Interacting Rényi holographic dark energy with parametrization on the interaction term

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Abstract

In the present work we study the Rényi holographic dark energy model (RHDE) in a flat FRW Universe where infrared cut-off is taken care by the Hubble horizon and also by taking three different parametrizations of the interaction term between the dark matter and the dark energy. Analysing graphically, the behaviour of some cosmological parameters in particular deceleration parameter, squared speed of sound and equation of state (EoS) parameter, in the process of the cosmic evolution, is found to be leading towards the late time accelerated expansion of RHDE model.

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1 Introduction

Our Universe is undergoing accelerated expansion which is marked by various cosmological observations like type-Ia supernova [1–4], the large-scale structure [5–8], cosmic microwave background (CMB) anisotropies [9–11]. For explaining this accelerated expansion of the cosmos the concept of dark energy (DE) was incorporated which is an extraordinary component with negative pressure [12, 13]. The late time acceleration of the Universe can be explained by two methods. First one is dynamical dark energy models in which we change the matter part of the Einstein field equation. Amongst a lot of theories and models the cosmological constant model is the simplest model, initially proposed by Einstein [14–18], which suggests that the equation of the state parameter (EoS) \( \omega = -1 \) and the cosmological constant are the most basic applicant for dark energy, and it is consistent with observations, besides the fine-tuning and coincidence problem [15, 19, 20]. To get relief from such problems, many dynamical DE models are given as an alternatives like k-essence [21], quintessence [22, 23], Chaplygin gas [24], phantom [25], tachyon [26], holographic dark energy (HDE) [27] and new agegraphic dark energy (NADE) [28]. One way to get feasible solution of the cosmic coincidence problem can be found by taking the interaction between the dark matter and the dark energy [29] and other methods are by \( f(R) \) theory [30], \( f(T) \) theory [31], Hořava-Lifshitz gravity [32, 33], Brans-Dicke theory [36], Gauss-Bonnet theory [37] and \( f(R, T) \) theory [38], which are obtained by changing the geometric part
of Einstein field equation.

The HDE has number of considerable features of the quantum gravity and has the traits of holographic principle \([39, 40]\), which states that degrees of freedom is dependent on bounding area instead of volume. The reason for flat FRW Universe was not known when HDE was considered in terms of Benkenstein entropy using infrared cut-off with the Hubble horizon \([41, 43]\). Physicists have taken various entropies with different cut-off scales like interaction between cold dark matter and dark energy or combination of the mentioned approaches \([44, 45]\).

In the literature \([27, 46–48]\), HDE model has been considered widely and examined as \(\rho_D \propto \Lambda^4\), while relation between the IR cutoff \(L\), UV cutoff \(\Lambda\) and the entropy \(S\) is \(\Lambda^3 L^3 \leq (S)^{\frac{3}{4}}\). So, the combination of the IR cut-offs with the entropy gives energy density of HDE model. The standard HDE model depends on Bekenstein-Hawking entropy \(S = \frac{A}{4G}\), where \(A = 4\pi L^2\), thus the density is \(\rho_D = \frac{3c^2}{8\pi G} L^{-2}\), where \(c\) is numerical constant. The focus must be on this declaration of \(\rho_D\) is achieve by consolidating the dimensional analysis and the holographic principle, rather than including a dark energy expression into the Lagrangian. Because of this extraordinary characteristic, HDE amazingly contrasts from some other theory of dark energy. The vacuum energy is associated with the UV cut-off and Ricci scalar, particle horizon, Hubble horizon, event horizon, etc. i.e. large scale structure of the Universe, is associated with the infrared (IR) cut-off. The HDE model endures the decision of IR cut-off problem. Numerous investigations of different (IR) cut-off’s has been done in Refs. \([43, 49–54]\).

