Non-parametric Probabilistic Load Flow using Gaussian Process Learning

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Abstract—In this work, we propose a non-parametric probabilistic load flow (NP-PLF) technique based on the Gaussian Process (GP) learning to understand the power system behavior under uncertainty for better operational decisions. The technique can provide "semi-explicit" power flow solutions by implementing the learning and testing steps which map control variables to inputs. The proposed NP-PLF leverages upon GP upper confidence bound (GP-UCB) sampling algorithm. The salient features of this NP-PLF method are: i) applicable for power flow problem having power injection uncertainty with an unknown class of distribution; ii) providing probabilistic learning bound (PLB) which further provides control over the error and convergence; iii) capable of handling intermittent distributed generation as well as load uncertainties, and iv) applicable to both balanced and unbalanced power flow with different type and size of power systems. The simulation results performed on the IEEE 30-bus and IEEE 118-bus system show that the proposed method can learn the voltage function over the power injection subspace using a small number of training samples. Further, the testing with different input uncertainty distributions indicates that complete statistical information can be obtained for the probabilistic load flow problem with average percentage relative error of order 10^{-5} % on 50000 test points.

I. INTRODUCTION

The decision-making process and control of power systems rely on the solution of optimal power flow (OPF) problem, solved by minimization of a variety of objectives such as real power generation cost. At the core of the OPF problem, there lies power balance equality constraints that govern the relationship between the voltages with the power injections, the so-called load flow problem. The non-convex load flow constraints pose obstacles in the decision-making process and in obtaining optimal setpoints, more so under uncertainty. Therefore, it is essential to understand such voltage-power relationship with the power input inside a uncertain space. With the increasing integration of renewable sources, such as wind power and photovoltaic (PV) energy, the generation parts of electrical systems may behave differently depending on the specific scenario [1]. On the other hand, there is also uncertainty in the load demand side, such as the impact of electric vehicles. Therefore, an essential analysis tool is probabilistic load flow (PLF), which provides an understanding of equality constraint and does mapping between voltage variables and the power inputs.

The PLF was first proposed in [2], considering the impact of the input variables with known uncertainty distribution characteristics. The PLF approach can provide the probability density functions (PDFs) of the output values such as the power flow and nodal voltages. In literature, the methods that have been applied to solve the PLF problem can be divided primarily based on the type of tools employed, i.e., numerical methods and analytical methods [3]. One of the most widely used numerical methods is Monte-Carlo simulation (MCS), which enters randomly generated variables to calculate the corresponding output by a large number of repetitive power flow calculations [4]. [5]. Although MCS can handle full nonlinear power flow equations, it requires a large number of iterations, which leads to a high computational cost and a long elapsed time. Further, many questions like how many points are sufficient and what can be the maximum error for any new point, cannot be answered using pure numerical approaches.

The other class, analytical methods for PLF, is mainly a set of statistical approaches aiming to obtain the results of some special moments or the mean, variance, and PDFs for the state and output variables [1]. Among them, convolution techniques follow mathematical assumptions in order to simplify the power flow problem [6]. But these methods do not apply to complex problems, such as AC power flow in unbalanced power distribution systems [7]. Further, for preserving power flow equations non-linearity, a set of deliberated operating conditions are used in point estimate methods [8]. However, their accuracy is low in estimating high order moments of probability distributions, especially for complex systems with many inputs [9]. The third analytical method is approximation expansions based on cumulants [10]. It is designed to obtain the cumulants of outputs from the cumulants of inputs through a simple mathematical process. These analytical methods can indeed reduce the computational costs, however, they suffer main drawbacks like: 1) requirement of model simplifications and adjustment of parameters, essentially losing information of tails of PDFs; 2) need of linearizing power flow equations for a specific operating condition, etc. With the high penetration of distributed energy resources, their accuracy declines due to ignoring non-linearity [9] of the power balance equation. Further, analytical methods are parametric and work on a fixed, specific type of uncertainty distribution only. Primarily, the numerical methods capture non-linearity but are slow while analytical methods need approximations leading to lower accuracy.

