A universal processing platform for ball shaped workpiece

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Abstract. This paper designs a mechanism which can be used to any precise operation on spherical surfaces. Usually, it is hard for ordinary processing device to mount and fix the spherical workpiece, and to cover whole surface area for any processing operation, so we developed this processing platform to solve this problem. It has the capacity to increase the quality and accuracy for manufacturing ball shaped workpieces. This platform can be widely used in surface treatment of spheres such as painting, coating, hydrophobic, engraving and laser pointing process. The platform is able to implement complex and continuous toolpaths for a workpiece by using a couple controllable wheels without adding any extra fixture to the workpiece, the processing facility on the top of the platform can process the whole surface of the workpiece. The platform can be adjusted to adapt to a wide range of radius of spherical workpieces. To perform any preprogrammed path on the surface, a control method for the platform by mathematical analyses of rotational movement of sphere has been developed and optimized.

1. Introduction

Ball shaped workpiece is widely used in industry robots [1-2]. Some work using inverted pendulum control system [3-5] via rotating a ball on the ground in bottom segment of their bodies [6-8]. Some have a ball shaped configuration and move by rotating themselves [9]. Larger friction coefficient of the ball’s surface would help the robot move smooth and continuously [10-11]. In industry, corrosion resistant coating on the surface of the spherical pressure vessels obviously increases their lifetime [12]. Hence, surface treatment on the ball shaped workpiece is very useful. The traditional processing methods could not fabricate ball shaped workpiece in one stage, because using the existing fixtures for holding the ball had some difficulties and sometimes the adjusted platform for movement of workpiece might be readjusted during processing and thereby errors occur. Because the existing processing equipment do not have access to the contact points where fixtures hold the workpiece [13] and sometimes, another (second or even more) fixture/mounting is required to keep the workpiece to release the initial supports for accessing those area. Any change of the fixture during operation reduces the accuracy. It would be better if all process has been implemented at one stage [14].

To solve this problem, we designed a universal processing platform particularly for operation on ball shaped workpiece. It is composed of three wheels to support the ball shaped workpiece, two of them are operation wheels. Each operation wheel has two motors that can control the moving speed of contacting point between workpiece and operation wheel. Hence, we can control two velocity vectors of contacting points on the surface of workpiece. Namely, we are able to control the ball’s rotation.
about any axis and cover the whole surface of the workpiece for operation. The processing facilities such as a laser device, painting sprayer, etc. would be installed on the top segment of the platform. This platform can be programmed to rotate the workpiece about any direction to perform an operation in a certain area of the workpiece without using a secondary mounting device because the mechanism is designed based on rotating the workpiece.

2. Mechanism of the processing platform

The processing platform for ball shaped workpiece is represented in figure 1(a). Laser surface treatment or such a related processing method for the ball’s surface can be utilized.

![Figure 1. Structure of the processing platform.](image)

There are two operation wheels to adjust the rotation of the workpiece. Each wheel is controlled by two motors shown in figure 1(b). These two motors control the rotation of the wheel about the axial direction and radial direction, individually.

In order to make a full contact between the wheel and the workpiece, the operation wheel has an adjustable base in such a way that by changing the angle of the base the platform will adapt for different radii of the workpiece. It adjusts the axis of the wheel along with the workpiece center in a line, shown in figure 1(c). Therefore, the relative velocity between contacting point and operation wheel will always tangent to the workpiece’s surface, it also simplifies the control method for workpiece’s movement.

2.1. Kinematics of operation

The platform has three wheels to hold the workpiece, they are evenly distributed around the workpiece. Using two operation wheels, this platform enforces the workpiece to rotate around any axis shown in figure 2.

![Figure 2. Coordinate system for the platform.](image)
xyz is ground coordinate, origin of the coordinate, \( O \), is the center of workpiece. \( \mathbf{r}_A \) and \( \mathbf{r}_B \) are the position vectors for the two contacting points between operation wheels and the workpiece. The angle between vector \( \mathbf{r}_A \) and \( xoy \) plane is \( \gamma \), so does the vector \( \mathbf{r}_B \). In ground coordinate, we have

\[
\mathbf{r}_A = \begin{bmatrix} R \cos(\gamma) \\ 0 \\ -R \sin(\gamma) \end{bmatrix}
\]

(1)

\[
\mathbf{r}_B = \begin{bmatrix} -R \cos(\gamma) \sin \left( \frac{\pi}{6} \right) \\ R \cos(\gamma) \cos \left( \frac{\pi}{6} \right) \\ -R \sin(\gamma) \end{bmatrix}
\]

