Domain wall network as QCD vacuum: confinement, chiral symmetry, hadronization

Sergei Nedelko, Vladimir Voronin

Bogoliubov Laboratory of Theoretical Physics, JINR

Hadron Structure’15
An overall task pursued by most of approaches to QCD vacuum structure is an identification of the properties of nonperturbative gauge field configurations able to provide a coherent resolution of the confinement, the chiral symmetry breaking, the $U_A(1)$ symmetry realization and the strong CP problems, both in terms of color-charged fields and colorless hadrons.

The other side of this task is identification of the conditions for deconfinement and chiral symmetry restoration.
• Confinement of both static and dynamical quarks
  \[ W(C') = \langle \text{Tr P} \ e^{i \int_C dz \hat{A}_\mu} \rangle \]
  \[ S(x, y) = \langle \psi(y) \bar{\psi}(x) \rangle \]

• Dynamical Breaking of chiral $SU_L(N_f) \times SU_R(N_f)$ symmetry
  \[ \langle \bar{\psi}(x) \psi(x) \rangle \]

• $U_A(1)$ Problem
  \[ \eta' (\chi, \text{Axial Anomaly}) \]

• Strong CP Problem
  \[ Z(\theta) \]

• Colorless Hadron Formation:
  Effective action for colorless collective modes:
  hadron masses and "wave functions", formfactors, scattering

  Light mesons and baryons, Regge spectrum of excited states of light hadrons,
  heavy-light hadrons, heavy quarkonia

What would be a formalism for coherent simultaneous description of all these nonperturbative features of QCD?

QCD vacuum as a medium characterized by certain condensates,
  quarks and gluons - elementary coloured excitations (confined),
  mesons and baryons - collective colourless excitations (masses, form factors, etc)

Quantum effective action of QCD!
An ensemble of almost everywhere (in $R^4$) homogeneous Abelian (anti-)self-dual gluon fields

P. Minkowski, Phys. Lett. B 76 (1978) 439.
H. Pagels, and E. Tomboulis, Nucl. Phys. B 143 (1978) 485.
P. Minkowski, Nucl. Phys. B 177 (1981) 203.
H. Leutwyler, Nucl. Phys. B 179 (1981) 129.

\[ \langle :g^2 F^2 : \rangle \neq 0, \quad \chi = \int d^4 x \langle Q(x) Q(0) \rangle \neq 0, \quad \langle Q(x) \rangle = 0 \]

Topological charge density $Q(x) = \frac{g^2}{32\pi^2} F_{\mu\nu}^a(x) \tilde{F}_{\mu\nu}^a(x)$

P.J. Moran, Derek B. Leinweber, arXiv:0805.4246v1 [hep-lat] 2008
| Meson | $\pi$ | $\rho$ | $K$ | $K^*$ | $\omega$ | $\phi$ |
|-------|------|------|-----|------|--------|--------|
| $M$   | 140  | 770  | 496 | 890  | 770    | 1034   |
| $M^{\exp}$ | 140  | 770  | 496 | 890  | 786    | 1020   |
| $f_P$ | 126  | -    | 145 | -    | -      | -      |
| $f_P^{\exp}$ | 132  | -    | 157 | -    | -      | -      |
| $h$   | 6.51 | 4.16 | 7.25| 4.48 | 4.16   | 4.94   |
| $M^*$ | 630  | 864  | 743 | 970  | 864    | 1087   |

- **Meson**
  - $D_e$, $D_s^*$, $D_{s1}^*$, $B$, $B^*$, $B_s$, $B_s^*$
  - $K^*$, $K_{1}(1270)$
- **$f_P$**
- **$h$**

| Meson | $\eta_c$ | $J/\psi$ | $\chi c_0$ | $\chi c_1$ | $\chi c_2$ | $\psi'$ | $\psi''$ |
|-------|----------|----------|-------------|-------------|------------|---------|---------|
| $n$   | 0        | 0        | 0           | 0           | 0          | 1       | 2       |
| $\ell$ | 0        | 0        | 1           | 1           | 0          | 0       | 0       |
| $j$   | 0        | 1        | 0           | 1           | 2          | 1       | 1       |
| $M$ (MeV) | 3000    | 3161    | 3452        | 3529        | 3531       | 3817    | 4120    |
| $M^{\exp}$ (MeV) | 2980    | 3096    | 3415        | 3510        | 3556       | 3770    | 4040    |

| Meson | $\gamma$ | $\chi b_0$ | $\chi b_1$ | $\chi b_2$ | $\gamma'$ | $\chi b_0'$ | $\chi b_1'$ | $\chi b_2'$ | $\gamma''$ |
|-------|-----------|-------------|-------------|-------------|-----------|-------------|-------------|-------------|-----------|
| $n$   | 0         | 0           | 0           | 0           | 1         | 1           | 1           | 1           | 2         |
| $\ell$ | 0        | 1           | 1           | 1           | 0         | 1           | 1           | 1           | 0         |
| $j$   | 1         | 0           | 1           | 2           | 1         | 0           | 1           | 2           | 1         |
| $M$ (MeV) | 9490    | 9767       | 9780        | 9780        | 10052     | 10212       | 10215       | 10215       | 10292     |
| $M^{\exp}$ (MeV) | 9460    | 9860       | 9892        | 9913        | 10230     | 10235       | 10255       | 10269       | 10355     |

$M_\eta = 640\ \text{MeV}$, $M_{\eta'} = 950\ \text{MeV}$, $h_\eta = 4.72$, $h_{\eta'} = 2.55$, $\sqrt{BR} = 1.56$.

**G.V. Efimov and S.N. Nedelko, Phys. Rev. D 51 (1995); Phys. Rev. D 54 (1996)**
\[ B^a_\mu = n^a B_{\mu\nu} x_\nu, \quad \tilde{B}_{\mu\nu} = \pm B_{\mu\nu}, \quad B_{\mu\alpha} B_{\alpha\nu} = \delta_{\mu\nu} B^2, \quad B^2 = \text{const} \]

\[ D^2(x) G(x, y) = -\delta(x - y), \quad G(x, y) = e^{ixB} y H(x - y) \]

\[ H(z) = \frac{e^{-B z^2}}{4\pi^2 z^2}, \quad \tilde{H}(P) = \frac{1 - e^{-P^2/B}}{P^2} \]

\[ \Phi^{aJ}(x, z) = \sum_{nl} (z^2)^{l/2} \varphi^{nl}_{\mu_1...\mu_l}(z) \Phi^{aJ\mu_1...\mu_l}(x), \quad \varphi^{nl}_{\mu_1...\mu_l} = L_{nl} (z^2)^l T^{(l)}_{\mu_1...\mu_l}(n_z), \quad n_z = \frac{z}{\sqrt{z}}. \]

Chiral symmetry realization is driven by the quasi-zero modes of the quark fields - (anti-)self-duality

Regge behaviour of the spectrum is due to nonlocality of the vertices and propagators.

\[ M_{aJ\ell n}^2 = \frac{8}{3} \ln \left( \frac{5}{2} \right) \cdot B \cdot n + O(\ln n), \quad \text{for } n \gg \ell, \quad M_{aJ\ell n}^2 = \frac{4}{3} \ln 5 \cdot B \cdot \ell + O(\ln \ell), \quad \text{for } \ell \gg n. \]

