Heavy exotic scalar meson $T_{bb,us}^-$

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The spectroscopic parameters and decay channels of the scalar tetraquark $T_{bb,us}^-$ (in what follows $T_{bb}$) are investigated in the framework of the QCD sum rule method. The mass and coupling of the $T_{bb}$ are calculated using the two-point sum rules by taking into account quark, gluon and mixed vacuum condensates up to dimension 10. Our result for its mass $m_0=(10250\pm 370)$ MeV demonstrates that $T_{bb}$ is stable against the strong and electromagnetic decays. Therefore to find the width and mean lifetime of the $T_{bb}$ we explore its dominant weak decays generated by the transition $b \rightarrow W^–c$. These channels embrace the semileptonic decays $T_{bb}^– \rightarrow Z_{bc}^0 \nu l$ and nonleptonic modes $T_{bb}^– \rightarrow Z_{bc}^0 b\bar c \pi^– (K^–, D^–, D_s^–)$, which at the final state contain the scalar tetraquark $Z_{bc}^0$. Key quantities to compute partial widths of the weak decays are the form factors $G_1(q^2)$ and $G_2(q^2)$: they determine differential rates $d\Gamma/dq^2$ of the semileptonic and partial widths of the nonleptonic processes, respectively. These form factors are extracted from relevant three-point sum rules at momentum transfers $q^2$ accessible for such analysis. By means of the fit functions $F_{1,2}(q^2)$ they are extrapolated to cover the whole integration region $m_0^2 \leq q^2 \leq (m–\bar m)^2$, where $\bar m$ is the mass of $Z_{bc}^0$. Predictions for the full width $\Gamma_{full}=(15.21\pm 2.59) \times 10^{-10}$ MeV and mean lifetime $4.33^{+0.63}_{-0.63} \times 10^{-13}$ s of the $T_{bb}$ are useful for experimental and theoretical investigation of this exotic meson.

I. INTRODUCTION

Investigation of exotic mesons that are composed of four quarks (tetraquarks) is among of the interesting topics of the high energy physics. Experimental information collected during last years by various collaborations and theoretical progress achieved in the framework of different methods and models form rapidly growing field of exotic studies [1, 2].

The states observed in experiments till now and interpreted as candidates to exotic mesons have different natures. Thus, some of them are neutral charmonium (bottomonium)-like resonances and may be considered as excited states of the charmonium. Others bear an electric charge and are free of these problems, but reside close to two-meson threshold permitting an interpretation as bound states of conventional mesons or dynamical effects. It is worth noting that all of the discovered tetraquarks have large full widths and decay strongly to two conventional mesons. Therefore, four-quark compounds stable against strong and electromagnetic interactions, and decaying only through weak transformations can provide valuable information on tetraquarks.

The stability of the tetraquarks $QQ’\bar q\bar q’$ were studied already in original articles [3, 4], in which it was proved that a heavy $Q’(f)$ and light $q’(l)$ quarks may form the stable exotic mesons provided the ratio $m_Q/m_q$ is large enough. Among such states tetraquarks $bbqq’$ are particles of special interest, because in this case the ratio $m_b/m_q(q’)$ reaches its maximum value. In fact, the isoscalar axial-vector tetraquark $T_{bb,us}^-$ with the mass lower than the $BB$ threshold is a strong-interaction stable state [5]. Calculations carried out by means of different approaches confirmed the strong- and electromagnetic-interaction stable nature of the tetraquark $T_{bb,us}^-$ [10, 13]. The full width and mean lifetime of the $T_{bb,us}^-$ were estimated in Ref. [13] (see, also Ref. [14]) by employing the semileptonic decays $T_{bb,us}^– \rightarrow Z_{bc}^0 \nu l$. The stability of the tetraquarks $T_{bb,us}^-$ and $T_{bb,us}^0$ was demonstrated in Ref. [12] using relations of the heavy-quark symmetry.

It turned out that not only exotic mesons containing $bb$ diquarks, but also tetraquarks built of $bc$ may be stable against the strong and electromagnetic decays. Relevant problems were addressed in numerous publications [11–13, 17, 16]. Thus, analysis performed in Ref. [13] demonstrated that the scalar tetraquark $Z_{bc}^0$ with the mass $m_Z=(6660\pm 150)$ MeV is considerably below thresholds for strong and electromagnetic decays. In other words, the $Z_{bc}^0$ transforms due to weak decays that allow us to estimate its full width and mean lifetime [16]. A situation with the axial-vector tetraquark $T_{bc,us}^0$ remains unclear: the mass of this state predicted in the range $(1705\pm 155)$ MeV admits a twofold explanation [17]. Indeed, using the central value of the mass one see that it lies below thresholds for the strong and electromagnetic decays, whereas the maximum estimate for the mass $7260$ MeV is higher than thresholds for strong and electromagnetic decays to $B^+–D^+//B^0-D^0$ and $D^+B^-/D^0\bar B^0\gamma$, respectively. In the first case the full width and lifetime of the tetraquark $T_{bc,us}^0$ are determined by its weak decays. In the second scenario the width of $T_{bc,us}^0$ is fixed mainly by strong modes, because widths of weak and electromagnetic processes are small and can be ignored [17].
It is worth noting that some of heavy exotic mesons containing diquarks $b\bar{s}$ may be stable as well. Thus, the scalar tetraquark $T_{b\bar{s}b\bar{s}}$ is strong- and electromagnetic-interaction stable particle: its spectroscopic parameters and semileptonic decays were explored in Ref. [18].

