The effect of a polymer material coating on the stress state of plate building structures with holes

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Abstract. Polymeric materials, in particular reinforced coatings, are widely used for the repair of worn and damaged plate building components of various types. Coatings made of polymeric materials not only increase the plate components’ thickness and smooth out the surface roughness, but also provide a reduction in stresses in the region of existing holes or perforations. The article presents a calculation technique that allows to increase the accuracy and reliability of determining the ability of coatings made of polymer materials to reduce stress concentration. Mathematical models for determining the stress state parameters of a plate building component with holes covered with a polymer material are obtained. The calculations confirmed that the use of polymeric materials as a coating reduces the normal and tangential stresses arising at the edge of the hole.

Introduction

Plate building structures are widely used in construction and by weight account for about 20% of all types of metal structures. Examples of their use include: tanks and bunkers for storing liquid and bulk solids; a casing; special technological installations of chemical and oil refining industries; spatial overlap, etc. [1–4].

A feature of the operation of plate structures is that they interact with the environment over a significant area. Under the influence of adverse environmental factors, such as atmospheric moisture, water, aggressive liquids and gases, oil products, etc., the plate metal structures undergo corrosion (rusting).

The most dangerous consequence of plate building structures’ corrosion is perforations. As a result of this, there is not only a deterioration in operational characteristics and a decrease in the bearing capacity of the actual plate structures, but also a premature decommissioning of the facility as a whole.

One of the effective methods for the restoration of perforated plate structures is the use of reinforced coatings made of polymer materials [5–7]. Despite numerous studies in this area [8–14], the functionality and stress-strain state of such coatings have not yet been studied sufficiently, especially for the specific types of corrosion.

Design scheme

Let us consider a metal plate building structure with perforation (hole) coated with a polymer material. When drawing up the design scheme, we select the area of the plate component (base) connection with a polymer coating at the boundary of the hole. The considered site is under the action of external...
forces located at some distance from this compound (Fig. 1). Let the following act near the adhesive layer in the coating: axial force $N$, cutting force $Q$ and bending moment $M$; and in metal, respectively $-N$, $Q$ and $M^*$. 

We introduce the following notation: $h_1$, $h_2$ – denote thickness of polymer coating and plate component (base); $E_1$, $E_1$ – define the elasticity modulus of the coating material and the base; $\mu_1$, $\mu_2$ – denote the Poisson’s ratio of coating material and base; $a$ – is the length of the interface between the coating and the base.

**Figure 1.** The design scheme for the connection of the base with the coating: 1 - polymer coating; 2 – plate component (base); 3 - hole.

Let us separate the elements of the plate composite structure and introduce the normal separation stresses in the adhesive layer $\sigma(x)$ and shear stresses $\tau(x)$. In addition, to maintain the equilibrium conditions and joint deformation of the base and coating elements, we introduce the concentrated forces at the ends of the adhesive layer $T_1$, $T_2$ and the focused moments $M_1$ and $M_2$ (Figure 2,a and 2,b).

**Figure 2.** The structural component of the base connection with the coating: a - element of the polymer coating; b – plate component (base).

**Mathematical model**

Bending moment $M^*$ is connected by the equilibrium condition of the plate composite structure with axial force $N$, cutting force $Q$ and bending moment $M$, which can be written as

$$M^* = M + Qa - 0.5N(h_1 + h_2).$$

(1)

The components of the concentrated efforts introduced at the ends of the metal-coating type composite structure adhesive layer are determined from the equilibrium conditions of this structure’s elements and the conditions of their joint deformation:

$$R_1 = \frac{1}{a} \left[ N \frac{h_1}{2} + M_1 - M - Qa + \int_0^a \sigma(t)(a-t)dt \right];$$

(2,a)
The conditions (2, a), (2, b) and (4, a), (4, b) correspond to the equilibrium conditions of the plate composite construction elements, and the conditions (3, a), (3, b) express the elements curvature radii equality at the ends of the adhesive layer.

In the cross sections of the structural elements that make up the plate composite structure, internal force factors (axial forces and bending moments) act.

We determine the magnitude of these internal force factors by applying the section method [15].

