Delocalization in Coupled Luttinger Liquids with Impurities

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We study effects of quenched disorder on coupled two-dimensional arrays of Luttinger liquids (LL) as a model for stripes in high-$T_c$ compounds. In the framework of a renormalization-group analysis, we find that weak inter-LL charge-density-wave couplings are always irrelevant as opposed to the pure system. By varying either disorder strength, intra- or inter-LL interactions, the system can undergo a delocalization transition between an insulator and a novel strongly anisotropic metallic state with LL-like transport. This state is characterized by short-ranged charge-density-wave order, the superconducting order is quasi long-ranged along the stripes and short-ranged in the transversal direction.

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I. INTRODUCTION

Quasi-one-dimensional electron liquids play a paradigmatic role in describing the conductive properties of a variety of physical systems such as organic conductors, quantum-Hall systems and striped phases in high-$T_c$ compounds. Recent studies of weakly coupled Luttinger liquids (LL) have provided evidence for the stability of non-Fermi-liquid behavior in more than one dimension as opposed to results for an isotropic 2D Fermi gas. This remarkable result is a consequence of the combined effect of single particle/Cooper pair tunneling and Coulomb interactions between the LLs. Earlier studies either excluded hopping treated all inter-LL interactions separately as weak perturbations or focused on single particle tunneling for strong repulsive intra-LL interactions only. In Refs. it was shown that backscattering and particle hopping processes between the LLs can be irrelevant for sufficiently strong inter-LL forward scattering. The resulting state was called “sliding Luttinger liquid” (SLL). For a large range of interactions, these processes can be partially relevant and lead to charge-density wave (CDW), transverse superconductor (SC) or Fermi Liquid (FL) phases. Experiments have provided evidence for 1D transport in high-$T_c$ compounds. Theoretically, novel and to date essentially unexplored behavior can arise from disorder, which is induced by doping in these materials.

Here, we examine the role of electron scattering by a random impurity potential. For a single LL it was shown that a delocalization transition can occur with increasing electron attraction and that repulsive interactions always lead to localization. On the other hand, for coupled LLs, a simple scaling analysis suggests that disorder would be irrelevant at least in the SLL phase. However, using a renormalization-group (RG) analysis, we show that disorder profoundly modifies the characteristic properties of these systems. It turns out that a delocalization transition persists in analogy to single LLs. Where Josephson inter-stripe couplings are irrelevant, the delocalized phase can be identified with a new state of matter, which we dub disordered stripe metal (DSM). In contrast to the SLL state of the pure system, even in this delocalized phase, there exists only short ranged longitudinal CDW order due to impurity forward scattering. Because of this scattering process, we also find a strong tendency towards the destruction of transverse CDW order. Thus, the novel DSM state combines short ranged CDW order and quasi long-ranged longitudinal superconducting order with LL-like transport properties. Interestingly, it has a much wider stability region in comparison to the pure system’s SLL state.

II. GENERAL MODEL AND RENORMALIZATION

We assume a spin gap in the LLs, as present in stripes in high-$T_c$ compounds. The low-energy charge excitations of noninteracting stripes (labeled by $j$) can be described by the bosonic phase fields $\Phi_j$ and their dual fields $\Theta_j$ with an action

$$S^0 = \frac{1}{2\pi} \sum_j \int_{x,\tau} \left[ v_J (\partial_x \Theta_j)^2 + v_N (\partial_x \Phi_j)^2 - 2i \partial_x \Phi_j \partial_\tau \Theta_j \right].$$

The characteristic velocities $v_N$ and $v_J$ include forward scattering by intra-stripe interactions, whereas backward scattering is assumed to be irrelevant.

Following Giamarchi and Schulz, forward and backward scattering by weak impurities (denoted by IFS and IBS, respectively) is described in terms of the action

$$S^{IFS} = -\frac{\sqrt{2}}{\pi} \sum_j \int_{x,\tau} \eta_j(x) \partial_x \Phi_j, \quad (2a)$$

$$S^{IBS} = \frac{1}{\pi \alpha} \sum_j \int_{x,\tau} \left\{ \xi_j(x)e^{i(\sqrt{2} \Phi_j - 2k_F x)} + h.c. \right\}. \quad (2b)$$

