Research of Reservoir Rock Properties in Violation of Darcy’s Linear Law

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Abstract: The paper presents the results of research to identify the dependence of reservoir rock properties in violation of the basic linear filtration law. The classification of macroroughness coefficient depending on the permeability is presented. Critical values of the coefficient of permeability when inertial force begins to exert are defined.

1. Introduction
Defining the filtration and capacity properties of geological materials (PES) obtained by laboratory research of cores is an important factor in developing oil and gas fields. Determination of porosity and absolute permeability plays a special role in such research. Quantitative determination of the filtration characteristics is crucial because the permeability of geological materials plays an important role in determining oil recovery factor.

While exploiting oil fields the conditions of occurrence of oil, water and gas vary. This process is accompanied by considerable changes in properties of geological material, stratum fluids, gases, gas-condensate mixes. Therefore, these properties are considered in dynamics depending on the change of formation pressure, temperature and other conditions in the fields.

In this situation, the relevance of the paper is connected with research of permeability and porosity properties of geological material to account for violations of Darcy’s linear law in the development of gas and gas condensate fields and more informative laboratory tests.

2. Research
Absolute permeability testing of rock samples using helium takes a special role in the research. Permeability is a key parameter characterizing conductivity of manifold. According to Darcy’s law, which establishes a linear relationship between mass filtration rate and pressure differential, absolute permeability is a factor of proportionality [1-3]:

\[ v = \frac{Q}{S} = k \left( \frac{\Delta p}{\Delta L} \right), \]

where \( v \) – linear filtration speed, m/s; \( k \) – coefficient of rock permeability; \( Q \) – volumetric flow rate, \( m^3/s \); \( S \) – cross-sectional area of the core, \( m^2 \); \( \mu \) – dynamic viscosity of gas, Pa s; \( \frac{\Delta p}{\Delta L} \) – pressure differential per unit length of the core, Pa/m.

In case of violation of linear filtration law Forchheimer equation is used for fluid in interstitial space:

\[ -\frac{dP}{dx} = \frac{\mu v}{k} + \beta \rho v^2, \]

for describing the gas flow:

\[ \frac{P_2^2 - P_1^2}{L} = \frac{2P_1Q}{kS} + \frac{2\beta P_1^2Q^2}{S^2}, \]
where \( P_a \) – pressure in atmospheric conditions, Pa; throughflow in atmospheric conditions, \( m^3/s \); \( \beta \) – Forchheimer coefficient (macroroughness coefficient), 1/m; \( P \) – pressure, Pa; \( L \) – sample length, m.

Zhengwen Zeng and Reid Grigg [4], S.A. Holditch, R.A.Morse, R.D. Evans, C.S. Hudson and J.E.Greenlee [5], [6] have studied the behavior of the gas flow in porous and fractured media in violation of linear filtration law. One of the conclusions is the fact that macroroughness coefficient is not only the quantity characterizing porous medium, but it is also flow coefficient under influence of inertial forces upon condition of high rates of filtration and increasing of pressure differential.

The most thoroughly studied deviations from Darcy’s law caused inertia effects in the time of increasing rates of filtration. In this case the criterion of applicability of the linear law is Reynolds number which characterizes the ratio of inertial forces and resistance forces (viscosity);

\[
Re = \frac{v_P d}{\mu},
\]

where \( d \) – some specific linear size of porous medium; \( \rho \) – fluid density, kg/m\(^3\).

The upper limit of applicability of Darcy’s law is usually associated with the critical (maximum) value of the Reynolds number. In this case, if Reynolds number in the task is less than the critical value \( Re < Re_{cr} \), then Darcy’s law is done and if \( Re \geq Re_{cr} \), then Darcy’s law is violated. Here are the expressions to calculate Reynolds number:

a) Shchelkachev’s formula:

\[
Re = \frac{10 \rho \sqrt{\kappa}}{m^{2.3} \mu}, \quad 1 < Re < 12;
\]

b) Millionshtchikov’s formula

\[
Re = \frac{\sqrt{k \rho v}}{\mu \sqrt{m m}}, \quad 0.022 < Re < 0.29.
\]

The results in papers [4], [5], [6] allowed us to assume that there is a relationship between Forchheimer coefficient, porosity and permeability.

Experimental researches were carried out with permeameter-porosimeter AP-608, which allows to measure intercommunicating porosity and absolute air permeability of rock samples. Samples for research were prepared in accordance with GOST 26450.0-85 “Rocks. General requirements for the selection and preparation of samples for determination of reservoir properties”. The measurements were conducted with AP-608 using helium as its properties are close to ideal gas. Measurements of samples’ porosity with AP-608 were implemented using volumetrical method applying Boyle-Mariotte’s law. 6000 core samples were measured; helium was passed through them under pressure. Output data obtained with AP-608 containing the information about fractional porosity, permeability and macroroughness coefficients are given below.
Figure 1. Density-porosity dependence.

