INTERSTELLAR SCINTILLATION OF THE DOUBLE PULSAR J0737–3039

B. J. Rickett1, W. A. Coles1, C. F. Nava1, M. A. McLaughlin2, 3, S. M. Ransom1, F. Camilo5, 6, R. D. Ferdman3, 7, P. C. C. Freire7, 8, M. Kramer8, 9, A. G. Lyne3, and I. H. Stairs9

1 ECE Dept., University of California San Diego, La Jolla, CA 92093-0407, USA; bjrickett@ucsd.edu
2 West Virginia University, Morgantown, WV 26505, USA
3 Jodrell Bank Center for Astrophysics, University of Manchester, M13 9PL, UK
4 National Radio Astronomy Observatory, Charlottesville, VA 22903, USA
5 National Astronomy and Ionosphere Center, Arecibo, PR 00612-8346, USA
6 Columbia Astrophysics Laboratory, Columbia University, New York, NY 10027, USA
7 Dept. of Physics, McGill University, Montréal, QC H3A 2T8, Canada
8 Max Planck Institut für Radioastronomie, Auf dem Hügel 69, D-53121 Bonn, Germany
9 Dept. of Physics and Astronomy, University of British Columbia, Vancouver, BC V6T 1Z1, Canada

Received 2014 February 3; accepted 2014 March 28; published 2014 May 15

ABSTRACT

We report a series of observations of the interstellar scintillation (ISS) of the double pulsar J0737–3039 over the course of 18 months. As in earlier work, the basic phenomenon is the variation in the ISS caused by the changing transverse velocities of each pulsar, the ionized interstellar medium (ISM), and the Earth. The transverse velocity of the binary system can be determined both by very long baseline interferometry and timing observations. The orbital velocity and inclination is almost completely determined from timing observations, but the direction of the orbital angular momentum is not known. Since the Earth’s velocity is known, and can be compared with the orbital velocity by its effect on the timescale of the ISS, we can determine the orientation Ω of the pulsar orbit with respect to equatorial coordinates (Ω = 65 ± 2°). We also resolve the ambiguity (i = 88:7 or 91:3) in the inclination of the orbit deduced from the measured Shapiro delay by our estimate i = 88:1 ± 0:5. This relies on the analysis of the ISS over both frequency and time, and provides a model for the location, anisotropy, turbulence level, and transverse phase gradient of the IISM. We find that the IISM can be well-modeled during each observation, typically of a few orbital periods, but its turbulence level and mean velocity vary significantly over the 18 months.

Key words: binaries: general – ISM: general – pulsars: general – pulsars: individual (J0737–3039)

Online-only material: color figures

1. INTRODUCTION

The double pulsar binary system J0737–3039 is in a highly relativistic orbit with significant eccentricity (Lyne et al. 2004). It is an eclipsing binary that is a wonderful laboratory for studies of general relativity (Kramer et al. 2006). Detailed measurements of the eclipses of A have been used to probe the magnetosphere of the B neutron star (McLaughlin et al. 2005, hereafter Paper 1). In this paper we found a correlation between the ISS of pulsars A and B near the time of A’s eclipse by B. From the correlation, we concluded that the orbital inclination angle was considerably closer to 90° than had been expected on the basis of the original measurements of the Shapiro delay (Lyne et al. 2004).

The main purpose of the observations reported here was to make use of the Earth’s orbital velocity to improve, calibrate, and align the earlier scintillation analyses. This would allow us to correct the center of mass velocity for the motion of Earth, to orient the binary orbit with respect to the celestial reference frame, and to locate the distance of the scattering region. The additional observations were also expected to improve the estimates of the inclination of the orbit, the anisotropy of the IISM, and the spatial spectrum of the electron density of the IISM. However, two factors made the original plan of analysis impossible. First, the phase at which emission from B is easily detectable drifted away from the time of A’s eclipse during the course of the 2004–2005 observations. This made the measurements of the correlation between the A and B pulsars much less consistent and reliable than had been expected. Second, we observed that although the turbulence in the IISM is homogeneous over several binary orbits, it is not stationary over a year nor is the velocity of the IISM constant over the year. This phenomenon was also observed by S. M. Ord (2010, private communication) when his group attempted to observe the effect of the Earth’s orbit on scintillations of PSR J1141−6545.

Subsequent pulse timing measurements have determined the Shapiro delay and proper motion with greater precision (Kramer et al. 2006). There have also been long baseline interferometry measurements of parallax and proper motion (Deller et al. 2009). We have altered our analysis of the timescale variations to take advantage of these observations, and we have modeled the entire
two-dimensional (2D) time–frequency correlation function of the scintillations rather than simply modeling its timescale. These changes have made it (just) possible to obtain a consistent interpretation. This provides the distance to the scattering region and the orientation of the pulsar orbit in celestial coordinates. It also provides an inclination estimate that is consistent with the Shapiro delay.

We now realize that the scattering is homogeneous over several binary orbits because the proper motion of the pulsar is low and the binary orbit remains entirely within the “scattering disk.” Since the measured intensity is a summation over waves that have traveled through all possible paths through the scattering disk, it is quite homogeneous over that area, even if the underlying turbulence is not. However, from month to month, as we repeated the observations, different realizations of the plasma turbulence occupied the scattering disk. The level of turbulence was clearly non-stationary on this timescale.

2. OBSERVATIONS AND REDUCTION TO DYNAMIC SPECTRA

Observations of the double pulsar system were made specifically for this project with the 100 m Robert C. Byrd Green Bank Telescope at intervals of 1–2 months from 2004 July to 2005 July. All the observations were made with the SPIGOT auto-correlation spectrometer, summing the polarizations every 81.92 μs (Kaplan et al. 2005). On 12 of the epochs, observations were made with 1024 frequency channels over an 800 MHz bandwidth centered near 1900 MHz, however only 600 MHz of the bandwidth was sufficiently free of radio frequency interference (RFI) to be useful. We have also been able to analyze data taken primarily for timing at 820 MHz with 1024 channels over a 50 MHz bandwidth on five epochs, the first of which was analyzed in Paper 1. In total, new and older observations span about 18 months (see Table 1 for dates). We also analyzed 1400 MHz dynamic spectra recorded earlier at Parkes (Manchester et al. 2005; Burgay et al. 2005).

The first step in the analysis of these data was to create dynamic spectra for each pulsar in each frequency band. We edited the raw SPIGOT data files for RFI and Fourier transformed the correlations with the Van Vleck corrections. Using SIGPROC (Lorimer 2001), we formed full pulse profiles at each frequency with 64 and 256 phase bins for pulsars A and B, respectively. Individual frequency channels were shifted with respect to each other using a dispersion measure of 48.9 cm$^{-3}$ pc (Lyne et al. 2004). These were added to create profiles at intervals of 10 s at 1900 MHz and 5 s or 10 s at 820 MHz. The pulse intensity from each profile was estimated by first subtracting the average in an off-pulse window, and then integrating the profile to create a dynamic spectrum of pulse intensity versus frequency and time. We estimated the gain in the passband using both the mean and rms in each frequency channel. We found substantial variation in both measures over the relatively broad bandwidths used. The rms appeared to be the better measure as it was less affected by RFI, which was a serious problem at 1900 MHz. Accordingly, we corrected the gain of each channel by dividing it by the rms in that channel.

Figure 1 shows the dynamic spectra of the A pulsar at 1900 MHz on MJD 53560 (top) and 820 MHz on MJD 53467 (bottom). The upper panel shows the mottled structure of ISS with islands of high intensity (scintles) whose typical timescale varies from a few samples up to tens of samples, repeating over the orbital period of 147 minutes. This observation should be interpreted as the motion of the line of sight through a quasi-stationary spatial pattern, as the pulsar and Earth move in their orbits and the ISM drifts in a linear fashion. The time variation is determined by the spatial structure and the velocity of the line of sight. The frequency correlation is only a few samples wide and does not vary over the orbit, as discussed in Section 3.1.

The lower panel shows 2.5 orbital periods at 820 MHz. There is minimal RFI in this band and the eclipse by the B pulsar can be seen as thin vertical lines of low intensities near 142 and 289 minutes. One can see that the ISS timescale is significantly shorter and the bandwidth is much narrower, as expected for stronger scattering at the lower frequency. There are many more scintles in this dynamic spectrum. This property and the absence of RFI make the analysis of the 820 MHz observations more satisfactory than the 1900 MHz observations. Sloping features, which are obvious in both plots, change sign over the orbital period. These are due to frequency dependent refraction as we discuss in Section 4.

The dynamic spectra at 1900 MHz were often contaminated by RFI. The full 800 MHz bandwidth was reduced to 600 MHz because of nearly continuous RFI in the remainder of the band. In other cases, RFI was short-lived and we flagged regions of the dynamic spectra where RFI was suspected and carried that flag array through subsequent processing. Flagged data were excluded from all subsequent analysis. In Figure 1, the flagged data were clipped at ±3σ for plotting only.

