Radiation back-reaction in relativistically strong and QED-strong laser fields

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The emission from an electron in the field of a relativistically strong laser pulse is analyzed. At high pulse intensities of $\geq 10^{22}$ W/cm$^2$ the emission from counter-propagating electrons is modified by the effects of Quantum ElectroDynamics (QED), as long as the electron energy is sufficiently high: $\mathcal{E} \geq 1$ GeV. The radiation force experienced by an electron is for the first time derived from the QED principles and its applicability range is extended towards the QED-strong fields.

In Quantum ElectroDynamics (QED) an electric field should be treated as strong if it exceeds the Schwinger limit: $E_S = m_e c^2/|e|\lambda_C$ [1]. Such field is potentially capable of separating a virtual electron-positron pair providing an energy, which exceeds the electron rest mass energy, $m_e c^2$, to a charge, $e$, over an acceleration length as small as the Compton wavelength, $\lambda_C = \hbar/|e|\mathcal{E}$. Particularly, a QED-strong Lorentz-transformed electric field, $E_0 = [\mathbf{p} \times \mathbf{B}]/(m_e c)$, may be exerted by a charged particle with the momentum $\mathbf{p}$, gyrating in the magnetic field, $\mathbf{B}$, if $|\mathbf{p}| \gg m_e c$ and/or the magnetic field is strong enough.

Consider QED-effects in a strong pulsed laser field [2]:

$$\sqrt{\alpha^2} \gg 1, \quad \alpha = \frac{eA}{m_e c^2}, \tag{1}$$

$A$ being the vector potential of the wave. In the laboratory frame of reference the electric field is not QED-strong for achieved laser intensities, $\sim 10^{22}$ W/cm$^2$ [3], and even for the $\sim 10^{25}$ W/cm$^2$ intensity projected [4]. Nonetheless, a counter-propagating particle in a 1D wave, $a(\xi), \xi = \omega t - (k \cdot x)$, may experience a QED-strong field, $E_0 = |dA/d\xi|\omega(\mathcal{E} - p_\|)|/c$, because the laser frequency, $\omega$, is Doppler upshifted in the frame of reference comoving with the electron. Herewith the electron dimensionless energy, $\mathcal{E}$, and its momentum are related to $m_e c^2, m_e c$ correspondingly, and subscript $\|$ herewith denotes the vector projection in the direction of the wave propagation. The Lorentz-transformed field exceeds the Schwinger limit, if $\mathcal{E} \sim E_0/E_S = \frac{2\mathcal{E}}{m_e^2}(\mathcal{E} - p_\|)|/2e\xi| \gg 1$,

where $\mathcal{E} = \frac{\xi}{2}$.

Within classical theory a signature for $E_0$ is a radiation loss rate, $I_{cl}(E_0) = 2e^2 E_0^2/(3m_e^2 c^5)$. Therefore, the QED-strength of the electromagnetic field may be determined in evaluating $I_{cl}$ and its ratio to $I_C = I_{cl}(2E_S/3)$:

$$\chi = \sqrt{\frac{I_{cl}}{I_C}}, \quad I_C = \frac{8e^2 c}{27\lambda_C^2}. \tag{2}$$

If $\chi \geq 1$ then the actual radiation loss rate differs from $I_{cl}$. The condition of $\chi > 1$ also separates the parameter range of the Compton effect from that of the Thomson effect, under the condition of Eq. (1). The distinctive feature of the Compton effect is an electron recoil, which is significant, if a typical emitted photon energy, $\hbar \omega_c$, is comparable with the electron energy $\mathcal{E}$. Their ratio, $\chi = \lambda_C \omega_c/(c\mathcal{E})$, equals $\chi$ as defined in Eq. (2) with the proper numerical factors (cf Eq. (13)).

Counter-propagating electrons can be generated in the course of laser pulse interaction with a solid target, that is why the radiation effects in the course of laser-plasma interaction are widely investigated (see [3, 4]). The principle matter in this paper is an account of the radiation back-reaction acting on a charged particle. This can be consistently done by solving the modified Lorentz-Abraham-Dirac equation as derived in [7], in which the radiation back-reaction on the electron motion is expressed in terms of the emission probability. The calculation of this probability in relativistically strong and QED-strong laser pulses is given in Section I. In Section II we discuss the radiation effect on the electron motion in strong fields.

I. ELECTRON IN A 1D WAVE: THE EMISSION PROBABILITY

The emission probability in the strong 1D wave field may be found in $\S\S 40,90,101$ in [8], as well as in [9, 10]. However, to simulate highly dynamical effects in pulsed
fields, one needs a reformulated emission probability, related to short time intervals (not \((-\infty, +\infty)\)).

