Combining parton showers and NNLO matrix elements

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In this talk, we discuss recent developments in combining parton showers and fixed-order calculations. We focus on the UN²LOPS method for matching next-to-next-to-leading order computations to the parton shower, and we present results from SHERPA for Drell-Yan lepton-pair and Higgs-boson production at the LHC.

1 Introduction

With the LHC experiments entering the second long phase of data collection after the upgrade period, we expect that the Standard Model (SM) of particle physics will be probed in exquisite detail while searching for hints of phenomena beyond our current knowledge. A major role in this endeavor is played by parton-shower Monte Carlo programs, which allow to predict the full final-state kinematics on an event-by-event basis.

In this talk, we will briefly describe the evolution and status of combining fixed-order calculations with parton shower (PS) resummation, followed by comments on which state-of-the-art merging schemes lend themselves to further improvements. We will then discuss how next-to-next-to-leading order (NNLO) accurate predictions can be included into event generators. Finally, we present results in the UN²LOPS scheme\textsuperscript{1,2} as implemented in SHERPA\textsuperscript{3}.

2 The story so far

Finding ways to combine accurate fixed-order calculations with parton showers has been a major topic in event generator development since the turn of the century. A decisive boost came from methods for merging multiple inclusive tree-level calculations by making them exclusive using Sudakov form factors derived from the parton shower\textsuperscript{6,7,8}. Another breakthrough was the development of algorithms for matching parton showers to NLO QCD calculations\textsuperscript{4}.

All these methods have ambiguities and uncertainties. A particularly striking example of differences between NLO+PS matched results was presented in\textsuperscript{5}: The prediction for the Higgs-boson transverse momentum distribution shown in this publication varies greatly with the matching scheme. Differences in the schemes are formally beyond the required NLO+PS accuracy. Their numerical size reveals, however, that more accurate and less variable calculations of the Higgs-boson + jet process must be included to make experimentally relevant predictions.

This can be achieved using methods for combining a sequence of multi-parton fixed-order calculations, often referred to as "multi-jet merging". Merging methods exist for tree-level\textsuperscript{6} and NLO calculations\textsuperscript{7,8}. They provide state-of-the-art predictions for LHC Run-II. A comparison of NLO merging schemes in\textsuperscript{9} has shown good agreement between different approaches. More importantly, the agreement between theory and experiment is improved, and theoretical uncertainties may be reduced.

\textsuperscript{a}Speaker
3 Moving towards NNLO accuracy

NLO multi-jet merging techniques have additional features compared to LO merging. For example, those real-emission corrections to $X + n$-jet production which lead to $n + 1$ well-separated jets above the merging scale need to be removed, since such configurations are already included by merging with the $n + 1$-jet calculation. In addition, the approximate virtual corrections included in the PS must, at $O(\alpha_s^{n+1})$, be replaced by the full NLO result. A more subtle issue arises from additionally demanding the stability of inclusive jet cross sections\textsuperscript{8,10}: In merged calculations, the emission probability is given by exact fixed-order matrix elements. In contrast, the resummed virtual corrections derive from the Sudakov factor of the parton shower. Upon integration over the radiative phase space, the two do not cancel, leading to a “unitarity violation”.

This discrepancy can be removed using unitary merging techniques\textsuperscript{8}. One of them is the so-called UNLOPS method. It allows, in a process-independent way, to add the precise difference between fixed-order real-emission matrix elements and their parton-shower approximations to the merged result. This is called the ”subtract what you add” philosophy. In the UNLOPS scheme, it is possible to combine arbitrarily many NLO calculations, and include tree-level results when NLO calculations are not available. UNLOPS retains the merging scale as a technical parameter, since low merging scales – while desirable to use higher-order calculations over most of the phase space – leads to inefficient event generation.

4 Combining NNLO calculations with parton showers

Although NLO merging yields accurate predictions for many multi-jet observables, it is desirable for some reactions to move beyond NLO accuracy. Such processes include reactions with large higher-order corrections, e.g. Higgs-boson production in gluon fusion, standard candles like Drell-Yan lepton pair production, and other phenomenologically important processes.

