CPT symmetry and properties of the exact and approximate effective Hamiltonians for the neutral kaon complex

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October 30, 2018

Abstract

We start from a discussion of the general form and general CP– and CPT– transformation properties of the Lee–Oehme–Yang (LOY) effective Hamiltonian for the neutral kaon complex. Next we show that there exists an approximation which is more accurate than the LOY, and which leads to an effective Hamiltonian whose diagonal matrix elements posses CPT transformation properties, which differ from those of the LOY effective Hamiltonian. These properties of the mentioned effective Hamiltonians are compared with the properties of the exact effective Hamiltonian for the neutral kaon complex. We show that the diagonal matrix elements of the exact effective Hamiltonian governing the time evolution in the subspace of states of an unstable particle and its antiparticle need not be equal at for \( t > t_0 \) (\( t_0 \) is the instant of creation of the pair) when the total system under consideration is CPT invariant but CP noninvariant. The unusual consequence of this result is that, contrary to the properties of stable particles, the masses of the unstable particle “1” and its antiparticle “2” need not be equal for \( t \gg t_0 \) in the case of preserved CPT and violated CP symmetries.

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*Talk given at the conference on "Symmetry in Nonlinear Mathematical Physics", June 23 — 29, 2003, Institute of Mathematics, National Academy of Sciences of Ukraine, Kiev, Ukraine.
1 Introduction.

The problem of testing CPT–invariance experimentally has attracted the attention of physicist, practically since the discovery of antiparticles. CPT symmetry is a fundamental theorem of axiomatic quantum field theory which follows from locality, Lorentz invariance, and unitarity [1]. Many tests of CPT–invariance consist in searching for decay process of neutral kaons. All known CP– and hypothetically possible CPT–violation effects in neutral kaon complex are described by solving the Schrödinger–like evolution equation [2] — [9] (we use $\hbar = c = 1$ units)

$$i \frac{\partial}{\partial t} |\psi; t >_{\|} = H_{\|} |\psi; t >_{\|}$$

(1)

for $|\psi; t >_{\|}$ belonging to the subspace $H_{\|} \subset H$ (where $H$ is the state space of the physical system under investigation), e.g., spanned by orthonormal neutral kaons states $|K_0 >$, $|\overline{K}_0 >$, and so on, (then states corresponding with the decay products belong to $H \oplus H_{\|}^{\text{def}} = H_{\perp}$), and nonhermitean effective Hamiltonian $H_{\|}$ obtained usually by means of Lee-Oehme–Yang (LOY) approach (within the Weisskopf–Wigner approximation (WW)) [2] — [5], [9]:

$$H_{\|} = M - \frac{i}{2} \Gamma,$$

(2)

where

$$M = M^+, \quad \Gamma = \Gamma^+, \quad (3)$$

are $(2 \times 2)$ matrices.

Solutions of Eq. (1) can be written in matrix form and such a matrix defines the evolution operator (which is usually nonunitary) $U_{\|}(t)$ acting in $H_{\|}$:

$$|\psi; t >_{\|} = U_{\|}(t)|\psi; t_0 = 0 >_{\|} \equiv U_{\|}(t)|\psi >_{\|},$$

(4)

where,

$$|\psi >_{\|} \equiv q_1 |1 > + q_2 |2 >,$$

(5)

and $|1 >$ stands for the vectors of the $|K_0 >$, $|\overline{B}_0 >$ type and $|2 >$ denotes antiparticles of the particle ”1”: $|\overline{K}_0 >$, $|\overline{B}_0 >$, $< j|k > = \delta_{jk}$, $j, k = 1, 2$.

In many papers it is assumed that the real parts, $\Re(.), \Re(h_{jj}) \equiv M_{jj}$, $(j = 1, 2)$,

(6)
where
\[ h_{jk} = \langle j | H_{\parallel} | k \rangle, \quad (j, k = 1, 2), \quad (7) \]
correspond to the masses of particle "1" and its antiparticle "2" respectively [2] — [9], (and such an interpretation of \( \Re(h_{11}) \) and \( \Re(h_{22}) \) will be used in this paper), whereas the imaginary parts, \( \Im(.) \),
\[ -2 \Im(h_{jj}) = \Gamma_{jj}, \quad (j = 1, 2), \quad (8) \]
are interpreted as the decay widths of these particles [2] — [9]. Such an interpretation seems to be consistent with the recent and the early experimental data for neutral kaon and similar complexes [10].

Relations between matrix elements of \( H_{\parallel} \) implied by CPT–transformation properties of the Hamiltonian \( H \) of the total system, containing neutral kaon complex as a subsystem, are crucial for designing CPT–invariance and CP–violation tests and for proper interpretation of their results. The aim of this paper is to examine the properties of the approximate and exact \( H_{\parallel} \) generated by the CPT–symmetry of the total system under consideration and independent of the approximation used.

