Secular evolution of galactic discs: constraints on phase-space density

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ABSTRACT
It was argued in the past that bulges of galaxies cannot be formed through collisionless secular evolution because that would violate constraints on the phase-space density: the phase-space density in bulges is several times larger than in the inner parts of discs. We show that these arguments against secular evolution are not correct. Observations give estimates of the coarsely grained phase-space densities of galaxies, \( f' = \rho_s / \sigma_R \sigma_{\phi} \sigma_z \), where \( \rho_s \) is stellar density and \( \sigma_R, \sigma_{\phi}, \sigma_z \) are the radial, tangential, and vertical rms velocities of stars. Using high-resolution N-body simulations, we study the evolution of \( f' \) in stellar discs of Galaxy-size models. During the secular evolution, the discs, which are embedded in live Cold Dark Matter haloes, form a bar and then a thick, dynamically hot, central mass concentration. In the course of evolution \( f' \) declines at all radii. However, the decline is different in different parts of the disc. In the inner disc, \( f'(R) \) develops a valley with a minimum around the end of the central mass concentration. The final result is that the values of \( f' \) in the central regions are significantly larger than those in the inner disc. The minimum, which gets deeper with time, seems to be due to a large phase mixing produced by the outer bar. We find that the shape and the amplitude of \( f'(R) \) for different simulations agree qualitatively with the observed \( f'(R) \) in our Galaxy. Curiously enough, the fact that the coarsely grained phase-space density of the bulge is significantly larger than the one of the inner disc turns out to be an argument in favor of secular formation of bulges, not against it.

Key words: Galaxy: evolution – Galaxy: structure – galaxies: kinematics and dynamics – galaxies: evolution.

1 INTRODUCTION
Formation of galactic spheroids remains as a major unsolved problem in astronomy. This is an important problem, especially if one takes into account that at least half of the stars in the local Universe are in spheroids: either bulges or ellipticals (e.g., Fukugita, Hoggan & Peebles 1998; Bell et al. 2003). The key question is how and where these stars formed. One possibility is that stars in present-day spheroids were formed in a self-regulated quiescent fashion characteristic of galactic discs, and then the disc stars were dynamically heated by mergers and/or secular disc processes. In this case the spheroid formation is predominantly collisionless. Another possibility is that spheroid star formation (SF) was highly dissipative and proceeded in a violent, possibly bursting and dust enshrouded mode during a dissipative disc merging event or during a phase of fast gas (monolithic) collapse. Although both possibilities happen certainly in the real Universe, it is important to evaluate the feasibility of each one as well as the physical/evolutionary context in which one or another possibility dominates. In the former case the SF rate is traced by UV/optical emission, while in the latter by FIR/submillimetre emission. Thus, understanding the mechanisms of spheroid formation and the regimes of formation of their stars is of crucial relevance for interpreting and modeling the contribution of present-day stars in spheroids to the cosmic SF rate history.

1.1 Secular bulge formation mechanism
In this paper we will study some aspects of the disc secular evolution. According to the secular scenario, the formation of a central mass concentration (a bar or a pseudobulge) happens in a predominantly dissipationless fashion in the course of development of gravitational instabilities in the central region of a galactic stellar disc. The evolution of the bar can give rise to a central component that is denser and thicker than the initial thin stellar disc (Kormendy 1979, 1982). In earlier simulations the bar in most of cases was dissolving,
leaving behind a pseudobulge (e.g., Combes & Sanders 1981; Pfenniger & Norman 1990; Combes et al. 1990; Raha et al. 1991; Norman, Sellwood & Hassan 1996). However, more recent simulations, which have many more particles and have more realistic setup, do not produce typically decaying bars (Debattista & SELLWOOD 2000; Athanassoula & Misiriotis 2002; O‘Neill & Dubinsky 2003; Vlaznuel & Klypin 2003, hereafter VK03; Shen & Sellwood 2003; Debattista et al. 2004). In those simulations bars typically slightly grow over billions of years.

In the VK03 simulations of discs inside live Cold Dark Matter (CDM) haloes, the redistribution of the angular momentum of the stellar disc is driven by the evolving bar and by interactions with the dark matter halo. This evolution produces a dense central mass concentration with nearly exponential profile, which resembles surface brightness profiles of late-type galaxy bulges (see also Athanassoula & Misiriotis 2002; O‘Neill & Dubinsky 2003). Shen & Sellwood (2003) and Debattista et al. (2004) argued that neither a small central mass concentration (e.g. black hole) nor the buckling instability are efficient enough to destroy a bar.

