Properties of $J^P = 1/2^+$ baryon octets at low energy

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The statistical model in combination with the detailed balance principle is able to phenomenologically calculate and analyze spin- and flavor-dependent properties like magnetic moments (with effective masses, with effective charge, or with both effective mass and effective charge), quark spin polarization and distribution, the strangeness suppression factor, and $d-u$ asymmetry incorporating the strange sea. The $s$ in the sea is said to be generated via the basic quark mechanism but suppressed by the strange quark mass factor $m_s > m_u, d$. The magnetic moments of the octet baryons are analyzed within the statistical model, by putting emphasis on the SU(3) symmetry-breaking effects generated by the mass difference between the strange and non-strange quarks. The work presented here assumes hadrons with a sea having an admixture of quark gluon Fock states. The results obtained have been compared with theoretical models and experimental data.

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1. Introduction

Understanding baryon spectroscopy and the internal structure of baryons is one of the main problems in hadron physics. Investigation of the properties of hadrons will provide a clear picture for unraveling the dynamics of their internal structure. A new state of matter called “pentaquarks” was reported in the LHCb experiment at the Large Hadron Collider, bringing a glorious triumph in the study of baryon spectroscopy, and “studying its properties may allow us to understand better how ordinary matter, the protons and neutrons from which we are all made, is constituted.” A pentaquark can be considered as a bound state of a two-particle system formed by a baryon and a meson. The baryonic properties in which we are interested are considered to be a set of valence ($qqq$) and a sea of $qar{q}$ pairs or gluons. It is assumed that the properties either for pentaquarks (which are recent) or baryons (valence) with sea can be correlated strongly for their low-energy properties.

The Brodsky, Hoyer, Peterson, and Sakai (BHPS) model [1,2] suggested that there are two types of quark contribution to the nucleonic sea: intrinsic and extrinsic. The intrinsic sea (valence-like) originates through non-perturbative fluctuations of the nucleonic state to a five-quark state ($uudqq$), and the extrinsic sea (sea-like) is produced perturbatively in the process of gluons splitting into $qar{q}$ pairs [3–5]. There is a wide range of results available for intrinsic light quark distributions from the meson cloud model [6–9], the chiral quark model [10,11], etc. Parton distribution functions (PDFs) has also been successful in extracting the probabilities of the intrinsic charm quark in the nucleon [12,13]. The QCD sum rule approach [14,15] for studying the properties of ground-state hadrons has been very successful. It is based on the fundamental QCD Lagrangian, and all non-perturbative effects are paramerized in terms of quark and gluon condensates. Predictions based on a number of theoretical formalisms have been made to calculate the various properties of octet baryons.
cloudy bag model [16], the approach of chiral perturbation theory [17], the $SU_F(3)$ flavor extension [18], the chiral quark–soliton model [19], the phenomenological quark model [20], the Lorentz-covariant chiral quark model [21], the QCD-based quark model [22], an unquenched quark model [23], a spin–flavor symmetry-based parametrization method of quantum chromodynamics [24,25], a covariant and confining Nambu–Jona-Lasinio model [26], the chiral constituent quark model [27] are just a few of them. Recently, the experimental masses and magnetic moments of the baryon octet were used to constrain the strength of the pion cloud contributions to the octet, and hence the nucleon, form factors at $Q^2 = 0$ [28].

The study of the internal structure of hadrons is still an unresolved issue, despite being studied extensively across the globe in the form of experiments and theoretical developments over the past 50 years. The study of hadrons is difficult for physicists due to the relativistic and non-perturbative nature of their constituents. The presence of quark–gluon interaction in valence quarks implies that $q\bar{q}$ pairs can be produced perturbatively by gluons emitted from valence quarks themselves, which are usually termed the “sea” (see Ref. [29] and references therein). The sea could consist of light quarks as well as heavy ones, and plays an important role in visualizing the hadronic structure. The distributions of $u$ and $d$ in the sea have a large asymmetry according to observations in deep inelastic scattering [30,31] and Drell–Yan experiments [32,33]. Hadrons made of valence quarks and gluonic degrees of freedom in terms of the “sea” have been modeled diversely to interpret their observed properties like magnetic moments, spin distributions, quadrupole moments, etc. This class of models assumes that a nucleon consist of valence quarks surrounded by a “sea” which, in general, contains gluons and virtual quark–antiquark pairs, and is characterized by its total quantum number [34]. The domains of validity and stability of the results obtained can be checked by calculating the maximum number of hadronic parameters or properties within these models. Various nucleonic parameters have been calculated theoretically using the chiral quark soliton model [35–37], the Skyrme model [38,39], sum rules, etc. [40–42]. An attempt by Zhang et al. [43] was made to understand the flavor asymmetry of the proton in the detailed balance model. The idea was that the proton is taken as an ensemble of quark gluon Fock states, and any two nearby Fock states should be balanced with each other under the principle of detailed balance [44,45]:

