Superstars and Giant Gravitons in M-Theory

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ABSTRACT

Following hep–th/0109127, we show that a certain class of BPS naked singularities (superstars) found in compactifications of M–theory can be interpreted as being composed of giant gravitons. More specifically, we study superstars which are asymptotically $AdS_7 \times S^4$ and $AdS_4 \times S^7$ and show that these field configurations can be interpreted as being sourced by continuous distributions of spherical M2– and M5–branes, respectively, which carry internal momenta and have expanded on the spherical component of the space–time.
1 Introduction

In a recent paper[1], convincing evidence was provided showing that certain supersymmetric
naked singularities, appearing in type IIB supergravity compactified on \(AdS_5 \times S^5\), can be interpreted in terms of distributions of giant gravitons[2]. These ‘superstar’ solutions correspond
to the supersymmetric limit of a certain family of black holes. The latter were originally found
when considering a consistent truncation of the type IIB supergravity theory dimensionally
reduced to \(AdS_7\) where they appear as charged black hole solutions. Once lifted back to the
full ten–dimensional theory, the solutions carry internal momentum along the three commuting
Killing angles on the five–sphere, and the supersymmetric limit still leaves a naked singularity.
However, within the context of ten–dimensional type IIB superstring theory, there is a physical
interpretation in which the singularities are generated by a distribution of giant gravitons, \(i.e.,\),
an ensemble of spherical D3–branes which carry internal angular momentum and have expanded
on the five–sphere[2]. This result was determined by examining the dipole field excited in the
Ramond–Ramond five–form near the singularity of the supergravity solutions.

In this note, we study the analogous superstar solutions in M–theory compactified on \(AdS_7 \times
S^4\) and \(AdS_4 \times S^7\). We show that these eleven–dimensional supergravity solutions can also
be interpreted as being sourced by distributions of giant gravitons. Hence in this case, the
constituent degrees of freedom are M2 and M5–branes, respectively. In the second part of this
note, we study the behavior of test–brane probes in the background geometry of the M–theory
superstars. By considering giant graviton probes, we are able to confirm the expansion of these
configurations. We conclude with a brief discussion of our results. We consider dual giant
graviton[3, 4] probes of superstars in an appendix.

2 Superstars in \(AdS_7 \times S^4\)

Eleven–dimensional supergravity appears as a particular low energy limit of M–theory[9]. The
bosonic sector of this theory is composed of the graviton and the four–form field strength \(F_{(4)}\).
The latter allows for a spontaneous compactification[6] of the theory on \(AdS_7 \times S^4\). Further
in this background, there is a consistent truncation of the full theory to \(N = 4\) gauged \(SO(5)\)
supergravity in seven dimensions[7, 8]. In this note, we focus on solutions of a specific
\(N = 2\) truncation of this theory where only the following seven–dimensional fields are retained: the
metric, two scalars labelled by \(X_i\) \((i = 1, 2)\) and two one–form gauge fields \(A_i^{(1)}\) associated
with the \(U(1)\) Cartan subgroup of \(SO(5)\) (see, \(e.g.,\) refs. [9, 10] for details). The correct
Kaluza–Klein \(S^4\) reduction ansatz, which is going to be used to lift seven–dimensional solutions
to solutions of the full eleven–dimensional supergravity equations of motion, is[9, 10]

\[
ds_{11}^2 = \tilde{\Delta}^{1/3} ds_7^2 + \frac{L^2}{\Delta^{2/3}} \left( \frac{1}{X_0} d\mu_0^2 + \sum_{i=1}^{2} \frac{1}{X_i} \left[ d\mu_i^2 + \mu_i^2 \left( d\phi_i + A_i^{(1)}/L \right)^2 \right] \right)
\]

for the metric, and

\[
* F_{(4)} = -\frac{2}{L} \sum_{\alpha=0}^{2} \left( X_\alpha^2 \mu_\alpha^2 - \tilde{\Delta} X_\alpha \right) \epsilon(7) - \frac{1}{L} \tilde{\Delta} X_0 \epsilon(7) - \frac{L}{2} \sum_{\alpha=0}^{2} \frac{1}{X_\alpha} \pi dX_\alpha \wedge d\mu_\alpha^2
\]

\[
- \frac{L^2}{2} \sum_{i=1}^{2} \frac{1}{X_i} d\mu_i^2 \wedge \left( d\phi_i + A_i^{(1)}/L \right) \wedge \pi F_{(2)}^i,
\]

for the \(F_{(4)}\) field strength.
for the field strength of the supergravity three–form. The internal four–sphere is parameterized by four angular variables: the \( \phi_i \)'s are azimuthal (Killing) angles and the other angles are defined through the direction–cosines

\[
\mu_0 = \sin \theta_1 \sin \theta_2 \quad \mu_1 = \cos \theta_1 \quad \mu_2 = \sin \theta_1 \cos \theta_2 ,
\]

such that \( \mu_0^2 + \mu_1^2 + \mu_2^2 = 1 \). The \( \ast \) denotes the Hodge dual with respect to the seven–dimensional metric \( ds_7^2 \). Finally, \( L \) is the (asymptotic) radius of curvature of the four–sphere, \( X_0 \equiv (X_1 X_2)^{-1} \) and \( \Delta = \sum_{\alpha=0}^2 X_\alpha \mu^2_\alpha \).