In the view of the aforementioned limitations in understanding power flow behavior under uncertainty, this work presents a non-parametric probabilistic load flow (NP-PLF) method using Gaussian process regression. The growing renewable generation (especially PV) and electric vehicles lack historical data and efficient forecasting. Therefore, it is challenging to obtain accurate uncertainty distributions for these uncertainties. This makes it imperative to develop non-parametric
methods which do not require input uncertainty distribution information. The proposed method deals with two inherent difficulties faced in obtaining PLF solutions: i) the non-linearity of the power flow equation set, and ii) lack of statistical information about uncertainty and complexity in modeling random power injection PDFs as input. We develop probabilistic learning bound (PLB) using regret bounds of the so-called GP upper confidence bound (GP-UCB) sampling algorithm. The PLB provides control over the desired accuracy and confidence level of learning. Hereafter, the learning step of proposed NP-PLF works without any PDF of input uncertainty and testing step allows obtaining the state and output variables (e.g., nodal voltages) for any class and type input uncertainty distribution. This two-step method makes the proposed approach “semi-explicit” in terms of the form of the power flow solution, while conventional numerical methods are implicit. By “semi-explicit”, we meant that we do not provide any fully analytical form of the voltage solutions, but the corresponding mean and error bound functions of power inputs expressed using the Gaussian process kernels such as those in (2).

The method can also serve as a multi-dimensional continuation power flow (CPF) (11) to estimate the power-voltage curve with a given confidence level as shown in fig.1. Further, this proposed uncertainty propagation framework could help in understanding other nonlinear balance equation behavior to facilitate the decision-making process, thus lend itself to smart buildings, microgrids, and control of electric transportation systems such as EVs.

The main contributions of the paper are as follows.

1) A novel non-parametric probabilistic load flow (NP-PLF) method, which handles the uncertain power injections. The method is generic as it does not rely on the class of uncertainty distribution. It is fast and captures the non-linearity of power flow equations. The learning step provides a “semi-explicit” form of power flow solutions, while the testing step can provide statistical information for the PLF solution.

2) Development of probabilistic learning bound (PLB) for the inverse power flow solution by GP learning and GP-UCB sampling. This PLB can serve as a convergence criterion for the learning stage algorithm and bounds the output solution within the specific range. Thus, PLB provides the confidence level for statistical information of output.

II. BACKGROUND

In this section, we introduce GP regression and GP-UCB algorithm for sampling training points for GP learning bounded by regret bounds. Hereafter, \( X = \{x^i \in \mathbb{R}^D \} \) represents matrix having \( N \) training points and \( y = \{y^i = y(x^i) \in \mathbb{R} \} \) indicates \( N \) observations over these training points for \( i = 1, \ldots, N \).

A. Gaussian Process Regression (GPR)

The Gaussian process is the backbone of the Bayesian optimization paradigm. The analogy extension of the multivariate Gaussian function and interpretation of GP as a distribution over random functions (12) is intuitive and very useful as a non-parametric method, for modeling PDFs over functions. In power system, various forecasting applications for wind power (13), solar power (15), and electricity demand (16) have been developed based on GP. Other than these forecasting works, the idea of using GP to learn the dynamics and stability index behavior has been explored recently in (17), (18).

The general non-parametric GP regression (GPR), with a training data set \( D = \{X, y\} \), attempts to infer the latent function \( f : \mathbb{R}^D \rightarrow \mathbb{R} \) (12) with i.i.d. noise \( \epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2) \):

\[
y(x^i) = f(x^i) + \epsilon, \quad i = 1 \ldots N.
\] (1)

The prior over this latent function, with zero mean GP, is placed as \( f(x) \sim \mathcal{GP}(0, k(x, x')) \). The kernel function \( k(x, x') \) incorporate our comprehension of unknown function into GP. We use squared exponential kernel for its smoothness property in this work. In the probabilistic load flow (PLF) problem, \( x \) can represent uncertain power injections such as those from intermittent renewable sources, and \( y(x) \) refers to the numerical voltage solution corresponding to a load injection point of \( x \). Interested reader can look into (12) for details of GP fundamentals. Following the extension of Bayesian rule (12), analytical formula set for posterior distribution for \( f \) are obtained as:

\[
\mu(x') = k(x', X)^T K^{-1} y
\] (2a)

\[
\sigma^2(x') = k(x', x') - k(x', X)^T K^{-1} k(x', X)
\] (2b)

Here, \( K = k(X, X) + \sigma^2 I \), \( \mu(x') \) is the inferred mean, \( k(x, x') \) is the covariance, and inferred variance is given as \( \sigma^2(x') \) at input \( x' \) (12). The random input vector \( x' \) can be drawn from any type of uncertainty distribution.