3. CHARACTERIZATION AND MODELLING OF THE SCINTILLATION

In many early ISS studies, the dynamic spectrum was characterized by only two parameters—the characteristic widths in time and frequency. These were usually estimated from auto-covariance functions (acf$s$) versus time and frequency, in each case averaged over the other coordinate. More generally, the acf can be computed in two dimensions with cuts along the two axes providing time and frequency widths. Recently, observers have analyzed the secondary spectrum, which is the (2D) Fourier transform of the acf (Stinebring et al. 2001). For nearby strong young pulsars with highly anisotropic scattering, these secondary spectra show a wealth of interesting information in the form of parabolic arcs. The secondary spectra of pulsar A do show parabolic arcs, but they are not sufficiently well-defined to assist with our analysis. However, we have found the 2D acf very useful in estimating both the anisotropy of the spatial structure and the mean phase gradient over the scattering disk. This is discussed in detail in Section 4.

A theoretical model for the 2D acf $ρ_2(τ, ν)$ is developed in Appendix A. It is actually a “cut” through a 3D acf $ρ_3(r = Vτ, ν)$, where $V$ is the velocity of the line of sight through the IISM and $r$ is the transverse coordinate. Thus the apparent timescale is the spatial scale in the direction of $V$ divided by the speed. The width in frequency is inversely proportional to the rms scattering angle, and thus to the strength of scattering. Here, as we want to study how the time and frequency scales of pulsar A vary over its orbit, we have computed the 2D acf from short blocks of the dynamic spectrum (which must be long enough to include at least one ISS timescale). An example is shown in the upper panel of Figure 2 for the data from MJD 53560 shown in Figure 1; the lower panel is a best-fit theoretical model that we discuss in Section 4.

3.1. ISS Characteristics and Model

The characteristic scales in frequency and time have been defined by where the auto-correlation functions fall to 0.5
and $e^{-1}$, respectively, and we will adhere to this convention. In our analysis, we estimate these by fitting an ISS model to each acf, since the fitting makes full use of the data. In deriving the model, we assume that the scintillations are dominated by scattering from a thin region a distance $z_0$ from the observer. The electron density fluctuations in the screen are described by a Kolmogorov wavenumber spectrum that is homogeneous, at least over the scattering disk. The pulsar is at a distance of $z_p$ from the observer and the fractional distance from the pulsar to the screen is $s = (z_p - z_0)/z_p$. Note that this thin scattering region can dominate the scattering, which is a path integral over density squared, but may not dominate the dispersion, which is a path integral over density. We also assume that refractive variations are negligible over the scales of interest (i.e., the scintillations are in the diffractive limit). This model is the starting point for many studies of ISS, which have been used to investigate fine-scale turbulence in the interstellar plasma (Armstrong et al. 1995; Cordes & Lazio 2005). These studies assumed an isotropic density spectrum, but recent observations such as Brisken et al. (2010) have shown evidence for anisotropy, so we also include it here.
Propagation through such a layer causes a phase modulation that is usefully characterized by the phase structure function, \( D_\phi(\sigma) = \langle (\phi_p(r) - \phi_p(r + \sigma))^2 \rangle \), where \( \phi_p(r) \) is the plasma phase contribution at transverse coordinate \( r \). The electric field correlation at the output of the phase screen is \( \Gamma_\phi(\sigma) = \exp(-0.5D_\phi(\sigma)) \), and it is invariant with distance.

We describe the anisotropy by two quantities, the axial ratio \( A_R \) and the orientation of the major axis \( \psi_{AR} \) of the inhomogeneities in the plasma density. In terms of the major (\( \sigma_{maj} \)) and minor axes (\( \sigma_{min} \)), the structure function can be written as

\[
D_\phi(\sigma) = \left( \frac{\sigma_{maj}^2}{s_0^2 A_R} + \frac{A_R \sigma_{min}^2}{s_0^2} \right)^{5/6}.
\]

In rotated coordinates (\( \sigma_x, \sigma_y \)) the quadratic form becomes

\[
D_\phi(\sigma) = Q(\sigma)^{5/6} s_0^{-5/3},
Q(\sigma) = a\sigma_x^2 + b\sigma_y^2 + c\sigma_x\sigma_y,
\]

with the major axis rotated by \( \psi_{AR} \) clockwise from the \( x \) axis. The mean diffraction scale \( s_0 \) decreases as the strength of the scattering increases. In this work we find it convenient to define anisotropy in terms of the bounded parameter \( R \),

\[
R = (A_R^2 - 1)/(A_R^2 + 1),
\]

which lies in the range 0 to 1. The coefficients of the quadratic form \( Q \) become

\[
a = [1 - R \cos(2\psi_{AR})]/\sqrt{1 - R^2},
\]

\[
b = [1 + R \cos(2\psi_{AR})]/\sqrt{1 - R^2},
\]

\[
c = -2R \sin(2\psi_{AR})/\sqrt{1 - R^2}.
\]

In the diffractive scintillation limit, the spatial correlation function for intensity is \( C_I(\sigma) = \Gamma_\phi^2(\sigma) \) at the Earth. There are no intensity fluctuations at the output of the screen. The temporal correlation function at the Earth is \( C_I(\tau) = \exp(-0.5D_\phi(V_{los}^2)) \), where \( V_{los} = (1 - s)V_{PA} + sV_E - V_{IS} \) is the transverse velocity of the “line of sight” with respect to the plasma. Here \( V_{E}, V_{IS} \), and \( V_{PA} \) are the velocities of the Earth, the plasma, and pulsar A, respectively, (Cordes & Rickett 1998), and are all defined with respect to the Sun.

The acf versus frequency in the diffractive limit is more complex (Lovelace 1970). It involves a Fourier-like integral that was solved by Armstrong & Rickett (1981) for the case of an isotropic Kolmogorov spectrum. We have done this integral numerically for the anisotropic model and fitted it to the acf of each block in each observing epoch. From this fit we obtain the parameters \( \Delta_{IS} \) and \( T_{IS} \) as a function of orbital phase \( \phi \). Details are given in Appendix A. The two scales \( \Delta_{IS}(\phi) \) and \( T_{IS}(\phi) \) are plotted in Figure 3 for the 820 MHz dynamic spectra shown in the lower panel of Figure 1 vs. true anomaly orbital phase \( \phi \).

The fit of a covariance model to the measured covariance. Most of the variance is due to white noise, so the error bars have not been corrected for the correlation between measured covariance points. These error bars are used in a subsequent weighted fit; however, the error estimates resulting from this subsequent fit are independent of the scaling of the errors in \( T_{IS} \).\(^{10}\)

\(^{10}\) Errors are one standard deviation throughout unless defined otherwise.
of freedom and one could expect, in an isotropic plasma, to require more information. This was first proposed by Lyne (1984) and used by Ord et al. (2002), but differs from Equations (4) and (5) of Bogdanov et al. (2002). Here $V_o$ is a mean orbital velocity given by $V_o = 2\pi a/(P_b\sqrt{1-e^2})$ in terms of $P_b$, the period, $a$ the semi-major axis of $A$, and $e$ the eccentricity of the orbit. We define the inclination $i$ as the angle between the orbital angular momentum $\mathbf{I}$ and the direction from the Earth toward the center of mass $\mathbf{s}$. We choose the $x$ coordinate along the line of nodes ($\mathbf{s} = \mathbf{x} \times \mathbf{I}/\sin i$ and $\mathbf{K} = \mathbf{s} \times \mathbf{x}$), which with an inclination near 90°, would make the angular momentum nearly anti-parallel to the $y$ axis.

We rewrite Equation (5) in the pulsar frame and model the data $T_{ISS}(\phi)$ by

$$(1/T_{ISS})^2 = Q(V_o^2)/s_p^2,$$

(5)

so here too $T_{ISS}$ depends inversely on $V_{oA}$.

In our case, pulsar $A$ is in a binary system so that $V_{PA}$ is the sum of the binary center of mass velocity $V_p$ and $A$’s orbital velocity $V_{oA}$ about the center of mass, which both have been measured accurately from pulsar timing (Kramer et al. 2006). It is the effect of the varying orbital velocity that is responsible for the variation in $T_{ISS}(\phi)$ over the orbital period in Figure 3. We can then fit a model to $T_{ISS}(\phi)$ and use the known $V_{oA}$ to calibrate a model for the other variables ($s_0$, $s$, and $V_{Ks}$). This was first proposed by Lyne (1984) and used by Ord et al. (2002) and Ransom et al. (2004) under isotropic scattering and generalized to anisotropic scattering in Paper 1. The technique is most valuable for short period binaries because the orbit lies entirely within the scattering disk and a homogeneous scattering model fits the observations over a few orbits very well. We will show that the time series $T_{ISS}(\phi)$ possesses five degrees of freedom and one could expect, in an isotropic plasma, to determine the scattering variables listed above as well as the orbital inclination $i$. When the plasma is anisotropic there are two more unknowns in $A_R$ and $\psi_{AR}$ and a complete solution requires more information.

