Neutrino Mass Hierarchy and $\nu - \bar{\nu}$ Oscillations from Baryogenesis

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Abstract

It has been recently proposed that the matter-antimatter asymmetry of the universe may have its origin in “post-sphaleron baryogenesis” (PSB). It is a TeV scale mechanism that is testable at the LHC and other low energy experiments. In this paper we present a theory of PSB within a quark-lepton unified scheme based on the gauge group $SU(2)_L \times SU(2)_R \times SU(4)_c$ that allows a direct connection between the baryon asymmetry and neutrino mass matrix. The flavor changing neutral current constraints on the model allow successful baryogenesis only for an inverted mass hierarchy for neutrinos, which can be tested in the proposed long base line neutrino experiments. The model also predicts observable neutron–antineutron oscillation accessible to the next generation of experiments as well as TeV scale colored scalars within reach of LHC.
1 Introduction

Since Sakharov first suggested the three conditions that would have to be satisfied by a microphysical theory to generate matter–antimatter asymmetry of the universe [1], many beyond the standard model scenarios have been constructed for this purpose. The very earliest ones that used proton decay in grand unified theories for this purpose run into difficulty on several counts: first is that successful inflation scenarios generally have reheating temperatures which are below the generic baryogenesis temperatures, especially in the context of supersymmetry, so that any GUT generated baryon number is erased by inflation; secondly, if baryogenesis is caused by $B - L$ conserving interactions as in $SU(5)$ models, they will be destroyed by electroweak sphalerons that are in equilibrium down to about 100 GeV.

In the mid 80’s, a new mechanism was suggested that uses baryogenesis via leptogenesis [2]. This mechanism is very attractive since it arises within the framework of the seesaw mechanism [3] that explains small neutrino masses. Here the initial lepton asymmetry is created far below the GUT scale and is then converted by the electroweak sphalerons [4] to a baryon asymmetry. This mechanism depends crucially on the properties of the electroweak sphaleron [4] which serves as the source of $B$ violation. While this is one of the most widely discussed schemes in literature today [5], it may also have problems since adequate leptogenesis in these models implies a lower bound on the leptogenesis scale [6] which is above the allowed reheating scale in supersymmetric models [7]. It is also not so easy to test by low energy experiments.

It is therefore important to explore alternative mechanisms that can explain the matter–antimatter asymmetry from particle decays around 100 GeV temperatures which do not conflict with the above bounds on reheating temperatures and at the same time yield testable consequences at LHC and other low energy experiments. Such a mechanism was proposed in
two recent papers [8, 9], where it was shown that with the use of higher dimensional baryon
violating operators, baryogenesis can occur after the electro-weak sphalerons have gone out
of thermal equilibrium. This mechanism was called Post-Sphaleron Baryogenesis (PSB).

In this paper, we propose a theory for this mechanism within a quark-lepton unified gauge
model for neutrino masses based on the gauge group $SU(2)_L \times SU(2)_R \times SU(4)_c$ [10] using
the symmetry breaking discussed in Ref. [11]. We show that in this model, quark-lepton
unification allows us to relate the baryon asymmetry directly to neutrino masses via the
type II seesaw mechanism [12]. The main result of our work is that successful baryogenesis
can occur only for an inverted mass hierarchy for neutrinos with a relatively large $\theta_{13}$. Both of
these predictions will be testable in ongoing neutrino experiments searching for neutrinoless
double beta decay as well as neutrino oscillation experiments.

The salient feature of PSB mechanism is that baryogenesis occurs via the direct decay
of a scalar boson $S_r$ having a weak scale mass and a higher dimensional baryon violating
coupling. $S_r$ is the real part of a baryon number carrying complex scalar $S$, which acquires
a vacuum expectation value (vev). In the context of the $SU(2)_L \times SU(2)_R \times SU(4)_c$ model,
$S_r$ is the real part of a Higgs scalar field belonging to $(1, 3, 10)$ representation of whose vev
breaks the $SU(2)_R \times SU(4)_c$ symmetry down to the $U(1)_Y$ of the standard model. The
decays $S_r \to 6q$ and $S_r \to 6\bar{q}$ provide the source for $B$ asymmetry. When the $S$-field has a
vev, the decay process generates an interaction that causes neutron-antineutron oscillation as
shown in ref. [11]. The parameter domain of our theory where adequate baryogenesis occurs
predicts that neutron-antineutron oscillation should occur at a rate observable in currently
available reactor facilities.
2 Basic ingredients of Post-Sphaleron Baryogenesis and its $SU(2)_L \times SU(2)_R \times SU(4)_c$ embedding

A starting Lagrangian for PSB that gives rise to the higher dimensional $B$-violating decay is given by \[8\]

\[
\mathcal{L}_I = \frac{h_{ij}}{2} \Delta_{de} \Delta_{de}^c d_i^c d_j^c + \frac{l_{ij}}{2} \Delta_{ue} \Delta_{de}^c u_i^c u_j^c + \frac{g_{ij}}{2} \Delta_{pd} \Delta_{de}^c (u_i^c d_j^c + u_j^c d_i^c) + \frac{\lambda_1}{2} S \Delta_{ue} \Delta_{de}^c \Delta_{de}^c + \frac{\lambda_2}{2} S \Delta_{de} \Delta_{de}^2 + h.c. \tag{1}
\]

From the above equation, we see that when the scalar field $S$, which has $B-L = 2$ is given a vev, it leads to cubic scalar field couplings of the type $\Delta_{ue} \Delta_{de}^c \Delta_{de}^c$ and $\Delta_{de} \Delta_{de}^2$ which break baryon number by two units.

