Search for $\psi(3770) \rightarrow \rho \pi$ at the BESII detector at the Beijing Electron-Positron Collider

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Abstract

Non-$D\bar{D}$ decay $\psi(3770) \to \rho\pi$ is searched for using a data sample of $(17.3 \pm 0.5) \text{ pb}^{-1}$ taken at the center-of-mass energy of 3.773 GeV by the BESII detector at the BEPC. No $\rho\pi$ signal is observed, and the upper limit of the cross section is measured to be $\sigma(e^+e^- \to \rho\pi) < 6.0 \text{ pb}$ at 90\% C. L. Considering the interference between the continuum amplitude and the $\psi(3770)$ resonance amplitude, the branching fraction of $\psi(3770)$ decays to $\rho\pi$ is determined to be $B(\psi(3770) \to \rho\pi) \in (6.0 \times 10^{-6}, 2.4 \times 10^{-3})$ at 90\% C. L. This is in agreement with the prediction of the $S$- and $D$-wave mixing scheme of the charmonium states for solving the “$\rho\pi$ puzzle” between $J/\psi$ and $\psi(2S)$ decays.

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I. INTRODUCTION

A. “ρπ puzzle” and ψ(3770) \(\rightarrow\) ρπ

Perturbative QCD (pQCD) predicts that the decays of \(J/\psi\) and \(\psi(2S)\) (shortened as \(\psi'\) below) into light hadrons are dominated by the annihilation of \(c\bar{c}\) into three gluons, with widths proportional to the square of the wave function at the origin [1]. This yields the pQCD “12% rule”, that is

\[
Q_h = \frac{B_{\psi' \rightarrow h}}{B_{\psi \rightarrow e^+ e^-}} = \frac{B_{\psi' \rightarrow e^+ e^-}}{B_{\psi \rightarrow e^+ e^-}} \approx 12%. \tag{1}
\]

The violation of the above rule was first observed in \(\rho\pi\) and \(K^{*+}K^- + c.c.\) modes by Mark II [2], known as the \(\rho\pi\) puzzle. Since then BES and CLEO-c have measured many two-body decay modes of \(\psi'\), among which some obey the 12% rule while others violate it [3, 4]. There have been many theoretical efforts trying to solve the puzzle [5]. A recent one is the \(S\)- and \(D\)-wave charmonia mixing model proposed by Rosner [6]. In this scheme, the mixing of the \(\psi(2^3S_1)\) and \(\psi(1^3D_1)\) states is in such a way that there is almost a complete cancellation of the decay amplitude of \(\psi' \rightarrow \rho\pi\), which instead shows up as an enhanced decay mode of \(\psi(3770)\) (shortened as \(\psi''\) below). A study shows that in \(e^+e^-\) experiments with \(B(\psi'' \rightarrow \rho\pi)\) predicted by the \(S\)- and \(D\)-wave mixing in Ref. [6], the destructive interference between the three-gluon decay amplitude of the \(\psi''\) resonance and the continuum one-photon amplitude leads to a very small \(\rho\pi\) cross section at the \(\psi''\) peak [7], which is in agreement with the unpublished upper limit of the \(\rho\pi\) cross section at the \(\psi''\) peak by Mark III [8]. Using a larger data sample to further study this channel will shed light on the understanding of the “\(\rho\pi\) puzzle” and the interference pattern between the resonance and the continuum amplitudes.

