Dynamics of zero-point energy and two-slit phenomena for photons

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Received 25 February 2019, revised 26 April 2019
Accepted for publication 1 May 2019
Published 14 August 2019

Abstract
An earlier forward and backward in time formalism developed by us to discuss non-relativistic electron diffraction is generalized to the relativistic case and here applied to photons. We show how naturally the zero-point energy emerges in the Planck black-body spectrum once symmetric in time motion—inherent in the Maxwell equations—is invoked for photons. Then, a detailed study is made of two-slit experiments for photons and some novel phenomena, amenable to experiments, are proposed, that arise due to the spin of the photon.

Keywords: zero-point energy, electromagnetic field, two-slit experiment

1. Introduction

In his study of the Brownian motion of a quantum oscillator, Schwinger introduced the notion of coordinates moving forward in time, $x_+(t)$, and coordinates $x_-(t)$ moving backward in time [1]. By using such a doubling of the degrees of freedom, Schwinger developed in full generality a mathematically complete formalism for dealing with quantum Brownian motion. The starting point in his analysis is that a quantum object may be viewed as splitting the single coordinate, say $x(t)$, into two coordinates $x_+(t)$ (going forward in time) and $x_-(t)$ (going backward in time). From the Schwinger quantum operator action principle it can be derived that the classical limit is obtained when both motions coincide $x(t) = x_+(t) = x_-(t)$. The impact of Schwinger’s notion of forward and backward in time coordinates on subsequent studies in stochastic mechanics, many-body physics, quantum dissipation and thermal quantum field theory (QFT) in general, has been enormous. Such a concept has been also used in order to illustrate the non-relativistic electron beam two-slit diffraction experiments in [2, 3]. The interference patterns were there computed with or without dissipation (described by a thermal bath). A dissipative interference phase, due to the inherent non-commutative geometry, closely analogous to the Aharanov–Bohm magnetic field induced phase, was also found. An experiment was proposed and its feasibility examined in [4].

Proceeding further, using Maxwell’s equations the photon Zitterbewegung motion along helical paths and the resulting non-commutative geometry of photon position was explored in [5]. There it was also shown that the distance between two photons in a polarized beam of a given helicity has a discrete spectrum that should become manifest in measurements of two photon coincidence counts.

In the present paper, we extend Schwinger’s formalism of forward and backward in time motions, used in [2, 3], to the relativistic case of the photon field.

Forward and backward in time motions have been of interest to the authors for a considerable period in particular as they concern the dynamics of dissipative systems. However, in this paper, we show how crucial such considerations are to the foundations of quantum mechanics in general. These are exhibited through considerations of Planck black-body energy distributions as well as for delineating fine structure in spatial distributions of photons in two-slit diffraction processes.

Our discussion will proceed actually in two parts. In the first part we will focus our analysis on the contribution to the zero-point energy of the forward and backward in time motions of field modes. In the second part, we will consider more specifically the two-slit photon experiments and propose a novel set of experiments.

In section 2, we review the Maxwell equations to emphasize that Maxwell’s theoretical construction is inherently time symmetric. This is illustrated by showing that the
zero-point energy in the Planck black-body spectrum finds its natural explanation once the forward and backward time-symmetry is enforced [6]. Since the photon is its own antiparticle, the notion of time-symmetry is often obscured. We illustrate it in section 3 by considering the case of a spin zero, charged (boson) field for which the two motions are distinct. Extension to any integer and half-integer spin is also considered.

While the emphasis in [4, 5] was upon the non-commutative photon field coordinates and on methods for its revelation through two-photon processes, here in section 4, turning to the second part of our discussion, we shall be focusing on the behavior of a single photon for two-slit arrangements to shed light on forward and backward in time propagation.

In section 4.1, we first discuss photons as scalar (spin zero) fields and deduce for it the well-known diffraction pattern as in classical optics. However, once non-commuting spins are introduced, the quantum and classical theories need not be equivalent. For example, the spin precession needs to be considered [5]. This is studied in section 4.2 and indeed a novel constraint—not present for scalar fields—is found when the slit width \( w < \lambda \), the wavelength of the radiation. Our proposal concerns experiments in such a limit which, as far as we know, has not been investigated. Further experimental issues are discussed in section 4.3.

Conclusions are presented in section 5 and some details of the formalism are given in the appendices.

2. Maxwell forward and backward in time motion and zero-point energy in Planck black-body radiation

The present section and the next are devoted to the zero-point energy generated by the forward and backward in time motions of field modes. For this purpose, we first review the time symmetric character of Maxwell equations. We show that the zero-point energy in the Planck black-body spectrum is due to the symmetric forward and backward in time motion of photons. Otherwise said, due to the symmetric distribution in the photon frequency \( \omega \leftrightarrow -\omega \). Our discussion is mostly based on derivations presented in [6].

For pure radiation, i.e. in a part of space-time that is devoid of charges and currents, the Maxwell field equations read

\[
\nabla \cdot \mathbf{E} = 0;  \\
\nabla \cdot \mathbf{B} = 0; \\
\n\nabla \times \mathbf{E} = \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}; \\
\n\nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}.
\]

As the (positive definite) Maxwell EM energy density [7] is proportional to \((\mathbf{E}^2 + \mathbf{B}^2)\), it is natural to associate it with \((\mathbf{F} \cdot \mathbf{F}^\dagger)\), where the complex vector fields are chosen as

\[
\mathbf{F} = \mathbf{E} + i\mathbf{B}; \quad \mathbf{F}^\dagger = \mathbf{E} - i\mathbf{B};
\]

\[
\nabla \cdot \mathbf{F} = 0;  \\
\nabla \cdot \mathbf{F}^\dagger = 0.  
\]

One also sees that

\[
\mathbf{F} \cdot \mathbf{F} = |\mathbf{E}|^2 - |\mathbf{B}|^2 + 2i\mathbf{E} \cdot \mathbf{B},
\]

which determines the Lorentz scalar \(|\mathbf{E}|^2 - |\mathbf{B}|^2\) and Lorentz pseudo-scalar \(\mathbf{E} \cdot \mathbf{B}\).