2 \( H_{LOY} \) and CPT–symmetry.

Now, let us consider briefly some properties of the LOY model. Let \( H \) be total (selfadjoint) Hamiltonian, acting in \( \mathcal{H} \) — then the total unitary evolution operator \( U(t) \) fulfills the Schrödinger equation
\[ i \frac{\partial}{\partial t} U(t) |\phi> = H U(t) |\phi>, \quad U(0) = I, \quad (9) \]
where \( I \) is the unit operator in \( \mathcal{H} \), \( |\phi> \equiv |\phi; t_0 = 0 \rangle \in \mathcal{H} \) is the initial state of the system:
\[ |\phi> \equiv |\psi>_{\parallel} \quad (10) \]
In our case \( U(t) |\phi> = |\phi; t \rangle \). Let \( P \) denote the projection operator onto the subspace \( \mathcal{H}_{\parallel} \):
\[ P \mathcal{H} = \mathcal{H}_{\parallel}, \quad P = P^2 = P^+, \quad (11) \]
then the subspace of decay products \( \mathcal{H}_{\perp} \) equals
\[ \mathcal{H}_{\perp} = (I - P) \mathcal{H} \overset{\text{def}}{=} Q \mathcal{H}, \quad Q \overset{\text{def}}{=} I - P. \quad (12) \]
For the case of neutral kaons or neutral $B$–mesons, etc., the projector $P$ can be chosen as follows:

$$P \equiv |1><1| + |2><2|. \quad (13)$$

We assume that time independent basis vectors $|K_0>$ and $|\bar{K}_0>$ are defined analogously to corresponding vectors used in LOY theory of time evolution in neutral kaon complex [2]. In the LOY approach it is assumed that vectors $|1>, |2>$ considered above are eigenstates of $H^{(0)}$ for 2-fold degenerate eigenvalue $m_0$:

$$H^{(0)}|j> = m_0|j>, \quad j = 1, 2, \quad (14)$$

where $H^{(0)}$ is a so called free Hamiltonian, $H^{(0)} \equiv H_{\text{strong}} = H - H_W$, and $H_W$ denotes weak and other interactions which are responsible for transitions between eigenvectors of $H^{(0)}$, i.e., for the decay process.

This means that

$$[P, H^{(0)}] = 0. \quad (15)$$

The condition guaranteeing the occurrence of transitions between subspaces $\mathcal{H}_\parallel$ and $\mathcal{H}_\perp$, i.e., a decay process of states in $\mathcal{H}_\parallel$, can be written as follows

$$[P, H_W] \neq 0, \quad (16)$$

that is

$$[P, H] \neq 0. \quad (17)$$

Usually, in LOY and related approaches, it is assumed that

$$\Theta H^{(0)} \Theta^{-1} = H^{(0)+} \equiv H^{(0)}, \quad (18)$$

where $\Theta$ is the antiunitary operator:

$$\Theta \overset{\text{def}}{=} CPT. \quad (19)$$

The subspace of neutral kaons $\mathcal{H}_\parallel$ is assumed to be invariant under $\Theta$:

$$\Theta P \Theta^{-1} = P^+ \equiv P. \quad (20)$$

In the kaon rest frame, the time evolution is governed by the Schrödinger equation [9], where the initial state of the system has the form [10], [3]. Within assumptions [11] — [16] the Weisskopf–Wigner approach, which is
the source of the LOY method, leads to the following formula for $H_{LOY}$ (e.g., see [2] [3] [4] [9]):

$$H_{LOY} = m_0 P - \Sigma(m_0) \equiv PHP - \Sigma(m_0),$$  \hspace{1cm} (21)

$$= M_{LOY} - \frac{i}{2} \Gamma_{LOY}$$  \hspace{1cm} (22)

where it has been assumed that $<1|H_W|2> = <1|H_W|2>^* = 0$ (see [2] — [9]),

$$\Sigma(\epsilon) = PHP\frac{1}{QHQ - \epsilon - i0 QHP}.$$  \hspace{1cm} (23)

The matrix elements $h_{jk}^{LOY}$ of $H_{LOY}$ are

$$h_{jk}^{LOY} = H_{jk} - \Sigma_{jk}(m_0), \hspace{1cm} (j,k = 1,2),$$  \hspace{1cm} (24)

$$= M_{jk}^{LOY} - \frac{i}{2} \Gamma_{jk}^{LOY}$$  \hspace{1cm} (25)

where, in this case,

$$H_{jk} = <j|H|k> \equiv <j|(H^{(0)} + H_W)|k> \equiv m_0 \delta_{jk} + <j|H_W|k>,$$  \hspace{1cm} (26)

and $\Sigma_{jk}(\epsilon) = <j|\Sigma(\epsilon)|k>$. 

