CKM matrix and CP violation in B-mesons

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Abstract

Planned originally as a review of CP violation (CPV) in B-mesons which covered recent B-factories results these lectures turned out to be a bit wider. It is not natural to be limited by CPV in decays and mixings of B-mesons and not to speak about the analogous phenomena in K-mesons since it is very useful and interesting to study what is common and what is different in these systems and why. CKM matrix elements are extracted from K and B mixings and decays and the deviation from unitarity of CKM matrix may become the place in which New Physics will show up. So we discuss this simple and elegant piece of Standard Model as well.

In order to follow these lectures you should be able to write the Lagrangian and to draw the Feynman diagrams in a Standard Model and to calculate the corresponding amplitudes.
Plan of the lectures

1. “The road map”
2. CKM matrix – where from?
3. CKM matrix: angles, phases, parametrization, unitarity triangles
4. $V_{us}, V_{cb}, V_{ub}$ – first circle
5. CPV: history; why phases are relevant
6. $M^0 - \bar{M}^0$ mixing, CPV in mixing ($|\frac{\alpha}{\rho}| \neq 1$)
7. Space-time pattern of $K^0 - \bar{K}^0$ ($B^0 - \bar{B}^0$, $\nu_{\mu} - \nu_{e}$) oscillations
8. $K^0 - \bar{K}^0$ mixing, $\Delta m_{LS}$
9. CPV in $K^0 - \bar{K}^0$ mixing, $K_L \to 2\pi$ decay, $\varepsilon_K$ - hyperbola
10. Direct CPV in K decays, $\varepsilon' \neq 0$ ($|\frac{\delta}{\rho_A}| \neq 1$)
11. $B^0_d - \bar{B}^0_d$, $B^0_s - \bar{B}^0_s$ mixings – two circles
12. CPV in $B^0 - \bar{B}^0$ mixing, $a_{SL}^{(B_s)}$ – too small effects
13. CPV in interference of mixing and decay ($Im(\frac{\delta}{\rho_A}) \neq 0$)
14. $B^0_d(\bar{B}^0_d) \to J/\psi K$, sin $2\beta$ - straight lines
15. $B \to \pi\pi$, sin $2\alpha$, penguin versus tree, $|\frac{\alpha}{\rho_A}| \neq 1$
16. Angle $\gamma$
17. CPV in $B \to \phi K_S$, $K^+K^-K_S, \eta'/K_S$: penguin domination
18. Conclusions: CKM fit and future prospects
1 “The road map”

In Fig.1 you can see a set of bounds on the parameters $\bar{\rho}$ and $\bar{\eta}$ ($\bar{\rho}$ and $\bar{\eta}$ are defined by eq.(3.10)) of the Cabibbo-Kobayashi-Maskawa [1, 2] quark mixing matrix (CKM). They comprise three circles, two branches of a hyperbola, and two straight lines. Three circles originate from the $V_{ub}$ (for definition of $V_{ub}$ see eq.(3.4)) measurement (the green one with the center at $\bar{\rho} = \bar{\eta} = 0$, see eq.(4.7)), the measurement of $\Delta m_{B_d}$ – the mass difference of two energy eigenstates of $B_d$ and $\bar{B}_d$ mesons (the red one, eq.(11.16) with the center at $\bar{\rho} = 1$, $\bar{\eta} = 0$) and from the lower bound on $\Delta m_{B_s}$ which is the same for $B_s$ and $\bar{B}_s$ mesons (the yellow one, eq.(11.20)). The hyperbola originates from the measurement of CP violation – CPV – in the mixing of K-mesons (see eq.(9.9)). Straight lines come from the measurement of CP asymmetry in $B^0_d(\bar{B}^0_d) \to J/\Psi K$ decays (see eq.(14.11)). The fact that all three circles, hyperbola and straight lines intersect at one and the same place means the triumph of Standard Model. Our aim is to explain in these lectures where all these bounds come from.

2 CKM matrix – where from?

In constructing the Standard Model Lagrangian the basic ingredients are 1. gauge group, 2. particle content and 3. renormalizability of the theory. There is no such a building block in Standard Model as CKM matrix in charged currents quark interactions. CKM matrix originates from Higgs field interactions with quarks. The piece of the Lagrangian from which the up quarks get their masses looks like:

$$\Delta \mathcal{L}_{up} = f_{ik}^{(u)} Q^i_L u^k_R H + c.c. , \quad i, k = 1, 2, 3 \quad (2.1)$$

where

$$Q^i_L = \begin{pmatrix} u^i_L \\ d^i_L \end{pmatrix} , \quad Q^2_L = \begin{pmatrix} c^i_L \\ s^i_L \end{pmatrix} , \quad Q^3_L = \begin{pmatrix} t^i_L \\ b^i_L \end{pmatrix} ;$$

$$u^i_R = u^i_R , \quad u^2_R = c^i_R , \quad u^3_R = t^i_R \quad (2.2)$$

and $H$ is the Higgs doublet:

$$H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix} \quad (2.3)$$
Figure 1: The domains at $(\bar{\rho}, \bar{\eta})$ plane allowed at 1$\sigma$ from $V_{ub}$, $\Delta m_{B_d}$, $\varepsilon_K$ and $\sin 2\beta$ measurements. 95% C.L. upper bound from the search of $\Delta m_{B_s}$ is shown as well.

The piece of the Lagrangian which is responsible for the down quark masses looks the same way:

$$\Delta \mathcal{L}_{\text{down}} = \sum_{ik} f_{ik}^{(d)} \bar{Q}_L i d'_{R} \widetilde{H} + \text{c.c.} ,$$  \hspace{1cm} (2.4)$$

where

$$d'_{R} = d'_{R} , \quad d''_{R} = s'_{R} , \quad d'''_{R} = b'_{R} \quad \text{and} \quad \widetilde{H}_a = \varepsilon_{ab} H^*_b ,$$  \hspace{1cm} (2.5)$$

$$\varepsilon_{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} .$$

After $SU(2) \times U(1)$ symmetry breaking by the Higgs field expectation value $< H^0 > = v$ from formulas (2.1) and (2.4) two mass matrices emerge:

$$M_{\text{up}}^{ik} \bar{u}_L^i u_R^k + M_{\text{down}}^{ik} \bar{d}_L^i d_R^k + \text{c.c.} . \hspace{1cm} (2.6)$$
The matrices $M_{\text{up}}$ and $M_{\text{down}}$ are arbitrary $3 \times 3$ matrices; their matrix elements are complex numbers. According to the well-known theorem an arbitrary matrix can be written as a product of the hermitian and unitary matrices:

$$M = UH,$$

where $H = H^+$, and $UU^+ = 1$, \hspace{1cm} (2.7)

(do not mix the hermitian matrix $H$ with the Higgs field doublet) which is analogous to the following representation of an arbitrary complex number:

$$a = e^{i\phi} |a|.$$

(2.8)

From eq. (2.7) it is evident that matrix $M$ can be diagonalized by 2 different unitary matrices acting from left and right:

$$U_LMU_R^+ = M_{\text{diag}} = \begin{pmatrix} m_u & 0 \\ 0 & m_c \\ 0 & m_t \end{pmatrix},$$

(2.9)

where $m_i$ are real numbers (if matrix $M$ is hermitian ($M = M^+$) then we will get $U_L = U_R$). Having these formulas in mind let us rewrite the up-quarks mass term from eq. (2.6):

$$\bar{u}_L^iM_{ik}u_R^k + \text{c.c.} \equiv \bar{u}_L^iU_L^+MU_R^+U_Ru_R^i + \text{c.c.} = \bar{u}_L^iM_{\text{diag}}u_R^i + \text{c.c.} = \bar{u}_L^iM_{\text{diag}}u_R^i,$$

(2.10)

where we introduce the fields $u_L$ and $u_R$ according to the following formulas:

$$u_L = U_Lu_L^i, \quad u_R = U_Ru_R^i.$$

(2.11)

Applying the same procedure to matrix $M_{\text{down}}$ we observe that it becomes diagonal as well in the rotated basis:

$$d_L = D_Ld_L^i, \quad d_R = D_Rd_R^i.$$

(2.12)

Thus we start from the primed quark fields and get that they should be rotated by 4 unitary matrices $U_L$, $U_R$, $D_L$ and $D_R$ in order to obtain unprimed fields with diagonal masses. Since kinetic energies and interactions with the vector fields $A_\mu^3$, $B_\mu$ and gluons are diagonal in the quark fields, these terms remain diagonal in a new unprimed basis. The only term in the SM Lagrangian where matrices $U$ and $D$ show up is charged current interaction with the emission of $W$-boson:

$$\Delta \mathcal{L} = gW_\mu^+\bar{u}_L^i\gamma_\mu d_L^i = gW_\mu^+\bar{u}_L^i\gamma_\mu U_L^+D_Ld_L,$$

(2.13)

and the unitary matrix $V \equiv U_L^+D_L$ is called Cabibbo-Kobayashi-Maskawa quark mixing matrix.
3 CKM matrix: angles, phases, parametrization, unitarity triangles

One can easily check that $n \times n$ unitary matrix has $n^2/2$ complex or $n^2$ real parameters. The orthogonal $n \times n$ matrix is specified by $n(n - 1)/2$ angles (3 Euler angles in case of $O(3)$). That is why the parameters of the unitary matrix are divided between phases and angles according to the following relation:

$$n^2 = \frac{n(n-1)}{2} + \frac{n(n+1)}{2}.$$ (3.1)

angles phases

The next question arises: are all these phases physical observables or, in other words, can they be measured experimentally. And the answer is “no” since we can perform phase rotations of quark fields ($u \to e^{i\xi}u$, $d \to e^{i\xi}d$ ...) removing in this way $2n - 1$ phases of the CKM matrix. The number of unphysical phases equals that of up and down quark fields minus one since the simultaneous rotation of all up-quarks on one and the same phase multiplies by (minus) this phase all the matrix elements of matrix $V$. The rotation of all down-quark fields on one and the same phase acts on $V$ in the same way. That is why the number of the “unremovable” phases of matrix $V$ is diminished by the number of possible rotations of up and down quarks minus one.

Finally for the number of observable phases we get:

$$\frac{n(n+1)}{2} - (2n - 1) = \frac{(n - 1)(n - 2)}{2}.$$ (3.2)

As you see for the first time one observable phase arrives in the case of 3 quark-lepton generations.

Now a bit of history. At the time when Cabibbo used the mixing angle $\theta_c$ [1] only three quarks ($u$, $d$ and $s$) were known, and his suggestion was to mix $d$- and $s$-quarks in the expression for the charged quark current:

$$J_{\mu}^+ = \bar{u}\gamma_{\mu}(1 + \gamma_5)\left[d\cos\theta_c + s\sin\theta_c\right].$$ (3.3)

In this way he related the suppression of the strange particles weak interactions to the smallness of angle $\theta_c$, $\sin^2\theta_c \approx 0.05$. After the establishment of Glashow-Salam-Weinberg theory of the weak interactions through the weak
neutral current discovery in 1973 and discovery of a charm quark in 1974 it became clear, that 2 quark-lepton generations exist. From the point of view of GSW theory the mixing of quark generations should be described by the unitary $2 \times 2$ matrix which according to eqs. (3.1, 3.2) has one angle and zero observable phases. This angle is Cabbibo angle. However even before the $c$-quark discovery Kobayashi and Maskawa noticed that in order to describe CP-violation (CPV) Standard Model needs at least 3 quark-lepton generations since for the first time the observable phase shows up for $n = 3$ [2]. At that time CPV was known only in $K^0$-mesons and to test KM mechanism one needed other systems. Finally almost 30 years after KM model of CP violation was suggested it was confirmed by the magnitude of CPV in neutral $B$-mesons.

In the case of three generations the matrix of charged currents looks like:

$$V = R_{23} \times R_{13} \times R_{12},$$

where the matrix elements $V_{ik}$ depend on four parameters: three angles and one phase. Let us present one possible way of the parametrization of matrix $V$, which is called “standard” parametrization (straightforwardly generalizable for $n > 3$ [3]). It is achieved by consequent rotations in planes (12), (13) and (23) and the rotation in plane (13) is accomplished by the phase rotation. Performed in such a way the phase rotation cannot be removed by $U(1)$ rotations of the quark fields:

$$R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix},$$

$$R_{13} = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}, \quad R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

and, finally:

$$V = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}.$$
As the next step we can take the experimental values for $V_{ik}$ and extract three angles $\theta_{12}$, $\theta_{23}$, $\theta_{13}$ and phase $\delta$ from them. The system is overconstrained; we have more than 4 experimental numbers (see below) and in this way one can check how good CKM model in describing data is. However it appeared to be useful to reparametrize $V_{ik}$ with the help of the so-called Wolfenstein parametrization. Let us introduce new parameters $\lambda$, $A$, $\rho$ and $\eta$ according to the following definitions:

$$\lambda \equiv s_{12}, \quad A \equiv \frac{s_{23}}{s_{12}}, \quad \rho = \frac{s_{13}}{s_{12}s_{23}} \cos \delta,$$

$$\eta = \frac{s_{13}}{s_{12}s_{23}} \sin \delta,$$

and get expressions for $V_{ik}$ through $\lambda$, $A$, $\rho$ and $\eta$:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta}) \\ -\lambda - iA\lambda^5\bar{\eta} & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 - iA\lambda^4\bar{\eta} & 1 \end{pmatrix},$$

(3.8)

where

$$\bar{\rho} \equiv \rho(1 - \frac{\lambda^2}{2}) \quad \bar{\eta} \equiv \eta(1 - \frac{\lambda^2}{2}),$$

(3.10)

Obtaining the last expression we take into account the following hierarchy of angles $\theta_{ij}$: $s_{13} \ll s_{23} \ll s_{12} \ll 1$ (see below). This last form of CKM matrix is very convenient for qualitative estimates and numerically is rather accurate.