B. Gaussian Process Upper Confidence Bound (GP-UCB)

Sampling schemes play an important role in learning latent function. This paper relies upon the GP-UCB, widely used in Bayesian optimization paradigm (19), for sampling of input vectors. The target is to obtain the mean \( \mu(x) \) for function \( f(x) \) with least standard deviation \( \sigma(x) \) and probability at least \( 1-\delta \) with \( \delta \in (0, 1) \). A joint multi-objective function balancing between exploration and exploitation is opted for obtaining next input point \( x^i \). With \( \beta_1 \) taken independent of state vector and \( S \) being uncertain input subspace, the sampling strategy will be (19):

\[
x^i = \arg\max_{x \in S} \left\{ \mu_i(x) + \sqrt{\beta_{i+1} \sigma_i(x)} \right\}. \quad (3)
\]

Intuitively, (3) means that next sampled input point will be the one where weighted sum, \( \mu_i(x) + \beta_{i+1} \sigma_i(x) \), of mean and variance is maximum. Interested readers can refer to (19) for more detail of this sampling strategy and GP-UCB.

III. NON-PARAMETRIC PROBABILISTIC LOAD FLOW (NP-PLF)

In this section, we present the proposed NP-PLF algorithm with the main result as a probabilistic guarantee bound on
power flow solution. First, we present the generic power flow formulation and inverse power flow analogy for obtaining uncertain state variables corresponding to uncertain input.

The relationship between the apparent power injection at \( l \)-th and node voltages for an \( n \)-bus network is given as:

\[
S_l = \sum_{m=1}^{n} [|V_l||V_m|\left(\cos\theta_{ml} + j\sin\theta_{ml}\right)(G_{ml} - jB_{ml})]. \tag{4}
\]

Here, \( S_l \) is the complex apparent power injection in node \( k \), \( V_l = |V_l|\angle\theta_l \) is node voltage while \( \theta_{ml} = \theta_m - \theta_l \) is angle difference between node \( m \) and \( l \). The \( G_{kl} \) and \( B_{kl} \) are conductance and susceptance of line connecting node \( m \) and \( l \). This complex non-linear power flow equation (4) is solved using iterative numerical methods such as Newton-Raphson in a deterministic manner for one set of input. There does not exist any explicit form for the relationship between the node voltage and power injections. We define an input vector \( \mathbf{x} = [S_1, \ldots, S_l, \ldots, S_n]^T \) for \( n \)-bus network then the family of power flow equation (4) can be written in compact form as \( \mathbf{x} = h(\mathbf{y}) \) with \( \mathbf{y} \) being collection of node voltage magnitude and angle.

Now, we want to learn the inverse power flow function, mapping the voltage and power injection relationship using GPR as \( |V_l| = f_l(\mathbf{x}) \) where, \( f_l: \mathbb{R}^n \rightarrow \mathbb{R} \) is inverse power flow function. In other words, we want to learn the relationship between voltage and power injection in uncertain subspace \( \mathbf{x} \in S \) of power injections.

Here, we assume that power flow is solvable in the considered space of random power injections \( \mathbf{x} \in S \). Therefore, the inverse power flow is well-defined. Once the voltage solution is known, other network quantities such as branch flows or power loss can be calculated. We also assume that the network structure is unchanged for power flow studies without contingencies.

The non-linearity of (4) mainly restrict efficient decision making under uncertainty as we cannot obtain the solution space tractably. More importantly, any individual state variable gets affected by the complete set of nonlinear equations as multiple voltages are present in (4) with \( V_l \). This non-linearity directly depends on the network graph as the values of \( G_{ml}, B_{ml} \) are zero when there is no connection between node \( m \) and \( l \). In such a case, \( V_m \) will not directly appear in (4). Thus, for different networks, the non-linearity affects various voltages differently. Therefore, the uncertainty propagation requires various approximations and complex formulations [20], [21] for a specific type of network and uncertainty distribution. Therefore, these methods cannot be generalized directly.

The dependence of various PLF methods on uncertainty distribution information is also a significant difficulty. The PV generation and EV penetration uncertainty distributions do not follow specific distributions. There is a big challenge in estimating the distribution associated with net power injection uncertainty, uniquely when multiple uncertain loads (EVs) and generation sources (PVs) are integrated into the system. To deal with these issues of non-linearity and lack of uncertainty information, we present the main result of GP based NP-PLF below.