Consider the classical motion of an electron in
a 1D field, \(a = a^2(\xi), \xi = (k \cdot x), (k \cdot a) = 0, a, k\) and \(x\) being the 4-vectors of the potential, the wave and the coordinates. Herewith the 4-dot-product is introduced in
a usual manner: \((k \cdot x) = \omega t - (k \cdot x)\) etc., 3-vectors in
contrast with 4-vectors being denoted in bold, 4-indices
are omitted below. Introduce a Transformed Space-Time
in the TST (spinors being denoted by \(\bar{\xi} = \xi + \frac{\omega t}{c}\))
noting the vector components orthogonal to \(k\). Note:
that: (1) \(dx^3 = \lambda d\xi/\sqrt{2}, x^2 = x_\perp = x_{\perp}, \lambda\)
denoting the subscript \(\parallel\) the vector components orthogonal to \(k\).
Note: (2) the momentum components, \(p^0 = 0\) and \(p_\perp = a\),
are conserved; (3) the metric tensor in the TST is: \(G^{01} = 1, G^{22} = G^{33} = 1\); and (4) \(p^\mu G_{\mu\nu} p^\nu = 1\) gives:
\(p^3 = (1 + p_\perp^2)/(2p_\perp^0)\). These properties allow us to find:

\[
I_{cl} = -\frac{2e^2c}{3} d\mathbf{p}_f G^{jk} dp_k \frac{dp_k}{ds} = \frac{2e^2c}{3} \left( \frac{dp_0}{d\xi} \right)^2, \tag{3}
\]
and to relate 4-momenta at different times:

\[
p(\xi) = p(\xi') - \alpha + \frac{2(\gamma(\xi') - \delta) \alpha}{2(k \cdot p)}, \tag{4}
\]
where \(ds = \sqrt{c^2 dt^2 - (\mathbf{p} \cdot d\mathbf{x}/c)^2}\) and \(\alpha = a(\xi) - a(\xi')\).

A QED solution of the Dirac equation in
the TST is given by a plane wave electron

\[
\psi = \frac{u(p(\xi))}{\sqrt{N}} C(\xi) \exp \left[ \frac{i(p_{1,0} \cdot \mathbf{x}_\perp)}{\lambda_C} - \frac{i(k \cdot p) x_\perp}{(\lambda C / \lambda)^2} \right]. \tag{5}
\]

The Dirac equation, \((\gamma \cdot i\lambda_C \frac{\partial}{\partial x^\mu} - a) - 1) \psi = 0\), is satisfied under the following conditions: \(u(p(\xi))\) is plane wave bi-spinor amplitude, \((\gamma \cdot p(\xi)) - 1) u(p(\xi)) = 0\), \(\bar{u} \cdot u = 2, \gamma^\mu\)
are the Dirac matrices, and \(C(\xi) = \exp \left( \frac{i}{\lambda C} \frac{1 + (p_\perp/2k \cdot p)}{2(k \cdot p)} \lambda_1 \right)\). Using Eq.\(5\), we find:

\[
u(p(\xi)) = \left[ 1 + \frac{(\gamma \cdot k)(\gamma \cdot \delta)}{2(k \cdot p)} \right] u(p(\xi)). \tag{6}
\]

The emission probability. Introduce domain,
\(\Delta x = (\Delta x^\perp \ast \Delta x^0)\), bounded by two hypersurfaces, \(\xi = \xi^-\) and \(\xi = \xi^+\) (see Fig.1). The difference \(\xi^+ - \xi^-\) is bounded as described below, so that \(\Delta x\)
 covers only a minor part of the pulse. A volume
\(V = S_{\perp} \lambda(\xi^+ - \xi^-)\), is a section of \(\Delta x\) subtended by a line
\(t = \text{const}\). With the choice of the coefficient in Eq.\(6\),
\(N = 2S_{\perp} \lambda(d\xi/\xi)\), the integral \(S_{\perp} \lambda \lambda^\perp d\xi^+ d\xi^-\)
is set to unity, i.e. there is a single electron in the
volume \(V\). The emission probability, \(dW\), for a photon
of wave vector, \(k'\), polarization vector, \(l\), and wave function,
\(\lambda = \exp[-i(k' \cdot x)]t, N_p = \frac{\lambda}{2\pi c}\), is given by an integral
over \(\Delta x\):

\[
dW = \frac{\alpha L_f L_p}{\hbar c} \left| \int \bar{\psi}(\gamma \cdot (A')) \psi \lambda^3 dx^0 dx^1 dx^2 dx^3 \right|^2, \tag{7}
\]

