H∞ Lower Order Controller for AC Servo System of Multiple Rockets

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Abstract. Design problem of a H∞ robust lower order controller of an AC servo system of multiple rockets is studied. It is pointed out that the problem is actually H∞ robust performance design problem, H∞ robust stability and performance requirements (enhance rapidity, shorten overshoot and static error). It is also recommended to represent the robust performance design as H∞ robust control problem and solve H∞ low-order controller, if the uncertainty weighting function and performance weighting function of system bandwidth and interference attenuation are properly modified. Solved the H∞ low-order controller. The simulation results show that the robust H∞ lower order control system has advantages of small overshoot and short setup time. It can also improve control system's anti-disturbance ability. The simulation example of the H∞ lower order control is also provided to demonstrate the effectively of the proposed design method. The H∞ lower order controller is easy for engineering implementation.

1. Introduction
The rocket launcher (RL) AC servo system is an effective way to achieve azimuth launching direction high-precision automatic aiming of and pitching angle high-precision automatic aiming for rocket launchers. However, rocket launchers have poor load characteristics [1]. The control of the RL AC servo system has become a hot research issue in recent years [1-3].

In order to build the model of and overcome the external interference with the RL AC servo system, effective control must be implemented. To improve the performance of the RL servo system and facilitate the adjustment of the controller, this paper proposes a design method of the low-order controller that can guarantee the robust performance of the RL servo system, in light of the characteristics of the control on the RL servo system. Robust performance design is an effective means for studying the design of uncertain system [4-8]. However, if the performance weight function is not selected properly, a good robust performance cannot be obtained. This paper proposes to combine the selection of the performance weight function with the performance of the RL AC servo system in the low-order control robust performance optimization design of the RL AC servo system, pointing out that the problem may be handled in a concentrated way under the H∞ robust framework.

In the paper, the selection of weight function is discussed, the H∞ robust low-order controller is solved, and the simulation study on the designed RL AC servo system is carried out. It can be concluded from the results that this design works well.
2. **H∞ Control of the RL AC Servo System**

The control of the RL AC servo system [1] is as shown in Figure 1. In the figure, $K(s)$ is controller, $G(s)$ is controlled object, $r$ is reference input, $u$ is control quantity, $d$ is output end disturbance, and $y$ is system output.

The controlled object of the RL AC servo system is:

$$G(s) = \frac{48.4}{s(0.0073s + 1)}e^{-0.0002s}$$  \hspace{1cm} (1)

The design problem discussed in this paper is to find the H∞ robust low-order controller $K(s)$, so that the system still has a good performance in case of perturbation, that means, a good robust performance is required.

3. **Performance Design of H∞ Robust Control System**

According to Literature [8], the H∞ robust design is to solve a stable controller, so that

$$\left\| \begin{bmatrix} W_1S \\ W_2T \end{bmatrix} \right\|_\infty \leq 1$$  \hspace{1cm} (2)

Where,

$$S = \left( I + G\hat{K} \right)^{-1}$$  \hspace{1cm} (3)

$$T = G\hat{K}\left( I + G\hat{K} \right)^{-1}$$  \hspace{1cm} (4)

$G$ and $K$ are the nominal controlled object of the RL AC servo system and the controller proposed to be designed in this paper. $W_1$ and $W_2$ are the uncertainty weight function and system performance weight function of the controlled object, respectively.

Formula (2) may be used for the performance analysis in the H∞ robust control of the RL AC servo system.

The robust performance may be analyzed by H∞ norm, and the H∞ norm solving formula of the SISO system is given in References [7].

$$\sqrt{\left\| W_1S \right\|^2 + \left\| W_2T \right\|^2} \leq 1$$  \hspace{1cm} (5)

4. **Selection of H∞ Robust Performance Index**

In the robust performance optimization design of the RL AC servo system, full attention should be paid to the performance index $W_2$, as it directly reflects the design index, improper selection will cause failure of the design. What should be clearly considered regarding robust performance is $W_2$, which should be determined based on the H∞ robust performance index after the RL AC servo system is perturbed. The $W_2$ should reflect the requirements on the robust performance index and require a higher gain $[W_2(0) \rightarrow \infty]$ in the low-frequency band.

