COEFFICIENT BOUNDS FOR CLOSE-TO-CONVEX FUNCTIONS ASSOCIATED WITH VERTICAL STRIP DOMAIN

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Abstract. By considering a certain univalent function in the open unit disk $\mathbb{U}$, that maps $\mathbb{U}$ onto a strip domain, we introduce a new class of analytic and close-to-convex functions by means of a certain non-homogeneous Cauchy-Euler-type differential equation. We determine the coefficient bounds for functions in this new class. Relevant connections of some of the results obtained with those in earlier works are also provided.

1. Introduction

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

(1.1)

which are analytic in the open unit disk $\mathbb{U} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$. We also denote by $\mathcal{S}$ the class of all functions in the normalized analytic function class $\mathcal{A}$ which are univalent in $\mathbb{U}$.

For two functions $f$ and $g$, analytic in $\mathbb{U}$, we say that the function $f$ is subordinate to $g$ in $\mathbb{U}$, and write

$$f(z) \prec g(z) \quad (z \in \mathbb{U}),$$

if there exists a Schwarz function $\omega$, analytic in $\mathbb{U}$, with

$$\omega(0) = 0 \quad \text{and} \quad |\omega(z)| < 1 \quad (z \in \mathbb{U})$$

such that

$$f(z) = g(\omega(z)) \quad (z \in \mathbb{U}).$$

Indeed, it is known that

$$f(z) \prec g(z) \quad (z \in \mathbb{U}) \Rightarrow f(0) = g(0) \quad \text{and} \quad f(\mathbb{U}) \subset g(\mathbb{U}).$$

Furthermore, if the function $g$ is univalent in $\mathbb{U}$, then we have the following equivalence

$$f(z) \prec g(z) \quad (z \in \mathbb{U}) \Leftrightarrow f(0) = g(0) \quad \text{and} \quad f(\mathbb{U}) \subset g(\mathbb{U}).$$

A function $f \in \mathcal{A}$ is said to be starlike of order $\alpha$ ($0 \leq \alpha < 1$), if it satisfies the inequality

$$\Re \left( \frac{zf'(z)}{f(z)} \right) > \alpha \quad (z \in \mathbb{U}).$$

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We denote the class which consists of all functions \( f \in \mathcal{A} \) that are starlike of order \( \alpha \) by \( \mathcal{S}^*(\alpha) \). It is well-known that \( \mathcal{S}^*(\alpha) \subset \mathcal{S}^*(0) = \mathcal{S}^* \subset \mathcal{S} \).

Let \( 0 \leq \alpha, \delta < 1 \). A function \( f \in \mathcal{A} \) is said to be close-to-convex of order \( \alpha \) and type \( \delta \) if there exists a function \( g \in \mathcal{S}^*(\delta) \) such that the inequality

\[
\Re \left( \frac{zf'(z)}{g(z)} \right) > \alpha \quad (z \in U)
\]

holds. We denote the class which consists of all functions \( f \in \mathcal{A} \) that are close-to-convex of order \( \alpha \) and type \( \delta \) by \( \mathcal{C}(\alpha, \delta) \). This class is introduced by Libera [5].

In particular, when \( \delta = 0 \) we have \( \mathcal{C}(\alpha, 0) = \mathcal{C}(\alpha) \) of close-to-convex functions of order \( \alpha \), and also we get \( \mathcal{C}(0, 0) = \mathcal{C} \) of close-to-convex functions introduced by Kaplan [3]. It is well-known that \( \mathcal{S}^* \subset \mathcal{C} \subset \mathcal{S} \).

Furthermore a function \( f \in \mathcal{A} \) is said to be in the class \( \mathcal{M}(\beta) (\beta > 1) \) if it satisfies the inequality

\[
\Re \left( \frac{zf'(z)}{f(z)} \right) < \beta \quad (z \in U).
\]

This class introduced by Uralegaddi et al. [11].

Motivated by the classes \( \mathcal{S}^*(\alpha) \) and \( \mathcal{M}(\beta) \), Kuroki and Owa [4] introduced the subclass \( \mathcal{S}(\alpha, \beta) \) of analytic functions \( f \in \mathcal{A} \) which is given by Definition 1 below.

**Definition 1.** (see [4]) Let \( \mathcal{S}(\alpha, \beta) \) be a class of functions \( f \in \mathcal{A} \) which satisfy the inequality

\[
\alpha < \Re \left( \frac{zf'(z)}{f(z)} \right) < \beta \quad (z \in U)
\]

for some real number \( \alpha \) \((\alpha < 1)\) and some real number \( \beta \) \((\beta > 1)\).

