Absence of evidence for pentaquarks on the lattice

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Abstract

We study the question of whether or not QCD predicts a pentaquark state $\Theta^+$. We use the improved, fixed point lattice QCD action which has very little sensitivity to the lattice spacing and also allows us to reach light quark masses. The analysis was performed on a single volume of size $(1.8 \text{ fm})^3 \times 3.6 \text{ fm}$ with lattice spacing of $a = 0.102 \text{ fm}$. We use the correlation matrix method to identify the ground and excited states in the isospin 0, negative parity channel. In the quenched approximation where dynamical quark effects are omitted, we do not find any evidence for a pentaquark resonance in QCD.

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1 Introduction

The initial observation \[1\] of a baryonic resonance $\Theta^+$ with strangeness $S = +1$ generated an enormous amount of experimental and theoretical activity. As the minimal quark content of the $\Theta^+$ is $uuudd\bar{s}$, this was the first evidence for exotic hadronic states with more than three quarks, which have long been conjectured to exist \[2\]. There followed a number of experimental results \[3\] consistent with a pentaquark resonance close to 1540 MeV, roughly 100 MeV above the Kaon-Nucleon threshold, with a surprisingly narrow width, possibly even as small as a few MeV. However, there have also been several null experimental results where the $\Theta^+$ resonance has not been found \[4\]. The uncertain experimental status of the pentaquark is an ideal situation for theoretical calculations to have an impact, particularly as the parity of the $\Theta^+$ has not yet been measured (it is expected to have isospin 0). In fact, previous to the experimental discovery, it was actually predicted \[5\] in the chiral soliton model that a narrow resonance should exist at approximately 1530 MeV, which was completely consistent with the first observations. There have since been a number of attempts \[6\] to model the behavior of the pentaquark, in particular its very narrow width, for example using quark models, Kaon-Nucleon bound states and QCD sum rules.

The only known method to derive hadronic properties \textit{ab initio} from a theory of strongly-interacting quarks and gluons is lattice QCD. Not surprisingly, there have been a number of lattice studies to see if the $\Theta^+$ can be predicted from QCD. The first attempts used standard hadron spectroscopy methods to explore pentaquark states in a number of isospin and parity channels, to determine for which quantum numbers could a pentaquark state be consistent with experiment \[8, 9, 10\]. It was consistently found that the lightest isospin 0 state is below the lightest isospin 1 state, but there was disagreement on the parity assignment of a possible $\Theta^+$ resonance. There were also studies which did not find a pentaquark resonance, only scattering states of weakly-interacting Kaons and Nucleons \[11, 12\]. More recent studies which try to disentangle a possible pentaquark from Kaon-Nucleon states indicate that a pentaquark resonance is not seen \[13, 14\]. However consensus has not yet been reached, as other work \[10, 15, 16\] continues to favor a single-particle resonance consistent with the experimental $\Theta^+$. The search for spin-$3/2$ pentaquarks on the lattice have also been conducted more recently by a couple of groups \[17, 18\]; however, also without agreement on their conclusions.

The purpose of this short paper is to try to add to the level of agreement among lattice QCD calculations. There are countless ways QCD can be regulated using the lattice as a space-time cutoff, but all possible formulations should agree in their physical predictions when the lattice spacing vanishes and the continuum limit is reached. We use a particular improved lattice QCD action, designed so that physical continuum results can be extracted even on relatively coarse lattices. Because our action is also chirally symmetric at vanishing quark mass (up to small parametrization errors), this allows us to reach light quark masses, which is not possible in many lattice formulations. We use the correlation matrix method to separate a possible pentaquark resonance from an interacting multiparticle system. As in the other studies, we use the
quenched approximation where dynamical quark effects are omitted, which is an enormous saving in computational difficulty. Although without justification, this approximation empirically gives qualitatively and even quantitatively the correct picture for the hadronic states whose width is narrow. The expectation is that this will also hold for any possible pentaquark states, if it is indeed as narrow as some experimental results claim.

The paper is organised as follows: in Section 2 we give a short description of the lattice action and the details of the numerical simulation. In Section 3 we discuss the correlation matrix analysis to separate ground and excited states in various channels. In Section 4 we present and discuss our numerical results, followed by our conclusions.

2 Simulation details

The lattice QCD action we use is based on the real space renormalization group [19]. The fixed point action is a close approximation to the exact renormalization group trajectory, so the theory has only a weak dependence on the lattice spacing and continuum results can be extracted even on relatively coarse lattices. If the continuum limit is reached earlier, this could bring a large reduction in the computational time required to work at a fixed physical volume. Fixed point actions have been constructed for 2- and 4-dimensional systems and in general have shown the expected good behavior [20]. In addition, the exact fixed point Dirac operator is a solution to the Ginsparg-Wilson relation [21], giving exact chiral symmetry at non-zero lattice spacing [22]. The fixed point action is one of the many possible lattice fermion formulations [23] that have all the desirable chiral properties of the continuum theory, such as massless pions at vanishing quark mass. The action we use in the simulations is close to an exact Ginsparg-Wilson solution, hence the chiral properties are almost completely intact.

