FNS-parameterization of non-stationarity effects in the solar activity dynamics

S A Demin¹, O Yu Panischev¹, R R Latypov¹ and S F Timashev²
¹ Kazan Federal University, Institute of Physics, Kazan, 420008 Russia
² Karpov Institute of Physical Chemistry, Moscow, 105064 Russia

E-mail: serge_demin@mail.ru

Abstract. In paper we present the findings of analyzing the non-stationary effects to a solar activity dynamics. Within the framework of Flicker-Noise Spectroscopy (FNS) we study the Zurich series of Wolf numbers from 1849 to 2009. According to the FNS methodology the properties of complex system evolution are manifested in the low frequency component (the “resonant” part of the signal) of time signal and contain in the specific sets of the frequencies. For another thing the signal also has the high frequency component (“chaotic” part) including the noise and the different types of short time irregularities. The FNS methodology allows to discover the intermittency phenomena in studied dynamics by analyzing the behavior of the nonstationarity factor. We will show that the maximum value of this parameter corresponds the maximum of Wolf number i.e. the solar activity.

1. Introduction
For a long time the Wolf numbers reflecting the solar activity are one of the most interesting object for astronomical and astrophysical studies. These data are registered since the invention of the telescope (from 1610). However there is no the whole model of the turbulent processes in sun spots which are the self-organizing long-lived magnetic structures. In paper within the framework of the Flicker-Noise Spectroscopy (FNS) we study the intermittency effects the solar activity dynamics. To study the nonstationarity effects we use the daily series of Wolf numbers from 1849 to 2009 which are published by Zurich observatory [1]. The series length (with no skips) is ~ 59 000 days. Daily spot number is calculated by \( W = k(n_s + 10n_g) \), where \( n_s \) is the number of observed spots, \( n_g \) – is the number of observed groups spots in the whole solar disc. The normalizing coefficients \( k \) are derived for each observer or a telescope. It allows to use the data from different observers and for different times.

The singularity of the FNS methodology is introducing the information parameters, characterizing the solar activity dynamics in different frequency ranges [2–4]. The irregularities “jumps” (long time alterations) and “spikes” (short time alterations) are described by power spectra and by difference moments of the second order. Also the FNS allows to reveal the most essential intermittency in solar activity by nonstationarity factor. This parameter is very useful to reveal the persistent or non-persistent character of studied dynamics [5, 6].

2. Basic relations of the Flicker-Noise Spectroscopy
In FNS, all introduced parameters for signal \( V(t) \), where \( t \) is time, are related to the autocorrelation function:
The dynamics of complex systems includes both stochastic components, i.e., spikes and jumps, and resonant internal and external resonances and th ... of an open system. All the specific frequencies and their interferential contributions, which manifest themselves as oscillations in the $S(f)$, will be further called “resonant”. It is assumed that $S(f)$ can be presented as a linear superposition of stochastic component $S_s(f)$ and resonant component $S_r(f)$:

$$S(f) = S_s(f) + S_r(f).$$
\[ S(f) = S_r(f) + S_s(f). \] (6)

Here, we assume that the resonant components are statistically stationary (they depend only on time lag \( \tau \)). This allows us to estimate \( \psi_r(\tau) \) as an “incomplete” cosine transform of \( S_r(f) \) by applying the Wiener–Khinchin Theorem:

\[ \psi_r(\tau) \approx 2 \int_{f_{\text{min}}}^{f_{\text{max}}} S_r(f) \cos(2\pi f \tau) \, df, \] (7)

where \( f_{\text{max}} = 0.5f_d, f_d \) is the sampling frequency. It should be noted that (7) is an approximation applied to a finite discrete time series assuming the wide-sense stationarity of the resonant signal component.

The resonant component \( \Phi_r^{(2)}(\tau) \) in this case is found by:

\[ \Phi_r^{(2)}(\tau) = 2[\psi_r(0) - \psi_r(\tau)]. \] (8)

The stochastic component of \( \Phi^{(2)}(\tau) \) can then be estimated as:

\[ \Phi_s^{(2)}(\tau) = \Phi^{(2)}(\tau) - \Phi_r^{(2)}(\tau). \] (9)

These equations allow one to sequentially separate out resonant and stochastic components of structure functions and power spectrum estimates for experimental time series and perform the parameterization of the components.

