Effect of population imbalance on the Berezinskii-Kosterlitz-Thouless phase transition in a superfluid Fermi gas

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The Berezinskii-Kosterlitz-Thouless (BKT) mechanism describes the breakdown of superfluidity in a two-dimensional Bose gas or a two-dimensional gas of paired fermions. In the latter case, a population imbalance between the two pairing partners in the Fermi mixture is known to influence pairing characteristics. Here, we investigate the effects of imbalance on the two-dimensional BKT superfluid transition and show that superfluidity is even more sensitive to imbalance than for three-dimensional systems. Finite-temperature phase diagrams are derived using the functional integral formalism in combination with a hydrodynamic action functional for the phase fluctuations. This allows to identify a phase-separation region and tricritical points due to imbalance. In contrast to superfluidity in the three-dimensional case, the effect of imbalance is also pronounced in the strong-coupling regime.

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I. INTRODUCTION

Quantum phenomena which occur at very low temperatures are a subject of intense experimental and theoretical study. Recent progress in the experimental investigation of ultracold atoms stimulated an unprecedented interest to the theoretical problems of condensation of cold bosons and pairing of interacting fermions (see, e.g., the review [1] and references therein). These phenomena are related to a variety of objects including stars, dense nuclear and quark matter, and plasma systems [2–4].

Phase transitions of quantum systems strongly depend on their dimensionality. Two-dimensional (2D) Fermi gases have remarkable features, which are not observed in three dimensions. The quasi-2D regime for cold atoms can be reached using a sufficiently strong confinement of atoms along one direction so that they occupy only the lowest size quantization subband. Advances in pairing of trapped cold atomic Fermi gases with a controlled geometry of a trapping potential allow experimentalists to realize systems of different (quasi)dimensionalities. Both condensation of bosonic atoms [5] and pairing of fermions [6] has been observed recently in optical lattices.

The Mermin-Wagner-Hohenberg theorem [7] shows that in a uniform two-dimensional (2D) system, the long-range order is destroyed by thermal fluctuations so that 2D Bose gases cannot undergo Bose-Einstein condensation at nonzero temperatures [8–10]. Nevertheless, these two-dimensional systems can form a “quasicondensate” and exhibit superfluidity [11]. Kosterlitz and Thouless [12] showed that the mechanism driving the superfluid-to-normal phase transition in this case is the proliferation of vortices and antivortices above a critical temperature $T_{\text{BKT}}$, spoiling the phase coher-

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they are theoretically analyzed by Botelho and de Melo [25]. They treated the fermion pairing in 2D by the path integral technique taking into account phase fluctuations. Within a similar approach, using an effective Hamiltonian which involves fermions interacting with each other and with dressed molecules, the BKT transition has been considered for a quasi-2D trapped Fermi gas [26].

In the present work, we extend the approach of Ref. [25] to investigate the effect of population imbalance on the BKT phase transition in a 2D Fermi gas. Using the Hubbard-Stratonovich transformation, we derive a hydrodynamic effective bosonic action, which in the limiting case of a balanced gas is reduced to the effective action of Ref. [25]. On the basis of the obtained effective action, we analyze phase diagrams for an interacting, imbalanced Fermi gas in 2D. The paper is organized as follows. In Sec. II, we describe the theoretical formalism for interacting fermions in 2D. In Sec. III, we analyze the dependence of the critical temperature of the BKT transition on the coupling strength and on the population imbalance. The section is followed by conclusions (Sec. IV).

II. FUNCTIONAL INTEGRAL DESCRIPTION

A. General formalism

We consider a two-component gas of interacting fermions in 2D, with the s-wave pairing and with a population imbalance. The partition function of the system of fermions in 2D is expressed as the path integral over Grassmann variables \[ \bar{\psi}_{\alpha,r}\sigma, \psi_{\alpha,r}\sigma \],

\[ Z = \int D[\bar{\psi}_{\alpha,r}](\tau), \psi_{\alpha,r}(\tau)] \exp(-S). \]  

