Eccentric Companions to Kepler-448b and Kepler-693b: Clues to the Formation of Warm Jupiters

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Abstract

I report the discovery of non-transiting close companions to two transiting warm Jupiters (WJs), Kepler-448/KOI-12b (orbital period \( P = 17.9 \) days, radius \( R_p = 1.23^{+0.05}_{-0.06} R_{Jup} \)) and Kepler-693/KOI-824b (\( P = 15.4 \) days, \( R_p = 0.91 \pm 0.05 R_{Jup} \)), via dynamical modeling of their transit timing and duration variations (TTVs and TDVs). The companions have masses of \( 22^{+7}_{-3} M_{Jup} \) (Kepler-448c) and \( 150^{+60}_{-40} M_{Jup} \) (Kepler-693c), and both are on eccentric orbits \((e = 0.65^{+0.13}_{-0.09} )\) for Kepler-448c and \( e = 0.47^{+0.11}_{-0.06} \) for Kepler-693c) with periastron distances of 1.5 au. Moderate eccentricities are detected for the inner orbits as well \((e = 0.34^{+0.08}_{-0.07} )\) for Kepler-448b and \( e = 0.2^{+0.2}_{-0.1} \) for Kepler-693b). In the Kepler-693 system, a large mutual inclination between the inner and outer orbits \((53^{\circ}_{9} \text{deg} \text{ or } 134^{\circ}_{10} \text{deg})\) is also revealed by the TDVs. This is likely to induce a secular oscillation in the eccentricity of the inner WJ that brings its periastron close enough to the host star for tidal star–planet interactions to be significant. In the Kepler-448 system, the mutual inclination is weakly constrained, and such an eccentricity oscillation is possible for a fraction of the solutions. Thus these WJs may be undergoing tidal migration to become hot Jupiters (HJs), although the migration via this process from beyond the snow line is disfavored by the close-in and massive nature of the companions. This may indicate that WJs can be formed in situ and could even evolve into HJs via high-eccentricity migration inside the snow line.

Key words: planets and satellites: individual (KOI-12, Kepler-448, KIC 5812701, KOI-824, Kepler-693, KIC 5164255)

Supporting material: machine-readable tables

1. Introduction

Warm Jupiters (WJs), giant planets in moderately close-in orbits (\( 7 \text{ days} < P < 100 \) days), pose a conundrum similar to that of hot Jupiters (HJs). A dozen WJs have been found to reside in circular orbits in multi-transiting systems (Huang et al. 2016), in which the orbital planes of the planets are likely well aligned. Such an architecture points to the formation via disk migration (Goldreich & Tremaine 1980; Lin et al. 1996), as originally proposed for HJs, or in situ formation inside the snow line (Batygin et al. 2016; Boley et al. 2016). Alignments of the planetary orbits with the equators of their host stars, as confirmed for some of them (e.g., Hirano et al. 2012; Sanchis-Ojeda et al. 2012), may also support the absence of past dynamical eccentricity/inclination excitation via planet–planet scattering (Rasio & Ford 1996) or secular chaos (Wu & Lithwick 2011).

On the other hand, roughly half of the WJs with measured masses from Doppler surveys have significant eccentricities that seem too high to result from disk migration or subsequent planet–planet scattering (Petrovich et al. 2014), but that are too low for their orbits to be tidally circularized (Socrates et al. 2012b; Dawson et al. 2015). A possible explanation is that these moderately eccentric WJs are experiencing “slow Kozai–Lidov migration” (Petrovich 2015); their eccentricities are still undergoing large oscillations driven by the secular perturbation from a close companion (Dong et al. 2014), without being suppressed by other short-range forces (Wu & Murray 2003), and their orbits shrink only at the high-eccentricity phase. This scenario may indeed reproduce the observed eccentricity distribution of WJs with outer companions (Petrovich & Tremaine 2016).

Observationally, long-period massive companions to WJs are nearly as common as such companions to HJs (Knutson et al. 2014), and their orbital properties might be consistent with what is expected from this scenario (Bryan et al. 2016). Indeed, the apsidal misalignments of some of those eccentric WJs with outer companions provide statistical evidence for the oscillating eccentricity due to a large mutual orbital inclination (Dawson & Chiang 2014). However, there has been no direct measurement of a large orbital misalignment between WJs and their outer companions such that it might induce a large eccentricity oscillation, partly because these systems are mostly detected with the radial velocity (RV) technique, which yields no information on the orbital direction. Notable exceptions include the Kepler–419 system (Dawson et al. 2014) and the doubly transiting giant-planet system Kepler-108 (Mills & Fabrycky 2017), where the mutual inclinations were constrained via dynamical modeling of transit timing and duration variations (TTVs and TDVs), although their mutual inclinations are likely too small to drive secular eccentricity oscillations.

Transiting WJs without transiting companions provide a unique opportunity to search for close and mutually inclined companions as direct evidence for the slow Kozai migration, because the full 3D architecture of the system can be dynamically constrained with a similar technique as used in the above systems. In addition, WJs on eccentric and intermediate orbits, unlike HJs, may still be strongly interacting with the companion, and their eccentricity can also help the TVT inversion for non-transiting objects by producing specific non-sinusoidal features. The TVT search for
the outer companions on such “hierarchical” orbits is also complementary to the TTV studies so far, which have mainly focused on nearly sinusoidal signals typical of planets near mean-motion resonances (Hadden & Lithwick 2017; Jontof-Hutter et al. 2016).

In this paper, I perform a search for non-transiting companions around transiting WJs using transit variations (Section 2) and report the discovery of outer companions to two transiting WJs, Kepler-448b (Bourrier et al. 2015) and Kepler-693b (Morton et al. 2016). Based on TTVs and TDVs of the WJs, I find that the companions are (sub-) stellar mass objects on highly eccentric, au-scale orbits (Sections 3–5). In particular, I confirm a large mutual orbital inclination between the inner WJ and the companion in the latter system, which can induce a large amplitude of the eccentricity oscillation and the tidal shrinkage of the inner orbit (Section 6)—exactly as predicted in the “slow Kozai” scenario by a close companion. The companions’ properties, however, are not fully compatible with such migration starting from beyond the snow line. Thus I also assess the possibility of in situ formation (Section 6.3). I discuss possible follow-up observations in Section 7, and Section 8 concludes the paper.

2. Systematic TTV Search for Singly Transiting WJs

To identify the signature of outer companions, I analyzed TTVs of 23 confirmed, singly transiting WJs (Section 2.1) with an orbital period of 7 days < \( P < 100 \) days and a radius of \( R_p > 8R_{\oplus} \) in the DR24 of the KOI catalog (Coughlin et al. 2016). Systems with multiple KOIs are all excluded, even though they consist of only one confirmed planet and false positives. I found clearly non-sinusoidal TTVs for Kepler-448/KOI-12b, Kepler-693/KOI-824b, and Kepler-419/KOI-1474b. The result is consistent with the TTV search by Holczer et al. (2016), who reported significant long-term TTVs for the same three KOIs in our sample.\(^4\)

Of these planets, the TTVs of Kepler-419b have previously been analyzed by Dawson et al. (2014) with RV data, and the companion planet Kepler-419c was found to be a super Jupiter on an eccentric and mutually aligned orbit with the inner one. Therefore in this paper, I focus on Kepler-448b and Kepler-693b, which both show clear non-sinusoidal TTVs, and masses and orbits of the perturbers can be well constrained.

For the dynamical modeling taking into account the possible orbital misalignment, I reanalyze the transit light curves of Kepler-448b and Kepler-693b to derive both TTVs and TDVs (Section 2.2), consistently with the other transit parameters (Tables 1–4). Here I also fit the planet-to-star radius ratio, \( R_p/R_\star \), so that the possible transit-depth modulation that is due to the different crowding depending on observing seasons does not mimic duration variations (cf. Mills & Fabrycky 2017). As shown in Figure 1, I find significant TTVs consisting of a long-term modulation and a short-term, sharp feature (top panels). For Kepler-448b, the spike-like feature is clearer than that reported in Holczer et al. (2016), presumably owing to a more careful treatment of the local baseline modulation. No significant correlation is found between TTVs and the local light-curve slope or the fitted \( R_p/R_\star \), which supports the hypothesis that the feature has a physical origin and is not due to star spots (Mazeh et al. 2015). I show in Sections 4 and 5 that this unusual feature is reproduced by a periastron passage of an outer non-transiting companion in an eccentric orbit. I also identify a significant (~5\( \sigma \)) linear trend in the transit duration of Kepler-693b, which points to a misalignment of the inner and outer orbits. The duration change is also clear in Figure 2, where each detrended and normalized transit is overplotted around its center, along with the best-fit model.

2.1. Iterative Determination of Transit Times

In the systematic TTV search for WJs, I analyze pre-search data conditioning simple aperture photometry (PDCSAP) fluxes obtained in long-cadence (LC) mode, retrieved from the NASA exoplanet archive. I follow the iterative method as described by Masuda (2015) to derive consistent transit times and light-curve shapes. The method consists of (1) the determination of the central time for each transit, and (2) the refinement of the transit parameters by fitting the mean transit light curve. The fitting in this subsection is performed by minimizing the standard \( \chi^2 \) (Markwardt 2009) defined as the squared sum of the difference between the model and data divided by the PDCSAP-flux error.

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\(^4\) Owing to the update in the stellar radius in the DR25 catalog, the WJ sample defined above now includes Kepler-522/KOI-318b and Kepler-827/KOI-1355b, for which Holczer et al. (2016) reported long-term TTVs. Interpretation of their TTVs is less clear than for the two systems discussed in the present paper because of the lack of clear non-sinusoidal features (Kepler-522b) and a low signal-to-noise ratio (Kepler-827b).

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### Table 1

| Transit Number | Transit Time (BJD$_{\text{TDB}}$ – 2454833) | Transit Duration (day) | Planet-to-star Radius Ratio |
|----------------|--------------------------------------------|-------------------------|-----------------------------|
| −1             | 128.7420 ± 0.0001                          | 0.2808 ± 0.0003         | 0.0920 ± 0.0004             |

**Note.** Quoted uncertainties are the standard errors derived from the covariance matrix scaled by $\sqrt{\chi^2/\text{dof}}$ of the fit. (This table is available in its entirety in machine-readable form.)

### Table 2

| Transit Number | Transit Time (BJD$_{\text{TDB}}$ – 2454833) | Transit Duration (day) | Planet-to-star Radius Ratio |
|----------------|--------------------------------------------|-------------------------|-----------------------------|
| 0              | 173.609 ± 0.002                           | 0.115 ± 0.007           | 0.115 ± 0.008               |

**Note.** Quoted uncertainties are the standard errors derived from the covariance matrix scaled by $\sqrt{\chi^2/\text{dof}}$ of the fit, or are based on the combination of the LC and SC data (see Section 2.2). (This table is available in its entirety in machine-readable form.)
In the first step, I fit the data segments of three times the total duration centered at each transit. I adopt the Mandel & Agol (2002) model for the quadratic limb-darkening law generated with the pyTransit package (Parviainen 2015), multiplied by a second-order polynomial function of time to take into account the local baseline modulation. The fitting is repeated iteratively removing 3σ outliers. Here a circular orbit is assumed and the values of central transit time \( t_c \) and three coefficients of the polynomial are fitted, while the other parameters are fixed (two limb-darkening coefficients \( q_1 \) and \( q_2 \) defined in Kipping (2013), mean stellar density \( \rho_* \), transit impact parameter \( b \), planet-to-star radius ratio \( R_p/R_* \), and orbital period \( P \)).

In the second step, I shift each transit by the value of \( t_c \) to align all the transits around time zero. Then the data are averaged into bins of 0.05 times the transit duration (to reduce the computational time), where the value and error of each bin is set to be the median and 1.4826 times the median absolute deviation of the flux values in the bin. The resulting “mean” light curve is again fitted with the same Mandel & Agol (2002) model for its central time \( t_0 \), normalization constant, \( q_1 \), \( q_2 \), \( R_p/R_* \), \( \rho_* \), and \( b \) assuming a circular orbit and fixing the period at the value refined in step (1); only in the first iteration, I adopt a quadratic function of time as a baseline and fit three coefficients rather than one normalization. The values of \( q_1 \), \( q_2 \), \( R_p/R_* \), \( \rho_* \), and \( b \) obtained from this fitting are used for the first step of the next iteration.