The class \( \mathcal{S}(\alpha, \beta) \) is non-empty. For example, the function \( f \in \mathcal{A} \) given by

\[
f(z) = z \exp \left\{ \frac{\beta - \alpha}{\pi} i \int_0^z \frac{1}{t} \log \left( \frac{1 - e^{2\pi i \frac{1 + \delta}{\beta - \alpha}} t}{1 - t} \right) dt \right\}
\]

is in the class \( \mathcal{S}(\alpha, \beta) \).

Also for \( f \in \mathcal{S}(\alpha, \beta) \), if \( \alpha \geq 0 \) then \( f \in \mathcal{S}^*(\alpha) \) in \( U \), which implies that \( f \in \mathcal{S} \).

**Lemma 1.** [4] Let \( f \in \mathcal{A} \) and \( \alpha < 1 < \beta \). Then \( f \in \mathcal{S}(\alpha, \beta) \) if and only if

\[
\frac{zf'(z)}{f(z)} < 1 + \frac{\beta - \alpha}{\pi} i \log \left( \frac{1 - e^{2\pi i \frac{1 + \delta}{\beta - \alpha}} z}{1 - z} \right) \quad (z \in U).
\]

Lemma 1 means that the function \( f_{\alpha, \beta} : U \rightarrow \mathbb{C} \) defined by

\[
f_{\alpha, \beta}(z) = 1 + \frac{\beta - \alpha}{\pi} i \log \left( \frac{1 - e^{2\pi i \frac{1 + \delta}{\beta - \alpha}} z}{1 - z} \right) \quad (1.2)
\]

is analytic in \( U \) with \( f_{\alpha, \beta}(0) = 1 \) and maps the unit disk \( U \) onto the vertical strip domain

\[
\Omega_{\alpha, \beta} = \{ w \in \mathbb{C} : \alpha < \Re(w) < \beta \} \quad (1.3)
\]
We note that the function \( f_{\alpha, \beta} \) defined by (1.2) is a convex univalent function in \( U \) and has the form

\[
f_{\alpha, \beta}(z) = 1 + \sum_{n=1}^{\infty} B_n z^n,
\]

where

\[
B_n = \frac{\beta - \alpha}{n\pi i} \left( 1 - e^{2n\pi i \frac{\beta - \alpha}{\beta - \alpha}} \right) \quad (n = 1, 2, \ldots).
\]

Making use of Definition 1, Kuroki and Owa [4] proved the following coefficient bounds for the Taylor-Maclaurin coefficients for functions in the subclass \( S(\alpha, \beta) \) of analytic functions \( f \in A \).

**Theorem 1.** [4, Theorem 2.1] Let the function \( f \in A \) be defined by (1.1). If \( f \in S(\alpha, \beta) \), then

\[
|a_n| \leq \prod_{k=2}^{n} \left[ \frac{k - 2 + \frac{2(\beta - \alpha)}{\pi} \sin \frac{\pi(1 - \alpha)}{\beta - \alpha}}{(n - 1)!} \right] \quad (n = 2, 3, \ldots).
\]

Here, in our present sequel to some of the aforecited works (especially [4]), we first introduce the following subclasses of analytic functions.

**Definition 2.** Let \( \alpha \) and \( \beta \) be real such that \( 0 \leq \alpha < 1 < \beta \). We denote by \( S_g(\alpha, \beta) \) the class of functions \( f \in A \) satisfying

\[
\alpha < \Re \left( \frac{zf'(z)}{g(z)} \right) < \beta \quad (z \in U),
\]

where \( g \in S(\delta, \beta) \) with \( 0 \leq \delta < 1 < \beta \).

Note that for given \( 0 \leq \alpha, \delta < 1 < \beta \), \( f \in S_g(\alpha, \beta) \) if and only if the following two subordination equations are satisfied:

\[
\frac{zf'(z)}{g(z)} < \frac{1 + (1 - 2\alpha)z}{1 - z} \quad \text{and} \quad \frac{zf'(z)}{g(z)} < \frac{1 - (1 - 2\beta)z}{1 + z}.
\]

**Remark 1.** (i) If we let \( \beta \to \infty \) in Definition 2 then the class \( S_g(\alpha, \beta) \) reduces to the class \( C(\alpha, \delta) \) of close-to-convex functions of order \( \alpha \) and type \( \delta \).

(ii) If we let \( \delta = 0, \beta \to \infty \) in Definition 2 then the class \( S_g(\alpha, \beta) \) reduces to the class \( C(\alpha) \) of close-to-convex functions of order \( \alpha \).

