Comparing the Broken $U_3 \times U_3$ Linear Sigma Model with Experiment.

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Abstract

The linear $\sigma$ model with broken $U_3 \times U_3$ is compared with data on the lightest scalar and pseudoscalar mesons. When 5 of the 6 parameters are fixed by the pseudoscalar masses and decay constants one finds that, already at the tree level, a reasonable description for the 4 scalar masses, mixing and up to 8 tri-linear couplings of lightest scalars, taken as $a_0(980)$, $f_0(980)$, $\sigma(\approx 500)$ and $K^*_0(1430)$. This clearly indicates that these scalars are the chiral partners of the $\pi$, $\eta$, $\eta'$, $K$ and strongly suggests that they like the latter are (unitarized) $q\bar{q}$ states.

I. INTRODUCTION

As is well known the naive quark models (NQM) fails badly in trying to understand the lightest scalars, the $a_0(980)$, $f_0(980)$, $K^*_0(1430)$ and the $\sigma(400 - 1200)$, which we shall here call $\sigma(500)$. Therefore today most authors want to give the $a_0(980)$, $f_0(980)$ and the $\sigma(500)$ other interpretations than being $q\bar{q}$ states. Popular alternative interpretations are $KK$ bound states, 4 quark states, or for the $\sigma$, a glueball. But there is also an obvious reason for why the NQM fails: Chiral symmetry is absent in the NQM, but is crucial for the scalars. Chiral symmery is widely believed to be broken in the vacuum, and two of the scalars ($\sigma$ and $f_0$) have the same quantum numbers as the vacuum. Thus to understand the scalar nonet in the same way as we believe we understand the vectors and heavier multiplets, and to make a sensible comparison with experiment, one must include chiral symmetry in
addition to flavour symmetry into the quark model.

The simplest such chiral quark model is the linear U3×U3 sigma model with 3 flavours. Then we can treat both the scalar and pseudoscalar nonets simultaneously, and on the same footing, getting automatically small masses for the pseudoscalar octet, and symmetry breaking through the vacuum expectation values (VEV’s) of the scalar fields.

As an extra bonus we have in principle a renormalizable theory, i.e. “unitarity corrections” are calculable. In fact, in the flavour symmetric \((u = d = s\) below) limit many of the unitarity corrections can be considered as being already included into the mass parameters of the theory, once the original 4-5 parameters are replaced by the 4 physical masses for the singlet and octet \(0^{-+}\) and \(0^{++}\) masses, and the pseudoscalar decay constant.

Unfortunately this over 30 years old model [1] has had very few phenomenological applications, although important exceptions are the intensive efforts of M. Scadron and collaborators [2]. The reestablishment [3,4] of the light and broad \(\sigma\) has also more recently revitalized the interest in the linear sigma model [5,6].

II. THE LINEAR SIGMA MODEL WITH 3 FLAVOURS

The well known linear sigma model [1] generalized to 3 flavours with complete scalar \((s_a)\) and pseudoscalar \((p_a)\) nonets has at the tree-level the Lagrangian the same flavour and chiral symmetries as massless QCD. The U3×U3 Lagrangian with a symmetry breaking term \(\mathcal{L}_{SB}\) is

\[
\mathcal{L} = \frac{1}{2} \text{Tr}[\partial_\mu \Sigma \partial_\mu \Sigma^\dagger] - \frac{1}{2} \mu^2 \text{Tr}[\Sigma \Sigma^\dagger] - \lambda \text{Tr}[\Sigma \Sigma^\dagger \Sigma \Sigma^\dagger] - \lambda' (\text{Tr}[\Sigma \Sigma^\dagger])^2 + \mathcal{L}_{SB}.
\]

(1)

