Quintessence with a localized scalar field on the brane

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Abstract

We study issues of the quintessence in the brane cosmology. The initial bulk spacetime consists of two 5D topological anti de Sitter black hole joined by the brane (moving domain wall). Here we do not introduce any conventional radiation and matter. Instead we include a localized scalar on the brane as a stress-energy tensor, and thus we find the quintessence which gives an accelerating universe. Importantly, we obtain a $\rho^2$-term as well as a holographic matter term of $\alpha/a^4$ from the masses of the topological black holes. We discuss a possibility that in the early universe, $\rho^2$-term makes a large kinetic term which induces a decelerating universe. This may provide a hint of avoiding from the perpetually accelerating universe of the present-day quintessence. If a holographic matter term exists, it will play the role of a CFT-radiation in the early universe.

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I. INTRODUCTION

Recently an accelerating universe has proposed to be a way to interpret the astronomical data of supernova [1–3]. Combining these data with the measurements of the cosmic microwave background, it may be led that the universe has the critical density which consists of 1/3 of ordinary matter and 2/3 of dark matter with a negative pressure as \( p < -\rho/3 \). At this moment a promising candidate for the dark matter is either a positive cosmological constant or a slowly evolving scalar known as “quintessence”. Also the quintessence can be considered as an alternative method to resolve the cosmological constant problem. This is possible because instead of the fine-tuning, the quintessence provides a model of slowly decaying cosmological constant.

However, there exists another question for the present-day quintessence. This is to ask whether the expansion will keep accelerating forever or it will decelerate again after some time. This is very similar to the exit problem in the inflationary cosmology. Some literature discussed on this issue within the string theory [4–6] and realistic cosmological context [7,8]. If the expansion keeps accelerating eternally, we have the universe with an event horizon which gives rise to a serious problem in defining the conventional S-matrix. Also it is hard to obtain de Sitter space (its asymptotic space) from the string theory.

On the other hand authors in ref. [9] also considered the possibility of quintessence in the dilatonic domain wall. After the integration over the dual holographic field theory, one obtained an effective dilaton gravity with the potential on the brane. They used the Hamilton-Jacobi method inspired by the holographic renormalization group to investigate the intrinsic Friedmann-Robertson-Walker (FRW) cosmology on the brane. It was shown that a holographic quintessence is allowed on the dilatonic brane because a single Liouville-type potential appears on the brane. However, using the zero-mode approach leads to the fact that the dilatonic domain wall including the Randall-Sundrum (RS) model [10] cannot accommodate the quintessence [11]. Although we found Liouville-type potentials, we failed to find any accelerating universe from the zero mode approach of the dilatonic domain wall. This is mainly because the unstable potential was found.

In this paper we wish to deal with the same issue within the brane cosmology context. More recently the related issues were discussed in [12,13]. The brane cosmology contains two important deviations from the standard Friedmann-Robertson-Walker (FRW) cosmology: The first is that \( \rho^2 \)-term appears in the Friedmann equations [14–16]. The second is that there exist a holographic matter term of \( \alpha/a^4 \), which is interpreted as either a dark matter [14,16,13] or a CFT-matter [17]. Finally the acceleration or deceleration of evolving universe will be determined by a more complicated fashion than a standard cosmology. Hence we expect to find something new to avoid the perpetually accelerating universe. Here we study mainly how the brane cosmology with a localized scalar and a holographic matter affects the quintessence of a standard scalar model.

There are two approaches in the brane cosmology. One approach is first to assume the 5D dynamic metric (that is, Binetruy-Deffayet-Longlois metric [14]) which is manifestly \( \mathbb{Z}_2 \)-symmetric under \( \hat{w} \rightarrow -\hat{w} \). Then one considers the Israel junction condition to take

\[ \hat{w} \]

