Stability Enforced Bandit Algorithms for Channel Selection in State Estimation and Control

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Abstract—In this paper we consider state estimation and control problems where a sensor or controller can, at each discrete time instant, transmit on one out of $M$ different communication channels. A key difficulty of the situation at hand is that the channel statistics are unknown. We study the case where both learning of the channel reception probabilities and state estimation / control is carried out simultaneously. Methods for choosing the channels based on techniques for multi-armed bandits are presented, and shown to provide stability. Furthermore, we define the performance notions of estimation and control regret, and derive bounds on how it scales with time for the considered algorithms.

I. INTRODUCTION

The use of machine learning techniques for estimation and control has gained increasing attention in recent years [1]–[3]. Learning based control holds the promise of enabling the solution of problems which are difficult or even intractable using traditional control design techniques.

In this paper we focus on remote state estimation and networked control problems where the channel statistics are unknown. As a motivating example, consider the use of unmanned aerial vehicles (UAVs) for providing wireless communication capabilities [4], [5], with the UAV acting as a mobile aerial base station collecting information from wireless sensors. The use of such systems can allow for monitoring of processes which are difficult to access otherwise, e.g., in a hostile or contested environment. The UAV would transmit collected information to a remote estimator or controller via one of $M$ different channels, e.g., using $M$ different frequency bands. Characteristics of the individual channels can vary substantially at different locations/environments [4] over complex terrain, which makes it difficult to have accurate knowledge of the channel statistics whenever the UAV moves to a different location.

Learning of transmission schedules for a single system over a channel with unknown packet reception probability has been studied in [6], while learning of power allocations for multiple control systems connected over an unknown non-stationary channel is considered in [7]. With multiple channels, channel allocation for multiple processes using deep reinforcement learning have been considered in [8]–[10]. While the use of deep reinforcement learning techniques can find schedules which perform well, in general stabilizing properties of the learned policies are not guaranteed. Furthermore, many training samples may be needed during learning. In this paper, we consider state estimation and control problems where, at every time instant, a sensor or controller can choose one of $M$ unknown channels for transmission. The aim is to learn the best channel while guaranteeing stability of the system and being sample efficient (via maintaining low regret).

We will view the channel selection task as a multi-armed bandit type problem [11], [12]. The multi-armed bandit problem is a simple example of a reinforcement learning problem which captures the exploration vs. exploitation tradeoff, between basing decisions only on knowledge currently obtained about a system (“exploit”) vs. trying out alternative decisions which may potentially be better (“explore”). In contrast to classical multi-armed bandit problems, here the underlying processes are dynamical systems, which raises additional challenges on stability and performance. In this paper, we will study the use of the $\varepsilon$-greedy algorithm [13], as well as several algorithms based on the Thompson Sampling technique [14], to carry out channel selection in the context of remote state estimation and networked control. Thompson Sampling is a sampling based algorithm for the multi-armed bandit problem, which has been shown to be optimal with respect to the accumulated regret in certain problems. Interestingly, Thompson sampling has been previously used to address problems in stochastic [15], [16] and linear quadratic control [17].

Summary of contributions: The main contributions of the current work are as follows:

- We propose the use of bandit algorithms for channel selection in the context of remote state estimation and networked control, including a new stability-aware Bayesian sampling algorithm.
- We prove that the considered algorithms can guarantee stability.
- We study performance of these algorithms by introducing the notions of estimation and control regret, and prove that the $\varepsilon$-greedy algorithm has linear regret, while the sampling based algorithms can achieve logarithmic regret.

The remainder of the paper is organized as follows: We first consider a remote state estimation problem, with the system model provided in Section II. A selection of bandit algorithms for channel selection are presented in Section III. Stability and performance bounds of these algorithms are studied in Section IV.
Sections [IV] and [V] respectively. Section [VI] extends the results to a networked control problem. Numerical studies are given in Section [VII] Section [VIII] draws conclusions.

Notation: Given a matrix $X$, we say that $X \geq 0$ if $X$ is positive semi-definite. For matrices $X$ and $Y$, we say that $X \leq Y$ if $Y - X \geq 0$. The spectral radius of a matrix $X$ is denoted by $\rho(X)$. The beta distribution with parameters $\alpha$ and $\beta$ is denoted by $\text{Beta}(\alpha, \beta)$, with expected value $\alpha / (\alpha + \beta)$. $\mathbb{I}(\cdot)$ is the indicator function that returns 1 if its argument is true and 0 otherwise.

II. REMOTE STATE ESTIMATION

Throughout the first part of this work, we will focus on a remote state estimation scenario as depicted in Fig. 1. Section [VI] will then outline how the results translate to networked systems with closed loop control.

A. System Model

We consider a discrete-time linear plant

$$x_{k+1} = Ax_k + w_k$$

with sensor measurements

$$y_k = Cx_k + v_k$$

where $w_k \sim \mathcal{N}(0, Q)$, $v_k \sim \mathcal{N}(0, R)$, see Fig. 1. The sensor runs a local Kalman filter, with local state estimates $\hat{x}_k^s \triangleq \mathbb{E}[x_k|y_0, \ldots, y_k]$ and estimation error covariances $P_k^s \triangleq \mathbb{E}[(x_k - \hat{x}_k^s)(x_k - \hat{x}_k^s)^\top|y_0, \ldots, y_k]$. We assume that the pair $(A, C)$ is observable and the pair $(A, Q^{1/2})$ is controllable, which implies that $P_k^s$ converges to some $\mathcal{P}$ as $k \to \infty$ [18]. We will assume steady state, so that $P_k^s = \mathcal{P}$, $\forall k$.

As depicted in Fig. 1, the local state estimates are transmitted over lossy channels to a remote estimator. There are $M$ such channels, each i.i.d. Bernoulli with packet reception probabilities $\gamma_m$, $m = 1, \ldots, M$. For notational simplicity, we will assume that all the packet reception probabilities are different, i.e., $\theta_i \neq \theta_j$, $\forall i \neq j$.

At every time step $k$, we can choose one of the $M$ channels to transmit over. This decision is determined by the remote estimator, which notifies the sensor via a transmission (Tx) management channel. As the information transmitted on the transmission management channel is normally only a few bits in length, we assume that these transmissions are error-free, see also [3].

A key issue is that none of the packet reception probabilities are known. Ideally, we would like to always use the best channel (i.e., the channel with the highest packet reception probability). But since the packet reception probabilities are unknown, this requires us to find/learn the best channel by trying out different channels and observing their outcomes.

The situation resembles a multi-armed bandit type problem [11], [12], with the channels acting as the different arms. The difference between the situation considered in this paper and classical multi-armed bandit problems is that the underlying estimation process has dynamics. This raises additional issues such as stability, as investigated in the current work.

Before proceeding, we recall (see, e.g., [19]) that the remote estimator has estimation error covariance

$$P_k = \begin{cases} \mathcal{P}, & \gamma_k = 1 \\ h(P_{k-1}), & \gamma_k = 0 \end{cases}$$

where

$$h(X) \triangleq AXA^\top + Q.$$

In the above, $\gamma_k = 1$ means that the transmission at time $k$ was successful, while $\gamma_k = 0$ means that the transmission was lost. We assume that $P_0 = \mathcal{P}$.

When $A$ is unstable, we know that if the same channel $m$ is chosen at every time step, then the remote estimator has bounded expected estimation error covariance for all $k$ if and only if its packet reception probability satisfies $\theta_m > \theta_c$ [20], [21], where

$$\theta_c \triangleq 1 - \frac{1}{\rho(A)^2}.$$  

Accordingly, in this paper, we will make the assumption that at least one of the channels has packet reception probability larger than $\theta_c$, i.e.,

$$\theta^* \triangleq \max_m \theta_m > \theta_c.$$  

B. Preliminaries

For later reference, we mention some properties of the operator $h(.)$ defined in (3).

**Lemma 1.** The operator $h(.)$ defined in (3) satisfies:

(i) $h(X) \leq h(Y)$ if $X \leq Y$

(ii) $h(\mathcal{P}) \geq \mathcal{P}$

(iii) $\text{tr}(h(\mathcal{P})) > \text{tr}(\mathcal{P})$.

**Proof.** The proof of (i) follows easily from the definition (3). Proofs of (ii) and (iii) can be found in [22].

Note that Lemma 1 implies that $P_k \geq \mathcal{P}$, $\forall k$, for $P_k$ evolving according to (2).