Here \(\Sigma\) is a \(3 \times 3\) complex matrix, \(\Sigma = S + iP = \sum_{a=0}^{8}(s_a + ip_a)\lambda_a/\sqrt{2}\), in which \(\lambda_a\) are the Gell-Mann matrices, normalized as \(\text{Tr}[\lambda_a \lambda_b] = 2\delta_{ab}\), and where for the singlet \(\lambda_0 = (2/N_f)^{1/2}\mathbb{1}\) is included. Each meson in Eq. (1) has a definite SU3f symmetry content, which in the quark model means that it has the same flavour structure as a \(q\bar{q}\) meson. Thus the fields \(s_a\) and \(p_a\) and potential terms in Eq. (1) can be given a conventional quark line structure [8] (Fig. 1).
The symmetry breaking terms are most simply:

$$\mathcal{L}_{SB} = \epsilon_\sigma \sigma_{u\bar{u}+d\bar{d}} + \epsilon_{s\bar{s}} \sigma_{s\bar{s}} + \beta [\det \Sigma + \det \Sigma^\dagger],$$

which give the pseudoscalars mass and break the flavour and $U_A(1)$ symmetries. The stability condition, that the linear terms in the fields must vanish after the shift of the scalar fields $(\Sigma \rightarrow \Sigma + V)$ determines the small parameters $\epsilon_i$ in terms of the pion and kaon masses and decay constants. One finds $\epsilon_\sigma = m^2_{\pi}\pi f_{\pi}$, $\epsilon_{s\bar{s}} = (2m^2_{K}f_{K} - m^2_{\pi}f_{\pi})/\sqrt{2}$, while $\beta$ in the $U_A(1)$ breaking term is determined by $m_{\eta'}$, or by $m^2_{\eta} + m^2_{\eta'}$.

My fit to the scalars with the unitarized quark model (UQM) \cite{9} is essentially a unitarization of eq.(1) with $\lambda \approx 16$ and $\lambda' = 0$, and with the main symmetry breaking generated by putting the pseudoscalar masses at their physical values. The model was used as an effective theory with a symmetric smooth 3-momentum cutoff 0.54 GeV/c given by a gaussian form factor. Such a form factor is natural, since physical mesons are of course not pointlike, but have a finite size of 0.7-0.8 fm. (See the discussion in connection to Eq.(37) below.) The fit included the Adler zeroes which follow from eq.(1), but only approximate crossing symmetry.

Here I shall study the theory at the tree level, leaving the detailed discussion of the unitarization for future work. In fact, when tadpole loops are included in the unitarization the “unitarity corrections” to the masses should not be too large, since the tadpole loops partly cancel the $(\log \Lambda$ divergence in) s-channel hadron loops. One expects the corrections to the mass spectrum to be at most of the same order as the flavour symmetry breaking, since because of the renormalizability, one can in the flavour symmetric limit, include the unitarity corrections into the mass parameters, fixed by experiment.

Eq. (1) without $\mathcal{L}_{SB}$ is clearly invariant under $\Sigma \rightarrow U_L \Sigma U_R^\dagger$ of $U_3 \times U_3$. After shifting the flavourless scalar fields by the VEV’s $(\Sigma \rightarrow \Sigma + V)$ to the minimum of the potential, the scalars acquire masses and also the pseudoscalars obtain a (small) mass because of $\mathcal{L}_{SB}$. Then
the $\lambda$ and $\lambda'$ terms generate trilinear $spp$ and $sss$ couplings, in addition to those coming from the $U_A(1)$ symmetry breaking determinant term. The $\lambda$ term, which turns out to be the largest, obeys the OZI rule, while the $\lambda'$ and $\beta$ terms violate this rule.

### III. TREE-LEVEL MASSES.

It is an ideal problem for a symbolic program like Maple V to calculate the predicted masses, and couplings from the Lagrangian, which has 6 parameters, $\mu, \lambda, \lambda', \beta, u = d$ and $s$, of which the last two define the diagonal matrix $V$ with the flavourless meson VEV’s: $V = \text{diag}[u, d, s]$. These are at the tree level related to the pion and kaon decay constants through $u = d = < \sigma_{u\bar{u},dd} > / \sqrt{2} = f_\pi / \sqrt{2}$ (assuming isospin exact) and $s = < \sigma_{ss} > = (2f_K - f_\pi) / \sqrt{2}$. One finds denoting the often occurring combination $u^2 + 4\lambda'(u^2 + d^2 + s^2)$ by $\bar{\mu}^2$, and expressing the flavourless mass matrices in the ideally mixed frame:

