Flat Parallelization

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Abstract

There are two intertwined factors that affect performance of concurrent data structures: the ability of processes to access the data in parallel and the cost of synchronization. It has been observed that for a large class of “concurrency-unfriendly” data structures, fine-grained parallelization does not pay off: an implementation based on a single global lock outperforms fine-grained solutions. The flat combining paradigm exploits this by ensuring that a thread holding the global lock sequentially combines requests and then executes the combined requests on behalf of concurrent threads.

In this paper, we propose a synchronization technique that unites flat combining and parallel bulk updates borrowed from parallel algorithms designed for the PRAM model. The idea is that the combiner thread assigns waiting threads to perform concurrent requests in parallel.

We foresee the technique to help in implementing efficient “concurrency-ambivalent” data structures, which can benefit from both parallelism and serialization, depending on the operational context. To validate the idea, we considered implementations of a priority queue. These data structures exhibit two important features: concurrent remove operations are likely to conflict and thus may benefit from combining, while concurrent insert operations can often be at least partly applied in parallel thus may benefit from parallel batching. We show that the resulting flat parallelization algorithm performs well compared to state-of-the-art priority queue implementations.

1 Introduction

There are two intertwined factors that affect performance of concurrent data structures: the ability of threads to access the data in parallel and the cost of synchronization. For many data structures, such as linked lists, skiplists, trees, etc., a high degree of parallelism can be achieved by fine-grained synchronization among concurrent threads. It has been observed, however, that for a large class of “concurrency-unfriendly” data structures, such as queues, stacks, and priority queues, fine-grained parallelization does not pay off. Operations on such data structures are likely to conflict on data “hotspots” and, as a result, the costs incurred by fine-grained synchronization mechanisms (low-level locks or lock-free synchronization primitives) do not allow us to overcome the performance of simple coarse-grained solutions based on a single global lock.

In the flat combining paradigm \cite{5}, a thread holding the global lock sequentially combines requests published by concurrent threads and then executes the combined requests on their behalf. Besides providing starvation-freedom (every thread makes progress), the approach enables an efficient execution of combined updates: e.g., a bunch of updates can often be performed by the combiner thread in a single pass over the data structure. Also, some updates, e.g., concurrent operations pop and push on a stack, can eliminate each other without touching the data structure.

However, while the combiner is busy performing the requests, other threads remain idle and the available multi-processor computational power remains unused. In this paper,
we propose a technique that leverages this power, while preserving the advantages of flat combining. In particular, we consider the bulk-update technique devised for the PRAM model of synchronous parallel computing. At a high level, the combiner (or the leader thread in our terminology) distributes “parallelizable” updates in a bulk among the waiting (worker) threads, so that they could perform the updates in a conflict-free way.

We foresee the technique to help in implementing efficient “concurrency-ambivalent” data structures, which can benefit from both parallelism and serialization, depending on the operational context. To validate the approach, we considered binary heap-based implementations of a priority queue. This data structure exhibits two important features: concurrent extractMin operations are likely to conflict and thus may benefit from combining, while concurrent insert operations can often be (at least partially) applied in parallel and thus may benefit from parallel batching. We show that the resulting flat-parallelization algorithm performs well compared to state-of-the-art concurrent priority queue implementations.

2 Algorithm

As a basis of our algorithm we took the binary heap based implementation of the priority queue. Despite the fact that the binary heap algorithm has one of the worst asymptotics compared to other heaps, it is easy to implement and its array-based implementation is fast because it does not induce overhead on memory management.

We briefly describe the sequential array-based binary heap implementation on which we based our parallel bulk-update algorithm. Note, that the implementation of insert operation differs from the one described in [3]. The heap is represented as a 1-indexed array $a$ with size $S$ where node $v$ has children $2v$ and $2v + 1$. The heap should satisfy the property that for any node $v$ the value in $v$ is less than the values in the children. The two operations extractMin and insert are performed as follows:

- ExtractMin operation swaps $a[size]$ with $a[1]$ and then performs a procedure sift down to restore the heap property. We start the procedure from the root. At each iteration, we are located in some node $v$. We compare $a[v]$ with the values in children and consider two cases. $a[v]$ is less than the values in the children then the property of the heap is satisfied and we stop our operation. Otherwise, we choose the child $c$, either $2v$ or $2v + 1$, with the smallest value, swap values $a[v]$ and $a[c]$, and continue with $c$.

- During insert(x) operation we consider the path from the root to node $S + 1$. The newly inserted value $x$ should be somewhere on that path, thus we only need to insert it in the proper place and shift the rest of the path one level down. We initialize a variable $val = x$ and start traversing the path from the root to $S + 1$. Suppose, that we are currently located in node $v$. We compare $val$ and $a[v]$ and consider two cases. If $val$ is less than $a[v]$ then we swap these values and continue with the corresponding children $c$, i.e., the next node on the path. Otherwise, $val$ is bigger than $a[v]$ and we simply continue with $c$.

