Improved 2D model of a ball bearing for the simulation of vibrations due to faults during run-up

Matej Tadina and Miha Boltežar
University of Ljubljana, Faculty of Mechanical Engineering, Aškerčeva 6, 1000 Ljubljana, SI-Slovenia
E-mail: matej.tadina@domel.si

Abstract. This paper present an improved 2D bearing model for investigation of the vibrations of a ball-bearing during run-up. The presented numerical model assumes deformable outer race, which is modelled with finite elements, centrifugal load effects and radial clearance. The contact force for the balls is described by a nonlinear Hertzian contact deformation. Various surface defects due to local deformations are introduced into the developed model. The detailed geometry of the local defects is modelled as an impressed ellipsoid on the races and as a flattened sphere for the rolling balls. The obtained equations of motion were solved numerically with a modified Newmark time-integration method for the increasing rotational frequency of the shaft. The simulated vibrational response of the bearing with different local faults was used to test the suitability of the continuous wavelet transformation for the bearing fault identification and classification.

1. Introduction
Ball bearings are among the most important and frequently used components the electric motors; however bearings may contain manufacturing errors or mounting defects. Such errors cause vibration, noise, and even failure of the whole system, which leads to expensive claims for damage. To avoid this and to ensure rapid and cheap production there is a need for a quick end-test of the electric motors to determine any bearing faults.

The first step in bearing-fault detection during run-up would be a numerical model for the bearing-vibration response due to faults during the run-up. A well-defined vibration signal during the run-up of a faulty bearing could be used to find a suitable method for the fault diagnostic. A lot of research work has been done to model the vibration response of a bearing due to faults at a constant rotational speed. The first mathematical model for modelling bearing vibrations was proposed by Sunnersjo [1]. The bearing was modelled as a 2 DOF system, which provides the load-deflection according to Hertzian contact theory, while ignoring the mass and the inertia of the rolling elements. The two orthogonal DOF are related to the inner race. Rafsanjani et al. [2] combined this 2 DOF model with an analytical approach to model the nonlinear dynamic behavior of a bearing due to localised surface defects. In this model the effect of the radial internal clearance was taken into account. Liew [3] presented four different bearing models in order to model bearing vibrations. The most comprehensive model includes the rolling-element centrifugal load, the angular contact and the radial clearance, and is a 5 DOF model. The 5 DOF bearing model includes not only the radial displacement of the inner race, but also the axial displacement and the rotation around the x and y axes, Figure 1. Building on the 2 DOF model developed by Liew, Feng [4] developed a bearing-pedestal model with 4
DOF, as it includes a pedestal 2 DOF. The model takes into consideration the slippage of the rolling elements, the effect of mass unbalance in the rotor and the possibility of introducing a localised fault in the inner or outer race. Arslan and Aktürk [5] developed a shaft-bearing system with bearing defects. The model has 3 DOF, 2 DOF for the radial displacement and 1 DOF for the axial displacement. The balls in the bearing have additional DOFs since they can vibrate in a radial direction. The effect of the centrifugal load on the balls is neglected. A new, comprehensive model was proposed by Sopanen and Mikkola [6, 7], which includes the effect of different geometrical faults, such as surface roughness, waviness and localised and distributed defects. The dynamic model is a 6 DOF model, which includes both the non-linear Hertzian contact deformation and the elasto-hydrodynamic fluid film. The model of the ball bearing was implemented and analysed using a commercial, multi-body software application (MSC.ADAMS).

A common feature of all these models is that the rotational speed of the shaft is constant, and that the outer ring is firmly fitted to a rigid housing. Although many researches have investigated the vibration characteristics of ball bearings due to local defects, no studies have been published on bearing vibration due to local defects during the run-up of the shaft. Hence, this paper will present an improved ball-bearing model with local faults, to simulate the vibration signal of a defective bearing during the run-up of the shaft, where the centrifugal load is taken into account and the outer ring is deformable in the radial direction. Such a simulated vibration signal of a faulty bearing will be used to test the suitability of the wavelet analysis for bearing-fault identification during run-up.

