Tachyonic $\delta$-Tsallis entropy of a thermal tachyonic BIon

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When a brane and an anti-brane come close to each other, the tachyonic potential between them increases and a tachyon wormhole is formed. This configuration, which consists of two branes and a tachyonic wormhole, is called a thermal tachyonic BIon. By considering the thermodynamic behaviour of this system, one finds that its entropy has the same form as that of the Tsallis one. By decreasing the separation between the branes, the tachyonic potential increases, and the entropy grows.

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I. INTRODUCTION

Recently, Tsallis and Cirto have proposed that the entropy of a gravitational system such as a black hole could be generalized to the non-additive entropy, which is given by $S = \gamma A^\beta$, where $A$ is the horizon area $^{[1]}$. There has been lots of discussion on this topic so far. For example, some authors have investigated the limited behaviour of the evolution of the Tsallis entropy in self-gravitating systems. They have argued that the Tsallis entropy generally exhibits a bounded property in a self-gravitating system. This shows the existence of a global maximum of the Tsallis entropy $^{[2]}$. Some other authors have proposed a coherence quantifier in terms of the Tsallis relative entropy, which lays the foundation for non-extensive thermo-statistics and plays the same role as the standard logarithmic entropy does in information theory $^{[3–5]}$. In another consideration, some authors have derived the entropic-force terms from a generalized black-hole entropy proposed by Tsallis and Cirto in order to examine entropic cosmology. Unlike the Bekenstein entropy, which is proportional to area, generalized entropy is proportional to volume because of appropriate nonadditive generalizations $^{[6]}$.

In another work, the relation between the Tsallis entropy and the exchange of energy between the bulk (the universe) and the boundary (the horizon of the universe) has been considered $^{[7]}$. In another investigation, Using the Tsallis entropy, the evolution of the universe in entropic cosmologies has been studied. In this model, the authors have considered an extended entropic-force model that includes a Hubble parameter (H) term and a constant term in entropic-force terms. The H term is derived from a volume entropy, whereas the constant term is derived from an entropy proportional to the square of an area $^{[8]}$. In another research, the evolution of the Tsallis entropy during non-adiabatic processes like the accelerated expansion of the late universe has been considered $^{[9]}$. In addition, the application of this entropy in other aspects of cosmology and physics has been investigated $^{[10–11]}$. And finally, employing the modified entropy-area relation suggested by Tsallis and Cirto and the holographic hypothesis, a new holographic dark energy (HDE) model was proposed $^{[12]}$.

In this paper, we will show that the Tsallis entropy could be produced by a tachyonic potential in a brane-anti-brane system. This potential produces a tachyonic wormhole between branes and leads to the formation of a BIon. A BIon is a configuration which is formed from a brane, an anti-brane and a wormhole which connects them $^{[13–15]}$. By increasing the tachyonic potential, thermodynamically, the behaviour of this BIon changes and its Tsallis entropy grows.

The outline of the paper is as follows. In section II, we will consider the dependency of the area of the BIon on tachyon fields. In section III, we will consider dependency on tachyon fields.

II. DEPENDENCY OF AREA OF A BION ON TACHYONS

In this section, we will show that tachyonic potential produces a wormhole between a brane and an anti-brane. This system which is formed by joining a brane, an anti-brane and a wormhole called a BIon. We obtain the area of

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this tachyonic BIon.

FIG. 1: A set of $D3\overline{D3}$-brane pairs which are placed at points $z_1 = l/2$ and $z_2 = -l/2$ respectively so that the separation between the brane and antibrane is $l$.

To obtain the tachyonic potential and construct a tachyonic black hole in this theory, we consider a set of $D3\overline{D3}$-brane pairs which are placed at points $z_1 = l/2$ and $z_2 = -l/2$, respectively, so that the separation between the brane and antibrane is $l$. Let $z$ be a transverse coordinate to the branes and $\sigma$ be the radius on the world-volume (See figure 1). The induced metric on the brane is:

$$\gamma_{ab} d\sigma^a d\sigma^b = -d\tau^2 + (1 + z'(\sigma)^2) d\sigma^2 + \sigma^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$  \hspace{1cm} (1)

