Optimal Parameter Scenario of Borehole Thermal Resistance for Vertical Ground Heat Exchanger Using Taguchi Method

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Abstract. The borehole thermal resistance is a significant design indicator for the ground heat exchanger (GHE) and careful design procedure is required for this indicator to decrease the length of GHE and capital investments. In this study, a modified two-dimension numerical model with the resistance model is first applied to analyze the effects of parameters including the borehole radius, pipe-pipe distance, pipe radius, soil thermal conductivity and grout thermal conductivity on the borehole thermal resistance. To implement the analysis, an L\textsuperscript{27} (3\textsuperscript{5}) is established. Then, the Taguchi method is carried out to obtain the optimal scenarios of parameters combination. Results using the optimal parameters set reveal that the maximum borehole thermal resistance decreased value and rate can be up to 0.146 m.K/W and 81.56%. The analysis of variance technique is applied lastly to analyze the relative significance of different parameters. The results indicate that the borehole radius, grout thermal conductivity, and pipe radius are important driving parameters in determining the borehole thermal resistance, while the pipe-pipe distance and ground thermal conductivity almost play no role in it.

1. Introduction
The ground source heat pump (GSHP) system is widely employed due to its high energy efficiency and eco-friendly. However, the high initial cost limits the development of such system [1]-[2]. The core component of the GSHP system is the ground heat exchanger (GHE). Compared with the horizontal GHE, the vertical one occupied less space area and has higher heat transfer efficiency. To improve the heat transfer performance of the GHE, parameters such as borehole thermal resistance, soil thermal conductivity, soil thermal diffusivity, and buried pipe length should be evaluated and calculated [3].

Many researchers have made researches on the borehole thermal resistance. Gu et al. [4]-[5] employed the equivalent diameter method to simplify the two-dimensional heat conduction problem into the one-dimensional one, and then the equivalent borehole thermal resistance was calculated. It should be noted that this method does not take the influence of the position of each pipe into consideration. Based on the experiment, Paul et al [6] obtained the empirical formula of the borehole thermal resistance. However, the accuracy of this mathematical expression is largely influenced by the number of experiments. Hellstrom et al. [7] investigated the borehole thermal resistance in different parameters set based on the virtual heat source method. This model made some ideal assumptions and the borehole thermal resistance may be underestimated or overestimated. Sharqawy et al. [8] established a two-dimensional steady-state heat transfer model by setting the Dirichlet boundary on
the inlet and outlet pipe walls and the borehole wall. Actually, the temperature of the borehole wall is not constant, and the result may make some errors.

Based on the above research, an improved two-dimension heat conduction numerical model is established by using the COMSOL Multiphysics 5.4 finite element software. Five parameters, including the pipe radius, borehole radius, pipe-pipe distance, soil thermal conductivity, and grout thermal conductivity along with different levels are taken into account and L_{27} orthogonal array is established as the input data of the numerical simulation. The borehole thermal resistance in different cases can be calculated by using the resistance model. Then, the Taguchi method is employed to obtain the optimal scenarios of parameters combination and the analysis of variance (ANOVA) technique is carried out lastly to figure out the relative significance of five tested parameters. The nomenclature, Greek symbols, and subscript used in this paper can be seen in Table 1.

### Table 1. Nomenclature, Greek symbols, and subscript used in this paper.

| Nomenclature | Subscript |
|--------------|-----------|
| $T$          | b         |
| $q$          | c         |
| $r$          | eff       |
| $R$          | g         |
| $x$          | in        |
| $m$          | out       |
| $n$          | p         |
| $l$          | s         |
| $Y$          | 2D        |
| Greek symbols | 1         |
| $\lambda$    | 2         |

2. **Two-dimension heat conduction numerical model**

2.1. **Heat transfer model**

The schematic diagram of the two-dimensional steady-state heat transfer model in GHE is shown in figure 1. This model ignores the convective heat transfer of the fluid in the pipe, which is a small part of total borehole thermal resistance. The position and size of the upward and downward pipes in the cross-section are considered.

