Using Tau Polarization to probe the Stau Co-annihilation Region of mSUGRA Model at the LHC

R.M. Godbole\textsuperscript{a}, Monoranjan Guchait\textsuperscript{b} and D. P. Roy\textsuperscript{c,d}

\textsuperscript{a} Center for High Energy physics, Indian Institute of Science, Bangalore 560 012, India.

\textsuperscript{b} Department of High Energy Physics Tata Institute of Fundamental Research Homi Bhabha Road, Mumbai-400005, India.

\textsuperscript{c} Homi Bhabha’s Centre for Science Education Tata Institute of Fundamental Research V. N. Purav Marg, Mumbai-400088, India.

\textsuperscript{d} AHEP group, Instituto de Fisica Corpuscular(IFIC), CSIC-U.de Valencia, Correos E-46071, Valencia, Spain

Abstract

The mSUGRA model predicts the polarization of the tau coming from the stau to bino decay in the co-annihilation region to be +1. This can be exploited to extract this soft tau signal at LHC and also to measure the tiny mass differences between the stau and the bino LSP. Moreover this strategy will be applicable for a wider class of bino LSP models, where the lighter stau has a right component at least of similar size as the left.
1. Introduction

The minimal supersymmetric standard model (MSSM) has been the most popular extension of the standard model (SM) for three reasons. It provides a natural solution to the hierarchy problem of the SM and a natural candidate for the cold dark matter in terms of the lightest superparticle (LSP) along with the unification of gauge couplings at the GUT scale. In particular there is a great deal of interest in the minimal supergravity (mSUGRA) model as a simple and well-motivated parametrization of the MSSM. This is described by the four and half parameters \[ m_{1/2}, m_0, A_0, \tan \beta \text{ and } \text{sgn}(\mu), \] the first three representing the common gaugino and scalar masses and trilinear coupling at the GUT scale. The \( \tan \beta \) stands for the ratio of the two Higgs vacuum expectation values, while the last one denotes the sign of the mixing parameter \( \mu \) between them. The magnitude of \( \mu \) is fixed by the radiative electroweak symmetry breaking condition.

Astrophysical constraints on dark matter (DM) require the LSP to be colorless and neutral, while direct DM search experiments strongly disfavor sneutrino LSP. Thus the favored candidate for LSP in the MSSM is the lightest neutralino, \( \tilde{\chi}_1^0 = N_{11}\tilde{B} + N_{12}\tilde{W}_3 + N_{13}\tilde{H}_1 + N_{14}\tilde{H}_2 \). (2)

In the mSUGRA model the \( \tilde{\chi}_1^0 \) is dominantly bino(\( \tilde{B} \)) over most of the allowed parameter space. Since bino has no gauge charge, it can pair-annihilate mainly via sfermion exchange. The large sfermion mass limits from LEP [2] makes this annihilation process inefficient, leading to an overabundance of DM over most of the mSUGRA parameter space. There are essentially two narrow strips of cosmologically compatible DM relic density [3] at the opposite edges of the parameter space, corresponding to the large and the small \( m_0 \) boundaries, called the focus point and the stau co-annihilation regions respectively [4, 5]. There is also a narrow strip in the middle called resonant annihilation region, but only at very large \( \tan \beta (> \sim 40) \). Out of these only the stau co-annihilation region is compatible with the muon anomalous magnetic moment constraint [6], and also the only one which can be completely covered at the LHC. Therefore the stau co-annihilation region is a region of special interest to the SUSY search programme at the LHC. In particular one is looking for a distinctive signature which will identify the SUSY signal at the LHC to this region and also enable us to measure the tiny mass difference \( \Delta M \) between the co-annihilating superparticles, which is predicted to be \( \sim 5\% \) by the DM relic density constraint. A distinctive feature of this region is that a large part of the SUSY cascade decay occurs via

\[ \tilde{\tau}_1 \to \tau \tilde{\chi}_1^0, \] (3)

leading to a \( \tau \) lepton along with the canonical missing \( E_T(E_T) \). Unfortunately, the \( \tau \) leptons are very soft because of the small mass difference between \( \tilde{\tau}_1 \) and \( \tilde{\chi}_1^0 \). But fortunately the
polarization is predicted to be very close to +1 in the mSUGRA model. In this work we hope to show with the help of a generator level Monte Carlo simulation that the positive polarization ($P_\tau = +1$) of this $\tau$ signal can be exploited to extract it from the negatively polarized $\tau$ ($P_\tau = -1$) background as well as the fake $\tau$ background from hadronic jets. Moreover, the steep $p_T$ dependence of the soft $\tau$ signal will provide a distinctive signature for the co-annihilation region as well as a measure of the tiny mass difference between the co-annihilating particles.

In section 2 we briefly describe how to use the $\tau$ polarization. In section 3 we describe the stau co-annihilation region with the help of an illustrative point in the parameter space of this region. We also identify the main channels of SUSY cascade decay. In section 4 we discuss the result of the simulated SUSY signal for this point. We show how the polarization cut retains most of the $P_\tau = +1$ signal $\tau$-jets while suppressing $P_\tau = -1$ background $\tau$-jets and practically eliminating the fake $\tau$ jets. We also estimate the significance level of the signal and the $\Delta M$ measurement from the slope of the soft $\tau$ jet signal. We conclude with a summary of our results in section 5.