Heavy-light mesons and heavy quarkonia

\[ m_Q \gg \sqrt{B}, \quad m_Q \gg m_q, \quad M_{Q\bar{q}} = m_Q + \Delta^{(J)}_{Q\bar{q}} + O(1/m_Q) \]

\[ m_Q \gg \sqrt{B}, \quad M_{Q\bar{Q}} = 2m_Q - \Delta_{Q\bar{Q}}, \quad \Delta^{(P)}_{Q\bar{Q}} = 2\Delta^{(V)}_{Q\bar{Q}} \]
QCD effective action and vacuum gluon configurations
Gluon condensates and domain wall network as QCD vacuum
Domain bulk - confinement
Domain wall junctions - deconfinement
The domain model of QCD vacuum
Testing the domain model - static characteristics of QCD vacuum
Hadronization: spectrum, decay constants
Summary
QCD effective action and vacuum gluon configurations

In Euclidean functional integral for YM theory one has to allow the gluon condensates to be nonzero:

\[
Z = N \int F_B DA \int \Psi D\psi D\bar{\psi} \exp\{ -S[A, \psi, \bar{\psi}] \}
\]

\[
F_B = \left\{ A : \lim_{V \to \infty} \frac{1}{V} \int_V d^4x g^2 F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) = B^2 \right\}.
\]

\[
A^a_\mu = B^a_\mu + Q^a_\mu, \quad \text{background gauge fixing condition } D(B)Q = 0:
\]

\[
1 = \int_\mathcal{B} DB \Phi[A, B] \int_\mathcal{Q} DQ \int_\Omega D\omega \delta[A^\omega = Q^\omega = B^\omega] \delta[D(B^\omega)Q^\omega]
\]

\[
Q^a_\mu - \text{local (perturbative) fluctuations of gluon field with zero gluon condensate: } Q \in \mathcal{Q};
\]

\[
B^a_\mu \text{ are long range field configurations with nonzero condensate: } B \in \mathcal{B}.
\]

\[
Z = N' \int_\mathcal{B} DB \int_\mathcal{Q} DQ \int \Psi D\psi D\bar{\psi} \det[D(B)D(B + Q)] \delta[D(B)Q] \exp\{ -S[B + Q, \psi, \bar{\psi}] \}
\]

B.V. Galilo and S.N. Nedelko, Phys. Rev. D84 (2011) 094017
L. D. Faddeev, [arXiv:0911.1013 [math-ph]]
H. Leutwyler, Nucl. Phys. B 179 (1981) 129
The character of long range fields has yet to be identified by the dynamics of fluctuations:

\[ Z = N' \int_B DB \int_\mathcal{D} \Phi \int_\Psi \mathcal{D} \Psi \mathcal{D} \bar{\Psi} \int_Q \mathcal{D} Q \det[D(B) D(B + Q)] \delta[D(B)Q] \exp\{-S_{\text{QCD}}[B + Q, \Phi, \bar{\Phi}]\} \]

\[ = \int_B DB \exp\{-S_{\text{eff}}[B]\} \]

Global minima of \( S_{\text{eff}}[B] \) – field configurations that are dominant in the thermodynamic limit \( V \to \infty \). Homogeneous Abelian (anti-)self-dual fields are of particular interest.

\[ \langle F^2 \rangle : \quad A_\mu = -\frac{1}{2} n F_{\mu \nu} x_\nu, \quad \tilde{F}_{\mu \nu} = \pm F_{\mu \nu} \]

\[ n = T^3 \cos \xi + T^8 \sin \xi. \]

\[ G(z^2) \sim \frac{e^{-B_{\text{vac}} z^2}}{z^2}, \quad \tilde{G}(p^2) \sim \frac{1}{p^2} \left(1 - e^{-p^2 / B_{\text{vac}}}\right) \]

Gluon propagator \( \Rightarrow \) Regge trajectories

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P. Minkowski, Nucl. Phys. B177 (1981) 203

H. Leutwyler, Nucl. Phys. B 179 (1981) 129

H. Leutwyler, Phys. Lett. B 96 (1980) 154

G.V. Efimov, and S.N. Nedelko, Phys. Rev. D 51 (1995)

A. Eichhorn, H. Gies and J. M. Pawlowski, Phys. Rev. D 83, 045014 (2011)
Gluon condensates and domain wall network

\[ \mathcal{L}_{\text{eff}} = -\frac{1}{4B^2} \left( D^{ab}_{\nu} F^b_{\rho \mu} D^{\mu \nu}_{\rho \nu} F^c_{\rho \mu} + D^a_{\mu} F^{\mu \nu}_{\mu \nu} D^{ac}_{\rho} F^c_{\rho \nu} \right) - U_{\text{eff}} \]

\[ U_{\text{eff}} = \frac{B^4}{12} \text{Tr} \left( C_1 F^2 + \frac{4}{3} C_2 F^4 - \frac{16}{9} C_3 F^6 \right) , \]

where

\[ D^{\mu}_{\nu} = \delta^{ab} \partial_{\mu} - i A^{ab}_{\mu} = \partial_{\mu} - i A^{c \mu} (T^c)^{ab} , \]
\[ F^{a}_{\mu \nu} = \partial_{\mu} A^a_{\nu} - \partial_{\nu} A^a_{\mu} - i f^{abc} A^b_{\mu} A^c_{\nu} , \]
\[ F^{a}_{\mu \nu} = F^a_{\mu \nu} T^a , \quad T^{a}_{bc} = -i f^{abc} \]

C_1 > 0, C_2 > 0, C_3 > 0.

B.V. Galilo, S.N. Nedelko, Phys. Part. Nucl. Lett., 8 (2011) 67
D. P. George, A. Ram, J. E. Thompson and R. R. Volkas, Phys. Rev. D 87, 105009 (2013) [arXiv:1203.1048 [hep-th]]
$U_{\text{eff}}$ possesses 12 degenerate discrete minima:

$$A_{\mu} = -\frac{1}{2} n_k F_{\mu\nu} x_\nu, \quad \tilde{F}_{\mu\nu} = \pm F_{\mu\nu},$$

where the matrix $n_k$ belongs to the Cartan subalgebra of $su(3)$

$$n_k = T^3 \cos (\xi_k) + T^8 \sin (\xi_k),$$

$$\xi_k = \frac{2k + 1}{6} \pi, \quad k = 0, 1, \ldots, 5.$$
Domain wall network

\[ \mathcal{L}_{\text{eff}} = -\frac{1}{2} \Lambda^2 b_{\text{vac}}^2 \partial_\mu \omega \partial_\mu \omega - b_{\text{vac}}^4 \Lambda^4 (C_2 + 3C_3 b_{\text{vac}}^2) \sin^2 \omega, \]

leads to sine-Gordon equation

\[ \partial^2 \omega = m_\omega^2 \sin 2\omega, \quad m_\omega^2 = b_{\text{vac}}^2 \Lambda^2 (C_2 + 3C_3 b_{\text{vac}}^2), \]

and the standard kink solution

\[ \omega(x_\nu) = 2 \arctg (\exp(\mu x_\nu)) \]
The general kink configuration can be parametrized as

$$\zeta(\mu_i, \eta^i, x_\nu - q^i) = \frac{2}{\pi} \arctan \exp(\mu_i (\eta^i, x_\nu - q^i)).$$