We note that not all of the $(\Delta_{ue} \Delta_{de}^c, \Delta_{de} \Delta_{de}^c)$ fields are needed for $B$ violation and $n \leftrightarrow \bar{n}$ oscillation: either $(\Delta_{ue} \Delta_{de}^c, \Delta_{de} \Delta_{de}^c)$ or $(\Delta_{ue} \Delta_{de}^c, \Delta_{de} \Delta_{de}^c)$ pair will do. In fact, consistency with flavor changing neutral current constraints and $n - \pi$ oscillation limits allow for only two of these three scalar states to be light near the TeV scale. The third state (in our case $\Delta_{ue} \Delta_{de}^c$, as we will see below) will have mass of order 100 TeV.

Baryon asymmetry arises from $W$-loop corrections to the $S_r$ decays and are therefore directly linked to CKM mixing [8].

The constraints on the parameter space of the model arise from the fact that the decay of $S_r$ occurs below 100 GeV and above 200 MeV or so – the former to ensure that the sphalerons do not play any role in baryogenesis and the latter so that quarks in the cosmic soup have not combined to form hadrons, which will affect the decay estimates and from the fact that the model must reproduce observed neutrino masses and mixings. If baryon asymmetry is created above the electroweak phase transition temperature, all of the baryon asymmetry will be washed out since there are both $B+L$ violating sphaleron interactions as
well as $W_R$ mediated $\Delta L = 2$ scatterings of right-handed Majorana neutrinos in equilibrium at that temperature.

Before discussing the constraints on the parameters of the model from low energy observations, let us discuss its embedding into the $SU(2)_L \times SU(2)_R \times SU(4)_c$ model \[10\]. The version of the model relevant to our discussion is not the one in the original Pati-Salam paper but rather the one considered in Ref. \[11\]. In this model \[11\], symmetry breaking from $SU(2)_R \times SU(4)_c$ to $U(1)_Y \times SU(3)_c$ is implemented by the Higgs fields belonging to the representation $\Delta_R(1, 3, 10) \oplus \Delta_L(3, 1, 10)$ under the $SU(2)_L \times SU(2)_R \times SU(4)_c$ group. Decomposing this field under the standard model group gives the various other fields in the model:

\[
\Delta_R(1, 3, 10) \equiv \Delta_{u,c}(1, \frac{8}{3}, 6^*) \oplus \Delta_{d,e}(1, \frac{2}{3}, 6^*) \oplus \Delta_{d,e}(1, \frac{4}{3}, 6^*) \oplus \\
\Delta_{\nu,\nu}(1, \frac{4}{3}, 3^*) \oplus \Delta_{\nu,\nu}(1, \frac{2}{3}, 3^*) \oplus \Delta_{\nu,\nu}(1, \frac{4}{3}, 3^*) \oplus \Delta_{\nu,\nu}(1, \frac{8}{3}, 3^*) \oplus \\
\Delta_{\nu,\nu}(1, 0, 1) \oplus \Delta_{\nu,\nu}(1, -2, 1) \oplus \Delta_{\nu,\nu}(1, -4, 1)
\]

The Yukawa Lagrangian of this model is given by

\[
\mathcal{L}_I = f_{ij} \Psi_i^{cT} C^{-1} \tau_2 \tau \tilde{\Delta}_R \Psi_j^{cT} + (R \leftrightarrow L) + H_i^{a} \Phi_i \Psi_j^{c} + \text{h.c.}
\]

where $\Psi = \begin{pmatrix} u_1 & u_2 & u_3 \\ d_1 & d_2 & d_3 \\ \nu \\ e \end{pmatrix}$. The $f$ couplings generate Majorana neutrino masses, while the couplings denoted $H^a$ generate the Dirac masses for fermions. Comparing Eq. (3) with Eq. (1), we see that exactly the interactions present are present in both cases. The $S$ field of Eq. (1) is the $\Delta_{\nu,\nu}$ whose vev breaks the gauge group of our model down to the SM gauge group \[11\]. We assume that the scale of this symmetry breaking is anywhere between $10 – 100$ TeV so that both the right–handed neutrinos as well as the gauge bosons belonging to $SU(4)_c/SU(3)_c$ have masses around these values.

We note that while breaking the gauge symmetry only by the $\Delta(1, 3, 10)$ makes the $W_R$ mass scale, $B – L$ breaking scale $v_{BL}$, and the $SU(4)_c$ breaking scales all equal. It would
also relate the $W_R^\pm$ mass with the $Z'$ mass. Some of the constraints we derive below require that $M_{W_R} \gg v_{BL}$. This can be achieved by including a $(1, 3, 1)$ Higgs field to break the symmetry, which will generate $W_R^\pm$ mass, but not $Z'$ mass and decouple $W_R^\pm$ mass from $v_{BL}$. Our results will be valid in the presence of such $(1, 3, 1)$ Higgs fields, or in their absence. In the latter case, the common scale of $B - L$ symmetry breaking will be required to be $> 100$ TeV or so.

