Short-time dynamics of random-bond Potts ferromagnet with continuous self-dual quenched disorders

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We present Monte Carlo simulation results of random-bond Potts ferromagnet with the Olson-Young self-dual distribution of quenched disorders in two-dimensions. By exploring the short-time scaling behaviors, we found universal power-law critical behavior of the magnetization and Binder cumulant at the critical point, and thus obtain estimates of the dynamic exponent \( z \) and magnetic exponent \( \eta \), as well as the exponent \( \theta \). Our special attention is paid to the dynamic process for the \( q=8 \) Potts model.

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I. INTRODUCTION

Ferromagnetic systems with quenched randomness are studied intensively for their critical properties affected by an addition of disorders. The Harris criterion was claimed 27 years ago that if the specific heat critical exponent \( \alpha \) of the pure system is positive, then quenched disorder is a relevant perturbation at the second-order critical point and it causes changes in critical exponents. Then, following the work of Imry and Wortis who argued that quenched disorder could smooth of a first-order phase transition and thus produce a new second-order phase transition, the introduction of randomness to pure systems originally undergoing a first-order transition has been comprehensively considered. The theory was firstly numerically checked with the Monte Carlo (MC) method by Chen, Ferrenberg and Landau (CFL) who studied the 8-state Potts model with a self-dual random-bond disorder (see Eqs. and ). Recently most of the work for such disordered systems is carried out by intensive MC simulations for examining how a phase transition is modified by the quenched disorder coupling to the local energy density for the \( q \)-state random-bond Potts ferromagnet (RBPF). The \( q \)-state RBPF, an interesting framework to study the influence of impurity on pure systems, is described by the Hamiltonian,

\[
-\beta H = \sum_{\langle i,j \rangle} K_{ij} \delta_{\sigma_i, \sigma_j}, \quad K_{ij} > 0, \quad (1)
\]

where \( \beta = 1/k_B T \) is the inverse temperature, the spin \( \sigma \) can take the values \( 1, \cdots, q \), \( \delta \) stands for the Kroneck \( \delta \) function, and the nearest-neighbor interactions \( < i, j > \) are considered. Usually the dimensionless couplings \( K_{ij} \) are selected from two possible (ferromagnetic) values \( K_1 \) and \( K_2 = rK_1 \), with a strong to weak coupling ratio \( r = K_2/K_1 \), called a disorder amplitude, according to a bimodal distribution,

\[
P(K) = p\delta(K - K_1) + (1-p)\delta(K - K_2). \quad (2)
\]

With \( p = 0.5 \), the system is self-dual and the exact critical point can be determined by \( \langle 1 \rangle \),

\[
(e^{Kc} - 1)(e^{Kc'} - 1) = q, \quad (3)
\]

where \( K_c \) and \( K_c' \) are the corresponding critical values of \( K_1 \) and \( K_2 \), respectively. The value \( r = 1 \) corresponds to the pure case where the critical point is located at \( K_c = \log(1 + \sqrt{q}) \) and the phase transitions are first-order for \( q > 4 \). As \( \alpha > 0 \) for all \( q > 2 \) the randomness acts as a relevant perturbation and, for \( q > 4 \), it even changes the nature of the transition from first to second order. Up to present, the MC studies are usually carried out in equilibrium and concentrated on disorder amplitudes in the range of \( r = 2-20 \) adapted to numerical analysis. They give good estimates of the disordered fixed point exponents. In Table 1 we present the magnetic scaling index \( \eta = 2\beta/\nu \) of the 2D 8-state RBPF obtained by different groups. In a recent work, Olson and Young (OY) used their special self-dual disorder distribution and performed the MC study of multiscaling properties of the correlation functions for several values of \( q \). Their results for the magnetic exponent are very interesting to examine the universality of the RBPF in the crossover regime from the pure fixed point to a percolation-like limit. Cardy and Jacobsen studied the RBPF based on the connectivity transfer matrix (TM) formalism, and their estimates of the critical exponents lead to a continuous variation of \( \eta \) with \( q \), which is in sharp disagreement with CFL’s results.

