METALLIC FINGERS AND METALlicity EXCESS IN EXOPLANETS’ HOST STARS: THE ACCRETION HYPOTHESIS REVISITED

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ABSTRACT

While the metallicity excess observed in the central stars of planetary systems is confirmed by all recent observations, the reason for this excess is still a subject of debate: is it primordial, or the result of accretion, or both? The basic argument against an accretion origin is related to the mass of the outer convective zones, which varies by more than 1 order of magnitude among the considered stars, while the observed overabundances of metals are similar. We show here that in previous discussions a fundamental process was forgotten: thermohaline convection induced by the inverse \( \mu \)-gradient. “Metallic fingers” might be created that dilute the accreted matter inside the star. Introducing this effect may reconcile the overabundances expected in cases of accretion with the observations of stars of different masses.

Subject headings: accretion, accretion disks — convection — diffusion — planetary systems

1. INTRODUCTION

The metallicity distribution of the central stars of planetary systems compared with other stars of the same spectral types clearly shows a metallicity excess of a factor of 2 on average, while the individual [Fe/H] values lie between \(-0.3\) and \(+0.4\) (Santos et al. 2003 and references therein). Although this excess is generally discussed in terms of iron abundances only (for most observers, [Fe/H] is synonymous with “metallicity”), spectroscopic observations show that the abundances of other heavy elements, beginning with carbon, are also enhanced (Santos, Israelian, & Mayor 2001).

The proposed possible explanations for this behavior are of two kinds:

1. Accretion hypothesis: this assumes that the star had normal abundances at its formation, but accreted metal-rich matter during the phases of planetary formation (Gonzalez 1998).

2. Primordial hypothesis: this assumes that the stellar gas out of which the planetary system formed was already metal enhanced. A corollary in this case would be that such a metal enrichment is necessary for planets to form around stars.

Other explanations have been proposed that mix the two previous scenarios.

The strongest argument invoked against the accretion hypothesis is related to the mass of the outer convective zone, which varies by more than 1 order of magnitude for stars between 1.0 and 1.4 \( M_\odot \), while the metallicity excess remains the same. On the other hand, the primordial hypothesis has to deal with the fact that some of the host stars have a subsolar metallicity, which is an argument against the assumption that metallicity excess is necessary for planetary formation.

Diffusion of accreted metals has been discussed by several authors (Pinsonneault, DePoy, & Coffee 2001; Murray & Chaboyer 2002). They show that gravitational settling is not sufficiently rapid to produce a decrease of the metal abundances and so lead to the observed values. They also discuss rotational and other mixing processes in relation to the observed lithium abundances. Since lithium is destroyed by thermonuclear reactions at a relatively low temperature (\( 3 \times 10^6 \) K) inside stars, it is often used as a test for mixing processes. However, such a comparison between the metal diffusion and the lithium depletion is highly model dependent, since it assumes a specific variation of the mixing effect with depth.

Here we show that the argument against accretion does not hold once we take into account the convective instability induced by inverse \( \mu \)-gradients, or thermohaline convection (also called “double-diffusive convection”). If freshly accreted metals accumulate in a small convective zone on top of radiative layers, they do not stay there but are diluted downward in “metallic fingers” similar to the “salt fingers” observed in the ocean, so that their overabundance rapidly decreases with time.

We discuss how this process should take place, and reach the conclusion that the final metallic abundances should not depend on the depth of the standard convective zone, as is usually assumed. However, the exact amount of metals that may remain in the stellar outer layers depends on parameters such as the size and depth of the metallic fingers, which cannot be precisely constrained in the framework of our present knowledge. We only show that a final overabundance by a factor of 2, as observed, can be obtained with plausible values of the unknown parameters. There are two paths to explore in furthering this study: numerical simulations of metallic fingers and asteroseismology of overmetallic stars.