The following recursion for the expected error covariances will also be used later. Let $\theta(k)$ be the reception probability at time $k$. Then, it follows from

$$\mathbb{E}[P_{k+1}|P_k] = \theta(k)\mathcal{P} + (1 - \theta(k))(AP_kA^\top + Q),$$

that

$$\mathbb{E}[P_{k+1}] = \theta(k)\mathcal{P} + (1 - \theta(k))h(\mathbb{E}[P_k]).$$  

Fig. 1: Remote state estimation over unknown channels
Algorithm 1 \( \varepsilon \)-greedy

1. Initialize: \( \alpha_{m,1} = 1, \beta_{m,1} = 1, \ m = 1, \ldots, M \)
2. for \( k = 1, 2, \ldots \) do
3. Set \( \hat{\theta}_{m,k} = \frac{\alpha_{m,k}}{\alpha_{m,k} + \beta_{m,k}} \), \( m = 1, \ldots, M \)
4. With probability \( \varepsilon > 0 \), choose \( m_k^* \) uniformly from \( \{1, \ldots, M\} \), else choose \( m_k^* = \arg\max_{m} \hat{\theta}_{m,k} \)
5. Observe \( \gamma_k \)
6. Update sample means
   \[
   \begin{align*}
   \alpha_{m,k+1} & = \alpha_{m,k} + \gamma_k, & \text{if} & m = m_k^* \\
   \beta_{m,k+1} & = \beta_{m,k} + 1 - \gamma_k, & \text{if} & m \neq m_k^* \\
   \alpha_{m,k+1} & = \alpha_{m,k}, \beta_{m,k+1} = \beta_{m,k}, & \text{if} & m \neq m_k^*
   \end{align*}
   \]

III. BANDIT ALGORITHMS FOR CHANNEL SELECTION

In this section we present a selection of multi-armed bandit algorithms for carrying out channel selection, mainly based on the \( \varepsilon \)-greedy and Thompson sampling approaches [14]. Thompson sampling is computationally efficient and can be easily implemented, requiring very little tuning apart from the choice of a prior distribution. There exist other bandit algorithms such as those based on the upper confidence bound (UCB) approach, which are not considered in the current work. Numerical studies however have shown that in many situations the performance of Thompson sampling based approaches is highly competitive even against well-tuned UCB approaches [23-25].

A. \( \varepsilon \)-greedy

We first consider the well-known \( \varepsilon \)-greedy algorithm [13], which with probability \( \varepsilon > 0 \) chooses a channel uniformly at random (“explore”), and with probability \( 1 - \varepsilon \) chooses the channel with the maximum estimated (via a sample mean) packet reception probability (“exploit”). The procedure is stated formally in Algorithm 1. In Algorithm 1 we have expressed the estimated packet reception probabilities in terms of the quantities \( \alpha_{m,k} \) and \( \beta_{m,k} \). This allows us to emphasize the similarities in the implementation with the other algorithms considered in this paper.

B. Thompson Sampling

Thompson sampling is a sampling based approach which draws samples \( \hat{\theta}_{m,k} \), \( m = 1, \ldots, M \) from the posterior distribution of the unknown packet reception probability of each arm/channel. The posterior distribution can be shown to have a beta distribution if the prior distribution is beta distributed [14], as we will assume here. In particular, the prior beta distribution with \( \alpha_{m,1} = 1, \beta_{m,1} = 1, \ m = 1, \ldots, M \) is uniformly distributed in \( [0, 1] \). The channel that is selected is then given by

\[ m_k^\text{TS} = \arg\max_m \hat{\theta}_{m,k} \]

Note that if multiple channels have the same maximum estimated packet reception probability, then a channel is chosen uniformly at random from these (maximizing) channels.

Algorithm 2 Thompson Sampling

1. Initialize: \( \alpha_{m,1} = 1, \beta_{m,1} = 1, \ m = 1, \ldots, M \)
2. for \( k = 1, 2, \ldots \) do
3. Sample \( \theta_{m,k} \sim \text{Beta}(\alpha_{m,k}, \beta_{m,k}) \), \( m = 1, \ldots, M \)
4. Choose \( m_k^\text{TS} = \arg\max_m \theta_{m,k} \) and observe \( \gamma_k \)
5. Update posterior distributions
   \[
   \begin{align*}
   \alpha_{m,k+1} & = \alpha_{m,k} + \gamma_k, & \text{if} & m = m_k^\text{TS} \\
   \beta_{m,k+1} & = \beta_{m,k} + 1 - \gamma_k, & \text{if} & m \neq m_k^\text{TS} \\
   \end{align*}
   \]

Algorithm 3 Optimistic Bayesian Sampling (OBS)

1. Initialize: \( \alpha_{m,1} = 1, \beta_{m,1} = 1, \ m = 1, \ldots, M \)
2. for \( k = 1, 2, \ldots \) do
3. Sample \( \theta_{m,k} \sim \text{Beta}(\alpha_{m,k}, \beta_{m,k}) \), \( m = 1, \ldots, M \)
4. Set \( \hat{\theta}_{m,k} = \frac{\alpha_{m,k}}{\alpha_{m,k} + \beta_{m,k}} \)
5. Choose \( m_k^\text{OBS} = \arg\max_m \hat{\theta}_{m,k} \) and observe \( \gamma_k \)
6. Update posterior distributions
   \[
   \begin{align*}
   \alpha_{m,k+1} & = \alpha_{m,k} + \gamma_k, & \text{if} & m = m_k^\text{OBS} \\
   \beta_{m,k+1} & = \beta_{m,k} + 1 - \gamma_k, & \text{if} & m \neq m_k^\text{OBS} \\
   \alpha_{m,k+1} & = \alpha_{m,k}, \beta_{m,k+1} = \beta_{m,k}, & \text{if} & m \neq m_k^\text{OBS}
   \end{align*}
   \]

After transmitting over this channel and observing \( \gamma_k \), i.e. whether the transmission was successful, the posterior distribution is then updated for the currently chosen channel \( m = m_k^\text{TS} \), using the formulas \( \alpha_{m,k+1} = \alpha_{m,k} + \gamma_k \) and \( \beta_{m,k+1} = \beta_{m,k} + 1 - \gamma_k \). The procedure is summarized in Algorithm 2.

C. Optimistic Bayesian Sampling

In [24] a modification of Thompson sampling called “Optimistic Bayesian sampling” (OBS) was proposed and studied. Recall that in Thompson sampling, at time \( k \) one would draw samples \( \hat{\theta}_{m,k} \), \( m = 1, \ldots, M \) and then select \( m_k^\text{TS} = \arg\max_m \hat{\theta}_{m,k} \). In OBS we would also draw samples \( \hat{\theta}_{m,k} \), \( m = 1, \ldots, M \), but now select

\[ m_k^\text{OBS} = \arg\max_m \hat{\theta}_{m,k}, \]

where

\[ \hat{\theta}_{m,k} = \max \left( \frac{\beta_{m,k}}{\beta_{m,k} + \alpha_{m,k}}, \frac{\alpha_{m,k}}{\alpha_{m,k} + \beta_{m,k}} \right). \]

The procedure is summarized in Algorithm 3. The motivation for OBS is to result in increased selection probabilities for arms which are uncertain [24]. For classical multi-armed bandit problems, simulations have shown that OBS can outperform Thompson sampling in certain situations [24, 26].

D. Stability-aware Bayesian Sampling

In the context of selecting channels for the purpose of remote state estimation of dynamical systems as in [4], it
The method is summarized in Algorithm 4.

Algorithm 4 Stability-aware Bayesian Sampling (SBS)

1: Initialize: $\alpha_{m,1} = 1, \beta_{m,1} = 1, \ m = 1, \ldots, M$
2: for $k = 1, 2, \ldots$ do
3: Sample $\tilde{\theta}_{m,k} \sim \text{Beta}(\alpha_{m,k}, \beta_{m,k}), \ m = 1, \ldots, M$
4: Set $\hat{\theta}_{m,k}^{\text{SBS}} = \eta_{m,k}\theta_{m,k}^{\text{obs}} + (1 - \eta_{m,k})\tilde{\theta}_{m,k}^{\text{TS}}, \ \text{where} \ \theta_{m,k}^{\text{obs}} = \max\left(\tilde{\theta}_{m,k}^{\text{TS}}, \frac{\alpha_{m,k}}{\alpha_{m,k} + \beta_{m,k}}\right)$ and $\eta_{m,k} = 1(\alpha_{m,k}/(\alpha_{m,k} + \beta_{m,k}) > \theta_c)$
5: Choose $m_k^{\text{SBS}} = \arg \max_m \hat{\theta}_{m,k}^{\text{SBS}}$ and observe $\gamma_k$
6: Update posterior distributions
   - $\alpha_{m,k+1} = \alpha_{m,k} + \gamma_k$, if $m = m_k^{\text{SBS}}$
   - $\beta_{m,k+1} = \beta_{m,k} + 1 - \gamma_k$, if $m = m_k^{\text{SBS}}$
   - $\alpha_{m,k+1} = \alpha_{m,k}, \ \beta_{m,k+1} = \beta_{m,k}$, if $m \neq m_k^{\text{SBS}}$

is important to take into account the long term estimation performance. In particular, the fundamental bound (4) suggests that, unlike when using OBS, one should not artificially stimulate the selection probabilities for channels which seem to be poor. More specifically, if the current mean-estimate $\alpha_{m,k}/(\alpha_{m,k} + \beta_{m,k})$ of the reception probability is less than the critical threshold $\theta_c$, then it is questionable whether this channel should be chosen with a probability higher than that prescribed by Thompson sampling.

