The Synthesis of Precise Rotating Machine Mathematical Model, Operating Natural Signals and Virtual Data

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Abstract. It is known that synchronous machines catalogue data are presented for the case of two-phase machine in rotating coordinate system, e.g. for their description with Park-Gorev’s equation system. Nevertheless, many problems require control of phase currents and voltages, for instance, in modeling of the systems, in which synchronous generators supply powerful rectifiers. Modeling of complex systems with synchronous generators, semiconductor convertors and etc. (with phase currents control necessary for power switch commutation algorithms) becomes achievable with the equation system described in this article. Given model can be used in digital control systems with internal model. It doesn’t require high capacity of computing resources and provides sufficient modeling accuracy.

1. Introduction
To analyze the processes in “synchronous generator – semiconductor converter” system it is necessary to have an adequate mathematical model. The simplifications made in the classic form of Park-Gorev’s equations don’t allow to derive this model, which will be shown further. Therefore, synchronous machines catalogue parameters are given right for the system describing those machines as two-phase machines in the rotating coordinate system.

Controlled rectifier (CR) and autonomous inverter (AI) are the ones considered nonlinear. Particular nonlinear properties of those elements are: relay nonlinearity; nonlinear parameters depend on time, which allows to classify them as the nonautonomous type; there are three and more nonlinearities in each CR and AI (for three-phase circuit).

Methods of analysis and synthesis of such systems are shown in the most accurate way in the nonlinear systems section in the theory of automatic control (TAC). With considering known nonlinear systems properties, the ones that lead to complicated equations, the amount of nonlinearities are usually limited by 1-2 [1, 2]. Despite lots of efforts to increase the accuracy of the calculations, it is noted, that “there are no universal analytical (mathematical) methods of calculating nonlinear systems” [3-4].

2. Problem statement
The synchronous generator (SG) model in Park-Gorev’s equations (PGE) is applied in marine electric station models based on SG. This SG model has a lot of advantages when working on linear load, which are:
- minimal amount of SG differential equations is: 5 with only electric and 7 with electromechanical processes considered;
models of SG voltage regulation due to excitation winding (EW) are the most simple, since PGE are reduced to EW;
- it is possible to get analytical equations (with some uncomplicated assumptions) for SG short-circuit mode, SG output voltage, etc.

Modeling of SG’s work on rectifier load with PGE doesn’t give good results for the following reasons:
- each stator phase winding is connected with its nonlinearity formed by controlled rectifier (CR) phase circuit. Reducing equations of stator three windings to the armature can be accomplished in 2 steps: first is using linear phase conversion (of three-phase stator winding signals in ABC axes into the two-phase stator winding signals in fixed α-β axes) and then nonlinear coordinates conversion (of two-phase stator winding signals in fixed α-β axes into the two-phase stator winding signals in d-q axes). This reducing also converts 3-6 CR nonlinearities (3 for half-bridge rectifier and 6 for full-bridge rectifier) into two huge nonlinearities in stator windings circuits in d-q axes;
- in SG model there will be 3-6 firstly nonlinear elements, originated from CR, and also nonlinearities of coordinate conversions from α-β into d-q;
- nonlinear elements (3 or 6) go through conversions, which represent the analytical form of structural conversions of nonlinear elements joints in the system;
- converted nonlinear joints’ input and output signals don’t equal the ones defining CR’s thyristor’s VS (diode’s) state, - voltage \( u_{VS} \), current \( i_{VS} \).

From the stated reasons it is clear, that the modeling of SG, working on CR, in PGE leads to the sphere of problems, which are not only unsolved in TAC, but also considered best not be addressed at all. Therefore, in the electric station model working on semiconductor converter, it is justifiable to neglect SG model in PGE in favor of a model with SG stator windings natural phase signals instead – voltages \( u_A, u_B, u_C \) and currents \( i_A, i_B, i_C \).

3. **Conversion of SG model**
The scheme of loaded SG with natural windings on stator and without neutral point output is shown on fig.1 [5].

![Figure 1](image-url)

**Figure 1.** The scheme of SG for three-wire circuit with physical stator and armature windings
On this scheme in all SG windings there are physical currents $i_A$, $i_B$, $i_C$, $i_J$, $i_d$ and $i_q$, armature rotates relative to stator with frequency $\omega_{EL}$. SG for the given scheme is described in the following system of equations:

\[
\begin{align*}
  i_d + i_g + i_c &= 0, \\
  0 &= (R_d + R_{ad})i_d + p\Psi_d, \\
  0 &= (R_g + R_{ag})i_g + p\Psi_g, \\
  0 &= (R_c + R_{ac})i_c + p\Psi_c, \\
  u_f &= R_f \cdot i_f + p\Psi_f, \\
  0 &= R_d \cdot i_d + p\Psi_d, \\
  0 &= R_q \cdot i_q + p\Psi_q,
\end{align*}
\]

System (1), which consists of 12 equations (without consideration of generator’s mechanical part), contains one independent variable – excitation voltage $u_f$ and 12 state variables – currents $i_A, i_B, i_C, i_J, i_d, i_q$ and linkages $\Psi_A, \Psi_B, \Psi_C, \Psi_j, \Psi_d, \Psi_q$. Therefore, the necessary condition of system’s solvability is satisfied.

When calculation of phase currents $i_A, i_B, i_C$ is done, phase voltages can be found from the equations

\[
u_a = (pL_{ad} + R_{ad})i_d.\]

However, self-inductances (with one-letter indexes) and mutual inductances (with two-letter indexes) $L$ in the equations (1) are periodical coefficients of rotation angle $\omega_{EL}$ of armature relative to stator. There is no information on $L$ constant values [6]. The property of $L$ constancy in the PGE model is achieved through the fact, that all SG windings, when reduced to the excitation winding, are motionless relative to each other.

