CARET analysis of multithreaded programs*

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Abstract. Dynamic Pushdown Networks (DPNs) are a natural model for multithreaded programs with (recursive) procedure calls and thread creation. On the other hand, CARET is a temporal logic that allows to write linear temporal formulas while taking into account the matching between calls and returns. We consider in this paper the model-checking problem of DPNs against CARET formulas. We show that this problem can be effectively solved by a reduction to the emptiness problem of Büchi Dynamic Pushdown Systems. We then show that CARET model checking is also decidable for DPNs communicating with locks. Our results can, in particular, be used for the detection of concurrent malware.

1 Introduction

Pushdown Systems (PDSs) are known to be a natural model for sequential programs [18]. Therefore, networks of pushdown systems are a natural model for concurrent programs where each PDS represents a sequential component of the system. In this context, Dynamic pushdown Networks (DPNs) [6] were introduced by Bouajjani et al. as a natural model of multithreaded programs with procedure calls and thread creation. Intuitively, a DPN is a network of pushdown processes \{P_1, ..., P_n\} where each process, represented by a Pushdown system (PDS), can perform basic pushdown actions, call procedures, as well as spawn new instances of pushdown processes. A lot of previous researches focused on investigating automated methods to verify DPNs. In [6,15,14,9], the reachability analysis of DPNs are considered. While the model-checking problem for DPNs against double-indexed properties is undecidable, i.e., the properties where the satisfiability of an atomic proposition depends on control states of two or more threads [10], it is decidable to model-check DPNs against the linear temporal logic (LTL) and the computation tree logic (CTL) with single-indexed properties [19], i.e., properties where the satisfiability of an atomic proposition depends on control states of only one thread.

CARET is a temporal logic of calls and returns [1]. This logic allows us to write linear temporal formulas while taking into account the matching between calls and returns. CARET is needed to describe several important properties such as malicious behaviors or API usage rules. Thus, to be able to analyse such properties for multithreaded programs, we need to be able to check CARET

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formulas for DPNs. We tackle this problem in this paper. As LTL is a subclass of CARET, CARET model-checking for DPNs with double-indexed properties is also undecidable. Thus, in this paper, we consider the model-checking problem for DPNs against single-indexed CARET formulas and show that it is decidable. A single-indexed CARET formula is a formula in the form $\bigwedge f_i$ where $f_i$ is a CARET formula over a certain PDS $P_i$. A DPN satisfies $\bigwedge f_i$ iff all instances of the PDS $P_i$ created in the network satisfy the subformula $f_i$.

The model-checking problem of DPNs against single-indexed CARET formulas is non-trivial because the number of instances of pushdown processes in DPNs can be unbounded. It is not sufficient to check if every PDS $P_i$ satisfies the corresponding formula $f_i$. Indeed, we need to ensure that all instances of $P_i$ created during a run of DPN satisfies the formula $f_i$. Also, it is not correct to check whether all possible instances of $P_i$ satisfy the formula $f_i$. Indeed, an instance of $P_i$ should not be checked if it is not created during the run of DPNs. In this paper, we solve these problems. We show that single-indexed CARET model checking is decidable for DPNs. To this end, we reduce the problem of checking whether Dynamic Pushdown Networks satisfy single-indexed CARET formulas to the membership problem for Büchi Dynamic Pushdown Networks (BDPNs). Finally, we show that single-indexed CARET model checking is decidable for Dynamic Pushdown Networks communicating via nested locks.

Related work.

[5,7,2,3] considered Pushdown networks with communications between processes. However, these works consider only networks with a fixed number of threads. The model-checking problem for pushdown networks where synchronization between threads is ensured by a set of nested locks is considered in [12,10,11] for single-indexed LTL/CTL and double-indexed LTL. These works do not handle dynamic thread creation.

Multi-pushdown systems were considered in [13,4] to represent multithreaded programs. These systems have only a finite number of stacks, and thus, they cannot handle dynamic thread creation.

Pushdown Networks with dynamic thread creation (DPNs) were introduced in [6]. The reachability problems of DPNs and its extensions are considered in [6,9,14,15,21]. [19] considers the model-checking problem of DPNs against single-indexed LTL and CTL, while [20] investigates the single-indexed LTL model checking problem for DPNs with locks.

[17,16] consider CARET model checking for pushdown systems and its application to malware detection. These works can only handle sequential programs. In this paper, we go one step further and extend these works [17,16] to DPNs and concurrent programs.

2 Linear Temporal Logic of Calls and Returns - CARET

In this section, we recall the definition of CARET [1]. A CARET formula is interpreted on an infinite path where each state on the path is associated with a tag in the set $\{\text{call, ret, int}\}$. A call-state denotes an invocation to a procedure of a program while the corresponding ret-state denotes the ret statement of that procedure. A simple statement (neither a call nor a ret statement) is called an internal statement and its associated state is called int-state.
Let $\omega = s_0s_1...$ be an infinite path where each state on the path is associated with a tag in the set \{call, ret, int\}. Over $\omega$, three kinds of successors are defined for every position $s_i$:
- **global-successor**: The global-successor of $s_i$ is $s_{i+1}$.
- **abstract-successor**: The abstract-successor of $s_i$ is determined by its associated tag.
  - If $s_i$ is a call, the abstract successor of $s_i$ is the matching return point.
  - If $s_i$ is a int, the abstract successor of $s_i$ is $s_{i+1}$.
  - If $s_i$ is a ret, the abstract successor of $s_i$ is defined as $\bot$.
- **caller-successor**: The caller-successor of $s_i$ is the most inner unmatched call if there is such a call. Otherwise, it is defined as $\bot$.

A **global-path** is obtained by applying repeatedly the global-successor operator. Similarly, an **abstract-path** or a **caller-path** are obtained by repeatedly applying the abstract-successor and caller-successor respectively.

**Formal Definition.** Given a finite set of atomic propositions $AP$. Let $AP' = AP \cup \{\text{call, ret, int}\}$. A CARET formula over $AP$ is defined as follows (where $e \in AP'$):
\[
\psi := e \mid \psi \lor \psi \mid \neg \psi \mid X^g\psi \mid X^a\psi \mid X^c\psi \mid \psi U^g\psi \mid \psi U^a\psi \mid \psi U^c\psi
\]

Let $\Sigma = 2^{AP'} \times \{\text{call, ret, int}\}$. Let $\pi = \pi(0)\pi(1)\pi(2)...$ be an $\omega$-word over $\Sigma$. Let $(\pi, i)$ be the suffix of $\pi$ starting from $\pi(i)$. Let $next_i^g$, $next_i^a$, $next_i^c$ be the global-successor, abstract-successor and caller-successor of $\pi(i)$ respectively.

