Student’s rigorous mathematical thinking based on cognitive style

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Abstract. The purpose of this research was to determine the rigorous mathematical thinking (RMT) of mathematics education students in solving math problems in terms of reflective and impulsive cognitive styles. The research used descriptive qualitative approach. Subjects in this research were 4 students of the reflective and impulsive cognitive style which was each consisting male and female subjects. Data collection techniques used problem-solving test and interview. Analysis of research data used Miles and Huberman model that was reduction of data, presentation of data, and conclusion. The results showed that impulsive male subjects used three levels of the cognitive function required for RMT that were qualitative thinking, quantitative thinking with precision, and relational thinking completely while the other three subjects were only able to use cognitive function at qualitative thinking level of RMT. Therefore the subject of impulsive male has a better RMT ability than the other three research subjects.

1. Introduction
Thinking is a routine activity that people do every day. Almost every issue of human life involves thinking activity to solve it. Thinking means mental activities which happen in the human mind involve remembering, considering, giving reason, reflecting, and deciding something tend to solving a problem. In the face of 21st century challenges, every individual should have a thinking skill to solve every problem. The thinking skills that have to be mastered at the moment are called high order thinking skill or HOTS. Higher order thinking (HOT) involves a variety of thinking processes which applied to complex situations and having multiple variables [1]. When an individual thinks of solving a mathematical problem, it does not rule out that he is doing a mathematical thinking. Katagiri [2] states that mathematical thinking is the ability of scholastic to think and make judgments independently. Furthermore, Sumarmo [3] defined mathematical thinking as a way of thinking with regard to the process of math (doing math) or in solving mathematical tasks both simple and complex.

In mathematical thinking, a person translates incoming external information into symbols for subsequent symbols to be processed according to the rules in mathematics already compiled. In learning and solving mathematical problems, need for precision, while the prerequisite to be precise and logical is rigor. Kinard [4] reveals that mathematical thinking integrates and harnesses cognitive processes that increase higher abstraction levels, and therefore they must be rigorous in nature. With
regard to the necessity of rigor in synthesizing and utilizing cognitive processes to increase the level of abstraction function it is necessary to have rigorous mathematical thinking (RMT). Furthermore, RMT is characterized by three levels of cognitive function such as cognitive function for qualitative thinking, cognitive function for quantitative thinking with accuracy and cognitive function for abstract relational thinking [4, 5]. Therefore RMT is a mathematical thinking for high level like HOT.

The first level of cognitive function for RMT consists of the general cognitive functions necessary for qualitative thinking. Before students engage in rigor conceptual reasoning, the cognitive process takes place at the concrete level and is dominated by existing natural psychological functions. The cognitive functions in this first level are labeling-visualizing, comparing, searching systematically to collect information clearly and completely, making use more than one source of information, and encoding-decoding. The second level consists of the cognitive functions necessary for quantitative thinking and accuracy. These functions are more structured than general cognitive functions. The cognitive functions in this second level are conserving constancy, quantifying space and spatial relationships, quantifying time and temporal relationships, analyzing-integrating, generalizing, and being precise. The third level of cognitive function integrates processes related to quantity and precision into unique structures and generalized abstract relational thinking. The cognitive functions in this third level consist of activating previous mathematically related knowledge, providing and articulating mathematical logical evidence, defining the problem, inferential-hypothetical thinking, projecting and restructuring relationships, Forming proportional quantitative relationships, forming a functional relationship, forming a unit functional relationship, mathematical inductive- deductive thinking, mathematical analogical thinking, mathematical syllogistic thinking, mathematical transitive relational thinking, and elaborating mathematical activity through cognitive categories [4, 5]. These three levels of cognitive function together define the mental processes of general cognitive skills to higher level specific mathematical cognitions [6].

Each individual is born with a unique characteristic attached to himself that is not owned by another individual. This uniqueness is included in terms of cognitive style. The cognitive style according to Warli [7] defined as individual characteristics in terms of feeling, remembering, organizing, processing, and solving problems, in an attempt to differentiate, understand, store, embody, and utilize information. Furthermore, Goldstein and Blackman in Amstrong [8] suggest that cognitive styles relate to the characters and habits in which individuals process and evaluate information, solve the problem and make decisions. Witkin [9] classify cognitive style in several types, one of which is reflective-impulsive cognitive style. The classify was in term of conceptual tempo. Conceptual tempo can be interpreted as the speed and accurate in thinking.

Warli [7] states that there are two important aspects to be considered in measuring reflective-impulsive cognitive style: 1) the level of the subject in describing the accuracy of the problem-solving allegation or the time to make a decision in solving the problem; 2) contains uncertain answers. When the time aspect is divided into two, namely fast and slow, then the uncertainty aspect is divided into two that is accurate and inaccurate then the students can be grouped into four categories: fast-accurate, slow and accurate (reflective), fast and inaccurate (impulsive), slow-inaccurate. In this research, only focus on reflective-impulsive categories because both of categories most dominant [7, 10-13] The characteristics of reflective cognitive-style students are slow in answering questions but the answers inclined to precise. whereas impulsive cognitive style students are fast in answering questions but the results tend to be inaccurate. The test to map reflective-impulsive cognitive style used MFFT (matching familiar figure test) which was develop by Warli [7]

2. Method
This research method used descriptive qualitative research. Subjects in this study were 4 first year students in the mathematics education study program of FKIP UAD, each consisting of 2 students of reflexive and impulsive cognitive style in different gender. The subjects were MR (male reflective), FR (female reflective), MI (male impulsive), and FI (female impulsive). Data collection techniques in this study using problem-solving test and interview. To see the consistency of research data used time
triangulation. Analysis of research data used Miles and Huberman [14] model that was reduction of data, presentation of data, and conclusion.