In the sections of the element from the polymer coating we will have:

\[ N_p = P_1 + N + \int_0^x \tau(t) dt; \]  
\[ M_p = M + P_1 \frac{h_1}{2} + (R_1 + Q)x - \int_0^x \sigma(t)(x-t) dt + \frac{h_1}{2} \int_0^x \tau(t) dt. \]  

In the sections of a metal plate component we get:

\[ N_M = -P_1 - \int_0^x \tau(t) dt; \]  
\[ M_M = M_1 + P_1 \frac{h_1}{2} - R_1 x + \int_0^x \sigma(t)(x-t) dt + \frac{h_1}{2} \int_0^x \tau(t) dt. \]  

In accordance with the values of internal factors, we write the expressions for determining the curvature radii of the plate composite structure elements as

\[ \frac{1}{12 \rho_p} = \frac{M_p}{E_1 h_1^3} \quad \text{and} \quad \frac{1}{12 \rho_M} = \frac{M_M}{E_2 h_2^3}. \]

To determine the linear deformations \( \varepsilon_x \), occurring in the adhesive layer, the expressions can be written as:

\[ \varepsilon_{x,p} = \frac{6M_p}{E_1 h_1^3} + \frac{N_p}{E_1 h_1} - \frac{\mu_1 \sigma(x)}{E_1} \quad \text{and} \quad \varepsilon_{x,M} = \frac{6M_M}{E_2 h_2^3} + \frac{N_M}{E_2 h_2} - \frac{\mu_2 \sigma(x)}{E_2}. \]

Equating the curvature radii of the compositional structures’ elements and linear deformations of these elements in the adhesive layer, we obtain the following system of equations:
Let us simplify the system of equations (8) by introducing the following notation:

\[
\begin{align*}
\alpha &= \frac{\mu_2 E_1 - \mu_1 E_2}{E_1 E_2} ; \\
\beta &= \frac{E_1 h_1^2 - E_2 h_2^2}{2 E_1 E_2 h_1^2 h_2^2} ; \\
\gamma &= \frac{4(E_1 h_1^3 - E_2 h_2^3)}{E_1 E_2 h_1 h_2} ; \\
\theta &= \frac{E_1 h_1^3 + E_2 h_2^3}{E_1 E_2 h_1^3 h_2^3} ; \\
A &= \frac{6M}{E_1 h_1^2} + \frac{6M}{E_2 h_2^2} + \frac{N}{E_1 h_1} .
\end{align*}
\]

Then the system of equations (10) takes the following form:

\[
\begin{align*}
\alpha \sigma(x) + 6 \beta \int_0^x \sigma(t)(x-t) dt + \gamma \int_0^x \tau(t) dt + 6x \left( \frac{Q}{Eh_1^2} - \beta R_1 \right) + A &= 0 ; \\
\theta \int_0^x \sigma(t)(x-t) dt + \frac{\beta}{2} \int_0^x \tau(t) dt - x \left( \frac{Q}{Eh_1^2} - \theta R_1 \right) &= 0 .
\end{align*}
\]

We exclude the tangential stresses from this system of equations \( \tau(x) \) and we obtain the integral equation for normal separation stresses \( \sigma(x) \)

\[
\frac{\alpha \beta}{2} \sigma(x) + (3\beta^2 - \gamma \theta) \int_0^x \sigma(t)(x-t) dt + x(\gamma \theta - 3\beta^2 R_1 + kQ) + \frac{\beta A}{2} = 0 ,
\]

where \( k = \frac{3\beta h_1}{E_1 h_1^3} - \gamma \).

We transform the equation (10) by introducing the following notation:
\[ K^2 = \frac{2(\gamma \theta - 3 \beta^2)}{\alpha \beta}, \quad B = \frac{A}{\alpha}. \]

As a result, we obtain the equation for normal stresses \( \sigma(x) \):

\[ \sigma(x) - K^2 \int_0^x \sigma(t)(x-t)dt + x(K^2 R_1 + kQ) + B = 0. \]  (11)

We differentiate the equation (11) with respect to \( x \):

\[ \sigma'(x) - K^2 \int_0^x \sigma(t)(x-t)dt + K^2 R_1 + kQ = 0. \]  (12)

After double differentiation of equation (11) we obtain a second-order differential equation with respect to \( x \):

\[ \sigma(x) : \sigma''(x) - K^2 \sigma(x) = 0. \]  (13)

The general solution of this equation has the form:

\[ \sigma(x) = C_1 e^{kx} + C_2 e^{-kx}. \]  (14)

Permanent \( C_1 \) and \( C_2 \) are determined from the equations (11) and (12). Substituting the general solution into equation (12) gives

\[ C_1 - C_2 = -KR_1 - \frac{kQ}{K}. \]  (15)