2. THERMOHALINE CONVECTION

2.1. The Salted-Water Case

Thermohaline convection is a well-known process in oceanography: warm salted layers on the top of cool unsalted layers rapidly diffuse downward, even in the presence of stabilizing temperature gradients. When a “blob” of salted water is displaced downward, it is pulled farther down because it is heavier than its surroundings, but at the same time its greater temperature opposes this tendency. When the salt gradient is large compared to the thermal gradient, salted water normally mixes down until the two effects compensate. Then thermohaline convection begins. While the medium is marginally stable, salted blobs fall down like fingers; unsalted matter rises around these. This process is commonly known as “salt fingers” (Stern 1960; Kato 1966; Veronis 1965; Turner
referred to as “double-diffusive convection.” In oceanography, the fastest diffusive component is conventionally referred to as \( T \), while the slowest one is \( S \), even when the components are different. Indeed, the effect is generally studied in the laboratory using water mixed with salt and sugar. In this case, sugar is the slowest component since it diffuses ~3 times slower than salt. The results can be well visualized, pictured, and studied (e.g., Wells 2001). The fingers have also been studied using two-dimensional and three-dimensional numerical simulations (e.g., Piacsek & Toomre 1980; Shen & Veronis 1997; Yoshida & Nagashima 2003).

The conditions necessary for the salt fingers to develop are related to the density variations induced by temperature and salinity perturbations. Two important characteristic numbers are defined as (1) the density anomaly ratio

\[
R_p = \frac{\alpha \nabla T}{\beta \nabla S},
\]

where \( \alpha = - (\rho^{-1} \partial \rho / \partial T)_{\rho,T} \), \( \beta = (\rho^{-1} \partial \rho / \partial S)_{\rho,T} \), and \( \nabla T \) and \( \nabla S \) are the average temperature and salinity gradients in the considered zone, respectively; and (2) the so-called Lewis number

\[
\tau = \frac{\kappa_S}{\kappa_T} = \frac{\tau_T}{\tau_S},
\]

where \( \kappa_S \) and \( \kappa_T \) are the saline and thermal diffusivities, respectively, and \( \tau_S \) and \( \tau_T \) are the saline- and thermal-diffusion timescales, respectively.

The density gradient is unstable and overturns into dynamical convection for \( R_p < 1 \), while the salt fingers grow for \( R_p \geq 1 \). On the other hand, they cannot form if \( R_p \) is larger than the ratio of the thermal to the saline diffusivities \( \tau^{-1} \), as in this case the salinity difference between the blobs and the surroundings is not large enough to overcome buoyancy (Huppert & Manins 1973; Gough & Toomre 1982; Kunze 2003).

Salt fingers can grow if the condition

\[
1 \leq R_p \leq \tau^{-1}
\]

is satisfied. In the ocean, \( \tau \) is typically 0.01, while it is 1/3 for a salt-sugar mixture. We see below that in solar-type stars where \( T \) is the temperature and \( S \) is the mean molecular weight, this ratio is \( \sim 10^{-10} \) if \( \kappa_S \) is the molecular (or “microscopic”) diffusion coefficient, but it can increase by many orders of magnitude when the shear flow instabilities that induce mixing between the edges of the fingers and the surroundings are taken into account.

### 2.2. The Stellar Case

Thermohaline convection may occur in stellar radiative zones when a layer with a larger mean molecular weight sits on top of layers with smaller ones (Kato 1966; Spiegel 1969; Ulrich 1972; Kippenhahn, Ruschpenplatt, & Thomas 1980, hereafter KRT80). In this case, \( \nabla \mu = d \ln \mu / d \ln P \) plays the role of the salinity gradient, and the difference \( \nabla_{ad} - \nabla \) (where \( \nabla_{ad} \) and \( \nabla \) are the usual adiabatic and local [radiative] gradients \( d \ln T / d \ln P \), respectively) plays the role of the temperature gradient. When \( \nabla_{ad} \) is smaller than \( \nabla \), the temperature gradient is unstable against convection (Schwarzschild criterion), which corresponds to warm water below cool water in oceanography. In the opposite case, the temperature gradient is stable but the medium can become convectively unstable if

\[
\nabla_{crit} = \frac{\phi}{\delta} \nabla_{ad} + \nabla - \nabla < 0,
\]

where \( \phi = (\partial \ln \rho / \partial \ln \mu) \) and \( \delta = (\partial \ln \rho / \partial \ln T) \). When this situation occurs, convection first takes place on a dynamical timescale and the \( \mu \)-enriched matter mixes down with the surroundings until \( \nabla_{crit} \) vanishes. Then marginal stability is achieved and thermohaline convection may begin as a “secular process,” namely on a thermal timescale (which is short compared to a stellar lifetime).

