Total hadronic and photonic cross sections and the nuclear configurational entropy concept

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Received: 18 May 2021 / Accepted: 30 September 2021
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Abstract In the framework of eikonalized mini-jet model, contributions from gluon recombination effects are studied for the free proton in $pp$ and $\bar{p}p$ reactions. The nuclear configurational entropy is then used to calculate the main parameters to estimate the total proton cross sections for the recent data LHC, also corroborating to previous developments. Photoproduction cross sections with vector meson dominance and saturation effects for both total hadronic and photonic cross sections at a high-energy regime are investigated. Extreme points of the configurational entropy can shed new light on some features of specific models of nuclear reactions, which are described by the corresponding reaction cross section, providing the latter by the necessary model parameters within an optimal root-mean-square deviation.

1 Introduction

The interest toward a study of strong interaction in the color-glass condensate (CGC) setup has been directed to how the configurational entropy (CE) approach can indicate directions for experiments and various methods in QCD [1–5]. Answering this question requires knowledge of the information of the total inelastic cross section that could lead to collective nuclear excitations from the point of view of the nuclear configurational entropy and its impact on the fundamental reaction mechanisms [6–8].

The color-glass condensate theory is a useful and convenient method to study the high-energy interaction in QCD. It opens the possibility to a deeper understanding of the basic features of highly excited nuclear configurations as, for example, the study of different reaction channels in deep inelastic scattering (DIS) of a lepton scattering off a hadronic target. In this case, the interacted hadron acts as the set of partons close to on-shell excitations and holding the fraction of the total hadron longitudinal momentum ($x$). At the lowest order of the perturbation theory, the inclusive DIS cross sections can be represented using the Lorentz-covariant quantities, such as $x$. In such a case, the corresponding longitudinal momentum portion, carried by a parton in the hadron, is the virtual photon four-momentum, exchanged between the electron and the hadron, at the center-of-mass energy squared $s$. The CGC can be considered as a special state of the highly excited matter with the gluon saturation at the typical momentum (saturation) scale, $Q_s$, which rises with the energy. It can be assumed as

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the critical interface that separates the linear and the saturation regimes of QCD dynamics [9–11].

Analytical and numerical studies have been already devoted to the deep investigation of such techniques concerning various aspects of interaction, including the production of mesons (scalar, vector, and tensorial) resonances [12–17], glueballs [18,19], charmonium and bottomonium [20], the quark-gluon plasma [21], baryonic fields [22], and also using the AdS/QCD model approach [23]. The configurational-entropic origin of the above mentioned studies can be found in Refs. [24–30]. Extreme points of the CE driving dominant states of physical systems have been also studied and analyzed in Refs. [31–40], also for AdS black holes and their quantum portrait as Bose–Einstein graviton condensates, emulating magnetic structures [41]. These critical points manifest the stability and the predominance of the system states, which also give the vision about the mechanism of interaction for different nuclear systems using, for instance, the CGC for parton saturation effects [3,6,23,42–44]. Such points have been successfully investigated for the calculation of the inelastic hadron cross sections. The probability of production of different reaction channels can be suggested as spatially localized configurations at high-energy regimes [3,6,43,44].

As it is well known, the total hadronic cross section rises with energy, which can be explained by QCD. There is a close relationship between the total hadronic cross section at the energy determined concerning the center-of-mass energy ($\sqrt{s}$) and the production of jets with the principal role of the interacted partons, comprising the so-called mini-jet model [45]. According to this model, the total hadronic cross sections at high energy can be represented as a sum of two terms: a nonperturbative energy-independent contribution, $\sigma_{0,a n d s e m i - h a r d}$, perturbative QCD ($\sigma_{pQCD}$), at low transverse momenta. A perturbative component at a high-energy regime is characterized by small fractional momentum (small Bjorken-$x$) of gluons. All calculations based on the mini-jet model confirm the rise of the hadron cross section with energy within the pQCD. The eikonal representation of the mini-jet model involves a comprehensive description of the dynamics of partons at high energies. Numerical experiments results of a deep inelastic scattering from HERA in a wide range of four-momentum transfer to the proton, $Q^2$, at small $x$, confirmed the theoretical calculations of a rapid gluon density rise with a falling of $x$ value as a result of nonlinear effects in gluon evolution equations. Such a growth at $x \lesssim 10^{-4}$ can be explained as gluons domination at this range, with the achievement of saturation afterward.

