Topology of the Electroweak Vacua

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In the Standard Model, the electroweak symmetry is broken by a complex, SU(2)-doublet Higgs field and the vacuum manifold SU(2) × U(1)/U(1) has the topology of a 3-sphere. We remark that there exist theoretical alternatives that are locally isomorphic, but in which the vacuum manifold is homeomorphic to an arbitrary non-trivial principal U(1)-bundle over a 2-sphere. These alternatives have non-trivial fundamental group and thus feature topologically-stable electroweak strings. An alternative based on the manifold RP³ (with fundamental group Z/2) allows custodial protection of gauge boson masses and their couplings to fermions, but, in common with all alternatives to S³, has a problem with fermion masses.

I. INTRODUCTION

Decades of experiment have confirmed that the weak nuclear force and the electromagnetic force are described by a gauge theory in which a group locally isomorphic to SU(1) is non-linearly realised in the vacuum, with only the electromagnetic subgroup U(1) < SU(2) × U(1) being linearly realised. Thus, the electroweak (EW) vacuum is degenerate and the vacua are described by a homogeneous space SU(2) × U(1)/U(1).

The starting point for this Letter is the observation that there are many ways to include U(1) in SU(2) × U(1); different ways lead to homogeneous spaces that can be topologically inequivalent. In the Standard Model (SM), the vacuum manifold arises due to a non-vanishing vacuum expectation value (VEV) of the Higgs field, carrying the doublet representation of SU(2), and is homeomorphic to the 3-sphere, S³. As is well-known, this is rather boring from a physicist’s point of view, since the vanishing of the homotopy groups π₁(S³) (respectively π₂(S³)) implies the absence of topologically-stable strings (respectively monopoles). Here, we investigate different inclusions of U(1), which lead to vacuum manifolds with fundamental group given by an arbitrary cyclic group; alternatives to the SM based on such inclusions thus feature topologically-stable strings, with potentially interesting consequences, a priori, for astrophysics, cosmology, and particle physics.

Given the recent discovery of a particle whose properties correspond rather closely to that of the Higgs boson, consideration of alternatives to the SM requires a willing suspension of disbelief on the part of the reader, and indeed we shall see that none of the topologically-distinct alternatives can be made consistent with data. Nevertheless, we feel that the very existence of such theoretical alternatives to the status quo is a noteworthy curiosity in its own right.

The outline is as follows. In [I] we discuss the topology of the vacuum manifold SU(2) × U(1)/U(1) and in [II] we show that an effective field theory based on a vacuum manifold with non-trivial topology can be consistent with data, apart from a problem with fermion masses. In [III] and [IV] we present explicit examples with non-trivial topology based on linear sigma models and composite Higgs models.

II. TOPOLOGY OF SU(2) × U(1)/U(1)

We begin our discussion by assuming that the EW gauge group really is G = SU(2) × U(1), deferring discussion of groups locally isomorphic thereto until the end. We write elements of G as (U,z), where U is a 2 × 2 unitary matrix with unit determinant and z is a unit complex number. For p,q ∈ Z there is a homomorphism φp,q : U(1) → G given by φp,q(z) = (diag(zᵖ, z⁻ᵖ), z⁰), and if (p,q) are coprime then φp,q is injective, in which case we write H_{p,q} ⊆ G for its image. (Any injective homomorphism φ : U(1) → G is conjugate to some φp,q, as its projection to the SU(2)-factor may be conjugated to land in the standard maximal torus.)

Our first goal is to investigate the topology of the homogeneous spaces G/H_{p,q}. An immediate result is that G/H_{p,q} cannot be homeomorphic for different p, because a loop wound once around H_{p,q} is wound p times around the U(1) factor of G. This implies, using the long exact sequence of homotopy groups π₁(H_{p,q}) ≅ Z → π₁(G) ≅ Z → π₁(G/H_{p,q}) → π₀(H_{p,q}) ≅ 0 of the fibre bundle H_{p,q} → G → G/H_{p,q} that π₁(G/H_{p,q}) ≅ Z/p. Moreover, we see that topologically-stable string configurations occur when p ≠ 1 [I].

To investigate the topology further, let K_{p,q} = H_{p,q} ∩ (SU(2) × {1}) ⊆ SU(2) and consider the function π : A → (A,1)H_{p,q} : SU(2) → G/H_{p,q}. This is a composition of smooth maps SU(2) → G → G/H_{p,q} and so smooth. The differential at the identity Dπ : su(2) → g/h_{p,q} is an isomorphism, and by homogeneity it follows that π is a submersion, and hence a local diffeomorphism. Fur-
thermore the right $K_{p,q}$-action on $SU(2)$ acts freely and transitively on the fibres of $\pi$, exhibiting it as a principal $K_{p,q}$-bundle, and hence giving a diffeomorphism $SU(2)/K_{p,q} \cong G/H_{p,q}$.