$$\rho_A R_{A\rightarrow B} = \rho_B R_{B\rightarrow A},$$

where $\rho_A$ and $\rho_B$ are the probabilities of finding the proton in state $A$ or $B$ respectively, and $R_{A\rightarrow B}$ and $R_{B\rightarrow A}$ are the transition probabilities of $A$ to $B$ and $B$ to $A$, respectively. The probabilities of finding every Fock state inside the proton were obtained in the same process. The detailed balance model was used to look into the statistical effects of the nucleon and $\bar{d}–\bar{u}$ asymmetry. The model generated $\bar{d}–\bar{u} = 0.124$, which was in agreement with the predictions of the E866/NuSea result of $0.118 \pm 0.012$.

We have computed the $\bar{d}–\bar{u}$ asymmetry for all other octet baryons using the relation [46]

$$\bar{d} – \bar{u} = \left( \sum_{j=0,k=0,i=0,l=0}^{j=2,k=3} \rho_{i,j,l,k} - \sum_{i=0,k=0,j=0,l=0}^{i=2,k=3} \rho_{i,j,l,k} \right), \quad (1.1)$$

where $i$ is the number of quark–antiquark $u\bar{u}$ pairs, $j$ is the number of quark–antiquark $d\bar{d}$ pairs, $l$ is the number of $s\bar{s}$ pairs, and $k$ is the number of gluons in the sea. The values for flavor asymmetry are shown in Table 1, along with a list of the average numbers of partons for all $J^P = \frac{1}{2}^+$ octet members. The total number of intrinsic partons inside every octet baryon can be calculated as
Table 1. $\bar{d}-\bar{u}$ asymmetry and the average numbers of partons for octet baryons.

| Octets | $\pi$ | $\bar{d}$ | $\bar{d}-\bar{u}$ | Data$^*$ | Avg. no. partons |
|--------|------|-----------|----------------|---------|-----------------|
| p      | 0.33 | 0.46      | 0.134          | 0.124 [39] | 4.84            |
| n      | 0.46 | 0.33      | $-0.134$       | —       | 4.84            |
| $\Lambda^0$ | 0.44 | 0.44      | 0              | 0       | 4.73            |
| $\Sigma^+$ | 0.31 | 0.71      | 0.39           | 0.410 [55] | 4.87            |
| $\Sigma^+$ | 0.44 | 0.44      | 0              | 0       | 4.74            |
| $\Sigma^-$ | 0.71 | 0.31      | $-0.39$        | —       | 4.87            |
| $\Xi^0$  | 0.43 | 0.70      | 0.26           | 0.27 [55] | 4.85            |
| $\Xi^-$  | 0.70 | 0.43      | $-0.26$        | —       | 4.85            |

Later, J. P. Singh et al. [34] constructed Fock states of nucleons having specific color and spin quantum numbers with definite symmetry properties, using statistical ideas. With this approximation, they studied the quarks' contribution to the spin of the nucleons, the ratio of the magnetic moments of the nucleons, their weak decay constant, and the ratio of SU(3) reduced matrix elements for the axial current. In this paper, we have used the concept for other octet particles to reproduce the flavor asymmetry, quark spin polarization and distribution, magnetic moments (with various modifications), and SU(3) symmetry breaking in magnetic moments for octet particles, which may help in unraveling the hadronic structure. We have also computed the strangeness suppression for octet particles in the framework of the statistical model. Because the masses of $u$ and $d$ quarks are fairly small compared with the typical energy scale in deep inelastic scattering, the splitting processes are expected to occur almost equally for these quarks, but a constraint needs to be applied for strange quark–antiquark pairs. The contribution of the $s\bar{s}$ pair is taken into account by applying a constraint resulting from the gluon free energy distribution [45]. The orbital motion of the three valence quarks has not been taken into account; however, some of the previous studies show that the inclusion of orbital motion preserves the success in describing the magnetic moments [47,48], the spin-averaged structure function, and the violation of the Gottfried sum rules [49]. The motivation of this paper is to generalize the conjecture of the detailed balance principle and the statistical model used earlier in the calculation of some low-energy properties of baryons.

