Abstract

This talk follows by a few months a talk by the same authors on nearly the same subject at the Coral Gables Conference. The ideas presented here are basically the same, but with some amplification, some change of viewpoint, and a number of new questions for the future. For our own convenience, we have transcribed the Coral Gables paper, but with an added ninth section, entitled “Problems of light cone current algebra”, dealing with our present views and emphasizing research topics that require study.

1. INTRODUCTION

We should like to show that a number of different ideas of the last few years on broken scale invariance, scaling in deep inelastic electron–nucleon scattering, operator product expansions on the light cone, “parton” models, and generalizations of current algebra, as well as some new ideas, form a coherent picture. One can fit together the parts of each approach that make sense and obtain a consistent view of scale invariance, broken by certain terms in the energy density, but restored in operator commutators on the light cone.

We begin in the next section with a review of the properties of the dilation operator $D$ obtained from the stress–energy–momentum tensor $\Theta_{\mu\nu}$ and the behavior of operators under equal–time commutation with $D$, which is described in terms of physical dimensions $l$ for the operators. We review the evidence on the relation between the violation of scale invariance and the violation of $SU_3 \times SU_3$ invariance.

Next, in Section 3, we describe something that may seem at first sight quite different, namely the Bjorken scaling of deep inelastic scattering cross sections of electrons on nucleons and the interpretation of this scaling in terms of the light cone commutator of two electromagnetic current operators. We use a generalization of Wilson's work$^1$, the light–cone expansion emphasized

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particularly by Brandt and Preparata\textsuperscript{2} and Frishman\textsuperscript{3}. A different definition \( l \) of physical dimension is thus introduced and the scaling implies a kind of conservation of \( l \) on the light cone. On the right–hand side of the expansions, the operators have \( l = -J - 2 \), where \( J \) is the leading angular momentum contained in each operator and \( l \) is the leading dimension.

In Section 4, we show that under simple assumptions the dimensions \( l \) and \( \bar{l} \) are essentially the same, and that the notions of scaling and conservation of dimension can be widely generalized. The essential assumption of the whole approach is seen to be that the dimension \( l \) or \( (\bar{l}) \) of any symmetry–breaking term in in the energy (whether violating scale invariance or \( SU_3 \times SU_3 \)) is higher than the dimension, \(-4\), of the completely invariant part of the energy density. The conservation of dimension on the light cone then assigns a lower singularity to symmetry–breaking terms than to symmetry–preserving terms, permitting the light–cone relations to be completely symmetrical under scale, \( SU_3 \times SU_3 \), and perhaps other symmetries.

In Section 5, the power series expansion on the light cone is formally summed to give bilocal operators (as briefly discussed by Frishman) and it is suggested that these bilocal light–cone operators may be very few in number and may form some very simple closed algebraic system. They are then the basic mathematical entities of the scheme.

It is pointed out that several features of the Stanford experiments, as interpreted according to the ideas of scaling, resemble the behavior on the light cone of free field theory or of interacting field theory with naive manipulations of operators, rather than the behavior of renormalized perturbation expansions of renormalizable field theories. Thus free field theory models may be studied for the purpose of abstracting algebraic relations that might be true on the light cone in the real world of hadrons. (Of course, matrix elements of operators in the real world would not in general resemble matrix elements in free field theory.) Thus in Section 6 we study the light–cone behavior of local and bilocal operators in free quark theory, the simplest interesting case. The relevant bilocal operators turn out to be extremely simple, namely just \( i/2 (\bar{q}(x) \lambda_i \gamma_\alpha q(y)) \) and \( i/2 (\bar{q}(x) \lambda_i \gamma_\alpha \gamma_5 q(y)) \), bilocal generalizations of \( V \) and \( A \) currents. The algebraic system to which they belong is also very simple.

In Section 7 we explore briefly what it would mean if these algebraic relations of free quark theory were really true on the light cone for hadrons. We see that we obtain, among other things, the sensible features of the so–called “parton” picture of Feynman\textsuperscript{4} and of Bjorken and Paschos\textsuperscript{5}, especially as formulated more exactly by Landshoff and Polkinghorne\textsuperscript{6}, Llewellyn Smith\textsuperscript{7}, and others. Many symmetry relations are true in such a theory, and can be checked by deep inelastic experiments with electrons and with neutrinos. Of course, some alleged results of the “parton” model depend not just on light cone commutators but on detailed additional assumptions about matrix elements, and about such results we have nothing to say.

The abstraction of free quark light cone commutation relations becomes more credible if we can show, as was done for equal time charge density commutation relations, that certain kinds of non–trivial interactions of quarks leave the relations undisturbed, according to the method of naive manipulation of operators, using equations of motion. There is evidence that in fact this is so, in a theory with a neutral scalar or pseudoscalar “gluon” having a Yukawa interaction with the quarks. (If the “gluon” is a vector boson, the commutation relations on the light cone might be disturbed for all we know.)

A special case is one in which we abstract from a model in which there are only quarks, with
some unspecified self–interaction, and no “gluons”. This corresponds to the pure quark case of the “parton” model. One additional constraint is added, namely the identification of the traceless part of $\Theta_{\mu\nu}$ with the analog of the traceless part of the symmetrized $\bar{q}\gamma_\mu \partial_\nu q$. This constraint leads to an additional sum rule for deep inelastic electron and neutrino experiments, a rule that provides a real test of the pure quark case.

We do not, in this paper, study the connection between scaling in electromagnetic and neutrino experiments on hadrons on the one hand and scaling in “inclusive” reactions of hadrons alone on the other hand. Some approaches, such as the intuition of the “parton” theorists, suggest such a connection, but we do not explore that idea here. It is worth reemphasizing, however, that any theory of pure hadron behavior that limits transverse momenta of particles produced at high energies has a chance of giving the Bjorken scaling when electromagnetism and weak interactions are introduced. (This point has been made in the cut–off models of Drell, Levy, and Yan\textsuperscript{8}).

2. DILATION OPERATOR AND BROKEN SCALE INVARIANCE\textsuperscript{9}

We assume that gravity theory (in first order perturbation approximation) applies to hadrons on a microscopic scale, although no way of checking that assertion is known. There is then a symmetrical, conserved, local stress–energy–momentum tensor $\Theta_{\mu\nu}(x)$ and in terms of it the translation operators $P_\mu$, obeying for any operator $O \ldots (x)$, the relation

$$[O \ldots (x), P_\mu] = \frac{1}{i} \partial_\mu O \ldots (x),$$

are given by

$$P_\mu = \int \Theta_{\mu0} d^3x. \tag{2.1}$$

Now we want to define a differential dilation operator $D(t)$ that corresponds to our intuitive notions of such an operator, i. e., one that on equal–time commutation with a local operator $O \ldots$ of definite physical dimension $\bar{l}_0$, gives

$$[O \ldots(x), D(t)] = ix_\mu \partial_\mu O \ldots (x) - i\bar{l}_0 O \ldots (x). \tag{2.3}$$

We suppose that gravity selects a $\Theta_{\mu\nu}$ such that this dilation operation $D$ is given by the expression

$$D = -\int x_\mu \Theta_{\mu0} d^3x. \tag{2.4}$$

It is known that for any renormalizable theory this is possible, and Callan, Coleman, and Jackiw have shown that in such a case the matrix elements of this $\Theta_{\mu\nu}$ are finite. From (2.4) we see that the violation of scale invariance is connected with the non–vanishing of $\Theta_{\mu\nu}$ since we have

$$\frac{dD}{dt} = -\int \Theta_{\mu\nu} q^3 x. \tag{2.5}$$
Another version of the same formula says that

$$[D, P_0] = -iP_0 - i \int \Theta_{\mu\nu} d^3x$$  \hspace{0.5cm} (2.6)

and we see from this and (2.3) that the energy density has a main scale–invariant term $\Theta_{00}$ (under the complete dilation operator $D$) with $l = -4$ (corresponding to the mathematical dimension of energy density) and other terms $w_n$ with other physical dimensions $\bar{l}_n$. The simplest assumption (true of most simple models) is that these terms are world scalars, in which case we obtain

$$- \Theta_{\mu\nu} = \sum_n (\bar{l}_n + 4) w_n$$  \hspace{0.5cm} (2.7)

along with the definition

$$\Theta_{00} = \tilde{\Theta}_{00} + \sum_n w_n.$$  \hspace{0.5cm} (2.8)

We note that the breaking of scale invariance prevents $D$ from being a world scalar and that equal–time commutation with $D$ leads to a non–covariant break–up of operators into pieces with different dimensions $\bar{l}$.

To investigate the relation between the violations of scale invariance and of chiral invariance, we make a still further simplifying assumption (true of many simple models such as the quark–gluon Lagrangian model), namely that there are two $q$–number $w$’s, the first violating scale invariance but not chiral invariance (like the gluon mass) and the second violating both (like the quark mass):

$$\Theta_{00} = \tilde{\Theta}_{00} + \delta + u + \text{const.},$$  \hspace{0.5cm} (2.9)

with $\delta$ transforming like $(1, 1)$ under $SU_3 \times SU_3$. Now how does $u$ transform? We shall start with the usual theory that it all belongs to a single $(3, \bar{3}) + (\bar{3}, 3)$ representation and that the smallness of $m^2_\pi$ is to be attributed, in the spirit of PCAC, to the small violation of $SU_2 \times SU_2$ invariance by $u$. In that case we have

$$u = -u_0 - cu_8,$$  \hspace{0.5cm} (2.10)

with $c$ not far from $-\sqrt{2}$, the value that gives $SU_2 \times SU_2$ invariance and $m^2_\pi = 0$ and corresponds in a quark scheme to giving a mass only to the $s$ quark. A small amount of $u_3$ may be present also, if there is a violation of isotopic spin conservation that is not directly electromagnetic; an expression containing $u_0$, $u_3$ and $u_8$ is the most general canonical form of a $CP$–conserving term violating $SU_3 \times SU_3$ invariance and transforming like $(3, \bar{3}) + (\bar{3}, 3)$.

According to all these simple assumptions, we have

$$- \Theta_{\mu\nu} = (\bar{l}_\delta + 4) \delta + (\bar{l}_u + 4) (-u_0 - cu_8) + 4 \text{ (const.)}$$  \hspace{0.5cm} (2.11)

and, since the expected value of $(-\Theta_{\mu\nu})$ is $2m^2$, we have

$$0 = (\bar{l}_\delta + 4) < \text{vac} | \delta | \text{vac} > + (\bar{l}_u + 4) < \text{vac} | u | \text{vac} > + 4 \text{ (const.)},$$  \hspace{0.5cm} (2.12)
\[ 2m_i^2(PS8) = (l_\delta + 4) (PS_i | \delta | PS_i) \]
\[ + (l_u + 4) <PS_i | u | PS_i>, \]

(2.13)

etc.

The question has often been raised whether \( \delta \) could vanish. Such a theory is very interesting, in that the same term \( u \) would break chiral and conformal symmetry. But is it possible?

It was pointed out a year or two ago to that for this idea to work, something would have to be wrong with the final result of von Hippel and Kim, who calculated approximately the \( \sigma \) terms in meson–baryon scattering and found, using our theory of \( SU_3 \times SU_3 \) violation, that \( <N | U | N> \) was very small compared to \( 2m_N^2 \). Given the variation of \( <B | u_8 | B> \) over the \( 1/2^+ \) baryon octet, the ratio of \( <\Xi | u | \Xi> \) to \( <N | u | N> \) would be huge if von Hippel and Kim were right, and this disagrees with the value \( m_\Xi^2/m_N^2 \) that obtains if \( \delta = 0 \).

Now, Ellis has shown that in fact the method of von Hippel and Kim should be modified and will produce different results, provided there is a dilation. A dilation is a neutral scalar meson that dominates the dispersion relations for matrix elements of \( \Theta_{\mu\nu} \) at low frequency, just as the pseudoscalar octet is supposed to dominate the relations for \( \partial_\alpha F_{5i}^\alpha \). We are dealing in the case of the dilation, with PCDC (partially conserved dilation current) along with PCAC (partially conserved axial vector current). If we have PCAC, PCDC, and \( \delta = 0 \), we may crudely describe the situation by saying that as \( u \to 0 \) we have chiral and scale invariance of the energy, the masses of a pseudoscalar octet and a scalar singlet go to zero, and the vacuum is not invariant under either chiral or scale transformations (though it is probably \( SU_3 \) invariant). With the dilation, we can have masses of other particles non–vanishing as \( u \to 0 \), even though that limit is scale invariant.

Dashen and Cheng have just finished a different calculation of the \( \sigma \) terms not subject to modification by dilation effects, and they find, using our description of the violation of chiral invariance, that \( <N | u | N> \) at rest is around \( 2m_N^2 \), a result perfectly compatible with the idea of vanishing \( \delta \) and yielding in that case a value \( l_u \approx -3 \) (as in a naive quark picture, where \( u \) is a quark mass term!).

An argument was given last year that if \( \delta = 0 \), the value of \( l_u \) would have to be \(-2\) in order to preserve the perturbation theory approach for \( m^2(PS8) \), which gives the right mass formula for the pseudoscalar octet. Ellis, Weisz, and Zumino have shown that this argument can be evaded if there is a dilation.

Thus at present there is nothing known against the idea that \( \delta = 0 \), with \( l_u \) probably equal to \(-3\). However, there is no strong evidence in favor of the idea either. Theories with non-vanishing \( \delta \) operators and various values of \( l_\delta \) and \( l_u \) are not excluded at all (although even here a dilation would be useful to explain why \( <N | u | N> \) is so large). It is a challenge to theorists to propose experimental means of checking whether the \( \delta \) operator is there or not.

It is also possible that the simple theory of chiral symmetry violation may be wrong. First of all, the expression \(-u_0 + \sqrt{2}u_8\) could be right for the \( SU_2 \times SU_2 \)-conserving but \( SU_3 \times SU_3 \)-violating part of \( \Theta_{00} \), while the \( SU_2 \times SU_2 \)-violation could be accomplished by something quite different from \((-c - \sqrt{2}) u_8\). Secondly, there can easily be an admixture of the eighth compo-
nent \( g_8 \) of an octet belonging to (1,8) and (8,1). Thirdly, the whole idea of explaining \( m_{\pi}^2 \approx 0 \) by near-conservation of \( SU_2 \times SU_2 \) might fail, as might the idea of octet violation of \( SU_3 \); it is those two hypotheses that give the result that for \( m_{\pi}^2 = 0 \) we have only \( u_0 - \sqrt{2} u_8 \) with a possible admixture of \( g_8 \). Here again there is a challenge to theoreticians to propose effective experimental tests of the theory of chiral symmetry violation.

3. LIGHT CONE COMMUTATIONS AND DEEP INELASTIC ELECTRON SCATTERING

We want ultimately to connect the above discussion of physical dimensions and broken scale invariance with the scaling described in connection with the Stanford experiments on deep inelastic electron scattering\(^{15} \). We must begin by presenting the Stanford scaling in suitable form. For the purpose of doing so, we shall assume for convenience that the experiments support certain popular conclusions, even though uncertainties really prevent us from saying more than that the experiments are consistent with such conclusions:

1) that the scaling formula of Bjorken is really correct, with no logarithmic factors, as the energy and virtual photon mass go to infinity with fixed ratio;

2) that in this limit the neutron and proton behave differently;

3) that in the limit the longitudinal cross section for virtual photons goes to zero compared to the transverse cross section.