Figure 2. Macroroughness coefficient-permeability coefficient dependence.
The results of measurement allow drawing the following conclusions:

- dependence of density on porosity has a pronounced linear character, as expected;
- dependence of macroroughness coefficient on permeability is a power function, which is observed in permeability ranging from 0.5 mD. It becomes difficult to determine the form of dependence to that value.
- dependence of porosity on permeability is difficult to determine, because it is necessary to include lithological properties.

Due to large number of samples corresponding to different medium parameters, we have divided them into groups according to Khanin’s classification of rock permeability [7] (Table 1):

| Group | Permeability k(mD) | Macro-roughness coefficient β (micrometers) |
|-------|-------------------|---------------------------------------------|
| 1     | k> 1000           | $\beta \sim \leq 10^7$                     |
| 2     | 500<k< 1000       | $\beta \sim \leq 10^7$                     |
| 3     | 100 < k < 500     | $10^7 < \beta \leq 10^8$                  |
| 4     | 10 < k < 100      | $10^8 < \beta \leq 10^{10}$               |
| 5     | 1 < k < 10        | $10^{10} < \beta \leq 10^{11}$            |
| 6     | 0.1 < k < 1       | $10^{11} < \beta \leq 10^{14}$            |
| 7     | k < 0.1           | $\beta \sim \geq 10^{15}$                 |

As an example, we present the research results for the fourth group of the classification, which is clearly seen as exponential dependence shown in figure 4, and, also the results for the seventh group, where the dependence $\beta(k)$ cannot be determined accurately.

Forchheimer in his equation used dimensionless value $\beta^*$, the formula is $\beta^* = \beta \sqrt{k}$. In design equation using AP-608 we took into account the usual macro-roughness coefficient $\beta$. But comparing the graphs for $\beta$ and $\beta^*$, we can conclude that they are very similar and the further calculations can be carried out using Forchheimer coefficient.
**Figure 4.** Changing Forchheimer coefficient* and Forchheimer coefficient for different permeability values of samples belonging to group 4 of the classification.

**Figure 5.** Changing Forchheimer coefficient for different values of porosity for samples belonging to group 4 of the classification.
As can be seen from the dependence, shown in the figure 4 macroroughness coefficient is a function of permeability and has power-law dependence. Using data obtained with AR-608 (Picture 5, 6), a certain kind of dependence between Forchheimer coefficient and porosity and also between porosity and permeability cannot be identified.

**Figure 6.** Dependence of porosity on permeability for the samples belonging to group 4 of the classification.

**Figure 7.** Changing Forchheimer coefficient* and Forchheimer coefficient for different permeability values of samples belonging to group 7 of the classification.
Calculating of Reynolds numbers using formulas (5) and (6) was carried out according to the results of laboratory tests of 6000 samples of rocks from various oil and gas fields in Western Siberia. The dependence of Reynolds number, calculated with Shchelkachev’s formula (5) on the absolute permeability of the samples is shown in figure 10. Similar dependence of Reynolds number, calculated with Millionshtchikov’s formula (6) is shown in figure 11: the minimum critical Reynolds number is indicated with the red line. The intersection of this line with the graph of dependence of Reynolds number permeability is the critical value of the absolute permeability, the excess of which leads to a violation Darcy’s law.
Figure 10. Changing permeability for different Reynolds numbers, calculated with Shehelkachev’s formula.

It should be noted that the critical Reynolds number, as shown by the authors of the formulas (5) and (6) is not unequivocal, but varies in a fairly wide range of values, and therefore the critical permeability may also vary.

Figure 11. Changing permeability for different Reynolds numbers, calculated with Millionshtchikov’s formula.
3. Conclusion

Thus, it was found that the determination of absolute permeability of rock samples in laboratory conditions using helium is significantly affected by inertial forces, characterized with Forchheimer coefficient. Also, it was determined that macroroughness coefficient depends on the absolute permeability: if permeability increases, macroroughness coefficient decreases. This dependence is expressed by a power formula with a high degree of correlation. The classification of macroroughness coefficient depending on permeability, according to Khanin’s classification was developed.

It was found that the violation of Darcy’s law occurs with the samples which permeability is more than 80 mD according to Shchelkachev’s formula and more than 0.7 mD according to Millionshtchikov’s formula. These values show that if rock samples’ absolute permeability exceeds the specified critical values, inertial forces start to exert. This causes using of the power filtration laws, for example, Forchheimer equation.

References
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