Various entropies are used for the investigation the cosmological and gravitational incidence. The Tsallis HDE \([55]\), RényiHDE \([56]\) and Sharma-Mittal HDE \([57]\) are in demand and are extensively studied in literature. Differing from usual HDE model with Bekenstein entropy, such models give late time accelerated Universe. Rényi HDE depicts better stability as its own, in a non-interacting Universe \([56]\). It is stable and Tsallis HDE \([58]\) is never stable, if Sharma-Mittal HDE become dominant in the Universe. So the inferences shows that Rényi and Tsallis entropies can be obtained by Sharma-Mittal entropy \([59, 61]\). By considering the Hubble horizon as the IR cutoff, Tsallis HDE in Brans-Dicke cosmology have been studied \([62]\), which demonstrate that both non-interacting and interacting cases are classically unstable. Recently Tsallis agegraphic dark energy model along with pressure-less dust was examined by Zadeh et al. \([63]\) and they observed that these models are classically unstable and shows late time acceleration in non-interacting case. Investigation of Sharma-Mittal, Rényi and Tsallis HDE, models has been done in \([64]\) by taking Loop Quantum Cosmology in consideration. HDE models generate late time acceleration using infrared cut-off with the Hubble horizon when there is some interaction between dark energy and dark matter \([27, 65–68]\). It can give late time acceleration with matter dominated decelerated expansion in the past. This work comprises of the reconstruction of Rényi HDE from three different parametrizations of the the interaction term \(Q\) \([69]\). The interaction function \(Q\) is supposed to be proportional to \(H\rho_D\), where \(H\) is the Hubble parameter and \(\rho_D\) is the Rényi HDE density. The strength of the interaction depends on the proportionality parameter \(\alpha\). Praseetha and Mathew checked at the apparent and event horizon in interacting holographic models whether the second law of thermodynamics is valid \([70]\).

These works are behind our motivation for investigating the cosmological consequence of Rényi HDE model by using infrared cut-off with the Hubble horizon and also by taking three
different parametrizations of the interaction function $Q$, in the context of interacting flat FRW Universe. The organization of the paper is as follows: In sect. 2, we discuss field equations in flat FRW Universe. In sects. 3 we study RHDE Model. In sects. 4 we have calculated some cosmological parameters in the interacting RHDE model and in sect. 5, we have given the observational data used in the analysis of RHDE model. In sects. 6, 7 and 8, we analysed the Cosmological behaviour of the interacting RHDE For model 1, model 2 and model 3. Finally in the last section we concluded outcomes.

2 Field equations in flat FRW Universe

The metric for an isotropic and homogeneous spatial flat FRW Universe is given by:

$$ds^2 = -dt^2 + a^2(t)\left(dr^2 + r^2 d\Omega^2\right),$$

where $a(t)$ is known as the scale factor. The Hubble parameter is determined as, $H = \frac{\dot{a}}{a}$, where dot represents derivative with respect to cosmic time. The Friedmann equations, in the form of Hubble parameter are given as,

$$H^2 = \frac{1}{3}(8\pi G) \left(\rho_D + \rho_M\right),$$

where $\Omega_D = \frac{1}{3}M_p^{-2}\rho_D H^{-2}$ and $\Omega_m = \frac{1}{3}M_p^{-2}\rho_m H^{-2}$ are the energy density parameter of RHDE and pressure less matter, respectively, expressed as fractions of critical density $\rho_c = 3M_p^2H^2$. Also, $\rho_m$ and $\rho_D$ denote the energy density of matter and RHDE, respectively, and $\rho_m/\rho_D = r$ represents the energy density ratio of two dark components [71,72]. Now Eq. (2) can be written as:

$$1 = \Omega_D + \Omega_m,$$

The conservation law to interacting RHDE and matter are found as:

$$\dot{\rho}_m + 3H\rho_m = Q,$$

$$\dot{\rho}_D + 3H(\rho_D + p_D) = -Q,$$

Here $Q$ denotes the interaction function and $\omega_D = p_D/\rho_D$ gives the equation of state. Equations ( Eq. [4] and (Eq.[5])) become decoupled for $Q = 0$ permitting the autonomous conservation of dark matter and dark energy. In this study, we have taken three different parametrizations of the interaction function $Q$. The common form of the interaction function is taken to be $Q = 3\alpha(z) H \rho_D$, where $\alpha$ represents the coupling term which is non other than a function of redshift $z$. Now the coincidence parameter ($r$) is defined as $r = \rho_m/\rho_D$, which is constant in case of HDE in a spatially flat Universe with Hubble horizon as the IR cut-off [73]. From Eq. [3] we get Hubble parameter $H \propto (z + 1)^{\frac{3}{2}(1-\omega)}$ for a constant $\alpha$. In this case the model does not let the transition to go to accelerated phase from decelerated one. Hence for the successful change to accelerated phase from decelerated one we need $\alpha(z)$ which is a time-varying coupling parameter. In this study to reconstruct the interaction function $Q$, we have taken three different
ansatzes which is given in [69] as:

Model (I)

\[ \alpha(z) = \alpha_1 + \alpha_2(1 + z), \]  
(6)

Model (II)

\[ \alpha(z) = \alpha_1 + \alpha_2 \left( \frac{z}{1 + z} \right), \]  
(7)

Model (III)

\[ \alpha(z) = \alpha_1 + \alpha_2 \left( \frac{1}{1 + z} \right), \]  
(8)

