Integrated Conditional Teleportation and Readout Circuit Based on a Photonic Crystal Single Chip

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We demonstrate the design of an integrated conditional quantum teleportation circuit and a readout circuit using a two-dimensional photonic crystal single chip. Fabrication and testing of the proposed quantum circuit can be accomplished with current or near future semiconductor process technology and experimental techniques. The readout part of our device can also be used on its own or integrated with other compatible optical circuits to achieve atomic state detection. Further improvement of the device in terms of compactness and robustness could be achieved by integrating it with sources and detectors in the optical regime.

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1. Introduction

It has been shown theoretically by Bennett\textsuperscript{1}, et al. that quantum teleportation of an unknown state can be achieved using Bell states. Since then several theoretical and experimental schemes have been proposed, which include implementation in optical, NMR, and cavity QED systems\textsuperscript{2-5}. Inspired by the recent paper of S.-B. Zheng\textsuperscript{2}, in this letter we propose a photonic crystal (PC) based cavity QED implementation of the scheme described therein, as well as the integration of the device with an atomic readout circuit to detect the teleported state.

Underlying our interest in Zheng's proposal is the vision of more sophisticated large-scale photonic crystal integration for compact, robust, and scalable quantum computation technology. Zheng's teleportation protocol is only conditional (i.e., the success of the protocol is conditioned on a specific measurement result contrary to unconditional teleportation), but the relative simplicity of the proposed scheme from the perspective of design and implementation is attractive for an initial step toward that vision. Compared to other cavity QED based teleportation schemes, it does not require a two-qubit Bell-state measurement and thus can be implemented, in principle, using only two atoms interacting with a single mode cavity. Briefly, atom $B$, initially prepared in the excited state, is first entangled with an initially empty cavity. Then atom $A$, whose state is to be teleported, interacts with the cavity. Subsequent detection of atom $A$ in the excited state collapses atom $B$ to the initial state of atom $A$.

2. Device Design

An illustration of the photonic crystal chip that we propose is shown in Fig. 1, where a single mode cavity is integrated with two parallel waveguides. The distances between various components of the circuit and the sizes are chosen in such a way that the possible sources of error due to the design of the PC are minimized. To achieve sufficiently strong and coherent atom-cavity interaction, negligibly small couplings between the cavity and the waveguide, and separated oscillatory waveguide modes we set the following size and distance parameters in our design: The actual size of the chip, which is designed in this letter, is larger than that of the illustration. The actual height (i.e., vertical) is $\frac{17\sqrt{3}a}{2}$ and the width (i.e., horizontal) is $79a$, where $a$ is the lattice constant. $x_1$, $x_2$, $x_3$, $x_4$ and $y$ are $9a$, $43a$, $18a$, $9a$, and $4\sqrt{3}a$, respectively. Because of its large size, we design the chip in a 2D lattice to reduce the high computational cost of 3D crystal analysis. This is, however, a good approximation due to the ability of 2D photonic crystal defect modes to emulate those of 3D photonic crystals. This emulation issue is considered in detail in Ref. [6] and its advantages for photonic crystal based quantum logic gate design are discussed in Ref. [7], where we have used exactly the same lattice parameters. Thus, our 2D PC structure is chosen to emulate its 3D counterpart.
and is based on a triangular lattice of silicon rods with dielectric constant of 12 and lattice constant of 2.202\text{mm} for the resonant atom-cavity interaction at 51GHz. The radii of the rods and the defects are 0.175\textit{a} and 0.071\textit{a}, respectively. The trajectories of the atoms through the PC are depicted by the differently dashed lines in Fig. 1.

Incorporated with detectors, the cavity part of the chip operates as a teleportation circuit and the parallel waveguides act as a readout circuit to detect the teleported atomic state (see Fig. 1). Below we use the terms teleportation circuit (TC) and readout circuit (RC), respectively, to refer those two integrated photonic crystal circuits. We choose the width of the former to be 43\textit{a} and the latter to be 36\textit{a} in order to avoid unwanted coupling between the two circuits. In the first part of this letter we describe the teleportation mechanism and in the second part we explain how to detect the atomic state\textsuperscript{8-11} within our designed PC quantum circuit (PCQC).

