Thin Film Flow of Couple Stress Magneto-Hydrodynamics Nanofluid with Convective Heat over an Inclined Exponentially Rotating Stretched Surface

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Abstract: In this article a couple stress magneto-hydrodynamic (MHD) nanofluid thin film flow over an exponential stretching sheet with joule heating and viscous dissipation is considered. Similarity transformations were used to obtain a non-linear coupled system of ordinary differential equations (ODEs) from a system of constitutive partial differential equations (PDEs). The system of ordinary differential equations of couple stress magneto-hydrodynamic (MHD) nanofluid flow was solved using the well-known Homotopy Analysis Method (HAM). Nusselt and Sherwood numbers were demonstrated in dimensionless forms. At zero Prandtl number the velocity profile was analytically described. Furthermore, the impact of different parameters over different state variables are presented with the help of graphs. Dimensionless numbers like magnetic parameter $M$, Brownian motion parameter $Nb$, Prandtl number $Pr$, thermophoretic parameter $Nt$, Schmidt number $Sc$, and rotation parameter $S$ were analyzed over the velocity, temperature, and concentration profiles. It was observed that the magnetic parameter $M$ increases the axial, radial, drainage, and induced profiles. It was also apparent that $Nu$ reduces with greater values of $Pr$. On increasing values of the Brownian motion parameter the concentration profile declines, while the thermophoresis parameter increases.

Keywords: MHD; nanofluid; stretching surface; rotating fluid; Homotopy Analysis Method (HAM)

1. Introduction

The word nanofluid denotes a mixture of nanoparticles and base fluids. Usually nanoparticles contain metals such as silver, copper, aluminum, nitrides like silicon nitride, carbides such as silicon carbides, oxides e.g., aluminum oxide and nonmetals such as graphite. The usual liquids are water,
oil, and ethylene glycol. The combination of nanoparticles with a base liquid greatly helps to develop the thermal qualities of the vile liquid. Choi et al. [1] introduced the term nanofluid and heat transfer features of vile fluids and studied the thermal conductivity enhancement. Wang et al. [2] investigated convective physiognomies of vile fluid and they found that these fluids are enriched by adding metal and non-metal atoms to them. Heat transfer enhancement and thermal conductivity variation of fluids by the addition of copper nanoparticles were studied by Eastman et al. [3,4]. Murshed et al. [5] found the thermal conductivity of vile liquid increases by adding sphere-shaped nanoparticles. Maiga et al. [6], scrutinized nanofluid flow in a uniformly heated tube with heat transfer. Bianco et al. [7] studied the implication between nanoparticles and liquid matrix in two-phase flow; further, nanofluids involuntary convection in circular tubes was deliberated. Buongiorno [8] introduced the two-phase model for convective transport in nanofluids. The single-phase model was studied by Tiwari et al. [9]. After these two models, several investigators considered nanofluids thermal attraction to investigate the actual fluid characteristics discussed in references [10–12]. Thin film Darcy–Forschheimer nanofluid flow with Joule dissipation and MHD effect were scrutinized by Jawad et al. [13]. Rotating flow in the existence of aqueous suspensions with the effect of non-linear thermal radiation was investigated by Jawad et al. [14]. Bhatti et al. [15] scrutinized Jeffrey nanofluid with immediate effects of variable magnetic field. MHD non-Newtonian nanofluid flow over a pipe with heat reliant viscosity was scrutinized by Ellahi et al. [16]. A Cu-water nanofluid applying porous media with a micro-channel heat sink was investigated by Hatami et al. [17]. Laminar nanofluid flow with heat transfer between rotating disks was studied by Hatami et al. [18]. Recently, Shah et al. [19–22] considered Hall current and thermal radiations of the nanofluid flow through a rotating system. Non-Newtonian fluids are complex in nature and various models have been constituted and developed for the purpose of defining the strain rate in these fluids. Recently, Ullah et al. [23] analyzed the Reiner–Philipoff fluid model analytically over a stretching surface. They studied the thermophoresis and Brownian motion impacts over the thin film. The heat transfer enhancement not only depends on the nanoparticles added, but also depends on the nature of the fluid. The couple stress impacts with joule heating and viscous dissipation have been studied and analyzed by different researchers over different surfaces [24–26]. Heat transfer enhancement and its detailed study with engineering applications can be found in the references [27–31].

