MAGNETICALLY TORQUED NEUTRINO-DOMINATED ACCRETION FLOWS FOR GAMMA-RAY BURSTS

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ABSTRACT

Recent observations and theoretical work on gamma-ray bursts favor the central engine model of a Kerr black hole (BH) surrounded by a magnetized neutrino-dominated accretion flow (NDAF). The magnetic coupling between the BH and the disk through a large-scale closed magnetic field exerts a torque on the disk and transports the rotational energy from the BH to the disk. We investigate the properties of the NDAF with this magnetic torque. For a rapidly spinning BH, the magnetic torque transfers enormous rotational energy from BH into the inner disk. There are two consequences: (1) the luminosity of neutrino annihilation is greatly augmented; (2) the disk becomes thermally and viscously unstable in the inner region, displaying s-shaped curves of the surface density versus accretion rate. It turns out that the magnetically torqued NDAF can be invoked to interpret the variability of gamma-ray luminosity. In addition, we discuss the possibility of restarting the central engine to produce the X-ray flares with required energy.

Key words: accretion, accretion disks – black hole physics – gamma rays: bursts – magnetic fields – neutrinos

1. INTRODUCTION

The nature of the central engine of gamma-ray bursts (GRBs) remains unclear. Currently favored models invoke a binary merger or a collapse of compact objects. These models lead to the formation of a transient hot and dense accretion torus/disk around a black hole (BH) of a few solar masses. The typical mass accretion rates in GRB models are extremely high, of the order of a fraction of solar mass up to a few solar masses per second. Under such conditions, the disk becomes dense and hot enough in the inner regions to cool via neutrino losses. For this reason, Popham et al. (1999, hereafter PWF99) named these disk neutrino-dominated accretion flows (NDAFs). Energy extraction from the BH-accretion disk for powering GRB is also possible, such as by Blandford–Znajek (BZ) process (Blandford & Znajek 1977; Lee et al. 2000), Blandford–Payne (BP) process and Parker instabilities in the disk (Blandford & Payne 1982; Narayan et al. 1992; Meszaros & Rees 1997). The NDAF has been extensively discussed by many authors, e.g., PWF99, Narayan et al. (2001, hereafter NPK01), Kohri & Mineshige (2002), Di Matteo et al. (2002, hereafter DPN02), Chen & Beloborodov (2007), Janiuk et al. (2004, 2007), and Gu et al. (2006, hereafter GLL06). However, due to its low energy conversion efficiencies and the effects of neutrino opacity, the power produced by neutrino–antineutrino annihilation can hardly match those of the energetic short-hard GRBs (e.g., GRB080913) and X-ray flares. Recently, some authors (e.g., DPN02; Fan et al. 2005; Shibata et al. 2006, 2007; Perez-Ramirez et al. 2008) suggested that the MHD process should be considered in the disk model. Moreover, as shown by Shibata et al. (2006, 2007), the magnetic braking and the magnetorotational instability (MRI; Balbus & Hawley 1991) in the disk play a role in angular momentum transporting, which causes turbulent motion, resultant shock heating, and mass accretion onto the BH. On the other hand, recent researches showed that the magnetic fields can be magnified up to $10^{15} \sim 10^{16}$ G by virtue of MRI or dynamo process (Pudritz & Fahlman 1982 and references therein) in hyperaccretion disk. These considerations stimulate us to discuss the magnetized NDAF.

Based on the work of NPK01 and DPN02, Xie et al. (2007) discussed the BZ and BP processes in the NDAF. They found that the jet of GRB may be magnetically dominated, which is also obtained by MHD simulations of Mizuno et al. (2004).

Recently, the magnetic coupling (MC) between the central spinning BH and their surrounding accretion disk has been paid much attention (e.g. Blandford 1999; van Putten 1999; Li & Paczynski 2000; Li 2002; Wang et al. 2002). As a variant of the BP process, the MC process exerts a torque on the disk and transports the rotational energy from the BH to the disk. The effects of MC torque have been discussed in some disk models, for example, Lai (1998) and Lee (1999) in a neutron star with slim disk; Li (2002), Wang et al. (2002, 2003), and Kluzniak & Rappaport (2007) in a compact object with thin disk; Ye et al. (2007) and Ma et al. (2007) in a BH with advection-dominated accretion flow (ADAF). It is found that the disk properties are greatly changed and their luminosity is augmented significantly due to the rotational energy of BH extracted in the MC process. Therefore, it is attractive for us to investigate the effects of MC torque on the NDAF. To highlight the effects of MC torque, we ignore other MHD processes, such as BZ and BP mechanisms, and refer to this model as MCNDAF.

