Influence of Truss Topology on Reliability Index

Wojciech Mochocki 1, Paulina Obara 1, Jan Turant 2

1 Kielce University of Technology, Faculty of Civil Engineering and Architecture, al. Tysiąclecia PP 7, 25-314 Kielce, Poland
2 Lodz University of Technology, Department of Technical Mechanics and Informatics, ul. Stefana Żeromskiego 116, 90-924 Łódź, Poland

wmochocki@tu.kielce.pl

Abstract. The reliability of steel trusses is calculated in the paper. The influence of truss topology on reliability is analysed. Five cases of truss are discussed. The reliability analysis concerns the formulation of the limit state, calculation of the failure probability and determination of the reliability index. There are four groups methods commonly used for the structural reliability assessment but only one of them, used in the paper, is definitely most complete. It is the system approach. This method contains both determining reliability models and reliability analysis. To determine reliability model, the spectral analysis of the linear stiffness matrix is used. To identify all possible non-repeatable combinations of removed rods transforming truss to a mechanism the program based on the application of the finite element method was created. For each truss the reliability index is estimated by evaluating its lower and upper bounds. The lower bound is evaluated by assuming that the truss behaves itself like a statically determinate truss, i.e., a failure of one element is equivalent to the failure of the whole structure (serial system). The upper bound is evaluated by assuming actual connections and interactions between the rods by using generated reliability model (mixed system). In this case the reliability index is computed using the program developed by the authors. Additionally, the approximation method is proposed as verification.

1. Introduction

In civil engineering the lowest possible material consumption is preferred. It is caused by economic favour, which an engineer should be taken into account in designer process. Economic design depends on the maximum using of the capacity of structural elements and the use of possibly simple assembly solutions. In case of large-area covers of industrial, warehouse and production halls the most preferred and economic is steel truss grids.

The designing of steel truss grids needs using some rules. It leads to create the rational and optimal construction. The most important rules are as following [1]:

- choice of geometry of truss depending on a load, a span and assembly conditions,
- optimization of the high of truss $h$ depending on a span $L$; for example for $L \leq 24$ m $h=(1.7+1.8)L$,
- choice of topology of truss – slope angle of bracings should be greater than 30°,
- selection of element of a top chord, a bottom chord and at last a bracings,
- optimization of profiles of truss elements depending on consumption, costs and assembly solutions (maximum five different profiles is preferred).
making unloaded elements, the so-called zero elements, from the same profile as the neighbouring chord and bracings, respectively.

Additionally, in engineering practice do not uses elements thinner than 4 mm and elements with the same widths and the different thicknesses, for example RK50x5 and RK50x4. The optimization problems were considered among others in [2-3].

An important issue in the case of pin-jointed multi-element steel structures is to determine and ensure adequate reliability. Reliability means the ability of the structure to perform functions assumed in the design process throughout the lifetime. The reliability analysis leads to the determining the probability of failure understood as crossing the limit state [4-6]. A usual measure of reliability is the reliability index.

The methods commonly used for determine the reliability index can be classified as follows [7]:

- methods of level I (semi-probabilistic methods) – the structure reliability is verified, but the exact failure probability is not determined,
- methods of level II (approximation (FORM, SORM) and simulation (Monte Carlo, Importance Sampling) methods) – the boundary probability distributions of individual random variables are used,
- methods of level III (fully probabilistic methods) – the cumulative probability distribution of random variables is known,
- system approach (serial, parallel and mixed systems) – interaction of the structural elements in the load-carrying system is taken into account [7-8].

In the paper, the reliability of steel trusses is calculated. The influence of truss topology on reliability index is analyzed. To determine the reliability index the system approach is used. This method is definitely more complete than others. Additionally the approximation method – FORM [6, 9] method is proposed as verification.

2. Material and methods

In case of a statically indeterminate trusses, there exist many possible failure modes and paths to complete failure of the structure. To generate all of them it is necessary considering a structural arrangement and determine so-called reliability model of structures [4, 10]. Only one of reliability methods contains both determining reliability models and reliability analysis. It is the system approach.