Various techniques have been used to analyze the problem constituted and modeled in the literature. Soleimani et al. [32] used the finite element method for natural convective nanofluid flow with transfer of heat, in a semi-annular object. They described the turning angle effect on the isotherms, streamlines, and local Nusselt number. Rudraiah et al. [33] studied numerically the natural convection inside a rectangular obstacle subject to magnetic field. They found that the heat transfer reduces with a magnetic field. For the simulation of magnetic drug targeting and ferrofluid flow, Sheikholeslami and Ellahi and Kandelousi [34] considered the lattice Boltzmann method (LBM). They found that both the magnetic parameter and the Reynolds number decrease the coefficient of skin friction. Ramzan et al. [24] found a series solution by using HAM for the flow of 3D nanofluid couple stress with joule heating. Recently, with the impacts of convective condition couple stress 3D MHD nanofluid flow in the presence of Cattaneo–Christov heat flux was explored by Hayat et al. [35,36]. Maxwell boundary layer flow of nanofluid was investigated by Hayat et al. [37]. Malik et al. [38] considered MHD flow through a stretching surface of Erying Powell nanofluid. Nadeem et al. [39] considered a vertical stretching surface and analyzed the flow of Maxwell’s liquid with nanoparticles. Raju et al. [40] investigated an MHD nanoliquid flow with free convective heat transfer through a cone. The impact of Lorentz forces and entropy generation for different nanofluids were numerically investigated by Sheikholeslami et al. [41–43]. They also used the control volume finite element method (CVFEM) and some new modified techniques for the analysis of the nanofluid flow through a square cavity by considering shape factors. More detailed studies on nanofluid investigation by considering different models can be found in the references [44–49].

Keeping in view the applications of nanofluid and its role in heat transfer enhancement various analyses have been made by researchers. Shah et al. [50] analyzed the Titanium nanofluid flow over a rotating surface analytically. In their work, they studied the impacts of the Hall current and the
magnetic parameter. Considering a similar approach for the problem geometry, in this work an inclined rotating surface is considered and is extended to the couple stress nanofluid MHD flow with convective heat transfer by ignoring the mass flux. Furthermore, the impacts of viscous dissipation and Joule heating are also considered in investigating the overall effects of the fluid parameters. The basic equations for the physical problem are constituted from the geometry of the problem and the assumptions made. The set of PDEs obtained from the fundamental equations of fluid dynamics and further assimilated from the boundary layer theory are transformed to a non-linear ODEs system, by using similarity transformations. The analytical Homotopy Analysis Method is applied to solve the set of ODEs.

2. Problem Formulation

Assume a three-dimensional nanofluid thin liquid flow through a steady rotating disk. The disk rotates on its own axis due to the angular velocity (Ω) as shown in Figure 1. With the horizontal axis the inclined disk creates an angle β. The radius of the disk is greater as it is associated to the thickness of the fluid film and therefore the last impact is unobserved. Here T₀ and Tw denote the temperatures of the surface film and the surface of the disk respectively. Similarly, at the film C₀, Ch are the concentrations on the disk surface respectively. The equations for the steady state flow are displayed as follows [50,55,56]:

\[ u_x + v_y + w_z = 0 \]  \hspace{1cm} (1)

\[ uu_x + vu_y + uu_z = nu_{xx} - v'w_{xxx} - \frac{\sigma B_z^2}{Q} \]  \hspace{1cm} (2)

\[ uv_x + vv_y + uv_z = nu_{xx} - v'w_{xxx} - \frac{\sigma B_z^2}{Q} \]  \hspace{1cm} (3)

\[ uw_x + vw_y + uv_z = nu_{xx} - v'w_{xxx} - \frac{\sigma B_z^2}{Q} \]  \hspace{1cm} (4)

\[ uT_x + vT_y + wT_z = \frac{k_{nf}}{(\rho \eta f)_{nf}} (T_{zz}) + \tau \left[ D_{nf} C_x T_z + \frac{D_{nf}}{T_0} (T_z)^2 \right] \]  \hspace{1cm} (5)

\[ uC_x + vC_y + wC_z = D_{p} (C_{xx}) + \frac{D_{w}}{T_0} (T_{zz}) \]  \hspace{1cm} (6)

