Metastable order via destructive many-body interference

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The phenomenon of metastability shapes dynamical processes ranging from radioactive decay to chemical reactions. Here, we present a mechanism for metastability in which a quantum gas self-stabilizes against relaxation towards thermal equilibrium by establishing a transient ordered state. In this state, the direct relaxation channel is suppressed by destructive interference, which derives from the chiral order of the transient state. In particular, we consider the dynamical evolution of an ultracold bosonic gas in an optical lattice, that is quenched into a higher band of the lattice, which triggers the dynamical evolution. Following this quench, the self-stabilization phenomenon manifests itself in three stages of relaxation, subsequent to the preparation of the incoherent excited state. In the first stage, the gas develops coherence resulting in the ordered state, during the second stage the gas forms a long-lived state with inhibited relaxation and slow loss of coherence, followed by the third stage of fast relaxation to the thermal ground state. We demonstrate this mechanism experimentally and theoretically, and discuss its broader implications.

When a many-body system is driven out of thermal equilibrium, and subsequently evolves freely, it typically relaxes to a thermal state at higher temperature [1, 2]. In certain cases, however, a metastable state inhibits relaxation leading to an excited state with prethermal properties on intermediate time scales. Only in the long-time limit the system relaxes to the true thermal equilibrium. A common mechanism that inhibits relaxation is a free-energy barrier that renders the system long-lived for sufficiently low temperatures [3–5]. Such a free-energy barrier is the origin of metastability in many classical systems but also in quantum systems such as quantum-tunneling scenarios [6–8]. Classically, the only relevant energy scales for metastability are the kinetic energy and the height of the free-energy barrier. In quantum mechanics, many-body phenomena such as Bose-Einstein condensation may dramatically alter this picture leading to metastability even in the regime where the kinetic energy by far exceeds the height of the free-energy barrier [9]. In fact, quantum-mechanical states, despite being unprotected by an energy barrier, can exhibit metastability due to some inherent property, e.g., a symmetry, which prevents dissipation [10–12], for example, via coherent feedback [13].

Metastability is not only helpful to advance our understanding of thermalization processes, but can also be used as a starting point to study states that are inaccessible in the equilibrium phase diagram. A well-defined environment to drive a many-body system far out of equilibrium are ultracold atomic gases in general and in particular, when prepared in excited Bloch bands of an optical lattice [14–20]. Exciting atoms to higher bands gives access to orbital physics, which also plays a crucial role for metal-insulator transitions, superconductivity and colossal magnetoresistance in transition-metal oxides [21, 22]. These excited states are naturally prone to relaxation towards the lowest band, however, their complex orbital nature opens a promising arena for studying scenarios of metastability.

In this work, we present a mechanism for metastability, where destructive interference between different relaxation channels inhibits relaxation to the ground state. A Bose-Einstein condensate (BEC) is excited to the second band of a bipartite square optical lattice. The staggered chiral order of the condensate, inhibits relaxation. Hence the condensate represents a dark state which leads to dramatically increased lifetimes. We identify three different relaxation stages, see Fig. 1. The first stage is characterized by the approach of the intermediate metastable state. In our case the metastable state is a BEC with a chiral periodic phase texture. The quasi-static properties of this long-lived state have been studied in Ref. [20]. Hence, during the first stage there is a significant increase of both short-range coherence and long-range order. In the second stage, the coherence inhibits relaxation to the thermal ground state. Only the incoherent fraction decays, which leads to heating of the atoms and therefore to a slow loss of coherence. At the beginning of the third stage, almost all coherence is lost. This leads to fast decay towards a thermal equilibrium ground state. For all stages we determine the underlying relaxation mechanisms and their role for the self-stabilization of the condensate.