### A. Main Result

Following the analogy presented in [1], the Newton-Raphson load flow (NRLF) solution obtained for input sample \( \mathbf{x}^i \) can be interpreted as \( y_l^i \) containing \( f_l(\mathbf{x}^i) \) for bus \( l \) with numerical computation noise as:

\[
y_l^i = f_l(\mathbf{x}^i) + \varepsilon. \tag{5}
\]

Based upon (5), using the GP learning (2), the posterior distribution parameters \( \mu(\mathbf{x}) \) and \( \sigma(\mathbf{x}) \) are obtained. Further, a straightforward method would be to keep obtaining more and more training samples \( (f_l(\mathbf{x}^i), \mathbf{x}^i) \), and keep updating the posterior distribution parameters. This approach has some major difficulties. This method does not provide any bound on the possible uncertainty of posterior distribution. Using this method, we have no criteria to stop learning, and it has the same issue as MCS with the question: How many points are sufficient for learning? How much is the confidence in the output predictions?

1) NP-PLF Learning: To overcome these difficulties, we present the probabilistic learning bound (PLB). The PLB defines a range within which the target state variable \( y_l \) will remain with the given probability. Now, based on GP-UCB regret bound [19], we present the PLB for any general random state variable \( y_l = f_l(\mathbf{x}) \) for bus \( l \), in the uncertain state subspace \( \mathbf{x} \in S \). If one concerns only the voltage magnitude and the real power injection \( \mathbf{P} = \{P_l\} \) where \( P_l = \text{real}(S_l) \), let’s learn function \( |V_l| = f_l(\mathbf{P}) \). However, the voltage-reactive power, and voltage-apparent power function can be learnt in the same manner.

**Theorem 1.** For a given \( \delta \in (0, 1) \), for any uncertain power system input vector \( \mathbf{P} \), the inverse power flow solution function \( |V_l|(\mathbf{P}) \) for bus \( l \) will be bounded with probability \( 1 - \delta \) as

\[
\mu_N(\mathbf{P}) - \xi_{max} \leq |V_l|(\mathbf{P}) \leq \mu_N(\mathbf{P}) + \xi_{max} \tag{6}
\]

where \( \xi_{max} = \max_{\mathbf{P} \in \mathcal{P} \subseteq \mathcal{P}} \{\sqrt{\beta_{N+1}}\sigma_N(\mathbf{P})\} \) is PLB, and \( \mathbf{P} \) lies in the uncertain state subspace \( \mathcal{P} \) which corresponds to the projection of \( S \) to the real space. \( N \) indicate number of training samples.

**Proof.** The proof follows Theorem 6 in [19] and Theorem 1 in [17]. The result on regret bound, after \( N \) GP-UCB sampled training points, provides the relation as [19]:

\[
|V_l|(\mathbf{P}) - \mu_N(\mathbf{P}) \leq \sqrt{\beta_{N+1}}\sigma_N(\mathbf{P}), \quad \forall \mathbf{P} \in \mathcal{P}. \tag{7}
\]

Upon opening the modules, we obtain upper and lower bounds as a function of \( \mathbf{P} \). Applying the maximum operator over the regret bound obtained after \( N \) sampling iterations of GP-UCB, \( \xi = \sqrt{\beta_{N+1}}\sigma_N(\mathbf{P}) \), we obtain the PLB [5].

Theorem 1 provides a probabilistic guarantee for bounds of power flow solution. Further, the probabilistic learning bound \( \xi_{max} \) can serve as convergence criteria for the NP-PLF algorithm. The NP-PLF algorithm, designed based on
CPF method for Therefore, Algorithm 1 can also be interpreted as a GP based inputs \((S, \mathcal{X})\). The CPF attempts to trace the proposed NP-PLF and the continuation power flow (CPF) class of probability distribution. Theorem 1, is given as Algorithm 1. The \(X\) represents set of training points and similar to description of matrix \(X\) given in section II. The \(\|f\|_t\) indicate reproducing kernel Hilbert space (RKHS) norm while \(\gamma_N\) is constant [17]. The prior mean \(\mu_o\), and covariance function (kernel) \(k(x, x')\) are selected based on our prior understanding about \(f_i\). The \(y_i(x^i)\) represents output at initial random input vector \(x^i\). As mentioned earlier, the input \(x\)

**Remark.** The proposed NP-PLF learning does not require any specific PDF of input variable \(P\). It can be seen as working with uniform probability distribution where every value in rectangular region, \(P = \{P | P_{min} \leq P \leq P_{max}\}\), has same probability of occurrence.