with boundary conditions:

\[ u = -\Omega y, v = \Omega x, w = 0, T = T_w, C = C_0 \quad \text{at} \quad z = 0 \]

\[ u_x = v_z = 0, w = 0, T = T_w, C = C_0 \quad \text{at} \quad z = h \]  \hspace{1cm} (7)

using the Similarity transformation [50]:

\[ u = -\Omega yg(\eta) + \Omega yf'(\eta) + \bar{g}(\eta) \sin \frac{\beta}{\Omega} \]

\[ v = \Omega xg(\eta) + \Omega yf'(\eta) + \bar{g}(\eta) \sin \frac{\beta}{\Omega} \]

\[ w = -2\sqrt{2\eta f(\eta)}, T = (T_0 - T_w) \theta(\eta) + T_w \]

\[ \eta \phi(\eta) = \frac{C - C_m}{C_a - C_m}, \eta = z \sqrt{\frac{\Omega}{v_{nf}}} \]  \hspace{1cm} (8)

we obtain:
\[ g'' - 2g'f + 2g^2 - K_g z^2 - Mg = 0 \]  
\[ f'' - f'^2 + g^2 + 2ff' - K_f x^2 - Mf = 0 \]  
\[ k'' + gk' + 2kk' - K_k z^2 - Mk = 0 \]  
\[ s'' - ks' + 2sf' - K_s z^2 - Ms = 0 \]

(9) \hspace{1cm} (10) \hspace{1cm} (11) \hspace{1cm} (12)

**Figure 1.** Geometrical description of the problem.

Now if \( \theta(\eta) \) and \( \phi(\eta) \) depend only on \( z \), then Equations (5) and (6) are reduced to the following forms:

\[ \theta'' + 2Pr \theta f + Pr Nb \eta \phi' \theta' + Pr \Omega Nt \theta' \phi'^2 = 0 \]  
\[ \phi'' + 2Scf \phi' + \frac{Nt}{Nb} \theta'' + \frac{S}{2} (\eta \phi' + \eta^2 \phi'^2) = 0 \]

(13) \hspace{1cm} (14)

\[ f(0) = 0, f'(0) = 0, f''(\delta) = 0 \]  
\[ g(0) = 0, g'(\delta) = 0 \]  
\[ k(0) = 0, k'(\delta) = 0 \]  
\[ s(0) = 0, s'(\delta) = 0 \]  
\[ \theta(0) = 0, \theta'(\delta) = 1 \]  
\[ \phi(0) = 0, \phi'(\delta) = 1 \]

(15)

while, \( Pr, M, Sc, Nb, K, S \) and \( Nb \) are defined as below [57]:

\[ Pr = \frac{\rho \nu}{k}, Sc = \frac{\mu}{\nu D}, Nb = \frac{\tau D}{\nu} (C_o - C_w) \]  
\[ Nt = \frac{\tau D}{\nu^2} (T_o - T_w), S = \frac{\alpha}{\Omega}, M = \frac{\sigma B_o}{\Omega \rho}, K = \frac{\nu \Omega}{\nu^2} \]

(16)

here, the constant of the normalized thickness is defined as:

\[ \delta = h \frac{\Omega}{\nu^2} \]

(17)

where the velocity of condensation is written as:
\[
f(\delta) = \frac{W}{2 \eta \Omega} = \alpha \tag{18}
\]

Direct integration of Equation (4) gives the pressure term. Using \( \Pr = 0 \) and \( \theta(\delta) = 1 \), we get

\[
\theta'(0) = \frac{1}{\delta} \tag{19}
\]

For small \( \delta \) an asymptotic limit is explained by Equation (17). The decrease of \( \theta'(0) \) for increasing \( \delta \) is not monotonic, thus \( Nu \) is signified as \( [57] \):

\[
Nu = \frac{k_w}{k_j} \frac{(T_s)_{in}}{(T_s - T_w)} = A_s \delta \theta'(0) \tag{20}
\]

Similarly, the Sherwood number is given by:

\[
Sh = \frac{(C_0)_{in}}{C_0 - C_w} = \delta \phi'(0) \tag{21}
\]

3. Solution by HAM

For the solution of three dimensional nanofluid thin layer flows through a steady rotating disk, the optimal approach is used. The obtained Equations (9)–(15) are solved by using HAM. The basic derivation and mechanism is explained below.