$\eta_j(x)$ and $\xi_j(x)$ are Gaussian random variables with zero mean and correlations $\eta_i(x) \eta_j(x') = \frac{1}{2} D_\eta \delta_{ij} \delta(x - x')$ and $\xi_i(x) \xi_j(x') = D_\xi \delta_{ij} \delta(x - x')$. $\alpha$ is the infinitesimal regularization length of bosonization.
In order to describe coupled arrays of stripes, we first include forward scattering due to density-density interactions between the stripes, such as screened Coulomb and electron-phonon interactions. The corresponding action reads

$$S^V = \frac{1}{2\pi} \sum_{i \neq j} \int_{x\tau} \partial_x \Phi_i V_{i-j} \partial_x \Phi_j.$$  \hspace{1cm} (3)

In principle, analogous couplings between $\partial_x \Theta_j$ can be added but are dropped for simplicity. Their inclusion can be achieved by an obvious generalization of our analysis, and will not modify our main results. Since $S^V$ is bilinear in $\Phi_i$, it can be included in the momentum-space representation of the action $S^0$ by replacing $v_N$ in Eq. (1) by the $q_\perp$ dependent velocity $\tilde{v}_N(q_\perp) = v_N + V_{q_\perp}$ where $V_{q_\perp}$ is the Fourier transform of $V_i$ (we use the spacing between stripes as transverse length unit). The restriction to low energy excitations of our analysis, and will not modify our main results.

The restriction to low energy excitations includes the following restrictions: placing $\tilde{v}_N(q_\perp)$ in order to include inter-stripe forward scattering.

Besides these forward scattering processes, we allow also for pairwise hopping between stripes, given by the transverse CDW and SC couplings

$$S^\text{CDW} = \sum_{i \neq j} C_{i-j} \int_{x\tau} \cos \left[ \sqrt{2} (\Phi_i - \Phi_j) \right],$$ \hspace{1cm} (5a)

$$S^\text{SC} = \sum_{i \neq j} J_{i-j} \int_{x\tau} \cos \left[ \sqrt{2} (\Theta_i - \Theta_j) \right].$$ \hspace{1cm} (5b)

Since we assume the presence of a spin gap, single electron hopping is irrelevant and can be ignored.

For the pure system without $S^\text{PS}$ and $S^\text{HS}$, a specific interaction $V_i$ in Eq. (3) can render the CDW and SC couplings irrelevant in an intermediate region of $K$, leading to the SSL phase found in Refs. [3]. The interaction must be sufficiently strong and has to include at least nearest and next-nearest neighbors.

In the presence of disorder, the scattering off impurities has to be taken into account. Let us first focus on the effect of impurity forward scattering [2] in the absence of any inter-stripe couplings of type (4). This process then changes the SSL phase as described by $S^0 + S^V$ into the disordered stripe metal (DSM). Introducing replicated fields and averaging over disorder still lead to a bilinear action with correlations

$$\langle \Phi^{a}_{\mathbf{q}} \Phi^{b}_{\mathbf{q}'\mathbf{q}} \rangle = \frac{\pi \delta^{ab}}{\omega^2 / v_N + \tilde{v}_N(q_{\perp}) q_{\parallel}^2} + \frac{D_g \delta(\omega)}{\tilde{v}_N(q_{\perp}) q_{\parallel}^2},$$ \hspace{1cm} (6a)

$$\langle \Theta^{a}_{\mathbf{q}} \Theta^{b}_{\mathbf{q}'\mathbf{q}} \rangle = \frac{\pi \delta^{ab} \tilde{v}_N(q_{\perp}) / v_J}{\omega^2 / v_J + \tilde{v}_N(q_{\perp}) q_{\parallel}^2}.$$ \hspace{1cm} (6b)

with $\mathbf{q} \equiv (\omega, q_{||}, q_{\perp})$ and upper indices $a, b$ as replica labels.

