Microscopic structure of electromagnetic whistler wave damping by kinetic mechanisms in hot magnetized Vlasov plasmas

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Abstract

Electromagnetic transverse perturbations propagating parallel to the external magnetic field in a warm electron plasma, specifically the warm electron whistler-mode waves, are simulated in Maxwellian as well as \( \kappa \) distributed (with energetic tail) electrons. The Vlasov-Maxwell phase-space continuum simulations are applied to the stable and unstable (i.e. isotropic and anisotropic) VDFs. The variation of real frequency from both numerical solution of dispersion relation and simulations show limited sensitivity to electron temperature in low wave-number regime as compared to high wave number regime, however the opposite holds for the imaginary frequency or the decay rate. The analytically predicted reduction in the decay rate of the whistler-mode with increasing electron temperature is recovered by the Vlasov-Maxwell simulations. The phase-space portraits of these cases show that after the linear damping phase of the evolution, the particles are trapped in the wave magnetic field leading to the wave amplitudes oscillating about a mean value which follow the theoretical analysis.

Palmadesso and Schmidt (1971) *Physics of Fluids* 14, 1411.

1. Introduction

The low frequency whistler-mode turbulence plays a key role in space plasmas and is believed to be operational also in the collisionless scattering of energetic electrons in magnetic confinement fusion-plasmas [2–6]. In case of coronal heating problem it is understood that the waves transport energy from sun surface to the corona while in the solar wind the turbulence operates by cascading the energy to smaller length scales. At the lower length scales the process is terminated by the collisionless wave damping, or the Landau damping [7–10] mechanism, the damping of whistler-mode remains an important kinetic process to be simulated in its full nonlinear form. Palmadesso and Schmidt [11] studied the collisionless damping of whistler-mode by means of nonlinear analytics and recovered residual amplitude oscillations of nonlinearly decaying whistler-mode. Husidic et al [12] and Summers et al [13] have solved the linear dispersion relation for regularized kappa VDF along with standard kappa and Maxwellian VDF numerically. Shaaban et al [14] treated the unstable whistler-mode for beam plasma interactions and solved the linear dispersion relation for beam plasma system. Mace et al [15] have analyzed the linear dispersion relation numerically for bikappa VDF to characterize the unstable whistler-mode. Schreiner et al [3] have performed the particle in cell simulations and discussed the cyclotron resonance of ions with different \( k \) values to characterize the temperature effects in the dispersion relation for the Maxwellian VDFs, where the high wave-vector regions could not be covered. Also, the particle in cell simulations in the low electron temperature regimes present significant challenges [16].

Whistler wave is one of the fundamental plasma waves which is heavily studied from space to laboratory plasma. In the Earth radiation belt, in solar wind, the observed whistler-mode are mostly have the frequency less than the electron cyclotron frequency, i.e. \( \omega \ll \omega_{ce} \), also the electron cyclotron frequency is less than the electron plasma oscillation frequency. In case of laboratory experiments, e.g. in the device like LVPD [17] the whistler-mode wave frequency is less than the electron cyclotron frequency. As most of the cases in space and linear...
devices have $\omega_{ce} < \omega_{pe}$ the study in this paper is done for the same conditions. In the papers [18, 19] the kinetic instability of whistler-mode is studied by means of Vlasov simulations in the range of whistler-mode frequency $\omega_{ce} < \omega_{pe}$ but for the unstable VDFs which is based on anisotropy and beam plasma systems. The property of whistler-mode or how its evolution behaves from low to high electron temperature in plasmas is analysed in this paper systematically. The most of the near-earth space plasmas and linear device plasmas whistler-mode operate in overdense regime which remains one of the objectives for this study and the parameter range adopted for paper systematically. The most of the near-earth space plasmas and linear device plasmas whistler-mode operate where

$$\text{directions with } \mathbf{e}.$$  

An externally applied constant magnetic field is well represented by the solutions of collisionless, fully nonlinear Vlasov equation for arbitrary species $s$,

$$\frac{\partial f_s}{\partial t} + v \frac{\partial f_s}{\partial x} + \frac{q_s}{m_s}(E + \frac{v \times B}{c}) \frac{\partial f_s}{\partial v} = 0.$$  

