Collective modes in light nuclei from first principles

T. Dytrych  
*Louisiana State University*

K. D. Launey  
*Louisiana State University*

J. P. Draayer  
*Louisiana State University*

P. Maris  
*Iowa State University*

J. P. Vary  
*Iowa State University*

*See next page for additional authors*

Follow this and additional works at: [https://digitalcommons.lsu.edu/physics_astronomy_pubs](https://digitalcommons.lsu.edu/physics_astronomy_pubs)

**Recommended Citation**

Dytrych, T., Launey, K., Draayer, J., Maris, P., Vary, J., Saule, E., Catalyurek, U., Sosonkina, M., Langr, D., & Caprio, M. (2013). Collective modes in light nuclei from first principles. *Physical Review Letters, 111* (25)  
[https://doi.org/10.1103/PhysRevLett.111.252501](https://doi.org/10.1103/PhysRevLett.111.252501)

This Article is brought to you for free and open access by the Department of Physics & Astronomy at LSU Digital Commons. It has been accepted for inclusion in Faculty Publications by an authorized administrator of LSU Digital Commons. For more information, please contact ir@lsu.edu.
Collective Modes in Light Nuclei from First Principles

T. Dytrych,1 K. D. Launey,1 J. P. Draayer,1 P. Maris,2 J. P. Vary,2 E. Saule,3 U. Catalyurek,3,4 M. Sosonkina,5 D. Langr,6 and M. A. Caprio7

1Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803, USA
2Department of Physics and Astronomy, Iowa State University, Ames, IA 50011, USA
3Department of Biomedical Informatics, The Ohio State University, Columbus, OH 43210, USA
4Department of Electrical and Computer Engineering, The Ohio State University, Columbus, OH 43210, USA
5Department of Modeling, Simulation and Visualization Engineering, Old Dominion University, Norfolk, VA 23529, USA
6Department of Computer Systems, Czech Technical University in Prague, Prague, Czech Republic
7Department of Physics, University of Notre Dame, Notre Dame, IN 46556, USA

Results for ab initio no-core shell model calculations in a symmetry-adapted SU(3)-based coupling scheme demonstrate that collective modes in light nuclei emerge from first principles. The low-lying states of 6Li, 9Be, and 6He are shown to exhibit orderly patterns that favor spatial configurations with strong quadrupole deformation and complementary low intrinsic spin values, a picture that is consistent with the nuclear symplectic model. The results also suggest a pragmatic path forward to accommodate deformation-driven collective features in ab initio analyses when they dominate the nuclear landscape.

Introduction. – Major progress in the development of realistic inter-nucleon interactions along with the utilization of massively parallel computing resources [1–3] have placed ab initio approaches [4–14] at the frontier of nuclear structure explorations. The ultimate goal of ab initio studies is to establish a link between underlying principles of quantum chromodynamics (quark/gluon considerations) and observed properties of atomic nuclei, including their structure and related reactions. The predictive potential of ab initio models hold [15, 16] makes them suitable for targeting short-lived nuclei that are inaccessible by experiment but essential to modeling, for example, of the dynamics of X-ray bursts and the path of nucleosynthesis (see, e.g., [17, 18]).

In this letter, we report on ab initio symmetry-adapted no-core shell model (SA-NCSM) results for the 6Li (odd-odd), 9Be (even-even), and 6He (halo) nuclei, using two realistic nucleon-nucleon (NN) interactions, the JISP16 [19] and chiral N3LO [20] potentials. The SA-NCSM framework exposes a remarkably simple physical feature that is typically masked in other ab initio approaches; the emergence, without a priori constraints, of simple orderly patterns that favor spatial configurations with strong quadrupole deformation and low intrinsic spin values. This feature, once exposed and understood, can be used to guide a truncation and augmentation of model spaces to ensure that important properties of atomic nuclei, like enhanced B(E2) strengths, nucleon cluster substructures, and others important in reactions, are appropriately accommodated in future ab initio studies.

