Integral-type non-local damage simulation of composites using mean-field homogenization methods

Peter Lenz1,* and Rolf Mahnken1
1 Chair of Engineering Mechanics (LTM), University of Paderborn, Warburger Str. 100, D-33098 Paderborn, Germany

This paper deals with non-local damage behaviour of composites subjected to eigenstrains. Examples for eigenstrains are thermal, chemical or plastic strains and any combination. Modern materials are often decomposed into several distinct materials, e.g. some reinforcements and a matrix material. A manufacturing process for these modern materials often induces eigenstrains, reversible and non reversible. The induced eigenstrains could lead to an initial damage. It is known that damage leads to mesh dependent simulation results. Therefore, non-local integral-type damage for multiphase composites subjected to eigenstrains is considered.

© 2021 The Authors. Proceedings in Applied Mathematics & Mechanics published by Wiley-VCH GmbH.

1 Integral-type non-local damage using mean-field homogenization methods

Complex hybrid structures are more and more common in daily live use. To form such complex hybrid structures, a complex manufacturing process is needed, e.g. the so called Resin-Transfer-Molding (RTM) process. Firstly a steel-part is inserted into a forming tool, secondly fibre layers are inserted, thirdly a matrix material is injected into the fibre layers after the forming tool is closed. During the RTM process the different materials are subjected to a thermal loading and the polymer based matrix material shrinks due to a chemical reaction. Both induces eigenstrains into the hybrid structure, where thermal strains are reversible. This is in contrast to curing strains, which are permanent. These eigenstrains can cause an initial damage in the hybrid structure, which could lead to a failure during use.

Consider a composite, decomposed into $n$ distinct material phases, where the macroscopic stress tensor is given by volume averaging

$$\sigma = \sum_{i=1}^{n} c_i \sigma_i = \sum_{i=1}^{n} c_i (1 - D_i [\hat{\epsilon}_i]) \mathbb{E}_i : (\hat{\epsilon}_i - \mu_i),$$

where $c_i$ is the volume fraction, $\mathbb{E}_i$ is the elasticity tensor, $D_i$ is the damage variable, $\mathbb{A}_i$ is the strain concentration tensor, $D_{iv}$ is the eigenstrain influence tensor [1], $\mu_i$ is the eigenstrain tensor of material phase $i$, respectively, and $\epsilon$ is the macroscopic strain. Note, that in this paper, strain based damage is considered, therefore, the damage variable $D_i [\hat{\epsilon}_i]$ depends on a equivalent strain $\hat{\epsilon}_i$, which in turns depends on the macroscopic strain. Differentiation of the macroscopic stress tensor in Eq. (1) w.r.t. the macroscopic strain $\epsilon$ tensor yields the macroscopic tangent modulus

$$C = \sum_{i=0}^{n} c_i (1 - D_i) \mathbb{E}_i : \mathbb{A}_i - c_i D'_i [\hat{\epsilon}_i] \mathbb{A}_i \mathbb{E}_i \mathbb{A}_i : \mathbb{E}_i : \mathbb{A}_i d\hat{\epsilon}_i,$$

where $D'_i [\hat{\epsilon}_i]$ is the derivative of the damage variable, $\hat{\sigma}_i = \mathbb{E}_i : (\hat{\epsilon}_i - \mu_i)$ is the effective stress, $\alpha_i [x, \xi]$ is the averaging function and depends only on the distance $r = ||x - \xi||$ between the source point $\xi$ and the receiver point $x$, e.g. [5], the derivative $\partial\hat{\sigma}_i/\partial\hat{\epsilon}_i$ depends on the used equivalent scalar function and is not given here. It is seen in Eq. (2) that for each material phase a local and a non-local part is given. The non-local part is divided into a receiver and a source part. The source part is integrated over a domain $V_i$ defined by an internal length, e.g. see [5] for more details.

2 Numerical examples

In the following two examples the Mori-Tanaka method [6] is used to determine the strain concentration tensor $\mathbb{A}_i$ in Eq. (1) and (2).

