Bounds on an energy-dependent and observer-independent speed of light from violations of locality

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We show that models with deformations of special relativity that have an energy-dependent speed of light have non-local effects. The requirement that the arising non-locality is not in conflict with known particle physics allows us to derive strong bounds on deformations of special relativity and rule out a modification to first order in energy over the Planck mass.

Modifications of special relativity have recently obtained increased attention since measurements of gamma ray bursts observed by the Fermi Space Telescope have now reached a precision high enough to test an energy-dependence of the speed of light to first order in the photon’s energy over the Planck mass [1-3]. While such modifications could also be caused by an actual breaking of Lorentz-invariance that introduces a preferred frame, Lorentz-invariance breaking is subject to many other constraints already [4]. This makes deformations of special relativity (DSR) [5-9], that preserve observer-independence and do not introduce a preferred frame, the prime candidate for an energy-dependent speed of light. We will here show that DSR necessitates violations of locality that put a bound on an energy-dependent speed of light that is 23 orders of magnitude stronger than the recent measurements of gamma ray bursts. We will use units in which $c = \hbar = 1$.

DSR is motivated by finding modified Lorentz-transformations that allow the energy associated to the Planck mass, $m_{Pl}$, to remain invariant under action of the transformations. In the cases of DSR we will examine, the speed of light is a function of energy $c(E)$, such that this function is the same for all observers. Thus, in a different restframe where $E$ was transformed into $E'$ under the deformed Lorentz-transformation, the speed of light would be $c'(E') = c(E')$. This invariance of the functional behavior of the new energy-dependent speed of light is the key point of DSR, and the modified transformations are constructed such that the function $c(E)$ can indeed remain invariant under a change of reference frame, which would not be the case under ordinary Lorentz-transformations.

The modified transformations are commonly derived by requiring the invariance of a modified dispersion relation [5] from which the speed of light can be obtained. These transformations were originally considered for momentum space. However, the claim that DSR makes predictions for the propagation of photons from gamma ray bursts clearly employs the energy-dependence of the speed of light in position space. We will in the following show that the requirement of an energy-dependent and observer-independent speed of light in position space results in an observer-dependence of what it means for two events to be at the same point in space and time. This results in a violation of locality in the sense that two observers in relative motion to each other cannot agree on whether two events at the same point, and this disagreement is macroscopic even for moderate relative velocities resulting in an inconsistent definition of space-time location that is in conflict with already established physics. It should be stressed that this problem does not occur in theories with an energy-dependent speed of light that actually break Lorentz-invariance. In this case the functional form of $c(E)$ will not remain invariant under a change of reference frame.

Consider a gamma ray burst (GRB) at distance $L \approx 4$ Gpc that, for simplicity, has no motion relative to a laboratory where it is detected. This source emits a photon with $E_{\gamma} \approx 10$ GeV which arrives in the lab restframe at $(0,0)$ inside a detector. Together with the 10 GeV photon there is a low energetic reference photon emitted. The energy of that photon can be as low as wanted.

In the DSR-scenario we are considering the phase velocity depends on the photons’ energy. To first order

$$c(E) \approx \left( 1 + \frac{\alpha E}{m_{Pl}} \right) + O \left( \frac{E^2}{m_{Pl}^2} \right),$$

where we will neglect corrections of order higher than $E_{\gamma}/m_{Pl}$ in the following. The important point is that Eq. (1) is supposed to be observer-independent, such that it has the same form in every reference frame. This then requires the non-linear, deformed Lorentz-transformations in momentum space. Four our purposes it is sufficient to know that the Lorentz-transformations receive to lowest order a correction in $E/m_{Pl}$. To ease the discussion we consider the case $\alpha < 0$ such that the speed of light decreases with increasing energy. The argument however does not depend on the sign.

