Probing Quark-Gluon Correlation Functions

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Abstract

We review applicability of QCD factorization theorem to multiple scattering in deeply inelastic lepton-nucleus scattering. We show why $A^{1/3}$-type nuclear enhancement can be calculated consistently in perturbative QCD. We derive the transverse momentum broadening of the leading pions in deeply inelastic lepton-nucleus collisions. We argue that the measurement of such transverse momentum broadening can provide direct information on the multiple-parton correlation functions inside a nucleus. We also estimate the numerical values of the broadening at different values of $x_B$ and $Q^2$.

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I. INTRODUCTION

Multiparton correlation functions inside a large nucleus are extremely important and useful for understanding nuclear dependence in relativistic heavy ion collisions. When a quark or a gluon passes through the nuclear matter, it loses energy via radiation, and it picks up extra transverse momentum because of multiple scattering [1–5]. The multiple scattering is directly associated with multiparton correlation functions [5]. Inside a large nucleus, multiple scattering can take place within one nucleon or between different nucleons. At high energy, a single hard scattering is always localized within one nucleon, and it cannot generate any large dependence on the nuclear size. Similarly, multiple scattering within one nucleon cannot provide much dependence on the nuclear size either. Therefore, the large dependence on the nuclear size is an unique signal of multiple scattering between nucleons. Measurements of such anomalous dependence on the nuclear size for any physical observable will provide direct information on multiparton correlation functions in a nucleus.

However, there are many multiparton correlation functions in QCD [5]. It is therefore important to identify physical observables which depend only on a small number of multiparton correlation functions. Otherwise, data will not be able to separate contributions from different correlation functions. It was pointed out recently in Ref. [6] that the $A^{1/3}$-type enhancement of the transverse momentum broadening of Drell-Yan pairs and the jet broadening in lepton-nucleus deeply inelastic scattering (DIS) are good observables, because at the leading order of $\alpha_s$, these two observables depend only on one type of multiparton correlation functions. It represents the correlations between hard quarks and soft-gluons, and has the following operator definition [5,6],

$$T_{qF}(x, \mu^2) = \int \frac{dy^-}{2\pi} e^{ixp^+y^-} \frac{dy_1^-dy_2^-}{2\pi} \theta(y_1^- - y^-)\theta(y_2^-) \times \langle PA|\bar{\psi}_q(0)\gamma^+ F_+^+(y^-_2) F^{+\alpha}(y^-_1) \psi_q(y^-)|PA \rangle,$$  \hspace{0.5cm} (1)

with $A$ the atomic weight, and $q$ is the quark flavor and $F$ is the gluon field strength. In Eq. (1), $x$ is the effective momentum fraction and $\mu$ represents the scale-dependence of the
correlation functions.

According to QCD factorization theorem [7,8], all information on the identity of the target is factorized into the matrix elements. The $A^{1/3}$-type enhancement must be a property of the relevant nuclear matrix elements, like the one in Eq. (1). Experimentally, the $A^{1/3}$-type nuclear enhancement have been observed in the transverse momentum broadening of Drell-Yan pairs in hadron-nucleus collisions [9]. Theoretically, Eq. (1) shows how such enhancement can occur, through integrals over the relative positions of the fields, $y_i$, that appear in the expectation values. For example, enhancement can occur when the two quark fields are close together and the two gluon fields are close together, and they pair off into color singlets that are separated by no more than a nucleon diameter in the $y_i$ integrals; but the two pairs are separated by a distance that varies up to the nuclear size. In this manner, aside from an overall factor of $A$ which reflects growth with the nuclear volume, we anticipate an $A^{1/3}$ nuclear enhancement [10].

Direct measurement of the multiparton correlation functions is of great importance for testing the generalized QCD factorization theorem [8], which allows us to apply QCD perturbation theory for studying the collisions involving nuclei. It was observed in Ref. [8] that measuring the jet transverse momentum broadening in DIS, $\Delta \langle p_T^2 \rangle$, provides a direct measurement of the quark-gluon correlation functions, $T_{qF}^A(x_B, Q^2)$, inside a nucleus. At the lowest order, the jet transverse momentum broadening in DIS can be expressed as [8]

$$\Delta \langle p_T^2 \rangle \equiv \langle p_T^2 \rangle^e_A - \langle p_T^2 \rangle^e_N \approx \frac{4\pi^2\alpha_s(Q^2)}{3} \sum_q e_q^2 T_{qF}^A(x_B, Q^2),$$

(2)

where $p_T$ is the transverse component of the jet momentum in the photon-nucleus center of mass frame in DIS. In Eq. (2), $x_B$ is the Bjorken variable and $Q^2 = -q^2$ with $q^2$ is the invariant mass of the virtual photon in DIS.

However, other than a future HERA with a nuclear beam, existing fixed target facilities can not provide good measurements on jets in lepton-nucleus DIS. It is the purpose of this paper to show that by measuring the leading pions in DIS and their transverse momentum
broadening, fixed target facilities, such as CEBAF \cite{11}, can provide good information on the quark-gluon correlation functions defined in Eq. (1).

This paper is organized as follows. In the next section, we derive the transverse momentum broadening of the leading pions in lepton-nucleus DIS. We demonstrate analytically the direct correspondence between the transverse momentum broadening of the leading pions and multiparton correlation functions. In Sec. III, we estimate the numerical values of such broadening at different values of $x_B$ and $Q^2$, by using the model of the quark-gluon correlation functions introduced in Ref. \cite{10}. We also derive explicit relations of the transverse momentum broadening for $\pi^\pm$ and $\pi^0$, and discuss the $x_B$ and $Q^2$ dependence of the correlation functions.

