Parameter estimation for dual-rate sampled Hammerstein systems with dead-zone nonlinearity

WANG Hongwei\textsuperscript{1,2,*} and CHEN Yuxiao\textsuperscript{1}

1. School of Electrical Engineering, Xinjiang University, Urumqi 830047, China;
2. School of Control Science and Engineering, Dalian University of Technology, Dalian 110024, China

Abstract: The identification of nonlinear systems with multiple sampled rates is a difficult task. The motivation of our paper is to study the parameter estimation problem of Hammerstein systems with dead-zone characteristics by using the dual-rate sampled data. Firstly, the auxiliary model identification principle is used to estimate the unmeasurable variables, and the recursive estimation algorithm is proposed to identify the parameters of the static nonlinear model with the dead-zone function and the parameters of the dynamic linear system model. Then, the convergence of the proposed identification algorithm is analyzed by using the martingale convergence theorem. It is proved theoretically that the estimated parameters can converge to the real values under the condition of continuous excitation. Finally, the validity of the proposed algorithm is proved by the identification of the dual-rate sampled nonlinear systems.

Keywords: dual-rate sampled data, dead-zone nonlinearity, Hammerstein model, system identification, convergence analysis.

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1. Introduction

Due to the hardware condition, economic costing or environment influence, the updating period of the input data and the sampled period of the output data are not always equal in the industrial production process [1–3]. If one system refreshes the input data at a fast rate and samples the output data at a slow rate, then that can be called the dual-rate sampled system [4–7]. Dual-rate sampled systems broadly exist in the chemical process, industrial process and biopharmaceutical process. To identify dual-rate sampled systems, there have been some methods to deal with the systems such as the polynomial transformation technique, lifting technique, auxiliary model principle, and so on. Based on lifting technique, Ding et al. presented two algorithms: the recursive least squared algorithm when the system states are measurable, and the hierarchical identification algorithm when the system states are unmeasurable [8]. Ding studied the extended recursive least squared algorithm with estimated residual for the dual-rate controlled auto regressive (CAR) system by using the polynomial transformation technique [9]. Aiming at the problem from the existence of unmeasurable variables in the identification process, based on the auxiliary model principle, the output value of the auxiliary model is used to replace the unmeasurable variable value in [10]. Further, Chen proposed the identification algorithm based on Kalman filtering to estimate the unmeasurable variables [11].

In actual industrial production, most systems are nonlinear in industrial processes. Thus, in recent years, more and more researchers have begun to study nonlinear systems. Nonlinear systems have none universal model to depict the dynamical process. In recent years, researchers have studied various nonlinear models, and Hammerstein is one of these models. For single-rate Hammerstein systems, many algorithms have been presented to identify methods such as the over-parameterization method [12], the hierarchical identification algorithm [13,14], the iterative method [15,16], the blind identification method [17,18] and so on. For single-rate Hammerstein systems with polynomial nonlinearity, Ding proposed the hierarchical multi-innovation stochastic gradient algorithm [19]; considering particle variation in particle swarms, Xu studied improved particle swarm optimization [20]; Ding et al. presented an auxiliary model based least squared algorithm [21]. For Hammerstein systems with continuous nonlinearity, Chen et al. simplified the nonlinearity into polynomial and presented two algorithms to identify the parameters: the stochastic gradient algorithm and the particle swarm optimization algorithm [22]. Ding et al. proposed the hierarchical least squared identification algorithm for Hammerstein CAR systems with the key term separation technique [23]. For extended Hammerstein syst-
tems, Fang et al. used the Preisach model to describe hysteresis nonlinearity and then the estimated parameters with the over-parameterized method [24]. However, these algorithms are the identification methods based on single-rate sampled data. Therefore, it is hard to apply these methods to dual-rate sampled Hammerstein systems directly. To identify dual-rate sampled Hammerstein systems, some researchers investigated the auxiliary model based recursive least squared algorithm [25,26]. Considering the Hammerstein CAR moving average (CARMA) systems, Wang et al. studied two algorithms: the hierarchical least squared algorithm and the key term separation based least squared algorithm [27]; Wang et al. studied the maximum likelihood estimation method [28]. For dual-rate sampled Hammerstein systems with polynomial nonlinearity, Li presented three different methods, including the recursive extended least squared algorithm for the dual-rate sampled nonlinear systems. Section 4 presents convergence analysis of the proposed algorithm. Section 5 provides the results of simulating one actual Hammerstein system to demonstrate the effectiveness of the proposed method. Section 6 draws some conclusions and remarks.

