The change of the Earth Pole oscillational mode under the Lunar-Solar perturbations

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Abstract. This article investigates conversion between different modes of the Earth Pole oscillational process which influence the precision of the Pole position forecast. It identifies the perturbing harmonic that leads to the saltatory change of the average velocity of Earth Pole motion and to the change of its oscillational mode. It also shows the qualitative conformity between the four-frequency model of the Pole motion and observation data based on the graph of phase variation of the polar angle of the Pole rotation around the average position.

Earlier in [1] we described the method of refinement of the Earth Pole motion model that uses the record of the additional harmonics with the frequencies that are close to Chandler and annual ones. The analysis of the measuring results of the Pole coordinates in the Earth system allows to make a conclusion about existence of specified oscillational process in its motion that is synchronized with the precession of the Lunar orbit. These new properties can also be used to refine the mathematical models of the Pole motion forecast.

The Earth Pole motion model with the additional summands $\Delta x_p$, $\Delta y_p$ obtained in [1] can be submitted in the following form:

$$
\begin{align*}
    x_p &= a_{ch} \cos w_{ch} + a_h \cos w_h + \Delta x_p \\
    y_p &= a_{ch} \sin w_{ch} + a_h \sin w_h + \Delta y_p \\
    \Delta x_p &= a_{ch/h} \cos(w_1 + \Omega) + a_{ch/h} \cos(w_1 - \Omega) + b_{ch/h} \cos(w_2 + \Omega) + b_{ch/h} \cos(w_2 - \Omega) \\
    \Delta y_p &= a_{ch/h} \sin(w_1 + \Omega) + a_{ch/h} \sin(w_1 - \Omega) + b_{ch/h} \sin(w_2 + \Omega) + b_{ch/h} \sin(w_2 - \Omega) \\
    \langle w_1 \rangle_T &= \begin{cases} w_h & \text{если } a_h < a_{ch} \\ w_{ch} & \text{если } a_{ch} < a_h \end{cases} \\
    \langle w_2 \rangle_T &= \begin{cases} w_{ch} & \text{если } a_{ch} < a_h \\ w_h & \text{если } a_h < a_{ch} \end{cases} \\
    \dot{w}_h &= 2\pi \omega, \quad \dot{w}_{ch} = 2\pi N \omega, \quad \Omega = \frac{2\pi \omega}{18.61}
\end{align*}
$$

Here $a_h$, $a_{ch}$, $w_h$, $w_{ch}$ are the amplitudes and phases of annual and Chandler oscillations accordingly. The frequencies $\dot{w}_h$, $\dot{w}_{ch}$, $\Omega$ are expressed through $\omega$ which is the Earth average orbital motion.
The additional summands in the (1) constitute the harmonics with the combinational frequencies. It must be noted that the celestial-mechanical approach to the creation of the Earth Pole motion model in the frames of the problem “the deformable Earth – the Moon in the field of gravitational center” allows to identify the existence of such oscillational process in the model problem. However, the explanation of this effect is connected with accounting of the perturbation from geophysical mobile environments which influence geopotential [2] and with the identification of the cause of these oscillations in geo-environments.

Although the amplitudes of additional harmonics appear to be significant for the problem of the Pole motion forecast (their accounting allows to increase the precision of definition of the Pole position on the Earth surface by 30 sm), their presence cannot fundamentally change the character of the motion itself. If we take their geophysical nature (which can be confirmed by the fuzziness of their harmonics “peaks” in the amplitude spectrum) into account as well as their celestial-mechanical interpretation, then we can assume possible increase of their amplitudes that will substantially affect dynamics of the Earth Pole motion.