The action functional

\[ S = S_0 + S_{int} \]  

is a sum of the free-fermion and interaction contributions,

\[ S_0 = \int_0^\beta d\tau \int d^2x \sum_{\alpha=r,\sigma} \left[ \bar{\psi}_{\alpha,r}(\tau) \left( \frac{\partial}{\partial \tau} - \nabla^2 - \mu_\sigma \right) \psi_{\alpha,r}(\tau) \right], \]  

\[ S_{int} = \int_0^\beta d\tau \int d^2x \int d^2y V(x - y) \bar{\psi}_{\sigma,1}(\tau) \bar{\psi}_{\alpha,1}(\tau) \psi_{\sigma,1}(\tau) \psi_{\alpha,1}(\tau), \]  

where \( \beta = \frac{1}{k_0} \) is the inverse to the temperature. We use the units in which \( \hbar = 1 \), the fermion mass \( m = 1/2 \), and the Fermi energy \( E_F = (\pi n_0)^{2/3} / (2m) = 1 \) (where \( n_0 \) is the fermion density in 2D). We express the results below in terms of the averaged chemical potential \( \mu = (\mu_\uparrow + \mu_\downarrow) / 2 \) determining the total number of fermions and the imbalance potential \( \xi = (\mu_\uparrow - \mu_\downarrow) / 2 \). For the interaction potential, we use the separable expression proposed in Refs. [25,27],

\[ V_{k,k'} = g^2 \Gamma_k \Gamma_{k'}. \]  

where \( g \) is the interaction strength. The factor \( \Gamma_k \) describes a finite-range potential,

\[ \Gamma_k = \left( 1 + \frac{k}{k_0} \right)^{-1/2}. \]  

where \( R \sim k_0^{-1} \) plays the role of the interaction range. The particular case of the contact interaction corresponds to \( k_0 \rightarrow \infty \) so that \( \Gamma_k \rightarrow 1 \). The interaction term \( S_{int} \) of the action functional is then given by

\[ S_{int} = \int_0^\beta d\tau \sum_{k,q} B_k(\tau) B_k(\tau), \]  

where \( L \) is the lateral size of the 2D system, and the collective coordinates \( B_k(\tau) \) are determined as

\[ B_k(\tau) = \sum_{\sigma} \bar{\Gamma}_k a_{k+q,\sigma}(\tau) a_{k+q,\sigma}(\tau). \]  

Further on, we apply the Hubbard-Stratonovich (HS) transformation. Introducing the extended action

\[ S_{ext} = S - \int_0^\beta d\tau \sum_{k,q} \bar{\phi}_k(\tau) \phi_k(\tau), \]  

with the auxiliary Bose field (HS field) \( \phi_k(\tau) \) and performing the shift of boson coordinates, which eliminates the fermion-fermion interaction term \( S_{int} \), the HS transformation results in the action

\[ S_{ext} = S_0 - \int_0^\beta d\tau \sum_{k,q} \left[ \bar{B}_k(\tau) \phi_k(\tau) + B_k(\tau) \bar{\phi}_k(\tau) \right] \]  

\[ - \int_0^\beta d\tau \sum_{k,q} \bar{\phi}_k(\tau) \phi_k(\tau). \]  

Because the phase fluctuations about the saddle point are, in general, not small, the boson (HS) and fermion variables in the coordinate representation are transformed as [28]

\[ \phi_{\sigma,1}(\tau) = \phi_{\sigma,1}(\tau) e^{i \mu_\sigma(\tau)}, \quad \psi_{\sigma,1}(\tau) = \psi_{\sigma,1}(\tau) e^{i \mu_\sigma(\tau)/2}. \]  

In the same formalism for a Fermi gas in 3D, the further step is the path integration over fermion variables and the expansion of the resulting bosonic action over fluctuations about the saddle point [29]. This method provides a description of a superfluid phase transition between the normal phase and the true condensate of fermion pairs. In the 2D case, at least when restricting the expansion by quadratic fluctuations, the superfluidity occurs only at \( T=0 \) [19]. Travens [30] showed that the interaction between fluctuations of the pairing field in a 2D attractive Fermi gas allows a superfluid phase transition at a very low temperature. However, the superfluid state can exist in a 2D Fermi gas at relatively high temperatures as a quasicondensate, i.e., a state with fermion pairing “where the density fluctuations are suppressed but the phase still fluctuates” [31]. In two dimensions, the quasicondensate can be realized through bound vortex-antivortex pairs [1].

