Matter Matters: Unphysical Properties of the $R_h = ct$ Universe

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ABSTRACT
It is generally agreed that there is matter in the universe, and in this paper, we show that the existence of matter is extremely problematic for the proposed $R_h = ct$ universe. Considering a dark energy component with an equation of state of $w = -\frac{1}{3}$, it is shown that the presence of matter destroys the strict expansion properties that define the evolution of $R_h = ct$ cosmologies, distorting the observational properties that are touted as its success. We further examine whether an evolving dark energy component can save this form of cosmological expansion in the presence of matter by resulting in an expansion consistent with a mean value of $\langle w \rangle = -\frac{1}{3}$, finding that the presence of mass requires unphysical forms of the dark energy component in the early universe. We conclude that matter in the universe significantly limits the fundamental properties of the $R_h = ct$ cosmology, and that novel, and unphysical, evolution of the matter component would be required to save it. Given this, $R_h = ct$ cosmology is not a simpler or more accurate description of the universe than prevailing cosmological models, and its presentation to date possesses significant flaws.

Key words: cosmology; theory

1 INTRODUCTION
While the ΛCDM picture is the favoured cosmological model, others have suggested that coincidences in the universe indicate that this picture is incorrect. Instead, it is claimed, the universe is in fact a simpler place, with a simple, linear evolution of the expansion of the cosmos. Referred to as the $R_h = ct$ universe, it is further claimed that within this picture, key observational features of the cosmos are more accurately explained.

In this paper, we demonstrate that the consideration of the matter content of the universe renders the central claims of the $R_h = ct$ universe as invalid, showing that its claimed benefits over ΛCDM are illusory, or at least very problematic.

The structure of this paper is as follows: Section 2 will briefly present the ideas behind the $R_h = ct$ universe, whereas Section 3 presents the influence of matter on the evolution of the cosmos. In Section 4 we discuss the observational consequences of the presence of matter, while in Section 5 we ask the question of whether the $R_h = ct$ universe can be saved by considering an evolving dark energy component. Our conclusions are presented in Section 6. Throughout we will assume that $c = 1$, and hence refer to the $R_h = ct$ cosmology simply as $R_h = t$.

2 THE $R_h = t$ UNIVERSE
The notion of the $R_h = t$ universe has grown in a series of papers over recent years. The foundation for this cosmological model is an apparent coincidence between the current age of the universe, $t_o$, and the current size of the Hubble Sphere, $R_h = \frac{1}{H_o}$, where $H_o$ is the present-day Hubble Constant, such that

$$t_o \sim \frac{1}{H_o}$$

(1)

This coincidence will not generally hold. In fact, within the ΛCDM cosmology, $R_h$ increased rapidly in the early epochs of the universe, before slowing as the influence of dark energy increases, before asymptoting to a particular value in the distant future; within this cosmology, there is a single moment where the relationship in Equation 1 holds, and we appear to be close to it.

One of the earliest comments on this coincidence was provided by Lima (2007) who noted that this implies that the mean acceleration of the universe must be close to zero, and that we should be surprised, if we are living in a ΛCDM cosmology, to find ourselves at this rare epoch (see van

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1 As an example, recall that in an Einstein-de Sitter universe, the age of the universe is related to the Hubble Constant through $t = \frac{2}{3H}$, so an equality in Equation 1 never occurs.
Oirschot, Kwan, & Lewis (2010). Given this, Lima (2007) further proposed that we can remove this coincidence if the universe had a simple linear expansion, such that Equation 1 becomes an exact equality at all times.

