Two Dimensional Stringy Black Holes with One Asymptotically Flat Domain

PETR HOŘAVA

Enrico Fermi Institute
University of Chicago
5640 South Ellis Avenue
Chicago, IL 60637, USA

ABSTRACT

The exact black hole solution of 2D closed string theory has, as any other maximally extended Schwarzschild-like geometry, two asymptotically flat space-time domains. One can get rid of the second domain by gauging the discrete symmetry on the $SL(2,\mathbb{R})/U(1)$ coset that interchanges the two asymptotic domains and preserves the Kruskal time orientation everywhere in the Kruskal plane. Here it is shown that upon performing this orbifold procedure, we obtain a theory of unoriented open and closed strings in a black hole background, with just one asymptotically flat domain and a time-like orbifold singularity at the origin. All of the open string states of the model are confined to the orbifold singularity. We also discuss various physical aspects of the truncated black hole, in particular its target duality – the model is dual to a conventional open string theory in the black hole geometry.

⋆ e-mail addresses: horava@yukawa.uchicago.edu or horava@curie.uchicago.edu
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Black holes are spacetimes of amazingly rich geometrical and topological structure. The recent discovery of exact black hole solutions of 2D string theory in terms of an \( SL(2, \mathbb{R})/U(1) \) coset CFT ([1,2], see also [3]) allows one to study some properties of quantum theory in the geometry of a black hole quite explicitly [4,5,6,7], and in particular to search for stringy effects, such as duality [8,9,10]. Another important aspect of the black hole solution is its expected relation to \( c = 1 \) matter coupled to Liouville [1,5,11,12].

One of the many interesting issues of quantum black hole physics is the role of various asymptotically flat domains in the black hole geometry. As any other maximally extended Schwarzschild-like black hole, the 2D black hole has two distinct asymptotically flat exterior regions (denoted by I and II throughout the paper) which cannot influence each other causally. In this paper we will address the question of getting rid of the second asymptotically flat domain of the stringy black hole in a consistent way. Actually, we will point out that the truncated black hole with just one asymptotically flat exterior domain is naturally inhabited by unoriented open and closed strings. As any difference between closed and open strings gets lost in the point particle limit, this is a typical stringy effect in the black hole physics. After establishing this result, we will analyze more closely some aspects of the truncated black hole, in particular its spectrum, spacetime duality, and the description of the black hole singularity in terms of an effective topological field theory. (Some other aspects of black holes in 2D open string theory have been studied recently in [15,16].)

In string theory, there is a particularly natural way of truncating spacetime manifolds, by using orbifold techniques. In fact, this strategy has been recommended for the 2D black holes quite explicitly by Witten [1], who has proposed to mod out the black hole coset by the \( \mathbb{Z}_2 \) symmetry that maps the two asymptotically flat domains to each other by mapping \( u \) to \( v \) in the Kruskal-Szekeres coordinate.

\[ \text{Considerable effort in this direction has been made in the 4D black hole physics since the middle of sixties [13] (for a review, see e.g. [14]). Most of the authors have identified points } (u, v) \equiv (-u, -v) \text{ in the Kruskal plane, which is not the strategy accepted in this paper.} \]
system. Once we try to do so throughout the Kruskal plane, we observe that we have to supplement the $\mathbb{Z}_2$ action on the target by an orientation reversal on the worldsheet. Orbifolds of this type are known to lead to the theory of open and closed unoriented strings [17]. In other words, the proposed orbifold model will represent a typical example of the worldsheet orbifold construction in the sense of [17], hence leading to a target with both closed and open strings in the spectrum.

This situation resembles somewhat the theory of topological sigma models with Kähler targets [18]: Once one attempts to mod out the topological target by an antiholomorphic involution, one has to act simultaneously by a complex conjugation on the worldsheet. Such a combined $\mathbb{Z}_2$ action can be gauged, and the resulting theory describes topological sigma models of open string theory (or $\mathbb{Z}_2$-equivariant topological sigma models in the terminology of [18]). This analogy is perhaps not accidental, as there are some indications [19] that two dimensional string theory, of which the black hole is a classical solution, might have an underlying topological phase.