2. Preliminaries

The structure of a hadron constitutes two parts: the valence part ($qqq$) and the sea part, which consists of quark–antiquark pairs mutually connected through gluons [50–55]. A $q^3$ state in a baryon is a one-, eight-, or ten-color state, which means that the sea should also be in corresponding states to form a color singlet baryon. The valence part of the hadronic wave function can be written as

$$\Psi = \Phi(|\phi\rangle|\chi\rangle|\psi\rangle)(|\xi\rangle),$$

(2.1)

where $|\phi\rangle$, $|\chi\rangle$, $|\psi\rangle$, and $|\xi\rangle$ denote flavor, spin, color, and space $q^3$ wave functions, and their contribution yields an antisymmetric total wave function. Here, the spatial part ($|\xi\rangle$) is symmetric under the exchange of any two quarks for the hadrons, and therefore the flavor–spin–color part
The total flavor–spin–color wave function of a spin-up baryon octet consisting of three valence
of each Fock state in spin and color space. These multiplicities are expressed in the form
\[ \langle \chi | \psi \rangle \]
where,
\[ N \]
here, \( N \) is the normalization constant. The first three terms in Eq. (2.3) are obtained by combining the
\[ a_8, a_{10}, b_1, b_8, b_{10}, c_8, d_8 \]
in Eq. (2.4) are to be determined statistically from the flavor, spin, and color probabilities for the study of low-energy properties of octet baryons. The wave function in Eq. (2.3) can also be written in the form \( \Phi_{\text{val}} \Phi_{\text{sea}} \). and the unknown parameters \( a_8, a_{10}, b_1, b_8, b_{10}, c_8, d_8 \) by a factor \( \sum n_{\mu \nu} c_{\text{sea}} \) such that the total wave function becomes \( |\Phi_{\frac{1}{2}}\rangle = \sum_{\mu \nu} (n_{\mu \nu} c_{\text{sea}}) \Phi_{\text{val}} \Phi_{\text{sea}} \) where \( \mu \) and \( \nu \) have values 0, 1, 2 and 1, 8, 10 respectively. All the \( n_{\mu \nu} s \) are calculated from multiplicities of each Fock state in spin and color space. These multiplicities are expressed in the form \( \rho_{p,q} \), the relative probability for a core part with spin \( p \) and sea with spin \( q \) such that the resultant should come out as 1/2. Similar probabilities could be calculated for the color space yielding a color singlet state. Each unknown parameter in the equation of the wave function will have a particular value of
\[ \sum n_{\mu \nu} c_{\text{sea}} \]
de pending on the Fock state [57]:
\[ a_8^2 = (n_{01} c_{\text{sea}})_{gg} + (n_{01} c_{\text{sea}})_{udg} + (n_{01} c_{\text{sea}})_{uddg} + (n_{01} c_{\text{sea}})_{ssg} + \cdots \]
\[ b_8^2 = (n_{11} c_{\text{sea}})_{gg} + (n_{11} c_{\text{sea}})_{uudg} + (n_{11} c_{\text{sea}})_{uuddg} + (n_{11} c_{\text{sea}})_{ssg} + \cdots \]
\[ d_8^2 = (n_{21} c_{\text{sea}})_{gg} + (n_{21} c_{\text{sea}})_{uudg} + (n_{21} c_{\text{sea}})_{uuddg} + (n_{21} c_{\text{sea}})_{ssg} + \cdots \]
\[ \vdots \]
Combinations for other unknown parameters can be written in a similar way. Though the set of different Fock states \(|gg\), |u\bar{u}g\), \(d\bar{d}g\), etc. is the same for all baryon octet members, the probability distribution is different for different baryons due to mass inherited from flavor, leading to different values of the unknown parameters. These calculations will give the value of the factor “nc” for Fock states which has a significant role in determining the low-energy properties. The method is based on counting the multiplicities in spin and color space for all possible sets of Fock states in valence as well as sea, and defining these multiplicities in the form of suitable ratios. Such a comparison of multiplicities in Fock states serves two purposes. The first is to find a common multiplier “c” for every combination of valence and sea which is multiplied with the multiplicity factor \(n\) for each Fock state. The second is to calculate the probability of each substate with specified spin and color quantum numbers. For instance, a simple two-gluon sea can have spin \((0, 1, 2)\) and color \((1, 8, \overline{10})\), and similarly for higher numbers of gluons. A general way to present the comparison of such probabilities is:

\[
\frac{\rho_{p,q}}{\rho_{p',q'}} = \frac{x(r_1)y(r_2)z(r_3)}{x'(r_1')y'(r_2')z'(r_3')}
\]

The coefficients in the octet wave function are nothing but the sum of products of multiplicities with a common factor for each feasible combination of the sea. Details of the calculation of probabilities are given in Ref. [34]. To calculate the low-energy properties like magnetic moments and spin distribution, it is convenient to write the contributions from the scalar, vector, and tensor sea in the form of two parameters \(\alpha, \beta\) [56]:

\[
\alpha = \frac{1}{N^2} \frac{4}{9} (2a + 2b + 3d + \sqrt{2}e) = \frac{2(6 + 3a_2^2 - 2b_1^2 - b_8^2 + 4b_8c_8 + 5c_8^2 - 3d_8^2)}{27(1 + a_8^2 + a_{10}^2 + b_1^2 + b_{10}^2 + b_8^2 + c_8^2 + d_8^2)},
\]

\[
\beta = \frac{1}{9N^2} (2a - 4b - 6c - 6d + 4\sqrt{2}e) = \frac{(3 - 9a_{10}^2 - 3a_8^2 - b_1^2 + 3b_{10}^2 + b_8^2 + 8b_8c_8 - 5c_8^2 + 3d_8^2)}{27(1 + a_8^2 + a_{10}^2 + b_1^2 + b_{10}^2 + b_8^2 + c_8^2 + d_8^2)}.
\]

All the properties that need to be calculated are written in terms of the parameters \(\alpha, \beta\), signifying the individual coefficients of each Fock state.

