Unsteady Convective MHD Flow and Heat Transfer of a Viscous Nanofluid across a Porous Stretching/Shrinking Surface: Existence of Multiple Solutions

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Abstract: The suspension of tiny solid particles inside the energy transport liquids could enhance their thermal conductivity as well as provide an efficient and inventive approach to significantly improve their properties of heat transport. Therefore, our aim is to explore the radiative two-dimensional unsteady flow of a viscous nanofluid about an aligned magnetic field that includes the joint effect of suction, velocity slip, and heat source across a porous convective stretching/shrinking surface. Initially, using non-dimensional variables, the nonlinear governing partial differential equations (PDEs) were transformed into ordinary differential equations (ODEs) which were subsequently solved with the help of bvp4c built-in package in MATLAB. The results declare that escalating the values of the unsteadiness parameter escalates the friction drag whereas it reduces with the escalation of the slip parameter. Furthermore, the heat transfer rate escalates with the escalation of radiation and concentration parameter, and the escalation of the heat source parameter causes to reduce the heat transfer rate. Finally, it is found that the rate of heat transfer and friction drag continuously improve and decline against the rising rates of stretching, respectively.

Keywords: nanofluid; heat source; unsteady flow; aligned magnetic field; thermal radiation; porous medium

1. Introduction

In physics, a substance that flows continuously during the exertion of an exterior force is called a fluid. It is possible to classify fluids based on their properties such as viscosity, density conductivity, compressibility, and the non-compressibility. Based on
the viscosity, the common categories of the fluids are real fluids, ideal fluids, Newtonian fluids, non-Newtonian fluids, and the ideal plastic fluids. Newtonian fluids are those that satisfy Newton’s law of viscosity, called viscous fluids. In recent times, the notion of a viscous nanofluid has been diverted into an additional broad field used for the scientist’s society in view of its vast range of importance in microelectronics, heat exchangers, transportation, biomedicine, engine cooling, cooling of electronic devices, food, fuel cells, double windowpanes, hybrid-powered engines, etc. A fluid consisting of small nano-scale particles is called nanofluid. The word “nanofluid” was initially presented by Choi [1] to define the composition of a base liquid and the nanoparticles. In order to augment the thermal conductivity of regular fluids; for instance, water, kerosene, engine oils, and ethylene glycol, it is necessary to incorporate several families of nanoparticles such as silver, carbon nanotubes, gold, oxides, graphene, alumina, silica, copper, etc., with the base fluids considering that nanofluids bear supreme thermal conductivity as compared to the base liquids [2–4]. Tamim et al. [5] explored the mixed convection in a time-dependent stagnation point flow of a nano liquid across a porous vertical surface. Aghamajidi et al. [6] used the Tiwari–Das model to explore the natural convection flow of a nanofluid associated with the magnetic field adjoined to a vertical cone. The flow of a hybrid nanofluid across an artery, including the applications of respiratory systems associated with the circulation and delivery of drugs, was explored by Chahregh et al. [7]. Nadeem et al. [8] explored the study of viscous nanofluid associated with stagnation point and magnetic field across a curved stretched surface. The comparative influence of different hybrid nanofluids on the heat and mass transport flow with the exterior aligned magnetic field was studied by Khan et al. [9] across a stretched surface associated with thermal radiation. Several other investigations have been carried out in literature that address boosting in the thermal conductivity of regular fluids through the suspension of numerous kinds of nanoparticles [10–22].

The analysis on flow and the rate of heat transport of a nano liquid is constantly developing under the observation of several facets. In addition, the nanofluid phenomenon attributed to convective cooling, including heat generation/absorption, have been addressed. Heat sink characterizes the absorbency of the surplus heat to the surface; however, the heat generation of the fluid is reflected by heat source. Awais et al. [23] described that the presence of heat sink provides a lower temperature, although the system temperature was larger, ensuring the availability of a heat source. Heat sink and source provide support to the dumping technique of the radioactive discarding substances as well as exothermal chemical advancement, system of operating of moisture compartment, cooling system of graphic processors, as well as thermal omission of the nuclear fuel rubbish. The consequences of heat generation and absorption in the magnetized flow of a Williamson nanofluid was checked by Li et al. [24] across a permeable exponential stretching surface. Zhao et al. [25] determined the entropy optimization including heat and mass transport in tangent hyperbolic stagnation flow of a nanofluid associated with magnetic field and heat generation/absorption. Some recent studies regarding the convective flow and heat generation/absorption phenomenon may be found in [26–31].