A single lump in two, three and four dimensions is given by

$$\omega(x) = \pi i \prod_{i=1}^{k} \zeta(\mu_i, \eta^i, x_\nu - q^i).$$

for $k = 4, 6, 8$, respectively. The general kink network is then given by the additive superposition of lumps

$$\omega = \pi \sum_{j=1}^{\infty} \prod_{i=1}^{k} \zeta(\mu_{i,j}, \eta_{i,j}^i, x_\nu - q_{i,j}^i)$$

S.N. Nedelko, V.E. Voronin, Eur.Phys.J. A51 (2015) 4

H. Pagels, and E. Tomboulis, Nucl. Phys. B 143 (1978) 485

P. Minkowski, Phys. Lett. B 76 (1978) 439

H. Leutwyler, Nucl. Phys. B 179 (1981);

G.V. Efimov, and S.N. Nedelko, Phys. Rev. D 51 (1995) 176

$$\langle F^2 \rangle = B^2$$
$$\langle |F\tilde{F}| \rangle = B^2$$

$$\langle F^2 \rangle = B^2$$
$$\langle |F\tilde{F}| \rangle \ll B^2$$
Domain bulk - confinement

The case of finite size spherical domains was considered in
A.C. Kalloniatis and S.N. Nedelko, Phys. Rev. D 64 (2001)

Elementary color charged excitations - fluctuations decaying in all four directions.

Eigenvalue problem for scalar field in $\mathbb{R}^4$:

$$B_\mu = B_{\mu\nu} x_\nu, \tilde{B}_{\mu\nu} = \pm B_{\mu\nu}, B_{\mu\alpha} B_{\nu\alpha} = B^2 \delta_{\mu\nu}.$$ 

$$- (\partial_\mu - iB_\mu)^2 G = \delta \quad \rightarrow \quad G(x - y) \sim \frac{e^{-B(x-y)^2/4}}{(x - y)^2}$$

$$- \left( \partial_\mu - i\tilde{B}_\mu \right)^2 \Phi = \lambda \Phi \quad \rightarrow \quad \left[ \beta_+^\pm \beta_-^\pm + \gamma_+^\pm \gamma_-^\pm + 1 \right] \Phi = \frac{\lambda}{4B} \Phi,$$

$$\beta_\pm = \frac{1}{2} (\alpha_1 \mp i\alpha_2), \quad \gamma_\pm = \frac{1}{2} (\alpha_3 \mp i\alpha_4), \quad \alpha_\mu = \frac{1}{\sqrt{B}} x_\mu + \partial_\mu,$$

$$\beta_+^\pm = \frac{1}{2} (\alpha_1^+ \pm i\alpha_2^+), \quad \gamma_+^\pm = \frac{1}{2} (\alpha_3^+ \pm i\alpha_4^+), \quad \alpha_\mu^+ = \frac{1}{\sqrt{B}} x_\mu - \partial_\mu.$$ 

The eigenfunctions and eigenvalues - 4-dim harmonic oscillator

$$\Phi_{nmkl}(x) = \frac{1}{\pi^2 \sqrt{n!m!k!!l!!}} \left( \beta_+^+ \right)^k \left( \beta_-^- \right)^l \left( \gamma_+^+ \right)^n \left( \gamma_-^- \right)^m \Phi_{0000}, \quad \Phi_{0000} = e^{-\frac{1}{2}Bx^2}$$

$$\lambda_r = 4B(r + 1), \quad r = k + n \text{ self-dual field, } \ r = l + n \text{ anti-self-dual field}$$
Domain wall junctions - deconfinement

The color charged scalar field inside junction:

\[- \left( \partial_\mu - i \tilde{B}_\mu \right)^2 \Phi = 0, \]

\[\Phi(x) = 0, \quad x \in \mathcal{T} = \{ x_1^2 + x_2^2 < R^2, (x_3, x_4) \in \mathbb{R}^2 \} \]

The solutions are quasi-particle excitations

\[\phi^a(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[ a_{alk}^+(p_3)e^{ix_0\omega_{alk} - ip_3x_3} + b_{alk}(p_3)e^{-ix_0\omega_{alk} + ip_3x_3} \right] e^{il\vartheta} \phi_{alk}(r), \]

\[\phi^{a\dagger}(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[ b_{alk}^+(p_3)e^{-ix_0\omega_{alk} + ip_3x_3} + a_{alk}(p_3)e^{ix_0\omega_{alk} - ip_3x_3} \right] e^{-il\vartheta} \phi_{alk}(r), \]

\[p_0^2 = p_3^2 + \mu^{2}_{alk}, \quad p_0 = \pm \omega_{alk}(p_3), \quad \omega_{alk} = \sqrt{p_3^2 + \mu^{2}_{alk}}, \]

\[k = 0, 1, \ldots, \infty, \quad l \in \mathbb{Z}, \]
Impact of electromagnetic fields on “QCD vacuum“.

- **Relativistic heavy ion collisions - strong electromagnetic fields**
  V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya,
  V. P. Konchakovski and S. A. Voloshin, Phys. Rev C 84 (2011)

**Strong electro-magnetic field plays catalyzing role for deconfinement and anisotropies!**
B.V. Galilo and S.N. Nedelko, Phys. Rev. D84 (2011) 094017.
M. D’Elia, M. Mariti and F. Negro, Phys. Rev. Lett. 110, 082002 (2013)
G. S. Bali, F. Bruckmann, G. Endrodi, F. Gruber and A. Schaefer, JHEP 1304, 130 (2013)
The domain model of QCD vacuum

A.C. Kalloniatis and S.N. Nedelko, Phys. Rev. D 64 (2001)

Euclidean partition function is defined as

$$ Z(\theta) = \lim_{V,N \to \infty} N \prod_{i=1}^{N} \int_{\mathcal{B}} dB_i \int_{\Omega_{\alpha,\beta}} d\Omega_{\alpha,\beta} \int_{\Psi_i} D\psi^{(i)} D\bar{\psi}^{(i)} \int_{Q_i} D\mu \left[ Q^i \right] $$

$$ \times e^{-S_{V_i}^{QCD} \left[ Q^{(i)} + B^{(i)}, \psi^{(i)}, \bar{\psi}^{(i)} \right]} - i\theta Q_{V_i} \left[ Q^{(i)} + B^{(i)} \right] $$

$$ D\mu = \delta[D(B^{(i)})Q^{(i)}] \Delta_{FP}[B^{(i)}, Q^{(i)}] $$

The thermodynamic limit: \( v^{-1} = N/V = \text{const} \), as \( V, N \to \infty \). Functional spaces \( Q^i \) and \( \Psi^i \) are specified by BCs at \( (x - z_i)^2 = R^2 \)

$$ \tilde{n}_i Q^{(i)}(x) = 0, $$

$$ i \gamma_i(x)e^{i(\alpha + \beta^a \gamma^a/2)\gamma_5} \psi^{(i)}(x) = \psi^{(i)}(x), $$

$$ \bar{\psi}^{(i)} e^{i(\alpha + \beta^a \gamma^a/2)\gamma_5 i \gamma_i(x) = -\bar{\psi}^{(i)}(x), $$

$$ \eta^\mu_i = \frac{(x - z_i)^\mu}{|x - z_i|}, \quad \tilde{n}_i = n_i^a T^a, T^a - \text{adjoint representation} $$
Testing the model - characteristics of the domain network ensemble