**Table 1.** FNS-parameterization of the solar activity dynamics (figure 1a). Daily total sunspot number derived by the formula: \( W=0.6(^{n}+10n_{g}) \), with \( n \) the number of spots and \( n_{g} \) the number of groups counted over the entire solar disk. The 0.6 scaling factor was determined to bring the modern counts to the scale of the original sunspot index derived by Rudolph Wolf in the mid-19th century. By definition, the index cannot take values between 0 (no spots) and 7 (single solitary spot, \( W=0.6 \cdot 11 \)).

| \( \sigma \), rel. un. | \( S_{n}(T_{0}^{-1}) \), (rel.un.)². day⁻¹ | \( T_{0} \) days | \( H_{1} \) | \( T_{j} \) days | \( n \) |
|------------------------|----------------------------------|----------------|---------|--------------|------|
| 32.6                   | 2.28 \( \times \) 10⁴            | 2513           | 0.14    | 1917         | 1.11 |

The proposed FNS parameters \( \sigma, S_{n}(T_{0}^{-1}), H_{1}, n, T_{j}, T_{0} \) describe the degree of chaoticity of the solar activity. First of all the parameter \( \sigma \) – the standard deviation of the value of the measured dynamic variable from the slowly varying resonant (regular) component, which is based solely on jump-like irregularities. The second is the “spikiness” factor \( S_{n}(f) \) – power spectrum (5) estimate at frequency \( f \sim T_{0}^{-1} \), which accounts for the “intensity” of jump- and spikelike irregularities in the highest-frequency interval. \( T_{0} \) is characteristic correlation time of high frequency irregularities. Also we use \( H_{1} \), the Hurst exponent (this estimate of the Hurst component is also referred to in literature as the Hausdorff exponent), which describes the rate at which the dynamic variable “forgets” its values on the time intervals that are less than \( T_{j} \) and \( n \), the flicker-noise parameter, which characterizes the rate of loss of correlations in the series of high-frequency irregularities in time intervals \( T_{0} \).

**3. FNS nonstationarity factor**

To reveal the non-stationarity effects the dynamics of the \( \Phi^{(2)}(\tau) \) functions changes at serial displacement of the \( [t_{k}, t_{k}+ T] \) trial period, where \( k = 0, 1, 2, 3, \ldots \) and \( t_{k} = k \Delta T \), along the entire length \( T_{\text{tot}} \) of the available set of experimental data \( (t_{k}+T<T_{\text{tot}}) \) is studied. The \( T \) and \( \Delta T \) time periods should be chosen on the basis of physical meaning of the studied problem with considering the process’ assumed characteristic time, most important for evolution of the studied system when its restructuring takes place.

Appearance of the restructuring precursor is related to the sharpest changes of \( \Phi^{(2)}(\tau) \) dependencies, when the upper bound of the averaging time period \( t_{k} \) approaches the systems’ restructuring moment \( t_{c} \).

It is clear that we can consider the restructuring characteristic times \( T_{\text{in}} \) only in that case, when the
precursor appearance time $t_k$ lags from the $t_c$ moment for less than the $\Delta T$ period, i.e. $\Delta T_{sn}=t_c-t_k>\Delta T$ at $\Delta T_{sn}<T_{tot}$. As the restructuring precursors of the system in FNS we consider splash values of nonstationarity indicators calculated on the basis of the difference moments $\Phi_i (\tau)$:

$$ C(t_k) = 2 \frac{Q_k - P_k}{Q_k + P_k} / \Delta T, \quad (10) $$

$$ Q_k = \frac{1}{\alpha T^2} \int_0^{\alpha T_k+\tau} \int_0^{\Delta T} [V(t)-V(t+\tau)]^2 \, d\tau \, d\tau, \quad (11) $$

$$ P_k = \frac{1}{\alpha T^2} \int_0^{\alpha T_k+\tau-\Delta T} \int_0^{\Delta T} [V(t)-V(t+\tau)]^2 \, d\tau \, d\tau. \quad (12) $$

Here it is taken into account that the $\Phi_i (\tau)$ dependencies can be reliably evaluated only for the $\tau$ interval of $[0, \alpha T]$ which is less than a half of the averaging interval $T$, i.e. $\alpha < 0.5$.

