Features of a mathematical model of heat transfer in a vacuum resistance furnace

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Abstract. The paper presents a mathematical model of heat transfer developed for the purposes of modeling the operation process in large-scale vacuum furnaces. It has been implemented on the basis of the author's software complex “SigmaFlow”.

1. Introduction

The object of the study is a vacuum electric resistance furnace with a working space volume of 14.7 m³, having ohmic heating elements, lining and internal equipment made of carbon materials. Inside the furnace there are two retorts – external and internal (figure 1). Within the second retort, the main technological process takes place.

The characteristic time of the furnace operation cycle is two days, the maximum heating temperature inside the working area is about 1800 °C. The pressure in the furnace is maintained at a level of several hPa by combining the evacuation of the furnace space with vacuum pumps and the dosed supply of argon. The dominant mechanism of heat transfer to the electric furnace is radiation. The flow of gases inside the electric furnace is laminar due to low density and low velocities (Re ~ 10). Numerical studies of heat exchange are conducted with the aim of controlling the temperature regime in the internal retort to ensure the conditions for the occurrence of physical and chemical processes in it in accordance with the requirements of the technology.

A mathematical model of heat transfer in a large-scale vacuum electric furnace and a grid generator were developed and implemented on the basis of the author’s software complex “SigmaFlow” [1]. In the “SigmaFlow” a control volume method is used to discretize the equations of hydrodynamics and heat transfer and parallel calculations are supported. The main problems associated with the formulation of the mathematical model of the heat transfer process in this electric furnace are due to the presence of essentially different-scale elements (the thickness of the heated parts can reach several millimeters with a characteristic size of the design area of several meters), complex geometry, the duration of the unsteady process, the presence of unconnected gas regions, different characteristic values of velocities and conjugate heat exchange. These problems lead to the complexity of grid construction and increased requirements for computational resources limited by capacities of single personal computers.
2. Computational mesh generation

The discretization of a computational domain is carried out on the base of a geometry imported from a CAD program. External and internal boundaries of a domain are determined by closed surfaces imported from files in the *stl* format. The process of a source data preparation for import into the grid generator involves the formation of several files in the *stl* format. Each of these files combines a group of elements that have the same properties. The grid generation is based on the preliminary octal partition with various discretization of a computational domain (according to geometry), otherwise called as is the octree mesh generation. A final grid is created from an octree by projecting vertexes of a preliminary grid to the boundaries of the numerical domain and forming cells that joint the areas with different discretization (transition areas). As a result, we obtain an unstructured hybrid grid (figure 2), which includes mostly hexahedral elements (cubes) and polyhedral elements (pyramids, prisms, and tetrahedrons) in the transition areas and on the boundaries of the object.

Figure 2. Mesh generation: a) an element of imported geometry; b) an octree; c) different types of control volumes (mesh cells).
3. Optimization of mathematical model of electric furnace

The mechanism of heat transfer by radiation inside of the resistance furnace is dominant, and the time expended to calculate radiation field can take more than 90% of the total calculation time (depending on the angular discretization). Therefore, first of all, first place, the mathematical model of radiation transfer based on the finite-volume method (FVM) must be optimized [2].

First, the standard approach have been used, which is based on reducing the number of iterations to calculate the radiation compared with the rest of the processes. In this case, volumetric and surface radiative terms are frozen for the energy equation on the specified iterative steps for which the radiation is not calculated. With increasing number of skipped radiative iterations the efficiency of the approach decreases, since the convergence of the solution slows down, and errors may to appear by calculation of the temperature for the unsteady task.

Secondly, the FTn modification of the FVM angular discretization of a solid angle [3] have been realized, which allows to provide a more uniform partition of the space, in comparison with the standard approach based on uniform discretization over spherical angles (figure 3). The FTn FVM allows the use of a smaller number of discrete solid angles than the standard angular discretization.

Thirdly, the realized octree mesh method allows using a marching scheme to calculate the radiation. Marching schemes require a mesh consisting of convex control volumes, and this condition is satisfied when using an octree mesh method. The essence of marching schemes is sequential direct calculation of the radiation intensity in the control volumes in a chosen direction. The marching scheme has shown its efficiency in comparison with the more universal iterative methods of solving the SLAE implemented in “SigmaFlow” such as the incomplete LU-factorization method (DILU), stabilized biconjugate gradient stabilized method (BiCGSTAB) etc. [4]. It allows to reduce approximately 3 times the time of the radiation calculation on arbitrary convex polyhedral meshes.