Such an effect has previously been studied for stars with a helium-rich accreted layer (KRT80). It was also invoked for helium-rich stars, in which helium is supposed to accumulate because of diffusion in a stellar wind (as proposed by Vauclair 1975), and for rapidly oscillating Ap (roAp) stars in cases in which some helium accumulation occurs (Vauclair, Dolez, & Gough 1991). Similar computations have been done in the case of accretion of matter on white dwarfs, in relation to novae explosions (e.g., Marks & Sama 1998).

As shown below, if hydrogen-poor matter is accreted on the top of a main-sequence type star with normal abundances, it creates an inverse \( \mu \)-gradient that can lead to thermohaline convection. Comparing the stellar case with the water case, we can guess that metallic fingers will form if the condition

\[
1 \leq \left| \frac{\delta (\nabla_{ad} - \nabla)}{\phi (\nabla_{ad})} \right| \leq \tau^{-1}
\]

is verified, with \( \tau = D_\mu / D_T = \tau_T / \tau_\mu \), where \( D_T \) and \( D_\mu \) are the thermal and molecular diffusion coefficients, respectively, and \( \tau_T \) and \( \tau_\mu \) are the corresponding timescales. This condition is similar to condition (3) for the stellar case. In the following, we neglect the deviations from perfect gas law, so that \( \phi = \delta = 1 \).

In the following section, we show computations of the timescales and orders of magnitude of this process.

### 3. THE FATE OF ACCRETED METALS IN SOLAR-TYPE STARS

We study the case of main-sequence solar-type stars that would have accreted hydrogen-poor material at the beginning of their lifetime. We assume, for simplicity, that the accretion occurred in a very short timescale compared to stellar evolution. We then study the fate of the accreted metals and choose 1.1 and 1.3 \( M_\odot \) stars as examples.

As the chemical composition of the accreted matter is not known, we assume that all elements are accreted with solar relative abundances except hydrogen and helium, which are assumed completely absent. We see that our general conclusion is unchanged if the relative abundances are modified; only the timescales are slightly different.
After accretion, the metal dilution occurs in two phases. In phase one, rapid convection takes place in a dynamical timescale until it reaches the marginal equilibrium obtained when \( \nabla_{\text{int}} \) vanishes (eq. [4]). In phase two, thermohaline mixing begins and dilutes the metal excess until condition (5) is no longer satisfied.

In the following, we first discuss the \( \mu \)-gradients induced by the original metal excess in the convective zone. We compute the situation at the end of phase one, according to the value of the accreted mass. We discuss the case of the same accretion occurring for the two different stellar masses. Finally, we study the timescales of thermohaline mixing and the remaining overabundances in the convective zone.

### 3.1. Dynamical Convection and Marginal Equilibrium

The main parameters of the two stellar models we have used as examples are given in Table 1. They correspond to 1.1 and 1.3 \( M_\odot \) both at an age of \( \sim 1.5 \) Gyr. We can see that the convective zone is 7 times more massive for the 1.1 \( M_\odot \) model than for the 1.3 \( M_\odot \) model, which does not change sensitively during main-sequence evolution. We thus expect that metal accretion leads to an original overabundance 7 times larger in the 1.3 \( M_\odot \) model than in the 1.1 \( M_\odot \) model.

For completely ionized hydrogen and helium, the mean molecular weight can be obtained with the expression

\[
\mu = \frac{1 + 4(\text{He}/\text{H}) + A(M/H)}{2 + 3(\text{He}/\text{H}) + x(M/H)}.
\]

Here, \( A \) represents an average mass of metals, \( (M/H) \) a relative abundance with respect to hydrogen, and \( x \) an averaged number of particles (ion and electrons) associated with the metals. In the following, we take \( (\text{He}/\text{H}) = 0.1 \) and \( (M/H)_0 = 1.4 \times 10^{-3} \) for the original abundances of helium and metals (Grevesse & Sauval 1998). We treat the metals as an average element to which we attribute the mass \( A = 20 \). We assume hydrogen and helium to be completely ionized and we neglect the term \( x(M/H) \). In these conditions, we obtain \( \mu_0 = 0.6 \) for the value of the mean molecular weight before accretion. If metals are added, \( \mu \) is modified by

\[
\Delta \mu \simeq 9\Delta \left( \frac{M}{H} \right).
\]

Let us now write

\[
\frac{M}{H} = \alpha \left( \frac{M}{H} \right)_0
\]

and define \( \alpha_i \) as the initial value of \( \alpha \), obtained just after the accretion process concludes (the accretion process is assumed to be rapid compared to the other timescales).

