Primordial and late-time inflation in Brans–Dicke cosmology

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Abstract. The basic motivation of this work is to attempt to explain the rapid primordial inflation and the observed slow late-time inflation by using the Brans–Dicke theory of gravity. We show that the ratio of these two inflation parameters is proportional to the square root of the Brans–Dicke parameter $\omega$ ($\omega \gg 1$). We also calculate the Hubble parameter $H$ and the time variation of the time dependent Newtonian gravitational constant $G$ for both regimes. The variation of the Hubble parameter predicted by Brans–Dicke cosmology is shown to be consistent with recent measurements: the value of $H$ in the late-time future is predicted as 0.86 times the present value of $H$.

Keywords: dark energy theory, inflation, gravity, cosmology of theories beyond the SM
1. Introduction

The inflationary universe model whose key feature is a finite period of primordial rapid exponential expansion has been proposed to resolve a number of cosmological puzzles, including the horizon, flatness and monopole problems. In the original or ‘old inflation model’ [1], the universe super-cools into a false vacuum phase and its energy density acts as an effective cosmological constant which causes an epoch of de-Sitter (exponential) expansion. In this old inflation model, the de-Sitter expansion never ends, and for a generic first order phase transition there appears an energy barrier between the false vacuum and the true vacuum phases. This problem is known as the ‘graceful exit’ problem. This problem was avoided with the invention of the new inflationary theory [2]. In this theory, inflation may begin either in the false vacuum, or in an unstable state at the top of the effective potential. Then the inflaton field $\phi$ slowly rolls down to the minimum of its effective potential. The density perturbations produced during the slow-roll inflation are inversely proportional to $\dot{\phi}$ [3, 4]. Thus the key difference between the new inflationary scenario and the old one is that the useful part of inflation in the new scenario, which is responsible for the homogeneity of our universe, does not occur in the false vacuum state, where $\dot{\phi} = 0$. Although this scenario was very popular at the beginning of the 1980s, it had its own problems. One of them, for example, is that the inflaton field has an extremely small coupling constant in most versions of this scenario, so it could not be in thermal equilibrium with other matter fields. The theory of cosmological phase transitions, which was the basis of old and new inflation, did not work in this situation. Furthermore, inflation in this theory begins very late, and during the preceding epoch the universe can easily collapse or become so inhomogeneous that inflation may never happen [5].

With the invention of the chaotic scenario all problems of old and new inflation were resolved. According to this scenario, inflation may occur even in theories with simple potentials such as $V(\phi) \sim \phi^n$. Inflation may begin even if there was no thermal equilibrium in the early universe, and it may start even at the Planckian density, in which case the problem of initial conditions for inflation can be easily resolved [5]. The field in this scenario evolved slowly and at this stage the energy density of the scalar field, unlike the density of ordinary matter, remained almost constant, and expansion of the universe
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continued with a much greater speed than in the old cosmological theory. Inflation does not require supercooling and tunnelling from the false vacuum [1], or rolling from an artificially flat top of the effective potential [2].

All models discussed so far have used field theories at very high energies to drive inflation. However, inflation may also be generated by changing the gravitational sector alone (\(R^2\) inflation) [6, 7] or both the gravitational and the matter sectors (extended inflation) [8], and the well known Jordan–Brans–Dicke theories are widely used in this third generation of inflation.

Jordan–Brans–Dicke theories are a class of theories in which the effective gravitational coupling evolves with time, and asymptotically attains a value of \(G\). The strength of the coupling is determined by a scalar field, \(\phi\), such that asymptotically it tends to a value \(G^{-1}\). The origins of Brans–Dicke theory are in Mach’s principle according to which the property of inertia of material bodies arises because of their interactions with the matter distributed in the universe. In the modern context, the Brans–Dicke theory attempted to rescue the inflationary scenario from some of its problems. The theory is parametrized by a dimensionless constant \(\omega\), where \(\omega \rightarrow \infty\) as Brans–Dicke theory goes over to the Einstein theory [9]. Present limits of the constant \(\omega\) based on time-delay [10]–[12] experiments require \(\omega > 500 \gg 1\). In the conventional inflationary scenario, the universe undergoes an exponential expansion for a brief period in its early phase. After the exponential phase is over the universe should transit to the normal cosmology phase. Within the framework of Einstein–Hilbert action, there is no satisfactory mechanism by which the universe transits to the normal phase. It was shown that within the framework of Brans–Dicke gravity a constant energy density leads to a rapid power-law expansion instead of exponential. This is rapid enough to solve the problems in standard cosmology and at the same time slow enough to make the transition to normal state possible after the inflationary phase. This has come to be known as extended inflation [13, 14]. Extended inflation constrains \(\omega\) to be less than 25. This bound comes from the fact that if it is more than 25 there will be much more anisotropy in the cosmic microwave background radiation [15] than is observed today. This is, however, incompatible with the bound which constrains \(\omega\) to be greater than 500 [10, 12]. A large number of inflationary models were proposed in the framework of multi-scalar tensor gravity to solve the problem. For instance, the introduction of a potential for the scalar field \(\phi\) and a scalar field dependent coupling constant \(\omega(\phi)\) solved some problems [16]–[22].