![Schematic diagram of two-dimensional steady-state heat transfer model in GHE.](image)
The governing equations of the model are listed as follows:

Heat conduction equation in the grout:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) = 0 \quad r = \sqrt{x^2 + y^2}
\]  

(1)

Heat conduction equation in the ground:

\[
\lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) = 0 \quad \sqrt{x^2 + y^2} \geq r_b
\]  

(2)

The borehole thermal resistance can be calculated by equation (3):

\[
R = \frac{(T_{in} + T_{out})/2 - T_b}{q_1 + q_2}
\]  

(3)

The initial temperature of ground and grout are set as 290.15 K and the temperature of inlet and outlet pipe walls are 305.15 K and 300.15 K respectively. The boundary area can hardly be influenced when the soil calculation area is large enough. In the actual engineering, the distance between the two adjacent boreholes is 3-6 m, so the circle boundary with a distance of 6 m away from the borehole center can be chosen as the far boundary.

2.2. Numerical simulation

The heat transfer model above is meshed by using COMSOL Multiphysics 5.4 finite element software, and the meshing diagram is shown in figure 2.

![Figure 2. Schematic diagram of the numerical model.](image)

Most literature [9]-[10] set the Dirichlet boundary or Neumann boundary on the borehole wall. However, the borehole wall temperature and heat transfer rate at each point may be different due to the discrepancies in inlet and outlet water temperature and shorter distance between the pipe wall and the borehole wall. By setting the domain point probe and the boundary probe on the borehole wall and pipe wall respectively, the mean heat transfer rate for upward and downward pipes and borehole wall
temperature can be attained by averaging the temperature value of each set point on the borehole wall and the heat transfer rate of each set boundary. The values of $T_b$, $q_1$, and $q_2$ can be calculated by using equation (4) and equation (5).

$$T_b = \frac{1}{m} \sum_{j=1}^{m} T_{mj}$$  \hspace{1cm} (4)

$$q_i = l_i q_{ij} / \sum_{j=1}^{n} l_{ij} \quad i = 1, 2$$  \hspace{1cm} (5)

2.3. Mesh and probe sensitivity analysis

To ensure that the numerical simulation works out the accurate results and save the calculation time meanwhile, 20 models with different meshes and probes are carried out. The input parameters used in these models can be seen in Table 2.

|          | $r_m$/m | $r_b$/m | $x_c$/m | $\lambda_s$/W.m$^{-1}$.K$^{-1}$ | $\lambda_g$/W.m$^{-1}$.K$^{-1}$ |
|----------|----------|----------|----------|-------------------------------|-------------------------------|
|          | 0.01     | 0.06     | 0.05     | 0.60                          | 1.20                          |

Figure 3 gives the results of the integral average borehole wall temperature evolution for various meshes and probes. The result shows that the root mean squared error (RMSE) and mean relative error (MRE) of $T_b$ for 70 probes and 100 probes are less than 0.01°C and 0.1% respectively. It also can be seen in figure 3 that the difference between the result of 654 meshes and 938 meshes is less than 3%. Hence, 70 probes and 938 meshes are selected. The analysis has also been carried out in other cases, but not reported here for the sake of brevity.

![Figure 3. Results of mesh and probe sensitivity analysis.](image-url)
2.4. Model validation
In this work, five different parameters set (summarized in Table 3) are chosen for the numerical model and they are validated with the results of Paul, Hellstrom, Bennet, and Sharqawy. Figure 4 illustrates the values of borehole thermal resistance calculated by using different methods. It can be seen that the simulation results show similar trends as the results of Paul, Hellstrom, Bennet, and Sharqawy. A reasonable agreement can be seen between the diagram of the values of borehole thermal resistance obtained by the numerical simulation model and those calculated by Paul, Hellstrom, Bennet, and Sharqawy.

Table 3. Thermophysical parameters used in the model validation.

| $r_p$/m | $r_b$/m | $x_c$/m | $\lambda_s$/W.m$^{-1}$.K$^{-1}$ | $\lambda_g$/W.m$^{-1}$.K$^{-1}$ |
|--------|--------|--------|-----------------|-----------------|
| 0.010  | 0.06   | 0.05   | 0.60            | 1.20            |
| 0.016  | 0.08   | 0.06   | 1.20            | 1.60            |
| 0.020  | 0.10   | 0.07   | 0.90            | 1.80            |
| 0.020  | 0.08   | 0.05   | 0.60            | 1.60            |
| 0.010  | 0.06   | 0.06   | 0.90            | 1.20            |

Figure 4. Borehole thermal resistance for different parameters combination.