For the simple case of a single $D3\overline{D3}$-brane pair with open string tachyon, the action is [16–19]:

$$S_{tot-extra} = -\tau_3 \int d^6\sigma \sum_{i=1}^{2} V(TA, l) e^{-\theta(\sqrt{-detA})}$$

$$\begin{align*}
(A_i)_{ab} &= \left( g_{MN} - \frac{TA^2}{Q} g_{Mz} g_{Nz} \right) \partial_a x_i^M \partial_b x_i^M + F_{ab} + \frac{1}{2Q} \left( (D_a TA)(D_b TA)^* + (D_a TA)^* (D_b TA) \right) \\
&+ il(\partial_a z_i g_{zz})(TA(D_b TA)^* - TA^*(D_b TA)) + il(TA(D_a TA)^* - TA^*(D_a TA))(\partial_b z_i g_{zz}), \hspace{1cm} (2)
\end{align*}$$
where

\[ Q = 1 + TA^2 l^2 g_{zz}, \]
\[ D_a TA = \partial_a TA - i(A_{2,a} - A_{1,a})TA, V(TA, l) = g_s V(TA)\sqrt{Q}, \]
\[ e^\phi = g_s (1 + \frac{R^4}{2})^{-\frac{1}{2}}, \]

(3)

The quantities \( \phi, A_{2,a} \) and \( D_{ab} \) are the dilaton field, gauge fields and field strengths on the world-volume of the non-BPS brane, respectively; \( TA \) is the tachyon field, \( \tau_3 \) is the brane tension and \( V(TA) \) is the tachyon potential. The indices \( a, b \) denote the tangent directions of the \( D \)-branes, while the indices \( M, N \) run over the background ten-dimensional space-time directions. The \( D_p \)-brane and the anti-\( D_p \)-brane are labeled by \( i = 1 \) and \( 2 \), respectively.

Then the separation between these \( D \)-branes is defined by \( z_2 - z_1 = l \). Also, in writing the above action, we are using the convention \( 2\pi \alpha' = 1 \).

Let us consider the action of a \( D_3 \)-brane and for simplicity, we consider only a \( \sigma \) dependence of the tachyon field \( TA \), and set the gauge fields to zero. In this case, the action (2) in the region that \( r > R \) and \( TA' \sim \text{constant} \) simplifies to

\[ S_{D3} \simeq -\frac{\tau_3}{g_s} \int dt \int d\sigma \sigma^2 V(TA)(\sqrt{D_{1,TA}} + \sqrt{D_{2,TA}}), \]

(4)

where \( D_{1,TA} = D_{2,TA} \equiv D_{TA}, V_3 = \frac{4\pi^2}{3} \) is the volume of a unit sphere \( S^3 \) and

\[ D_{TA} = 1 + \frac{l'(\sigma)^2}{4} + TA^2 l^2, \]

(5)

where the prime denotes a derivative with respect to \( \sigma \). A useful potential that can be used is [16]:

\[ V(TA) = \frac{\tau_3}{\cosh \sqrt{\pi TA}}. \]

(6)

The energy momentum tensor is obtained from the action by calculating its functional derivative with respect to the ten-dimensional background metric \( g_{MN} \). The variation is \( T^{MN} = \frac{2}{\sqrt{-\det g}} \frac{\delta S}{\delta g_{MN}} \). We get [15],

\[ T^0_0^{brane} = V(TA)\sqrt{D_{TA}}, \]

(7)

After doing some calculations and using some approximations, we obtain:

\[ T^0_0^{brane} = \tau_3 + V_{brane}, \]

(8)

where

\[ V_{brane} = \tau_3\left[\frac{\sqrt{\pi TA}}{2}\right][1 + e^{-2\sqrt{\pi TA}}]^{-1} \times \]
\[ \frac{l'(\sigma)^2}{4} + TA^2 l^2 \]

(9)

This potential depends on the tachyon and the separation distance between the two branes. To obtain the dependence of these parameters in terms of time, we should regard the effects of other branes. We will show that when branes come close to each other, the tachyons produce a wormhole which connects the branes and transmits energy from the extra dimensions into our black hole.