Fourthly, since in the resistance furnace the dominant mechanism of heat transfer in an optically transparent gaseous medium is radiation, the numerical simulation of the gas flow outside the internal retort was not carried out. Such a simplification of the physical model is possible due to that the gas flow outside the internal retort doesn’t have influence on the main technological processes. It has allowed to improve the stability of the calculations and the convergence by reducing the requirements for the grid generation and by excluding from the calculations regions with different scale flow rates of gases. In particular, such regions are in the volume of the internal retort and in the volume between the insulation and the outer retort.

Test calculations have shown that the optimized numerical model of radiation transfer made it possible to reduce by an order of magnitude the calculation time without loss of accuracy. Further optimization of the model aimed at reducing the computational resource consumption is carried out by selecting constant or rarely changed elements of the electric furnace and describing the properties of these elements with the help of economical submodels derived from the analysis of the problem in its complete formulation. A part of the considered problem, which is subject to change relatively rarely, is

![Figure 3. Discretization of a solid angle for the FVM method: a) a standard discretization; b) FTn modification.](image)
the space and equipment of the furnace in the interval from the outer walls cooled by water to the wall of the outer retort (the dimensions of the latter are fixed, while the dimensions of the inner retort can vary from one operating mode to the other). The composition of this part includes the thermal insulation of the furnace, the internal lining of its walls, the columns of heaters and the current leads to them. The submodel for its description is chosen based on an analysis of spatial and temporal scales. We give the most significant results of the analysis.

The heat transfer coefficient at the furnace surfaces is estimated via thermal radiation relations:

\[ \alpha = \frac{4 \sigma T^3}{\pi} = 5 \div 2000 \]

The Biot number for the heater columns \( Bi = 0.001 \div 0.1 \), which is followed by that the temperature differences between the heaters and other elements of the furnace are much greater than the temperature differences in the heaters themselves. The characteristic time of establishment of the heaters’ temperature as a result of heat exchange with other elements of the furnace \( t_h = 10^2 \div 10^4 \) s, which is followed by that the temperature field in the heaters can be considered homogeneous, but unsteady. Therefore, heaters can be described by a non-stationary balance thermal model based on the energy conservation equation

\[ C_h(T_h) m_h \frac{dT_h}{dt} = Q_{\text{in}} + Q_{W H} + Q_{W R}, \tag{1} \]

where \( C_h \) is the specific heat capacity of the heater material, \( m_h \) is the heater mass, \( T_h \) is the heater temperature, \( Q_{\text{in}} \) is the Joule power, \( Q_{W H} \) is the power of radiative heat exchange with the inner surface of furnace lining, \( Q_{W R} \) is the same for the wall of outer retort.

The temperature difference in the thermal insulation and lining is inevitable. During the formulation of the model it is necessary to determine whether the temperature field in the heat-insulating layer can be considered steady. At specified values of the characteristic time, known thermophysical properties of the heat-insulating material (soot), and the thickness of the heat-insulating layer (0.3 m), the Fourier number is of the order of 0.1, which means an essentially unsteady nature of the temperature field in the heat-insulating layer. Consequently, the maximally simplified model should assume a one-dimensional nonstationary temperature field in the outer wall of the furnace, described by the equation of thermal conductivity:

\[ C_h(T) \rho(T) \frac{\partial T}{\partial t} - \frac{\partial}{\partial x} \left( \lambda(T, x) \frac{\partial T}{\partial x} \right) = 0, \tag{2} \]

Its domain covers both the internal graphite lining of the furnace and the thermal insulation layer. A boundary condition for the heat conduction equation at one of the boundaries corresponding to the external, water-cooled surface of the furnace wall is the known temperature value. At the second boundary, corresponding to the inner surface of the lining, the boundary condition is the given value of the heat flux generated by the emitted and falling radiation flux; the latter is created by the radiation of the heater and the wall of the outer retort.