Considering that the projection \( R_1 \) is dependent on voltage \( \sigma(x) \), we obtain the following equation for constants \( C_1 \) and \( C_2 \):

\[
\begin{align*}
C_1 & \left( \frac{e^{ka}}{k} - \frac{1}{ak} + \frac{kBD}{2\theta} e^{ka} \right) \frac{1}{a} + C_2 \left( \frac{e^{-ka}}{k} - \frac{1}{ak} + \frac{kBD}{2\theta} e^{-ka} \right) \frac{1}{a} = \\
& = -k \left( \frac{Qa}{\partial E_2 h_2^2} + Qa + \frac{BF}{2\theta} \right) \frac{1}{a} - \frac{KQ}{k} \\
& = -k \left( \frac{Qa}{\partial E_2 h_2^2} + Qa + \frac{BF}{2\theta} \right) \frac{1}{a} - \frac{KQ}{k} \\
& = \frac{1}{\partial E_2 h_2^2} \left[ G + 6Qa \left( \frac{1}{E_1 h_1^2} - \frac{\beta}{\partial E_2 h_2^2} \right) \right],
\end{align*}
\]  (16)

where \( D = -\frac{\alpha \partial E_1 h_1^2}{3\beta \left( 3 - \frac{\gamma}{\beta h_1} \right)} \); \( F = \frac{\partial E_1 h_1^2}{3\beta \left( 3 - \frac{\gamma}{\beta h_1} \right)} \);

\[
G = \frac{2M}{E_1 h_1^2} \left( 3 - \frac{\gamma}{\beta h_1} \right) + \frac{N}{E_1 h_1} + \frac{2}{E_2 h_2^2} \left( 3 - \frac{\gamma}{\beta h_1} \right) \left( M - \frac{h_1}{2} N + \frac{Qa}{\partial E_2 h_2^2} \right).
\]

We obtain the second equation for constants \( C_1 \) and \( C_2 \) after substituting the solution of equation (14) into the integral equation (11) in the form \( C_1 + C_2 = B \).

Considering that the quantity \( D \) depends on \( \int_0^a \tau(t)dt \) and \( \int_0^a \tau(t)dt = D\sigma(a) + F \), we get the second equation to determine the integration constants \( C_1 \) and \( C_2 \):
\[
C_1 \left( 1 + \frac{BDJ}{2 \alpha \theta} e^{\frac{\gamma}{\beta h_2}} \right) - C_2 \left( 1 + \frac{BDJ}{2 \alpha \theta} e^{-\frac{\gamma}{\beta h_2}} \right) = -\frac{1}{\alpha} \left( G - \frac{\rho FJ}{2 \theta} \right), \tag{17}
\]

where \( J = C_1 \left( 3 - \frac{\gamma}{\beta h_2} \right) \).

The equations (16) and (17) determine the constants \( C_1 \) and \( C_2 \). After solving them, the value of normal stresses \( \sigma(x) \) is determined by the formula (14), and shear stresses \( \tau(x) \) can be determined after differentiating the second equation from system (9)

\[
\tau(x) = \frac{2}{\beta} \int_0^\frac{\pi}{\alpha} \sigma(t) dt + \frac{2}{\beta} \left( \phi R_1 + \frac{Q}{E_i h_1^2} \right) = 0. \tag{18}
\]

**Calculation Results and Discussion**

Figure 3 shows the curves characterizing the influence of the overlapping polymer coating length of the hole contour \( a \) to maximum normal \( \sigma_{\text{max}} \) and tangents \( \tau_{\text{max}} \) voltage. As shown by the calculation results, the highest stresses occur at the edge of the hole. An increase in the overlap length leads to a nonlinear decrease in normal stresses by 3.8 ... 4.9 times, and the tangential stresses tend to zero.

Figure 4 presents the curves showing the distribution nature of the normal \( \sigma_0 \) and tangents \( \tau_0 \) stresses along the length of the polymer overlap with its constant length. As it can be seen from the results obtained, the normal and tangential stresses nonlinearly decrease along the overlap length to insignificant values of the 0.6 ... 7.0 MPa order.

**Figure 3.** The dependence of the maximum normal and tangential stresses on the overlap length by coating the contour of the hole

**Figure 4.** Distribution of normal and shear stresses along the overlap length at \( a = 140 \) mm

Thus, the stress values decrease significantly with increasing the overlap length (Fig. 3) and with increasing the distance from the edge of the hole (Fig. 4).

The calculations were performed according to the mathematical dependencies (9) - (18). Dotted lines correspond to the case of the distributed load action, solid lines correspond to the concentrated load action.

**Summary**

The methodology for determining normal and shear stresses in a plate component having a hole molded with a reinforced polymer coating is developed and an algorithm is presented (mathematical
dependsences (9) - (18)). This makes it possible to assess the stress state in the connection area between the base and the coating, as well as to establish the optimal amount of the hole contour overlap with a polymer coating and its required thickness.

It is shown that the use of a polymer coating for sealing perforations in plate metal building structures allows reducing stresses at their hole edges by increasing the overlap length.

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