As the orbital velocity of the pulsar is the reference, we use Equation (5) in the pulsar frame where the spatial scale is $s_p = s_0/(1 - s)$. The appropriate “scintillation velocity” is then $V_A = V_{oA}/(1 - s) = V_C + V_{oA}$, where

$$V_C = V_P + V_E s/(1 - s) - V_{KS}/(1 - s),$$

(6)

which is constant during an observation, but obviously varies over the year. We can write the transverse scintillation velocity of $A$ $V_A$, in terms of $V_C$, its true anomaly $\theta$, and its orbital phase from the line of nodes $\phi = \omega + \theta$, where $\omega$ is the longitude of periastron, as follows:

$$V_{Ax} = V_{Cx} + V_o[\epsilon \sin \theta \cos \phi - (1 + e \cos \theta) \sin \phi]$$

$$= V_{Cx} - V_o \epsilon \cos \omega - V_o \sin \phi$$

$$V_{Ay} = V_{Cy} + \cos i[V_o \epsilon \sin \phi + V_o(1 + e \cos \theta) \cos \phi]$$

$$= V_{Cy} + \cos i[V_o \epsilon \cos \omega + V_o \cos \phi]$$

(7)

Here ($V_{Ax}$, $V_{Ay}$) is in agreement with the equations given by Ord et al. (2002), but differs from Equations (4) and (5) of Bogdanov et al. (2002). Here $V_o$ is a mean orbital velocity given by $V_o = 2\pi a/(P_b\sqrt{1-e^2})$ in terms of $P_b$, the period, $a$ the semi-major axis of $A$, and $e$ the eccentricity of the orbit.

We rewrite Equation (5) in the pulsar frame and model the data $T_{ISS}(\phi)$ by

$$Q(V_A^2)/s_p^2 = (a V_{Ax}^2 + b V_{Ay}^2 + c V_{Ax} V_{Ay})/s_p^2.$$ 

(8)

When the velocities from Equation (7) are substituted into Equation (8), we obtain the expression as a sum of five harmonics.

$$(1/T_{ISS}(\phi))^2 = K_0 + K_s \sin \phi + K_C \cos \phi + K_{S2} \sin 2\phi + K_{C2} \cos 2\phi,$$

(9)

where the harmonic coefficients are shown below.

$$K_0 = [0.5V_o^2(a + b \cos^2 i) + a(V_{Cx} - V_o \epsilon \sin \omega)^2 + b(V_{Cy} + V_o \epsilon \cos \omega \cos i)^2 + c(V_{Cx} - V_o \epsilon \sin \omega)(V_{Cy} + V_o \epsilon \cos \omega \cos i)]/s_p^2$$

$$K_s = -V_o[2a(V_{Cx} - V_o \epsilon \sin \omega) + c(V_{Cy} + V_o \epsilon \cos \omega \cos i)]/s_p^2$$

$$K_C = V_o \epsilon i [c(V_{Cx} - V_o \epsilon \sin \omega) + 2b(V_{Cy} + V_o \epsilon \cos \omega \cos i)]/s_p^2$$

$$K_{S2} = -0.5c V_o^2 \cos i/s_p^2$$

$$K_{C2} = 0.5V_o^2 (-a + b \cos^2 i)/s_p^2.$$ 

(10)

These five harmonics carry all the information in the data set, that is, they are “sufficient statistics” for the ISS with a general (eccentric) orbit.

The ISS can be characterized by five quantities ($V_{Cx}$, $V_{Cy}$, $s_0$, $R$, and $\psi_{AR}$), so that the five measurable parameters are insufficient to determine all the ISS parameters and $i$, unless $R$ is known to be zero. Further, the double pulsar is nearly edge-on so $K_{S2}$ and $K_C$ are small and the signal-to-noise ratio is marginal. One could still solve for $s_0$, $V_{Cx}$, and $V_{Cy}$ if $R$ and $\psi_{AR}$ were known. But since the ISS anisotropy is not known, extra information is needed, as discussed in Section 3.3. Thus we estimate the five harmonic coefficients as a first step in extracting the full ISS parameter set and the inclination of the orbit.

In estimating $T_{ISS}$, we split the dynamic spectrum of $A$ into time blocks of length 310 or 630 s, subtracted the mean from each channel, and computed the temporal autocorrelation function $\rho(t)$ of each block averaged over all channels. We fitted a

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**Table 1** The Five Orbital Harmonic Coefficients versus Date

| Day | $K_0$ | $K_s$ | $K_C$ | $K_{S2}$ | $K_{C2}$ |
|-----|-------|-------|-------|----------|----------|
| −3  | ±0.8  | −0.1  | ±0.3  | 0.1  | ±0.5  | ±0.8  |
| 202 | 0.4  | ±0.1  | ±1.1  | 0.12  | ±0.1  | ±1.1  |
| 203 | ±0.1  | ±0.2  | ±0.7  | 0.00  | ±0.4  | ±0.8  |
| 247 | ±0.4  | ±0.2  | ±0.9  | 0.02  | ±0.0  | ±0.8  |
| 315 | ±0.3  | ±0.1  | ±0.6  | 0.04  | ±0.0  | ±0.4  |
| 319 | ±0.4  | ±0.1  | ±0.9  | 0.05  | ±0.4  | ±1.1  |
| 374 | ±0.1  | ±0.2  | −0.9  | ±0.0  | ±0.6  | ±0.9  |
| 378 | ±0.4  | ±0.2  | ±0.5  | −0.0  | ±0.2  | ±0.6  |
| 380 | ±0.3  | ±0.4  | −0.0  | ±0.0  | ±0.0  | ±0.4  |
| 467 | ±0.2  | ±0.5  | ±0.1  | ±0.0  | ±0.1  | ±0.6  |

**Notes.** The first five lines are 820 MHz and the latter 12 lines are at 1900 MHz. The units are $10^{-4}$ s$^{-2}$. Day = MJD-53000.
theoretical model to the autocorrelation of each block, beginning at the first non-zero time lag to avoid the “noise spike.” We used the theoretical form for diffractive ISS from a Kolmogorov scattering medium \( \rho(t) = \exp\left(-t/T_{\text{ISS}}\right)^{3/2} \). Because the time resolution in the dynamic spectrum is 5 or 10 sec, we convolved the model acf with the appropriate triangular resolution function. Also, since the block autocorrelation estimates are weighted with a triangle function, we applied that same triangle weight to the model before fitting.

In choosing the block length, there is a trade-off between the need for short block lengths, as the timescale is continuously changing, and the need to keep the block length longer than the timescale. At 820 MHz, \( T_{\text{ISS}} \) was always shorter than 310 s, so we fixed the blocks at that length. However, at 1900 MHz, the diffractive scintillation scale is larger, which means that the longest timescales are sometimes greater than 310 s. So at the higher frequency, we computed \( T_{\text{ISS}} \) for blocks of both 310 s and 630 s (one-fourteenth of the orbital period). We then selected the estimates from the shorter blocks, except where the longer block estimate gave \( T_{\text{ISS}} > 310 \) s, in which case we replaced the two nearest shorter blocks by the single block of 630 s. We note that when \( T_{\text{ISS}} \) is large, it is quite sensitive to the form of the correlation function used in the model and is accompanied by larger errors. This makes it difficult to exploit the fact that near the times of slow ISS the scintillation velocity vector rotates rapidly and so has the potential to determine the anisotropy of the ISS. At 1900 MHz we omitted the occasional blocks that were contaminated by RFI in all channels.

We then fitted the five harmonic model versus orbital phase (\( \phi \)) to the estimates of \( T_{\text{ISS}} \) from each block. Because Equation (9) is linear in the five harmonic coefficients, we started by fitting to \( T_{\text{ISS}}(\phi)^{-2} \) directly. However, for a weighted fit we need errors in \( T_{\text{ISS}}^{-2} \), which could not be reliably obtained from the error (\( \sigma_{T_{\text{ISS}}} \)) estimated for each \( T_{\text{ISS}} \). So we used the reciprocal of the square root of that equation to model \( T_{\text{ISS}}(\phi) \), after first smoothing it over the range of phases in each block. We performed a non-linear least squares fit with residuals weighted by \( 1/\sigma_{T_{\text{ISS}}} \) starting from the model parameters of the \( T_{\text{ISS}}^{-2} \) fit. The fits for the five coefficients and their standard errors are listed in Table 1. As expected the coefficients \( K_C \) and \( K_{S2} \) are small for all epochs. Examples of the fits are shown in Figure 4.

Figure 4. Plots of \( T_{\text{ISS}}(\phi) \) vs. true anomaly orbital phase \( \phi \) at 1900 MHz (left to right) from MJD 53378 & 53560 and 820 MHz from MJD 53211. Data are open symbols with error bars (magenta for the longer blocks). The 3 and 5 parameter fits are both plotted but are barely distinguishable.

(A color version of this figure is available in the online journal.)
be done for six of the best dynamic spectra, including all five observations at 820 MHz. The process is complicated by the need to also fit for a transverse gradient in the interstellar phase, because the phase gradient causes a first-order effect on $\rho_I$ off the time axis. This process is discussed in Section 4. The result is that we are able to estimate the five IISM parameters plus the phase gradient at each of these six epochs. This is very useful in searching for time variations in parameters otherwise assumed to be constant.

Accordingly we use a hybrid procedure. First, we assume $\cos i = 0$ and fit the annual variation of the three strong orbital harmonics. From this we can estimate the five IISM parameter plus $s$ and $\Omega$. Second, we include $\cos i$ in the parameters and add the two weaker orbital harmonics to the data to be fit. This provides an estimate of $\cos i$ and also improves the estimate of the anisotropy. Then we fit the frequency–time acfs for the six best dynamic spectra and obtain estimates of the time variation of the IISM parameters. Finally, we include these time variations in the fit of the annual variation of the five orbital harmonics.

