Testing the anisotropy of the universe with the distance duality relation

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The distance duality relation (DDR) is a deduction of the standard cosmological model, which is based on the assumption that the universe is homogeneous and isotropic on large scale. However, the astronomical data hint that the universe may have certain preferred direction. If indeed the case, the anisotropy would be likely to impact the DDR. In this paper, we test the anisotropy of the universe by performing two anisotropic DDR parametrizations with the dipole and quadrupole structures. The DDR is tested by comparing the luminosity distance from type-Ia supernovae (Union 2.1 and JLA compilations) and the angular diameter distance from strong gravitational lensing (SL) systems at the same redshift. It is shown that, with the Union2.1 compilation, the DDR is valid in dipole parametrization, but it is violated in quadrupole parametrization at 1σ confidence level. If JLA compilation is applied, the result is completely reversed, i.e. the DDR is valid in quadrupole parametrization, but it is violated in dipole parametrization at 2σ confidence level. Additionally, we try to incorporate galaxy clusters into SL systems to strengthen the results, but find that it is inconsistent with the results of pure SL systems. Due to the large uncertainty of available data, no strong evidence for the anisotropy of the universe is found.

Keywords: cosmology: distance scale - galaxies: clusters: general - supernovae: general

I. INTRODUCTION

The relation between luminosity distance $D_L(z)$ and angular diameter distance $D_A(z)$ at given redshift $z$, i.e. the so-called distance duality relation (DDR), has aroused enormous interests in astronomy and cosmology, since it was first introduced by Etherington [1, 2]. In the standard cosmological model, the DDR takes the simple form $D_A(z)(1+z)^2/D_L(z) = 1$. Ellis [3] proved this equation and showed that the DDR relation holds true in the conditions that, a) the spacetime is depicted by a metric theory of gravity, b) the photons travel along null geodesics and the number of photons is conserved. There will be new physics beyond the standard model if the DDR is violated. The violation of DDR could be induced by the coupling of photon with particles beyond the standard model of particle physics [4], dust extinction [5], or varying fundamental constants [6]. Hence, numerous works have been devoted to testing the validity of DDR. The most popular way is to compare the luminosity distance estimated from type-Ia supernovae (SNe Ia) and angular diameter distance estimated from strong gravitational lensing (SL) or galaxy clusters at the same redshift [7–13]. Up to now, no strong evidence for the violation of the DDR has been found. Therefore, the DDR has been used to probe the gas mass density profiles [14] and the shapes of galaxy cluster [15].

Most of previous studies are based on the homogeneous and isotropic universe, which is a crucial fundamental principle in the standard model of cosmology, namely the ΛCDM model. Although ΛCDM model is well compatible with accurate astronomical data from the Wilkinson Microwave Anisotropy Probe [16, 17] and the Planck satellite [18, 19], it also encounters many challenges. Recently, it was found that the electromagnetic fine-structure constant varies with cosmological distance from earth by the analysis of a large sample of quasar absorption-line spectra [20, 21]. Additionally, the SNe Ia data hint that the universe may have certain preferred direction [22, 24]. The recent Planck data show that the CMB temperature map possesses power asymmetry [23, 20]. All above indicate the possible violation of the cosmological isotropy, and motivate physicists to propose new models to describe the cosmology, such as the Bianchi cosmology [27, 30], the Finsler cosmology [31–34], the ΛCDM with a scalar perturbation [35], and so on. At present, the tests of DDR usually involve the measurement of $D_L$ and $D_A$ from two different objects locating at the same redshift. For example, $D_L$ can be extracted from SNe Ia, and $D_A$ can be obtained from strong gravitational lensing (SL). However, the two objects usually locate in different direction in the sky, so couldn’t be compared directly if the universe is anisotropic. Inspired by these, in this paper we try to investigate the effect of anisotropy on the DDR. We parameterize the DDR similarly to previous works [8, 13] but with the form involving the direction: $D_A(z)(1+z)^2/D_L(z) = 1 + A\cos^n\theta$, where $\theta$ represents the angle between two objects, $A$ is the amplitude of anisotropy, and $n = 1, 2$ represents the dipole and quadrupole structures of the DDR, respectively. The SL sample used in this work is compiled in [36]. We also add galaxy clusters to SL systems to enlarge the sample. For SNe Ia sample, we use two different datasets, i.e. Union2.1 [37] and JLA [38]. Lin et al. [39] searched for the anisotropic signal in two compilations of SNe Ia and found that the results are not consistent. Thus,...
is interesting to test the DDR using two different SNe Ia compilations.