There are two main cases concerning the momentum transfer, $k_\perp$, and the transverse size of a gluon during $g \rightarrow gg$ interactions: on the one hand, at large momentum transfer one can expect a large number of small size gluons per unit of rapidity. Another case is that small momentum transfer gluons, which are originated during the interaction, overlap themselves in the transverse zone via the fusion process. The fusion case has a low probability for values $k^2_\perp > Q^2_s$, where $Q_s$ is a saturation scale. And the probability of the fusion rise at momentum transfer $k^2_\perp < Q^2_s$ as the gluon density is large and rises with small $x$. Within a CGC, a system is limited by the maximum phase-space parton density. This condition is described by the hadron wave function and the energy-dependent momentum scale, $Q_s(x)$.

In this paper, the nuclear CE concept is used to study the influence of the gluon recombination during high-energy photonic and hadronic interactions, namely, to reconstruct experimentally obtained total reaction cross sections. In Sect. 2, we describe the eikonalized mini-jet model for hadronic and photonic cross sections as well as present data analysis for the inelastic interaction within the Glauber formalism. In Sect. 3, we compare the results of our calculations using the CE approach and give the basic onset for the probability of the production cross section in photon and hadron collisions from the point of view on the nuclear system configuration. Section 4 is devoted to the conclusion.
2 Eikonalized mini-jet model

The main idea of the hadronic cross section within the QCD is a unitarity constraint formalism. In the so-called eikonal formulation, the mini-jet cross sections undergo the unitarization procedure after sequential multiple scattering process. Here, the total, elastic, and inelastic \( pp(\bar{p}) \) cross sections can be, respectively, represented as

\[
\sigma_{\text{tot}}^{pp(\bar{p})}(s) = 2 \int d^2b \left[ 1 - e^{-3} \chi(b,s) \cos[\Re \chi(b,s)] \right],
\]

(1)

\[
\sigma_{\text{el}}^{pp(\bar{p})}(s) = \int d^2b \left[ 1 - e^{i} \chi(b,s) \right]^2,
\]

(2)

\[
\sigma_{\text{inel}}^{pp(\bar{p})}(s) = \int d^2b \left[ 1 - e^{-2.3} \chi(b,s) \right].
\]

(3)

where \( b \) is the impact parameter of the collision; \( \chi(b,s) \) is the eikonal function, which depends on the energy and the transverse momentum of the nuclear matter distribution and can be determined as \( \chi(b,s) = \Re \chi(b,s) + i \Im \chi(b,s) \). In the case of for \( pp(\bar{p}) \) reactions, the real part of eikonal function, \( \Re \chi(b,s) \) is assumed to be zero as an approximation since it was observed to be small as in the ratio of the real to the imaginary part of the forward elastic amplitude the real part of \( \chi(b,s) \) consist of 4%. The interaction of the partons obey a Poisson distribution and the average number of the inelastic collisions can be represented as a sum of soft and hard as \( n(b,s) \equiv 2 \Im \chi(b,s) = n_{\text{soft}}(b,s) + n_{\text{hard}}(b,s) \) concerning such parameters as \( b \) and \( s \):

\[
n(b,s) = W(b, \mu_{\text{soft}}) \sigma_{\text{soft}}(s) + \sum_{k,l} W(b, \mu_{\text{hard}}) \sigma_{k,l}^{\text{hard}}(s).
\]

(4)

In Eq. (4), \( W(b, \mu_{\text{soft}}) \) and \( W(b, \mu_{\text{hard}}) \) are the effective overlap functions of the nucleons, which contain information about the nucleon form factor and usually are normalized according to \( \int W(b, \mu) d^2b = 1 \); \( \sigma_{\text{soft}}(s) \) and \( \sigma_{k,l}^{\text{hard}}(s) \) are the excitation functions of the reactions, measured in millibarns (mb).