A.C. Kalloniatis and S.N. Nedelko, Phys. Rev. D 64 (2001); Phys. Rev. D 69 (2004); Phys. Rev. D 71 (2005); Phys. Rev. D 73 (2006)

Area law
Spontaneous chiral symmetry breaking
$U_A(1)$ is broken by anomaly
There is no strong CP violation
\[ Z = \int dB \int \Psi \mathcal{D}\psi \mathcal{D}\bar{\psi} \int_Q \mathcal{D}Q \delta[D(B)Q] \Delta_{FP}[B, Q] e^{-S_{QCD}[Q+B, \psi, \bar{\psi}]} = \]

\[ \int dB \int \Psi \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp \left\{ \int dx \bar{\psi} (i \slashed{\partial} + gB - m) \psi \right\} W[j] \]

\[ W[j] = \int_Q \mathcal{D}Q \delta[D(B)Q] \Delta_{FP}[B, Q] \exp \left\{ -\frac{1}{2} \int dx \text{ Tr} G^2 [B + Q] + g \int dx \, Q^a j^a \right\}, \]

\[ j^a_\mu = \bar{\psi} \gamma_\mu t^a \psi \]

Recalling the definition of Green’s functions

\[ G^{a_1 \ldots a_n}_{\mu_1 \ldots \mu_n}(x_1, \ldots, x_n|B) = \frac{1}{g^n} \frac{\delta^n \ln W[j]}{\delta j^{a_1}_{\mu_1}(x_1) \ldots \delta j^{a_n}_{\mu_n}(x_n)}, \]

we obtain

\[ W[j] = \exp \left\{ \sum_n \frac{g^n}{n!} \int dx_1 \ldots \int dx_n j^{a_1}_{\mu_1}(x_1) \ldots j^{a_n}_{\mu_n}(x_n) G^{a_1 \ldots a_n}_{\mu_1 \ldots \mu_n}(x_1, \ldots, x_n|B) \right\} \]

\( W[j] \) is truncated up to the term including two-point gluon correlation function
\[ Z = \int dB \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp \left\{ \int dx \bar{\psi} \left( i \not{\partial} + g \not{B} - m \right) \psi \\
+ \frac{g^2}{2} \int dx_1 dx_2 G_{\mu_1 \mu_2}^{a_1 a_2} (x_1, x_2 | B) j_{\mu_1}^{a_1} (x_1) j_{\mu_2}^{a_2} (x_2) \right\} \\
\int dz dx G(z | B) J^a J(x, z) = \int dx \sum_{a J l n} J^a J(x, z) \]

\[ \alpha_s = \alpha_s (0) \left[ 1 + \sum_{R} (p^2) \right]; \quad \Pi^R \left( 0 \right) = 0 \]

\[ 0 \xrightarrow{z} e^{-\frac{1}{4} B z^2} \]

\[ \int dx_1 dx_2 \xrightarrow{x_1 \xrightarrow{x_2}} = \int dx \sum_{a J l n} \]

\[ J^a J(x, z) = \sum_{n l} \left( z^2 \right)^{l/2} f_{\mu_1 \ldots \mu_l}^{n l} (z) J_{\mu_1 \ldots \mu_l}^{a J l n} (x), \quad J_{\mu_1 \ldots \mu_l}^{a J l n} (x) = \bar{q}(x) V_{\mu_1 \ldots \mu_l}^{a J l n} \left( \frac{D(x)}{B} \right) q(x), \]

\[ f_{\mu_1 \ldots \mu_l}^{n l} = L_{n l} \left( z^2 \right) T_{\mu_1 \ldots \mu_l}^{(l)} (n_z), \quad n_z = \frac{z}{\sqrt{z}}. \]

\[ T_{\mu_1 \ldots \mu_l}^{(l)} \text{ are irreducible tensors of four-dimensional rotational group} \]

\[ T_{\mu_1 \ldots \mu_\nu \ldots \mu_l}^{(l)} (n_z) = T_{\mu_1 \ldots \nu \ldots \mu_l}^{(l)} (n_z), \quad T_{\mu_\nu \ldots \mu_l}^{(l)} (n_z) = 0, \]

\[ \int_0^\infty du \rho_l (u) L_{n l} (u) L_{n' l} (u) = \delta_{n n'}, \quad \rho_l (u) = u^l e^{-u} \leftrightarrow \frac{e^{-B z^2}}{z^2} \text{ gluon propagator} \]
Effective meson action for composite colorless fields:

\[
Z = \mathcal{N} \lim_{V \to \infty} \int D\Phi \exp \left\{ -\frac{B}{2} \frac{h_Q^2}{g^2 C_Q} \int dx \Phi_Q^2(x) - \sum_k \frac{1}{k} W_k[\Phi] \right\}, \quad Q = (aJln)
\]

\[
1 = \frac{g^2 C_Q}{B} \tilde{\Gamma}_Q^{(2)}(-M_Q^2|B), \quad h_Q^{-2} = \frac{d}{dp^2} \tilde{\Gamma}_Q^{(2)}(p^2)|_{p^2=-M_Q^2}.
\]

\[
W_k[\Phi] = \sum_{Q_1 \ldots Q_k} h_{Q_1} \ldots h_{Q_k} \int dx_1 \ldots \int dx_k \Phi_{Q_1}(x_1) \ldots \Phi_{Q_k}(x_k) \Gamma_{Q_1 \ldots Q_k}^{(k)}(x_1, \ldots, x_k|B)
\]

\[
\Gamma_{Q_1 Q_2}^{(2)} = G_{Q_1 Q_2}^{(2)}(x_1, x_2) - \Xi_2(x_1 - x_2)G_{Q_1}^{(1)}G_{Q_2}^{(1)},
\]

\[
\Gamma_{Q_1 Q_2 Q_3}^{(3)} = G_{Q_1 Q_2 Q_3}^{(3)}(x_1, x_2, x_3) - \frac{3}{2} \Xi_2(x_1 - x_3)G_{Q_1 Q_2}^{(2)}(x_1, x_2)G_{Q_3}^{(1)}(x_3)
\]

\[
+ \frac{1}{2} \Xi_3(x_1, x_2, x_3)G_{Q_1}^{(1)}(x_1)G_{Q_2}^{(1)}(x_2)G_{Q_3}^{(1)}(x_3),
\]

\[
\Gamma_{Q_1 Q_2 Q_3 Q_4}^{(4)} = G_{Q_1 Q_2 Q_3 Q_4}^{(4)}(x_1, x_2, x_3, x_4) - \frac{4}{3} \Xi_2(x_1 - x_2)G_{Q_1}^{(1)}(x_1)G_{Q_2}^{(3)}Q_3 Q_4(x_2, x_3, x_4)
\]

