A Kinetics of non-equilibrium Universe. III. Stability of non-equilibrium scenario.
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Abstract
An influence of initial distribution nonequilibrium parameters on thermodynamical equilibrium recovery processes in early Universe under the assumption of elementary particles’ scaling of interactions in range of superhigh energies is researched.

1 Introduction
In previous papers of the Authors the model of superheat particles’ cosmological evolution in conditions of scaling of interactions was built up [1, 2]. For elementary particles’ cut set of interactions at that the Universal Asymptotic Cut Set of Scattering, introduced in papers [3, 4], was used:

\[ \Sigma_0(s) = \frac{2\pi}{s \left(1 + \ln^2 \frac{s}{s_0}\right)} = \frac{2\pi}{8\Lambda(s)}, \]  

where \( s \) is a kinematic invariant of four-piece reaction (details see in [1]), \( s_0 = 4 \) is a square of two colliding Planck masses’ total energy,

\[ \Lambda(s) = 1 + \ln^2 \frac{s}{s_0} \approx \text{Const}, \]  

In [2] a relaxation of cosmological plasma’s superheat component on equilibrium component was researching under the assumption that particles number in superheat component is far less than the equilibrium particles number. In particular, it was shown that the solution of kinetic equation describing an evolution of the ultrarelativistic superheat component in the equilibrium cosmological plasma has a form:

\[ \Delta f_a(t, \mathcal{P}) = \Delta f_a^0(\mathcal{P}) \exp \left[ \frac{-\xi(t, \mathcal{P})}{\mathcal{P}} \int_0^t \frac{y^2(t')dt'}{\sqrt{t'}} \right], \]  

where \( \Delta f_a^0(\mathcal{P}) = \Delta f_a(0, \mathcal{P}) \) is an initial deviation from equilibrium (detail see in [2]) and the dimensionless function is introduced:

\[ y(t) = \frac{T(t)}{T_0(t)}; \quad \sigma \equiv y^4; \quad \sigma_0 \equiv y^4(0). \]  

\(^1\text{system of units } G = \hbar = c = 1.\)
where \( T(t) \) is a temperature of plasma’s equilibrium component, \( T_0(t) \) is a temperature in the same point of time in completely equilibrium Universe and the dimensionless momentum variable \( \mathcal{P} \) (conformal momentum) is introduced:

\[
p = \mathcal{P} \mathcal{N}^{1/4} T_0(t). \tag{5}
\]

Farther \( \mathcal{N} \) is an efficient number of particles’ equilibrium types:

\[
\xi(t, \mathcal{P}) = \frac{\pi \tilde{N}}{3\sqrt{\mathcal{N}}} \left( \frac{45}{32\pi^3} \right)^{1/4} \frac{1}{\Lambda(\mathcal{P} T T_0/2)}; \tag{6}
\]

is a parameter, weakly dependent on variables \( t, \mathcal{P}, \)

\[
\tilde{N} = \frac{1}{2} \left[ \sum_B (2S + 1) + \frac{1}{2} \sum_F (2S + 1) \right] = N_B + \frac{1}{2} N_F;
\]

\( N_B \) is a number of sorts of equilibrium bosons, \( N_F \) is a number of sorts of equilibrium fermions. Relative temperature \( y(t) \) of plasma’s equilibrium component, presenting a parameter of the nonequilibrium distribution \( \xi(t, \mathcal{P}) \), is determined via the integral equation of energy-balance:

\[
y^4 + \frac{15}{\pi^4} \sum_a (2S_a + 1) \int_0^\infty \mathcal{P}^3 \Delta f_a^0(\mathcal{P}) \times \\
\times \exp \left[ - \frac{\xi(t, \mathcal{P})}{\mathcal{P}} \int_0^t \frac{y^2(t')dt'}{\sqrt{t'}} \right] d\mathcal{P} = 1, \tag{7}
\]

where \((2S_a + 1)\) is a statistical factor. From \( \xi(t, \mathcal{P}) \) in zero point of time follows the relation \[2\] :

\[
\frac{15}{\pi^4} \sum_a (2S_a + 1) \int_0^\infty \mathcal{P}^3 \Delta f_a^0(\mathcal{P}) d\mathcal{P} = 1 - \sigma_0. \tag{8}
\]

