CPA-laser effect and exceptional points in $\mathcal{PT}$-symmetric multilayer structures

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Abstract

The simultaneous existence of coherent perfect absorption (CPA) and lasing is one of the most intriguing features of non-Hermitian photonics. However, the link between CPA lasing and $\mathcal{PT}$ symmetry breaking at the exceptional point (EP) needs clarification. In this paper, we study the manifestations of the CPA-laser effect in a $\mathcal{PT}$-symmetric multilayer loss-gain structure using both the transfer-matrix method and numerical simulations of the Maxwell-Bloch equations. We show that the maximal contrast between absorption and amplification at different phase relations between the input waves is reached well below the EP and therefore is not connected to true lasing. In this regime, there is a good qualitative agreement between both computational approaches. Above the EP, the system demonstrates lasing regardless of the parameters of the input waves. Thus, the maximal contrast between the absorption and amplification rather corresponds to the CPA amplifier than to the CPA laser.

Keywords: $\mathcal{PT}$ symmetric optical systems, coherent perfect absorption, light amplification, multilayer structure

(Some figures may appear in colour only in the online journal)

1. Introduction

Non-Hermitian optics is a remarkable concept allowing us to look at loss and gain in optical systems from a different, somewhat unexpected point of view. This concept has turned out to be extremely fruitful in active photonics and generated a multitude of effects in many systems. One of the most popular implementations of optical non-Hermiticity is to use the so-called $\mathcal{PT}$-symmetric structures characterized by the permittivity distribution invariant with respect to both parity and time inversion [1–3]. Not pretending to name all of the interesting properties of such systems, we mention the observation in $\mathcal{PT}$-symmetric structures of the asymmetric light transmission and beam power oscillations [4], anisotropic transmission resonances [5], unidirectional ‘invisibility’ [6], negative refraction and focusing [7], topologically protected bound states [8], light stopping [9], Talbot effect [10, 11], etc.

Among many publications on optical $\mathcal{PT}$ symmetry, perhaps the most intriguing are those devoted to the simultaneous existence of lasing and antilasing in such systems. The idea of an antilaser, or coherent perfect absorber (CPA), was proposed in 2010 by Chong et al [12]. The CPA considered as a time-reversed version of a laser is based on using both absorption (instead of gain) and interference to trap the incident radiation. The idea was soon generalized by Longhi [13] who showed that both CPA and laser can be realized in the same $\mathcal{PT}$-symmetric multilayer containing balanced loss and gain slabs. In fact, the interference plays an important part in this case, as well as allowing either to fully absorb two incoming waves or to generate two outgoing waves. As demonstrated experimentally [14, 15], the key parameter of this scheme is the phase difference $\Delta \phi$ between the waves incident on the $\mathcal{PT}$-symmetric multilayer containing balanced loss and gain slabs. Later developments in this field allowed us to demonstrate the CPA-laser effect in other types of $\mathcal{PT}$-symmetric structures,
such as microrings [16], plasmonic cavity [17], coupled resonators [18], and graphene-containing multilayers [19]. The existence of laser-absorber modes was also connected to the broken-symmetry phase which exists above the exceptional point [20].

We should note that the strict \( PT \) symmetry is not a necessary condition for the lasing/antilasing effect. Other loss-gain profiles are also possible, if they provide proper distribution of loss-gain and interference of electromagnetic field resulting in enhanced absorption or amplification. The examples of such non-\( PT \)-symmetric CPA-lasers include the so-called zero-index media [21], purely imaginary metamaterials [22], and systems with the generalized PT symmetry [23].

Although there are many studies of the CPA-laser effect, it is still not clear how it is connected to the exceptional point (EP) of the structure, i.e. the parameter set at which \( PT \)-symmetry breaking occurs. It is worth noting that there are different definitions of the EP according to different notations of the scattering matrix. One of the definitions implies that the EP coincides with the point of unitary transmission [6]. In this case, the CPA-laser effect is perhaps not connected to the EP position [14]. However, this definition is problematic as shown by Ge et al [5] who advanced another one and demonstrated its advantages in explanation of the symmetry-breaking conditions. Using this definition, it was shown that the CPA laser can be observed above the EP [20]. On the other hand, the experimental verification of the CPA lasing was performed just below the EP [15]. Further, we adopt Ge et al’s definition of the EP. It is also indirectly supported by our recent calculations in the framework of resonant loss and gain [24].