II. TRANSVERSE MOMENTUM BROADENING OF PIONS IN DIS

Consider leading pion production in the deeply inelastic lepton-nucleus scattering,

\begin{equation}
  e(k_1) + A(p) \rightarrow e(k_2) + \pi(\ell) + X,
\end{equation}

where $k_1$ and $k_2$ are the four momenta of the incoming and the outgoing leptons respectively, $p$ is the momentum per nucleon for the nucleus with the atomic number $A$, and $\ell$ is the observed pion momentum. In order to extract useful information on multiple scattering from the pion production, it is natural to think that the $A$-dependence of the differential cross section $d\sigma_{eA\rightarrow e\pi X}/dx_BdQ^2d\ell_T^2$ is the physical observable to study, because the transverse momentum spectrum is more sensitive to multiple scattering \cite{5}. But, other than a future HERA with a nuclear beam, existing fixed target facilities can not produce good data on the production of leading pions at large transverse momenta in DIS. On the other hand, at the small transverse momentum $\ell_T^2$, the differential cross section $d\sigma/dx_BdQ^2d\ell_T^2$ is proportional to $1/\ell_T^2$, and is not perturbatively stable. A nontrivial all order resummation of the large logarithms $\log(Q^2/\ell_T^2)$ is necessary in order to get a reliable prediction of the transverse momentum spectrum at small $\ell_T^2$ \cite{12}. However, the inclusive moments of the transverse
momentum spectrum \( \int d\ell_T^2 (\ell_T^2)^n d\sigma_{eA \to eX} / dx_B dQ^2 d\ell_T^2 \) with \( n \geq 1 \) are perturbatively stable, and also enhance the information in the region of large \( \ell_T^2 \), where the multiple scattering is most relevant. Therefore, the \( A \)-dependence for the moments of the transverse momentum spectrum of the leading pions is a good observable for extracting information on multiple scattering.

We define the first moment of the transverse momentum spectrum - the averaged pion transverse momentum squared, as

\[
\langle \ell_T^2 \rangle^{eA} = \int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eA \to eX}}{dx_B dQ^2 d\ell_T^2} \frac{d\sigma_{eA \to eX}}{dx_B dQ^2} \ , \tag{4}
\]

where \( d\sigma_{eA \to eX} / dx_B dQ^2 \) is the total inclusive DIS cross section normalized by the atomic weight \( A \). In Eq. (4), the Bjorken variable \( x_B = Q^2 / (2p \cdot q) \), and \( q = k_1 - k_2 \) is the four-momentum of the virtual photon. The transverse momentum \( \ell_T \) of the leading pion depends on our choice of the frame. In this paper, we choose the photon-nucleus frame, and choose the \( z \)-axis along the momentum line of the nucleus and the virtual photon.

To separate the multiple scattering contribution from the single scattering contribution, we define the nuclear broadening of the transverse momentum square as

\[
\Delta \langle \ell_T^2 \rangle \equiv \langle \ell_T^2 \rangle^{eA} - \langle \ell_T^2 \rangle^{eN} \ . \tag{5}
\]

As we will demonstrate below, the nuclear broadening of the transverse momentum square defined in Eq. (5) can be parameterized as \( [13,14] \)

\[
\Delta \langle \ell_T^2 \rangle = a + b A^{1/3} \ , \tag{6}
\]

where \( b A^{1/3} \) represents the contribution directly from the multiple scattering which is explicitly proportional to the nuclear size \( (\propto A^{1/3}) \) and term \( a \) includes all contributions from the localized hard scattering as well as those suppressed by high power of \( 1/Q^2 \). In principle, the parameter \( a \) in Eq. (6) can also depend on the atomic weight \( A \). But, as we will explain below, its dependence on \( A \) should be very weak (e.g., \( A^\alpha \) dependence with \( \alpha \simeq \pm 0.02 \)). In this paper, we are interested in the second term in Eq. (6), and we show that experimental
measurement of the constant $b$ in Eq. (4) will provide direct information on the quark-gluon correlation functions shown in Eq. (1).

To derive explicit expressions for the $a$ and $b$ in Eq. (4), we expand both numerator and the denominator in Eq. (1) in terms of a power series of $1/Q^2$. In addition to the terms of leading power, we keep only the power suppressed terms that are explicitly enhanced by a factor $A^{1/3}$. Since the denominator, $d\sigma_{eA\rightarrow eX}/dx_BdQ^2$, is the total inclusive DIS cross section, the operator product expansion (OPE) allows us to expand it in terms of the power series of $1/Q^2$ [15,16],

$$
\frac{d\sigma_{eA\rightarrow eX}}{dx_BdQ^2} = \sum_a \int dx \phi_{a/A}(x,\mu^2) \frac{d\hat{\sigma}^{(0)}_{ea\rightarrow eX}}{dx_BdQ^2}(x_B/x,\mu^2/Q^2,\alpha_s(\mu^2)) \left[1 + O\left(\frac{1}{Q^2}\right)\right] = D_A^{(0)} \left[1 + O\left(\frac{1}{Q^2}\right)\right],
$$

where $\mu$ is the factorization and/or renormalization scale, $\phi_{a/A}(x,\mu^2)$ is the leading twist parton distribution of flavor $a$ in a nucleus normalized by the atomic weight $A$, and $d\hat{\sigma}^{(0)}_{ea\rightarrow eX}/dx_BdQ^2$ is a perturbatively calculable short-distance hard part, which is independent of the nuclear medium. The total DIS cross section normalized by the atomic weight $A$ is an inclusive quantity and depends only on one hard scale $Q^2$. The QCD factorization theorem [15,16] allows us to factorize each term in Eq. (4) into a convolution of a short-distance coefficient function and a corresponding matrix element. The $1/Q^2$ term in Eq. (7) can be expressed in terms of twist-4 matrix elements [16]. As demonstrated in Refs. [8,17], integration of all position variables ($y_i$) of the field operators, which define these twist-4 matrix elements, are bounded by $1/x_Bp$ due to the oscillating exponential. Let $m$ and $r$ be nucleon mass and radius, respectively. If $1/x_Bp < 2r(m/p)$ (i.e. $x_B > 1/2mr \approx 0.1$), all the fields in these twist-4 matrix elements are bounded within the size of individual nucleon. Therefore, the power suppressed terms in Eq. (7) is of the order of $O(1/Q^2)$ if $x_B > 0.1$, not $O(A^{1/3}/Q^2)$.

