A Note on Agegraphic Dark Energy

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Abstract

Recently a new model of dynamical dark energy, or time-varying $\Lambda$, was proposed by Cai[arXiv:0707.4049] by relating the energy density of quantum fluctuations in a Minkowski spacetime, namely $\rho_q \equiv 3n^2m_p^2/t^2$, where $n \sim O(1)$ and $t$ is the cosmic time, to the present day dark energy density. In this note, we show that the model can be adjusted to the present values of dark energy density parameter $\Omega_q (\sim 0.73)$ and the equation of state $w_q (\sim -1)$ only if the numerical coefficient $n$ takes a reasonably large value ($n \gtrsim 3$) or the present value of the gravitational coupling of $q$-field to (dark) matter is also nonzero, namely, $\tilde{Q} \simeq \frac{3n}{2}(\Omega_{q0})^{3/2} > 0$ where $\Omega_{q0}$ is the present value of dark energy density fraction. We also discuss some of the difficulties of this proposal as a viable dark energy model with a constant $n$; especially, the bound imposed on the dark energy density parameter $\Omega_q < 0.1$ during big bang nucleosynthesis (BBN) requires $n < 1/6$. To overcome this drawback, we outline a few modifications where such constraints can be weakened or relaxed. Finally, by establishing a correspondence between the agegraphic dark energy scenario and the standard scalar-field model, we also point out some interesting features of an agegraphic quintessence model.
1 Introduction

Dark energy, or a mysterious force propelling the universe, is one of the deepest mysteries in all of science. This mysterious force now thought to account for about 73% of the density of the entire universe \cite{1} came to many’s surprise in 1998, when the Supernova Cosmology Project and the High-Z Supernova Search teams \cite{2} independently announced their discovery that the expansion of the universe is currently accelerating. One possible source of this late-time cosmic acceleration is a form of energy known as the Einstein’s cosmological constant $\Lambda$ - a vacuum energy of empty space, which acts like a perfect fluid with an equation of state $w_\Lambda = p_\Lambda/\rho_\Lambda = -1$.

In physics, it may be true that we do not have to go around a very complicated (cosmological) model to explain the concurrent universe. By somehow consistent with this idea, it has often been argued by many that the mysterious dark energy we see today may well be the manifestation of the Einstein’s cosmological constant. However, although appealing, this simplest explanation is in blatant contradiction with all known calculations of zero-point (vacuum) energy in quantum field theories \cite{3}. No theoretical model, not even the most sophisticated, such as supersymmetry or string theory \cite{4}, is able to explain the presence of a small positive cosmological constant in the amount that our observations require \cite{1}, $\rho_\Lambda \sim 10^{-47}$ GeV$^4$. If $\rho_\Lambda$ is to be interpreted as the present-day dark energy density, then the most pressing issue would be an understanding of why $\rho_\Lambda^{1/4}$ is fifteen orders of magnitude smaller than the electroweak scale ($M_{EW} \sim 10^{12}$ eV) - the energy domain of major elementary particles in standard model physics, and also why $\Omega_\Lambda \sim 3\Omega_{\text{matter}}$ now.

Needless to say, that the most popular alternative to the cosmological constant, which uses a dynamical scalar field $\phi$ with a suitably defined scalar field potential $V(\phi)$ \cite{5,6}, predicts a small (but still an appreciable) deviation from the central prediction of Einstein’s cosmological constant, i.e. $w_{DE} = -1$. Also, the models of holographic dark energy \cite{7} and agegraphic dark energy \cite{8}, which both appear to be consistent with quantum kinematics, in the sense that these models obey the Heisenberg type uncertainty relation, predict a time-varying dark energy equation of state, $w_{DE} > -1$. The cosmological observations only suggest that $w_{DE} < -0.82$ (see, for example, Ref. \cite{9}). Clearly, there remains the possibility that the gravitational vacuum energy is fundamentally variable. In this Letter we discuss about this possibility in a framework of the model of ‘agegraphic’ dark energy recently proposed by Cai \cite{8}. By adopting the viewpoint that the standard scalar field models are effective theories of an underlying theory of dark energy, we also establish a correspondence between the agegraphic dark energy model and the standard scalar field cosmology.
2 Agegraphic dark energy

Based on an intuitive idea developed by C. Mead in 1960s and its generalization by Károlyházy [10], Ng and van Dam [11], Maziaashvili [12], Sasakura [13] and others, Cai recently proposed a model of dark energy, which he called agegraphic [8]. In this proposal, the present-day vacuum energy density is represented by the energy density of metric fluctuations in a Minkowski space-time

\[ \rho_q = \rho_\Lambda \propto \frac{m_P^2}{t^2} = \frac{3n^2m_P^2}{t^2}, \]  

(1)

where the numerical coefficient \( n \sim \mathcal{O}(1) \) and \( l_P \) is Planck’s scale. For the derivation of Eq. (1), we refer to the original papers [10–13]. This idea per se is not totally new; many cosmological models which involve discussion of a time-varying vacuum energy either predict or demand similar scaling solutions. Although the expression (1) is based on a limit on the accuracy of quantum measurements [10, 12], or thought experiments, it can also be motivated by various field theoretic arguments, see, e.g. [5, 14]. According to [10–13] the total quantum fluctuations in the geometry of space-time can be non-negligible (as compared to the critical mass-energy density of the universe) when one measures them on long distances, like the present linear size of our universe!