Whether the bar is destroyed or not, the heating of the central parts of the stellar disc and accumulation of mass at the centre are common features in all models of secular evolution. Further exploration, including a wide range of realistic initial conditions and inclusion of processes such as gas infall (e.g., Bournaud & Combes 2002), minor mergers and satellites (Aguerri et al. 2001), hydrodynamics, SF and feedback are certainly necessary. All the processes are likely to play some role in evolution of galaxies. Here we intentionally do not include these complex processes in order to isolate the effects of the secular evolution: we include only stellar and dark matter components and do not consider any external effects.

Models of secular disc evolution gradually find their place in the theory of galaxy formation. Encouraging results were obtained when a prescription for secular bulge formation was incorporated in CDM semi-analytical models of disc galaxy formation and evolution (Avila-Reese & Firmani 2000,1999; see also van den Bosch 1998). These models successfully reproduce the observed correlations of the bulge-to-disc ratio with other global properties for late-type galaxies. Secular disc evolution should be considered as a complementary path of spheroid formation rather than a concurring alternative to the dissipative merging mechanism.

From the observational side, an increasing evidence shows that structural, kinematic, and chemical properties of the bulges of late-type galaxies are tightly related with properties of the inner discs (for reviews and references see Wyse, Gilmore & Franx 1997; MacArthur, Courteau & Holtzmann 2003; Carrollo 2004; Kormendy & Kennicutt 2004). Besides, bars – signature of secular evolution – are observed in a large fraction of spiral galaxies. These pieces of evidence strongly favor the secular evolution scenario.

1.2 Do phase-space constraints pose a difficulty for the secular mechanism?

According to the Liouville’s theorem, in a collisionless system the phase-space density $f(x,v)$ is preserved along trajectories of individual stars. Thus, one expects that a collisionless system “remembers” its initial distribution of $f(x,v)$, and this can be used to test the secular evolution scenario. In fact what is “observed” is not $f(x,v)$, but a rough estimate of the coarsely grained phase-space density $f' = \frac{\rho_s}{\sigma_R \sigma_\phi \sigma_z}$.

where, $\rho_s$ is stellar density and $\sigma_R, \sigma_\phi, \sigma_z$ are radial, tangential, and vertical rms velocities of stars. The coarse-grained phase-space density is not preserved. Still, there are significant constraints on the evolution of $f'$, which are imposed by the mixing theorem (Tremaine, Henon, Lynden-Bell 1986). The process that changes $f'$ is the mixing. Bringing and mixing together two patches of stars with different fine-grained phase-density results in $f'$, which is lower than the maximum phase-space density of the two patches. In other words, the mixing results in reducing $f'$. The only way to increase $f'$ is to bring in stars with initially large $f'$. Indeed, when a bar forms, there is a substantial radial infall of mass to the central region. This does not help much because the bar is formed from the central region of the disc where the initial $f'$ is low. Additional mixing produced by the bar seems to make the things even worse by lowering down already low $f'$.

Simple estimates for elliptical and spiral galaxies indicate that the coarse-grained phase-space-density in spirals are lower than in ellipticals (Carlberg 1986), making difficult to produce elliptical galaxies by merging of stellar discs (Hernquist, Spergel & Heyl 1993). Wyse (1998) discussed similar arguments but for the Galactic disc in the context of the secular bulge formation scenario. For an exponential disc with constant height $h_z$ and with an isotropic velocity-dispersion tensor one finds that $f'(R) \propto \rho_s/\sigma_z^2 \propto (\Sigma_z/2h_z)/(\Sigma_z h_z)^{3/2} \propto \exp(R/2h_z)$, where $\Sigma_z$ and $h_z$ are the disc central surface brightness and scale-length radius, respectively. Therefore, $f'(R)$ is lower toward the centre. Wyse (1998) then states: “one should not find a higher phase-space density in stellar progeny, formed by a collisionless process, than in its stellar progenitor”. Observational inferences for our Galaxy show actually that $f'$ is higher in the bulge than in the inner disc. This discrepancy led Wyse (1998) to conclude that the secular scenario has a serious difficulty, unless dissipative physics is included.

The problem is partially mitigated if one considers that the Toomre parameter $Q$ is constant along the initial disc. For $Q = \text{const}$, the rms velocity is $\sigma_R \propto \exp(-R/h_d)/\kappa(R)$, where the epicycle frequency $\kappa$ increases as $R$ decreases. In this case $f'(R)$ has an $U$–shaped profile with a maximum at the centre (Lake 1989) and a valley in the inner regions. How steep or shallow is this valley depends on the inner behavior of $\kappa(R)$.

