Investigation of the electromagnetically induced transparency in the era of cosmological hydrogen recombination

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Abstract

Investigation of cosmic microwave background formation processes is one of the most compelling problems at the present time. In this paper we analyse the response of the hydrogen atom to external photon fields. Field characteristics are defined via conditions corresponding to the recombination era of the universe. Approximation of the three-level atom is used to describe the ‘atom–field’ interaction. It is found that the phenomena of the electromagnetically induced transparency (EIT) take place in this case. Consideration of EIT phenomena makes it necessary to update the astrophysical description of the processes of cosmic microwave background formation and, in particular, the Sobolev escape probability. Additional terms to the optical depth entering in the Sobolev escape probability are found to contribute on the level about 1%.

(Some figures may appear in colour only in the online journal)

1. Introduction

In view of recent success in the theoretical description and experimental observations of the cosmic microwave background (CMB), a detailed analysis of all the processes occurring in the cosmological recombination era is required. For a precise description of CMB in astrophysical investigations, the different photon emission and absorption, photon–electron scattering, etc, processes should be included. The spectral characteristics of the emitted radiation due to different phenomena, the polarization state and the cross section are particularly relevant from the astrophysical point of view. In standard calculations for such kinds of tasks, the quantum mechanical approach for the isolated atom is applied.

However, consideration of the ‘atom–field’ interaction becomes important in the case of astrophysical experiments with an accuracy of about 1% [1, 2] and with the expectation of even increasing the accuracy up to the level of ∼ 0.1%. The atom’s interaction with an external field can lead to effects such as population inversion, steady-state solution, line strengths, cross sections, susceptibility and polarization, etc [3–8]. Apart from other powerful methods (the Green function approach, for example), the statistical operator theory can be applied for the study of ‘atom–field’ systems. Application of the density matrix formalism in a three-level approximation seems to be more simple and appropriate in this case. The clear description of the density matrix theory and its applications, such as spontaneous emission and line broadening (power broadening and saturation, collision line broadening, Doppler broadening and the Voigt profile), can be found, for example, in [9].

The radiation transfer theory that is usually applied in CMB research was suggested in [10, 11]. In particular, it
was established that the $2s \leftrightarrow 1s$ transition is able to substantially control the dynamics of cosmological hydrogen recombination. Moreover, distortions of the order $10^{-6}$ were predicted [10]. Recently, the radiation transfer theory for the recombination era of the early universe was intensively re-examined due to precise observations of the CMB [12, 13]. In [14, 15] corrections to the ionization history were found that exceed the per cent level. In order to achieve this accuracy, the multi-photon decays in atoms involving set of states should be included in the evaluation of CMB formation [16–24]. As a rule the two-photon decays of the excited states are considered for the evaluation of CMB. Two-photon emission processes were evaluated accurately in [25–27]. In our recent works [28, 29], three- and four-photon transitions with separation out of the two-photon links were considered from the astrophysical point of view. In [30, 31] we considered one- and two-photon transitions in external electric fields. Such modifications should have a strong impact on the determination of key cosmological parameters [32].

Generally, the absorption coefficient calculated per atom is used for the study of the radiative transfer in spectral lines. The influence of powerful high-frequency electromagnetic radiation on the absorption coefficient in the low-frequency line in a three-level $\Lambda$ atom was considered in [33]. In this paper we consider the other kind of multi-photon process, namely electromagnetically induced transparency (EIT) phenomena. The nature of EIT phenomena can be examined by the evaluation of the response of the multi-level system to the presence of an external radiation field. EIT leads to a significant modification of the absorption profile of the system. Description of EIT phenomena for a three-level ladder system interacting with two near-resonant monochromatic fields can be found, for example, in [34–36]. A complete and detailed treatment of a three-level ladder atom, absorption and emission spectra, as the transient and steady-state responses of the $\Xi$ atom is given in [34]. We study the response of the three-level ladder system on the external fields originating from photons emitted during the recombination and evaluate the absorption coefficient, which we apply to radiation transfer theory. The physics of an atom interacting with photon fields can be understood on the basis of the ‘interfering-pathway’ description, which corresponds to the multi-photon process defined in terms of a power series expansion over field amplitudes (see e.g. [34–36]).