Motivated by the above, in this paper, we propose a stability-aware modification of OBS which uses the OBS sample $\theta_{m,k}^{\text{obs}}$ only if $\alpha_{m,k}/(\alpha_{m,k} + \beta_{m,k}) > \theta_c$, while using the Thompson sample $\tilde{\theta}_{m,k}^{\text{TS}}$ otherwise. The algorithm, here named Stability-aware Bayesian Sampling (SBS), can thus be characterized by:

$$\hat{\theta}_{m,k}^{\text{SBS}} = \eta_{m,k}\theta_{m,k}^{\text{obs}} + (1 - \eta_{m,k})\tilde{\theta}_{m,k}^{\text{TS}}, \ m = 1, \ldots, M$$ (7)

and

$$m_k^{\text{SBS}} = \arg \max_m \hat{\theta}_{m,k}^{\text{SBS}},$$

where

$$\eta_{m,k} = \begin{cases} 1 & \text{if } \alpha_{m,k}/(\alpha_{m,k} + \beta_{m,k}) > \theta_c \\ 0 & \text{otherwise} \end{cases}$$ (8)

The method is summarized in Algorithm 4.

IV. STABILITY OF REMOTE STATE ESTIMATION WITH CHANNEL SELECTION

In this section, we show that the $\varepsilon$-greedy algorithm will ensure stability of the remote estimator, provided the exploration rate $\varepsilon$ is not too high, while all of the sampling based channel selection schemes presented in the previous section (Thompson sampling, OBS, and SBS) will ensure stability. It is important to emphasize that Theorems 1 and 2 stated below, cover the challenging case where channel qualities are unknown, and where channel selection and learning are done in real-time.

Let $m_k$ be the channel chosen at time $k$, and define

$$m^* \triangleq \arg \max_m \theta_m.$$ (9)

Theorem 1. Suppose condition (5) holds. Then under the $\varepsilon$-greedy algorithm, the expected estimation error covariance $\mathbb{E}[P_k]$ is bounded for all $k$ if and only if

$$0 < \varepsilon < \frac{\theta^* - \theta_c}{\theta^* - \frac{1}{M} \sum_{m=1}^M \theta_m}.$$ (10)

Proof. Because of the exploration phase, where with probability $\varepsilon > 0$ a channel is chosen randomly, it is easy to see that each channel will be accessed infinitely often. Hence the estimates of the packet reception probabilities for each of the channels will converge to their true values as $k \to \infty$. Thus, during the exploitation phase we have

$$\mathbb{P}(m_k = m^*|\text{exploit}) \to 1 \text{ as } k \to \infty,$$

while during the exploration phase we have

$$\mathbb{P}(m_k = m^*|\text{explore}) = \frac{1}{M}.$$

Then, given any $\delta > 0$, there exists a $K(\delta) < \infty$ such that

$$\mathbb{P}(\text{reception at time } k) \geq (1-\varepsilon)\theta^*(1-\delta) + \frac{\varepsilon}{M} \sum_{m=1}^M \theta_m$$

for all $k > K(\delta)$. Now let

$$\theta' \triangleq (1-\varepsilon)\theta^*(1-\delta) + \frac{\varepsilon}{M} \sum_{m=1}^M \theta_m.$$ (11)

For $k > K(\delta)$, we have by Lemma 1 and a similar derivation to (5) that

$$\mathbb{E}[P_{k+1}] \leq \theta' \mathbb{P} + (1 - \theta')h(\mathbb{E}[P_k]).$$

By induction, it follows that $\{\mathbb{E}[P_k]\}$ will be upper bounded by a sequence $\{V_k\}$ defined by

$$V_k = \mathbb{E}[P_k], \ k \leq K(\delta)$$

$$V_{k+1} = \theta' \mathbb{P} + (1 - \theta')h(V_k), \ k \geq K(\delta).$$

The sequence $\{V_k\}$ converges if and only if $\theta' > \theta_c$. If

$$(1 - \varepsilon)\theta^* + \frac{\varepsilon}{M} \sum_{m=1}^M \theta_m > \theta_c,$$ (12)

then one can always find a sufficiently small $\delta > 0$ to satisfy $\theta' > \theta_c$. As (12) is equivalent to the condition (9), this proves the “if” direction of the theorem.

For the “only if” direction, first define

$$\tilde{\theta} \triangleq (1-\varepsilon)\theta^* + \frac{\varepsilon}{M} \sum_{m=1}^M \theta_m,$$ (13)

and note that, for a given $\varepsilon$,

$$\mathbb{P}(\text{reception at time } k) \leq \tilde{\theta}$$

holds, because $\tilde{\theta}$ is the reception probability assuming the optimal $m^*$ is always chosen during the exploitation phase. If $\varepsilon \geq \theta^* - \theta_c)/\theta^* - \frac{1}{M} \sum_{m=1}^M \theta_m$, then $\mathbb{P}(\text{reception at time } k) \leq \tilde{\theta} \leq \theta_c$, and thus

$$\mathbb{E}[P_{k+1}] \geq \theta_c + (1 - \theta_c)h(\mathbb{E}[P_k]).$$
Define a sequence \( \{ \hat{V}_k \} \) by
\[
\hat{V}_{k+1} = \theta_c \mathcal{P} + (1 - \theta_c)h(\hat{V}_k),
\]
which from the definition of \( \theta_c \) is a divergent sequence. An induction argument shows that \( \mathbb{E}[P_k] \geq \hat{V}_k, \forall k \). This implies that \( \mathbb{E}[P_k] \) also diverges and completes the proof. \( \square \)

**Remark 1.** If \( (\theta^* - \theta_c)/(\theta^* - \frac{1}{M} \sum_{m=1}^{M} \theta_m) > 1 \), then Theorem 7 establishes that the \( \varepsilon \)-greedy algorithm provides estimation stability for all \( \varepsilon > 0 \).

We next consider stability of the sampling based algorithms. We first give a preliminary result.

**Lemma 2.** Under i) Thompson sampling, ii) OBS, and iii) SBS, all channels will be used infinitely often, and
\[
P(m_k = m^*) \to 1 \quad \text{as} \quad k \to \infty. \tag{12}
\]

**Proof.** For Thompson sampling and OBS, this is proved in [24] (see also [23] for the two armed bandit case under Thompson sampling). For SBS we note that, in view of (7), it holds that \( \hat{\theta}_{TS}^{m,k} \leq \hat{\theta}_{SBS}^{m,k} \leq \hat{\theta}_{OBS}^{m,k} \). Thus, intuitively (12) should also hold, as SBS in a sense lies in between Thompson sampling and OBS (and convergence holds for both of these schemes). A rigorous proof of Lemma 2 for SBS can be given using similar arguments as in [24]. The details are omitted for brevity. \( \square \)

**Theorem 2.** Suppose condition 5 holds. Then under i) Thompson sampling, ii) OBS, and iii) SBS, the expected error covariance \( \mathbb{E}[P_k] \) is bounded for all \( k \).

**Proof.** From Lemma 2 we know that under all three sampling based schemes, given any \( \varepsilon > 0 \), there exists a \( K(\varepsilon) < \infty \) such that
\[
P(\text{reception at time } k) \geq \theta^*(1 - \varepsilon), \forall k > K(\varepsilon).
\]
Pick a sufficiently small \( \delta \) such that \( \theta^* \leq \theta^*(1 - \delta) > \theta_c \) still holds. Then for \( k > K(\delta) \), we have by Lemma 1 and a similar derivation to (6) that
\[
\mathbb{E}[P_{k+1}] \leq \theta^* \mathcal{P} + (1 - \theta^*)h(\mathbb{E}[P_k]).
\]
Define a sequence \( \{ V_k \} \) by
\[
V_k = \mathbb{E}[P_k], \quad k \leq K(\delta)
\]
\[
V_{k+1} = \theta^* \mathcal{P} + (1 - \theta^*)h(V_k), \quad k \geq K(\delta).
\]
Using an induction argument, we can show that \( V_k \) upper bounds \( \mathbb{E}[P_k] \) for all \( k \). Since \( \theta^* > \theta_c \), the sequence \( \{ V_k \} \) converges, and in particular \( V_k \) is bounded for all \( k \). Hence \( \mathbb{E}[P_k] \) is also bounded for all \( k \).

**V. Performance Bounds for Remote State Estimation**

In the multi-armed bandit literature, it is common to analyze the performance of an algorithm via the notion of regret [11], [12], which is defined as the difference between the expected cumulative reward from always playing the optimal arm and the expected cumulative reward using a particular bandit algorithm.