The Higgs fields belonging to the $\Delta$ multiplet will have the following mass pattern: $\Delta_c^d$, $\Delta_{u^c_d^c}$ will have mass near a TeV, whereas $\Delta_{u^c_u^c}$ will have mass near 100 TeV. Such a mass pattern is consistent, since as noted above, we could have the $SU(4)_c/SU(3)_c \times U(1)_{B-L}$ boson and the $W_R$ mass different from the $Z'$ mass and the right–handed neutrino mass scale.

One important point to note is that due to $SU(4)_c$ gauge symmetry, all three couplings in Eq. (1) become equal to each other i.e., $h_{ij} = g_{ij} = l_{ij} = f_{ij}$ of Eq. (3).

In general the neutrino mass in this model is given by a combination of type I and type II seesaw contributions:

$$M_\nu = \gamma \frac{v_{BL}^2}{v_{BL}^2} f - M_{\nu}^{\text{Dirac}}(v_{BL}f)^{-1} \left(M_{\nu}^{\text{Dirac}}\right)^T$$

The coupling matrix $f$ that appears in the neutrino mass formula above is related to the diquark couplings, which lead to FCNC effects, $n - \bar{n}$ oscillations as well as the baryon asymmetry. They are therefore very highly constrained.

In what follows, we assume that $M_{\nu}^{\text{Dirac}} = 0$ or very small by an appropriate choice of the Yukawa couplings of $\Phi_1 \sim (2, 2, 1)$ and $\Phi_{15} \sim (2, 2, 15)$ fields (see Eq. (3)). The details of this are not relevant to the main point of our paper. Since $M_D$ depends on the same parameters as the quark and charged lepton masses, it is useful to point out that setting $M_D = 0$ does not lead to any conflict with realistic fermion mass and mixing patterns. To see this note that the two bi-doublet fields $\Phi_1$ and $\Phi_{15}$ are both complex scalars; therefore
each field will have two independent Yukawa couplings with fermions:

\[ \mathcal{L}_{\text{Yukaa}} = Y_1 \bar{\psi}_L \Phi_1 \psi_R + \bar{Y}_1 \bar{\psi}_L \bar{\Phi}_1 \psi_R + Y_{15} \bar{\psi}_L \Phi_{15} \psi_R + \bar{Y}_{15} \bar{\psi}_L \bar{\Phi}_{15} \psi_R + h.c. \]  \hspace{1cm} (5)

where \( \psi_L \sim (2, 1, 4) \) and \( \psi_R \sim (1, 2, 4^*) \) fermions, \( \bar{\Phi}_i \sim \tau_2 \Phi_i^* \tau_2 \). From Eq. (5), it follows that the Dirac mass matrices of the up quark, down quark, charged lepton, and the neutrino are all independent.

Once we set \( M^\text{Dirac}_\nu = 0 \), we can directly link the neutrino mass matrix to the coupling matrix \( f \). The advantage of this is that the requirement of adequate baryogenesis as well as FCNC and other constraints fix not only the neutrino mass matrix, but also the mass spectrum of the theory. The FCNC constraints come from the fact that \( \Delta_u^u, \Delta_d^d \) fields have masses in the multi-TeV range and can lead to sizable \( K^0 - \bar{K}^0 \), \( B_{d,s} - \bar{B}_{d,s} \) mixings. They in turn severely constrain the pattern of the Yukawa couplings \( f_{ij} \) and thereby the neutrino mass matrix.

### 3 Low energy constraints on the model

In this section, we discuss the tree level flavor changing neutral current contributions to processes such as \( K - \bar{K}, B_{d,s} - \bar{B}_{d,s}, D - \bar{D} \) mixings from the di-quark Higgs field exchanges. We have to make sure that they are not in conflict with observations. One cannot make the di-quark scalar masses very large to satisfy the FCNC constraints, since successful post-sphaleron baryogenesis requires the masses of at least two of these scalars to be not more than about a TeV. Similarly, the doubly charged scalar bosons from the same multiplet will contribute to rare processes such as \( \mu \rightarrow 3e \) via tree level diagrams. Neutrino oscillation data, on the other hand, suggests specific form of the \( f \) matrix. We need to examine if these dual requirements can be simultaneously met. We have found that indeed this can be satisfied, but only for an inverted mass hierarchy spectrum for the neutrinos.
To discuss the constraints on the couplings $f_{ij}$ and masses of $\Delta_{uc,dc}$ implied by these considerations, we first note that above the $SU(4)_c$ scale, all couplings to diquarks and dileptons are given by a single matrix $f_{ij}$. The form of this matrix can be specified in any basis without loss of generality and we specify them in the basis in which the down quarks are mass eigenstates. In this basis, the $f_{ij}$ couplings split up into the following depending on which quarks they couple to: $f_{dd}$, $f_{ud}$ and $f_{uu}$, where $f_{dd}$ indicates the coupling to $d^cd^c$, etc. Assuming for simplicity that CP is not broken by the vacuum expectation values of the bidoublet fields (so that the left–handed and right–handed CKM matrices are equal to each other), we get

\begin{align}
    f_{ud} &= U_{CKM} f_{dd} \\
    f_{uu} &= U_{CKM} f_{dd} U_{CKM}^T \\
    f_{\nu\nu} &= U_l f_{dd} U_l^T = f_{ee}
\end{align}

(6)

where $U_{CKM}$ is the quark rotation matrix and $U_l$ is the matrix that makes the charged leptons diagonal. Clearly, it is $f_{\nu\nu}$ which determines the neutrino mass matrix in the type II seesaw case.

