Hierarchical Expert Networks for Meta-Learning

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Abstract

The goal of meta-learning is to train a model on a variety of learning tasks, such that it can adapt to new problems within only a few iterations. Here we propose a principled information-theoretic model that optimally partitions the underlying problem space such that the resulting partitions are processed by specialized expert decision-makers. To drive this specialization we impose the same kind of information processing constraints both on the partitioning and the expert decision-makers. We argue that this specialization leads to efficient adaptation to new tasks. To demonstrate the generality of our approach we evaluate on three meta-learning domains: image classification, regression, and reinforcement learning.

1 Introduction

Recent machine learning research has shown impressive results on incredibly diverse tasks from problem classes such as pattern recognition, reinforcement learning, and generative model learning [Devlin et al., 2018, Mnih et al., 2015, Schmidhuber, 2015]. These success stories typically have two computational luxuries in common: a large data base with thousands or even millions of training samples and a very long and extensive training period. However, applying these pre-trained models to new tasks naively usually leads to very poor performance, as with each new incoming batch of data, expensive and slow re-learning is required. In contrast to this, humans are able to learn from very few examples and excel at adapting quickly [Jankowski et al., 2011], for example in motor tasks [Braun et al., 2009] or at learning new visual concepts [Lake et al., 2015].

Sample-efficient adaptation to new tasks can be regarded as a form of meta-learning or “learning to learn” [Thrun and Pratt, 2012, Schmidhuber et al., 1997, Caruana, 1997] and is an ongoing and active field of research—see e.g. [Koch et al., 2015, Vinyals et al., 2016, Finn et al., 2017, Ravi and Larochele, 2017, Ortega et al., 2019, Botvinick et al., 2019, Yao et al., 2019]. Meta-learning can be defined in different ways, but a common point is that the system learns on two levels, each with different time scales: slow learning across different tasks on a meta-level, and fast learning to adapt to each task individually.

Here, we propose a novel learning paradigm for hierarchical meta learning systems. Our method finds an optimal soft partitioning of the problem space by imposing information-theoretic constraints on both the process of expert selection and on the expert specialization. We argue that these constraints drive an efficient division of labor in systems that are bounded in their respective information processing power, where we make use of information-theoretic bounded rationality [Ortega and Braun, 2013]. When the model is presented with previously unseen tasks it assigns them to experts specialized on similar tasks – see Figure 1. Additionally, expert networks specializing on only a subset of the problem space allows for smaller neural network architectures with only few units per layer. In order to split the problem space and to assign the partitions to experts, we learn to represent tasks through a common latent embedding, that is then used by a selector network to distribute the tasks to the experts.

The outline of this paper is as follows: first we introduce bounded rationality and meta learning, next we introduce our novel approach and derive applications to classification, regression, and reinforcement learning. Finally, we conclude.

Preprint, work in progress.
2 Background

2.1 Bounded Rational Decision Making

An important concept in decision making is the notion of utility [Von Neumann and Morgenstern, 2007], where an agent picks an action \( a^*_s \in A \) such that it maximizes their utility in some context \( s \in S \), i.e. \( a^*_s = \arg \max_a U(s, a) \), where the utility is given by a function \( U(s, a) \) and the states distribution \( p(s) \) is known and fixed. Trying to solve this optimization problem naively leads to an exhaustive search over all possible \((a, s)\) pairs, which is in general a prohibitive strategy. Instead of finding an optimal strategy, a bounded-rational decision-maker optimally trades off expected utility and the processing costs required to adapt. In this study we consider the information-theoretic free-energy principle [Ortega and Braun, 2013] of bounded rationality, where the decision-maker’s resources are modeled by an upper bound on the Kullback-Leibler divergence \( D_{KL}(p(a|s)||p(a)) = \sum_{a} p(a) \log \frac{p(a|s)}{p(a)} \) between the agent’s prior distribution \( p(a) \) and the posterior policy \( p(a|s) \), resulting in the following constrained optimization problem:

\[
\max_{p(a|s)} \sum_{s,a} p(s)p(a|s)U(s, a)
\text{s.t. } \mathbb{E}_{p(a|s)}[D_{KL}(p(a|s)||p(a))] \leq B.
\]