\[
- \frac{1}{2} \Xi_2(x_1 - x_3)G_{Q_1 Q_2}^{(2)}(x_1, x_2)G_{Q_3 Q_4}^{(2)}(x_3, x_4)
\]

\[
+ \Xi_3(x_1, x_2, x_3)G_{Q_1}^{(1)}(x_1)G_{Q_2}^{(1)}(x_2)G_{Q_3 Q_4}^{(2)}(x_3, x_4)
\]

\[
- \frac{1}{6} \Xi_4(x_1, x_2, x_3, x_4)G_{Q_1}^{(1)}(x_1)G_{Q_2}^{(1)}(x_2)G_{Q_3}^{(1)}(x_3)G_{Q_4}^{(1)}(x_4).
\]
\[ G_{Q_1 \ldots Q_k}^{(k)}(x_1, \ldots, x_k) = \int dB_j \text{Tr} V_{Q_1} \left( x_1 | B^{(j)} \right) S \left( x_1, x_2 | B^{(j)} \right) \ldots \]

\[ \ldots V_{Q_k} \left( x_k | B^{(j)} \right) S \left( x_k, x_1 | B^{(j)} \right) \]

\[ G_{Q_1 \ldots Q_l}^{(l)}(x_1, \ldots, x_l) G_{Q_{l+1} \ldots Q_k}^{(k)}(x_{l+1}, \ldots, x_k) = \]

\[ \int dB_j \text{Tr} \left\{ V_{Q_1} \left( x_1 | B^{(j)} \right) S \left( x_1, x_2 | B^{(j)} \right) \ldots V_{Q_k} \left( x_k | B^{(j)} \right) S \left( x_k, x_1 | B^{(j)} \right) \right\} \]

\[ \times \text{Tr} \left\{ V_{Q_{l+1}} \left( x_{l+1} | B^{(j)} \right) S \left( x_{l+1}, x_{l+2} | B^{(j)} \right) \ldots V_{Q_k} \left( x_k | B^{(j)} \right) S \left( x_k, x_{l+1} | B^{(j)} \right) \right\} , \]

Bar denotes integration over all configurations of the background field with measure \( dB_j \).

\[ \Gamma_{Q_1 Q_2}^{(2)} = \]

\[ \Gamma_{Q_1 Q_2 \ldots Q_n}^{(n)} = \]

\[ + \ldots + \]

\[ + \ldots + \]

\[ + \ldots \]
Meson-quark vertex operators \( \Leftarrow J_{\mu_1...\mu_l}^{aJln} = \tilde{q}(x)V_{\mu_1...\mu_l}^{aJln}q(x) \)

\[
V_{\mu_1...\mu_l}^{aJln}(x) = M^a \Gamma^J \left\{ F_{nl} \left( \frac{\leftrightarrow^2 D(x)}{B^2} \right) T_{\mu_1...\mu_l}^{(l)} \left( \frac{1}{i} \frac{\leftrightarrow D(x)}{B} \right) \right\},
\]

\[
F_{nl}(s) = s^n \int_0^1 dt t^{n+l} \exp(st) = \int_0^1 dt t^{n+l} \frac{\partial^n}{\partial t^n} \exp(st),
\]

\[
\leftrightarrow \leftrightarrow D = \leftrightarrow D \xi_f, \quad \rightarrow \xi_f = \frac{m_f}{m_f + m_{f'}}
\]

Quark propagator in homogeneous Abelian (anti-)self-dual field

\[
\frac{1}{m(0)} \left[ 1 + \Sigma^R(p^2) \right]; \quad \Sigma^R(0) = 0 \quad S(x, y) = \exp \left( -\frac{i}{2} x_\mu B_{\mu\nu} y_\nu \right) H(x - y),
\]

\[
\tilde{H}_f(p|B) = \frac{1}{\nu B^2} \int_0^1 ds e^{-p^2/\nu B^2} \left( \frac{1 - s}{1 + s} \right)^{m_f^2/2\nu B^2} \left[ p_\alpha \gamma_\alpha \pm is \gamma_5 \gamma_\alpha \frac{B_{\alpha\beta}}{\nu B^2} p_\beta + m_f \left( P_+ + P_- \frac{1 + s^2}{1 - s^2} - \frac{i}{2} \gamma_\alpha \frac{B_{\alpha\beta}}{\nu B^2} \gamma_\beta \frac{s}{1 - s^2} \right) \right]
\]

The parameters of the model are

\[
\alpha_s(0) \quad m_{u/d}(0) \quad m_s(0) \quad m_c(0) \quad m_b(0) \quad B \quad R
\]

\[
\langle \alpha_s F^2 \rangle = \frac{B^2}{\pi} \quad \chi_{YM} = \frac{B^4 R^4}{128\pi^2}
\]
Quadratic part of the effective action for colorless composite fields is

\[ I_2 = -\frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \varphi_k^\mu(p) \left[ B \delta^{aa'} \delta_{\mu\mu'} - G_{Jln} G_{J'l'n'} \Pi^{kk'}_{\mu\mu'}(p^2) \right] \varphi_{\mu'}^k(p). \]

\[ k = (aJln), \quad \mu = (\mu_1 \ldots \mu_l), \quad G_{Jln} = g \sqrt{C_J \frac{l + 1}{2l n!(n + l)!}}, \quad C_{V/A} = \frac{1}{18}, \quad C_{S/P} = \frac{1}{9}. \]

\[ \tilde{\varphi}^n(p) = O^{nn'}(p) \varphi^{n'}(p), \quad \tilde{\Pi}^{nn'}(p^2) = \delta^{nn'} \tilde{\Pi}^{n'}(p^2). \]

Mass and coupling constant of a meson with radial number \( n \) are found from

\[ 1 = \frac{g^2 \tilde{\Pi}^n(-M^2)}{B}, \quad h_{aJln}^{-2} = \frac{d}{dp^2} \tilde{\Pi}^n(p^2)|_{p^2=-M^2}. \]
Polarization operator

Polarization operation for \( l = 0 \):

\[
\Pi_{J J'}^{n n'} (-M^2; m_f, m_{f'}; B) =
\]
\[
\frac{B}{4\pi^2} \text{Tr}_\nu \int_0^1 dt_1 \int_0^1 dt_2 \int_0^1 ds_1 \int_0^1 ds_2 \left( \frac{1 - s_1}{1 + s_1} \right)^{m_f^2/4\nu B} \left( \frac{1 - s_2}{1 + s_2} \right)^{m_{f'}^2/4\nu B} \times
\]
\[
\times t_1^n t_2^{n'} \frac{\partial^n}{\partial t_1^n} \frac{\partial^{n'}}{\partial t_2^{n'}} \frac{1}{\Phi_2^2} \left[ \frac{M^2}{B} \frac{F_1^{(J)}}{\Phi_2} + \frac{m_f m_{f'}}{B} \frac{F_2^{(J)}}{(1 - s_1^2)(1 - s_2^2)} + \frac{F_3^{(J)}}{\Phi_2} \right] \exp \left\{ \frac{M^2 \Phi_1}{2\nu B \Phi_2} \right\}.
\]