The rest of the paper is organized as follows: In Section III, we introduce two anisotropic parametrizations (dipole and quadrupole parametrizations) of the DDR and describe the methodology to constrain the anisotropic parameters. In Section IV, we combine the SNe Ia, SL systems and galaxy clusters to give a constraint on the anisotropic parameters. Finally, discussion and summary are given in Section V.

II. THEORY AND METHODOLOGY

The main idea of testing the DDR is to compare the luminosity distance \( D_L \) and angular diameter distance \( D_A \) at the same redshift. If we can measure both \( D_L \) and \( D_A \) of a specific object, we can compare them directly. However, it is difficult to measure both \( D_L \) and \( D_A \). In practice, \( D_L \) and \( D_A \) are usually measured from different kinds of objects. For example, it is easy to measure \( D_L \) from SNe Ia, and \( D_A \) from gravitational lensing.

Strong gravitational lensing plays a significant role in constraining cosmological parameters [[10] [11]], testing cosmology models [[42–44] and the structure of galaxy cluster [[11] [13]]. The Einstein radius (\( \theta_E \)) is a characteristic angle for gravitational lensing, which depends on the angular diameter distances between the lens and source \( D_{A,ls} \), and between the observer and source \( D_{A,s} \). In a singular isothermal sphere (SIS) lens model, Einstein radius can be written as

\[
\theta_E = 4\pi \frac{D_{A,ls} \sigma^2_{SIS}}{D_{A,s} c^2},
\]

where \( \sigma_{SIS} \) is the velocity dispersion due to lens mass profile, and \( c \) is the speed of light. White & Davis [[45]] pointed out that \( \sigma_{SIS} \) does not necessary equal to the observed stellar velocity dispersion \( \sigma_0 \). Therefore, a phenomenological parameter \( f \) is introduced to account for the difference between these two velocity dispersions, i.e., \( \sigma_{SIS} = f \sigma_0 \).

From equation (1), we couldn’t obtain \( D_{A,ls} \) and \( D_{A,s} \) separately. However, we can derive the ratio of \( D_{A,ls} \) and \( D_{A,s} \) if both \( \theta_E \) and \( \sigma_{SIS} \) are measured, i.e.,

\[
R_A \equiv \frac{D_{A,ls}}{D_{A,s}} = \frac{\rho^2 \theta_E}{4\pi \sigma^2_{SIS}}.
\]

The uncertainty of \( R_A \) is propagated from that of \( \theta_E \) and \( \sigma_{SIS} \).

\[
\Delta R_A = R_A \sqrt{\left( \frac{\Delta \sigma_{SIS}}{\sigma_{SIS}} \right)^2 + \left( \frac{\Delta \theta_E}{\theta_E} \right)^2}.
\]

Due to the approximately constant absolute luminosity, SNe Ia are usually used as the distance indicators in cosmology. The luminosity distance can be extracted directly from the light curve of SNe Ia. The distance modulus of a SN Ia at redshift \( z \) is given by

\[
\mu_B(z; \alpha, B, \beta) = 5\log_{10} D_L(z) + 25 = m_B - M_B + \alpha x(z) - \beta c(z),
\]

where \( D_L(z) \) is the luminosity distance in unit of Mpc, \( m_B \) is the apparent magnitude observed in rest frame B band, \( x \) and \( c \) are the stretch factor and color parameter, respectively. \( M_B, \alpha \) and \( \beta \) are nuisance parameters which can be derived using the least-\( \chi^2 \) method or be marginalized. In a flat universe, the relation between the comoving distance \( r(z) \) and angular diameter distance \( D_A(z) \) is given by

\[
r(z) = (1 + z) D_A(z),
\]

and the comoving distance from lens to source is simplified to \( r_{ls} = r_s - r_l \) [[49]]. Therefore, \( R_A \) can be expressed as

\[
R_A(z_l, z_s) = \frac{D_{A,ls}}{D_{A,s}} = 1 - \frac{(1 + z_l) D_{A,ls}}{(1 + z_s) D_{A,s}}.
\]

From equation (6), the ratio of \( D_{A,ls} \) and \( D_{A,s} \) can be converted to the ratio of \( D_{A,ls} \) and \( D_{A,s} \), which can be further converted to the ratio of \( D_L(z) \) and \( D_A(z) \) using the DDR.