B. Data samples and detector

The data used for this analysis are taken with the BESII detector at the BEPC storage ring at the center-of-mass energy of 3.773 GeV. The integral luminosity of the data sample is \((17.3 \pm 0.5) \text{ pb}^{-1}\) as measured using large angle Bhabha events. To study the continuum process, BESII also collected \((6.42 \pm 0.24) \text{ pb}^{-1}\) data at \(\sqrt{s} = 3.65\) GeV [9]. We analyze these two data samples simultaneously to get the numbers of events at the \(\psi''\) peak and at the continuum.
BESII is a conventional solenoidal magnet detector that is described in detail in Refs. [10, 11]. A 12-layer vertex chamber (VC) surrounding the beam pipe provides trigger information. A 40-layer main drift chamber (MDC), located radially outside the VC, provides trajectory and energy loss (dE/dx) information for tracks over 85% of the total solid angle. The momentum resolution is \( \sigma_p/p = 0.017\sqrt{1+p^2} \) (\( p \) in GeV/c), and the dE/dx resolution for hadron tracks is \( \sim 8\% \). An array of 48 scintillation counters surrounding the MDC measures the time-of-flight (TOF) of tracks with a resolution of \( \sim 200 \) ps for hadrons. Radially outside the TOF system is a 12 radiation length, lead-gas barrel shower counter (BSC). This measures the energies of electrons and photons over \( \sim 80\% \) of the total solid angle with an energy resolution of \( \sigma_E/E = 22%/\sqrt{E} \) (\( E \) in GeV). Outside of the solenoidal coil, which provides a 0.4 Tesla magnetic field over the tracking volume, is an iron flux return that is instrumented with three double layers of counters that identify muons of momentum greater than 0.5 GeV/c.

C. Monte Carlo Simulation

A Monte Carlo simulation is used for the determinations of the mass resolutions and the detection efficiencies. This program (SIMBES), which is Geant3 based, simulates the detector response, including the interactions of secondary particles with the detector material. Reasonable agreement between data and Monte Carlo simulation has been observed in various channels tested [12], including 

\[ e^+e^- \rightarrow (\gamma)e^+e^-, \quad e^+e^- \rightarrow (\gamma)\mu^+\mu^-, \quad J/\psi \rightarrow p\bar{p} \]

\[ \psi' \rightarrow J/\psi\pi^+\pi^-, \quad J/\psi \rightarrow \ell^+\ell^- \quad (\ell = e, \mu). \]

For the signal process, \( e^+e^- \rightarrow \rho\pi \), the Monte Carlo events are generated with an angular distribution of \( \sin^2\theta_1(1+\cos^2\theta_v+\sin^2\theta_v\cos(2\phi_1)) \), where \( \theta_v \) is the angle between the \( \rho \) and the positron direction, and \( \theta_1 \) and \( \phi_1 \) are the polar and azimuthal angles of the pion in the \( \rho \) helicity frame. A Monte Carlo sample of \( \psi'' \rightarrow \pi^+\pi^-\pi^0 \) is also generated with the same angular distribution, taking into account the available phase-space of the two-pion system. The generators include the effects of initial state radiation (ISR), and the \( \rho\pi \) or \( \pi^+\pi^-\pi^0 \) form factor varies as a function of \( s \) (\( s \) denotes the square of the center-of-mass energy), where a \( 1/s \) dependence is assumed. The generator PPCON is also used to study the interference between the continuum and resonant \( \psi'' \) amplitudes. In this generator, the branching fraction of \( \psi'' \rightarrow \rho\pi \) can be set to be any number between 0 and 1, the
relative phase between the $\psi''$ strong and electromagnetic decay amplitudes can be set to be any possible value from $-180^\circ$ to $180^\circ$ and the measurement of $\sigma^{\text{Born}}(e^+e^- \rightarrow \rho\pi)$ at $\sqrt{s} = 3.67$ GeV measured by CLEO-c [4] is used to normalize the contribution of the continuum cross section at the $\psi''$ peak.

Monte Carlo samples of Bhabha, dimuon, $D\bar{D}$ and inclusive hadronic events generated with Lundcrm [13] are used for the background study.

**II. EVENT SELECTION**

The final states of the study include two charged pions and one neutral pion which is reconstructed from two photons. Event selection includes photon identification and charged particle identification.

A neutral cluster is considered to be a photon candidate when the deposited energy in the BSC is greater than 80 MeV, the angle between the nearest charged track and the cluster is greater than $16^\circ$, and the angle between the two nearest photons is larger than $7^\circ$. The first hit of the cluster is in the beginning 6 radiation lengths, and the angle between the cluster development direction in the BSC and the photon emission direction must be less than $37^\circ$. The number of photon candidates after the above selection is required to be two.

