Damped structural dynamics models of large wind-turbine blades including material and structural damping

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Abstract. The paper presents a brief description of composite damping mechanics for blade sections of arbitrary lamination and geometry. A damped 3-D shear beam element is presented enabling the assembly of damped structural dynamic models of blades with hollow multi-cell tubular laminated sections. Emphasis is placed to the inclusion of composite material coupling effects, first in the blade section stiffness and damping matrices and finally into the stiffness and damping matrices of the finite element. Evaluations of the beam element are presented, to quantify the material coupling effect on composite beams of simple box sections. Correlations between predicted and measured modal frequencies and damping values in small model Glass/Epoxy are also shown. Finally, the damped modal characteristics of a 35m realistic wind-turbine blade model design, are predicted.

1. Introduction

The establishment of wind-turbine rotors as one of the major and most auspicious recoverable sources of energy, necessitates the maximization of their performance through longer and more flexible composite blade configurations. Large wind-turbine blades, however, introduce new technical challenges associated with their increased flexibility and deterioration in vibrational and aeroelastic response and fatigue life. The inevitable utilization of advanced carbon-epoxy composite systems in long blades is another related issue, as it improves stiffness/mass and strength/mass ratios, but reduces the damping and increases the extension-shear coupling of the material. All these issues require, among other things, the understanding and improved prediction capabilities of damping and stiffness in a composite blade structure, especially in the case of large blades. In order to aim to that target, composite material and structural coupling effects are included into a new beam finite element for composite blades within the activities of the UPWIND project, capable of predicting blade structural damping and stiffness. The proposed paper will outline the analytical, numerical and computational background required for predicting damped structural dynamic response of large composite blades with non-negligible coupling effects.

Most of the reported work in the area of composite damping modeling has been focused on damping mechanics of composite materials and laminates [1-3]. There are also analytical solutions and finite elements for the prediction of damping of laminated plate and shell structures [4-8]. In addition,
various beam formulations and finite elements for predicting the static and undamped dynamic response of composite blades have been also reported [9-13].

In the following sections the damping mechanics of a beam finite element developed by Saravanos et al. [14], for predicting the equivalent damping of composite blade sections of arbitrary lamination and geometry are expanded to include material coupling effects. Evaluation cases of the developed damp beam element are shown to quantify the material coupling effect on box sections with anti-symmetric angle-ply laminations. Comparisons between predicted and measured modal frequencies and damping ratios in small model Glass/Epoxy blades with various structures are also shown. Finally, a large 35m wind-turbine blade is modeled and damped modal analysis results are presented.

2. Mechanics of composite blade section

A hollow tubular laminated beam section (figure 1a) is assumed with arbitrary geometry and skin of arbitrary lamination defined about its mid-plane. The beams are assumed to be neither curved nor pre-twisted. The equivalent cross-section properties are expressed in terms of the coordinate system Oxyz; whereas, the skin lamination, ply properties and skin laminate properties are defined in terms of a local curvilinear system Oxsζ attached on the mid-plane of the skin, as shown in figure 1.

2.1. Section kinematics

The assumed section deformation admits extension along x-axis, bending in y and z directions, torsion about the x-axis and shear in y and z directions. The transverse normal and shear laminate stresses Nss, Nζ and transverse moment Mζ, along the local coordinate axes Oxsζ, are assumed to be negligible; s denotes hoop direction and ζ the normal to the skin mid-plane. In the curvilinear system Oxsζ the kinematic assumptions take the following form [14].

\[
\begin{align*}
  u(x, s, \zeta) &= u^o(x) + \beta_x(x)(z_0 + y^o_0 \zeta) + \beta_y(x)(y_0 - z_0 \zeta) + \theta_z(x)(-r^0_\zeta + \Psi^o(s)) \\
  v(x, s, \zeta) &= v^o(x) - \theta(x)(r^0_\zeta + \zeta) \\
  w(x, s, \zeta) &= w^o(x) + \theta(x)r^0_\zeta
\end{align*}
\]

where: u^o, v^o, w^o are the displacements of the section at the origin of the coordinate system Oxyz; β_x and β_y are bending rotation angles about axes y and z, respectively; θ is the twisting angle and \( \Psi(s, \zeta) = -r^0_\zeta + \Psi^o(s) \) is the secondary warping of the section; the comma in the subscripts indicates differentiation \( (y^0, x^0) \) and \( (r^0_\zeta, r^0_\zeta) \) are the projections of the vector \( r^0 \) describing the distance between a point O’ on the skin mid-surface from point O on the section axis-x, on the respective axes of
coordinate systems \(Oxyz\) and \(O'x's\). The assumed section deformation yields the normal and shear strains acting on the cross-section.

