Non-spherical collapse of a two fluid star

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We obtain the analogue of collapsing Vaidya-like solution to include both a null fluid and a string fluid, with a linear equation of state \((p_\perp = k \rho)\), in non-spherical (plane symmetric and cylindrically symmetric) anti-de Sitter space-times. It turns out that the non-spherical collapse of two fluid in anti-de Sitter space-times, in accordance with cosmic censorship, proceed to form black holes, i.e., on naked singularity ever forms, violating hoop conjecture.

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I. INTRODUCTION

It is well known that the general relativity admits solutions with singularity, and that such solutions can be produced by non-singular initial data. The singularity theorems revealed that the occurrence of singularities is a generic property of space-times in classical general relativity (GR). These theorems however can not predict the final state of such a singularity. However, there is common belief that a cosmic censor exists who safely hides the singularity inside a black hole, actually this is a famous cosmic censorship conjecture (CCC), first formulated by Penrose. The CCC remains as one of the most outstanding unresolved question in GR. However, there are many known examples in the literature showing that both naked singularities and black holes can form in gravitational collapse. The central shell focusing singularity can be naked or covered depending upon the choice of initial data. There is a critical branch of solution where a transition from naked singularity to black hole occurs. In particular gravitational collapse of spherical matter in the form of radiation (null fluid) described by Vaidya metric is well studied for investigating CCC in four dimension as well as in higher dimensions. These counterexamples are restricted to spherical symmetry. Are these singularities an accident of spherical symmetry?

For the non-spherical collapse, Thorne has proposed hoop conjecture: that collapse will yield a black hole only if a mass \(M\) is compressed to a region with circumference \(C \leq 4\pi M\) in all directions. If hoop conjecture is true, naked singularities may form if collapse can yield \(C > 4\pi M\) in some direction. Thus, planar or cylindrical matter will not form a black hole (black plane or black string). Indeed, Shapiro and Teukolsky showed that collapse of a prolate spheroid leads to a spindle singularity without horizon, i.e., a naked singularity may form in non-spherical relativistic collapse. Also, hoop conjecture was given for space-times with a zero cosmological term and in the presence of negative cosmological term one can expect the occurrence of major changes. It is clear that, in spherically symmetric, the effect of adding negative cosmological term does not alter the final fate of collapse. However, Lemos has shown that planar or cylindrical black holes form rather than naked singularity from gravitational collapse of a planar or cylindrical matter distribution (null fluid) in an anti-de Sitter space-time, violating in this way the hoop conjecture but not CCC. Here the negative cosmological constant plays crucial role as in the BTZ black holes.

Several generalization of the Vaidya solution in which the source is a mixture of fluids and radiation have been obtained. Recently, Glass and Krisch came up with a generalized Vaidya solution for a two fluids: a null fluid with a string fluid. They show that by allowing the Swarszchild mass as a function of retarded time creates an atmosphere for two fluid: Vaidya radiation fluid in addition to a string fluid. The solution is very important in view of recent links between black holes and string theories. Further, the string is very important ingredient in many physical theories and idea of string is fundamental to superstring theories. The solution has been employed to look into the consequence of string fluid on the formation of naked singularities in Vaidya collapse. The effect of string is a shrinkage of the naked singularity initial data space, or an enlargement of the black hole initial data space of Vaidya collapse.

In this paper, we shall first obtain the general solution for a two-fluid: a null fluid and a string fluid, with the equation of state \((p_\perp = k \rho \text{ and } \rho = \rho_0)\), in non-spherical (plane symmetric and cylindrical symmetric) anti-de Sitter space-times. We then see how the results that were presented in get modified in the presence of the string fluid.

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We find that the non-spherical collapse, in contrast to spherically symmetric collapse where naked singularity inevitable, lead to black holes, an accordance with CCC, giving explicit counterexample to hoop conjecture.