**Experiment**

We start with a BEC of $10^5 \ ^{87}\text{Rb}$ atoms in the $|F = 2, m_F = 2\rangle$ hyperfine state in an isotropic magnetic trap. The BEC is adiabatically loaded into the lowest band of a bipartite square optical lattice with wavelength $\lambda = 1064\, \text{nm}$, for details see Methods and also [14–20]. The key tool is the tunability of the potential offset $\Delta V$ between the two non-equivalent sublattice sites $A$ and

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Figure 1: Snapshots of the field in real-space, obtained from a numerical simulation. We characterize four states connected by three stages of relaxation (I), (II) and (III). After loading the atoms into the upper band they form an unordered, incoherent cloud (denoted incoherent excited state). (I) The atoms condense into a metastable state, characterized by a chiral phase pattern, which inhibits relaxation (coherent metastable state). (II) Only the thermal fraction of the atoms decays. Decay processes lead to heating, a resulting loss of coherence and, hence, an increasingly populated thermal excited state. (III) The loss of coherence leads to fast relaxation and eventually all atoms relax to the thermal ground state. For all states the plots show the occupations of s-orbitals (circles) and p-orbitals (dumbbells) for a snapshot of a single x-y-layer of our numerical simulation. The phase of the field is depicted as shown in the color wheel. Low amplitudes are indicated as gray. We show the idealized case of very low initial temperature ($T = 0.5\, \text{nK}$) that leads to nearly perfect phase coherence.

B, see Fig. 2. We load the BEC at large negative offset $\Delta V < 0$, such that the resulting ground state has vanishing occupation on the $B$-sublattice sites. The atoms, which exclusively reside on the $A$-sublattice, are phase-incoherent even for temperatures as low as 50 nK due to the deep trapping potential in the $x$-$y$-plane. In a rapid quench to $\Delta V > 0$, a significant fraction of atoms is excited to the second Bloch band and we study the resulting relaxation dynamics. We compare the experimental results to numerical simulations using a tight-binding model and solving the time evolution within classical-field theory, for details see Methods and [23–28]. Experimentally, the atoms are observed via a band mapping technique, which allows to resolve the band populations in momentum space. We extract the condensed and thermal fractions in the second band by defining appropriate regions of interest, see Methods. In our simulations, we model the band mapping process by projecting the wave function onto the Bloch functions of the tight-binding lattice.

Relaxation dynamics

Immediately after the quench, all available Bloch modes in the second band are nearly equally populated. The weak harmonic confinement in the $z$-direction gives rise to tubular sites, which act as a reservoir for thermalization for the subsequent relaxation dynamics. This dynamics begins with a condensation stage, see Fig. 3(a,b), stage I, where during several tens of milliseconds a significant fraction of atoms accumulates in the two inequivalent minima of the second band, i.e., the $X_{\pm}$-points, see Fig. 3(c), second panel. Here the atoms form a phase-coherent condensate and interactions lead to a staggered chiral order of p-orbitals [19], see Fig. 1 coherent metastable state. The chiral phase pattern stabilizes the atoms against decay to the first Bloch band through destructive interference of the dominant decay channels, see Fig. 3 stage II. In fact, the condensate itself shows perfect destructive interference and hence represents a dark state with infinite lifetime. Decay arises exclusively due to the thermal fraction of atoms. The inhibited decay stage is characterized by a large condensate fraction and slower than exponential decay. Due to the large energy difference between the first excited and the lowest Bloch band of several recoil energies, decaying atoms gain a large amount of energy that leads to heating. Collisions between atoms in the lowest and excited bands transfer the extra heat also into the excited bands. This effect together with the lower number of atoms in the excited bands leads to a lower phase-space density and hence lower condensate fraction. The resulting loss
Figure 3: Time evolution of the total number of atoms in the second band as well as its condensate and thermal fraction after a quench to $\Delta V = 0.43 V_0$. Panel (a) shows experimental and panel (b) numerical data. We identify three main decay stages: the coherence buildup stage (I), the inhibited relaxation stage (II) and the fast relaxation stage (III). Black lines show exponential fits to the data points in stage (III). (c) Time of flight images for experimental data points marked by black squares in (a). The outermost right panel illustrates the first (blue) and second (red) Brillouin zone and the $X_{\pm}$-points of the lattice. The temperature of the initial state for both experiments and simulation is $T \approx 0.5 E_{\text{rec}}/k_B \approx 50 \, \text{nK}$.

of coherence induces a chain reaction of faster decay and even stronger reduction in phase-space density. At some point we cross the critical temperature and only thermal atoms remain. Here, we enter the fast decay stage, where the atoms decay exponentially, see Fig. 3 stage III. Higher initial temperature with a subsequent larger thermal fraction leads to an earlier onset of stage III and hence a significantly faster decay, see App. C.