\[
e_x(x,s,\zeta) = e_x^0(x) + k_x(x)(y_s + y_s'\zeta) + k_{xs}(x)(y_o - z_o\zeta) + \theta_{xs}(x)(-e_{x_s}^0\zeta + \Psi'(s))
\]

\[
e_{xs}(x,s,\zeta) = e_{xs}^0(x)x_s + e_{xy}^0(x)y_o + e_{x_s}(x,s) - 2\theta_{xs}\zeta
\]

\[
e_{sx}(x,s,\zeta) = e_{sx}^0(x)y_s - e_{sy}^0(x)x_s
\]

Based on the above equations (1), the generalized strains, which equivalently describe the deformation of the section, include the axial strain \(\varepsilon_x = \varepsilon_x^0\), the transverse shear strains \(\varepsilon_{xy} = \varepsilon_{xy}^0 + \varepsilon_{sxy}\), the bending curvatures \(k_y = k_{ys}, k_x = k_{zs}\) and the twisting curvatures \(k_\theta = \theta_{s}\) and \(k_{\theta} = \theta_{sx}\). The second twisting curvature \(k_{\theta}\) expresses second-order variation and is assumed to be negligible with respect to other generalized strains. The torsional strain on the mid-surface \(\varepsilon_{ss}^0\) is evaluated by considering and solving the torsion stress equilibrium equation on the \(s\zeta\) plane using a properly chosen torsional strain function \(\Phi\).

2.2. Equations of Motion

The equations of motion of the beam can be described by a variational form

\[
\int_0^L dx \int_A \left( -\delta H + \delta T - \delta W_d \right) dsd\zeta + \int_\Gamma \delta \bar{T} \bar{d} d\Gamma = 0
\]

where \(H\) and \(T\) are the strain and kinetic energy, \(W_d\) is the dissipated energy, \(\bar{T}\) overbar is surface tractions on the free surface \(\Gamma\), \(A\) is the cross-sectional areas covered by material and \(L\) is the length of the beam. The variation of the strain and kinetic energy of the section is represented by the respective integrals over the cross-section area.

\[
\delta H_s = \int_A \delta \varepsilon_s^T \sigma_s dsd\zeta = \int_h ds \int_H \delta \varepsilon_s^T [Q_c] \varepsilon_s d\zeta
\]

\[
\delta T_s = \int_A -\delta u^T \text{diag}(\rho) \bar{u} d\zeta = \int_h ds \int_H -\delta u^T \text{diag}(\rho) \bar{u} d\zeta
\]

The variation of the dissipated strain energy due to composite damping during a vibration cycle will be:

\[
\delta W_{dc} = \int_A \delta \varepsilon_s^T [Q_c] \eta_c \varepsilon_s dsd\zeta = \int_h ds \int_H \delta \varepsilon_s^T [Q_c] \eta_c \varepsilon_s d\zeta
\]

In the above equations (4, 5), \(\varepsilon_s = \{\varepsilon_{s1}, \varepsilon_{s2}, \varepsilon_{s3}\} = \{\varepsilon_{xs}, \varepsilon_{xc}, \varepsilon_{cs}\}\) and \(\sigma_s = \{\sigma_{s1}, \sigma_{s2}, \sigma_{s3}\} = \{\sigma_{ss}, \sigma_{sc}, \sigma_{cs}\}\) are the off-axis strains and stresses of a composite ply; \([Q_c]\) and \(\eta_c\) are the equivalent off-axis stiffness and damping (loss-factor) matrices of the composite ply with respect to the system \(O'x's\zeta\); \(\rho\) is the density of the ply and \(h\) is the thickness of the skin laminate. The damping matrix \(\eta_c\) is related to 3 in-plane damping coefficients of the composite ply, which are the longitudinal, the transverse and the in-plane shear loss-factor, respectively [4]. It is pointed out that both ply stiffness and damping matrices include axial-in plane shear coupling terms, \(Q_{16}, \eta_{16}\), respectively.
2.3. Section stiffness