We now examine the relevance of CDW and SC couplings and of impurity backward scattering (IBS) with respect to the DSM state using an RG analysis similar to that of Ref. [3]. To first order in $D_\xi, C_m,$ and $J_m$ we obtain the RG flow equations

$$\frac{dD_\xi}{dl} = (3 - \Delta^\text{IBS}) D_\xi,$$ \hspace{1cm} (7a)

$$\frac{dD_\eta}{dl} = D_\eta,$$ \hspace{1cm} (7b)

$$\frac{dK}{dl} = -\frac{2}{\pi \alpha^2 \Lambda^3} K D_\xi \int_{q_{\perp}} \tilde{v}_N^{-2}(q_{\perp}),$$ \hspace{1cm} (7c)

$$\frac{dC_m}{dl} = (2 - \Delta^\text{CDW}) C_m,$$ \hspace{1cm} (7d)

$$\frac{dJ_m}{dl} = (2 - \Delta^\text{SC}) J_m,$$ \hspace{1cm} (7e)

with $\int_{q_{\perp}} = \int_{q_{\perp}} \frac{dq_{\perp}}{2\pi} \tilde{v}_N$ is not renormalized to first order. The scaling dimensions are

$$\Delta^\text{IBS} = \int_{q_{\perp}} K(q_{\perp}),$$ \hspace{1cm} (8a)

$$\Delta^\text{CDW} = \int_{q_{\perp}} \left[ 1 - \cos(q_{\perp}) \right] \left[ K(q_{\perp}) + \frac{2D_\eta}{\pi \Lambda \tilde{v}_N^2(q_{\perp})} \right],$$ \hspace{1cm} (8b)

$$\Delta^\text{SC} = \int_{q_{\perp}} \left[ 1 - \cos(q_{\perp}) \right] K^{-1}(q_{\perp}),$$ \hspace{1cm} (8c)

with longitudinal momentum cutoff $\Lambda \sim 1/\alpha$.

### III. PHASE DIAGRAMS

Before we analyze specific models for the inter-stripe interaction, we discuss the general picture emerging from renormalization group.

#### A. General Topology

In the absence of disorder ($D_\xi = D_\eta = 0$), $K$ preserves its unrenormalized value $\sqrt{v_J/v_N}$. Then the scaling dimensions [3] reproduce the expressions given in Ref. [3]. For weak inter-stripe interactions $V_{q_{\perp}} \ll v_N$ the system is in the SC phase for $K \gtrsim 1$, whereas it is in the CDW phase for $K \lesssim 1$. For a suitable choice of a strong inter-action $V_{q_{\perp}}$, an intermediate range of $K$ can exist where all SC and CDW couplings are irrelevant and the system is in the SSL phase. The stability of the SSL phase is determined by the conditions $K > K^\text{CDW} = 2/\min_m \left\{ \epsilon_m^+ \right\}$ and $K < K^\text{SC} = \min_m \left\{ \epsilon_m^- \right\} / 2$. Hereby we define

$$\epsilon_m^+ \equiv \int_{q_{\perp}} \left[ 1 - \cos(q_{\perp}) \right] \left( \frac{v_N}{\tilde{v}_N} \right)^{\pm 1/2}.$$ \hspace{1cm} (9)

In the presence of disorder, the strength of impurity forward scattering $D_g$ increases exponentially under the
RG flow. This has two important consequences. First, the CDW order along the stripes becomes now short ranged as can be easily seen from the second term in Eq. (10). Second, it implies an exponential increase of $\Delta_{\text{CDW}}^m$ for all $m$, i.e., the irrelevance of weak CDW couplings. Thus, impurity scattering transforms the SLL and CDW phases of the pure system into different phases. If impurity backward scattering (IBS) is irrelevant—this is the case in the entire stability region of the SLL—a novel phase is present which we call DSM phase. Unlike for the SLL, the stability of the DSM for small $K$ is no longer limited by the CDW couplings but by IBS. Its phase boundary is determined by the relevance of SC couplings at large $K$ and the relevance of IBS at small $K$. IBS leads to localization for a bare IBS strength $D_\xi$ larger than a critical value $D_{\xi,c}$ (that depends on intra- and inter-stripe interactions). In this case $D_\xi$ diverges and $K$ goes to zero under renormalization. For $D_\xi < D_{\xi,c}$, the system is delocalized, $D_\xi \to 0$ and $K$ saturates at a finite value $K^* = K^*(D_\xi)$. For $K$ below the critical value $K_c = \frac{3}{c_\infty}$, infinitesimal disorder produces localization ($D_{\xi,c} = 0$). For $K > K_c$, the system remains delocalized at finite disorder strength, $0 < D_\xi < D_{\xi,c}$.