3. Magnetic moments using constituent quark masses and spin distribution of hyperons

Out of the many approaches and models, such as lattice QCD, the quark–diquark model, the chiral constituent model, the potential model, QCD sum rules, etc., that are available for three-body systems, we employ here the statistical approach to study the baryons in the \(J^P = \frac{1}{2}^+\) state. Magnetic moments are a low-energy and long-distance phenomenon. The magnetic moments of \(J^P = \frac{1}{2}^+\) baryons are computed using the spin–flavor wave functions of the constituting quarks. Here, we have incorporated the effect of (a) quark effective masses, (b) quark effective charges, and (c) both, i.e. quark effective mass plus effective charge, to compute the magnetic moments. The quark magnetic moments in terms of effective quark masses can be written as [58]:

\[
\mu_{u,\text{eff}} = 2[1 - (\Delta M/M_B)]\mu_N,
\]

\[
\mu_{d,\text{eff}} = -[1 - (\Delta M/M_B)]\mu_N,
\]

\[
\mu_{s,\text{eff}} = -M_u/M_s[1 - (\Delta M/M_B)]\mu_N,
\]

\[\text{(3.1)}\]
where $M_B$ is the mass of the baryon obtained additively from the quark masses, and $\Delta M$ is the mass difference between the experimental value and $M_B$. The values of $\alpha$ and $\beta$ are calculated statistically by including the strange quark–antiquark pairs in the sea [59]. The magnetic moments for baryons in the ground state are written as a vector sum of quark magnetic moments,

$$
\mu_{\text{baryon}} = \sum_{i=1,2,3} \mu_i \sigma_i,
$$

(3.2)

where

$$
\mu_i = \frac{e_i}{2m_i}
$$

(3.3)

for $i = u, d, s$, and $e_i$ represents the quark charge. Here, $\sigma_j$ is the Pauli matrix representing the spin term of the $i$th quark, and $\mu_i$ represents the magnitude of the quark magnetic moments, and therefore the values of the magnetic moments are different for different baryons. We have applied the magnetic moment operator $\hat{O} = \mu_i \sigma_i$ to the octet wave function in Eq. (2.3) in the following way:

$$
\mu_B = \sum_{u,d,s} \left\langle \Psi_B \left| \frac{e_i \sigma_i^j}{2m_{\text{eff}}} \right| \Psi_B \right\rangle.
$$

(3.4)

In general [56],

$$
\langle \Phi_{1/2}^{(1)} | \hat{O} | \Phi_{1/2}^{(1)} \rangle = \frac{1}{N^2} \left[ \langle \Phi_{1/2}^{(1)} | \hat{O} | \Phi_{1/2}^{(1)} \rangle + \sum_{i=1,8,10} a_i^2 \langle \Phi_{1/2}^{(1)} | \hat{O} | \Phi_{1/2}^{(1)} \rangle + \sum_{i=1,8,10} b_i^2 \langle \Phi_{1/2}^{(1)} | \hat{O} | \Phi_{1/2}^{(1)} \rangle 
+ 2b_8c_8 \langle \Phi_{1/2}^{(1)} | \hat{O} | \Phi_{1/2}^{(1)} \rangle + c_8^2 \langle \Phi_{1/2}^{(1)} | \hat{O} | \Phi_{1/2}^{(1)} \rangle + a_8^2 \langle \Phi_{1/2}^{(1)} | \hat{O} | \Phi_{1/2}^{(1)} \rangle \right].
$$

(3.5)

The magnetic moment relations obtained after applying the operator for $J^P = \frac{1}{2}^+$ particles in terms of the parameters $\alpha$ and $\beta$ and the quark effective masses $\mu_{u}^{\text{eff}}, \mu_{d}^{\text{eff}}, \mu_{s}^{\text{eff}}$ are shown in column 1 of Table 3. We repeat our computations by varying the effective masses of quarks (in MeV) from 370 to 390 for $u$ and $d$ quarks and from 500 to 530 for the strange quark, to determine the most suitable set of effective quark masses that yield the quark magnetic moments and hence the magnetic moments of baryons. As the values of the effective masses are model dependent, so the magnetic moments of quarks are also model dependent and one has to take their values consistently with the constituent quark masses. Also, the masses used for the above calculations for all baryons (in the case of two gluons) are shown in detail in Table 2.

