1. Introduction

In the last decade the interest in concrete products manufactured by vibration under a surcharge load, such as paving blocks, kerbs, etc., has increased significantly. In the compaction by vibration under a surcharge load the mixture is subjected to the dynamic effect of vibration and static surcharge load. This combined effect must overcome the internal resistance of the mixture, which is made up of frictional, viscosity and cohesion resistance. As a result the mixture volume decreases and its apparent density increases.

![Fig. 1. Changes of volume and apparent density of mixture under loads](image)

The method of vibration under a surcharge load is a combined method which does not cause a change of the W/C ratio during compaction. The frictional and viscosity resistance is overcome mainly by the effect of vibration, that is applying a multiplied force of gravity. Due to vibration the concrete mixture behaves like a dense, tixotropic fluid and the internal friction decreases. The effect of pressure exerted on the mixture enhances the process of compaction. What is important in this technology is the proportion of the two effects. The most favourable proportion of contribution of the two effects differs for different mix composition (different aggregate grading, different cement paste content, different W/C ratio, different consistency).

The technology of vibration under a surcharge load, described briefly below, requires a particularly careful selection of mix composition which ensures both the concrete assumed strength and proper consistency and workability.

2. Description of two useful methods of concrete mix compacted by vibration under a surcharge load

The methods that have recently proved very useful in concrete mix design for the technology of surcharge vibration are: the method of heaping up the aggregate void spaces with cement paste and the method of coating the aggregate grains. In Poland these methods have been known for years as the Kopyciński method and the Paszkowski method [2].
In Germany the method based on heaping up the aggregate void spaces with paste was proposed by Zipelius [3] in the 1960s. The method was later used to construct the so-called Econom apparatus [4], due to which it is possible to empirically define the most economical paste content in concrete mix. The empirical design of concrete mix by means of the Econom apparatus takes into account the real conditions of compacting by surcharge vibration, that is, the characteristics of vibration and surcharge pressure are taken into account.

The other method is based on the determination of indispensable quantity of cement paste as a function of aggregate surface area and thickness of paste layer coating its grains.

2.1 Method of heaping up aggregate void spaces with cement paste Econom apparatus method [4]

The method is based on the assumption that in order to make a concrete mixture from the adopted aggregate, the void spaces between the grains have to be heaped up, to some indispensable degree, with cement paste (Fig. 3).

Before we start the empirical design it is obviously necessary to select both the type and class of cement and W/C ratio as well. The value of W/C ratio can be defined from any dependence between concrete strength and its composition, for example from Bolomey’s formula. It is also necessary to decide on the aggregate type and design its grading such that the grading curve was within the regions shown in Fig. 2 [5].

Initially we determine the volume of a set portion of aggregate $V_{agg}$ [dm$^3$] (in the Econom apparatus method portion of 10 kg is used) compacted in conditions similar to those of the mixture to be compacted (vibration under surcharge load). The next step is to determine the volume of voids in the aggregate. It can be done easily by infusion of water in the amount of $V_{water}$ [dm$^3$] to the aggregate compacted in proper conditions so that the water fills all the void spaces between the aggregate grains. Thus the infused water volume $V_{water}$ is equal to the volume of the voids in the tested portion of aggregate $V_{av}$ (Fig. 3a).

The next step is to prepare the cement paste of a certain W/C ratio, its volume $V_{paste}$ equal to water volume $V_{water}$ (equal to the volume of voids $V_{av}$) is mixed with 10 kg of aggregate identical as before. The concrete mix obtained in this way is compacted in identical conditions as the dry aggregate was compacted initially. Its volume after compaction $V_{concrete max}$ [dm$^3$] is determined. This volume will be slightly larger than the volume of compacted dry aggregate $V_{agg}$ because the aggregate grains will be coated with a thin layer of the paste and will be separated (Fig. 3b). The difference between the volume of the mix composed in this way and the volume of the dry aggregate itself ($\Delta V = V_{concrete max} - V_{agg}$) is the volume of void spaces in the prepared mix. It is this value by which the volume of cement paste should be enlarged to obtain a mix with no pores. The final indispensable volume of paste is thus $V_{paste def}$.