To determine the phase boundary $D_{\xi,c}$ of the localization transition for $K > K_c$, we integrate the flow Eqs. (7a) and (7d). In terms of the dimensionless disorder strength $D = D_\xi/(\pi^2\Lambda_0 K)$ we obtain

$$D(K) = D_0 + \frac{c_\infty}{c} (K - K_0) - \frac{3}{c} \ln \frac{K}{K_0}$$

with

$$c \equiv \int_{q=0}^{q_*} \left( \frac{v_N}{v_N} \right)^2.$$ (11)

The critical disorder strength then follows from the condition that $D(K) = 0$ at its minimum at $K = 3/c_{\infty} = K_c$:

$$D_{0,c} = \frac{c_\infty}{c} (K_0 - K_0) - \frac{3}{c} \ln \frac{K_c}{K_0}.$$ (12)

In the delocalized phase, the renormalized value $K^*$ of $K$ can be obtained from Eq. (10) with $D(K^*) = 0$. The phase boundary at large $K$ between DSM and SC phase is given by the condition $K^* < K^\text{SC} = \min_m \{ c_m \}/2$, and the boundary at small $K$ between the DSM and the localized phase is described by Eq. (12).

The actual form of the phase diagram and, more importantly, the stability range of the SLL or DSM phase depends on the interaction $V_{q_\perp}$ under consideration. To be specific, we will consider in the following two models for this interaction.

**B. Model A**

A minimal model that renders simultaneously all SC- and CDW-couplings irrelevant for some parameter region was suggested by Vishwanath et al. It assumes an inter-stripe interaction forward scattering leading to

$$\tilde{K}(q_\perp) = \kappa/[1 + \lambda_1 \cos(q_\perp) + \lambda_2 \cos(2q_\perp)]$$ (13)

Within this model, the three parameters $\kappa$, $\lambda_1$ and $\lambda_2$ implicitly determine the intra- and inter-stripe interaction. The corresponding Luttinger parameter is given by $K^2 = \kappa^2/(1 + (\lambda_1^2 + \lambda_2^2)/2)$, and the inter-stripe potential $V_\perp$ has a range of four stripe spacings. In analogy to Ref. 17 we rewrite $\lambda_1$ and $\lambda_2$ as

$$\lambda_1 = \frac{4(1 - \Delta) \cos q_0}{1 + 2 \cos^2 q_0},$$ (14a)

$$\lambda_2 = \frac{1 - \Delta}{1 + 2 \cos^2 q_0},$$ (14b)

such that $\tilde{K}(q_\perp)$ takes its maximal value $\kappa/\Delta$ at $q_\perp = q_0$. By adjusting $\Delta$ to small values, CDW couplings are suppressed, while small $\kappa$ gives negative scaling dimensions for SC-couplings, as pointed out by Mukhopadhyay et al. One thus indeed finds regions in $(q_0, \kappa, \Delta)$-phase space, where the system is stable against all inter-stripe couplings of type (\(14\)) and is thus in the SLL phase. This is demonstrated in Fig. 1, which may be compared to similar plots in Refs. 17, 18. For sufficiently small $\Delta$, windows of $q_0$ exist where the system evolves from a phase coherent SC through the 2D metallic SLL to a charge ordered CDW state with decreasing parameter $\kappa$. Here and in the following, when both SC and the CDW couplings compete, the one that is most strongly relevant is assumed to determine the phase. The actual boundary between two such strong coupling phases might differ within a narrow corridor. Note however, that boundaries to either the DSM or SLL phase are obtained also quantitatively correctly.

![Phase diagram of model A for $\Delta = 10^{-3}$ in the absence of disorder. For large $K$, the system forms a 2D superconductor, while it is a 2D CDW for small $K$. The intermediate SLL exists only for suitable values of the parameter $q_0$.](image)
Now disorder is added while all other parameters are unchanged. The CDW and the SLL phases of the pure system become indistinguishable and merge to the metallic, short-range CDW-ordered ’disordered stripe metal’ (DSM). Backscattering off impurities leads to localization in a large portion of the former CDW phase. The SC phase shrinks through downward renormalization of \( \kappa \) by disorder (note that \( \kappa \) and \( K \) differ by a factor that is not renormalized). In Fig. 2, the boundaries between the three phases are given both for infinitesimal and for finite disorder. The latter shifts the boundaries to larger \( \kappa \).