In the present article we study the scalar tetraquark $T_{b\bar{s}}$ with the quark content $bb\bar{s}\bar{s}$ and compute its spectroscopic parameters, full width and mean lifetime. The mass $m$ and coupling $f$ of $T_{b\bar{s}}$ are extracted from the QCD two-point sum rules by taking into account vacuum expectation values of the local quark, gluon and mixed operators up to dimension ten. The information on the mass of this state is crucial to determine whether $T_{b\bar{s}}$ is strong- and electromagnetic-interaction stable particle or not. It is not difficult to see that dissociation to a pair of conventional pseudoscalar mesons $B^0\bar{B}^0$ is the first S-wave strong decay channel for the unstable $T_{b\bar{s}}$. Therefore if the mass of $T_{b\bar{s}}$ is higher than the $B^0\bar{B}^0$ threshold 10646 MeV then one should calculate the width of the process $T_{b\bar{s}} \rightarrow B^0\bar{B}^0$. But, our investigations show (see, below) that the mass of the tetraquark $T_{b\bar{s}}$ is equal to $m = (10250 \pm 270)$ MeV, and it lies below this bound. The $T_{b\bar{s}}$ is stable against the possible electromagnetic transition $T_{b\bar{s}} \rightarrow B^0\bar{B}^0\pi$ as well, because for realization of this process the mass of the master particle should exceed 11108 MeV which is not a case. Therefore to evaluate the full width and lifetime of $T_{b\bar{s}}$ one has to explore its weak decays.

The weak transformations of the $T_{b\bar{s}}$ may run due to the subprocesses $b \rightarrow W^- c$, and $b \rightarrow W^- u$ which generate its semileptonic dissociation to the scalar four-quark mesons $Z_{b\bar{s}b\bar{s}}^0$ (hereafter $Z_{b\bar{s}}^0$) and $Z_{b\bar{s}u\bar{d}}^0$. The process $T_{b\bar{s}} \rightarrow Z_{b\bar{s}b\bar{s}}^0$ is dominant weak channel for the tetraquark, because the decay $T_{b\bar{s}} \rightarrow Z_{b\bar{s}u\bar{d}}^0\pi$ is suppressed relative to first one by a factor $|V_{ub}|^2/|V_{cb}|^2 \approx 0.01$ with $|V_{cb}|$ being the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements.

But the subprocess $b \rightarrow W^- c$ can also give rise to nonleptonic weak decays of the $T_{b\bar{s}}$. Indeed, the vector boson $W^-$ instead of a lepton pair $\pi\pi$ can produce $d\bar{u}$, $s\bar{d}$, $d\bar{c}$, and $s\bar{c}$ quarks as well. These quarks afterwards form one of the conventional mesons $M = \pi^-, K^-, D^-$ and $D_s^-$ leading to the nonleptonic final states $Z_{b\bar{s}M}^0$. Depending on the difference $m - \bar{m}$, where $\bar{m}$ is the $Z_{b\bar{s}M}^0$ tetraquark’s mass, some or all of these nonleptonic weak decays become kinematically allowed.

We calculate the full width of the $T_{b\bar{s}}$ by taking into account its semileptonic and nonleptonic decay modes. To this end, employing the QCD three-point sum rule approach, we determine the weak form factors $G_{1(2)}(q^2)$ necessary to evaluate the differential rates of the semileptonic decays. Partial width of the processes $T_{b\bar{s}} \rightarrow Z_{b\bar{s}M}^0$, $l = e^-, \mu^-$ and $\tau^-$ can be found by integrating the differential rates over kinematically allowed momentum transfers $q^2$, whereas width of the nonleptonic decays $T_{b\bar{s}} \rightarrow Z_{b\bar{s}M}^0$ are fixed by values of the $G_{1(2)}(q^2)$ at $q^2 = m_M^2$, where $m_M$ is the mass of a produced meson.

This article is structured in the following way: In Section II we calculate the spectroscopic parameters of the scalar tetraquarks $T_{b\bar{s}}$ and $Z_{b\bar{s}}^0$. For these purposes, we derive two-point sum rules from analysis of corresponding correlation functions and include into calculations the qark, gluon and mixed condensates up to dimension ten. In Section III we derive three-point sum rules for the weak form factors $G_{1(2)}(q^2)$ and compute them in regions of the momentum transfer, where the method gives reliable predictions. We extrapolate $G_{1(2)}(q^2)$ to the whole integration region by means of fit functions and find partial widths of the semileptonic decays $T_{b\bar{s}} \rightarrow Z_{b\bar{s}M}^0$. In Section IV we analyze the nonleptonic weak decays $T_{b\bar{s}} \rightarrow Z_{b\bar{s}M}^0$. Here we also present our final estimate for the full width and mean lifetime of the $T_{b\bar{s}}$. Section V is reserved for discussion and concluding notes.

II. MASS AND COUPLING OF THE SCALAR TETRAQUARKS $T_{b\bar{s}}$ AND $Z_{b\bar{s}}^0$

The spectroscopic parameters of the tetraquark $T_{b\bar{s}}$ are necessary to reveal its nature and answer questions about its stability. The mass and coupling of $Z_{b\bar{s}}^0$ are important to explore the weak decays of the master particle $T_{b\bar{s}}$. It is worth noting that the $T_{b\bar{s}}$ and $Z_{b\bar{s}}^0$ have the same heavy diquark-light antidiquark organization.