Where \( \alpha_1 \) and \( \alpha_2 \) are constant parameters. Model I, II and III has a linear, mixed and inverse dependence on \( z \). So, aforementioned three models lead us to a pure CDM model after reduction, if \( \alpha_1 \) and \( \alpha_2 \) are taken as zero. Here we find two parameters \( \beta_1 = \frac{\alpha_1}{r} \) and \( \beta_2 = \frac{\alpha_2}{r} \), since for these three models \( r \) is constant. We also scale the Hubble constant (\( H_0 \)) by \( 100 km/sec^{-1} Mpc^{-1} \) for demonstrating it by \( h_0 \) which is a dimensionless way. The signature of the parameters \( \alpha_1 \) and \( \alpha_2 \) decides the path for the energy flow between the dark matter and the dark energy since the interaction function \( Q \) depends on the parameters \( \alpha_1 \) and \( \alpha_2 \). A negative \( Q \) shows the flow of energy from the dark matter to the dark energy and a positive \( Q \) shows the reverse.

3 Rényi Holographic Dark Energy Model

The form of the Bekenstein entropy of a system is \( S = \frac{A}{4} \), where \( A = 4\pi L^2 \) and \( L \) is the IR cut-off. Another modified form of the Rényi entropy [56] is given as:

\[ S = \frac{1}{\delta} \log \left( \frac{\delta}{4} A + 1 \right) = S = \frac{1}{\delta} \log \left( \pi \delta L^2 + 1 \right), \]  
(9)

Rényi HDE density, by considering the assumption \( \rho_d \, dV \propto T dS \), takes the following form:

\[ \rho_D = \frac{3c^2}{8\pi L^2} \left( \pi \delta L^2 + 1 \right)^{-1}, \]  
(10)

By taking Hubble horizon as an IR cut-off \( L = \frac{1}{H} \), we obtained:

\[ \rho_D = \frac{3c^2 H^2}{8\pi \left( \frac{\delta}{H^2} + 1 \right)^2}, \]  
(11)

where \( c^2 \) is a numerical constant as usual.
4 Evolution of cosmological parameters in the interacting RHDE model

Combined with the definition of $r$, we obtain:

$$r = \frac{1}{\Omega_D} - 1,$$  \hspace{1cm} (12)

Now, inserting the time derivative of Eq. (2) in Eq. (5), and combining the result with Eq. (3) and Eq. (4), we obtain

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}\Omega_D (\omega_D + r + 1),$$  \hspace{1cm} (13)

Now using Eq. (13) we get deceleration parameter $q$

$$q = \frac{3}{2}\Omega_D (\omega_D + r + 1) - 1,$$  \hspace{1cm} (14)

Combining the time derivative of Eq. (11) with Eq. (13), we get

$$\rho_D = -3H\rho_D \Omega_D \left(\frac{\pi\delta}{\pi\delta + H^2} + 1\right) (\omega_D + r + 1),$$  \hspace{1cm} (15)

Now substituting Eq. (19) in Eq.(5) and combining with Eq. (13). We get

$$\omega_D = \frac{H^2 (-r + 1)\Omega_D + \alpha(z) + 1 + \pi\delta (-2(r + 1)\Omega_D + \alpha(z) + 1)}{\pi\delta (2\Omega_D - 1) + H^2 (\Omega_D - 1)},$$  \hspace{1cm} (16)

Finally, we explore the stability of the RHDE model as:

$$v_s^2 = \frac{dp_D}{d\rho_D} = \frac{\rho_D}{\dot{\rho}_D} \dot{\omega}_D + \omega_D$$  \hspace{1cm} (17)

$$v_s^2 = \left(\frac{1}{3(\pi\delta + H^2)(2\pi\delta + H^2)((\alpha + 1)\Omega_D - 1)(\pi\delta (2\Omega_D - 1) + H^2 (\Omega_D - 1))}\right)$$

$$\times (\pi\delta H^6 (2(z + 1)\alpha' (\Omega_D - 1) (3\Omega_D - 2 - 3\alpha (c^2 - 7\Omega_D + 5)) ((\alpha + 1)\Omega_D - 1)) + \pi^2 \delta^2 H^4 ((z + 1)\alpha' ((13\Omega_D - 18) \Omega_D + 6) - 3 ((\alpha + 1)\Omega_D - 1) (3\alpha (c^2 - 5\Omega_D + 3) - c^2 + 3\Omega_D - 2))$$

$$+ \pi^3 \delta^3 H^2 (2(z + 1)\alpha' ((6\Omega_D - 7) \Omega_D + 2) - 3 ((\alpha + 1)\Omega_D - 1) (\alpha (2c^2 - 13\Omega_D + 7) - 2c^2 + 7\Omega_D - 4))$$

$$+ H^8 (\Omega_D - 1) (3\alpha ((\alpha + 1)\Omega_D - 1) + (z + 1)\alpha' (\Omega_D - 1)) + \pi^4 \delta^4 2\Omega_D - 1 (6(\alpha - 1) ((\alpha + 1)\Omega_D - 1))$$

$$+ (z + 1)\alpha' (2\Omega_D - 1))$$  \hspace{1cm} (18)