(2)

where \( R \) is the radius of the workpiece. \( \mathbf{v}_A \) and \( \mathbf{v}_B \) are the relative speed from the operation wheel to workpiece. \( \mathbf{p} \) is a unit vector along the rotational axis. This platform can change the rotating axis \( \mathbf{p} \) by changing the direction and value of \( \mathbf{v}_A \) and \( \mathbf{v}_B \). Because the processing facility is fixed with ground, the manufacturing point is always on the top of the workpiece. We can always find a vector \( \mathbf{p} \) in the \( xoy \) plane when this platform makes the workpiece to rotate to another manufacturing point on the top of the workpiece. After finishing this point, the platform moves to the next one in a similar way. Therefore, the processing facility goes through any area on the workpiece to cover whole surface.

2.1.1. Calculation for processing any area in workpiece. Any manufacturing operation on a ball’s surface can be simplified as a task for tracing a curve; the processing facility goes through the area along this curve. And any curve can be simplified as a procedure for generating a series of points along this curve, which is illustrated in figure 3. As long as the density of points is high enough, the whole surface would be processed with points. \( x'y'z' \) is ball coordinate which is fixed with workpiece. We choose a series of points to cover the processing area, assuming that the position vectors for those points in the ball coordinate are \( q_1, q_2, q_3, \ldots, q_{i+1}, \ldots, q_N \).

\[ \begin{aligned}
\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3, \ldots, \mathbf{c}_i, \mathbf{c}_{i+1}, \ldots, \mathbf{c}_N \\
\end{aligned} \]

\( \mathbf{c}_i \) are the position vectors, \( \mathbf{c} \) represents the position of \( \mathbf{Q} \) in ground coordinate. Because the processing facility is fixed with ground coordinate frame, the coordinates of processing point will always be \[ \begin{bmatrix} 0 \\ 0 \\ R \end{bmatrix} \] in the ground coordinate frame, showed in figure 4.
Figure 4. Discretized coordinates for processing points.

Assuming that the platform has just finished processing $Q_i$, and goes to process $Q_{i+1}$, we choose a unit vector $p_i$ in the $xoy$ plane. The workpiece rotates about $p_i$ by angle $\varphi_i$ and makes $c_{i+1}$ move to $c_i$, the position vector $s_{Ci}$ indicates that the moving trace of $q_{i+1}$, showed in figure 5(a). After the rotation, the spherical workpiece contacting points, $r_A$ and $r_B$, will move to $r_{Ai}$ and $r_{Bi}$. $s_{Ai}$ and $s_{Bi}$ indicate their moving traces.

Therefore, we have

$$c_i = c_{i+1} + s_{Ci}$$

(a) (b)

Figure 5. Movement for processing the $(i+1)$st point.

Therefore, in ground coordinate, there is

$$c_i = \begin{bmatrix} 0 \\ 0 \\ R \end{bmatrix}$$

Then $s_{Ci}$ is expressed by
\[ s_{c_i} = c_i - c_{i+1} = \begin{bmatrix} 0 \\ 0 \\ R \end{bmatrix} - c_{i+1} \]  

(5)

\[ p_i \] is displayed by

\[ p_i = \frac{c_i \times c_{i+1}}{|c_i \times c_{i+1}|} \]  

(6)

Because \( p \) is perpendicular to \( c_i, c_{i+1} \) and \( s_{c_i} \), showed in figure 5(b). We have

\[ \sin\left(\frac{1}{2} \varphi_i\right) = \frac{1}{2} \frac{|s_{c_i}|}{|c_i|} = \frac{|s_{c_i}|}{2R} \]  

(7)

Hence, \( \varphi_i \) can be showed by

\[ \varphi_i = 2 \arcsin\left(\frac{|s_{c_i}|}{2R}\right) \]  

(8)

After rotating, \( r_d \) will move to \( r_{d_i} \). Assuming \( M(p, \varphi) \) is a transformation matrix for a vector, rotates about \( p \) by angle \( \varphi \). We have

\[ r_{d_i} = M(p, \varphi)r_d \]  

(9)

This rotation is accomplished through several steps. First, rotate \( p \) and \( r_d \) around \( y \)-axis by angle \( \alpha \), then rotate them around \( z \)-axis by \( \beta \) to coincide \( p \) with \( y \)-axis. After that, rotate \( r_d \) around \( y \)-axis by \( \varphi \), then apply the inverse process to move \( p \) and \( r_d \) back together, \( p \) will go back to the initial position and \( r_d \) will become \( r_{d_i} \) illustrated by figure 6.