In this paper, we apply the resonant description of both loss and gain [24] to the analysis of coherent absorption and amplification in \( PT \)-symmetric multilayers. This approach based on numerical simulations of the Maxwell-Bloch equations allows us to self-consistently describe dynamics of both light field and two-level loss-gain media. Using both this approach and the stationary transfer matrix method, we analyze the conditions for the CPA-laser effect in the multilayer illuminated from both sides by the counter-propagating plane monochromatic waves. For this geometry, which corresponds to the experimentally studied one [15], we calculate the output coefficient and the contrast ratio between the absorption and amplification and study their dependence on the phase difference and amplitude ratio of the waves. We assume that the CPA-laser effect corresponds to the maximal contrast between absorption and amplification under changing phase difference and leaving the other parameters (such as loss-gain level) unaltered.

Our aim is to find out whether the optimal contrast between the absorption and amplification can be associated with the EP (in our case, it is the value of pump, or imaginary part of the permittivity, where the \( PT \) symmetry gets broken). We show that the maximal contrast can be reached well below the exceptional point and hence does not require the breaking of \( PT \) symmetry. Above the phase transition point, lasing occurs for any phase of the input waves. Our results mean that one cannot directly associate the conditions of maximal contrast between absorption and amplification regimes with the EP and hence lasing per se. It is true that there is a possibility to reach both the CPA effect and lasing in the same \( PT \)-symmetric structure, but these effects can be reached at different levels of pump. If we take the same pumping and change only the phase difference between the incoming waves, then the maximal contrast can be reached well below the EP. Thus, the maximal contrast rather corresponds to switching between absorption and amplification, not lasing, which can be reached only above the EP.

2. Theoretical description of resonant loss and gain

As proposed in our recent paper [24], we describe both loss and gain as a homogeneously-broadened two-level medium. Then, the Maxwell-Bloch equations for the microscopic polarization amplitude \( \mu \), population difference of ground and excited states \( w \) and electric field amplitude \( A \) can be written as [25]

\[
\frac{d\rho}{d\tau} = i\Omega w + i\rho\delta - \gamma_2 \rho, \tag{1}
\]

\[
\frac{dw}{d\tau} = 2i(\rho^2\Omega^2 - \rho^2\Omega - \gamma_1(w - w_{eq})), \tag{2}
\]

\[
\frac{\partial^2 \Omega}{\partial \xi^2} = \frac{n_d^2}{\omega} \frac{\partial^2 \Omega}{\partial \xi^2} + 2i\frac{\partial \Omega}{\partial \xi} + 2\frac{\partial^2 \Omega}{\partial \xi \partial \tau} + (n_d^2 - 1)\Omega
\]

\[
= 3\alpha \Omega \left( \frac{\partial^2 \rho}{\partial \tau^2} - 2\frac{\partial \rho}{\partial \tau} - \rho \right), \tag{3}
\]

where \( \tau = \omega t \) and \( \xi = k z \) are respectively the dimensionless time and distance, \( \Omega = (\mu / \hbar \omega) \) is the normalized Rabi frequency, \( k = \omega / c \) is the wavenumber in vacuum, \( c \) is the speed of light, \( h \) is the reduced Planck constant, \( \mu \) is the dipole moment of the quantum transition, and \( \delta = (\omega_0 - \omega) / \omega \) is the detuning of the light frequency \( \omega \) from the resonance frequency \( \omega_0 \). The dimensionless parameter \( \alpha = \omega_L / \omega = 4\pi \mu^2 C^2 / 3\hbar \omega \) is the strength of light-matter coupling, where \( \omega_L \) is the Lorentz frequency and \( C \) is the concentration of two-level particles. The normalized relaxation rates of population \( \gamma_1 = 1 / (\omega T_1) \) and polarization \( \gamma_2 = 1 / (\omega T_2) \) are expressed by means of the longitudinal \( T_1 \) and transverse \( T_2 \) relaxation times. The local-field enhancement factor \( l = (n_d^2 + 2) / 3 \) takes into account the influence of the polarization of the host dielectric with real-valued refractive index \( n_d \) on the embedded active particles [26, 27].

