Does locality plus perfect correlation imply determinism?

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Abstract A 1964 paper by John Bell gave the first demonstration that quantum mechanics is incompatible with local hidden variables. There is an ongoing and vigorous debate on whether he relied on an assumption of determinism, or instead, as he later claimed, derived determinism from assumptions of locality and perfect correlation. This paper aims to bring clarity to the debate via simple examples and rigorous results. It is shown that the weak form of locality used in Bell’s 1964 paper (parameter independence) is insufficient for such a derivation, whereas an independent form called outcome independence is sufficient even when weak locality does not hold. It further follows that outcome independence, by itself, implies that standard quantum mechanics is incomplete. It is also shown that an appeal by Bell to the Einstein-Rosen-Podolsky argument to support his claim fails, via examples that expose logical gaps in this argument. However, replacing the reality criterion underpinning the argument by a stronger criterion enables a rigorous derivation of both weak locality and determinism, as required for Bell’s 1964 paper. Consequences for quantum interpretations, locality, and classical common causes are briefly discussed, with reference to an example of local classical indeterminism.

Keywords locality · perfect correlation · determinism · outcome independence · Einstein-Podolsky-Rosen argument · classical common causes

1 Introduction

Bell’s theorem, that there are no local hidden variable models that can reproduce all quantum correlations [1][2], is one of the most surprising results of twentieth century physics. It has important ramifications for both physics and metaphysics, in not only underlying the security of a number of device-independent communication protocols such as quantum key distribution and random number generation, but also ruling out naive classical interpretations of quantum probability [2].
The mathematics required to prove Bell’s theorem is remarkably straightforward. However, there is one aspect of the assumptions used in Bell’s first exposition that continues to generate differing opinions and vigorous debate: do the assumptions of locality and perfect correlation made in his 1964 paper necessarily imply that certain measurement outcomes must be predetermined, or does such determinism represent a third assumption?

It may be noted that the answer to this question is unimportant in one sense: later generalisations of Bell’s theorem do not rely on assuming perfect correlation nor determinism. Nevertheless, the question remains of strong interest for several reasons. First, there is no general consensus on the answer. For example, while Wigner wrote that Bell postulated both deterministic hidden variables and locality (see also [4–5]), Bell himself later claimed that determinism was in fact inferred rather than assumed in his 1964 paper [6], and the debate has only intensified since then. Second, an appeal by Bell to the Einstein-Podolsky-Rosen (EPR) incompleteness argument to support his claim puts the latter argument itself into question. Finally, the validity of Bell’s claim would suggest that there is no choice between giving up determinism or giving up locality in interpreting quantum phenomena, i.e., that it would be compulsory to give up locality—a conclusion that is strongly contested in its own right (see, e.g., [29–36]).

With the aim of making a clear and concise contribution to these issues, this paper is guided by the following quote from an excellent early discussion on the subject by van Fraassen:

“I have made an effort to present the deduction . . . shorn of all superfluous mathematical technicalities and woolly interpretative commentary. (A reader as yet unfamiliar with the literature will be astounded to see the incredible metaphysical extravaganzas to which this subject has led.)” [7]

In particular, attention will be focused on what can be proved in a simple yet rigorous manner, and what can be disproved via simple counterexamples, whilst avoiding woolly assertions and metaphysical extravaganzas.

In section 2 the only explicit sense of locality supported by Bell’s 1964 paper is recalled, and shown to be too weak for deriving determinism for perfectly correlated measurement outcomes (Proposition 1). In contrast, an assumption of outcome independence (related to the existence of classical common causes) is sufficient to derive determinism, whether or not weak locality (or even experimental free will) is valid, generalising a result of van Fraassen [7] (Proposition 2). Outcome independence is also sufficient, by itself, to imply the incompleteness of standard quantum mechanics (Proposition 3).

In section 3 the degree of support afforded to Bell’s claim by the EPR incompleteness argument is examined. Two gaps in the EPR logic are identified via simple counterexamples, implying that the argument is itself incomplete (Proposition 4). However, a suitable strengthening of the EPR reality criterion closes these gaps (Proposition 5), and further enables a rigorous derivation of both weak locality and determinism as required for Bell’s 1964 paper (Proposition 6).

Implications for quantum interpretations are noted in section 4, and the necessity or otherwise of nonlocality in quantum mechanics is discussed with reference to a classical example of nondeterministic evolution. Conclusions are given in section 5.
2 Locality, perfect correlation, and outcome independence

2.1 Weak locality (parameter independence)

Bell spells out his intended sense of locality in the introduction to his 1964 paper [1]:

“It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system, with which it has interacted in the past, . . .”.

It is applied in his section 2, in the context of Stern-Gerlach measurements, as

“Now we make the hypothesis, and it seems one at least worth considering, that if the two measurements are made at places remote from one another, the orientation of one magnet does not influence the result obtained with the other.”

Finally, it is the combination of determinism with this sense of locality that he argues is incompatible with quantum mechanics, as per the first sentence of his Conclusion:

“In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one device can influence the reading of another instrument, however remote.”

The last quote might suggest that Bell assumes both determinism and locality to obtain a contradiction with quantum mechanics. However, he later claimed that determinism is derived in his 1964 paper as a direct consequence of locality for the case of perfect correlations (giving rise to the debate mentioned in section 1). Bell’s claim is examined further below. For now, we need only note it must be possible to mathematically formulate Bell’s sense of locality without reference to determinism, if the latter is to be inferred rather than assumed. That is, a statistical formulation of locality is required.

