Study on $B_s \to D_{sJ}(2317, 2460)\ell \bar{\nu}$ Semileptonic Decays in the CQM Model

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Assuming $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ to be the $(0^+, 1^+)$ chiral partners of regular $D_s(1968)$ and $D_s^*(2112)$, we calculate the semileptonic decays of $B_s$ to $D_s(1968)$, $D_s^*(2112)$, $D_{sJ}(2317)$, $D_{sJ}(2460)$ in terms of the Constituent Quark Meson (CQM) model. The large branching ratios of the semileptonic decays of $B_s$ to $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ indicate that those two semileptonic decays should be seen in future experiments.

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I. INTRODUCTION

The discoveries of exotic mesons $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ [1, 2, 3], whose spin-parity structure are respectively $0^+$ and $1^+$, have attracted great interests of both theorists and experimentalists of high energy physics. $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ are supposed to be $(0^+, 1^+)$ chiral partners of $D_s$ and $D_s^*$ [4], i.e., p-wave excited states of $D_s$ and $D_s^*$. Beveren and Rupp suggested that $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ are made up of $c$ and $\bar{s}$ by studying the mass spectra [5]. With the QCD spectral sum rules, Narison calculated the masses of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ by assuming them as quark-antiquark states and obtained results which are consistent with the experiment data within a wide error range [6]. Very recently, considering the contribution of DK continuum in QCD sum rules, Dai et al. obtained the mass of $D_{sJ}^*(2317)$ which is consistent with the mass given by the experiments [8]. Meanwhile, some authors suggested that $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ may be of four-quark structure [5, 7, 11, 12]. Thus one needs to use various theoretical approaches to clarify the mist of the structures of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$. The studies of the productions and decays of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ are very interesting topics.

The semileptonic decays of $B_s$ are one of the ideal platforms to study the productions of $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$. Especially the Large Hadron Collider (LHC) will be run in the coming 2007, which can produce large amounts of data of $B_s$. Thus the measurements of $B_s \to D_{sJ}(2317, 2460)\ell \bar{\nu}$ would be realistic. In Ref. [12], author calculated $B_s \to D_{sJ}(2317, 2460)\ell \bar{\nu}$ decays by the QCD sum rules in HQET. Recently, authors of Ref. [13, 14] completed the calculations of $B_s \to D_{sJ}(2317, 2460)\ell \bar{\nu}$ semileptonic decays in the QCD sum rules and obtained large branching ratios. However, the results obtained by Ref. [13, 14] are one order smaller than those given by Ref. [12]. Thus studies on $B_s \to D_{sJ}(2317, 2460)\ell \bar{\nu}$ with other plausible models would be helpful. It not only deepen our understanding about the properties of these states but also test the reliability of models which are applied to calculate the semileptonic decays.

In this work, we study the $B_s \to D_s(1968)\ell \bar{\nu}$, $B_s \to D_s^*(2112)\ell \bar{\nu}$ and $B_s \to D_{sJ}(2317, 2460)\ell \bar{\nu}$ semileptonic decays in terms of the Constituent Quark Meson (CQM) model. In order to complete the calculations of the semileptonic decays of $B_s$, we firstly base on the assumption that $(D_s(1968), D_s^*(2112))$ with spin-parity $(0^-, 1^-)$ and $(D_{sJ}(2317), D_{sJ}(2460))$ with spin-parity $(0^+, 1^+)$ can be respectively categorized as $H(0^-, 1^-)$ and $S(0^+, 1^+)$ doublets in HQET.

CQM model was proposed by Polosa et al. [15] and has been well developed later based on the work of Ebert et al. [16] (See the Ref. [15] for a review). The model is based on an effective Lagrangian which incorporates the flavor-spin symmetry for heavy quarks with the chiral symmetry for light quarks. Employing the CQM model to study the phenomenology of heavy meson physics, reasonable results have been achieved [17, 18]. Therefore, we believe that the model is applicable to our processes and expect to get relatively reliable conclusion.

This paper is organized as follow: After the introduction, in Sect. II, we formulate the semileptonic decays of $B_s$ to $D_s(1968)$, $D_s^*(2112)$, $D_{sJ}(2317)$ and $D_{sJ}(2460)$. The numerical results along with all the input parameters are presented in Sect III. Section IV is devoted to the discussion and the conclusion. Some detailed expressions are collected in the appendix.