Let us introduce corresponding to \[2\] a new dimensionless time variable \( \tau \) :

\[
\tau = \frac{\xi}{\mathcal{P}_0} \sqrt{t}; \tag{9}
\]

where \( \xi = \xi(\mathcal{P}_0) \):

\[
\mathcal{P}_0 = \frac{\sum_a (2S_a + 1) \int_0^\infty d\mathcal{P} \mathcal{P}^3 \Delta f_a^0(\mathcal{P})}{\sum_a (2S_a + 1) \int_0^\infty d\mathcal{P} \mathcal{P}^2 \Delta f_a^0(\mathcal{P})}. \tag{10}
\]
is an average value of momentum variable $\mathcal{P}$ in point of time $t = 0$, $Z(\tau)$:

$$Z(\tau) = 2 \int_0^\tau y^2(\tau') d\tau'.$$

Then subject to (8) equation (7) is reduced to the form (12):

$$y = [1 - (1 - \sigma_0)\Phi(Z)]^{1/4},$$

where

$$\Phi(Z) = \frac{\mathcal{P}(t)}{\mathcal{P}(0)} = \frac{\sum_a (2S_a + 1) \int_0^\infty d\mathcal{P} \mathcal{P}^3 \Delta f_a^0(\mathcal{P}) e^{-2\mathcal{P}_0/\mathcal{P}}}{\sum_a (2S_a + 1) \int_0^\infty d\mathcal{P} \mathcal{P}^3 \Delta f_a^0(\mathcal{P})}.$$  

## 2 Numerical Model

In paper [2] distribution function’s initial deviation from the equilibrium was represented in the most plain form:

$$\Delta f^0(x) = \frac{A}{\mathcal{P}_0^3(k^2 + x^2)^{3/2}} \chi(1 - x), \quad k \to 0,$$

where $\chi(z)$ is a Heaviside function (a staircase function), $x = \mathcal{P}/\mathcal{P}_0$ is a dimensionless momentum variable, $A$, $\mathcal{P}_0$ and $k$ are certain parameters; $k$ parameter is introduced to provide a convergence of distribution function’s all moments in range of momentum’s small values. A distribution function of superheat particles’ energy density at that has a form, close to the spectrum of the so-called white noise, when all energy’s values are equiprobable up to the certain critical value $\mathcal{P}_0$, after which distribution breaks. From the qualitative point of view an evolution of the initial distribution (14) is reduced to the “corrosion” of nonequilibrium distribution spectrum’s low-energy part while energy density’s constancy in high-energy part conserves, what did not allow to make predictions about the form of distribution’s tail area. In given paper we select a distribution function’s deviation in more realistic form, allowing to carry out a research of the relaxation process dependence on parameters of the initial distribution. The paper’s main goal at that is a stability’s research of the nonequilibrium cosmological scenario [2] with respect to the parameters of the superheat particles’ initial distribution. So, let us specify the initial distribution in form:

$$\Delta f^0(\mathcal{P}) = A e^{-\alpha \mathcal{P}},$$

so that:

$$\Delta \tilde{N} = \sum_a \frac{1}{\pi^2} \int_0^\infty \Delta f^0(\mathcal{P}) \mathcal{P}^2 d\mathcal{P} = \frac{2A}{\pi^2 \alpha^3} N.$$
is an initial conformal density of nonequilibrium particles’ number. Let us calculate an initial conformal energy density of nonequilibrium particles:

$$\tilde{\varepsilon}_1 = \sum_a \frac{1}{\pi^2} \int_0^\infty \Delta f_0 \mathcal{P}^3 d\mathcal{P} = \frac{3!A}{\pi^2 \alpha^2} N. \tag{17}$$

Thus, we obtain for the initial average conformal energy:

$$\overline{\varepsilon}_0 = \frac{\Delta \tilde{\varepsilon}}{\Delta N} = \frac{3}{\alpha}. \tag{18}$$