is the fifth one of Gaussian normal coordinates \( \{ \hat{\tau}, \hat{x}_i, \hat{w} \} \).
into account a localized matter distribution on the brane [18] and solves the bulk Einstein equation to find the behavior of the scale factor on the brane. We call this the BDL approach. The other approach starts with a static bulk configuration which consists of two 5D anti de Sitter-Schwarzschild (AdSS\(_5\)) black hole spaces joined by the domain wall. In this case, embedding the moving domain wall (MDW) into the bulk spacetime is possible by choosing an appropriate normal vector \(n_M\) and tangent vector \(t_M\) [16]. The domain wall separating two such bulk spaces is taken to be located at \(r = a(\tau)\), where \(a(\tau)\) will determined by solving solely the Israel junction condition. Then observers on the MDW will interpret their motion through the static bulk background as cosmological expansion or contraction. This is called the MDW approach. Although these two approaches seem to be different, these give us the same result. Actually Mukhoyama et al. [19] performed a coordinate transformation \(\{\hat{\tau}, \hat{x}_i, \hat{w}\} \to \{t(\tau), a(\tau), \chi, \theta, \phi\}\) in order to bring the BDL metric into the AdSS\(_5\)-metric.

The organization of our paper is as follows. In Sec. II we briefly review a simple model for the quintesence. We study issues of the quintessence within the brane cosmology in Sec. III. Finally we discuss our results in Sec.IV.

II. QUINTESSENCE

A minimally coupled scalar field with a potential that decreases as the field increases is usually introduced for the quintessence [7]. The corresponding action is given by

\[
S_Q = \frac{1}{2} \int d^4x \sqrt{-g} \left[ \frac{1}{\kappa^2_4} R - (\partial \phi)^2 - 2V(\phi) \right]
\]

with the 4D gravitational constant \(\kappa^2_4 = 8\pi G_N\). Hereafter we choose \(\kappa^2_4 = 1\) for simplicity. This is a canonically normalized scalar action coupled to gravity. The Einstein equation is

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu}
\]

with

\[
T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} (\partial \phi)^2 g_{\mu\nu} - V(\phi) g_{\mu\nu}.
\]

Considering the FRW universe of \(ds^2_{FRW} = -dt^2 + a^2(t)d\Sigma_k^2\), the equation of motion for \(\phi\) and the conservation law of \(\nabla_\mu T^{\mu\phi} = 0\) lead to the the same equation as

\[
\ddot{\phi} + 3 \frac{\dot{a}}{a} \dot{\phi} + V'(\phi) = 0.
\]

The two FRW equations are given by

\[
\frac{\dot{a}^2}{a^2} = -\frac{k}{a^2} + \frac{\rho}{3}, \quad \frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p).
\]

\(^2\)Our convention is (\(+-++\)).
Assuming $\phi = \phi(t)$ for cosmological purpose, then the energy density and pressure are given by, respectively,

$$\rho = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad p = \frac{1}{2} \dot{\phi}^2 - V(\phi). \tag{6}$$

The corresponding equation of state takes the form

$$\omega \equiv \frac{p}{\rho} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}. \tag{7}$$

The causality restricts $\omega$ to be $|\omega| \leq 1$. For $\dot{\phi}^2 = 0$, we have a trivial case of $p = -\rho$, which implies that $\rho$ is independent of $a$. Alternatively $V(\phi)$ plays the role of a cosmological constant. On the other hand, if $V(\phi) = 0$, one finds an extreme case of a massless scalar which plays the role of a stiff matter as $\rho$ drops as $1/a^6$.

The relevant equation of state ranges over $-1 < \omega < -1/3$, depending on the dynamics of the field. We note that when $\dot{\phi}^2 < V(\phi)$ on later time, we obtain an accelerating universe of $p < -\rho/3$ from the second equation in Eq.(5). It was shown that for a flat spacetime $k = 0$, $\phi$ and $\rho$ scale as

$$\frac{\partial \phi}{\partial a} = \sqrt{3(1 + \omega)} \frac{\rho}{a}, \quad \rho \sim \frac{1}{a^{3(1+\omega)}}. \tag{8}$$