This constraint can also be interpreted as a regularization on \( p(a|s) \). We can transform this into an unconstrained variational problem by introducing a Lagrange multiplier \( \beta \in \mathbb{R}^+ \):

\[
\max_{p(a|s)} \mathbb{E}_{p(s,a)}[U(s, a)] - \frac{1}{\beta} \mathbb{E}_{p(a|s)}[D_{KL}(p(a|s)||p(a))].
\]

For \( \beta \to \infty \) we recover the maximum utility solution and for \( \beta \to 0 \) the agent can only act according to the prior. The optimal prior in this case is given by the marginal \( p(a) = \sum_{s \in S} p(s)p(a|s) \) [Ortega and Braun, 2013].

2.1.1 Hierarchical Decision Making

Aggregating several bounded-rational agents by a selection policy allows for solving optimization problems that exceed the capabilities of the individual decision-makers [Genewein et al., 2015]. To achieve this, the search space is split into partitions such that each partition can be solved by a decision-maker. A two stage mechanism is introduced: The first stage is an expert selection policy \( p(x|s) \) that chooses an expert \( x \) given a state \( s \) and the second stage chooses an action according to the expert’s posterior policy \( p(a|x, s) \). The optimization problem given by (3) can be extended to incorporate a trade-off between computational costs and utility in both stages:

\[
\max_{p(a|x, s), p(x|s)} \mathbb{E}[U(s, a)] - \frac{1}{\beta_1} I(S; X) - \frac{1}{\beta_2} I(S; A|X)
\]

where \( \beta_1 \) is the resource parameter for the expert selection stage and \( \beta_2 \) for the experts. \( I(\cdot; \cdot) \) is the mutual information between the two random variables. The solution can be found by iterating the following set of equations [Genewein et al., 2015]:

\[
\begin{align*}
    p(x|s) &= \frac{1}{Z(s)} p(x) \exp(\beta_1 \Delta F_{\text{par}}(s, x)) \\
    p(x) &= \sum_s p(s)p(x|s) \\
    p(a|x, s) &= \frac{1}{Z(s,x)} p(a|x) \exp(\beta_2 U(s, a)) \\
    p(a|x) &= \sum_s p(s|x)p(a|s, x)
\end{align*}
\]

where \( Z(s) \) and \( Z(s,x) \) are normalization factors and \( \Delta F_{\text{par}}(s, x) = \mathbb{E}_{p(a|x, s)}[U(s, a)] - \frac{1}{\beta_2} D_{KL}(p(a|x, s)||p(a|x)) \) is the free energy of the action selection stage. Thus the marginal distribution \( p(a|x) \) defines a mixture-of-experts policy given by the posterior distributions \( p(a|s, x) \) weighted by the responsibilities determined by the Bayesian posterior \( p(s|x) \). Note that \( p(s|x) \) is not determined by a given likelihood model, but is the result of the optimization process (1).

2.2 Meta Learning

Meta-learning algorithms can be divided roughly into Metric-Learning [Koch et al., 2015], Vinyals et al., 2016, Snell et al., 2017, Optimizer Learning [Ravi and Larochelle, 2017], Finn et al., 2017, Zintgraf et al., 2018, and Task Decomposition Models [Lan et al., 2019, Vezhnevets et al., 2019]. Our approach depicted in Figures 2 can be seen as a member of the latter group.
2.2.1 Meta Supervised Learning

In a supervised learning task we are usually interested in a dataset consisting of multiple input and output pairs $\mathcal{D} = \{(x_i, y_i)\}_{i=1}^N$ and the learner is tasked with finding a function $f(x)$ that maps from input to output, for example through a deep neural network. To do this, we split the dataset into training and test sets and fit a set of parameters $\theta$ to the training data and evaluate on test data using the learned function $f_\theta(x)$. In meta-learning, we are instead working with meta-datasets $\mathcal{D} \in \mathcal{D}$, each containing regular datasets split into training and test sets. We thus have different meta-sets for meta-training, meta-validation, and meta-test ($\mathcal{D}_{meta-train}, \mathcal{D}_{meta-val}$, and $\mathcal{D}_{meta-test}$, respectively). On $\mathcal{D}_{meta-train}$, we are interested in training a learning procedure (the meta-learner) that can take as input one of its training sets $\mathcal{D}_{train}$ and produce a classifier (the learner) that achieves low prediction error on its corresponding test set $\mathcal{D}_{test}$.