The unitarity of the matrix $V$ leads to the following six equations that can be drawn as triangles on a complex plane (under each term in these equations the power of $\lambda$ entering it, is shown):

$$V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} \sim 0,$$

(3.11)

$$V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} \sim 0,$$

(3.12)

$$V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} \sim 0,$$

(3.13)
\begin{align}
V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* &= 0, \\
\sim \lambda & \sim \lambda \sim \lambda^5 \tag{3.14} \\
V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* &= 0, \\
\sim \lambda^3 & \sim \lambda^3 \sim \lambda^3 \tag{3.15} \\
V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* &= 0. \\
\sim \lambda^4 & \sim \lambda^2 \sim \lambda^2 \tag{3.16}
\end{align}

Among these triangles the four are almost degenerate: one side is much shorter than two others, and two triangles, expressed by equations (3.12) and (3.15), have all three sides of more or less equal lengths, of the order of $\lambda^3$. These two nondegenerate triangles almost coincide. To prove this statement let us note that with very good accuracy $V_{ud} = V_{tb} = 1$, that is why two sides of these triangles have equal lengths and directions. That is why the triangles coincide and their third sides should also be equal:

\begin{align}
V_{cd}^*V_{cb} = V_{us}V_{ts}^* \tag{3.17}
\end{align}

Now, since $V_{us}$ and $-V_{cd}$ are almost equal to each other (and to $\lambda$) we come to the following result:

\begin{align}
V_{ts}^* = -V_{cb} \tag{3.18}
\end{align}

the validity of which can be checked by eq.(3.9).

So, as a result we have only one nondegenerate unitarity triangle; it is usually described by a complex conjugate of our equation (3.12):

\begin{align}
V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* &= 0 \tag{3.19}
\end{align}

and it is shown in Fig.2. It has the angles which are called $\beta$, $\alpha$ and $\gamma$ (according to BaBar collaboration) or $\phi_1$, $\phi_2$ and $\phi_3$ (according to Belle collaboration). They are determined from CPV asymmetries in $B$-mesons decays.

Looking at Figure 2 one can easily obtain the following formulas:

\begin{align}
\beta &= \pi - \arg \frac{V_{ub}^*V_{id}}{V_{cb}^*V_{cd}} = \phi_1 \tag{3.20} \\
\alpha &= \arg \frac{V_{tb}^*V_{id}}{-V_{ub}^*V_{ud}} = \phi_2 \tag{3.21}
\end{align}

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Figure 2: Unitarity triangle

\[ \gamma = \arg \left( \frac{V_{ub}^* V_{ud}}{-V_{cb}^* V_{cd}} \right) = \phi_3. \] (3.22)

Angle $\beta$ ($\phi_1$) was directly measured through time dependent CPV asymmetry in $B_d \to J/\Psi K$ decays, $\alpha$ ($\phi_2$) has been measured recently with a rather poor accuracy from CPV asymmetries in $B_d \to \pi^+\pi^-$ decays and, finally, $B_s$ decays could be important to determine angle $\gamma$ ($\phi_3$).

Let us make three final remarks about the unitarity triangle:

1. In standard parametrization which we use $V_{cd}V_{cb}^*$ is almost real;

2. Multiplication of any up quark field on a phase does not change the unitarity triangle while multiplying $d$- or $b$- quark field on a phase we rotate it as a whole not changing its angles which are physical observables;

3. Usually a rescaled triangle is used. We get it dividing all three sides by $|V_{cb}^* V_{cd}| \approx \lambda^3$. In this way the length of the triangle basement becomes equal to one while two other sides have the length of the order of one.

Four quantities are needed to specify CKM matrix: $s_{12}, s_{13}, s_{23}$ and $\delta$, or $\lambda, A, \rho, \eta$. Knowing more we are checking Standard Model and looking for New Physics.
4 \( V_{us}, V_{cb}, V_{ub} - \text{first circle} \)

The most precise value for the quantity \( V_{us} \) follows from the extrapolation of the formfactor of \( K \rightarrow \pi e \nu \) decay \( f_+(q^2) \) to the point \( q^2 = 0 \), where \( q \) is the lepton pair momentum. Due to the Ademollo-Gatto theorem corrections to the CVC value \( f_+(0) = 1 \) are of the second order of flavor SU(3) violation, and these small terms were calculated in [4]. As a result of this analysis Particle Data Group gives the following value [5]:

\[
V_{us} \equiv \lambda = 0.2196 \pm 0.0026 .
\]  

(4.1)

The accuracy of \( \lambda \) is high: the other parameters of CKM matrix are known much worse.

The value of \( V_{cb} \) is determined from the inclusive and exclusive semileptonic decays of \( B \)-mesons with charmed particles production. At the level of quarks \( b \rightarrow ce \nu \) transition is responsible for these decays.

According to PDG [5]:

\[
V_{cb} = (41.2 \pm 2.0)10^{-3} ;
\]  

(4.2)

and the error is dominated by a theoretical one.

From eq.(3.9) with the help of (4.2) for parameter \( A \) we get:

\[
A = \frac{V_{cb}}{\lambda^2} = 0.85 \pm 0.04 .
\]  

(4.3)

From the formula for the semileptonic width:

\[
\Gamma_{SL} \sim |V_{cb}|^2 (m_b - m_c)^5
\]  

(4.4)

it is clearly seen that 4\% error in \( V_{cb} \) corresponds to the knowledge of quark mass difference with the very high accuracy:

\[
m_b - m_c = 3 \text{ GeV} \pm 60 \text{ MeV} .
\]  

(4.5)

Vast literature is devoted to the different aspects of the \( V_{cb} \) determination. Not going into details let us only note that at the moment the error in \( V_{cb} \) is not a “bottle neck” since the errors in other relevant quantities (\( V_{ub}, V_{td}, \sin 2\beta \)) are significantly larger.

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The value of $|V_{ub}|$ is extracted from the semileptonic $B$-mesons decays without the charmed particles in the final state which originated from $b \to ul\nu$ transition. $b \to cl\nu$ decays are approximately 100 times more probable than $b \to ul\nu$ and for their suppression the high energy charged lepton tail is examined (the energetic leptons cannot accompany heavy $D$-mesons). Theoretical analysis of such seminclusive decays is highly involved and it leads to a large theoretical uncertainty [5]:

$$| \frac{V_{ub}}{\lambda V_{cb}} | = 0.40 \pm 0.08 . \quad (4.6)$$

There are hopes to considerably diminish this error using the exclusive modes $B \to \pi e\nu$ and $B \to \omega e\nu$. (The recent study of the $B \to \pi e\nu$ and $B \to \rho e\nu$ decays by CLEO collaboration gives 10% smaller central value of $V_{ub}$ with practically the same error [6].

Using equation (3.9) we obtain a bound on the parameters $\tilde{\rho}$ and $\tilde{\eta}$:

$$\sqrt{\tilde{\rho}^2 + \tilde{\eta}^2} = 0.40 \pm 0.08 , \quad (4.7)$$

which produces a circle on $(\tilde{\rho}, \tilde{\eta})$ plane with the center at the point $(0,0)$. The area between such two circles in Fig.1 corresponds to the domain allowed at one sigma.

The detailed articles on $V_{ub}$ and $V_{cb}$ determination with bibliography can be found in [5]; for the recent review see [7].

To finish this section let us look at the value of $|V_{ud}|$. Its value extracted from the neutron decays has the smallest theoretical uncertainty [8]:

$$| V_{ud} | = 0.9713(13) ; \quad (4.8)$$

using this value as well as (4.1), (4.2) and (4.6) we obtain:

$$| V_{ud} |^2 + | V_{us} |^2 + | V_{ub} |^2 = 0.9434(25) + 0.0482(11) + 0.00001 =$$

$$= 0.9916(27) , \quad (4.9)$$

and the unitarity of CKM matrix is violated at $3\sigma$ level. This can be a signal for New Physics, while it may be a statistical fluctuation as well. PDG discussing the value of $V_{ud}$ takes into account the nuclear beta decays as well resulting in [5]:

$$| V_{ud} | = 0.9734(8) , \quad (4.10)$$

which diminishes the violation of unitarity to $2\sigma$. 

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5 CPV: history; why phases are relevant

In 1956 Lee and Yang in order to solve $\theta - \tau$ problem suggested that P-parity is broken in weak interactions [9]. Soon it was noted that C-invariance is also broken in a proposed theory [10].

In 1957 looking for a way to resurrect P-invariance L.D. Landau stated that weak interactions should be invariant under the product of P reflection and C conjugation. He called this product the combined inversion and according to him it should substitute $P$-inversion broken in weak interactions [11]. In this way the theory should be invariant when together with changing sign of the coordinate, $\vec{r} \rightarrow -\vec{r}$, one changes an electron to positron, proton to antiproton and so on.

It is clearly seen from paper [11] that according to Landau CP-invariance should become a basic symmetry for physics in general and weak interactions in particular.

This beautiful picture did not stop experimentalists (or may be even stimulated) and in 1964 CP violating decay of the long-lived neutral kaon on two pions was discovered [12]. The question of the violation of P-parity in weak interactions which stimulated Landau was: why is P violated? Landau’s answer to the question “Why is parity violated in weak interactions” was: because CP, not P is the fundamental symmetry of nature. A modern answer to the same question is: because in P-invariant theory with the Dirac fermions the gauge invariant mass terms can be written for quarks and leptons which are not protected of being of the order of $M_{\text{GUT}}$ or $M_{\text{Planck}}$. So in order to have our world made from light particles P-parity should be violated.

$K_L \rightarrow 2\pi$ decay discovered in 1964 occurs due to CPV in the mixing of neutral kaons ($\varepsilon \neq 0$). Only thirty years later the second major step was done: direct CPV was observed in kaon decays [13]:

$$\frac{\Gamma(K_L \rightarrow \pi^+\pi^-)}{\Gamma(K_S \rightarrow \pi^+\pi^-)} \neq \frac{\Gamma(K_L \rightarrow \pi^0\pi^0)}{\Gamma(K_S \rightarrow \pi^0\pi^0)}, \quad \varepsilon' \neq 0 \quad (5.1)$$

Finally, in the year 2001 CPV was for the first time observed out of the decays of neutral kaons: the time dependent CP-violating asymmetry in $B^0$ decays was measured [14]:

$$a(t) = \frac{N(B^0 \rightarrow J/\Psi K_{S(L)}) - N(\bar{B}^0 \rightarrow J/\Psi \bar{K}_{S(L)})}{N(B^0 \rightarrow J/\Psi K_{S(L)}) + N(\bar{B}^0 \rightarrow J/\Psi \bar{K}_{S(L)})} \neq 0. \quad (5.2)$$
Starting from the year 1964 we know that there is no symmetry between particles and antiparticles. In particular, the $C$-conjugated partial widths are different:

$$\Gamma(A \rightarrow BC) \neq \Gamma(\bar{A} \rightarrow \bar{B}\bar{C}) . \quad (5.3)$$

However CPT (deduced from the invariance of the theory under 4-dimensional rotations) remains intact. That is why the total widths as well as the masses of particles and antiparticles are equal:

$$M_A = M_{\bar{A}}, \quad \Gamma_A = \Gamma_{\bar{A}} \quad \text{(CPT)} . \quad (5.4)$$

The consequences of CPV can be divided into macroscopic and microscopic. CPV is one of the three famous Sakharov’s necessary conditions to get as a result of evolution of charge symmetric Universe a charge nonsymmetric one [15]. In these lectures we will not discuss this very interesting and well developed branch of physics, but will deal with CPV in particle physics where the data obtained up to now confirm Kobayashi-Maskawa model of CPV. New data which should become available in coming years could well disprove it clearly demonstrating the necessity of physics beyond the Standard Model. Let us note that there exist many excellent reviews on CPV; a partial list of them can be found in ref. [16], which starts from the lectures on this topic given at ITEP Winter Schools in a chronological order.

The next question we wish to discuss is why the phases are relevant for CPV. Let us take the simplest example of one scalar particle decaying in two scalar particles. The interaction Lagrangian is:

$$\mathcal{L} = \lambda AB^*C^* + \lambda^*A^*BC . \quad (5.5)$$

Performing CP conjugation (changing particles to antiparticles and reflecting space coordinates) we obtain:

$$\mathcal{L}_{CP} = \lambda A^*BC + \lambda^*AB^*C^* , \quad (5.6)$$

which coincides with the original Lagrangian only if $\lambda$ is real, $\lambda^* = \lambda$. That is why the complex coupling constants are necessary for CPV. However, this complexity is not enough: in our example we can redefine field $A$ by phase rotation, $A = e^{i\alpha}A'$, in this way making $\lambda$ real. However, adding one new field (say $D$) we introduce many new couplings ($\lambda_1D_{BC} + \lambda_2DAB + \lambda_3DAC$) while by rotation $D = e^{i\alpha}D'$ a phase of only one coupling constant can be
eliminated. Thus to get CPV several fields are needed. The last statement is illustrated in Standard Model by the fact that at least three generations are needed to get CPV through the phases in the quark mixing matrix.

6 $M^0 - \bar{M}^0$ mixing; CPV in mixing

In this part the general formulas for the meson-antimeson mixing will be derived. In order to mix the mesons must be neutral, not coincide with its antiparticle and decay due to the weak interactions. There are four such pairs:

- $K^0(\bar{s}d) - \bar{K}^0(s\bar{d})$
- $D^0(c\bar{u}) - \bar{D}^0(\bar{c}u)$
- $B^0_d(\bar{b}d) - \bar{B}^0_d(b\bar{d})$
- $B^0_s(\bar{b}s) - \bar{B}^0_s(bs)$

Fast $t \to bW$ decay prevents forming $t$-quark containing hadrons.

