KAPPA DISTRIBUTION MODEL FOR HARD X-RAY CORONAL SOURCES OF SOLAR FLARES

M. Oka¹, S. Ishikawa¹, S. Saint-Hilaire¹, S. Krucker¹,³, and R. P. Lin¹,４

¹ Space Sciences Laboratory, University of California Berkeley, USA
² National Astronomical Observatory of Japan, Japan
³ 4Dx, University of Applied Sciences and Arts Northwestern Switzerland, Switzerland
⁴ School of Space Research, Kyung Hee University, Republic of Korea

Received 2012 May 15; accepted 2012 December 10; published 2013 January 18

ABSTRACT

Solar flares produce hard X-ray emission, the photon spectrum of which is often represented by a combination of thermal and power-law distributions. However, the estimates of the number and total energy of non-thermal electrons are sensitive to the determination of the power-law cutoff energy. Here, we revisit an “above-the-loop” coronal source observed by RHESSI on 2007 December 31 and show that a kappa distribution model can also be used to fit its spectrum. Because the kappa distribution has a Maxwellian-like core in addition to a high-energy power-law tail, the emission measure and temperature of the instantaneous electrons can be derived without assuming the cutoff energy. Moreover, the non-thermal fractions of electron number/energy densities can be uniquely estimated because they are functions of only the power-law index. With the kappa distribution model, we estimated that the total electron density of the coronal source region was $\sim 2.4 \times 10^{10}$ cm$^{-3}$. We also estimated without assuming the source volume that a moderate fraction ($\sim 20\%$) of electrons in the source region was non-thermal and carried $\sim 52\%$ of the total electron energy. The temperature was 28 MK, and the power-law index $\delta$ of the electron density distribution was $\sim 4.3$. These results are compared to the conventional power-law models with and without a thermal core component.

Key words: Sun: flares – Sun: particle emission – Sun: X-rays, gamma rays

Online-only material: color figures

1. INTRODUCTION

A solar flare is an explosive energy release phenomenon on the Sun and accelerates a large number of electrons up to tens of MeV (e.g., Brown 1971; Lin & Hudson 1976; Miller et al. 1997; Holman et al. 2003). To diagnose accelerated electrons, the hard X-ray (HXR) observations of electron bremsstrahlung emission have been used.

In general, the spatially integrated HXR photon spectrum exhibits a relatively flat, non-thermal tail in addition to an intense and steep thermal component (Lin et al. 1981). Although a model with multiple temperatures can often fit the entire spectra (Emslie & Brown 1980), the non-thermal tail can typically be described as a power law or a double power law that is connected to the thermal component, typically in the 15–30 keV range.

When viewed as an image, the intense thermal emission is dominated by an arcade loop structure, whereas the less intense but high-energy tail of the HXR emission is usually detected at the chromospheric footpoint of the loop (e.g., Hoyng et al. 1981; Brown et al. 1983). The non-thermal tail can also originate from the corona (e.g., Frost & Dennis 1971; Palmer & Smerd 1972), and the source is sometimes located “above-the-loop” (e.g., Masuda et al. 1994; Krucker et al. 2008; Ishikawa et al. 2011).

A caveat of studying the non-thermal HXR emission is that the thermal emission from a loop is so bright that it masks spectral features of non-thermal sources (in either the footpoint or “above-the-loop”), especially in the lower energy range. Therefore, it is difficult to clarify how far the power-law spectrum extends in the lower energy direction. As such, a low-energy cutoff $E_c$ of the power law has been considered, typically in the 15–30 keV range, to estimate the number and total energy of non-thermal electrons in the source, although the estimates can be sensitive to the choice of $E_c$.

Thus, efforts have been made to understand properties of the HXR emission around $E_c$ (e.g., Holman & Benka 1992; Sui et al. 2005). In particular, since the launch of RHESSI, it has been successfully shown that a range of values for $E_c$ fit the data equally well and that the highest value of $E_c$ that still fits the data can be used to derive the lower limit for the non-thermal number and energy densities (Holman et al. 2003; Emslie et al. 2004; Saint-Hilaire & Benz 2005; Kontar et al. 2008). As for the physical meaning of the low-energy cutoff, it has been argued that the cutoff represents the critical velocity above which electrons run away and are freely accelerated by the reconnection electric field (Holman & Benka 1992). If a sharp cutoff existed, however, then plasma instabilities would lead to a flattening of the distribution around and below the cutoff energy (as reviewed by Holman et al. 2011).

