Simplified Analysis of a Multiproduct Newsboy Problem Using Simulation

J. Vijayarangam1*, B. Navin Kumar2, A. Vasudevan2, Pandiyarajan3

1GF, BITS PILANI-WILP, Chennai, India
2Department Of Mechanical Engineering, Saveetha School of Engineering, SIMATS, Chennai-602105, Tamilnadu, India.
3Department of Mechanical Engineering, K.L.N. College of Engineering, Sivagangai, India-630612

* Corresponding author: jvijayarangam@yahoo.com

Abstract. A newsboy problem is a specialized form of a general inventory problem which discusses the decision making skill of a trader, a retailer to be specific, who is facing uncertainty in the form of the demand of one or more perishable products. The lifetime of a perishable product depends on the domain. Products like newspapers, flowers are single day products as they are invalid or less useful after one day. Products like vegetables, fruits may be considered for two or three days depending on the storage procedure employed. So, the retailer has to dispense off the products within their lifetime, either by selling it or by salvaging it. This has been one of the most interesting and sought after problem for discussion as it provides with a simplistic platform to start the discussions and hence the literature is huge for such problems. It has been analyzed using very many avenues in the literature. This paper discusses the problem of the retailer, dealing in two perishable products using Monte Carlo Simulation and compare its efficiency with standard theoretical Fractile method.

Keywords: Inventory, Fractile, Newspaper.

1. Introduction

This paper addresses a two product, perishable, newsboy problem using two models, one simple method proposed based on Monte Carlo Simulation and the other one, for comparing, based on Littlewood’s Fractile method.

A newsboy problem is a special form an inventory problem of a trader, in this paper a retailer, who is selling two perishable products with a lifetime of one day like a newspaper, with uncertain demand. If we assume the product is a newspaper for simplicity, it is like buying the paper for ‘c’ rupees each, selling each for ‘s’ rupees. In addition there are two other significant quantities, one a salvage value ‘sa’ which...
any unsold paper will be sold for at the end of a day, its lifetime , and a penalty ‘p’ rupees for each unmet demand during the day. There are many versions of analyzing this in the literature. [5] Cachon and Fischer give a good discussion about supply chain inventory . [2] Arrow KJ et al., have obtained for finished goods inventories, optimal rules. Initially, they obtained an optimal policy for a known and constant demand flow. Then, they extended that to uncertain demand models which is also a dynamic one but with a known probability distribution for the random variable representing the demand flow. [3] Moon I, Yoo DK and Saha S solves a newsboy problem by considering discounts, extending it to a multiproduct model with a distribution-free approach. [5] Petruzzi et al, have extended this to a dynamic one. They initially solved a mixed newsboy problem whose characterizations are multiple discounts with upgrades and then they extended that to a multiproduct newsboy problem which also addresses storage constraints. [1] Vijayarangam Jayapalan has performed simulation based analysis for the newsboy problem with a single perishable product. In this paper that work is extended to a multiproduct scenario.

The standard newsboy profit function is \( E[\text{profit}] = E[s \times \min(q,D)] - c q \), where ‘s’ is the selling price of a single product whose cost price is ‘c’. ‘q’ denotes the EOQ value, the number of products purchased on any given day and D is the random variable representing the demand on any day whose probability distribution is generated or assumed. A simplified formula for calculating profit is

\[ \text{Profit} = \text{Revenue} - \text{cost price} + \text{salvage} - \text{Penalty}. \]

The fractile formula to obtain the EOQ ‘q’ is given by (1)

\[ q = F^{-1} \left( \frac{s-c}{s} \right) \]

(1)

where the \( F^{-1} \) is the inverse of the demand distribution. For different assumptions of the demand distributions, we obtain the varying inverse and hence a possibly different EOQ.

For a uniformly distributed demand, the fractile formula is given by (2),

\[ q_{opt} = F^{-1} \left( \frac{s-c}{s} \right) = F^{-1}(x) = D_{\min} + (D_{\max} - D_{\min})^* x \]

(2)

For a normally distributed demand, with \( N(\mu, \sigma) \), then the fractile formula is given by (3),

\[ q_{opt} = F^{-1} \left( \frac{s-c}{s} \right) = F^{-1}(x) = \mu + \sigma Z^{-1}(x) \]

(3)

where Z is the standard normal variate.