On the other hand, the numerator in Eq. (4) depends on two physically measured hadrons: the pion and the nucleus, and therefore, the OPE alone cannot be used to expand the numerator. However, the generalized factorization theorem introduced in Ref. [8]
can be used to factorize the numerator up to the $1/Q^2$ power corrections. Similarly to the Eq. (1) of Ref. [10], we expand the numerator in Eq. (4) as

$$
\int d\ell_T^2 \ell_T^2 \frac{d\sigma_{eA\to e\pi X}}{d x_B dQ^2 d\ell_T^2} = \sum_{a,c} \phi_{a/A}(x) \otimes C_{ea\to e\pi X}^{(0)}(x_B/x, z = \ell/p_c, Q^2) \otimes D_{c\to \pi}(z)
$$

$$+ \frac{1}{Q^2} \sum_{a,c} \left[ T_{a/A}(x) \otimes C_{ea\to e\pi X}^{(2)}(x_B/x, z = \ell/p_c, Q^2) \otimes D_{c\to \pi}(z) \right.
$$

$$+ \phi_{a/A}(x) \otimes \bar{C}_{ea\to e\pi X}^{(2)}(x_B/x, z = \ell/p_c, Q^2) \otimes d_{c\to \pi}(z) \bigg] + \ldots
$$

$$\equiv H_A^{(0)} + H_A^{(2)} + \bar{H}_A^{(2)} + \ldots,$$

where $\ldots$ represents terms further suppressed in $1/Q^2$, $\otimes$ represents the convolution over partons’ momentum fractions, and explicit dependence on the factorization and/or renormalization scale is suppressed for simplicity. In Eq. (8), $C^{(0)}$, $C^{(2)}$, and $\bar{C}^{(2)}$ are perturbatively calculable short-distance hard parts [8], and $D_{c\to \pi}$ and $d_{c\to \pi}$ are twist-2 and twist-4 parton-to-pion fragmentation functions, respectively. $\phi_{a/A}$ are nuclear parton distributions, and the $T_{a/A}$ in Eq. (8) represents the twist-4 quark-gluon correlation functions (e.g., the one defined in Eq. (1)). Both $\phi_{a/A}$ and $T_{a/A}$ are normalized by the atomic weight $A$.

Effective nuclear parton distributions $\phi_{a/A}(x)$ have been measured [18–21] and are known to have the nuclear shadowing for small $x$ ($x \simeq 0.1$), the EMC effect for intermediate $x$ values ($0.3 \simeq x \simeq 0.7$), and Fermi motion effect for the large $x$ region. By fitting recent high precision DIS and Drell-Yan data on various nuclear targets, Eskola et al. [22] extracted a set of effective scale dependent nuclear parton distributions. In order to estimate the $A$-dependence of the nuclear parton distributions, we introduce a parameter $\alpha(A, x_B)$ as

$$F_2^A(x_B, Q^2) \equiv A^{\alpha(A, x_B)} F_2^N(x_B, Q^2),$$

$$\text{or} \quad \alpha(A, x_B) = \frac{\ln \left[ F_2^A(x_B, Q^2) / F_2^N(x_B, Q^2) \right]}{\ln [A]},$$

where $F_2^A(x_B, Q^2)$ is a nuclear structure function normalized by $A$ and $F_2^N(x_B, Q^2)$ is an isoscalar nucleon structure function. Using the lowest order expression for the structure function: $F_2(x_B, Q^2) = \sum_q e_q^2 x_B \phi_q(x_B, Q^2)$ with $e_q$ being a quark’s fractional charge and $\phi_q$
the effective nuclear parton distributions of Ref. [22], we plot the parameter \( \alpha(A, x_B) \) as a function of \( x_B \) for \( A=12, 64, \) and \( 207 \) at \( Q^2 = 4 \) and \( 9 \) GeV\(^2\) in Fig. [a] and [b], respectively. Fig. [a] shows that although the exact \( x_B \)-dependence and the \( A \)-dependence of the structure functions are non-trivial, the overall \( A \)-dependence of the structure functions is limited to \( A^\alpha \) with a value of \( \alpha \simeq \pm 0.02 \) for \( x_B \) values between two vertical lines in Fig. [a], which are relevant to pion production in this paper. Actually, as shown in Fig. [a], the absolute value of \( \alpha \) is limited to 0.05 for any practical value of \( x_B \) accessible at fixed target energies.

Such \( A \)-dependence in the structure function as well as in the \( \phi_{a/A}(x) \) is much weaker than the \( A^{1/3} \)-dependence caused by multiple scattering. Since the parton-to-pion fragmentation functions, \( D_{c\to\pi} \) and \( d_{c\to\pi} \), should not have explicit \( A^{1/3} \)-dependence, the term \( \bar{H}^{(2)}_A \) is of the order of \( O(1/Q^2) \), not \( O(A^{1/3}/Q^2) \). On the other hand, the term \( H^{(2)}_A \) depends on the multiparton correlation function in a nucleus \( T_{a/A}(x) \), and therefore, it will have an \( A^{1/3} \)-type enhancement [10].

Substituting Eqs. (7) and (8) into Eq. (4), and keeping terms up to \( O(A^{1/3}/Q^2) \), we obtain

\[
\langle \ell_T^2 \rangle^{eA} \approx \frac{H^{(0)}_A + H^{(2)}_A}{D^{(0)}_A} \mathcal{D}^{(0)}_A + O \left( \frac{A^0}{Q^2} \right). \tag{10}
\]

Similarly, for a nucleon target, we have

\[
\langle \ell_T^2 \rangle^{eN} \approx \frac{H^{(0)}_N}{D^{(0)}_N} + O \left( \frac{A^0}{Q^2} \right). \tag{11}
\]

Substituting above Eqs. (10) and (11) into our definition of the nuclear broadening of the transverse momentum square in Eq. (3), we derive

\[
\Delta \langle \ell_T^2 \rangle \approx \left[ \frac{H^{(0)}_A}{D^{(0)}_A} - \frac{H^{(0)}_N}{D^{(0)}_N} \right] + \frac{H^{(2)}_A}{D^{(0)}_A} + O \left( \frac{A^0}{Q^2} \right), \tag{12}
\]

where the power suppressed terms \( O(A^0/Q^2) \) should be very small due to the cancellation between \( \langle \ell_T^2 \rangle^{eA} \) and \( \langle \ell_T^2 \rangle^{eN} \) and the fact that they are not enhanced by the \( A^{1/3} \). The first term in Eq. (12) would be exactly equal to zero if the normalized effective nuclear parton distributions \( \phi_{a/A}(x) = \phi_{a/N}(x) \). However, due to the well-known nuclear effects in the

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parton distributions, the first term in Eq. (12) does not have to vanish. But, because of very weak $A$-dependence of nuclear parton distributions shown in Fig. 1, we identify this term with the $a$ in Eq. (6),

$$a \approx \frac{H^{(0)}_A}{D^{(0)}_A} - \frac{H^{(0)}_N}{D^{(0)}_N}.$$  \hspace{1cm} (13)

It is the second term in Eq. (12) that is responsible for the $A^{1/3}$-type dependence of the nuclear broadening.