In our work, we start up with a strong link between inflation and the Brans–Dicke [23] theory of gravity. The proposed model in this work is simple in that no other phenomenon, such as the domination of the false vacuum over the scalar field energy density as in the extended inflation model, is used. Since the recent progress in observational cosmology shifted attention towards experimental verification of various inflationary theories, the motivation of this work is accelerated with the recent measurements of the dependence of the Hubble parameter \(H = \dot{a}/a\) on the scale size \(a(t)\) of the universe as

\[
\left( \frac{H}{H_0} \right)^2 = \Omega_\Lambda + \Omega_M \left( \frac{a_0}{a} \right)^3
\]

where \(\Omega_\Lambda \cong 0.75\) and \(\Omega_M \cong 0.25\) [24]. In standard cosmology the \(\Omega_\Lambda\) term would be induced by a cosmological constant. An immediate question which arises is the physical reason for this cosmological constant. A universe expanding under the sole influence of
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a cosmological constant \( \Lambda = \lambda^2 \) inflates as \( a(t) \sim e^{\lambda t} \). For the present day expansion \( \lambda \simeq H_0 \), whereas for the primordial expansion responsible for the present large size of the universe \( \lambda \) is much bigger.

In this paper, we will show a natural model where the large ratio of primordial inflation to present day inflation can be explained by Brans–Dicke theory which effectively replaces the Newtonian gravitational constant \( G_N \) in the Einstein–Hilbert action by a power of the Brans–Dicke scalar field. The additional kinetic and potential terms of this scalar field in the action behave effectively as time dependent cosmological constants. We choose the Brans–Dicke field \( \phi \) where the right-hand side of the equation below is set to be zero in accordance with (dot denotes \( d/dt \))

\[
\frac{3}{4\omega} \ddot{\phi} \left( \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) - \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} m^2 \phi^2 + \frac{3}{2\omega} \frac{\dot{a}}{a} \dot{\phi} = \rho
\]

(4)

\[
-\frac{1}{4\omega} \ddot{\phi} \left( 2 \frac{\dot{a}^2}{a^2} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) - \frac{1}{\omega} \dot{\phi}^2 - \frac{1}{2\omega} \ddot{\phi} - \left( \frac{1}{2} + \frac{1}{2\omega} \right) \dot{\phi}^2 + \frac{1}{2} m^2 \phi^2 = p
\]

(5)

\[
\ddot{\phi} + \frac{3}{\omega} \dot{\phi} + \left[ m^2 - \frac{3}{2\omega} \left( \frac{\dot{a}^2}{a^2} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) \right] \phi = 0.
\]

(6)
2. Primordial inflation

In the primordial inflation analysis, we start to solve the field equations (4)–(6) for an empty–static universe by setting \( \dot{a} = 0 \) and \( p = \rho = 0 \) and get the following vacuum solutions:

\[
\phi = \phi_0 e^{\alpha t} \tag{7}
\]

\[
a = a_* = \text{constant} \tag{8}
\]

\[
k = 1 \text{ (closed universe)} \tag{9}
\]

where

\[
\alpha^2 = m^2 \left( \frac{\omega}{2\omega + 3} \right) \tag{10}
\]

\[
a_*^2 = \frac{1}{m^2} \left( \frac{2\omega + 3}{3\omega + 3} \right) \left( \frac{3}{2\omega} \right). \tag{11}
\]