The major interest of our research is to explore the two-dimensional time dependent aligned magnetohydrodynamics (MHD) movement of a nanofluid across a porous stretching/shrinking surface by introducing the influence of radiation, heat generation, and convective condition. The fluid is electrically conductive, influenced by an aligned magnetic field with acute angle β, and is created using Al₂O₃ nanoparticles in the base liquid (H₂O). The attentive survey of the previously published work specifies that the study investigated here is novel and is not addressed earlier according to the knowledge and information of the authors. Moreover, the innovations of the present problem can also be noted by providing dual solutions. The study presented here possesses steep implementations in various manufacturing and engineering processes and is providing responses to the questions below:
(a) How the rate of heat transfer improves with the addition of alumina (Al₂O₃) nanoparticles in the base fluid water (H₂O).
(b) What is the effect of the angle of inclination of the external magnetic field on the skin friction coefficient?
(c) What is the behavior of the heat transfer rate and skin friction coefficient against the rising rates of both surface stretching and shrinking?
(d) What is the difference between the stable and unstable results of the study and at what point the two solutions meet.

The same pattern of logical research questions has been provided by [32] and [33] which are relevant and applicable to their problem of consideration.

2. Mathematical Modeling

The two-dimensional time dependent aligned MHD flow of a nanofluid across a porous stretching/shrinking surface placed at the origin (x = 0, y = 0) is considered as seen in Figure 1. With the consideration of high velocity, it is necessary to mention that the relationship between the pressure and the flow rate is presumed to be nonlinear [34]. The coordinates are settled in such a manner that the stretching/shrinking surface is located adjacent to the x-axis, and the y-axis is measured perpendicular to the surface. The fluid is electrically conductive, influenced by an unsteady aligned magnetic field of strength \( B(t) = \frac{B_0}{(1-\delta t)^{1/2}} \) with acute angle \( \beta \), as well as the surface velocity and the free stream velocity which are assumed to be varying from a stagnation point, respectively, represented by \( U_w(x,t) = \frac{ax}{1-\delta t} \) and \( U_e(x,t) = \frac{bx}{1-\delta t} \), where \( a, b \) and \( \delta \) are positive constant. The dimension of \( a + b \) is \( t^{-1} \), \( a \) is initial stretching/shrinking, and \( \frac{a}{1-\delta t} \) is the efficient rate of the stretching/shrinking escalating with time. The term \( \frac{\partial u}{\partial y} \) and \( V_w(x,t) = \frac{V_0}{(1-\delta t)^{3/2}} \), respectively, used to signify the velocity slip and the suction velocity as well as the surface which is supposed to have an interior heat generation coefficient \( Q_t = \frac{Q_0}{1-\delta t} \) and the radiative heat flux \( q_r \), in which \( Q_0 \) indicates the heat generation coefficient (heat source). The nanofluid is created using alumina (Al₂O₃) nanoparticles in the base fluid water (H₂O) and their mathematical expressions with the thermophysical properties have been, respectively, displayed in Tables 1 and 2. Considering the foregoing pre-conditions, the boundary layer governing flow equations have been addressed as [35–37].

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} + \frac{\mu}{\rho} \frac{u}{\delta t} - \frac{\sigma_n B^2(t)}{\rho \alpha} \sin^2 \beta u, \tag{2}
\]

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{(\rho c_p)_{inf}} \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho c_p)_{inf}} \frac{\partial q_r}{\partial y} + \frac{q_r}{(\rho c_p)_{inf}} - (T - T_w). \tag{3}
\]

The assigned boundary conditions to the above equations are

\[
\begin{align*}
\frac{\partial u}{\partial y} & = 0, \\
v & = V_w(x, t), \\
-k \frac{\partial T}{\partial y} & = h_f(T_w - T) \text{ at } y = 0, \\
u & \to U_w(x, t), T \to T_w \text{ as } y \to \infty.
\end{align*} \tag{4}
\]

