Non-integer conformal dimensions for type IIA flux vacua

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We study the conformal dimensions of the would-be dual operators to the stabilized moduli for two non-supersymmetric DGKT vacua. For the first of them, related to the standard SUSY DGKT vacuum by \( G_4 \neq G_4^{\text{SUSY}} \), we obtain integer conformal dimensions. For the second of them, which has a non-zero harmonic component in \( G_2 \), we obtain both integers and real numbers.

I. INTRODUCTION

Scale separation and moduli stabilisation are two indispensable requirements in any EFT constructed from string theory that aims to describe our universe. As of today, some of the most studied models featuring both characteristics are the so-called DGKT vacua [1, 2], AdS\(_3\) flux compactifications of massive IIA in Calabi-Yau orientifolds.

Under the approach of the swampland program [3] -for reviews see [4–7]- these scenarios have received a renewal interest and are now being scrutinised, specially since it was conjectured in [8] that (SUSY) scale separated vacua may lie in the swampland -see [9] for previous discussion and [10] for a refinement of the conjecture. In this regard, its 10d uplift was addressed in [11], whereas more discussion on scale separation in IIA can be found in [12–16].

The study of the holographic description of these vacua was initiated recently in [22, 23, 27]. Scale separation has a clear interpretation in terms of parametric gaps in the spectrum of masses of the dual theory, and it may be clarified if DGKT vacua are consistent by looking at the kind of CFT that they would correspond to. Surprisingly, it was first noticed in [26] for the original toroidal example, and then generalized in [27] for any CY orientifold, that the conformal dimension for the low-lying scalar primaries in the dual CFT of SUSY DGKT vacua is always an integer, independently of the details of the compactification. In the same spirit, [22] found that this is also true for the toroidal non-SUSY DGKT models examined there.

In this letter, we will continue in this direction by examining two of the non-SUSY branches of general (no specific CY orientifold is assumed) DGKT vacua derived in [28] and [24]. As we will show below, for the non-SUSY branch related to the SUSY one by changing \( G_4^{\text{non-SUSY}} = G_4^{\text{SUSY}} \), the conformal dimension of the scalars dual to the stabilised moduli are integers as well.

Before presenting the results, it is worth pointing out that non-SUSY AdS vacua of this kind are conjectured to be unstable [29]. Regarding the status of the two branches at hand, its perturbative stability was verified in [28, 30]. In addition to this, the non-perturbative stability of the first one (\( G_4^{\text{SUSY}} \)) was first studied in [30], where decays were found to be at best marginal, and then in [31], where a D8 domain wall instability was found if space-time filling D6s are used to cancel the tadpole.

II. DGKT VACUA IN A NUTSHELL

Consider massive type IIA string theory compactified on an orientifold of \( R^{1,3} \times \mathcal{M}_6 \) with \( \mathcal{M}_6 \) a compact Calabi-Yau threefold. Dimensional reduction leads to a \( \mathcal{N} = 1 \) 4d supergravity theory whose massless scalar field content is organised as follows [32]. On the one hand, there are the complexified Kähler moduli, coming from integrating the Kähler 2-form \( J = \omega_a \omega_a \) and the \( B = b^a \omega_a \) field

\[
T^a = b^a + it^a, \quad a \in \{1, \ldots, h^{1,1}_+\},
\]

where \( l_s^{-2} \omega_a \) are harmonic representatives of \( H^2(\mathcal{M}_6, \mathbb{Z}) \) and \( l_s \) the string length. The metric appearing in the kinetic terms of these moduli is obtained from the Kähler potential

\[
K_K = -\log \left( \frac{4}{3} K_{abc} t^a t^b t^c \right) = -\log \left( \frac{4}{3} \mathcal{K} \right),
\]