We introduce $\Delta \langle \ell^2_T \rangle_{1/3}$ as

$$\Delta \langle \ell^2_T \rangle_{1/3} \equiv b A^{1/3} \approx \frac{H^{(2)}_A}{D^{(0)}_A} = \frac{\int d\ell^2_T \ell^2_T d\sigma^{(D)}_{eA\to e\pi X}}{dx BDQ^2 d\ell^2_T},$$  \hspace{1cm} (14)

where superscript “(D)” represents the double scattering contribution, and “(0)” stands for the leading power contribution. Our formalism in Eqs. (13) and (14) can be verified from the $A$-dependence of the measured transverse momentum broadening.

From the factorized formula in Eq. (8), the double scattering contribution $H^{(2)}_A$ depends only on the twist-2 parton-to-pion fragmentation function $D(z)$. Therefore, the differential cross section in Eq. (14) for the pion production in DIS can be expressed as

$$\frac{d\sigma^{(D)}_{eA\to e\pi X}}{dx BDQ^2 dp^2_{ct}} = \sum_c \int d\sigma^{(D)}_{eA\to e\pi X \rightarrow c\pi} \cdot D_{c\to \pi}(z) \cdot \frac{dz}{z^2},$$  \hspace{1cm} (15)

where $\sum_c$ runs over all parton flavors $c$, $z = \ell/p_c$ is the momentum fraction carried by the produced pion, and $D_{c\to \pi}(z)$ is the fragmentation function for the parton of momentum $p_c$ to become a pion of momentum $\ell$. In Eq. (15), $d\sigma^{(D)}_{eA\to e\pi X}/dx BDQ^2 dp^2_{ct}$ is the double scattering contributions to the differential cross section to produce a parton of momentum $p_c$ in DIS, and it can be written as

$$\left[ \frac{d\sigma^{(D)}_{eA\to e\pi X}}{dx BDQ^2 dp^2_{ct}} \right] dp^2_{ct} = \frac{1}{8\pi} \frac{e^2}{x^2 B s^2 Q^2} L^{\mu\nu}(k_1, k_2) W^{(D)}_{\mu\nu}(x_B, Q^2, p_c),$$  \hspace{1cm} (16)

where $s = (p + k_1)^2$ is the total invariant mass of the lepton-nucleon system. In Eq. (16), the leptonic tensor $L^{\mu\nu}$ is given by the diagram in Fig. 3a,
\[ L^{\mu\nu}(k_1, k_2) = \frac{1}{2} \text{Tr}(\gamma \cdot k_1 \gamma^\mu \gamma \cdot k_2 \gamma^\nu) \, , \]  

(17)

and \( W^{(D)}_{\mu\nu} \) is the hadronic tensor due to double scattering, which is given by the diagram shown in Fig. 2b. For comparison, the lowest order differential cross section for producing a parton of momentum \( p_c \) due to single scattering is given by

\[ \left( \frac{d\sigma^{(0)}_{eA\rightarrow ecX}}{dx_B dQ^2 dp^2_{ct}} \right) d\sigma^{(0)}_{eA\rightarrow ecX} = \left( \frac{d\sigma^{(0)}_{eA\rightarrow ecX}}{dx_B dQ^2 \delta(p^2_{ct})} \right) \delta(p^2_{ct}) \]  

(18)

where \( d\sigma^{(0)}/dx_B dQ^2 \) is the leading order inclusive DIS cross section, which appears in the definition of \( \Delta \langle \ell_T^2 \rangle_{1/3} \) in Eq. (14).

The complete double scattering contributions to the production of a leading quark of momentum \( p_c \) in DIS at the leading order of \( \alpha_s \) come from the diagrams shown in Fig. 3, which represent the final-state interactions, plus the same order diagrams involving initial-state interactions shown in Fig. 4. In the following, we first discuss the role of initial-state interactions, and argue that although required by gauge invariance, they contribute to the \( a \) term of Eq. (6) only.

After the collinear expansion of the parton momenta, the gluon interactions on the initial quark lines, as shown in Fig. 4, can be reduced into two categories: the long-distance and the short-distance contributions due to the fact that every “on-shell” propagator can have a pole contribution as well as a contact contribution [17]. For example, a quark propagator of momentum \( k \) can always be written as

\[ \frac{i\gamma \cdot k}{k^2} = \frac{i\gamma \cdot \hat{k}}{k^2} + \frac{i\gamma \cdot n}{2k \cdot n} \, , \]  

(19)

where \( \hat{k}^2 = 0 \) and \( n^\mu \) is any auxiliary vector with \( k \cdot n \neq 0 \). The first term in the right-hand-side of Eq. (13) corresponds the pole contribution, while the second term is the contact contribution [17]. Attaching one gluon to the initial quark line introduces a quark propagator, and this propagator will have both the pole and contact contributions. The pole contribution is long-distance in nature, representing the interactions between the quark and the gluon long before the hard collision between the quark and the virtual photon. The
pole contribution is partially responsible for the \( A \)-dependence of the leading-twist parton distributions in a nucleus \[23\], and therefore, is one of the sources for the weak \( A \)-dependence appeared in the \( a \) term in Eq. \((\mathcal{F})\).

Because of the nature of the contact term, its contribution is localized in space, and therefore, it does not contribute to the \( A^{1/3} \)-type of nuclear enhancement \[8\]. Since the short distance contributions of Feynman diagrams in Fig. \( \mathcal{H} \) have at least one propagator given by the contact term, these diagrams do not contribute to the \( A^{1/3} \)-type of nuclear enhancement. At the same time, the contact term of the initial-state interaction is important for the gauge invariance of the complete double scattering (twist-4) process \[17\], but only at order \( A^0 \).