The parameters of these states can be extracted from the two-point correlation function

$$\Pi(p) = i \int d^4x e^{ipx} \langle 0| T\{J(x)J^\dagger(0)\}|0\rangle,$$  \hspace{1cm} (1)

where $J(x)$ is the interpolating current for a scalar particle. In the case of $T_{b\bar{s}}$ it has the following form [19]

$$J(x) = [b\bar{c}(x)C\gamma_5 b\bar{b}(x)] [\overline{\tau}\tau(x)\gamma_5 C\sigma_\tau^T(x)],$$  \hspace{1cm} (2)

whereas for the $Z_{b\bar{s}}^0$ we use [20]

$$J(x) = [b\bar{c}(x)C\gamma_5 b\bar{b}(x)] [\overline{\tau}\tau(x)\gamma_5 C\sigma_\tau^T(x)]$$
$$+ \overline{u}(x)\gamma_5 C\sigma_u^T(x).$$  \hspace{1cm} (3)

In the expressions above $a$, and $b$ are color indices and $C$ is the charge-conjugation operator.

The currents $J(x)$ and $J(x)$ have the symmetric color organizations $[6_c]_b \otimes [6_c]_{b\bar{s}}$ and $[6_c]_{b\bar{s}} \otimes [6_c]_{b\bar{s}}$, and are composed of the scalar diquark and antidiquark. They describe the particles containing two heavy quarks of the same and four quarks of different flavors, respectively. It is known that the scalar diquarks are most attractive and stable structures [21], therefore these currents correspond to the lowest lying ground-state tetraquarks in each class.

Here, we consider in a detailed form computation of the $T_{b\bar{s}}$ tetraquark’s mass $m$ and coupling $f$, and provide final results for $Z_{b\bar{s}}^0$. To derive the sum rules for $m$ and $f$, we need first to find the phenomenological expression of the correlation function $\Pi^{\text{Phys}}(p)$, which should...
be written down in terms of the spectroscopic parameters of $T^-_{b\pi}$. Since $T^-_{b\pi}$ is a ground-state particle, we use the "ground-state + continuum" scheme. Then separating contribution of the tetraquark $T^-_{b\pi}$ from effects of the higher resonances and continuum states, we can write

$$\Pi^{\text{phys}}(p) = \frac{\langle 0|J[T^-_{b\pi}(p)]T^-_{b\pi}(p)]J[1]|0\rangle}{m^2 - p^2} + \ldots$$

The phenomenological function $\Pi^{\text{phys}}(p)$ is obtained by inserting into $\Pi(p)$ a full set of scalar four-quark states and performing integration over $x$. Calculation of $\Pi^{\text{phys}}(p)$ can be finished by employing the matrix element

$$\langle 0|J[T^-_{b\pi}(p)] = fm.$$  

After simple manipulations we get

$$\Pi^{\text{phys}}(p) = \frac{f^2 m^2}{m^2 - p^2} + \ldots$$

The correlation function $\Pi^{\text{phys}}(p)$ has a trivial Lorentz structure which is proportional to $\sim I$. Hence, the only term in Eq. (4) is nothing more than the invariant amplitude $\Pi^{\text{phys}}(p^2)$ corresponding to this structure. Now, we have to fix the second component of the sum rule analysis, and express $\Pi(p)$ in terms of the quark propagators. To this end, we utilize the explicit expression of the interpolating current $J(x)$, and contract relevant heavy and light quark fields to get $\Pi^{\text{OPE}}(p)$. After these manipulations, we find

$$\Pi^{\text{OPE}}(p) = i \int d^4xe^{ipx} \text{Tr} \left[ \gamma_5 S^b(x)\gamma_5 S^a(x) \right] \times \left\{ \text{Tr} \left[ \gamma_5 S^b(x)\gamma_5 S^b(x) \right] + \text{Tr} \left[ \gamma_5 S^a(x)\gamma_5 S^b(x) \right] \right\},$$

where $S_b(x)$ and $S_u(x)$ are the $b$- and $u(s)$-quark propagators, respectively. Here we also use the shorthand notation

$$\tilde{S}_{b(u,s)}(x) = CS^T_{b(u,s)}(x)C.$$  

The explicit expressions of the heavy and light quark propagators were presented in Ref. [22], for instance. The nonperturbative part of these propagators contains vacuum expectation values of various quark, gluon, and mixed operators which generate a dependence of $\Pi^{\text{OPE}}(p)$ on nonperturbative quantities. To derive the sum rules, we equate the amplitudes $\Pi^{\text{phys}}(p^2)$ and $\Pi^{\text{OPE}}(p^2)$, and apply to both sides of the obtained equality the Borel transformation. This operation is necessary to suppress contributions of higher resonances and continuum states. Afterwards, we carry out the continuum subtraction using the assumption on the quark-hadron duality. The expression found by this way, and an equality obtained by applying the operator $d/d(-1/M^2)$ to the first one form a system which is enough to obtain the sum rules for $m$

$$m^2 = \frac{\int_{M^2}^{\infty} ds \rho^{\text{OPE}}(s)e^{-s/M^2}}{\int_{M^2}^{\infty} ds \rho^{\text{OPE}}(s)e^{-s/M^2}},$$

and $f$

$$f^2 = \frac{1}{M^2} \int_{M^2}^{\infty} ds \rho^{\text{OPE}}(s)e^{(m^2-s)/M^2}.$$}

