The correlation between the sizes of globular cluster systems and their host dark matter haloes

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ABSTRACT

The sizes of entire systems of globular clusters (GCs) depend on the formation and destruction histories of the GCs themselves, but also on the assembly, merger and accretion history of the dark matter (DM) haloes that they inhabit. Recent work has shown a linear relation between total mass of globular clusters in the globular cluster system and the mass of its host dark matter halo, calibrated from weak lensing. Here we extend this to GC system sizes, by studying the radial density profiles of GCs around galaxies in nearby galaxy groups. We find that radial density profiles of the GC systems are well fit with a de Vaucouleurs profile. Combining our results with those from the literature, we find tight relationship (∼ 0.2 dex scatter) between the effective radius of the GC system and the virial radius (or mass) of its host DM halo. The steep non-linear dependence of this relationship (\( R_{\text{GCS}} \propto R^{1.2}_{\text{vir}} \)) is currently not well understood, but is an important clue regarding the assembly history of DM haloes and of the GC systems that they host.

Key words: galaxies: groups – galaxies: haloes – globular clusters

1 INTRODUCTION

Globular clusters (GCs) trace the formation and evolution of galaxies and of their dark matter (DM) haloes. Recent studies have shown that the mass of the entire GC system (GCS) is correlated linearly with the DM mass of the host halo (Hudson et al. 2014; Harris et al. 2015; Forbes et al. 2016), extending and updating previous work (Blakeslee et al. 1997; Spitler & Forbes 2009; see Harris et al. 2015 and references therein). This correlation is surprising, because the mass in stars in the host galaxy is not linearly related to its DM halo mass, but rather has a break around the luminosity of an \( L_\star \) galaxy, or equivalently, a \( \sim 10^{12} M_{\odot} \) halo mass (Marinoni & Hudson 2002; Behroozi et al. 2013). The linear correlation between GCS mass and DM halo mass may reflect the early formation epoch of GCs, before feedback regulated the formation of stars in the host galaxy.

The GCS of a halo will be affected by the accretion and tidal stripping of its satellite galaxies, as first suggested by Searle & Zinn (1978) for the Milky Way halo, and subsequently updated to galaxy group and cluster-scale halo environments in the context of hierarchical cosmological models (West et al. 1995; Forbes et al. 1997; Cote et al. 1998; Beasley et al. 2002; Tonini 2013). When a satellite galaxy falls into a larger halo, its DM is stripped and becomes part of the host halo. A statistical detection of this has been observed via weak lensing in galaxy clusters (Limousin et al. 2007; Natarajan et al. 2009; Li et al. 2016) and in galaxy groups (Gillis et al. 2013; Li et al. 2014). Tidal stripping is most likely to affect the least bound objects, and so, because the GC system as a whole is less spatially extended than the DM, it is likely to be stripped to a lesser degree than the DM (Yahagi & Bekki 2005; Smith et al. 2013, 2015; Ramos et al. 2015). Nevertheless, some of a satellite’s GCs may be stripped from the satellite’s halo, in which case these GCs will join the GCS of the host dark matter halo. There is possible evidence of tidal stripping of GC systems, in the form of arcs or tails of GCs around satellite galaxies in clusters (Romanowsky et al. 2012; Blom et al. 2012; Cho et al. 2016; Voggel et al. 2016) as well as around galaxies such as M31 (Mackey et al. 2010). The indirect evidence for tidal stripping is also quite strong. First, recent studies have uncovered large, spatially-extended populations of mostly blue/metal-poor intracluster GCs in nearby galaxy clusters such as Coma (Peng et al. 2011) and Virgo (Lee et al. 2010; Durrell et al. 2014). Second, a number of studies have found evidence that satellite galaxies in clusters have lower GC specific frequencies than dominant central galaxies (Fleming et al. 1995; Forbes et al. 1997; Peng et al. 2008; Wehner et al. 2008; Coenda et al. 2009).

After cluster pericentric passage, the “backsplash” orbit of a satellite galaxy may reach an apocentre distance of more than twice the virial radius (Balogh et al. 2000; Gill et al. 2005; Ludlow et al. 2009; Oman et al. 2013). Objects that are tidally stripped from the satellite – such as GCs – will follow approximately the same orbit as the satellite galaxy but either leading or lagging. As a result, one may also expect to find tidally-stripped GCs at large cluster-centric radii. To detect this population, imaging should extend beyond the virial radius of the host halo, which may be as much as 2 Mpc for a rich galaxy cluster.

Accretion and tidal stripping of satellites is also expected to occur in lower mass haloes such as those hosting galaxy groups,
but to date there have been no detections of “intra-group” GCs, including in the Local Group (Mackey et al. 2016). Indeed galaxy
groups, with halo masses \( \sim 10^{13} M_\odot \), contribute the most of all
DM haloes to the overall abundance of GCs in the Universe (Harris
2016). The primary goal of this paper is to study the large-scale spatial distribution of the GC systems around group galaxies and
within galaxy groups. The virial radius of \( 10^{13} M_\odot \) galaxy group is
\( \sim 500 \) kpc, which at a distance of 30 Mpc corresponds to a
degree on the sky. Hence, to study the distribution of GCs on large scales,
wide-field imaging is required. In recent years, a number of authors
have conducted wide-field imaging of GCs reaching galactocentric
distances of \( \sim 100 \) kpc (Rhode & Zepf 2004; Bassino et al. 2006;
Rhode et al. 2007; Harris 2009; Harris et al. 2012; Pota et al. 2013;
Rejkuba et al. 2014; Kartha et al. 2014; Hargis & Rhode 2014;
Kartha et al. 2016)

It has long been known that more luminous galaxies have a GCS with a shallower radial profiles (Harris 1986; Kissler-Patig
1997; Ashman & Zepf 1998). Early work on the spatial distribution of
GCs is reviewed in Brodie & Strader (2006). More recent work
has compared the “extent” of the GCS, where extent is defined as
the radius at which the GC density is consistent with zero, to the
stellar mass of the host galaxy (Rhode et al. 2007). There have been
no attempts to compare the overall spatial scale of the GCS to the
size or mass of its host dark matter halo.

In this paper, we identify nearby galaxy groups within the
footprint of the CFHT Legacy Survey (CFHTLS) fields and identify
candidate GCs around the group galaxies. An outline of this paper
is as follows: in Section 2 we describe the data sets and sample
selection used to compile samples of candidate GCs. Section 3
discusses the radial profiles of GCs and compares the total GC counts
to previous work. In Section 4, we combine the results from this
paper with other measurements from the literature, and explore the
correlations between the physical size of the GCS system with the
effective radius of the stellar light and the virial radius and/or mass
of the host halo. We discuss the implications of these results in Sec-
section 5 and conclude in Section 6

2 DATA

2.1 Group Selection and Host Galaxy Data

We targeted galaxies in groups that were in CFHTLS fields. Four
groups with \( cz < 5000 \) km/s (\( dz \lesssim 70 \) Mpc) that overlapped
the CFHTLS-Wide footprint were selected from the 2MASS-based
2M++ group catalogue of Lavaux & Hudson (2011). For compari-
son with the properties of its GCS, we require, for each galaxy, its
distance, stellar mass, effective radius and host halo mass. These
were determined as follows:

