Some investigation on continuous functions in almost contra semi-generalized star \( b \) in topological spaces

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Abstract
This manuscript’s main aim is to present some class of functions in topological spaces called almost contra semi generalized star \( b \)-continuous function (briefly almost contra \( sg^*b \)-continuous). Some characterizations and a variety of characteristics are obtained related to almost contra \( sg^*b \)-continuous functions.

Keywords
\( sg^*b \)-closed sets; \( sg^*b \)-closed map; \( sg^*b \)-continuous map; contra \( sg^*b \)-continuity.

AMS Subject Classification
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1 Introduction ......................................................... 173
2 Continuous functions Almost contra semi generalised star \( b \) ......................................................... 173
3 Semi generalised star \( b \)- regular graphs and strongly contra semi generalised star \( b \)- closed graphs . . 176
4 Connectedness ..................................................... 176
5 Separation axioms .................................................. 177
References .............................................................. 177

1. Introduction

A novel class was organized and investigated by Jafari and Noiri in 2002 of functions called continuous contra-pre functions. The purpose of this paper is to implement and research almost against semi generalised star \( b \)-continuous functions through the semi generalised star \( b \)-closed sets. Also, near contra semi generalised star \( b \)-continuity characteristics are discussed about. In addition, fundamental properties and preservation are obtained. semi generalised star \( b \)-continuous theorems This paper defines the \((X, \tau)\) and \((Y, \sigma)\) describing the non-empty topological spaces without axioms of separation Unless otherwise mentioned, this is presumed. Let \( A \subseteq X, \) the \( cl(A) \) and will denote the closure of \( A \) and the interior of an \( A int(A) \) and the union of all \( X \) semi generalised star \( b \)-open sets found in \( A \) is classified as \( sg^*b \)-interior of \( A \) and is marked by \( sg^* bint(A) \), The intersection of all semi generalised star \( b \)-closed sets of \( X \) containing \( A \) is called semi generalised star \( b \)-closure of \( A \) and it is denoted by \( sg^* bcl(A) \) [5].

2. Continuous functions Almost contra semi generalised star \( b \)

In this section, we present nearly contra semi generalised in this section Star \( b \)-continuous features and analyzed of some of their functions axioms.

Definition 2.1. [11] A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is almost called contra semi generalised star \( b \)-continuous if \( f^{-1}(V) \) is semi generalised star \( b \)-closed in \((X, \tau)\) for each regular open set \( V \) in \((Y, \sigma)\).

Example 2.2. [11] Let \( X = Y = \{1, m, n\} \) with \( \tau = \{X, \varnothing, \{l\}, \{l, m\}\} \) and \( \sigma = \{Y, \varnothing, \{l\}, \{m\}, \{l, m\}, \{l, n\}\} \). Define a function \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(l) = m, f(m) = n, f(n) = l \). Clearly \( f \) is almost contra semi generalised star \( b \)-continuous.

Theorem 2.3. If \( f : X \rightarrow Y \) is contra semi generalised star \( b \)-continuous then it is almost contra semi generalised star \( b \)-continuous.
Proof. It’s simple, because every open regular set is an open set.

Remark 2.4. Alternatively, the theorem alluded to above need not be valid as shown in the subsequent case in common.

Example 2.5. Let $X = Y = \{l, m, n\}$ with $\tau = \{X, \varphi, \{l\}\}$ and $\sigma = \{Y, \varphi, \{l\}, \{m\}, \{l, m\}\}$. Defining a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(l) = m, f(m) = n, f(n) = l$. Then $f$ is almost contra semi generalised star $b$ - continuous but not contra function semi generalised star $b$ - continuous, because for the open set $\{1\}$ in $Y$ and $f^{-1}\{1\} = \{n\}$ is not semi generalised star $b$ - closed in $X$.

Theorem 2.6. 1) Almost every contra pre - continuous function is nearly contra semi generalised star $b$ - continuous function.

2) Each semi-continuous almost contra function is almost contra semi generalised star $b$ - continuous function.

3) Almost every contra $\alpha$ - the continuous function is almost contra semi generalised star $b$ - continuous function.

4) Almost every contra $\alpha g$ - the continuous function is almost contra semi generalised star $b$ - continuous function.

5) Almost every contra semi generalised star $b$ - the continuous function is almost contra gsp - continuous function.

6) Almost every contra semi generalised star $b$ - the continuous function is almost contra semi generalised star $b$ - continuous function.

Remark 2.7. As can be seen in the following example, the reverse of the above claims is not true.

