FRAGMENTATION OF GRAVITATIONALLY UNSTABLE GASEOUS PROTOPLANETARY DISKS WITH RADIATIVE TRANSFER

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ABSTRACT

We report on the results of the first 3D SPH simulations of gravitationally unstable protoplanetary disks with radiative transfer. We adopt a flux-limited diffusion scheme justified by the high opacity of most of the disk. The optically thin surface of the disk cools as a blackbody. We find that gravitationally bound clumps with masses close to a Jupiter mass can arise. Fragmentation appears to be driven by vertical convective-like motions capable of transporting the heat from the disk midplane to its surface on a timescale of only about 40 years at 10 AU. A larger or smaller cooling efficiency of the disk at the optically thin surface can promote or stifle fragmentation by affecting the vertical temperature profile, which determines whether convection can happen or not, and by regulating accretion from optically thin regions toward overdense regions. We also find that the chances of fragmentation increase for a higher mean molecular weight, \( \mu \), since compressional heating is reduced. Only disks with masses \( \geq 0.12 \, M_\odot \) and with \( \mu \geq 2.4 \), as expected for gas with a metallicity comparable to solar or higher, can fragment.

Subject headings: accretion, accretion disks — hydrodynamics — methods: n-body simulations — planetary systems: formation — solar system: formation

1. INTRODUCTION

Long-lived clumps can form in a massive gravitationally unstable protoplanetary disk if the gas is isothermal (Mayer et al. 2002) or cools on a timescale comparable to the local orbital time (Gammie 2001; Rice et al. 2003; Mayer et al. 2004b, 2005; Mejia et al. 2005). Fragmentation under such favorable conditions is a numerically robust result obtained with a variety of codes (Durisen et al. 2007; L. Mayer et al. 2007, in preparation). One important open question is whether the required short cooling times can be obtained in the disks. Shock heating along spiral arms is intense during the phase of strong gravitational instability (Pickett et al. 2000, 2003; Mayer et al. 2004a) and will tend to erase clumps exactly when they have the best chance to collapse. Boss (2002a, 2002b, 2004) has used a grid-based code that solves radiation transport with the diffusion approximation. He found that the disk cools rapidly via convection instead of radiation and forms long-lasting clumps. Cai et al. (2006) adopt flux-limited diffusion in their cylindrical grid code. They find long cooling times, no evidence of convection, and no fragmentation. While Boss does not use a flux limiter but embeds the disk in a thermal bath at a constant temperature in the range 40–50 K, Cai et al. add an Eddington-like atmosphere on top of the optically thick layer, explicitly matching the flaxes at the boundary. Nelson et al. (2000) also did not find fragmentation in a 2D SPH simulation to which a plane-parallel atmosphere radiating as a blackbody was added assuming that rapid vertical cooling was achieved through convection. Here we present the first 3D SPH simulations of protoplanetary disks with flux-limited diffusion.

2. THE SIMULATIONS

The disk models are set up as described in Mayer et al. (2004a). Initially, disks extend from 4 to 20 AU, have a power-law surface density profile, and a minimum temperature of 40 K at the outermost radius. The simulations employ \( 10^6 \) gas particles with a gravitational softening of 0.06 AU, while the central star is represented by a particle with mass \( 1 \, M_\odot \) and softening 0.2 AU. Disks are grown in mass from a stable state with a Toomre parameter (Toomre 1964) \( Q_{\mathrm{min}} > 4 \). The mass of each particle is increased by the same constant factor while its specific internal energy is kept fixed. Mass growth is stopped either when the disk fragments or when it reaches a mass equal to \( 0.2 \, M_\odot \). Our simulations fulfill the criterion of Bate & Burkert (1997) to avoid spurious fragmentation as well as the stricter criterion based on the local Toomre mass (Nelson 2006), while the convergence of the results with increasing vertical resolution requires further testing. The simulations were run with the parallel SPH code GASOLINE (Wadsley et al. 2004). We solve the energy equation in asymmetric form accounting for irreversible shock heating via the standard Monaghan artificial viscosity with the Balsara correction term to reduce unwanted shear viscosity.

The radiation transport is implemented using the diffusion approximation and the flux limiter of Bodenheimer et al. (1990). A similar method has been used by Whitehouse & Bate (2006) to study the collapse of molecular cloud cores. The flux limiter appears as a coefficient of a diffusive term in the energy equation. As in Cleary & Monaghan (1999), the diffusive term reads

\[
\dot{U}_a = \sum_b \frac{4m_p}{\rho_a \rho_b} \frac{k_b}{k_a + k_b} \left( T_a - T_b \right) \frac{r_{ab} \cdot \nabla W}{r_{ab}^2},
\]

where the summation is over neighboring particles, \( W \) is the smoothing kernel, \( r_{ab} \) is the vector from the position of particle
a to particle \( b \), and \( k_e \) takes the form of a thermal conductivity term

\[
k_e = \frac{16 \alpha}{\rho_a \kappa_a} \lambda_a T_a^4,
\]

where \( \kappa_a \) is the opacity, \( \alpha \) is the Stefan-Boltzmann constant, and \( \lambda_a \) is the flux limiter. This is a stable method that relaxes the noise of second derivatives.