What may be particularly interesting in Cai’s discussion [8] is that one may take the cosmic time

\[ t = \int_0^a \frac{da}{Ha} = \int H^{-1} d\ln a \]

(2)

(up to an arbitrary constant) as the age of our universe, where \( a(t) \) is the scale factor of a Friedmann-Robertson-Walker universe and \( H \equiv \dot{a}/a \) is the Hubble parameter (the dot denotes a derivative with respect to cosmic time \( t \)). This implies \( dt/d\ln a = 1/H \). Then, using the definition

\[ \Omega_q \equiv \frac{\kappa^2 \rho_q}{3H^2} = \frac{n^2}{t^2H^2}, \]

(3)

(where \( \kappa \) is the inverse Planck mass \( m_P^{-1} = (8\pi G_N)^{1/2} \)) and differentiating it with respect to e-folding time \( \mathcal{N} \equiv \ln a \), we get

\[ \Omega_q' + 2\varepsilon \Omega_q + \frac{2}{H^2} \Omega_q = 0. \]

(4)

where the prime denotes a derivative with respect to e-folding time, \( N = \ln a \), and \( \varepsilon \equiv \frac{\dot{H}}{H^2} \). Although \( n \) can take either sign, we take \( n > 0 \) and \( t = \frac{n}{H \sqrt{\Omega_q}} > 0 \). Hence

\[ \Omega_q' + 2\varepsilon \Omega_q + \frac{2}{n} \left( \Omega_q \right)^{3/2} = 0, \]

(5)

Eq. (5) may be supplemented by the conservation equation for the field \( q \):

\[ \dot{\rho}_q + 3H\rho_q(1 + w_q) = 0, \]

(6)
or, equivalently,
\[ \Omega_q' + 2\varepsilon \Omega_q + 3(1 + w_q) \Omega_q = 0. \]  
(7)

By comparing eqs. (5) and (7) we get
\[ w_q = -1 + \frac{2}{3n} \sqrt{\Omega_q}. \]  
(8)

This shows that the energy density \( \rho_q \) emanating from the space-time itself may act as a source of gravitational repulsion, provided that \( \sqrt{\Omega_q} < n \). This can be seen by considering a pure de Sitter solution for which \( 3H^2 = \rho_q \). By inverting the relation (11), i.e. \( t \equiv \frac{n}{H \sqrt{\Omega_q}} \), and using Eq. (2), we find
\[ a(t) = \left( \frac{c_1 t + c_2}{n} \right)^n, \]  
(9)

where \( c_1, c_2 \) are arbitrary constants. For \( n > 1 \), the \( q \)-field behaves like a standard scalar field (or an inflaton), leading to an accelerated expansion. However, this is just an ideal situation; in practice \( \Omega_m \) is never zero. Moreover, since \( \rho_q \) is decreasing with the cosmic time \( t \), the ratio \( \rho_m/\rho_q \) could be relevant for all times! That is to say, in the present universe, a small \( n \) in a close proximity of being unity cannot give an accelerated expansion. Specifically, with the input \( \Omega_q = 0.73 \), we get \( w_q < -0.82 \) only for \( n > 3.16 \). With such a large value of \( n \), however, the model cannot satisfy the bound \( \Omega_q(1\text{MeV}) < 0.1 \) imposed during the big bang nucleosynthesis (BBN) epoch unless one modifies certain premises of the standard model cosmology (see below). That is to say, the form of the so-called agegraphic dark energy as presented in (11) is problematic, if the present universe consists of only matter and this “dark energy”. Does this mean that the model is already inconsistent with observations? The answer is probably not. The present model may yield some desirable cosmological features with some simple modifications, such as
\[ \Omega_q \equiv \frac{n^2}{H^2(t + \delta)^2}, \quad t + \delta \equiv \frac{n}{H \sqrt{\Omega_q}}, \quad t \equiv \int_0^a \frac{da}{aH}. \]  
(10)

The rescaling \( t \rightarrow t + \delta \) does not affect the equations like (5) and (8). For brevity, we shall assume that \( \delta \geq 0 \) unless specifically specified.

Regardless of the choice of \( \delta \), it is not sufficient to concentrate only on the gravitational sector of the theory when studying the concurrent cosmology. In order to study the transition between deceleration and acceleration, one has to consider the ordinary matter field, which is also the constituent that we know dominated the universe in the past. To this end, one supplements the evolution equation (5) by conservation equations for the ordinary fields (matter and radiation). With the standard

\footnote{Note that \( tH = 1/2, 2/3 \) and \( > 1 \), respectively, during radiation, matter and dark energy dominated phases.}
assumption that matter is approximated by a non-relativistic pressureless fluid component \((w_m \approx 0)\), and using the Friedmann constraint \(\Omega_m + \Omega_q + \Omega_r = 1\), we find

\[
\Omega_q = 1 - \left(1 + c_0 e^{\ln a}\right) \Omega_r, \quad \Omega_m = \Omega_r c_0 e^{\ln a},
\]

(11)

where we have used the conservation equations \(\dot{\rho}_m + 3H(1+w_m)\rho_m = 0\) and \(\dot{\rho}_r + 4H\rho_r = 0\). Thus, if an explicit functional form of \(\Omega_q\) is known, then \(\Omega_r\) and \(\Omega_m\) can be known. The numerical coefficient \(c_0\) in Eq. (11) can be fixed using observational inputs: ideally, \(\Omega_{m0} \approx 0.27\) and \(\Omega_r \approx 5 \times 10^{-5}\) at the present epoch \((a \approx a_0 \equiv 1)\) implies that \(c_0 \approx 5400\). For future use, we also define \(e^{\ln a} = (1+z)^{-1}\), so that \(a = a_0 \equiv 1\) at \(z = 0\) \((a_0\) is the present value of scale factor). All the examinations so far have been in a rather general way, i.e. without making additional assumptions, except that \(w_m \approx 0\). For sure this is not really satisfying, as one might be interested in analytic solutions of the system of equations (5) and (11). To this end, we take \(\Omega_r \approx 0\), which is also a reasonable approximation valid at late times. From eqs. (5) and (11), we find

\[
\varepsilon = -\frac{1}{2} \frac{\Omega_q'}{\Omega_q} - \frac{1}{n} \sqrt{\Omega_q}, \quad \Omega_m + \Omega_q = 1
\]

subject to the constraint

\[
\ln a + C = \frac{8}{3} \ln \left|3n - 2\sqrt{\Omega_q}\right| - \frac{n}{3n+2} \ln \left|1 - \sqrt{\Omega_q}\right| - \frac{n}{3n+2} \ln \left(\sqrt{\Omega_q} + 1\right) + \frac{2}{3} \ln \sqrt{\Omega_q},
\]