Using the relation of $V(\phi) = (1 - \omega)\rho/2$ with $\phi = \sqrt{3(1 + \omega)} \ln a$, one of relevant cases which give the quintessence is an exponential potential

$$V(\phi) = V_0 e^{-\sqrt{3(1+\omega)}\phi}, \quad 0 < 3(1 + \omega) < 2 \tag{9}$$

which is a kind of Liouville-type potential that decreases as $\phi$ increases. According to the theory of quintessence, the dark energy of the universe is dominated by the potential of a scalar field $\phi$ which is still rolling to its minimum at $V = 0$. In addition we require its minimum at $\phi = \infty$. The above Liouville-type potential is suited well for the quintessence. For example, if one takes $\omega = -\frac{1}{2} < -\frac{1}{3}$, $V(\phi) = V_0 e^{-\sqrt{3/2}\phi}$ derives an accelerating universe. However, this model has a serious drawback which shows that it leads to an eternally accelerating universe and accordingly to the universe with an event horizon.

### III. QUINTESSENCE WITHIN BRANE COSMOLOGY

Now we wish to discuss issues of the quintessence within the brane cosmology. For simplicity we choose the MDW approach. For the cosmological embedding, a relevant solution

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3. Here our case of $\frac{\partial \phi}{\partial a}$ differs from $\frac{\partial \phi}{\partial a} = \sqrt{6(1+\omega)} \frac{\rho}{a}$ in [7] because our action Eq.(1) is different from [7].

4. There are three types of quintessence model: (1) an inverse-power-law potential, (2) an exponential potential, and (3) kinetic-term quintessence model [13].
is initially chosen as the 5D topological anti de Sitter (TAdS) black holes [20] for right hand side (+) and left hand side (−),

\[ ds_5^2 = g_{MN} dx^M dx^N = -h_\pm(r) dt^2 + \frac{1}{h_\pm(r)} dr^2 + r^2 \gamma_{ij} dx^i dx^j, \]  

where the metric function \( h_\pm \) is given by

\[ h_\pm(r) = k - \frac{\alpha_\pm}{r^2} + \frac{r^2}{\ell^2}, \] 

with \( \ell \) the curvature radius of AdS\(_5\) spaces. \( \gamma_{ij} \) is the horizon metric of a constant curvature manifold \( M^3 \) with volume \( Vol(M^3) = \int d^3x \sqrt{\gamma} \). The horizon geometry of the TAdS\(_5\) black hole is elliptic, flat, and hyperbolic for \( k = 1, 0, -1 \), respectively:

\[ \gamma_{ij} dx^i dx^j = \left[ d\chi^2 + f_k(\chi)^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] \] 

where \( f_k(\chi) = \sin \chi \) for \( k = 1 \), \( f_k(\chi) = \chi \) for \( k = 0 \), and \( f_k(\chi) = \sinh \chi \) for \( k = -1 \).

In the case of \( \alpha_\pm = 0 \), we have two exact AdS\(_5\)-spaces with the same cosmological constants \( \Lambda_\pm = -6/\ell^2 \). However, \( \alpha_\pm \neq 0 \) generates the electric part of the Weyl tensor \( C_{MNPQ}^\pm \) on each side. Its presence means that the bulk spacetime has the two black holes with horizon located at \( r = r_{\pm h} \), where \( r_{\pm h}^2 = \ell^2(-k + \sqrt{k^2 + 4\alpha_\pm/\ell^2})/2 \) [10]. Now we introduce the location of brane (moving domain wall) in the form of the radial, timelike geodesics parametrized by the proper time \( \tau : (t, r, \chi, \theta, \phi) \rightarrow (t(\tau), a(\tau), \chi, \theta, \phi) \). Then the induced metric of dynamical domain wall will be given by the conventional FRW-type. In this case \( \tau \) and \( a(\tau) \) denote the cosmic time and scale factor of the FRW universe, respectively. The tangent vectors of this brane can be expressed as

\[ u_\pm = \dot{t}_\pm \frac{\partial}{\partial t_\pm} + a \frac{\partial}{\partial a}, \] 

where the overdot means differentiation with respect to \( \tau \). These satisfy \( u_\pm M u_\pm N g^{MN} = -1 \). Further we need the normal 1-forms directed toward to each side: \( n_\pm M n_\pm N g^{MN} = 1 \). Here we choose these as

\[ n_\pm = \pm \dot{a} dt_\pm \mp \dot{t}_\pm da. \] 

