Small Object Signal Formation Model for Optical System with Matrix Photodetector When Processed by the Laplace Operator

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Abstract. The authors review the formation of a small object signal by an optical system with a matrix photodetector. They develop a mathematical model of signal formation and test it. The authors calculate the signal/noise proportion for various small object signal shapes and amplitudes processed by the Laplace operator.

1. Introduction
Currently, matrix photodetectors are one of the main types of data recorders in the optoelectronic system (OES).

One of the key applications of OES is the detection of point objects whose image sizes in the photodetector can be compared to the size of photosensors, and lighting distribution is described by the point-spread function (PSF) of the optical system.

The impossibility of spatial discretization of the registered image in increments smaller than the size of the photosensor is typical of matrix viewing devices.

Besides, it is necessary to maintain a specific proportion between the size of the object image and the size of the photosensor to have the best signal/noise (S/N) proportion.

The shape and amplitude of registered point object signals can shift and depend on the positioning of the image on the matrix [1-3].

This research work considers the impact of changing the point object signal shape and amplitude parameters during the Laplace operator processing on the S/N proportion.

2. Model
Review the process of receiving data for their further processing.

The rays from the object go through the lens that blurs the point and hit the matrix photodetector that discretizes the image and adds noises [13]. The output signal of the matrix photodetector is processed by the scanning operator to identify the object, suppress the background, and reduce noises.
Generally, this process can be represented as follows:

\[
F(i,j) = \begin{cases} 
B(i,j) + N(i,j) + T(i,j) & \text{target detected} \\
B(i,j) + N(i,j) & \text{target not detected}
\end{cases}
\]  

where \(i,j\) are pixel coordinates, \(F(i,j)\) is the signal amplitude for the respective pixel, \(B(i,j)\) is the background, \(N(i,j)\) is the noise, and \(T(i,j)\) is the target.

Firstly, the image is formed in the object space plane. This image consists of three key elements: the background, the target, and the noises. Each of the elements of this image can be expressed as follows:

\[
F(i,j) = B(i,j) + N(i,j) + T(i,j) - \text{target detected}
\]

During registration, the analog input object image is convoluted with the optical system FRT and then accumulated in discrete photosensitive elements of the detector. The mathematical representation of \(f\) with sizes \(M \times N\) and the distortion function \(h\) with sizes \(m \times n\) is expressed as \([1, 2, 11]\):

\[
g(x, y) = h(x, y) * f(x, y) = \sum_{i=-a}^{a} \sum_{j=-b}^{b} h(i, j) f(x+i, y+j)
\]

where:
- \(f(x, y)\) is the original image,
- \(h(x, y)\) is the distortion function,
- \(g(x, y)\) is the resulting image,
- \(a=(m-1)/2\),
- \(b=(n-1)/2\).

The next stage is image discretization. For various options of the signal center hitting the photosensor, there are different signal representation options in the photodetector plane. The shape and amplitude of the signal change in this case, which makes it difficult to detect the object \([4-7]\).

Thus, after the discretization, we get the resulting (blurred) image at the output of the matrix photodetector.

After the output frame is generated, various scanning window operators are used for its improvement and object detection.

Applying various operators to the image produced helps improve the S/N proportion.
3. Scanning operator

Generally, object detection against the background with noises is performed using the “processing window” or “processing operator” of constant sizes that calculate the contrast between the signal and background areas.

To design processing systems, the signal and background areas of the processing windows are formed using the apriori data.

Based on the signal and background area calculations, the resolving statistics (function) is generated that produces the optimum assessment of area contrast [4, 5]. The “processing window” scans the entire image. If the signal/noise proportion exceeds the threshold, the object is detected.

To detect the signal, we used a sliding window with signal and background areas [12] sized 3x3 and 1x1 respectively.

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1/8 1/8 1/8
1/8 1 1/8
1/8 1/8 1/8
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Figure 3. The processing window for the three- and two-dimensional cases and its matrix view.

The sizes of the signal area correspond to the observed object projection sizes. The background area of the “processing window” is generated around the signal area (Figure 3).

4. Experiment

Review the filter operation using several signals with different shapes and amplitudes.

![Point signals: a) point (1x1), b) line (3x1), c) blurred signal with a normal distribution (3x3), d) square with constant amplitude (2x2)](image)

Figure 4. Point signals: a) point (1x1), b) line (3x1), c) blurred signal with a normal distribution (3x3), d) square with constant amplitude (2x2)

During the experiments, we reviewed three types of point signals. The 1x1 signal, the 3x1 blurred line signal, the 3x3 signal with normal brightness distribution, and the 2x2 signal with a constant amplitude. All three signals can be considered point signals corresponding to different observation conditions [8, 9].

This filter extracts the averaged background from the signal, which results in background suppression and point signal detection. However, when the signal is blurred (Figures 3b, 3c), the central area is suppressed due to the high amplitude of adjacent pixels, which results in the reduced S/N proportion. Determine the impact the amplitude of adjacent pixels have on signal suppression against a constant background for 1x1 and 2x2 point signals.
As we can see from the experiment, during the processing of a point-signal image, the background is successfully suppressed, which results in signal detection. However, suppression occurs for small signals (larger than 1x1) due to the high amplitudes of adjacent pixels.