Before actually discussing our truncated black hole, let us mention some more motivation. First, the role of various spacetime geometries in 2D string theory should be probably addressed in a non-perturbative framework, such as string field theory. Up to now, string field theory of open strings is still conceptually much better understood than its closed string counterpart. Second motivation comes from the fact that, as has been argued some time ago by Shenker [20], the leading nonperturbative effects in matrix models are of order $e^{-C/\kappa}$, where $\kappa^2$ is the closed string coupling constant. This is a stringy effect, as in quantum field theory one rather expects the leading nonperturbative effects to behave like $e^{-C/\kappa^2}$. Recalling that $\kappa$ is the open string coupling constant, it is tempting to speculate that open strings are hidden somehow inside the closed string theory, and generate the leading nonperturbative effects of order $e^{-C/\kappa}$. It is thus natural to search for a mechanism in which open strings naturally emerge in low dimensional closed string theories. We hope to offer one in this paper.
To answer the physical question of whether one can forget consistently about the second asymptotically flat domain of the stringy black hole, we will construct the truncated black hole by gauging a $\mathbb{Z}_2$ extension of $U(1)$ (or of its non-compact version $SO(1,1)$) in the black hole coset. First, after summarizing some basic facts to fix our notation, we will identify the corresponding $\mathbb{Z}_2$ action.

The Lagrangian for the $SL(2,\mathbb{R})$ WZW model

$$\mathcal{L}_{WZW} = \frac{k}{8\pi} \int_{\Sigma} d^2z \; \text{Tr} \left( g^{-1} \partial g \; g^{-1} \partial g \right) + \frac{ik}{12\pi} \int_{\Sigma} \text{Tr} \left( g^{-1} dg \wedge g^{-1} dg \wedge g^{-1} dg \right)$$

is by construction symmetric under the left-moving and right-moving current al-
gebras of $SL(2,\mathbb{R})$, acting on an element $g$ of $SL(2,\mathbb{R})$ by

$$g(z, \bar{z}) \to \Omega_L(z) g(z, \bar{z}) \Omega_R(\bar{z}). \quad (2)$$

For the study of the maximally extended Minkowski black hole in Schwarzschild coordinates, the following faithful parametrization [21] of $SL(2,\mathbb{R})$ is most suitable:

$$g = \begin{pmatrix} e^{t_L/2} & 0 \\ 0 & e^{-t_L/2} \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}^{\epsilon_1} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}^{\epsilon_2} \mathcal{M} \begin{pmatrix} e^{-t_R/2} & 0 \\ 0 & e^{t_R/2} \end{pmatrix}, \quad (3)$$

where $t_L, t_R \in \mathbb{R}$, $\epsilon_1, \epsilon_2 \in \{0, 1\}$ and $\mathcal{M}$ is one of the following two matrices:

$$\mathcal{M} = \begin{cases} \begin{pmatrix} \cosh \frac{r}{2} & \sinh \frac{r}{2} \\ \sinh \frac{r}{2} & \cosh \frac{r}{2} \end{pmatrix} & r \in \mathbb{R}, \\ \begin{pmatrix} \cos \frac{r}{2} & -\sin \frac{r}{2} \\ \sin \frac{r}{2} & \cos \frac{r}{2} \end{pmatrix} & r \in (-\pi/2, \pi/2). \end{cases} \quad (4)$$

$\epsilon_1$ parametrizes the centrum of $SL(2,\mathbb{R})$, and is irrelevant on $SO(2,1)$, while $\epsilon_2$ is related to the (self) duality of the black hole.
We can gauge any non-anomalous subgroup of the symmetry group. The requirement of non-anomalousness leaves just two choices for gauging Abelian symmetries [9], namely the vector symmetry \( g \to hgh^{-1} \) and the axial symmetry \( g \to hgh \). Upon gauging the axial \( U(1) \) generated by \( \sigma_3 = \text{diag}(1, -1) \) and choosing the unitary gauge,

\[
t_L = t_R \equiv t, \tag{5}
\]

the first choice of \( \mathcal{M} \) as indicated in (4) describes regions I and II (and regions V and VI behind the singularities, depending on the value of \( \epsilon_2 \)), while the second choice of \( \mathcal{M} \) describes the interior of the black hole between the horizon and the singularity (regions III and IV). The conventional Kruskal coordinates, \( u \) and \( v \), are related to the Schwarzschild coordinates \( r, t \) by

\[
g = \begin{pmatrix} a & u \\ -v & b \end{pmatrix}, \quad ab + uv = 1, \tag{6}
\]

and the unitary gauge (5) is now \( a = \pm b \).