Within the mini-jet leading order (LO) pQCD model, when the partons are produced back to back in the transverse plane, the differential production cross section can be given by:

\[
\frac{d\sigma}{dy}^{mj}_{k,l}(s) = \kappa \int d\hat{t} d\hat{u} \sum_{i,j} \sum_{x_1} f_{i/h_1}(x_1, Q^2) x_2 f_{j/h_2}(x_2, Q^2)
\]

\[
\times \left[ \delta_{f_k} \frac{d\hat{\sigma}^{ij\rightarrow kl}_{\hat{t},\hat{u}}}{d\hat{t}}(\hat{t}, \hat{u}) + \delta_{f_l} \frac{d\hat{\sigma}^{ij\rightarrow kl}_{\hat{t},\hat{u}}}{d\hat{u}}(\hat{u}, \hat{t}) \right],
\]

(5)

where the constant \( \kappa \), \( h_1 \) and \( h_2 \) are the interacting hadrons and by \( d\hat{\sigma}^{ij\rightarrow kl}/d\hat{t} \) one labeled the subprocess cross sections \( (gg \rightarrow gg, gq(\bar{q}) \rightarrow gq(\bar{q}) \) and \( gg \rightarrow q\bar{q} \ (q \equiv u, d, s)) \). In such a case, the hard term of the eikonal functions have the following form: \( W(b, \mu_{gg}) \), \( W(b, \mu_{gq} = \sqrt{\mu_{gq} \mu_{gg}}) \) and \( W(b, \mu_{qq}) \) with free mass parameters \( \mu_{gq} \) and \( \mu_{gg} \), which determine the effective area occupied by quarks and gluons. For each final state of the partons, \( k \) and \( l \) correspond the parameter of rapidity, \( y \equiv y_1 \) and \( y_2 \), while the transverse momentum of a parton is denoted by \( p_T \geq p_{T_{\text{min}}} \), where \( T_{\text{min}} \) is the smallest transverse momentum of a parton with the corresponding density \( f_{i,j/h_1,2}(x_{1,2}, Q^2) \). Moreover, for two colliding partons, \( i \) and \( j \), the fractional momenta are \( x_{1,2} = p_T / \sqrt{s} (e^{\pm y_1} + e^{\pm y_2}) \) and the ratio \( 1/(1 + \delta_{kl}) \) determines a statistical weight of the particles in the final state.
Within the eikonal limit, which implies the straight-line trajectories of colliding nucleons, one can use the Glauber multiple collision approximation \([46, 47]\) in order to estimate the inelastic proton–nucleus cross section, \(\sigma^{pA}_{\text{inel}}(s)\). Such cross section is immediately derived from the corresponding inelastic nucleon–nucleon \((NN)\) cross section, \(\sigma^{NN}_{\text{inel}}(s)\) as:

\[
\sigma^{pA}_{\text{inel}}(s) = \int d^2b \left[ 1 - e^{-\sigma^{NN}_{\text{inel}}(s) T_A(b)} \right],
\]

where \(T_A(b) = \int dz \rho_A(b, z)\) is the thickness function with \(\int d^2b T_A(b) = A\), which determines the number of nucleons in the nucleus \(A\) per area concerning the \(z\)-axis.