(13)

where \(C\) is an integration constant. Differentiating this last equation with respect to \(\ln a\), we get

\[
\Omega_q' = (1 - \Omega_q)(3 - 2n \sqrt{\Omega_q}) \Omega_q.
\]

(14)

Substituting this expression back into Eq. (12), or Eq. (5), we find

\[
\varepsilon = -\frac{3n(1 - \Omega_q) - 2\Omega_q^{3/2}}{2n}. \quad \Omega_q \approx 0.73, \quad \varepsilon > -1 \quad \text{and} \quad w_q < -0.82, \quad \text{only if} \quad n \gtrsim 3.16.
\]

(15)

This expression shows that the model can be consistent with concordance cosmology, for which \(\Omega_q \approx 0.73, \quad \varepsilon > -1\) and \(w_q < -0.82\), only if \(n \gtrsim 3.16\). From the plots in Fig. 1, we can see that during the matter dominated phase, \(\Omega_m/\Omega_m \approx \text{const}\), the present value of scale factor.

Next we study the system of equations with nonzero radiation component. From equations (5) and (11), along with conservations equations \(\dot{\Omega}_m + (2\varepsilon + 3)\Omega_m = 0\) and \(\dot{\Omega}_r + (2\varepsilon + 4)\Omega_r = 0\), we get

\[
\varepsilon = -\frac{1}{2} \frac{\Omega_r'}{\Omega_r} - 2, \quad \frac{\Omega_r'}{\Omega_r} = \frac{3n\Omega_m - 4n(1 - \Omega_r) + 2\Omega_q^{3/2}}{n}. \quad \Omega_r \approx 0.02, \quad \varepsilon > -1 \quad \text{and} \quad w_q < -0.82, \quad \text{only if} \quad n \gtrsim 3.16.
\]

(16)

\footnote{As already noted in [8,15], Eqs. (5) and (11) hold not only for the form \(t = \frac{n}{H \sqrt{\Omega_q}}\), but also for \(t = \frac{n}{H \sqrt{\Omega_q}} + \text{const.}\)

The assumption that \(a \approx 0\) in the matter dominated phase was, however, not necessary, which led to an apparent contradiction in [8]. In the limit \(\Omega_q \rightarrow 0\), Eq. (13) gives \(\Omega_q \propto a^3[8]\). However, this solution may not correspond to the matter dominated epoch; in any consistent model, one should actually allow a nonzero \(\Omega_q\) in the limit \(\Omega_q \rightarrow 0\). During matter dominance one has \(a \propto t^{2/3}\), \(H^2 = 4/9g^2\) and hence \(\Omega_q = 9n^2/4(1 + \delta t)^2\). With \(\delta \gtrsim \mathcal{O}(10) \times t_{BBN}\), the present model could lead to some desirable features even for \(n \sim \mathcal{O}(1)\), thus the extra parameter \(\delta\) is a mixed blessing.}
The acceleration parameter $\varepsilon \equiv \dot{H}/H^2$ (left plot) and the dark energy equation of state $w_q$ (right plot) as functions $\Omega_q$, with $n = 1, 2, 3, 4$ (bottom to top (left plot) or top to bottom (right plot)). Cosmic acceleration occurs for $\varepsilon > -1$ or $w_{\text{eff}} < -1/3$.

To solve the system of equations analytically, we need an extra condition. Here we just want to check consistency of the model by considering the following simplest solution

$$ a(t) = (c_1 t + c_2)^m, \quad \sqrt{\Omega_q} = \frac{n}{m + n c_0 a^{-1/m}}, $$

where $m$ is arbitrary. It should be emphasized that this solution is valid for any value of $\delta$ in eqn. [10]. The integration constant $c_0$ may be fixed such that $\Omega_q = \Omega_{q0} \simeq 0.73$ at $a = a_0 = 1$. Fig. 2 shows the behaviour of the acceleration parameter $\varepsilon$ and the dark energy equation of state $w_q$. With input $\Omega_{q0} \simeq 0.73$, we clearly require $n > 3$ to get $w_q < -0.82$ at the present epoch. This discussion is consistent with the best-fit cosmological values of $n$ given in Ref. [16,17]. Below we will consider the case of interacting dark energy, for which the putative dark energy field $q$ interacts non-minimally with (dark) matter.

### 3 Interacting agegraphic dark energy

In the non-minimal coupling case, the energy conservation equations can be modified as

$$ 0 = \Omega_q' + 2\varepsilon \Omega_q + 3(1 + w_q)\Omega_q + \tilde{Q}, $$

$$ 0 = \Omega_m' + 2\varepsilon \Omega_m + 3(1 + w_m)\Omega_m - \tilde{Q}, $$

$$ 0 = \Omega_r' + 2\varepsilon \Omega_r + 4\Omega_r, $$

where $\tilde{Q}$ measures the strength of the gravitational coupling of $q$-field to matter. In general, $\tilde{Q} = Q_q \Omega_m$, where $Q_q \equiv \frac{d \ln A(q)}{d \ln a}$ and $A(q)$ is a coupling function. In the minimal coupling or noninteracting

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4This is a fairly good approximation at a given epoch, such as $m \simeq 2/3$ during the matter-dominated epoch.
case $A(q) = 1$. For simplicity, we will take $w_m \approx 0$ so that the matter is approximated by a pressureless non-relativistic dust. Eqs. (5) and (18)-(19) can then be written as

$$
\frac{\dot{\Omega}}{2 \Omega} = -2, \\
\frac{\dot{\Omega}}{2 \Omega} = -2 \frac{\Omega_q}{\Omega_r}, \\
w_q = -1 + 2 \frac{3n}{\Omega_q} - \frac{Q}{3 \Omega_q}.
$$

Although $\Omega_r \approx 0$ at the present epoch, the ratio $\Omega'_r/\Omega_r$ is non-negligible; in fact, the value of $\Omega'_r/\Omega_r$ should be less than $-2$ so as to allow an accelerated expansion ($\epsilon > -1$). Note that, with $\dot{Q} \neq 0$ the EoS parameter $w_q$ does not explicitly depend on $n$ rather on the values of $\Omega_q$ and $\Omega'_r/\Omega_r$.