Now, if $\Theta H_W \Theta^{-1} = H_W^+ \equiv H_W$, that is if

$$[\Theta, H] = 0,$$  \hspace{1cm} (27)

then using, e.g., the following phase convention [3] — [9]

$$\Theta|1>^{\text{def}} = -|2>, \hspace{1cm} \Theta|2>^{\text{def}} = -|1>,$$  \hspace{1cm} (28)

and taking into account that $<\psi|\varphi> = <\Theta \varphi|\Theta \psi>$, one easily finds from (21) — (26) that

$$h_{11}^{LOY\Theta} - h_{22}^{LOY\Theta} = 0,$$  \hspace{1cm} (29)

and thus

$$M_{11}^{LOY} = M_{22}^{LOY},$$  \hspace{1cm} (30)

(where $h_{jk}^{LOY\Theta}$ denotes the matrix elements of $H_{LOY}^{\Theta}$ — of the LOY effective Hamiltonian when the relation (27) holds), in the CPT–invariant system. This is the standard result of the LOY approach and this is the picture which one meets in the literature [2] — [8].
If to assume that CPT–symmetry is not conserved in the physical system under consideration, i.e., that

$$[\Theta, H] \neq 0,$$

then $$h_{11}^{LOY} \neq h_{22}^{LOY}$$.

It is convenient to express difference between $$H_{LOY}^\Theta$$ and the effective Hamiltonian $$H_{LOY}$$ appearing within the LOY approach in the case of non-conserved CPT–symmetry as follows

$$H_{LOY} \equiv H_{LOY}^\Theta + \delta H_{LOY}$$ \quad (32)

$$= \left( \begin{array}{cc} (M_0 + \frac{1}{2}\delta M) - \frac{i}{2}(\Gamma_0 + \frac{1}{2}\delta \Gamma), & M_{12} - \frac{1}{2}\Gamma_{12} \\ M_{12}^* - \frac{1}{2}\Gamma_{12}^* & (M_0 - \frac{1}{2}\delta M) - \frac{i}{2}(\Gamma_0 - \frac{1}{2}\delta \Gamma) \end{array} \right).$$

In other words

$$h_{jk}^{LOY} = h_{jk}^{LOY^\Theta} + \Delta h_{jk}^{LOY},$$ \quad (33)

where

$$\Delta h_{jk}^{LOY} = (-1)^{j+1}\frac{1}{2}(\delta M - \frac{i}{2}\delta \Gamma)\delta_{jk},$$ \quad (34)

and $$j, k = 1, 2$$. Within this approach the $$\delta M$$ and $$\delta \Gamma$$ terms violate CPT–symmetry.

### 3 Beyond the LOY approximation

The more accurate approximate formulae for $$H_{\parallel}(t)$$ have been derived in [16, 17] using the Krolikowski–Rzewuski equation for the projection of a state vector [18], which results from the Schrödinger equation [19] for the total system under consideration, and, in the case of initial conditions of the type [20], takes the following form

$$(i\frac{\partial}{\partial t} - PHP)U_{\parallel}(t) = -i \int_0^\infty K(t - \tau)U_{\parallel}(\tau)d\tau,$$ \quad (35)

where $$U_{\parallel}(0) = P$$,

$$K(t) = \Theta(t)PHQ\exp(-itQHQ)QHP,$$ \quad (36)

and $$\Theta(t) = \{1 \text{ for } t \geq 0, \quad 0 \text{ for } t < 0\}.$$
The integro–differential equation (35) can be replaced by the following differential one (see [11] — [18])

\[(i \frac{\partial}{\partial t} - PHP - V_\parallel(t))U_\parallel(t) = 0,\]  

(37)

where

\[PHP + V_\parallel(t) \overset{\text{def}}{=} H_\parallel(t).\]  

(38)

Taking into account (35) and (37) or (1) one finds from (4) and (35) 

\[V_\parallel(t)U_\parallel(t) = -i \int_0^\infty K(t - \tau)U_\parallel(\tau)d\tau \overset{\text{def}}{=} -iK * U_\parallel(t).\]  

(39)

(Here the asterix, *, denotes the convolution: \(f * g(t) = \int_0^\infty f(t - \tau)g(\tau) d\tau\).)