In addition to the calculation of magnetic moments with effective masses, we have also computed magnetic moments with effective charge. Here, we have applied the magnetic moment operator $\hat{O} = \mu_i \sigma_i$ to the octet wave function in the following manner:

$$
\mu_B = \sum_{u,d,s} \left\langle \Psi_B \left| \frac{e_i^{\text{eff}} \sigma_i^j}{2m_{\text{eff}}} \right| \Psi_B \right\rangle.
$$

(3.6)

We have taken the effective charge to depend linearly on the charge of the shielding quarks. Thus, the effective charge $e_a$ of quark $a$ in the baryon $B(a, b, c)$ is written as [61]:

$$
e_a^B = e_a + \alpha_{ab} e_b + \alpha_{ac} e_c,
$$

(3.7)
Table 2. The values of individual effective quark masses and their experimental masses for every baryon octet.

| Baryon octet | $M_\text{e}$ (MeV) | $\Delta M$ (MeV) [60] |
|-------------|-------------------|----------------------|
| p $(uud)$   | 370 + 370 + 378   | 938.27               |
| n $(udd)$   | 370 + 386 + 386   | 939.56               |
| $\Lambda^0$ $(uds)$ | 370 + 370 + 530 | 1115.683             |
| $\Sigma^+$ $(uus)$ | 370 + 370 + 500 | 1189.37              |
| $\Sigma^0$ $(uds)$ | 370 + 370 + 510 | 1192.642             |
| $\Sigma^-$ $(dds)$ | 380 + 380 + 530 | 1197.449             |
| $\Xi^0$ $(uss)$ | 370 + 500 + 500 | 1314.86              |
| $\Xi^-$ $(dss)$ | 370 + 500 + 500 | 1321.71              |

where $e_a$ is the bare charge of quark $a$. Taking isospin symmetry, we have

$$\alpha_{uu} = \alpha_{ud} = \alpha_{dd} = \beta,$$

$$\alpha_{us} = \alpha_{ds} = \alpha,$$

$$\alpha_{ss} = \gamma.$$

The screened quark charges for various baryons in terms of the three parameters $\alpha', \beta', \gamma'$ (not to be confused with the statistical parameters, i.e. $\alpha, \beta$) can be expressed as:

$$e_u^0 = \frac{2}{3}(1 + \frac{1}{2}\beta'), \quad e_d^0 = -\frac{1}{3}(1 - 4\beta'),$$

$$e_u^p = \frac{2}{3}(1 - \beta'), \quad e_d^p = -\frac{1}{3}(1 - \beta'), \ldots$$  \hspace{1cm} (3.8)

Substituting these values of effective charge and applying the magnetic moment operator to the wave function as in Eq. (2.3), and using as input the unknown parameters in the effective charge relations, i.e. $\alpha' = 0.248$, $\beta' = 0.025$, and $\gamma'$ varying from 0.018 to 0.029, we determine the magnetic moments, which are shown in Table 3. SU(3) symmetry-breaking effects are also applied in the sea and valence quarks. This breaking is due to the mass difference between strange and non-strange light quarks. Symmetry breaking is applied to the magnetic moments calculated with the effective masses (with two gluons), in the form of the parameter $r = m/m_s$ [56], where $m$ is the mass of the $u$ and $d$ quarks. The value of $r$ lies in the range 0.70 to 0.78, depending upon the respective effective quark masses. The results for magnetic moments with SU(3) broken symmetry for hyperons are shown in Table 5; the results obtained under SU(3) symmetry are shown in the second column of Table 3. Recently, the CLAS collaboration [62] reported strangeness suppression in the proton from the production rates of baryon–meson states in exclusive reactions, i.e. without the production of an intermediate baryon resonance. We are also interested in measuring how often, compared with pairs of light quarks, strange quarks are made. For this purpose, we define the strangeness suppression factor as $\lambda_s = \frac{2\langle s s \rangle}{\langle uu + dd \rangle}$. This ratio implies the existence of strange quarks in the sea. So, we have calculated the strangeness suppression factor for all particles in the $J^P = \frac{1}{2}^+$ state in the framework of the principle of detail balance. The calculation of the strangeness suppression factor includes various subprocesses like $g \leftrightarrow q\bar{q}$, $g \leftrightarrow gg$, $q \leftrightarrow gq$. The results are presented in Table 6. The value for the strangeness suppression factor is in good agreement with the values determined in both exclusive reactions and in high-energy production.
Table 3. Magnetic moments of $J^P = \frac{1}{2}^+$ baryons with (a) effective quark masses, (b) quark effective charge, and (c) both, i.e. quark effective mass plus quark effective charge. The results for magnetic moments with various modifications are shown, taking two and three gluons in the sea respectively.

