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Discounted Cash Flow Methods in Lease versus Purchase Analysis—A Technical Note

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Abstract
The earlier framework uses the before-tax cost of debt as the discount rate in valuation of lease contracts for the reason that such framework explicitly includes the interest tax credits as a component of each period’s cash flows. The short-cut or modern standard textbook approaches use after-tax cost of debt as the discount rate for the reason that it ignores the interest tax credits. Some existing literatures state the two approaches are equivalent without exploring the reasons analytically. This note provides a mathematical demonstration showing, the two approaches are not equivalent accompanied by numerical examples.

Keywords
leasing, purchasing, tax credit, discounted cash flow, discount rate

1. Introduction
When a firm wishes to obtain the use of an equipment, it can either purchase it or lease it. An equipment lease is an agreement in which one party (the “lessor”) gives the other party (the “lessee”) the right to use the lessor’s equipment for a certain period of time. In exchange, the lessee will make payments to the lessor during the lease period. At the end of the period, depending on the lease agreement, the lessee can walk away, or may be given the option to buy the equipment that it has been leasing, or may be given the option to renew the lease for another term.

Purchasing an expensive equipment can use up available funds and may saddle a firm with an outdated asset and leasing can be an excellent way to update the business without significant upfront costs. The other advantages to lessees mentioned in mainstream managerial finance textbooks (e.g., Ross, Westerfield, & Jordan, 2019; Brealey, Myers, & Allen, 2020) include high operating flexibility, risk reduction of technological obsolete and uncertain demand, and most importantly, the tax advantage: A
A firm that leases equipment or real estate, for example, will be able to deduct its lease payments from its taxable income immediately rather than deducting the cost of purchasing equipment as depreciation over time. Lessors also benefit from leasing in a number of ways. Compared to lessees, they are often able to acquire equipment at a low cost, and obtain acceptable financing terms. These advantages result from various economies of scale, such as increased buying power with sellers of equipment. Lessors are also better positioned to take advantage of certain tax laws, such as depreciation allowances and investment tax credit. The tax advantages to both lessee and lessor largely depend on the tax bracket each is in: the higher marginal tax rate, the larger the tax benefit, other things being equal. The conventional wisdom is that the firm needing to use the equipment lets a firm with a higher effective tax rate purchase it and take the depreciation tax benefit. The asset’s ultimate user then leases it from the purchaser under a contract which allows the tax benefit to be shared between two firms.

2. Review of Lease versus Purchase Valuation

The modern agreed-upon conceptual framework of lease vs. purchase analysis is formulated on the notion that a lease contract is equivalent to a loan. Because the asset user’s operating cash flows are not affected by lease or purchase decision therefore may be ignored in a lease vs. purchase analysis. The Net Advantage to Lease (NAL) is the difference between the saved initial investment cash outlay to pay the asset and the present value of the Lease’s Net Cash Outflows (LNCOs), where each period’s LNCO is the sum of the following three components: (1) the after-tax lease payment; (2) the foregone depreciation tax credit; and (3) the foregone interest tax credit on the debt that the lease is presumed to displace. If NAL>0, the asset user would prefer leasing to borrowing-to-buy.

A lessor’s analysis of a proposed lease is a capital budgeting valuation problem. The Net Present Value (NPV) of the project is the difference between the present value of the lessor’s net cash inflows and initial investment outlay. The lessor’s net cash inflows in each period is the following three components: (1) the after-tax lease revenue; (2) the depreciation tax credit; (3) the interest tax credit on the debt. If NPV>0, lessor benefits from the leasing.

To evaluate lessee’s NAL and lessor’s NPV, the discounted cash flow (DCF) methods are used. DCF is commonly used to estimate the value of an investment, here the leasing contract, based on its future cash flows. DCF analysis attempts to figure out the value of an investment today, based on projections of how much money it will generate in the future.

A considerable amount of work has been done on the analysis of leasing as a financial decision of the firm. A comprehensive paper by Myers, Dill, and Bautista (1976) discusses the fundamental issues in leasing and presents a thorough analysis of the valuation of leasing contracts based upon fundamental financial principles. Their framework, along with Lewellen et al. (1976), use the before-tax cost of debt as the discount rate because they explicitly include the interest tax credits as a component of each period’s cash flows for lessee’s NAL calculation. A simpler, “short-cut” approach ignores the interest tax credits and use after-tax cost of debt as the discount rate for lessee’s NAL and lessor’s NPV.
calculations, and this short-cut approach has later also become the modern standard textbook lease/purchase framework. A natural question arises: are these two approaches equivalent? Early literatures, such as, Long (1980) and Ezzell and Miles (1983), state that under appropriate assumptions on debt displaced, the two approaches are equivalent. And, recently, Musumeci and O’Brien (2019), also state the two approaches yield the same lessee’s NAL, thus are equivalent without exploring the reasons analytically. In next section, we will present a mathematical demonstration showing that the two approaches are not equivalent and material errors can arise accompanying with numerical examples.

