A new method for measuring the spectral emissivity of heated bodies

V P Khodunkov\textsuperscript{1} and Yu P Zarichnyak\textsuperscript{2}

\textsuperscript{1}D.I. Mendeleyev Institute for Metrology (VNIIM), St. Petersburg 190005, Moscow Avenue, 19, Russia
\textsuperscript{2}ITMO National Research University (ITMO), St. Petersburg 197101, Kronverksky Avenue, 49, Russia

Email: walkerearth@mail.ru

Abstract. A new spectral method for measuring the emissivity of solids at high temperatures is considered. The method is based on a linear dependence of the spectral energy brightness of a body on wavelength in a narrow spectral range. The theoretical basis and the results of experimental verification of the method are presented.

1. Introduction

Emissivity is an important thermophysical characteristic of anybody or object, without its knowledge it is impossible to accurately measure the thermodynamic temperature. This is especially true for high temperatures. Existing methods for measuring the emissivity are diverse, among them we can distinguish methods based on measuring the spectral distribution of the intensity of the object’s own radiation [1], multi-wave methods and other methods. All methods have their advantages and disadvantages and can be used under certain conditions and with certain limitations. Therefore, their further development is constantly in demand and practically significant.

This article discusses a method based on measuring the spectral distribution of the intrinsic radiation intensity of an object. According to the authors of the method, it can find successful application in the practice of scientific research.

Here is a brief background to the development of this method. In the department of standards and scientific research in the field of thermodynamics, D.I. Mendeleyev Institute for Metrology (Russia), operates the primary standard of temperature unit [2]. With the help of this standard, the unit of thermodynamic temperature is transferred from the model of an absolutely black body (MABB) to temperature lamps. The transfer is performed by equalizing the spectral energy brightness of the emitting cavity of the blackbody and the incandescent tape of the temperature lamp, in this case, if the brightness is equal, the brightness temperature of the incandescent lamp is taken equal to the thermodynamic temperature of the blackbody, and its actual temperature is recalculated according to a known ratio taking into account the value of the spectral emissivity of the incandescent tape, which is taken from the reference data. In a specific case, for a lamp with a tungsten tape, data from the source are used [3]. Despite the high accuracy of the literature data, where the spectral emissivity of tungsten...
is represented by four significant figures, we set out to double-check them for a specific lamp and establish the value of the possible error when transmitting a temperature unit. Since the standard has a double monochromator in its composition, the measurement of the emission spectra of the blackbody and the incandescent filament of the temperature lamp was not particularly difficult. Thus, we assumed that by measuring the spectra of radiation from the blackbody and lamps with a monochromator, we can find the desired values of the spectral emissivity according to its classical definition [4]: “the spectral emissivity of the body \( \varepsilon(\lambda, T) \) is the ratio of the radiation energy of the source (of the body) to the energy emitted by the absolute black body at the same temperature”. In addition, we pursued another goal - to experimentally measure the spectral emissivity of the blackbody itself, which, as a rule, is determined by calculation, for example, using the Monte Carlo simulation method [5].

2. Description of the experimental setup

The radiation spectra of the blackbody and the incandescent temperature lamp were measured using equipment included in the complex of the state primary temperature standard GET 34-2007 (high-temperature part of the standard), the generalized structural diagram of which is shown in Fig. 1. Equipment used: MABB - BB3500YY - model of absolutely blackbody with an ampoule of pure copper (melting point \( T_m=1357.78 \) K); temperature lamp with a tungsten filament; optical focusing system of mirror type; dual monochromator, model MSA-130 (Solar Systems), photodetector - silicon photodiode, model S1337-33BR (Hamamatsu), photodetector signal recorder femtoampermet, model B2983A (Keysight Technologies).With the specified equipment in the wavelength range \( 646 \div 662 \) nm. The radiation spectra of the emitting cavity (copper ampoules) of the MABB and the incandescent tape of a temperature lamp were measured. The measurements were carried out according to the following algorithm.