The rest of this paper is organized as follows: in section 2 we shall briefly review essentials of the density matrix approach when deriving the density matrices’ element $\rho_{21}$ and present its series expansion. In section 3 we shall define the absorption coefficient and illustrate its application to astrophysics on the basis of [13]. In the subsequent sections we shall provide numerical calculations together with a discussion of the results. We conclude this paper with a summary.

2. Three-level $\Xi$ atom and density matrix

In this section we employ the three-level ladder scheme for the description of the hydrogen atom. We assume that hydrogen atoms formed during the recombination epoch in the earlier universe reach their ground states via emission of photons of all the spectral lines corresponding to atomic continuum–bound and bound–bound transitions. All of the emitted photons generate the (coherent) external field environment, which feeds back onto the hydrogen atom. Investigation of this self-consistent scenario under the conditions of cosmic expansion is the aim of this paper. We confine our consideration to the spontaneous emission rates. Collisional excitation and ionization can be omitted because at the relevant temperatures and densities, they are negligible for a three-level hydrogen atom [13].

To identify the possible effect we discuss atoms subjected to an external field which mainly consists of two adjacent spectral lines, namely $Ly_\alpha$ and $H_\alpha$ lines, with the initial condition corresponding to full population of the atomic ground state. Then we use the standard density matrix formalism which can be found, for example, in [36].

The generic three-level ladder system is depicted in figure 1. Solving the density matrix equations employing the steady-state and the rotating-wave approximations yields the following set of equations for the relevant density matrix elements [36]:

\[
\begin{align*}
\rho_{21} & = \frac{i/2[\Omega_\alpha(\rho_{22} - \rho_{11}) - \Omega_\beta^\ast \rho_{12}]}{\gamma_{21} - i(\Delta_\alpha + \Delta_\beta)}, \\
\rho_{32} & = \frac{i/2[\Omega_\beta(\rho_{33} - \rho_{22}) + \Omega_\alpha^\ast \rho_{21}]}{\gamma_{32} - i\Delta_\beta}, \\
\rho_{31} & = \frac{i/2[\Omega_\alpha \rho_{32} - \Omega_\beta \rho_{21}]}{\gamma_{21} - i(\Delta_\alpha + \Delta_\beta)}, \\
\rho_{22} & = \frac{i}{2\Gamma_2} (\Omega_\alpha^\ast \rho_{21} - \Omega_\alpha \rho_{12}), \\
\rho_{33} & = \frac{i}{2\Gamma_3} (\Omega_\beta^\ast \rho_{32} - \Omega_\beta \rho_{23}).
\end{align*}
\]

The levels of the system are specified with the following hydrogenic states $|1\rangle = |1s\rangle$, $|2\rangle = |2p\rangle$ and $|3\rangle = |3s\rangle$. The frequencies $\omega_{21}$ and $\omega_{32}$ correspond to the transitions $|1\rangle \rightarrow |2\rangle$ and $|2\rangle \rightarrow |3\rangle$, respectively. The system is driven to a significant modification of the absorption profile of the system. Description of EIT phenomena can be examined by the evaluation of the response of the multi-level system to the presence of an external radiation field. EIT leads to a significant modification of the absorption profile of the system. Description of EIT phenomena for a three-level ladder system interacting with two near-resonant monochromatic fields can be found, for example, in [34–36]. A complete and detailed treatment of a three-level ladder atom, absorption and emission spectra, as the transient and steady-state responses of the $\Xi$ atom is given in [34]. We study the response of the three-level ladder system on the external fields originating from photons emitted during the recombination and evaluate the absorption coefficient, which we apply to radiation transfer theory. The physics of an atom interacting with photon fields can be understood on the basis of the ‘interfering-pathway’ description, which corresponds to the multi-photon process defined in terms of a power series expansion over field amplitudes (see e.g. [34–36]).