To obtain such a formulation, let $x$ and $y$ label possible measurements which may be made in two separate regions of spacetime, having respective outcomes labeled by $a$ and $b$, and let $p(a,b|x,y)$ denote the joint probability density for these outcomes. If $\lambda$ denotes any further statistical variables of interest—arising, e.g., from a physical or mathematical model of the measurements—then it follows from the basic rules of probability that

$$p(a,b|x,y) = \sum_\lambda p(a,b,\lambda|x,y) = \sum_\lambda p(a,b|x,y,\lambda) p(\lambda|x,y).$$  \hspace{1cm} (1)

Here summation is replaced by integration over any continuous range of $\lambda$. The locality of such a model, in the sense spelled out in the introduction to Bell’s 1964 paper and quoted above, can now be rigorously formulated as

**Definition 1 (Weak locality)**: The probability distribution of the result of a measurement in one region is unaffected by measurement operations in a distant region, i.e.,

$$p(a|x,y,\lambda) = p(a|x,\lambda), \quad p(b|x,y,\lambda) = p(b|y,\lambda),$$

for all $x,y,\lambda$. 


This property is called weak locality here to distinguish it from other possible notions of locality, and is most easily justified when the regions are spacelike separated. It is also known as ‘hidden locality’ [7], ‘locality’ [37] and, more commonly, ‘parameter independence’ [35]. Importantly, it is the only statistical formulation of locality that is explicitly supported by Bell’s 1964 paper.

2.2 Determinism does not follow from weak locality and perfect correlation

As mentioned above, Bell claimed in 1981 that determinism was not assumed in his 1964 paper, but was logically inferred [6]:

“My own first paper on this subject . . . starts with a summary of the EPR argument from locality to deterministic hidden variables. But the commentators have almost universally reported that it begins with deterministic hidden variables.”

However, the only explicit argument for determinism in that first paper is supplied by just two sentences, in the context of measurements on two perfectly correlated spins $\sigma_1$ and $\sigma_2$ [1]:

“Now we make the hypothesis, and it seems one at least worth considering, that if the two measurements are made at places remote from one another, the orientation of one magnet does not influence the result obtained with the other. Since we can predict in advance the result of measuring any component of $\sigma_2$, by previously measuring the same component of $\sigma_1$, it follows that the result of any such measurement must actually be predetermined.”

Thus, it would appear that determinism is inferred as a logical consequence of weak locality and perfect correlation. The ensuing debate in the literature arises, at least in part, from a simple observation that undermines this argument [15,25].

**Proposition 1:**

weak locality + perfect correlation $\not\Rightarrow$ determinism.  

(3)

That is, Bell’s claim certainly does not hold for the only form of locality explicitly supported in his 1964 paper: weak locality is too weak.

To demonstrate the above proposition, consider two particular measurements, $x_0$ and $y_0$, that have a common range of possible values for their respective outcomes $a$ and $b$. Perfect correlation of these outcomes then corresponds to

$$p(A = B|x_0, y_0) = 1,$$  

(4)

where $A$ and $B$ denote the random variables corresponding to outcomes $a$ and $b$. It follows that proposition can be demonstrated via any model of correlations satisfying Eqs. (1), (2) and (4) such that the outcomes of measurements $x_0$ and $y_0$ are not predetermined by the model. The simplest such model is one for which $\lambda$ takes a single fixed value, $\lambda_0$ say, and only one measurement is possible in each region, $x_0$ and $y_0$ say, each with $n$ possible outcomes. Then the joint measurement distribution defined by

$$p(a, b|x_0, y_0) = p(a, b|x_0, y_0, \lambda_0) := n^{-1}\delta_{ab}$$  

(5)
trivially satisfies Eqs. (1), (2) and (4) (with \(p(\lambda_0|x_0, y_0) = 1\)), and yet is clearly not deterministic for any \(n > 1\).

Note that it is irrelevant whether or not the above counterexample has an underlying model which is deterministic (indeed, this is trivially the case for any given joint probability distribution \(p(a, b|x_0, y_0)\), perfectly correlated or otherwise): Bell’s *prima facie* claim is that determinism, rather than its mere possibility, is inferred from locality and perfect correlation alone. Counterexamples can also be easily constructed for continuous outcome ranges (e.g., with \(x_0\) and \(y_0\) corresponding to perfectly correlated classical phase space measurements), and for multiple measurement settings (e.g., based on simple models of ‘no-signalling’ correlations [39,40]).

2.3 Determinism does follow from outcome independence and perfect correlation

We have from Proposition 1 that determinism can only be inferred from perfect correlation via some assumption other than, or possibly in addition to, weak locality. For example, in his 1981 paper Bell interchangeably uses the terms ‘local causality’, ‘local explicable’, and ‘action-at-a-distance’ for the condition

\[
p(a, b|x, y, \lambda) = p(a|x, \lambda)p(b|y, \lambda)
\]

on models of correlations between distant regions (see also [41,42]). This condition is strictly stronger than weak locality in Eq. (2) (the latter follows by summing Eq. (6) over each of \(a\) and \(b\)), and is indeed sufficiently strong to derive determinism for the case of perfect correlation [9,12]. Thus local causality is a candidate for a missing additional assumption in Bell’s 1964 paper, albeit not actually supported in any guise therein.

However, local causality is a far stronger assumption than is necessary for the purpose of deriving determinism from perfect correlations: physical models need in fact only satisfy the much weaker condition

**Definition 2 (Outcome independence):** Any observed correlations between measurement outcomes arise from ignorance of some further variable(s) \(\lambda\), i.e.,

\[
p(a, b|x, y, \lambda) = p(a|x, y, \lambda)p(b|x, y\lambda).
\]

for all \(a, b, x, y\) and \(\lambda\).

This property has also been called, for example, ‘causality’ [7] and ‘complete-ness’ [37], and implies that any correlation between observed outcomes is due solely to the average over \(\lambda\) in Eq. (1). In particular, \(\lambda\) can be interpreted as a classical common cause for the correlations. Outcome independence is logically independent of weak locality, and clearly weaker than local causality in Eq. (6). Indeed, simple substitution shows that local causality is equivalent to the combination of weak locality and outcome independence [37].