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TABLE I: Magnetic exponent $\eta$ estimated by different groups for the 2D 8-state RBPF.

| Authors                  | $r$  | $\eta$ | Technique |
|--------------------------|------|--------|-----------|
| CFL                      | 2.10 | 0.236(4) | MC        |
| Olson and Young          | 0.319(7) | MC    |
| Cardy and Jacobsen       | 2.0 | 0.284(4) | TM        |
| Chatelain and Berche     | 0.304(6) | MC    |
| Chatelain and Berche     | 0.301(1) | TM    |
| Picco                    | 0.306(2) | MC    |
| Ying and Harada          | 0.302(5) | STD   |

In this paper, we present a MC study to verify the dynamic scaling of the RBPF and estimate the critical exponents in the short-time dynamics (STD) for the OY’s continuous distribution of the disorder interactions. It is a self-dual scheme to introduce quenched randomness and defined by the following probability

$$P_X(x) = \frac{2\sqrt{q}}{\pi(1-x^2+q x^2)},$$

for the Boltzmann factor $x = e^{-K}$. It is obvious that the model has strong disorder as the distribution function is nonzero everywhere at $x \in (0, 1)$ ($K \to \infty$, $K \to 0$) and has a finite weight at its two limit points $x = 0$ and $x = 1$. Another advantage of this distribution is that it is easy to generate random numbers with the probability $P_X(x)$ through

$$x = \frac{1}{1 + \sqrt{q} \tan(\pi R_x/2)},$$

where $R_x$ is a random number with uniform distribution between 0 and 1.

In particular, we will investigate the critical behavior affected by introduction of such continuous quenched randomness to clarify the universality class of the RBPF in the crossover regime. In simulations we mostly choose the model with $q = 8$ which is known to have a strong first-order phase transition without disorders. We hope to confirm the new second-order phase transition induced by the random disorder in order to show the influence of quenched impurities on the first-order systems and to check whether an Ising-like universality would be satisfied from the STD approach.

### II. SHORT-TIME DYNAMICS

For a long time, it was believed that universality and scaling relations can be found only in equilibrium or in the long-time regime. In Refs. [24, 25], however it was discovered that for a $O(N)$ vector magnetic system in states with a very high temperature $T \gg T_c$, when it is suddenly quenched to the critical temperature $T_c$, and evolves according to a dynamics of model A, a universal dynamic scaling behavior emerges already within the short-time regime,

$$M^{(k)}(t, \tau, L, m_0) = b^{-k\beta/\nu} M^{(k)}(b^{-z} t, b^{1/\nu} \tau, b^{-1} L, b^{\nu} m_0),$$

where $M^{(k)}$ is the $k$th moment of magnetization, $t, \tau = (T - T_c)/T_c, L$ and $b$ are time, reduced temperature, lattice size and scaling factor, respectively. $\beta$ and $\nu$ are the well known static critical exponents. The $x_0$, a new independent exponent, is the scaling dimension of initial magnetization $m_0$. This dynamic scaling form is generalized from finite size scaling in the equilibrium stages. The dynamic MC simulations have been successfully performed in such non-equilibrium processes to estimate the critical point and the critical exponents $(K_c, z, \beta, \nu, \theta)$.

The MC simulation begins with a study of the time evolution in the initial stage of the dynamic relaxation starting at very high temperature and small magnetization $(m_0 \sim 0)$. For a sufficiently large lattice $(L \to \infty)$, from Eq.(6) by setting $\tau = 0$ and $b = t^{1/z}$, it is easy to derive that

$$M^{(k)}(t, m_0) = t^{-k\beta/\nu z} M^{(k)}(1, t^{x_0/2} m_0).$$

When $k = 1$ we get the most important scaling relation on which our measurements of the critical exponent $\theta$ are based,

$$M(t) \sim m_0 t^{\theta}, \quad \theta = (x_0 - \beta/\nu)/z.$$  

In most cases the magnetization undergoes an initial increase at the critical point $K_c$ after a microscopic time $t_{mic}$.

In our MC sweeps, the time evolution of $M(t)$ is calculated through the definition

$$M(t) = \frac{1}{N} \left[ \frac{<q M(t) - N}{q-1} \right].$$

Here $M_O = \max(M_1, M_2, \ldots, M_N)$ with $M_i$ being the number of spins in the $i$th state among $q$ states. $N = L^2$ is the number of spins on the square lattice. We use $L$ up to 128. $< \cdots >$ denotes thermal averages over independent initial states and random number sequences, and [· · ·] the disorder averages over quenched randomness distributions. The unit of time $t$ is defined as a MC sweeps over all spins on the lattice.

