CP VIOLATING METASTABLE STATES AND BARYOGENESIS IN THE HOT STANDARD MODEL

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Abstract

We discuss a novel form of CP violation in the standard model. It takes place at temperatures of the order of the electroweak transition, when two regions with different values of the Wilson line are juxtaposed. This CP violation is maximal. A sufficient condition is simply the existence of a long-lived metastable state; this can occur for fewer than three generations, and also in the minimal susy standard model. It leads to baryogenesis in all of these models.

Key-Words: standard model, metastable states, CP violation, baryogenesis

Number of figures: 2

June 1995
CPT-94/P.3099

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One of the outstanding problems in cosmology is the understanding of the observed baryon asymmetry of the universe (BAU) [1]. As the electroweak phase transition is most probably the last occasion during the cooling of the universe at which this asymmetry may have come about [2], it is only natural to try to use the standard model (SM) to explain BAU [2, 3]. The conventional approach involves the consideration just below the critical temperature $T_{ew}$ of the nascent bubbles of normal vacuum where the Higgs expectation value is non-zero. The basic idea is that, as the bubbles expand, the scattering of fermions from the walls differs from that of their CP conjugates as a result of the SM CP violation encoded in the Kobayashi-Maskawa [4] mixing among generations. As a consequence, a net baryon-antibaryon separation builds up. However, actual calculations give conflicting results: while the authors of Ref. [3] concluded that the observed BAU may just be explained within the SM in this way, the authors of Ref. [5] concluded that the separation is orders of magnitude too small. Part of the problem is the smallness of the SM KM CP violation effects. To solve this problem, many authors have proposed variations of the SM with a sufficient amount of microscopic CP violation built in [6].

However, there exists an independent possible source of CP violation in the SM due to thermal effects described by the phase of the Wilson line [7]. This source of CP violation depends on the existence of very long-lived metastable minima of the free energy of the universe as a function of the Wilson line. These metastable minima are not CP self-conjugate states, and so break CP invariance spontaneously. For this source of CP violation to be operational, the only assumption required is that, rather than falling into the absolute minimum, the universe has fallen into one of these long-lived metastable minima at a temperature somewhere below the GUT scale (we should however add the caveat that the thermodynamic admissibility of these metastable states has been questioned [8, 9]). The electroweak transition is supposed to be first order. In this scenario, the bubbles of normal vacuum which nucleate at the electroweak phase transition then have walls that separate two regions not only of the usual ordered and disordered Higgs phase but also of ordered phases parameterized by the Wilson line.

In this letter, we describe both surface and volume effects due to this alternative scenario. It is shown how the juxtaposed ordered phases of the Wilson line lead to the localisation on the wall of fermions, but not their CP conjugates. This static, localised fermion density therefore violates CP invariance maximally. Also, it is argued that the baryon-antibaryon separation current arising from the different interactions with the wall of left- and right-handed fermions leads to a baryon asymmetry of the required order of magnitude. Furthermore, these two effects occur not just in the SM, but in fact are generic effects occurring in any model with sufficiently long-lived CP violating metastable...
states. Such models include the SM with fewer than three generations and the minimal supersymmetric SM. We wish to emphasize that this CP violation is spontaneous and entirely independent of the KM mechanism [4].

The order parameter in hot gauge theories is the Wilson line $\Omega(x)$. It is defined in terms of the imaginary time formulation of $T \neq 0$ field theory as the path-ordered product of gauge group elements along a line in the $\tau$ direction:

$$\Omega(x) = \mathcal{P} \exp i \int_0^\tau V_0(x, \tau) d\tau$$

where $V_0(x, \tau)$ is the $\tau$ component of the vector potential of the theory. In the standard model, the gauge group has SU(3) generators $A_0$ with coupling $g_{st}$, SU(2) generators $A_0$ with coupling $g$ and U(1) generators $B_0$ with coupling $g'$. In this notation we have

$$V_0 \equiv g_{st} A_0 + g A_0 + \frac{1}{6} g' B_0.$$ 

From the transformation properties of the Wilson line we know that only its eigenvalues are gauge-invariant. We are thus lead to the following parametrisation of $V_0$:

$$A_0 = \frac{2\pi T}{g_{st}} \text{diag} \left( \frac{q}{3} + \frac{r}{2} \frac{q}{3} - \frac{r}{2} \frac{2}{3} q \right), \quad A_0 = \frac{2\pi T}{g} \text{diag} \left( \frac{s}{2} - \frac{s}{2} \right), \quad B_0 = \frac{2\pi T}{g'} t.$$ 

The real numbers $q$, $r$, $s$ and $t$ are in the directions of colour hypercharge, colour isospin, weak isospin and weak hypercharge respectively. This parametrisation is not gauge-invariant.