2. Bearing modelling

Based on previous studies [3, 8], improved 2D model of a radial ball bearing was developed, shown in Figure 1, to determine the vibrations of a faulty bearing during run-up.

![Figure 1. Schematic Diagram of a ball bearing.](image)

Lagrange equations are applied to derive the equations of motion of the bearing model. Using the Lagrange equations it is necessary to calculate the total potential and kinetic energy of the system, shown in Figure 2. In order to calculate the potential energy of the contact deformation, first the contact deformation of the \( j \)-th ball at the inner and outer races has to be calculated. Since the outer race is not fixed and is deformable in the radial direction, the inner and outer contact deformations are expressed as [8]:

\[
\delta_{i,j} = r + \rho_i - \chi_j, \\
\delta_{o,j} = (\rho_j + \rho_b) - R(\theta_j),
\]

![Figure 2. Positions of race and balls centres and deflection of the \( j \)-th ball-race contact.](image)
The variable $\theta_j$ defines the angular position of the $j$th ball centre with respect to the centre of the outer race $x_o$ and $y_o$, $r$ is the radius of the inner race, $\rho_j$ is the radial position of the $j$-th ball from the centre of the outer race, $\rho_b$ is the radius of the ball, $x_i$ and $y_i$ are the positions of the inner race and $\chi_j$ is radial position of the $j$-th ball from the centre of the inner race.

Since the outer ring in the proposed model is deformable, the local radius of the outer race is:

$$R(\theta_j) = R_0 + N_j^e \cdot d^e,$$

where $R_0$ is the radius of the non-deformed outer race, $N_j^e$ is the vector of the interpolation functions evaluated at the position in the element where the contact occurs and $d^e$ is the vector of the nodal displacements for the element.

The contact force between the inner race and the ball or the outer race and the ball arise only when there is a compression in the contact, which means that $\delta_i$ and $\delta_o$ have to be greater than 0:

$$\delta_{i,j^+} = \begin{cases} r + \rho_b - \chi_j, & \text{if } r + \rho_b - \chi_j > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\delta_{o,j^+} = \begin{cases} (\rho_j + \rho_b) - R(\theta_j), & \text{if } (\rho_j + \rho_b) - R(\theta_j) > 0 \\ 0, & \text{otherwise} \end{cases}$$

With a known contact deformation and following the procedure described in [9] the total potential and kinetic energy are calculated. Applying the Lagrange equation, the governing equations of motion of the inner ring and the rolling elements are derived as:

$$m_s \ddot{x}_s + k_i \sum_{j=1}^{N} \frac{\delta_{i,j^+}^3 \rho_j \cos \theta_j + x_o - x_i}{\chi_j} = F_u \cos \theta_s$$

$$m_s \ddot{y}_s + k_i \sum_{j=1}^{N} \frac{\delta_{i,j^+}^3 \rho_j \sin \theta_j + y_o - y_i}{\chi_j} = m_s g + F_u \sin \theta_s$$

$$m_b \ddot{\rho}_j - \rho_j \omega^2 + k_o \delta_{o,j^+}^3 - k_i \delta_{i,j^+}^3 \rho + \frac{(x_o - x_i) \cos \theta_j + (y_o - y_i) \sin \theta_j}{\chi_j}$$

$$- m_b g \sin \theta_j - m_b (\ddot{x}_o \cos \theta_j + \ddot{y}_o \sin \theta_j)$$

And for the outer ring, since the finite elements are used:

$$M \ddot{d} + C \dot{d} + K d = F_m$$

where $M$ is the structural mass matrix, $K$ is the structural stiffness matrix and $F_m$ are the nodal forces in the finite-element model and $d$ is the displacement vector, and dot notation is used to denote the differentiation with respect to time. $C$ is the structural damping matrix, which is additionally introduced as the equivalent Rayleigh damping.