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Figure 1. Scheme of the three-level ladder system under consideration. The three levels correspond to the hydrogenic states: $|1\rangle \rightarrow |1s\rangle$, $|2\rangle \rightarrow |2p\rangle$ and $|3\rangle \rightarrow |3s\rangle$, respectively. The frequencies $\omega_\alpha$, $\omega_\beta$ are the frequencies of external fields (which correspond to probe and controlled laser fields in [36]). The fields $E_\alpha$, $E_\beta$ stimulate transitions $1s \rightarrow 2p$ and $2p \rightarrow 3s$ (Lyman-$\alpha$ and Balmer-$\alpha$ lines). Possible detunings $\Delta_\alpha$ and $\Delta_\beta$ for the amplitudes of the fields are also indicated.
by a ‘probe’ field with amplitude $E_a$ at frequency $\omega_a$ and by a ‘control’ field with amplitude $E_\beta$ and frequency $\omega_\beta$. $\Delta_\alpha = \omega_a - \omega_{21}, \Delta_\beta = \omega_\beta - \omega_{32}$ define the corresponding detunings, $\Omega_\alpha = 2d_{21}E_\beta$ and $\Omega_\beta = 2d_{32}E_a$ are the Rabi frequencies, which depend on the atomic dipole-matrix element $d_{ij}$. All of these expressions are given in atomic units. Neglecting collisional dephasing effects, the decay rate is given by $\gamma_i = (\Gamma_i + \Gamma_j)/2$, where $\Gamma_i$ is the natural decay rate of the level $|i\rangle$.

In the limit of a weak probe field and $\rho_{11} \approx 1$, $\rho_{22} \approx \rho_{33} \approx 0$ (full population of the ground state of the atom), the solution of equations (1) for $\rho_{21}$ to first order in the probe field and to all orders in the control field was found in [35, 36]. But the total solution of equation (1) for $\rho_{21}$ is

$$\rho_{21} = \frac{i\Omega_\alpha/2}{(\frac{\Omega_\alpha^2}{4} - B) - \left(-\frac{\Omega_\beta^2}{4} + \gamma_{21} - i\Delta_1\right)(\frac{\Omega_\beta^2}{4} + A)},$$

$$A = (\gamma_{31} - i(\Delta_1 + \Delta_2)),\quad B = \frac{\Omega_\beta^2}{4\Gamma_3} + \gamma_{32} - i\Delta_2,$$ (2)

which can be easily transformed in the approximation of weak fields to expression (2) in [36].

The underlying physics of the system response on external fields can be understood considering the power series expansion of the solution (2) for the matrix element $\rho_{21}$ with respect to the variables $\Omega_\alpha$ and $\Omega_\beta$. Wieland and Gaeta [36] derived a series that contains terms up to third order in $\Omega_\alpha$ and $\Omega_\beta$ at zero detuning. For our purposes it is important to keep non-zero detuning and, as before, in the approximation of the weak field, expression (2) will be expanded into a power series with respect to $\Omega_\alpha$ and $\Omega_\beta$.

The resulting expression for $\rho_{21}$ simplifies significantly in the case of exact two-photon resonance, i.e. when the frequencies of two external fields coincide exactly with the transition frequency $\omega_{31} = E_3 - E_1$. In this case the equality $\Delta_\alpha + \Delta_\beta = 0$ holds and the series expansion looks like

$$\rho_{21} = \frac{\Omega_\alpha/2}{\gamma_{21} - i\Delta_\alpha} \left[ 1 - \frac{\Omega_\beta^2/4}{\gamma_{21}(\gamma_{21} - i\Delta_\alpha)} + \frac{\Omega_\beta^2/4}{\gamma_{21}(\gamma_{21} - i\Delta_\alpha)} \left( \gamma_{32} - i\Delta_\beta \right) \left( \frac{\gamma_{32} - i\Delta_\beta}{\Gamma_2\gamma_{31}} \right) \right],$$ (3)

where the dots imply the higher order terms in $\Omega_\alpha, \Omega_\beta$ and product $\Omega_\alpha \cdot \Omega_\beta$. The series expansion is performed under the conditions $\Omega_\beta/\gamma_i \ll 1$ and $\Omega_\alpha/\gamma_i \ll 1$.