Mixing occurs in the second order in weak interactions through the box diagram which is shown in Fig. 3 for $K^0 - \bar{K}^0$ pair.

```
Figure 3: $K^0 - \bar{K}^0$ mixing
```

The effective $2 \times 2$ hamiltonian $H$ is used to describe the meson-antimeson mixing. It is most easily written in the following basis: $M^0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\bar{M}^0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. The meson-antimeson system evolves according to the Shroedinger equation with this effective hamiltonian which is not hermitian since it takes...
meson decays into account. So, \( H = M - \frac{i}{2} \Gamma \), where both \( M \) and \( \Gamma \) are hermitian.

According to CPT invariance the diagonal elements of \( H \) are equal:

\[
< M^0 | H | M^0 > = < \bar{M}^0 | H | \bar{M}^0 > .
\]  

(6.1)

Substituting into the Schrödinger equation

\[
i \frac{\partial \psi}{\partial t} = H \psi
\]  

(6.2)

\( \psi \) - function in the following form:

\[
\psi = \begin{pmatrix} p \\ q \end{pmatrix} e^{-i \lambda t}
\]  

(6.3)

we come to the following equation:

\[
\begin{pmatrix}
M - \frac{i}{2} \Gamma & M_{12} - \frac{i}{2} \Gamma_{12} \\
M_{12}^* - \frac{i}{2} \Gamma_{12}^* & M - \frac{i}{2} \Gamma
\end{pmatrix}
\begin{pmatrix} p \\ q \end{pmatrix} = \lambda
\begin{pmatrix} p \\ q \end{pmatrix}
\]  

(6.4)

from which for eigenvalues \((\lambda_{\pm})\) and eigenvectors \((M_{\pm})\) we obtain:

\[
\lambda_{\pm} = M - \frac{i}{2} \Gamma \pm \sqrt{(M_{12} - \frac{i}{2} \Gamma_{12})(M_{12}^* - \frac{i}{2} \Gamma_{12}^*)},
\]  

(6.5)

\[
\begin{cases}
M_+ = p M^0 + q \bar{M}^0 \\
M_- = p M^0 - q \bar{M}^0
\end{cases}, \quad \frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}}}.
\]  

(6.6)

Multiplying \( M^0 \) by \( e^{i \phi} \) we are changing the phase of the ratio \( q/p \), that is why \( \text{arg} \left( \frac{q}{p} \right) \) is not a physical observable.

CP transforms field \( M^0 \) in the following way:

\[
C P M^0 = e^{i \alpha} M^0,
\]  

(6.7)

being defined up to the arbitrary phase \( \alpha \). We can rotate field \( M^0 \) by phase \( \alpha/2 \), removing phase \( \alpha \) from the definition of CP transformation. In this way we come to the standard definition of CP transformation:

\[
C P M^0 = \bar{M}^0,
\]  

(6.8)

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simultaneously loosing freedom of the arbitrary phase rotation of field \( M^0 \).

The eigenstates \( M_+ \) and \( M_- \) are not orthogonal in general case:

\[
\langle M_+ | M_- \rangle = \frac{p}{|q|} \neq 0.
\]  
(6.9)

However if there is no CPV in mixing, then:

\[
\langle M^0 | H | \bar{M}^0 \rangle = \langle \bar{M}^0 | H | M^0 \rangle,
\]  
(6.10)

\[
M_{12} - \frac{i}{2} \Gamma_{12} = M_{12}^* - \frac{i}{2} \Gamma_{12}^*,
\]  
(6.11)

that is why:

\[
\frac{p}{q} = 1, \quad \langle M_+ | M_- \rangle = 0.
\]  
(6.12)

We observe the one-to-one correspondence between CPV in mixing and nonorthogonality of the eigenstates \( M_+ \) and \( M_- \). According to Quantum Mechanics if two hermitian matrices \( M \) and \( \Gamma \) commute, they have a common orthonormal basis. Let us calculate the commutator of \( M \) and \( \Gamma \):

\[
[M, \Gamma] = \begin{pmatrix}
M_{12} \Gamma_{12}^* - M_{12}^* \Gamma_{12} & 0 \\
0 & M_{12}^* \Gamma_{12} - M_{12} \Gamma_{12}^*
\end{pmatrix}.
\]  
(6.13)

It equals zero if the phases of \( M_{12} \) and \( \Gamma_{12} \) coincide modulo \( \pi \). So, for \( [M, \Gamma] = 0 \) we get \( |q/p| = 1, \langle M_+ | M_- \rangle = 0 \) and there is no CPV in the meson-antimeson mixing. And vice versa.

Introducing quantity \( \tilde{\varepsilon} \) according to the following definition:

\[
\frac{q}{p} = \frac{1 - \tilde{\varepsilon}}{1 + \tilde{\varepsilon}},
\]  
(6.14)

we see that if \( \text{Re} \, \tilde{\varepsilon} \neq 0 \), then CP is violated. From (6.6) for the eigenstates we obtain:

\[
M_+ = \frac{1}{\sqrt{1 + |\tilde{\varepsilon}|^2}} \left[ \frac{M^0 + \bar{M}^0}{\sqrt{2}} + \tilde{\varepsilon} \frac{M^0 - \bar{M}^0}{\sqrt{2}} \right],
\]  

\[
M_- = \frac{1}{\sqrt{1 + |\tilde{\varepsilon}|^2}} \left[ \frac{M^0 - \bar{M}^0}{\sqrt{2}} + \tilde{\varepsilon} \frac{M^0 + \bar{M}^0}{\sqrt{2}} \right].
\]  
(6.15)

If CP is conserved, then \( \text{Re} \, \tilde{\varepsilon} = 0 \), \( M_+ \) is CP even and \( M_- \) is CP odd (CP transformation should be defined according to (6.7) - (6.8)). If CP is violated in mixing, then \( \text{Re} \, \tilde{\varepsilon} \neq 0 \) and \( M_+ \) and \( M_- \) get admixtures of the opposite CP parity and become nonorthogonal.
7 Space-time pattern of $K^0 - \bar{K}^0$
$(B^0 - \bar{B}^0, \nu_\mu - \nu_e)$ oscillations

Neglecting CPV for the states with the definite masses and width we obtain:

\[
\begin{align*}
K_1 &= \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0), \\
K_2 &= \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0),
\end{align*}
\]

where CP$|K^0>$ = $|\bar{K}^0>$, CP$|\bar{K}^0>$ = $|K^0>$.

A plain wave which describes the propagation of the superposition of $K_1$ and $K_2$ with definite energies and momenta is:

\[
\Psi(x, t) = e^{-iE_1 t + ip_1 x} |K_1 > + e^{-iE_2 t + ip_2 x} |K_2 > ,
\]

\[
E_1^2 - p_1^2 = m_1^2, \quad E_2^2 - p_2^2 = m_2^2,
\]

and we consider the propagation in the direction of axis $x$. We should impose a boundary condition stating that at point $x = 0$ continuously in time $K^0$ is produced (or $B^0$, or $\nu_e$). The only way to get this is by putting $E_1 = E_2$, having at $x = 0$:

\[
e^{-iE t} |K_1 > + e^{-iE t} |K_2 > = e^{-iE t} [|K_1 > + |K_2 >] = e^{-iE t} |K^0 > .
\]

If we do not impose this boundary condition and take $E_1 \neq E_2$, then after time $\tau \sim \frac{1}{E_1 - E_2}$ at the point $x = 0$ $\bar{K}^0$ will emerge, while in the experiment only the events when $\bar{K}^0$ at $x = 0$ is produced are selected. For example, in CPLEAR we have: $p\bar{p} \rightarrow \pi K^- \bar{K}^0$ and $K^0$ is tagged by $K^-$, showing that $K^0$ was produced in the interaction point, and not $\bar{K}^0$. Another example: in the reaction $pn \rightarrow p\Lambda K^0$ together with $\Lambda$ hyperon $K^0$ is produced and not $\bar{K}^0$ because of strangeness conservation in strong interactions. In the case of antineutrino production in nuclear reactor initiated by $n \rightarrow pe^-\bar{\nu}_e$ transition it is always electron antineutrino and never muon antineutrino.

Substituting $E_1 = E_2 = E$ in equation (7.2) we get:

\[
\Psi(x, t) = e^{-iE t + ip_1 x} [|K_1 > + e^{i(p_2 - p_1)x} |K_2 >] .
\]

For the phase factor in brackets we have:

\[
p_2 - p_1 = \sqrt{E^2 - m_2^2} - \sqrt{E^2 - m_1^2} = \]

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\[
\sqrt{E^2 - m_1^2 + m_1^2 - m_2^2} - \sqrt{E^2 - m_1^2} = \frac{m_1^2 - m_2^2}{2p}, \quad (7.5)
\]

where we take into account that for mesons (as well as for neutrinos) always \(E^2 - m_1^2 \gg m_1^2 - m_2^2\). Therefore for a probability to detect \(K^0\) at a distance \(x\) from a production point we have:

\[
P_{K^0K^0} = \frac{1}{2} \left[ 1 + \cos \left( \frac{m_1^2 - m_2^2}{2p} x \right) \right] \bigg|_{\Delta m < m} = \frac{1}{2} \left[ 1 + \cos \left( \frac{\Delta m}{\beta \gamma} x \right) \right], \quad (7.6)
\]

\[\beta = v/c, \quad \gamma = 1/\sqrt{1 - \beta^2},\]

where the second equality in eq.(7.6) holds for mesons, but (may be) not for neutrinos.

There is no surprise that both neutrino and meson oscillations are described by the identical formulas. In the first paper where the neutrino oscillations were considered [17], B.M. Pontecorvo did it in full analogy with \(K^0 - \bar{K}^0\) oscillation analysis of Gell-Mann and Pais. Since at that time the second (muon) neutrino was not yet discovered Pontecorvo considered \(\nu_e - \bar{\nu}_e\) oscillations which were allowed because \(V - A\) theory was not established yet (in \(V - A\) theory \(\nu - \bar{\nu}\) oscillations are forbidden by chirality conservation).

In this particular case the diagonal elements of \(\nu - \bar{\nu}\) mixing matrix should be equal due to CPT and mixing is maximal, \(\theta_\nu = \pi/4\), just as for \(K^0(B_d, B_s)\) mesons.

### 8 \(K^0 - \bar{K}^0\) mixing, \(\Delta m_{LS}\)

\(\Gamma_{12}\) for the \(K^0 - \bar{K}^0\) system is given by the diagram shown in Fig. 4. With our choice of CKM matrix \(V_{us}\) and \(V_{ud}\) are real, so \(\Gamma_{12}\) is real.

\(M_{12}\) is given by dispersion part of the diagram shown in Fig.5. Now all three up quarks should be taken into account.

To calculate this diagram it is convenient to implement Glashow-Illiopulos-Maiani compensation mechanism from the very beginning, subtracting zero from the sum of the fermion propagators:

\[
\frac{V_{us}V_{ud}^*}{\hat{p} - m_u} + \frac{V_{cs}V_{cd}^*}{\hat{p} - m_c} + \frac{V_{is}V_{id}^*}{\hat{p} - m_t} - \frac{\sum_i V_{is}V_{id}^*}{\hat{p}}. \quad (8.1)
\]
Figure 4: Quark diagram responsible for $\Gamma_{12}$ in $K^0 - \bar{K}^0$ system

Figure 5: Quark diagram responsible for $M_{12}$ in $K^0 - \bar{K}^0$ system

Since $u$-quark is with good accuracy massless, $m_u \approx 0$, its propagator drops out and we are left with the modified $c$- and $t$-quark propagators:

$$\frac{1}{p - m_{c,t}} \rightarrow \frac{m_{c,t}^2}{(p^2 - m_{c,t}^2)p}.$$  \hspace{1cm} (8.2)

The modified fermion propagators decrease in ultraviolet so rapidly that one can calculate the box diagrams in the unitary gauge, where $W$-boson propagator does not decrease (one can demonstrate that the diagrams with the charged higgs exchange which occur in the renormalizable $R$ gauges, and for which eq.(8.1) does not work since the vertex of higgs emission is proportional to the quark masses, becomes zero in the limit $\xi \rightarrow \infty$, which
corresponds to the unitary gauge).

We easily get the following estimates for three remaining diagram contributions in $M_{12}$:

\begin{align}
(cc) & : \quad \lambda^2(1 + 2i\bar{\eta}A^4\lambda^4)G_F^2m_c^2, \\
(ct) & : \quad \lambda^6(1 - \bar{\rho} - i\bar{\eta})G_F^2m_c^2\ln\left(\frac{m_t}{m_c}\right)^2, \\
(tt) & : \quad \lambda^{10}(1 - \bar{\rho} - i\bar{\eta})^2G_F^2m_t^2.
\end{align}

(8.3)

(8.4)

(8.5)

Since $m_c \approx 1.3$ GeV and $m_t \approx 175$ GeV we observe that the $cc$ diagram dominates in $ReM_{12}$ while $ImM_{12}$ is dominated by $(tt)$ diagram. The real part dominates in $M_{12}$:

$$\frac{ImM_{12}}{ReM_{12}} \sim \lambda^8 \left(\frac{m_t}{m_c}\right)^2 \sim 0.1.$$  

(8.6)

The explicit calculation of the $cc$ exchange diagram gives:

$$L_{\Delta s=2}^{\text{eff}} = -\frac{g^4}{2^9\pi^2M_W^4}(\bar{s}\gamma_\alpha(1 + \gamma_5)d)^2\eta_1 m_c^2 V_{cs}^2 V_{cd}^*,$$

(8.7)

where $g$ is SU(2) gauge coupling constant, $g^2/8M_W^2 = G_F/\sqrt{2}$, and the factor $\eta_1$ takes into account the hard gluon exchanges. Since

$$M_{12} - \frac{i}{2}\Gamma_{12} = <K^0 | H^{eff} | \bar{K}^0 > /2m_K,$$

(8.8)

(here $H^{eff} = -L_{\Delta s=2}^{eff}$) we should calculate the matrix element of the product of two $V - A$ quark currents between $\bar{K}^0$ and $K^0$ states. Using the vacuum insertion we obtain:

$$< K^0 | \bar{s}\gamma_\alpha(1 + \gamma_5)d\bar{s}\gamma_\alpha(1 + \gamma_5)d | \bar{K}^0 > =$$

\footnote{Factor $1/2m_K$ appears when taking a square root from the quadratic in meson masses mesonic Hamiltonian in order to obtain linear in meson masses Hamiltonian which enters the Schrödinger equation:}

$$\left[ \frac{(M - \frac{i}{2}\Gamma)^2}{<\bar{K}^0 | H^{eff} | K^0 >} \right]^{1/2} = \left( \frac{M - \frac{i}{2}\Gamma}{<\bar{K}^0 | H^{eff} | K^0 >} \right) \left( \frac{<K^0 | H^{eff} | K^0 >}{2m_K} \right),$$

here $H^{eff} = -L_{\Delta s=2}^{eff}$.  