Two-fluid model

We compare the experimental results and the results of the classical-field theory simulations with a two-fluid model. We split the total number of atoms into a thermal and a condensed part $N_{\text{tot}} = N_{\text{th}} + N_c$. The thermal part decays exponentially with constant $\gamma_{\text{dec}}$, while the condensed part is stable. Each decaying atom gains an energy $\Delta E$ proportional to the band gap between the first excited and lowest band. This extra energy is assumed to thermalize among all atoms and hence increase the temperature $T$. Additionally, we assume that thermalization between the thermal and the condensed fraction occurs at fixed total energy on a time-scale $\Gamma_{\text{eq}}$. The resulting differential equations are

$$\dot{N}_{\text{th}} = -\gamma_{\text{dec}} N_{\text{th}} + \Gamma_{\text{eq}} (T^\alpha - N_{\text{th}})$$

$$\dot{N}_{\text{tot}} = -\gamma_{\text{dec}} N_{\text{th}}$$

Figure 4: Two-fluid model for self-stabilization. We show the total, thermal and condensed particle number as well as temperature computed within a two-fluid model, for details see App. B. The model shows qualitative agreement with experimental and simulated data shown in Fig. 3. All quantities within the two-fluid model are unitless. We use $\gamma_{\text{dec}} = 1$, $\Gamma_{\text{eq}} = 3$, $\alpha = 2$ and $\Delta E = 3$.  

$$\dot{T} = \gamma_{\text{dec}} \Delta E - \frac{\Gamma_{\text{eq}} T}{N_{\text{th}}} (T^\alpha - N_{\text{th}})$$

where $T$ is scaled in terms of an arbitrary unit temperature, an exponent $\alpha$ has been introduced and further details are described in App. B. We find that the resulting time evolution, shown in Fig. 4, captures the different decay stages and hence stresses the general applicabil-
In order to understand why the phase-coherence of the condensate inhibits decay, we analyze the contributing decay mechanisms. The character of the dominant decay channels depends on the value of the potential offset $\Delta V$. We consider values of $\Delta V$ between the first two band crossings in Fig. 2. For $\Delta V = 0$ the sublattice sites have equal depth and hence their $s$-orbitals are degenerate, while at the second band crossing for $\Delta V = 0.86 V_0$ the $p$-orbitals on $B$-sublattice sites are degenerate with $s$-orbitals on $A$-sublattice sites. Since only the thermal fraction of atoms decays, we utilize the exponential decay during the fast relaxation stage to determine the decay times observed for different final values of $\Delta V$ shown in Fig. 3(a). The decay is slow for intermediate values of $\Delta V$, while becoming increasingly fast towards the two band crossings at $\Delta V = 0$ and $\Delta V = 0.86 V_0$. It has been proposed in Ref. [29] that the main decay channel close to $\Delta V = 0.86 V_0$ is an interaction-driven decay. In this regime the $p$-orbitals on $B$-sites are nearly degenerate with $s$-orbitals on $A$-sites and hence are strongly occupied. Therefore, a significant interaction term is associated with the collision of two $p_x$- or $p_y$-atoms on a $B$-site, which are both scattered into the $s$-orbital on the same site:

$$a_{i,s}^+ a_{i,s}^\dagger (a_{i,x} a_{i,x} + a_{i,y} a_{i,y}) .$$

Here $a_{i,s}^\dagger (a_{i,x,y,s})$ creates (annihilates) an atom on site $i$ in orbital $\{p_{x,y}, s\}$. We confirm that this is the dominant decay channel for $\Delta V > 0.43 V_0$ by artificially switching off the above interaction term in our numerical simulations, see Fig. 3(b). We find that this leads to dramatically increased life times for $\Delta V > 0.43 V_0$, while life times are almost unchanged for $\Delta V < 0.43 V_0$.

Next we highlight the stability of the condensate with respect to this decay channel by showing that the above interaction term is subject to destructive interference in this case. The condensation mode is characterized by the interaction-stabilized relative phase $\pm i$ and equal populations $n_p$ of $p_x$- and $p_y$-orbitals [17], which leads to

$$\langle \psi_{px+ipy} | a_{i,s}^\dagger a_{i,s}^\dagger (a_{i,x} a_{i,x} + a_{i,y} a_{i,y}) | \psi_{px+ipy} \rangle = \langle \psi_{px+ipy} | a_{i,s}^\dagger a_{i,s}^\dagger (n_p e^{i\phi} - n_p e^{-i\phi}) | \psi_{px+ipy} \rangle = 0$$

and hence perfect destructive interference.

