The number of chained lumped-circuits for the physical analogy of half-wavelength power transmission lines

Rongrong Zhan 1, Yurong Li 2, Chongqing Jiao 2, Yue Yu 1, Jiangwen Meng 1, Bei Wang 2

1. State Key Laboratory of Power Grid Safety and Energy Conservation, Relay Protection Research Department, China Electric Power Research Institute, Beijing 100192, China;
2. State Key Laboratory of Alternate Electrical Power System With Renewable Energy Sources, North China Electric Power University, Beijing 102206, China.

zhanrr@epri.sgcc.com.cn, lyrncepu@163.com

Abstract. Half-wavelength AC power transmission (HWACT) technology is a mode for long-distance and large-capacity power transmission, has broad application prospect in ultra-distance point-to-point transmission mode. In order to implement physical analogy of HWACT lines, in general, the equivalent lumped-circuits consisting of some chained π-type circuits or T-type circuits are used in laboratory. The number of the chained circuits is the most important aspects in the physical analogy of HWACT lines. In this paper, the voltage waveform along π-type equivalent circuit is analysed for different number of the chained circuits. Next, the relationship between the number of chained circuits and the wave frequencies is discussed based on the relative error of the cascaded matrix (relative to the transfer matrix). For 50Hz, 32 chained π-type circuits can ensure the oscillation at the end of the waveform is small enough and the sending-to-receiving voltage ratio is approximately 1. The greater frequency is, the more the number of chained circuits required to establish good equivalence between the lumped-circuits and the transmission line. These works are instructive for the physical analogy of HWACT lines.

1. Introduction

Half-wavelength AC power transmission (HWACT) technology is a long-distance and large-capacity power transmission technology with a transmission distance of 3000 km (50Hz) or 2500 km (60 Hz). Compared with high voltage direct current (HVDC) transmissions, HWACT transmission has good economic efficiency, low loss and low cost, as well as good social benefits [1].

In 1940s, the study about half-wavelength transmission was first addressed by Soviet experts [2]. In 1965, American experts studied the tuning scheme for half-wavelength transmission [3]. Later, India, Italy and other countries have carried out research on the theory of half-wavelength transmission lines [4-7]. A detailed theoretical study and simulation calculation of the half-wavelength technology had been carried out in Brazil in order to send the large hydropower from the Amazon basin to the load center. In recent years, scholars in China have launched a series of research projects on on HWACT [8-9]. These research mainly focused on its steady state and transient operational characteristics, power transmission ability, tuned half-transmission lines, reactive balance, relay protection of HWACT, etc [10-20]. However, the relationship between the number of chained circuits and the waveform voltage as well as the wave frequencies has not been discussed [21]. As a result, it is necessary to make a study on determining the reasonable number according to the actual needs of physical analogy.
In this paper, the EMTP (Electromagnetic Transients Program) is used to simulate the equivalent lumped-circuit of HWACT line. The voltage waveform along π-type equivalent circuit is discussed for different number of the chained circuits, as well as the relationship between the number of chained circuits and the wave frequencies. It is concluded that for 50Hz, 32 chained π-type circuits can ensure the oscillation at the end of the waveform is small enough and the sending-to-receiving voltage ratio is approximately 1. The greater frequency is, the more the number of chained circuits required to establish good equivalence between the lumped-circuits and the transmission line. These works are instructive for the physical analogy of HWACT lines.