The fixed point QCD action has been tested in a number of quenched QCD studies [24], including hadron spectroscopy, $\pi-\pi$ scattering lengths, the chiral condensate, glueball masses, the topological susceptibility, unphysical zero-mode effects in quenched QCD, and properties of the Dirac operator eigenvalues. Large-scale hadron spectroscopy studies have shown the improved behavior of the action [25, 26]. Hadron masses calculated at lattice spacings of 0.153 and 0.102 fm have shown only a weak dependence on the lattice cutoff and are closer to continuum-extrapolated values than using the Wilson action at 0.05 fm or the improved staggered action at 0.09 fm. Pion masses as light as 220 MeV could be reached without any indication of exceptional configurations (i.e. gauge field configurations for which the Dirac operator has unphysical accidentally small eigenvalues due to large fluctuations), which limit the lightest possible quark mass that can be reached. This is an indication of the good chiral behavior of the lattice action.

In this paper we study an ensemble of 176 gauge configurations\(^2\) of lattice size

\(^2\)Four random configurations were irretrievable from the backup tapes making the total
$18^3 \times 36$ which have already been generated for standard hadron spectroscopy \cite{26}. From the Sommer parameter $r_0 = 0.49$ fm, the lattice spacing is found to be $a(r_0) = 0.102$ fm, where the systematic error is at the $\%$ level and the statistical error is less than $1\%$. The gauge configurations are separated by 500 alternating Metropolis and pseudo-over-relaxation sweeps. The gauge configurations are fixed to the Coulomb gauge and Gaussian-smeared sources of width $\sigma/a \sim 2.3$ were used for the quark propagators. The ensemble covers a range of quark masses, reaching pseudoscalar meson masses as light as $m_{PS} = 300$ MeV.

3 Analysis

The usual goal in standard hadron spectroscopy studies is to extract particle masses for the various possible quantum numbers. The typical method used is to calculate the correlation functions in Euclidean time of two- and three-quark operators with the desired quantum numbers. For example, the pion correlation function is $C_{\pi}(t) = \langle \Omega | O(t) O^\dagger(0) | \Omega \rangle$, where the pion operator is $O = \bar{\psi} \gamma_5 \psi$. For large Euclidean time, the correlation function is dominated by the lowest energy state

$$C(t) \propto | \langle \Omega | O | 0 \rangle |^2 \exp(-E_0 t).$$

(1)

If the correlation function can be measured for sufficiently large time separations, the higher energy states are suppressed and this is a safe method to extract the ground state energy. This can be seen in the effective mass

$$m_{\text{eff}}(t) = -\ln[C(t + 1)/C(t)],$$

(2)

which develops a plateau when the higher states give only a negligible contribution to the correlation function. However, if the energy separations are small, the higher energy states die out slowly and contaminate the lowest state. In addition, if the overlap of the operator with the ground state is accidentally small, it might be necessary to measure the correlation function for very large time separations to suppress the higher states. In these situations, the time separation required for the effective mass to develop a plateau might be unfeasibly large. In principle, one can extract the energy of the excited states by including their contribution to the correlation function. However, in practice, this requires measuring $C(t)$ to greater accuracy than is normally possible.

A more reliable method to extract the energy states is to form a matrix of correlation functions $C_{ij}$ of operators with the same desired quantum numbers \cite{27, 28}. For an $n \times n$ correlation matrix, the solution of the generalized eigenvalue problem

$$\sum_j C_{ij}(1) b_j^{(n)} = \lambda_n \sum_j C_{ij}(0) b_j^{(n)}$$

(3)

between timeslices $t = 0$ and $t = 1$ is used to construct the correlators

$$\tilde{C}_n(t) = \sum_{i,j} b_i^{(n)} C_{ij}(t) b_j^{(n)}$$

(4)

number of configurations 176 as opposed to the 180 which were used in Ref. \cite{26}. The conclusions are not affected by this loss in any way.
whose overlap with the $n$ lowest energy states is optimal. The contamination of the ground state by higher states is minimized and the excited states themselves can also be extracted if the correlation matrix is measured to sufficient accuracy. The correlation matrix analysis is more reliable for extracting the ground state and it is necessary for getting higher energy states for multiparticle systems. If the $\Theta^+$ state does exist, its quantum numbers will be the same as a tower of Kaon-Nucleon interacting states. If the pentaquark is not the lightest state in that channel, at the very least the two lightest states must be extracted to positively identify the $\Theta^+$.