In fact, a theoretical study pointed out that the non-thermal electron distribution could seamlessly merge into a thermal distribution, removing the need for a low-energy cutoff (Emslie 2003). Moreover, in situ observations of electrons in planetary and interplanetary space often show that the higher energy tail of a thermal core component smoothly extends into a power-law distribution. Examples can be found at Earth’s bow shock (e.g., Gosling et al. 1989) and the magnetotail reconnection (e.g., Øieroset et al. 2002).

In some cases of in situ observations, the kappa distribution model (Vasyliunas 1968) has been used to represent the entire electron distribution because it is a composite of a Maxwellian-like core and a power-law tail (e.g., Christon et al. 1988, 1989, 1991; Onsager & Thomsen 1991; Leubner 2004; Imada et al. 2011). While the kappa distribution was first introduced as an empirical model (Vasyliunas 1968), recent theoretical and computational studies have suggested that self-consistent formation of the electron kappa distribution is made possible by the beam–plasma interactions that involve the Langmuir/ion-sound...
turbulence (Yoon et al. 2006; Rhee et al. 2006; Ryu et al. 2007). The origin of the kappa distribution has also been discussed in terms of Gibbsian theory (Treumann & Jaroschek 2008) and Tsallis Statistical Mechanics (Livadiotis & McComas 2011 and references therein). From a solar physics point of view, Kašparová & Karlický (2009) have already suggested that the kappa distribution may also be useful for interpreting solar HXR sources. They reported that a kappa distribution fits the spectrum of a coronal loop-top source but does not fit the spatially integrated spectrum (coronal and footpoint sources) as well.

The purpose of this paper is to complement the work of Kašparová & Karlický (2009) by examining the kappa distribution model in the recently reported RHESSI event of 2007 December 31 (Kruker et al. 2010). We studied this event because an unusually intense HXR emission was detected from an “above-the-loop” coronal source. We show that the spatially integrated HXR spectrum can be fitted by not only a combination of thermal and non-thermal power-law distributions, but also a combination of thermal and kappa distributions. The introduction of the core distribution in the non-thermal source via the kappa distribution enables us to estimate the number and energy densities without assuming the cutoff energy.

2. KAPPA DISTRIBUTION

The isotropic, three-dimensional (3D) form of the kappa distribution function $f_k(v)$ ($v^3$ cm$^{-6}$) is written as

$$f_k(v) = \frac{N_k}{(\pi \theta^2)^{3/2}} \frac{\Gamma(\kappa + 1)}{\Gamma(1 - 1/2)} \left(1 + \frac{v^2}{\kappa \theta^2}\right)^{-\kappa},$$

(1)

where $v$ is the particle speed, $\kappa$ is the power-law index, $\theta$ is the most probable particle speed, $\Gamma$ is the Gamma function, and $N_k$ is the number density. The coefficient is such that $\int f_k(v) dv = N_k$. If $\kappa$ is sufficiently large, then the distribution approaches a single Maxwellian distribution. The most probable energy is $E_{mp} = (1/2)m\theta^2$ at which the differential flux ($= (v^2/m) f_k(v)$) becomes maximum. However, the temperature is defined as $kBT_k = (1/2)m\theta^2 [\kappa/(\kappa - 3/2)]$ so that the average energy of particles can be expressed as $E_{avg} = (3/2) kBT_k$. Note that $E_{avg} = (3/2) kBT_M$ for the isotropic 3D Maxwellian distribution $f_M(v)$ with $k_BT_M = (1/2)mv_{th}^2$, where $v_{th}$ is the thermal speed.

By using the kappa temperature $k_BT_k$ and introducing particle energy $E = (1/2)m\theta^2$, we can convert $f_k(v)$ into the density distribution $F_k(E)$ (cm$^{-3}$ keV$^{-1}$) as

$$F_k(E) = \frac{2N_k \sqrt{E}}{\pi(k_BT_k)^3} \frac{\Gamma(\kappa + 1)}{\Gamma(1 - 1/2)} \frac{E}{k_BT_k} \left[1 + \left(\frac{E}{k_BT_k}\right)^{-(\kappa + 1)}\right]^{-\kappa},$$

(2)

so that $\int F_k(E) dE = N_k$. The thin-target formula of this expression has been incorporated into the Solar Soft Ware (SSW) by Kašparová & Karlický (2009) and can be used as OSPEX fitting function f_thin_kappa.pro.