For values of $\Delta V < 0.43 V_0$ we see in Fig. 3 that switching off the interaction-induced decay leaves the decay time scale nearly unchanged. Here we identify the main decay channel to be a hopping-induced decay between $s$-orbitals on neighboring sites. This is expected...
to be strong close to $\Delta V = 0$, where the neighboring $s$-orbitals are degenerate. The responsible decay term is

$$\sum_{(i,j)} a_{i,s}^\dagger a_{j,s},$$

where $(i,j)$ denotes the sum over nearest-neighbor sites. Again we confirm this process as the main decay channel when switching on the respective hopping term and observing dramatically increased decay time scales for $\Delta V < 0.43 V_0$, while those near $\Delta V = 0.86 V_0$ are nearly unchanged. The condensate is also stable with respect to this decay channel, since it has opposite phase on $s$-orbitals on opposite sides of a $B$-site and hence the hopping-type decay shows destructive interference for the condensate, see Fig. 3. For intermediate values of $\Delta V$ there is a competition between the two decay channels. Depending on the relative size of the respective decay constants the longest life time is obtained close to the center point $\Delta V = 0.43 V_0$. We do observe, however, that it shifts slightly depending on e.g. temperature, see App. D.

**Coherence-buildup stage**

Finally we take a closer look at the coherence-buildup stage. In the beginning of this stage, there is a large thermal fraction, and hence low coherence, and therefore the atoms should decay quickly. This effect is not seen in Fig. 3. We observe it, however, at significantly lower temperatures shown in Fig. 7. Here the atoms initially decay fast until the condensate fraction becomes large and the total amount of atoms in the upper bands reaches a plateau with negligible decay. At significantly longer times, the condensate fraction reduces and we enter the fast decay stage. We therefore confirm the initial fast decay in the condensation stage and conclude that it is only a minor effect at the experimentally realized temperature and therefore not observed in Fig. 3.

**Summary and outlook**

We have demonstrated a mechanism for metastability, experimentally and theoretically, in which the emergence of a metastable ordered state inhibits relaxation to thermal equilibrium. The inhibition of the decay of the metastable state is due to destructive quantum interference of different relaxation channels. More specifically, it is the chiral order of a Bose-Einstein condensate in the first excited band of an optical lattice that leads to destructive interference between the two relevant relaxation channels and hence to dramatically increased lifetimes. Depending on the parameter regime, we identify nearest-neighbor hopping or flavor-changing interactions as the main decay channels. For both channels, we show that the chiral condensate is protected from relaxation due to perfect destructive interference. An alternative view on the inhibition of relaxation due the proximity of a metastable state is the critical slowdown of dynamics near fixed points of renormalization group theory discussed in Ref. [30, 31]. The connection between the approach presented here and renormalization group theory is an interesting direction for future work. Furthermore, we emphasize that the mechanism described here is not only conceptually interesting, but provides a framework to create non-equilibrium ordered states in solids. By optically pumping electrons into higher bands, an ordered state can be long-lived if this ordered state is a dark state with regards to particle relaxation to the lower band, as we have argued in this work.

**Methods**

**Lattice geometry**

The light-shift potential for the bipartite square optical lattice reads

$$V(x, y) = -V_0 \left[ \cos^2(kx) + \cos^2(ky) \ight.$$  

$$\left. + 2 \cos(\theta) \cos(kx) \cos(ky) \right],$$

where $V_0(1 + 2\cos(\theta))$ is the total depth of the lattice and $k = 2\pi/\lambda$ with wavelength $\lambda = 1064 \text{ nm}$. The lattice is composed of two wells (denoted $A$ and $B$) with different depth arranged according to the black and white fields of a chequerboard (cf. Fig. 8(a)). The control parameter $\theta$, that can be experimentally adjusted on a few-ten-microsecond time-scale with better than $\pi/300$ precision by interferometric techniques [32], determines the relative depth of the $A$- and $B$-sublattice sites, which is given by $\Delta V = -4V_0 \cos(\theta)$. In its second Bloch band this lattice provides two inequivalent degenerate energy

![Figure 7: Simulated time evolution of the total number of atoms in the second band as well as its condensate and thermal fractions after a quench to $\Delta V = 0.25 V_0$ at a very low initial temperature $T = 0.05 E_{\text{rec}}/k_B = 4.8 \text{ nK}$](image)
minima (denoted $X_+$ and $X_-$) at the edge of the first Brillouin zone (BZ) (cf. Fig. 8(b)).