In this basis, first there are constraints from flavor changing processes such as $K - \bar{K}$, $B_{s,d} - \bar{B}_{s,d}$ and $D - \bar{D}$ mixings. Below we list the constraints [13] and their implications for the parameters of the model:

(i) $D^0(uc) - \bar{D}^0(\bar{u}\bar{c})$ constraint:

\begin{equation}
    \frac{f_{uu,11}f_{uu,22}}{[m_{\Delta_{uc}}/\text{TeV}]^2} \lesssim 2 \times 10^{-6}
\end{equation}

(7)

where we have taken $m_D = 1.865$ GeV and $f_D = 180$ MeV.

(ii) $K^0(d\bar{s}) - \bar{K}^0(\bar{d}s)$ constraint:

\begin{equation}
    \frac{f_{dd,11}f_{dd,22}}{[m_{\Delta_{dc}}/\text{TeV}]^2} \lesssim 3.3 \times 10^{-6}
\end{equation}

(8)
taking \( m_K = 497.6 \) MeV and \( f_K = 113 \) MeV.

Turning to the \( B \) system, we get from

(iii) \( B_s^0(s\bar{b}) - \bar{B}_s^0(\bar{s}b) \) constraint:

\[
\frac{f_{dd,22} f_{dd,33}}{[m_{\Delta e,e}/\text{TeV}]^2} \lesssim 2.0 \times 10^{-4}
\]  

(9)

with \( m_{B_s} = 5.366 \) GeV and \( f_{B_s} = 260 \) MeV.

(iv) \( B_d^0(d\bar{b}) - \bar{B}_d^0(\bar{d}b) \) constraint:

\[
\frac{f_{11,dd} f_{33,dd}}{[m_{\Delta e,e}/\text{TeV}]^2} \lesssim 7.6 \times 10^{-6}
\]  

(10)

with \( m_{B_d} = 5.28 \) GeV and \( f_{B_d} = 216 \) MeV.

In addition, lepton family number violating modes\[14\], such as \( \mu \rightarrow 3e \) imply

\[
\frac{f_{ee,11} f_{ee,12}}{[m_{\Delta e,e}/\text{TeV}]^2} \lesssim 3.3 \times 10^{-5}.
\]  

(11)

This can be satisfied by requiring the \( \Delta^{++} \) mass to be in the 100 TeV range for our choice of \( f_{12,11} \) as we see below. The constraints from the various \( \tau \) decay modes can then be easily satisfied for this limit on the \( \Delta^{++} \) mass and we do not give those constraints here.

Another constraint on the parameters of the theory comes from the present limits on \( n - \bar{n} \) oscillation period. \( \tau_{n-\bar{n}} \geq 10^8 \) sec. \[15, 16\] implies that the strength \( G_{n-\bar{n}} \) of the \( \Delta B = 2 \) transition is \( \leq 10^{-28} \) GeV\(^{-5} \). In a generic model of this type, \( n - \bar{n} \) oscillations arise from the diagram in Fig. 2 and we find that

\[
G_{n-\bar{n}} \sim \frac{\lambda_1 \langle S \rangle f_{dd,11}^2 f_{uu,11}}{M_{\Delta_e,e}^4 M_{\Delta_e,e}^2} + \frac{\lambda_2 \langle S \rangle f_{dd,11}^2 f_{ud,11}^2}{M_{\Delta_e,e}^2 M_{\Delta_e,e}^4} \leq 10^{-28} \text{ GeV}^{-5}.
\]  

(12)

We discuss this further in our model in Section 6.

The nontrivial aspect of this model is that the same set of parameters responsible for baryogenesis are directly related to neutrino masses and mixings and must be such that they
satisfy the strong FCNC constraints listed above. Note that we cannot suppress the FCNC effects by simply raising the masses of $\Delta_{d^c,d^c}$, $\Delta_{u^c,u^c}$ particles since in that case we cannot satisfy the desired constraints for adequate baryogenesis.

4 Inverted neutrino mass hierarchy from the FCNC constraints

In this section, we address the question of how we satisfy these constraints and yet obtain concordance with neutrino oscillation observations. It turns out that if we choose $f_{dd}$ matrix as

$$ f_{dd} = \begin{pmatrix} 0 & 0.95 & 1 \\ 0.95 & 0 & 0.01 \\ 1 & 0.01 & -0.0627357 \end{pmatrix}, \quad (13) $$

then for TeV scale $\Delta_{dd}$ and $\Delta_{ud}$ and 100 TeV mass for $\Delta_{uu}$, we can satisfy all the hadronic constraints. Such a choice will automatically satisfy $K - \overline{K}$ and $B_{d,s} - \overline{B}_{d,s}$ mixing constraints, owing to the zeros in the diagonal entries. In the leptonic sector, as already noted, the most stringent constraint comes from $\mu \rightarrow 3e$ and it requires that $\Delta^{++}$ mass also be of order 100 TeV or so.