The satisfiability relation is defined inductively as follows:
- $(\pi, i) \models e$, where $e \in AP'$, iff $\pi(0) = (Y, d)$ and $e \in Y$ or $e = d$
- $(\pi, i) \models \psi_1 \lor \psi_2$ iff $(\pi, i) \models \psi_1$ or $(\pi, i) \models \psi_2$
- $(\pi, i) \models \neg \psi$ iff $(\pi, i) \not\models \psi$
- $(\pi, i) \models X^g\psi$ iff $(\pi, next_i^g) \models \psi$
- $(\pi, i) \models X^a\psi$ iff $next_i^a \not\models \bot$ and $(\pi, next_i^a) \models \psi$
- $(\pi, i) \models X^c\psi$ iff $next_i^c \not\models \bot$ and $(\pi, next_i^c) \models \psi$
- $(\pi, i) \models \psi_1 U^b\psi_2$ (with $b \in \{g, a, c\}$) iff there exists a sequence of positions $h_0, h_1, ..., h_k$ where $h_0 = i$, for every $0 \leq j \leq k - 1 : h_{j+1} = next_i^b$, $(\pi, h_j) \models \psi_1$ and $(\pi, h_k) \models \psi_2$

Then, $\pi \models \psi$ iff $(\pi, 0) \models \psi$. Other CARET operators can be expressed by the above operators: $F^g\psi = \text{true} U^g\psi$, $G^g\psi = \neg(\text{true} U^g\neg\psi)$, $F^a\psi = \text{true} U^a\psi$, ...

**Closure.** Let $\psi$ be a CARET formula over $AP$. The closure of $\psi$, denoted $Cl(\psi)$, is the smallest set that contains $\psi$, call, ret and int and satisfies the following properties:
- if $\psi' \in Cl(\psi)$, then $\psi' \in Cl(\psi)$
- if $X^b\psi' \in Cl(\psi)$ (with $b \in \{g, a, c\}$), then $\psi' \in Cl(\psi)$
- if $\psi_1 \lor \psi_2 \in Cl(\psi)$, then $\psi_1 \in Cl(\psi)$, $\psi_2 \in Cl(\psi)$
- if $\psi_1 U^b\psi_2 \in Cl(\psi)$ (with $b \in \{g, a, c\}$), then $\psi_1 \in Cl(\psi)$, $\psi_2 \in Cl(\psi)$, $X^b(\psi_1 U^b\psi_2) \in Cl(\psi)$
- if $\psi' \in Cl(\psi)$, and $\psi'$ is not in the form $\neg \psi''$ then $\neg \psi' \in Cl(\psi)$

**Atoms.** A set $A \subseteq Cl(\psi)$ is an atom of $\psi$ if it satisfies the following properties:
- $\forall \psi' \in Cl(\psi), \psi' \in A \iff \neg \psi' \notin A$
- $\forall \psi' \lor \psi'' \in Cl(\psi), \psi' \lor \psi'' \in A \iff \psi' \in A$ or $\psi'' \in A$
- $\forall \psi' U^b\psi'' \in Cl(\psi)$, where $b \in \{g, a, c\}, \psi' U^b\psi'' \in A \iff \psi'' \in A$ or (\psi' \in A and $X^b(\psi' U^b\psi'') \in A$)
A Dynamic Pushdown Network (DPN) is a natural model for multithreaded programs [6]. To be able to define CARET formulas over DPNs, we must extend this model to record whether a transition rule corresponds to a call, ret or a simple statement (neither call nor ret).

**Definition 1.** A Dynamic Pushdown Network (DPN) \( \mathcal{M} \) is a set \( \{ P_1, ..., P_n \} \) s.t. for every \( 1 \leq i \leq n \), \( P_i = (P_i, I_i, \Delta_i) \) is a Labelled Dynamic Pushdown System (DPDS), where \( P_i \) is a finite set of control locations, \( P_i \cap P_j = \emptyset \) for all \( j \neq i \), \( I_i \) is a finite set of stack alphabet, and \( \Delta_i \) is a finite set of transition rules. Rules of \( \Delta_i \) are of the following form, where \( p, p_1 \in P_i, \gamma, \gamma_1, \gamma_2 \in I_i, \omega_1 \in \Gamma_i^* \):

- \( (r_1) \) \( p \gamma \xrightarrow{\text{call}} p_1 \gamma_1 \gamma_2 \triangleright d \)
- \( (r_2) \) \( p \gamma \xrightarrow{\text{ret}} p_1 \triangleright d \)
- \( (r_3) \) \( p \gamma \xrightarrow{\text{int}} p_1 \omega_1 \triangleright d \)

Intuitively, there are two kinds of transition rules depending on the nature of \( d \). A rule with a suffix of the form \( \triangleright \Box \) is a nonspawn rule (does not spawn a new process), while a rule with a suffix \( \triangleright p_\omega \omega_x \) describes a spawn rule (a new process is spawned). A nonspawn step describes pushdown operations of one single process in the network. Roughly speaking, a call statement is described by a rule in the form \( p \gamma \xrightarrow{\text{call}} p_1 \gamma_1 \gamma_2 \triangleright d \) in \( \Delta_i \). This rule usually models a statement of the form \( \gamma \xrightarrow{\text{call}} \proces \gamma_2 \) where \( \gamma \) is the control point of the program where the function call is made, \( \gamma_1 \) is the entry point of the called procedure \( \proces \), and \( \gamma_2 \) is the return point of the call; \( p \) and \( p_1 \) can be used to encode various information, such as the return values of functions, shared data between procedures, etc. A return statement is modeled by a rule \( (r_2) \), while a rule \( (r_3) \) is used to model a simple statement (neither a call nor a return). A spawn step allows in addition the creation of a new process. For instance, a rule of the form \( p \gamma \xrightarrow{\text{call}} p_1 \omega_1 \triangleright p_\omega \omega_x \in \Delta_i \) where \( t \in \{ \text{call, ret, int} \} \) describes that a process \( P_i \) at control location \( p \) and having \( \gamma \) on top of the stack can (1) change the control location to \( p_1 \) and modify the stack by replacing \( \gamma \) with \( \omega_1 \) and also (2) create a new instance of a process \( P_j \) (\( 1 \leq j \leq n \)) starting at \( p_\omega \omega_x \). Note that in this case, if \( t \) is call, then \( \omega_1 = \gamma_1 \gamma_2 \), and if \( t \) is ret, then \( \omega_1 = \epsilon \).
A DPDS $P_i$ can be seen as a Pushdown System (PDS) if there are no spawn rules in $\Delta_i$. Generally speaking, a DPN consists of a set of PDSs $\{P_1, ..., P_n\}$ running in parallel where each PDS can dynamically spawn new instances of PDSs in the set $\{P_1, ..., P_n\}$ during the run. An initial local configuration of a newly created instance $p_\omega \omega_s$ is called a Dynamically Created Local Initial Configuration (DCLIC). For every $i \in \{1, n\}$, let $D_i = \{p_\omega \omega_s \in \bigcup_{1 \leq j \leq n} P_j \times \Gamma^*_j \mid p_\gamma \xrightarrow{t_i} p_1 \omega_1 \triangleright p_\omega \omega_s \in \Delta_i\}$ be the set of DCLICs that can be created by the DPDS $P_i$.