3. Teleportation Circuit

The TC involves the interaction of two Rb atoms with a resonant high-Q (assumed to be larger than 10\textsuperscript{8} to prevent cavity decoherence) single mode cavity and the detection of one of the atoms using a detector. The total Hamiltonian (Jaynes-Cummings Model) for the TC under the dipole and rotating wave approximations is given by\textsuperscript{7}:

\[
H_{\text{atom-cavity}} = \frac{\hbar \omega}{2} \sum_j \sigma_z^j + \hbar \omega \alpha \alpha + \hbar \sum_j \langle g(r_j) | (\alpha^\dagger \sigma_z^j + \alpha \sigma_+^j) \rangle
\]  

(1)

where the summation is carried over two atoms, \(A\) and \(B\), \(\omega\) is the resonant frequency, \(\sigma_z\) is the \(z\)-component of the Pauli spin operator and \(\sigma_{\pm}\) are the atomic raising and lowering operators. \(\alpha\) and \(\alpha^\dagger\) are the annihilation and creation operators, respectively. \(g(r_j)\) is the atom-cavity coupling parameter at the position of atom \(j\) and can be written as

\[
g(r_j) = g_0 E(r_j) / |E(r_m)|. 
\]

(2)

\(g_0\) denotes the vacuum Rabi frequency and \(r_m\) is the position in the dielectric where the electric field energy density is maximum. Note that we implicitly assume in Eq. (2) that the atomic dipole moments, \(\mu_0\), of the atoms are aligned with the photon polarization.

In the first stage of the conditional teleportation protocol, atom \(B\) (whose trajectory is horizontal in Fig. 1), initially in the excited state \(|1\rangle_B\), traverses the cavity, which is initially in the vacuum state \(|0\rangle\), with velocity \(V_B\). Since at this stage atom \(A\), whose state is to be teleported, is not in play, index \(j\) only takes value \(B\) in Eq. (1). The composite state for atom \(B\) and the cavity at time \(t\) then can be written as

\[
|\Psi(t)\rangle = \cos G_B(t) |1\rangle_B |0\rangle_0 - i \sin G_B(t) |0\rangle_B |1\rangle_0,
\]

(3)

3
where

\[ G_j(t) \equiv \int_{t_0}^{t} |g(r_j)| \, dt. \quad (4) \]

The cavity we have designed in this letter supports a transverse magnetic (TM) polarized monopole mode as illustrated in Fig. 1. Based on the block-iterative plane wave expansion method\textsuperscript{12}, with 32 grid points per \( a \), we find the normalized frequency of the cavity mode to be 0.3733c/\( a \). By setting \( a = 2.202 \)mm, we tune the resonant wavelength to 5.9mm. At this wavelength, the atomic dipole moment for the Rb atom is taken to be \( 2 \times 10^{-26} \)Cm.\textsuperscript{7}

Using these parameters, we obtain \( G_B(t) \approx 9\pi/4 \) at time \( t_1 = 51.6 \)\( \mu s \) at 18\( a \) from the left edge of the TC, if \( V_B \) is set to 767.7m/s, and hence atom-cavity entanglement in Eq. (3). In other words, this asymptotic (constant) value of \( G_B(t) \) is the necessary condition for the completion of the entanglement process [see Eq. (3)] and can be satisfied only under certain circumstances at the end of the atom-cavity interaction. As atom B travels across the cavity, the time-dependent atom-cavity coupling parameter and the evolution of the probability amplitudes of states \( |1\rangle_B|0\rangle \) (initially at 1; shown in blue online) and \(-i|0\rangle_B|1\rangle \) (initially at 0; shown in pink online) in Eq. (3) are displayed in Fig. 2. In our calculations we neglect the insignificant effect of the mode tail on the probability amplitudes of the entangled state less than 1\% beyond 18\( a \). Another even less significant source of error in our design is the coupling of the waveguides in the RC to the cavity\textsuperscript{13,14}, which is simply circumvented by keeping \( x_2 \) in Fig. 1 sufficiently large (i.e., 43\( a \)).

