The Growth of Dark-Matter Halos. The Effects of Accretion and Mergers

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\textbf{Abstract.} High resolution cosmological N-body simulations show that the density profiles of dark matter halos in hierarchical cosmogonies are universal, with low mass halos typically denser than more massive ones. This mass-density correlation is interpreted as reflecting the earlier formation of less massive objects. We investigate this hypothesis in the light of formation times defined as the epoch at which halos experience their last major merger. We find that the characteristic density and the scale radius of halos are essentially proportional, respectively, to the critical density of the universe and the virial radius at the time of their formation. These two relations are consistent with the following simple evolutionary picture. Violent relaxation caused by major mergers rearrange the structure of halos leading to a universal dimensionless density profile. Between major mergers, halos gradually grow through the accretion of surrounding layers by keeping the central part steady and only expanding their virial radius as the critical density of the universe diminishes.

1. \textbf{Introduction}

Despite the big efforts made in the last two decades in trying to understand the origin of the density profile of galaxies and clusters, the situation is still very confused. Is the typical density profile of halos mainly the result of accretion (spherical secondary infall) or of repetitive major mergers (strong violent relaxation)? Does it arise, on the contrary, from secular evolution due to the effects of dynamical friction plus the tidal stripping of captured satellites? Or is it some combination of the preceding processes?

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Recently, two empirical facts have emerged from high resolution cosmological N-body simulations which seem to have important implications on this long-standing debate. First, the spherically averaged density profile of dark-matter halos is universal (Navarro, Frenk, & White 1997, hereafter NFW; Cole & Lacey 1997; Tormen, Bouchet, & White 1997), its exact form being however controversial (see Moore et al. 1998). The various laws proposed can be encapsulated within the general expression

$$\frac{\rho(\xi)}{\rho_{\text{crit}}} = \frac{\delta_c}{\xi^\alpha(1 + \xi^\beta)^{(\gamma-\alpha)/\beta}},$$

(1)

with $\alpha = 1$, $\beta = 1$, and $\gamma = 3$ for the NFW profile, and $\alpha = 1.4$, $\beta = 1.4$, and $\gamma = 2.8$ for the Moore et al. (1998) profile. The classical King law and the Hernquist (1990) profile also admit this general form for $\alpha = 0$, $\beta = 2$, and $\gamma = 3$, and $\alpha = 1$, $\beta = 1$, and $\gamma = 4$, respectively. In equation (1), $\xi = r/r_s$ is the radial distance to the halo center in units of the scale radius $r_s$, and $\delta_c = \rho_c/\rho_{\text{crit}}$ is the characteristic halo density in units of the critical density of the universe. The dimensional parameters $\rho_c$ and $r_s$ are linked by the condition that the mean density within the virial radius $R$ of a halo of a given mass is a constant factor $a$ times $\rho_{\text{crit}}$; here we adopt $a = 200$. Therefore, the density profiles of halos at a given epoch depend on their mass $M$ through a unique parameter, $\delta_c$ or $x_s = r_s/R$. Second, the smaller the mass of halos, the denser they are (NFW, Cole & Lacey 1997) or, equivalently, $\delta_c$ is a decreasing function of $M$.

This mass-density correlation is interpreted as reflecting the fact that, in hierarchical clustering, less massive halos form typically earlier when the mean density of the universe is higher. According to this interpretation the structural properties of halos would be fixed at their time of formation. As major mergers yield a substantial rearrangement of the system, while accretion only causes its smooth evolution, this would point to the fact that violent relaxation taking place after major mergers is the main responsible for the halo density profiles. NFW have shown, indeed, that the characteristic density of halos with a given mass at a given epoch is proportional to the mean cosmic density when they form. This connection between the characteristic density of halos and their formation time strongly favors the previous interpretation of the mass-density correlation. However, the physics behind such a proportionality is unclear. Moreover, the halo formation time adopted by NFW does not correspond to the last major merger experienced by these systems.

Here we report the results of a similar analysis of the mass-density correlation recently performed by Salvador-Solé, Solanes, & Manrique (1998, SSM) using a better suited formation time estimate. Not only does our analysis confirm the origin proposed by NFW for the mass-density correlation, but also provides a physical base for the proportionality between halo characteristic density and cosmic density at formation. Furthermore, our results point to a complete picture for the evolution of halo structure as a consequence of the interplay of accretion and major mergers.
2. A Better Suited Halo Formation Time Estimate

To follow the formation and evolution of halos, SSM have used a slightly modified version of the extended Press-Schechter (1974, PS) clustering model, known to agree with N-body simulations. The only change introduced consists on including a schematic distinction between minor and major mergers. Such a distinction does not obviously alter the success of the basic model while it allows one to define the formation of a halo as the last major merger it experiences.