To look for a possible pentaquark state $\Theta^+$, some five-quark operator with the desired quantum numbers must be chosen. To construct a correlation matrix, more than one such operator is required. Without much guidance, it is difficult to tell which will have the best overlap with the $\Theta^+$ wave function. A number of operators have been tried, including ones based on a Kaon-Nucleon-type construction or using the diquark-diquark-antiquark structure proposed by Wilczek and Jaffe. Many possible five-quark operators can be related to one another by Fierz transformations and all of them must have some overlap with Kaon-Nucleon states. Even with the experience of the previous lattice QCD studies, it is not clear what the best choice is. To allow us to make a direct comparison with other work, we use the operator originally suggested in \cite{7} and first used in a lattice simulation in \cite{8}

\[ \Theta = e^{abc}[u_a^T C \gamma_5 d_b]\{u_c \bar{s}_e i \gamma_5 d_c \mp (u \leftrightarrow d)\}, \quad (5) \]

as well as the color-rearranged combination

\[ KN = e^{abc}[u_a^T C \gamma_5 d_b]\{u_c \bar{s}_e i \gamma_5 d_c \mp (u \leftrightarrow d)\}, \quad (6) \]

where $C = \gamma_2 \gamma_4$ is the charge-conjugation matrix and the minus and plus signs correspond to the isospin 0 and 1 combinations respectively. These operators have intrinsic negative parity, however, it is well-known that the correlation functions constructed from these operators receive contributions from negative and positive parity states. To project a particular parity state, an additional factor $(1 \pm \gamma_4)$ is included in the correlation function.

The quark propagators were generated from Gaussian-smeared sources. Using a smeared instead of point source has previously been shown to accelerate the decay of excited states in correlation functions and to generate effective masses whose plateaux extend to smaller time separation, allowing a more accurate determination of the effective mass. Smearing both the source and the sink in the correlation function has not in general shown much additional improvement. In our study, we use smeared sources and point sinks. This asymmetry means the correlation matrix is not Hermitian and so left and right eigenvectors must be determined in the generalized eigenvalue problem to construct the optimal
correlators:
\[
\sum_j C_{ij}(1)b_j^{(n)} = \lambda_n \sum_j C_{ij}(0)b_j^{(n)},
\]
\[
\sum_i a_i^{(n)\dagger}C_{ij}(1) = \lambda_n \sum_i a_i^{(n)\dagger}C_{ij}(0),
\]
\[
\tilde{C}_n(t) = \sum_{i,j} a_i^{(n)\dagger}C_{ij}(t)b_j^{(n)}.
\]  

(7)

4 Numerical results

Previous lattice studies have examined a variety of isospin and parity channels to find a state compatible with the observed $\Theta^+$. With the exception of one study, a possible signal of a genuine pentaquark state has been found only in the isospin 0 negative parity channel. For the purpose of this short paper, we will concentrate on these quantum numbers.

For this ensemble of gauge configurations, quark propagators were calculated for 10 quark masses, with pseudoscalar meson masses ranging from $m_{PS} = 300$ MeV to 1390 MeV. Although one is ultimately interested in reaching the physical quark masses, it is also instructive to study the mass dependence of hadronic states. To calculate a correlation function of five-quark operators, the number of quark propagator contractions is much greater than for two- or three-quark operators. Hence we consider only some of the possible combinations of up-down and strange masses, $m_{ud}$ and $m_s$, for this ensemble. In quenched QCD, where dynamical quark effects are omitted, gauge configurations with topological zero modes of the Dirac operator are not suppressed in the ensemble when the quark masses become small. The zero modes can give large unphysical $O(1/m^n_q)$ contributions to correlation functions, which are only suppressed as the volume becomes large. This effect and possible solutions have been studied extensively for two- and three-quark correlation functions, but not yet for the pentaquark case. Since our physical volume is relatively small, roughly 1.8 fm, this might prevent us from extracting useful information from the lightest quark masses.

We calculate the $2 \times 2$ correlation matrix of the operators $\Theta$ and $KN$ and construct the optimal correlators $\tilde{C}_{ij}(t)$. Near the region where a plateau in the effective mass is observed, we fit the diagonal entries of the optimal correlators to a two-exponential ansatz

\[
\tilde{C}_{00}(t) = A_0 e^{-E_0 t}(1 + B_0 e^{-\Delta_0 t})
\]

\[
\tilde{C}_{11}(t) = A_1 e^{-E_1 t}(1 + B_1 e^{-\Delta_1 t}),
\]

(8)
giving us the ground and first excited energies $E_0$ and $E_1$. The best fit values were obtained using the standard correlated $\chi^2$ method using the measured covariance to estimate the covariance matrix. The errors were obtained by a 1024-point bootstrap sampling procedure. The different quark mass combinations were fitted independently and so correlations between the different quark
mass values were not taken into account. We should note that there are also backward-propagating contributions to the correlators, but these are suppressed by a factor of $\sim 100$ in the region where the two-exponential fits are performed. In Figures 1 and 2 we show typical effective mass plots and the corresponding two-exponential fits for the pentaquark ground and first excited state. A single-exponential fit can also be used, but only over a shorter time range, which limits the precision of the fitted parameters. The single- and two-exponential fitting procedures give completely consistent results, as shown in Figure 3. To demonstrate the stability of the two-exponential fits, we show in Figure 4 the fitted energies for a particular quark mass combination as a function of the minimum timeslice included in the fits. The dotted lines are the $\chi^2$ per degree of freedom from the fits and the horizontal solid and dashed lines indicate the Kaon-Nucleon threshold and lowest non-zero momentum scattering states. The fitting procedure clearly works well.