(1)

The evolution of electric field $E$ and magnetic field $B$ for the electromagnetic processes follows the Maxwell’s equations,

$$\nabla \times B = \frac{1}{c} \frac{\partial E}{\partial t} + \frac{4\pi}{c} \sum_s \int_{-\infty}^{\infty} dv \, q_s v f_s,$$

$$\nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t},$$

$$\nabla \cdot B = 0.$$  

(2)

(3)

(4)

An externally applied constant magnetic field $B_0$ is used, electrons are mobile and infinitely massive ions maintains the quasineutrality of the system. The subscript for species $s$ therefore only representing the electrons (and henceforth omitted), which are the only mobile species and contributing to the perturbation. The full nonlinear kinetic model (1–4) is implemented for the case of waves propagating parallel to an applied magnetic field $(B_0||k)$. The simulations presented in this analysis is by evolving the magnetized plasma VDF according to the dynamics of the phase-fluid flow [20], which is governed by the collisionless Vlasov equation (1), and associated Maxwell’s equations (2)–(4). The phase space simulations are performed using an advanced flux balance technique [21, 22] to simulate the electromagnetic plasma modes in a large range of magnetization in 1X-3Y dimension. The results of simulations are characterized first against the analytical dispersion relation and cyclotron damping descriptions [2, 20, 23, 24].

In order to simulate the waves propagating parallel to an ambient magnetic field $B_0$, in a wide range of frequency $\omega$ and wave vector $k$, a setup is assumed where both the $B_0$ and $k$ are aligned to $z$-axis and the periodic boundary condition is used at both the boundaries of the one-dimensional simulation zone located between $z = 0$ and $L$. The setup therefore assumes symmetry along both $\hat{v}_x$ and $\hat{v}_y$ directions with finite spread of electron velocities along these dimensions, besides the direction $\hat{z}$. The fully resolved VDF for the electron species is considered to be a Maxwellian and Kappa as follows,

$$f_M(z, v) = \frac{1}{(2\pi)^{3/2} v_{th}^3} \exp \left(-\frac{v^2}{2 v_{th}^2}\right),$$

$$f_K(z, v) = \frac{1}{\pi^{3/2} \theta^3} \Gamma(\kappa + 1) \left(\frac{1 + \frac{v^2}{\kappa \theta^2}}{\kappa^{3/2}}\right)^{-(\kappa + 1)}$$

(5)

(6)

where $v_{th} = \left(T_e/m_e\right)^{1/2}$ is electron thermal velocity, $T_e$ is electron temperature in energy units and $m_e$ is electron mass and $\theta = \left[2^{(\kappa - 1/2)}/\kappa\right]^{1/2} v$ is the generalised temperature, $\kappa$ is a parameter the value of $\kappa$ should be $>1.5$ The initial perturbation [25, 26] to the VDF is given in such a way that it can generate coherent whistler-mode [27, 28]. Initially the wavevector $k$ is chosen which has to be simulated for the periodic box along $z$ axis along which the external magnetic field is applied. The transverse component of the electric and magnetic field of the wave, varies along the $z$ direction. The initial excitation given to the system to generate the exact whistler-mode in the simulation is given by,