II. FORMULATION

For the convenience of readers, we give a brief introduction of the CQM model [16]. The model is relativistic and based on an effective Lagrangian which combines the
HQET and the chiral symmetry for light quarks

\[ \mathcal{L}_{CQM} = \bar{x}[\gamma \cdot (i\partial + V)]x + \bar{x} \cdot A_\gamma \gamma x - m_q \bar{x} x \]

\[
+ \frac{f_s^2}{8} \text{Tr}[\partial^\mu \Sigma^\nu \partial_\nu \Sigma^\mu] + h_v(i\nu \cdot \partial)h_v
\]

\[-[\bar{x}(H + S + \frac{i\tilde{T}_\mu S_\mu}{\Lambda}h_v + \text{h.c.})]
\]

\[
+ \frac{1}{2G_3} \text{Tr}[(\bar{H} + S)(H - S)] + \frac{1}{2G_4} \text{Tr}[\tilde{T}_\mu T_\mu].
\]

where the fifth term is the kinetic term of heavy quarks with \( \bar{h}_v = h_v \) and \( H \) and \( S \) denote the super-fields corresponding to doublets \((0^-,1^-)\) and \((0^+,1^+)\) respectively. The explicit matrix representations of \( H \) and \( S \) read as \[ H = \frac{1 + \not{p} - \not{P}_\mu \gamma_\mu - P_5}{2}, \]

\[ S = \frac{1 + \not{p} - \not{P}_1 \gamma_5 + P_0}{2}, \]

where \( P, P_\mu, P_0 \) and \( P_1 \) are the annihilation operators of pseudoscalar, vector, scalar and axial vector mesons which are normalized as

\[
(0|P|0) = \sqrt{M_H}, \quad (0|P_\mu |0) = \sqrt{M_H} \not{e}_\mu, \\
(0|P_0 |0) = \sqrt{M_S} \gamma_5, \quad (0|P_1 |0) = \sqrt{M_S} \gamma_5 \not{e}. \]

\( T \) is the super-field corresponding to the doublet \((1^+, 2^+)\)

\[
T^\mu = \frac{1 + \not{p}}{2\sqrt{2}} \left[ \frac{P_\mu^\nu \gamma_\nu}{2} - \frac{1}{2} \gamma^\nu \not{e}_\nu - \frac{1}{2} \gamma^\nu \not{e}_\nu (\gamma^\mu - \gamma^\nu) \right].
\]

\[ \chi = \xi(q = u, d, s) \) is the light quark field and \( \xi = e_i \), and \( M \) is the octet pseudoscalar matrix. We also have

\[ V^\mu = \frac{1}{2} \left( \xi^i \partial^\mu \xi + \xi \partial^\mu \xi^i \right), \]

\[ A^\mu = \frac{-i}{2} \left( \xi^i \partial^\mu \xi + \xi \partial^\mu \xi^i \right). \]

Because the spin-parity of \( D_s(1968) \) and \( D_s^*(2112) \) are respectively \( 0^- \) and \( 1^- \), whereas \( D_s^*(2317) \) and \( D_{sJ}(2460) \) belong to the \( S \)-type doublet \((0^+, 1^-)\). Thus we can calculate the semileptonic decays of \( B_s \) to \( D_s(1968), D_s^*(2112), D_s^{*(s)}(2317) \) and \( D_{sJ}(2460) \).

**A. The calculations of \( B_s \to D_s(1968) \) and \( B_s \to D_s^*(2112) \) in the CQM model.**

The four fermion operator of \( b \to c + \bar{\nu} \bar{\nu} \) which is relevant to the semileptonic decays of \( B_s \) to \( D_s \) mesons reads as \[ \mathcal{O} = \frac{G_F V_{cb}}{\sqrt{2}} \bar{c} \gamma^\mu (1 - \gamma_5) b \bar{\nu} \gamma_\mu (1 - \gamma_5) \bar{\nu}. \]