2. Using $\tau$ Polarization

The best channel for $\tau$ identification is its 1-prong hadronic decay channel, accounting for 50% of its decay width. Over 90% of this comes from

$$\tau \rightarrow \pi^\pm \nu(12.5\%), \rho^\pm \nu(26\%), a_1^\pm \nu(7.5\%),$$

where the branching fractions for $\pi$ and $\rho$ include the small $K$ and $K^*$ contributions respectively, which have identical polarization effects [2]. The CM angular distribution of $\tau$ decay into $\pi$ or a vector meson $v(\rho, a_1)$ is simply given in terms of its polarization as,

$$\frac{1}{\Gamma_\pi} \frac{d\Gamma_\pi}{d \cos \theta} = \frac{1}{2} (1 + P_\tau \cos \theta)$$

$$\frac{1}{\Gamma_v} \frac{d\Gamma_{vL,T}}{d \cos \theta} = \frac{1}{2} \frac{m_\tau^2}{m_v^2} \frac{m_v^2}{m_\tau^2 + 2m_v^2} (1 \pm P_\tau \cos \theta)$$

where $L,T$ denote the longitudinal and transverse polarization states of the vector meson. The fraction $x$ of the $\tau$ laboratory momentum carried by its decay meson i.e. the (visible) $\tau$-jet, is related to the angle $\theta$ via

$$x = \frac{1}{2} (1 + \cos \theta) + \frac{m_\tau^2}{2m_v^2} (1 - \cos \theta),$$

in the collinear approximation ($p_\tau \gg m_\tau$). It is clear from eqs. 5 and 6 that the relatively hard part of the signal ($P_\tau = +1$) $\tau$-jet comes from the $\pi$, $\rho_L$ and $a_{1L}$ contributions; while for the background ($P_\tau = -1$) $\tau$-jet it comes from $\rho_T$ and $a_{1T}$ contributions [7]. This is the important part that would pass the $p_T$ threshold for $\tau$ jets.
Now the $\rho_T$ and $a_{1T}$ decays favor even sharing of the momentum among the decay pions, while the $\rho_L$ and $a_{1L}$ decays favor uneven distributions where the charged pion carries either very little or most of the momentum. Thus plotted as a function of the momentum fraction carried by the charged pion

$$R = \frac{p_{\pi^{\pm}}}{p_{\tau-jet}},$$

the longitudinal $\rho$ and $a_1$ contributions peak at very low or very high $R(\lesssim 0.2$ or $\gtrsim 0.8)$, while the transverse contributions peak in the middle [7, 8]. The low $R$ peak of the longitudinal $\rho$ and $a_1$ contributions are not detectable because of the minimum $p_T$ requirement on the charged track for $\tau$ identification ($R \geq 0.2$). Now moving the $R$ cut from 0.2 to 0.8 one cuts out the transverse $\rho$ and $a_1$ peaks while retaining the detectable longitudinal peak along with the single $\pi^\pm$ contribution. Thanks to the complementarity of these two sets of contributions, one can effectively suppress the former while retaining most of the latter by a simple cut on the ratio

$$R > 0.8.$$ 

Thus one can suppress the hard part of the $\tau$-jet background ($P_\tau = -1$), while retaining most of it for the signal ($P_\tau = +1$), even without separating the different meson contributions from one another [8]. This is a simple but very powerful result particularly for the hadron colliders, where one cannot isolate the different meson contributions to the $\tau$-jet in eq. (4). This has been used to enhance the $P_\tau = +1$ signal of charged Higgs boson at Tevatron and the LHC [8, 9]. It can be also used with effect in the investigation of the $P_\tau = +1$ SUSY signal coming from eq.(3) at Tevatron and the LHC [10] as well as the ILC [11].

This is the first application of the $\tau$ polarization effect, however, in the context of probing the stau co-annihilation region. As we shall see below the simple polarization cut of eq. (8) helps to suppress not only the ($P_\tau = -1$) $\tau$-jet backgrounds, but simultaneously the fake $\tau$ jet background from hadronic jets as well. We shall also see that the conclusion of this section remains true even after we add the non-resonant contributions to the $\tau$ decay of eq.(4).

3. Stau co-annihilation region of the mSUGRA model

For illustration we have chosen one point in the stau co-annihilation region of the mSUGRA model. We expect the results to hold equally for any other point in this region. The input mSUGRA parameters are shown in Table-1 along with the resulting weak scale superparticle spectrum calculated using ISAJET(v7.74)[12]. For simplicity we have taken $A_0 = 0$ and positive sign for $\mu$ since our results are not sensitive to them. The corresponding LHC cross sections for the three strong processes at LO are,

$$\sigma(\tilde{g}\tilde{g}, \tilde{g}\tilde{q}, \tilde{q}\tilde{q} + \tilde{q}\tilde{q}^*) = (0.46, 2.4, 1.3) \text{pb}.$$
These are evaluated with the CTEQ3L[13] structure functions setting a common factorization and renormalization scale at the average mass of the superparticle pair. But the results are insensitive to either of these choices. We have also checked that changing from CTEQ3L to the more recent structure functions of CTEQ5L only change the total signal cross-section of eq.(9) from 4.16 pb to 4.11 pb, although there are somewhat larger variations for the three individual processes. The largest contribution to the third process of eq.(9) comes from $qq \rightarrow \tilde{q}\tilde{q}$ because of the dominance of the valence quark flux over gluon. Therefore, we expect uncorrelated production of singlet(S) and doublet(D) squarks in all the three processes i.e. SS:SD:DD=1/4:1/2:1/4. Thus 3/4 of the events contain one or two doublet squarks. The doublet squarks mainly undergo cascade decay via the $\tilde{W}$ dominated chargino and neutralino states $\tilde{\chi}_1^{\pm}, \tilde{\chi}_2^0$, into,

\begin{align}
\tilde{\chi}_1^\pm &\rightarrow \tilde{\tau}_1 \nu \rightarrow \tau \nu \tilde{\chi}_1^0, \\
\tilde{\chi}_2^0 &\rightarrow \tau' \tilde{\tau}_1 \rightarrow \tau' \tau \tilde{\chi}_1^0.
\end{align}