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\[ \frac{8}{3} B_K < K^0 | \bar{s}\gamma_\alpha(1 + \gamma_5)d | 0 > \times  \\
< 0 | \bar{s}\gamma_\alpha(1 + \gamma_5)d | \bar{K}^0 > = -\frac{8}{3} B_K f_K^2 m_K^2 , \]

where \( B_K = 1 \) if the vacuum insertion saturates this matrix element.

With the help of eq. (6.5) we obtain:

\[ m_S - m_L - \frac{i}{2}(\Gamma_S - \Gamma_L) = 2[Re M_{12} + \frac{i}{2}\Gamma_{12}] , \]

where \( S \) and \( L \) are the abbreviations for \( K_S \) and \( K_L \), short and long-lived neutral \( K \)-mesons respectively. For the difference of masses from (8.7 - 8.10) we get:

\[ m_L - m_S = \Delta m_{LS} = \frac{G_F^2 B_K f_K^2 m_K}{6\pi^2}\eta_1 m_c V_{cs} V_{cd}^* . \]

Constant \( f_K \) is known from \( K \to l\nu \) decays, \( f_K = 160 \) MeV. Gluon dressing of the box diagrams in 4 quark model in the leading logarithmic (LO) approximation was calculated in [18], \( \eta_1^{LO} = 0.6 \). It appears that the subleading logarithms are numerically very important [19], \( \eta_1^{NLO} = 1.3 \pm 0.2 \), the number which we will use in our estimates. We take \( B_K = 1 \pm 0.1 \) assuming that the vacuum insertion is good numerically, though the smaller values of \( B_K \) can be found in literature as well (see [20]).

Experimentally the difference of masses is:

\[ \Delta m_{LS}^{exp} = 0.5303(9) \cdot 10^{10} \text{ sec}^{-1} ; \]

Substituting the numbers in eq. (8.11) we get:

\[ \frac{\Delta m_{LS}^{theor}}{\Delta m_{LS}^{exp}} = 0.6 \pm 0.2 \]

and we almost get an experimental number from the short-distance contribution described by the box diagram with \( c \)-quarks. As \( V_{cs} \) and \( V_{cd} \) are already known nothing new for CKM matrix elements can be extracted from \( \Delta m_{LS} \).

Concerning the neutral kaon decays we have:

\[ \Gamma_S - \Gamma_L = 2\Gamma_{12} \approx \Gamma_S = 1.1 \cdot 10^{10} \text{ sec}^{-1} \ (\Delta m_{LS} \approx \Gamma_S/2) , \]
since $\Gamma_L \ll \Gamma_S$, $\Gamma_L = 2 \cdot 10^7 \text{ sec}^{-1}$. $K_L$ is so long-lived because it can decay only into 3 particles finite states (neglecting CPV) and for the decays into 3 pions the energy release is small:

$$m_K - m_{3\pi} = 490 \text{ MeV} - 3 \cdot 140 \text{ MeV} = 70 \text{ MeV}.$$  \hspace{1cm} (8.15)

$K_S$ rapidly decays to two pions which have CP= +1.

### 9 CPV in $K^0 - \bar{K}^0 : K_L \rightarrow 2\pi$, $\varepsilon_K$-hyperbola

CPV in $K^0 - \bar{K}^0$ mixing is proportional to the deviation of $|q/p|$ from one; so let us calculate this ratio according to eq. (6.6) taking into account that $\Gamma_{12}$ is real, while $M_{12}$ is mostly real:

$$\frac{q}{p} = 1 - i \frac{Im M_{12}}{M_{12} - \frac{i}{2} \Gamma_{12}} = 1 + \frac{2i Im M_{12}}{m_L - m_S + \frac{i}{2} \Gamma_S}. \hspace{1cm} (9.1)$$

In this way for the quantity $\tilde{\varepsilon}$ introduced in eq. (6.14) we obtain:

$$\tilde{\varepsilon} = -i \frac{Im M_{12}}{\Delta m_{LS} + \frac{i}{2} \Gamma_S}. \hspace{1cm} (9.2)$$

Branching of CP-violating $K_L \rightarrow 2\pi$ decay equals:

$$Br(K_L \rightarrow 2\pi^0) + Br(K_L \rightarrow \pi^+ \pi^-) = \frac{\Gamma(K_L \rightarrow 2\pi)}{\Gamma_{K_L}} = \frac{\Gamma_{K_L \rightarrow 2\pi} \Gamma(K_S)}{\Gamma_{K_S \rightarrow 2\pi} \Gamma(K_L)} =$$

$$\approx \frac{|\eta_{00}|^2 \Gamma(K_S \rightarrow 2\pi^0) + |\eta_{+-}|^2 \Gamma(K_S \rightarrow \pi^+ \pi^-) \Gamma(K_S)}{\Gamma(K_S \rightarrow 2\pi^0) + \Gamma(K_S \rightarrow \pi^+ \pi^-) \Gamma(K_S)} \approx$$

$$\approx |\eta_{00}|^2 \frac{\Gamma(K_S)}{\Gamma(K_L)} \approx |\varepsilon|^2 \frac{\Gamma(K_S)}{\Gamma(K_L)} \approx \hspace{1cm} (9.3)$$

$$\approx |\tilde{\varepsilon}|^2 \left| 1 - \sqrt{2} \frac{\varepsilon' ReA_0}{\tilde{\varepsilon} ReA_2} \left(1 + i\right) \right|^2 \approx 5.17(4) \cdot 10^{-8} \text{ sec} \approx 0.894(1) \cdot 10^{-10} \text{ sec} \approx$$

$$\approx 578(1 - 0.08) |\tilde{\varepsilon}|^2 = 3.02(3) \cdot 10^{-3},$$

where the last number is the sum of $K_L \rightarrow \pi^+ \pi^-$ and $K_L \rightarrow \pi^0 \pi^0$ branching ratios and experimental values of the $K_S$ and $K_L$ widths are used. We also use
the approximate relation \( \delta_0 - \delta_2 \approx 45^0 \) which follows from the analysis of \( \pi - \pi \) scattering. In eq. (9.3) the small factors of the order of \( \varepsilon'/\bar{\varepsilon} \) are neglected while the enhanced term \( \frac{ReA_0}{ReA_2} \varepsilon' \approx 22.2 \varepsilon'/\bar{\varepsilon} \) is taken into account (\( \varepsilon'/\bar{\varepsilon} \) is almost real). This term originates from direct CPV in kaon decays, see the next section. We used the following approximate formula: \( \eta_{00} = \bar{\varepsilon} + i \frac{ImA_0}{ReA_0} \), and estimated the ratio of amplitudes using eq. (10.10), in which only induced by QCD penguin term \( \sim ImA_0/ReA_0 \) is taken into account. The electroweak penguins partially cancel QCD penguin; omitting them we underestimate \( |\bar{\varepsilon}| \) a bit.

In this way the experimental value of \( |\bar{\varepsilon}| \) is determined, and for our theoretical result described by eq. (9.2) we should have:

\[
|\bar{\varepsilon}| = \frac{|ImM_{12}|}{\sqrt{2 \Delta m_{LS}}} = 2.38(1) \cdot 10^{-3} \quad (9.4)
\]

To calculate \( ImM_{12} \) one should use eq. (8.8) and the expression for \( \mathcal{L}_{\Delta S=2}^{\text{eff}} \) which follows from the calculation of the box diagrams. As we already demonstrated \( (tt) \) box gives the main contribution in \( ImM_{12} \). For the first time it was calculated explicitly not supposing that \( m_t < m_W \) in the paper [21] (a year later the same result was independently obtained in [22]):

\[
ImM_{12} = -\frac{G_F^2 B_K f_K^2 m_K}{12\pi^2} m_t^2 \eta_2 Im(V_{ts}^2 V_{td}^*) \times I(\xi),
\]

\[
I(\xi) = \left\{ \frac{\xi^2 - 11\xi + 4}{4(\xi - 1)^2} - \frac{3\xi^2 \ln \xi}{2(1 - \xi)^2} \right\}, \quad \xi = \left( \frac{m_t}{m_W} \right)^2
\]

where factor \( \eta_2 \) which takes into account the gluon exchanges in the box diagram with \( (tt) \) quarks was found in the same paper [21] in the leading logarithmic approximation: \( \eta_2^{LO} = 0.6 \). This factor is not changed substantially by subleading logs [19]: \( \eta_2^{NLO} = 0.57(1) \). Concerning factor \( \eta_3 \) which is responsible for gluon dressing of \( (ct) \) box and which was as well found in [21]: \( \eta_3^{LO} = 0.4 \), the subsequent approximation is [19]: \( \eta_3^{NLO} = 0.47(4) \).

Let us present the numerical values for the expression in figure brackets in eq. (9.5) for several values of the top quark mass (this factor was obtained in paper [21] and later in literature was named Inami-Lim factor, an evident example of the famous Arnold principle: “If a notion bears a personal name,
then this name is not the name of the discoverer”):

\[
\begin{align*}
1, & \quad m_t = 0, \quad \xi = 0 \\
\{ \} = & \quad 0.55, \quad \xi = 4.7, \quad \text{which corresponds to } m_t = 175 \text{ GeV} \quad (9.6) \\
0.25, & \quad m_t = \xi = \infty
\end{align*}
\]

It is clearly seen from (9.5) and (9.6) that the top contribution to the box diagram is not decoupled (it does not vanish) in the limit \( m_t \to \infty \). Three years earlier the analogous non-decoupling of \( t \)-quark contribution through loops to the quantity \( \rho = \frac{g^2/M_W^2}{g^2/M_W^2} \) was observed in [23].

One can easily get where this enhanced at \( m_t \to \infty \) behaviour originates from the estimating box diagram in ’t Hooft-Feynman gauge. In the limit \( m_t \gg m_W \) the diagram with two charged higgs exchanges shown in Fig. 6 dominates, since each vertex of higgs boson emission is proportional to \( m_t \).

![Diagram](image)

**Figure 6:** **Box diagram the contribution of which is enhanced as** \( m_t^2 \) **in the limit** \( m_t \gg m_W \)

For the factor which multiplies the four-quark operator from the diagram shown in Fig. 6 we get:

\[
\sim \left( \frac{m_t}{v} \right)^4 \int \frac{d^4 p}{p^2 - M_W^2} \left[ \frac{\hat{p}}{p^2 - m_t^2} \right]^2 \sim \left( \frac{m_t}{v} \right)^4 \frac{1}{m_t^2} = G_F m_t^2 \quad ,
\]

(9.7)

where \( v \) is the Higgs boson expectation value.

Substituting eq. (9.5) into (9.4) and substituting the numbers we obtain:

\[
| \bar{\varepsilon} |_{\text{theor}} = 0.0075 \eta (1 - \bar{\rho})(1 \pm 0.1) \quad ,
\]

(9.8)
where 10\% uncertainty in the value of $B_K = 1 \pm 0.1$ dominates in the error. Taking into account $(ct)$ and $(cc)$ boxes we get the following equation:

$$\bar{\eta}(1.52 - \bar{\rho}) = 0.32(3) ,$$

which determines the hyperbola in Fig. 1.