- Distances are based on the recession velocity from NED, adopting a value of 70 km/s/Mpc for the Hubble parameter (\( H_0 \)).
- Stellar masses are derived from 2MASS K-band magnitudes.
  We adopt a solar K-band magnitude of 3.28 (Binney & Merrifield
1998), to calculate the luminosity of each galaxy in units of
\( L_\odot \). The galaxy’s stellar mass is then obtained via the K-band
stellar-mass-to-light ratio (\( M_\star / L_K \)) from Bell et al. (2003), using
the \( B - V \) colour of each galaxy from the Third Reference Catalog
of Bright Galaxies (de Vaucouleurs et al. 1991). In cases where the
colours were not available in the catalogue, the average colour of
the galaxy’s morphological type (Fukugita et al. 1995) was used.
- Effective radii of the galaxy light in the K-band will be used

in Section 4. These were kindly provided to us by J. Lucey (priv.
comm.), using the methods described in Campbell et al. (2014).
- The halo mass (\( M_{200} \)) and halo virial radius (\( R_{200} \)) are critical
parameters in this study. Unfortunately, it is impossible to measure
dark matter halo masses out to the virial radius directly in individ-
ual galaxies. Therefore we use the mean relationship between dark
matter halo mass and stellar mass, calibrated via weak gravitational
lensing (Hudson et al. 2015). Specifically, we use the fits described
in Appendix C of that paper, extrapolated to \( z \approx 0 \) to obtain halo
masses (and hence virial radii). Note also that the intrinsic scatter
in \( M_{200} \) at fixed \( M_\star \) is estimated to be in the range 0.15-0.20 dex.

It is important to understand that the \( M_{200} \) masses determined
from this relationship refer to the mass of the entire halo of which
the galaxy is assumed to be the central member. For massive galax-
ies that are the dominant galaxy of a group or cluster, the halo mass
is therefore the total mass of the group or cluster. For example, the
stellar mass of NGC 883 is \( 2.5 \times 10^{13} M_\odot \), typical of the domi-
nant “brightest group galaxies” (hereafter BGG). The correspon-
ding halo mass of its group is then \( 2.7 \times 10^{13} M_\odot \).

Parameters of massive galaxies in groups are summarized in
Table 1. There is a special case in Table 1: the galaxies in the pair
NGC 942+943 are too close to separate their GC systems. We there-
to combine their GC counts and their stellar masses and treat the
pair as a single “BGG”.

2.2 Globular Clusters: Catalogues and Selection Criteria

2.2.1 Catalogues

The photometric data used to select GC candidates were obtained from
the Wide component of the Canada-France-Hawaii Telescope
Legacy Survey (CFHTLS). This survey used the MegaCam instru-
ment, with a \( 1^\circ \times 1^\circ \) field of view, and a scale of 0.187 per pixel.
CFHTLS-Wide covers 155 deg\(^2\) across four patches: W1, W2, W3,
and W4. Each of these patches comprises several fields with cov-
erage in five Megacam filters: \( u', g', r', i', z' \). The last four are
similar to SDSS filters. Hereafter we drop the prime notation. The
limiting magnitude in the \( i\)-band is \( \sim 24.7 \) for a point source. Cali-
brated images and source catalogues were produced by Terapix and
made available through the CFHTLS-T0007 release. For more in-
formation on the calibration and other technical details, see Hudelot
et al. (2012)

2.2.2 Size and magnitude selection

In order to distinguish globular clusters from the other sources, we
impose several selection criteria. To avoid contamination from
bright stars, nearby bright galaxies, defects and edge effects, masks
were created and applied to each image. We then select GC can-
didates based on apparent \( i\) magnitude, half-flux radius (\( r_{1/2} \)) and
(\( g - i \)) colour.

At the distances of the galaxies in this paper, we would expect
GC candidates to appear as unresolved point sources. We first select
point sources by setting an upper limit on \( r_{1/2} \). Figure 1 shows a
representative plot of \( r_{1/2} \) vs \( i \) for one of the CFHTLS fields. The
line in the bottom left portion of the figure are point sources, most
of which are foreground stars but also including GCs. Because the
seeing varies from exposure, the cut in \( r_{1/2} \) used to define point
sources varies from field to field. In Fig. 1, background galaxies

1 http://www.cfht.hawaii.edu/Science/CFHTLS/
dominate at magnitudes fainter than $i \sim 24$, so we limit GCs to $i < 24$. This criterion is shown in Fig. 1 as a vertical dashed blue line.

2.2.3 Globular Cluster Luminosity Function and Corrections for Incompleteness

The globular cluster luminosity function (GCLF) is the number of clusters per unit magnitude, and is well described by a Gaussian whose mean and standard deviation depend on the absolute magnitude of the host galaxy. The full details of the adopted GCLFs are given in Section A1. The GCLF allows a correction for undetected GCs below the flux limit.

If only a faint $i$-band magnitude selection limit were imposed, with no bright limit, then a large number of stars would also be selected by the criteria and treated as GC candidates. In order to minimize the stellar contamination and to maximize the number of GCs, we use the GCLFs (Section A1) to determine the bright $i$-band magnitude limit ($i_{\text{limit}} < i < 24$) on a galaxy-by-galaxy basis. Specifically, we select $i_{\text{limit}}$ in such a way that we lose no more than 10\% of the GCLF visible at $i < 24$.

Using the GCLF we obtained for each galaxy in Section A1, we calculate the fraction of the total GCLF that lies within our observed magnitude range ($i_{\text{limit}} < i < 24$). This factor allows us to correct the raw counts for the GCs that lie outside the magnitude range.

2.2.4 Colour Selection

We determine the range of $(g - i)$ colour in which we would expect to find globular clusters by plotting a colour-magnitude diagram (CMD) of $i$ vs $(g - i)$ colour (Durrell et al. 2014; Escudero et al. 2015; Salinas et al. 2015), shown in Fig. 2. The dense column at $(g - i) = 0.5$ is made up mostly of stars. The area at the base of the column is likely to contain globular clusters. Boxes have been placed around this region to represent our selection criteria within 0.15 $(g - i)$ of the $i$-band magnitude, so we limit GCs to $i < 24$. This area and the blue box surrounds the region of blue GCs. The distinction is made at $(g - i)$ colour of 0.8.

Using the GCLF we obtained for each galaxy in Section A1, we calculate the fraction of the total GCLF that lies within our observed magnitude range ($i_{\text{limit}} < i < 24$). This factor allows us to correct the raw counts for the GCs that lie outside the magnitude range.