Importantly, in contrast to Proposition 1 one has

**Proposition 2:**

\[
\text{outcome independence + perfect correlation } \implies \text{ determinism}
\]

with probability 1 (i.e., for a set of \(\lambda\) over which \(p(\lambda|x_0, y_0)\) sums or integrates to unity).
This result was first given for the case of discrete outcomes by van Fraassen [7], but it does not appear to be well known. It is not discussed, for example, in any of the contributions [8]–[26] to the debate other than [8,14]. It is extended here to include the case of continuous outcomes, as proved at the end of this section.

Significantly, Proposition 2 is independent of whether or not weak locality as per Eq. (2) is satisfied (although the latter is required for the further task of deriving Bell inequalities). Thus locality, in the only sense that is explicitly discussed in Bell’s 1964 paper, is in fact irrelevant for the derivation of determinism from perfectly correlated measurement outcomes! It is also worth noting the proposition does not rely on making an assumption of measurement independence or experimental free will (i.e., it is not assumed that \( p(\lambda|x, y) = p(\lambda) \)), nor even that the ‘common cause’ \( \lambda \) lies in the past.

As further observed by van Fraassen [7], Eq. (8) has a simple corollary of fundamental interest, also proved below.

**Proposition 3 :**

\[
\text{outcome independence } \implies \text{incompleteness of standard QM.}
\]

Thus an assumption of outcome independence by itself is sufficiently strong to imply the incompleteness of the standard Hilbert space formulation of quantum theory—regardless of whether or not weak locality or local causality or measurement independence holds. Noting the discussion following Eq. (7), this result can also be informally stated as *some quantum correlations have no classical common cause*. Further, although not recognised by van Fraassen, it is independent of the EPR incompleteness argument [27] (discussed in section 3), as is demonstrated by the proof below.

In particular, Proposition 3 follows via consideration of any quantum wave function that predicts perfect correlation between two measurements. Perhaps the simplest example is one given by Einstein at the 1927 Solvay Congress and later streamlined [43,44]: a superposition of a single particle in two regions, R1 and R2 say, with a wave function of the form

\[
|\psi\rangle = \frac{1}{\sqrt{2}} (|R1\rangle + |R2\rangle)
\]  (10)

Hence, if \( x_0 \) is a measurement detecting whether the particle is in R1, and \( y_0 \) is a measurement of whether the particle is in R2, it follows that the outcomes (suitably labeled) are perfectly correlated. It follows from Proposition 2 that any outcome-independent model of the detection statistics for this example must be deterministic. But quantum theory predicts a nondeterministic (50%) probability for either outcome. Hence, the wave function description is incomplete under an assumption of outcome independence, as claimed.

Finally, to demonstrate Proposition 2 note that perfect correlation for the outcomes of measurements \( x_0 \) and \( y_0 \) implies \( \sum_\lambda p(A = B|x_0, y_0, \lambda)p(\lambda|x_0, y_0) = 1 \) via Eqs. (1) and (4). Hence \( \sum_\lambda [1 - p(A = B|x_0, y_0, \lambda)]p(\lambda|x_0, y_0) = 0 \), and so, noting each factor is nonnegative,

\[
p(A = B|x_0, y_0, \lambda) = 1
\]  (11)
with probability 1. Now, for the case of discrete outcomes we have
\[ p(A = B|x_0, y_0, \lambda) = \sum_a p(a, a|x_0, y_0, \lambda). \] (12)

It follows via Eq. (11) and outcome independence as per Eq. (7) that
\[ \sum_a p(A = a|x_0, y_0, \lambda) [1 - p(B = a|x_0, y_0, \lambda)] = 1 - 1 = 0. \]

Thus, since each factor is nonnegative,
\[ p(A = a|x_0, y_0, \lambda) [1 - p(B = a|x_0, y_0, \lambda)] = 0 \] (13)
with probability 1. But \( p(A = a|x_0, y_0, \lambda) > 0 \) for at least one value, \( a = a' \) say, and therefore \( p(B = a'|x_0, y_0, \lambda) = 1 \), implying \( p(B = b|x_0, y_0, \lambda) = \delta_{b,b'}. \) This vanishes for \( b \neq a' \), and so \( p(A = a|x_0, y_0, \lambda) = 0 \) for \( a \neq a' \) from Eq. (13), yielding \( p(A = a|x_0, y_0, \lambda) = \delta_{a,a} \). Thus the measurement outcomes are deterministic, as claimed in Proposition 2.

The case of continuous outcomes is more subtle, noting that Eq. (12) does not generalise to \( p(A = B|x_0, y_0, \lambda) = \int da p(a, a|x_0, y_0, \lambda) \): the right hand side is not a probability, and indeed it diverges for perfectly correlated cases such as \( p(a, b|x_0, y_0, \lambda) = \delta(a)\delta(b) \). Instead, one has
\[ p(A = B|x_0, y_0, \lambda) := \lim_{\epsilon \to 0} p(|A - B| < \epsilon|x_0, y_0, \lambda) \]
\[ = \int da p(a|x_0, y_0, \lambda) \lim_{\epsilon \to 0} \int_{\{b : |b - a| < \epsilon\}} db p(b|x_0, y_0, \lambda), \] (14)
where the last line follows assuming outcome independence as per Eq. (7). Equation (13) therefore generalises to
\[ p(a|x_0, y_0, \lambda) \left[ 1 - \lim_{\epsilon \to 0} \int_{\{b : |b - a| < \epsilon\}} db p(b|x_0, y_0, \lambda) \right] = 0 \] (15)
with probability 1. Essentially the same argument as for the discrete case may now be applied. First, \( p(a|x_0, y_0, \lambda) > 0 \) for at least one value, and therefore the limit in Eq. (15) must be unity for this value, \( a = a' \) say, implying that \( p(b|x_0, y_0, \lambda) \) is fully supported on \( \{b : |b - a'| < \epsilon\} \) with probability 1, for all \( \epsilon > 0 \), i.e., \( p(b|x_0, y_0, \lambda) = \delta(b - a') \). But this vanishes for \( b \neq a' \), implying from Eq. (15) that \( p(a|x_0, y_0, \lambda) = 0 \) for \( a \neq a' \), and hence that \( p(a|x_0, y_0, \lambda) = \delta(a - a') \). Thus, again, the measurement outcomes are deterministic with probability 1, and Proposition 2 follows.