$$m_{\pi^+}^2 = \bar{\mu}^2 + 4\lambda(u^2 + d^2 - ud) + 2\beta s$$
$$m_{K^+}^2 = \bar{\mu}^2 + 4\lambda(u^2 + s^2 - su) + 2\beta d$$
$$m_{\eta'}^2 = e_{igv} \begin{pmatrix} \bar{\mu}^2 + 2\lambda(u^2 + d^2) - 2\beta s & -\beta \sqrt{2}(u + d) \\ -\beta \sqrt{2}(u + d) & \bar{\mu}^2 + 4\lambda s^2 \end{pmatrix}$$
$$m_{\eta'_s}^2 = \bar{\mu}^2 + 4\lambda(u^2 + d^2 + ud) - 2\beta s$$
$$m_{\eta'_o}^2 = \bar{\mu}^2 + 4\lambda(u^2 + s^2 + su) - 2\beta d$$
$$m_{\sigma}^2 = e_{igv} \begin{pmatrix} \bar{\mu}^2 + 4\lambda'(u + d)^2 + 6\lambda(u^2 + d^2) + 2 + \beta s & (4\lambda' + \beta)\sqrt{2}(u + d) \\ (4\lambda' + \beta)\sqrt{2}(u + d) & \bar{\mu}^2 + 8\lambda' s^2 + 12\lambda s^2 \end{pmatrix}$$
$$\phi^{s\bar{s} - \eta'} = \frac{1}{2} \arctan \frac{-2\sqrt{2}\beta(u + d)}{2\lambda(u^2 + d^2 - 2s^2) - 2\beta s}$$
$$\phi^{s\bar{s} - f_0} = \frac{1}{2} \arctan \frac{2\sqrt{2}(4\lambda' + \beta)(u + d)}{4\lambda'(u + d)^2 - 2s^2 + 6\lambda(u^2 + d^2 - 2s^2) + 2\beta s}$$

where $e_{igv}$ means the eigenvalues of the matrix which follows.

Let us first discuss the flavour symmetric limit $u = d = s$. Then the pseudoscalar decay constants are equal $f_P = \sqrt{2}u = f_\pi = f_K$, while the mixing angles $\Theta_{\eta' - \text{singlet}} = \phi^{s\bar{s} - \eta'} - 54.73^\circ$ and $\Theta_{\sigma - \text{singlet}} = \phi^{s\bar{s} - \sigma} + 35.26^\circ$ vanish, and one has 4 nondegenerate physical masses for the octet and singlet mesons $m_{P_S}, m_{S_S}, m_{P_0}, m_{S_0}$. Then there are also simple
relations between the 5 model parameters $\lambda, \lambda', \bar{\mu}^2, \beta$ and $u = d = s$ and the 4 physical masses and the decay constant $f_P$:

$$\lambda = \frac{(3m_{S8}^2 - 2m_{P0}^2 - m_{P8}^2)}{(12f_P^2)}$$
(11)

$$\lambda' = \frac{(m_{S0}^2 - m_{S8}^2 - m_{P8}^2 + m_{P0}^2)}{(12f_P^2)}$$
(12)

$$\bar{\mu}^2 = \frac{(-3m_{S8}^2 + 5m_{P8}^2 + 4m_{P0}^2)}{6}$$
(13)

$$\beta = \frac{(m_{P8}^2 - m_{P0}^2)}{(3\sqrt{2}f_P)}$$
(14)

$$u = d = s = <\sigma_{u\bar{u}}>=<\sigma_{d\bar{d}}>=<\sigma_{s\bar{s}}>=f_P/\sqrt{2}$$
(15)

It is obvious that we can reparametrize the theory in terms of these physical quantities, which can be kept fixed in the renormalization, as long as flavour symmetry is exact. (If one chooses the same tree-level values for the parameters $\lambda, \mu^2, \beta$ as found below in Eq.(16) and $\lambda' = 1$ but for $u = d = s$ the average value, 75.05 MeV, or $f_P = 106$ MeV, one would have $m_{P8} = 384$ MeV, $m_{P0} = 956$ MeV, $m_{S8} = 1086$ MeV, $m_{S0} = 741$ MeV.) Thus these masses can be thought of as already “unitarized”. On the other hand the original parameters and induced tri-meson couplings will be renormalized from the tree level values.

