INTERNAL-CYCLE VARIATION OF SOLAR DIFFERENTIAL ROTATION

K. J. Li1,2, J. L. Xie1,3, and X. J. Shi1,3

1 National Astronomical Observatories/Yunnan Observatory, CAS, Kunming 650011, China; ljk@ynao.ac.cn
2 Key Laboratory of Solar Activity, National Astronomical Observatories, CAS, Beijing 100012, China
3 Graduate School of CAS, Beijing 100086, China

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ABSTRACT

The latitudinal distributions of the yearly mean rotation rates measured by Suzuki in 1998 and 2012 and Pulkkinen & Tuominen in 1998 are utilized to investigate internal-cycle variation of solar differential rotation. The rotation rate at the solar equator seems to have decreased since cycle 10 onward. The coefficient $B$ of solar differential rotation, which represents the latitudinal gradient of rotation, is found to be smaller in the several years after the minimum of a solar cycle than in the several years after the maximum time of the cycle, and it peaks several years after the maximum time of the solar cycle. The internal-cycle variation of the solar rotation rates looks similar in profile to that of the coefficient $B$. A new explanation is proposed to address such a solar-cycle-related variation of the solar rotation rates. Weak magnetic fields may more effectively reflect differentiation at low latitudes with high rotation rates than at high latitudes with low rotation rates, and strong magnetic fields may more effectively repress differentiation at relatively low latitudes than at high latitudes. The internal-cycle variation is inferred as the result of both the latitudinal migration of the surface torsional pattern and the repression of strong magnetic activity in differentiation.

Key words: Sun: activity – Sun: rotation – sunspots

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1. INTRODUCTION

The Sun’s atmosphere is found to rotate faster at the equatorial region than at higher latitude regions. Specifically, it rotates in a circular course every 26 days at the solar equator but 30 days at 60° latitude, which is the so-called differential rotation (Balthasar & Wöhl 1980; Gilman & Howard 1984; Sheeley et al. 1992; Rybak 1994; Altröck 2003; Song & Wang 2005; Chu et al. 2010; Wöhl et al. 2010; Li et al. 2013). Two main methods have been used to measure the rotation velocity of the solar atmosphere: the tracer method and the spectroscopic method (Howard et al. 1984; Pulkkinen & Tuominen 1998a; Brájša et al. 2000, 2002; Wöhl & Schmidt 2000; Le Mouël et al. 2007; Li et al. 2012). The solar rotation rate is also determined in the solar interior but via a specialized method—the helioseismology measurement method (Howe et al. 2000a, 2000b; Antia & Basu 2001)—and the latitudinal migration of rotation angular velocity in the solar interior (Howe et al. 2009) is found to be similar to the torsional oscillation pattern of the solar atmosphere measured by the spectroscopic method (Howard & LaBonte 1980; LaBonte & Howard 1982; Schröter 1985). Observations and studies of solar differential rotation have made great progress (Howard 1984; Schröter 1985; Snodgrass 1992; Paterno 2010; Li et al. 2012); however, there are still some aspects, for example, the solar-cycle-related and long-term variations of the solar rotation rate, that remain unknown (Komm et al. 1993; Ulrich & Bertello 1996; Stix 2002; Li et al. 2011a, 2011b).

In this study, we will investigate the internal-cycle variation of solar differential rotation using measurements of rotation rates taken by Suzuki (1998, 2012) and Pulkkinen & Tuominen (1998a). A new explanation is proposed to address such a solar-cycle-related variation of the solar rotation rates.

2. INTERNAL-CYCLE VARIATION OF SOLAR DIFFERENTIAL ROTATION

2.1. Revisiting the Measurements of Solar Differential Rotation Taken by Suzuki (1998, 2012)

Daily photographic observations of sunspots at the solar full disk have been made with a refractor of 102 mm aperture and 1200 mm focal length by Suzuki since 1988, and the positions of sunspot groups at the solar disk have been obtained (Suzuki 1998, 2012). Suzuki measured the annual mean rotation rates of sunspots in a 5° latitude bin during cycles 22 to 23 (from 1998 to 2006) by analyzing observational data of sunspots. Measurements for 1988–1995 are given in Table 1 of Suzuki (1998) and for 1996–2006 in Table 1 of Suzuki (2012).