Until now, we have considered that the tachyon field grows slowly \( (TA \sim t^4/t^3 = t) \), and we ignored \( TA' = \frac{\partial TA}{\partial \sigma} \) and \( T' = \frac{\partial TA}{\partial t} \) in our calculations. In this section, we show that, with the decrease of the distance between the brane and antibrane black holes, the tachyon field grows very fast and \( TA' \) and \( TA \) cannot be discarded. These dynamics lead to the formation of a new wormhole. In this stage, the black hole evolves from non-phantom phase to a new
phantom phase and consequently, the phantom-dominated era of the black hole accelerates and ends up in the big-rip singularity. In this case, the action (2) is given by the following Lagrangian $L$:

$$L \simeq -\frac{\tau^3}{g_5} \int d\sigma \sigma^2 V(TA)(\sqrt{D_{1,T_A}} + \sqrt{D_{2,T_A}}),$$  \hspace{1cm} (10)$$

where

$$D_{1,T_A} = D_{2,T_A} \equiv D_{TA} = 1 + \frac{\nu'(\sigma)^2}{4} + T'A^2 - T'A^2 + TA'^2,$$  \hspace{1cm} (11)$$

and we assume that $TAl \ll TA'$. Now, we study the Hamiltonian corresponding to the above Lagrangian. In order to derive such a Hamiltonian, we need the canonical momentum density $\Pi = \frac{\partial L}{\partial \dot{T_A}}$ associated with the tachyon, i.e.,

$$\Pi = \frac{V(TA)T'A}{\sqrt{1 + \frac{\nu'(\sigma)^2}{4} + T'A^2 - T'A^2}},$$  \hspace{1cm} (12)$$

so that the Hamiltonian can be obtained as:

$$H_{DBI} = 4\pi \int d\sigma \sigma^2 \Pi T'A - L.$$  \hspace{1cm} (13)$$

By choosing $T'A = 2TA'$, this gives:

$$H_{DBI} = 4\pi \int d\sigma \sigma^2 \left[\Pi(T'A - \frac{1}{2}TA') + \frac{1}{2}TA\partial_\sigma(\Pi\sigma^2) - L\right].$$  \hspace{1cm} (14)$$

In this equation, we have, in the second step, integrated by parts the term proportional to $T'A$, indicating that the tachyon can be studied as a Lagrange multiplier imposing the constraint $\partial_\sigma(\Pi\sigma^2V(TA)) = 0$ on the canonical momentum. Solving this equation yields:

$$\Pi = \frac{\beta}{4\pi\sigma^2},$$  \hspace{1cm} (15)$$

where $\beta$ is a constant.

Using equations (12 and 15), and assuming $(\nu' \ll TA')$, we can obtain $\sigma$ in terms of tachyons:

$$\sigma = \left[\frac{4\pi\sqrt{1 + T'A^2 - T'A'^2}}{\beta V(TA)T'A}\right]^\frac{1}{2}.$$  \hspace{1cm} (16)$$

Taking the derivative of the above equation with respect to time, we obtain the acceleration of system:

$$a = \frac{d^2}{dt^2}\sigma = \frac{d^2}{dt^2}\left[\frac{4\pi\sqrt{1 + T'A^2 - T'A'^2}}{\beta V(TA)T'A}\right]^\frac{1}{2}.$$  \hspace{1cm} (17)$$

The above equation shows that acceleration of the BIon has a direct relation with tachyonic fields which live on it. This acceleration leads to the emergence of a Rindler space-time (See figure 2). In these conditions, the relation between the world volume coordinates of the BIon $(\tau, \sigma)$ and the coordinates of Minkowski space-time $(t, r)$ are [20]:

$$at = e^{\alpha\sigma} \sinh(\alpha\tau) \hspace{1cm} ar = e^{\alpha\sigma} \cosh(\alpha\tau) \hspace{1cm} \text{In Region I}$$

$$at = -e^{-\alpha\sigma} \sinh(\alpha\tau) \hspace{1cm} ar = e^{-\alpha\sigma} \cosh(\alpha\tau) \hspace{1cm} \text{In Region II}$$  \hspace{1cm} (18)$$

Now, we can obtain the metric of a thermal BIon in non-flat space-time. Replacing the acceleration by tachyonic fields in equation (17), we can rewrite equation (18) as:
The above equation shows that tachyonic fields change the coordinates of space-time, leading to acceleration and produce two different regions in a new Rindler space-time. Thus, the metric changes, and a new metric in regions I and II emerge.