3 Can the EPR argument be considered complete?

Bell’s 1964 two-sentence derivation of determinism from locality and perfect correlation, as quoted in section 2 above, is certainly incomplete. In particular, as evidenced by Proposition 1 it is not supported by the only explicit form of locality discussed in that paper, and must therefore rely on some other unstated assumption. While either of local causality or outcome independence would do the job, as implied by Proposition 2 it is natural to ask, given that Bell claims his derivation is a version of the EPR argument, whether his unstated assumption can in fact be deduced from the original EPR paper. This is the subject of this section.
3.1 Logic of the EPR argument

The derivation of determinism via perfect correlation in the EPR paper begins with a simple sufficient condition for an element of reality, quoted here [27]:

**Definition 3 (EPR reality criterion)**: If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

This condition is then applied to cases of perfect correlation as per Eq. (4) (and in particular to perfect position and momentum correlations), using the following logic (see also section 2 of [45]):

1. Assume measurement \( x_0 \) is made in a first region, with result \( a \) (assumption).
2. The outcome of a measurement \( y_0 \) in a distant second region can then be predicted as \( b = a \) with certainty (perfect correlation).
3. This prediction can be obtained without disturbing the distant region in any way (assumption).
4. Hence, the value of \( b \) is an element of physical reality, prior to any actual measurement of \( y_0 \) in the distant region (EPR reality criterion).

The above steps are all that EPR explicitly use to derive reality/determinism for any given perfect correlation. However, when one tries to make the above steps mathematically rigorous, a gap shows up in the above logic. Indeed, as will be shown via counterexamples below, the EPR reality criterion is in fact insufficient for concluding the reality of \( b \) via these steps: a further assumption is required.

Moreover, EPR go on to consider what can be said if there are two or more perfectly correlated pairs of measurements, and add the following further step:

5. If \( x_1 \) and \( y_1 \) are a second pair of perfectly correlated measurements, for the first and second regions respectively, then applying the same steps as above implies that the outcomes of both \( y_0 \) and \( y_1 \) are real and predetermined.

EPR rely on this last step to conclude that the incompleteness of quantum mechanics follows from their reality criterion [27]. However, as is also shown via a counterexample below, Step 5 does not strictly follow as a logical consequence of Steps 1–4: there is a second missing assumption. It follows that

**Proposition 4**: The logic of the EPR argument for the incompleteness of quantum mechanics is itself incomplete, i.e.,

\[
\text{EPR reality criterion} \not\Rightarrow \text{incompleteness of QM.}
\]  

Thus, comparing with Proposition 3, it is seen that the EPR reality criterion is not as strong as outcome independence in this regard. Not all is lost, however. In particular, it is not difficult to formulate a stronger form of the reality criterion that allows the EPR argument to go through.

3.2 Counterexamples to the EPR logic

The EPR logic, as summarised above, has two important gaps which prevent Steps 4 and 5, respectively, from going through without additional assumptions, and lead to Proposition 4. These gaps relate to symmetry and joint measurements, and are best illuminated via counterexamples.
3.2.1 The asymmetry gap

The first gap in the EPR logic is relatively minor, and arises from the inherent asymmetry of the argument: it requires only that a measurement made in the first region does not disturb the system in the second region in any way, as per Step 3 (e.g., “at the time of measurement . . . no real change can take place in the second system in consequence of anything that may be done to the first system” [27]). This does not rule out, however, the possibility that making a measurement in the second region can disturb the system in the first region. Note this is consistent with no direct interaction between the two systems if it is the measurement device that is responsible for the disturbance.

One class of counterexamples that exploits this gap is to allow (faster-than-light) signalling from the second region to the first region, but not vice versa:

Example 1: Suppose that each region contains a single spin-\(\frac{1}{2}\) particle in a totally mixed state, and that if a device measures spin in the \(y\) direction of the second particle, with result \(b\), it sends a signal (e.g., superluminally or along its backward lightcone) that puts the first particle into a \(-b\) eigenstate of spin in the \(y\) direction. Hence, if a measurement of spin in the \(x\) direction is made in the first region, the statistics of the singlet state are reproduced, i.e., \(p(a, b|x, y) = \frac{1}{4}(1 - ab x \cdot y)\).

Note for this example, choosing \(y_0 = \pm x_0\), that the outcome in the second region can be predicted with probability unity from the result of a measurement in the first region, and that there is no disturbance of the second region by any measurement carried out in the first region, thus fulfilling the conditions for the EPR reality criterion. Nevertheless, contrary to Step 4 of the EPR logic, there is no pre-existing element of reality for the outcome of a measurement in the second region: a totally random and unpredictable result \(b = \pm 1\) is obtained for any spin direction \(y\). Thus, use of the EPR reality criterion fails for this example, implying that the EPR logic is incomplete without some further assumption. Note that if the spin measurements are replaced by linear polarisations, for a two-photon state, then the two initial states can be replaced by eigenstates of circular polarisation.

