Abstract

The \( \psi DD \) form factor is evaluated in a QCD sum rule calculation for both \( D \) and \( \psi \) off-shell mesons. We study the double Borel sum rule for the three point function of two pseudoscalar and one vector meson current. We find that the momentum dependence of the form factors is different if the \( D \) or the \( \psi \) meson is off-shell, but they lead to the same coupling constant in the \( \psi DD \) vertex.

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The coupling constant of the strong interactions at the most fundamental level, i.e., of quarks and gluons, has been studied both theoretically in lattice QCD calculations and experimentally in \( e^+e^- \) colliders. Our knowledge on \( \alpha_s \) has significantly increased during the last decade [1]. Unfortunately, these advances did not bring a better understanding of the seemingly more accessible quantities: the coupling constants between hadrons. Indeed, on this next level of the strong interactions there has been little progress in what regards the determination of the “hadron strong charges”. Even in the most studied case, the \( NN\pi \) coupling, there is still some controversy. In the strange sector, the couplings between nucleons, hyperons and strange mesons have been strongly constrained by the use of SU(3) symmetry and intense phenomenological analyses of low energy hadronic reactions [2]. In the charm sector, neither it is reasonable to use SU(4) symmetry since it is badly broken, nor experimental information is available. Of course, there are, here and there, exceptions as, for example, the \( D^*D\pi \) coupling, which was recently determined by the CLEO collaboration from \( D^* \) decays [3].

In a curious development of the recent history of hadron physics, the couplings involving charm mesons became a very important building block in the construction of effective lagrangian theories designed to describe low energy reactions such as, for example:

\[
\pi + \psi \rightarrow D^* + d
\]  

(1)
theories may be used to compute cross sections between light and charmed mesons, which, in turn may be used to understand charm hadron production (and suppression) in the nucleus-nucleus collisions performed at RHIC [4–13].

The work of calculating these form factors is rewarded with an interesting side product: information about the size of charm mesons. Of course, the size of a hadron depends on how we “look” at it. The most extensively studied particle is the nucleon, which has been probed mainly by photons. In lower energy experiments, where also the four momentum transfer \( q^2 \) is low, it was possible to determine the electromagnetic form factor (and the charge radius) of the nucleon. In higher energies experiments and very large values of \( Q^2 (Q^2 = -q^2) \) a very different picture of the nucleon emerged, in which it is made of pointlike particles, the quarks. From these observations one may conclude that, when probing the nucleon, nearly on-shell photons \( (q^2 \simeq 0) \) recognize sizes whereas highly off-shell photons \( (q^2 \ll 0) \) do not. This statement is supported by the phenomenologically very successful vector meson dominance hypothesis, according to which real photons are with a large probability converted to vector mesons (which are extended objects) and then interact with the nucleon.

A couple of years ago we started our program of computing the above mentioned quantities in the framework of QCD sum rules (QCDSR). In [14] we calculated the coupling constant in the vertex \( NDA_c \) with the \( D \) meson off-shell. In [15] we did a similar calculation for strange coupling constants. In [16] we extended the calculation performed in [14] and computed the \( Q^2 \) dependent form factors of the \( NDA_c \) and \( ND^*A_c \) vertices. We have also studied the \( D^*D\pi \), \( B^*B\pi \) [17,18] and \( DD\rho \) [19] vertices. One of the conclusions of these works is that when the off-shell particle in the vertex is heavy, the form factor tends to be harder as a function of \( Q^2 \), which means larger cut-off parameters and smaller associated sizes.

In the present work we will further investigate form factors involving heavy mesons in order to extend our previous conclusions. We also want to better estimate the uncertainties in the procedure of determining coupling constants with our techniques. For these purposes we consider the vertex \( J/\psi DD \) and compute form factors and coupling constants for the cases where the \( D \) is off-shell, the \( J/\psi \) is off-shell and then compare the results.

