Massive Majorana neutrinos in nuclear processes

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Abstract. The observation of neutrino oscillations has opened a new excited era in neutrino physics and represents a big step forward in our knowledge of neutrino properties. The observed small neutrino masses have profound implications for our understanding of the Universe and are now a major focus in astro, particle and nuclear physics and in cosmology. Of crucial importance is the problem of the nature of neutrinos (Dirac or Majorana). In this context the experimental and theoretical study of the neutrinoless double-beta decay and double-electron capture is at the forefront.

1. Introduction
Neutrinos are an enigma, billions of them come streaming out of the Sun’s core and some neutrinos are created by high-energy cosmic rays from deep space striking air molecules in the Earth’s upper atmosphere. Terrestrial neutrinos are produced at nuclear reactors, accelerators and due to natural radioactivity.

Neutrinos are electrically neutral, and are able to pass through ordinary matter almost undisturbed. This makes neutrinos extremely difficult to detect and to learn more about their basic properties. The recent neutrino oscillation experiments have convinced us that neutrinos are massive particles. However, the problem of absolute scale of neutrino masses, CP violation in neutrino sector, magnetic moments and the nature of neutrinos (Dirac or Majorana), possible existence of sterile neutrinos is still waiting for a solution.

Precise determinations of the neutrino properties will shed light on the breaking of the SM symmetries associated with total baryon and lepton number and lepton flavors. Moreover, neutrinos are potentially a window (through the so-called see-saw mechanism, where the smallness of neutrino masses is explained by the interplay of weak-scale Dirac masses with much larger Majorana masses) to very high mass scales, directly inaccessible to foreseeable colliders.

At present the attention of physics community is concentrated on challenging problem, finding whether neutrinos are indeed Majorana particles (i.e. identical to its own antiparticle) as many particle models suggest or Dirac particles (i.e. is different from its antiparticle).

2. Neutrinoless double-beta decay
The total lepton number (LN) violating neutrinoless double beta decay (0$\nu\beta\beta$-decay) [1],

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^-, \quad (1)$$

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Table 1. Averaged $0\nu\beta\beta$ nuclear matrix elements $\langle M^{0\nu\beta\beta}\rangle$ and their variance $\sigma$ (in parentheses) evaluated in the RQRPA and QRPA [7]. $M^{2\nu}_{GT}$ denotes the $2\nu\beta\beta$-decay nuclear matrix element deduced from $T_{1/2}^{2\nu-exp}$. In column 5 the $0\nu\beta\beta$ half-lives evaluated with the RQRPA average nuclear matrix element and for assumed $m_{\beta\beta} = 50$ meV are shown. For $^{136}$Xe there are two entries: the upper two use the upper limit of the $2\nu$ matrix element while the lower two use the ultimate limit, vanishing $2\nu$ matrix element. $g_A = 1.25$ is assumed.

| Nuclear transition | $M^{2\nu}_{GT}$ [MeV$^{-1}$] | $\langle M^{0\nu\beta\beta}\rangle$ | $T_{1/2}^{0\nu\beta\beta}$ ($\langle m_{\beta\beta}\rangle = 50$ meV) |
|--------------------|---------------------|------------------|------------------|
| Ge$^{76}$ → Se$^{82}$ | 0.15 ± 0.006 | 5.44(0.23) | 4.46$^{+0.22}_{-0.24} \times 10^{26}$ |
| Se$^{82}$ → Kr$^{82}$ | 0.10 ± 0.009 | 4.86(0.20) | 1.26$^{+0.17}_{-0.14} \times 10^{26}$ |
| Zr$^{90}$ → Mo$^{90}$ | 0.11$^{+0.06}_{-0.06}$ | 2.01(0.20) | 3.49$^{+0.48}_{-0.93} \times 10^{26}$ |
| Mo$^{100}$ → Ru$^{100}$ | 0.22 ± 0.01 | 4.28(0.28) | 9.99$^{+1.32}_{-1.55} \times 10^{25}$ |
| Sn$^{116}$ → Sn$^{116}$ | 0.12 ± 0.006 | 3.41(0.24) | 1.47$^{+0.26}_{-0.21} \times 10^{26}$ |
| Te$^{128}$ → Te$^{128}$ | 0.034 ± 0.012 | 4.82(0.15) | 2.04$^{+0.24}_{-0.20} \times 10^{27}$ |
| Te$^{128}$ → Te$^{128}$ | 0.036$^{+0.03}_{-0.09}$ | 4.40(0.13) | 9.70$^{+1.05}_{-1.65} \times 10^{25}$ |
| Xe$^{130}$ → Xe$^{130}$ | 0.030 | 2.89(0.17) | 2.11$^{+0.27}_{-0.23} \times 10^{26}$ |
| Xe$^{136}$ → Ba$^{136}$ | 0.0 | 2.53(0.17) | 2.76$^{+0.30}_{-0.33} \times 10^{26}$ |