Then we use the following unitary transformation to rotate $f_{dd}$ to get the neutrino mass matrix:

$$ U_l = \begin{pmatrix} \cos \Theta & \sin \Theta & 0 \\ -\sin \Theta & \cos \Theta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (14) $$

with $\Theta = 0.23$. This gives

$$ f_\nu = U_l f_{dd} U_l^T = \begin{pmatrix} 0.421751 & 0.85125 & 0.975946 \\ 0.85125 & -0.421751 & -0.218241 \\ 0.975946 & -0.218241 & -0.0627357 \end{pmatrix} \quad (15) $$

This matrix must be multiplied by the $SU(2)_L$ triplet vev $v_L$ (which is much smaller than the electroweak vev in Type II seesaw) to give the neutrino mass matrix. For $v_L = 0.03$
eV, we get the three neutrino masses to be

\[ m_1 = 0.048 \text{ eV}, \ m_2 = -0.041 \text{ eV}, \ m_3 = -0.001 \text{ eV} \]

which yields the correct solar-to-atmospheric neutrino mass square difference of about 30
and the PMNS mixing angles \( \theta_{12} = 35.6^0 \), \( \theta_{23} = 46^0 \) and \( \theta_{13} = 8^0 \). This value of \( \theta_{13} \) is
observable in the ongoing (Double CHOOZ) \cite{17} and planned (Daya Bay) \cite{18} experiments
and should provide a test of the model. Secondly we predict that the neutrinoless double
\( \beta \)-decay experiments should observe a Majorana neutrino mass at the 10-20 meV level which
is perhaps within reach of the next round of neutrinoless double beta decay experiments. In
Fig. 1 we present two scatter plots that display the preference of oscillation parameters in
our model We clearly see the lower bound on the \( \theta_{13} \) from them.

![Figure 1: We give the predictions for neutrino oscillation parameters for the allowed ranges
of the diquark scalar couplings in our model. Note the lower limit on the \( \theta_{13} \) of about 0.1.](image)

We will see below that this form of the \( f \) matrix satisfies the baryon asymmetry con-
straints as well as the \( n - \bar{n} \) constraints.

5 Origin of matter

Before proceeding to the discussion of how baryon asymmetry arises in this model, let us first
sketch the cosmological sequence of events starting at the \( SU(4)_c \) scale that leads up to this.
For temperatures above the \( SU(4)_c \) scale of about 100 TeV, there is no \( B - L \) violation. The
Figure 2: Tree level diagrams contributing to $S_r$ decays into 6 anti-quarks. There are other diagrams where $S_r$ decays into 6 quarks, obtained from the above by reversing the arrows of the quark fields.

Sphalerons are active and therefore erase any pre-existing $B + L$ asymmetry in the universe. So if there was a primordial GUT scale generated baryon asymmetry that conserved $B - L$ (like that in most $SU(5)$ and some $SO(10)$ models), it will be erased by sphalerons. Any baryon asymmetry residing in $B - L$ violating interactions will however survive.

Below the $SU(4)_c$ scale, $B - L$ violating interactions arise e.g. $HH \to e^-e^+$, and will be in equilibrium together with the $\Delta B = 2$ interactions. So together they will erase any pre-existing baryon or lepton asymmetry. Thus in models of this kind, baryon asymmetry of the universe must be generated fresh below the sphaleron decoupling temperature.

In order to sketch how fresh baryon asymmetry arises in our model, we assume the following mass hierarchy between the $S_r$ field and the $\Delta_d^d d^c, \Delta_u^c u^c, \Delta_u^d u^d$ fields:

$$m_t < M_S (\sim 400 - 500 \text{ GeV}) \leq M_{\Delta_d^d d^c} \sim M_{\Delta_u^d u^d} (\sim \text{ TeV}) \ll M_{\Delta_u^c u^c} (\sim 100 \text{ TeV}),$$

where $m_t$ is the top quark mass.

Between $1 \leq T \leq 100$ TeV, the $\Delta B = 2$ interaction rates go like

$$\Gamma(\Delta B = 2) \sim \frac{f_{11}^6}{(2\pi)^9} T$$

(16)
and are therefore in equilibrium if some of the $f_{ij}$’s are above 0.3 as is the case as we see below.

Below $T \sim 1$ TeV, the $\Delta B = 2$ processes such as the decay $S_r \rightarrow 6q + 6\bar{q}$, $(\bar{q}, q) + S_r \rightarrow 5(q, \bar{q})$ occur at a rate given by

$$\Gamma(\Delta B = 2) \sim \frac{100 f_{ud,12}^6 T^{13}}{(2\pi)^9 (6M)^{12}}$$

where $M \sim$ TeV, the average mass of the $\Delta_{d,d',u,d',d'}$ particles; in our model and are in equilibrium. The $\Delta_{u,d'}$ is about 100 TeV and hence its contribution to these processes is more suppressed compared to that of $\Delta_{d,d',u,d',d'}$. This decay then goes out of equilibrium somewhat below the TeV temperature range. One impact of this is that these interactions being in equilibrium above $T \sim$ TeV erase any pre-existing baryon asymmetry as discussed above.

By the time the universe cools to temperature near or slightly below $M_S$, its decay channels can start if the rates are faster compared to the Hubble expansion rate. Let us therefore estimate the various decay rates:

There are four decay modes which are competitive with each other: (i) $S_r \rightarrow 6q$; (ii) $S_r \rightarrow Zq\bar{q}$; (iii) $S_r \rightarrow ZZ$ and (iv) $S_r \rightarrow \tau e$.\footnote{The $S_r \rightarrow WW$ is suppressed by $W_L - W_R$ mixing parameter which can be adjusted to be small.} We discuss them below.

