A Comparison of the Power Spectrum Area and Multifractal Singular Value Decomposition Methods for Extraction of IP Anomaly (A Cause Study: Hamyj Copper Deposit, Iran)

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Abstract: Enhancement of anomaly plays such a critical role in mineral exploration that this work has been carried out by removed noise in geophysical data. Power spectrum area (S-A) and multifractal singular value decomposition (MSVD) methods are application methods in enhancement of anomaly that are usually used in the geochemical exploration process. However, these methods have not been widely used in the geoelectrical data so far. Hamyj deposit is located about 80 kilometers west of Birjand city, South Khorasan province, Iran. In this area, resistivity and induced polarization data have been surveyed by dipole- dipole array. In this paper, IP data has been inverted by the linear method. Then, enhancement of anomaly IP data has been carried out by S-A and MSVD methods. Results of S-A and MSVD methods were compared with each other. Results indicate that the S-A method has determined the location of anomaly IP data better than the MSVD method.

Keywords: Power spectrum area; Multifractal singularity value decomposition; Induced polarization; Hamyj Copper deposit

1. Introduction

Integrated geophysical methods are commonly used in mineral exploration to obtain qualified results [1,2]. Induced polarization (IP) is a method in geophysics which is applied to detect geological bodies with metallic content and to a minor extent in groundwater exploration. The voltage across the potential electrodes generally does not drop to zero instantaneously, but decays rather slowly, after an initial large decrease from the original steady-state value [3]. Although induced polarization was known in the 1920s, it was not until the 1950s that IP for economic mineralization was used routinely [4]. Measurements of induced polarization are common in both frequency and time domains. Measurements of the time domain lead to the frequency domain due to quick measurement and saving time. Due to the rise in value time of exploration mining, the measurement time domain has progressed rapidly [5].

In the past decades, various methods have been identified for enhancing anomalies in geochemical and geophysical data processing. Both the power spectrum area (S-A) method and multifractal singular value decomposition (MSVD) are used for enhancement of anomalies [6,7]. The S-A method has been vastly used in geochemical data processing [8] and the MSVD method has been used for development in seismic and gravity data processing [9,10].

In this paper, S-A and MSVD methods are compared with each other for enhancing anomaly induced polarization data. In addition, this can create an innovative method to enhancing anomaly in geoelectrical interpretation as well.

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as geochemical and seismic data processing. Results have shown that S-A and MSVD methods are acceptable for highlighting anomalies IP data. However, the S-A method further highlighted the location of anomalies IP data.

1.1 Power spectrum area method (S-A)

The S-A method, based on the Fourier spectral analysis, is a fractal filtering technique used for identifying the anomalous data. Cheng et al. (2000)\(^6\) proposed the power spectrum-area model to represent the power-law frequency distribution of the power spectrum density, which is useful to separate anomalies from background.

The S-A fractal model is based on the power-law relationships between areas of sets of data, consisting the wave number \((k)\) with spectral energy density \(S[A (> S)]\) on the 2D frequency domain, and it is determined as:

\[
A(\geq S) \propto S^{-2d/\beta}\quad \text{Eq.(1)}
\]

Where \(\beta\) is the anisotropic scaling exponent. For a 2D linear case, \(d=1\)\(^{8,11}\). Power spectrum-area is as follows:

\[
\begin{align*}
F(K_x, K_y) &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) \cos(K_x x, K_y y) \, dx \, dy - i \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) \sin(K_x x, K_y y) \, dx \, dy \\
K_x \text{ and } K_y \text{ are wave number with respect to x (horizontal axes) and y (vertical axes), respectively}^{12}.
\end{align*}
\]

The power spectrum is calculated based on equation 3 determined in the following:

\[
E(K_x, K_y) = R^2(K_x, K_y) + I^2(K_x, K_y)\quad \text{Eq.(3)}
\]

Based on the power spectrum, the data can be separated into different populations (threshold \((S_0)\)). This process is performed similar to the concentration-area method except for the fact that the utilized method in this paper is the power spectrum.

Finally, digital filter design is formed based on the threshold calculated in step (B). This filter can be defined as:

\[
G_B(\xi) = 1 \text{ if } E > S_0, \text{ otherwise } G_B(\xi) = 0 \quad \text{equation 4)}/
\]

The above equation \(F^{-1}\) is an inverted Fourier transform\(^{12}\).

1.2 Multifractal singular value decomposition method (MSVD)

The multifractal singular value decomposition method is based on both singular value decomposition (SVD) and fractal/ multifractal theories. The SVD has been used in geophysical data processing, e.g., for compressing seismic reflection profiles\(^{13}\), radar processing\(^{14}\), and enhancing weak signals in vertical seismic profiles\(^{9}\). It has also been used as noise separation\(^{15}\).