As was established in [36], the common pre-factor in (3) corresponds to the one-photon absorption processes, while the squared terms are associated with the two-photon absorption and subsequent emission processes. The products $\Omega_\alpha^2 \cdot \Omega_\beta^2$ represent the ‘interfering-pathway’ terms; see figure 2. Thus the matrix element $\rho_{21}$ describes the multi-photon processes of the coherent ‘atom–field’ interaction. For a detailed analysis of equation (3), we refer to [36].

3. The absorption coefficient and Sobolev escape probability

The theory of radiation transfer for multilevel atoms utilizing the concept of the Sobolev escape probability has been described in [13]. With the method of escape probability, a simple solution to the radiative transfer problem for all bound–bound transitions can be found. The Sobolev escape probability $p_{ij}$ ($j$ refers to the upper level and $i$ to the lower level of a multilevel atom) is the probability that photons associated with this transition will ‘escape’ without being further scattered or absorbed. If $p_{ij} = 1$, the photons produced in the line transition can escape to infinity—they do not give rise to distortions of the radiation field. If $p_{ij} = 0$, the photons cannot escape to infinity; all of them get re-absorbed, and the line is optically thick. In general, $p_{ij} \ll 1$ for the Lyman lines and $p_{ij} = 1$ for all other line transitions. The Sobolev escape probability is included in direct astrophysical equations (see, for example, [13, equation (25)]) of radiation transfer.

Following [13, section 2.3.3] the Sobolev escape probability can be presented in the form

$$p_{ij} = \frac{1 - \exp(-\tau_S)}{\tau_S},$$ (4)

where $\tau_S$ is the Sobolev optical depth. The optical depth is a measure of the extinction coefficient or absorptivity up to a specific ‘depth’. In other words, the optical depth expresses the quantity of light removed from a beam by scattering or absorption during its path through a medium. The Sobolev optical depth can be defined as

$$\tau_S = \frac{\lambda_{ij}k}{|v'|},$$ (5)
where \( k \) is the integrated line absorption coefficient and \( \lambda^i_{ij} \) is the central line wavelength. The monochromatic absorption coefficient or opacity is \( k = k\phi(v_{ij}) \) \( (v_{ij} \) is the frequency for a given line transition and \( \phi(v_{ij}) \) is the normalized line profile); \( v' \) is the velocity gradient which is given by the Hubble expansion rate \( H(z) \).

The absorption coefficient depends strongly on the external conditions and requires particular consideration for each case. In the presence of an external field, the opacity can be related to the imaginary part of the density matrix elements \( \rho_{ij} \) as follows:

\[
k = \frac{N\delta_{ij} \omega_{ij}}{2\varepsilon_0 \Omega_{ij}} \text{Im}\{\rho_{ij}\},
\]

where \( \varepsilon_0 \) is the permittivity of the vacuum and \( N \) is the number of atoms.

Using expression (3) for the definition of the imaginary part of \( \rho_{21} \), we obtain

\[
\text{Im}\{\rho_{21}\} = \frac{\gamma_{21} \Omega_{ij}^2}{\Delta_0^2 + \gamma_{21}^2} \left[ 1 + f(\Omega_{ij}^2, \Omega_{ij}^2, \Delta_0, \Delta_\beta) \right]
\]

together with the dimensionless function

\[
f(\Omega_{ij}^2, \Omega_{ij}^2, \Delta_0, \Delta_\beta) = \frac{\Delta_0^2 - \gamma_{21}^2}{4\gamma_{21}\Delta_0^2} - \frac{\gamma_{21}^2}{4\gamma_{21}} \left[ \frac{\gamma_{21}^2}{\Delta_0^2} \frac{\beta_0^2 \nu_0^2}{\Delta_0^2 + \gamma_{21}^2} \right] + \left[ \frac{\gamma_{21}^2 \beta_0^2 \nu_0^2}{\Delta_0^2 + \gamma_{21}^2} + \frac{\gamma_{21}^2 \beta_0^2 \nu_0^2}{\Delta_0^2 + \gamma_{21}^2} \right] \left[ \frac{\Delta_0^2 + \gamma_{21}^2}{\Delta_0^2 + \gamma_{21}^2} \right] \quad (\text{7})
\]