For the remote estimation problem at hand, it is convenient to regard the trace of the estimation error covariance \( trP_k \) as a cost, or alternatively regard \(-trP_k\) as a reward, see also [8]. Motivated by the notion of regret in multi-armed bandit problems, in this paper we define the estimation regret over a horizon \( T \) for the remote state estimation problem as
\[
\text{regret}^E(T) \triangleq \sum_{k=1}^{T} ( -tr\mathbb{E}[P_k^*] - tr\mathbb{E}[P_k])
\]
\[
= \sum_{k=1}^{T} tr\mathbb{E}[P_k - \mathbb{E}[P_k^*]] \tag{13}
\]
In (13), \( \mathbb{E}[P_k^*] \) is the expected error covariance assuming that the optimal channel \( m^* \) is always chosen, and \( \mathbb{E}[P_k] \) is the expected error covariance when using a particular channel selection algorithm. Thus, (13) constitutes a measure of the degree of suboptimality incurred from using a particular algorithm. Stationary and transient performance of an algorithm can be quantified by inspecting how fast the regret grows in relation to the horizon length \( T \).

To state our results, we first recall some order notation.

**Definition 1.** Given functions \( f(\cdot) \) and \( g(\cdot) \), we say that \( f(T) = O(g(T)) \) if there exist constants \( c \) and \( T_0 \), such that \( |f(T)| \leq cg(T), \forall T \geq T_0 \). We say that \( f(T) = \Omega(g(T)) \) if \( g(T) = O(f(T)) \), and that \( f(T) = \Theta(g(T)) \) if both \( f(T) = O(g(T)) \) and \( f(T) = \Omega(g(T)) \). We say that \( f(T) = o(g(T)) \) if \( \lim_{T \to \infty} f(T)/g(T) = 0 \).

For \( \varepsilon \)-greedy, the regret scales linearly with \( T \).

**Theorem 3.** Suppose conditions 5 and 9 hold. Then under the \( \varepsilon \)-greedy algorithm, we have
\[
\text{regret}^E(T) = \Theta(T). \tag{14}
\]

**Proof.** We first show that \( \text{regret}^E(T) = O(T) \). By Theorem 1 there exists some bounded \( P^* \) such that
\[
\mathbb{E}[P_k] \leq P^*, \quad \forall k.
\]
We have
\[
\text{regret}^E(T) = \sum_{k=1}^{T} tr\mathbb{E}[P_k - \mathbb{E}[P_k^*])
\]
\[
\leq \sum_{k=1}^{T} tr\mathbb{E}[P_k]
\]
\[
\leq tr(P^*)T = O(T).
\]
We now show that \( \text{regret}^E(T) = \Omega(T) \). Recall the definition \( \hat{\theta} \) from (11). Note that we have
\[
\mathbb{E}[P_k] \geq \hat{\theta} \mathcal{P} + (1 - \hat{\theta})h(\mathbb{E}[P_{k-1}])
\]
\[
\geq \hat{\theta} \mathcal{P} + (1 - \hat{\theta})h(\mathbb{E}[P^*_{k-1}])
\]
\[
= \theta^* \mathcal{P} + (1 - \theta^*)h(\mathbb{E}[P^*_{k-1}] - \mathcal{P})
\]
\[
= \min_{k \geq 1} tr(h(\mathbb{E}[P_{k-1}] - \mathcal{P}) \triangleq r. \tag{14}
\]

\[
\text{tr}(\mathbb{E}[P_k] - \mathbb{E}[P_k^*]) \geq (\theta^* - \hat{\theta}) \min_{k \geq 1} tr(h(\mathbb{E}[P_{k-1}] - \mathcal{P})
\]
\[
\geq (\theta^* - \hat{\theta}) r.
\]
By the definition (11) and Lemma 1, we have $r > 0$. Hence
\[
\text{regret}^E(T) = \sum_{k=1}^{T} \text{tr}(E[P_k] - E[P'_k]) \\
\geq rT = \Omega(T).
\]

For the Bernoulli bandit problem with per stage rewards in \{0, 1\}, it is known that the regret scales logarithmically with $T$ under Thompson sampling \cite{27,28}. An “age-of-information regret” measure was recently considered in \cite{25}, where the per stage cost is unbounded, and can increase by (at most) one at every time step. It is shown that this age-of-information regret also scales logarithmically with $T$. In the current work, it follows from \cite{25} that the per stage cost $E[P_k]$ (or reward $-E[P_k]$) may also become unbounded, and furthermore can increase at an exponential rate. Interestingly, for the estimation regret introduced in \cite{13}, a bound that is logarithmic in $T$ still holds.

Let us refer to sub-optimal channels as those whose packet reception probabilities are not equal to the optimal $\theta^*$. We begin with a preliminary result.

**Lemma 3.** Let $N_{\text{sub}}(T)$ denote the number of uses of sub-optimal channels over horizon $T$. Then, under i) Thompson sampling, ii) OBS, and iii) SBS, we have
\[
E[N_{\text{sub}}(T)] = \Theta(\log T).
\]

**Proof.** The property that $E[N_{\text{sub}}(T)] = O(\log T)$ is shown for Thompson sampling in \cite{27,28}. The corresponding result for OBS is proved in \cite{26}, based on similar arguments as in \cite{27}. For SBS, $E[N_{\text{sub}}(T)] = O(\log T)$ can be shown by making use of the relation $\theta_{\text{sub}} \leq \theta_{\text{SBS}} \leq \theta_{\text{OBS}}$ and the arguments of \cite{26}. The (rather lengthy) details are omitted for brevity.

We next recall a result from \cite{29}, namely that any policy with $R(T) = o(T^a)$ for every $a > 0$ satisfies $E[N_{\text{sub}}(T)] = \Omega(\log T)$, where $R(T) \triangleq \sum_{k=1}^{T} (\theta^* - \theta(k))$ is the classical notion of regret for the Bernoulli bandit problem, see e.g. \cite{27,28}. Since for Thompson sampling, OBS, and SBS:
\[
R(T) = O(E[N_{\text{sub}}(T)]) = O(\log T) = o(T^a), \forall a > 0,
\]
where the first equality follows from standard arguments \cite{27,28} and the second equality is what we have just shown, the property $E[N_{\text{sub}}(T)] = \Omega(\log T)$ therefore holds for all three schemes.

**Theorem 4.** Suppose that condition (5) holds. Then under i) Thompson sampling, ii) OBS, and iii) SBS, we have
\[
\text{regret}^E(T) = \Theta(\log T).
\]

**Proof.** Before giving the formal proof of this result, we first provide a sketch of the proof strategy for the upper bound $\text{regret}^E(T) = O(\log T)$. Following the idea of \cite{25}, which showed a logarithmic bound for the age-of-information regret, we will upper bound the estimation regret with the regret that is achieved using an alternative schedule, say $A$, that replaces all uses of sub-optimal channels with the worst channel. In \cite{25}, this schedule $A$ is then further upper bounded by a “worst-case” schedule $B$ that groups all the uses of the worst channel together. However, for the problem at hand, as the expected error covariance can increase exponentially fast if the worst channel does not stabilize the remote estimator, this argument will give the desired logarithmic regret bound only in problem instances where the worst channel (and hence all channels) is stabilizing. To cover more interesting and challenging situations, to establish Theorem 4 we will consider a division of the time interval into “epochs” separated by successive uses of the worst channel, see Fig. 2. Through a careful analysis, we will show that the accumulated regret within each epoch is bounded, while the expected number of epochs is logarithmic in $T$. As a consequence the required logarithmic regret upper bound is established.

The detailed proof can be found in Appendix A.

**Remark 2.** We note that some works on learning based control have made assumptions that the underlying system is stable \cite{30}, or can be stabilized for all possible controls \cite{17}, when carrying out a regret analysis. This stands in contrast to Theorem 7 stated above, which merely requires that at least one of the $M$ channels is stabilizing for the remote estimator. Our result thus considers scenarios where some of the sub-optimal channels can cause the expected error covariance to diverge if they are used too often.

**VI. Extension to Networked Control.** We will next show how the analysis presented in the preceding sections can be applied to closed loop control systems with random dropouts in the links between controller and plant.

---

Fig. 2: Channel selection based on schedule $A$, where ‘w’ and ‘b’ refer to the worst and the best channel selection, respectively.

Fig. 3: Networked control over unknown channels.
Let \( e \) be an error vector that is the difference between the true state \( x_k \) and the estimated state \( \hat{x}_k \). For state estimation, the controller uses an estimator \( \hat{x}_k \) for state estimation as
\[
\hat{x}_{k+1} = Ax_k + Bu_k + w_k,
\]
where \( u_k \) is the control input at time \( k \), and \( w_k \sim \mathcal{N}(0, Q) \). We assume that the pair \( (A, B) \) is controllable. For the networked control problem considered in this section, we will assume that the sensor’s measurement is noise-free, i.e., \( x_k \) is known to the sensor.