Table 1: Masses of superparticles(in GeV), for an illustrative mSUGRA point in the stau-coannihilation region.

| $m_0$ | $m_{1/2}$ | $\tan \beta$ | $\tilde{g}$ | $\tilde{q}_L$ | $\tilde{q}_R$ | $\tilde{\tau}_1$ | $\tilde{\tau}_2$ | $\tilde{\chi}_1^\pm$ | $\tilde{\chi}_2^0$ | $\tilde{\chi}_3^0$ | $\tilde{\chi}_4^0$ |
|-------|-----------|---------------|-------------|--------------|--------------|---------------|---------------|----------------|----------------|----------------|----------------|
| 223   | 400       | 40            | 942         | 884          | 856          | 172.5         | 356.8         | 526            | 163            | 307            | 511.5          |
|       |           |               |             |              |              |               |               |                |                |                |                |

Thus one expects to see one soft $\tau$ in most cascade decays, sometimes accompanied by a relatively hard $\tau'$. The lighter stau

\begin{equation}
\tilde{\tau}_1 = \tilde{\tau}_R \sin \theta_\tau + \tilde{\tau}_L \cos \theta_\tau,
\end{equation}

is dominated by $\tilde{\tau}_R$, since there is no SU(2) contribution to the RGE. The resulting polarization of the soft $\tau$ from $\tilde{\tau}_1 \rightarrow \tau \chi$ is given by \[10, 14\],

\begin{align}
P_\tau &= \frac{\Gamma(\tau_R) - \Gamma(\tau_L)}{\Gamma(\tau_R) + \Gamma(\tau_L)} = \frac{(a_{11}^R)^2 - (a_{11}^L)^2}{(a_{11}^R)^2 + (a_{11}^L)^2} \\
a_{11}^R &= -\frac{g}{\sqrt{2}}N_{11} \tan \theta_W \sin \theta_\tau - \frac{g m_\tau}{\sqrt{2}m_W \cos \beta}N_{13} \cos \theta_\tau \\
a_{11}^L &= \frac{g}{\sqrt{2}}[N_{12} + N_{11} \tan \theta_W] \cos \theta_\tau - \frac{g m_\tau}{\sqrt{2}m_W \cos \beta}N_{13} \sin \theta_\tau.
\end{align}

The first subscript of $a_{11}^{LR}$ refers to the $\tilde{\tau}_1$ and the second to the neutralino $\tilde{\chi}_1^0$. Thus the dominant term is $a_{11}^R \simeq -\frac{g}{\sqrt{2}}N_{11} \tan \theta_W \sin \theta_\tau$, implying $P_\tau \simeq +1$. In fact in the mSUGRA model there is a cancellation between the sub-dominant terms, so that one gets $P_\tau \simeq 0.9$ throughout the allowed parameter space \[10\]. In the stau co-annihilation region of our interest $P_\tau \simeq 0.98$. So one can safely set $P_\tau = +1$.

The polarization of the hard $\tau'$ from eq.(11) is obtained from eq.(13) replacing $a_{11}^{LR}$ by $a_{12}^{LR}$. The dominant contribution comes from $a_{12}^L \simeq \frac{g}{\sqrt{2}}N_{22} \cos \theta_\tau$, implying $P_{\tau'} \simeq -1$. There is
a similar cancellation of the sub-dominant contributions, leading to \( P_{\tau'} \leq -0.98 \) in the stau co-annihilation region. Thus one can set \( P_{\tau'} = -1 \).

Finally it should be noted that probing the stau co-annihilation region via the \( \tau \) polarization cut of eq. (8) should be applicable in a wider class of MSSM with a \( \tilde{B} \) LSP. Since the coupling of \( \tilde{B} \) to \( \tilde{\tau}_R \) is 2 times larger than to \( \tilde{\tau}_L \) in eq. (13) one gets \( P_{\tau} \approx 0.6 \) as long as the \( \tilde{\tau}_1 \) has a right component at least of similar size as the left component \([15]\). In that case the \( R > 0.8 \) cut will keep at least half of the soft tau signal.

4. Event simulation, results and discussion

The SUSY signal events are generated using the event generator \textsc{Pythia}(v6.23) \([16]\), which simulates superparticle pair production and cascade decay for the spectrum shown in Table 1. The generated \( \tau \) leptons are then passed through the \textsc{Tauola}\(^1\) package \([17]\) to simulate \( \tau \) decay, which includes the effect of \( \tau \) polarization in the hadronic decay channels of eq. (4) along with the small non-resonant contribution. The generated events are then passed through the CMSJET package \([18]\) for jet reconstruction. The jets are constructed using the cone algorithm in CMSJET with a cone size of \( \Delta R = 0.5 \). The kinematic cuts for jet reconstruction are \( E_T > 15 \text{ GeV} \) and \( |\eta| < 4.5 \). Finally, the missing \( E_T (E_T^m) \) is reconstructed by a vector summation of calorimetric energies.