An example of $F_k(E)$ is plotted in Figure 1(a). The Maxwellian distribution $F_M(E)$ with $k_BT_M = E_{mp} = k_BT_k [\kappa/(\kappa - 3/2)]$ is also plotted for comparison. Here, the density $N_M$ of the Maxwellian distribution $F_M(E)$ has been adjusted so that $F_k(E_{mp}) = F_M(E_{mp})$. Such $N_M$ is derived as

$$\frac{N_M}{N_k} = 2.718 \frac{\Gamma(\kappa + 1)}{\Gamma(1 - 1/2)} \left(1 + \frac{1}{\kappa}\right)^{-(\kappa + 1)}.$$

Then, the difference between $F_k(E)$ and the adjusted Maxwellian distribution $F_M(E)$ represents the non-thermal particles, as indicated by the shaded region. A slight difference remains in the lower energy range ($E < E_{mp}$), but the total difference in this range is negligible compared to the total difference in the higher energy range ($E > E_{mp}$). Therefore, the ratio $R_N$ of the non-thermal electron density to the total electron density in a source can be approximated by $R_N \equiv 1 - N_M/N_k$. The ratio $R_k$ of the non-thermal electron energy to the total electron energy can also be calculated using $E_{avg}$.

For the example case of $\kappa = 4$ (i.e., Figure 1(a)), the non-thermal electrons constitute ~20% of the total electrons and such non-thermal electrons carry ~50% of the total electron energy. Note that we do not need to assume the cutoff energy or source volume to estimate the values. Also, these ratios are much larger than what is generally assumed in an electron beam model where beam density is much less than the ambient density (“diluted beam”). The presence of a kappa distribution in a coronal source region may indicate that a significant number of electrons are locally accelerated.

3. ANALYSIS

To test the kappa distribution model, we performed imaging spectroscopy for a partially disk-occulted solar flare of 2007 December 31 observed by RHESSI. Following the work by Kruker et al. (2010), our focus shifts to the time of HXR peak flux, 00:47:42–00:47:50 UT.

Figure 2 shows the spatial structure of the HXR sources during the peak time in eight different energy ranges. There were mainly two separate sources. The northern source at $(X, Y) \sim (−980, −150)$ arcsec was dominated by low-energy (<10 keV) X-ray emission (Figures 2(a) and (b)), whereas the southern source at $(X, Y) \sim (−970, −165)$ arcsec was dominated by high-energy (>20 keV) X-ray emission (Figures 2(g) and (h)).

The northern source in the 6–8 and 8–10 keV ranges was so bright that the flux of the southern source should be less than what is calculated in, for example, the blue polygons of Figures 2(a) and (b). Conversely, the fluxes in the red polygons of Figures 2(g) and (h) would be the upper limits of the northern sources in the 20–25 and 25–30 keV ranges.

Note that, soon after the HXR peak time, the main thermal loop of the southern source was identified along the limb on
the western side of the southern source. Thus, the northern source has been considered to be a separate thermal source, although the precise relation between the northern and southern sources remains unclear (Krucker et al. 2010). Below, we focus on the spectral features of the sources rather than on a possible relationship between the two sources.

Figures 2(c)–(f) show the details of the HXR sources in the intermediate energy ranges. In the 10–12 and 12–14 keV ranges (Figures 2(c) and (d), respectively), both northern and southern sources appear together, indicating that both sources had comparable fluxes. In the 14–17 and 17–20 keV ranges (Figures 2(e) and (f), respectively), only one source can be identified somewhere between the northern and southern sources and it is not clear to which of the two sources it belongs. Because of the unclear nature of the source structure, we took the sum of the fluxes in the dashed polygons and considered it to be the upper limit of both the northern and southern sources. The sum, however, is essentially the same as the values in the spatially integrated spectrum shown below and so we do not use the data from Figures 2(e) and (f) in the following analysis. Then, assuming that the red and blue polygons represent the northern and southern sources, respectively, we calculated the total flux within each polygon to be compared with spectrum models.

It must be noted that, while we chose the polygons so that the double sources in Figure 2(c) can be separated, another choice of boundary indicated by the white line in Figure 2(d) could also be used to separate the double sources better in Figure 2(d). We found that such modification to the polygons leads to a less than 30% flux change in all energy ranges. Thus, we will use this number as the uncertainty of the measured fluxes.

Figure 3 shows the result of the imaging spectroscopy. The light red and light blue squares indicate the fluxes from red and blue polygons in Figure 2, and the fluxes are compared to the spatially integrated photon spectrum (histograms) as well as four different model distributions (colored curves; see Tables 1 and 2 for model parameters). The steeper and flatter components are evident in the integrated spectrum and we will use thermal, power law, and kappa distributions to represent these two components. To represent the non-thermal tail, Models A and B use the power-law distributions whereas Models C and C’ use the kappa distribution.