All these conclusions are easy to accept if we draw our intuition from certain field theories without interactions or from certain field theories with naive manipulation of operators. However, detailed calculations using the renormalized perturbation expansion in renormalizable field theories do not reveal any of these forms of behavior, unless of course the sum of all orders of perturbation theory somehow restores the simple situation. If we accept the conclusions, therefore, we should probably not think in terms of the renormalized perturbation expansion, but rather conclude, so to speak that Nature reads books on free field theory, as far as the Bjorken limit is concerned.

To discuss the Stanford results, we employ a more or less conventional notation. The structure functions of the nucleon are defined by matrix elements averaged over nucleon spin,

\[
\frac{1}{4\pi} \int d^4x < N, p \mid [j_\mu(x), j_\nu(y)] \mid N, p > e^{-iq(x-y)}
\]

\[
= \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) W_1 \left( q^2, p \cdot q \right) \\
+ \left( p_\mu - \frac{p \cdot q}{p^2} q_\mu \right) \left( p_\nu - \frac{p \cdot q}{q^2} q_\nu \right) W_2 \left( q^2, p \cdot q \right) \\
= \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \left( W_1 - \frac{(p \cdot q)^2}{q^2} W_2 + \frac{\delta_{\mu\nu}(p \cdot q)^2 + p_\mu p_\nu q^2 - (p_\mu q_\nu + q_\mu p_\nu) p \cdot q}{q^2} W_2 \right) \tag{3.1}
\]
where \( p \) is the nucleon four-momentum and \( q \) the four-momentum of the virtual photon. As \( q^2 \) and \( q \cdot p \) become infinite with fixed ratio, averaging over the nucleon spin and assuming \( \sigma_L/\sigma_T \to 0 \), we can write the Bjorken scaling in the form

\[
\frac{1}{4\pi} \int d^4x < N, p \left| [j_\mu(x), j_\nu(y)] \right| N, p > e^{-i q \cdot (x-y)} \rightarrow \frac{(p_\mu p_\nu + p_\nu q_\mu) p \cdot q - \delta_{\mu\nu}(p \cdot q)^2 - p_\mu p_\nu q^2}{q^2(q \cdot p)} F_2(\xi),
\]

(3.2)

where \( \xi = -q^2/2p \cdot q \) and \( F_2(\xi) \) is the scaling function in the deep inelastic region.

In coordinate space, this limit is achieved by approaching the light cone \((x - y)^2 = 0\), and we employ a method, used by Frishman\(^3\) and by Brandt and Preparata\(^2\), generalizing earlier work of Wilson, that starts with an expansion for commutators or operator products valid near \((x - y)^2 = 0\). (The symbol \( \hat{=} \) will be employed for equality in the vicinity of the light cone.) After the expansion is made, then the matrix element is taken between nucleons. To simplify matters, let us introduce the “barred product” of two operators, which means that we average over the mean position \( R \equiv (x + y)/2 \), leaving a function of \( z \equiv x - y \) only (as appropriate for matrix elements with no change of momentum) and that we retain in the expansion only totally symmetric Lorentz tensor operators (as appropriate for matrix elements averaged over spin). Then the assumed light-cone expansion of the barred commutator \([j_\mu(x), j_\nu(y)]\) tells us that we have, as \( z^2 \to 0 \),

\[
\frac{1}{\pi i} \left( \frac{2\delta_{\mu\nu} \partial_\mu \partial_\sigma - \delta_{\mu\sigma} \partial_\nu \partial_\sigma - \delta_{\nu\sigma} \partial_\mu \partial_\sigma - \delta_{\sigma\mu} \partial_\nu \partial_\sigma - \delta_{\sigma\nu} \partial_\mu \partial_\sigma - \delta_{\mu\sigma} \delta_{\nu\rho} \partial_\rho}{\partial^2} - \partial_{\nu\sigma} \partial_{\mu\rho} - \partial_{\mu\rho} \partial_{\nu\sigma} \right)
\]

and the second term, the one that gives \( \sigma_L \), will be ignored for simplicity in our further work.

In order to obtain the Bjorken limit, we have only to examine the matrix elements between \( | Np > \) and itself of the operators \( 0_{\alpha\beta}, 0_{\alpha\beta\gamma\delta}, 0_{\alpha\beta\gamma\delta}^\varepsilon \), etc. The leading tensors in the matrix elements have the form \( c_2 p_\alpha p_\beta, c_4 p_\alpha p_\beta p_\gamma p_\delta \), etc., where the \( c \)'s are dimensionless constants. The lower tensors, such as \( \delta_{\alpha\beta} \), have coefficients that are positive powers of masses, and these tensors give negligible contributions in the Bjorken limit. All we need is the very weak assumption that \( c_2, c_4, c_8 \), etc., are not all zero, and we obtain the Bjorken limit.
We define the function
\[ \tilde{F}(p \cdot z) = c_2 + \frac{1}{2!} c_4 (p \cdot c)^2 + \cdots. \]  

(3.4)

Taking the Fourier transform of the matrix elements of (3.3), we get in the Bjorken limit
\[
W_2 \to \frac{1}{2\pi^2 i} \int d^4 z e^{-iz \cdot z} \tilde{F}(p \cdot z) \varepsilon(z_0) \delta(z^2)
\]
\[
= \frac{1}{2\pi^2 i} \int F(\xi) d\xi \int d^4 z e^{-i(q+\xi p) \cdot z} \varepsilon(z_0) \delta(z^2)
\]
\[
= 2 \int F(\xi) d\xi \varepsilon(-q \cdot p) \delta(q^2 + 2q \cdot p) \xi
\]
\[
= \frac{1}{-q \cdot p} F(\xi)
\]

(3.5)

where function \( F(\xi) \) is \(-q^2/2q \cdot p\) and \( F(\xi) \) is the Fourier transform of \( \tilde{F}(p \cdot z) \):
\[ F(\xi) = \frac{1}{2\pi} \int e^{i\xi(p \cdot z)} \tilde{F}(p \cdot z) d(p \cdot z). \]  

(3.6)

The function \( F(\xi) \) is therefore the Bjorken scaling function in the deep inelastic limit and is defined only for \(-1 < \xi < 1\). We can write (3.6) in the form
\[ F(\xi) = c_2 \cdot \delta(\xi) - c_4 \frac{1}{2!} \delta''(\xi) + c_6 \frac{1}{4!} \delta^{(4)}(\xi) - \cdots. \]  

(3.7)

The dimensionless numbers \( c_i \) defined by the matrix elements of the expansion operators can be written as
\[ c_2 = \int_{-1}^{1} F(\xi) d\xi, \quad c_4 = -\int_{-1}^{1} F(\xi) \xi^2 d\xi \cdots. \]  

(3.8)

This shows the connection between the matrix elements of the expansion operators and the moments of the scaling function. The Bjorken limit is seen to be a special case (the matrix element between single nucleon states of fixed momentum) of the light cone expansion.\(^{17}\)

Now the derivation of the Bjorken limit from the light cone expansion can be described in terms of a kind of physical dimension \( l \) for operators. (We shall see in the next section that these dimensions \( l \) are essentially the same as the physical dimensions \( l \) we described in Section 2.) We define the expansion to conserve dimension on the light cone and assign to each current \( l = -3 \) while counting each power of \( z \) as having an \( l \)-value equal to the power. We see then that on the right-hand side we are assigning to each \( J \)-th rank Lorentz tensor (with maximum spin \( J \)) the dimension \( l = -J - 2 \). Furthermore, the physical dimension equals the mathematical dimension in all of these cases.
4. GENERALIZED LIGHT CONE SCALING AND BROKEN SCALE INVARIANCE

We have outlined a situation in which scale invariance is broken by a non–vanishing $\Theta_{\mu\nu}$ but restored in the most singular terms of current commutators on the light cone. There is no reason to suppose that such a restoration is restricted to commutators of electromagnetic currents. We may extend the idea to all the vector currents $F_{i\mu}$ and axial vector currents $F_{i\mu}^A$, to the scalar and pseudoscalar operators $u_i$ and $v_i$ that comprise the $(3,\bar{3})$ and $(\bar{3},3)$ representation thought to be involved in chiral symmetry breaking, to the whole stress–energy momentum tensor $\Theta_{\mu\nu}$, to any other local operators of physical significance, and finally to all the local operators occurring in the light cone expansions of commutators of all these quantities with one another. Let us suppose that in fact conservation of dimension applies to leading terms in the light cone in the commutators of all these quantities and that finally a closed algebraic system with an infinite number of local operators is attained, such that the light cone commutator of any two of the operators is expressible as a linear combination of operators in the algebra. We devote this section and the next one to discussing such a situation.

If there is to be an analog of Bjorken scaling in all these situations, then on the right–hand side of the light cone commutation relations we want operators with $l = -J - 2$, as above for electromagnetic current commutators, so that we get leading matrix elements between one–particle states going like $c p_\alpha p_\beta \cdots$, where the $c$ are dimensionless constants.

Of course, there might be cases in which, for some reason, all the $c$’s have to vanish, and the next–to–leading term on the light cone becomes the leading term. Then the coefficients would have the dimensions of positive powers of mass. We want to avoid, however, situations in which coefficients with the dimension of negative powers of mass occur; that means on the right–hand side we want $l \leq -J - 2$ in any case, and $l = -J - 2$ when there is nothing to prevent it.

This idea might have to be modified, as in a quark model with a scalar or pseudoscalar “gluon” field, to allow for a single operator $\phi$, with $l = -1$ and $J = 0$, that can occur in a barred product, but without a sequence of higher tensors with $l = -J - 1$ that could occur in such a product; gradients of $\phi$ would, of course, average out in a barred product. However, even this modification is probably unnecessary, since preliminary indications are that, in the light cone commutator of any two physically interesting operators, the operator $\phi$ with $l = -1$ would not appear on the right–hand side.

Now, on the left–hand side, we want the non–conserved currents among $F_{i\mu}$ and $F_{i\mu}^A$ to act as if they have dimension $-3$ just like the conserved ones, as far as leading singularities on the light cone are concerned, even though the non–conservation implies the admixture of terms that may have other dimensions $l$, dimensions that become $l - 1$ in the divergences, and correspond to dimensions $l - 1$ in the $SU_3 \times SU_3$ breaking terms in the energy density. But the idea of conservation of dimension on the light cone tells us that we are dealing with lower singularities when the dimensions of the operators on the left are greater. What is needed, then, is for the dimensions $l$ to be $>-3$, i. e., for the chiral symmetry breaking terms in $\Theta_{\mu\nu}$ to have dimension $>-4$. Likewise, if we want the stress–energy–momentum tensor itself to obey simple light cone scaling, we need to have the dimension of all scale breaking parts of $\Theta_{\mu\nu}$ restricted to values $>-4$. In general, we can have symmetry on the light cone if the symmetry breaking terms in
\(\Theta_{\mu\nu}\) have dimension greater than \(-4\). (See Appendix 1.)

Now we can have \(F_{i\mu}\) and \(F_{i\mu}^5\) behaving, as far as leading singularities on the light cone are concerned, like conserved currents with \(l = -3, \Theta_{\mu\nu}\) behaving like a chiral and scale invariant quantity with \(l = -4\), and so forth. To pick out the subsidiary dimensions associated with the non-conservation of \(SU_3 \times SU_3\) and dilation, we can study light cone commutators involving, \(\partial_\alpha F_{i\alpha}, \partial_\alpha F_{i\alpha}^5\), and \(\Theta_{\mu\nu}\). (If the \((3,3) + (\bar{3},\bar{3})\) hypothesis is correct, that means studying commutators involving \(u\)’s and \(v\)’s and also \(\delta\), if \(\delta \neq 0\).

In our enormous closed light cone algebra, we have all the operators under consideration occurring on the left-hand side, the ones with \(l = -J - 2\) on the right-hand side, and coefficients that are functions of \(z\) behaving like powers according to the conservation of dimension. But are there restrictions on these powers? And are there restrictions on the dimensions occurring among the operators?

If, for example, the functions of \(z\) have to be like powers of \(z^2\) (or \(\delta (z^2)\), \(\delta' (z^2)\) etc.) multiplied by tensors \(z_\alpha z_\beta z_\gamma \cdots\), and if \(l + J\) for some operators is allowed to be non-integral or even odd integral, then we cannot always have \(l = -J - 2\) on the right, i.e., the coefficients of all such operators would vanish in certain commutators, and for those commutators we would have to be content with operators with \(l < -J - 2\) on the right, and coefficients of leading tensors that act like positive powers of a mass.

Let us consider the example:

\[
[\Theta_{\mu\nu}(x), u(y)] = E_{\mu\nu}(z) \cdot 0(y) + z_\rho O_\rho(y) + \cdots + \cdots,
\]

where \(u(y)\) has the dimension \(-3\). In this case we cannot have the Bjorken scaling. Because of the relation

\[
[D(0), u(0)] = -3iu(0),
\]

the operator \(0(y)\) has to be proportional to \(u(y)\). The operator series fulfilling the condition \(l = -J - 2\) is forbidden in this case on the right-hand side.

We have already emphasized that Nature seems to imitate the algebraic properties of free field theory rather than renormalized perturbation theory. (We could also say that Nature is imitating a super-renormalizable theory, evan though no sensible theory of that kind exists, with the usual methods of renormalization, in four dimensions.) This suggests that we should have in our general expansion framework finite equal-time commutators for all possible operators and their time derivatives.

Such a requirement means that all functions of \(z\) multiplying operators in light cone expansion must have the behavior described just above, i.e., the scalar functions involved behave like integral forces of \(z^2\) or like derivatives of delta functions with \(z^2\) as the argument. The formula

\[
\frac{1}{(z^2 + i\varepsilon)^\alpha} - \frac{1}{(z^2 - i\varepsilon)^\alpha} \underset{z_0 \to 0}{\to} \text{const. } z_0^{-2\alpha + 3} \delta(z)
\]

shows the sort of thing we mean. It also shows that \(\alpha\) must not be too large. That can result in lower limits on the tensorial rank of the first operator in the light cone expansion in higher and higher tensors; to put it differently, the first few operators in a particular light cone expansion
may have to be zero in order to give finiteness of equal time commutators with all time derivatives.

Now, on the right–hand side of a light cone commutator of two physically interesting operators, when rules such as we have just discussed do not forbid it, we obtain operators with definite \( SU_3 \times SU_3 \) and other symmetry properties, of various tensor ranks, and with \( l = -J - 2 \). Now, for a given set of quantum numbers, how many such operators are there? Wilson\(^1\) suggested a long time ago that there may be very few, sometimes only one, and others none. Thus no matter what we have on the left, we always would get the same old operators on the right (when not forbidden and less singular terms with dimensional coefficients occurring instead.)

This is very important, since the matrix elements of these universal \( l = -J - 2 \) operators are then natural constants occurring in many problems. Wilson presumably went a little too far in guessing that the only Lorentz tensor operator in the light cone expansion of \[ j_\mu(x), j_\nu(y) \] would be the stress–energy–momentum tensor \( \Theta_{\mu\nu} \), with no provision for an accompanying octet of \( l = -4 \) tensors. That radical suggestion, as shown by Mack,\(^{17}\) would make \( \int F_2^{\text{en}}(\xi) d\xi \) equal to \( \int F_2^{\text{ep}}(\xi) d\xi \), which does not appear to be the case. However, it is still possible that one singlet and one octet of tensors may do the job. (See the discussion in Section 7 of the “pure quark” case.)