The informational value of such an approach in terms of revealing precursors of catastrophic changes of system was demonstrated, when analyzing geodynamical phenomena preceding the major earthquakes [6].

**Figure 1.** (а) – initial time series (abscissa is the day numbers from January, 1, 1849); (b) nonstationarity factor for daily Wolf numbers at $T = 1000$ ($\Delta T = 1$ days). The predictors (dotted lines), calculated for $T = 1000$ days, are manifested on the eve the maximum values of Wolf numbers, starting from the extreme left maximum, from: 989, 663, 1216, 756, 924, 605, 686, 677, 673, 726, 642, 1220, 1041 days.

The time-dependent nonstationarity factor has been obtained at averaging interval $T = 1000$ days (figure 1b). This dependence demonstrates that the peak values of $C(t_k)$ outstrip the maximums of Wolf numbers.
4. Conclusions
Solar activity is an amazing astronomical phenomenon, available for studying by many methods of observing and registering. In the middle of the XIX century, G. Shvabe and R. Wolf discovered an 11-year periodicity in changing the number of sunspots on the visible solar disk. Since then, the Wolf numbers have been used as the main parameter to estimate the solar activity and to study the characteristics of the Sun. Sunspots are indicators of the magnetic activity of the Sun, which covers its atmosphere and, moreover, manifests itself in the form of dark fibers and protuberances, flares in the chromosphere, coronal holes through which streams of high-speed charged particles are ejected.

In paper we have demonstrated that the FNS nonstationarity factor is the predictor of maximum of the solar activity. Depending on the length of the averaging interval, we identify the nonstationarity effects, associated both with 11-year periods and processes, within each of these cycles. The results will be of interest to study the formation processes of sunspots characterized the Sun magnetic activity, which in turn affects the radiation energy and the strength of the solar wind [9].

The proposed methodology may be useful to parameterize the dynamics of the X-ray radiation from microquasars [10, 11], radio emission from quasars [12], and other complex systems [13, 14].

Acknowledgments
The work is performed according to the Russian Government Program of Competitive Growth of Kazan Federal University.

References
[1] http://sidc.oma.be/sunspot-data/dailyssn.php
[2] Timashev S F 2007 Flicker-Noise Spectroscopy: Information in Chaotic Signals (Moscow: Fizmatlit) p 248
[3] Timashev S F and Polyakov Y S 2007 Fluct. Noise Lett. 7 R15
[4] Yulmetyev R M, Demin S A, Panischev O Yu, Hänggi P, Timashev S F and Vstovsky G V 2006 Physica A 369 655
[5] Borog V V, Ivanov I O, Kryanev A V and Timashev S F 2015 Phys. Procedia 74 336
[6] Polyakov Y S, Ryabinin G V, Solovyeva A B and Timashev S F 2015 Pure Appl. Geophys. 172 1945
[7] Vladimirov V S 1971 Equations of Mathematical Physics (New York: Marcel Dekker) p 426
[8] Schuster H G 1984 Deterministic Chaos: An Introduction (Weinheim: Physik-Verlag) p 220
[9] Demin S A, Nefedyev Y A, Andreev A O, Demina N Y and Timashev S F 2018 Adv. Space Res. 61 639
[10] Demin S A, Panischev O Y and Nefedyev Y A 2014 Kinemat. Phys. Celest. Bodies 30 63
[11] Demin S A, Panischev O Y and Nefedyev Y A 2014 Nonl. Phen. Compl. Syst. 17 177
[12] Demin S A, Panischev O Y and Nefedyev Y A 2015 J. Phys. Conf. Ser. 661 012003
[13] Panischev O Y, Demin S A and Bhattacharya J 2010 Physica A 389 4958
[14] Demin S A, Yulmetyev R M, Panischev O Y and Hänggi P 2008 Physica A 387 2100