Example 2.8. i) Let $X = Y = \{l, m, n\}$ with $\tau = \{X, \varphi, \{l\}, \{m\}, \{l, m\}\}$ and $\sigma = \{Y, \varphi, \{l\}, \{n\}, \{l, n\}\}$. Defining a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(l) = m, f(m) = l, f(n) = n$. Clearly $f$ is almost contra semi generalised star $b$ - continuous but $f$ is not nearly contra pre - continuous. Because $f^{-1}\{n\} = \{m\}$ is not pre - closed in $(X, \tau)$ where $\{n\}$ is regular - open in $(Y, \sigma)$.

ii) Let $X = Y = \{l, m, n\}$ with $\tau = \{X, \varphi, \{l\}, \{m\}\}$ and $\sigma = \{Y, \varphi, \{l\}, \{n\}, \{l, n\}\}$. Defining a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(l) = n, f(m) = l, f(n) = m$. It’s obvious that $f$ is almost contra semi generalised star $b$ - continuous function but $f$ is not almost contra semi - continuous. Because $f^{-1}\{l\} = \{m\}$ is not semi - closed in $(X, \tau)$ where $\{l\}$ is regular - open in $(Y, \sigma)$.

iii) Let $X = Y = \{l, m, n\}$ with $\tau = \{X, \varphi, \{l\}, \{m\}\}$ and $\sigma = \{Y, \varphi, \{l\}, \{n\}, \{l, n\}\}$. Defining a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(l) = m, f(m) = l, f(n) = n$. It’s obvious that $f$ is almost contra semi generalised star $b$ - continuous but $f$ is not almost contra $\alpha$ - continuous. Because $f^{-1}\{l, n\} = \{m, n\}$ is not $\alpha$ - closed in $(X, \tau)$ where $\{l, n\}$ is regular - open in $(Y, \sigma)$.

iv) Let $X = Y = \{l, m, n\}$ with $\tau = \{X, \varphi, \{l, m\}\}$ and $\sigma = \{Y, \varphi, \{l\}, \{m\}, \{l, m\}\}$. Defining a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(l) = n, f(m) = l, f(n) = m$. It’s obvious that $f$ is almost contra semi generalised star $b$ - continuous but $f$ is not almost contra $\alpha g$ - continuous. Because $f^{-1}\{m\}$ is not semi generalised star $b$ - closed in $(X, \tau)$ where $\{m\}$ is regular - open in $(Y, \sigma)$.

Example 2.10. Let $X = Y = \{l, m, n\}$ with $\tau = \{X, \varphi, \{l\}, \{m\}, \{l, m\}\}$ and $\sigma = \{Y, \varphi, \{l\}, \{m\}, \{l, m\}\}$. Defining a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(l) = n, f(m) = m, f(n) = l$. It’s obvious that $f$ is almost contra semi generalised star $b$ - continuous but $f$ is not almost contra semi generalised $b$ - continuous. Because $f^{-1}\{m\}$ is not $sgb$ - closed in $(X, \tau)$ where $\{m\}$ is regular - open in $(Y, \sigma)$.

Example 2.11. Let $X = Y = \{l, m, n\}$ with $\tau = \{X, \varphi, \{l\}, \{n\}, \{l, n\}\}$ and $\sigma = \{Y, \varphi, \{m\}, \{n\}, \{m, n\}\}$. Defining a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(l) = n, f(m) = l, f(n) = m$. It’s obvious that $f$ is almost contra semi generalised $b$ - continuous but $f$ is not almost contra semi generalised star $b$ - continuous. Because $f^{-1}\{m\}$ is not semi generalised star $b$ - closed in $(X, \tau)$ where $\{m\}$ is regular - open in $(Y, \sigma)$.

Theorem 2.12. The following are equivalent for a function $f : X \rightarrow Y$.

(a) $f$ is almost contra semi generalised star $b$ - continuous.

(b) for every regular closed set $F$ of $Y$, $f^{-1}(F)$ is semi generalised star $b$ - open set of $X$.

(c) for each $x \in X$ and each regular closed set $F$ of $Y$ containing $f(x)$ there exists semi generalised star $b$ - open $U$ containing $x$ such that $U \subset F$.

(d) for each $x \in X$ and each regular open set $V$ of $Y$ not containing $f(x)$, there exists semi generalised star $b$ - closed set $K$ not containing $x$ such that $f^{-1}(V) \subset K$. 