The events are then subject to the following selection cuts for triggering and suppression of SM background:

\[
\begin{align*}
\text{Number of jets} & \geq 2 \quad \text{with} \quad E_T^{j_{1,2}} > 100 \text{ GeV}, \ |\eta_{j_{1,2}}| < 4.5 \\
E_T & > 250 \text{ GeV}.
\end{align*}
\]

From the events passing this cut we select those containing at least one \( \tau \) jet at the generator level. We then require the softest \( \tau \) jet in each event to satisfy

\[
15 \text{ GeV} \leq p_T^{\tau-jet} \leq 40 \text{ GeV}.
\]

This is our sample of soft \( \tau \) jet events. We try to simulate the efficiency of tracker isolation on the \( \tau \) jet by requiring that (i) it has one and only one charged track (leading track) of \( p_T^{tr} > 6 \text{ GeV} \) within narrow signal cone of \( \Delta R_s = 0.1 \) measured with respect to its calorimetric energy deposit, and (ii) there is no other charged track in a surrounding isolation cone of \( \Delta R_I = 0.4 \) with \( p_T^{ch} > 3(1) \text{ GeV} \). The \( p_T^{ch} > 1 \text{ GeV} \) isolation cut ensures higher purity of \( \tau \)-jets at the cost of a lower efficiency of \( \tau \) identification, as genuine \( \tau \) jets can be often accompanied by such a charged track in the environment of the LHC.

It should be noted here that in a full simulation the tracker isolation cut is supplemented with calorimetric cuts for complete \( \tau \) identification \([19]\), which is beyond the scope of the

\(^1\)The current version of \textsc{Tauola} cannot simulate \( \tau \) decay when it comes from sparticle decay. One of the authors(MG) has modified it to include these decay modes.
present work. Since the efficiency of these supplementary cuts is quite high ($\sim 0.8$) we shall simply assume it to be 1. We know from full simulation studies\[19\] that $\tau$-jets identified via tracker isolation and calorimetric cuts match very well (in direction and energy) with the generator level $\tau$-jets. Since we are unable to incorporate the calorimetric cuts, however, we shall work only with generator level $\tau$-jets, satisfying the tracker isolation, as in the case of ref.\[20\]. We shall add to these the fake $\tau$-jets coming from the generator level hadronic jets, which satisfy the tracker isolation cut. Thus we approximately incorporate the tau identification efficiency and purity in our analysis. Working with generator level $\tau$-jets allows us to tag it to its 'mother'. Thus we can separate the soft $\tau$ jet signal of the stau decay of eq.(3) from the background coming from the various other sources in the cascade decay; and look at the effect of the polarization cut of eq.(8) on each of them. This will help us to understand the major contributors to the soft $\tau$ jet background. We shall also consider the largest SM background to the soft $\tau$-jet signal, coming from $t\overline{t}$ and $W+$multijet channels.

Table 2: Number of events for the process $\tilde{q}\tilde{g}$ with and without tracker isolation. "Mothers" of each $\tau$ jet, shown below the 3rd row are found by direct tagging in the event generator. (I) no tracker isolation (II) with tracker isolation $p_{T}^{ch}>3$ GeV and $p_{T}^{tr}>6$ GeV (III) with tracker isolation $p_{T}^{ch}>1$ GeV and $p_{T}^{tr}>6$ GeV. Below the 3rd row the parenthetic entry in each block is obtained with the $R>0.8$ cut.

| Selection                              | I     | II    | II    |
|----------------------------------------|-------|-------|-------|
| No. of events simulated                | 1500k | 1500K | 1500K |
| At least one $\tau$-jet               | 431283| 320207| 254150|
| With $15$ GeV < $p_{T}^{\tau}$-jet < 40 GeV | 156555| 108398| 86137 |
| $\tilde{\tau}_1$                      | 87061 (42259) | 61925 (35239) | 48415 (28178) |
| $\tilde{\chi}_2^0$                    | 27073 (10852) | 22813 (9475) | 19178 (7913) |
| $\tilde{\chi}_3^0$                    | 644 (345) | 573 (315) | 509 (283) |
| $\tilde{\chi}_4^0$                    | 712 (377) | 639 (353) | 548 (304) |
| $W$                                    | 22419 (6770) | 16532 (5358) | 13033 (4227) |
| $Z$                                    | 787 (320) | 611 (265) | 494 (212) |
| $h$                                    | 2994 (1189) | 2366 (1033) | 1925 (831) |
| $\tilde{\tau}_2$                      | 98 (40) | 79 (37) | 59 (30) |
| None of the above 'mothers'            | 14767 (5287) | 2860 (1265) | 1976 (939) |

