RELATIONS BETWEEN MONOPOLES, INSTANTONS
AND CHIRAL CONDENSATE *

S. THURNER, M. FEURSTEIN, H. MARKUM

Institut für Kernphysik, TU Wien, A-1040 Vienna, Austria

ABSTRACT

We analyze the interplay of topological objects in four dimensional QCD. The distributions of color magnetic monopoles obtained in the maximum abelian gauge are computed around instantons in both pure and full QCD. We find an enhanced probability of encountering monopoles inside the core of an instanton on gauge field average. For specific gauge field configurations we visualize the situation graphically. Moreover we investigate how monopole loops and instantons are locally correlated with the chiral condensate.

1. Introduction and Theory

There are two different kinds of topological objects which seem to be important for the confinement mechanism: color magnetic monopoles and instantons. Color magnetic monopoles play the main role in the dual superconductor hypothesis where confinement occurs by condensation of abelian monopoles via the dual Meissner effect. There is strong evidence from lattice calculations that the idea of dual superconductivity is in essence correct. On the other hand the role of instantons with respect to confinement is not so clear. It is assumed that instantons can only cause confinement in QCD if they form a so-called instanton liquid. In our lattice calculations we demonstrate that color magnetic monopoles and instantons are highly correlated. This might explain that both pictures of the confinement mechanism have the same topological origin and that both approaches can be united.

It is believed that both instantons and monopoles can explain chiral symmetry breaking. In this contribution we present first results on the local correlation functions of the chiral condensate, the topological charge density, and the monopole density.

In order to investigate monopole currents one has to project $SU(N)$ onto its abelian degrees of freedom, such that an abelian $U(1)^{N-1}$ theory remains. This aim can be achieved by various gauge fixing procedures. We employ the so-called maximum abelian gauge which is most favorable for our purposes. For the definition of the monopole currents $m(x, \mu)$ we use the standard method. To extract abelian parallel transporters $u(x, \mu)$ after imposing the maximum abelian gauge one has to perform the decomposition

$$\tilde{U}(x, \mu) = c(x, \mu)u(x, \mu), \quad \text{with} \quad (N = 3)$$

\[\text{(1)}\]

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\[ u(x, \mu) = \text{diag} \left[ u_1(x, \mu), u_2(x, \mu), u_3(x, \mu) \right], \]
\[ u_i(x, \mu) = \exp \left[ i \arg \tilde{U}_{ii}(x, \mu) - \frac{1}{3} i \phi(x, \mu) \right], \]
\[ \phi(x, \mu) = \sum_i \arg \tilde{U}_{ii}(x, \mu) \bmod 2\pi \in (-\pi, \pi]. \]

Since the maximum abelian subgroup \( U(1)^{N-1} \) is compact, there exist topological excitations. These are color magnetic monopoles which have integer-valued magnetic currents on the links of the dual lattice:
\[ m_i(x, \mu) = \frac{1}{2\pi} \sum_{\Box \ni \partial f(x+\hat{\mu}, \mu)} \arg u_i(\Box), \quad (2) \]

where \( u_i(\Box) \) denotes a product of abelian links \( u_i(x, \mu) \) around a plaquette \( \Box \) and \( f(x+\hat{\mu}, \mu) \) is an elementary cube perpendicular to the \( \mu \) direction with origin \( x+\hat{\mu} \). The magnetic currents form closed loops on the dual lattice as a consequence of monopole current conservation. From the monopole currents we define the local monopole density as \( \rho(x) = \frac{1}{4V} \sum_{\mu,i} |m_i(x, \mu)|. \)

