Indicator of reliability of power grids and networks for environmental monitoring

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Abstract. The energy supply of the mining enterprises includes power networks in particular. Environmental monitoring relies on the data network between the observers and the facilitators. Weather and conditions of their work change over time randomly. Temperature, humidity, wind strength and other stochastic processes are interconnecting in different segments of the power grid. The article presents analytical expressions for the probability of failure of the power grid as a whole or its particular segment. These expressions can contain one or more parameters of the operating conditions, simulated by Monte Carlo. In some cases, one can get the ultimate mathematical formula for calculation on the computer. In conclusion, the expression, including the probability characteristic function of one random parameter, for example, wind, temperature or humidity, is given. The parameters of this characteristic function can be given by retrospective or special observations (measurements).

1. Introduction
The energy supply of the mining enterprises has a network structure. At the same time, its technical projects are evaluated by the particular parameter of reliability or probability of failure within the required period time. Often, this assessment used the binomial distribution or the Poisson distribution of the number of failures or the number of failed components. This implies the independent functioning of the system over time components. In real life, this leads to inaccuracies, entailing additional costs for implementation, maintenance work or compensation risks. Here it is possible to use Markov’s chains.

Below, let us discuss another approach, in some cases, more adequate for the analyzed system. The system components operation dependence is introduced in the operating model by using a common parameter, e.g. random traffic intensity in branches of a network transmission or attenuation coefficient of the transmission line, underground engineering networks under the effect of random environmental humidity. This allows getting expressions for evaluating the probability of undesirable vectors: some errors in the communication channel or particular undesirable state of a data network etc. The following ideas were used: a method for analytical representation of the Bernoulli’s independent trials scheme (the binomial distribution) through outcome numbers of ordered subsets (1) and expression of the event probability in a component as a function of some system parameter (2).

The Bernoulli's scheme of independent trials or the binomial distribution still enjoys the attention of researchers [1]. Its modifications, often have been called as generalizations of the binomial
distribution, are dictated by real research in different fields of science and practice. The dependence of the experiments results is presented in this probability distribution by different ways.

So in [2], the success probability is expressed in the j-th test as follows:

$$P (x_j) = P_{(0)} (x_j) \cdot f (x_{j_1}, x_{j_2} \cdots x_{j_m})$$.

Here, $P_{(0)} (x_j)$ is the event probability in the scheme of independent experiments.

Further, the author introduces variables:

$$z_k = (x_k - p)/[p (1 - p)]^{1/2}$$,

where $p$ is the occurrence probability of the event series $x_{j_1}, x_{j_2} \cdots, x_{j_m}$, $k = 1, 2, \ldots, n$. Then the correction factor is:

$$f (x_{j_1}, x_{j_2}, \ldots, x_{j_m}) = 1 + \sum_{k < k'} E [z_k z_{k'}] + \sum_{k < k' < k''} E [z_k z_{k'} z_{k''}] + \ldots + E [z_{j_1}, z_{j_2}, \ldots, z_{j_m} z_{j_1} z_{j_2} \cdots z_{j_m}]$$.

Thus, $f (x_{j_1}, x_{j_2}, \ldots, x_{j_m})$ is a function of the pairwise (or second-order) correlations among the $x_{j_k}$'s, the third-order correlations, etc., up to the $n$-th order correlation, as well as a function of $p$ itself. For example, if all correlations are taken to be zero, then one is back to the standard mutually uncorrelated case, and thus, one could say that $P_{(0)} (x_j)$ is a "first-order" approximation to $P (x_j) [2, 3].

Authors of [4] involved the measure of association which they called a "dependence ratio" (DR). In notation of [1], a dependence ratio of order $k - 1$ is the ratio:

$$\tau_{j_1,j_2,\ldots,j_k} = P_{(1)} (x_{j_1}, x_{j_2} \cdots x_{j_k}) / P_{(0)} (x_{j_1} x_{j_2} \cdots x_{j_k})$$.

DR is the ratio of a joint probability to the value it would have if the marginal components r.v.'s were independent. A DR must be always referred to as a specific category. Following [4], the author of [1] chooses "category '1', that is, 'success', as a reference and writes:

$$\tau_{1,1,1,\ldots,1} = P_{(1)} (x_{j_1} x_{j_2} \cdots x_{j_k}) / P_{(0)} (x_{j_1} x_{j_2} \cdots x_{j_k})$$,

where $[11\ldots1]^T$ is now the index vector with $k$ 1's, and the dependence ratio can be interpreted as the ratio of the probability of the event 'k units responses, which are successes, to the probability of the same event if the k units were independent" [4].

