Mathematical solution of the stone column effect on the load bearing capacity and settlement using numerical analysis

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Abstract. The most important application of various geotechnical construction techniques is for ground improvement. Many soil improvement project had been developed due to the ongoing increase in urban and industrial growth and the need for greater access to lands. Stone columns are one of the best effective and feasible techniques for soft clay soil improvement. Stone columns increase the bearing capacity and reduce the settlement of soil. Finite element analyses were performed using the program PLAXIS 2D. An elastic-perfectly plastic constitutive relation, based on the Mohr–Coulomb criterion, governs the soft clay and stone column behaviour. This paper presents on how the response surface methodology (RSM) software is used to optimize the effect of the diameters and lengths of column on the load bearing capacity and settlement of soft clay. Load tests through the numerical modelling using Plaxis 2D were carried out on the loading plate at 66 mm. Stone column load bearing capacity increases with the increasing diameter of the column and settlement decreases with the increasing length of the column. Results revealed that the bigger column diameter, the higher load bearing capacity of soil while the longer column length, the lower settlement of soil. However, the optimum design of stone column was varied with each factor (diameter and length) separately for improvement.

1. Introduction

The suitable construction area is not always available, thus an engineer needs to modify the ground based on the technical requirements of each project [1]. The soft ground needs to remediate to achieve its suitability for construction if the soil has low shear strength less than 40 kN/m² [2]. During the last 25 years, significant new soil remediation techniques have been developed to stabilise the ground in cost-effective solutions for construction on marginal or difficult site [3][4]. The selection of a suitable technique is based on a number of factors, in terms of improving the ground condition and to control the cost, social and environmental aspects of the projects [5][6]. Among the wide range of ground
improvement techniques is stone columns, which is often considered as environmentally friendly [7]. The first use of stone columns was possibly for military purposes in Bayonne, France in the 1830s [8]. This technique was not mentioned again until the 1930s when Serzey Steuerman, an employee of the Keller Company in Germany, revolutionized this technique by inventing the simple vibratory machine that could improve the ground by using a poker vibrator [9]. Ground improvement by stone columns offers three key benefits to the soil in terms of increasing bearing capacity, reducing consolidation times and reducing total settlement of cohesive soils [10]. The usage of this technique has become more prominent in recent times. Hence, this method is currently the most common ground improvement method used in the UK, which is a relatively economical alternative to the conventional piling methods for less settlement sensitive structures such as road and residential or commercial parking lot [11][12].

Over the last 20 years, numerical analysis has been the preferred method for studying and designing stone column systems. Numerical analysis is a powerful technique that can be used to understand the complexity of these systems due to the different behaviours and responses of clay and granular material [9]. It is capable of performing very complex calculations in a relatively short time with very flexible tools in finding the solution [7]. Similar to the analytical solutions, different concepts, assumptions, and idealizations have been adopted in numerical analysis to study the behaviour of reinforced ground [9]. Balaam and Booker [13] analysed the settlement behaviour of rafts supported by stone columns installed in soft clay for three different column arrangements using the unit cell method. Balaam and Poulos [14] examined the effect of the ratio of column to soil modulus, which indicates that the replacement ratio is an important element influencing the load bearing capacity. Narasimha Roa et al. [15] performed a series of laboratory load studies on model gravel columns used to stabilise soft marine clays in a large triaxial cell. Marine clays were utilised during testing because of their low shear strength. Three different column diameters with different lengths were selected for testing. It was found that the ultimate load carrying capacity of the stone columns becomes asymptotic with column lengths, such that beyond a column length of 5 to 8 diameters. The load transfer of a column does not increase which suggesting an optimum length as compared well by Sivakumar et al. [16] and Meghzili et al. [17] who suggested that the optimum length is 5 diameters beyond where no increment in bearing capacity is observed. Hence, it appears to be an optimum length for a stone column beyond where the bearing capacity does not increase but further settlement reduction is possible. In addition, Ambily and Gandhi [18] and Narasimha Roa et al [15] successfully validated the performance of stone column via comparing the finite element analysis using Plaxis 2D and the laboratory testing via adopting the unit cell approach and the Mohr-Coulomb criterion for both column and clay. So, the aim of this paper is to evaluate the diameter and length of the stone column effects on the load bearing capacity and settlement.