The transition amplitudes of \( B_s \to D_s(1968) \) and \( B_s \to D_s^*(2112) \) can be written as

\[ \mathcal{M} = \langle D_s^{(*)} | \mathcal{O} | B_s \rangle \]

\[
= \frac{G_F V_{cb}}{\sqrt{2}} \langle D_s^{(*)} | \bar{c} \gamma^\mu (1 - \gamma_5) b | B_s \rangle \langle l \bar{\nu} \gamma_\mu (1 - \gamma_5) | l \bar{\nu} \rangle,
\]

where the hadronic matrix element is related to non-perturbative QCD effects. In the HQET symmetries, the hadronic matrix element can be expressed with the following form

\[ \langle D_s(v') | \bar{c} \gamma_\mu (1 - \gamma_5) b | D_s(v) \rangle \]

\[ = \sqrt{M_{B_s} M_{D_s}} \langle v'_\mu + v_\mu \rangle \xi(\omega), \]

\[ \langle D_s^{(*)}(v', \epsilon') | \bar{c} \gamma_\mu (1 - \gamma_5) b | D_s(v) \rangle \]

\[ = \sqrt{M_{B_s} M_{D_s}} \langle \epsilon_\mu (1 - \gamma_5) b | D_s(v) \rangle \]

\[ + \langle \epsilon'_\mu (v' | v_\mu \rangle \xi(\omega), \]

where \( \omega = v \cdot v' \). In HQET, \( \xi(\omega) \) is the Isgur-Wise function which is a dimensionless probability function. In the following, the central task is how to extract the Isgur-Wise function in the calculation of the CQM model.

In the CQM model, the Feynman diagram corresponding to the hadronic matrix element \( \langle D_s^{(*)} | \bar{c} \gamma^\mu (1 - \gamma_5) b | B_s \rangle \) can be depicted in Fig. 1.

![FIG. 1: The Feynman diagrams depict the decays of \( B_s \to D_s^{(*)} \) l\( \bar{\nu} \). The thick-line denotes the heavy quark propagator.](image)

According to the CQM model [13], the couplings of \( D_s(1968), D_s^*(2112) \) and \( B_s \) with light and heavy quarks are expressed as

\[ \frac{1 + \not{p}}{2} \sqrt{Z_H M_{D_s}} \gamma_5, \]

\[ \frac{1 + \not{p}}{2} \sqrt{Z_H M_{D_s}} \not{f}, \]

\[ \frac{1 + \not{p}}{2} \sqrt{Z_H M_{B_s}} \gamma_5. \]

where \( \not{e} \) denotes the polarization vector of \( D_s^*(2112) \). The concrete expression of \( Z_H \) is given in [18] as

\[ Z_H^{-1} = (\Delta_H + m_s) \frac{\partial \mathcal{I}_3(\Delta_H)}{\partial \Delta_H} + \mathcal{I}_3(\Delta_H), \]

\[ \mathcal{I}_3(a) = \frac{i N_c}{16 \pi^2} \int_{1/\Lambda^2}^{1/m_s^2} \frac{dy}{\sqrt{y}} \text{exp}[-y(m_s^2 - a^2)] \]

\[ \times (1 + \text{erf}(a \sqrt{y})), \]
where erf is the error function. \( m_s \) is the mass of the \( s \) quark.

Now we can write out the hadronic transition matrix element

\[
\langle D_s^{(*)}|\bar{c}_\gamma\mu(1-\gamma_5)b|B_s \rangle \\
\frac{(i^5)}{4}\sqrt{M_{B_s}M_{D_s^{(*)}}}\ Z_H N_c \\
\times \int d^4k \ \text{Tr}[(k+m)\Gamma(1+\gamma\nu(1+\gamma)^{-1})(2\pi)^4\left(k^2-m_\omega^2\right)(k^\nu\cdot k + \Delta_H)(k^\nu \cdot k + \Delta_H)]
\]

(15)

with \( N_c = 3 \), where \( \Gamma \) should be taken as \( \gamma_5 \) and \( \ell \) corresponding to \( D_s \) and \( D_s^{(*)} \) respectively.

One omits the technical details in the text for saving space and finally gets the Isgur-Wise function \( \xi(\omega) \) by comparing the results of eqs. (15) to eqs. (8) and (9)

\[
\xi(\omega) = \frac{Z_H}{1 + \omega} \left[ \frac{2}{\omega} I_3(\Delta_H) + \left( \frac{m_s + 2\Delta_H}{1 + \omega} \right) I_5(\Delta_H, \Delta_H, \omega) \right]
\]

(16)

where the definitions of \( I_{1,5} \) are listed in the appendix.