Using Maxwell’s equations, it is easy to show that \(\mathbf{F}, \mathbf{F}^\dagger\) obey the Schrödinger equation, along with the transversality condition:

\[
i\hbar \frac{\partial \mathbf{F}_i}{\partial t} = (\hbar c) \epsilon_{ijk} \partial_k \mathbf{F}_i = H^{(\dagger)}_{jk} F_j;  \\
i\hbar \frac{\partial \mathbf{F}^\dagger_j}{\partial t} = -(\hbar c) \epsilon_{ijk} \partial_k \mathbf{F}^\dagger_j = H^{-1}_{jk} F^\dagger_k; \\
\partial_j F_j = 0; \quad \partial_j F^\dagger_j = 0.
\]

Defining the momentum operator \(p_j = -i\hbar \partial_j\) and a spin-one operator \(\mathbf{S}\) with matrix elements \((S)_{ij} = -i\epsilon_{ijk}\), with \(S^2 = s(s + 1) = 2\), we may rewrite equations (8) and (9) as matrix equations

\[
i\hbar \frac{\partial \mathbf{F}}{\partial t} = c (\mathbf{p} \cdot \mathbf{S}) \mathbf{F} = H^{(\dagger)} \mathbf{F};  \\
i\hbar \frac{\partial \mathbf{F}^\dagger}{\partial t} = -c (\mathbf{p} \cdot \mathbf{S}) \mathbf{F}^\dagger = H^{-1} \mathbf{F}^\dagger.
\]

Physically, in this (Maxwell) representation, \(\mathbf{F}\) goes forward in time and \(\mathbf{F}^\dagger\) goes backward in time.

Equations (11), (12) may be written more compactly as a Schrödinger equation in a six-component form by putting

\[
\Psi = \left( \begin{array}{c} \mathbf{F} \\ \mathbf{F}^\dagger \end{array} \right), \quad \mathbf{v} = \beta \mathbf{S}, \quad \mathbf{v} = \beta \mathbf{S},
\]

and

\[
\beta = \frac{1}{0}; \quad \mathbf{v} = \beta \mathbf{S},
\]

so that

\[
i\hbar \frac{\partial \Psi}{\partial t} = c (\mathbf{p} \cdot \mathbf{v}) \Psi = H \Psi.
\]

It is clear that the eigenvalues \pm 1 of \(\beta\) distinguish the forward versus backward in time motions. Explicitly

\[
\Psi = \frac{1 + \beta}{2} \Psi + \frac{1 - \beta}{2} \Psi = H^{(+)} \Psi;  \\
\Psi = \Psi + \Psi; \quad H^{(+)} = -H^{(-)} = c (\mathbf{p} \cdot \mathbf{S}).
\]

Much of the above formalism can be found in [5].

A symmetric treatment of forward and backward in time motions is part and parcel of the Maxwell field theory. In the following, we shall show that once this intrinsic time symmetry in the Maxwell equation is enforced, the zero-point energy in the Planck black-body thermal radiation follows.

Let us recall that Planck originally [8] discussed the mean number of photons of frequency \(\omega\) in the thermal vacuum

\[
\bar{n} = \frac{1}{e^{\hbar \omega/k_B T} - 1}.
\]
The mean thermal energy of an electromagnetic oscillator was thereby taken to be

\[ E(\omega) = \hbar \omega n = \frac{\hbar \omega}{e^{\hbar \omega/k_B T} - 1}. \]  

Later [9] Planck arbitrarily added the zero-point energy:

\[ E_T(\omega) = \hbar \omega \left( n + \frac{1}{2} \right) = \frac{\hbar \omega}{2} \coth \left( \frac{\hbar \omega}{2k_B T} \right). \]  

Remarkably, equation (21) is symmetric in \( \omega \leftrightarrow -\omega \). Of course, as well known, the zero-point energy is obtained by actually solving the quantum mechanical harmonic oscillator.

Einstein and Stern [10] noted that the excess energy over and above the equipartition value obeyed

\[ \lim_{T \to \infty} \left( E(\omega) + \frac{\hbar \omega}{2} - k_B T \right) = 0, \]  

that might theoretically be regarded as slight evidence of a zero temperature energy of \( \hbar \omega/2 \).

We now remark that

\[ E_T(\omega) = \frac{1}{2} [E(\omega) + E(-\omega)]. \]  

Equation (23) is indeed true in virtue of equations (20) and (21). Also, note the zero-point energy

\[ E_0(\omega) \equiv \lim_{T \to 0^+} E_T(\omega) = \frac{\hbar |\omega|}{2}. \]  

The relevance of equations (23) and (24) relies in the fact that they exhibit the contributions to the zero-point energy by the positive and negative frequency modes (forward and backward in time, respectively). Of course, if one expresses this result in terms of the photon creation operator \( a^\dagger \) and destruction operator \( a \) with \( [a, a^\dagger] = 1 \), then the photon number operator \( n = a^\dagger a \) enters into the Hamiltonian via the symmetrized product \( (aa^\dagger + a^\dagger a) \) as

\[ \mathcal{H} = \frac{\hbar |\omega|}{2} (a^\dagger a + aa^\dagger) = \hbar |\omega| (n + 1/2). \]  

Equation (25) leads directly to equation (24).