But there are the harmonics with the frequencies that are close to the Chandler frequency which is the Earth free nutation frequency. The proximity of these harmonics to resonance frequency leads to the increase of their amplitudes in the resulting process. However their account in the models of the Pole motion forecast does not reasonable usually. Their account increases the quantity of basic functions of the model and increases by several times the interval of interpolation for separation of close frequencies that leads to fundamental decreasing of forecast precision on relatively short time intervals (till 3 years). But their existence can change the dynamics of the Pole motion with high probability yet. The occurrences of change of the Pole motion oscillational mode (when the average velocity of its motion changes saltatory [3]) are observed several times for last 120 years yet. The standard short-parameters model gives the largest discrepancy between the forecast and the measurements data with such changes what is requires a modification of the forecast model. The increasing of basic function quantity does not lead to positive result in this case due to non-stationarity of this process. Thus the correction of the model is connected with the definition of new structural properties of mathematical model (dependence the parameters of the model) at transition from one oscillations mode to another.

To use such structural properties allowed to adapt the model to new mode it is necessary to know when this change is happened. This change may be detected with the help of numerical processing of the measurements data in real time with sufficiently long delay only. The selection of the perturbation leaded to change of the Pole motion mode allows to decrease a delay time and to make a forecast of transition of the motion to another mode.
Let suggest that harmonics which leads to change of the Pole oscillation mode have the combinational nature and is connected with the motion of the Lunar orbit. Then let use the equations of the form (1) where as additional summands let account the harmonics with frequencies $\frac{\nu + N}{2} - \dot{\Omega}$ and $\nu - \dot{\Omega} - \dot{\nu}$ (0.868 and 0.833 cycles annual accordingly):

$$x_p = a_{ch} \cos \omega_{ch} + a_{h} \cos \omega_{h} + \Delta x_p,$$

$$y_p = a_{ch} \sin \omega_{ch} + a_{h} \sin \omega_{h} + \Delta y_p,$$

$$\Delta x_p = a_s \cos \left( \frac{\omega_h + \omega_{ch}}{2} - \Omega \right) + a_z \cos \left( \omega_h - \Omega - \nu \right)$$

$$\Delta y_p = a_s \sin \left( \frac{\omega_h + \omega_{ch}}{2} - \Omega \right) + a_z \sin \left( \omega_h - \Omega - \nu \right)$$

Here through $\dot{\nu}$ the frequency of the Lunar orbit perigee motion with 8.85 years period is denoted. The average values of amplitudes and phases of the Chandler and annual components according to interpolation on long time interval are taken as parameters in (2). The amplitude and the phase of harmonic with frequency $\frac{\nu + N}{2} - \dot{\Omega}$ are chosen due to spectrum analysis on the same interval. The parameters of harmonic with frequency $\nu - \dot{\Omega} - \dot{\nu}$ have been adjusted during numerical simulation. It is shown in result of numerical calculations that if amplitude of harmonic with frequency $\nu - \dot{\Omega} - \dot{\nu}$ is
3 times less than amplitude of Chandler component then the Pole resulting motion on 120 years interval corresponds to observations qualitatively. In particular the time moments of transitions between different oscillations modes are coincided. The comparison of observed (zigzag line) and calculated (smooth line) phases $\psi$ of the Earth Pole motion after deduction of linear part on 120 years interval (which is denoted as $\varphi$ and is measured in radians) is given on Fig. 1. The transitions between two different modes of the Pole oscillations are well visible on the Fig. The average frequencies of the Pole rotation around the central point correspond to the Chandler frequency (intervals with monotonic increase on the graph) or to the annual frequency (intervals with monotonic decrease on the graph). Here one can note that theoretical calculated graph on the interval with average annual frequency has more broken character comparing with the graph which has been constructed according to observed motion. This effect connects with more close amplitude values of the Chandler and annual components during calculations than their observed ones. The change of the Chandler frequency does not consider in representation (2) which leads to the change of modulation period of the Chandler and annual harmonics. This leads to the shift of calculated modulation phase on the graph in the beginning of simulation interval.

The processing of IERS observation data shows that this process is not stationary. But available observation interval is not enough to answer the question, can this process be considered as stable one with slowly-varying parameters. Thus if used harmonic appears to be stable then the moment of the next oscillation mode change can be suggested easily.

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