with \( K_{abc} = l_s^{-6} \int \omega_a \wedge \omega_b \wedge \omega_b \) and \( \mathcal{K} = 6 \text{Vol}_{\mathcal{M}_6} \). On the other hand, there are the complex structure moduli, coming from the complex 3-form \( \Omega \), the axio-dilaton and the RR 3-form potential \( C_3 \). Introducing \( \Omega_c \equiv C_3 + i \text{Re}(\mathcal{C}) \) where \( \mathcal{C} = e^{-\phi} e^{\frac{i}{6}(K_C - K_K)} \) is a compensator,
with \( K_{cs} = -\log \left( -i \ell^{-6} \int \Omega \wedge \bar{\Omega} \right) \) and \( \phi \) the 10d dilaton, the complex structure moduli are defined as
\[
U^\mu = \xi^\mu + i u^\mu = l_s^{-3} \int \Omega_c \wedge \beta^\mu, \quad \mu \in \{0, \ldots, h^2 + 1\},
\]
where we are taking a symplectic basis \( \beta^\mu \in H^3 (\mathcal{M}_6, \mathbb{Z}) \). Finally, the metric appearing in the kinetic terms of this sector is constructed from the following Kähler potential
\[
K_Q = 4 \log \left( \frac{e^\phi}{\sqrt{\text{Vol} \mathcal{M}_6}} \right) = - \log (e^{-4D}).
\]

On top of this background one can add RR and NSNS background fluxes. Following the conventions of [28] their flux quanta are
\[
l_s \check{G}_0 = -m, \quad \frac{1}{l_s^2} \int_{\pi_a} \check{G}_2 = m^a, \quad \frac{1}{l_s^2} \int_{\pi_a} \check{G}_4 = -e_a,
\]
\[
\frac{1}{l_s^2} \int_{\mathcal{M}_6} \check{G}_6 = e_0, \quad \frac{1}{l_s^2} \int_{B_{\mu}} \check{H} = h_{\mu}, \quad \mu \in \{0, \ldots, h^2 + 1\},
\]
with \([\pi_a] \in H^2_+ (\mathcal{M}_6, \mathbb{Z})\) Poincaré dual to \([l_s^{-2} \omega_a]\) and \([\bar{\pi}^a] \in H^2 (\mathcal{M}_6, \mathbb{Z})\) Poincaré dual to \([l_s^{-4} \omega_a]\), where \(l_s^{-6} \int_{\mathcal{X}} \omega^a \wedge \bar{\omega}^b = \delta^b_a\). \(B_{\mu}\) is the 3-cycle de Rham dual to \(\beta^\mu\). In the 4d action the presence of fluxes is encoded through the superpotential
\[
l_s W = e_0 + e_a T^a + \frac{1}{2} K_{abc} m^a T^b T^c + \frac{m}{6} K_{abc} T^a T^b T^c + h_{\mu} U^\mu,
\]
which involves both the Kähler and the complex structure moduli. As shown in [33], the F-term scalar potential generated by this superpotential exhibits a remarkable factorisation between the saxionic and the axionic components. Namely, the potential can be written in full generality as
\[
V = \frac{1}{\kappa^2} \check{\rho}^\dagger Z \check{\rho},
\]
where the vector \(\check{\rho}\) depends only on the flux quanta and the axions \((b, \xi)\):
\[
l_s \rho_0 = e_0 + e_a b^a + \frac{1}{2} K_{abc} m^a b^b c^c + \frac{m}{6} K_{abc} b^a b^b c^c + h_{\mu} \xi^\mu,
\]
\[
l_s \rho_a = e_a + K_{abc} m^b c^c + \frac{m}{2} K_{abc} b^a b^b c^c,
\]
\[
l_s \check{\rho}^a = m^a + m b^a,
\]
\[
l_s \check{\rho} = m,
\]
\[
l_s \check{\rho}_{\mu} = h_{\mu},
\]
whereas the matrix \(Z\) depends only on the saxions \((t, u)\):
\[
Z = e^K \begin{pmatrix}
4 K_{ab} & \frac{1}{2} K^2 K_{ab} & \frac{1}{2} K^2 K_{ab} \\
\frac{1}{2} K^2 K_{ab} & \frac{1}{2} K^2 K_{ab} \\
\frac{1}{2} K^2 K_{ab} & \frac{1}{2} K^2 K_{ab} \\
\end{pmatrix}
\]
with \(K = K_K + K_Q\), \(K_{ab} = \frac{1}{2} \partial_{\mu} \partial_{\nu} K_K\) and \(K_{\mu \nu} = \frac{1}{2} \partial_{\mu} \partial_{\nu} K_Q\). This factorisation is maintained even when D6-brane moduli [34] and \(\alpha'\) corrections [35] are included.