A local configuration of an instance of a DPDS $P_i$ is a tuple $p_\omega$ where $p \in P_i$ is the control location, $\omega \in \Gamma^*_i$ is the stack content. A global configuration of $M$ is a multiset over $\bigcup_{1 \leq i \leq n} P_i \times \Gamma^*_i$, in which $p_\omega \in P_i \times \Gamma^*_i$ is a local configuration of an instance of $P_i$ which is running in parallel in the network $M$.

A DPDS $P_i$ defines a transition relation $\Rightarrow_i$ as follows: if $p_\gamma \xrightarrow{t_i} p_1 \omega_1 \triangleright d$ then $p_\omega \Rightarrow_i p_1 \omega_1 \triangleright D$ for every $\omega \in \Gamma^*_i$ where $D = \emptyset$ if $d = \emptyset$, $D = \{p_\omega \omega_s\}$ if $d = p_\omega \omega_s$. Let $\Rightarrow^*_i$ be the transitive and reflexive closure of $\Rightarrow_i$, then, for every $p_\omega \in P_i \times \Gamma^*_i$:
- $p_\omega \Rightarrow^*_i p_\omega \triangleright \emptyset$
- if $p_\omega \Rightarrow^*_i p_1 \omega_1 \triangleright D_1$ and $p_1 \omega_1 \Rightarrow^*_i p_2 \omega_2 \triangleright D_2$, then $p_\omega \Rightarrow^*_i p_2 \omega_2 \triangleright D_1 \cup D_2$

A local run of an instance of a DPDS $P_i$ starting at a local configuration $c_0$ is a sequence $c_0 c_1 ...$ s.t. for every $x \geq 0$, $c_x \in P_i \times \Gamma^*_i$ is a local configuration of $P_i$, $c_x \Rightarrow_i c_{x+1} \triangleright D$ for some $D$. A global run $\rho$ of $M$ from a global configuration $G = \{p_0 \omega_0, ..., p_k \omega_k\}$ is a set of local runs (possibly infinite) where each local run describes the execution of one instance of a certain DPDS $P_i$. Initially, $\rho$ consists of $k$ local runs of $k$ instances starting from $\{p_0 \omega_0, ..., p_k \omega_k\}$, when a new instance is created, a new local run of this instance is added to $\rho$. For example, when a DCLIC $c$ is created by a certain local run of $\rho$, a new local run that starts at $c$ is added to $\rho$. Note that from a global configuration, we can obtain a set of global runs because from a local configuration, we can have different local runs.

### 3.2 Single-indexed CARET for DPNs

Given a DPN $M = \{P_1, ..., P_n\}$, a single-indexed CARET formula $f$ is a formula in the form $\bigwedge_{i=1}^n f_i$, s.t. for every $1 \leq i \leq n$, $f_i$ is a CARET formula in which the satisfiability of its atomic propositions depends only on the DPDS $P_i$.

Given a set of atomic propositions $AP$, let $\lambda : \bigcup_{i=1}^n P_i \rightarrow 2^{AP}$ be a labeling function that associates each control location with a set of atomic propositions.

Let $\pi = p_0 \omega_0 p_1 \omega_1 ...$ be a local run of the DPDS $P_i$. We associate to each local configuration $p_x \omega_x$ of $\pi$ a tag $t_x$ in $\{call, int, ret\}$ as follows, where $D = \emptyset$ or $D = \{p_x \omega_x\}$:
- If $p_x \omega_x \Rightarrow_i p_{x+1} \omega_{x+1} \triangleright D$ corresponds to a transition rule $p_\gamma \xrightarrow{t_i} p_1 \omega_1 \triangleright d$, then $t_x = t$.

Then, we say that $\pi$ satisfies $f_i$ iff the $\omega$-word $(\lambda(p_0), t_0)(\lambda(p_1), t_1)...$ satisfies $f_i$. A local configuration $c$ of $P_i$ satisfies $f_i$ (denoted $c \vDash f_i$) iff there exists a local run $\pi$ starting from $c$ such that $\pi$ satisfies $f_i$. If $D$ is the set of DCLICs created during the run $\pi$, then, we write $c \vDash_D f_i$. A DPN $M$ satisfies a single-indexed CARET formula $f$ iff there exist a global run $\rho$ s.t. for every $1 \leq i \leq n$, each local run of $P_i$ in $\rho$ satisfies the formula $f_i$. 

4 Applications

We show in this section how model-checking single-indexed CARET for DPNs is necessary for concurrent malware detection.

Malware detection is nowadays a big challenge. Several malwares are multi-threaded programs that involve recursive procedures and dynamic thread creation. Therefore, DPNs can be used to model such programs. We show in what follows how single-indexed CARET for DPNs can describe malicious behaviors of concurrent malwares.

More precisely, we show how this logic can specify email worms. To this aim, let us consider a typical email worm: the worm Bagle. Bagle is a multi-threaded email worm. In the main thread, one of the first things the worm does is to register itself into the registry listing to be started at the boot time. Then, it does some different actions to hide itself from users. After this, the malware creates one thread (named Thread2) that listens on the port 6777 to receive different commands and also allow the attacker to upload a new file and execute it. This grants the attacker the ability to update new versions for his malware. In addition, the attacker can send a crafted byte sequence to this port to force the malware to kill itself and delete it from the system. Thus, the attacker can remove his malware remotely. In the next step, the malware creates one more thread (named Thread3) which contacts a list of websites every 10 minutes to announce the infection of the current machine. The malware sends the port it is listening to as well as the IP of the infected machine to these sites. At some point in the program, the malware continues to spawn a thread named Thread4 to search on local drives to look for valid email addresses. In this thread, for each email address found, the malware attaches itself and sends itself to this email address.

Thus, you can see that Bagle is a multi-threaded malware with dynamic thread creation, i.e., the main process can create threads to fulfill various tasks. To model Bagle, DPNs is a good candidate since DPNs allow dynamic thread creation. Let \( M = \{ P_1, P_2, P_3, P_4 \} \) be a model of Bagle where \( P_1 \) is a PDS that represents the main process of the malware; \( P_2, P_3, P_4 \) are PDSs that model the code segments corresponding to Thread1, Thread2, Thread3 respectively. Note that \( P_2, P_3, P_4 \) are designed to execute specific tasks, while \( P_1 \) is a main process able to dynamically create an arbitrary number of instances of \( P_2, P_3, P_4 \) to fulfill tasks in need.

We show now how the malicious behavior of the different threads can be described by a CARET formula. Let us start with the main process. The typical behaviour of this process is to add its own executable name to the registry listing so that it can be started at the boot time. To do this, the malware needs to invoke the API function \( GetModuleFileNameA \) with 0 and \( x \) as parameters. \( GetModuleFileNameA \) will put the file name of its current executable on the memory address pointed by \( x \). After that, the malware calls the API function \( RegSetValueExA \) with the same \( x \) as parameter. \( RegSetValueExA \) will use the file name stored at \( x \) to add itself into the registry key listing. This malicious behaviour can be specified by CARET as follows:

\[
\psi_1 = \bigvee_{x \in K} F^n(\text{call}(\text{GetModuleFileNameA}) \land 0 \land x \Gamma^\ast \land F^n(\text{call}(\text{RegSetValueExA}) \land x \Gamma^\ast))
\]

where the \( \bigvee \) is taken over all possible memory addresses \( x \) over domain \( K \).
Note that parameters are passed via the stack in binary programs. For succinctness, we use regular variable expression \( xI^* \) (resp. \( 0Ix^* \)) to describe the requirement that \( x \) (resp. \( 0x \)) is on top of the stack. Then, this formula states that there is a call to the API \( GetModuleFileNameA \) with 0 and \( x \) on the top of the stack (i.e., with 0 and \( x \) as parameters), followed by a call to the API \( RegSetValueExA \) with \( x \) on the top of the stack. Using the operator \( P^m \) guarantees that \( RegSetValueExA \) is called after \( GetModuleFileNameA \) terminates.

