Self-organization of dissipative and coherent vortex structures in non-equilibrium magnetized two-dimensional plasmas

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Abstract
The properties of non-equilibrium magnetized plasmas confined in planar geometry are studied on the basis of first-principle microscopic Langevin dynamics computer simulations. The non-equilibrium state of plasmas is maintained due to the recombination and generation of charges. The intrinsic microscopic structure of non-equilibrium steady-state magnetized plasmas, in particular the inter-particle correlations and self-organization of vortex structures, are examined. The simulations have been performed for a wide range of parameters including strong plasma coupling, high charge recombination and generation rates and intense magnetic field. As is shown in simulations, the non-equilibrium recombination and generation processes trigger the formation of ordered dissipative or coherent drift vortex states in 2D plasmas with distinctly spatially separated components, which are far from thermal equilibrium. This is evident from the unusual properties of binary distributions and behavior of the Coulomb energy of the system, which turn out to be quite different from the ones typical for the equilibrium state of plasmas under the same conditions.

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1. Introduction
Cooperative phenomena in non-equilibrium open systems have, since the works by Belousov [1] and Zhabotinsky [2], attracted considerable interest of researchers. The general theoretical basis for the description of ordering in such systems has been given by Prigogine and co-workers [3, 4] and Haken [5]. The theory of self-organization in open systems proposed by Haken, namely synergetics, suggests the consistent probabilistic description of such systems based on Fokker–Planck equations, which in turn originate from stochastic Langevin dynamics (LD). However, in actual practice, the theoretical models for ordered dissipative structures (DS) occurring in particular open systems incorporate, as a rule, additional phenomenological assumptions or approximations (see, for instance, [6]).

Evidently, this fact is associated with the complexity of real systems, which makes consistent accurate implementation of the theory rather difficult. In view of this, the outstanding challenge in this field would be to describe cooperative phenomena in more or less simple specific systems on the basis of Langevin particle dynamics by means of computer simulations. In particular, this would provide the possibility to verify the validity of the basic principles of synergetics in direct numerical simulations.

In this work, we study the properties of vortex formation in two-dimensional (2D) magnetized plasmas, where the non-equilibrium is maintained due to the processes of recombination and generation of charges, by means of computer simulations based on first-principle microscopic particle dynamics. Vortex structures occurring in magnetized plasmas have attracted considerable attention of researchers
in the last decades. The vast majority of theoretical works in this area are based on the continuous models, such as the classical Hasegawa–Mima theory [7] or the drift-Poisson model [8]. Due to the computer simulations based on these models, which are performed predominantly for the case of cylindrical geometry, significant progress has been achieved, in particular, in the description of the evolution of vortex structures from the initial turbulent state, and the explanation of formation of vortex crystals and other experiments with non-neutral magnetized electron plasmas in the electron columns [9–11].

The above-mentioned continuous models originate in the end from the plasma hydrodynamics based on the mean-field concept. As the result, one of the restrictions for them is that the plasma must be ideal (i.e. weakly coupled). This means that the continuous approaches completely neglect the effects associated with inter-particle correlations in plasmas, and provide no way to study intrinsic microscopic plasma structure, particularly, in the case when the plasma coupling is not weak. At the same time, as has been demonstrated by recent experimental and theoretical studies of strongly coupled plasmas, the latter may differ drastically in properties from the weakly coupled plasmas [12].

The microscopic computer simulations based on first-principle particle dynamics are not subject to the above drawback and provide tools to examine the correlations and structure of vortices in magnetized plasmas. As far as we know, such simulations up to this time have not yet been performed.

As a relevant physical system associated with this problem, we consider the planar hetero-structures in semiconductors such as quantum wells, where the electron–hole plasma is confined in planar 2D geometry. The properties of such systems have been intensively investigated in recent years from both the experimental and theoretical points of view [13, 14]. The crystal lattice responsible for the dissipation due to scattering of charge carriers represents in this case a natural heat bath, the presence of which is taken into account in Gibbs equations.

Since the external magnetic field does not contribute to the Gibbs distribution for a system of point charges, the equilibrium properties (thermodynamic averages) of such a system do not depend on its presence. The presence of the magnetic field manifests itself only in non-equilibrium processes, in particular, in the kinetic properties. Note that the microscopic description of two-component plasmas (TCP) suggests a natural source of non-equilibrium, namely, recombination. The need to take into consideration the processes of recombination is connected with the fact that the TCP consisting of point charges cannot be in thermal equilibrium because of the classical collapse of oppositely charged components [15]. By the way, this is one more distinguishing feature of it compared to continuous models. For this reason, our microscopic model includes, along with the point plasma charges interacting via Coulomb forces and the transversal magnetic field, the processes of charge recombination (and generation, if steady states have to be examined).