2. Model

In this paper, the transmission line investigated is a 1000kV three-phase transmission line of 3000km. The geometric parameter of the half-wavelength line is shown in Table 1, and its line arrangement is shown in Figure 1. The impedance per unit length and the admittance per unit length of the line with different frequencies are shown in Table 2. For 50Hz, the positive sequence component of the impedance per unit length is \( Z_0 = (0.0081864 + 0.26164j) \times 10^{-3} (\Omega/m) \), the positive sequence component of the admittance per unit length is \( Y_0 = 0.43334j \times 10^{-8} (S/m) \), the resistance per unit length is \( R_0 = 8.1864 \times 10^{-7} (\Omega/m) \), the inductance per unit length is \( L_0 = 8.3283 \times 10^{-7} (H/m) \), the capacitance per unit length is \( C_0 = 1.3794 \times 10^{-11} (F/m) \), the characteristic impedance is \( Z_C \approx 246 \Omega \), the propagation constant is \( \gamma = (0.01666 + 1.0649j) \times 10^{-6} (\text{rad/m}) \). As shown in Table 2, it is easy to see that \( Z_0 \) and \( Y_0 \) increase as the frequency increases. For 1000Hz, \( Z_0 \) increases to \( (0.031623 + 5.19002j) \times 10^{-3} (\Omega/m) \), \( Y_0 \) increases to \( 8.6668j \times 10^{-8} (S/m) \).

### Table 1. The Geometric Parameter of The Half-Wavelength Line

| Line type     | 8×LGJ-500/35 |
|---------------|--------------|
| DC resistance | 0.0581 Ω/km  |
| Conductor diameter | 30 mm       |
| Splitting distance | 400 mm     |
| Horizontal distance (to the centre line) |  |
| the upper conductor: | 0 m          |
| the lower conductor (two-phase): | 16 m        |
| Tower height sag | 18.5 m       |
| sag            | 30 mm       |
|                | 16 mm       |
|                | 45 mm       |
|                | 65 mm       |

### Table 2. The Impedance Per Unit Length and The Admittance Per Unit Length of HWACT Line With Different Frequencies

| frequency (Hz) | \( Z_0 \) \((\times 10^{-3} \Omega/m)\) | \( Y_0 \) \((\times 10^{-8} S/m)\) |
|----------------|----------------------------------------|----------------------------------|
| 50             | 0.0081864+0.26164j                     | 0.43334j                         |
| 100            | 0.0091935+0.52246j                     | 0.86668j                         |
Figure 2 shows a transmission line of $l$. The left end represents the sending end, and the right end represents the receiving end.

For the two port network of the transmission line shown in Figure 2, the transfer relationship between the voltage of receiving end $U_2$, the current of receiving end $I_2$, the voltage of sending end $U_1$ and the current of sending end $I_1$ is:

\[
\begin{bmatrix}
U_1 \\
I_1
\end{bmatrix} = \begin{bmatrix}
\cosh(\gamma l) & Z_l \sinh(\gamma l) \\
\frac{1}{Z_l} \sinh(\gamma l) & \cosh(\gamma l)
\end{bmatrix} \begin{bmatrix}
U_2 \\
I_2
\end{bmatrix} = \begin{bmatrix}
A & B \\
C & D
\end{bmatrix} \begin{bmatrix}
U_2 \\
I_2
\end{bmatrix}
\]

(1)

Figure 3 is a π-type equivalent circuit of a transmission line, assuming $Z_l = R_l + j\omega L_l$, $Z_2 = \frac{2}{j\omega C_2 l}$.

The equation of the voltage $U_1$ and the current $I_1$ are given by (2) and (3):

\[
U_1 = U_1 + \left( \frac{U_2}{Z_2} + I_2 \right) \times Z_1 = \left( \frac{Z_1 + 1}{Z_2} \right) \times U_2 + Z_1 I_2
\]

(2)

\[
I_1 = \frac{U_1}{Z_2} + \frac{U_2}{Z_2} + I_2 = \frac{2Z_1 + Z_2}{Z_2} U_2 + \left( \frac{Z_1 + 1}{Z_2} \right) I_2
\]

(3)

The transfer relationship between the sending end and the receiving end is described in equation (4):

\[
\begin{bmatrix}
U_1 \\
I_1
\end{bmatrix} = \begin{bmatrix}
\frac{Z_1 + 1}{Z_2} & Z_1 \\
\frac{2Z_1 + Z_2}{Z_2} & \frac{Z_1}{Z_2} + 1
\end{bmatrix} \begin{bmatrix}
U_2 \\
I_2
\end{bmatrix} = \begin{bmatrix}
A & B \\
C & D
\end{bmatrix} \begin{bmatrix}
U_2 \\
I_2
\end{bmatrix}
\]