Similarly, the malicious behaviors of the Threads 2, 3 and 4 can be described by CARET formulas \( \psi_2, \psi_3 \) and \( \psi_4 \) respectively.

Thus, the malicious behavior of the concurrent worm Bagle can be described by the single-indexed CARET formula \( \psi = \psi_1 \wedge \psi_2 \wedge \psi_3 \wedge \psi_4 \).

### 5 Single-indexed CARET model-checking for DPNs

In this section, we consider the CARET model-checking problem of DPNs. Let \( \lambda : \bigcup_{i=1}^n P_i \rightarrow 2^{AP} \) be a labeling function that associates each control location with a set of atomic propositions. Let \( M = \{ P_1, \ldots, P_n \} \) be a DPN, \( f = \bigwedge_{i=1}^n f_i \) be a single-indexed CARET formula.

#### 5.1 Büchi DPNs (BDPNs)

**Definition 2.** A Büchi DPDS (BDPDS) is a tuple \( \mathcal{BP}_i = (P_i, \Gamma_i, \Delta_i, F_i) \) s.t. \( P_i = (P_i, \Gamma_i, \Delta_i) \) is a DPDS, \( F_i \subseteq P_i \) is the set of accepting control locations. A run of a BDPDS is accepted iff it visits infinitely often some control locations in \( F_i \).

**Definition 3.** A Generalized Büchi DPDS (GBDPDS) is a tuple \( \mathcal{BP}_i = (P_i, \Gamma_i, \Delta_i, F_i) \), where \( P_i = (P_i, \Gamma_i, \Delta_i) \) is a DPDS and \( F_i = \{ F_1, \ldots, F_k \} \) is a set of sets of accepting control locations. A run of a GBDPDS is accepted iff it visits infinitely often some control locations in \( F_j \) for every \( 1 \leq j \leq k \).

Given a BDPDS or a GBDPDS \( \mathcal{BP}_i = (P_i, \Gamma_i, \Delta_i, F_i) \), let \( c \in P_i \times \Gamma_i^* \) be a local configuration of \( \mathcal{BP}_i \). Then, let \( \mathcal{L}(\mathcal{BP}_i) \) be the set of all pairs \((c, D) \in P_i \times \Gamma_i^* \times 2^{D} \) s.t. \( \mathcal{BP}_i \) has an accepting run from \( c \) and \( D \) is the set of DCLICs generated during that run. We get the following properties:

**Proposition 1.** Given a GBDPDS \( \mathcal{BP}_i \), we can effectively compute a BDPDS \( \mathcal{BP}'_i \) s.t. \( \mathcal{L}(\mathcal{BP}_i) = \mathcal{L}(\mathcal{BP}'_i) \).

This result comes from the fact that we can translate a GBDPDS to a corresponding BDPDS by applying the similar approach as the translation from a Generalized Büchi automaton to a corresponding Büchi automaton [8].

**Definition 4.** A Büchi Dynamic Pushdown Network (BDPN) is a set \( \{ \mathcal{BP}_1, \ldots, \mathcal{BP}_n \} \) s.t. for every \( 1 \leq i \leq n \), \( \mathcal{BP}_i = (P_i, \Gamma_i, \Delta_i, F_i) \) is a BDPDS. A (global) run \( \rho \) of a BDPN is accepted iff all local runs in \( \rho \) are accepting (local) runs.

**Definition 5.** A Generalized Büchi Dynamic Pushdown Network (GBDPN) is a set \( \{ \mathcal{BP}_1, \ldots, \mathcal{BP}_n \} \) s.t. for every \( 1 \leq i \leq n \), \( \mathcal{BP}_i = (P_i, \Gamma_i, \Delta_i, F_i) \) is a GBDPDS. A (global) run \( \rho \) of a GBDPN is accepted iff all local runs in \( \rho \) are accepting (local) runs.

Given a BDPN or a GBDPN \( \mathcal{BM} = \{ \mathcal{BP}_1, \ldots, \mathcal{BP}_n \} \), let \( \mathcal{L}(\mathcal{BM}) \) be the set of all global configurations \( \mathcal{G} \) s.t. \( \mathcal{BM} \) has an accepting run from \( \mathcal{G} \). We get the following properties:
Proposition 2. Given a GBDPN $BM$, we can effectively compute a BDPN $BM'$ s.t. $L(BM) = L(BM')$.

This result is obtained due to the fact that we can translate each GBDPDS in $BM$ to a corresponding BDPDS in $BM'$.

Given a BDPN $BM = \{BP_1, ..., BP_n\}$ where $BP_i = (P_i, I_i, \Delta_i, F_i)$. Let $I(c)$ be the index $i$ of the local configuration $c \in P_i \times I_i^*$. Let $D = \bigcup_{i=1}^n \Delta_i$. Then, we get the following theorem:

**Theorem 1.** [19,30] The membership problem of a BDPN is decidable in time $O(\Sigma_{i=1}^n |\Delta_i| |P_i| I_i^* |D|^4 + \Sigma_{c \in D} (|\varsigma| P_{I(c)}^i |I_{I(c)}^i | 2^{2 I_{I(c)}} + |D|^2 2^{|D|} )$.

Thus, from Proposition 2 and Theorem 1, we get that the membership problem of a GBDPN is decidable.

**Theorem 2.** The membership problem of GBDPNs is decidable.

5.2 From CARET model checking of DPNs to the membership problem in BDPNs

Given a local run $\pi$, let $\delta(\pi)$ be the index of the DPDS corresponding to $\pi$. Let $G$ be an initial global configuration of the DPNS $M$, then we say that $G$ satisfies $f$ iff $M$ has a global run $\rho$ starting from $G$ s.t. every local run $\pi$ in $\rho$ satisfies $f_{\delta(\pi)}$.

Determining whether $G$ satisfies $f$ is a non-trivial problem since the number of global runs can be unbounded and the number of local runs of each global run can also be unbounded. Note that it is not sufficient to check whether every pushdown process $P_i$ satisfies the corresponding CARET formula $f_i$. Indeed, we need to ensure that all instances of $P_i$ created during a global run satisfy the formula $f_i$. Also, it is not correct to check whether all possible instances of $P_i$ satisfy the formula $f_i$. Indeed, an instance of $P_i$ should not be checked if it is not created during a global run. To solve these problems, we reduce the CARET model-checking problem for DPNs to the membership problem for GBDPNs.