2. Model equations

The magnetized electromagnetic plasma mode simulated in this paper follows pure kinetic formulation and are well represented by the solutions of collisionless, fully nonlinear Vlasov equation for arbitrary species $s$,
\[
\delta v_x(z) = -\frac{|q| \epsilon e \sin(kz)}{|q|B_0 - cm_e \omega} \quad \delta v_y(z) = -\frac{|q| \epsilon e \cos(kz)}{|q|B_0 - cm_e \omega}
\] (7)

\[
\delta B_x(z) = \frac{ck}{\omega} \epsilon \sin(kz); \quad \delta B_y(z) = \frac{ck}{\omega} \epsilon \cos(kz)
\] (8)

\[
\delta E_x(z) = \epsilon \cos(kz); \quad \delta E_y(z) = -\epsilon \sin(kz)
\] (9)

\[
\delta v_z(z) = \delta B_z(z) = \delta E_z(z) = 0
\] (10)

where \( \epsilon \) is the initial perturbation amplitude, \( q, B_0 \) and \( c \) are the electronic charge, external magnetic field and light velocity in free space respectively. The \( \omega \) in the above equations corresponds to the particular \( k \) to be excited in the simulation.

The initial perturbation in velocity distribution generate the conduction current in the system. The conduction current is connected to the Amperes law, which is connected to the wave magnetic field. Again the wave magnetic field is connected to the wave electric field by the Faradays law. This is how the self consistence system is generated. But if one looks at the linear dispersion plot, where for a given \( k \) there is other frequency apart from whistler-mode wave. The initial perturbation is calculated with proper perturbation amplitude can eliminate the other frequency and can excite the coherent whistler-mode wave initially, maintaining the quasineutrality by, \( n_e = n_i \).

### 2.1. Linear dispersion relations

By using the kinetic dispersion relation for the electromagnetic waves [27, 29–31] given by,

\[
\omega^2 + \frac{8\pi^2 q_e^2}{m} \int_{-\infty}^{\infty} \int_{0}^{\infty} \left( \frac{\omega - kv_z}{\omega - kv_z \pm \omega_e} \right) v_z^2 dv_z \, dv_r = 0
\] (11)

Using (5) and (6) in (11) one can get the dispersion relation,

\[
\omega^2 + \omega^2 \frac{\varepsilon_e}{\sqrt{2} kv_{th}} Z(\zeta) - \epsilon^2 k^2 = 0.
\] (12)

\[
\omega^2 + \omega^2 \frac{\varepsilon_e}{\sqrt{2} k\theta} U_e(\zeta_e) - \epsilon^2 k^2 = 0.
\] (13)

where \( Z(\zeta) \) and \( U_e(\zeta_e) \) are the plasma dispersion function (PDF) and modified PDF defined as,

\[
Z(\zeta) = \int_{-\infty}^{\infty} f(t) dt
\] (14)

\[
U_e = i \left( \frac{\kappa - 0.5}{\kappa^3 / 2} \right) F_1[1, 2\kappa; \kappa + 1; \frac{1}{2} \left( 1 - \frac{\zeta}{iv_e} \right)]
\] (15)

where \( F_1 \) is called Gauss Hypergeometric function. With the argument,

\[
\zeta \equiv \frac{\omega - \omega_e}{\sqrt{2} kv_{th} \epsilon},
\]

\[
\zeta_e = \frac{\omega - \omega_e}{\sqrt{2} k\theta}
\] (17)

respectively. In general the generalised plasma dispersion function (GPDF) is defined as [15, 26, 32–35],

\[
Z_{\theta}(\zeta_{\theta}) = \int_{-\infty}^{\infty} f(t) dt
\] (18)

Where \( Z_{\theta}(\zeta_{\theta}) \) is the GPDF and it’s argument is the generalised zeta function for Kappa and Maxwellian cases, defined as,

\[
\zeta_{\theta} = \frac{\omega - \omega_e}{\sqrt{2} kv_{th, \theta}}
\] (19)

for Kappa VDFs, \( v_{th, \theta} = \theta \) and for Maxwellian VDFs, \( v_{th, \theta} = v_{th} \). Now, if one put equation (5) and (6) in (18) will get the PDF and modified PDF respectively.
Under the cold plasma approximation i.e. $v_{th} \ll v_p$ equation (12) takes the form as,

$$\omega^2 + \frac{\omega_{pe}^2}{k^2 - \omega^2 - c^2k^2} = 0,$$

which is well known cold plasma whistler-mode dispersion relation, is independent of temperature.