Table 4. RMSE and MAE between the simulation and calculation values.

|        | Paul   | Hellstrom | Bennet | Sharqawy |
|--------|--------|-----------|--------|----------|
| RMSE   | 1.13%  | 0.13%     | 0.26%  | 0.23%    |
| MAE    | 0.86%  | 0.10%     | 0.24%  | 0.16%    |
Table 4 shows the RMSE and MRE of borehole thermal resistance between the numerical simulation model and those calculated by Paul, Hellstrom, Bennet, and Sharqawy. It can be seen that the difference between the simulation value and those calculated by Paul, Hellstrom, Bennet, and Sharqawy is small and the numerical model shows high reliability and accuracy. It is worth noting that the simulated values deviate significantly from Paul’s value in some cases, and the RMSE and MAE values are 1.13% and 0.86%, respectively. This is mainly because the value of borehole thermal resistance calculated by Paul is based on experiments and its accuracy is highly correlated with the number of experiments.

3. Taguchi method

Taguchi method [11]-[12] is an engineering optimization method, which emphasizes that the improvement of product quality is through design, rather than test. This method uses a standard orthogonal table and signal-to-noise (S/N) ratios to obtain the maximum information and best level of parameter combination with a minimum number of experiments. The S/N ratio, derived from the communication system, is an important concept in the Taguchi method. The larger the value of the S/N ratio, the better the performance of the system. There are three types of objection function to calculate the S/N ratio: the larger-the-better, the nominal-the-better, and the smaller-the-better. The objectives of the present research are to find the optimal scenarios for the value of borehole thermal resistance. The smaller value of borehole thermal resistance is recommended. Therefore, the smaller-the-better objection function shown in equation (6) is chosen for the optimal scenarios of borehole thermal resistance.

\[
S/N = -10 \times \log_{10} \left( \frac{1}{n} \sum_{j=1}^{n} Y_j^2 \right)
\]  

(6)

Three typical pipe size (DN20, DN32, and DN40) and three ground materials are chosen in the numerical model. This study takes three borehole radii of 0.06 m, 0.08 m, and 0.10 m and three grout materials into consideration. Three pipe-pipe distance is also selected in the numerical simulation. Table 5 lists the parameters influencing the value of borehole thermal resistance and their corresponding levels.

| Label | Parameter | Level |
|-------|-----------|-------|
| A     | \( r_p \) | 0.010 | 0.016 | 0.020 |
| B     | \( r_b \) | 0.06  | 0.08  | 0.10  |
| C     | \( x_c \) | 0.05  | 0.06  | 0.07  |
| D     | \( \lambda_s \) | 0.60  | 0.90  | 1.20  |
| E     | \( \lambda_g \) | 1.20  | 1.60  | 2.00  |

Five parameters and three levels are taken into consideration in the numerical simulations. To minimize the number of experiments and lessen the experiment effort without influencing the accuracy of the result, an L^{27} (3^5) orthogonal array is built for analyzing the value of S/N ratio and borehole thermal resistance under different parameters combination. Table 6 lists the 27 parameters combination cases and corresponding values of borehole thermal resistance. The S/N ratios using smaller-the-better objection function are also shown in Table 6.
Table 6. Taguchi L$_2^7$($3^5$) array with the corresponding borehole thermal resistance and S/N ratios.

