Nonlinear Finite Element Analysis of Rod String in Rod Pumping System

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Abstract: Rod pumping is the main oil recovery method at home and abroad. The corresponding displacement and load of the sucker rod section are important parameters in the system. This paper uses the advantages of the finite element method to solve the three nonlinear problems in the rod string model, and establishes its finite element analysis model. According to the upper and lower boundary conditions and initial conditions, the software is used to program and calculate. The surface dynamometer card of multi-level steel rod columns and glass fiber reinforced plastics-steel hybrid rod string is predicted, and the calculation results with and without considering mass nonlinearity are compared and analyzed. The feasibility of predicting the surface dynamometer card by the finite element method is verified. It is proved that the coupling has a great influence on the mechanical properties of the glass fiber reinforced plastics-steel hybrid rod string.

1. Introduction
The rod pumping method is currently the most widely used one at home and abroad. Because the rod string is subjected to complicated forces in the slender wellbore, and it is very difficult to actually measure the downhole specific parameters, it is important to predict the dynamic parameters of the rod string in the rod pumping system for designing the combination and optimizing the parameter in the system[1]. In the 1960s, S G Gibbs of the United States proposed a finite difference method to solve the one-dimensional damped wave equation for vertical wells [2], and later other scholars conducted the further research [3]. In this paper, the three "non-linear" problems generated in the prediction of the mechanical behavior of the rod string in the rod pumping system are analyzed, the application example is used to obtain the prediction results under the combination of pure steel rod columns and combined glass fiber reinforced plastics-steel hybrid rod columns.

2. Rod pumping system and its combined rod string
2.1. Rod pumping system
The schematic diagram of the rod pumping system is shown in Figure 1, it is mainly composed of three parts of the pumping unit, the pump, and the rod string. The reciprocating sucker rod string transmits the motion and power of the ground drive equipment to the oil pump in the lower part of the oil well and draws it for pumping work, the oil in the oil layer is lifted to the surface.
2.2. Combined sucker rod string
Steel sucker rods are widely used in oil wells, with the continuous development of oil well extraction technology, fiberglass sucker rod have also been developed. The sucker rod has low density, light weight, good tensile properties and corrosion resistance, and may cause pump overstroke when used. The fiberglass sucker rod can not be compressed, generally used in combination [4,5].

3. Non-linear problems in rod pumping system
(1) **Nonlinearity of sucker rod string mass** The coupling is an important part of the sucker rod string. The diameter of the coupling is larger than the diameter of the sucker rod, which makes the radial dimension of the coupling abrupt. However, due to the short length of the coupling and the limitation of the calculation method, people omit the coupling quality when calculating. Using the finite element method, the existence of the coupling can be considered.

(2) **Non-linearity of pump load** For the pump work diagram under normal operating conditions, the load on the pump is linear over time, which can be defined as a nonlinear problem as a whole. For the prediction of the rod string, it is necessary to process the load of the pump in sections, and it is very important to determine when the load at the pump changes. When the finite element method is used to solve the calculation, the displacement and load of the lowest point of the pole can be obtained according to the elements in the matrix.

(3) **Non-linearity of pump displacement** Due to the uncertainty of the corresponding time of the piecewise linearly varying pump load, the calculated pump displacement also changes in a jumping manner, which is the nonlinearity of the pump displacement. Generally, the initial pump displacement is set to zero, and a new initial pump displacement will be obtained after a stroke. Using this new pump displacement as the initial value, continue iterative calculation until the convergence condition is satisfied.

There are many ways to solve the wave equation. Because the finite element method is not limited by the calculation process, it is easy to solve these three kinds of nonlinear problems

4. Establish the finite element dynamic equation
Based on certain assumption and analysis, the force analysis of the sucker rod string L is carried out from the perspective of structural dynamics, which is formed by connecting sucker rods through couplings, so each rod is regarded as a secondary rod unit. The coupling is treated as a concentrated mass B. The establishment of the dynamic model of the sucker rod string is shown in Figure 2(a): The upper end of the rod is fixed at the base position A, and A has a certain motion law. The external force
received by the rod is variable, expressed by \( F(t) \). In order to meet the situation of multi-stage rods, the rod diameter and rod material of the long rod \( L \) in the model can be changed in multiple stages \([6,7,8]\).