The Euclidean black hole geometry, on the other hand, is most transparent in the Euler angle parametrization of \( SU(1,1) \) (isomorphic to \( SL(2,\mathbb{R}) \)):

\[
h = \begin{pmatrix} \alpha & \beta \\ \bar{\beta} & \bar{\alpha} \end{pmatrix} = \begin{pmatrix} \cosh \frac{r}{2} e^{\frac{i}{2}(\theta_R-\theta_L)} & \sinh \frac{r}{2} e^{\frac{i}{2}(\theta_L+\theta_R)} \\ \sinh \frac{r}{2} e^{-\frac{i}{2}(\theta_L+\theta_R)} & \cosh \frac{r}{2} e^{\frac{i}{2}(\theta_L-\theta_R)} \end{pmatrix}, \tag{7}
\]

with

\[
\theta_L \in \mathbb{R}, \quad r \in [0, \infty), \quad \theta_R \in [-2\pi, 2\pi]. \tag{8}
\]

Note that we can obtain (7) from (3) by an analytic continuation, setting \( t_{L,R} = i\theta_{L,R} \). In the unitary gauge, this is the Euclidean continuation of the black hole, in which the Schwarzschild time coordinate is continued analytically. Later on we will be using another Euclidean continuation of the Minkowski black hole, used e.g. by Hartle and Hawking in [22], in which the coordinate to be continued is the Kruskal time.
At least two discrete symmetries of the coset are worth mentioning. First, the theory is symmetric under the $\mathbb{Z}_2$ that multiplies $g$ by the non-trivial central element of $SL(2, \mathbb{R})$. What is usually done in this situation is the obvious modification of the WZW model to the orbifold model based on the centerless factor of $SL(2, \mathbb{R})$, isomorphic to $SO(2, 1)$. This procedure is profitable because it eventually takes away the two-sheeted degeneration of the Minkowski black hole geometry mentioned first in [1]. We will interpret the black hole geometry in precisely this sense.* Another important $\mathbb{Z}_2$ symmetry of the coset model is generated by the inversion of $g$, 

$$\Theta' : g \rightarrow g^{-1}. \quad (9)$$

This is an anti-automorphism of the group, hence it sends $\Omega_{L,R}$ to $\Omega^{-1}_{R,L}$ in (2). As $\Omega_L$ is purely left moving and $\Omega_R$ purely right moving, $\Theta'$ has to act simultaneously as a parity reversal on the worldsheet in order to become a symmetry of the coset.

Now we are going to find the orbifold action that will map one asymptotically flat region to the other by mapping $u$ to $v$ and vice versa. To lowest order in $1/k$, the spacetime metric and dilaton field are given by

$$ds^2 = \frac{du\, dv}{1 - uv}, \quad \Phi = \ln(1 - uv) + \text{const.} \quad (10)$$

This geometry solves the low-energy effective approximation to classical string theory in two dimensions. Clearly, the intended orbifold action is a symmetry of (10); the crucial point is to extend the symmetry to the full-fledged classical string solution, represented by the exact coset.

* The two-sheeted geometry of the $SL(2, \mathbb{R})/SO(1, 1)$ black hole can be interpreted alternatively as follows. Generic points in the Penrose diagram of a $D$-dimensional black hole describe $(D - 2)$-spheres, isomorphic in two dimensions to the set of two elements, $\{1, -1\}$. In this sense, the black hole will have four asymptotically flat domains, two of them describing the two sides of one “universe” surrounding the black hole, and the remaining two describing the “mirror universe.” We will not accept this interpretation in this paper, however.
The corresponding $\mathbb{Z}_2$ action can be identified e.g. by requiring the correspondence with the flat limit of the black hole geometry, where the model reduces to free fields, or (with some guesswork) even directly; the action we need is given by

$$\Theta: \quad g \rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} g^{-1} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \equiv sg^{-1}s.$$ (11)

To be a symmetry of the coset, $\Theta$ has to act simultaneously on the worldsheet as the parity flip. $\Theta$ extends the Abelian $SO(1,1)$ gauge group to the direct product $\mathbb{Z}_2 \times SO(1,1)$. As the gauge group now acts on the worldsheet as well as in the target, we have arrived at a typical example of orbifold models studied in [17], and expect open strings to emerge in the twisted sector of the model. As we will see below, this is indeed true, and establishes the central result of the paper.