Because the location of the contact forces between the outer ring and the rolling element moves on the beam element, the moving load formulation within the element has to be used. The moving load vector $F_m$ for the outer ring is formulated as:

$$F_m = \sum_{j=1}^{N} N_j^T F_{o,j},$$

where $N_j^T$ is the transposed vector of the interpolation functions, $F_{o,j}$ is the contact force on the outer race at the $j$-th ball.
3. Modelling of faults
At an early stage of the bearing’s operation almost only local defects are present due to the improper mounting of the bearing, which will cause a dent due to the plastic deformation of the rolling surface. Most of the presented models of bearing faults [2, 4, 5] assume that the rolling element will lose contact suddenly once it enters the dent region, and will regain contact instantly when exiting that area. In this way the modelling results in very large impulsive forces in the system and as a consequence the acceleration increases sharply in order to maintain the balance within system. In order to model the behaviour of the rolling element so as to reflect the actual path that the rolling element takes while rolling into and out of the dent, the dent was modelled as an ellipsoidal depression on the inner and outer races, while the dent on the rolling element was modelled as a flattened sphere. Another important aspect of this model is that due to the different geometry of the contact between the dent and the bearing component the contact-stiffness changes due to the different geometrical properties in the contact.

3.1. Outer-race defect
Let there be a defect on the surface of the outer race at an angle $\varphi$ from the horizontal axis, as shown in Figure 3. The dent has an angle length of $\varphi_d$. If the rolling-element position $\theta_j$ coincides with the dent-angle range $\varphi < \theta_j < \varphi + \varphi_D$ the contact deformation will be smaller for the dent depth $D_d$ at the position $\varphi_c$ where is the contact point $C$. The contact deformation between the rolling element and the dent is now calculated as:

$$\delta_{Do+} = \begin{cases} \|\rho_j + \rho_{BC}\| - (R(\varphi_C) + D_{do}(\varphi_C)) & \text{if } \|\rho_j + \rho_{BC}\| - (R(\varphi_C) + D_d(\varphi_C)) > 0 \\ 0 & \text{otherwise} \end{cases}$$ (11)

where $D_d$ is the depth of the dent at the contact position $\varphi_c$. Another important aspect of the detailed dent modelling is changing stiffness in the contact. According to Hertzian contact theory the contact stiffness depends on the curvatures in the contact. When the rolling element strikes the dent, the curvature properties are different to those on the race. Not only that the contact stiffness change in the dent, but also the direction of the contact force change, as is shown in Figure 3. The contact force does not act in the direction of $\rho_j$ under the angle $\theta_j$ but under the angle $\theta_C$, and thus the contact force in the radial direction is:

$$F_{c,d} = k_{d,o} \delta_{Do+}^2 \cos(\theta_j - \theta_C)$$ (12)

The tangential component of the contact force in the dent is neglected because the cage ensures the angular position of the rolling elements, but is taken into account as the loading on the outer race.

3.2. Inner-race defect
Let there be a defect on the surface of the inner race as shown in Figure 4. This defect will rotate with the angular speed of the shaft $\omega_S$. If the defect angle $[\varphi, \varphi + \varphi_d]$ coincides with one of the balls the deflection on that ball will be:

$$\delta_{Di+} = \begin{cases} (r - D_d(\varphi_C)) - \|\chi_j + \rho_{BC}\| & \text{if } (r - D_d(\varphi_C)) - \|\chi_j + \rho_{BC}\| > 0 \\ 0 & \text{otherwise} \end{cases}$$ (13)

As for the dent on the outer race, the stiffness in the contact and the direction of the contact force changes in the contact between the dent and the ball. The component of the force in the radial direction of the ball is given as:

$$F_{c,di} = k_{d,i} \delta_{Di+}^2 \cos(\theta_C - \theta_j)$$ (14)

$$F_{c,di} = \frac{\rho + (x_o - x_i) \cos \theta_j + (y_o - y_i) \sin \theta_j}{\chi_j}$$