(iii) If we let \( \alpha = \delta = 0, \beta \to \infty \) in Definition 2 then the class \( S_g(\alpha, \beta) \) reduces to the close-to-convex functions class \( C \).

Using (1.3) and by the principle of subordination, we can immediately obtain Lemma 2

**Lemma 2.** Let \( \alpha, \beta \) and \( \delta \) be real numbers such that \( 0 \leq \alpha, \delta < 1 < \beta \) and let the function \( f \in A \) be defined by (1.1). Then \( f \in S_g(\alpha, \beta) \) if and only if

\[
\frac{zf'(z)}{g(z)} < f_{\alpha, \beta}(z)
\]
where \( f_{\alpha,\beta}(z) \) is defined by (1.2).

**Definition 3.** A function \( f \in \mathcal{A} \) is said to be in the class \( \mathcal{B}_g(\alpha, \beta; \rho) \) if it satisfies the following non-homogenous Cauchy-Euler differential equation:

\[
z^2 \frac{d^2 w}{dz^2} + 2(1 + \rho) z \frac{dw}{dz} + \rho (1 + \rho) w = (1 + \rho)(2 + \rho) \varphi(z)
\]

\((w = f(z) \in \mathcal{A}, \varphi \in \mathcal{S}_g(\alpha, \beta), g \in \mathcal{S}(\delta, \beta), \quad 0 \leq \alpha, \delta < 1 < \beta, \quad \rho \in \mathbb{R} \setminus (-\infty, -1])\).

**Remark 2.**

(i) If we let \( \beta \to \infty \) in Definition 3 then we get the class \( \mathcal{B}_g(\alpha; \rho) \) which consists of functions \( f \in \mathcal{A} \) satisfying

\[
z^2 \frac{d^2 w}{dz^2} + 2(1 + \rho) z \frac{dw}{dz} + \rho (1 + \rho) w = (1 + \rho)(2 + \rho) \varphi(z)
\]

\((\varphi \in C(\alpha, \delta), \quad 0 \leq \alpha, \delta < 1, \quad \rho \in \mathbb{R} \setminus (-\infty, -1])\).

(ii) If we let \( \delta = 0, \beta \to \infty \) in Definition 3 then we get the class \( \mathcal{H}_g(\alpha; \rho) \) which consists of functions \( f \in \mathcal{A} \) satisfying

\[
z^2 \frac{d^2 w}{dz^2} + 2(1 + \rho) z \frac{dw}{dz} + \rho (1 + \rho) w = (1 + \rho)(2 + \rho) \varphi(z)
\]

\((\varphi \in C(\alpha), \quad 0 \leq \alpha < 1, \quad \rho \in \mathbb{R} \setminus (-\infty, -1])\).

(iii) If we let \( \alpha = \delta = 0, \beta \to \infty \) in Definition 3 then we get the class \( \mathcal{M}_g(\rho) \) which consists of functions \( f \in \mathcal{A} \) satisfying

\[
z^2 \frac{d^2 w}{dz^2} + 2(1 + \rho) z \frac{dw}{dz} + \rho (1 + \rho) w = (1 + \rho)(2 + \rho) \varphi(z)
\]

\((\varphi \in C, \quad \rho \in \mathbb{R} \setminus (-\infty, -1])\).

The coefficient problem for close-to-convex functions studied many authors in recent years, (see, for example [1, 2, 7, 9, 10, 12, 13]). Upon inspiration from the recent work of Kuroki and Owa [4] the aim of this paper is to obtain coefficient bounds for the Taylor-Maclaurin coefficients for functions in the function classes \( \mathcal{S}_g(\alpha, \beta) \) and \( \mathcal{B}_g(\alpha, \beta; \rho) \) of analytic functions which we have introduced here. Also we investigate Fekete-Szegö problem for functions belong to the function class \( \mathcal{S}_g(\alpha, \beta) \).

2. **Coefficient bounds**

In order to prove our main results (Theorems 2 and 3 below), we first recall the following lemma due to Rogosinski [8].

**Lemma 3.** Let the function \( g \) given by

\[
g(z) = \sum_{k=1}^{\infty} b_k z^k \quad (z \in U)
\]
be convex in \( U \). Also let the function \( f \) given by
\[
f(z) = \sum_{k=1}^{\infty} a_k z^k \quad (z \in U)
\]
be holomorphic in \( U \). If
\[
f(z) \prec g(z) \quad (z \in U),
\]
then
\[
|a_k| \leq |b_1| \quad (k = 1, 2, \ldots).
\]

We now state and prove each of our main results given by Theorems 2 and 3 below.