| Baryon octet | Magnetic moments (μN) | Effective quark mass | Effective quark charge | Effective quark mass and quark charge | Expected results [60] |
|-------------|------------------------|----------------------|------------------------|--------------------------------------|-----------------------|
|             |                        | 2 gluons             | 3 gluons              | 2 gluons                             | 3 gluons              |                          |
| $\mu_p$    | $3(\mu_u^{\text{eff}} - \mu_d^{\text{eff}} \beta)$ | 2.79                 | 2.29                  | 1.81                                 | 1.73                  | 2.74                    | 2.62                    | 2.79                    |
| $\mu_n$    | $3(\mu_d^{\text{eff}} - \mu_u^{\text{eff}} \beta)$ | -1.83                | -1.50                 | -0.85                                | -0.84                 | -1.38                   | -1.37                   | -1.91                   |
| $\mu_A$    | $\frac{1}{2}(\alpha - 4\beta)(\mu_u^{\text{eff}} + \mu_d^{\text{eff}}) + (2\alpha + \beta)\mu_s^{\text{eff}}$ | -0.634               | -0.60                 | -0.41                                | -0.36                 | -0.59                   | -0.52                   | -0.613                  |
| $\mu_{\Sigma^+}^0$ | $3(\mu_u^{\text{eff}} - \mu_d^{\text{eff}} \beta)$ | 2.464                | 2.11                  | 1.99                                 | 1.78                  | 3.0                     | 2.77                    | 2.458                   |
| $\mu_{\Sigma^0}$ | $\frac{1}{2}(\mu_u^{\text{eff}} + \mu_d^{\text{eff}} \alpha - 2\mu_s^{\text{eff}} \beta)$ | 0.775                | 0.680                 | 0.82                                 | 0.76                  | 1.31                    | 1.19                    | 0.775                   |
| $\mu_{\Sigma^-}$ | $3(\mu_u^{\text{eff}} - \mu_d^{\text{eff}} \beta)$ | -0.974               | -0.82                 | -0.83                                | -0.75                 | -1.29                   | -1.17                   | -1.160                  |
| $\mu_{\Xi^0}$ | $3(\mu_s^{\text{eff}} - \mu_u^{\text{eff}} \beta)$ | -1.38                | -1.203                | -0.933                               | -0.84                 | -1.36                   | -1.23                   | -1.250                  |
| $\mu_{\Xi^-}$ | $3(\mu_s^{\text{eff}} - \mu_d^{\text{eff}} \beta)$ | -0.615               | -0.53                 | -0.422                                | -0.403                | -0.57                   | -0.55                   | -0.6507                |

Table 4. Final predictions of magnetic moments using the statistical model.

| Baryon octet | Magnetic moments (μN) with effective quark mass and effective quark charge | Expected results [60] |
|-------------|--------------------------------------------------------------------------------|-----------------------|
| p           | 2.74                                                                 | 2.79                  |
| n           | -1.38                                                                | -1.91                 |
| $\Lambda^0$ | -0.59                                                             | -0.613                |
| $\Sigma^+$  | 3.0                                                                  | 2.458                 |
| $\Sigma^0$  | 1.31                                                                  | 0.775                 |
| $\Sigma^-$  | -1.29                                                               | -1.160                |
| $\Xi^0$     | -1.36                                                               | -1.250                |
| $\Xi^-$     | -0.57                                                               | -0.6507               |

Table 5. Magnetic moments of particles with SU(3) symmetry breaking, with SU(3) symmetry, and a comparison with experimental data. The symmetry breaking is applied to the magnetic moments obtained with effective quark masses. The results shown under the third column are the magnetic moments with effective quark masses (with two gluons) from the second column of Table 3.

| Baryon octet | Magnetic moments (μN) | SU(3) symmetry breaking | SU(3) symmetry | Expected results [60] |
|-------------|-----------------------|-------------------------|----------------|-----------------------|
| $\Lambda^0$ | -0.44                  | -0.634                  | -0.613         |
| $\Sigma^+$  | 2.40                   | 2.464                   | 2.458          |
| $\Sigma^0$  | 0.644                  | 0.775                   | 0.775          |
| $\Sigma^-$  | -1.018                 | -0.974                  | -1.160         |
| $\Xi^0$     | -1.063                 | -1.388                  | -1.250         |
| $\Xi^-$     | -0.35                  | -0.615                  | -0.6507        |
Table 6. Strangeness suppression for all particles in octets.

| Baryon octet | $\frac{S}{d^0}$ | $\frac{d}{u^0}$ | $\frac{\lambda_s}{(u^0+d^0)}$ |
|-------------|-----------------|----------------|-------------------------------|
| p           | 0.32/0.26       | 0.71/0.57      | 0.38/0.34                    |
| n           | 0.46            | 1.40           | 0.38                         |
| $\Lambda^0$ | 0.28            | 1              | 0.29                         |
| $\Sigma^+$  | 0.19            | 0.43           | 0.277                        |
| $\Sigma^0$  | 0.28            | 1              | 0.28                         |
| $\Sigma^-$  | 0.45            | 2.26           | 0.276                        |
| $\Xi^0$     | 0.17            | 0.62           | 0.21                         |
| $\Xi^-$     | 0.28            | 1.61           | 0.21                         |

* Results from unquenched quark model [65].