3. Result and discussion

The RMT process requires the use of cognitive function from low-level to high-level cognitive functions. The cognitive function is categorized in three levels that are general cognitive functions for qualitative thinking, cognitive functions for quantitative thinking in the manner of precision, and cognitive functions for generalized, logical abstract relational thinking in the mathematics culture. The following is a summary of the use of cognitive function of the subjects.

![Figure 1. Percentage of the using of cognitive function of subject for RMT](image-url)

3.1 Characteristics of cognitive function of reflective cognitive style subject for RMT

Reflective subject consist of male and female reflective cognitive style which labeled by MR and FR respectively. The use of cognitive function for RMT at first level is indicated by the ability of the both subjects in giving something a name (labeling) when given the picture by taking the critical attributes of the object but MR looks still unsure of the explanation. Subjects make constructed of cone, sphere, equilateral triangles, cubes, cuboid when names given but the picture made only sketches without using ruler. The subjects compared the cube and cuboid in terms of the diagonal, the shape of the sides and their dimensions. Subjects paid attention to the picture carefully to collect and complete the information, even the subjects added streaks on the image to gather information. Furthermore, subjects used more than one source of information by mentally working with several concepts at the same time when solving a problem, but have not been able to discover the right answer.

Subjects able to symbolize radius with \( r \); the measure of cuboid edge with \( p, l, t \); right angle with \( \square \); same length with \( / \); angle symbol but the symbol of angle size was less precise. MR paid less attention to the symbol of the vertex with capital letters. Subjects were able interpreting the symbols \( / \), \( >> \), and measure of segment accurately. The subject defined the symbol \( / \) as the same long side symbol; \( \square \) as a right angle symbol; \( >> \) on some segments as symbols of two parallel lines; MR interpreted the XY symbol on the picture as the length of the X segment plus the length of the Y line segment. However, he was able to understand the symbol of measurement of segment of the constructed picture itself. This shown the subject's lack of understanding about the symbol of the segment and its size. Overall, reflective subject able to use all of cognitive function at the level qualitative thinking for RMT.

At second level cognitive function for RMT, both subject only able to use one cognitive function appropriately. FR used cognitive function of integrating by building rectangles by combining 3 congruent square, trapezoid with respect to two parallel edge while MR used cognitive function of generalizing by stated the orthocenter of the equilateral triangle divided the altitude into 2 parts by 2: 1 ratio and cuts the right side of it in the middle. Generally, MR comprehended that three special lines of equilateral triangle were coincide. When given the problem of explaining the relationship between the cube and the cuboid, both subjects were unable to name the cuboid when the size of the edge was

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the same length. FR identified that the edge of the cuboid and cubes were congruent to differentiate were their dimensions and constituents (the cube was hollow and in 3 dimension, the cuboid was solid and in 2 dimension). Subjects were less able to use internal references as a guide to analyze the relationship between cone, cone meridians and sphere in cone, and the relationship between cubes and cuboid. Subjects were not able to analyze the critical attributes of equilateral triangles, right triangles, cubes and cuboid appropriately. Subjects can not distinguish the terms face and edge on the context of cubes and cuboid. FR was less able to generalize the features of cubes and cuboid without referring to the picture. Both subjects were less focused and meticulous when working on the problem. Therefore, reflective subject still not fulfilled second level of cognitive function for RMT yet.

As the previous level of cognitive function for RMT, reflective subject not able to use all cognitive function at the third level of cognitive functions for generalized, logical abstract relational thinking in the mathematics culture. While solve on the problem, MR has activated prior mathematically related knowledge but the solution was not quite right. In addition, MR has also been able to use cognitive function of mathematical deductive thinking by applying formulas in problem solving though not yet precise. While FR was only capable of doing mathematical inductive thinking by determining the height of the cone using the height of the triangle as its meridian slices using a ruler. In addition, mathematical inductive thinking was done by assuming that the angles of the right triangle were 90°, 45°, 45° or 90°, 30°, 60°. It was constructed by FR from her experience that often saw a right triangle with such an angle size. Others cognitive function for RMT still not appeared in the both of subject. Therefore, reflective subject still not fulfilled third level of cognitive function for RMT. Although both of the reflective cognitive style subjects have different gender, but their RMT process almost the same.