For each charged track, the TOF and $dE/dx$ measurements are used to calculate $\chi^2$ values and the corresponding confidence levels to the hypotheses that the particle is a pion, a kaon or a proton ($\text{Prob}_\pi, \text{Prob}_K, \text{Prob}_p$). A track is considered to be a pion when the confidence level of the pion hypothesis is greater than the confidence levels of the kaon and proton hypotheses. At least one charged track is required to be identified as a pion.

For the decay channel of interest, the candidate events must satisfy the following selection criteria:

1. An event is required to have only two oppositely charged tracks in the MDC, each with a good helix fit. The closest approach of the track to the interaction point is required to be within 2 cm in the transverse plane and within 20 cm in the beam direction, and the transverse momentum $P_{xy} > 0.06$ GeV/c is used to remove the beam associated background.

2. A four-constraint kinematic fit is performed under the hypothesis $e^+e^- \rightarrow \gamma\gamma\pi^+\pi^-$. 
and the confidence level of the fit is required to be greater than 1%. A four-constraint kinematic fit is also performed under the hypothesis of $e^+e^- \rightarrow \gamma\gamma K^+K^-$, and $\chi^2_{\gamma\gamma\pi\pi} < \chi^2_{\gamma\gamma KK}$ is required to remove the $K^+K^-\pi^0$ events.

3. To remove the di-muon background and the backgrounds produced by the ISR process $e^+e^- \rightarrow \gamma\psi'$, with $\psi' \rightarrow$ neutral $+ J/\psi$, $J/\psi \rightarrow \mu^+\mu^-$, two tracks should be in the $|\cos \theta| < 0.80$ region ($\theta$ is the polar angle of the track in MDC) and at least one track is required to be in the coverage of muon counter, in which $N_{\pi^+}^{hit} + N_{\pi^-}^{hit} < 3$ is required. Here, $N^{hit}$ is the number of muon counter layers with matched hits and ranges from 0 to 3, indicating not a muon (0), a weakly (1), medianly (2), or strongly (3) identified muon track [14].

4. After the four-constraint kinematic fit, the energy of the higher momentum photon candidate is required to be less than 1.5 GeV to remove the $\rho^0(770)$ background produced by ISR.

5. After the above selection, the radiative Bhabha background can still be seen clearly from Figure 1, where the $dE/dx$ separation from pion hypothesis ($\chi^2_{dE/dx}$) and the energy deposited in the BSC of the charged track ($E_{BSC}$) are shown. The cluster of events in the top right corner are electron tracks and can be removed by requiring $\chi^2_{dE/dx} < -2E_{BSC} + 3$ with high efficiency for the signal events.

After applying all of the above selection criteria, the invariant mass distributions of the two photons after the kinematic fit are shown in Figures 2a and 2b for the $\psi''$ and the continuum data samples, respectively. It can be seen that there are $\pi^0$ signals in the $\psi''$ and $\sqrt{s} = 3.65$ GeV data samples.

Analyses using the Monte Carlo samples of Bhabha, dimuon, $D\bar{D}$, inclusive hadronic events and ISR production of $J/\psi$ and $\psi'$ decays show that the background contaminations to the two-photon invariant mass spectra are small or in a random distribution.

III. FITS TO THE INvariant MASS SPECTRA

The invariant mass spectra of the two photons are fitted with the Monte Carlo simulated $\pi^0$ invariant mass distribution (where the $\pi^0$ mass is fixed to the PDG value [15] in the
FIG. 1: Scatter plot of the $dE/dx$ separation from the pion hypothesis versus the deposited energy in BSC of the charged track. The cluster at the top right corner is radiative Bhabha electrons. The events above the straight line are removed as Bhabha candidates.

FIG. 2: Invariant mass distributions of the two photons after final selection for (a) $\psi''$ data sample, and for (b) $\sqrt{s} = 3.65$ GeV data. The histograms are data, and the curves show the best fits.