• Composite consisting of three different spherical inclusions

In the first example a composite consisting of three different spherical inclusions is simulated, illustrated in Fig. 1.a). In each material phase $\omega_i$ non-local damage is considered and material parameters are different for each material phase. Additionally, different local equivalent scalar strain functions are used. This example demonstrates the reduction of the mesh sensitivity. To reduce the complexity, no eigenstrains are considered in this example. As seen in Fig. 1 medium and fine mesh are almost identical and the difference of maximum load between coarse and fine mesh is about 5N. In general, the results are mostly mesh independent.

* Corresponding author: e-mail lenz@ltm.upb.de
**Fibre reinforced composite subjected to curing**

In the second example a fibre reinforced composite subjected to curing is simulated. The representative-volume-element (RVE) is illustrated in Fig. 1.f). The RVE is decomposed into three different materials: a fibre $\Omega_F$, an interface $\Omega_I$ and a matrix material $\Omega_M$. The following eigenstrains are considered in the individual material phase, respectively,

\[
\begin{align*}
1. & \quad \boldsymbol{\mu}_F = \alpha_F \theta \mathbf{1}, \\
2. & \quad \boldsymbol{\mu}_M = \alpha_M [z] \theta \mathbf{1} + \beta_M [z] z \mathbf{1}, \\
3. & \quad \boldsymbol{\mu}_I = \gamma_F \alpha_F \theta \mathbf{1} + (1 - \gamma_F) \alpha_M [z] \theta \mathbf{1} + \beta_M [z] z \mathbf{1},
\end{align*}
\]  

where $\alpha_i$ is the heat-dilatation coefficient, $\beta_i$ is the curing-dilatation coefficient, $\theta$ is the temperature, $z$ is the degree of cure and $\gamma_F = 0.6$ is the influence of the fibre. The eigenstrains in the matrix and in the interface material are dependent on the degree of cure $z$. The curing process of the matrix material is idealized as a system of multiple growing spheres [2, 3]. Evolution function of curing $\dot{z} [\theta]$ is given in [4] as a $n$-th order autocatalytic reaction and is integrated following a backward-euler scheme. The coarse mesh of the tensile specimen is used, illustrated in Fig. 1.c). The tensile specimen is thermal loaded for $0 < t < 2500$ s with the top- and the bottom-side clamped in all directions. The thermal loading is given by the brown curve in Fig. 1.g)-i), respectively. After $t > 2500$ s displacement is applied at the top-side, given by the red curve in Fig. 1.g)-i), respectively. The resulting degree of cure $z$ is shown as the black curve, damage in the interface gray, in the matrix yellow and in the fibre blue. Damage in matrix and interface occurs only for $z \geq 0.8$. It is seen in Fig. 1.g)-i), that the thermal loading has an influence on the damage distribution in all three phases.

![Fig. 1: Composite consisting of three different spherical inclusions: meso-RVE is shown in a), resulting reaction–force–displacement curve for three different meshes is given in b). The coarse mesh with 494 triangles and 562 dofs is shown in d), the medium mesh with 1976 triangles 2110 dofs in d) and the fine mesh with 7904 triangles and 8170 dofs in e). Fibre reinforced composite subjected to curing: meso-RVE is illustrated in f), results for three different thermal loadings: max temperature 180°C in g), max temperature 200°C in h), max temperature 220°C in i).](image-url)

**Acknowledgements**  Open access funding enabled and organized by Projekt DEAL.

**References**

[1] G. Dvorak, Micromechanics of Composite Materials (Springer Netherlands, 2013).
[2] R. Mahnken, P. Lenz, C. Dammann, Arch. Appl. Mech. **11**, (2018).
[3] R. Mahnken, C. Dammann, P. Lenz, Int. J. Mult. Comp. Eng. **15**, 295-322, (2017)
[4] P. I. Karkanas, I. K. Partridge and D. Attwood, Polymer International **41**, 183-191, (1996).
[5] M. Jirásek and B. Patzák, Comp. and Struct. **80**, 1279-1293, (2002).
[6] T. Mori and K. Tanaka, Acta metallurgica, **21**(5), 571–574, (1973).