Note that the hybrid analysis is necessarily a compromise. We are estimating diffractive scintillation parameters from a few hours of data at each epoch, in a situation where we expect statistical variations in these parameters on the timescale for refractive scintillations (several days). Thus the estimates are in the “snapshot” regime, as discussed by Romani et al. (1986), who give theoretical predictions for the rms variation expected for various diffractive ISS parameters, such as the anisotropy, see also the simulations by Coles et al. (2010). Thus an ideal model could allow the ISS parameters to vary from one epoch to the next, as in the five observations at 820 MHz. However, there is insufficient information to include such variations in the analysis at 1900 MHz of $T_{\text{iss}}(\phi)$ and its harmonic coefficients.

### 3.3.2. Non-stationarity in the Level of Turbulence

The harmonic coefficients $K_{C2}$ and $K_{S2}$ are independent of $V_E$ and should be constant over the year. Although $K_{S2}$ is too weak to be useful in testing this hypothesis, $K_{C2}$ is accurately measured. It is inversely proportional to the square of the spatial scale in the $x$ direction (i.e., $s^2_p/\alpha$) and to the frequency decorrelation width $\Delta \nu_{\text{iss}}$.

We found that both $K_{C2}$ and $\Delta \nu_{\text{iss}}$ varied significantly over the 18 months of our observations. We used $K_{C2}$ to infer $\Delta \nu_{\text{iss}}$ and plotted both the inferred and directly measured $\Delta \nu_{\text{iss}}$ in Figure 5 (upper and lower panels, respectively). The equation used is

$$
\Delta \nu_{\text{iss}} = \frac{-a V_0^2}{2K_{C2}} \frac{s}{1-s} \frac{2\pi v_m^2}{c L}, \tag{11}
$$

where $L$ is the distance from the Earth to the pulsar and $v_m$ is the center frequency. One can see very significant variations in both, although they are not identical. The difference is most probably because they have different dependence on anisotropy. In particular, only $K_{C2}$ depends on $\psi_{AR}$ and this parameter can vary between different realizations of the same random process.

We can remove the effect of these changes in $s_p$ on the other parameters by normalizing all the harmonic coefficients by $K_{C2}$, but this will not correct for minor variations in the anisotropy. This variation in the diffractive scale $s_p$ seems greater than expected from refractive effects, but as in other pulsar observations it may indicate that the scattering medium is not well represented as homogeneous turbulence on a timescale of weeks to months (Hemberger & Stinebring 2008). Thus we might also expect to see some variation in the anisotropy reflected in the harmonic analysis. We return to this question in Section 4.

### 3.3.3. Annual Variation in Harmonic Coefficients

The transverse velocity of the Earth and the proper motion of the pulsar are defined in celestial coordinates, so we rotate them by the unknown angle $\Omega$ into $(x,y)$ coordinates defined by the pulsar orbital plane (i.e., $V_{xy} = MV_{\alpha\delta}$) where $M$ is

$$
M = \begin{pmatrix}
\sin \Omega & \cos \Omega \\
-\cos \Omega & \sin \Omega
\end{pmatrix}.
$$

It remains convenient to define the unknown interstellar velocity in $x, y$ coordinates. We then substitute $V_C$ into Equation (10), and knowing the Earth’s velocity, obtain model equations for the harmonic coefficients at each epoch. We expect $s_p$ to depend on the observing frequency, but we have removed its influence by normalizing the two remaining coefficients by $K_{C2}$ (i.e., $k_i = K_s/K_{C2}$ and $k_0 = K_0/K_{C2}$). Thus we can combine the data from all frequency bands (820, 1400, and 1900 MHz) by
plotting $k_s$ and $k_0$ versus date in Figure 6. The overlap of values from different frequencies confirms the validity of this approach.

The equations for the two normalized coefficients versus date now involve $V_C$, $R$, $\psi_{AR}$, $s$, and $\Omega$, but it is convenient to group them into the following three combinations:

$$u_x = (V_{C_x} - e V_o \sin \omega)/V_o,$$
$$u_y = \sqrt{b/a}(V_{C_y} + e V_o \cos \omega \cos \omega)/V_o,$$
$$w = c/\sqrt{ab},$$

then we have

$$k_s = 4u_x + 2wu_y,$$
$$k_0 = -1 - 2u_x^2 - 2wu_xu_y - 2u_y^2.$$  

One can see that $k_s$ is linear in velocity, whereas $k_0$ is quadratic. The known sinusoidal variation in $V_x$ will appear in $k_s$ scaled by $s/(1 - s)$ and shifted in phase due to the rotation of coordinates by $\Omega$. So the annual variation of $k_s$ has three degrees of freedom (including the constant). The additional information provided by $k_0$ is the square of a known sine wave plus an unknown constant. The semianharmonic will provide an estimate of the axial ratio and the annual an estimate of $V_{IS}$. Thus including the constant term and $K_{c2}$ there are seven degrees of freedom in total. This is sufficient to estimate $s_0$, $s$, $\Omega$, $V_{IS}$, $R$, and $\psi_{AR}$. The remaining two coefficients, $k_{s2}$ and $k_s$, are highly correlated and add only two degrees of freedom. However, with $\omega$ and/or $e$ known from timing, one can estimate $\cos i$.

In terms of the physical parameters we have

$$u_x = p_1 + p_2 V_{Ex}, \quad u_y = p_3 + p_4 V_{Ey},$$

where

$$p_1 = [V_p - V_{IS}]/(1 - s) - e V_o \sin \omega]/V_o,$$
$$p_2 = s/[V_o(1 - s)],$$
$$p_3 = \sqrt{b/a}[V_{Py} - V_{ISy}]/(1 - s) + e V_o \cos \omega \cos \omega]/V_o,$$
$$p_4 = p_2 \sqrt{b/a}.$$

In fitting the model, we use the parameters obtained from the pulsar timing solution of Kramer et al. (2006) for eccentricity $e$ and for the longitude of periastron $\omega$ as a function of date; for the proper motion velocity we use $V_{Pa} = -17.8$, $V_{Pp} = 11.6 \, \text{km s}^{-1}$ from Deller et al. (2009). This depends on the VLBI parallax distance of 1.15$^{+0.16}_{-0.11}$ kpc to the pulsar, which is larger than 0.5 kpc based on the Cordes & Lazio (2005) Galactic electron model. This smaller distance and the most recent proper motion estimated from pulse timing (M. Kramer 2014, private communication) give a slower pulsar proper motion velocity: $V_{Pa} = -5.3$, $V_{Pp} = 6.2 \, \text{km s}^{-1}$. In Section 4.1, we discuss the small effect of using this lower pulsar velocity.

We initially optimized all six physical parameters $s$, $\Omega$, $V_{IS}$, $R$, and $\psi_{AR}$ (under the assumption that $\cos i = 0$) to fit the data for $k_s$ and $k_0$ at 820 and 1900 MHz. The result is shown in Figure 6 by a thin cyan line. This fit has a high reduced $\chi^2 \sim 9$, largely due to the obvious outlier at day $-3$ (MJD 52997). This is an 820 MHz observation with very small error bars but it is obviously discrepant because it does not agree with a similar observation one year later. The dynamic spectrum for these data were at 5 s intervals, in contrast to all the others, which used 10 s intervals. Although this would change the pulse intensity and could influence $T_{ISS}$, it should not distort the estimation of the harmonic coefficients in $T_{ISS}$. It suggests an error in our assumptions that the velocity and anisotropy of the ISM were constant over this period, so we continued the analysis with the data point at MJD 52997 excluded. The results are shown as a blue line in Figure 6 and tabulated in the first column of Table 2. The $\chi^2$ is reduced to a more reasonable 2.6, but the errors on the velocity and the anisotropy are still high. We conclude that the location of the scattering medium $s$ is quite accurately estimated and the orientation of the pulsar orbit in celestial coordinates $\Omega$ is adequately determined, but the parameters of the ISM are weakly constrained.

### 3.3.4. Constraining the orbital inclination and scattering anisotropy

In view of the difficulties in fitting a fixed set of parameters to $k_s$, $k_0$ and with the hope of determining the sign of $\cos i$, we tried fitting all four normalized harmonic coefficients with $\cos i$ restricted to $i = 88:7$ or $91:3$. The measured Shapiro delay (Kramer et al. 2006) provides estimates of $\sin i$, leaving an ambiguity in the sign of $\cos i$, bounded by $|\cos i| = 0.023^{+0.013}_{-0.009}$. The observed harmonics $k_1$, $k_{s2}$ are plotted in Figure 7 versus date and the best fit models for these two inclinations are overplotted. Even with the relatively large errors in these coefficients, it is clear that the solid curves for $i = 88:7$ fit better than the dashed curves for $i = 91:3$. These two fits are tabulated in the third and fourth columns of Table 2. It is remarkable how good the fits are with fixed $\cos i$, and the model can be made even better with this fixed $\cos i$. This shows that the $\chi^2$ is not a good measure of the goodness of fit, and that $\cos i$ is not well constrained from the timing data alone.