174/177
Some investigation on continuous functions in almost contra semi-generalised star $b$ in topological spaces — 175/177

Proof. (a) $\Rightarrow$ (b): Let $F$ be a regular closed set in $Y$, then $Y - F$ is a regular open set in $Y$. By (1), $f^{-1}(Y - F) = X - f^{-1}(F)$ is semi generalised star $b$ - closed set in $X$. This implies $f^{-1}(F)$ is semi generalised star $b$ - open set in $X$. Therefore, (b) holds.

(b) $\Rightarrow$ (a): Let $G$ be a regular open set of $Y$. Then $Y - G$ is a regular closed set in $Y$. By (b), $f^{-1}(Y - G)$ is semi generalised star $b$ - open set in $X$. This implies $X - f^{-1}(G)$ is semi generalised star $b$ - open set in $X$, which implies $f^{-1}(G)$ is semi generalised star $b$ - closed set in $X$. Therefore, (a) hold.

Theorem 2.13. The following are equivalent for a function $f : X \rightarrow Y$.

(I) $f$ is almost contra semi generalised star $b$ - continuous.

(II) $f^{-1}(Int(Cl(G)))$ is semi generalised star $b$ - closed set in $X$ to each open subset $G$ of $Y$.

(III) $f^{-1}(Cl(Int(G)))$ is semi generalised star $b$ - open set in $X$ to each closed subset $F$ of $Y$.

Proof. (I) $\Rightarrow$ (II): Let $G$ be an open set in $Y$. Then $Int(Cl(G))$ is regular open set in $Y$. By (I), $f^{-1}(Int(Cl(G))) \in sg^b - C(X)$.

(II) $\Rightarrow$ (I): Proof is obvious.

(I) $\Rightarrow$ (III): Let $F$ be a closed set in $Y$. Then $Cl(Int(G))$ is regular closed set in $Y$. By (I), $f^{-1}(Cl(Int(G))) \in sg^b - O(X)$.

(III) $\Rightarrow$ (I): Proof is obvious. □

Theorem 2.14. Let $f : X \rightarrow Y$ be a contra semi generalised star $b$ - continuous and $g : Y \rightarrow Z$ be semi generalised star $b$ - continuous. If $Y$ is $Tsg^b$ - space, then $g \circ f : X \rightarrow Z$ is an almost contra semi generalised star $b$ - continuous.

Proof. Let $V$ be any regular open and hence open set in $Z$. Since $g$ is semi generalised star $b$ - continuous $g^{-1}(V)$ is semi generalised star $b$ - open in $Y$ and $Y$ is $Tsg^b$ - space implies $g^{-1}(V)$ open in $Y$. Since $f$ is contra semi generalised star $b$ - continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is semi generalised star $b$ - closed set in $X$. Therefore, $g \circ f$ is an almost contra semi generalised star $b$ - continuous. □

Theorem 2.15. If $f : X \rightarrow Y$ is onto strongly semi generalised star $b$ - open (or strongly semi generalised star $b$ - closed) and $g : Y \rightarrow Z$ is a function such that $g \circ f : X \rightarrow Z$ is an almost contra semi generalised star $b$ - continuous, then $g$ is an almost contra semi generalised star $b$ - continuous.

Proof. Let $V$ be any regular closed (resp. regular open) set in $Z$. Since $g \circ f$ is an almost contra semi generalised star $b$ - continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is semi generalised star $b$ - open (resp. semi generalised star $b$ - closed) in $X$. Since $f$ is surjective and strongly semi generalised star $b$ - open (or strongly semi generalised star $b$ - closed), $(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is semi generalised star $b$ - open (or semi generalised star $b$ - closed). Therefore $g$ is an almost contra semi generalised star $b$ - continuous. □

Definition 2.16. A function $f : X \rightarrow Y$ is called weakly semi generalised star $b$ - continuous if for every $x \in X$ and each open set $V$ of $Y$ containing $f(x)$, there exists $U \in sg^b - O(X; x)$ such that $f(U) \in cl(V)$.

Theorem 2.17. If a function $f : X \rightarrow Y$ is an almost contra semi generalised star $b$ - continuous, then $f$ is weakly semi generalised star $b$ - continuous function.

Proof. Let $x \in X$ and $V$ be an open set in $Y$ containing $f(x)$. Then $cl(V)$ is regular closed in $Y$ containing $f(x)$. Since $f$ is an almost contra semi generalised star $b$ - continuous function by Theorem 3 (2), $f^{-1}(cl(V))$ is semi generalised star $b$ - open set in $X$ containing $x$. Set $U = f^{-1}(cl(V))$, then $f(U) \subset f^{-1}(cl(V))) \subset cl(V)$. This shows that $f$ is weakly semi generalised star $b$ - continuous function. □

Definition 2.18. A space $X$ is called locally semi generalised star $b$ - indiscrete if every semi generalised star $b$ - open set is closed in $X$.