\[
\Phi_1 = s_1 s_2 + 2 \left( \xi_1^2 s_1 + \xi_2^2 s_2 \right) (t_1 + t_2) \nu,
\]

\[
\Phi_2 = s_1 + s_2 + 2(1 + s_1 s_2)(t_1 + t_2) \nu + 16(\xi_1^2 s_1 + \xi_2^2 s_2)t_1 t_2 \nu^2,
\]

\[
F_1^{(P)} = (1 + s_1 s_2) \left[ 2(\xi_1 s_1 + \xi_2 s_2)(t_1 + t_2) \nu + 4\xi_1 \xi_2 (1 + s_1 s_2)(t_1 + t_2)^2 \nu^2 + s_1 s_2 (1 - 16\xi_1 \xi_2 t_1 t_2 \nu^2) \right],
\]

\[
F_1^{(V)} = \left( 1 - \frac{1}{3} s_1 s_2 \right) \left[ s_1 s_2 + 16\xi_1 \xi_2 t_1 t_2 \nu^2 + 2(\xi_1 s_1 + \xi_2 s_2)(t_1 + t_2) \nu + 4\xi_1 \xi_2 (1 - s_1^2 s_2^2)(t_1 - t_2)^2 \nu^2, \right.
\]

\[
F_2^{(P)} = (1 + s_1 s_2)^2, \quad F_2^{(V)} = (1 - s_1^2 s_2^2),
\]

\[
F_3^{(P)} = 4\nu (1 + s_1 s_2)(1 - 16\xi_1 \xi_2 t_1 t_2 \nu^2), \quad F_3^{(V)} = 2\nu (1 - s_1 s_2)(1 - 16\xi_1 \xi_2 t_1 t_2 \nu^2).
\]
Masses of radially excited mesons: light mesons

| $m_{u/d}$, MeV | $m_s$, MeV | $m_c$, MeV | $m_b$, MeV | $\Lambda$, MeV | $\alpha_s$ | $R$, fm |
|----------------|------------|------------|------------|--------------|------------|--------|
| 145            | 376        | 1566       | 4879       | 416          | 3.45       | 1.12   |

$$\chi_{YM} = \frac{B^4 R^4}{128 \pi^2} = (604 \text{ MeV})^4$$

$$\frac{\alpha_s}{\pi} \langle F^2 \rangle = \frac{B^2}{\pi^2} = 0.04 \text{ GeV}^4$$

$$q = \frac{B^2 R^4}{16} = 0.2$$

Asymptotic relation for spectrum (Regge trajectories):

$$M_n^2 \sim B n, \quad n \gg 1$$

$$M_l^2 \sim B l, \quad l \gg 1$$

G.V. Efimov and S.N. Nedelko, Phys. Rev. D 51 (1995)

$$\tilde{m} = 136 \text{ MeV}$$

$$\mu_{u/d} = m_{u/d} - \tilde{m}$$

$$\mu_s = m_s - \tilde{m}$$

$$\frac{\mu_s}{\mu_{u/d}} = 26.7$$
Masses of radially excited mesons: light mesons

| meson   | $n$ | $M$, MeV [*] | $\tilde{M}$, MeV | $M$, MeV |
|---------|-----|--------------|------------------|----------|
| $\pi$   | 0   | 140          | 140              | 0        |
| $\pi(1300)$ | 1   | 1300         | 1310             | 1301     |
| $\pi(1800)$ | 1   | 1812         | 1503             | 1466     |
| $K$     | 0   | 494          | 494              | 0        |
| $K(1460)$ | 1   | 1460         | 1302             | 1301     |
| $K$     | 2   | 1655         | 1466             |          |
| $\rho$  | 0   | 775          | 775              | 769      |
| $\rho(1450)$ | 1   | 1450         | 1571             | 1576     |
| $\rho$  | 2   | 1720         | 1946             | 2098     |
| $K^*$   | 0   | 892          | 892              | 769      |
| $K^*(1410)$ | 1   | 1410         | 1443             | 1576     |
| $K^*(1717)$ | 1   | 1717         | 1781             | 2098     |
| $\phi$  | 0   | 1019         | 1039             | 769      |
| $\phi(1680)$ | 1   | 1680         | 1686             | 1576     |
| $\phi$  | 2   | 2175         | 1897             | 2098     |

[*] K.A. Olive et al. (Particle Data Group) Chinese Phys. C 38, 090001, 2014
Masses of radially excited mesons: $\eta$ and $\eta'$

Resolution of $U_A(1)$ problem.

$\Xi\left(\frac{|x-y|}{R}\right)$

| meson     | $n$ | $M$, MeV [*] | $\bar{M}$, MeV | $\tilde{M}$, MeV |
|------------|-----|--------------|-----------------|------------------|
| $\eta$     | 0   | 548          | 621             | 0                |
| $\eta'$    | 0   | 958          | 958             | 872              |
| $\eta(1295)$ | 1   | 1294         | 1138            | 1361             |
| $\eta(1475)$ | 1   | 1476         | 1297            | 1516             |

[*] K.A. Olive et al. (Particle Data Group) Chinese Phys. C 38,090001, 2014
Masses of radially excited mesons: heavy-light mesons

Asymptotic formula:

\[ M_{Q\bar{q}} = m_Q + \Delta_{Q\bar{q}}^{(J)} + O(1/m_Q), \quad m_Q \gg \sqrt{B}, \quad m_Q \gg m_q \]

G.V. Efimov and S.N. Nedelko, Phys. Rev. D 51 (1995)

| meson | \( n \) | \( M, \text{MeV} \) | \( M, \text{MeV} \) |
|-------|-------|----------------|----------------|
| \( D \) | 0     | 1864 [*]       | 1715           |
| \( D \) | 1     | 2579 [†]       | 2274           |
| \( D \) | 2     | 2508           | 2274           |
| \( D_s \) | 0   | 1968 [*]       | 1827           |
| \( D_s \) | 1   | 2670 [†]       | 2521           |
| \( D_s \) | 2   | 2808           | 2521           |
| \( B \) | 0    | 5279 [*]       | 5041           |
| \( B \) | 1    | 5883 [†]       | 5535           |
| \( B \) | 2    | 5746           | 5535           |
| \( B_s \) | 0   | 5366 [*]       | 5135           |
| \( B_s \) | 1   | 5971 [†]       | 5746           |
| \( B_s \) | 2   | 5988           | 5746           |
| \( B_c \) | 0   | 6277 [*]       | 5952           |
| \( B_c \) | 1   | 6842 [†]       | 6904           |
| \( B_c \) | 2   | 7233           | 6904           |

| meson | \( n \) | \( M, \text{MeV} \) | \( M, \text{MeV} \) |
|-------|-------|----------------|----------------|
| \( D^* \) | 0    | 2010 [*]       | 1944           |
| \( D^* \) | 1    | 2629 [†]       | 2341           |
| \( D^* \) | 2    | 2564           | 2341           |
| \( D_s^* \) | 0   | 2112 [*]       | 2092           |
| \( D_s^* \) | 1   | 2716 [†]       | 2578           |
| \( D_s^* \) | 2   | 2859           | 2578           |
| \( B^* \) | 0    | 5325 [*]       | 5215           |
| \( B^* \) | 1    | 5898 [†]       | 5578           |
| \( B^* \) | 2    | 5781           | 5578           |
| \( B_s^* \) | 0   | 5415 [*]       | 5355           |
| \( B_s^* \) | 1   | 5984 [†]       | 5783           |
| \( B_s^* \) | 2   | 6021           | 5783           |