An interesting second class of counterexamples exploiting the asymmetry gap is temporal in nature, and applies to the case of a single particle:

Example 2: Suppose, a single spin-\(\frac{1}{2}\) particle is initially described by a maximally mixed state, and that the second region lies within the past lightcone of the first region. Suppose further that a measurement of spin in some direction, if made in the second region, acts to leave the particle in an eigenstate of spin in that direction. Hence, if the same spin direction is subsequently measured in the first region, the results are perfectly correlated.

Thus, the outcome in the second region can be predicted from the result of a measurement made in the first region and, assuming no retrocausality, it is impossible for any measurement in the first region to disturb the second region in any way, as required by the EPR reality criterion. Yet there is no pre-existing element of reality for the outcome in the second region, contrary to Step 4 of the EPR argument.

Classical examples can also be obtained by replacing the particles in the above examples by classical models thereof [1]. One way to close the asymmetry gap is
simply to symmetrise the EPR logic, by further assuming that measurement $y_0$ is made in the second region, without disturbing the first region in any way (which has the additional advantage that one can also obtain the reality of the outcome of $x_0$, via Steps 1–4 with the roles of $x_0$ and $y_0$ reversed). It turns out, however, that this assumption is too strong in that it prevents closure of a second gap.

### 3.2.2 The joint measurement gap

To expose the second (and most important) gap in the EPR argument (see also section 2 of [45]), observe that the logic starts with the assumption in Step 1 that $x_0$ is measured. Without this assumption the value of $b$ cannot be predicted with certainty, as required for Steps 2 and 4. It follows that, even ignoring (or in some way closing) the asymmetry gap, one cannot proceed directly from Step 4 to Step 5. In particular, for a second pair of perfectly correlated measurements $x_1$ and $y_1$, one can only conclude that the outcomes of both $y_0$ and $y_1$ are real and predetermined if both $x_0$ and $x_1$ are measured without disturbing the predicted perfect correlations. Conversely, if they cannot both be so measured, then Step 5 does not logically follow (without some additional assumption). Note that this issue is only exacerbated if the asymmetry gap is closed as per the method of the previous subsection, as Step 5 would then require that both $y_0$ and $y_1$ are also measured, without disturbing the correlations.

In his response to the EPR paper, Bohr famously gave an explicit quantum counterexample exploiting this joint measurement gap, by showing how perfect momentum and position correlations between two quantum particles cannot both be maintained as soon as either the position or momentum of one particle is measured [46] (note that the fourth and fifth pages, describing the counterexample, are printed in reverse order). The singlet state measurements considered by Bell supply a similar counterexample (since a Stern-Gerlach magnet cannot simultaneously have two measurement orientations), and one can also construct classical counterexamples [47].

EPR allude to the joint measurement gap in the penultimate paragraph of their paper, when they reject a possible replacement of their reality criterion by a more restrictive criterion for the reality of the momentum $P$ and position $Q$ of a particle in the second region [27]:

"Indeed, one would not arrive at our conclusion if one insisted that two or more physical quantities can be regarded as simultaneous elements of reality only when they can be simultaneously measured or predicted. ... This makes the reality of $P$ and $Q$ depend upon the process of measurement carried out on the first system, which does not disturb the second system in any way. No reasonable definition of reality could be expected to permit this."

This quote suggests a possible method for closing the joint measurement gap: by making the ‘reasonable’ assumption that the reality of a physical quantity in a given region is independent of any measurement that does not in any way disturb that region (note that this assumption would represent a necessary condition for an element of reality, in contrast to the sufficient condition given by the EPR reality criterion). Thus, if Steps 1–4 are taken to be valid, then the reality of both $y_0$ and $y_1$ must still hold even if neither of the non-disturbing measurements of $x_0$ and $x_1$ are actually made. However, this assumption is too weak to also close the
Does locality plus perfect correlation imply determinism? 11

asymmetry gap, which prevents Step 4 from being used to actually conclude the reality of \( y_0 \) (or \( y_1 \)) even in the case that \( x_0 \) (or \( x_1 \)) is measured. In particular, the assumption can only be applied once the reality of the outcome of \( y_0 \) (or \( y_1 \)) has actually been established.

3.3 Closing the gaps: a stronger reality criterion

It is clear from the counterexamples of the previous subsection that the EPR reality criterion is not strong enough to derive the incompleteness of quantum mechanics, as per Proposition 4. It is further clear that the common issue arising from the gaps identified by these counterexamples is the need for stronger notions of non-disturbance and reality. A natural solution to this issue is to minimally strengthen the EPR reality criterion as follows.

**Definition 4 (Local reality criterion)**: If, without in any way disturbing or being disturbed by a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists, prior to the prediction, an element of physical reality corresponding to this physical quantity.

The italicised phrases indicate additions to the original EPR reality criterion in Definition 3, and close the asymmetry and joint measurement gaps respectively. Note that they correspond to a weakening of the method of closing the asymmetry gap considered in section 3.2.1 (a direct measurement of \( y_0 \) is not required), and to a strengthening of the method of closing the joint measurement gap considered in section 3.2.2 (the reality of the outcome does not require the prediction to have actually been established by some suitable measurement).

It is shown here that this stronger criterion of local reality restores the desired logical completeness of the EPR argument, i.e., in contrast to Proposition 4.

**Proposition 5**

\[
\text{local reality criterion } \implies \text{incompleteness of standard QM,} \tag{17}
\]

It is further shown that one can rigorously obtain:

**Proposition 6**

\[
\text{local reality criterion } + \text{perfect correlation } \implies \text{weak locality } + \text{determinism,} \tag{18}
\]

as required by Bell for his 1964 paper. Hence it is natural to identify the unstated assumption in that paper with the local reality criterion. Note that Proposition 6 is also valid if the local reality criterion is replaced by local causality as per Eq. 10. Thus this criterion is also closely related to the assumption used to derive general Bell inequalities [2,41,42].

