Investigation of Langdon effect on the stimulated backward Raman and Brillouin scattering

Jie Qiu¹, Liang Hao¹,∗, Lihua Cao¹,², and Shiyang Zou¹

¹ Institute of Applied Physics and Computational Mathematics, Beijing 100094, People’s Republic of China
² HEDPS, Center for Applied Physics and Technology, Peking University, Beijing 100871, People’s Republic of China

E-mail: hao_liang@iapcm.ac.cn

Received 21 June 2021, revised 24 August 2021
Accepted for publication 10 October 2021
Published 10 November 2021

Abstract
In a laser-irradiated plasma, the Langdon effect makes the electron energy distribution function tend to a super-Gaussian distribution, which has important influence on laser plasma instabilities. In this work, the influence of a super-Gaussian electron energy distribution function on the convective stimulated backward Raman scattering and stimulated backward Brillouin scattering is studied systematically for a wide range of typical plasma parameters in the inertial confinement fusion. Distinct behaviors are found for stimulated Raman scattering and stimulated Brillouin scattering in the variation trend of the peak spatial growth rate and the corresponding wavelength of the scattered light. Especially, the Langdon effect on the stimulated Brillouin scattering in plasmas with different ion species and isotopes is analyzed in detail, and the parameter boundary for judging the variation trend of the peak spatial growth rate of stimulated Brillouin scattering with the super-Gaussian exponent is presented for the first time. In certain plasma parameter region, it is found that the Langdon effect could enhance stimulated Brillouin scattering in mixed plasma, which may attenuate the improvement in suppressing stimulated Brillouin scattering by mixing low-Z ions into the high-Z plasma. The comprehension of Langdon effect on stimulated Raman scattering and stimulated Brillouin scattering would contribute to a better understanding of laser plasma instabilities in experiments.

Keywords: Langdon effect, EEDF, SRS, SBS, spectra

(Some figures may appear in colour only in the online journal)

1. Introduction

In laser-driven inertial confinement fusion (ICF), the dominant heating mechanism is inverse bremsstrahlung (IB) heating. When the heating rate of IB exceeds the electron thermalization rate, the electron energy distribution function (EEDF) would deviate from a Maxwellian EEDF and become a super-Gaussian one [1, 2]. This so-called Langdon effect is tightly related to the laser intensity $I_0$, electron temperature $T_e$ and charge state $Z$ of plasma, and has important influence on the processes of electron conduction, IB absorption, laser plasma instabilities (LPIs). Recent study showed that with the increasing intensity, the Langdon effect occurred when $Z\nu_{os}^2/\nu_{th}^2 \gg 1$ but would be suppressed by the nonlinear effect at high laser intensity when $\nu_{os} \gg \nu_{th}$, where $\nu_{os}$ and $\nu_{th}$ are the electron quiver velocity and electron thermal velocity respectively [3]. In ICF, the Langdon effect should be important in high-Z plasma ablated from the hohlraum wall in indirect-drive experiments. Besides, recent experimental results revealed the

* Author to whom any correspondence should be addressed.
presence of super-Gaussian EEDFs when multibeamsl irradiate at the low-Z gas plasma target [4] via the Thomson scattering technique [5, 6]. The power transfer of crossed beams was found to be overestimated by the standard Maxwellian calculations, and the impact of Langdon effect on the crossed beam energy transfer (CBET) was considered to be a possible reason for the discrepancy between the Maxwellian calculations and experimental results on National Ignition Facility (NIF) [4].

Besides CBET, backward stimulated Raman scattering (SRS) and backward simulated Brillouin scattering (SBS) are also important LPs in ICF [7–11]. Commonly, SRS and SBS are three-wave interaction processes where an incident electromagnetic wave (EMW) decays into a backscattered EMW and a forward propagating electron plasma wave (EPW) or ion acoustic wave (IAW), respectively. They are detrimental to ICF, since the backscattered light can take energy away from the incident laser, and the hot electrons generated by SRS can preheat the capsule [7]. Besides, the strong backscattered light of SBS also has a potential risk to damage the optical device within the laser facility. In experiments, the spectra and reflectivity of the backscattered light are important diagnostics for SRS and SBS [12]. Currently, the investigation of the scattered spectra and reflectivity usually assumes a Maxwellian EEDF [13, 14]. However, there was some discrepancy between the experimentally measured SRS spectra and the simulated result by the ray-tracing method [15, 16]. Furthermore, to match the experimental reflectivity of SRS, an artificial large seed of backward SRS light was also needed in calculations [16]. Because the SRS and SBS are very sensitive to the EEDF, one possible source of the discrepancy is the assumption of a Maxwellian EEDF in the current physical model of the ray-tracing method. Although the impact of super-Gaussian EEDF on the threshold of SRS and the ion acoustic frequency of SBS was studied for some specific plasma conditions [17, 18], for practical interest, it is necessary to study the effects of super-Gaussian EEDF on the gain and spectra of SRS and SBS systematically for a wide range of typical plasma conditions and plasma compositions in ICF.

To help resolve these issues, in this work, the influence of super-Gaussian EEDF with different exponents on the SRS and SBS for a wide range of plasma conditions is studied systematically based on the linear gain coefficient calculation. Besides the distinct behaviors in wavelength of scattered light and the growth rate of SRS and SBS, some of which are consistent with previous works, the effects of super-Gaussian EEDF on the peak value of spatial grow rate of SBS are found to be different in plasmas with different ion compositions. For each typical plasma composition, the boundary of plasma parameters for the variation trend of the peak spatial growth rate of SBS with the super-Gaussian exponent is presented for the first time, which is useful for the convenient quick judgment of the variation trend of SBS. One interesting result is that the Langdon effect can weaken SBS in single species plasma but enhance SBS in mixed plasma under certain plasma parameter conditions. For such parameters, although mixing low-Z ion species is often used to suppress SBS [19, 20], Langdon effect may attenuate this improvement. Furthermore, it is also found that the isotopic type of the low-Z mixture can have a significant impact on SBS. This work not only advances the further physical modeling and ray-tracing simulations of backscattering instabilities with the consideration of Langdon effect, but also helps to better understand the spectra and growth of SRS and SBS in experiments.

This paper is organized as follows: In section 2, the theoretical analysis method is specified. In section 3, the influence of a super-Gaussian EEDF on the linear gain coefficient versus wavelength is discussed for SRS and SBS processes under various plasma conditions. In section 4, the conclusions as well as some discussions are given.

2. Theoretical analysis method

Due to the IB heating effect in a laser-irradiated plasma, the slow-variation component of the EEDF can deviate from the Maxwellian [1]. This distorted EEDF can be described by an isotropic super-Gaussian function [2],

\[ f_{\text{se}}(v) = \frac{n_{e} m}{4 \pi v_{\text{th}, e}^{3} \beta_{m}^{2} m!} \exp \left[ - \left( \frac{v}{\beta_{m} v_{\text{th}, e}} \right)^{m} \right]. \]  

where the distortion is characterized by the super-Gaussian exponent \( m \), the factor \( \beta_{m} = \sqrt{3 \Gamma(3/m)/\Gamma(5/m)} \), \( \Gamma \) is the gamma function, and \( v_{\text{th}, e} = \sqrt{T_{e}/m_{e}} \) is the electron thermal velocity. The super-Gaussian exponent \( m \) is determined by the competition between Langdon heating and electron–electron collisions. By fitting to the Fokker–Planck simulation, a formula of \( m \) which is valid from low to very high intensity is given as [3]

\[ m = 2 + \frac{3}{1 + 0.62/\alpha_{\text{ln}}}, \]  

where \( \alpha_{\text{ln}} = Z_{\text{eff}}(v_{\text{on}}^{3}/v_{\text{th}, e}^{3})/(v_{\text{on}}^{3}/v_{\text{th}, e}^{3} + 1)^{3} \), \( Z_{\text{eff}} = Z_{Z}^{1/3} \), and \( v_{\text{on}} \equiv E_{0}/m_{e} \omega_{0} \) is the electron quiver velocity with \( E_{0} \) and \( \omega_{0} \) as the amplitude and frequency of the incident laser, respectively.