Following the QCDSR formalism described in our previous works we write the three-point function associated with a \( J/\psi DD \) vertex, which is given by

\[
\Gamma^{(D)}(p,p') = \int d^4x \ d^4y \langle 0\{j_D(x)j_D(y)j_D^\dagger(0)\}|0\rangle \ e^{ip'.x} \ e^{-i(p'-p).y} \ ,
\]  

for a \( D \) off-shell meson, and by

\[
\Gamma^{(J/\psi)}(p,p') = \int d^4x \ d^4y \langle 0\{j_{J/\psi}(x)j_D(y)j_D^\dagger(0)\}|0\rangle \ e^{ip'.x} \ e^{-i(p'-p).y} \ ,
\]

for a \( J/\psi \) off-shell meson, where \( j_D = i\bar{d}\gamma_5 c \), \( j_D^\dagger = i\bar{c}\gamma_5 d \) and \( j_{J/\psi} = \bar{c}\gamma_\mu c \) are the interpolating fields for the outgoing \( D^+ \), incoming \( D^+ \) and \( J/\psi \) respectively with \( d \) and \( c \) being the down, and charm quark fields respectively.

The general expression for the vertex function, or correlator, in Eqs. (2) and (3) has two independent structures. We can write \( \Gamma_\mu \) in terms of the invariant amplitudes associated with these two structures:

\[\text{(2)}\]
\[\text{(3)}\]
\[ \Gamma_\mu(p, p') = \Gamma_1(p^2, p'^2, q^2)p_\mu + \Gamma_2(p^2, p'^2, q^2)p'_\mu \]  

(4)

As usual we shall write the correlators (2) and (3) in terms of hadron and quark-gluon degrees of freedom. These two representations, also called phenomenological and theoretical side, are then indentified one with the other yielding a sum rule. On the theoretical side we make an operator product expansion (OPE), keeping the first terms, which are represented in Fig. 1a and 1b for the correlators (2) and (3) respectively. The diagram of Fig. 1a and the first of Fig. 1b may be evaluated giving the perturbative contributions. Applying Cutkosky’s rule to these contributions we can write a double dispersion relation for each one of the invariant amplitudes \( \Gamma_i^{pert} (i = 1, 2) \), over the virtualities \( p^2 \) and \( p'^2 \) holding \( Q^2 = -q^2 \) ( with \( q = p' - p \) ) fixed:

\[ \Gamma_i^{pert}(p^2, p'^2, Q^2) = -\frac{1}{4\pi^2} \int_{s_{min}}^{\infty} ds \int_{m_c^2}^{\infty} du \frac{\rho_i^{pert}(s, u, Q^2)}{(s - p^2)(u - p'^2)}, \]

(5)

where \( \rho_i^{pert}(s, u, Q^2) \) equals the double discontinuity of the amplitude \( \Gamma_i(p^2, p'^2, Q^2) \) on the cuts \( s_{min} \leq s \leq \infty, m_c^2 \leq u \leq \infty \), and where \( s_{min} = 4m_c^2 \) in the case of the \( D \) off-shell, where the dispersion relation is written in terms of the two \( D \) mesons’ momenta, and \( s_{min} = m_c^2 \) in the case of \( J/\psi \) off-shell, the dispersion relation being now written in terms of the \( J/\psi \) and the \( D \) meson momenta. In terms of the \( s, u \) and \( t \) invariants, the double discontinuities are:

\[ \rho_1^{pert(D)} = \frac{3}{4\sqrt{\lambda}} \left[ 2(u - m_c^2) + (s - t - u + 2m_c^2)(1 - \frac{(s - t - u - 2m_c^2)(s - t + u)}{\lambda}) \right] \]

(6)

\[ \rho_2^{pert(D)} = \frac{3s}{2\sqrt{\lambda}} \left[ -1 + \frac{(s - t - u + 2m_c^2)(s - t - u - 2m_c^2)}{\lambda} \right] \]

(7)

for a \( D \) off-shell meson, with the integration limits for \( u \):

\[ u_{max, min} = \frac{1}{2m_c^2} \left[ -st + m_c^2(s + 2t) \pm \sqrt{(s - 4m_c^2)s(m_c^2 - t)^2} \right] \]

(8)

and

\[ \rho_1^{pert(J/\psi)} = \frac{3}{\lambda^{3/2}}(s - t - u)(m_c^4 - su) \]

