Optimal Design of Electrical Machines Assisted by Hybrid Surrogate Model Based Algorithm

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Abstract—In this paper, for design of large-scale electromagnetic problems, a novel robust global optimization algorithm based on surrogate models is presented. The proposed algorithm can automatically select a proper meta-model technique among multiple alternatives. In this paper, three representative meta-modeling techniques including ordinary Kriging, universal Kriging, and response surface method with multi-quadratic radial basis functions are applied. In each optimization iteration, the above three models are used for parallel calculation. The proposed hybrid surrogate model optimization algorithm synthesizes advantages of these different meta-models. Without verification of a specific meta-model, a suitable one for the engineering problem to be analyzed is automatically selected. Therefore, the proposed algorithm intends to make a better trade-off between numerical efficiency and searching accuracy for solving engineering problems, which are characterized by stronger non-linearity, higher complexity, non-convex feasible region, and expensive performance analysis.

Index Terms—Electromagnetic problem, global optimization, hybrid surrogate model.

I. INTRODUCTION

The design of electromagnetic problems normally involves expensive performance analysis by the finite element method. Especially when intelligent optimal algorithm is adopted, the huge computational cost makes it impractical and impossible to find global optimum [1],[2]. For guaranteeing global optimum, the sub-domain strategy has been applied to the design of electrical machines [3],[4]. Normally, even in the subdomain, the expensive performance analysis is a big burden for optimization of practical engineering problems. Nowadays, the design optimization against uncertainties is attracted more attentions. The reliability-based optimal design and robust optimal design are two kinds of methods to deal with uncertainties [5],[6]. For each optimization iteration, these methods need many times of performance analysis to evaluate reliability and robustness, which definitely increase the computational cost of optimum searching.

Due to the development of meta-modeling technique such as Kriging and response surface method, the optimal design of electromagnetic devices has obtained tremendous progress [7]-[9]. However, the different meta-models tend to perform quite differences for different design problems due to their own underlying characteristics. For example, some models are available for low-dimension linear problems, while other models may show better prediction for nonlinear problems. Until now, in the field of electrical engineering, for a specific practical problem, there is no generalized criterion to select a proper meta-model. The engineers usually try different strategies to guarantee a better design such as increasing sampling points, reducing design space, increasing searching iterations, dimension reduction, and attempting different meta-models. All the current strategies cannot improve optimization efficiency effectively.

Nowadays, many engineers are trying to solve the bottleneck of optimization problem in engineering field. Some researchers focus on parallel strategy [10] and distributed parallel optimizer such as particle swarm optimization [11]. However, in these literatures, only single surrogate model is applied. Recently, multiple surrogate assisted decomposition based optimizer is proposed for expensive multi-objective optimization [12], while the method is only tested on a wide range of unconstrained standard problems. Some literatures combine several surrogate models into a new single model by allocating different weighting coefficients to each surrogate model [13],[14]. However, it is difficult to decide proper coefficients. Through application of pumping optimization of coastal aquifers, the performances of multiple and single surrogate models are compared in Ref. [15]. Furthermore, in the field of mechanical engineering, attempts of using multiple surrogates in meta-modeling have been made [16],[17]. The competitive performance of multiple surrogate-assisted optimizations is proved by vehicle design problem. It is potential for real-life engineering problems involving computationally expensive evaluations [18]. However, until now there is hardly any related research in electrical engineering.
In order to extend stable and robust optimizer into various engineering problems, it is urgent to develop an efficient and widely used optimization algorithm based on surrogate models. Based on the existing meta-modeling techniques, the ordinary from foregoing researches, it is known that Kriging is suitable for simple low-dimension problems; the universal Kriging shows better performance for more complex low-dimension problems [19]; and the multi-quadratic radial basis function response surface model is fit for high-dimension problems [20].

With the help of these three surrogate models, this paper presents a hybrid meta-model robust optimization algorithm (HMRO). The particle swarm optimization is selected as the optimizer for optimum searching. By applying the HMRO algorithm to several benchmark test problems, the performance is investigated. Finally, the cogging torque of one permanent magnet machine is optimized by the proposed algorithm.

II. CONSTRUCTION OF HYBRID SURROGATE MODEL

A. Fundamental of Hybrid Meta-model (HMM)

The basic idea of the proposed HMRO algorithm is to call multiple surrogate models with different characteristics simultaneously in optimization searching process. The surrogate models are constructed by different techniques at the same time, which avoids the analyzed problem not being completely fitted due to limitation of construction technology. In addition, it can guarantee that more useful information of the analyzed problem is obtained. As long as there is one surrogate model working, the proposed HMRO can find optimal solution.