B. Fluctuating phase

After integrating out the fermion (Grassmann) variables, an effective action in the Bose field \( \phi_\tau \) is obtained. The re-
maining functional integral over this Bose field cannot be taken in general. Several levels of approximation can be made to get results. The crudest approximation is the mean-field approximation which replaces the field by a constant, $\phi_\tau \to |\Delta|$, the saddle point. To improve on this, fluctuations around the saddle point can still be taken into account; this can be done in an exact way only up to quadratic order in the fluctuation. One can choose to write the fluctuations as $\phi_\tau \to |\Delta| + \delta_\tau$, with $\delta_\tau$ being complex; this corresponds to the NSR [29] approach that investigates the presence of a real condensate rather than a quasicondensate. Alternatively the 2D case, we have to focus on phase fluctuations: that corresponds to setting $\phi_\tau \to |\Delta|e^{i\theta_\tau}$. We will simplify the notation and write $|\Delta|=\Delta$ for the energy gap parameter of the Bogoliubov excitations. Moreover, we will assume that the remaining fluctuation field $\theta_\tau$ varies slowly as a function of position and time with respect to the variations in the fermion fields. A similar assumption was used in Refs. [26,28]. The resulting hydrodynamic action is structurally similar to the saddle-point action for imbalanced fermions in 2D [32]

$$S_{\text{eff}} = -\int_0^\beta d\tau \int d^2r \frac{1}{L^2} \sum_k \frac{1}{\beta} \ln(2 \cosh \beta \varepsilon_k) + 2 \cosh \beta E_k - \frac{B L^2}{g} \Delta^2, \quad (11)$$

in which the fermion energy $\epsilon_k = k^2 - \mu$, the Bogoliubov excitation energy $E_k = \sqrt{\epsilon_k^2 + \Delta^2} \gamma_k$, and the imbalance potential $\zeta$ are replaced by expressions depending on boson coordinates:

$$\tilde{\epsilon}_k = k^2 - \bar{\mu}, \quad \tilde{E}_k = \sqrt{\tilde{\epsilon}_k^2 + \Delta^2} \gamma_k.$$  \(12\)

$$\bar{\mu} = \mu - \frac{i}{2} \frac{\partial \theta}{\partial \tau} - \frac{1}{4} \left( \nabla \theta \right)^2 + \frac{i}{2} \nabla^2 \theta,$$ \(13\)

$$\zeta_k = \zeta - \nabla \theta \cdot k.$$ \(14\)

Keeping the quadratic-order fluctuation terms we arrive at the action $S_{\text{eff}}$ as the sum of the saddle-point action $S_{\text{sp}}$, which coincides with expression (5) of Ref. [32], and the diagonal quadratic form of the phase fluctuations

$$S_{\text{fl}} = \int_0^\beta d\tau \int d^2r \left[ A \left( \frac{\partial \theta}{\partial \tau} \right)^2 + \rho_s (\nabla \theta)^2 \right]. \quad (15)$$

The coefficients in the fluctuation action are the pair superfluid density

$$\rho_s = L^2 \sum_k \frac{1}{2} \left( 1 - \frac{\epsilon_k}{E_k} \right) X_k,$$ \(16\)

and the constant

$$A = \frac{1}{4L^2} \sum_k \left( \frac{\gamma_k^2 \Delta^2}{E_k} + \frac{\tilde{\epsilon}_k^2}{E_k} \gamma_k \right),$$

where $X_k$ is the function

$$X_k = \frac{\sinh \beta E_k}{\cosh \beta \zeta + \cosh \beta E_k}$$

and $Y_k$ is the extension of the Yoshida distribution to imbalanced fermions:

$$Y_k = \beta \cosh \beta \zeta \cosh \beta E_k + 1 \left( \cosh \beta \zeta + \cosh \beta E_k \right)^2.$$ \(17\)

Note that expression (16) for $\rho_s$ agrees with the definition of superfluid density used for a 3D balanced Fermi gas [33].