A cut through Figs. 1 and 2 at fixed \( q_0 \) but with varying disorder is shown in Fig. 3. The SLL and CDW phases exist only for \( D = 0 \). A delocalized phase can exist – due to inter-stripe forward scattering – even for purely repulsive interactions (for example, \( q_0 = 0.85\pi, \Delta = 10^{-3} \) and \( \kappa \lesssim 1.42 \) corresponds to repulsive on-stripe and repulsive inter-stripe interactions), as opposed to the strictly one dimensional electron gas with delocalization for \( K > 3 \) corresponding to strongly attractive interactions. However, the inter-stripe interactions corresponding to the values of \( \Delta, \kappa, q_0 \) where the SLL or DSM exist may not be very realistic because of their strength.

C. Model B

Model A is constructed specifically in a way such that the inter-stripe forward scattering interactions \( \tilde{K}(q_L) \) give rise to a nonmonotonous \( \tilde{K}(q_L) \) which allows for the simultaneous irrelevance of CDW and SC couplings in the absence of disorder. For a large range of parameters \( q_0 \) and \( \Delta \), this potential has oscillatory character in real space, which also may not be very realistic.

A physically motivated choice for a potential that is monotonous both in real and Fourier space may be the screened Coulomb potential

\[
V(r) = \frac{A}{r} e^{-\mu r}
\]

which we consider as model B. In Fourier space this model reads

\[
V_{q_L} = -A \ln \left[ 1 + e^{-2\mu} - 2e^{-\mu} \cos q_L \right].
\]

Due to the stability condition \( V_{q_L}/v_N > -1 \), see Eq. (4), there is a critical amplitude \( A_c(\mu) \), above which the model breaks down.

Fig. 4 displays the stability of the model with respect to weak inter-stripe CDW and SC couplings in the absence of disorder. No SLL is found, the system shows for all \( \mu \) and \( A \) a direct transition from the SC to the CDW phase for decreasing \( K \). The addition of disorder leads to the phase diagram in Fig. 2. As opposed to model A, impurity backscattering completely covers the CDW phase and thus leaves only two phases, the localized one and the SC phase. In contrast to the competition between CDW and SC couplings, the localization boundary is not given by the most relevant bare coupling. Since even weakly relevant IBS renormalizes \( K \) to small values, the disorder scaling dimension decreases while the Josephson coupling, provided a small enough bare value, ultimately becomes irrelevant, see Eqs. (15). Hence the boundary is given by the onset of relevance of IBS with respect to the pure system. It moves to larger \( K \) for increasing disorder. No DSM phase is found now. Formally, the absence of a minimum of \( V_{q_L} \) inside the interval \((0, \pi)\) makes up for this latter qualitative difference in models A and B.
eral, one has to use the renormalized but unrescaled amplitude described by the correlations (6) with the bare IFS am-

delocalized DSM phase

D. Correlations

Having established the generic topology of phase diagrams, we now address the nature of the possible phases. First we consider the delocalized DSM phase. It is described by the correlations \( \Phi \) with the bare IFS amplitude \( D_0 \) and the renormalized effective \( K^* \) (in general, one has to use the renormalized but unrescaled quantities). We find a linear growth of the fluctuations of \( \Phi \) with longitudinal system size \( L \), \( \langle \Phi^2(x, \tau) \rangle_L = (c/2\pi^2)\nu_{N}^{-2}D_0L \), which leads to short-ranged longitudinal CDW correlations like for single LLs. On the other hand, IFS does not affect the quasi long-ranged longitudinal superconducting order (fluctuations of \( \Theta \)) of the pure system. Equally, the conductivity along the stripes is not affected by IFS since \( \eta_j(x) \) is time independent.

From a linear-response calculation we obtain the LL-like conductivity (which also determines the conductance)

\[
\sigma(\omega, q_\parallel, q_\perp) = \frac{2e^2}{\pi \hbar (\omega + i0^+)^2/v_j^*} - v_N(q_\perp)q_\parallel^2,
\]

representing a longitudinal metal. Here, \( \omega \) represents a real frequency in contrast to Matsubara frequencies in Eqs. (6). Notice that \( v_j^* = v_NK^* \). Since \( K^* \) jumps from a finite value to zero at the localization transition, \( \sigma(q) \) behaves discontinuously there. In the transverse direction, CDW and SC correlations will be short-ranged since the corresponding couplings are irrelevant. In the presence of a spin gap (which suppresses single particle hopping) the irrelevance of the couplings also signals that the DSM is a transverse insulator.