We have just identified our orbifold group action, leading to a worldsheet orbifold model. One can note, however, that the coset enjoys also a purely target $\mathbb{Z}_2$ symmetry that maps region I to region II by $u \leftrightarrow v$, or in terms of the group variable, acts by

$$g \rightarrow \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} g \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad \quad \quad (12)$$

This discrete symmetry can be gauged, leading to a target orbifold model with one asymptotic region, orbifold singularity at $u = v$ in regions III and IV, and just closed strings in the spectrum. Does the existence of this orbifold model endanger our claim that the truncated black hole is inevitably a theory of open strings?

The answer to this question results from a careful analysis of the orbifold action on the full-fledged coset. In the Kruskal parametrization, (12) becomes

$$\begin{pmatrix} a & u \\ -v & b \end{pmatrix} \rightarrow \begin{pmatrix} b & v \\ -u & a \end{pmatrix}. \quad \quad \quad (13)$$

In the unitary gauge we set $a = b$ in regions I–IV and $a = -b$ in regions V, VI. Recalling now that two elements of $SL(2, \mathbb{R})$ that differ by an overall minus sign
describe actually the same point on $SO(2,1)$, we define Kruskal coordinates $u,v$ on the one-sheeted black hole geometry by requiring non-negative $a$ in (13). (This amounts to a gauge fixing of the discrete gauge symmetry that makes the $SO(2,1)$ WZW model out of the $SL(2,\mathbb{R})$ WZW model.) Keeping in mind these gauge fixing conditions, one can see that the target orbifold action (13) does indeed map $u$ to $v$ in regions I–IV, but interchanges regions V and VI (transforming $(u,v)$ to $(-u,-v)$ there). This target orbifold might represent an attractive possibility for getting rid of the second asymptotic region, as the action (12) on the target is everywhere space-like. Nevertheless, the worldsheet orbifold (11) remains the only possibility for modding out by $u \leftrightarrow v$ everywhere, and will be the only orbifold analyzed in this paper.

Let us now analyze the spectrum of the truncated black hole, first in the Euclidean signature.\footnote*{Since our orbifold action mixes different causal regions of the Minkowski black hole, it is crucial – in order to keep the physical interpretation of the orbifold unchanged – to use the Hartle-Hawking Euclidean continuation [22], which covers the whole Kruskal plane.} Physical states of the black hole are defined as cohomology classes of the BRST charge for the coupled system of the coset matter and diffeomorphism ghosts on the worldsheet,

$$Q = Q_{\text{coset}} + Q_{\text{gravity}}, \quad Q_{\text{coset}} = \oint_C c(J^3 + ikA) + \text{h.c.}$$  \hfill (14)

(here $c$ is the spin-zero $U(1)$ ghost, and $A$ is the $U(1)$ gauge field). The orbifold group action defines its own condition of physicality, as it projects the physical states of the full black hole to the $\mathbb{Z}_2$ invariant sector (and introduces twisted states). This can be reformulated in terms of equivariant cohomology of the BRST charge, thus defining the physical states of the model as the cohomology of $Q$, equivariant with respect to the antighost field action [11] as well as the orbifold $\mathbb{Z}_2$ action. In the closed string sector, these conditions are

$$Q|\text{phys}\rangle = 0, \quad (b_0 - \bar{b}_0)|\text{phys}\rangle = 0, \quad (\Theta - 1)|\text{phys}\rangle = 0.$$  \hfill (15)