The mini-jet model is used also for the calculation of \(\gamma p\) and \(\gamma \gamma\) cross sections. One can use \(pp\) forward scattering amplitude through the vector meson dominance (VMD) to derive the cross section for reactions as well as the additive quark model, in which one uses a probability \((P^{\gamma p(\gamma)}_{\text{had}})\) that the photon interacts as a hadron. Thus, if we suggest that the interaction of high-energy photon as a hadron, consisting of two quarks \((\sigma^{s,h} \mapsto \frac{2}{3}\sigma^{s,h}\) and \(\mu_{s,h} \mapsto \sqrt{\frac{3}{2}}\mu_{s,h}\) in both, soft and hard components of Eq. (4)), one can derive the simple equation for \(\gamma p\) cross section in the following form:

\[
\sigma^{\gamma p}_{\text{tot}}(s) = 2P^{\gamma p}_{\text{had}} \int d^2b \{1 - e^{-3\chi^{\gamma p}(b, s)} \cos[\Re \chi^{\gamma p}(b, s)]\}.
\]

And in the case of \(\gamma \gamma\) cross section \((\sigma^{s,h} \mapsto \frac{4}{3}\sigma^{s,h}\) and \(\mu_{s,h} \mapsto \frac{3}{2}\mu_{s,h}\), one has correspondingly:

\[
\sigma^{\gamma \gamma}_{\text{tot}}(s) = 2P^{\gamma \gamma}_{\text{had}} \int d^2b \{1 - e^{-3\chi^{\gamma \gamma}(b, s)} \cos[\Re \chi^{\gamma \gamma}(b, s)]\}
\]

where \(P^{\gamma \gamma}_{\text{had}}\) is a free parameter.

### 3 The model parametrization and configurational entropy

During the analysis, for instance, of the latest data concerning reaction cross section for \(pp\) and \(p\bar{p}\) scattering as well as for \(\gamma\)-proton, and \(\gamma \gamma\) reaction, one uses the fitted parameters for the soft term of the model at low energies, and the fixed parameters for the hard term at higher energies. Then using Eqs. (6), (7), and (8), the inelastic \(p\)-Air, \(\gamma p\), and \(\gamma \gamma\) cross sections for \(pp\) and \(p\bar{p}\) reactions can be obtained, which is related to the energy dependence of the total cross sections for the \(pp\) and \(p\bar{p}\) reactions (3). The fixed hard parameters for nonlinear evolution (EHKQS) were studied in Ref. [45] at

\[
p^2_{\text{min}} = 1.51 \text{ GeV}^2, \quad \mu_{gg} = 2.00 \text{ GeV}, \quad \mu_{q\bar{q}} = 0.70 \text{ GeV},
\]

whereas the parameter \(\mu_{s,\text{soft}}^2 = 0.7 \text{ GeV}^2\) was used [45].

The analysis in Refs. [48–53] yields the increment of cross sections at high energies induced by the increasing of gluon densities, at small \(x\). The nonlinear evolution leads to a small softening of these cross section due gluon fusion \(gg \to g\) processes. Above \(\sqrt{s} \sim 6\) GeV, the cross section in Ref. [45] was fitted by Froissart-like parametrizations,

\[
\sigma(s) = (30.5 \pm 0.8) + \alpha_1 s^{\beta_1} - \alpha_2 s^{\beta_2} + (0.2014 \pm 0.0035) \log^2(s),
\]

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where $\alpha_1 = 42.53 \pm 1.35$ mb, $\alpha_2 = 33.34 \pm 1.04$ mb, $\beta_1 = -0.458 \pm 0.017$, and $\beta_2 = -0.545 \pm 0.007$.

In our analysis, the mini-jet model of QCD can be then employed for the photoproduction cross sections with the inclusion of the vector meson dominance and the saturation effects for total hadronic and photonic cross sections at a high-energy regime.