Suppose the performance weight function is selected as:

$$W_2(s) = 0, 7 \frac{s + \rho}{s}$$  \hspace{1cm} (6)
Where $\rho$ reflects the frequency of $W_2(j\omega)$ while it was passing 0dB, and indicates the bandwidth of the RL AC servo system. In the $H_\infty$ robust low-order control optimization design of the RL AC servo system, Formula (2) will be solved by adjusting the $\rho$ value from large to small.

During the design of the RL AC servo system using the method proposed in this paper. The main steps below should be followed:

1. Select a parameter $\rho$ that reflects the bandwidth of the RL AC servo system with in a certain range, according to the design requirements of the RL servo system.

2. Solve the $H_\infty$ robust low-order controller that satisfies Formula (2) by adjusting the value of $\rho$ from large to small.

3. Evaluate the performance of the designed RL AC servo system by simulation, end the evaluation if the design requirements (adjustment time, overshoot, etc.) are met, otherwise, return to Step (1) for a redesign.

5. Design of $H_\infty$ Robust Low-order Controller of the RL AC Servo System

To expound this method, the robust performance optimization scheme of the RL AC servo system is discussed below.

Due to the error generated by the establishment of the mathematical model, the controlled object as the actual physical object is not likely to be sufficiently accurate. The control system design for a mathematical model cannot guarantee a good performance is obtained in practice. Therefore, the uncertainties that may exist in the system should be considered.

In this paper, according to the form of the controlled object in Formula (1), the nominally controlled object of the $H_\infty$ design is selected as

$$G(s) = \frac{48.4}{s(0.0073s + 1)}$$

The design problem of the RL AC servo system is to find the $H_\infty$ robust low-order controller $K(s)$, so that the system still has a good performance in case of perturbation (delay actually in this paper), that means, good robust performance is required.

To satisfy the robust performance index of the RL AC servo system shown in Figure 1

$$\min_{\Delta K(s)} \left[ \left\| W_2(s) \Delta \right\|^2 + \left\| T(s) \right\|^2 \right] \leq 1$$

Formula (8) may be used for the solution of parameter optimization of the $H_\infty$ robust low-order controller.

According to Figure 1, the $H_\infty$ robust performance design may be conducted for the RL AC servo system.

According to the form and the range of perturbation of the controlled object, the selected uncertainty weight function should reflect the influence caused by the delayed perturbation of the controlled object. On the basis of the above requirements, the controlled object of the RL AC servo system is:

$$G(s) = \frac{48.4}{s(0.0073s + 1)}e^{-0.0002s}$$

The nominally controlled object may be selected as:

$$G(s) = \frac{48.4}{s(0.0073s + 1)}$$

Therefore, the multiplicative uncertainty weight function may be finally selected.

$$\Delta(s) = \frac{0.00028s + 0.14}{0.0001s + 1}$$

The performance weight function may be selected as:

$$W(s) = 0.7 \frac{s + \rho}{s}$$

Where, $\rho$ is a parameter that can be adjusted, and its optimized possible value should be determined in the design of the $H_\infty$ robust controller.
Suppose the expression of the $H_\infty$ robust low-order controller

$$ K(s) = \frac{as + b}{cs + 1} \quad (13) $$

The means, a first-order controller simple and practical in engineering should be selected.

With respect to the design method, $\rho$ should be adjusted repeatedly within a certain range, to obtain the optimized $a$, $b$ and $c$ of Formula (8). Due to limited space, the relevant discussion is simplified, and the design results of the RL AC servo system are directly given here.