For the implementation of the topological charge on the lattice there exists no unique discretization. In this work we restrict ourselves to the so-called field theoretic definitions which approximate the topological charge in the continuum, \( q(x) = \frac{g^2}{32\pi^2} \epsilon_{\mu
u\rho\sigma} \text{Tr} \left( F_{\mu\nu}(x) F_{\rho\sigma}(x) \right), \) in the following ways:
\[ q^{(P,H)}(x) = -\frac{1}{2^4 32\pi^2} \sum_{\mu,...=\pm 1} \tilde{\epsilon}_{\mu
u\rho\sigma} \text{Tr} O^{(P,H)}_{\mu\nu\rho\sigma}, \quad (3) \]
with
\[ O^{(P)}_{\mu\nu\rho\sigma} = U_{\mu\nu}(x) U_{\rho\sigma}(x), \quad (4) \]
for the plaquette prescription and
\[ O^{(H)}_{\mu\nu\rho\sigma} = U(x, \mu) U(x+\hat{\mu}, \nu) U(x+\hat{\mu}+\hat{\nu}, \rho) U(x+\hat{\mu}+\hat{\nu}+\hat{\rho}, \sigma) \]
\[ \times \ U^\dagger(x+\hat{\nu}+\hat{\rho}+\hat{\sigma}, \mu) U^\dagger(x+\hat{\rho}+\hat{\sigma}, \nu) U^\dagger(x+\hat{\sigma}, \rho) U^\dagger(x, \sigma), \quad (5) \]
for the hypercube prescription. We mention here that the topological charges employed are locally gauge invariant, whereas the monopole currents are not. The lattice and continuum versions of the theory represent different renormalized quantum field theories, which differ by finite, non-negligible renormalization factors. A simple procedure to get rid of renormalization constants, while preserving physical information contained in lattice configurations, is the cooling method. The cooling procedure systematically reduces quantum fluctuations, and suppresses differences between the different definitions of the topological charge. In our investigation we have employed the so-called “Cabbibo–Marinari method”.
To measure correlations between topological quantities we calculate the functions
\[ \langle q(0)q(d) \rangle, \langle \rho(0)\rho(d) \rangle, \langle \rho(0)q^2(d) \rangle, \langle q^2(0)\bar{\psi}\psi(d) \rangle, \langle \rho(0)\bar{\psi}\psi(d) \rangle, \]
which are normalized after subtracting the corresponding cluster values. Since topological objects with opposite sign are equally distributed, we correlate the monopole density and the local chiral condensate with the square of the topological charge density.

2. Results and Discussion

![Graphs](image)

Figure 1. Correlation functions between topological charge densities and monopole densities in the confinement phase ($\beta = 5.6$) for 0, 1, 2, 6, 11 cooling steps. The instanton autocorrelations (a) grow with cooling reflecting the existence of extended instantons whereas the monopole autocorrelations (b) decrease since monopoles become diluted. The correlations between monopoles and instantons (c) are hardly affected by cooling with a range of approximately two lattice spacings indicating a nontrivial relation between these topological objects.

Our simulations for the $SU(3)$ case were performed on an $8^3 \times 4$ lattice with periodic boundary conditions using the Metropolis algorithm. For pure QCD we evaluated the path integral with standard Wilson action in the confinement phase at inverse gluon coupling $\beta = 6/g^2 = 5.6$. The measurements were taken on 1000 configurations separated by 50 sweeps. Each configuration was cooled and then subjected to 300 gauge fixing steps enforcing the maximum abelian gauge. Switching on dynamical fermions we simulated full QCD with 3 flavors of Kogut-Susskind quarks of equal mass $m_a = 0.1$. We performed runs in the confinement at $\beta = 5.2$ and measured on 1000 configurations separated by 50 sweeps.

The correlation functions between topological quantities in pure QCD (hypercube definition for $q$) are shown in Fig. 1 for several cooling steps. The range of the instanton autocorrelation $qq$ (a) being originally $\delta$-peaked grows rapidly with cooling reflecting the occurrence of extended instantons. In contrast the $\rho\rho$-correlation (b) decreases since monopole loops become dilute with cooling. The $\rho q^2$-correlation (c) seems rather insensitive to cooling and clearly extends over more than two lattice spacings.
spacings, indicating some nontrivial local correlation between monopoles and topological charges.