Substituting equation (19) in equation (18), we obtain:

\[
\frac{d^2}{dt^2} \left[ \frac{4\pi \sqrt{1 + T^2 - TA^2}}{\beta V(TA)TA} \right]^\frac{1}{2} \sigma = e^{\frac{\triangle}{d\tau} \left[ \frac{4\pi \sqrt{1 + T^2 - TA^2}}{\beta V(TA)TA} \right]^\frac{1}{2} \sigma} \sinh(\alpha \tau) \\
\frac{d^2}{dt^2} \left[ \frac{4\pi \sqrt{1 + T^2 - TA^2}}{\beta V(TA)TA} \right]^\frac{1}{2} \tau = e^{\frac{\triangle}{d\tau} \left[ \frac{4\pi \sqrt{1 + T^2 - TA^2}}{\beta V(TA)TA} \right]^\frac{1}{2} \tau} \cosh(\alpha \tau) \quad \text{In Region I} \\
\frac{d^2}{dt^2} \left[ \frac{4\pi \sqrt{1 + T^2 - TA^2}}{\beta V(TA)TA} \right]^\frac{1}{2} \sigma = e^{-\frac{\triangle}{d\tau} \left[ \frac{4\pi \sqrt{1 + T^2 - TA^2}}{\beta V(TA)TA} \right]^\frac{1}{2} \sigma} \sinh(\alpha \tau) \\
\frac{d^2}{dt^2} \left[ \frac{4\pi \sqrt{1 + T^2 - TA^2}}{\beta V(TA)TA} \right]^\frac{1}{2} \tau = e^{-\frac{\triangle}{d\tau} \left[ \frac{4\pi \sqrt{1 + T^2 - TA^2}}{\beta V(TA)TA} \right]^\frac{1}{2} \tau} \cosh(\alpha \tau) \quad \text{In Region II} \\
\quad \text{(19)}
\]

\[
ds^2_{1A,\text{thermal}} = D_1^{\frac{1}{2}} - A H_{I-A}^{\frac{1}{2}} f_{I-A} \times \\
\left( e^{\frac{\triangle}{d\tau} \left[ \frac{4\pi \sqrt{1 + T^2 - TA^2}}{\beta V(TA)TA} \right]^\frac{1}{2} \sigma} \sinh^2 \left( (\frac{d\tau}{d\sigma})^2 \left[ \frac{4\pi \sqrt{1 + T^2 - TA^2}}{\beta V(TA)TA} \right]^\frac{1}{2} \right) \right) (dz)^2 (d\tau)^2 - \\
\left( e^{\frac{\triangle}{d\tau} \left[ \frac{4\pi \sqrt{1 + T^2 - TA^2}}{\beta V(TA)TA} \right]^\frac{1}{2} \sigma} \cosh^2 \left( (\frac{d\tau}{d\sigma})^2 \left[ \frac{4\pi \sqrt{1 + T^2 - TA^2}}{\beta V(TA)TA} \right]^\frac{1}{2} \right) \right) (dz)^2 (d\sigma)^2 + \\
\left( \frac{d\sigma}{d\tau} \right)^2 (dz) (d\sigma) (d\tau) (d\sigma) + \\
D_1^{\frac{1}{2}} - A H_{I-A} \left( \frac{1}{\beta V(TA)TA} \right) e^{\frac{\triangle}{d\tau} \left[ \frac{4\pi \sqrt{1 + T^2 - TA^2}}{\beta V(TA)TA} \right]^\frac{1}{2} \sigma} \cosh \left( (\frac{d\tau}{d\sigma})^2 \left[ \frac{4\pi \sqrt{1 + T^2 - TA^2}}{\beta V(TA)TA} \right]^\frac{1}{2} \right) \right)^2 \\
\left( d\theta^2 + \sin^2 \theta d\phi^2 \right) + \\
D_1^{\frac{1}{2}} - A H_{I-A} \sum_{i=1}^5 dz_i \\
\quad \text{(20)}
\]
\[ D_{II^{-A}}^{-\frac{1}{2}} H_{II^{-A}}^{-\frac{1}{2}} (\frac{1}{d^2} \frac{d^2}{dt^2} \frac{[4\pi \sqrt{1+T A^2 - T A'^2}]}{\beta V(TA)TA} \frac{1}{2}) \cosh((\frac{d^2}{dt^2} \frac{[4\pi \sqrt{1+T A^2 - T A'^2}]}{\beta V(TA)TA} \frac{1}{2}) \tau)^2 (d\theta^2 + \sin^2 \theta d\phi^2) + \]