The seven–dimensional \( N = 2 \) supergravity admits doubly–charged \( AdS \) black hole solutions of the form \[11\],

\[
ds_7^2 = -\frac{f}{(H_1 H_2)^{4/5}} dt^2 + (H_1 H_2)^{1/5} \left( \frac{1}{f} dr^2 + r^2 d\Omega_5^2 \right),
\]

\[
A_{(1)}^i = \left( \frac{1}{H_i} - 1 \right) dt \quad i = 1, 2 ,
\]

\[
X_i = \frac{(H_1 H_2)^{2/5}}{H_i},
\]

where we have introduced

\[
f = 1 - \frac{\mu}{r^4} + \frac{r^2}{4L^2} H_1 H_2 ,
\]

\[
H_i = 1 + \frac{q_i}{r^4} ,
\]

with \( \mu \) a mass parameter and the \( q_i \)'s related to the physical charges of the black hole. We will use the following expression for the metric on the five–dimensional unit sphere,

\[
d\Omega_5^2 = d\alpha_1^2 + \sin^2 \alpha_1 \left( d\alpha_2^2 + \sin^2 \alpha_2 \left[ d\alpha_3^2 + \sin^2 \alpha_3 \left( d\alpha_4^2 + \sin^2 \alpha_4 d\alpha_5^2 \right) \right] \right). \]

The mass of this \( AdS_7 \) black hole is \[12\]

\[
M = \frac{\pi^2}{4G_7} \left( \frac{5}{4} \mu + \sum_i q_i \right) ,
\]

a quantity that will later be compared with the energy of the suggested ensemble of source branes.

The horizon structure of both the metric eq. \[1\] and its lift to eleven–dimensional supergravity using ansatz eq. \[1\] are essentially the same as can be seen by analysing the \((rr)\) component of the corresponding metrics. The supersymmetric limit corresponds to \( \mu = 0 \) where the event horizon vanishes to reveal a naked singularity. Hence we denote these BPS configurations as ‘superstars.’ For \( \mu = 0 \), the entropy vanishes which means that the system should have a very simple physical interpretation in terms of a single arrangement of fundamental degrees of freedom. It is natural to conjecture that these degrees of freedom are giant gravitons, just as in the type IIB case. The \( \mu \neq 0 \) black holes of finite entropy are likely still related to a similar ensemble of expanded branes but non–trivial interactions in the non–extremal case complicate the analysis. Hence we restrict ourselves to studying the BPS objects.

Following ref. \[1\] then, we make the conjecture that the superstars can be given a reasonable physical interpretation as the external fields around an ensemble of giant gravitons. The first evidence of this conjecture is simply that the off–diagonal form of the eleven–dimensional metric
(1) shows that the superstars carry internal momentum along the \( \phi_i \) directions, just as giant gravitons would. However if the \( q_i \)’s are to be macroscopic distance scales (much larger than the eleven–dimensional Planck scale), then the mass of the superstar must be significantly larger than the energy of an individual giant graviton. This fact suggests that superstars must be composed of a collection of these fundamental objects.

For the detailed analysis, we begin for simplicity by considering superstars that are singly charged: \( q_1 \neq 0 \) and \( q_2 = 0 \). Since the source M2–branes must carry internal momentum along \( \phi_1 \), they should span the two–sphere parameterized by \( \theta_2 \) and \( \phi_2 \). The collection of giant gravitons each with varying momenta should be distributed along the \( \theta_1 \)–direction. Using a test–brane analysis [2], one finds that for a given internal momentum \( P_{\phi_1} \) (the canonical conjugate to \( \phi_1 \)), a spherical M2–brane configuration on \( AdS_7 \times S^4 \) has two minima, one at \( \sin \theta_1 = 0 \) and the other at \( \sin \theta_1 = P_{\phi_1} / N \), where \( N \) is the number of four–form flux quanta on the four–sphere. The former corresponds to a point–like graviton with angular momentum \( P_{\phi_1} \), and the latter to the giant graviton, with the same angular momentum, that has expanded to a fixed size on a two–dimensional submanifold of the \( S^4 \). The point–like and giant gravitons preserve the same number of supersymmetries [3, 4] and have the same energy [2]:

\[
E = \frac{P_{\phi_1}}{L} = \frac{N}{L} \sin \theta_1.
\]  

(11)

It is natural to assume that the superstar source must correspond to the expanded giant gravitons as the singularity in the metric extends over the entire four–sphere at \( r = 0 \).

A M2–brane is electrically charged with respect to the supergravity three–form potential. However, the spherical M2–branes will carry no net charge, but they will locally excite this field. If one considers a small (seven–dimensional) surface that encloses a portion of the sphere, one will find a net flux proportional to the number of M2–branes enclosed. In fact,

\[
\int_{M_7} \ast F_4 = 16\pi G_{11} T_{M2} n_1,
\]  

(12)

where \( G_{11} \) is the eleven–dimensional gravitational constant, \( T_{M2} \) the tension of a M2–brane and \( n_1 \) the total number of giant gravitons enclosed. We are using the following conventions,

\[
G_{11} = 16\pi^7 l_p^9, \quad G_7 = \frac{G_{11}}{V_{S^4}} = \frac{3}{8\pi^2 L^4} G_{11}, \quad T_{M2} = \frac{1}{4\pi^2 l_p^5},
\]  

(13)

where \( l_p \) is the fundamental eleven–dimensional Planck length. Given the orientation of the giant gravitons described above, the surface \( M_7 \) is a manifold in the subspace spanned by \( \{ \theta_1, \phi_1, \alpha_i \} \). If we wish to consider the entire distribution, we naturally fix \( r \) and integrate over the remaining angles [3]. The \( \phi_1 \) integration is of course trivial because the supergravity solution is smeared along this direction. In evaluating the expression eq. (12), we find that the only term in eq. (2) which gives a nonvanishing contribution is

\[
- \frac{L^2}{2X_1^2} d\mu_1^2 \wedge d\phi_1 \wedge \ast F(2),
\]  