2. Problem formulation

Consider dual-rate sampled Hammerstein systems with dead-zone nonlinearities as shown in Fig. 1. It consists of the dynamic linear part and the static nonlinear part. In Fig. 1, $H_k$ means the zero-order holder whose holding period is $h$. $S_{qh}$ is the sampler with sampling period as $q h$. $u(k)$ and $y(kq)$ are the discrete input data and the output data of systems, respectively. $u(t)$ and $y(t)$ are the corresponding continuous input data and output data, respectively. $v(t)$ is uncorrelated white noise. $\pi(t)$ and $x(t)$ are the intermediate variables.

![Fig. 1 Dual-rate sampled Hammerstein system with dead-zone nonlinearities](image)

According to Fig. 1, the Hammerstein system consists of a static nonlinear part and the dynamic linear part. The dynamic linear model of the controlled object can be modeled as

$$
y(t) = x(t) + v(t) \tag{1}
$$

$$
x(t) = B(z) \pi(t). \tag{2}
$$

where $A(z)$ and $B(z)$ are polynomials of the unit delay operator $z^{-1}(z^{-1}x(t) = x(t-1)), A(z) = a_0 + a_1 z^{-1} + \cdots + a_n z^{-n}$. $B(z) = b_0 + b_1 z^{-1} + \cdots + b_m z^{-m}$.

The static nonlinear part $f(\cdot)$ is the function of the dead-zone characteristic. The input/output relationship can be shown as Fig. 2.

The function $f(\cdot)$ can be described by

$$
\pi(t) = \begin{cases}
    m_1(u(t) - d_1), & u(t) > d_1 \\
    0, & d_2 \leq u(t) \leq d_1 \\
    m_2(u(t) - d_2), & u(t) < d_2
\end{cases} \tag{3}
$$

![Fig. 2 Dead-zone function of the static nonlinear part](image)

where $m_1$ and $m_2$ are the segment slopes of the nonlinear function.
part; $d_1$ and $d_2$ are the dead-zone break-points, $d_1 \geq 0$, $d_2 \leq 0$. Equation (3) can also be written as

\[
\overline{\mu}(t) = m_1 u(t) h(u(t)) - m_1 d_1 h(u(t)) + m_2 u(t) h(-u(t)) - m_2 d_2 h(-u(t)) - m_1 (u(t) - d_1) h(u(t)) h(d_1 - u(t)) - m_2 (u(t) - d_2) h(-u(t)) h(u(t) - d_2) = \varphi_f^T(t) \theta_f
\]

where

\[
\varphi_f(t) = [u(t) h(u(t)), -h(u(t)), u(t) h(-u(t)), -h(-u(t)), (u(t) - d_1) h(u(t)) h(d_1 - u(t)), (u(t) - d_2) h(-u(t)) h(u(t) - d_2)]^T
\]

\[
\theta_f = [m_1, m_1 d_1, m_2, m_2 d_2, m_1, m_2]^T, \quad \text{and } h \text{ is the switching function, i.e.,}
\]

\[
h(u(t)) = \begin{cases} 
1, & u(t) > 0 \\
0, & u(t) \leq 0 
\end{cases}
\]

In this paper, the following transfer function is used to describe the dynamic linear part in Fig. 1.

\[
\frac{B(z)}{A(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_n z^{-n}}{a_0 + a_1 z^{-1} + \cdots + a_n z^{-n}}
\]

In order to have correct and sole parameters, we assume $b_0 = 1$. After that, substitute (5) into (2), and we have

\[
x(t) = -a_1 x(t-1) - \cdots - a_n x(t-n) + \overline{\mu}(t) + b_1 \overline{\mu}(t-1) + \cdots + b_n \overline{\mu}(t-n) + v(t) = [a_1 \ a_2 \ \cdots \ a_n \ b_1 \ b_2 \ \cdots \ b_n] \begin{bmatrix} -x(t-1) \\
-x(t-2) \\
\vdots \\
-x(t-n) \\
\overline{\mu}(t-1) \\
\overline{\mu}(t-2) \\
\vdots \\
\overline{\mu}(t-n) \end{bmatrix} + \overline{\mu}(t) + v(t).
\]