In conclusion, the physical meaning of equation (23) is that both the positive frequency \( \omega > 0 \), a particle moving forward in time, and the negative frequency \( \omega < 0 \), an anti-particle moving backward in time, contribute to the zero-point energy. Since the photon is its own anti-particle, the physical meaning of equations (23) and (24) may be somewhat obscured. In order to make the particle content in the zero-point energy more evident, we consider in the following section 3, a case wherein the particle and anti-particle are distinct.

3. Charged fields

In this section we discuss first spinless charged boson oscillator energies. Extension to the non-zero spin boson and fermion fields are taken up later in the second part of section 3.1 (see equation (48)).

The energy of a spinless charged boson field in a uniform magnetic field \( B = (0, 0, B) = (0, 0, |B|) \) is given by [11]

\[ \epsilon_\pm(n, p, B) = \pm c \sqrt{\frac{m^2 c^2}{\hbar^2} + p^2} + (2n + 1)/|eB|/c, \]  

wherein the integer \( n = 0, 1, 2, \ldots \) is the label for the circular Landau orbit, the momentum along the magnetic field axis is \( p = (0, 0, p) \), \( p \equiv /k \) and \( \kappa = (mc/\hbar) \) is the mass in inverse length units. Thus

\[ \omega(n, k, B) = c \sqrt{\kappa^2 + k^2 + (2n + 1)|eB|/\hbar c}. \]  

The zero-point charged boson oscillator energies per unit volume counting the particle and anti-particle separately in virtue of the different charge \( \pm e \) is determined by

\[ U_0(B) = 2 \times \frac{eB}{2\pi \hbar c} \sum_{n=0}^\infty \int_0^\infty \frac{dk}{2\pi} \frac{\hbar \omega(n, k, B)}{2}. \]  

This vacuum energy per unit volume in a magnetic field is clearly divergent so one must regularize and renormalize. After doing both exercises, a finite vacuum boson energy per unit volume in a magnetic field \( U(B) \) arises. We present in appendix A some of the Gamma function regularization formalism and we briefly comment on the charge renormalization procedure.

The physical fields are defined so that the normal vacuum magnetic energy density is \( |B|^2/8\pi \). This can be realized by a charge renormalization subtraction in equation (A9). The physical justification for and the precise formalism used for charge renormalization used here is due to Schwinger. See equation (3.47) of [12].

Thus, for scalar boson fields the vacuum energy density is obtained as

\[ U(B) = \frac{hc}{16\pi^2} \int_0^\infty \frac{ds}{s^3} e^{-s^2} \times \left[ 1 - \frac{(eBs/\hbar c)}{\sinh(eBs/\hbar c)} - \frac{(eBs/\hbar c)^2}{6} \right]. \]  

Equation (29) is both finite and exact for the sum of zero-point oscillations of charged boson spin zero systems. The vacuum boson magnetization is thereby

\[ M = -\frac{\partial U}{\partial B}. \]  

In [13] and [14] the vacuum energy for a scalar field in the presence of an external magnetic field was previously investigated for the Casimir effect. However, our analysis is over all space, whereas more specialized analyses geared for the Casimir effect deal with restricted boundaries (e.g. parallel plates) and thus the vacua are necessarily different.

To consider now what happens in an external electric field, we recall that to go from a pure external magnetic field to a pure external electric field one takes \( B^2 \rightarrow -E^2 \). This allows us to obtain the boson pair production rate \( \Gamma \) per unit time per unit volume in an external electric field. This may be computed from

\[ \Gamma = -\frac{2}{\hbar} 3m U(B \rightarrow -iE). \]
Equations (29) and (31) imply
\[ \Gamma = \frac{c}{8\pi^2} \left( \frac{eE}{\hbar c} \right)^2 \times \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \exp \left( -\pi n \left( \frac{m^2c^3}{\hbar^2E} \right) \right). \] (32)

A uniform electric field can thereby excite the charged boson oscillators emitting pairs ($\pi^+\pi^-$) from the vacuum. The electric field does the work required to break down the vacuum.5

It should be mentioned that the effective Lagrange density for a charged spin zero field in a constant electric and magnetic field was derived independently in 1936 by Weisskopf [15] and by Heisenberg and Euler [16]. In 1951, Schwinger [12] used gauge-invariant proper time methods to derive the same both for spin-zero and spin-half fields. These results agree with each other and for spin-zero our equation (32) agrees with Schwinger’s equations (6.40–6.41) (apart from a typo in his equation (6.41)).

3.1. Charged particle paths

Let us here consider how the zero-point energy is expressed in terms of paths forward in time (particle) and backward in time (anti-particle). Let us at first work in one space and one time (1 + 1) dimensions. With a small modification, this leads to a correct description in physical three space and one time (3 + 1) dimensions.

In (1+1) dimensions, the energy-momentum relation reads
\[ \mathcal{E}^2 - c^2p^2 = (mc^2)^2. \] (34)

A particle of charge $q$ and rest mass $m$ subjected to a (constant) electric field $E$ has the following equation of motion
\[ \frac{dp}{dt} = qE \] (35)

with $p = (mc)\beta \gamma$ and the Lorentz factor \( \gamma = (1 - \beta^2)^{-1/2} \). Defining $a = \frac{qE}{m}$ and taking $\beta(t = 0) = 0$, we get:
\[ \beta(t) = \frac{a(t/c)}{\sqrt{1 + (at/c)^2}} = \frac{dx(t)}{dt}, \] (36)
\[ x(t) - x(0) = \frac{c^2}{a} \left( \sqrt{1 + (at/c)^2} - 1 \right). \] (37)

Thus, the motion of the particle on the right hyperbola, (i.e. \( x_+(t) \)) is along the direction of time (particle comes from \( x \to \infty \) in the far past \( t \to -\infty \), upto \( x = c^2/a \) at \( t = 0 \) and then goes out to \( x = +\infty \) in the far future \( t \to +\infty \)). whereas that of the anti-particle (i.e. \( x_-(t) \)) is opposite to the direction of time \( t \) (the anti-particle comes from \( x \to -\infty \) in the far future \( t \to +\infty \), upto \( x = -c^2/a \) at \( t = 0 \) and then goes out to \( x = -\infty \) in the far past \( t \to -\infty \)). From the above is derived the nomenclature regarding the arrow of time: (i) \( x_+ \) goes forward in time and (ii) \( x_- \) backward in time.