3. A Mathematical Presentation

Let P denote the equipment’s purchasing price, $D_k$ depreciation in year k, $L_k$ lease payment in year k, $d_R$ lessor (buyer)’s before-tax cost of debt, $d_E$ lessee’s before-tax cost of debt, $T_R$ lessor’s tax rate and $T_E$ lessee’s. As mentioned earlier, the modern agreed-upon conceptual textbook framework of lease vs. purchase analysis is formulated on the notion that a lease contract is equivalent to a loan agreement. Thus, “lease versus borrow-and-buy” is a more accurate description of the modern textbook framework than the simple term of “lease versus purchase” which implies that the lessor uses 100% debt to finance its purchase. The standard textbook framework further assumes the equipment has no salvage value at the end of the lease term. Under these assumptions, using the original framework (MBD/LLM), the lessee’s NAL can be expressed as

$$NAL = P - \sum_{k=0}^{n} \frac{L_k (1-T_E)+D_k T_E+P d_E T_E}{(1+d_E)^k}$$  \(1\)

While when the short-cut or standard text book framework is used, the lessee’s NAL becomes

$$NAL = P - \sum_{k=0}^{n} \frac{L_k (1-T_E)+D_k T_E}{(1+d_E(1-T_E))^k}$$  \(1')\)

Where $L_k (1 - T_E)$ is lessee’s after-tax lease payment, $D_k T_E$ depreciation tax credit and $P d_E T_E$ interest tax credit.

For lessor’s NPV, theearly framework (MBD/LLM) gives

$$NPV = \sum_{k=0}^{n} \frac{L_k (1-T_R)+D_k T_R+P d_R T_R}{(1+d_R)^k} - P$$  \(2\)

While the short-cut or standard text book framework gives

$$NPV = \sum_{k=0}^{n} \frac{L_k (1-T_R)+D_k T_R}{(1+d_R(1-T_R))^k} - P$$  \(2')\)

Where $L_k (1 - T_R)$ is lessor’s after-tax lease payment, $D_k T_R$ depreciation tax credit and $P d_R T_R$ interest tax credit.

Mathematically speaking, unless the tax rates are zero, equations (1) and (1’) are not equivalent, neither are equations (2) and (2’).
To illustrate our point, we first use the same example seen in Musumeci and O’Brien (2019) (also in Ross, Westerfield, Jaffe, & Jordan, 2019). In that scenario, the user wants an equipment that costs $10,000 and that may be depreciated straight-line for 5 years ($D_n=$2,000) with salvage value of zero. A leasing company offers to lease the equipment to the user for an annual lease payment of $2393 per year. Lessor and lessee are assumed to have the same interest rates of 6.329% which implies annual interest payment $I_m=$632.9, but their tax rates vary: lessee $T_l=0\%$ and lessor $T_p=21\%$. Since the lessee’s tax rate is assumed to be zero, equations (1) and (1’) merge, and it is easy to calculate lessee’s NAL=$9.40. However, the two approaches yield different NAL if the lessee’s tax rate is not zero, for example, 5%, we can calculate the lessee’s after-tax cash outflow including interest tax credit in each year:

$$L_k(1-T_E) + D_k T_E + Pd_E T_E = 2393(1-5\%) + 2000x5\% + 10000x6.329\%x5\% = 2405$$

And the present value of this five-year annuity discounted at 6.329% is 10040.70, thus NAL=-40.70. When using the short-cut or standard textbook framework, the lessee’s after-tax cash outflow without including interest tax credit in each year:

$$L_k(1-T_E) + D_k T_E = 2393(1-5\%) + 2000x5\% = 2373.75$$

And the present value of this five-year annuity of 2373.75 discounted at after-tax rate of 6.01% (6.329%(1-5%)) is 9.996.38, and NAL=3.62. And the two approaches yield different lessee’s NAL.

For lessor’s NPV, using the original MDB/LLM framework, lessor’s after-tax cash inflow in each period is the sum of after-tax lease payment, depreciation tax credit and interest tax credit which is

$$L_k(1-T_R) + D_k T_R + Pd_R T_R = 2393(1-21\%) + 2000x21\% + 10000x6.329\%x21\% = 2943.37$$

And the present value of five-year annuity of 2943.37 discounted at lessor’s before tax debt rate of 6.329% is 12288.36, thus NPV=2288.36. However if we use the standard textbook framework in which the interest tax credit of 632.9 is not included in lessor’s cash inflow in each period and the after-tax discount rate is 6.329%(1-21%)=5%. The present value of the five-year annuity of 2310.47 (=2943.37-632.9) discounted at 5% is 10003.13, thus the lessee’s NPV=3.13 which is the same as shown in Ross, Westerfield, Jaffe and Jordan (2019).

4. Conclusion

This note examines the two widely used frameworks in the valuation of financial lease contracts: the early framework which includes the tax credit in lessee and lessor’s annual cash flows and uses the pre-tax discount rate; the short-cut or standard textbook framework ignores the tax credit in lessee and lessor’s annual cash flows and uses after-tax discount rate. This note challenges existing literatures’ claim that the two approaches are equivalent and demonstrates that there are fundamental difference between the two framework analytically accompanied by numerical examples.
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