Let us now formulate the major distinctive features that distinguish our approach from the previous works in this field. Firstly, we employ the microscopic description based on LD, allowing for dissipation, which is applicable in the case of strong correlations. Secondly, we take into account the recombination and generation of charges, which enables one to examine the associated non-equilibrium processes. Finally, we investigate the case of 2D planar geometry rather than the cylindrical one.

2. Formulation of the dynamic equations

Thus, we consider the TCP consisting of point charges interacting via Coulomb interaction. It is confined in 2D planar (in the $XOY$ plane) geometry and exposed to an external constant magnetic field oriented perpendicularly, along the $Z$-axis. The equations of motion for the particles we employ are the LD equations [16]

$$\frac{m_i \, \ddot{r}_i}{\ddt^2} = \vec{K}_i - \gamma_i v_i + \vec{F}_i(t). \tag{1.1}$$

Here $\vec{r}_i$ and $m_i$ are the radius vector and the mass of the $i$th particle, respectively, $\vec{v}_i \equiv \ddot{r}_i / \ddt$ is its velocity and

$$\vec{K}_i = -\nabla_i U_{\text{Coul}} + \vec{K}_{i,\text{ext}} \tag{1.2}$$

is the regular force acting on the $i$th particle. The quantity $U_{\text{Coul}} = \frac{1}{2} \sum_{i \neq j} z_i z_j e^2 / |\vec{r}_i - \vec{r}_j|)$ is the Coulomb energy of the system of particles (where $e$ is the absolute value of the electron charge; $z_i = \pm 1$ is the charge number of the $i$th particle) and $\vec{F}_i$ is the stochastic Langevin force (acting on the $i$th particle) due to the presence of a heat bath defined by the correlator

$$\langle F_{i,\text{st}}(t) F_{j,\text{st}}(t') \rangle = 2 \gamma_i k_B T \delta_{i,j} \delta_{\alpha,\beta} \delta(t - t') \tag{1.3}$$

with $\gamma_i$ being the relevant friction coefficient. Thus, we assume that the heat bath is in thermal equilibrium at a temperature $T$. Subscripts $\alpha, \beta \equiv x, y, z$ denote the Cartesian components of corresponding vectors, and $k_B$ is the Boltzmann constant. In our case the external force $\vec{K}_{i,\text{ext}}$ includes, along with possible external potential fields, the Lorentz force $\vec{K}_{i,L} = (e z_i / c)[\vec{v}_i \times \vec{B}]$, where $\vec{B}$ is the magnetic induction and $c$ is the velocity of light.

Since in this work we study the self-organization processes at long time scales, $t \gg m / \gamma$, we may simplify the LD equations by setting the inertial term equal to zero. After simple transformations, equation (1.1) may be reduced to the overdamped form

$$\vec{v}_i = \frac{1}{\gamma_i} \{ \vec{K}_i + \vec{F}_i(t) \}. \tag{1.4}$$

We see that the overall velocity naturally splits into two parts, the drift $\vec{v}_{i,\text{drift}} = (1 / \gamma_i) \vec{K}_i$ associated with the regular part of the forces and the diffusion $\vec{v}_{i,\text{diff}} = (1 / \gamma_i) \vec{F}_i(t)$ associated with stochastic forces.

The equations of motion (1.4), which we shall refer to as Brownian dynamics (BD) hereafter, determine the particle dynamics in the drift–diffusion limit. By explicitly taking into account the magnetic field $\vec{B} \parallel OZ$, one can reformulate the BD equations into the form

$$\vec{v}_i = \frac{1}{\gamma_i (1 + \beta_i^2)} \{ \vec{P}_i + \beta_i [\vec{P}_i \times \vec{e}_z] \}. \tag{1.5}$$


where $\vec{e}_z$ is the unit vector giving the orientation of the OZ axis. We introduced here the dimensionless parameter $\beta_i = \varepsilon_{ik} B_i / \varepsilon_{ik}$ specifying the field strength in relation to the friction, and the total force acting on the $i$th particle

$$\vec{F}_i = -\nabla U + \vec{F}_i(t),$$

(1.6)

where the potential energy $U = U_{\text{Coul}} + U_{\text{ext}}$ of the many-particle system includes, along with Coulomb energy, the energy in possible external potential fields. Equations (1.5) determine the overdamped molecular dynamics (MD) of charged particles in a thermostat in the presence of a magnetic field. Exactly the same equations (in the case where the 2D particle dynamics is confined within the XOY plane) were employed predominantly in our computer simulations in order to investigate the specific problems under consideration. Note that equations (1.5) are quite equivalent to the ones obtained in the paper [17] dealing with closely related problems.

