Effect of free-stream turbulence characteristics on boundary layer transition

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Abstract. The present measurement campaign on the free-stream turbulence induced boundary layer transition scenario has provided a unique set of experimental data. This new set of data has the potential to enhance the understanding of the effect of the free-stream turbulence characteristic length scales on the transition location and not only the turbulence intensity, which has been the focus in most previous studies. Recent investigations where the turbulence intensity has been kept essentially constant, while the integral length scale has been changed, show that the transition location is advanced for increasing length scale. The present data confirms previous results for low turbulence intensities, but shows the opposite behavior for high turbulence intensities, i.e. that the transition location is advanced for decreasing integral length scales. Important to underline here is that the integral length scale has a relatively small influence on the transition location as compared to the turbulence intensity and data analyses are now directed towards enhanced understanding of how the different parts of the incoming energy spectrum affects the energy growth inside the boundary layer.

1. Introduction

Considering the numerous cases where a solid body and a fluid move in contact with each other (cars, airplanes, turbines, pipes etc.) one realizes the significance of boundary layer research. The simplest case is when a fluid passes over a flat plate with zero pressure gradient. Although this case is rarely found in practical applications, it is a well defined and the idealized geometry for fundamental investigations. On the other hand, in real flow situations there is always some degree of background noise, denoted disturbances. These disturbances can enter into the laminar boundary layer, grow in amplitude and cause modulations of the base flow. The source of disturbances can vary, common sources are surface roughness and velocity fluctuations in a broad frequency spectrum, in the free-stream, which is called free-stream turbulence (FST). The receptivity of FST to boundary layer flows has been studied for decades, but today there is still no reliable prediction model for the onset of the transition location that can be used with confidence in engineering design works. There exists a numerous amount of empirical relationships between the location of transition onset and the turbulence intensity ($Tu$) in a flat plate boundary layer, but more recent investigations have shown that the $Tu$-level is not the only dependent variable. An increase of the FST integral length scale ($\Lambda_x$) has shown, both in experiments and numerical simulations (Jonáš et al., 2000; Brandt et al., 2004; Ovchinnikov et al., 2004), to advance the transition location. However, so far too little data has been available to draw any conclusions on how the transitional Reynolds number correlates with $\Lambda_x$. Fransson
et al. (2005) performed an extensive experiment on the FST transition scenario by varying the turbulence intensity. Detailed analyses of the transition location and length of the transitional zone were reported from measurements carried out in the streamwise direction. The present investigation partly aims at characterizing the importance of the FST length scales on the transition scenario. Totally 42 unique FST conditions have been generated which gives a good statistical basis for reliable data analyses. Furthermore, measurements have been performed in all three directions, i.e. streamwise, wall-normal and spanwise directions, using two hot-wire probes.

2. Experimental setup and methods

2.1. Facility and measurement technique

All the experiments were performed in the Minimum Turbulence Level (MTL) wind tunnel located at the Royal Institute of Technology (KTH) in Stockholm. MTL is a closed circuit wind tunnel with a 7 metre long test section and $0.8 \times 1.2$ m$^2$ (height x width) cross-sectional area. An axial fan (DC 85 kW) can produce airflow in the empty test section with a speed up to 70 m s$^{-1}$. To keep the airflow temperature constant (20 °C), a cooling system was employed with the accuracy of ±0.1 °C at a free-stream velocity of 6 m s$^{-1}$, which was the velocity throughout the present investigation. The experiments were carried out over a 5 metre-long flat plate. To minimize the leading edge effect, a trailing flap with a $6^\circ$ angel was used along with an asymmetric leading edge with an aspect ratio of 12. The wind-tunnel ceiling contains 6 adjustable parts providing the ability of obtaining a zero pressure gradient. A hot-wire anemometry system in the constant temperature (CTA) mode was employed. For the calibration, the modified King’s law (Johansson & Alfredsson, 1982), which has an extra term compensating for natural heat convection from the wire at low velocities, was used. The single wire sensor was made in-house of a Wollaston Platinum wire of 2.54 µm in diameter and 0.7 mm long. In all the experiments, a DANTEC Dynamics$^\text{TM}$ StreamLine 90N10 Frame anemometer system was employed and the signals were acquired by a National Instruments$^\text{TM}$ convertor board (NI PCI-6259, 16-Bit) with
a sampling frequency of 10 kHz. Besides the hot-wire anemometer, a Prandtl tube was connected to a manometer (Furness FC0510) to measure the dynamic pressure during the calibration. This probe was also used to record the reference velocity inside the test section. Furthermore, the manometer used external probes for registering the temperature and the total pressure inside the test section.