**Theorem 2.** Let \( \alpha, \beta \) and \( \delta \) be real numbers such that \( 0 \leq \alpha, \delta < 1 < \beta \) and let the function \( f \in A \) be defined by (1.1). If \( f \in S_g(\alpha, \beta) \), then
\[
|a_n| \leq \prod_{k=2}^{n} \left[ k - 2 + \frac{2(\beta-\delta)}{\pi} \sin \frac{\pi(1-\delta)}{\beta-\delta} \right] \frac{n!}{n^n} \left( 1 + \sum_{j=1}^{n-2} \prod_{k=2}^{n-j} \left[ k - 2 + \frac{2(\beta-\delta)}{\pi} \sin \frac{\pi(1-\delta)}{\beta-\delta} \right] \right) (n = 2, 3, \ldots),
\]
where \( g \in S(\delta, \beta) \).

**Proof.** Let the function \( f \in S_g(\alpha, \beta) \) be of the form (1.1). Therefore, there exists a function
\[
g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in S(\delta, \beta)
\]
so that
\[
\alpha < \Re \left( \frac{zf'(z)}{g(z)} \right) < \beta.
\]
Note that by Theorem 1 we have
\[
|b_n| \leq \prod_{k=2}^{n} \left[ k - 2 + \frac{2(\beta-\delta)}{\pi} \sin \frac{\pi(1-\delta)}{\beta-\delta} \right] \frac{(n-1)!}{n^n} (n = 2, 3, \ldots).
\]
Let us define the function \( p(z) \) by
\[
p(z) = \frac{zf'(z)}{g(z)} \quad (z \in U).
\]
Then according to the assertion of Lemma 2 we get
\[
p(z) \prec f_{\alpha,\beta}(z) \quad (z \in U),
\]
where \( f_{\alpha,\beta}(z) \) is defined by (1.2). Hence, using Lemma 3 we obtain
\[
\left| \frac{p^{(m)}(0)}{m!} \right| = |c_m| \leq |B_1| \quad (m = 1, 2, \ldots),
\]
where
\[ p(z) = 1 + c_1 z + c_2 z^2 + \cdots \quad (z \in \mathbb{U}) \tag{2.7} \]
and (by (1.4))
\[ |B_1| = \left| \frac{\beta - \alpha}{i \pi} \left(1 - e^{2\pi i \frac{\alpha - \pi}{\beta - \alpha}}\right)\right| = \frac{2 (\beta - \alpha)}{\pi} \sin \frac{\pi (1 - \alpha)}{\beta - \alpha}. \tag{2.8} \]
Also from (2.4), we find
\[ zf'(z) = p(z)g(z). \tag{2.9} \]
Since \( a_1 = b_1 = 1 \), in view of (2.9), we obtain
\[ na_n - b_n = c_{n-1} + c_{n-2} b_2 + \cdots + c_1 b_{n-1} = c_{n-1} + \sum_{j=1}^{n-2} c_j b_{n-j} \quad (n = 2, 3, \ldots). \tag{2.10} \]
Now we get from (2.8), (2.9) and (2.10),
\[
|a_n| \leq \prod_{k=2}^{n} \frac{k - 2 + \frac{2(\beta - \delta)}{\pi} \sin \frac{\pi (1 - \delta)}{\beta - \delta}}{n!} + \frac{|B_1|}{n} \left(1 + \sum_{j=1}^{n-2} \prod_{k=2}^{n-j} \frac{k - 2 + \frac{2(\beta - \delta)}{\pi} \sin \frac{\pi (1 - \delta)}{\beta - \delta}}{(n - j - 1)!}\right) \quad (n = 2, 3, \ldots). 
\]
This evidently completes the proof of Theorem 2. \( \square \)

**Remark 3.** It is worthy to note that the inequality obtained for \( |a_n| \) in Theorem 2 is also valid when \( \alpha, \delta < 1 < \beta \) by Theorem 1.

Letting \( \beta \to \infty \) in Theorem 2, we have the coefficient bounds for close-to-convex functions of order \( \alpha \) and type \( \delta \).

**Corollary 1.** [5] Let \( \alpha \) and \( \delta \) be real numbers such that 0 \( \leq \alpha, \delta < 1 \) and let the function \( f \in \mathcal{A} \) be defined by (1.1). If \( f \in \mathcal{C}(\alpha, \delta) \), then
\[ |a_n| \leq \frac{2 (3 - 2\delta) (4 - 2\delta) \cdots (n - 2\delta)}{n!} \frac{n (1 - \alpha) + (\alpha - \delta)}{[n (1 - \alpha) + (\alpha - \delta)]} \quad (n = 2, 3, \ldots). \]

Letting \( \delta = 0, \beta \to \infty \) in Theorem 2 we have the following coefficient bounds for close-to-convex functions of order \( \alpha \).