Next, using this relation and a retarded Green’s operator \(G(t)\) for the equation (35)

\[G(t) = -i\Theta(t) \exp(-itPHP)P,\]  

(40)

one obtains [16, 17]

\[U_\parallel(t) = \left[1 + \sum_{n=1}^{\infty} (-i)^n L * \ldots * L\right] * U_\parallel^{(0)}(t),\]  

(41)

and thus from (39)

\[V_\parallel(t) U_\parallel(t) = -iK * \left[1 + \sum_{n=1}^{\infty} (-i)^n L * \ldots * L\right] * U_\parallel^{(0)}(t),\]  

(42)

where \(L\) is convoluted \(n\) times, \(1 \equiv 1(t) \equiv \delta(t),\)

\[L(t) = G * K(t),\]  

(43)

\[U_\parallel^{(0)} = \exp(-itPHP) P\]  

(44)

is a "free" solution of Eq. (35). Of course, the series (42) is convergent if \(\parallel L(t) \parallel < 1\). If for every \(t \geq 0\)

\[\parallel L(t) \parallel \ll 1,\]  

(45)

then, to the lowest order of \(L(t)\), one finds from [12] [16] [17]

\[V_\parallel(t) \equiv V_\parallel^{(1)}(t) \overset{\text{def}}{=} -i \int_0^\infty K(t - \tau) \exp [i(t - \tau)PHP]d\tau.\]  

(46)
Thus \[ H_{\parallel}(0) \equiv PHP, \quad V_{\parallel}(0) = 0, \quad V_{\parallel}(t \to 0) \simeq -itPHQHP. \] (47)

In the case of \([13]\) of the projector \(P\) and such \(H\) that

\[ PHP \equiv m_0 P, \] (48)

that is for

\[ H_{12} = H_{21} = 0, \] (49)

the operator \(Pe^{itPHP}\) takes the following form,

\[ Pe^{itPHP} = Pe^{itm_0}, \] (50)

and thus the approximate formula \([46]\) for \(V_{\parallel}(t)\) leads to

\[ V_{\parallel}^{(1)}(t) = -PHQe^{-it(QHQ - m_0)} - \frac{1}{QHQ - m_0}QHP, \] (51)

which leads to \(V_{\parallel} \overset{\text{def}}{=} \lim_{t \to \infty} V_{\parallel}^{(1)}(t),\)

\[ V_{\parallel} = -\Sigma(m_0). \] (52)

This means that in the case \([48]\)

\[ H_{\parallel} = m_0 P - \Sigma(m_0), \] (53)

and \(H_{\parallel} = H_{LOY}.\)

On the other hand, in the case

\[ H_{12} = H_{21}^* \neq 0, \] (54)

the form of \(Pe^{itPHP}\) is more complicated. For example in the case of conserved CPT, formula \([13]\) leads to the following form for \(V_{\parallel} \overset{\text{def}}{=} \lim_{t \to \infty} V_{\parallel}^{(1)}(t)\) \([9, 19]\)

\[ V^{\Theta}_{\parallel} = -\frac{1}{2} \Sigma(H_0 + |H_{12}|) \left[ (1 - \frac{H_0}{|H_{12}|})P + \frac{1}{|H_{12}|}PHP \right] \]
\[ -\frac{1}{2} \Sigma(H_0 - |H_{12}|) \left[ (1 + \frac{H_0}{|H_{12}|})P - \frac{1}{|H_{12}|}PHP \right], \] (55)
where
\[ H_0 \overset{\text{def}}{=} \frac{1}{2}(H_{11} + H_{22}), \]  
and \( V_0^\Theta \) denotes \( V_\parallel \) when \( \Theta \) occurs.

In the general case \((54)\), when there are not any assumptions about symmetries of the type CP–, T–, or CPT–symmetry for the total Hamiltonian \( H \) of the system considered, the form of \( V_\parallel \) \((t \to \infty) \cong V_\parallel^{(1)}(\infty) \) is a yet more complicated. In such a case one finds the following expressions for the matrix elements \( v_{jk}(t \to \infty) \overset{\text{def}}{=} v_{jk} \) of \( V_\parallel \) \([16, 17]\),

\[ v_{j1} = - \frac{1}{2} \left( 1 + \frac{H_z}{\kappa} \right) \Sigma_{j1}(H_0 + \kappa) - \frac{1}{2} \left( 1 - \frac{H_z}{\kappa} \right) \Sigma_{j1}(H_0 - \kappa) - \frac{H_{21}}{2\kappa} \Sigma_{j2}(H_0 + \kappa) + \frac{H_{21}}{2\kappa} \Sigma_{j2}(H_0 - \kappa), \]

\[ v_{j2} = - \frac{1}{2} \left( 1 - \frac{H_z}{\kappa} \right) \Sigma_{j2}(H_0 + \kappa) - \frac{1}{2} \left( 1 + \frac{H_z}{\kappa} \right) \Sigma_{j2}(H_0 - \kappa) - \frac{H_{12}}{2\kappa} \Sigma_{j1}(H_0 + \kappa) + \frac{H_{12}}{2\kappa} \Sigma_{j1}(H_0 - \kappa), \]

where \( j, k = 1, 2, \)

\[ H_z = \frac{1}{2}(H_{11} - H_{22}), \]

and

\[ \kappa = (|H_{12}|^2 + H_z^2)^{1/2}. \]

Hence, by \([58]\)

\[ h_{jk} = H_{jk} + v_{jk}. \]

It should be emphasized that all components of the expressions \([57]\) have the same order with respect to \( \Sigma(\varepsilon) \).