and

\[
f(\Omega_{ij}^2, \Omega_{ij}^2, \Delta_0, \Delta_\beta) = \frac{\Delta_0^2 - \gamma_{21}^2}{4\gamma_{21}\Delta_0^2} \left[ \frac{\gamma_{21}^2}{\Delta_0^2} \frac{\beta_0^2 \nu_0^2}{\Delta_0^2 + \gamma_{21}^2} \right] + \left[ \frac{\gamma_{21}^2 \beta_0^2 \nu_0^2}{\Delta_0^2 + \gamma_{21}^2} + \frac{\gamma_{21}^2 \beta_0^2 \nu_0^2}{\Delta_0^2 + \gamma_{21}^2} \right] \left[ \frac{\Delta_0^2 + \gamma_{21}^2}{\Delta_0^2 + \gamma_{21}^2} \right] \quad (\text{8})
\]

To obtain information about the physical meaning of the function \( f \), i.e., about the absorption and subsequent emission processes, one can proceed as earlier for the density matrix element \( \rho_{21} \). For the definition of the integrated line absorption coefficient from equations (6)–(8), the line profile is separated out. It is assumed that the line profile appearing in equation (7) corresponds to the monochromatic absorption coefficient; see [13, equation (31)]. Here the function \( f \) depends on the fixed parameters \( \Delta_0 \) and \( \Delta_\beta \), although the common factor represents the Lorentz line profile, where \( \Delta_0 = \omega_{ij} - \omega_{21} \).

Thus the integrated line absorption coefficient can be presented in the form

\[
\tilde{k}_{21} = \frac{\pi d_2^2 N\omega_{21}}{4\delta_0} \left[ 1 + f(\Omega_{ij}^2, \Omega_{ij}^2, \Delta_0, \Delta_\beta) \right]
\]

with the line profile \( \phi(v_{21}) = \gamma_{21} / \left[ \left( \omega_{ij} - \omega_{21} \right)^2 + \gamma_{21}^2 \right] \).

In accordance with the theory described in [13], the line profile should be normalized within the interval \([0, \infty]\) and the coefficient \( \pi \) arises in equation (9). Thus the Sobolev escape probability extends to the expression

\[
p_{12} = \frac{1 - \exp(-\tau_s \left[ 1 + f(\Omega_{ij}^2, \Omega_{ij}^2, \Delta_0, \Delta_\beta) \right])}{\tau_s \left[ 1 + f(\Omega_{ij}^2, \Omega_{ij}^2, \Delta_0, \Delta_\beta) \right]},
\]

where \( \tau_s \) can be taken in the form (5); see [13, equations (39) and (40)]. In principle, expression (10) should be employed in further astrophysical evaluations. We reserve this for forthcoming research and restrict ourselves to the consideration of the function \( f(\Omega_{ij}^2, \Omega_{ij}^2, \Delta_0, \Delta_\beta) \).

The function \( f \) depends strongly on the parameters \( \Omega_{ij}^2, \Omega_{ij}^2, \Delta_0, \Delta_\beta \). The applicability of the power series expansion equation (3) with respect to \( \Omega_{ij} \) and \( \Omega_{ij} \) is limited by the field amplitudes and can be found in [35] but beyond in [36]. Estimates for the field amplitudes can be deduced from the CMB distribution corresponding to the hydrogen recombination era in the earlier universe.

4. Numerical results and discussion

While the universe further expanded and cooled down, the electrons and protons tended towards the formation of hydrogen atoms. The temperature at this era is very well known from laboratory physics, \( T \approx 4500–3000 \text{ K} \). After ‘recombination’, the photons released were able to travel through the universe relatively undisturbed, and formed the primordial background radiation. However, such a photon environment (background) should have an influence on the hydrogen atom. The field amplitudes for a circular polarized wave can be obtained from the (thermal-averaged) spectral energy density

\[
c_{\nu_0}|E|^2 = \frac{2\nu_0^3}{c^2} \frac{1}{e^{\nu_0/c} - 1},
\]

where \( c \) is the speed of light, \( k_B \) is the Boltzmann constant, \( h \) is Planck’s constant and in further calculations we use \( T_c = 3000 \text{ K} \). The right-hand side of the equation above corresponds to the black-body distribution of the CMB, while the left-hand side defines the (electrical) energy density.