Solar differential rotation is usually expressed by the standard formula (Newton & Nunn 1951)

$$\omega(\phi) = A + B \sin^2 \phi,$$

where $\omega(\phi)$ is the solar sidereal angular velocity at latitude $\phi$ and the coefficients $A$ and $B$ represent the equatorial rotation rate and the latitudinal gradient of the rotation, respectively (Howard 1984). The latitudinal distribution of annual mean sidereal rotation rates measured by Suzuki (1998, 2012) is fitted by the formula for each year. Figure 1 shows the cross-correlation coefficient of the formula fitting to the latitudinal distribution of the annual mean rotation rates, and the corresponding tabulated value at the 95% confidence level is also given. The calculated correlation coefficient is larger than the corresponding tabulated value for all years except the years of 1993 and 1994, indicating that the formula can statistically significantly give a good fitting to Suzuki’s measurements of annual mean rotation rates. In these two years (1993 and 1994), both correlation coefficients are less
than the corresponding tabulated values at the 95% confidence level. Thus for these two years, the fitting values of \( A \) and \( B \) are replaced by the linearly extrapolated values of their neighboring two points. Figure 2 shows the obtained coefficients \( A \) and \( B \). A linear fitting is taken to the coefficient \( A \) varying with time \( t \) (in years), and resultantly, \( A = 39.0713 - 0.0122 \times t \), and the correlation coefficient is 0.4246, which is statistically significant at the 92% confidence level. There is a decreasing trend for \( A \), at a rate of 0.0122 day\(^{-1}\) yr\(^{-1}\). A five-point smoothing is performed on the coefficient \( B \), and is shown in the figure. A special feature for \( B \) is that its absolute value is larger in the several years after the minimum of a solar cycle than in the several years after the maximum time of the cycle, and the absolute value of \( B \) clearly decreases and then increases within the declining phase of a sunspot cycle. As the figure shows, a short-term effect can be seen for \( B \) varying within a solar cycle, and a long-term effect for \( A \) tending to decrease. The correlation coefficient of yearly sunspot numbers with \( B \) is calculated to be 0.521, which is statistically significant at a confidence level of 95% while the correlation coefficient of yearly sunspot numbers with \( A \) is −0.384, which is of no significance. For the solar surface rotation rate at the solar equator (coefficient \( A \)), a secular decrease of statistical significance has existed from cycle 12 onward (Javaraiah et al. 2005a, 2005b; Li et al. 2013).

Based on the obtained coefficients \( A \) and \( B \), we calculate the latitudinal distribution of rotation rates in each year from 1988 to 2006 using the standard formula, which is shown in Figure 3. Rotation rates are found to obviously show a migration within a solar cycle, but such a migration is different at the falling part of a solar cycle from both the latitudinal migration of sunspots and the torsional oscillation pattern of solar surface differential rotation (Snodgrass 1987; Li et al. 2008). Figure 4 shows four isopleth lines of rotation rates and their corresponding five-point smoothing lines. The isopleth values of rotation rates are 14.4, 14.2, 14.0, and 13.8 day\(^{-1}\) in turn from low to high latitudes. Also shown in the figure are the minimum and maximum times of sunspot cycles. As Figures 3 and 4 show, the migration of rotation rates seems thwarted at the declining phase of a solar cycle compared with the oscillation pattern of solar surface differential rotation (Snodgrass 1987; Li et al. 2008).

It is inferred that strong magnetic fields should repress solar differential rotation, in agreement with Brajša et al. (2006) and Wöhl et al. (2010).