In the case of preserved CPT–symmetry \([27]\), one finds \( H_{11} = H_{22} \) which implies that \( \kappa \equiv |H_{12}|, H_z \equiv 0 \) and \( H_0 \equiv H_{11} \equiv H_{22} \), and \([16, 17]\)

\[ \Sigma_{11}(\varepsilon = \varepsilon^*) \equiv \Sigma_{22}(\varepsilon = \varepsilon^*) \overset{\text{def}}{=} \Sigma_0(\varepsilon = \varepsilon^*). \]

Therefore matrix elements \( v_{jk}^\Theta \) of operator \( V_\parallel^\Theta \) take the following form

\[ v_{j1}^\Theta = - \frac{1}{2} \left( \Sigma_{j1}(H_0 + |H_{12}|) + \Sigma_{j1}(H_0 - |H_{12}|) \right). \]
\[ v_{j_2}^\theta = - \frac{1}{2} \left\{ \Sigma_{j_2}(H_0 + |H_{12}|) + \Sigma_{j_2}(H_0 - |H_{12}|) \right\} + \frac{H_{12}}{|H_{12}|} \Sigma_{j_1}(H_0 + |H_{12}|) - \frac{H_{12}}{|H_{12}|} \Sigma_{j_1}(H_0 - |H_{12}|) \right\}, \]  
(62)

Assuming \[ |H_{12}| \ll |H_0|, \]  
(63)
we find

\[ v_{j_1}^\theta \approx - \Sigma_{j_1}(H_0) - H_{21} \frac{\partial \Sigma_{j_2}(x)}{\partial x} \bigg|_{x=H_0}, \]  
(64)

\[ v_{j_2}^\theta \approx - \Sigma_{j_2}(H_0) - H_{12} \frac{\partial \Sigma_{j_1}(x)}{\partial x} \bigg|_{x=H_0}, \]  
(65)

where \( j = 1, 2 \). One should stress that due to a presence of resonance terms, derivatives \( \frac{\partial}{\partial x} \Sigma_{jk}(x) \) need not be small and neither need the products \( H_{jk} \frac{\partial}{\partial x} \Sigma_{jk}(x) \) in (62), (63).

Finally, assuming that (63) holds and using relations (64), (65), (7) and the expression (24), we obtain for the CPT–invariant system \[ 20, 21 \]

\[ h_{j_1}^\theta \approx h_{j_1}^{\text{LOY}} - H_{21} \frac{\partial \Sigma_{j_2}(x)}{\partial x} \bigg|_{x=H_0} \overset{\text{def}}{=} h_{j_1}^{\text{LOY}} + \delta h_{j_1}, \]  
(66)

\[ h_{j_2}^\theta \approx h_{j_2}^{\text{LOY}} - H_{12} \frac{\partial \Sigma_{j_1}(x)}{\partial x} \bigg|_{x=H_0} \overset{\text{def}}{=} h_{j_2}^{\text{LOY}} + \delta h_{j_2}, \]  
(67)

where \( j = 1, 2 \). From these formulae we conclude that, e.g., the difference between diagonal matrix elements of \( H^\theta_\parallel \), which plays an important role in designing CPT–invariance tests for the neutral kaons system, equals

\[ \Delta h \overset{\text{def}}{=} h_{11} - h_{22} \approx H_{12} \frac{\partial \Sigma_{21}(x)}{\partial x} \bigg|_{x=H_0} - H_{21} \frac{\partial \Sigma_{12}(x)}{\partial x} \bigg|_{x=H_0} \neq 0, \]  
(68)

which differs from the L0Y results (29), (30).
4 CPT and the exact effective Hamiltonian

The aim of this Section is to show, that contrary to the LOY conclusion (29), diagonal matrix elements of the exact effective Hamiltonian $H_{\parallel}$ can not be equal when the total system under consideration is CPT invariant but CP noninvariant. This will be done by means of the method used in [22].