Although the Feynman diagrams with the final-state interactions in Fig. \( \mathcal{E} \) are not gauge invariant in general, their contributions to the \( A^{1/3} \)-behavior of nuclear enhancement are observable and hence gauge invariant \[10\]. The three lowest order diagrams in Fig. \( \mathcal{E} \) are all convoluted with the same two-quark-two-gluon matrix element through three independent parton momenta. The leading power contributions of these diagrams come from the region phase space when all partons are collinear to the direction of the target, as shown in Fig. \( \mathcal{E} \). In order to convert the gluon field \( A^+ \) to the corresponding field strength \( F^{+\perp} \) in covariant gauge, we expand the gluon momenta in an extra transverse component \( k_T \) in Fig. \( \mathcal{E} \) \[10\]. All three Feynman diagrams in Fig. \( \mathcal{E} \) have two potential poles due to the interactions between the final-state quark and gluons. These two poles (which are not pinched) can be used to carry out the momentum fraction \( dx_1 \) and \( dx_2 \) integrals \[5,10\]. After taking the poles, the hard-scattering factor (the interaction between the virtual photon and the initial quark) and the final-state interaction between the “on-shell” outgoing quark and the gluon are separately gauge invariant.

To derive the leading contribution from the double scattering, we follow the derivation in Ref. \[3\]. By taking the poles to fix the integrations of momentum fractions \( x_1 \) and \( x_2 \), we derive contributions of all three diagrams in Fig. \( \mathcal{E} \) to the hadronic tensor \( W^{(D)}_{\mu\nu} \) in Eq. \((\mathcal{E})\) as
In Eq. (21),
\[ W^{(D)}_{\mu\nu}(x_B, Q^2, p_c) = \frac{1}{4\pi} \int \frac{dy_1 dy_2}{2\pi^2} d^2 y_T \int d^2 k_T e^{ik_T \cdot y} e^{\frac{k_T^2}{2\gamma^2 x_B}} (y_1 - y_2) \]
\[ \times \langle p_A | \bar{\psi}_q(y_1) \gamma^+ A^+(y_1, y_T) A^+(y_2, y_T) \psi_q(y_2) | p_A \rangle \]
\[ \times (2\pi) \theta(y_1 - y_2) \]
\[ \times \frac{2\pi}{(2p \cdot q)^3} \cdot \hat{H}_{\mu\nu}^{(D)}(p_c) \cdot \left[ \delta(p^2_{cT}) - k_T^2 \delta(p^2_{cT}) - \delta(p^2_{cT}) \right] dp^2_{cT} \]
\[ \approx \left[ \frac{\pi}{(2p \cdot q)^3} \hat{H}_{\mu\nu}^{(D)}(p_c) \right] \left[ -\delta'(p^2_{cT}) \right] T^{A}_{q\bar{T}}(x_B, \mu^2) \right] dp^2_{cT}. \quad (20) \]

In Eq. (21), \( T^{A}_{q\bar{T}}(x_B, \mu^2) \) is defined by Eq. (11) with the factorization scale \( \mu^2 \). \( \hat{H}_{\mu\nu}^{(D)}(p_c) \) represents the spinor trace of the partonic double scattering and is given by
\[ \hat{H}_{\mu\nu}^{(D)}(p_c) = \frac{4}{3} \pi^2 \alpha_s \alpha_em^2_q Tr \left[ \gamma \cdot p \gamma \cdot q \gamma \cdot (xp + q) \gamma \cdot p_c \gamma \cdot q \gamma \cdot (xp + q) \gamma \cdot \mu \right] p^\mu p_\nu, \quad (22) \]
where a color factor 1/2\( N_c \) with \( N_c = 3 \) was included. Using the definition \( z = \ell/p_c \), see Eq. (23) below, we can reexpress the momentum \( p_c \) in \( \delta'(p^2_{cT}) \) in terms of the momentum \( \ell \) of the observed pion,
\[ -\delta'(p^2_{cT}) = -\frac{d}{d(\ell^2_T)} \left( \delta(\ell^2_T) \right) = -z^4 \delta'(\ell^2_T) \].
\[ (23) \]

Combining Eqs. (15), (16), (21), and (23), we obtain
\[ \frac{d\sigma^{(D)}_{e\Lambda \rightarrow e\pi X}}{dx_B dQ^2 d\ell^2_T} = e^2 \pi \frac{\alpha_s}{8\pi x_B^2 s^2 Q^2} \sum_q \int dz z^2 D_{q\rightarrow \pi}(z) T^{A}_{q\bar{T}}(x_B, Q^2) L^{\mu\nu} \hat{H}_{\mu\nu}^{(D)}(\ell/z) [-\delta'(\ell^2_T)], \quad (24) \]

where the \( \sum_q \) runs over all quark and antiquark flavors, and \( z = \ell/p_q \) is the momentum fraction of the quark flavor \( q \) carried by the observed pion. At the lowest order in \( \alpha_s \), hard gluon initiated double scattering, which is proportional to the four-gluon correlation function \( T_{gF} \), does not contribute [10,10].

In the photon-nucleus frame, we choose the target momentum \( p \) along the \(-\vec{z}\)-axis, such that \( p^\mu = (p^0, p^x, p^y, p^z) = (P, 0, 0, -P) \), and only \( p^- = (p^0 - p^z)/\sqrt{2} \) is nonvanishing. In this frame, we have the following expression for the momentum fraction \( z = \ell/p_c \),
\[ z \equiv \frac{\ell}{p_c} = \frac{\ell^+}{p^+_c} = \frac{p \cdot \ell}{p \cdot p_c} \approx \frac{p \cdot \ell}{p \cdot q}. \quad (25) \]
In deriving the last equation, we used \( p_c = x_B p + q \) and \( p^2 \approx 0 \).

After working out the algebra for \( L^{\mu \nu} \hat{H}_{\mu \nu}^{(D)} \), and substituting Eq. (24) into Eq. (14), we obtain

\[
\Delta \langle \ell^2_T \rangle_{1/3} (\ell_{\min}) = \frac{4\pi^2 \alpha_s(\mu^2)}{3} \sum_q e_q^2 \int_{z_{\min}}^{1} dz z^2 D_{q \to \pi}(z) T_{qF}^A(x_B, \mu^2) \sum_q e_q^2 q^A(x_B, \mu^2),
\]

where \( D_{q \to \pi}(z) \) are the quark-to-pion fragmentation functions, and \( q^A(x_B, \mu^2) = \phi_{q/A}(x_B, \mu^2) \) are the effective quark distributions in the nucleus normalized by the atomic weight \( A \). In Eq. (26), \( z_{\min} = p \cdot \ell_{\min}/(p \cdot q) \) is defined in terms of \( \ell_{\min} \), which is the momentum cut for the pions measured in the experiment. One can choose \( \ell_{\min} \) to be large enough to ensure that the observed pions are from the fragmentation of energetic quarks.