From these (7)–(9) solutions, we see that \( \phi \) evolves exponentially with expansion parameter \( \alpha \) and on the other hand \( a_* \) is the constant size of this static universe. We regard the fact that only the closed universe solution is possible as a positive aspect of this solution since homogeneity of the universe only makes sense if a closed universe undergoes the big bang. Let us note that, since two variables \( \phi(t) \) and \( a(t) \) satisfy the three equations (4)–(6), these solutions (7) and (8) are expected to be stable. To prove this stability we impose the size of the universe \( a \) and the field \( \phi \) to be a function of time \( t \) as follows:

\[
a = a_*(1 + \epsilon b(t)) \tag{12}
\]

\[
\phi = e^{\alpha t}(1 + \epsilon \psi(t)) \tag{13}
\]

where \( \epsilon \) is the perturbation factor (\( \epsilon \ll 1 \)) and \( b(t), \psi(t) \) are perturbation functions of \( a(t) \) and \( \phi(t) \) respectively. We get the following differential equations by using (4)–(6) and equalities (10), (11)

\[
\dot{\psi}(t) - \frac{3}{2\omega} \dot{b}(t) + \frac{3}{2\omega \alpha a_*^2} b(t) = 0 \tag{14}
\]

\[
\ddot{\psi}(t) + (4 + 2\omega)\alpha \dot{\psi}(t) + \ddot{b}(t) + 2\alpha \dot{b}(t) - \frac{\dot{b}(t)}{a_*^2} = 0 \tag{15}
\]

\[
\ddot{\psi}(t) + 2\alpha \dot{\psi}(t) + 3\alpha \dot{b}(t) - \frac{3}{2\omega} \dddot{b} + \frac{3}{\omega a_*^2} b = 0. \tag{16}
\]

Solving (14)–(16) simultaneously gives us that \( \dot{b} = 0 \) is the only solution which implies \( \ddot{b} = 0 \) and \( \dddot{b} = 0 \) and \( \psi = 0 \). Namely, this closed universe vacuum solution where \( a = a_* = \text{cst} \) and \( \phi \sim e^{\alpha t} \) is stable.

We now investigate how the presence of radiation changes the behaviour of the universe compared to this stable solution. Since solving the equations (4)–(6) for the primordial equation of state \( p = \frac{4}{3}\rho \) is hard, we solve (6) for \( a(t) \) by keeping \( \phi \sim e^{\alpha t} \). By
changing to the variable $a^2(t) = \theta(t)$, (6) turns out to be the second order differential equation

$$-\frac{3}{4\omega} \ddot{\theta} + \frac{3\alpha}{2} \dot{\theta} + (\alpha^2 + m^2)\theta = \frac{3}{2\omega} \theta$$

(17)

with the following solution for $\theta(t)$:

$$\theta(t) = a^2(t) = \left( \frac{3}{2\omega} \right) \left( \frac{1}{\alpha^2 + m^2} \right) + c_1 e^{\beta_1 t} + c_2 e^{\beta_2 t}$$

(18)

where

$$\beta_{1,2} = \left( \frac{3\alpha}{2} \right) \pm \sqrt{\left( \frac{9}{4} \right) \alpha^2 + \left( \frac{3}{\omega} \right) \left( \alpha^2 + m^2 \right)}$$

(19)

and $\beta_1 < 0$, $\beta_2 > 0$, $c_1$ and $c_2$ are integration constants. Now, if we define the big-bang time as the limit when $t \to 0$ and $\omega \gg 1$, we get $\alpha \simeq m/\sqrt{2}$, $\beta_1 \simeq -2\alpha$ and $\beta_2 \simeq 2\omega\alpha$.

Equation (18) when expanded about $t = 0$ becomes

$$\theta(t) = a^2(t) = a_*^2(1 + c_1 + c_2 - 2c_1\alpha t + 2c_2\omega t)$$

(20)

with the constraint $1 + c_1 + c_2 = 0$ since we want $a^2 \sim t$ as $t \to 0$. Thus, we end up with the general solution for the scale size of the universe in the primordial inflation regime with $\omega \gg 1$ as

$$a^2(t) = a_*^2[1 - (1 + c)e^{-2\alpha t} + ce^{2\omega t}]$$

(21)

This general solution is important for at least three reasons.