(14)

where

\[
[\ast F(2)]_{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5} = -\frac{4}{H_1^{6/5}} \sin^4 \alpha_1 \sin^3 \alpha_2 \sin^2 \alpha_3 \sin \alpha_4.
\]  

(15)
Hence the total number of giant gravitons $n_1$ is found to be

$$n_1 = \frac{q_1 L^2}{4 \pi G_{11} T_{M2}} \int d\theta_1 d\phi_1 d^5\alpha_i \cos \theta_1 \sin \theta_1 \sin^4 \alpha_1 \sin^3 \alpha_2 \sin^2 \alpha_3 \sin \alpha_4. \quad (16)$$

Dropping the $\theta_1$ integration (along which the giant gravitons are assumed to be distributed) from eq. (16) and integrating over the other angles we find an expression for the density of giant gravitons as a function of $\theta_1$,

$$\frac{dn_1}{d\theta_1} = \frac{q_1 N^2}{8 L^4} \cos \theta_1 \sin \theta_1. \quad (17)$$

Given that we have detected a distribution of ‘electric dipole’ sources for $F_{(4)}$ at the singularity is certainly evidence of the presence of giant gravitons.

Further support for this result comes by considering the energy of the above distribution\[1]. The test–brane analysis\[2] found that the size and internal momentum of an individual giant graviton were related by: $P_{\phi_1} = N \sin \theta_1$. Combining this result with eq. (17), we find the total angular momentum of the distribution to be

$$\overline{P}_{\phi_1} = N \int_0^{\pi/2} d\theta_1 \sin \theta_1 \frac{dn_1}{d\theta_1} = \frac{1}{24} \frac{q_1 N^3}{L^4}. \quad (18)$$

According to eq. (11), the total energy is then

$$\overline{E} = \frac{1}{24} \frac{q_1 N^3}{L^5}. \quad (19)$$

The mass of the superstar (10), calculated by conventional supergravity means, is

$$M = \frac{\pi^2 q_1}{4 G_7}. \quad (20)$$

Using eqs. (13) to write this mass in terms of $N$ and $L$, one shows that it agrees exactly with the total energy eq. (19) of a distribution of giant gravitons. As it stands this result is somewhat of a curiosity. It is not clear why the result for the size of the giant graviton derived from a test–brane propagating in the $AdS_7 \times S^4$ background should apply here. However, we will find supporting evidence for this fact from our probe analysis in section 4.

It is straightforward to generalize the calculation to the case of superstars with two non–vanishing charges: $q_1 \neq 0$ and $q_2 \neq 0$. We consider the following embedding of $S^4$ in $\mathbb{R}^5$ with coordinates $x^{0,1,2,3,4}$,

$$x^0 = L\mu_0, \quad x^{2i-1} = L\mu_i \cos \phi_i, \quad x^{2i} = L\mu_i \sin \phi_i, \quad (21)$$

where $i = 1, 2$. Consequently, a giant graviton moving along $\phi_i$ has a radius

$$\rho_i = L\sqrt{1 - \mu_i^2}. \quad (22)$$

In analogy with the single charge calculation, the density of gravitons of a certain radius involves the $*F_{(4)}^{(4)}$,

$$[*F_{\rho_i}]_{\rho_i \alpha_1 \alpha_2 \alpha_3 \alpha_4} = \frac{d\mu_i^2}{d\rho_i} \left[ *F_{\rho_i}^{(4)} \right]_{i \phi_i \alpha_1 \alpha_2 \alpha_3 \alpha_4} = 4 q_i \rho_i \sin^4 \alpha_1 \sin^3 \alpha_2 \sin^2 \alpha_3 \sin \alpha_4. \quad (23)$$
The corresponding density of giant gravitons for each direction is then

$$\frac{dn_i}{d\rho_i} = \frac{q_i N^2 \rho_i}{L^6}. \quad (24)$$

Treating this as two independent distributions, it is found that the total angular momentum carried by each set of giant gravitons is

$$P_{\phi_i} = \frac{1}{24} q_i N^3 \frac{\rho_i}{L^4}. \quad (25)$$

These results match the total angular momentum calculated for the superstar solution, and we have complete agreement between the BPS mass of the superstar and the total energy of the giant gravitons, \( E = \sum P_{\phi_i} / L \).

3 Superstars in \( AdS_4 \times S^7 \)

In this section we consider superstars which are asymptotically \( AdS_4 \times S^7 \). Their description is hypothesized to be in terms of M5–branes carrying internal momentum that have expanded on the \( S^7 \). Eleven–dimensional supergravity admits a spontaneous compactification on \( AdS_4 \times S^7 \). The Kaluza–Klein reduction of this theory on \( S^7 \) leads to \( N = 8 \) gauged \( SO(8) \) supergravity in four dimensions. We focus on a specific \( N = 2 \) truncation where only the following fields are retained: the metric, four scalars labelled by \( X_i \) (\( i = 1, 2, 3, 4 \)) \((X_1 X_2 X_3 X_4 = 1)\) and four one–form gauge fields \( A_{i(1)}^i \) (see, e.g., refs. \([9, 10]\) for details). The correct Kaluza–Klein \( S^7 \) reduction ansatz, which is going to be used to lift four–dimensional solutions to solutions of the full eleven–dimensional supergravity equations of motion, is\([9, 10]\)