The situation is actually more complicated than the simple picture outlined above because of the spatial and dynamical properties of the system evolve. Formation of bars is a complex process that affects a large fraction of the disc – not just the central region. Thus, to study the overall evolution of $f'(R)$ one needs to turn to numerical simulations.

The main question, which we address in this paper is how the macroscopic (observational) phase-space density profile, $f'$, of stellar discs inside CDM haloes evolves during the formation (and potential dissolution) of a bar, and whether the shape of this profile agrees with estimates from
Table 1. Parameters of models

| Parameter                  | C  | A1 | D_ha | D_cs |
|---------------------------|----|----|------|------|
| Disk mass (10^10M_☉)      | 4.8| 4.3| 4.35 | 5.0  |
| Total mass (10^12M_☉)     | 1.0| 2.0| 1.22 | 1.4  |
| Initial disc scale-length (kpc) | 2.9| 3.5| 2.25 | 2.57 |
| Initial Toomre parameter Q | 1.2| <1.2| >1.8 | 1.3  |
| Initial disc scale-height (kpc) | 0.14| 0.25| 0.17 | 0.20 |
| Halo concentration c_{NFW}| 19.0| 15| 18 | 17 |
| Number of disc particles (10^5) | 12.9| 2.0| 4.6 | 2.3 |
| Number of halo particles (10^5) | 8.48| 3.3| 3.3 | 2.2 |
| Particle mass (10^5M_☉)   | 0.37| 2.14| 0.93 | 2.14 |
| Formal force resolution (pc)| 100| 22| 19 | 22 |

The particle mass refers to the mass of “stellar” disc particles. This is also the mass of the least massive halo particles.

We observe disc/bar/bulge galaxy systems. We analyze state-of-the-art high-resolution N-body simulations of Galaxy-like discs embedded in live CDM haloes. The secular evolution of the disc in these simulations yields bars that redistribute particles and produce a dynamically hot mass central concentration.

In §2 we present a brief description of the simulations and the procedure to estimate the phase-space density. The results are given in §3, and in §4 we discuss some aspects of the simulations. In §5 a comparison with observational estimates is presented. Our summarizing conclusions are given in §6.

2 MODELS AND SIMULATIONS

We study four N-body simulations for evolution of bars in stellar discs embedded in live CDM haloes. Two simulations, A1 and C, are taken from VK03 and the other two, D_ha and D_cs, are from Klypin et al. (2005). The models were chosen to cover some range of initial conditions and parameters, with the aim to test the sensitivity of the results to them. For example, the initial Toomre parameter Q is constant along the disc for models C, D_ha, and D_cs. The Q parameter is variable for model A1 (increases in the central regions). Instead, this model is initially set to have a radial velocity dispersion as \( \sigma_R^2(R) \propto \exp(-R/h_d) \). As the result, all the models have different initial profiles of the azimuthally averaged coarsely grained phase-space density. Parameters of the models and details of simulations are presented in Table 1.

The initial conditions are generated using the method introduced by Hernquist (1993). The galaxy models initially have exponential discs in equilibrium inside a dark matter halo with a density profile consistent with CDM cosmological simulations (Navarro, Frenk & White 1997). The halo concentrations are set somewhat larger than the expected concentration c_{NFW} \approx 12 for a halo without baryons, which should host our Galaxy (Klypin et al. 2002). This is done to mimic the adiabatic compression of the dark matter produced by baryons sinking to the centre of the halo in the process of formation of the galaxy.

The haloes are sampled with particles of different masses: particle mass increases with distance. The lightest dark matter particles have the same mass as the disc particles. At any time there are very few large particles in the central 20-30 kpc region. The time steps of simulations were forced to be short. For example for model C, the minimum time step is 1.2 \( 10^3 \) yrs, while for model D_ha it is 1.5 \( 10^2 \) yrs. For a reference, in model C the typical time that a star requires to travel the vertical disc extension of the disc at the radius of 8 kpc requires 372 steps, and the orbital period in the disc plane at the same distance takes 1900 steps. Model simulations C, A1, D_cs, and D_ha were followed for \~4.4, 4, and 4 Gyrs, respectively, using the Adaptive Refinement Tree (ART) code (Kravtsov et al. 1997).

In the models C and A1 the bar is strong even at the end of simulations without any indication that it is going to die. In the model D_ha the bar is gradually getting weaker, but it is still clearly visible. The bars typically buckle at some stage producing a thick and dynamically hot central mass concentration with a peanut shape. The model D cs is only model where bar dissolved completely and produced a (pseudo)bulge.