(1) It is a natural solution in the sense that it does not need any special equation of state for the matter. It is solely deduced from the theory by using the stable–empty universe solution (7) in equation (6).

(2) When we examine this inflationary solution concerning $t \to 0$ and $t > 0$ but not too much, we see that (21) is both consistent with $a(t) \sim \sqrt{t}$ as $t \to 0$ and also with primordial rapid inflation described by $a(t) \sim e^{\alpha t}$ for $\omega \gg 1$.

(3) We also check for $\omega \gg 1$ that if we substitute $\phi \sim e^{\alpha t}$ and $a \sim \sqrt{t}$ into (4)–(6) then the equation of state $p = \frac{1}{3}\rho$ is satisfied automatically as expected in this regime.

3. Late-time inflation

In this section, we analyse how far today’s universe is from late-time inflation by considering the case of a slowly expanding empty universe ($\rho = p = 0$) except for the $\phi$ field in it. Since the considered universe should be big, we ignore the curvature parameter $k/a^2$ as $a(t)$ increases with the expansion of the universe. Under these considerations, in analogy with the previous section, we put $a = e^{\tilde{\beta} t}$ and $\phi = e^{\tilde{\alpha} t}$ into (4)–(6) where $\tilde{\beta}, \tilde{\alpha}$ are new constants to be determined and search for a stable solution. We get the following
coupled equations for $\beta$ and $\alpha$:

$$
\ddot{\beta}^2 - \frac{2}{3} \omega \dot{\alpha}^2 + 2 \beta \ddot{\alpha} - \frac{2\omega}{3} m^2 = 0 
$$
(22)

$$
\ddot{\beta} + \left( \frac{2}{3} \omega + \frac{4}{3} \right) \dot{\alpha}^2 + 4 \beta \ddot{\alpha} - \frac{2\omega}{3} m^2 = 0 
$$
(23)

$$
\ddot{\beta} - \frac{\omega}{3} \dot{\alpha}^2 - \omega \dot{\beta} \ddot{\alpha} - \frac{\omega}{3} m^2 = 0. 
$$
(24)

These equations have the solution

$$
\ddot{\beta} = 2(\omega + 1) \left( \frac{\omega}{6\omega^2 + 17\omega + 12} \right)^{1/2} m \approx 0.8 \sqrt{\omega} m
$$
(25)

$$
\ddot{\alpha} = \left( \frac{\omega}{6\omega^2 + 17\omega + 12} \right)^{1/2} m \approx \frac{0.4}{\sqrt{\omega}} m
$$
(26)

where the approximations are again for $\omega \gg 1$ so that $m \approx \frac{1}{\sqrt{\omega} \alpha}$.

We see that although the primordial inflation parameter is $0.7 \omega m$, the late-time inflation parameter is found to be $0.8 \sqrt{\omega} m$, namely, a factor $\sqrt{\omega}$ less than the primordial inflation parameter. This is a very important result in the sense that although there is an experimental lower bound on $\omega$, there is no upper bound [10] on it, hence in Brans–Dicke cosmology the late-time inflation can be as small as one wishes compared to the primordial inflation.

Now, we consider the case where the universe is closed ($k = 1$) and matter dominated $p \approx 0$. Since solving the field equations (4)–(6) for $a(t)$ and $\phi(t)$ under $p \approx 0$ is hard enough, we proceed to work by defining the rate of change in $\phi$ as $F(a) = \dot{\phi}/\phi$ and the Hubble parameter as $H(a) = \dot{a}/a$, and rewriting the right-hand side of the field equations (4)–(6) in terms of $H$, $F$ and their derivatives with respect to $a$ (prime denotes $\text{d}/\text{d}a$),

$$
H^2 - \frac{2\omega}{3} F^2 + 2HF + \frac{1}{a^2} - \frac{2\omega}{3} m^2 = \left( \frac{4\omega}{3} \right) \frac{\rho}{\phi^2}
$$
(27)

$$
H^2 + \left( \frac{2\omega}{3} + \frac{4}{3} \right) F^2 + \frac{4}{3} HF + \frac{2a}{3} (H \dot{H} + H \dot{F}) + \frac{1}{3a^2} - \frac{2\omega}{3} m^2 = \left( \frac{-4\omega}{3} \right) \frac{p}{\phi^2} \approx 0
$$
(28)

$$
H^2 - \frac{\omega}{3} F^2 - \omega HF + a \left( \frac{H \dot{H}}{2} - \frac{\omega}{3} H \dot{F} \right) + \frac{1}{2a^2} - \frac{\omega}{3} m^2 = 0.
$$
(29)