According to the solution step of the finite element method, the mechanical model obtained in Figure 2 is divided into individual units, and the finite element model of the sucker rod string in Figure 2(b) can be obtained. In this paper, a two-degree-of-freedom primary rod unit is used for division, and there are \( n \) rod units and \((n+1)\) nodes. The displacement, velocity, and acceleration at the donkey head are \( u_b, \dot{u}_b, \ddot{u}_b \). The elastic displacement, velocity, and acceleration vector of each node relative to the donkey head are \( \{u_b\}, \{\dot{u}_b\}, \{\ddot{u}_b\} \), and \( \{u\} = \{u_1, u_2, \ldots, u_3\}^T \). Take the \( i \)-th unit from the model for analysis, as shown in Figure 2(c).

![Fig 2(a), pole and column dynamic model](image1)
![Fig 2(b), pole and column finite element model diagram](image2)
![Figure 2 (c), unit model diagram](image3)

According to the finite element principle, the finite element dynamic equation corresponding to the sucker rod string can be listed as follows:

\[
[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F\} \quad (1)
\]

In the above three formulas: 
- \([M]\) — the overall mass matrix of the rod, the matrix order is \((N+1)\times(N+1)\);
- \([C]\) — the overall damping matrix of the rod string, the matrix order is \((N+1)\times(N+1)\);
- \([K]\) — the overall stiffness matrix of the rod, the matrix order is \((N+1)\times(N+1)\).

\[
\{F\} = -\ddot{u}_b [M]\{1\} - \dot{u}_b [C]\{1\} + \{f\} + g[M]\{1\} \quad (2)
\]

\(\{1\} = \{1, 1, \ldots, 1\}\), \(\{f\} = \{f_1, 0, \ldots, 0, f_N\}\).

For the total load on the rod \( \{F\} \), the expressions of the suspended point velocity and acceleration in it need to be multiplied by \(-1\) to ensure that the same positive direction is defined. The formula of \( g[M]\{1\} \) represents the gravity of the rod, due to it is a constant, it can be omitted and just add it at the end. The symbol of \( f_1 \) expressed as hanging point load; The symbol of \( f_N \) expressed as pump load. Thus:

\[
[M]\{\ddot{u}_b + \ddot{u}\} + [C]\{\dot{u}_b + \dot{u}\} + [K]\{u\} = \{f\} \quad (3)
\]

One of the most commonly used methods for calculating the dynamic response of mechanical structures is the mode superposition method and the other is the direct integration method. The direct integration method solves the kinetic equation step by step in time segments, without simplifying the equation before solving. Therefore, the method named Wilson-\(\theta\) in the direct integration method is used for calculation.
5. Example analysis

5.1. Example analysis of combined steel rod string

A two-stage steel rod column in a well in Changqing is taken as an example. The specific parameters are: the pumping unit model is CYJ10-3-37HB, the rod string combination is 22mm×770m+19mm×880m, the pump diameter is 28mm, the working fluid level is 1550m, the stroke times is 2.5min⁻¹, the stroke is 2.3m. The measured maximum load of the suspension point is 44.7KN, the minimum load of the suspension point is 36.2KN, and the effective stroke of the pump is 2.23m. The measured and predicted indicator graphs of the oil well rod string are shown in Fig. 3(a) and Fig. 3(b). The results of the suspension point load are shown in Table 1.

![Figure 3(a). Actual measured surface dynamometer of a well](image1)

![Figure 3(b). Predicted surface dynamometer of a well](image2)

| Pole load       | Maximum load (KN) | Minimum load (KN) | error(%)   |
|-----------------|-------------------|-------------------|------------|
| Measured        | 44.7              | 36.2              |            |
| Predicted coupling | 44.2              | 37                | -1.12      |
| Predicting without coupling | 43.6              | 36.6              | -2.46      |
| Predicting with coupling | 44.2              | 37                | 2.21       |
| Predicting without coupling | 43.6              | 36.6              | 1.11       |

It can be seen, whether the coupling is considered to have little effect on the load on the suspension point.

5.2. Example analysis of FRP-steel hybrid rod column

The dynamic behavior of the FRP-steel hybrid sucker rod string is predicted, and the Yumen well is selected for analysis. The specific parameters are as follows: the total length of the rod string is 960m, the FRP rod and the steel rod each account for 50%, and the FRP Rod diameter is 25mm, steel rod diameter is 19mm, stroke is 1.5m, punching time is 6min⁻¹. The measured and predicted indicator diagrams of the oil well rod string are shown in Figure 4(a) and Figure 4(b), and the comparative analysis of the results of the suspension point load is shown in Table 2.