If we allow \( z_0 \) to approach zero in a light cone commutator, we obtain an equal time commutator. If Wilson’s principle (suitably weakened) is admitted, then all physically interesting operator must obey some equal time commutation relations, with well–known operators on the right–hand side, and presumably there are fairly small algebraic systems to which these equal time commutators belong. The dimensions of the operators constrain severely the nature of the algebra involved. For example, suppose \( SU_3 \times SU_3 \) is broken by a quantity \( u \) belonging to the representation \( (3, \bar{3}) \otimes (3, \bar{3}) \) and having a singe dimension \( l_u \). Then, if \( l_u = -3 \), we may well have the algebraic system proposed years ago by one of us (M.G.–M.) in which \( F_i, F_5^5, \int u_i d^3 x \) and \( \int v_i d^3 x \) obey the E.T.C. relations of \( U_6 \), as in the quark model. If \( l_u = -2 \), however, then we would have \( \int u_i d^3 x \) and \( d/dt \int u_i d^3 x \) commuting to give a set of quantities including \( \int u_i d^3 x \) and so forth.

We have described scaling in this section as if the dimensions \( l \) were closely related to the dimensions \( l \) obtained by equal time commutation with the dilation operator \( D \) in Section 2. Let us now demonstrate that this is so.

To take a simple case, suppose that in the light cone commutator of an operator \( 0 \cdots \) with itself, the same operator \( 0 \cdots \) occurs in the expansion on the right–hand side. Then we have a situation crudely described by the equation.

\[
[O \cdots (z), O \cdots (0)] \doteq + (z)^l O \cdots (0) + \cdots ,
\]

(4.1)

where \( l \) is the principal dimension of \( O \cdots \). Here \( (z)^l \) means any function of \( z \) with dimension \( l \), and we must have that because of conservation of dimension. Now under equal time commutation with \( D \), say \( O \cdots \) exhibits dimension \( \bar{l} \). Let \( z_0 \to 0 \) and perform the equal time commutation, according to Eq. (2.3). We obtain

\[
(i z \cdot \nabla - 2i \bar{l}) [O \cdots (z), O \cdots (0)] = -i \bar{l} (z)^l O \cdots (0)
\]

(4.2)
so that \( l = \bar{l} \), as we would like.

Now to generalize the demonstration, we consider the infinite closed algebra of light cone commutators, construct commutators like (4.1) involving different operators, and from commutation with \( D \) as in (4.2) obtain equations

\[
l_1 + l_2 - l_3 = \bar{l}_1 + \bar{l}_2 - \bar{l}_3 ,
\]

where \( O \ldots ^{(1)} \) and \( O \ldots ^{(2)} \) are commuted and yield a term containing \( O \ldots ^{(3)} \) on the right. Chains of such relations can then be used to demonstrate finally that \( l = \bar{l} \) for the various operators in which we are interested.

The subsidiary dimensions associated with symmetry breaking have not been treated here. They can be dealt with in part by isolating the expressions \( \partial_\mu \mathcal{F}_{\mu\nu}^0, \Theta_{\mu\nu}, \) etc., that exhibit only the subsidiary dimensions and applying similar arguments to them. In that way we learn that also for subsidiary dimensions \( l = \bar{l} \).

However, the subsidiary dimensions, while numerically equal for the two definitions of dimension, do not enter in the same way for the two definitions. The physical dimension \( \bar{l} \) defined by light cone commutation always enters covariantly, while \( \bar{l} \) is defined by equal time commutation with the quantity \( D \) and enters non–covariantly, as in the break–up of \( \Theta_{\mu\nu} \) into the leading term \( \bar{\Theta}_{\mu\nu} \) of dimension \(-4\) and the subsidiary ones of higher dimensions. If these others come from world scalars \( w_n \) of dimensions \( \bar{l}_n \), then we have we have

\[
\Theta_{\mu\nu} = \bar{\Theta}_{\mu\nu} + \sum_n \left\{ (3 + l) \delta_{\mu\nu} + (4 + l) \delta_{\mu0} \delta_{\nu0} \right\} \frac{w_n}{3} ,
\]

so that we agree with the relations

\[
\Theta_{00} = \bar{\Theta}_{00} + \sum_n w_n , \quad (2.8)
\]

\[
-\Theta_{\mu\mu} = \sum_n (l_n + 4) w_n . \quad (2.9)
\]

Clearly, \( \bar{\Theta}_{\mu\nu} \) is non–covariant.

To obtain the non–covariant formula from the covariant one, the best method is to write the light cone commutator of an operator with \( \Theta_{\mu\nu} \), involving physical dimensions \( l \), and then construct \( D = -\int x_\mu \Theta_{\mu0} d^3x \) out of \( \Theta_{\mu\nu} \) and allow the light cone commutator to approach an equal time commutator. The non–covariant formula involving \( \bar{l} \) must then result.

As an example of non–covariant behavior of equal time commutation with \( D \), consider such a commutator involving an arbitrary tensor operator \( O_{\rho\sigma} \) of dimension \(-4\). We may pick up non–covariant contributions that arise from lower order terms near the light cone than those that give the dominant scaling behavior. We may have

\[
[\Theta_{\mu\nu}(x), O_{\rho\sigma}(y)] = \text{leading term} + \partial_\mu \partial_\nu \partial_\rho \partial_\sigma \left\{ \varepsilon(z_0) \delta \left( z^2 \right) [O(y) + \cdots] \right\} + \cdots
\]
giving the result

\[ [D, O_{\rho\sigma}(0)] = 4i\Theta_{\rho\sigma}(0) + \text{const.} \delta_{\rho0}\delta_{\sigma0}O(0) + \cdots. \]

For commutation of \( D \) with a scalar operator, there is no analog of this situation.

5. BILOCAL OPERATORS

So far, in commuting two currents at points separated by a four-dimensional vector \( z_\mu \), we have expanded the right-hand side on the light cone in powers of \( z_\mu \). It is very convenient for many purposes to sum the series and obtain a single operator of low Lorentz tensor rank that is a function of \( z \). In a barred commutator, it is a function of \( z \) only, but in an ordinary unbarred commutator, it is a function of \( z \) and \( R \equiv (x + y)/2 \), in other words, a function of \( x \) and \( y \). We call such an operator a bilocal operator and write it as \( O(\cdot \cdot \cdot (x, y)) \) or, in barred form, \[ \bar{O}(\cdot \cdot \cdot (x, y)). \]

We can, for example, write Eq. (3.3) in the form

\[ [j_\mu(x), j_\nu(y)] = t_{\mu\nu\rho\sigma} \left\{ \varepsilon(z_0) \delta(z^2) \bar{O}_{\rho\sigma}(x, y) \right\} + \text{longitudinal term}, \]

using the barred form of a bilocal operator \( O_{\rho\sigma}(x, y) \) that sums up all the tensors of higher and higher rank in Eq. (3.3).

Now in terms of bilocal operators we can formulate a much stronger hypothesis than the modified Wilson hypothesis mentioned in the last section. There we supposed that on the right-hand side of any light-cone commutators (unless the leading terms were forbidden for some reason) we would always have operators with \( l = -J - 2 \) and that for a given \( J \) and a given set of quantum numbers there would be very few of these, perhaps only one, and that the quantum numbers themselves would be greatly restricted (for example, to \( SU_3 \) octets and singlets). Here we can state the much stronger conjecture that for a given set of quantum numbers the bilocal operators appearing on the right are very few in number (and perhaps there is only one in each case), with the quantum numbers greatly restricted. That means that instead of an arbitrary series \( O_{\rho\sigma} + \text{const.} O_{\rho\sigma\lambda\mu} z_\lambda z_\mu O_{\alpha\beta\rho\sigma\lambda\mu} + \cdots \), we have a unique sum \( O_{\rho\sigma}(x, y) \) with all the constants determined. The same bilocal operator will appear in many commutators, then, and its matrix elements (for example, between proton and proton with no change of momentum) will give universal deep inelastic form factors.

Let us express in terms of bilocal operators the idea mentioned in the last section that all tensor operators appearing on the right-hand side of the light cone current commutators may themselves be commuted according to conservation of dimension on the light cone, but lead to the same set of operators, giving a closed light cone algebra of an infinite number of local operators of all tensor ranks. We can sum up all these operators to make bilocal operators and commute those, obtaining, on the right-hand side according to the principle mentioned above, the same bilocal operators. Thus we obtain a light cone algebra generated by a small finite number of bilocal operators. These are the bilocal operators that give the most singular terms on the light cone in any commutator of local operators, the terms that give scaling behavior. (As we have said, in certain cases they may be forbidden to occur and positive powers of masses would then appear instead of dimensionless coefficients.)
This idea of a universal light cone algebra of bilocal operators with $l = -J - 2$ is a very elegant hypothesis, but one that goes far beyond present experimental evidence. We can hope to check it some day if we can find situations in which limiting cases of experiments involve the light cone commutators of light cone commutators. Attempts have been made to connect differential cross sections for the Compton effect with such mathematical quantities;\textsuperscript{5} it will be interesting to see what comes of that and other such efforts.

A very important technical question arises in connection with the light cone algebra of bilocal operators. When we talk about the commutators of the individual local operators of all tensor ranks, we are dealing with just two points $x$ and $y$ and with the limit $(x − y)^2 → 0$. but when we treat the commutator of bilocal operators $O(x, u)$ and $O(y, v)$, what are the space–time relationships of $x, u, y,$ and $v$ in the case to which the commutation relations apply? We must be careful, because if we give too liberal a prescription for these relationships we may be assuming more than could be true in any realistic picture of hadrons.

The bilocal operators arise originally in commutators of local operators on the light cone, and therefore we are interested in them when $(x − u)^2 → 0$ and $(y − v)^2 → 0$. In the light cone algebra of bilocal operators, we are interested in singularities that are picked up when $(x − y)^2$ or when $(u − v)^2$ → 0 or when $(x − v)^2$ → 0 or when $(u − y)^2$ → 0. But do we have to have all six quantities simultaneously brought near to zero? That is not yet clear. In order to be save, let us assume here that all six quantities do got to zero.

6. LIGHT CONE ALGEBRA ABSTRACTED FROM A QUARK PICTURE

Can we postulate a particular form for the light cone algebra of bilocal operators?

We have indicated above that if the Stanford experiments, when extended and refined, still suggest the absence of logarithmic terms, the vanishing of the longitudinal cross section, and a difference between neutron and proton in the deep inelastic limit, then it looks as if in this limit Nature is following free field theory, or interacting field theory with naive manipulation of operators, rather than what we know about the perturbation expansions of renormalised field theory. We might, therefore, look at a simple relativistic field theory model and abstract from it a light cone algebra that we could postulate as being true of the real system of hadrons. The simplest such model would be that of free quarks.

In the same way, the idea of an algebra of equal–time commutators of charges or charge densities was abstracted ten years ago from a relativistic Lagrangian model of a free spin 1/2 triplet, what would nowadays be called the quark triplet. The essential feature in this abstraction was the remark that turning on certain kinds of strong interaction in such a model would not affect the equal time commutation relations, even when all orders of perturbation theory were included; likewise, mass differences breaking the symmetry under $SU_3$ would not disturb the equal time commutation relations of $SU_3$.

We are faced, then, with the following question. Are there non–trivial field theory models of quarks with interactions such that the light cone algebra of free quarks remains undisturbed to all orders of naive perturbation theory? Of course, the interactions will make great changes in
the operator commutators inside the light cone; the question is whether the leading singularity on the light cone is unaffected. Let us assume, for purposes of our discussion, that the answer is affirmative. Then we can feel somewhat safe from absurdity in postulating for real hadrons the light cone algebras of free quarks, and indeed of massless free quarks (since the masses do not affect the light cone singularity).

Actually, it is easy to construct an example of an interacting field theory in which our condition seems to be fulfilled, namely a theory in which the quark field interacts with a neutral scalar or pseudoscalar “gluon” field $\phi$. We note the fact that the only operator series in such a theory that fulfills $l = -J - 2$ and contains $\phi(x)$ is the following: $\phi(x)\phi(x), \phi(x)\partial_\mu \phi(x) \cdots$. But these operators do not seem to appear in light cone expansions of products of local operators consisting only of quark fields, like the currents. A different situation prevails in a theory in which the “gluon” is a vector meson, since in that case we can have the operator series $\bar{q}(x)\gamma_\mu B_\nu(x)\gamma(x), \bar{q}(x)\gamma_\mu B_\nu B_\rho q(x), \cdots$, contributing to the Bjorken limit. The detailed behavior of the various “gluon” models is being studied by Llewellyn Smith.\textsuperscript{18}. 
In the following, we consider the light cone algebra suggested by the quark model. We obtain for the commutator of two currents on the light cone (connected part only):

\[
[F_{\mu}^{\nu}(x), F_{\rho}^{\nu}(y)] = \frac{1}{4\pi} \partial_\rho \left[ \varepsilon(z_0) \delta(z^2) \right] \{i f_{ijk} [s_{\mu\nu\rho\sigma} (F_{k\sigma}(x,y) + F_{k\sigma}(y,x)) \\
+ i \varepsilon_{\mu\nu\rho\sigma} (F_{k\sigma}(y,x) - F_{k\sigma}(x,y)) \} + d_{ijk} [s_{\mu\nu\rho\sigma} (F_{k\sigma}(x,y) \\
- (F_{k\sigma}(y,x)) - i \varepsilon_{\mu\nu\rho\sigma} (F_{k\sigma}^5(y,x) + F_{k\sigma}^5(x,y)) \} \right],
\]

\[
[F_{5\mu}^{\nu}(x), F_{5\nu}^{\rho}(y)] = \frac{1}{4\pi} \partial_\rho \left[ \varepsilon(z_0) \delta(z^2) \right] \{i f_{ijk} [s_{\mu\nu\rho\sigma} (F_{5k\sigma}(x,y) + F_{5k\sigma}(y,x)) \\
+ i \varepsilon_{\mu\nu\rho\sigma} (F_{5k\sigma}(y,x) - F_{5k\sigma}(x,y)) \} + d_{ijk} [s_{\mu\nu\rho\sigma} (F_{5k\sigma}(x,y) \\
- (F_{5k\sigma}(y,x)) - i \varepsilon_{\mu\nu\rho\sigma} (F_{5k\sigma}^5(y,x) + F_{5k\sigma}^5(x,y)) \} \right],
\]

(6.1)

\[
s_{\mu\nu\rho\sigma} = \delta_{\mu\sigma} \delta_{\nu\rho} + \delta_{\nu\rho} \delta_{\mu\sigma} - \delta_{\mu\nu} \delta_{\rho\sigma}, \quad z = x - y.
\]

If we go to the equal time limit in (6.1) we pick up the current algebra relations for the currents; in fact we obtain, for the space integrals of all components of nine vector and nine axial–vector currents, the algebra\textsuperscript{19} of $U_6 \times U_6$. Note that we can get similar relations for the current anti–commutators or for the products of currents on the light cone, just be replacing

\[
\frac{1}{4\pi} \partial_\rho \left[ \varepsilon(z_0) \delta(z^2) \right] \text{ by } \frac{i}{4\pi^2} \partial_\rho \frac{1}{z^2} \text{ or by } \frac{i}{8\pi^2} \partial_\rho \frac{1}{z^2} + i \varepsilon z_0
\]

respectively. Perhaps we can abstract these relations also and use them for hadron theory.
In (6.1) we have introduced bilocal generalizations of the vector and axial–vector currents, which in a quark model correspond to products of quark fields:

\[ F_{k\sigma}(x, y) \sim \bar{q}(x)\frac{i}{2} \lambda_k \gamma_\sigma q(y), \]
\[ F_{5k\sigma}(x, y) \sim \bar{q}(x)\frac{i}{2} \lambda_k \gamma_\sigma \gamma_5 q(y). \]

(6.2)

Note that the products in (6.2) have to be understood as “generalized Wick products”. The \( c \)–number part in the product of two quark fields is already excluded, since it does not contribute to the connected current commutator. The \( c \)–number part is measured by vacuum processes like \( e^+e^- \) annihilation. Assuming that the disconnected part of the commutator on the light cone is also dictated by the quark model, we would obtain

\[ \sigma_{tot} e^+e^- \sim \text{const.}/s \text{ for } e^+e^- \text{ annihilation}, \]

where \( s \) is as usually defined: \( s = -(p_1 + p_2)^2 \). In particular, we would get

\[ \sigma_{tot} (e^+e^- \text{ into hadrons}) \rightarrow (\sum Q^2) \sigma_{tot} (e^+e^- \text{ into muons}) \]

with \( \sum Q^2 = (2/3)^2 + (1/3)^2 = 2/3 \).