The higher energetic photon is slowed down and arrives later than the lower energetic one. One has for the difference $\Delta T$ between the arrival times of the high and low energetic photon

$$\Delta T = L \left( \frac{1}{c(E_{\gamma})} - 1 \right) = L \frac{E_{\gamma}}{m_{Pl}} + O \left( \frac{E_{\gamma}^2}{m_{Pl}^2} \right).$$

With $4 \text{ Gpc} \approx 10^{26}$ m, $E_{\gamma} \approx 10^{-18} m_{Pl}$, the delay is of the order 1 second. Strictly speaking, we should take into account the cosmological redshift since the photon propagates in a time-dependent background. However, the purpose of estimating the effect it will suffice to consider a static background, since using the proper general relativistic expression does not change the result by more than an order of magnitude [10,11].

We further consider an electron at $E_e \approx 10$ MeV emitted from a source in the detector’s vicinity such that it arrives together with the high-energetic photon at $(0,0)$ inside the detector. The low-energetic photon leaves the GRB together with the high-energetic photon at $(x_e,t_e) = (-L,-L/c(E_{\gamma}))$. It arrives in the detector at $(x_a,t_a) = (0,L(1-1/c(E_{\gamma})))$, by $-t_a$ earlier than the electron. We have chosen the emission time...
such that $-t_a = \Delta T$ in the lab frame and the electron arrives with the same delay after the low energetic photon as the high energetic photon. With an energy of 10 MeV, the electron is relativistic already, but any possible energy-dependent DSR effect is at least 3 orders of magnitude smaller than that of the photon, and due to the electron’s nearby emission the effects cannot accumulate over a long distance. The electron’s velocity is $v_e \approx 1 - 10^{-3}$.

Inside the detector at $x = 0$ the photon scatters off the electron. The photon changes the momentum of the electron, which triggers a bomb and the lab blows up. That is of course completely irrelevant. It only matters that the elementary scattering process can cause an irreversible and macroscopic change. This setup is depicted in Figure 1.

Now let us consider a team of physicists in a satellite moving towards the GRB who observe and try to describe the propagation of a particle in a background with quantum gravitational effects. Yet the question is whether this modification should be understood as one for the electron at arrival.

With higher energy, the speed of the electron increases. The speed of the photon also changes but, and here is the problem, according to DSR the function $\tilde{c}$ is observer-independent. In the satellite frame one then has

$$\tilde{c}(E') = 1 - \frac{E'_r}{m_{Pl}} = 1 - \sqrt{1 - \frac{1}{1 + v_s} E'_r} + \mathcal{O}\left(\frac{E'^2_r}{m_{Pl}^2}\right), \quad (5)$$

and the distance the photons travel is $L' = \gamma_S (v_S/\tilde{c}(E'_r) - 1) L$. Thus, the time passing between the arrival of the low-energetic and the high-energetic photon in the satellite is

$$\Delta T' = \frac{E'_r}{m_{Pl}} L' = \left(\frac{1 - v_S}{1 + v_S} - \frac{1}{\gamma_S(1 - v_S)}\right) \Delta T + \mathcal{O}\left(\frac{E'^2_r}{m_{Pl}^2}\right). \quad (6)$$

The question arises whether there could also be some energy-dependence in the transformation of $L$ in position-space. We will discuss this possibility later. With the above, in the satellite frame the high energetic photon arrives later than the electron by

$$\Delta T' - t_a' = \left(\frac{1 - v_S}{1 + v_S} - \frac{1}{\gamma_S(1 - v_S)}\right) \Delta T + \mathcal{O}\left(\frac{E'^2_r}{m_{Pl}^2}\right). \quad (7)$$

Inserting $(1/\gamma_S) \approx 1 - (v_S^2)/2$ for $v_S \ll 1$, one finds

$$\Delta T' - t_a' \approx -3 \Delta T \left(\frac{v_S - v_S^3/2}{\gamma_S}\right) \approx 10^{-5} \Delta T. \quad (8)$$

In the satellite frame, depicted in Figure 2, the high energetic photon thus misses the electron by $\approx 10^{-5}$ seconds, and is still lagging behind as much as a kilometer when it arrives in the detector. The photon then cannot scatter off the electron in the detector, and the electron cannot trigger the bomb to blow up the lab. The physicists in the satellite are puzzled.

An assumption we implicitly made for this derivation was that the quantum mechanical space- and time-uncertainties $\Delta x, \Delta x'$ are not modified in DSR, such that the GeV photon can be considered peaked to a $\Delta x$ smaller than the distance to the electron at arrival.