As resonant instabilities, SRS and SBS are sensitive to the EEDF and background plasma parameters. To investigate the impact of super-Gaussian EEDF on SRS and SBS under different plasma parameter conditions, we start from the linear gain exponent which is an important quantity to characterize the convective amplification of SRS and SBS [7]. In WKB approximation, the spectrum of linear gain exponent, which had been widely used to calculate the backscattered SRS and SBS spectra in the ray-tracing method [14], is defined as

\[ G_{R,B}(\omega_{s}) = \int_{\text{path}} K_{R,B}(z, \omega_{s}) \, dz, \]  

where the subscript R or B denotes the SRS or SBS process, and \( \omega_{s} \) is the radian frequency of the scattered light. The integration is along one specified laser ray, and the hot electrons generated by the IB heating effect in a laser-irradiated plasma, the slow-variation component of the EEDF can deviate from the Maxwellian [1]. This distorted EEDF can be described by an isotropic super-Gaussian function [2],

\[ f_{\text{se}}(v) = \frac{n_{e} m}{4 \pi v_{\text{th}, e}^{3} \beta_{m}^{2} m!} \exp \left[ - \left( \frac{v}{\beta_{m} v_{\text{th}, e}} \right)^{m} \right]. \]  

where the distortion is characterized by the super-Gaussian exponent \( m \), the factor \( \beta_{m} = \sqrt{3 \Gamma(3/m)/\Gamma(5/m)} \), \( \Gamma \) is the gamma function, and \( v_{\text{th}, e} = \sqrt{T_{e}/m_{e}} \) is the electron thermal velocity. The super-Gaussian exponent \( m \) is determined by the competition between Langdon heating and electron–electron collisions. By fitting to the Fokker–Planck simulation, a formula of \( m \) which is valid from low to very high intensity is given as [3]

\[ m = 2 + \frac{3}{1 + 0.62/\alpha_{\text{ln}}}, \]  

where \( \alpha_{\text{ln}} = Z_{\text{eff}}(v_{\text{on}}^{3}/v_{\text{th}, e}^{3})/(v_{\text{on}}^{3}/v_{\text{th}, e}^{3} + 1)^{3} \), \( Z_{\text{eff}} = Z_{Z}^{1/3} \), and \( v_{\text{on}} \equiv E_{0}/m_{e} \omega_{0} \) is the electron quiver velocity with \( E_{0} \) and \( \omega_{0} \) as the amplitude and frequency of the incident laser, respectively.

As resonant instabilities, SRS and SBS are sensitive to the EEDF and background plasma parameters. To investigate the impact of super-Gaussian EEDF on SRS and SBS under different plasma parameter conditions, we start from the linear gain exponent which is an important quantity to characterize the convective amplification of SRS and SBS [7]. In WKB approximation, the spectrum of linear gain exponent, which had been widely used to calculate the backscattered SRS and SBS spectra in the ray-tracing method [14], is defined as

\[ G_{R,B}(\omega_{s}) = \int_{\text{path}} K_{R,B}(z, \omega_{s}) \, dz, \]  

where the subscript R or B denotes the SRS or SBS process, and \( \omega_{s} \) is the radian frequency of the scattered light. The integration is along one specified laser ray, and the local spatial growth rate \( K_{R,B} \) obtained by the kinetic theory [21] is given by,
\[ K_{R,B} (\omega_s) = \frac{1}{4} k^2 \frac{m}{e^2} \text{Im} \left[ \frac{\chi_e (1 + \chi_{\text{ion}})}{\epsilon} \right] \]  

(4)

where the subscript 1 or a is for EPW or IAW respectively, \( \chi_e (\omega_{1,a}, k_{1,a}) \) is the electron susceptibility, \( \chi_{\text{ion}} (\omega_{1,a}, k_{1,a}) \) is the total susceptibility of ions, and \( \epsilon = 1 + \chi_e + \chi_{\text{ion}} \) is the dielectric function. The frequency \( \omega_{sa} \) and wave-number \( k_{a} \) for the scattered wave are related to \( \omega_{1,a} \) and \( k_{1,a} \) for EPW or IAW by the frequency and wave-number matching conditions

\[
\omega_0 = \omega_s + \omega_{1,a}, \\
 k_0 = k_s + k_{1,a}
\]

(5)

where \( k_0 \) is the wavenumber of the incident laser. The incident laser and scattered light satisfy the dispersion relation for the EMWs,

\[
 \omega_{sa}^2 = \omega_{pe}^2 + c^2 k_{0,s}^2.
\]

(6)

The linear kinetic electron susceptibility can be derived from the perturbed Vlasov equation [22]

\[
\frac{\partial \delta f_e}{\partial t} + \mathbf{E} \cdot \nabla \delta f_e = \frac{e}{m} \frac{\partial \delta f_e}{\partial \mathbf{v}} \cdot \nabla \mathbf{E}
\]

\[
\delta n_e = \int \delta f_e dv = \left( \chi_e / \epsilon \right) e_0 \nabla \cdot \mathbf{E}
\]

(7)

where \( f_e \) is the EEDF of the background plasma, \( \delta f_e \) and \( \delta n_e \) are the perturbation of EEDF and electron density respectively, \( \mathbf{E} \) is the electrostatic field, \( e \) is the charge of electron, and \( e_0 \) is the vacuum permittivity. Notice that the background EEDF includes both a slow-variation component \( f_{0\alpha} \) and high frequency components such as \( f_{1\alpha} \) which oscillates at \( \omega_0 \). However, only \( f_{0\alpha} \) can participate in the generation of EPW or IAW with both the perturbation \( \delta n_e \) and \( \mathbf{E} \) at frequency \( \omega_{1,a} \), since the Gauss’s law \( e_0 \nabla \cdot \mathbf{E} = -\epsilon_0 \delta n_e + \int Z_\alpha \delta n_{\alpha\alpha} dv \), which requires resonance of \( \mathbf{E} \) and \( \delta n_{\alpha\alpha} \) can not be satisfied for the density perturbation at the frequency \( \omega_0 \pm \omega_{1,a} \) driven by \( \mathbf{E} \) and the high frequency component \( f_{1\alpha} \). Here, \( Z_\alpha \) and \( \delta n_{\alpha\alpha} \) are charge state and perturbation of ion species \( \alpha \). In practical ray-tracing simulations, the main pulse is divided into different time slices with intervals such as 100 ps (~10^5 laser period), and the gain \( G_{R,B} \) calculated at an instantaneous step within each time slice is used to evaluate the average level of SRS and SBS in that corresponding time slice [13, 15, 16]. Consequently, \( \chi_e \) can be substituted by \( \chi_{0\alpha} \) which can be assumed constant within each time slice when calculating \( \chi_e \). Then, equation (7) yields

\[
\chi_e (\omega, k) = \frac{\omega_{pe}^2}{k^2 \int f_{0\alpha} dv} \int \frac{k \cdot \partial f_{0\alpha} / \partial \mathbf{v}}{\omega - k \cdot \mathbf{v}} dv
\]

\[
= \frac{\omega_{pe}^2}{k^2 \int f_{0\alpha} dv} \int_{-\infty}^{\infty} \frac{\partial f_{0\alpha} / \partial \nu_v}{\omega / k - \nu_v} dv_v
\]

(8)

where \( \omega_{pe} \) is the electron plasma frequency, and the 1D distribution function \( f_{0\alpha} (v_x) = \int f_{0\alpha} (v_x, v_y, v_z) dv_y dv_z \). For the isotropic super-Gaussian EEDF given in equation (1), it can be obtained

\[
\chi_e (\omega, k, m) = \frac{1}{k^2 \chi_{0e}} \frac{Z_e}{k v_{th\alpha}} m
\]

(9)

where

\[
Z_e [x, m] = \frac{1}{\Lambda_m} \left[ 1 + \frac{m}{2 \Gamma (1/m) \beta_m} \int_{-\infty}^{\infty} \exp (-u^m) du \right]
\]

(10)

where \( \Lambda_m = 3 \Gamma^2 (3/m) / \Gamma (1/m) \Gamma (5/m) \).