For the power-law distribution fits used in Models A and B, we used a formula that calculates thin-target Bremsstrahlung X-ray spectrum from a power-law electron flux density distribution (cm$^{-2}$ s$^{-1}$ keV$^{-1}$). This formula is implemented as f_thin.pro in the SSW/OSPEX software and contains three free parameters: $\alpha_{SSW}, \delta_{FD},$ and $E_{c}$. $\alpha_{SSW}$ (10$^{55}$ electrons cm$^{-2}$ s$^{-1}$) is the normalization factor and is a product of the number density of plasma ions, the flux density of non-thermal electrons, and the volume of the radiating source region. $\delta_{FD}$ is the power-law index of the flux density distribution. Throughout this paper, however, we use the power-law index $\delta$ of the number density distribution when comparing different models. The two indices can be converted to one another by $\delta = \delta_{FD} + 0.5$. $E_{c}$ (keV) is the low-energy cutoff of the power-law distribution.

For the kappa distribution fits used in Models C and C’, we used the SSW/OSPEX procedure f_thin_kappa.pro, but it does not contain line emissions. We added line emissions in the analysis and imposed that the same values of emission measure (EM) and temperature ($T$) are used in the kappa distribution and line emissions. It is to be noted that the assumed values for the fits were derived for a Maxwellian distribution, and the temperature inferred under the assumption that electron distribution is Maxwellian may be an overestimate of the actual temperature of a distribution with a non-thermal tail (Owocki & Scudder 1983). As for the spectral index, the program f_thin_kappa.pro assumes a kappa distribution for the electron density and the spectral index $\kappa$ can be converted to $\delta$ by $\delta = \kappa + 0.5$.

Figure 3 Model A uses the thermal (red curve) and power-law (blue curve) distributions for the steeper and flatter components,
Figure 3. Comparison of the observed spatially integrated photon spectrum (histograms) with modeled distributions (solid curves) as well as the imaging spectroscopy result (light red and light blue squares for the northern and southern sources, respectively). Four different sets of models were used to fit the observed spectrum: (A) thermal + power law, (B) thermal + thermal + power law, (C) thermal + kappa, and (C’) kappa + kappa. See Tables 1 and 2 for the resulting parameter values of the steeper (red) and flatter (blue) component, respectively. The gray curve is the background. The histograms in the lower panels are the residuals of the fit in units of photons cm$^{-2}$ s$^{-1}$ keV$^{-1}$. The color version of this figure is available in the online journal.

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Table 1

| Fit Parameters       | Model A                | Model B                | Model C                | Model C’               |
|----------------------|------------------------|------------------------|------------------------|------------------------|
|                      | Thermal                | Thermal                | Thermal                | Kappa                  |
| Emission measure EM, cm$^{-3}$ | (1.2 ± 0.1) × 10$^{48}$ | (8.6 ± 1.0) × 10$^{47}$ | (4.8 ± 0.2) × 10$^{47}$ | (2.5 ± 1.0) × 10$^{46}$ |
| Total density$^a$ $N_{tot}$, cm$^{-3}$ | (3.9 ± 1.3) × 10$^{10}$ | (3.3 ± 1.1) × 10$^{10}$ | (2.4 ± 0.6) × 10$^{10}$ | (1.8 ± 1.1) × 10$^{11}$ |
| Temperature$^b$ $T$, MK | 17 ± 0.5               | 18 ± 0.6               | 21 ± 0.4               | 10 ± 0.7               |
| Power-law index$^c$ $\delta$ | …                     | …                     | …                     | 12 ± 0.8               |

Notes.
$^a$ The density is derived from the emission measure by assuming a source volume of $\sim 8 \times 10^{26}$ cm$^3$ (Krucker et al. 2010).
$^b$ The kappa temperature $k_{\delta} T_s$ is used for the kappa distribution fit.
$^c$ The power-law index $\delta (= \kappa + 0.5)$ is for the density distribution $F(E) \propto E^{-\delta}$.