Pair production at time zero requires a space-like transition from \( x_-(0) = -(c^2/a) \) to \( x_+(0) = (c^2/a) \) along the semicircle in Euclidean time \( \tau \), i.e. equation (38) reads in Euclidean time
\[ \mathcal{E}^2 + c^2t^2 = \left( \frac{c^2}{a} \right)^2. \] (41)

The arc length of the semicircle is \( s = \pi (c^2/a) \) giving rise to the Euclidean action
\[ W = mc^2 + \frac{mc^3}{a} = \frac{m^2c^3}{cE}. \] (42)

The boson weight of such pair production processes summed over the number \( k \) of pairs produced is related to the partition function
\[ Z = \sum_{k=0}^{\infty} (-1)^k e^{-kW/\hbar} = \frac{1}{1 + e^{-W/\hbar}}. \] (43)

For details, see [12] and [17]. The factor of \(-1\) for each semicircle means a Bose factor of one for each circle. Since the rate of change of momentum is equal to the force, \( dp/dt = eE \), the transition rate per unit time per unit length \( \Gamma \) is given by
\[ \Gamma dt = \frac{dp}{2\pi\hbar} (-\ln Z), \] (44)
\[ i = \frac{eE}{2\pi\hbar} \ln\left[ 1 + e^{-(\varepsilon_{m}c^{2}/\hbar E)} \right] \]
\[ = \frac{eE}{2\pi\hbar} \sum_{n=1}^{\infty} \frac{(-1)^{n} + 1}{n} e^{-n(\varepsilon_{m}c^{2}/\hbar E)}. \quad (45) \]

By taking the momentum perpendicular to the electric field into account, the \((3 + 1)\) dimensional result follows from equation \((45)\)

\[ \Gamma = \frac{eE}{2\pi\hbar} \int \frac{d^{3}p}{(2\pi\hbar)^{3}} \times \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \exp \left( -\pi n \frac{m^{2}c^{4} + cp^{2}}{\hbar E} \right) \]
\[ = \frac{e}{8\pi\hbar} \frac{(eE)^{2}}{\hbar} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^{2}} \exp \left( -\pi n \frac{m^{2}c^{4}}{\hbar E} \right). \quad (46) \]

in agreement with equation \((32)\).

It is not difficult to write the transition rate for producing pairs wherein the charged particles have spin \(s\), i.e. \(s = 0, 1, 2, 3, \cdots\) for bosons, and \(s = 1/2, 3/2, 5/2, \cdots\) for fermions. The statistical index may be defined as \(\eta_{s} = \exp(\pi (2s + 1))\), \(\eta_{s} = -1\) for bosons, \(\eta_{s} = +1\) for fermions. From a QFT viewpoint, the statistical index is related to the commutation or anti-commutation relation between creation and destruction operators

\[ [a, a^{\dagger}]_{\eta_{s}} = ad^{\eta_{s}} + \eta_{s}a^{\dagger}a = 1. \quad (47) \]

For arbitrary spin, equation \((46)\) may be argued from the factor \(-\eta_{s}\) for each closed circle loop to be

\[ \Gamma = \frac{(2s + 1)c}{8\pi^{3}\hbar} \left( \frac{eE}{\hbar} \right)^{2} \sum_{n=1}^{\infty} \frac{\eta_{s}^{n+1}}{n^{2}} \exp \left( -\pi n \frac{m^{2}c^{4}}{\hbar E} \right) \quad (48) \]

Equation \((48)\) has been discussed in the literature \([17]\). Finally, the Euclidean action \(W\) may be associated with an entropy \(S\) via

\[ \frac{W}{\hbar} = \frac{S}{k_{B}} = \frac{m^{2}c^{4}}{\hbar E}. \quad (49) \]

The derivative of the entropy with respect to the rest energy determines the reciprocal temperature

\[ \frac{1}{c^{2}} \frac{dS}{dm} = \frac{1}{T} \quad \Rightarrow \quad k_{B}T = \frac{\hbar E}{2\pi mc}. \quad (50) \]

In terms of the acceleration of the charged bosons, there exists an effective temperature \([18]\)

\[ k_{B}T = \frac{\hbar a}{2\pi mc} \quad (51) \]

of the environment inducing position fluctuations equivalent to the energy fluctuations in the rest frame of the applied electric field (the Unruh effect or Unruh temperature).

Let us close by observing that the central results of this and the previous section are not new. For example, the spin zero charged boson pair production rate in equation \((32)\), as well as its generalization to the general spin \(s\) charged particle pair production rate in equation \((48)\) are well known. However, the derivations, physical pictures and consequences of zero-point oscillations are to our knowledge original. The notion of zero-point energy in relativistic QFT is made real by the particle and anti-particle content of the theory.

### 4. Photon two-slit

We turn now to the second part of our discussion focusing on the forward and backward in time motion formalism for two-slit photon phenomena. In particular we generalize the forward and backward in time motion formalism developed in \([2]\) for non-relativistic two-slit interference processes to the relativistic case of two-slit processes for photons.

There are several motivations for such an extension:

- There is a well defined radiation QFT with a classical limit called the Maxwell theory.
- There is no mass gap for photons contrary to the electrons. A photon is its own anti-particle. Thus, both forward and backward motions must be there anyway, as discussed in section \(2\).
- Experimentally, both classical and quantum optics are amongst the most studied subjects in physics; and not only theoretically.