In Eq. (26), the \( \mu^2 \) in \( \alpha_s(\mu^2) \) is the renormalization scale, and the \( \mu^2 \) in \( T_{qF}^A(x_B, \mu^2) \) and \( q^A(x_B, \mu^2) \) are the factorization scale. Since the short-distance part of the transverse momentum broadening defined in this paper is an inclusive and infrared safe calculable quantity, the scale \( \mu^2 \) should be chosen to be the order of the physically measured momentum scales. As shown in Eq. (26), the only physically measured momentum scales for the transverse momentum broadening are \( Q \) and \( \ell_{\min} \). Since \( |\ell_{\min}| \) are of the same order as \( \sqrt{Q^2} \), we choose \( \mu^2 = Q^2 \) for the numerical calculations below, and we believe that such a choice will not result into the large logarithmic high order corrections.

Since the quark-to-pion fragmentation functions have been measured [24], Eq. (26) shows that the transverse momentum broadening of pions provides direct information on the quark-gluon correlation functions inside a nucleus. The measured size of the transverse momentum broadening, \( \Delta \langle \ell^2_T \rangle_{1/3} \), depends on the choice of the momentum cut \( \ell_{\min} \) (or equivalently \( z_{\min} \)). Our predictions should be more reliable for the leading pions, or pions with relatively large momenta. By measuring the transverse momentum broadening for \( \pi^\pm \) and \( \pi^0 \), and keeping a reasonable large value of \( z_{\min} \), we can extract the quark flavor dependence of the correlation functions.
III. NUMERICAL RESULTS AND DISCUSSIONS

The numerical values of the pion transverse momentum broadening, $\Delta (\ell_T^2)_{1/3}(\ell_{\text{min}})$, depend on the explicit functional form of the quark-to-pion fragmentation functions and the quark-gluon correlation functions. In this section, without assuming any specific form for the quark-gluon correlation functions, we derive relations for the transverse momentum broadening between $\pi^\pm$ and $\pi^0$ based on isospin symmetry and the charge conjugation invariance. Furthermore, by using the simple model proposed in Ref. [3] for the quark-gluon correlation functions, we explore both the normalization and functional dependence of the momentum broadening.

Although new parameterizations of the quark-to-pion fragmentation functions were obtained recently [24], we will use the simple parameterizations of Ref. [25] for the following analytical derivations and discussions on the general features of the momentum broadening. Later, when we present our figures for the numerical values of the momentum broadening, we will use the parameterizations from both Refs. [25] and [26], and demonstrate the similarities and differences.

Like the parton distributions, the quark-to-pion fragmentation functions have scaling violation (or $Q^2$-dependence). To simplify our discussion on the general features of transverse momentum broadening, we ignore the scaling violation of the fragmentation functions, and take for the fragmentation functions the input distributions of Ref. [25], which are given as:

\begin{align}
D^+(z) &= D^{\pi^+}_u = D^{\pi^+}_d = D^{\pi^-}_u = D^{\pi^-}_d = \frac{1 - z^2}{4z}; \\
D^-(z) &= D^{\pi^-}_u = D^{\pi^+}_d = D^{\pi^+}_u = D^{\pi^-}_d = \frac{(1 - z)^2}{4z}; \\
D^0(z) &= D^{\pi^0}_u = D^{\pi^0}_d = D^{\pi^0}_u = D^{\pi^0}_d = \frac{1 - z}{4z};
\end{align}

with $D^0(z) = [D^+(z) + D^-(z)]/2$. For the strange quark fragmentation functions, we use $D^{\pi^+}_s = D^{\pi^-}_s = D^{\pi^0}_s = D^-$. Notice that the fragmentation functions given in Eq. (27), as well as those given in Ref. [24][26], violate Gribov-Lipatov reciprocity, which requires
that the power of \((1 - z)\) must be even. However, since it is not our purpose to invent better fragmentation functions in this paper, we will first use the fragmentation functions in Eq. (27) in our numerical calculation to illustrate the size and the general features of the transverse momentum broadening. Our predictions can be easily adjusted for other sets of fragmentation functions.

In order to evaluate the transverse momentum broadening, Eq. (26) requires the moments of the fragmentation functions. We therefore introduce

\[
D^{(i)}(n, z_{\text{min}}) \equiv \int_{z_{\text{min}}}^{1} d z \ z^n \ D^{(i)}(z) ,
\]

where \(i = +, -\) and 0 for \(\pi^+, \pi^-\) and \(\pi^0\), respectively. Using the fragmentation functions defined in Eq. (27), we obtain the following identity,

\[
D^+(n, z_{\text{min}}) - D^0(n, z_{\text{min}}) = D^0(n, z_{\text{min}}) - D^-(n, z_{\text{min}}) ,
\]

for all \(n\). This identity is useful for the following discussions on the general features of the transverse momentum broadening. From Eq. (26), the transverse momentum broadening, \(\Delta \langle \ell_T^2 \rangle_{1/3}^{(n)}(\ell_{\text{min}})\), depends on the second moments \(D^{(i)}(2, z_{\text{min}})\), which are plotted in Fig. 5.

Given the quark-to-pion fragmentation functions, we derive the following general relations between the transverse momentum broadening of \(\pi^+, \pi^-\), and \(\pi^0\) particles:

\[
\Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^+} - \Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^0} = \Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^0} - \Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^-},
\]

and

\[
\Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^+} > \Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^0} > \Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^-},
\]

independent of the details of the correlation functions \(T_{qF}^A\). Eq. (30a) is a result of the isospin symmetry of the fragmentation functions given in Eq. (27) and the identity given in Eq. (29). The inequality given in Eq. (30b) results from the fact that the contribution of each quark flavor is weighted by the square of the quark’s fractional charge, \(e_q^2\), and the relation \(D^+(z) > D^0(z) > D^-(z)\).
Since the correlation functions \( T_{qF}^A \) are not known, in order to obtain numerical estimates of the transverse momentum broadening of pions, we adopt the following model for quark-gluon correlation functions [5,10]:

\[
T_{qF}^A(x, Q^2) = \lambda^2 A^{1/3} q^A(x, Q^2),
\]

where \( q^A(x) \) is the effective twist-2 quark distribution of a nucleus normalized by the atomic weight \( A \), and \( \lambda \) is a free parameter to be fixed by experimental data. This model was proposed after comparing the operator definitions of the four-parton correlation functions and the definitions of the normal twist-2 parton distributions. Substituting Eq. (31) into Eq. (26), we have

\[
\Delta \langle \ell_T^2 \rangle_{1/3} = \frac{4\pi^2 \alpha_s(Q^2)}{3} A^{1/3} \lambda^2 \sum_q e_q^2 q^A(x_B, Q^2) D_{q\rightarrow\pi}(2, z_{\text{min}}).
\]