II. NON-SPHERICAL TWO FLUID MODEL

In this section, we extend the Glass and Krisch solutions [15] to non-spherical (plane symmetric and cylindrical symmetric) anti-de Sitter space-times. Let us first consider the case of plane symmetry. The metric of general plane symmetric space-time, expressed in terms of Eddington advanced time coordinate (outgoing coordinate) \( v \), reads:

\[
\begin{align*}
\mathbf{ds}^2 &= -e^{\psi(v,r)} dv \left[ e^{\psi(v,r)} f(v,r) dv + 2 dr \right] + r^2 \alpha^2 (dx^2 + dy^2),
\end{align*}
\]

where \(-\infty \leq x, y \leq \infty\) are coordinate which describe two-dimensional zero-curvature space which has topology \( R \times R \), \(-\infty \leq v \leq \infty\) is null coordinate called the retarded time, and \( 0 \leq r \leq \infty\) is the radial coordinate. Further, \( e^{\psi(v,r)}\) is an arbitrary function and where \( 3\alpha^2 = -\Lambda > 0\) denote negative cosmological constant. It is useful to introduce a local mass function \( m(v,r) \) defined by \( f = 1 - 2m(v,r)/r \). For \( f = m(v)/r \) and \( \psi = 0\), the metric reduces to the plane symmetric Vaidya-like metric [22, 23]. Initially \( f = M_0/r \) (with \( \psi = 0\)) provides the vacuum Taub solution [24].

It is the field equation \( G^a_0 = 0\) that leads to \( e^{\psi(v,r)} = g(v)\). However, by introducing another null coordinate \( e^{\psi(v,r)} = g(v)\), we can always set without the loss of generality, \( \psi(v,r) = 0\). Hence, the metric takes the form:

\[
\mathbf{ds}^2 = - \left[ 1 - \frac{2m(v,r)}{r} \right] dv^2 + 2dvdr + \alpha^2 r^2 (dx^2 + dy^2).
\]

The use of a Newman-Penrose null tetrad formulism leads to Einstein tensor of the form [15, 16]:

\[
\Sigma_{ab} = -2\Psi_{11}(l_a n_b + l_b n_a + m_a \overline{n}_b + \overline{m}_a n_b) - 2\Psi_{11} l_a n_b - 6\Lambda..
\]

Here the null tetrad Ricci scalars are

\[
\Psi_{11} = \frac{1}{r^2} \left[ 2 \frac{\partial m}{\partial r} - r \frac{\partial^2 m}{\partial r^2} - 1 \right],
\]

\[
\Psi_{22} = -\frac{1}{r^2} \frac{\partial m}{\partial v},
\]

\[
\Lambda = \frac{1}{r^2} \left[ 2r \frac{\partial^2 m}{\partial r^2} - 2 \frac{\partial m}{\partial r} - 1 \right],
\]

the principal null geodesic vectors are \( l_a, n_a \) of the form

\[
l_a = \delta^u_a, \quad n_a = f/2\delta^u_a - \delta^r_a,
\]

where \( l_a l^a = n_a n^a = 0\), \( l_a n^a = -1\). The metric [22] admits an orthonormal basis defined by four unit vectors

\[
\hat{u}_a = f^{1/2} \delta^u_a - f^{-1/2} \delta^r_a, \quad \hat{r}_a = f^{-1/2} \delta^r_a,
\]

\[
\hat{x}_a = \alpha x \delta^x_a, \quad \hat{y}_a = \alpha y \delta^y_a,
\]

where \( \hat{u}_a \) is a timelike unit vector and \( \hat{r}_a, \hat{x}_a, \hat{y}_b \) are unit space-like vector such that

\[
g_{ab} = \hat{u}_a \hat{u}_b - \hat{r}_a \hat{r}_b - \hat{x}_a \hat{x}_b - \hat{y}_a \hat{y}_b.
\]