Next, knowing the W/C = $\omega$ characterising the paste, we can calculate its density $\rho_{paste}$ which is:

\[
\rho_{paste} = \frac{\omega + 1}{\rho_c} \ [kg/dm^3]
\]  

where: $\rho_c$ - cement density, commonly adopted as 3.1 [kg/dm$^3$]

Knowing that the mass of paste of volume $V_{paste def}$ is:

\[
m_{paste def} = V_{paste def} \cdot \rho_{paste} \ [kg]
\]  

Fig. 2. Approximate areas of good grading of aggregates 0/8 mm and 0/16 mm for concretes compacted by vibration under a surcharge load [5] (the area between upper and lower broken curve - for rounded aggregate, the area between the upper and lower solid curve - for crushed aggregate).

Fig. 3. The idea of heaping up the aggregate with paste; the Econom apparatus version (description in the text)
the content of cement $C$ and water $W$ in concrete mix containing aggregate quantity $A = 10$ kg can be calculated from formulae:

$$C = \frac{m_{\text{paste def}}}{1 + \omega} [\text{kg}]$$  \hspace{1cm} (3)

$$W = m_{\text{paste def}} - C [\text{kg}]$$  \hspace{1cm} (4)

Since from the quantities of components given above we have obtained the compacted concrete mix of the volume $V_{\text{concrete mix}}$, the composition of $1$ m$^3$ of the mix is:

$$C = \frac{C}{V_{\text{concrete mix}}} 1000 \ [\text{kg/m}^3]$$  \hspace{1cm} (5)

$$W = \frac{W}{V_{\text{concrete mix}}} 1000 \ [\text{kg/m}^3]$$  \hspace{1cm} (6)

$$A = \frac{10}{V_{\text{concrete mix}}} 1000 \ [\text{kg/m}^3]$$  \hspace{1cm} (7)

2.2 Method of coating aggregate grains with paste

As in any design method, also in our case it is necessary to start with the assumptions as to the quantitative selection of ingredients, i.e. type and class of cement and aggregate grading as well as the value of $W/C$ ratio that ensures the proper strength of concrete.

In the presented method the calculations are based on the aggregate surface area $F_{\text{aggr}}$, i.e. the total surface of aggregate grains of $1$ kg mass. Since aggregate is a mixture of grains of various sieve size $f_i$, its surface area can be calculated as a weighted average of the surfaces $F_{ai}$ of all the sieve sizes found in it:

$$F_{\text{aggr}} = \frac{\sum_{i=1}^{i=n} F_{ai} f_i}{100} \ [\text{dm}^2/\text{kg}]$$  \hspace{1cm} (8)

where: $F_{ai}$ – the surface of $i$th sieve size [dm$^2$/kg],

$$f_i$$ – the content of $i$th sieve size [% mass]

The values of surface areas of particular size of grains of rounded aggregate nearly spherical in shape and density $\rho_a = 2.65$ kg/dm$^3$ have been presented in Table 1.

When crushed aggregate is used, the values in Table 1 should be multiplied by 1.15. For aggregate from material of density $\rho_a \neq 2.65$ kg/dm$^3$ the values in Table 1 should be additionally multiplied by quotient $\rho_a/2.65$.

When the surface area of the aggregate used $F_{\text{aggr}}$ is known, it is possible to calculate the paste demand for $1$ kg of aggregate $v_{\text{paste}}/1 \text{ kg}$:

$$v_{\text{paste}}/1 \text{ kg} = F_{\text{aggr}} t \ [\text{dm}^3/\text{kg}]$$  \hspace{1cm} (9)