A Maxwellian distribution with an ion temperature \( T_i \) is assumed for all the ion species, yielding the ion susceptibility

\[
\chi_{\text{ion}} = \sum_{\alpha} \chi_{0\alpha} (\omega, k) = \sum_{\alpha} \frac{1}{k^2 \chi_{0\alpha \alpha}} [1 + c_{\alpha} Z (\zeta_{\alpha})]
\]

(11)

summed over the ion species \( \alpha \), where \( Z (\zeta_{\alpha}) \) is the plasma dispersion function with \( \zeta_{\alpha} = \omega / k v_{th\alpha} \), the thermal velocity \( v_{th\alpha} = \sqrt{T_i / m_{\alpha}} \), the Debye length \( \lambda_{\alpha \alpha} = v_{th\alpha} / \omega_{pe\alpha} \), and the ion plasma frequency \( \omega_{pe\alpha} = \sqrt{e^2 Z_{\alpha} n_{\alpha} / e_0 n_{th\alpha}} \) with charge state \( Z_{\alpha} \) and ion mass \( m_{\alpha} \).

Typically in experimental analysis, the profile of plasma parameters is provided by the radiation hydrodynamic simulation, and the time-resolved spectra of backscattered light of SRS and SBS are obtained by the ray-tracing LPI simulation [13, 14, 23], where the linear gains of SRS and SBS are calculated by integrating the spatial growth rates \( K_{R,B} \) along the ray path. As a consequence, the effects of super-Gaussian EEDFs on SRS and SBS are essentially determined by the impacts of \( m \) on \( K_{R,B} \) under different plasma parameters and plasma compositions, which are mainly discussed in the following sections.

### 3. Influences of Langdon effect on SRS and SBS

In ICF hohlraum, the evolution of background plasma is determined by all the irradiating beams. The typical ion species include the initial filled gas like mid-Z CsH_{2} or CO_{2}, or low-Z He or the mixture of H_{2} and He (labeled as HHe in following paper), the mid-Z CH plasma from the ablation off the capsule, and the high-Z plasma such as Au or AuB ablated from hohlraum wall [7, 19, 20]. According to the empirical formula given by equation (2), the super-Gaussian exponent \( m \) depends on the laser intensities \( I_{15} \) and \( I_{15} / v_{th\alpha}^{2} \propto I_{15} / T_{e,k ev} (1 - n_{e} / n_{c})^{1/2} \), where \( I_{15} = I_{0} [W \text{cm}^{-2}] / 10^{15} \), \( T_{e,k ev} \) is the value of \( T_{e} \) in unit of keV, and \( n_{c} \) is the critical density of incident laser with wavelength \( \lambda_{0} = 0.351 \mu \text{m} \) in this paper. In hohlraum experiment, \( T_{e} \) is typically a few kiloelectronvolts. Considering the complex beam overlapping [24] and the high intensity speckles in the laser spot [7], the overlapped local intensity for IB heating can range from \( 10^{14} \) to about \( 10^{16} \) W cm^{-2}. Consequently, the possible values of \( I_{15} / T_{e,k ev} (1 - n_{e} / n_{c})^{1/2} \) approximately cover the range of 0.1–10 in ICF hohlraum, over which the variation of \( m \) with \( I_{15} / T_{e,k ev} (1 - n_{e} / n_{c})^{1/2} \) is shown in figure 1.
for typical ion compositions in hohlraum. It can be seen that the typical value of the super-Gaussian exponent \( m \) can vary from 2 to about 3, from 2 to about 4 and from 2 to about 5 for the low-Z, mid-Z and high-Z plasmas, respectively. Consequently, although the plasma parameters vary at different locations on different rays in hohlraum, the Langdon effect should be prevalent and important to LPIs.

3.1 SRS process

In hohlraum, SRS mainly occurs in the gas region with low-Z or mid-Z plasma [7], for which \( m \) can vary from 2 to about 4 as shown in figure 1. Through a systematic analysis of the influence of \( m \) on \( K_R \) for various plasma conditions, it is found that the wavelength of backward SRS light corresponding to the peak value of \( K_R \) decreases with \( m \) in the regime of high \( n_e \) and low \( T_e \) while increases with \( m \) in the regime of low \( n_e \) and high \( T_e \), but the peak value of \( K_R \) always increases with \( m \), irrespective of the plasma condition. Besides, the half width of \( K_R \) versus \( \lambda_R \) is anti-correlated to the peak value of \( K_R \), implying that it always decreases with \( m \). To illustrate these effects, as two typical examples in different regimes of \( n_e \) and \( T_e \), case I and case II are shown in figures 2(a) and (b), respectively. Notice that \( K_R \) is proportional to the intensity of the single beam that stimulates the SRS, which may be unequal to the total intensity contributing to the IB heating, due to effects
such as beam overlapping. For the universality, $K_R/I_{fs}$ which is independent of the single beam intensity is used to characterize the physics trends in these figures.

Commonly, $K_R$ peaks approximately at the naturally resonant frequency of EPW, which satisfies the dispersion equation
\[ \chi_e(\omega_i - i\nu, k_i, m) + 1 = 0, \]
where $k_i$, $\omega_i$ and $\nu$ are the wavenumber, frequency and Landau damping of EPW, respectively; while the wavelength of backward SRS light is related to $\omega_i$ by $\lambda_R = \omega_i / (\omega_0 - \omega_i)$. To understand the different variation trends of peak wavelength with $m$ at different plasma conditions, a normalized dispersion relation
\[ Z_e \left[ \frac{\omega_i - \bar{\nu}_l}{\omega_{pe}(k_i\lambda_{De})-m} \right] + k_i^2 \lambda_{De}^2 = 0 \]  
(12)
of $\omega_i/\omega_{pe}$ versus $k_i\lambda_{De}$ for EPW is calculated at different $m$ as shown in figure 3. For a given $k_i\lambda_{De}$, there are roughly three regimes as labeled in figure 3(a). In regime I, where $k_i\lambda_{De} < 0.43$, $\omega_i$ decreases monotonously with $m$, and the dependence of the dispersion relation on $m$ is rather weak at about $k_i\lambda_{De} < 0.2$. In regime II, where $0.43 < k_i\lambda_{De} < 0.55$, the variation trend of $\omega_i$ with $m$ becomes non-monotonous. In regime III, where $k_i\lambda_{De} > 0.55$, $\omega_i$ increases monotonously with $m$. In fact, for a fixed $k_i\lambda_{De}$, $\Delta\omega_i/\Delta m = -(\partial Z_e/\partial m)/(\partial Z_e/\partial \omega_i)$ can be obtained from equation (12). It can be proven $\partial Z_e/\partial \omega_i > 0$ at the EPW frequency, but the sign of $\partial Z_e/\partial m$ depends on the values of $k_i\lambda_{De}$ and $m$, which results in the three regimes of different $k_i\lambda_{De}$. For case I and case II shown in figure 2, by solving the dispersion relations equations (6) and (12) together with the matching conditions equation (5), $k_i\lambda_{De}$ can be obtained to be about 0.36 for case I and 0.68 for case II as indicated in figure 3(a). As expected, case I and case II are located in the regimes where $\lambda_R$ is blue-shifted and red-shifted with $m$, respectively.