To do this, we compute a GBDPN $BM = \{BP_1, ..., BP_n\}$ where $BP_i (i \in \{1..n\})$ is a GBDPDS s.t. (1) the problem of checking whether each instance of $P_i$ satisfies a CARET formula $f_i$ can be reduced to the membership problem of $BP_i$; (2) if $P_i$ creates a new instance of $P_j$ starting from $p_\omega$, which requires that $p_\omega \models f_j$; $BP_i$ must also create an instance of $BP_j$ starting from a certain configuration (computed from $p_\omega$) from which $BP_j$ has an accepting run. In what follows, we present how to compute such GBDPDSs.

Let $Label = \{exit, unexit\}$ (we explain later the need to these labels). Given a DPNS $P_i (i \in \{1..n\})$, a corresponding CARET formula $f_i$, we define $Initial_i$, as the set of atoms $A (A \in Atoms(f_i))$ such that $f_i \in A$ and $NextCallerFormulas(A) = \emptyset$. Our goal is that for every $P_i (i \in \{1..n\})$, we compute a GBDPDS $BP_i$ s.t. for every $p_\omega \in P_i \times I_i^*$, $p_\omega \models f_i$ if there exists an atom $A$ where $A \in Initial_i$ s.t. $BP_i$ has an accepting run from $(\rho, A, unexit) \omega$.

**GBDPDSs Computation.**

Let us fix a DPNS $P_i = (P, I, \Delta)$ in the DPNS $M$, a CARET formula $f_i$ in $f = \bigwedge_{i=1}^n f_i$ corresponding to the DPNS $P_i$. In this section, we show how to compute such a GBDPDS $BP_i$ corresponding to $P_i$. Given a local configuration $p_\omega$, let $\delta(p_\omega)$ be the index of the DPNS corresponding to $p_\omega$. We define $BP_i = (P'\omega, I'\omega, \Delta'F)$ as follows:
The transition relation $\Delta'$ of $\mathcal{BP}_i$ is the smallest set of transition rules satisfying the following:

- $(\alpha_1)$ for every $p \gamma \xrightarrow{\text{call}} q \gamma' q'' \triangleright d \in \Delta: \{p, A, l\} \gamma \rightarrow_1 \{q, A', l', q''\} \triangleright d_0 \in \Delta'$ for every $A, A' \in \text{Atoms}(f_i); l, l' \in \text{Label}$ such that:
  - $(\beta_0) A \cap \{\text{call}, \text{ret}, \text{int}\} = \{\text{call}\}$
  - $(\beta_1) A \cap \text{AP} = \lambda(p)$
  - $(\beta_2) A' \cap \text{AP} = \lambda(q)$
  - $(\beta_3) \text{GlNext}(A, A')$
  - $(\beta_4) \text{CallerNext}(A', A)$
  - $(\beta_5) l' = \text{unexit}$ implies $(l = \text{unexit} \text{ and } \text{NexAbsForms}(A) = \emptyset)$
  - $(\beta_6) d_0 = \emptyset$ if $d = \emptyset; d_0 = \{p_s, A_0, \text{unexit}\}_{\omega_s}$ where $A_0 \in \text{Initial}_{\delta(p, \omega_s)}$
    if $d = p_s \omega_s$

- $(\alpha_2)$ for every $p \gamma \xrightarrow{\text{ret}} q \gamma' q'' \triangleright d \in \Delta: \{p, A, l\} \gamma \rightarrow_1 \{q, A', l', q''\} \triangleright d_0 \in \Delta'$ for every $A, A' \in \text{Atoms}(f_i); l, l' \in \text{Label}$ such that:
  - $(\beta_0) \text{AbsNext}(A_0, A')$
  - $(\beta_1) \text{NexCallerForms}(A) = \text{NexCallerForms}(A_0)$
  - $(\beta_2) A' \cap \text{AP} = \lambda(q)$
  - $(\beta_3) d_0 = l'$

- $(\alpha_3)$ for every $p \gamma \xrightarrow{\text{int}} q \gamma' q'' \triangleright d \in \Delta: \{p, A, l\} \gamma \rightarrow_1 \{q, A', l', q''\} \triangleright d_0 \in \Delta'$ for every $A, A' \in \text{Atoms}(f_i); l \in \text{Label}$ such that:
  - $(\beta_0) A \cap \{\text{call}, \text{ret}, \text{int}\} = \{\text{int}\}$
  - $(\beta_1) A \cap \text{AP} = \lambda(p)$
  - $(\beta_2) A' \cap \text{AP} = \lambda(q)$
  - $(\beta_3) \text{GlNext}(A, A')$
  - $(\beta_4) \text{AbsNext}(A, A')$
  - $(\beta_5) \text{NexCallerForms}(A) = \text{NexCallerForms}(A')$
  - $(\beta_6) d_0 = \emptyset$ if $d = \emptyset; d_0 = \{p_s, A_0, \text{unexit}\}_{\omega_s}$ where $A_0 \in \text{Initial}_{\delta(p, \omega_s)}$
    if $d = p_s \omega_s$

Let $d_{U^g}(f_i) = \{\phi_1 U^g \chi_1, \ldots, \phi_k U^g \chi_k\}$ and $d_{U^a}(f_i) = \{\xi_1 U^a \tau_1, \ldots, \xi_k U^a \tau_k\}$ be the set of $U^g$-formulas and $U^a$-formulas of $Cl(f_i)$ respectively. The generalized Büchi accepting condition $F$ of $\mathcal{BP}_i$ is defined as: $F = \{F_1\} \cup F_2 \cup F_3$ where

- $F_1 = P \times \text{Atoms}(f_i) \times \{\text{unexit}\}$
- $F_2 = \{F_1^0, \ldots, F_1^k\}$ where $F_1^k = \{P \times F_2 \times \text{Label}\}$ where $F_2 = \{A \in \text{Atoms}(f_i) \mid \text{if } \phi_2 U^a \chi_x \in A \text{ then } \chi_x \in A\}$ for every $1 \leq x \leq k$.
Given a configuration $p\omega$, let $\mathcal{P}(p\omega)$ be the procedure to which $p\omega$ belongs. For example, in Figure 1, $\mathcal{P}(p_{x+1}\omega_{x+1}) = \text{proc} ... \mathcal{P}(p_{y-1}\omega_{y-1}) = \text{proc}$. Intuitively, we compute $\mathcal{BP}_i$ as a kind of product of $P_i$ and $f_i$ which ensures that: for every $p\omega \in P_i \times I_i^*$, $p\omega$ satisfies $f_i$ iff there exists an atom $A \in \text{Initial}$, s.t. $\mathcal{BP}_i$ has an accepting run from $(p, A, unexit)$. To do this, we encode atoms of $f_i$ into control locations of $P_i$. The form of control locations of $\mathcal{BP}_i$ is $(p, A, l)$ where $A$ contains all sub formulas of $f_i$ which are satisfied at the configuration $p\omega$. $l$ is a label to determine whether the execution of the procedure of $p\omega$, $\mathcal{P}(p\omega)$, terminates in the path $\pi$. A configuration $p\omega$ labeled with $\text{exit}$ means that the execution of $\mathcal{P}(p\omega)$ is finished in $\pi$, i.e., the run $\pi$ will run through the procedure $\mathcal{P}(p\omega)$, reaches its ret statement and exits $\mathcal{P}(p\omega)$ after that. On the contrary, $p\omega$ labeled with $\text{unexit}$ means that in $\pi$, the execution of the procedure $\mathcal{P}(p\omega)$ never terminates, i.e., the run $\pi$ will be stuck in and never exits the procedure $\mathcal{P}(p\omega)$. Let $\pi = p_0\omega_0p_1\omega_1...$ be a run of $P_i$ and $(p_0, A_0, l_0)\omega_0(p_1, A_1, l_1)\omega_1...$ be a corresponding run of $\mathcal{BP}_i$. We give in what follows the intuition behind our construction.