The response surface method (RSM) originated by Box and Wilson [19] is a collection of statistical and mathematical techniques which is helpful for developing, improving and optimizing processes through the empirical model building. Response surface methodology is the practice of adjusting predictor variables to move the response in a desired direction to an optimum by iteration [20]. The method generally engages a combination of both computation and visualization. The Response surface method consists of design of experiments and response surface analysis. They typically arise in the design of experiments, where they are used to determine a set of design variables that optimize a response [21].
2. Methodology and schedule

The modelling was performed using the Plaxis 2-dimensional program. The model was based on the scaled-down at 25 times, which is similar to the study conducted by Kelly [22]. The model size of 105 mm wide and 450 mm high represents the unit cell (De) containing a soft clay as shown in Figure 1(a). It was deemed to represent a field scenario where gravelly sand column was installed in a triangular pattern (De=1.05S) where the spacing (S) at 2.5 m as shown in Figure 1(b). For that, the use of the 2.5 m spacing was important for avoiding boundary effects. Table 1 summarises the parameters that were adopted in this study. In the present study, response surface methodology (RSM) software was used to optimize the effect of the diameters and lengths of the column on the load bearing capacity and settlement. Each independent variable of diameter (20 to 44 mm) and length (100 to 400 mm) for design of stone column was selected based on a scale down from the typical design of stone column diameter of 0.5 m to 1.1 m, and for the lengths of 2.5 m to 10 m in the field. Therefore, 12 randomized trials design with variations in diameter and length of stone column were carried out as shown in Table 2. The Design Expert software was used to analyse the result data and investigate the first order response surface equations of the model. The significance of 2 variables on the load bearing capacity and settlement was analysed using the analysis of variance (ANOVA, p<0.05). The adjusted coefficient of determination (R2 adj) was used for checking the fit of the quadratic model. The interactions between the factors and their effects were presented using a three-dimensional graphical representation of the system behaviour, called Response surface methodology (RSM). All the tests were adopted the loading plate at constant length of 66 mm. The maximum load represents the total reaction force corresponding to the applied prescribed vertical displacement, which corresponds to the total force of 1.0 radian of the circular plate. In order to obtain the total force, the value of the load should be multiplied by 2π [23].

![Figure 1: Geometry unit cell of model](image)

**Table 1:** diameter and length of stone column of the independent variables

| Factor     | Symbol | Level   |
|------------|--------|---------|
| Diameter, mm | D      | 20, 32, 44 |
| Length, mm  | L      | 100, 250, 400 |
3. Material properties

Kaolin was obtained from Associated Kaolin Industries used as the base material. Kaolin was mixed with 42 ± 2% water content to achieve 15 ± 2 kPa undrained shear strength. Meanwhile, the material for constructing the column was obtained by sieving a fine aggregate. The stone sizes were based on the scaled-down typical stone diameter of 25, which were between 0.4 mm and 1.6 mm, according to a typical stone diameter of 10 to 40 mm in the field. Properties of the kaolin and gravelly sand were determined in the laboratory and summarized in Table 2.

| Properties               | Unit  | Soft clay | Gravelly sand |
|--------------------------|-------|-----------|---------------|
| Specific gravity         | No unit | 2.65      | 2.70          |
| Moisture content         | %     | 42        | 9             |
| Undrained shear strength | kPa   | 15        | NG            |
| Cohesion                 | kPa   | 8.87      | 3.76          |
| Young modulus            | kPa   | 1800      | 40000         |
| Poisson ratio            | No unit | 0.35      | 0.30          |
| Saturated unit weight    | kN/m² | 17.38     | 21.38         |
| Unsaturated unit weight  | kN/m² | 12.24     | 20.04         |

4. Results and Discussion

The results of numerical analysis at different stone column diameters and lengths on the load bearing capacity and settlement were summarised in Table 3. The result indicates the load bearing capacity increased with the stone column diameter and the settlement decreased with the stone column length. Therefore, both parameters, i.e. diameter and length can be combined to optimise the design of the stone column. The evaluation of the response data showed that these 13 trials provided sufficient information to describe the correlations of the independent variables adequately. Response surface methodology (RSM) was used for the generation of response surface plots, which is a helpful tool to better understand the link between each factor and its response. However, a central composite design (CCD) with two independent variables (diameter and length) was used in order to get the optimal conditions for the load bearing capacity and settlement. 13 runs were conducted in order to cover the possible combinations of factor levels as shown in Table 3.
Table 3: Effect of independent variables of diameters and lengths on the design responses of load bearing capacity and settlement.