Let’s make a system of equations describing SG as a two-phased machine in random axes in $u-v$ coordinates, which are common for all SG windings (both armature and stator) and rotating in space with random velocity $\omega_K$.

The result of it is a system of equations describing SG as two-phase machine in random axes $u-v$, rotating with velocity $\omega_K$. With any value of velocity $\omega_K$ the inductances $L$ and mutual inductances $M$, through which $\Psi_A, \Psi_B, \Psi_C, \Psi_J, \Psi_d, \Psi_q$ are calculated, will have constant values, since all SG model windings are motionless relative to each other. Moreover, and this is very important, with any choice of coordinate axes $u-v$ and their frequency $\omega_{EL}$, values do not change, only voltage frequencies $(u_{ir}, u_{ir}, u_{qr}, u_{qr})$ do.

Let’s consider coordinate axes $u-v$ incorporated with axes $d-q$ of the excitation winding. Then their rotation frequency $\omega_K$ becomes close to the armature frequency $\omega_{EL}$ (fig.1) [8-9]. After placing $\omega_K = \omega_{EL}$ into the system (2) and changing of indexes $u \rightarrow d$ and $v \rightarrow q$ we derive

\[
\begin{align*}
  u_{id} &= R_d \cdot i_d + p\Psi_{id} - \omega_{2q} \cdot \Psi_q, \\
  u_{iq} &= R_q \cdot i_q + p\Psi_{iq} + \omega_{2q} \cdot \Psi_{id}, \\
  u_i &= R_f \cdot i_q + p\Psi_f, \\
  0 &= R_d \cdot i_d + p\Psi_{2d}, \\
  0 &= R_q \cdot i_q + p\Psi_{2q},
\end{align*}
\]
in which the equation for the excitation winding on the $q$ axis is neglected, since there is literally no such winding, and index $d$ is neglected in the equation for the excitation winding on the $d$ axis.

System (2) is a PGE system, in which SG catalogue data can be used for calculation of the linkages.

Let’s consider [10-12] that coordinate axes $u–v$ coincide with axes $\alpha-\beta$, also axis $\alpha$ coincides with axis $A$. Therefore, rotation frequency $\omega_K$ of coordinate axes $\alpha-\beta$ equals zero. Now the SG model contains fictitious voltages $u_{1\alpha}, u_{1\beta}$ and currents $i_{1\alpha}, i_{1\beta}$ of stator with two windings in $\alpha-\beta$ axes. For modeling of semiconductor converter, working due to SG, it is necessary to operate real voltages $u_A, u_B, u_C$ and currents $i_A, i_B, i_C$ in stator circuit. Since that, it is necessary to convert system, which describes two-phase machine, into three-phase machine through the stator. With conversion of a two-phase machine into three-phase we solve a problem of conversion variables of equivalent two-phase machine stator in coordinate system $\alpha-\beta$ into the real variables of three-phase machine stator in axes $A-B-C$.

The formulas of direct conversion of currents $i_{1\alpha}, i_{1\beta}$ into currents $i_A, i_B, i_C$, which originate from the condition of invariability of SG power, calculated in axes $\alpha-\beta$ and $A-B-C$, are viewed as [4,11]. Let’s change currents $i_{1\alpha}, i_{1\beta}$ and voltages $u_{1\alpha}, u_{1\beta}$ (in $\alpha-\beta$ axes) on currents $i_A, i_B, i_C$ and voltages $u_A, u_B, u_C$ (in axes $A-B-C$) in equations of the system and linkages equations. Considering mechanical part, we derive the following system of SG equations:

\[
\begin{align*}
\Psi_A &= L_{iA} + \frac{2}{3} L_{i2} (t_{2a} + i_{1a}) \\
\Psi_B &= L_{iB} + \frac{1}{\sqrt{2}} L_{i2} (t_{2\beta} + i_{1\beta}) - \frac{1}{\sqrt{6}} L_{i2} (t_{2a} + i_{1a}) \\
\Psi_C &= L_{iC} - \frac{1}{\sqrt{2}} L_{i2} (t_{2\beta} + i_{1\beta}) - \frac{1}{\sqrt{6}} L_{i2} (t_{2a} + i_{1a}) \\
\Psi_{2a} &= L_{i2a} + L_{i2} \left( \frac{3}{2} i_{1a} + i_{1\alpha} \right) \\
\Psi_{2\beta} &= L_{i2\beta} + L_{i2} \left( \frac{1}{\sqrt{2}} (i_{1\beta} - i_{1\alpha}) + i_{1\beta} \right) \\
\Psi_{fA} &= L_{iFA} + L_{i2} \left( i_{2a} + \frac{3}{2} i_{1a} \right) \\
\Psi_{f\beta} &= L_{iFB} + L_{i2} \left( i_{2\beta} + \frac{1}{\sqrt{2}} (i_{1\beta} - i_{1\alpha}) \right) \\
\Delta \omega(t) &= \frac{1}{2 J_e} \int (M_{hex} - M_{el}) dt \\
\omega_{el}(t) &= \Delta \omega(t) + \omega_s
\end{align*}
\]
where $\Delta \omega(t)$ is a difference between armature rotation speed and synchronous speed; $J$ is armature moment of inertia; $M_M$ is mechanical moment; $M_{EL}$ is electrical moment; $\omega_{EL}(t)$ is armature rotation speed; $\omega_S$ is synchronous speed.

**Conclusion**

We derived a full system of equations describing synchronous generator in still axes with the usage of catalogue parameters and operating real currents and voltages of stator circuit (three-phase machine).

This model can be applied in digital control systems with internal model, giving advantage on information processing speed and modeling accuracy, which will be shown in further publications.

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