From Eq. (32), we can deduce a very simple formula for the transverse momentum broadening of \( \pi^0 \) by using the fragmentation functions given in Eq. (27c):

\[
\Delta \langle \ell_T^2 \rangle_{1/3} \approx \frac{4\pi^2 \alpha_s(Q^2)}{3} A^{1/3} \lambda^2 D^0(2, z_{\text{min}}).
\]

In deriving Eq. (33), we neglected the strange quark contribution which is much smaller. This simple expression shows that \( \Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^0} \) has a scaling behavior as \( x_B \) varies. The approximate \( x_B \)-scaling behavior of \( \Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^0} \) is a direct consequence of the model for \( T_{qF}^A(x) \), given in Eq. (31). If the future experimental data on the transverse momentum broadening of \( \pi^0 \) shows a strong violation of the \( x_B \)-scaling, the model for the quark-gluon correlation function \( T_{qF}^A(x_B) \), given in Eq. (31), will have to be modified. In addition, Eq. (33) also shows that \( Q^2 \) dependence of \( \alpha_s(Q^2) \) is already known, we can learn the \( Q^2 \) dependence of the correlation function \( T_{qF}^A(x_B, Q^2) \) by measuring the \( Q^2 \) dependence of \( \Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^0} \). If the measured \( Q^2 \) dependence of \( \Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^0} \) is consistent with the \( Q^2 \) dependence of \( \alpha_s(Q^2) \), it means the correlation function \( T_{qF}^A(x_B, Q^2) \) has a similar scale-dependence as a normal parton distribution. By examining the \( x_B \)-scaling property and \( Q^2 \) dependence of \( \Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^0} \), we can test whether the model for \( T_{qF}^A(x_B, Q^2) \), given in Eq. (31), is reasonable.
It should be emphasized that our general conclusions given in last paragraph, as well as numerical estimates given below, are the consequences of our lowest order calculations. High order corrections in $\alpha_s$ and/or in inverse powers of $Q^2$ can certainly change the dependence on $x_B$ and $Q^2$, as well as the absolute values of our numerical estimates. But, because of the inclusivity and the infrared safety of the transverse momentum broadening, and the lack of the large logarithms of the ratios of the physically measured scales, we believe that the change should not be very dramatic, and that our predictions should not be off by orders of magnitudes.

In the following, we obtain our numerical estimates of transverse momentum broadening for $\pi^+$, $\pi^-$, and $\pi^0$ particles by evaluating Eq. (32). We normalize $\Delta\langle \ell_T^2 \rangle_{1/3}$ by $A^{1/3}$, and plot our results in Fig. 6 to Fig. 7. In view of the limited range of $Q^2$ available at CEBAF, we neglect the scaling violation of the fragmentation functions for our numerical estimates. We used the simple fragmentation functions given in Eq. (27) to produce Fig. 6 to Fig. 7. More realistic parameterizations for quark-to-pion fragmentation functions were obtained in Refs. [24] and [26]. In particular, those in Ref. [26] are extracted from DIS data. We present both results for $\Delta\langle \ell_T^2 \rangle_{1/3} / A^{1/3}$ by using two different sets of the fragmentation functions at $Q^2 = 4 \text{ GeV}^2$ in Fig. 8 to illustrate the uncertainties in our numerical estimates due to different choices of the fragmentation functions. Although numerical details of the $A^{1/3}$-type transverse momentum broadening depend on the fragmentation functions used, the overall shape and magnitude of the broadening are consistent. The most important feature is that the size of the $A^{1/3}$-type broadening is large enough to be measured experimentally [13,14].

In obtaining our numerical results, we used CTEQ3L parton distributions of Ref. [27] for the quark distributions in the nucleons. We took an average of the quark distributions of the proton and the neutron for the effective quark distribution in the nucleus, $q^A(x_B)$. If the $A$-dependence of the nuclear quark distributions is independent of the flavor, from Eq. (12), such nuclear dependence can be factorized and canceled between the numerator and denominator. However, there is no obvious reason why the nuclear dependence of quark distributions is flavor independent. But, as we discussed above, in the range of $x$ probed
in these experiments the effective nuclear parton distribution should have extremely weak nuclear dependence in comparison with the $A^{1/3}$-type enhancement that we discussed in this paper. In order to verify this feature, we used both $q^A(x_B) = q^N(x_B)$ and the realistic parameterizations for $q^A(x_B)$ given in Ref. [22] to calculate the transverse momentum broadening $\Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^\pm}/A^{1/3}$. We found no noticeable difference for the transverse momentum broadening presented in Figs. 3a, 4, and 5 which were obtained with $q^A(x_B) = q^N(x_B)$.

Figs. 6a and 7b shows $\Delta \langle \ell_T^2 \rangle_{1/3}/A^{1/3}$ as functions of $z_{\min}$ at $Q^2 = 4$ GeV$^2$, and $x_B = 0.2$ and 0.3, respectively. Figs. 6b and 7a show $\Delta \langle \ell_T^2 \rangle_{1/3}/A^{1/3}$ as functions of $z_{\min}$ at $Q^2 = 9$ GeV$^2$, and $x_B = 0.2$ and 0.3, respectively. In these figures, the relations $\Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^+} > \Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^-} > \Delta \langle \ell_T^2 \rangle_{1/3}$ and $\Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^+} - \Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^-} = \Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^0} - \Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^-}$ are clearly demonstrated. Furthermore, we have $x_B$-scaling for $\Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^0}$, and $\Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^+/\pi^-}$ at $z_{\min} \approx 2$ for small $z_{\min}$. The approximate ratio, $\Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^+}/\Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^-} \approx 2$, is a result of the isospin averaged targets, and can be easily verified by using the fragmentation functions in Eq. (27), keeping only the valence quark contributions and $z_{\min}^2 \ll 1$.