Having entangled atom B and the cavity, the second stage of the conditional teleportation protocol is to inject atom A (whose trajectory is diagonal in Fig. 1), which is initially (i.e., at time \( t_1 \)) in an arbitrary (or an unknown) quantum state,

\[ |\phi\rangle_A = c_0|0\rangle_A + c_1|1\rangle_A. \quad (5) \]

Thus at time \( t_1 \), the state of the whole system is written as

\[ |\varphi(t_1)\rangle \approx \frac{1}{\sqrt{2}} (c_0|0\rangle_A + c_1|1\rangle_A)\left(|1\rangle_B|0\rangle - i|0\rangle_B|1\rangle\right). \quad (6) \]

Once atom A arrives at the detector, right before the detection, say at time \( t_1 + t_2 \), the state of the whole has evolved into

\[ |\varphi(t_1 + t_2)\rangle \approx \frac{1}{\sqrt{2}} \left[ c_0|0\rangle_A|1\rangle_B|0\rangle - i c_0|0\rangle_B[\cos G_A(t_2)|0\rangle_A|1\rangle - i \sin G_A(t_2)|1\rangle_A|0\rangle] \right. \\
+ c_1|1\rangle_B[\cos G_A(t_2)|1\rangle_A|0\rangle - i \sin G_A(t_2)|0\rangle_A|1\rangle] \\
- i c_1|0\rangle_B[\cos \sqrt{2} G_A(t_2)|1\rangle_A|1\rangle - i \sin \sqrt{2} G_A(t_2)|0\rangle_A|2\rangle] \right], \quad (7) \]

which would reduce to Eq. (6) of Ref. (2), if \( |g(r_j)| \) were spatially uniform in Eq. (1).

In the derivation of Eq. (7), the contribution of \( |g(r_B)| \) in Eq. (1) is neglected, since atom B is sufficiently far from the cavity when atom A is injected. At times close to \( t_1 \), although
\[ |g_r(B)| \approx |g_r(A)|, \] neither atom alone nor combined can start the Rabi oscillation.\(^7\) On the other hand, as atom \(A\) proceeds its coupling to the cavity mode increases, while the coupling of atom \(B\) decreases. This means that the effect of \( |g_r(B)| \) becomes even less significant for times greater than \(t_1\).

The third stage of the conditional teleportation protocol is the measurement of the state of atom \(A\). If the result is \( |1\rangle_A \), the combined state of atom \(B\) and the cavity becomes

\[
\Psi(t_1 + t_2) \cong \frac{-c_0 \sin G_A(t_2) |0\rangle_B |0\rangle + c_1 \cos G_A(t_2) |1\rangle_B |0\rangle - ic_1 \cos \sqrt{2} G_A(t_2) |0\rangle_B |1\rangle}{|c_0|^2 \sin^2 G_A(t_2) + |c_1|^2 \cos^2 G_A(t_2) + |c_1|^2 \cos^2 \sqrt{2} G_A(t_2)^{1/2}}.\tag{8}
\]

The denominator in Eq. (8) is simply the norm of the unnormalized state in the numerator. We obtain \(G_A(t_2) \cong 7\pi/4\) \[i.e.,\] the necessary condition to be satisfied for the completion of the teleportation process, see Eq. (8)] in Eq. (8) if we set \(V_A = 987\text{m/s}\), and thus

\[ |\phi\rangle_B \cong c_0 |0\rangle_B + c_1 |1\rangle_B.\tag{9}\]

That is, the initial arbitrary (or unknown) state of atom \(A\) has been teleported to atom \(B\), conditional on detecting atom \(A\) in state \( |1\rangle_A \), which occurs with probability \(1/4\). The time-dependent coupling parameter for atom \(A\) and the evolution of the probability amplitudes of states \( -c_0 |0\rangle_B |0\rangle \) (initially at 0; shown in pink online), \( c_1 |1\rangle_B |0\rangle \) (dark curve initially at 1; shown in blue online), and \(-ic_1 |0\rangle_B |1\rangle \) (light curve initially at 1; shown in yellow online) for the unnormalized state (i.e., the numerator) in Eq. (8) are shown in Fig. 3. Note that as time elapses Eq. (8) transforms into Eq. (9). Note also that atom \(B\) arrives at \(31a\) at time \(t_1 + t_2\) (i.e., \(88.9\mu s\)). After an additional distance of \(12a\), the fourth stage of our conditional teleportation and detection protocol begins as atom \(B\) enters the RC at time \(t_1 + t_2 + t_3\) (i.e., \(123.3\mu s\)). Next we study how to detect its state.