Let us make two comments before finishing this section.

a) Why is \( \varepsilon_K \) small? From eqs. (9.4), (9.5) and (8.11) we obtain the following estimate for \( \varepsilon_K \):

$$\varepsilon_K \sim \frac{m_2^4 \lambda^1_0 \eta(1 - \rho)}{m_2^2 \lambda^2} .$$

It means that \( \varepsilon_K \) is small not because CKM phase is small, but because 2 \( \times \) 2 part of CKM which describes the mixing of the first two generations is almost unitary and the third generation almost decouples. We are lucky that the top quark is so heavy; for \( m_t \sim 10 \text{ GeV} \) CPV would not be discovered up to now (the last statement is true only if the mixing angles of the first two generations with the third one are not proportional to \( 1/\sqrt{m_t} \) – in the opposite case \( \varepsilon_K \) would not depend on \( m_t \) and its smallness would not have any qualitative explanation).

b) If the masses of any 2 up (or down) quarks are equal, then CPV escapes from CKM matrix. Indeed, for \( m_c = m_t \) we have:

$$\text{Im} M_{12} \sim \text{Im} [V_{ts} V_{td}^* + V_{cs} V_{cd}^*] = \text{Im} [V_{us} V_{ud}^*] = 0 .$$

10 Direct CPV in K decays, \( \varepsilon' \neq 0 \) (| \( \frac{A}{\bar{A}} \) | \( \neq 1 \))

Let us consider the neutral kaon decays into two pions. It is convenient to deal with the amplitudes of the decays into the states with definite isospin:

$$A(K^0 \to \pi^+ \pi^-) = \frac{a_2}{\sqrt{3}} e^{i \xi_2} e^{i \delta_2} + \frac{a_0}{\sqrt{3}} \sqrt{2} e^{i \xi_0} e^{i \delta_0} ,$$

$$A(\bar{K}^0 \to \pi^+ \pi^-) = \frac{a_2}{\sqrt{3}} e^{-i \xi_2} e^{i \delta_2} + \frac{a_0}{\sqrt{3}} \sqrt{2} e^{-i \xi_0} e^{i \delta_0} ,$$
\[
A(K^0 \rightarrow \pi^0 \pi^0) = \sqrt{2} a_2 e^{i\xi_2} e^{i\delta_2} - \frac{a_0}{\sqrt{3}} e^{i\xi_0} e^{i\delta_0},
\]
(10.3)

\[
A(\bar{K}^0 \rightarrow \pi^0 \pi^0) = \sqrt{2} a_2 e^{-i\xi_2} e^{i\delta_2} - \frac{a_0}{\sqrt{3}} e^{-i\xi_0} e^{i\delta_0},
\]
(10.4)

where “2” and “0” are the values of \((\pi\pi)\) isospin, \(\xi_{2,0}\) are the weak phases which originate from CKM matrix and \(\delta_{2,0}\) are the strong phases of \(\pi\pi\)-rescattering. If the only quark diagram responsible for \(K \rightarrow 2\pi\) decays were charged current tree diagram which describes \(s \rightarrow u\bar{d}d\) transition through \(W\)-boson exchange, then the phases would be zero and it would be no CPV in the decay amplitudes (the so-called direct CPV). All CPV would originate from \(K^0 - \bar{K}^0\) mixing. Such indirect CPV was called superweak. However in Standard Model CKM phase penetrates into the amplitudes of \(K \rightarrow 2\pi\) decays through the so-called “penguin” diagram shown in Fig. 7, and \(\xi_{0,2}\) are nonzero leading to direct CPV as well.

![Diagram of the penguin diagram responsible for direct CPV in Standard Model](image)

Figure 7: Penguin diagram responsible for direct CPV in Standard Model

From eqs. (10.1) and (10.2) we get:

\[
\Gamma(K^0 \rightarrow \pi^+\pi^-) - \Gamma(\bar{K}^0 \rightarrow \pi^+\pi^-) = -4\sqrt{2} \frac{a_0 a_2}{3} \sin(\xi_2 - \xi_0) \sin(\delta_2 - \delta_0),
\]

so for direct CPV to occur through the difference of \(K^0\) and \(\bar{K}^0\) widths at least two decay amplitudes with different CKM and strong phases should exist.

In the decays of \(K_L\) and \(K_S\) mesons violation of CP occurs due to that in mixing (indirect CPV) and in decay amplitudes of \(K^0\) and \(\bar{K}^0\) (direct
CPV). The first effect is taken into account in the expression for $K_L$ and $K_S$ eigenvectors through $K^0$ and $\bar{K}^0$. With the help of eq. (6.15) we readily obtain:

$$K_S = \frac{K^0 + \bar{K}^0}{\sqrt{2}} + \tilde{\varepsilon} \frac{K^0 - \bar{K}^0}{\sqrt{2}},$$

$$K_L = \frac{K^0 - \bar{K}^0}{\sqrt{2}} + \tilde{\varepsilon} \frac{K^0 + \bar{K}^0}{\sqrt{2}},$$

where we neglect $\sim \varepsilon^2$ terms. For the amplitudes of $K_L$ and $K_S$ decays into $\pi^+\pi^-$ with the help of eqs. (10.1), (10.2) we obtain:

$$A(K_L \to \pi^+\pi^-) = \frac{1}{\sqrt{2}} \left[ \frac{a_2}{\sqrt{3}} e^{i\delta_2} 2i \sin \xi_2 + \frac{a_0}{\sqrt{3}} \sqrt{2} e^{i\delta_0} 2i \sin \xi_0 \right] +$$

$$+ \tilde{\varepsilon} \frac{1}{\sqrt{2}} \left[ \frac{a_2}{\sqrt{3}} e^{i\delta_2} \cos \xi_2 + \frac{a_0}{\sqrt{3}} \sqrt{2} e^{i\delta_0} \cos \xi_0 \right],$$

$$A(K_S \to \pi^+\pi^-) = \frac{1}{\sqrt{2}} \left[ \frac{a_2}{\sqrt{3}} e^{i\delta_2} \cos \xi_2 + \frac{a_0}{\sqrt{3}} \sqrt{2} e^{i\delta_0} \cos \xi_0 \right],$$

where in the last equation we omit the terms which are proportional to the product of two small factors, $\varepsilon$ and $\sin \xi_{0,2}$. For the ratio of these amplitudes we get:

$$\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \tilde{\varepsilon} + i \frac{\sin \xi_0}{\cos \xi_0} + \frac{i e^{i(\delta_2-\delta_0)} a_2}{\sqrt{2} a_0} \frac{\cos \xi_2}{\cos \xi_0} \left[ \frac{\sin \xi_2}{\cos \xi_2} - \frac{\sin \xi_0}{\cos \xi_0} \right],$$

where we neglect the terms of the order of $(a_2/a_0)^2 \sin \xi_{0,2}$ because from the $\Delta T = 1/2$ rule in $K$-meson decays it is known that $a_2/a_0 \approx 1/22$.

The analogous treatment of $K_{L,S} \to \pi^0\pi^0$ decay amplitudes leads to:

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \tilde{\varepsilon} + i \frac{\sin \xi_0}{\cos \xi_0} - i e^{i(\delta_2-\delta_0)} \frac{\sqrt{2} a_2}{a_0} \frac{\cos \xi_2}{\cos \xi_0} \left[ \frac{\sin \xi_2}{\cos \xi_2} - \frac{\sin \xi_0}{\cos \xi_0} \right].$$

The difference of $\eta_{\pm}$ and $\eta_{00}$ is proportional to $\varepsilon'$:

$$\varepsilon' = \frac{1}{\sqrt{2}} e^{i(\delta_2-\delta_0)} \frac{a_2}{a_0} \frac{\cos \xi_2}{\cos \xi_0} \left[ \frac{\sin \xi_2}{\cos \xi_2} - \frac{\sin \xi_0}{\cos \xi_0} \right] =$$

28
$$\frac{i}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \frac{ReA_2}{ReA_0} \left[ \frac{ImA_2}{ReA_2} - \frac{ImA_0}{ReA_0} \right] = \frac{i}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \frac{1}{ReA_0} \left[ ImA_2 - \frac{1}{22} ImA_0 \right] ,$$

where $A_{2,0} \equiv e^{i\xi_{2,0}}a_{2,0}$.

Introducing quantity $\varepsilon$ according to the standard definition:

$$\varepsilon = \bar{\varepsilon} + i \frac{ImA_0}{ReA_0} ,$$

we obtain:

$$\eta_{+} = \varepsilon + \varepsilon', \quad \eta_{00} = \varepsilon - 2\varepsilon' . \quad (10.11)$$

The double ratio $\eta_{+}/\eta_{00}$ was measured in the experiment and its difference from 1 demonstrates direct CPV in kaon decays:

$$\left( \frac{\varepsilon'}{\bar{\varepsilon}} \right)^{\text{exp}} = (1.8 \pm 0.4) 10^{-3} . \quad (10.12)$$

The smallness of this ratio is due to (1) the smallness of the phases produced by the penguin diagrams shown in Fig. 7 and (2) smallness of the ratio of $a_2/a_0 \approx ReA_2/ReA_0$.

Let us estimate the value of $\varepsilon'$. The penguin diagram with the gluon exchange generates $K \rightarrow 2\pi$ transition with $\Delta I = 1/2$; those with $\gamma$- and $Z$-exchanges contribute to $\Delta I = 3/2$ transitions as well. The contribution of electroweak penguins being smaller by the ratio of squares of coupling constants is enhanced by the factor $ReA_0/ReA_2 = 22$, see the last part in eq. (10.10). As a result a partial compensation of QCD and electroweak penguins occur. In order to obtain an order of magnitude estimate let us take into account only QCD penguins. From eq. (10.10) we obtain the following estimate for the sum of the loops with $t$- and $c$-quarks:

$$| \varepsilon' | \approx \frac{1}{22\sqrt{2}} \frac{\sin \xi_0}{\cos \xi_0} = \frac{1}{22\sqrt{2}} \frac{\alpha_s(m_c)}{12\pi} \ln\left( \frac{m_t}{m_c} \right)^2 A^2 \lambda^5 \approx$$

$$\approx 10^{-5} \frac{\alpha_s(m_c)}{12\pi} \ln\left( \frac{m_t}{m_c} \right)^2 . \quad (10.13)$$

Taking into account that $| \varepsilon | \approx 2.4 \cdot 10^{-3}$ we see that the smallness of the ratio of $\varepsilon'/\varepsilon$ can be readily understood.

In order to make an accurate calculation of $\varepsilon'/\varepsilon$ one should know the matrix elements of the quark operators between $K$-meson and two $\pi$-mesons.
which with the present knowledge of low-energy QCD is not possible. That is why a horizontal strip in Fig. 1 which should correspond to eq. (10.13) has too large width and is not shown. Nevertheless we discussed direct CPV in this section since it will be important for $B$-mesons. For more details about $\varepsilon'/\varepsilon$ estimations see review papers [24].

11 $B^0_d - \bar{B}^0_d$ and $B^0_s - \bar{B}^0_s$ mixing – two circles

The $B$-meson semileptonic decays are induced by a semileptonic $b$-quark decay, $b \rightarrow l^-\nu c(l^-\nu u)$. In this way in the decays of $\bar{B}^0$ mesons $l^-$ are produced, while in the decays of $B^0$ mesons $l^+$ are produced. However $B^0$ and $\bar{B}^0$ are not the mass eigenstates and being produced at $t = 0$ they start to oscillate according to the following formulas:

$$
B^0(t) = \frac{e^{-i\lambda_+ t} + e^{-i\lambda_- t}}{2} B^0 + \frac{q}{p} \frac{e^{-i\lambda_+ t} - e^{-i\lambda_- t}}{2} \bar{B}^0,
$$

$$
\bar{B}^0(t) = \frac{e^{-i\lambda_+ t} + e^{-i\lambda_- t}}{2} \bar{B}^0 + \frac{p}{q} \frac{e^{-i\lambda_+ t} - e^{-i\lambda_- t}}{2} B^0;
$$

for derivation use eq. (6.5 - 6.6).

That is why in their semileptonic decays the “wrong sign leptons” are sometimes produced, $l^-$ in the decays of the particles born as $B^0$ and $l^+$ in the decays of the particles born as $\bar{B}^0$. The amount of these “wrong sign” events depends on the ratio of the oscillation frequency $\Delta m$ and $B$-meson lifetime $\Gamma$ (unlike the case of $K$-mesons for $B$-mesons $\Delta \Gamma \ll \Gamma$, see below). For $\Delta m/\Gamma \gg 1$ a large number of oscillations occurs, and the number of “the wrong sign leptons” equals that of a normal sign. If $\Delta m \ll \Gamma$, then $B$-mesons decay before they start to oscillate, and the number of “the wrong sign events” is power suppressed. The pioneering detection of “the wrong sign events” by ARGUS collaboration [25] in 1987 demonstrates that $\Delta m$ is of the order of $\Gamma$, which in the framework of Standard Model could be understood only if a top quark is unusually heavy, $m_t \gtrsim 100$ GeV. Fast $B^0 - \bar{B}^0$ oscillations made possible the construction of asymmetric $B$-factories where CPV in $B^0$ decays was observed. (At the end of this introduction let us mention that UA1 collaboration saw the events which were interpreted as a possible manifestation of $B^0_s - \bar{B}^0_s$ oscillations [26].)
Integrating the probabilities of $B^0$ decays in $l^+$ and $l^-$ over $t$, we obtain for “the wrong sign lepton” probability:

$$W_{B^0 \rightarrow B^0} \equiv \frac{N_{B^0 \rightarrow l^- x}}{N_{B^0 \rightarrow l^- x} + N_{B^0 \rightarrow l^+ x}} = \frac{\left| \frac{q}{p} \right|^2 \left( \frac{\Delta m}{\Gamma} \right)^2}{2 + (\frac{\Delta m}{\Gamma})^2 + \left| \frac{q}{p} \right|^2 \left( \frac{\Delta m}{\Gamma} \right)^2}$$

where we neglect $\Delta \Gamma$, the difference of $B_+ -$ and $B_- -$ mesons lifetimes. Precisely according to our discussion for $\Delta m/\Gamma \gg 1$ we have $W = 1/2$, while for $\Delta m/\Gamma \ll 1$ we have $W = 1/2(\Delta m/\Gamma)^2$ (with high accuracy $|p/q| = 1$, see below).

For $\bar{B}^0$ decays we get the same formula with the interchange of $q$ and $p$:

$$W_{\bar{B}^0 \rightarrow B^0} \equiv \frac{N_{\bar{B}^0 \rightarrow l^+ x}}{N_{\bar{B}^0 \rightarrow l^+ x} + N_{\bar{B}^0 \rightarrow l^- x}} = \frac{\left| \frac{p}{q} \right|^2 \left( \frac{\Delta m}{\Gamma} \right)^2}{2 + (\frac{\Delta m}{\Gamma})^2 + \left| \frac{p}{q} \right|^2 \left( \frac{\Delta m}{\Gamma} \right)^2}$$

In ARGUS experiment $B$-mesons were produced in $\Upsilon(4S)$ decays: $\Upsilon(4S) \rightarrow B\bar{B}$. For $\Upsilon$ resonances $J^{PC} = 1^{--}$, that is why (pseudo)scalar $B$-mesons are produced in $P$-wave. It means that $B\bar{B}$ wave function is antisymmetric at the interchange of $B$ and $\bar{B}$. This fact forbids the configurations in which due to $B - \bar{B}$ oscillations both mesons became $B$, or both became $\bar{B}$. However after one of the $B$-meson decays the flavor of the remaining is tagged, and it oscillates according to eqs. (11.1).