5 Observational Data

The present section deals with the observational data which was used to analyse the RHDE model with Hubble horizon cut-off. In the present analysis, the distance modulus measurements of type Ia supernova from the Joint Light-curve Analysis (JLA) \cite{74} and the observational measurements of Hubble parameter (OHD) have been used. Cosmic Chronometer method \cite{75}, measurements from galaxy distribution \cite{76} and from Lyman $-\alpha$ forest distribution \cite{77}.
Table 1: The parameters used in the models with JLA+OHD

| Model  | $h_0$ (±0.01) | $\beta_1$ (±0.005) | $\beta_2$ (±0.004) |
|--------|---------------|---------------------|---------------------|
| Model I | 0.696 ±0.007  | 0.942 ±0.066        | −0.304 ±0.035       |
| Model II| 0.700 ±0.008  | 0.737 ±0.042        | −0.906 ±0.102       |
| Model III| 0.700 ±0.008 | −0.170 ±0.004       | 0.907 ±0.103        |

Figure 1: The evolution of deceleration parameter ($q$) in RHDE model (I) versus red shift $z$ for different values of model parameter $\delta$ in flat Universe where $\alpha_1 = 0.41919$, $\alpha_2 = -0.135828$, $r = 0.445$, $H_0 = 69.6$.

methods are used to measure the OHD. Table 1 which represents the values of the parameters used with OHD+JLA in the analysis. The values of the model parameters $\beta_1$ and $\beta_2$ are scaled by $r$ which is the value of the coincident parameter. According to the Planck measurement of $\Omega_\Lambda$, the value of is $r$ is $0.445±0.010$. HDE in addition to Hubble scale cut-off in a spatially flat FRW Universe which possibly addresses the problem related to the coincidence problem of the standard model of cosmology.

6 Cosmological behaviour of the interacting RHDE For model 1

The deceleration parameter takes the form

$$q = \frac{3(\Omega_D (\alpha_1 + \alpha_2(z + 1) + 1) - 1) \left(\pi \delta + H_0^2 e^{-\frac{3\alpha_2 z}{r}} (z + 1)^{3(1-\alpha_1)}\right)}{2\pi \delta (2\Omega_D - 1) + 2H_0^2 (\Omega_D - 1) e^{-\frac{3\alpha_2 z}{r}} (z + 1)^{3(1-\alpha_1)}} - 1$$

(19)
Figure 2: The evolution of EOS parameter $\omega_D$ in RHDE model (I) versus red shift $z$ for different values of model parameter $\delta$ in flat Universe where $\alpha_1 = 0.41919$, $\alpha_2 = -0.135828$, $r = 0.445$, $H_0 = 69.6$.

Figure 3: The evolution of energy density parameter $\Omega_D$ in RHDE model (I) versus red shift $z$ for different values of model parameter $\delta$ in flat Universe where $\alpha_1 = 0.41919$, $\alpha_2 = -0.135828$, $r = 0.445$, $H_0 = 69.6$. 
The EOS parameter takes the form

$$\omega_D = \frac{(\alpha_1 + \alpha_2(z + 1)) \left( \pi \delta + H_0^2 e^{\frac{3\alpha_2 z}{r}} \frac{(z + 1)^3(1 - \frac{\Omega_D}{3})}{(\pi \delta e^{\frac{3\alpha_2 z}{r}} (z + 1)^3(1 - \frac{\Omega_D}{3}))} \right)}{\pi \delta (2\Omega_D - 1) + H_0^2 (\Omega_D - 1) e^{-\frac{3\alpha_2 z}{r}} (z + 1)^3(1 - \frac{\Omega_D}{3})}$$

(20)

The dark energy density parameter takes the form

$$\Omega_D = \frac{0.7 \left( \frac{\pi \delta}{H_0^2} + 1 \right)}{\pi \delta e^{\frac{3\alpha_2 z}{r}} (z + 1)^3(1 - \frac{\Omega_D}{3})} + 1$$