3.2 Characteristics of cognitive function of impulsive cognitive style subject for RMT
Impulsive subject consist of male and female impulsive cognitive style which labeled by MI and FI respectively. Both of subjects were able to give name the object based on their critical attributes such as symbols of equal edge, size of angle, and parallel. Subjects construct cone, sphere, equilateral triangles, cubes, cuboid, spheres in cones and meridian cones in equilateral triangles when the name was given. MI was able to determine the similarities of cubes and cuboid appropriately while FI determined the similarities of cubes and cuboid on the same edge, whereas the difference lied in its dimensions and constructions. MI has a well concept understanding so that he be able to compare both objects geometry properly. When solving the problem, beside to paying attention to the picture carefully to collect and complete the information, both of subjects also mentally worked with some concepts at the same time but there were concepts used FI was still not quite right. In the use of cognitive function of encoding, both of subjects were able to interpret the object by using a particular symbol, such as the area of the triangle with L, the length of the triangle with a, half of the circumference of the triangle with S, the surface area of the sphere with Lp, the same length of the segment by "/ ", the radius by “r”, the right angle with" □ ", congruent, the name of the object by encoding the vertex with capital letters, angle notation but not appropriate in the use of notation angle size. While for the cognitive decoding function, the subjects were able to interpret the mathematical symbol correctly, but FI failed to interpret the symbol of the size of the segment. This was because she was used to denoting the size of a segment with segment notation. Even though the notation was totally different.

There was the difference of using cognitive function at the second level of cognitive function for RMT. FI failed using cognitive function at this level. Such as impulsive subject characteristics, FI tend to fast in solve the given problem but the answer tend to inaccurate. FI did not able to identify and describe what stays the same in terms of an attribute and concept of cube and cuboid while some other things were changing. FI did not being precise in solve the problem. FI cannot quantify space and spatial relationships between cube and cuboid, radius of sphere in cone and high of cone, cannot analyze the cone and sphere in cone, cube and cuboid, cannot integrate the square and quadrilateral, and cannot describe the nature of cube and cuboid without referring to specific details. While MI can
using all of cognitive functions at second level of cognitive function for RMT. MI was able to identify cuboid even though its similar edges, use internal referential as an internal of reference as a guide to analyze parts relationships based on whole-to-parts relationships, do decomposing a quantity into its critical attributes and constructing a whole by merging its parts. MI was able to generalize that altitude of equilateral triangle will divide the edge in front of it into two equal lengths, without regard to critical attributes of equilateral triangle. Beside it, MI also can generalize that cube was cuboid with equal edges. During solve the problem, MI strived to be focused and exact.

On the third level of cognitive function for RMT, only MI was capable of using all of cognitive functions needed for RMT. MI able to do mobilizing previously acquired mathematical knowledge by searching through past experiences to solve the problem, explain the logical supporting details to prove the truth of his preposition. MI can define the problem by analyzing and seeing relationships to figure out precisely what has to be done mathematically. MI also can form a mathematical proposition that a cube was a cuboid and search for mathematical logical evidence to support the proposition. He also able to develop valid generalizations and proofs based on a number of mathematical events. MI has formed connections between congruent triangles and reconstructing existing connections between triangles to find the measurement of its edges. In solve the problem, MI also took aspects from various mathematical details that seem to form a pattern, categorizing them into general relationships of attributes, and organizing the results to form a general mathematical rule; applying some formulas ie Pythagorean theorem, area of triangle theorem, the length of radius of sphere in triangle formula, area and volume of the sphere, the rule of congruent triangle to a specific situation that connect only with the rule in terms of belonging to categories of attributes by the rule. MI used cognitive function mathematical relational thinking by considering relationship among some objects in geometry ie relationship between parallelepipedum, cube and cuboid. In the whole of solving problem, MI able to elaborate mathematical activity through cognitive categories by reflecting on and analyzing mathematical activity and discovering, labeling, and articulating, orally and in writing, underlying mathematical principles and concepts using the language of mathematics and cognitive functions. Therefore, both of the impulsive subjects have different ability in using cognitive function needed for RMT. Male impulsive subject have better ability than female impulsive subject.

Based on the above exposure, from the fourth research subjects obtained that only male impulsive subject was capable to use all of cognitive function needed for RMT, while the other three subjects were only capable to use cognitive function at first level (qualitative thinking). Although if noted the characteristics of reflective-impulsive cognitive style, reflective subject need more time to answer and tend to accurate. Therefore, it should be more careful and thorough in solve the problem with the result that the answer was more accurate. But in this research the opposite was true, impulsive subject was more accurate and thorough in solve the problem so that cognitive function needed to solve the problem can used properly. Therefore the finding of this research was in accordance with the findings of Witkin[9], Taram [15], Fitiyani [11], Agoestanto [16], and Warli [7] that the difference of cognitive style influences how the student solve the problem. The problem in this research was geometry problem. The research result shown that students have difficulties to solve the given geometry problem. Students still have difficulty in construct concepts of geometry, make relation between the concepts, etc [17-19]. This finding was suitable with the finding of Hardianti [20].

4. Conclusions
Based on the result and discussion, impulsive male subjects used three levels of the cognitive function required for RMT that were qualitative thinking, quantitative thinking with precision, and relational thinking completely while the other three subjects were only able to use cognitive function at the level of qualitative thinking for RMT. Therefore the subject of impulsive male has a better RMT ability than the other three research subjects.
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