- **Table 1.** Important parameters for each galaxy analyzed in this paper. The CFHTLS field and group where the galaxy can be found as well as the morphological type (from NED) are given. The $K$ magnitude, $M_K$, as well as $L_*$, $M_*$, and $M_{200}$ from Section 2.1 are also given.

| Galaxy | Field | Group | Distance (Mpc) | Morphology | $M_K$ | $L_*$ ($M_\odot$) | $M_*$ ($M_\odot$) | $M_{200}$ ($M_\odot$) |
|--------|-------|-------|---------------|------------|-------|------------------|------------------|------------------|
| IC 219 | W1-0-0 | IC 219 | 72.7 | E | -24.33 | 1.112e+11 | 9.35e+10 | 5.020e+12 |
| NGC 883 | W1-0-0 | | | | -25.42 | 3.020e+11 | 2.509e+11 | 2.704e+13 |
| NGC 942+943 | W1+3-4 | NGC 943 | 67.0 | S0 | -25.16 | 2.373e+11 | 1.972e+11 | 1.744e+13 |
| NGC 2695 | W2-0+1 | NGC 2695 | 26.5 | S0 | -23.26 | 4.140e+10 | 3.461e+10 | 1.407e+12 |
| NGC 2699 | W2-0+1 | | | | -22.74 | 2.569e+10 | 2.044e+10 | 8.624e+11 |
| NGC 5473 | W3-2-0 | NGC 5473 | 26.2 | S0 | -23.74 | 6.398e+10 | 5.267e+10 | 2.267e+12 |
| NGC 5475 | W3-2+1 | | | Sa | -22.69 | 4.182e+10 | 3.475e+10 | 1.413e+12 |
| NGC 5485 | W3-2-0 | | | | -23.69 | 6.155e+10 | 5.067e+10 | 2.162e+12 |

- **Figure 1.** Half-flux radius $r_{1/2}$ plotted against $i$-band magnitude for the W1-2-1 field. The half flux radius value is the radius of the object (in pixels on the image) that encloses half of the object’s total flux. The large region in the bottom, right corner is mostly unresolved background galaxies. To avoid selecting these, we impose a criterion of $i < 24$, which is represented by a vertical dashed blue line. The thin focus on the left side are point sources, mostly stars but also some GCs. The adopted cut on flux radius for this field is shown by a solid blue line.

- **Figure 2.** Colour-magnitude diagram that combines point sources from every field. The dense column at $(g - i) \sim 0.5$ is made up mostly of stars. The area at the base of the column is likely to contain globular clusters. Boxes have been placed around this region to represent our selection criteria of $h_{\text{limit}} < i < 24$ and $0.6 < (g - i) < 1.1$. Although $h_{\text{limit}}$ takes a different value for each galaxy, in the figure the box extends upwards to $i = 22$, a typical value of $h_{\text{limit}}$. The red box surrounds the region of red GCs and the blue box surrounds the region of blue GCs. The distinction is made at $(g - i)$ colour of 0.8.
there is background contamination which may vary as a function of colour. In order to correct for this, we subtract the background estimated from unmasked areas of the field, far from galaxies. The background-subtracted histograms of each galaxy co-added to estimate from unmasked areas of the field, far from galaxies. The separate blue from red GCs. dashed vertical lines demonstrate the range of colour assumed to be dominated by GCs. The blue and red dashed lines show the colours adopted to separate blue from red GCs.

Figure 3. Stacked histogram of the \((g-i)\) colours of all GCs within 3 \(R_e\) of all galaxies. The dotted line represents the observed histogram, while the solid black line represents the background-subtracted histogram. The dashed vertical lines demonstrate the range of colour assumed to be dominated by GCs. The blue and red dashed lines show the colours adopted to separate blue from red GCs.

3 GCS DENSITY PROFILES

We now turn to the number and density profile of GCs around their parent galaxies. We consider common functional forms used to model the surface density of GCs and apply these models to the well-studied Milky Way GCS. We then fit the models to the group galaxies discussed in this paper.

3.1 Sérsic and Power Law Models

By modelling GC number density as a function of galactocentric distance, it is possible to extrapolate to the number of GCs around each galaxy. GC density is often modelled with a Sérsic profile and a power law (e.g. Rhode & Zepf 2004; Faifer et al. 2011; Kartha et al. 2014). We consider both functional forms in this paper.

A Sérsic intensity profile (Sérsic 1963; Sersic 1968) is often used to model the surface brightness profiles of galaxies. It is represented by the function

\[
\Sigma_e(R) = \Sigma_0 \exp \left( -b_n \left[ \left( \frac{R}{R_e} \right)^{1/n} - 1 \right] \right),
\]

(1)

where \(\Sigma_e\) is the surface number density of GCs (\(\Sigma_{GC}\)) at the effective radius \(R_e\), the radius that encloses half of the total GC density. The parameter \(n\) describes the shape of the curve, and \(b_n\) is a constant that is dependent on \(n\). The constant, \(b_n\), is calculated using \(b_n = 1.9992n - 0.3271\) (Graham & Driver 2005). The Sérsic fit can be used to calculate the total number of GCs (\(N_{GC}\)) around the galaxy by integrating equation (1) over a projected 2D area to obtain

\[
N_{GC} = \Sigma_e R^2 \frac{2\pi n}{(b_n)^{2n}} \Gamma(2n).
\]

(2)

In addition to a Sérsic profile, the galaxies were also fit with a power law of the form

\[
\Sigma_{PL}(R) = \Sigma_0 \left( \frac{R}{R_0} \right)^\gamma.
\]

(3)

\(\Sigma_0\) and \(\gamma\) were treated as free parameters. To avoid degeneracy, \(R_0\) was set to a fixed value of 100 kpc. Normally, this model has an additional term that represents a core within which the number density begins to flatten. However, we will be limiting our GC detection aperture with a certain inner radius (discussed in Section 3.2) that is much larger than the core radius (Forbes et al. 1996). Consequently the core radius term is omitted in this power law model. The power law model can also be used to calculate the number of GCs belonging to each galaxy. This is done simply by integrating equation (3) over a projected 2D area to obtain

\[
N_{GC} = \int_{R_{min}}^{R_{max}} 2\pi R \Sigma_{PL}(R) dR = \frac{2\pi \Sigma_0}{R_0^\gamma (\gamma + 2)} (R_{max}^{\gamma + 2} - R_{min}^{\gamma + 2})
\]

(4)

3.2 GCS profiles and fits

In this work, our primary goal is to understand the spatial extent of the GC system, particularly on large scales. Because the hosts are often not smooth early types, we do not attempt to model and subtract the galaxy light in order to identify GCs close to the galaxy centre. Instead we first adopt an inner radius, \(R_{in}\), which is taken to be the \(K\)-band 20\(^{th}\) magnitude isophotal radius from the 2MASS All-Sky Extended Source Catalog (Jarrett et al. 2000). We then calculate surface density profiles using concentric, logarithmically-scaled annular bins extending from \(R_{in}\) to the edge of the image. As seen in Fig. 6, \(R_{in}\) ranges from 5 – 15 kpc. To avoid contamination from the globular cluster systems of other galaxies, circular masks of radius 0.15 \(\times R_{2500}\) were placed around each other galaxy and added to the overall mask for the galaxy being studied. The number of GCs in each annulus was divided by the unmasked area of the annulus to obtain the number density, yielding the profile of GC surface number density versus galactocentric distance for each galaxy (as Faifer et al. 2011; Salinas et al. 2015), based on the distribution of GCs within the annulus.
on the distance to each galaxy from Table 1. Surface density profiles for globular clusters are shown in Fig. 6. The number of red and blue GCs were also counted in each annulus separately, using \((q - i) = 0.80\) to separate the subpopulations (Section 2.2.4). Red and blue surface density profiles are shown in Fig. 6 as red squares and blue triangles, respectively.

Three different radial profiles were fit to the surface density of GCs as a function of projected radius. These models are (1) a de Vaucouleurs \((n = 4)\) profile where \(R_e\) was allowed to vary, and (2) a power law, and (3) de Vaucouleurs profile where \(R_e\) was kept constant. A constant background term was included in the fit. After subtracting the background, we use the fraction of GCs within the magnitude limits (as discussed in Section 2.2) and divide the raw GC density in each bin by this fraction to obtain surface density profiles that are corrected for background contamination and incompleteness.