To understand the growth of the peak value of $K_R$ with $m$ and the anti-correlation between the peak value and the half width of $K_R$,
\[ K_R \sim -\text{Im} \left[ \frac{1}{1 + \chi_e} \right] = \frac{\chi_e^2}{(1 + \chi_e^2)^2 + (\chi_e^2)^2} \]  
(13)
can be deduced from equation (4), where the superscripts $R$ and $I$ denote the real and imaginary parts, respectively. According to equation (13), the half width is determined by $|1 + \chi_e^2| \approx \chi_e^2$, so a higher peak value of $K_R$ is always related to a smaller half width of it. At the peak, $\chi_e^2 + 1 \approx 0$, and the peak value of $K_R$ is proportional to $1/\chi_e^2$. Since $\chi_e^2 = \nu_l(\partial \bar{e}_R / \partial \omega) \approx \nu_l(\partial \bar{e}_I / \partial \omega)$ [14], it is $\nu_l$ that primarily determines the trend of the peak value of $K_R$ with $m$. As shown in figure 3(b), $\nu_l$ decreases with $m$ for all $k_i\lambda_{De}$, so the peak value of $K_R$ always increases with $m$. The trend of Landau damping with $m$ and $k_i\lambda_{De}$ can be understood from the feature of the EEDF near the matched phase velocity $v_{ph}$ at $\omega_i/k_i$. For $\nu_l \ll \omega_i$, $\chi_e^2$ from equation (8) can be written as
\[ \chi_e^2 \approx \frac{\omega_{pe}^2}{k_i^2} \int_{-\infty}^{\infty} \frac{\partial \bar{e}_R}{\partial \nu} \text{Im} \left[ \frac{1}{\omega_i/k_i - \nu_i} \right] d\nu_i \]
\[ = \frac{-\pi \omega_{pe}^2}{k_i^2} \int_{-\infty}^{\infty} \frac{\partial \bar{e}_R}{\partial \nu} \bigg|_{\nu_i = \nu_{ph}}, \]  
(14)
where the second equality follows by using $\text{Im}[1/(\omega/k - \nu_i)] = -\pi \delta(\omega/k - \nu_i)$ [22]. As shown in figure 4, in case I of small $k_i\lambda_{De}$, the matched $v_{ph}$ is located in the tail of the EEDF where $\chi_e^2 \propto e^{-\nu_{ph}^2}$, and $\nu_l$ decreases with $m$ rapidly. While in case II of large $k_i\lambda_{De}$, the matched $v_{ph}$ is in the bulk region of the EEDF where $\chi_e^2(\nu_{ph})$ at different $m$ is close. For such a case, it is the increase of the matched phase velocity with $m$ (see figure 3(a)) that leads to the decrease of Landau damping with $m$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{(a) The dispersion relations and (b) Landau damping of EPW at different $m$. The matching points for each $m$ are indicated by the diamond and asterisk symbols for case I and case II, respectively.}
\end{figure}
To comprehend the behavior of $K_B$ with $m$, equation (4) can be written as

$$K_B = \frac{1}{4} \frac{k^2 c_0^2 \mathcal{G}^R}{k c_0^2 (\mathcal{G}^R)^2 + (\mathcal{G}^R)^2},$$  \hspace{1cm} (15)$$

where

$$\mathcal{G} \equiv \frac{1}{\lambda e} + \frac{1}{1 + \chi_{ion}}$$ \hspace{1cm} (16)$$

and superscripts R and I denote the real and imaginary parts, respectively. Near the peak of $K_B$, $\mathcal{G}^R \approx 0$ is consistent with the dispersion relation of IAW $\epsilon \approx 0$ for weak damping $|\lambda_{e}^{R}| \ll |\lambda_{e}^{I}|$ and $|\lambda_{ion}^{R}| \ll |1 + \chi_{ion}|$, implying $K_B$ peaks near the natural resonance mode. Thus, the peak value of the spatial growth rate approximately satisfies

$$K_B \approx -\frac{1}{4} \frac{k^2 c_0^2}{k c_0^2} \mathcal{G}^R.$$ \hspace{1cm} (17)$$

The redshift of SBS backscattered light with $m$ can be understood from the dispersion relation of IAW. Under the approximation $\omega_{a0}/k_0 \ll v_{th0}$, for the super-Gaussian EEDF

$$\chi_{e}^{R} \approx \frac{1}{A_{m}k_0^2 \lambda_{e}^{R}} \approx \frac{1}{k_0^2 A_{m}T_{e}}$$ \hspace{1cm} (18)$$

Because $A_{m}$ increases with $m$ monotonically, $\chi_{e}^{R}$ decreases with $m$, implying a reduced number of slow electrons to shield the ions due to the Langdon effect [18]. For such $\chi_{e}^{R}$, the ion acoustic velocity $c_{s} = \omega_{a0}/k_0$ in plasma composed of one or two species [25, 26], is given by

$$c_{s}^2 = \frac{1}{2} (A + \sqrt{B})$$ \hspace{1cm} (19)$$

where ‘+’ corresponds to the fast mode, and ‘−’ corresponds to the slow mode, and

$$A \equiv \left( \frac{\gamma_{1}}{A_{1}} + \frac{\gamma_{2}}{A_{2}} \right) \frac{T_{i}}{M_{p}} + \frac{Z^2/A_{m}T_{e}}{ZM_{p}q}$$ \hspace{1cm} (20)$$

and

$$B \equiv \left( \frac{\gamma_{1}}{A_{1}} - \frac{\gamma_{2}}{A_{2}} \right)^2 \left( \frac{T_{i}}{M_{p}} \right)^2 + \left( \frac{Z^2/A_{m}T_{e}}{ZM_{p}q} \right)^2 + 2 \left( \frac{\gamma_{1}}{A_{1}} - \frac{\gamma_{2}}{A_{2}} \right) \left( \frac{f_{1}Z_{1}^2}{A_{1}} - \frac{f_{2}Z_{2}^2}{A_{2}} \right) \frac{T_{m}T_{e}}{M_{p}Zq}.$$ \hspace{1cm} (21)$$

Here $q \equiv 1 + A_{m}k_0^2 \lambda_{e}^{R}$, $M_{p}$ is the proton mass, the ion species is indicated by the subscript $\alpha = 1, 2$, $\gamma_{\alpha}$ is the adiabatic exponent, $A_{\alpha}$ is the mass number, and $f_{\alpha}$ is the number fraction of ion species $\alpha$. The bars denote the average, i.e.

$$\overline{Z} = f_{1}Z_{1} + f_{2}Z_{2}$$

$$\overline{Z^2/A} = f_{1}Z_{1}^2/A_{1} + f_{2}Z_{2}^2/A_{2}.$$
effectively boosted to $A_m T_e$. In appendix A, it is proven that $c_s$ increases with $A_m T_e$ for both the fast and slow modes, when all other parameters such as $T_i$ and $n_e$ are kept constant. So it can be expected that due to the effective increase of Debye shielding electron temperature caused by a super-Gaussian EEDF, the IAW frequency $\omega_a = k_a c_s$ increases with $m$. Consequently, the scattered wavelength $\lambda_B = \omega_0 \lambda_0 / (\omega_0 - \omega_a)$ increases as $m$ increases.

From equation (17), the behavior of the peak value of $K_B$ with $m$ is primarily determined by the value of $-\mathcal{P}$ near the peak wavelength. Since at the peak $\epsilon \approx 0$ and $|\chi_e| \approx |1 + \chi_{ion}|$, it is found $K_B \propto -1/|\mathcal{P}| \approx |\chi_e| |1 + \chi_{ion}| / (\chi_e + \chi_{ion})$. Considering that $\chi_{ion} = \text{Im} \{1/(1 + \chi_{ion})\} = -\chi_{ion}/|1 + \chi_{ion}|^2$ (23), and $\chi_e = \text{Im} \{1/\chi_e\} = -\chi_e/|\chi_e|^2$ (24), the relative contribution of electron and ion to $\mathcal{P}$ is determined by the relative importance of the electron Landau damping versus the ion Landau damping. As shown in figure 4, the electron Landau damping reaches maximum when the phase velocity
Figure 6. The convective spatial growth rate ($K_B/I_{15}$) of SBS versus the scattered wavelength shift ($\lambda_B - \lambda_0$) at different super-Gaussian exponents $m$ in (a) Au plasma and (b) Au$^{10}$B and Au$^{11}$B plasmas. The condition of case ‘d’ as specified in table 2 is taken.