It is worth mentioning that from equation (1.5) one can easily obtain the mean-square displacement for a charged particle moving in the presence of magnetic field $\vec{B}||OZ$ on the 2D XOY plane in a thermostat,

$$\langle r^2 \rangle = \frac{4Dt}{(1 + \beta^2)}$$

(1.7)

recovering the known result that the diffusivity in this case reduces by the factor $(1 + \beta^2)$ [18]. To establish the correspondence between friction $\gamma$ and diffusivity $D$, we used herewith the Einstein relation

$$\gamma D = k_B T.$$  

(1.8)

Let us now say a few words on the continuous probabilistic description of the problem under consideration associated with BD particle dynamics. To obtain the relevant drift–diffusion equations for the positive and negative plasma densities $n_{p,n}$, one can use the method given in the work [16]. Explicit allowance for the presence of a magnetic field yields

$$\frac{\partial n_{p,n}}{\partial t} = -\frac{1}{y_{p,n}(1 + \beta_{p,n}^2)} \text{div}\{n_{p,n} (\vec{K}_{p,n} + \beta_{p,n} [\vec{K}_{p,n} \times \vec{e}_z])\} + \frac{D_{p,n}}{(1 + \beta_{p,n}^2)} \Delta n_{p,n},$$

(1.9)

where $\vec{K}_{p,n} = -\varepsilon_{p,n} \nabla \phi$, and for the potential $\phi$, one needs to solve the associated Poisson problem

$$\Delta \phi = -4\pi e (\varepsilon_p n_p + \varepsilon_n n_n).$$

(1.10)

As we can see, the essential difference between the continuous drift–diffusion and microscopic BD approaches is that the former is based on the mean-field concept for the potential $\phi$ and provides no possibility to study the inter-particle correlations.

Note that, as one would expect, in the absence of magnetic field ($\vec{K} \neq 0$, $\beta = 0$), equation (1.9) reduces to the conventional Smolukhovsky equation (i.e. the Fokker–Planck equation in the coordinate space) [16], and in the absence of Coulomb forces but the presence of a magnetic field ($\vec{K} = 0$, $\beta \neq 0$) we obtain the diffusion equation with modified diffusivity, i.e. the result consistent with equation (1.7) for a single charged particle. At the same time, in the limit $\beta \rightarrow \infty$ one can easily obtain from equation (1.9) the drift-Poisson model mentioned above. We stress these points, since they are evidence for the correctness of the basic dynamic equations (1.5) used in our computer simulations.

3. Description of numerical simulations

Note that, in actual simulations, we used the dimensionless form of equations obtained by employing the physical units relevant to the problem and dimensionless parameters. In what follows, we use size $L$ of the simulation square on a 2D plane as a length unit, and $T_0 = L^2 / D_p$ (typical diffusion time over the distance $L$) as a time unit. The associated dimensionless variables are length $\tilde{r} \equiv r / L$ and time $\tau \equiv t / T_0$, respectively. The other dimensionless parameters relevant to the problem, which were used in simulations, are as follows. We define the field strength parameters for positive and negative plasma components as

$$\beta_{p,n} \equiv \frac{\varepsilon_{p,n} B}{c y_{p,n}},$$

(2.1)

where $\varepsilon_{p,n} = \pm 1$. The coupling parameter,

$$\chi \equiv \frac{e^2}{k_B T L},$$

(2.2)

specifies the potential-to-kinetic energy ratio in the system. In the 2D case it relates to the plasma coupling constant $\Gamma$ commonly used in physics of strongly coupled plasmas as $\Gamma \approx \sqrt{2\pi N \chi}$ (here $N = N_p + N_n$ is the number of particles in the simulation square for plasma components). Also, we used in simulations the dimensionless recombination coefficient, $\sigma \equiv c_i / D_p$, with $c_i$ being the given 2D recombination coefficient (note that in the 2D case recombination and diffusivity are of the same dimensionality) and the ratio of diffusivities $\theta = D_n / D_p$.