Through a series of papers (Melia 2007, 2009; Bikwa, Melia, & Shevchuk 2012; Melia & Abdelqader 2009; Melia 2012a), this idea was refined further, firstly with the claim that the Hubble Sphere, $R_0$, is in fact an unrecognised, yet fundamental, property of the universe, labelled the “Cosmic Horizon”; this is a surprise, as horizons within the standard cosmological model have been understood for many decades (see Rindler 1956; Harrison 1991; Ellis & Rothman 1993). The initial claim was that $R_0$ represents an infinite redshift surface, limiting our view of the universe, although this was shown to be incorrect by van Oirschot, Kwan, & Lewis (2010). The debate of the Cosmic Horizon has continued in the literature, but its claimed properties as a fundamental horizon have been shown to be incorrect (Lewis & van Oirschot 2012; Lewis 2013). More recently, Melia & Shevchuk (2012) argue that the coincidence given by Equation 1 is based upon a deeper significance relating to the “Weyl Postulate” and the “corollary to Birkhoff’s theorem” which demands that Equation 1 is a strict equality that must hold at all times (although these arguments were developed through previous papers). If that is the case, then the universe has a linear expansion and the Hubble Constant, $H = t^{-1}$. This is very unlike the decelerating and then accelerating expansion experienced in the ΛCDM cosmology, and hence the constituent energy densities in a $R_0 = t$ universe must be significantly different to the matter and dark energy underlying ΛCDM.

The cosmological implications of an equality of Equation 1 were further considered, especially with regards to its impact how we observe the universe (e.g. Melia 2012b,c). As well as claiming that it provides a better fit to cosmological observations, one of the key claims about the $R_0 = t$ universe is an apparent underlying simplicity when compared to ΛCDM (Melia 2012d). However, some of the claims of efficacy of the $R_0 = t$ in explaining cosmological observations have been called into question (Bilicki & Seikel 2012).

This question of the make-up of the $R_0 = t$ universe has been examined in several papers, most recently by Melia & Shevchuk (2012), who concluded that the density energy driving the expansion must possess a time-averaged equation of state of $w = -\frac{1}{3}$, although an equality in Equation 1 is required, then the equation of state must be $w = -\frac{1}{3}$ at all times.

For a universe with a critical density of a single fluid, an equation of state of $w = -\frac{1}{4}$ represents a dividing line between energy densities that decelerate cosmic expansion ($w > -\frac{1}{4}$), and those that result in universal acceleration ($w < -\frac{1}{4}$), and hence results in a uniform cosmic expansion.

The real universe, however, possesses a mix of energy densities; the ΛCDM universe, currently possess two dominant components, matter (with $w = 0$) and a cosmological constant ($w = -1$). Irrespective of the fact that we may not fully understand the properties of the dark energy component, it is pretty uncontroversial that there is matter in the universe, either the substantial quantity of dark matter inferred from dynamical arguments, or the more modest baryonic component we can directly observe. Hence, how is the requirement of $w = -\frac{1}{3}$ to be interpreted in the presence of matter? This is the subject of the remainder of this paper.

In the following, we will adopt the tenet of the $R_0 = t$ universe, namely that it is simple when compared to other cosmologies, such as ΛCDM (Melia 2012d). We will assume that the expansion of the universe is governed by Einstein’s general relativity through the Friedmann equations; we note that there is potential for the universe experiencing $R_0 = t$ evolution if gravity acts only locally, without a long-range interaction, although this would not be consistent with the standard cosmological model, and is, therefore, beyond the scope of this paper. As well as ΛCDM, we also consider universes that contain a matter component and a dark energy component with an equation of state $w = -\frac{1}{3}$; at late times, when the density of matter has dropped to negligible amounts, these cosmologies will asymptote to $R_0 = t$ universes. Later, we will relax this assumption and allow the equation of state of dark energy to evolve over cosmic history, such that the accelerated expansion of the dark energy counterbalances the deceleration due to the presence of matter, and the result expansion is that demanded by the $R_0 = t$ cosmology. We will examine the consequences of this on the resultant required form of the dark energy equation of state.

3 THE INFLUENCE OF MATTER

To understand the influence of matter on the expansion of the universe, we start with the Friedmann-Robertson-Walker (FRW) invariant interval for a spatially flat cosmological space-time, given by:

$$ds^2 = -dt^2 + a(t)^2 (dr^2 + r^2d\Omega^2)$$  \hspace{1cm} (2)$$

where $t$ is the cosmic time, $r$ is the radial comoving coordinate and $d\Omega$ accounts for angular terms. The normalised
scale-factor, \( a(t) \) evolves with cosmic time and embodies the evolving cosmos.

