Critical Opalescence around the QCD Critical Point and Second-order Relativistic Hydrodynamic Equations Compatible with Boltzmann Equation

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Abstract
The dynamical density fluctuations around the QCD critical point (CP) are analyzed using relativistic dissipative fluid dynamics, and we show that the sound mode around the QCD CP is strongly attenuated whereas the thermal fluctuation stands out there. We speculate that if possible suppression or disappearance of a Mach cone, which seems to be created by the partonic jets at RHIC, is observed as the incident energy of the heavy-ion collisions is decreased, it can be a signal of the existence of the QCD CP. We have presented the Israel-Stewart type fluid dynamic equations that are derived rigorously on the basis of the (dynamical) renormalization group method in the second part of the talk, which we omit here because of a lack of space.

1. Introduction

A unique feature of the QCD phase diagram is the existence of a critical point. At the QCD CP, the first order phase transition terminates and turns to a second order phase transition. Around a critical point of a second order transition, we can expect large fluctuations of various quantities, and more importantly there should exist a soft mode associated to the CP. The QCD CP belongs to the same universality class as the liquid-gas phase transition point, and, hence, the density fluctuating mode in the space-like region is a softening mode at the CP; The would-be soft mode of the chiral transition, the $\sigma$ mode, is coupled to the density fluctuation[1] and becomes a slaving mode of the density variable[2]; see [3] for another argument on the fate of the $\sigma$ mode around the CP.

The density fluctuation depends on the transport as well as thermodynamic quantities that show an anomalous behavior around the critical point. In particular, we should note that the density-temperature coupling which was not explicitly taken into account can be important. In fact, the dynamical density fluctuations are analyzed in the non-relativistic case with use of the Navier-Stokes equation, which shows that the Rayleigh peak due to the thermal fluctuation would overwhelm the Brillouin peak due to the sound modes[4].

We apply for the first time relativistic fluid dynamic equations to analyze the spectral properties of density fluctuations, and examine possible critical phenomena. We shall show that even the so called first-order relativistic fluid dynamic equations have generically no problem to describe fluid dynamical phenomena with long wave lengths contrary to naive expectation. In this report[5], we shall show that the genuine and remaining soft mode at the QCD CP is not a sound mode but the diffusive thermal mode that is coupled to the sound mode, and that the possible...
divergent behavior of the viscosities might not be observed through the density fluctuations because the sound modes are attenuated around the CP and would eventually almost die out at the CP.

2. Relativistic fluid dynamic equations for a viscous system

The fluid dynamic equations are the balance equations for energy-momentum and particle number, \( \partial_{\mu}T^{\mu\nu} = 0, \partial_{\mu}N^{\mu} = 0 \), where \( T^{\mu\nu} \) is the energy-momentum tensor and \( N^{\mu} \) the particle current, respectively. They are expressed as \( T^{\mu\nu} = (\epsilon + P)u^{\mu}u^{\nu} - P g^{\mu\nu} + \tau^{\mu\nu} \) and \( N^{\mu} = nu^{\mu} + \nu^{\mu} \), where \( \epsilon \) is the energy density, \( P \) the pressure, \( u^{\mu} \) the flow velocity, and \( n \) the particle density, the dissipative part of the energy-momentum tensor and the particle current are denoted by \( \tau^{\mu\nu} \) and \( \nu^{\mu} \), respectively.

The so called first order equations such as Landau[6] and Eckart[7] equations are parabolic and formally violates the causality, and are hence called acausal. The causality problem is circumvented in the Israel-Stewart equation[8], which is a second-order equation with relaxation times incorporated. One should, however, note that the problem of the causality is only encountered when one tries to describe phenomena with small wave lengths beyond the valid region of the fluid dynamics: The phenomena which the fluid dynamics should describe are slowly varying ones with the wave lengths much larger than the mean free path. Indeed, the results for fluid dynamical modes with long wave lengths are qualitatively the same irrespective whether the second-order or first-order equations are used or not[5]. As for the instability seen in the Eckart equation[9], a new first-order equation in the particle frame constructed by Tsumura, Kunihiro and Ohnishi (TKO) [10] has no such a pathological behavior. We employ Landau[6], Eckart[7], Israel-Stewart(I-S)[8] and TKO equation.

3. Spectral function of the dynamical density fluctuation

By linearizing the fluid dynamic equation around the equilibrium, we can obtain the spectral function of the density fluctuation. The calculational procedure is an extension of the non-relativistic case described in the text book [4].