2.2. Base flow characteristics

When experimentally studying boundary layer stability and transition to turbulence, it is important to reduce the number of influencing parameters. It is well known that with an asymmetric leading edge of the flat plate one may avoid the adverse pressure gradient region and minimize the favorable pressure gradient region in the streamwise direction, which arises on any symmetric body (Klingmann et al., 1993; Fransson, 2004). The ideal developing boundary layer for stability and transition experiments is the Blasius boundary layer. In figure 1 seven profiles taken at different streamwise locations along the plate are shown with the solid line corresponding to the Blasius solution. An even more qualitative validation would be to compare the boundary layer integral length scales and their ratio, which is done in figure 2. Note that the solid lines correspond to the theoretical values of the displacement thickness, $\delta_1 = 1.721 \sqrt{x \nu / U_\infty}$, the momentum thickness, $\delta_2 = 0.664 \sqrt{x \nu / U_\infty}$, and the shape factor, $H = \delta_1 / \delta_2 = 2.59$, of the Blasius boundary layer. Here, $x$ and $\nu$ correspond to the downstream location from the leading edge and the kinematic viscosity, respectively.

2.3. Turbulence generating grids

The free-stream turbulence (FST) may be characterized by means of its turbulence intensity ($T_u$), integral ($\Lambda_x$), Taylor ($\lambda_x$) and Kolmogorov length scales, where the index $x$ denotes the longitudinal component. By mounting different grids in front of the leading edge of a flat plate, different FST characteristics may be obtained. In order to get a wide range of FST characteristics, six new grids were designed and manufactured for the present investigation. They are manufactured with different grid parameters, such as the mesh width ($M$) and bar diameter ($d$), which sets the so-called porosity ($\beta$), defined as the ratio between open and total area of the grid. Here, we have taken advantage of the fact that a higher pressure drop over the grid leads to a higher turbulence intensity (Gad-El-Hak & Corrsin, 1974), by applying a secondary counter-flow, relative to the free-stream, by means of upstream pointing air jets from the grid. The counter-flow injection is accomplished by pressurizing the new grids, which have

![Figure 2. Shape factor ($H_{12}$), normalized displacement thickness ($\delta_1$) and momentum thickness ($\delta_2$) by $\delta = \sqrt{x \nu / U_\infty}$ along $x$. Solid lines show the theoretical values of the Blasius parameters.](image-url)
Table 1. Geometrical data of all grids. \( d \), \( M \) and \( \sigma \) \( (= 1 - \beta) \) are the bar diameter, the mesh width and the solidity, respectively.

| Grid | \( d \) (mm) | \( M \) (mm) | \( \sigma \) | Bar geometry | Type | Symbol |
|------|--------------|--------------|----------|--------------|------|--------|
| \( G_1 \) | 8            | 40           | 0.360    | Circle       | Active | ○      |
| \( G_2 \) | 10           | 50           | 0.360    | Circle       | Active | □      |
| \( G_3 \) | 12           | 60           | 0.360    | Circle       | Active | ◆      |
| \( G_4 \) | 12           | 50           | 0.422    | Circle       | Active | △      |
| \( G_5 \) | 8            | 50           | 0.294    | Circle       | Active | ▽      |
| \( G_6 \) | 12           | 70           | 0.313    | Circle       | Active | ▼      |
| \( G_7 \) | 6            | 36           | 0.305    | Circle       | Passive | ⊿      |
| \( G_8 \) | 10           | 50           | 0.360    | Square       | Passive | ⋆      |

been manufactured using copper tubes as grid bars. This type of grid we here call \textit{active} in contrary to \textit{passive} when the grid may not be pressurized. Totally eight different grids were used and all of them are summarized in table 1.