Let us calculate the parameter $\sigma_0 = \varepsilon_0 / (\varepsilon_0 + \varepsilon_1)$, where $\varepsilon_0$ is an energy density of plasma’s equilibrium component and $\varepsilon_1$ is a nonequilibrium component’s energy density. Plasma’s equilibrium component is described via the distribution:

$$f_0 = B \exp \left( -\frac{p}{T} \right) = B \exp \left( -\frac{PN_1^{1/4}T_0}{T} \right) = B \exp \left( -\frac{PN_1^{1/4}}{y} \right). \tag{19}$$

Calculating conformal particles number densities and theirs energies relative to this distribution, we obtain:

$$\tilde{N}_0 = \frac{y^3B2!}{\pi^2} N^{1/4}, \quad \tilde{\varepsilon}_0 = \frac{y^4B3!}{\pi^2}. \tag{20}$$

Comparing (20) with an energy density of equilibrium plasma, which is determined via its temperature by means of the relation (see [1]):

$$\varepsilon_0 = N \frac{\pi^2 T^4}{15}, \tag{21}$$

we find:

$$B = \frac{\pi^4}{90} N. \tag{22}$$

Then:

$$N_0 = \frac{\pi^2}{45} y^3 N^{5/4}. \tag{23}$$

Using (17), (20) and (22) we obtain:

$$A = (1 - \sigma_0) \frac{\pi^4 \alpha^4}{90}. \tag{24}$$

Then according to the referred above nonequilibrium particles’ condition of smallness:

$$n_1(t) \ll n_0(t), \tag{25}$$

it has to be:

$$\alpha(1 - \sigma_0) \ll y_0^3 N^{1/4}. \tag{26}$$

On Fig.7-9 the graphs of the initial staircase and exponential distributions at equal energy densities and particles’ average energies for various nonequilibrium parameters are shown in comparison.
Figure 1: Comparison of staircase and exponential initial distributions at equal in both cases energy densities $\tilde{\varepsilon}_1$ and average energies of particles $\bar{\varepsilon}_0$: $\alpha = 0.9$. Staircase distribution is a thin line, exponential distribution is a heavy line.

Figure 2: Comparison of staircase and exponential initial distributions at equal in both cases energy densities $\tilde{\varepsilon}_1$ and average energies of particles $\bar{\varepsilon}_0$: $\alpha = 0.5$. Staircase distribution is a thin line, exponential distribution is a heavy line.
Figure 3: Comparison of staircase and exponential initial distributions at equal in both cases energy densities $\tilde{\varepsilon}_1$ and average energies of particles $\bar{E}_0$, $\alpha = 0.1$. Staircase distribution is a thin line, exponential distribution is a heavy line.

3 Research of nonequilibrium distribution’s relaxation

Calculating function $\Phi(Z)$ relative to the distribution (15) according to the formula (13), we obtain:

$$
\Phi(Z) = \frac{9Z^3}{2} \left( \frac{2(3Z + 6)K_0(2\sqrt{3Z})}{9Z^2} + \frac{4\sqrt{3}(3 + 6Z)K_1(2\sqrt{3Z})}{27Z^{3/2}} \right),
$$

(27)

where $K_\nu(z)$ is a Bessel function of the second sort (McDonald function) [5]:

$$
K_\nu(z) = \frac{\sqrt{n\pi} y}{2^\nu \Gamma(\nu + \frac{1}{2})} \int_0^\infty e^{-z \cosh \nu} \sinh^{2\nu} t dt,
$$

$\Re(z) > 0, \Re(\nu) > -\frac{1}{2}.
$$

(28)
Relation of $\tau$ and $Z$ is determined via the following formula

$$\frac{1}{2} \int_{0}^{Z} \frac{dU}{\sqrt{1 - (1 - \sigma_0)\Phi(U)}} = \tau,$$

(29)

in which an obtained value of $\Phi(Z)$ is to be substituted. On Fig. 4-7 the graphs showing an influence of initial distribution’s parameters on plasma’s temperature relaxation process are represented.