**Corollary 2.** Let \( \alpha \) be a real number such that 0 \( \leq \alpha < 1 \) and let the function \( f \in \mathcal{A} \) be defined by (1.1). If \( f \in \mathcal{C}(\alpha) \), then
\[ |a_n| \leq n (1 - \alpha) + \alpha \quad (n = 2, 3, \ldots). \]

Letting \( \alpha = \delta = 0, \beta \to \infty \) in Theorem 2 we have the well-known coefficient bounds for close-to-convex functions.
Corollary 3. [6] Let the function $f \in A$ be defined by (1.1). If $f \in C$, then

$$|a_n| \leq n \quad (n = 2, 3, \ldots).$$

Theorem 3. Let $\alpha, \beta$ and $\delta$ be real numbers such that $0 \leq \alpha, \delta < 1 < \beta$ and let the function $f \in A$ be defined by (1.1). If $f \in B \left( \alpha, \beta; \rho \right)$, then

$$|a_n| \leq \left\{ \prod_{k=2}^{n} \frac{k - 2 + \frac{2(\beta - \delta)}{\pi} \sin \frac{\pi(1-\delta)}{\beta - \delta}}{n!} \right. + \left. \frac{2(\beta - \alpha)}{n\pi} \sin \frac{\pi(1-\alpha)}{\beta - \alpha} \left( 1 + \sum_{j=1}^{n-2} \frac{\prod_{k=2}^{n-j} \frac{k - 2 + \frac{2(\beta - \delta)}{\pi} \sin \frac{\pi(1-\delta)}{\beta - \delta}}{n-j-1)!} {(n-j)!} \right) \right\} \times \frac{(1+\rho)(2+\rho)}{(n+\rho)(n+1+\rho)} \quad (n = 2, 3, \ldots), \quad (2.11)$$

where $\rho \in \mathbb{R} \setminus (-\infty, -1]$.

Proof. Let the function $f \in A$ be given by (1.1). Also let

$$\varphi(z) = z + \sum_{n=2}^{\infty} \varphi_n z^n \in S_{g} (\alpha, \beta).$$

We then deduce from Definition 3 that

$$a_n = \frac{(1+\rho)(2+\rho)}{(n+\rho)(n+1+\rho)} \varphi_n \quad (n = 2, 3, \ldots; \rho \in \mathbb{R} \setminus (-\infty, -1]).$$

Thus, by using Theorem 2 in conjunction with the above equality, we have assertion (2.11) of Theorem 3.

Letting $\beta \to \infty$ in Theorem 3, we have the following consequence.

Corollary 4. Let $\alpha$ and $\delta$ be real numbers such that $0 \leq \alpha, \delta < 1$ and let the function $f \in A$ be defined by (1.1). If $f \in B_g (\alpha; \rho)$, then

$$|a_n| \leq \left\{ \prod_{k=2}^{n} \frac{k - 2\delta}{n!} + \frac{2(1-\alpha)}{n} \left( 1 + \sum_{j=1}^{n-2} \frac{\prod_{k=2}^{n-j} \frac{k - 2\delta}{n-j-1)!} {(n-j)!} \right) \right\} \frac{(1+\rho)(2+\rho)}{(n+\rho)(n+1+\rho)} \quad (n = 2, 3, \ldots),$$

where $\rho \in \mathbb{R} \setminus (-\infty, -1]$.

Letting $\delta = 0$, $\beta \to \infty$ in Theorem 3, we have the following consequence.
Corollary 5. Let $\alpha$ be a real number such that $0 \leq \alpha < 1$ and let the function $f \in \mathcal{A}$ be defined by (1.1). If $f \in \mathcal{H}_g(\alpha; \rho)$, then

$$|a_n| \leq [n (1 - \alpha) + \alpha] \frac{(1 + \rho) (2 + \rho)}{(n + \rho) (n + 1 + \rho)} \quad (n = 2, 3, \ldots),$$

where $\rho \in \mathbb{R} \setminus (-\infty, -1]$.

Letting $\alpha = \delta = 0$, $\beta \to \infty$ in Theorem 3, we have the following consequence.

Corollary 6. Let the function $f \in \mathcal{A}$ be defined by (1.1). If $f \in \mathcal{M}_g(\rho)$, then

$$|a_n| \leq n \frac{(1 + \rho) (2 + \rho)}{(n + \rho) (n + 1 + \rho)} \quad (n = 2, 3, \ldots),$$

where $\rho \in \mathbb{R} \setminus (-\infty, -1]$.

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