Thus, in numerical simulations, BD equations (1.5) were solved for overall neutral systems consisting of equal numbers of oppositely charged point particles within a simulation square of unit size, with periodically repeating image configurations in the $X$ and $Y$ directions. To allow for the periodic boundary conditions, the nearest image approximation has been used, which is usual for particle MD or Monte Carlo simulations [19]. The pair Coulomb interaction between particles has been slightly modified in simulations at short distances (for $\rho \leq \rho_c$),

$$V_{\text{Coul}}(\rho_{ik}) = \chi \frac{\rho_{ik} \rho_{ik}}{\rho_{ik}^2} \text{erf}(\rho_{ik} / \rho_c),$$

(2.3)

in order to eliminate the singularity. In simulations we choose $\rho_c$ to be rather small, at least by an order less than the typical radius of recombination (see below).

As concerns the description of recombination, the main requirement is to reproduce with good accuracy the overall recombination process in accordance with the well-known relation

$$\frac{dN_{p,n}}{d\tau} = -\sigma N_p N_n$$

(2.4)
For this purpose, we specify the probability for a pair of opposite charges placed at a distance $\rho$ to recombine during the time span $\Delta \tau$ as

$$W(\rho, \Delta \tau) = \exp\left\{-\left(\rho/\rho_0\right)^2\right\}. \quad (2.5)$$

Here, we introduced the recombination radius $\rho_0 = \sqrt{2D\sigma\Delta \tau}$, where $D = 1/(1 + \beta^2/\sigma)$ has the meaning of modified diffusivity. The expression for the probability (2.5) can be obtained as an approximate solution of the relevant recombination–diffusion equation for a single pair of particles for short time intervals $\Delta \tau$. As was tested in computer simulations, this approach allows one to describe with satisfactory accuracy the recombination process given by equation (2.4).

Along with recombination, we took into consideration the process of particle generation, for instance, induced by UV radiation or as a result of charge injection. In all the simulations, we assumed the relevant source intensity $I$ to be uniformly distributed over the simulation square, without spatial correlations and constant with time. It was supposed that the particles are created randomly in time and space and in pairs, so that overall charge neutrality is conserved.

The random Langevin forces were generated in simulations so as to satisfy relations (1.3), in a manner given in [20]. The amplitude of the random force $F(t)$ acting on a particle during the time span $\Delta \tau$ is specified in the 2D case by the Gaussian probability distribution

$$w(\vec{M}) = \frac{1}{4\pi \gamma^2 D \Delta \tau} \exp\left(-\frac{M^2}{4\gamma^2 D \Delta \tau}\right) \quad (2.6)$$

for the momentum $\vec{M}$

$$\vec{M}(\Delta \tau) = \int_0^{\Delta \tau} F(t) \, dt \quad (2.7)$$

to be transferred to this particle. We omitted here the obvious subscripts for simplicity.

The general method of simulations was as follows. The overall time interval of a simulation run is divided into sufficiently short time steps ($\Delta \tau = 10^{-1} - 10^{-4}$). Beginning with the initial given particle configuration and set of parameters, equations (1.5) are solved in each step with the current number of particles and current (generated for this time step) amplitudes of random Langevin forces. At the end of the time step, the probabilities for each pair of particles in the configuration to recombine and for new pairs of particles to be created are evaluated. Then, according to this, the changes in the configuration are introduced. Therefore, the total number of particles and, respectively, the number of equations of motion (1.5) change with time. After that, the procedure repeats until the final time point is achieved. Note that the cooperative phenomena, which we are going to examine, manifest themselves at rather long time intervals, of the order of $\tau \approx 10^2 - 10^3$. For this reason, one should expect that the results of simulations should not be sensitive to the choice of time step, which is an arbitrary parameter of simulations. In actual fact, the magnitude of a time step is determined by the requirement that the displacement of each particle must be much less than the typical spatial irregularity of fields. Accordingly, higher magnitudes of magnetic field admit longer time steps (as evidenced by equation (1.7)), whereas the strong coupling makes shorter time steps necessary. The configurations obtained in the above way are then written successively into a file for subsequent analysis.