In the following, we will consider a universe containing two energy density components, one matter, with an equation of state of zero, and a dark energy component with an equation of state of \( w \). The normalised present-day densities of these components will be given \( \Omega_m \) and \( \Omega_{de} \) respectively (Linder 1998); in the following we will neglect the influence of the photon energy density which dominates in the earliest epochs of the universe, but as this results in a larger deceleration during the time it dominates it only exacerbates the problems outlined in this paper.

The Friedmann equations tell us that the scale-factor in a spatially-flat universe will evolve according to

\[
H \equiv \frac{\dot{a}}{a} = H_o \sqrt{\Omega_m a^{-3} + \Omega_{de} a^{-3(1+w)}}
\]  

where \( H \) is the Hubble constant, with a present day value of \( H_o \). It is straightforward to integrate Equation 3 and determine the evolution of \( a \) and then calculate \( R_h \) as a function of cosmic time, and in Figure 1 we present this for several fiducial cosmologies. Note that for this plot, zero on the time axis corresponds to the present day, and we integrate backwards to the Big Bang in each model.

The blue curve presents the pure \( R_h = t \) universe, with no matter, so \( \Omega_m = 0 \), and consisting of only a dark energy component, with \( \Omega_{de} = 1 \), with an equation of state of \( w = -\frac{1}{3} \). As expected, this possesses a linear evolution from zero, at a time of \( H_o t = -1 \) to a value of unity today. For comparison, we also present the evolution of \( R_h \) in \( \Lambda \)CDM (in cyan); clearly this does not possess such a linear evolution.

The remaining two lines replace some of the dark energy in the \( R_h = t \) cosmology with matter. In the first case (red line) we adopt the currently favoured present day matter density of \( \Omega_m = 0.27 \), such that \( \Omega_{de} = 0.73 \) but assuming an equation of state of \( w = -\frac{1}{3} \), rather than \( w = -1 \) of \( \Lambda \)CDM. While this trend is roughly linear, clearly it is not the required \( R_h = t \) evolution of the Cosmic Horizon that is apparently demanded by the “Weyl Postulate” and the corollary of “Birkhoff’s theorem” (Melia & Shevchuk 2012).

With the final green line in Figure 1, we banish dark matter and assume that the only matter present is the baryonic component we can observe, adopting an \( \Omega_m = 0.05 \). Again, the evolution of the Cosmic Horizon is roughly linear, and, while it approximately coincides with \( R_h = t \) at the present time, it clearly deviates from this relation in the earlier epochs of cosmic history.

One conclusion one can immediately make from an examination of Figure 1 is that if the dark energy component of the universe possesses an equation of state of \( w = -\frac{1}{3} \) then the presence of any quantity of matter will result in a deviation from the strictly required evolution of the \( R_h = t \) universe. While in Section 5 we will consider if an evolving equation of state of the dark energy component can result in a \( R_h = t \) expansion history, we will now examine the observational consequence of matter in dark energy universes with \( w = -\frac{1}{3} \).

4 OBSERVATIONAL CONSEQUENCES

This paper will consider only two of the recent claims of the observational superiority of the \( R_h = t \) universe over other cosmological models. This will be the relation between lookback time and redshift, and the lack of the requirement on an inflationary period to explain the observed causal properties of the Cosmic Microwave Background (CMB).

4.1 Look-back time verses redshift

A recent claim of the success of the \( R_h = t \) is that it solves the problem of the existence of high redshift quasars, given the limited time for the growth of supermassive back holes since the Big Bang (Melia 2013). Here we present an identical analysis for the cosmologies considered in Section 3.

The two key ingredients for this analysis are the age of the universe at the epoch of quasar activity, given by the look-back time verses redshift given in Figure 2 \(^2\), while the other is the time-scales for the formation of supermassive black holes. As with Melia (2013), rather than black holes being primordial, we will assume that supermassive black holes grow from seed black holes of masses of either \( 5M_\odot \) or \( 20M_\odot \), resulting from supernovae in the first generation of stars. Following Melia (2013) we adopt the relation between the seed mass, \( M_\bullet \), and the time, \( \tau_{M_\bullet} \), for the growth to a mass of \( M \) to be

\[
M = M_\bullet \exp \left( \frac{\tau_{M_\bullet}}{45 \text{ Myrs}} \right)
\]

Hence, given the time that we see a quasar, and an estimate of its black hole mass, it is straight forward to calculate the time at which appropriate seed mass black holes were required.