The galaxy models used in the simulations are scaled to roughly mimic the Galaxy. For example, they have realistic disc scale lengths (\~3 kpc), scale heights (\~200 – 300 pc), and they have nearly flat rotation curves with \( V_c \approx 220 \) km/s. Yet, we do not make an effort to reproduce detailed structure of our Galaxy. For example, the radius of the bar in model C is 5 – 5.5 kpc – too large as compared with real bar which has radius 3 – 3.5 kpc.

Model D_ha presents a shorter bar (3 kpc) and it is a better model in that respect\(^1\), and we describe it in more detail to show the reliability of our approach. With the exception of disc mass (which is somewhat small), model D_ha makes a reasonable match for our Galaxy. Its stellar surface density at the “solar” distance of 8 kpc is \( \Sigma_s = 54 M_☉/pc^2 \). For comparison, for our Galaxy Kuijken & Gilmore (1989) find \( \Sigma_s = 48 ± 8 M_☉/pc^2 \) while Siebert et al. (2003) find \( \Sigma_s = 67 M_☉/pc^2 \). Stellar rms velocities in radial and vertical directions in the model are 47 km s\(^{-1}\) and 17 km s\(^{-1}\). Dehnen & Binney (1998) give for the old thin disc stellar population of our Galaxy 40 km s\(^{-1}\) and 20 km s\(^{-1}\), respectively. Within the solar radius the model has a ratio of dark matter to total mass of \( M_{DM}/M_{tot} = 0.6 \). This ratio is significantly lower inside the bar radius of 3 kpc: \( M_{DM}/M_{tot} = 0.35 \). The bar pattern speed is \( \Omega_p = 54 \) Gyr\(^{-1}\). Bissantz et al. (2003) give \( \Omega_p = 60 ± 5 \) Gyr\(^{-1}\) for our Galaxy although their estimate is also based on a model.

In order to make a more detailed comparison of the mass distribution in the model D_ha with that of our Galaxy, we mimic the position-velocity (P-V) diagram for neutral hydrogen and CO in the plane of our Galaxy. Observations of Doppler-shifted 21-cm and CO emission along a line-of-sight performed at different galactic longitudes \( l \) provide the P-V diagram. Because the gas is cold, it provides a good probe for mass profile. There are two especially interesting features in the P-V diagram. Envelops of the diagram in the first quadrant (\( 0 < l < 90, V > 0 \)) and in the third quadrant (\( 0 > l > -90, V < 0 \)) are the terminal velocities.

\(^1\) In order to make a better match of the model with the Milky Way, we rescaled the model: all coordinates and masses were scaled down by factor 1.15. As any pure gravitational system, it can be arbitrary scaled using two free independent scaling factors. In this case we chose mass and distance: time, velocity, surface density, and so on are scaled accordingly.
(Knapp et al. 1985; Kerr et al. 1986). Data in the second and forth quadrants are coming for regions outside of Solar radius (Blitz & Spergel 1991). They have information on the motion in the outer part of our galaxy. For distances larger than the radius of the bar 3-4 kpc (|l| < 30°) the terminal velocities can be converted into the circular velocity curve (assuming a distance to the Galactic center). At smaller distances, perturbations produced by the Galactic bar are large and cannot be taken into account in a model-independent way. This is why we use the P-V diagram, not the rotation curve.

We mimic the P-V diagram for the $D_{hs}$ model by selecting “stellar” particles, that are close to the plane $|z| < 300$ pc and have small velocities relative to their local environment. A particle velocity must deviate not more than 20 km s$^{-1}$ from the velocity of its background defined by the nearest 50-70 particles. This gives the rms line-of-sight velocity of 8 km s$^{-1}$, which is compatible with the random velocities of the cold gas. We place an “observer” in the plane of the disc at the distance 8 kpc. Its position was chosen so that the bar major axis is 20 degrees away from the line joining the “observer” and the galactic center. The observer has the same velocity as the local flow of “stars” at that distance. We then measure the line-of-sight velocity of each cold particle and plot the particles in the longitude-velocity coordinates.

With the procedure just described we are selecting a population of “stellar” particles, which has small asymmetric drift and yet has the bulk flows induced by the bar. The procedure is insensitive to particular set of parameters as long as rms velocities stay significantly smaller than the rotational velocity 220 km s$^{-1}$ and as long as the radius for the background particles is significantly smaller than the distance to the center. Figure 1 shows the P-V diagram for the cold “stellar” particles in model $D_{hs}$. The envelop of the diagram very closely follows that of our Galaxy, indicating that the mass distribution in the inner part of the model is compatible with the data on our Galaxy. Small deviations in the outer part of the galaxy are due to lopsidedness of our Galaxy (Blitz & Spergel 1993), which our model cannot reproduce.