(21)

$$v_s^2 = \left( \frac{3(\pi \delta + H^2)[2\pi \delta + H^2]^{\frac{1}{2}}(\pi \delta (2\Omega_D - 1) + H^2 (\Omega_D - 1))^2(\Omega_D (\alpha_1 + \alpha_2 (z + 1) + 1))}{H^2} \right)$$

$$\times (\pi^4 \delta^4 (2\Omega_D - 1) (\alpha_2 (z + 1)) (2\Omega_D - 1) + 6 (\alpha_1 + \alpha_2 (z + 1)) (\Omega_D (\alpha_1 + \alpha_2 (z + 1) + 1) - 1))$$

$$+ \delta H^8 (\Omega_D - 1) (\alpha_2 (z + 1)) (\Omega_D - 1) + 3 (\alpha_1 + \alpha_2 (z + 1)) (\Omega_D (\alpha_1 + \alpha_2 (z + 1) + 1) - 1))$$

$$+ \pi \delta H^6 (2\alpha_2 (z + 1)) (\Omega_D - 1) (3\Omega_D - 2) - 3 (\alpha_1 + \alpha_2 (z + 1)) (c^2 - 7\Omega_D + 5)$$

$$\times \Omega_D (\alpha_1 + \alpha_2 (z + 1) + 1) - 1 + \pi^3 \delta^3 H^2 (-3 (-2c^2 + (\alpha_1 + \alpha_2 (z + 1)) - 4$$

$$\times (2c^2 - 13\Omega_D + 7) + 7\Omega_D) \Omega_D (\alpha_1 + \alpha_2 (z + 1) + 1) - 1 + 2\alpha_2 (z + 1) (\Omega_D (6\Omega_D - 7) + 2)$$

$$+ \pi^2 \delta^2 H^4 (-3 (3 (\alpha_1 + \alpha_2 (z + 1)) (c^2 - 5\Omega_D + 3) - c^2 + 3\Omega_D - 2)$$

$$\times (\Omega_D (\alpha_1 + \alpha_2 (z + 1) + 1) - 1) + \alpha_2 (z + 1) (\Omega_D (13\Omega_D - 18) + 6))$$

(22)

For analysis of RHDE models, model parameter $\delta$ have been taken three different values. fig. 1. depicts the behaviour of the deceleration parameter $q$ versus redshift $z$. It shows that $q$ changes it’s sign from positive to negative. Hence model 1 shows a transition from early
decelerated phase to present accelerating phase of the Universe. Fig. 2 shows the evaluation of the EoS parameter $\omega$ versus redshift $z$ for model I. Which depicts that EoS parameter $\omega$ varies from the quintessence era $\omega > -1$ to the phantom era $\omega < -1$ as time increases. Finally converges to quintessence era $\omega > -1$ at late time. Fig. 3 describe the behaviour of dark energy density parameter $\Omega_D$ with redshift $z$. We observe that $\Omega_D$ approaches to 1 at late time. Hence our model I predicts that for sufficiently large time the anistropy will vanish and Universe will become isotropic. So at late time the Universe will become flat. The squared speed of the sound $v_s^2$ of model I has been given by equation 22. It is plotted in fig. 4 versus redshift $z$, which depicts that the Rényi HDE model I with Hubble cutoff is classically stable initially for all model parameter $\delta$. Model I become unstable $v_s^2 < 0$ at different redshifts but sharply recoverse and presently it is stable $v_s^2 > 0$ for all model parameter $\delta$. It is also be noted that model I becomes unstable early for lower values of model parameter $\delta$.

7 Cosmological behaviour of the interacting RHDE For model 2

Similarly we obtain for model 2:

$$q = \frac{3 \left( \Omega_D \left( \alpha_1 + \frac{\alpha_2}{z+1} + 1 \right) - 1 \right) \left( \pi \delta + H_0^2 e^{\frac{3\alpha_2}{\alpha_1+1}} (z+1)^3 \left( \frac{-\alpha_1 - \alpha_2}{\alpha_1 + \alpha_2} + 1 \right) \right)}{2 \pi \delta (2 \Omega_D - 1) + 2 H_0^2 (\Omega_D - 1) e^{\frac{3\alpha_2}{\alpha_1+1}} (z+1)^3 \left( \frac{-\alpha_1 - \alpha_2}{\alpha_1 + \alpha_2} + 1 \right)} - 1$$