The SA-NCSM joins a no-core shell model (NCSM) theory [4] with a multi-shell, SU(3)-based coupling scheme [21, 22]. Specifically, nuclear wavefunctions are represented as a superposition of many-particle configurations carrying a particular intrinsic quadrupole deformation linked to the irreducible representation (irrep) labels (λμ) of SU(3) [23–25], and specific intrinsic spins (S_pS_nS) for protons, neutrons, and total spin, respectively (proton-neutron formalism). The fact that SU(3) plays a key role, e.g., in the microscopic description of the experimentally observed collectivity of ds-shell nuclei [26–30], and for heavy deformed systems [31], tracks from the seminal work of Elliott [21] and is reinforced by the fact that it is the underpinning symmetry of the microscopic symplectic model [32, 33], which provides a comprehensive theoretical foundation for understanding the dominant symmetries of nuclear collective motion [32, 34].

The outcome further suggests a symmetry-guided basis selection that yields results that are nearly indistinguishable from the complete basis counterparts. This is illustrated for 6Li and 6He for a range of harmonic oscillator (HO) energies ℏΩ, and Nmax=12 model spaces, where Nmax is the maximum number of HO quanta included in the basis states above the Pauli allowed minimum for a given nucleus. An overarching long-term objective is to extend the reach of the standard NCSM scheme by exploiting symmetry-guided principles that enable one to include configurations beyond the Nmax cutoff, while capturing the essence of long-range correlations that often dominate the nuclear landscape.

Ab initio realization of collective modes. – The expansion of eigenstates in the physically relevant SU(3) basis unveils salient features that emerge from the complex dynamics of these strongly interacting many-particle systems. To explore the nature of the most important correlations, we analyze the probability distribution across (S_pS_nS) and (λμ) configurations of the four lowest-lying isospin-zero (T = 0) states of 6Li (1^+_2, 3^+_1, 5^+_1, 7^+_1).
mixing of \( S_p S_n S \) and \((\lambda \mu)\) values (horizontal axis) for the calculated \( I^+_\text{gs} \) of \( ^6\text{Li} \) obtained for \( N_{\text{max}} = 10 \) and \( h\Omega = 20 \) MeV with the JISP16 interaction (left) and the \( O^+_\text{gs} \) of \( ^8\text{Be} \) obtained for \( N_{\text{max}} = 8 \) and \( h\Omega = 25 \) MeV with the chiral \( N^3\text{LO} \) interaction (right). The total probability for each \( Nh\Omega \) subspace is given in the upper left-hand corner of each histogram. The concentration of strengths to the far right demonstrates the dominance of collectivity.

2\(^+\), and 1\(^+\)) along with the ground-state rotational bands of \( ^8\text{Be} \) and \( ^6\text{He} \). Results for the ground-state of \( ^6\text{Li} \) and \( ^8\text{Be} \), obtained with the JISP16 and chiral \( N^3\text{LO} \) interactions, respectively, are shown in Figure 1. This figure illustrates a feature common to all the low-energy solutions considered; namely, a highly structured and regular mix of intrinsic spins and SU(3) spatial quantum numbers that has heretofore gone unrecognized in other \textit{ab initio} studies, and which does not seem to depend on the particular choice of realistic \( NN \) potential.

First, consider the spin content. The calculated eigenstates project at a 99% level onto a comparatively small subset of intrinsic spin combinations. For instance, the lowest-lying eigenstates in \( ^6\text{Li} \) are almost entirely realized in terms of configurations characterized by the following intrinsic spin \((S_p S_n S)\) triplets: \((\frac{3}{2} \frac{3}{2} 3), (\frac{1}{2} \frac{1}{2} 2), (\frac{3}{2} \frac{1}{2} 2), \) and \((\frac{1}{2} \frac{1}{2} 1)\), with the last one carrying over 90% of each eigenstate. Similarly, the ground-state bands of \( ^8\text{Be} \) and \( ^6\text{He} \) are found to be dominated by configurations carrying total spin of the protons and neutrons equal to zero and one, with the largest contributions due to \((S_p S_n S) = (000)\) and \((112)\) configurations.