With $\dot{Q} \neq 0$, one may get $w_q \approx -1$ by taking

$$
\dot{Q}_0 \approx \frac{2}{n} (\Omega_{q0})^{3/2},
$$

where $\Omega_{q0}$ is the present value of $\Omega_q$. With the input $\Omega_{q0} = 0.73$ and $n \sim \mathcal{O}(1)$, the coupling $\dot{Q}_0$ is relatively large. For this to happen, the $q$-field should interact strongly at least with invisible or dark matter. With further input that $\Omega_r \approx 0$ and $\Omega_{m0} \approx 0.27$, we get $\Omega'_r/\Omega_r = -3.19$ and hence $\epsilon = -0.405$, which leads to an accelerated expansion, i.e. $a(t) \propto t^{2.47}$.

\footnote{It is a simplification when we say the $q$-field couples to matter, when actually it is meant that $q$-field couples to dark matter and that the baryonic component is negligible. This discussion can easily be generalized to the case where $Q_b = 0$, in which case the $q$-field is coupled only to dark matter, which then automatically satisfies possible local gravity constraints.}
Figure 3: The evolution of the EoS parameter \( w_q \) and the coupling function \( \tilde{Q} \).

The above discussion shows that in the case of a nontrivial coupling between the \( q \)-field and matter, so that \( \tilde{Q} \neq 0 \), the model proposed in [8] may be adjusted to present-day dark energy parameters \( \Omega_q \simeq 0.73 \) and \( w_q \simeq -1 \), if the present value of \( \tilde{Q} \) is large, \( \tilde{Q} \sim \mathcal{O}(1) \) (see Fig.3). However, this is not end of the story. As mentioned above, with \( \delta = 0 \) in (10), the present model finds stringent constraints in the early universe, including the bound imposed on \( \Omega_q \) during the BBN. To quantify this, let us consider an epoch of cosmological expansion where \( tH \approx \text{const} \equiv \alpha \). This then implies that

\[
\Omega_q \equiv \frac{\rho_q}{3m_p^2H^2} = \frac{n^2}{\alpha^2},
\]

where we have used the relation (1). The explicit solution is then given by

\[
\Omega_r = \Omega_r^{(0)}e^{-\frac{4\ln a+(2/\alpha)\ln a}{\alpha}}\Omega_r, \quad \varepsilon = -\frac{1}{\alpha}, \quad \Omega_m = 1 - \Omega_q - \Omega_r,
\]

\[
\tilde{Q} = \frac{2(2\alpha - 1)}{\alpha}\Omega_r + \frac{6(1 + w) - 2}{\alpha}\Omega_m,
\]

where \( w = 0 \) (\( w = 1/3 \)) for matter (radiation). During the radiation dominance, one would expect that \( a \propto t^{1/2} \), implying that \( \alpha = 1/2 \) and thus \( \Omega_q \simeq 4n^2 \). If so, the above solution can satisfy the bound \( \Omega_q(1\text{MeV}) < 0.1 \) during BBN only if \( n < 1/6 \), indicating a small value of \( n \) for which there would be no cosmic acceleration at late times, satisfying \( \Omega_{q0} \simeq 0.73 \) and \( w_q < -0.82 \). For a consistent model cosmology, perhaps one needs to satisfy during radiation-dominated epoch the both conditions \( \Omega_q \ll 1 \) and \( tH \simeq 1/2 \), simultaneously. Clearly, with \( \delta = 0 \), the model of agegraphic dark energy, which may be called age-mapping, cannot describe both the present and far past eras (including the radiation-dominated universe) with a constant \( n \), see also the discussion in [12]. Nevertheless, as advertised above, with some simple modifications the present model could lead to a viable cosmological scenario. Let us in turn briefly discuss them.

(1) A natural modification for which the numerical coefficient \( n \) appearing in (1) varies slowly
(actually, increases) with time, such that \( n(t_1) \ll n(t_2) \) where \( t_2 \gg t_1 \), could be compatible with concordance cosmology, giving rise to standard conventional results, such as \( \Omega_0 \ll 1 \) and \( tH \simeq 1/2 \) during the radiation-domination epoch, and \( \Omega_q \simeq 0.73 \) and \( t_q H_0 \simeq 1 \) at the present epoch.

(2) Perhaps the most interesting possibility is to replace the cosmic time \( t \) by a conformal time \( \eta \), as discussed recently by Cai and Wei [18], and in more detail in [19], for which \( dt \equiv ad\eta \) and

\[
w_q = -1 + \frac{2}{3n} \sqrt{\Omega_q} e^{-\ln a} - \frac{\bar{Q}}{3\Omega_q}.
\]

By setting \( \bar{Q} = 0 \), and then comparing this equation with the standard expression

\[
w_q = -1 - \frac{1}{3} \frac{\Omega_q'}{\Omega_q} - \frac{2\varepsilon}{3},
\]

we get

\[
\sqrt{\Omega_q} = \frac{ne^{-\int \varepsilon \ln a}}{c + \int e^{-\ln a[e^{-\varepsilon \ln a}]} d\ln a},
\]

where \( c \) is an integration constant. This yields

\[
\Omega_q = a^2 \left( \frac{1}{n} + \frac{c}{a} \right)^{-2} \quad \text{(RD)}, \quad \Omega_q = a^2 \left( \frac{2}{n} + \frac{c}{\sqrt{a}} \right)^{-2} \quad \text{(MD),}
\]

respectively, for the radiation and matter dominated epochs. The discussion in Ref. [18] corresponds to the choice \( c = 0 \). Especially, in the case \( \Omega_q \propto a^2 \), the limit \( a \to 0 \) can be regular, since \( w_q \to \text{finite} \) as \( a \to 0 \). The equation of state parameter \( w_q \) takes a finite value also in the early universe, provided that the coupling term \( \bar{Q} \) approaches zero faster than \( \Omega_q \).