Table 2

| Fit Parameters       | Model A                | Model B                | Model C                | Model C’               |
|----------------------|------------------------|------------------------|------------------------|------------------------|
|                      | Power Law              | Thermal                | Power Law              | Kappa                  |
| Emission measure EM, cm$^{-3}$ | …                     | (1.6 ± 0.3) × 10$^{46}$ | …                     | (8.6 ± 6.2) × 10$^{46}$ |
| Total density$^a$ $N_{tot}$, cm$^{-3}$ | …                     | (4.5 ± 2.2) × 10$^{10}$ | …                     | (1.0 ± 0.9) × 10$^{10}$ |
| Temperature $T$, MK | 52 ± 4                 | …                     | 28 ± 9                 | 29 ± 18                |
| Power-law index $\delta$ | 3.9 ± 0.04             | …                     | 3.8 ± 0.2              | 4.3 ± 0.2              |

Notes. Same format as Table 1. An error range (sigma level) is not shown when it exceeded the parameter value. The power-law index $\delta$ is for the density distribution $F(E) \propto E^{-\delta}$ and can be converted from $\delta_{15}$ of the flux density distribution used in f_thin.pro (Models A & B; $\delta = \delta_{15} + 0.5$) and from $\kappa$ of the number density distribution used in f_thin_kappa.pro (Models C and D; $\delta = \kappa + 0.5$).

respectively. The light red and light blue squares are consistent with the red and blue curves, respectively, indicating that the northern source was producing the steeper component, whereas the southern source was producing the flatter component. Note that a range of values for the low-energy cutoff $E_c$ fits the data equally well. We found that the highest $E_c$ that still fits the data is 16 keV with $\chi^2 = 1.0$ (not shown). However, this model underestimates the flux in the 10–12 keV range by $46_{-21}^{+51}$% of what was measured in the image (light blue square). Then, we restricted $E_c$ to be in the range $11$ keV < $E_c$ < $13$ keV in order to look for a solution consistent with the imaging spectroscopy result. We found that, as shown in Figure 3 Model A, $E_c = 12$ keV would best fit the data, although $\chi^2$ became relatively large ($\chi^2 = 1.3$).

Figure 3 Model B uses the thermal (red curve) and a combination of thermal and power-law (blue curve) distributions for the steeper and flatter components, respectively. This is basically the same as Model A, but an additional thermal distribution is introduced to account for the possible core component of the southern source (blue curve). A similar model...
is used by Caspi & Lin (2010). This set of distributions can also fit the data nicely ($\chi^2 = 0.9$) including the 10–12 keV range, suggesting that a core distribution could have existed in the southern source. Again, a range of values for the low-energy cutoff $E_c$ fits the data equally well and we chose the highest $E_c$ that still fits the data ($E_c = 35$ keV). Note that the model curves (generated to fit the integrated photon spectrum) have a higher energy resolution so that the modeled line emissions partially exceed the fluxes obtained from images in the 6–8 and 8–10 keV ranges. We confirmed that the modeled values averaged over the same energy ranges are consistent with the values from the imaging spectroscopy. In Model B, the thermal core component has a relatively large temperature of $\sim 52$ MK and this is comparable to the temperature $\sim 44$ MK of a super-hot coronal source reported by Caspi & Lin (2010).

Figure 3 Model C uses the thermal (red curve) and kappa distributions (blue curve) for the steeper and flatter components, respectively. Although the reduced $\chi^2$ is slightly larger ($\chi^2 = 1.2$), this set of models can also represent nicely the data including the 10–12 keV range. Again, the modeled line emissions partially exceed the fluxes obtained from images in the 6–8 and 8–10 keV ranges, but we confirmed that the modeled values averaged over the same energy ranges are consistent with the values from the imaging spectroscopy. The number of free parameters, five, is still the same as that of Model A (thermal + power law), whereas Model B (thermal + thermal + power law) needed seven parameters to have a thermal core distribution in the flatter component (blue curve).

Figure 3 Model C’ uses the kappa distributions for both steeper and flatter components. It is evident that the non-thermal tail of the steeper component is still below the upper limits (light red arrows). The slope is quite soft ($\kappa \sim 12$), however, indicating that the steeper component (northern source) was mostly thermal. Note also that the core temperature of the steeper component 10 MK is reduced by 50% compared to the temperature from the thermal fit, 21 MK. While such a low temperature of the steeper component is still comparable to the temperature of 15 MK measured by GOES during the same interval (00:47:42–00:47:50), the EM obtained by the fit was unrealistically high, $2.5 \times 10^{49}$ cm$^{-3}$, and it is an order of magnitude higher than what was measured by GOES, $1.5 \times 10^{48}$ cm$^{-3}$. Therefore, this set of two kappa distributions is not favorable for representing the observation.

To better visualize the above comparisons, Figure 4 uses the model curves in the 5–31 keV range and takes the ratio of the flatter component (blue curves in Figure 3) to the steeper component (red curves in Figure 3). The ratios are compared to the flux ratios from images (black squares with error bars). The peak spectra in Figure 3 were taken during the eight-second interval of 00:47:42–00:47:50 UT (indicated by the dashed lines), we fitted the data every four seconds over the nearly two-minute interval of 00:47:15–00:49:10 UT in this figure.