The susceptibility plays an important role in the equilibrium. Its finite size behavior has been often used to determine the critical temperature and the critical exponents $\gamma/\nu$ and $\beta/\nu$. For the STD approach, the time-dependent susceptibility (the second moment of the magnetization) is also interesting and important. For the random-bond Potts model, it is defined as

$$M^{(2)}(t) = \frac{1}{N} \left[ < M^2(t) > - < M(t) >^2 \right].$$  


One studies its scaling behavior from $m_0 = 0$ in short-time relaxation. Because the spatial correlation length in the beginning of the evolution is small compared with the lattice size $L^d$ the second moment behaves as $M^{(2)}(t, L) \sim L^{-d}$. Then the finite size scaling Eq.(1) induces a power-law behavior at the critical temperature, 

$$M^{(2)}(t) \sim t^y, \quad y = (d - \eta)/z. \quad (11)$$

In the above considerations the dynamic relaxation was assumed to start from disordered states with $m_0$ small enough. Another interesting and important process is the dynamic relaxation from a completely ordered state. The initial magnetization is taken exactly at its other fixed point $m_0 = 1$, where a critical scaling form

$$M^{(k)}(t, \tau, L) = b^{-k\beta/\nu} M^{(k)}(b^{-\nu\tau}, b^{-1}L) \quad (12)$$

is expected [24, 28]. This scaling form looks to be the same as the dynamic scaling form in the long-time regime, however, it is now assumed already valid in the macroscopic short-time regime. For the magnetization itself, $b = t^{1/z}$ yields, for a sufficiently large lattice and at the critical point ($\tau = 0$), a power-law decay behavior of

$$M(t, \tau) = t^{-c_1}, \quad c_1 = \beta/\nu z. \quad (13)$$

The formula can be used to calculate the critical exponents $\eta = 2\beta/\nu$ or $z$.

We further observe an evolution of the Binder cumulant $U(t, L) = M^{(4)}(t, L)/M^2(t, L) - 1$. A similar power-law behavior at criticality induced from the Eq.(12) shows that

$$U(t, L) \sim t^{-c_u}, \quad c_u = d/z \quad (14)$$

on large enough lattices. Here, unlike the relaxation from the disordered state, the fluctuations caused by the initial configurations are much smaller. In practical simulations, these measurements of the critical exponents and critical temperature are better in quality than those from the dynamical relaxation starting from disordered states.

### III. MC RESULTS

In simulations, the heat-bath algorithm is used for MC updating, and up to 500 samples are chosen as disorder averages and 200-500 initial configurations and/or random number sequences are taken as thermal averages for each disorder realization (so total MC averages are over 100,000). Statistical errors are simply estimated by performing three groups of averages with different random number sequences as well as independent initial states. We first focus our attention on evolutions of the magnetization both from the initial states with small magnetization ($m_0 \sim 0$) and complete ordered states ($m_0 = 1$).

At the critical point ($\tau = 0$), achieved by generating the Boltzmann factors according to the distribution of Eq.(1), the curves $M(t)$ should show power-law behavior as described by Eq.(6) or Eq.(13) dependent on the initial states. They, however, should deviate from the power-law scaling behavior for a small but nonzero $\tau$ characterized by $M(t, \tau) = m_0 t^\delta M(t^{1/\nu \tau})$ or $M(t, \tau) = t^{-c_u} F(t^{1/\nu \tau})$ with respect to the $m_0 \sim 0$ or $m_0 = 1$ initial states respectively. Now a question arises how to introduce a distribution which deviates from the critical point with distance $\tau$? We can first generate the Boltzmann factors, $x \equiv e^{-K}$, from the self-dual distribution $P_N(x)$ in Eq.(4), then modify them by the replacement $x \rightarrow x^{1/(1+\tau)}$ away from criticality [15]. We observe that the best power-law behavior appeared at the same critical point for both the random and ordered initial states. This fact indicates a second-order phase transition induced at $K_{c\sigma}$ as argued by Schülke and Zheng [30] since $K_{order} < K_c < K_{random}$ if a first-order phase transition happened. We exhibit in Fig.1 and Fig.2 the double-logarithmic plots of magneti-
TABLE II: The values of indices $c_1$, $c_\eta$, $\theta$ and $y$ and estimated results $z$ and $\eta$ estimated from the $m_0 = 1$ state for the $q = 8$ on lattice sizes $L^2$.

| index | $m_0 = 1$ | $m_0 \sim 0$ | results |
|-------|-----------|--------------|---------|
|       | $c_1$     | $c_\eta$    | $\theta$ | $y$ | $z$ | $\eta$ |
| $L = 32$ | .0468(6) | .603(7) | .254(3) | .597(6) | 3.32(5) | .310(5) |
| $L = 64$ | .0450(6) | .597(7) | .238(3) | .526(6) | 3.35(5) | .303(5) |
| $L = 96$ | .0449(5) | .593(7) | .234(4) | .508(5) | 3.37(5) | .302(5) |
| $L = 128$ | .0445(5) | .586(7) | .230(4) | .497(5) | 3.41(6) | .304(5) |

zation versus the MC time on a $L = 64$ lattice to show this behavior obviously.