This paper is organized as follows. In Section 2, we describe the MCNDAF model, which is a relativistic steady state thin disk. The effects of MHD stress are described by the dimensionless parameter $\alpha$. The main equations are based on DPN02 and NPK01. Recently, GLL06, Chen & Beloborodov (2007), and Shibata et al. (2006, 2007) argued that the general relativistic (GR) effects are important for the NDAF; so we introduce GR correction factors to the equations. The MC torque appears in the angular momentum equation. We solve the set of equations for the solutions in the MCNDAF in Section 3 and compute the neutrino and neutrino annihilation luminosities. Following GLL06, we include the neutrino radiation from the optically thick region in the computation for the neutrino luminosity. To show the effects of MC torque, we compare it with previous results. In Section 4, we discuss the stability of the accretion flow. We also discuss the physical origin of the instabilities in the MCNDAF. Finally, we summarize our results and discuss some related issues in Section 5.
2. NEUTRINO-DOMINATED ACCRETION FLOWS WITH MC EFFECTS

Considering that central BHs are rapidly rotating in most candidate GRB engines, we discuss a model of a steady state disk around a Kerr BH, in which neutrino loss and transfer are taken into account. Our model is presented in the context given by DPN02, and the GR corrections are adopted from Riffert & Herold (1995).

As mentioned by DPN02 and PWF99, although the accretion rate may vary in GRB central engines, it is expected to vary significantly only in the outer disk. Hence, it seems reasonable to study the main properties in the inner neutrino-cooled disk by assuming a constant accretion rate.

Because the gas cools efficiently, we are entitled to discuss the MCNDAF model in the context of a thin disk (Shakura & Sunyaev 1973). The accuracy of the thin-disk approximation is not perfect at large radii, where the disk is thick. Fortunately, the details of the outer region have little effect on the solution for the neutrino-cooled disk (Chen & Beloborodov 2007).

The MCNDAF model is a relativistic steady thin disk, and the large-scale magnetic field contributing to the MC process and the small-scale tangled magnetic field related to the viscosity is included. We assume that these two kinds of fields work independently and the large-scale magnetic field remains constant at the BH horizon. Following Blandford (1976), we assume that the magnetic field $B$, on the disk varies as $B \propto \xi^{-n}$, where $\xi \equiv r/r_m$ is the radius in terms of the marginally stable orbit $r_m$ (Novikov & Thorne 1973), and $n$ is the power-law index indicating the degree of concentration of the magnetic field in the central region of the disk.

Based on equipartition relation, the magnetic field at the horizon is related to the mass density at the inner disk as follows (McKinney 2005):

$$
\frac{B^2_{\text{H}}}{8\pi} = \rho_{0,\text{disk}}c^2, \\
(1)
$$

where $\rho_{0,\text{disk}} \equiv M_T/r_0^3/t_x = GM/c^2$, and $r_x = GM/c^2$.

The MC torque is derived from Wang et al. (2002) based on an equivalent circuit given by Macdonald & Thorne (1982) as follows:

$$
\frac{T_{\text{MC}}}{T_0} = 4a_x(1 + q) \int_0^{\pi/2} (1 - \beta) \sin^3 \theta d\theta \\
- 2 - (1 - q) \sin^2 \theta, \\
(2)
$$

where $T_0 \approx 3.26 \times 10^{45}(\frac{M}{10^9 M_{\odot}}) \frac{\mu_0 \gamma^2 \rho}{10^7 G} \frac{r_0^4}{a_x^3} \frac{c}{(G M_{\text{disk}})}$ is the dimensionless BH spin parameter defined by the BH mass $M$ and the angular momentum $J$, $q = \sqrt{1 - a_x^2}$ and $\beta \equiv \Omega_d/\Omega_H$ is the ratio of the angular velocity of the disk $\Omega_d = (r_x^3/G M)^{1/2} + a_x GM/c^3 + 1$ to that of the horizon $\Omega_H = a_x c^3/[2GM(1 + q)]$.