Now $K_{p,q} = \{\text{diag}(e^{2\pi i k/p}, e^{-2\pi i k/p}) : k \in \mathbb{Z}\}$ is the same subgroup of $SU(2)$ as $K_{p,1}$, because $(p,q)$ are coprime, and as $K_{p,1}$; thus we shall suppose $p > 0$. It follows that $G/H_{p,q}$ is diffeomorphic to $SU(2)/K_{p,1}$, which is further diffeomorphic, as we now show, to a lens space $\mathbb{L}$. These spaces are of great historical importance in mathematics, providing the first examples of which is further diffeomorphic, as we now show, to a lens space $\mathbb{L}$. The lens spaces $L(n,m)$ is defined for $(n,m)$ coprime as the quotient of the unit sphere, $S^3 \subset \mathbb{C}^2$ by the free $\mathbb{Z}/n$-action generated by $(z_1, z_2) \mapsto (e^{2\pi i/n}z_1, e^{2\pi i m/n}z_2)$. Identifying $SU(2)$ with the unit sphere $S^3 \subset \mathbb{C}^2$, $SU(2)/K_{p,1}$ is thus identified with the lens space $L(p,1)$.

The lens spaces $L(p,1)$ are precisely those 3-manifolds that arise as principal $U(1)$-bundles over the 2-sphere (except for $S^2 \times U(1)$). Indeed, the clutching construction shows that such bundles are in bijection with $\pi_1(U(1)) = \mathbb{Z}$, and this bijection may be given by assigning to a principal $U(1)$-bundle over the 2-sphere its Euler number. Writing $U(1) = \{\text{diag}(e^{i\theta}, e^{-i\theta}) : \theta \in [0, 2\pi]\} \leq SU(2)$, the Hopf bundle $h_1 : SU(2) \rightarrow SU(2)/U(1) = S^2$ is the principal $U(1)$-bundle with Euler number 1. As $K_{p,1} \leq U(1)$ the map $h_1$ is the composition

$$SU(2) \rightarrow SU(2)/K_{p,1} \xrightarrow{h_p} SU(2)/U(1) = S^2$$

of a $p$-fold covering map and a principal $(U(1)/K_{p,1} \cong U(1))$-bundle $h_p$, whose Euler number is therefore $p$ and whose total space is $G/H_{p,q} \cong SU(2)/K_{p,1} \cong L(p,1)$.

From this perspective, we may use standard results to read off the algebraic topological invariants of $G/H_{p,q}$: the homotopy groups are given by $\pi_1 = \mathbb{Z}/p$, $\pi_{i>1} = \pi_{i>1}(S^3)$ (so $\pi_2 = 0$, $\pi_3 = \mathbb{Z}$, $\pi_4 = \mathbb{Z}/2$, &c.); the integral cohomology is given by $H^0 = \mathbb{Z}$, $H^1 = 0$, $H^2 = \mathbb{Z}/p$, $H^3 = \mathbb{Z}$. Most interesting among these, for physicists, is $\pi_1 = \mathbb{Z}/p$.

How do these results relate to the SM? In that case, we postulate the existence of a Higgs field, that is a matter field $\phi$ whose potential is such that it acquires an non-vanishing VEV. It carries the doublet irreducible representation of $SU(2)$ and its charge $q \in \mathbb{Z}$ under $U(1)$ is non-vanishing, but otherwise arbitrary. The $G$-action is then $G : \phi \mapsto U\phi$. Without loss of generality, we may write the Higgs VEV as $\langle \phi \rangle = (0, v)^T$, such that the unbroken subgroup is $H_{1,q} = \{\text{diag}(z^q, z^{-q}), z\}$. The discussion above then shows that, as expected, the SM EW vacuum manifold is homeomorphic to $S^3$ and does not feature topologically-stable strings.

III. NON LINEAR SIGMA MODEL

Having established the existence of homogeneous spaces with non-trivial topology, we now construct physical theories based upon them, beginning with non-linear sigma models (NLSMs). These represent the most general low energy-effective field theories consistent with the non-linearly realised symmetry $G$ [8] and we would like to examine whether such theories can also be consistent with experimental data for any $p, q$.