Let us first consider a massless spin zero (scalar wave). If Planck, Einstein and Bose could invoke it for the black-body radiation for example, and then, after computing the radiation energy density, they multiplied their results by \(2\) to take care of the two polarizations, we are in good company. However, as we shall see later, spin is neither harmless nor a trivial complication.

#### 4.1. Spin zero, massless radiation

Under the assumptions of our earlier paper \([2]\), the diffraction limit formula (equation \((27)\) of \([2]\)) would still read, *mutatis mutandis*, for photons (see appendix \(B\))

\[ P_{\gamma}(x; D) \approx \frac{4}{\pi \beta K x^{2}} \cos^{2}(K x) \sin^{2}() \]

with the following replacement for the definition of \(K_{\gamma}\) compared to \(K_{\text{electron}}\) (a particle of mass \(M\))

\[ K_{\text{electron}} = \frac{M v d}{\hbar D}, \quad (53) \]

\[ \Longrightarrow K_{\gamma} = \frac{p d}{\hbar D} = \frac{2\pi d}{\lambda D}, \quad (54) \]

where \(p = 2\pi \hbar/\lambda\) is the (mean)-momentum and \(\lambda\) the wavelength of the photon. Also, \(w\) is the size of the slit, \(2d\) is the distance between the two slits, \(\beta = w/d\) and \(D\) is the...
distance of the screen from the source. Thus, the diffraction pattern remains exactly as before.

It is worthy of note that in the extreme limit \( \beta d / \lambda = w / \lambda \to 0 \), the quantity \( P(x, D) / \beta K \gamma \) has a finite limit:

\[
P_{\gamma K}(x, D) = \frac{4}{\pi} \cos^2(K, x),
\]

there remains just the expected Young’s interference pattern showing maxima at \( x_{\text{max}} \) (constructive interference) and minima at \( x_{\text{min}} \) (destructive interference) according to the path difference (see equation (54))

\[
\sqrt{D^2 + (x + d)^2} - \sqrt{D^2 + (x - d)^2} \approx 2x \frac{d}{D},
\]

so that (with integer \( n = 0, 1, 2, \ldots \))

\[
2x \frac{d}{D} = \lambda n \implies x_{\text{max}} = \frac{\lambda D}{2} n,
\]

\[
2x \frac{d}{D} = \lambda (n + \frac{1}{2}) \implies x_{\text{min}} = \frac{\lambda D}{2} \left( n + \frac{1}{2} \right).
\]

Let us pause here and note that we have derived—in general—the diffraction pattern for a scalar photon, under the hypothesis of symmetric forward and backward time motion. There is no visible trace of quantum mechanics left, i.e. there are no factors of \( \gamma \) showing maxima at \( x_{\text{max}} \) (constructive interference) and minima at \( x_{\text{min}} \) (destructive interference) according to the path difference (see equation (54)).

4.2. Spin-one radiation and Maxwell theory

Much of the formalism discussed below can be found in [5]. While the emphasis in [5] was upon the non-commutative photon coordinates and on methods for its revelation through two photon processes, here we shall be focussing on the behavior of a single photon for two-slit arrangements to shed light on forward and backward in time photon propagation.

There are two sets of (non-commuting) coordinates and velocity operators \( \mathbf{V} \) that are given by

\[
\dot{X} = \mathbf{V} = \frac{\partial \mathcal{H}}{\partial \mathbf{p}} = c \beta \mathbf{S}.
\]

Due to the divergence condition equation (1), the motion of the field \( \mathbf{S} \) is confined to the plane perpendicular to the momentum. Thus, only the motion of coordinates and velocities in the plane perpendicular to \( \mathbf{p} \) are of physical relevance here. For example, if the momentum is directed along the \( z \)-axis, equation (61) tells us that the \( x \) - and \( y \) -components of the velocities do not commute

\[
[V_{x,1}, V_{x,2}] = i c^2 \lambda = [V_{y,1}, V_{y,2}],
\]

where \( \Lambda = \pm 1 \) is the helicity of the photon. The mixed commutator

\[
[V_{x,1}, V_{y,2}] = i c^2 \beta \lambda,
\]

shall not be discussed here, as it does not enter the discussions to follow.

The non-commutativity of the photon position coordinates lying in a plane orthogonal to the direction of its motion equation (61) has been thoroughly discussed in [4, 5]. In particular, for a photon of helicity \(+1\) moving along the \( z \)-axis, the \( X_1, X_2 \) coordinates of the photon precess about the \( z \)-axis with a frequency \( pc / h \) and the radius \( R^2 = (X_1^2 + X_2^2) \) is quantized:

\[
R^2 = \left( \frac{\lambda}{p} \right)^2 (2n + 1) = \left( \frac{\lambda}{2\pi} \right)^2 (2n + 1); \quad n = 0, 1, \ldots
\]

Of course, the center is unspecified, that is why two, parallel, same helicity, photons were needed in [5] to allow for a measurement of the quantization in the difference between the (transversal) positions of the two photons. Incidentally, Maxwell was well aware of two opposite screw motions (corresponding to the two helicities) about the axis of propagation. He was only missing the names photon and spin (and possible quantization conditions) for the EM waves6.