(3) Another interesting possibility is to modify the expression for \( \rho_q \), Eq. (11), itself, such that

\[
\rho_q = \frac{3n^2 m_p^2}{(t + \delta)^2},
\]

where now \( \delta > 0 \). This yields

\[
\frac{n}{\sqrt{\Omega_q}} = tH \left( 1 + \frac{\delta}{t} \right).
\]

In the radiation-dominated universe \( a(t) \propto t^{1/2} \) and hence \( Ht \simeq 1/2 \). Now, the BBN bound \( \Omega_q(1 \text{ MeV}) < 0.1 \) can be satisfied by choosing \( \delta \) such that \( 40n^2 < (1 + \delta/t)^2 \). As a typical example, let us take \( n = 3 \), then the BBN bound \( \Omega_q(1 \text{ MeV}) \lesssim 0.1 \) is satisfied for \( \delta \gtrsim 18 \times t_{BBN} \). Although the choice \( \delta = 0 \), being the most canonical, allows one to solve the field equations analytically, the consistency of the model with concordance cosmological requires \( \delta > 0 \).

One may reconstruct an explicit observationally acceptable model of evolution from the big bang nucleosynthesis to the present epoch, by considering a general exponential potential [20]

\[
V(\phi) = V_0 \exp \left( -\lambda \phi/m_p \right)
\]
where $\lambda \equiv \lambda(\phi)$. In the present model, this again translates to the condition that the numerical coefficient $n$ (appearing in Eq. (1)) also becomes a slowly varying function of cosmic time $t$ (or the age of the universe). An explicit construction of such a model is beyond the scope of this Letter.

4 Agegraphic quintessence

The *agegraphic* dark energy model discussed above can be analysed also by considering the standard scalar field plus matter Lagrangians

$$L = \sqrt{-g} \left( \frac{R}{2\kappa^2} - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right) + L_m. \quad (31)$$

Without loss of generality, we will relate the putative dark energy field $q$ (appearing in Eq. (1)) with the standard scalar field $\phi$ by defining $\phi \equiv \phi(q)$. For simplicity, let us first drop the matter part of the Lagrangian, which will be considered later anyway. With the standard flat, homogeneous FRW metric: $ds^2 = -dt^2 + a^2(t)dx^2$, we find that the two independent equations of motion following from Eq. (31) are given by

$$2\dot{H} + \kappa^2 \dot{\phi}^2 = 0, \quad (32)$$

$$\ddot{\phi} + 3H \dot{\phi} = -\frac{dV(\phi)}{d\phi}. \quad (33)$$

Eq. (33) can be written as

$$\dot{\rho}_\phi + 3H \rho_\phi (1 + w_\phi) = 0, \quad (34)$$

where $w_\phi \equiv p_\phi/\rho_\phi$ and $\rho_\phi \equiv \frac{1}{2} \dot{\phi}^2 + V(\phi)$. Using the definitions

$$\varepsilon \equiv \frac{\dot{H}}{H^2}, \quad \Omega_\phi = \frac{\kappa^2 \rho_\phi}{3H^2}, \quad (35)$$

we arrive at

$$0 = \Omega_\phi' + 2\varepsilon \Omega_\phi + 3(1 + w_\phi)\Omega_\phi, \quad (36)$$

$$0 = 3w_\phi \Omega_\phi + 2\varepsilon + 3. \quad (37)$$

These equations may be solved analytically only by imposing one extra condition, since the number of degrees of freedom exceeds the number of independent equations.

For completeness, we write down the equations of motion by considering the case where the putative dark energy field $\phi$ interacts with ordinary matter. The set of equations (36)-(37) are then modified as (see Appendix for the details)

$$0 = \Omega_\phi' + 2\varepsilon \Omega_\phi + 3(1 + w_\phi)\Omega_\phi + \bar{Q}, \quad (38)$$

$$0 = \Omega_m' + 2\varepsilon \Omega_m + 3(1 + w_m)\Omega_m - \bar{Q}, \quad (39)$$

$$0 = \Omega_r + 3w_\phi \Omega_\phi + 3w_m \Omega_m + 2\varepsilon + 3. \quad (40)$$

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6Here $a(t)$, $\phi(t)$ and $V(\phi)$ are primary variables, while $\Omega_\phi$, $\varepsilon$ and $w_\phi$ are secondary (derived) variables.
Here $\tilde{Q}$ measures the strength of a gravitational coupling of $\phi$-field to matter. Without any restriction on $\Omega_\phi$, or the potential $V(\phi)$, we find that the dark energy EoS $w_\phi$ is given by

$$w_\phi = \frac{-2\varepsilon + 3 + 3 \sum_i w_i \Omega_i + \Omega_r}{3\Omega_\phi},$$

(41)

where $i = m$ (matter) includes all forms of matter fields, such as pressureless dust ($w = 0$), stiff fluid ($w = 1$) and cosmic strings ($w = -1/3$). Note that the universe accelerates when the effective equation of state $w_{\text{eff}}$ becomes less than $-1/3$ (where $w_{\text{eff}} \equiv -1 - 2\varepsilon/3$), not when $w_\phi < -1/3$; it is because, for a cosmic acceleration to occur, a gravitationally repulsive force or dark energy must overcome a gravitational attraction caused by ordinary matter and radiation.

