ERRATA TO LOCAL CONTRIBUTION TO THE LEFSCHETZ FIXED POINT FORMULA

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ABSTRACT. This note contains a correction to the paper, “Local contribution to the Lefschetz fixed point formula”, Inv. Math. 111 (1993), 1-33.

This note concerns the paper, “Local contribution to the Lefschetz fixed point formula”, Inv. Math. 111 (1993), 1-33. We are grateful to S. Morel for pointing out two errors in this paper. The first error occurs in the proof of Lemma 5.10 (which appears in §5.11). The second error occurs in the proof of Theorem 4.7 for the case $j = 4$ (which appears in §7.3). Indeed, Theorem 4.7 is false as stated.

The two errors are related. The correction consists of adding the following hypothesis to the definition (3.1) of a weakly hyperbolic neighborhood $W$,

$$(d) \quad c_1^{-1}(F') \cap c_2^{-1}(F') \cap W = F$$

where $F' = c_1(F) = c_2(F)$ as in §3.1. This condition holds automatically if the correspondence $C$ is the graph of a function.

The addition of condition (d) makes Lemma 5.10 obvious. Lemma 5.10 is used in §5.13 (the proof of Proposition 5.7) which in turn is needed for the proof of Theorem 4.7. The error in §7.3 occurs in §7.3 Step 2 (the construction of a morphism $\Phi_4 : c_2^1A_4^* \to c_1^1A_4^*$). This step should be replaced by the following construction, which also uses condition (d) above:

Consider the following diagram of spaces and mappings in which $\diamond$ denotes a Cartesian square. Recall that $W \subset C$ is a neighborhood of the fixed point set $F$ and that $F' = c_1(F) = c_2(F)$. Write $\tilde{R}_{Lk} = c_k^{-1}t^{-1}(R_L)$ for $k = 1, 2$ and define

$$\tilde{F}_k = F' \times_{R_L} (\tilde{R}_{L1} \cap \tilde{R}_{L2})$$

to be the fiber product in the lower left hand square. (The mapping $a : t^{-1}(R_L) \to t^{-1}(R_L)$ in the lower middle square is the identity.) The top row is obtained from the middle row by intersecting with $W$.
The new condition (d) implies the following slightly weaker condition: 

\[
\begin{array}{c}
F \xrightarrow{h} R_{L1} \cap R_{L2} \xrightarrow{a_k} R_{Lk} \xrightarrow{j_k} W \\
\downarrow i_k \quad \downarrow i \quad \downarrow i_k \quad \downarrow i \quad \downarrow i \\
\widetilde{F}_k \quad \bigodot \quad \widetilde{R}_{L1} \cap \widetilde{R}_{L2} \xrightarrow{\tilde{a}_k} \widetilde{R}_{Lk} \xrightarrow{j_k} C \\
\downarrow c_k \quad \downarrow c_k \quad \downarrow c_k \quad \downarrow c_k \\
F' \xrightarrow{h_L} t^{-1}(R_L) \xrightarrow{a} t^{-1}(R_L) \xrightarrow{j_L} X
\end{array}
\]

We claim that the left-hand rectangle, denoted \( \bigodot \), is Cartesian when \( k = 1 \), that is, \( c_1^{-1}(F') \cap c_1^{-1}t^{-1}(R_L) \cap c_2^{-1}t^{-1}(R_L) \cap W = F \). This follows from the hyperbolic hypothesis together with the (newly added) hypothesis (d) above.

We now construct a morphism \( \phi_4 : c_2^*(A^*_4) \to c_1^*(A^*_4) \) where \( A^*_4 = j_{L*}h_{L*}h_{L}^*j_{L}A^* \):

\[
\begin{align*}
c_2^*j_{L*}a_1h_{L*}h_{L}^*a_1^*j_{L}A^* & \to i_*c_2^*j_{L*}a_1h_{L*}h_{L}^*a_1^*j_{L}A^* \to (2.2a) \\
i_*j_{2*}(i_2^*c_2^*)a_1h_{L*}h_{L}^*a_1^*j_{L}A^* & \to i_*j_{2*}i_2^*a_1h_{L*}h_{L}^*a_1^*j_{L}A^* \to (2.2a) \\
i_*j_{2*}i_2^*a_1h_{L*}h_{L}^*a_1^*j_{L}A^* & \to i_*j_{2*}a_2(i_1^*h_{L*}h_{L}^*a_1^*j_{L}A^*) \to (2.5a) \\
i_*j_{2*}a_2h_{L*}h_{L}^*a_1^*j_{L}A^* & \to i_*j_{2*}a_2h_{L*}h_{L}^*a_1^*j_{L}A^* \to (2.6a) \\
i_*[j_{1*}a_1h_{L*}h_{L}^*a_1^*j_{L}A^*] & \quad \text{as desired.}
\end{align*}
\]

We also have a morphism \( c_2^*A^* \to c_1^*A^* \). Putting these together gives a morphism to

\[
\begin{align*}
i_*[j_{1*}a_1h_{L*}h_{L}^*a_1^*j_{L}A^*] & \quad \text{as desired.}
\end{align*}
\]

Remarks. Let \( \overline{R}(x_0, y_0) = [0, x_0] \times [0, y_0] \) denote the closure of the rectangle defined in §4.1.

The new condition (d) implies the following slightly weaker condition:

(d') For some (and hence for any sufficiently small) \( x_0, y_0 > 0 \) the set

\[
S = W \cap c_1^{-1}t^{-1}(\overline{R}(x_0, y_0)) \cap c_2^{-1}t^{-1}(\overline{R}(x_0, y_0)) \subset C
\]
is closed (and hence compact),
which is equivalent to the statement that the closure of $S$ is contained in $W$. The (erroneous)
proof of Lemma 5.10, which appears in §5.11, actually shows that (d) implies (d'). In fact,
this weaker condition (d') suffices for the case $j = 1$ of Theorem 4.7, although the stronger
condition (d) is needed for the other cases.