In the above equations, (2)–(4), \( K = k_1(1 - \delta t) \) provides the porous medium permeability, where \( k_1 \) is the initial permeability. The terms \( \mu, \mu, \rho, \rho, \sigma_n, k_n, C_p, (\rho c_p)_{inf} \) accordingly refer to the dynamic viscosity of base liquid and nanofluid, density, electrical conductivity, coefficient of thermal conductivity, specific heat, and heat capacity of the nanofluid. Additionally, \( \lambda_1 = \lambda \sqrt{T - \delta t}, T, T_w \) represents the slip factor of velocity, the temperature of the surface, and the free stream temperature, respectively. The third boundary condition provides the convection of hot fluid that heats up the surface at a temperature \( T_w \), and introduces the coefficient of heat transfer \( h_f \). Neglecting the viscous
dissipation term and using the Rosseland approximation, we obtain \( q_r = \frac{-4\sigma \partial T^4}{3k^*} \), where \( k^* \) and \( \sigma^* \), respectively, defines the coefficient of mean absorption and the Stefan–Boltzmann constant. We imagine that within the flow, the difference of the temperature is in such a way that \( T^4 \) can be extended in a Taylor series. Consequently, expanding \( T^4 \) about \( T_\infty \) and ignoring the terms of high order, we reach at \( T^4 \approx 4T_\infty^2 T - 3\sigma^* T_\infty^4 \). Eventually, we find \( q_r = \frac{-16\sigma^* T_\infty^2 \partial T}{3k^*} \). This is based on the assumption that within the flow, the difference of the temperature is small [38]. In the case of a higher temperature difference, it is also possible to simplify the radiative term with the use of implicit differentiation [39].

Thus, Equation (3) becomes:

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{nt}}{(\rho c_p)_n T} \frac{\partial^2 T}{\partial y^2} = \frac{1}{(\rho c_p)_n T} \frac{16\sigma^* T_\infty^2 \partial T}{3k^*} + \frac{Q_t}{(\rho c_p)_n T} (T_w - T_\infty).
\]

The relations for \( u, v, \) and \( \theta \) can be introduced as:

\[
\frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad \frac{\theta - T_\infty}{T_w - T_\infty}.
\]

where \( \psi \) is the stream function.

![Figure 1](image)

**Figure 1.** Physical representation and coordinate system.

The relations in Equation (6) have been utilized in Equations (2) and (6) to obtain the subsequent equations:

\[
\frac{\partial^2 \psi}{\partial r^2} + \frac{\partial \psi}{\partial r} = \frac{\nu_n \partial^2 \psi}{(\rho c_p)_n T} \frac{\partial^2 \psi}{\partial y^2} = \frac{1}{(\rho c_p)_n T} \frac{16\sigma^* T_\infty^2 \partial T}{3k^*} + \frac{Q_t}{(\rho c_p)_n T} (T - T_\infty),
\]

and

\[
\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \frac{k_{nt}}{(\rho c_p)_n T} \frac{\partial^2 T}{\partial y^2} + \frac{1}{(\rho c_p)_n T} \frac{16\sigma^* T_\infty^2 \partial T}{3k^*} + \frac{Q_t}{(\rho c_p)_n T} (T - T_\infty),
\]

The similarity variable \( \eta, f, \) and \( \theta \) can be defined as:

\[
\eta = \sqrt{\frac{b}{v_n(1-\delta t)}} y, \quad \psi = \frac{b v_n}{(1-\delta t)} f(\eta), T_w = T_\infty + T_0 \frac{b x}{2v_n(1-\delta t)} \theta(\eta).
\]

In view of the relations (9), the Equations (7) and (8) become:

\[
\frac{\nu_n \epsilon f'''' - f'''}{\rho_t \eta} - \frac{P}{2} \left( \frac{\eta f'''}{2} + f' \right) - \frac{1}{2} f' - M^2 \frac{\eta}{\sigma_t \eta} \sin^2 \theta f' = 0,
\]

\[
\frac{1}{\epsilon} \frac{k_{nt}}{\rho_t \eta} \left( 1 + \frac{4}{3} \text{Rd} \right) \theta''' - \frac{(\rho c_p)_n P}{(\rho c_p)_n} \left( \frac{P}{2} (\eta \theta' + 40) + f' \theta - f \theta' - Q \theta \right) = 0.
\]
The related boundary conditions are changed into the non-dimensional boundary conditions which are given below:

\[ f'(0) = \gamma + \lambda_2 f''(0), \quad f(0) = S, \quad \theta'(0) = -Bi(1 - \theta(0)) \]
\[ f'(\eta) = 1, \quad f''(\eta) = 0, \quad \theta(\eta) = 0, \quad \text{as } \eta \to \infty \] \hspace{1cm} (12)