Substituting (4) and (6) into (1), (1) can be rewritten as

\[
y(t) = \varphi(t)^T \theta + v(t)
\]
The outputs of auxiliary models are described as
\[
\hat{\varphi}(kq - q + i) = \hat{\varphi}_T^T(kq - q + i)\hat{\theta}_f(kq - q),
\]
\[i = 0, 1, \ldots, q - 1\]
\[
\hat{x}(kq - q + i) = \hat{\varphi}_T^T(kq - q + i)\hat{\theta}(kq - q),
\]
\[i = 0, 1, \ldots, q - 1.\]

Thus, the recursive least squared estimation algorithm can be given as
\[
\hat{\theta}(kq) = \hat{\theta}(kq - q) + P(kq)\hat{\varphi}(kq).
\]

According to the estimation values of \(\hat{\theta}(kq)\), we can get
\[
\hat{m}_1(kq) = \frac{\hat{\theta}_{n_a+n_b+1}(kq) + \hat{\theta}_{n_a+n_b+5}(kq)}{2}
\]
\[
\hat{m}_2(kq) = \frac{\hat{\theta}_{n_a+n_b+3}(kq) + \hat{\theta}_{n_a+n_b+6}(kq)}{2}
\]
\[
\hat{d}_1(kq) = \frac{\hat{\theta}_{n_a+n_b+2}(kq)}{m_1}
\]
\[
\hat{d}_2(kq) = \frac{\hat{\theta}_{n_a+n_b+4}(kq)}{m_2}.
\]

4. Convergence analysis

To understand convergence analysis of the proposed algorithm, some mathematical notations are given. \(||X||\) is the norm of matrix \(X\), it is defined as
\[
||X|| = \text{tr}[XX^T] = \text{tr}[X^TX].
\]

\(I_n\) means the identity matrix of the order \(n\) by \(n\); \(\xi_{\max}[X]\) and \(\xi_{\min}[X]\) are maximum and minimum singular values of the matrix \(X\), respectively. \(|X| = \text{det}[X]\) is the determinant of one matrix. \(f(k) = o(g(k))\) means that \(g(k) > 0 \text{ and } \lim_{k \to \infty} \frac{f(k)}{g(k)} = 0\); \(f(k) = O(g(k))\) means that \(g(k) > 0 \text{ and there exists constants } \varepsilon_1 \text{ and } k_0 \text{ to satisfy } |f(k)| \leq \varepsilon_1|g(k)|, k \geq k_0, \text{ where } \varepsilon_1 > 0.\)

Let \(\tilde{x}(kq - q + i) = -\hat{x}(kq - q + i), \ldots, -\hat{x}(kq - n_a)\).

Suppose \(\{v(kq), F_{kq}\}\) is the martingale-difference sequence in the probability space \((\Omega, F, P)\), where \(\{F_{kq}\}\) is the \(\sigma\) algebra sequence generated from all measurable data till time \(kq\). The noise sequence \(\{v(kq)\}\) satisfies the following conditions:

\[E[v(kq)F_{kq-\tau}q] = 0, \text{ a.s.} \quad (A1)\]

\[E[||v(kq)||F_{kq-\tau}q] = a_0^2 \leq \sigma^2, \text{ a.s.} \quad (A2)\]

After assuming statistical conditions in (21) and (22) of the noise signal \(\{v(kq)\}\), the following lemma can be given.

Lemma 1 For the proposed algorithm in (10)–(20), the following inequalities hold:

\[\left(\sum_{i=1}^{\infty} \hat{\varphi}_T^T(iq)P(iq)\hat{\varphi}(iq) \leq \ln |P^{-1}(kq)| + n_0 \ln p_0 \right. \quad (i)\]

\[\left. \sum_{k=1}^{\infty} \hat{\varphi}_T^T(kq)P(kq)\hat{\varphi}(kq) \leq \frac{c}{\ln |P^{-1}(kq)|}\right. \quad (ii)\]

\[\text{a.s., or any } c > 1.\]
Proof According to the definition of $P(kq)$ in (12), we can have

$$P^{-1}(kq - g) = P^{-1}(kq) - \hat{\varphi}(kq)\hat{\varphi}^T(kq) = P^{-1}(kq)[I_{n_0} - P(kq)\hat{\varphi}(kq)\hat{\varphi}^T(kq)].$$

(23)

Take the determinant on both sides of the above equation, and use the determinant relationship $|I_n + DE| = |I_n + ED|$, $D \in \mathbb{R}^{n \times n}$, $E \in \mathbb{R}^{n \times m}$, then

$$|P^{-1}(kq - g)| = |P^{-1}(kq)||I_{n_0} - P(kq)\hat{\varphi}(kq)\hat{\varphi}^T(kq)| = |P^{-1}(kq)||1 - \hat{\varphi}^T(kq)P(kq)\hat{\varphi}(kq)|.