(4)

Figure 4 is the equivalent lumped-circuit of the multi-stage π-type for 3000 km transmission line. As shown in Figure 4, the transmission line is evenly divided into $N$ segments, the length of each...
Each segment is equivalent to a π-type lumped-circuit as shown in Figure 3, with the resistance is $R_0l$, the inductance is $L_0l$, and the capacitance is $C_0l/2$.

\[ \text{Figure 4. A 3000 km power transmission line and its chained π-type equivalent circuit.} \]

3. Results and discussion
In this section, The EMTP is used to simulate the lumped-circuits, and the time step is $1 \times 10^{-5}$ s. The characteristic impedance of the transmission line is added to the receiving end. That is the load impedance $Z_L$ is equal to the characteristic impedance $Z_C$ (i.e. the line is matched).

3.1. The Voltage Waveform Along π-Type Equivalent Circuit
At $t=5$ ms, the sine wave voltage source is added to the sending end, and the voltage amplitude is 10kV. The time domain simulation results of the chained π-type circuits with 50 Hz and different $N$ are presented in Figure 5. $U_1$, $U_2$, $U_3$, $U_4$ respectively represent the voltage waveforms at 750 km, 1500 km, 2250 km, and 3000 km with different $N$. 

(a) Voltage waveform at 750km
(b) Voltage waveform at 1500km
(c) Voltage waveform at 2500km
(d) Voltage waveform at 3000km
The local enlarged drawing of voltage waveform at 3000km

**Figure 5.** Voltage waveforms along the π-type equivalent circuit with 50 Hz and different N: (a) At 750 km; (b) At 1500 km; (c) At 2250 km; (d) At 3000 km; (e) The local enlarged drawing at 3000km.

As can be seen from Figure 5, the closer to the sending end, the less the difference of the voltage waveforms between different N is. For 50Hz, the difference between the voltage waveforms with N=32 and N=64 is very small. Hence for 50Hz, 32 chained π-type circuits can ensure to establish good equivalence between the lumped-circuits and the transmission line.

### 3.2. The Voltage Waveform Under Sinusoidal Excitation

In this section, the sine wave voltage source is added to the sending end at t=5 ms and ended in 15 ms. For the chained π-type lumped-circuits with 50 Hz and different N, Figure 6 respectively shows waveforms of the sending end (green line) and the receiving end (red line).

**Figure 6.** The voltage waveforms of the sending end (green line) and the receiving end (red line) of the 50Hz π-type lumped-circuits: (a) N=4; (b) N=8; (c) N=16; (d) N=32

As can be seen from Figure 6, The larger the number of the chained circuits is, the smaller oscillation at the end of the waveform has, and the sending-to-receiving voltage ratio is closer to 1. For 50Hz, 32 chained π-type circuits can ensure the oscillation is small enough and the sending-to-receiving voltage ratio is approximately 1.
3.3. The Cascaded Matrix Calculation

Next, the relationship between the number of the chained circuits and the wave frequencies is discussed. For 50Hz, Table 3 respectively shows the specific values of $A$, $B$, $C$, $D$ in the transfer matrix and in the cascaded matrix. Table 4 gives the relative error of the cascaded matrix (relative to the transfer matrix), including the error of real part $\varepsilon_r$ and the error of imaginary part $\varepsilon_i$. In this paper, for 3000km transmission line, the $A$, $B$, $C$, $D$ in transfer matrix are obtained by using transmission line theory to analysis, while the $A$, $B$, $C$, $D$ in cascaded matrix are obtained by using some chained $\pi$-type circuits to equivalent the transmission line. 