The fits yield $11.4 \pm 4.7$ and $10.0 \pm 3.8$ $\pi^0$s for the $\psi''$ and the $\sqrt{s} = 3.65$ GeV data samples respectively, and the corresponding signal significance are $3.1\sigma$ and $4.6\sigma$. The fit results are shown in Figures 2a and 2b. The efficiency of detecting $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ at $\psi''$ is $(7.65 \pm 0.12)\%$ and that at $\sqrt{s} = 3.65$ GeV is $(7.88 \pm 0.12)\%$ according to the Monte Carlo simulation by assuming a phase space distribution, where the errors are statistical due to limited statistics of the Monte Carlo samples. Here in the generator, only the continuum amplitude is considered.

In order to evaluate the contributions of $\rho\pi$ in the data samples, the Dalitz plots of the $\pi^+\pi^-\pi^0$ system are shown in Figures 3a and 3b after requiring the invariant mass of the two
photons lies between 0.10 and 0.17 GeV/c\(^2\) (about 2\(\sigma\) around the \(\pi^0\) nominal mass). The Monte Carlo predicted \(\rho\pi\) events are also shown in the plots as small black dots. No clear \(\rho(770)\) signal can be seen either in the \(\psi''\) or in the continuum data sample.

FIG. 3: Dalitz plots of \(e^+e^- \rightarrow \pi^+\pi^-\pi^0\) at (a) \(\psi''\), and (b) \(\sqrt{s} = 3.65\) GeV after the final selection. The big black dots are for data, and the small black dots are the Monte Carlo simulated \(\rho\pi\) events. There are still non-\(\pi^0\) backgrounds in both plots for data.

Since the \(\rho\pi\) signals are not significant, we try to set the upper limits for the \(e^+e^- \rightarrow \rho\pi\) cross sections at \(\psi''\) and at \(\sqrt{s} = 3.65\) GeV. This is done by fitting the two photon invariant mass distributions (shown in Figure 4) after requiring the \(\pi^+\pi^-\) or \(\gamma\gamma\pi^\pm\) invariant mass in the range of 0.626 to 0.926 GeV/c\(^2\). The fits yield 2.9 \(\pm\) 2.8 and 4.9 \(\pm\) 2.7 \(\pi^0\)s at \(\psi''\) and \(\sqrt{s} = 3.65\) GeV respectively. The upper limits at 90\% C. L. on the numbers of events are 5.1 and 8.3 at \(\psi''\) and \(\sqrt{s} = 3.65\) GeV respectively. Here the systematic errors are considered, see below. The number of \(\rho\pi\) events are overestimated since there are, in general, non-\(\rho\pi\) contributions to events containing \(\pi^0\)s. The detection efficiency is \((4.87 \pm 0.09)\%\) at \(\psi''\) and \((5.14 \pm 0.09)\%\) at \(\sqrt{s} = 3.65\) GeV from the Monte Carlo simulations assuming a pure continuum contribution.

IV. SYSTEMATIC ERRORS

Many sources of systematic error are considered in the cross section measurement. Systematic errors associated with the efficiency are determined by comparing \(J/\psi\) and \(\psi'\) data and Monte Carlo simulation for very clean decay channels, such as \(J/\psi \rightarrow \pi^+\pi^-\pi^0\), \(\psi' \rightarrow \pi^+\pi^-J/\psi\), which allow the determination of systematic errors associated with the MDC tracking, trigger, kinematic fitting, photon identification efficiency, requirement of photon number and particle identification [16, 17]. The uncertainty due to the generator for
FIG. 4: Invariant mass distributions of the two photons of $e^+e^- \to \rho\pi$ at (a) $\psi''$ and (b) $\sqrt{s} = 3.65$ GeV after making the $\rho$ mass requirement. The histograms are data, and the curves show the best fits.

the $\pi^+\pi^-\pi^0$ mode is estimated by the efficiency difference between the phase space and the $\rho\pi$ generators. The systematic errors from the background estimation are determined by comparing the fitting result between the 2nd-order polynomial background and a 1st-order polynomial background and different fitting ranges in the fits of the two-photon invariant mass spectra. Uncertainty of the integral luminosity of the data sample is also a source of the systematic error.