Table 1. The representative cases for He and HHe (1:1) plasmas. In all cases, the flow velocity is assumed to be zero.

| Case | $n_e/n_c$ | $T_e$ (keV) | $T_i$ (keV) | $I_{15}^a$ |
|------|-----------|-------------|-------------|-----------|
| ‘a’-He | 0.08 | 2.5 | 0.5 | m = 2.3 | 1.9 | 4.8 | 9.8 |
| ‘b’-He | 0.06 | 2 | 1 | m = 2.6 | 1.6 | 3.9 | 7.9 |
| ‘a’-HHe | 0.08 | 2.5 | 0.5 | m = 2.9 | 2.4 | 6.0 | 12.9 |
| ‘c’-HHe | 0.06 | 1 | 0.12 | | 0.95 | 2.4 | 5.2 |

The required $I_{15}$ for each $m$ is estimated using equation (2).

Table 2. The representative cases for Au and AuB (1:1) plasmas. In all cases, the flow velocity is assumed to be zero.

| Case | $Z_{\alpha}$ | $n_e/n_c$ | $T_e$ (keV) | $T_i$ (keV) | $I_{15}^a$ |
|------|-------------|-----------|-------------|-------------|-----------|
| ‘d’-Au | Z$_{Au}$ = 50 | 0.2 | 5 | m = 3 | 0.9 | 0.61 | 2.5 | 6.9 |
| ‘d’-AuB | Z$_{Au}$ = 50, Z$_{B}$ = 5 | 0.2 | 5 | m = 4 | 0.9 | 0.67 | 2.8 | 7.6 |

The required $I_{15}$ for each $m$ is estimated using equation (2).

$v_{ph}$ is close to $v_{the}$. Similarly, the ion damping is maximum when the phase velocity $v_{ph}$ is close to the ion thermal velocity $v_{the}$. For SBS, typically $c_s \ll v_{the}$. When $Z_{\alpha}$ increases, the ion acoustic velocity $c_s$ also increases, leading to the increase of electron damping $\nu_e$ and the decrease of ion damping $\nu_{ion}$ when $v_{the} < c_s$. The contribution of electron damping or ion damping can be quite different in low-Z and high-Z plasma. Furthermore, the possible super-Gaussian exponent $m$ due to Langdon effect is smaller in low-Z plasma than in high-Z plasma. Considering these differences in low-Z plasma and high-Z plasma, in the following, we discuss them separately.

In low-Z He and HHe plasmas, taking case ‘a’ as example, we show $-\mathcal{D}$ in figures 7(a) and 8(a) for He and HHe plasmas, respectively. In proximity to the peak wavelength, $-\mathcal{D}$ is mainly contributed by ions, because the ion damping which is proportional to $\lambda_{ion}^4$ is much higher than the electron damping as shown in figures 7(b) and 8(b). Since the Maxwellian distribution of ions is assumed, $\mathcal{D}_{ion}$ itself does not depend on $m$. However, since the phase velocity of IAW corresponding to the peak wavelength $\lambda_B$ increases with $m$, the decrease of $-\mathcal{D}_{ion}$ with the peak wavelength $\lambda_B$ at different $m$ in He plasma leads to the increase of the peak value of $K_B$ with $m$. Similarly in HHe plasma, the increase of $-\mathcal{D}_{ion}$ with the peak wavelength $\lambda_B$ at different $m$ results in the decrease of the peak $K_B$ with $m$. It can be concluded that the variation trend of peak $K_B$ with $m$ is determined by the behavior of $-\mathcal{D}$ with $v_{ph}$ near the phase velocity of IAW corresponding to the peak wavelength. Accordingly, two parameter regimes can be defined. In regime 1, $\partial \mathcal{D}/\partial v_{ph} < 0$ near the peak point and thus the peak $K_B$ decrease with $m$, while $\partial \mathcal{D}/\partial v_{ph} > 0$ near the peak point and...
the peak $K_B$ increase with $m$ in regime II. So, the boundary between regime I and II satisfies $\partial \mathcal{D}_I / \partial \nu_{\text{ph}} = 0$ at the peak point which maintains $\mathcal{R}^H = 0$ [27]. These two equations determine a relation between $A_m T_e / T_i$ and $T_i (1 - n_e / n_i) / n_e$ for a given ion composition. Solving the two equations numerically yields the boundary between regime I and II for He and HHe plasma as shown in figure 9, where the weak dependence of $-\mathcal{D}_I$ on $m$ is ignored, and the different cases shown in figure 5 are labeled as asterisks. As expected, case 'b' is located in regime I of He plasma, and case 'c' is located in regime II of HHe plasma. Case 'a' of He and HHe are located in the special parameter range that is in regime II of He plasma yet in regime I of HHe plasma, thus the variation trend of peak $K_B$ with $m$ are opposite in case 'a' of He and HHe plasmas even under the same parameter condition.

Now we take a closer look into the case 'a' to comprehend the physical mechanism for the effects of mixture. Since $-\mathcal{D}_I = \chi_{\text{ion}}^1 / |1 + \chi_{\text{ion}}^2|$ dominates $-\mathcal{D}_I$ near the matched $c_s$ as discussed above, and both $\chi_{\text{ion}}^1$ and $|1 + \chi_{\text{ion}}^2|$ drop with $\nu_{\text{ph}}$, the relative dropping rates of $\chi_{\text{ion}}^1$ and $|1 + \chi_{\text{ion}}^2|$ with $\nu_{\text{ph}}$ determine the sign of $\partial \mathcal{D}_I / \partial \nu_{\text{ph}}$ and thus the regime. In figure 10, the dropping rates of $\chi_{\text{ion}}^1$ and $|1 + \chi_{\text{ion}}^2|$, which are
Figure 9. (a) The boundary between regime I and II as specified by $A_m T_e/T_i$ versus $T_i(n_e - n_i)/n_e$ for He (red line) and HHe (1:1) (blue line) plasmas. (b) Zoomed in boundary for HHe (1:1) (blue line) plasmas. The asterisk symbols indicate different cases with conditions specified in table 1, with the four points corresponding to $m = 2$, $m = 2.3$, $m = 2.6$ and $m = 2.9$ from bottom to top for each case. The boundaries with only ion damping considered are also shown in gray and black lines for He and HHe plasmas, respectively.

Figure 10. The dropping rates of $\chi_{\text{ion}}^1$ (solid line) and $|1 + \chi_{\text{ion}}^2|$ (dashed line) versus $v_{\text{ph}}$ for fully ionized H (purple), He (blue) and HHe (1:1) (black) plasmas. The asterisk symbols indicate locations of the matched phase velocity corresponding to the peak of $K_B$ at $m = 2$ (red), $m = 2.3$ (blue), $m = 2.6$ (green) and $m = 2.9$ (magenta) in case ‘a’.

The dropping rate is defined as $\partial(\ln \chi_{\text{ion}}^1)/\partial v_{\text{ph}}$ and $\partial(\ln |1 + \chi_{\text{ion}}^2|)/\partial v_{\text{ph}}$ respectively, as shown as the function of $v_{\text{ph}}$ for H, He and HHe (1:1) with the same plasma parameters in case ‘a’, as well as the matched phase velocity corresponding to the peak of $K_B$ at different $m$ for each ion composition. Notice that $v_{\text{ph}}$ and the dropping rate are normalized by $v_{\text{thHe}}$ (thermal velocity of helium) and $1/v_{\text{thHe}}$, respectively. As expected, in HHe plasma, the matched $C_{HHe}$ is between the matched $C_{iH}$ in single-species H plasma and $C_{iHHe}$ in single-species He plasma, with $C_{iHHe}/v_{\text{thHe}} \sim 4$ and $C_{iHHe}/v_{\text{thHe}} \sim 2$. Because $C_{iHHe}$ is closer to $v_{\text{thHe}}$, the Landau damping is mainly contributed by H ions, and the dropping rate of $\chi_{\text{HHe}}$ is closer to $\chi_{\text{H}}$ in case of single H ions, as shown in figure 10. However, due to the contributions of He ions, the dropping rate of $|1 + \chi_{\text{HHe}}^2|$ is larger than $|1 + \chi_{\text{H}}^2|$ of the single H case. As a consequence, the decrease of $\chi_{\text{HHe}}$ with $v_{\text{ph}}$ is slower than the decrease of $|1 + \chi_{\text{HHe}}^2|$ in HHe plasma near $C_{iHHe}$, which leads to $\partial\mathcal{D}/\partial v_{\text{ph}} < 0$ and thus case ‘a’ is located in regime I.