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We have generated 1.5 million events for the $\tilde{q}\tilde{g}$ production, which has the largest SUSY cross-section of eq.(9) i.e. 2.4 pb. Thus it corresponds to an effective luminosity of about 600 fb$^{-1}$. This will ensure that the predicted events have greater statistical accuracy than the data, so that our error estimates are primarily controlled by the latter. Table 2 shows the effects of the above mentioned cuts on these events. The 2nd row shows that about 1/3rd of the events contain at least one $\tau$-jet, where the suppression factor includes the effects of the selection cut(14) and the 1-prong hadronic branching ratio of $\tau$ lepton. It also shows the effect of the tracker isolation cuts in the three columns. We get an efficiency factor of about 2/3(1/2) corresponding to the $p_T^{ch}$ >3(1) GeV on the accompanying charged tracks. The 3rd row shows that about 1/3 of these events have the softest $\tau$ jet in the $p_T$ range of eq.(15). The subsequent rows tag these $\tau$ jets to their 'mother' and show the effect of the polarization cut of eq. (8) in each of these cases in parentheses. We see that about half of these soft $\tau$ jets come from the signal process of eq. (3) having $P_\tau$ =+1. About 60% of them survive the polarization cut. The background is dominated by the $P_\tau$ =-1 $\tau$-jets coming from $\tilde{\chi}_0^2$ of eq. (11), and the W bosons produced in cascade decay. About 30%(40%) of the $\tau$-jets from $W(\tilde{\chi}_0^2)$ decay survive the polarization cut(8). The reason for the higher survival probability of the $\tau$ jets from the $\tilde{\chi}_0^2$ decay(11) is that they are quite hard to start with. Hence even the relatively soft part of these $\tau$-jets, coming from the $\rho_L$ and $\pi$ decay channels given in eq. (5), survive the modest $p_T$ cut of eq.(15). The $\tau$ jets tagged to none of the listed 'mothers' are expected to come mainly from b hadron decay.

Table 3: Estimation of faking efficiency for QCD and SUSY ($\tilde{q}\tilde{g}$) jet events; QCD jet events are for $\hat{p}_T$ range of 15-40 GeV, where $\hat{p}_T$ denotes the transeverse momentum in the partonic centre of mass frame.

| Selection | $p_T^{ch}$ >1 GeV, $p_T^{ltr}$ >6 GeV | $p_T^{ch}$ >3 GeV, $p_T^{ltr}$ >6 GeV |
|-----------|-----------------------------------|-----------------------------------|
| No. of events simulated | QCD | SUSY | QCD | SUSY |
| Total no. of jets | 181377 | 349989 | 827424 | 347303 |
| Total no. of (fake)$\tau$ jets | 6486 | 9183 | 123100 | 38473 |
| Total no. of (fake)$\tau$-jets with R cut | 376 | 313 | 2496 | 651 |
| Faking efficiency | 0.034 | 0.026 | 0.15 | 0.11 |
| Faking efficiency with R cut | 0.0015 | 0.0009 | 0.003 | 0.0019 |

Table 3 demonstrates the effect of polarization cut on the fake $\tau$ jets. It presents the fake $\tau$ jets coming from the hadronic jets produced in SUSY($\tilde{q}\tilde{g}$) and QCD processes without the selection cuts of eq. (14). It shows that the fake $\tau$-jets background from the hadronic jets in this SUSY cascade decay are quite large, particularly for the tracker isolation cut of $p_T^{ch}$ >3 GeV. But it gets practically eliminated by the R cut of eq.(8). The faking efficiency of 11(3)% for the isolation cut of $p_T^{ch}$ >3(1) GeV falls to 0.2(0.1)% after the R cut. For comparison we also show in this table the faking efficiencies of hadronic jets for 1 million simulated QCD events. The faking efficiencies before and after the R cut are seen to be very
similar to those of the SUSY events. This is evidently a very powerful result, which is not unexpected though. A hadronic jet can fake an one prong \( \tau \)-jet by a rare fluctuation, when all but one of the constituent particles (mostly pions) are neutral. Then requiring the single charged particle to carry more than 80% of the total jet energy requires a second fluctuation which is even rarer. Normally identification of \( \tau \) jets down to \( p_T = 15 \) GeV would be difficult at the LHC because of the fake backgrounds, which is alleviated now by the \( R > 0.8 \) cut. Note that the \( R \) cut automatically raises the \( p_T \) threshold of the leading track to 12 GeV.

Table 4: The SM backgrounds from \( t\bar{t} \) and W+multijet processes. In the latter case separate simulations are done for the two \( \hat{p}_T \) ranges of W, as shown in the table.

| Process→ | \( t\bar{t} \) | \( W+\text{multijets} \) |
|----------|----------------|-----------------|
| \( \hat{p}_T = 100-300 \) GeV | 10\(^6\) | 10\(^6\) |
| \( \hat{p}_T = 300-1000 \) GeV | 10\(^6\) | 10\(^6\) |

| \( \hat{E}_T \) > 250 GeV | 15318 | 1996 | 150073 |
| Jet cut | 15318 | 1996 | 150072 |
| \( M_{\text{eff}} > 750 \) GeV | 9277 | 136 | 73841 |
| One \( \tau \)-jet | 1250 | 13 | 12931 |
| \( M_T > 50 \) GeV | 417 | 2 | 2651 |
| 15 GeV < \( p_T \) < 40 GeV | 150 | 0 | 602 |
| From W | 148 | 0 | 601 |
| With R cut | (40) | 0 | (143) |