Simulation is a multidisciplinary domain which indulges in solving problems using a combination of ideas and concepts from different fields and heuristics usually are a dominating ones in such efforts. It could be seen as a process of imitating the functioning of a real life system to understand its characteristics. So, in a way we are generating an artificial history of a system to draw inferences about the system so that we can make suitable changes in the system to make it a better one. The general methodology of simulation is as given by Table 1 but it is flexible based on the problem in hand.
Table 1. Simulation Procedure

|   | Statement of problem. |   | Design of the experiments; |
|---|-----------------------|---|-----------------------------|
| 1 | Objectives and plan.  |   | Simulation runs & analysis; |
| 2 | Simulation model development. |   | Decision on no. of runs; |
| 3 | Translating the model. |   | Documentation & reporting; |
| 4 | Model Verification and Validation. |   | Implementation |

2. Methodology

The study is about multiproduct analysis, in specific two perishable products with single day lifetime like newspapers. Let us say a person has P rupees to invest on two newspapers whose demand is random and also depends on the type of news day, assumed to be of three types: Fine, Moderate or Ordinary. He wishes to find out how much amount he should invest in each to optimize his profit. The proposed method is a very simple one based on Monte Carlo Simulation technique.

Step 1: Generate demand distributions for the two newspapers, six in total accommodating three news-days each.

Step 2: Calculate the Expected demand for all six distributions, using the simple Expectation procedure.

Step 3: Simulate using the expected demand (for all six days separately) and obtain EOQ for the six cases.

Step 4: Assuming the news-days to be equally likely, calculate a single EOQ for both the newspapers.

Step 5: Calculate EOQ using the Fractile formula, assuming Uniform and Normal distributions.

Step 6: Compare EOQ obtained using Monte Carlo Simulation (step 4) and Standard Fractile method (step 5) and infer.

Once he is convinced that the EOQ calculated using the proposed method is a reasonably good one, invest money according to the optimal EOQ.

3. Results and Discussion

The six probability distributions are in Table 2.

Table 2. Probability distributions of the three news days: fine day, Moderate day and Ordinary day.

| Daily Demand | Newspaper 1-Probability distribution | Newspaper 2-Probability distribution |
|--------------|--------------------------------------|--------------------------------------|
|              | Fine       | Moderate | Ordinary | Fine       | Moderate | Ordinary |
| 40           | 0.03       | 0.1      | 0.44     | 0.04402    | 0.10108  | 0.16091  |
| 50           | 0.05       | 0.18     | 0.22     | 0.09246    | 0.12713  | 0.20728  |
| 60           | 0.15       | 0.4      | 0.16     | 0.14264    | 0.35342  | 0.23685  |
| 70           | 0.2        | 0.2      | 0.12     | 0.25524    | 0.19904  | 0.22835  |
| 80           | 0.35       | 0.08     | 0.06     | 0.17417    | 0.05022  | 0.08134  |
| 90           | 0.15       | 0.04     | 0        | 0.23508    | 0.05368  | 0.05734  |
| 100          | 0.07       | 0        | 0        | 0.05639    | 0.11543  | 0.02793  |

Just to confirm the different natures of the two newspapers and also to compare, we get the picture1
Then from the probability distributions generated, the expected demand is calculated for each of the six possible combination of news-days (three for each paper resulting in six for the two together). Then simulation is performed for each of these simulated demands to obtain expected profit. Also from the calculated EOQ for the three cases for each newspaper, assuming equal probability for the three cases, the expected EOQ is calculated, table 3-Picture2. These are compared with Littlewood's Fractile formula based EOQ, for uniform and Normally distributed assumptions.