The model to be orbifolded is the $SL(2,\mathbb{R})/U(1)$ coset, which is a target for
closed string propagation. Hence, the untwisted sector on the orbifold consists of the closed strings of the original model. The only thing that remains to be identified in the untwisted sector is the action of the orbifold group on it. It is illuminating to weaken (15) temporarily, and study primaries of the coset. Parametrizing the group by (7), vertex operators of the $SL(2, \mathbb{R})/U(1)$ primaries of the closed-string black hole CFT are given by matrix elements of (unitary) $SL(2, \mathbb{R})$ representations:

$$T^\ell_{mn}(r, \theta_L, \theta_R) = \langle \ell, m | g(r, \theta_L, \theta_R) | \ell, n \rangle.$$  \hspace{1cm} (16)

This expression can be written with the use of Jacobi functions as

$$T_{mn}^\ell(\theta_L, r, \theta_R) = P_{mn}^\ell(\cosh r) e^{im\theta_L + in\theta_R}. \hspace{1cm} (17)$$

$m$ and $n$ can take only integer values on $SO(2, 1)$, and their physical interpretation can be extracted from the asymptotic behavior at infinity, leading to

$$m = \frac{1}{2}(M + kN), \quad n = \frac{1}{2}(M - kN). \hspace{1cm} (18)$$

Here $M$ and $N$ are the discrete momentum and winding number in the asymptotically compact direction, respectively; both $N$ and $M$ are in $\mathbb{Z}$.

The orbifold group action (13) corresponds in the parametrization (7) to

$$\Theta : \quad r \rightarrow r, \quad \theta_L \rightarrow \pi - \theta_R, \quad \theta_R \rightarrow \pi - \theta_L. \hspace{1cm} (19)$$

Under the orbifold action, vertex operators with quantum numbers $m, n$ are mapped to vertex operators with quantum numbers $-n, -m$, which clearly corresponds to the expected behavior of the momentum and winding states under the $\mathbb{Z}_2$. The Jacobi functions $P_{mn}^\ell$ transform under the $m \leftrightarrow -n$ interchange as follows
\[ \mathcal{P}_{mn}(\cosh r) = \frac{\Gamma(\ell + m + 1)\Gamma(\ell - n + 1)}{\Gamma(\ell + m + 1)\Gamma(\ell - m + 1)} \mathcal{P}_{-n,-m}(\cosh r), \tag{20} \]

and we arrive at the following picture. The orbifold group \( \mathbb{Z}_2 \) acts on the untwisted sector, consisting of closed strings in the original black hole background, by

\[ \Theta : |T_{\ell}^{mn}\rangle \rightarrow (-1)^{m+n} \frac{\Gamma(\ell + n + 1)\Gamma(\ell - n + 1)}{\Gamma(\ell + m + 1)\Gamma(\ell - m + 1)} |T_{\ell}^{\ell-n,-m}\rangle; \tag{21} \]

consequently, the only surviving tachyons are the following \( \mathbb{Z}_2 \)-even linear combinations of the tachyons with sharp momentum and winding number,

\[ \frac{\Gamma(\ell + m + 1)}{\Gamma(\ell + n + 1)} |T_{mn}^\ell\rangle + (-1)^{m+n} \frac{\Gamma(\ell - n + 1)}{\Gamma(\ell - m + 1)} |T_{-n,-m}^\ell\rangle. \tag{22} \]

Recalling the indications obtained within \( c = 1 \) matrix models that amplitudes simplify considerably when expressed in terms of renormalized vertex operators, it may be natural to redefine e.g.

\[ T_{mn}^\ell(r, \theta_L, \theta_R) \rightarrow T_{mn}^\ell \equiv \frac{\Gamma(\ell + m + 1)}{\Gamma(\ell + n + 1)} T_{mn}^\ell(r, \theta_L, \theta_R). \tag{23} \]

On these renormalized vertex operators, (20) can be rewritten as

\[ \Theta : T_{mn}^\ell(r, \theta_L, \theta_R) \rightarrow (-1)^{m+n} T_{-n,-m}^\ell(r, \theta_L, \theta_R). \tag{24} \]

