A version for intense γ-ray radiation based on the multiphoton scattering of strong laser radiation on relativistic particle beam channeled in a crystal is proposed. The scheme is considered when the incident laser beam and charged particles beam are counter-propagating and the laser radiation is resonant to the energy levels of transversal motion of channeled particles.

1. INTRODUCTION

As is known channeling occurs if a charged particle enters a crystal at an angle to a crystallographic axis or plane smaller than Linchard angle $\theta_L = \sqrt{2U_0/\varepsilon}$, where $U_0$ is the depth of a transverse potential well, and $\varepsilon$ is the particle’s energy [1]. The spontaneous radiation of channeled particles [2] has some significant properties that open possibilities for implementation of short-wave radiation sources since due to their large Doppler shift and the high oscillation frequency in the channel ($\Omega \sim 10^{14} - 10^{16} s^{-1}$ for particles energies $\varepsilon \sim 10$GeV–10MeV) particles emit quanta mainly in the X-ray and γ-ray domain with an intensity much higher than the intensity of other types of radiation (see [1] and references therein).

The spontaneous radiation of channeled particles has been comprehensively studied both theoretically and experimentally (see, e.g., Refs [1], [3], while induced channeling radiation, that is radiation in the presence of external electromagnetic wave (EMW), has been studied mainly in the linear regime of interaction [4], [5]. To achieve a considerable amplification in the single pass non-linear regime of X-ray amplification has been studied in [6], but these investigations show that implementation of short-wave coherent radiation sources due to stimulated channeling radiation is far from being realized yet. One reason is that because of short lifetime of a particle transverse-motion levels the length of coherent interaction of a channeled particle with EMW is quite short (e.g., of the order of one micrometer at positron energies $\sim 10$MeV) compared to the interaction length in other versions of Free Electron Laser (such as the undulator and Cherenkov lasers). There is another problem connected with the controlling of the channeled particles overpopulation [7].

Two component laser assisted schemes for radiation enhancement have also been studied. One of those is based on stimulated photon scattering by channeled particles [8]. Since absorption of a photon by a channeled particle is a resonant process, the cross section of photon scattering by a channeled particle is $10^4$ times larger than the free electron Compton scattering cross section [9]. Nevertheless, the gain of a EMW in stimulated Compton scattering on channeled particles does not exceed the gain of stimulated emission in the channel from the initially inverse populated states. The second scheme concerns the quantum mode of interaction, that is the coherent radiation of quantum modulated beam at the frequency of the stimulating wave [10] and its harmonics [11].

In this paper we investigate multiphoton scattering of strong laser radiation on relativistic particle beam channeled in a crystal which can serve as a possible scheme for γ-ray generation. The scheme is considered when averaged potential for a plane channeled particles is good enough described by the harmonic potential. Then it is assumed that the incident laser beam and charged particles beam are counter-propagating and the laser radiation is resonant to the energy levels of transversal motion of channeled particles. In the result of the multiphoton scattering the hard quanta are generated. The discussed scheme has several advantages in respect to the known ones. First of all, the cross section of the process is resonantly enhanced in respect to the Compton scattering process. At the second, the multiphoton processes arise at the much lower laser intensities than in the case of the Compton scattering. Besides, the scheme enables to use the forward channeling radiation arising due to transitions from short living, high excited states of a particle to the ground state, that could not be achievable in the process of spontaneous channeling radiation.

This paper is organized as follows. In Section II the wave function of a plane channeled particle in a EMW is obtained. In Sec. III within the scope of Quantum Electrodynamics the spectral intensity of multiphoton Compton scattering in a strong laser field is obtained. The first-order Feynman diagram, where the electron/positron lines correspond to the wave functions in the strong laser field is calculated and the resonant case of interaction is discussed.
II. WAVE FUNCTION OF A PLANE CHANNELED PARTICLE IN THE FIELD OF TRANSVERSE ELECTROMAGNETIC WAVE

We will consider the case when averaged potential of the crystal for a plane channeled particle is good enough described by the harmonic potential

\[ U(x) = \kappa \frac{x^2}{2} \]  \hspace{1cm} (2.1)

for plane channeled positron

\[ \kappa = \frac{2U_0}{d^2} \]  \hspace{1cm} (2.2)

where \( U_0 \) is the transverse potential holes depth, and \( d \) is the interspace distance \([4]\). For plane channeled electrons the approximate potential is actually not harmonic, but for the high energies it can be approximated by (2.1). As it is known, for the channeled particles the depth of the potential hole is \( U_0 << E \), where \( E \) is the particle energy. The spin interaction which is \( \tilde{\gamma} \cdot \nabla U(x) \) is again less than \( E \). For this reason the transversal motion is good enough describes by the Schrödinger equation \([3]\) with effective mass \( m_{eff} = E_\parallel \) (the natural units \( \hbar = c = 1 \) will be used throughout this paper), where \( E_\parallel = \sqrt{p_x^2 + m^2} \) is the energy of longitudinal motion, \( m \) is the particle mass.