In the free \(R_e\) de Vaucouleurs fits there is considerable degeneracy between the fitted \(R_e\) and fitted \(\Sigma_m\), leading to large uncertainty in the total counts. Therefore, we also perform fits fixing \(R_e = 0.05R_{200}\). We compare the corrected GC counts within an annulus \(R_{in} < R < R_{out}\) = 100 kpc to the integral of the models over the same annulus. The comparison of these aperture counts is given in Table 3. In most cases, the direct, corrected counts agree well with the models within the aperture. This indicates that our models are reasonable within the range galactocentric radii \(R_{in} < R < R_{out} = 100\) kpc.

The fit parameters, as well as the \(\chi^2\) of the fits are given in Table 2 for the free-\(R_e\) fits, and in Appendix B for the other profiles. The \(\chi^2\) allows us to compare these three models. The variable-\(R_e\) de Vaucouleurs fit and the power law fit both have the same number of free parameters. In most cases, the de Vaucouleurs fit has a slightly lower \(\chi^2\), but the difference is usually negligible (\(\lesssim 1\)) suggesting that our data do not easily distinguish these two profiles. In the case of NGC 942+943, the variable-\(R_e\)-de Vaucouleurs fit does offer a significantly better \(\chi^2\) and appears to be the better fit. The blue GCs of NGC 942+943 seem to be slightly better represented by the Sérsic model. Faiter et al. (2011) also fit both a de Vaucouleurs profile and a power law to GC surface density distributions and also found the two models yield very similar quality of fit. However, they found that the inner regions are represented slightly better by the de Vaucouleurs profile. The power law fit has 13 degrees of freedom, while the fixed-\(R_e\)-de Vaucouleurs fit has 14. In most cases there is not a significant difference in the quality of the fit of the two models. NGC 883 is slightly better fit by the power law, but this difference is less significant than the difference between the fixed \(R_e\) and the variable \(R_e\) de Vaucouleurs fits for this galaxy. Overall, the GC surface density distributions are well represented by all three models.

### 3.3 Total Number of Globular Clusters: Comparison to Previous Results

The main purpose of this paper is to compare the spatial sizes of GC systems. However, in order to compare our results with previous work, it is useful to also obtain total GC counts and red GC fractions. To determine the total number of GCs across all galactocentric radii, the fixed-\(R_e\) de Vaucouleurs fits GCs in the inner region was calculated by evaluating the de Vaucouleurs integral from \(R_{min} = 0\) kpc to \(R_{max} = R_{in}\). The number of GCs in the outer region was calculated by integrating the de Vaucouleurs fit from \(R_{min} = R_{out}\) to \(R_{max} = \infty\) and integrating the extrapolated inner regions, the aperture counts, and the extrapolated outer regions were added together to create total “combined” counts. The counts determined through the various methods in this section are shown in Table 3, including the combined counts for red and blue GCs.

As discussed in the introduction, the total mass of the GCS is related to both the stellar mass of the host galaxy and to its halo mass. The total mass of the GCS system for our galaxies, \(M_{GCS}\), was obtained by multiplying the combined counts by the average GC mass of \(2.4 \times 10^5M_\odot\) (Durrell et al. 2014). In Fig. 4, \(M_{GCS}\) of each GCS is plotted against \(M_*\) and \(M_{200}\) of its host galaxy and these results are compared to galaxies from the compilation of Harris et al. (2013).

We also compare the ratio of red GCs to total GCs, or red fraction \((f_{red})\), between our galaxies and the Harris catalogue galaxies. \(f_{red}\) was calculated using \(NGC\) determined by the combined counts method of de Vaucouleurs extrapolation. In Fig. 5, \(f_{red}\) of each GCS is plotted against \(M_*\) and \(M_{200}\) of the parent galaxy. The galaxies in this paper generally follow the overall trends from Harris et al. (2013), although a couple (NGC 2695 and NGC 883) are at red end of the distribution for their stellar or halo mass.

### 4 Spatial Extent of Globular Clusters

In this section, we investigate the radial distribution of the GCS as a function of their host galaxy or halo properties. We will combine GCS sizes from the CFHTLS galaxies studied in this paper with data from the literature.

#### 4.1 Fits to the CFHTLS galaxy sample

We have fit a free \(R_e\) to the GCS of galaxies in our sample, assuming de Vaucouleurs profile. The results are tabulated in Table 2 for all GCs, including for red and blue GCs separately. The GCSs of two of our galaxies, NGC 5473 and NGC 5475, do not have a well-defined \(R_e\). Therefore, we omit these from subsequent discussion in this section. We also fit the power law model: results are in Table B2. The power-law and de Vaucouleurs fits are compared in Fig. 7. As expected, more extended galaxies with a larger \(R_e\) have a less negative \(\gamma\) and the two parameters are correlated.

It is also interesting to study spatial profiles of the the red and blue GC populations. In most previous studies, the blue GC distribution is more extended that the red GCS distribution (Rhode & Zepf 2004; Bassino et al. 2006; Faiter et al. 2011; Kartha et al. 2014; Cho et al. 2016). A comparison of the red and blue GC profiles for the galaxies studied in this paper is shown in Fig. 8. The upper panel compares \(R_e/R_{200}\) ratios of both populations. The lower panel of the figure compares \(\gamma\) from the power law fit for both populations. If the blue population is more extended, one would expect \(R_e/R_{200}\) to be higher for blue GCs and \(\gamma\) to have a larger negative value for red GCs. This is the sense of the observed trend, but the uncertainties on the parameters of the red and blue parameters are large, and so within the errors, they are also consistent with being equal.

#### 4.2 Data for other GCS

Our CFHTLS sample contains only 7 galaxies with usable GCS \(R_e\) measurements. Here we describe additional measurements of GCS sizes that allow us to extend the dynamical range of the sample. Specifically, we describe our analysis of the Milky Way, M31 and M87/Virgo, as well as additional galaxies from the literature.
Figure 4. The relationship between $M_{\text{GCS}}$ and $M_*$ (left hand panel) or $M_{200}$ (right hand panel). $M_{\text{GCS}}$ was determined using the extrapolated counts described in Section 3.3. Harris et al. (2013) galaxies are represented by small gray points. Galaxies from this paper are represented as circles labelled according to their name. Galaxies that are members of the same group are given the same colour. A line of slope 1.04 is plotted on the left side of Fig. 4 to represent this relationship. In the right panel, the blue line indicates the relation of Hudson et al. (2014): $\eta = M_{\text{GCS}}/M_{200} = 4 \times 10^{-5}$.

Figure 5. The red fraction $f_{\text{red}}$ for galaxies in this paper compared with data from Harris et al. (2015). Red fractions were calculated using the de Vaucouleurs fit. $f_{\text{red}}$ is plotted against $M_*$ in the left panel and $M_{200}$ in the right panel.