![Graph showing the effective equation of state $w_{\text{eff}}$ and dark energy equation of state $w_\phi$ as a function of $\Phi$, with $n = 1, 2, 3, 4$ (from top to bottom) and $\Omega_\phi = (0, 0.73)$. The end point of each curve (or line) corresponds to the value $\Omega_\phi = 0.73$, while $\Omega_\phi = 0$ at $|\Phi| = 0$.](image)

Figure 4: The effective equation of state $w_{\text{eff}} (\equiv -1 - 2\varepsilon/3)$ and dark energy equation of state $w_{\text{DE}} \equiv w_\phi$ as a function of $\Phi$, with $n = 1, 2, 3, 4$ (from top to bottom) and $\Omega_\phi = (0, 0.73)$. The end point of each curve (or line) corresponds to the value $\Omega_\phi = 0.73$, while $\Omega_\phi = 0$ at $|\Phi| = 0$.

In the particular case that $w_i \approx 0$ and $\Omega_r \approx 0$, the universe accelerates when $w_\phi \Omega_\phi < -1/3$, or when $\varepsilon > -1$, where

$$\varepsilon = -\frac{3(1 + w_\phi \Omega_\phi)}{2}.$$

(42)

From the relations given below Eq. (34), we can easily derive

$$\Phi^2 \equiv \frac{\dot{\phi}^2}{m_p^2 H^2} = 3\Omega_\phi \left(1 + w_\phi\right), \quad U \equiv \frac{V(\phi)}{m_p^2 H^2} = \frac{3}{2}\Omega_\phi \left(1 - w_\phi\right).$$

(43)

In order to reconstruct a model of agegraphic quintessence, one may supplement these relations by the EoS of agegraphic dark energy, $w_\phi = -1 + (2/3n)\sqrt{\Omega_\phi}$. From Eq. (12), we then find

$$\varepsilon \equiv \frac{\dot{H}}{H^2} = -\frac{3}{2}(1 - \Omega_\phi) - \frac{\Omega_\phi^{3/2}}{n}.$$

(44)

As expected, this expression of $\varepsilon$ matches with that obtained from eqs. (12) and (14). In Fig. 4 we show the behaviour of $w_{\text{eff}}$ and $w_\phi$ with respect to a dimensionless parameter, $\Phi (\equiv |\dot{\phi}|/(m_p H))$. 

11
The plots there show that the universe can accelerate ($w_{\text{eff}} < -1/3$) only if $n \gtrsim 2$, and $\Omega_{\phi}$ may evolve from zero to higher values as the $\phi$-field starts to roll. The $\phi$-field is almost frozen, i.e. $\dot{\phi} \simeq 0$, during the matter-dominated phase where $w_{\text{eff}} \simeq 0$ or $\epsilon \simeq -3/2$, while $\dot{\phi}$ is nonzero during an accelerating (or dark energy dominated) regime, leading to $U(\phi) \equiv V(\phi)/(m_P^2 H^2) > 0$ at present.

![Figure 5](image1.png)

*Figure 5:* (Left plot) Evolution of a normalised agegraphic potential $U(\phi)$ with respect to $\Phi$, for $n = 1, 2, 3, 4$ (bottom to top). (Right plot) The ratio $r \equiv \dot{\phi}^2/2V$ with respect to $\epsilon$, for $n = 1, 2, 3, 4$ (top to bottom), which usually measures the value of $(1 + w_{\phi})/(1 - w_{\phi})$. Acceleration occurs when $\epsilon > -1$. The end point of each curve corresponds to $\Omega_{\phi} = 0.73$. The potential $V(\phi)$ vanishes at $|\Phi| = 0$ (where $\Omega_{\phi} = 0$), while it increases as the density parameter $\Omega_{\phi}$ grows.

![Figure 6](image2.png)

*Figure 6:* The reconstructed potential $V(\phi)$ and the time-derivative of $\phi$ as functions of $\Omega_{\phi}$ and scale factor $a$. We have set $\sqrt{\Omega_{m0} m_P H_0} = 1$.

*Fig. 6* shows that the normalised agegraphic potential $U(\phi)$ vanishes at $\Phi = 0$. This feature is clearly different from that of the standard quintessence model, for which, generally, $V(\phi) = \text{const}$ at $\dot{\phi} = 0$. Another crucial difference is that as the universe evolves from a matter-dominated epoch ($\epsilon \simeq -3/2$) towards a dark energy dominated epoch ($\epsilon > -1$), the ratio $\dot{\phi}^2/V(\phi)$ increases.
with respect to dark energy density fraction $\Omega_\phi$, as well as with $\varepsilon$, implying that the agegraphic quintessence model constructed above falls into the ‘thawing’ model [21], rather than the ‘freezing’ model for which $\dot{\phi} = 0$ corresponds to an analytic minimum of the potential. This behaviour is seen also from the ratio $\dot{\phi}/H \propto \Omega_\phi^{3/2}$, which increases as $\Omega_\phi$ increases.

To evaluate $V(\phi)$ we also need an analytic expression of $H(a(\phi))$. From the Friedmann constraint, $\Omega_\phi + \Omega_m = 1$, we obtain

$$1 - \Omega_\phi = \frac{\rho_m}{3H^2m_P^2} = \frac{\rho_m}{3H^2m_P^2a^3H^2} \equiv \frac{\Omega_m H_0}{a^3 H^2} \equiv \Omega_m^0$$

$$\Rightarrow \quad H(a) = H_0 \left( \frac{\Omega_m^0}{(1 - \Omega_\phi)a^3} \right)^{1/2}, \quad (45)$$

where for $1 > \Omega_\phi > 0$. From eqs. (43), we then find

$$V(\phi) = \frac{3}{2} \Omega_m^0 m_P^2 H_0^2 \sqrt{\Omega_\phi (1 - w_\phi)} \frac{(1 - w_\phi)}{(1 - \Omega_\phi)a^3}, \quad \dot{\phi} = \sqrt{\Omega_m^0 m_P H_0} \sqrt{3 \Omega_\phi (1 + w_\phi)} \frac{(1 - \Omega_\phi)}{(1 - \Omega_\phi)a^3}. \quad (46)$$

We plot these quantities in Fig. 6. The left plot in Fig. 6 shows that $\Omega_\phi$ tends to increase the potential while a growth in scale factor tends to decrease it. Using the relation $\dot{\phi}/H \propto \Omega_\phi^{3/2}$, we find that the potential is a slowly increasing exponential function of $\phi$. Thus it is not surprising that the agegraphic quintessence model draws a parallel with the simplest solution of an exponential potential $V(\phi) \propto e^{-\sqrt{\frac{2}{\lambda}}(\phi/m_P)}$, i.e. $\phi/m_P = (\sqrt{2}/\lambda)\ln(t + t_1)$ and $\rho_\phi \equiv \frac{1}{2} \dot{\phi}^2 + V(\phi) \propto m_P^2/(t + t_1)^2$.