On the other hand, the spin interaction can play a role in a spontaneous radiation process if the radiated photons energy is \( \omega \cdot E \). But if the particles energy is not enough high, i.e.

\[ E << \frac{m^2}{E_\parallel} \]  \hspace{1cm} (2.3)

where \( E_\parallel \) is the energy of transversal motion, then \( \omega << E \) and the spin effects are not substantial. Let us mention that (2.3) can be re-written in the following way

\[ E_\perp << \frac{E}{\gamma^2} \]  \hspace{1cm} (2.2')

(where \( \gamma \) is the Lorenz factor) which is the condition allowing to neglect the impact of the transversal oscillations on the longitude motion. We will consider that not only but also the change of the transversal motions energy which is resulted from the interaction with the external EM field is small

\[ \Delta E_\perp << \frac{E}{\gamma^2} \]  \hspace{1cm} (2.4)

As there is a relation between the total \( E \) and transverse energy changes \( \Delta E_\perp \):

\[ \Delta E = \gamma^2 \Delta E_\perp \]

then due to the Doppler-shift of the emitted radiation, the condition (2.4) actually restricts the total energy change

\[ \Delta E << E_0, \]  \hspace{1cm} (2.3r)

which means that anyway for the frequency of the external electromagnetic radiation \( \omega_0 << E \). For this reason in the considering process of interaction of the channeled particles with the external EM radiation the spin interaction will not play any role. So we will use the Klein-Gordon equation

\[ \left[ i \frac{\partial}{\partial t} - U(x) \right]^2 \Psi = \left[ (\hat{\mathbf{p}} - e\mathbf{A})^2 + m^2 \right] \Psi, \]  \hspace{1cm} (2.5)

where \( e \) is the particle charge and

\[ \mathbf{A} = \{A_0 \cos \omega_0(t + nz), 0, 0\} \]  \hspace{1cm} (2.6)

is the vector potential of the plane EMW. Here \( n \) is the crystal refracting index on \( \omega_0 \), So, as the external EM field depends only on the \( \tau = t + nz \) then raising from the problem symmetry, the wave function can be found in the following form:

\[ \Psi(r, t) = f(x, \tau) \exp [i p_y y + i p_z z - i E t] \]  \hspace{1cm} (2.7)

Taking into account (2.4) we can consider \( f(x, \tau) \) as a slowly varying function of \( \tau \) and neglect the second derivative compared with the first order. So for \( f(x, \tau) \) we will have the following equation:

\[ \left[ \partial_{xx} + 2E_\parallel (E_\perp - U(x)) + 2i\tilde{p}\partial_\tau \right. \]

\[ -2iA(\tau)\partial_z - e^2A^2(\tau) \left] f = 0 \right. \]  \hspace{1cm} (2.8)

where

\[ \tilde{p} = E + np_z. \]

In Eq. (2.8) transverse and longitudinal motions are not separated. But after a certain Unitary transformation in the equation for the transformed function the variables are separated \([10]\) and for wave function we obtain

\[ \Psi = \frac{1}{\sqrt{2\Pi}} \exp \{i\Pi_y y + i\Pi_z z - i\Pi t\} \]

\[ \times \exp \left\{ -i\frac{\tilde{\omega}(\tilde{\omega}^2 + \Omega^2)}{8E_\parallel E^2} \tilde{e} \tilde{A}_0^2 \sin 2\omega_0 \tau - i\frac{\Omega^2}{\Delta} \tilde{e} A_0 \cos \omega_0 \tau \right\} \]

\[ \times V_\epsilon \left[ x + \frac{\tilde{\omega}}{E_\parallel E} \tilde{e} A_0 \sin \omega_0 \tau \right], \]  \hspace{1cm} (2.9)

where the state of the particle in EM fields (2.1, 2.0) is characterized by the average energy and momentum ("quazimomentum") defining via free particle energy-momentum by the following equations.

\[ \Pi_y = p_y, \quad \Pi_z = p_z - n \frac{\tilde{\omega}^2}{4p_\Delta} \tilde{e}^2 \tilde{A}_0^2, \]
Here

\[ \tilde{\omega} = \frac{\bar{p}}{E_{||}} \omega_0; \quad \Delta = \tilde{\omega}^2 - \Omega^2 \]  

(2.11)

and

\[ V_s(x) = \frac{1}{\pi^2} \sqrt{\frac{\chi}{2^{2s!}}} \exp \left[ -\frac{\chi^2 x^2}{2} \right] H_s(\chi x), \]

\[ \chi = \sqrt{E_{||}/\Omega}, \quad \Omega = \sqrt{\frac{k}{E_{||}}} \]

are the wave functions of the harmonic oscillator with Hermit polynomials \( H_s(\chi x). \) In \( \Theta \) it is assumed that initial state of the channeled particle is \( \{p_y, p_z, s\} \) (before the interaction with EMW).