Associated with the string world sheet we have the string bivector defined by

\[
\Sigma^{ab} = \epsilon^{AB} \frac{dx^a}{dk^A} \frac{dx^b}{dk^B},
\]

where \( \epsilon^{AB} \) is two dimensional Levi-Civita symbol. It is useful to write the bivector, in terms of the unit vectors, as

\[
\Sigma^{ab} = \hat{r}^a \hat{u}^b - \hat{u}^a \hat{r}^b,
\]

and the condition that the worldsheet are timelike, i.e., \( \gamma = 1/2 \Sigma^{ab} \Sigma_{ab} < 0 \) implies that only the \( \Sigma^{ur} \) component is non-zero, therefore one obtains:

\[
\Sigma^{ur} \Sigma_{ur} = \hat{u}^a \hat{u}^b - \hat{r}^a \hat{r}^b.
\]
The string energy-momentum tensor for a cloud of string, by analogy with the one for the perfect fluid, is written as 

\[ T^{(s)}_{ab} = \rho \Sigma^c \Sigma_{cb} - p \perp h_{ab}, \]  

(14)

The energy-momentum of two fluid system is \( T_{ab} = T^{(n)}_{ab} + T^{(s)}_{ab} \), where

\[ T^{(n)}_{ab} = \psi l_a l_b. \]  

(15)

It is the null fluid tensor corresponding to the component of the matter field that moves along the null hypersurfaces \( v = \text{const} \). The effective energy momentum tensor for two fluid system, in terms of the unit vectors, can be cast as:

\[ T_{ab} = \psi l_a l_b + \rho \hat{a} u_b + p v \hat{r} + p \perp (\hat{x}_a \hat{x}_b + \hat{y}_a \hat{y}_b). \]  

(16)

For \( \rho = p_r = p_\perp = 0 \), Eq. (16) reduces to stress-energy tensor which gives Vaidya metric \[22, 23\]. Comparing Eq. (3) and Eq. (16), the Einstein field equations now take the form:

\[ \psi = \frac{1}{4\pi r^2} \frac{\partial m}{\partial v}, \]  

(17)

\[ \rho = -p_r = -\frac{1}{8\pi r^2} + \frac{1}{4\pi r^2} \frac{\partial m}{\partial r} + \frac{1}{8\pi} 3\alpha^2, \]  

(18)

\[ p_\perp = -\frac{1}{8\pi r} \frac{\partial^2 m}{\partial r^2} - \frac{1}{8\pi} 3\alpha^2. \]  

(19)

Next, we construct a model by assuming the tangential pressure \( P_\perp \) to energy density \( \rho \) by a linear equation of state \( p_\perp = k\rho \) \[22\], which leads to

\[ \frac{\partial^2 m}{\partial r^2} + \frac{2k}{r} \frac{\partial m}{\partial r} + 3\alpha^2(1 + k) = \frac{k}{r}. \]  

(20)

The integration of the this equation reads:

\[ m(v, r) = \begin{cases} r/2 + M(v) + \frac{S(v) r^{1-2k}}{1-2k} - \alpha^2 r^3/2, & \text{if } k \neq 1/2, \\ r/2 + M(v) + S(v) \ln r - \alpha^2 r^3/2 & \text{if } k = 1/2. \end{cases} \]  

(21)

where \( M(v) \) and \( S(v) \) are arbitrary functions of integration and may not be related, and \( M(u) \) can be treated as Vaidya-mass and \( S(v) \) as contribution from string fluid. The class of solution discussed above, in general, belongs to Type II fluid defined in \[1\]. When \( m = m(r) \), we have \( \psi = 0 \), and the matter field degenerates to type I fluid \[12\]. In the rest frame associated with the observer, the energy-density of the matter will be given by,

\[ \psi = T^r_v, \quad \rho = -T^i_i = -T^r_r, \]  

(22)

and the principal pressures are \( P_i = T^i_i \) (no sum convention). Therefore \( P_r = T^r_r = -\rho \) and \( P_x = P_y = k\rho \).