Universal properties of the (unstable) particle–antiparticle subsystem of the system described by the Hamiltonian $H$, for which the relation (27) holds, can be extracted from the matrix elements of the exact $U_{\parallel}(t)$ appearing in (4). Such $U_{\parallel}(t)$ has the following form

$$U_{\parallel}(t) = PU(t)P,$$  \hfill (69)

where $P$ is defined by the relation (13), and $U(t)$ is the total unitary evolution operator $U(t)$, which solves the Schrödinger equation (9). Operator $U_{\parallel}(t)$ acts in the subspace of unstable states $\mathcal{H}_{\parallel} \equiv P\mathcal{H}$. Of course, $U_{\parallel}(t)$ has nontrivial form only if (17) holds, and only then transitions of states from $\mathcal{H}_{\parallel}$ into $\mathcal{H}_{\perp}$ and vice versa, i.e., decay and regeneration processes, are allowed.

Using the matrix representation one finds

$$U_{\parallel}(t) \equiv \begin{pmatrix} A(t) & 0 \\ 0 & 0 \end{pmatrix}$$ \hfill (70)

where $0$ denotes the suitable zero submatrices and a submatrix $A(t)$ is the $2 \times 2$ matrix acting in $\mathcal{H}_{\parallel}$

$$A(t) = \begin{pmatrix} A_{11}(t) & A_{12}(t) \\ A_{21}(t) & A_{22}(t) \end{pmatrix}$$ \hfill (71)

and $A_{jk}(t) = \langle j|U_{\parallel}(t)|k \rangle \equiv \langle j|U(t)|k \rangle$, $(j,k = 1,2)$.

Now assuming (27) and using the phase convention (28), [2] — [5], one easily finds that [7], [23] — [26]

$$A_{11}(t) = A_{22}(t).$$ \hfill (72)

Note that assumptions (27) and (28) give no relations between $A_{12}(t)$ and $A_{21}(t)$.

The important relation between amplitudes $A_{12}(t)$ and $A_{21}(t)$ follows from the famous Khalfin’s Theorem [7], [21] — [26]. This Theorem states that in...
the case of unstable states, if amplitudes $A_{12}(t)$ and $A_{21}(t)$ have the same time dependence
\[ r(t) \equiv \frac{A_{12}(t)}{A_{21}(t)} = \text{const} \equiv r, \tag{73} \]
then it must be $|r| = 1$.

For unstable particles the relation (72) means that decay laws
\[ p_j(t) \equiv |A_{jj}(t)|^2, \tag{74} \]
(where $j = 1, 2$), of the particle $|1\rangle$ and its antiparticle $|2\rangle$ are equal,
\[ p_1(t) \equiv p_2(t). \tag{75} \]
The consequence of this last property is that the decay rates of the particle $|1\rangle$ and its antiparticle $|2\rangle$ must be equal too.

From (72) it does not follow that the masses of the particle "1" and the antiparticle "2" should be equal.

More conclusions about the properties of the matrix elements of $H_\parallel$ one can infer analyzing the following identity [27], [11] — [15]
\[ H_\parallel \equiv H_\parallel(t) = i \frac{\partial U_\parallel(t)}{\partial t} [U_\parallel(t)]^{-1}, \tag{76} \]
where $[U_\parallel(t)]^{-1}$ is defined as follows
\[ U_\parallel(t) [U_\parallel(t)]^{-1} = [U_\parallel(t)]^{-1} U_\parallel(t) = P. \tag{77} \]
(Note that the identity (76) holds, independent of whether $[P, H] \neq 0$ or $[P, H] = 0$). The expression (76) can be rewritten using the matrix $A(t)$
\[ H_\parallel(t) \equiv i \frac{\partial A(t)}{\partial t} [A(t)]^{-1}. \tag{78} \]
Relations (76), (78) must be fulfilled by the exact as well as by every approximate effective Hamiltonian governing the time evolution in every two dimensional subspace $H_\parallel$ of states $H$ [27], [11] — [15].

It is easy to find from (76) the general formulae for the diagonal matrix elements, $h_{jj}$, of $H_\parallel(t)$, in which we are interested. We have
\[ h_{11}(t) = \frac{i}{\det A(t)} \left( \frac{\partial A_{11}(t)}{\partial t} A_{22}(t) - \frac{\partial A_{12}(t)}{\partial t} A_{21}(t) \right), \tag{79} \]
\[ h_{22}(t) = \frac{i}{\det A(t)} \left( - \frac{\partial A_{21}(t)}{\partial t} A_{12}(t) + \frac{\partial A_{22}(t)}{\partial t} A_{11}(t) \right). \tag{80} \]
Now, assuming (27) and using the consequence (72) of this assumption, one finds

\[ h_{11}(t) - h_{22}(t) = \frac{i}{\det \mathbf{A}(t)} \left( \frac{\partial A_{21}(t)}{\partial t} A_{12}(t) - \frac{\partial A_{12}(t)}{\partial t} A_{21}(t) \right). \]  