The controller works in a time-division-duplex manner, i.e., either the uplink (sensor-controller) or the downlink (controller-actuator) transmission is scheduled in each time slot. There are \( M \) unknown i.i.d. Bernoulli channels available for uplink and downlink data transmissions. Since the sensor and the actuator are co-located, and due to the channel reciprocity property, uplink and downlink transmissions at channel \( m \) are assumed to have the same channel quality and hence have identical packet reception probabilities \( \theta_m, m = 1, \ldots, M \). Let \( \gamma^U_k = 1 \) and \( \gamma^D_k = 1 \) denote a transmission success at time \( k \) on the uplink and downlink, respectively.

Furthermore, we assume that the controller schedules the uplink and downlink transmissions using a persistent scheduling policy \([10], [31]\). Such a policy keeps scheduling the sensor’s transmission until it is successful, and then keeps scheduling the controller’s transmission until success, and so on. Let \( \nu_k = 0 \) and \( \nu_k = 1 \) denote the schedule action for the uplink and downlink, respectively. The persistent policy can be formally written as (see Fig. 4)
\[
\nu_{k+1} = \begin{cases} 
1 & \text{if } \nu_k = 0 \text{ and } \gamma^U_k = 1 \\
0 & \text{if } \nu_k = 1 \text{ and } \gamma^D_k = 1 \\
\nu_k & \text{otherwise}.
\end{cases}
\]

(16)

Without loss of generality, we assume the initial scheduling action is for sensing, i.e., \( \nu_0 = 1 \) when \( k = 1 \).

Due to packet dropouts, the controller may not receive the sensor measurement in each time, and thus applies an MMSE estimator for state estimation as
\[
\hat{x}_{k+1} = \begin{cases} 
Ax_k + Bu_k & \text{if } \gamma^U_k = 1 \\
A\hat{x}_k + Bu_k & \text{otherwise}.
\end{cases}
\]

(17)

Let \( e_t \triangleq x_t - \hat{x}_t \) denote the estimation error. It can be shown that
\[
e_{k+1} = \begin{cases} 
w_k & \text{if } \gamma^U_k = 1 \\
Ae_k + w_k & \text{otherwise}.
\end{cases}
\]

(18)

The controller adopts a linear control law for generating a control signal once scheduled, as per:
\[
a_k = K\hat{x}_k, \quad \text{if } \nu_k = 1,
\]
(19)

where \( K \) is a constant controller gain. Due to the transmission scheduling and the downlink packet dropouts, the actuator may at times not receive a control packet. We assume that the actuator adopts a hold-input strategy, which applies the most recently received control signal as an input:
\[
u_k = \begin{cases} 
a_k & \text{if } \nu_k \gamma^D_k = 1, k > 1 \\
u_{k-1} & \text{if } \nu_k \gamma^D_k = 0, k > 1 \\
0 & \text{if } k = 1.
\end{cases}
\]

(20)

Different from the remote estimation scenario considered in previous sections, in the networked control scenario the controller uses both the uplink and downlink transmissions to learn the channels. The controller generates the scheduling action and the channel selection action, and informs the sensor and actuator via a transmission management channel. In addition, the actuator sends an acknowledgment (ACK) to the controller on the success/failure of the control packet reception via an ACK channel, see Fig. 3. Since the transmission management commands and the ACK normally require only a few bits of information, we assume that these transmissions are error-free.

The \( \varepsilon \)-greedy, Thompson sampling, OBS, and SBS algorithms presented in Section III can be directly applied at the controller to learn the best channel.

B. Stability of Networked Control with Channel Selection

We note that the stability condition of the networked control system described in Section VI-A with a single channel has not been previously investigated in the literature. In the following, we will first derive the stability condition with \( M = 1 \), and then extend the result to the multi-channel case.

We first define the control system stability as:

**Definition 2.** The system (15) is stable in the mean-square sense if
\[
\limsup_{T \to \infty} \frac{\sum_{k=1}^T \mathbb{E}[x_k^\top x_k]}{T} < \infty, \forall x_0.
\]
(21)

Then, we derive necessary and sufficient stability conditions of the system as below.

**Theorem 5.** Consider the networked control system described in Section VI-A over a single channel with packet reception probability \( \theta \). Then:
(i) A sufficient condition for mean-square stability is
\[
\rho(A)^2(1 - \theta) < 1
\]
(22)

**and**
\[
\mathbb{E}[trH_0H_0^\top] < 1,
\]
(23)

where the random matrix \( H_0 \) is defined as
\[
H_0 \triangleq \begin{bmatrix}
(A + BK)A^{\ell_U} + A^{\ell_D - 1} & (A + BK)\sum_{\ell=0}^{\ell_U + \ell_D - 2} A^\ell \\
KA^{\ell_U + \ell_D - 1}B & K\sum_{\ell=0}^{\ell_U + \ell_D - 2} A^\ell B
\end{bmatrix}
\]
and the random variables $\ell_U$ and $\ell_D$ have independent geometric distributions with parameter $\theta$, i.e.

$$\mathbb{P}(\ell_c = \ell) = \theta(1 - \theta)^{\ell - 1}, c = U \text{ or } D, \ell = 1, 2, \ldots$$  \hfill (24)

(ii) A necessary condition for mean-square stability is

$$\rho(A)^2(1 - \theta) < 1$$  \hfill (25)

$$\rho(\mathbb{E}[H_0]) < 1.$$  \hfill (26)

Proof. Due to the stochastic scheduling and the control operation \(17\), \(19\), and \(20\), the next plant state $x_{k+1}$ depends not only on the current state $x_k$ and scheduling action $\nu_k$, but also on the most recently received plant state measurement at the controller. In addition, the time elapsed since the last received measurement is a random variable. Thus, the process \(\{x_k\}\) is not a Markov process, making the direct analysis of \(\mathbb{E}[x_k^T x_k]\) difficult. We will instead develop a stochastic control cycle-based approach to analyze the stability. The detailed proof can be found in Appendix B.

Based on similar methods as in the proofs of Theorems 1 and 2, we can directly extend Theorem 5 to the multi-channel scenario as below.

**Theorem 6.** Consider the networked control system described in Section 4.1 with $M$ unknown channels and the stochastic stability notion of Definition 2.

(i) Under the $\varepsilon$-greedy algorithm, if

$$\theta^* \triangleq (1 - \varepsilon)\theta^* + \varepsilon \frac{1}{M} \sum_{m=1}^{M} \theta_m$$

satisfies (23) and (24), then the control system is stable; if $\theta^*$ does not satisfy any of (23) and (24), then the system is unstable.

(ii) Under the Thompson sampling, OBS, and SBS algorithms, if $\theta^*$ satisfies (23) and (25), the control system is stable; if $\theta^*$ does not satisfy any of (23) and (25), the system is unstable.

**C. Performance Bounds**

We consider the regret of the control system as the difference between the expected cumulative control cost from always using the best channel and the expected cumulative control cost using a particular bandit algorithm. Given a positive semidefinite matrix $F$, the control cost at time step $k$ under a particular bandit algorithm is

$$J_k \triangleq \bar{x}_k^T F \bar{x}_k,$$

where

$$\bar{x}_k \triangleq \begin{bmatrix} x_k \\ u_k \end{bmatrix}.$$

Let $\bar{x}^*_k \triangleq [x^*_k, u^*_k]^T$ denote the augmented control system state assuming the best channel is always chosen, and $J^*_k \triangleq (\bar{x}^*_k)^T F \bar{x}^*_k$ denote the corresponding cost. The control regret over horizon $T$ is defined as

$$\text{regret}^C(T) \triangleq \mathbb{E}\left[ \sum_{k=1}^{T} (J_k - J^*_k) \right].$$  \hfill (27)

Fig. 5: Learning using Stability-aware Bayesian Sampling

We have the following regret upper bounds.

**Theorem 7.** Suppose the sufficient stability condition in Theorem 6(i) holds. Then under the $\varepsilon$-greedy algorithm, we have

$$\text{regret}^C(T) = O(T).$$

**Theorem 8.** Suppose the sufficient stability condition in Theorem 6(ii) holds. Then under i) Thompson sampling, ii) OBS, and iii) SBS, we have

$$\text{regret}^C(T) = O(\log T).$$

Proof. The proof of Theorem 8 can be found in Appendix C. Theorem 7 for the $\varepsilon$-greedy algorithm can be proved in a similar way and is omitted.

Recall that in the remote estimation scenario, the lower bound on the estimation regret was derived based on the property that $\{\mathbb{E}[P_k]\}$ is a Markov process with a countable state space, together with properties of the $h(\cdot)$ operator given in Lemma 1. However, in the control scenario, similar properties do not hold, mainly because $\{\mathbb{E}[\bar{x}_k^T F \bar{x}_k]\}$ is non-Markov and has an uncountable state space. As such, obtaining lower bounds on the control regret appears to be difficult and is left for future work.

