Calculations on the Threshold Anomaly of weakly bound projectiles with São Paulo and Woods-Saxon polarization potentials

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Abstract. A thorough study of the energy dependence of the nuclear optical potential in reactions involving the weakly bound projectiles ⁸B, ⁷Be and ⁶Li on the target ⁵⁸Ni and ⁹Be on ²⁷Al is carried out by performing a χ²-analysis of recent measurements of elastic scattering cross sections for energies around and above the Coulomb barrier. For this purpose two different potential types are used: the double folding São Paulo potential and the Woods-Saxon potential. The calculations performed for the energy dependence of the real and imaginary parts of the polarization potentials show that these potentials besides satisfying the dispersion relation, for some nuclear systems the uncertainties on the energy dependence of the polarization potentials allow to conclude that these systems present a behavior consistent with the Breakup Threshold Anomaly. In other cases, due to the large uncertainties, it is not possible to make a definitive conclusion about the anomalies.

1. Introduction

Lately, special interest has been focused on the study of the threshold anomaly (TA) at energies around the Coulomb barrier in reactions involving weakly bound projectiles. This interest is due to the fact that the study of the energy dependence of the nuclear optical potential at near barrier energies is one of the tools to investigate the influence of the breakup, nucleon transfer and other reaction mechanisms upon the elastic and fusion channels for reactions with small breakup threshold energies. The TA, usually present in reactions with heavy nuclei, implies a peculiar behavior of the real and imaginary parts of the optical potential around the Coulomb barrier. Both the real and imaginary parts of the optical potential are nearly energy independent at energies well above the Coulomb barrier, but as the energy decreases towards the barrier, the imaginary part W(E) sharply decreases while the real part V(E) presents a localized peak. This correlated behavior is attributed to a strong coupling between the elastic channel and other reaction mechanisms. As the collision energy is lowered, reaction channels such as inelastic scattering, nucleon or cluster transfer, fusion and fission become effectively less important.
the other hand, reaction mechanisms may produce real polarization potentials of different signs, attractive or repulsive, that dynamically decrease or increase the Coulomb barrier [1, 2]. The net real polarization potential depends on the importance and strength of the different specific polarization potentials. This threshold link between the real and imaginary components of the polarization potential is connected by the dispersion relation which has been shown to be a manifestation of the principle of causality [3, 4, 5]. For the case of tightly bound nuclei, for which the most important channel couplings are inelastic excitations of projectile (target) or cluster (nucleon) transfer, the net real polarization potential results attractive at energies near and below the nominal Coulomb barrier. Consequently, these nuclei show enhancement of the fusion cross section at sub-barrier energies when compared to predictions from one dimensional barrier penetration models.

The situation may be different when weakly bound nuclei are involved, due to the strong coupling between the elastic and the breakup channel [6], even at energies well below the Coulomb barrier. As a consequence, the breakup channel remains open to account for the observed large break-up cross sections that overcome the fusion cross section at this energy regime [7, 8]. Accordingly, part of the absorptive imaginary potential $W(E)$ must account for the non-vanishing breakup process and this potential does not vanish at sub-barrier energies. Recent studies suggest that the usual TA is not longer present when weakly bound nuclei are involved [9, 10]. The reason is that as the energy is lowered towards the barrier, a net repulsive polarization potential is produced by the coupling to the continuum breakup states. Accordingly, part of the absorptive imaginary potential $W(E)$ keeps increasing or at least does not decrease. This effect has recently been observed for several systems involving weakly bound stable projectiles like $^6$Li and $^9$Be [11, 12]. The conjugate behavior of the real and imaginary polarization potentials is known as the Breakup Threshold Anomaly (BTA) [11]. The BTA has also been observed in the nuclear optical potential of the halo $^6$He projectile impinging on heavy targets $^{209}$Bi [13] and $^{208}$Pb [14] at near barrier energies.