![Figure 6](image_url) Normalized harmonic coefficients $k_s$ and $k_0$ vs. day. Symbols indicate the observing frequency in GHz. Points at 1.4 GHz were not included in the fit, since their error bars were substantially larger. Three models are shown as solid lines for $k_s$ and dashed lines for $k_0$: the thinnest (cyan) lines are constant velocity fits, including the point at day $-3$; the medium wide (black) lines are the same, but excluding day $-3$. The widest (magenta) lines include day $-3$, but are fitted with variable velocity as discussed in Section 4.

(A color version of this figure is available in the online journal.)

### Table 2

Parameters Estimated from Fitting to the Annual Variation of the Normalized Orbital Harmonic Coefficients at 0.8 and 1.9 GHz, Excluding MJD 52997

| Parameter | $i = 90^\circ$ | $i = 91:3$ | $i = 88:7$ |
|-----------|---------------|------------|------------|
| $s$       | 0.71 ± 0.03   | 0.70 ± 0.02| 0.70 ± 0.02|
| $\Omega$ (deg) | 69 ± 31     | 111 ± 8   | 61 ± 8    |
| $R$       | 0.76 ± 0.27   | 0.96 ± 0.11| 0.71 ± 0.21|
| $\psi_{AR}$ (deg) | 72 ± 36   | 118 ± 8   | 61 ± 9   |
| $V_{ISx}$ (km s$^{-1}$) | $-12 ± 29$ | $-79 ± 68$ | $-9 ± 11$ |
| $V_{ISy}$ (km s$^{-1}$) | 50 ± 32 | $\geq 100$ | 42 ± 22 |
| $\Delta$ | 26            | 58         | 58         |
| Reduced $\chi^2$ | 2.6          | 3.2        | 2.8        |

Note. The first column is a two-harmonic fit, the others are four-harmonic fits.
much the inclusion of the fit for cos $i$ and the two extra harmonics improved the error bars on all the fitted parameters. The results for $i = 91^\circ 3$ have a higher $\chi^2$, the parameters do not match those of the two harmonic fit well, and the IISM velocities are well outside the expected range. We conclude that the location of the screen and the orientation of the orbital plane are now accurately estimated, and the estimates of the anisotropy and velocity of the IISM are now useful. Encouraged by this result we included cos $i$ as a fitted parameter, obtaining $i = 88^\circ 6 \pm 0^\circ 4$.

To show the improvement in fitting four harmonics, we redo the three fits discussed (i.e., the fit for two harmonics with cos $i = 0$ and the fits for four harmonics with cos $i = \pm 0.023$). Here we step $R$ and 2$\psi_{AR}$ over the ranges: 0 $\leq R \leq$ 1; 0 $\leq 2\psi_{AR} \leq 2\pi$, while fitting for the other four parameters at each grid point. The left panel of Figure 8 shows the sum of the squares of the residuals as a polar plot in this space, which could be called a Poincaré circle; see also a similar plot by Grall et al. (1994), as well as simulations of pulsar scattering (Coles et al. 2010). They occur due to dispersive refraction in the interstellar medium, which causes a frequency dependent spatial shift in the ISS pattern that is mapped to a frequency dependent time shift by the pulsar velocity. The effect can be seen very clearly in upper panel of Figure 2 where the observed frequency–time (2D) acfs show regular changes in slope that are synchronous with the orbital period. We find that this variation can be modeled well with a constant phase gradient through which the line of sight moves during the pulsar orbit. The modeled acfs are shown in the right panel and are discussed below.

We computed 2D acfs for all the observing epochs at both 820 and 1900 MHz. For the 820 MHz observations we were also able to fit models to the 2D acfs in all the blocks in each of the five epochs, providing estimates for the velocity, anisotropy, spatial scale, and the phase gradient for those five epochs. The block length was 320 sec for MJD 52997, which was sampled at 5 s intervals; for the four remaining epochs, the sample interval

The left panel shows the result for the two harmonic fit with $i = 90^\circ$. The mean squared error surface is very broad, covering the left half of the Poincaré circle. The middle panel is for a four harmonic fit with $i = 88^\circ 7$. It is much more compact, but consistent with the left panel. The right panel is for a four harmonic fit with $i = 91^\circ 3$. It is completely disjointed with the other panels and shows a much higher axial ratio. We conclude that cos $i > 0$ and that the six parameters are reasonably well constrained.

The fit is not completely satisfactory because we have not explained the discrepancies at day $-3$ and day $560$. It seems most likely that in addition to the level of turbulence, which we know to vary, that the velocity of the IISM or its anisotropy must also vary with time. In searching for time variations, we need to obtain more information at each epoch because the three primary harmonic coefficients are only capable of constraining $\delta p$ and two other parameters. However we can assume that we know $s$, $\Omega$, and cos $i$. So we only need to constrain two more parameters to obtain an independent fit at each epoch. We will show in the following section that this can be done using the full 2D frequency–time acf.

4. THE FREQUENCY-TIME STRUCTURE AND LARGE SCALE PHASE GRADIENTS

The dynamic spectra in Figure 1 show striking features that are tilted in the frequency–time plane. Such tilted structures have often been seen in pulsar ISS observations (Hewish et al. 1985; Gupta et al. 1994), as well as simulations of pulsar scattering (Coles et al. 2010). They occur due to dispersive refraction in the interstellar medium, which causes a frequency dependent spatial shift in the ISS pattern that is mapped to a frequency dependent time shift by the pulsar velocity. The effect can be seen very clearly in upper panel of Figure 2 where the observed frequency–time (2D) acfs show regular changes in slope that are synchronous with the orbital period. We find that this variation can be modeled well with a constant phase gradient through which the line of sight moves during the pulsar orbit. The modeled acfs are shown in the right panel and are discussed below.

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![Figure 7](image1.png)

Figure 7. Normalized harmonic coefficients $k_s$ and $k_{s2}$ vs. day (plot symbols for observations at 820 and 1900 MHz as in the legend of Figure 6). Models were fit to all four harmonic coefficients, excluding those on day $-3$. The solid medium-wide (black) lines are constant velocity models with $i = 88^\circ 7$ and the dashed medium-wide (black) lines are the same with $i = 91^\circ 3$. The widest (magenta) line is from a variable velocity fit discussed in Section 4, which includes day $-3$.

![Figure 8](image2.png)

Figure 8. Residuals ($\chi^2$) from fitting harmonic coefficients vs. date (excluding MJD 52997) in a “Poincaré” polar plot of anisotropy ($R$, 2$\psi_{AR}$). Residuals are normalized to the global minimum. Left: fitting two coefficients $k_s$, $k_0$. Center: fitting four coefficients $k_s$, $k_0$, $k_1$, $k_{s2}$ with $i = 88^\circ 7$. Right: fitting four coefficients $k_s$, $k_0$, $k_1$, $k_{s2}$ with $i = 91^\circ 3$. Looking toward the pulsars their orbit line of nodes ($x$ axis) is to the right in the plot. The orientation of the major axis ($\psi_{AR}$) is defined clockwise from $x$. 

![Figure 198x92 to 203x220](image3.png)

![Figure 204x89 to 209x220](image4.png)

![Figure 210x92 to 215x220](image5.png)

![Figure 216x92 to 221x220](image6.png)

![Figure 222x92 to 227x220](image7.png)

![Figure 228x92 to 233x220](image8.png)

![Figure 234x92 to 239x220](image9.png)

![Figure 240x92 to 245x220](image10.png)

![Figure 246x92 to 251x220](image11.png)

![Figure 252x92 to 257x220](image12.png)

![Figure 258x92 to 263x220](image13.png)

![Figure 264x92 to 269x220](image14.png)

![Figure 270x92 to 275x220](image15.png)

![Figure 276x92 to 281x220](image16.png)

![Figure 282x92 to 287x220](image17.png)

![Figure 288x92 to 293x220](image18.png)

![Figure 294x92 to 299x220](image19.png)

![Figure 300x92 to 305x220](image20.png)
was ≈10 s and the blocks were 63 samples. At 1900 MHz the blocks were 630 s, but the fitting was much less successful due to sporadic narrow band RFI, and only the observations on MJD 53560, shown in Figures 1 and 2, provided a useful set of acfs.

The computation of the 2D acf from each block of data is such that it is tapered by a triangle in time lag, but is essentially unbiased versus frequency lag, because the recorded bandwidth is much wider than that of the ISS. In addition, the acf contains a contribution from white noise as a spike at the origin. As described in Section 3.2, we fit a theoretical model to the temporal acf, which is well sampled, beginning at the first time lag. Then we extrapolate the model back to zero lag to estimate the variance of the ISS. This is used both to replace the acf at zero lag and to normalize the acfs at all frequency and time lags. Correct normalization is essential in obtaining good fit to the frequency variation because the frequency axis is rather undersampled and the point at the origin is very important in the fit. We computed the theoretical model of the acf with higher resolution than the observations, filtered it to include the effect of finite resolutions in time and frequency, and also multiplied it by the triangular taper in time lag.