The complexity of classical and described implementations are $O(\log n)$. Nevertheless, the original approach works faster on average, because it generally does not need to traverse the whole height of the heap.

Now, we are ready to describe our parallel bulk-update algorithm. Our algorithm separates extractMin and insert and processes, firstly, a batch of extractMin requests and then a batch of insert requests. In a few words, for $k$ extractMin operations we swap $k$ smallest values with $a[S - k + 1], a[S - k + 2], \ldots, a[S]$ and perform $k$ parallel sift down procedures, each in a separate thread. The algorithm benefits from the fact that sift downs are in general
operate on different subtrees. Note, that for this algorithm we have to find \( k \) smallest values first. For \( k \) \textbf{insert} operations one of the threads starts inserting all the values simultaneously and traverse the heap towards nodes \( S + 1, \ldots, S + k \). Sometimes, it splits its set of values and gives the tasks to other threads. Now, we describe each bulk-update operation in more details.

- \( k \) \textbf{extractMin} operations. As discussed earlier we are provided with the nodes with the smallest \( k \) values. At first, we set the special field \textit{locked} in each of these nodes. Then, we swap these values with the \( k \) latest values in the heap, i.e., \( a[S - k + 1], \ldots, a[S] \). The nodes where were the smallest values now become the start positions of \( k \) parallel sift down procedures, each performed by a separate thread. The \textit{locked} field specifies if the value in the node is currently under swap. Each thread starts its own sift down from the corresponding node. In each iteration it considers some node \( v \). The thread waits while the children of \( v \) has \textit{locked} field set. When all the children becomes unlocked, the thread compares the value \( a[v] \) with the values in the children. If \( a[v] \) is smaller then we unset the \textit{locked} field of \( v \) and stop the procedure. Otherwise, we choose the child \( c \) with the smallest value, we set \textit{locked} field of \( c \), swap \( a[v] \) with \( a[c] \), unset \textit{locked} field of \( v \) and continue the next iteration with \( c \).

- \( k \) \textbf{insert} operations. As in the sequential implementation of \textbf{insert}, \( k \) newly inserted values should be on the paths from the root to nodes \( S + 1, \ldots, S + k \). Primarily, we set the special field \textit{split} in the nodes that have some of the nodes \( S + 1, \ldots, S + k \) in both subtrees. Then one thread starts from the root while other threads wait on the nodes with \textit{split} set. Note, that there are exactly \( k - 1 \) \textit{split} nodes. The thread that starts from the root is provided with the set of \( k \) inserted values. Each thread on each iteration is located at some node \( v \). If the smallest value from its set is less than \( a[v] \), it puts the smallest value from the set to \( a[v] \) while inserting \( a[v] \) into set, otherwise, the thread does nothing. After, it checks whether the \textit{split} field is set in node \( v \). If it is not set, then the thread continues with the corresponding child. Otherwise, the thread splits the set into two parts, in proportion of how much nodes from \( S + 1, \ldots, S + k \) are in the left subtree and in the right subtree. The thread gives the set for the right subtree to the thread that waits at the node \( v \) and unsets \textit{split}. By itself, the thread continues with the left children, while the waken up thread starts with the right child.

We argue, that the described algorithm, given the preliminary work done, performs a bulk-update on \( k \) operations in \( O(k \log n) \) work and \( O(k + \log n) \) span. Thus, providing us with linear speedup when \( k \leq \log n \).

Now, given the parallel batching algorithm we explain how we incorporate it together with the flat combining technique, probably omitting some optimizations.

- A thread puts a request of the operation in the queue. Each request consists of the type of the operations, either \textbf{extractMin} or \textbf{insert}, and the status of the request. The status could be one of the three types: \textbf{PUSHED}, \textbf{SIFT} and \textbf{FINISHED}. The freshly pushed request has \textbf{PUSHED} state.

- The thread tries to take a lock on the data structure. If it succeeds then it becomes a leader.

- If the thread becomes a leader it performs the following:
  - The leader takes all non-performed operations from a queue. Sorts them by the type and the value. Let \( E \) be the number of \textbf{extractMin} requests and \( I \) be the number of \textbf{insert} requests.
  - The leader finds \( E \) nodes with the smallest values and sets their \textit{locked} fields. Then he swaps their values with the \( \min(E, I) \) newly inserted values and the rest with the
latest $E - \min(E, I)$ values in the heap. That is how we combine two types of requests. And finally, he sets the state of \texttt{extractMin} requests to SIFT and the state of first \texttt{insert} requests to FINISHED.