**Table 3.** Simulated Optimal Order quantities.

| Newspaper | Fine | Moderate | Ordinary | EOQ   |
|-----------|------|----------|----------|-------|
| 1         | 75   | 61       | 51       | 62.33333 |
| 2         | 74   | 66       | 61       | 67    |

**Figure 1.** Probability distribution histograms of the two papers, Newspaper 1 on Left and Newspaper 2 on right.

**Figure 2.** Average EOQ (Blue) and those in Fine (Blue), Moderate (Red) and Ordinary (green) days.
Using the standard critical formula (or) Littlewood’s rule with uniformly distributed demand assumption, we obtained EOQ as given by (4) for newspaper 1 and (5) for newspaper 2,

\[ q_{opt} = F^{-1}\left(\frac{6 - 4}{6}\right) = F^{-1}(0.33) = D_{\min} + \left(D_{\max} - D_{\min}\right)0.33 = 40 + (100 - 40)0.33 = 60 \]  
\[ q_{opt} = F^{-1}\left(\frac{3.5 - 2}{3.5}\right) = F^{-1}(0.428) = D_{\min} + \left(D_{\max} - D_{\min}\right)0.428 = 40 + (100 - 40)0.428 = 65.71 \]  

Using the standard critical formula (or) Littlewood’s rule with Normally distributed demand assumption, we obtained EOQ for the two newspapers by (6) for Newspaper 1 and (7) for newspaper 2,

\[ q_{opt} = F^{-1}\left(\frac{6 - 4}{6}\right) = F^{-1}(0.33) = \mu + \sigma Z^{-1}(0.33) = 70 + 21.6 * Z^{-1}(0.33) = 70 + 21.6 * -0.45 = 60.28 \]  
\[ q_{opt} = F^{-1}\left(\frac{3.5 - 2}{3.5}\right) = F^{-1}(0.428) = \mu + \sigma Z^{-1}(0.428) = 70 + 21.6 * Z^{-1}(0.428) = 70 + 21.6 * -0.18 = 66.112 \]

The three computed EOQ’s [Table4,Figure3] are

| Newspaper | cost | selling | salvage | penalty | Simulated | LW-Uniform | LW-Normal | Avg profit |
|-----------|------|---------|---------|---------|-----------|------------|-----------|------------|
| 1         | 4    | 6       | 1       | 2       | 62.33     | 60         | 60.28     | 83.67      |
| 2         | 2    | 3.5     | 0.75    | 1.5     | 67        | 65.7142    | 66.112    | 93         |

**Table 4. EOQ’s using Simulation and Fractile methods and the profits.**

![Bar chart showing EOQs for newspapers 1 and 2 using different methods]

**Figure 3.** EOQ’s using Simulation and Fractile methods and the profits
From the table [4] and figure [3], we could infer that the Monte Carlo Simulation based on Jerry Banks is very much in alignment with the Littlewood’s theoretical forms as far as the value is concerned and in addition this is relatively easy, computationally and theoretically.

Simulation suggests that buying 62 papers of Newspaper 1 fetches an average profit of Rs.83.67 and buying 67 papers of Newspaper2 would yield an average profit of Rs.93. This could be rephrased as, (62*4=) Rs.248 spent on newspaper1 fetches an average profit of 83.67 while (67*2=) Rs.134 spent on newspaper2 fetches an average profit of Rs.93. So, investing 248+134=Rs.382 in buying the two newspapers in a morning would on an average yield a profit of 83.67+93=Rs.176.67 by evening which is approximately an expected profit of 46%.

Also, we observe that the average profit for the simulated period is higher for the cheaper newspaper which could be due to many reasons like, a quantity based on simulation which could be found out with many more runs or could be an effect of the lesser penalty or simply could be a factor of the actual population demand in which case this needs a more rigorous study taking other possible avenues which paves the way for our future study.

4. Conclusion

The above modeling, analysis and discussion of the two product newsboy problem, a specific form of an inventory problem provides enough evidence to conclude that we can employ the proposed method based on Monte Carlo simulation to address the issues regarding a general multiproduct inventory problem. This also means that we do not have to make a lot of theoretical assumptions we usually make when we follow traditional methods like Fractile method. But we should be cautious while obtaining the basic probability distribution, the basement on which the entire procedure is built. Once we have empirical data with respect to any number of newspapers, we can find the empirical distribution and just plug it into the distribution stage of the proposed method and just follow the remaining simple steps to obtain reasonably good measures. Even for those who are not fully convinced with the procedure in spite of the simplicity or the answers, we could say that these can be good starting approximations at least for a more robust theoretical study. So, in every possible way, the methodology developed should be a good, easy and fruitful one a multiproduct inventory analysis.

5. References

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