The effect of subgrid-scale model on prediction of flow around a surface-mounted finite square cylinder

M Einian, DJ Bergstrom, D Sumner

Department of Mechanical Engineering, University of Saskatchewan, 57 Campus Drive, Saskatoon, Saskatchewan, Canada S7N 5A9

E-mail: don.bergstrom@usask.ca

Abstract. The present study investigates the effect of the subgrid-scale (SGS) stress model on large eddy simulation of flow over a finite-length square cylinder mounted vertically on a ground plane. The study specifically considers a cylinder of aspect ratio $AR = 5$ immersed in a thin boundary layer for a Reynolds number of $Re = 500$ (based on the free stream velocity and cylinder width.) Three different subgrid-scale models are considered: a Smagorinsky model, a dynamic Smagorinsky model and a dynamic nonlinear model. The mean flow structure is similar for all SGS models on both grids, although noticeable differences are observed in the size of the recirculation zone, level of resolved-scale turbulence and vortical structures for different SGS models. The dynamic Smagorinsky model appears to be especially sensitive to the grid resolution.

1. Introduction

A surface-mounted cylinder of diameter $D$ and height $H$ immersed in a boundary layer of thickness $\delta(x)$ results in a turbulent wake which is three-dimensional and complex in structure (Wang and Zhou, 2009). The boundary layer flow around the base of the cylinder and the flow over the free end cause distinct changes in the flow pattern along the cylinder height, in particular the formation of a counter-rotating pair of tip vortices at the free end. The tip vortices extend in the streamwise direction into the wake and interact in a complex manner with the von Kármán vortex shedding from the sides of the cylinder. They are responsible for a downward-directed local velocity field near the free end referred to as “downwash.” In addition, a pair of weaker base vortex structures (distinct from the horseshoe vortex) may be found in the wake region close to the ground plane. The structure of the flow field is strongly influenced by the cylinder’s aspect ratio, $AR = H/D$, as well as the relative thickness of the boundary layer developed on the ground plane, $\delta/H$ (Okamoto et al., 1995). Although most experimental studies have considered a surface-mounted finite circular cylinder, many of the same flow features are observed for the case of a finite cylinder (or prism) of square cross-section.

Bluff body flows such as that over a finite square cylinder are nominally well suited to large eddy simulation (LES) in which the large-scale motions are calculated directly, while the effect of the small-scale or subgrid-scale (SGS) motions is modeled. For example, LES can resolve the large-scale unsteady motions which characterise the near-wake region of such flows, including the shedding of shear layers from the sides of the cylinder. On the other hand, the fidelity in reproducing the effect of
the small-scale motions mostly depends on the SGS model. Many SGS models were originally developed in the context of simulating turbulent boundary layers and channel flows. Flows over bluff bodies, which involve flow separation and recirculation regions, clearly involve different physical behaviour. Furthermore, the performance of a SGS model inherently depends on the grid resolution, which typically defines the cut-off length scale for the LES filter. Many SGS models assume that the cut-off length scale is within the inertial subrange, although for most complex flows it is practically impossible to design a grid to ensure this condition is satisfied. Often, LES will compensate for potential deficiencies associated with a simpler SGS model by refining the grid to reduce the significance of the contributions of the unresolved scales. However, an overly resolved LES loses the principal advantage of this methodology relative to a direct numerical simulation (DNS).

The present paper reports LES predictions of the flow over a finite square cylinder mounted on a ground plane using three different SGS models on two different grids. As expected, the performance of each SGS model improves with grid refinement; however, even on a relatively fine grid, the effect of the SGS model is evident in the predictions for both the time-averaged and fluctuating velocity fields.

2. Subgrid-scale models

Perhaps the most popular SGS model is the Smagorinsky model (SM), which adopts a linear eddy viscosity formulation for the constitutive relation between the SGS stress and resolved-scale velocity field, i.e.

\[ r_{ij} = -2C_S \frac{1}{\Delta} \delta_{ij} \| \mathbf{S} \|_S \]  