In Eqs. (9) and (10) $M = 2m_b + m_s$, and $\rho^{\text{OPE}}(s)$ is the two-point spectral density. The $\rho^{\text{OPE}}(s)$ is proportional to the imaginary part of the correlation function $\Pi^{\text{OPE}}(p)$: its explicit expression is rather cumbersome, therefore we do not provide it here. The sum rules for $m$ and $f$ depend on the Borel and threshold parameters $M^2$ and $s_0$, which appear after the Borel transformation and continuum subtraction procedures, respectively. Both of $M^2$ and $s_0$ are the auxiliary parameters a proper choice of which depends on the problem under analysis, and is one of the important problems in the sum rule computations. Apart from $M^2$ and $s_0$, the sum rules contain also the universal vacuum condensates and the mass of $b$ and $s$ quarks:

$$\langle \bar{q}q \rangle = -(0.24 \pm 0.01)^3 \text{GeV}^3, \quad \langle s\bar{s} \rangle = 0.8 \langle \bar{q}q \rangle, \quad \langle \bar{q}g_s\sigma Gq \rangle = m_0^2 \langle \bar{q}q \rangle, \quad \langle \bar{q}g_s\sigma Gs \rangle = m_0^2 \langle s\bar{s} \rangle,$$

$$m_b^2 = (0.8 \pm 0.1) \text{ GeV}^2,$$

$$\langle \alpha_s G^2 \rangle = (0.012 \pm 0.004) \text{ GeV}^4;$$

$$\langle g_s^2 G^3 \rangle = (0.57 \pm 0.29) \text{ GeV}^6, \quad m_s = 93^{+11}_{-5} \text{ MeV},\quad m_c = 1.27 \pm 0.2 \text{ GeV}, \quad m_b = 4.18^{+0.03}_{-0.02} \text{ GeV}. $$

The working windows for the auxiliary parameters $M^2$ and $s_0$ have to satisfy some essential constraints. Thus, at maximum of $M^2$ the pole contribution (PC) should

**FIG. 1:** The mass $m$ of the tetraquark $T^-_{b\pi}$ as a function of the Borel $M^2$ and continuum threshold $s_0$ parameters.
exceed a fixed value, which for the multiquark systems is chosen in the form

\[ PC = \frac{\Pi(M^2, s_0)}{\Pi(M^2, \infty)} > 0.2, \quad (12) \]

The function \( \Pi(M^2, s_0) \) in Eq. (12) is the Borel-transformed and subtracted invariant amplitude \( \Pi^{\text{OPE}}(p^2) \). The minimum of \( M^2 \) is extracted from analysis of the ratio

\[ R(M^2) = \frac{\Pi^{\text{DimN}}(M^2, s_0)}{\Pi(M^2, s_0)} \leq 0.01. \quad (13) \]

Fulfilment of Eq. (13) implies the convergence of the operator product expansion (OPE) and obtained sum rules. Here, \( \Pi^{\text{DimN}}(M^2, s_0) \) denotes a contribution to the correlation function coming from the last term (or a sum of last few terms) in the expansion. In the present calculations we use a sum of last three terms, and hence \( \text{DimN} \) means \( \text{Dim}(8 + 9 + 10) \).

The numerical analysis proves that the working regions for the parameters \( M^2 \) and \( s_0 \)

\[ M^2 \in [9, 12] \text{ GeV}^2, \quad s_0 \in [115, 120] \text{ GeV}^2, \quad (14) \]

satisfy all aforementioned constraints on \( M^2 \) and \( s_0 \). Namely, at \( M^2 = 12 \text{ GeV}^2 \) the pole contribution is 0.22, whereas at \( M^2 = 9 \text{ GeV}^2 \) it amounts to 0.56. These two values of \( M^2 \) determine the boundaries of a window within of which the Borel parameter can be varied. At the minimum of \( M^2 = 9 \text{ GeV}^2 \) we get \( R \approx 0.001 \). Apart from that, at the minimum of \( M^2 \) the perturbative contribution amounts to 85% of the whole result overshooting significantly the nonperturbative terms.

Our results for \( m \) and \( f \) are

\[ m = (10250 \pm 270) \text{ MeV}, \]
\[ f = (2.69 \pm 0.58) \times 10^{-2} \text{ GeV}^4, \quad (15) \]

where we indicate also uncertainties of the computations. These theoretical errors stem mainly from variation of the parameters \( M^2 \) and \( s_0 \) within allowed limits. It is seen, that for the mass these uncertainties equal to \( \pm 2.6\% \) of its central value, whereas for the coupling \( f \) they are larger and amount to \( \pm 22\% \). In other words, the result for the mass is less sensitive to the choice of the parameters than the coupling \( f \). The reason is that the sum rule for the mass \( \rho \) is given as a ratio of two integrals of the function \( \rho^{\text{OPE}}(s) \) which stabilizes undesired effects, but even in the situation with the coupling \( f \) uncertainties do not exceed limits accepted in sum rule computations. In Fig. 1 we plot the sum rule’s prediction for \( m \) as a function of the parameters \( M^2 \) and \( s_0 \), where one can see its residual dependence on them.

The mass \( \tilde{m} \) and coupling \( \tilde{f} \) of the scalar tetraquark \( Z^0_{b\pi} \) can be found from Eqs. (9) and (10) after replacing \( \rho^{\text{OPE}}(s) \) by a relevant spectral density \( \tilde{\rho}^{\text{OPE}}(s) \) and using \( \tilde{M} = m_0 + m_c + m_s \) instead of \( M \). Predictions for \( \tilde{m} \) and \( \tilde{f} \) read

\[ \tilde{m} = (6830 \pm 160) \text{ MeV}, \]
\[ \tilde{f} = (7.1 \pm 1.8) \times 10^{-3} \text{ GeV}^4. \quad (17) \]

The \( \tilde{m} \) and \( \tilde{f} \) are extracted using the following regions for the parameters \( M^2 \) and \( s_0 \)

\[ M^2 \in [5.5, 6.5] \text{ GeV}^2, \quad s_0 \in [53, 55] \text{ GeV}^2. \quad (18) \]

These working windows meet standard requirements of the sum rule computations which have been discussed above. In fact, since at \( M^2 = 5.5 \text{ GeV}^2 \) the ratio \( R \) is equal to 0.008, the convergence of the obtained sum rules is guaranteed. The pole contribution at maximum of the Borel parameter \( M^2 = 6.5 \text{ GeV}^2 \) amounts to \( PC = 0.25 \), which is in accord with the restriction (12), and reaches \( PC = 0.64 \) at \( M^2 = 5.5 \text{ GeV}^2 \). Theoretical uncertainties of calculations for the mass \( \pm 2.3\% \) are considerably smaller than ambiguities of the coupling \( \pm 25\% \) due to reasons explained above. In Fig. 2 we depict our prediction for the mass of the tetraquark \( Z^0_{b\pi} \) and show its dependence on \( M^2 \) and \( s_0 \).