Theorem 2.19. If a function $f : X \rightarrow Y$ is almost contra semi generalised star $b$ - continuous and $X$ is locally semi generalised star $b$ - indiscrete space, then $f$ is almost continuous.

Proof. Let $U$ be a regular open set in $Y$. Since $f$ is almost contra semi generalised star $b$ - continuous $f^{-1}(U)$ is semi generalised star $b$ - closed set in $X$ and $X$ is locally semi generalised star $b$ - indiscrete space, which implies $f^{-1}(U)$ is an open set in $X$. Therefore $f$ is almost continuous. □

Lemma 2.20. Let $A$ and $X_0$ be subsets of a space $X$. If $A \in sg^b - O(X)$ and $X_0 \in s\alpha$, then $A \cap X_0 \in sg^b - O(X_0)$.  

175
Theorem 2.21. If \( f : X \to Y \) is almost contra semi generalised star \( b \) - continuous and \( X_0 \in \tau^b \) then the restriction \( f|X_0 : X_0 \to Y \) is almost contra semi generalised star \( b \) - continuous. 

Proof. Let \( V \) be any regular open set of \( Y \). By Theorem, we have \( f^{-1}(V) \in sg^b O(X) \) and hence \( (f|X_0)^{-1}(V) = f^{-1}(V) \cap X_0 \in sg^b O(X_0) \). By Lemma 1, it follows that \( f|X_0 \) is almost contra semi generalised star \( b \) - continuous. \( \square \)

Theorem 2.22. If \( f : X \to \prod Y_\lambda \) is almost contra semi generalised star \( b \) - continuous, then \( P_\lambda \circ f : X \to Y_\lambda \) is almost contra semi generalised star \( b \) - continuous for each \( \lambda \in \mathcal{V} \), where \( P_\lambda \) is the projection of \( \prod Y_\lambda \) onto \( Y_\lambda \).

Proof. Let \( Y_\lambda \) be any regular open set of \( Y \). Since \( P_\lambda \) is continuous open, it is an \( R \) - map and hence \( (P_\lambda)^{-1} \in RO(\prod Y_\lambda) \). By theorem, \( f^{-1}(P_\lambda^{-1}(V)) = (P_\lambda \circ f)^{-1} \in sg^b O(X) \). Hence \( P_\lambda \circ f \) is almost contra semi generalised star \( b \) - continuous. \( \square \)

3. Semi generalised star \( b \) - regular graphs and strongly contra semi generalised star \( b \) - closed graphs

Definition 3.1. A graph \( G_f \) of a function \( f : X \to Y \) is said to be semi generalised star \( b \) - regular (strongly contra semi generalised star \( b \) - closed) if for each \( (x, y) \in (X \times Y) \setminus G_f \), there exist a semi generalised star \( b \) - closed set \( U \) in \( X \) containing \( x \) and \( V \in R - O(Y) \) such that \( U \cap G_f = \emptyset \).

Theorem 3.2. If \( f : X \to Y \) is almost contra semi generalised star \( b \) - continuous and \( Y = T_2 \), then \( G_f \) is semi generalised star \( b \) - regular in \( X \times Y \). 

Proof. Let \( (x, y) \in (X \times Y) \setminus G_f \). It is obvious that \( f(x) \neq y \). Since \( Y = T_2 \), there exist \( V, W \in RO(Y) \) such that \( f(x) \in V \), \( y \in W \) and \( V \cap W = \emptyset \). Since \( f \) is almost contra semi generalised star \( b \) - continuous, \( f^{-1}(V) \) is a semi generalised star \( b \) - closed set in \( X \) containing \( x \). If we take \( U = f^{-1}(V) \), we have \( f(U) \subset V \). Hence, \( f(U) \cap W = \emptyset \) and \( G_f \) is semi generalised star \( b \) - regular. \( \square \)

Theorem 3.3. Let \( f : (X, \tau) \to (Y, \sigma) \) be a function and \( g : (X, \tau) \to (X \times Y, \tau \times \sigma) \) the graph function defined by \( g(x) = (x, f(x)) \) to each \( x \in X \). Then \( f \) is almost semi generalised star \( b \) - continuous iff \( g \) is almost semi generalised star \( b \) - continuous.