Figure 6 shows that, as the small object signal increases, the amplitude in the central and adjacent elements increases too. After this signal is processed by the Laplace operator, the S/N proportion is reduced 2-2.5 times because, despite the general increase of signal amplitude, the amplitudes of adjacent elements increase faster than in the central element, which results in increasing signal suppression.
We assessed the impact of blurring width on the S/N proportion in experiments. During the experiments, we constructed a gaussoid whose width at the mid-height [10] corresponds with the width of one photosensor.

The assessment of the number of photosensors covered by the gaussoid when its width changed (and amplitude reduced respectively) and the assessment of the number of photosensors covered by the gaussoid when its amplitude changes. At each of the stages, we used the Laplace operator for processing.

Figure 8. Dependence of the S/N ratio on the width and amplitude. Orange is the number of elements, blue is the maximum amplitude, black is the difference.

Experiments showed that when the gaussoid width increases, the signal generated by the Laplace operator reduces up to 0. The increase in the amplitude (almost) does not impact the signal generated by the Laplace operator.
Figure 9. Changes in the S/N proportion when the signal center shifts from one pixel to another. Blue - 1st pixel, orange - 2nd pixel, black - S/N for the 1st pixel, green - S/N for the 2nd pixel.

Similarly, we shifted the gaussoid centered on one photosensor to the adjacent one by increasing the mathematical expectation. The amplitude in these photosensors changed as well (reduced in the first sensor and increased in the second). The experiment showed that when the center of the gaussoid (signal) shifted, the signal produced by the Laplace operator was reduced for the first photosensor and increased for the second photosensor.

As we can see from the experimental data when the background is constant and the amplitude of the pixels adjacent to the signal increases, the central signal is suppressed more and more. Thus, if the amplitude of adjacent pixels is high enough, the central signal can be suppressed completely.

5. Conclusions

The authors review the formation of a small object signal by an optical system with a matrix photodetector based on the constructed mathematical model.

For the constructed mathematical model of S/N proportion generation, we showed that the distortion of signal shape and amplitude is an important factor for the detection of point objects in the images generated by matrix photodetectors.

The conducted experiments show that if the center of the photosensor and the center of the point object do not correspond in the matrix photodetector, the image is blurred, which results in a 2-2.5 S/N proportion reduction because after the Laplace operator processing the signal is suppressed due to the high amplitudes of adjacent pixels.

6. References

[1] Pratt W 1978 Digital Image Processing Interscience Publication (New York, NY, USA) 807
[2] Rafael Gonzalez, Richard Woods 2017 Digital Image Processing (4ed Edition) Pearson Edition 1192
[3] Kolbanev M O 2010 Optimization of the selection of a useful signal in multi-mode information systems Questions of radio electronics, ser. RLT issue 1 pp 92-101
[4] Rogachev V A 2011 Features of signal processing in precision television systems Questions of radio electronics. Series: Television Technology 21 pp 81-86
[5] Berezin V V, Tsytstulin A K 2008 Detection and estimation of coordinates of images of point objects in problems of astronavigation and adaptive optics Vestn. TOGU 1(8) pp 11–20
[6] Samson V, Champagnat F, Giovannelli J-F 2004 Point target detection and subpixel position estimation in optical imagery Appl. Opt. 43 2 pp 257-263
[7] Kirichuk V S, Kosykh V P, Kurmanbek uulu T 2006 Adaptive filtering with subpixel estimation of coordinates of point objects *Avtometriya* 42 1 pp 3–12

[8] Tsytsulin A K, Rogachev V A, Kamenev A A, Merzlyakov M A, Shirobokov V V, Zakutaev A A 2018 Threshold characteristics of detection of precision optoelectronic systems *Questions of radio electronics, ser. Television technology* 4 pp 49-58

[9] Ivanov V G, Kamenev A A, Gusakov V M, Naishuler D V 2017 Quantum-optical devices Physical foundations of functioning and principles of construction of radiation detectors (SPb) VKA them. M A Mozhasiky p 297

[10] Kokhirova G I, Bakhtigaraev N S 2019 Results of space debris research based on observations with the Zeiss-1000 telescope at the Sangloh observatory Proceedings Space debris: fundamental and practical aspects of the threat (Moscow: IKI RAN) pp 78–82

[11] Rogachev V A, Mikhailov A A, Emelyanov A V 2020 Analysis of Point Signal Restorretion Methods 2020 International Multi-Conference on Industrial Engineering and Modern Technologies (FarEastCon 2020) 921593

[12] Sytko I I, Smirnova E E 2020 Instrumental virtual tools for measuring random signal characteristics *Journal of Physics: Conference Series* 1384 Vol 1 pp 1-6

[13] Porfinev L F 1989 Fundamentals of the theory of signal conversion in optoelectronic systems (L.: Mashinostroenie) p 387