In order to avoid a dephasing problem we should choose \( \Delta_\nu_{ij} \sim \Gamma_r \). Hence for the spectral lines \( \nu_{12} = \nu_a (L_\alpha \text{ line}) \) and \( \nu_{32} = \nu_\beta (H_\alpha \text{ line}) \), we obtain

\[
E_a \approx 0.000 \, 068 \, 802 \text{ V m}^{-1} = 1.337 \, 99 \times 10^{-16} \text{ au},
\]

\[
E_\beta \approx 52.8636 \text{ V m}^{-1} = 1.028 \, 03 \times 10^{-10} \text{ au}.
\]

The magnitudes (12) of the field are small; we should compare the Rabi frequencies with the corresponding level widths. The one-photon transition rates, which yield the major contributions to level widths, can be easily evaluated and are well known. For the hydrogen atom the dominant one is the Ly-\( \alpha \) transition rate, \( \Gamma_{2p} \sim 10^{-8} \) in atomic units and, therefore, the power series (3) is valid. Moreover, the estimates given in equation (12) reveal that we can neglect all terms of the order \( \Omega_{ij}^2 \) and higher.

Furthermore we evaluate the function \( f(\Omega_{ij}^2, \Omega_{ij}^2, \Delta_0, \Delta_\beta) \) numerically. In the case if detuning is defined by \( \Delta_\sigma \equiv \Gamma_{2p} \) and \( \Delta_\beta \equiv \Gamma_{2p} + \Gamma_\beta \), and using equation (2) we can present the function \( f \) in the form

\[
f(\Omega_{ij}^2, \Omega_{ij}^2, \Delta_0, \Delta_\beta) \approx -1.304 \, 94 \times 10^{15} \Omega_{ij}^2
\]

\[
-2.081 \times 10^{30} \Omega_{ij}^4 + 8.672 \times 10^{14} \Omega_{ij}^2 + 5.155 \, 73
\]

\[
\times 10^9 \Omega_{ij}^4 + 2.426 \, 65 \times 10^{14} \Omega_{ij}^2
\]

\[
+ 3.229 \, 16 \times 10^{26} \Omega_{ij}^2 - 1.089 \, 79
\]

\[
\times 10^{17} \Omega_{ij}^2 \Omega_{ij}^2 - 6.186 \, 61 \times 10^{14} \Omega_{ij}^2 \Omega_{ij}^2
\]

\[
- 1.635 \, 88 \times 10^{62} \Omega_{ij}^4 \Omega_{ij}^2 + 2.926 \, 29
\]

\[
\times 10^{63} \Omega_{ij}^2 \Omega_{ij}^2 - 4.885 \, 45 \times 10^{63} \Omega_{ij}^2 \Omega_{ij}^2.
\]
Table 1. The numerical results of the function $f(\Omega_a, \Omega_\beta, \Delta_a, \Delta_\beta)$ for the different magnitudes of detunings are presented. In the first column different values for $f(\Omega_a, \Omega_\beta, \Delta_a, \Delta_\beta)$ are listed; in the second and third columns the detunings and field amplitudes are compiled.