2.1. Reliability model

In the reliability theory, three models are distinguished, namely serial, parallel and mixed (parallel-serial and serial-parallel) systems. Defining a reliability model is the most difficult part of the system reliability analysis. Reliability model can be defined as the structure eigenvalue, which depends on its geometry and boundary conditions. In principle, it is not numeric approach. To define the reliability model, the kinematically admissible failure mechanisms which contain minimal critical sets of elements (MCSEs) are specified. Exhaustion of the capacity of all the elements included in the MCSE leads to the transformation of the safe structural system into a geometrically variable system (mechanism). To determine reliability model, the spectral analysis [10, 11] of the linear stiffness matrix is used:

\[
(K - \lambda I)q = 0
\]

where \( K \) is the linear stiffness matrix, \( \lambda \) is the eigenvalues and \( q \) is the displacement vector.

The eigenvalues \( \lambda \) describe the system energy state, whereas the eigenvectors corresponding to eigenvalues describe the strain modes. The system is not a mechanism if all eigenvalues \( \lambda \) have a positive value. Zero eigenvalues are related to the finite or infinitesimal mechanisms. To establish if the mechanism is infinitesimal it is necessary to apply the nonlinear analysis with the use of geometric stiffness matrix [12]. Mechanism form is defined by the eigenvector corresponding to zero eigenvalue.
To identify all possible non-repeatable combinations of removed rods transforming truss to a mechanism the program based on the application of the finite element method was created.

2.2. Reliability of the system
After determining reliability models, the reliability of the system and the reliability index, which is a measure of safety, are calculated. In order to determine the reliability of the system $R_s$, it is necessary to compute the reliability of a single element $R_i$. It is assumed that both the capacity of an element $N_i$, and the effect of actions $E_i$, have normal distribution and are characterized by standard deviation ($\sigma_{N_i}$, $\sigma_{E_i}$) and expected value ($\mu_{N_i}$, $\mu_{E_i}$). As a result, the safety margin $Z_i$ is also a random variable described by means of the normal distribution. The parameters of this system are the expected value of the safety margin $\mu_{Z_i}$ and standard deviation of the safety margin $\sigma_{Z_i}$. Based on these parameters the reliability index for a single element $\beta_i$ is computed and next the probability of the element failure $p_{fi}$. Finally the reliability of a single element $R_i$ is estimated. Scheme of estimates of reliabilities of individual elements for $n$-element truss is shown in Table 1.

| No. | $E_i$ | $\mu_{E_i}$ | $\sigma_{E_i}$ (6%) | $N_i$ | $\mu_{N_i}$ | $\sigma_{N_i}$ (10%) | $\mu_{Z_i}$ | $\beta_i$ | $p_{fi}$ | $R_i$ |
|-----|------|-------------|-------------------|------|-------------|------------------|-------------|---------|--------|------|
| 1   | $E_1$ | 0.06$E_1$   | $N_1$             | 0.1$N_1$ | $N_1 - E_1$ | $\frac{\mu_{Z_1}}{\sigma_{E_1}}$ | $\Phi(-\beta_1)$ | 1 - $p_{f1}$ |
| ... | ...   | ...         | ...              | ...   | ...         | ...              | ...         | ...      | ...    | ...   |
| n   | $E_n$ | 0.06$E_n$   | $N_n$             | 0.1$N_n$ | $N_n - E_n$ | $\frac{\mu_{Z_n}}{\sigma_{E_n}}$ | $\Phi(-\beta_n)$ | 1 - $p_{fn}$ |

When the reliabilities of all elements $R_i$ are known, on the basis of the defined reliability model, it is possible to estimate the reliability of the whole structure (system) $R$. For analyzed trusses the reliability $R$ is estimated by evaluating its lower and upper bounds. The lower bound is evaluated by assuming that the truss behaves itself like a statically determinate truss, i.e., a failure of one element is equivalent to the failure of the whole structure (serial system):

$$R = \prod_{i=1}^{n} R_i$$  \hspace{1cm} (2)