Whether or not the wave function spreads in DSR depends on the interpretation of the modified dispersion relation. It is supposed to describe the propagation of a particle in a background with quantum gravitational effects. Yet the question is whether this modification should be understood as one for a plane wave or for a localized superposition of plane waves already. In the first case a wave-packet would experience enhanced dispersion, in the latter case not. In the absence of a derivation, both seems plausible, so let us just examine the possibilities. There either is a modification, or there is not. The above covered the case without modification.

In case there is a modification caused by a dispersion of the wave-packet, then the uncertainty of the slowed down, high energetic photon at arrival would be vastly larger than the maximal localization of the Heisenberg limit allows. If one starts with a Gaussian wave-packet localized to a width of $\sigma_0$ at emission and tracks its spread with the modified dispersion...
relation, one finds that to first order the now time-dependent width is

$$\sigma(t) = \sigma_0 \sqrt{1 + \left( \frac{2t}{m_p \sigma_0^2} \right)^2}. \quad (9)$$

If we start with a width of $\sigma_0 \approx 1/E_p$, then for $t \gg m_p \sigma_0^2$ one has $\sigma(t) \approx 2tE_p/m_p$. Or, in other words, in the worst case the uncertainty of the wave-packet at arrival was about the same size as the time delay $\Delta t \approx \Delta T$ and the photon would at arrival be smeared out over some hundred thousand kilometers. A delay of $\Delta T'$ with an uncertainty of $\Delta T'$ is hard to detect, but it would also be impossible to find out whether or not the center of the wave-packet had been dislocated by a factor five orders of magnitude smaller than the width of the wave-packet.

However, the problem was caused by the unusual transformation behavior of $\Delta T$. To entirely hide this behavior, the quantum mechanical uncertainty $\Delta t$ needed to be much larger than the delay $\Delta T - t_a$ in all restframes, such that it was practically unfeasible to ever detect a tiny difference in probability with the photons we can receive, say, in the lifetime of the universe. Therefore, let us boost into a reference frame with $v = 1 - \epsilon$, such that $\gamma \approx 1/\sqrt{2}\epsilon$. The inequality that needs to be fulfilled to hide the delay $|\Delta T' - t_a'| \ll |\Delta T'|$ is then equivalent to $|\epsilon - \sqrt{2}/|\epsilon| \ll \epsilon$, which is clearly violated without requiring extreme boosts. To put in some numbers, for an observer in rest with the electron one has $\epsilon = 10^{-3}$, $\gamma \approx 20$ and $|\Delta T' - t_a'| \approx 10^4\Delta t'$. Similarly, for $v = -1 + \epsilon$, the requirement to hide the delay takes the form $|2/\epsilon - \sqrt{2}/|\epsilon| | \ll 2/\epsilon$, which is also clearly violated. Though in this case the delay does not actually get much larger than the uncertainty, they both approach the same value. We would then be comparing the probability of interaction at the center of the wave-packet with one at a distance comparable to its width. This would require several photons to get a proper statistic, but it is a difference in probability that is feasible to measure within the lifetime of the universe, and thus is still in conflict with observer-independence. Let us point out that we have considered here a photon whose approximate momentum uncertainty at emission is comparable to the mean value, which is quite large already. If the photon’s momentum had instead a smaller uncertainty, e.g. $\approx 100$ MeV only, then the mismatch in timescales was by two orders of magnitude larger.

To keep track of our assumptions, we used normal Lorentz-boosts to calculate the time span $t_a'$. This is based on already available data since the transformation behavior under Lorentz-boosts has been tested to high precision in particle physics experiments ($12$). For the time-dilatation in particular, the decay-time of muons is known to transform as $\Delta t' = \gamma \Delta t$ up to a $\gamma$-factor of 30 to a precision of one per mille ($13$).