We say that a halo of mass $M$ experiences a major merger and is destroyed when the relative mass captured by a halo $\Delta M/M$, with $\Delta M = M' - M$ the increment of mass, exceeds a certain threshold $\Delta_m$. Otherwise, the event is regarded as an accretion and the capturing halo keeps its identity. Consequently, the mass-accretion and destruction rates of halos of mass $M$ at $t$ are respectively given by

$$r_{\text{mass}}^a(M, t) = \int_M^{M(\Delta_m + 1)} \Delta M r_{\text{LC}}^m(M \rightarrow M', t) \, dM'$$

and

$$r^d(M, t) = \int_M^{\infty} \int_M^{M(\Delta_m + 1)} r_{\text{LC}}^m(M \rightarrow M', t) \, dM' ,$$

where $r_{\text{LC}}^m(M \rightarrow M', t)$ is the specific merger rate in the usual extended PS model (Lacey & Cole 1993).

The distinction between minor and major mergers does not modify the mass function of halos at $t$, $N(M, t)$, which therefore takes the standard PS form.

As a result of major mergers, some halos are destroyed and other halos form. The formation rate of halos with mass $M = M(t)$ can then be obtained from the conservation equation

$$r^f[M(t), t] = \frac{d \ln N[M(t), t]}{dt} + r^d[M(t), t] + \partial_M r^a_{\text{mass}}(M, t) |_{M=M(t)}$$

for the number density of halos per unit mass along mean accretion tracks, $M(t)$, solution of the differential equation $dM/dt = r_{\text{mass}}^a[M(t), t]$.

Finally, from this formation rate, one can compute the distribution of formation times for halos of mass $M_0$ at $t_0 > t$ (see SSM for details)

$$\Phi_t(t) = r^f[M(t), t] \exp \left\{ - \int_t^{t_0} r^f[M(t'), t'] \, dt' \right\} ,$$

with $M(t)$ the mean accretion track satisfying $M(t_0) = M_0$. The median value for this distribution function is then taken as the typical formation time $t_f(M_0, t_0)$ of such halos.

At this stage, the value of $\Delta_m$ establishing in a simple way the frontier between accretion and major mergers must be regarded as a phenomenological parameter of the model. Its value will be fixed in the next section through fits to the empirical mass-density (or mass-radius) correlation, the only data which seems to be connected with the distinction between merger and accretion.
Figure 1. Predicted $\delta_c(M_0)$ correlations compared with the empirical data from NFW’s N-body simulations (filled circles). Dotted and short-dashed curves show the predictions for two extreme values of $\Delta_m$, while the solid curves correspond to the value of this parameter that gives the best overall fit. Cosmogonies with $\Omega_0 < 1$ contain a fourth long-dashed curve which shows, for $\Delta_m = 0.6$, the predictions arising from the assumption that $\rho_c$ is proportional to $\rho_{\text{crit}}[z_f(M_0)]$, instead of to the mean density of the universe at halo formation.
3. The Mass-Density and Mass-Radius Correlations

As shown in Figure 1, the empirical $\delta_c(M_0)$ correlation for present halos is well fitted, in all the cosmogonies analyzed, by the simple proportionality between the halo characteristic density $\rho_c$ and the mean density $\bar{\rho}(z_f)$ of the universe when halos form proposed by NFW

$$\delta_c = C\Omega_0[1 + z_f(M_0)]^3,$$

provided that the mass threshold for merger takes the very natural value of 0.6.

Similarly good fits are also obtained for this same value of $\Delta_m$ in low $\Omega$ cosmogonies if $\rho_c$ is taken instead proportional to the critical density for closure $\rho_{\text{crit}}(z_f)$ at the time of halo formation. This new model, which assumes that the dimensionless characteristic density of halos at their time of formation is equal to the cosmogony-dependent constant $\delta_{\text{cf}}$, while the value of its dimensional counterpart $\rho_c$ remains fixed since that epoch, is described by the relation

$$\delta_c = \delta_{\text{cf}} \frac{\Omega_0}{\Omega[z_f(M_0)]} [1 + z_f(M_0)]^3.$$

In fact, the similarity between the empirical distribution function of halo characteristic densities and the theoretical one implied by this latter relation, suggests that eq. (7) holds also for individual halos. More importantly, in contrast with the relation (6) proposed by NFW, eq. (7) has a clear physical interpretation: violent relaxation occurring after major mergers produces virialized halos with identical dimensionless density profiles, which remain essentially unaltered during the accretion phase, except for the continuous stretching of their outer boundaries caused by the secular decrement of the critical density of the universe.