In figure 9, regime II is constrained between a limited range of $A_m T_e/T_i$ when $T_i(1 - n_e/n_i)/n_e$ is small, while it is totally in regime I when $T_i(1 - n_e/n_i)/n_e$ is large enough. This general feature of the partition between regime I and II can be explained as follows. When $T_i(1 - n_e/n_i)/n_e$ is large enough, $-\mathcal{D}_{\text{ion}}$ increases with $v_{\text{ph}}$ for all possible $v_{\text{ph}}$ of IAW, as shown in figure 11(b). Since $-\mathcal{D}_{\text{ion}}$ also increases (linearly) with $v_{\text{ph}}$ for $v_{\text{ph}} \ll v_{\text{thHe}}$, $-\mathcal{D}_{\text{ion}}$ increases with $v_{\text{ph}}$ over the entire range of all possible $c_s$, rendering such $T_i(1 - n_e/n_i)/n_e$ belonging to regime I for all $A_m T_e/T_i$. When $T_i(1 - n_e/n_i)/n_e$ is smaller, as $v_{\text{ph}}$ increases, $-\mathcal{D}_{\text{ion}}$ firstly increases to a maximum then decreases to a minimum, because $\chi_{\text{ion}}^1$ drops with $v_{\text{ph}}$ faster and faster after $v_{\text{ph}} > v_{\text{thHe}}$, and its dropping rate exceeds the dropping rate of $|1 + \chi_{\text{ion}}^2|$ beyond this maximum point, as shown in figure 11(a). The minimum point arises because $-\mathcal{D}_{\text{ion}}$ rises sharply toward a peak at large $v_{\text{ph}}$ where $\chi_{\text{ion}}^1 + 1$ approaches zero. This peak corresponds to the greatest possible matched $c_s$ when $A_m T_e \gg T_i$, and $\chi_e \ll 1$ is negligible in the dispersion equation $\epsilon \approx 1 + \chi_{\text{ion}} = 0$. For a given $T_i$, the matched $c_s$ increases with increasing $A_m T_e/T_i$, consequently, the ion damping decreases while the electron damping increases, as shown in figures 7(b) and 8(b). When $A_m T_e/T_i$ is small
enough, the contribution of $-\mathcal{D}_{\text{ion}}^1$ dominates. As shown in figure 9, the boundary with both ion and electron damping considered coincides with the boundary with only the ion damping considered. The lower boundary corresponds to the maximum point of $-\mathcal{D}_{\text{ion}}^1$. For a parameter point with $A_m T_e / T_i$ located below the lower boundary, the matched $c_s$ is on the left side of the phase velocity corresponding to the maximum point of $-\mathcal{D}_{\text{ion}}^1$, thus $-\mathcal{D}_e^1$ increases with $v_{ph}$ and this parameter point is in regime I. While for a parameter point with $A_m T_e / T_i$ located above the lower boundary, the matched $c_s$ is on the right side of the maximum point, thus $-\mathcal{D}_e^1$ decreases with $v_{ph}$ and this parameter point is in regime II. With the increase of $A_m T_e / T_i$, the matched $c_s$ becomes closer and closer to the minimum point of $-\mathcal{D}_{\text{ion}}^1$, meanwhile, $-\mathcal{D}_{\text{ion}}^1$ which increases with $v_{ph}$ becomes more and more important. At the upper right boundary of regime II with a relatively large $T_i (1 - n_e / n_i) / n_e$, $c_s$ reaches the minimum point before $-\mathcal{D}_{\text{ion}}^1$ becomes sufficiently important. While for smaller $T_i (1 - n_e / n_i) / n_e$, the increase of $-\mathcal{D}_e^1$ with $v_{ph}$ completely cancels the decrease of $-\mathcal{D}_{\text{ion}}^1$ with $v_{ph}$ before the phase velocity approaches the minimum point of $-\mathcal{D}_{\text{ion}}^1$ as shown in figure 11(a), where the asterisk symbol indicates the location of the phase velocity corresponding to the minimum point of $-\mathcal{D}_e^1$ with consideration of electron damping. This minimum point of $-\mathcal{D}_e^1$ corresponds to the upper boundary of regime II at small $T_i (1 - n_e / n_i) / n_e$ in figure 9.

In high-Z plasmas, the Langdon effect can be more significant and the super-Gaussian exponent can approach about 5. With the same definition of parameter regimes discussed above, the boundary between regime I and II is shown in figure 12 for Au and AuB plasma (including both Au$^{10}$B and Au$^{11}$B), respectively. Due to the weak dependence of $-\mathcal{D}_e^1$ on $m$, the parameter boundary shifts with $m$ indistinctly as shown in figure 12(b). Here, only the boundary at $m = 2$ is shown for Au plasma. It is noticed that when the ion damping alone is considered, the regime II of AuB plasma is enclosed in the regime II of Au plasma. In fact, this holds generally for mixing low-Z species into the single ion species plasma with higher charge state. However, taking into account contributions from the electron damping, the parameter region of regime II of Au plasma is reduced, and regime II of AuB plasma locates inside regime I of Au plasma. An interesting point of figure 12 is that the boundary of regime II for Au$^{10}$B plasma is upwardly shifted relative to that for Au$^{11}$B plasma. Such an isotope effect should be embodied especially near the boundary of regime II. As shown in figure 12, case ‘d’ belongs to regime I in Au plasma, and spans both regime I and regime II in AuB plasma. Thus, as expected, the peak $K_B$ decreases with $m$ monotonically in Au plasma, while the trend of the peak $K_B$ with $m$ becomes non-monotonic and depends sensitively on the isotopic type of the low-Z species boron in AuB plasma, as shown in figure 6.

Using case ‘d’ as example, we take a closer look into the effects of the super-Gaussian EEDF and mixture near the upper boundary between regime I and regime II. In Au plasma, the electron damping dominates over the negligible ion damping since the matched $c_s$ corresponding to the peak $K_B$ is much larger than ion thermal velocity $v_{th} \text{Au}$, $-\mathcal{D}_e^1 \approx -\mathcal{D}_e^1$ increases with the matched phase velocity $c_s$ and hence $\lambda_B$, but decreases with $m$ as shown in figure 13(b). The first effect has a greater impact in case ‘d’, causing the peak $K_B$ to decrease with $m$ as shown in figure 13(a). After mixing the low-Z boron species into Au plasma, the variation of the ion damping with $v_{ph}$ due to B ions is not negligible compared to the variation of electron damping with $v_{ph}$, as shown in figure 14(b). Furthermore, $-\mathcal{D}_e^1$ decreases with $\lambda_B$ whereas $-\mathcal{D}_e^1$ increases with $\lambda_B$. The trend of the peak $K_B$ with $m$ depends on the competition of the opposite change of $-\mathcal{D}_e^1$ and $-\mathcal{D}_{\text{ion}}^1$ with $\lambda_B$. In

---

**Figure 11.** The typical variation of $-\mathcal{D}_{\text{ion}}^1 (1 + |\chi_{\text{ion}}|^2)$ and $\chi_{\text{ion}}^1$ with $v_{ph}$ for He plasma with $T_i (1 - n_e / n_i) / n_e = 7.7$ (a) and 17.9 (b). In (a), $\mathcal{D}_e^1 = \mathcal{D}_{\text{ion}}^1 + \mathcal{D}_{\text{el}}^1$ with $T_i / T_e = 32.7$ corresponding to the upper boundary of regime II at $T_i (1 - n_e / n_i) / n_e = 7.7$ is also shown with its minimum point indicated by the asterisk symbol.
Figure 12. (a) The boundary between regime I and II as specified by $A_m T_e / T_i$ versus $T_i (n_c - n_e) / n_e$ at $m = 2$ for Au (gold solid line), Au$^{10}$B (brown dot-dashed line) and Au$^{11}$B (red solid line) plasmas. The boundaries with only ion damping considered are also shown in gray solid, black dot-dashed, and black solid lines for Au, Au$^{10}$B, and Au$^{11}$B plasmas, respectively. (b) Zoomed boundaries for Au$^{10}$B (brown, light-blue, and purple dot-dashed lines for $m = 2, 3, 4$ respectively) and Au$^{11}$B (red, blue and magenta solid lines for $m = 2, 3, 4$ respectively) plasmas. The asterisk symbols indicate the case ‘d’ for Au, Au$^{10}$B and Au$^{11}$B with the same condition of electron density, electron temperature and ion temperature as specified in table 2, with the four points corresponding to $m = 2, m = 3, m = 4$ and $m = 4.5$ from bottom to top.