Now we go on to close the algebraic system of (6.1), where local currents occur on the left–hand side and bilocal ones on the right.

Let us assume that the bilocal generalizations of the vector and axial vector currents are the basic entities of the scheme. Again using the quark model as a guideline on the light cone, we obtain the following closed algebraic system for these bilocal operators:

\[
\begin{align*}
[F_{ij}(x, u), F_{j\nu}(y, v)] & \equiv \frac{1}{4\pi} \partial_\rho \left\{ \epsilon (x_0 - v_0) \delta \left[(x - v)^2\right] (if_{ijk} - d_{ijk}) (s_{\mu\nu\rho\sigma} F_{k\sigma}(y, u)) \\
& + i \epsilon_{\mu\nu\rho\sigma} F_{5k\sigma}(y, u) \right\} + \frac{1}{4\pi} \partial_\rho \left\{ \epsilon (u_0 - y_0) \partial \left[(u - y)^2\right] (if_{ijk} + d_{ijk}) \\
& \cdot (s_{\mu\nu\rho\sigma} F_{k\sigma}(x, v) - i \epsilon_{\mu\nu\rho\sigma} F_{5k\sigma}(x, v)) \right\},
\end{align*}
\]

(6.3)
\[
\left[ F^5_{i\mu}(x,u), F^5_{j\nu}(y,v) \right] \\
\cong \frac{1}{4\pi} \partial_\rho \left\{ \varepsilon (x_0 - v_0) \partial \left[ (x - v)^2 \right] \right\} (i f_{ijk} - d_{ijk}) \\
\left( s_{\mu\nu\rho\sigma} F^5_{k\sigma}(y,u) + i \varepsilon_{\mu\nu\rho\sigma} F_{k\sigma}(y,u) \right) \\
+ \frac{1}{4\pi} \partial_\rho \left\{ \varepsilon (u_0 - y_0) \delta \left[ (u - y)^2 \right] \right\} (i f_{ijk} + d_{ijk}) \\
\cdot \left( s_{\mu\nu\rho\sigma} F^5_{k\sigma}(x,y) - i \varepsilon_{\mu\nu\rho\sigma} F_{k\sigma}(x,v) \right),
\]

\[
\left[ F^5_{i\mu}(x,u), F^5_{j\nu}(y,v) \right] \cong \left[ F_{i\mu}(x,u), F_{j\nu}(y,v) \right].
\]

Similar relations might be abstracted for the anticommutators and products of two bilocal currents near the light cone. The relations (6.3) are assumed to be true if

\[
(x - u)^2 \approx 0, \quad (u - y)^2 \approx 0, \\
(u - v)^2 \approx 0, \quad (x - y)^2 \approx 0, \\
(x - v)^2 \approx 0, \quad (u - v)^2 \approx 0.
\]

This condition is obviously fulfilled if the four points \(x, u, y, v\) are distributed on a straight line on the light cone. The algebraic relations (6.3) can be used, for example, to determine the light cone commutator of two light cone commutators and relate this more complicated case to the simpler case of a light cone commutator. It would be interesting to propose experiments in order to test the relations (6.3).

7. LIGHT CONE ALGEBRA AND DEEP INELASTIC SCATTERING

In the last section we have emphasized that perhaps the light cone is a region of very high symmetry (scale and \(SU_3 \times SU_3\) invariance). Furthermore, we have abstracted from the quark model certain algebraic properties that might be right on the light cone. Now we should like to mention some general relations that we can obtain using this light cone algebra. But let us first consider the weak interactions in the deep inelastic region.
We introduce the weak currents $J^+_{\mu}(x), J^-_{\nu}(x)$ and consider the following expression:

$$W_{\mu\nu}(q) = \frac{1}{4\pi} \int d^4 z e^{-iq\cdot z} <\left[ J^+_{\mu}(z), J^-_{\nu}(0) \right] | p >$$

$$= \left( \delta_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2} \right) \left( W^1_+ - \frac{p\cdot q}{q^2} W^2_+ \right) - \frac{i}{2} \varepsilon_{\mu\nu\alpha\beta} p_\alpha p_\beta W^3_+$$

$$\delta_{\mu\nu} (p\cdot q)^2 + p_\mu p_\nu q^2 - (p_\mu p_\nu + p_\nu p_\mu) p\cdot q W^2_+ + q^2 q^\nu W^4_+$$

$$+ (q_\mu p_\nu + q_\nu p_\mu) W^5_+ + i (q_\mu p_\nu + q_\nu p_\mu) W_6. \quad (7.1)$$

In general, we have to describe the inelastic neutrino hadron processes by six structure functions. From naive scaling arguments we would expect in the deep inelastic limit:

$$W^1_+ \to F_1(\xi), \quad -q\cdot pW^2_+ \to F_2(\xi),$$

$$-q\cdot pW^3_+ \to F_3(\xi), \quad -q\cdot pW^4_+ \to F_4(\xi), \quad (7.2)$$

$$-q\cdot pW^5_+ \to F_5(\xi), \quad -q\cdot pW^6_+ \to F_6(\xi).$$

The formulae above have the most general form, valid for arbitrary vectors $J_\mu(x)$. We neglect the $T$–violating effects, which may in any case be 0 on the light cone: $F_6 = 0$. We have already stressed that the weak currents are conserved on the light cone, and we conclude:

$$F_4(\xi) = F_5(\xi) = 0. \quad (7.3)$$

Equation (7.3) is an experimental consequence of the $SU_3 \times SU_3$ symmetry on the light cone, which may be tested by experiment. In the deep inelastic limit we have only three non–vanishing structure functions, corresponding to a conserved current.

It is interesting to note that there is the possibility of testing the dimension $l$ of the divergence of the axial vector current, if our scaling hypothesis is right. We write, for the weak axial vector current,

$$\partial_\mu F^5_{\pm\mu} = c \cdot v_\pm(x) \quad (7.4)$$

where $v_\pm(x)$ is a local operator of dimension $l$. and $c$ is a parameter with non–zero dimension.
According to our assumptions about symmetry breaking, \( c \) can be written as a positive power of a mass. Using (7.1), we obtain

\[
q^\mu q^\nu W_{\mu\nu}^+(q) = \frac{e^2}{4\pi} \int d^4z e^{-iq\cdot z} <p | [v_+(z), v_-(0)] | p >
\]

\[
= (q^2)^2 W_4^+ - 2q^2 q \cdot p W_5^+ .
\]

We define:

\[
D(q^2, q \cdot p) = \frac{1}{4\pi} \int d^4z e^{-iq\cdot z} <p | [v_+(z), v_-(0)] | p > .
\]

If we assume that \( D \) scales in the deep inelastic region according to the dimension \( l \) of \( v_\pm(x) \), we obtain

\[
\lim_{b_j} (-p \cdot q)^{-l-3} D(q^2, q \cdot p) = \phi(\xi)
\]

where \( \phi(\xi) \) denotes the deep inelastic structure function for the matrix element (7.6). Using (7.5) we obtain

\[
\lim_{b_j} (-p \cdot q)^{5+l} (\xi^2 W_4^+ - 2\xi W_5^+) = c^2 \phi(\xi) .
\]

If we determine experimentally the scaling properties of \( W_4 \) and \( W_5 \), then we can deduce from (7.8) the dimension \( l \) of \( v_\pm(x) \). This \( l \) is the same quantity as the dimension \( \hat{l}_u \) discussed in Section 2, provided the \( SU_3 \times SU_3 \) violating term in the energy has a definite dimension.

In order to apply the light cone algebra of Section 6, we have to relate the expectation values of the bilocal operators appearing there to the structure function in question. This is done in Appendix II, where we give this connection for arbitrary currents. We use Eqs. (A.12) and (A.13), where the functions \( S_k(\xi), A_k(\xi) \) are given by the expectation value of the symmetric and antisymmetric bilocal currents (Eq. (A.8)), and obtain:

(a) for deep inelastic electron–hadron scattering:

\[
F_2^{ep}(\xi) = \xi \left( \frac{2}{3} \sqrt{\frac{2}{3}} A^0(\xi) + \frac{1}{3\sqrt{3}} A^8(\xi) + \frac{1}{3} A^3(\xi) \right)
\]

(b) for deep inelastic neutrino–hadron scattering:

\[
F_2^{\nu p}(\xi) = \xi \left( 2S^3(\xi) + 2 \sqrt{\frac{2}{3}} A^0(\xi) + \frac{2}{\sqrt{3}} A^8(\xi) \right)
\]

\[
F_3^{\nu p}(\xi) = 2A^3(\xi) - 2 \sqrt{\frac{2}{3}} S^0(\xi) - \frac{2}{\sqrt{3}} S^8(\xi) .
\]

In (7.5) and (7.6) we have neglected the Cabibbo angle, since \( \sin^2 \Theta_c = 0.05 \approx 0 \). Both in (7.4) and (7.6), \( A^3(\xi) \) occurs as the only isospin dependent part, and we can simply
derive relations between the structure functions of different members of an isospin multiplet, e.g., for neutron and proton:

$$6 \cdot (F_2^{en} - F_2^{ep}) = \xi \cdot (F_3^{vp} - F_3^{vn}) .$$

(7.13)

This relation was first obtained by C. H. Llewellyn Smith within the “parton” model. One can derive similar relations for other isospin multiplets.

In the symmetric bilocal current appear certain operators that we know. The operator

$$j_\mu(x) = i\bar{q}(x)\gamma_\mu q(x)$$

has to be identical with the hadron current (we suppress internal indices) in order to give current algebra. But we know their expectation values, which are given by the corresponding quantum number. In such a way we can derive a large set of sum rules relating certain moments of the structure functions to their well-known expectation values.

We give only the following two examples, which follow immediately from (7.10), (7.11), (7.12):

$$\int_{-1}^{1} \frac{d\xi}{\xi} (F_2^{vp}(\xi) - F_2^{vn}(\xi)) = \int_{-1}^{1} \frac{d\xi}{\xi} (F_2^{vp}(\xi) - F_2^{vp}(-\xi))$$

$$= 4s_3^1(p) = 4.$$  

(7.14)

Here $s_3^1(p)$ means, as in Appendix II, the proton expectation value of $2F_3$. This is the Adler sum rule, usually written as

$$\int_{0}^{1} \frac{d\xi}{\xi} (F_2^{vp}(\xi) - F_2^{vn}(\xi)) = 2.$$  

(7.15)

From (7.11) we obtain:

$$\int_{-1}^{1} (F_3^{vp} + F_3^{vn}) d\xi = -2 (2s_0^0(p) + s_8^8(p)) = -12$$  

(7.16)

or

$$\int_{0}^{1} (F_3^{vp} + F_3^{vn}) d\xi = -6,$$  

(7.17)

which is the sum rule first derived by Gross and Llewellyn Smith.

If we make the special assumption that we are abstracting our light cone relations from a pure quark model with no “gluon field” and non-derivative couplings, we can get a further set of relations.

Of course, no such model is known to exist in four dimensions that is even renormalizable, much less super-renormalizable as we would prefer to fit in with the ideas presented here. Nevertheless, it may be worthwhile to examine sum rules that test whether Nature imitates the “pure quark” case.

The point is that when we expand the bilocal quantity $F_{0\alpha}(x,y)$ to first order in $y-x$, we pick up a Lorentz tensor operator, a singlet under $SU_3$, that corresponds in the quark picture to the operator $1/2 \{ \bar{q}(x)\gamma_\mu \partial_\nu q(x) - \partial_\nu \bar{q}(x)\gamma_\mu q(x) \}$, which, if we symmetrize in $\mu$ and $\nu$ and ignore the
trace, is the same as the stress–energy–momentum tensor $\Theta_{\mu\nu}$ in the pure quark picture. But the expected value of $\Theta_{\mu\nu}$ in any state of momentum $p$ is just $2p_{\mu}p_{\nu}$, and so we obtain sum rules for the pure quark case.

We consider the isospin averaged expressions:

$$
(F_2^{ep}(\xi) + F_2^{en}(\xi)) = 2\xi \left\{ \frac{2}{3} \sqrt{\frac{2}{3}} A_0(\xi) + \frac{1}{3} \sqrt{\frac{2}{3}} A_8(\xi) \right\}
$$

$$
(F_2^{np}(\xi) + F_2^{an}(\xi)) = 2\xi \left\{ 2 \sqrt{\frac{2}{3}} A_0(\xi) + \frac{2}{\sqrt{3}} A_8(\xi) \right\}
$$

and obtain

$$
6 (F_2^{ep} + F_2^{en}) - (F_2^{np} + F_2^{an}) = 4 \sqrt{\frac{2}{3}} A_0(\xi)
$$

$$
= 4 \sqrt{\frac{2}{3}} \left( a_0^0 \delta(\xi) - \frac{1}{2!} a_0^0 \delta''(\xi) \cdots \right)
$$

In pure quark theories we have $a_0^0 = \sqrt{2/3}$ and we obtain

$$
6 \int_{-1}^{1} (F_2^{ep} + F_2^{en}) d(\xi) - \int_{-1}^{1} (F_2^{np}(\xi) + F_2^{an}(\xi)) d\xi = 8/3
$$

or, for the physical region $0 \leq \xi \leq 1$:

$$
6 \int_{0}^{1} (F_2^{ep} + F_2^{en}) d\xi - \int_{0}^{1} (F_2^{np}(\xi) + F_2^{an}(\xi)) e\xi = 4/3.
$$

(7.18)

The sum rule (7.18) can be tested by experiment. This will test whether one can describe the real world of hadrons by a theory resembling one with only quarks, interacting in some unknown non–linear fashion.

The scaling behavior in the deep inelastic region may be described by the "parton model".\footnote{In the deep inelastic region, the electron is viewed as scattering in the impulse approximation off point–like constituents of the hadrons ("partons"). In this case the scaling function $F_2^s(\xi)$ can be written as

$$
F_2^s(\xi) = \sum_N P(N) \left( \sum_i Q_i^2 \right)_N \xi f_N(\xi)
$$

where we sum up over all “partons” ($\sum_i$) and all the possibilities of having $N$ partons ($\sum_N$). The momentum distribution function of the “partons” is denoted by $f_s(\xi)$, the charge of the i-th “partin” by $Q_i$. We compare (7.9) with (7.18):

$$
F_2^s(\xi) = \xi \left( \frac{2}{3} A_0(\xi) + \frac{1}{6} A^8(\xi) + \frac{1}{3} A^3(\xi) \right)
$$

(7.20)
As long as we do not specify the functions \( f_N(\xi) \) and \( P(N) \), the “parton model” gives us no more information than the generalization of current algebra to the light cone as described in the last sections. If one assumes special properties of these functions, one goes beyond the light cone algebra of the currents, that means beyond the properties of the operator products on the light cone. Such additional assumptions, e. g., statistical assumptions about the distributions of the “partons” in relativistic phase space, appear in the light cone algebra approach as specific assumptions about the matrix elements of the expansion operators on the light cone. These additional assumptions are seen, in our approach, to be model dependent and somewhat arbitrary, as compared to results of the light cone algebra. Our results can, of course, be obtained by “parton” methods and are mostly well-known in that connection.