(A color version of this figure is available in the online journal.)
law with thermal core) is systematically larger than that of Model A (power law without thermal core) because of the presence of the thermal core distribution in the flatter component (southern source). As for the parameters of the core distribution, the emission measure EM and the temperature $T$ of Model B (thermal + power law) are systematically lower and higher, respectively, than those of Model C (kappa). Therefore, Model B suggests the presence of a super-hot thermal core distribution in the flatter component (southern source) whereas Model C suggests the presence of a larger number of non-thermal electrons. The reduced $\chi^2$ fluctuated around $\sim 1$, indicating that Models A, B, and C fitted the data fairly well. The averages in the interval shown (00:47:15–00:49:10 UT) are Model A—1.22, Model B—0.88, and Model C—0.94.

4. DISCUSSION

We now discuss the implications of the results based on Models A, B, and C for the peak flux interval (00:47:42–00:47:50 UT). We will discuss in particular the non-thermal fractions of electron number/energy densities in the southern source. The estimated non-thermal fractions are summarized in Table 3.

Model A (thermal + power law) assumes that the southern source (“above-the-loop” coronal source) contains a negligible amount of thermal electrons and uses the power law with no thermal core to represent the flatter component. To estimate the number density of non-thermal electrons (“instantaneous” density), we assumed a source volume $V = 8 \times 10^{26} \text{ cm}^3$ (Krucker et al. 2010) and applied the formula by Lin (1974) to the power-law part of the photon spectrum. To estimate the number density of thermal electrons (“ambient density”), we assumed that the ambient environment should be similar to that of the nearby thermal source. Following the derivation by Krucker et al. (2010), the ratio $N_{\text{nt}}/N_{\text{th}}$ can be expressed as

$$N_{\text{nt}}/N_{\text{th}} = 0.05 \left( \frac{N_{\text{th}}^{\text{upper}}}{N_{\text{th}}} \right)^2 \left( \frac{E_c}{12 \text{ keV}} \right)^{-2.9},$$

where $N_{\text{th}}^{\text{upper}} = 8 \times 10^9 \text{ cm}^{-3}$ is the upper limit of the ambient density and we used $\gamma = 4.4$, the flux at 50 keV of 0.16 ph s$^{-1}$ cm$^{-2}$ keV$^{-1}$, and the low-energy cutoff $E_c = 12 \text{ keV}$. If we use $N_{\text{th}} = N_{\text{th}}^{\text{upper}}$, then we obtain $R_N = N_{\text{nt}}/N_{\text{th}} \sim 0.05$. If we use the best estimate of $N_{\text{th}} = 2 \times 10^9 \text{ cm}^{-3}$ (Krucker et al. 2010), then we obtain $R_N \sim 0.44$. Similarly, the ratio $\varepsilon_{\text{nt}}/\varepsilon_{\text{th}}$ can be expressed as

$$\varepsilon_{\text{nt}}/\varepsilon_{\text{th}} = 0.47 \left( \frac{N_{\text{th}}^{\text{upper}}}{N_{\text{th}}} \right)^2 \left( \frac{E_c}{12 \text{ keV}} \right)^{-2.9},$$

where we assumed that the temperature of the ambient plasma is 22 MK (Krucker et al. 2010). Then, if we use $N_{\text{th}} = N_{\text{th}}^{\text{upper}}$, we obtain $R_c \sim 0.32$. If we use the best estimate of $N_{\text{th}} = 2 \times 10^9 \text{ cm}^{-3}$, then we obtain $R_c \sim 0.88$. Because of this large fraction of non-thermal electrons, Model A implies that the non-thermal electrons are not simply a tail on the thermal distribution and that electrons are accelerated locally in the southern source (i.e., “above-the-loop” coronal source). However, Model A resulted in a relatively large $\chi^2 (\sim 1.3$ at the flux peak time and $\sim 1.2$ on average). Therefore, we explored other possible models as described below.

Model B (thermal + thermal + power law) assumes that the southern source (“above-the-loop” coronal source) contains a significant amount of thermal electrons and uses the power law with a hot (52 MK) thermal core to represent the flatter component. To estimate the non-thermal fraction of electron number density, we can use the obtained normalization factor $\alpha_b (\sim 0.3 \times 10^{55} \text{ cm}^{-3} \text{ s}^{-1})$ because it is actually a product of the number density of plasma ions, the flux density of non-thermal electrons, and the volume of the radiating source region. Using the plasma density $4.5 \times 10^3 \text{ cm}^{-3}$ (Table 2) and an assumed source volume of $8 \times 10^{26} \text{ cm}^3$ (Krucker et al. 2010), the electron flux density is estimated to be $8 \times 10^{13} \text{ cm}^{-2} \text{ s}^{-1}$. Based on the low-energy cutoff energy $E_c = 35 \text{ keV}$, the mean speed of the accelerated electrons can be estimated on the order of $10^{10} \text{ cm}^{-3}$ and the number density of the non-thermal electrons in the southern X-ray source is estimated to be $8 \times 10^7 \text{ cm}^{-3}$. This is only 2% of the thermal electron density.