Experimental protocol

A Bose-Einstein Condensate (BEC) of 100 000 $^{87}$Rb atoms in the $|F = 2, m_F = 2 \rangle$ hyperfine state is prepared in an isotropic magnetic trap with trap frequencies $(\omega_x, \omega_y, \omega_z) = (39, 42, 35)$ Hz. By adiabatically ramping up the lattice depth in 100 ms to $V_0 = 7.2 E_{\text{rec}}$, the atoms are loaded into the lowest band of the optical lattice. The time phase at this stage is set to $\theta = 0.4 \pi$ corresponding to a potential difference $\Delta V = -1.2 V_0$. After a short waiting time of 10 ms, $\theta$ is tuned in 300 $\mu$s to a final value $\theta_f \in [0.5 \pi, 0.57 \pi]$ i.e. $\Delta V \in [0, 0.86 V_0]$ and the atoms are subsequently allowed to relax during a variable hold time of up to 900 ms. Subsequently, the atoms are observed via a band-mapping technique: The lattice potential is adiabatically ramped down to zero in 2 ms followed by a ballistic expansion during $t = 30$ ms. This maps the quasimomenta of the Bloch functions onto the momenta of the free particles in the Brillouin zones (BZ) of the lattice, which is recorded via an absorption image. To determine the condensed part of the atomic sample we define four disk-shaped regions of interest (ROIs) and four ring-shaped ROIs, all enclosing the same area, centered around the $X_\pm$-points (white-dashed lines in Figure 8(c)). By subtracting the number of atoms found within the outer ring from the atoms within the inner disk, we get rid of the thermal part and remain with the number of condensed atoms. The number of thermal atoms in the first and second band are determined by counting the atoms within ROIs comprising the first and second BZs, respectively, however, with the intersections with the disk-shaped ROIs around $X_\pm$ cut out and with the intersections with the ring-shaped ROIs counted twice. For the total population in the second band we add up the population of the thermal atoms in the second BZ and the condensed atoms.

Decay time measurement

For extracting the decay time scale shown in Fig. 5 we perform an exponential fit to the data points in stage (III). We identify the onset of stage (III) as the time where the condensate fraction, i.e. the number of condensed atoms divided by the number of total atoms, drops below a certain threshold value. This threshold is chosen to be 20% for the experimental data and 5% for the simulated data shown in Fig. 5. Due to dephasing effects we expect the experimental data to overestimate the actual condensate fraction and therefore use the higher threshold value. We fit an exponential of the type $f(t) = a e^{-t/T_d} + b$ and extract the decay time scale $T_d$.

Details on classical-field theory simulations

For our numerical simulations we employ a tight-binding model. In the Hamiltonian we include all nearest- and next-nearest-neighbor hopping terms as well as on-site interaction terms, see App. A. For our simulations we use a lattice with $12 \times 12$ sites with periodic boundary conditions in the $x$-$y$- and a harmonic trap in the $z$-direction. We discretize the $z$-direction and adjust the interaction strength such that the mean-field interaction energy in the center of the trap $g_{\text{d}n(r = 0)}$ equals the tight-binding interaction times the mean occupation $\bar{n}$. We assume 300 atoms per tube and obtain for a discretization length $d_z = 0.13 \mu m$ a hopping constant $J_z = 1.7 E_{\text{rec}}$ and interaction parameters $U_A \approx U_B \approx 0.4 E_{\text{rec}}$. We initialize the system at $\theta = 0.35 \pi$ corresponding to $\Delta V \approx -1.8 V_0$. For the time evolution we use a classical-field approach, for reviews see [23,28]. For large occupation of modes it is a good approximation to replace creation and annihilation operators by their expectation values and solve the resulting Heisenberg equations of motion for these operators numerically. We initialize the system using Monte-Carlo minimization, starting from an empty lattice and working at fixed chemical potential. For the Monte-Carlo procedure we use the Hamiltonian as the minimization functional and adjust the Monte-Carlo temperature to the desired temperature of the lattice. For the low temperature data we perform annealing starting at higher temperature and reducing the temperature to the desired one during the Monte-Carlo procedure. For our simulations we repeat this procedure and the subsequent time evolution 300–600 times. We average all observables over all of these Monte-Carlo trajectories. This accounts for thermal fluctuations.
that a rather low number of Monte-Carlo trajectories is sufficient due to the self-averaging along the $z$-direction.