Table 3. For 50Hz, the specific values of the cascaded matrix (with different $N$) and the transfer matrix

| 50Hz | the transfer matrix | $N=16$ | $N=32$ |
|------|---------------------|--------|--------|
| $A$  | -0.9998-0.0027j     | -0.9995-0.0029j | -0.9998-0.0027j |
| $B$  | -12.4756-12.8887j   | -12.6193-14.2660j | -12.5113-13.2307j |
| $C$  | (-1.9975-2.1972j)×10^{-4} | (-1.99332.4023j)×10^{-4} | (-1.9965-2.2485j)×10^{-4} |
| $D$  | -0.9998-0.0027j     | -0.9995-0.0029j | -0.9998-0.0027j |

Table 4. The relative error of the cascaded matrix with 50Hz and different $N$

| 50Hz | $\varepsilon_r$ (%) | $\varepsilon_i$ (%) | $\varepsilon_r$ (%) | $\varepsilon_i$ (%) |
|------|-------------------|-------------------|-------------------|-------------------|
| $A$  | 0.028523          | 10.56             | 6.8427×10^{-3}    | 2.62              |
| $B$  | 1.15              | 10.69             | 0.29              | 2.65              |
| $C$  | 0.21              | 9.33              | 0.051672          | 2.33              |
| $D$  | 0.028523          | 10.56             | 6.8427×10^{-3}    | 2.62              |

As can be seen from Table 3 and Table 4, The larger the number of the chained circuits is, the smaller the relative error of the cascaded matrix becomes. For 50Hz, 32 chained $\pi$-type circuits can ensure that the relative error of the cascaded matrix is less than 3%.

Next, the relationship between the number of chained circuits and the wave frequencies is discussed. The length of the transmission line is fixed at 3000km. Table 5 to Table 10 respectively shows the specific values of the transfer matrix, as well as the cascaded matrix and its relative error at different wave frequencies.

Table 5. For 100Hz, the specific values of the cascaded matrix and its relative error with $N=64$

| 100Hz | the transfer matrix | $\varepsilon_r$ (%) | $\varepsilon_i$ (%) |
|-------|---------------------|-------------------|-------------------|
| $A$   | 0.9965+0.0057j      | 0.9962+0.0058j    | 0.026764          | 2.75              |
| $B$   | 13.9452+24.6373j    | 13.9822+25.3165j  | 0.27              | 2.76              |
| $C$   | (2.2407+4.1263j)×10^{-4} | (2.2390+4.2286j)×10^{-4} | 0.071746          | 2.48              |
| $D$   | 0.9965+0.0057j      | 0.9962+0.0058j    | 0.026764          | 2.75              |

Table 6. For 200Hz, the specific values of the cascaded matrix and its relative error with $N=128$

| 200Hz | the transfer matrix | $\varepsilon_r$ (%) | $\varepsilon_i$ (%) |
|-------|---------------------|-------------------|-------------------|
| $A$   | 0.9843+0.0135j      | 0.9833+0.0139j    | 0.10              | 2.86              |
|   | \(17.4072+46.6195j\) | \(17.4402+47.9549j\) |
|---|---|---|
| \(C\) | \((2.8055+7.7771j)\times10^{-4}\) | \((2.8013+7.9793j)\times10^{-4}\) |
| \(D\) | \(0.9843+0.0135j\) | \(0.9833+0.0139j\) |

Table 7. For 400Hz, the specific values of the cascaded matrix and its relative error with \(N=256\)

|   | \(\text{transfer matrix}\) | \(\text{cascaded matrix}\) | \(\varepsilon_r\) (%) | \(\varepsilon_i\) (%) |
|---|---|---|---|---|
| \(A\) | \(0.9429+0.0361j\) | \(0.9391+0.0372j\) | 0.39 | 2.97 |
| \(B\) | \(24.2347+85.4398j\) | \(24.2106+87.9801j\) | 0.099294 | 2.97 |
| \(C\) | \(0.0004 + 0.0014j\) | \(0.0004 + 0.0015j\) | 0.44 | 2.71 |
| \(D\) | \(0.9429 + 0.0361j\) | \(0.9391 + 0.0372j\) | 0.39 | 2.97 |