We assumed that cold “stellar” particles resemble neutral and molecular gas in the P-V diagram. To test this assumption we use a simulation presented elsewhere (Valenzuela et al. 2005, in preparation), which includes not only collisionless particles ("stellar" and dark matter), but also gas. This simulation has been run with the Gasoline N-body+SPH code (Wadsley, Stadel & Quinn 2004). The galaxy model is similar to the one discussed above. The simulation had $5 \times 10^5$ dark matter particles, $2 \times 10^6$ disc (“stellar”) particles, and $5 \times 10^5$ gas particles. The force resolution was 200 pc for gas and “stars”, and 600 pc for dark matter. The simulation was run until 1.6 Gyr. The simulated “galaxy” develops a strong bar with a radius $\approx 3$ kpc. We selected an “observer” at distance 8 kpc from the center at the angle of 20° relative to the major axis of the bar. Figure 2 shows P-V diagrams for different components. P-V diagram for the cold gas with $T < 10^4$K shows remarkable complexity: there are lumps and filaments. Those are due to spiral arms and shock waves. The cold “stellar” particles do not have those details, yet they remarkably well follow the same envelopes in the P-V diagram as the cold gas. This is exactly what we want to demonstrate. The whole stellar population (the top panel) shows large velocities in the central region – well in excess of cold gas motions. In this case, significant (20-30 percent) corrections are indeed required to account for the asymmetric drift.

2.1 Measuring the phase space density

The phase-space density is defined as the number of stars in a region of phase space around a point $(x, v)$ divided by the volume in phase space of that region as this volume tends to zero (e.g., Binney & Tremaine 1987; Wyse 1998). The measurable quantity is the coarsely grained phase-space density that is defined in finite phase space volumes. However, this quantity is still difficult to infer observationally. The measure of the phase-space density commonly derived from observations is the stellar spatial density, $\rho_s$, divided by the cube of the stellar velocity dispersion (eq. 1). For the latter, one typically uses either the projected line of sight velocity dispersion, $\sigma_p$, or the inferred radial velocity dispersion, or the product of the three components of the velocity dispersions, $\sigma_R, \sigma_\phi, \sigma_z$ if they are known. We will refer to $f'$ as the azimuthally averaged “observational” phase-space density as defined above.
Figure 2. Position-velocity diagrams for the N-body+SPH simulation carried out with the Gasoline code. Upper, medium and lower panels are for all the “stellar” particles, for only cold “stellar” particles, and for cold gas, respectively. The full curves in every panel are the same. They show the envelopes for the cold gas.

Because our main aim is to analyze the coarsely grained phase-space density evolution in the N-body simulations in a way that mimics observations, we do the following. We calculate the average density of “stellar” particles, \( \rho_s(R) \), and their velocity dispersions within cylindrical (equatorial) rings of width \( \Delta R \) and thickness \( \Delta Z \). Thus, \( f' \) is estimated in a representative region above and below the disc plane. Nonetheless, we have checked that qualitatively similar results are obtained when using other binning or even other geometries, in particular the spherical one for the centre. The thickness \( \Delta Z \) and \( \Delta R \) were assumed to be 2\( h_s \) and 200 pc, respectively. The results do not change significantly for a large range of assumed values for \( \Delta Z \) and \( \Delta R \).

3 EVOLUTION OF THE COARSELY GRAINED PHASE-SPACE DENSITY

Figure 3 shows the evolution of the surface and the volume densities averaged azimuthally for the simulation of model C, as an example. Dotted lines correspond to the beginning of the simulation, and the short dashed and solid lines are for moments of time separated approximately by 2 Gyr. The effect of bar evolution on the disc surface density is significant: matter is accumulated at the centre while the slope of the outer disc becomes shallower than the initial slope, with the disc scale-length increasing by \( \sim 30\% \) (VK03). The volume density shows similar behavior, but including also the disc vertical expansion with time.

The disc gets hotter and thicker as clearly demonstrated in Fig. 4, which shows the evolution of the three components of the velocity dispersion as well as \( h_s (R) \) for the same model C. The disc heating is very large in the central 2 – 4 kpc region where the bar forms. The heating happens also in the outer disc but to a much lesser degree. Here the heating is due to spiral waves, which form in the initially unstable disc. The waves gradually decay and heat the disc, but most of the heating occurs in the plane of the disc: the radial and tangential rms velocities increase substantially more than the vertical rms velocity, which changes by 25-30 percent over 4 Gyr (see also VK03).