I perform five iterations for each KOI, starting with the second step based on the transit light curve phase-folded at the period given in the KOI catalog (Coughlin et al. 2016). I fit the resulting transit times with straight lines to search for any TTVs and identify the candidates mentioned above.

### 2.2. Transit Times and Durations of Kepler-448b and Kepler-693b

For Kepler-448b and Kepler-693b, I perform more intensive analyses taking into account possible variations in transit durations using the short-cadence (SC) data if available. For Kepler-448, I use the SC data for all the quarters, while for Kepler-693 I combine both the LC (Q2, Q7, Q8, Q10–Q12, Q14–16) and SC (Q14–Q16) data.

The method is the same as in Section 2.1, except for the following differences. I additionally fit \( R_p/R_* \) and \( b \) in the first step and repeated 10 iterations. Here I fit \( R_p/R_* \) so that the seasonal variation in the transit depths is not misinterpreted as the duration variation (e.g., Van Eylen et al. 2013; Masuda 2015). The resulting \( b \) and \( R_p/R_* \) are combined with \( \rho_* \) and \( P \) to yield the transit duration \( T \) as the average of the total and full durations (see Equations (14) and (15) of Winn 2011). The errors in \( t_c \) and \( T \) are scaled by the square root of the reduced chi-squared \( \chi^2 \) of the best-fit model to enforce \( \chi^2 = 1 \). When the mean light curve is fitted, the \( \chi^2 \) minimization is complemented by a short Markov chain Monte Carlo (MCMC) chain (Foreman-Mackey et al. 2013), and the data are averaged into one-minute bins for the SC data.

I also tuned the parameters specifically to each system as follows:

**Kepler-448b:** I fit the data of 1.6 times duration for each transit. When more than 10% of the data were missing in the segment, the transit was excluded from the analysis. I used fourth-order polynomial because the light curve shows short-term wiggles that are likely due to the stellar rotation (1.245 ± 0.124 days; McQuillan et al. 2013).

**Kepler-693b:** I fit the data of twice the duration for each transit and used a second-order polynomial because the light curve shows less variability than Kepler-448. I omitted the transits with more than 10% gaps for the SC data, while for the LC data I omitted those with more than 30%. The SC and LC data were analyzed independently, and the resulting transit times and durations were averaged to give one measurement when both were available. For

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### Table 3

| Parameter | Value |
|-----------|-------|
| \( q_1 = (u_1 + u_2)^2 \) | 0.199(7) |
| \( q_2 = \sqrt{u_1 u_2} \) | 0.39(2) |
| Center of the mean transit (day) | -0.00001(2) |
| \( R_p/R_* \) | 0.09044(7) |
| Normalization | 0.9999993 |
| Transit impact parameter | 0.373(6) |
| Mean stellar density (g cm\(^{-3}\)) | 0.393(3) |
| Photometric jitter | 0.0000362(2) |
| \( (a/R_*)_{<0} \) | 18.77(4) |
| \( u_1 \) | 0.348(9) |
| \( u_2 \) | 0.10(2) |
| \( t_0 \) (BJDTDB) | 2454979.5961(2) |
| \( P \) (day) | 17.855234(4) |

**Notes.** Parentheses after values denote uncertainties in the last digit.

* Median and 68% credible interval of the MCMC posteriors from the mean transit light curve. Here the circular orbit is assumed to relate mean stellar density to the semimajor axis over stellar radius, \( a/R_* \). The limb-darkening coefficients \( u_1 \) and \( u_2 \) are converted from \( q_1 \) and \( q_2 \).

**Table 4**

| Parameter | Value |
|-----------|-------|
| \( q_1 = (u_1 + u_2)^2 \) | 0.62(0.3) |
| \( q_2 = \sqrt{u_1 u_2} \) | 0.4(0.3) |
| Center of the mean transit (day) | 0.00000(3) |
| \( R_p/R_* \) | 0.117(1) |
| Normalization | 1.00000(8) |
| Transit impact parameter | 0.63(2.1) |
| Mean stellar density (g cm\(^{-3}\)) | 3.0(0.3) |
| Photometric jitter | 0.00074(7) |
| \( (a/R_*)_{<0} \) | 34.2(2) |
| \( u_1 \) | 0.7(0.2) |
| \( u_2 \) | 0.1(0.2) |
| \( t_0 \) (BJDTDB) | 2455006.613(1) |
| \( P \) (day) | 15.37566(3) |

**Notes.** Parentheses after values denote uncertainties in the last digit.

* Median and 68% credible interval of the MCMC posteriors from the mean transit light curve. Here the circular orbit is assumed to relate mean stellar density to the semimajor axis over stellar radius, \( a/R_* \). The limb-darkening coefficients \( u_1 \) and \( u_2 \) are converted from \( q_1 \) and \( q_2 \).

**Table Parameters of Kepler-693b Based on Long-cadence Data**
each data, I computed $\sqrt{\sigma_{LC}^2 + \sigma_{SC}^2/2}$ and half of the difference between the LC and SC values, and assigned the larger of the two as its error. The former was the case for most of the points, while the latter process helped mitigate the effect of a few outliers that were presumably caused by local features in the light curve.

The resulting transit times, durations, and radius ratios are summarized in Figure 1 and Tables 1 and 2. I also fit the mean transit light curve from the final iteration (binned version of those in Figure 2) with an MCMC including an additional Gaussian error in quadrature (denoted as “photometric jitter”) and derived the posteriors for the transit parameters that are summarized in Tables 3 and 4. For Kepler-448b, the result agreed well with those from the least-squares fit to the observed transit times (i.e., TTVs) are shown for clarity.

The TTVs observed in the two systems (Figure 1) are clearly different from the sinusoidal signal because of a companion near a mean-motion resonance ( Lithwick et al. 2012). In particular, rapid timing variations on a short timescale suggest that the orbits of the perturbing companions are eccentric. In addition, such a feature is observed only once for each system, and so the companions must be far outside the WJs’ orbits. Thus I assume a hierarchical three-body system as the only viable configuration and model the observed TTVs and TDVs to derive the outer companions’ masses and orbits.

I only consider the Newtonian gravity between the three bodies regarded as point masses (see Section 3.5.1 for justification), as well as the finite light-travel time in computing timings. To better take into account the hierarchy of the system, I adopt the Jacobi coordinates to describe their orbits: the inner orbit (denoted by the subscript 1) is defined by the relative motion of the inner planet around the host star, while the outer orbit (denoted by the subscript 2) is the motion of the outer companion relative to the center of mass of the inner two bodies. The sky plane is chosen to be the reference plane, to which arguments of periastron and the line of nodes are referred. The direction of the $+Z$-axis (which matters for the definition of the “ascending” node) is taken toward the observer.

### 3. Dynamical Modeling of TTVs and TDVs: Method

#### 3.1. Model Assumptions

The TTVs observed in the two systems (Figure 1) are clearly different from the sinusoidal signal because of a companion near a mean-motion resonance ( Lithwick et al. 2012). In particular, rapid timing variations on a short timescale suggest that the orbits of the perturbing companions are eccentric. In addition, such a feature is observed only once for each system, and so the companions must be far outside the WJs’ orbits. Thus I assume a hierarchical three-body system as the only viable configuration and model the observed TTVs and TDVs to derive the outer companions’ masses and orbits.

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#### 3.2. Bayesian Framework

I derive the posterior probability density function (PDF) for the set of system parameters $\theta$ conditioned on the observed data $d$:

$$p(\theta | d, I) = \frac{1}{Z} p(d | \theta, I) p(\theta | I),$$

(1)
where $L$ denotes the prior information. The normalization factor

$$Z = \int p(d|\theta, I)p(\theta|I)d\theta$$

is called the global likelihood or evidence, which represents the plausibility of the model. The prior PDF $p(\theta|I)$ is given as a product of the prior PDFs for each parameter, which are assumed to be independent; they are presented in Tables 5 and 6. The likelihood $L \equiv p(d|\theta, I)$ is defined and computed as described in Sections 3.4 and 3.5.

To invert the observed signals, I use the nested-sampling algorithm MultiNest (Feroz & Hobson 2008; Feroz et al. 2009, 2013) and its python interface PyMultiNest (Buchner et al. 2014), which allows us to sample the whole prior volume to identify multiple modes, if any. I typically use 4800 live points and a default sampling efficiency of 0.8, keep updating the live points until the evidence tolerance of 0.5 is achieved, and allow for the detection of multiple posterior modes. When multiple modes are detected, MultiNest also computes evidence corresponding to each mode, which is referred to as “local” evidence.

### 3.3. Procedure for Finding Solutions

To check the reliability of our numerical scheme and to find all the possible posterior modes, I adopt the following procedure. First, I use the analytic TTV formula for a hierarchical triple system (Borkovits et al. 2015, Appendix A) to fit only the TTV signal. This allows us to search a wide region of the parameter space and resulted in one global mode containing two peaks. The resulting solution was also found to be consistent with a rough analytical estimate based on the observed TTV features (see, e.g., Section 4.2). Then, I fit the same TTVs numerically by adopting a slightly narrower prior range that well incorporates the global mode found from the analytic fit. The resulting posterior was found to be consistent with the one derived from the analytic fit. This agreement validates the numerical scheme I rely on. Finally, the same numerical method is used to model both TTVs and TDVs simultaneously to determine the physical and geometric properties of the outer companions. In Sections 4 and 5, I mainly report and discuss the results from the final fit, while the analytic and numerical posteriors from TTVs alone are found in Appendix B for comparison.

### 3.4. TTV Modeling

The likelihood for the TTV modeling is defined as follows:

$$L_{\text{TTV}} \equiv \prod_i \frac{1}{\sqrt{2\pi(\sigma^2_{i,t} + \sigma^2_{\text{TTV}})}} \exp \left[ -\frac{(t_i - t_i^{\text{model}})^2}{2(\sigma^2_{i,t} + \sigma^2_{\text{TTV}})} \right],$$

where $t_i$ and $\sigma_{i,t}$ are the transit times and their errors obtained in Section 2.2 (Tables 1 and 2). I also include $\sigma_{\text{TTV}}$ as a model parameter that takes into account the additional scatter and marginalize over it to obtain a more conservative constraint; it turns out that this parameter is not correlated with any other physical parameters (Figures 13 and 14) and hence does not affect the global shape of the joint PDF.