The tangential component of the contact force on the ball is neglected but is taken into account as loading on outer race.
3.3. Defect on the ball surface

The defect on the ball surface is modelled as a flattened ball, as shown in Figure 5 and Figure 6, where the flattened region is a sphere with a larger radius. The loss of contact will happen twice per complete rotation of the damaged rolling element, i.e. when the dent is in contact with the inner race and the outer race. The dent has the form of a sphere with the radius $R_2$ and $R_2 > \rho_b$. Due to the constant radius of the dent the contact stiffness is a constant, although it is different to that in the contact between the races and the rolling element.

When the dent of the rolling element is in contact with the inner ring, the contact deformation is defined as:

$$\delta_{DB_i} = \begin{cases} 
  r - (\|\chi_j + \rho_{12}\| - R_2), & \text{if } r - \|\chi_j + \rho_{12}\| + R_2 > 0 \\
  0, & \text{else}
\end{cases} \quad (15)$$

where $\rho_{12}$ is the vector from the centre of the rolling element to the centre of the radii of the flattened curvature. The projection of the contact force to the radial direction of the rolling
element is:

\[ F_{DCi} = k_{DCi} \delta_{DBi}^2 \rho + (x_o - x_i) \cos \theta_j + (y_o - y_i) \sin \theta_j \times \cos(\theta_x - \phi_C), \]  

(16)

where \( k_{DCi} \) is the contact stiffness between the dent on the ball and the inner race.

The contact deformation between the dent on the ball and the outer race is given by:

\[ \delta_{DBo} = \begin{cases} \| \rho_j + \rho_{12} \| + R_2 - R(\theta), & \text{if } \| \rho_j + \rho_{12} \| + R_2 - R(\theta) > 0 \\ 0, & \text{else} \end{cases} \]  

(17)

and the contact force in the radial direction of the ball position is given by:

\[ F_{DCo} = k_{DCo} \delta_{DBo}^2 \rho + (x_o - x_i) \cos \theta_j + (y_o - y_i) \sin \theta_j \times \cos(\theta_j - \phi_C) \]  

(18)

4. Numerical results

In this section the presented model was used to simulate the vibration response of a bearing with different faults during run-up. Whenever the fault is in contact with its mating surface, one of the equations (12)-(18) is used to replace the appropriate equations for the contact force in equations (6)-(9).

The bearing under investigation is NMB R-1240KK1, shown in Figure 7, whose parameters are given in Table 1. The outer ring is assumed to be closely fitted into another aluminium ring, which represents the housing of the bearing and the under half of the aluminium ring is connected with springs to the fixed pedestal, as shown in Figure 8. The aluminium ring is 3mm thick, and is connected with 36 linear springs to the fixed pedestal. The stiffness of the connecting spring is \( k_s = 100000 \) N/m. The bearing loads are assumed to be the gravitational load and the unbalanced force. The unbalance of the rotor is \( me_o = 2 \times 10^{-5} \) kgm². The dynamic response of the bearing will be taken from point P1, which is on the top of the aluminium ring, Figure 8.

Table 1. Geometrical and physical properties used for the rolling-element bearing shown in Figure 7.

| Property                           | Value            |
|-----------------------------------|------------------|
| Inner-race radius \( r \)         | 2.754 mm         |
| Outer-race radius \( R \)         | 4.7565 mm        |
| Ball radius \( \rho_b \)          | 1 mm             |
| Number of balls \( N_b \)         | 7                |
| Ball mass \( m_b \)               | 0.0000329 kg     |
| Mass of inner ring and shaft \( m_S \) | 0.15 kg     |
| Radial clearance \( c \)          | 5 \( \mu \)m    |
| Inner contact stiffness \( k_i \) | \( 3.828 \times 10^9 \) N/m |
| Outer contact stiffness \( k_o \)  | \( 3.867 \times 10^9 \) N/m |