Second, consider the spatial degrees of freedom. The mixing of \((\lambda \mu)\) quantum numbers exhibits a remarkably simple pattern. One of its key features is the preponderance of a single \( 0\Omega \) SU(3) irrep. This so-called leading irrep, is characterized by the largest value of the intrinsic quadrupole deformation \( \lambda \). For instance, the low-lying states of \( ^6\text{Li} \) project at a 40%-70% level onto the prolate \( 0\Omega \) SU(3) irrep \((20)\), as illustrated in Fig. 1. For the ground-state band of \( ^8\text{Be} \) and \( ^6\text{He} \), qualitatively similar dominance of the leading \( 0\Omega \) SU(3) irreps is observed. The dominance of the most deformed \( 0\Omega \) configuration indicates that the quadrupole–quadrupole interaction of the Elliott SU(3) model \([21]\) is realized naturally within an \textit{ab initio} framework.

The analysis also reveals that the dominant SU(3) basis states at each \( Nh\Omega \) subspace \((N = 0, 2, 4, \ldots)\) are typically those with \((\lambda \mu)\) quantum numbers given by

\[
\lambda + 2\mu = \lambda_0 + 2\mu_0 + N
\]  

where \( \lambda_0 \) and \( \mu_0 \) denote labels of the leading SU(3) irrep in the \( 0\Omega \) \((N = 0)\) subspace. We conjecture that this regular pattern of SU(3) quantum numbers reflects the presence of an underlying symplectic \( Sp(3, \mathbb{R}) \) symmetry of microscopic nuclear collective motion \([32]\) that governs the low-energy structure of both even-even and odd-odd \( p\)-shell nuclei. This can be seen from the fact that \((\lambda \mu)\)
configurations that satisfy condition \( \{1\} \) can be determined from the leading SU(3) irrep \((\lambda_0, \mu_0)\) through a successive application of a specific subset of the Sp(3, \(\mathbb{R}\)) symplectic \(2\hbar\Omega\) raising operators. This subset is composed of the three operators, \(\hat{A}_{zz}, \hat{A}_{xx},\) and \(\hat{A}_{xx}\), that distribute two oscillator quanta in \(z\) and \(x\) directions, but none in \(y\) direction, thereby inducing SU(3) configurations with ever-increasing intrinsic quadrupole deformation. These three operators are the generators of the Sp\((2, \mathbb{R})\subset\text{Sp}(3, \mathbb{R})\) subgroup [35], and give rise to deformed shapes that are energetically favored by an attractive quadrupole-quadrupole interaction [34]. This is consistent with our earlier findings of a clear symplectic Sp\((3, \mathbb{R})\) structure with the same patterns \(\{1\}\) in \textit{ab initio} eigensolutions for \(^{12}\text{C}\) and \(^{18}\text{O}\) [36].

Furthermore, the \(\mathcal{N}\hbar\Omega\) configurations with \((\lambda_0+N, \mu_0)\), the so-called stretched states, carry a noticeably higher probability than the others. For instance, the \((2+N, 0)\) stretched states contribute at the 85\% level to the ground-state of \(^{6}\text{Li}\), as can be readily seen in Fig. 1. The sequence of the stretched states is formed by consecutive applications of the \(\hat{A}_{zz}\) operator, the generator of Sp\((1, \mathbb{R})\subset\text{Sp}(2, \mathbb{R})\subset\text{Sp}(3, \mathbb{R})\) subgroup, over the leading SU(3) irrep. This translates into distributing \(N\) oscillator quanta along the direction of the \(z\)-axis only and hence rendering the largest possible deformation.