The variations of \( (M/H) \) and \( \mu \) are then related by

\[
\Delta \mu = 9(\alpha - 1) \left( \frac{M}{H} \right)_0.
\]

We have computed, as a function of the initial overabundance ratio \( \alpha_i \), the depth at which metal-enriched material has been diluted when it reaches the marginal-equilibrium phase, and the actual overabundance ratio \( \alpha \) in the convective zone. This has been obtained by integrating in all cases the mass of metals diluted by this process.

Suppose, for example, that the metal-excess ratio in a 1.1 \( M_\odot \) star is \( \alpha = 2 \) after dilution by dynamical convection. This corresponds to an original overabundance factor of 2.05. The same accretion mass would lead to an original metal-excess ratio \( \alpha_1 = 14.3 \) in a 1.3 \( M_\odot \) star. Our computations show that such an original metal excess is diluted in a “transition zone” down to \( r_{\text{TZ}} = 7.5 \times 10^{10} \) cm, leading to an overabundance after dilution \( \alpha = 8.5 \). Figure 1 displays the abundance profiles obtained in this case in the 1.3 \( M_\odot \) star. The parameters at the bottom of the transition zones as we have defined above are shown in Table 1 for the two models. Figures 2 and 3 display, for the two models, the values of \( \alpha_i \) and the corresponding \( \alpha \) as a function of the width of the transition zone.

### 3.2. Thermohaline Mixing, Metallic Fingers, and Timescales

The study of thermohaline mixing in stars is far from trivial. Detailed comparisons of numerical simulations and laboratory experiments in the water case have recently been published (Gargett & Ruddick 2003), but the stellar case may differ, since mixing occurs in a compressible stratified fluid.

In the following, we use the formalism proposed by KRT80, who discussed the timescale of thermohaline mixing in stars

| Model     | \( r \) | \( r/R_\odot \) | \( \Delta M_\odot \) | \( T_\odot \) | \( P_\odot \) | \( \kappa_\odot \) | \( \tau_{\text{dil}} \) (yr) |
|-----------|--------|----------------|---------------------|-------------|-------------|-----------------|------------------|
| CZ        |        |                |                     |             |             |                 |                  |
| 1.1 \( M_\odot \) | 5.2E10 | 0.75           | 3.7E31              | 2.0E6       | 0.128       | 3.5E13          | ...              |
| 1.3 \( M_\odot \) | 8.0E10 | 0.84           | 5.1E30              | 9.0E5       | 0.007       | 1.2E12          | ...              |
| TZ        |        |                |                     |             |             |                 |                  |
| 1.1 \( M_\odot \) | 4.9E10 | 0.71           | 4.6E31              | 2.3E6       | 0.195       | 6.2E13          | 15.8             | 4200 |
| 1.3 \( M_\odot \) | 7.5E10 | 0.79           | 8.2E30              | 1.2E6       | 0.013       | 2.2E12          | 12.6             | 800  |

Note.—The values are given at the bottom of the convective zone (CZ) and at the bottom of the “transition zone” (TZ), defined as the region of metal dilution after dynamical convection, for the example discussed in the text (metal excess \( \alpha = 2 \) in the 1.1 \( M_\odot \) model); the metal abundances decrease with depth inside this region until reaching a normal value at \( r_{\text{TZ}} \) (see Fig. 1).
in a simplified way and applied it to the case of helium-rich layers standing on the top of hydrogen-rich ones.