| $\Delta_a$, s$^{-1}$ | $\Delta_\beta$, s$^{-1}$ | $E_v$, V m$^{-1}$ | $|E_\beta|$, V m$^{-1}$ |
|------------------------|------------------------|------------------|------------------|
| 6.486 35 $\times$ 10$^{-7}$ | $\Delta v_i$ = $|\Delta_a|/|\Delta_\beta|$ in equation (11) | 6.8802 $\times$ 10$^{-5}$ | 4.068 52 $\times$ 10$^{-6}$ |
| 6 $\times$ 10$^{-7}$ | $W_{21}$ = 6.2682 26 $\times$ 10$^8$ | 6.8802 $\times$ 10$^{-5}$ | 10W$_{21}$ = 6.2682 26 $\times$ 10$^8$ |
| 6 $\times$ 10$^{-7}$ | $W_{32}$ = 6.3169 96 $\times$ 10$^6$ | 5.30686 | 10W$_{21}$ + W$_{32}$ = 6.3313 43 $\times$ 10$^8$ |
| 0.000 032 6097 | $W_{21}$ = 6.2682 26 $\times$ 10$^8$ | 6.8802 $\times$ 10$^{-5}$ | 0.000 032 4474 |
| 0.002 527 43 | $W_{21}$ = 6.2682 26 $\times$ 10$^8$ | 5.31293 | 0.000 032 6097 |
| 0.002 527 43 | $W_{21}$ = 6.2682 26 $\times$ 10$^8$ | 5.28636 | 0.000 032 4474 |
| 0.002 527 43 | $W_{21}$ = 6.2682 26 $\times$ 10$^8$ | 5.28636 | -W$_{21}$ = -6.2682 26 $\times$ 10$^8$ |
| 0.002 527 43 | $W_{21}$ = 6.2682 26 $\times$ 10$^8$ | 5.28636 | -W$_{21}$ = -6.2682 26 $\times$ 10$^8$ |
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Taking into account the estimates (12), we receive

$\Omega_a \approx 1.993 43 \times 10^{-16}$ au,
$\Omega_\beta \approx 1.929 42 \times 10^{-10}$ au,
$f \approx 0.000 032 2844$. (14)

In the case of exact resonances from equation (2) and (12) we obtain

$\Delta_a = \Delta_\beta \equiv 0$,
$f \approx -0.015 814$. (15)

Finally, in the case of an exact two-photon resonance, where $\Delta_a + \Delta_\beta = 0$ and together with $\Delta_a = \Gamma_2p$, the function $f$ takes the value

$\Delta_a = \Gamma_2p = -\Delta_\beta \sim 10^{-8}$ au
$f \approx 0.009 527 43$. (16)

Thus the magnitude of the function $f(\Omega^2_a, \Omega^2_\beta, \Delta_a, \Delta_\beta)$ is about 1% ($10^{-2}$), in the case of an exact one-photon resonance it is about 1.5%, respectively, about 0.95% if the detuning is non-zero but of opposite sign. In quantum optics this effect is well known when the EIT is investigated for different kinds of systems (two-, three- or four-level systems with $A-$, $V-$ or $V-$scheme of levels). The results of calculations for different values for detuning are compiled in table 1 and shown graphically in figures 3 and 4. In astrophysical investigations of the CMB formation processes, the Lorentz line profile is used for defining the absorption coefficient. This line profile represents the dominant term in $\text{Im} \{\rho_{21}\}$ and is separated out in equation (7). But in addition the function $f(\Omega^2_a, \Omega^2_\beta, \Delta_a, \Delta_\beta)$ in equation (7) has to be included in the astrophysical evaluation of CMB.

The radiation transfer theory is based on the elementary electron scattering processes on the atom at rest. We evaluated this process for the hydrogen atom with the purpose of their use in the tasks of the radiation scattering in the earlier universe. In fact, dependence of the escape probability, $p_{ij}(\tau_5)$, for the photon in the line wing due to the expanding universe can have the complex form. The Sobolev approximation works at a certain phase which is well known. In more complicated cases the diffuson approximation has to be applied [14]. However, we restrict ourselves by the task of taking into account the external fields in the single scattering of external photons, while we leave the investigation of the radiation transfer processes for future works. As an example of application of our calculations, we wrote down the expression (equation (4)) (the most widely used approximation) and, consequently, equation (10). In the more common case we can write $p_{ij} = p_{ij}(\tau_5)$, which should be replaced by $p_{ij}(\tau_5) \rightarrow p_{ij}(\tau^5_{ij}(1 + f(\Omega^2_a, \Omega^2_\beta, \Delta_a, \Delta_\beta)))$, where the optical depth $\tau^5_{ij}$ corresponds to the ‘standard’ definition (equation (5)).