The configurational entropy approach provides a nuclear system with the necessary information that defines interconnections among its constituents. In other words, several features of localized nuclear systems encoded in particle properties, which include different regimes of the wave space, can be evaluated by the CE. When the system under study is described by the uniform probability distribution, the CE reaches its maximum value. In the case of the continuous limit, when the information dimension tends to zero, the so-called differential configurational entropy (DCE) mode, which has nat\(^{-1}\)/unit volume in the way as nat\(^{-1}\) = log 2/bits, can be used instead of the CE. In such a case, DCE estimates the information obtained in the interval of the real line between the origin and the Euler’s number for the uniform probability distribution [27]. Once the nuclear configuration is recognized as a localized scalar field, comprised by the cross section, the DCE approach can be used to describe such a system. Indeed, the main aim of the DCE is to correctly describe the correlated nuclear configuration. In order to characterize quantitative the excited nuclear system, one can apply the interaction cross section, $\sigma(s)$, as a measurable Lebesgue-integrable localized scalar field. The decomposition of the Fourier function into the number of weighted component modes, associated with hadronic cross sections was introduced in Ref. [13]. The total inelastic cross section can be transformed through the energy-weighted correlation Fourier transform of the cross section, considered as a spatially localized function,

$$\sigma(k) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} \sigma(s)e^{iks}ds,$$  \hspace{1cm} (11)

where $k$ denotes the momentum space coordinate. It is worth emphasizing that the cross section $\sigma$ is a function of the parameters $p_{\text{min}}$, $\mu_{gg}$, and $\mu_{q\bar{q}}$. The important role of nuclear CE-based techniques is to try to derive from the theoretical point of view the values assumed by the parameters listed in Eq. (9), obtained by experiments, as the ones which can extremize the nuclear CE of the physical system. The expression [12]

$$\Gamma(s) = \int_{\mathbb{R}} ds' \sigma(s')\sigma(s + s')$$  \hspace{1cm} (12)

formally emulates the power spectrum for the case of the cross section and probes the amount of correlations among the nuclear system constituents, together with the information entropy. Wave modes correspond to a probability distribution, $P$, describing the power spectrum associated with the wave mode, [27] $P(k) \propto |\sigma(k)|^2 dk$. Thus, we calculate the modal fraction by the following equation:

$$f_{\sigma(k)} = \frac{|\sigma(k)|^2}{\int_{\mathbb{R}} |\sigma(k)|^2dk}.$$  \hspace{1cm} (13)

The modal fraction constituting the nuclear CE specifies the probability of the reaction and thus can give us all the information we need concerning the given reaction at a given energy regime. The mechanism of the high-energy collision can be completely determined by the number of critical points of the nuclear CE to obtain the parameters, which describe any

\footnote{Natural unit of entropy.}
localized system [24,26]. After all, we apply appropriate expression for the nuclear CE [24] in order to calculate the corresponding critical points:

$$\text{CE} = -\int_{\mathbb{R}} f_{\sigma(k)} \log f_{\sigma(k)} \, dk.$$  \hspace{1cm} (14)

The nuclear CE has units of nat (the natural unit of information). Thus, having as a starting point the total inelastic cross section for hadronic and photonic reaction [45] given by Eqs. (7, 8), we compute the nuclear CE using Eqs. (11)–(14). The results for the inelastic \( pp \) and \( \bar{p}p \) cross sections, in Fig. 1, represent the results of our calculation for the total inelastic cross section in hadron–hadron and photon–hadron or photon–photon collisions.

Figure 1a shows the prevalence of quantum states with parameters \( p_{T_{\text{min}}}^2 = 1.531 \text{ GeV}^2 \) and \( \mu_{gg} = 2.009 \text{ GeV} \). Such a dominance of states fades away in the space of parameters, as one moves away from the center \( p_{T_{\text{min}}}^2 = 1.531 \text{ GeV}^2 \) and \( \mu_{gg} = 2.009 \text{ GeV} \) that corresponds to the global minimum of the nuclear CE. In Fig. 1b, the hotter [colder] the color, the lower [higher] the probability of dominant states is. The numerical analysis involving Fig.
1a is a direct result of the nuclear CE computation, yielding a global minimum point at $p_{T_{\text{min}}}^2 = 1.531 \text{GeV}^2$ and $\mu_{gg} = 2.009 \text{GeV}$.