The spectroscopic parameters \( m, f \) of the tetraquark \( T_{b}\pi \) allow us, first of all, to analyze its possible decay channels. Together with the mass and coupling of the \( Z^0_{b\pi} \) they are also necessary to compute partial widths of these decays.
III. SEMILEPTONIC DECAYS $T^0_{b\pi} \rightarrow Z^0_{b\pi}l\bar{v}$

Our result $m = (10250 \pm 270)$ MeV for the mass of the tetraquark $T^0_{b\pi}$ demonstrates its stability against the strong and electromagnetic decays to final states $B^+\bar{B}^0$ and $B^+\bar{B}^0_{1s}$ (5830) $\gamma$, respectively. In fact, the central value of the mass $m = 10250$ MeV is 396 MeV lower than the threshold for strong decay to the conventional mesons $B^+\bar{B}^0$. Even its maximal value 10520 MeV obtained by taking into account uncertainties of the method is 126 MeV below this limit. Because the threshold 11108 MeV for electromagnetic dissociation of the $T^0_{b\pi}$ is considerably higher than $m$ the similar arguments hold for the corresponding process as well.

Therefore, the full width and lifetime of the $T^0_{b\pi}$ are determined by its weak transitions. In this section we concentrate on the dominant semileptonic decays $T^0_{b\pi} \rightarrow Z^0_{b\pi}l\bar{v}$ generated by the weak transition $b \rightarrow W^-c \rightarrow cl\bar{v}$ of the heavy $b$-quark. It is clear, that due to large mass difference $m - \bar{m} \approx 3420$ MeV decays $T^0_{b\pi} \rightarrow Z^0_{b\pi}l\bar{v}$ are kinematically allowed for all lepton species $l = e, \mu$ and $\tau$. We do not consider processes $b \rightarrow W^-u$ because they are suppressed relative to dominant ones by a factor $|V_{bu}|^2/|V_{bc}|^2 \simeq 0.01$.

The transition $b \rightarrow W^-c$ at the tree-level can be described by means of the effective Hamiltonian

$$\mathcal{H}^{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{bc} \gamma_\mu (1 - \gamma_5) \not{d} \gamma^\mu (1 - \gamma_5) q_l,$$

(19)

where $G_F$ and $V_{bc}$ are the Fermi coupling constant and the relevant CKM matrix element, respectively:

$$G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2},$$

$$|V_{bc}| = (42.2 \pm 0.08) \times 10^{-3}.$$  (20)

After sandwiching $\mathcal{H}^{\text{eff}}$ between the initial and final tetraquark fields, and removing a leptonic part from an obtained expression, we get the matrix element of the current

$$J^{\mu}_{\mu} = \bar{c} \gamma_\mu (1 - \gamma_5) b.$$  (21)

The latter can be written down using the form factors $G_i(q^2)$ ($i = 1, 2$) which parametrize the long-distance dynamics of the weak transition. In terms of $G_1(2)(q^2)$ the matrix element of the current $J^{\mu}_{\mu}$ has the form

$$(Z^0_{b\pi}(p')|J^{\mu}_{\mu}|T^0_{b\pi}(p)) = G_1(q^2) P_\mu + G_2(q^2) q_\mu,$$

(22)

where $p$ and $p'$ are the momenta of the initial and final tetraquarks, respectively. Here we also introduce variables $P_\mu = p'_\mu + p_\mu$ and $q_\mu = p_\mu - p'_\mu$. The $q_\mu$ is the momentum transferred to the leptons, and $q^2$ changes within the limits $m_l^2 \leq q^2 \leq (m - \bar{m})^2$ with $m_l$ being the mass of a lepton $l$.

To derive the sum rules for the form factors $G_1(2)(q^2)$, we begin from analysis of the three-point correlation function

$$\Pi_\mu(p, p') = i^2 \int d^4xd^4y e^{i(p'y - px)} \times \langle 0| T\{ J(x) J^{\mu\dagger}(0) J^\dagger(x)\} |0\rangle.$$  (23)

In accordance with standard prescriptions, we write the correlation function $\Pi_\mu(p, p')$ using the spectroscopic parameters of the tetraquarks, and get the physical side of the sum rule $\Pi^{\text{phys}}_\mu(p, p')$. The function $\Pi^{\text{phys}}_\mu(p, p')$ can be presented in the following form

$$\Pi^{\text{phys}}_\mu(p, p') = \left(0| \tilde{J} Z^0_{b\pi}(p')\rangle \langle Z^0_{b\pi}(p')| J^{\mu\dagger}\not{T} Z^0_{b\pi}(p)\langle 0\right| \not{P} (p^2 - m^2)(p^2 - \bar{m}^2)$$

$$\times (T^0_{b\pi}(p)| J^\dagger|0\rangle + \ldots,$$  (24)

where the contribution of the ground-state particles is shown explicitly, whereas effects of excited resonances and continuum states are denoted by dots.