\(^2\) It appears several numerical errors have crept into the analysis of Melia (2013). For example, for \( H_o = 72 \text{ km/s/Mpc} \), the age of the universe in the \( R_h = t \) cosmology is 13.58 GYrs, and therefore the age of the universe for \( z = 7.085 \) (Mortlock et al. 2011) should be 13.58 GYrs \((1+7.085)\) = 1680 Myrs, not the 1634 Myrs listed in the paper.
Figure 3. The initial seed period for the formation of super-massive black holes for the universes considered in this paper, high-lighting the period of reionization ($z \sim 5 - 16$). This presents a subsample of quasars from Melia (2013), but uses the same formation time-scales for 5 M$_\odot$ and 20 M$_\odot$ seed black holes. The Big Bang is marked as a solid black vertical line, whereas the end of the Dark Ages is given by a dashed line. The red horizontal bars are the formation epochs for seed black holes as described above. The left-hand lower panel, with $\Omega_m = 0.27$ and $w = -1$, corresponds to a $\Lambda$CDM universe, whereas the right-hand panel, with $\Omega_m = 0.00$ and $w = -\frac{1}{3}$, corresponds to the $R_h = t$ universe. The upper panels present cosmologies with a dark energy component with $w = -\frac{1}{3}$, but with $\Omega_m = 0.27$ (left-hand panel) and $\Omega_m = 0.05$ (right-hand panel).

Directly taking a subsample of the tabulated 3 values from Melia (2013), the lower left-hand panel of Figure 3 illustrates the problem in $\Lambda$CDM; the red bar corresponds to the epoch in which seed black holes of masses between 20M$_\odot$ and 5M$_\odot$ need to be formed to result in the quasars we see (this is equivalent to Figure 1 in Melia (2013)). Clearly, in a $\Lambda$CDM universe, seed black holes need to be formed in the “Dark Ages”, before the supernovae required to create them. The situation is even more extreme for the highest redshift quasar, ULAS J1120+0641 at $z = 7.085$ (Mortlock et al. 2011), which requires seed black holes to be formed before the Big Bang. Various solutions, such as super-Eddington accretion (e.g. Volonteri & Rees 2006), have been proposed to allow supermassive black holes to grow in the limited time available.

The lower right-hand panel in Figure 3 presents the same situation in the $R_h = t$ universe. With the different relationship between cosmic time and redshift (Figure 2), the seed formation time for quasars now occurs later in the universe, most occurring well after dark ages, during reionization when massive stars are readily available to produce black hole seeds.

While this may appear to be a success for the $R_h = t$ universe, it is clear that from Figure 2 that the look-back time verses redshift relations are significantly different when matter is added to a $w = -\frac{1}{3}$ universe. In the upper right-hand panel of Figure 3 we present the case when the present matter density is $\Omega_m = 0.05$, and as can be seen, this moves epoch for the production of black hole seed back to just

3 While it is not clearly described in Melia (2013), the Seed/Myr columns of their Table 1 corresponds to the ages that you would require appropriate mass seed black holes in the $R_h = t$ universe. The values for $\Lambda$CDM are not given.
after the dark ages which, while better than the $\Lambda$CDM universe, reduces some of the claimed advances of a pure $R_h = t$ universe.

The situation is exacerbated when considering a more realistic present day matter density of $\Omega_m = 0.27$ (upper left-hand panel of Figure 3), as the conversion time for the black hole seeds are now pushed back well into the dark ages, with the requirement that some seed black holes are required at the epoch of, or even before, the Big Bang. Hence, we can conclude that presence of matter significantly curtails the claimed advantage of the $R_h = t$ universe in providing ample time for the production of supermassive black holes in the early universe, and that the question of how these grew in the limited time available is still to be answered.