Expanding $F(a)$ and $H(a)$ in powers of $(a_0/a)$ up to third order, where $a_0$ is the present size of the universe,

$$
H(a) = H_\infty + H_2 \left( \frac{a_0}{a} \right)^2 + H_3 \left( \frac{a_0}{a} \right)^3
$$
(30)

$$
F(a) = F_\infty + F_2 \left( \frac{a_0}{a} \right)^2 + F_3 \left( \frac{a_0}{a} \right)^3
$$
(31)
and putting them into (27)–(29), we get the perturbation constants defined above for ($\omega \gg 1$):

\[ H_\infty = \tilde{\beta} \approx 0.8\sqrt{\omega m} \]  
\[ H_2 \approx -\frac{1}{2a_0^2 H_\infty} \approx -\frac{0.6}{\sqrt{\omega a_0^2 m}} \approx 0 \]  
\[ F_\infty = \tilde{\alpha} \approx 0.4 \frac{\sqrt{\omega}}{m} \]  
\[ F_2 \approx \frac{3}{4\omega a_0^2 H_\infty} \approx \frac{0.9}{\omega^{3/2} a_0^2 m} \approx 0 \]  
\[ H_3 \approx 2\omega \frac{F_3}{3} \]  
\[ H_3 \approx -F_3. \]  

(32)  
(33)  
(34)  
(35)  
(36)  
(37)

Up to now, we note that all the constants required in our assumption for $H(a)$ and $F(a)$ in the late-inflation regime are almost determined from the theory except $H_3$ and $F_3$. Indeed, solving equations (36) and (37) simultaneously gives us $H_3$ and $F_3$ as zero. But to explain the late-time universe we may assume that $p/\rho \ll 1$ rather than $p$ being exactly zero. To overcome this problem, we use the relation (36) between $H_3$ and $F_3$ coming from equation (29) which is more exact than relation (37) coming from equation (28). We also use the classical Friedman formula which is used for fitting observations of Hubble parameter to density parameter $\Omega$:

\[ \frac{H^2}{H_0^2} = \Omega_\Lambda + \Omega_R \left( \frac{a_0}{a} \right)^2 + \Omega_M \left( \frac{a_0}{a} \right)^3 \]  

(38)

where $\Omega_\Lambda$ is the vacuum density parameter, $\Omega_R$ is the curvature density parameter, $\Omega_M$ is the matter density parameter and $H_0$ is the present Hubble parameter. Using (32) and (33), we rearrange (30) leaving $H_3$ as a free parameter and put it into (38) to get

\[ \Omega_\Lambda \approx \frac{H_\infty^2}{H_0^2} \]  
\[ \Omega_R \approx \frac{2H_\infty H_2}{H_0^2} \]  
\[ \Omega_M \approx \frac{2H_\infty H_3}{H_0^2}. \]  

(39)  
(40)  
(41)

Using the present observational result [24] $\Omega_M \approx 0.25$, $\Omega_\Lambda \approx 0.75$ and $\Omega_R \approx 0$, we find $H_3$ and $F_3$ to be

\[ H_3 \approx 0.13\sqrt{\omega m} \quad (\omega \gg 1) \]  
\[ F_3 \approx 1.41 \frac{\sqrt{\omega}}{m} \quad (\omega \gg 1). \]  

(42)  
(43)

After finding the perturbation constants explicitly for $H$ and $F$, we also find it worthwhile to determine how the Hubble parameter $H(a) = \dot{a}/a$ and the time variation of $G$, where $G$ is the time dependent value of the gravitational constant, change in the primordial and late-time regimes compared to their present values. To do so, we use the fact that since
Brans–Dicke gravity becomes identical to Einstein gravity as $\omega$ approaches infinity the kinetic term for the scalar field $(1/8\omega)\dot{\phi}^2$ in the action (2) will be the same as that of the term $1/16\pi G$ in the Hilbert–Einstein action. Using this fact, we get the relation between the scalar field $\phi$ and $G$ as