\(L_p = d^2k_f \lambda d(k \cdot k') \hbar c N_p/((2\pi)^2(k \cdot k'))\) is the number
of states for the emitted photon, a subscript \(i, f\)
denotes the electron in the initial (i) or final (f) state, and \(\alpha = \frac{\pi}{2c} \approx \frac{\hbar c}{\lambda}\). The number of electron states, \(L_{i,f}\), per 
\(d^2p_{i,f} = \frac{d^2k_{i,f} \lambda d(k \cdot p_i) \lambda}{(2\pi)^2(k \cdot p_i)}\) \(E_{i,f}(\xi)\),
in the wave field can be found by calculating the Hamiltonian invariant, \(d^3d\psi \sim E(\xi)\),
with no field: \(L_{i,f} = \int \lambda^3 d(k \cdot k') \hbar c N_p/((2\pi)^2(k \cdot k'))\).

Conservation laws. The integration by \(dx^1 dx^2 dx^3 = c^2 dtdx^3\)
results in three \(\delta\)–functions, expressing the conservation
of totals of \(p_\perp\) and \(k \cdot p\), for particles in
initial and final states. Twice integrated with respect to
\(dx^1\), the probability \(dW\) is proportional to a long time
interval, \(\Delta t = \Delta x^1/(c\sqrt{2})\), if the boundary condition
for the electron wave at \(\xi = \xi^-\) is maintained within
that long time. For a single electron, which locates between
the wave fronts \(\xi = \xi^-\) and \(\xi = \xi^+\) during a shorter time,

\[
\delta t(\xi^- - \xi^+) = (1/c) \int_{\xi^-}^{\xi^+} \epsilon_{i,f}(\xi) d\xi^2/(k \cdot p_i), \tag{8}
\]
the emission probability is: \(dW_{i,f}(\xi^- - \xi^+) = \delta t dW/\Delta t\).
Using \(\delta\)–functions we integrate Eq.\(7\) over \(d^2p_{i,f} d(k \cdot p_f)\):

\[
dW_{i,f}(\xi^- - \xi^+) = \frac{\alpha}{(4\pi)^2} \int_0^{\xi^+} \lambda^3 F(\xi) \lambda^3 u(p_i) \lambda^3 d\xi^2/(k \cdot p_f), \tag{9}
\]
where \(F(\xi) = \exp[i \lambda^3 (k' \cdot p_i(\xi)) d\xi^2/(k \cdot p_f)]\) and

\[
\lambda^3 = \frac{\lambda^3}{(k \cdot p_i(\xi))}, \tag{10}
\]

To integrate Eq.\(9\), we re-write it as the double integral over \(d\xi^2\) and reduce the matrices \(u(p_{i,f}(\xi)) \otimes \bar{u}(p_{i,f}(\xi))\) in the integrand to the polarization matrices
of the electron at \(\xi\) or at \(\xi_1\) using Eq.\(8\). Although in
a strong wave electrons may be polarized (see [11]),
we then average over electron and photon polarizations and find:

\[
\frac{dW_{i,f}}{d(k \cdot k') \hbar c N_p} = \frac{\alpha}{(4\pi)^2} \int_0^{\xi^+} \overset{\xi^-} \lambda^3 F(\xi, \xi_1) d\xi^2/(k \cdot p_i(\xi)), \tag{11}
\]

\[
\lambda^3 = \frac{\lambda^3}{(k \cdot p_i(\xi))}, \tag{12}
\]

\[
\frac{dW_{i,f}}{d(k \cdot k') \hbar c N_p} = \frac{\alpha}{(4\pi)^2} \int_0^{\xi^+} \overset{\xi^-} \lambda^3 F(\xi, \xi_1) d\xi^2/(k \cdot p_i(\xi)), \tag{13}
\]

\[
\lambda^3 = \frac{\lambda^3}{(k \cdot p_i(\xi))}, \tag{14}
\]
where $D = -\left(\frac{|a(\xi) - a(\xi'_1)|^2 + |(k' - p_i)|^2}{4|k' - p_i|^2} + 1\right)$. Now we develop the dot-product, $(k' - p_i)$, in $F(\xi)$ in the TST metric and find: $F(\xi)F(-\xi_1) = \exp[i(F_1 + F_2)]$, where

$$F_1 = \frac{(k' - p_i)}{2(k' - k)(k - p_f)} \left(\frac{k'}{k - p_i}\right)(p_\perp) - K_\perp \right)^2 (\xi - \xi'),$$

$$F_2 = \frac{(k' - k)\{\xi - \xi_1' + \int_\xi^\xi' a(\xi) - a(\xi')\,d\xi\}}{2(k' - p_i)(k - p_f)} - \xi - \xi_1.'$$

$$dW_{f_i}(\xi, \xi' \pm) = \frac{\alpha^2}{2\pi\xi' \xi_{\pm}} \int_{\xi}^{\xi' \pm} \frac{\xi_{\pm} e^{i\varphi(i)}}{\xi_{\pm}} \exp \left(\frac{\xi_{\pm}}{\xi_{\pm}}\right) d(\xi, \xi) d\xi d\xi',$$