In the MABB, a phase transition (melting-solidification) of pure copper (\( T_m=1357.78 \) K) was carried out. When the phase transition was reached, the spectral brightnesses of the blackbody and lamp were first equalized. The brightness equalization consisted in the selection of the electric current supplying the lamp, in which the photocurrents of the photodiode generated from the radiation of the blackbody and the lamps were equal. To do this, the central wavelength of the studied range \( \lambda_0=654 \) nm, the bandwidth \( \Delta \lambda=3 \) nm were established on the monochromator, the radiation from the cavity blackbody was focused on the entrance slit of the monochromator using the optical system, and the measured photocurrent \( I_1 \) was recorded. Then, using the optical system, the radiation from the lamp was focused on the entrance slit of the monochromator, and the measured photocurrent \( I_2 \) was recorded. By changing the lamp current, the equality of currents \( I_1=I_2 \) was achieved, i.e. equality of brightness. Uncertainty in the equalization of brightness in terms of temperature did not exceed 10 mK. Then, during the phase transition with the help of an optical system, the radiation from the cavity blackbody was focused on the entrance slit of the monochromator and the spectrum of its radiation was measured. To do this, using a monochromator in the range 652–660 nm, a radiation spectrum was scanned and the photodiode photocurrent was recorded for each fixed wavelength. The scanning step along the wavelength was set equal to \( \Delta \lambda=2 \) nm. Since the duration of the phase transition was at least 30 minutes, this time was quite sufficient for measurements.

Then, the previously obtained current \( I_2=I_1 \) was installed on the lamp, the radiation from the lamp was focused on the entrance slit of the monochromator using the optical system, and after the lamp reached the stationary mode (≈30 min), a similar scanning of the lamp emission spectrum was performed.

Figure 1. Generalized structural diagram of the experimental setup:
1 - a heated body (MABB), 2 - optical focusing system of mirror type, 3 - a dual monochromator, 4 - photodetector, 5 - femtoamperometer.
As a result of the measurements, the emission spectra of the blackbody and lamp were obtained, i.e. dependences of the photodiode photocurrent on the wavelength in the range \( \lambda = 652–660 \) nm, which were then reduced to a normalized form (Fig. 2). In Fig. 2, in addition, the calculated graphic dependence of the dimensionless (normalized) photocurrent, which should have been generated from the radiation of an ideal absolutely blackbody is plotted. Normalization according to the photocurrent was performed by assigning the current measured photocurrent \( I \) to the value of the photocurrent \( I_{\lambda_1} \), measured at the lower boundary of the wavelength range \( \lambda_1 \), and then subtracting 1.0, i.e. by the ratio: \( \frac{I}{I_{\lambda_1}}-1 \). The wavelength normalization was performed by assigning the current value of the wavelength \( \lambda_i \) to the wavelength of the lower boundary of the wavelength range \( \lambda_1 \), i.e. by the ratio: \( \lambda_i/\lambda_1 (\lambda_i=646 \text{ nm}) \). The normalization operation was used to provide the ability to compare the spectra of an ideal blackbody and a real body. As you know, it is impossible to measure the photocurrent from an ideal blackbody - it can only be calculated, therefore normalization was necessary to compare the spectra, their subsequent processing and calculations.

### 3. The calculated ratio

The calculated ratio for the spectral emissivity of the studied object (body) was obtained by us on the basis of the well-known ratio for the photocurrent generated by the photodiode from the focused radiation incident on it, which has the form:

\[
I_F = \int_{\lambda_1}^{\lambda_2} \eta_{OS}(\lambda) S_\lambda(\lambda) \varepsilon(\lambda, T) \varphi L_{BB}(\lambda, T) \, d\lambda ,
\]

where \( I_F \) - the generated photocurrent, A; \( \eta_{OS}(\lambda) \) - the transmittance of radiation by the optical system; \( S_\lambda(\lambda) \) is the spectral sensitivity of the photodiode, A W⁻¹; \( \varepsilon(\lambda, T) \) - the spectral emissivity of the body; \( \varphi \) - correction factor, characterizing the solid angle of radiation of the body - for a sphere, cylinder, plate, parallelepiped uniformly distributed over the entire radiation surface \( \varphi = 2 \); for bodies radiating into half-space (for example, a model of a completely black body), the specified coefficient is \( \varphi = 1 \); \( L_{BB}(\lambda, T) \) - the spectral energy brightness of an ideal blackbody calculated according to the Planck formula, W m⁻³; \( F \) - the cross-sectional area of the radiation beam at the entrance to the photodiode, m²; \( \lambda_1 \) - the lower boundary of the emission spectrum band; \( \lambda_2 \) - the upper boundary of the emission spectrum band, m; \( \Delta \lambda = \lambda_2 - \lambda_1 \) - the polychromatic radiation bandwidth (bandwidth of the monochromator), m; \( \varepsilon(\lambda, T) \varphi L_{BB}(\lambda, T) \) - the product is the spectral energy brightness of the body.