The operators \( L_j, L_f \) and \( L_s \) are defined as \([13,51]\):

\[
L_j(f) = \hat{f}^*, L_j(\hat{k}) = \hat{k}^*, L_j(\hat{g}) = \hat{g}^*,
\]

\[
L_s(s) = s^*, L_s(\hat{\theta}) = \hat{\theta}^*, L_s(\hat{\phi}) = \hat{\phi}^* \tag{22}
\]

where,

\[
L_j(e_1 + e_2 \eta + e_3 \eta^2) = 0, L_s(e_1 + e_3 \eta) = 0, L_s(e_2 + e_4 \eta) = 0,
\]

\[
L_s(e_4 + e_10 \eta) = 0, L_s(e_11 + e_12 \eta) = 0, L_s(e_13 + e_14 \eta) = 0 \tag{23}
\]

The consistent non-linear operators are reasonably selected as \( N_j, N_f, N_s, N_\theta, N_\phi \) and \( N_s \), and are recognized in the system \([13,22]\):

\[
N_j \left[ f(\eta; \zeta), g(\eta; \zeta) \right] = f_{\eta \eta} - \hat{f}^2 + \hat{g}^2 + 2 \hat{f} \hat{g} - K \hat{f}_{\eta\eta\eta\eta} - M \hat{f} \tag{24}
\]

\[
N_s \left[ \hat{g}(\eta; \zeta), \hat{f}(\eta; \zeta) \right] = \hat{s}_{\eta \eta} - 2 \hat{g} \hat{f} + 2 \hat{g} \hat{f} - K \hat{g}_{\eta\eta\eta\eta} - M \hat{g} \tag{25}
\]

\[
N_s \left[ \hat{g}(\eta; \zeta), \hat{f}(\eta; \zeta) \right] = \hat{s}_{\eta \eta} - 2 \hat{g} \hat{f} + 2 \hat{g} \hat{f} - K \hat{g}_{\eta\eta\eta\eta} - M \hat{g} \tag{26}
\]

\[
N_\theta \left[ \hat{\theta}(\eta; \zeta), \hat{f}(\eta; \zeta), \hat{k}(\eta; \zeta) \right] = \hat{s}_{\eta \eta} - \hat{K} - \hat{s}_{\eta \eta} + 2 \hat{s}_{\eta \eta} \hat{f} - K \hat{S}_{\eta\eta\eta\eta} - M \hat{S} \tag{27}
\]

\[
N_\phi \left[ \hat{\phi}(\eta; \zeta), \hat{f}(\eta; \zeta), \hat{\theta}(\eta; \zeta) \right] = \hat{\phi}_{\eta \eta} + 2 \hat{N}_{\eta \eta} \hat{\phi}_{\eta} + \hat{\phi}_{\eta \eta} + \hat{\phi}_{\eta \eta} + \hat{\phi}_{\eta \eta} \tag{28}
\]

For Equations (8)–(10) the 0th-order system is written as \([21]\):

\[
(1 - \zeta) L_j \left[ f(\eta; \zeta), \hat{k}(\eta; \zeta) \right] = \phi_{\eta \eta} N_j \left[ f(\eta; \zeta), \hat{g}(\eta; \zeta) \right] \tag{30}
\]

\[
(1 - \zeta) L_s \left[ \hat{k}(\eta; \zeta), \hat{g}(\eta; \zeta) \right] = \rho \tilde{\eta} N_j \left[ f(\eta; \zeta), \hat{g}(\eta; \zeta), \hat{f}(\eta; \zeta), \hat{s}(\eta; \zeta) \right] \tag{31}
\]
while the embedding constraint is:

\[ (1 - \zeta) L_c \left[ \phi(\eta; \zeta) - \phi_0(\eta) \right] = p h_4 N_2 \left[ \phi(\eta; \zeta), \phi(\eta; \zeta) \right] \]

where the boundary conditions are [22]:

\[ \left. \frac{\partial \hat{f}(\eta; \zeta)}{\partial \eta} \right|_{\eta=0} = 0, \quad \left. \frac{\partial^2 \hat{f}(\eta; \zeta)}{\partial \eta^2} \right|_{\eta=0} = 0, \quad \left. \frac{\partial \hat{g}(\eta; \zeta)}{\partial \eta} \right|_{\eta=0} = 0 \]