By eqs. (7) - (9), we obtain the explicit expressions of the semileptonic decays \( B_s \rightarrow D_s(1688)\bar{l}\bar{v} \) and \( B_s \rightarrow D_s^{(*)}(2112)\bar{l}\bar{v} \).

\[
d\Gamma(B_s \rightarrow D_s(1688)\bar{l}\bar{v}) = \frac{G_F^2|V_{cb}|^2}{48\pi^3} (M_{B_s} + M_{D_s})^2 M_{D_s^{(*)}}^3 (\omega^2 - 1)^{3/2} \xi^2(\omega) d\omega,
\]

\[
d\Gamma(B_s \rightarrow D_s^{(*)}(2112)\bar{l}\bar{v}) = \frac{G_F^2|V_{cb}|^2}{48\pi^3} (M_{B_s} - M_{D_s})^2 M_{D_s^{(*)}}^3 \sqrt{\omega^2 - 1(\omega + 1)^2}
\]

\[
\left[ 1 + \frac{4\omega}{(1 + \omega)} \left( M_{B_s} - M_{D_s} \right)^2 \right] \xi(\omega)^2 d\omega,
\]

(17)

where \( q^2 = M_B^2 + M_D^2 - 2M_B M_D \).

B. The calculations of \( B_s \rightarrow D_s^{(*)}(2317)\bar{l}\bar{v} \) and \( B_s \rightarrow D_{s,1}(2460)\bar{l}\bar{v} \) in the CQM model

The hadronic matrix elements of the decays \( B_s \rightarrow D_s^{(*)}(2317)\bar{l}\bar{v} \) and \( B_s \rightarrow D_{s,1}(2460)\bar{l}\bar{v} \) in HQET read as [21]

\[
\langle D_s^{(*)}(2317)|\bar{c}_\gamma\mu(1-\gamma_5)b|B_s \rangle
\]

\[
= 2\sqrt{M_{B_s}M_{D_s^{(*)}}(2317)(\omega^2 - 1)^{3/2} \xi^2(\omega),
\]

(18)

\[
\langle D_{s,1}(2460)(\omega', \nu')|\bar{c}_\gamma\mu(1-\gamma_5)b|B_s \rangle
\]

\[
= \sqrt{M_{B_s}M_{D_{s,1}(2460)}} \left[ 2\epsilon^{\mu\rho\sigma\beta}\epsilon^{*\alpha\nu\beta}\nu^\gamma
\right.
\]

\[
\left. + 2[(1 - \omega)\epsilon^{\mu\nu}_{s}\epsilon^{*\alpha} + (\epsilon^{*\cdot} \cdot \nu')_{s}] \right] \xi(\omega),
\]

(19)

where \( \xi(\omega) \) denote the the Isgur-Wise function.

We use the same treatment in subsection A to get \( \zeta(\omega) \). In the CQM model, the couplings of \( D_s^{(*)}(2317)(0^+) \) and \( D_{s,1}(2460)(1^+) \) with light and heavy quarks are respectively

\[
\frac{1 + \frac{\gamma}{2}}{2} \sqrt{Z_SM_{D_s^{(*)}(2317)}},
\]

(20)

\[
\frac{1 + \frac{\gamma}{2}}{2} \sqrt{Z_SM_{D_{s,1}(2460)}(\ell\gamma_5)}
\]

(21)

with \( \zeta_s^{-1} = (\Delta_S + m_s)\frac{\partial \sqrt{Z_S}}{\partial \Delta_S} + \sqrt{Z_S} \).

(22)

where \( \epsilon' \) is the polarization vector of \( D_{s,1}(2460) \).

Thus, in the CQM model, the hadronic transition matrix elements of \( \langle D_{s,1}(2317)|\bar{c}_\gamma\mu(1-\gamma_5)b|B_s(v) \rangle \) and \( \langle D_{s,1}(2460)|\bar{c}_\gamma\mu(1-\gamma_5)b|B_s(v) \rangle \) can be obtained by replacing \( \Gamma \) in eq. (15) with 1 and \( \ell\gamma_5 \) respectively. Meanwhile, \( Z_H \) should be replaced by \( \sqrt{Z_HZ_S} \).