It is interesting to consider the different sum rules within the “parton model”. The sum rules (7.15) and (7.17) are valid in any “quark–parton” model; so is the symmetry relation (7.18). The sum rule (7.18) is a specific property of a model consisting only of quarks. If there is a “gluon” present, we obtain a deviation from 4/3 on the right-hand side, which measures the “gluon” contribution to the energy–momentum tensor.

Our closed algebra of bilocal operators on the light cone has, of course, a parallel in the “parton” model. However it is again much easier using our approach to disentangle what may be exactly true (formulae for light cone commutators of light cone commutators) from what depends on specific matrix elements and is therefore model dependent. It would be profitable to apply such an analysis to the work of Bjorken and Paschos, in the context of “partons”, on scaling in the Compton effect on protons.

As an example of a “parton model” relation that mingles specific assumptions about matrix elements with more general ideas of light cone algebra and abstraction from a pure quark model, we may take the allegation that in the pure quark case we have \( \int F_2^q(\xi) d\xi = 2/9 \). Light cone algebra and the pure quark assumption do not imply this.

8. CONCLUDING REMARKS

There are many observations that we would like to make and many unanswered questions that we would like to raise about light cone algebra. But we shall content ourselves with just a few remarks.

First comes the question of whether we can distinguish in a well-defined mathematical way, using physical quantities, between a theory that makes use of \( SU_3 \) triplet representations locally and one that does not. If we can, we must then ask whether a theory that has triplets locally necessarily implies the existence of real triplets (say real quarks) asymptotically. Dashen (private communication) raises these two questions by constructing local charge operators \( \int_V F_{\mu\nu} d^3x \) over a finite volume. (This construction is somewhat illegitimate, since test functions in field theory have to be multiplied by \( \delta \) functions in equal time charge density commutators and should...
therefore have all derivatives, not like the function that Dashen uses, which is unity inside $V$ and zero outside.) If his quantities $F_i^V$ make sense, they obey the commutation rules of $SU_3$ and we can ask whether for any $V$ our states contain triplet (or other triality $\neq 0$) representation of this $SU_3$. Dashen then suggests that our bilocal algebra probably implies that local triplets in this sense are present; if the procedure and the conclusion are correct, we must ask whether real quarks are then implied.

The question of quark statistics is another interesting one. If quarks are real, then we cannot assign them para–Fermi statistics of rank 3, since that is said to violate the factoring of the $S$–matrix for distant subsystems. However, if somehow our quarks are permanently bound in oscillators (and our theory is thus perhaps equivalent to a bootstrap theory with no real quarks), then they could be parafermions of rank 3. They can be bosons, too, if they are not real, but only if there is a spinless fermion (the “soul” of a baryon) that accompanies the three quarks in each baryon.

Another topic is the algebra of $U_6 \times U_6 \times O_3$ that is implied at equal times for the integrals of the current component and the angular momentum. Is that algebra really correct or is it too strong an assumption? Should it be replaced at $P_s = \infty$ by only the “good–good” part of the algebra?

If we do have the full algebra, then the quark kinetic part of the energy density is uniquely defined as the part behaving like \((35, 1)\) and \((1, 35)\) with $L = 1$, i.e. like $\alpha \cdot \nabla$.

If we abstract relations from a pure quark picture without gradient couplings, then this quark kinetic part of $\Theta_{\mu\nu}$ is all there is apart from the trace contribution. In that case, we have the equal time commutation relation for the whole energy operator:

$$
\sum_r \sum_r \left[ \mathcal{F}_{ir} d^3 x, \left[ \int \mathcal{F}_{ir} d^3 x, P_0 \right] \right] = 16/3 P_0 + \text{scale violating terms}.
$$

This relation, in the pure quark case, can be looked at in another way. It is an equal time consequence of the relation

$$
\Theta_{\mu\nu} = \lim_{y \to x} \frac{3\pi^2}{32} \partial_\mu \partial_\nu \left\{ \left( z^2 \right)^2 \mathcal{F}_{i\alpha}(x)\mathcal{F}_{i\alpha}(y) \right\} + \text{scale violating terms}
$$

that holds when the singlet tensor term in the light cone expansion of $\mathcal{F}_{i\mu}(x)\mathcal{F}_{j\nu}(y)$ is just proportional to $\Theta_{\mu\nu}$ as in the pure quark case. This relation is what, in the pure quark version of the light cone algebra (extended to light cone products), replaces the Sugawara model, in which $\Theta_{\mu\nu}$ is proportional to $\mathcal{F}_{i\mu}\mathcal{F}_{i\nu}$, with dimension - 6. Our expression is much more civilized, having $l = -4$ as it should. A more general equal time commutator than the one above, also implied by the pure quark case, is the following:

$$
\sum_r \left[ \mathcal{F}_{i\mu}(x), \partial_\mu \mathcal{F}_{i\nu}(y) \right] = 16i/3 \Theta_{00} \delta(x - y) + \text{scale breaking terms}.
$$
Another important point that should be emphasized is that the \( U_6 \times U_6 \) algebra requires the inclusion of a ninth vector current \( \mathcal{F}_{0\alpha} \) and a ninth axial vector current \( \mathcal{F}_{50\alpha} \), and that the Latin index for \( SU_3 \) representation components in Appendix II has to run from 0 to 8. Now if the term in the energy density that breaks \( SU_3 \times SU_3 \) follows our usual conjecture and behaves like \(-u_0 - cu_8\) with \( c \) near \(-\sqrt{2}\) and if the chiral symmetry preserving but scale breaking term \( \delta \) is just a constant, then as \( u \to 0 \) scale invariance and chiral invariance become good, but the mass formula for the pseudoscalar mesons indicates that we do not want \( \partial_\alpha \mathcal{F}_{0\alpha} \) to be zero in that limit. Yet \( \mathcal{F}_{0\alpha} \) is supposed to be conserved on the light cone. Does this raise a problem for the idea of \( \delta = \text{const.} \) or does it really raise the whole question of the relation of the light cone limit and the formal limit \( u \to 0, \delta \to 0 \)?

If there are dilations, with \( m^2 \to 0 \) in the limit of scale invariance while other masses stay finite, how does that jibe with the light cone limit in which all masses act as if they go to zero? Presumably there is no contradiction here, but the situation should be explored further.

Finally, let us recall that in the specific application of scaling to deep inelastic scattering, the functions \( F(\xi) \) connect up with two important parts of particle physics. As \( \xi \to 0 \), if we can interchange this limit with the Bjorken limit, we are dealing with fixed \( q^2 \) and with \( p \cdot q \to \infty \) and the behavior of the \( F \)'s comes directly from the Regge behavior of the corresponding exchanged channel. If \( \alpha_p(0) = 1 \), then \( F_2^{ep}(\xi) + F_2^{en}(\xi) \) goes like a constant at \( \xi = 0 \), i.e., \( \xi^{1-\alpha_p(0)} \), while \( F_2^{ep}(\xi) - F_2^{en}(\xi) \) goes like \( 1 - \alpha_p(0) \), etc.

As \( \xi \to 1 \), as emphasized by Drell and Yan, there seems to be a connection between the dependence of \( F(\xi) \) on \( 1 - \xi \) and the dependence of the elastic form factors of the nucleons on \( t \) at large \( t \).

9. PROBLEMS OF LIGHT CONE CURRENT ALGEBRA

If we take the notion of current algebra on the light cone seriously we are faced with a number of important theoretical questions, to most of which we have already alluded. We shall attempt to summarize them here and to comment on them.

We have exhibited in Eqs. (6.3) a closed algebraic system of light cone commutators of the connected parts of the 72 components of nine vector and nine axial vector bilocal currents, valid in the limit where all four points tend to lie on a straight line on the light cone (all six invariant intervals approaching zero). We shall refer to this system as the basic light cone algebra. The bilocal operators involved we may rename, in an obvious notation, \( D \left( x, y, \left( \frac{\alpha_\mu}{2} \right) \gamma_\mu \right) \) and \( D \left( x, y, \left( \frac{\alpha_5}{2} \right) \gamma_\mu \gamma_5 \right) \). They are well defined as \( (x-y)^2 \to 0 \) and their local limits are \( \mathcal{F}_{\mu\nu}(x) \) and \( \mathcal{F}_{5\mu}(x) \) respectively. We may ask the following questions about the basic light cone algebra:

a) Assuming that further refinement of the SLAC experiments and work on corresponding neutrino experiments continue to support the basic algebra, what further practical experimental tests can be designed? We want to generalize the tests of light cone commutators
of local currents to spin–flip matrix elements, to matrix elements with momentum transfer \( \neq 0 \), and to matrix elements between different numbers of particles. (We note, by the way, that as soon as we depart from matrix elements between 1 particle and 1 particle, the notion that mathematical dimension = physical dimension for the amplitudes is seen to be arbitrary, the mathematical dimension of the amplitude depending on the number of particles in a way that varies with our normalization. What must be preserved is the existence of a well–defined Bjorken limit for the commutator matrix elements, even though, with a given normalization convention, powers of masses occur in the final answer.)

When many hadron momenta \( p \) are present in the problem (all finite and timelike), we need a generalization of the Bjorken limit in momentum space, which corresponds to the light cone in co–ordinate space. Presumably, we choose a fixed light–vector \( e \) and a fixed timelike vector \( a \) and write the current momentum \( q \) as \( ue + a \), where the variable \( u \) is allowed to approach \( \infty \). Then for any hadron momentum \( p \), we have \( 2q \cdot p \to 2ue \cdot p \), while \( q^2 \to 2ue \cdot a \), and the ratios are all finite as \( u \to \infty \) (since a timelike vector dotted into a light–like one is non–zero).

b) Can tests be designed for the commutators of bilocal operators in the basic algebra, that is to say for light cone commutators of light cone commutators of currents?

First, we should generalize the Bjorken limit further to cover more than one current momentum \( q \). A possible way to do that may be to let \( q_j = u_je + a \), with fixed \( e \) and \( a \) as above. Then \( q_j^2 \to 2u_je \cdot a \), \( (q_j + q_k)^2 \to 2(u_j + u_k)e \cdot a \\
2q_j \cdot p_i \to 2u_je \cdot p_i \), etc. If all the \( u \)'s \( \to \infty \), then there is a fixed ratio between any \( q^2 \) and the corresponding \( 2q \cdot p_i \) in the limit.

Next, we have to consider if we can really measure the light cone commutator of light cone commutators. Actually that is very difficult, and the tests may be practical only if we generalize, as discussed in i) below, from commutators of currents on the light cone to physical ordered products of currents as well.

Tests of bilocal commutators are important not only for verifying that the bilocal algebra makes sense, but also because they involve the fourth powers of the quark charges, and therefore make possible comparison with the squares of the charges so as to check whether the fractional values are really correct. Other tests of the fractional charges are conceivable if the algebra is generalized to disconnected parts (hadron vacuum expectation values of commutators) as discussed in k) below, but there several questions arise that make a test within the basic algebra desirable.

c) To what extent can we abstract the basic algebra from a quark field theory model with interactions? It is, of course, all right in a free quark model but so are a great many results that we would not dream of abstracting for real hadrons. Recent work of Llewellyn Smith,18 Cornwall and Jackiw,24 and Gross and Treiman25 has confirmed that in a quark field theory model with neutral gluons, using formal manipulation of operators and not renormalized perturbation theory term by term, the basic algebra comes out all right in the presence of interactions. When the gluon is vector, the correspondence between the
\( D \)'s and quark expressions must be modified by the presence of the factor \( \exp \int_x^y B_\mu dl_\mu \), where \( B_\mu \) is the gluon field, \( g \) its coupling constant, and the integral is along a straight line.

The renormalized perturbation theory, taken term by term, reveals various pathologies in commutators of currents. Not only are there in each order logarithmic singularities on the light cone, which destroy scaling, and violations of the rule that \( \sigma_L/\sigma_T \to 0 \) in the Bjorken limit, but also a careful perturbation theory treatment shows the existence of higher singularities on the light cone, multiplied by the gluon fields, such as we worried about earlier on the basis of dimensional analysis. For example,\(^{26} \) in vector gluon theory we meet a term of the form

\[
g(x - y)\varepsilon^{\alpha\beta\gamma\delta} \partial_\gamma B_\delta/(x - y)^2
\]

occurring where we would expect from the basic algebra the finite operator \( F_0^\alpha(x, y) \): the gluon field strength, having in lowest order dimension \( l = -2 \), can appear multiplied by a more singular function than can a finite operator \( F_0^\alpha(x, y) \) of dimension \(-3\). Such a term would ruin the basic algebra as a closed system and even wreck the equal time algebra of charge densities by introducing a \( \nabla \partial \) term), although leaving untouched particular commutators, such as those involved in the SLAC experiments, and in fact any matrix elements with \( \Delta p = 0 \). A term involving the gluon field strength would also elevate that operator to the level of a physical quantity, occurring in the light cone commutator of real local currents.

If we wish to preserve the abstraction of the basic algebra, we must reject these “anomalous” singularities just as we do the logarithmic singularities in each order of renormalized perturbation theory and the occurrence of asymptotic longitudinal cross–sections. If, however, we blindly accept for hadrons the abstraction of any property of the gluon model that follows from naive manipulation of operators, we risk making some unwise generalizations of the basic algebra. It would be desirable to have some definite point of view about the relation of the abstracted results to the renormalized perturbation theory. Such a point of view, if available, would replace the transverse momentum cut–off of Drell and collaborators as a way of forcing the barely renormalizable gluon models into the mold of a super–renormalizable theory.

We note, for example, that if we take vector gluon theory at all seriously, we must deal with the fact that the vector baryon current \( F_0^\mu \) and the gluon exist in the same channel and are coupled, so that a string of vacuum polarization bubbles contributes to the unrenormalized current. But all the currents have fixed normalization, since their charges are well–defined quantum numbers, and it must be the unrenormalized currents that obey the algebra if the algebra is right. Hence, if the renormalized coupling constant \( g_1 \), of the gluon is to be non–zero, its renormalization constant \( Z_3^{-1} \) must be finite and we must imagine that the sum of perturbation theory yields the special case of a “finite vector theory”\(^{27} \), if we are to bring the vector gluon theory and the basic algebra into harmony. Perhaps this picture of a “finite theory” (assuming it is consistent, and we note that it involves finding roots of a particular equation for the coupling constant, an equation which may
not have roots!) leads, when the perturbation theory is summed, to canonical scaling and
the disappearance of the “anomalous” light cone singularities, so that the basic algebra is
preserved. But, if that is so and we lean on the “finite theory” for our abstraction of the
algebra, we have trouble with the possible generalization of the algebra to disconnected
parts with the accompanying naive or free quark behaviour at high momentum of the
vacuum expectation values of bilocal operators [as discussed in \( k \)] below. The reason
is that in the “finite theory” the asymptotic behavior of the vacuum expectation value
of current products or current commutators has a reduced singularity compared to naive
or free quark behaviour; this is evident in the case of two baryon currents in order to
make \( Z_3^{-1} \) finite. Thus the logic of the “finite theory”, while it might preserve the basic
algebra, excludes the simplest generalization to disconnected parts and may exclude other
generalizations.

d) Is a generalization possible to a connected light cone algebra of 144 components of
\( V,A,S,T, \) and \( P \) densities as in the free quark model, with divergences of the vector
and axial vector currents given by \( S \) and \( P \) densities with definite coefficients (effective
quark masses) and with divergence and curl of the tensor current given by well–defined
quantities in the theory?