$$ds_{11}^2 = \tilde{\Delta}^{2/3} ds_4^2 + \frac{L^2}{\Delta^{1/3}} \sum_{i=1}^4 \frac{1}{X_i} \left( d\mu_i^2 + \mu_i^2 \left( d\phi_i + A_{i(1)}^i / L \right)^2 \right) \quad (26)$$

for the metric, and

$$F_{(4)} = \frac{2}{L} \sum_{i=1}^4 \left( X_i^2 \mu_i^2 - \tilde{\Delta} X_i \right) \epsilon_{(4)} + \frac{L}{2} \sum_{i=1}^4 \frac{1}{X_i} \tilde{\pi} dX_i \wedge d\mu_i^2 - \frac{L^2}{2} \sum_{i=1}^4 \frac{1}{X_i^2} \tilde{\pi} \mu_i^2 \wedge \left( d\phi_i + A_{i(1)}^i / L \right) \wedge \tilde{\pi} F_{(2)}, \quad (27)$$

for the field strength of the supergravity three–form. The space transverse to \( ds_4^2 \) is characterized by seven angular variables: the \( \phi_i \)’s are azimuthal (Killing) angles and the other angular variables are defined through the direction–cosines

$$\mu_1 = \cos \theta_1 \quad \mu_2 = \sin \theta_1 \cos \theta_2 \quad \mu_3 = \sin \theta_1 \sin \theta_2 \sin \theta_3 \quad \mu_4 = \sin \theta_1 \cos \theta_2 \cos \theta_3, \quad (28)$$

such that \( \mu_1^2 + \mu_2^2 + \mu_3^2 + \mu_4^2 = 1 \). Finally, \( L \) is the (asymptotic) radius of curvature of the seven–sphere and \( \Delta = \sum_{i=1}^4 \mu_i^2 \).

The corresponding four–dimensional action for the graviton, the scalars \( X^i \) and the one–form fields \( A_{i(1)}^i \) admit a quadruply charged \( AdS \) black hole solution\([13, 14]\),

$$ds_4^2 = - \frac{f}{(H_1 H_2 H_3 H_4)^{1/2}} dt^2 + (H_1 H_2 H_3 H_4)^{1/2} \left( \frac{1}{f} dr^2 + r^2 d\Omega_2^2 \right), \quad (29)$$
\[ A^i_{(1)} = \left( \frac{1}{H_i} - 1 \right) dt \quad i = 1, 2, 3, 4 , \quad (30) \]

\[ X_i = \frac{(H_1 H_2 H_3 H_4)^{1/4}}{H_i}, \quad (31) \]

where we have introduced

\[ f = 1 - \frac{\mu}{r} + \frac{4 r^2}{L^2} H_1 H_2 H_3 H_4, \quad (32) \]

\[ H_i = 1 + \frac{q_i}{r}, \quad (33) \]

with \( \mu \) a mass parameter and the \( q_i \)'s related to the physical charges of the black hole. We use the following expression for the metric on the two–dimensional unit sphere,

\[ d\Omega_2^2 = d\alpha_1^2 + \sin^2 \alpha_1 d\alpha_2^2. \quad (34) \]

The mass of this \( AdS_4 \) black hole is [12]

\[ M = \frac{1}{4 G_4} \left( 2 \mu + \sum_i q_i \right), \quad (35) \]

a quantity that is be compared with the energy of the hypothesized equivalent system composed of spherical M5–branes. Once again, the horizon structure of the lower dimensional realisation of the black hole and its lift to eleven–dimensional supergravity using ansatz eq. (26) are essentially the same. We assume that

\[ q_1 \geq q_2 \geq q_3 \geq q_4 . \quad (36) \]

For \( q_1 \neq 0 \) with all other charges vanishing, we find that for \( \mu = 0 \) there is no horizon behind which the curvature singularity at \( r = 0 \) is hidden. Whenever two charges or more have a finite value, there is a critical value for the non–extremality parameter, \( \mu = \mu_{\text{crit}} \), corresponding to dissapearing horizons. In other words, for \( \mu > \mu_{\text{crit}} \) the geometry corresponds to a regular black hole and for \( \mu < \mu_{\text{crit}} \), it corresponds to a naked singularity i.e., to the so–called superstars. In any case, a generic feature is that for \( \mu = 0 \) (BPS limit) the area of the spherical horizon is zero.

Following the discussion of section 4 we consider superstars that are singly charged (\( q_1 \neq 0 \) and \( q_2 = q_3 = q_4 = 0 \)) and assume that they are composed of M5–branes spanning a five–sphere parametrized by \( \theta_2, \theta_3 \) and \( \phi_2, \phi_3, \phi_4 \). The giant gravitons have non–zero internal momentum along \( \phi_1 \), are distributed along the \( \theta_1 \)–direction and have an energy given by [2]

\[ E = \frac{P_{\phi_1}}{L} = \frac{N}{L} \sin^4 \theta_1, \quad (37) \]

where \( N \) is the number of seven–form (the dual of \( F^{(4)} \) flux quanta on the seven–sphere. A M5–brane is magnetically charged with respect to the supergravity three–form potential. The spherical M5–branes carry no net charge but nevertheless excite the field locally. If one considers a small surface enclosing a portion of the sphere, one finds a net flux proportional to the number of M5–branes enclosed,

\[ \int_{M_4} F_4 = 16 \pi G_{11} T_{M5} n_1, \quad (38) \]
where $G_{11}$ is the eleven–dimensional gravitational constant, $T_{M5}$ is the tension of a M5–brane and $n_1$ is the total number of giant gravitons making the superstar. Given the orientation of the giant gravitons described above, $M_4$ is a manifold in the subspace spanned by \{\theta_1, \phi_1, \alpha_i\}. The $\phi_1$ integration is trivial because the supergravity solution is smeared along that direction.