$F$, the free energy divided by the factor $\pi^2 T^4$ to be dimensionless, has been computed as a function of the order parameter in perturbation theory [8]. An interesting feature of $F$ is the occurrence of metastable states. These can be present when the charges of the various particle species are multiples of each other. This is the case for the weak hypercharges in the standard model. And indeed, $F$ in the weak hypercharge direction $(0,0,0,t)$ shows two metastable states $t_1, t_2 \neq 0$ for two and three generations (Fig. 1), with life times that exceed by orders of magnitude the advent of the electroweak transition [7]. These metastable states have distinct CP conjugates, as follows from $V_0$ being a CP-odd quantity. Thus we have “spontaneously broken CP invariance”. The question we wish to address is, how will the spontaneous CP breaking manifest itself at the electroweak transition?

We consider first surface effects. The first order character of the transition will show up in undercooling of the Higgs symmetric phase. Eventually drops of broken phase will form. It is easy to show that a non-zero value of the Higgs field forces the phase of the Wilson line to be either zero or to be in a metastable minimum of the effective potential.
built up in the broken phase \[7\]. For the purpose of the discussion we will assume that the former is true. Fig. 2 shows the wall of the drop, which consists of the Higgs profile between \( \langle H \rangle \neq 0 \) and \( \langle H \rangle = 0 \) and the hypercharge profile between \( t = 0 \) (i.e. \( \Omega = 1 \)) and, say, \( t = t_1 \) (i.e. \( \Omega = \exp i2\pi \frac{1}{6}t_1 \)).

Let us look at the Dirac equation around the wall. The wall is given by the profile, and couples through the \( \tau \) component of the covariant derivative. The fermion fields are antiperiodic. We have

\[
(\gamma_0 D_0(B) + i\gamma_2 \partial_2)\psi(z, \tau) = 0. \tag{4}
\]

The covariant derivative is given by \( D_0(B) = \partial_0 + ig'YB_0(z) \) for a particle with hypercharge \( Y \). Using a chiral representation for the \( \gamma \)'s, \( Y \) acts as \( Y_R \) (\( Y_L \)) on upper (lower) pairs of components of \( \psi \). We have neglected the Higgs contribution, which is allowed for fermions with a mass much less than \( T_{ew} \). By the same token, we can look at a fermion with a given handedness, say the right-handed one. Our gauge choice in Eq. (3) rendered \( B_0 \) \( \tau \)-independent, and hence \( \psi \) can be Fourier analysed in terms of \( \exp i(n - \frac{1}{2})2\pi T, n \) integral. Then the two normalizable solutions of this equation can be written as

\[
\psi_{1,2}(z, \tau) = \left(\exp -\gamma_0 \gamma_2 2\pi T \int^z [Y t(\zeta) - \frac{1}{2}]d\zeta\right)\left(\exp -i\pi T \tau\right)\phi_{1,2} \tag{5}
\]

where \( n = 0 \) and \( \phi_1 = (1, 0, 0, 0)^T, \phi_2 = (0, 0, 0, 1)^T \). Thus, the hypercharge profile localises fermions with wavefunctions proportional to these two spinors. The crucial point is that, if a given fermion species localises, its CP conjugate \( \gamma_0 \gamma_2 \psi^* \) cannot. The residual \( \tau \) dependence in the localised wavefunction disappears in the gauge-invariant combination \( \psi^\dagger \psi \). Thus, the density is static and localised.

We want to point out that the juxtaposition of two ordered phases \( (t = 0 \) and \( t = t_{1,2} \)) is at the root of the localisation process. The Higgs profile cannot possibly localise fermions, as it separates ordered and disordered phases. This is a quite general phenomenon: one needs the order parameter to be non-zero on both sides of the wall in order to localise the fermion.\(^4\) Although the numerical contribution of the localised modes to baryon asymmetry will turn out to be very small, their presence shows clearly the CP violating properties of the wall, and how this CP violation is maximal.

In the context of the strong interactions alone, similar modes have been found \[12\]. These modes break C invariance. But their existence has no physical consequences, since

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\(^4\)Of course, in the seminal work of Jackiw and C. Rebbi \[10\] the Higgs wall separated ordered phases, and they found localised modes that were C eigenstates. As in that case, one may think here of possible applications in solid state physics. Physically, there is a comparison between the localised modes and the “surfactants” (amphiphiles) of molecular surface physics (see ref. \[11\]). These surfactants lower the surface tension.
the strong interaction metastable point \[13\] destabilises in the presence of the electroweak forces \[7\].