**VII. Numerical Studies**

**A. Remote State Estimation**

We consider a situation with $M = 4$ channels. Figure 5 illustrates how Stability-aware Bayesian sampling (SBS) learns, and increasingly uses, the optimal channel, for a situation where

$$A = \begin{bmatrix} 1.5 & 0.2 \\ 0.3 & 0.9 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad Q = I, \quad R = 1$$

and the (unknown) channel success probabilities are given by

$$\theta = (\theta_1, \theta_2, \theta_3, \theta_4) = (0.8, 0.7, 0.6, 0.5).$$

Table II illustrates the empirical estimation regret for a number of different channel probabilities

$$\theta = (\theta_1, \theta_2, \theta_3, \theta_4)$$

and system matrices $A$, with $C$, $Q$, and $R$ as above. All bandit algorithms studied in this work, namely $\varepsilon$-greedy, Thompson sampling (TS), OBS, and SBS, are considered. The regret is computed over horizon $T = 1000$, and averaged over 10000
runs. For \( \varepsilon \)-greedy, we considered different values of \( \varepsilon \) in steps of 0.02, with the value giving the smallest simulated regret presented in Table I.

As expected, the \( \varepsilon \)-greedy algorithm is outperformed by the sampling based algorithms. It also seems that OBS and SBS can achieve some improvement (on the order of 5-10\%) in performance over Thompson sampling. In our simulation study the performance of OBS and SBS are quite close to each other; in some scenarios SBS performs slightly better, while in other scenarios OBS performs better.

In Fig. 6 we plot the estimation regret over time for the \( \varepsilon \)-greedy and Thompson sampling algorithms. Plots for OBS and SBS are qualitatively similar to Thompson sampling and are omitted. The parameters used are the same as those in the fifth scenario of Table I. We see a linear scaling (at larger times) for \( \varepsilon \)-greedy and a logarithmic scaling in the case of Thompson sampling, in agreement with Theorems 3 and 4.

**B. Networked Control**

We fix \( \theta = (0.9, 0.8, 0.7, 0.6) \) and

\[
A = \begin{bmatrix} 1.1 & 0.2 \\ 0.1 & 0.8 \end{bmatrix} ; \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} ; \quad Q = 0.1I.
\]

The controller gain in (19) is chosen as \( K = [-0.7, 0.2] \). For the control regret (27), we choose \( F = I \).

In Fig. 7 we plot the simulated control regret over time for the \( \varepsilon \)-greedy and Thompson sampling algorithms, where we use \( \varepsilon = 0.2 \) for \( \varepsilon \)-greedy. The control regret is averaged over 10000 runs. Plots for OBS and SBS are again qualitatively similar to Thompson sampling and omitted.

**VIII. CONCLUSION**

We have studied state estimation and control problems where a sensor can choose between a number of channels with unknown channel statistics, with which to transmit over. We have made use of bandit algorithms to learn the statistics of the channels, while simultaneously carrying out the estimation or control procedure. Stability of the resulting processes using these algorithms has been shown, and bounds on the estimation and control regret have been derived. Future work will include the use of bandit algorithms in the allocation of channels to multiple processes.

**APPENDIX**

**A. Proof of Theorem 2**

We will first prove that regret\( F(T) = O(\log T) \). Define the reception probability of the worst channel as

\[
\theta_w \triangleq \min_{m} \theta_m.
\]

Given any schedule of channel selections, let schedule \( \mathcal{A} \) denote an alternative schedule which replaces all uses of sub-optimal channels with the worst channel. Let \( \mathbb{E}[P_k^A] \) be the expected error covariance under this replacement procedure. Then from (6) and Lemma I we can easily show that

\[
\mathbb{E}[P_k] \leq \mathbb{E}[P_k^A], \quad \forall k.
\]

In particular, this means that

\[
\sum_{k=1}^{T} \text{tr}[\mathbb{E}[P_k]] \leq \sum_{k=1}^{T} \text{tr}[\mathbb{E}[P_k^A]].
\]

Then, we have

\[
\text{regret}^F(T) \leq \text{regret}^A(T) \triangleq \sum_{k=1}^{T} \text{tr} \left( \mathbb{E}[P_k^A] - \mathbb{E}[P_k^*] \right). \tag{29}
\]

Thus, we only need to prove regret\( A(T) = O(\log T) \).

Let \( k_i^w \in \mathbb{N}, i \geq 1 \) denote the time index of the \( i \)th usage of the worst channel based on schedule \( \mathcal{A} \). The time horizon is divided by \( \{k_i^w\} \) into epochs, as illustrated in Fig. 2. The \( i \)th epoch starts from \( k_i^w \) and ends before \( k_{i+1}^w \), and is of epoch length \( \ell_i \triangleq k_{i+1}^w - k_i^w \geq 1 \). Note that \( \ell_i \) is a random process with a time-varying distribution due to the bandit algorithms. We define

\[
\Delta_i^w \triangleq \mathbb{E}[P_{k_{i+1}^w}^f] - \mathbb{E}[P_{k_{i+1}^w}^*]. \tag{30}
\]
as the gap between the expected estimation error covariances obtained that

Let \( k_{i,j}^b \in \mathbb{N}, i,j \geq 1 \) denote the time index based on schedule \( A \) of the \( j \)th usage of the best channel in epoch \( i \), when the epoch length \( \ell_i \geq 1 \) (see Fig. 2). Let \( k_{i,j}^b \in \mathbb{N}, i,j \geq 1 \) denote the time index of the \( j \)th usage of the best channel before the first epoch and \( \ell_0 \) denote the total number of time slots before the first epoch. Then, we define

\[
\Delta_{i,j}^b = \mathbb{E} [ P_{i,j}^{A_k^b} ] - \mathbb{E} [ P_{i,j}^{A_k^*} ], i \geq 0, j \geq 1, \text{ if } \ell_i > 1. \tag{31}
\]

In particular, we have \( \Delta_{i,j}^b = 0 \) when \( i = 0 \) since schedule \( A \) is identical to the persistent schedule of the best channel before the first epoch.

We use random variable \( N_w \) to denote the number of uses of the worst channel within a time interval of length \( T \). Then, regret\(^A\)\((T)\) in (29) can be upper bounded as

\[
\text{regret}^A(T) \leq \mathbb{E} \left[ \sum_{j=1}^{\ell_0-1} \sum_{i=1}^{N_w} \left( \text{tr} \Delta_{i,j}^w + \sum_{j=1}^{\ell_i-1} \text{tr} \Delta_{i,j}^b \right) \right],
\]

where the expectation is over \( N_w \) and \( \{ \ell_i \} \). To prove the desired result, we will analyze \( \text{tr} \Delta_{i,j}^w + \sum_{j=1}^{\ell_i-1} \text{tr} \Delta_{i,j}^b \) first.

**Analysis of per epoch regret:** By applying the estimation error covariance updating rule (2) into (31), we have

\[
\Delta_{i,j}^b = (1 - \theta^*) h(\mathbb{E} [ P_{i,j}^{A_k^b} ] + \theta^* T) - (1 - \theta^*) h(\mathbb{E} [ P_{i,j}^{A_k^*} ] + \theta^* T)
\]

\[
= (1 - \theta^*) A \left( \mathbb{E} [ P_{i,j}^{A_k^b} ] - \mathbb{E} [ P_{i,j}^{A_k^*} ] \right) A^\top.
\]

For the time slots in epoch \( i \) when \( \ell_i > 1 \), it directly follows that

\[
\Delta_{i,j}^b = (1 - \theta^*) A \Delta_i^w (A^\top)^\top, 1 \leq j \leq \ell_i - 1. \tag{34}
\]

Applying the trace operator to inequality (34), it can be obtained that

\[
\text{tr} \Delta_{i,j}^b = (1 - \theta^*)^2 \text{tr} \left( A \Delta_i^w (A^\top)^\top \right)
\]

\[
\leq (1 - \theta^*)^2 \text{tr} \left( A (A^\top)^\top \right) \text{tr} \Delta_i^w \]

\[
\leq \kappa_1(\epsilon)(1 - \theta^*)^2 (\rho(A) + \epsilon)^2 \text{tr} \Delta_i^w \tag{37}
\]

where \( \epsilon > 0 \) is arbitrary and \( \kappa_1(\epsilon) > 0 \). Inequality (36) is due to the property that \( \text{tr} (A V) \leq \text{tr}(U) \text{tr}(V) \) for any positive semidefinite matrices \( U \) and \( V \), see e.g. [32, p.445]. Inequality (37) is due to the fact that \( \text{tr}((A^\top)^\top A^\top) \) is the sum of squares of all elements of \( A^\top \), and is obtained based on the lemma below about the element-wise bounds of matrix powers [33, Lemma 2]:

**Lemma 4.** Consider a \( z \)-by- \( z \) matrix \( Z \), and let \( [Z]_{i,i'} \) denote the element at the \( i \)th row and \( i' \)th column of \( Z \).