The model transit times $r_i^{\text{model}}$ are computed both analytically and numerically as mentioned in Section 3.3.; this is for cross-validation and to obtain insights into the way the parameters are determined. In both cases, the model includes 14 parameters in addition to $\sigma_{\text{TTV}}$ (see Tables 8 and 9): orbital period, orbital phase, eccentricity, and argument of periastron for both inner and outer orbits; cosine of the outer orbit inclination (the inner one is fixed to be 0 for simplicity; this does not affect the result); difference in the longitudes of the ascending node; and masses of the host star and outer companion (here I fix the inner planet mass to be $1M_{\text{Jup}}$ as it is not determined from TTVs). While I use the time of inferior conjunction $t_P$ and orbital period $P$ for the inner orbit, I choose the time of periastron passage $\tau_P$ and periastron distance relative to the inner semimajor axis $q_2/a_1$ to specify the outer orbital phase and period. This is because the latter two parameters are more directly related to the position and duration of the “spike” in the observed TTVs than $t_P$ and $P$, and thus they are better determined. I fit the mass scale of the whole system as well because I include the light-travel time effect (LTTE); in practice, however, the observed TTVs are insensitive to this parameter, and its value is solely constrained by the prior knowledge.
### Fitted Parameters

**Inner Orbit**

| Parameter | Solution 1 | Solution 2 | Combined | Prior |
|-----------|-------------|-------------|----------|-------|
| 1. Time of inferior conjunction | \(128.7418^{+2}_{-3}\) | \(128.7413^{+2}_{-2}\) | \(128.7415^{+4}_{-3}\) | \(\mathcal{U}(128.73, 128.75)\) |
| 2. Orbital period | \(17.85518^{+6}_{-6}\) | \(17.85518^{+7}_{-5}\) | \(17.85518^{+8}_{-7}\) | \(\mathcal{U}_{\text{sys}}(17.8551, 17.8553)\) |
| 3. Orbital eccentricity \(e_1\) | \(0.34^{+0.07}_{-0.06}\) | \(0.35^{+0.09}_{-0.08}\) | \(0.34^{+0.07}_{-0.07}\) | \(\mathcal{U}(0, 0.7)\) |
| 4. Argument of periastron \(\omega_1\) (deg) | \(-99^{+2}_{-3}\) | \(-131^{+1}_{-2}\) | \(\cdots\) | \(\mathcal{U}(-180, 180)\) |
| 5. Cosine of orbital inclination \(\cos i_1^a\) | 0.013(1) | 0.013(1) | 0.013(1) | \(\mathcal{U}(0, 0.02)^a\) |

**Outer Orbit**

| Parameter | Solution 1 | Solution 2 | Combined | Prior |
|-----------|-------------|-------------|----------|-------|
| 6. Time of the periastron passage | \(1076_{-10}^{+20}\) | \(1080_{-11}^{+10}\) | \(1078_{-11}^{+10}\) | \(\mathcal{U}(900, 1200)\) |
| 7. Periastron distance over inner semimajor axis \(q/a_1\) | \(9.5_{-0.3}^{+0.4}\) | \(9.4_{-0.4}^{+0.4}\) | \(9.4_{-0.3}^{+0.4}\) | \(\mathcal{U}_{\text{bb}}(5, 20)\) |
| 8. Orbital eccentricity \(e_2\) | \(0.61^{+0.11}_{-0.07}\) | \(0.69^{+0.13}_{-0.10}\) | \(0.65^{+0.13}_{-0.09}\) | \(\mathcal{U}(0, 0.95)\) |
| 9. Argument of periastron \(\omega_2\) (deg) | \(-89^{+5}_{-6}\) | \(-89^{+8}_{-6}\) | \(-89^{+6}_{-6}\) | \(\mathcal{U}(-180, 180)\) |
| 10. Cosine of orbital inclination \(\cos i_2^a\) | \(-0.3^{+0.3}_{-0.2}\) | \(-0.6^{+0.3}_{-0.2}\) | \(-0.5^{+0.4}_{-0.3}\) | \(\mathcal{U}(-1, 1)\) |
| 11. Relative longitude of ascending node \(\Omega_2\) (deg) | \(3^{+4}_{-4}\) | \(-178^{+5}_{-5}\) | \(\cdots\) | \(\mathcal{U}(-180, 180)\) |

### Physical Properties

| Parameter | Solution 1 | Solution 2 | Combined | Prior |
|-----------|-------------|-------------|----------|-------|
| 12. Mass of Kepler-448b \(M_\ast\) (M\(_\odot\)) | \(1.5 \pm 0.1\) | \(1.5 \pm 0.1\) | \(1.5 \pm 0.1\) | \(\mathcal{G}(1.51, 0.09, 0.14)\) |
| 13. Mean density of Kepler-448b \(\rho_\ast\) (g cm\(^{-3}\)) | \(0.79^{+0.09}_{-0.09}\) | \(0.81^{+0.10}_{-0.09}\) | \(0.80^{+0.10}_{-0.09}\) | \(\mathcal{G}(0.76, 0.11, 0.12)\) |
| 14. Mass of Kepler-448b \(m_b\) (M\(_{\text{Jup}}\)) | \(1.1^{+3.5}_{-0.8}\) | \(1.1^{+3.5}_{-0.8}\) | \(1.1^{+3.5}_{-0.8}\) | \(\mathcal{U}_{\text{bb}}(0.1, 1)\) |
| 15. Mass of Kepler-448c \(m_c\) (M\(_{\text{Jup}}\)) | \(21^{+6}_{-5}\) | \(24^{+6}_{-5}\) | \(22^{+7}_{-5}\) | \(\mathcal{U}_{\text{bb}}(0.001M_\ast, 0.1M_\ast)\) |

### Jitters

| Parameter | Solution 1 | Solution 2 | Combined | Prior |
|-----------|-------------|-------------|----------|-------|
| 16. Transit-time jitter \(\sigma_{\text{TTV}}\) (10^-4 day) | \(2.2 \pm 0.3\) | \(2.2 \pm 0.3\) | \(2.2 \pm 0.3\) | \(\mathcal{U}_{\text{bb}}(5 \times 10^{-2}, 5)\) |
| 17. Transit-duration jitter \(\sigma_{\text{TDJ}}\) (10^-4 day) | \(3.8 \pm 0.6\) | \(3.7 \pm 0.6\) | \(3.7 \pm 0.6\) | \(\mathcal{U}_{\text{bb}}(0.1, 10)\) |

### Derived Parameters

- Outer orbital period \(P_2\) (day)
- Inner semimajor axis \(a_1\) (au)
- Outer semimajor axis \(a_2\) (au)
- Periastron distance of the inner orbit
- Mutual orbital inclination \(i_2\) (deg)
- Physical radius of Kepler-448b \(R_b\)
- Physical radius of Kepler-448c \(R_c\)
- Transit impact parameter of Kepler-448b
- Occultation impact parameter of Kepler-448b
- Log evidence \(\ln Z\) from Multinest

**Notes**

The elements of the inner and outer orbits listed here are Jacobian osculating elements defined at the epoch BJD = 2454833 + 120. The quoted values in the “Solution” columns are the median and 68% credible interval of the marginal posterior. Parentheses after values denote uncertainties in the last digit. The “combined” column shows the values from the marginal posterior combining the two solutions; no value is shown when the combined marginal posterior is multimodal. In the prior column, \(\mathcal{U}(a, b)\) and \(\mathcal{U}_{\text{bb}}(a, b)\) denote the (log-uniform) priors between \(a\) and \(b\), \(f(x) = 1/(b − a)\) and \(f(x) = x^{−1}/(\ln b − \ln a)\), respectively; \(\mathcal{G}(a, b, c)\) means the asymmetric Gaussian prior with the central value \(a\) and lower and upper widths \(b\) and \(c\).

- \(a\) There also exists a solution with negative \(\cos i_2\). The solution is statistically indistinguishable from the one reported here, except that the signs of \(\cos i_2\) and \(\Omega_2\) are flipped (and thus \(i_2\) remains the same; see Equation (23)). In principle, the TVVs for the solutions with \(i_2 > 0\) and \(i_2 < 0\) are not completely identical (see, e.g., A15 of Borkovits et al. 2015). However, the difference is in practice negligibly small for a transiting system because the effect is proportional to \(\cot i_2\). I confirmed that this is indeed the case by performing the same numerical fit with the prior on \(\cos i_2\) replaced by \(\mathcal{U}(-0.02, 0)\).

### I first compute analytic transit times using the formula by Borkovits et al. (2011, 2015) developed for hierarchical planetary systems and triple-star systems (Appendix A). I include the LLTE and \(P_2\)-timescale dynamical effect that is due to the quadrupole component of the perturbing potential; I did not find notable changes even with the octupole terms. The former is due to the motion of the inner binary (here the central star orbited by the inner planet), which changes finite time for the light emitted from the star and blocked by the planet to travel to us. The latter process involves the actual variation in the orbital period of the inner binary that is due to the tidal gravitational field exerted by the companion.
Table 6
Parameters of the Kepler-693 System from the Dynamical TTV and TDV Analysis

| Parameter | Solution 1 | Solution 2 | Combined | Prior |
|-----------|------------|------------|----------|-------|
| (Inner Orbit) |             |            |          |       |
| 1. Time of inferior conjunction | 173.611±0.1 | 173.610±0.2 | 173.610±0.1 | 173.58, 173.64 |
| \(T_{\text{lc,1}}\) (BJD$_{TDB}$ − 2454833) |            |            |          |       |
| 2. Orbital period \(P_1\) (day) | 15.37541±0.008 | 15.37534±0.009 | 15.37537±0.010 | 15.375, 15.376 |
| 3. Orbital eccentricity \(e_1\) | 0.14±0.04 | 0.13±0.02 | 0.13±0.02 | 0.0, 0.7 |
| 4. Argument of periastron \(\omega_1\) (deg) | 41±80 | 174±90 | ... | \(\sim -180, 180\) |
| 5. Cosine of orbital inclination \(\cos i_1\) | 0.021(2) | 0.022(3) | 0.022(3) | 0.0, 0.04 |
| (Outer Orbit) |             |            |          |       |
| 6. Time of the periastron passage | 640±27 | 640±27 | 640±27 | \(\sim 500, 900\) |
| \(T_2\) (BJD$_{TDB}$ − 2454833) |            |            |          |       |
| 7. Periastron distance over inner semimajor axis \(a_1/a_1\) | 13±2 | 13±2 | 13±2 | \(\sim 6, 25\) |
| 8. Orbital eccentricity \(e_2\) | 0.48±0.12 | 0.46±0.10 | 0.47±0.11 | 0.0, 0.95 |
| 9. Argument of periastron \(\omega_2\) (deg) | 41±20 | 25±23 | 30±24 | \(\sim -180, 180\) |
| 10. Cosine of orbital inclination \(\cos i_2\) | \(-0.2±0.05\) | \(-0.3±0.11\) | \(-0.3±0.02\) | \(\sim -1, 1\) |
| 11. Relative longitude of ascending node \(\Omega_2\) (deg) | \(51±8\) | \(-138±11\) | ... | \(\sim -180, 180\) |
| (Physical Properties) |             |            |          |       |
| 12. Mass of Kepler-693 \(m_s\), \(M_s\) | 0.80±0.03 | 0.80±0.04 | 0.80±0.04 | \(0.79, 0.03, 0.15\) |
| 13. Mean density of Kepler-693 \(\rho_s\) (g cm$^{-3}$) | 2.2±0.3 | 2.2±0.3 | 2.2±0.3 | \(1.93, 0.18, 0.53\) |
| 14. Mass of Kepler-693 \(m_b\), \(M_b\) | 0.8±2.7 | 1.0±3.2 | 0.9±1.0 | \(\sim 0.1, 10\) |
| 15. Mass of Kepler-693 \(m_c\), \(M_c\) | 167.5±59 | 136.6±34 | 145.3±62 | \(\sim 0.001M_\odot, 0.3M_\odot\) |
| (Jitters) |             |            |          |       |
| 16. Transit-time jitter \(\sigma_{\text{TTV}}\) (10$^{-4}$ day) | 2.4±1 | 1.4±1 | 1.4±1 | \(\sim 5 \times 10^{-2}, 50\) |
| 17. Transit-duration jitter \(\sigma_{\text{TD}}\) (10$^{-4}$ day) | 9.1±14 | 9.1±13 | 9.1±14 | \(\sim 0.1, 10^{2}\) |
| Derived Parameters |             |            |          |       |
| Outer orbital period \(P_2\) (day) | \((1.8_{-0.6}^{+0.2}) \times 10^{3}\) | \((1.8_{-0.6}^{+0.2}) \times 10^{3}\) | \((1.8_{-0.6}^{+0.2}) \times 10^{3}\) | ... |
| Inner semimajor axis \(a_2\) (au) | \(0.112_{-0.01}^{+0.01}\) | \(0.112_{-0.01}^{+0.01}\) | \(0.112_{-0.01}^{+0.01}\) | ... |
| Outer semimajor axis \(a_2\) (au) | 2.9±11 | 2.8±07 | 2.8±08 | ... |
| Periastron distance of the outer orbit \(a_2(1 - e_2)\) (au) | 1.5±0.2 | 1.5±0.2 | 1.5±0.2 | ... |
| Mutual orbital inclination \(i_2\) (deg) | 53±7 | 134±10 | ... | ... |
| Physical radius of Kepler-693 \(R_s\) | 0.80±0.03 | 0.80±0.04 | 0.80±0.04 | ... |
| Physical radius of Kepler-693 \(R_b\) (\(R_{\text{Jup}}\)) | 0.91±0.05 | 0.91±0.05 | 0.91±0.05 | ... |
| Transit impact parameter of Kepler-693b | 0.58±0.04 | 0.59±0.05 | 0.59±0.05 | ... |
| Occultation impact parameter of Kepler-693b | 0.68±0.06 | 0.67±0.09 | 0.64±0.08 | ... |
| Log evidence \(\ln Z\) from Multinest | \(-90.30±0.08\) | \(-89.57±0.08\) | \(-89.18±0.08\) | ... |

Notes. The elements of the inner and outer orbits listed here are Jacobian osculating elements defined at the epoch BJJD = 2454833 + 170. The quoted values in the “Solution” columns are the median and 68% credible interval of the marginal posteriors. Parentheses after values denote uncertainties in the last digit. The “combined” column shows the values from the marginal posterior combining the two solutions; no value is shown when the combined marginal posterior is multimodal. In the prior column, \(u(a, b)\) and \(U_{\text{FT}}(a, b)\) denote the (log-junior) priors between \(a\) and \(b\), \(f(x) = 1/(b - a)\) and \(f(x) = x^{-1}/(\ln b - \ln a)\), respectively; \(\sigma(a, b, c)\) means the asymmetric Gaussian prior with the central value \(a\) and lower and upper widths \(b\) and \(c\).