This embedding reduces to the Randall-Sundrum case when \( \alpha_\pm = 0 \) [10]. Using these, we can express \( \dot{t} \) in terms of \( \dot{a} \) as

\[ \dot{t} = \frac{\dot{a}^2 + h_\pm(a)^{1/2}}{h_\pm(a)}. \] 

From the bulk metric Eq.(10) together with Eq.(14), we arrive at the 4D induced metric

\[ ds_4^2 = -dr^2 + a(\tau)^2 \left[ d\chi^2 + f_k(\chi)^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] \equiv h_{\mu\nu} dx^\mu dx^\nu. \] 

[5] On later, \( k \) will turn out to be the spatial curvature of the universe.
Hereafter we use the Greek indices (\(\mu, \nu, \cdots\)) for physics of the brane. The extrinsic curvatures for cosmological embedding are given by

\[
(K_{\pm})_{\tau\tau} \equiv (K_{\pm})_{MN} u^M_{\pm} u^N_{\pm} = \pm (h_{\pm} \dot{t}_{\pm})^{-1} (\ddot{a} + h_{\pm} / 2),
\]

\[
(K_{\pm})_{\chi} = (K_{\pm})_{\theta} = (K_{\pm})_{\phi} = \mp h_{\pm} \dot{t}_{\pm} / a,
\]

where the prime stands for derivative with respect to \(a\). The above equation implies that the extrinsic curvature will jump across the brane if a localized matter resides on the brane. This jump is easily realized through the Israel junction condition. Here we need only its 4D induced version defined through Eqs. (16) and (17)

\[
\Delta K_{\mu\nu} = -\kappa_5^2 \left( T_{\mu\nu} - \frac{1}{3} T^\lambda_{\lambda \mu\nu} \right)
\]

with \(\Delta K_{\mu\nu} \equiv (K_+)_{\mu\nu} - (K_-)_{\mu\nu}\) and the 5D gravitational constant \(\kappa^2_\ell \equiv \kappa^2_4 \ell = \ell\) when \(\kappa^2_4 = 1\). We usually choose the localized stress-energy tensor on the brane as the 4D perfect fluid

\[
T_{\mu\nu} = (\rho + p) u^\mu_u u^\nu + p h_{\mu\nu}
\]

with \(\rho = \rho_m + \sigma (p = p_m - \sigma)\). \(\rho_m (p_m)\) is the energy density (pressure) of the localized matter and \(\sigma\) is the brane tension (or vacuum energy like cosmological constant). In the absence of a localized matter, Eq.(18) takes the form of \(-\frac{\sigma \kappa^2_5}{3} h_{\mu\nu}\) as in the RS case. We stress again that \(u^\mu\) are defined through the 4D induced metric \(h_{\mu\nu}\) of Eq.(15). In addition, we need the Gauss-Codazzi equations with the 4D induced Einstein tensor \(G_{\mu\nu}\) with respect to \(h_{\mu\nu}\)

\[
\frac{\kappa^2_5}{2} [(K_+)_{\mu\nu} + (K_-)_{\mu\nu}] T^\mu\nu = \Delta G_{\mu\nu} n^\mu n^\nu,
\]

\[
\kappa^2_5 h^\mu_{\lambda} \nabla^\nu T^\mu_{\lambda} = h^\mu_{\lambda} \Delta G^\nu_{\lambda} n^\nu,
\]

where the last one is here nothing but the conservation law

\[
\frac{d}{d\tau} \left( \rho a^3 \right) + p \frac{d}{d\tau} \left( a^3 \right) = 0.
\]