The surviving primaries of the untwisted sector are thus given by (22). To obtain the string spectrum, one has to impose the equivariant BRST conditions (15). Imposing the on-shell condition, we get \([11]\) \( m = \pm n = \pm (3\ell + \frac{3}{2}) \). This completes our analysis of the closed-string tachyons. The orbifold group action can be extended easily to the discrete modes of [11], as we know the action on the primaries as well as on the Kac-Moody currents.
The twisted sector of our orbifold consists of open strings. The orbifold group action on the target fixes the boundary conditions on the worldsheet, and determines the spectrum. Primary states of the twisted sector are built up from the representation theory of $SL(2, \mathbb{R})$; the left and right Kac-Moody symmetries are now identified. As $\theta_L = \pi - \theta_R$ at the boundary, the open string will have only winding modes, moving in $\tilde{\theta} \equiv \frac{1}{2}(\theta_L - \theta_R)$:

$$\tilde{T}_m^\ell(r, \tilde{\theta}) = \langle \ell, m | g(r, \frac{\pi}{2} + \tilde{\theta}, \frac{\pi}{2} - \tilde{\theta}) | \ell, -m \rangle$$  \hspace{1cm} (25)

This is not unusual, an open string model with only winding modes in a flat toroidal target has been constructed in [17,23]. In the coordinate system $r, \theta$, the open-string modes (25) are effectively one-dimensional, since they are restricted to

$$\theta = \pm \pi/2, \quad r \text{ arbitrary.}$$  \hspace{1cm} (26)

Far away from the black hole, the free sign in (26) becomes a new discrete quantum number of the open string, which is not conserved in interactions with the black hole. Analogously as in the untwisted sector, we have to project to $\mathbb{Z}_2$ invariant states, and impose the on-shell condition $m = \pm (3\ell + \frac{3}{2})$. Analyzing the equivariant BRST cohomology in the open sector, we also obtain an infinite tower of discrete states restricted effectively to (26).

In worldsheet orbifold models, much information is gathered in the boundary and crosscap states [24,17,25], which allow one e.g. to find the BRST invariant Chan-Paton factors of the open spectrum, hence determining the Yang-Mills gauge symmetry. Equations of motion for the open-string background are determined from the condition of conformal invariance of the boundary state,

$$\left( L_n - \bar{L}_{-n} \right) |B\rangle = 0, \quad n \in \mathbb{Z}. \hspace{1cm} (27)$$

Setting $n = 0$ in (27), we get a simple equation for the Kac-Moody primary piece
of the boundary operator:

\[
\left( \frac{\partial^2}{\partial \theta_L^2} - \frac{\partial^2}{\partial \theta_R^2} \right) |B\rangle_0 = 0.
\]

(28)

Consequently, the primary part of the boundary operator can be written as a sum of the contributions from the purely winding modes and purely momentum modes,

\[
|B\rangle_0 = |B(\theta, r)\rangle + |\tilde{B}(\tilde{\theta}, r)\rangle.
\]

(29)

We cannot, however, construct the full boundary and crosscap states and find the proper Chan-Paton symmetry group of the black hole coset, as this issue requires detailed knowledge of the closed string model, which is not yet available.

If we trust the procedure of inferring the temperature of the black hole from its Euclidean continuation by measuring the periodicity of $\theta$ far away from the black hole, our orbifold truncation of the black hole cannot change its temperature, which is thus the same as in the closed string coset. On the other hand, what we do expect is a change in the temperature behavior of the system in the vicinity of the Hagedorn temperature. Indeed, there is a serious difference between the formal high-temperature behavior of open and closed string theories even in flat spacetime: The nonzero contribution of the first winding mode to the boundary state is not compensated for by an analogous contribution to the crosscap state, and causes BRST anomalies and infinities already at the top of the Hagedorn temperature, where the closed string model [26] is still formally finite.

Now let us turn to the Minkowski signature. The orbifold truncates the Minkowski black hole at $u = v$, where a time-like orbifold singularity is formed. Open strings of the model are confined to the orbifold singularity. In the Euclidean case we have learnt that the open-string tachyon has only winding modes. As there are no winding modes allowed in the Minkowski signature, the whole spectrum of open strings will consist of discrete states, which can be constructed precisely as in [11]. Since the open strings are confined with their center of mass
to the orbifold singularity, their corresponding background fields are restricted to
the singularity as well. Hence, the effective low-energy Lagrangian is expected to
acquire the following form:

$$\mathcal{L}_{\text{eff}} = \int_{M} du \, dv \, e^{\Phi} \sqrt{G} \{ R + (\nabla \Phi)^2 + (\nabla T)^2 - 2T^2 - 8 \} + \int_{\partial M} d\tau \, e^{\Phi/2} \sqrt{G_{\tau \tau}} \{ K + \ldots \}. \tag{30}$$