Given the tight relation between the characteristic density and the scale radius of halos of a given mass at a given epoch, if the evolutionary scheme we are proposing is correct, the value at formation of the dimensionless scale radius $x_s$ of halos should also be universal, while the associated dimensional parameter $r_s$ should keep its value invariant between major mergers. Under these circumstances, the scale radius $r_s$ for current halos of mass $M_0$ should be proportional to the virial radius of the halos when they formed, which implies that the empirical mass-radius correlation associated with the mass-density one should be well fitted by the expression

$$x_s = x_{\text{sf}} \frac{R[M[z_f(M_0), M_0]]}{R(M_0)}.$$

We want to stress that this is a new relation, independent from the two previous ones (eqs. (6) and (7)). Indeed, to verify it we need to estimate the mass of halos at the time of their formation (i.e., when they experienced their last major merger), so a clustering model such as the one developed in the preceding section is necessary. In contrast, the verification of the two previous relations does not require any clustering model.

As can be seen from Figure 2, relation (8) gives very good fits, indeed, to the empirical correlations for all cosmogonies, provided, once again, that $\Delta_m$ is
Figure 2. Predicted $x_s(M_0)$ correlations compared with the empirical data from NFW’s N-body simulations (solid circles).
equal to 0.6. Furthermore, as shown in Table \( \text{[I]} \), the best fitting values of \( x_{sf} \) are in very good agreement with the values of this parameter implied independently by the best fits obtained from relation \( \text{[II]} \) above. These results therefore give strong support our physical interpretation of the fitting formulae.

| \( P(k) \) | \( \Omega_0 \) | \( \lambda_0 \) | \( \sigma_8 \) | \( C \) | \( \delta_{cf} \) | \( x_{sf}^a \) | \( x_{sf}^b \) |
|-----------|--------|------|------|-----|-------|------|------|
| SCDM      | 1.0    | 0.0  | 0.63 | 1.21\times10^4 | 1.21\times10^4 | 0.173 | 0.229 |
| \( \Lambda \)CDM | 0.25   | 0.75 | 1.3  | 4.21\times10^3 | 3.77\times10^3 | 0.291 | 0.285 |
| \( n = -1.5 \) | 1.0    | 0.0  | 1.0  | 8.30\times10^3 | 8.30\times10^3 | 0.204 | 0.223 |
| \( n = -1.0 \) | 1.0    | 0.0  | 1.0  | 1.28\times10^4 | 1.28\times10^4 | 0.169 | 0.181 |
| \( n = -0.5 \) | 1.0    | 0.0  | 1.0  | 2.65\times10^4 | 1.00\times10^4 | 0.188 | 0.184 |
| \( n = 0.0 \) | 1.0    | 0.0  | 1.0  | 6.19\times10^4 | 6.19\times10^4 | 0.088 | 0.096 |
| 0.1       | 0.0    | 1.0  | 5.77\times10^5 | 1.33\times10^5 | 0.064 | 0.065 |

\( ^a \)implied by \( \delta_{cf} \)

\( ^b \)from direct fits to the empirical mass-radius relation

4. **Summary and Discussion**

For \( \Delta_m \sim 0.6 \), the empirical mass-density obtained in a number of different cosmogonies is very well fitted by the simple relations \( \text{[III]} \) or \( \text{[IV]} \). This confirms, with a physically motivated halo formation time estimate, the previous claim by NFW that the characteristic density shown by dark halos in equilibrium is proportional to the mean or critical density of the universe at the time they form. A relation like eq. \( \text{[V]} \) is naturally expected in the case that halos show a universal dimensionless density profile at the time of formation, and then keep the central part steady while the virial radius expands as the density of the universe (and its critical value) diminishes. This simple interpretation of the mass-density correlation is supported by the fact that relation \( \text{[VI]} \) also gives, for \( \Delta_m \sim 0.6 \), very good fits to the empirical mass-radius correlation, with values of \( x_{sf} \) fully consistent with those of \( \delta_{cf} \) found in the previous fit.

Such a behavior for the scaling parameters of halo density profiles is very well understood as the combined result of the different dynamical effects of major mergers and accretion. The strong violent relaxation due to the former would produce an important rearrangement of the halo structure leading to essentially a universal dimensionless density profile only dependent on the particular cosmogony considered. On the contrary, the smooth growth via spherical secondary infall due to accretion would keep the halo central body unaltered and gradually build an extended envelope following the slow decrease of the mean density of the universe.

In another talk in this meeting (see González-Casado, Raig, & Salvador-Solé 1998) we report the results of the detailed check of such an evolutionary scheme.
for halos. As it is shown there, the values of parameters $\delta_{cf}$ or $x_{sf}$ are completely fixed in such a scenario, and it turns out that they are consistent with the values found (see Table 1) in the fits to the empirical mass-density and/or mass-radius correlations.

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