In simulations, we monitored the total number of particles, the Coulomb energy per particle and binary distribution functions. Also, we evaluated the kinetic energy of drift motion per particle for both components, which we define as

$$E_{\text{kin}}^{\text{p,n}}(t) = \frac{1}{2} (\vec{v}_{\text{p,n}}^2(t)), \quad (2.8)$$

and the spectral distribution of the above kinetic energy over a given time interval, defined as

$$S_{\text{p,n}}(\omega) = A_{\text{p,n}} (\vec{v}_{\text{p,n}}^2(\omega)), \quad (2.9)$$

where $\vec{v}_{\text{p,n}}(\omega)$ are the Fourier amplitudes for velocities of drift motion over that time interval, and $A_{\text{p,n}} = 1/\sum_\omega (\vec{v}_{\text{p,n}}^2(\omega))$ is the relevant normalization coefficient. The brackets denote as usual the averaging over the ensemble of particles. The spectral density $S$ enables one to estimate the extent of coherency of rotational motion of particles.

It should be mentioned that, in order to check the accuracy of simulations, we performed a large number of tests. Most important of them, which have physical meaning, are the drift and diffusion tests, recombination tests mentioned above and comparative studies based on LD and BD simulations. In the drift and diffusion tests, we checked the consistency of the results for average drift and mean-square displacement of a single charged particle in magnetic field obtained within LD, BD and genuine microscopic MD, where the presence of the heat bath was simulated by random scattering of particles. In particular, the validity of equation (1.7) in the drift–diffusion limit has been verified. In comparative LD and BD tests, agreement between the results obtained within these approaches for many-particle TCP in magnetic field has been checked.

In addition, we performed a number of tests of the properties of equilibrium one-component plasmas (OCP). As is known, OCP are capable of forming Coulomb liquids or crystal lattices at strong coupling. The usual methods of computer simulation for these are Monte Carlo or MD simulations (see, for instance, [21]). The BD simulations demonstrated the possibility of reproducing these properties (we checked the formation of a 2D hexagonal lattice, binary distributions, etc). This means that the strong correlations are correctly described within the BD approach.

4. Numerical simulations: discussion of the results

BD simulations for the above formulated system have been performed with the total number of particles $N_{tot} \approx 100–2000$ for the following range of parameters: field strength parameter $\beta = 10–500$; ratio of diffusivities $\theta = 0, 1$ and $10$; coupling parameter $\chi = 0.01–20$ (which corresponds approximately to the range of plasma coupling $\Gamma \approx 0.1–500$); recombination $\sigma_1 \approx 1–10^2$; source intensity $I = 0–1000$. The most important, from the theoretical point of view, results have been obtained for rather extreme parameters as regards their relevance to the
real physical planar semiconductor systems. Below, we will discuss this issue in more detail, but first we present the results of a number of runs with the most significant typical features.

The first series of runs has been performed for the following parameters: $I = 200$, $\sigma = 200$, $\beta_p = 500$, $\theta = 10$ and $\chi = 2 \times 10^{-2}$. The simulations started from the initial particle configuration with $N_p = N_n = 510$ and random uniform spatial distribution. The results are given in the figures. In figures 1 and 2, a series of subsequent snapshots of plasma configurations is given illustrating the formation of a vortex structure. The associated behavior of the particle number and the Coulomb energy per particle is given in figures 3 and 4. The formation of a steady state occurs during the time span of the order $\tau \approx 100$–500 as a result of the build up of fluctuations accompanied by the creation of small vortices, which then merge together. The examination of the configuration in figure 2 reveals that the opposite plasma components in the final steady state are to a considerable extent spatially separated. The behavior of spectral density for the drift kinetic energy manifests the peaks which correspond to the rotation frequency of the majority of plasma particles (figure 5). Note that the more mobile negative component (with higher diffusivity) rotates at a higher speed and forms a more compact and pronounced vortex than the slow positive component. This is also evident from the behavior of the kinetic energy of drift motion in figure 6, where an acceleration of plasma components in the process of self-organization is observed. In figure 7, we give the behavior of binary distribution functions, which show the tendency for like-charged particles to form compact clusters whereas oppositely charged particles tend to be spatially separated. Remarkably, these distributions indicate the high probability for formation of like-charged pairs of particles at close separations, which, in actual fact, turn out to be micro-vortices. Note that this behavior of pair distributions is quite opposite to that typical for the equilibrium TCP. The unusual behavior of the Coulomb energy should be pointed out as well. Typically, the Coulomb energy per particle in equilibrium TCP is $\Xi = -\Gamma$, while for the vortex structures observed in these simulations it is positive and considerably exceeds the coupling constant (for the final steady state, its average value was estimated as $\Gamma \approx 1.5$). Obviously, all the

Figure 1. Successive plasma configurations for the time points $\tau = 0, 100$ and 200 (from left to right). Open circles denote the positive plasma component, and filled circles denote the negative plasma component. The process of spatial separation of plasma components is accompanied by the growth in rotation of plasma clusters.