2.2. Revisiting the Measurements of Solar Differential Rotation Taken by Pulkkinen & Tuominen (1998a)

The Royal Observatory in Greenwich began conducting sunspot observations under the Greenwich Photoheliographic Results (GPR) in 1874 and it stopped in 1976, lasting about 103 yr. After that, one of the continuing records of sunspots has been made by the Solar Optical Observing Network (SOON) of the U.S. Air Force together with the U.S. National Oceanic and Atmospheric Administration (NOAA; Pulkkinen & Tuominen 1998a). The GPR and SOON/NOAA data sets can be found on the Web.4 Pulkkinen & Tuominen (1998a) used these GPR and SOON/NOAA data from cycles 10 to 22, namely, at the time interval of 1874 to 1996 to study velocity structures from sunspot statistics. In their Figure 3 they gave the rotation profiles of the data at the equatorial range between latitudes ±20° at different phases of a cycle. These rotational velocity values are the sidereal values. The figure was enlarged to triple size, then all points of the rotation profiles in the figure were measured independently by each of the three authors of this study, and finally, the individual measurements were averaged. Three times of measurements ensure the obtained data matches the original well. The obtained rotation profiles are fitted here using the standard formula of differential rotation for each year within a solar cycle, corresponding to different phases of a Schwabe cycle. A profile gives a set of fitting parameters, reducing the effect of measurement errors of individual points on the fitting parameters. Figure 5 shows the cross-correlation coefficients of the formula fitting to the rotation profiles and the corresponding tabulated values at the 98.5% confidence level. The calculated correlation coefficients are larger than the corresponding tabulated values for all years, indicating that the formula can statistically significantly give a good

4 http://solarscience.msfc.nasa.gov/greenwch.shtml
fitting to the annual rotation profile. The correlation coefficient is lowest in and around solar activity minima due to the lower number of sunspots on the Sun in those time intervals. Figure 6 shows the obtained coefficients $A$ and $B$ at different phases of a Schwabe cycle. A linear fitting is taken to the coefficient $A$ varying with time $t$ of a solar cycle, and resultantly, $A = 14.6261 - 0.0107 \times t$, and the correlation coefficient is 0.6029, which is statistically significant at the 93% confidence level. There is a decreasing trend for coefficient $A$ at a rate of $0.0107 \text{ day}^{-1} \text{ yr}^{-1}$. A three-point smoothing is taken to the coefficient $B$, which is shown in the figure. The same special feature for $B$ is obtained as that obtained through analyzing the data of Suzuki (1998, 2012).

Based on the fitting values of coefficients $A$ and $B$, we also calculate the latitudinal distribution of rotation rates in each year of a solar cycle through the standard formula, which is shown in Figure 7. Rotation rates are found to obviously show the same migration within a solar cycle as mentioned above. Figure 8 shows four isopleth lines of rotation rates and their corresponding three-point smoothing lines. The isopleth values of rotation rates are 14:2, 13:9, 13:6, and 13:3 day$^{-1}$, respectively, from low to high latitudes. As Figures 7 and 8 show, large sunspots seem to hinder the torsional oscillation pattern shifting toward the equator 1–2 yr after the start of a solar cycle (Snodgrass 1987; Li et al. 2008). The strong magnetic fields should obviously repress the differentiation of rotation rates.
3. CONCLUSIONS AND DISCUSSION

The latitudinal distributions of the yearly mean rotation rates measured respectively by Suzuki (1998, 2012) and Pulkkinen & Tuominen (1998a) are exploited to investigate the internal-cycle variation of solar differential rotation. First, they are fitted by the standard formula of solar differential rotation. As a result, the rotation rate at the solar equator is found to have decreased from cycle 10 onward at a rate of about 0°.011 day\(^{-1}\) yr\(^{-1}\) within a solar cycle. For the coefficient \(B\), its absolute value is found to be larger in the several years after the minimum of a sunspot cycle than in the several years after the maximum time of the cycle, and the absolute value clearly decreases and then increases within the declining phase of the sunspot cycle, namely, coefficient \(B\) peaks several years after the maximum time of the solar cycle. Such a profile of coefficient \(B\) in a solar cycle was also given in Figure 5 of Javaraiah (2003). Although the variations of coefficients \(A\) and \(B\) within a solar cycle obtained by analyzing the data of Pulkkinen & Tuominen (1998a) are similar to those obtained by analyzing the data of Suzuki (1998, 2012), the former are more plausible because the data utilized by Pulkkinen & Tuominen (1998a) are more reliable in observations and much longer in time than those of Suzuki (1998, 2012). Thus in the following we mainly focus on the former.
The differential rotation of the solar atmosphere has a periodical pattern of change. Such a pattern can be described by the so-called torsional oscillation, in which the solar differential rotation should be cyclically speeded up or slowed down in certain zones of latitude while elsewhere the rotation remains essentially steady (Snodgrass & Howard 1985; Li et al. 2008, 2012). The zones of anomalous rotation move on the Sun in wavelike fashion, keeping pace with and flanking the zones of magnetic activity (LaBonte & Howard 1982; Snodgrass & Howard 1985). The surface torsional pattern, and perhaps the magnetic activity as well, are only the shadows of another unknown phenomenon occurring within the convection zone (Snodgrass 1987; Li et al. 2008). In this investigation, the internal-cycle variation of solar differential rotation can be explained as follows on the framework of the surface torsional pattern, which is briefly stated above, and by the magnetic activity together.