In this paper, another dependency type is proposed in the binomial model.

Appropriate notations [5] extended to all possible modifications of the independent tests scheme are available in the literature (e.g. [6,7]): the probability of various alternative outcomes is in trials, several possible outcomes with equal and different probabilities of their implementation are in the tests.

Thus, our objective is to obtain a formula for calculating the value of the complex system quality on the base of a dependent trials sequence. This approach differs from the Markov chains theory.

2. New Symbols for the Independent Experiments Scheme

2.1. Our Symbols

Let us introduce the following notation:

$$I_n = \{1, 2, \ldots, n\}$$.

Also there are the ordered subsets for $m < n$ and $k \in \{1, 2, \ldots, C_n^m\}$:

$$I_m^{(k)} = \{i_{1}^{(k)}, i_{2}^{(k)}, \ldots, i_{m}^{(k)}\}$$,

$$I_{n-m}^{(k)} = \{j_{1}^{(k)}, j_{2}^{(k)}, \ldots, j_{n-m}^{(k)}\}.$$ (2)

2.2. Bernoulli Independent Trials

From (1), (2) let us have the following formula:

$$P_n (m) = \sum_{k=1}^{C_n^m} \prod_{i \in I_m^{(k)}} p_i \prod_{j \in I_{n-m}^{(k)}} (1 - p_j)$$ (3)

instead of:

$$P_n (m) = C_n^m p^m (1 - p)^{n-m}$$

This is the probability of $m$ successes in $n$ independent trials in an unchanged condition (a Bernoulli scheme) ([6] for example).
2.3. The Different Probabilities of Success in the Experiments

Now let us consider the case of the experiment conditions change, that is, the success probabilities are not equal in a variety tests. Then let us have \( \{p_i\}, i \in \{1, 2, \ldots, n\} \). The \( m \) successes probability equals to the coefficient of \( z^m \) degree argument of the generating function \( f_n(z) = \prod_{i=1}^{n} (q_i + p_i, z) \), where \( q_i = 1 - p_i \) [7, Chapter 4]. The corresponding probability looks the same as (3) in the above-proposed notation (2).

2.4. Several Possible Test Results with the same Probabilities

If the test conditions are constant and the result of each test has \( s \) possible alternatives with probabilities \( p_{ij}, j \in \{1, 2, \ldots, s\} \), then the probability of \( m_i \) times the \( i \)-th result, \( \ldots \) and \( m_s \) times the \( s \)-th result in \( n \) trials equal to [7, Chapter 4]:

\[
P_n(m_1, m_2, \ldots, m_s) = n! \prod_{j=1}^{s} p_{ij}^{m_j} m_j!
\]

This probability has the following form in our notation:

\[
P_n(m_1, m_2, \ldots, m_s) = n! \prod_{j=1}^{s} (\prod_{i=1}^{m_j} p_{ij}) m_j!,
\]

where \( \sum_{j=1}^{s} m_j = I_n \) — the probability of the \( j \)-th success in the \( i \)-th experience, \( I_{mj} \) — the ordered subset of power \( m_j \).

2.5. Several Possible Outcomes of Experiments in Heterogeneous Conditions

If the experiment conditions are various, i.e. event \( A_i \) has a probability \( p_{ij} \) in the \( i \)-th experience, \( i \in \{1, \ldots, n\}, j \in \{1, \ldots, s\} \), then \( P_n(m_1, m_2, \ldots, m_s) \) equals the coefficient of the product \( z^{m_i} \times \ldots \times z^{m_s} \) in the expansion of the generating function \( f_n(z) = \prod_{i=1}^{n} \times \sum_{j=1}^{s} p_{ij} z^j \) [7]. Now one can simply express only the probability of outcomes particular configuration \( \{I_{m_1}, \ldots, I_{m_s}\} \) in the \( n \) trials:

\[
P_n(I_{m_1}, \ldots, I_{m_s}) = \prod_{j=1}^{s} \prod_{i=1}^{m_j} p_{ij}.
\]

Then the probability is:

\[
P_n(m_1, m_2, \ldots, m_s) = \sum_{I_{m_1}} \ldots \sum_{I_{m_s}} P_n(I_{m_1}, \ldots, I_{m_s}).
\]

The components number equals the value of \( n! / \prod_{j=1}^{s} m_j! \) in this multiple sum (7). The summation is over all subsets possible configurations of kind \( I_{mj} \) with indexes \( m_1, \ldots, m_s \). It should be remembered that all the subsets are interconnected and changed in (7) synchronously in conjunction \( \sum_{i=1}^{s} I_{mj} = I_n \).