At the local level at least, this is almost a triviality. Indeed, even in the ungauged theory, physics which depends only on local properties of the vacuum manifold can only depend on $p$ and $q$ through their quotient. But even the quotient is unphysical (locally) in the gauged theory, because differing values of $p$ and $q$ can be absorbed by redefinitions of the gauge coupling constants.

As an explicit example, consider quark masses. The necessary and sufficient condition for writing these in the NLSM is that one can form $H_{p,q}$-invariants out of the the quark fields $Q$, $U^c$, and $D^c$. Writing the corresponding $U(1)$ hypercharges as $y_Q, y_D$, and $y_D$, the $SU(2) \times U(1)$ action on the $SU(2)$-doublet Q restricts under $H_{p,q}$ to $Q \mapsto \text{diag}(z^{y_Q + y_D}, z^{-y_Q + y_D})Q$, while the action on the $SU(2)$-singlets $U^c$ and $D^c$ restricts to $U^c, D^c \mapsto z^{y_Q} U^c, z^{y_D} D^c$. Thus we require that $y_Q + y_D = -y_D - y_Q = \pm \frac{2}{p}$. Locally, these are precisely the same relations that we require in the SM, up to an unphysical overall rescaling.

Thus, locally, all such models are equivalent to models with $p = 1$. But models with $p = 1$ (with the Higgs boson treated as an additional, CP-even, singlet, scalar matter field with arbitrary couplings [3]) contain the SM (or rather a low-energy limit thereof) as a special case and thus can be consistent with data. So for any $p$, at the local level, there exists a choice of parameters in the corresponding NLSM that is consistent with particle physics data.

A. Custodial symmetries

Though local considerations suggest that a NLSM based on $G/H_{p,q}$ can fit the data for any $p$, it is clearly far less satisfactory than the SM as regards its predictivity. Most seriously, while the Higgs boson is an integral part of the SM, in an NLSM description we are forced to arbitrarily include an additional scalar matter field whose couplings are tuned to be close to those of the SM Higgs boson. Even if we are willing to overlook these issues regarding the Higgs, there are two more successful predictions of the SM which, though they can be accommodated by any model, are not predicted in a generic NLSM. The first of these is the $W - Z$ boson mass ratio, which is fixed in the SM, but is arbitrary in the $G/H_{p,q}$ NLSM. Indeed, the gauge boson masses are determined by specifying a $G$-invariant metric on $G/H_{p,q}$. In the SM, the metric is fixed to be the round metric on $S^3$, which is
unique up to an overall normalization, so that the $W - Z$ mass ratio is fixed. But in a generic NL, we may pick an arbitrary $G$-invariant metric on $G/H_{p,q}$. Such metrics may be classified as follows [12]: as $G$ acts almost effectively on $G/H_{p,q}$ (i.e. the subgroup of $G$ that fixes all elements of $G/H_{p,q}$ is discrete), the $G$-invariant metrics on $G/H_{p,q}$ are in 1-1 correspondence with those inner products on $g/H_{p,q}$ which are invariant for the adjoint action of $H_{p,q}$. The adjoint representation of $SU(2) \times U(1)$ restricts to $H_{p,q}$ as $3 \oplus 1 \to 2q \oplus 2q \oplus 0 \oplus 0$, where we denote representations on the LHS by their dimension and on the RHS by their $H_{p,q}$ charge. There are thus 2 such independent inner products, each with an arbitrary overall normalization, leading to an arbitrary $W - Z$ mass ratio.

As is well-known, the particular mass ratio that obtains in the SM can be understood via custodial symmetry [13]: the vacuum manifold $S^4$ is invariant under a larger action of $SU(2) \times SU(2)$ (with the original $U(1)$ included in the second $SU(2)$), broken to the diagonal $SU(2)$ by the Higgs VEV. Since the adjoint representation of $SU(2) \times SU(2)$ restricts as $(3,1) \oplus (1,3) \to 3 \oplus 3$, there is just one invariant inner product, up to an overall normalization.