Therefore,

$$\hat{\varphi}^T(kq)P(kq)\hat{\varphi}(kq) = \frac{|P^{-1}(kq)| - |P^{-1}(kq - g)|}{|P^{-1}(kq)|}. (24)$$

(i) Replacing $k$ with $i$ in the above equation and summing from $i = 1$ to $i = k$ on both sides of the above equation, we can get

$$\sum_{i=1}^{k} \hat{\varphi}^T(iq)P(iq)\hat{\varphi}(iq) = \sum_{i=1}^{k} \frac{|P^{-1}(iq)| - |P^{-1}(iq - g)|}{|P^{-1}(iq)|} = \sum_{i=1}^{k} \int_{|P^{-1}(iq - g)|}^{|P^{-1}(iq)|} \frac{dx}{|P^{-1}(iq)|} \leq \ln |P^{-1}(kq)| - \ln |P^{-1}(0)| = \ln |P^{-1}(kq)| + n_0 \ln n_0.

(ii) Divide both sides of (24) by $[\ln |P^{-1}(kq))]|^c$, and sum from $k=1$ to $k = \infty$ on both sides of the above equation. Thus,

$$\sum_{k=1}^{\infty} \hat{\varphi}^T(kq)P(kq)\hat{\varphi}(kq) = \sum_{k=1}^{\infty} \frac{|P^{-1}(kq)| - |P^{-1}(kq - g)|}{|P^{-1}(kq)|\ln |P^{-1}(kq)|}|^c \leq \int_{|P^{-1}(0)|}^{|P^{-1}(\infty)|} \frac{dx}{x\ln x}|^c = \frac{1}{c - 1} \left[ \frac{1}{\ln |P^{-1}(0)|}|^c - \frac{1}{\ln |P^{-1}(\infty)|}|^c \right] < \infty, \text{a.s.}\]
\[ \gamma_T(kq)P(kq)\gamma(kq)[v^2(kq) + \eta^2(kq) + 2\eta(kq)v(kq)]. \]  
(28)

Besides, the following inequality holds:

\[
1 - \gamma_T(kq)P(kq)\gamma(kq) = [1 + \gamma_T(kq)P(kq - q)\gamma(kq)]^{-1} \geq 0.
\]

Using the above inequality, (28) can be written as

\[
V(kq) \leq \gamma_T(kq - q)P^{-1}(kq - q)\gamma(kq - q) + 2[\eta(kq) + v(kq) + \gamma_T(kq)P(kq)\gamma(kq)].
\]

\[
[v^2(kq) + \eta^2(kq) + 2\eta(kq)v(kq)]. \]  
(29)

In addition, \( \gamma_T(kq - q)P^{-1}(kq - q)\gamma(kq - q), \gamma_T(kq)P(kq)\gamma(kq), \gamma(kq) \) and \( \eta(kq) \) and \( v(kq) \) are uncorrelated. All of them are measurable in \( F_{kq-q} \). Supposing \( \eta^2(t) \leq \varepsilon \leq \infty, \) taking the condition expectation on both sides of (29) and using the conditions of (A1) and (A2), we have

\[
E[V(kq)] = E[V(kq - q)] + 0 + E[\gamma_T(kq)P(kq)\gamma(kq)[v^2(kq) + \eta^2(kq)]] \leq E[V(kq - q)] + E[\gamma_T(kq)P(kq)\gamma(kq)[\sigma^2 + \varepsilon]] \leq E[V(0)] + E[\sum_{i=1}^{k} \gamma_T(iq)P(iq)\gamma(iq)[\sigma^2 + \varepsilon]].
\]

Using (23), we can get

\[
P^{-1}(kq) = P^{-1}(kq - q) + \gamma(kq)\gamma_T(kq)
\]

\[
P(0) = p_0I.
\]

Once persistent excitation condition (A3) holds, condition (A4) also holds, and \( c_1 \leq c_3, c_2 \geq c_4. \) Using condition (A3), the above equation is calculated iteratively and is written as

\[
P^{-1}(kq) = \sum_{i=1}^{k} \gamma_T(iq)\gamma_T(q) + P^{-1}(0) \leq c_2kI_{n_0} + I_{n_0}/p_0
\]

\[
P^{-1}(kq) \geq c_1kI_{n_0} + I_{n_0}/p_0.
\]