(9)

\[ \rho_2^{pert(J/\psi)} = \frac{3}{\lambda^{3/2}}(s + t - u)(m_c^4 - su) \]

(10)

for a \( J/\psi \) off-shell meson, with the corresponding integration limits for \( u \):

\[ u_{max, min} = \frac{1}{2m_c^2} \left[ -st + m_c^2(2s + t) \pm (s - m_c^2)\sqrt{-t(4m_c^2 - t)} \right] \]

(11)

with \( \lambda = \lambda(s, t, u) = s^2 + t^2 + u^2 - 2st - 2su - 2tu \) and \( t = -Q^2 \). For this last limit we remark that the condition \( u \geq t - m_c^2 \) must be satisfied and that, as it can be seen from the square root, in the timelike region, \( t = q^2 > 0 \), we must have \( t \geq 4m_c^2 \).
Since we are dealing with heavy quarks, we expect the perturbative contribution to be dominant on the OPE side. However, for the \(J/\psi\) off-shell it turns out that the contribution of the quark condensate, shown in the second diagram of Fig. 1b, is also important and must be included:

\[
\Gamma_{\mu}^{<\bar{q}q>(J/\psi)}(p, p') = \frac{m_c \langle \bar{q}q \rangle}{(p^2 - m_c^2)(p'^2 - m_c^2)} (p_\mu + p'_\mu) = \Gamma^{<\bar{q}q>}(p_\mu + p'_\mu)
\]  

(12)

The phenomenological side of the vertex function is obtained by considering the contribution of the \(J/\psi\) and one \(D\) meson, or the two \(D\) mesons states to the matrix element in Eqs. (2) and (3) respectively. In doing so, we introduce the decay constants \(f_D\) and \(f_{J/\psi}\), which are defined by the matrix elements

\[
\langle 0 | j_D | D \rangle = m_D^2 f_D \frac{m_c}{m_c}
\]

(13)

and

\[
\langle J/\psi | j^\mu_{\psi} | 0 \rangle = m_{J/\psi} f_{J/\psi} \epsilon_\mu
\]

(14)

where \(\epsilon_\mu\) is the polarization of the vector meson. We also make use of the matrix element:

\[
\langle J/\psi(q) D(p') | D(p) \rangle = -g_{J/\psi DD}(Q^2)(p_\mu + p'_\mu) \epsilon^\mu
\]

(15)

Altogether these steps lead to:

\[
\Gamma_{\mu}^{phen(D)}(p, p') = -\frac{m_D^2 f_D}{m_c} m_{J/\psi} f_{\psi} g_D(Q^2) \frac{1}{p^2 - m_{J/\psi}^2} \frac{1}{p'^2 - m_D^2} \times
\]

\[
\left(-2p'_\mu + \frac{m_D^2 + m_{J/\psi}^2 + Q^2}{m_J^2_{J/\psi}} p_\mu \right) + \text{higher resonances}
\]

(16)

\[
\Gamma_{\mu}^{phen(J/\psi)}(p, p') = -\frac{m_{J/\psi}^4 f_{J/\psi}^2}{m_c^2} g_{J/\psi}(Q^2) \frac{1}{p^2 - m_D^2} \frac{1}{p'^2 - m_D^2} \left(p'_\mu + p_\mu \right) + \text{higher resonances}
\]

(17)

where

\[
g_D(Q^2) = \frac{m_D^2 f_D g_{DDJ/\psi}^{(D)}(Q^2)}{Q^2 + m_D^2},
\]

(18)

and

\[
g_{J/\psi}(Q^2) = m_{J/\psi} f_{\psi} g_{DDJ/\psi}^{(J/\psi)}(Q^2) \frac{Q^2}{Q^2 + m_{J/\psi}^2},
\]

(19)

where \(m_D, m_{J/\psi}\) are the masses of the mesons \(D\) and \(J/\psi\) respectively, and \(g_{DDJ/\psi}^{(M)}(Q^2)\) is the form factor at the \(DDJ/\psi\) vertex when the meson \(M\) is off-shell. The contribution of
higher resonances in Eqs. (16) and (17) will be taken into account in the standard form of continuum contribution from the thresholds $s_0$ and $u_0$ \[20\].