(i) $S_r \rightarrow 6q$ decay: The diagram for this is given in Fig. (2). Since $M_S \gg m_t$, in its decay all modes will participate. Including all the modes, we find the decay rate to be:

$$\Gamma_{S_r \rightarrow 6q} \simeq \frac{36\text{Tr}(f^\dagger f)^3 \lambda^2 M_S^{13}}{(2\pi)^9 (6M)^{12}}$$

Taking as an example a typical set of parameters $M_\Delta \simeq 2M_S \sim$ TeV and taking the parameters for the $f$ matrix elements from Eq.(13), we get $\Gamma(S_r \rightarrow 6q) \sim 2.6 \times 10^{-16}$ GeV.

(ii) $S_r \rightarrow Z + q\bar{q}$: This arise from the $S\bar{S}$ coupling to $Z'Z'$ with one of the $Z$’s mixing with $Z'$ (Fig. 3) and the virtual $Z'$ decaying to $q\bar{q}$. This occurs only for $T \leq v_{wk}$. This is
Figure 3: Feynman diagram for $S_r$ decay to $Z f^c \bar{f}^c$ via $Z - Z'$ mixing

because for $T \geq v_{wk}$, $Z - Z'$ mixing disappears. Below the electroweak symmetry breaking temperature, this mixing denoted below by $g_{ZZ'}$ becomes effective and is given by

$$g_{ZZ'} = \frac{g^2 \cos^2 \theta_W}{\sqrt{\cos 2\theta_W}} \left( \frac{M_Z}{M_{Z'}} \right)^2 v_{BL} \tag{19}$$

which leads to the new $S_r$ decay mode (Fig. 3) given above. This decay rate is given by

$$\Gamma(S_r \rightarrow Z f^c \bar{f}^c) \simeq \frac{7.0 \times 10^{-2}}{M_s M_{Z'}^6} \left[ M_s \sqrt{M_s^2 - M_Z^2} \left( 6M_s^4 - 19M_s^2M_Z^2 + 28M_Z^4 \right) - 3M_Z^2(M_s^2 + 4M_Z^2) \log \left( \frac{M_s + \sqrt{M_s^2 - M_Z^2}}{M_Z} \right) \right] \tag{20}$$

For our choice of parameters and $M_{Z'} \sim 100$ TeV, we find that $\Gamma(S_r \rightarrow Zq\bar{q}) \simeq 5 \times 10^{-18}$ GeV and is therefore slower than the $6q$ mode. (iii) $S_r \rightarrow ZZ$ decay: This decay mode arises from $Z - Z'$ mixing with the decay width given by

$$\Gamma(S_r \rightarrow ZZ) = \frac{g_{ZZ}^2 M_s^3}{128\pi M_Z^2} \left( 1 - \frac{4M_Z^2}{M_s^2} \right)^{1/2} \left[ 1 - \frac{4M_Z^2}{M_s^2} + \frac{12M_Z^4}{M_s^4} \right], \tag{21}$$

where the $S_rZZ$ vertex is given by

$$g_{ZZ} = \frac{1}{2} g^2 \cos^2 \theta_W v_{BL} \left( \frac{M_Z}{M_{Z'}} \right)^2 \tag{22}$$

In Fig. 4, this decay rate is plotted against the mass of the scalar field for various values of $v_{BL}$. Note that $M_{Z'}$ is related to $v_{BL}$ as follows:

$$M_{Z'}^2 \simeq \frac{2g^2 v_{BL} \cos^2 \theta_W}{\cos 2\theta_W}$$

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Figure 4: The $S_r \rightarrow Z f^c \bar{f}^c$ (thin solid lines) and $S_r \rightarrow ZZ$ (thin dashed lines) decay rates for various values of $v_{BL}$ (in TeV). The thick solid line corresponds to the $S_r \rightarrow 6q^c$ decay rate for $M_{\Delta u c d c} / M_S = 2$. We see that for $v_{BL} \geq 50$ TeV, the six quark decay mode dominates for a large range of $M_S$.

(iv) $S \rightarrow \tau + e$ decay mode: This decay mode arises from the Feynman diagram in Fig. 5 and its rate can be estimated to be:

$$\Gamma(S_r \rightarrow \tau + e) \simeq \frac{f^2_{13} g^4 (m_\tau M_{\nu_R})^2 M_S}{12\pi (256\pi^4) M^4_{WR}}$$

This is estimated to be $\Gamma(S_r \rightarrow \tau + e) \simeq 5 \times 10^{-20}$ GeV. Therefore this is also much smaller than the decay rate to six quark modes.

At the time the universe has a temperature of $\sim M_S$ or slightly below so that it is out of equilibrium from the cosmic soup, the Hubble expansion rate is $\sim \sqrt{g_*} M_S^2 / M_{Pl} \sim 2.5 \times 10^{-13}$ GeV implying that all the above decay modes are out of equilibrium. Since the decay rate remains constant below $T \sim M_S$, but the expansion rate of the universe is slowing down as it expands, there will come a time (or temperature $T_d$) when the dominant decay $\Gamma(S_r \rightarrow 6q) \simeq H(T_d)$. At that point the $S_r$ particle will start decaying and produce the baryon asymmetry as in ref. [8].
Figure 5: $S_r \to e\tau$ decay

At this temperature which is far below the masses of the $\Delta_{u^c d^c d^c d^c}$ particles, the decay processes $\Delta_{q^c q^c} \to q^c q^c$ being very fast have depleted all the diquark Higgses and have left only the $S_r$ particles to survive along with the usual standard model particles. The primary decay modes of $S_r$ are $S_r \to u^c d^c u^c d^c$ and $S_r \to \bar{u}^c \bar{d}^c \bar{u}^c \bar{d}^c$ as already noted (Fig. 2). Other decay modes are negligible as discussed.