Recently, elastic scattering measurements of $^8$B, its core $^7$Be and $^6$Li on the target $^{58}$Ni have been performed at energies near and well below the Coulomb barrier [15]. Regarding the system $^6$Li + $^{58}$Ni, other elastic scattering measurements for energies above the barrier have been carried out by Pfeiffer et al [16]. Also, for the $^6$Li scattering, the recent work by Biswas et al [17] suggests dubious conclusions about the presence of the TA. The purpose of the present work is to investigate whether the usual TA or the BTA occurs for this weakly bound projectile $^6$Li, as well as for the proton halo projectile $^8$B and its core nucleus $^7$Be reacting on the medium size target $^{58}$Ni. Also, the reaction of the weakly bound nucleus $^9$Be with the target $^{27}$Al will be analyzed by means of the polarization potentials as described in the last paragraph. In this case, the analysis will be performed by using the elastic scattering angular distributions and fusion cross sections of Refs. [18, 19]. In order to make a simultaneous fitting of elastic and fusion data, in the case of the Woods-Saxon polarization potential, the imaginary part is split as $W(E, r) = W_F(E, r) + W_{DR}(E, r)$. Being, $W_F$ responsible for fusion reactions and $W_{DR}$ for direct reactions as it will be described below.

Thus, the basic purpose of the present work is to determine by the use of two independent potentials whether or not the threshold anomaly or breakup threshold anomaly appears. That is, in order to obtain potential independent conclusions about the presence of the TA or BTA, we analyze the energy dependence of the real and imaginary parts of the dynamic polarization potential for energies above and below the barrier by two different types of potentials: the double-folding S˜ao Paulo potential (SPP) and the Woods-Saxon potential. The barrier energy $V_B$ for the different nuclear systems is calculated with barrier parameters given by the SPP. The S˜ao Paulo potential has been widely studied with success in the last years for a variety of systems and reaction mechanisms, including scattering and fusion of weakly bound nuclei, fusion and quasi-elastic barrier distributions and CDCC calculations [20, 21, 22, 23]. In this model the
barrier parameters are obtained by approximating the barrier by a parabola at distances near the maximum of the Coulomb and nuclear potentials. The ECIS94 code [24] has been used to perform the $\chi^2$ analyses with the SPP, whereas the JUPITOR code that has been successfully used for many nuclear systems [25], was adopted in the analyses using Woods-Saxon potential. A very brief theoretical description of these potentials is given in section II. The optical potential parameters adequate for each system and for each energy are extracted by fitting the elastic scattering angular distributions for each reaction. The details of this fitting procedure as well as the calculations will be given in the next sections. Finally, we present a short summary and final conclusions.

2. Theoretical Framework
The Hamiltonian $H$ for the nuclear system is of the form,

$$H_a = T_a + V_a,$$

(1)

where the distorted wave $\chi_a^{(+)}$ in the incident elastic channel $a$ satisfies the expression,

$$(T_a + V_a)\chi_a^{(+)} = E_a\chi_a^{(+)}.$$

(2)

The potential $V_a$ is defined by,

$$V(r, E) = V_{Coul}(r) - U_N(r, E),$$

(3)

where the subindex $a$ has been suppressed. Here, $V_{Coul}(r)$ is the Coulomb potential and $U_N(r, E)$ is the energy dependent nuclear potential which is the sum of a bare potential with a slow energy dependence, plus the dynamic polarization potential which may have rapid variations at near barrier energies.

For the nuclear potential, different approaches can be used. As for the bare potential the best physical approach might be to use a double-folding potential for the real part with realistic matter densities and any reasonable approach for the imaginary part. This imaginary part can be constructed with the same radial dependence as the real part (double-folding potential) with a strength coefficient obtained at energies well above the barrier. However, when one studies the optical potential at near barrier energies, strength coefficients are needed to account for dynamic polarizations. A normalized version of the São Paulo optical potential [26, 27] reads as,

$$U_{SP}(r, E) = [N_R(E) + iN_I(E)]F(r, E),$$

(4)

where $F(r, E)$ is the double-folding potential

$$F(r, E) = V_F(r)e^{-4v^2/c^2}.$$

(5)