### 4.2 The need for inflation?

Another grand claim of the $R_h = t$ cosmology is that does away with the need for inflation in explaining the causal contact of differing patches during the earliest stages of the universe (Melia 2012c,e,f). This is more clearly understood in examining the expansion of the universe in a conformal representation (see Harrison 1991, for a full explanation). Essentially, for this representation the abcissa is the comoving radial coordinate, while the ordinate is the conformal time, given by

$$\tau = \int_{t_0}^{t} \frac{dt}{a(t)}$$

where $t_0$ is the present epoch and $t_e$ is a different epoch in the universe. In this coordinate system, light-rays have the same 45° paths as in flat Minkowski space-time, making the understanding of causal connexions straightforward.

The first key property to understand with regards to the need for an inflationary epoch in the universe is whether the cosmic evolution results in a finite or infinite ‘conformal horizon’ when integrating back to the Big Bang, $t_e = 0$. If we consider a spatially flat universe consisting of a single fluid with an equation of state, $w$, then it is simple to show that

$$a(t) \propto t^{\frac{2}{3(w+1)}}$$

and the integral in Equation 5 in the limit of $t_e \to 0$ becomes

$$\tau = \kappa \int_{t_0}^{t_e} \left(1 - \frac{2}{3(w+1)}\right)^0 dt_0$$

where $\kappa$ is a constant and $t_0$ is the present age of the universe. The zero limit of the integral, corresponding to the Big Bang at $t_e = 0$, means that this integral converges iff $w > -\frac{1}{3}$. When considering an observer in such a universe, this convergence of the integral implies that any past light cone can be extended backwards a finite conformal time before they reach the Big Bang. However, in universes where $w \leq -\frac{1}{3}$, the conformal integral does not converge, and past light cones can be extended backwards an arbitrarily large amount of conformal time, encountering the Big Bang at $\tau = -\infty$. Such universes do not possess particle horizons, with immediate consequences for causality (Harrison 1991).\footnote{The situation is reversed when we consider future conformal horizons, with Equation 5 converging for $t_e \to \infty$ for $w < -\frac{1}{3}$, and these universes possess an “event horizon”. Hence, the $R_h = t$ universe possesses neither a particle or event horizon. However, a $\Lambda$CDM universe possess both, being matter (and radiation) dominated at early epochs, and by dark energy with $w = -1$ in the future (see panel 3 of Figure 1 in Davis & Lineweaver (2004) for a conformal representation of this cosmology).}

In examining the importance of the existence of a finite conformal horizon, it is useful to look at the $\Lambda$CDM cosmology, as can be seen in the lower left-hand panel of Figure 4 which presents the conformal structure of a universe with a present day matter density of $\Omega_m = 0.27$ and an equation of state of dark energy given by $w = -1$. While dark energy is the dominant component today, in the past matter dominated, and this universe possesses a finite conformal horizon at the Big Bang, denoted by the blue line. The red line corresponds to the epoch of recombination, the source of the Cosmic Microwave Background at a redshift of $z \sim 1100$ with respect to a current observer. The large black triangle corresponds to the past light cone of an observer today, stretching back to the Big Bang, whereas the smaller black triangles at the are the past light cones for emitters at recombination. These smaller light cones do not overlap, and hence these two points on the CMB were never in causal contact, and have no reason (other than fine tuning) to be of the same temperature; this is simply a restatement of the well known Horizon Problem.

How does this picture change if we consider the $R_h = t$ universe? As mentioned previously, a universe containing a single fluid at critical density possess a past conformal horizon if $w > -\frac{1}{3}$, and hence, if the universe were solely a fluid with $w = -\frac{1}{3}$ then all past light cones can become infinite in extent. Recombination, however, occurs at a finite conformal time in the past, and hence all light cones from this epoch can be extended backward by an arbitrary amount and all overlap; in such a model, all points at recombination were in causal contact and hence an inflationary epoch is not required; this is represented in the lower right-hand panel of Figure 4.