3. Characterization of whistler-mode in Maxwellian and Kappa VDFs

The Vlasov simulations adopted in the present study present an effective alternative as a simulation procedure that covers low to high $k$ regimes over a large electron temperature range, including very small temperature cases. The grid based Vlasov-Maxwell simulations are performed both for Maxwellian and kappa VDF for a wide variety of parameters (for example different $k$, kappa, different magnetic field and temperatures) also the linear dispersion relation is solved numerically to show good agreement between the analytical and simulation results. The simulations recover the finite temperature effect on both the real and imaginary frequency both numerically and from simulations. The simulated solutions are well resolved over both temperature and wave-vector ranges of relevance. Among significant results, it is shown that for high $\kappa$ values the change in maximum damping of whistler-mode is very low and for low $\kappa$ values the change in maximum damping increases. Overall the maximum damping increases with the decrease of the intensity of external magnetic field. In a recent work [18] the authors have also presented the detailed Vlasov simulations of system having separate core and beam electrons and the resulting whistler-mode instability, the present work however focuses on the whistler-mode damping in the warm electron plasmas.

3.1. Linear analysis

The result of linear analysis are presented here with their comparison with the results of simulations which are described in section 3.2 figures 1 and 2 are the numerical solutions of dispersion relation for low and high anisotropy respectively. It is clear from the the figure itself that for low anisotropy the maximum growth rate increases with $\kappa$ and decreases for higher anisotropy. The figure 3 is for higher anisotropy and the maximum growth rate with the variation $\kappa$ shows that the maximum growth rate decreases with increase of $\kappa$. The anisotropy of the system is defined as both for Maxwellian and Kappa is,

$$A = \frac{v_{th,\perp}^2}{v_{th,\parallel}^2} - 1 = \frac{\theta_{th,\perp}^2}{\theta_{th,\parallel}^2} - 1.$$

(21)

Figure 4 shows the maximum growth rate variation with $\kappa^{-1}$ values. It shows that with increase of external magnetic field maximum growth rate decreases. The maximum growth rate also decreases with decrease of $\kappa$ value.

The figure 5 is the variation of growth rate with $\zeta$. This shows that there is some lower and upper bound of the resonant factor which increases with increase of perpendicular temperature (red to blue line in Fig.) and decreases with increase of the external magnetic field. Figure 6 shows the temperature effect on the linear whistler-mode dispersion (solid lines) and its comparison with the simulation data (markers) at certain selected...
Figure 2. Numerical solution of dispersion relation with $\omega_{ce} = 0.02 \omega_{pe}$ and $v_{th} = 0.02 c$ and $v_{th} = 0.04 c$.

Figure 3. The maximum of the growth rate is calculated from the figure 2 and plotted with the variation of $\kappa$ for two different anisotropy.

Figure 4. The maximum of the growth rate is calculated from the figure 2 and plotted with the variation of $\kappa$ for two different external magnetic field.
$k$ values which show resonable agreement. In the temperature range covered in this comparison is from 0.5 eV (cold) to 200 eV (hot). It is clear from this characerization that for the high temperature case there is a finite reduction in real frequency, further discussed in the section 3.2 where simulations are described. The profiles for smaller values of $v_{th}$ in figure 6 have negligible resonant damping as for these cases of rather large $ck/\omega_{pe} = 1.0$ the resonant electron population drops to negligible values for these cases. However, for $ck/\omega_{pe} = 5.0$ the electrons are strongly resonant so the resonant damping is present for higher $k$ value (figure 6) at higher $v_{th}$ or temperatures.
3.2. Vlasov simulation

An advanced Vlasov simulation set up [18, 19], with phase-space grid size of dimension 64 points along x and $64 \times 64 \times 64$ points along $v_x$, $v_y$, and $v_z$, respectively, is implemented in the following sets of simulations. The thermal velocity range is used such that the nonrelativistic limit is suitably applicable.