We read out the occupations of individual bands by projecting onto the Bloch functions of the tight-binding lattice. As an estimate for the condensed atoms we use the number of atoms occupying the $X$-points of the lattice.

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Appendix A: Tight-binding Hamiltonian

For our numerical simulations we use the following tight-binding Hamiltonian

\[
H = H_{ss} + H_{sp_x} + H_{sp_y} + H_{p_x p_x} + H_{p_y p_y} + H_{p_x p_y} + H_{d} + H_{d} + H_{sp_d} + H_{int}
\]

where

\[
H_{ss} = -J_{ss} \sum_{R_i \in A, R_j = \pm e_x, \pm e_y} b_i^\dagger R_i b_j R_j + V_s \sum_{R_i \in A} b_i^\dagger R_i b_i R_i
\]

\[
H_{sp_x} = \sum_{R_i \in B} -J_{sp_x} b_i^\dagger R_i \pm e'_x b_x R_i + J_{sp_x} b_i R_i \pm e'_x b_x R_i + h.c.
\]

\[
H_{sp_y} = \sum_{R_i \in B} -J_{sp_y} b_i^\dagger R_i \pm e'_y b_y R_i + J_{sp_y} b_i R_i \pm e'_y b_y R_i + h.c.
\]

\[
H_{p_x p_x} = -J_{p_x} \sum_{R_i \in B} b_i^\dagger R_i + b_i R_i + V_x \sum_{R_i \in B} b_i^\dagger R_i b_i R_i
\]

\[
H_{p_y p_y} = -J_{p_y} \sum_{R_i \in B} b_i^\dagger R_i + b_i R_i + V_y \sum_{R_i \in B} b_i^\dagger R_i b_i R_i
\]

\[
H_{d} = -J_{decay} \sum_{R_i \in \pm e_x, \pm e_y} b_i^\dagger R_i b_i R_i
\]

\[
H_{sp_d} = \sum_{R_i \in B} J_{sp_d} \left( b_i^\dagger R_i \pm e'_x b_x R_i + b_i^\dagger R_i \pm e'_y b_y R_i \right) b_x R_i + h.c.
\]

\[
H_{sp_d} = \sum_{R_i \in B} J_{sp_d} \left( b_i^\dagger R_i \pm e'_x b_x R_i + b_i^\dagger R_i \pm e'_y b_y R_i \right) b_y R_i + h.c.
\]

\[
H_{int} = \frac{U_A}{2} \sum_{R_i \in A} b_i^\dagger R_i b_i^\dagger R_i b_i R_i b_i R_i + \frac{U_B}{2} \sum_{R_i \in B} b_i^\dagger R_i b_i^\dagger R_i b_i R_i b_i R_i
\]

Here the operator \( b_{s,x,y}^\dagger R_i \) (\( b_{s,x,y} R_i \)) creates (annihilates) an atom on the site \( R_i \) in the \( s \) - \( p_x \) or \( p_y \)-orbital, respectively. The unit vectors \( e_x \) and \( e_y \) connect nearest-neighbor \( A \)- (or equivalently \( B \)-) sites, \( e'_x = e_x + e_y \) and \( e'_y = e_y - e_x \). We determine the relative prefactors of the interaction terms from the integrals of 2d-harmonic-oscillator wave functions. For each value of \( \theta \) we determine the Bloch bands and adjust the hopping parameters and lattice site potentials such that we obtain a best fit of the resulting tight-binding bands to the Bloch bands. We do so by considering analytic equations for symmetry points for some of the tight-binding parameters, while we fit others using a numerical Monte-Carlo-type optimiza-
tion routine. The resulting tight-binding parameters for 
$V_0 = 7E_{\text{rec}}$ are shown in Fig. 9. We determine the 1d-
interaction strength of each tube by assuming harmonic-
oscillator wave functions in the $x$-$y$-plane.