In first approximation, they treated thermohaline mixing as a diffusion theory. The whole picture can be described as follows: blobs of metal-enriched matter begin to fall down from the convective zone and exchange heat and heavy elements with their surroundings; chemicals diffuse more slowly than heat, so the blobs continue falling until they are completely disrupted, thereby creating finger shapes. The most efficient processes for element diffusion out of the blobs are the shear flow instabilities at the edge of the fingers: as the falling matter undergoes friction with rising matter, turbulence occurs and mixes part of the fingers with their surroundings, on a horizontal length scale that is a fraction $\frac{L}{C_{15}}$ of the horizontal length $L$ of the blobs. The blobs disappear when they have traveled down a distance $W$ long enough for this mixing to disrupt them completely.

KRT80 define a diffusion coefficient $D_{th}$ as the product of the blobs’ vertical velocity $v$ and their size $L$. They evaluate $v$ as

$$v = \frac{H_p}{(\nabla_{ad} - \nabla)\tau_{KH}} \left( \frac{D_{\mu}}{\mu} \right),$$

where $D_{\mu}$ represents the $\mu$ difference between the inside and the outside of the blobs. The diffusion coefficient $D_{th}$ is then obtained as

$$D_{th} = \frac{H_p}{(\nabla_{ad} - \nabla)} \frac{L^2}{\tau_{KH}} \left| \frac{d \ln \mu}{dr} \right|,$$

where $H_p$ is the pressure scale height and $\tau_{KH}$ the local thermal timescale given by

$$\tau_{KH} = \frac{C_p \rho \kappa \rho^2 L^2}{16 \alpha c T^3},$$

where the parameters have their usual meanings. We can see that $L^2$ vanishes in this diffusion coefficient, which does not depend on the size of the blobs.

In this simple picture, $D_{th}$ is not exactly the local mixing coefficient, since it involves the whole blobs while local mixing involves only the edges of the blobs. The local mixing coefficient becomes correctly represented by $D_{th}$ only at the bottom of the fingers, where the blobs disappear. For this reason, KRT80 define the thermohaline diffusion timescale as $\tau_{th} = W^2/D_{th}$, where $W$ is the depth of the fingers (it also represents the size of the transition zone in which $\mu$ varies by $\Delta \mu$, between its two extreme values). This diffusion timescale can be written

$$\tau_{th} = \frac{\mu}{\Delta \mu} \frac{C_p \kappa \rho^2 W^3}{4ac T^3} \frac{\nabla_{ad} - \nabla}{H_p}.$$

Note that if the mixing scale at the finger edges increases linearly while the blobs fall down, the coefficient $\epsilon$ is nothing other than the ratio of the horizontal and vertical length scales of the fingers, namely $L/W$. 
In Table 1 we provide the thermohaline diffusion timescales corresponding to the example given in the previous section, with a metallic excess of a factor of 2 in 1.1 $M_\odot$. Computations done with a metal excess of 1.5 instead of 2 give similar results for the thermohaline diffusion timescales. They are always very short compared to the stellar lifetime. If nothing prevented these metallic fingers from growing, the convective zone would be emptied of its metallic matter a short time after it is accreted.

We must now take into account the second condition for the formation of the fingers (eq. [5]), namely

$$\left| \nabla_{\text{ad}} - \nabla_{\mu} \right| \leq \frac{D_T}{D_\mu}. \quad (14)$$

At the end of the dynamical convection phase, the $\left| \nabla_{\text{ad}} - \nabla_{\mu} \right|$ ratio is 1. Then, during thermohaline convection, the local $\mu$-gradient decreases so that the $\left| \nabla_{\text{ad}} - \nabla_{\mu} \right|$ ratio increases. Thermohaline convection should stop when this ratio reaches the value of the $D_T/D_\mu$ ratio. The remaining overabundances in the stellar outer layers strongly depend on the efficiency of the turbulent mixing at the edge of the fingers, which is nonlinear and not well understood. Here we can only give a tentative discussion of this process.

The thermal diffusion coefficient $D_T$ can be evaluated as the square of a relevant length scale (e.g., the size of the falling blobs) divided by the local thermal timescale $\tau_{\text{KHI}}$, so

$$D_T = \frac{L^2}{\tau_{\text{KHI}}} = \frac{16aC^3}{\mu^2r^2}. \quad (15)$$

Below the convective zone in the 1.1 $M_\odot$, $D_T$ is typically $3.0 \times 10^8$ cm$^2$ s$^{-1}$, while it is $8 \times 10^9$ cm$^2$ s$^{-1}$ in the 1.3 $M_\odot$ model. Meanwhile, the microscopic element diffusion coefficient is of the order of 1 cm$^2$ s$^{-1}$: if no turbulence occurred, the metallic fingers would extend far down and deplete the overmetallic material until an extremely small $\mu$-gradient would be reached; in this case, no metallic excess would be left in the convective zone.