- The leader, probably, performs \texttt{extractMin} operation by itself and then waits while the workers with \texttt{extractMin} request set their status to \texttt{FINISHED}.
- The leader sets the \texttt{split} field of the nodes that have nodes $S+1, \ldots, S+(I-\min(E, I))$ in both subtrees. Then it sets the state of remaining \texttt{insert} requests to SIFT.
- The leader, probably, performs \texttt{insert} operation by itself and then waits while the workers with \texttt{insert} request set their status to \texttt{FINISHED}.
- If the thread is not a leader, then it spins until the status of its request becomes SIFT. Then it performs its operation and sets the status to \texttt{FINISHED}.

One could calculate the number of remote memory accesses during one operation. It does not exceed $O(P + \log S)$, where $P$ is the number of threads working on the priority queue and $S$ is the current size of the heap.

3 Experiments

For our experiments, we used 4-processor AMD Opteron 6378 2.4 GHz server with 16 threads per processor (yielding 64 threads in total), 512 Gb of RAM, running Ubuntu 14.04.5. It has Java 1.8.0_111-b14 and HotSpot JVM 25.111-b14.
We compare our algorithm (FC Parallel) against five implementations: flat combining with binary heap (FC Binary), flat combining with pairing heap (FC Pairing), lazy lock-based skip-list (Lazy SL), lock-free skip-list similar to implementation in java.concurrency package (Lock-free SL) and and coarse-grained binary heap (Coarse Binary). We are aware of Linden-Johnson algorithm, but we do not have its Java implementation. For more information about concurrent priority queue implementations we refer the reader to survey. The code is available at https://github.com/Aksenov239/FC-heap.

We provide results for four different settings: the queue is prepopulated with $8 \cdot 10^5$ or $8 \cdot 10^6$ random integer values, and the inserted values are from $[0, 1000]$ or $[0, 2^{31} − 1]$. For the depicted plots we assumed a workload with 50% extractMin and 50% insert operations. Each point is averaged over 5 runs of 10 seconds with the warmup of 10 seconds.

Our algorithm performs badly on the small number of processors for two reasons. First, the algorithm induces overhead on the preparation and combining parts of flat combining. Second, our insert operation incurs overhead with respect to the state-of-the-art algorithm. But when the number of processors increases, the parallelization start overwhelming these disadvantages. Starting from 20 threads our algorithm outperforms all other algorithms.

At the same time, the results of coarse-grained algorithm are surprisingly good in compare to flat combining approaches. But by common sense, flat combining implementations should perform better. We link this to the fact that the coarse-grained algorithm is implemented using the reentrant lock from Java’s concurrent package while our flat combining implementation were written from scratch identically to the original implementation in C++.

There two reasons why our algorithm slower than the coarse-grained on almost all settings. First, with the increase of the initial size the ratio of the time spent on the preparation work of the leader and the time spent during the parallel algorithm decreases. That is why the gap between algorithms shrinks when size increases. Second, the insert algorithm of the classical implementation of the binary heap spent less time on average on smaller range of values rather than on bigger range, while the execution of our algorithm does not change. Thus, when the range increases the gap between algorithms decreases.

Anyway, we find this very encouraging that our algorithm on the settings of initial size $8 \cdot 10^6$ with values from $[0, 2^{31} − 1]$ outperforms the coarse-grained implementation on 63 processors.

4 Conclusion

In this paper, we studied new design of concurrent data structures that unites together flat combining and parallel bulk-update. We applied this approach to the priority queue data structure. The evaluation suggests that proposed technique could be an adequate design principle since the demonstrated performance is comparable to the state-of-the-art algorithms.

Besides performance gains obtained by exploiting idle threads, there might be other benefits of our flat parallelization technique. First, given a parallel bulk-update algorithm the technique may automatically produce an efficient concurrent counterpart. For example, one can devise a concurrent dynamic forest with insert- and remove-edge operations, given the bulk-update algorithm described in [1]. Second, the technique can be used to maintain certain non-trivial properties of data structures, e.g., the strict balancing condition in binary search trees. Indeed, there exist strictly-balanced parallel bulk-update implementations, while up-to-date concurrent algorithms only maintain relaxed AVL condition [2]. Finally, we argue that complexity bounds of the data structures designed using flat parallelization can
be computed easier using the notions of work and span, well-established in the PRAM model.

A more thorough evaluation analysis of our approach is indispensable. We should run our experiments on a larger scale, to check if the performance gap with considered algorithms continues to grow. Also, we need to enable a comparison with C++ implementations, e.g., the one by Lindén and Jonsson [6], believed to be the best concurrent priority queue to date [4]. Finally, we should explore the potential of the technique on other data structures.

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