**Figure 6.**: Rotating \( r_d \) about \( p \) by \( \varphi \).

So this process can be described by

\[ r_{d_i} = R_y^{-1}(\alpha)R_z^{-1}(\beta)R_y(\varphi)R_z(\beta)R_y(\alpha)r_d \]  

(10)

\( R_y \) and \( R_z \) are the rotational matrices about \( y \)- and \( z \)-axes.

\[ R_y(\alpha) = \begin{bmatrix} \cos(\alpha) & 0 & \sin(\alpha) \\ 0 & 1 & 0 \\ -\sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix} \]  

(11)

\[ R_z(\beta) = \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \]  

(12)

Assuming the position vector for \( P \) is
\[
\mathbf{p} = \begin{bmatrix}
    p_x \\
    p_y \\
    p_z
\end{bmatrix}
\]  

We have

\[
\cos(\alpha) = \frac{p_x}{\sqrt{p_x^2 + p_z^2}}
\]  

(14)

\[
\sin(\alpha) = \frac{p_z}{\sqrt{p_x^2 + p_z^2}}
\]  

(15)

\[
\cos(\beta) = \frac{p_y}{\sqrt{p_x^2 + p_y^2 + p_z^2}} = p_y
\]  

(16)

\[
\sin(\beta) = \frac{\sqrt{p_x^2 + p_y^2}}{\sqrt{p_x^2 + p_y^2 + p_z^2}} = \sqrt{p_x^2 + p_y^2}
\]  

(17)

So expression for \( \mathbf{M}(\mathbf{p}, \varphi) \) is

\[
\mathbf{M} = \mathbf{R}_z^{-1}(\alpha) \mathbf{R}_z^{-1}(\beta) \mathbf{R}_y(\varphi) \mathbf{R}_z(\beta) \mathbf{R}_y(\alpha)
\]

\[
= \begin{bmatrix}
    p_x^2 + (p_y^2 + p_z^2) \cos(\varphi) & P_x P_y - P_y P_z \cos(\varphi) - P_z \sin(\varphi) & P_x P_y - P_y P_z \cos(\varphi) + P_z \sin(\varphi) \\
    P_x P_y - P_y P_z \cos(\varphi) + P_z \sin(\varphi) & P_x^2 + (p_y^2 + p_z^2) \cos(\varphi) & P_x P_y - P_y P_z \cos(\varphi) - P_z \sin(\varphi) \\
    P_x P_y - P_y P_z \cos(\varphi) - P_z \sin(\varphi) & P_x P_y - P_y P_z \cos(\varphi) + P_z \sin(\varphi) & P_x^2 + (p_y^2 + p_z^2) \cos(\varphi)
\end{bmatrix}
\]  

(18)

After this rotation the processing point changes from \( \mathbf{c}_i \) to \( \mathbf{c}_{i+1} \), \( \mathbf{p}_{Ai} \) and \( \mathbf{p}_{Bi} \) are unit vectors that indicate the moving directions of operation wheels, A and B, and scalar \( N_{Ai} \) and \( N_{Bi} \) are operation wheel’s relative displacements with respect to workpiece shown in figure 5(b).

We have

\[
\mathbf{p}_{Ai} = \frac{\mathbf{p}_i \times \mathbf{r}_A}{|\mathbf{p}_i \times \mathbf{r}_A|}
\]  

(19)

\[
\mathbf{p}_{Bi} = \frac{\mathbf{p}_i \times \mathbf{r}_B}{|\mathbf{p}_i \times \mathbf{r}_B|}
\]  

(20)

\[
N_{Ai} = \varphi_i |\mathbf{p}_i \times \mathbf{r}_A|
\]  

(21)

\[
N_{Bi} = \varphi_i |\mathbf{p}_i \times \mathbf{r}_B|
\]  

(22)

There are two motors to operate wheel A. The green motor in figure 7(a) controls the moving direction of workpiece. The blue one controls the moving angle of the workpiece. Using \( \eta_{Ai} \) and \( \psi_{Ai} \) as the angle signals to operate wheel A, and \( \eta_{Bi} \) and \( \psi_{Bi} \) as angle signals to operate wheel B, which is illustrated by figure 7(b). The radius of the operation wheels is \( r \).
We have

$$\psi_{A_i} = \frac{N_{A_i}}{r}$$

(23)

$$\eta_{A_i} = \begin{cases} 
\cos^{-1} \left( \frac{r_A \times \hat{y}}{r_A \cdot \hat{y}} \cdot p_{A_i} \right), & \text{when } p_{A_i} \cdot \hat{y} \geq 0 \\
2\pi - \cos^{-1} \left( \frac{r_A \times \hat{y}}{r_A \cdot \hat{y}} \cdot p_{A_i} \right), & \text{when } p_{A_i} \cdot \hat{y} < 0
\end{cases}$$