Here $c$ is the speed of light, $v$ the relative velocity of the colliding ions given by

$$v^2(r, E) = \frac{2}{\mu}[E - V_{Coul}(r) - U_N(r, E)],$$

(6)

and $V_F(r)$ is the folding potential given in terms of the matter densities of the ions

$$V_F(r) = \int \rho(r_1)\rho(r_2)V_0\delta(r - r_1 + r_2)dr_1dr_2.$$

(7)

A zero-range approximation is assumed with strength $V_0 = -456$ MeV-fm$^3$. In order to provide a parameter-free description of the interaction an extensive systematics of nuclear densities
has been proposed by L.C. Chamon [27]. It has been found within the two-parameter Fermi distribution approach to describe the densities that the radii of the distributions are given by,

$$R_0 = 1.31 A^{1/3} - 0.84,$$

where \( A \) is the number of nucleons in the nucleus. The systematics also shows that the matter diffuseness \( a \) of the distributions has an average value \( a = 0.56 \) fm throughout the periodic table. The normalization of the double-folding potential \( e^{-a^2/r^2} \) arising from the Pauli non-locality due to exchange of the nucleons vanishes at near barrier energies, and therefore the S\( \tilde{a} \)o Paulo potential becomes an usual double-folding potential.

The energy dependent normalization parameters \( N_R(E) \) and \( N_I(E) \) of Eq. (4) take into account the effects of the dynamic polarization potentials (DPP) arising from direct channel couplings. The real and imaginary parts of the DPPs should be related by the dispersion relation [11]

$$N_R(E) = N_{R0} + \Delta N_R(E),$$

while

$$\Delta N_R(E) = \frac{1}{\pi} \mathcal{P} \int_0^\infty \frac{N_I(E')}{E' - E} dE'.$$

\( N_{R0} \) is energy independent and is determined at some high enough energy for which the values of \( \Delta N_R(E) \) and \( N_I \) are known. The energy dependence of the parameters \( N_R \) and \( N_I \) are calculated by fitting the elastic scattering angular distributions.

In order to obtain model independent conclusions about the threshold anomaly, a combined use of different approaches for determining the energy dependence of the optical potential is desirable. For this reason, we also use Woods-Saxon shape polarization potentials to calculate the energy dependence of such nuclear polarization potential [28, 29]. In this case, the total interaction between projectile and target of Eq. (2) is

$$V(r, E) = V_{Coul}(r) - V_0(r) - U_N(r, E),$$

where \( V_0(r) \) is the energy independent average nuclear potential and \( U_N(r, E) \) is the nuclear dynamic polarization potential given by

$$U_N(r, E) = V(r, E) + iW(r, E).$$

The strength of the real polarization potential \( V(E) \) is connected to the imaginary part \( W(E) \) by the dispersion relation [3, 4, 5]

$$V(E) = V(E_s) + (E - E_s) \frac{1}{\pi} \mathcal{P} \int_0^\infty \frac{W(r, E')}{(E' - E_s)(E' - E)} dE',$$

where \( V(E_s) \) is the value of the potential at the reference energy \( E_s \). The energy independent average nuclear potential \( V_0(r) \) and the absorption potential \( W(r, E) \) are assumed to have volume geometrical forms. The radial dependence of \( V(r, E) \) will be assumed to be the same as that for \( W(r, E) \) with the same radial and diffuseness parameters. Therefore, the strengths \( W(E) \) and \( V(E) \) and the corresponding radial and diffuseness parameters will be extracted from the \( \chi^2 \)-analysis of the elastic angular distribution data as it will be shown in the next section.