In the first set of simulations two different values of magnetic field corresponding to $\omega_{ce} = 0.04$ and $0.1 \omega_{pe}$ are used with the wave vector value $c k / \omega_{pe} = 1$. A finite anisotropy $A = 3$ is used showing that growing solutions are duly recovered for anisotropic electron distribution. The variation of the amplitude of growing whistler-mode with variation in $\kappa$ values is shown in figure 7(a) and the corresponding saturated amplitudes are presented in figure 7(b). The saturation amplitude is found to remain constant with $\kappa$ parameter. Also, the saturation level decreases with increase of the external magnetic field.

3.2.1. Recovery of temperature dependent frequency drop

The whistler-mode undergo a damping in the limit of zero anisotropy (isotropic distribution). The corresponding simulations results for the case of Maxwellian distribution function are presented in figures 8 and 9 with two different temperature cases with $v_{th} - 0.001c$ and $0.02c$, i.e. $T_e = 0.5$ eV (cold) to 200 eV (hot). The reduction in real frequency with temperature in simulated distribution is recovered in its plots in the perpendicular velocity plane. In figures 8 and 9 the contours of distribution function perturbation ($\Delta f = f - f_0$) obtained from the simulations with and without whistler-mode perturbation are plotted in this plane for cold and warm electron cases respectively. The column 1-3 correspond to three different parallel velocity values. For warm plasma (figure 9), the rotation of the $B_1$ (red) and $v$ (black) vectors in time (top to bottom) is smaller, indicating a reduction in the wave frequency in agreement with the analytical result. In addition, the contours of the $\Delta f$ in high temperature case show finite distortion at phase velocity and also at resonant velocity in comparison to the cold electron case. In the cold electron case at both these velocities the contours of the
perturbation undergo a regular rotation corresponding to the rotation of the polarization in a right-handed circularly polarized wave.

3.2.2. Recovery of residual amplitude oscillations

We now present the results where a residual nonlinear oscillations of the amplitude of a damping whistler-mode wave are recovered \[1\] rather than a whistler-mode wave linearly damping to asymptotically small amplitude by the linear Landau damping. The full nonlinear evolution of magnetic field is simulated for a given wave vector, \(ck/\omega_{pe}=5.0\) and external magnetic field, \(\omega_{ce}=0.05 \omega_{pe}\). The simulations are performed for temperatures from low to high values for isotropic Maxwellian (\(\kappa \to \infty\)) and kappa distribution (\(\kappa = 2\)) and the long time evolution of the wave amplitude is plotted for them in figure 10 and figure 11, respectively. From the existing theoretical analysis of the oscillations \[36-38\] the trapping time is calculated using the definition, \(T \approx (eB_0 k_{\parallel} / mc)^{-1/2}\) \[1\]. The trapping time calculated from simulation results is shown in figure 10 for different temperatures. The simulation results are calculated from the time evolution of the wave magnetic field, where the periodic oscillations give the trapping time, calculated from the peak to peak value of the time evolved wave magnetic field. The simulation done using isotropic kappa distribution as shown in figure 11 show the similar evolution. The comparison of analytical and simulation trapping time is listed in the table 1. It is observed that order of the theoretical and simulated trapping is comparable. The agreement is good in high temperature cases however finite deviation is present at low temperatures.

Figure 12 is the evolution of the strength of the wave magnetic field compared with the Bessel function \(J_0(x)\) for isotropic Maxwellian VDFs showing a qualitative similarity with the simulation results. One can find from the theoretical expression of the trapping time, for example that obtained by Palmadesso and Schmidt \[1\], that as
the electron temperature increases the trapping time tends to reduce. This results in the frequency of the oscillations increasing with temperature as in Figure 12. According to the linear theory if the unperturbed distribution is isotropic, the wave will damp exponentially. The time after which the wave reaches to the half

**Figure 9.** Evolution of the electrons VDF (normalised to the peak value) perturbation in the transverse velocity-space plane $v_x,v_y$ for the warm electrons case, with temperature 200 eV. Frames from top to bottom correspond to equal intervals over one whistler-mode cycle from $t = 0$, while those from left to right are for $v_z = v_{\text{mz}}, 0$, and $v_{\text{phase}}$, respectively. Red and black arrows indicate, $B_1$ and $\langle v \rangle_1$, respectively.