![Figure 4(a). Actual measured surface dynamometer of a well](image3)

![Figure 4(b). Predicted surface dynamometer of a well](image4)
Table 2. Comparison of suspension center load results

| Pole load                  | Maximum load (KN) | Minimum load (KN) | error(%) |
|----------------------------|-------------------|-------------------|----------|
| Measured                   | 33.4              | 20                |          |
| Predicted                  | 33                | 20.5              | -1.19    |
| Predicting without coupling| 32                | 21.2              | -4.19    |

Table 3 shows the non-linear time interval of FRP-steel hybrid rod string.

Table 3. Non-linear time interval of FRP-steel hybrid rod string

| Pump load change time period (downward is positive, unit: s) | $t_1 - t_2$ | $t_2 - t_3$ | $t_3 - t_4$ | $t_2 - t_1'$ |
|-------------------------------------------------------------|-------------|-------------|-------------|--------------|
| 0s-1.45s                                                    | 1.45s-4.8s  | 4.8s-6.35s  | 6.35s-10s   |
| Corresponding pump load (KN)                                | /           | ≈9.5        | /           | ≈0           |
| Corresponding pump displacement (m)                         | 0           | /           | 1.9         | /            |

$t_1$ – The time corresponding to the bottom dead center, $t_2$ – The time at which the sucker rod string is fully lifted, $t_3$ – The end of up stroke, $t_4$ – The time at which the sucker rod string as a whole begins to move down, $t_1'$ – The time at which the lower stroke ends and a new cycle begins.

It can be seen, for the FRP-steel hybrid rod string, whether the coupling is considered to have a significant impact on the load on the suspension point. Therefore, the influence of the coupling should be considered in the prediction of the dynamic behavior of the hybrid rod string. The following is the time period corresponding to the nonlinear change of the pump load calculated using the Wilson method and the displacement of the pump calculated by the iterative method.
6. Conclusion

(1) In the longitudinal vibration model of sucker rod string, the load prediction result after considering the mass nonlinearity is closer to the measured value than that obtained without considering the mass nonlinearity. The coupling of the FRP-steel hybrid rod column has a greater influence on the prediction results.

(2) The finite element method can well solve the three nonlinear problems in the prediction of the rod string of the rod pumping system.

(3) Due to the good elasticity of glass fiber reinforced plastics, when the sucker rod string just finishes the down stroke and starts upstroke, the proportion of the total time that the mixing rod string is fully lifted is greater than the proportion of the pure steel rod string.

Acknowledgement

Thanks for the financial support from the research group of the 7th generation ultra-deep water drilling platform (ship) Innovation special drilling package integration and some key equipment application research project (No : Joint Installation [2016] No. 24, Ministry of Industry and Information Technology) of Xi’an Shiyou University.

Reference

[1] Cui Zhenhua, Yu Guoan, etc. Rod pumping system [M]. Beijing: Petroleum Industry Press, 1994.
[2] S.G. Gibbs. Predicting the Behavior of Sucker Rod Pumping Systems [J]. JPT. July 1963: 769-778.
[3] Li Zifeng. General situation and development trend of research on impact dynamics of oil and gas well string [J]. Journal of Petroleum, 2019, 40 (05): 604-610.
[4] Wang Wenchang, Di Qinfeng, Yao Jianlin et al. A new method for finite element analysis of the mechanical properties of sucker rod strings in three-dimensional directional wells [J]. Journal of Petroleum, 2010, 31 (06): 1018-23.
[5] Peng Huifen, Wang Baokun, Wang Cheng. Transient dynamic analysis of carbon fiber composite sucker rod [J]. Petroleum Mine Machinery, 2018, 47 (5): 24-28.
[6] Peng Yong, Yu Guoan, Gao Guohua. Finite element model of sucker rod string and diagnostic solution [J]. Petroleum Machinery, 1993, 21 (12): 44-48.
[7] Peng Yong, Wang Hongfei, Yu Guoan, et al. Difference decomposition and convergence of finite element dynamic equations for sucker rod strings [J]. Journal of Xi’an Shiyou Institute, 1994, 9(3): 42-46.
[8] Peng Yong. Application of pattern recognition technology in petroleum industry [M]. Shaanxi: Shaanxi Science and Technology Press, 1998.
[9] Xu Jun. Research on the analytical method of the dynamic model of the hybrid sucker rod string of FRP rod and steel rod [J]. Petroleum Machinery, 1994, 22 (01): 39-45.