For the nuclear systems under study, the energy variation of the parameters \( N_R(E) \) and \( N_I(E) \) of the SPP is thus studied as the collision energy approaches the Coulomb barrier. These parameters are obtained by fitting the experimental elastic scattering angular distributions for each system. On the other hand, the same energy dependence is searched for \( V(E) \) and \( W(E) \) of the Woods-Saxon potential whose optical potential parameters are also determined by a \( \chi^2 \)-fitting of the data. Conclusions about the presence of the usual Threshold Anomaly or the Breakup Threshold Anomaly can be drawn from the energy dependence of these functions. All of these calculations are presented in detail in the next section.
Figure 1. (a) Real $N_R$ parameter of the polarization potential as fitted with the SPP (squares) and the dispersion relation calculation as obtained from the $a$ and $b$ values. (b) Imaginary $N_I$ values as fitted with the SPP (squares) using the data of [15].

3. The Threshold Anomaly Investigation
The recently published data of Aguilera et al [15] have been used for the $^7$Be, $^6$Li and $^8$B with $^{58}$Ni systems. Fig. 1 shows the values of the $N_R$ and $N_I$ parameters (filled squares) that best fit the elastic experimental data.

The lines in Fig. 1 represent possible behaviors of $N_R(E)$ and $N_I(E)$ that are compatible with the dispersion relation. Different behaviors of $N_I$ can be used in the dispersion relation that follow the “data” obtained from the $\chi^2$-fitting of the elastic angular distributions. The possible linear representations of $N_I$ are given by,

$$ N_I = \begin{cases} 
0, & E_{c.m.} < E_1 \\
\frac{a(E - E_1)}{E_1 - E_2}, & E_1 < E < E_2 \\
\frac{a(E_2 - E_1) + b(E - E_2)}{E_2 - E_3}, & E_2 < E < E_3 \\
\frac{a(E_2 - E_1) + b(E_3 - E_2)}{E_3 - E_1}, & E > E_3 \\
N_\infty & E > E_3
\end{cases} $$

The integration of these linear forms have been explicitly given elsewhere [11]. The solid line of Fig. 1a is the result for $N_R(E)$ obtained from Eq. (9) corresponding to the linear fits of Fig. 1b where $E_1 = 12.45$ MeV, $E_2 = 13.4$ MeV and $E_3 = 16.7$ MeV and the $a$ and $b$ values shown in Fig. 1b.

There is a big uncertainty in the determination of the values for $E_1$, $E_2$ and $E_3$. Since, for the nuclear systems under study there are no experimental data in the low energy region to support any extrapolation, thus for this energy region there are several possibilities for the behavior of the imaginary potential. The only thing that we can say is that below a certain energy, the total reaction cross section must vanish and so does the imaginary potential. From the systematic behavior established by Stelson et al [30], the quantity, $S = \sqrt{E_{c.m.} \sigma}$ is very close to a linear behavior at low energies, thus it can be represented by $S = \alpha(E - E_{thre})$, $\alpha$ being the slope of
Figure 2. Energy dependence of the Woods-Saxon real and imaginary polarization potentials at the strong absorption radius $R_{sa}$ for the system $^{7}$Be + $^{58}$Ni. The data for this system correspond to [15].

the linear function and $E_{thre}$ the threshold energy where $S$ or $\sigma$ become zero. Although, this systematics has been derived from reactions with tightly bound stable nuclei, for the case of weakly bound systems, it can give us an upper limit for the threshold energy. So, the threshold energy $E_1$ can be determined in this way. On the other hand, $E_2$ is chosen close to the first data point and $E_3$ in the region where the values of the imaginary potential $N_I$ seem to stabilize.

The uncertainties of $N_R$ and $N_I$ shown in Fig. 1 have been obtained by calculating the range where these quantities could change by varying the $\chi^2/N$-value in one unity relative to the corresponding minimum value. As observed in Fig. 1b, the error bars of $N_I$ for the lowest energies (up to 3 MeV below the barrier) are too large, not allowing us to draw a definitive conclusion about the presence of the BTA. The reasons for the large uncertainties obtained are possibly due to the poor statistics of the data for this nuclear system, as usually happens when radioactive beams are used, and also to the fact that the SPP has only two normalization parameters with which the data are fitted. The Woods-Saxon potential should give more and the SPP has only two normalization parameters. The Woods-Saxon potential should give more reliable results in such a situation, although both potentials lead to the same conclusions and similar uncertainties when stable projectiles are used [1, 12, 31, 32].