Returning to our two-slit arrangement for a single photon, we can try to obtain some information from the above

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6 For example, in section 813 of Maxwell’s Treatise, vol 2, he writes: ‘any undulation, the motion of which at any point is circular, may be represented by a helix or a screw’. In his figure 67, he depicts the two opposite helical motions (helicities, in modern parlance). Further on, in section 820, for motion along the \( z \)-axis, he describes a circular velocity in the \( xy \) plane. In section 821, he writes: ‘whatever light is, at each point of space there is something going on, whether displacement, or rotation, or something not yet imagined, but which is certainly of the nature of a vector or directed quantity, the direction of which is normal to the ray. This is completely proved by the phenomena of interference’. Maxwell the genius, is describing in words, what in modern parlance is called the spin-vector of the photon.
quantization condition. As in the constant magnetic field Landau level problem, there is a huge degeneracy introduced by the uncertainty in the center of the coordinates. The number of states/area for the magnetic case is well known to be \( eB/(2\pi\hbar c) \). The density of transversal states is given by

\[
\rho = \frac{1}{2\pi} \left( \frac{2\pi}{\lambda} \right)^2 = \frac{2\pi}{\lambda^2}.
\]  

(65)

In our problem, each of the slits of total width \( w \) can be considered as a circle (in the \( x, y \) plane) of area \( A = \pi (w/2)^2 \). Hence, we can estimate (semi-classically) the total number traversing each slit to be

\[
N = \rho A = \frac{\pi^2}{2} \left( \frac{w}{\lambda} \right)^2.
\]  

(66)

Thus, at least theoretically it would appear as if we can confine the transversal photon coordinates to be in its ground state \( n = 0 \) for \( w < (\lambda/\pi) \).

There is a matter of principle involved here. For a scalar wave diffraction pattern, equations (52), (53), there appears to be no theoretical lower limit to \( w \) apart from \( w \ll d \ll D \). On the other hand, for a spin-one photon, there is a quantum constraint. Can it be measured?

With micro/nano technology, both the fabrication of apertures small enough as well as procurement of polarized light of wavelengths smaller than the size of the apertures should be possible. With such setups, the very interesting fine structure in the diffraction pattern can be investigated as the width \( w \) is lowered for a fixed wavelength \( \lambda \). It seems to us, on the basis of the discussion in this paper, that an experiment in such a setup might be very worthwhile.

What one can find in classic texts such as [19] are diffraction patterns in the limit where \( \lambda \ll w \ll d \ll D \). What is interesting, and to us at least intriguing, is that even for mercury light of wavelength \( \lambda \sim 5.79 \times 10^{-7} \) cm passing through a single aperture \( w \sim 0.6 \) cm, four or five diffraction minima are clearly visible. Thus, in the diffractive part of the spectrum

\[
S(x) = \left( \frac{\sin \eta}{\eta} \right)^2; \quad \eta = \frac{2\pi x w}{\lambda D}.
\]  

(67)

At the first minimum say, \( \eta_1 = \pi \). Translated into the vertical distance \( x \) on the screen to the distance of the screen \( D \) from the source, one finds \( x_1 \sim 10^{-3}D \). Thus, even a meter away, the value of \( x_1 \sim 10^{-2} \) cm. To us it is remarkable that it can be measured so well.

In any event, if small apertures of size \( w \sim (10^{-5}\text{--}10^{-4}) \) cm, can be fabricated, then \( x_1 \sim D \) and measurements might be easier. A discussion on experimental issues is undertaken in the next section.

4.3. Some experimental issues

In this section we shall discuss a few interference and diffraction experiments done in the past by way of comparison to the proposed two-slit experiments for photons in the present paper.

4.3.1. Magnetic fields and the quantum Hall effect. Let us begin by recalling the well-known fact that once a magnetic field is introduced via a vector potential, the components of velocity \( \mathbf{v} = (\mathbf{p} - e\mathbf{A}/c)/M \)—even for a non-relativistic electron—do not commute:

\[
[v_x, v_y] = \frac{ie\hbar}{M^2c} \varepsilon_{ijk} B_k = -\frac{e\hbar}{M^2c} (\mathbf{S} \cdot \mathbf{B}).
\]  

(68)

In particular, for \( \mathbf{B} = B \hat{\mathbf{k}} \), we have

\[
\begin{bmatrix} v_1 / c & v_2 / c \end{bmatrix} = -i \omega_B \hbar c^2 \Delta \equiv -\Delta,
\]  

(69)

with \( \omega_B = eB/(Mc) \). If one compares equation (68) for the non-commuting components of the electron velocity (that are perpendicular to the magnetic field), with the corresponding non-commuting components of the photon velocity (that are perpendicular to the direction of motion of the photon) given in equation (62), one finds the right hand side of the commutator for the electron a rather small value \( \Delta \ll 1 \), whereas for the photon the factor is unity. It is for this reason that (for the case of an electron) one needs high magnetic fields and low temperatures so that thermal fluctuations do not wash out the quantum effects for an electron. For example, integer quantum Hall steps were made visible experimentally [20], with \( B = 18 \) Tesla and at a low temperature \( T = 1.5 \) K. (The value of \( \Delta \sim 10^{-9} \) for this experiment.)

To observe fractional quantum Hall steps [21], even higher fields (\( B \sim 35 \) Tesla) and milli-Kelvin temperatures were necessary. For a detailed derivation of quantum Hall steps in the context of QED, see [22] and for a review see [23].

The relevant point of the above discussion for the present paper is that for the photon case there are no small factors such as \( \Delta \), and thus visibility of the proposed quantum effects for the photon are not afflicted by background thermal fluctuations and experiments at room temperatures should be adequate.

4.3.2. Cold neutron experiments. Several very cold neutron diffraction experiments have been performed and they have been excellently reviewed in [24]. The wavelength of the neutrons in such experiments is typically \( \lambda_{\text{neutron}} \approx 20 \) Å, to be compared with 3900 Å \( \leq \lambda_{\text{visible light}} \leq 7000 \) Å. The relevant slit widths in such experiments were about \( w \sim 20 \mu\text{m} = 10^4 \lambda_{\text{neutron}} \). In the experimentally covered regime \( w \gg \lambda \), the observed diffraction patterns are in good agreement with their theoretical expectations. The prospect of future cold neutron experiments in the opposite regime \( w \ll \lambda \) is rather remote. On the other hand, as we outline below, for photons in the visible spectrum with wavelengths over two hundred times larger than the cold neutron wavelengths, fabrication of needed slit widths of sufficiently small size (say \( \leq 0.1 \mu\text{m} \)) may not be technically so daunting.