Brajiša et al. (2006) investigated solar-cycle-related variations in the solar rotation rate. They found a higher than average rotation velocity in the minimum time of a Schwabe cycle, and a plausible interpretation was then given. When magnetic fields are weaker, one can expect a more pronounced differential rotation yielding a higher rotation velocity at low latitudes on average (Brajiša et al. 2006). As Figures 2 and 6 show, more pronounced differentiation of rotation rates appears in the first four years of a solar cycle rather than in the second four years, due to weaker magnetic fields appearing in the first four years. Strong magnetic fields should repress differentiation, but weak magnetic fields seem to just reflect differentiation of rotation rates. Further, weak magnetic fields may more effectively reflect differentiation at low latitudes with high rotation rates than at high latitudes with low rotation rates, and strong magnetic fields may more effectively repress differentiation at relatively low latitudes than at high latitudes. As Figures 2 and 6 show, coefficient $B$ may be divided into three parts. Phase one spans from the start to the fourth year of a solar cycle. The absolute $B$ is approximately a constant or slightly fluctuates. Relatively high latitudes and relatively weak magnetic fields at this time interval make the repress action of sunspots less obvious than after this interval. Phase two spans from the fourth to the seventh year and absolute $B$ decreases. When solar activity is progressing into this phase, sunspots appear at increasingly lower latitudes, magnetic fields repress differentiation increasingly more effectively, and differentiation appears increasingly less conspicuously, thus the absolute $B$ decreases within this phase. Phase three spans from the seventh year to the end of a solar cycle. Within this phase, magnetic fields become weaker, and they repress differentiation less effectively. Sunspots appearing at increasingly lower latitudes lead to the conclusion that differentiation reflected by latitudinal migration should be more conspicuous, thus, the absolute $B$ increases (Li et al. 2012). In sum, the internal-cycle variation of solar differential rotation is inferred to be the result of both latitudinal migration of the surface torsional pattern and repression of strong magnetic activity. This means that measurements of differential rotation should be different at different phases of a Schwabe cycle and/or at different latitudes (different spatial positions of observed objects on the solar disk), which is the main reason why too many different results about solar differential rotation exist at present (Howard 1984; Schröter 1985; Snodgrass 1992; Paterno 2010; Li et al. 2012).