(11)

**In the strong field as in Eq. (1)** the formulae simplify. In $F_2$ we estimate: $|\xi - \xi_1| \sim |da/d\xi|^{-1}$, and $(k' - k) \sim (k' - p_i)(k - p_i) |da/d\xi|$. Now we can consistently introduce the bounds for $\xi_+ - \xi_-,$:

$$|da/d\xi|^{-1} \ll \xi_+ - \xi_- \ll \min\left(\alpha^{-1} |da/d\xi|^{-1}, 1\right).$$

Under these bounds, the emission probability: (1) is linear in $\xi_+ - \xi_-$: $dW_{f_i}(\xi, \xi' \pm) = \langle dW_{f_i}/d\xi \rangle \langle \xi_+ - \xi_- \rangle$; (2) is less than unity: $\int dW_{f_i} < 1$; and (3) can be expressed in terms of the local electric field. By introducing $\xi' = (\xi + \xi_1)/2, \theta = (\xi - \xi_1)|da/d\xi|^2/2$, $(a(\xi) - a(\xi')) \approx 4\theta^2$ and expressing the integral over $\theta$ in terms of the MacDonald functions we find:

$$dW_{f_i} = \frac{\alpha \chi}{2\pi k' k} \int_{-\infty}^{\infty} r_\chi \frac{K_\pm(y)dy + r_0 r_\chi^2 K_\pm(r_\chi)}{\sqrt{\pi}k' k} \chi_\perp,$$

(12)

$$r_\chi = \frac{r_0}{1 - \chi r_0}, \quad \chi = \frac{3}{2} \frac{k' k}{k - p_i} \left|\frac{da}{d\xi}\right| \chi_\perp = \sqrt{\frac{\chi_\perp}{k_\perp}},$$

(13)

Probability (similar to that found in [9]) is expressed in terms of functions of $r_0 = \frac{2}{3} \frac{k' k}{k - p_i} |da/d\xi|$, and related to interval of $d\phi$. Below we demonstrate the way to use this probability to describe the electron motion and emission.

**II. RADIATION AND ITS BACK-REACTION**

At $p_\perp^2 \gg 1$ the wave vector of the emitted photon is almost parallel to the electron momentum. For colinear $k'$ and $p_i$, one has $\omega'/\xi_1 \approx c(k' - k')/(k - p_i)$, therefore,

$$r_0 = \frac{\omega'}{\omega_c}, \quad \omega_c = \frac{3}{2} \varepsilon_c (k - p_i) \left|\frac{da}{d\xi}\right| \chi_\perp \varepsilon_c \chi = \sqrt{\frac{\varepsilon_c}{\chi}}.$$

(13)

The assumption $p_\perp^2 \gg 1$ also allows us to find the momentum of the emitted radiation, which we relate to the interval of the electron proper time, using Eqs. (12):

$$\frac{dp_{\text{rad}}}{d\tau} = \int \chi C k' \frac{c(k - p_i) dW}{d(k' - k) d^2 k_\perp d\xi} (d(k' - k) d^2 k_\perp d\xi =$$

$$= [p + k O((k - p_i)^{-1})) \int \chi C(k' - k') \frac{dW}{d\xi} d\xi d\xi].$$

(14)

As with other 4-momenta, $p_{\text{rad}}$ is related to $m_e c$. To prove the 4-vector relationship [14], we expand its components in the TST metric and integrate them over $k_\perp$ using the symmetry of $F_1$. The small term, $O(1/(kp_i))$, arises from the electron rest mass energy and from the small ($\sim 1/(kp_i)$) but finite width of the photon angular distribution. Below we neglect this term and find: $dp_{\text{rad}}/d\tau = p_i m_e c^2 (dF_{\text{QED}}/d\tau) d\phi d\phi = \chi_{\perp} c^2 k_\perp d\phi$ is the radiation loss rate.