If the first decay is semileptonic with $l^+$ emission indicating that a decaying particle was $B^0$, then the second particle was initially $\bar{B}^0$. With the help of eqs. (11.2 - 11.3) and taking $|p/q| = 1$ we get for the relative number of the same sign dileptons born in semileptonic decays of $B$-mesons, produced in $\Upsilon(4S) \rightarrow B\bar{B}$ decays:

$$\frac{N_{l^+l^+} + N_{l^-l^-}}{N_{l^+l^-}} = \frac{W}{1 - W} = \frac{x^2}{2 + x^2} \Rightarrow x = \frac{\Delta m}{\Gamma}.$$  \hspace{1cm} (11.4)

Let us note that if $B^0$ and $\bar{B}^0$ are produced incoherently (say, in hadron collisions) a different formula should be used:

$$\frac{N_{l^+l^+} + N_{l^-l^-}}{N_{l^+l^-}} = \frac{2W - 2W^2}{1 - 2W + 2W^2} = \frac{x^2(2 + x^2)}{2 + 2x^2 + x^4}.$$  \hspace{1cm} (11.5)
In the absence of oscillations \((x = 0)\) both (11.4) and (11.5) are zero; for high frequency oscillations \((x \gg 1)\) both of them are one.

From the time integrated data of ARGUS and CLEO, the time-dependent analysis of \(B\)-decays at the high energy colliders (LEP II, Tevatron, SLC) and the time-dependent analysis at the asymmetric \(B\)-factories Belle and BaBar the following result was obtained [5]:

\[
x_d = 0.755 \pm 0.015 .
\]  
(11.6)

By using the life time of \(B_d\)-mesons [5]:  
\[
\Gamma_{B_d} = [1.54(1) \cdot 10^{-12} \text{ sec}]^{-1} \equiv [1.54(1) \text{ ps}]^{-1}
\]
we get for the mass difference of \(B_d\) mesons [5]:

\[
\Delta m_{B_d} = 0.489(8) \text{ ps}^{-1} .
\]  
(11.7)

In Standard Model \(B_d - \bar{B}_d\) transition occurs through the box diagram shown in Fig. 8.

![Diagram responsible for \(B_d - \bar{B}_d\) mixing](image)

Figure 8: Diagram responsible for \(B_d - \bar{B}_d\) mixing

Unlike the case of \(K^0 - \bar{K}^0\) transition the power of \(\lambda\) is the same for \(u, c\) and \(t\) quarks inside a loop, so the diagram with \(t\)-quarks dominates. \(M_{12}\) is the same as calculated in [21, 22] for the \(K\)-meson case.

From eq. (9.5) substituting \(B_K f_K^2 V_{ts}^2\) by \(B_{B_d} f_B^2 V_{tb}^2\) and removing “Im” we obtain:

\[
M_{12} = -\frac{G_F^2 B_{B_d} f_B^2}{12\pi^2} m_B m_t^2 \eta_B V_{tb}^2 V_{td}^* I(\xi) ,
\]  
(11.8)
where $I(\xi)$ is the same function as that for $K$-mesons, eq. (9.5). $\eta_B$ with the account of NLO corrections is [27]:

$$\eta_B^{NLO} = 0.55 \pm 0.01 \ . \quad (11.9)$$

$\Gamma_{12}$ is determined by the diagram analogous to that shown in Fig. 4 (where $s$-quark should be substituted by $b$-quark) but inside the loop $c$-quark can propagate as well (so, 4 diagrams altogether: $uu, uc, cu, cc$ quarks in the inner lines). According to [28]:

$$\Gamma_{12} = \frac{G_F^2 B_{B_d} f_B^2 m_B^3}{8\pi} [V_{cb} V_{cd}^* (1 + O(\frac{m_c^2}{m_b^2})) + V_{ub} V_{ud}^*]^2 , \quad (11.10)$$

where the term $O(\frac{m_c^2}{m_b^2})$ accounts for nonzero $c$-quark mass.

Using the unitarity of CKM matrix we get:

$$\Gamma_{12} = \frac{G_F^2 B_{B_d} f_B^2 m_B^3}{8\pi} [-V_{tb} V_{td}^* + O(\frac{m_c^2}{m_b^2}) V_{cb} V_{cd}^*]^2 , \quad (11.11)$$

and the main term in $\Gamma_{12}$ has the same phase as the main term in $M_{12}$, eq. (11.8). That is why CPV in mixing of $B$-mesons is suppressed by an extra factor $(m_c/m_b)^2$. Postponing the discussion of CPV in $B - \bar{B}$ mixing till the next section, from eq. (6.5)we obtain:

$$M_+ - M_- - \frac{i}{2}(\Gamma_+ - \Gamma_-) = 2[M_{12} \mid - \frac{i}{2} \mid \Gamma_{12} \mid] \ , \quad (11.12)$$

and with the help of eq. (11.8) for the difference of masses of the two eigenstates we obtain:

$$\Delta m_{B_d} = \frac{G_F^2 B_{B_d} f_B^2}{6\pi^2} m_B m_B^2 \eta_B \mid V_{tb} V_{td}^* \mid I(\xi) \ , \quad (11.13)$$

and $\Delta m_{B_d}$ is negative as well as in the kaon system.

Comparing this theoretical formula with the experimental result (11.7) we obtain the bound on $\mid V_{td} \mid$, which forms the circle in Fig.1 with the center at the point $\eta = 0$, $\rho = 1$:

$$\mid V_{td} \mid^2 = A^2 \lambda^6 [(1 - \bar{\rho})^2 + \bar{\eta}^2] \ . \quad (11.14)$$

\footnote{The dependence of $\Gamma_{12}$ on $(\frac{m_c^2}{m_b^2})$ is more involved since the contributions of the diagrams with $uc$ and $cc$ quarks depend differently on $(m_c/m_b)^2$. However this is not important for what follows.}
The main uncertainties in this bound on $\bar{\eta}$, $\bar{\rho}$ plane are theoretical [5]:

$$B_{B_d}f_{B_d}^2 = (1.3 \pm 0.1)(200 \pm 30 \text{ MeV})^2,$$  \hspace{1cm} (11.15)

where the lattice results are used.

Substituting the numbers in (11.13) using (11.7) we get:

$$(1 - \bar{\rho})^2 + \bar{\eta}^2 \bigg|_{B_d-\bar{B}_d} = (0.85 \pm 0.15)^2.$$

(11.16)

If in the diagram shown in Fig. 8 we substitute $s$-quark instead of $d$-quark we will get $B_s - \bar{B}_s$ transition. Making straightforward replacements in eq. (11.13) we obtain:

$$\Delta m_{B_s} = \Delta m_{B_d} \frac{B_{B_s}f_{B_s}^2}{B_{B_d}f_{B_d}^2} \left| \frac{V_{ts}^*}{V_{td}^*} \right|^2 \approx \Delta m_{B_d}(1.2 \pm 0.1)^2 \times\,$$

$$\times \frac{1}{\lambda^2[(1 - \bar{\rho})^2 + \bar{\eta}^2]} = (45 \pm 7)\Delta m_{B_d} = (22 \pm 4) \text{ps}^{-1},$$

(11.17)

where for the hadron parameters we use a lattice result from [5], the numerical values of $\rho$ and $\eta$ from the general fit (see Conclusions) and the experimental result for $\Delta m_{B_d}$. Since the lifetimes of $B_d$- and $B_s$-mesons are almost equal, from (11.6) and (11.17) we get:

$$x_{B_s} = 34 \pm 5,$$

(11.18)

which means very fast oscillations. That is why $W_{B_s}$ equals $1/2$ with very high accuracy and one cannot extract $x_{B_s}$ from the time integrated measurements. Performed up to now searches were not sensitive enough to measure $x_{B_s}$; only the lower bound was obtained [5]:

$$\Delta m_{B_s} > 13.1 \text{ps}^{-1} \text{ at } 95\% \text{ CL }.$$

(11.19)

New Tevatron run should be able to find the value of $\Delta m_{B_s}$ if it is close to Standard Model prediction, eq. (11.17).

However even the lower bound (11.19) appears to be powerful enough to bound possible values of $\rho$ and $\eta$ quite effectively. From (11.17) and (11.19) we get:

$$(1 - \bar{\rho})^2 + \bar{\eta}^2 < 1.1 \text{ at } 95\% \text{ CL },$$

(11.20)
which produces $\Delta m_{B_s}$ circle in Fig. 1. One can see that this circle is already inside the outer circle from (11.16) (recalculated to $2\sigma$ deviation). This progress is possible because in the ratio $(B_{B_s}f_{B_s}^2)/(B_{B_d}f_{B_d}^2)$ theoretical uncertainty diminishes.

For the difference of the width of $B_d^+$ and $B_d^-$ we obtain:

$$\Delta \Gamma_{B_d} = \frac{|\Gamma_{12}|}{2} \approx \frac{G_F^2 B_{B_d} f_{B_d}^2 m_B^3}{16\pi} |V_{td}|^2 ,$$  \hspace{1cm} (11.21)

which is very small:

$$\frac{\Delta \Gamma_{B_d}}{\Gamma_{B_d}} \approx 0.3\% ,$$  \hspace{1cm} (11.22)

as opposite to $K$-meson case, where $K_S$ and $K_L$ lifetimes differ strongly.

In the $B_s$-meson system a larger time difference is expected; substituting $V_{ts}$ instead of $V_{td}$ in (11.21) we obtain:

$$\frac{\Delta \Gamma_{B_s}}{\Gamma_{B_s}} \sim 10\% .$$  \hspace{1cm} (11.23)

12 CPV in $B^0 - \bar{B}^0$ mixing, $a_{SL}^B$ — too small

For a long time CPV in $K$-mesons was observed only in $K^0 - \bar{K}^0$ mixing. That is why it seems reasonable to start studying CPV in $B$-mesons from their mixing:

$$\left| \frac{q}{p} \right| = \sqrt{1 + \frac{i}{2} \left( \frac{\Gamma_{12}}{M_{12}} - \frac{\Gamma_{12}^*}{M_{12}^*} \right)} = 1 + \frac{i}{4} \left( \frac{\Gamma_{12}}{M_{12}} - \frac{\Gamma_{12}^*}{M_{12}^*} \right) =$$

$$= 1 - \frac{1}{2} Im \left( \frac{\Gamma_{12}}{M_{12}} \right) \approx 1 - \frac{m_t^2}{m_t^2} Im \frac{V_{cb} V_{cd}^*}{V_{tb} V_{td}^*} \approx 1 - O(10^{-4}) ,$$  \hspace{1cm} (12.1)

where the expressions for $M_{12}$ (eq. (11.8)) and $\Gamma_{12}$ (eq. (11.11)) were used. We see that CPV in $B_d - \bar{B}_d$ mixing is very small because $t$-quark is very heavy and is even smaller in $B_s - \bar{B}_s$ mixing. The experimental observation of $B_d - \bar{B}_d$ mixing comes from the detection of the same sign leptons produced in the semileptonic decays of $B_d - \bar{B}_d$ pair from $\Upsilon(4S)$ decay. Due to CPV in the mixing the number of $l^-l^-$ events will differ from that of $l^+l^+$ and this difference is proportional to $|q/p| - 1 \sim 10^{-4}$. So, one needs more than $10^8$
semileptonic decays of both $B$ and $\bar{B}$ to detect this effect (statistical error $\sim \sqrt{N}$). Taking into account that semileptonic branching ratio of $B$ to $e\nu X$ or $\mu\nu X$ is 20%, we see that about $10^{10}$ $\Upsilon(4S) \rightarrow B^0\bar{B}^0$ decays are needed to observe CPV in mixing according to the Standard Model. This is completely hopeless now taking into account that $B$-factories have collected about $10^8$ $B\bar{B}$ pairs produced in $\Upsilon(4S)$ decays.

Another type of experiment is possible as well. Let us suppose that in some reactions when $B^+\bar{B}^0$ pair is produced $B^+$ is detected, and if $B^-\bar{B}^0$ is produced $B^-$ is detected. In this way a flavor state is tagged: it is known what particle ($B^0$ or $\bar{B}^0$) was produced. If neutral $B$ decays semileptonically one can look for CPV charge asymmetry:

$$a_{SL}^B = \frac{N_{B^0 \rightarrow l^-} - N_{\bar{B}^0 \rightarrow l^+}}{N_{B^0 \rightarrow l^-} + N_{\bar{B}^0 \rightarrow l^+}} = O(10^{-4}) \ , \quad (12.2)$$

while the experimental result is (in [5] it is denoted by $a_{CP}$):

$$a_{SL}^B = -0.002 \pm 0.009{\,(\text{stat})} \pm 0.008{\,(\text{syst})} \quad (12.3)$$

– two orders of magnitude weaker bound on $a_{SL}^B$ than the theoretical prediction.

13 CPV in interference of mixing and decay

($Im \frac{qA}{pA} \neq 0$)

As soon as it became clear that CPV in $B - \bar{B}$ mixing is small theoreticians started to look for another way to find CPV in $B$ decays. The evident alternative is the direct CPV. It is very small in $K$-mesons because: a) the third generation almost decouples in $K$ decays; b) due to $\Delta T = 1/2$ rule. Since in $B$-meson decays all three quark generations are involved and there are many different final states one can hope to have large direct CPV there [29] - [32]. An evident drawback of this strategy: a branching ratio of $B$-meson decays into any particular exclusive hadronic mode is very small (just because there are many modes available), so large number of $B$-meson decays is needed. The specially constructed asymmetric $e^+e^-$-factories Belle and BaBar collected about thirty million $BB$ pairs produced in $\Upsilon(4S)$ decays.
each and discovered in 2001 CPV in $B(B)$ decays [14] (at the end of 2002 statistics is almost 3 times larger in each experiment).

