$ and $\sum m_\nu$ (less than 10% and 30%, respectively). The agreement with $N_{\text{eff}} = 3.046$ improves with the possibility of an early dark energy component, itself constrained to be less than 5% of the critical density (95% CL) in our maximally expanded parameter space.

Added to a minimal $\Lambda$CDM universe, $\sum m_\nu < 0.45$ eV (95% CL), dominantly from WMAP+HST. The sum of neutrino masses is bounded at 1.2 eV (95% CL) when jointly allowing for a constant dark energy equation of state, curvature, and running to vary as free parameters. The upper bound degrades to 2.0 eV (95% CL) when further including the primordial helium abundance and early dark energy density as additional degrees of freedom. The single parameter that most strongly increases the upper bound on the sum of neutrino masses when added to “vanilla+$\sum m_\nu$” is the curvature of the universe, which weakens the bound by more than a factor of 2 to $\sum m_\nu < 1.0$ eV (95% CL) in “vanilla+$\sum m_\nu+\Omega_k$.”

In extensions of the standard cosmological model that minimally allow for nonzero neutrino masses and additional light degrees of freedom, the derived amplitude of linear matter fluctuations $8\sigma$ is found consistent with low-redshift cluster abundance measurements to within $1\sigma$, and the spectral index agrees with unity to within 1 to 2 $\sigma$. Moreover, larger values of the dark matter density are preferred, as $\Omega_\Lambda h^2$ generally lives around $0.13 \pm 0.01$. When allowing the primordial helium abundance to vary as a free parameter, we consistently find $1\sigma$ agreements with estimates from observations of low-metallicity extragalactic H II re-
gions.

With the advent of increasingly sensitive CMB and large-scale structure data, our understanding of the neutrino sector depends critically on the ability to distinguish its signatures from other cosmological parameters. Fortunately, even for extended parameter spaces, Planck alone could determine the possible existence of extra relativistic species at the 4σ level and constrain the sum of neutrino masses to 0.2 eV (68% CL). Next-generation probes of the expansion history and large-scale structure hold the key to further improving these estimates.

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