The meson and nucleon masses for degenerate $m_s$ and $m_{ud}$ quarks have previously been calculated for this ensemble and are tabulated in [26]. In Tables 1 and 2 we give the results for the five-quark ground and first excited state energies, and the pseudoscalar meson mass for non-degenerate quarks. Most of the data are for equal up-down and strange quark masses. The physical value of the strange quark mass is very close to $am_s = 0.078$, so we also consider non-degenerate quark masses with the strange quark mass fixed at this value.

Previous lattice QCD and experimental studies have shown that the Kaon-Nucleon interaction in the isospin 0 s-wave channel is weak [29, 30]. Therefore the energy of a Kaon-Nucleon scattering state with equal back-to-back momentum is closely approximated by

$$E_{KN}(p) = \sqrt{M_K^2 + |\vec{p}|^2} + \sqrt{M_N^2 + |\vec{p}|^2},$$

where the momentum on a finite lattice is quantized $\vec{p} = 2\pi \vec{n}/L$. In Figure 5 we plot the five-quark ground and first excited state energies and the three lowest Kaon-Nucleon scattering state energies as a function of the sum of the nucleon and pseudoscalar meson masses. Over a large range of quark masses, we see that the five-quark energies are in very good agreement with the weakly-interacting scattering states. As the quark masses are reduced, the five-quark excited state is more difficult to determine, but the ground state is still accurately measured. With $m_s$ close to the physical value and $m_{ud} < m_s$, the five-quark ground state agrees well with the Kaon-Nucleon ground state but we cannot make a strong statement about the excited state. The most important observation is that, for heavier quarks where our results are most accurate, we do not see any indication of a pentaquark resonance state, which should lie between the scattering state energy levels. The picture for light quarks in consistent with this, where the first excited state lies between the first and second excited Kaon-Nucleon scattering states, with much larger statistical errors. In principle, one would like to extrapolate the results to the physical quark masses. However, as we mentioned previously, quenched QCD correlation functions suffer from unphysical large contributions from topological zero modes at small quark mass.

This complication might obstruct taking the chiral limit on this lattice volume. In addition, one would like to repeat this calculation for a range of lattice
Let us compare our results with other studies. The first attempts\(^3\) at pentaquark spectroscopy found an energy state slightly above the Kaon-Nucleon threshold in the isospin 0 negative parity channel, which looked consistent with the experimental \(\Theta^+\). Further studies using Bayesian techniques to extract ground and excited states found energies consistent with K-N scattering only \(11\). Scattering states can also be distinguished from resonances via the volume-dependence of the energies, which can also be raised with hybrid boundary conditions \(12\). The amplitudes \(W_n\) of the energy states contributing to the correlation functions also have a distinctive volume-dependence for two-particle scattering. The correlation matrix method has also been used previously to extract the pentaquark ground and excited state energies \(13,14,16\).

\(^3\)One of the referees of this paper has requested to emphasize at this point that the quark-exchange diagram between diquark pairs have been omitted in Ref. \(9\).
There is growing evidence that the isospin 0 negative parity channel, the one thought most likely to describe the experimentally observed $\Theta^+$, only has only weakly-interacting scattering states and no resonance. A stronger claim is made in [13], that a genuine pentaquark resonance should become a bound state at heavy quark mass, which is clearly not seen in any study. However, the picture is not completely consistent. In [16] the pentaquark first excited energy state determined from the correlation matrix does not have the expected volume-dependence of two-particle scattering in small volumes. In [16, 15] the amplitudes $W_n$ for the excited state appear to be independent of volume, signalling a resonance rather than a two-particle state, in contradiction to the findings in [11]. There is also the competing claim [10] that the $\Theta^+$ is actually in the isospin 0 positive parity channel.
5 Conclusions

Our conclusions are relatively straightforward. The correlation matrix technique is designed to extract a number of energy levels in a given channel. Previous evidence suggests that the physical $\Theta^+$, if it exists, is in the isospin 0 negative parity channel, above the Kaon-Nucleon threshold. In that case, the minimum requirement from a lattice study is to identify the two lowest energy states. Doing this, we can accurately determine the five-quark energies for a range of quark masses. We find that the ground and first excited states are completely consistent with weakly-interacting Kaon-Nucleon scattering states, with no indication of a pentaquark resonance which should lie between them. Although we work at a finite lattice spacing, previous work has shown that this lattice action reproduces extremely well continuum properties of hadronic states, so we do not expect our observation to change significantly in the continuum limit.