Several models have been proposed to describe the anisotropy of the universe. The Bianchi cosmology is one of the most popular models to describe anisotropic cosmology [[27] [30]]. Campanelli et al. [[30]] combined an anisotropic Bianchi type I cosmological model and a dark-energy fluid with anisotropic equation of state to show that, it is possible for the existence of a large level anisotropy. The redshift is expressed with a quadrupole structure in Bianchi-I model. While in the anisotropic universe described by Finsler geometry, the redshift is expressed as a dipole structure [[31] [32] [54]]. Furthermore, the low multipole models have been developed to analyze the anisotropic cosmology [[51] [54]]. Therefore, considering the anisotropy of cosmology, we tentatively parameterize the DDR as the multipole form

\[
\frac{D_A(1 + z)^2}{D_L} = 1 + A \cos^n \theta,
\]

where \( \theta \) is the angle between two objects for which \( D_L \) and \( D_A \) are measured (here it is the angle between SNe Ia and SL), \( A \) is the anisotropic amplitude, \( n = 1 \) and \( n = 2 \) represent the dipole and quadrupole parametrization, respectively.

Combining equations (6) and (7), we can obtain

\[
R_A(z_l, z_s) = 1 - R_L(z_l, z_s) q,
\]

where

\[
q = \frac{(1 + z_s)(1 + A \cos^n \theta_l)}{(1 + z_l)(1 + A \cos^n \theta_s)}.
\]
The standard error propagation technique, equation (9), is used to calibrate the uncertainty of \( R \) in this paper. The distance moduli of Union2.1 presented in [37] is obtained from the SNe Ia data according to equation (2). The parameters are obtained using the least-\( \chi^2 \) method.

\[
\chi^2_{\text{SL}} = \sum_{i=1}^{N} \left[ \frac{\Delta R_L(z_i, z_s) - 1 + q R_L(z_i, z_s)}{\sigma_i} \right]^2, \tag{11}
\]

where \( \sigma = \sqrt{\Delta R^2 + q^2 \Delta R^2} \) is the measurement error propagated from the uncertainties of \( R_L \) and \( R_A \). The free parameters are \( A \) and \( f \).

We list the best-fitting parameters and the reduced chi-square \( \chi^2 / \text{dof} \) in the second column of Table II for the dipole model and of Table III for the quadrupole model, respectively, where \( \text{dof} = N - p \) is the degree of freedom, \( N \) is the number of data points and \( p \) is the number of free parameters. We can see that the DDR is consistent with isotropy in the dipole parametrization. However, in the quadrupole parametrization, the significance of anisotropic signal is at the level of about 1\( \sigma \). From the reduced-\( \chi^2 \), it seems that the quadrupole model fits the data slightly better than the dipole model.

Additionally, the angular diameter distance can also be obtained from the galaxy cluster using the Sunyaev-Zeldovich effect. The derived angular diameter distance depends on the mass model of the cluster. Two models are often used. One is the spherical symmetry model and the other is ellipsoidal model. For the former there are 38 clusters presented in [53], and for the latter there are 25 clusters presented in [50]. Following Cao et al. [14], we also add the galaxy clusters to SL system to test the anisotropic DDR. Using equation (7), the distance modulus of cluster is given by

\[
\mu_{\text{cluster}}(z) = 5 \log_{10} \left[ \frac{D_A(z)}{1 + A \cos \theta_c} \right] + 25, \tag{12}
\]

where \( \theta_c \) is the angle between the cluster and SNe Ia matched at the same redshift \( z \). We filter the SNe Ia data as above criteria, then fit the distance modulus of the filtered SNe Ia and cluster with the least-\( \chi^2 \),

\[
\chi^2_{\text{cluster}} = \sum_{i=1}^{N} \left[ \frac{\mu_{\text{cluster}}(z_i) - \mu_{\text{sn}}(z_i)}{\sigma_i} \right]^2, \tag{13}
\]

where \( \mu_{\text{sn}} \) is the distance modulus of SNe Ia matched with galaxy cluster at the same redshift, \( z \).