A special case of meta-learning for classification are $K$-Shot $N$-way tasks. In this setting, we are given for each dataset $\mathcal{D}$ a training set consisting of $K$ labeled examples of each of the $N$ classes ($K \cdot N$ examples per dataset) and corresponding test sets. In our study, we focus on the following variation of $K$-Shot 2-Way tasks: the meta-learner is presented with 2$K$ samples ($K$ positive and $K$ negative examples) and must assign this dataset to an expert learner. Note that the negative examples may be drawn from any of the remaining classes.

2.2.2 Meta Reinforcement Learning

We model sequential decision problems by defining a Markov Decision Process as a tuple $(\mathcal{S}, \mathcal{A}, P, r)$, where $\mathcal{S}$ is the set of states, $\mathcal{A}$ the set of actions, $P : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to [0, 1]$ is the transition probability, and $r : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$ is a reward function. The aim is to find the parameter $\theta$ of a policy $\pi_\theta$ that maximizes the expected reward:

$$
\theta^* = \arg \max_{\theta} \mathbb{E}_{\pi_\theta} \left[ \sum_{t=0}^{\infty} r(s_t, a_t) \right].
$$

We define $r(\tau) = \sum_{t=0}^{\infty} r(s_t, a_t)$ as the cumulative reward of trajectory $\tau = \{(s_t, a_t)\}_{t=0}^{\infty}$, which is sampled by acting according to the policy $\pi$, i.e. $(s, a) \sim \pi(\cdot | s)$, and $s_{t+1} \sim P(\cdot | s_t, a_t)$. Learning in this environment can then be modeled by reinforcement learning [Sutton and Barto, 2018], where an agent interacts with an environment over a number of (discrete) time steps $t$. At each time step $t$, the agent finds itself in a state $s_t$ and selects an action $a_t$ according to the policy $\pi(a_t | s_t)$. In return, the environment transitions to the next state $s_{t+1}$ and generates a scalar reward $r_t$. This process continues until the agent reaches a terminal state after which the process restarts. The goal of the agent is to maximize the expected return from each state $s_t$, which is typically defined as the infinite horizon discounted sum of the rewards. A common choice to achieving this is Q-Learning [Watkins and Dayan, 1992], where we make use of an action-value function that is defined as the discounted sum of rewards $Q(\tau) = \sum_{t=0}^{\infty} \gamma^t r(s_t, a_t)$, where $\gamma \in (0, 1)$ is a discount factor. Learning the optimal policy can be achieved in many ways. Here, we consider Policy gradient methods [Sutton et al., 2000] which are a popular choice to tackle continuous reinforcement learning problems. The main idea is to directly manipulate the parameters $\theta$ of the policy in order to maximize the objective $J(\pi_\theta)$ by taking steps in the direction of the gradient $\nabla_\theta J(\pi_\theta)$.

In meta reinforcement learning the problem is given by a set of tasks $t_i \in T$, where each task $t_i$ is defined by an MDP $t_i = (\mathcal{S}, \mathcal{A}, P_i, r_i)$ as described earlier. We are now interested in finding a set of policies $\Theta$ that maximizes the average cumulative reward across all tasks in $T$ and generalizes well to new tasks sampled from a different set of tasks $T'$. 
Algorithm 1 Expert Networks for Supervised Meta-Learning