The mapping relation between the angular coordinate $\theta$ on the horizon and the radial coordinate $\xi$ on the disk is derived based on the conservation of magnetic flux as follows (Wang et al. 2003):

$$
\cos \theta = \int_1^\xi \Theta(a_x, \xi, n) d\xi, \\
(3)
$$

where in Equations (3) and (4) $\xi = (r/r_m)^{1/2}$ and $\chi_{ms} = (r_m/r_0)^{1/2}$.

The basic equations of the MCNDAF are given as follows.

1. The continuity equation:

$$
M = -2\pi r v_x, \Sigma, \\
(11)
$$

2. The total pressure consists of five terms—radiation pressure; gas pressure; degeneracy pressure; neutrino pressure; and magnetic pressure:

$$
P = \frac{11}{12} a T^4 + \frac{\rho k T}{m_p} \left( \frac{1 + 3 \Xi_{\text{nu}}}{4} \right) \\
+ 2\pi h c \frac{3}{8 \pi m_p} \left( 3 \left( \frac{\rho}{\mu_e} \right)^{4/3} + \frac{u_v}{3} + P_{\text{mag}} \right), \\
(12)
$$

where $P_{\text{mag}} = B_0^2/8\pi$ is the magnetic pressure contributed by the tangled magnetic field in the disk and $\beta$ is the ratio of the magnetic pressure to the total pressure. The $u_v$ is the neutrino energy density defined as (Popham & Narayan 1995)

$$
u_v = (7/8)a T^4 \sum \frac{\tau_v}{\tau_v/2 + 1/\sqrt{3} + 1/(3a_v)} , \\
(13)
$$

In Equation (13), $\tau_v = \tau_{v_{\nu_e}} + \tau_{v_{\nu_{\mu}}} + \tau_{v_{\nu_{\tau}}}$ is the sum of absorptive and scattering optical depths calculated for each neutrino flavor ($v_{\nu_e}$, $v_{\nu_{\mu}}$, $v_{\nu_{\tau}}$). The absorptive optical depths for the three neutrino flavors are (Kohri et al. 2005)

$$
\tau_{v_{\nu_e}} \approx 2.5 \times 10^{-7} T_{10}^2 H + 4.5 \times 10^{-7} T_{10}^2 X_{\nu_{e}} \rho_{10} H, \\
(14)
$$
\[ \tau_{\alpha, \nu} = \tau_{\alpha, \nu} \simeq 2.5 \times 10^{-7} T_1^2 H, \]

where \( X_{\text{nucl}} \) is the mass fraction of free nucleons approximately given by (e.g., PWF99; Qian & Woosley 1996)

\[ X_{\text{nucl}} \simeq 34.8 \rho_1^{3/4} T_1^{9/8} \exp(-0.61/T_1). \]

The total scattering optical depth is given by DPN02 as

\[ \tau_{\alpha, \nu} \simeq 2.7 \times 10^{-7} T_1^2 \rho_{10} H. \]

3. Combining the conservation of the angular momentum with Equation (11), we have

\[ \frac{d}{dr} (\dot{M} l) + 4\pi r H_{\text{MC}} = \frac{d}{dr} g = -\frac{d}{dr} (4\pi r^2 t_{\nu} H). \]

where \( l \) is the specific angular momentum of the accreting gas. The flux of angular momentum transferred magnetically from the BH to the disk, \( H_{\text{MC}} \), is related to the MC torque \( T_{\text{MC}} \) by

\[ T_{\text{MC}} = 4\pi \int_{r_{\text{in}}} r H_{\text{MC}} r\, dr. \]

Vanishing of \( t_{\nu} \) (or \( g \)) at \( r_{\text{ms}} \) leads to

\[ M r^2 \frac{2GM M}{r^3} + T_{\text{MC}} = \dot{M} = 4\pi r^2 t_{\nu} H = 4\pi r^2 H \alpha P \frac{A}{BC}. \]