In addition, by locating the grids at different distances from the leading edge, different turbulence intensities and integral length scales were obtained. Setting a grid close to the leading edge higher turbulence intensities would be available, but there is a rule of thumb of about 20-mesh-widths, which is required in order to obtain a close to homogeneous turbulence. Figure 3 (a, b and c) shows the streamwise distribution of intensities, integral and Taylor length scales, respectively, for all 42 studied cases. This figure really illustrates the wide range of FST conditions at the leading edge where the receptivity takes place. All the FST evaluations have been performed using the numerical tools developed in Kurian & Fransson (2009).

3. Results

The intermittency factor \((\gamma)\) of a velocity signal, taken inside the boundary layer, is a valuable measure of where in the transition process the signal is observed. Here, we have used the same method to calculate \(\gamma\) as proposed in Fransson et al. (2005). With different FST characteristics at the leading edge different intermittency distributions are obtained. Figure 4 presents the intermittency distribution for different grids without injection. As observed, the location of transition onset and the length of the transition zone vary among the different cases. Obviously, the range of transition regions would be even wider by considering different grid locations relative to the leading edge and different injection rates along with different grids. One way to scale the data in the streamwise direction is to introduce the non-dimensional coordinate \((\xi)\), which is defined as

\[
\xi = \frac{x - x_{\gamma=0.5}}{x_{\gamma=0.9} - x_{\gamma=0.1}}. \tag{1}
\]

Defining the start and end of transition as where \(\gamma = 0.1\) and \(0.9\), respectively, and consequently the length of the transition zone as \(\Delta x_{tr} = x_{\gamma=0.9} - x_{\gamma=0.1}\) with the transition location \(x_{tr}\) corresponding to the location where \(\gamma = 0.5\), the expression for \(\xi\) may be rewritten as \(\xi = (Re_x - Re_{tr})/\Delta Re_{tr}\), using the Reynolds number \(Re_x = U_{\infty}x/\nu\), the transitional Reynolds number \(Re_{tr} = U_{\infty}x_{tr}/\nu\) and the transition zone Reynolds number \(\Delta Re_{tr} = U_{\infty}\Delta x_{tr}/\nu\).

Considering this new coordinate, all the intermittency distributions collapse on an unique curve as depicted in figure 5. There have been many attempts, based on theoretical arguments, to form the seemingly universal curve in figure 5 (see e.g. Narasimha, 1957; Dhawan & Narasimha,
Figure 3. Turbulence intensity decay, longitudinal integral and Taylor length scales in the streamwise direction are shown in (a), (b) and (c), respectively. $x_L$ corresponds to the distance between the grid and the leading edge in each case.

1957; Johanson & Fashifar, 1994). Johanson & Fashifar (1994) presented a relation for the curve as

$$\gamma(x) = 1 - \exp[-A(\xi + B)^3], \quad (2)$$

where $A$ and $B$ are constants. The values of these constants determined in a least square fit sense to the present data become 0.67 and 1.02, respectively. These values are in good agreement with the Fransson et al. (2005) results which were reported as 0.60 and 1.05, respectively.

For employing the equation 2 or plotting figure 5, $\Delta Re_{tr}$ and $Re_{tr}$ are needed. For this
Figure 4. Typical intermittency distribution in the streamwise direction. No injection has been applied here. Triangle and star symbols represent G7 and G8 mounted 1710 mm and 1000 mm before the leading edge, respectively. Circular symbols shows G1 mounted 800 mm and 1400 mm before the leading edge, respectively.