Such a small fraction of non-thermal electron density is consistent with an electron beam scenario in which electrons are accelerated above the hot flare loops and stream through the source region to produce a super-hot thermal plasma. Model B is also consistent with our imaging spectroscopy result especially in the 10–12 keV range, justifying our assumption that a thermal core distribution may have existed in the southern X-ray source (“above-the-loop” coronal source). In fact, Model B gives the least $\chi^2 (\sim 0.9)$ and, as such, seems to be a plausible model.

A caveat is that, because it contains an additional thermal distribution, the number of free parameters, seven, is relatively larger compared to the five in Models A and C. In general, a larger number of free parameters contributes to decreasing the reduced $\chi^2$. Therefore, Model B may have resulted in the lower $\chi^2$ partly because of the smaller number of parameters, although the number of free parameters alone does not explain the $\chi^2$ difference. It also must be noted that a range of values for $E_c$ fits the data equally well and the highest value of $E_c$ that still fits the data has been used in Model B. As such, the above estimation of the fraction of non-thermal electrons, 2%, should only be considered as a lower limit.

In both power-law Models A and B, we needed to assume the source volume $V$ as well as the low-energy cutoff $E_c$ to estimate...
the fraction of non-thermal electrons. Note that we can only obtain a very rough estimate of $V$ and the estimated fraction of non-thermal electrons can be sensitive to the choice of $E_c$. Then, we consider the kappa distribution as an alternative because it contains a thermal core component that seamlessly extends to a power-law distribution. The kappa distribution allows us to estimate the fraction of non-thermal electron density/energy without assuming the source volume (Section 2).

In Model C (thermal + kappa), the ratio of non-thermal electron density to the total electron density in the southern source was $R_N = 0.20^{+0.01}_{-0.01}$, and the ratio of non-thermal energy to the total electron energy in the southern source was $R_c = 0.52^{+0.03}_{-0.02}$. It is to be emphasized that the non-thermal fractions of number/energy densities have been derived less ambiguously than in Model A and that the result does not invoke the possibility of non-thermal electrons outnumbering thermal electrons. On the other hand, the derived estimate of $R_N = 0.20^{+0.01}_{-0.01}$ is much larger than the $R_N \sim 0.02$ of Model B as derived above. This implies that not all electrons are thermalized in the southern (“above-the-loop”) source region and that there may have been local acceleration of electrons in this region.

If we assume a source volume of $8 \times 10^{26}$ cm$^3$ (Krucker et al. 2010), then the total density of the southern source $N_{tot}$ can be estimated as $(1.0 \pm 0.9) \times 10^{10}$ cm$^{-3}$. Within the error range, the estimated density is consistent with the upper limit of $8 \times 10^{10}$ cm$^{-3}$ derived by Krucker et al. (2010). The estimated density is also consistent with what was estimated in Model B, $(4.5 \pm 2.2) \times 10^{9}$ cm$^{-3}$ (Table 2).

As for temperature, the “above-the-loop” region (i.e., southern source) had a temperature of 28 ± 9 MK. This is $\sim 1.3$ times larger than the temperature 21 ± 0.4 MK of the nearby thermal source but is $\sim 0.6$ times the temperature of the super-hot component discussed in Model B or Časpi & Lin (2010). We speculate that the released magnetic field energy was converted to both thermal and non-thermal energies of electrons, but a significant fraction ($R_c \sim 0.5$) went to non-thermal electrons so that the temperature did not increase considerably.

As for the effective plasma beta $\beta$, Krucker et al. (2010) estimated the magnetic field strength $B$ to be 30–50 G and derived $\beta$ between $\sim 0.005$ and $\sim 0.02$ for a pre-flare plasma with a density of $2 \times 10^9$ cm$^{-3}$ and a temperature of 2 MK. They argued that the pre-flare thermal plasma could be replaced with non-thermal electrons (the low-energy cutoff at 16 keV) so that the effective plasma beta becomes $\beta \sim 1$. If we use the density $\sim 10^{10}$ cm$^{-3}$ and the kappa temperature $\sim 28$ MK as derived from Model C, then the effective plasma beta falls between $\sim 1$ and $\sim 3$.