In the above two equations, \( M^2 = \frac{\sigma \nu}{b \rho_f} \) identifies the Hartmann number, \( D = Da_x Re_x = \frac{b \nu}{u_f} \), \( Da_x = \frac{K}{x^2} = \frac{k_x(1-x^2)}{x^2} \) is the local Darcy number, \( P = \frac{\nu}{b} \) is the unsteadiness parameter, \( Rd = 4 \sigma \nu \frac{b}{k} \) defines the radiation parameter, \( Pr = \frac{\nu}{b} \) is a Prandtl number, and \( Q = \frac{q_o}{b \rho_f} \) is the heat source parameter. Similarly, \( S = \left( \frac{v_0}{\sqrt{b \nu}} \right) > 0, \gamma = \frac{b}{a} \) and \( \lambda_2 = \lambda \sqrt{b \nu} \), respectively, embodies the suction, stretching, and the velocity slip parameter, and \( Bi = \frac{b \nu}{k_f \sqrt{b}} \) is the Biot number.

The skin friction coefficient and the local Nusselt number have been defined as

\[ C_f = \frac{\tau_w}{\rho_f u_f^2}, \quad Nu = \frac{\theta_w}{\kappa(T_w-T_0)} \] \hspace{1cm} (13)

In the above equation, \( \tau_w \) and \( q_w \) defines the respective local wall shear stress and heat flux which can be defined as:

\[ \tau_w = \mu_{nf} \frac{\partial u}{\partial y} \bigg|_{y=0} \quad \text{and} \quad q_w = -k_{nf} \frac{\partial \theta}{\partial y} \bigg|_{y=0} \] \hspace{1cm} (14)

In non-dimensional form, Equation (13) can be written as:

\[ \left( \frac{1}{2} (Re_x)^{1/2} C_f = \frac{\nu}{b} f''(0), \right) \]
\[ (Re_x)^{-1/2} Nu = \frac{-k_{nf}}{k_f}(1 + \frac{4}{3} \frac{Rd}{b}) \theta'(0). \] \hspace{1cm} (15)

In the above Equation (15), \( Re_x = \frac{v_0^2}{a \nu} \) symbolizes the Reynolds number.

**Table 1.** The thermophysical properties of \( Al_2O_3 - H_2O \) in mathematical expressions [40].

| Properties          | Nanofluid (\( Al_2O_3 \) - \( H_2O \)) |
|---------------------|----------------------------------------|
| Density             | \( \rho_{nf} = \phi \rho + (1 - \phi) \rho_f \) |
| Heat capacity       | \( \left( \rho C_p \right)_{nf} = \phi \left( \rho C_p \right) + (1 - \phi) \left( \rho C_p \right)_f \) |
| Electrical conductivity | \( \sigma_{nf} = (1 - \phi) \sigma_f + \phi \sigma_s \) |
| Viscosity           | \( \mu_{nf} = \frac{\nu}{b} \) |
| Thermal diffusivity | \( \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \) |
| Thermal conductivity | \( k_{nf} = \frac{(k_s + 2) \phi (1 - \phi)}{(k_f + 2) \phi (1 - \phi)} \) |

**Table 2.** Thermal properties of water and alumina (\( Al_2O_3 \)) [2,8].

| Thermophysical Properties | Water (\( H_2O \)) | Alumina (\( Al_2O_3 \)) |
|---------------------------|-------------------|------------------------|
| \( C_p^\prime/\text{(J/kgK)}\) | 4179              | 765                    |
| \( \rho/\text{(kg/m}^3)\)       | 997.1             | 3970                   |
| \( \sigma/\text{(S/m)}\)         | \(5.5 \times 10^{-6}\) | \(35 \times 10^6\) |
| \( k/\text{(W/mK)}\)          | 0.613             | 40                     |