The spectral function derived from the Landau equation is found to be

\[
S_{nn}(k, \omega) = \langle (\delta n(k, t = 0))^2 \rangle \left[ 1 - \frac{1}{\gamma} \frac{2\Gamma_B k^2}{\omega^2 + \Gamma_B^2 k^4} \right] \quad \text{(1)}
\]

Here, the first factor represent the static spectral function, which would show a divergent behavior in the forward angle \( (k = 0) \) at the CP; this is known as the critical opalescence. The first term in the square bracket represents the thermal mode called Rayleigh mode, whereas the second and the third the sound mode or Brillouin mode.

The Eckart equation in the particle frame does not give a sensible result for the dynamical density fluctuation, in accord with its pathological property [6]. It is noteworthy that newly proposed equation, the TKO equation[10], in the particle frame gives a sensible result even thou it is a first-order equation. We have also applied the Israel-Stewart equation[8] in the particle frame to obtain the spectral function for the dynamical density fluctuation. The result is the same as that of Landau equation; this tells us that the modified part to circumvent the causality problem does not affect the dynamics in the proper fluid dynamic regime.
4. Critical behavior of the dynamical density fluctuations

We examine the critical behavior of the spectral function of the density fluctuations around the QCD CP. We introduce the static critical exponents \( \tilde{\gamma} \) and \( \tilde{\alpha} \) which are defined as follows

\[
\tilde{c}_n = c_0 t^{-\tilde{\gamma}}, \quad \tilde{K}_T = K_0 t^{-\tilde{\alpha}},
\]

where \( t = |(T - T_c)/T_c| \) is a reduced temperature, \( c_0 \) and \( K_0 \) are constants and \( \tilde{K}_T = (1/n_0)(\partial n/\partial P)_T \) is the isothermal compressibility. We also denote the exponent of the thermal conductivity by \( \tilde{a}_\kappa \), i.e.,

\[
\tilde{\kappa} = \kappa_0 t^{-\tilde{a}_\kappa},
\]

where \( \kappa_0 \) is a constant. It is known that \( \tilde{a}_\kappa \sim 0.6 \) around the liquid-gas phase transition point.

Unfortunately or fortunately, these singular behaviors of the width of the Brillouin peaks around the QCD CP may not be observed. The strengths of the Rayleigh and the Brillouin peaks are given in terms of \( \gamma \) as seen from eq. (1), the ratio of the specific heats, which behaves like

\[
\gamma = \tilde{c}_p/\tilde{c}_n \sim t^{\tilde{\gamma}+\tilde{\alpha}} \to \infty,
\]

in the critical region. Then the strength of the Brillouin peaks is attenuated and only the Rayleigh peak stands out in the critical region, as shown in Fig. 1.

Let \( \xi = \xi_0 t^{-\nu} \) be the correlation length which diverges as the critical point is approached. If we write the wave length of the sound mode by \( \lambda_s \), the fluid dynamic regime is expressed as

\[
\xi \ll \lambda_s,
\]

with which condition the sound mode can develop. However, in the vicinity of the critical point, the correlation length \( \xi \) becomes very large and eventually becomes infinity, so the above inequality can not be satisfied, and the sound mode can not be developed in the vicinity of the critical point.

From this argument, we can speculate about the fate of the possible Mach cone formation by the particle passing through the medium with a speed larger than the sound velocity \( c_s \). Such a Mach-cone like particle correlations are observed in the RHIC experiment. Then the disappearance or suppression of the Mach cone according to the lowering of the incident energy by RHIC would be a signal of the existence of the QCD critical point provided that the incident energy is large enough to make parton jets.
5. Concluding remarks

In this report, the density fluctuations is analyzed using the relativistic fluid dynamic equations\cite{5}. We have suggested that a suppression or disappearance of the Mach cone formation with lowering the incident energy at RHIC can be a signal of the detection of the QCD CP. Although we have presented the Israel-Stewart type fluid dynamic equations that are derived rigorously on the basis of the (dynamical) renormalization group method, we omit them here because of a lack of space. For the details, we refer to the submitted paper\cite{13}, where it is shown that the transport coefficients have no frame dependence while the relaxation times are generically frame-dependent in the derived equations.

Acknowledgments

This work was partially supported by a Grant-in-Aid for Scientific Research by the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan (No. 20540265), and by the Grant-in-Aid for the global COE program "The Next Generation of Physics, Spun from Universality and Emergence" from MEXT.

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