One can see from Fig. 1a that the concentric curves are configurationally isentropic, consisting of the states in the space of parameters that present the similar values of nuclear CE. This is accomplished in the picture of the interaction the calculation for nuclear CE for the total inelastic cross sections where the minimum on the curve coincides with the fitted data from Ref. [45]. If compared to the parameters fitted to experimental data, $p_{T_{\text{min}}}^2 = 1.51 \text{GeV}^2$, $\mu_{gg} = 2.00 \text{GeV}$, the differences are at the level of $\Delta p_{T_{\text{min}}}^2 = 1.39\%$ and $\Delta \mu_{gg} = 0.45\%$.

We have been able thus to determine the critical point of the nuclear CE, corresponding to the natural choice of the total inelastic cross section for the free parameters $p_{T_{\text{min}}}^2$ and $\mu_{gg}$, corroborating to experimental data to a high degree of precision. When computing the nuclear CE, the global minimum value CE $(1.531, 2.009) = 1.0031$ nat was derived, at the point $p_{T_{\text{min}}}^2 = 1.531 \text{GeV}^2$ and $\mu_{gg} = 2.009 \text{GeV}$. Such absolute critical value reflects the configurational stability and dominance of system states that are characterized by $p_{T_{\text{min}}}^2 = 1.531 \text{GeV}^2$ and $\mu_{gg} = 2.009 \text{GeV}$. The main results of our calculation of high-energy collision via hadron or/and photon interaction are successfully confirmed by the global minimum of the nuclear CE. Such a global minimum of the CE refers to the most dominant state of the nuclear configuration. The outer regions in Fig. 1a show that are no other global minima of total inelastic cross sections at the observed parameters $p_{T_{\text{min}}}^2$ and $\mu_{gg}$. One can also see in Fig. 1a the ellipsis-like closed strips which are exactly the configurational isentropic subdomains. The center of the marine blue ellipsis-like curve corresponds to the values of the free parameters $p_{T_{\text{min}}}^2 = 1.531 \text{GeV}^2$ and $\mu_{gg} = 2.009 \text{GeV}$, for the global minimum CE $(1.531, 2.009) = 1.0031$ nat. The inner regions of the plots reflect the lower values of the nuclear CE, and regions which lie far from the center correspond to higher values of the nuclear CE. We intend to investigate further and deeper other types of nuclear configurations, with other field theoretical effects and other wave functions, including new fermionic ones, like the ones proposed in Ref. [54,55].

4 Conclusions

In the framework of the eikonalized mini-jet model, the total hadronic and photonic cross sections were studied via the nuclear configurational entropy approach. The total inelastic $pp$ and $p \bar{p}$ scattering and $\gamma p$ and $\gamma \gamma$ cross sections were estimated from the corresponding inelastic nucleon–nucleon cross section and the global minimum of the nuclear configurational entropy have been derived. Such minima show the critical point which can define within the uncertainty of the calculation the total inelastic production cross section for hadrons and/or $\gamma$ induced reactions. Within the color-glass condensate mini-jet model, it has been possible to provide the natural choice used on the complex theoretical analysis. Our calculations predicted the values of the free fitted parameters $p_{T_{\text{min}}}^2 = 1.531 \text{GeV}^2$ and $\mu_{gg} = 2.009 \text{GeV}$, for the global minimum CE $(1.531, 2.009) = 1.0031$ nat, within the uncertainties of calculation, $\Delta p_{T_{\text{min}}}^2 = 0.7\%$, whereas $\Delta \mu_{gg} = 0.45\%$. The errors represent its upper limits for the given fitted parameters for the given center-of-mass energy and impact parameter value. The deep investigation of the critical point of CE the interacted system gives us information about the stability of the system and the number of nuclear states available. Such a result can be successfully used to obtain the set of the trustable parameters which can adequately describe interaction as a whole.
Acknowledgements

GK thanks The São Paulo Research Foundation—FAPESP (Grant No. 2018/19943-6).

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