We note that Eqs. (20) and (21) describe effectively the metric junction condition. In these equations, right-hand sides become zero because we choose the same cosmological constant for the two sides. From Eqs.(18), (19) and (20), one finds

\[
(h_+ \dot{t}_+)^{-1} (\ddot{a} + h_+ / 2) + (h_- \dot{t}_-)^{-1} (\ddot{a} + h_- / 2) = -\kappa^2_5 \left( p + \frac{2}{3} \rho \right),
\]

\[
h_+ \dot{t}_+ + h_- \dot{t}_- = \frac{\kappa^2_5}{3} \rho a,
\]

\[
(h_+ \dot{t}_+)^{-1} (\ddot{a} + h'_R / 2) - (h_- \dot{t}_-)^{-1} (\ddot{a} + h'_L / 2) = 3 \frac{p}{\rho a} (h_+ \dot{t}_+ - h_- \dot{t}_-).
\]

Equation (23) corresponds to the acceleration of the moving domain wall, while Eq.(24) corresponds to the velocity of the moving domain wall. We solve the above equations simultaneously to give the second Friedmann equation

\[
\frac{d}{d\tau} \left( \rho a^3 \right) + p \frac{d}{d\tau} \left( a^3 \right) = 0.
\]
\[ \frac{\ddot{a}}{a} = -\frac{1}{\ell^2} - \frac{\alpha_+ + \alpha_-}{2a^4} - \frac{\kappa_5^4}{18} \phi (\phi + \frac{3}{2}p) - \frac{27 (\alpha_+ - \alpha_-)^2 p}{4 \kappa_5^4 g^2 a^8}. \] (26)

and the first Friedmann equation with the spatial curvature of the universe \( k = 1, 0, -1 \)
\[ \frac{\dot{a}^2}{a^2} = -\frac{k}{a^2} + \frac{\alpha_+ + \alpha_-}{2a^4} + \frac{1}{36} \kappa_5^4 g^2 + \frac{9 (\alpha_+ - \alpha_-)^2}{4 \kappa_5^4 g^2 a^8}. \] (27)

For simplicity, we choose hereafter the \( Z_2 \)-symmetric case with \( \alpha_+ = \alpha_- \equiv \alpha \). Furthermore we require the fine-tuning of \( \sigma = 6/\kappa_5^2 \ell^2 \) for two-sided brane world scenario. Then considering \( \rho = \rho_m + \sigma (p = p_m - \sigma) \) with \( \kappa_5^2 = \ell \), Eq.(27) leads to
\[ \frac{a^2}{a^2} = -\frac{k}{a^2} + \frac{\rho_m}{3} + \frac{\alpha}{a^4} + \frac{\ell^2}{36} \rho_m^2. \] (28)

The second Friedmann equation takes an interesting form with \( p_m = \omega \rho_m \)
\[ \frac{\dot{a}}{a} = -\frac{\rho_m}{6} (1 + 3\omega) - \frac{\alpha}{a^4} - \frac{\ell^2}{36} \rho_m^2 (2 + 3\omega). \] (29)

A. \( \alpha = 0 \) case

In the case of \( \alpha = 0 \), the bulk spacetime consists of two exact AdS_5 spacetimes. Let us first discuss its cosmological implication by introducing a localized scalar with the potential \( V(\phi) \). The corresponding energy-stress tensor is also given by Eq.(3). In this case the energy density (pressure) of the localized matter are given by the same form as in Eq.(3)
\[ \rho_m = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad p_m = \frac{1}{2} \dot{\phi}^2 - V(\phi). \] (30)

Expressing Eq.(29) in terms of Eq.(31) leads to
\[ \frac{\dot{a}}{a} = -\frac{1}{3} (\dot{\phi}^2 - V(\phi)) - \frac{\ell^2}{4 \cdot 36} (\dot{\phi}^2 + 2V(\phi))(5\dot{\phi}^2 - 2V(\phi)). \] (31)

Here we classify three cases.
(i) \( 5\dot{\phi}^2/2 \leq V(\phi) \): dominance of the potential energy.