Using the strain expressions provided by equation (2) in combination with equation (4), integrating over the skin thickness and around the skin midline, the stored strain energy of the section is finally expressed in terms of generalized strains and equivalent section stiffness terms, as follows:

\[
\delta H_s = \begin{bmatrix} \delta \varepsilon^0_x \\ \delta \varepsilon^0_y \\ \delta \varepsilon^0_z \end{bmatrix} \begin{bmatrix} A^0 \\ B^0 \\ C^0 \end{bmatrix} \begin{bmatrix} \varepsilon^0_x \\ \varepsilon^0_y \\ \varepsilon^0_z \end{bmatrix}
\]

(6)

where, \(\varepsilon^0=\{\varepsilon^0_x, \varepsilon^0_y, \varepsilon^0_z\}\) and \(k=\{k_{xx}, k_{yy}, k_{zz}\}\) represent the average strains and curvatures of the section, respectively. The previous equations yield the equivalent extensional–shear coupling and flexural–torsional stiffness matrices \([A^0],[B^0]\text{ and }[D^0]\), of the cross-section, having the form:

\[
[A^0] = \begin{bmatrix}
A_{11}^0 & A_{12}^0 & A_{16}^0 \\
A_{21}^0 & A_{22}^0 & A_{26}^0 \\
A_{61}^0 & A_{62}^0 & A_{66}^0 \\
\end{bmatrix}
[\begin{bmatrix}
A_{11}^0 & A_{12}^0 & A_{16}^0 \\
A_{21}^0 & A_{22}^0 & A_{26}^0 \\
A_{61}^0 & A_{62}^0 & A_{66}^0 \\
\end{bmatrix}]
[\begin{bmatrix}
B_{11}^0 & B_{12}^0 & B_{16}^0 \\
B_{21}^0 & B_{22}^0 & B_{26}^0 \\
B_{61}^0 & B_{62}^0 & B_{66}^0 \\
\end{bmatrix}]
[\begin{bmatrix}
D_{11}^0 & D_{12}^0 & D_{16}^0 \\
D_{21}^0 & D_{22}^0 & D_{26}^0 \\
D_{61}^0 & D_{62}^0 & D_{66}^0 \\
\end{bmatrix}]
\]

(7)

In the previous matrices the overbar indicates the section stiffness terms, which depend directly to the extension-shear coupling stiffness components of the skin laminate \(A_{16}, B_{16}, D_{16}\). These terms induce material coupling between bending and torsion (terms \(D_{16}^0\) and \(D_{26}^0\)), coupling between extension and torsion (term \(B_{16}^0\)) and so forth. Conversely, these coupling terms vanish if the respective skin laminate coupling stiffness is negligible or zero.

2.4. Section damping

Similarly, substituting the strain relations, equation (2) into the dissipated energy equations (5) and integrating over the cross-sectional area, we arrive at the dissipated energy of the beam expressed in terms of generalized strain amplitudes

\[
\delta W_{ds} = \begin{bmatrix} \delta \varepsilon^0 \\ \delta H_s \\ \delta \varepsilon^0 \\ \delta C^0 \\ \delta D^0 \\ \delta \theta \end{bmatrix} \begin{bmatrix} A_d^0 & B_d^0 & C_d^0 & D_d^0 \end{bmatrix} \begin{bmatrix} \varepsilon^0 \\ \varepsilon^0 \\ \varepsilon^0 \\ \varepsilon^0 \\ \varepsilon^0 \\ \varepsilon^0 \end{bmatrix}
\]