Table 8. For 600Hz, the specific values of the cascaded matrix and its relative error with \(N=384\)

|   | \(\text{transfer matrix}\) | \(\text{cascaded matrix}\) | \(\varepsilon_r\) (%) | \(\varepsilon_i\) (%) |
|---|---|---|---|---|
| \(A\) | \(0.8824+0.0654j\) | \(0.8746+0.0673j\) | 0.89 | 2.96 |
| \(B\) | \(29.284+119.76j\) | \(29.113+123.31j\) | 0.59 | 2.96 |
| \(C\) | \(0.0005 + 0.0020j\) | \(0.0005 + 0.0021j\) | 0.94 | 2.70 |
| \(D\) | \(0.8824+0.0654j\) | \(0.8746+0.0673j\) | 0.89 | 2.96 |

Table 9. For 800Hz, the specific values of the cascaded matrix and its relative error with \(N=512\)

|   | \(\text{transfer matrix}\) | \(\text{cascaded matrix}\) | \(\varepsilon_r\) (%) | \(\varepsilon_i\) (%) |
|---|---|---|---|---|
| \(A\) | \(0.8053+0.1001i\) | \(0.7922+0.1029i\) | 1.62 | 2.85 |
| \(B\) | \(32.545 +150.54i\) | \(32.117+154.85i\) | 1.32 | 2.86 |
| \(C\) | \(0.0005+0.0025i\) | \(0.0005+0.0026i\) | 1.70 | 2.60 |
| \(D\) | \(0.8053+0.1001i\) | \(0.7922+0.1029i\) | 1.62 | 2.85 |

Table 10. For 1000Hz, the specific values of the cascaded matrix and its relative error with \(N=640\)

|   | \(\text{transfer matrix}\) | \(\text{cascaded matrix}\) | \(\varepsilon_r\) (%) | \(\varepsilon_i\) (%) |
|---|---|---|---|---|
| \(A\) | \(0.7139+0.1392j\) | \(0.6946+0.1429j\) | 2.70 | 2.67 |
| \(B\) | \(33.987+177.79j\) | \(33.180+182.54j\) | 2.70 | 2.67 |
From Table 5 to Table 10, it is easy to find that the greater frequency is, the more the number of chained circuits required to establish good equivalence between the lumped-circuits and the transmission line. The number $N=32$ for 50 Hz, $N=64$ for 100 Hz, $N=128$ for 200 Hz, $N=256$ for 400 Hz, $N=384$ for 600 Hz, $N=512$ for 800 Hz, $N=640$ for 1000 Hz can ensure that the relative error of the cascaded matrix is less than 3%.

4. Conclusion

In this paper, the voltage waveform along π-type equivalent circuits is analysed for different number of the chained circuits. Furthermore, the relationship between the number of chained circuits and the wave frequencies is discussed based on the cascaded matrix. The main conclusions are as follows:

1) For 50Hz, the closer to the sending end, the less the difference of the voltage waveforms between different $N$ is.

2) The larger the number of the chained circuits is, the smaller oscillation at the end of the waveform has, and the sending-to-receiving voltage ratio is closer to 1. For 50Hz, 32 chained π-type circuits can ensure the oscillation at the end of the waveform is small enough and the sending-to-receiving voltage ratio is approximately 1.

3) The greater frequency is, the more the number of chained circuits required to establish good equivalence between the lumped-circuits and the transmission line. The number of chained circuits $N=32$ for 50 Hz, $N=64$ for 100 Hz, $N=128$ for 200 Hz, $N=256$ for 400 Hz, $N=384$ for 600 Hz, $N=512$ for 800 Hz, $N=640$ for 1000 Hz can ensure that the relative error of the cascaded matrix is less than 3%.

Acknowledgments

This project is supported by the science and technology project of State Grid Corporation of China, Simulation and Experimental Study on EHV half wavelength transmission.