Now, we analyze the same data by using Woods-Saxon potentials. The average nuclear potential used in these calculations is $V_0 = 120$ MeV, $a_0 = 0.56$ fm and $r_0 = 1.2$ fm, these parameters are obtained from the angular distribution at the highest energy. The reduced radius of the imaginary polarization potential has been fixed at 1.4 fm, since the calculated values from the $\chi^2$-analysis become around this value. Therefore, only the strengths of the polarization potential and the diffuseness $a_W$ are fitted. Fig.2 shows the calculations for the polarization potentials $V(E)$ and $W(E)$ at the strong absorption radius $R_{sa}$.

The dashed line in Fig. 2 is the dispersion relation calculation corresponding to the linear
Figure 3. (a) Real $N_R$ parameter of the polarization potential as fitted with the SPP (circles and stars) and the dispersion relation calculation corresponding to the $a$ and $b$ values. (b) Imaginary $N_I$ values as fitted with the SPP (circles and stars). The data represented by circles are taken from [15] and those represented by stars from [16].
Figure 4. Energy dependence of the Woods-Saxon real and imaginary polarization potentials at the strong absorption radius $R_{sa}$ for the system $^6\text{Li} + ^{58}\text{Ni}$. (color online) The data represented with black squares correspond to [15] while those with blue squares to [16].

up. This tells us that an important reaction channel should remain open for energies well below $V_B$, that is the breakup channel corresponding to the $^6\text{Li} \rightarrow d + \alpha$ channel with small breakup threshold energy of 1.47 MeV. The three lines of Fig. 3a are the results of the dispersion relation integration corresponding to the linear representations of $N_I$ of Eq. (14) with, $E_1 = 7.9$ MeV, $E_2 = 9.5$ MeV, $E_3 = 14.0$ MeV. The anomalous behavior of the imaginary potential at sub-barrier energies corresponds to the presence of the BTA. We would like to mention that although the BTA behavior is usually attributed to break-up, it may be also contain contributions from transfer of nucleons.

As for the previous system, we also performed the analysis using Woods-Saxon potential. An energy independent average nuclear potential $V_0 = 102$ MeV, $r_0 = 1.3$ fm and $a_0 = 0.513$ fm has been obtained by fitting the elastic scattering angular distribution at the highest energy. As for the imaginary potential $W$, we set $r_W$ at 1.3 fm, thus, only the strengths $V$, $W$ and the diffuseness $a_W$ are fitted in the analysis. Fig. 4 shows the results for the energy dependence of the polarization potentials $V$ and $W$.

The error bars have, as before, been calculated by requiring $\chi^2_{\text{min}}/N$ to vary in one unit i.e., $[(\chi^2_{\text{min}}/N) + 1]$. One can observe that $W(E)$ becomes rather flat for most of the energies while $V(E)$ becomes repulsive and follows well the dispersion relation calculation shown by the line in Fig. 4a. In the dispersion relation, we have set the reference energy at $E_s = 9.0$ MeV with reference energy $V(E_s) = 0.1$ MeV. Similar to the analysis using double folding potential (SPP), $W(E)$ does not decrease around the barrier. Indeed, it maintains an almost constant value for most of the energy interval and has a slightly increase only for the lowest energies. As it keeps non-decreasing values around the barrier, this is a manifestation of the presence of the BTA for this system.

It has been recently shown that for neutron halo projectiles, like $^6\text{He}$, the use of double
Real (a) and imaginary (b) Woods-Saxon energy dependent strengths of the polarization potential for the system $^8\text{B} + ^{58}\text{Ni}$ at the strong absorption radius $R_{sa}$. Elastic scattering angular distributions of [15] have been used in the fitting analysis.