2) NP-PLF Testing : In this step, once the Algorithm 1 converges, the state variable \(y_i\) can be obtained for distribution of test points using \(P'\) in (2). This means that for obtaining a probabilistic distribution of state variable against any input PDF, we use input PDF as test points over the output of the Algorithm 1 \(\mu_N(P)\) and \(\sigma_N(P)\). Further, from Theorem 1 the state PDF will be bounded by error \(\xi_{max}\) with probability \(1 - \delta\). Therefore, the testing step can be performed for any class of probability distribution.

Here, it is important to highlight the difference between the proposed NP-PLF and the continuation power flow (CPF) method. The CPF attempts to trace the \(P - V\) curve by increasing the load, generally in one dimension [11]. The proposed method can learn the voltage variation curve in multiple dimensions as \(x \in \mathbb{R}^{2n}\) for \(n\)-bus network. The input dimension taken as \(2n\) because complex power injection \((S_i)\) is separated as real \((P_i)\) and reactive power \((Q_i)\) injection inputs \((S_i = P_i + jQ_i)\) to capture their effects separately. Therefore, Algorithm 1 can also be interpreted as a GP based CPF method for \(2n - \text{dimensional}\) input space. Nevertheless, it still suffers from the same issue of Jacobian ill-conditioning near the critical point [22–25]. Eliminating this assumption and finding the critical loading point is an objective of our ongoing research.

More importantly, Algorithm 1 can work in parallel to learn many state variables simultaneously. Each such effort will sample points to learn the particular \(y_i\) using GP-UCB (5) in parallel execution. Nevertheless, Multi-linear Gaussian

**Algorithm 1 NP-PLF**

**Input:** \(S, \mathcal{X} \in \mathbb{R}^{n}, \delta, \xi_{max}, \mu_o, k(x, x'), x^i, y_i(x^i)\)

**Output:** \(x^i, y(x^i), \mu_i(x), \sigma_i(x), \beta_{i+1} ; i \in \{1, \ldots, N\}\)

\[
\text{while} \; (\max_{x \in \mathcal{S}} \{\sqrt{\beta_{i+1}} \sigma_i(x)\} \leq \xi_{max}) \; \text{do} \\
\text{Select} \; x^i = \arg\max_{x \in \mathcal{X}} \mu_i(x) + \sqrt{\beta_{i+1}} \sigma_i(x) \\
\text{if} \; x^i \in \mathcal{X} \; \text{then} \\
\text{Select randomly} \; x^i \\
\text{end if} \\
\text{Sample} \; y(x^i) = f(x^{(i)}) + \varepsilon \text{ by NRLF} \\
\text{Update} \; \mu_i(x) \text{ and } \sigma_i(x) \text{ from (2)} \\
\text{Update} \; \beta_{i+1} = 2||f||_t^2 + 300\gamma_N \ln^3 (i/\delta) \\
\text{Update} \; \mathcal{X} = \mathcal{X} \cup x^i \text{ and } i = i + 1 \\
\text{end while}
\]

At first, we consider a situation when the renewable generator replaces the conventional generator into the IEEE 30-Bus system [28]. Later, the load uncertainties are considered in 30-Bus and 118-Bus systems [28] at different buses. The biggest challenge in PLF involves the difficulty in modeling and obtaining the statistical distribution of random power generation variable, more so in case of solar power. Therefore, for the NP-PLF learning stage, we consider uniform distribution between zero to the maximum limit of renewable generator. However, other distributions can be considered in a similar way. In the table I number of training samples \((N)\) required, to achieve \(\xi_{max} \leq 1\%\) with probability \(\geq 0.99\), is given with computation time. It is important to note here that as we are not using any specific distribution of uncertainty, we need not define and fix the type of renewable generator and NP-PLF can work with any type of source.

![Fig. 1](image)

It is clear from table I that for various cases, the proposed NP-PLF has been able to achieve the higher accuracy results in very less time when compared to the MCS method. Further, the fig.1 shows the voltage variation obtained as “semi-explicit” form via learning Algorithm 1 for one dimensional input subspace where \(P_{g4}\) indicate random real power generator by generator 4 connected at 27-th bus. The curve also indicates that learning regret is higher at locations where training samples are not obtained. Thus, regret bound region can further be decreased using more samples, especially with a higher value of \(P_{g4}\). Most importantly, upon completion of the NP-PLF learning stage, the complete \(P - V\) curve is obtained. The

| Uncertain Renewable Generator | Variable | \% \(\epsilon_v\) | \(N\) | Time (sec.) |
|-------------------------------|----------|-----------------|-------|------------|
| \(P_{g3}\)                   | \(V_{21}\) | 0.0014          | 8     | 2.15       |
| At bus 22                    | \(V_{22}\) | 0.0001          | 7     | 1.39       |
| \(P_{g4}\)                   | \(V_{25}\) | 0.0082          | 7     | 1.66       |
| At bus 27                    | \(V_{28}\) | 0.0055          | 8     | 1.88       |
| \(P_{g5}\)                   | \(V_{35}\) | 0.0006          | 13    | 2.92       |
| At bus 13                    | \(V_{24}\) | 0.0049          | 7     | 1.89       |