Since the \( R > 0.8 \) cut practically eliminates the fake \( \tau \) jet background for both the isolation cuts, we choose to work with the \( \hat{p}_T > 3 \) GeV cut for two reasons. Firstly it is less demanding on the tracker momentum resolution; and secondly it has a higher efficiency for identifying genuine \( \tau \) jet, as seen above. We have found that imposing the selection cuts of eq.(14) suppresses the QCD process very effectively in agreement with ref.[20]. Supplementing this with the faking efficiency with \( R > 0.8 \) cut makes the QCD background to the SUSY signal completely negligible. The largest SM background comes from the \( t\bar{t} \) and W+multijet processes. In order to control these backgrounds we supplement the selection cuts of eqs.(14),(15), with two more kinematic cuts,

\[
M_{\text{eff}} = \hat{E}_T + E_{T}^{j_1} + E_{T}^{j_2} > 750 \text{ GeV}, \quad M_T(\tau - \text{jet}, \hat{E}_T) > 50 \text{ GeV},
\]

where \( M_T = \sqrt{2p_T^{\tau-\text{jet}}\hat{E}_T(1 - \cos \phi(p_T^{\tau-\text{jet}}, \hat{E}_T))} \) and \( \phi \) is the azimuthal angle between \( p_T^{\tau-\text{jet}} \) and \( \hat{E}_T \). Table 4 summarizes the effect of all these kinematic cuts on the simulated \( t\bar{t} \) and W+multijet events. In both cases the \( \tau \) coming from W decay has \( P_T = -1 \). The \( R > 0.8 \) cut is
seen to further reduce these backgrounds to the 25% level. The last two row show the number of background events for luminosity $L=10$ fb$^{-1}$. Here we have used the LO cross-sections for these SM processes for consistency with the SUSY signal.

Table 5: Events from three SUSY processes for tracker isolation, $p_T^{ch}>3$ GeV, $p_T^{tr}>6$ GeV. In the last column the total number of events are presented after normalizing the contributions from various sub-processes with their respective cross sections for luminosity of 10 fb$^{-1}$. As in Table 2, the parenthetic entry in each block is obtained with the $R>0.8$ cut.

| Process→ | $\tilde{g}\tilde{g}$ | $\tilde{g}\tilde{q} + \tilde{q}\tilde{q}$ | $\tilde{q}\tilde{q}$ | a+b+c |
|----------|-----------------|-----------------|-----------------|-------|
| No. of event simulated | 300K | 800K | 1000K | |
| $B_T>250$ GeV | 183548 | 624350 | 744535 | |
| Jet cut | 183548 | 624350 | 744532 | |
| $M_{eff}>750$ GeV | 155364 | 596914 | 685045 | |
| One $\tau$-jet | 35876 | 114016 | 152093 | |
| $M_T>50$ GeV | 25083 | 76437 | 105108 | |
| $15$ GeV$<p_T<40$ GeV | 8065 | 25618 | 34558 | |
| $\tilde{\tau}_1$ | 3209 | 17143 | 16827 | 731 |
| | (1811) | (9681) | (9544) | (414) |
| $\tilde{\chi}^0_2$ | 1991 | 7043 | 8927 | 358 |
| | (776) | (2841) | (3651) | (146) |
| $\tilde{\chi}^0_3$ | 84 | 10 | 235 | 7 |
| | (46) | (5) | (112) | (3) |
| $\tilde{\chi}^0_4$ | 80 | 57 | 223 | 7 |
| | (45) | (31) | (130) | (4) |
| W | 2049 | 873 | 6084 | 191 |
| | (656) | (283) | (1937) | (61) |
| Z | 52 | 31 | 230 | 7 |
| | (22) | (12) | (99) | (3) |
| h | 219 | 396 | 861 | 30 |
| | (94) | (153) | (343) | (12) |
| $\tilde{\tau}_2$ | 13 | 1 | 24 | <1 |
| | (5) | (1) | (14) | (<1) |
| None of the above mothers | 368 | 64 | 1147 | 34 |
| | (166) | (30) | (508) | (15) |
| Fake as a $\tau$ jet | 1681 | 2006 | 5096 | 180 |
| With R cut | (40) | (62) | (126) | (5) |
| Total number of events: Signal SUSY Background SM Background | 731(414) | 814(249) | 427(110) | |

9
Table 5 shows the simulated SUSY events for all the three processes of eq.(9). The effects of kinematic cuts on the soft $\tau$ jet events and their distributions to the various sources are very similar for the three production processes. The last column gives the total number of events from the sum of the three processes normalized by their respective cross sections, for a luminosity of $10\text{fb}^{-1}$. We see from Table 5 that without the R cut the total background is almost twice as large as the signal. Imposing the R cut eliminates the fake $\tau$ background and reduces the total background to below the signal size.

Fig.1 shows the $p_T$ distributions of the soft $\tau$ jet signal and the backgrounds before the R cut, where the contribution from the fake $\tau$ jet background is indicated separately by the dotted line. The vertical scale on left gives the cross section in fb/0.5 GeV, while that on right gives the number of events/0.5 GeV for a luminosity($L$) of $10\text{fb}^{-1}$. Fig.2 gives the corresponding distributions after the R cut of eq. (8). It clearly shows a steep rise of the soft $\tau$ jet signal above the background at the low $p_T$ end.