The weak energy conditions (WEC): The energy momentum tensor obeys inequality \( T_{ab} u^a u^b \geq 0 \) for any timelike vector, i.e.,

\[ \psi \geq 0, \quad \rho \geq 0, \quad P_x \geq 0, \quad P_y \geq 0. \]  

(23)

However, \( \psi > 0 \) gives the restriction on the choice of the functions \( M(v) \) and \( S(v) \). We observe \( \psi > 0 \) requires,

\[ \frac{\partial M}{\partial v} + \frac{\partial S}{\partial v} r^{1-2k} > 0. \]  

(24)

This, in general, is satisfied as long as both \( \partial M/\partial v \) and \( \partial S/\partial v \) are greater than zero. Furthermore, to guarantee the positivity of the energy density, we must have \( k < 1/2 \) and \( S(v) > 0 \), or \( k > 1/2 \) and \( S(v) < 0 \). For \( k = 1/2 \), weak energy condition can not be satisfied for all \( r \) \[13\]. Therefore, we shall not consider this case further.
In summary, we have shown that the metric:

\[ ds^2 = -\left(\alpha^2 r^2 - \frac{2M(v)}{r} - \frac{2S(v)}{(1 - 2k)r^{2k}}\right)dv^2 + 2dvdr + \alpha^2 r^2(dx^2 + dy^2). \] (25)

is solution of the Einstein equations for the energy momentum tensor \( T_{\mu\nu} \) with equation of state \( p_\perp = k\rho \). Thus we have extended the collapsing plane symmetric Vaidya-like metric \[22, 23\] to include both a null fluid and a string fluid and we shall call it generalized plane symmetric Vaidya metric. The metric \[10\] is solution of Einstein equation for two fluid system: a null fluid and a string fluid in plane symmetric anti-de Sitter space-times. This metric was also obtained by Cai et al. \[24\] by treating the null fluid to behave like perfect fluid. The physical quantities for this metric are given by

\[ \psi = \frac{1}{4\pi r^2} \left( \frac{\partial M}{\partial v} + \frac{\partial S}{\partial v} \right) r^{1-2k}, \] (26)

\[ \rho = \frac{1}{4\pi} \frac{S(v)}{r^{2k}} = -p_r, \] (27)

\[ p_\perp = k\rho \] (28)

There are special cases of this solution which are already known: One is plane symmetric anti-de Sitter Vaidya metric \[30\], which arises for \( S(v) = 0 \) (vanishing \( \rho \) and \( p_\perp \)). When \( k = 1 \) and \( 2S(v) = -Q^2(v) \), the solution \[24\] reduces to plane symmetric Bonnor-Vaidya anti-de Sitter metric \[24\]. The Riemann anti-de Sitter metric arises when, in addition to \( S(v) = 0 \), \( M(v) = M_0 \) is a constant. The vacuum background (\( M = S = 0 \)) to Eq. (25) is anti-de Sitter space-time.

**Horizons:** As demonstrated by York \[27\], horizons can be obtained by noting that (i) apparent horizons are defined as surface such that \( \Theta \approx 0 \) and (ii) event horizons are surfaces such that \( d\Theta/dv \approx 0 \), where \( \Theta \) is expansion. For the metric \[24\], we have

\[ \Theta = \frac{1}{r} \left[ \alpha^2 r^2 - \left( \frac{2M(v)}{r} - \frac{2S(v)}{(1 - 2k)r^{2k}} \right) \right]. \] (29)

Since the York conditions require that at apparent horizons \( \Theta \) vanish, it follows form the Eq. (29) that apparent horizons will satisfy

\[ \alpha^2 r^3 - \frac{2S(v)}{1 - 2k} - 2M = 0, \] (30)

which in general has positive solutions and hence final configuration would be of multiple horizon depending on the value of \( k \).

**Cylindrically symmetric space-time:** We turn our attention to the cylindrical symmetric anti-de Sitter space-time. The metric \[24\] in the Cylindrical space-time has the form:

\[ ds^2 = -\left(\alpha^2 r^2 - \frac{2M(v)}{r} - \frac{S(v)}{(1 - 2k)r^{2k}}\right)dv^2 + 2dvdr + r^2 d\theta^2 + \alpha^2 r^2 dz^2, \] (31)

where \( \infty < v, z < \infty, 0 \leq r < \infty \) and \( 0 \leq \theta \leq 2\pi \). The two dimensional surface has topology of \( R \times S^1 \). The analysis given above, with suitable modifications, is valid in the cylindrical anti-de Sitter space-time as well. Hence, to conserve space, we avoid repetition of the detailed analysis (being similar to plane symmetric case). Hence, we shall limit ourself to plane symmetric space-time in the further analysis.