The phenomenological side of the sum rules can be detailed by expressing the matrix elements in terms of the tetraquarks’ mass and coupling, and weak transition form factors. For these purposes, we use Eqs. (19) and (22), and employ the matrix element of the state $Z^0_{b\pi}$

$$(0| \tilde{J} Z^0_{b\pi}(p')\rangle = \bar{f}m.$$  (25)

Then it is not difficult to find that

$$\Pi^{\text{phys}}_\mu(p, p') = \frac{f m \bar{f} m}{(p^2 - m^2)(p^2 - \bar{m}^2)} \times \left[ G_1(q^2) P_\mu + G_2(q^2) q_\mu \right] + \ldots.$$  (26)

We determine $\Pi_\mu(p, p')$ also by utilizing the interpolating currents and quark propagators, which lead to QCD side of the sum rules

$$\Pi^{\text{QCD}}_\mu(p, p') = i^2 \int d^4xd^4y e^{i(p'y - px)} \left(Tr\left[ \gamma_5 \bar{S}^{\mu\nu}_{b'}(x - y) \right] \gamma_5 \bar{S}^{\nu\alpha}_{u}(x - y) + Tr\left[ \gamma_5 \bar{S}^{\alpha\nu}_{u}(x - y) \gamma_5 \bar{S}^{\mu\nu}_{b'}(y) \gamma_5 \bar{S}^{\nu\alpha}_{b'}(y) \right] \right)$$

$$\times \left( Tr\left[ \gamma_5 \bar{S}^{\nu\alpha}_{b'}(y - x) \gamma_5 \bar{S}^{\mu\nu}_{b}(y) \gamma_5 \bar{S}^{\nu\alpha}_{b}(y - x) \right] + Tr\left[ \gamma_5 \bar{S}^{\nu\alpha}_{b}(y - x) \gamma_5 \bar{S}^{\mu\nu}_{b'}(y) \gamma_5 \bar{S}^{\nu\alpha}_{b'}(y - x) \right] \right).$$  (27)

One can obtain the sum rules for the form factors $G_1(2)(q^2)$ by equating invariant amplitudes corresponding to structures $P_\mu$ and $q_\mu$ from $\Pi^{\text{phys}}_\mu(p, p')$ and $\Pi^{\text{QCD}}_\mu(p, p')$. It is known that, these invariant amplitudes depend on $p^2$ and $p'^2$, and therefore in order to suppress contributions of higher resonances and continuum states we have to apply the double Borel transformation over these variables. As a result, the final expressions contain a set of Borel parameters $M^2 = (M_1^2, M_2^2)$. The continuum subtraction should be carried out in two channels...
which generates a dependence on the threshold parameters $s_0 = (s_0, s_0')$.

These operations lead to the sum rules

$$G_i(M^2, s_0, q^2) = \frac{1}{f_{im}} \int_{s_0}^{s'} ds \int_{M^2}^{s} ds' \rho_i(s, s', q^2) e^{(m^2-s)/M^2} e^{(\bar{m}^2-s')/M'^2},$$

(28)

where $\rho_{1(2)}(s, s', q^2)$ are the spectral densities calculated as the imaginary part of the correlation function $\Pi^{\text{OPE}}_q(p, p')$ with dimension-7 accuracy. The first pair of parameters $(M_i^2, s_0)$ in Eq. (28) is related to the initial state $T^c_{b\tau}$ whereas the second set $(M_i^2, s_0')$ corresponds to the final particle $Z_0^{b\tau}$.

In numerical computations of $G_{1(2)}(q^2)$ the working regions for the parameters $M_i^2$ and $s_0$ are chosen exactly as in the corresponding mass calculations. Values of the vacuum condensates are collected in Eq. (11), whereas the masses and couplings of the tetraquarks $T^c_{b\tau}$ and $Z_0^{b\tau}$ have been calculated in the present work and written down in Eqs. (15) and (18), respectively. Obtained sum rule predictions for the form factors $G_1(q^2)$ and $G_2(q^2)$ are shown in Fig. 3.

The sum rules give reliable results for $G_{1(2)}(q^2)$ in the region $m_i^2 \leq q^2 \leq 9$ GeV$^2$. But this is not enough to calculate the partial width of the decay $T^c_{b\tau} \to Z_0^{b\tau} \tau_l$ under analysis. Indeed, the form factors determine the differential decay rate $dT/dq^2$ of the process through the following expression

$$\frac{dT}{dq^2} = \frac{G_i^2 |V_{bc}|^2}{64 \pi^3 m^3} \lambda \left( m^2, \bar{m}^2, q^2 \right) \left( \frac{q^2 - m_i^2}{q^2} \right)^2 \times \left\{ \begin{array}{l}
(2q^2 + m_i^2) \left[ G_1^2(q^2) \left( \frac{q^2}{2} - m^2 - \bar{m}^2 \right) \right. \\
- G_2^2(q^2) \frac{q^2}{2} + (m^2 - m_i^2) G_1(q^2) G_2(q^2) \\
\left. + \frac{q^2 + m_i^2 \bar{m}^2}{q^2} \left[ G_1(q^2) (m^2 - \bar{m}^2) + G_2(q^2) q^2 \right] \right\},
\end{array} \right.
$$

(29)

where

$$\lambda \left( m^2, \bar{m}^2, q^2 \right) = \left[ m^4 + \bar{m}^4 + q^4 \right. - 2 \left( m^2 \bar{m}^2 + m^2 q^2 + \bar{m}^2 q^2 \right)]^{1/2}.
$$

(30)

To find the partial width of the semileptonic decay, $dT/dq^2$ should be integrated over $q^2$ in the limits $m_i^2 \leq q^2 \leq (m - \bar{m})^2$. But the region $m_i^2 \leq q^2 \leq 11.7$ GeV$^2$ is wider than a domain where the sum rules lead to strong predictions. This problem can be solved by introducing model functions $F_i(q^2)$ which at the momentum transfers $q^2$ accessible for the sum rule computations coincide with $G_i(q^2)$, but can be extrapolated to the whole integration region. These functions should have a simple form and be suitable to perform integrations over $q^2$.