Proof. Necessary : Let \( x \in X \) and \( V \in sg^b O(Y) \) containing \( f(x) \). Then, we have \( g(x) = (x, f(x)) \in R \setminus O(X \times Y) \). Since \( f \) is almost semi generalised star \( b \) - continuous, there exists a semi generalised star \( b \) - open set \( U \) of \( X \) containing \( x \) such that \( g(U) \subset X \times Y \). Therefore, we obtain \( f(U) \subset V \). Hence \( f \) is almost semi generalised star \( b \) - continuous.

Sufficiency : Let \( x \in X \) and \( w \) be a regular open set of \( X \times Y \) containing \( g(x) \). There exists \( U_1 \in RO(X, \tau) \) and \( V \in RO(Y, \sigma) \) such that \( (x, f(x)) \in (U_1 \times V) \subset W \). Since \( f \) is almost semi generalised star \( b \) - continuous, there exists \( U_2 \in sg^b O(X, \tau) \) such that \( x \in U_2 \) and \( f(U_2) \subset V \). Set \( U = U_1 \cap U_2 \). We have \( x \in U_2 \in sg^b O(X, \tau) \) and \( g(U) \subset (U_1 \times V) \subset W \). This shows that \( g \) is almost semi generalised star \( b \) - continuous. \( \square \)

4. Connectedness

Definition 4.1. A space \( X \) is called semi generalised star \( b \) - connected if \( X \) cannot be written as a disjoint union of two non - empty semi generalised star \( b \) - open sets.

Theorem 4.2. If \( f : X \to Y \) is an almost contra semi generalised star \( b \) - continuous onto and \( X \) is semi generalised star \( b \) - connected, then \( Y \) is connected.

Proof. Suppose that \( Y \) is not a connected space. Then \( Y \) can be written as \( Y = U_0 \cup U_1 \) such that \( U_0 \) and \( U_1 \) are disjoint non - empty open sets. Let \( U = int(cl(U_0)) \) and \( V = int(cl(U_1)) \). Then \( U \) and \( V \) are disjoint nonempty regular open sets such that \( Y = U \cup V \). Since \( f \) is almost contra semi generalised star \( b \) - continuous, \( f^{-1}(U) \) and \( f^{-1}(V) \) are semi generalised star \( b \) - open sets of \( X \). We have \( X = f^{-1}(U) \cup f^{-1}(V) \) such that \( f^{-1}(U) \) and \( f^{-1}(V) \) are disjoint. Since \( f \) is onto, this shows that \( X \) is not semi generalised star \( b \) - connected. Hence \( Y \) is connected. \( \square \)

Theorem 4.3. The almost contra semi generalised star \( b \) - continuous image of semi generalised star \( b \) - connected space is connected.

Proof. Let \( f : X \to Y \) be an almost contra semi generalised star \( b \) - continuous function of a semi generalised star \( b \) - connected space \( X \) onto a topological space \( Y \). Suppose that \( Y \) is not a connected space. There exist non - empty disjoint open sets \( V_1 \) and \( V_2 \) such that \( Y = V_1 \cup V_2 \). Therefore, \( V_1 \) and \( V_2 \) are clopen in \( Y \). Since \( f \) is almost contra semi generalised star \( b \) - continuous, \( f^{-1}(V_1) \) and \( f^{-1}(V_2) \) are semi generalised star \( b \) - open in \( X \). Moreover, \( f^{-1}(V_1) \) and \( f^{-1}(V_2) \) are non - empty disjoint and \( X = f^{-1}(V_1) \cup f^{-1}(V_2) \). This shows that \( X \) is not semi generalised star \( b \) - connected. This is a contradiction and hence \( Y \) is connected. \( \square \)

Definition 4.4. A topological space \( X \) is said to be semi generalised star \( b \) - ultra connected if every two non - empty semi generalised star \( b \) - closed subsets of \( X \) intersect.

A topological space \( X \) is said to be hyper connected if every open set is dense.
Theorem 4.5. If \( X \) is semi generalised star \( b \)-ultra connected and \( f : X \rightarrow Y \) is an almost contra semi generalised star \( b \)-continuous onto, then \( Y \) is hyper connected.