Using formal manipulation of operators, all of this seems to happen in the quark theory
model with vector gluons. (If the gluons are scalar or pseudoscalar, the various divergen-
ciies do not come out in terms of densities in the algebra.)

The resulting generalized system has densities

\[
\mathcal{D}(x, y, \frac{i\lambda_i}{2}\gamma_5) \; ; \; \mathcal{D}(x, y, \frac{\lambda_i}{2}) \quad \text{and} \quad \mathcal{D}(x, y, \frac{\lambda_i}{2}\sigma_{\mu\nu})
\]

as well as the vector and axial vector ones of the basic algebra, and the system closes
algebraically under commutation, with the same rules as the free quark theory. Besides
the familiar divergence equations

\[
\frac{\partial}{\partial x_\mu}(x, x, \frac{i\lambda_i}{2}\gamma_\mu) = \mathcal{D}(x, x, \frac{i [M, \lambda_i]}{2}), \quad (9.1)
\]

\[
\frac{\partial}{\partial x_\mu}\mathcal{D}(x, x, \frac{i\lambda_i}{2}\gamma_\mu\gamma_5) = \mathcal{D}(x, x, \frac{i [M, \lambda_i]}{2}\gamma_5), \quad (9.2)
\]

where \( M \) is the “quark mass” matrix, diagonal for the three quarks \( u,d, \) and \( s, \) we have
in addition relations for the tensor currents:

\[
\frac{\partial}{\partial x_\mu}\mathcal{D}(x, x, \frac{\lambda_i}{2}\sigma_{\mu\nu}) = -\mathcal{D}(x, x, i\{M, \lambda_i/2\}\gamma_\nu) + \left[ \left( \frac{\partial}{\partial x_\nu} \frac{\partial}{\partial \gamma_\nu} \right) \mathcal{D}(x, y, i\lambda_i/2) \right]_{x=y} \quad (9.3)
\]
\[ \frac{\partial}{\partial x_\mu} \mathcal{D} \left( x, x, \frac{\lambda_i}{2} \gamma_\mu \gamma_5 \right) \]

\[ = -\mathcal{D} \left( x, x, \frac{i}{2} M, \frac{\lambda_i}{2} \gamma_\mu \gamma_5 \right) + \left[ \left( \frac{\partial}{\partial x_\nu} \frac{\partial}{\partial y_\nu} \right) \mathcal{D} \left( x, y, \frac{i\lambda_i}{2} \gamma_5 \right) \right]_{x=y} \]

In the first of these we see the generalization of the famous Gordon break-up of the Dirac vector current into a “convective current” and the divergence of a tensor “spin current”. In the second, we see appearing on the right-hand side the axial vector analogue of the “convective current” and we note that it is a “second-class current” that may some day play a rôle in a theory of CP violation. It is fascinating that these convective currents occur in the generalized algebra as first internal derivatives of the bilocal quantities.

It is interesting to look into the divergences not only of the local currents but also of their internal derivatives. In a free quark model the bilocal vector currents corresponding to conserved local vector currents are themselves conserved (with respect to \( x + y \)); in other words, all their internal derivatives are conserved. This is an example of an outrageously strong result that we presumably do not wish to abstract, and indeed it fails in a quark model with interactions.

Let us look in detail at the divergence of the first internal derivative of the baryon current in the vector gluon model, putting \( R = (x + y)/2 \) and \( z = x - y \). These first internal derivatives are light cone quantities and defined in the basic algebra. The equations of motion yield:

\[ \frac{\partial}{\partial R_\mu} \left[ \frac{\partial}{\partial z_\nu} \mathcal{D} \left( R, z, i\gamma_\mu \right) \right]_{z=0} = -i g \left( \frac{\partial B_\mu}{\partial R_\nu} - \frac{\partial B_\nu}{\partial R_\mu} \right) \mathcal{D} \left( R, 0, i\gamma_\mu \right). \]  

(9.5)

We may also look at the first nonlocal correction to Eq. (9.2):

\[ \frac{\partial}{\partial R_\mu} \mathcal{D} \left( R, z, i\frac{\lambda_i}{2} \gamma_\mu \gamma_5 \right) = \mathcal{D} \left( R, z, i\frac{\lambda_i}{2} \gamma_\mu \gamma_5 \right) \]

\[ -i g z_\nu \left( \frac{\partial B_\mu}{\partial R_\nu} - \frac{\partial B_\nu}{\partial R_\mu} \right) \mathcal{D} \left( R, z, i\frac{\lambda_i}{2} \gamma_\mu \gamma_5 \right) \]

\[ + \cdots \cdots \]

(9.6)

The first of these relations shows how, in the vector, gluon model the integral of the first internal derivative of the baryon current fails to be conserved and is therefore not equal to the total momentum that corresponds to the failure of the sum rule (7.16), which is now being tested by neutrino experiments. It will be exciting to see whether experiment leaves room for gluons or not in our abstraction of algebraic results from models.

The second relation is important in a different way, since it shows how, if an anomalous linear singularity on the light cone is introduced into \( \mathcal{D} \left( R, 0, i\gamma_\mu \gamma_5 \right) \propto \mathcal{F}_{0\mu}(R) \), an
“anomalous divergence term” $\propto g^2\epsilon_{\mu\nu k\lambda} \times B_{\mu\nu}B_{k\lambda}$ appears to be introduced into the divergence of the ninth axial vector current. This anomalous divergence has dimension $-4$ (which we supposed could not be present in $\partial_\mu F_{5i\mu}$) and spoils the situation in which the divergences or currents are contained in the generalized algebra.

It is unclear whether the mathematical relation between the high energy anomalous singularity and the low energy anomalous divergence is real or apparent, when operator products are carefully handled. Like the Adler term discussed in j) below, the anomalous divergence may be obtainable as a kind of low energy theorem and might survive a treatment in which the anomalous singularity is gotten rid of.

Summarizing what we have just examined, we add two more questions to our list:

e) Is there a failure of the sum rule (7.17) and thus room in the algebraic structure for abstraction from a model with gluons?

Naïve manipulation of operators in the vector gluon model seems to give the enlarged light cone algebra of the connected parts of 144 densities, with no anomalies singularities and no anomalous divergences.

The renormalized perturbation theory, taken term by term, contains a large number of anomalous that spoil even the basic algebra, though not necessarily in direct conflict with experimental results so far.

A “finite theory” approach is needed if the sum of renormalized perturbation theory is to be brought into any sort of correspondence with the light cone algebra. However, it is not at all clear how many anomalies are cured in this manner. (We note, by the way, that for the scalar and pseudoscalar densities to have canonical dimensions and for their unrenormalized versions to be finite, so that they can obey the algebra and allow finite bare quark masses $M$ as coefficients, another function of the coupling constant must vanish, namely, the exponent that appears in the mass renormalization in the finite theory.)

In any case, certain anomalies, like the anomalous divergence of $F_{5i\mu}^5$ come out only in lowest order of renormalized perturbation theory and appear difficult if not impossible to fix by the “finite theory” approach, even if other diseases are cured.

We are left, then, with four possible attitudes:

A) The whole system, including scaling, is wrong as in renormalized perturbation theory term by term.

B) A “finite theory” approach is to be used, from which certain features of canonical scaling can be abstracted, but in which a number of anomalies are left that wreck either the basic or the enlarged algebra as a closed system, while also destroying the possibility of abstracting the behaviour of disconnected parts from free quark theory or naïve considerations.

C) The naïve approach is right, and the basic algebra can be abstracted, with probably the enlarged algebra as well, and perhaps even the behaviour of disconnected parts. The gluon field is not necessarily directly observable, but its effect is felt indirectly, for example, i. e., the failure of the sum rule (7.16). In this case, what happens to the Adler anomaly, discussed in j) below, which formally resembles the corresponding
gluon anomalies, but involves the real electromagnetic field and real electric charges, instead of the presumably fictitious gluon quantities?

D) The naïve approach is right, but we are forced to have the sum rule (7.16) and the corresponding conservation of the quark momentum alone, as if we had a super-renormalizable self-interaction of quark currents. This last situation seems attractive, as we have indicated in earlier Sections, but is it right? What will experiments have to say about it? Is it consistent theoretically?

f) Are there anomalous divergences in hadron theory or do these go away if the anomalous singularities disappear?

We might remark, by the way, that an anomalous divergence term in $F^0_{\mu 5}$ looks at first sight like a welcome addition, since it distinguishes the ninth axial vector current from the other eight and seems to provide an excuse for the apparent failure of the ninth one to have a zero mass pseudoscalar meson in the approximation in which $M$ is neglected, while the other eight have the pseudoscalar octet; the distribution of mass squared of the nine pseudoscalar mesons certainly suggests some sort of distinction. However, in fact an anomalous divergence is not needed to provide such an excuse, since the algebra $U_3 \times U_3$ of the vector and axial vector current charges already allows for a distinction. Since the charge $F^0_5$ commutes with all the others, there is no reason for it not to vanish when $M$ is neglected, unlike the other eight, which are prevented from vanishing by their commutation rules. Thus $F^5_0$ escapes the choice, in the approximation of its conservation, between having a zero mass pseudoscalar meson and causing degeneracy of opposite parities, while the other $F^5_i$ do not escape the choice and apparently have the massless pseudoscalar octet in the limit $M \to 0$.

g) Is there any practical way of testing the enlarged algebra including the divergences of vector and axial vector currents?

We have alluded to this matter in previous sections when we discussed tests of the dimensionalities of these divergences, which are here equal to -3. Weak interaction tests are perfectly possible, but they are very difficult, especially since the amplitudes of leptonic processes involving the current divergences vanish with the lepton masses.

h) Assuming the extended algebra, are we right in our understanding of the relation between high energy pion elastic scattering and the Bjorken limit of the matrix element of the commutator of two pseudoscalar densities?

We commute a pseudoscalar density, say $\mathcal{D}(x, x, ((i\lambda_3/2) \gamma_5))$, with itself at two points near the light cone and obtain at the righthand side a term proportional to $d_{33} \partial_\mu \left[ \varepsilon (x_0 - y_0) \delta \left((x - y)^2\right)\right] \mathcal{D}(x, y, (i\lambda_j/2) \gamma_\mu)$, which, between two proton states of equal momenta, gives just the SLAC form factors and the related one for neutrino experiments, provided we take the Bjorken limit. We then utilize the principle invoked in Section 8 that we may interchange the Bjorken limit and the limit $\xi \to 0$ to obtain the high energy limit for fixed large $q^2$ and the connection with Regge behaviour. Assuming, as before, that $\alpha_p(0) = 1$, we get a
form factor $\sim \xi^{-1}$ at small $\xi$, compatible with the SLAC results, and the amplitude for the commutator at high energy for fixed large $q^2$ goes like $(2p \cdot q)/q^2$, or $s^1/q^2$.

Now, for any $q^2$, we expect this matrix element of the commutator of pseudoscalar densities to go like $s^1 \varphi(q^2)$, since $\alpha_s(0) = 1$. At $q^2 = -m^2$, $\varphi$ should have a double pole corresponding to the pion scattering. If PCAC is useful here, then the double pole should dominate the behaviour of $\varphi$ near $q^2 = 0$ and in that region we can calculate $\varphi$ from the asymptotic elastic pion scattering amplitude (i.e., the total cross-section), the Goldberger–Treiman constant, and the “quark masses” in the diagonal matrix $M$, which have a definite physical significance in the extended algebra, since they relate the divergences of the axial vector currents to the pseudoscalar densities in the algebra.

Thus we know the behaviour of $\phi(q^2)$ at large $q^2$ (proportionality to $1/q^2$ with a coefficient obtainable from the usual deep inelastic form factors) and we know it at small $q^2$ in terms of the “quark masses” and the total asymptotic pion cross-section. Unfortunately we do not know any reliable way to connect the two regions, but some day this insight may be helpful. Anyway, we see that the extended algebra is perfectly compatible with Regge behaviour and the interchange of limits.

i) To what extent can we generalize the algebra to a set of relations for the connected parts of products, or of physical ordered products, of operators near the light cone?

First of all, if the commutator algebra is correct, the generalization to ordinary binary products near the light cone seems straightforward; we need only exclude catastrophic singularities in the anticommutator (or real part in momentum space) near $z = 0$. Then the operator product near the light cone looks like the commutator, but with $\varepsilon(z_0) \delta(z^2)$ replaced by $(2\pi i)^{-1} (z^2 - iz_0 \varepsilon)^{-1}$.

Next, we go on to binary ordered products of currents, as in Feynman amplitudes. To clarify the ideas, let us look at the ordered product of two electromagnetic currents and see what would happen if we were abstracting our formula from a model containing a scalar charged field $\phi$. Then there would be a non–vanishing asymptotic longitudinal cross-section $\sigma_L$, and terms in $\delta_{\mu\nu} - (q_\mu q_\nu)/q^2$ would survive in the Bjorken limit. Multiplying such terms in co–ordinate space by $\varepsilon(z_0)$ would be non–covariant, and it would be necessary to add non–covariant terms to the ordered product to restore the covariance; these correspond to operator Schwinger terms and they would be proportional to $\Phi^+ \Phi$. In addition, to make up the physical two–photon amplitude, it would be necessary to add in the second order physical “sea–gull” interaction $e^2 A_\mu A^\mu \Phi^+ \Phi$, a covariant term of second degree in the electromagnetic potentials and also proportional to $\Phi^+ \Phi$.

The essential point to be learned from this example is that it is only the complete amplitude, or physical ordered product, including all the possible types of contribution mentioned above, that matters. For electromagnetism, that represents the actual coupling to two virtual photons to order $e^2$.

Given our picture of the ordinary product of two electromagnetic currents near the light cone, is it trivial to construct the physical ordered product, just replacing $(z^2 - iz_0 \varepsilon)^{-1}$ by $(z^2 - i \varepsilon)^{-1}$? We can presumably dispense with the complications just mentioned for the abstraction from charged scalar theory, since we have no asymptotic $\sigma_L$, no operator.
Schwinger terms, and presumably no explicit “sea–gulls” of the type encountered there. However, we must be careful about the possibility of some subtler type of subtraction term in the dispersion relation connecting absorptive and dispersive parts of the physical amplitude. Further investigation of that point would be very useful, and should soon clear up the matter.

If the connected part of the physical ordered product of two electromagnetic currents is simply understood as we have indicated, then we are in a position to propose experimental tests of the bilocal algebra by experiment. For example, we examine the reaction $e^- + p \rightarrow e^- + X + \mu^+ \mu^-$, where $X$ is any hadronic system, and consider the cross–section summed over $X$, which gives us the amplitude of a fourth order electromagnetic process, with a proton as initial and final state and $\Delta p = 0$. There are two variables $q$, one for the electrons and one for the muon, and we go to the generalized Bjorken limits, as sketched above. We are dealing with the light cone commutator of two light cone–physical ordered products, and if the latter are indeed simple, we have the light cone commutator of two bilocal currents, with all four points tending to lie on a straight line. The right–hand side then involves the same matrix elements of bilocal currents as in the SLAC and corresponding neutrino experiments, and a test of bilocal algebra and of quark charges becomes possible in principle, as suggested above under b).