* There also exists a solution with negative \(\cos i_2\). The solution is statistically indistinguishable from the one reported here, except that the signs of \(\cos i_2\) and \(i_2\) are flipped (and thus \(i_2\) remains the same; see Equation (23)). In principle, the TTVs for the solutions with \(\cos i_2 > 0\) and \(\cos i_2 < 0\) are not completely identical (see, e.g., A15 of Borkovits et al. 2015). However, the difference is in practice negligibly small for a transiting system because the effect is proportional to \(\cos i_2\). I confirmed that this is indeed the case by performing the same numerical fit with the prior on \(\cos i_2\) replaced by \(\sigma(-0.04, 0)\).
* \(\sigma\) Referenced to the ascending node of the inner orbit, whose direction is arbitrary.
* This parameter is not constrained by the data; the posterior is identical to the log-uniform prior.
* Derived from the posterior of \(R_s\) and that of \(R_b/R_s\) from the mean transit light curve (Table 4).

Effects are routinely observed in triple-star systems containing eclipsing binaries (e.g., Rappaport et al. 2013; Masuda et al. 2015).

Numerical TTVs are computed using the TTVFast code (Deck et al. 2014) by integrating the orbits of the three bodies and finding the times at which the sky-projected distance between the star and the inner planet, \(d_{\text{sky}}\), becomes minimum. I choose the time steps to be 0.3 days for Kepler-448 and 0.1 days for Kepler-693, which are roughly 1/60 and 1/150 of the inner orbital periods, respectively. To compute transit
times, I modify the default output of the TTVFast code to take into account the effect of light-travel time using the line-of-sight coordinate of the center of mass of the inner binary (i.e., central star and inner planet).

3.5. Joint TTV and TDV Modeling

The joint TTV and TDV modeling was performed using the following likelihood:

\[
\mathcal{L} = \mathcal{L}_{\text{TTV}} \times \prod_j \frac{1}{\sqrt{2\pi(\sigma_{T,j}^2 + \sigma_{\text{dur}}^2)}} \exp \left[ -\frac{(T_j - T_{j,\text{model}})^2}{2(\sigma_{T,j}^2 + \sigma_{\text{dur}}^2)} \right] \\
\times \frac{1}{\sqrt{2\pi\sigma_b^2}} \exp \left[ -\frac{(b_{\text{mean}} - b_{\text{model}})^2}{2\sigma_b^2} \right],
\]

where the second row is defined analogously to \(\mathcal{L}_{\text{TTV}}\) with the transit time \(t\) replaced by the transit duration \(T\). I also include the constraint on the transit impact parameter \(b_{\text{mean}}\) from the mean transit light curve.\(^5\) The TDV modeling additionally requires the mean density of the host star \(\rho_c\) (equivalently, the stellar radius and the “jitter” for durations \(\sigma_{\text{dur}}\), and \(\theta\) consists of the 17 parameters listed in Tables 5 and 6 as fitted parameters. Here I fit the inclination of the inner orbit and the mass of the inner planet, although the latter is not constrained at all by the data.

This joint modeling is performed only numerically using TTVFast. In addition to the transit times above, I use another default output, \(d_{\text{sky}}\) at each transit center, to obtain the model transit impact parameter \(b = d_{\text{sky}}/R_*\), where the stellar radius \(R_*\) is given by \((3m_*/4\pi\rho_c)^{1/3}\); here I adopt \(b\) at the transit around the center of the observing duration as \(b_{\text{model}}\). The transit durations are obtained from yet another default output of the code \(v_{\text{sky}}\), the sky-projected relative star–planet velocity at the transit center, via \(T = 2\sqrt{1 - b^2}R_*/v_{\text{sky}}\).

3.5.1. Additional TDV Sources

While I only consider the Newtonian gravitational interaction between point masses, transit durations may also be affected by secular orbit variation due to stellar quadrupole moment and/or general relativistic effect. Indeed, we expect that Kepler-448 has a relatively large quadrupole moment \(J_2\) because of its rapid rotation (McQuillan et al. 2013; Bourrier et al. 2015), and the inner orbits are moderately eccentric in both systems, as we show below. Here we show that those effects on transit durations are negligible (the effects on transit times are even smaller; see Miralda-Escudé 2002).

The duration drift due to the nodal precession is given by (Miralda-Escudé 2002)

\[
[\dot{T}] \approx 3J_2 \left(\frac{a}{R_*}\right)^2 \frac{b}{\sqrt{1 - b^2}} |\sin \lambda|,
\]

where \(\lambda\) is the sky-projected stellar obliquity. For Kepler-448b with \(b \approx 0.4\), \(a/R_* \approx 20\), and \(\lambda \approx 10^9\), this results in a duration change over four years

\[
\Delta T \lesssim 2 \times 10^{-3} \text{ hr} \left(\frac{J_2}{10^{-7}}\right)^{1/2} \lesssim 2 \times 10^{-3} \text{ hr} \left(\frac{e}{1 - e^2}\right)
\]

where the quoted value of \(J_2\) is motivated by theoretical modeling and observational inference for a star with similar parameters (Szabó et al. 2012; Masuda 2015). This \(\Delta T\) is smaller than the measurement precision. The same is also true for Kepler-693b, for which \(J_2\) is likely smaller and measurements of \(T\) are less precise.

The drift caused by general relativistic apsidal precession is (Miralda-Escudé 2002)

\[
T \lesssim 4e \left(\frac{a}{R_*}\right)^{-1} \sqrt{1 - b^2} \times \frac{3}{1 - e^2} \left(\frac{na}{c}\right)^2
\]

or

\[
\Delta T \lesssim 2 \times 10^{-3} \text{ hr} \left(\frac{e}{1 - e^2}\right)
\]

over four years; this is also negligible. Note that possible apsidal precession induced by the gravitational perturbation from the outer companion is already taken into account in our Newtonian model.

4. Results: Kepler-448/KOI-12

The resulting posterior PDF from the MultiNest analysis and the corresponding models are shown in Figure 3 (red solid lines) and in Figure 13. The summary statistics (median and 68% credible interval) of the marginal posteriors as well as the priors adopted for those parameters are given in Table 5. The solution consists of two separated posterior modes (denoted as Solution 1 and Solution 2) that reproduce the observed TTVs and TDVs equally well, without any significant difference in the local evidence computed for each mode.

In both solutions, the outer companion (we tentatively call it Kepler-448c) is likely to have a mass in the brown dwarf regime (22–37 \(M_{\text{up}}\)) and resides in a highly eccentric orbit close to the inner WJ. In spite of the small outer orbit, its pericenter distance \(1.46^{+0.07}_{-0.06}\) au is more than nine times larger than the inner semimajor axis and well within the stable regime (Mardling & Aarseth 2001). The system has one of the smallest binary separations compared to the planet aphelion \(0.21^{+0.05}_{-0.04}\) au of the known planetary systems with (sub-) stellar companions; see Figure 8 of Triaud et al. (2017).

I also find a modest eccentricity \(0.34^{+0.08}_{-0.07}\) for the inner WJ from the combination of timing and duration information. While the TTVs alone favor an even higher value (Appendix B), transit duration combined with the spectroscopic prior on \(\rho_c\) lowers it. The inner WJ mass is floated between 0.1–10 \(M_{\text{up}}\) but no constraint better than that from the RV data (<10 \(M_{\text{up}}\) as the 3\(\sigma\) upper bound; Bourrier et al. 2015) is found. This is because the TTVs of the inner WJ do not depend on its own mass to the first order.

While the above constraints essentially come from TTVs, TDVs possibly allow us to constrain the mutual inclination \(i_{21}\) between the inner and outer orbits. In fact, the two solutions are different in this respect: Solution 1 is “prograde,” i.e., the two orbits are in the same directions, while Solution 2 corresponds to a “retrograde” configuration. Nonzero mutual inclinations are slightly favored in both solutions, with the posterior probability...
that $39.2^\circ < i_21 < 140.8^\circ$ being 27%. However, it is more or less due to the large prior volume corresponding to the misaligned solution, and a wide range of the mutual inclination is allowed by the data (Figure 4). In particular, the data are totally consistent with the coplanar configuration, as visually illustrated in the right panels of Figure 3. I found that the evidence for the coplanar model is not significantly different from the misaligned case. Only the configuration with $i_90 = 21^\circ$ is slightly disfavored due to the lack of a large TDV expected from the strong perturbation in the direction perpendicular to the orbital plane.

The joint TTV/TDV analysis also allows for constraining the impact parameter during a possible occultation, $b_{oc}$, based on that during the transit as well as on the eccentricity and argument of periastron of the inner orbit. I find $b_{oc} = 0.22 \pm 0.02$, which means that the secondary eclipse should have been detected if Kepler-448b were a star. This strengthens the planetary interpretation by Bourrier et al. (2015), who confirmed $m_b < 10 M_{Jup}$ at the $3\sigma$ level.

The following subsections detail the prior information and interpretation of TTVs and TDVs.

4.1. Adopted Parameters

I adopt $m_* = 1.51^{+0.14}_{-0.09} M_\odot$ and $\rho_* = 0.758^{+0.12}_{-0.07} g \text{ cm}^{-3}$ (converted from the radius $r_* = 1.41 \pm 0.06 R_\odot$) as the priors based on the spectroscopic values by Bourrier et al. (2015). The values agree with, and are more precise than, the latest KIC values (Mathur et al. 2016): $m_* = 1.386^{+0.009}_{-0.007} M_\odot$, $r_* = 1.367^{+0.027}_{-0.014} R_\odot$, and $\rho_* = 0.7647^{+0.0039}_{-0.0032} g \text{ cm}^{-3}$. Note that I do not use the values from the joint analysis in Bourrier et al. (2015) because they are derived assuming a circular orbit for the inner transiting planet, which is unlikely based on our dynamical modeling. I adopt $b_{mean} = 0.373$ and $e_0 = 0.006$ based on the posterior from the mean transit light curve (Table 3).

4.2. Constraints from TTVs

The mass and some elements of the outer orbit ($\gamma_2$, $q_2 / a_1$, $e_2$) are well determined from TTVs in spite of the non-transiting nature of the companion. The relationship between these parameters and observed TTV features can roughly be

![Figure 3. Dynamical models of the observed TTVs and TDVs of Kepler-448b. The red solid lines show 50 models randomly sampled from the joint posterior distribution obtained by fitting the data (black circles with error bars). The left column corresponds to the model that allows the mutual inclination to vary, while the right column shows the result when it is fixed to zero.](image-url)
understood as follows, with the help of the analytic formula in Appendix A.