To this end, we use the functions of the form

$$F_i(q^2) = F_i^0 \exp \left[ c_i \frac{q^2}{m^2} + c_i^2 \left( \frac{q^2}{m^2} \right)^2 \right],
$$

(31)

where $F_i^0$, $c_i$, and $c_i^2$ are constants which have to be fixed by comparing $F_i(q^2)$ and $G_i(q^2)$ at common regions of validity. Numerical analysis allows us to fix

$$F_1^0 = -0.30, c_1 = 9.98, c_1^2 = -10.07, F_1^1 = 0.41, c_1 = 8.67, c_1^2 = -7.15.
$$

(32)

The functions $F_i(q^2)$ are plotted in Fig. 3 where one can see their nice agreement with the sum rule predictions.

Other input information to calculate the partial width of the process $T^c_{b\tau} \to Z_0^{b\tau} \tau_l$, namely the masses of the leptons $m_e = 0.511$ MeV, $m_\mu = 105.658$ MeV, and $m_\tau = (1776.82 \pm 0.16)$ MeV are borrowed from Ref. [23].

Our results for the partial widths of the semileptonic decay channels are presented below:

$$\Gamma(T^c_{b\tau} \to Z_0^{b\tau} e^- \tau_e) = (6.16 \pm 1.74) \times 10^{-10} \text{ MeV},$$
$$\Gamma(T^c_{b\tau} \to Z_0^{b\tau} \mu^- \tau_\mu) = (6.15 \pm 1.74) \times 10^{-10} \text{ MeV},$$
$$\Gamma(T^c_{b\tau} \to Z_0^{b\tau} \tau^- \tau_\tau) = (2.85 \pm 0.81) \times 10^{-10} \text{ MeV}.
$$

(33)

As we shall see below, the semileptonic decays $T^c_{b\tau} \to Z_0^{b\tau} \tau_l$ establish an essential part of the full width of $T^c_{b\tau}$.

IV. NONLEPTONIC DECAYS

$T^c_{b\tau} \to Z_0^{b\tau} \pi^- (K^-, D^-, D_s^-)$

In this section, we investigate the nonleptonic weak decays $T^c_{b\tau} \to Z_0^{b\tau} \pi^- (K^-, D^-, D_s^-)$ of the tetraquark.
T_{b \pi}^- in the framework of the QCD factorization method, which allows us to calculate partial widths of these processes. This approach was applied to investigate nonleptonic decays of the conventional mesons \[ 24 \, 25 \], and used to study nonleptonic decays of the scalar and axial-vector tetraquarks $Z_{b;u\bar{u}d}^0$ and $T_{b;u\bar{u}d}^0$ in Refs. \[ 16 \, 17 \], respectively.

Here, we consider in a detailed form the decay $T_{b;u\bar{u}d}^- \to Z_{b;u\bar{u}d}^0\pi^-\pi^-$, and write down final predictions for remaining processes. This approach was applied to investigate non-leptonic decays of $K$, $D$, and $D_s$, and implementing substitutions $|V_{ud}| \to |V_{us}|$, $|V_{cd}|$, and $|V_{cs}|$, respectively.

The masses and decay constants of the final-state pseudoscalar mesons, as well as values of the CKM matrix elements used in computations are collected in Table I.

| Quantity                  | Value          |
|---------------------------|----------------|
| $m_{\pi}$                 | 139.570 MeV    |
| $m_{K}$                   | (493.677 ± 0.016) MeV |
| $m_{D}$                   | (1869.61 ± 0.10) MeV |
| $m_{D_s}$                 | (1968.30 ± 0.11) MeV |
| $f_{\pi}$                 | 131 MeV        |
| $f_{K}$                   | (155.72 ± 0.51) MeV |
| $f_{D}$                   | (203.7 ± 4.7) MeV |
| $f_{D_s}$                 | (257.8 ± 4.1) MeV |
| $|V_{ud}|$                 | 0.97420 ± 0.00021 |
| $|V_{us}|$                 | 0.2243 ± 0.0005 |
| $|V_{cd}|$                 | 0.218 ± 0.004 |
| $|V_{cs}|$                 | 0.997 ± 0.017 |

The similar analysis can be performed for the decay modes $T_{b;u\bar{u}d}^- \to Z_{b;u\bar{u}d}^0K^-D^-$ as well. The partial width of these channels can be obtained from Eq. \[ 41 \] by replacing $(m_{\pi}, f_{\pi})$ with the spectroscopic parameters of the mesons $K$, $D$, and $D_s$, and implementing substitutions $|V_{ud}| \to |V_{us}|$, $|V_{cd}|$, and $|V_{cs}|$, respectively.

The Wilson coefficients $c_1(m_b)$, and $c_2(m_b)$ with next-to-leading order QCD corrections can be found in Refs. \[ 26 \, 27 \].