4.2.1 The Milky Way GCS in projection

To obtain the GCS effective radius for the Milky Way (MW), we use the catalogue of MW GCs that has been compiled by Harris (1996) and updated in Harris (2010). We retrieve the Galactic Cartesian $X$, $Y$, and $Z$ positions of each GC relative to the Galactic centre, and use these to create 2D projections of the GCS in the $XY$, $XZ$, and $YZ$ planes. The GCs were placed into 15 concentric annular logarithmically-space bins by projected galactocentric distance. The three projections of surface density are shown in Fig. 9. The fit to a de Vaucouleurs profile is shown in Fig. 9 as the dashed green line. The average $n$ was 3.3 but in the $YZ$ plane $n = 4.82$. The $\chi^2$ of this fit is not significantly better than $n = 4$. We conclude there is no strong evidence for deviations from $n = 4$ in the MW GCS. With free $n$, the $R_e$ for the $YZ$ projection is 3.86 kpc. Finally, a cored power law model, with the functional form

$$\Sigma_{\text{PL}}(R) = \Sigma_0 \left( R_c^2 + R^2 \right)^{\gamma/2}.$$
Sizes of globular cluster systems

\[
\Sigma_{GC} \left( \frac{GC}{kpc^2} \right)
\]

De Vaucouleurs Fit
Power Law Fit
Total GCs
Red GCs
Blue GCs

\[
IC 219 \quad NGC 883 \quad NGC 942+943
\]

\[
NGC 2695 \quad NGC 2698
\]

\[
NGC 2699 \quad NGC 5473
\]

\[
R (kpc)
\]

\[
10^{-3} 
10^{-2} 
10^{-1} 
\Sigma_{GC} \left( \frac{GC}{kpc^2} \right) 
\]

\[
10^1 
10^2 
R (kpc)
\]
The fit parameters of free-\(R_c\) de Vaucouleurs fits for galaxies studied in this paper, as well as the \(\chi^2\) of the fit. In each fit there are 13 degrees of freedom.

| Galaxy       | Total \(\Sigma_c\) \((10^{-2} \, \text{GC}/\text{kpc}^2)\) | Total \(R_c\) (kpc) | Total \(\chi^2\) | Red \(\Sigma_c\) \((10^{-2} \, \text{GC}/\text{kpc}^2)\) | Red \(R_c\) (kpc) | Red \(\chi^2\) | Blue \(\Sigma_c\) \((10^{-2} \, \text{GC}/\text{kpc}^2)\) | Blue \(R_c\) (kpc) | Blue \(\chi^2\) |
|--------------|---------------------------------|------------------|----------------|-------------------|----------------|----------------|-------------------|----------------|----------------|
| IC 219       | 11.84±8.17                      | 25.89±8.83       | 5.99           | 16.11±19.72       | 15.70±8.72     | 6.75           | 8.71±11.41        | 19.46±12.15    | 8.67           |
| NGC 883      | 0.28±0.17                       | 177.49±71.17     | 4.39           | 0.10±0.09         | 259.53±167.55  | 6.96           | 0.12±0.21         | 125.87±136.78  | 6.21           |
| NGC 942+943  | 1.04±1.07                       | 77.53±44.90      | 13.81          | 0.36±0.53         | 89.71±77.66    | 11.58          | 0.30±0.34         | 106.82±69.42   | 10.80          |
| NGC 2695     | 5.05±6.16                       | 10.32±6.08       | 11.06          | 14.39±26.82       | 5.30±4.23      | 15.01          | 26.82±117.77      | 2.90±4.82      | 16.92          |
| NGC 2698     | 3.50±6.82                       | 10.35±9.86       | 9.85           | 15.25±58.53       | 3.52±5.31      | 7.53           | 9.81±26.13        | 4.87±5.47      | 9.03           |
| NGC 2699     | 77.93±162.14                    | 2.01±1.62        | 6.34           | 62.15±186.21      | 1.83±2.08      | 7.85           | 0.15±0.34         | 32.85±47.00    | 6.31           |
| NGC 5473     | 539.84±2.55e3                   | 1.55±2.37        | 18.99          | 1.07e6±1.29e7     | 0.18±0.43      | 18.82          | 4.93±10.65        | 7.86±7.79      | 5.85           |
| NGC 5475     | 0.01±0.01                       | 423.42±567.03    | 6.04           | 28.06±87.50       | 2.93±3.52      | 5.68           | 0.05±0.15         | 50.05±89.11    | 7.76           |
| NGC 5485     | 8.52±9.89                       | 12.85±7.27       | 11.29          | 0.01±0.02         | 449.30±501.26  | 6.80           | 24.67±36.41       | 6.52±4.09      | 12.32          |

\[ \Sigma = \text{Surface density of GCs, } \Sigma_{\text{GC}}, \text{ in } \frac{\text{GC}}{\text{kpc}^2}, \text{ versus galactocentric distance (} R, \text{ in kpc) for all galaxies. Total GC counts are displayed as black circles, red GCs as red triangles, and blue GCs as blue squares. The de Vaucouleurs fit appears a solid green line, while the power law fit appears as a dashed purple line. The red tick mark at the top of each plot corresponds to a galactocentric distance of } 3 R_c \text{ or } 0.15 \times R_{200}. \]

\[ \gamma = \text{the power-law } \gamma \text{ was fit to the MW GCS data. In the above equation, } R_c \text{ represents the core radius. The core radius, } R_c, \text{ was found to be } 1.3 \text{kpc and the power-law } \gamma \text{ was } -2.6, \text{ again for the } YZ \text{ plane. This is a slightly poorer fit than the de Vaucouleurs profile.} \]

\[ \text{Finally, the stellar mass of the Milky Way (} 3 \pm 1 \times 10^10 M_\odot \text{) was obtained from } \text{Bland-Hawthorn & Gerhard (2016).} \]

\[ \text{4.2.2 M31 GCS} \]

We have also produced an updated fit to the M31 GCS effective radius. Specifically, we use the GC compilation from Caldwell & Romanowsky (2016). The GC surface density profile is shown in Fig. 10. Huxor et al. (2011) noted that M31 was not well fit by a single profile. Specifically they studied power laws with three components and Sersic profiles with two components. Our goal here to treat M31 in a similar way to the other galaxies in our sample. Hence we restrict the fit to counts at radii larger than the isophotal radius at 20th mag per square arcsecond in the \(K\)-band, which at the adopted distance of 780 kpc corresponds to 6.14 kpc (Jarrett et al. 2003).

Neither power law nor Sersic profiles are very good fits to the entire range: the data appear to oscillate around any smooth function. It is not clear whether this is a result of incompleteness in the GC catalogues at some radii. The best fit to the outer regions

\[ \text{was fit to the MW GCS data. In the above equation, } R_c \text{ represents the core radius. The core radius, } R_c, \text{ was found to be } 1.3 \text{kpc and the power-law } \gamma \text{ was } -2.6, \text{ again for the } YZ \text{ plane. This is a slightly poorer fit than the de Vaucouleurs profile.} \]

\[ \text{Finally, the stellar mass of the Milky Way (} 3 \pm 1 \times 10^10 M_\odot \text{) was obtained from } \text{Bland-Hawthorn & Gerhard (2016).} \]
Table 3. Globular cluster counts determined using different methods. Within the aperture, the counts are calculated by summing annuli over the specified range, using the de Vaucouleurs fit, and by using the power law fit. The combined extrapolated fit is presented here for total, red, and blue GCs. The red fraction for combined counts is also presented here.