Most of the information comes from the spike-like feature around BJD \( \sim 2454833 + 1050 \), which is caused by the close encounter of the outer body and thus defines the time of its periastron passage \( \tau_2 \) (the effect is represented by the \( S \) and \( C \) functions in Equations (17) and (18); see also Figure 12). In addition, its duration \( \Delta t \) is sensitive to both \( a_2 \) and \( e_2 \); the former determines the overall orbital timescale, and the latter changes the fraction of time spent around the periastron. The \( \Delta t \) may be roughly estimated using Kepler’s second law as follows:

\[
\Delta t \sim \frac{P_1 (1 - e_2^2)^{3/2}}{2 \pi} \int_{-\pi/2}^{+\pi/2} \frac{dv_2}{(1 + e_2 \cos v_2)^2} \\
\simeq \frac{P_1}{2} \left( 1 - \frac{4}{\pi} e_2 \right) \\
\simeq \frac{P_1}{2} \left[ \frac{a_1 (1 - e_2^2)^{8/3}}{a_1} \right]^{3/2} \sim \frac{P_1}{2} \left( \frac{q_2}{a_1} \right)^{3/2},
\]

where its order of magnitude is not so sensitive to the rather arbitrary choice of the interval of integration (\(-\pi/2 \) to \( \pi/2 \)). Since \( \Delta t \sim 300 \) days and \( P_1 \sim 20 \) days, this estimate gives \( q_2/a_1 \sim 10 \). Finally, its amplitude \( \Delta A \) is given by (Equations (10) and (11))

\[
\Delta A \simeq \frac{P_1}{2 \pi} A_{\text{L1}} \simeq \frac{P_1}{16 \pi} \left[ \frac{q_2 (1 + e_2)}{a_1} \right]^{-3/2} \frac{m_c}{m_*},
\]

assuming \( m_b, m_c \ll m_* \). Combining \( \Delta A \sim 2 \) min with the above estimate \( q_2/a_1 \sim 10 \), we find \( m_c/m_* \sim 10^{-2} \). These numbers are in reasonable agreement with those from the full dynamical modeling.

In addition to the spike, a long-term modulation due to “tidal delay” (Borkovits et al. 2003; Agol et al. 2005 represented by the \( \mathcal{M} \) function in Equation (16)) is also visible (blue curves in Figure 12); the tidal force from the outer companion delays the inner binary period, depending on the distance between the two. This effect, combined with the short-term spike, allows for further constraints on \( \tau_2, \omega_2, e_2 \), and \( P_2 \). In principle, this additional constraint enables the separate determination of \( a_2 \) and \( e_2 \) rather than only \( q_2 \), although \( a_2 \) is not well constrained because it is not clear whether the whole cycle of the outer orbit is covered given only the one observed periastron passage.

The analytic expressions (10)–(15) show that the short-term modulation represented by the \( S \) or \( C \) function is produced only when \( e_1 \neq 0 \) or \( i_2 \neq 0 \). In fact, the observed \( \text{TTVs} \) alone are found to be fitted well either by the nonzero \( e_1, e_2, \) or noncoplanar \( \theta_{\text{noncopl}} \) effects. This causes the anticorrelated degeneracy between \( e_1 \) and \( i_2 \). Since \( e_1 \) also affects the spike amplitude, we have two classes of solutions: the high-\( e_1 \)-low-\( i_2 \)-low-\( m_2 \) solution, and the low-\( e_1 \)-high-\( i_2 \)-high-\( m_2 \) solution. The joint \( \text{TTV}/\text{TDV} \) model slightly favors the latter.

The directions of the orbits, \( \omega_1, \omega_2, \cos i_2 \), and \( \Omega_2 \), are not well constrained from the \( \text{TTVs} \) alone. The complex degeneracy between \( \omega_2 \) and \( \Omega_2 \) comes from the fact that \( \text{TTVs} \) are rather sensitive to such “dynamical” angles as \( n_1 \) and \( n_2 \) in Figure 1 of Borkovits et al. (2015), which are referred to the invariant plane. Nevertheless, I use \( \cos i_2 \) and \( \Omega_2 \) because of the simplicity in implementing the isotropic prior. The relationship between the two sets of angles is summarized in Appendix A.2.

Finally, the LTTE effect (Equation (9)) is about 0.1 min/au for \( m_c/m_* \sim 10^{-2} \). This term does therefore not play a major role, as also seen in Figure 12.

4.3. Constraints from TTVs and TDVs

In the joint analysis, I find a smaller \( e_1 \) than that derived from the \( \text{TTVs} \) alone because the combination of \( T \), \( b \), and \( \rho_2 \) favors a small \( e_1 \sin \omega_1 \) (see Equations 16, 18, and 19 of Winn 2011). This constraint is combined with those from \( \text{TTVs} \) to better determine \( e_1 \) and \( \omega_1 \). The distribution of \( \omega_1 \) is still bimodal because no constraint is available for \( \omega_1 \) from \( \text{TTVs} \).

These additional constraints on \( e_1 \) and \( \omega_1 \) partly solve the degeneracies with other angles mentioned above. In particular, the value of \( m_c \) from the joint \( \text{TTV}/\text{TDV} \) analysis is higher than from the \( \text{TTVs} \) alone because the low-\( e_1 \)-high-\( i_2 \)-high-\( m_2 \) solution is favored.

Absence of significant \( \text{TDVs} \) only weakly constrains the mutual inclination. As mentioned, the solution with \( i_2 \sim 90^\circ \) is disfavored; if this were the case, a large tangential perturbation should have produced significant \( \text{TDVs} \) around the periastron passage, but none are present in the data. The solutions with \( \Omega_2 \sim 0^\circ \) and \( \Omega_2 \sim 180^\circ \) are basically indistinguishable and the log-likelihood is not sensitive to \( \cos i_2 \) either. In the \( \text{TDV} \) models (bottom left panel of Figure 3), both the positive and negative bumps are seen around the outer periastron because the inner inclination is perturbed in the opposite way, depending on the sign of \( \cos i_2 \). The current data do not significantly favor either of the cases.

5. Results: Kepler-693/KOI-824

The resulting posterior PDF from the MultiNest analysis and the corresponding models are shown in Figure 5 (red solid lines) and in Figure 14. The summary statistics (median and 68% credible region) of the marginal posteriors as well as the priors adopted for these parameters are given in Table 6. Again I found two almost identical solutions with prograde and retrograde orbits.

Similarly to Kepler-448c, the outer companion (we tentatively call it Kepler-693c) orbits close to the inner WJ on a highly eccentric orbit, except that it has a higher mass \( 1.5^{+0.6}_{-0.4} \times 10^2 M_{\oplus} \) and can be a low-mass star. The outer pericenter distance of
1.5 ± 0.2 au is also similar to Kepler-448c and satisfies the stability condition \((Mardling & Aarseth 2001)\). Again, nonzero eccentricity is favored for the inner orbit \(e = 0.21^{+0.2}_{-0.1}\), and its mass is not constrained at all from the data.

A notable difference from the previous case is the clear TDV signal, which tightly constrains the mutual inclination to be \(i = 90.1\pm2.1^\circ\) (Figure 4, bottom). I checked that the data can never be explained by the aligned configuration (right column of Figure 5). I find the Bayesian evidence for the mutually inclined model \((\ln Z = -89.18 \pm 0.08)\) is larger than that of the coplanar model \((\ln Z_{\text{copl}} = -103.1 \pm 0.07)\) by a factor of \(10^6\). The observed mutual inclination is the highest of the dynamically measured inclinations for planetary systems, and it is likely above the critical angle for the Kozai oscillation \((\text{posterior probability that } i > 90^\circ \text{ is } 80\%)\).

In Section 6, I show that the perturbation from the inclined companion does cause a large eccentricity oscillation of the inner WJ.

The mass of Kepler-693b is not measured, and the planet was statistically validated by Morton et al. (2016). A possible concern is that the absence of secondary eclipse, which the statistical validation partly relies on (in addition to other factors, including the transit shape and non-detection of the companion via the AO imaging by Baranec et al. 2016) may lose its meaning if \(b_{\text{occ}} > 1\) due to the nonzero inner eccentricity. However, our dynamical modeling finds \(b_{\text{occ}} = 0.64^{+0.08}_{-0.09}\), which excludes the possibility. In addition, the derived mean stellar density is also compatible with the KIC value. Therefore, the low false-positive probability (lower than \(1/300\)) derived by Morton et al. (2016) is still valid in the light of our new constraints.

### 5.1. Adopted Parameters

I adopt \(m_0 = 0.793^{+0.054}_{-0.039} \ M_\odot\) and \(\rho_0 = 1.931^{+0.526}_{-0.292} \ \text{g cm}^{-3}\) (corresponding to \(r_0 = 0.833^{+0.032}_{-0.062} \ R_\odot\) from KIC as the priors. I adopt \(b_{\text{mean}} = 0.6\) and \(\sigma_b = 0.1\), which roughly incorporate the 68% credible region of the posterior from the mean transit light curve (Table 4).
fixed value of \(i_{21}\) by optimizing the other parameters, and examine its form as a function of \(i_{21}\). This has been done by searching for minimum \(\chi^2\) solutions for a grid of \(\cos i_{21} \in [-1, 1]\) and \(\Omega_{21} \in [-90^\circ, 90^\circ]\) (prograde solutions). Here the constraints on \(m_s\) and \(\rho_e\) are also incorporated in \(\chi^2\), and \(\sigma_{TTV}\) and \(\sigma_{TDV}\) were fixed to zero.

Figure 6 shows the resulting profile of \(-2 \ln(\hat{\mathcal{L}}(i_{21})/\mathcal{L})\), where \(\hat{\mathcal{L}}\) denotes the maximum likelihood found by optimizing all the model parameters, including \(i_{21}\). This is equivalent to the chi-squared difference from its minimum value, \(\Delta \chi^2\), in our current setting. Here the \(\Delta \chi^2\) value is scaled so that the minimum \(\chi^2\) solution has \(\chi^2 = 1\). The resulting 1σ “confidence” interval is \(i_{21} = 47^\circ \pm 17^\circ\), and the 2σ lower bound is found to be \(i_{21} = 18^\circ\). Thus I conclude that a high mutual inclination is indeed favored by the data, and the conclusion is insensitive to the prior information on \(i_{21}\).

6. Long-term Orbital Evolution

A high mutual inclination confirmed for the Kepler-693 system may result in a large eccentricity oscillation of the inner orbit as long as the perturbation is strong enough to overcome other short-range forces. If this is indeed the case, the system may serve as direct evidence that some WJs are undergoing eccentricity oscillations. Even in the Kepler-448 system, where highly inclined solutions are not necessarily favored, significant eccentricities of both the inner and outer orbits may still lead to an eccentricity excitation that is due to the octupole-level interaction.

Given the full set of parameters constrained from the dynamical analysis, we can assess the future of the systems rather reliably by extrapolating the dynamical solutions. In this section, we explore the effect of the secular perturbation due to the outer companions on the inner planet’s orbit and its tidal evolution.

6.1. Oscillation of the Inner Eccentricity

I compute secular evolution of both the inner and outer orbits along with the spins of the star and inner planet. I use the code developed and used in Xue & Suto (2016) and Xue et al. (2017), which takes into account (i) gravitational interaction up to the octupole order, and (ii) precessions due to general relativity as well as tidal and rotational deformation of the bodies. Here I neglect magnetic braking of the star and tidal dissipation inside the bodies, and assume zero stellar and planetary obliquities for simplicity. The rotation periods are set to be 1.25 days for Kepler-448, 10 days for Kepler-693, and 1 day for the inner planets, although the spin evolution does not significantly affect the result. I adopt standard values for the dimensionless moments of inertia (0.059 and 0.25 for the star and inner planet) and the tidal Love numbers (0.028 and 0.5). The orbits are integrated for 10 Myr (sufficiently longer than the oscillation timescale; see below), starting from 1000 random sets of parameters sampled from the posterior distribution from the dynamical analyses (Sections 4 and 5).