Au$^{11}$B plasma, this competition effect is the main reason for the non-monotonic change of the peak $K_B$ with $m$. Nevertheless, the obviously high $-D^I$ at $m = 2$ as shown in figure 14(a) leads to a significantly lower value of the peak $K_B$ at $m = 2$ than $m = 3, 4, 5$ as shown in figure 6(b). Compared to $^{11}$B ion, the lighter $^{10}$B ion has a larger thermal velocity as shown in figure 14(b). Hence, near the peak wavelength, the ion damping contributed by $^{10}$B is larger than $^{11}$B. Consequently, the contribution of $-D^I_{ion}$ to $-D^I$ become larger in Au$^{10}$B plasma, which leads to a stronger decreasing trend of $-D^I$ with $\lambda_B$ as shown in figure 14(a). Thus, the peak $K_B$ increase with $m$ in Au$^{10}$B plasma as shown in figure 6(b). For the suppression of SBS, mixing the lighter $^{10}$B ion into Au is better than mixing $^{11}$B ion, because of the lower peak $K_B$ with the same $m$. However, this advantage of $^{10}$B relative to $^{11}$B becomes inapparent with increasing $m$ as shown in figure 6(b), due to the different trends of $K_B$ with $m$ in Au$^{10}$B and Au$^{11}$B plasmas.

Figure 13. (a) $-D^I$ (solid line) versus wavelength shift ($\lambda_B - \lambda_0$) for Au plasma at $m = 2$ (red), $m = 3$ (blue), $m = 4$ (green) and $m = 4.5$ (magenta). The asterisk symbols indicate the wavelength locations corresponding to the peak $K_B$ in case ‘d’. (b) $\chi^I_0 \propto -\partial f_{ex} / \partial v_{ex}$ of electrons at different $m$ and Au ions. The IAW phase velocities corresponding to the peak wavelengths for each $m$ in case ‘d’ are indicated by the vertical dotted lines.

Au$^{10}$B plasma, this competition effect is the main reason for the non-monotonic change of the peak $K_B$ with $m$. Nevertheless, the obviously high $-D^I$ at $m = 2$ as shown in figure 14(a) leads to a significantly lower value of the peak $K_B$ at $m = 2$ than $m = 3, 4, 5$ as shown in figure 6(b). Compared to $^{11}$B ion, the lighter $^{10}$B ion has a larger thermal velocity as shown in figure 14(b). Hence, near the peak wavelength, the ion damping contributed by $^{10}$B is larger than $^{11}$B. Consequently, the contribution of $-D^I_{ion}$ to $-D^I$ become larger in Au$^{10}$B plasma, which leads to a stronger decreasing trend of $-D^I$ with $\lambda_B$ as shown in figure 14(a). Thus, the peak $K_B$ increase with $m$ in Au$^{10}$B plasma as shown in figure 6(b). For the suppression of SBS, mixing the lighter $^{10}$B ion into Au is better than mixing $^{11}$B ion, because of the lower peak $K_B$ with the same $m$. However, this advantage of $^{10}$B relative to $^{11}$B becomes inapparent with increasing $m$ as shown in figure 6(b), due to the different trends of $K_B$ with $m$ in Au$^{10}$B and Au$^{11}$B plasmas.
Figure 14. (a) $-D_I$ (solid and dash-dotted lines) and $-D_{ion}$ (dashed line) versus wavelength shift ($\lambda_B - \lambda_0$) for Au$^{10}$B and Au$^{11}$B plasmas at different $m$. The asterisk symbols indicate the wavelength locations corresponding to the peak $K_B$ at each $m$ in case ‘d’.

(b) $\chi_I \propto -\partial f_{x\alpha}/\partial v_x$ of electrons at different $m$, Au ions, $^{10}$B ions, and $^{11}$B ions. The IAW phase velocities corresponding to the peak wavelengths for each $m$ in case ‘d’ are indicated by the vertical dot-dashed lines for Au$^{10}$B plasma and by the vertical solid lines for Au$^{11}$B plasma.

4. Discussion and summary

It is worthwhile to mention that in the high-density gas-filled hohlraum experiments on NIF, the peak wavelength of SRS is shorter than the simulated results obtained by the inline ray-tracing model and an artificial large seed of SRS backscattered light is needed to match the experimental reflectivity [16]. Commonly, the SRS of the inner cone is mainly stimulated in the high density region with $n_e > 0.1$ $n_c$ and $T_e < 5$ keV located in the small $k_l\lambda_D$ regime where the wavelength of backscattered light of SRS decreases with the super-Gaussian exponent $m$. As a result, the simulated peak wavelength of SRS light is expected to be shorter with the consideration of the Langdon effect, which could be a possible explanation for this discrepancy in SRS spectra. Besides, considering the enhancement of SRS due to the Langdon effect in ray-tracing calculations may also ameliorate the current problem of the underestimated SRS reflectivity without the artificial seed. For SBS, the typical parameters $T_e/T_i \sim 1.4–6$ and $T_i \sim$ keV in plasma ablated from hohlraum wall in experiments are roughly located in regime II of the high-Z plasma mixed with low-Z ions like AuB, but located in regime I of the single high-Z plasma like Au. The Langdon effect itself can decrease the convective growth of SBS in single high-Z plasma but enhance SBS in mixed plasma, which may attenuate the improvement in suppression of SBS by mixing low-Z ion species into the high-Z plasma. Here, the influence of Langdon effect on SRS and SBS is mainly investigated based on the linear convective gain obtained from Vlasov model. In actual experiments, some possible nonlinear effects like electron trapping and the three-dimensional distribution of the high intensity speckles within the overlapped laser beams make the construction process of LPIs more complex [24], which still needs deeper investigations in future.

In summary, the Langdon effect should be prevalent in ICF hohlraum experiments and important to LPIs due to the sensitivity of LPIs on EEDF. Based on a linear analysis, it is found that the peak wavelength of the scattered wave and peak value of the spatial growth rate of both SRS and SBS processes are observably influenced by the Langdon effect which induces a super-Gaussian EEDF. For SRS, a super-Gaussian EEDF modifies the dispersion relation of EPW, yielding a redshift or blueshift of the peak wavelength. However, the Landau damping of EPW always drops with increasing super-Gaussian exponent $m$. Consequently, the peak spatial growth rate of SRS always increases with $m$. For SBS, by reducing the low energy electron number to shield the ions, a super-Gaussian EEDF always increases the ion acoustic velocity, leading to a redshift of SBS scattered wavelength. However, the effects of a super-Gaussian EEDF on the peak spatial growth rate of SBS depend on the plasma conditions such as electron density, electron temperature and ion temperature, and the ion composition (including the isotopic type) in a complex way. The boundary between different regimes of the enhancement or reduction of SBS by Langdon effect is given, and distinct behaviors are presented for low-Z and high-Z plasma with typical parameters in hohlraum plasmas. The clarification of the Langdon effect on SRS and SBS can make us better understand the experimentally observed spectra and growth of SRS and SBS. Also it provides valuable references for improvement of the physical modeling and simulations of the LPI processes.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.
Acknowledgments

This work was supported by the National Key R&D Program of China (Grant No. 2017YFA0403204), the Science Challenge Project (Grant No. TZ2016005), the National Natural Science Foundation of China (Grant Nos. 11875093 and 11875091), and the Project supported by CAEP Foundation (Grant No. CX20210040).