\[ D_{II^{-A}}^{-\frac{1}{2}} H_{II^{-A}}^{-\frac{1}{2}} \sum_{i=1}^{5} \alpha_i^2 \]

where

\[ f_{I^{-A}} = 1 - \left( e^{\frac{1}{dt^2} \frac{[4\pi \sqrt{1+T A^2 - T A'^2}]}{\beta V(TA)TA} \frac{1}{2}} \sigma_0 \cosh((\frac{d^2}{dt^2} \frac{[4\pi \sqrt{1+T A^2 - T A'^2}]}{\beta V(TA)TA} \frac{1}{2}) \tau_0)^4 \right) \]

\[ f_{III^{-A}} = 1 - \left( e^{\frac{1}{dt^2} \frac{[4\pi \sqrt{1+T A^2 - T A'^2}]}{\beta V(TA)TA} \frac{1}{2}} \sigma_0 \cosh((\frac{d^2}{dt^2} \frac{[4\pi \sqrt{1+T A^2 - T A'^2}]}{\beta V(TA)TA} \frac{1}{2}) \tau_0)^4 \right) \]

\[ H_{I^{-A}} = 1 + \left( e^{\frac{1}{dt^2} \frac{[4\pi \sqrt{1+T A^2 - T A'^2}]}{\beta V(TA)TA} \frac{1}{2}} \sigma_0 \cosh((\frac{d^2}{dt^2} \frac{[4\pi \sqrt{1+T A^2 - T A'^2}]}{\beta V(TA)TA} \frac{1}{2}) \tau_0)^4 \right) \]

\[ H_{III^{-A}} = 1 + \left( e^{\frac{1}{dt^2} \frac{[4\pi \sqrt{1+T A^2 - T A'^2}]}{\beta V(TA)TA} \frac{1}{2}} \sigma_0 \cosh((\frac{d^2}{dt^2} \frac{[4\pi \sqrt{1+T A^2 - T A'^2}]}{\beta V(TA)TA} \frac{1}{2}) \tau_0)^4 \right) \]

\[ D_{I^{-A}} = \cos^2 \epsilon_{I^{-A}} + \sin^2 \epsilon_{I^{-A}} H_{I^{-A}}^{-\frac{1}{2}} \]

\[ D_{III^{-A}} = \cos^2 \epsilon_{III^{-A}} + \sin^2 \epsilon_{III^{-A}} H_{III^{-A}}^{-\frac{1}{2}} \]

and

\[ \cosh^2 \alpha_{I^{-A}} = \frac{3 \cos \frac{\delta_{I^{-A}}}{3} + \sqrt{3} \cos \frac{\delta_{I^{-A}}}{3}}{2} \]

\[ \cos \epsilon_{I^{-A}} = \frac{1}{\sqrt{1 + \left( e^{\frac{1}{dt^2} \frac{[4\pi \sqrt{1+T A^2 - T A'^2}]}{\beta V(TA)TA} \frac{1}{2}} \sigma_0 \cosh((\frac{d^2}{dt^2} \frac{[4\pi \sqrt{1+T A^2 - T A'^2}]}{\beta V(TA)TA} \frac{1}{2}) \tau_0)^4 \right) \}} \]

\[ \cosh^2 \alpha_{III^{-A}} = \frac{3 \cos \frac{\delta_{III^{-A}}}{3} + \sqrt{3} \cos \frac{\delta_{III^{-A}}}{3}}{2} \]

\[ \cos \epsilon_{III^{-A}} = \frac{1}{\sqrt{1 + \left( e^{\frac{1}{dt^2} \frac{[4\pi \sqrt{1+T A^2 - T A'^2}]}{\beta V(TA)TA} \frac{1}{2}} \sigma_0 \cosh((\frac{d^2}{dt^2} \frac{[4\pi \sqrt{1+T A^2 - T A'^2}]}{\beta V(TA)TA} \frac{1}{2}) \tau_0)^4 \right) \}} \]