Comparing Figs. 3 and 4, it is evident that the absolute sizes of the $\Delta \langle \ell_T^2 \rangle_{1/3}$ decrease as $Q^2$ increases. This is caused by the $Q^2$ dependence of the $\alpha_s(Q^2)$ in the overall factor of the transverse momentum broadening. Due to the available energy at CEBAF, we plotted the broadening at $Q^2 = 4$ GeV$^2$ in Fig. 3. Even though $Q^2 = 4$ GeV$^2$ may not be large enough to apply for the fragmentation analysis, our results can be tested at CEBAF with its future upgrade, and can be easily tested at RHIC with its future option of electron-ion collider.

From Figs. 6 and 7, we noticed that the identity, $\Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^+} - \Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^-} = \Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^0} - \Delta \langle \ell_T^2 \rangle_{1/3}^{\pi^-}$, has some weak violation on the value of $x_B$, which is due to the fact that the number of sea quark increases at smaller $x_B$ in comparison with the valence quarks.

The value of $\lambda^2$ for the quark-gluon correlation function $T_{qF}^A$, given in Eq. (31), has not been well determined. It was estimated in Ref. [10] by using the measured nuclear enhancement of the momentum imbalance of two jets in photon-nucleus collisions [13,14], and was found to be the order of $\lambda^2 \sim 0.05 - 0.1$ GeV$^2$. However, the nuclear enhancement of the Drell-Yan transverse momentum [28,29] indicates a much smaller value of $\lambda^2$ [3].
Although the momentum imbalance of the di-jet data depends on the final-state multiple scattering, and the Drell-Yan data is an effect of initial-state multiple scattering, the leading order calculation indicates that $\lambda^2$ from the two data sets to be the same [6]. Therefore, this work on the transverse momentum broadening in pion productions should provide a new and independent measurement of the size of the quark-gluon correlation functions, and the value of the $\lambda^2$, which is extremely important for understanding the nuclear dependence and the event rates in the relativistic heavy ion collisions. Since both di-jet momentum imbalance and the transverse momentum broadening are caused by the final state multiple scattering, we used $\lambda^2 = 0.05 \text{ GeV}^2$ for Figs. 6 to 8. Any change in $\lambda^2$ will only change the overall normalization factor in $\Delta \langle \ell_T^2 \rangle_{1/3}/A^{1/3}$, and hence does not affect the general features of $\Delta \langle \ell_T^2 \rangle_{1/3}/A^{1/3}$.

We conclude that the transverse momentum broadening of leading pions in deeply inelastic lepton-nucleus scattering is an excellent observable to probe the parton correlation functions in the nucleus. Measuring the $A^{1/3}$-type transverse momentum broadening $\Delta \langle \ell_T^2 \rangle_{1/3}$ in the leading pion production in DIS provides an independent test for the existing model of the quark-gluon correlation functions [4]. More importantly, by measuring such broadening, we can directly measure the $x$ and $Q^2$ dependence of the correlation functions, which is very useful for predicting the event rates [30] and for understanding the nuclear dependence in relativistic heavy ion collisions.

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FIGURES

FIG. 1. The $\alpha(A,x_B)$ defined in Eq. (9) as a function of $x_B$ at (a) $Q^2 = 4 \text{ GeV}^2$ and (b) $Q^2 = 9 \text{ GeV}^2$.

FIG. 2. (a) Diagram representing $L^{\mu\nu}$; (b) Diagram representing $W_{\mu\nu}^{(D)}$.

FIG. 3. Lowest order double scattering contribution to the broadening of the parton $c$: (a) symmetric diagram; (b) and (c): interference diagrams.

FIG. 4. Lowest order double scattering diagrams which do not contribute to the broadening of the parton $c$.

FIG. 5. $D^+(2,z_{\text{min}})$, $D^-(2,z_{\text{min}})$, and $D^0(2,z_{\text{min}})$ as functions of $z_{\text{min}}$.

FIG. 6. Transverse momentum broadening of pions, $\Delta \langle \ell_T^2 \rangle_{1/3}/A^{1/3}$, as a function of $z_{\text{min}}$ at $Q^2 = 4 \text{ GeV}^2$ and (a) $x_B = 0.2$, and (b) $x_B = 0.3$.

FIG. 7. Transverse momentum broadening of pions, $\Delta \langle \ell_T^2 \rangle_{1/3}/A^{1/3}$, as a function of $z_{\text{min}}$ at $Q^2 = 9 \text{ GeV}^2$ and (a) $x_B = 0.2$, and (b) $x_B = 0.3$.

FIG. 8. Transverse momentum broadening of pions, $\Delta \langle \ell_T^2 \rangle_{1/3}/A^{1/3}$, at $Q^2 = 4 \text{ GeV}^2$ and $x_B = 0.2$ with different $D_{q\rightarrow\pi}(z)$. The solid lines are obtained by using the fragmentation functions of Ref. 25, and the dashed lines are obtained by using the fragmentation functions of Ref. 26.
Fig. 1

$Q^2 = 4 \text{ GeV}^2$

- Dotted line: $A = 12$
- Solid line: $A = 64$
- Dash-dotted line: $A = 207$

Graph showing $\alpha$ vs $x_B$ for different values of $A$. The graph includes a vertical dotted line at $x_B = 0.2$. The plot is labeled as (a) and is part of Fig. 1.
Fig. 1

\( Q^2 = 9 \text{ GeV}^2 \)

- Dotted line: \( A = 12 \)
- Solid line: \( A = 64 \)
- Dashed line: \( A = 207 \)

\( x_B \) vs \( \alpha \)
Fig. 2
Fig. 3
Fig. 3
Fig. 5
Fig. 6

\[ Q^2 = 4 \text{ GeV}^2 \]

\[ x_B = 0.2 \]

\[ \Delta \langle l_T^2 \rangle / \langle l \rangle \text{ (GeV)}^2 \]

\[ \pi^+ \]

\[ \pi^0 \]

\[ \pi^- \]
Fig. 6

\[ Q^2 = 4 \text{ GeV}^2 \]
\[ x_B = 0.3 \]
Fig. 7

\( Q^2 = 9 \text{ GeV}^2 \)

\( x_B = 0.2 \)

\( \Delta \langle l_T^2 \rangle / A \) (GeV²)

\( Z_{\text{min}} \)

(a)
Fig. 7 (b)
Fig. 8

\[
Q^2 = 4 \text{ GeV}^2
\]

\[
x_B = 0.2
\]