The geometrical properties of the local faults are shown in Figures 9-11. The faults on the inner race and on the ball will rotate and their position will move in and out of the loading zone in the bearing. The fault on the outer race is stationary, and the position of the dent on outer race is very important. Analyses will be made for the dent in the middle of the loading zone, at the bottom of the outer race.
4.1. Vibration response of the bearing due to local faults during the run-up of the shaft
With the presented model, the vibration response of the roller-element bearing due to a local defect is simulated during the run-up of the shaft, with angular acceleration \( \alpha_S = 20\pi \text{ rad/s}^2 \), which will produce non-stationary vibrational signals of the bearing. Non-stationary signals can be analysed by applying time-frequency domain techniques such as the short-time Fourier transform, the Wavelet transform (WT) and the Hilbert-Huang transform [10, 11]. In fault diagnostics, the WT is the most popular time-frequency domain technique, because it can provide a more flexible, multi-resolution solution than the short-time FT. For the detection of the present bearing fault the continuous WT (CWT) transform will be used. The damaged bearing will produce small amplitudes of vibration in the high frequency band as the response of the bearing and the housing to the impact that is caused by the fault. With known instantaneous shaft rotational frequency \( f_S \), the instantaneous characteristic frequencies of bearing faults can be calculated with well established equations [12]. The characteristic defect frequencies for the analysed bearing are listed in Table 2, where \( f_S \) is the rotational speed of the shaft, and \( f_b \) is the ball’s rotational frequency. The high-frequency band of structural vibrations has to be known prior to the wavelet analysis. In the analysed case, Figure 8, the high-frequency response will be at the eigenfrequency of the bearing/housing-pedestal system, which is around 8.2 kHz; the eigenfrequency of the shaft is around 11.3 kHz, although this frequency varies due to the changing position of the balls, and the first eigenfrequency of the outer ring with the aluminium ring is around 36 kHz. The existence of the vibrations in these frequency bands will be used to identify the bearing faults. The classification of the bearing faults will be made based on the time interval between the repetitive vibrations in the high-frequency band. The time interval between the vibrations in the high-frequency band is determined by the instantaneous rotational frequency of the shaft and the type of bearing fault. The CWT analysis will be made for the simulated time signal between 2.2 s and 2.5 s, and the rotational speed of the shaft is linearly changing from 20 Hz to 23 Hz.
4.1.1. Healthy bearing  The time-frequency plot of the CWT is shown in Figure 12. There are small amplitudes of the vibration in the high-frequency band, most of them around 11 kHz. The time intervals $T_1 \approx 0.0125$ s and $T_2 \approx 0.0099$ s between two consecutive amplitudes indicate the excitation at a high frequency. The time interval $T_1$ corresponds to the frequency $f_1 = 85.5$ Hz, which is the 4th harmonic of the shaft’s instantaneous rotating frequency, which is at $t = 2.24$ s $f_{S1} = 20.3$ Hz. The time interval $T_2$ corresponds frequency to the $f_2 = 101$ Hz, which would indicate bearing inner-race damage, since the characteristic inner-race defect frequency at an instantaneous shaft speed $f_{S1} = 22.2$ Hz is $f_{iD} = 4.432 \times 22.3 = 98.8$ Hz. Although the bearing is healthy, there is the presence of the characteristic frequency of the inner race fault. This presence is due to the varying position of balls in the bearing and due to the centrifugal loading of the shaft. From the time signal it is clear that the amplitude of the bearing vibration is very small. The conclusion can be made that the bearing has no fault, although in the spectra there are detectable characteristic fault frequencies.