3. The Dependent Experiments and their Further Development and Applicability

3.1. The Introduction of a Parameter that Affects the Experiments Results

Real experiments and systems are such that every event depends on the other one in their components. Let us build this relationship by introducing a functional dependence of the event probability from any parameter \( \mu \): \( p_i = p_i(\mu_i) = \varphi(\mu_i) \). Function \( \varphi(\mu_i) \) can be the real trend of a certain indicator or a parameter of the system and may be hypothetical or estimated experimentally. The parameter or indicator \( \mu \) can be a random value or a random process. Then there is the problem of determining the mathematical expectation of the corresponding probabilities, for example, \( E\{P_n(I_{mj})\} = Q_n(I_{mj}) \).

Its solution can be obtained in the analytical form in the case of analytical transformations possibility or can be estimated by Monte Carlo simulation if analytic transformations are not possible partially or completely.
Further, from this point of view, let us consider the sub-items 2.3 and 2.4. Subparagraphs 2.2 and 2.4 are only applicable solutions to the persistent probabilities of the outcome in experiments series.

### 3.2. Different Success Probabilities in Experiments

Let us get (8) from (3) by replacing \( p_i \) by \( p (\mu_i) \) and \( p (\mu_i) \):

\[
P_n (m) = \sum_{i=1}^{C_n^m} \prod_{i \in I_m^{(k)}} p (\mu_i) \prod_{j \in I_{n-m}^{(k)}} (1 - p (\mu_j)). \tag{8}
\]

Let us replace the second product:

\[
\prod_{j \in I_{n-m}^{(k)}} (1 - p (\mu_j)) = 1 + (-1)^{s-1} \sum_{j=1}^{C_{n-m}^1} p (\mu_j) + (-1)^{s-2} \sum_{j=1}^{C_{n-m}^2} \prod_{j \in I_2^{(kl)}} p (\mu_j) + (-1)^{s-3} \ldots +
\]

\[
+ (-1)^{s-n-1} \sum_{j=1}^{C_{n-m}^{n-m}} \prod_{j \in I_{s}^{(kl)}} p (\mu_j).
\]

Here \( I_1^{(kl)} \) — the \( i \)-th combination of \( s \)-element subsets \( I_{n-m}^{(k)} \). The first term (9) is equal to 1 when \( s = 0 \). Thus, one obtained the probability of \( m \) successes depending on some parameter \( \mu \) in the case of two experiment results:

\[
P_n (m) = \sum_{i=1}^{C_n^m} \prod_{i \in I_m^{(k)}} p (\mu_i) \sum_{s=0}^{C_{n-m}^s} (-1)^s \sum_{j=1}^{C_{n-m}^s} \prod_{j \in I_s^{(kl)}} p (\mu_j). \tag{10}
\]

### 3.3. Several Possible Outcomes of Experiments in Heterogeneous Conditions

Here let us consider the probability of specific configuration successes. Introducing the notation \( p_j = p_j (\mu_j) \), it is possible to get:

\[
P_n (I_{m_1}, \ldots, I_{m_o}) = \prod_{j=1}^s \prod_{i \in I_m^{(j)}} p_j (\mu_j). \tag{11}
\]

And finally:

\[
P_n (m_1, m_2, \ldots, m_s) = \sum_{\{I_m^{(1)}, \ldots, I_m^{(s)}\}} \prod_{(I_{m_1}^{(j)})} \prod_{i \in I_m^{(j)}} p_j (\mu_i). \tag{12}
\]

Here, \( I_m^{(s)} \) is the subset of two tests numbers corresponding to, for example, particular outcomes. The total number of these subsets is \( C_n^2 \).

Let us note that the specific configuration in a trials sequence (for example, malfunction of the system certain components) has meaning and interest from both engineering and monitoring points of view. The mathematical expectation of probabilities (9)-(12) has a value in the evaluation of the fault tolerance characteristics and technical solutions quality in complex and large systems with dependent components.

### 3.4. Stochastic Parameter that Accompanies the Tests

Next, let us consider the case when the parameter, which determines the tests outcomes probability, is random and the vector \( \mu = (\mu_1, \ldots, \mu_n) \) has a finite-dimensional function of the distribution or the probability density. Then the researcher may be interested in the mathematical expectation of the probabilities (9), (11) or (12). Let us restrict here expression (9) to illustrate this operation by the example:
\[ E\{P_n(m)\} = \sum_{k=1}^{C_n^m} \sum_{s=0}^{C_{n-m}^s} \sum_{i=0}^{l} \sum_{j=0}^{m-s} E\{\prod_{i \in I_{m(k)}^{(k)}} \prod_{j \in I_{s(l)}^{(l)}} p(\mu_i) p(\mu_j)\}. \] (13)

If the probabilities depend on the parameter exponentially, then the expectation in (13) becomes a finite-dimensional characteristic function of a random parameter. If the probabilities depend on the parameter as a degree with an integer index, then one will deal with mixed moments of the random parameter. These and other options can be the subject of a stochastic systems research. This model system can be represented by many components which behavior is random and depends on some vector parameter.