4.1. Model Fitting of the Frequency–time acf

As noted in Section 3.1, the spatial correlation of intensity $C_I(\mathbf{r}) = \exp(-D_\sigma(\mathbf{r}))$. However, $C_I(\mathbf{r}, \Delta \nu)$ is more complex, as described in Appendix A. The 2D (frequency–time) acf of a dynamic spectrum is $C_I(\mathbf{r} = V_{\text{los}}(\tau, \Delta \nu)$. If $V_{\text{los}}$ is constant, this 2D acf carries no information on spatial anisotropy, but in a binary orbit $V_{\text{los}}$ varies over a considerable angular range and it becomes possible to estimate both the anisotropy and the velocity. However, we must also consider the effects of a transverse gradient in $\phi_p$, which can have a significant effect on $C_I(\mathbf{r}, \Delta \nu)$, as commonly seen in the ISS of some pulsars. Such a gradient causes refraction by an angle:

$$\theta_p = \nabla \phi_p / k \propto \nu^{-2}. \quad (16)$$

The refraction displaces the ISS pattern by a transverse vector $\sigma_p$, which, due to dispersion, is $\propto \nu^{-2}$. In Appendices A and A.1 we include this refractive shift, assumed to be constant over the pulsar orbit, in the theoretical model.

The model involved the following eight parameters: $\nabla \phi_p$, characterized by $\sigma_p$, relative to the orbital $x$ axis; $R$, and $\psi_{AR}$; $\Delta V_{\text{iss}}$ and the timescaling factor $V_0/s_0$; and the normalized velocities $u_x, u_y$, defined in Equation (14). The fitting is constrained by the harmonic coefficients $k_i$ and $k_0$ on the date in question. As $s$ and $\Omega$ are known, and $R$ and $\psi_{AR}$ are fitting parameters, we can use $k_i$ and $k_0$ to determine $u_x$ and $u_y$. Thus only six parameters need to be fit. However, there is a sign ambiguity in $u_x$, which we resolved by matching its sign to that of the annual variation model shown in Figure 6 on that date.

The 820 MHz acf observations on the fifth day are shown in the upper panel of Figure 9 in the format of Figure 2 with the fitted model in the lower panel. The data on all five days at 820 MHz were RFI free and gave good fits to the acf model, as we describe below.

We precomputed a grid of models with anisotropies ($R = 0.05 \pm 0.02 0.38 \pm 0.6 0.8 0.88$), and at each $R$ we fitted the remaining five parameters with uniform weighting in the frequency–time plane. From the sums-of-squares of the residuals versus $R$, we fitted a parabola to estimate the best-fitting value and its error. In Table 3, we list these with the other fitted parameters and their errors determined at the nearest grid value of $R$. We also give an estimate of the reduced $\chi^2_{\text{red}}$ (row 8), which requires the typical error at each point in the 2D acf, which was calculated as follows. There are contributions from system noise ($V_{\text{noise}}/\sqrt{N_{\text{noise}}}$) and statistical variations in the ISS ($V_{\text{iss}}/\sqrt{N_{\text{iss}}}$), which sum in quadrature. While the noise term is independent in each of the pixels of the acf, the ISS term is correlated over the characteristic scales in frequency and time. In the 820 MHz data the ratio of the variance in the ISS to the variance in the noise was in the range 0.1–0.2, such that the ISS and noise made roughly equal contributions to the acf error, which was typically ~0.03. The resulting $\chi^2_{\text{red}}$ are near unity, indicating that the fits are satisfactory.

| Day   | −3   | 211  | 311  | 379  | 467  |
|-------|------|------|------|------|------|
| $R$   | .43  | .40  | .17  | .49  | .19  |
| $\psi_{AR}$ | .04 | .19 | .17 | .19 | .03 |
| $\sigma_p$(10$^6$) m | 6.5 | 1.1 | 1.1 | 1.1 | 1.1 |
| $\psi_p$ | −156 | 16 | 16 | 16 | 16 |
| $\Delta V_{\text{iss}}$ (kHz) | 47 | 2 | 2 | 2 | 2 |
| $s_0$(10$^6$) m | 3.7 | 3.7 | 3.7 | 3.7 | 3.7 |
| $\chi^2_{\text{red}}$ | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 |
| $V_{\text{iss}}$ (km s$^{-1}$) | −1 | 1 | 1 | 1 | 1 |
| $V_{\text{iss}}$ (km s$^{-1}$) | 32 | 39 | 39 | 39 | 39 |
| $s_{\text{red}}$(10$^6$m) | 6.5 | 5.7 | 5.7 | 5.7 | 5.7 |
| $\theta_p/\theta_{\text{iss}}$ | 0.10 | 0.19 | 0.14 | 0.15 | 0.27 |

Notes. Errors in $V_{\text{iss}}$, $V_{\text{iss}}$ do not include the small systematic errors due to uncertainty in $s$ and in the proper motion of the pulsar binary system; Rows 11 and 12 are derived quantities.
The five anisotropies are somewhat lower than those estimated from the annual variation of the harmonics in Table 2, but are consistent at the 1.5σ level. However, there is an indication of temporal changes. The Δσ estimates are smaller than or comparable to the 49 kHz channel bandwidth of the spectrometer, so they should be considered upper bounds. The σθ estimates come directly from the model fit. The interstellar velocities are derived from \( u_x, u_y \) in the model fit via Equation (15), where the listed velocity errors do not include the effects of uncertainty in \( s \) and \( \Omega \). As discussed in Section 3.3.3 the proper motion velocity of the pulsar system that we used could be too large. However, even the lower estimate of \( V_p = -5.3, V_p = 6.2 \, \text{km s}^{-1} \), would change the interstellar velocity by their constant vector difference (times 1 \(-\sigma \)) in magnitude. The last two quantities in Table 3 are calculated from those above. The acf model at 1900 MHz, in the lower panel of Figure 2, was fitted in a similar fashion, except that the axial ratio was held constant at \( A_R = 2 (R = 0.6) \) and the other five parameters were fit. The other observations at 1900 MHz also show ISS slopes and similar behavior. However, the data are occasionally corrupted by RFI, which made the fitting unreliable.

4.2. Reanalysis of Annual Modulation by the Earth’s Motion

The acf analysis indicates that \( V_{IS} \) and the anisotropy is somewhat time variable. Accordingly we modified our annual fitting routine for the harmonics to take variable \( V_{IS} \) and anisotropy. The model interpolates linearly between the values found above, from the five 820 MHz acf epochs, to obtain the appropriate values for 1900 MHz harmonics. We included the formerly discrepant data from MJD 52997 in the fits.

With variable anisotropy the reduced \( \chi^2 \) (3.6) was significantly worse than for fits with constant anisotropy. This probably reflects the fact that the mean \( R \) from the 820 MHz acfs is somewhat lower than the best-fit value of the harmonic fit with constant \( R \). Thus, the evidence for variable anisotropy on a timescale of a year, as shown in Table 3, is only marginal.

With variable \( V_{IS} \) the fits showed a large improvement. The fits with variable \( V_{IS} \) are shown as magenta lines in Figure 6. The revised model now fits the first and last points much better and is equally good in the other points. We conclude that the evidence for variation in \( V_{IS} \) on this timescale is strong, and is the cause of the apparent discrepancy on MJD 52997.

We have redone the four harmonic fitting including MJD 52997 using variable \( V_{IS} \) to produce “best available” values. In each case we fit \( s, \Omega, R, \) and \( \psi_{AR} \). We used the variable velocity model, or fixed velocities from the previous constant velocity fit. In two cases, we held \( \cos i = \pm 0.0227 \) and in the third case we fit \( \cos i \), which yielded \( i = 88:1 \pm 0.5 \). The results are given in Table 4. The case for \( \cos i > 0 \) is compelling as the alternative location doubles the \( \chi^2 \). It is remarkable how well the location of the scattering medium and the alignment of the pulsar orbital plane are determined. This is because these parameters are not correlated with the inclination.

However, in the fit for \( i \), the estimated anisotropy \( R \) is significantly larger than the mean from Table 3, and as noted, the fit is worsened by allowing it to vary. This suggests that the uncertainty on the anisotropy is underestimated and weakens the indication of temporal variation in the anisotropy. However, the velocity variation must be real and the variations, which are of the order of \( \pm 10 \, \text{km s}^{-1} \), are probably not super-Alfvénic. However, they suggest that the scattering medium is much less homogeneous than has been assumed.

### Table 4

| Parameter | Fit i | \( i = 91:3 \) | \( i = 88:7 \) |
|-----------|-------|---------------|---------------|
| \( \cos i \) | 0.033 ± 0.009 | -0.0227 | 0.0227 |
| \( s \) | 0.73 ± 0.01 | 0.74 ± 0.02 | 0.72 ± 0.01 |
| \( \Omega \) (deg) | 62 ± 2 | 63 ± 3 | 62 ± 2 |
| \( R \) | 0.58 ± 0.08 | 0.78 ± 0.12 | 0.54 ± 0.06 |
| \( \psi_{AR} \) (deg) | 68 ± 3 | 75 ± 3 | 66 ± 3 |
| \( V_{IS} \) (km s\(^{-1}\)) | Variable | Variable | Variable |
| \( V_{R} \) (km s\(^{-1}\)) | Variable | Variable | Variable |
| \( N_{ref} \) | 63 | 64 | 64 |
| Reduced \( \chi^2 \) | 2.6 | 5.0 | 2.7 |

The fit for the inclination \( i = 88:1 ± 0.5 \) from the variable velocity model agrees at the 1σ level with that assuming a fixed velocity in Section 3.3.4. It is also consistent with the confidence limits derived from the Shapiro delay at the 1 σ level for \( i < 90^\circ \). However, the inconsistency in estimating \( R, \psi_{AR} \), discussed above, indicates a weakness in the model, which may contribute a systematic error that is not included in our estimate of \( i \) and its standard error.