(8)

where, \(W_{ds}\) is the dissipated energy per unit length of the beam subject to an arbitrary combination of cyclic strain and bending curvatures and twisting angles. The damping matrices \([A_d^0],[B_d^0]\text{ and }[D_d^0]\) have the following form,

\[
[A_d^0] = \begin{bmatrix}
A_{d11}^0 & A_{d12}^0 & A_{d16}^0 \\
A_{d21}^0 & A_{d22}^0 & A_{d26}^0 \\
A_{d61}^0 & A_{d62}^0 & A_{d66}^0 \\
\end{bmatrix}
[\begin{bmatrix}
B_{d11}^0 & B_{d12}^0 & B_{d16}^0 \\
B_{d21}^0 & B_{d22}^0 & B_{d26}^0 \\
B_{d61}^0 & B_{d62}^0 & B_{d66}^0 \\
\end{bmatrix}]
[\begin{bmatrix}
D_{d11}^0 & D_{d12}^0 & D_{d16}^0 \\
D_{d21}^0 & D_{d22}^0 & D_{d26}^0 \\
D_{d61}^0 & D_{d62}^0 & D_{d66}^0 \\
\end{bmatrix}]
\]

(9)

The elements in the \([A_d^0],[B_d^0]\text{ and }[D_d^0]\) damping matrices express the equivalent damping and energy dissipation per unit length of the beam due to extension-shear, extension-bending coupling and bending-torsion deformation, respectively. In the previous stiffness and damping matrices in equations (7,9), the terms with overbar are associated with extension-shear ply coupling, induced by nonzero ply stiffness \(Q_{c16}\) and damping \(\eta_{c16}\) terms.

The equivalent section mass matrices are not affected by the coupling mechanism and they have already been estimated [14].
3. Finite element formulation
A three-dimensional shear beam finite element was developed for the damped dynamic analysis of
tubular composite beam structures (figure 1b) such as wind-turbine blades and helicopter rotor blades.
The beam finite element formulation is based on the kinematic assumptions and shear damping beam
theory described in the previous section. The element has 6 DOFs at each node (indicated with superscript i),

\[ \mathbf{U}_i = \{u^{li}, v^{li}, w^{li}, \beta_f^{i}, \beta_t^{i}, \theta^{i}\} \]  

which are respectively: the three displacements \( u, v, w \) at the origin \( Oxyz \) of the section; the two
bending rotation angles; and the twisting angle. The displacements along the axis of the beam, are
approximated by \( c^0 \) continuous interpolation functions. The same interpolation functions are used for
each DOF. By applying the previous displacement and strain approximation equations (1, 2) into the
equations of motion (3, 4, 5) and collecting the common coefficients, the total stiffness \([K]\), mass \([M]\)
and damping \([C]\) matrices of the beam finite element are derived similarly by Saravanos et al. [14].

For the case of free vibration, the equations of motion take the final form:

\[ -\omega^2[M]\mathbf{U} + j[C]\mathbf{U} + [K]\mathbf{U} = 0 \]  

(11)

Solution of this equation yields the complex eigenvalues of the problem. An alternative procedure,
used herein, relies on the calculation of undamped modal frequencies and mode shapes of the beam
structure (\([C]=0\)). Using the dissipated energy method, the modal loss factors of the beam are
calculated from the ratio of the dissipated energy over the maximum stored modal strain energy of the
respective undamped mode shape, which have the form:

\[ \eta_m = \frac{1}{2\pi} \frac{\mathbf{U}_m^T[C]\mathbf{U}_m}{\mathbf{U}_m^T[K]\mathbf{U}_m} \]  

(12)

where \( \mathbf{U}_m \) is the mode deflection. A beam element with 2 nodes and linear shape functions was
developed and encoded into a research code DAMPBEAM, which predicts the damped dynamic
characteristics (natural frequencies and modal damping) of the beam model, using the energy method
described above.

4. Small-Scale Model Blade Testing
Two small model Glass/Epoxy blades with uniform sections have been fabricated and modally tested
as part of the DAMPBLADE project [15]. The blades have uniform Glass/Epoxy skins of uniform
thickness but they differ to each other in terms of their sectional structure. Both blades (figure 2) entail
two [0\( \psi/(45/-45)_2\)] girder sections and [(+45/-45)_5] laminations at the leading (LE) and trailing edge
(TE) sections. The first blade has a [(45/-45)_3 /FOAM/(45/-45)_3] shear web (figure 2a), while the
second blade has no shear web (figure 2b).