Figure 2. Steady-state vortex DS formed at $\tau = 300$ as a result of the evolution given in figure 1.

Figure 3. Number of particles in the simulation square as a function of time.
above mentioned are evidence that the plasma system is far from equilibrium, which is related in the end to the spatial self-organization (separation of components). Note that this plasma state can be identified as rather non-ideal, since for the ideal plasma it must be $\Gamma \ll 1$.

Similar structures have been observed in numerical experiments at different plasma parameters. The most essential requirement for the above vortex self-organization is that the recombination must be rather intensive ($\sigma \gg 1$) and the magnetic field strong ($\beta \gg 1$). We regard the vortex structures obtained in these runs as DS, since they appear as a result of the process of self-organization in open dissipative systems, where non-equilibrium is maintained due to the recombination and generation of particles, and exhibit marked signs of ordering. Note that we can regard the pair distributions or spectral densities for velocities as the order parameters for these DS. One of the most distinguishing signs of vortex growth is the above-mentioned unusual behavior of the Coulomb energy of the system.

In the next series of runs, we examined the relaxation of DS obtained in previous runs. We here call as relaxation a free decay of an initial plasma state in the absence of sources of particle generation (i.e. with no pumping). Naturally, this non-equilibrium process is induced, in particular, by plasma recombination and dissipation. The simulations started from the final configurations of a steady-state DS obtained in the manner described above, with the same parameters, except for the source intensity, $I = 0$. The results presented in figures 8–13 indicate that, as a result of relaxation, the magnetized TCP forms an almost steady collective vortex state with very distinctly spatially separated plasma

Figure 4. Behavior of the Coulomb energy of a 2D plasma system in the process of the formation of vortex DS.

Figure 5. Spectral distributions for kinetic energy of drift motion for positive (left) and negative (right) plasma components evaluated for the steady vortex DS over the time interval $\Delta \tau = 250–300$.

Figure 6. Kinetic energy of drift motion for positive (solid) and negative (dashed) plasma components during the process of self-organization of vortex DS. The sharp peaks occur due to the Coulomb collisions at close inter-particle distances.
components and a high extent of coherency. A snapshot of the configuration for this state is given in figure 8. As can be seen from the behavior of the particle number and Coulomb energy (figures 9 and 10), during a short time span $\Delta \tau = 300-310$, the processes of intensive recombination dominate accompanied by the increase in Coulomb energy. After that, the system acquires a state of relatively high coherency, which is evidenced, in particular, by the direct visual observations of particle configurations. Then, the system slowly loses its kinetic energy due to dissipation (figure 11). In particular, the retardation manifests itself in lowering the rotation frequencies of plasma components (as compared to the above case of steady-state DS). However, the extent of coherency of rotational mode increases therewith, and the kinetic energy tends to concentrate in a narrow frequency band of the spectral distributions (figure 12). At this stage the recombination becomes rather insignificant, obviously due to the spatial separation of plasma components. We performed a number of test runs starting from this coherent vortex state after making the recombination negligibly small ($\sigma \simeq 10^{-1}, 10^{-3}$ and $10^{-5}$). These runs have shown that the behavior of the coherent vortex mode does not essentially depend on the intensity of recombination. The radial distributions exhibit qualitatively the same features as for the DS (figure 13). In general, the properties of vortex DS and free coherent vortex modes in 2D magnetized plasmas turned out to be similar. The main difference is that the plasma rotation in DS is much less coherent.