(i) For any \( \epsilon > 0 \), there exists \( \kappa_1(\epsilon) > 0 \) such that

\[
||[Z]_{i,i'}||^2 \leq \kappa_1(\epsilon)(\rho(Z) + \epsilon)^2, \forall i,i' \in \{1, \ldots, z\}.
\]

(ii) There exist \( \kappa_2 > 0 \) and \( L \in \mathbb{N} \) such that

\[
||[Z]_{i,i'}||^2 \leq \kappa_2 \rho(Z)^2 \text{ with period } L.
\]

From assumption (5), we can find a sufficiently small \( \epsilon \) such that \( (1 - \theta^*) (\rho(A) + \epsilon)^2 < 1 \). Using the inequality (37), it can be obtained that

\[
\sum_{j=1}^{\ell_i-1} \text{tr} \Delta_{i,j}^w \leq \kappa_1(\epsilon) \text{tr} \Delta_i^w \leq \frac{\kappa_1(\epsilon) \text{tr} \Delta_i^w}{1 - (1 - \theta^*) (\rho(A) + \epsilon)^2}.
\]

Therefore, the regret of epoch \( i \) is bounded as

\[
\text{tr} \Delta_i^w + \sum_{j=1}^{\ell_i-1} \text{tr} \Delta_{i,j}^b \leq \kappa(\epsilon) \text{tr} \Delta_i^w, \tag{39}
\]

where \( \kappa(\epsilon) = 1 + \frac{\kappa_1(\epsilon)}{1 - (1 - \theta^*) (\rho(A) + \epsilon)^2} > 0 \). Inequality (38) is due to the property that \( \text{tr}(A^\top P) \leq \text{tr}(U) \text{tr}(V) \) for any positive semidefinite matrices \( U \) and \( V \), see e.g. [32, p.445].
it directly follows that $\Delta_W^N \leq (1 - \theta_\omega) A P' A^T + G$. Then, we can find a positive constant $\xi$ such that

$$\text{tr} \Delta_W^N \leq \xi, \forall i.$$  \hfill (41)

By taking (39) and (41) into (32) and using Lemma 3, it follows that

$$\text{regret}^E(T) \leq \text{regret}^A(T) \leq E \left[ \sum_{i=1}^{N} \kappa(e) \xi \right] = \kappa(e) \xi E[N_u] = O(\log T).$$

To conclude, we will now prove that $\text{regret}^E(T) = \Omega(\log T)$. We have

$$\text{regret}^E(T) = \sum_{k=1}^{T} \text{tr}(E[P_k] - E[P_k^*]) \leq E \left[ \sum_{k=1}^{T} \text{tr}(E[P_k] - E[P_k^*]) \right] \leq E \left[ \sum_{k=1}^{T} \text{tr}(E[P_k] - E[P_k^*]) \right].$$

Let $\theta^*$ denote the second largest packet reception probability. When $\theta(k) \neq \theta^*$, we can show similar to (14) that

$$\text{tr}(E[P_k] - E[P_k^*]) \geq (\theta^* - \theta) \min_{k \geq 1} \text{tr}(h(E[P_k^*] - E[P_k]) - P).$$

Thus

$$\text{regret}^E(T) \geq r^* \sum_{k=1}^{T} \text{tr}(E[P_k] - E[P_k^*]) \geq r^* E[N_{\text{sub}}(T)] \geq \Omega(\log T),$$

where the last line follows from Lemma 3. This completes the proof that $\text{regret}^E(T) = \Omega(\log T)$, and hence establishes Theorem 4.

B. Proof of Theorem 5

We will develop a stochastic control cycle-based approach to convert the stability requirement (21) into a form that is easier to analyze. Specifically, the time domain is divided into effective control cycles, where each cycle starts after the controller’s reception of a control signal and ends at the next control signal update, as shown in Fig. 8. Let $x_{t,i}$, $u_{t,i}$, and $w_{t,i}$ denote the plant state, control input, and disturbance in the i-th time slot of the i-th cycle, respectively. Note that the control input is identical from the last time slot of the t-1th cycle to the second last time slot of the t-th cycle. Let $\ell^U_t$ and $\ell^D_t \in \mathbb{N}$ denote the number of uplink and downlink transmissions, respectively, in control cycle t. Then, the number of time slots of cycle t is $\ell_t = \ell^U_t + \ell^D_t \geq 2$.

![Fig. 8: An illustration of stochastic control cycles, where U and D denote uplink and downlink transmissions.](image)

Based on the persistent scheduling policy (16) and the fixed packet reception probability $\theta$, it is straightforward to show that $\{\ell^U_t\}$ and $\{\ell^D_t\}$ are two i.i.d. processes following the geometric distribution (23). Therefore, we have

$$E[\ell_t] = \ell < \infty, \forall t$$

where $\ell = 1/\theta$. Furthermore, we define the sum cost per cycle as

$$c_t = \sum_{i=1}^{\ell_t} x_{t,i} u_{t,i}.$$

From (17), (19), and (20), it can be derived that

$$\hat{x}_{t,\ell^U_t + \ell^D_t} = A^\ell_t + \ell^D_t - 1 \hat{x}_{t,1} + \left( A^\ell_t + \ell^D_t - 3 \right) w_{t,2} + \cdots + A^\ell_t w_{t,\ell_t - 1}.$$

Knowing the last control input in cycle t is

$$u_{t,\ell^U_t + \ell^D_t} = K \hat{x}_{t,\ell^U_t + \ell^D_t} + \tilde{w}_t,$$

and taking it into (15), we have

$$x_{t+1,1} = (A + BK) \hat{x}_{t,\ell^U_t + \ell^D_t} + \tilde{w}_t,$$

where

$$\tilde{w}_t \triangleq A^\ell_t w_{t,\ell^U_t + \ell^D_t - 1} + A^{\ell_t + \ell^D_t - 3} w_{t,2} + \cdots + A^{\ell_t} w_{t,\ell^D_t - 1}.$$

Let

$$\hat{x}_t \triangleq \left[ x_{t,1} \right].$$

From (42) and (43), it directly follows that

$$\hat{x}_{t+1} = H_t \hat{x}_t + W_t,$$

where

$$H_t = \begin{bmatrix} (A + BK) A^\ell_t + \ell^D_t - 1 & (A + BK) \sum_{\ell=0}^{\ell^D_t - 2} A^\ell \sum_{\ell=0}^{\ell^D_t - 2} B K \sum_{\ell=0}^{\ell^D_t - 2} A^\ell B \end{bmatrix}$$

and

$$W_t = \begin{bmatrix} (A + BK) \tilde{w}_t + \hat{w}_t \end{bmatrix}.$$

Since both $\{H_t\}$ and $\{W_t\}$ are i.i.d. processes, $\{\hat{x}_t\}_{t \in \mathbb{N}}$ generated by (45) is a Markov process. In addition, $\hat{x}_t, \forall t > 1$ can take any value as $W_t$ is Gaussian. Therefore, $\{\tilde{x}_t\}_{t \in \mathbb{N}}$, i.e., the head states of the control cycles, form an ergodic Markov process. Then, we can easily show that $\{c_t\}_{t \in \mathbb{N}}$ is Markovian and ergodic, and hence has a unique steady state distribution. Hence, there exists some $\tilde{c}$ such that

$$E[c_t] = \tilde{c}, \forall t$$

in steady state.
Due to the linear control law, it can be proved that the average plant state cost is bounded if and only if the average control input cost is. Therefore, the stability requirement \( (21) \) can be rewritten as
\[
\limsup_{T \to \infty} \frac{\sum_{k=1}^{T} \mathbb{E}[x_k^T x_k]}{T} = \limsup_{N \to \infty} \frac{\sum_{t=1}^{N} \mathbb{E}[c_t]}{\bar{c}T} = \frac{\bar{c}}{\bar{c}} < \infty, \forall x_0.
\]
(46)

Since \( \bar{c} \) is a positive constant, the stability requirement is equivalent to \( \frac{\bar{c}}{\bar{c}} < \infty \). In the following, we derive necessary and sufficient stability conditions by analyzing the per cycle sum cost \( \mathbb{E}[c_t] \).

**Proof of sufficiency:** We will first analyze the sufficient condition making the expected state cost bounded, and then the condition for the sum cost.