Folding potential with strength coefficients for the real and imaginary parts is not a suitable method to simulate the polarization potentials at near barrier energies [33]. Therefore, we do not attempt to investigate the presence of BTA for the proton-halo projectile $^8\text{B}$ on $^{58}\text{Ni}$ system using the double-folding potential, but rather only use the Woods-Saxon potential. We calculate $V(r, E)$ and $W(r, E)$ of Eq. (13) at the strong absorption radius $R_{sa}$. The parameter of the energy independent potential $V_0(r)$ are set as $V_0 = 150$ MeV, $a_0 = 0.56$ fm and $r_0 = 1.2$ fm. These values fit the elastic angular distribution at the highest energy. The parameters of the absorption potential $W(r, E)$ and the strength $V(E)$ are then determined by the $\chi^2$-analysis. In preliminary calculations we have found that the calculated reduced radius $r_I$ becomes around 1.4 fm. So, in the following calculations we have fixed this value. Fig. 5 shows the resulting $V(E)$ and $W(E)$ at the strong absorption radius $R_{sa}$.

In Fig. 5a, $V(E)$ has been normalized at the values $E_s = 17$ MeV, $V(E_s) = 0.92$ MeV and the lines shown correspond to the dispersion relation integration of the linear fits to the extracted potentials $W(E)$ shown by the full circles in Fig. 5b. Now, for this system $^8\text{B} + ^{58}\text{Ni}$, there is only one experimental point at sub-barrier energies to support these calculations, since measurements of elastic scattering at this energy regime are rather difficult and the cross sections show only small deviations from the Rutherford cross section. However from Fig. 5b, it seems clear to us that $W(E)$ does not start to decrease at the barrier energy $V_B$. On the other hand, the results of the integration of the dispersion relation are compatible with the results obtained from the $\chi^2$-fitting including the lowest energy value for $V(E)$ with large uncertainty. In spite of the fact that there are very few data for energies below the barrier $V_B$, the results of this calculation show that the conjugated behavior of $V(E)$ and $W(E)$ around $V_B$ indicate that the BTA is present for the system $^8\text{B} + ^{58}\text{Ni}$.

The calculations for the system $^9\text{Be} + ^{27}\text{Al}$ with the São Paulo potential $U_{SP}(r, E)$ are...
Figure 6. Real $N_R$ and imaginary $N_I$ parameters of the SSP for the reaction $^9$Be + $^{27}$Al. These parameters have been obtained from the best fit to the angular distributions of elastic scattering data of [18].

shown in Fig. 6. Elastic scattering angular distributions are those of [18], where there are not measurements for energies below the barrier. $N_R$ and $N_I$ are the normalization parameters of the real and imaginary parts of the SPP that are determined in the $\chi^2$-fitting to the data. Although the values of $N_I$ show some anomalous increase at two energies near but above the barrier, it is clear that the values of $N_I$ do not decrease as the energy approaches the barrier $V_B$ from high energies. On the other side, as the energy decreases from high energies, the values of $N_R$ show an increasing strength but precisely at the closest energies to $V_B$ this strength decreases. This might be an indication that just around the barrier, the energy variation of $N_R$ manifests a contribution from a repulsive potential that arises from coupling to the breakup channel of $^9$Be.