In this case we have the equation of state \(-1 < \omega \leq -\frac{2}{3} \). Two terms in right hand side give us positive accelerations. And thus we have more acceleration than the standard quintessence. Here it is important to remind the reader that the observational evidence for a cosmological constant is really the same bound of \(-1 < \omega_{observed} \leq -\frac{2}{3} \) as is here [3]. A borderline of \( \omega = -1 \) can be achieved when \( \dot{\phi}^2 = 0 \). In this case the potential term \( V(\phi) \) plays the role of a cosmological constant. Hence we expect presumably that the quintessence within the brane cosmology has something to tell us how to resolve the cosmological constant problem.

(ii) \( \dot{\phi}^2 < V(\phi) < 5\dot{\phi}^2/2 \).

Here we have \(-\frac{3}{5} < \omega < -\frac{1}{3} \). This is a very interesting case because the first term induces positive acceleration, while the second induces negative acceleration. So there exists
competition between two terms and accordingly it may have a chance to exit from the accelerating universe in the early universe.

(iii) \( \dot{\phi}^2 > V(\phi) \): dominance of the kinetic energy.

The corresponding equation of state is \(-\frac{1}{3} < \omega < 1\). This corresponds to the deceleration epoch because both two terms induce negative accelerations. The case of \( \dot{\phi}^2 = V(\phi) \) corresponds to a borderline of \( \omega = -1/3 \).

We consider a case that the quadratic term \( (\rho_m^2) \) in right hand side of Eq. (28) becomes significant in comparison with the linear term \( (\rho_m) \) in the early universe. This is possible when \( \rho_m > 12/\ell^2 \). Considering \( k = \alpha = 0 \) and \( \rho_m^2 \)-term only with \( \rho_m \sim 1/a^{3(1+\omega)} \), Eq. (28) is solved to give \( a(\tau) \sim \tau^q \) with \( q = \frac{1}{3(1+\omega)} > \frac{1}{2} \). On the other hand, the conventional quintessence with \( \rho_m \) leads to \( a(\tau) \sim \tau^p \) with \( p = \frac{2}{3(1+\omega)} > 1 \). Hence when the quadratic term is dominated in the early universe, the Hubble expansion rate decreases. In this case the friction term (the second one) in Eq.(34) becomes small and the dynamics of the scalar field will be drastically changed from the conventional quintessence even for the same potential [13]. It is easily expected that a kinetic term will play a more important role than the potential. Assuming this scenario, we could have a chance to meet the case (iii): dominance of the kinetic term. Then it is proposed that the universe may show a nice transition from an accelerating universe to a decelerating one in the early universe [6].

\section*{B. \( \alpha \neq 0 \) case}

In this case we have the holographic matter term from the bulk configuration[1]. This term of \( \alpha/a^4 \) in Eqs. (28) and (29) originates from the masses of the topological black holes. It always behaves like radiation for either a dark matter candidate [16,13] or a CFT-matter [17]. Here we are interested especially in a flat universe (\( k = 0 \)) with a CFT-radiation matter [21]. We have \( \alpha = \omega_4M \), where \( \omega_4 = \frac{2\kappa_5^2}{Vol(M^3)} \), \( M = \frac{4}{7}E, \kappa_5^2 = \ell \). Here \( M \) is the ADM mass of the TAdS\(_5\) black hole and \( E \) is its holographic energy on the brane. Further \( V = a^3Vol(M^3) \) is the size of the universe. Then one gets a universe filled partly by the CFT-radiation and the scalar matter. Its acceleration is given by

\[
\frac{\ddot{a}}{a} = -\frac{\rho_{CFT}}{6} - \frac{\rho_m}{6}(1 + 3\omega) - \frac{\ell^2}{36}\rho_m^2(2 + 3\omega),
\]

where \( \rho_{CFT} = E/V \) is the energy density of the CFT-holographic matter. Another form of the above equation is

\[
\frac{\ddot{a}}{a} = -\frac{1}{3}\left( \frac{1}{2}\rho_{CFT} + \dot{\phi}^2 - V(\phi) \right) - \frac{\ell^2}{4 \cdot 36}(\dot{\phi}^2 + 2V(\phi))(5\dot{\phi}^2 - V(\phi)).
\]