Now breaking the flavour symmetry ($s \neq u = d$) we have only one more parameter, given by $f_K - f_\pi$, and it is evident that this breaking splits the degeneracy in the mass spectrum from 4 independent masses to 8 masses and generates two mixing angles. Thus one gets several tree-level predictions in particular for the scalars (Table I). Of course now we expect these tree-level predictions to receive corrections from the unitarization, but for small symmetry breaking (experimentally $(s - u)/s \approx 0.308$) one would expect these corrections not to be larger than this, i.e. $<30\%$.

We can fix 5 of the 6 parameters, leaving $\lambda'$ free, by the 5 experimental quantities from the pseudoscalar sector alone: $m_\pi, m_K, m_\eta^2 + m_{\eta'}^2, f_\pi = 92.42$ MeV and $f_K = 113$ MeV \cite{3}, which all are accurately known from experiment. One finds that at the tree level

$$\lambda = 11.57, \bar{\mu}^2 = 0.1424 \text{ GeV}^2, \beta = -1701 \text{ MeV}, u = d = 65.35 \text{ MeV}, s = 94.45 \text{ MeV}.$$  
(16)

The remaining $\lambda'$ parameter changes only the $\sigma$ and $f_0$ masses and their trilinear couplings, not those of the pseudoscalars. Gavin et. al. \cite{6} calls for a group theoretical reason for this
simplification. In fact, graphically it is easy to see that the $\lambda'$ term can only break the OZI rule for the scalars, which can couple to the vacuum, but for the pseudoscalars the $\lambda'$ term must leave the OZI rule intact. Therefore $\lambda'$ and $\mu$ affects the pseudoscalars only through the combination $\hat{\mu}^2 = \mu^2 + 4\lambda'(u^2 + d^2 + s^2)$.

It is of some interest that the simple Gell-Mann–Okubo mass formula for the mixing does not give the same result for the mixing angle between $\eta$ and $\eta'$ as our model. This is because there is flavour symmetry breaking also in the anomaly terms $\beta_d$ and $\beta_s$ in Eqs. (3-10) above. E.g. for the octet pseudoscalar mass one gets from Eqs. (3-10) $(4m_K^2 - m_\pi^2)/3 = 542.5$ MeV and a mixing angle of $\Theta^{ss-\eta'} = -12.7^\circ$. This would be closer to the conventional mixing angle ($-10^\circ$ to $-23^\circ$), than our model $-5.0^\circ$, where the octet $\eta_8$ mass is 566.1 MeV.

Some of the couplings of $\sigma$ and $f_0$ depend sensitively on $\lambda'$, since $\lambda'$ changes the small ideal mixing angle, $\phi^{ss-f_0}$. It turns out below that $\lambda'$ must be small, compared to $\lambda$, in order to fit the tri-linear couplings. By putting $\lambda' = 1$ one gets a reasonable compromise for most of these couplings. With $\lambda' \approx 3.75$ one almost cancels the OZI rule breaking coming from the determinant term, and the scalar mixing becomes near ideal (for $\lambda' \approx -\beta/(4s) = 4.5$ the cancellation is exact).

As can be seen from Table I the predictions are not far from the experimental masses taken as $a_0(980)$, $f_0(980)$, $K_0^*(1430)$, and $\sigma(500)$. For a discussion of the existence of the $\sigma(500)$ see my recent Frascati talk [4], which also includes some preliminary results of the present paper. Considering that one expects from our previous UQM analysis of the scalars [9] and our discussion above that unitarity corrections can be up to 30% , and should go in the right direction compared to experiment, one must conclude that these results are even better than expected. Similar mass analyses as in Table I have been done in Refs. [5,7], although with somewhat different input data.