**Figure 10.** The variation of the wave magnetic field evolution with temperature for isotropic Maxwellian VDF is shown in the figure.
period, the particles which were initially absorbing the energy are emitting and the wave amplitude start to oscillate [1].

Figure 13 and the figure 14 are the time evolution of $v_x - v_z$ plane analysis of the Maxwellian VDFs for temperatures 2 eV and 200 eV, respectively, which shows that the perpendicular heating is happening for both the cases in linear limit and it remains almost constant for further time evolution because of the particles are trapped. The heating is more in higher temperatures as compared to the low temperature, where it can be seen that the perpendicular broadening in the distribution is larger for the high temperature case compare to the low temperature case.

Table 1. Parameters used in the presented simulation cases. Five sets corresponding to different VDF types are separated by lines.

| Cases | $T_e$ | Theoretical trapping time | Simulated trapping time |
|-------|-------|---------------------------|-------------------------|
| 1     | 2 eV  | 71                        | —                       |
| 2     | 30 eV | 32                        | —                       |
| 3     | 112.5 eV | 26                  | $45 \omega_{pe}^{-1}$  |
| 4     | 200 eV | 22                        | $32 \omega_{pe}^{-1}$  |
| 5     | 1250 eV | 14                      | $13 \omega_{pe}^{-1}$  |

Figure 11. The variation of the wave magnetic field evolution with temperature for isotropic kappa VDF is shown in the figure.

Figure 12. The scaling of the magnetic field evolution with the Bessel function is shown in the figure.
4. Summary and conclusions

The linear and nonlinear analysis of whistler-mode waves is performed using fully nonlinear Vlasov-Maxwell kinetic simulation in 4-dimensional phase-space configuration (1X-3V). In the linear limit the reduction of real frequency, of low frequency transverse perturbation propagating parallel to the external magnetic field in a magnetized warm plasma is recovered in the cases of both Maxwellian and Kappa VDFs. The impact of finite electron temperature on the wave frequency (both real and imaginary) is described analytically and recovered in the kinetic simulations. The variation of linear growth rate with $\kappa$ is shown and it is observed that for higher $\kappa$ the damping is very low and it is independent of external magnetic field. The anisotropic Maxwellian and Kappa VDFs are also analysed and the systematic study of nonlinear (finite amplitude) whistler-mode propagating in isotropic VDFs for Maxwellian and Kappa is carried out. The trapping time is calculated from the magnetic field evolution and it is observed that the simulation results closely agree with the analytical results for high electron temperatures apart from recovering the standard low temperature results of damping and growth. The corresponding phase-space evolutions are also shown at resonant and phase velocities with recovery of reduction in the frequency of the rotation of the polarization. Also the decay of the wave magnetic field with time for different temperature leads to development of nonlinear residual whistler-mode amplitude oscillations. These results are quantitatively compared with the analytical results of the oscillation time obtained by Palmadesso and Schmidt\cite{1, 11}.

**Figure 13.** The time evolution of the distribution function for isotropic maxwellian in $V_x - V_z$ plane is shown with temperature 2 eV and the external magnetic field is $\omega_{ce} = 0.05 \omega_{pe}$.

**Figure 14.** The time evolution of the distribution function for isotropic Maxwellian in $V_x - V_z$ plane is shown with temperature 200 eV and the external magnetic field is $\omega_{ce} = 0.05 \omega_{pe}$. 