For consistency we use in our analysis the QCDSR expressions for the decay constants appearing in Eqs. (16) and (17) up to dimension four:

\[
f_D^2 = \frac{3m_c^2}{8\pi^2m_D^2} \int_{m_D^2}^{u_0} du \frac{(u-m_c^2)^2}{u} e^{(m_D^2-u)/M_{m^2}} - \frac{m_c^6}{m_D^4} (\bar{q}q) e^{(m_D^2-m_c^2)/M_{m^2}},
\]

(20)

\[
f_\psi^2 = \frac{1}{4\pi^2} \int_{4m_c^2}^{r_0} dr \frac{(r+2m_c^2)\sqrt{r-4m_c^2}}{r^{3/2}} e^{(m_{J/\psi}^2-r)/M_{m^2}}
\]

(21)

where $u_0 = (m_D + \Delta_u)^2$ GeV$^2$, $r_0 = (m_{J/\psi} + \Delta_r)^2$ GeV$^2$ and $M_{m^2}$ is the Borel mass used in the two-point functions given above.

We have omitted the numerically insignificant contribution of the gluon condensate.

Eq. (17) shows that when the $J/\psi$ is off-shell, the sum rules in both structures give the same results. On the other hand, according to Eq. (16), when the $D$ is off-shell the two structures give different results. In this last case we will concentrate in the $p'_\mu$ structure, which we found to be the more stable one. The sum rules in this structure read:

\[
\Gamma_{2}^{p_{\mu}(D)}(p^2, p'^2, q^2) = \Gamma_{2}^{pert(D)}(p^2, p'^2, q^2)
\]

(22)

for a $D$ off-shell and

\[
\Gamma_{2}^{p_{\mu}(J/\psi)}(p^2, p'^2, q^2) = \Gamma_{2}^{pert(J/\psi)}(p^2, p'^2, q^2) + \Gamma_{<\bar{q}q>}(p^2, p'^2, q^2)
\]

(23)

for a $J/\psi$ off-shell.

Inserting in these equations the corresponding expressions for the perturbative, quark condensate and phenomenological terms and performing a double Borel transformation \[20\] in both variables $P^2 = -p^2 \rightarrow M^2$ and $P'^2 = -p'^2 \rightarrow M'^2$ we obtain the final expressions for the sum rules. In order to allow for different values of $M^2$ and $M'^2$ we take them proportional to the respective meson masses, which leads us to study the sum rules as a function of $M^2$ at a fixed ratio:

\[
\frac{M^2}{M'^2} = \frac{m_{J/\psi}^2}{m_D^2}
\]

(24)

for (22) and $M^2 = M'^2$ for (23).

In refs. \[21,22\] it was found that relating the Borel parameters in the two- ($M_{m^2}$) and three-point functions ($M^2$) as

\[
2M_{m^2}^2 = M^2,
\]

(25)

is a crucial ingredient for the incorporation of the HQET symmetries, and leads to a considerable reduction of the sensitivity to input parameters, such as continuum thresholds $s_0$ and $u_0$, and to radiative corrections. Therefore, in this work we will use Eq. (23) to relate the Borel masses.
The parameter values used in all calculations are $m_q = (m_u + m_d)/2 = 7$ MeV, $m_c = 1.3$ GeV, $m_D = 1.87$ GeV, $m_{J/\psi} = 3.1$ GeV, $(\bar{q}q) = -(0.23)^3$ GeV$^3$. We parametrize the continuum thresholds as

$$s_0 = (m_M + \Delta_s)^2,$$

where $m_M = m_D(m_{J/\psi})$ for the case that the $J/\psi(D)$ meson is off-shell, and

$$u_0 = (m_D + \Delta_u)^2.$$

Using $\Delta_s = \Delta_u = \Delta_r \sim 0.5$ GeV for the continuum thresholds and fixing $Q^2$ we found good stability of the sum rule for $g_{DDJ/\psi}^{(D)}(Q^2)$ as a function of $M^2$ in the interval $4 < M^2 < 16$ GeV$^2$ and also for $g_{DDJ/\psi}^{(J/\psi)}(Q^2)$ in the interval $2 < M^2 < 10$ GeV$^2$.