A similar photocurrent for an ideal absolutely blackbody \( \varepsilon(\lambda, T) = 1 \), according to (1), is equal:

\[
I_{BB} = \int_{\lambda_1}^{\lambda_2} \eta_{OS}(\lambda) S_\lambda(\lambda) \varphi_{BB} L_{BB}(\lambda, T) \, d\lambda .
\]

We take the derivatives of (1), (2) with respect to the wavelength and find their ratio, we obtain:

\[
\frac{dI_F}{d\lambda} \bigg|_{\lambda_1}^{\lambda_2} = \frac{\eta_{OS}(\lambda) S_\lambda(\lambda) \varepsilon(\lambda, T) \varphi L_{BB}(\lambda, T) \, d\lambda}{\eta_{OS}(\lambda) S_\lambda(\lambda) \varphi_{BB} L_{BB}(\lambda, T) \, d\lambda} = \varepsilon(\lambda, T) \frac{\varphi}{\varphi_{BB}} ,
\]

\( \varphi \) - correction factor, characterizing the solid angle of radiation of the absolutely blackbody.

![Figure 2. Dependence of the dimensionless (normalized) photocurrent from normalized wavelength (linear approximation of experimental data): 1 - absolutely blackbody (calculated on relative (2)), \( dI/\lambda^2 = 13.29509 \); 2 – MABB, \( dI/\lambda^2 = 13.2899 \); 3 – temperature lamp, \( dI/\lambda^2 = 5.8679 \).](image-url)
Relation (3) is the measurement equation for the spectral emissivity of an object \( \varepsilon(\lambda, T) \). It follows from (3) that for measuring \( \varepsilon(\lambda, T) \) it is necessary to experimentally find the derivative of the photocurrent with respect to the wavelength for the object under study and relate it to the analogous derivative for an ideal absolutely blackbody, calculated for the same thermodynamic temperature at which the object is located. A detailed justification of the measurement equation and theory are given in [6]. Here, the assumptions made for relation (3) should be noted.

Firstly, an assumption was made on the linear dependence of the spectral energy brightness of an ideal blackbody on wavelength. Indeed, for a fixed stationary thermodynamic body temperature and for a narrow wavelength range, for example, for the range \( \lambda=640–660 \) nm (range width 20 nm), the dependence of the spectral energy brightness of an ideal absolutely blackbody \( L_{BB}(\lambda, T) \) on the wavelength is linear and with high accuracy can be approximated by an equation of the form:

\[
L_{BB}(\lambda, T) = K_{BB} \lambda + B_{1}.
\]

As a proof of this linearity approximation, Fig. 3 shows the dependence of the spectral energy brightness of an ideal absolutely black body \( L_{BB}(\lambda, T) \) on the wavelength calculated by the Planck’s formula for the wavelength range 640–660 nm at a temperature \( T=2000 \) K, and the table shows the calculated values of the coefficient of determination, which characterizes the deviation of the true dependence on the linear one (for different widths of the approximation interval and different temperatures).

| spectrum range (width of the approximation Interval \( \Delta \lambda \text{ nm} \)) | Coefficient of determination, \( R^2 \) |
|---|---|---|
| | \( T = 3000 \) K | \( T = 2000 \) K | \( T = 1000 \) K |
| 646-655 (10 nm) | 0.99994 | 0.99994 | 0.9989 |
| 646-656 (12 nm) | 0.99998 | 0.99992 | 0.9985 |
| 644-654 (8 nm) | 0.99999 | 0.99996 | 0.9993 |
| 646-652 (6 nm) | 0.99999 | 0.99998 | 0.9996 |
| 648-652 (4 nm) | 0.99999 | 0.99999 | 0.9998 |
| 648-650 (2 nm) | 1.00000 | 1.00000 | 1.0000 |