Finally, the evolution of the “observational” radial phase-space density profile, \( f' \), of model C is shown in Fig. 5(a). Because the initial disc of this model has a constant \( Q \), the initial \( f'(R) \) profile is \( U \)-shaped: the maximum at the centre is followed by a minimum at the inner disk. The steepness of the central maximum depends on the behavior of \( \kappa(R) \) at small radii. For model C, the inner \( f'(R) \) profile is almost flat. In Fig. 5 (panels b, c and d) are shown also the evolution of \( f'(R) \) for models \( D_{a\alpha} \), \( D_{ha} \), and \( A_1 \), respectively. The initial inner \( f' \) profile of the first two models are significantly steeper than for model C, while for model \( A_1 \), \( f' \) decreases toward the centre. In the latter case, \( \sigma_{R}^{2}(R) \propto \exp(-R/h_d) \) was assumed initially instead of \( Q = const \) (see §1.1).

One clearly sees that the macroscopic (observational) azimuthally averaged phase-space density decreases with time along the whole disc of all the models. In the outer parts of the disc the bar and the spiral arms heat and thicken the disc. Here the surface density at each radius remains almost constant. The disc heating explains why \( f'(R) \) decreases at all radii. In the inner disc, the \( f' \) profile of all the models develops a valley whose depth increases with time. For example, for model C the minimum of this valley is at \( \sim 1.5–2 \) kpc and this is close to the radius where the central mass concentration ends. Around this radius one observes the maximum radial mass exchange as well as the maximum vertical heating and disc thickening due to the bar. Here the outer region
of the bar produces a large phase-space mixing – larger than in the centre. As the result, $f'$ at some radius inside the central mass concentration is higher than $f$ measured at the inner disc. The largest changes in the $f'(R)$ profile, as well as in other quantities (such as $h_z$ and rms velocities) occur during typically the first $\sim 1$ Gyr of evolution. The evolution continues, but much slower at later moments.

4 BULGE-LIKE STRUCTURE FORMATION AND ROBUSTNESS OF THE RESULTS

The bar-driven evolution produces a dense central concentration (or even a pseudobulge as is the case of model $D_{a}$) with a slope steeper than the original one. Figure 3 clearly illustrates this point for model $C$. For almost all of our simulations the surface density in the inner $\sim 2kpc$ region is well approximated by a Sérsic profile (Sérsic 1968) with slope index $n < 4$. For example, for model $C$, $n \approx 1$. This is similar to what is observed for bars and bulges of late type galaxies (e.g., de Jong 1996; Graham 2001; Mac Arthur et al. 2003; Hunt et al. 2004).

Comparison of different models indicate remarkable similarities in the evolution and the shape of the phase-space density profile $f'(R)$. In all the models $f'(R)$ has a maximum at the centre followed by a deep minimum at $\sim 1 - 2kpc$, where the corresponding outer bars live (see Fig. 5). It seems that the results do not depend on the numerical resolution and are not particularly sensitive to initial conditions. For example, for model $A_1$ the resolution is lower than for model $C$, and $Q$ was not assumed constant initially. Still, the evolution and shape of the $f'$ profile are similar to those of the model $C$ (panels a and d in Fig. 5). Although resolution is an important factor in simulations aimed to explore gravitational instabilities of thin discs embedded in large hot haloes (O’Neill & Dubinsky 2003; VK03), we find that the shape and evolution of $f'(R)$ is qualitatively the same in simulations with different resolutions.

When we look closely at the results, we clearly see differences between models. For example, the minimum of $f'(R)$ is at different radii and the depth of the minimum varies from model to model. The smallest radius and the deepest minimum are attained by model $D_{as}$, where the bar is dissolved and a pseudobulge forms. The differences in the evolution of $f'(R)$ are expected because the lengths and strengths of bars are different in different models. Yet, it seems that the overall generic shape of $f'$ after evolution is a robust prediction of the secular collisionless scenario.