The action in Eq. (15) describes a Bose gas of spin waves [25] with the energy spectrum $\omega_k$ given by

$$\omega_k = ck, \quad c = \sqrt{\frac{\rho_s}{A}}.$$ \(18\)

The spin-wave contribution to the thermodynamic potential is given by the expression

$$\Omega_{\text{sw}} = \frac{1}{\beta} \sum_k \ln(1 - e^{-\beta \omega_k}).$$ \(19\)

The total thermodynamic potential of the fermion system taking into account phase fluctuations is the sum of the spin-wave term [Eq. (19)] and the saddle-point contribution, which is provided by the saddle-point action,

$$\Omega_{\text{sp}} = \frac{1}{\beta} \sum_k \left[ \frac{1}{\beta} \ln(2 \cosh \beta \zeta + 2 \cosh \beta E_k - \tilde{\epsilon}_k) - \frac{B L^2}{g} \Delta^2 \right].$$ \(20\)

For an imbalanced quasi-2D Fermi gas in an optical potential, the mean-field zero-temperature phase diagrams were analyzed in Ref. [32] on the basis of this saddle-point action (neglecting spin-wave contributions).

The gap parameter $\Delta$ for an imbalanced Fermi gas is determined through the minimization of the saddle-point thermodynamic potential $\Omega_{\text{sp}}$ as a function of $\Delta$ at given $\beta, \mu, \zeta$:

$$\left( \frac{\partial \Omega_{\text{sp}}}{\partial \Delta} \right)_{\beta, \mu, \zeta} = 0.$$ \(21\)

For a complete determination of thermodynamic parameters at a given temperature, the minimum condition [Eq. (21)] is solved jointly with the number equations:

$$n = -\left( \frac{\partial}{\partial \mu L^2} \frac{\Omega}{\beta \zeta \Delta} \right)_{\beta, \mu, \Delta} = \frac{1}{2\pi}, \quad \delta n = -\left( \frac{\partial}{\partial \zeta L^2} \frac{\Omega}{\beta \mu \Delta} \right)_{\beta, \mu, \Delta} = \frac{1}{2\pi} n.$$ \(22\)

where $n$ and $\delta n$ are the total fermion density $n$ and the density difference $\delta n$, respectively.

### C. BKT transition temperature

The coupled gap and number Eqs. (21) and (22) have to be solved together. In Eq. (22), three different levels of approximation can be made, in analogy to the approximations for the Bose field $\phi_\tau (\tau)$ as discussed in the beginning of the previous subsection.
The first and simplest case is the mean-field approximation (as described in Refs. [25,26]), where we use \( \Omega = \Omega_{sp} \). This corresponds to setting the (Hubbard-Stratonovic) Bose field equal to a constant \( \phi = \Delta_{MF} \) (both constant in amplitude and in phase). The constant can be determined by minimizing \( \Omega_{sp} \) and allows to determine the phase-transition line between the normal state, in which \( \Delta_{MF} = 0 \), and the quasicondensate state in which \( \Delta_{MF} \neq 0 \). The temperature separating the aforementioned phases will be denoted by \( T_{MF} \). For a balanced gas, the phase transition between normal and paired states is of the second order. For an imbalanced gas, also the first-order phase transition between the normal and paired states is possible.

The paired state below \( T_{MF} \) is not always superfluid. Proliferating vortices and antivortices destroys phase coherence in the quasicondensate and suppresses superfluidity [12]. The mean-field approximation does not allow us to determine the temperature \( T_{BKT} \) separating the superfluid quasicondensate state from the nonsuperfluid paired state. To investigate the question of superfluidity and determine the BKT temperature, we need to use \( \Omega = \Omega_{sp} + \Omega_{sw} \) in Eq. (22), where \( \Omega_{sw} \) is the free energy contribution [Eq. (19)] from the phase fluctuations. This corresponds to giving the (Hubbard-Stratonovic) Bose field a constant amplitude, but allowing its phase to fluctuate freely. The BKT transition temperature is then a root of the universal Nelson-Kosterlitz equation \([34]\)

\[
T_{BKT} - \frac{\pi}{2} \mu(T_{BKT}) = 0.
\]

As distinct from the aforesaid phase transitions at \( T \leq T_{MF} \), the BKT phase transition at \( T = T_{BKT} \) is characterized by an abrupt change in the superfluid density from zero to a finite value satisfying Eq. (16). The phase-fluctuation contribution to the density vanishes at the phase boundary between the paired state and the normal state because at \( \Delta_{MF} = 0 \) the superfluid density \( \rho_s \) turns to zero.