As the title of this paper suggests, absence of evidence is not evidence of
Figure 4: The fitted energies for the pentaquark ground and first excited states and the quality of the fits as a function of $t_{\text{min}}$, the smallest time separation included in the fit range. The horizontal solid lines are the Kaon-Nucleon threshold (including the error) and the horizontal dashed lines are the energy levels of Kaon-Nucleon scattering states.

absence and there are many caveats one can include. This and all other lattice QCD pentaquark studies have ignored the effect of dynamical fermions, which is an unjustified approximation. The most accurate results ruling out a pentaquark resonance are at unphysically heavy quark masses, requiring an extrapolation to the physical values. We work at a finite lattice spacing and do not extrapolate to the continuum. However we do not expect these effects to qualitatively change our conclusions. We believe a more serious issue is our choice of operators in the correlation matrix, which may have little overlap with a genuine $\Theta^+$ state, if it exists. We cannot rule out this possibility, and there may even be other reasons why we do not see a pentaquark resonance. However we can only conclude that we find no evidence that QCD predicts the $\Theta^+$ state.
Figure 5: The fitted pentaquark ground and first excited state energies as a function of the sum of the nucleon and pseudoscalar meson mass. The degenerate quark mass data are the dark blue and red circles, the non-degenerate quark mass data the light blue and orange squares. The hollow symbols are the three lowest Kaon-Nucleon scattering state energies. The dashed lines indicate the physical values of $M_K + M_N$.

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Table 1: The fitted energy values for the ground and first excited states. The two states were fitted separately to two exponentials after the diagonalization of the $2 \times 2$ correlation matrix.

| $am_{ud}$ | $am_s$ | $t_{\text{min}}$ | $t_{\text{max}}$ | $\chi^2_{\text{dof}}$ | $Q$ | $E_0$ | $E_1$ |
|-----------|--------|------------------|------------------|-----------------------|-----|-------|-------|
| 0.029     | 0.078  | 3                | 12               | 0.84                  | 0.54| 0.889(87) | -     |
|           |        | 2                | 7                | 1.58                  | 0.21| - 1.31(20) |
| 0.032     | 0.078  | 3                | 13               | 0.74                  | 0.64| 0.903(67) | -     |
|           |        | 2                | 7                | 3.02                  | 0.05| - 1.35(15) |
| 0.037     | 0.078  | 2                | 14               | 0.60                  | 0.79| 0.967(30) | -     |
|           |        | 2                | 8                | 2.57                  | 0.05| - 1.37(12) |
| 0.045     | 0.078  | 4                | 15               | 0.45                  | 0.89| 0.981(35) | -     |
|           |        | 2                | 9                | 1.90                  | 0.11| - 1.360(91) |
| 0.037     | 0.037  | 2                | 11               | 0.60                  | 0.61| 0.844(45) | -     |
|           |        | 2                | 7                | 0.57                  | 0.57| - 1.35(20) |
| 0.045     | 0.045  | 4                | 15               | 0.47                  | 0.88| 0.912(36) | -     |
|           |        | 2                | 9                | 0.78                  | 0.53| - 1.34(11) |
| 0.058     | 0.058  | 5                | 16               | 0.50                  | 0.86| 1.027(28) | -     |
|           |        | 2                | 11               | 1.74                  | 0.11| - 1.324(60) |
| 0.078     | 0.078  | 4                | 14               | 0.20                  | 0.99| 1.140(20) | -     |
|           |        | 5                | 11               | 0.43                  | 0.73| - 1.388(75) |
| 0.100     | 0.100  | 4                | 14               | 0.33                  | 0.94| 1.273(17) | -     |
|           |        | 4                | 11               | 0.69                  | 0.60| - 1.438(36) |
| 0.100     | 0.140  | 4                | 14               | 0.47                  | 0.86| 1.329(16) | -     |
|           |        | 4                | 11               | 0.71                  | 0.59| - 1.527(42) |
| 0.140     | 0.140  | 5                | 14               | 0.64                  | 0.70| 1.498(18) | -     |
|           |        | 4                | 12               | 0.41                  | 0.84| - 1.646(34) |
| 0.180     | 0.180  | 4                | 14               | 0.64                  | 0.72| 1.716(13) | -     |
|           |        | 4                | 12               | 0.10                  | 0.99| - 1.863(35) |
| 0.240     | 0.240  | 4                | 15               | 1.08                  | 0.37| 2.011(16) | -     |
|           |        | 4                | 13               | 1.12                  | 0.35| - 2.120(43) |

Table 2: The pseudoscalar meson mass for non-degenerate quark masses.
References

[1] T. Nakano et al. [LEPS Collaboration], Phys. Rev. Lett. 91, 012002 (2003) arXiv:hep-ex/0301020.

[2] R. L. Jaffe, SLAC-PUB-1774 (1976); D. Strottman, Phys. Rev. D 20, 748 (1979); A. V. Manohar, Nucl. Phys. B 248, 19 (1984); M. Chemtob, Nucl. Phys. B 256 (1985) 600.