The time evolution of states produced at $t = 0$ as $B^0$ or $\bar{B}^0$ is described by eqs. (11.1).

It is convenient to present these formulae in a little bit different form:

$$|B^0(t)\rangle = e^{-i\frac{M_+ + M_- - \Gamma}{2} t} |B^0\rangle + i\frac{q}{p} \sin\left(\frac{\Delta m}{2} t\right) |\bar{B}^0\rangle,$$

$$|\bar{B}^0(t)\rangle = e^{-i\frac{M_+ + M_- - \Gamma}{2} t} + i\frac{p}{q} \sin\left(\frac{\Delta m}{2} t\right) |B^0\rangle + \cos\left(\frac{\Delta m}{2} t\right) |\bar{B}^0\rangle,$$

where $\Delta m \equiv M_- - M_+ > 0$, and we put $\Gamma_+ = \Gamma_- = \Gamma$ neglecting their small difference.

Let us consider a decay in some final state $f$. Introducing the decay amplitudes according to the following definitions:

$$A_f = A(B^0 \to f), \quad \bar{A}_f = A(\bar{B}^0 \to f),$$

$$A_{\bar{f}} = A(B^0 \to \bar{f}), \quad \bar{A}_{\bar{f}} = A(\bar{B}^0 \to \bar{f}),$$

for the decay probabilities as functions of time we obtain:

$$P_{B^0 \to f}(t) = e^{-\Gamma t} |A_f|^2 \left[\cos^2\left(\frac{\Delta m}{2} t\right) + \left|\frac{q\bar{A}_f}{pA_f}\right|^2 \sin^2\left(\frac{\Delta m}{2} t\right) - \text{Im}\left(\frac{q\bar{A}_f}{pA_f}\right) \sin(\Delta m t)\right],$$

$$P_{\bar{B}^0 \to f}(t) = e^{-\Gamma t} |\bar{A}_f|^2 \left[\cos^2\left(\frac{\Delta m}{2} t\right) + \left|\frac{pA_{\bar{f}}}{q\bar{A}_{\bar{f}}}\right|^2 \sin^2\left(\frac{\Delta m}{2} t\right) - \text{Im}\left(\frac{pA_{\bar{f}}}{q\bar{A}_{\bar{f}}}\right) \sin(\Delta m t)\right].$$

The difference of these two probabilities signal different types of CPV: the difference in the first two terms in brackets appears due to direct CPV; that in the last term – due to CPV in interference of $B^0 - \bar{B}^0$ mixing and decays.

Let $f$ be a CP eigenstate: $\bar{f} = \eta_f f$, where $\eta_f = +(-)$ for CP even (odd) $f$. (Two examples of such decays: $B^0 \to J/\Psi K_{S(L)}$ and $B^0 \to \pi^+\pi^-$ are described by the quark diagrams shown in Fig. 9. The analogous diagrams describe $B^0$ decays in the same final states.) The following equalities can be
easily obtained:  

\[ A_f = \eta_f A_f , \quad \bar{A}_f = \eta_f \bar{A}_f . \]

(13.4)

\[ \lambda \equiv \frac{q \bar{A}_f}{p A_f} . \]

(13.6)

\( a_{CP}(t) \equiv \frac{P_{B^0 \to f} - P_{B^0 \to f}}{P_{B^0 \to f} + P_{B^0 \to f}} = \frac{|\lambda|^2 - 1}{|\lambda|^2 + 1} \cos(\Delta mt) + \frac{2Im\lambda}{|\lambda|^2 + 1} \sin(\Delta mt) \equiv -C_f \cos(\Delta mt) + S_f \sin(\Delta mt) , \]

(13.5)

Figure 9: *Quark diagrams describing \( B^0 \to J/\Psi K_{S(L)} \) (a)) and \( B^0 \to \pi^+\pi^- \) (b)) decays*

In the absence of CPV the expressions in brackets in (13.3) are equal and formulas (13.3) describe the exponential particle decay without oscillations. Taking CPV into account and neglecting a small deviation of \( |p/q| \) from one, for CPV asymmetry of the decays into CP eigenstate we obtain:

(38)
more than one amplitude contribute to the decay. For extraction of CPV parameters (angles of unitarity triangle) in this case the knowledge of strong rescattering phases is necessary. The nonvanishing $S_f$ describes CPV in interference of mixing and decay. It is nonzero even when there is only one decay amplitude, and $|\lambda| = 1$. Such decays are of special interest since the extraction of CPV parameters becomes independent on poorly known strong phases of the final particles scattering.

The decays of $\Upsilon(4S)$ resonance produced in $e^+e^-$ annihilation are a powerful source of $B^0\bar{B}^0$ pairs. A semileptonic decay of one of the $B$'s tags “beauty” of the partner at the moment of decay thus making it possible to study CPV. However the time-integrated asymmetry is zero for decays were $C_f$ is zero. This happens since we do not know which of the two $B$-mesons decays earlier, and asymmetry is proportional to:

$$I = \int_{-\infty}^{\infty} e^{-\Gamma|t|} \sin(\Delta mt) dt = 0.$$  

The asymmetric $B$-factories provide possibility to measure the time-dependence: $\Upsilon(4S)$ moves in a laboratory system, and since energy release in $\Upsilon(4S) \to B\bar{B}$ decay is very small both $B$ and $\bar{B}$ move with the same velocity as the original $\Upsilon(4S)$. This makes the resolution of $B$ decay vertices possible unlike the case of $\Upsilon(4S)$ decay at rest, when $B$ and $\bar{B}$ being non-relativistic decays at almost the same point (for the detailed description of the experiments see the lecture of A. Bondar in these proceedings) [33]. The implementation of the time-dependent analysis for the search of CPV in $B$-mesons was suggested in papers [34, 35].

14. $B^0_d(\bar{B}^0_d) \to J/\Psi K_{S(L)}, \sin 2\beta$ – straight lines

Currently this is the only decay channel where both $B$-factories observe statistically significant CPV.

The tree diagram contributing to this decay is shown in Fig. 9a). The product of the corresponding CKM matrix elements is: $V_{cb}^*V_{cs} \simeq A\lambda^2$. Also the penguin diagram $b \to sg$ with the subsequent $g \to c\bar{c}$ decay contributes to the decay amplitude. Its contribution is proportional to:

$$P \sim V_{us}V_{ub}^*f(m_u) + V_{cs}V_{cb}^*f(m_c) + V_{ts}V_{tb}^*f(m_t) =$$
\[ V_{us}V_{ub}^*(f(m_u) - f(m_t)) + V_{cs}V_{cb}^*(f(m_c) - f(m_t)), \quad (14.1) \]

where function \( f \) describes the contribution of quark loop and we subtracted zero from the expression on the first line just as we did when calculating the box diagram in Section 8. The last term on the second line has the same weak phase as the tree amplitude, while the first term has a CKM factor \( V_{us}V_{ub}\sim \lambda^4(\rho - i\eta)A \). Since (one-loop) penguin amplitude should be in any case smaller than the tree one we get that with 1% accuracy there is only one weak amplitude governing \( B_0 \to \bar{B}_0 \to J/\Psi K_{S(L)} \) decays. This is the reason why this mode is called a “gold-plated mode” – the accuracy of the theoretical prediction of the CP-asymmetry is very high, and \( Br(B_d \to J/\Psi K^0) \approx 10^{-3} \) is large enough to detect CPV. Substituting in Eq.(13.5) \( |\lambda| = 1 \) we obtain:

\[ a_{CP}(t) = Im\lambda \sin(\Delta m\Delta t), \quad (14.2) \]

where \( \Delta t \) is the time difference between the semileptonic decay of one of \( B \)-mesons produced in \( \Upsilon(4S) \) decay and that of the second one to \( J/\Psi K_{S(L)} \).

Using the following equation:

\[ \bar{A}_f = \eta_f \bar{A}_f, \quad (14.3) \]

where \( \eta_f \) is CP parity of the final state, we obtain:

\[ \lambda = \left( \frac{q}{p} \right)_{B_d} \frac{A_{B^0 \to J/\Psi K_{S(L)}}}{A_{\bar{B}^0 \to J/\Psi K_{S(L)}}} = \left( \frac{q}{p} \right)_{B_d} \eta_f \frac{A_{\bar{B}^0 \to J/\Psi K_{S(L)}}}{A_{B^0 \to J/\Psi K_{S(L)}}}. \quad (14.4) \]

The amplitude in the nominator contains \( \bar{K}^0 \) production. To project it on \( K_{S(L)} \) we should use:

\[ \bar{K}^0 = \frac{K_S - K_L}{(q)_K} = \frac{\bar{K}_S + \bar{K}_L}{(q)_K}, \quad (14.5) \]

getting \( (q)_K \) in denominator. The amplitude in the denominator contains \( K^0 \) production, and using:

\[ K^0 = \frac{K_S + K_L}{(p)_K} \quad (14.6) \]

we obtain the factor \( (p)_K \) in the nominator. Collecting all the factors together and substituting CKM matrix elements for \( \bar{A}_f/A_f \) ratio we get:

\[ \lambda = \eta_{S(L)} \left( \frac{q}{p} \right)_{B_d} \frac{V_{cb}V_{cs}^*}{V_{cb}V_{cs}} \left( \frac{p}{q} \right)_K. \quad (14.7) \]

40
Since in $B$ decays $J/\Psi$ and $K_{S(L)}$ are produced in $P$-wave, $\eta_{S(L)} = -(+)$ (CP of $J/\Psi$ is $+$, that of $K_S$ is $+$ as well, and $(-1)^l = -1$ comes from $P$-wave; CP of $K_L$ is $-$).

Substituting the expressions for $(q/p)_{Bd}$ and $(p/q)_K$ in (14.4) and taking into account $\eta_{S(L)}$ we obtain:

$$\lambda(J/\Psi K_{S(L)}) = -(+) \frac{V_{td} V_{tb}^* V_{cb} V_{cs}^* V_{cd}^* V_{cs}}{V_{td}^* V_{tb} V_{cb}^* V_{cs} V_{cd} V_{cs}^*} ,$$

which is invariant under the phase rotation of any quark field. From eq. (3.20) and Fig. 2 we have:

$$\arg(V_{td}^* V_{tb}) = 2\pi - \beta ,$$

and we finally obtain:

$$a_{CP}(t)\bigg|_{J/\Psi K_{S(L)}} = (-) \sin(2\beta) \sin(\Delta m \Delta t) ,$$

which is a simple prediction of Standard Model. In this way the measurement of this asymmetry at $B$-factories provides the value of the angle $\beta$ of the unitarity triangle. The results of Belle and BaBar are consistent; their average is:

$$\sin 2\beta = 0.73 \pm 0.05\text{ (stat)} \pm 0.035\text{ (syst)} .$$

This result is based on the analysis of approximately $80 \cdot 10^6$ pairs of $B\bar{B}$ produced in $\Upsilon(4S)$ decays per collaboration [36]. As a final state not only $J/\Psi K_{S(L)}$ were selected, but also the other states with hidden charm ($\Psi' K_S, \eta_c K_S, \chi_c K_S$). The value of $|\lambda|$ was also determined from the absence of $\cos(\Delta m \Delta t)$ term in asymmetry:

$$|\lambda| = 0.95 \pm 0.035\text{ (stat)} \pm 0.025\text{ (syst)}$$

in accordance with Standard Model prediction. From eq. (14.11) we obtain 2 possible solutions with angle $2\beta$ in the first or second quadrant. Both are shown in Fig. 1 by straight lines and the first one coincides nicely with Standard Model expectations.

Returning to formula (14.8) let us note that the decay amplitudes and $K^0 - \bar{K}^0$ mixing do not contain complex phases, that is why the only source of it in CP-asymmetry in $B^0 \to J/\Psi K$ decays is $B^0 - \bar{B}^0$ mixing:
\[
\left( \frac{q}{p} \right)_{B_d} = \sqrt{\frac{M_{12}^*}{M_{12}}} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*},
\]

thus the phase comes from \( V_{td} \), that is why the final expression (14.10) contains the angle \( 2\beta \) – the phase of \( V_{td}/V_{td}^* \).