**Encoding atoms to control locations.** Firstly, we need to ensure that $\mathcal{BP}_i$ has an accepting (local) run from $(p_x, A_x)\omega_x$ iff $p_x\omega_x$ satisfies $\phi$ (denoted $p_x\omega_x \models \phi$) for every $\phi \in A_x$. To ensure this, in rules $(\alpha_1)$, $(\alpha_2)$ and $(\alpha_3)$, the first class of conditions $(\beta_0)$ ensures that the tags $\{\text{call}, \text{ret}, \text{int}\}$ assigned to each configuration of the run are guessed correctly. The second class of conditions $(\beta_1)$ and $(\beta_2)$ expresses that for every $e \in AP$, $(\pi, x) \models e$ iff $e \in \lambda(p_x)$, and the class of conditions $(\beta_4)$ expresses that $(\pi, x) \models X^a\phi'$ iff $(\pi, x + 1) \models \phi'$. Now, let us consider the most delicate case $\phi = X^a\phi' \in A_x$. There are two possibilities:

\[ A_x \xrightarrow{A} A_x+1 \xrightarrow{A_x+1} \text{proc} \xrightarrow{A_y+1} A_y \]

- $p_x\omega_x \Rightarrow q p_{x+1}\omega_{x+1} \triangleright d_0$ corresponds to a call statement. Let us consider Figure 1 to explain this case. Let $p_y\omega_y$ be the abstract-successor of $p_x\omega_x$. $(\pi, x) \models X^a\phi'$ iff $(\pi, y) \models \phi'$. Thus, we must have $\phi' \in A_y$. This is ensured by rules $(\alpha_1)$ and $(\alpha_2)$: rules $(\alpha_1)$ allow to record $X^a\phi'$ in the return point of the call, and rules $(\alpha_2)$ allow to extract and validate $\phi'$ when the return-point is reached. In what follows, we show in more details how this works: Let $p_x\gamma \xrightarrow{\text{call}} q, p_{x+1}\gamma' \triangleright d$ be the rule associated with the transition $p_x\omega_x \Rightarrow q, p_{x+1}\omega_{x+1} \triangleright d_0$, then we have $\omega_x = \gamma \omega'$ and $\omega_{x+1} = \gamma' \omega'$. Let $p_{y-1}\omega_{y-1} \Rightarrow q, p_{y}\omega_y \triangleright d_0$ be the transition that corresponds to the $\text{ret}$ statement of this call. Let then $p_{y-1}\beta \xrightarrow{\text{ret}} q, p_{y}\epsilon \triangleright d \in \Delta$ be the corresponding return rule. Then, we have necessarily $\omega_{y-1} = \beta \gamma'' \omega'$, since as explained in Section 3.1,
$\gamma''$ is the return address of the call. After applying this rule, $\omega_y = \gamma'' \omega'$. In other words, $\gamma''$ will be the topmost stack symbol at the corresponding return point of the call. So, in order to recover $\omega'$ in $A_y$, we proceed as follows: At the call $p_x \gamma \longrightarrow_1 p_{x+1} \gamma' \gamma'' \triangleright d$, we encode $A_x$ into $\gamma''$ by the rule $(\alpha_1)$ stating that $\langle p_x, A_x, l \rangle \gamma \longrightarrow_1 \langle p_{x+1}, A_{x+1}, l' \rangle \gamma' \langle \gamma'', A_x, l \rangle \triangleright d_0 \in \Delta'$. This allows us to record $X^a \phi'$ in the corresponding return point of the stack. After that, $\langle \gamma'', A_x, l \rangle$ will be the topmost stack symbol at the corresponding return-point of this call. At the return-point, the condition $(\beta_0)$ in $(\alpha_2)$ stating that $AbsNext(A_x, A_y)$ and the fact that $\phi = X^a \phi' \in A_2$ imply that $\omega' \in A_y$.

Let us consider Figure 1. $p_{x} \omega_{x} \Rightarrow_{1} p_{x+1} \omega_{x+1} \triangleright d_0$ corresponds to a call statement. Note that our explanation above makes implicitly the assumption that along the run $\pi$, every call to a procedure proc will eventually reach its corresponding return point, i.e., the run $\pi$ will finally exit proc, then, we can encode formulas at the call and validate them at its corresponding return-point. However, it might be the case that at a certain point in the procedure proc, there will be a loop, and $\pi$ never exits proc. To solve this problem, we annotate the control states by the label $l \in \{exit, unexit\}$ to determine whether $\pi$ can complete the execution of the procedure $\mathcal{P}(p_x)$. In the following, we explain three cases corresponding to three kinds of statements:

1. Let us consider Figure 1. $p_{x+1} \omega_{x+1} \triangleright d_0$ corresponds to a call statement. Note that $\mathcal{P}(p_{x+1}) = proc$ in this case. There are two possibilities.

   If proc terminates, then the call at $p_{x+1} \omega_{x+1}$ will reach its corresponding return-point. In this case, $p_{x+1} \omega_{x+1}$ is labelled by $exit$. If proc never terminates, then the call at $p_{x+1} \omega_{x+1}$ will never reach its corresponding return-point. Thus, $p_{x+1} \omega_{x+1}$ does not satisfy any formula in the form $X^a \phi$. This is ensured by the condition $(l' = unexit)$ in the rule $(\alpha_1)$. In addition, if $p_{x+1} \omega_{x+1} \triangleright d_0$ is labelled by $unexit$, then $p_{x+1} \omega_{x+1}$ never reaches its corresponding return-point. Thus, $p_{x+1} \omega_{x+1}$ does not satisfy any formula in the form $X^a \phi$. This is ensured by the condition $(l' = unexit)$ in the rule $(\alpha_1)$.

2. Again, let us consider Figure 1. $p_{y-1} \omega_{y-1} \Rightarrow_{1} p_y \omega_y \triangleright d_0$ corresponds to a ret statement. At $p_{y-1} \omega_{y-1}$, we are sure that proc will terminate. In this case, $p_{y-1} \omega_{y-1}$ must be always labelled by $exit$ and $p_y \omega_y$ can be labelled by $exit$ or $unexit$. This is ensured by the rule $(\alpha_2)$. Also, the abstract-successor of $p_{y-1} \omega_{y-1}$ is $\perp$, then, $p_{y-1} \omega_{y-1}$ does not satisfy any formula in the form $X^a \phi$. This is ensured by the condition $(NexAbsForms(A) = \emptyset)$ in the rule $(\alpha_2)$.