We have denoted by $\partial M$ the orbifold singularity in the truncated black hole, $K$
denotes the exterior curvature of $\partial M$ in $M$, and "$\ldots"$ denote contributions from
the open-string background fields. In higher dimensions, the boundary terms in
the effective action would turn the orbifold singularity to a dynamical membrane
[27,28]. In two dimensions, the (massive) vector boson is one of the discrete states
of the open sector, carries the adjoint representation of the Chan-Paton symmetry
group, and its background plays the role of the Yang-Mills gauge field (sitting at
the orbifold singularity). Hence, it is natural to expect that a class of deformations
of the truncated black hole will exist, related to charged black holes (compare
[29,16]). Note that in the dual spacetime, the background gauge field is no longer
restricted to the orbifold singularity, as a result of general properties of open-string
target duality [23,27] (see also our discussion of duality below).

The unitary gauge choice we have used up to now ceases to be valid in the
vicinity of the singularity at $uv = 1$. Witten has shown [1] that in that region,
the black hole is effectively described by a topological field theory, namely the
$U(1)$ Chern-Simons-Witten (CSW) theory, dimensionally reduced from 3D to 2D.
Indeed, upon parametrizing the spacetime manifold in the vicinity of $uv = 1$ by
$u = e^w$ and $v = e^{-w}$, the gauged WZW Lagrangian reduces to

$$\mathcal{L}_{\text{sing}} = -\frac{k}{4\pi} \int d^2 z \, \sqrt{h} \, h^{ij} D_i a \, D_j b + \frac{ik}{4\pi} \int d^2 z \, \sqrt{h} \, w F, \tag{31}$$

where $F = e^{ij}(\partial_i A_j - \partial_j A_i)$, $D_i$ is the covariant derivative for $A$, and $a,b$ carry
$U(1)$ charge $\pm 1$ respectively. The second term in (31) is obviously the dimensional
reduction of an abelian CSW theory, $w$ being the third component of the 3D gauge
field. On our truncated black hole, \( w \) gets mapped by the orbifold group to minus itself. Recalling that \( A_z \) is mapped to \( A_z \), we easily find the full action of the orbifold group on the CSW theory, which fits nicely to the picture already known in CSW theory on 3D \( \mathbb{Z}_2 \) orbifolds [25]. Hence, we conclude that the vicinity of the singularity in the truncated black hole is effectively described by a dimensionally reduced, \( \mathbb{Z}_2 \)-equivariant CSW theory. The fact that topological degrees of freedom become important in the vicinity of the spacetime singularity, i.e. in the regime of the strong coupling, is indeed very interesting conceptually [30]. In our case, however, the correspondence just discussed may have an even more direct impact, as CSW theory on \( \mathbb{Z}_2 \) orbifolds is helpful in clarifying such issues of open string theory as the structure of the non-anomalous Chan-Paton factors for the model, relating them to 3D algebraic topology [25].

The truncated black hole model inherits a spacetime duality symmetry from its closed string ancestor. Here we will establish a simple, yet striking result, that the truncated black hole is dual to a conventional open string theory in the black hole background. So far we have considered the model in the axial gauging representation, in which the \( \mathbb{Z}_2 \) orbifold group acts by \( g \rightarrow s g^{-1} s \) (see (11)). In the dual model, the only difference is in the action of the \( U(1) \) gauge group, which is now

\[
g \rightarrow h g h^{-1},
\]

while the \( \mathbb{Z}_2 \) group action remains unchanged. While in the truncated black hole model the \( \mathbb{Z}_2 \) orbifold action preserves the generator of the \( SO(1, 1) \) gauge symmetry, and extends thus the gauge group to \( \mathbb{Z}_2 \times SO(1, 1) \), in the dual model the discrete group action reverses the generator of the gauge group. Consequently, the dual geometry is described by a gauged WZW model, in which the abelian gauge group is extended to the semi-direct product \( \mathbb{Z}_2 \ltimes SO(1, 1) \).