We now turn to volume effects. We will follow the procedure as laid out in refs. \[2, 3\]. The CP violating wall gives rise to a baryonic separation current \(J_{CP}\). This current sends a net flow of CP conjugate fermions into the unbroken phase, where baryon non-conservation \[14\] is still effective through the sphaleron activity \(f(\rho)\). Inside the bubble the sphaleron activity is frozen. The net baryon number \(n_B\) is therefore given by \(n_B = -J_{CP}f(\rho)\).

The net flow of antiparticles into the unbroken phase comes from comparing the reflection from the wall of particles and antiparticles from the outside of the drop, and from comparing the transmission from the inside to the outside of the drop. We work in a one dimensional picture\[4\] and obtain in the wall restframe \[3\]

\[
\begin{align*}
n_B &= \frac{1}{3} \int_{0}^{\infty} \frac{d\omega}{2\pi} n_{L}^{v}(\omega) \left( -R_{LR}^{\dagger}R_{LR} + R_{RL}^{\dagger}R_{RL} \right) f(\rho) + L \leftrightarrow R \\
&= \frac{1}{3} \int_{0}^{\infty} \frac{d\omega}{2\pi} \left( n_{L}^{v}(\omega) - n_{R}^{v}(\omega) \right) R_{RL}^{\dagger}R_{RL} f(\rho).
\end{align*}
\]

(6)

Here \(R_{LR}\) is the reflection amplitude for right-handed into left-handed quarks, scattering off the wall from the unbroken phase, \(R_{RL}\) is that of their CP conjugates and \(n_{L}^{v}(\omega)\) is the equilibrium distribution for the left-handed quarks in the wall rest frame, \(n_{L}^{v}(\omega) = n[\gamma(\omega + \vec{v}.\vec{p}_{L})]\). Eq. \[3\] follows from CPT invariance, which relates amplitudes for particles to amplitudes for CP conjugates with left and right exchanged.

As is well known \[3, 5\] the contribution to \(n_B\) from transmitted quarks is related by unitarity and CPT invariance to the same expression Eq. \[6\] only with \(v\) changed into \(-v\). For small enough \(v\) one expands in \(v\) and retains as lowest order contribution the quadratic term. One then observes \[3, 5\] that at one loop order the left- and right-handed quasiparticles have different dispersion relations due to their different interactions with the weak vector bosons. These dispersion relations are easily obtained in the presence of the vev in the metastable phase and are slightly shifted \[15\]. It then follows that the integration over \(\omega\) in Eq. \[3\] gives a result of the order of \(\alpha_W T\), as in the standard treatment \[3, 5\].

Now we turn to the difference of matrix elements in Eq. \[4\]. Our CP violation is spontaneous, and not subject to the GIM mechanism \[16\]. From Fig. 1 we see that the two generation standard model has a metastable state, and it easily survives until the electroweak transition. Decoupling one more generation leaves us with an even more pronounced dip, and an even longer life time. So our CP violation does not require scattering of one quark generation into another one.

\[5\]The phase space difference between one and three dimensions is not subject to suppression factors \((m_s/T)^2\) as in the conventional approach.
Hence we can draw three important conclusions: first, from the discussion of the localised mode we learned that baryon-antibaryon separation is of order one and hence we gain a factor $10^{-5}$ in comparison to the traditional CP violation in the standard model \[3\]; second, the typical momentum of the quarks in Eq. 6 is of order $T \leq T_{ew}$ (not of the order of the strange quark mass as with KM CP violation \[3\]), resulting in a gain of another factor of $m_s/T \approx 10^{-3}$; third, the decoherence mechanism of Gavela et al. \[3\] is based precisely on multiple scattering between generations with a “coherence length” of order $(120 \text{ Gev})^{-1}$, and so does not apply to quarks with momentum of order $T \leq T_{ew}$.

Lastly we discuss the sphaleron efficiency factor $f$ in the metastable, symmetric phase. Its value depends on the typical number $\rho$ of sphaleron transitions felt by the quark \[3\]. This is given by the combination $\rho = 3D_B \Gamma / v^2$ of the diffusion length $D_B$ of the quarks, the sphaleron rate $\Gamma = 9\Gamma_{sph}/T^3$ and the velocity $v$ of the wall, all in the symmetric metastable phase\[3\].