is the most powerful tool to clarify if the neutrino is a Dirac or a Majorana particle. From the experimental point of view LN violation in $0\nu\beta\beta$-decay is observed through the appearance of two electrons in the final state with no missing energy.

The inverse value of the $0\nu\beta\beta$-decay half-life for a given isotope (A,Z) is a product of the effective mass of Majorana neutrinos $m_{\beta\beta}$, the known phase-space factor $G_{0\nu}(Q_{\beta\beta}, Z)$ (depending on nuclear charge $Z$ and the energy release $Q_{\beta\beta}$ of the reaction) and the nuclear matrix element $M^{0\nu}$, which depends on the nuclear structure of the particular isotope under study [1]:

$$
(T_{1/2}^{0\nu\beta\beta})^{-1} = G^{0\nu}(Q_{\beta\beta}, Z) |M^{0\nu\beta\beta}|^2 |m_{\beta\beta}|^2,
$$

(2)

where nuclear matrix element $M^{0\nu\beta\beta}$ consists of Fermi, Gamow-Teller and tensor parts as

$$
M^{0\nu\beta\beta} = -\frac{M_F}{g_A} + M_{GT} + M_T. \tag{3}
$$

Experimentally, half-lives are measured or constrained, and the effective Majorana neutrino mass $m_{\beta\beta}$,

$$
m_{\beta\beta} = U^2_{e1} m_1 + U^2_{e2} m_2 + U^2_{e3} m_3, \tag{4}
$$

is the ultimate goal. Here, $U_{ei}$ and $m_i$ ($i=1,2,3$) are elements of Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix [2, 3] and masses of neutrinos, respectively.

From the most precise experiments on the search for $0\nu\beta\beta$-decay the following bounds were inferred: $|m_{\beta\beta}| < (0.20 - 0.32) \text{ eV}$ ($^{76}$Ge [4]), $(0.30 - 0.71) \text{ eV}$ ($^{130}$Te [5]) and $(0.50 - 0.96) \text{ eV}$ ($^{136}$Mo [6]). Here, the $0\nu\beta\beta$-decay NMEs of calculated with Brueckner two-nucleon short-range correlations are used [7]. A new generation of the $0\nu\beta\beta$-decay experiments, in particular GERDA/MAJORANA($^{76}$Ge), SuperNEMO($^{82}$Se), CUORE($^{130}$Te), MOON($^{100}$Mo), COBRA($^{116}$Cd), LUCIFER($^{82}$Se, $^{116}$Cd), EXO($^{136}$Xe), Kamland-ZEN($^{136}$Xe), SNO+($^{150}$Nd) and other experiments, hope to probe $m_{\beta\beta}$ down to 10-50 meV [1], what is the value predicted by making assumption about the inverted hierarchy of neutrino masses. These experiments with about up to 1 ton of radioactive isotope would require 5-10 years of measurements.
Table 2. The calculated $0\nu$ECEC half-lives of $^{152}$Gd for $m_{\beta\beta} = 50$ meV. The second and third columns show the quantum numbers of the electron holes. Here, $n$ is the principal quantum number, $j$ is the total angular momentum, and $l$ is the orbital momentum. The column four shows the radiative widths of the excited electron shells. The column five shows the mass difference of the initial and final atoms. The columns six and seven present the minimum and maximum half-lives of the $0\nu$ECEC transitions obtained within the spherical QRPA ($M^{0\nu ECEC} = 7.6$). The last two columns show the minimum and maximum half-lives calculated by assuming the nuclear matrix element of the deformed QRPA ($M^{0\nu ECEC} = 3.2$).