III. COMPTON SCATTERING ON CHANNELED PARTICLES IN A CRYSTAL

A. Feynman first order diagram

As we saw in Sec. II \( \Pi_y, \Pi_z \) and \( s \) are the quantum numbers (neglecting spin interaction) describing the state of a particle moving in EM fields \( (2.1, 2.6). \) It is clear that between them there will be a spontaneous transitions which causes spontaneous radiation. The spontaneous radiation may be considered by the theory of perturbation. In this case first order Feynman diagram describes spontaneous radiation where wave functions \( \{2.9\} \) correspond to electron/positron lines. The probability amplitude of transition from the state \( \{\Pi_{0y}, \Pi_{0z}, s_0\} \) to the state \( \{\Pi_y, \Pi_z, s\} \) with emission of a photon with the frequency \( \omega \) and momentum \( k \) will be \( \Theta \)

\[ M_{s_0,s}^{(\nu)} = e \sqrt{4\pi f_{\nu}} \frac{e_{\nu}^\mu}{\sqrt{2\omega}} \]

(3.1)

where \( e_{\nu}^\mu \) is the four-dimensional polarization vector, index \( \nu \) corresponds to the emitted photons of two possible polarizations \( \nu = 1, 2. \) Here

\[ j^\mu = \int d^4x j^\mu f_{\nu}(r,t) e^{i(\omega t - kr)} \]

(3.2)

where \( j^\mu f_{\nu}(r,t) \) is the four-dimensional transition current.

As is known the polarization vector may always be chosen in a way that \( e_0^0 = (0, e_0); e_0 k = 0 \) (three dimension gage). The differential probability calculated in unit volume and unit time will be

\[ dW_{s_0,s}^{(\nu)} = \frac{\left| M_{s_0,s}^{(\nu)} \right|^2}{L_y L_z T} \frac{dk d\Pi_y d\Pi_z}{(2\pi)^3} \]

(3.3)

where \( L_y, L_z \) are quantization length \( T \) is the interaction time. If we are not interested in the dependence of process on the photons polarization then the probabilities must be summed by all possible polarizations

\[ |M_{s_0,s}^{(\nu)}|^2 = \left| M_{s_0,s}^{(1)} \right|^2 + \left| M_{s_0,s}^{(2)} \right|^2 \]

(3.4)

Taking into consideration that transition current \( j^\mu \) satisfies to the continuity equation

\[ j_k = \omega j_0 \]

the probabilities \( \Theta \) can be presented in the following form

\[ |M_{s_0,s}^{(\nu)}|^2 = |j|^2 - |j_0|^2 \]

(3.5)

that corresponds to the summation by the photons polarizations for \( \nu = 1, \ldots, 4 \) in general case.

As long as in our case \( \omega << E \) and the condition \( (2.4) \) must be satisfied then the transition currents may be calculated using the solution of the Klein-Gordon equation. The transition currents will be

\[ j_{\nu,i} = i \left( \Psi_i \nabla \Psi_\nu^* - \Psi_\nu^* \nabla \Psi_i \right) - 2eA \Psi_i \Psi_\nu^* \]

(3.6)

Using this expression for transition currents and taking into account \( (3.2), (3.3) \) and \( (3.6) \) we will arrive to the following expression for the differential probability

\[ dW_{s_0,s}^{(\nu)} = \frac{e^2}{8\pi^2 \Omega_\Pi^2} \sum_{\ell = -\infty}^{\infty} W_{s_0,0}^{(\ell)} \delta(\Pi_z - \Pi_{0z} + k_z + \ell \omega) \]

\[ \times \delta(\Pi_y - \Pi_{0y} + k_y) \delta(\Pi - \Pi_0 + \omega - \ell \omega) dkd\Pi_y d\Pi_z \]

(3.7)

In general the expressions of the partial probabilities are very complicated and we will not bring here. The \( \delta \) functions presenting in the expression \( (3.7) \) for differential probability express the quazimomentum and quazienergy conservation laws in the given process. Different \( \ell \) correspond to different partial processes with fixed photon numbers and \( W_{s_0,0}^{(\ell)} \) are the partial probabilities. Let us find the emitted photon’s energy rising from the conservation laws. Taking into account \( (2.4) \) and \( \omega << E \) we will have the following expression for \( \omega \)

\[ \omega = \frac{1 + n\bar{v}_{0z}}{1 - \frac{k\bar{v}_{0z}}{\omega}} \left[ \omega_0 + \Omega'(s_0 - s) \right] \]

(3.8)

where

\[ \Omega' = \frac{\Omega}{1 + n\bar{v}_{0z}} \]

and

\[ \bar{v}_{0z} = \frac{\Pi_z}{\Pi_0} \]
is the mean longitudinal velocity. When \( \Omega = 0 \) and \( n = 1 \) the formula (3.8) is reduced to the well known Compton effect one for the scattered frequency (neglected quantum recoil).