We are using the following conventions,

$$G_{11} = 16\pi^7 l_p^9, \quad G_4 = \frac{G_{11}}{V_{S7}} = \frac{3}{\pi^4 L^7} G_{11}, \quad T_{M5} = \frac{1}{2^5 \pi^5 l_p^6},$$

(39)

where $l_p$ is the fundamental eleven–dimensional Planck length. In evaluating the expression eq. (38) we find that all except the last term of eq. (27) lead to a vanishing integral. The following term of $F_{(4)}$ is therefore the only one that is relevant to the present analysis,

$$F_{(4)} = - \frac{L^2}{2} \frac{1}{X} d(\mu_1^2) \wedge d\phi_1 \wedge \star F_{(2)},$$

(40)

where

$$\left[ \star F_{(2)} \right]_{\alpha_1\alpha_2} = - \frac{q_1}{H^{3/2}} \sin \alpha_1.$$  

(41)

In the end, the total number of giant gravitons $n_1$ is found to be

$$n_1 = \frac{q_1 L^2}{4\pi G_{11} T_{M5}} \int d\theta_1 d\phi_1 d\alpha_1 d\alpha_2 \cos \theta_1 \sin \theta_1 \sin \alpha_1.$$  

(42)

Dropping the $\theta_1$ integration (along which the giant gravitons are assumed to be distributed) from eq. (42) and integrating over the other angles we find an expression for the density of giant gravitons as a function $\theta_1$,

$$\frac{dn_1}{d\theta_1} = \frac{8q_1 N^{1/2}}{\sqrt{2} L} \cos \theta_1 \sin \theta_1.$$  

(43)

As in section 2, we have detected a distribution of 'electric dipole' sources for $F_{(4)}$ at the singularity, which is certainly evidence of the presence of giant gravitons.

In order to give further support for this result, we calculate the energy of the above distribution. A test–brane analysis showed[2] that a single giant graviton is associated with an internal momentum given by

$$P_{\phi_1} = N \sin^4 \theta_1.$$  

(44)

Combining this result with eq. (42), we find the total angular momentum of the distribution,

$$\mathcal{P}_{\phi_1} = N \int_0^{\pi/2} d\theta_1 \sin \theta_1 \frac{dn_1}{d\theta_1} = \frac{4\sqrt{2} q_1 N^{3/2}}{3} \frac{L}{L},$$

(45)

which corresponds, according to eq. (11), to a total energy

$$\bar{E} = \frac{4\sqrt{2} q_1 N^{3/2}}{3} \frac{L}{L^2}.$$  

(46)

The mass of the superstar (33), calculated by conventional supergravity means, is

$$M = \frac{q_1}{4G_4},$$

(47)
Using eqs. (39) to write the mass of the superstar in terms of $N$ and $L$, one shows that it agrees exactly with the total energy eq. (46) of a distribution of giant gravitons. This is strong evidence that the asymptotically $AdS_4 \times S^7$ superstars are in fact M–theory objects composed of microscopic spherical M5–branes that have expanded on the seven–sphere.

Following the procedure of the previous section, we can easily generalize to the case of superstars with none of the four charges vanishing: $q_i \neq 0 \ (i = 1, 2, 3, 4)$. We consider the following embedding of $S^7$ in $\mathbb{R}^8$ with coordinates $x^{1,...,8}$,

$$x^{2i-1} = L\mu_i \cos \phi_i, \quad x^{2i} = L\mu_i \sin \phi_i. \quad (48)$$

Consequently, a giant graviton moving along $\phi_i$ has a radius of

$$\rho_i = L\sqrt{1 - \mu_i^2}. \quad (49)$$

In analogy with the single charge calculation, the density of gravitons of a certain radius involves the $F^{(4)}$,

$$F^{(4)}_{\mu_i \phi_i \alpha_1 \alpha_2} = \frac{d\mu_i^2}{d\rho_i} F^{(4)}_{\mu_i \phi_i \alpha_1 \alpha_2} = q_i \rho_i \sin \alpha_1. \quad (50)$$

The corresponding density of giant gravitons for each direction is then

$$\frac{dn_i}{d\rho_i} = \frac{8q_i \sqrt{N} \rho_i}{\sqrt{2} L^3}. \quad (51)$$

Just as above, it is found that the total angular momentum carried by each set of giant gravitons is

$$P_{\phi_i} = \frac{4\sqrt{2} q_i N^{3/2}}{3 L}. \quad (52)$$

These results correspond to the total angular momentum calculated for the superstar solution, and we have complete agreement between the BPS mass of the superstar (35) and the total energy of the giant gravitons, $E = \sum P_{\phi_i}/L$.