Firstly of all, the proper number of surrogate models should be decided. Since two surrogate models have certain limitations in quantity, if one surrogate model fails to find optimal solution, only the other will limit the search efficiency. However, four or more surrogate models working simultaneously will definitely increase calculation cost and reduce optimization efficiency. Therefore, in the proposed HMM, number of surrogate models is decided as three. Based on the literatures, the ordinary Kriging (OK), the universal Kriging (UK) and the multi-quadratic radial basis function response surface (MQ-RBF) model are selected.

During optimization process, response values can be predicted by the constructed three surrogate models. Among them the multiple optimal values can be selected. Therefore, in the current iteration, design with better response values can be selected as new sampling points and added to the next iteration. In this way, the optimal solution is updated step by step, so verification analysis of constructed surrogate model is not required. To establish hybrid meta-model more accurately, the optimal Latin hypercube design is applied to obtain initial samples with uniform distribution and better randomness.

B. Procedure of Constructing HMMs

The mathematical model of optimization problem considered in this paper is formulated as follows:

\[
\min \quad f(x) \\
\text{s.t.} \quad g_i(x) \leq 0, i = 1, 2, \ldots, m \quad (1)
\]

\[
x_l \leq x \leq x_u
\]

where \( x \) is design variable vector with lower and upper limits, \( x_l \) and \( x_u \) respectively; \( f(x) \) and \( g(x) \) are objective and constraint functions, respectively. The flowchart of the proposed HMRO for problem (1) is summarized as follows.

Step 1: Generate \( m \) initial sampling points by the optimal Latin hypercube design in the global design space. Obtain the true objective/constraint data by the finite element method or analytic functions.

Step 2: Construct three initial surrogate models by \( m \) samples: the OK model is represented by \( \hat{e}(x) \), the UK model is represented by \( \hat{g}(x) \), and the MQ-RBF model is represented by \( \hat{h}(x) \), Fig. 1 (a) is a schematic diagram of model construction.

Step 3: Generate \( N \) testing points by uniform design. \( N \) should be a large number to guarantee higher accuracy. In this paper, \( N \) is set as \( 10^4 \).

Step 4: The three surrogate models constructed in Step 2 are used to predict response values at \( N \) testing points. For (1), after objective function sorting in descending order, \( n \) test points with smaller function values are selected in each of the three models to construct three groups marked as E: \( \hat{e}(x) \), G: \( \hat{g}(x) \) and H: \( \hat{h}(x) \). In this paper, \( n \) is set as 100. So, the \( 3n \) testing points with superior selection are treated as candidate sampling points used for next iteration.

Step 5: The \( 3n \) candidate sampling points are divided into 7 groups as shown in Fig. 1(b). The groups are marked as A1, A2, …, A7, and the number of corresponding points locating in each group are marked as n1, n2, …, n7, respectively. The number of points belongs to each group can be described by mathematical set operation as:

\[
A1 = E \cap G \cap H; \\
A2 = E \cap G \cap A1; \\
A3 = E \cap H \cap A1; \\
A4 = G \cap H \cap A1; \\
A5 = E - (A1UA2UA3); \\
A6 = G - (A1UA2UA4); \\
A7 = H - (A1UA3UA4);
\]

where the symbol \( \cap \) represents intersection, and the symbol \( \cup \) represents union. For example, in set A1, all samples belong to three groups E, G, and H. The purpose of the above grouping is to find the point which is most likely to represent the optimal value. Therefore, the points with all smaller values in the three selected models are considered to be the most important points.
In other words, the importance of each testing point is directly proportional to how many models the points can be found.

**Step 6:** From the perspective of surrogate model construction, choosing more promising candidates to participate in the surrogate model reconstruction will help to find the global optimal solution of (1). The number of selected points \(k_i\) in each subspace is decided as follows:

\[
k_i = \text{int}\{\omega_i \times 7\}, \quad i = 1, \ldots, 7
\]

\(i\) denotes the group number, \(\omega_i\) is the weighting factor. In the ideal case, when after rounding off, each of the 7 groups will select one testing point as new sample to reconstruct the surrogate model. Totally seven sample points are selected. In order to avoid unbalance between initial samples and reconstructed samples, the number of initial sample points in **Step 1** is set eight.

**Step 7:** The real values of the selected seven sampling points are obtained by finite element analysis. Combining the selected seven sampling points with previous samples, three surrogate models are reconstructed.

**Step 8:** Repeat **Step 3 - Step 7** until finding the optimal solution with a required convergence.

In the HMM, the number of sampling points in each group and the importance of this group should be considered [16], the weighting coefficient is calculated as:

\[
\omega_i = n_i \times p_m / K, \quad i = 1, \ldots, 7, \quad m = 1, 2, 3
\]

where \(n_i\) is the number of candidate points selected in the \(i\)th group, \(p_m\) is the number of sharing surrogate models for candidate points in the \(i\)th group, and \(K\) is the total number of sample points to be selected. In this paper, \(K\) is equal to 3n.