Though we will use the \(\check{\rho}\) language in this note, it may be useful to recall how their components are related with the more familiar gauge invariant field strengths [33]. Using the democratic formulation and calling \(C = C_1 + C_2 + C_5 + C_7 + C_9\) and \(G = dC - H + C + \mathbf{G} + \mathbf{e}^B\), where \(\mathbf{G}\) is just the sum of the previously introduced flux quanta, then
\[
\rho_0 = \int_{\mathcal{M}_6} G_6, \quad \rho_a = \int_{\pi_a} G_4, \quad \check{\rho}^a = \int_{\bar{\pi}^a} G_2,
\]
\[
\check{\rho} = G_0, \quad \check{\rho}_{\mu} = \int_{B_{\mu}} H_3.
\]

III. NON-SUSY VACUA AND CONFORMAL DIMENSIONS

Writing the potential as a bilinear expression is a very useful tool in the search for vacua, as it was exploited in [34, 35] and especially in [28], where a systematic search of extrema for this setup was performed. Among the different branches of vacua found in that paper we will focus on two cases:

A) Branch Non-SUSY\(G_4\), characterised by \(\rho_a = -\rho_a^{\text{SUSY}}\). In this vacuum
\[
\check{\rho}_{\mu} = \frac{1}{15} \beta K \partial_{\mu} \kappa, \quad \rho_a = -\frac{3}{10} \check{\beta} \kappa_a,
\]
\[
\check{\rho}^a = 0, \quad \rho_0 = 0,
\]
\[
V = -\frac{4 e^K}{15} \kappa^2 \check{\rho}^2;
\]

B) Branch Non-SUSY\(G_2\), characterised by \(\check{\rho}^a \neq 0\). In this vacuum
\[
\check{\rho}_{\mu} = \frac{1}{12} \beta K \partial_{\mu} \kappa, \quad \rho_a = -\frac{1}{4} \check{\beta} \kappa_a,
\]
\[
\check{\rho}^a = \pm \frac{1}{2} \check{\beta} t^a, \quad \rho_0 = 0,
\]
\[
V = -\frac{e^K}{18} \kappa^2 \check{\rho}^2;
\]
with \(\kappa_a = K_{abc} t^b c^c\). In addition to this, the SUSY branch was studied recently in [27], agreeing with what we obtain. We will not discuss it here.
A. Branch Non-SUSY\(_{G_4}\)

All the necessary features of this solution, including the physical mass of the stabilized moduli, were calculated in \([28]\), namely in appendix B. We can limit ourselves to compute the conformal dimension \(\Delta (\Delta - d) = m^2 R^2_{AdS}\) of the correspondent fields in the would-be CFT\(_3\) dual. Regarding the saxions of the compactification, its conformal dimension would be

\[
\Delta = 10, \quad \Delta_i = 1 \text{ or } 2, \quad \Delta_a = 6, \quad (13)
\]

with \(i = 1, \ldots, h^{2,1}\) and \(a = 1, \ldots, h^{-1,1}_\omega\) taking into account that some fields acquire the same mass. Notice that, as expected, these same results were obtained in \([27]\) for the saxionic spectrum of the supersymmetric case, since the mass matrix in both cases is block diagonal and shares the entries of this part. The difference comes from the axions, whose conformal dimensions in the dual theory would be

\[
\Delta = 1 \text{ or } 2, \quad \Delta_i = 3, \quad \Delta_a = 8, \quad (14)
\]

with again \(i = 1, \ldots, h^{2,1}\) and \(a = 1, \ldots, h^{-1,1}_\omega\). The \(\Delta_i = 3\) comes from the fact that only a linear combination of axions appear in the superpotential, so \(h^{2,1}\) of them remain massless. These results agree and generalize the work of \([22]\), who looked at this kind of vacua but only in toroidal examples.