| Nucl. | $n$ | $2j^l_1$ | $n$ | $2j^l_1$ | $\Gamma_{ab}$ [keV] | $\Delta$ [eV] | $T_{1/2}^{\text{min}}$ [y] | $T_{1/2}^{\text{max}}$ [y] | $T_{1/2}^{\text{min}}$ [y] | $T_{1/2}^{\text{max}}$ [y] |
|-------|-----|-----------|-----|-----------|-------------------|-------------|-------------------|-------------------|-------------------|-------------------|
| $^{152}$Gd | 110 | 210 | 23 | $-0.8 \pm 0.2$ | $8.5 \times 10^{27}$ | $8.7 \times 10^{28}$ | $4.7 \times 10^{28}$ | $4.8 \times 10^{29}$ |
| 110 | 211 | 23 | $-1.3 \pm 0.2$ | $7.6 \times 10^{30}$ | $2.0 \times 10^{31}$ | $4.2 \times 10^{31}$ | $1.1 \times 10^{32}$ |
| 110 | 310 | 32 | $-7.1 \pm 0.2$ | $1.7 \times 10^{31}$ | $2.0 \times 10^{31}$ | $9.4 \times 10^{31}$ | $1.1 \times 10^{32}$ |

For $m_{\beta\beta}$ equal to 50 meV the half-lives for double $\beta$-decaying nuclei of interest are presented in Table 1. The associated NMEs were calculated in the framework of the Quasiparticle Random Phase Approximation (QRPA) and renormalized QRPA by assuming Brueckner short-range correlations and the weak coupling constant of free nucleon $g_A = 1.25$ [7]. We see that half-lives are of $10^{26}$ years and more.

3. Resonant neutrinoless double-electron capture
There has been an increased theoretical and experimental interest [8, 9, 10, 11, 12, 13, 14, 15, 16] to another LN violating process, namely to the neutrinoless double-electron capture ($0\nu$ECEC) [17]:

\[(A, Z) + e^- + e^- \rightarrow (A, Z - 2)^*\].

(5)

Here, the two asterisks denote the possibility of leaving the system in an excited nuclear and/or atomic state, the latter being characterized by two vacancies in the electron shell of the otherwise neutral atom.

The process of the $0\nu$ECEC has been revisited for those cases where the two participating atoms are nearly degenerate in mass [10]. The reaction (5) could in principle be detected by monitoring the X-rays or Auger-electrons emitted from excited electron shell of the atom and the electromagnetic decay of the excited nucleus (in case of a non-ground-state transition). However, the main question remains, which atomic systems are favorable for the detection of the $0\nu$ECEC, what would mean that neutrino is a Majorana particle.

A new theoretical approach developed by Šimkovic and Krivoruchenko [9] and Krivoruchenko et al. [10] allowed a unified description of the oscillations of stable and quasi-stationary atoms, which take place with violation of the total lepton number conservation and are followed by de-excitation with emission of photons.

The $0\nu$ECEC transition rate near the resonance is of Breit-Wigner form [9, 10, 17],

\[\Gamma_{ab}^{0\nu ECEC} = \frac{|V_{ab}|^2}{\Delta^2 + 4|\Gamma_{ab}|^2}\].

(6)

The degeneracy parameter $\Delta$ can be expressed as $\Delta = Q - B_{ab} - E_\gamma$. $Q$ stands for a difference between the initial and final atomic masses in ground states and $E_\gamma$ is an excitation energy of
the daughter nucleus. $B_{ab} = E_a + E_b + E_C$ is the energy of two electron holes, whose quantum numbers $(n, j, l)$ are denoted by indices $a$ and $b$ and $E_C$ is the interaction energy of the two holes. The binding energies of single electron holes $E_{a,b}$ are known with accuracy with few eV [18]. The width of the excited final atom with the electron holes is given by

$$\Gamma_{ab} = \Gamma_a + \Gamma_b + \Gamma^*. \quad (7)$$

Here, $\Gamma_{a,b}$ is one-hole atomic width and $\Gamma^*$ is the de-excitation width of daughter nucleus, which can be neglected. Numerical values of $\Gamma_{ab}$ are about up to few tens eV [19].