Thus, we observed in simulations the formation of DS in the open system, namely the magnetized 2D
The essence of the phenomenon is that due to non-equilibrium recombination and generation processes, predominantly the drift vortex (rotational) collective mode is excited. Apparently, this steady state of DS can exist as long as pumping and recombination are present in the system. However, after being excited, the above collective mode can exist for rather long by itself even in the case where the sources of non-equilibrium (pumping and recombination) are removed. The lifetime of this free mode is determined evidently by the dissipation present in the system. In this connection, we find it appropriate to draw an important analogy with the classic example of an open DS, the laser. In a laser, also there exists a collective mode represented by a standing electromagnetic wave in a mirror resonator. Once excited, it decays due to dissipation, whereas in the absence of dissipation and losses (e.g. via mirrors), it would exist for an infinitely long time. However, in order to trigger the process of self-organization, i.e. to excite this mode, pumping and dissipation in an active medium are needed. In our case, the role of a dominating collective mode is played by the vortex rotational mode, and generation and recombination are responsible for its excitation.

Let us say a few words on the possibility for experimental observation of the vortex DS predicted in our computer simulations in planar semiconductor structures. We performed simple estimates for a range of parameters that are more or less plausible for semiconductors and the problem as a whole, namely square size $L \approx 1–3 \mu m$; surface charge density $n \approx 10^{10} \text{cm}^{-2}$; electron/hole diffusivity $D \approx 10^2–10^3 \text{cm}^2 \text{s}^{-1}$; temperature $kT \approx 0.001–0.05 \text{eV}$; magnetic field $B \approx 10–20 \text{Tl}$; source intensity (pumping) $I \approx 10^{18}–10^{19} \text{cm}^{-2} \text{s}^{-1}$; typical time scale $t \approx 10^{-9}–10^{-10} \text{s}$.

**Figure 11.** Kinetic energy of drift motion for positive (solid) and negative (dashed) plasma components in the process of relaxation.

**Figure 12.** Spectral distributions of kinetic energy of drift motion for positive (left) and negative (right) plasma components for the coherent vortex state evaluated over the time interval $\Delta \tau = 350–400$.

**Figure 13.** Radial distributions for the coherent vortex state formed after the period of relaxation averaged over the time span $\tau = 350–400$. 

\[ \frac{\psi_{\Delta r}}{2N_{\theta}T} \]

\[ g(\rho) \]

Let us say a few words on the possibility for experimental observation of the vortex DS predicted in our computer simulations in planar semiconductor structures. We performed simple estimates for a range of parameters that are more or less plausible for semiconductors and the problem as a whole, namely square size $L \approx 1–3 \mu m$; surface charge density $n \approx 10^{10} \text{cm}^{-2}$; electron/hole diffusivity $D \approx 10^2–10^3 \text{cm}^2 \text{s}^{-1}$; temperature $kT \approx 0.001–0.05 \text{eV}$; magnetic field $B \approx 10–20 \text{Tl}$; source intensity (pumping) $I \approx 10^{18}–10^{19} \text{cm}^{-2} \text{s}^{-1}$; typical time scale $t \approx 10^{-9}–10^{-10} \text{s}$.
They have shown that most of the parameters, despite being rather extreme, can still be achieved by employing strong magnetic fields and semiconductors with a high carrier mobility (diffusivity) such as GaAs. The problem may arise with the 2D recombination coefficient, which in real semiconductors seems to be much less than that needed to initiate the process of vortex formation. However, we expect that the implementation of a special experimental processing such as doping with recombination-enhancing traps-impurities could remedy this issue.

5. Conclusions

To conclude, we study the formation of vortex structures in non-equilibrium magnetized plasmas, with allowance for charge recombination and generation, confined in a planar geometry, on the basis of microscopic first-principle LD computer simulations. In simulations, we examined the behavior of the Coulomb energy, kinetic energy of drift motion and its spectral distributions, and the intrinsic microscopic structure of vortices. The simulations have shown that, for sufficiently strong magnetic fields and intensive recombination and pumping, the magnetized 2D plasma forms steady-state vortex DS maintained due to non-equilibrium recombination and generation processes. The plasma state in these DS is far from equilibrium, which is evident from the properties of binary distributions and behavior of the Coulomb energy of the system. The latter are distinctly different from the ones typical for equilibrium plasmas and indicate pronounced spatial separation of positive and negative plasma components. The investigation of the relaxation of vortex DS in the absence of pumping reveals the formation of slowly decaying rotational vortex mode with a high extent of coherency. Due to the spatial separation of components, the recombination in this coherent state is negligibly small. We expect that the predicted dissipative and coherent vortex structures could be observed in such physical systems as the electron–hole plasmas in semiconductor hetero-structures, e.g. quantum wells or other planar plasma systems with dissipation.

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