where the Smagorinsky eddy viscosity is constructed from a local grid scale \( \Delta \) and the magnitude of the resolved strain rate tensor \( S_{ij} \). Two significant deficiencies of the SM are the need to adjust the model coefficient, \( C_S \), for different flows, and the failure of the model to relax to zero in regions of negligible turbulence. The latter issue is often dealt with by using empirical wall-damping functions. Several model formulations have been developed to address the deficiencies of the Smagorinsky model; foremost among these is the so called dynamic model formulation (Germano et al., 1991; Lilly, 1992). In the dynamic Smagorinsky model (DSM), the coefficient \( C_S \) is determined as a variable of space and time, by sampling the smallest resolved-scale motions utilizing two filters with different characteristic scales: the grid filter and a test filter. The value of \( C_S \) automatically goes to zero in regions of laminar flow, including the region immediately next to a solid wall, which then eliminates the need for any ad hoc wall treatment. Unfortunately, the model coefficient can become unbounded when the denominator goes to zero. This potential singular condition is typically avoided by averaging the model coefficient over homogeneous planes, which then eliminates the theoretical advantage of the calculation of a local value of \( C_S \). Notwithstanding the improvements of the DSM, it retains the same linear constitutive formulation as the SM. Wang and Bergstrom (2005) proposed a dynamic nonlinear SGS stress model (DNM) based on Speziale’s quadratic nonlinear constitutive relation. This model uses both the symmetric and asymmetric parts of the resolved-scale velocity gradient to model the SGS stress tensor and includes the conventional DSM as its first-order approximation. Unlike Smagorinsky-type modeling approaches, it does not postulate any arrangement between the SGS stress and the resolved strain rate tensors. Finally, it does not require any plane averaging to avoid erroneous backscatter of SGS turbulence kinetic energy and potential singularity problems, and thus retains the goal of a model coefficient calibrated to the local flow conditions. For details of the DNM formulation and its comparison to the linear DSM, the reader is referred to the paper by Wang and Bergstrom (2005).
3. LES results
In the present study, LES was used to predict the flow around a surface-mounted finite square cylinder of \( AR = 5 \) and a Reynolds number of \( Re = 500 \). The boundary layer on the ground plane was laminar and relatively thin. The finite-volume method was used to discretise the filtered Navier-Stokes equations on a non-uniform grid, and the discrete equation set was solved using a fractional step method. The solution domain extended 20, 10 and 15 cylinder widths in the streamwise \((x)\), spanwise \((y)\) and vertical \((z)\) directions, respectively. Two different grid resolutions were considered: a coarse grid of \(64 \times 72 \times 48\) and a fine grid of \(128 \times 144 \times 96\) control volumes.

Figure 1 shows the time-averaged streamlines in a vertical plane on the flow centreline for the three different SGS models on both the coarse and fine grids. All cases tend to capture the strong downwash originating from the free end and a weaker upwash originating from the ground plane. For each SGS model, the streamline pattern differs with the grid resolution; this variation is observed to be much more significant for the DSM compared to the other two SGS models. Frohlich and Rodi (2004) attributed this behaviour to the fact that for the DSM the test filter operation projects the velocity onto a coarser grid, which can be problematic on a grid with minimal resolution. Surprisingly, for the DNM, which also uses a test filter operation, the insufficient resolution only results in a slightly larger recirculation region. On the fine grid, compared to the SM results, the DSM and the DNM predict a somewhat smaller recirculation zone and a weaker vortex structure immediately behind the tip of the cylinder. The recirculation zones predicted by the two dynamic SGS models on the fine mesh are closer to that shown by the measurements of Wang et al. (2006).

Figure 2 presents contours of the resolved-scale streamwise stress \(<u'u'>/U^2\) (normalised by the freestream velocity) in the vertical plane on the flow centreline for the three different SGS models and two different grids. In general, finite levels of the stress are observed in the shear layer separating from the top of the cylinder and in the recirculation zone behind the cylinder. Immediately behind the cylinder the fluctuating velocity is minimal, while the peak magnitude tends to be located close to the downstream edge of the recirculation zone. There is a clear dependence on both the grid and SGS model. All of the distributions are weaker on the coarse grid, which would then require the SGS model to cover a wider range of small-scale motions. For both the coarse and fine grids, the DSM predicts the highest values for the resolved-scale streamwise velocity fluctuations; these are likely erroneously high for the coarse grid. The SM predicts relatively low levels for the velocity fluctuations, even on the fine grid. The DNM predicts peak levels to occur in the separating shear
layer on top of the cylinder and at two different locations within the recirculation zone behind cylinder.

![Figure 2](image1)

**Figure 2.** Resolved-scale streamwise normal stress $<uw>/U^2$ on the flow centreline: a) SM coarse grid, b) DSM coarse grid, c) NDM coarse grid, d) SM fine grid, e) DSM fine grid, f) NDM fine grid.

Figure 3 shows contours of the mean streamwise vorticity in a vertical plane located at $x/D = 2$. In each case, a pair of tip vortices is formed near the free end and a much weaker pair of base vortices (with opposite sign) is formed near the ground plane. The tip vortices are aligned with the edges of the cylinder and on the fine grid extend almost to the ground plane. For all three SGS models, the vortex patterns on the fine grid exhibit more small-scale features, compared to the relatively smoother contours observed on the coarse grid. Note that for the coarse grid, the tip vortices predicted by the DSM are noticeably shorter and rounder than for the other two SGS models, while the base vortices are larger. Comparing predictions on the fine grid, although the overall pattern of contours is similar for all SGS models, for the SM the contours are smoother and the base vortex on the left hand side extends up along the outer edge of the tip vortex. On the fine grid, both the SM and DNM resolve an additional pair of vortices just above the top of the cylinder.