The upper bound is evaluated by assuming actual connections and interactions between the rods by using generated reliability model (mixed system). The parallel-serial systems is analysed:

$$R = \prod_{j=1}^{k} \left[ 1 - \prod_{i=1}^{n} (1 - R_{ji}) \right]$$  \hspace{1cm} (3)

In this case the reliability is computed using the program developed by the authors. Knowing (2) and (3) the probability of failure and the reliability index is calculated respectively:

$$p_f = 1 - R$$  \hspace{1cm} (4)

$$\beta = -\Phi^{-1}(p_f)$$  \hspace{1cm} (5)

where $\Phi$ is the Laplace function.

To verify calculations, the FORM method is used. The FORM method is one of most effective approximate methods of the calculation of reliability measures. In this method the surface of limit state is approximated by a hyperplane tangential to it at the design point, which is nearest to the beginning of the coordinate system. Finding the design point is thus reduced to the solution of the optimization
problem. A number of algorithms have been developed to this end. The earliest ones, which originated from the works [13-14] are based on gradient procedures.

**Figure 1.** Truss topology: a) T1, b) T2, c) T3, d) T4, e) T5
3. Results and discussions

The paper concerns the reliability analysis of flat steel truss. The influence of truss topology on reliability is analyzed. Five types of truss are discussed (figure 1). The first truss (figure 1a – truss T1 with 33 elements) is statically determinate. The others are created by adding next two bracings. These trusses are statically indeterminate and the degree of statically indeterminacy (DSI) is respectively: DSI=2 (figure 1b – truss T2 with 35 elements), DSI=4 (figure 1c – truss T3 with 37 elements), DSI=6 (figure 1d – truss T4 with 39 elements), DSI=8 (figure 1e – truss T5 with 41 elements).

The analyzed trusses are designed according to the standard [15]. The concentrated vertical forces (figure 1a) is considered. Elements of trusses are designed from S235 steel with the yield strength $f_y=235$ MPa. It is assumed the maximal stress intensity level is less than 80%. The profiles of elements (chords, bracings, vertical elements) are presented in Table 2. Additionally, table 2 shows numbers of most stressed elements with their stress intensity level and steel consumption.

Should be noted that the main purpose of the paper was not to obtain the intended reliability index of the structure, but the analysis the influence of truss topology on reliability. The profiles of elements are adopted so as to the largest possible number of elements have the efforts of elements relevant from the point of view of reliability analysis. The greater the number of elements with high efforts, the reliability analysis is effective.

The change of truss topology by adding next bracings does not cause a big increase a steel consumption. Maximal increase is 4.5%. The next added elements cause decrease forces in the compression elements. The cross-sections of these elements may be smaller (table 2).

Table 2. Main characteristics of trusses T1-T5

|                      | T1  | T2  | T3  | T4  | T5  |
|----------------------|-----|-----|-----|-----|-----|
| Profile of element:  |     |     |     |     |     |
| bottom chord         | HEAA140 |     |     |     |     |
| top chord            | HEAA160 |     |     |     |     |
| bracings             | RK70x6 RK40x4 | RK60x6 RK40x4 | RK60x6 RK50x4 RK40x4 | RK60x6 RK50x4 RK40x4 | RK60x6 RK50x4 RK40x4 |
| vertical elements    | RK40x4 RK50x4 RK40x4 | RK40x4 | RK40x4 | RK40x4 | RK40x4 |
| Most stressed element| number of element | 18 | 27 | 26 | 26 | 26 |
|                      | efforts of elements | 76 % | 75 % | 76 % | 76 % | 76 % |
| Steel consumption    | 731 kg | 729 kg | 744 kg | 745 kg | 764 kg |

For all cases, kinematically admissible failure mechanisms (KAFM) which contain minimal critical sets of elements are determined. Two variant of support are take into account – the trusses with 3 constraints (figure 1) and 4 constraints (it was assumed that in node 17 there is hinged support). Table 3 and table 4 shows the number of all mechanisms depending on truss topology, for both variant of support. In figure 2 the KAFMs for the truss T4 with 3 constraints are presented for example.