B. Branch Non-SUSY\(_{G_2}\)

Unlike the previous case, the mass acquired by the stabilized moduli was not explicitly computed in the original reference, so some intermedia steps have to be done. We relegate the diagonalization of the Hessian and all the details needed to appendix A). Using again the relation \(\Delta (\Delta - d) = m^2 R^2_{AdS}\), the conformal dimension of the dual operators for this branch should be

\[
\Delta = \frac{1}{2} \left(3 + \sqrt{393}\right), \quad \Delta_a = \frac{1}{2} \left(3 + \sqrt{201}\right), \quad \Delta_i = 3, \\
\Delta = \frac{1}{2} \left(3 + \sqrt{33}\right), \quad \Delta_a = 6, \quad \Delta_i = 3, \quad (15)
\]

with again \(i = 1, \ldots, h^{2,1}\) and \(a = 1, \ldots, h^{-1,1}_\omega\). In this case there are \(2h^{2,1}\) fields with conformal dimension \(\Delta = 3\), since the saxionic partners of the usual massless axions do not acquire a mass.\(^3\)

IV. CONCLUSIONS

In this work we have computed the conformal dimension of the low-lying operators of the putative CFT\(_3\) dual of two different non-supersymmetric DGKT vacua.

In the first place, we focused on what we called the Non-SUSY\(_{G_4}\) vacuum, which has the property of being related to the SUSY one by \(G_4 = -G^{\text{SUSY}}_4\). We obtained that these dimensions are always integers -see expressions \([13]\) for the saxionic sector and \([13]\) for the axionic sector-, and totally independent of the details of the compactification. With respect to the saxions this is not new, since this part of the mass matrix is shared with the SUSY branch and the same numbers were obtained in \([27]\). For the axions, this result extends the work of \([22]\) who only looked at toroidal models- to any CY orientifold.

In the second place, we studied what we named the Non-SUSY\(_{G_2}\) vacuum, called in this way by having \(G^\text{harmonic}_2 \neq 0\). Unlike the SUSY and the Non-SUSY\(_{G_4}\) vacua, the conformal dimensions of the operators of the dual theory would not be only integers, see \([15]\). The result is again quite simple and does not depend on the details of the compactification. The 10d uplift of this branch has not been studied in detail and it could happen that problems arise when looked at from the 10d point of view. Following the analysis of \([15]\), there should not be any obstructions in constructing first, a smearing uplift, and then, expanding the solution and localising the source at first order. We have already checked explicitly that indeed the smearing uplift exists. These and more details are being studied in an upcoming work \([30]\).

Non-supersymmetric vacua of the kind studied here are conjectured to be unstable \([24]\). This could imply that it would not make much sense to study their CFT duals, since these theories could be sick or ill-defined. Indeed in \([31]\) we showed that for the Non-SUSY\(_{G_4}\) branch there seems to be an instability if space-time filling D6s are used to cancel the tadpole. Opposite to this reasoning, studying these non-supersymmetric vacua from their putative CFT dual could be useful to show that they are unstable, which makes this analysis interesting per se.

In fact, constructing the would-be CFT duals of scale separated AdS vacua could be a way of understanding if scenarios of this type are consistent or if they lie in the swampland. In this note, we have shown that not only integers appear when we study the conformal dimensions of the low-lying operators of DGKT vacua. But we still do not understand why they do appear for the SUSY case or even if the conformal theories we are trying to construct really exist. We hope that both questions could be answered in the not too distant future.

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\(^3\) They develop a quartic potential
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Appendix A: Mass spectrum for the non-SUSY $G_2$ vacua

The Hessian for this branch was obtained explicitly in appendix B of [28], see equation (B.6). To compute the physical mass spectrum one has to express the fields on a canonical basis. As explained there, for this we decompose the Kähler metrics for the Kähler and complex structure fields as

$$K_{ab} = \frac{3}{2K} \left( \frac{3K_aK_b}{K} - K_{ab} \right) = \frac{3}{4} \frac{K_aK_b}{K^2} + \frac{3}{2K} \left( \frac{K_aK_b}{K} - K_{ab} \right) = K_{NP} + K_{ab},$$

$$K_{\mu\nu} = \frac{1}{16} \frac{\partial_\mu G \partial_\nu G}{G^2} + \frac{1}{4} \left( \frac{3 \partial_\mu G \partial_\nu G}{G^2} - \frac{\partial_\mu \partial_\nu G}{G} \right)$$

$$= K_{NP} + K_{\mu\nu}, \quad (A1)$$

with $K_{ab} = K_{abc} t^c$, $K = K_K + K_Q = -\log (G)$ and $\partial_\mu \equiv \partial_{\mu}$. Here $K^P$ and $K^{NP}$ refers to the primitive and non-primitive parts of the metric, which act on orthogonal subspaces. One can then express the fields in the canonically normalised basis