$$G^{-1} = \frac{2\pi\dot{\phi}^2}{\omega}. \quad (44)$$

Then, putting equations (32), (33) and (42) into (30) for the present value of the Hubble constant $H_0$ and for the present value of the scale factor of the universe $a_0$ gives us the magnitude of expansion parameter $\alpha$ and mass $m$ of the scalar field when $\omega \gg 1$ as

$$\alpha \approx \frac{0.70}{\sqrt{\omega}} H_0 \quad (45)$$

$$m \approx \frac{1.08}{\sqrt{\omega}} H_0. \quad (46)$$

Similarly, using (44) and putting equations (34), (35) and (43) into (31) for the present value of the scale factor of the universe $a = a_0$ gives us the magnitude of the present value of the parameter $\dot{G}/G$ when $\omega \gg 1$ as

$$\left(\frac{\dot{G}}{G}\right)_0 \approx -\frac{1.81}{\sqrt{\omega}} m \approx -\frac{1.95}{\sqrt{\omega}} H_0. \quad (47)$$

On the other hand, since $\phi \approx e^{0.7mt}$, $a \approx e^{0.7m\omega t}$ and $\phi \approx e^{(0.4/\sqrt{\omega})mt}$, $a \approx e^{0.8\sqrt{\omega}mt}$ in the primordial and late-time regimes respectively, using (44) we get the parameter $\dot{G}/G$ and the Hubble parameter $H = \dot{a}/a$ in these regimes as

$$\left(\frac{\dot{G}}{G}\right)_{\text{primordial}} \approx -1.4m \approx -\frac{1.51}{\sqrt{\omega}} H_0 \quad (48)$$

$$\left(\frac{\dot{G}}{G}\right)_{\text{late-time}} \approx -\frac{0.81}{\sqrt{\omega}} m \approx -\frac{0.88}{\omega} H_0 \quad (49)$$

$$(H)_{\text{primordial}} \approx 0.7m\omega \approx 0.75\sqrt{\omega}H_0 \quad (50)$$

$$(H)_{\text{late-time}} \approx 0.8\sqrt{\omega}m \approx 0.86H_0. \quad (51)$$

Lastly, we investigate the ratio $\nu = p/\rho$ where $p$ and $\rho$ are the pressure and energy density of the late-time universe respectively as $\omega \to \infty$,

$$\nu = \frac{p}{\rho} = \frac{-(\omega + 6)/(\omega(6\omega + 6))a_0^{-2} - H_\infty H_3((20\omega + 21)/(3\omega + 3))}{((\omega + 3)/(\omega(2\omega + 2))a_0^{-2} + 2H_\infty H_3} \approx 0 \quad (\omega \gg 1) \quad (52)$$

and find it approaching a value of zero as expected.
4. Discussion and conclusion

In this work we have investigated the nature of the simplest chaotic inflation-style potential energy density \( V(\phi) = \frac{1}{2} m^2 \phi^2 \) which is composed only of the scalar field mass term. We have assumed that \( \phi \) evolves with time in Brans–Dicke cosmology with a perfect fluid distribution. We have found a general solution in exponential form for the size of the universe in the primordial regime inflating with an expansion parameter \( \omega_\alpha \). We also calculated how the Hubble parameter \( H(a) = \dot{a}/a \) and the time variation of \( G \), where \( G \) is the time dependent value of the gravitational constant, change in the primordial and late-time regimes compared to their present values. We note that the newest measurement [24] of \( \Omega_\Lambda \) and \( \Omega_M \) has been used as input to derive these results. One interesting feature is that the predicted present day and primordial values of \( |\dot{G}/G| \) are comparable whereas the asymptotic value is much smaller. In any case, a measurement of \( \dot{G}/G \) will be crucial in determining the Brans–Dicke parameter \( \omega \). On the other hand, the Hubble parameters predicted by the theory in both regimes have yielded interesting results. Besides this, the ratio \( \nu = p/\rho \) is found to be zero as universe approaches late-time inflation (\( \omega \gg 1 \)).

In the end we can say that the fact that the ratio of the primordial and late-time inflation parameters is proportional to \( \sqrt{\omega} \) is the most appealing feature of Brans–Dicke cosmology. Thus, recent measurements, which imply that in today’s universe \( \Omega_\Lambda \neq 0 \), require \( 1/\omega \neq 0 \) and make this model attractive.

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