15 \( B_d \to \pi^+\pi^- \), \( \sin 2\alpha \), Penguin versus tree, \( |\bar{A}/A| \neq 1 \)

The pair \( \pi^+\pi^- \) produced in \( B_d \) decay has positive CP: \( CP(\pi\pi)_{t=0} = +1 \). The tree level diagram contributing to this decay is shown in Fig. 9b. Let us suppose for a moment that it dominates in the decay probability analogously to \( B \to J/\Psi K \) case. Since CKM matrix element \( V_{ub} \) has a nonzero phase the CP-asymmetry should be different from that in \( J/\Psi K \) decays. Let us calculate it:

\[
\lambda = \left( \frac{q}{p} \right)_{B_d} \frac{\bar{A}}{A} = \frac{V_{td} V_{tb}^*}{V_{tb}^* V_{td}} \cdot \frac{V_{ub} V_{ud}^*}{V_{ud}^* V_{ub}} = e^{-i(2\beta + 2\gamma)},
\]

(15.1)

\[
\text{Im} \lambda = \sin 2\alpha,
\]

\[
a_{CP}(t) \bigg|_{\pi^+\pi^-} = \sin(2\alpha) \sin(\Delta m t)
\]

where the triangle relation \( \alpha + \beta + \gamma = \pi \) is used. So, the study of \( t \)-dependent CP asymmetry in \( \Upsilon(4S) \to BB \to l^\pm X \pi^+\pi^- \) decay would measure angle \( \alpha \) if tree diagram dominates in \( B \to \pi\pi \) decay. The penguin diagrams producing the transition \( b \to d g \to d\bar{u}u \) also contribute to the \( \pi^+\pi^- \) decay mode:

\[
P \sim V_{ud} V_{ub}^* f(m_u) + V_{cd} V_{cb}^* f(m_c) + V_{td} V_{tb}^* f(m_t) =
\]

\[
= V_{ud} V_{ub}^* [f(m_u) - f(m_t)] + V_{cd} V_{cb}^* [f(m_c) - f(m_t)],
\]

(15.2)

and while the first term should be added to the tree diagram (CKM phase is the same), the second one has a different phase and is of the order of \( \lambda^3 \), just like the tree diagram. Naively one would expect that the penguin contribution should be nevertheless much suppressed. Feynman diagram calculation leads to the following damping factor:

\[
P/T \sim \frac{\alpha s(m_b)}{12\pi} \ln \left( \frac{m_t}{m_b} \right)^2 \approx 0.04.
\]

(15.3)
However the comparison of $B \to \pi\pi$ and $B \to \pi K$ branching ratios demonstrates that the naive estimate is not valid and the penguin contributions are very much enhanced. The amplitudes of $B \to \pi\pi$ decays are of the order of:

$$A_{B \to \pi\pi} \sim \lambda^3 T + \lambda^3 P ,$$

(15.4)

(here $\lambda$ is the CKM parameter, $\lambda \approx 0.22$), while that for $B \to \pi K$ decays are:

$$A_{B \to \pi K} \sim \lambda^4 T + \lambda^2 P ,$$

(15.5)

and if estimate (15.3) were correct, the decay probabilities with $K$-meson production should be suppressed as $\lambda^2 \sim 5 \cdot 10^{-2}$. Experimentally a number of branchings of these two types of decays were measured, or upper bounds were established (different decay modes have different charges of $B$, $K$ and $\pi$). It appears that the decays to $\pi K$ are even more probable than to $\pi\pi$ [5]. This can happen only for $P/T > \lambda$; it follows from the experimental data that:

$$(P/T)_{\text{exp}} \sim \sqrt{\lambda} .$$

(15.6)

Taking into account the penguin contribution instead of eq. (15.1) we get [37]:

$$\lambda = \left[ e^{2i\alpha} \frac{1 + |P/T| e^{i(\delta+\gamma)}}{1 + |T| e^{i(\delta-\gamma)}} \right] ,
$$

(15.7)

where the part of the penguin contribution proportional to $V_{ud}V_{ub}^*$ is included into the tree amplitude and $\delta \equiv \delta_P - \delta_T$ is the difference of strong phases of the penguin and tree amplitudes. $\alpha$ and $\gamma$ are the angles of unitarity triangle.

The experimental data on CP asymmetries in $B \to \pi^+\pi^-$ decay are currently controversial. According to Belle [38]:

$$S_{\pi\pi} = -1.23 \pm 0.41 \pm 0.08$$

$$C_{\pi\pi} = -0.77 \pm 0.27 \pm 0.08 ,$$

(15.8)

where the first errors are statistical, the second – systematical. Both numbers are different from zero at the level of 3 sigma. Since $C_{\pi\pi}$ is different from zero the penguin contributions are not negligible. Extracting the penguin amplitude from the $B \to \pi K$ decay branching ratios with the help of flavor SU(3) and the tree amplitude from the factorization hypothesis applied to $B \to \pi e\nu$ decay the authors of [39] conclude that $|P/T| \sim 0.3$. Using this
result and the triangular relation $\gamma = \pi - \beta - \alpha$ one can extract the values of $\alpha$ and $\delta$ from (15.8). The result is [38]:

$$78^0 < \alpha < 152^0$$

(15.9)

for $\beta = 23^0$ and $0.15 < |P/T| < 0.45$.

BaBar Collaboration did not observe CP violation in $B_d \to \pi^+\pi^-$ decay [40]:

$$S_{\pi\pi} = 0.02 \pm 0.34 \pm 0.05$$

$$C_{\pi\pi} = -0.30 \pm 0.25 \pm 0.04$$

(15.10)

We can hope that with more data available and corresponding diminishing of statistical error the Belle-BaBar controversy on CPV in $B_d \to \pi^+\pi^-$ decay will be resolved.

16 Angle $\gamma$

Angle $\beta$ of the unitarity triangle is already measured with good accuracy; angle $\alpha$ will be determined from CP asymmetry in $B_d \to \pi^+\pi^-$ decays with better accuracy when more statistics will become available. Evidently the next task is to measure angle $\gamma$, or the phase of $V_{ub}$. In $B_d$ decays angle $\beta$ enters the game through $B_d - \bar{B_d}$ mixing. To avoid it in order to single out the angle $\gamma$ let us turn to $B_s$ decays: CKM matrix elements $V_{ts}$ and $V_{tb}$ which participate in $B_s - \bar{B_s}$ mixing are real. Selecting the decays where $b \to u$ transition dominates and looking for CP asymmetry we can measure angle $\gamma$. $B_s$ decays to CP eigenstates which occur through $b \to u$ transition would provide the necessary information: $B_s \to K_S\rho, K_S\pi^0$. However the penguin transition $b \to dg \to d\bar{u}u$ is proportional to $\lambda^3$ as well as the tree decay $b \to wu\bar{d}$. That is why the extraction of $\gamma$ from these decays will be as involved as that of $\alpha$ from $B_d \to \pi^+\pi^-$ decays.

The analogs of “golden” $B_d \to J/\Psi K$ mode in the case of $B_s$ are $B_s \to J/\Psi\phi, J/\Psi\eta'$ decays. In the framework of Standard Model the CP asymmetries in these decays should be zero: CKM phase is absent in $B_s - \bar{B_s}$ mixing and in $b \to c\bar{c}s$ decay amplitude (CP $(J/\Psi\eta') = +, J/\Psi\phi$ is a mixture of CP odd and even states, but this fact only dilutes asymmetry). That is why the

\footnote{Another way to avoid $B_d - \bar{B_d}$ mixing is to look for direct CPV in $B^\pm$ decays.}
search of this asymmetry is very interesting: a nonzero result discovers New Physics (most probably in $B_s - \bar{B}_s$ mixing).

An interesting strategy for the measurement of angle $\gamma$ was suggested in paper [41]: the study of the time-dependent decay asymmetries of $\bar{B}_s \to D^+_s K^-$, $B_s \to D^+_s K^-$ as well as $\bar{B}_s \to D^-_s K^+$, $B_s \to D^-_s K^+$ decays. The diagrams contributing to the first pair of decays are shown in Fig. 10. The final states are not CP eigenstates (unlike the previously discussed $B_d(\bar{B}_d)$ decays).

![Diagram](image)

Figure 10: Dominant diagrams contributing to $\bar{B}_s \to D^+_s K^-$ (a) and $B_s \to D^+_s K^-$ (b) decays

Tagging a parent meson we can study the time-dependent asymmetry of $\bar{B}_s \to D^+_s K^-$ and $B_s \to D^+_s K^-$ decays and extract from it the following quantity:

$$\lambda_{D^+_s K^-} = \left( \frac{q}{p} \right)_{B_s} \frac{A_1 V_{us}^* V_{cb}}{A_2 V_{ub}^* V_{cs}},$$

(16.1)

see eqs. (13.6), (14.4), where amplitude $A_1$ corresponds to Fig. 10a, and amplitude $A_2$ corresponds to Fig. 10b (CKM matrix elements are written separately). Analogously from the study of $\bar{B}_s \to D^-_s K^+$, $B_s \to D^-_s K^+$ decays we extract:

$$\lambda_{D^-_s K^+} = \left( \frac{q}{p} \right)_{B_s} \frac{A_2 V_{ub}^* V_{cs}}{A_1 V_{us}^* V_{cb}}.$$

(16.2)
Multiplying these two lambdas we get:

\[ \lambda_{D_s^+ K^-} \times \lambda_{D_s^- K^+} = \left( \frac{q}{p} \right)_B \frac{V_{ub} V_{cs}^*}{V_{ub} V_{cs}} \frac{V_{us} V_{cb}^*}{V_{us} V_{cb}} = e^{-2i\gamma}, \]  

(16.3)

the unknown hadronic amplitudes \( A_i \) cancel, \( (q/p)_B \) is real and phase \( \gamma \) will be measured – the “only” problem is to collect enough statistics of tagged \( B_s(B_s) \to D_s^+ K^- (D_s^- K^+) \) decays and to measure its time-dependent asymmetries.

Another way to determine \( \gamma \) is through \( B_d \) decays into \( DK \) final states \([42]\), see also \([43]\).

Study of \( B^\pm \) and \( B_d \) decays into \( \pi K \) final states also allows to determine angle \( \gamma \) \([44]\); the present experimental uncertainties do not allow to draw definite conclusions.

Other strategies to measure (or constrain) angle \( \gamma \) can be found in literature; for the list of references look at a recent review \([45]\).

17 CPV in \( B \to \phi K_S, K^+ K^- K_S, \eta' K_S \): penguin domination

These decays are dominated by penguin diagrams \( b \to sg \to ss\bar{s} \). The diagram with an intermediate \( u \)-quark is proportional to \( \lambda^4 \), while those with intermediate \( c \)- and \( t \)-quarks are proportional to \( \lambda^2 \). In this way the main part of the decay amplitude is free of CKM phase, just like in case of \( B_d \to J/\Psi K \) decays. A nonzero phase which leads to time-dependent CP asymmetry comes from \( B_d - \bar{B}_d \) transition:

\[ a_{CP}(t) = -\eta_f \sin(2\beta) \sin(\Delta m \Delta t), \]  

(17.1)

analogously to \( B_d \to J/\Psi K \) decays, eq. (14.10) \([46]\), see also \([47]\). \( \phi K_S \) and \( \eta' K_S \) final states are CP-odd, while in case of the decay into three kaons final state is a mixture of CP-even and odd states. According to \([48]\) CP-even final states dominate. Here are Belle results \([48]\):

\[ \begin{align*}
\text{Mode} & \quad \sin 2\beta \\
\phi K_S & \quad -0.73 \pm 0.64 \pm 0.22 \\
K^+ K^- K_S & \quad 0.49 \pm 0.43 \pm 0.11^{+0.33}_{-0.90} \\
\eta' K_S & \quad +0.71 \pm 0.37 \pm 0.05 
\end{align*} \]  

(17.2)
where the third error for the $K^+K^-K_S$ mode arises from uncertainty in the fraction of the CP-odd component. The value of $\sin 2\beta$ obtained from CP asymmetry in $B_d \to \phi K_S$ decay deviates from the Standard Model expectation (eq. (14.1)) by more than 2$\sigma$. According to BaBar [49]:

$$\sin 2\beta(\phi K_S) = -0.19 \pm 0.52 \pm 0.09,$$

(17.3)

and the deviation is more moderate. A number of New Physics contributions to the penguin diagram were immediately suggested which explain the deviation from Standard Model in $\phi K_S$ mode observed by Belle Collaboration. More statistics is needed to clarify the present situation: perfect SM description of CP asymmetry in $B_d \to J/\Psi K$ decays which occurs at the tree level and possible New Physics contribution to loop-induced $B_d \to \phi K_S$ decay. Belle and BaBar differ not only in $S_{\phi K_S}$, but in $C_{\phi K_S}$ as well:

$$C_{\phi K_S}(\text{Belle}) = 0.56 \pm 0.41 \pm 0.16$$

$$C_{\phi K_S}(\text{BaBar}) = -0.80 \pm 0.38 \pm 0.12$$

(17.4)

### 18 Conclusions: CKM fit and future prospects

Four parameters of CKM matrix are fitted from the following experimental data: $|V_{ud}| = 0.9734 \pm 0.0008$ [5], $|V_{us}| = 0.2196 \pm 0.00261$ [5], $|V_{cd}| = 0.224 \pm 0.016$ [5], $|V_{cs}| = 0.996 \pm 0.013$ [5], $|V_{cb}| = 0.041 \pm 0.002$ [5], $|V_{ub}| = 0.0036 \pm 0.0007$ [5], $\sin 2\beta = 0.73 \pm 0.06$, $|\varepsilon_K| = (2.282 \pm 0.017) \cdot 10^{-3}$, $\Delta M_{B_d} = 0.489 \pm 0.008$ ps$^{-1}$. Here are the results of the fit:

$$\lambda = 0.223 \pm 0.002$$

$$\delta = 0.82 \pm 0.04$$

$$\bar{\eta} = 0.32 \pm 0.04$$

$$\bar{\rho} = 0.24 \pm 0.08$$

$$\chi^2/n.d.o.f. = 7.8/5.$$  

The good quality of the fit is caused partly by avoiding controversial data, like eq. (17.2).
For the angles of a unitarity triangle we obtain:

\[ \beta = 23^0 \pm 3^0 \]
\[ \alpha = 103^0 \pm 9^0 \] \hspace{1cm} (18.2)
\[ \gamma = 54^0 \pm 9^0 \]

Future prospects are:

1. Diminishing the experimental errors in \( \beta \) from \( B \to J/\Psi K \) and in \( \alpha \) from \( B \to \pi^+\pi^- \) CPV asymmetries – the better accuracy in CKM parameters;

2. Measurement of asymmetries in \( B \to \phi K_S \) decay with better accuracy – check of SM;

3. Measurement of \( \Delta M_{B_s} \) through \( B_s - \bar{B}_s \) oscillation – check of SM;

4. Measurement of the angle \( \gamma \) – check of Standard Model.

The rare \( K \)-meson decay \( K^+ \to \pi^+\nu\nu \) and \( K^0 \to \pi^0\nu\nu \) are also sensitive to the values of CKM parameters \( \rho \) and \( \eta \) which make the measurements of the corresponding decay probabilities interesting: the deviations from SM predictions would signal New Physics.

In this way the study of flavor physics in pre-LHC era could put an end to thirty years success of Standard Model.

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