![Figure 3](image2)

**Figure 3.** Time-averaged streamwise vorticity contours in a plane located at $x/D = 2$: a) SM coarse grid, b) DSM coarse grid, c) DNM coarse grid, d) SM fine grid, e) DSM fine grid, f) DNM fine grid.
The second invariant of the velocity gradient tensor can be used to analyse the three-dimensional vortical structures in the near-wake region of the cylinder. The second invariant is defined as follows:

\[ Q = \frac{1}{2} (\Omega \cdot \Omega - S \cdot S) \quad (2) \]

where \( S \) is the strain rate tensor and \( \Omega \) is the vorticity tensor (which are the symmetric and asymmetric parts of the velocity gradient tensor.) As such, \( Q \) can be considered to represent the local balance between rotation and strain. Positive \( Q \) iso-surfaces represent areas where the strength of rotation dominates strain, indicating a vortex core. An advantage of the second invariant over the vorticity for vortex visualization is that unlike the vorticity, \( Q \) becomes zero at the wall (Dubief and Delcayre, 2004). Figure 4 shows iso-surfaces of the second invariant for the instantaneous flow-field, with the colour indicating the local value of the streamwise velocity. A weak horseshoe vortex is observed in front of the cylinder on the ground plane for all six cases. Typically the shear layers detach from the top and sides of the cylinder in the form of an arch structure, which then breaks down as it is swept into the wake and convected downward towards the ground plane. Anti-symmetric vortex structures, perhaps associated with alternate, von Kármán-like vortex shedding, begin to appear in the lower region of the wake, and the lateral extent of the structures also increases. In general, the solutions with the higher grid resolution exhibit more refined structures, with rounder tube-like shapes. The fine grid appears to enable the continued development and interaction of the vortex structures as they are swept downstream. For example, the anti-symmetric vortices are more clearly exhibited by the fine grid simulations. On both grids, the DSM predicts larger structures, with less small-scale vortices than the other two SGS models. On the other hand, both dynamic models show more instabilities, e.g. wave-like perturbations, on the long vertical shear layers as they detach from the side walls compared to the SM predictions.

![Figure 4](image)

**Figure 4.** Instantaneous \( Q \) iso-surfaces of the instantaneous flow field: a) SM coarse grid, b) DSM coarse grid, c) DNM coarse grid, d) SM fine grid, e) DSM fine grid, f) DNM fine grid.

4. Conclusion

This paper compares LES predictions for flow over a square cylinder with \( AR = 5 \) at \( Re = 500 \) mounted on a ground plane for three different SGS models, i.e. SM, DSM and DNM, on two different grids. The flow pattern is surprisingly complex for a reasonably simple geometry: it includes
separation from both the top and sides of the cylinder, and strong interaction between the flow over the free end and that around the base of the cylinder. There is a complicated instantaneous vortex structure which further evolves as it moves downstream. As would be expected, for all three SGS models, the predictions improved with grid resolution, although some features were consistent on both the coarse and fine grid. The DSM exhibited the greatest sensitivity to grid resolution. For example, on the coarse grid, it predicted an erroneously narrow recirculation pattern as well as stronger resolved-scale velocity fluctuations than the other two models. The predictions of the SM were in general similar on both the coarse and fine grid. One deficiency of this model may be the low level of resolved-scale velocity fluctuations predicted in the recirculation zone behind the cylinder. The DNM predicted surprisingly consistent features on both grids. Although like the DSM it uses a dynamic procedure involving a test-grid, it did not exhibit the excessive grid sensitivity of the DSM. The DNM also captured some flow features that were not evident in the SM predictions.

Acknowledgements
The research support of the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged.

References
DUBIEF, Y. & DELCAYRE, F. 2000 On coherent-vortex identification in turbulence. J. of Turbulence, 1, 011.
FROHLICH, J. & RODI, W. 2004 LES of the flow around a circular cylinder of finite height. Intl. J. Heat Fluid Flow, 25, 537-548.
GERMANO, M., PIOMELLI, P., MOIN, P. & CABOT, W.H. 1991 A dynamic subgrid-scale eddy viscosity model. Physics of Fluids A, 3, 1760-1765.
LILLY, D.K. 1992 A proposed modification of the Germano subgrid-scale closure method. Physics of Fluids A, 4, 633-635.
OKAMOTO, S., TSUNODA, K. & TAKAGI, T. 1995 Turbulent near wake behind square cylinder of finite length on ground plane. Proceedings of the 1995 ASME/JSME Fluids Engineering and Laser Anemometry Conference and Exhibition, August 13-18, 1995, ASME FED vol. 229, pp. 195-216.
WANG, B.C. & BERGSTROM, D.J. 2005 A dynamic nonlinear subgrid-scale stress model. Physics of Fluids, 17, 035109.
WANG, H.F. & ZHOU, Y. 2009 The finite-length square cylinder near wake. J. Fluid Mechanics, 638, 453-490.
WANG, H.F., ZHOU, Y., CHAN, C.K. & LAM, K.S. 2006 Effect of initial conditions on interaction between a boundary layer and a wall-mounted finite-length-cylinder wake. Physics of Fluids, 18, 065106.