In the stationary approximation, one can obtain the effective permittivity of a two-level medium. At the exact resonance \( \delta = 0 \) (this condition holds throughout the paper) and for low-intensity external radiation, \( |\Omega| \ll \Omega_{sat} = \sqrt{\gamma_1 (\gamma_2^2 + \delta^2) / 4\Omega^2 \gamma_2} \), the final expression is [28]

\[
\varepsilon_{eff} \approx n_d^2 + 3i\Omega^2 \omega_L T_2 w_{eq}.
\]

From this equation, one can easily see that the equilibrium population difference \( w_{eq} \) is the key parameter, which allows us to describe both gain and loss materials with the
Maxwell-Bloch equations (1)–(3). The value and sign of this parameter is governed by the external pump and, therefore, it can be called a pumping parameter. Indeed, when it is positive, we have the case of absorbing medium corresponding to low pumping. On the contrary, if \( w_{\text{eq}} \) is negative, this is the case of gain medium with strong external pumping. The negativity of the equilibrium population difference in equation (2) means that the external excitation tends to invert the medium and place more particles to the excited level than are on the ground one. As is well-known, the pumping cannot be fully described in the framework of the two-level model and requires consideration of other levels of the quantum particles. However, since we do not deal with the pumping processes (such as pump depletion), the two-level approximation with the phenomenological account of pumping is enough for calculation of light propagation through the medium with gain already created on the transition between the two levels of interest. The two-level approach to amplifying media is well-known in laser physics [29], including the use of the \( w_{\text{eq}} \)-like values to take pump into account [30, 31].

As shown in [24], it is straightforward to compose a \( \mathcal{PT} \)-symmetric structure from alternating layers with balanced loss (\( \varepsilon_{\text{eff}} \)) and gain (\( \varepsilon_{\text{eff}} \)), where

\[
\varepsilon_{\text{eff}} \approx n_d^2 \pm 3i\omega_l T_2|w_{\text{eq}}|.
\]

Since the magnitude of the pumping parameter is the same for loss and gain layers, the necessary condition for \( \mathcal{PT} \) symmetry \( \varepsilon(z) = \varepsilon^*(-z) \) is fulfilled, providing even (odd) function of \( z \) for the real (imaginary) part of the permittivity.

Further, we first employ the transfer-matrix method (TMM) with equation (5) for the permittivities of loss and gain layers to obtain the main conditions for a CPA-laser. Then we compare the TMM results with the numerical simulations of the full set of equations (1)–(3) which are solved with the finite-difference approach developed in our previous publication [32]. As an initial value of the population difference, we employ its equilibrium value, i.e. \( \omega(t = 0) = w_{\text{eq}} \).

In this paper, we use semiconductor doped with quantum dots as an active material with the following parameters [33, 34]: \( n_d = 3.4, \omega_l = 10^{11} \text{s}^{-1}, T_1 = 1 \text{ns}, \) and \( T_2 = 0.5 \text{ps} \). The estimation of the gain coefficient \( g = 4\pi \text{Im}(\sqrt{\varepsilon_{\text{eff}}})/\lambda \lesssim 10^4 \text{cm}^{-1} \) for \( \lambda \sim 1.5 \mu m \) and \( |w_{\text{eq}}| \lesssim 0.2 \) shows that it can be realized in practice [35]. This choice of materials is not unique, since the multilayer parameters and light wavelength can be easily adjusted to obtain similar results with different materials. The multilayer structure contains \( N = 20 \) unit cells with both loss and gain layers having the same thickness \( d = 1 \mu m \).

### 3. CPA laser via transfer-matrix calculations

The scheme of the one-dimensional loss-gain multilayer is shown in figure 1. To excite the gain layers, one can employ the side pumping scheme similar to that realized in [15]. It was shown previously [24] that the structure with the parameters given above demonstrates the characteristic features of \( \mathcal{PT} \)-symmetric system, such as anisotropic transmission resonances and symmetry-breaking phase transition (at \( |w_{\text{eq}}| > 0.22 \)). Those results were obtained for the single input monochromatic wave with \( \lambda = 1.513 \mu m \). For the CPA-laser effect, it is of fundamental importance to have two coherent input waves from both sides of the structure, since the key ingredient is the interference between the phase-shifted waves. In this section, we analyze the main conditions for CPA-laser by using TMM calculations with the stationary permittivities equation (5).