Finally we get the Isgur-Wise function \( \xi(\omega) \) with the following form

\[
\xi(\omega) = \frac{\sqrt{Z_HZ_S}}{2(1 - \omega)} I_3(\Delta_S) - I_5(\Delta_H)
\]

\[
+ (\Delta_H - \Delta_S + m_s(1 - \omega)) I_5(\Delta_H, \Delta_S).
\]

(23)

Using eqs. (7), (18) and (19), we deduce the decay widths of \( B_s \rightarrow D_s^{(*)}(2317)l\bar{v} \) and \( B_s \rightarrow D_{s,1}(2460)l\bar{v} \)

\[
d\Gamma(B_s \rightarrow D_s^{(*)}(2317)l\bar{v})
\]

\[
= \frac{G_F^2|V_{cb}|^2}{12\pi^3} M_{D_s^{(*)}(2317)}^3 \left( M_{B_s} - M_{D_s^{(*)}(2317)} \right)^2
\]

\[
\times (\omega^2 - 1)^{3/2} \xi^2(\omega) d\omega,
\]

(24)

\[
d\Gamma(B_s \rightarrow D_{s,1}(2460)l\bar{v})
\]

\[
= \frac{G_F^2|V_{cb}|^2}{12\pi^3} M_{D_{s,1}(2460)}^3 \left( M_{B_s}^2 + M_{D_{s,1}(2460)}^2 \right)^{(5\omega^2 - 6\omega + 1)}
\]

\[
- 2M_{B_s}M_{D_{s,1}(2460)}(4\omega^3 - 5\omega^2 + 2\omega - 1) d\omega.
\]

(25)

III. NUMERICAL RESULTS

The semileptonic decays \( B^+ \rightarrow D^{(*)}l^+\nu \) and \( B^0 \rightarrow D^{(*)}l^-\nu \) are measured well [22]. The results calculated by the CQM model [17] and measured by the experiments are collected in Table I. By comparing the theoretical results with that measured by the experiments, one believes that the CQM model is applicable to our processes and expects to get relatively reliable result.
TABLE I: The numerical results are taken from Ref. [15].

| $BR[B^+ \to D^0 l^+ \bar{\nu}]$ | CQM model [15] | Experiment [22] |
|---------------------------------|-----------------|-----------------|
| (2.2±3.0)\%                    | (2.15 ± 0.22)\% |
| $BR[B^0 \to D^- l^+ \bar{\nu}]$ | (2.12 ± 0.20)\% |
| $BR[B^+ \to D^{*0} l^+ \bar{\nu}]$ | (5.9±7.6)\%     |
| $BR[B^0 \to D^{*-} l^+ \bar{\nu}]$ | (5.35 ± 0.20)\% |

With the formulation derived from the last section, one numerically evaluate the corresponding decay rates.

In Fig. 2, we show the dependence of Isgur-Wise function $\zeta(\omega)$ on $\omega$ in $B \to D^{(*)} l \bar{\nu}$ and $B_s \to D_s^{(*)} l \bar{\nu}$ decays. The dependence of $\zeta(\omega)$ on $\omega$ is shown in Fig. 3.

For a comparison, we also put the values of the branching ratios of $B_s \to D_{sJ}(2317) l \bar{\nu}$ and $B_s \to D_{sJ}(2460) l \bar{\nu}$, which are calculated in Ref. [13]-[14], in Table I.

| $\Delta_H$ (GeV) | $\Delta_S$ (GeV) | $Z_H$ (GeV)$^{-1}$ | $Z_S$ (GeV)$^{-1}$ | $BR_1$ | $BR_2$ | $BR_3$ | $BR_4$ |
|------------------|------------------|-------------------|-------------------|--------|--------|--------|--------|
| 0.5              | 0.86             | 3.99              | 2.02              | 2.95%  | 7.66%  | 5.71 x 10^{-3} | 8.69 x 10^{-3} |
| 0.6              | 0.91             | 2.69              | 1.47              | 2.86%  | 7.53%  | 5.25 x 10^{-3} | 7.91 x 10^{-3} |
| 0.7              | 0.97             | 1.74              | 0.98              | 2.73%  | 7.49%  | 4.90 x 10^{-3} | 7.52 x 10^{-3} |

We present the branching ratios of $B_s \to D_s(1968) l \bar{\nu}$, $B_s \to D_s^{*}(2112) l \bar{\nu}$, $B_s \to D_{sJ}(2317) l \bar{\nu}$ and $B_s \to D_{sJ}(2460) l \bar{\nu}$ in Table III.