Theoretical investigation should be extended to the physical ordered light cone product of any number of electromagnetic currents, to see if surprises turn up.

Finally, let us allude to the generalization from electromagnetic currents to others in the system, when we take physical ordered products. Except for PCAC considerations, as mentioned below under j), we can attach meaning to physical ordered current products only if we discuss the actual physical interactions to which they refer. In other words, we must consider products of weak currents or mixed products of weak and electromagnetic currents and ask about the actual amplitudes for weak processes or weak electromagnetic processes, to the lowest order in $G$ and $e$ in which these occur. (Indeed, if $G$ is really like $e^2 m_X^{-2}$, where $m_X$ is an intermediate boson mass, then we may have to treat weak and electromagnetic orders as interchangeable.) Such discussions contain considerable uncertainties, since the amplitudes may contain intermediate boson propagators, electromagnetic vertices of intermediate bosons, and more complicated Yang–Mills type interactions of intermediate bosons. We would have to base our work on a definite picture of higher-order weak and weak–electromagnetic processes in order to make it fully meaningful and understand the significance of any subtraction terms that arise. The same statement may be turned around, however, to sound more hopeful: a study of the physical ordered products near the light cone of weak and electromagnetic currents can help in the construction of a skeleton theory of higher order weak and weak–electromagnetic processes.

j) What are the implications for light cone current algebra of the “Adler anomaly”?

Here we must turn our attention to the physical amplitude for two photons to turn into the divergence of the axial vector current $F_{3a}^5$, where the physical significance of the last is given not by the weak interaction but the PCAC hypothesis, treating the pion mass as small and obtaining an approximation to the decay amplitude $\pi^0 \rightarrow 2\gamma$. The photon
frequencies may be treated as small, also, and the whole problem can be phrased as a search for a low energy limit.

In the renormalized perturbation theory approach to the vector gluon model, a sophisticated treatment shows that Adler’s “anomalous divergence” term in $\partial_\alpha F^5_{3\alpha}$, of the form (const.) $e^2 F_{\mu\nu} F^{\mu\nu}$, shows up only in the zeroth order of the renormalized perturbation expansion in $g^2$, and thus the PCAC approximation to $\pi^0 \to 2\gamma$ can be calculated exactly in terms of a simple triangular quark loop, which gives the value of the constant. If we look at the Adler calculation in terms of a vacuum closed loop it seems to belong with our discussion under k) of disconnected parts, but if we think of it as concerning the matrix element between vacuum and a low mass $\pi^0$ of the physical ordered product of two electromagnetic currents, it is seen to be related to a connected part. Again, the Adler result is a “low energy theorem”, but it is connected with a possible high energy singularity arising through electromagnetic effects, in a way that parallels the apparent relation between anomalous divergence and high energy singularity discussed for the vector gluon model, with the difference noted above that photons are real and gluons presumably fictitious.

Now the actual calculation of the $\pi^0 \to 2\gamma$ decay amplitude by the Adler method gives, for quarks with Fermi–Dirac statistics, an amplitude about three times too small to agree with observation, while “parastatistics of rank three” gives a factor of three and good agreement with experiment.

We note that when using quarks as constituents of hadrons in the simple phenomenological $3q$ picture of the baryon, those “constituent quarks” look as if they should be assigned para–Fermi statistics of rank three, so that we can have for the ground state a totally symmetric rather than a totally antisymmetric spatial wave function, the latter being rather bizarre. We discuss below under m) the complicated transformation connecting these “constituent quarks” (non–relativistic for a hadron at rest and with low probability for pairs) with the relativistic “current quarks” of the quark–gluon field theory model (in which a hadron bristles with $q\bar{q}$ pairs). That transformation presumably does not affect the statistics. Thus, including the Adler result, we have an argument in each case for parastatistics.

We may describe the “paraquarks” in the following way. We start with three kinds of $s$ quark, three kinds of $u$ quark, and three kinds of $d$ quark, nine in all, obeying conventional Fermi–Dirac statistics, and then apply a supplementary condition that any physical hadron system is in a singlet state of the new $SU_3$ spin. This supplementary condition is presumably not allowed if the quarks are real, since it does not factor when a system is divided into two distant subsystems. Thus we are dealing with three fictitious “paraquarks” $u, d,$ and $s$.

If we insist on having real quarks, then the Adler argument leads us to nine real particles, giving us a so-called “three–triplet” situation.

The paraquarks always give us a factor of 3 in a vacuum loop compared to Fermi–Dirac quarks. This is important when we go on to our next generalization, which is to disconnected parts of amplitudes.
k) Apart from the “Adler anomaly”, to what extent can we use here quark theory on the light cone for the algebra of disconnected parts of the currents and for the vacuum expectation values of bilocal currents?

We have mentioned briefly in Section 6 the possibility that free quark behaviour might characterize not only the algebraic structure of connected amplitudes but also the high frequency limits of expectation values of current products or commutators in the hadron vacuum. We still do not know whether that makes sense or not, and whether, if it makes sense, it is experimentally correct.

We mentioned the simplest consequence of using abstracting light cone formulae between vacuum and vacuum, namely the prediction of the asymptotic total cross-section to order $e^4$ for $e^+ + e^- \to \text{hadrons}$ divided by the same for $e^+ + e^- \to \mu^+ + \mu^-$. With Fermi–Dirac quarks, we would get an asymptotic ratio of $(2/3)^2 + (-1/3)^2 + (-1/3)^2 = 2/3$, but with paraquarks we get three times as much, namely 2. The explicit check on the fractional charges in the model is, of course, less convincing now that one predicts 2, and the rôle of experimental tests depending on connecting parts becomes more important.

A fourth order test of the disconnected commutator of ordered light cone products is provided by the cross-section for $e^+ + e^- \to \mu^+ + \mu^- + X$, summed over $X$, as suggested by Gross and Treiman. Again the result should be multiplied by 3 for parastatistics.

We must consider here the possibility that the vacuum expectation values of current products are less singular at high frequencies than in the free quark model; such a situation obtains, for example, in the “finite theory” approach to the vector gluon model. In such a case, the cross-section ratio

$$\sigma (e^+ + e^- \to \text{hadrons}) / \sigma (e^+ + e^- \to \mu^+ + \mu^-)$$

would tend asymptotically to zero in hadrons divided by the same for $e^+ + e^- \to \mu^+ + \mu^-$. These considerations make the possible experimental investigation of the high energy behaviour $\sigma (e^+ + e^- \to \text{hadrons})$ especially interesting. Unfortunately, the energy of colliding beam experiments now envisaged is limited to a total of 7 GeV. Furthermore, there is a practical limitation at sufficiently high energy, when higher order electromagnetic effects make the one–photon annihilation difficult to measure.

We note that the high energy behaviour of the vacuum expectation value of the product or commutator of two scalar or pseudoscalar densities is important, as well as that of two vector or axial vector currents. Much recent theoretical work on the $K_{e3}$ and $K_{\mu3}$ decays has been based on the notion that the Fourier transform of the vacuum expectation value of the ordered product of two such densities obeys an unsubtracted dispersion relation, whereas free quark theory would suggest two subtractions. Where does the truth lie?

l) If we assume the basic bilocal algebra, or go further and assume some of the generalizations discussed here, do we at some point abstract so much from a quark model that we end up with the necessity of having real quarks (or three real triplets)?

We have alluded to this all–important question in Section 8. It is still not cleared up. If the bilocal algebra is to be maintained without real triplets, we must somehow benefit from what is effectively the asymptotic form $(i\gamma \cdot p)^{-1}$ of free quark propagators without
having any actual propagation of particles with single quark quantum numbers; instead
singularities occur only for mesons and baryons, etc., with quark number divisible by 3.
No one knows how to write down explicitly a field theory in which the quarks are perma-
nently bound and nevertheless act free at large momenta, but that sort of thing is what
we seem to require of the abstract hadron theory.
Meanwhile, it would be useful to investigate further whether any of the assumptions dis-
cussed here can be shown to lead to real triplets.

m) Do the considerations discussed here throw any light on the nature of the transformation
connecting “constituent quarks” and “current quarks”?
Let us first put the question into a more physical form, quarks being after all probably
fictitious entities. We note that the constituent quark model has a rough symmetry
under a group $SU_6 \times SU_6 \times O_3$, representing, respectively, the spin and unitary spin
of quarks, and the relative orbital angular momentum. For collinear processes (with all
particles moving in the z direction, say) the approximate symmetry reduces to the famous
subgroup $(SU_6)_W \times O_2$. We may examine the special case of $P_z = \infty$ and the resulting
$(SU_6)_W$ group $(SU_6)_W_{-\infty,\text{strong}}$.
Now by studying the charges associated with various currents at $P_z = \infty$ (looking at
matrix elements between finite mass states), we arrive at the algebra of another $(SU_6)_W$,
which we may call $(SU_6)_W_{\infty,\text{currents}}$. Between finite mass states at $P_z = \infty$ we have

$$F_i = \int \mathcal{D} \left( x, x, \frac{\beta \lambda_i}{2} \sigma_z \right) d^3x = \int \mathcal{D} \left( x, x, \frac{\beta \lambda_i}{2} \alpha_z \right) d^3x,$$

$$-F_i^5 = \int \mathcal{D} \left( x, x, \frac{\beta \lambda_i}{2} \sigma_z \right) d^3x = -\int \mathcal{D} \left( x, x, \frac{\beta \lambda_i}{2} \gamma_5 \right) d^3x,$$

$$F_{ix} \equiv \int \mathcal{D} \left( x, x, \frac{\beta \lambda_i}{2} \sigma_x \right) d^3x = -\int \mathcal{D} \left( x, x, \frac{\beta \lambda_i}{2} i \alpha_y \right) d^3x,$$

$$F_{iy} \equiv \int \mathcal{D} \left( x, x, \frac{\beta \lambda_i}{2} \sigma_y \right) d^3x = \int \mathcal{D} \left( x, x, \frac{\beta \lambda_i}{2} i \alpha_x \right) d^3x.$$

(7.7)

[If we do not wish to include tensor currents, we may discuss instead just the subgroups
$(SU_3 \times SU_3)_W_{\infty,\text{strong}}$ and $(SU_3 \times SU_3)_W_{\infty,\text{currents}}$.]
[If we do not like to work at $P_z = \infty$, we can switch to the construction of light–like
charges and construct an $(SU_6)_W_{\text{currents}}$ or an $(SU_3 \times SU_3)_W_{\text{currents}}$ out of those.]
What is the relation between $(SU_6)_W_{\infty,\text{strong}}$ and $(SU_6)_W_{\infty,\text{currents}}$? That is a physical
question that can replace the question about the relation of constituent quarks to current
quarks. There must be a transformation, perhaps a unitary transformation, taking the
generators of one $(SU_6)_W$ into those of the other. We know that this transformation is
very different from the unit transformation, since $(SU_6)_W_{\infty,\text{strong}}$ is approximately con-
served, while $(SU_6)_W_{\infty,\text{currents}}$ is very far from conserved. We know that baryon and
meson eigenstates of mass are very impure with respect to \((SU_6)^W, \infty, \text{currents}\). If they were pure, there would be no anomalous magnetic moments for neutron and proton, \(-G_A/G_V\) would be 5/3, etc. Furthermore, we can see from many arguments, for example the one about anomalous moments, that the transformation between the two \((SU_6)^W\)'s mixes up orbital angular momenta. It mixes 56, \(L = 0^+\) with 70, \(L = 1^-\), for example, at \(P_z = \infty\). In “first approximation”, so to speak, the correction to unity in the transformation behaves like 35, \(L = 1\) under either \((SU_6)^W\). We note, to avoid confusion, that the charges \(F_i\) are not much affected by the transformation, and \(I\) and \(Y\) not at all.

Now a hint about the transformation is provided by PCAC. (We are indebted to Mr. H. J. Melosh and Mr. J. Amarante for discussions of this point.) Assuming the generalized algebra of 144 densities, we use PCAC to tell us that at low frequencies the pseudoscalar densities act like fields for pseudoscalar mesons. Now under commutation with appropriate generators of \((SU_6)^W, \infty, \text{currents}\), the pseudoscalar densities are transformed into components of vector currents \(F_{i\mu}\). But under commutation with analogous generators of \((SU_6)^W, \infty, \text{strong}\), the effective “pseudoscalar meson fields” are transformed into components of effective “vector meson fields”. In order for the transformation to be different from unity, the effective “vector meson fields” must be different from the regular vector currents \(F_{i\mu}\). Suppose the transformation is unitary; then if we make an expansion of it about 1 as \(U = 1 + iA + \cdots\), we can discuss some properties of \(A\). A possible expansion would involve dimensionality. The regular vector current would go into itself, with \(l = -3\), plus a correction with \(l = -4\), plus another correction with \(l = -5\), etc. In a model where effective “bare quark masses” \(M\) have real physical meaning (relating divergences of vector and axial vector currents to scalar and pseudoscalar densities of the algebra), we might think of the expansion as one in inverse powers of those masses, even though the masses are probably small and the expansion very bad in practice: we may still learn something from it.

We have, then, \(A\) as an operator that commutes with the pseudoscalar densities \(D(x, x, (i\lambda_i/2) \gamma_5)\) to give correction with \(l = -4\) in the effective “vector meson field” to the vector currents \(D(x, x, (i\lambda_i/2) \gamma_\mu)\). What can these corrections be? The logical candidates are the convective currents \(\partial/\partial z \mu \frac{\partial}{\partial z} D\left( R, z, (i\lambda_i/2) \right)\big|_{z=0}\), where \(R = (x + y)/2\) and \(z = x - y\). These have just the right properties. The operator \(A\) behaves like 35, \(L = 1\), has the right charge conjugation behaviour, and so forth.

If we look explicitly at the vector gluon model, we see that the first order transformation \(1 + iA\) corresponds to the first order expansion of the Foldy–Wouthuysen transformation, and this may be an important clue to the nature of the whole transformation that connects constituent quarks and current quarks.

Confusion has existed for many years between the two kinds of quarks, and study of the transformation may help to clear up such confusion. Many theorists have been surprised to find that the current quarks (or “partons”) in the deep inelastic scattering analysis of the proton show indefinitely large numbers of pairs, while the constituent quarks in the quark model of the proton are three in number, with little allowance for pairs. The fact that the transformation is very far from unity, of course, explains the difference.
What can we do to explore the connection, if any, between scaling in high energy hadronic processes and scaling in electromagnetic and weak processes?

High energy hadronic scaling has been interpreted by Mueller\(^28\) as coming from the applicability of Regge theory to many particle processes at high energy, with the leading Regge exchange being that of a Pomeranchuk pole with \(a_p(0) = 1\). (It is not yet certain whether this pole has to be a moving one; there is perhaps still a possibility that it might be a fixed pole and the Gribov–Pomeranchuk difficulty overcome by the existence of a moving singularity that passes \(a = 1\) between \(t = 0\) and the lowest hadron threshold \(t = 4m_h^2\) for the \(P\) channel.) Mueller then gets, for an inclusive reaction with incoming momenta \(p\) and \(p'\) and an outgoing momentum \(q\) for the particle observed forward scaling (say) when \(q \cdot p\) and \(p \cdot p'\) go to \(\infty\) proportionately with \(q \cdot p\) finite, backward scaling when \(q \cdot p'\) and \(p \cdot p'\) go to \(\infty\) proportionately with \(q \cdot p\) finite, and “pionization” when \(q \cdot p\) and \(q \cdot p'\) both go to infinity like \(\sqrt{p \cdot p'}\). Here \(p^2, p'^2\), and \(q^2\) are, of course, fixed.