(24)

$$\psi_{B_i} = \frac{N_{B_i}}{r}$$

(25)

$$\eta_{B_i} = \begin{cases} 
\cos^{-1} \left( \frac{r_B \times \hat{z} \times r_B}{r_B \cdot \hat{z} \times r_B} \cdot p_{B_i} \right), & \text{when } p_{B_i} \cdot (\hat{z} \times r_B) \geq 0 \\
2\pi - \cos^{-1} \left( \frac{r_B \times \hat{z} \times r_B}{r_B \cdot \hat{z} \times r_B} \cdot p_{B_i} \right), & \text{when } p_{B_i} \cdot (\hat{z} \times r_B) < 0
\end{cases}$$

(26)

where $\hat{y}$ and $\hat{z}$ are unit vectors along $y$- and $z$-axes. With the calculation above, the platform can control the workpiece to move from the processing point $Q_i$ to $Q_{i+1}$ using the control signal $\psi_{A_i}$, $\eta_{A_i}$, $\psi_{B_i}$, and $\eta_{B_i}$. After setting a series coordinates of processing points $Q_1, Q_2, Q_3, \ldots, Q_i, Q_{i+1}, \ldots, Q_N$, the platform can manufacture the points one by one.

3. Application

To show how this platform works, we manufacture the whole surface of a spherical workpiece as an example. Assuming that the processing points are in a spiral surface on workpiece.

$\theta_{qi}$ and $\varphi_{qi}$ describe the position for processing point $q_i$ in the ball coordinate frame shown in figure 8.

$$\theta_{qi} = \pi \frac{i}{N}$$

(27)

$$\varphi_{qi} = 2\pi n \frac{i}{N}$$

(28)
Choose 1001 points to cover the whole surface of the workpiece. \( i = 0, 1, 2, \ldots, 1000 \). \( N = 1000 \). 

\( n \) is the number of revolutions of the spiral, \( n = 20 \). Therefore, the coordinates for processing point \( Q_i \) are expressed in vector form

\[
\mathbf{r}_i = R \begin{bmatrix} \sin \theta_i \cos \phi_i \\
\sin \theta_i \sin \phi_i \\
\cos \theta_i \end{bmatrix} = R \begin{bmatrix} \sin \left( \frac{i \pi}{1000} \right) \cos \left( \frac{40i \pi}{1000} \right) \\
\sin \left( \frac{i \pi}{1000} \right) \sin \left( \frac{40i \pi}{1000} \right) \\
\cos \left( \frac{i \pi}{1000} \right) \end{bmatrix}
\]

(29)

Now a series of coordinates of processing points are defined, assume that \( R = 5r \), using the aforementioned calculation method, the control signals for moving the workpiece to the next point can be generated, which is illustrated in figure 9.

Figure 9. Control signal for the driving wheels.

Figure 9 shows that the operation wheel, A, keeps rotating around its radial direction, and operation wheel, B, keeps swinging around its radial direction by the amplitude of 100\(^\circ\). The manufacturing trajectory on the workpiece is illustrated in figure 10.
Every blue point in figure 10 is a processing point. The surface of workpiece is covered by the processing points we have set up above.

4. Conclusions
Processing on spherical surface has been always a significant research topic of advanced manufacturing technology. Any mechanical processing on spherical surface attracts more attentions because of its large range of application. Therefore, manufacturers have been pursuing valuable finished surface and implementation accuracy. The new design has been dedicated for processing the spherical surface and can be utilized as an accessory to increase the capability of existing machines or solely by attaching tool mechanism on itself. This platform is particularly designed for processing the ball shaped workpiece. It controls two degrees of freedom of the workpiece by four motors. Different kinds of processing facility can be installed on this platform to implement various processing function. A simulation of the processing function on a whole ball’s surface shows that this platform is capable to process the ball’s surface along any points where we have chosen. This platform has different kinds of application in surface treatment area, as its processing facility can be changed easily and utilized in many areas such as hydrophobic coating treatments, laser surface hardening, etc. It doesn’t need a fixture on the ball shaped workpiece, hence, this platform can manufacture the workpiece in one stage with high efficiency and good quality.

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