The proof of Proposition 5 is straightforward. First, the EPR logic for pairs of perfectly correlated measurements \( x_0, y_0 \) and \( x_1, y_1 \) is modified as follows (with primes and italics indicating modified steps):

1. Assume measurement \( x_0 \) is made in a first region, with result \( a \) (assumption).
2. The outcome of a measurement \( y_0 \) in a distant second region can then be predicted as \( b = a \) with certainty (perfect correlation).
3’. This prediction can be obtained without disturbing or being disturbed by the distant region in any way (assumption).

4’. Hence, the value of \( b \) is an element of physical reality, prior to any actual measurement of \( y_0 \) in the distant region, and prior to obtaining the prediction via an actual measurement of \( x_0 \) (local reality criterion).

5’. If \( x_1 \) and \( y_1 \) are a second pair of perfectly correlated measurements, for the first and second regions respectively, then applying the same steps as above implies that the outcomes of both \( y_0 \) and \( y_1 \) are real and predetermined prior to any actual measurement of \( x_0 \) or \( x_1 \).

Note that Step 3’ rules out the counterexamples in section 3.2.1 and Step 4’ rules out those in section 3.2.2.

The above logic is sufficient to obtain Proposition 6 using either the perfectly correlated momentum and position wave function considered by EPR or the singlet state considered by Bell. It is also sufficient for the determinism component of Proposition 5. However, the weak locality component of the latter requires a little more work, including several uses of the following Lemma.

**Lemma 1:** If the conditional distribution of a random variable \( j \) is deterministic for given prior information \( k \) (i.e., \( p(j|k) = 0 \) for all but one value of \( j \)), then it remains deterministic when conditioned on any further information \( l \) compatible with \( k \) (i.e., \( p(j|k,l) = 0 \) for all but one value of \( j \)).

**Proof:** We have \( p(j|k) = 0 \) for all \( j \neq j_0 \), for some \( j_0 \), and \( p(l|k) > 0 \). Hence \( 0 \leq p(j,l|k) \leq p(j|k) = 0 \) for \( j \neq j_0 \), and so, using the standard rules of probability, \( p(j|k,l) = p(j,l|k)/p(l|k) = 0 \) for all \( j \neq j_0 \).

To prove Proposition 6, note first from Step 4’ above that the outcome of \( y_0 \) is predetermined prior to any measurement of \( x_0 \) or \( y_0 \). Let \( \lambda_0' \) denote any additional variables needed to determine this outcome, i.e., \( p(b|a,x_0,y_0,\lambda_0') = 0 \) for all but the predetermined value of \( b \). Further, from Step 3’, actually making a measurement of \( x_0 \) in the first region cannot affect this predetermined outcome, i.e., \( x_0 \) and \( a \) are redundant in determining its value, and hence \( p(b|a,x_0,y_0,\lambda_0) = p(b|y_0,\lambda_0') = 0 \) for all but one value of \( b \).

Second, suppose that in fact some other measurement, \( x \) say, rather than \( x_0 \), is actually made in the first region, without disturbing the second region in any way. Then, since we have \( p(b|y_0,\lambda_0') = 0 \) for all but one value of \( b \), and \( x \) must be compatible with \( y_0 \) and \( \lambda_0' \) if it is actually made, it follows from the Lemma that

\[
p(b|x_0,y_0,\lambda_0') = p(b|y_0,\lambda_0') = 0
\]

for all but one value of \( b \) and any such measurement. Similarly, reversing the roles of the first and second regions, and letting \( \lambda_0'' \) denote any additional information needed to determine the outcome of \( x_0 \), it also follows that

\[
p(a|x_0,y,\lambda_0'') = p(a|x_0,\lambda_0'') = 0
\]

for all but one value of \( a \) and any measurement \( y \) made in the second region that does not disturb the first region.

Third, suppose there is some set of perfectly correlated pairs of measurements, \( \{(x_s,y_s) : s \in S\} \), for some index set \( S \), where the measurement in each region
Does locality plus perfect correlation imply determinism?

... not disturb and is not disturbed in any way by the measurement in the other region. Using an obvious notation, the above two equations then generalise to

\[
p(b|x_s, y_t, \lambda'_s) = p(b|y_t, \lambda'_s) = 0, \quad p(a|x_s, y_t, \lambda''_s) = p(a|y_t, \lambda''_s) = 0
\]

for all but one value of \(a\) and \(b\) and all \(s, t \in S\). Hence, letting \(\lambda\) denote the set of pairs \(\{(\lambda'_s, \lambda''_s) : s \in S\}\), application of the above Lemma immediately gives

\[
p(a|x_s, y_t, \lambda) = p(a|x_s, \lambda) = 0, \quad p(b|x_s, y_t, \lambda) = p(b|y_t, \lambda) = 0
\]

(21)

for all but one value of \(a\) and \(b\) and all \(s, t \in S\). Thus, the outcomes of all pairs \((x_s, y_t)\) are deterministic, and satisfy weak locality as per Eq. (2), thereby proving Proposition (6).

4 Implications for quantum interpretations and locality

4.1 Completeness of quantum interpretations

While not as strong as Bell inequalities in their physical consequences, Propositions 3 and 5 are nevertheless significant for various interpretations of quantum phenomena, as briefly noted below.

First, any interpretations that rely exclusively on the standard Hilbert space description, such as the Copenhagen [48], many worlds [19,50], consistent histories [51], spontaneous collapse [52,53], relational quantum mechanics [54] and QBist [55] interpretations, necessarily reject outcome independence (and thus classical common causes): probabilities calculated from states on Hilbert space do not factorise as per Eq. (7). Thus these interpretations are not subject to a charge of incompleteness via Proposition 3.