3. Results and Discussion

3.1. Analysis of Result

In order to discuss the numerical results of the current unsteady MHD flow of a nanofluid across a permeable medium under the joint effect of suction, velocity slip, heat generation, thermal radiation, convective conditions, and the stretching surface, the numerical bvp4c technique through MATLAB was operated to solve the resulting Equations (11) and (12) of the problem connected to the boundary conditions (13). This technique is
thoroughly described by Shampine et al. [41]. In order to obtain the graphical results of heat transfer $\text{Nu}$ and the friction drag $C_f$, some appropriate values are assigned to the parameters arising from several of the above effects such as the Hartmann number $M$, the aligned magnetic field angle $\beta$, the local Darcy number $D$, the unsteadiness parameter $P$, the radiation parameter $R_d$, the Prandtl number $Pr$, the heat source parameter $Q$, the suction parameter $S$, the stretching parameter $\gamma$, the velocity slip parameter $\lambda_2$, the Biot number $Bi$, and the nanoparticles concentration $\phi$.

3.2. Discussion of Results

3.2.1. Skin Friction Coefficient

Figures 2–4 represents the influence of $C_f$ which are, respectively, plotted in the light of different values of $D$, $\lambda_2$, and $\beta$ along a stretching/shrinking surface. It clearly seems that the escalation of $D$ and $\beta$ escalating only the first solution plot of $C_f$, and for the plot of the second solution of $C_f$, a reverse phenomenon is detected. This result could lead us to the fact that the features of a porous medium associated with the intensity of the exterior magnetic field can show an opposite effect against the fluid velocity as stated by [42]. Furthermore, the growth of $\lambda_2$ values diminishes the plot of $C_f$ for both solutions. It should be noted that for the positive values of $\gamma$, the plot of the $C_f$ always declines, and for the negatively rising values of $\gamma$, it continuously enhances. The black spot in each of the graphical results indicates the critical points of the dual branches of solutions. At all critical points we have only a unique solution.

In the same way, Figure 5 exhibits the properties of $C_f$ along a stretching/shrinking surface based on the modifications of parameter $P$. In the same manner, it reduces for the positive values of $\gamma$ and boosts for the negatively rising values of $\gamma$ however, it appears that the escalating values of $P$ escalates the skin friction. In this figure, $C_f$ escalates only for the original solution, and for second solution it reduces with the negatively rising values of $P$.

![Figure 2. Variation of $C_f$ with $D$.](image-url)
3.2.2. Nusselt Number (Heat Transfer Rate)

In the case of the graphical results of Nu, there are some singularities arising in the curves of second solutions. For this reason, the graphical outcomes of Nu are solely limited to the plots of first solutions. The two graphical results elucidated in Figures 6 and 7, respectively, presents the variations in Nu with ϕ and Rd along a stretching/shrinking surface. These figures make it clear that Nu escalates with the escalating values of both ϕ
and Rd. It is also noteworthy to mention that Nu slightly mounts with a bigger value of $\gamma$ and vice versa.

Moreover, Figure 8 presents the influence of Nu which is designed in accordance with the different values of $Q$ along a stretching/shrinking surface. It clearly seems that the escalation of $Q$ reduces the rate of Nu. It should be noted that for the lower values of the $\gamma$, the plot of Nu always declines as well as for the rising values of $\gamma$, it continuously enhances.

![Figure 6. Variation of Nu with $\phi$.](image1)

![Figure 7. Variation of Nu with Rd.](image2)

![Figure 8. Variation of Nu with $Q$.](image3)
4. Conclusions

The radiative two-dimensional unsteady flow of a viscous nanofluid about an aligned magnetic field that includes the joint effect of suction, velocity slip, and heat source across a porous convective stretching/shrinking surface is studied. The obtained result of this study is concluded in the following remarks:

- The escalation of $D$ and $\beta$ escalates $C_f$ only for the original solution, and for the second solution, reverse phenomenon is detected.
- The escalating values of $P$ and $\lambda_2$, respectively, escalates and reduces the friction drag.
- For the positive values of $\gamma$, $C_f$ always declines, and for the negatively rising values of $\gamma$, it continuously enhances.
- The rate of $Nu$ escalates and reduces, respectively, with the escalating values of $\phi, Rd$, and $Q$.
- The lower values of $\gamma$, continuously reduce the rate of $Nu$, and for the ascent values of $\gamma$, it always enhances.

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