We claim that a similar construction can be applied to the $G/H_{p,q}$ NL only in the cases $p \in \{1,2\}$. Indeed, to do so requires us to find a metric on $G/H_{p,q}$ which is invariant not just under $SU(2) \times SU(2)$, but rather under the larger $SU(2) \times SU(2)$. Now, this larger group need not act effectively on $G/H_{p,q}$, but the subgroup $N = \{g \in SU(2) \times SU(2) : g.x = x \forall x \in G/H_{p,q}\}$, being the kernel of a homomorphism from $SU(2) \times SU(2)$ to Sym$(G/H_{p,q})$, will necessarily be normal, and furthermore $(SU(2) \times SU(2))/N$ will act effectively on $G/H_{p,q}$. Now, the normal subgroups of $SU(2)$ are $\{e\}$, $2$ such that the quotient project $N$ to either $SU(2)$-factor must give one of these. But neither projection can be $SU(2)$, as otherwise $(SU(2) \times SU(2))/N$ would have dimension at most 3, which is incompatible with the known restriction of the action to the subgroup $SU(2) \times U(1)$ (in which both factors have a non-trivial action), so in fact $N$ must be a subgroup of the centre $Z/2 \times Z/2$. Thus the $SU(2) \times SU(2)$-action is almost effective and the isometry group of the desired metric has dimension at most 6. But it is a theorem [14] that the isometry group of a compact Riemannian 3-manifold has dimension at most 6, equaling 6 only if it is $S^3$ or $\mathbb{RP}^3$, corresponding to $p = 1$ or $p = 2$ respectively.

Thus, we see that a custodial symmetry protecting $W$ and $Z$ masses may also be imposed in the topologically non-trivial case $p = 2$. Here $G/H_{2,1} \cong \mathbb{RP}^3 \cong SO(3)$ has an action of $SO(3) \times SO(3)$ by $(g,h).x = ghx^{-1}$, with the $SU(2) \times SU(2)$-action being the one induced by the covering map from $SU(2) \to SO(3)$; the LSM model (respectively composite Higgs model) in §IV (respectively §V) give explicit realisations.

The second successful prediction of the SM involves the couplings of gauge bosons to fermions. Again, for a generic $G/H_{p,q}$ NL, deviations in these couplings may occur. In a concrete theory of flavour such as partial compositeness (which has the desiderata of explaining much of the hierarchical structure of Yukawa couplings whilst suppressing potential flavour-changing effects) [15] this is not so much of a problem, because the SM fermions are largely elementary, being weakly mixed with the strong sector. Deviations are therefore expected to be small. The biggest problem arises in the coupling of the $Z$ boson to left-handed bottom quarks, since the latter belongs to the same $SU(2)$ doublet as the top quark, and is forced to be sizably mixed with the sigma model sector in order to accommodate the large top quark mass. But this too can be protected by a symmetry in the $p = 1$ case [16]. The required group is $(SU(2) \times SU(2)) \times \mathbb{Z}/2$, where $\mathbb{Z}/2$ permutes the two $SU(2)$s [17]. A similar protection can also be achieved when $p = 2$, by using the action of $(SO(3) \times SO(3)) \times \mathbb{Z}/2$ on $G/H_{2,1} \cong \mathbb{RP}^3 \cong SO(3)$, where the $\mathbb{Z}/2$ acts by inversion on the group $SO(3)$.

B. Global properties

Problems with consistency with the data arise when we consider the global properties of a $G/H_{p,q}$ NL. The first obvious question is whether the presence of topologically-string solutions is consistent with astrophysical data. In fact, this issue is hard to settle. One would expect a network of strings to form in the early Universe during the cosmological EW phase transition [10], but the purely gravitational effects of such strings are of order $v^2/M_P^2 \sim 10^{-34}$ and are utterly negligible. However, such a string necessarily features quark and lepton zero modes localised on its core, which lead to the formation of superconducting currents (as well as baryon- and lepton-number violation) in the presence of astrophysical magnetic fields [9]. A number of resulting astrophysical signatures have been discussed (in, among others, the microwave background, radio bursts, cosmic rays, and galactic and stellar dynamics; for a review, see [10]) but there seems to be no consensus that they lead to robust constraints on EW-scale strings.

A much more serious (and easier to settle) problem occurs when we consider fermion masses. We saw above that, for quarks say, these require us to satisfy the conditions $y_Q + y_D = -y_Q - y_U = \pm \frac{2}{p}$, which, locally, are no different from the conditions one obtains in the SM. But there is a problem globally, which is that the hypercharges must be integers, in order that the fermions carry bona fide representations of $SU(2) \times U(1)$. This, together with the requirement that $p,q$ be coprime, implies that one cannot write $H_{p,q}$-invariant fermion mass terms unless $p = 1$. Hence, one obtains a gross conflict with data for $p \neq 1$.

There is a simple way to avoid this unfortunate conclusion, which is to change the representation content of the fermion fields. In particular, mass terms are allowed if the fermions are assigned to higher-dimensional repre-
sentations of $SU(2) \times U(1)$. But this has the undesirable knock-on effect of either requiring additional unobserved fermion states (to fill out the higher-dimensional multiplets), or of modifying the observed couplings of existing fermions to gauge bosons. Neither is phenomenologically viable.