(23)

$$\omega_D = \frac{\left( \alpha_1 + \frac{\alpha_2}{z+1} \right) \left( \pi \delta + H_0^2 e^{\frac{3\alpha_2}{\alpha_1+1}} (z+1)^3 \left( \frac{-\alpha_1 - \alpha_2}{\alpha_1 + \alpha_2} + 1 \right) \right) - \pi \delta}{\pi \delta (2 \Omega_D - 1) + H_0^2 (\Omega_D - 1) e^{\frac{3\alpha_2}{\alpha_1+1}} (z+1)^3 \left( \frac{-\alpha_1 - \alpha_2}{\alpha_1 + \alpha_2} + 1 \right)}$$

(24)
Figure 6: The evolution of EOS parameter $\omega_D$ in RHDE model (II) versus red shift $z$ for different values of model parameter $\delta$ in flat Universe where $\alpha_1 = 0.327965$, $\alpha_2 = -0.40317$, $r = 0.445$, $H_0 = 70$.

Figure 7: The evolution of energy density parameter $\Omega_D$ in RHDE model (II) versus red shift $z$ for different values of model parameter $\delta$ in flat Universe where $\alpha_1 = 0.327965$, $\alpha_2 = -0.40317$, $r = 0.445$, $H_0 = 70$. 
Figure 8: The evolution of square of the sound speed parameter $v_s^2$ in RHDE model (II) versus red shift $z$ for different values of model parameter $\delta$ in flat FRW Universe where $\alpha_1 = 0.327965$, $\alpha_2 = -0.40317$, $r = 0.445$, $H_0 = 70$.

\[ \Omega_D = \frac{0.7 \left( \frac{\pi \delta}{H_0} + 1 \right)}{\frac{\pi \delta}{H_0} e^{-\frac{3\alpha_2}{z}(z+1)+3\left(-\frac{\alpha_1}{z} - \frac{\alpha_2}{z+1} + 1\right)}} + 1 \]  

\[ v_s^2 = \frac{\left( \frac{\pi \delta}{H_0} + 1 \right)}{\frac{3(\pi \delta + H^2)(2\pi \delta + H^2)(\pi \delta (\Omega_D - 1) + H^2(\Omega_D - 1))^2}{H_0^2} (\Omega_D (\alpha_1 + \frac{\alpha_2}{z+1} + 1) - 1)} \times \left( \pi^4 \delta^4 (2\Omega_D - 1) (z + 1) (2\Omega_D - 1) \left( \frac{\alpha_2}{z+1} - \frac{\alpha_2}{(z+1)^2} \right) + 6 \left( \alpha_1 + \frac{\alpha_2}{z+1} - 1 \right) \Omega_D \left( \alpha_1 + \frac{\alpha_2}{z+1} + 1 \right) - 1 \right) \right) \]

\[ + H^8 \left( \Omega_D - 1 \right) (z + 1) (\Omega_D - 1) \left( \frac{\alpha_2}{z+1} - \frac{\alpha_2}{(z+1)^2} \right) + 3 \left( \alpha_1 + \frac{\alpha_2}{z+1} \right) \Omega_D \left( \alpha_1 + \frac{\alpha_2}{z+1} + 1 \right) - 1 \right) \]

\[ + \pi \delta H^6 \left( 2(z + 1) (\Omega_D - 1) (3\Omega_D - 2) \left( \frac{\alpha_2}{z+1} - \frac{\alpha_2}{(z+1)^2} \right) - 3 \left( \alpha_1 + \frac{\alpha_2}{z+1} \right) (c^2 - 7\Omega_D + 5) \right) \times \left( \Omega_D \left( \alpha_1 + \frac{\alpha_2}{z+1} + 1 \right) - 1 \right) + 2(z + 1) (\Omega_D (6\Omega_D - 7) + 2) \left( \frac{\alpha_2}{z+1} - \frac{\alpha_2}{(z+1)^2} \right) \]

\[ + \pi^2 \delta^2 H^4 \left( -3 \left( \alpha_1 + \frac{\alpha_2}{z+1} \right) (c^2 - 5\Omega_D + 3) - c^2 + 3\Omega_D - 2 \right) \]

\[ \times \left( \Omega_D \left( \alpha_1 + \frac{\alpha_2}{z+1} + 1 \right) - 1 \right) + (z + 1) (\Omega_D (13\Omega_D - 18) + 6) \left( \frac{\alpha_2}{z+1} - \frac{\alpha_2}{(z+1)^2} \right) \right) \]  