[3] S. Stepanyan et al. [CLAS Collaboration], Phys. Rev. Lett. 91, 252001 (2003) arXiv:hep-ex/0307018; J. Barth et al. [SAPHIR Collaboration], arXiv:hep-ex/0307083; V. V. Barmin et al. [DIANA Collaboration], Phys. Atom. Nucl. 66, 1715 (2003) [Yad. Fiz. 66, 1763 (2003)] arXiv:hep-ex/0304040; A. E. Asratyan, A. G. Dolgolenko and M. A. Kubantsev, Phys. Atom. Nucl. 67, 682 (2004) [Yad. Fiz. 67, 704 (2004)] arXiv:hep-ex/0309042; V. Kubarovsky et al. [CLAS Collaboration], Phys. Rev. Lett. 92, 032001 (2004) [Erratum-ibid. 92, 049902 (2004)] arXiv:hep-ex/0311046; A. Airapetian et al. [HERMES Collaboration], Phys. Rev. D 70, 012004 (2004) arXiv:hep-ex/0402012; K. T. Knopfle, M. Zavertyaev and T. Zivko [HERA-B Collaboration], J. Phys. G 30, S1363 (2004) arXiv:hep-ex/0403020; C. Pinkenburg [PHENIX Collaboration], J. Phys. G 30, S1201 (2004) arXiv:nucl-ex/0404001; M. J. Longo et al. [HyperCP Collaboration], Phys. Rev. D 70, 111101 (2004) arXiv:hep-ex/0410027; D. O. Litvintsev [CDF Collaboration], Nucl. Phys. Proc. Suppl. 142, 374 (2005) arXiv:hep-ex/0410024; T. Berger-Hryn’ova [BaBar Collaboration], arXiv:hep-ex/0411017; K. Abe et al. [Belle Collaboration], arXiv:hep-ex/0411008; S. R. Armstrong, Nucl. Phys. Proc. Suppl. 142, 364 (2005) arXiv:hep-ex/0410080; P. Rossi [CLAS Collaboration], arXiv:hep-ex/0409057.

[4] J. Z. Bai et al. [BES Collaboration], Phys. Rev. D 70, 012004 (2004) arXiv:hep-ex/0402012; K. T. Knopfle, M. Zavertyaev and T. Zivko [HERA-B Collaboration], J. Phys. G 30, S1363 (2004) arXiv:hep-ex/0403020; C. Pinkenburg [PHENIX Collaboration], J. Phys. G 30, S1201 (2004) arXiv:nucl-ex/0404001; M. J. Longo et al. [HyperCP Collaboration], Phys. Rev. D 70, 111101 (2004) arXiv:hep-ex/0410027; D. O. Litvintsev [CDF Collaboration], Nucl. Phys. Proc. Suppl. 142, 374 (2005) arXiv:hep-ex/0410024; T. Berger-Hryn’ova [BaBar Collaboration], arXiv:hep-ex/0411017; K. Abe et al. [Belle Collaboration], arXiv:hep-ex/0411008; S. R. Armstrong, Nucl. Phys. Proc. Suppl. 142, 364 (2005) arXiv:hep-ex/0410080; P. Rossi [CLAS Collaboration], arXiv:hep-ex/0409057.

[5] D. Diakonov, V. Petrov and M. V. Polyakov, Z. Phys. A 359, 305 (1997) arXiv:hep-ph/9703373.

[6] R. L. Jaffe and F. Wilczek, Phys. Rev. Lett. 91, 232003 (2003) arXiv:hep-ph/0307341; Phys. Rev. D 69, 114017 (2004) arXiv:hep-ph/0312369; K. Cheung, Phys. Rev. D 69, 094029 (2004) arXiv:hep-ph/0308176; R. D. Matheus, F. S. Navarra, M. Nielsen, R. Rodrigues da Silva and S. H. Lee, Phys. Lett. B 578, 323 (2004) arXiv:hep-ph/0309001; F. Stancu and D. O. Riska, Phys. Lett. B 575, 242 (2003) arXiv:hep-ph/0307010; M. Karliner and H. J. Lipkin, Phys.
Lett. B 575, 249 (2003) [arXiv:hep-ph/0402260]; L. Y. Glozman, Phys. Lett. B 575, 18 (2003) [arXiv:hep-ph/0308232]; S. Capstick, P. R. Page and W. Roberts, Phys. Lett. B 570, 185 (2003) [arXiv:hep-ph/0307019]; A. Hosaka, Phys. Lett. B 571, 55 (2003) [arXiv:hep-ph/0307232]; B. K. Jennings and K. Maltman, Phys. Rev. D 69, 094020 (2004) [arXiv:hep-ph/0308286]; C. E. Carlson, C. D. Carone, H. J. Kwee and V. Nazaryan, Phys. Lett. B 573, 101 (2003) [arXiv:hep-ph/0307396]; Phys. Lett. B 579, 52 (2004) [arXiv:hep-ph/0310038], Phys. Rev. D 70, 037501 (2004) [arXiv:hep-ph/0312325]; F. Huang, Z. Y. Zhang, Y. W. Yu and B. S. Zou, Phys. Lett. B 586, 69 (2004) [arXiv:hep-ph/0310040]; R. Bijker, M. M. Giannini and E. Santopinto, Eur. Phys. J. A 22, 319 (2004) [arXiv:hep-ph/0310281]; N. Itzhaki, I. R. Klebanov, P. Ouyang and L. Rastelli, Nucl. Phys. B 684, 264 (2004) [arXiv:hep-ph/0309305]; D. E. Kahana and S. H. Kahana, Phys. Rev. D 69, 117502 (2004) [arXiv:hep-ph/0310026]; F. J. Llanes-Estrada, E. Oset and V. Mateu, Phys. Rev. C 69, 055203 (2004) [arXiv:nucl-th/0311020]; J. Sugiyama, T. Doi and M. Oka, Phys. Lett. B 581, 167 (2004) [arXiv:hep-ph/0309271].