| No. | A   | B    | C    | D    | E    | R    | S/N   |
|-----|-----|------|------|------|------|------|-------|
| 1   | 5   | 0.06 | 0.05 | 0.60 | 1.20 | 0.131| 17.624|
| 2   | 0.010| 0.06 | 0.05 | 0.60 | 1.60 | 0.098| 20.158|
| 3   | 0.010| 0.06 | 0.05 | 0.60 | 2.00 | 0.079| 22.087|
| 4   | 0.010| 0.08 | 0.06 | 0.90 | 1.20 | 0.159| 15.968|
| 5   | 0.010| 0.08 | 0.06 | 0.90 | 1.60 | 0.119| 18.459|
| 6   | 0.010| 0.08 | 0.06 | 0.90 | 2.00 | 0.096| 20.392|
| 7   | 0.010| 0.10 | 0.07 | 1.20 | 1.20 | 0.180| 14.889|
| 8   | 0.010| 0.10 | 0.07 | 1.20 | 1.60 | 0.135| 17.382|
| 9   | 0.010| 0.10 | 0.07 | 1.20 | 2.00 | 0.108| 19.317|
| 10  | 0.016| 0.06 | 0.06 | 1.20 | 1.20 | 0.082| 21.762|
| 11  | 0.016| 0.06 | 0.06 | 1.20 | 1.60 | 0.062| 24.204|
| 12  | 0.016| 0.06 | 0.06 | 1.20 | 2.00 | 0.050| 26.101|
| 13  | 0.016| 0.08 | 0.07 | 0.60 | 1.20 | 0.114| 18.880|
| 14  | 0.016| 0.08 | 0.07 | 0.60 | 1.60 | 0.086| 21.361|
| 15  | 0.016| 0.08 | 0.07 | 0.60 | 2.00 | 0.069| 23.286|
| 16  | 0.016| 0.10 | 0.05 | 0.90 | 1.20 | 0.166| 15.619|
| 17  | 0.016| 0.10 | 0.05 | 0.90 | 1.60 | 0.124| 18.116|
| 18  | 0.016| 0.10 | 0.05 | 0.90 | 2.00 | 0.099| 20.053|
| 19  | 0.020| 0.06 | 0.07 | 0.90 | 1.20 | 0.054| 25.376|
| 20  | 0.020| 0.06 | 0.07 | 0.90 | 1.60 | 0.041| 27.718|
| 21  | 0.020| 0.06 | 0.07 | 0.90 | 2.00 | 0.033| 29.547|
| 22  | 0.020| 0.08 | 0.05 | 1.20 | 1.20 | 0.118| 18.569|
| 23  | 0.020| 0.08 | 0.05 | 1.20 | 1.60 | 0.089| 21.060|
| 24  | 0.020| 0.08 | 0.05 | 1.20 | 2.00 | 0.071| 22.993|
| 25  | 0.020| 0.10 | 0.06 | 0.60 | 1.20 | 0.138| 17.209|
| 26  | 0.020| 0.10 | 0.06 | 0.60 | 1.60 | 0.103| 19.704|
| 27  | 0.020| 0.10 | 0.06 | 0.60 | 2.00 | 0.083| 21.639|

Table 7 shows the results of the average S/N ratios for five parameters at various levels. On the basis of results shown in Table 7, the optimal parameter combination for borehole thermal resistance is obtained by selecting the largest S/N ratio for each parameter: $A_3$ ($r_p=0.020$ m), $B_1$ ($r_b=0.06$ m), $C_3$ ($x_c=0.07$ m), $D_2$ ($\lambda_s=0.90$ W.m$^{-1}$.K$^{-1}$), $E_3$ ($\lambda_g=2.00$ W.m$^{-1}$.K$^{-1}$). An extra numerical simulation is
operated by using the optimal parameter combination of borehole thermal resistance and the optimal value is 0.033 m.K/W. The maximum borehole thermal resistance decreased value and decrement rate can be up to 0.146 m.K/W and 81.56% respectively.

Table 7. Average S/N ratio response table.

| Level | A  | B  | C  | D  | E  |
|-------|----|----|----|----|----|
| 1     | 18.48 | 23.84 | 19.59 | 20.22 | 18.43 |
| 2     | 21.04 | 20.11 | 20.60 | 21.25 | 20.91 |
| 3     | 22.65 | 18.21 | 21.97 | 20.70 | 22.82 |
| Delta | 4.17 | 5.63 | 2.39 | 1.03 | 4.39 |
| Rank  | 3   | 1  | 4  | 5  | 2  |

4. Analysis of variance

The relative significance of various parameters and the contribution rate of different parameters can be obtained by employing the analysis of variance (ANOVA) method. The parameter with higher F value indicates a higher contribution to the optimal scenarios of borehole thermal resistance. Table 8 shows the degree of freedom (DOF), the sum of squares (SS), mean of squares (MS), F value, P value, and the contribution rate of different parameters for borehole thermal resistance. Table 8 reveals that the contribution rate order from largest to smallest is B (borehole radius), E (grout thermal conductivity), A (pipe radius), C (pipe-pipe distance), and D (ground thermal conductivity). The borehole radius is a significant contributory factor to the development of borehole thermal resistance whereas the pipe-pipe distance and ground thermal conductivity almost do not affect the borehole thermal resistance.