Fig. 2 is an overall flowchart of the proposed HMRO method. All the time-consuming parts marked in dash rectangles can be carried out independently. Even through the proposed method considers three surrogate models, it owns higher efficiency and better stability than its single-surrogate counterparts. The algorithm is very flexible. Any optimizer such as genetic algorithm and differential evolution algorithm can be combined with the proposed hybrid surrogate model. In this paper, the particle swarm optimization is applied as one example for optimum searching.

### III. ALGORITHM VALIDATION BY BENCHMARK PROBLEMS

#### A. Mathematical Examples

Three benchmark mathematical functions are selected as:

\[
f_1(x) = 3(1 - x_1)^2 \exp[-x_1^2 - (x_2 + 1)^2] - 10(0.2x_1 - x_2^3 - x_3^5) \exp(-x_1^2 - x_2^2) \tag{4}
\]

\[
f_2(x) = -20 \exp\left(-0.2 \sqrt{\frac{1}{m} \sum_{i=1}^{m} x_i^2}\right) - \exp\left(\frac{1}{m} \sum_{i=1}^{m} \cos(2\pi x_i)\right) + 20 + \exp(1), \quad -32.768 \leq x_i \leq 32.768 \tag{5}
\]

\[
f_3(x) = 0.5 \sum_{i=1}^{5} (x_i^4 - 16x_i^2 + 5x_i), \quad -5 \leq x_i \leq 5 \tag{6}
\]

Function \(f_1(x)\) has several local maximums and local minimums [20]. The global minimum exists at \((0.2282, -1.6256)\), with object value of -195.8308. For \(f_2(x)\) (Ackley Function), there are multiple local maximums and minimums around global minimum value 0 at \((0, 0, 0)\). Function \(f_3(x)\) (Styblinski-Tang Function) is a typical valley function with several local minimums. The global minimum exists at \((-2.9035, ..., -2.9035)\), with object value of -195.8308.

The optimization results of the HMM and other single meta-models are compared in Table I. Without loss of generality, the results are obtained after 10 independent runs, the worst and best represent maximum and minimum object values respectively. \(Nf\) is the average number of samples.
among 10 runs, and NIter is the average iterations. For single or hybrid surrogate model, the update criterion of sample points is same as described in Fig. 2.

It can be seen, for these three different test functions, the proposed HMM can find better solutions than any other three single surrogate models. Furthermore, the HMRO method can converges faster than its counterparts.

B. Superconducting Magnetic Energy Storage System

The superconducting magnetic energy storage system problem is the 22nd benchmark problem for testing of electromagnetic analysis method (TEAM 22). There are two versions defined by three design variables and eight design variables as shown in Fig.3. The design target is to obtain an optimal design, which can give minimum stray field evaluated along line a and line b, and stored energy of 180 MJ. The design variables are geometric parameters of inside and outside superconducting coils as defined in Fig.3 $x=[D_1, D_2, H_1, H_2, R_1, R_2, J_1, J_2]^T$. The objective functions and constraint functions are defined as follows:

Objective: $f(x) = \frac{B_{\text{stray}}^2}{B_{\text{norm}}^2} + \frac{|E - E_0|}{E_0}$  \hspace{1cm} (7a)

Constraint:

$g_1(x) = 6.4 |B_{m,1}| + |J_1| - 54 \leq 0$  \hspace{1cm} (7b)

$g_2(x) = 6.4 |B_{m,2}| + |J_2| - 54 \leq 0$  \hspace{1cm} (7b)

$g_3(x) = (R_1 + D_1/2) - (R_2 - D_2/2) \leq 0$  \hspace{1cm} (7b)

Stray field: $B_{\text{stray}}^2 = \sum_{i=1}^{22} |B_{\text{stray},i}|^2 / 22$  \hspace{1cm} (7c)

where reference values of stray field and energy are $B_{\text{norm}}=3$ mT for 3D, 200 μT for 8D, and $E_0=180$ MJ, respectively. $B_{m,i}$ is the maximum magnetic flux density in the $i$th coil. $B_{\text{stray},i}$ is the magnetic flux density of the $i$th test point.

To avoid occasionality of optimal searching, each optimization also runs 10 times. Results are compared in Table II. Both in the case of 3D and 8D, taking results given in [21] as a reference, the best result of the objective function is obtained by the HMM model. Its iteration and required number of sampling points are less than those required by other three models.