Fixing $M^2 = 11$ GeV$^2$ we calculate the momentum dependence of the form factor $g_{DDJ/\psi}^{(D)}(Q^2)$ in the interval $-0.5 \leq Q^2 \leq 5.0$ GeV$^2$, where we expect the sum rules to be valid (since in this case the cut in the $t$ channel starts at $t \sim m_c^2$ and thus the Euclidian region stretches up to that threshold). Our numerical calculations can be well reproduced by the gaussian parametrization:

$$g_{DDJ/\psi}^{(D)}(Q^2) = 16.4 e^{-\frac{(Q^2+16.2)^2}{228}}$$

We stress here that it was not possible to fit our results with a monopole form!

As in ref. [6], we define the coupling constant as the value of the form factor at $Q^2 = -m_M^2$, where $m_M$ is the mass of the off-shell meson. In the case of an off-shell $D$, this leads to

$$g_{DDJ/\psi}^{(D)} = 8.05$$

Along the same lines we choose $M^2 = 8$ GeV$^2$ and calculate the momentum dependence of the form factor $g_{DDJ/\psi}^{(J/\psi)}(Q^2)$. The numerical results can be fitted by the monopole parametrization:

$$g_{DDJ/\psi}^{(J/\psi)}(Q^2) = \frac{1069.76}{Q^2 + 143.18}$$

which, at the $J/\psi$ pole leads to:

$$g_{DDJ/\psi}^{(J/\psi)} = 7.98$$

All our numerical results and parametrizations are shown in Fig. 2. The circles and squares correspond to the numerical results for the $D$ and $J/\psi$ off-shell respectively. These points are fitted by the dashed (Eq. (28)) and solid lines (Eq. (30)) respectively and extrapolated to the meson poles which are represented by the triangles, which give the numbers quoted in (29) and (31).

A closer look into Fig. 2 reveals that, whereas the circles are well fitted by the dashed line, this is not the case of the squares, which are not very accurately described by the solid line. Due to this reason and also because the $J/\psi$ pole is farther away, the estimate (31)
is much less reliable than (24). In fact, to reduce the uncertainty in the extrapolation of the $J/\psi$ off-shell form factor, we have to impose the condition that both coupling constants $g^{(D)}_{DDJ/\psi}$ and $g^{(J/\psi)}_{DDJ/\psi}$ must coincide. We therefore use the former as a guide in the fitting procedure (and subsequent extrapolation) leading to the latter. This condition also imposes a severe constraint in the minimum $Q^2$ used in the calculation. We have used the interval $5.5 < Q^2 < 9.5 \text{ GeV}^2$. Including smaller values of $Q^2$ in the calculation would increase the curvature of the solid line, which, when extrapolated to lower $Q^2$ values, would give coupling constants very different from $g^{(D)}_{DDJ/\psi}$. In other words, it would become impossible to obtain the two triangles at the same height.

The star in Fig. 2 indicates the $J/\psi$ off-shell form factor taken at $Q^2 = 0$. As it can be immediately obtained from (30), $g^{(J/\psi)}_{DDJ/\psi} = 7.47$ at $Q^2 = 0$. This coincides with the estimate made in Ref. [4] using the vector meson dominance model.

The information presented in Fig. 2 may now be crossed with those obtained in [19], about the size of the $D$ meson. In Fig. 3 we compare $g^{(J/\psi)}_{DDJ/\psi}(Q^2)$, given by (30) with $g^{(\rho)}_{DD\rho}(Q^2)$, calculated in [19] and parametrized by:

$$g^{(\rho)}_{DD\rho}(Q^2) = 2.53 \ e^{-\frac{Q^2}{0.98}}$$

(32)

The $DDJ/\psi$ form factor is represented by a solid line whereas the $DD\rho$ one is shown with a dashed line.