$$\begin{pmatrix} \xi^\mu & b^a & u^\mu & t^a \end{pmatrix} \rightarrow \begin{pmatrix} \xi^\mu & b^a & u^\mu & t^a \end{pmatrix} \begin{pmatrix} \hat{\xi} & \hat{b} & \hat{u} & \hat{t} \end{pmatrix},$$

$$\begin{pmatrix} \hat{\xi} & \hat{b} & \hat{u} & \hat{t} \end{pmatrix} = \frac{1}{\sqrt{3}} \left( \begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & \pm \frac{8}{9} & 0 \\ 0 & -\frac{4}{3\sqrt{3}} & \frac{26}{9} & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \begin{pmatrix} \xi^\mu & b^a & u^\mu & t^a \end{pmatrix},$$

$$H = K^{NP} \hat{\rho}^2 \quad (A4)$$

and the physical masses can be obtained straightforwardly by diagonalising this matrix and dividing by two. Expressed in terms of $R^2_{AdS} = \frac{1}{|m|}$ they are

$$m^2 = R^2_{AdS} \{ 96, \quad 6, \quad 48, \quad 18, \quad 0 \}, \quad (A5)$$

with multiplicity $\{ 1, \quad 1, \quad h_{-1}^{1,1}, \quad h_{-1}^{1,1}, \quad 2h^{2,1} \}$ respectively.

[1] O. DeWolfe, A. Giryavets, S. Kachru, and W. Taylor, Type IIA moduli stabilization, JHEP 07, 066, arXiv:hep-th/0505100

[2] P. G. Camara, A. Font, and L. E. Ibanez, Fluxes, moduli fixing and MSSM-like vacua in a simple IIA orientifold, JHEP 09, 013, arXiv:hep-th/0506066

[3] C. Vafa, The String landscape and the swampland, (2005), arXiv:hep-th/0509212

[4] T. D. Brennan, F. Carta, and C. Vafa, The String Landscape, the Swampland, and the Missing Corner, Proceedings, Theoretical Advanced Study Institute in Elementary Particle Physics: Physics at the Fundamental Frontier (TASI 2017): Boulder, CO, USA, June 5-30, 2017, PoS TASI2017, 015 (2017), arXiv:1711.00864 [hep-th]

[5] E. Palti, The Swampland: Introduction and Review, Fortsch. Phys. 67, 1900037 (2019), arXiv:1903.06239 [hep-th]

[6] M. van Beest, J. Calderón-Infante, D. Mirfendereski, and I. Valenzuela, Lectures on the Swampland Program in String Compactifications, (2021), arXiv:2102.01111 [hep-th]

[7] M. Graña and A. Herráez, The Swampland Conjectures: A Bridge from Quantum Gravity to Particle Physics, Universe 7, 273 (2021), arXiv:2107.00087 [hep-th]

[8] D. Lüst, E. Palti, and C. Vafa, AdS and the Swampland, JHEP 10, 016 (2019), arXiv:1906.05225 [hep-th]

[9] F. F. Gautason, M. Schillo, T. Van Riet, and M. Williams, Remarks on scale separation in flux vacua, JHEP 03, 061 (2019), arXiv:1512.00457 [hep-th]

[10] G. Buratti, J. Calderon, A. Mininno, and A. M. Uranga, Discrete Symmetries, Weak Coupling Conjecture and Scale Separation in AdS Vaca, JHEP 06, 083, arXiv:2003.09740 [hep-th]

[11] F. F. Gautason, V. Van Hemelryck, and T. Van Riet, The Tension between 10D Supergravity and dS Uplifts, Fortsch. Phys. 67, 1800091 (2019), arXiv:1810.08518 [hep-th]

[12] B. S. Acharya, F. Benini, and R. Vandalro, Fixing moduli in exact type IIA flux vacua, JHEP 02, 018, arXiv:hep-th/0607223
[13] F. Saracco and A. Tomasiello, Localized O6-plane solutions with Romans mass, *JHEP* **07**, 077, arXiv:1201.5378 [hep-th]

[14] J. McOrist and S. Sethi, M-theory and Type IIA Flux Compactifications, *JHEP* **12**, 122, arXiv:1208.0261 [hep-th]