[*] K.A. Olive et al. (Particle Data Group) Chinese Phys. C 38,090001, 2014.
[†] D. Ebert, V. O. Galkin and R. N. Faustov, Phys. Rev. D 57, 5663 (1998) [Erratum-ibid. D 59, 019902 (1999)] [hep-ph/9712318]
[‡] D. Ebert, R. N. Faustov and V. O. Galkin, Eur. Phys. J. C 71, 1825 (2011) [arXiv:1111.0454 [hep-ph]]
Masses of radially excited mesons: heavy quarkonia

Asymptotic spectrum:

\[ M_{Q\bar{Q}} = 2m_Q - \Delta_{Q\bar{Q}}^{(J)} + O(1/m_Q), \quad m_Q \gg \sqrt{B} \]

G.V. Efimov and S.N. Nedelko, Phys. Rev. D 51 (1995)

| meson          | n  | \( M, \text{MeV} \) [*] | \( M, \text{MeV} \) |
|----------------|----|--------------------------|----------------------|
| \( \eta_c(1S) \) | 0  | 2981                     | 2751                 |
| \( \eta_c(2S) \) | 1  | 3639                     | 3620                 |
| \( \eta_c \)    | 2  | 3882                     |                       |
| \( J/\psi(1S) \) | 0  | 3097                     | 3097                 |
| \( \psi(2S) \)  | 1  | 3686                     | 3665                 |
| \( \psi(3770) \) | 2  | 3773                     | 3810                 |
| \( \Upsilon(1S) \) | 0  | 9460                     | 9460                 |
| \( \Upsilon(2S) \) | 1  | 10023                    | 10102                |
| \( \Upsilon(3S) \) | 2  | 10355                    | 10249                |

[*] K.A. Olive et al. (Particle Data Group) Chinese Phys. C 38,090001, 2014
### Leptonic decay constants of pseudoscalar mesons

![Diagram showing lepton decay constants](image)

| meson       | $n$ | $f_P$, MeV | $f_P$, MeV | $\tilde{f}_P$, MeV |
|-------------|-----|------------|------------|---------------------|
| $\pi$       | 0   | 130 [*]    | 140        | 128                 |
| $\pi(1300)$ | 1   |           | 29         |                     |
| $K$         | 0   | 156 [*]    | 175        | 128                 |
| $K(1460)$   | 1   |           | 27         | 29                  |
| $D$         | 0   | 205 [*]    | 212        |                     |
| $D$         | 1   |           | 51         |                     |
| $D_{s}$     | 0   | 258 [*]    | 274        |                     |
| $D_{s}$     | 1   |           | 57         |                     |
| $B$         | 0   | 191 [*]    | 187        |                     |
| $B$         | 1   |           | 55         |                     |
| $B_{s}$     | 0   | 253 [†]    | 248        |                     |
| $B_{s}$     | 1   |           | 68         |                     |
| $B_{c}$     | 0   | 489 [†]    | 434        |                     |
| $B_{c}$     | 1   |           | 135        |                     |

[*] K.A. Olive et al. (Particle Data Group) Chinese Phys. C 38,090001, 2014

[†] T. W. Chiu et al. [TWQCD Collaboration], PoS LAT 2006, 180 (2007) [arXiv:0704.3495 [hep-lat]]
\( g_{V\gamma} \)

\[ \begin{array}{|c|c|c|c|}
\hline
\text{meson} & n & g_{V\gamma} & g_{V\gamma} \\
\hline
\rho & 0 & 0.2 & 0.2 \\
\rho & 1 & \text{—} & 0.034 \\
\omega & 0 & 0.059 & 0.067 \\
\omega & 1 & \text{—} & 0.011 \\
\phi & 0 & 0.074 & 0.069 \\
\phi & 1 & \text{—} & 0.025 \\
J/\psi & 0 & 0.09 & 0.057 \\
J/\psi & 1 & \text{—} & 0.024 \\
\Upsilon & 0 & 0.025 & 0.011 \\
\Upsilon & 1 & \text{—} & 0.0039 \\
\hline
\end{array} \]

\[ \text{[*] K.A. Olive et al. (Particle Data Group) Chinese Phys. C 38,090001, 2014} \]


\[ \bar{g} V P P \]

\[
\begin{array}{c}
\rho^0 \rightarrow \pi^+ \pi^- \\
\omega \rightarrow \pi^+ \pi^- \\
K^{*\pm} \rightarrow K^\pm \pi^0 \\
K^{*\pm} \rightarrow K^0 \pi^\pm \\
\phi \rightarrow K^+ K^- \\
D^{*\pm} \rightarrow D^0 \pi^\pm \\
D^{*\pm} \rightarrow D^\pm \pi^0
\end{array}
\]

\begin{tabular}{|c|c|c|}
\hline
Decay & \( g_{VPP} \) [*] & \( g_{VPP} \) \\
\hline
\( \rho^0 \rightarrow \pi^+ \pi^- \) & 5.95 & 7.58 \\
\( \omega \rightarrow \pi^+ \pi^- \) & 0.17 & 0 \\
\( K^{*\pm} \rightarrow K^\pm \pi^0 \) & 3.23 & 3.54 \\
\( K^{*\pm} \rightarrow K^0 \pi^\pm \) & 4.57 & 5.01 \\
\( \phi \rightarrow K^+ K^- \) & 4.47 & 5.02 \\
\( D^{*\pm} \rightarrow D^0 \pi^\pm \) & 8.41 & 7.9 \\
\( D^{*\pm} \rightarrow D^\pm \pi^0 \) & 5.66 & 5.59 \\
\hline
\end{tabular}

[*] K.A. Olive et al. (Particle Data Group) Chinese Phys. C 38,090001, 2014

local gauge invariance
Overall accuracy of this "tree level" approximation is about 10% (with a couple of exceptions).

The method for calculation of corrections to the tree level is defined

Effective meson action describes strong, electromagnetic and weak interactions of mesons - form factors, etc.
Comparison with Bethe-Salpeter approach

S. Kubrak, C. S. Fischer and R. Williams, arXiv:1412.5395 [hep-ph]
C. S. Fischer, S. Kubrak and R. Williams, Eur. Phys. J. A 51, no. 1, 10 (2015) [arXiv:1409.5076 [hep-ph]]
C. S. Fischer, S. Kubrak and R. Williams, Eur. Phys. J. A 50, 126 (2014) [arXiv:1406.4370 [hep-ph]].