4.3.3. Photon double slit experiments. As we have discussed at length in section 4.2, there is a fundamental difference between the propagation of a massless scalar (spin zero) wave through two slits as compared to that of a massless vector (spin one) wave. This should not be surprising as the former has ‘no directional pointers’, whereas the latter does have one through the
direction of the spin. In practice, there is a precession of the spin at a frequency \( \omega = Pe/\hbar \) in a plane perpendicular to the direction of motion of the photon. Since the transversal velocities of the photon do not commute (see equation (62)), the transversal positions of the photon satisfy a quantum Pythagoras theorem (see equation (64)) with an arbitrary center of the circular orbits. Thus, in contrast to a scalar wave, there is degeneracy constraint for a physical EM vector-wave traversing a slit of width \( w \). The number of states is given by equation (66) to be \( N = (\pi^2/2)(w/\lambda)^2 \). Thus, for small enough slits \( w \leq \lambda/\pi \), we can limit the transversal quantum number \( n \) to the ground state \( (n = 0) \).

To be concrete, let us consider mercury yellow light of wavelength \( \lambda = 0.58 \mu m \). The standard diffraction pattern as expected have been confirmed for ‘large’ slits of width \( w \sim 0.6 \text{ cm} \) [19]. Our proposal is to vary the slit width \( w \) and observe the change in the diffraction pattern specially once it is reduced to 0.1 \( \mu m \) or even lower. (Of course, respecting \( w \ll d \), the distance between the two slits.) Such a region to our knowledge has not been explored previously and that is our suggestion.

5. Conclusions

In the present paper, as in our earlier papers on the subject, we have shown that both forward and backward motions in time are essential for a proper description of a particle’s motion from its classical to its quantum counterpart. For the important case of a photon, it plays a particularly decisive role. A free photon described by the Maxwell equations in its inherent time symmetric aspect has been shown to be essential for obtaining the correct Planck thermal radiation distribution with the zero-point energy. We have stressed that, as a photon is its own antiparticle for which forward and backward in time formalism but often overlooked. Thus, various aspects of the dynamics of a charged particle for which forward and backward in time motions are distinct have been considered in detail.

When applied to the case of a photon, considered first as a scalar field, standard expressions for interference and diffraction have been obtained. On the other hand, when extended to the realistic case of a spin-1 photon, the non-commutativity of the spin components, induce non-commutativity in the components of the photon position coordinates. As the commutator between two such coordinates is proportional to the square of the wavelength, the intrinsic uncertainty in the position of a photon is proportional to its wavelength. We have shown here that it can manifest itself through changes in the interference pattern of a two-slit photon experiment as the width of a slit is lowered below the wavelength of the photon.

Our formalism also provides an understanding of why a strict localization of a photon—to better than its wavelength—runs into serious difficulties [25]’. The experiments suggested in the present paper could shed light not only on the validity of the forward and backward motions in time formalism but also on the fundamental subject of photon localizability specially in so far that the Zitterbewegung in the plane perpendicular to the direction of motion of the photon distorts the diffraction pattern for slit widths \( w \) smaller than its wavelength \( \lambda \).

Acknowledgments

Y S would like to thank the Department of Physics and Geology at the University of Perugia for its hospitality.

Appendix A. Gamma function regulation

The Gamma function is defined in the \( \Re(z) > 0 \) part of the complex plane as

\[
\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt, \quad (A1)
\]

from which we find for \( a > 0 \) the identity

\[
a^{-z} = \frac{1}{\Gamma(z)} \int_0^\infty s^{z-1}e^{-as}ds. \quad (A2)
\]

In other regimes, \( \Gamma(z) \) is defined by analytic continuation. This analysis is often assisted by the identity

\[
\Gamma(1 + z) = z\Gamma(z). \quad (A3)
\]

For example, by putting \( z = -(1/2) \) in equation (A3), one finds

\[
\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} \quad \Rightarrow \quad \Gamma\left(-\frac{1}{2}\right) = -2\sqrt{\pi}. \quad (A4)
\]

Equations (A2) and (A4) lead to a formally divergent integral

\[
\sqrt{a} - \sqrt{b} = \frac{1}{2\sqrt{\pi}} \int_0^\infty e^{-as}e^{-bs} \left(\frac{ds}{s^{3/2}}\right). \quad (A5)
\]

For those readers who find it strange in equation (A5) to put a finite positive quantity equal to a negative infinite quantity, we invite the reader to prove the following theorem. For any \( a > 0 \) and \( b > 0 \)

\[
\sqrt{a} - \sqrt{b} = \frac{1}{2\sqrt{\pi}} \int_0^\infty \left(e^{-as} - e^{-bs}\right) \left(\frac{ds}{s^{3/2}}\right). \quad (A6)
\]

Subtractions will be made below. Equations (28) and (A5) yield

\[
U_0(B) = -\frac{eB}{8\pi^5/2} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} dk \int_0^\infty \left(\frac{ds}{s^{3/2}}\right) \times \exp\left[-k^2s - k^2s - (2n + 1) \left|\frac{eB}{\hbar c}\right| s\right]
\]

\[
= -\frac{\hbar c}{16\pi^2} \int_0^\infty \frac{ds}{s^2} e^{-s^2} \left(\frac{eBs}{\hbar c}\right) \sinh(eBs/\hbar c). \quad (A7)
\]
that is still divergent. One then subtracts the vacuum zero-
point oscillations when the magnetic field is zero

\[ \hat{U}(B) = U_0(B) - U_0(0). \] (A8)

The zero-point oscillation energy per unit volume due to
vacuum particle anti-particle pairs, say (\(\pi^+\pi^-\)), virtual
magnetic moments are thereby

\[ \hat{U}(B) = \frac{\hbar c}{16\pi^2} \int_0^\infty \frac{dx}{x^3} e^{-x^2} \left[ 1 - \frac{eBs/\hbar c}{\sinh(eBs/\hbar c)} \right]. \] (A9)

which still is divergent but only as a logarithm at small
distance squared as \(s \to 0^+\). Once the divergences are only
logarithmic, one may pass from regularization to
renormalization.