Thus, the angular distribution can be represented as $\delta(\Omega - p/\sqrt{p^2}) d\Omega$, with $\Omega$ being the solid angle of the photon direction. The photon energy spectrum, $dF_{\text{QED}}/d\tau$, is described as function only of the random scalar, $r_\tau = \omega'/\omega_c$, using only the parameter, $\chi$ (see Fig.2). The latter may be parameterized in terms of the radiation loss rate, evaluated within the framework of classical theory (see Eq. (2) and Fig. 3). The expressions for $q(\xi_1) = F_{\text{QED}}/F_1$ and for the normalized spectrum function, $Q(\xi_1, \chi)$, coincide with formulae known from the gyroscopic emission theory (see §90 in [9]):

$$Q = \frac{9}{8\pi} \int_0^\infty \left(\int_{-\infty}^\infty K_\pm(y)dy + r_0 r_\chi^2 K_\pm(r_\chi)\right),$$

$$Q(r_\chi, \chi) = \frac{9}{8\pi} \int_0^\infty \left(\int_{-\infty}^\infty K_\pm(y)dy + r_0 r_\chi^2 K_\pm(r_\chi)\right)$$
Radiation back-reaction. While emitting a photon, an electron also acquires 4-momentum from the external field, equal to \( dp_F = \Omega_{ik} p_k - \frac{I_{QED}}{m_e c^2} p_i \approx k \lambda_C (k' \cdot p_i)/(k \cdot p_i) \) (see Eq. (10)). Usually this is small compared to \( dp_{\text{rad}} \). However, the account for the interaction with the field ensures that the total effect of emission on the electron not to break the entity \((p_f \cdot p_f) = 1\). The choices of near-unity correction coefficients in \( dp_F \) are somewhat different in the cases \( \chi \leq 1 \) and \( \chi \gg 1 \). For moderate values of \( \chi \) the radiation force, \( (dp - dp_{\text{rad}})/dt \), may be found by integrating both \( dp_F \) and \( dp_{\text{rad}} \) over \( dk' \):

\[
\frac{dp_f - p_i}{dt} = \left( k \frac{p_i \cdot p_i}{(k \cdot p_i)} - p_i \right) \frac{I_{QED}}{m_e c^2},
\]

where the choice of the coefficient in \( dp_F \), first, ensures that the radiation force maintains the abovementioned entity (since \((p_f \cdot d(p_f - p_i)/dt) = 0\)), and, second, makes Eq. (15) applicable with dimensional momenta as well.

We already mentioned in [7], that QED is not compatible with the traditional approach to the radiation force in classical electrodynamics and suggested an alternative equation of motion for a radiating electron:

\[
\frac{dp_i}{d\tau} = \Omega_{ij} p_k - \frac{I_{QED}}{m_e c^2} p_i + r_0 \frac{I_{QED}}{I_{cl}} \Omega_{ik} \Omega_{kl} p_i,
\]

where \( \Omega_{ij} = e F_{ij}/(m_e c) \), \( F_{ij} \) is the field tensor and \( r_0 = 2 e^2/(3m_e c^3) \). In the 1D plane wave \( r_0 \Omega_{ik} \Omega_{kl} p^2 = k^2 (p \cdot p) / (m_e c^2 (k \cdot p)) \), so that the radiation force in Eq. (16) is the same as its QED formulation in Eq. (15). This proves that the earlier derived Eq. (16) has a wide range of applicability including an electron quasi-classical motion in QED strong fields. The way to solve Eq. (16) and integrate the emission is described in [3].

In Fig. 4, we show the numerical result for an electron interacting with a laser pulse. We see that the QED effects essentially modify the radiation spectrum even with laser intensities which are already achieved.

We conclude that in a wide range of applications, including the case of very strong laser fields with essential QED effects, the electron motion may be successfully described within the radiation force approximation. The necessary corrections in the radiation force and the emission spectra to account for the QED effects are parameterized by the sole parameter, \( I_{cl} \).

The future application to QED Monte-Carlo simulations may be based on the total probability of emission per interval of proper time: \( W = \Delta \tau (I_{QED}/(m_e c^2)) \omega^{-1} \), where \( \omega^{-1} = \int Q(r_0, \chi)/(\chi r_0) dr_0 \). The expression of the only scalar to gamble, \( \lambda_{\chi} \omega'/\epsilon c \), in terms of a random number, \( 0 \leq R < 1 \), is given by an integral equation as follows:

\[
\int_0^{\lambda_{\chi} \omega'/\epsilon c} Q/(r_0 \chi) dr_0 = R < \lambda_{\chi} \omega'/\epsilon c >^{-1}.
\]

This method will be described in a forthcoming publication in detail, including the pair production (see [12] regarding the latter effect).

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