### 4 Giant graviton probes

In the previous sections, we showed that the two eleven–dimensional superstar geometries have an interpretation as the external fields for a collection of giant gravitons. That is, we should regard these configurations as solutions of the eleven-dimensional supergravity coupled to the brane actions of the corresponding giant gravitons. The primary evidence is the fact that the naked singularities behave as sources for the appropriate dipoles of the four-form field strength. However, to complete the analysis, we should also show that we also have a solution of the brane equations of motion. As a step in this direction, we consider a giant graviton probe in the superstar background. We will indeed find expanded configurations in the limit as $r \to 0$. Further for these configurations, we will confirm the result: $P_{\phi_1} = N \sin^{p-1} \theta_1$ (where $p + 1$ is the dimension of the M-brane’s worldvolume). It was with this result that we showed that the distribution of giant gravitons derived from flux considerations matched the total internal momentum of the superstar.
4.1 Probing superstars in $AdS_7 \times S^4$

In the following, we present a brief summary of the M2-brane probe calculations for the superstars presented in section 2. We will restrict our attention to the simplest case of a singly charged superstar (i.e. only $q_1$ is nonvanishing). The results of the doubly charged case are less clear — we will return to this point in the discussion.

Our giant graviton probe will be a spherical M2-brane inside the $S^4$ at constant $\theta_1$ and moving on a circle in the $\phi_1$ direction. We will first consider this configuration at a finite radius in the anti–de Sitter space (i.e. away from the singularity) and then consider taking the test–brane to $r=0$. The M2-brane couples to the three form potential $A^{(3)}$ satisfying $dA^{(3)} = F^{(4)}$. Hence to evaluate the worldvolume action, we need to dualize $*F^{(4)}$ given in eq. (2) and integrate to find $A^{(3)}$. The result is

$$F^{(4)} = -\frac{1}{L} H_1 \left( \frac{\Delta + 2}{\Delta^2} \sin^2 \theta_1 \cos \theta_1 L \partial_1 \wedge \sin \theta_2 \cos \partial_2 \bigwedge_j [L \partial_j + A^{(4)}_{1j}] \right)$$

$$= d \left( -H_1 \frac{\sin^3 \theta_1}{\Delta} \wedge \sin \theta_2 \cos \partial_2 \bigwedge_j [L \partial_j + A^{(4)}_{1j}] \right)$$

$$= dA^{(3)}. \quad (54)$$

The probe action is then

$$S_2 = -T_{M2} \int dtd\theta_2 d\phi_2 \left[ \sqrt{-g} - A^{(6)}_{\theta_2 \phi_2} - \dot{\phi}_1 A^{(4)}_{\phi_1 \theta_2 \phi_2} \right]$$

$$= \frac{N}{L} \int dt \left[ -\Delta^{-1/2} \sin^2 \theta_1 \left( \frac{f}{H_1} - \Delta^{-1} H_1 \mu_1^2 [L \dot{\phi}_1 + (H_1^{-1} - 1)]^2 \right)^{1/2} \right.$$

$$\left. + \frac{H_1}{\Delta} \sin^3 \theta_1 (L \dot{\phi}_1 + H_1^{-1} - 1) \right]. \quad (55)$$

Now fixing the momentum $p \equiv P_{\phi_1}/N$, we find the Hamiltonian

$$\mathcal{H} = \frac{N}{L} \left[ \sqrt{\frac{f}{H_1} \left( \frac{p^2}{H_1} + \tan^2 \theta_1 (p - \sin \theta_1)^2 \right) + (1 - H_1^{-1}) p} \right]. \quad (56)$$

Minimizing $\mathcal{H}(r, \theta_1)$ with respect to $\theta_1$ yields $\theta_1 = 0$ or $\sin \theta_1 = p$ for arbitrary values of $r$. However, to find a true solution of the equations of motion, we must also minimize with respect to the radius. Setting $\sin \theta_1 = p$, a short calculation shows that the minimum energy configuration is at $r=0$, and that the energy of this configuration satisfies the BPS relation:

$$\mathcal{H}(r = 0, \sin \theta_1 = p) = \frac{N}{L} p = \frac{P_{\phi_1}}{L}. \quad (57)$$

4.2 Probing superstars in $AdS_4 \times S^7$

The four–form and metric corresponding to the superstars in $AdS_4 \times S^7$ were given in section 3. The probe which we shall be studying in this case is a spherical M5-brane inside the $S^7$ at constant $\theta_1$ and moving on a circle in the $\phi_1$ direction. Again, we begin by placing the
probe away from the singularity and consider the limit as $r \to 0$. The M5-brane couples to the six–form potential $A^{(6)}$ satisfying $dA^{(6)} = *F^{(4)}$ and so our first task is to dualize $F^{(4)}$ given in eq. (27) and integrate to find $A^{(6)}$. We find

$$\ast F^{(4)} = -\frac{2}{L} H_1 \frac{(2\Delta + 1)}{\Delta^2} \sin^5 \theta_1 \cos \theta_1 Ld\theta_1 \wedge \sin^3 \theta_2 \cos \theta_2 Ld\theta_2 \wedge \sin \theta_3 \cos \theta_3 Ld\theta_3 \bigwedge_j [Ld\phi_j + A_{(1)}^j] + \ldots$$

(58)

$$= d \left( -H_1 \frac{\sin^6 \theta_1}{\Delta} \wedge \sin^3 \theta_2 \cos \theta_2 Ld\theta_2 \wedge \sin \theta_3 \cos \theta_3 Ld\theta_3 \bigwedge_j [Ld\phi_j + A_{(1)}^j] \right)$$

$$= dA^{(6)}.$$  

(59)

Evaluating the probe action for the configuration described above yields

$$S_5 = -T_{M5} \int dt d\theta_2 d\theta_3 d\phi_2 d\phi_3 d\phi_4 \left[ \sqrt{-g} + A^{(6)}_{\theta_2 \theta_3 \phi_2 \phi_3 \phi_4} + \dot{\phi}_1 A^{(4)}_{\phi_1 \theta_2 \phi_2 \phi_3 \phi_4} \right]$$

$$= \frac{N}{L} \int dt \left[ -\Delta^{-1/2} \sin^5 \theta_1 \left( \frac{f}{H_1} - \Delta^{-1} H_1 \mu_1^2 [L\dot{\phi}_1 + (H_1^{-1} - 1)]^2 \right)^{1/2} + \frac{H_1}{\Delta} \sin^6 \theta_1 (L\dot{\phi}_1 + H_1^{-1} - 1) \right].$$