[7] S. L. Zhu, Phys. Rev. Lett. 91, 232002 (2003) [arXiv:hep-ph/0307345].

[8] F. Csikor, Z. Fodor, S. D. Katz and T. G. Kovacs, JHEP 0311, 070 (2003) [arXiv:hep-lat/0309090].

[9] S. Sasaki, Phys. Rev. Lett. 93, 152001 (2004) [arXiv:hep-lat/0310014].

[10] T. W. Chiu and T. H. Hsieh, Phys. Rev. D 72, 034505 (2005) [arXiv:hep-ph/0403020].

[11] N. Mathur et al., Phys. Rev. D 70, 074508 (2004) [arXiv:hep-ph/0403198].

[12] N. Ishii, T. Doi, H. Iida, M. Oka, F. Okiharu and H. Suganuma, Phys. Rev. D 71, 034001 (2005) [arXiv:hep-lat/0408030].

[13] B. G. Lasscock et al., Phys. Rev. D 72, 014502 (2005) [arXiv:hep-lat/0503008].

[14] F. Csikor, Z. Fodor, S. D. Katz, T. G. Kovacs and B. C. Toth, Phys. Rev. D 73, 034506 (2006) [arXiv:hep-lat/0503012].

[15] C. Alexandrou and A. Tsapalis, Phys. Rev. D 73, 014507 (2006) [arXiv:hep-lat/0503013].

[16] T. T. Takahashi, T. Umeda, T. Onogi and T. Kunihiro, Phys. Rev. D 71, 114509 (2005) [arXiv:hep-lat/0503019].

[17] N. Ishii, T. Doi, Y. Nemoto, M. Oka and H. Suganuma, Phys. Rev. D 72, 074503 (2005) [arXiv:hep-lat/0506022].

[18] B. G. Lasscock, D. B. Leinweber, W. Melnitchouk, A. W. Thomas, A. G. Williams, R. D. Young and J. M. Zanotti, Phys. Rev. D 72, 074507 (2005) [arXiv:hep-lat/0504015].
[19] P. Hasenfratz and F. Niedermayer, Nucl. Phys. B 414 (1994) 785,
    arXiv:hep-lat/9308004; P. Hasenfratz, arXiv:hep-lat/9803027.

[20] U. J. Wiese, Phys. Lett. B 315, 417 (1993), arXiv:hep-lat/9306003;
    W. Bietenholz and U. J. Wiese, Phys. Lett. B 378, 222 (1996),
    arXiv:hep-lat/9503022; T. DeGrand, A. Hasenfratz, P. Hasenfratz and
    F. Niedermayer, Nucl. Phys. B 454, 587 (1995), arXiv:hep-lat/9506030;
    T. DeGrand, A. Hasenfratz, P. Hasenfratz and F. Niedermayer, Nucl. Phys.
    B 454, 615 (1995), arXiv:hep-lat/9506031; T. DeGrand, A. Hasenfratz,
    P. Hasenfratz and F. Niedermayer, Phys. Lett. B 365, 233 (1996),
    arXiv:hep-lat/9508024; M. Blatter, R. Burkhalter, P. Hasenfratz and
    F. Niedermayer, Phys. Rev. D 53, 923 (1996), arXiv:hep-lat/9508028;
    W. Bietenholz and U. J. Wiese, Nucl. Phys. B 464, 319 (1996),
    arXiv:hep-lat/9510026; T. DeGrand, A. Hasenfratz and D. c. Zhu,
    Nucl. Phys. B 475, 321 (1996), arXiv:hep-lat/9603015; T. DeGrand, A. Hasenfratz and
    D. c. Zhu, Nucl. Phys. B 478, 349 (1996), arXiv:hep-lat/9604018;
    T. DeGrand, A. Hasenfratz, P. Hasenfratz, P. Kunszt and F. Niedermayer,
    Nucl. Phys. Proc. Suppl. 53, 942 (1997), arXiv:hep-lat/9608056;
    W. Bietenholz, R. Brower, S. Chandrasekharan and U. J. Wiese,
    Nucl. Phys. Proc. Suppl. 53, 921 (1997), arXiv:hep-lat/9608068;
    T. DeGrand, A. Hasenfratz and T. G. Kovacs, Nucl. Phys. B 505, 417 (1997),
    arXiv:hep-lat/9705009; C. B. Lang and T. K. Pany, Nucl. Phys. B 513, 645 (1998),
    arXiv:hep-lat/9707024; F. Farchioni and V. Laliena, Nucl. Phys. B
    521, 337 (1998), arXiv:hep-lat/9709040; K. Orginos, W. Bietenholz,
    R. Brower, S. Chandrasekharan and U. J. Wiese, Nucl. Phys. Proc. Suppl.
    63, 904 (1998), arXiv:hep-lat/9709100; F. Farchioni and V. Laliena,
    Phys. Rev. D 58, 054501 (1998), arXiv:hep-lat/9802009; F. Farchioni,
    I. Hip, C. B. Lang and M. Wohlgenannt, Nucl. Phys. Proc. Suppl. 73, 939 (1999),
    arXiv:hep-lat/9809049; W. Bietenholz and H. Dilger, Nucl.
    Phys. B 549, 335 (1999), arXiv:hep-lat/9812016; T. Bhattacharya,
    R. Gupta and W. J. Lee, Nucl. Phys. Proc. Suppl. 83, 860 (2000),
    arXiv:hep-lat/9910046.