$c_1(m_b) = 1.117$, $c_2(m_b) = -0.257$. \[ 42 \]

For the decay $T_{b;u\bar{u}d}^- \to Z_{b;u\bar{u}d}^0\pi^-$ calculations lead to the result
\[
\Gamma(T_{b;u\bar{u}d}^- \to Z_{b;u\bar{u}d}^0\pi^-) = (6.97 ± 1.99) \times 10^{-13} \text{ MeV},
\]
\[ 43 \]

For the remaining nonleptonic decays of the tetraquark $T_{b;u\bar{u}d}^-$, we get
\[
\Gamma(T_{b;u\bar{u}d}^- \to Z_{b;u\bar{u}d}^0K^-) = (5.33 ± 1.47) \times 10^{-14} \text{ MeV},
\]
\[
\Gamma(T_{b;u\bar{u}d}^- \to Z_{b;u\bar{u}d}^0D^-) = (1.13 ± 0.31) \times 10^{-13} \text{ MeV},
\]
\[
\Gamma(T_{b;u\bar{u}d}^- \to Z_{b;u\bar{u}d}^0D_s^-) = (3.88 ± 1.01) \times 10^{-12} \text{ MeV}.
\]
\[ 44 \]

It is seen that partial widths of the nonleptonic decays are negligibly smaller than widths of the semileptonic decays. Only widths of the processes $T_{b;u\bar{u}d}^- \to Z_{b;u\bar{u}d}^0\pi^-$ and $T_{b;u\bar{u}d}^0 \to Z_{b;u\bar{u}d}^0D_s^-$ affect the final result for $\Gamma_{\text{full}}$.

Collected information on the partial widths of the weak decays of the tetraquark $T_{b;u\bar{u}d}^-$ allow us to find its full width and mean lifetime:
proved that the exotic meson
pling of the scalar tetraquark
work.
Predictions for $\Gamma_{Z}$ and scalar
and light antidiquark
is enough to find the branching ratios of the various decay
s, and corresponding branching ratios.

$$\Gamma_{\text{full}} = (15.21 \pm 2.59) \times 10^{-10} \text{ MeV},$$
$$\tau = 4.33^{+0.89}_{-0.63} \times 10^{-13} \text{ s.} \quad (45)$$

Predictions for $\Gamma_{\text{full}}$ and $\tau$ are main results of the present work.

V. DISCUSSION AND CONCLUDING NOTES

In this article we have evaluated the mass and coupling
of the scalar tetraquark $T_{b\pi}$ Our analysis has
proved that the exotic meson $T_{b\pi}$ composed of the heavy
diquark $bb$ and light antidiquark $\overline{\tau}\tau$ is the strong- and
electromagnetic-interaction stable state, and dissociates
to conventional mesons only through the weak decays.
This fact places it to a list of stable axial-vector $T_{bb,\overline{\tau}\tau}$,
and scalar $Z_{bc,\overline{\tau}\tau}^{0}$ and $T_{bb,\overline{\tau}\tau}^{0}$
tetraquarks.

We have investigated also the dominant weak decay modes of the $T_{b\pi}$, and computed their partial widths.
These results have allowed us to estimate the full width and
mean lifetime of the $T_{b\pi}^{-}$. The collected information
is enough to find the branching ratios of the various decay
modes as well (see Table II). It is worth noting that only
the semileptonic decays of the $T_{b\pi}^{-}$ play a dominant role
in forming of $\Gamma_{\text{full}}$.

The tetraquark $T_{b\pi}^{-}$ can be considered as "s" member
of a multiplet of the scalar $bb\overline{\tau}\tau$ states with $q$ being one
of the light quarks. In Ref. [13], we studied the stable
axial-vector particle $T_{bb,\overline{\tau}\tau}^{-}$. It will be very interesting
to investigate the scalar partner of $T_{bb,\overline{\tau}\tau}^{-}$ as well as the
axial-vector state $bc\overline{\tau}\tau$ which may shed light on others members of scalar and axial-vector multiplets $bb\overline{\tau}\tau$.

We have computed the mass and coupling of the scalar
tetraquark $Z_{bc,\overline{\tau}\tau}^{0}$: these parameters are required to explore
the weak decays of the $T_{b\pi}^{-}$. The state $Z_{bc,\overline{\tau}\tau}^{0}$ with the
content $bc\overline{\tau}\tau$ belongs to a famous class of exotic mesons
composed of four different quarks [27]. A simple analysis
confirms that it is a strong-interaction stable particle. Indeed,
the scalar tetraquark $Z_{bc,\overline{\tau}\tau}^{0}$ in $S$-wave may decay to a pair
of pseudoscalar mesons $B^{-}D_{s}^{+}$ and $B_{s}^{-}D^{0}$. Thresholds
for production of these pairs are 7248 MeV and 7237 MeV, respectively. Because the maximum allowed
value of the $Z_{bc,\overline{\tau}\tau}^{0}$'s mass is $m = 6990$ MeV, it is
stable against to these strong decays. The $\overline{\tau}\tau$ member
of the scalar multiplet $bc\overline{\tau}\tau$ was investigated in Ref. [16],
in which it was found that this particle is a strong- and
electromagnetic-interaction stable state. It seems scalar
particles with such diquark-antidiquark structures are
among real candidates to stable four-quark compounds.

The revealed features of the $Z_{bc,\overline{\tau}\tau}^{0}$ determine a decay
pattern of the master particle $T_{b\pi}^{-}$. Indeed, the
tetraquark $Z_{bc,\overline{\tau}\tau}^{0}$ created at the first stage of the
decays, at the next step due to subprocesses $b \rightarrow W^{-}c$
and $c \rightarrow W^{+}s$ should undergone weak transformations.
In other words, decays of $T_{b\pi}^{-}$ to final states that con-
tain only conventional mesons and leptons run through,
the least, three phases. Such cascade picture of decays
was encountered in theoretical investigations of other
tetraquarks [13, 16], and studied in a detailed form in Ref.
[18]. A comprehensive analysis of the $T_{b\pi}^{-}$
tetraquark's decays will be finished in our forthcoming publications.

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