It can be observed that \( \{\tilde{x}_t\}_{t \in \mathbb{N}} \) in \( (45) \) is a Markov jump linear system (MJLS) with the i.i.d. jump process \( \{\ell'_t, \ell''_t\}_{t \in \mathbb{N}} \), which has infinite state space. We note that the stability condition of a MJLS with infinitely many jump states has no explicit formula. In \( (34) \), the condition was converted to the existence of a MJLS with infinitely many jump states has no explicit formula. From \( (44) \), it can be derived that
\[
\limsup_{N \to \infty} \frac{\sum_{t=1}^{N} \mathbb{E}[c_t]}{\bar{c}T} = \frac{\bar{c}}{\bar{c}} < \infty, \forall x_0.
\]
(47)

The necessary condition \( (25) \) is bounded, i.e.,
\[
\mathbb{E}\left[ \sum_{i=1}^{\ell_{t-1}} \left( x_{t,i}^T x_{t,i} + u_{t,i}^T u_{t,i} \right) \right] < \infty, \forall t.
\]
(52)

We now investigate the expected cost of the end state per cycle. The control input can be written as
\[
\begin{align*}
u_{t,i} &= K(\hat{x}_{t,i} - e_{t,i})
&= K(Ax_{t,i} + Bu_{t,i} + w_{t,i} - e_{t,i})
&= K(Ax_{t,i} + Bu_{t,i} + w_{t,i} - e_{t,i})
\end{align*}
\]
(53)

By taking the expectation on both sides of \( (45) \), we have
\[
\mathbb{E}\left[ \tilde{x}_{t,i} \right] = \mathbb{E}\left[ \hat{x}_{t,i} \right] - \mathbb{E}\left[ e_{t,i} \right].
\]
Thus, if \( \mathbb{E}[\hat{x}_t] < \infty, \forall \hat{x}_0 \), one should have \( \rho(\mathbb{E} [H_t]) < 1. \) This completes the proof of \( (26) \).

### C. Proof of Theorem 8

Similar to the remote estimation scenario, to prove the desired result, we only need to show that the per epoch regret is bounded. We assume that each epoch starts from a slot that does not choose the optimal channel for transmission and ends before the next one, as illustrated in Fig. 7.

Assume that an epoch of length \( T_0 \) starts at time slot \( k_0 \). Considering the scheduling starts at the beginning of the epoch, the regret of the epoch can be written as
\[
\mathbb{E} \left[ \sum_{k=k_0}^{k_0+T_0-1} (J_k - J_k^*) \right] = \mathbb{E}\left[ R_0(k_0, T_0) \right] \mathbb{P}(\nu_{k_0} = 0, \nu_{k_0}^* = 0) + \mathbb{E}\left[ R_0(k_0, T_0) \right] \mathbb{P}(\nu_{k_0} = 0, \nu_{k_0}^* = 1) + \mathbb{E}\left[ R_0(k_0, T_0) \right] \mathbb{P}(\nu_{k_0} = 1, \nu_{k_0}^* = 0) + \mathbb{E}\left[ R_0(k_0, T_0) \right] \mathbb{P}(\nu_{k_0} = 1, \nu_{k_0}^* = 1),
\]
(54)
Fig. 9: An illustration of the scheduling process and the epoch for regret for regret analysis, where U and D denote uplink and downlink transmissions and the highlighted slots denote the starts of epochs.

where

\[ R_{j,j'}(k_0, T_0) \triangleq \mathbb{E} \left[ \sum_{k=k_0}^{k_0+T_0-1} (J_k - J^*_k) | \nu_{k_0} = j, \nu^*_{k_0} = j' \right], \]

and \( \nu_{k_0} \in \{0, 1\} \) is the scheduling action at \( k \) when the optimal channel is always chosen. Next, we will prove that the expected per epoch regret is bounded with the initial scheduling action \( \nu_{k_0} = 0, \nu^*_{k_0} = 0 \), i.e.,

\[ R_{00}(k_0, T_0) < \infty, \forall k_0, T_0, \]

and the rest of the cases can be proved using the same method.

Since the channel selections of the two algorithms only differs at the first time slot of the epoch, we rewrite \( R_{00}(k_0, T_0) \) consisting of terms with \( k = k_0 \) and \( k > k_0 \) as

\[ R_{00}(k_0, T_0) = \mathbb{E}[(J_{k_0} - J^*_{k_0}) + \hat{R}_{01}(\nu_{k_0+1} = 0, \nu^*_{k_0+1} = 0) + \hat{R}_{11}(\nu_{k_0+1} = 1, \nu^*_{k_0+1} = 1)], \]

where \( \hat{R}_{j,j'} \) denotes the event \( \nu_{k_0+1} = j, \nu^*_{k_0+1} = j' \) for brevity. Building on the stability analysis in Section VI-B where we proved that the average per cycle sum cost is bounded and the head state per cycle forms an ergodic Markov process, and due to the fact that \( \nu_k \) and \( \nu^*_k \) are ergodic Markovian as well, it is easy to verify that the expected cost is bounded at all times, i.e.,

\[ \mathbb{E}[J_k | \nu_{j,j'}], \mathbb{E}[J^*_k | \nu_{j,j'}] < \infty, \forall k, j, j'. \]

(54)

Then, one needs to prove that \( \hat{R}_{j,j'} < \infty, \forall j, j' \).

In the following, we only prove the representative case where the bandit and the optimal algorithms schedule uplink and downlink transmissions, respectively, i.e.,

\[ \hat{R}_{01} < \infty, \]

since the other three cases can be proved similarly. Let \( \ell_0^* \) denote the number of slots to reach the first uplink transmission block of the optimal algorithm, as illustrated in Fig. 9. Then, we have

\[ \hat{R}_{01} \leq \mathbb{E} \left[ \sum_{k=k_0}^{k_0+T_0-1-\ell_0^*} (J_k - J^*_k) + \sum_{k=k_0+T_0-\ell_0^*}^{k_0+T_0-1} J_k | \nu_{01} \right]. \]

(55)

From (54) and the fact that \( \mathbb{E}[\ell_0^*] < \infty \), it follows that the second summation on the right-hand side of (55) satisfies

\[ \mathbb{E} \left[ \sum_{k=k_0+T_0-\ell_0^*}^{k_0+T_0-1} J_k | \nu_{01} \right] < \infty. \]

Next, we will prove

\[ \mathbb{E} \left[ \sum_{k=k_0+T_0-1-\ell_0^*}^{k_0+T_0-1} (J_k - J^*_k) | \nu_{k_0+1} = \nu^*_{k_0+1}, \ell_0^* = 0 \right] < \infty. \]

(56)

We note that both the sequences \( \{J_k\} \) and \( \{J^*_k\} \) in (56) start from sensing transmissions (i.e., the beginning of control cycles), and \( \{\nu_k\} \) and \( \{\nu^*_k\} \) follow identical distributions due to the same channel selection actions. We adopt the control cycle-based analytical approach of Section VI-B. For notational simplicity, we replace \( J^*_k \) with \( J^*_0 \). Let \( L \) denote the number of control cycles in the time period from \( k_0 + 1 \) to \( k_0 + T_0 - 1 - \ell_0^* \). Then (56) is equivalent to

\[ \mathbb{E} \left[ \sum_{t=1}^{L} (J_t - J^*_t) \right] < \infty, \]

(57)

where

\[ \tilde{J}_t \triangleq \sum_{i=1}^{\ell_t} \tilde{x}^T_{t,i} F \tilde{x}_{t,i}, \tilde{J}^*_t \triangleq \sum_{i=1}^{\ell_t} (\tilde{x}^*_t, i - 1) F \tilde{x}^*_t, \]

(58)

From (51) and (53), we have

\[ \mathbb{E} \left[ \tilde{x}^T_{t,i} F \tilde{x}_{t,i} \right] = \mathbb{E} \left[ \tilde{x}^T_{t,i} \right] F \mathbb{E} \left[ \tilde{x}_{t,i} \right], \]

\[ = \left[ \begin{array}{c} \text{tr} F A_1^{-1} \mathbb{E} \left[ \text{tr} \tilde{x}^T_{t,i} \tilde{x}_{t,i} - \tilde{x}^T_{t,i} \tilde{x}^*_t, i \right] A_1^{-1} \right], 1 \leq i < \ell_t \]

\[ \text{tr} F A_2 A_1^{-1} \mathbb{E} \left[ \text{tr} \tilde{x}^T_{t,i} \tilde{x}_{t,i} - \tilde{x}^T_{t,i} \tilde{x}^*_t, i \right] (A_2 A_1^{-1})^T, i = \ell_t \]

From (59) and (53), we have

\[ \mathbb{E} \left[ \tilde{x}^T_{t,i} F \tilde{x}_{t,i} \right] \leq \Delta_{t,i}, \]

(60)

\[ \text{tr} F A_1^{-1} (A_1^{-1})^T \text{tr} \tilde{x}_{t,i} \tilde{x}_{t,i}^T, 1 \leq i < \ell_t \]

\[ \text{tr} F A_2 A_1^{-1} (A_2 A_1^{-1})^T \text{tr} \tilde{x}_{t,i} \tilde{x}_{t,i}^T, i = \ell_t \]

It is clear that the left-hand side of (57) is smaller than

\[ \sum_{t=1}^{\infty} \sum_{i=1}^{\ell_t} | \Delta_{t,i} | . \]

When the conditions (22) and (23) hold, similar to the stability analysis of Section VI-B and the regret analysis of Section V, it is readily obtained that

\[ \sum_{t=1}^{\infty} \sum_{i=1}^{\ell_t} | \Delta_{t,i} | < \infty. \]

This completes the proof of Theorem 8.

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