4. Readout Circuit

We can write the atom-maser Hamiltonian, under the dipole and rotating wave approximations, as\(^{15}\)

\[
H_{atom-maser} = \frac{\hbar \omega}{2} \sigma_z + \hbar \frac{\Omega(r)}{2} (e^{i \omega_m t} \sigma_- + e^{-i \omega_m t} \sigma_+)\tag{10}
\]

where

\[
\Omega(r) = \frac{2 \mu_{10} |E(r)|}{\hbar}\tag{11}
\]

is the Rabi frequency and \( \omega_m \) is the frequency of the maser, whose polarization is also matched with the atomic dipole moment, \( \mu_{10} \), of the atom. In the following we will first assume a constant \( \Omega(r) = \Omega_0 \) to simplify the explanation of the detection mechanism, and then we will modify the analysis to apply to our proposed PCQC, where \( \Omega(r) \) is not constant.
Assume that atom $B$ in state $|\phi\rangle_B$ [see Eq. (9)] enters a uniform field region at time $t$ and interacts during a time interval $t_4$. Then its probability amplitudes at time $t + t_4$ can be computed to be

$$c_0(t + t_4) = \{[i \cos \Theta \sin (\frac{\Lambda t_4}{2}) + \cos (\frac{\Lambda t_4}{2})]c_0 + i \sin \Theta \sin (\frac{\Lambda t_4}{2})e^{i\omega_m t_4}c_1\}e^{i\omega_m t_4/2},$$

$$c_1(t + t_4) = \{i \sin \Theta \sin (\frac{\Lambda t_4}{2})e^{-i\omega_m t_0}c_0 + [-i \cos \Theta \sin (\frac{\Lambda t_4}{2}) + \cos (\frac{\Lambda t_4}{2})]c_1\}e^{-i\omega_m t_4/2},$$

where $\cos \Theta$, $\sin \Theta$, and $\Lambda$ are defined as follows.

$$\Lambda \equiv \sqrt{(\omega - \omega_m)^2 + \Omega_0^2},$$

$$\cos \Theta \equiv \frac{\omega - \omega_m}{\Lambda},$$

$$\sin \Theta \equiv -\frac{\Omega_0}{\Lambda}.$$

After this interaction, atom $B$ interacts with a second uniform field region for the same time interval $t_4$ starting at time $t + t_4$. The probability amplitudes, $c_0(t + 2t_4)$ and $c_1(t + 2t_4)$, in this case can be also found from equations (12) and (13) by replacing $c_0$, $c_1$, and $t$ with $c_0(t + t_4)$, $c_1(t + t_4)$, and $t + t_4$, respectively.

Note that one can also treat these two uniform field regions as a single uniform field region with an interaction time of $2t_4$. In this letter, however, we also intend to show that our RC has the potential to be an atomic interferometer, like a Ramsey interferometer. In order to hint how this could work, we design two separate oscillating field regions based on two parallel waveguides as discussed below. Implementation of a fully working interferometer, however, requires further consideration.

After atom $B$ has interacted with the two uniform field regions, we can implicitly write the final state of atom $B$ at time $t + 2t_4$, before detection as

$$|\phi(t + 2t_4)\rangle_B \cong (c_0c_{00} + c_1c_{11})|0\rangle_B + (c_0c_{01} + c_1c_{11})|1\rangle_B$$

where $c_{ij}$ is the probability amplitude of the atom being in state $|j\rangle$ at time $t + t_4$ if it is initially prepared in state $|i\rangle.$ For example if the atom is initially in state $|1\rangle$ then it evolves into $c_{10}|0\rangle + c_{11}|1\rangle$ at time $t + t_4$.

Having interacted with both waveguides, atom $B$ is finally detected by an ionization detector, as shown in Fig. 1. The probability of finding it in the excited state is

$$P_i = |c_0|^2|c_{01}|^2 + |c_1|^2|c_{11}|^2 + [c_0^{(r)}c_1^{(r)} + c_0^{(i)}c_1^{(i)}]^2(2c_{11}c_{i1}^*)^2(r) + [c_0^{(r)}c_1^{(i)} + c_0^{(i)}c_1^{(r)}]^2(2c_{11}c_{i1}^*)^2(i).$$

Superscripts $(r)$ and $(i)$ represent the real and imaginary part, respectively.
We can simplify Eq. (18) by writing Eq. (9) in the form

$$|\phi\rangle_B \cong \cos(\theta/2)|0\rangle_B + \sin(\theta/2)e^{i\varphi}|1\rangle_B$$

where we have implicitly set $c_0^{(r)} = \cos(\theta/2)$, $c_0^{(i)} = 0$, $c_1^{(r)} = \sin(\theta/2)\cos\varphi$, and $c_1^{(i)} = \sin(\theta/2)\sin\varphi$.