| Run | Factor 1 | Factor 2 | Response 1 | Response 2 |
|-----|----------|----------|------------|------------|
|     | Diameter (D), mm | Typical diameter, m | Length (L), mm | Typical length, m | Load bearing capacity, N | Settlement, mm |
| 1   | 20       | 0.5      | 400        | 10         | 1914.96       | 1859.07       |
| 2   | 32       | 0.8      | 250        | 6.25       | 2540          | 2446.42       |
| 3   | 32       | 0.8      | 250        | 6.25       | 2346.1        | 2446.42       |
| 4   | 48.97    | 1.22     | 250        | 6.25       | 3652.44       | 3466.9        |
| 5   | 44       | 1.1      | 100        | 2.5        | 3054.2        | 3260.70       |
| 6   | 32       | 0.8      | 250        | 6.25       | 2495          | 2446.42       |
| 7   | 44       | 1.1      | 400        | 10         | 2785.24       | 2885.24       |
| 8   | 32       | 0.8      | 250        | 6.25       | 2545          | 2446.42       |
| 9   | 15.02    | 0.38     | 250        | 6.25       | 1942.47       | 1977.4        |
| 10  | 32       | 0.8      | 400        | 10         | 2237.5        | 2239          |
| 11  | 32       | 0.8      | 250        | 6.25       | 2306          | 2446.42       |
| 12  | 20       | 0.5      | 100        | 2.5        | 2129.8        | 2180.41       |
| 13  | 32       | 0.8      | 37.87      | 0.95       | 2767.53       | 2616.92       |

The experiments were performed in a random and with triplicate in order to reduce the unexpected variability effects for the observed responses. The results revealed that the investigated factors evidently induced the increments of load bearing capacity and the reduction of settlement. Moreover, the optimum operation for the increment and reduction percentage was varied with each factor separately. The collected data was analysed using the analysis of variance (ANOVA, p<0.05). The independent factors which included diameter (20 mm to 44 mm) and length (100 mm to 400 mm) were selected in order to examine the comprehensive factors which might have effects on the load bearing capacity and settlement. The summary of the analysis of variance (ANOVA) for the quadratic model is presented in Table 4, 5. It was noted that the regression model for the increment and reduction of load bearing capacity and settlement was significant at a confidence level of 95% (p<0.05) with determination coefficients ($R^2$ adj) equal to 0.88 and 0.68 for load bearing capacity and settlement respectively, indicating the aptness of the model. The lack of fit for the model was not significant (P>0.05) which indicates that the model was predicted for increment of load bearing capacity and reduction of settlement and adequate for the prediction within the range of the variables studied. The low residual values are an indication of the good agreement of the experimental data with the mathematical model [24].

The calculation of the regression coefficients of the quadratic, as well as the interaction between the factors in the model, was conducted using the least square method. The effect of each independent factor was considered significant at P-value <0.05 and 95% of the confidence level. The results found that two examined factors have a positive significant quadratic effect on the increment of load bearing capacity and reduction of settlement. In contrast, two factors including diameter and length have positive significant effects on load bearing capacity and settlement respectively. The interactions between the variable factors with the investigated range at the actual factor of the centre point were performed using the response surface methodology (RSM) analysis. A significant statistical interaction (P<0.01) was recorded between diameter and length of stone column, which improved the load bearing capacity and settlement by 68.5 % as shown in Table 6. Therefore, the effect of the two factors—the diameter and length of column on two specific responses were displayed in three-dimensional views. Two selected surface plots are presented in Figure 2.
Table 4: Summary of the analysis of variance (ANOVA) for the quadratic model of load bearing capacity

| Source   | Sum of Squares | Mean Square | F Value | Prob > F |
|----------|----------------|-------------|---------|----------|
| Model    | 2456945        | 491389      | 17.62557| 0.0016   |
| D        | 2218611        | 2218611     | 79.57908| 0.0001   |
| L        | 145658         | 145658      | 5.224589| 0.0623   |
| D^2      | 123286.2       | 123286.2    | 4.422138| 0.0802   |
| L^2      | 6057.152       | 6057.152    | 0.217263| 0.6576   |
| DL       | 732.2436       | 732.2436    | 0.026265| 0.8766   |
| Residual | 167275.9       | 27879.32    |         |          |
| Lack of Fit | 116658.8   | 58329.41    | 4.60946 | 0.0916   |
| Pure Error | 50617.13     | 12654.28    |         |          |
| Cor Total| 2624221        |             |         |          |

R-Squared 0.9363, Adj R-Squared 0.8831

Table 5: Summary of the analysis of variance (ANOVA) for the quadratic model of settlement

| Source   | Sum of Squares | Mean Square | F Value | Prob > F |
|----------|----------------|-------------|---------|----------|
| Model    | 274.7094       | 54.94188    | 5.735368| 0.0276   |
| D        | 43.55274       | 43.55274    | 4.546459| 0.0770   |
| L        | 154.4098       | 154.4098    | 16.1188 | 0.0070   |
| D^2      | 7.03747        | 7.03747     | 0.73464 | 0.4243   |
| L^2      | 8.284278       | 8.284278    | 0.864794| 0.3883   |
| DL       | 3.822025       | 3.822025    | 0.39898 | 0.5509   |
| Residual | 57.47691       | 9.579485    |         |          |
| Lack of Fit | 26.36491    | 13.18246    | 1.694839| 0.2930   |
| Pure Error | 31.112       | 7.778       |         |          |
| Cor Total| 332.1863       |             |         |          |