In model II, $q$ the deceleration parameter is plotted as function $z$ in fig. 5 by considering three different values of model parameter $\delta$. It also shows that $q$ goes towards south from from positive to negative region which depicts the transition of the Universe from early decelerated phase to present accelerating phase. Presently model II is more inflating in comparison to model I. Fig. 6 shows the evaluation of of the EoS parameter $\omega$ versus redshift $z$ for model II. Which
\( q = \frac{3 (\Omega_D (\alpha_1 + \frac{\alpha_2}{z+1} + 1) - 1) \left( \pi \delta + H_0^2 e^{-\frac{3\alpha_2}{r(z+1)}} (z+1)^3(1-\frac{\alpha_1}{r}) \right)}{2\pi \delta (2\Omega_D - 1) + 2H_0^2 (\Omega_D - 1) e^{-\frac{3\alpha_2}{r(z+1)}} (z+1)^3(1-\frac{\alpha_1}{r})} - 1 \) \hspace{1cm} (27)

\( \omega_D = \frac{(\alpha_1 + \frac{\alpha_2}{z+1}) \left( \pi \delta + H_0^2 e^{-\frac{3\alpha_2}{r(z+1)}} (z+1)^3(1-\frac{\alpha_1}{r}) \right) - \pi \delta}{\pi \delta (2\Omega_D - 1) + H_0^2 (\Omega_D - 1) e^{-\frac{3\alpha_2}{r(z+1)}} (z+1)^3(1-\frac{\alpha_1}{r})} \) \hspace{1cm} (28)

\( \Omega_D = \frac{0.7 \left( \frac{\pi \delta}{H_0^2} + 1 \right)}{\pi \delta e^{-\frac{3\alpha_2}{r(z+1)}} (z+1)^3(1-\frac{\alpha_1}{r}) H_0^2} + 1 \) \hspace{1cm} (29)

Figure 9: The evolution of deceleration parameter \( (q) \) in RHDE model (III) versus red shift \( z \) for different values of model parameter \( \delta \) in flat Universe where \( \alpha_1 = -0.07565, \alpha_2 = 0.403615, r = 0.445, H_0 = 70 \).

depicts that EoS parameter \( \omega \) varies from the quintessence era \( \omega > -1 \) to the phantom era \( \omega < -1 \) as time increases. Model II always lies in phantom era \( \omega < -1 \) once it crosses the phantom divided line \( \omega = -1 \). From fig. 7 and fig. 8, we observe that behaviour of square of the sound speed parameter \( v_s^2 \) in RHDE model (II) versus red shift \( z \) and energy density parameter \( \Omega_D \) in RHDE model (II) versus red shift \( z \) for different values of model parameter \( \delta \) in flat FRW Universe for \( \alpha_1 = -0.327965, \alpha_2 = -0.40317, r = 0.445 \) and \( H_0 = 70 \) is same as of model I.

8 Cosmological behaviour of the interacting RHDE For model 3

We obtain for model 3:

\( q = \frac{3 (\Omega_D (\alpha_1 + \frac{\alpha_2}{z+1} + 1) - 1) \left( \pi \delta + H_0^2 e^{-\frac{3\alpha_2}{r(z+1)}} (z+1)^3(1-\frac{\alpha_1}{r}) \right)}{2\pi \delta (2\Omega_D - 1) + 2H_0^2 (\Omega_D - 1) e^{-\frac{3\alpha_2}{r(z+1)}} (z+1)^3(1-\frac{\alpha_1}{r})} - 1 \) \hspace{1cm} (27)

\( \omega_D = \frac{(\alpha_1 + \frac{\alpha_2}{z+1}) \left( \pi \delta + H_0^2 e^{-\frac{3\alpha_2}{r(z+1)}} (z+1)^3(1-\frac{\alpha_1}{r}) \right) - \pi \delta}{\pi \delta (2\Omega_D - 1) + H_0^2 (\Omega_D - 1) e^{-\frac{3\alpha_2}{r(z+1)}} (z+1)^3(1-\frac{\alpha_1}{r})} \) \hspace{1cm} (28)

\( \Omega_D = \frac{0.7 \left( \frac{\pi \delta}{H_0^2} + 1 \right)}{\pi \delta e^{-\frac{3\alpha_2}{r(z+1)}} (z+1)^3(1-\frac{\alpha_1}{r}) H_0^2} + 1 \) \hspace{1cm} (29)
Figure 10: The evolution of EOS parameter $\omega_D$ in RHDE model (III) versus red shift $z$ for different values of model parameter $\delta$ in flat Universe where $\alpha_1 = -0.07565$, $\alpha_2 = 0.403615$, $r = 0.445$, $H_0 = 70$.