[15] D. Junghans, O-Plane Backreaction and Scale Separation in Type IIA Flux Vacua, *Fortsch. Phys.* **68**, 2000040 (2020), arXiv:2003.06274 [hep-th]

[16] F. Marchesano, E. Palti, J. Quirant, and A. Tomasiello, On supersymmetric AdS$_4$ orientifold vacua, *JHEP* **08**, 087, arXiv:2003.13578 [hep-th]

[17] A. Font, A. Herrera, and L. E. Ibanez, On scale separation in type II AdS flux vacua, *JHEP* **03**, 013, arXiv:1912.03317 [hep-th]

[18] F. Farakos, G. Tringas, and T. Van Riet, No-scale and scale-separated flux vacua from IIA on G2 orientifolds, *Eur. Phys. J. C* **80**, 659 (2020), arXiv:2005.05246 [hep-th]

[19] S. Baines and T. Van Riet, Smearing orientifolds in flux compactifications can be OK, *Class. Quant. Grav.* **37**, 195015 (2020), arXiv:2005.09501 [hep-th]

[20] G. B. De Luca and A. Tomasiello, Leaps and bounds towards scale separation, *JHEP* **12**, 086, arXiv:2104.12773 [hep-th]

[21] N. Cribiori, D. Junghans, V. Van Hemelryck, T. Van Riet, and T. Wrase, Scale-separated AdS4 vacua of IIA orientifolds and M-theory, *Phys. Rev. D* **104**, 126014 (2021), arXiv:2107.00019 [hep-th]

[22] F. Apers, M. Montero, T. Van Riet, and T. Wrase, Comments on classical AdS flux vacua with scale separation, (2022), arXiv:2202.00682 [hep-th]

[23] M. Emelin, F. Farakos, and G. Tringas, O6-plane backreaction on scale-separated Type IIA AdS$_3$ vacua, (2022), arXiv:2202.13431 [hep-th]

[24] N. Cribiori and G. D’ell’ Agata, Weak gravity versus scale separation, (2022), arXiv:2203.05559 [hep-th]

[25] O. Aharony, Y. E. Antebi, and M. Berkooz, On the Conformal Field Theory Duals of type IIA AdS(4) Flux Compactifications, *JHEP* **02**, 093, arXiv:0801.3326 [hep-th]

[26] J. P. Conlon, S. Ning, and F. Revello, Exploring The Holographic Swampland, (2021), arXiv:2110.06245 [hep-th]

[27] F. Apers, J. P. Conlon, S. Ning, and F. Revello, Integer Conformal Dimensions for Type IIA Flux Vacua, (2022), arXiv:2202.09330 [hep-th]

[28] F. Marchesano and J. Quirant, A Landscape of AdS Flux Vacua, *JHEP* **12**, 110, arXiv:1908.11386 [hep-th]

[29] H. Ooguri and C. Vafa, Non-supersymmetric AdS and the Swampland, *Adv. Theor. Math. Phys.* **21**, 1787 (2017), arXiv:1610.01533 [hep-th]

[30] F. Narayan and S. P. Trivedi, On The Stability Of Non-Supersymmetric AdS Vacua, *JHEP* **07**, 089, arXiv:1002.4498 [hep-th]

[31] F. Marchesano, D. Prieto, and J. Quirant, Bionic membranes and AdS instabilities, (2021), arXiv:2110.11370 [hep-th]

[32] T. W. Grimm and J. Louis, The Effective action of type IIA Calabi-Yau orientifolds, *Nucl. Phys. B* **718**, 153 (2005), arXiv:hep-th/0412277

[33] A. Herrera, L. E. Ibáñez, F. Marchesano, and G. Zoccarato, The Type IIA Flux Potential, 4-forms and Freed-Witten anomalies, *JHEP* **09**, 018, arXiv:1802.05771 [hep-th]

[34] D. Escobar, F. Marchesano, and W. Staessens, Type IIA Flux Vacua with Mobile D6-branes, *JHEP* **01**, 096, arXiv:1811.09282 [hep-th]

[35] D. Escobar, F. Marchesano, and W. Staessens, Type IIA flux vacua and $\alpha'$-corrections, *JHEP* **06**, 129, arXiv:1812.08735 [hep-th]

[36] F. Marchesano, J. Quirant, and M. Zatti, Work in progress, .