![Figure 2](image_url)

**Figure 2.** Cross sections of manufactured model blades: (a) blade (i); (b) blade (ii).
The blades were suspended by chords in a nearly free-free configuration, thus minimizing the effects of supports on overall modal damping of the tested blades. The blades were dynamically excited using an electromagnetic shaker with an instrumented tip applying a continuous swept-sine waveform point force. The acceleration of the vibrating blade was measured using a tri-axial accelerometer at a grid of locations over the span of the blade. The signals from the force and accelerometer sensors were conditioned and then acquired by a workstation based data acquisition and frequency analyzer system implementing a high-speed DAQ card and customized Labview® software. The acquired time signals were transformed to frequency domain and the accelerance (acceleration/force) frequency functions were calculated and correlated with a parametric model of complex exponentials using a least-squares fit to extract the modal characteristics of the tested blade.

5. Numerical results

5.1. Glass/Epoxy small blades
The DAMPBEAM element was used to model the two fabricated small Glass/Epoxy blades. The results, in tables 1 and 2 consist of comparison between the experimental modal frequencies and damping values with the predicted values from the DAMPBEAM code.

| Mode         | Natural Frequency (Hz) | Modal Loss Factor (%) |
|--------------|------------------------|-----------------------|
|              | Beam Element | Experiment | Beam Element | Experiment |
| 1st Flapping | 79.60        | 78.89       | 0.60         | 0.55       |
| 1st Sweeping | 268.5        | 262.6       | 0.93         | 1.2        |
| 1st Torsional| 198.5        | 198.3       | 0.79         | 2.0        |

| Mode         | Natural Frequency (Hz) | Modal Loss Factor (%) |
|--------------|------------------------|-----------------------|
|              | Beam Element | Experiment | Beam Element | Experiment |
| 1st Flapping | 82.3         | 79.9        | 0.58         | 0.44       |
| 1st Sweeping | 280.9        | 306.1       | 0.93         | 0.98       |
| 1st Torsional| 193.4        | 193.7       | 0.79         | 0.94       |

It is obvious that the present damped element seems to provide reasonably good predictions of both natural frequencies and modal damping of the two blades. The modeling difficulties imposed by the short length of the blades, which deviate from ideal beam behavior should be considered in assessing the quality of the beam element predictions, together with the effects of possible imperfections imposed by the hand lay-up fabrication method and of the extra non-structural mass of adhesive putty which existed in the tested blades. More importantly, the present model did not aim to model the shearing of the adhesive layers which was observed to contribute in the case of blade damping.

Moreover, due to the short length, transverse shearing of the cross section appeared significant. While the present beam element assumes uniform shearing of the cross-section, which is a step in the right direction, yet, future improvements may be required. Especially, for the tested blade with shear web, the predicted damping loss factor is much smaller from the measured value. This difference is primarily due to the fact that while the adhesive and the shear web contributes to the total structural damping, the DAMPBEAM code can only evaluate an average influence of shear web to the total
structural damping. We finally point out that the model is deterministic, so there is no capability to model effect of uncertainties on the blade response. But, nevertheless, the shown correlations also quantify possible deviation between the model and actual model blade possibly uncertainties. Overall, the DAMPBEAM finite element has captured both values and trends seen in the measured data, which leads to the conclusion that the accuracy of the predictions will increase for longer, more slender blade configurations. The successful comparison quantifies the expected trends in the damping behavior of composite blades.

5.2. Box beam with material coupling
In these cases, emphasis is given on the effect of material coupling on Carbon/Epoxy beams of box sections and on the capability of finite element to model it. Static and modal analysis results are provided for a L=0.762m long box beam with uniform sections having \([0/0]/3\), \([-0/0]/3\), \([0]_6\) and \([-0]_6\) skin laminations at the left, right, top and bottom side, respectively, proposed by Volovoi and Hodges [12, 13] which exhibits high material coupling.

Figures 3, 4 and 5 show the predicted tip displacement, tip slope and tip twist for the above beam for a tip load of 4.45N, for the respective cases of including and neglecting the material coupling terms equations (7, 9). There is a substantial improvement in the predicted values of tip displacement and bending angle when material coupling is included in the finite element, in the range of ply orientations yielding non-negligible shear-extension coupling. Moreover, as illustrated in figure 5, the inclusion of material coupling terms in the model captures the twisting of the beam, while the finite element neglecting coupling yields fails to do so. The effect of material coupling on the statically deflected slope of the beam with laminations having ply angles \(\theta=20^0\) and \(\theta=45^0\), respectively, is shown in figures 6a, 6b and 6c.