(60)

Again fixing $p \equiv P_{\phi_1}/N$, we find the Hamiltonian

$$\mathcal{H} = \frac{N}{L} \left[ \sqrt{\frac{f}{H_1} \left( \frac{p^2}{H_1} + \tan^2 \theta_1 (p - \sin^4 \theta_1)^2 \right) + (1 - H_1^{-1})p} \right].$$

(61)

Extremizing $\mathcal{H}$ with respect to $\theta_1$ yields minima at $\theta_1 = 0$ and $\sin^4 \theta_1 = p$ independent of the radius. The $r$-dependence of the Hamiltonian is essentially as before and we once again find a solution to the equations of motion at $r = 0$ satisfying the BPS relation:

$$\mathcal{H}(r = 0, \sin^4 \theta_1 = p) = \frac{P_{\phi_1}}{L}.$$  

(62)

5 Discussion

In this note, we have extended the analysis of ref. [1] to the context of M–theory. In particular, we have studied superstar solutions of eleven–dimensional supergravity in asymptotically $AdS_7 \times S^4$ and $AdS_4 \times S^7$ space–times. While these BPS configurations contain naked singularities, we have argued that M–theory provides a natural external source in the form of an ensemble of giant gravitons. Hence we should regard the superstars as solutions of the combined equations of motion of eleven–dimensional supergravity coupled to the worldvolume actions of the corresponding giant gravitons.

In section 4, we considered the dynamics of giant gravitons in the superstar backgrounds. With this analysis, we were able to recover the result that the expanded branes satisfy:

$$P_{\phi_1} = N \sin^{p-1} \theta_1$$

(63)

where $p + 1$ is the dimension of the M-brane’s worldvolume. We have also confirmed that this result applies for a giant graviton probe in the type IIB superstar background considered
in ref. [1]. It is curious that these relations between the internal momentum of the various
giant gravitons and their expansion in the internal space is identical to that derived in the
 corresponding $AdS_m \times S^n$ background. Certainly the energy–momentum relations for the BPS
 configurations must hold in the superstar backgrounds, as this is dictated by supersymmetry.
However, the latter does not require the dynamical details such as the size to remain unper-
turbed in the superstar backgrounds. It would be interesting to investigate whether this result
is simply a coincidence or whether there is a deeper reason why the expansion of the giant
 gravitons matches in these different backgrounds.

One should also question the validity of the result presented in eq. (63) since it relies on
 pushing past the expected regime of validity of the test–brane analysis. We expect, e.g., that
 the usual Born–Infeld form of the action is only a leading order result that should be valid to
describe the low energy dynamics of branes in slowly varying background fields. We must expect
 that there are higher derivative corrections that must become important in rapidly varying
 backgrounds, e.g., in regions of strong curvature. For instance the leading order corrections
to the worldvolume action can be systematically calculated in superstring theory [15]. In the
 present analysis, we begin by placing the giant gravitons away from the origin but then find
that minimizing the energy requires that we set $r = 0$. However, this is the position of the
naked singularity in the singly charged superstar, and so in fact, it is a location where the
curvatures are diverging. The fact that we get a simple and consistent result seems to indicate
 that the higher curvature corrections must cancel out in the final analysis. However, clearly
this point deserves a better understanding. In particular, one should note that the analysis
in section 4 only considers the case of a singly charged superstar. We were unable to confirm
the same results for the general case, but it may well be that strong curvature effects play an
important role in these cases. Certainly, the analysis for the type IIB superstars [11] show the
multiply charged systems to be more exotic.

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2 and 3.

A Dual giant graviton probes

We have also considered probing the M–theory superstars with dual giant graviton probes, in
analogy with the analysis presented in ref. [1].

A.1 Dual giants in $AdS_7 \times S^4$

In $AdS_7 \times S^4$, the dual giants are spherical M5-branes, spanning the $S^5$ of the AdS$_7$
 space at constant r. They orbit on the $S^4$ at constant $\theta_1$ and $\theta_2$ with fixed angular momentum $P_{\phi_i}$
conjugate to the angles $\phi_i$.

The probe Lagrangian takes the following form (after integrating over the 5-sphere):

$$
\mathcal{L} = \frac{\tilde{N}}{L} \left[ -\Delta^{1/2} \left( \frac{r}{L} \right)^5 \left( f \left( \mu_0^2 + \sum_i \frac{\mu_i^2}{H_i} \right) - \sum_i H_i \mu_i^2 \left[ L \dot{\phi}_i + (H_i^{-1} - 1) \right]^2 \right)^{1/2} 
+ \frac{r^6}{L^6} \Delta + \sum_i \frac{\mu_i^2 q_i}{L^4} (L \dot{\phi}_i - 1) \right] 
$$

(64)