Table 8. ANOVA table for the borehole thermal resistance.

| Level | DOF | SS   | MS   | F   | P   | Contribution rate |
|-------|-----|------|------|-----|-----|--------------------|
| A     | 2   | 79.67| 39.84| 19620.72 | 0       | 23.08%            |
| B     | 2   | 147.61| 73.81| 36351.93 | 0       | 42.76%            |
| C     | 2   | 25.81| 12.91| 6356.67  | 0       | 7.48%             |
| D     | 2   | 4.81 | 2.41 | 1185.07  | 0       | 1.39%             |
| E     | 2   | 87.23| 43.61| 21481.76 | 0       | 25.27%            |
| Error | 16  | 0.03 | 0.00 |       |       |                    |
| Total | 26  | 345.17|      |       |       | 100%              |

5. Conclusions

A combination of improved 2D numerical simulation and Taguchi method is applied to optimize the value of borehole thermal resistance in this study. The ANOVA technique is also employed to find out the relative importance of different parameters. The conclusions are listed as follows.

1. By implementing the Taguchi method for the borehole thermal resistance, the optimal parameter combination for $R_{b,3D}$ is obtained as $A_3$ ($r_p=0.020$ m), $B_1$ ($r_b=0.06$ m), $C_3$ ($x_c=0.07$ m), $D_2$ ($\lambda_s=0.90$ W.m$^{-1}$.K$^{-1}$), $E_3$ ($\lambda_g=2.00$ W.m$^{-1}$.K$^{-1}$).

2. The minimum borehole thermal resistance is 0.033 m.K/W by using the optimal parameters set and the maximum borehole thermal resistance decreased value and decrement rate can be up to 0.146 m.K/W and 81.56% respectively.
ANOVA technique infers that the borehole radius, grout thermal conductivity, and pipe radius play vital roles in optimizing the value of borehole thermal resistance with 42.76%, 25.27% and 23.08% of the contribution rate. The impact of pipe-pipe distance and ground thermal conductivity can be nearly neglected.

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References
[1] Ghoreishi-Madiseh SA, Kuyuk AF and Rodrigues de Brito MA. An Analytical Model for Transient Heat Transfer in Ground-Coupled Heat Exchangers of Closed-Loop Geothermal Systems. *Appl Therm Eng* 2019;150:696–705.
[2] Gao J, Li A, Xu X, Gang W and Yan T. Ground Heat Exchangers: Applications, Technology Integration and Potentials for Zero Energy Buildings. *Renew Energy* 2018;128:337–49.
[3] Zhang L, Zhang Q, Huang G and Du Y. A p(t)-Linear Average Method to Estimate the Thermal Parameters of the Borehole Heat Exchangers for In Situ Thermal Response Test. *Appl Energy* 2014;131:211–21.
[4] Gu Y and O’Neal DL. Development of an Equivalent Diameter Expression for Vertical U-Tubes Used in Ground-Coupled Heat Pumps. *ASHRAE Trans* 1998;104:347–55.
[5] Shonder JA and Beck JV. Determining Effective Soil Temperature Thermal Properties from Field Data Using Parameter Estimation Technique. *ASHRAE Trans* 1999;105:458–66.
[6] Paul ND. The Effect of Grout Thermal Conductivity on Vertical Geothermal Heat Exchanger Design and Performance. South Dakota State University, Vermillion, the United States, 1996.
[7] Hellstrom G. Thermal Analysis of Duct Storage Systems: Part I. Theory. Department of Mathematical Physics, University of Lund, Lund, Sweden, 1991.
[8] Sharqawy MH, Mokheimer EM and Badr HM. Effective Pipe-to-Borehole Thermal Resistance for Vertical Ground Heat Exchangers. *Geothermics* 2009;38:271–7.
[9] Zeng H, Diao N and Fang Z. Efficiency of Vertical Geothermal Heat Exchangers in the Ground Source Heat Pump System. *J Therm Sci* 2003;12:77–81.
[10] Loveridge F and Powrie W. 2D Thermal Resistance of Pile Heat Exchangers. *Geothermics* 2014;50:122–35.
[11] Ji D, Wei Z, Mazzoni S, Mengarelli M, Rajoo S and Zhao J. Thermoelectric Generation for Waste Heat Recovery: Application of a System Level Design Optimization Approach via Taguchi Method. *Energy Convers Manag* 2018;172:507–16.
[12] Adewale P, Vithanage LN and Christopher L. Optimization of Enzyme-Catalyzed Biodiesel Production from Crude Tall Oil Using Taguchi Method. *Energy Convers Manag* 2017;154:81–91.