If we take into account the shear flow instabilities that result from the motion of the blobs, $D_\mu$ has to be replaced by the local turbulent diffusion coefficient, which is not exactly $D_h$ but $\epsilon^2D_{\text{th}}$, where $\epsilon$ represents the $L/W$ ratio. Using equation (11) for $D_{\text{th}}$, we find from condition (14) that the fingers disappear when $W$ is of the same order as $L$, which makes sense but is not helpful in evaluating the amount of heavy elements that remain in the outer layers at the end of the whole process; the KRT80 approximations are too rough to be useful in this respect.

Another approach consists of estimating the value of $D_\mu$ that would be needed to account for the observations. Our aim is that a metallicity excess of the order of 2 remains in the convective zone after these processes occur. From equation (9), we find that it corresponds to $\Delta \mu \simeq 0.0126$ or, with $\mu = 0.6$ as computed in these stellar layers, $\Delta \mu/\mu \simeq 0.02$. If we suppose that the mixing region extends over 1 pressure scale height (which is by no means proved!), we find that the $D_T/D_\mu$ ratio should be of the order of 10, which seems reasonable compared to the water case. Such a value would be obtained with a turbulent diffusion coefficient $D_\mu$ at the edge of the fingers of the order of $10^7$–$10^9$ cm$^2$ s$^{-1}$.

More computations are needed for a better understanding of the shear flow turbulence induced by the fingers. Furthermore, the growing of the metallic fingers may depend on the other processes at work in the star: rotation-induced turbulence, internal waves, etc. (compare with similar problems in salted water in Gargett & Ruddick 2003).

In any case, the computations presented here show that the metallic matter accreted onto a star does not stay inside the standard convective zone: it first turns over because of dynamical convection and then continues diffusing because of thermohaline convection. The observed overabundances in the host stars of planetary systems can be obtained with realistic values of the unknown parameters.

4. CONCLUSION AND FUTURE PROSPECTS

The important conclusion of this paper is that the strongest argument against accretion as an explanation of the metallic excess observed in planetary systems’ host stars has to be revised: if hydrogen-poor matter is accreted in the early phases of stellar evolution, during planet formation, the metal excess in the convective zone leads to a stabilizing $\mu$-gradient that induces metal dilution, first by dynamical convection, then by thermohaline mixing, on a timescale much shorter than the stellar lifetime (typically 1000 yr). The remaining overabundances in the convective zones depend on the physical conditions inside the star. We do not expect, in the case of accretion, to end up with metallic overabundances increasing with decreasing convective mass, as obtained in the standard models. Furthermore, the region mixed by the thermohaline process is localized below the convective zones and should not go down to the lithium-destruction layers.

In the present study, we have supposed that the accreted material had a relative solar abundance, except for hydrogen and helium, which were assumed completely absent. Changing these relative abundances would modify the relation between the $\mu$-value and the metallicity (eq. [9]). The general conclusions would still hold, only the numerical values and timescales would be changed accordingly.

We did not discuss the amount of matter needed to explain the observed abundances by accretion and how this accretion could have proceeded. In the example of $\frac{3}{2}$ 3.1, a metal excess of 2.05 in a 1.1 $M_\odot$ star would be obtained with 145 $M_\odot$ of all metals included, corresponding to $\sim 11 M_\odot$ of iron, which is a large value. A metal excess of 1.5 would be obtained with 5 $M_\odot$ of iron or 66 $M_\odot$ of all metals. The difficulty of explaining such a large accretion from protoplanetary disks represents a second argument against the accretion hypothesis, which we do not discuss here: this is a different subject, out of the scope of the present paper.

Detailed spectroscopic observations and precise abundance determinations will be of great interest in testing the accretion processes against the primordial scenario for exoplanets’ hosts stars. Asteroseismic studies of these stars will also help derive whether they are overmetallic down to the center or only in their outer layers and give an important hint for a better understanding of the formation of planetary systems.

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