All the sources considered are listed in Table I. The total systematic errors for $e^+e^- \to \rho\pi$ are 20.8\% and 16.4\% for the $\psi''$ and the continuum data, respectively, and for $e^+e^- \to \pi^+\pi^-\pi^0$ the corresponding numbers are 23.8\% and 19.3\% for $\psi''$ and continuum data, respectively.

V. RESULTS AND DISCUSSION

We give the measurement of the cross section of $e^+e^- \to \pi^+\pi^-\pi^0$ and the upper limit of the cross section of $e^+e^- \to \rho\pi$ at $\psi''$ and $\sqrt{s} = 3.65$ GeV. For the process $e^+e^- \to \pi^+\pi^-\pi^0$, the Born order cross section is calculated with

$$\sigma^B (e^+e^- \to \pi^+\pi^-\pi^0) = \frac{N^{obs}}{\varepsilon \cdot \mathcal{L} \cdot \mathcal{B}(\pi^0 \to \gamma\gamma)(1 + \delta)},$$

and for $e^+e^- \to \rho\pi$, the upper limit of the cross section is calculated with

$$\sigma^B (e^+e^- \to \rho\pi) < \frac{N_{UL}^{obs}}{\varepsilon \cdot \mathcal{L} \cdot \mathcal{B}(\pi^0 \to \gamma\gamma)(1 + \delta)}.$$
TABLE I: Systematic errors for $\rho\pi$ and $\pi^+\pi^-\pi^0$ cross section measurements at $\psi''$ and $\sqrt{s} = 3.65$ GeV (%).

| Source                      | $\sqrt{s} = 3.65$ GeV | $\psi''$ |
|-----------------------------|------------------------|----------|
| Monte Carlo statistics      | 1.7                    | 1.7      |
| Trigger                     | 1.0                    |          |
| MDC tracking                | 4.0                    |          |
| Kinematic fit               | 6.0                    |          |
| Photon efficiency           | 4.0                    |          |
| Number of photons           | 2.0                    |          |
| Particle ID                 | negligible             |          |
| Background                  | 13.4                   | 18.7     |
| Luminosity                  | 3.7                    | 3.0      |
| Generator (for $\pi^+\pi^-\pi^0$ only) | 10.2 | 11.5 |
| Total $[\rho\pi (\pi^+\pi^-\pi^0)]$ | 16.4 (19.3) | 20.8 (23.8) |

Here the initial state radiative correction factor, $1 + \delta$, and the efficiency, $\varepsilon$, are obtained from the Monte Carlo simulation assuming a pure continuum contribution. Using the numbers obtained from the above analysis (listed in Table II), one gets

$$\sigma^B(e^+e^- \to \pi^+\pi^-\pi^0) = (8.4 \pm 3.5 \pm 2.0) \text{ pb},$$

$$\sigma^B(e^+e^- \to \rho\pi) < 6.0 \text{ pb}$$

at $\psi''$ and

$$\sigma(e^+e^- \to \pi^+\pi^-\pi^0) = (19.3 \pm 7.3 \pm 3.7) \text{ pb},$$

$$\sigma(e^+e^- \to \rho\pi) < 25 \text{ pb}$$

at $\sqrt{s} = 3.65$ GeV, where the first errors are statistical and the second ones are systematic, and the upper limits are at 90% C. L.

The upper limit of $e^+e^- \to \rho\pi$ cross section at $\sqrt{s} = 3.65$ GeV is consistent with the measurement at $\sqrt{s} = 3.67$ GeV by CLEO-c [4] and the calculation from Ref. [7]. The upper
TABLE II: Numbers used in the calculations of $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ and $\rho\pi$ cross sections.