Figure 7 shows the initial (black circles) and minimum (red diamonds) values of \(a_1(1 - e_1^2)\) over the course of evolution, the latter of which correspond to the maxima of \(e_1\). In both systems, significant eccentricity oscillations occur at least for some of the solutions. When we adopt \(a_1(1 - e_1^2) < 0.1\) au as a conventional threshold for the migrating WJs (Socrates et al. 2012b; Dong et al. 2014; Dawson et al. 2015), the periastra become close enough to drive the tidal migration for 12% of the solutions for Kepler-448b and 96% for Kepler-693b, excluding the tidally disrupted cases, which are shown with
transparent colors. If we instead choose $a \langle 1 - e^2 \rangle \leq 0.05$ au as a threshold (e.g., Anderson et al. 2016), the fractions become 5% and 33% for Kepler-448b and Kepler-693b, respectively. Large eccentricity oscillations (and hence small minimum periastra) are observed mainly for $i_{21} \lesssim 40^\circ$; this explains why a larger portion of the solutions has sufficiently small $a \langle 1 - e^2 \rangle$ for the Kepler-693 system.

The current eccentricities of the two WJs (1σ region shown with vertical dotted lines in Figure 8) also turn out to be the most likely values to be observed over the course of an oscillation if we are random observers in time; they are around the peaks of the inner eccentricity distribution during the 10 Myr evolution averaged over the dynamical solutions (gray histograms in Figure 8).

For highly inclined solutions, both libration and circulation of the argument of periastron (Kozai 1962), modified by the octupole effects, are observed. In either of the two systems, the Kozai–Lidov timescale $\tau_{KL}$ (Kiseleva et al. 1998) and the octupole timescale $\tau_{Oct}$ are both found to be much shorter than the timescale of general relativistic apsidal precession $\tau_{GR}$ for any $i_{21} \lesssim 40^\circ$.

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**Figure 7.** Effect of secular eccentricity oscillation on the semi-latus rectum of the inner orbit, $a_1(1 - e_1^2)$. This distance corresponds to the final semimajor axis of the HJ if the orbit is circularized at a fixed angular momentum. The black circles and histograms are randomly sampled from the posterior of the dynamical TTV/TDV modeling and represent the current values. The red diamonds and histograms are the minimum values over many oscillation timescales (Section 6.1). Solutions with a minimum periastron distance $a_1(1 - e_1^2)$ smaller than the Roche limit (here chosen to be $2.7 R_1$, based on Guillochon et al. 2011), simply assuming the planetary mass $10^{-3} M_\odot$ and radius $0.1 R_\star$ are plotted with transparent colors and are not included in the histogram. The horizontal dashed line indicates $a_1(1 - e_1^2) = 0.1$ au, which is the conventional threshold for possible tidal circularization accepted by Socrates et al. (2012b), Dong et al. (2014), and Dawson et al. (2015).

**Figure 8.** Distributions of the inner eccentricities ($e_1$) during the secular oscillation averaged over the dynamical solutions. The vertical dotted lines (shaded region) show the median and 68% credible interval of the current $e_1$ measured from the TTVs and TDVs.

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5 On the other hand, the libration of the difference in the apsidal longitudes $\Delta \omega_{inv}$, as discussed in Dawson & Chiang (2014), was not observed in the current simulations.
solution found from TTVs and TDVs: for reference, I find typical $\tau_{KL} \sim 10^{-2}$ Myr, $\tau_{Oct} \sim 10^{-1}$ Myr, and $\tau_{GR} \sim 10^{0.5}$ Myr for Kepler-448, and $\tau_{KL} \sim 10^{-3}$ Myr, $\tau_{Oct} \sim 10^{-1.5}$ Myr, and $\tau_{GR} \sim 10^{0.5}$ Myr for Kepler-693. Thus the perturbation from the “close friends” is indeed strong enough to overcome general relativity (Dong et al. 2014). Note that this is the case for the octupole effect as well, which explains why some low-$i_{31}$ solutions in the Kepler-448 system lead to a large eccentricity oscillation.

6.2. Migration Timescales

Are the two WJs currently migrating into HJs as a result of the eccentricity oscillation? If this were the case, the current migration timescale $\tau_{\text{mig}}$ would need to be comparable to the system age $\tau_{\text{age}}$. If $\tau_{\text{mig}} \gg \tau_{\text{age}}$, they are unlikely to be migrating, while if $\tau_{\text{mig}} \ll \tau_{\text{age}}$, the WJs should have rapidly evolved into HJs and we are unlikely to observe the system in the current state. Here we perform an order-of-magnitude comparison between the two timescales, given their large observational and theoretical uncertainties.

Figure 9 shows the distribution of the tidal migration timescale given by Equation (2) of Petrovich & Tremaine (2016) at the minimum periastron distance computed in Section 6.1. Strictly speaking, the relevant timescale is expected to be longer because oscillation of $e_1$ slows the migration down (Petrovich 2015), but in our case, the maximum eccentricity is not so close to unity (since $a_1$ is already small) that the modification is unlikely large (cf. Figure 2 of Petrovich 2015); thus we simply neglect the correction. The timescales are computed for three different values of the viscous time of the planet, which gives a characteristic timescale for dissipation: $t_V = 0.015$ year, $t_V = 1.5$ year, and $t_V = 150$ year, while the dissipation inside the star is neglected (see Socrates et al. 2012a for comparison with different parameterizations). The three values roughly correspond to (i) very efficient tidal dissipation required for some high-eccentricity migration scenarios to explain the observed HJs (Petrovich 2015; Hamers et al. 2017), (ii) dissipation required to circularize the orbits of HJs with $P \lesssim 5$ days within 10 Gyr (Socrates et al. 2012a), and (iii) values calibrated based on a sample of eccentric planetary systems (Hansen 2010; Quinn et al. 2014), while the limit from the Jupiter–Io system ($t_V \gtrsim 15$ year) lies in between the latter two. I also indicate (rough) estimates for the ages of the two systems with vertical dashed lines: 1.5 Gyr for Kepler-448 based on spectroscopy (Bourrier et al. 2015), and 5 Gyr for Kepler-693 as a tentative value given that the host star has dimensions of a K dwarf.

The comparison between the histogram and the dashed line shows that the migrating solutions with $\tau_{\text{mig}} \sim \tau_{\text{age}}$ exist for a wide range of $t_V$ for the Kepler-693 system. In particular, the eccentricity oscillation plays a crucial role for $t_V \gtrsim 1.5$ year so that such solutions realize. The migrating solutions also exist for the Kepler-448 system, although they seem plausible only for a small fraction of significantly misaligned solutions that lead to the large eccentricity oscillation, or they require efficient tidal dissipation with $t_V \lesssim 1.5$ year.

6.2.1. Possible Fates of the Inner Planets

The timescale arguments above indicate the inner WJs may evolve into HJs within the lifetime of the system. If this is the case, HJ systems with close substellar companions as found by a long-term RV monitoring (Triaud et al. 2017) may have been WJs like ours in the past.

As a proof of concept, I compute the evolution of the two systems for 1 Gyr, including the tidal dissipation with the planetary viscous timescale $t_V = 1.5$ year for illustration. I fix $t_V = 50$ year for the star. I stop the calculation when the inner orbit is circularized (both $a_1 < 0.1$ au and $e_1 < 0.01$ are achieved) before 1 Gyr. If we change $t_V$, we expect that the systems evolve merely on a different timescale corresponding to the change.

Figure 10 compares the initial and final semimajor axes from these simulations. We see that some solutions do survive the tidal disruption and evolve into HJs within 1 Gyr. Such an outcome is
rarer in the Kepler-693 system than in the Kepler-448 system. This is consistent with the expectation that the former system is likely older than the latter at least by a factor of a few. In fact, this type of path may be even rarer than it seems in the right histograms of Figure 10, because some of the surviving HJs have experienced the circularization on a much shorter timescale compared to the system age because of the rapid eccentricity surge: if we are random observers in time, it is a priori unlikely to observe a system with such a short remaining lifetime compared to the current age (Gott 1993). However, I do not attempt to correct for this effect here, given that the outcome will be sensitive to the uncertain tidal parameter in any case. If correctly taken into account, this type of argument will potentially allow for better constraints on the system parameters.

6.3. Implications for the In Situ Formation Scenario

While the observed properties of the two inner planets are consistent with those of migrating WJs, the presence of the outer (sub-)stellar companions at as close as 1.5 au challenges the high-eccentricity migration scenario from beyond the snow line. Planets in S-type orbits around tight binaries with periasteron distances smaller than 10 au have also been reported around KOI-1257 (Santerne et al. 2014), Kepler-444 (Dupuy et al. 2016), HD 59686 (Ortiz et al. 2016), and possibly ν Octantis (Ramm et al. 2016). If confirmed to be a low-mass star, Kepler-693c has the smallest peristion of these stellar companions.

A similar issue has also been discussed for WJs with outer planetary-mass companions (Antonini et al. 2016): the outer orbits in these systems, if primordial, are in most cases too small for the inner WJs to have migrated from ≥1 au. In addition, population synthesis simulations of high-eccentricity migration from ≥1 au, either via the companion on a wide orbit (Anderson et al. 2016; Petrovich & Tremaine 2016) or via secular chaos in multiple systems (Hamers et al. 2017), have difficulty in producing a sufficient number of WJs relative to HJs. These may also argue for the WJ formation via disk migration or in situ. Considering the prevalence of compact super-Earth systems revealed by Kepler, the latter can well be possible theoretically (Lee et al. 2014) and may also have observational supports (Huang et al. 2016).

The companions discovered in the Kepler-448 and Kepler-693 systems may further argue for the in situ origin. Such low-mass stellar or brown dwarf companions on au-scale orbits may be formed via fragmentation at a larger separation followed by the orbital decay due to dissipative dynamical interactions involving gas accretion and disks, which proceed in ≤1 Myr (Bate et al. 2002; Stamatellos & Whitworth 2009; Bate 2012). This implies that giant-planet formation and migration must have occurred very quickly if they preceded those of the outer companion. Alternatively, the companions may have arrived at the current orbit well after disk migration and disk dispersal via chaotic dissolution of an initial triple-star system or an impulsive encounter with a passing star (Marzari & Barbieri 2007; Martí & Beaugé 2012). While these scenarios are compatible with the eccentric and inclined outer orbit, they may suffer from the fine-tuning problem. In the former scenario, for example, a binary orbit typically shrinks only by a factor of a few, limited by energy conservation (Marzari & Barbieri 2007). The outcomes are likely more diverse in the latter, but only a small fraction of them usually constitutes a suitable solution (Martí & Beaugé 2012), and such close encounters as to alter the binary orbit significantly are likely rare when the planet formation is completed, even in a cluster with ~10^3 stars (Adams et al. 2006). We also note that tidal friction associated with the close encounter with the primary (e.g., Kiseleva et al. 1998) is unavailable to shrink the binary orbit in the presence of the inner planet. Considering these possible difficulties of the alternative scenarios, in situ formation in a tight binary seems to be an attractive possibility that provides simple solutions both for our two systems and for other theoretical and observational issues, although disk migration followed by rearrangement of the outer orbit cannot be excluded.

This motivates us to examine whether the moderate eccentricities of our WJs can be explained in the in situ framework, in which a near-circular orbit is normally expected. Here we consider a specific form of question, in the same spirit as Anderson & Lai (2017). Suppose that the inner WJs were produced into circular
orbits with the current semimajor axes, can their observed nonzero eccentricities be explained by the perturbation from the detected companions? To answer this, I perform a similar set of simulations as in Section 6.1 by setting $e_i = \omega_i = 0$ initially, while sampling the other parameters from the posterior. Note that this experiment is applicable to the disk migration case as well, whose outcome is also a short-period planet on a circular orbit.