### Table 1

| Sieve size $f_i$ [mm] | Specific surface $F_{ai}$ [dm$^2$/kg] | Sieve size $f_i$ [mm] | Specific surface $F_{ai}$ [dm$^2$/kg] |
|----------------------|--------------------------------------|----------------------|--------------------------------------|
| 0/0.125              | 4000                                 | 2/4                  | 100                                  |
| 0.125/0.25           | 1600                                 | 4/8                  | 50                                   |
| 0.25/0.5             | 800                                  | 8/16                 | 35                                   |
| 0.5/1                | 400                                  | 16/31.5              | 12.5                                 |
| 1/2                  | 200                                  | 31.5/63              | 6.2                                   |

where: $t$ – required thickness of paste layer coating the aggregate grains [dm], after [3] to obtain the consistency proper for compacting by vibration under a surcharge load the values of $t$ are:

- for $W/C = 0.8 \rightarrow t = 0.0004$ dm (0.04 mm),
- $W/C = 0.6 \rightarrow t = 0.0005$ dm,
- $W/C = 0.4 \rightarrow t = 0.0006$ dm

The thickness values of the paste layer coating the grains have been selected in such a way that the necessity of heaping up the voids between the grains has been taken into account.

The total volume of $1$ kg of aggregate and quantity of paste necessary for its grain coating is:

$$v_t = \frac{1}{\rho_a} + \frac{v_{\text{paste}}/1 \text{ kg}}{\rho_c} \ [\text{dm}^3]$$  \hspace{1cm} (10)

while the necessary paste volume in $1$ m$^3$ of the mix is:

$$V_{\text{paste}} = \frac{1000}{v_t} v_{\text{paste}}/1 \text{ kg} \ [\text{dm}^3/\text{m}^3]$$  \hspace{1cm} (11)

It should be noted that the $1000/v_t$ quotient gives the necessary quantity of aggregate $A$ in $1$ m$^3$ of concrete mix.

In the following sequence of a set of equations:

$$W = \frac{C}{\omega}$$  \hspace{1cm} (12)

$$W + \frac{C}{\rho_c} = V_{\text{paste}}$$  \hspace{1cm} (13)

cement and water content in $1$ m$^3$ of the mix can be calculated:

$$C = \frac{V_{\text{paste}} \rho_c}{\rho_c \omega + 1} \ [\text{kg/m}^3]$$  \hspace{1cm} (14)

$$W = \omega C \ [\text{kg/m}^3]$$  \hspace{1cm} (15)

The aggregate content, as mentioned above, is calculated as:

$$A = \frac{1000}{v_t} \ [\text{kg/m}^3]$$  \hspace{1cm} (16)

or from the volume balance:

$$A = (1000 - V_{\text{paste}}) \rho_a \ [\text{kg/m}^3]$$  \hspace{1cm} (17)
Similarly as in the first method, the final step is empirical verification of the designed concrete mix in which the compaction process and properties of hardened concrete in product are checked.

3. Additional remarks

The technology of concrete products manufacture by vibration under a surcharge load makes the designer of concrete mix face many specific requirements. Besides the obvious necessity to obtain concrete of assumed technical properties, these requirements include proper consistency and workability of concrete mixture. Even minor changes in mix composition can make it useless in product manufacture by this method. The two presented methods of concrete mix design, used for ordinary concretes for a long time, have proved very useful also in the design of concretes vibrated under a surcharge load.

References

[1] MIKOŚ, J.: Wybrane zagadnienia prefabrykacji, PWN, Warszawa 1987.
[2] ŚLIWINSKI, J.: Beton zwykły, projektowanie i podstawowe właściwości, wyd. Polski Cement Kraków, 1999.
[3] ZIPELIUS, R.: Zementleimermittlung für beliebige Zuschlagstoffgemische; Empirische Methode für hochfesten Beton grösster Wirtschaftlichkeit; Betonsteinzeitung 33, 1967, H.5; Bauverlag GmbH, Wiesbaden.
[4] DRINKGERN, G.: Abweichungen von der "klassischen" Mischungsberechnung für Beton, Betonwerk+Fertigteil-Technik, H.11/1994, 45-57
[5] BRYLICKI, W.: Zasady kształtowania jakości betonów wibroprasowanych, Materiały Sympozjum Naukowo-Technicznego, Technologia produkcji i badania właściwości betonowej kostki brukowej, kwiecień 1999, 39-61.