The angles \( \delta_{I^{-A}} \) and \( \delta_{III^{-A}} \) are defined by:

\[ \cos \delta_{I^{-A}} = T_{0,I^{-A}} \frac{K^2}{\alpha \left( e^{\frac{1}{dt^2} \frac{[4\pi \sqrt{1+T A^2 - T A'^2}]}{\beta V(TA)TA} \frac{1}{2}} \sigma_0 \cosh((\frac{d^2}{dt^2} \frac{[4\pi \sqrt{1+T A^2 - T A'^2}]}{\beta V(TA)TA} \frac{1}{2}) \tau_0)^4 \right) \}

\[ T_{0,I^{-A}} = \frac{9\pi^2 N}{4\sqrt{3}T_D^3} \]

(26)
\[ \cos \delta_{II-A} = T_{0,II-A}^4 \left[ 1 + \frac{K^2}{\left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}}} - 1 \right] e^{\frac{K^2}{\beta V(TA)TA}} \cosh\left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right) \right)^4 \]

\[ T_{0,II-A} = \left( \frac{9\pi^2 N}{4\sqrt{3}T_3} \right)^{\frac{1}{2}} T_{0,II-A} \]  

where \( T_0 \) is the temperature of the Blon in non-Rindler space-time. The above equations show that the metric of the thermal Blon depends on the evolutions of the tachyonic fields. In fact, the evolution of tachyonic fields has a direct effect on the thermodynamics of the Blon. Following the method in [20], we can obtain the separation distance between two manifolds in a 5-dimensional Blon as:

\[
d z_{I-A} = d z_{I-B} \approx \left( e^{-4\left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}}} \sinh^2 \left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}} \right) \cosh^2 \left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}} \right) \times \]

\[
F_{DBI,I,A}(\tau, \sigma) \left( F_{DBI,I,A}(\tau, \sigma_0) - e^{-4\left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}}} \sinh^2 \left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}} \right) \cosh^2 \left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}} \right) \times \]

\[
F_{DBI,I,A}(\tau_0, \sigma) \left( F_{DBI,I,A}(\tau_0, \sigma_0) - e^{-4\left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}}} \sinh^2 \left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}} \right) \cosh^2 \left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}} \right) \times \]

\[
\sinh^2 \left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}} \left( \tau - \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}} \right) \left( \sinh^2 \left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}} \right) \times \]

\[
(28)
\]

or

\[
d z_{I-B} = d z_{II-A} \approx \left( e^{-4\left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}}} \sinh^2 \left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}} \right) \cosh^2 \left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}} \right) \times \]

\[
F_{DBI,I,A}(\tau, \sigma) \left( F_{DBI,I,A}(\tau, \sigma_0) - e^{-4\left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}}} \sinh^2 \left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}} \right) \cosh^2 \left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}} \right) \times \]

\[
F_{DBI,I,A}(\tau_0, \sigma) \left( F_{DBI,I,A}(\tau_0, \sigma_0) - e^{-4\left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}}} \sinh^2 \left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}} \right) \cosh^2 \left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}} \right) \times \]

\[
\sinh^2 \left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}} \left( \tau - \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}} \right) \left( \sinh^2 \left( \frac{\beta V(TA)TA}{\beta V(TA)TA} \right)^{\frac{1}{2}} \right)^{\frac{1}{2}} \right) \times \]

\[
(29)
\]

with the definition of \( F_{DBI,I,A} \) given below:

\[
F_{DBI,I,A} = F_{DBI,I,B} = \left( d^2 \left[ 4\pi \sqrt{1+TA^2-TA^2} \right]^2 \right) - 1 \right] e^{\frac{4\pi \sqrt{1+TA^2-TA^2}}{\beta V(TA)TA}} \cosh\left( \frac{4\pi \sqrt{1+TA^2-TA^2}}{\beta V(TA)TA} \right) \right)^4 \times
\]

\[
F_{DBI,II,A} = F_{DBI,II,B} = \left( d^2 \left[ 4\pi \sqrt{1+TA^2-TA^2} \right]^2 \right) - 1 \right] e^{\frac{4\pi \sqrt{1+TA^2-TA^2}}{\beta V(TA)TA}} \cosh\left( \frac{4\pi \sqrt{1+TA^2-TA^2}}{\beta V(TA)TA} \right) \right)^4 \times
\]