No one seems really to understand the connection between this kind of scaling and the light cone scaling for weak and electromagnetic currents that we have discussed, with the corresponding Bjorken limits in which various quantities \(q^2 \to \infty\) for current momenta \(q\).

This is true despite the fact that by interchanging limits one can relate light cone scaling as \(\xi \to 0\) Regge behaviour for large \(q^2\).

One common feature of hadronic and light cone scaling is the effective transverse momentum cut–off in both cases, and that may provide a clue to a possible connection when we understand better the way in which the theory cuts itself off.

The study of mixed processes, in which a current scaling limit and a hadron scaling limit are taken at the same time, is being undertaken by several theorists, including Bjorken.\(^29\).

Such studies may lead us to a guess as to the general systematics of mixed scaling, that would include current scaling and hadronic scaling as special cases. That might well be useful for understanding the connection, if any, between the two.

In the course of such work further attention will no doubt be paid to the hypothesis of scaling in the hadronic production of lepton pairs; that is an example of a conjecture about mixed scaling, since the \(s\) value of the initial hadron system and the \(q^2\) of the lepton pair are both supposed to go to infinity, and in proportion.

In conclusion, let us express our hope that this summary of problems and difficulties may encourage some theoretical research and perhaps some experimental work that will reduce the number of mysteries facing us and allow the beauty and simplicity of the merging picture of hadrons to stand out more clearly.
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APPENDIX K. SCALING HYPOTHESIS
FOR THE ENERGY MOMENTUM TENSOR

The underlying physical process is the interaction of an off–shell graviton with a hadronic target, with no momentum transfer and an average taken over spins. The corresponding matrix element is:

$$W_{\mu\nu\rho\sigma} \equiv \frac{1}{4\pi} \int \frac{e^{-iq\cdot z}}{p^4} \left[ \Theta_{\mu\nu}(z), \Theta_{\rho\sigma}(0) \right] \mid p > d^4 z.$$

We can describe the process by five structure functions.

$$W_{\mu \nu ; \rho \sigma} \left( q^2, q \cdot p \right) = \left( \delta_{\mu \sigma} - \frac{q_\mu q_\sigma}{q^2} \right) \left( \delta_{\rho \sigma} - \frac{q_\rho q_\sigma}{q^2} \right) T_1 \left( q^2, q \cdot p \right) + \left( \delta_{(\mu \rho} - \frac{q_{(\mu}q_{\rho)}}{q^2} \right) \left( \delta_{\nu)\sigma} - \frac{q_{(\nu)}q_{\sigma)}}{q^2} \right) T_2 \left( q^2, q \cdot p \right) + P_\mu P_\nu \left( q_{\rho} - \frac{q_{\rho \nu}q_{\nu}}{q^2} \right) T_3 \left( q^2, q \cdot p \right) + \left\{ P_\mu P_\nu \left( \delta_{\rho \sigma} - \frac{q_{\rho}q_{\sigma}}{q^2} \right) + P_\rho P_\sigma \left( \delta_{\mu \nu} - \frac{q_{\mu}q_{\nu}}{q^2} \right) \right\} T_4 \left( q^2, q \cdot p \right) + P_{\mu} P_{(\rho} \left( \delta_{\nu)\sigma} - \frac{q_{\nu q_{\sigma}}}{q^2} \right) T_5 \left( q^2, q \cdot p \right)$$

where

$$P_\mu \equiv \frac{1}{\sqrt{-q \cdot p}} \left( p_\mu - \frac{q \cdot p}{q^2} q_\mu \right).$$

The symbol $()$ means symmetrization.

According to the scaling hypothesis and our principle of higher dimensions of the symmetry breaking terms, we expect:
1) At least one of the dimensionless structure functions $T_i(q^2, q \cdot p)$ behaves in the deep inelastic region like

$$T_i(q^2, q \cdot p) \rightarrow G_i(\xi).$$

$$\zeta = -\frac{q^2}{2q \cdot p}$$

2) No structure function diverges in the deep inelastic region. This is a specific consequence of our postulate about symmetry breaking effects. Note that (2) is not true in certain Lagrangian models, e.g., in a theory with a formal interaction term of dimension-6 like $\bar{\psi} \psi \bar{\psi} \psi$.

3) The trace terms

$$\delta_{\mu \nu} W_{\mu \nu \rho \sigma}, \delta_{\rho \sigma} W_{\mu \nu \rho \sigma}, \text{ and } \delta_{\mu \nu} \delta_{\rho \sigma} W_{\mu \nu \rho \sigma}$$

are connected with the trace of the energy momentum tensor. The corresponding structure functions, which can be calculated in terms of the five functions $T_i$, have to vanish in the deep inelastic region. That means that we can express the $G_i(\rho)$ in terms of three non-vanishing structure functions.

**APPENDIX II**

We consider deep inelastic current hadron processes in general. Define:

$$W_{\mu \nu}^{ij}(q) = \frac{1}{4\pi} \int d^4 z e^{-iq \cdot z} <p | \left[ \bar{F}_{i\mu}(x), F_{j\nu}(y) \right] | p >$$

$$= \left( \delta_{\mu \nu} - \frac{q_{\mu} q_{\nu}}{q^2} \right) W_{1}^{ij}(q^2, p \cdot q) - \frac{(q \cdot p)^2}{q^2} W_{2}^{ij}(q^2, p \cdot q)$$

$$+ \frac{\delta_{\mu \nu} (p \cdot q)^2 + p_{\mu} p_{\nu} q^2 - (p_{\mu} q_{\nu} + p_{\nu} q_{\mu}) (p \cdot q)}{q^2} W_{2}^{ij}(q^2, pq) + \cdots$$

(A.1)

($p$: arbitrary one-particle state; $z = x - y$).

$$W_{\mu \nu}^{5ij}(q) = \frac{1}{4\pi} \int d^4 z e^{-iq \cdot z} <p | \left[ \bar{F}_{i\mu}^5(x), F_{j\nu}(y) \right] | p >$$

$$= -\frac{i}{2} \varepsilon_{\mu \nu \alpha \beta} p_{\alpha} q_{\beta} W_{3}^{5ij}(q^2, p \cdot q) + \cdots$$

(A.2)
where \( \cdots \) denotes terms which destroy the conservation. In the Bjorken limit, we obtain:

\[
\lim_{b_j} W_{1}^{ij} = F_1^{ij}(\xi), \quad \lim_{b_j} (-p \cdot q) W_{2}^{ij} = F_2^{ij}(\xi),
\]

\[
\lim_{b_j} (-p \cdot q) W_{3}^{5ij} \left( q^2, q \cdot p \right) = F_3^{5ij}(\xi).
\]

We assume \( \sigma_L \to 0 \) and get:

\[
W_{ij}^{\mu \nu} \to \frac{(p_\mu q_\nu + p_\nu q_\mu)(p \cdot q) - \delta_{\mu \nu}(p \cdot q)^2 - p_\mu p_\nu q^2}{q^2(q \cdot p)} F_2^{ij}(\xi)
\]

\[
\to \frac{(p_\mu q_\nu + p_\nu q_\mu) - \delta_{\mu \nu}(p \cdot q) + 2p_\mu p_\nu \xi}{q^2} F_2^{ij}(\xi)
\]

\[
\to \frac{s_{\mu \nu \rho \sigma} p_\rho (q_\sigma + \xi p_\sigma) F_2^{ij}(\xi)}{2(q \cdot p)} \xi . \quad (A.3)
\]

Similarly, we find

\[
W_{5ij}^{\mu \nu} \to \frac{1}{2(q \cdot p)} \varepsilon_{\mu \nu \sigma \beta} p_\sigma q_\beta F_3^{5ij}(\xi) . \quad (A.4)
\]

We use the formula of Section V in order to relate \( F_3^{ij} \) and \( F_3^{5ij} \) to the bilocal operators appearing there. We find:

\[
W_{\mu \nu}^{ij}(q) \to \frac{s_{\mu \nu \rho \sigma}}{16\pi^2} \int e^{iq \cdot z} d^4z \partial_\rho \left( \varepsilon(z_0) \delta \left( z^2 \right) \right)
\]

\[
< p | i f_{ijk} (\mathcal{F}_{k\sigma}(x, y) + \mathcal{F}_{k\sigma}(y, x)) + d_{ijk} (\mathcal{F}_{k\sigma}(x, y) - \mathcal{F}_{k\sigma}(y, x)) \ | p > .
\]

and

\[
W_{\mu \nu}^{5ij}(q) \to \frac{i\varepsilon_{\mu \nu \rho \sigma}}{16\pi^3} \int e^{-iq \cdot z} d^4z \partial_\rho \left( \varepsilon(z_0) \delta \left( z^2 \right) \right)
\]

\[
< p | i f_{ijk} (\mathcal{F}_{k\sigma}(y, x) - \mathcal{F}_{k\sigma}(x, y)) - d_{ijk} (\mathcal{F}_{k\sigma}(x, y) + \mathcal{F}_{k\sigma}(y, x)) \ | p > .
\]

We define:

\[
< p | \mathcal{F}_{k\rho}(y, x) - \mathcal{F}_{k\rho}(x, y) \ | p > \equiv +2\tilde{A}_k^k(p \cdot z)p_\rho + \text{trace terms}, \quad (A.6)
\]

\[
< p | \mathcal{F}_{k\rho}(y, x) + \mathcal{F}_{k\rho}(x, y) \ | p > \equiv +2\tilde{S}_k^k(p \cdot z)p_\rho + \text{trace terms}
\]
where \( z = x - y \) is light–like,
\[
\begin{align*}
\tilde{A}^k(p, z) &\equiv \int e^{-i\xi(p \cdot z)} A^k(\xi) d\xi, \\
\tilde{S}^k(p \cdot z) &\equiv \int e^{-i\xi(p \cdot z)} S^k(\xi) d\xi. 
\end{align*}
\] (A.7)

Inserting (A.9) into (A.8) and (A.7), we obtain
\[
W_{\mu\nu}^{ij}(q) \rightarrow +\frac{s_{\mu\nu\rho\sigma} p_{\rho}}{8\pi^2} \int d^4 z e^{-i q \cdot z} \partial_{\rho} \left( \varepsilon(z_0) \delta(z^2) \right) \left\{ if_{ijk} S^k(p \cdot z) + d_{ijk} \tilde{A}^k(p \cdot z) \right\}, \]
\[
W_{\mu\nu}^{5i,j}(q) \rightarrow +\frac{i\varepsilon_{\mu\nu\rho\sigma}}{8\pi^2} \int d^4 z e^{-i q \cdot z} \partial_{\rho} \left( \varepsilon(z_0) \delta(z^2) \right) \left\{ if_{ijk} \tilde{A}^k(p \cdot z) - d_{ijk} S^k(p \cdot z) \right\},
\]
and get further, using (A.9),
\[
W_{\mu\nu}^{ij} \rightarrow +\frac{s_{\mu\nu\rho\sigma} p_{\rho}}{8\pi^2} \int d\alpha \int d^4 z e^{-i (q + \alpha \cdot p) \cdot z} \partial_{\rho} \left( \varepsilon(z_0) \delta(z^2) \right) \left\{ if_{ijk} S^k(a) + d_{ijk} A^k(a) \right\} 
\]
\[
= -\frac{i s_{\mu\nu\rho\sigma} p_{\rho} q_{\sigma}}{8\pi^2} \int d\alpha \int d^4 z (q + \alpha \cdot p)_\rho e^{-i (q + \alpha \cdot p) \cdot z} \varepsilon(z_0) \delta(z^2) \left\{ if_{ijk} S^k(a) + d_{ijk} A^k(a) \right\},
\]
\[
\xi = -\frac{q^2}{2(q \cdot p)}. \tag{A.8}
\]

Similarly, we obtain
\[
W_{\mu\nu}^{5i,j}(q) \rightarrow +\frac{i\varepsilon_{\mu\nu\rho\sigma} p_{\rho} q_{\sigma}}{4(q \cdot p)} \left\{ if_{ijk} A^k(\xi) - d_{ijk} S^k(\xi) \right\}. \tag{A.9}
\]

We compare (A.12) with (A.6) and find
\[
F_2^{ij}(\xi) = +\frac{1}{2} \xi \left\{ if_{ijk} S^k(\xi) + d_{ijk} A^k(\xi) \right\} \tag{A.10}
\]
and also (13) with (7)

\[ F_3^{5ij}(\xi) = + \frac{1}{2} \left\{ i f_{ijk} A^k(\xi) - d_{ijk} S^k(\xi) \right\}. \]  

(A.11)

Further, we should like to demonstrate how one can compute the functions \( S^k(\xi) \) and \( A^k(\xi) \) in terms of the expectation values of the local operators appearing in the Taylor expansion of the bilocal operators.

Using the definitions (A.8) and neglecting internal indices, we get:

\[ F_\rho(y,x) + F_\rho(x,y) \sim i \bar{q}(y) \gamma_\rho q(x) + i \bar{q}(x) \gamma_\rho q(y) \]

\[ \sim 2 i \bar{q}(y) \gamma_\rho q(x) - z^\alpha i (\bar{q}(x) \gamma_\rho \partial_\alpha q(x) + \partial_\alpha \bar{q}(x) \gamma_\rho (x)) \]

\[ + \frac{i}{2!} z^\alpha z^\beta (\bar{q}(x) \gamma_\rho \partial_\alpha \partial_\beta q(x) + \partial_\alpha \partial_\beta \bar{q}(x) \gamma_\rho q(x)) + \cdots \]

We define the numbers \( s_i \):

\[ < p |'' i \bar{q}(x) \gamma_\rho q(x)'' | p >= s_1 p_\rho, \]

\[ \frac{1}{2} < p |'' \bar{q}(x) \gamma_\rho \partial_\alpha \partial_\beta q(x) + \partial_\alpha \partial_\beta \bar{q}(x) \gamma_\rho q(x)'' | p >= s_3 p_\rho p_\alpha p_\beta + \text{trace terms, etc.} \]  

(A.12)

and find

\[ \tilde{S}^k(p \cdot z) = + \left( s_1^k + \frac{s_3^k}{2!} (p \cdot z)^2 + \cdots \right). \]  

(A.13)

Similarly we define the dimensionless number \( \alpha_i \):

\[ \frac{1}{2} < p |'' \bar{q}(x) \gamma_\rho \partial_\alpha q(x) - \partial_\alpha \bar{q}(x) \gamma_\rho q(x)'' | p >= a_2 p_\rho p_\alpha \cdots \]

and find

\[ \tilde{A}^k(p \cdot z) = + i \left( a_2^k (p \cdot z) + \frac{1}{3!} a_4^k (p \cdot z)^3 + \cdots \right). \]  

(A.14)

If we carry out the Fourier transform, we obtain, restoring \( SU_3 \) indices,

\[ S^k(\xi) = s_1^k(\xi) - \frac{1}{2!} s_3^k \delta''(\xi) + \cdots, \]  

(A.15)

\[ A^k(\xi) = \left( a_2^k \delta''(\xi) - \frac{1}{3!} a_4^k \delta'''(\xi) + \cdots \right). \]  

(A.16)
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