(81)

Next, after some algebra one obtains

\[ h_{11}(t) - h_{22}(t) = -i \frac{A_{12}(t)}{\det \mathbf{A}(t)} \frac{\partial}{\partial t} \ln \left( \frac{A_{12}(t)}{A_{21}(t)} \right). \]  

(82)

This result means that in the considered case for \( t > 0 \) the following Theorem holds:

\[ h_{11}(t) - h_{22}(t) = 0 \iff \frac{A_{12}(t)}{A_{21}(t)} = \text{const., (} t > 0). \]  

(83)

Thus for \( t > 0 \) the problem under studies is reduced to the Khalfin’s Theorem (see the relation (73)).

From (79) and (80) it is easy to see that at \( t = 0 \)

\[ h_{jj}(0) = <j|H|j>, \quad (j = 1, 2), \]  

(84)

which means that in a CPT invariant system (27) in the case of pairs of unstable particles, for which transformations of type (28) hold

\[ M_{11}(0) = M_{22}(0) \equiv <1|H|1>, \]  

(85)

the unstable particles "1" and "2" are created at \( t = t_0 \equiv 0 \) as particles with equal masses.

Now let us go on to analyze the conclusions following from the Khalfin’s Theorem. CP noninvariance requires that \( |r| \neq 1 \) (see also [7, 23, 24, 26]). This means that in such a case it must be \( r = r(t) \neq \text{const} \). So, if in the system considered the property (27) holds but

\[ [\mathcal{CP}, H] \neq 0, \]  

(86)

and the unstable states "1" and "2" are connected by a relation of type (28), then at \( t > 0 \) it must be \( (h_{11}(t) - h_{22}(t)) \neq 0 \) in this system. This means that for \( t \gg 0 \) it can be \( \Re (h_{11}(t) - h_{22}(t)) \neq 0 \) and \( \Im (h_{11}(t) - h_{22}(t)) = 0 \) or \( \Re (h_{11}(t) - h_{22}(t)) = 0 \) and \( \Im (h_{11}(t) - h_{22}(t)) \neq 0 \) or \( \Re (h_{11}(t) - h_{22}(t)) \neq 0 \) and \( \Im (h_{11}(t) - h_{22}(t)) \neq 0 \) both.
Let us focus our attention on \( \Re(h_{11}(t) - h_{22}(t)) \). Following the method used in [14] and using assumption (20) and the identity (76), after some algebra, one finds [28].

\[
[\Theta, H_{\parallel}(t)] = \mathcal{A}(t) + \mathcal{B}(t),
\]

where:

\[
\mathcal{A}(t) = P[\Theta, H]U(t)P(U_{\parallel}(t))^{-1},
\]

\[
\mathcal{B}(t) = \left\{ P\mathcal{H} - H_{\parallel}(t)P \right\} [\Theta, U(t)]P(U_{\parallel}(t))^{-1}
\]

(87)

We observe that \( \mathcal{A}(0) \equiv P[\Theta, H]P \) and \( \mathcal{B}(0) \equiv 0 \). From definitions and general properties of operators \( C, P \) and \( T \), it is known that \( T U(t\neq 0) = U_{\parallel}(t\neq 0) T \neq U(t\neq 0) T \) (Wigner’s definition for \( T \) is used), and thereby \( \Theta U(t \neq 0) = U_{CPT}(t \neq 0) \Theta \) i.e. \( [\Theta, U(t \neq 0)] \neq 0 \). So, the component \( \mathcal{B}(t) \) in (87) is nonzero for \( t \neq 0 \) and it is obvious that there is a chance for \( \Theta \)-operator to commute with the effective Hamiltonian \( H_{\parallel}(t) \) only if \( [\Theta, H] \neq 0 \). On the other hand, the property \( [\Theta, H] \neq 0 \) does not imply that \( [\Theta, H_{\parallel}(0)] = 0 \) or \( [\Theta, H_{\parallel}(0)] \neq 0 \). These two possibilities are admissible, but if \( [\Theta, H] = 0 \) then there is only one possibility: \( [\Theta, H_{\parallel}(0)] = 0 \) [14].