Figure 5. Intermittency distribution for all the 42 cases. Streamwise distance is normalized to $\xi$. The solid line shows the curve fit according to 2.

reason the location of the three different intermittency values ($\gamma = 0.1, 0.5, 0.9$) were determined through interpolation of the data in each case. By knowing the location of $\gamma = 0.1$ and $\gamma = 0.9$ the length of the transition zone ($\Delta Re_{tr}$) is obtained. Figure 6 shows the length of transition versus $Re_{\gamma=0.1,0.5,0.9}$. Circles, squares, and triangles represent intermittency of 0.1, 0.5 and 0.9, respectively, for all 42 cases. By passing straight lines through the data, the minimum possible length of the transition region corresponding to different $Re_{x,\gamma}$ can be obtained. It should be mentioned that there is a limitation for them based on physical reasoning. Even at high intensities, a minimum length is needed for the boundary layer to become turbulent. By assuming high turbulence intensity, the transition region starts at the leading edge (i.e. $\gamma = 0.1$) so the length of the transition region is either equal to the location where $\gamma = 0.9$ or to twice the distance of the location where $\gamma = 0.5$. This minimum transition length is plotted in figure 6(a) on the one hand versus $Re_{\gamma=0.5}$ ($\Delta Re_{tr} = 2 \cdot Re_{\gamma=0.5}$) with a dashed line through the origin and on the other hand versus $Re_{\gamma=0.9}$ ($\Delta Re_{tr} = Re_{\gamma=0.9}$) with a solid line. The left hand sides of these lines are not allowed when considering above assumption of a minimum transitional region. The crosses of these two lines with the solid black and solid gray lines corresponding to
Figure 6. The relation between the length of transition and location of transition. Circles, squares and triangles represent intermittency of 0.1, 0.5 and 0.9, respectively (a). A closer view shows the detail of minimum length of the transition. Three star symbols correspond to three minimum distance and the dashed horizontal line ($\Delta Re_{tr}^{min} = 3.49 \times 10^4$) shows the average of them (b).

Figure 7. Transition region in dark. the symbols are the locations of $\gamma = 0.5$. The upper and lower solid lines correspond to $\gamma = 0.9$ and $\gamma = 0.1$ respectively. The dash line belongs to $\gamma = 0.5$.

the empirical $Re_{x,\gamma=0.9}$ and $Re_{x,\gamma=0.5}$, based on the present data, provide two values of $\Delta Re_{tr}$. In addition, the intersection location of the dashed gray line ($Re_{x,\gamma=0.1}$) with the ordinate axis ($Re_{x,\gamma=0.1}=0$) gives a third value of $\Delta Re_{tr}$. These three $\Delta Re_{tr}$ are almost the same (star symbols in figure 6b) with an average of $\Delta Re_{tr}^{min} = 3.49 \times 10^4$, which gives the minimum transitional Reynolds number of $Re_{tr}^{min} = 1.74 \times 10^4$. 


Assuming that when the energy inside the boundary layer reaches a certain level, the streaks break-down to turbulent spots, leads to the hypothesis that the transition location is a function of the input energy (i.e. the turbulence intensity at the leading edge, which from here on is denoted $Tu$). By considering a minimum receptivity distance as suggested above, the relation between the transition location and the turbulence intensity at the leading edge can be written as

$$Re_{x,\gamma} = C_\gamma \cdot Tu^{-2} + Re_{\gamma}^{min},$$

with the exponent of $-2$ on $Tu$ coming from the argumentation used in Andersson et al. (1999). A least square curve fit to the data gives $C_\gamma = (1.21, 1.63, 1.96) \times 10^2$ for $\gamma = 0.1, 0.5$ and 0.9, respectively. Furthermore, as described above, $Re_{\gamma}^{min} = (0, 1.74, 3.49) \times 10^4$ for $\gamma = 0.1, 0.5$ and 0.9, respectively. By knowing these coefficients, it is possible to estimate the transition region, which is shown in gray color in figure 7. The symbols depict the transition location ($\gamma = 0.5$). This figure reveals that by increasing the intensity at the leading edge, the transition

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**Figure 8.** $Re_{tr}$ filled contour plot based on the turbulence intensity ($Tu$) and integral length scale ($\Lambda_x$) and the Taylor length scale ($\lambda_x$) in (a) and (b), respectively, at the leading edge.