### III. GRAVITATIONAL COLLAPSE

To determine the solution completely, the boundary condition must be imposed, as well as the specification of the space-time in the vacuum region exterior to star. The initial radius of star from which collapse begin would be given by \( p_r = 0 \) which would demand that the function \( S(v) = 0 \). The Kretschmann scalar \( K = R_{abcd}R^{abcd} \), \( R_{abcd} \) is the Riemann tensor for the metric \[11\] reduces to

\[ K = \frac{48}{r^6} M^2(v) + \frac{16K_1}{r^{4k+4}} S^2(v) + \frac{32K_2}{r^{5+2k}} M(v) S(v) + \frac{16\alpha^2 K_3}{r^{2k+2}} S(v) + 24\alpha^4. \] (32)
Here, $K_1 = (4k^4 + 4k^3 + 5k^2 + 1)/(2k - 1)^2$, $K_2 = (-4k^3 - 4k^2 + k + 1)/(2k - 1)^2$ and $K_3 = (4k^3 - 8k^2 + 5k - 1)$. So the Kretschmann scalar diverges only on the plane $r = 0$ for $M$ and $S \neq 0$, establishing that metric (11) is scalar polynomial singular.

The physical situation is that of a radial influx of two fluid in the region of the anti-de Sitter universe. The first shell arrives at $r = 0$ at time $v = 0$ and the final at $v = T$. A central singularity of growing mass developed at $r = 0$. For $v < 0$ we have $M(v) = S(v) = 0$, i.e., the anti-de Sitter metric, and for $v > T$, $M(v) = S(v) = 0$, $M(v)$ is positive definite. The metric for $v = 0$ to $v = T$ is generalized plane symmetric Vaidya-anti-de Sitter, and for $v > T$ we have the exterior background space-time, called Reimann anti-de Sitter solution (30). Thus the metric (11) is not asymptotically flat.

Radial ($\theta$ and $\phi = \text{const.}$) null geodesics of the metric (1) must satisfy the null condition

$$\frac{dr}{dv} = \frac{1}{2} \left[ \frac{\alpha^2 r^2}{\gamma} - \frac{2M(v)}{\gamma} - \frac{2S(v)}{\gamma^{1/2}} \right].$$

(33)

The nature (a naked singularity or a black hole) of the collapsing solutions can be characterized by the existence of radial null geodesics coming out from the singularity. The nature of the singularity can be analyzed by techniques in (3). In order to get an analytical solution, we choose

$$M(v) = \lambda v (\lambda > 0) \quad \text{and} \quad S(v) = \mu^2 v^{2k} (\mu^2 > 0)$$

(34)

for $0 \leq v \leq T$ (24). Let $y = v/r$ be the tangent to a possible outgoing geodesic from the singularity. In order to determine the nature of the limiting value of $y$ at $r = 0$, $v = 0$ on a singular geodesic, we let

$$y_0 = \lim_{r \to 0 \atop v \to 0} y = \lim_{r \to 0 \atop v \to 0} \frac{v}{r}.$$

Using Eqs. (35), (24) and L’Hôpital’s rule we get

$$y_0 = \lim_{r \to 0 \atop v \to 0} y = \lim_{r \to 0 \atop v \to 0} \frac{v}{r} = \lim_{r \to 0 \atop v \to 0} \frac{dv}{dr} = \frac{2}{\alpha^2 r^2 - 2\lambda y_0 - \mu^2 y_0^{2k}}$$

(36)

which implies,

$$\mu^2 y_0^{2k+1} + 2\lambda y_0^2 + 2 = 0.$$ 

(37)

If Eq. (37) admits one or more positive real roots then the central shell focusing singularity is at least locally naked. In the absence of positive roots of (37), the central singularity is not naked because in that case there are no outgoing future directed null geodesics from the singularity (for more details, see (3)). Since there are no positive roots to (37), for all $\lambda$ and $\mu$, the collapse will always lead to a black hole, i.e., no radial future null geodesics terminate at the singularity. This is contrast to spherically symmetric collapse where a naked singularity is inventible (7).