Next, we carry out simulations at the exact critical point $K_c$ to investigate the magnetization and Binder cumulant, starting from $m_0 = 1$ ordered state. Such state is a ground state where all the spins orient in the same direction ($e.g.$, $\sigma_i = 1, i = 1, N$). In Figs.3 and 4, $M(t)$ and $U(t)$ are presented and values of the scaling indices $c_1 = \beta / \nu z$ and $c_\eta = d / z$ can be estimated from slopes of the curves in $t = (100,1000)$. The results are listed in Table 2. Here we find that, unlike for relaxations from the disordered state, the fluctuations caused by ordered initial configurations are much smaller. As a result, these measurements of the critical exponents based on Eqs.(13) and (14) are better in quality than those from disordered states on Eqs.($8$) and ($9$). So we take only 200 MC configurations for the thermal averages in the MC simulations. Now, from the results of $c_1$ and $c_\eta$, the critical exponents $\eta$ and $z$ can be calculated based on Eqs.(13) and (14), and their values are summarized in Table 2. To estimate the critical exponent $\theta$, the random initial states are prepared with small magnetizations $m_0 = 0.04 - 0.01$. The curves of $M(t)$ for different $m_0$ on $128^2$ lattice with a double-logarithmic scale are displayed in Fig.5 which exhibits the finite $m_0$ effect on $M(t)$ and a very nice power-law behavior at $m_0 \to 0$. Thus the exponent $\theta$ can be estimated from slopes of the curves in $t = (50,1000)$ by using Eq.($8$) after an extrapolation to $m_0 \to 0$. Its values are shown also in Table 2.

As mentioned above, for a determination of the dynamic exponent $z$ and the magnetic exponent $\eta$ a dynamic process starting from the ordered state ($m_0 = 1$) is more favorable since the thermal fluctuation is much smaller than for random states. Therefore we have mainly studied the short-time dynamic process from such an ordered state to calculate $z$ and $\eta$. On the other hand it is also desirable to measure these exponents from the $m_0 = 0$ initial states (24). In our work we now take the evolution for the second moment of magnetization $M^2(t)$ from the $m_0 = 0$ to verify the results of $\eta$ and $z$. By input of values of $\eta$ and $z$ already obtained from the indices $c_1$ and $c_\eta$, we check if they also satisfy the relation $y = (d - \eta) / z$ in Eq.($14$). As expected we find they are consistent with less than 2% error for $L \geq 96$.

FIG. 3: Time evolution of magnetization for different lattice sizes from the ordered state at the critical point, plotted on a double-log scale. It shows the finite size effect on the $M(t)$.

FIG. 4: Time evolution of Binder cumulant for different lattice sizes from the ordered state at the critical point, plotted on a double-log scale. The $t_{mic}$ is extended to about one hundred.

IV. CONCLUSIONS

We have made a first attempt to apply the STD approach to the 2D RBPF with a continuous distribution of quenched disorders. Our simulations verified that a second order phase transition has been induced. Then the universal dynamic scaling behavior in the short-time dynamics is used to estimate the exponents $\theta$ and $z$, as well the magnetic critical exponent $\eta$. It is found that they really violate the Ising-like universality class where $\theta = 0.191(1)$ and $\eta = 0.240(15)$ (24). Our result of the dynamical exponent $z = 3.41(6)$ is much bigger than that for systems without disorder, and it causes the microscopic time $t_{mic}$ to be extended up to one hundred for the Binder cumulant from the ordered initial state. The value for the exponent $\eta$ nearly completely overlaps with those listed in Table 1, except that it is somewhat smaller


FIG. 5: Time evolution of magnetization for different initial magnetizations $m_0$, plotted on a log-log scale. The curves of $m_0 = 0.01$ and $m_0 = 0.00$ show nearly complete overlap.

than that given in Ref.[14]. More accurate calculation remains to be done to solve this controversy.

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