Thus,

\[
(c_1k + 1/p_0)^{n_0} \leq |P^{-1}(kq)| \leq (c_2k + 1/p_0)^{n_0}.
\]

Using the conclusion (1) in Lemma 1, we can get

\[
\sum_{i=1}^{k} \gamma_T(iq)P(iq)\gamma(iq) \leq 
\]

\[
\ln |P^{-1}(kq)| + n_0 \ln p_0 \leq n_0 \ln (c_2k + 1/p_0) + n_0 \ln p_0.
\]

Taking the condition expectation on both sides of (27), the following inequality holds:

\[
E\{V(kq)\} = E[\gamma_T(kq)P^{-1}(kq)\gamma(kq)] \geq (c_1k + 1/p_0)E\|\gamma(kq)\|^2].
\]

Using

\[
E\{V(0)\} = E[\gamma_T(0)P^{-1}(0)\gamma(0)] = n_0/p_0^3,
\]

we can get

\[
(c_1k + 1/p_0)E\|\gamma(kq)\|^2 \leq E\{V(kq)\} \leq E\{V(0)\} + E\sum_{i=1}^{k} \gamma_T(iq)P^{-1}(iq)\gamma(iq)\|\gamma^2 + \varepsilon\| \leq n_0/p_0^3 + n_0 \ln (c_2k + 1/p_0) + n_0 \ln p_0 \|\gamma^2 + \varepsilon\|
\]

Taking the extremum of the above inequality, we get

\[
\lim_{k \to \infty} E\|\gamma(kq)\|^2 \leq \lim_{k \to \infty} \left[ \frac{n_0/p_0^3}{(c_1k + 1/p_0)} + \frac{n_0 \ln (c_2k + 1/p_0) + n_0 \ln p_0 \|\gamma^2 + \varepsilon\|}{(c_1k + 1/p_0)} \right] = 0.
\]

5. Simulation

In this paper, we use an actual Hammerstein system with dead-zone nonlinearity as an example simulation. The function of dead-zone nonlinearity is described as follows:

\[
\pi(k) = \begin{cases} 
0.5(u(k) - 0.4), & u(k) > 0.4 \\
0, & -0.3 \leq u(k) \leq 0.4 \\
0.7(u(k) + 0.3), & u(k) < -0.3
\end{cases}
\]

The linear part model is shown as

\[
x(k) = G(z)\pi(k) = \frac{B(z)}{A(z)}\pi(k)
\]

\[
y(k) = x(k) + v(k)
\]

where

\[
A(z) := 1 + 0.4z^{-1} + 0.3z^{-2}
\]

\[
B(z) := 0.5z^{-1} + 0.6z^{-2}.
\]

\( v(k) \) is white noise with zero mean and different variances, \( \sigma^2 = 0.1^2, \sigma^2 = 0.15^2, \sigma^2 = 0.25^2. \)

For the model of the simulation example, the real parameter vector is shown as follows:

\[
\theta = [\alpha_1, \alpha_2, \beta_1, \beta_2, m_1, m_3, d_1,]
\]
In the process of the simulation, the input signal $u(k)$ is a stochastic signal sequence which is independent, uncorrelated, evenly distributed with zero mean and unit variance. The noise-to-signal ratio values of the system are $\delta_{ns} = 13.135 \%$, $\delta_{ns} = 32.403 \%$, $\delta_{ns} = 54.768 \%$, respectively. The parameter estimation error is $\delta := \| \hat{\theta}(t) - \theta \| / \| \theta \|$. We use the proposed algorithm to identify simulated example.

The estimation results are shown in Tables 1 – 3, and the changing diagram of relative parameter estimation errors is shown as Fig. 3. Observing these tables and the figure, it can be obviously found that the estimation results are close to the true values. At the same data length, it is obviously shown by the three sets of identification results corresponding to the three systems noise variances that with the increasing noise variance the parameter estimation error becomes bigger. Furthermore, observing the identification results from each set respectively, it can be found that the parameter estimation errors $\delta$ decrease when increasing the data length $t$.