\[
(30)
\]
These separation distances depend on the tachyonic fields and temperature. When the separation distance in one region grows, the separation distance in the other region shrinks. Now, we calculate the area of a thermal Blon by using equations (28), (29) and (19):

\[
A_{I-A} = A_{II-B} = \int \pi r_{I-A}^2 dz_{I-A} = \int \pi r_{I-B}^2 dz_{II-B} = \\
\int d\sigma \left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right) \cosh \left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right) \times \\
\left( e^{-4\left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right)} \sinh^2 \left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right) \right) \\
F_{DBI,I,A}(\tau, \sigma) \left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right) = \\
\left( e^{-4\left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right)} \sinh^2 \left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right) \right) \\
F_{DBI,I,A}(\tau_0, \sigma) = \\
\left( e^{-4\left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right)} \sinh^2 \left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right) \right) \sinh^2 \left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right) \right) \\
(30)
\]

or

\[
A_{I-A} = A_{I-B} = \int \pi r_{I-A}^2 dz_{I-A} = \int \pi r_{I-B}^2 dz_{I-B} = \\
\int d\sigma \left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right) \cosh \left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right) \times \\
\left( e^{-4\left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right)} \sinh^2 \left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right) \right) \\
F_{DBI,I,A}(\tau, \sigma) \left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right) = \\
\left( e^{-4\left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right)} \sinh^2 \left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right) \right) \\
F_{DBI,I,A}(\tau_0, \sigma) = \\
\left( e^{-4\left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right)} \sinh^2 \left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right) \right) \sinh^2 \left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right) \right) \sinh^2 \left( \frac{d^2}{dt^2} \left[ 4\pi \sqrt{1 + T^2 A^2 - TA^2} \right] \beta V(T)TA \right) \right) \\
(32)
\]

The above equations show that the area of the thermal accelerating Blons depends on the tachyonic fields which live on them. These fields lead to the acceleration of the Blon. This acceleration produces a Rindler space-time with two regions. By increasing the strength of the tachyonic fields, the area of a Blon in region I expands, while the area of a Blon in region II contracts.

III. DEPENDENCY OF THE TSALLIS ENTROPY OF A BION ON TACHYONS

In this section, we will consider the effect of tachyonic fields on the entropy of the Blon. We will show that tachyonic fields lead to the expansion of the Blon and increasing the entropy of the Blon in one region and decreasing the entropy in the other region. Previously, thermodynamical parameters like the entropy have been obtained in [13] [15] [20]. Using those relations and replacing the acceleration by tachyonic fields in equation (19), we obtain:
similar to the Tsallis entropy and could be written as regions in each of which a tachyonic BIon lives. We have shown that the entropy of these BIons includes some terms potential leads to the acceleration of branes and emergence of a Rindler space-time. This space-time includes two

...potential and temperature of the system. In one region, by increasing the area, the entropy increases, while in another region by increasing the area, the entropy decreases. In fact, thermodynamically, the behaviour of the BIon in each region is opposite to the behaviour of a BIon in another region.

\[ dS_{1-A} = dS_{1-B} = \frac{4T_{D3}^2}{\pi T_{0,I-A}^5} F_{DBI, I,A}(\sigma, \tau) \left( \frac{1}{\left[ \frac{d^2}{d\tau^2} \left[ \frac{4 \pi \sqrt{1 + T A^2 - T A^2}}{\beta V(TA) TA} \right] \right]^{\frac{1}{2}}} \right) e^{-\left[ \frac{d^2}{d\tau^2} \left[ \frac{4 \pi \sqrt{1 + T A^2 - T A^2}}{\beta V(TA) TA} \right] \right]^{\frac{1}{2}}} \sigma \cosh(\alpha \tau)^2 \times \]

\[ \left( \sinh^2(\left[ \frac{d^2}{d\tau^2} \left[ \frac{4 \pi \sqrt{1 + T A^2 - T A^2}}{\beta V(TA) TA} \right] \right]^{\frac{1}{2}}) + \cosh^2(\left[ \frac{d^2}{d\tau^2} \left[ \frac{4 \pi \sqrt{1 + T A^2 - T A^2}}{\beta V(TA) TA} \right] \right]^{\frac{1}{2}}) \right) \times \]

\[ \sqrt{F_{DBI, I,A}(\sigma, \tau) - F_{DBI, I,A}(\sigma_0, \tau)} \]