while the embedding constraint is \( \zeta \in [0, 1] \), to regulate for the solution convergence \( h_f, h_L, h_g, h_4 \), \( h_\theta \) and \( h_\phi \) are used. When \( \zeta = 0 \) and \( \zeta = 1 \) we have:

\[ \hat{f}(\eta; \zeta) = \hat{f}(\eta; 1) = \hat{f}(\eta) \quad \hat{g}(\eta; \zeta) = \hat{g}(\eta) \]

expand the \( \hat{f}(\eta; \zeta), \hat{g}(\eta; \zeta), \hat{s}(\eta; \zeta), \hat{\phi}(\eta; \zeta) \) and \( \hat{\phi}(\eta; \zeta) \) through Taylor’s series for \( \zeta = 0 \):

\[ \hat{f}(\eta; \zeta) = \hat{f}(\eta; 0) + \sum_{n=1}^\infty \hat{f}_n(\eta) \zeta^n, \quad \hat{g}(\eta; \zeta) = \hat{g}(\eta; 0) + \sum_{n=1}^\infty \hat{g}_n(\eta) \zeta^n, \quad \hat{s}(\eta; \zeta) = \hat{s}(\eta; 0) + \sum_{n=1}^\infty \hat{s}_n(\eta) \zeta^n, \quad \hat{\phi}(\eta; \zeta) = \hat{\phi}(\eta; 0) + \sum_{n=1}^\infty \hat{\phi}_n(\eta) \zeta^n \]

where the boundary restrictions are:

\[ \hat{f}(0) = 0, \quad \hat{f}'(0) = 0, \quad \hat{f}''(0) = 0, \quad \hat{g}(0) = 0, \quad \hat{g}'(0) = 0, \quad \hat{k}(0) = 0, \quad \hat{s}(0) = 0, \quad \hat{s}'(0) = 0, \quad \hat{\phi}(0) = 0, \quad \hat{\phi}'(0) = 0, \quad \hat{\phi}''(0) = 0 \]

now define

\[ \mathcal{R}_f^j(\eta) = \hat{f}''''_{\eta=1} - \sum_{j=0}^{n-1} \hat{f}_{\eta=1}''\hat{f}_{\eta=1} + \sum_{j=0}^{n-2} \hat{k}_{\eta=1}'' - \sum_{j=0}^{n-2} \hat{f}_{\eta=1}''^{\eta=1} - M \hat{f}''^{\eta=1} \]

\[ \mathcal{R}_g^j(\eta) = \hat{g}''''_{\eta=1} - \sum_{j=0}^{n-1} \hat{g}_{\eta=1}''\hat{g}_{\eta=1} + \sum_{j=0}^{n-2} \hat{k}_{\eta=1}'' - \sum_{j=0}^{n-2} \hat{f}_{\eta=1}''^{\eta=1} - K \hat{g}''^{\eta=1} - M \hat{g}''^{\eta=1} \]

\[ \mathcal{R}_s^j(\eta) = \hat{s}''''_{\eta=1} - \sum_{j=0}^{n-1} \hat{s}_{\eta=1}''\hat{s}_{\eta=1} + \sum_{j=0}^{n-2} \hat{k}_{\eta=1}'' - \sum_{j=0}^{n-2} \hat{f}_{\eta=1}''^{\eta=1} - K \hat{s}''^{\eta=1} - M \hat{s}''^{\eta=1} \]

\[ \mathcal{R}_\phi^j(\eta) = \hat{\phi}''''_{\eta=1} - \sum_{j=0}^{n-1} \hat{\phi}_{\eta=1}''\hat{\phi}_{\eta=1} + \sum_{j=0}^{n-2} \hat{k}_{\eta=1}'' - \sum_{j=0}^{n-2} \hat{f}_{\eta=1}''^{\eta=1} - K \hat{\phi}''^{\eta=1} - M \hat{\phi}''^{\eta=1} \]
\begin{equation}
\mathcal{R}_w^S(\eta) = \hat{s}_{n-1} - \sum_{j=0}^{n-1} k_{n-1-j} \hat{\phi}_{j} - \sum_{j=0}^{n-1} \hat{s}_{n-j-1} \hat{f}_{j} + 2 \sum_{j=0}^{n-1} \hat{s}_{n-j-1} \hat{f}_{j} - K \hat{s}_{n-1} \quad (44)
\end{equation}