**Figure 9.** $Re_{tr}$ distribution when the $\Lambda_x$ varies at $Tu \approx 2.6\%$ (a) and $Tu \approx 3.5\%$ (b).
occurs closer to the leading edge but with a limitation of a minimum $Re_{tr}$ of $1.74 \times 10^4$. In addition, at lower turbulence intensities ($Tu < 1\%$) the transition occurs far away from the leading edge. The transitional Reynolds number ($Re_{tr}$) can be evaluated based on the integral length scale ($\Lambda_x$) and the Taylor length scale ($\tau_x$) at the leading edge along with the $Tu$. As shown in figure 8, the main parameter affecting the $Re_{tr}$ is the turbulence intensity at the leading edge. By focusing on a constant $Tu$ (practically a narrow band of $Tu$), one can see the effect of transition location on the integral length scale at the leading edge. As seen in figure 9 (a), at a chosen low turbulence intensity ($Tu = 2.6 \pm 0.1\%$), the transition occurs closer to the leading edge for increasing $\Lambda_x$. On the other hand, at a chosen high turbulence intensity ($Tu = 3.5 \pm 0.1\%$), the transition location moves downstream for an increase in $\Lambda_x$ (figure 9b). Note, the effect of $\Lambda_x$ on the transition location is stronger at low $Tu$. The derivative of the transition location with respect to the integral length scale ($d\Delta x_{tr} / d\Lambda_x$) for low and high $Tu$ is $-125$ and $+8$, respectively. Jonás et al. (2000) and Nagarajan et al. (2007) reported that the transition location moves downstream with increasing free-stream turbulence length scale both at $Tu=3\%$. This result has also been confirmed in direct numerical simulations by Brandt et al. (2004). However, the previous data has been inconclusive for any firm length scale dependence on the transition location. With the present parameter study we are able, for the first time, to show a convincing dependence of the FST integral length scale on the transition location.

4. Summary and conclusions

The hot-wire anemometry technique was employed to investigate the transitional boundary layer in presence of free-stream turbulence (FST). In this investigation a parameter study has been performed by varying both the turbulence intensity and FST length scales and the experimental setup was validated and the natural boundary layer developing without FST was close to the Blasius boundary layer. The most important results of this investigation are summarized below point-by-point.

- The turbulence intensity at the leading edge, generated with the new active grids, is a function of the pressure inside the grids and therefore the counter flow jet velocity from the active grids. The higher the jet velocity the larger is the pressure drop over the grid, which in turn leads to a higher turbulence intensity.

- The result by Fransson et al. (2005) of a universal intermittency distribution in the streamwise direction, using the non-dimensional coordinate $\xi$, is here reinforced by showing the universality even though the FST integral length scale spans from 15 to 25 mm, i.e. an increase of over 70%.

- The transition zone is proportional to the onset of transition and a minimum distance from the leading edge is required for the receptivity process to be completed, even for high $Tu$s. It should be noted that the relative length of the transition zone increases with increasing turbulence intensity. The minimum transition region length is $\Delta Re_{tr}^\text{min} = 3.49 \times 10^4$ considering the present data, which in turn leads to a minimum transitional Reynolds number, $Re_{tr}^\text{min} = 1.74 \times 10^4$, required for self-sustained turbulence.

- The turbulence intensity is clearly the most important parameter for the transition location. In this investigation it is shown that the FST length scales have a week but peculiar effect on the transition location. For low $Tu$ levels, longer length scales advance transition, in agreement with previous investigations, but for high $Tu$ levels, the result is the opposite as well as weaker.
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