We have to make sure that the decay of $S_r$ starts below the sphaleron decoupling temperature and above the QCD phase transition temperature. To check if this indeed happens in our model, let us calculate the $T_d$: This decay goes out of equilibrium around $T_d$:

$$T_d \approx \left[ \frac{18P\lambda_2^3 \text{Tr}(f^\dagger f)^3 M_{Pl}M^{13}_{S_r}}{(2\pi)^9 1.66g_e^{1/2} (6M_{\Delta_{u^c d^c d^c d^c}})^{12}} \right]^{1/2}$$

$$\approx 60.81 \text{ GeV}^{1/2} \left( \frac{M^{13}_{S_r}}{M^{12}_{\Delta_{u^c d^c d^c d^c}}} \right)^{1/2}$$

Here $P = 2.05$ is a phase space factor. For $M_S \sim 500$ GeV and $M_{\Delta_{u^c d^c d^c d^c}} \sim 1$ TeV, we get $T_d \approx 0.3$ GeV which is comfortably above the QCD phase transition temperature.

It is worth emphasizing that if we increased the sextet scalar masses arbitrarily to satisfy the FCNC constraints, this will lower the $T_d$ to undesirable values below the QCD temperature. One may think that we could simultaneously increase the value of $M_S$ but as we will see below, the magnitude of the baryon asymmetry goes inversely like the square of $M_S$ and

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increasing it above 500-600 GeV will suppress the baryon asymmetry to a level below the observations.

The calculation of the baryon asymmetry is same as in Ref. [8] and we do not repeat it here except to give a brief summary. The vertex correction via the $W$ boson exchange (Fig. 6) dominates the baryon asymmetry given by

$$\frac{\epsilon_B^{\text{vertex}}}{\text{Br}} \simeq -\frac{\alpha_2}{4} \frac{\text{Im} \left[ 36 f^T \tilde{M}_u V M_d f^\dagger \tilde{M}_d^* V^* \right]}{\text{Tr} \left( f^\dagger f \right) M_w^2 M_S^2}.$$  \hspace{1cm} (25)

Here we have assumed that $M_S \gg m_t$. Note that if we increased $M_S$ above 500 GeV or so, the generated baryon asymmetry will fall short of the observed values.

![Figure 6: One loop vertex correction diagram for the $B$-violating decay $S_r \to 6q^c$. There are also wave function corrections involving the exchange of $W^\pm$ gauge bosons, but they are much smaller.](image)

This gives for the baryon asymmetry at $T = T_d$: $\epsilon_B \sim (2 - 3) \times 10^{-8}$. To compare it with the observed $\eta_B$, we divide this by $g_s(200 \text{ MeV})/g_s(1 \text{ eV}) \sim 62.75/5.5 = 11.4$ and apply an additional dilution factor of 0.25 (see discussion below) which gives us the desired value. Note that the observed value of $\eta_B^{\text{CMB}} \sim 6 \times 10^{-10}$ [19].

Since the $S_r$-particle decays far below its mass to generate the baryon asymmetry, we have to take into account the effect of its decay, which as we explain below amounts to a dilution of the original baryon asymmetry calculated. In order to estimate the dilution

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factor, we note that $S_r$-decay will release all the energy in its mass to lighter relativistic particles which will thermalize with the rest of the cosmic fluid and in the process raise its temperature which will increase the entropy and hence dilute the net baryon asymmetry. Suppose the decay temperature is $T_d$. Energy conservation then gives

$$\rho_S + \rho_{\text{rel}}|_{T_d} \simeq \rho_{\text{rel}, T_d}$$

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Solving this one finds for the dilution factor $D$ that

$$d \equiv \frac{T_d^3}{T^3} \simeq \frac{0.32 g_* T_d}{0.12 M_S + 0.32 g_* T_d}$$

For $M_S \sim 1 \text{ TeV}$ and $T_d \sim 1 \text{ GeV}$, $d \simeq 22\%$.

There is another factor coming from the fact that the baryon asymmetry generated is at $T_d \sim 1 \text{ GeV}$ where the $g_* \sim 62$ whereas the measured value is at $T_{\text{rec}}$ where $g_* = 5.5$. All these dilution factors have been taken into account in our estimate of final baryon asymmetry, which is in agreement with observations.

6 Prediction of observable neutron-antineutron oscillation time

In this section, we discuss the prediction of the model for neutron-anti-neutron oscillation. In order to estimate the $n - \bar{n}$ oscillation, let us first recall that the only contribution to this process comes from the right handed sector since the vev of the $\Delta_{\nu e, \nu e}$ is in the 100 TeV range and that of its left-handed counterpart is in the eV range.