The $H_\infty$ robust method is adopted for the design of the controlled object of the RL AC servo system. Let’s substitute Formula (13) into Formula (8), solve Formula (8) with the simplex evolutionary method, and adjust $\rho=81$ of Formula (12), then, we will obtain the $H_\infty$ robust low-order controller:

$$ K(s) = \frac{0.0368s + 1.1972}{0.0167s + 1} \quad (14) $$

6. Simulation research Study

To prove the effectiveness of the design method proposed in this paper for the $H_\infty$ robust low-order controller of the RL AC servo system, a simulation study is carried out on the results of the design method.

**Figure 2** Simulation block diagram for output end d during unit step disturbance

**Figure 3** Response curve $y$ of the system during unit step disturbance of output end d

**Figure 4** Error response curve $e$ during unit step disturbance of output end d
Figure 5 - Figure 5 shows the curves of response y, error e and control quantity of the system for the output end d during unit step disturbance.

As seen from Figure 3 to Figure 5:

1. In terms of system output, during the unit step response of the output end d, the system response y attenuates and tends to 0 within 0.2 seconds, suppressing the unit step disturbance quickly and effectively.

2. In terms of the error response, the error response e attenuates and tends to zero within 0.2 seconds, showing the effective, quick and smooth characteristics.

3. In terms of control quantity, the designed system control quantity has appropriate amplitude and good smoothness, which is conductive to engineering implementation.

Figure 6 - Figure 6 Simulation Block Diagram during Unit Step Response of the System

Figure 7 - Figure 7 System Unit Step Response y

Figure 8 - Figure 8 System Error Response e
As seen from Figure 7-9:

1. In terms of system output, the system response $y$ tends to 1 within 0.2 seconds, achieving the tracking on unit step input effectively and quickly.

2. In terms of error response, the error response $e$ attenuates and tends to zero within 0.2 seconds, showing the effective, quick and smooth characteristics.

3. In terms of control quantity, the designed system control quantity has appropriate amplitude and good smoothness, which is conductive to engineering implementation.

7. Conclusion

In the paper, the problem of how to select the performance weight function in the H∞ robust performance optimization design of the RL AC servo system is solved, and the H∞ robust low-order controller is proposed; the simulation analysis of the RL servo system is carried out, and the simulation results show that the H∞ robust control system designed here not only satisfies the simple structure and easy adjustment demands of the H∞ robust low-order controller but also meets the requirements in terms of rapidity, overshoot and magnitude of the control quantity, easy for implementation. The simulation results show that the RL AC servo system designed in this article has a good robust performance.

References

[1] Hu J., Ma D. W., Guo Y. J., Adaptive rocket launcher position AC servo system based on Elman network, Journal of Projectiles, Rockets, Missiles and Guidance. 5 (2010) 181-184.

[2] Chai H. W., Ma D. W., Li Z. G., IMC—PID control of AC servo system for multiple rockets, Electrical Automation, 5 (2006) 17-19.

[3] Chen G. H., Jia Y. B., Li Y. X., Robust multiple internal model control of rocket launcher position servo system, Fire Control & Command Control. 9 (2014) 139-141.

[4] Ngoc A. N., Sorin O., Pedro R. A., Michal K., Convex liftings-based robust control design, Automatica. 17 (2017) 206-213.

[5] John A., Robust multi-objective control of hybrid renewable microgeneration systems with energy storage, Applied Thermal Engineering. 114 (2017) 1498-1506.

[6] Konstantin S., Jannik H., Maksim W., Robust control for voltage and transient stability of power grids relying on wind power, Control Engineering Practice. 60 (2017) 7-17.

[7] Qinghai Heng, Jing Lu, Huan Cao, Xiaoshan Lu. "Robust H∞ PI controller for a fan system", The 27th Chinese Control and Decision Conference (2015 CCDC), 2015, pp. 723–727.

[8] Postlethwaite I., Turner M. C., Herrmann Guido, Robust control applications, Annual Reviews in Control. 31 (2007) 27-39.