4. The equation for the energy balance is

\[ Q^+ = Q^-, \]

where \( Q^+ = Q_{\text{vis}} \) represents the viscous dissipation and \( Q^- = Q_{\nu} + Q_{\text{photo}} \). \( Q_{\text{adv}} \) is the total cooling rate due to neutrino losses \( Q_{\nu} \), photodisintegration \( Q_{\text{photo}} \), and advection \( Q_{\text{adv}} \). We employ a bridging formula for calculating \( Q_{\nu} \), which is valid in both the optically thin and thick cases. The expressions for \( Q_{\nu} \), \( Q_{\text{photo}} \), and \( Q_{\text{adv}} \) are (DPN02; GLL06)

\[
\begin{align*}
Q_{\nu} &= \frac{7(8\pi T^4)}{(3/4)(\tau_{\nu}/2 + 1/\sqrt{3} + 1/3\tau_{\alpha,\nu}}), \\
Q_{\text{photo}} &= 10^{29} \rho_1^{1/2} \frac{dX_{\text{nucl}}}{dr} H \text{ erg cm}^{-2} \text{s}^{-1}, \\
Q_{\text{adv}} &= \nu_r \frac{H}{r} \left( \frac{11}{3} 4 T^4 + \frac{3}{2} \frac{\rho kT}{m_p} + \frac{X_{\text{nucl}}}{4} + \frac{4\nu_r}{3} \right),
\end{align*}
\]

where \( 4\nu_r/3 \) is the entropy density of neutrinos. Note that the cooling function given by the bridging formula reduces to the optically thin expression for small optical depths (as adopted in PWF99) but differs significantly from the latter at optical depths \( \sim 1 \).

By considering the MC effects, the heating rate \( Q_{\text{vis}} \) is expressed as

\[ Q_{\text{vis}} = -\frac{g\Omega_0}{4\pi r} = \frac{3GM M}{8\pi r^3} \frac{\dot{M}}{B} = \frac{T_{\text{MC}} \Omega'_D}{4\pi r}, \]

where the second term is the MC contribution.

As we can see from Equation (20), the magnetic torque may deposit angular momentum in the inner disk, and this extra angular momentum must be transported outward by the viscous torque in the disk, resulting in energy dissipation and increasing the disk luminosity based on Equation (25).

Defining \( Q_G = GM M D/(8\pi r^3 B) \) and \( Q_{\text{MC}} = -T_{\text{MC}} Q_D/(4\pi r) \) as the contributions due to the gravitational release and the MC process, respectively, we have the ratio \( \eta = Q_{\text{MC}}/Q_G \) versus the disk radius \( R \equiv r/r_g \), as shown in Figure 1.

From Figure 1 we find that \( Q_{\text{MC}} \) is much greater than \( Q_G \) in the inner disk, where the neutrino cooling dominates. The ratio \( \eta \) is very sensitive to the value of \( a_n \) and \( n \), and it increases monotonically with the increasing \( a_n \) and \( n \). This implies that the MC effects are more important for the greater \( a_n \) and \( n \). For simplicity, we choose \( a_n = 0.9 \) and \( n = 3 \) in the calculations and discuss the influence of their values in Section 5.

We solve numerically Equations (12), (20), and (21) to find the disk temperature \( T \) and density \( \rho \) versus the disk radius with the given \( a_n \), \( n \), and \( m \) (where \( m \) is the accretion rate in units of \( M_\odot\text{s}^{-1} \)). We take \( X_{\text{nucl}} = 1 \) for the fully photodisintegrated nuclear, which is appropriate in the inner disk. In the calculation, we do not include the cooling term arising from the photodisintegration because it is much less than the neutrino cooling rate in the inner disk (Janiuk et al. 2004). Furthermore, \( \alpha = 0.1 \), \( M = 7 M_\odot \), and \( \beta_r = 0.1 \) are adopted in calculations.

3. EFFECTS OF THE MC TORQUE ON NEUTRINO ANNIHILATION LUMINOSITY

As discussed in Section 2, the MC process applies a strong torque on the disk, resulting in huge viscous dissipation. This would lead to a more powerful neutrino radiation and neutrino annihilation luminosity. Here, we will show the effects of MC torque on the neutrino annihilation luminosity.