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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References

[1] Palmadesso P and Schmidt G 1971 The Physics of Fluids 14 1411
[2] Chen L, Thorne R M, Shpritz V and Ni B 2013 Journal of Geophysical Research: Space Physics 118 2185
[3] Schreiner C, Kilian P and Spanier F 2017 Communications in Computational Physics 21 947
[4] Schreiner C 2016 Numerical modelling of the microphysical foundation of astrophysical particle acceleration Ph.D. Thesis North-West University (South Africa), Potchefstroom Campus
[5] Shuster J et al 2021 Nat. Phys. 17 1056
[6] Fülöp T, Smith H and P outline G 2009 Phys. Plasmas 16 022502
[7] Landau L D 1944 C. R. Acad. Sci. U. R. S. S. 44 311
[8] Landau L D 1946 J. Phys. U.S.S.R. 10 23
[9] Stenzel R 1999 Journal of Geophysical Research: Space Physics 104 14379
[10] Stenzel R 2016 Advances in Physics: X 1 687
[11] Palmadesso P and Schmidt G 1972 The Physics of Fluids 15 485
[12] Husidic E, Lazar M, Fichtner H, Scherer K and Astfalk P 2020 Phys. Plasmas 27 042110
[13] Summers D and Tang R 2021 Journal of Geophysical Research: Space Physics 126 e2020JA028276
[14] Shaaban S, Lazar M, Yoon P and Poefts S 2018 Phys. Plasmas 25 082105
[15] Mace R and Sydora R 2010 Journal of Geophysical Research: Space Physics 115 A07206
[16] Hughes R S, Wang J, Deyuk V K and Gary S P 2016 Phys. Plasmas 23 042106
[17] Sanyasi A, Srivastav L, Srivastava P, Sugandhi R and Sharma D 2021 Plasma Phys. Controlled Fusion 63 085008
[18] Paul A and Sharma D 2023 Phys. Plasmas 30 102104
[19] Paul A and Sharma D 2024 Phys. Plasmas 31 032117
[20] Kral N A and Trivelpiece A W 1986 Principles of Plasma Physics (San Francisco Press Inc.)
[21] Fijalik E 1999 Comput. Phys. Commun. 116 329
[22] Boris J F, Landsberg A M, Oran E S and Gardner J H 1993 LCPFCT-A flux-corrected transport algorithm for solving generalized continuity equations Tech. Rep. Naval Research Lab Washington DC
[23] Gary S P 1993 Theory of Space Plasma Microinstabilities (Cambridge University Press)
[24] Gurnett D A and Bhattacharjee A 2005 Introduction to Plasma Physics: With Space and Laboratory Applications (Cambridge University Press)
[25] Schreiner C and Spanier F 2014 Comput. Phys. Commun. 185 1981
[26] Sonnerup B O and Su S-Y 1967 The Physics of Fluids 10 462
[27] Chen F F 2012 Introduction to Plasma Physics (Springer Science & Business Media)
[28] Hellwell R A 1965 Whistlers and Related Ionospheric Phenomena (Stanford Univ. Press)
[29] Bittencourt J A 2013 Fundamentals of Plasma Physics (Springer Science & Business Media)
[30] Stix T H 1992 Waves in Plasmas (Springer Science & Business Media)
[31] Swanson D G 2008 Plasma Kinetic Theory (Crc Press)
[32] Abramowitz M and Stegun I A 1964 Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, IX GPO printing, X gpo printing ed. (Dover)
[33] Fried B D and Conte S D 2015 The Plasma Dispersion Function: The Hilbert Transform of the Gaussian (Academic Press)
[34] Summers D and Thorne R M 1991 Physics of Fluids B: Plasma Physics 3 1835
[35] Hellberg M and Mace R 2002 Phys. Plasmas 9 1495
[36] Ossakow S, Haber I and Sudan R 1972 The Physics of Fluids 15 935
[37] Karpman V, Istomin J N and Shklyar D 1974 Plasma Phys. 16 685
[38] Latomirski R F and Sudan R 1966 Phys. Rev. 147 156