Here $\tilde{L} = 2L$ is the radius of the AdS space and $\tilde{N} = L\tilde{L}^2 A_2 T_2$, where $A_2$ is the area of a unit $S^2$ and $T_2$ is the tension of an M2 brane. Fixing the value of $\tilde{p}_i \equiv P_{\phi_i}/\tilde{N}$ we find the Hamiltonian:

$$
\mathcal{H} = \frac{\tilde{N}}{L} \left[ \frac{1}{L} f \left( \mu_0^2 + \sum_i \frac{\mu_i^2}{H_i} \right) \left( \sum_i \left( \tilde{p}_i - \frac{\mu_i^2 q_i}{L} \right)^2 \right) + \Delta \frac{r^{10}}{L^{10}} 
+ \sum_i (1 - H_i^{-1}) \left( \tilde{p}_i - \frac{\mu_i^2 q_i}{L} \right) - \Delta \frac{r^6}{L^6} + \sum_i \frac{\mu_i^2 q_i}{L^4} \right] 
$$

(65)

Minimima of this action saturating the BPS bound $\mathcal{H} = (N/L) \sum p_i$ occur whenever the following equations are satisfied:

$$
\frac{r^4}{L^4} \mu_i^2 = \tilde{p}_i - \frac{q_i \mu_i^2}{L^4}. 
$$

(66)

Using the constraint that $\sum \mu_i^2 = 1$, one can reduce these equations to the same form as presented in eqs. (28) and (29) of ref. [1].

### A.2 Dual giants in $AdS_4 \times S^7$

In $AdS_4 \times S^7$, the giant gravitons are M5-branes whereas the dual giants are spherical M2-branes, spanning the $S^2$ of the AdS$_4$ space at constant $r$. They orbit on the $S^7$ at constant $\theta_1$, $\theta_2$ and $\theta_3$ with fixed angular momentum $P_{\phi_i}$ conjugate to the angles $\phi_i$.

The probe Lagrangian takes the following form (after integrating over the 2-sphere):

$$
\mathcal{L} = \frac{\tilde{N}}{L} \left[ -\Delta^{1/2} \left( \frac{r}{L} \right)^5 \left( \mu_0^2 f - \sum_i H_i \mu_i^2 \left[ L \dot{\phi}_i + (H_i^{-1} - 1) \right]^2 \right)^{1/2} 
+ \frac{r^3}{L^3} \Delta + \sum_i \frac{\mu_i^2 q_i}{L} (L \dot{\phi}_i - 1) \right] 
$$

(67)

Here we have introduced $\tilde{L} = L/2$ as the radius of the AdS space and $\tilde{N} = L\tilde{L}^5 A_5 T_5$ where $A_5$ is the area of a unit $S^5$ and $T_5$ is the tension of an M5-brane. Fixing the value of $\tilde{p}_i \equiv P_{\phi_i}/\tilde{N}$ we find the Hamiltonian:

$$
\mathcal{H} = \frac{\tilde{N}}{L} \left[ \sqrt{\left( \sum_i \frac{\mu_i^2}{H_i} \right) \left( \sum_i \left( \tilde{p}_i - \frac{\mu_i^2 q_i}{L} \right)^2 \right) + \Delta \frac{r^4}{L^4}} 
+ \sum_i (1 - H_i^{-1}) \left( \tilde{p}_i - \frac{\mu_i^2 q_i}{L} \right) - \Delta \frac{r^3}{L^3} + \sum_i \frac{\mu_i^2 q_i}{L} \right] 
$$

(68)
Minimima of this action saturating the BPS bound $\mathcal{H} = (N/L) \sum p_i$ occur whenever the following equations are satisfied:

$$\frac{r_i}{L} \mu_i^2 = \tilde{p}_i - \frac{g \mu_i^2}{L}. \quad (69)$$

References

[1] R.C. Myers and O. Tafjord, *Superstars and Giant Gravitons*, hep–th/0109127, to appear in JHEP.

[2] J. McGreevy, L. Susskind and N. Toumbas, JHEP 0006, 008 (2000) [hep–th/0003073].

[3] M.T. Grisaru, R.C. Myers and O. Tafjord, JHEP 0008, 040 (2000) [hep–th/0008015].

[4] A. Hashimoto, S. Hirano and N. Itzhaki, JHEP 0008, 051 (2000) [hep–th/0008016].

[5] E. Witten, Nucl. Phys. B 443, 85 (1995) [hep–th/9503124].

[6] P.G. Freund and M.A. Rubin, Phys. Lett. B 97, 233 (1980).

[7] H. Nastase, D. Vaman and P. van Nieuwenhuizen, Phys. Lett. B 469, 96 (1999) [hep–th/9905078].

[8] H. Nastase, D. Vaman and P. van Nieuwenhuizen, Nucl. Phys. B 581, 179 (2000) [hep–th/9911238].

[9] M. Cvetic, M.J. Duff, P. Hoxha, J.T. Liu, H. Lu, J.X. Lu, R. Martinez-Acosta, C.N. Pope, H. Sati, T.A. Tran, Nucl. Phys. B 558, 96 (1999) [hep–th/9903214].

[10] M.J. Duff, *TASI lectures on branes, black holes and anti-de Sitter space*, hep–th/9912164.

[11] M. Cvetic and S.S. Gubser, JHEP 9904, 024 (1999) [hep–th/9902195].

[12] R.G. Cai, Phys. Rev. D 63, 124018 (2001) [hep–th/0102113].

[13] M.J. Duff and J.T. Liu, Nucl. Phys. B 554, 237 (1999) [hep–th/9901149].

[14] W.A. Sabra, Phys. Lett. B 458, 36 (1999) [hep–th/9903143].

[15] C.P. Bachas, P. Bain and M.B. Green, JHEP 9905, 011 (1999) [hep–th/9903210].

[16] V. Balasubramanian and A. Naqvi, *Giant Gravitons and a Correspondence Principle*, hep–th/0111163.