1: **Input:** Data Distribution \( p(D) \), number of samples \( K \), batch-size \( M \), training episodes \( N \)
2: **Hyper-parameters:** resource parameters \( \beta_1, \beta_2 \), learning rates \( \eta_s, \eta_x \) for selector and experts
3: Initialize parameters \( \theta, \vartheta \)
4: for \( i = 0, 1, 2, ..., N \) do
5: Sample batch of \( M \) datasets \( D_i \sim p(D) \), each consisting of a training dataset \( D_{\text{meta-train}} \) and a meta-validation dataset \( D_{\text{meta-val}} \) with \( 2K \) samples each
6: for \( D \in D_i \) do
7: Find Latent Embedding \( z(D_{\text{meta-train}}) \)
8: Select expert \( x \sim p_0(x|z(D_{\text{meta-train}})) \)
9: Compute \( \Delta F_{D,x} \) of \( x \) on \( D_{\text{meta-val}} \)
10: Update selection parameters \( \theta \) with \( \Delta F_{D,x} \)
11: Update Autoencoder with pos. samples in \( D_i \)
12: Update experts \( x \) with assigned \( D_{\text{meta-train}} \)
13: return \( \theta, \vartheta \)

3 Expert Networks for Meta-Learning

Information-theoretic bounded rationality postulates that hierarchies and abstractions emerge when agents have only limited access computational resources [Genewein et al., 2015]. [Gottwald and Braun, 2019b] [Gottwald and Braun, 2019a] e.g. limited sampling complexity [Hihn et al., 2018] or limited representational power [Hihn et al., 2019]. We will show that forming such abstractions equips an agent with the ability of learning the underlying problem structure and thus enables learning of unseen but similar concepts. The method we propose comes out of a unified optimization principle and has the following important features:

1. A regularization mechanism to enforce the emergence of expert policies.
2. A task compression mechanism to extract relevant task information.
3. A selection mechanism to find the most efficient expert for a given task.
4. A regularization mechanism to improve generalization capabilities.

3.1 Latent Task Embeddings

Note that the selector assigns a complete dataset to an expert and that this can be seen as a meta-learning task, as described in [Ravi and Larochelle, 2017]. To do so, we must find a feature vector \( z(d) \) of the dataset \( d \). This feature vector must fulfill the following desiderata: 1) invariance against permutation of data points in \( d \), 2) high representational capacity, 3) efficient computability, and 4) constant dimensionality regardless of sample size \( K \). In the following we propose such features for image classification, regression, and reinforcement learning problems.

For image classification we propose to pass the positive images in the dataset through a convolutional autoencoder and use the respective outputs of the bottleneck layer. Convolutional Autoencoders are generative models that learn to reconstruct their inputs by minimizing the Mean-Squared-Error between the input and the reconstructed image (see e.g. [Chen et al., 2019]). In this way we get similar embeddings \( z(d) \) for similar inputs belonging to the same class. The latent representation is computed for each positive sample in \( d \) and then passed through a pooling function \( h(z(d)) \) to find a single embedding for the complete dataset—see figure 2 for an overview of our proposed model. While in principle functions such as max, mean, and min can be used, we found that max pooling yields the best results. The authors of [Yao et al., 2019] propose a similar feature set.

For regression we define a similar feature vector. The \( K \) training data points are transformed into a feature vector \( z(d) \) by binning the points into \( N \) bins according to their respective \( x \) value and collecting the respective \( y \) value. If more than one point falls into the same bin the \( y \) values are averaged, thus providing invariance against the order of the data points in \( D_{\text{meta-train}} \). We use this feature vector to assign each data set to an expert according to \( p_0(x|z(d)) \).

In the reinforcement learning setting we use a dynamic recurrent neural network (RNN) with LSTM units [Hochreiter and Schmidhuber, 1997] to classify trajectories. We feed the RNN with \((s_t, a_t, r_t, t)\) tuples to describe the underlying Markov Decision Process describing the task. At \( t = 0 \) we sample the expert \( x \) according to the learned prior distribution \( p(x) \), as there is no information available so far. The authors of [Lan et al., 2019] propose a similar feature set.