Analysing the expression of the elementary process probability, it is necessary to stress that the additional function $f$ arises to the standard Lorentz profile. The physical meaning of this supplement for the other atomic systems is well known and described in a lot of works; see, for example, [34–36]. Namely in the presence of the ‘probe’ alone ($\Omega_a$) there is the conventional one-photon pathway directly from $|1\rangle$ to $|2\rangle$ in figure 2(a). If one adds a strong ‘control’ field ($\Omega_\beta$), then an additional pathway results from the three-photon transition shown in figure 2(b), which entails the absorption and subsequent re-emission of a ‘control’ photon. If the ‘probe’ field is allowed to be strong, then additional multi-photon pathways exist from $|1\rangle$ to $|2\rangle$. In particular, there are three-photon and five-photon transitions shown in figures 2(c) and (d) involving three ‘probe’ photons and three ‘probe’ photons with two ‘control’ photons, respectively. These terms and other higher order terms become important as the strength of $\Omega_a$ is increased and interfere with the one-photon and three-photon terms, which can result in a deterioration of the induced transparency and even in induced absorption. This ‘interfering-pathway’ description is illustrated by performing an iterative solution to the system of equations (1) for resonant ‘probe’
the detunings $\Delta_1\alpha, \Delta_1\beta$ fields (12), i.e. confronted with the consideration of the phenomena of EIT. Thus, we are the universe and formed the CMB. Generated by this way lines. After recombination, photons were able to travel through the ground state via emission of photons from all of the spectral. During the recombination era the hydrogen atoms reach their total detuning is equal to zero, we obtained $f \approx 0.95\%$. Figures 3 and 4 also reveal that additional transparency of the medium yields contribution at the level about 1%. We expect that the modifications of this magnitude should definitely be relevant in determinations of the key cosmological parameters. The problem of dephasing appears when defining the magnitude of the field. The effect of dephasing leads to the spectral line broadening. To prevent dephasing phenomena we confined the definition of the field by a narrow strip. In this case the width of the corresponding line appears as a natural parameter. In our calculations we used the following relation $\Delta\nu_{ij} = \Delta_a (\Delta_\rho)$. In addition we should note the exponential behaviour over temperature for the field amplitudes. With the increase in the temperature $T_e$, larger values for the amplitudes could be obtained; see equation (11). Accordingly, the contribution of the EIT effect will become more significant for higher temperatures.

5. Conclusions

The aim of our paper is to study the electromagnetically induced transparency (EIT) phenomenon and its influence on CMB formation. The phenomenon of EIT consists in the investigation of the system response on the external field. During the recombination era the hydrogen atoms reach their ground state via emission of photons from all of the spectral lines. After recombination, photons were able to travel through the universe and formed the CMB. Generated by this way the field should affect the primordial atoms. Thus, we are confronted with the consideration of the phenomena of EIT.

We employed the quantum optical evaluation of the integrated line absorption coefficient that assumes application of photon beams (laser). In astrophysics the photon beam diffusion is accounted for via the Sobolev optical depth. Usually, the standard line profile is employed for the solution of the recombination problem and description of the CMB formation processes and the integrated line absorption coefficient is defined via Einstein coefficients in frames of the Sobolev approximation. But the determination of the integrated line absorption coefficient based on the quantum optical techniques allows one to take into account the influence of external fields on the atom. Accordingly, we derive the absorption coefficient from the imaginary part of the density matrix element and deduce the magnitudes of fields from the CMB distribution.

We are led to the additional function $f$ which depends on external conditions. The values of function $f$ are listed in table 1 for different values of detuning. The dependence on detuning is depicted in figures 3 and 4. The maximal value of $f$ was found and amounts to about 1.5% in the case of exact resonances (when both detunings are equal to zero). In the case of exact two-photon resonance, when frequencies of fields are close but differ slightly to the corresponding resonances and the total detuning is equal to zero, we obtained $f \approx 0.95\%$. Figures 3 and 4 also reveal that additional transparency of the medium yields contribution at the level about 1%. We expect that the modifications of this magnitude should definitely be relevant in determinations of the key cosmological parameters.

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