For what concerns the charge renormalization procedure
we observe that in quantum electrodynamics one starts with
charges and fields described for the problem at hand by \(e_0\) and \(B_0\). Both the fields and charges have to be renormalized by
considering the vacuum polarization contributions \([12, 26]\) in
such a way that \(e_0B_0 = eB\) and thus divergent logarithms are
buried. See equation (3.47) etc of Schwinger \([12]\). The vacuum
energy density for scalar boson fields is reported in
section 3.

Appendix B. Derivation of equation (52)

Here we exhibit the simple calculations through which
equation (52) is obtained for a scalar (spin 0) massless pho-
ton. It fixes the precise assumptions and the notation. Using
the formalism and notation of \([12]\), we have equation (18) of
\([2]\), but with \(H = (1/2) \text{MV}^2\) for a non-relativistic particle
replaced by \(H_s = E_s = pc = 2\pi\hbar c/\lambda\). So that, the classical
action for a scalar photon reads \(A_s = pet\). Thus, equation
(19) of \([2]\), for the amplitude is replaced by

\[ P_s(x, t) = \frac{1}{\lambda x} \int_{-\infty}^{+\infty} dx_x \int_{-\infty}^{+\infty} dx_x' e^{iS_s(x_t|x_{t-})}, \] (B1)

where

\[ S_s = S_s - S_n, \] (B2)

\[ S_n = H_{s,0}(t/\hbar) = \frac{pc}{\hbar} |\mathbf{x} - \mathbf{x}_1|/c; \] (B3)

\[ S_n = H_{n,0}(t/\hbar) = \frac{pc}{\hbar} |\mathbf{x} - \mathbf{x}_1|/c. \] (B4)

\[ S = 2\pi \lambda \left[ |\mathbf{x} - \mathbf{x}_1| - |\mathbf{x} - \mathbf{x}_1| \right]. \] (B5)

Using \(|\mathbf{x} - \mathbf{x}_1| = (D^2 + (x - x_1)^2)^{1/2}\), in the diffraction limit of
large \(D\) (the Fraunhoffer limit valid up to quadratic order
\([19]\)), the effective action reduces to

\[ S_s \approx -\frac{2\pi x}{\lambda D} (x_t - x_{t-}). \] (B6)

The choice and construction of the initial density matrix
for the two-slit arrangement with each slit of size \(w\) placed a
distance \(2d\) apart, proceeds identically as in (equations (15),
(22) (23), (24) of \([2]\)):

\[ (x_t|x_{t-}) = \psi^{(0)}(x_t)\psi(x_{t-}); \]

\[ \psi(x) = \frac{1}{\sqrt{2}} [\phi(x - d) + \phi(x + d)]; \]

\[ \phi(x) = \frac{1}{\sqrt{w}}; |x| < \frac{w}{2}, \text{ and } 0 \text{ otherwise}, \] (B7)

so that as in our original paper \([2]\)

\[ (x_t|x_{t-}) = \frac{1}{2} [\phi(x_t - d)\phi(x_{t-} - d) \]

\[ + \phi(x_t + d)\phi(x_{t-} + d) + \phi(x_t - d)\phi(x_{t-} + d) \]

\[ + \phi(x_t + d)\phi(x_{t-} - d)]. \] (B8)

Using equations (B6)–(B8), the integrals involved in
equation (B1) for the computation of \(P_s(x, D)\) are finite-range
Fourier transforms that lead to a product of (Young’s inter-
fERENCE factor) \(Y(Kx)\) with (Fraunhoffer diffraction factor)
\(F(\beta Kx)\), where \(K = (2\pi/\lambda)d/D\) and \(\beta = w/d:\)

\[ P_s(x, D) = \frac{4\beta K}{\pi} Y(Kx) F(\beta Kx); \] (B9)

\[ Y(Kx) = \cos^2(Kx); \] (B10)

\[ F(\zeta) = \frac{\sin^2(\zeta)}{\zeta^2}; \zeta = \beta Kx; \] (B11)

the result quoted in equation (52).

The dimensionless quantity \(P_s(\beta K)\)

\[ \frac{P_s(x, D)}{\beta K} = \frac{4}{\pi} Y(Kx) F(\beta Kx), \] (B12)

has a smooth limit as the size of each slit \(w \to 0\). In this limit,
the diffraction pattern disappears, leaving behind just the
Young’s interference pattern. Of course, for visible light this
is only a deceptive limit unless—as discussed in the text—
apertures can be constructed for which \(w \ll \lambda\), an ardu-
ous task.

We mention in passing that if one assumes two circular
apertures each of (radius \(w/2\)), \(P(x, D)\) can be computed
exactly as above: the only change is in the Fraunhoffer dif-
frac tion function that now reads

\[ F_{\text{circle}} = \left[ \frac{2J_1(\eta)}{\eta} \right]^2; \eta = \frac{2\pi w}{\lambda D} R. \] (B13)

In this case, we expect circular (bright and dark) rings, with a
maximum at \(R = 0\), and minima at various \(R\), corresponding
to the zeroes of the Bessel function of the first kind \(J_1(\eta)\); see
\([19]\).
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