From (87) we find

\[
\Theta H_{\parallel}(t)\Theta^{-1} - H_{\parallel}(t) \equiv (\mathcal{A}(t) + \mathcal{B}(t))\Theta^{-1}.
\]

(90)

Now, keeping in mind that \( \mid 2 > \equiv \mid K_0 > \) is the antiparticle for \( \mid 1 > \equiv \mid K_0 > \) and that, by definition, the (anti–unitary) \( \Theta \)-operator transforms \( \mid 1 > \) in \( \mid 2 > \) according to formulae (28), we obtain from (90)

\[
h_{11}(t)^* - h_{22}(t) = \langle 2 | (\mathcal{A}(t) + \mathcal{B}(t))\Theta^{-1}|2 > ,
\]

(91)

Adding expression (91) to its complex conjugate one gets

\[
\Re(h_{11}(t) - h_{22}(t)) = \Re < 2 | (\mathcal{A}(t) + \mathcal{B}(t))\Theta^{-1}|2 > .
\]

(92)

Note that if to replace the the requirement (17) for the projector \( P \) (13) by the following one:

\[
[P, H] = 0,
\]

i.e., if to consider only stationary states instead of unstable states, then one immediately obtains from (88), (89):

\[
\mathcal{A}(t) = P[\Theta, H]P,
\]

(94)

\[
\mathcal{B}(t) = 0.
\]

(95)
Let us assume that the property (27) holds. For the stationary states (28), this assumption, relations (29), (30) and (31) yield $\Re (h_{11}(t) - h_{22}(t)) = 0$.

Now let us consider the case of unstable states, i.e., states $|1\rangle, |2\rangle$, which lead to such projection operator $P$ (33) that condition (34) holds. If in this case (27) also holds then $A(t) \equiv 0$ and thus $[\Theta, H_\parallel(0)] = 0$, which is in agreement with (35) and with an earlier, similar result (14). In this case we have $\Theta U(t) = U^+(t)\Theta$, which gives $\Theta U_\parallel(t) = U^+_\parallel(t)\Theta, \Theta U_\parallel^{-1}(t) = (U^+_\parallel(t))^{-1}\Theta$, and

$$[\Theta, U(t)] = -2i(\Im U(t))\Theta$$

This relation leads to the following result in the case of the conserved CPT–symmetry

$$B(t) = -2iP\{H - H_\parallel(t), P\}(\Im U(t))P(U^+_\parallel(t))^{-1}\Theta$$

Formula (37) allow us to conclude that $<2|B(0)\Theta^{-1}|2> = 0$ and $\Re (<2|B(t > 0)\Theta^{-1}|2>) \neq 0$, if condition (27) holds. This means that in this case it must be $\Re (h_{11}(t)) \neq \Re (h_{22}(t))$ for $t > 0$. So, there is no possibility for $\Re (h_{11})$ to equal $\Re (h_{22})$ for $t > 0$ in the considered case of $P$ fulfilling the condition (17) (i.e., for unstable states) when CPT–symmetry is conserved: It must be $\Re (h_{11}) \neq \Re (h_{22})$.

Assuming the LOY interpretation of $\Re (h_{jj}(t))$, ($j = 1, 2$), one can conclude from the Khalfin’s Theorem and from the properties (38), (39), (40) that if $A_{12}(t), A_{21}(t) \neq 0$ for $t > 0$ and if the total system considered is CPT–invariant, but CP–noninvariant, then $M_{11}(t) \neq M_{22}(t)$ for $t > 0$, that is, that contrary to the case of stable particles (the bound states), the masses of the simultaneously created unstable particle ”1” and its antiparticle ”2”, which are connected by the relation (38), need not be equal for $t > t_0 = 0$. Of course, such a conclusion contradicts the standard LOY result (29), (30). However, one should remember that the LOY description of neutral $K$ mesons and similar complexes is only an approximate one, and that the LOY approximation is not perfect. On the other hand the relation (38) and the Khalfin’s Theorem follow from the basic principles of the quantum theory and are rigorous. Consequently, their implications should also be considered as rigorous.

5 Final remarks

Note that properties of the more accurate approximation described in Sec. 3 are consistent with the general properties and conclusions obtained in Sec.
4 for the exact effective Hamiltonian — compare (83) and (84) and relations
(83) with (68).

From the result (68) it follows that in the case of the approximate $H_{||}$,
$\Delta h = 0$ can be achieved only if $H_{12} = H_{21} = 0$. This means that if the
first order $|\Delta S| = 2$ interactions are forbidden in the $K_0, \bar{K}_0$ complex then
predictions following from the use of the mentioned more accurate approxi-

mation and from the LOY theory should lead to the the same masses for $K_0$
and for $\bar{K}_0$. This does not contradict the results of Sec. 3 derived for the
exact $H_{||}$; the mass difference is very, very small and should arise at higher
orders of the more accurate approximation.

On the other hand from (68) it follows that $\Delta h \neq 0$ if and only if $H_{12} \neq 0$.
This means that if measurable deviations from the LOY predictions concern-
ing the masses of, e.g. $K_0, \bar{K}_0$ mesons are ever detected, then the most plau-
sible interpretation of this result will be the existence of first order $|\Delta S| = 2$
interactions in the system considered.

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