Comment on “The third Zemach moment of the proton”, by Clöet and Miller.

A. De Rújula$^{a,b,c}$

$^a$ Instituto de Física Teórica, Univ. Autónoma de Madrid, Madrid, and CIEMAT, Madrid, Spain,
$^b$ Physics Dept., Boston University, Boston, MA 02215,
$^c$Physics Department, CERN, CH 1211 Geneva 23, Switzerland

(Dated: August 27, 2010)

Clöet and Miller, in arXiv:1008.4345, state that “existing data rule out a value of the third Zemach moment large enough to explain the current puzzle with the proton charge radius determined from the Lamb shift in muonic Hydrogen. This is in contrast with the recent claim of De Rújula”. To be more precise: it is not. It is, however, contrary to what they claim that I claim. Clöet and Miller have simply misinterpreted my claims.

PACS numbers: 31.30.jr, 12.20.-m, 32.30.-r, 21.10.Ft

I. MY MAIN CLAIM

Let “model-independent” mean “independent of a particular parametrization of the charge distribution of the proton, $\rho_p(r)$”. The most precise model-independent measurements of the corresponding mean square radius, $\langle r_p^2 \rangle$, are mainly based on the theory [2] and observations [3] of Hydrogen. The result, compiled in CODATA [4], is

$$\langle r_p^2 \rangle / \text{(CODATA)} = (0.8768 \pm 0.0069 \text{ fm})^2$$

(1)

Consider next the $2P_{3/2} \rightarrow 2S_{1/2}$ Lamb shift in a $\mu p$ atom. In meV units for energy and fermi units for the radii, the predicted value [5] is of the form

$$L^\text{th} (\langle r_p^2 \rangle, \langle r_p^3 \rangle / (2)) = 209.9779(49) - 5.2262 \langle r_p^2 \rangle + 0.00913 \langle r_p^3 \rangle / (2),$$

(2)

which is also model-independent. The first two coefficients are best estimates of many contributions [6] while the third stems from the $n = 2$ value of an addend [7]

$$\Delta E_3(n) = \frac{\alpha^5}{3n^2} m^2 \delta_0 (\langle r_p^3 \rangle / (2)),$$

(3)

proportional to the third Zemach moment

$$\langle r_p^3 \rangle / (2) \equiv \int d^3 r_1 d^3 r_2 \rho(r_1)\rho(r_2) |r_1 - r_2|^3$$

(4)

The quoted Lamb shift has been measured [6] to be

$$L_\text{exp} = 206.2949 \pm 0.0032 \text{ meV}.$$  

(5)

Assume that the theory and experiments quoted so far are correct. Use Eqs. (2,5) to write $L^\text{th} = L_\text{exp}$. Introduce into the expression for $L^\text{th}$ the observed value of $\langle r_p^2 \rangle$ given in Eq. (1). Solve for $\langle r_p^3 \rangle / (2)$. The result is:

$$[\langle r_p^3 \rangle / (2)]^{1/3} = 3.32 \pm 0.22 \text{ fm}$$

(6)

with the error dominated by the CODATA uncertainty.

My main claim is to have done the algebra leading to Eq. (6), which is model-independent.

II. MY ALLEGED CLAIM

Clöet and Miller state “In the simple monopole model of De Rújula, which claims to account for the proton radius puzzle...”.

I have not claimed that a monopole model (which, incidentally, is not “mine”) accounts for the proton radius puzzle. On the contrary, I explicitly constructed a “toy model” with two added monopole form factors to illustrate how the non-vanishing contributions of no fewer than two monopoles were needed to reconcile the data.

The comment that may have led Clöet and Miller to misinterpret me is the last paragraph in arXiv:1008.3861:

“The result Eq. (6) is $\rho_p(r)$-independent and to be treated with due respect. Right after offering excuses, I shall break this rule. The third Zemach moment is very sensitive to the long-distance part of $\rho(r)$... A correct question to ask is the scale, $m$, to which this tail corresponds. Not to be confused with the toy model fit to both $\langle r_p^2 \rangle$ and $\langle r_p^3 \rangle / (2)$, the simplest naive –but consistent– answer is provided by Eq. (6) in the single-pole approximation i.e. $m \approx 261 \text{ MeV}$, tantalizingly close to the threshold of the proton form factor’s cut at $2m_{a\pi} \approx 278 \text{ MeV}$”.