Solar differential rotation is not a fossil but is proposed to be generated and continuously maintained by the angular momentum transport from higher latitudes toward the equator (Pulkkinen & Tuominen 1998a). Measurements from the GPR by Ward (1965) revealed the existence of this transport, mostly through the Reynolds stress. Vršnak et al. (2003) found a statistically significant correlation between rotation residuals and meridional motions, through tracing “point-like structures” (predominantly young coronal bright points, CBPs), indicating the existence of the equatorward transport of angular momentum. Tracing of CBPs provides an extension of the Reynolds stress analysis to high latitudes which could be an important tool for investigating the dependence of the Reynolds stress on the latitude (Vršnak et al. 2003). CBPs are interesting features to be used for the rotation estimation along a solar cycle since they appear even on the full disk and at both minimum and maximum cycle phases (Zaatri et al. 2009; Wöhl et al. 2010). The torsional oscillation that was found by Howard & LaBonte (1980) indicated that latitudinal motions should exist, and they may result from both hydrodynamic circulation and the solar magnetic cycle, therefore the Reynolds stress is strongly present (Tuominen et al. 1983; Rüdiger et al. 1986; Pulkkinen et al. 1993; Pulkkinen & Tuominen 1998a). The horizontal Reynolds stress should be a function (dependence) of not only the gradients of rotation but also rotation itself (Pulkkinen & Tuominen 1999b). We propose here that the spacial variations of the solar magnetic activity act on differential rotation in different ways (see the explanation in Figures 2 and 6). The latitudinal shift velocity should be related to differential rotation, accordingly supporting this dependence.

A comparison of Figure 8 with Figure 6 shows that the internal-cycle variation (isopleth lines) of the solar rotation rates looks similar in profile to that of coefficient $B$, and the former is mainly reflected by the latter. The strong magnetic fields should hinder the torsional oscillation pattern migrating toward the equator, and a block is thus formed in the migration “river.” The block peaks when $B$ peaks. The internal-cycle variation of the solar rotation rates is inferred to be the result of both latitudinal migration of the surface torsional pattern and repression of strong magnetic activity for differentiation. Indeed the surface torsional pattern, and perhaps the magnetic activity as well, are only the shadows of another unknown phenomenon occurring within the convection zone (Snodgrass 1987; Li et al. 2008).

Based on the above explanation about the variation profile of $B$ within a solar cycle, the temporal distribution of magnetic activity strength in a solar cycle should give rise to $B$ periodically distributed in a solar cycle, but the spacial distribution of magnetic activity strength should disturb $B$ to form such a solar-cycle-period distribution. Therefore, the variation profile of the coefficient $B$ in a solar cycle (see Figures 2 and 6) is similar to but obviously different from the variation profile of sunspot numbers in a solar cycle. This seemingly implies that the coefficient $B$ should possibly have a not-too-strong or weak relation with sunspot activity at the scale of solar cycles. We calculated the correlation coefficient of yearly sunspot numbers respectively with the coefficients $A$ and $B$ determined through the data of Suzuki (1998, 2012), and resultantly it is $0.521$ for $B$, which is statistically significant at a confidence level of 95%. For $A$ it is $-0.384$, which is of no significance. However, Jurdana-Šepić et al. (2011) recently found that coefficient $A$ is significantly related to sunspot activity, while $B$ is not related, by tracing small bright coronal structures. Chandra et al. (2010) found that $B$ should not show any systematic variations for the soft X-ray corona, which is in agreement with the results of Jurdana-Šepić et al. (2011). The correlation of sunspot activity with the rotation coefficients determined by tracing sunspots contradicts
the idea that they are determined by tracing coronal activity events. The possible reason for is inferred to be that the coronal magnetic field is much weaker than the sunspot magnetic field. For timescales longer than solar cycles, coefficient $A$ is found to be negatively correlated with sunspot activity, while $B$ should be barely correlated (Javaraiah et al. 2005a, 2005b; Javaraiah & Ulrich 2006; Chandra & Vats 2011; Li et al. 2012).

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REFERENCES

Altrock, R. C. 2003, SoPh, 213, 23
Antia, H. M., & Basu, S. 2001, ApJL, 559, L67
Balthasar, H., & Wöhl, H. 1980, A&A, 92, 111
Brajša, R., Ruždjak, D., & Wöhl, H. 2006, SoPh, 237, 365
Brajša, R., Ruždjak, V., Vršnak, B., et al. 2000, SoPh, 196, 279
Brajša, R., Wöhl, H., Vršnak, B., et al. 2002, SoPh, 206, 229
Chandra, S., & Vats, H. O. 2011, MNras, 414, 3158
Chandra, S., Vats, H. O., & Iyer, K. N. 2010, MNRAS, 407, 1108
Chu, Z., Zhang, J., Nie, Q. H., & Li, T. 2010, SoPh, 264, 1
Gilman, P., & Howard, R. 1984, ApJ, 283, 385
Howard, R. 1984, ARA&A, 22, 151
Howard, R., Gilman, P. A., & Gilman, P. I. 1984, ApJ, 283, 373
Howe, R., & LaBonte, B. J. 1980, ApJL, 239, L33
Howe, R., Christensen-Dalsgaard, J., Hill, F., et al. 2000a, Sci, 287, 2456
Howe, R., Christensen-Dalsgaard, J., Hill, F., et al. 2000b, ApJL, 533, L163
Howe, R., Christensen-Dalsgaard, J., Hill, F., et al. 2009, ApJL, 701, L87