4.3. Anisotropy and Phase Gradient Variations

The phase gradient for each of the five 820 MHz observations is plotted as a solid line vector in Figure 10. The anisotropy is plotted as a dashed line along its major axis on the same figure. There is some variation in both quantities. Furthermore the mean direction of the phase gradient is nearly parallel or anti-parallel to the mean major axis of the anisotropy. Both quantities would vary randomly, even if the IISM were a uniformly turbulent Kolmogorov plasma from refractive effects due to the finite number of scintles in the scattering disk. Thus the variations in both quantities in Figure 10 could be due to such statistical variations. However, the phase gradient does show a persistent mean, which is not expected if the turbulence was statistically uniform.

4.4. Theoretical Model for Phase Gradient Variations

Since the phase gradient \( \nabla \phi_p \) for the five epochs at 820 MHz is well estimated, one can ask if the gradients are typical of what one would expect of a Kolmogorov random process, or if one must invoke a deterministic structure. This is easily done for an isotropic medium, and the anisotropy we have measured is not large enough to make a significant difference. The rms phase difference \( \phi_{rms}(x) \) over a distance \( x \) can be obtained from the structure function of the phase directly, as \( \phi_{rms}(x) = D_\phi(x)^{0.5} \) and the rms gradient would be \( \phi_{rms}(x)/x \). Here we have measured \( \nabla \phi_p \) over \( s_{ref} \), so we can compare it with \( D_\phi(s_{ref})^{0.5}/s_{ref} \). However it is more intuitive to compare the resulting angular displacement \( \theta_p \) with the rms scattering angle \( \theta_0 = 1/k s_0 \) where \( k = 2\pi/\lambda \) is the propagation constant. Substitution yields

\[
\text{rms}(\theta_p)/\theta_0 = (s_0/s_{ref})^{1/6}. \tag{17}
\]

The scales \( s_0 \) and \( s_{ref} \) are listed in Table 3, where the bottom row shows that \( \theta_p/\theta_0 \sim 0.15 \pm 0.05 \). With \( s_0/s_{ref} \sim 5 \times 10^{-5} \), the predicted rms value is 0.2 on a refractive timescale 10–20 days. Thus, with observations separated by 60 to 100 days, we are seeing independent samples of a phase gradient that is close
Figure 10. Vector representations of the IISM anisotropy, phase gradient, and velocity in the plane of the sky for the five epochs observed at 820 MHz. The orbital x axis (toward the line of nodes) is horizontally to the right and the y axis is vertically down. Note that with the orbit inclination near 90° its angular momentum vector points upward in the figure. Celestial north and east are marked by arrows on day 1. The major axis of scattering is shown as a black dashed line of length \( R \) at angle \( \psi_{AR} \) to the x axis. The black solid arrow is the refractive displacement vector \( \sigma_p, \psi_p \) (parallel to the transverse phase gradient), as defined in the Appendix (and arbitrarily scaled). The dashed arrows show the interstellar velocity vector. The system proper motion velocity is shown by the red arrow on day 1. The scaling of all velocity vectors is in 100 km s\(^{-1}\) units. The Earth’s motion around the Sun makes a cycloidal trajectory through the IISM, which complicates the spatial mapping of the five epochs.

(A color version of this figure is available in the online journal.)

Figure 11. Correlations \( \rho(t_A, t_B) \) between the ISS of pulsars A and B on MJDs 52984, 52997, 53203, 53312, 53374, and 53560 at frequencies of 1420, 820, 1950, 1950, 1950, and 1950 MHz, working left to right and top to bottom. The plotted coordinates are observing times \((t_A, t_B)\) relative to the time of A’s eclipse. The 50% contour of the theoretical models described in the text are shown as white and black ellipses for \( i = 89\,^\circ \) and \( i = 91\,^\circ \), respectively. The models will be reduced by the factors, shown in the marginal plots, due to the changing signal-to-noise ratios for A and B. The scale in the marginal plots is 0 to 1.0.

(A color version of this figure is available in the online journal.)

to the rms expected from a Kolmogorov spectrum. However it should be noted that comparable phase gradients, lasting for decades, have been observed in other pulsars (Keith et al. 2013). These are due to transverse gradients in the electron column density (dispersion measure) that may be due to a breakdown in the homogeneity of the turbulence, or to the presence of a discrete plasma structure somewhere along the line of sight.

4.5. Theoretical Model for Anisotropy Variations

The scattering from an isotropic Kolmogorov phase screen will, in any particular realization, appear slightly anisotropic simply because there are a finite number of “scintles” in the scattering disk. This effect has been discussed and quantified by Romani et al. (1986). From their Table 1, one can find an rms value \( R \sim 0.7(S_0/s_{rec})^{-1/6} \). If the IISM were isotropic,
then from our Table 3 (lines 6 & 7), we would expect an rms $R \sim 0.14$. We actually observe $0.2 < R < 0.5$, which suggests that $R = 0.3 \pm 0.14$ would be a realistic estimate. In this case the variation of $\pm 0.14$ is simply statistical variation in a Kolmogorov medium with a constant anisotropy.

5. THE CROSS CORRELATION BETWEEN THE ISS OF THE TWO PULSARS

In Paper 1, we measured the correlation between the ISS of the two pulsars near the time of A’s eclipse, which led to an estimate of the orbital inclination that was even closer to edge-on than found from the timing observations Kramer et al. (2006). However, as Perera et al. (2010) have shown, the time windows during each orbit where B is observable have shifted and shrunk due to relativistic precession of B’s spin about the orbital angular momentum vector. By 2008 the B pulsar had become undetectable.

We were only able to measure the cross-correlation in the ISS of the pulsars for six of the observing epochs. In our interpretation we assume that, where the projected paths of the two pulsars cross, their lines of sight through the IISM to the Earth are identical. In such a situation, ISS would be highly correlated if the IISM changes slowly enough. The time offsets ($t_A$ and $t_B$), relative to the center of A’s eclipse. The results are shown in Figure 11 for the six days that showed significant correlation. $\rho(t_A, t_B)$ is a measure of the correlation between the ISS of the two pulsars $\rho(s = r_A - r_B)$ where $r_A = V_A t_A$ and $r_B = V_B t_B + (0, y_B(0))$, with the velocities projected transverse to the path through the IISM. Here A’s position at its eclipse defines the origin of the x, y coordinates. $(0, y_B(0))$ is the projected offset of B, where $y_B(0) = -d_{AB} \cos i$ with $d_{AB}$ their separation at the eclipse. The time offsets ($t_A$ and $t_B$) are short enough that $V_A \& V_B$ are effectively constant. The observations show the greatest correlation when A is before the eclipse and when B is after the eclipse.

We have overplotted two 50% correlation contours derived from the model developed in previous sections. The two models correspond to orbital inclinations of $89^0$ and $91^0$ and are plotted as overlapping white and black lines, respectively. Since the observed correlations are normalized by the square root of the product of the total variances in the A and B spectra, the observed correlations are normalized by the square root of the product of the total variances in the A and B spectra, the model correlations must be reduced by factors that depend on the signal to noise ratios. The marginal plots of these factors involve tracing the path of the binary system back to possible birthplaces in the Galactic Plane, evolving the orbital eccentricity and semi-major axis to the appropriate age, and determining which set of kick magnitudes and directions is compatible with the known properties of the system. Most of these studies point to a small kick and therefore likely very little tilt of the post-supernova orbital plane relative to the pre-supernova one; this is reinforced by the finding that the spin axis of the A pulsar is nearly aligned with the orbital angular momentum (Ferdman et al. 2013). Previous work on the evolution of this system did not use a constraint on $\Omega$, as the anisotropy of the IISM prevented a robust determination of the angle at the time (Stairs et al. 2006). Recent modeling (Wong et al. 2010) has found that kicks in the plane of the pre-supernova orbit are preferred over polar kicks, although the rather large angle ($60^0$) found here between the Line of Nodes and the proper motion may be at odds with this result. Revised modeling incorporating the constraint on $\Omega$ will be presented elsewhere.

The determination of $\Omega$, plus the resolution of the sign ambiguity in $\cos i$, permits a strengthening of the double-pulsar test of preferred-frame effects in semi-conservative theories of gravity (Wex & Kramer 2007). Such effects would produce periodic changes in the longitude of periastron and the eccentricity of the system, in a manner that depends on the orientation of the system relative to the coordinates of the preferred frame. Therefore, the knowledge of the orientation will allow a more precise limit to be set on the parameters of this theory.