**IV. LINEAR SIGMA MODELS**

Even though we have seen that models with topologically non-trivial vacuum manifold are incompatible with data, it is interesting to see how they could have arisen from physically-sensible, UV-complete models. As a first, example, we consider linear sigma models. A model in which the Higgs field of the SM is replaced by a scalar, $\Phi$, carrying a bi-triplet, $(\mathbf{3}_L, \mathbf{3}_R)$ action of $(\mathbf{3}, \mathbf{3})$, is labelled by its dimension $2j + 1$ leads (for a suitable choice of orbit for the VEV) to a vacuum manifold homeomorphic to $L(p, 1)$, with $p = 2j/\gcd(2j, q)$. Either the arguments given in [11] or an explicit calculation shows that the pattern of couplings of gauge bosons to themselves and to fermions is exactly as in the SM, but, since custodial symmetry is not respected, there is a gross violation of the $W - Z$ mass ratio for $2j \neq 1$, with $m_W^2/m_Z^2 = \frac{g_2^2}{2g_1^2 + g_1^2}$ at tree-level, where $g_2$ (respectively $g_1$) denotes the SM value of the $SU(2)$ (resp. $U(1)$) gauge coupling.

For a model yielding the correct tree-level value of $W - Z$ mass ratio, we replace the SM Higgs field by a real scalar, $\Phi$, carrying a bi-triplet, $(\mathbf{3}_L, \mathbf{3}_R)$, of modifying the observed couplings of existing fermions to gauge bosons. Neither is phenomenologically viable.

**V. A COMPOSITE HIGGS MODEL**

It is also possible to achieve a topologically non-trivial vacuum manifold in models in which the hierarchy problem is solved by making the Higgs boson a composite of some new TeV-scale dynamics. Indeed, in the most favoured among such models [10], the Higgs is a pseudo-Goldstone boson, taking values in a homogeneous space $SO(5)/O(4) \cong \mathbb{R}P^4$. Due to the presence of couplings to gauge bosons and fermions, the dynamics cannot be fully $SO(5)$ invariant and so the low-energy effective Lagrangian contains a potential for the Higgs field. This is a real-valued, $SU(2) \times U(1)$-invariant function on $\mathbb{R}P^4$, with the vacuum manifold being given by the level set of the minimum. The specific form of the function is determined by the uncancellable strong dynamics, but let us suppose, for the sake of illustration, that it takes the form $V = \frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5}$, where $x_i, i \in \{1, 2, 3, 4, 5\}$ are coordinates in $\mathbb{R}^5$. This is well-defined on $\mathbb{R}P^4$, the space of lines through the origin in $\mathbb{R}^5$, and is, moreover, smooth and invariant under the larger group $O(4)$. We have that $0 \leq V \leq 1$, with the maximum at the point $x_3 = x_4 = 0$ and the minimum at $x_1 = 0$, corresponding to the level set $\mathbb{R}P^3$.

In accordance with our general result, choosing such a minimum as vacuum must imply vanishing fermion masses; this can be confirmed by comparing with, e.g., formulae for the top quark mass in explicit models in [19].

**VI. GROUPS LOCALLY ISOMORPHIC TO $SU(2) \times U(1)$**

Finally, we consider gauge groups that are locally, but not globally, isomorphic to $SU(2) \times U(1)$. Such groups need not be connected, in which case the possibilities are infinite, but all feature domain-wall type solutions that are potentially dangerous from a cosmological point of view. If we restrict our attention to the component connected to the identity, then the possibilities are finite in number, given by quotients of the universal cover $SU(2) \times \mathbb{R}$ by a discrete subgroup of the centre $\mathbb{Z}/2 \times \mathbb{R}$. Of the five possibilities, the only ones other than $SU(2) \times U(1)$ admitting doublet irreducible
representations (as carried by quarks and leptons) are $SU(2) \times \mathbb{R}$ and $U(2)$. The former has subgroups isomorphic to $\mathbb{R}$ and is disfavoured by the apparent quantization of hypercharge; similar arguments to those given in §II show that the vacuum manifold is always homeomorphic to $S^3$ in this case. The latter has $U(1)$ subgroups given by $H_{r,s} = \{ \text{diag}(e^{i r \theta}, e^{i s \theta}) \}$, with $r, s \in \mathbb{Z}$ and coprime, is homeomorphic to $L(r+s,1)$, and also admits topologically-stable strings.

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