The influence additional constrain on the number of mechanisms is significant. For the truss T1 the number of mechanisms increase by 61% and for T5 – by 1195%. Additionally it should be noted that the time of determination KAFMs for second variant change significantly – from 10 hours to 26 hours for truss T5.
Table 3. Kinematically admissible failure mechanisms for 3 constraints

| Type of truss | T1 | T2 | T3 | T4 | T5 |
|---------------|----|----|----|----|----|
| Number of elements | 33 | 35 | 37 | 39 | 41 |
| DSI           | 0  | 2  | 4  | 6  | 8  |
| l=1           | 33 | 23 | 15 | 7  | 0  |
| l=2           | -  | 30 | 40 | 52 | 56 |
| l=3           | -  | -  | 50 | 80 | 120|
| l=4           | -  | -  | -  | 50 | 104|
| l=5           | -  | -  | -  | -  | 88 |
| l=6           | -  | -  | -  | -  | 72 |
| l=7           | -  | -  | -  | -  | 56 |
| l=8           | -  | -  | -  | -  | 40 |
| l=9           | -  | -  | -  | -  | 25 |
| Number of all MCSEs | 33 | 53 | 105| 189| 561|

Table 4. Kinematically admissible failure mechanisms for 4 constraints

| Type of truss | T1 | T2 | T3 | T4 | T5 |
|---------------|----|----|----|----|----|
| Number of elements | 1  | 3  | 5  | 7  | 9  |
| DSI           | 25 | 17 | 11 | 5  | 0  |
| l=1           | 28 | 35 | 30 | 31 | 30 |
| l=2           | -  | 60 | 96 | 92 | 69 |
| l=3           | -  | 25 | 194| 327| 405|
| l=4           | -  | -  | 144| 528| 1093|
| l=5           | -  | -  | -  | 256| 1533|
| l=6           | -  | -  | -  | -  | 1321|
| l=7           | -  | -  | -  | -  | 838 |
| l=8           | -  | -  | -  | -  | 396 |
| l=9           | -  | -  | -  | -  | -  |
| Number of all MCSEs | 53 | 137| 546| 1735| 7263|

Figure 2. Kinematically admissible failure mechanisms for the truss T4
In next steep, the reliability indexes $\beta$ for all cases of trusses topology is calculated. The four approach are applied. The first approach is based on the assumption that all truss elements are connected in series (serial system). In the next two approaches reliability index is computed for the assumption that mixed connections are considered. Only the parallel-serial systems are analysed (parallel-serial system with 3 constrains and parallel-serial system with 4 constrains). This approach allows full automation of the computational process. In the case of well-designed structures the analysis of the series-parallel systems is not necessary. The reliability analysis for first three approaches is performed using the program created by the authors. In the fourth approach, the FORM approximation method is used to calculate the reliability indexes. These calculations are made using the NumpressExplore program [16]. Results are shown in figure 3.

Figure 3. Reliability indexes of the structure

Based on the assumption that all truss elements are connected in series the lower bound is evaluated. In this case the reliability index remains at a similar level with the decreasing trend with the increase in the number of truss elements. Only for truss T2 the increase about 18% is occurred. The upper bound is evaluated by assuming actual connections and interactions between the rods by using generated reliability model. The parallel-serial systems are used. It is observed that additional constrains has no significant effect on the value of the reliability index. Using parallel-serial model the value of the reliability index for truss T5 increase above 100% compared with serial model.

The reliability index calculated by the approximation FORM method remains at a similar level for all trusses. This method is based on the reliability of the most stressed element. The number of element do not influence on the reliability index. It is related to the assumption of the maximum effort of elements at the level of 80%.

Additionally, it should be noted that along with a significant increase the reliability index the minimal increase of steel consumption is occurred.