Figure 4: Relaxation of plasma’s temperature to the equilibrium:
$y = T(\tau)/T_0(\tau)$ subject to the parameter $\sigma_0$: bottom-up
$\sigma_0 = 0, 0.1; 0.1, 0.3; 0.5, 0.9$. Values of dimensionless time variable $\tau$ are put on the abscissa axis.

\textsuperscript{2}details see in [2].
Figure 5: Relaxation of plasma’s temperature to the equilibrium: 
\[ y = \frac{T(\tau)}{T_0(\tau)} \] for the parameter \( \sigma_0 = 0.9 \) for staircase function (dotted line) 
and \( Ae^{-\alpha \tau} \) function (firm line). Values of dimensionless time variable \( \tau \) are 
put on the abscissa axis.

Figure 6: Relaxation of plasma’s temperature to the equilibrium: 
\[ y = \frac{T(\tau)}{T_0(\tau)} \] for the parameter \( \sigma_0 = 0.3 \) for staircase function (dotted line) 
and \( Ae^{-\alpha \tau} \) function (firm line). Values of dimensionless time variable \( \tau \) are 
put on the abscissa axis.
Figure 7: Relaxation of plasma’s temperature to the equilibrium: 
\[ y = \frac{T(\tau)}{T_0(\tau)} \] for the parameter \( \sigma_0 = 0.01 \) for staircase function (dotted line) and \( Ae^{-\alpha p} \) function (firm line). Values of dimensionless time variable \( \tau \) are put on the abscissa axis.

On Fig. 4 the results of numerical modelling of equilibrium component’s heating process by superheat particles for the initial distribution (15) in terms of equation (29) subject to the initial distribution’s parameters are shown. It is obvious from this picture that temperature of plasma’s equilibrium component is saturated up to the value \( T_0(\tau) \) at \( \tau \sim 3 - 4 \). On Fig. 5-7 temperature’s relaxation process of the staircase and exponential distributions for various values of initial distribution’s nonequilibrium parameter \( \sigma_0 \) is shown. It is clear from these pictures, that in the case of the exponential distribution temperature relaxes to the equilibrium rather slower than in the case of the staircase function, however qualitative behavior of \( y(\tau) \) function’s graphs coincides with each other.

Using the results of equation (29) numerical integration in formula for the relaxation of the nonequilibrium distribution (3), we obtain time dependence of distribution function’s deviation from the equilibrium. On Fig. 8 the results of equations’ (29) and (3) simultaneous numerical integration subject to the value of initial distribution’s nonequilibrium parameter are shown.

Thus, an evolution of distribution in time variable \( Z \) does not depend explicitly from the nonequilibrium parameter \( \sigma_0 \), but the relation of real time \( \tau \) and time variable \( Z \) do. This relation is given by formula (29) and is represented in graphical form on Fig. 8.
Figure 8: Relaxation of superheat component for distribution (15) \((A = 1)\) subject to time parameter \(Z\): top-down - \(Z = 0.5, Z = 1, Z = 2, Z = 4\). Value of dimensionless time variable \(x\) is put on the abscissa axis, value \(\lg(1 + f)\) is put on the ordinate axis. 

Figure 9: Dependence of distribution’s maximum on dimensionless time \(\tau(Z)\), calculated by formula (3) relative to the initial distribution (15) subject to initial distribution’s nonequilibrium parameter \(\sigma_0\). Top-down: \(\sigma_0 = 0.01, \sigma_0 = 0.1, \sigma_0 = 0.3, \sigma_0 = 5, \sigma_0 = 0.9\).
4 Conclusion

Thus, carried out research has shown from the one hand though certain differ-
ences the stability of nonequilibrium distribution’s relaxation scenario relative
to parameters of the superheat particles’ initial distribution and from the other
hand has allowed to reveal the dependence of initial distribution’s average pa-
rameters on Universe’s evolution time. As evident from aforecited pictures,
maximum of nonequilibrium particles’ energy spectrum shifts with time by the
approximate law:

\[ P_{\text{max}} \approx \sqrt{\tau}. \]

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