Then, at four different $\Delta \equiv \omega_m - \omega$ values we calculate $|c_{00}|^2$, $|c_{11}|^2$, $2(c_{01}c_{10})^{(r)}$, and $2(c_{01}c_{10})^{(i)}$ directly from the system parameters using Eqs. (12) and (13); and measure only $P_1$ using field ionization detector. Thus having obtained four equations, we can find $c_0^2$, $|c_1|^2$, $[c_0^{(r)}c_1^{(r)} + c_0^{(i)}c_1^{(i)}]$, $[c_0^{(r)}c_1^{(i)} - c_0^{(i)}c_1^{(r)}]$ and hence $|\phi\rangle_B$. Note that, although we have only two unknowns in Eq. (19), it is convenient to use four equations due to the form of Eq. (18). This demonstrates that one could detect a teleported (or an arbitrary) unknown state with our proposed device, provided that $P_1$ is measured by employing an atomic beam of identically prepared single atoms, since multiple observations are necessary to estimate the probability. Since the TC only has a 25 percent conditional success rate, incoming atoms to the RC will not be identical. Thus if atom $A$ of the relevant pair is not detected in the excited state, the corresponding atom $B$ measurement is discarded from the calculation of $P_1$.

Finally, we describe how to apply these ideas to our proposed PCQC. Remember that atom $B$ in state $|\phi\rangle_B$ enters the RC at time $t_1 + t_2 + t_3$. The steady-state mode profiles of the waveguides are calculated by two-dimensional-finite-difference-time-domain (2D-FDTD) method with discretizations of $\frac{a}{13}$ and $\frac{a\sqrt{2}}{13}$ in the horizontal and vertical directions, respectively. We have designed the waveguides as coupled-cavity-waveguides\textsuperscript{17,18} to allow the atoms sufficiently large void regions through which to travel freely without a resonant-dipole-dipole-interaction or the Casimir-Polder effect (see Fig. 1). The steady-state cross section of the electric field magnitude, for $\Delta = 0$, along the path of atom $B$ is shown in Fig. 4. At the steady state the coupling between the two parallel waveguides\textsuperscript{19–22} is not significant.

In our calculations we assume that atom $B$ interacts with two separate nonuniform waveguide modes of total width 36$a$, shown by the part of the curve in Fig. 4 to the right of approximately 95mm (shown in pink online). In other words we truncated their tails (the part of the curve to the left of approximately 95mm in Fig. 4; shown in blue online), which overlap with the cavity mode. The effect of this truncation in our design is observed to be negligibly small. We have observed that the coupling of waveguide modes on the cavity mode is less than the cavity coupling to waveguides, which does not effect the probability amplitudes more than 0.6%.

Since there is no feedback mechanism, as there is in the atom-cavity interaction, the atom-waveguide interaction can be described by a Markoff approximation. In other words, when an atom emits a photon into the waveguide, it leaves the interaction region immediately and cannot act back on the atom. Then the waveguides must be designted in such a way that total rate of spontaneous emission into the waveguide and the free-space lossy modes
is sufficiently low as atom $B$ traverses them. In our design considering estimated effective
transverse cross section of corresponding three-dimensional waveguide mode the maximum
probability of spontaneous emission for a given atom is estimated to be on the order of
$10^{-4}$.

Thus, it is safe to assume that the Bloch vector of atom $B$ can be manipulated
without being correlated with the defect mode of the waveguides.

The total interaction time with the parallel waveguides is also $2t_4$, as in uniform case above,
with $t_4$ (i.e., 51.6 µs) for each. Note that, since the Rabi frequency, $\Omega(r)$, is proportional to
$|E(r)|$, the part of the curve in Fig. 4 to the right of approximately 95mm (shown in pink 
online) also describes the position dependence of the Rabi frequency if multiplied by $2\mu\text{10} / \hbar$.

Thus the evolution of the teleported state, as it interacts with the parallel waveguides, can
be determined numerically by exploiting equations (12) and (13) successively, $t_4$ replaced
with the maximum tolerated time-step, which is 44ns in our case (i.e. time for atom $B$ to travel $a/65$).

Thus, right before the detection, the final state is $|\phi(t_1 + t_2 + t_3 + 2t_4)\rangle_B$, which can be
written exactly in the form of Eq. (17). The probability of detecting atom $B$ in the excited
state is then given by Eq. (18), and following the same method as in the uniform case above,
we can easily detect $|\phi\rangle_B$, the state output by TC.