To show this, we compare the results of the MCNDAF with the NDAF model without MC (hereafter NDAF refers to the model without MC). GLL06 pointed out that the GR effects and the neutrino radiation from the optically thick region are important for the NDAF luminosity. Therefore, we include these two effects in our calculations for both MCNDAF and NDAF.
The neutrino luminosity from the accretion flow is

\[ L_\nu = 4\pi \int_{r_{\text{in}}}^{r_{\text{out}}} Q_\nu r dr. \]  

We are interested primarily in the properties of the inner accretion flow, where neutrino processes are important. As argued by PWF99, NPK01, and DPN02, the flows are fully advection-dominated for \( r > 100r_g \), since neutrino cooling is not important and photons are completely trapped. Thus, we concentrate the discussion in the region from \( r_{\text{in}} = 100r_g \) to \( r_{\text{max}} = 10^5r_g \).

Our method for calculating neutrino annihilation is similar to that of PWF99 and Rosswog et al. (2003). The disk is modeled as a grid of cells in the equatorial plane. A cell \( k \) has its neutrino mean energy \( \bar{\epsilon}_\nu \) and luminosity \( l_{\nu} \), and the height above (or below) the disk is \( d_k \). The angle at which neutrinos from cell \( k \) encounter antineutrinos from another cell \( k' \) at that point is denoted by \( \theta_{kk'} \). Then the neutrino annihilation luminosity at that point is given by the summation over all pairs of cells:

\[ l_{\nu\bar{\nu}} = A_1 \sum_k \frac{l_{\nu}^{k}}{d_k} \sum_{k'} \frac{l_{\nu^*}^{k'}}{d_{k'}} (\bar{\epsilon}_\nu^{k} + \bar{\epsilon}_{\bar{\nu}}^{k'})(1 - \cos \theta_{kk'})^2 + A_2 \sum_k \frac{l_{\nu}^{k}}{d_k} \sum_{k'} \frac{l_{\nu^*}^{k'}}{d_{k'}} (\bar{\epsilon}_\nu^{k} + \bar{\epsilon}_{\bar{\nu}}^{k'})(1 - \cos \theta_{kk'}) \]  

(27)

where \( A_1 \approx 1.7 \times 10^{-44} \, \text{cm} \, \text{erg}^{-2} \, \text{s}^{-1} \) and \( A_2 \approx 1.6 \times 10^{-56} \, \text{cm} \, \text{erg}^{-2} \, \text{s}^{-1} \).

The total neutrino annihilation luminosity is obtained by integrating over the whole space outside the BH and the disk:

\[ L_{\nu\bar{\nu}} = 4\pi \int \int l_{\nu\bar{\nu}} r dr dz. \]  

(28)

According to our calculations for the MCNDAF, \( L_{\nu\bar{\nu}} \) varies from \( 3.7 \times 10^{49} \, \text{erg} \, \text{s}^{-1} \) to \( 1.4 \times 10^{54} \, \text{erg} \, \text{s}^{-1} \) for \( 0.01 < \dot{m} < 10 \). We find that \( L_{\nu\bar{\nu}} \) nearly stays constant around \( 10^{54} \, \text{erg} \, \text{s}^{-1} \) for the accretion rate above \( \dot{m} \approx 0.5 \). This implies that the effect of neutrino optical depth becomes important. Our results for the NDAF are in good agreement with those given by PWF99 and GLL06, but larger than those in DPN02. This is because the GR effects are taken into account in this paper as well as in PWF99 and GLL06.

4. STABILITY ANALYSIS: THERMAL–VISCOUS INSTABILITY

DPN02 discussed the thermal, viscous, and gravitational stability properties of the NDAF solutions. They found that NDAF is stable in most cases. But this result is not consistent with the variability in GRB light curve. To explain the X-ray flares, it is required that after the prompt gamma-ray emission has ceased, the central engine can be restarted (Fan & Wei 2005; Zhang et al. 2006). Based on this scenario, Perna et al. (2006) suggested that the X-ray flares could be produced by accretion of matter after the breaking of the disk due to the setting up of various instabilities either gravitational or viscous. Therefore, it is interesting to examine whether the MCNDAF solution is stable.

The condition for viscous stability is given by

\[ \frac{dM}{d\Sigma} > 0. \]  

(29)

The stability curves for several radii in the disk are shown in Figure 3.

From Figure 3, we find that the \( \dot{m} - \Sigma \) curves show s-shape, in which the branch of solutions with negative slope is viscously unstable. It is shown that the viscous instability occurs at larger radius for larger accretion rate.