Javaraiah, J. 2003, SoPh, 212, 23
Javaraiah, J., Bertello, L., & Ulrich, R. K. 2005a, ApJ, 626, 579
Javaraiah, J., Bertello, L., & Ulrich, R. K. 2005b, SoPh, 232, 25
Javaraiah, J., & Ulrich, R. K. 2006, SoPh, 237, 245
Jurdana-Sepić, R., Brajša, R., Wöhl, H., et al. 2011, A&A, 534, A17
Komm, R. W., Howard, R. F., & Harvey, J. W. 1993, SoPh, 143, 19
LaBonte, B. J., & Howard, R. 1982, SoPh, 75, 161
Le Mouël, J. L., Shnirman, M. G., & Blanter, E. 2007, SoPh, 246, 295
Li, K. J., Feng, W., Shi, X. J., et al. 2013, SoPh, 246, 295
Li, K. J., Li, Q. X., Gao, P. X., & Shi, X. J. 2008, JGR, 113, A11108
Li, K. J., Liang, H. F., Feng, W., & Zhan, L. S. 2011a, ApSS, 331, 441
Li, K. J., Shi, X. J., Feng, W., et al. 2012, MNRAS, 423, 3584
Li, K. J., Shi, X. J., Liang, H. F., et al. 2011b, ApJL, 730, 49
Newton, H. W., & Nunn, M. L. 1951, MNRAS, 111, 413
Paterno, L. 2010, Ap&SS, 328, 269
Pulkkinen, P., & Tuominen, I. 1998a, A&A, 332, 748
Pulkkinen, P., & Tuominen, I. 1998b, A&A, 332, 755
Pulkkinen, P., Tuominen, I., Brandenburg, A., Nordlund, Å., & Stein, R. F. 1993, A&A, 267, 265
Rüdiger, G., Tuominen, I., Krause, F., & Virtanen, H. 1986, A&A, 166, 306
Rybak, J. 1994, SoPh, 152, 161
Schröter, E. H. 1985, SoPh, 100, 141
Sheeley, N. R., Jr., Wang, Y. M., & Nash, A. G. 1992, ApJ, 401, 378
Snodgrass, H. B. 1987, SoPh, 110, 35
Snodgrass, H. B. 1992, in ASP Conf. Ser. 27, The Solar Cycle, ed. K. L. Harvey (San Francisco, CA: ASP), 205
Snodgrass, H. B., & Howard, R. 1985, Sci, 228, 945
Song, W. B., & Wang, J. X. 2005, ApJL, 624, L137
Stix, M. 2002, The Sun (Berlin: Springer), 277
Suzuki, M. 1998, SoPh, 176, 259
Suzuki, M. 2012, SoPh, 278, 257
Tuominen, J., Tuominen, I., & Kyröläinen, J. 1983, MNRAS, 205, 691
Ulrich, R. K., & Bertello, L. 1996, ApJL, 465, L65
Vršnak, B., Brajša, R., Wöhl, H., et al. 2003, A&A, 404, 1117
Ward, F. 1965, ApJ, 141, 534
Wöhl, H., Brajša, R., Hanslmeier, A., & Gissot, S. F. 2010, A&A, 520, 29
Wöhl, H., & Schmidt, W. 2000, A&A, 357, 763
Zaaatri, A., Wöhl, H., Roth, M., Corbard, T., & Brajša, R. 2009, A&A, 504, 589