IV. TRI-LINEAR COUPLINGS AT THE TREE LEVEL.

The trilinear coupling constants follow from the Lagrangian after one has made the shift $\Sigma \rightarrow \Sigma + V$. The most important spp couplings are at the tree level, when expressed in terms of the original parameters:
\[
g_{\kappa^+K^0\pi^+} = 4\lambda(d - u + s) - 2\beta
\]
\[
g_{\kappa^+K^0\eta} = -4\lambda u \sin \phi^{s\bar{s}-\eta'} + (2\sqrt{2}\lambda s + \beta \sqrt{2}) \cos \phi^{s\bar{s}-\eta'}
\]
\[
g_{\kappa^+K^0\eta'} = 4\lambda u \cos \phi^{s\bar{s}-\eta'} + (2\sqrt{2}\lambda s + \beta \sqrt{2}) \sin \phi^{s\bar{s}-\eta'}
\]
\[
g_{\sigma^\pi^+\pi^-} = 2\sqrt{2} \cos \phi^{s\bar{s}-f_0}(u + d)[\lambda + 2\lambda'] - \sin \phi^{s\bar{s}-f_0}(8\lambda's + 2\beta)
\]
\[
g_{\sigma^\pi^0\pi^-} = \cos \phi^{s\bar{s}-f_0}\sqrt{2}[\lambda(4u - 2s) + 4\lambda'(u + d) + \beta] + 4\sin \phi^{s\bar{s}-f_0}[\lambda(u - 2s) - 2\lambda's] \]
\[
g_{f_0^\pi^+\pi^-} = 2\sqrt{2} \sin \phi^{s\bar{s}-f_0}(u + d)[\lambda + 2\lambda'] + \cos \phi^{s\bar{s}-f_0}(8\lambda's + 2\beta)
\]
\[
g_{f_0^\pi^0\pi^-} = \sin \phi^{s\bar{s}-f_0}\sqrt{2}[\lambda(4u - 2s) + 4\lambda'(u + d) + \beta] - 4\cos \phi^{s\bar{s}-f_0}[\lambda(u - 2s) - 2\lambda's]
\]
\[
g_{a_0^\pi\eta} = \cos \phi^{s\bar{s}-\eta'}2\sqrt{2}\lambda(u + d) - 2\beta \sin \phi^{s\bar{s}-\eta'}
\]
\[
g_{a_0^\pi\eta'} = \sin \phi^{s\bar{s}-\eta'}2\sqrt{2}\lambda(u + d) + 2\beta \cos \phi^{s\bar{s}-\eta'}
\]
\[
g_{a_0^\kappa^+\pi^-} = \sqrt{2}[\lambda(4u - 2s) - 4\lambda'(d - u) - \beta]
\]

In fact these can be written in more useful forms in terms of the predicted physical masses and mixing angles and decay constants:

\[
g_{\kappa^+K^0\pi^+} = (m^2_\kappa - m^2_\pi)/(\sqrt{2} f_K)
\]
\[
g_{\kappa^+K^0\eta} = -\sqrt{3} \sin(\phi^{s\bar{s}-\eta'} - 35.26^\circ)(m^2_\kappa - m^2_\eta)/(2f_K)
\]
\[
g_{\kappa^+K^0\eta'} = \sqrt{3} \cos(\phi^{s\bar{s}-\eta'} - 35.26^\circ)(m^2_\kappa - m^2_\eta')/(2f_K)
\]
\[
g_{\sigma^\pi^+\pi^-} = \cos \phi^{s\bar{s}-f_0}(m^2_\sigma - m^2_\pi)/f_\pi
\]
\[
g_{\sigma^\pi^0\pi^-} = -\sqrt{3} \sin(\phi^{s\bar{s}-f_0} - 35.26^\circ)(m^2_\sigma - m^2_K)/(2f_K)
\]
\[
g_{f_0^\pi^+\pi^-} = \sin \phi^{s\bar{s}-f_0}(m^2_{f_0} - m^2_\pi)/f_\pi
\]
\[
g_{f_0^\pi^0\pi^-} = \sqrt{3} \cos(\phi^{s\bar{s}-f_0} - 35.26^\circ)(m^2_{f_0} - m^2_K)/(2f_K)
\]
\[
g_{a_0^\pi\eta} = \cos \phi^{s\bar{s}-\eta'}(m^2_{a_0} - m^2_\eta)/f_\pi
\]
\[
g_{a_0^\pi\eta'} = \sin \phi^{s\bar{s}-\eta'}(m^2_{a_0} - m^2_\eta')/f_\pi
\]
\[
g_{a_0^\kappa^+\pi^-} = (m^2_{a_0} - m^2_\kappa)/f_K
\]