| Galaxy      | Aperture | $R_{de} < R < 100$ kpc | Power Law Fit | Combined |
|-------------|----------|------------------------|---------------|----------|
|             |          | de Vaucouleurs Fit     |               |          |
|             |          | $R_e$                   | $f_{red}$     | Total    |
| IC 219      | 104±192  | 1032±120               | 1041±115      | 937±151  |
| NGC 883     | 720±157  | 532±87                 | 553±75        | 806±138  |
| NGC 942+943 | 647±124  | 641±130               | 624±129       | 532±103  |
| NGC 2695    | 119±25   | 79±13                  | 86±18         | 110±21   |
| NGC 2698    | 59±20    | 52±11                  | 55±18         | 36±15    |
| NGC 2699    | 21±18    | 27±10                  | 26±11         | 7±13     |
| NGC 5473    | 38±27    | 66±21                  | 49±22         | 22±21    |
| NGC 5475    | 50±22    | 12±5                   | 22±11         | 41±17    |
| NGC 5485    | 130±26   | 193±29                 | 208±38        | 77±21    |

4.2.3 M87/Virgo

To extend the dynamic range of the GCS sample to include dark matter haloes with the highest masses, we include the GCS of M87. Recall that, as discussed in Section 2.1, for central cluster galaxies such as M87, the “host halo” is the entire galaxy cluster. Virgo is a galaxy cluster for which there are GC data spanning the entire cluster. Specifically, Lee et al. (2010) studied the GCS of Virgo, and fit its GC density profile with a broken power law. We have used their parametric fit to estimate the projected radius which encloses half of the Virgo GCs within 5 degrees of M87. We obtain a GCS effective radius of 84 ± 50 kpc, where the uncertainty is estimated via the scatter in the power law parameters. We take the stellar mass of M87 to be $5.5 \times 10^{11} M_{\odot}$ (Agnello et al. 2014).

4.2.4 Other GCS

We supplement the above with additional data from the literature: 11 galaxies from Rhode and collaborators (Rhode et al. 2007; Hargis & Rhode 2012; Young et al. 2012) and 12 galaxies from Kartha and collaborators (Kartha et al. 2014, 2016). For the data from Rhode and collaborators, we refit their GC radial density profile data to obtain $R_e$ in a consistent way.

For all of these galaxies, $K$-band magnitudes, $B-V$ colours, and distances were compiled from NED in order to calculate stellar masses using the methods outlined in Section 2.1. $M_{200}$ and $R_{200}$ of all the above galaxies were obtained using the method introduced in Section 2.1.

4.3 Results

In this section, we focus on the de Vaucouleurs fits and compare the $R_e$ of the GCS with properties of the host galaxy, such as the $R_e$ of its light, or the halo of which it is a central galaxy. Table 4 summarizes the properties of all the galaxies and their GCS used in this section.

Fig. 11 (left panel) shows $R_e$ of the GCS is plotted against $R_e$ of the light for group galaxies studied in this paper, and additional galaxies as described above. The effective radii of CFHTLS galaxies are measured in the $K$-band, we multiply these by a factor 1.33 for consistency with optically-determined effective radii (Ko & Im 2005). We perform a fit allowing for scatter in both the independent and dependent variables. For galaxies with no quoted errors in their effective radii, we assume an error of 0.1 dex. The line of best fit appears as a dotted black line and is given by

$$\log_{10} R_{e,GCS} = 1.27 \pm 0.33 \log_{10} R_{e, host} + 0.39 \pm 0.22$$
The slope is not significantly different from unity, so if we fix it we find that the data are consistent with a constant ratio between the GCS $R_e$ and the galaxy light $R_e$. Specifically, $R_e(GCS) \sim 3.5R_e\text{,light}$. This ratio is lower than the value 6.5 ± 1.3 found by Kartha et al. (2016) for twelve galaxies, which are a subset of our sample.

It is also interesting to consider the scatter in the relationship. We measure this by adding a fractional intrinsic scatter, $\sigma_{int}$, to the observational errors in quadrature and fit for this intrinsic scatter via maximum likelihood. From these fits, we find $\sigma_{int} = 0.33 \pm 0.08$ dex.

Galaxies are embedded within a dark matter halo and so it is interesting to compare the size of the GCS with that of the halo. The plot on the right side of Fig. 11 is similar to the plot on the left, but here GCS $R_e$ is plotted against the $R_{200}$ of the dark matter halo instead of the effective radius of the galaxy light. We assume that there is an uncertainty of 0.15 dex in the estimated $M_{200}$ at fixed $M_*$, which becomes 0.05 in $R_{200}$. In the $R_{200}$ plot, the slope is much steeper so that the GCS is not a fixed fraction of the virial radius. Specifically, we find

$$\log_{10} R_e(GCS) = 2.63 \pm 0.38 \log_{10} R_{200} - 5.6 \pm 0.97$$

More importantly, this comparison has less intrinsic scatter: $\sigma_{int} = 0.17 \pm 0.07$ dex. This tighter relationship suggests that the $R_e$ of the GCS is more closely related to $R_{200}$, and therefore $M_{200}$, than it is to the $R_e$ of the host galaxy light.

However, since $M_{200}$ and $R_{200}$ are based on the stellar mass, it is also interesting to see which property correlates best with $R_e\text{,GCS}$. We have calculated $M_*$ for each galaxy using its $B - V$ colour (Bell et al. 2003). The scatter in this relationship between stellar mass at fixed colour is 0.1-0.2 dex, and so we conservatively adopt a scatter in $M_*$ of 0.1 dex. Moreover, $M_*$ is used to calculate $M_{200}$, in the manner described by Hudson et al. (2015), who adopt a scatter of 0.15 dex in this relationship as we do here. Fig. 12 shows these comparisons for a larger sample of galaxies. For the relation between $R_e\text{,GCS}$ and $M_*$ we find:

$$\log_{10} R_e(GCS) = 1.30 \pm 0.14 \log_{10}(M_*/(10^{11} M_\odot)) + 1.17 \pm 0.05$$

whereas for the correlation with $M_{200}$ we obtain

$$\log_{10} R_e(GCS) = 0.88\pm0.10 \log_{10}(M_{200}/(10^{13} M_\odot))+1.35\pm0.06.$$
sizes of globular cluster systems

of GCS effective radii from CFHTLS, we find that varying α from 1 to 4 causes the slope of the log \( R_e \sim \log M_* \) relation to change from 0.97 to 1.58 and the slope of the log \( R_e \sim \log M_{200} \) relation varies from 0.70 to 1.15. Note also that the CFHTLS effective radii appear to be systematically larger than those from the literature. This may be due to our fitting method which fits only the outer region of the GCS. For example, for the case of M31, we did indeed find a larger \( R_e \) when the outer regions of the GCS. Alternatively, the differences may be due to differences in the galaxy morphologies of different subsamples. Our CFHTLS sample is mostly early types whereas other samples, notably Young et al. (2012) contain relatively more spirals. A larger, more homogeneous sample is needed to understand the systematics in fitting methods.

The slope of the power law fit, \( \gamma \), was plotted against \( M_* \) and \( M_{200} \) in Fig. 13. The weighted average of \( \gamma \) was found to be \( \gamma = -1.95 \). The MW is significantly lower on this plots than the other galaxies. Note, however, that the MW power law fit included a core, whereas the power law fits of the other galaxies did not. This could possibly have an effect on the \( \gamma \) of the power law. While in the previous plots there was a clear trend of increasing \( R_e \), in this plot the trend is not as convincing. Kissler-Patig (1997) observes a trend of increasing density profile slope with galaxy luminosity, and therefore mass. However, he argues that it is not a continuous trend, but evidence for the existence of two distinct types of GCSs.