The (Rosseland mean) opacity of each particle is interpolated from a table of \( \kappa(T, \mu) \) (D’Alessio et al. 1997). The next step is to allow particles on the boundary of the disk to radiate their energy away to infinity. To find particles that are “on the edge” of the disk, we examine the directions to all of the neighbors used in smoothing sums. If a particle has no neighbors within a certain fraction of a solid angle from a preferred direction, it is considered an edge particle. From the geometry of a disk, the preferred directions (treated independently) are out of the plane of the disk (both up and down) and radially outward.

We let edge particles radiate as blackbodies, adding the following term to the energy equation of each particle

\[
\dot{U}_a = f_e S \sigma a T_a^4 m_a,
\]

where \( S = 4\pi h_a^2 \) is the surface through which the particle radiates, with \( h_a \) the smoothing length of the particle. The “edgeness factor” \( f_e \) represents the fraction of their surface area over which a particle radiates. It is usually zero and takes value \( \frac{1}{2} \) for particles on one of the up, down, or out boundaries, \( \frac{1}{2} \) for those on the edge in both the up, down, and out directions, and \( \frac{1}{4} \) for those on the edge in the out and either up or down direction.

In practice, along each direction we consider particles comprised within an edge detection angle (EDT) measured as a linear angle in a cone centered on a given particle. We consider EDT values in the range 30°–60° so that particles identified as “edge” do lie in regions where the optical depth is \( \tau < 1 \). Different choices for the EDT correspond to a different atmospheric cooling efficiency since both the number of particles identified as edge and the edgeness factor for each particle will change. The smallest possible size of the resulting radiating surface area is that of the geometric surface area of the disk, which corresponds to an EDT of 50° when the disk becomes gravitationally unstable. Within the range of angles considered the radiative surface area changes by about a factor of 2.

A near solar opacity is used in most of the simulations, and we adopt \( \gamma = 7/5 \) (Nelson et al. 2000). The (mean) molecular weight of the gas is by default equal to the solar metallicity value (\( \mu = 2.4 \)) but is varied between \( \mu = 2 \) (as in Mayer et al. 2004a, 2004b, 2005) and \( \mu = 3 \). A molecular weight higher than solar could result from the combination of three factors, i.e., a higher mean metallicity of the gas, an increased dust-to-gas ratio in the spiral arms, and the vaporization of dust grains. Water ice can be vaporized in the spiral shocks of a gravitationally unstable disk, where temperatures can rise above 150 K (Nelson et al. 2000; Mayer et al. 2005) and accounts for about 30%–40% of the dust content. The dust/gas ratio is expected to increase by an order of magnitude in the spiral arms as grains larger than micron size rapidly migrate toward gaseous overdensities (Rice et al. 2004; Haghhipour & Boss 2003). An order of magnitude enhancement in the gas-to-dust ratio and the vaporization of water ice would produce \( \mu \sim 2.5 \) in a disk with mean solar metallicity, while \( \mu \sim 2.85 \) would be achieved in a disk with a mean metallicity 3 times larger than solar. We have run about 15 simulations with various molecular weights and opacities.

3. COOLING, HEATING, AND FRAGMENTATION

The disk grows uniformly in mass at the rate of \( \sim 10^{-4} M_\odot \text{ yr}^{-1} \), approaching 0.1 \( M_\odot \) after about \( 10^7 \) yr. When its mass grows above 0.05 \( M_\odot \), the Toomre parameter \( Q \) drops below 2 in the outer part of the disk and strong spiral patterns begin to appear. In order to minimize the computational time, simulations with different molecular weights or different opacities are run by restarting the simulation from this point (the base simulation has \( \mu = 2.4 \) and an opacity consistent with solar metallicity). For the molecular weight this is indeed a rather sensible choice since variations toward higher values might occur only after strong spiral shocks begin to develop (see § 2). The shocks occurring along the spiral arms limit the growth of their amplitude as the increasing pressure counteracts self-gravity. Yet fragmentation occurs in some of the simulations once the disk mass is in the range 0.12–0.15 \( M_\odot \) (Fig. 1). Whether the disk fragments or not depends on the details of thermodynamics in the disks, in particular the molecular weight of the gas and the cooling efficiency at the boundary (see Fig. 1).