| Decay Channel | $\sqrt{s} = 3.65$ GeV | $\psi''$ |
|---------------|------------------------|-----------|
|               | $\pi^+\pi^-\pi^0$ | $\rho\pi$ | $\pi^+\pi^-\pi^0$ | $\rho\pi$ |
| $N^{obs}(N^{UL})$ | 10.0 ± 3.8 | 4.9 ± 2.7 (8.3) | 11.4 ± 4.7 | 2.9 ± 2.8 (5.1) |
| $\varepsilon(\%)$ | 7.88 ± 0.12 | 5.14 ± 0.09 | 7.65 ± 0.12 | 4.87 ± 0.09 |
| $1 + \delta$ | 1.033 | 1.025 | 1.033 | 1.026 |
| $\mathcal{L}(pb^{-1})$ | 6.42 ± 0.24 | 17.3 ± 0.5 |
| $\mathcal{B}(\pi^0 \rightarrow \gamma\gamma)$ [15] | | 0.988 |

limit of $\sigma(e^+e^- \rightarrow \rho\pi)$ at $\psi''$ is a little lower than the upper limit of 6.3 $pb$ determined by Mark III [8] and is lower than the measurement of $\sigma(e^+e^- \rightarrow \rho\pi) = 8.0^{+1.7}_{-1.4} \pm 0.9$ $pb$ off the $\psi''$ peak at $\sqrt{s} = 3.67$ GeV by CLEO-c [4], which indicates that there must be a non-zero $\psi'' \rightarrow \rho\pi$ amplitude at the $\psi''$ energy.

The measurement of $\sigma(e^+e^- \rightarrow \rho\pi)$ at $\psi''$ also supports the postulation in Ref. [7] that the relative phase between the strong and electromagnetic decays of $\psi''$ into light hadrons is around $-90^\circ$. In this scheme [7], the number of observed $\rho\pi$ events at the $\psi''$ peak depends on both the $\psi'' \rightarrow \rho\pi$ branching fraction $\mathcal{B}(\psi'' \rightarrow \rho\pi)$ and the relative phase, $\phi$, between the $\psi''$ strong and electromagnetic decay amplitudes. Using the measurement of $\rho\pi$ at the $\psi''$ peak in this experiment, and the $e^+e^- \rightarrow \rho\pi$ cross section at the continuum by the CLEO-c experiment [4], a 2-dimensional scan indicates that the physical region of $\mathcal{B}(\psi'' \rightarrow \rho\pi)$ and $\phi$ is restricted in the hatched area as shown in Figure 5 at 90% C. L. If the correlation between $\mathcal{B}(\psi'' \rightarrow \rho\pi)$ and $\phi$ is neglected, one gets $\mathcal{B}(\psi'' \rightarrow \rho\pi) \in (6.0 \times 10^{-6}, 2.4 \times 10^{-3})$, and $\phi \in (-150^\circ, -20^\circ)$ at 90% C. L. This result is consistent with the calculation in Refs. [6, 7] that the $\psi'' \rightarrow \rho\pi$ branching fraction is at the $10^{-4}$ level, and supports the explanation of the “$\rho\pi$ puzzle” observed between $J/\psi$ and $\psi'$ decays by the $S$- and $D$-wave mixing model. We also expect CLEO-c, with a few $fb^{-1}$ of $\psi''$ data [18], and BESIII, with even more $\psi''$ data [19], to be able to produce tighter constraints on $\mathcal{B}(\psi'' \rightarrow \rho\pi)$ and $\phi$, and give a better test of this scenario.
VI. SUMMARY

The processes $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ and $\rho\pi$ are searched for at $\psi''$ and $\sqrt{s} = 3.65$ GeV. We observe $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ signals at the 3.1$\sigma$ and 4.6$\sigma$ levels for $\sqrt{s} = 3.773$ GeV and $\sqrt{s} = 3.65$ GeV, respectively. No significant $\rho\pi$ signal is observed, and the upper limit of the $e^+e^- \rightarrow \rho\pi$ cross section at $\psi''$ is measured to be 6.0 $pb$ at 90% C. L. assuming no $\psi''$ contribution. Considering the interference between the continuum amplitude and the $\psi''$ resonance amplitude, the branching fraction of $\psi''$ decays to $\rho\pi$ is determined to be $B(\psi'' \rightarrow \rho\pi) \in (6.0 \times 10^{-6}, 2.4 \times 10^{-3})$ at 90% C. L.

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