Figure 11 summarizes the maximum inner eccentricities achieved during the 10 Myr evolution against the current mutual inclination. The maximum value exceeds the current best-fit value (horizontal dashed line) for 14% and 76% of the solutions that did not lead to tidal disruption in the Kepler-448 and Kepler-693 systems, respectively. Thus the current architecture can indeed be compatible with the initially circular orbits. The sequences of the maximum $e_1$ ($e_{1,\text{max}}$) and initial $i_2$ ($i_{21,\text{init}}$) roughly follows the relation $\sqrt{1 - e_{1,\text{max}}^2} \sqrt{3/5} = \cos i_{21,\text{init}}$.

Considering the arguments in Section 6.2, it is also conceivable that the inner WJs were formed into circular orbits at larger semimajor axes than observed now (but still inside the snow line) and have migrated to the current orbits via tidal migration. In this case, excitation of eccentricity should have been easier because the gravitational interaction with the companion was initially stronger. This process might serve as yet another path of HJ formation: some HJs may have been isolated WJs formed in situ or via disk migration, whose orbits were shrunk as a result of the tidal high-eccentricity migration driven by a close companion.

7. Discussions

7.1. Follow-up Observations

While the future observations of transit times and durations will surely improve the constraint on the outer period and eccentricity, I did not find any systematic dependence of the future TTV and TDV evolutions on the current mutual orbital inclination at least within about 10 years. Moreover, it is challenging to observe a full transit from the ground because of the long transit duration ($\approx 7$ hr) for Kepler-448b and faintness ($V = 16.8$) of the host star for Kepler-693b.

What can be achieved with RV observations? Based on the constraints from TTVs and TDVs, the RV semi-amplitude due to the outer companion is expected to be $2.7^{+0.7}_{-0.6} \times 10^2$ m s$^{-1}$ for Kepler-448 and $2.7^{+0.9}_{-0.6}$ km s$^{-1}$ for Kepler-693. The variation may be observable for Kepler-693 with a large telescope, while the detection is not plausible for Kepler-448 with $v \sin i_c \approx 60$ km s$^{-1}$.

Instead, Gaia astrometry (Perryman et al. 2001) is promising to detect the outer binary motion and better determine the mutual inclination. Since the misalignment in the sky plane $\Omega_{21}$ is dynamically well constrained in both systems (and the inner planets are transiting), inclinations of the outer orbits $i_2$, if measured, significantly improve the constraint on $\Omega_{21}$. Based on the dynamical modeling, the expected astrometric displacements due to the outer companions are $1.4^{+1.2}_{-0.5} \times 10^2 \mu$as for Kepler-448 with $V = 11.4$ (assuming the distance $d = 426 \pm 40$ pc from Bourrier et al. 2015) and $4.4^{+3.4}_{-1.3} \times 10^2 \mu$as for Kepler-693 with $V = 16.8$ (assuming $d = 1110$ pc from the isochrone fit). They are both well above the astrometric precision expected after the nominal five-year mission (Perryman et al. 2014).

7.2. Frequency of Close and Massive Companions to WJs

It is admittedly difficult to evaluate the completeness of our TTV search because of the complex dependence of the signal on the system parameters. Nevertheless, our detection of the close and massive companions in two systems, among the sample of 23 WJs, suggests such an architecture is not extremely uncommon. We also need to take into account that the two systems would not have been detected if the periastron passage had not occurred during the Kepler mission. The ratios of the outer orbital periods ($P_2 \sim 2500$ days for Kepler-448c and $P_2 \sim 1800$ days for Kepler-693c) to the Kepler observing duration ($\sim 1400$ days) suggest that there may be one or two more WJs with similar companions in the Kepler sample that have evaded our search. This crude estimate seems compatible with the theoretical
argument by Petrovich & Tremaine (2016) that roughly 20% of WJs can be accounted for by high-eccentricity migration, although the observed architectures of our systems may not support the migration from $\geq 1$ au via this process, as assumed in Petrovich & Tremaine (2016).

8. Summary and Conclusion

This paper reported the discovery of close companions to two transiting WJs via their TTVs and TDVs. The companions have masses comparable to a brown dwarf or a low-mass star ($22^{+7}_{-3} M_{\text{Jup}}$ and $150^{+66}_{-40} M_{\text{Jup}}$), and they are on highly eccentric orbits ($e \geq 0.5$) with small periastron distances ($\approx 1.5$ au). For the companion of Kepler-693b, a large mutual orbital inclination ($\approx 50^\circ$) with respect to the inner planetary orbit is indicated by TDVs, while the constraint on the mutual inclination is weak for the Kepler-448 system. They are among the few systems with constraints on mutual inclinations, and the inclination inferred for the Kepler-693 system is the highest ever determined dynamically for planetary systems. The value is indeed high enough for the eccentricity oscillation via the Kozai mechanism to occur: more than 90% of the solutions inferred for Kepler-693b (and some 10% for Kepler-448b) imply that the eccentricities of the inner WJs exhibit strong enough oscillations for tidal dissipation to significantly affect the inner orbits, by bringing $e(1-e^2)$ to less than 0.1 au. The corresponding migration timescales can be compatible with the hypothesis that the inner WJs are tidally migrating to evolve into HJs for a wide range of viscous timescales.

The architectures of the two systems support the scenario that eccentric WJs are currently undergoing eccentricity oscillation induced by a close companion and are experiencing slow tidal migration where the orbit shrinks only at the high-eccentricity phase. On the other hand, the origin of the current highly eccentric/inclined configuration is still unclear. Specifically, they may not fit into the classical picture that close-in gas giants have migrated from beyond the snow line, given the close and (sub-) stellar nature of the outer companions. Formation of gas giants within the snow line onto circular orbits, followed by eccentricity excitation by the companion, therefore seems another viable option to be pursued, although the companion may instead have arrived at the current orbit after disk migration of the inner WJ. Regardless of the origin of the current configuration, the long-term evolution simulation demonstrates a new pathway of HJ formation via “high-eccentricity” migration of a WJ that formed in situ or via disk migration.

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Appendix A

Analytic TTV Formula for Hierarchical Triple Systems

In a part of the TTV analysis, we adopt the analytic timing formula for hierarchical three-body systems by Borkovits et al. (2015), which takes into account the eccentricities of both inner and outer orbits and the arbitrary mutual inclination between them (see also Borkovits et al. 2011, 2015, 2016, for its applications). Of the various effects that could possibly affect the observed transit times, we include two of the most important effects: LTTE and $P_2$-timescale dynamical effects up to the quadrupole order. The other effects including the octupole-level dynamical effects and short-term perturbations are at least an order-of-magnitude smaller than the quadrupole terms and are thus neglected (see Borkovits et al. 2015, for a complete discussion of these effects). Note that in this appendix, $+Z$-axis is taken away from the observer’s direction; this definition is opposite to the definition in the main text, and arguments of periastron in the formulae below therefore differ by $\pi$ from those in the main text.

A.1. Formula

We model the timing variations from the linear ephemeris that is due to the LTTE effect and the $P_2$-timescale dynamical effect. The LTTE term is given by

$$\Delta_{\text{LTTE}} = \frac{a_2 \sin i_2}{c} \frac{m_c}{m_\star + m_b + m_c} (1 - e_2^2) \sin u_2,$$

where $u_2$ is the true anomaly and $u_2 \equiv \omega_2 + \nu_2$ is the true longitude. The $P_2$-timescale dynamical effect is modeled up to the quadrupole order as follows (Borkovits et al. 2015, Equation (5)):

$$\Delta_l = \frac{P_1}{2\pi} A_{l1} L_{11} (1 - e_i^2)^{1/2} [\delta_{\text{idal}} + \delta_{\text{eccl}} + \delta_{\text{ecc2}} + \delta_{\text{noncopl}}].$$

Here the overall amplitude is fixed by the factor

$$A_{l1} = \frac{15}{8} \frac{m_c}{m_\star + m_b + m_c} \frac{P_1}{P_2} (1 - e_2^2)^{-3/2},$$

and each TTV component is given by

$$\delta_{\text{idal}} = \left[ \frac{8}{15} f_1 + \frac{4}{5} K_1 \right] M,$$
\[ \delta_{\text{ecc1}} = (1 + \cos i_2) \{ K_{11} S[2u_2 - 2(n_2 - n_1)] - K_{12} C[2u_2 - 2(n_2 - n_1)] \}, \]  
\[ \delta_{\text{ecc2}} = (1 - \cos i_2) \{ K_{11} S[2u_2 - 2(n_2 + n_1)] + K_{12} C[2u_2 - 2(n_2 + n_1)] \}, \]  
\[ \delta_{\text{noncopl}} = \sin^2 i_2 (K_{11} \cos 2n_1 + K_{12} \sin 2n_1 - \frac{2}{5} f_0^2 - \frac{3}{5} K_i) [2M - S(2u_2 - 2n_2)], \]

where 
\[ M = v_2 - l_2 + e_2 \sin v_2, \]
\[ S(2u_2) = \sin 2u_2 + e_2 \left[ \sin(u_2 + \omega_2) + \frac{1}{3} \sin(3u_2 - \omega_2) \right], \]
\[ C(2u_2) = \cos 2u_2 + e_2 \left[ \cos(u_2 + \omega_2) + \frac{1}{3} \cos(3u_2 - \omega_2) \right], \]
\[ f_i = 1 + \frac{25}{8} e_i^2 + \frac{15}{8} e_i^4 + \frac{95}{64} e_i^6 + O(e_i^8), \]
\[ K_i(e_1, \omega_1) = -e_1 \sin \omega_1 + \left( \frac{3}{4} e_1^2 + \frac{1}{8} e_1^4 + \frac{3}{64} e_1^6 \right) \cos 2\omega_1 + \left( \frac{1}{2} e_1^2 + \frac{3}{16} e_1^4 \right) \sin 3\omega_1 - \left( \frac{5}{16} e_1^4 + \frac{3}{16} e_1^6 \right) \cos 4\omega_1 - \frac{3}{16} e_1^6 \sin 5\omega_1 + \frac{7}{64} e_1^6 \cos 6\omega_1 + O(e_1^7), \]
\[ K_{1i}(e_1, \omega_1) = \frac{3}{4} e_1^2 + \frac{3}{16} e_1^4 + \frac{3}{32} e_1^6 + \left( e_1 + \frac{1}{2} e_1^3 + \frac{1}{4} e_1^5 \right) \sin \omega_1 + \left( \frac{51}{40} e_1^2 + \frac{37}{80} e_1^4 + \frac{241}{640} e_1^6 \right) \cos 2\omega_1 - \frac{3}{16} e_1^3 \sin 3\omega_1 - \left( \frac{1}{16} e_1^4 - \frac{1}{16} e_1^6 \right) \cos 4\omega_1 - \frac{1}{16} e_1^5 \sin 5\omega_1 + \frac{3}{64} e_1^6 \cos 6\omega_1 + O(e_1^7), \]
\[ K_{2i}(e_1, \omega_1) = -\left( e_1 - \frac{1}{2} e_1^3 - \frac{1}{4} e_1^5 \right) \cos \omega_1 + \left( \frac{51}{40} e_1^2 + \frac{87}{80} e_1^4 + \frac{541}{640} e_1^6 \right) \sin 2\omega_1 - \frac{3}{16} e_1^3 \cos 3\omega_1 - \left( \frac{1}{16} e_1^4 + \frac{5}{32} e_1^6 \right) \sin 4\omega_1 + \frac{1}{16} e_1^5 \cos 5\omega_1 + \frac{3}{64} e_1^6 \sin 6\omega_1 + O(e_1^7). \]

The angles \( n_1 \) and \( n_2 \) are the directions of \( Z > 0 \) part of the line of intersection of the inner and outer orbits measured from the ascending nodes of the two orbits, defined as in Figure 1 of Borkovits et al. (2015) between \([0, \pi]\), and \( l_2 \) is the mean anomaly. Note that \( K_{1i}(e_1, \pi - \omega_1) = K_{1i}(e_1, \omega_1) \) and \( K_{12}(e_1, \pi - \omega_1) = -K_{12}(e_1, \omega_1) \).