[21] P. H. Ginsparg and K. G. Wilson, Phys. Rev. D 25 (1982).

[22] P. Hasenfratz, Nucl. Phys. Proc. Suppl. 63, 53 (1998),
    arXiv:hep-lat/9709110; P. Hasenfratz, V. Laliena and F. Niedermayer,
    Phys. Lett. B 427, 125 (1998), arXiv:hep-lat/9801021;
    P. Hasenfratz, Nucl. Phys. B 525, 401 (1998), arXiv:hep-lat/9802007.

[23] R. Narayanan and H. Neuberger, Nucl. Phys. B 443, 305 (1995),
    arXiv:hep-th/9411108; Nucl. Phys. B 412, 574 (1994),
    arXiv:hep-lat/9307006; Phys. Rev. Lett. 71, 3251 (1993),
    arXiv:hep-lat/9308011; Phys. Lett. B 302, 62 (1993),
    arXiv:hep-lat/9212019; D. B. Kaplan, Phys. Lett. B 288, 342 (1992),
    arXiv:hep-lat/9206013; Y. Shamir, Nucl. Phys. B 406, 90 (1993),
    arXiv:hep-lat/9303005; V. Furman and Y. Shamir, Nucl.
    Phys. B 439, 54 (1995), arXiv:hep-lat/9405004; T. Blum et al.,
    Phys. Rev. D 69, 074502 (2004), arXiv:hep-lat/0007038; C. Gattringer,
    Phys. Rev. D 63, 114501 (2001), arXiv:hep-lat/0003005;
C. Gattringer, I. Hip and C. B. Lang, Nucl. Phys. B 597, 451 (2001), arXiv:hep-lat/0007042; C. Gattringer, Nucl. Phys. Proc. Suppl. 119, 122 (2003), arXiv:hep-lat/0208058.

[24] P. Hasenfratz, S. Hauswirth, K. Holland, T. Jorg, F. Niedermayer and U. Wenger, Int. J. Mod. Phys. C 12, 691 (2001), arXiv:hep-lat/0003013; F. Niedermayer, P. Rufenacht and U. Wenger, Nucl. Phys. B 597, 413 (2001), arXiv:hep-lat/0007007; P. Hasenfratz, S. Hauswirth, K. Holland, T. Jorg, F. Niedermayer and U. Wenger, Nucl. Phys. Proc. Suppl. 94, 627 (2001), arXiv:hep-lat/0010061; F. Niedermayer, P. Rufenacht and U. Wenger, Nucl. Phys. Proc. Suppl. 94, 636 (2001), arXiv:hep-lat/0011041; P. Rufenacht and U. Wenger, Nucl. Phys. B 616, 163 (2001), arXiv:hep-lat/0108005; P. Hasenfratz, S. Hauswirth, K. Holland, T. Jorg and F. Niedermayer, Nucl. Phys. Proc. Suppl. 106, 799 (2002), arXiv:hep-lat/0109004; P. Hasenfratz, S. Hauswirth, K. Holland, T. Jorg and F. Niedermayer, Nucl. Phys. Proc. Suppl. 106, 751 (2002) arXiv:hep-lat/0109007; S. Hauswirth, arXiv:hep-lat/0204015; T. Jorg, arXiv:hep-lat/0206025; P. Hasenfratz, S. Hauswirth, T. Jorg, F. Niedermayer and K. Holland, Nucl. Phys. B 643, 280 (2002), arXiv:hep-lat/0205010; C. Gattringer et al. [Bern-Graz-Regensburg Collaboration], Nucl. Phys. Proc. Suppl. 119, 796 (2003), arXiv:hep-lat/0209099.

[25] C. Gattringer et al. [BGR Collaboration], Nucl. Phys. B 677, 3 (2004) arXiv:hep-lat/0307013.

[26] P. Hasenfratz, K. J. Juge and F. Niedermayer [Bern-Graz-Regensburg Collaboration], JHEP 0412, 030 (2004) arXiv:hep-lat/0411034.

[27] C. Michael, Nucl. Phys. B 259, 58 (1985).

[28] M. Luscher and U. Wolff, Nucl. Phys. B 339, 222 (1990).

[29] M. Fukugita, Y. Kuramashi, M. Okawa, H. Mino and A. Ukawa, Phys. Rev. D 52, 3003 (1995), arXiv:hep-lat/9501024.

[30] J. S. Hyslop, R. A. Arndt, L. D. Roper and R. L. Workman, Phys. Rev. D 46, 961 (1992).