Appendix A. Proof that the ion acoustic velocity is monotonically increasing with $A_m T_e$

The ion acoustic velocity is given by equation (19) as $c_s^2 = (A ± \sqrt{B})/2$, with the plus for the fast mode, and the minus for the slow mode. Firstly we define

$$s_e = \frac{A_m T_e}{2 M_p q} \equiv \frac{A_m T_e}{ZM_p(1 + A_m k^2 \lambda_{D_e}^2)} \times \frac{A_m T_e}{1 + e_0 k^2 A_m T_e / n e^2}.$$  
(A1)

It can be seen $s_e$ is monotonically increasing with $A_m T_e$. On the other hand,

$$\frac{\partial A}{\partial s_e} = \frac{Z^2}{A} > 0$$  
(A2)

and

$$\frac{\partial \sqrt{B}}{\partial s_e} = \frac{1}{\sqrt{B}} \left[ \frac{\gamma_1}{A_1} - \frac{\gamma_2}{A_2} \right] \left( \frac{f_1 Z_1^2}{A_1} - \frac{f_2 Z_2^2}{A_2} \right) \frac{T_e}{M_p} + s_e \left( \frac{Z^2}{A} \right)^2.$$  
(A3)

Using equations (21), (A2) and (A3), we obtain

$$B \left( \frac{\partial A}{\partial s_e} \right)^2 - \left( \frac{\partial \sqrt{B}}{\partial s_e} \right)^2 = \left[ 2Z_1 Z_2 \left( \frac{\gamma_1}{A_1} - \frac{\gamma_2}{A_2} \right) \frac{T_e}{M_p} \right] f_1 f_2 \frac{A_1 A_2}{A_1 A_2} \geq 0.$$  
(A4)

Therefore,

$$\frac{\partial A}{\partial s_e} \geq \left| \frac{\partial \sqrt{B}}{\partial s_e} \right|.$$  
(A5)

In other words,

$$\partial c_s^2 / \partial s_e = \frac{1}{2} \left[ \frac{\partial A}{\partial s_e} \pm \frac{\partial \sqrt{B}}{\partial s_e} \right] \geq 0.$$  
(A6)

So $c_s$ is monotonically increasing with $s_e$, which is itself monotonically increasing with $A_m T_e$.

ORCID iDs

Jie Qiu https://orcid.org/0000-0003-1066-6140
Liang Hao https://orcid.org/0000-0003-4162-5309
Lihua Cao https://orcid.org/0000-0003-0263-4106

References

[1] Langdon A B 1980 Nonlinear inverse bremsstrahlung and heated-electron distributions Phys. Rev. Lett. 44 575–5
[2] Matte J P, Lamoureux M, Moller C, Yin R Y, Delettrez J, Virmont J and Johnston T W 1988 Non-Maxwellian electron distributions and continuum x-ray emission in inverse bremsstrahlung heated plasmas Plasma Phys. Control. Fusion 30 1665–89
[3] Weng S M, Sheng Z M and Zhang J 2009 Inverse bremsstrahlung absorption with nonlinear effects of high laser intensity and non-Maxwellian distribution Phys. Rev. E 80 056406
[4] Turnbull D et al 2020 Impact of the Langdon effect on crossed-beam energy transfer Nat. Phys. 16 181–5
[5] Zheng J, Yu C X and Zheng Z J 1997 Effects of non-Maxwellian (super-Gaussian) electron velocity distribution on the spectrum of Thomson scattering Phys. Plasmas 4 2736–40
[6] Milder A L et al 2020 Evolution of the electron distribution function in the presence of inverse bremsstrahlung heating and collisional ionization Phys. Rev. Lett. 124 025001
[7] Lindl J D, Amendt P, Berger R L, Glendinning S G, Glienzer S H, Haan S W, Kauffman R L, Landen O L and Suter L J 2004 The physics basis for ignition using indirect-drive targets on the national ignition facility Phys. Plasmas 11 339–491
[8] Tikhonchuk V, Gu Y J, Klimo O, Limpouch J and Weber S 2019 Studies of laser-plasma interaction physics with low-density targets for direct-drive inertial confinement schemes Matter Radiat. Extremes 4 045402
[9] Ji Y et al 2021 Convective amplification of stimulated Raman rescattering in a picosecond laser plasma interaction regime Matter Radiat. Extremes 6 015001
[10] Hao L, Huo W Y, Liu Z J, Li J, Zheng C Y and Ren C 2021 A frequency filter of backscattered light of stimulated Raman scattering due to the Raman rescattering in the gas-filled hohlraums Nucl. Fusion 61 036041
[11] Feng Q S, Cao L H, Liu Z J, Zheng C Y and He X T 2020 Stimulated Brillouin scattering of backward stimulated Raman scattering Sci. Rep. 10 3492
[12] Gong T et al 2019 Recent research progress of laser plasma interactions in Shenguang laser facilities Matter Radiat. Extremes 4 055202
[13] Hao L et al 2014 Analysis of stimulated Raman backscatter and stimulated Brillouin backscatter in experiments performed on SG-III prototype facility with a spectral analysis code Phys. Plasmas 21 072705
[14] Strozzi D J, Williams E A, Hinkel D E, Froula D H, London R A and Callahan D A 2008 Ray-based calculations of backscatter in laser fusion targets Phys. Plasmas 15 102703
[15] Hall G N et al 2017 The relationship between gas fill density and hohlraum drive performance at the national ignition facility Phys. Plasmas 24 082706
[16] Strozzi D J et al 2017 Interplay of laser-plasma interactions and inertial fusion hydrodynamics Phys. Rev. Lett. 118 025002
[17] Bychenkov V Y, Rozmus W and Tikhonchuk V T 1997 Stimulated Raman scattering in non-Maxwellian plasmas Phys. Plasmas 4 1481
[18] Afeyan B B, Chou A E, Matte J P, Town R P J and Kruer W J 1998 Kinetic theory of electron-plasma and ion-acoustic waves in nonuniformly heated laser plasmas Phys. Rev. Lett. 80 2322–5
[19] Neumayer P et al 2008 Suppression of stimulated Brillouin scattering by increased landau damping in...
multiple-ion-species hohlraum plasmas Phys. Rev. Lett. 100 105001

[20] Neumayer P et al 2008 Energetics of multiple-ion species hohlraum plasmas Phys. Plasmas 15 056307

[21] Drake J F, Kaw P K, Lee Y C, Schmid G, Liu C S and Rosenbluth M N 1974 Parametric instabilities of electromagnetic waves in plasmas Phys. Fluids 17 778–85

[22] Chen F F 1984 Introduction to Plasma Physics and Controlled Fusion vol 1 (Berlin: Springer)

[23] Hao L et al 2019 Investigation on laser plasma instability of the outer ring beams on SGIII laser facility AIP Adv. 9 095201

[24] Kirkwood R K et al 2013 A review of laser-plasma interaction physics of indirect-drive fusion Plasma Phys. Control. Fusion 55 103001

[25] Williams E A, Berger R L, Drake R P, Rubenchik A M, Bauer B S, Meyerhofer D D, Gaeris A C and Johnston T W 1995 The frequency and damping of ion acoustic waves in hydrocarbon (CH) and two-ion-species plasmas Phys. Plasmas 2 129–38

[26] Feng Q S, Liu Z J, Cao L H, Xiao C Z, Hao L, Zheng C Y, Ning C and He X T 2020 Interaction of parametric instabilities from 3ω and 2ω lasers in large-scale inhomogeneous plasmas Nucl. Fusion 60 066012

[27] In fact, it can be proven under the condition \( \frac{\partial \mathcal{P}}{\partial \omega_b} = 0 \) and \( \mathcal{R}_b = 0 \), \( K_B \) given in equation (15) satisfies \( \frac{\partial K_B}{\partial \omega_b} = 0 \). That is, the peak condition of \( K_B \) is exactly satisfied. So when the weak dependence of \( -\mathcal{R}_b \) on \( m \) can be neglected, these two conditions give the exact conditions for the boundary where the trend of the peak \( K_B \) with \( m \) changes.