Appendix B: Two-fluid model

A simple two-fluid model captures the main aspects of
the three different decay stages and qualitatively shows
the same behavior as the experimental data and our full
simulation. We consider only the atoms in the upper
band and assume that we have a thermal fraction $N_{\text{th}}$
and a condensed fraction $N_c$ of atoms. The total number
of atoms is $N_{\text{tot}} = N_{\text{th}} + N_c$. Furthermore we assume
that the equilibration within the thermal and condensed
atoms happens on a much faster time scale than the equi-
libration between the two, such that we can assume the
equilibration within each fraction to be instantaneous.
Without loss of generality we assume that the condensed
atoms have zero energy. Following the description in
chapter 2 of Ref. [33] we assume a generic density of
states $g(\epsilon) = c_\alpha \epsilon^{\alpha - 1}$.

We can then compute the total number of thermal atoms
and the total energy $E$ as

$$N_{\text{th}} = \int d\epsilon \, g(\epsilon) \frac{1}{e^{\epsilon/(k_B T)} - 1} \propto T^{\alpha}$$

$$E = \int d\epsilon \, g(\epsilon) \frac{\epsilon}{e^{\epsilon/(k_B T)} - 1} \propto T^{\alpha + 1}.$$ 

We absorb the proportionality constants into the units of
temperature and energy and therefore obtain

$$N_{\text{th}} = T^\alpha \quad N_{\text{th}} T = E.$$ 

As we have seen above the condensed atoms do not decay
due to perfect destructive interference. We therefore as-
sume that only the thermal atoms decay with time-scale
$1/\gamma_{\text{dec}}$. Furthermore we assume that the thermal and
condensed fraction equilibrate on a time scale of $1/\Gamma_{\text{eq}}$.
On average whenever a thermal atom decays the total en-
ergy is decreased by the mean energy of a thermal atom
$E/N_{\text{th}}$. Additionally the atom gains an energy $\Delta E$
corresponding to the energy difference between the upper
and the lower band. We assume that this energy is redis-
tributed to the atoms in the upper band and hence the
energy of these atoms is increased by this amount. Hence
the equations of motion for our model system are

$$\dot{N}_{\text{th}} = -\gamma_{\text{dec}} N_{\text{th}} + \Gamma_{\text{eq}} (T^\alpha - N_{\text{th}})$$

$$\dot{N}_{\text{tot}} = -\gamma_{\text{dec}} N_{\text{th}}$$

$$\dot{E} = (\Delta E - \frac{E}{N_{\text{th}}}) \gamma_{\text{dec}} N_{\text{th}}.$$ 

We derive the equation for the temperature of the sys-
tem by taking the derivative of $T = E/N_{\text{th}}$. We obtain

$$\dot{T} = \gamma_{\text{dec}} \Delta E - \frac{\Gamma_{\text{eq}} T}{N_{\text{th}}} (T^\alpha - N_{\text{th}}).$$ 

Figure 9: Tight-binding parameters as a function of
$\theta$ for $V_0 = 7E_{\text{rec}}$, $\lambda_L = 1064 \text{ nm}$ and the rubidium mass $m = 87m_p$. All
parameters are given in units of $E_{\text{rec}}$. 
Figure 10: Two-fluid model for self-stabilization at low initial temperature (a) and high initial temperature (b). We show the total, thermal and condensed particle number as well as temperature computed within a two-fluid model, for details see App. B. The model shows qualitative agreement with experimental and simulation data shown in Fig. 3 of the main text. All quantities within the two-fluid model are unitless. We use $\gamma_{\text{dec}} = 1$, $\Gamma_{\text{eq}} = 3$, $\alpha = 2$ and $\Delta E = 3$.

Figure 11: Time evolution of the total number of atoms in the second band as well as its condensate and thermal fraction after a quench to $\Delta V = 0.43 V_0$. Panel (a) shows experimental and panel (b) numerical data. We identify three main decay stages: the coherence build-up stage (I), the inhibited relaxation stage (II) and the fast relaxation stage (III). The blue lines show exponential fits to the data points in stage (III). (c) Time of flight images for experimental data points marked by black squares in (a). The temperature of the initial state for both experiments and simulation is $T \approx 1.1 E_{\text{rec}}/k_B \approx 110 \text{nK}$.