![Fig. 3 Parameter estimation errors diagram with different variances](image_url)

### Table 1

| $t$    | $\alpha_1$ | $\alpha_2$ | $\beta_1$ | $\beta_2$ | $m_1$ | $m_1d_1$ | $m_2$ | $m_2d_2$ | $m_1$ | $m_2$ | $\delta(\%)$ |
|--------|-------------|-------------|-----------|-----------|-------|-----------|-------|-----------|-------|-------|---------------|
| 100    | 0.3981      | 0.3047      | 0.5239    | 0.6187    | 0.4435 | 0.1549    | 0.7291 | −0.2401   | 0.4741 | 0.9225 | 15.4799        |
| 1000   | 0.3994      | 0.3021      | 0.4984    | 0.6014    | 0.4995 | 0.2067    | 0.7014 | −0.2210   | 0.5190 | 0.8022 | 6.7293         |
| 2000   | 0.3992      | 0.3022      | 0.5016    | 0.6033    | 0.5012 | 0.2022    | 0.7083 | −0.2213   | 0.5093 | 0.7743 | 4.9024         |
| 3000   | 0.3989      | 0.3028      | 0.5038    | 0.6075    | 0.5007 | 0.2014    | 0.7080 | −0.2205   | 0.5041 | 0.7650 | 4.3070         |
| 4000   | 0.3987      | 0.3031      | 0.5039    | 0.6059    | 0.5024 | 0.2022    | 0.7039 | −0.2174   | 0.5017 | 0.7492 | 3.2527         |
| 5000   | 0.3988      | 0.3030      | 0.5008    | 0.6047    | 0.5010 | 0.2008    | 0.7043 | −0.2160   | 0.5020 | 0.7274 | 1.8675         |
| True value | 0.4000 | 0.3000 | 0.5000 | 0.6000 | 0.5000 | 0.2000 | 0.7000 | −0.2100 | 0.5000 | 0.7000 | 0.0000         |

### Table 2

| $t$    | $\alpha_1$ | $\alpha_2$ | $\beta_1$ | $\beta_2$ | $m_1$ | $m_1d_1$ | $m_2$ | $m_2d_2$ | $m_1$ | $m_2$ | $\delta(\%)$ |
|--------|-------------|-------------|-----------|-----------|-------|-----------|-------|-----------|-------|-------|---------------|
| 100    | 0.3656      | 0.3314      | 0.4670    | 0.6368    | 0.4361 | 0.1276    | 0.6959 | −0.2065   | 0.3030 | 0.6122 | 15.7959        |
| 1000   | 0.4612      | 0.2986      | 0.5489    | 0.6008    | 0.4899 | 0.1919    | 0.7121 | −0.2209   | 0.3950 | 0.7395 | 8.8882         |
| 2000   | 0.4454      | 0.2986      | 0.5334    | 0.6025    | 0.5098 | 0.2194    | 0.7114 | −0.2161   | 0.5280 | 0.6702 | 4.7606         |
| 3000   | 0.4425      | 0.3029      | 0.5225    | 0.6098    | 0.5010 | 0.2056    | 0.7092 | −0.2115   | 0.5051 | 0.6426 | 4.9161         |
| 4000   | 0.4269      | 0.2970      | 0.5144    | 0.6005    | 0.5010 | 0.2035    | 0.7008 | −0.2053   | 0.5059 | 0.6550 | 3.5410         |
| 5000   | 0.4257      | 0.2929      | 0.5158    | 0.5984    | 0.5041 | 0.2070    | 0.7056 | −0.2118   | 0.5247 | 0.7025 | 2.6330         |
| True value | 0.4000 | 0.3000 | 0.5000 | 0.6000 | 0.5000 | 0.2000 | 0.7000 | −0.2100 | 0.5000 | 0.7000 | 0.0000         |
6. Conclusions

Nonlinear systems have no certain, common models to depict until now. Nonlinearities do not just include dead-zone characteristic. Their uncertainty and complexity increase the difficulty of research. In our paper, the auxiliary model idea and the recursive least squared algorithm are combined to deal with the difficulty. For algorithm convergence, the martingale convergence theorem is employed to prove convergence performance of the proposed method. It can only apply to dual-rate sampled Hammerstein system identification with the dead-zone characteristic. Therefore, it is worth for further research about the identification issues of the dual-rate sampled nonlinear system with different nonlinearities.

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Biographies

WANG Hongwei was born in 1969. He is a Ph.D. and a professor. He is sent to help Xinjiang University by Dalian University of Technology. His research interests are switched system identification, nonlinear system identification and fuzzy modeling.
E-mail: 1195201627@qq.com

CHEN Yuxiao was born in 1995. She is now pursuing her M.S. degree at School of Electrical Engineering, Xinjiang University. Her research interests are nonlinear system modeling and system identification.
E-mail: cyxling@126.com