For favorable cases of a capture of $s_{1/2}$ and $p_{1/2}$ electrons the explicit form of lepton number violating amplitude associated with nuclear transitions $0^+ \to J^\pi = 0^\pm, 1^\pm$ can be find in [10]. After factorization the electron shell structure and nuclear matrix element and by considering the $0^+ \to 0^+$ nuclear transition we have

$$V_{ab} = \frac{1}{4\pi} G^2_{\beta\beta} m_{\beta\beta} \frac{g_A^2}{R} < F_{ab} > M^{0\nu ECEC}. \quad (8)$$

Here, $< F_{ab} >$ is a combination of averaged upper and lower bispinor components of the atomic electron wave functions [10] and $M^{0\nu ECEC}$ is the nuclear matrix element. We note that by neglecting the lower bispinor components $M^{0\nu ECEC}$ takes the form of the $0\nu\beta\beta$-decay NME for ground state to ground state transition after replacing isospin operators $\tau^-$ by $\tau^+$. Based on the most recent data and realistic evaluation of the decay half-lives, a complete list of the most perspective isotopes for which the $0\nu ECEC$ capture may have the resonance enhancement was provided for further experimental study [10]. Some isotopes such as $^{156}$Gd were found to admit several closely lying resonance levels. Assuming an effective mass for the Majorana neutrino of 50 meV and appropriate value of nuclear matrix element, some half-lives were found to be as low as $10^{25}$ years in the unitary limit.

A detailed calculation of the $0\nu ECEC$ of $^{152}$Gd associated with the ground-state to ground-state nuclear transitions was performed in [20]. Improved measurements of Q-value for this transitions with accuracy of about 100 eV [15] was considered. The nuclear matrix element of $^{152}$Gd $\to$ $^{152}$Sm transition was calculated within spherical QRPA.

However, nuclei participating in this $0\nu ECEC$ ground state to ground state nuclear transition are deformed. The deformation parameter $\beta$ can be extracted from values on measured E2 probability $(Q_p = \sqrt{16\pi B(E2)/5e^2}$, the sign cannot be extracted). Within the deformed QRPA [21, 22, 23] nuclear matrix element of the $^{152}$Gd $\to$ $^{152}$Sm transition becomes by factor 2.3 smaller when compared with the spherical case. From Table 2 we see that the effect of nuclear deformation has a strong impact on the $0\nu ECEC$ half-life of $^{152}$Gd. Currently, it is the smallest known half-life among known $0\nu ECEC$ transitions, which is 2-3 orders of magnitude longer than the half-life of $0\nu\beta\beta$ decay of $^{76}$Ge (see Table 1).

Precise measurements of Q-values between the initial and final atomic states [15, 24, 25, 26], additional spectroscopic information on the excited nuclear states (energy, spin and parity) and reliable calculation of corresponding NMEs are highly required to improve predictions of half-lives of the resonant $0\nu ECEC$. It is expected that the accuracy of 10 eV in the measurement of atomic masses will be achievable in the near future.

4. Conclusions

The search for the $0\nu\beta\beta$-decay represents the new frontiers of neutrino physics, allowing in principle to fix the neutrino mass scale, the neutrino nature and possible CP violation effects. Many next generation $0\nu\beta\beta$-decay experiments are in preparation or under consideration. In the case nature prefers inverted hierarchy of neutrino masses they have a chance to observe this rare process. It would prove that the total LN is not a conserved quantity and that neutrinos are massive Majorana fermions.
It is reasonable to hope that the search for the $0\nu\text{ECEC}$ of atoms, which are sufficiently long lived to conduct a practical experiment, may also established the Majorana nature of neutrinos. This possibility is considered as alternative and complementary to searches for the $0\nu\beta\beta$-decay.

To interpret the data from LN violating nuclear processes accurately better understanding of the nuclear structure effects important for the description of the corresponding nuclear matrix elements is needed. In this connection it is crucial to develop theoretical methods capable to evaluate reliably the nuclear matrix elements, and to realistically asses their uncertainties. Nuclear matrix elements are as important as data.

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