\begin{equation}
\mathcal{R}_ \eta^0(\eta) = \left( \hat{\theta}_{n-1}^r \right) + 2 \frac{Pr}{ \chi} \sum_{j=0}^{n-1} \hat{\theta}_{n-1-j} \hat{f}_{j} + Pr N \eta \sum_{j=0}^{n-1} \hat{\phi}_{n-1-j} \hat{\phi}_{j} + Pr \Omega N / \hat{\phi}_{n-1}^2 \quad (45)
\end{equation}

\begin{equation}
\mathcal{R}_w^+ = \hat{\phi}_{n-1}^r + 2 c \sum_{j=0}^{n-1} \hat{f}_{n-1-j} \hat{\phi}_{j} + \frac{N \text{t}}{N \text{t}} \hat{\phi}_{n-1}^r + S \left( \eta \hat{\phi}_{n-1}^r + \eta^2 \hat{\phi}_{n-1}^r \right) \quad (46)
\end{equation}

where,
\begin{equation}
\chi_\eta = \begin{cases} 0, & \text{if } \zeta \leq 1 \\ 1, & \text{if } \zeta > 1 \end{cases} \quad (47)
\end{equation}

4. Results

A magneto-hydrodynamic nanofluid flow over an exponential stretching sheet with joule heating and viscous dissipation effects was modeled. The reduced modeled Equations (9)–(15) were solved by HAM. The graphical interpretation of the modeled problem is articulated in Figure 1. The impacts of important physical parameters such as $M, S, \Omega, Sc, Nt, Nb$ and $Pr$ are discussed with the help of Figures 2–17.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Impact of $M$ on $f(\eta)$ when $k = 1$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.png}
\caption{Impact of $M$ on $g(\eta)$ when $k = 2$.}
\end{figure}
Figure 4. Impact of $M$ on $k(\eta)$ when $k = 1$. 

Figure 5. The effect of $M$ on $s(\eta)$ when $k = 0.5$. 

Figure 6. Effect of $K$ on $f(\eta)$ while $M = 0.3$. 
Figure 7. Effect of $K$ on $g(\eta)$ when $M = 0.5$.

Figure 8. Effect of $K$ on $k(\eta)$ for $M = 0.3$.

Figure 9. Impact of $K$ on $s(\eta)$ for $M = 0.5$. 
Figure 10. Impact of Pr on $\theta_{(\eta)}$ for $M = 0.3, \Omega = 0.4, S = 0.5, Nt = 0.8, Sc = 0.7, Nb = 0.6$.

Figure 11. The effect of Nb on $\theta_{(\eta)}$ when $M = 0.3, \Omega = 1, S = 0.5, Pr = 2, Sc = 0.7, Nt = 0.5$.

Figure 12. The influence of Nt on $\theta_{(\eta)}$ when $Sc = 0.7, S = 0.5, \Omega = 1, Nb = 0.6, M = 0.3, Pr = 0.6$. 
Figure 13. The impact of $\Omega$ on $\theta_{(n)}$ while $Nt = 0.8, Pr = 0.5, S = 0.5, Nb = 0.6, Sc = 0.7, M = 0.3$.

Figure 14. The effect of $Nb$ on $\theta_{(n)}$ when $M = 0.3, Nt = 0.7, S = 0.5, Pr = 1, Sc = 0.5, \Omega = 1$.

Figure 15. Impact of $Nt$ on $\theta_{(n)}$ for $M = 0.3, Nb = 0.7, S = 0.5, Pr = 1, Sc = 0.5, \Omega = 1$. 
5. Discussion