In the non-minimal coupling case, the energy conservation equations can be modified as

$$\frac{d\rho_m}{da} + \frac{3}{a}\rho_m = +\dot{\alpha}\rho_m, \quad \frac{d\rho_\phi}{da} + \frac{3}{a}\rho_\phi(1 + w_\phi) = -\dot{\alpha}\rho_m, \quad (47)$$

where $\dot{\alpha} \equiv -Q(\beta(\phi)\frac{d(\kappa\phi)}{da} = -\frac{3\beta\phi}{a^2} + Q \equiv d\ln \beta(\phi)/d(\kappa\phi)$ (see Appendix A) and we have taken $w_m = 0$. The local gravity experiments provide some constrains on the value of $Q$ [24] and, presumably, also on $\dot{\alpha}$. In the particular case that $\dot{\alpha} \simeq \text{const}$, or $\beta(\phi) \propto a(\phi) + \beta_0$, we get

$$\rho_m(a) = \frac{\rho_m^0}{a^3} e^{-\dot{\alpha}a}, \quad \rho_\phi = \left( \rho_\phi^0 - \dot{\alpha} \int \rho_m \exp \left[ 3 \int \frac{1 + w_\phi(a)}{a} da \right] da \right) \exp \left[ -3 \int \frac{1 + w_\phi(a)}{a} da \right]. \quad (48)$$

The above two equations can be inverted to give

$$w_\phi(a) = -1 - \frac{a}{3} \frac{d\ln \rho_\phi}{da} - \frac{\dot{\alpha}\rho_m a}{3\rho_\phi}. \quad (49)$$

It is interesting to note that, for $\dot{\alpha} > 0$, the dark energy equation of state becomes more negative as compared to the $\dot{\alpha} = 0$ case. It is also plausible that $w_\phi(a) < -1$, if $\dot{\alpha} \gtrsim O(1)$ is allowed.

\footnote{More precisely, $|Q| < 0.1778$ or $Q^2 = (1 - \gamma)/(1 + \gamma) < 0.0313$, or equivalently $|1 - \gamma| < 2 \times 10^{-3}$, where $\gamma$ is the PPN parameter.}
Although it may not be essential, one can modify the conservation equations, for example, as
\[
\frac{d\rho_m}{da} + \frac{3}{a}\rho_m = +\tilde{\alpha}\rho_\phi, \quad \frac{d\rho_\phi}{da} + \frac{3}{a}(1 + w_\phi)\rho_\phi = -\tilde{\alpha}\rho_\phi,
\]
in which case
\[
\rho_m = \frac{\rho_0}{a^3} + \frac{\tilde{\alpha}}{a^3} \int \rho_\phi a^3 da, \quad w_\phi(a) = -1 - a\left(\frac{d\rho_\phi/da}{3\rho_\phi}\right) - \frac{a\tilde{\alpha}}{3}.
\]
Now, the last term in the expression of \(w_\phi\) does not depend on the ratio \(\rho_m/\rho_\phi\), but only on the product \(\tilde{\alpha}a\), which can therefore be negligibly small in the early universe, where \(a \ll 1\).

Finally, as one more alternative, let us suppose that \(\beta(\phi) \propto \ln a(\phi) + \beta_0\). This implies
\[
\tilde{\beta} \equiv a\tilde{\alpha} = -a\frac{d(\kappa\phi)}{da}\left(\frac{d\beta}{d(\kappa\phi)}\right) \equiv \text{const.}
\]
Further, as a phenomenological input, following [8], we assume that
\[
\rho_\phi = \frac{3n^2m_p^2}{t^2}, \quad t \equiv \int_0^a \frac{da}{H a}
\]
where \(t > 0\). The parameters \(\epsilon\) and \(w_q\) of the agegraphic quintessence are now given by
\[
\epsilon = -\frac{3}{2}(1 - \Omega_\phi) + \frac{\Omega_\phi^{3/2}}{n} - \frac{\tilde{\beta}}{2}(1 - \Omega_\phi), \quad w_\phi = -1 + \frac{2}{3n}\sqrt{\Omega_\phi} - \frac{(1 - \Omega_\phi)\tilde{\beta}}{3\Omega_\phi},
\]
where \(1 > \Omega_\phi > 0\). To reconstruct an agegraphic quintessence potential, we now clearly need an extra input, which is the value of the coupling \(\tilde{\beta}\). With a reasonable choice of the coupling, say \(\tilde{\beta} \lesssim 0.8\), we find that the shape of the potential \(V(\phi)\) is qualitatively similar to that shown in Fig. [6]. But we find some other differences (as compared to the \(\tilde{\beta} = 0\) case); notably, the universe can accelerate even if \(n \sim O(1)\), and the normalised potential \(U(\phi)\) may not vanish at \(\Phi = 0\) (cf. Fig. [7]).

We conclude the Letter with some remarks.