Figure 11: The evolution of energy density parameter $\Omega_D$ in RHDE model (III) versus red shift $z$ for different values of model parameter $\delta$ in flat Universe where $\alpha_1 = -0.07565$, $\alpha_2 = 0.403615$, $r = 0.445$, $H_0 = 70$. 

\[ v_s^2 = \left( \frac{3(\pi \delta + H^2)(2\pi \delta + H^2)(\pi \delta(2\Omega_D - 1) + H^2(\Omega_D - 1))^2(\Omega_D(\alpha_1 + \frac{\alpha_2}{z+1} + 1) - 1)}{\Omega^6} \right) \]
\[ \times \left( 4^4 \delta^4 (2\Omega_D - 1) \left[ 6 \left( \alpha_1 + \frac{\alpha_2}{z+1} + 1 \right) \left( \Omega_D(\alpha_1 + \frac{\alpha_2}{z+1} + 1) - 1 \right) - \frac{\alpha_2(2\Omega_D - 1)}{\left( \frac{\alpha_2}{z+1} - 1 \right)} \right] \right) \]
\[ + H^8 \left( \Omega_D - 1 \right) \left[ 3 \left( \alpha_1 + \frac{\alpha_2}{z+1} \right) \Omega_D \left( \alpha_1 + \frac{\alpha_2}{z+1} + 1 \right) - 1 - \frac{\alpha_2(\Omega_D - 1)}{\left( \frac{\alpha_2}{z+1} - 1 \right)} \right] \]
\[ + \pi \delta H^6 \left( -3 \left( \alpha_1 + \frac{\alpha_2}{z+1} \right) \left( e^2 - 7\Omega_D + 5 \right) - \frac{2\alpha_2(\Omega_D - 1)(3\Omega_D - 2)}{\left( \frac{\alpha_2}{z+1} - 1 \right)} \right) \]
\[ \times \Omega_D \left( \alpha_1 + \frac{\alpha_2}{z+1} + 1 \right) \left( 2e^2 - 13\Omega_D + 7 \right) - 2e^2 + 7\Omega_D - 4 \]
\[ \times \left( \Omega_D \left( \alpha_1 + \frac{\alpha_2}{z+1} + 1 \right) - 1 - \frac{2\alpha_2(\Omega_D(6\Omega_D - 7))}{\left( \frac{\alpha_2}{z+1} + 1 \right)} \right) + \pi^2 \delta^6 H^4 \left( -3 \left( e^2 - 3 \left( \alpha_1 + \frac{\alpha_2}{z+1} \right) \right) - 2 \right) \]
\[ \times \left( e^2 - 5\Omega_D + 3 \right) + 3\Omega_D \right) \Omega_D \left( \alpha_1 + \frac{\alpha_2}{z+1} + 1 \right) - 1 - \frac{\alpha_2(\Omega_D(13\Omega_D - 18) + 6)}{\left( \frac{\alpha_2}{z+1} + 1 \right)} \right) \]

\[ (30) \]

Figure 12: The evolution of square of the sound speed parameter \( v_s^2 \) in RHDE model (III) versus red shift \( z \) for different values of model parameter \( \delta \) in flat Universe where \( \alpha_1 = -0.07565 \), \( \alpha_2 = 0.403615 \), \( r = 0.445 \), \( H_0 = 70 \).

\[ \delta = -1000 \]
\[ \delta = -1200 \]
\[ \delta = -1400 \]

\[ \text{z vs } v_s^2 \]

9 Conclusion

This work comprises of the study of the RHDE model where Hubble horizon is taken as the infrared cut-off by taking three different parametrizations of the interaction term in the context of flat FRW Universe. Three different values of the model parameter \( \delta \) are taken for non-linear interaction of dark matter and dark energy models. Following are results which we obtained on the basis of the graphical analysis:

* The sign of deceleration parameter \( q \) indicates whether the model inflates or not. The deceleration parameter \( q \) of all three models decreases from positive to negative region. Which
shows a transition from early decelerated phase to present accelerating phase of the Universe. Presently model II and model III are more inflating in comparison to model I.

* The trajectories of the EOS parameter $\Omega_D$ for RHDE model I behave like quintessence for all model parameter $\delta$, while model II and model III shows an aggressive phantom regime for Hubble horizon as IR cutoff at late time.

* We observe that dark energy density parameter $\Omega_D$ approaches to 1 at late time for all the three model I, II and III.

* The graphical behaviour of the squared speed of sound are used to analyse the stability of the RHDE models. We have noticed that the Rényi HDE models I, II and III with Hubble horizon as IR cutoff are classically stable initially for all model parameter $\delta$. These becomes unstable $v_s^2 < 0$ at different redshifts but sharply recovers and presently all models are stable $v_s^2 > 0$ for all model parameter $\delta$.

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