### A.2. Conversion of the Angles

In the main body of the paper, we did not use the physically most natural parametrization of the angles because it complicates the assignment of the prior in the MultiNest fitting. Here we summarize how our set of fitted angles \((i_1, i_2, \Omega_{21})\) can be related to that of physical angles \((i_{21}, n_1, n_2)\) in the analytic formula by Borkovits et al. (2015), since this case is not covered in their Appendix D.

The mutual inclination \( i_{21} \) can be computed as usual:
\[ \cos i_{21} = \cos i_1 \cos i_2 + \sin i_1 \sin i_2 \cos \Omega_{21}. \]  
Note that \( \sin i_{21} = 0 \) for \( \Omega_{21} \neq 0 \) since \((i_1, i_2) = (0, 0)\) nor \((\pi, \pi)\) for transiting systems, as discussed in this paper. For \( \cos \Omega_{21} = \pm 1 \), the above equation reduces to
\[ \cos i_{21} = \cos(i_2 \mp i_1). \]

We first consider the case when \( \sin i_{21} = 0 \). If \( \sin i_{21} = 0 \), the sine and cosine rules of the spherical trigonometry yield
\[ \frac{\sin i_{21}}{\sin \Omega_{21}} = \frac{\sin i_1}{\sin n_1} = \frac{\sin i_2}{\sin n_2} \]
and
\[ \cos n_1 = \cos \Omega_{21} \cos n_2 + |\sin \Omega_{21}| \sin n_2 \cos i_2, \]
\[ \cos n_2 = \cos \Omega_{21} \cos n_1 + |\sin \Omega_{21}| \sin n_1 \cos (\pi - i_1), \]
for \( \sin \Omega_{21} < 0 \) case. For \( \sin \Omega_{21} > 0 \), the cosine rules are replaced by
\[ \cos n_1 = \cos \Omega_{21} \cos n_2 + \sin \Omega_{21} \sin n_2 \cos (\pi - i_2), \]
\[ \cos n_2 = \cos \Omega_{21} \cos n_1 + \sin \Omega_{21} \sin n_1 \cos i_1. \]

In fact, the two cases can be written in a single form as
\[ \cos n_1 = \cos \Omega_{21} \cos n_2 - \sin \Omega_{21} \sin n_2 \cos i_2, \]
\[ \cos n_2 = \cos \Omega_{21} \cos n_1 + \sin \Omega_{21} \sin n_1 \cos i_1. \]

Thus, for a nonzero mutual inclination, we have
\[ \sin n_{12} = \text{sgn}(\sin \Omega_{21}) \frac{\sin i_{21}}{\sin i_{21}} \sin \Omega_{21} = \frac{\sin i_{21}}{\sin i_{21}} |\sin \Omega_{21}|. \]
and

\[ \cos n_1 = \frac{\text{sgn}(\sin \Omega_{21})}{\sin i_{21}} (-\sin i_1 \cos i_2 + \cos \Omega_{21} \cos i_1 \sin i_2), \]

\[ \cos n_2 = \frac{\text{sgn}(\sin \Omega_{21})}{\sin i_{21}} (\cos i_1 \sin i_2 - \cos \Omega_{21} \sin i_1 \cos i_2). \]

If \( \sin i_{21} = 0 \), we may just define \( n_2 - n_1 = 0 \) for \( i_{21} = 0 \) and \( n_2 + n_1 = \pi \) for \( i_{21} = \pi \) to correctly compute the non-vanishing terms of TTVs. Except for \( i_1 = i_2 = 0 \) or \( \pi \), this includes the two cases: (i) \( i_1 - i_2 = 0 \) and \( \Omega_{21} = 0 \) and (ii) \( i_1 - i_2 = \pi \) and \( \Omega_{21} = \pi \). As shown in Table 7, we have \( n_1 - n_2 = 0 \) for either \( i_2 - i_1 \to \pm \pi \) in case (i). Similarly, in case (ii), we have \( n_1 + n_2 = \pi \) for either \( i_2 + i_1 \to \pm \pi \). Although the other combination is ambiguous, it appears only in the vanishing terms of the TTV formula.

**Appendix B**

**Results of Analytic and Numerical TTV Analyses**

**B.1. Comparison of the Two Results**

For TTVs, I performed both an analytic fit with a wider prior range and a numerical fit with a narrower prior range. As shown in Tables 8 and 9, I found consistent posteriors from the two analyses; this agreement validates our numerical procedure. The best-fit models are basically indistinguishable from those in Figures 3 and 5.

**Table 8**  
Parameters of the Kepler-448 System from the Analytical and Numerical TTV Analyses

| Parameter                                           | Fitted Parameters | Derived Parameters |
|-----------------------------------------------------|-------------------|--------------------|
| **(Inner Orbit)**                                  |                   |                    |
| 1. Time of inferior conjunction                     | Analytic          | Prior              |
| \( t_{\text{in}} \) (BJD)                            | 128.7414(\pm 14)  | \( t_{\log} \) (128.7, 128.8) |
| 2. Orbital period \( P_1 \) (day)                    | 17.85522(\pm 60)  | \( t_{\log} \) (17.85, 17.86) |
| 3. Orbital eccentricity \( e_1 \)                    | 0.5(\pm 0.3)      | \( t_{\log} \) (0, 0.95) |
| 4. Argument of periastron \( \omega_1 \) (deg)       | -43\( ^\circ \)104 | \( t_{\log} \) (-180, 180) |
| 5. Cosine of orbital inclination \( \cos i_1 \)       | 0 (fixed)         | \( t_{\log} \) (180, 180) |
| **(Outer Orbit)**                                   |                   |                    |
| 6. Time of the periastron passage \( t_2 \) (BJD)    | 1076.10(\pm 10)   | \( t_{\log} \) (120, 1600) |
| 7. Periastron distance over inner semimajor axis \( q_2/a_1 \) | 9.3(\pm 0.4)     | \( t_{\log} \) (1, 50) |
| **(Physical Properties)**                           |                   |                    |
| 12. Mass of Kepler-448 \( m_1 \) \( (M_\odot) \)    | 1.54(\pm 0.11)    | \( t_{\log} \) (1, 0.09) |
| 13. Mass of Kepler-448 \( m_2 \) \( (M_\odot) \)    | 1 (fixed)         | \( t_{\log} \) (1, 1) |
| 14. Mass of Kepler-448c \( m_3 \) \( (M_\odot) \)   | 12(\pm 3)         | \( t_{\log} \) (10^{-4} M_\odot, 0.3 M_\odot) |
| **(Jitter)**                                        |                   |                    |
| 15. Transit time jitter \( \sigma_{\text{TV}} \) (10^{-4} day) | 2.2(\pm 0.3)     | \( t_{\log} \) (0.05, 5) |

**Note.** The elements of the inner and outer orbits listed here are Jacobian osculating elements defined at the epoch BJD = 2454833 + 120. The quoted values in the “Solution” columns are the median and 68% credible interval of the marginal posteriors. Parentheses after values denote uncertainties in the last digit. The “combined” column shows the values from the marginal posterior combining the two solutions; no value is shown when the combined marginal posterior is multimodal. In the prior column, \( t(a,b) \) and \( t_{\log}(a,b) \) denote the (log-) uniform priors between \( a \) and \( b \), \( f(x) = 1/(b - a) \) and \( f(x) = x^{-1}/(\ln b - \ln a) \), respectively; \( \mathcal{G}(a, b, c) \) means the asymmetric Gaussian prior with the central value \( a \) and lower and upper widths \( b \) and \( c \).

\( ^a \) Referenced to the ascending node of the inner orbit, whose direction is arbitrary.
Figure 12. Decomposition of the analytic TTV models into individual terms in Equations (9) and (10).

Table 9
Parameters of the Kepler-693 System from the Analytical and Numerical TTV Analyses

| Parameter                                      | Analytic |       | Numerical |       |
|------------------------------------------------|----------|-------|-----------|-------|
|                                                 | Posterior| Prior | Posterior | Prior |
| **Fitted Parameters**                           |          |       |           |       |
| (Inner Orbit)                                   |          |       |           |       |
| 1. Time of inferior conjunction                 | 173.611±1.377 |       | 173.609±1.796 |       |
| \( t_e,1 \) (BJD_{TDB} − 2454833)               | \( t_e,1 \) (173.55, 173.65) |       | \( t_e,1 \) (173.58, 173.64) |       |
| 2. Orbital period \( P_1 \) (day)               | 15.3756±0.0005 |       | 15.37549±0.0011 |       |
| 3. Orbital eccentricity \( e_1 \) (deg)         | 0.6 ± 0.2 |       | 0.5 ± 0.1 |       |
| 4. Argument of periastron \( \omega_1 \) (deg)  | 64.7±0.1 |       | 84.5±0.3 |       |
| 5. Cosine of orbital inclination \( \cos h_1 \) | 0 (fixed) |       | 0 (fixed) |       |
| (Outer Orbit)                                   |          |       |           |       |
| 6. Time of the periastron passage \( \tau_2 \)  | 636.1±0.1 |       | 660±0.3 |       |
| (BJD_{TDB} − 2454833)                           | \( \tau_2 \) (170, 1560) |       | \( \tau_2 \) (500, 900) |       |
| 7. Periastron distance over inner semimajor axis \( q_1/m_1 \) | 13.2±2 |       | 15±2 |       |
| (Physical Properties)                          |          |       |           |       |
| 12. Mass of Kepler-693 \( m_1 \) (\( M_\oplus \)) | 0.80±0.03 |       | 0.80±0.03 |       |
| 13. Mass of Kepler-693b \( m_0 \) (\( M_\oplus \)) | 1 (fixed) |       | 1 (fixed) |       |
| 14. Mass of Kepler-693c \( m_2 \) (\( M_\oplus \)) | 57±4 |       | 94±17 |       |
| (Jitter)                                       |          |       |           |       |
| 15. Transit time jitter \( \sigma_{TTV} \) (0.04 day) | 6.4±3 |       | 6.4±3 |       |

**Note.** The elements of the inner and outer orbits listed here are Jacobian osculating elements defined at the epoch BJD = 2454833 + 170. The quoted values in the “Solution” columns are the median and 68% credible interval of the marginal posteriors. Parentheses after values denote uncertainties in the last digit. The “combined” column shows the values from the marginal posterior combining the two solutions; no value is shown when the combined marginal posterior is multimodal. In the prior column, \( \mathcal{U}(a, b) \) and \( \mathcal{U}_{log}(a, b) \) denote the (log-) uniform priors between \( a \) and \( b \), \( f(x) = 1/(b - a) \) and \( f(x) = e^{-x}/(\ln b - \ln a) \), respectively; \( \mathcal{G}(a, b, c) \) means the asymmetric Gaussian prior with the central value \( a \) and lower and upper widths \( b \) and \( c \).

* Referenced to the ascending node of the inner orbit, whose direction is arbitrary.
B.2. Decomposition of the TTV Solutions
Using the Analytic Formula

The analytic formula allows us to understand how each physical effect in Equation (10) contributes to the observed TTVs. Figure 12 shows the decomposed signals for each of the (i) “LTTE” $\Delta_{\text{LTTE}}$, (ii) “tidal” $\delta_{\text{tidal}}$, (iii) “eccentric” $\delta_{\text{ecc1}} + \delta_{\text{ecc2}}$, and (iv) “non-coplanar” $\delta_{\text{noncopl}}$ terms for 10 solutions randomly sampled from the posterior obtained in the previous section. The plot shows that the $\delta_{\text{ecc}}$ terms play a crucial role in producing the short-term feature, especially for Kepler-448b.

Appendix C
Corner Plots for the Posteriors from Dynamical Analyses

Figures 13 and 14 show the corner plots of the posterior distributions obtained from the dynamical TTV and TDV
analyses in Sections 4 and 5. The figures are generated using corner.py by Foreman-Mackey (2016).

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Figure 14. Joint posterior distribution from the dynamical analysis of the TTVs and TDVs of Kepler-693b (Table 6 and Figure 5).