**Table 2.** Characteristic defect frequencies for the stationary outer ring.

| Characteristic | Frequency |
|----------------|-----------|
| Outer race defect frequency, $f_{oD}$ | $2.568 f_S$ |
| Inner race defect frequency, $f_{iD}$ | $4.432 f_S$ |
| Ball defect frequency, $f_{bD}$ | $3.488 f_S$ |

**Figure 12.** Vibration response of the healthy bearing. (a) Time domain vibration response at point P1. (b) Plot of the CWT.

4.1.2. Outer-race fault  From the time-frequency plot of the CWT, Figure 13, the existence of high-frequency components is detected and their occurrence is repetitive, which indicates the presence of the fault. The high-frequency response is observed at the first eigenfrequency of the outer race and the eigenfrequency of the outer ring-pedestal system, which is higher in amplitude. The time intervals $T_1 = 0.019$ s and $T_2 = 0.0179$ s correspond to the frequencies $f_1 = 52.6$ Hz and $f_2 = 55.9$ Hz. These two frequencies are correlated with the characteristic frequencies of the outer-race defect, at the shaft’s instantaneous rotational frequencies $f_{S11} = 20.7$ Hz and $f_{S12} = 21.5$ Hz.

4.1.3. Inner-race fault  From the time-frequency plot of the CWT, Figure 14, the existence of high-frequency components is detected and their occurrence is repetitive, which indicates the presence of the fault. The vibrations of the bearing/pedestal structure appear in the spectrum
only when the inner-race fault is in the loading zone. Due to this the interval between two consecutive components of the vibration in the high-frequency band has to be carefully chosen. The time intervals $T_1 = 0.0108$ s and $T_2 = 0.0102$ s correspond to the frequencies $f_1 = 92.6$ Hz and $f_2 = 98.3$ Hz. These two frequencies correlate very well with the characteristic frequencies of the outer-race defect at the instantaneous shaft speeds $f_{S11} = 20.7$ at $t_1 = 2.27$ s and $f_{S12} = 22.5$ Hz at $t_2 = 2.45$ s.

Figure 13. Vibration response of the bearing with the outer-race defect. (a) Time domain vibration response at point P1. (b) Plot of the CWT.

4.1.4. Rolling-element fault To investigate the vibration response of the bearing to the ball fault, it is assumed that the dent is on the 1st ball. The size and the geometry of the flattened ball are shown in Figure 11.

From the time-frequency plot of the CWT, Figure 15 the existence of high-frequency components is detected only when the ball is in the load zone. The time interval between two consecutive amplitudes in the high-frequency band determines the presence of the ball faults. The time intervals $T_1 = 0.0134$ s and $T_2 = 0.0125$ s correspond to the frequencies $f_1 = 74.6$ Hz and $f_2 = 80$ Hz, which are characteristic frequencies of the ball defect at the shaft’s instantaneous

Figure 14. Vibration response of the bearing with the inner-race defect. (a) Time domain vibration response at point P1. (b) Plot of the CWT.
rotational frequencies $f_{Si1} = 21$ Hz at $t_1 = 2.3$ s and $f_{Si2} = 22.5$ Hz at $t_2 = 2.45$ s.

Figure 15. Vibration response of the bearing with the ball defect. (a) Time domain vibration response at point P1. (b) Plot of the CWT.

5. Conclusion
A comprehensive model of a ball bearing is developed to obtain the vibration response due to localised defects. For the purpose of simplification, the inner ring of the bearing has only 2 DOF and the centrifugal load effects are included. The proposed model includes several new considerations. The outer ring is deformable and is modelled with finite elements. The contact properties are described with detailed geometrical properties and small changes in the contact stiffness are taken into account. The combination of the rolling elements as rigid bodies, where contact is modelled as an analytical Hertz contact, and with the outer ring modelled as finite elements is very efficient from numerical point of view. Presented bearing model is used to simulate the vibration response of the bearing/pedestal system due to different local faults, while the shaft’s rotational frequency is increasing. The continuous wavelet analysis was successfully applied to identify and to characterise bearing faults on simulated faulty bearing vibration signal.

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