### IV. Results and Discussion
average percentage relative error index $\% \varepsilon_v$, for $N_s$ testing samples, is defined as \[ (8) \]

$\% \varepsilon_v = \frac{1}{N_s} \sum_{k=1}^{N_s} \left( \frac{V_{MCS} - V_{GP}}{V_{MCS}} \right) \times 100$

As mentioned before, the proposed NP-PLF method can work with any class of input uncertainty distribution through testing phase. The fig. 2 and fig. 3 are obtained by varying the $P_{g4}$ with normal and gamma distribution respectively. The comparison between histograms obtained using MCS and proposed NP-PLF method validates that proposed method can calculate statistical features, mean and standard deviation, of PLF output for any type of input uncertainty distribution. Here, in fig. 2 the distribution is one sided because the $|V_{25}|_{max} = 99.02kV$ as indicative in the fig.1. The maximum number of samples in $P_{g4}$ is around the mean (28 MW) which leads to voltage near maximum value. Thus one sided distribution of $|V_{25}|_{max}$ is obtained in fig.2

For indicating the effect of load variation, different node voltage magnitudes are expressed in “semi-explicit” form using the learning step and shown in fig. 4. As indicated, the $|V_{30}|$ gets affected maximum with variations in load demand at node 30 while as we move away, the effect decreases largely. Yet, it is clear that the proposed method has been able to record the complete non-linearity of power flow equations and effect on all node voltages. Table II shows that low values of $\xi_{max}$ are obtained with a very less number of training samples. These results show the higher accuracy and speed of the proposed method. The fig. 5 shows $|V_{75}|$ variation in 2-dimensional, $P_{d75} - Q_{d75}$ space. It is important to understand that for this learning, MCS would require very large number of points while proposed method has been able to do this with very less points. The fig.5 is drown with $\xi_{max} \leq 1\%$ and probability $\geq 0.99$. 

In the following, we report the time consumption for the testing stage with unoptimized codes. The GPML toolbox \[29\] with MATLAB 2018b on PC having Intel Xeon E5-1630v4@3.70 GHz, 16.0 GB RAM is used for simulations. The time consumption increases with increment in the input subspace size. For larger problems, works on approximation methods (chapter 8 \[12\], \[27\]) can be used for future works. Also, the proposed NP-PLF is divided into stages where the learning stage can be done offline, improving the overall computational performance. Testing is very less time consuming; and 50000 samples take 0.073563 seconds only to test in fig.5 while 50000 samples take 108.15 seconds with MCS.

V. Conclusion

In this paper, a novel non-parametric probabilistic load flow (NP-PLF) is presented to estimate the nodal voltages for uncertain power injections. The proposed method attempts to understand the power balance equality constraint under uncertain input space to improve the power system’s decision-making process. The proposed NP-PLF consists of two steps. The learning step has been built on GP regression, and voltage solution has been learned as a function of arbitrary power injections providing a “semi-explicit” form with probabilistic learning bound (PLB). Then, the testing step has shown to approach the final voltage distribution, in terms of inverse power flow solutions, for any class of power injection uncertainty distribution. The proposed algorithm was tested in the IEEE 30-bus and IEEE 118-bus systems. The simulation results prove that the NP-PLF method can obtain statistical information using very few sampling points. Also, the relative error-index is sufficiently small, while the computational time is much less when compared with the traditional MCS method. Future works involve the development of multiple applications based on the “semi-explicit” learning method developed in this work.

ACKNOWLEDGEMENT

Parikshit Pareek, Wang Chuang, and Hung D. Nguyen are supported by NTU SUG, MOE, EMA and NRF fundings.
Fig. 4. $|V|$ as a function of uncertain load $P_{d30}$ in 30-Bus system

Fig. 5. $|V_{75}|$ as a function of uncertain load $S_{d75}$ in 118-Bus system

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