3.2 Hierarchical On-line Meta-Learning

As discussed in section 2.1, the aim of the selection network is to find an optimal partition of the experts \( p_0(x|s) \), such that the selector’s expected utility \( \sum_x p_0(x|s) \Delta F_{\text{par}}(s, x) \) is maximized under an information-theoretic constraint \( D_{\text{KL}}(p_0(x|s)||p(x)) \), where \( \theta \) are the selector’s parameters (e.g. weights in a neural network), \( x \) the expert and \( s \) is an input. Each expert \( x \) follows a policy \( p_{\theta}(a|s, x) \) that maximizes their expected utility \( \sum p(s|x) \sum p_{\theta}(a|s, x) U(s, a) \).
We introduce our gradient based on-line learning algorithm to find the optimal partitioning and the expert parameters in the following. Rewriting the optimization problem \[ \text{(4)} \] as

\[
\max_{p_{\theta}(a|x,s), p_{\vartheta}(x|s)} p(s)p_{\theta}(x|s)p_{\vartheta}(a|x) J(s,x,a) \quad (7)
\]

where the objective \( J(s,x,a) \) is given by

\[
J(s,x,a) = U(s,a) - \frac{1}{\beta_1} \log \frac{p_\theta(x|s)}{p(x)} - \frac{1}{\beta_2} \log \frac{p_\vartheta(a|x)}{p(a|x)}, \quad (8)
\]

and \( \vartheta, \theta \) are the parameters of the selection policy and the expert policies, respectively. Note that each expert policy has a distinct set of parameters \( \vartheta_x \), i.e. \( \vartheta = \{ \vartheta_x \} \), but we drop the \( x \) index for readability.

In the following we will show how we can apply this formulation to classification, regression and reinforcement learning.

### 3.2.1 Application to Supervised Learning

Combining multiple experts can often be beneficial [Kuncheva, 2004], e.g. in Mixture-of-Experts [Ynakel et al., 2012] or Multiple Classifier Systems [Belmann et al., 2018]. Our method can be interpreted as a member of this family of algorithms.

In accordance with Section 2.2.1 we define the utility as the negative prediction loss, i.e. \( U(f_x(d), y) = -\mathcal{L}(f_x(d), y) \), where \( f_x(d) \) is the prediction of the expert \( x \) given the input data point \( d \) (in the following we will use the shorthand \( \hat{y}_x \) and \( y \) is the ground truth. We define the cross-entropy loss \( \mathcal{L}(\hat{y}_x, y) = -\sum_i y_i \log \hat{y}_{ix} \) as a performance measure for classification and the mean squared error \( \mathcal{L}(\hat{y}_x, y) = \sum_i (y_i - \hat{y}_i)^2 \) for regression. The objective for expert selection thus is given by

\[
\max_{\vartheta} \mathbb{E}_{p_{\theta}(x|d)} \left[ \frac{f\theta - 1}{\beta_1} \log \frac{p_\theta(x|d)}{p(x)} \right], \quad (9)
\]

where \( \hat{f} = \mathbb{E}_{p_{\theta}(\hat{y}_x|x,s)} \left[ -\mathcal{L}(\hat{y}_x, y) - \frac{1}{\beta_2} \log \frac{p_\vartheta(\hat{y}_x|x)}{p(\hat{y}_x|x)} \right], \) i.e. the free energy of the expert and \( \vartheta, \theta \) are the parameters of the selection policy and the expert policies, respectively. Analogously, the action selection objective for each expert \( x \) is defined by

\[
\max_{\vartheta} \mathbb{E}_{p_{\theta}(\hat{y}_x|x,s)} \left[ -\mathcal{L}(\hat{y}_x, y) - \frac{1}{\beta_2} \log \frac{p_\vartheta(\hat{y}_x|x)}{p(\hat{y}_x|x)} \right]. \quad (10)
\]

### 3.2.2 Application to Reinforcement Learning

In the reinforcement learning setup the utility \( U(s,a) \) is given by the reward function \( r(s,a) \). In maximum entropy RL the regularization penalizes deviation from a fixed uniform prior, but in a more general setting we can discourage deviation from an arbitrary prior policy by determining the optimal policy \( p^*(a|s) \) as

\[
\arg\max_{p} \mathbb{E}_{p} \left[ \sum_{t=0}^{\infty} \gamma^t \left( r(s_t,a_t) - \frac{1}{\beta} \log \frac{p(a_t|s_t)}{p(a_t)} \right) \right]. \quad (11)
\]

As discussed in Section 2.1 the optimal prior is the marginal of the posterior policy given by \( p(a) = \sum_s p(s)p(a|s) \). We approximate the prior distributions \( p(x) \) and \( p(a|x) \) by exponential running mean averages of the posterior policies.