** Experimental results [62] for strangeness suppression in proton.

Table 4 shows the values of the magnetic moments of octet baryons calculated by including the effect of the effective quark mass plus the effective quark charge. The results of the statistical model are found to be better with two gluons than three gluons, because we have suppressed higher multiplicities to include three gluons. The magnetic moments are compared with the experimental data from the Particle Data Group (PDG) within the error bar 2%–30%.

The individual spin polarizations due to quarks for hyperons are calculated and compared with data of other available models. In this application, the individual spin polarization is defined as

$$\Delta q = n(q \uparrow) - n(q \downarrow) + n(\bar{q} \uparrow) - n(\bar{q} \downarrow)$$

for $q = u, d, s$, where $n(q \uparrow)$ is the number of spin-up and $n(q \downarrow)$ is the number of spin-down quarks of flavor $q$ for both quarks and antiquarks. In addition, the total spin distribution of the baryon is also determined by applying the operator $I_B^1 = \frac{1}{2} e_i \sigma_i^Z$, where $e_i$ and $\sigma_i^Z$ are the charge of quark and the spin projection operator, respectively, to the baryonic wave function. The results from the statistical model and a comparison with other models are shown in Table 7.

A well-known and important problem for physicists over the last 20 years, the proton spin crisis, suggested that the proton’s spin is built from its constituent quarks plus a sea of quark–antiquark pairs and gluons. Deep inelastic experiments and the European Muon Collaboration predicted that the total spin of the proton has very little contribution from the quark’s intrinsic spin, which was contrary to the results of the non-relativistic quark model. The whole story led both phenomenologists and experimentalists to think beyond the already-known facts. Various experiments [63,64] measured the spin structure function $g_1$ at $x = 0.1$ to $x = 0.01$, and summarized that the proton is a system of three massive constituent quarks interacting self-consistently with a cloud of virtual pions and condensates generated from spontaneous breaking of chiral symmetry between left- and right-handed quarks. On the other hand, when probed at high resolution, the structure of the proton seems to be a combination of three valence quarks plus a sea of quark–antiquark pairs and gluons. Thus, we conclude that nucleonic spin is distributed among gluons, valence, and sea quarks plus their angular momenta.

4. Discussion of result and conclusion

Statistical models provide physical simplicity in describing the various properties of the baryonic states, which includes the “sea.” Baryonic structure is considered to consist of valence quarks and a sea limited by a few quark–antiquark pairs multiconnected non-perturbatively through gluons. In the present article, the baryon octet wave function is studied by looking at the concept of effective mass and effective charge to analyze various baryonic properties like magnetic moments, flavor asymmetry,
Table 7. Spin distribution from individual quarks, and the total spin distribution computed in the statistical model. We have defined all the properties in terms of the parameters $\alpha$ and $\beta$.

| Baryon | Quark spin polarizations and distribution | Calculated values | Theoretical predictions [66] |
|--------|-----------------------------------------|------------------|-----------------------------|
| $\Lambda$ | $\Delta u = \frac{2}{3} - 2\beta$ | $-0.02$ | $-0.03$ |
|        | $\Delta d = \frac{2}{3} - 2\beta$ | $-0.02$ | $-0.03$ |
|        | $\Delta s = 2\alpha + \beta$ | $0.70$ | $0.74$ |
|        | $I_{1}^{\Lambda} = \frac{1}{3}(2\alpha - 2\beta)$ | $0.041$ | $0.027$ |
| $\Sigma^+$ | $\Delta u = 3\alpha$ | $0.91$ | $0.98$ |
|        | $\Delta d = 0$ | $-7.40 \times 10^{-17}$ | $-0.02$ |
|        | $\Delta s = -3\beta$ | $-0.21$ | $-0.29$ |
|        | $I_{1}^{\Sigma^+} = \frac{2}{3}\alpha - \frac{1}{6}\beta$ | $0.191$ | --- |
| $\Sigma^0$ | $\Delta u = \frac{2}{3}\alpha$ | $0.46$ | $0.48$ |
|        | $\Delta d = 3\alpha$ | $0.46$ | $0.48$ |
|        | $\Delta s = -3\beta$ | $-0.22$ | $-0.29$ |
|        | $I_{1}^{\Sigma^0} = \frac{2}{3}\alpha - \frac{1}{6}\beta$ | $0.117$ | --- |
| $\Sigma^-$ | $\Delta u = \frac{2}{3}\alpha - \frac{1}{6}\beta$ | $0.03$ | $-0.02$ |
|        | $\Delta d = 3\alpha$ | $0.91$ | $0.98$ |
|        | $\Delta s = -3\beta$ | $-0.22$ | $-0.29$ |
|        | $I_{1}^{\Sigma^-} = \frac{1}{6}\alpha - \frac{1}{2}\beta$ | $0.0389$ | --- |
| $\Xi^0$ | $\Delta u = -3\beta$ | $-0.22$ | $-0.29$ |
|        | $\Delta d = 0$ | $0$ | $-0.02$ |
|        | $\Delta s = 3\alpha$ | $0.95$ | $0.98$ |
|        | $I_{1}^{\Xi^0} = \frac{1}{2}\alpha - \beta$ | $0.0838$ | --- |
| $\Xi^-$ | $\Delta u = -3\beta$ | $-0.2$ | $-0.020$ |
|        | $\Delta d = \alpha - 4\beta$ | $0.017$ | $-0.29$ |
|        | $\Delta s = 3\alpha$ | $0.95$ | $0.98$ |
|        | $I_{1}^{\Xi^-} = \frac{1}{8}\alpha - \frac{1}{5}\beta$ | $0.0404$ | --- |