NNLO accurate matching to the parton shower has been achieved first in the MINLO approach\textsuperscript{11}. The MINLO method\textsuperscript{10} is based on matching the hard process plus one-jet NLO calculation to the parton shower, and supplement it with Sudakov form factors that account for the resummed virtual and unresolved higher-order corrections between the hard scale and the resolution scale of the jet. In its current implementation it uses analytic Sudakov factors derived for $q_T$ resummation, which limits its applicability to hard processes with no light QCD jets in the final state. The genuine NNLO corrections are included through pre-tabulated phase-space dependent K-factors, which leads to fast event generation but makes the extension to processes with more complicated final states challenging.

Within the UN\textsuperscript{2}LOPS approach\textsuperscript{1,2}, a variant of UNLOPS, NNLO corrections associated with the emission of resolvable QCD radiation are treated as the hard process plus one additional jet at NLO. The remainder of the phase space is filled by a calculation for the hard process at NNLO, with a corresponding veto on any QCD activity. Both parts are separately finite, and parton shower matching is only needed for the first. To make the result physically meaningful, the separation cut must be smaller than the infrared cutoff of the parton shower. This requires very stable NLO matched calculations for the one-jet process. In contrast to the MINLO method, real-emission configurations do not receive a contribution from the NNLO K-factor.

Neither NNLOPS nor UN\textsuperscript{2}LOPS should be considered final a answer to NNLO+PS matching, but rather as a first step towards more general methods.

5 NNLO+PS matched results in SHERPA

We will now discuss some phenomenologically relevant results obtained with the UN\textsuperscript{2}LOPS matching as implemented in the SHERPA event generator. In order to control all aspects of the matched calculation, the full NNLO calculation using a $q_\perp$ cutoff method has been implemented
in Sherpa itself. This technique is limited to processes without light jets in the hard process, a shortcoming that can in principle be remedied by using different techniques for performing the fixed-order NNLO calculation. The following plots, and the SHERPA plug-in containing the UNLOPS implementation are publicly available\textsuperscript{14}.

Figure 1 highlights an interesting feature of the NNLO corrections to neutral and charged current Drell-Yan lepton pair production. For inclusive observables, using a NNLO PDF for a NLO calculation reproduces the full NNLO calculation very well, both in normalization and in shape. This is clearly a very process-dependent statement, and it breaks down once an observable depends not only on the Born degrees of freedom, as shown in the lower right panel of Figure 1: In the phase space region which can only be accessed by giving the lepton-pair system transverse momentum ($p_T > 40$ GeV), the NNLO result cannot be mimicked by a NLO calculation. In this region the improvement obtained from UNLOPS is apparent.

The UNLOPS prescription has also been applied to $^2$. Figure 2 exemplifies the residual uncertainties of the NNLO matched calculation in Higgs-boson production through gluon fusion. We use two different ways to include the Wilson coefficient for the $ggh$ vertex $^2$: A factorized matching scheme which is reminiscent of the POWHEG strategy, and an individual matching scheme that somewhat mimics the MC@NLO procedure. The results are as expected: The factorized approach leads to a harder tail in the $q_T$ distribution, whereas the individual matching has a softer tail and a small enhancement for medium $q_T$ values. The individual matching shows better agreement with the NNLO+NNLL result of the HqT program\textsuperscript{13}. The uncertainty due to varying the parton shower starting scale becomes appreciable for small $q_T$ values, and is significantly larger than the resummation scale variation in HqT. This might be taken as indication that a more accurate parton shower would be beneficial.
6 Conclusions

We have reviewed the current status of matching and merging parton shower resummation and fixed order calculations. Some state-of-the-art NLO merging methods have recently been molded into NNLO matching methods. The prerequisite for these extensions was a well-defined one-jet cross section, which was then updated to NNLO accuracy for the inclusive process. Results of the UNLOPS scheme as implemented in SHERPA have been presented. This implementation includes new NNLO fixed-order calculations for (neutral and charged current) Drell-Yan lepton pair and (gluon-fusion initiated) Higgs-boson production. When applied to the Drell-Yan process, we find that the NLO results, when computed with NNLO PDFs, reproduce the full NNLO results for (gluon-fusion initiated) Higgs-boson production. When applied to the Drell-Yan process, we find that the NLO results, when computed with NNLO PDFs, reproduce the full NNLO results for inclusive observables. For Higgs-boson production at NNLO+PS accuracy, two schemes were presented, highlighting some residual uncertainties of the matching.

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