Table III: In this table, we list our results of the semileptonic decays of $B_s$ to $D_s(1968)$, $D_s^{*}(2112)$, $D_{sJ}(2317)$ and $D_{sJ}(2460)$ and that obtained by other approaches [12]-[14].

| processes                  | CQM    | QSR in HQET ($m_Q \to \infty$) [12] | QSR                  |
|----------------------------|--------|-----------------------------------|----------------------|
| $BR[B_s \to D_s(1968) l \bar{\nu}]$ | (2.73 ± 3.00)% | -                                 | -                    |
| $BR[B_s \to D_s^{*}(2112) l \bar{\nu}]$ | (7.49 ± 7.66)% | -                                 | -                    |
| $BR[B_s \to D_{sJ}(2317) l \bar{\nu}]$ | (4.90 ± 5.71) x 10^{-3} | 0.09 | $\sim 10^{-3}$ [13] |
| $BR[B_s \to D_{sJ}(2460) l \bar{\nu}]$ | (7.52 ± 8.69) x 10^{-3} | 0.08 | 4.9 x 10^{-3} [14] |

IV. DISCUSSION AND CONCLUSION

By the comparison between the theoretical results and experimental results listed in Table I, we have reason to believe that the CQM model can be applied well to study these semileptonic decays related to this work. In this work, we study the semileptonic decays of $B_s$ to $D_s(1968)$, $D_s^{*}(2112)$, $D_{sJ}(2317)$ and $D_{sJ}(2460)$. In the

The input parameters include $G_F = 1.1664 \times 10^{-5}$ GeV$^{-2}$, $V_{cb} = 0.043$, $M_B = 5.3696$ GeV, $M_{D_s} = 1.968$ GeV, $M_{D_s^{(*)}} = 2.112$ GeV, $M_{D_{sJ}(2317)} = 2.317$ GeV, $M_{D_{sJ}(2460)} = 2.46$ GeV [22]. $m_s = 0.5$ GeV, $\Lambda = 1.25$ GeV, the infrared cutoff $\mu = 0.593$ GeV and $\Delta S - \Delta H = 335 \pm 35$ MeV [18].

At present the experiments only give $BR[D_s^0 \to D_s^{*-} l^+ \nu_{\text{anything}}] = (7.9 ± 2.4)\%$ [22] and the information of $B_s \to D_s(1968) l \bar{\nu}$ is still absent. Thus the re-
predictions are far smaller than those given by Ref. [12].
Anyway, at present both our calculations and the analyses from other groups all indicate that the semileptonic decays $B_s \to D_{sJ}^*(2317, 2460)\bar{\nu}$ own large branching ratios. Therefore we urge our experimental colleagues to measure those semileptonic channels in the CDF experiment and in the future LHCb experiment. It will help us to further understand the nature of those exotic $D_{sJ}$ mesons. And more future experiments will also improve our understanding about the models applied to calculate the semileptonic decays of $B_s$ to $D_{sJ}$ mesons.

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Appendix

The explicit expressions of $I_{1,5}$ which are related to our calculation are listed

\begin{equation}
I_1 = \frac{iN_c}{16\pi^3} \int_{\Delta}^{m^2} \frac{d^4k}{k^2 - m_s^2} = \frac{N_c m_s^2}{16\pi^2} \Gamma \left( -1, \frac{m_s^2}{\Lambda^2}, \frac{m_s^2}{\mu^2} \right) \tag{26}
I_5(\alpha_1, \alpha_2, \omega) = \frac{iN_c}{16\pi^4} \int_{\Delta}^{m^2} \frac{d^4k}{(k^2 - m_s^2)(v \cdot k + \alpha_1 + i\epsilon)(v' \cdot k + \alpha_2 + i\epsilon)}
= \int_{0}^{1} \frac{dx}{1 + 2x^2(1 - \omega) + 2x(\omega - 1)} \times \left[ \frac{6}{16\pi^3/2} \int_{1/\Lambda^2}^{1/\mu^2} ds \, e^{-s(m_s^2 - \epsilon^2)s^{-1/2}(1 + erf(\sqrt{s}))} + \frac{6}{16\pi^2} \int_{1/\Lambda^2}^{1/\mu^2} ds \, e^{-s(m_s^2 - 2\epsilon^2)s^{-1}} \right]. \tag{27}
\end{equation}

where the definitions of $erf(z)$ and $\varrho$ are

\begin{equation}
\text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_{0}^{z} dx \, e^{-x^2}, \tag{28}
\end{equation}

\begin{equation}
\varrho = \frac{\alpha_1(1 - x) + \alpha_2 x}{\sqrt{1 + 2(\omega - 1)x + 2(1 - \omega)x^2}}. \tag{29}
\end{equation}
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