The calculations for this system $^9$Be + $^{27}$Al with the Woods-Saxon polarization potentials for the same elastic scattering data are shown in Fig. 7. In this case a simultaneous fitting of elastic scattering with fusion data is performed, where total fusion data are taken from [19]. The polarization potentials $V(r, E)$ and $W(r, E)$ of Eq. (12) are split into fusion parts $V_F$, $W_F$ and direct reaction parts $V_{DR}$, $W_{DR}$ respectively. The parameters of these potentials are determined by fitting the fusion $\sigma_F$ and direct reaction $\sigma_{DR} = \sigma_R - \sigma_F$ cross section data. It is assumed that for this nuclear system, incomplete fusion is negligible as compared to complete fusion in the whole range of energies considered in this work. Also, fusion data at $E_{c.m.}$ = 13.5, 18.75, 21.0, 24.0, 24.75, 26.25, 30.0 and 32.2 MeV are those values obtained from a barrier penetration model calculation with parameters $V_B$ = 8.05 MeV, $h\omega$ = 5.1 MeV and $R_B$ = 8.39 fm. The energy independent nuclear potential has parameters $V_0$ = 60 MeV, $r_0$ = 1.1 fm and $a_0$ = 0.52 fm. Regarding the parameters of the imaginary fusion $W_F$ and direct reaction $W_{DR}$ potentials, we set $r_F$ and $r_{DR}$ at 1.4 fm and 1.8 fm respectively. So, the strengths $W_F(E)$, $W_{DR}(E)$ and the diffusenesses $\alpha_F$ and $\alpha_{DR}$ are calculated by the $\chi^2$-fitting procedure. As seen in Fig. 7, the values of $V_F(E)$ and $W_F(E)$ show that the fusion potential satisfies the usual threshold anomaly. Also, the values of $W_{DR}(E)$ dominate over $W_F(E)$ at all energies but oscillate very strongly throughout the
Figure 7. Real and imaginary Woods-Saxon energy dependent strengths of the polarization potential at the strong absorption radius $R_{sa}$ for the $^9\text{Be} + ^{27}\text{Al}$ system.

energy range of the data. Evidently, the total absorption potential $W(E) = W_F(E) + W_{DR}(E)$ is also affected by these oscillations. From these results, we can not draw any conclusion about the energy behavior of the polarization potentials and therefore about the usual or Breakup Threshold Anomaly. It is possible that the origin of these strong variations on $W_{DR}$ might be related to a) the values used for the fusion cross section at the energies where there are no measurements and that were obtained from a barrier penetration model calculation, or b) that there are some problems with the elastic scattering angular distribution measurements.

4. Summary and conclusions

We have analyzed elastic scattering angular distributions for the weakly bound systems, $^6\text{Li}$, $^7\text{Be}$, $^8\text{B}$ + $^{58}\text{Ni}$ at energies around and above the Coulomb barrier. Also, a simultaneous fitting of elastic scattering and fusion cross section data was performed for $^9\text{Be} + ^{27}\text{Al}$. The presence of the usual TA, observed for tightly bound systems or the BTA found in some weakly bound systems was investigated through two different approaches, namely, the double folding São Paulo potential and the Woods-Saxon potential. In particular, the energy dependence of the real and imaginary parts of the polarization potentials was calculated around the barrier energy for each system. In this way a potential independent conclusion about the TA or Breakup Threshold Anomaly can be achieved for each system. For some of the systems studied, the determination of the TA or BTA was complicated by the large uncertainties of the real and imaginary energy dependent polarization potentials, particularly when the SPP was used. The calculations of $N_R$ and $N_I$ for $^7\text{Be} + ^{58}\text{Ni}$ with the SPP show large uncertainties that is not possible to arrive to a definite conclusion about the presence of the BTA. However, the calculation with the Woods-Saxon potential presents less uncertainties so as to conclude that for this system the BTA shows up. Both calculations agree in that the system $^6\text{Li} + ^{58}\text{Ni}$ shows the BTA. Since the SPP can not be applied to systems with halo projectiles the calculation for $^8\text{B} + ^{58}\text{Ni}$ was performed only.
with the Woods-Saxon potential for which the BTA was found. A definite conclusion about the Threshold Anomalies was not possible to make for $^9$Be + $^{27}$Al system with any polarization potential. The difficulties in establishing the presence of the TA or BTA are basically due to the strong variations of the energy dependent polarization potentials $N_R$ and $N_I$ of the SPP as well as the total imaginary part $W(E)$ of the Woods-Saxon potential for energies above the Coulomb barrier. In the latter case, although the direct reaction absorption potential $W_{DR}$ dominates over the fusion potential $W_F$ in the energy region studied. The oscillations of $W_{DR}$ are such that is not possible to establish a certain energy behavior. This is further complicated by the absence of data for energies below the barrier $V_B$. Certainly, some measurements below the barrier energy and a better statistics above the barrier would allow us to arrive to better conclusions regarding the TA or BTA for the system $^9$Be + $^{27}$Al.

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