However, any such interpretation must further reject the local reality criterion in Definition 4 if it is to also maintain completeness in the light of Proposition 5. A common approach is to reject the existence of an element of reality prior to being able to make a prediction with probability unity. For example, in the many worlds interpretation such a prediction cannot be made until a measurement establishes the relevant branch (or memory sequence) of the observer making the prediction [50], while in the Copenhagen interpretation Bohr emphasises there is “an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system . . . [where] these conditions constitute an inherent element of the description of any phenomenon to which the term ‘physical reality’ can be properly attached” [46] (his italics). Another approach is to assert the reality of wave function collapse, thus implying a disturbance of one region by measurement in another, as in spontaneous collapse interpretations [52,53] (and in some variants of the Copenhagen interpretation, albeit explicitly rejected by Bohr [46]: “Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation”—see also [50]).

In contrast, interpretations which either supplement or replace standard quantum theory with a deterministic description of outcomes, such as the deBroglie-Bohm [57,58], many-interacting-worlds [59,60] and superdeterministic [61,62] interpretations, treat standard quantum mechanics as incomplete from the outset,
and hence are untroubled by Propositions 3 and 5. Note that any such interpretation automatically satisfies outcome independence (deterministic models necessarily satisfy Eq. (7)), and may or may not satisfy the local reality criterion. For example, for the case of perfect position and momentum correlations considered by EPR, both the deBroglie-Bohm and many-interacting-worlds interpretations satisfy this criterion for position measurements (the positions of all particles are ‘real’), but not for momentum measurements (such measurements on one particle necessarily disturb the other particle, directly via a nonlocal influence in the deBroglie-Bohm interpretation [58], and indirectly via interactions with other worlds in the many-interacting-worlds approach [60]). In contrast, the local reality criterion is always satisfied in superdeterministic interpretations (as is local causality), as these give a measurement-dependent explanation of quantum correlations in terms of local classical common causes, which not only determine the measurement outcomes but the measurement selections themselves [61,62].

4.2 Is outcome independence a locality property?

As noted in the Introduction, Bell’s claim that determinism follows from locality and perfect correlations would further suggest, in the light of his 1964 inequality [1], that quantum mechanics is necessarily nonlocal. While debate on this question is also extensive (see, e.g., [9,15,28]–[36]), it largely centres around different possible definitions of ‘locality’ [15] and so, in the spirit of avoiding ‘woolly interpretive commentary’ as per the Introduction, will not be reviewed here. Nevertheless, some basic points are made below that help clarify some aspects, including an example of local classical indeterminism with perfect correlations.

First, standard quantum mechanics is certainly local in the sense of weak locality: operations carried out in one region cannot affect the statistics in a distant region [63,64]. Any claim of nonlocality must therefore rely on a different sense (or on an interpretation that goes beyond the standard quantum description). Second, the local reality criterion in section 3.3 clearly involves notions of both locality and reality, and hence its failure cannot unambiguously rule out locality. Third, the violation of Bell inequalities by quantum correlations do not necessarily rule out locality in the strong sense of local causality as per Eq. (6), unless one further assumes measurement independence [65]. Fourth, even under the assumption of measurement independence, local causality is equivalent to the combination of weak locality and outcome independence, and hence its failure corresponds to an unambiguous signature of nonlocality if and only if the failure of outcome independence is also such a signature. But is this the case?

The answer to the above question is, prima facie, negative. Outcome independence can be interpreted purely in terms of classical common causes [4] or completeness [47], and hence any violation can be taken to correspond to, for example, the existence of nonclassical common causes, rather than to some type of nonlocality. Further, while Shimony refers to the failure of outcome independence as an ‘uncontrollable nonlocality’ [28], the nature of this nonlocality is only in the vague and unexplained sense of ‘passion at a distance’ (amusingly criticised by Mermin as ‘fashion at a distance’ [30]). These points do not, however, strictly rule out the possibility that some overlooked and unambiguous nonlocal effect is associated with the failure of outcome independence. In this context it is of inter-
est to note the existence of classical examples that further curtail this possibility, based on systems in classical mechanics and electrodynamics that are inherently nondeterministic [66, 67, 68, 69].

For example, consider two classical particles of mass \( m \) moving in one dimension under a potential of the form

\[
V(x_1, x_2) = \begin{cases} 
-\gamma |x_1 - x_2|^{3/2}, & |x_1 - x_2| \leq d, \\
-\gamma d^{3/2}, & |x_1 - x_2| > d,
\end{cases}
\]

(22)

where \( d \) is some fixed separation distance beyond which the particles do not interact. If the particles are initially at rest at the origin, i.e., \( \dot{x}_1(0) = \dot{x}_2(0) = 0 \), then conservation of energy in the centre of mass frame corresponds to

\[
\frac{1}{4}m \dot{x}_2^2 - \gamma \left| x \right|^{3/2} = 0
\]

for separations \( x = x_1 - x_2 \) less than \( d \), and free motion for separations greater than \( d \). Remarkably, this system has the non-unique set of solutions:

\[
x_R(t) = -x_L(t) = \begin{cases} 
0, & 0 \leq t \leq T, \\
\frac{\gamma^2}{m} \left( t - T \right)^4, & T < t \leq T + \tau, \\
\frac{d^2 + \sqrt{\gamma m} d^{3/4} (t - T - \tau)}{2 m}, & t > T + \tau,
\end{cases}
\]

(23)

where \( x_L \) and \( x_R \) label the left-moving and right-moving particles, \( T \geq 0 \) is arbitrary, and \( \tau := 2(\frac{dm^2}{\gamma})^{1/4} \). Thus the particles remain at the origin for time \( T \), then repel each other for a fixed duration \( \tau \), and are noninteracting thereafter. The evolution is therefore nondeterministic, since the ‘pause time’ \( T \) is not uniquely fixed despite all forces and initial conditions being specified and well-defined.