C. Die Press Model

The model of die press with electromagnet for orientation of magnetic power is shown in Fig.4. It is mainly applied for producing anisotropic permanent magnet [22]. In this problem, the target is to optimize the shape of the die molds with $x=[R_1, L_2, L_3, L_4]^T$. The optimization model is formulated as:

$$
\min \ f(x) = \sum_{i=1}^{N} (B_{\text{stray}} - B_{\text{ref}})^2 + (B_{\text{stray}} - B_{\text{stray}})^2
$$

$$
B_{\text{stray}} = 0.35 \cos \theta_i
$$

$$
B_{\text{stray}} = 0.35 \sin \theta_i
$$

where $N$ is number of specified test point (N=10) on curve e-f in Fig.4, subscripts p and o mean calculated and specified values respectively. Other descriptions can be found in [22].

The proposed HMRO is compared with its counterparts in Table III, where $\theta_{\text{max}}$ and $\theta_{\text{max}}$ are maximum errors of
amplitude and angle of magnetic flux density, respectively [22]. The solution obtained by the PSO with 50 particles and 300 maximum iterations is taken as the reference optimal solution. It can be seen that the multi-surrogate model (HMRO) owns better searching ability for optimum than the single surrogate model. It can find an optimal design much closer to the reference. Furthermore, the efficiency of the HMRO is not worse than its counterparts.

Through different electromagnetic applications in sections B and C, the necessity of the HMM model is verified.

### IV. OPTIMIZATION OF ELECTRICAL MACHINE

In order to verify the engineering utility of the proposed optimization algorithm based on the HMM, one brushless DC motor is selected as an application example. The simulation model by the FEM and basic specifications are shown in Fig. 5 and Table IV, respectively. In this problem, the target is to optimize the cogging torque while the back-EMF should not be worse than the original design. Based on above requirements and design theory of electrical machine, the six parameters of stator slot defined in Fig. 5(b), the width of permanent magnet (h), and the pole-arc coefficient \( \alpha_p \) are selected as design variables \( x=[hs0, hs1, hs2, hs0, hs1, hs2, \alpha_p, h]^T \). The pole-arc coefficient is calculated as,

\[
\alpha_r = \frac{b_p}{\tau} = \frac{\theta \pi r/180^\circ}{2 \pi r/(2p)} = \frac{\theta p}{180^\circ}
\]

where \( b_p \) is the length of the pole arc, \( \tau \) is the pole distance, \( r \) is the outer radius of the rotor, \( \theta \) is the center angle of pole arc, and \( p \) is the number of pole pairs.

The mathematic optimization model is formulated as:

\[
\begin{align*}
\min & \quad f(x) = T_{cog}/T_{initial} \\
\text{s.t.} & \quad x_L \leq x \leq x_U
\end{align*}
\]

where \( T_{initial} \) is initial value of cogging torque \( T_{cog} \); \( x_L \) and \( x_U \) are lower and upper limits of each variables, respectively. The design space is listed in Table V.

![BH curve of iron core.](image)

For performance analysis of the electrical machine, the BH curve of the stator core as shown in Fig.6 is applied. After the searching program converges, the total number of sampling points simulated by the finite element method is 505. The optimal results are listed in Table V.

The stator slot shapes and cogging torques before and after optimizations are compared in Fig.7 and Fig.8, respectively. Compared with the initial design the optimized cogging torque is obviously decreased. From the specific value, the cogging torque decreases 36.97% (from 567.7 mNm to 357.8 mNm).

In problem (10), the target is to minimize the cogging torque. However, other performances of the electrical machine should not become worse. For further validation, the no-load back EMF and air-gap magnetic flux density are compared in Fig.9. From Fig.9 (a) and (b), the waveform and maximum values of no-load back EMF after optimization are almost same with the initial design.
Furthermore, for the optimal design, the fundamental amplitude of no-load air-gap magnetic flux density (0.764 T) is slightly bigger than the original design of 0.758 T. While other harmonic components do not become worse. The total harmonic distortion is reduced from 26.52% to 25.92%.

From above discussion, it can be concluded that the optimal design of electrical machine can satisfy the design requirements and keep other performances at the acceptable level. In a word, the utility and efficiency of the proposed optimization based on HMM are verified. In addition, it also can be widely used in optimal design of practical electromagnetic devices.

V. CONCLUSION

From the viewpoint of improving stability and reliability of surrogate model-based optimization algorithm, this paper introduces one hybrid meta-model-based algorithm. For different practical problems to be optimized, the proposed method can dynamically select the proper surrogate model during each optimum searching loop among ordinary Kriging, universal Kriging, and multi-quadratic radial basis function response surface model. However, the surrogate model used in the proposed algorithm is not limited in the above three models, it can also be replaced by other counterparts. In a sense, due to parallel computation, the proposed hybrid meta-model robust optimization (HMRO) algorithm doesn't make a burden of computing cost. Through several applications, it is shown that the HMRO is especially suitable for complex and time-consuming problems.

The main contribution of this paper is to provide a powerful help for the research of surrogate model optimization algorithm in the future. It also has important significance in the optimal design of electromagnetic devices. In the future research, the different kinds of surrogate models used for the HMRO will be further investigated.

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