To optimize the objective we define two separate value functions: one to estimate the discounted sum of rewards and one to estimate the free energy of the expert policies. The discounted reward for the experts is \( R_t = \sum_{t=0}^{T} \gamma^t r(s_{t+1},a_{t+1}) \), which we learn by parameterizing the value function with a neural network. Similar to the discounted reward \( R_t \) we can now define the discounted free energy \( F_t \) as \( F_t = \sum_{t=0}^{T} \gamma^t f(s_{t+1},a_{t+1}) \), where \( f(s,x,a) = r(s,a) - \frac{1}{\beta_2} \log \frac{p_\vartheta(a|x)}{p(a|x)} \). The value function \( F_t \) is learned by parameterizing it with a neural network and performing regression on the mean-squared-error.

### 3.2.3 Expert Selection

The selector network learns a policy \( p_{\theta}(x|s) \) that assigns states \( s \) to expert policies \( x \) optimally. The

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### Omniglot Few-Shot Classification Results

| K     | 2     | % Acc | I(X|W) | 4     | % Acc | I(X|W) | 8     | % Acc | I(X|W) | 16    | % Acc | I(X|W) |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1     | 76.2 (± 0.02) | 0.99 (± 0.01) | 86.7 (± 0.02) | 1.96 (± 0.01) | 90.1 (± 0.01) | 2.5 (± 0.20) | 92.9 (± 0.01) | 3.2 (± 0.3) |
| 5     | 67.3 (± 0.01) | 0.93 (± 0.01) | 75.5 (± 0.01) | 1.95 (± 0.10) | 78.4 (± 0.01) | 2.7 (± 0.10) | 81.2 (± 0.01) | 3.3 (± 0.2) |
| 10    | 66.4 (± 0.04) | 0.95 (± 0.30) | 75.8 (± 0.01) | 1.90 (± 0.03) | 77.3 (± 0.01) | 2.8 (± 0.15) | 77.8 (± 0.01) | 3.1 (± 0.2) |

Lake et al., 2011. We evaluate our system by splitting the dataset into training and validation data (80% - 20%) and train the system as described in Algorithm 1 and report the classification accuracy on the validation, i.e. classes and samples that are novel to the model. We trained for 50,000 episodes each with a batch of 32 datasets and set \( \beta_1 = 15 \) and \( \beta_2 = 5 \).
resource parameter $\beta_1$ constrains the information-processing in this step. For $\beta_1 \to 0$ the selection assigns each state completely randomly to an expert, while for $\beta_1 \to \infty$ the selection becomes deterministic, always choosing the most promising expert $x$. The selector optimizes the following objective:

$$\max_{\theta} \mathbb{E}_{p_\theta(s|x)} \left[ \hat{f}(s, x) - \frac{1}{\beta_1} \log \frac{p_\theta(x|s)}{p(x)} \right],$$

(12)

where $\hat{f}(s, x) = \mathbb{E}_{p_\theta(a|s,x)}[f(s, x, a)]$, which is the free energy of the expert. The gradient of $J(\theta)$ is then given (up to an additive constant) by

$$\mathbb{E} \left[ \nabla_\theta \log p_\theta(x|s) \left( \hat{f}(s, x) - \frac{1}{\beta_1} \log \frac{p_\theta(x|s)}{p(x)} \right) \right].$$

(13)