Here it is important to note that energy densities in Eq.(32) drop like

\[
\rho_{CFT} = \frac{\alpha}{a^4}, \quad \rho_m \sim \frac{1}{a^{3(1+\omega)}}, \quad \rho_m^2 \sim \frac{1}{a^{6(1+\omega)}}, \quad 0 < 3(1 + \omega) < 2.
\]

\[6\]In the BDL approach, we can also obtain this term [22].
Considering the CFT-term \((a_{CFT}(\tau) \sim \tau^{1/2})\), this term dominates in the very early universe \(\text{[6]}\). This is because \(a_{\rho m} \sim \tau^p\) with \(p > 1\) and \(a_{\rho m}^2 \sim \tau^q\) with \(q > 1/2\). On later the quadratic term will be significant because we expect a transition from radiation to matter: \(1/2 \rightarrow q \rightarrow p\) as the universe evolves. If we have a phase of dominant kinetic term during this transition, we can apply the same scenario as in the \(\alpha = 0\) to this case to have a nice exist from the accelerating universe.

**IV. DISCUSSIONS**

We discuss issues of the quintessence in the brane cosmology. The initial bulk spacetime consists of the two 5D topological anti de Sitter black hole joined by the brane. Considering the embedding of a moving domain wall into this background, we find the FRW universe with a scale factor \(a(\tau)\). Here this factor is determined by not the Einstein equation in Eq.(2) but the Israel junction condition Eq.(18) mainly. For our purpose we do not introduce any conventional radiation and matter. Instead we include a localized scalar with a potential on the brane as a stress-energy tensor, and thus we find the quintessence which gives an accelerating universe. Importantly, we obtain both a quadratic term of \(\rho^2\), as a representative of the brane cosmology and a holographic matter term of \(\alpha/a^4\) from the topological black holes. We suggest a possible scenario that in the early universe, \(\rho^2\)-term makes a large kinetic term which induces a decelerating universe. This may provide a way to exit from the accelerating universe in the early universe. Furthermore, one finds that if a holographic CFT-matter from the TAdS\(_5\) black holes includes, it will plays the role of a radiation in the very early universe.

In this work we do not choose any explicit form of the potential \(V(\phi)\) which is suitable for explaining the early accelerating universe. There exist three kinds of potential for the present-day accelerating universe: (i) Liouville-type potential : \(V(\phi) \sim \exp(-\sqrt{3(1+\omega)}\phi)\). (ii) inverse power law potential : \(V(\phi) \sim 1/\phi^n\). (iii) exponential potential with \(1/\phi : V(\phi) \sim (\exp(M_P/\phi)-1)\). According to authors in \(\text{[6]}\), solutions of all these models asymptote towards the equation of state \(p = -\rho\), showing eternally accelerating universe. This type of the universe (like de Sitter spacetime) has the future horizon and thus is not suited to an S-matrix or S-vector description \(\text{[5]}\).

We compare our model with Mizuno and Maeda (MM) case \(\text{[13]}\). As we were almost finishing our work, we are informed that their paper discussed similar issue in the brane cosmology. Both two models are suitable for explaining a transition from an accelerating universe to the standard one. But there exist differences between MM and our cases. The first one is that we do not include any conventional radiation and matter. Also we introduce an explicit form of a holographic matter from the TAdS\(_5\) black holes. This is regarded as a CFT-radiation matter \(\text{[17]}\). But they neglected this in favor of conventional radiation and matter.

Finally we summarize our result. Initially this work aims for resolving an eventually accelerating universe of the present-day quintessence. However, the brane cosmology provides a mechanism to exit from an accelerating universe only in the early universe. In this case the relevant term is \(\rho^2_m\)-term which makes a large kinetic term. This induces a transition from an accelerating universe to a decelerating universe in the early universe. Although we fail
to resolve the problem of the present-day quintessence, we expect that this approach provide a hint of avoiding an eternally accelerating universe, assuming our universe is presently accelerating.

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