Finally, the approximation of setting \( \Omega = \Omega_{sp} + \Omega_{sw} \) in Eq. (22) does not allow us to determine whether the superfluid quasicondensate can form a true condensate below some critical temperature \( T_{BEC} \). Note that the emergence of a true condensate in 2D is only possible in a finite system, as \( T_{BEC} \rightarrow 0 \) for \( L \rightarrow \infty \). To determine the critical temperature we should use \( \Omega = \Omega_{sp} + \Omega_{flucts} \) in Eq. (22), where \( \Omega_{flucts} \) contains contributions from both amplitude and phase fluctuations in the Hubbard-Stratonovic Bose field.

III. RESULTS AND DISCUSSION

When \( T_{MF} > 0 \), Eq. (23) can be satisfied if the superfluid density \( \rho_s \neq 0 \). At \( T = T_{MF} \), the superfluid density for a balanced Fermi gas turns to zero. Therefore for a balanced Fermi gas \( T_{BKT} \) is always lower than \( T_{MF} \). On the contrary, in the case of unequal “spin-up” and “spin-down” populations, for a sufficiently high population imbalance the phase transition at \( T = T_{MF} \) can be of the first order when the gap parameter \( \Delta \) changes discontinuously from zero to a finite value. In this case, as shown below, \( T_{BKT} \) and \( T_{MF} \) coincide in a definite range of the coupling strength.

In the region between \( T_{BKT} \) and \( T_{MF} \), the superfluidity is destroyed owing to a proliferation of free vortices. However, the phase of a 2D Fermi gas in this region is not a uniform normal state because the gap parameter for \( T_{BKT} < T < T_{MF} \) can be other than zero so that fermion pairing can occur in that temperature region. The region between \( T_{BKT} \) and \( T_c \) can be therefore attributed to a state in which pairing can occur but the phase coherence is destroyed. For slightly higher temperatures with respect to \( T_{BKT} \), vortices form a disordered gas of phase defects [1]. For higher temperatures the concept of vortices is inapplicable due to density fluctuations.

In general, there are the following regions in phase diagrams for an imbalanced Fermi gas in 2D: (1) the normal state in which the gap parameter \( \Delta = 0 \), (2) the superfluid quasicondensate state in which \( \Delta \neq 0 \) and \( \rho_s \neq 0 \), (3) the state in which pairing is possible but without phase coherence, and (4) the phase-separation region, which appears due to a population imbalance. In the latter one, no solution exist for the set of Eqs. (21) and (22). Therefore, in the phase-separation region the system will unmix in a phase with lower (or no) imbalance and the normal phase at higher imbalance. At the first-order phase transition, the system abruptly passes through the phase-separation region.

In Fig. 1, we represent phase diagrams in the variables \([T, E_b]\), where \( T \) is the temperature and \( E_b \) is the binding energy of a two-particle bound state in 2D. The phase diagrams in Fig. 1 are obtained at a given value of the imbalance chemical potential \( \xi \). The top panel shows the balanced case \( (\xi = 0) \), and the middle and lower panels illustrate how the diagram changes with increasing difference between the chemical potentials of up and down species. The energy \( E_b \) is the parameter which characterizes the coupling strength of the fermion-fermion attractive interaction. In two dimensions, the strength \( g \) of the contact interaction is related to this binding energy through \([20]\)

\[
\frac{1}{g} = \frac{m}{4\hbar^2} \left[ 1 - \ln \left( \frac{E_b}{\mu} \right) \right].
\]

The binding energy itself can be related to the experimental parameters. The two-dimensional system is created through a strong confinement of the third direction; in general this strong confinement can be associated with a harmonic potential with frequency \( \omega_L \) (and oscillator length \( \ell_L \)). The two-particle bound state exists for all values of the (3D) \( s \)-wave scattering length \( a_s \) of the fermionic atoms and its binding energy is given by

\[
E_b = \frac{\hbar^2}{2m} \exp \left( \frac{\ell_L}{2\pi a_s} \right),
\]

with \( C \approx 0.915 \) (cf. Ref. [35]).