The double expectation can be replaced by Monte Carlo estimates, where in practice we use a single $(s, x, a)$ tuple for $\hat{f}(s, x)$. This formulation is known as the policy gradient method [Sutton et al., 2000] and is prone to producing high variance gradients, but can be reduced by using an advantage function instead of the reward [Schulman et al., 2015]. The advantage function $A(a_t, s_t)$ is a measure of how well a certain action $a$ performs in a state $s$ compared to the average performance in that state, i.e. $A(a, s) = Q(s, a) - V(s)$. Here, $V(s)$ is called the value function and captures the expected cumulative reward when in state $s$, and $Q(s, a)$ is an estimate of the expected cumulative reward achieved in state $s$ when choosing a particular action $a$. Thus the advantage is an estimate of how advantageous it is to pick $a$ in state $s$ in relation to a baseline performance $V(s)$. Instead of learning $V$ and $Q$, we can approximate the advantage function

$$\hat{A}_t^F = \frac{f(s_t, x_t, a_t) + \gamma V_\phi(s_{t+1}) - V_\phi(s_t)}{\approx Q(s_t, a_t)},$$

(14)

such that we can get away with just learning a single value function $V_\phi(s_t)$. Both the selection network and the selector value network are implemented as recurrent neural networks with LSTM cells [Hochreiter and Schmidhuber, 1997]. Both networks share the recurrent cell followed by independent feed forward layers.

3.2.4 Action Selection

The actions $a$ is sampled from the posterior action distribution of the experts. Each expert $x$ maintains a policy for each of the world states $s$ and updates those according to the utility/cost trade-off. The advantage function for each expert is given as

$$\hat{A}_t = r(s_t, a_t) + \gamma V_\phi(s_{t+1}) - V_\phi(s_t).$$

(15)

The objective of this stage is then to maximize the expected advantage $\mathbb{E}_{p_\theta(a|s,x)}[\hat{A}_t]$.

4 Empirical Results

4.1 Sinusoid Regression

We adopt this task from [Finn et al., 2017]. In this $K$-shot problem, each task consists of learning to predict a function of the form $y = a \cdot \sin(x + b)$, with both $a \in [0.1, 5]$ and $b \in [0, 2\pi]$ chosen uniformly, and the goal of the learner is to find $y$ given $x$ based on only $K$ pairs of $(x, y)$. Given that the underlying function changes in each iteration it is impossible to solve this problem with a single learner. Our results show that by combining expert networks, we are able to reduce the generalization error iteratively as we add more experts to our system—see Figures 5 for $K = 5$ and $K = 10$ settings. In Figure 4 we show how the system is able to capture the underlying problem structure as we add more experts and in Figure 3 we visualize how the selector’s partition of the problem space looks like.
Few-Shot Regression

Figure 4: The single expert system is not able to learn the underlying structure of the sine wave, where the two expert system is already able to capture the periodic structure. Adding more experts improves adaption further, as the results show. We trained for 10,000 episodes each with a batch of 32 data sets.

Figure 5: Analogously to the rate-distortion curve in rate-distortion theory [Blahut, 1972] [Arimoto, 1972], we can interpret this curve as the rate-utility showing the trade-off between information processing and expected utility (transparent area represents the standard deviation). Increasing the processing power of the selection stage \( I(W; X) \) (i.e. adding more experts) improves adaption.

4.2 Few-Shot Classification

The Omniglot dataset [Lake et al., 2011] consists of over 1600 characters from 50 alphabets. As each character has merely 20 samples each drawn by a different person, this forms a difficult learning task and is thus often referred to as the "transposed MNIST" dataset. The Omniglot dataset is regarded as a standard meta learning benchmark, see e.g. [Finn et al., 2017] [Vinyals et al., 2016] [Ravi and Larochelle, 2017].

We train the learner on a subset of the dataset (~80%, i.e. ~1300 classes) and evaluate on the remaining ~300 classes, thus investigating the ability to generalize to new samples of the target class and distinguish them from negative examples. Using this optimization scheme, we train the expert networks to become experts in recognizing a subset of classes. After a suitable expert is selected we train that expert using the 2\( K \) samples from the training dataset—see Figure 5 and Table 1 for results. To generate this figure, we ran a 10-fold cross-validation on the whole dataset and show the averaged performance metric and the respective standard-deviation across the folds. In both settings "0 bits" corresponds to a single expert, i.e. a single neural network trained on the task.