A three-dimensional nanofluid film flow with heat and mass transfer through a steady rotating inclined surface was analyzed. The impact of $M$, $S$, $\Omega$, $Sc$, $Nt$, $Nb$, $Pr$ and $K$ were explored for radial velocity $k(\eta)$, axial velocity $f(\eta)$, induced flow $s(\eta)$, and drainage flow $g(\eta)$ respectively. Furthermore, the impact of these parameters was also described for the temperature profile $\theta(\eta)$ and concentration profile $\theta'(\eta)$. The impact of the magnetic parameter $M$ over the velocity profiles $f(\eta)$, $k(\eta)$, $g(\eta)$, and $s(\eta)$ are presented in Figures 2–5. The impact of the magnetic parameter on the axial and the radial profiles of the velocity look similar for up to $\eta = 2.0$ as shown in Figures 2 and 4. For greater values of the magnetic parameter these profiles decline sharply and the peak point is at $\eta = 4.0$. Physically, the larger values of the magnetic parameter reduce the rotational parameter, and as a result both the profiles decline. Figures 3 and 5 both show an increasing trend in the drainage and induced flow with the smaller and larger values of the magnetic parameter respectively. For $\eta = 2.3$ all the curves coincide, and after this point the induced velocity jumps in increasing order as shown in Figure 5, while on the other hand smaller values of the magnetic parameter enhance the drainage velocity as presented in Figure 3. The variation in Figures 3 and 5 as a consequence of layer thickness and internal velocity of nanofluid definitely reduce. The direction of this force is perpendicular to both the fields. Also $M$ endorses the ratio of viscous and hydromagnetic body forces, the fluid flow is decreased due to greater values of $M$ which require further hydromagnetic body forces. Lorentz force theory defines that $M$ has a reverse influence on velocity function. Figures 6–9 show the effect of $K$ on $f(\eta)$, $k(\eta)$, $g(\eta)$, and $s(\eta)$. Figure 6 displays the axial velocity which accelerates in response to an increase in the couple stress parameter. This is mainly due to the decrease in friction, which rises from the particle
(i.e., base-fluid particles) additives that create a size-dependent influence in couple stress liquids. The influence of $K$ on $k(\eta)$ is demonstrated in Figure 7. The disk rotation increases $K$, due to which the flow velocity also rises. For $K = 0$ the flow retains Newtonian fluid. Figure 8 displays the influence of $K$ on drainage flow $g(\eta)$. The flow is perceived to rise with rising values of $K$, since the rise in $K$ causes a reduction in the dynamic viscosity of the liquid and, hence a rise in the molecular distance among the liquid particles. Figure 9 displays the influence of $K$ on induced flow $s(\eta)$. Here a rise in $K$ leads to a rise in the stress among all the coupled fluid atoms, which further leads to a reduction in the induced velocity. Figure 10 depicts the Pr impact on the temperature profile $\theta(\eta)$ since Pr has an inverse relation to the thermal diffusivity and direct relation to the momentum diffusivity. Greater values of Pr mean that there is robust momentum diffusivity which is associated with the thermal diffusivity and as a result the thermal diffusivity decreases the temperature profile $\theta(\eta)$. These variations almost look similar up to $\eta = 1.8$. Hence, the greater values of $Pr$ drop the boundary layer of heat. The influence of $Nb$ on $\theta(\eta)$ is demonstrated in Figure 11. It is clear that larger values of $Nb$ enhance the thermal boundary layer after $\eta = 2.7$. Physically, when Brownian motion increases the interaction between the particles enhances and as a result the energy transfers rapidly from one point to another, which as a result increase the thermal boundary layer. The impact of the thermophoresis parameter $Nt$ over $\theta(\eta)$ is given in Figure 12. It can be seen that a rise in $Nt$ leads to augment the liquid temperature. Thermophoresis forces generate temperature gradient that further causes a degenerate flow away from the surface. The influence of $\Omega$ on $\theta(\eta)$ is demonstrated in Figure 13. The larger values of the rotation parameter $\Omega$ enhance the temperature profile $\theta(\eta)$. This is due to the larger values of $\Omega$, the fluid temperature increases. Physically, greater values of $\Omega$ increase the kinetic energy, which in consequence increases $\theta(\eta)$. Influence of $Nb$ on $\theta(\eta)$ is demonstrated in Figure 14. For greater values of $Nb$ the concentration profile $\Phi(\eta)$ increases. This variation is very effective for greater values of $\eta$. In practice, rising values of $Nb$ cause an enhancement of the random motion amongst the nanoparticles, and consequently reduces $\theta(\eta)$ of the liquid. Higher values of $Nb$ decrease the boundary layer thicknesses, which as a result reduces the concentration profile $\Phi(\eta)$. Figure 15 displays the performance of the $Nt$ on $\Phi(\eta)$. Increasing values of $Nt$ push the nanoparticles far away from the hot sheet, and consequently enhance the concentration profile $\Phi(\eta)$. Since $Nt$ varies with the gradient of the temperature of the nanofluids, the kinetic energy increases of the nanofluids due to the increase of $Nt$, which further increases $\Phi(\eta)$. The effect of $S$ on $\Phi(\eta)$ is presented in Figure 16. It is seen that $\Phi(\eta)$ varies directly with $S$. Augmenting $S$ increases the concentration, which as a result raises the kinetic energy of the liquid, which further enhances the speed of fluid film. It is evident from the graph that the rising values of $S$ reduce $\Phi(\eta)$. This impact of $S$ over $\Phi(\eta)$ is due to the stretching sheet, steady flow, and the greater concentration, and hence rising $S$ shows a converse influence. The effect of the Schmidt number $Sc$ on the concentration field is presented in Figure 17. The concentration boundary layer reduces due to the rise in $Sc$. In practice $Sc$ decreases the molecular diffusivity, which reduces the concentration boundary layer. It is observed that the reduction in heat transfer at the sheet leads to a rise in the values of $Sc$