The dashed curves in Fig. 1 correspond to the phase-transition curves, \( T_{MF}(E_b) \), within the mean-field approach. When the system is imbalanced, two changes occur with respect to the balanced case: (i) below a certain value of the binding energy, \( T_{MF} \rightarrow 0 \), and (ii) a tricritical point appears on this curve. The mean-field phase-transition curve splits below this tricritical point and the phase-separation region opens up. In this tricritical point, there is coexistence of the
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FIG. 1. (Color online) Phase diagrams for a 2D imbalanced Fermi gas in the variables \( (T, E_b) \) for different values of the imbalance potential \( \zeta \). The dotted curve indicates the BKT transition to a metastable superfluid state.

The dotted curve in the region of the normal state separates two regimes: (i) the regime where the thermodynamic potential contains only one minimum at \( \Delta = 0 \) above the dotted curve and (ii) the regime where the thermodynamic potential contains two minima: the lower minimum at \( \Delta = 0 \) and a higher one at a value \( \Delta \neq 0 \). This second minimum suggests the existence of a metastable superfluid state.

In Fig. 2, we choose to fix \( \delta n/n \), the relative population imbalance itself, rather than the imbalance potential \( \zeta \), and study the phase diagram in the variables \( (E_b, , n) \). The inset shows a slice of this phase diagram at \( E_b = 0.01 \). At this binding energy, the BKT transition temperature is a non-single-valued function of \( \delta n/n \). This result is not unexpected, because a similar non-single-valued behavior of the critical temperature exists also for the superfluid phase transition in a 3D Fermi gas [36]. In the 3D case this can be related to the Sarma mechanism [37]: at nonzero temperatures, a balanced superfluid system coexists with an imbalanced gas of thermal excitations—these excitations carry some of the excess majority component of the imbalanced gas.

The BKT phase transition for a Fermi gas in 2D is suppressed at lower values of the population imbalance than the superfluid phase transition for a Fermi gas in 3D. Furthermore, this suppression is more strongly revealed at higher binding energies. The reason of such a dependence of the BKT phase transition as a function of the binding energy consists in the following. Let us consider first a part of the phase diagram in the variables \( (E_b, , n) \) taking into attention only the mean-field phase transition. Similar phase diagrams were calculated for a 3D Fermi gas in Ref. [36]. An
ciently high binding energy, the phase-separation region can increase with an increasing coupling strength. As follows from the BKT transition temperature does not unrestrictedly in- restricted both from lower and higher couplings. For the com-parison, the regular superfluid phase transition in a 3D im-balanced Fermi gas is restricted only from lower couplings [38]. We can see from Fig. 3(b) that for a higher (but rela-tively low) imbalance $\delta n/\bar{n}=0.1$, the higher-temperature bound of the phase-separation region lies higher than $T_{BKT}$ for all considered values of the binding energy $E_b$. As a result, for $\delta n/\bar{n}=0.1$ the superfluid state is absent. The obtained behavior of phase diagrams for a 2D Fermi gas confirms our suggestions above.

IV. CONCLUSIONS

In summary, we have described the effects of imbalance on the Berezinskii-Kosterlitz-Thouless superfluid transition in a 2D Fermi gas through the functional integral formalism. Owing to a population imbalance, the superfluid state cannot exist for all values of the coupling strength, but only above a certain critical binding energy which depends on the imbalance. The larger the imbalance potential, the higher is this critical binding energy. As distinct from the balanced case, there is a phase-separated region in phase diagrams, in which no uniform state can exist. As a result, tricritical points appear at the phase diagrams, in which three different regimes coexist. The area of the phase-separated region increases with increasing the population imbalance. Due to the rise of the upper-temperature bound of the instability region, the area of the superfluid state at a fixed relative population im-bal ance decreases with increasing the binding energy. Therefore a population imbalance is a factor destroying superfluid-ity in 2D systems, especially at high binding energies. The BKT transition can be experimentally observable for an im-balanced Fermi gas through the phase separation in a quasi-2D trap. As follows from the obtained results, the par-ameters of the state of that system (e.g., critical tempera-tures and/or density profiles) are expected to be substantially more sensitive to the population imbalance than the corre-sponding parameters for a 3D gas.

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FIG. 3. (Color online) Phase diagram for an imbalanced 2D gas of cold fermions in the variables $\left( T, E_b \right)$ at the relative population imbalance (a) $\delta n/\bar{n}=0.03$ and (b) $\delta n/\bar{n}=0.1$. The denotations are the same as in Fig. 1.
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