5 COMPARISON WITH OBSERVATIONS

Wyse (1998) presented estimates of $f'_b$ and $f'_{d,inn}$ for the Galaxy from available observational information. We use more recent data to give updated estimates. We measure $f'_b$ and $f'_{d,inn}$ at the typical radii $r_e/2$ ($r_e$ is the effective bulge radius, $r_e \approx 0.7$ kpc, see Tremaine et al. 2002) and $R_{d,inn} = 2.5$ kpc, respectively. The bulge and inner disc stellar volume densities are estimated from the galaxy model used in Bissantz & Gerhard (2002), where the parameters were fixed from fittings to the dust-corrected COBE-DIRBE $L$-band maps (Spergel et al. 1995). Taking into account the bulge ellipticity and averaging vertically the disc density within $h_z = 330$ pc (inner disc scale height, Chen et al. 2001), we obtain $\rho_b(r_e/2) = 6.7 M_\odot pc^{-3}$ and $\rho(R_{d,inn}) = 0.31 M_\odot pc^{-3}$.

Regarding velocity dispersions, for the bulge we use an interpolation of measured $\sigma_p$’s at different projected galactocentric distances as compiled by Tremaine et al. (2002) ($\sigma_p(r_e/2) = 116.7$ km/s, corrected for ellipticity), and we assume velocity isotropy and that $\sigma_c \approx \sigma_p$. This approximation is valid for Sérsic profiles within $0.1 \lesssim r/r_e \lesssim 10$ (Ciotti 1991).

For the disc, the three velocity dispersions at $R_{d,inn}$ are calculated by using the radial profiles given in Lewis & Freeman (1989). Thus, at $R_{d,inn}$, $\sigma_R = 78.9$ km/s, $\sigma_\phi = 73.1$ km/s, and $\sigma_z = 41.8$ ($\sigma_z = 0.53\sigma_R$ was assumed). For completeness, we also calculate $f'$ in the outer bulge (at 1.5 kpc) and in the solar neighborhood (8.5 kpc). For the former, the same Bissantz & Gerhard (2002) bulge model and the Tremaine et al. (2002) recompilation for $\sigma_p$ were used. For the latter, we use the local estimate for the stellar density, $\rho_\odot = 0.09 M_\odot pc^{-3}$ (Hollenberg & Flynn 2000), and the velocity dispersion profiles from Lewis & Friedmann (1989).

The values of $f'_b(r_e/2)$, $f'_b(1.5kpc)$, $f'_{d,inn}$ and $f'_d$ (4.3 $10^{-6}$, 7.8 $10^{-7}$, 9.6 $10^{-7}$, and 2.8 $10^{-6}$ $M_\odot$ pc km/s $^{-3}$, respectively) are indicated with empty squares in Fig. 5. The qualitative agreement in the shape of $f'$ between observations and numerical predictions is remarkable, in spite of the fact that the observations have large uncertainties and the models do not include gas, SF processes or gas infall and minor mergers. Note also that $f'(R)$ tends to stabilize after $1 - 2$ Gyr, although one sees still changes after this period. We consider that the collisionless secular scenario (not including dissipative physics) is a good approximation...
for Milky Way-like galaxies for their last 4-7 Gyr. Before this time, the discs were much more gaseous and dissipative phenomena should be taken into account.

The models predict that the phase-space density decreases with time even at large radii. This decline of \( f' \) in outer regions is produced by spiral waves, which develop in the unstable disc. In real galaxies the same effect should be produced by real spiral waves and possibly by molecular clouds. It is interesting to compare the models with what is observed for our Galaxy at the solar neighborhood. At \( R \sim 8 \) kpc in the models \( f' \) roughly decreases by a factor 5-10 during 4-7 Gyr of evolution. Studies in the solar neighborhood show that the components of the stellar velocity dispersion increase with the age of stars. This is the so called age-velocity relation (for recent estimates see Rocha-Pinto et al. 2004 and the references therein). Observations indicate that each of the three velocity dispersion components increases by 20 - 60\% during the last \( \sim 6-7 \) Gyr of galactic evolution. If this is an indication how the whole stellar population evolves, we can make rough estimates of the evolution of \( f' \). For an equilibrium disc with stellar surface density \( \Sigma \) and total rms velocity \( \sigma \), the phase-space density scales as \( f' \propto \Sigma^2/\sigma^3 \). If for the sake of argument we assume that the rms velocity of the whole stellar population increased by factor of 2 during the last 6 Gyrs without substantial change in the stellar surface density, then we expect decline in \( f' \) by 32 times. Likely increase in the stellar surface density \( \Sigma \) (e.g., Hernández, Avila-Reese & Firman 2001) should reduce this very large factor. Yet, naively one does expect for our Galaxy a large drop in \( f' \) just as our models predict.