It is clearly shown in Figure 3 that the disk is unstable at \( \dot{m} = 0.5 \) for \( R = 3 \), \( \dot{m} = 2 \) for \( R = 10 \), and \( \dot{m} = 10 \) for \( R = 20 \). To understand this s-shape, we draw Figure 4. In addition, we find that this instability can occur only at \( \dot{m} > 0.086 \) as shown in Figure 5.
In Figures 3 and 4, we find that the viscous instability occurs when the disk is neutrino cooling and the radiation pressure dominates. For \( \dot{m} < 0.086 \), the MC torque will become very small, and the disk is optically thin to neutrinos. As discussed in NPK01, an optically thin NDAF is viscously stable for all pressure cases. Therefore, we find no viscous instability when \( \dot{m} < 0.086 \). If the accretion rate is beyond 0.086 \( M_\odot \text{s}^{-1} \), the MC torque transports enormous energy into the inner disk and makes the inner disk optically thick to neutrinos. In the meantime, we have \( \dot{m} \propto \Sigma^{-1} \) for neutrino cooling and radiation pressure dominated cases, and the disk is viscously unstable. If gas pressure dominates, we have \( \dot{m} \propto \Sigma \) and \( \dot{m} \propto \Sigma^3 \) for optically thin and thick NDAF, respectively. In the advection-dominated region, the disk is viscously stable, which is the well known property of the slim disk.

From the above analysis, we conclude that the appearance of the viscous instability is due to the MC torque. First, the MC torque results in an optically thick neutrino-cooling dominated flow in the inner disk for high accretion rates. Second, the high opacity will drop the neutrino-cooling rate and leave a hot and thick disk. In this region, the gas pressure drops and the radiation pressure becomes important. We checked that the unstable solutions appear for black hole spin \( a_* > 0.35 \) and magnetic pressure \( \beta_\ell < 0.8 \).

The disk is thermally unstable if \( (d \ln Q^+ / d \ln T) |_{\Sigma} > (d \ln Q^- / d \ln T) |_{\Sigma} \). Then, a small increase (decrease) in temperature leads to heating rate, which is more (less) than the cooling rate, and as a consequence a further increase (decrease) in the temperature. For an optically thick NDAF in which neutrino cooling and radiation pressure dominates, we have \( Q^+ \propto T^8 / \Sigma \) and \( Q^- \propto T^4 \). Therefore, the disk is also thermally unstable at the negative slope of the s-shape \( m - \Sigma \) curves shown in Figure 3.

Janiuk et al. (2007) obtain the viscous instability occurring at \( \dot{m} > 10 \) without the s-shape curves, which is caused by the behavior distribution for \( X_{\text{nuc}} \) and photodisintegration terms. However, in the MCNDAF, the photodisintegration term is not included and \( X_{\text{nuc}} = 1 \) is assumed for simplicity. Therefore, we infer that the s-shape curves in the MCNDAF arise from the MC effects.

Finally, we check the gravitational stability condition, for which the Toomre parameter \( Q_T \) should be larger than unity. For Keplerian disk, \( Q_T \) is given by \( Q_T = \frac{c_s \kappa}{(\pi G \Sigma)} = \frac{\Omega_s^2}{(\pi G \rho)} \). \( Q_T \) decrease with increasing \( r \) so that outside the flow is most unstable.

5. SUMMARY AND DISCUSSION

In this paper, we investigate some properties of MCNDAF. The angular momentum deposited in the disk by the magnetic torque exerted by the BH leads to a substantial additional dissipation of energy in the disk, which is greater than that
from the NDAF. A series of the MCNDAF solutions that are significantly different expected from gravitational release alone. Therefore, we obtain Figure 5.

The main results are summarized as follows.