The metastable phase has $SU(2) \times U(1)$ unbroken, like the stable symmetric phase, since the Higgs vev vanishes. The non-zero vev of the weak hypercharge potential $B_0$ does not introduce any other mass scales, even in the zero Matsubara frequency sector. Had the vev had a component in, say, the weak isospin Lie algebra $A_0$, then the fourth component of the vector potential would couple as a “Higgs scalar” to the spatial components of the gauge fields in that sector and its vev would have generated a mass. Obviously this does not happen for the abelian component. Hence one expects no order of magnitude difference in the sphaleron rates $\Gamma_{sph}$ in the metastable and stable symmetric Higgs phases, i.e. $\Gamma_{sph} = (10^{-2} - 1)(\alpha_W T)^4 [17]$.

However the diffusion lengths and wall velocity may be much larger in the metastable phase because of the higher free energy and lower pressure there. The calculation of the sphaleron activity was done in the thin wall approximation \[3\]. However, the wall has a typical width \[18\] given by the Debye screening length $(g'T)^{-1}$, so necessitates a sufficiently large diffusion length to guarantee that the quarks penetrate deep enough into the symmetric phase to feel the sphaleron activity. For quarks the diffusion length is determined by the strong interactions and is estimated to be $(4 - 5)/T \leq 3$ in the stable phase, which is of the order of the thickness of the wall. The uncertainty in the parameter $\rho$ is therefore larger than its counterpart in the stable phase, leading to a sphaleron activity $f(\rho) \sim 10^{-5} - 1$.

Thus we find a ratio

$$\eta \equiv \frac{n_B}{s} \sim 10^{-12} - 10^{-7}$$

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6 This dependence is slightly changed with respect to the corresponding one in Ref. \[3\], because our CP violation extends also to the lepton sector.
where $s$ is the 1-d entropy density taken at the time of the phase transition, $s = \frac{73}{4} \pi T$. The first figure for $\eta$ is determined essentially by the lower limit on the sphaleron activity, the second figure by the weak coupling $\alpha_W$ due to the difference in left- and right-handed dispersion relations, the sphaleron rate and the number of massless species at $T_{ew}$. However $s$ increases due to reheating by an order of magnitude at least. So our ratio diminishes by that same order of magnitude.

Where bubbles coalesce there is a very small matter contrast due to the zero mode condensate travelling with the wall. The radius of the bubbles at coalescence time is of order $R \sim 10^{12}/T$, so the ratio of the contribution of the zero modes $\eta_{zm}$ to the ratio Eq. 7 is of order $10^{-12}$, hence a very small contrast.

What if the standard model turns out to be incomplete? In Fig. 1, we have plotted its minimal SUSY version as well. It is to a very good approximation just twice as high. The life times of the metastable states are sufficiently long [15], so our mechanism works again.

In summary, once part of the universe has fallen into a metastable state there is baryogenesis of the above order of magnitude, both for the standard model and its minimal SUSY extension, consistent with the experimental number $\eta = (4 - 6) \times 10^{-10}$ [2].

The metastable states are possible in any theory with a non-trivial center in the gauge group. If they have a large enough life time, CP violation will be of order one at $T_{ew}$, and baryogenesis will be a generic feature of such theories.

How can part of the universe “fall” into the metastable state? The answer is that GUT theories have metastable states as well (we have checked this for SU(5) and SUSY-SU(5)). If we assume that these metastable states are realized at the Planck scale, then it is energetically possible to fall at $T_{GUT}$ into $t_1$ or $t_2$. Of course the question is then pushed to the Planck scale, where, without any microscopic CP violation, the universe would be divided into equal numbers of causally disconnected regions of conjugate minima. It may be that microscopic CP violation plays a role in the choice of metastable state, or in how the various metastable states develop.

One of us (C.P.K.A.) wishes to acknowledge the Niels Bohr Institute and the Danish Research Council for hospitality and for support, the other (N.J.W.) to acknowledge the financial support of EC grant ERB4001GT933989. The comments and criticisms of Rob Pisarski, Kimyeong Lee, Holger Nielsen, Vladimir Eletsky, Owe Philipsen, Vadim Kuzmin, Valerie Rubakov and Misha Shaposhnikov have been very useful.
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Figure Captions

Fig. 1. The normalized free energy $F$ in the weak hypercharge direction $(0,0,0,t)$. The continuous line is the SM with three generations, the dashed line the SM with two generations and the dot-dash line the MSSM. The Planck free energy $F(0,0,0,0)$ is set to zero.

Fig. 2. The profile of $t \equiv (g'/2\pi T)B_0$ and the Higgs vacuum expectation value in the $z$ spatial direction, perpendicular to the bubble wall.
Figure 2