In table II 8 different spp couplings are compared with quoted experimental numbers. In some of the channels of table II the resonance is below threshold and the widths therefore vanish at the resonance mass. However, the coupling constants have recently been determined through a loop diagram from \(\phi \to K\bar{K} \to \gamma\pi\pi\) and \(\phi \to K\bar{K} \to \gamma\pi\eta\) (albeit in a
somewhat model dependent way) by the Novosibirsk group [11, 12]. For channels where the phase space is large, it is important that one includes a form factor related to the finite size of physical mesons. In the $^3P_0$ quark pair creation model a radius of $\approx 0.8$ fm leads to a gaussian form factor, as in the formula below, where $k_0 \approx 0.56$ GeV/c (as was found in the UQM [9]). Thus the widths are computed from the formula:

$$\Gamma(m) = \sum_{\text{isospin}} \frac{g_i^2}{8\pi} \frac{k_{cm}^2(m)}{m^2} e^{-\left[k_{cm}(m)/k_0\right]^2}.$$ (37)

As can be seen from table II most of the couplings are not far from experiment. Only the $f_0 \rightarrow \pi\pi$ and $a_0 \rightarrow \pi\eta$ couplings and widths come out a bit large, but these are very sensitive to higher order loop corrections due to the $K\bar{K}$ threshold, and $f_0 \rightarrow \pi\pi$ is extremely sensitive to the scalar near-ideal mixing angle and $\lambda'$. If one choses $\lambda' = 3.75$ this mixing angle nearly vanishes ($\phi^{s8-f_0} = -3.0^\circ$) together with the $f_0 \rightarrow \pi\pi$ coupling (c.f. eq.(32)). From our experience with the UQM [9] the $a_0 \rightarrow K\bar{K}$ peak width, when unitarized, is reduced, because of the $K\bar{K}$ theshold, by up to a factor 5. Therefore one cannot expect that the tree level couplings should agree better with data than what those of Table II do. After all, this is a very strong coupling model ($\lambda = 11.57$, leading to large $g_i^2/4\pi$) and higher order effects should be important.

V. CONCLUSIONS.

In summary, I find that the linear sigma model with three flavours, at the tree level, works much better than what is generally believed. When the 6 model parameters are fixed mainly by the pseudoscalar masses and decay constants, one predicts the 4 scalar masses and mixing angle to be near those of the experimentally observed nonet $a_0(980)$, $f_0(980)$, $\sigma(500)$, $K_0^*(1430)$. Also 8 couplings/widths of the scalars to two pseudoscalars are predicted reasonably close to their presently known, rather uncertain experimental values. The agreement is good enough considering that some of these are expected to have large higher order corrections. The model works, in my opinion, just as well as the naive quark model works for the heavier nonets. A more detailed data comparison would become meaningful, after one has included higher order effects, i.e. after one has unitarized the model, e.g., along the lines of the UQM [9].
Those working on chiral perturbation theory and nonlinear sigma models usually point out that the linear model does not predict all low energy constants correctly. However, one should remember that the energy regions of validity are different for the two approaches. Chiral perturbation theory usually breaks down when one approaches the first scalar resonance. The linear sigma model, on the other hand, includes the scalars from the start and can be a reasonable interpolating model in the intermediate energy region near 1 GeV, where QCD is too difficult to solve.