S. M. Dorkin, L. P. Kaptari and B. Kampfer, arXiv:1412.3345 [hep-ph]
S. M. Dorkin, L. P. Kaptari, T. Hilger and B. Kampfer, Phys. Rev. C 89, no. 3, 034005 (2014) [arXiv:1312.2721 [hep-ph]]

\[
S^{-1}(p) = Z_2 S^{-1}_0(p) + 4\pi Z_2^2 C_F \int \frac{d^4 k}{(2\pi)^4} \gamma^\mu S(k+p) \gamma^\nu (\delta_{\mu\nu} - k_\mu k_\nu/k^2) \frac{\alpha_{\text{eff}}(k^2)}{k^2},
\]

\[
\alpha_{\text{eff}}(q^2) = \pi \eta^7 x^2 e^{-\eta^2 x} + \frac{2\pi \gamma_m (1 - e^{-y})}{\ln [e^2 - 1 + (1 + z)^2]}, \quad x = q^2/\Lambda^2, \quad y = q^2/\Lambda_t^2, \quad z = q^2/\Lambda^2_{QCD}
\]

S. Kubrak, C. S. Fischer and R. Williams, arXiv:1412.5395 [hep-ph]
Bridge to Harmonic confinement – Light-Front Holography – soft-wall AdS/QCD

H. Leutwyler and J. Stern, Phys. Lett. B 73 (1978) 75; Annals Phys. 112 (1978) 94.
G. F. de Teramond and S. J. Brodsky, Phys. Rev. Lett. 94 (2005) 201601; Phys. Rev. Lett. 96 (2006) 201601; Phys. Rev. Lett. 102 (2009) 081601;
A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D 74, 015005 (2006) [hep-ph/0602229]

\[ S_\Phi = \frac{(-1)^J}{2} \int d^d x \, dz \left( \frac{R}{z} \right)^{d+1} e^{-\kappa^2 z^2} \left( \partial_N \Phi_J \partial^N \Phi_J - \mu^2_J(z) \Phi_J \Phi_J \right) \]

\[ M_{nJ} = 4\kappa^2 \left( n + \frac{l+J}{2} \right) \]

G.V. Efimov and S.N. Nedelko, Phys. Rev. D 51, 176 (1995)

\[ J^{aJ}(x, z) = \sum_{nl} (z^2)^{l/2} \varphi^{nl}(z) J^{aJln}(x), \quad J^{aJln}(x) = \bar{q}(x) V^{aJln}(x) q(x) \]

\[ n \gg 1: M^2_n \propto Bn \]
\[ l \gg 1: M^2_l \propto Bl \]

\[ \varphi^{nl}_{\mu_1 \ldots \mu_l} = L^l_n \left( z^2 \right) T^{(l)}_{\mu_1 \ldots \mu_l}(n_z), \quad n_z = \frac{z}{\sqrt{z}}, \]

\[ \int_0^\infty du \rho_l(u) L^l_n(u) L^l_{n'}(u) = \delta_{nn'}, \quad \rho_l(u) = u^l e^{-u}. \]
Bosonization of the four-quark interaction

\[ \mathcal{L}_2 = \frac{g^2}{2} \int d^4x \int d^4z D(z|B) J^\dagger_{J\sigma}(x, z|B) J_{J\sigma}(x, z|B). \]

in terms of bi-local colorless meson-like fields \( \Phi_{J\sigma}(x, z) \) leads to the functional integral

\[ Z = \int_{\mathcal{B}} dB \int D\Phi_{J\sigma}(x, z) \exp \left\{ -\frac{1}{2} \int d^4x \int d^4z D(z|B) \Phi^2_{J\sigma}(x, z) + \text{Tr} \ln (1 + 2\sqrt{g} D(z|B) \Phi_{J\sigma}(x, z) V_{J\sigma}(x, z) S(x, y|B)) \right\} \]

\[ S(x, y|B) = \left[i \nabla + gB - m\right]^{-1} \delta(x - y), \quad V_{J\sigma}(x, z) = \Gamma_{J\sigma} \exp \left\{ iz_{\mu} \overleftrightarrow{D_{\mu}}(x) \right\}. \]

The quadratic part of the nonlinear meson action reads

\[ S_2 = -\frac{1}{2} \int d^4x \int d^4z D(z|B) \Phi^2_{J\sigma}(x, z) \]

\[ -2g^2 \int d^4xd^4x'd^4zd^4z' D(z|B) D(z'|B) \Phi_{J\sigma}(x, z) \Pi_{J\sigma, J'\sigma'}(x, x'; z, z') \Phi_{J'\sigma'}(x', z'), \]

\[ \Pi_{J\sigma, J'\sigma'}(x, x'; z, z') = \text{Tr} V_{J\sigma}(x, z) S(x, x'|B) V_{J'\sigma'}(x', z') S(x', x|B) \]

**Integral equation for the meson wave functions \( \varphi_{\mu_1...\mu_l}^{nl}(z) \)!**

\[ \Phi^a_{J\sigma}(x, z) = \sum_{nl} (z^2)^{l/2} \varphi_{\mu_1...\mu_l}^{nl}(z) \Phi^a_{J\sigma\mu_1...\mu_l}(x) \]
Summary

Starting with
\[ \lim_{V \to \infty} \frac{1}{V} \int_V d^4x g^2 F^a_{\mu \nu}(x) F^a_{\mu \nu}(x) \neq 0. \]
once one arrives at the importance of the lumpy structured gluon configurations (almost everywhere homogeneous abelian (anti-)self-dual field) and correctly implemented:

- **Domain wall network as QCD vacuum**: almost everywhere homogeneous Abelian (anti-)selfdual gluon fields.
- **Domain wall network as QCD vacuum**: deconfinement occurs in two stages:
  \[ \langle |\tilde{F}F| \rangle = \langle FF \rangle \to \langle |\tilde{F}F| \rangle \ll \langle FF \rangle \to \langle |\tilde{F}F| \rangle = \langle FF \rangle = 0. \]
- **Confinement of both static and dynamical quarks** \[ \to W(C) = \langle \text{Tr} P e^{i \int_C dz_\mu \hat{A}_\mu} \rangle, \]
  \[ S(x,y) = \langle \psi(y) P e^{i \int_0^y dz_\mu \hat{A}_\mu} \bar{\psi}(x) \rangle \]
- **Dynamical Breaking of** \( SU_L(N_f) \times SU_R(N_f) \) \[ \to \langle \bar{\psi}(x)\psi(x) \rangle \]
- **\( U_A(1) \) Problem** \[ \to \eta', \chi, \text{Axial Anomaly} \]
- **Strong CP Problem** \[ \to \lim_{V \to \infty} \frac{\partial^n}{\partial \theta^n} Z_V(\theta) \neq \partial^n \frac{\partial^n}{\partial \theta^n} Z_V(\theta) \]
- **Colorless Hadron Formation**: \[ \to \text{Effective action for colorless collective modes: spectrum, formfactors} \]
  (Light mesons and baryons, **Regge spectrum** of excited states of light hadrons, **heavy-light** hadrons, **heavy quarkonia**)
- **QCD vacuum is characterized as heterophase mixed state with corresponding phase transition mechanism.**
- **Impact of a strong electromagnetic field as a trigger of deconfinement is indicated.**
- **Basic meson wavefunctions in the domain model practically coincide with meson wavefunctions in soft-wall AdS/QCD with dilaton field** \( \phi = \kappa^2 z^2 \).