and quark spin distribution. The explicit numerical values of quark effective mass and quark effective charge contributing to the magnetic moments of $J^P = \frac{1}{2}^+$ octet baryons are calculated. Effective quark masses and effective quark charges for quarks $u, d,$ and $s$ are calculated using fixed inputs for baryon masses (PDG) and statistical parameters $(\alpha, \beta)$ as input to the respective formulae. These effective masses and effective charges of $u, d,$ and $s$ are acting as inputs to the magnetic moment of the $J^P = \frac{1}{2}^+$ baryon octet.

SU(3) breaking is studied for the magnetic moment by using a symmetry breaking parameter $r$, which plays an important role by providing the basis to understand the extent to which sea quarks contribute to the structure of the baryon. We have also investigated the flavor asymmetry of all the octet baryons. It can be seen from Table 1 that there exist simple relations between the flavor asymmetries, e.g., the excess of $d$ over $u$ in the proton is equal to the excess of $u$ over $d$ in the neutron, and similarly for other hyperons. The isospin symmetry leads to these relations among the flavor asymmetries of octet baryons. The magnetic moments of the baryon octet are studied in the framework of the statistical model along with principle of detailed balance, in which the effect of the “sea” is taken into account via inclusion of quark effective mass and quark effective charge. It is interesting to observe that our results for the magnetic moments of $J^P = \frac{1}{2}^+$ octet particles give a good match with the experimental values, specifically when calculated with quark effective mass (with two gluons), whereas magnetic moments deviate when quark effective charge is considered,
as seen in Table 3. In all the cases, though, the contribution of the quark sea is quite significant. The detailed justification and calculation of the importance of higher-order Fock states can be seen in Ref. [40–42]. The importance of the sea with $q\bar{q}$ pairs and gluons can be seen in Refs. [21,40–42,44]. Here, each Fock state has a certain probability which contributes to the coefficients in the wave function used to calculate the effect of effective quark charges or effective quark masses on the magnetic moments by including a range of different parameters. The parameters discussed here are highly dependent on the coefficients $a_0, b_1, b_8, \ldots$ of the octet wave function. The calculated values of magnetic moments with SU(3) breaking are compared with magnetic moments when SU(3) symmetry is taken in Table 5. The listed values show that the strange quark contribution to the magnetic moment due to its mass is almost an order of magnitude smaller than the up and down quarks, leading to a very small contribution from the heavy quarks when compared with the contribution coming from the light quarks.

The results with SU(3) symmetry breaking are not much in agreement with the experimentally observed values. Plausibly, at energies of 1 GeV$^2$, the results are better in column 3 of Table 5 due to the conservation of SU(3) symmetry where $s = u = d$ is applicable. Energies of 1 GeV$^2$ are considered to be low energies in high-energy physics. We study all the properties such as magnetic moments and spin distribution at such low energies. To appreciate the strange quarks in the sea, a factor called the strangeness suppression factor is calculated for all the octet particles (see Table 6). The strangeness suppression factor discussed in this paper suggests the strange quark contribution for all baryons in the sea. It also suggests the importance of strangeness in the sea. Though the data for this factor is experimentally available only for the proton, we have calculated this strangeness suppression factor for all the particles in the octet in our model. Hence, this suppression factor suggests that $s\bar{s}$ sea accommodability enhances for particles with higher strangeness in their valence part.

The importance of the sea with effective mass and charge of the quark content has been studied, and its various effects have been shown through the above properties, which provide rich information about the structure of all the octets thereby motivating experiments for further inspection. Most of the results presented are dependent on the effective quark masses, which are model dependent. To conclude, the statistical model is able to phenomenologically estimate the sea contributions to the low-energy properties of the $J^P = \frac{1}{2}^+$ baryons.

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