4. Conclusions
Steel trusses are one of the most common structural solutions for roofs of steel halls. Due to the fact that these are pin-jointed multi-element steel structures it is important to pay special attention to reliability. For example, increasing the reliability of the structure can be obtained by increasing the cross-sections of elements. In the era of economic construction, structural solutions allowing for the lowest possible material consumption should be considered.

The analyses carried out for the study lead to some important conclusions presented below.
The system approach is the most complete analysis of reliability. Only this approach allows to take into account structural arrangement. Using the parallel-serial model the value of the reliability index for trusses increases with growth degree of statically indeterminacy (DSI) and the number of elements with parallel-serial connections. Using serial model reliability index remains at a similar level with the decreasing trend with the increase in the number of truss elements.

Additional constrains (assumption that in both nodes there is hinged support) has no significant effect on the value of the reliability index. However, it has an impact on the calculation time due to the significant increase in the number of mechanisms.

Along with a significant increase the reliability index the minimal increase of steel consumption is occurred (maximal increase is 4.5%).

Taking into account the full load combinations, determining the required reliability index and considering the scenario of the failure may be the next stage of research.

References
[1] Z. Kurzawa, and M. Chybiński, „Designing steel structures. Connections and selected elements,” Publishing House of the Poznan University of Technology, 2008 (in Polish).
[2] M. Major, and P. Adamski, „Comparative analysis of the economical performance of the cover of steel hangars on the example of a truss frame and arch truss,” Construction of optimized energy potential, vol. 8, pp. 171-178, 2011 (in Polish).
[3] B. Blachowski, and W. Gutkowski, „Discrete structural optimization controlled by state variables,” Modelling in Engineering, 5 (36), pp. 27-34, 2008 (in Polish).
[4] J. Murzewski, „Reliability of engineering structures,” Arkady, Warsaw 1989 (in Polish).
[5] K. Kubicka, and U. Radoń, “Proposal for the assessment of steel truss reliability under fire conditions,” Archives of Civil Engineering, vol. 61 (4), pp. 141-154, 2015.
[6] A. Dudzik, and U. Radoń, “The reliability assessment for steel industrial building,” Advances In Mechanics: Theoretical, Computational And Interdisciplinary Issues, pp. 163-166, 2016.
[7] O. Ditlevsen, and H. O. Madsen, “Structural Reliability Methods,” Department of Mechanical Engineering Technical University of Denmark, June-September, 2007.
[8] S. Wolński and K. Wróbel, “Reliability of building structures,” Publishing House Rzeszow University of Technology, 2001 (in Polish).
[9] A. Dudzik, and U. Radoń, “The evaluation of algorithms for determination of the reliability index,” Archives of Civil Engineering, vol. 61(3), pp. 133-147, 2015.
[10] J. Kłosowska, P. Obara and J. Turant, “Kinematically admissible failure mechanisms for plane trusses,” IOP Conference Series: Materials Science and Engineering, 245, 2017, doi:10.1088/1757-899X/245/2/022022.
[11] K. Dems, and J. Turant, “Structural damage identification using frequency and modal changes,” Bulletin of the Polish Academy of Sciences: Technical Sciences, vol. 59 (1), pp. 27-32, 2011.
[12] W. Gilewski, J. Klosowska, and P. Obara, “Form finding of tensegrity structures via Singular Value Decomposition of compatibility matrix,” Advances In Mechanics: Theoretical, Computational And Interdisciplinary Issues, pp. 191-195, 2016.
[13] A. M. Hasofer, and N. C. Lind, “Exact and invariant second moment code format,” Journal of the Engineering Mechanics Division, ASCE, vol. 100, pp. 111-121, 1974.
[14] R. Rackwitz, and B. Fiessler, “Structural reliability under combined random load sequences,” Computers & Structures, vol. 9 (5), pp. 489-494, 1978.
[15] PN-EN 1993-1-1 Eurocode:2006 3. Design of steel structures. Part 1-1: General rules and rules for buildings.
[16] http://numpress.ippt.pan.pl/ access: 05/2018