As an example, assume that the final state of the TC [see Eq. (19)] is

$$|\phi\rangle_B = \cos(\pi/8)\ 0\rangle_B + \sin(\pi/8)e^{-i\pi/6}|1\rangle_B$$  \hspace{1cm} (20)

The corresponding calculated values of $|c_{01}|^2$ (solid line; shown in green online), $|c_{11}|^2$ (‘$\times$’;
shown in red online), $2(c_{01}c_{11}^*)$ (dotted line; shown in light blue online), $2(c_{01}c_{11}^*)$ (‘$o$’;
shown in purple online), and the value of $P_1$ (‘$+$’; shown in dark blue online), which will be
measured, are shown in Fig. 5. Picking four different values of $\Delta$, we can easily determine
that $\theta = \pi/4$ and $\varphi = -\pi/6$.

5. Discussions

Our readout scheme does not require 100% detection efficiency. Finite detection efficiency
with a relatively larger atomic ensemble can lead to an arbitrarily accurate result. For ex-
ample, with a detection efficiency of 40%,\textsuperscript{9} our detection scheme gives about ±4% error
(confidence level 99%) in $P_1$ out of $10^4$ events. Increasing the number of events to $10^5$ reduces
the error to about ±1% with the same confidence level. Using the latter number, we have estimated that the maximum error in the probability amplitudes (i.e., $c_0^{(r)}$, $c_1^{(r)}$, and
$c_1^{(i)}$) is about 0.1; for most states the error is at least an order of magnitude smaller.

The total operation time in our circuit is 226.5 µs compared to the 30 ms lifetime of the Rb
atoms, which are prepared one at a time in a circular Rydberg state with principal quantum
number 50 (ground) or 51 (excited).\textsuperscript{25}
Note that we need a maser (as assumed in §4) or a coherent microwave source operating around the atomic transition frequency (i.e., 51 GHz), which allows measurements at four different detunings of 10s of kHz as shown in Fig. 5. We can achieve this task using, for example, continuously tunable backward wave oscillators (BWO) which can generate coherent, highly monochromatic and linearly polarized radiation. Minimum frequency steps of 100 Hz and frequency stability of up to $10^{-11}$ can be obtained for BWOs, using an external 5 MHz signal from a rubidium standard. This would be more than sufficient for our purpose.

Due to the electromagnetic scalability of photonic crystals one could also implement this device in more compact form in the optical regime. In this case, however, the electric field gradient and the momentum recoil of the atom would also have to be analyzed. In the microwave regime studied in this letter, the small momentum recoil due to a microwave photon does not cause external degrees of freedom (i.e., the position or momentum) of the atom to couple with its internal degrees of freedom (i.e., the ground or excited state).

To implement our PCQC, one should also engineer the surface states, which reside at the air-photonic crystal interfaces. These surface modes may also become correlated with the atoms exiting the crystal. These modes can be handled, however, by appropriate termination of the crystal.

6. Conclusions

In conclusion, we have designed an integrated conditional teleportation circuit with readout in a single PC chip. Our proposed PCQC can be implemented by current or near future semiconductor processing technologies and experimental techniques. Even the detectors and sources illustrated in Fig. 1 could also be integrated. The feasibility of single atom detection on a PC chip has already been demonstrated, as have PC based lasers. In Ref. [32], for example, the detection of single atoms by exploiting a highly efficient integrated optical fiber-PC waveguide coupling is proposed. It could be envisioned that a compatible integrated atomic detection device based on such an experimental system might lead to more compact and robust PCQCs in general. The RC part of our device can not only be used with the TC but also can be used independently, or integrated with other compatible optical circuits to read out atomic states. It also hints at the potential for implementation of atomic interferometers, such as the Ramsey interferometer, in photonic crystals.

Achieving good mode-matching stability becomes quite cumbersome for many multiply nested interferometers in optical quantum computing. Photonic crystals, however, may be an especially promising paradigm as robust quantum circuit boards for delicate, next generation, scalable optical quantum computing and networking technologies, as well as scalable quantum dynamic random access memory, where qubits are refreshed using PC based cyclical quantum circuits. Integrated PC devices, such as planar waveguides and elementary
interferometer modules, could replace the current bulky elements in linear optical quantum circuits. Furthermore, such devices could be made quite compact using PC integrated with single photon sources.36

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