On a qualitative level, the comparison between solid and dashed lines both in Fig. 2 and Fig. 3 shows that the form factor is harder if the off-shell meson is heavier. In particular, remembering that there is an overlap between the photon and the vector mesons $\rho$ and $J/\psi$, we check in Fig. 3 an empirical formula regarding the resolving power of the photon, used some time ago by experimentalists. HERA data on electron-proton reactions could be well understood introducing a “transverse radius of the photon”, parametrized as [23]:

$$r_\gamma \simeq 1/\sqrt{Q^2 + m^2}$$

(33)

where $m$ is the mass of the vector meson considered. This empirical formula tells us that for $Q^2 \to \infty$ the photon is pointlike and “resolves” the nucleon target, i.e., identifies its pointlike constituents and does not “see” the size of the nucleon. Moreover this formula indicates that for $Q^2 \simeq 0$ and for light mesons (like the $\rho^0$) the photon has appreciable transverse radius and therefore also identifies the global nucleon extension. Finally, in the above formula we may have a heavy vector meson ($J/\psi$ or $\Upsilon$) which will, either real or virtual, resolve the nucleon into pointlike constituents. This conjecture can be applied to a $D$ target probed by a $J/\psi$ and seems to be supported by Fig. 3 when we consider the vicinity of $Q^2 \simeq 0$. There, we see (again) that the slope of the dashed line is stronger than that of the solid line. This means that also around the “real photon” region, the $J/\psi$ resolves smaller scales than the $\rho$.

In Fig. 4 we compare $g^{(D)}_{DDJ/\psi}(Q^2)$, given by (28) (solid line), with $g^{(D)}_{DD\rho}(Q^2)$ (dashed line), calculated in [19] and parametrized by:

$$g^{(D)}_{DD\rho}(Q^2) = \frac{37.5}{Q^2 + 12.12}$$

(34)
We can clearly observe that the $D + \rho \rightarrow D$ transition has a much harder form factor than the $D + J/\psi \rightarrow D$ one.

In the limiting case $r_{J/\psi} << r_D << r_\rho$ these results can be understood in a simple picture: when the $D$ hits the smaller $J/\psi$ it can “see” a size, it can measure it! On the other hand, when the $D$ hits the much larger $\rho$ meson, it does not see any size, in the same way as large $Q^2$ photons (in DIS measurements) do not see any size in the proton. Rather, they will interact with pointlike partons! In real life the mentioned radii are not so different from each other and therefore the differences in the curves in Fig. 4 are not so pronounced.

Our calculations contain uncertainties in the QCD parameters (masses and vacuum condensates), in the OPE (because we neglect higher order operators), in the model of the continuum (the uncertainty in the values of $s_0$ and $u_0$) and the systematic error in the extrapolation procedure to obtain the couplings. These errors are present in most of the QCDSR calculations and are to a certain extent unavoidable. Due to them most of QCDSR results are plagued by a 20% error. Therefore our numbers for the couplings have this same uncertainty. On the other hand, the curves shown in the figures are so dramatically different that they confirm beyond any doubt our previous suspicion [16,17,19] regarding transition vertices and, most of all, as one can see clearly in Fig. 3, they show in the case of “charge form factors” (where the same meson goes in and out the vertex), that the $D$ meson “seen” by a $\rho$ is much larger than when it is probed by the $J/\psi$.

Our program will continue and studies of other vertices are in progress.

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FIG. 1. a) diagrams that contribute to $g_{DDJ/\psi}(Q^2)$ b) diagrams that contribute to $g_{DDJ/\psi}(Q^2)$.

FIG. 2. Momentum dependence of the $DDJ\psi$ form factor. Circles and squares represent our numerical calculations for the $D$ and $J/\psi$ off-shell respectively. The dashed and solid lines give the parametrization of the QCDSR results through Eq. (28) for the circles and Eq. (30) for the squares. The triangles give the form factors at the poles of the particles (which we indentify with the coupling constant). The star shows the form factor at $Q^2 = 0$. 
FIG. 3. Momentum dependence of the $DDJ\psi$ (solid line) and $DD\rho$ (dashed line) form factors. In both cases the vector mesons are off-shell.

FIG. 4. Momentum dependence of the $DDJ\psi$ (solid line) and $DD\rho$ (dashed line) form factors. In both cases the $D$ meson is off-shell.