1. The neutrino annihilation luminosity in the MCNDAF varies from $3.7 \times 10^{49}$ erg s$^{-1}$ to $1.4 \times 10^{54}$ erg s$^{-1}$ for $0.01 < \dot{m} < 10$, while for the NDAF the value range is from $1.2 \times 10^{45}$ erg s$^{-1}$ to $2.6 \times 10^{53}$ erg s$^{-1}$, i.e., it is greatly improved by the MC torque. Recently, observations show that half of the Swift bursts exhibit X-ray flares. Fan et al. (2005) pointed out that the energy from the NDAF cannot match the X-ray flares detected in GRB 050724 of $\sim 100$ s, which is also the time scale of the central engine. The time-averaged isotropic luminosity of the X-ray flare component is $L_X \sim 10^{48}$ erg s$^{-1}$. If we assume that the total mass available for accretion is $\sim 1 M_\odot$ (a typical value for the compact object merger scenarios and the massive star collapse scenario) and that most of the mass is accreted during the X-ray flare phase, the time-averaged accretion rate is about $0.01 M_\odot$ s$^{-1}$. At this accretion rate, the jet luminosity powered by neutrino annihilation is $L_{\nu} \sim 10^{45}$ erg s$^{-1}$ for the NDAF without the MC torque, and it is insufficient to power the X-ray flares. But the power produced by the MCNDAF can satisfy this requirement. Very recently, the new observation of the highest redshift ($z = 6.7$) swift source GRB080913 puts a very strong constraint on the central engine (Perez-Ramirez et al. 2008). The duration of this short burst is $T_\text{90} = 8 s$, and the isotropic energy required is $E_{\nu} \approx 7 \times 10^{52}$ ergs. If the central engine is NDAF without the MC torque, the jet collimation factor obeys $f_{\text{coll}} < 7 \times 10^{-4}$, i.e., the jet should be strongly collimated. If we invoke the MC torque in the NDAF, the observed energy can be easily reached.

2. The disk becomes thermally and viscously unstable in its inner region for $\dot{m} > 0.086$. It is interesting to obtain the $\dot{m} - \Sigma$ curves behaving s-shape, which may produce a limit-cycle activity. The disk is rather thick in the inner region, and therefore the thermal and viscous timescales are close to each other. The timescale for the unstable disk can be estimated by the viscous timescale, $t_{\text{vis}} = [1/(\alpha \Omega)](r/H)^2$, which is about 10 ms at the inner disk. Following the discussions in Janiuk et al. (2007), these instabilities will lead to a variable energy output on millisecond timescales, which may correspond to the variability in the gamma-ray luminosity. The irregularity in the overall outflow can also help produce internal shocks. On the other hand, the thermal–viscous instability may be accompanied by the disk breaking, which can lead to the several episodic accretion events and explain the long-time activity accounting for the X-ray flares.

Therefore, the MCNDAF can easily power both GRBs and their X-ray flares, naturally interpret the variability in the gamma-ray luminosity, and explain the production of X-ray flares. However, there are several issues that should be addressed.

First, in all of the MCNDAF solutions, we assume $n = 3$ and a large BH spin $a_\ast = 0.9$, but does not give any reason. From Figure 1 we find that the MC contribution is sensitive to $a_\ast$ and $n$. Thus, for very small BH spin and value of $n$, the MC effects may be ignored, and the solutions return to the NDAF solutions. Considering that the BH is spun-down in the MC process, while it is spun-up in the accretion process, the two processes with opposite effects result in a state with an equilibrium spin $a_\ast^\text{eq}$. Calculation shows that $a_\ast^\text{eq}$ is greater than 0.85 for $n > 3$. This result implies that the MC effects are dominant in the whole duration of GRB, for which a fast-spinning BH is the central engine.

Second, we made many simplifications in the MCNDAF model, such as we omitted the photodisintegration term in the cooling rates, assumed that the disk is thin, and made a very simple magnetic configuration. Recently, Liu et al. (2007) took into account more realistic microphysics. Chen & Beloborodov (2007) worked out the NDAF solution under the full Kerr metric. It is necessary to combine these effects with the MC process and work out a more detailed model in the future.

Finally, our MCNDAF is steady. Recently, Janiuk et al. (2004, 2007) computed the time evolution of the NDAF that proceeds during the burst. It is interesting to investigate a time-dependent MCNDAF.

Recently, Zhang & Dai (2007) proposed a hyperaccretion disk around a neutron star. They found that compared with a BH disk, the hyperaccretion disk around a neutron star can be cooled more efficiently and produce a much higher neutrino luminosity. As discussed by Kluzniak & Rappaport (2007), the magnetic dipole of the neutron star can also torque the disk. Therefore, it is also important to consider the effects of the magnetic torque in the context of hyperaccretion disk around a neutron star.

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