Both incident waves are assumed to have the same wavelength \( \lambda = 1.513 \mu m \) and are shifted in phase by \( \Delta \phi \), so that the ratio of field strengths for right- and left-incident fields is given by \( E_{\text{right}}/E_{\text{left}} = \sigma e^{i\Delta \phi} \), where \( \sigma \) is the real number showing the ratio of field amplitudes. This means that the total input intensity is \( I = I_L + I_R = I_L(1 + \sigma^2) \). The left-output field is formed as a sum of reflection of the left-incident wave \( r_L \) and transmission of the right-incident one \( t_R \). The analogous condition is valid for the right-output field. The amplitude reflection and transmission coefficients can be easily expressed through corresponding elements of the structure’s transfer matrix, so that we can write the formulas for the output intensities as follows:

\[
O_L = |r_L + t_R|^2 = I_L \left| M_{21} + |M_1|\sigma e^{i\Delta \phi} \right|^2 = I_L \Phi_L, \quad (6)
\]

\[
O_R = |r_L + t_R|^2 = I_L \left| 1 - |M_{12}|\sigma e^{i\Delta \phi} \right|^2 = I_L \Phi_R. \quad (7)
\]

The transfer matrix of a multilayer structure \( M \) can be obtained in a standard way, see e.g. [36]. Since we deal with the case of normal incidence, the transfer matrix of the multilayer can be represented in the especially simple form [37],

\[
M = \Delta_{01}(\Pi_1 \Delta_{12} \Pi_2 \Delta_{21})^{N} \Delta_{00},
\]

where the matrices \( \Delta_{lm} \) and \( \Pi_l \) describe the reflection and propagation of light, respectively.

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**Figure 1.** Schematic of the multilayered \( \mathcal{PT} \)-symmetric structure under consideration. Blue color indicates loss layers, whereas red color is for gain ones.
is the axis. 

\( \Delta \) 

Figure 2. Dependence of the output coefficient \( \Theta \) on the phase difference \( \Delta \varphi \). Other parameters: \(|w_{eq}| = 0.2, \sigma = 1\).

\[ \Delta_{lm} = \left\{ \begin{array}{ll} \delta_{lm}^- \delta_{lm}^+ & \delta_{lm}^\pm = \frac{1}{2} \left( 1 \pm \frac{n_l}{n_i} \right), \\ \exp^{-i\alpha/c} & 0 \\ \exp^{i\alpha/c} & \end{array} \right\} \]

Here, \( n_l \) is the refractive index of the \( l \)th layer (\( l = 1, 2 \)), \( n_0 \) is the refractive index of the ambient medium. \(|M|\) is the determinant of the transfer matrix. The total output intensity is \( O = O_L + O_R \).

As a main parameter, we use the output coefficient of the CPA-laser [15]:

\[ \Theta = 2 \frac{O}{I} = \frac{2 \Phi_L + \Phi_R}{1 + \sigma^2}. \]  

A factor 2 means that equation (8) gives the output intensity per one input channel. We search for the conditions, when \( \Theta \) reaches minimum (CPA) and maximum (lasing). The contrast ratio between these maxima and minima

\[ R = \Theta_{\text{max}} / \Theta_{\text{min}} \]  

demonstrated in figure 3. \( \Theta_{\text{max}} \) increases with growing pump and reaches the maximum \( |w_{eq}| \approx 0.23 \) (just above the EP), whereas \( \Theta_{\text{min}} \) decreases, has the minimal value \( |w_{eq}| \approx 0.20 \) and then rapidly grows approaching the EP. As a result, the peak value of \( R \) (up to about 700) occurs at the same \( |w_{eq}| \approx 0.20 \), where the dip of \( \Theta_{\text{min}} \) occurs. This means that the optimal (from the contrast maximization point of view) value of pumping is reached significantly below the EP and cannot be attributed simply to the effects of \( \mathcal{P} \mathcal{F} \) symmetry breaking (such as the onset of lasing [24]).

Similar analysis can be performed for the dependence on the amplitude ratio \( \sigma \) shown in figure 4 (at \(|w_{eq}| = 0.2\)). \( \Theta_{\text{max}} \) has very weak dependence on \( \sigma \), therefore the contrast \( R \) is fully determined by the behavior of \( \Theta_{\text{min}} \). As could be expected, the maximum of \( R \) corresponds to the symmetric situation of two waves with equal amplitudes \( (\sigma = 1) \). It is also worth mentioning that the asymmetry \( (\sigma \neq 1) \) does not change the phase dependence shown in figure 2, but shifts the position of \( R \) maximum and the \( \Theta_{\text{min}} \) dip along the \(|w_{eq}| \) axis.
This can be viewed as an instrument for tuning the $R$ peak position with respect to the EP—closer ($\sigma < 1$) or farther ($\sigma > 1$) from it.