The logic of the here presented argument is as follows. If there was a delay of the order 1 second for the 10 GeV photon caused by an energy-dependent speed of light, then the requirement of observer-independence results in violations of locality that are in conflict with experiment already. Note that it is not necessary to actually perform the above sketched experiment in all reference frames, since observer-independence allows us to use cross-sections measured in our laboratories. This means in turn one can now use the knowledge of QED processes combined with the measurements of Lorentz-boosts to constrain the possibility of there being such a DSR modification by requiring the resulting mismatch in arrival times not to result in any conflict with existing measurements.

The distance $L = \text{some Gpc}$ is as high as we can plausibly get in our universe, and the 10 GeV photon is as high as we have reliable observational data from particles traveling that far. The center of mass energy of the electron and the high energetic photon is $\sqrt{s} \approx 15$ MeV. The process thus probes distances of $\approx 10$ fm. If the photon and the electron were closer already than the distance their scattering process probes, we would not have a problem. Requiring $|\Delta T' - t_a'| < 10$ fm for boosts up to $\gamma = 30$ leads to a bound on the delay between the low and high energetic photon of

$$\Delta T < 10^{-23}\text{s} \quad , \quad (10)$$

or, if we reinsert $\alpha$ from Eq. ($1$), $|\alpha| < 10^{-23}$. Note that this covers both cases, the one with and without spread of the wave-packet.

We have here not discussed all possible constraints that one could consider, for example different scattering processes and their exact precision. We see now that this is not necessary, since the ratio $E_p/m_\gamma$ is $\approx 10^{-18}$. With the above constraint, we are thus already in the regime where second order modifications would become important. The here used analysis however made use of the scaling in Eq. ($6$) and thus does not in this way apply to the second order modifications. We can conclude however that present-day observations do already rule out a modification in the speed of light to first order in the energy over Planck mass.

To retrace our steps, the problem stems from the transformation behavior of $\Delta T'$ in Eq. ($5$). This behavior is a direct consequence of requiring the energy-dependent speed of light
\( \tilde{c} \) to be observer-independent, together with applying a normal, passive, Lorentz-transformation to convert the distance \( L \) into the satellite restframe.

The formulation of DSR in position space has been under debate. It has been argued that the space-time metric and also the Lorentz-transformations in position space should become energy-dependent \([14-18]\). Now if one would use a modified transformation also on the coordinates, a transformation depending on the energy of the photon, then \( \Delta T' \) might transform properly and both particles would meet also in the satellite frame. This would require that the transformation of the distance \( L \) was modified such that it converted the troublesome transformation behavior of \( \Delta T' \) back into a normal Lorentz-transformation.

This would imply that the distance between any two objects would depend on the energy of a photon that happened to propagate between them. The distance between the GRB and the detector was then energy-dependent such that it got shortened in the right amount to allow the slower photon to arrive in time together with the electron. That however would mean that the speed of the photon would not depend on its energy when expressed in our usual low-energetic and energy-independent coordinates. The confusion here stems from having defined a speed from the dispersion relation without that speed a priori having any meaning in position space. This possibility thus just reaffirms that observer-independence requires the speed of light to be constant.

That DSR implies a frame-dependent meaning of what is “near” was mentioned already in \([19]\). Serious conceptual problems arising from this were pointed out in \([18, 20]\), and here we have demonstrated a conflict with experiment to very high precision. If DSR was indeed the origin of time-delays of highly energetic photons from GRBs, then it would also lead to macroscopic effects we would long have observed. Consequently, DSR cannot be cause of observable effects in GRB spectra.

DSR is motivated by the idea that the Planck energy should be observer-independent which then leads to deformed Lorentz-transformations and an energy-dependent speed of light. We have here seen that such an energy-dependent speed of light that is also observer-independent implies violations of locality that are strongly constrained by experiment. It has however been argued in \([21]\) that the requirement of the Planck scale being observer-independent does not necessitate it to be an invariant of Lorentz boosts, since the result of such a boost does not itself constitute an observation. It is sufficient that experiments made are in agreement over that scale. In particular if the Planck length plays the role of a fundamentally minimal length no process should be able to resolve shorter distances. This does require a modification of interactions in quantum field theory at very high center-of-mass energies and small impact parameters, but it does not necessitate a modification of Lorentz boosts for free particles. In these models the speed of light remains constant.

An extended version of the here presented argument and further discussion can be found in \([22]\).

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