As we see from Tables 4 and 5, the SUSY background is larger than the SM background because the former is not suppressed by the selection cuts. Moreover while one can independently estimate the SM background, the SUSY background depends on the relevant SUSY masses. It will be very hard therefore to disentangle this background from the SUSY signal. It is for this reason that the $\tau$ polarization plays a very important role in signal extraction. The $P_\tau = +1$ signal and the $P_\tau = -1$ background(SUSY and SM) are known a priori to have very distinct R dependences. Likewise one knows the distinct R dependence of the fake $\tau$ background from hadronic jets. Thus for a given $p_T$ range of the $\tau$-jet events one can make
use of the observed $R$ dependence to separate the $P_\tau = +1$ $\tau$ signal from the $P_\tau = -1$ $\tau$ as well as the fake $\tau$ backgrounds. We shall assume that when real data becomes available, all the important SUSY masses, except the small mass difference $\Delta M$ between $\tilde{\tau}_1$ and $\tilde{\chi}_1^0$, can be roughly estimated (to $\sim$25% accuracy, say), via mass reconstructions using the hard jets along with the hard $\tau$-jet of $p_T > 40$ GeV, as widely discussed in the literature. Then one can estimate the size of the SUSY background from this hard $\tau$-jet region of the data and parametrize it in terms of a cascade decay fit to this data using these SUSY masses. One can then extrapolate it to the $p_T^{\tau-jet} < 40$ GeV region using this parametrization. This will provide a fairly reliable estimate of the SUSY background. An observed excess of low $p_T^{\tau-jet}$ events over the (SUSY+SM) background, which has been suppressed using the polarization cut of $R > 0.8$ (Fig.2), will constitute the soft $\tau$ signal from eq.(3). Thus the procedure for estimating the SUSY background from real data is quite clear. In the absence of real data, however, we see no better alternative to estimate the SUSY background than the simple one adopted here, i.e., take it directly from the SUSY event generator along with the signal for the illustrative mSUGRA point in the stau-coannihilation region of interest. We feel it suffices for our main purpose of demonstrating the importance of the polarization cut in extracting the soft $\tau$ SUSY signal from background. The importance of this cut for signal extraction is evident from a comparison of figures 1 and 2.

The steep slope of the soft $\tau$ jet signal from the $\tilde{\tau}_1$ decay of eq.(3) over the low $p_T$ region can be used to extract the signal as well as to measure the tiny mass difference $\Delta M$, responsible for this steep slope. For this purpose, we divide the $p_T$ range of the soft $\tau$-jet of Fig.2 into two parts, $p_T = 15-25$ GeV and $25-40$ GeV. Then we consider the ratio of the numbers of events coming from the two parts, i.e.

$$D_s = \frac{N_{15-25}}{N_{25-40}} = \frac{288}{126} = 2.3$$ \hspace{1cm} (17)

as a distinctive parameter for extracting the signal and measuring the mass difference $\Delta M$. Being a ratio of cross sections this quantity should be dominated by the statistical error. From Fig.2 we can estimate this ratio for the signal+background and for the background only. We get

$$D_{S+B} = \frac{467}{306} = 1.52 \pm 0.14;$$ \hspace{1cm} (18)

$$D_B = \frac{178}{180} = 0.98 \pm 0.14;$$ \hspace{1cm} (19)

where the statistical errors shown are for $10\text{fb}^{-1}$ luminosity run of the LHC. We see that this luminosity will be enough to extract the signal at the $3\sigma$ level. This will go up to $\sim 10\sigma$ level at the $100\text{fb}^{-1}$ luminosity since statistical error goes down like the $\sqrt{L}$.

To estimate the accuracy in the determination of the small mass difference $\Delta M$ between $\tilde{\tau}_1$ and $\tilde{\chi}_1^0$, we have chosen a second mSUGRA point near that shown in Table 1, increasing $m_0$ by 6 GeV. It increases $\Delta M$ from nearly 10 GeV to 15 GeV without practically changing any other mass. Fig.3 compares the resulting soft $\tau$ jet cross sections(signal and background) for
Figure 2: Same as Fig.1, but with R cut of eq.(8).

Figure 3: $p_T$ (in GeV) of softest tau jet for signal(solid) and background(dashed) processes. The upper solid and dashed histograms are for signal and background processes for $\Delta M = 15$ GeV and the lower ones are for $\Delta M = 10$ GeV. These are subject to R cut of eq.(8).
the two points, the RHS scale again corresponding to the number of events for a luminosity of 10 fb$^{-1}$. One can see a flattening of the low $p_T$ slope with increasing mass difference. The ratio (18) for the upper curves is

$$D_{(S+B)}(\Delta M = 15 \text{ GeV}) = \frac{552}{411} = 1.34 \pm 0.10.$$  \hspace{1cm} (20)

Thus one can estimate $\Delta M$ to 50\% accuracy at $\sim 1.5\sigma$ level with a luminosity of 10 fb$^{-1}$. The significance level goes up to 5\% level for a luminosity of 100 fb$^{-1}$, which makes the statistical error smaller by factor of 3.2.