Suppose now that two detectors are placed at equal distances greater than \( d/2 \) from the origin, and switched on at time \( t = 0 \). It follows that there is a perfect correlation between their readouts at all times, irrespective of the value of \( T \) (there is also a perfect correlation between the particle positions and momenta, up until detection). We thus have a classical example of perfect correlations between two noninteracting regions, in a nondeterministic context.

It follows via Proposition 6 that the local reality criterion in section 3.3 fails for this example (as does the EPR reality criterion). Further, while outcome independence cannot be directly assessed (the example is nonstatistical), it nevertheless fails in the broad sense of being incompatible with Proposition 2 for this example. It is clear that these failures are not due to any nonlocal effect, but to the lack of a classical cause for the pause time \( T \), where this lack may itself be regarded as form of incompleteness.

The above example of classical indeterminism corresponds to reinterpreting the single-particle model in [68] as describing the relative displacement of two identical particles initially at rest, and switching off the inter-particle interaction above a specified separation distance to ensure locality. Similar examples can be obtained via the single-particle models in [67], while an inherently local and relativistic example is provided by a weakly bound system of a classical electromagnetic field and two identical classical particles in two-dimensional spacetime [66, 69]. It would be of interest to determine whether the model of indeterministic classical motion suggested by Gisin can provide further examples [70] (although it is not clear how to satisfactorily obtain perfect correlations via conservation laws in this model).
5 Conclusions

Bell’s “insistence that the determinism was inferred rather than assumed” [6], via the combination of locality and perfect correlation in his 1964 paper, is not supported by the only form of locality considered in that paper (weak locality is simply too weak), nor by an appeal to the EPR paper (the incompleteness argument therein is itself incomplete), as per Propositions 1 and 4. It is interesting in this regard that Bell did not back up his claim by spelling out any details of how determinism might be rigorously derived. Indeed, when revisiting the example of perfect spin correlations in section 3 of his 1981 paper [6], he merely states that “we seem obliged to admit that the results on both sides are determined in advance” (my italics). Likewise, in earlier papers he states only, for example, that “This strongly suggests that the outcomes of such measurements, along arbitrary directions, are actually determined in advance” [71]; or asks “Is it not more reasonable to assume that the result was somehow predetermined all along?” [72] (my italics).

It follows in any case, from Proposition 1, that an extra assumption of some sort, either additional to or in place of weak locality, is required to derive determinism from perfect correlation (see also [15,25]). Further, as first noted by van Fraassen [7], the assumption of outcome independence is sufficient for this purpose (whether or not weak locality or local causality or even measurement independence holds) and, moreover, is even sufficient to derive the incompleteness of quantum mechanics, as reviewed in Propositions 2 and 3.

It similarly follows that the EPR incompleteness argument requires an additional assumption of some sort, to avoid the asymmetry and joint measurement gaps identified in section 3.2. Strengthening the EPR reality criterion to the local reality criterion in Definition 3 is sufficient for this purpose, as per Proposition 5. Moreover, the strengthened criterion is also sufficient, when combined with perfect correlation, to imply both weak locality and determinism as per Proposition 6, as required for deriving Bell’s original inequality. Given Bell’s appeal to the EPR argument to support his claim, the local reality criterion is therefore a natural candidate for the unstated assumption in his 1964 paper.

As noted in section 4, the completeness of quantum mechanics can be maintained by all interpretations that rely exclusively on the standard Hilbert space description. Locality can also be maintained, in the absence of any evidence that the failure of outcome independence corresponds to an unambiguous signature of nonlocality. Indeed, if we follow van Fraassen in interpreting outcome independence as corresponding to classical common causes, it follows that its failure in standard quantum mechanics instead corresponds to the existence of nonclassical common causes. In this regard it is of interest to note that some quantitative distinctions between classical, quantum, and nonclassical common causes have been recently identified [73].

Finally, it is well known that the experimental violation of Bell inequalities requires that models of quantum correlations give up at least one of weak locality, outcome independence or measurement independence. But stronger conclusions can be obtained by finding experimental tests that relax at least one of these assumptions. For example, a relaxed Bell inequality has recently been given [74] and experimentally tested [75] for models satisfying measurement independence.
but which relax weak locality and outcome independence to the weaker condition
\[
p(a, b|x, y, \lambda) = w_{A \rightarrow B}^\lambda p(a|x, \lambda) p(b|a, y, \lambda) + w_{B \rightarrow A}^\lambda p(a|b, x, \lambda) p(b|y, \lambda)
\]  
for all \(x, y, \lambda\), where \(w_{A \rightarrow B}^\lambda + w_{B \rightarrow A}^\lambda = 1\). Thus, the measurement \(x\) in a first region can influence the statistics of \(b\) in a second region via its outcome \(a\) (with some probability \(w_{A \rightarrow B}^\lambda\)), while the measurement \(y\) in the second region can influence the statistics of \(a\) in the first region via its outcome \(b\) (with probability \(w_{B \rightarrow A}^\lambda\)). Violations of a relaxed Bell inequality for such models, by some quantum systems, show that some quantum correlations cannot be described even if weak locality and outcome independence are relaxed to this extent \([74, 75]\). In contrast, the simple one-particle example used to prove Proposition 3 suggests the possibility of semi-device independent tests of models that are only constrained by outcome and measurement independence (i.e., allowing weak locality to be fully relaxed), based on modifying the example to allow approximate perfect correlations. This will be investigated elsewhere.

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