Fig. 2 shows correlation functions of full QCD in the confinement region. The $\rho q^2$ correlation (a) looks similar to the corresponding function in pure QCD (Fig. 1a). The same holds for the instanton and monopole autocorrelation functions (not shown). In the case of the $\bar{\psi}\psi q^2$ correlation (b) exponential fits show that an increasing number of cooling steps results in a narrower correlation function. The $\bar{\psi}\psi \rho$ correlation (c) on the other hand is not sensitive to cooling and has the same exponential decay as the $\bar{\psi}\psi q^2$ correlation after some cooling steps.

Figure 2. Correlation functions in the presence of dynamical quarks in the confinement ($\beta = 5.2$). The monopole-instanton correlation (a) is almost the same as in pure $SU(3)$. The correlation of the quark condensate and the topological charge (b) is cooling dependent, whereas the correlation between the condensate and the monopole density is not. All correlations extend over two lattice spacings and indicate local correlations of the chiral condensate and topological objects.

To obtain some insight into the topological correlations, Fig. 3 presents a cooling history of an $SU(2)$ gluon field at a fixed time slice on a $12^3 \times 4$ lattice. The topological charge density using the plaquette and the hypercube definition is displayed for cooling steps 0, 15 and 25. A dot is plotted if $|q(x)| > 0.01$. The lines represent the monopole loops. Without cooling the topological charge distribution cannot be resolved from the noise. Also the monopole loops do not exhibit a structure. After 15-20 cooling steps one can identify clusters of topological charge with instantons. At cooling steps 35-40 the instanton and antiinstanton begin to approach each other until they annihilate several cooling steps after (not shown). Monopole loops also thin out with cooling, but they survive in the presence of instantons. In general, there is an enhanced probability that monopole loops exist in the vicinity of instantons.

In summary, our calculations of correlation functions between topological objects and the chiral condensate yield a range of about two lattice spacings. This suggests that the chiral condensate takes a nonvanishing value predominantly in the regions of instantons and monopole loops. To our knowledge this observation is the first direct indication that chiral symmetry breaking occurs locally in the vicinity of non-trivial topological structure. Our calculations are in agreement with other studies in
Cooling step 0  Cooling step 15  Cooling step 25

Figure 3. Cooling history for a time slice of a single gauge field configuration. The dots represent the topological charge distribution. Monopole loops are represented by lines. It can be seen that with cooling instantons evolve from noise accompanied by monopole loops in almost all cases. Note that the black-and-white pictures do not present the situation so clearly as color plots.

\[SU(2)\] With the visualization of instantons and monopole loops in specific gauge field configurations we show directly that at the sites of instantons monopole loops are present. This confirms our conjecture that monopoles and instantons might be two faces of a more subtle fundamental topological object, which even might carry an electric charge.\[8\]

3. References

1. G. 't Hooft, in High Energy Physics, Proceedings of the EPS International Conference, Palermo 1975, ed. A. Zichichi (Editrice Compositori, Bologna, 1976); S. Mandelstam, Phys. Rep. 23C (1976) 245.
2. E.V. Shuryak, Nucl. Phys. B302 (1988) 559.
3. S. Thurner, M. Feurstein, H. Markum and W. Sakuler, Phys. Rev. D54 (1996) in print; S. Thurner, H. Markum and W. Sakuler, in Proceedings of Confinement 95, Osaka 1995, eds. H. Toki et al. (World Scientific, 1996) 77, hep-th/9506123; H. Markum, W. Sakuler and S. Thurner, Nucl. Phys. B (Proc. Suppl.) 47 (1996) 254.
4. O. Miyamura, Nucl. Phys. B (Proc. Suppl.) 42 (1995) 538.
5. G. 't Hooft, Nucl. Phys. B190 (1981) 455.
6. A.S. Kronfeld, G. Schierholz and U.-J. Wiese, Nucl. Phys. B293 (1987) 461.
7. P. Di Vecchia, K. Fabricius, G.C. Rossi and G. Veneziano, Nucl. Phys. B192 (1981) 392; Phys. Lett. B108 (1982) 323; Phys. Lett. B249 (1990) 490.
8. V. Bornyakov and G. Schierholz, Preprint, DESY 96-069, HLRZ 96-22, hep-lat/9605019.