There are two types of contributions to $n - \bar{n}$ oscillation from the right handed sector: the one involving two $d^c d^c$ type and one $u^c u^c$ type bosons of the right handed sector and another which involves two $u^c d^c$ and one $d^c d^c$ type $\Delta$ boson. Since the diagonal 11 and 22
entries of \( f_{dd} \) are close to zero, the first contribution is actually much smaller than the second one. The second type generates an effective operator of the form \( u^c d^c b^c u^c d^c b^c \). To get \( n - \bar{n} \) oscillation, we will have to change the two \( b^c \) quarks to 2 \( d^c \) quarks by second order weak interactions (see Fig. 7 below).

![Figure 7: One-loop diagram for \( n - \bar{n} \) oscillation](image)

From the above figure, we estimate

\[
G_{n-\bar{n}} \approx \frac{f_{dd,12}^2 f_{uu,11} \lambda_{vBL} g^4 V_{ud}^2 m_b^2 m_l^2}{M_1^2 \Delta_{udc}^2 M_2^2 \Delta_{dsc}^2 (16\pi^2)^2 m_W^4} \quad (28)
\]

This gives \( G_{n-\bar{n}} \sim 10^{-31} \) GeV\(^{-5} \) corresponding to an \( n - \bar{n} \) transition time of \( 10^{10-11} \) sec. given the uncertainties in the parameters. The present lower limit on this transition time is \( 10^8 \) sec. from the Grenoble experiment [15] as well nuclear decay experiments [16]. Our predicted value is accessible to current experiments under discussion at DUSEL as well as other facilities [20].

### 7 Other implications and tests of the model

(i) As noted in Sec. 2, a crucial prediction of our quark-lepton unified model of post-sphaleron baryogenesis is that neutrinos must be Majorana fermions and exhibit an inverted mass hierarchy form with large value for \( \theta_{13} \). This should be testable in long base line experiments
as well as the ongoing and planned reactor experiments searching for $\theta_{13}$ and neutrinoless double beta decay searches \cite{21}. It is perhaps worth noting that there is indication of a non-zero $\theta_{13}$ from already existing neutrino oscillation data \cite{22}.

(ii) Our theory is also testable in collider experiments such as the LHC since we have colored diquark scalar fields with masses in the TeV range. It is clear from the form of the $f_{ud}$ matrix that in a $pp$ collision, the valence quarks in the two protons could produce the $\Delta_{u^c d^c}$ field which could then decay to $t^+ \text{jets}$. This could either be an s-channel single production \cite{23} or Drell-Yan pair production \cite{24}. The s-channel process will have a resonant enhancement which can give a signal above the standard model background. The Drell-Yan pair production could give signals of type $bbl^\pm l^\mp jj + \text{missing } E_T$. Unlike the s-channel process, the Drell-Yan pair production has the advantage of not being dependent on the specific flavor texture of the two quark couplings $f$ and is promising for color sextet masses upto a TeV \cite{24}. It would therefore be important to search for these fields at LHC. Their discovery will signal a completely different direction for unification beyond the standard model than the conventional SUSYGUT theories.

(iii) In our model, since there is a mass hierarchy between the $\Delta_{u^c d^c \cd ^c,d^c} \text{ and } \Delta_{u^c u^c}$ masses i.e. $M_{\Delta_{u^c d^c \cd ^c,d^c}} \ll M_{\Delta_{u^c u^c}}$, a one loop level box graph induces by the trilinear coupling $\lambda v_{BL} \Delta_{u^c u^c} \Delta_{u^c d^c \cd ^c,d^c} \Delta_{u^c d^c \cd ^c,d^c}$ a $(\Delta_{u^c u^c} \Delta_{u^c d^c \cd ^c,d^c})^2$ coupling with a strength $\lambda_{eff} \simeq - \left( \frac{\lambda v_{BL}}{M_{\Delta_{u^c d^c \cd ^c,d^c}}} \right)^4$. This can lead to color breaking unless $\lambda_{eff} \leq 1$. This can be satisfied by lowering the $v_{BL}$ scale to about 30 TeV with $\lambda \sim 0.1$. In order to reconcile this lower value with constraints from $\mu \rightarrow 3e$, we can introduce a multiplet of type $(1,3,1)$ with a vev in the 100 TeV range which gives mass to the $W_R$ and the $\Delta^{++}$. This vev decouples the B-L breaking scale $v_{BL}$ from the masses of the $W_R$ and $\Delta^{++}$ fields. In this case, one can keep the $B - L$ breaking scale in the 30 TeV while keeping the $W_R$ and $\Delta^{++}$ mass around 100 TeV as required by the $\mu \rightarrow 3e$ and $S_{\tau} \rightarrow \tau + e$ constraints.
(iv) Finally, note that a priori in the model there could be a coupling of type $\Delta^\dagger \Delta \mathrm{Tr} \Phi^\dagger \Phi$, which will induce a $S_r$ decay to two SM Higgs fields. We assume that this parameter is very small. This assumption could be justified in supersymmetric extensions of the model where such terms are forbidden by holomorphy of the superpotential.

8 Conclusion

In summary, we have pointed out that the post-sphaleron baryogenesis mechanism proposed in Ref. [8, 9] can be naturally embedded into a 100 TeV scale quark-lepton unified $SU(2)_L \times SU(2)_R \times SU(4)_c$ model. If we further assume that the neutrino masses in this model arise via the type II seesaw mechanism, then the couplings responsible for baryogenesis and neutrino masses get intimately linked to one another. In this case adequate baryogenesis predicts that neutrino mass ordering must be inverted with large $\theta_{13}$, a prediction that can be tested in ongoing neutrino experiments.

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