All I did in the previous paragraph is to use a monopole to extract a rough estimate of the slope of $\rho_p(r)$ at large $r$, and to express it as an inverse mass. I did not contend that a single monopole representation of $G_E(-q^2)$ “accounts for the proton radius puzzle” at all relevant values of $q^2$, nor that its Fourier transform accurately describes $\rho_p(r)$ at all relevant $r$.

The penultimate paragraph above, from v2 of arXiv:1008.3861 (posted one day after v1) may have been more obscure in v1. No doubt this is what mislead Clöet and Miller, regarding what my claims really were.

III. CLÖET AND MILLER’S CLAIMS

I use the word “claims” here for literary consistency. But I mean “statements”.

CERN-PH-TH/2010-202
Clòet and Miller study the proton-radius puzzle by use of three models—the dipole and the ones in [8, 9]—that describe $G_E(q^2)$ as measured in ep scattering experiments. They find that the model for which $G_E(q^2)$ falls fastest at large $q^2$ gives the largest value of $\langle r_p^3 \rangle^2 / \langle r_p^2 \rangle^3$, but is insufficient to resolve the “puzzle”.

In the models in [8, 9] the numerical parameters can be combined with their single mass scale ($m_p$) to be reinterpreted as different mass scales. This is in agreement with a claim that I have indeed made: “Any simple one-parameter description of the proton’s non-relativistic Sacks form factor, $G_E(-q^2)$, in terms of only one mass parameter is inaccurate: the proton is not so simple”.

The crucial problem was adroitly emphasized by Sick [10]. It is very difficult to extract reliable information on $\rho(r)$ (such as its moments) from its Fourier transform, $G_E(q^2)$. The radius of convergence of the expansion in $r$ from which one directly extracts $\langle r_p^2 \rangle$ from the ep data is so small, that one must use numerical simulations and a continued-fraction expansion to skirt the uncertainties associated with data normalization at small $q^2$ (and their systematic errors) and to obtain a stable, numerically-meaningful result not contaminated, for instance, by the term in $\langle r_p^4 \rangle$. Clearly, if extracting $\langle r_p^2 \rangle$ is delicate, the more so it is to infer $\langle r_p^3 \rangle^2 / \langle r_p^2 \rangle^3$.

The authors of [9] are aware of the above-mentioned practicalities. But they do not need to face them à la Sick, since they analyze data at much larger $q^2$ than the range with which we are concerned. Clòet and Miller simply extrapolate the high-$q^2$ models in [8, 9] to the $|q|$ values of $O(\alpha m_p)$ of interest here. In this way, not surprisingly, they obtain a model- and extrapolation-dependent value of $\langle r_p^3 \rangle^2$ in disagreement with Eq. (6).

Acknowledgments

I am indebted to York Schroeder and to Ian Clòet and Gerald Miller for having independently found a mistake in my explicit $\langle r_p^3 \rangle^2$ numerical expressions, which I shall correct. This error does not alter my claims.

[1] “To claim: to state or assert that something is the case, typically without providing evidence or proof.”
[2] M. I. Eides, H. Grotch & V. A. Shelyuto, Springer Tracts in Mod. Phys. 222 (Springer, Berlin Heidelberg, 2007). S. G. Karshenboim, Phys. Rep. 422 (2005) 1.
[3] M. Niering, Phys. Rev. Lett. 84 (2000) 5496. B. de Beauvoir, Eur. Phys. J. D12 (2000) 61. C. Schwob Phys. Rev. Lett. 82 (1999) 4960.
[4] P. J. Mohr, B. N. Taylor & D. B. Newell, Rev. Mod. Phys. 80 (2008) 633.
[5] S. G. Karshenboim, Phys. Rep. 422 (2005) 1. K. Pachucki, Phys. Rev. A60 (1999) 3593. E. Borie, Phys. Rev. A71 (2005) 032508. A. P. Martynenko, Phys. Rev. A71 (2005) 022506. A. P. Martynenko, Phys. At. Nucl. 71 (2008) 125. K. Pachucki, & U. D. Jentschura, Phys. Rev. Lett. 91 (2003) 113005.
[6] R. Pohl, et al. Nature 466 213 (2010) 213.
[7] J. L. Friar, Annals of Physics 122 (1979) 151.
[8] J. J. Kelly, Phys. Rev. C70 (2004) 062802.
[9] W. M. Alberico et al. Phys. Rev. C79 (2009) 065204.
[10] I. Sick, Phys. Lett. B576 (2003) 62.