However, the situation is different with the addition of matter. In the upper left-hand panel of Figure 4, the matter density remains the same as in the $\Lambda$CDM universe, but now the equation of state of dark energy is given by $w = -\frac{1}{3}$. The overall structure in this cosmology is very similar to that seen in the lower right-hand panel of Figure 4, with the existence of a finite conformal horizon in the Big Bang, due to matter becoming dominant in the early universe, and two relatively small light cones connecting this with the epoch of recombination. Again, these past light cones do not overlap and hence the points on the surface of recombination were not in causal contact, and so a mechanism similar to inflation is required to solve the Horizon Problem.

The situation is not significantly improved by reducing the present day matter density further to $\Omega_m = 0.05$, as presented in the upper right-hand panel of Figure 4. Again, the structure of the conformal representation of the universe is relatively unchanged, with very similar features observed previously, and again, the past light cones from recombination back to the Big Bang do not overlap, revealing no previous causal contact and hence still requiring a solution to the Horizon Problem.
Figure 4. The history of the universes considered in this paper in conformal coordinates. The abscissa presents the comoving distance, while the left-hand ordinate presents the conformal time, integrated backwards from the present time ($\tau = 0$). The large black triangle represent our past light cone, with us at the apex. The blue horizontal line represents the Big Bang (a finite conformal horizon; see Equation 5), whereas the red line represents the epoch of recombination. The dotted lines represent epochs corresponding to $(1 + z) = 2, 10, 100 & 1000$ (from top to bottom). As in Figure 3, the left-hand lower panel, with $\Omega_m = 0.27$ and $w = -1$, corresponds to a $\Lambda$CDM universe, whereas the right-hand panel, with $\Omega_m = 0.00$ and $w = -\frac{1}{3}$ corresponds to the $R_h = t$ universe. The black triangles at the base represent the past light cones for the surface of recombination; for three of the universes, these do not overlap before they hit conformal horizon, the Big Bang, these points at recombination were never in causal contact, but for the $R_h = t$ universe, where the Big Bang occurs at a conformal time of $\tau = -\infty$, these light cones overlap and all points were in causal contact before recombination. Note the difference in axes scales for the panel presenting the $R_h = t$ universe.

The universal behaviour for the universes with any matter in Figure 4 is easy to understand as when we look at earlier and earlier epochs in the universe, matter becomes more and more dominant, significantly more than the influence of the dark energy component. In all of these cases, this dominant dark energy component results in a finite conformal horizon when integrating Equation 5 back in time, and if there is not sufficient conformal time between the Big Bang and recombination, past light cones will not have the opportunity to overlap; this is precisely what inflation does (see Figure 2 in Harrison 1991).

5 EVOLVING DARK ENERGY?

Can the $R_h = t$ universe be saved if we allow the equation of state of the dark energy component to differ from $w = -\frac{1}{3}$? Clearly, $w$ cannot be a constant over cosmic history, because if $w < 0$, then the universe will always become matter dominated in its earliest epochs, and if $w > 0$, the resulting dominant energy component in the early universe will result in more significant deceleration and deviation from $R_h = t$. Hence, in the following, we will search for a evolving $w$ such that the required expansion is achieved.

Remembering that for $R_h = t$, then $\dot{a} = H_o a$ and $a = H_o t$ (which is found for a spatially flat universe with a single dark energy component with $w = -\frac{1}{3}$), then
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6 CONCLUSIONS

In this paper, we have examined the properties of the proposed $R_h = ct$ universe, considering the evolution for the Cosmic Horizon in the presence of matter. It is found that for any density of matter, the resulting evolution deviated from the strictly required form demanded by the $R_h = ct$ universe. This is imply a manifestation of universes becoming matter dominated in their earliest epochs.

When considering the observational consequences of the presence of matter in a $R_h = ct$ universe, it is clear that this deviation significantly influence the key properties touted as being evidence for the efficacy of this cosmology, showing claims about the simplicity of the redshift-lookback time and the removal of the need for inflation are, in fact, incorrect.

Finally, we considered whether allowing an evolving equation of state for the dark energy component can save the $R_h = t$ universe, finding that the presence of any matter results in the need for of an unphysical equation of state in the early universe. While we could imagine some contrived models, where the equation of state of normal matter is forced to evolve, we conclude that the presence of matter in the universe is a blow to the proposed $R_h = t$ cosmology.

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