Secondly, the operation of differentiating the integrals (1), (2) with their subsequent assignment to each other also implies a strict linear dependence of the spectral sensitivity \( S(\lambda) \) of the photodiode in a given spectral range and the spectral emissivity \( \varepsilon(\lambda) \) itself.
Under these conditions, the obtained value of $\varepsilon(\lambda)$ is actually considered constant in the wavelength under study and is attributed to the wavelength of the center of the range, in our case, to the wavelength $\lambda_0=656$ nm. The assumption for the spectral sensitivity of the photodiode in a narrow spectral band is absolutely accurate, which is confirmed by the well-known classical dependence of spectral sensitivity on the wavelength, which has the form:

$$S(\lambda) = QED \kappa e/\hbar c,$$

where, $QED$ - the quantum efficiency of the photodiode, there is a constant value for a narrow spectral range (as practice shows, the width is not more than 50 nm), $e$ - the electron charge, $h$ - the Planck’s constant, $c$ - the speed of light, $\lambda$ - the wavelength.

Thirdly, it is believed that the thermodynamic temperature of the MABB, which is equal to the brightness temperature of the object, is known for sure. This, in turn, implies that the measurements use the MABB, in which the spectral emissivity is very close to or almost equal to the emissivity of an ideal blackbody. As the estimates made show, for example, for tungsten, to obtain the values of $\varepsilon(\lambda,T)$ with a relative uncertainty of no worse than $\delta \varepsilon =10^{-3}$, the thermodynamic temperature of the MABB should be known with an error of no worse than $\pm 0.5\%$. For example, for the melting (solidification) temperature of copper ($T_m=1357.78$ K), this corresponds to a deviation of $\Delta T = \pm 6.8$ K, which is quite achievable.

4. Experimental results
As a result of measurements performed at a thermodynamic temperature $T=1357.78$ K and at a central wavelength of $\lambda_0=656$ nm, the following spectral emissivity values were obtained:
- for a tungsten filament ribbon $\varepsilon =5.8679/13.295=0.4413$ (Fig.2). The reference given from [3] is $\varepsilon=0.4428$, the discrepancy is $\delta \varepsilon =\Delta \varepsilon/\varepsilon=(0.4428-0.4413)/0.4428=0.0016=0.15\%$;
- for the model of an absolutely black body BB3500YY $\varepsilon=13.290/13.295=0.9996$ (Fig.2). The reference data from [7] is $\varepsilon=0.998$, the discrepancy with the data of the authors-developers of the MABB is $\delta \varepsilon =\Delta \varepsilon/\varepsilon=(0.9996-0.998)/0.998=0.0016=0.16\%$.

5. Estimation of the error of the method
The estimation of the non-excluded systematic error of the method is carried out according to the general rule for calculating the error in indirect measurements and, as applied to the measurement equation of the method, has the following form:

$$\delta \varepsilon=1.414 \sqrt{(\Delta I/I)^2 + (\Delta \lambda/\lambda)^2},$$

where $\Delta I$ - the uncertainty in measuring the photocurrent, $\Delta \lambda$ - the uncertainty in setting the wavelength, $I$ - the measured photocurrent and $\lambda$ is the wavelength specified by the monochromator. In real measurements for tungsten, the following values of the parameters of relation (5) were obtained: $\Delta I=4 \times 10^{-6}$ A, $\Delta \lambda=0.15$ nm, $I=0.008$ A, $\lambda=656$ nm. Calculation of the non-excluded systematic error for these values gives $\delta \varepsilon=7.8 \times 10^{-4}$ relative units or 0.078%, in absolute units $\Delta \varepsilon=\varepsilon \delta \varepsilon=7.8 \times 10^{-4} \times 0.443=3.5 \times 10^{-4}$.

The standard deviation (SD) observed in the experiments was $\text{SD}=0.15\%$. So, the expanded uncertainty (coverage factor $k=2$) of the proposed method for measuring emissivity is $u_{\varepsilon, \text{SD}}=2\sqrt{\delta \varepsilon^2 + \text{SD}^2} = 0.34\%$.

6. Conclusions
As we established experimentally, the uncertainty of measuring the spectral emission coefficients by the method considered above does not exceed 0.5%. Proceeding from this, it is quite legitimate to assert that the method allows measuring the spectral emissivity with an accuracy close to the reference one and provides the possibility of measurements in a wide temperature and wave range for a wide range of solids, including absolutely black body models for which the applicability of other known methods is limited.
It should be noted that the method considered above, for the first time in world practice, makes it possible to experimentally measure the spectral emissivity of absolutely black body models - this is its uniqueness.

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