![Figure 3. Tip displacement of the box beam.](image1)

![Figure 4. Tip slope of the box beam.](image2)
Figure 5. Tip twist of the box beam.

Figure 6. Deflected shape of Carbon/Epoxy box beam for $\theta=20^0$ and $\theta=45^0$: (a) Transverse displacement; (b) bending angle; (c) twisting angle.

The modal damping and natural frequencies of Glass/Epoxy box section beams with similar laminations to the previous case were also predicted. Figures 7 and 8 show the first modal flapping frequency and modal damping loss factor values for a range of ply angles from $\theta=0^0$ to $90^0$. 
It is obvious that inclusion of coupling yields lower natural frequency values, which is attributed to the softening of the beam due to the material coupling. This difference is eliminated at $\theta=0^\circ$ and $90^\circ$, where, indeed, the material coupling effect is physically eliminated. Similarly, the inclusion of coupling provides higher predictions of modal damping and dissipated energy, for a ply angle range from $\theta=5^\circ$ to $50^\circ$, of modal structure damping.

5.3. 35m wind-turbine blade
The finite element code was finally applied for the prediction of the modal characteristics of a realistic 35m wind-turbine blade design (figure 9), in terms of external/internal geometry, skin laminations, materials and structural details, and so forth. The blade is made of Glass/Epoxy composite sandwich laminates, which are supported by two shear webs along its length.

Table 4 presents the predicted modal frequencies and loss factors for the first three flapwise, edgewise and torsional modes of the blade, with and without coupling consideration. There is no difference in the results between these two cases, probably due to the fact that the material coupling effects are insignificant in the 35m wind-turbine blade. The main reason for this result is the symmetric lay-up fabrication of the blade’s structure, which in addition to the high number of plies,
eliminates the extension-shear skin coupling term $A_{16}$, the bending-shear skin coupling term $B_{16}$ and the bending-torsion skin coupling term $D_{16}$.

Table 4. Predicted modal damping and natural frequencies of 35m Glass/Epoxy blade.

| Mode          | Natural Frequency (Hz) | Modal Loss Factor $\eta$ (%) |
|---------------|------------------------|-----------------------------|
|               | Coupling | NO Coupling | Coupling | NO Coupling |
| 1st Flapping  | 1.113    | 1.113       | 0.854    | 0.854 |
| 2nd Flapping  | 3.138    | 3.138       | 0.882    | 0.882 |
| 3rd Flapping  | 6.512    | 6.512       | 0.921    | 0.921 |
| 1st Sweeping  | 1.654    | 1.654       | 1.695    | 1.695 |
| 2nd Sweeping  | 5.251    | 5.251       | 1.606    | 1.606 |
| 3rd Sweeping  | 12.336   | 12.336      | 1.677    | 1.677 |
| 1st Torsional | 16.332   | 16.332      | 1.734    | 1.734 |
| 2nd Torsional | 26.633   | 26.633      | 1.642    | 1.642 |
| 3rd Torsional | 37.811   | 37.811      | 1.700    | 1.700 |

6. Conclusions

The thrust of this paper was to present the capability of a new blade beam finite element to predict the damped structure dynamics of composite wind-turbine blades. New stiffness and damping terms encompassing material coupling effects were included. The damped beam element was encoded in the DAMPBEAM code and used to calculate the natural frequencies and modal damping of two small fabricated model blades with different design configurations. Good correlation between measured data and predicted DAMPBEAM values was achieved for the first model despite the difficulties imposed by the small length aspect ratio of the model blades. Furthermore, the effect of material coupling on laminated Graphite/Epoxy and Glass/Epoxy cantilever box-section beams was investigated. Notable differences in predictions were observed to the static characteristics of the blade with the coupling inclusion. In addition, similar differences were present in modal damping and natural frequency predictions of the box beam for a range of ply angles. Finally, the modal characteristics of a current design of a 35m wind-turbine blade were investigated. The results did not show any noticeable gains when material coupling terms are included, due to the symmetry in blade section construction and the high number of plies, which apparently eliminate material coupling. Asymmetric section configurations will be investigated in the near future. Additional, future work will address the non-linear effects and present in the damped structural dynamics of large wind-turbine blades.

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