5 DISCUSSION

5.1 Scaling with halo properties

The tight scaling of the sizes of GCSs with halo mass or virial radius suggests that the same accretion and stripping mechanisms that builds the DM halo may also build the bulk of the GCS. The origin of the steep non-linear dependence of the effective radius of GCS on the virial radius of the halo (\( R_e \sim R_{200} \)) is less clear. It is likely that this is a product of the hierarchical assembly of haloes in ΛCDM. Consider, for example, a \( \sim L_* \)-galaxy, like the MW or M31. As discussed above, the GCS of such a galaxy may be a combination of “in-situ” GCs associated with the disk and bulge, plus, as suggested by Searle & Zinn (1978), GCs associated with tidally-stripped infalling satellite galaxies. The stripped GCs would be found at large galactocentric radii, leading or lagging the orbit of the infalling satellite. This is the case for outer halo GCs associated with the Sagittarius dwarf spheroidal (Bellazzini et al. 2003).

Now consider what happens when such an \( L_* \)-galaxy merges...
with a galaxy group, and is itself tidally stripped. The least-bound GCs around the infalling galaxy, i.e. those that are most distant from it, are most easily stripped. They will become associated with the GCS of the group as a whole and their orbits will now be of order of the virial radius of the group into which they have been accreted. At the same time, some of the more tightly bound GCs may not be stripped until a later pericentric passage, at which time dynamical friction has reduced the orbit of the satellite. These GCs would be deposited at smaller group-centric radii. Finally, when that group merges into a larger galaxy cluster, the process repeats again. In this way, a considerable fraction of the GC population may be stripped and orbit the new halo at ever increasing radii. This provides a mechanism to explain the steep scaling relation between GCS size and halo size.

5.2 Comparison with predictions from models

There have been few theoretical models that have studied how the spatial distribution of GCs develops in hierarchical models of structure formation. A notable exception is Bekki et al. (2008), who followed GC-like particles through a cosmological simulation. They identified DM haloes at $z \gtrsim 6$ and tagged the central particles in those haloes as GC particles. They then tracked the GC particles to the present day. They predict a scaling of the effective radii of these metal-poor (blue) GCs with $R_e \propto M_{200}^{0.18}$. At face value, this is correlation is much flatter than the value found in this paper. However, their treatment of GCs is different from that adopted here. First, they only consider metal-poor/blue GCs. Second, they exclude “in-
troupe" GCs, whereas in our data any “intragroup” GCs are assigned to galaxies.

Most other models, whether theoretical (Kuijken 2015) or semi-analytic coupled to merger trees (Tonini 2013; Li & Gnedin 2014), predict abundances and metallicities, but do not predict the spatial distribution of GCs. For example, Boylan-Kolchin (2017) has proposed that the linear scaling of GC number with halo mass is largely due to a linear scaling of the blue GC number with halo mass for haloes with $M \gtrsim 10^9 \, M_\odot$ at $z \gtrsim 6$. At face value, this appears to be similar to the assumptions in Bekki et al. (2008). It would be interesting to see if the observed radial scaling is also predicted in this simple halo-based model.

5.3 Red and blue GCs

Most previous studies have found that the metal-poor/blue GC population is more extended than the metal-rich/red one. For the galaxies studied in this paper, the uncertainties on the individual subpopulation sizes are large and so, for most galaxies, the sizes of the two subpopulations are consistent with being equal. One outlier is NGC 2699 which has a larger red GC population than its blue population. Examination of its radial GC density profile in Fig. 6 suggests that the red fit may be biased up by a "bump" in the red GC counts in the range $R \sim 40$–50 kpc. NGC 2699 has the poorest GCS of galaxies studied in this paper, and so it may be susceptible to contamination from the richer GCS around NGC 2698, which is $\sim 35$ kpc away on the plane of the sky. The other outlier is NGC 5485, which has a much smaller red GC $R_e$ than that of its blue GC population.

Previous work has shown that metallicity of the red GC subpopulation is similar to the metallicity of the starlight measured the same galactocentric radius (Pastorello et al. 2015). Moreover, while we have shown that the $R_e$ of the GCS is larger than that of the galaxy light by a factor $\sim 3$, if the red GCS $R_e$ is smaller then it may be close to that of the galaxy light, as it is in some galaxies (Kartha et al. 2016).

These correlations suggest a physical connection between the red GCs and the stars in the host galaxy. More specifically, this population may have been formed "in-situ," with the blue GCs accreted during hierarchical assembly. In this scenario, one might expect the effective radius of the red GCS to be more tightly linked to the galaxy light $R_e$ while the blue GCS might scale more tightly with the halo mass or virial radius. With larger, deeper and more homogeneous samples of GCS, it should be possible to test these predictions.

5.4 Environmental Effects

Assuming that stripping is the dominant astrophysical process responsible for the scaling of the GCS size with halo mass, one would expect significant environmental effects. Specifically, GCs that have been stripped from satellite galaxies and orbit the host halo are then assigned to the GCS of the central galaxy in the halo. Consequently, we expect that, all other things being equal, satellite galaxies should have both lower GCS counts and masses and a smaller GCS $R_e$, whereas the reverse should be true of centrals, particularly if they are central ellipticals. For the galaxies studied in this paper, there is a slight hint that the more massive galaxy (presumed to be the central) in the group lies somewhat higher above the mean $R_e - M_{200}$ relation than subdominant (presumably satellite) galaxies. For the galaxies collected from the literature, most of these are central galaxies in groups, or isolated field galaxies and so again, the samples are not sufficient to make this distinction. Clearly as measurements of the GCS $R_e$ improve in quality and quantity, it will be possible to test for environmental effects.

5.5 Future prospects

To make significant progress in this area, a systematic multi-band survey with sufficient depth to detect most of the GC populations in nearby galaxies is required. The traditional approach has been targeted photometry of individual galaxies or galaxy clusters (e.g. NGVS). In this paper, we adopted a different approach, targetting foreground galaxies in “blank fields” chosen originally for weak gravitational lensing. In the current era, deep, multiband surveys covering significant fractions of the sky will are planned or underway, including the Dark Energy Survey (Dark Energy Survey Collaboration et al. 2016), the Canada-France Imaging Survey (Ibata et al. 2016).
6 CONCLUSIONS

We have shown that the size of the GCS is more closely linked to the halo properties of its host dark matter halo (or, equivalently, to the total stellar mass of the central galaxy) than it is to the effective radius of the galaxy star light. The GCS size is not simply a fixed fraction of the virial radius but rather scales steeply with the virial radius of the halo: $R \propto R_{200}^{2.5 - 3}$.

Dark matter haloes are built hierarchically by the accretion and tidal stripping of smaller units. A similar hierarchical assembly of GCSs (that are increasingly less bound as one moves up the hierarchy) likely results in the steep dependence of GCS size on halo mass.

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The above equations are based on magnitudes in the F850LP ($\approx$ Sloan $z$) passband of the Advanced Camera for Surveys (ACS) on the Hubble Space Telescope. In order to convert our galaxy $K$ magnitudes into Sloan $z$ magnitudes, we use average galaxy colours given in Girardi et al. (2003), adopting an average $B - K$ colour of 3.8, and Fukugita et al. (1995). Using these $z$ magnitudes, we obtain the peak and deviation of the GCLF for each galaxy.