6. Conclusions

The flow past an exponential stretching sheet of couple stress nanofluid with the impacts of MHD, viscous dissipation, and Joule heating was investigated analytically. The influences of zero mass flux and convective heat condition were also considered. HAM was applied for the solution of the non-linear differential equations. The effects of the numerous constraints on velocity field, temperature profile, and concentration are portrayed by the graphs. The important results of the current study are described below as follows:

- Higher values of $S$ provide coolness to the boundary layer, while $Nu$ increases the profile.
- A larger value of $Sc$ reduces the Sherwood number, which as a result enhances the concentration and kinematic viscosity of the fluid.
- The higher values of the Prandtl number reduce the Nusselt number.
- The higher values of the Brownian motion parameter increase the concentration distribution and decline with increasing values of the thermophoresis parameters.
Nomenclature:

- \( u, v, w \): Velocity components in \( x, y, z \) directions (m/s)
- \( x, y, z \): Space coordinates (m)
- \( T \): Temperature of fluid (K)
- \( T_w \): Temperature at the surface (K)
- \( T_\infty \): Ambient temperature (K)
- \( B_0 \): Uniform magnetic field (Tesla)
- \( C \): Concentration of fluid (kg/m\(^3\))
- \( C_w \): Concentration at the surface (kg/m\(^3\))
- \( D_B \): Brownian diffusion coefficient
- \( D_T \): Thermophoresis diffusion coefficient
- \( C_\infty \): Ambient concentration (kg/m\(^3\))
- \( U_w \): Stretching sheet velocity (m/s)
- \( g \): Gravity (m/s\(^2\))
- \( \alpha \): Thermal diffusivity (m\(^2\)/s)
- \( \beta \): Angle of inclination
- \( f \): Dimensionless stream function
- \( N_t \): Thermophoresis parameter
- \( Q \): Heat generation/absorption coefficient (W/m\(^2\)·K)
- \( S_r \): Soret number
- \( \delta \): Electrical conductivity of the fluid (m/s)
- \( K_0 \): Chemical reaction coefficient
- \( C_P \): Specific heat at constant pressure (J/kg K)
- \( N_B \): Brownian motion parameter
- \( P_r \): Prandtl number
- \( C_{fs} \): Skin friction coefficient
- \( N_{fs} \): Local Nusselt number
- \( Re \): Reynolds number
- \( \varrho \): Fluid density (kg/m\(^3\))
- \( q_w \): Surface heat flux
- \( \mu_0 \): Zero shear viscosity (s\(^{-1}\))
- \( \mu_\infty \): Infinite shear viscosity (s\(^{-1}\))
- \( \nu \): Kinematic viscosity (m\(^2\)/s)
- \( \mu \): Dynamic viscosity (N s/m\(^2\))
- \( \Phi, \theta \): Dimensionless concentration and temperature

Subscripts
- \( \infty \): Condition for away from the surface
- \( w \): Condition at the sheet

Superscript
- \( ' \): Derivative with respect to \( \eta \)

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