Note that for estimating the small mass differences $\Delta M$ from the $p_T$ slope of the soft $\tau$-jet one has to assume some knowledge of the other SUSY masses. It follows from simple kinematics that in the rest frame of the decaying $\tilde{\tau}_1$ the $p_T^{\tau-jet} \sim \Delta M$. But the boost factor relating it to the $p_T^{\tau-jet}$ in the laboratory frame, $p_{\tilde{\tau}_1}/M$, depends on the other SUSY masses. As mentioned above, we assume that these masses will be known to $\sim 25\%$ accuracy, say, from the mass reconstruction programme. Therefore, we need to check the robustness of our result to a $\sim 25\%$ variation in the SUSY masses, other than $\Delta M$. For this purpose we have chosen a higher point on the stau-coannihilation strip with $m_0$ and $m_{1/2}$ values $\sim 25\%$ higher than those of Table 1. The SUSY mass parameter for this point are shown in Table 6. The resulting ratio for this point is

$$D_{S+B}(\Delta M = 13 \text{ GeV}) = \frac{641}{444} = 1.44 \pm 0.11.$$  \hspace{1cm} (21)

Note that this mass difference, $\Delta M = 13 \text{ GeV}$, is approximately midway between those of the two points represented in Fig.3, $\Delta M = 10 \text{ GeV}$ and 15 GeV. So we expect the resulting ratio (21) to be nearly midway between those of eqs.(18) and (20), which is indeed the case. Of course there is room for a more detailed investigation of the sensitivity of the $\Delta M$ estimate to the other SUSY masses, which is beyond the scope of this illustrative work.

Table 6: Masses of superparticles (in GeV), for a higher mSUGRA point in the stau-coannihilation region compared to that of Table 1.

| $m_0$ | $m_{1/2}$ | $\tan \beta$ | $\tilde{g}$ | $\tilde{q}_L$ | $\tilde{q}_R$ | $\tilde{\tau}_1$ | $\tilde{\tau}_2$ | $\tilde{\chi}^0_1$ | $\tilde{\chi}^0_2$ | $\tilde{\chi}^0_3$ | $\tilde{\chi}^0_4$ |
|-------|-----------|---------------|------------|-------------|-------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 262   | 500       | 40            | 1156       | $\sim 1083$| $\sim 1043$| 219           | 425            | 390            | 635            | 206            | 390            |
|       |           |               |            |             |             | 622           | 635            |                |                |                |                |

It should be mentioned here that we have selected a large value of $\tan \beta$ for our illustrative mSUGRA point of Table 1 simply because it gives stau-coannihilation region for comparable values of $m_0$ and $m_{1/2}$. But we have also checked the results for a low values $\tan \beta = 10$. It gives stau-coannihilation region for $m_0 << m_{1/2}$. However, the size of the soft $\tau$-jet SUSY signal and background are similar to those of the present analysis.

Finally let us note that an independent method of probing the stau co-annihilation region at the LHC has been investigated in [21] via a ditau-jet signal coming from $\tilde{\chi}^0_2$ decay given in eq. (11). It contains a hard ($p_T > 40 \text{ GeV}$) $\tau$ jet along with the soft one, corresponding to the $\tau'$ and $\tau$ of eq.(11) respectively. The background is suppressed by taking the difference
of opposite-sign and same-sign ditau events. The upper edge of this di-taujet invariant mass plot gives an estimate of the tiny mass difference $\Delta M$, again assuming some knowledge of the other SUSY masses. We find in our simulation, however, that the number of such soft and hard di-taujet events are suppressed by an order of magnitude compared to the soft taujet events analyzed here. Therefore, the present analysis has the benefit of utilizing a much larger fraction of the SUSY events, triggered via the selection cut of eq. (14). But, of course, the di-taujet channel offers an independent probe of this signal and hence should be pursued independently. Let us point out that the $\tau$ polarization effect can be also exploited to improve the result of this di-taujet analysis. In particular imposing the $R >0.8$ cut of eq.(8) on the soft $\tau$ jet and the complementarity cut on the hard one will ensure that both the $\tau$ jets are leading ones - i.e they carry most of the momentum of the respective $\tau$ leptons, with accompanying soft neutrinos. This will steepen the upper edge of the di-taujet invariant mass distribution and improve the resulting estimate of $\Delta M$, apart from suppressing the soft $\tau$-jet background.

5. Conclusion

The stau co-annihilation region of the mSUGRA model is a region of special interests to the SUSY search programme at the LHC. In particular one is looking for a distinctive signature, which will identify the SUSY signal at the LHC to this region and also enable us to measure the tiny mass difference $\Delta M$ between the co-annihilating superparticles, which is predicted to be $\sim 5\%$ by the DM relic density constraint. A distinctive feature of this region is that a large part of the SUSY cascade decay occurs via $\tilde{\tau}_1 \rightarrow \tau \tilde{\chi}_1^0$, leading to a $\tau$ lepton along with the canonical missing $E_T(E_T)$. Admittedly the resulting $\tau$-jets are very soft because of the small mass difference between $\tilde{\tau}_1$ and $\tilde{\chi}_1^0$ states. On the other hand its polarization ($P_\tau$) is predicted to be very close to $+1$. We have shown here with the help of generator level Monte Carlo simulation that the positive polarization($P_\tau=+1$) of this signal can be exploited to extract it from the negatively polarized($P_\tau=-1$) $\tau$-jet background as well as the fake $\tau$ background from hadronic jets. Moreover, the steep $p_T$ dependence of the soft $\tau$-jet signal is shown to provide a distinctive signature for this co-annihilation region as well as a measure of the tiny mass difference $\Delta M$ between the co-annihilating superparticles. The significance levels of this signal and $\Delta M$ measurement are estimated for the LHC luminosities of $10\text{fb}^{-1}$ and $100\text{fb}^{-1}$.

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