Similar to the above discussion is also valid in Cylindrical case as well. Therefore, we can conclude that the gravitational collapse in the cylindrical symmetric anti-de Sitter space-time also forms a black hole.

**IV. CONSTANT DENSITY STRING FLUID ($\rho = \rho_0$)**

Next, we consider that the string fluid is of constant density ($\rho_0$). Although strictly constant density is not completely realistic, the particular analytic solution to Einstein field equation with this equation of state has provided some insights concerning stars in general relativity. For example, the star represented by this solution has property that it cannot remain in equilibrium if the size-to-mass ratio is less than $9/4$ (28). To obtain the constant density solution, we take $\rho = \rho_0$. With this it can be seen that (11) can be easily integrated to yield:

$$ds^2 = - \left[ \beta^2 r^2 - \frac{2M(v)}{r} \right] dv^2 + 2dvdr + \alpha^2 r^2(dx^2 + dy^2).$$

(38)

where $\beta^2 = (3\alpha^2 - 8\pi\rho_0)/3$ is a constant. Because $\rho$ is the constant, the physical parameter can also be trivially evaluated to give

$$\psi = \frac{1}{4\pi r^2} M(v), \quad p_r = p_\perp = -\rho_0.$$

(39)
The results obtained suggest that constant string density star can have negative pressure or tension, this is consistent with Katz and Lynden Bell [28]. This can happen particularly in the hybrid star, which have ordinary positive pressure in the outermost layer and tension in the interior.

This is singular star solution. The reason for calling singular can be seen by examining the various scalars that can be built form Reimann tensor at \( r = 0 \), e.g., the Kretschmann for the metric (11), has the form:

\[
K = \frac{48M^2(v)}{r^6} + \frac{512\pi^2 \rho^2}{3} + 24\alpha^2 - 128\alpha^2 \pi \rho
\]  

So the Kretschmann scalar diverges along \( r = 0 \) for \( M(v) \neq 0 \), establishing that metric (1) is scalar polynomial singular [1]. Proceeding as above the analysis of structure of this singularity is initiated by study of the radial null geodesics that satisfy

\[
\frac{dv}{dr} = \frac{2r}{\beta^2 r^3 - M(v)}
\]  

The Eq. (11) has a singular point at \( r = 0 \) and \( v = 0 \) and we write

\[
M(v) = \lambda v \quad (\lambda > 0)
\]  

Following the standard techniques one finds that the singularity is center for all \( \lambda \), no radial outgoing geodesic that will terminate in the past at the singularity. Hence the collapse will always lead to a black hole in accordance with CCC.

V. CONCLUDING REMARKS

In conclusion, we have given a class of non-spherical collapse solutions (25) and (38), for two fluid, of the Einstein field equations for the energy momentum tensor (10). The solutions are asymptotically anti-de Sitter black hole solutions \((1/2 < k < 1)\) with multiple horizon. The general metric (25) depends on one parameter \( k \), two arbitrary function of \( v \) subject to energy condition. The class of the solutions discussed above, in general, belongs to Type II fluid defined in [1]. When \( m = m(r) \), we have \( \psi = 0 \), and the matter field degenerates to type I fluid [12]. We have used the solutions to study string effects in the non-spherical null fluid collapse. It turns out that, non-spherical collapse of a two-fluid in anti-de Sitter background, naked singularities do not form, in accord with CCC. On the other hand planar and cylindrical black-holes form, giving explicit counter example of the hoop conjecture. Note that this conjecture is not valid when \( \alpha = 0 \). This shows that the non-spherical collapse in anti-de Sitter back ground may violate hoop conjecture but not CCC.

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