The localized phase is less amenable to an analytic description since the divergence of IBS would necessitate a strong-coupling analysis. However, if the SC coupling is irrelevant, the localization transition and the localized phase share the qualitative properties of their 1D counterparts. The inter-stripe couplings will lead to merely quantitative renormalization effects. Whether the localized phase is a random antiferromagnet or a pinned CDW depends on the mechanism generating the spin gap. The longitudinal localization length \( L_{loc} \) can be estimated analytically when the transition line is approached from the localized side. For \( K < K_c \), \( L_{loc} \approx \Lambda^{-1}D_0^{-1/2(1-K/K_c)} \).

For \( K > K_c \), \( L_{loc} \approx \Lambda^{-1}\exp(c/\sqrt{D_0 - D_{0,c}}r) \) with \( c \) a numerical factor of order unity. Thus, the inter-stripe interactions influence the localization length quantitatively, but the qualitative behavior found[24] for the localization transition in a single LL persists.

IV. DISCUSSION AND SUMMARY

In summary, we have examined impurity effects in arrays of coupled LLs. The competition between impurity backscattering, CDW and SC couplings allows for three different phases: a localized phase, a superconducting phase, and the disordered stripe metal. The latter two phases are delocalized since IBS scattering is irrelevant. While for a single stripe delocalization occurs only for strongly attractive on-stripe interactions (\( K \lesssim 3 \)), for a coupled stripe array delocalization is possible also for purely repulsive on-stripe and inter-stripe interactions (forward scattering).

The delocalized DSM phase is metallic in longitudinal direction and insulating in transverse directions. Its correlations for CDW order are short-ranged in all directions whereas superconducting correlations are quasi long-ranged along the stripes and short-ranged in the transversal direction. These experimentally accessible features should allow to identify the disordered stripe metal and to distinguish it from the SLL phase.

In the above analysis we have determined the phase diagram from a stability analysis of a Gaussian fixed point.
– representing the stripe array with forward scattering by interactions and impurities – with respect to CDW and SC couplings as well as impurity backward scattering. This stability analysis, reflected by the flow equations (12) which are linear in $D_m$, $C_m$, and $J_m$, requires the weakness of these couplings. In principle, this approach does not cover strong-coupling phenomena: sufficiently strong couplings might drive transitions into SC or CDW phases which are less susceptible to disorder than the SLL.

Although we cannot consistently access such strong coupling phenomena via our flow equations, they nevertheless can be used to determine crossovers related to the relative strength of non-Gaussian couplings. Since all CDW couplings are irrelevant at the DSM fixed point, we raise the question of whether a CDW phase can be reestablished if CDW couplings are sufficiently strong in comparison to disorder. We focus on the region where a CDW coupling $C_m$ is relevant in the absence of disorder and where IBS is irrelevant (i.e., $\delta \equiv 2 - \Delta_{CDW}^m > 0$ and $\Delta_{BS} > 3$ for the bare parameters).

Irrespective of the relative strength of CDW couplings and IBS, the presence of disorder implies a continuous growth of $D_\eta$ and thus also of $\Delta_{CDW}^m$ which implies the irrelevance of $C_m$ only on sufficiently large scales. Thus the question is, whether $D_\eta$ increases fast enough to achieve $\Delta_{CDW}^m > 2$ before a strong-CDW coupling regime is entered. This regime is entered when the dimensionless coupling $\hat{C}_m \equiv C_1/(\Lambda^2 v_N)$ becomes of order unity under renormalization before the disorder contribution to $\Delta_{CDW}^m$ becomes of order $\delta$. For weak $V_{\perp}$ or $m \gg 1$, this is the case if

$$\hat{C}_m \gtrsim \left( \frac{2\pi C_D}{\pi \Lambda v_N^2 \delta} \right)^\delta.$$

(18)

For CDWs (quasi-)long-ranged charge correlations can exist in the presence of disorder only in $D > 2$ dimensions like for vortex lattices. Then the fermions would form a pinned (localized) Wigner crystal. However, in $D = 2$, the formation of CDW order is prohibited by the proliferation of dislocations which ultimately render the CDW coupling irrelevant on sufficiently large scales.

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