The definition \(\Phi\), which is, in fact, the central premise of the agegraphic dark energy proposal, reveals the possibility that the dark energy density, or gravitational vacuum energy, at late times is approximated by \(\rho_{DE} \propto t_p^{-2}t_0^{-2} \sim m_p^2H_0^2\), where \(t_0\) is mapped to a linear size of the maximum observable patch of the universe and \(H_0\) is the present value of the Hubble expansion rate. The form of the agegraphic dark energy as presented in Eq. \(\Phi\) is problematic if the present universe consists of only matter and this “dark energy”, possibly for two reasons. One of which is that one might need a variable \(n\) in order to reconcile the model with the early universe as well as with dark energy dominance at late times. The other is that the matter energy density fraction may exhibit some unusual behavior in the limit \(\Omega_q \to 0\). However, both these shortfalls may be overcome by modifying the ansatz \(\Phi\), as in Eq. \(\Phi\), and then considering a nonzero radiation component in the early universe, or in the limit \(\Omega_q \to 0\). It is interesting to note that eqn. \(\Phi\) is valid with an
arbitrary rescaling in the definition of agegraphic time $t$, i.e. for both definitions $\rho_q \propto n^2/t^2$ and $\rho_q \propto n^2/(t+\delta)^2$. The extra parameter $\delta$ is a kind of mixed blessing, which should be nonzero in order to satisfy BBN constraints.

For some phenomenologically motivated solutions, like $a \propto t^m$ (where $m = 2/3$ during matter dominance and $m > 1$ during dark energy dominance), the matter energy density $\rho_m$ could be varying as $\rho_m \propto 1/a^3 \propto 1/t^2$ and $\rho_m \propto 1/t^{3m} \ll 1/t^2 \sim \rho_q$, respectively, during the matter and dark energy dominated epochs. Thus, for a suitable choice of $n$, the “agegraphic” dark energy density may exceed the matter energy density (at late times), leading to a regime of dark energy dominance.

We have shown that in the case of a non-minimal coupling between the $q$-field and matter, the model proposed in [8] can be adjusted to present-day dark energy parameters $\Omega_q \simeq 0.73$ and $w_q \simeq -1$, by allowing a relatively large coupling between the $q$-field and (dark) matter. Although the model does not explain much about the dynamics or the origin of dark energy, it provides an interesting kinematic approach to dark energy equation of state by outlining a possible time growth of dark energy component (at late times). The model naturally predicts an interesting value for the dark energy equation of state, which is $-1 \leq w_q < -1/3$ in the minimal coupling case. It can be hoped that future cosmological observations will provide new constraints on this model, via a more precise measurement of the dark energy equation of state, which is currently constrained to be $-1.38 < w_q < -0.82$ at zero redshift. The model deserves further investigations, especially, in the case of a non-minimal interaction between the $q$-field and (dark) matter.

**Note added:** After the first submission of this Letter to the archive, there have appeared some generalisations of the original agegraphic dark energy model, including the $w$–$w'$ phase-space
analysis [25], the study of instability of agegraphic dark energy [26] and reconstructions of agegraphic quintessence models [27].

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Appendix A

Here we write the matter Lagrangian $\mathcal{L}_m$ in a general form [22]:

$$\mathcal{L}_m \equiv \mathcal{L}(\beta^2(\phi)g_{\mu\nu}, \psi_m) = \sqrt{-g} \beta^4(\phi) \sum \rho_i,$$

where $\psi_m$ denotes collectively the matter degrees of freedom and $\beta(q)$ is a general function of $q$. The radiation term $\rho_r$ ($i = r$) does not contribute to the effective potential or the Klein-Gordon equation. As a result, the effect of the coupling $\beta(\phi)$ can be negligibly small during the epoch where $(\rho_m \ll \rho_r)$. However, as explained in [23], the coupling $\beta(\phi)$ between the dynamical field $\phi$ and the matter can be relevant especially in a background where $\rho_m \gtrsim \rho_r$ (see, for example, Refs. [24]).

Einstein’s equations following from Eqs. (31) and (A.1) are

$$3H^2 = \kappa^2 \left(\frac{1}{2} \dot{\phi}^2 + V(\phi) + \beta^4 (\rho_m + \rho_r)\right),$$

$$-2\dot{H} = \kappa^2 \left(\dot{\phi}^2 + \beta^4 (1 + w_m) \rho_m + \frac{4}{3} \beta^4 \rho_r\right),$$

where $w_i \equiv p_i/\rho_i$ and $\rho_i \propto (a\beta)^{-3(1+w_i)}$. The equation of motion for $\phi$ is

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{dV(\phi)}{d\phi} + \eta Q \beta^4 \rho_i,$$

and the fluid equation of motion for matter ($m$) or radiation ($r$) is:

$$\dot{\rho}_i + 3H \rho_i (1 + w_i) = -\dot{\phi} \eta Q \beta^4 \rho_i, \quad (i = m, r),$$

where $\eta \equiv (1 - 3w_i)$ and $Q \equiv \frac{d\ln \beta(\phi)}{d(\kappa \phi)}$. Eq. (A.4) can be written as

$$\dot{\rho}_\phi + 3H \rho_\phi (1 + w_\phi) = \dot{\phi} \eta Q \beta^4 \rho_m,$$

where $w_\phi \equiv p_\phi/\rho_\phi$, $\rho_\phi \equiv \frac{1}{2} \dot{\phi}^2 + V(\phi)$ and $p_\phi \equiv \frac{1}{2} \dot{\phi}^2 - V(\phi)$. This last equation along with (A.5) guarantees the conservation of total energy: $\dot{\rho}_{tot} + 3H(\rho_{tot} + p_{tot}) = 0$, where $\rho_{tot} = \rho_m + \rho_r + \rho_\phi$.

Using the following definitions

$$\varepsilon \equiv \frac{\dot{H}}{H^2}, \quad \Omega_i \equiv \kappa^2 \frac{\beta^4 \rho_i}{3H^2}, \quad \Omega_\phi = \kappa^2 \frac{\rho_\phi}{3H^2},$$

we arrive at the system of equations (38)-(40).
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