References

[1] Peng Qian, Wang Xiaoning, Zhang Youquan, et al. Study on electrical unbalance and transposition mode of UHV half-wavelength AC transmission line [J]. Electric Technology, 2016, 35(6): 56-60.
[2] Wolf A A, Shcherbachev O V. On normal working conditions of compensated lines with half-wave characteristics [J]. Elektrichesvto, 1940(1): 147-158 (in Russian).
[3] Hubert F J, Gent M R. Half-wavelength power transmission lines [J]. IEEE Spectrum, 1940, 2(1): 87-92.
[4] Prabhakara F S, Arthesarathy K P, Rao H N R. Analysis of Natural Half-wavelength Power Transmission Lines [J]. IEEE Transactions on Power Apparatus and Systems, 1969, 88(12): 1787-1794.
[5] Gatta F M, Iliceto F. Analysis of some operation problems of half-wave length power transmission lines [C]. Ezulwini Valley: AFRICON '92 Proceedings, 1992: 59-64.
[6] Iliceto F, Cinieri E. Analysis of half-wavelength transmission lines with simulation of corona losses [J]. IEEE Transactions on Power Delivery, 1988, 3(4): 2081-2091.
[7] Aredes, M. Static series compensators applied to very long distance transmission lines [J]. Electrical Engineering, 2004, 2(28): 69-76.
[8] Dias R, Santos G, Aredes M. Analysis of a Series Tap for Half-Wavelength Transmission Lines Using Active Filters [C]. Recife: Power Electronics Specialists Conference, 2005: 1894-1900.
[9] Tavares M C, Portela C M. Half-Wave Length Line Energization Case Test - Proposition of a
Real Test[C].Chongqing:High Voltage Engineering and Application,2008: 261-264.

[10] Wang Lingtao,Cui Xiang.Research on Steady-State Operation Characteristics of UHV Half-wavelength AC Power Transmission Line [J].Power System Technology,2011,35(9):7-12.

[11] Han Bin,Lin Jiming,Ban Liangeng,et al.Analysis on Electromagnetic Transient Characteristics of UHV Half-wavelength AC Power Transmission System[J].Power System Technology,2011,35(9):22-27.

[12] Qin Xiaohui,Zhang Zhiqiang,Xu Zhengxiong,et al.Study on the Steady State Characteristic and Transient Stability of UHV AC Half-wave-length Transmission System Based on Quasi-steady Model,2011,31(31):66-70.

[13] JIAO Chongqing,QI Lei,CUI Xiang.Compensation Technology for Electrical Length of Half-wavelength AC Power Transmission Lines [J].Power System Technology,2011,09:17-21(in Chinese).

[14] DAI Wuchang,CHEN Dongkui,ZHANG Wei,et al.Reactive Power Balance and Steady Voltage Control of Large-Scale Long-Distance AC Transmission System[J].Power System Technology,2013,04:1101-1105 (in Chinese).

[15] XIAO Shiwu,CHENG Yanjie, WANG Ya.A Bergeron Model Based Current Differential Protection Principle for UHV Half-wavelength AC Transmission Line[J].Power System Technology,2011,09:46-50 (in Chinese).

[16] SHI Wei, SUN Qiuqin,LI Qingmin,et al.Research on Characteristics of Transient Overvoltages Caused by Faults Occurred in Half-Wave Length AC Transmission Lines [J].Power System Technology,2012,02:43-47 (in Chinese).

[17] XING Jinyuan,LI Qingmin,CONG Haoxi,et al. Research on Multi-field Coupled Dynamic Modeling of Secondary Arcs With Half-wavelength Transmission Lines[J].Proceedings of the CSEE, 2015,09:2351-2359 (in Chinese).

[18] CONG Haoxi,LI Qingmin,SUN Qiuqin,et al.Experimental Study on Arc Root Motion and Extinction Characteristics of the Secondary Arcs with Half-wavelength Power Transmission Lines[J].Proceedings of the CSEE,2014,24:4171-4178 (inChinese).

[19] SHU Hongchun,CAO Pulin,DONG Jun.Analysis of Traveling Wave in UHV Half-wavelength Transmission Lines Considering Frequency Dependent Characteristics [J].High voltage engineering, 2015,03:716-723 (in Chinese).

[20] MA Lixin ,FEI Shaoshuai,MU Qinglun. Analysis on Capacitance Tuning System of Half-wavelength AC Power Transmission Lines[J].Proceedings of the CSU-EPSA,2015,09:2351-2359 (in Chinese).

[21] В.А.Веников.Physical simulation of power system[M].Beijing:China Industry Press,1962.