(33)

and

\[ dS_{1-A} = dS_{1-B} = \frac{4T_{D3}^2}{\pi T_{0,II-A}^5} F_{DBI, II,A}(\sigma, \tau) \left( \frac{1}{\left[ \frac{d^2}{d\tau^2} \left[ \frac{4 \pi \sqrt{1 + T A^2 - T A^2}}{\beta V(TA) TA} \right] \right]^{\frac{1}{2}}} \right) e^{-\left[ \frac{d^2}{d\tau^2} \left[ \frac{4 \pi \sqrt{1 + T A^2 - T A^2}}{\beta V(TA) TA} \right] \right]^{\frac{1}{2}}} \sigma \cosh(\alpha \tau)^2 \times \]

\[ \left( \sinh^2(\left[ \frac{d^2}{d\tau^2} \left[ \frac{4 \pi \sqrt{1 + T A^2 - T A^2}}{\beta V(TA) TA} \right] \right]^{\frac{1}{2}}) + \cosh^2(\left[ \frac{d^2}{d\tau^2} \left[ \frac{4 \pi \sqrt{1 + T A^2 - T A^2}}{\beta V(TA) TA} \right] \right]^{\frac{1}{2}}) \right) \times \]

\[ \sqrt{F_{DBI, II,A}(\sigma, \tau) - F_{DBI, II,A}(\sigma_0, \tau)} \]

(34)

The above equations show that the entropy of a thermal BIon depends on tachyonic fields which live on it. These fields lead to the acceleration of a thermal BIon. Due to this acceleration, a Rindler horizon emerges and two regions appear. The entropy of the BIon in one region is opposite to that in another region. This means that by increasing the entropy of the BIon in one region, the entropy of a BIon in another region decreases. Also, the entropy of the BIon in one end is opposite to the entropy of the BIon in the other end in that region. Consequently, the entropy of brane A in region I acts oppositely to the entropy of brane A in region II, and also acts similar to the entropy of brane B in region II.

Now, comparing the entropies in equations (31) and (32) with the areas in equations (33) and (34), we obtain the relation between entropy and area:

\[ S_{I-A} = S_{I-B} = [1 + \left( \frac{4T_{D3}^2}{\pi T_{0,I-A}^5} \right)^{-1} \tanh^2(\left[ \frac{d^2}{d\tau^2} \left[ \frac{4 \pi \sqrt{1 + T A^2 - T A^2}}{\beta V(TA) TA} \right] \right]^{\frac{1}{2}}) \frac{\sigma}{2} A^2 \]

(35)

\[ S_{II-A} = S_{II-B} = [1 + \left( \frac{4T_{D3}^2}{\pi T_{0,II-A}^5} \right)^{-1} \coth^2(\left[ \frac{d^2}{d\tau^2} \left[ \frac{4 \pi \sqrt{1 + T A^2 - T A^2}}{\beta V(TA) TA} \right] \right]^{\frac{1}{2}}) \frac{\sigma}{2} A^{-\frac{5}{2}} \]

(36)

The above equations show that entropies have direct relations with the areas of a BIon. This is in good agreement with the prediction of Tsallis whereby entropy should have the form \( S = \gamma A^3 \). These entropies depend on the tachyonic potential and temperature of the system. In one region, by increasing the area, the entropy increases, while in another region by increasing the area, the entropy decreases. In fact, thermodynamically, the behaviour of the BIon in each region is opposite to the behaviour of a BIon in another region.

IV. SUMMARY

In this research, we have shown that the tachyonic potential between branes and anti-branes leads to the emergence of a wormhole between them. This wormhole and two branes form a tachyonic BIon. In this type of BIon, the tachyonic potential leads to the acceleration of branes and emergence of a Rindler space-time. This space-time includes two regions in each of which a tachyonic BIon lives. We have shown that the entropy of these BIons includes some terms similar to the Tsallisis entropy and could be written as \( S = \gamma A^3 \), where A is the area of the BIon. By increasing the tachyonic potential, the entropy of a BIon in one region increases, while the entropy of a BIon in another region decreases.
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