3. Finally, let us consider Figure 1. $p_{y-2} \omega_{y-2} \Rightarrow_{1} p_{y-1} \omega_{y-1} \triangleright d_0$ corresponds to a simple statement. Then, $p_{y-2} \omega_{y-2}$ and $p_{y-1} \omega_{y-1}$ are in the same procedure proc. Thus, the labels assigned to $p_{y-2} \omega_{y-2}$ and $p_{y-1} \omega_{y-1}$ should be the same. This is ensured by the transition rule $(\alpha_3)$.
The accepting conditions. The generalized Büchi accepting condition \( F \) of \( \mathcal{BP}_1 \) consists of three families of accepting conditions \( F_1, F_2 \) and \( F_3 \). The first set \( F_1 \) guarantees that an accepting run should go infinitely often through the label \textit{unexit}. Each set of \( F_2 \) ensures that the liveness requirement \( \phi_2 \) in \( \phi_1U^\omega \phi_2 \) is eventually satisfied in \( \mathcal{P} \). The idea behind the set \( F_3 \) is similar to the set \( F_2 \) except that the liveness requirement for a \( U^\omega \)-formula \( \phi_1U^\omega \phi_2 \) is only required on the (unique) infinite abstract path (labelled by \textit{unexit}). With respect to caller-until formulas, note that caller paths are always finite, so we do not need to consider this case in \( F \). The liveness requirements of caller-until formulas are ensured by the condition \( \textit{NexCallerForms}(A) = \emptyset \) since \( \pi(0) \) have no caller successors.

Lemma 1. Given a DPDS \( \mathcal{P}_1 = (P, \Gamma, \Delta) \), and a CARET formula \( f_i \), we can construct a GBDPN \( \mathcal{BP}_1 = (P', \Gamma', \Delta', F) \) such that for every configuration \( p\omega \in P_1 \times I_1^\omega \), \( p\omega \vDash f_i \) iff there exists an atom \( A \in \text{Initial}_i \) s.t. \( \mathcal{BP}_1 \) has an accepting run from \( \langle p, A, \text{unexit} \rangle \omega \).

Spawning new instances. Lemma 1 guarantees that the problem of checking whether an instance of \( \mathcal{P}_1 \), starting from \( p\omega \), satisfies \( f_i \) can be reduced to the problem of checking if \( \mathcal{BP}_1 \) has an accepting run from \( \langle p, A, \text{unexit} \rangle \omega \) where \( A \in \text{Initial}_i \). Now, we need to ensure the satisfiability on instances created dynamically. Suppose that \( \mathcal{P}_i \) spawns a new instance of \( \mathcal{P}_j \) starting from \( p_i\omega_s \), this means that we need to guarantee that \( p_i\omega_s \vDash f_j \). Note that by applying Lemma 1 for the DPDS \( \mathcal{P}_j \), we get that \( p_i\omega_s \vDash f_j \) iff there exists an atom \( A \in \text{Initial}_j \) s.t. \( \mathcal{BP}_j \) has an accepting run from \( \langle p_i, A, \text{unexit} \rangle \omega_s \). Then, the requirement \( p_i\omega_s \vDash f_j \) is ensured by the conditions (\( \beta_0 \)) in (\( \alpha_1 \)), (\( \beta_5 \)) in (\( \alpha_2 \)) and (\( \beta_6 \)) in (\( \alpha_3 \)) stating that for every \( p\gamma \xrightarrow{\gamma} q\omega \xrightarrow{d} \Delta \) (\( t \in \{\text{call, ret, int}\} \)), we have \( \langle p, A, t \rangle \gamma \xrightarrow{\gamma} \langle q, A', t' \rangle \omega \xrightarrow{d_0} \Delta' \) such that if \( d = p_i\omega_s \), then, \( d_0 = \langle p_i, A_0, \text{unexit} \rangle \omega_s \) where \( A_0 \in \text{Initial}_j \) (since \( \delta(p_i\omega_s) = j \) in this case).

Thus, we can show that:

Theorem 3. Given a DPN \( \mathcal{M} = \{\mathcal{P}_1, ..., \mathcal{P}_n\} \), a single-indexed CARET formula \( f = \bigwedge_{i=1}^n f_i \), we can compute a GBDPN \( \mathcal{BM} = \{\mathcal{BP}_1, ..., \mathcal{BP}_n\} \) such that a global configuration \( \mathcal{G} \) of \( \mathcal{M} \) satisfies \( f \) iff \( \mathcal{G}' \in \mathcal{L}(\mathcal{BM}) \) where \( \mathcal{G}' \) is a global configuration of \( \mathcal{BM} \) that corresponds to the configuration \( \mathcal{G} \).

6 Single-indexed CARET model-checking for DPNs with regular valuations

In this section, we consider the single-indexed CARET model-checking problem for DPNs with regular valuations, in which the set of configurations where an atomic proposition is satisfied is a regular language.

Definition 6. Let \( \mathcal{M} = \{\mathcal{P}_1, ..., \mathcal{P}_n\} \) be a DPN. For every \( i \in \{1..n\} \), a set of configurations of a pushdown process \( \mathcal{P}_i = (P_i, \Delta_i, I_i) \) is regular if it can be written as the union of sets of the form \( E_p \), where \( p \in P_i \) and \( E_p = \{(p, w) | w \in L_p \} \), where \( L_p \) is a regular set over \( I_i^\omega \).

Definition 7. Let \( \mathcal{M} = \{\mathcal{P}_1, ..., \mathcal{P}_n\} \) be a DPN. Let \( AP \) be a finite set of atomic propositions. Let \( \nu : AP \rightarrow 2(U_1 \times I_1^\omega) \) be a valuation. \( \nu \) is called regular if for every \( e \in AP \), \( \nu(e) \) is a regular set of configurations.
Let $\nu : AP \to 2^{\bigcup_{i=1}^{n} P_i \times \Gamma_i^e}$ be a regular valuation. We define $\lambda_\nu : P \times \Gamma^* \to 2^{AP}$ such that $\lambda_\nu(p\omega) = \{ e \in AP \mid \omega \in \nu(e) \}$. Let $\pi = p_0\omega_0p_1\omega_1\ldots$ be a local path of $P_i$. We associate each configuration $p_\omega\omega_x$ of $\pi$ with a tag $t_x$ in \{call, int, ret\} as presented in Section 3.2. Let $f_i$ be a CARET formula over $AP$. The satisfaction relation w.r.t. the regular valuation $\nu$ is defined as follows:

$$\pi \models f_i \text{ iff } (\lambda_\nu(p_0\omega_0), t_0)(\lambda_\nu(p_1\omega_1), t_1) \cdots \models f_i$$

**Theorem 4.** [19] Single-indexed LTL model-checking with regular valuations for DPNs can be reduced to standard LTL model checking for DPNs.