We can get some insight into the dual geometry by switching to its alternative description in the axial gauging representation. The \( O(1, 1) \equiv \mathbb{Z}_2 \ltimes SO(1, 1) \)
gauge group now acts by

\[ SO(2) : \begin{pmatrix} a & u \\ -v & b \end{pmatrix} \rightarrow h \begin{pmatrix} a & u \\ -v & b \end{pmatrix} h, \quad Z_2 : \begin{pmatrix} a & u \\ -v & b \end{pmatrix} \rightarrow \begin{pmatrix} b & u \\ -v & a \end{pmatrix}. \]

Recalling the gauge fixing conditions discussed above, we can see that the \( Z_2 \) acts on the Minkowski black hole (in the Kruskal coordinate system) by

\[
(u, v) \rightarrow \begin{cases} 
(u, v), & \text{regions I – IV}, \\
(-u, -v), & \text{regions V, VI}.
\end{cases}
\]

In Euclidean signature the situation is analogous. The truncated semi-infinite cigar, on which the orbifold group acts by \((r, \theta) \rightarrow (r, \pi - \theta)\), is dual to the infinite funnel, on which the target part of the orbifold group acts trivially.

A few remarks on the dual description of the truncated black hole are in order. First, the \( Z_2 \) group action of (33) leaves points of regions I–IV fixed, and acts only on the worldsheet – the model describes in these regions the conventional open string theory in the black hole geometry. Indeed, the open-string winding modes, which are fixed to \( \theta = \pm \pi/2 \) by (26), become momentum modes in the dual picture, and can move freely in the dual coordinate \( \tilde{\theta} \). Second, it is natural that while in the truncated black hole we gauge the direct product \( Z_2 \times SO(1, 1) \), in the dual model we have to gauge the semi-direct product \( Z_2 \ltimes SO(1, 1) \). Indeed, the duality transformation exchanges the generator of the gauge group action on the group manifold with the Killing vector of the model, and the \( Z_2 \) action of worldsheet orbifold models changes the orientation of precisely one of these two vector fields, as can be seen e.g. in the semiclassical limit. Finally, note that this pattern of dual models can be extended easily along the lines of [10] to any target for open string propagation interpretable as a sigma model with a Killing vector.

Let us now turn to possible relations of our truncated 2D black hole to \( c = 1 \) string theory. It was realized in the free-fermion approach to \( c = 1 \) string theory [31] that the eigenvalue density fluctuations (i.e. the spacetime tachyon) can escape
into the world beyond the mirror, once exposed to not very extreme conditions. This was related in [5] to a possibility for the string in the black hole background to fall into the black hole, be scattered off the singularity, and enter the second asymptotically flat domain in the direction of the increasing Schwarzschild time.

In the free-fermion framework one can start from the free fermionic system in the upside-down harmonic oscillator potential, with infinite walls at distances \( \pm A \) before the double-scaling limit. There is a simple variation of this theory [32] in which one fixes an alternative set of boundary conditions on the potential by moving one of the two walls (say) to the origin. We believe that such a modification of the standard double-scaled theory of free fermions can be related to our truncated black hole. If this conjecture is true, one can possibly learn something about the wall of \( c = 1 \) string theory from the fact that in the analogous situation of the black hole geometry, the wall is inhabited by open strings. This might be a challenge for the matrix model approach to \( c = 1 \) string theory.

In this paper we have found a class of exact black-hole solutions of two dimensional (closed) string theory, with open strings in the spectrum. One lesson we can draw from our results is a reconfirmation of the general observation that in string theory, the worldsheet and target geometries are tightly entangled with each other. Indeed, asking for a black hole geometry with one asymptotically flat domain, we end up with a theory of open and closed unoriented strings, i.e. we are in some sense forced to change the topology of the string. In this paper we have limited ourselves to establishing this result, and analyzed only briefly the physics and geometry of the truncated black hole. Since many aspects of the closed-string black hole itself, as well as its precise relation to \( c = 1 \) matrix models are still unclear, many of the exciting questions one can ask about the role of the truncated black hole in two dimensional string theory cannot be answered until we gain some more insight into these issues, and represent an interesting task for the future.

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