One of the appropriate methods in the algebra fields for linearly numerical modelling is SVD\(^{16}\). SVD is a factorization of the rectangular matrix \(A \ (m,n)\) into orthogonal matrices, i.e.

\[
A = USV^T\quad \text{Eq.(5)}
\]

Where \(U\) is a left eigenvector matrix, \(S\) is a diagonal matrix called singular value matrix, \(V\) is a right eigenvector matrix and \(T\) is the transpose matrix\(^{16}\).

The singular value decomposition of the \(A\) matrix (eigen image) can also be written as follows:

\[
A = \sum_{i=1}^{r} \sigma_i u_i v_i^T\quad \text{Eq.(6)}
\]

Where \(r\) is rank of \(A\), \(u_i\) is the i-th left eigenvector, \(v_i\) is the i-th right eigenvector and \(\sigma_i\) is i-th singular value of \(A\) (or weigh with \(\sigma_1 \geq \sigma_2 \geq \cdots \sigma_n \geq 0\))^\(^7\). The \(\sigma_i\) and eigenvalue \((\lambda_i)\) can be written as follows:

\[
\sigma_i = \sqrt{\lambda_i}\quad \text{Eq.(7)}
\]
Equation 3 corresponds to the spectral energy densities of the eigen image \[^{[16]}\]. Therefore, the total energy squared SVs are larger than \(\lambda_i\) and can be written as

\[
E \left( SV_i \leq \lambda_k^2 \right) = \sum_{i=1}^{r} \lambda_i, 1 \leq k \leq r \tag{8}
\]

And its relevant energy proportion (P) is:

\[
P \left( SV_i \leq \lambda_k^2 \right) = \frac{1}{\sum_{i=1}^{r} \lambda_i} \sum_{i=1}^{r} \lambda_i, 1 \leq k \leq r \tag{9}
\]

Based on equation 8 and 9, energy within the radius of the corresponding energy can also be expressed as an energy spectrum for certain scales. \(\lambda\) and \(E\) (or \(P\)) may represent a fractal or multifractal \[^{[16]}\]

\[
E \propto \lambda^\alpha \tag{10}
\]

Or

\[
P \left( SV_i \leq \lambda_k^2 \right) \propto \lambda^\alpha \tag{11}
\]

Due to the power law distribution, the curve in the log- log plot of \(\lambda - P\) can be separated into several segments due to different slopes. As the break point is \(p\) with the multifractal SVD method, \(p\) in equations 12 and 13 can be determined.

\[
X_{LP} = \sum_{i=1}^{p-1} \sigma_i u_i v_i^T \tag{12}
\]

\[
X_{HP} = \sum_{i=p}^{r} \sigma_i u_i v_i^T \tag{13}
\]

\(X_{LP}\) is low pass and \(X_{HP}\) is high pass. \(p\) is the eigen image index of breakpoint \[^{[10]}\].

2. Study area

Hamij copper deposit is a porphyry deposit situated eastern Iran, Birjand province (Figure 1). A promising mineralization has been confirmed by remote sensing methods (band ratio methods) and geological field prospecting\[^{[17]}\]. Gabbro, Altered Andesite, Dacite, and Andesite as well as sedimentary rocks like old Gravel are the main lithological outcrops in the region. The ages of mineralization are Cretaceous and paleogene periods for gabbro and andesite, respectively. Figure 2 shows the geological map of this deposit.

3. Methodology

Dipole-dipole array was utilized for collecting the RS (resistivity) and IP data. Spacing of current electrodes were 10 m. Figure 3 shows the situation of the profiles used for this study. The azimuth of the profiles A, B, D and F are close to N-S and the azimuth of the profiles C and E are approximately perpendicular on the profiles A, and B. the geological feature of the outcrops and geomorphology of the area played a significant part in designing the profiles. Syscal R2 made in France was a IP/RS equipment used for obtaining the data. Profile D has been compiled in this paper.
4. Discussion

The induced polarization was inversed by linear inversed using the equation as constraints \(^{[18]}\). In this paper, polarization data were inversed by Res2dinv software.

4.1 Power spectrum area method

The inversed IP data were transferred by 2D Fourier transform to the frequency domain. Secondly, power spectrum data were calculated by equation 3. Table 1 shows the statistical properties of the results, achieved by the power spectrum method.
According to Table 1, the power spectrum of data shows a wide range. Therefore, the logarithm of the power spectrum of the data was used. According to Sturge’s Rule, the logarithmic power spectrums were classified to 8 classes. Sturge’s Rule determined the number of class based on the number of data. The distance between points is 10 m in the cross section. Therefore, each point covers an area of 10x10m in which the power spectrum of data is determined in the center of each cell.