These results strongly favour the interpretation that the $a_0(980)$, $f_0(980)$, $\sigma(500)$, $K^*_0(1430)$ belong to the same nonet, and that they are the chiral partners of the $\pi$, $\eta$, $\eta'$, $K$. If the latter are believed to be unitarized $q\bar{q}$ states, so are the light scalars $a_0(980)$, $f_0(980)$, $\sigma(500)$, $K^*_0(1430)$, and the broad $\sigma(500)$ should be interpreted as an existing resonance. The $\sigma$ is a very important hadron indeed, as is evident in the sigma model, because this is the boson which gives the constituent quarks most of their mass and thereby it gives also the light hadrons most of their mass. Therefore it is natural to consider the $\sigma(500)$ as the Higgs boson of strong interactions.

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TABLE I. Predicted masses in MeV and mixing angles for two values of the $\lambda'$ parameter. The asterix means that $m_\pi, m_K$ and $m_\eta^2 + m_\eta^2$ are fixed by experiment together with $f_\pi = 92.42$ MeV and $f_K = 113$ MeV.

| Quantity       | Model $\lambda' = 1$ | Model $\lambda' = 3.75$ | Experiment |
|----------------|-----------------------|--------------------------|------------|
| $m_\pi$        | 137*                  | 137*                     | 137        |
| $m_K$          | 495*                  | 495*                     | 495        |
| $m_\eta$       | 538*                  | 538*                     | 547.3      |
| $m_\eta'$      | 963*                  | 963*                     | 957.8      |
| $\Theta_{\eta'-singlet}$ | -5.0°                | -5.0°                    | (-16.5±6.5)° |
| $m_{a_0}$      | 1028                  | 1028                     | 983        |
| $m_\kappa$     | 1123                  | 1123                     | 1430       |
| $m_\sigma$     | 651                   | 619                      | 400-1200   |
| $m_{f_0}$      | 1229                  | 1188                     | 980        |
| $\Theta_{\sigma-singlet}$ | 21.9°                | 32.3°                    | (28-i8.5)° |
TABLE II. Predicted couplings $\sum_i g_i^2$ (in GeV$^2$), when $\lambda' = 1$, compared with experiment and predicted widths with experiment (in MeV). (We have used isospin invariance to get the sum over charge channels, when there is data for one channel only.) The predicted $f_0 \rightarrow \pi\pi$ width is extremely sensitive to the value of $\lambda'$ (for $\lambda' = 3.75$ it nearly vanishes) and unitarity effects as discussed in the text. Also the $a_0\pi\eta$ coupling is very sensitive to loop corrections due to the $K\bar{K}$ threshold.

| Process                  | $\sum_i g_i^2$ in model | $\sum_i g_i^2$ in experiment | $\sum_i \Gamma_i$ model | $\sum_i \Gamma_i$ experiment |
|--------------------------|--------------------------|-------------------------------|--------------------------|-----------------------------|
| $\kappa^+ \rightarrow K^0\pi^+ + K^+\pi^0$ | 7.22                     | -                             | 678                      | 278 ± 23 [10]               |
| $\kappa^+ \rightarrow K^+\eta$              | 0.28                     | $\approx 0$ [10]              | 13                       | < 26 [10]                   |
| $\sigma \rightarrow \pi^+\pi^- + \pi^0\pi^0$ | 2.17                     | 1.95 [11]                     | 574                      | 300-1000 [3]                |
| $\sigma \rightarrow K^+K^- + K^0\bar{K}^0$  | 0.16                     | 0.004 [11]                    | 0                        | 0                           |
| $f_0 \rightarrow \pi^+\pi^- + \pi^0\pi^0$  | 1.67                     | $0.765^{+0.20}_{-0.14}$ [12]  | see text                 | 40 - 100 [3]                |
| $f_0 \rightarrow K^+K^- + K^0\bar{K}^0$     | 6.54                     | $4.26^{+1.78}_{-1.12}$ [12]   | 0                        | 0                           |
| $a_0^+ \rightarrow \pi^+\eta$               | 2.29                     | 0.57 [12]                     | 273 see text             | 50 - 100 [3]                |
| $a_0^+ \rightarrow K^+\bar{K}^0$            | 2.05                     | $1.34^{+0.36}_{-0.28}$ [12]   | 0                        | 0                           |