From the spectral fit, we obtained $\kappa \sim 3.8$, which leads to $\delta \sim 4.3$ for the density distribution $F(E) \propto E^{-\delta}$ (see Equation (2)). This is somewhat smaller compared to $\delta \sim 3.9$ derived from Model A (thermal + power law) and $\delta \sim 3.8$ derived from Model B (thermal + thermal + power law). The $\kappa$-value is not too large, however, and the kappa distribution is far from a single Maxwellian. Thus, an electron acceleration theory still needs to reproduce a power-law tail for this event. Note again that the power-law index $\delta$ can be converted from $\delta_{FP}$ of the flux density distribution used in f_thin.pro (Models A and B; $\delta = \delta_{FP} + 0.5$) and from $\kappa$ of the number density distribution used in f_thin_kappa.pro (Models C and D; $\delta = \kappa + 0.5$).

It must be mentioned that Model C also has caveats and disadvantages. First, $\chi^2$ was relatively larger ($\chi^2 \sim 1.2$ at the peak flux interval 00:47:42–00:47:50). The relatively large $\chi^2$ may imply that the kappa distribution is still not the best functional form to represent the HXR spectrum from the southern source. However, the reduced $\chi^2$ averaged over the larger interval 00:47:15–00:49:10 was smaller, $\sim 0.9$, and this is comparable to the average $\chi^2$ of $\sim 0.9$ of Model B. The time variation of $\chi^2$ was also similar between Models B and C (Figure 5). Thus, Model C is as acceptable as Model B in terms of spectral fitting. Second, the spectral index at the time of peak flux $\delta \sim 4.3$ is even more inconsistent with that deduced from the radio observations ($\delta \sim 3.4$; Krucker et al. (2010)) compared with that deduced from the other models ($\delta \sim 3.9$ in Model A and $\delta \sim 3.8$ in Model B). However, the radio emission represents electrons with energies larger than $\sim 100$ keV, whereas our analysis was made in the $<100$ keV range. To further understand whether or not these disadvantages are common in other solar flare events, it is important to test the kappa distribution in a larger number of solar flare events.

5. CONCLUSION

The kappa distribution does not require a low-energy cutoff $E_c$ to represent non-thermal electrons, and the thermal core component can seamlessly extend to a power-law distribution. Furthermore, the non-thermal fractions of electron number/energy densities can be uniquely estimated because they are functions of the power-law index $\kappa$ only. While Kašparová & Karlický (2009) applied the kappa distribution to loop-top coronal sources, we examined the kappa distribution model in an unusually bright “above-the-loop” coronal source obtained on 2007 December 31 (Krucker et al. 2010). For comparison, we also examined the conventional power-law models with and without a thermal core distribution in the source.

Model A, the power law with no thermal core component, was consistent with the imaging spectroscopy result when we chose $E_c = 12$ keV, although the reduced $\chi^2$ was relatively large ($\sim 1.2$ on average). This model implies that non-thermal electrons can outnumber thermal electrons.

Model B, the power law combined with a super-hot (52 MK) thermal core component, could fit the observed spectrum well ($\chi^2 \sim 0.9$ on average) and was consistent with the imaging spectroscopy. This model implies that at least 2% of the source electrons carried non-thermal energies.

However, both Models A and B require a low-energy cutoff $E_c$ to represent the non-thermal tail, and the estimates of the electron number/energy densities can be sensitive to the choice of $E_c$. Furthermore, a source volume $V$ had to be assumed for the estimates, but we can only obtain a rough estimate of $V$.

Thus, we examined Model C (the kappa distribution model). We found that it can fit the observed spectrum well ($\chi^2 \sim 0.9$ on average) and is consistent with the imaging spectroscopy result. Without assuming the source volume $V$ and the lower-energy cutoff $E_c$, we estimated that a moderate fraction (20%) of the source electrons had non-thermal energies and carried 52% of the total electron energy in the “above-the-loop” coronal source region. The temperature was 28 MK and the power-law index of the electron density distribution was $\sim 4.3$. It is important to examine a larger number of events in order to verify the generality of the kappa distribution model.

We acknowledge helpful comments by the anonymous referee and L. Glesener, T. D. Phan, and M. Hoshino. M.O. was
supported by NASA grant NNX08AO83G at UC Berkeley. P.S.H. was supported by NASA grant NASA-98033. R.P.L. was supported by the WCU grant (R31-10016) funded by the Korean Ministry of Education, Science and Technology.

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