We find that \( \mu \geq 2.4 \) is a necessary condition for fragmentation (Fig. 1). Larger molecular weights can have two effects; they lower the value of the pressure gradients in the adiabatic compression term of the internal energy equation, since for an ideal gas \( P \sim T \rho \mu \) when \( \mu \) is increased while holding \( T \) fixed, and they increase the cooling rate associated with the blackbody atmosphere (eq. [3]) since \( T \sim \mu \) when \( P \) is held fixed. We verified that it is the first effect that dominates; in a simulation restarted with \( \mu = 2.7 \) we artificially increased the initial temperature by a factor such that the pressure term equals that in a \( \mu = 2.4 \) simulation and found the same result of the latter (no fragmentation) despite the increased cooling rate at the surface. This suggests that reducing compressional heating is...
disks are very large, near the disk midplane. Convection short timescales thanks to convection. The optical depths in our order of the orbital time, for fragmentation to happen (Rice et al. 2003; Mayer et al. 2004b). Such short radiative cooling times are significantly longer than the orbital time, and hence no fragmentation is expected in such conditions for (Ruden & Pollack 1993). The pressure scale height, whose scale should be comparable to the mixing length in the convective motions, is 

The overdensities along the spiral shocks can engulf material from neighboring cold regions between spiral arms (Fig. 3). Such regions are optically thin down to a height at least half of that in the spiral arms and are naturally produced as the arms unwind. The accretion flow from such regions is more prominent, and thus the overdensities grow faster, for lower temperatures of the interarm gas, hence for a higher radiative cooling rate as controlled by the EDA parameter. This is an example of the complex coupling between the dynamics and thermodynamics seen in these simulations. Nevertheless, we notice that the typical midplane temperatures (>200 K) and midplane surface densities (>2000 g cm$^{-2}$) of our disks where clumps first appear (at about 15 AU) are admitted as fragmenting solutions by the analytical model of Rafikov (2005), given a Toomre parameter $Q = 1.4$ and a cooling time $t_{cool} = 1.5T_{sh}$ as the criteria for fragmentation (see Mayer et al. 2004b). On the other end, more recently Rafikov (2007) extended his analytical model to include the effect of convection and surface cooling, and predicted cooling times significantly longer than the orbital time, and hence no fragmentation, at distances less than 20 AU.

![Fig. 2.—Top: Evolution of the vertical temperature profile of an overdense region that collapses and becomes a gravitationally bound clump in the run with $\mu = 2.7$ and EDA of 40° (same clump as in Fig. 3). The profiles are taken at $T = 1220$ (solid line), $T = 1221$ (dashed line), $T = 1222$ (long-dashed line), and $T = 1223$ (dot-dashed line) yr and are azimuthally averaged. As the profile steepens, the gas becomes convectively unstable (based on the Schwarzschild criterion) almost everywhere for heights above 0.05 AU. It then relaxes to a shallower, more stable profile, with an average temperature lower than at the start (dot-dashed line).](image1)

![Fig. 3.—Color-coded logarithmic temperature plots of a region of size about 4.5 AU centered on one of the gravitationally bound clumps in the run with $\mu = 2.7$ and EDA of 40°. Left: Smooth temperature map is shown (the temperature range goes from 10 K (dark blue) to 250 K (dark red) at time $t = 1248$ yr showing the cold, low-density regions adjacent to the hot gas in the spiral shock and clump. Right: Temperature map of the gas particles surrounding the same clump about half an orbit earlier with the particles that will accrete by $t = 1248$ yr marked in green. The brighter the color, the higher the temperature. Most of the accreted mass appears to come from the colder regions outside the spiral shock.](image2)
Once formed, gravitationally bound clumps have masses that range from one to a few Jupiter masses, are differentially rotating, and have densities a million times higher than the background density. They reach $\tau \sim 1000$ near the resolution limit and temperatures $T > 300$ K.

4. DISCUSSION

We have found that disks can fragment in simulations that account for radiative transfer. They appear to cool fast enough via convection, although other explanations for the observed turbulence, such as shock bores (Boley & Durisen 2006), cannot be excluded. Fragmentation requires a disk more massive than those in previous works, relying on simple equations of state or via convection, although other explanations for the observed account for radiative transfer. They appear to cool fast enough stars or the protostellar envelope, or from shocks produced by gas accreting from the envelope. These missing ingredients could erase the steep temperature gradient necessary to drive convection. Convection might be artificially enhanced if our recipe for blackbody emission overestimates the cooling rate near the boundary, steepening the temperature gradient exessively. This might be the case for particles identified as edge particles based on our scheme but which have $\tau \sim \frac{1}{2}$. Resolution tests will clarify the extent of the problem since the edge will become sharper with increasing resolution. Excessive cooling might also boost the amount of cold material in the interarm regions that feed the clumps.

The discovered dependence on molecular weight establishes a possible link between the disk instability mechanism and the metallicity of the gas. We varied the molecular weight everywhere in the disk rather than just locally in the spiral shocks. However, in a test we raised the molecular weight from 2.4 to 2.7 very late in the evolution of the disk, which then fragmented just a third of an orbital time later as in the original run with $\mu = 2.7$. Since over such a short timescale the gas can only respond to changes in the local conditions, future simulations with locally varying molecular weights might confirm the trend seen here for runs with globally different molecular weights. Surely a major drawback is that opacity and molecular weight do not vary self-consistently here. A more metal-rich disk will also have a higher cooling rate in the optically thin region (Cai et al. 2006), which also appears to promote gravitational instability, while the associated increase of opacity in the optically thick regions seems to have no effect. Ultimately, a definitive answer on the feasibility of the disk instability model for giant planet formation will require incorporation of all aspects of radiation physics in the disks and in their surrounding environment together with the details of the disk formation process.

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