\[
\sigma = \sqrt{\Delta \mu^2_{\text{cluster}} + \Delta \mu^2_{\text{sn}}}, \tag{14}
\]

In the case of the sample containing SL systems and galaxy clusters, the total \( \chi^2 \) is expressed by

\[
\chi^2 = \chi^2_{\text{SL}} + \chi^2_{\text{cluster}}. \tag{15}
\]

The best-fitting parameters are obtained by minimize equation (15).
The angular diameter distance is derived from three cases: strong lensing (SL), strong lensing + elliptical cluster model (E), and strong lensing + spherical cluster model (S).

| SL       | SL+25 Clusters(E) | SL+38 Clusters(S) |
|----------|------------------|------------------|
| $A$      | $0.0795 \pm 0.0717$ | $0.0461 \pm 0.0521$ | $-0.1219 \pm 0.0323$ |
| $f$      | $1.0299 \pm 0.0118$ | $1.0299 \pm 0.0118$ | $1.0292 \pm 0.0118$ |
| $\chi^2$/dof | $1.7226$ | $1.5477$ | $2.0978$ |

The results of adding galaxy clusters to SL systems are presented in the third and fourth columns of Table II for the dipole case and of Table III for the quadrupole case, respectively. In the elliptical cluster model, the DDR is consistent with isotropy both in the dipole and quadrupole parametrizations. However, in the spherical cluster model, the anisotropic signal is more than $3\sigma$ significant in both parametrizations. From the reduced-$\chi^2$, we can see that the quadrupole parametrization fits the data slightly better than the dipole parametrization.

IV. DISCUSSION AND SUMMARY

The variation of the DDR implies that there are new physics beyond the standard cosmological model. If the universe is anisotropic, both the luminosity distance $D_L$ and the angular diameter distance $D_A$ are direction-dependent. In this paper, we combined the SNe Ia, SL systems and galaxy clusters to test the anisotropy of the universe with DDR. Differing from previous works, the directions of SNe Ia and SL systems (or galaxy clusters) should be considered in the anisotropic cosmology. We phenomenologically parameterized the DDR to the dipole and quadrupole forms, and constrain the anisotropic parameters using the combined dataset. The luminosity distance is measured from two different compilations of SNe Ia (Union2.1 and JLA), and the angular diameter distance is measured from SL systems and galaxy clusters.

Comparing results shown in Table II and III within Union2.1 compilation, for pure SL system, the DDR validity is verified at $1\sigma$ in the dipole parametrization, but it is excluded at $1\sigma$ confidence level in the quadrupole parametrization. For the elliptical cluster model, the validity of DDR holds within $1\sigma$ uncertainty in both dipole and quadrupole parametrization. For the spherical cluster model, the DDR is violated at $\sim 3\sigma$ confidence level in both parametrizations. Filtering SNe Ia data from JLA compilation, the results are presented in Table III and IV.

The analysis for pure SL system excludes the DDR validity at $2\sigma$ confidence level in dipole parametrization, while...
in quadrupole parametrization the DDR is still valid. For two different cluster profile models added, within the dipole structure, the DDR validity is unsatisfied in 1.5σ for elliptical model, but is verified at the 1σ for spherical model. However, in the quadrupole parametrization, the results show the DDR violation at 1σ confidence level for elliptical model and 3σ confidence level for the spherical model. In summary, due to the small data sample and large uncertainty, it is still premature to make a convincing conclusion. The number of available SL system is no more than one half of the total SL systems due to the lack of matched SNe Ia. The available SL sample can be enlarged by adopting other techniques, such as using the polynomial fitting method to calculate the luminosity distance at any redshift, adding GRB data to the SNe Ia sample.

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