Given a DPN $M = \{P_1, \ldots, P_n\}$ and a regular valuation $\nu : AP \to 2^{\bigcup_{i=1}^{n} P_i \times \Gamma_i^e}$, this result is based on translating every DPDS $P_i$ ($i \in \{1..n\}$) into a DPDS $P'_i = (P_i, \Gamma'_i, \Delta'_i)$ where the regular valuation requirements are encoded in $\Gamma'_i$. The same reduction is still true for single-indexed CARET with regular valuations. For details about this reduction, we refer readers to [19]. We can show that:

**Theorem 5.** Single-indexed CARET model-checking with regular valuations for DPNs can be reduced to standard single-indexed CARET model checking for DPNs.

## 7 DPNs Communicating via Locks

Dynamic Pushdown Network with Locks (L-DPNs) is a natural formalism for multithreaded programs communicating via locks [14,20]:

**Definition 8.** A Dynamic Pushdown Network with Locks (L-DPN) $M$ is a set \{L, Act, P_1, \ldots, P_n\} where L is a set of locks, Act = \{acq(l), rel(l), \tau \mid l \in L\} is a set of actions on locks s.t. acq(l) (resp. rel(l)) for $l \in L$ represents an acquisition (resp. release) of the lock $l$ and the action $\tau$ describes internal actions (neither acquire nor release locks); for every $1 \leq i \leq n$, $P_i = (\Gamma_i, \Delta_i)$ is a Labelled Dynamic Pushdown System with Locks (L-DPDS), where $P_i$ is a finite set of control locations and $P_i \ni P_j = \emptyset$ for all $j \neq i$, $\Gamma_i$ is a finite set of stack alphabets, and $\Delta_i$ is a finite set of transitions rules. Rules of $\Delta_i$ are of the following form, where $a \in Act$, $p, p_1 \in P_i$, $\gamma \in \Gamma_i$, $\omega \in \Gamma_i^e$, $d \in \{\Box, p_\omega, p_\omega s \mid p_\omega s \in \bigcup_{1 \leq j \leq n} P_j \times \Gamma_j^e\}$:

- $(r_1) \quad p_\gamma \xrightarrow{(a, call)} p_1\gamma_1\gamma_2 \triangleright d$
- $(r_2) \quad p_\gamma \xrightarrow{(a, ret)} p_1\epsilon \triangleright d$
- $(r_3) \quad p_\gamma \xrightarrow{(a, init)} p_1\omega \triangleright d$

Intuitively, a L-DPN is a DPN where processes communicate via locks. The difference is that each transition rule of L-DPNs is assigned to one additional action $a \in Act$. Depending on the nature of the associated action $a$, each transition step of L-DPDSs include one additional operation on a given lock $l$. acq(l) (resp. rel(l)) represents an acquisition (resp. release) of the lock $l$ and the action $\tau$ describes internal actions (neither acquire nor release locks).

A local configuration of an instance of a L-DPDS $P_i$ is a tuple $(p, \omega, L)$ where $p \in P_i$ is the control location, $\omega \in \Gamma_i^e$ is the stack content and $L \subseteq L$ is a set of locks owned by the instance. A global configuration of $M$ is a multiset
over $\bigcup_{i<j<n} P_i \times \Gamma_i^* \times 2^L$, in which $(p\omega,L) \in P_i \times \Gamma_i^* \times 2^L$ represents the local configuration of an instance of a pushdown process $P_i$, which is running in the network.

A L-DPDS $\mathcal{P}_i$ defines a transition relation $\Rightarrow_i$ as follows where $t \in \{\text{call, ret, int}\}$:

- if $p\gamma \xrightarrow{\langle \text{act}, t \rangle_i} p_1\omega_1 \triangleright d$ then $(p\gamma\omega,L) \Rightarrow_i (p_1\omega_1\omega,L) \triangleright D_0$ where $D_0 = \emptyset$ if $d = \Box$, $D_0 = \{(p\omega_s,\emptyset)\}$ if $d = p\omega_s$ for every $\omega \in \Gamma_i^*$, $L \subseteq L$. This expresses that the current instance can move from $(p\gamma\omega,L)$ to $(p_1\omega_1\omega,L \cup \{l\})$. This ensures that the current instance owns the lock $l$ after the action $\text{act}(l)$.

- if $p\gamma \xrightarrow{\langle \text{rel}, t \rangle_i} p_1\omega_1 \triangleright d$ then $(p\gamma\omega,L) \Rightarrow_i (p_1\omega_1\omega,L \cup \{l\}) \triangleright D_0$ where $D_0 = \emptyset$ if $d = \Box$, $D_0 = \{(p\omega_s,\emptyset)\}$ if $d = p\omega_s$ for every $\omega \in \Gamma_i^*$, $L \subseteq L$. This means that the current instance can move from $(p\gamma\omega,L)$ to $(p_1\omega_1\omega,L \cup \{l\})$.

Roughly speaking, if $d = p\omega_s$, then the current instance not only does local move but also creates a new instance of the pushdown process $P_j$ starting at $(p\omega_s,\emptyset)$. Note that we suppose that the new instance holds no locks when it is created.

A local run of an instance of a L-DPDS $\mathcal{P}_i$ starting at a local configuration $c_0$ is a sequence $c_0,c_1,\ldots$ such that for every $j \geq 0$, $c_j \in P_i \times \Gamma_i^* \times 2^L$ is a local configuration of $P_i$, $c_j \Rightarrow_i c_{j+1} \triangleright D_0$. The definition of global run of a L-DPNS $\mathcal{M}$ is similar to the one for DPNs.

**Nested Lock Access.** In this work, we suppose that in all local runs, the locks are accessed in a well-nested and no-reentrant manner, i.e. a local run can only release the latest lock it acquired that is not released yet. Indeed, if we allow arbitrary locks, then reachability becomes undecidable [10].

**Theorem 6.** [20] Single-indexed LTL model-checking for L-DPNs can be reduced to single-indexed LTL model checking for DPNs.

Given a L-DPN $\mathcal{M} = \{\mathcal{M}, \text{Act}, \mathcal{P}_1, \ldots, \mathcal{P}_n\}$, this result is based on translating every $\mathcal{P}_i$ ($i \in \{1,n\}$) into a DPDS $\mathcal{P}_i' = (P_i', \Gamma_i', \Delta_i')$ s.t. $\mathcal{P}_i'$ is a kind of product between the DPDS $\mathcal{P}_i$ and the acquisition structure, where an acquisition structure (encoded in control locations of $\mathcal{P}_i'$) stores information about how locks are used such as the number of held locks, the order of acquisition and release of locks. We can compute a DPN $\mathcal{M}' = \{\mathcal{P}_1', \ldots, \mathcal{P}_n'\}$ s.t. the global runs of $\mathcal{M}'$ mimic the global runs of $\mathcal{M}$ and the acquisition structures reflect the lock usages. Thus, the global runs of $\mathcal{M}'$ correspond to global runs of $\mathcal{M}$ in which the locks are accessed in a nested manner. The same reduction is still true for single-indexed CARET formulas. For details of this reduction, we refer readers to [20]. We can show that:

**Theorem 7.** Single-indexed CARET model-checking for L-DPNs can be reduced to single-indexed CARET model checking for DPNs.
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