Finding \( f' \) for external galaxies is not easy, specially when it concerns the velocity dispersion. Here we estimate the ratios of \( f'_d \), calculated at \( r_e/2 \), to \( f'_d, \text{inn} \), calculated at 0.8h_{\odot} \) for four spiral galaxies of different morphological types (Sa - Sbc) studied by Shapiro et al. (2003). We use the \( K \) - and \( I \)-band surface brightness profiles and the \( \sigma_p \) profiles reported in Shapiro et al. (2004). The disc volume density is assumed to be proportional to the surface brightness divided by 2h_{\odot} (h_{\odot} is set equal to 0.125h_{\odot}, Kregel 2003). We also use the disc (\( \sigma_R, \sigma_\phi, \sigma_z \)) profiles, which Shapiro et al. fit to their spectroscopic observations. The bulge luminosity density profile is estimated as follows: (i) the surface brightness profiles reported in Shapiro et al. are decomposed in a Sérisc bulge and an exponential disk, (ii) spherical symmetry is assumed for the bulge, (iii) its luminosity density profile is calculated with the found Sérisc parameters by using the approximations given in Lima-Neto, Gerbal & Marquez (1999). We use \( \sigma_p^2 \) for the bulge, assuming an isotropic velocity-dispersion tensor and that \( \sigma_\phi = \sigma_p \); therefore, our \( f_b \) is a lower limit. To pass from luminosity to mass, we assume that the M/L ratios in the \( I \) and \( K \) band are 2 and 1.5 times larger for the bulge than for the disc, respectively.

Table 2 shows the estimates of \( f'_d(r_e/2)/f'_d, \text{inn}(0.8h_{\odot}) \) for the spirals from Shapiro et al. (2004). The \( f'_d(r_e/2)/f'_d, \text{inn}(0.8h_{\odot}) \) ratio is indeed > 1 for three galaxies and close to one for the last one. NGC 4030 has the latest type (Sbc) among the four and probably it was not affected significantly by the secular evolution. Overall, the results are consistent with what we find for the Galaxy.

### Table 2. Bulge Sérsic index \( n \) and phase-space density bulge-to-disc ratio for four galaxies.

| Name    | Type | \( n \) | \( f'_d(r_e/2)/f'_d, \text{inn}(0.8h_{\odot}) \) |
|---------|------|--------|---------------------------------|
| NGC 1068 | Sb   | 2.1    | > 6.7                           |
| NGC 2460 | Sa   | 1.7    | > 3.2                           |
| NGC 2775 | Sab  | 1.8    | > 1.3                           |
| NGC 4030 | Sbc  | 2.0    | > 0.9                           |

#### 6 CONCLUSIONS

We studied the evolution of the observational measure of the coarsely grained phase-space density, \( f' \), in high-resolution N-body simulations of Galaxy-like models embedded in live
CDM halos. In our models the initially thin stellar disc is unstable. As the system evolves, a bar with almost exponential density profile is produced. The bar redistributes matter in such a way that the disc ends with a high accumulation of mass in the centre and and extended outer disc with a density profile shallower than the exponential law. During the secular evolution, the disc is also dynamically heated and thickened, mainly in the inner parts where a bulge-like structure (peanut-shaped bar or pseudobulge) arises.

The secular evolution produces dramatic changes in the radial distribution of the coarsely grained phase-space density $f'(R)$. As the disc is heated and expanded vertically, $f'(R)$ decreases at every radius $R$. The outer region of the bar produces a large phase mixing in the inner disc –larger than at the centre. As the result, the $f'(R)$ profile develops an increasing with time valley, with a pronounced minimum in the inner disc, where the central, bulge-like mass concentration ends. In this region the vertical heating and the radial mass exchange in the disc are maximum. Our results on the evolution and shape of the $f'$ profile are qualitatively robust against initial conditions and assumptions, numerical resolution, and the way of measuring the volume density and dispersion velocities.

We conclude that the secular evolution of a collisionless galactic disc is able to form a thick, dynamically hot, central mass concentration (eventually a pseudobulge), where the phase-space density is much higher than in the inner disc. Using observational data we have estimated $f'$ at several radii for the Galaxy. In particular we estimated the $f'$ bulge-to-inner disc ratio. The qualitative agreement with our numerical results is remarkable. Therefore, the secular evolution of a collisionless disc yields a radial coarsely grained phase-space density profile in agreement with that it is observationally inferred for the Galaxy. The phase-space density constraints favor the bulge secular formation scenario. The inclusion of other important physical ingredients, as gas dissipative effects and satellite accretion, will likely enhance the secular evolution of disc models.

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