4.3 Meta Reinforcement Learning

| Parameter                  | Task Distribution | \( T \)  | \( T' \) |
|----------------------------|-------------------|---------|---------|
| Distance Penalty           | \([10^{-3}, 10^{-1}]\) | \([10^{-3}, 10^{-2}]\) |
| Goal Position              | \([0.3, 0.4]\)     | \([0, 3]\)       |
| Start Position             | \([-0.15, 0.15]\)  | \([-0.25, 0.25]\) |
| Motor Torques              | \([0, 5]\)         | \([0, 3]\)       |
| Motor Actuation            | \([185, 215]\)     | \([175, 225]\)   |
| Inverted Control           | \(p = 0.5\)        | \(p = 0.5\)      |
| Gravity                    | \([0.01, 4.9]\)    | \([4.9, 9.8]\)   |

Table 2: All parameters are sampled uniformly from the specified range for each environment. \( T \) is used for training and \( T' \) for meta evaluation.

We create a set of RL tasks by sampling the parameters for the Inverted Double Pendulum problem [Sutton, 1996] implemented in OpenAI Gym [Brockman et al., 2016]. The task is to balance a two-link pendulum in an upward position. We modify inertia, motor torques, reward function, goal position and invert the control signal—see Table 2 for details. The control signal \( a \) is continuous in the interval [-1,1] is generated by neural network that outputs \( \mu \) and \( \log(\sigma) \) of a gaussian. The action is sampled by re-parameterizing the distribution to \( p(a) = \mu + \exp(\sigma)\epsilon \), where \( \epsilon \sim \mathcal{N}(0, 1) \), so that the distribution is differentiable w.r.t to the network outputs.
The meta task set $T'$ is based on the same environment, but the parameter distribution and range is different, providing new but similar reinforcement learning problems. In each episode $M$ environments are sampled and the system is updated accordingly. After training is concluded the system is evaluated on tasks sampled from $T'$. We trained the system for 1000 Episodes with 64 tasks from $T$ and evaluate for 100 system updates on tasks from $T'$. We report the results in Figure 6, where we can see improving performance as more experts are added and the mutual information in the selection stage indicates that the tasks can be assigned to their respective expert policy.

5 Discussion

We have introduced and evaluated a novel information-theoretic approach to meta learning. In particular we leveraged an information-theoretic approach to bounded rationality [Leibfried et al., 2017, Grau-Moya et al., 2019, Hihn et al., 2019, Schach et al., 2018, Gottwald and Braun, 2019b, Lindig-Leon et al., 2019]. Our results show that our method is able to identify sub-regions of the problem set with expert networks. In effect, this equips the system with several initializations covering the problem space and thus enables it to adapt quickly to new but similar tasks. To reliably identify such tasks, we have proposed feature extraction methods for classification, regression and reinforcement learning, that could be simply be replaced and improved in the future. The strength of our model is that it follows from simple principles that can be applied to a large range of problems. Moreover, the system performance can be interpreted in terms of the information processing of the selection stage and the expert decision-makers.

Most other methods for meta learning such as [Finn et al., 2017] and [Ravi and Larochelle, 2017] try to find a initial parametrization of a single learner, such that it is able to adapt quickly to new problems. This initialization can be interpreted as compression of the most common task properties over all tasks. Our method however learns to identify task properties over a subset of tasks and provide several initializations. Task specific information is thus directly available instead of a delayed availability after several iterations as in [Finn et al., 2017] and [Ravi and Larochelle, 2017]. In principle, this can help to adapt within fewer iterations. Thus our method can be seen as the general case of such monolithic meta-learning algorithms.

Another hierarchical approach to meta-learning is the work of [Yao et al., 2019], where the focus is on learning similarities between completely different problems (e.g. different classification datasets). In this way the portioning is largely governed by the different tasks. Our study however focuses on discovering meta-information within the same task family, where the meta-partitioning is determined solely by the optimization process and can thus potentially discover unknown dynamics and relations within a task family.

Although our method is widely applicable, it suffers from low sample efficiency in the RL domain. An interesting research direction would be to combine our system with model-based RL which is known improve sample efficiency. Another research direction would be to investigate our systems performance in continual adaption tasks, such as in [Yao et al., 2019]. There the system is continuously provided with data sets (e.g. additional classes and samples). Another limitation is the restriction to binary meta classification tasks, which we leave for feature work.

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