These equations are of cause only valid as long as there is a condensed fraction, otherwise $N_{\text{tot}} = N_{\text{th}}$ and

$$\dot{N}_{\text{th}} = -\gamma_{\text{dec}} N_{\text{th}}$$
$$\dot{T} = \gamma_{\text{dec}} \Delta E.$$

In our case, having one harmonically-trapped and two free dimensions, we obtain $\alpha \approx 1/2 + 1/2 + 1 = 2$.

From Eq. B2 we can see that moving an atom from the condensate to the thermal cloud effectively cools the atoms. The reason is that the mean kinetic energy of thermal atoms is larger than the mean kinetic energy of condensed atoms. This cooling process counteracts the heating effect due to the decay of thermal atoms.

Within the two-fluid model we can also obtain an approximation for the scaling of the decay of total atoms. To this end we assume that the equilibration time scale $\Gamma_{\text{eq}}$ is much faster than all other time scales, such that $N_{\text{th}} \approx T^\alpha$. We find that this approximation is reasonable during stage II. Within this approximation we can solve Eq. B1 and derive the corresponding time dependence of
For completeness we also show the results of the two-fluid model for lower and higher initial temperature in Fig. 10. By construction, the decay of the two-fluid model is exponential when the condensate fraction vanishes. We also see from Fig. 10 that the inflection point of the curve for the total number of excited atoms is exactly the point where the condensate fraction vanishes.

Appendix C: Decay of a thermal state

As a cross-check that the coherence of the chiral condensate is indeed the origin of the inhibition of decay, we consider a cloud of atoms at higher initial temperature in Fig. 11. As a result we obtain a lower condensate fraction and larger thermal fraction. The decay is now dominated by the thermal fraction of the atoms. As a result we find that an exponential fit shows good agreement even for short times. This is in contrast to the low temperature sample discussed in Fig. 3 of the main text.

Appendix D: Decay-time-scale for different temperatures

Within our numerical simulations we can determine the decay time for the exponential decay in stage (III) for a range of different temperatures. The results are shown in Fig. 12(a). The decay time shows no strong dependence on the initial temperature of the atomic cloud. Only the maximum of the decay times shifts to slightly lower values of $\Delta V$ for higher temperatures. This can be explained as follows: the initial temperature of the cloud determines the condensate fraction at the beginning of stage (II). Hence we expect larger coherence and slower decay during stage (II). Subsequently heating leads to increasing temperature and hence reducing phase-space density. The onset of stage (III) is essentially determined by the time when the phase-space density has reduced below the critical value for condensation. Hence independent of the initial temperature the phase-space density in the beginning of stage (III) is always the same. A lower initial temperature only leads to a later cross-over from stage (II) to (III). We confirm this by showing the cross-over time point from stage (II) to (III) in Fig. 12, which indeed changes dramatically with temperature, indicating significantly longer durations of stage (II) and hence longer life-times of the condensate for lower initial temperature.

Figure 12: (a) Simulated decay time scale as a function of final potential offset $\Delta V$ at several different temperatures as indicated in the legend. After loading the atoms into the second Bloch band they relax back to the lowest band. During stage III the decay is exponential and we show the associated time scale versus $\Delta V$. (b) Cross-over time from stage (II)-(III). We extract the time where the condensate fraction drops below 3%, which indicates the cross-over from stage (II) to (III) and is the starting point for the exponential fits. We see that the cross-over time crucially depends on temperature while the temperature dependence of the decay time scale is not as strong.

the temperature

$$T(t) = (\Delta E - (\Delta E - T_0)e^{-\frac{\pi}{\alpha} \gamma T_0})^\alpha$$

Hence the temperature always equilibrates at $\Delta E^\alpha$. For any fixed temperature we can now solve the equation of motion for the total number of atoms

$$N_{\text{tot}}(t) = N_{\text{tot}}(0) - \gamma_{\text{dec}} T_0^\alpha t,$$

and see that it scales linearly. We note that for large $\Delta E$ the condensate fraction may be depleted before the constant temperature is reached and hence linear scaling of the total number of atoms is not always observed.

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