Next, we convert the GCLF peak magnitude from SDSS $z$ to SDSS $i$. Strader et al. (2005) give equations for average $g - z$ colours for red and blue GCs. Averaging these equations will yield an average $g - z$ colour for GCs. Durrell et al. (2014) define GCs as existing within a CFHT $g - i$ colour range of $0.55 < g - i < 1.15$. We will define the average $g - i$ of GCs as the centre of this range at $g - i = 0.85$. To convert this colour from the CFHT filter set to the ACS filter set we use the CFHT to SDSS reverse transformations. Using the average $g - z$ and $g - i$ colours, we convert the GCLF from SDSS $z$ to SDSS $i$.

We need to convert the GCLF from SDSS $i$ to CFHT $i$. Using the models of Worthey (1994)$^2$, we determine the average $r - i$ colour of a sample of an equal amount of red and blue GCs. Red GCs were treated as having an age of 12 Gyr and [Fe/H] = -1.5. Red GCs were treated as having an age of 10 Gyr and [Fe/H] = -0.5. The average $r - i$ colour of a GC was calculated as 0.267. We can use this colour and the transformations from Gwyn (2008) to transform our GCLF model from the ACS/SDSS filter system to the CFHT filter system.

Galaxy and GC magnitudes are corrected for extinction using Schlafly & Finkbeiner (2011).

APPENDIX A: CORRECTIONS

A1 GCLF, photometry and completeness corrections

To make corrections for the incompleteness of the observed GCs, we adopt the Gaussian GCLF of Villegas et al. (2010):

$$\frac{dN}{dm} = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[ -\frac{(m - \mu)^2}{2\sigma^2} \right]$$

(A1)

where the peak ($\mu$) and standard deviation ($\sigma$) are functions of the $z$-band magnitude of the parent galaxy:

$$\sigma_z = (1.07 \pm 0.02) - (0.10 \pm 0.01) (M_{z,gal} + 22)$$

(A2)

$$\mu_z = (-7.66 \pm 0.18) + (0.04 \pm 0.01) (M_{z,gal} + 22).$$

(A3)

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### APPENDIX B: DE VAUCOULEURS AND POWER LAW FIT PARAMETERS

Table B1. The fit parameters of fixed $R_e$ de Vaucouleurs fits for galaxies studied in this paper, as well as the $\chi^2$ of the fit. In each fit there are 14 degrees of freedom.

| Galaxy      | Field | Total $\Sigma_e$ $(10^{-2} \text{ Gpc}^{-2})$ | $\chi^2$ | Red $\Sigma_e$ $(10^{-2} \text{ Gpc}^{-2})$ | $\chi^2$ | Blue $\Sigma_e$ $(10^{-2} \text{ Gpc}^{-2})$ | $\chi^2$ |
|-------------|-------|---------------------------------------------|----------|---------------------------------------------|----------|---------------------------------------------|----------|
| IC 219      | W1-0-0 | 25.67±2.99                                  | 6.44     | 12.49±2.17                                  | 6.78     | 10.65±2.27                                  | 8.72     |
| NGC 883     | W1-0-0 | 4.85±0.80                                   | 7.67     | 2.68±0.67                                    | 9.95     | 1.27±0.44                                    | 6.58     |
| NGC 942+943 | W1+3-4 | 7.25±1.47                                   | 16.76    | 2.98±0.88                                    | 13.14    | 3.64±0.89                                    | 13.56    |
| NGC 2695    | W2-0+1 | 4.01±0.70                                   | 11.17    | 2.56±0.69                                    | 16.22    | 0.95±0.54                                    | 18.04    |
| NGC 2698    | W2-0+1 | 2.80±0.63                                   | 9.92     | 1.04±0.36                                    | 7.63     | 1.44±0.44                                    | 9.61     |
| NGC 2699    | W2-0+1 | 1.70±0.69                                   | 7.98     | 1.08±0.56                                    | 8.76     | 1.14±0.46                                    | 6.51     |
| NGC 5473    | W3-2-0 | 2.57±0.83                                   | 19.80    | 0.80±0.55                                    | 20.61    | 1.61±0.33                                    | 5.99     |
| NGC 5475    | W3-2+1 | 1.01±0.47                                   | 7.77     | 1.44±0.70                                    | 6.19     | 0.97±0.46                                    | 8.46     |
| NGC 5485    | W3-2-0 | 7.90±1.21                                   | 11.47    | 2.25±0.67                                    | 8.20     | 5.00±1.02                                    | 13.62    |

Table B2. The fit parameters of power law fits for galaxies studied in this paper, as well as the $\chi^2$ of the fit. In each fit there are 13 degrees of freedom.

| Galaxy      | Field | Total $\Sigma_0$ $(10^{-6} \text{ Gpc}^{-2})$ | Total $\gamma$ | Total $\chi^2$ | Red $\Sigma_0$ $(10^{-6} \text{ Gpc}^{-2})$ | Red $\gamma$ | Red $\chi^2$ | Blue $\Sigma_0$ $(10^{-6} \text{ Gpc}^{-2})$ | Blue $\gamma$ | Blue $\chi^2$ |
|-------------|-------|---------------------------------------------|----------------|---------------|---------------------------------------------|--------------|------------|---------------------------------------------|--------------|------------|
| IC 219      | W1-0-0 | 6450±1305                                    | -2.07±0.14     | 5.53          | 2695±956                                    | -2.20±0.24   | 6.05       | 2246±1030                                    | -2.17±0.29   | 9.14       |
| NGC 883     | W1-0-0 | 5837±1070                                    | -1.72±0.16     | 5.27          | 3484±999                                    | -1.67±0.26   | 8.02       | 1482±735                                    | -1.77±0.41   | 6.18       |
| NGC 942+943 | W1+3-4 | 5311±1437                                    | -1.87±0.24     | 15.36         | 2433±958                                    | -1.78±0.35   | 12.44      | 2631±791                                    | -1.89±0.28   | 12.18      |
| NGC 2695    | W2-0+1 | 399±216                                      | -2.10±0.26     | 10.64         | 143±138                                     | -2.39±0.43   | 15.64      | 24±59                                       | -2.78±1.01   | 17.22      |
| NGC 2698    | W2-0+1 | 228±218                                      | -2.21±0.44     | 9.81          | 48±89                                       | -2.49±0.82   | 7.32       | 57±84                                       | -2.55±0.65   | 9.42       |
| NGC 2699    | W2-0+1 | 28±44                                        | -2.73±0.54     | 6.58          | 15±35                                       | -2.77±0.77   | 7.92       | 157±180                                     | -1.81±0.48   | 6.31       |
| NGC 5473    | W3-2-0 | 54±127                                       | -3.01±1.03     | 18.54         | 0±1                                         | -5.00±2.87   | 18.83      | 104±119                                     | -2.49±0.53   | 5.75       |
| NGC 5475    | W3-2+1 | 219±224                                      | -1.58±0.56     | 7.14          | 26±47                                       | -2.77±0.75   | 5.73       | 99±139                                      | -1.92±0.79   | 8.26       |
| NGC 5485    | W3-2-0 | 1056±506                                     | -2.12±0.25     | 10.73         | 452±363                                     | -1.92±0.44   | 7.48       | 375±263                                     | -2.43±0.35   | 12.88      |