It follows from (3.8) that \( \ell > 0 \) corresponds to the multiphoton absorption and \( \ell < 0 \) to the multiphoton emission of a wave quanta. It is noteworthy to mention that in the nonlinear Compton process on free electrons \([19]\) only the case of \( \ell > 0 \), i.e. the multiphoton absorption process in the strong EM wave takes place. In contrary, at the scattering of a strong wave on channeling particles the multiphoton emission (\( \ell < 0 \)) of quanta of EMW takes place.

B. Resonance case

Let us consider the resonance case which is of more interest and expression for differential probability may be simplified. We will consider the case when

\[
\frac{\bar{\omega}_0 - \Omega}{\Omega} \ll 1; \quad \frac{\bar{\omega} - \Omega}{\Omega} \ll 1 \tag{3.9}
\]

where \( \bar{\omega}_0 \) and \( \bar{\omega} \) are initial and final Doppler shifted frequencies (3.9). Besides we will assume that

\[
\xi \equiv \frac{eA_0}{m} >> |\delta|; \quad \delta = \frac{\bar{\omega}_0 - \Omega}{\Omega} \tag{3.10}
\]

Here \( \xi \) is the relativistic invariant parameter of the wave intensity.

Considering (3.9) it is possible that despite \( \xi << 1 \) but the condition (3.10) may be satisfied.

To obtain the total cross section of nonlinear scattering the expression (3.7) must be summed by all discrete states of transverse motion in the channel. After integrating by \( \Pi_y \) and \( \Pi_z \), then summing by \( \ell \), using the \( \delta \) functions, and taking into account the (3.4), (3.10) for differential cross section we will have

\[
dW = \frac{m^2e^2}{2\pi\bar{\omega}\omega_0 \Pi_\Pi} \left[ -\Lambda_0^2(N) \right. \\
+ \left. \left( \frac{\xi}{2\delta} \right)^2 [\Lambda_1^2(N) - \Lambda_0(N)\Lambda_2(N)] \right] d\mathbf{k} \tag{3.11}
\]

Here

\[
\Lambda_r(N, \alpha, \beta) = (2\pi)^{-1} \int d\theta \cos^r \theta \exp \left[ i (\alpha \sin \theta - \beta \sin 2\theta - \delta \cos \theta) \right] \tag{3.12}
\]

are known functions \([19]\) and represent non linear processes in the field of linear polarized wave (multiphoton Compton effect, pair production, etc.).

\[
\alpha = \frac{2m^2k_{\bar{\omega}}}{E_\parallel \Delta \Delta_0} (\bar{\omega}_0 \bar{\omega} - \Omega^2) \tag{3.13}
\]

\[
\beta = \frac{\xi^2 m^2(\bar{\omega}_0 - \bar{\omega})}{8E_\parallel \Delta \Delta_0} (\bar{\omega}_0 \bar{\omega} - \Omega^2) \tag{3.14}
\]

and \( N \) is fixed by the conservation law which in the resonant case is

\[
\omega = \frac{1 + n\bar{\omega}_0}{1 - \frac{k\bar{\omega}_0}{\omega}} N\omega_0, \tag{3.15}
\]

The formula (3.11) defines the spectral intensity of an one-photon emission (if product to \( \omega \)) in the crystal at simultaneously nonlinear ”Compton” scattering of a strong EM wave on the channeled particle at the resonance. Instead of parameter nonlinearity \( \xi^2 \) in the Compton effect on free electrons the effective nonlinearity in the channeling process is determined by the resonance parameter \( \left( \frac{\xi}{2\delta} \right)^2 \), increasing the cross sections of the multiphoton ”Compton” scattering. For the actual cases \( \delta \sim 10^{-2} \div 10^{-1} \) \([7]\), and consequently the parameter of nonlinearity increases \( ^*10^3 \) times. As the number of absorbed photons should be restricted by condition \( N\omega_0 << U_0 \) (\( U_0 \) being the depth of the channeling potential well) to avoid the dechanneling effects, so for forward radiation of a particle with the energy \( E_\parallel \sim 50MeV \), maximum of emitted quanta energies up to \( h\omega \sim 1MeV \) are achievable.

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