In the next step, the area of the classified power spectrum of data should be calculated, (the graphs of all logarithms are traced so that the power spectrum of data is on the horizontal axis as against the area on the vertical axis) (Figure 4).

Since the two lines with different slopes have been fitted statistically by excel software, they can be considered as two distinct populations. The break point in the graph (Figure 4) was considered as a threshold of power spectrum of data (S0). The threshold of power spectrum is equal to $10^{5.6}=398107$ (back transformation). The correlation coefficient and the equation of the line are given in equations 14 and 15.

$$y=-0.6342x+5.4855 \quad R^2=0.97 \quad \text{Eq.(14)}$$
This threshold (398107) was applied to design digital filters. Digital filter is carried out by equation 4. This filter can be defined as: \( G_D = 1 \) if power spectrum (E) > 398107, otherwise \( G_B = 0 \). We have written the related code in MATLAB software. Finally, using digital filters, the anomaly map for induced polarization data is obtained by the Kriging method using Golden software Surfer v.10 (Figure 5).

![Figure 5](image.png)

**Figure 5.** Map of the distribution of anomaly areas, using low-pass filter

According Figure 5, anomaly IP data is identified in a depth of 25 meters and at a distances of 160 and 220 meters from the beginning of the profile.

### 4.2 Multifractal singular value decomposition

Initially, IP data of the profile D is considered as the A(m,n) matrix. Then, the left eigenvector, right eigenvector and singular value matrix are calculated by MATLAB software. Eigenvalue and singular value are shown in Table 2, for which the eigenvalue is six and maximum eigenvalue is 27193.

| eigenvalue  | 27193 | 2078.313 | 529.8 | 32.27 | 1.68 |
|-------------|-------|----------|-------|-------|------|
| Singular value | 164.9 | 45.58 | 23.0  | 5.68  | 1.29 |

**Table 2.** Value of eigenvalue and singular value for the profile D

Then, energy proportion (P) is calculated by equation 9. In the next step, the graph of all logarithms is traced so that the eigenvalue is on the horizontal axis as against energy proportion on the vertical axis. Figure 6 is shown in the graph.

![Figure 6](image.png)

**Figure 6.** All logarithmic plot eigenvalues in horizontal axes versus energy proportion in vertical axes

The break point in the graph (Figure 6) was considered as a threshold (p). The curve log-log plot of \( \lambda \)-P of induced polarization data can be separated into two segments according to the break point between straight lines with
different slopes. The correlation coefficient and the equation of the line are given in equations 16 and 17.

\[
y = -0.0002x + 0.045 \quad \text{Eq.}(16)
\]
\[
y = -0.024x + 0.11 \quad \text{Eq.}(17)
\]

The right segment is made of \( \lambda_1 \) and \( \lambda_2 \), and the percentage of its energy is about 90% total energy. The reconstructed image of IP data is shown in Figure 7 with the sum of the 1st and 2nd eigen image.

![Figure 7](image)

**Figure 7.** Reconstructed image of the 1st and 2nd eigen image

According Figure 7, anomaly IP data is identified in a depth of 25 meters and at distances of 160 and 220 meters from the beginning of the profile.

By comparing Figure 5 and Figure 7, we find that S-A and MSVD methods are acceptable for highlighting anomalies IP data. However, the S-A method highlighted the location of anomalies IP data better than the SVD method with a comparison in figures 5 and 7. The S-A method removed noise better than the SVD method but each of the two methods are acceptable. Therefore, the S-A method may be better than the SVD method for enhancement of anomaly induced polarization data.

**5. Conclusion**

In this paper, induced polarization data was inversed by the linear method. Then anomalies IP data is ed by the power spectrum area (S-A) and multifractal singular value decomposition (MSVD) methods. By comparing Figure 5 and Figure 7, we find that the S-A method further highlights the location of anomalies induced polarization data. The S-A method is vastly used in geochemical data processing and the MSVD method has been developed in the seismic data. Therefore, this can create an innovative method in geoelectrical interpretation as well as geochemical and seismic data processing. Also, the S-A method removed noise better than the MSVD method for induced polarization data.

The results of using these methods are expressed as follows:

These methods (S-A and SVD) are based on a strong mathematical foundation which are used in order to highlight anomalies more accurately.

The amount of noise and interference is removed as a result of applying these methods. Also, it makes anomalies significantly clear and visible. But the S-A method makes anomalies more clear than the MSVD method.

Also, the scientific fundamental of this paper can be used to interpret and survey other geophysical data.

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