R-Squared 0.8270, Adj R-Squared 0.6828

Table 6: Solutions for design of interactions factors

| N | Diameter (D), mm | Length (L), mm | Load bearing capacity, N | Settlement, mm | Desirability, % |
|---|------------------|----------------|--------------------------|----------------|-----------------|
| 1 | 44               | 395.26         | 2893.11                  | 28.71          | 68.5 Selected   |
| 2 | 44               | 377.28         | 2922.38                  | 29.20          | 68.4            |
The optimization of the diameter and length of stone column in increasing and reducing of load bearing capacity and settlement respectively was conducted separately using the point optimization technique by the Design Expert software. The condition for the best operation of the design process was performed based on the results obtained from the screening of independent factors which revealed the possible direction for maximizing the design process of the stone column. The optimal operation parameter of the design process for the increment of load bearing capacity was recorded at a diameter of 44 mm with a length of 100 mm to achieve 3260.7 N of load bearing capacity for 77.5 % of desirability as shown in Table 7. Moreover, the optimal operation parameter of the design process for the reduction of settlement was recorded at a diameter of 24.18 mm with a length of 400 mm to achieve 25.75 mm of settlement for 99.2 % of desirability as shown in Table 8. Under these conditions, the independent factors exhibited strong interactions with 95% confidence level for both load bearing capacity and settlement improvement.

**Table 7:** Solutions for design of diameter factor separate with length factor

| N  | Diameter (D), mm | Length (L), mm | Load bearing capacity, N | Desirability, % |
|----|------------------|----------------|--------------------------|-----------------|
| 1  | 44               | 100            | 3260.7                   | 77.5 selected   |
| 2  | 44               | 120.59         | 3244.63                  | 76.5            |

**Table 8:** Solutions for design of length factor separate with diameter factor

| N  | Diameter (D), mm | Length (L), mm | Settlement, mm | Desirability, % |
|----|------------------|----------------|---------------|-----------------|
| 1  | 24.18            | 400            | 25.7498       | 99.2 selected   |
| 2  | 24.34            | 400            | 25.7499       | 99.2            |
| 3  | 24.75            | 400            | 25.7521       | 99.2            |

The model describes the significant relationship between the increment and reduction of load bearing capacity and settlement selected in terms of factors as given by Equations (1), (2).
Load bearing capacity $= 2147.32 - 15.51D - 0.078L + 0.96D^2 - 1.69 \times 10^{-3}L^2 - 7.52 \times 10^{-3}DL$ (1)

Settlement $= 40.65 - 0.13D - 0.052L + 7.23 \times 10^{-3}D^2 + 6.23 \times 10^{-5}L^2 - 5.43 \times 10^{-4}DL$ (2)

Where: $D$ is the diameter of column and $L$ is the length of stone column.

5 Conclusion

Dimensions of stone column were analysed for various column lengths and diameters consistent with current practice. The effect of column diameter and length on loading capacity and settlement was examined. The optimum design for the diameter and length of column was determined by superimposing the surface plots for all response variables using the Design Expert software. The regions that best satisfy all the quality requirements were selected as optimum conditions. The response surface methodology (RSM) model does not include all possible external factors that might influence the design and therefore, it is only an approximation. The key findings in terms of the influence of the diameter and length of a column on loading capacity and settlement performance are summarised as follows:

- Increasing the diameter of the column has shown to increase the loading capacity. Hence, diameter of stone column is responsible for increasing the bearing capacity of soil. This is seen for the highest value of the loading which has the biggest diameter of the column.
- Increasing the length of stone column is able to reduce the settlement of soil. This suggests that with the increasing of column length, the settlement reduces. Thus, the length of stone column reduces the settlement of soil.
- The optimum design of stone column according to response surface methodology (RSM) is 44 mm diameter and 100 mm length of stone column to achieve 3260.7 N of load bearing capacity for 77.5 % of desirability and 24.18 mm diameter with a length of 400 mm of stone column to achieve 25.75 mm of settlement for 99.2 % of desirability.
- The condition for the best operation of the design process was performed based on the results obtained from response surface methodology (RSM), which revealed the possible direction for maximizing the design process of the stone column diameter and length. Hence, for further design of stone column must be incorporated with a diameter combination to achieve realistic results with reduction of amount of stone.

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