Proof. Suppose that \( Y \) is not hyper connected. Then, there exists an open set \( V \) such that \( V \) is not dense in \( Y \). So, there exist non-empty regular open subsets \( B_1 = \text{int}(\text{cl}(V)) \) and \( B_2 = Y - \text{cl}(V) \) in \( Y \). Since \( f \) is almost contra semi generalised star \( b \)-continuous, \( f^{-1}(B_1) \) and \( f^{-1}(B_2) \) are disjoint semi generalised star \( b \)-closed. This is contrary to the semi generalised star \( b \)-ultra-connectedness of \( X \). Therefore, \( Y \) is hyper connected. \( \square \)

5. Separation axioms

Definition 5.1. A topological space \( X \) is said to be \( sg^* b - T_1 \) space if for any pair of distinct points \( x \) and \( y \), there exist a semi generalised star \( b \)-open sets \( G \) and \( H \) such that \( x \in G \) and \( y \in H \).

Theorem 5.2. If \( f : X \rightarrow Y \) is an almost contra semi generalised star \( b \)-continuous one to one and \( Y \) is weakly Hausdorff, then \( X \) is \( sg^* b - T_1 \).

Proof. Suppose \( Y \) is weakly Hausdorff. For any distinct points \( x \) and \( y \) in \( X \), there exist \( V \) and \( W \) regular closed sets in \( Y \) such that \( f(x) \in V \), \( f(y) \notin V \), \( f(y) \in W \) and \( f(x) \notin W \). Since \( f \) is almost contra semi generalised star \( b \)-continuous, \( f^{-1}(V) \) and \( f^{-1}(W) \) are semi generalised star \( b \)-open subsets of \( X \) such that \( x \notin f^{-1}(V) \), \( y \notin f^{-1}(V) \), \( y \in f^{-1}(W) \) and \( x \notin f^{-1}(W) \). This shows that \( X \) is \( sg^* b - T_1 \).

Corollary 5.3. If \( f : X \rightarrow Y \) is a contra semi generalised star \( b \)-continuous one to one and \( Y \) is weakly Hausdorff, then \( X \) is \( sg^* b - T_1 \).

Theorem 5.4. If \( f : X \rightarrow Y \) is an almost contra semi generalised star \( b \)-continuous one to one function from space \( X \) into a Ultra Hausdorff space \( Y \), then \( X \) is \( sg^* b - T_2 \).

Proof. Let \( x \) and \( y \) be any two distinct points in \( X \). Since \( f \) is an one to one \( f(x) \neq f(y) \) and \( Y \) is Ultra Hausdorff space, there exist disjoint clopen sets \( U \) and \( V \) of \( Y \) containing \( f(x) \) and \( f(y) \) respectively. Then \( x \in f^{-1}(U) \) and \( y \notin f^{-1}(V) \), where \( f^{-1}(U) \) and \( f^{-1}(V) \) are disjoint semi generalised star \( b \)-open sets in \( X \). Therefore \( X \) is \( sg^* b - T_2 \).

Theorem 5.5. If \( f : X \rightarrow Y \) is an almost contra semi generalised star \( b \)-continuous closed injection and \( Y \) is ultra normal, then \( X \) is semi generalised star \( b \)-normal.

Proof. Let \( E \) and \( F \) be disjoint closed subsets of \( X \). Since \( f \) is closed and one to one \( f(E) \) and \( f(F) \) are disjoint closed sets in \( Y \). Since \( Y \) is ultra normal there exist disjoint clopen sets \( U \) and \( V \) in \( Y \) such that \( f(E) \subset U \) and \( f(F) \subset V \). This implies \( E \subset f^{-1}(U) \) and \( F \subset f^{-1}(V) \). Since \( f \) is an almost contra semi generalised star \( b \)-continuous injection, \( f^{-1}(U) \) and \( f^{-1}(V) \) are disjoint semi generalised star \( b \)-open sets in \( X \). This shows \( X \) is semi generalised star \( b \)-normal. \( \square \)

Theorem 5.6. If \( f : X \rightarrow Y \) is an almost contra semi generalised star \( b \)-continuous and \( Y \) is semi-regular, then \( f \) is semi generalised star \( b \)-continuous.

Proof. Let \( x \in X \) and \( V \) be an open set of \( Y \) containing \( f(x) \). By definition of semi-regularity of \( Y \), there exists a regular open set \( G \) of \( Y \) such that \( f(x) \in G \subset V \). Since \( f \) is almost contra semi generalised star \( b \)-continuous, there exists \( U \in sg^* b - O(X, x) \) such that \( f(U) \subset G \). Hence we have \( f(U) \subset G \subset V \). This shows that \( f \) is semi generalised star \( b \)-continuous function. \( \square \)

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