Now let \( p(\mu_i) = \alpha_i \exp(-\beta_i \mu_i), \alpha_i \in [0, 1], \beta_i > 0, \mu_i \geq 0, i \in [1, ..., n]. \) The approximation accuracy of the real events probability can be ensured by appropriate selection of values \( \alpha_i \) and \( \beta_i. \) And let there be a probability density finite-dimensional function of the random vector \( \mu = (\mu_1, ..., \mu_n). \) Then the terms in (13) are equal to

\[ E\{\prod_{i \in I_{m(k)}^{(k)}} \prod_{j \in I_{s(l)}^{(l)}} p(\mu_i) p(\mu_j)\} = \Theta j\beta (I_{m(k)}^{(k)}, I_{s(l)}^{(l)}) \prod_{i \in I_{m(k)}^{(k)}} \prod_{j \in I_{s(l)}^{(l)}} \alpha_i \alpha_j. \] (14)

Here \( j = (-1)^{ls}. \) The vector \( \beta (I_{m(k)}^{(k)}, I_{s(l)}^{(l)}) \) consists of components from \( \{\beta_i\} \) corresponding to the ordered subsets elements \( I_{m(k)}^{(k)}, I_{s(l)}^{(l)}. \) \( \Theta [... ] \) — the Laplace \( (m + s)-\)dimensional characteristic function [8, 9].

4. Some Examples

Thus, the author sets out an approach to estimating the probability of a given state of a complex system or process with interrelated components. It tested the practical calculations in the analysis of the properties and the quality of transmission channels in random media ([9-11] and references therein). The probability of a single error of binary signals reception in the considered conditions is equal to \( p(\mu_i) = 0.5 \exp(-\beta_i \mu_i^2). \)

The random attenuation \( \mu_i \) of the signals propagation medium is described by the \( m \)-generalized distribution [11]. Its special cases are known as the Rice's, the \( m \)- and the Rice-Nakagami's distributions [9, 12]. These results have academic expertise at seminars and in dissertation councils, where author's pupils V.V. Myasnikova, S.A. Panov and V.Y. Iskam defended the relevant dissertations.

5. Discussion

Thus, the analytical results are a generalization of the previously applied approach.

If one talks about the practical application of the above-mentioned expressions in the context of the supercomputers that are used in important tasks, then its computational complexity is not the deciding factor. In addition, it is possible to use the event "all components are functioning" (event \( \neg A \) --- not A) to assess the probability of failure of at least one system component (event A). Then \( P(A) = 1 - P(\neg A). \) The calculation of the probability \( P(\neg A) \) does not require the summation of many possible configurations of vectors \( I_{m(k)}^{(k)} \) and \( I_{s(l)}^{(l)} \) in (10)--(13) [11].

The section III results can be used for evaluation of the reliability of the network objects displayed by graphs with random edges: underground engineering networks, networks communication, data transmission networks, pipeline networks, for example, networks of electric wires, and so on [13]. It is possible to use these expressions when evaluating the quality of data transmission via communication channels with random parameters, probability of error-free transmission of messages in the command systems, where there are dependent errors.

It is planned to develop a software complex on the supercomputer for further researches. Numerical methods or Monte Carlo simulation can be used to produce probability values of the desired or predicted states of network architecture systems with interconnected components. Such problem can
be solved as the choice of architecture alternatives in the design and in the monitoring mode of system operation.

6. About practical tasks formulation
Let the power network of the mining enterprises (or monitoring system) consist of \( N \) distributed components. Each sector of the grid has \( N_k \) power lines (or data transmission lines), \( k = 1, \ldots, m \):

\[
N = \sum_{k=1}^{m} N_k.
\]

in the service manual, the network must know: a) the failure probability of at least one network component; b) the failure probability of a network specific sector; c) the failure probability of a particular path on the graph that displays the network. Those probabilities can be calculated using the above deriving formulas. It is possible to consider full and partial independence of the behavior of network components. The dependence is caused by the correlations of humidity, temperature or/and wind variances in the relevant areas of the mining enterprise (parameter \( \mu \) in (12), (13), etc.).

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