We model the dominant interstellar scattering plasma as a thin layer located at a distance from the pulsar of $73 \pm 1$% of the distance to the Earth. The success of this “thin screen” model emphasizes the highly localized distribution of scattering plasma along the line of sight. We also measured its velocity
and scattering parameters. The velocity is about 40 km s\(^{-1}\) with respect to the Sun, with variations of about 10 km s\(^{-1}\), which are comparable with expected Alfvén speeds. The level of turbulence varied by a factor of two on a timescale of months, which is much greater than the statistical variation expected due to refractive effects in a homogeneous Kolmogorov random process. At five epochs we measured its anisotropy (axial ratio 1.2–1.7) and phase gradient (due to a transverse gradient in the electron column density). There is some significant variability between epochs in both of these parameters, however, the variations are at the level one might expect in different refractive realizations of a homogeneous Kolmogorov random process concentrated in a thin layer.

Our results add to the evidence suggesting that the IISM model of homogeneous isotropic Kolmogorov turbulence is no longer adequate. There is accumulating evidence for anisotropy and intermittency in the turbulence on sub-AU scales (Rickett et al. 2002; Dennett-Thorpe & DeBruyn 2003; Tunstov et al. 2003; Hill et al. 2005; Brisken et al. 2010) and for persistent phase gradients (Keith et al. 2013). Evidently, this default model of turbulence in the IISM will need to be modified. Apart from the light this throws on the interstellar plasma, turbulence in the IISM is a problem for accurate pulsar timing, which is limited in precision by dispersion and scattering (Keith et al. 2013).

The National Radio Astronomy Observatory is facility of the National Science Foundation operated under cooperative agreement by Associated Universities, Inc. Coles and Rickett acknowledge partial support from the NSF under grant AST 0507713. Rickett thanks the Cavendish Astrophysics Group at Cambridge University for their hospitality. Pulsar research at UBC is supported by an NSERC Discovery Grant.

APPENDIX A

SPACE-FREQUENCY CORRELATIONS IN THE LIMIT OF STRONG DIFFRACTIVE SCINTILLATION

The theory for \(C_{I}(\mathbf{\sigma}, \Delta \nu)\) was reviewed by Lambert & Rickett (1999) and we follow their treatment here. The correlation of intensity is a fourth order moment of scattered electric field. While in general, fourth moments cannot be solved analytically, they simplify in the limit of strong diffractive scintillation to the square of second order moments of electric field versus spatial offset \(\mathbf{\sigma}\) and frequency difference \(\Delta \nu\). Here we assume scattering in a thin layer of plasma (i.e., a “phase screen”). As noted \(C_{I}(\mathbf{\sigma}, \Delta \nu)\) is given by

\[
C_{I}(\mathbf{\sigma}, \Delta \nu) = \left| \Gamma_{D}(\mathbf{\sigma}, \Delta \nu) \right|^2, \tag{A1}
\]

where \(\Gamma_{D}(\mathbf{\sigma}, \Delta \nu)\) is the diffractive part of the electric field covariance. Here \(\mathbf{\sigma}\) is defined as a transverse distance measured at the screen. The definition of “diffractive” involves factoring out a term that corresponds to fluctuations of the pulse arrival time. We assume spherical waves from a pulsar at distance \(z_{p}\) beyond a scattering screen that is at a distance \(z_{o}\) from the observer. The pulsar distance is thus \(L = z_{o} + z_{p}\). Equation (59) of Lambert & Rickett (1999) gives

\[
\Gamma_{D}(\mathbf{\sigma}, z_{o}, v_{m}, \Delta \nu) = \frac{v_{m}^2}{2 \pi i z_{o} c} \int_{-\infty}^{\infty} \exp[-0.5 D_{\nu}(\mathbf{\sigma}'; v_{m})] \times \exp \left[ i \frac{v_{m}^2}{2 z_{o} c \Delta \nu} |\mathbf{\sigma} — \mathbf{\sigma}'|^2 \right] d^2 \mathbf{\sigma}'. \tag{A2}
\]

Here \(z_{o} = z_{p} z_{o}/L\) and \(D_{\nu}(\mathbf{\sigma}'; v_{m})\) is the structure function of the plasma phase caused by the screen at frequency \(v_{m}\) and spatial separation \(\mathbf{\sigma}'\), as given in Equation (2).

We change the spatial coordinate to \(\mathbf{\sigma}'' = \mathbf{\sigma} — \mathbf{\sigma}'\) and introduce normalized variables:

\[
p = [(\mathbf{\sigma}'')/s_{0}]^2 \quad \text{and} \quad \nu = (\Delta \nu/v_{m})(r_{Fe}/s_{0})^2
\]

where \(r_{Fe} = \sqrt{z_{o}/v_{m}}\).

The second moment can then be written in circular coordinates as

\[
\Gamma_{D}(\mathbf{\sigma}, z_{o}, v_{m}, \Delta \nu) = \frac{1}{i 4 \pi v} \int_{0}^{\infty} e^{iv/2c} \int_{0}^{2\pi} \exp[-0.5(\alpha + \beta \sqrt{\nu} + \gamma \nu)] d\theta'' d\nu,
\]

where

\[
\begin{align*}
\alpha &= (2a_{\sigma}^2 + b_{\sigma}^2 + c_{\sigma,\sigma})/s_{0}^2, \\
\beta &= - 2(a_{\sigma} \cos \theta'' + 2b_{\sigma} \sin \theta''), \\
\gamma &= a \cos \theta'' + b \sin \theta'' + c \sin \theta' \cos \theta''.
\end{align*}
\]

The \(\theta''\) integral is simple to do numerically, but the Fourier-like integral over \(\nu\) requires care as the normalized frequency offset \(\nu\) approaches zero.

Note that the \(\nu\) acf versus frequency offset at a single antenna is included as \(\mathbf{\sigma} = 0, \alpha = 0, \beta = 0\). So we used Equation (A4) to compute the normalized frequency decorrelation bandwidth, \(v_{o.5}\), versus axial ratio, holding \(s_{0}\) constant. The result is that the higher the axial ratio the narrower the width \(v_{o.5}\); for example, an axial ratio 4:1 reduces \(v_{o.5}\) to 0.43 compared to 0.96 for an axial ratio of 1:1. Nevertheless, the shape of the acf is only weakly dependent on the axial ratio. Consequently, in reporting the decorrelation bandwidths we fitted the isotropic model and record the frequency offset for a 50% reduction in the acf, since the bias by unknown axial ratio is less than the fitting error.

A.1. Effect of Refraction and Application to the ISS of Pulsar A

A constant phase gradient over the scattering disk will cause a refractive shift of the entire diffraction pattern by an angle \(\theta_{p}\) as given in Equation (16). This gives rise to a displacement \(\mathbf{\sigma}_{p} = z_{o} \theta_{p} \propto v^{-2}\). The frequency derivative \(d\mathbf{\sigma}_{p}/d\nu = -2\mathbf{\sigma}_{p}/v\). So for a small refractive shift \(\Delta \mathbf{\sigma}_{p}\) over a small frequency range \(\Delta \nu\), we can write \(\Delta \mathbf{\sigma}_{p} \sim -2\mathbf{\sigma}_{p}(\Delta \nu/v)\). The time/frequency correlation \(C_{I}(\tau, \delta \nu)\) can then be written

\[
C_{I}(\tau, \delta \nu) = C_{I}(\mathbf{\sigma}) = V_{St}(z_{o}/L) - 2\mathbf{\sigma}_{p}(\Delta \nu/v, \Delta \nu). \tag{A6}
\]

Models computed in this way are shown in the 28 sub-panels of the lower plot of Figure 2. Time lag is plotted horizontally and frequency lag vertically. The slopes come from a constant refractive shift that appears to reverse in sign due the changing pulsar velocity over its 2.45 hr orbit.

APPENDIX B

INFLUENCE OF ANISOTROPY ON FREQUENCY–TIME ACF

The earlier work of Lambert & Rickett (1999) was done for thick and thin screens, but only for isotropic scattering.
Here, we extend their work to a thin anisotropic screen with a refractive gradient that is constant over the scattering disk. They showed that the shape of contours of constant correlation changed significantly with the level of correlation and this effect was more prominent in thin screens than in thick screens. Here we find that the effect is even more prominent as the anisotropy increases. While the higher level contours near the peak are found that the effect is even more prominent as the anisotropy was more prominent in thin screens than in thick screens. Here the x component varies linearly with $\sin\phi$, but the y component is independent of the orbital phase because with $\cos i$ is so close to zero that $V_{\phi y}$ depends only on the center of mass velocity. Thus the combination of the orbital and center of mass motion of the pulsar gives a spatial offset vector $(\sigma_x, \sigma_y)$ that swings over a range in angles governed by parameters $u_x, u_y$. This is what provides the sensitivity of the frequency–time acf to spatial orientations.

Then we simply substitute the $\sigma$ obtained above in Equation (B1). To speed execution, we precompute the 3D $C_f(\sigma, \Delta \nu)$ over a 3D grid and find the necessary values of $C_f(\sigma(\tau, \Delta \nu), \Delta \nu)$ by interpolation. The eight parameters to be determined are $\Delta t_{\text{ffs}}$ that connects $\Delta \nu$ to $v$ in Equation (A3); $u_x, u_y, R; \psi_{AR}, \sigma_p, \psi_p$ & $V_o/s_0$. $u_x, u_y$ are partially constrained by the fitting of harmonic coefficients to $T_{\text{ffs}}(\phi)$. We proceeded in an iterative fashion also constrained by the results of the annual fitting described in Section 3.3.

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