Note that the curves for $R$ in figures 3 and 4 have very narrow resonance (this may be not obvious due to the logarithmic scale). The same is true if we plot the spectral dependence changing $\lambda$ and leaving all the parameters of the media unaltered (not shown here). Though this procedure (which gives the peak at our chosen $\lambda = 1.513 \, \mu m$) cannot be strictly justified (one should take into account the linewidth of the particles resonance as well), it allows us to feel the importance of the subtle match between the structure geometry and the wavelength to observe the optimal CPA-laser effect.

4. CPA laser via simulations of the Maxwell-Bloch equations

In this section, we compare the stationary analysis given above with the full numerical simulations of the
Maxwell-Bloch equations. As previously, we take two counter-propagating waves of the same intensity ($\sigma = 1$, the absolute amplitude is $I_0 = 10^{-5} \text{w}$) and the phase difference $\Delta \varphi$. The dynamics of output intensities calculated for $|w_{0j}| = 0.2$ is shown in figure 5(a). One can see the rapid establishment of the stationary level of the output radiation. These dynamics are characteristic for the $\mathcal{PT}$-symmetric state. With respect to the single-wave case, the stationary output intensities are much greater for $\Delta \varphi = -\pi/2$ and much smaller for $\Delta \varphi = \pi/2$. This is in qualitative conformity with the discussion in the previous section. However, the results do no coincide quantitatively. One can see this in figure 6(a). Although the output intensity for both $\Delta \varphi = -\pi/2$ and $\Delta \varphi = \pi/2$ behaves similar to the curves in figure 3, the maximum in the first case is not so high and the minimum in the second one is not so deep. In addition, the minimum is reached at $|w_{0j}| = 0.16$, not at 0.20 as in figure 3.

We stop in figure 6(a) at the pumping parameter $|w_{0j}| = 0.23$, since above this value the system jumps into the broken-symmetry state. In this latter state, the system generates powerful light pulses as shown in figure 5(b) for $|w_{0j}| = 0.24$. One can see that the phase difference between the incident waves does not influence the intensity of this lasing pulses. Only the time of pulse appearance can be controlled with $\Delta \varphi$. This implies that the CPA-laser effect should be searched for only below the EP.

The contrast ratios shown in figure 6(b) corroborate that the Maxwell-Bloch simulations strongly underestimate the value of $R$ (only about 22) in comparison to the calculations within TMM ($R \approx 700$). The possible reason is the narrowness of the resonance pointed out in the end of the previous section. However, the results do not coincide quantitatively. Nevertheless, the position of the contrast-ratio peak ($|w_{0j}| = 0.20$) is identical, according to both approaches which can be considered as complementary.

Finally, in figure 7, we plot the phase dependence of the output intensity obtained via the Maxwell-Bloch simulations at $|w_{0j}| = 0.20$. Perhaps due to the reasons discussed above, the dependence is shifted in comparison to the analogous relationship obtained within TMM (figure 2); the minimum is observed here at $\Delta \varphi = 0.4 \pi$, whereas the maximum is at $\Delta \varphi = -0.6 \pi$. Taking this into account and calculating the contrast ratio for these shifted phase differences, we have $R = \Theta_{\text{max}}/\Theta_{\text{min}} = \Theta(-0.6 \pi)/\Theta(0.4 \pi) \approx 230$, which is much greater than only 22 reported in figure 6(b) and better corresponds to the TMM values.

5. Conclusion

In this paper, we have analyzed the conditions for the CPA-laser effect in the $\mathcal{PT}$-symmetric multilayer structure with resonant loss and gain illuminated by two counter-propagating waves. We employed two methods—the standard transfer-matrix method in the steady-state approximation and the numerical simulations of the full set of the Maxwell-Bloch equations. The results (the pump- and phase-dependencies of the output coefficient and the contrast ratios of the maximal and minimal outputs) given by both methods are in good correspondence, in particular, the position of the contrast-ratio peak is reliably determined. The quantitative discrepancy between the approaches is perhaps due to the narrow spectral resonance and the proximity to the EP. We should emphasize that according to our calculations, the maximum of the contrast ratio in the case of equal-amplitude incident waves is located well below the EP and hence does not require $\mathcal{PT}$ symmetry breaking and lasing per se. Therefore, it would be more correct to say about CPA-amplifier, but not CPA-laser in these conditions. We believe that our results will be helpful to clarify the properties of $\mathcal{PT}$-symmetric or similar loss-gain structures.

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