The boundary of the domain of the spatial heat transfer problem is the outer wall of the outer retort. The boundary condition on it is the magnitude of the heat flux – emitted and falling, formed by the heater and the inner wall surface of the furnace. The heat flux from the heater depends on the temperature of the heater (as a fourth power) and on the coordinate of the point on the surface of the outer retort. To clarify the form of the spatial dependence, it was necessary to calculate the heat transfer in a complete geometric setting without simplification, including the heater columns and other elements outside the retort space. In this case, the temperature of the heater is artificially set equal to a certain value, much greater than the temperature of all other elements, as a result of which the radiative heat flux on the remaining elements of the problem is related only to the radiation of the heater. After solving the heat radiation transfer problem, the dependence of the radiation flux incident on the external retort on the coordinate was determined. For the lateral surface of the retort, the obtained dependence was averaged over the sections:

\[ i \cdot h_z < z < (i + 1)h_z, \]

where \( h_z \) is the coordinate step that is about 0.01 of the retort height and OZ axis is oriented along the retort axiz. In this way we obtain a tabulated distribution of the radiation flux incident on the side of the
heater, which is used in solving the problem in the reduced formulation (taking into account the proportionality of the heat flux to $T_h^4$). The flow to the upper surface of the retort is averaged and tabulated along the intervals of the coordinate $r$ - the distance from the axis of the retort. A similar procedure is performed to find and further use the heat flux incident on the surface of the retort and heater from the inner surface of the furnace lining.

One of the essential elements of the model is the dependence of the thermal conductivity of the heat-insulating material (soot) on temperature. There are no reliable measurement data on this issue. In addition, the change in material properties during the operation of the furnace is largely uncertain. In this connection, it is expedient to determine the required dependence on the materials of measurements carried out during the operation of the furnace. The most convenient for this are the data on the time dependence of the retort temperature during the cooling of the furnace, i.e. in the absence of electrical heating. As the results of preliminary methodological calculations show, the retort and its contents can be approximately regarded as a single isothermal body. This makes it possible to supplement the above model of the furnace space, external with respect to the retort, by the nonstationary balance equation of retort energy conservation, similar to (1):

$$C_R(T_R)m_R \frac{dT_R}{dt} = Q_{HR} + Q_{WR},$$

where $Q_{HR}, Q_{WR}$ are the powers of the retort radiative heat exchange with the heater and the inner surface of furnace lining. The numerical solution of the system (1-3) makes it possible to obtain a time dependence of the retort temperature for a given dependence of the thermal conductivity on temperature. Introducing a two-parameter power-law dependence of the thermal conductivity coefficient on the temperature

$$\lambda(T) = LT^p, \ L, p = const,$$

and performing a numerical solution of the system (1-3), we obtain an integral square deviation of the calculated results from the experimental ones, which is a function of $L$ and $p$:

$$\int \left( T_R^{calc}(t) - T_R^{exp}(t) \right)^2 dt = f(L, p)$$

The required parameters of the dependence of the thermal conductivity on temperature are those that provide the minimum of $f(L, p)$. As calculations have shown, at the minimum of $f(L, p)$, the deviation of the calculated temperature from the experimental temperature does not exceed 10 K, which indicates that the form of the $\lambda(T)$ dependence chosen for optimization is close to the exact one.

The formulated model of radiative heat transfer processes, including the sub-model of heat exchange in space outside the outer retort, was used to calculate the operating mode of the SSHVH furnace 25.30 / 21. The operating mode includes a heating stage with four periods of constant temperature (the last is at a maximum temperature of 1800 °C) and a cooling stage. A comparison of the calculated results with the results of measurements carried out during the implementation of the operating regime is shown in figure 4. As can be seen, a fairly good agreement between the calculated and experimental results is achieved. The use of the sub-model of heat exchange in space outside the outer retort makes it possible to reduce 3-4 times the grid dimensions and, consequently, computational costs.
4. Conclusion
The complexity of the geometry of the electric furnace creates difficulties in solving practical problems of optimizing the technological process, involving multivariate numerical analysis at a reasonable time without the use of powerful computing clusters. Optimization of the mathematical model, including the separation of zero-dimensional and one-dimensional submodels, allows solving the problem of heat exchange in an electric furnace with acceptable accuracy at relatively small computing powers (starting with 1-2 processor computing systems). Using the model in the simplest form, when considering the electric furnace retort and its content in the zero-dimensional approximation, it was possible to determine the important closing relation of the model – the dependence of the thermal conductivity of the thermal insulation on temperature, based on the comparison of the calculated and experimental time dependence of the retort temperature in the cooling mode.

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