\textbf{Symmetry-guided framework.} – The observed patterns of intrinsic spin and deformation mixing supports the symmetry-guided basis selection philosophy referenced above. Specifically, one can take advantage of dominant symmetries to relax and refine the definition of the SA-NCSM model space, which for the NCSM is fixed by simply specifying the \(N_{\text{max}}\) cutoff. In particular, SA-NCSM model spaces can be characterized by a pair of numbers, \(\langle N_{\text{max}} \rangle_{\mathcal{N}^T_{\text{max}}}\), which implies inclusion of the complete space up through \(N_{\text{max}}^T\), and a subset of the complete set of \(\langle \lambda \rangle \mu\) and \((S_pS_pS)\) irreps between \(N_{\text{max}}^T\) and \(N_{\text{max}}\). Though not a primary focus of this paper, an ultimate goal is to be able to carry out SA-NCSM investigations in deformed nuclei with \(N_{\text{max}}\) values that go beyond the highest \(N_{\text{max}}\) for which complete NCSM results can be provided.

The SA-NCSM concept focuses on retaining the most important configurations that support the strong many-nucleon correlations of a nuclear system using underlying Sp\((1, \mathbb{R})\subset\text{Sp}(2, \mathbb{R})\subset\text{Sp}(3, \mathbb{R})\) symmetry considerations. It is important to note that for model spaces truncated according to \(\langle \lambda \rangle \mu\) and \((S_pS_pS)\) irreps, the spurious center-of-mass motion can be factored out exactly [37], which represents an important advantage of this scheme.

The efficacy of the symmetry-guided concept is illustrated for SA-NCSM results obtained in model spaces which are expanded beyond a complete \(N_{\text{max}}^T\) space with irreps that span a relatively few dominant intrinsic spin components and carry quadrupole deformation specified by \(\{1\}\). Specifically, we vary \(N_{\text{max}}^T\) from 2 to 10 with only the subspaces determined by \(\{1\}\) included beyond \(N_{\text{max}}^T\). This allows us to study convergence of spectroscopic properties towards results obtained in the complete \(N_{\text{max}}=12\) space and hence, probes the efficacy of the SA-NCSM symmetry-guided model space selection concept. In the present study, a Coulomb plus JISP16 NN interaction for \(\hbar\Omega\) values ranging from 17.5 up to 25 MeV is used, along with the Gloeckner-Lawson prescription [38] for elimination of spurious center-of-mass excitations. SA-NCSM eigenstates are used to determine spectroscopic properties of low-lying \(T = 0\) states of \(^{6}\text{Li}\) and the ground-state band of \(^{6}\text{He}\) for \(\langle N_{\text{max}} \rangle_{\mathcal{N}^T_{\text{max}}}\)12 model spaces.

The results indicate that the observables obtained in the \(\langle N_{\text{max}} \rangle_{\mathcal{N}^T_{\text{max}}}\)12 symmetry-guided truncated spaces are excellent approximations to the corresponding \(N_{\text{max}}=12\) complete-space counterparts. Furthermore, the level of agreement achieved is only marginally dependent on \(N_{\text{max}}^T\). In particular, the ground-state binding energies obtained in a (2)12 model space represent approximately 97\% of the complete-space \(N_{\text{max}}=12\) binding energy in the case of \(^{6}\text{Li}\) and reach over 98\% for \(^{6}\text{He}\) [Fig. 2 (a) and (b)]. The excitation energies differ only by 5 keV to a few hundred keV from the corresponding complete-space \(N_{\text{max}}=12\) results [see Fig. 2 (c) and (d)], and the agreement with known experimental data is reasonable over a broad range of \(\hbar\Omega\) values.

The number of basis states used, e.g., for each \(^{6}\text{Li}\) state, is only about 10-12\% for \((2)12, (4)12, (6)12, 14\% for \(8)12, \) and 30\% for \((10)12\) as compared to the num-
TABLE I: Magnetic dipole moments $\mu$, [\mu N] and point-particle rms matter radii $r_m$, [fm] of $T = 0$ states of $^6$Li calculated in the complete $N_{\text{max}} = 12$ space and the (6)12 subspace for JISP16 and $\hbar \Omega = 20$ MeV. The experimental value for the $1^+$ ground-state is known to be $\mu = +0.822 \mu_N$ [40].

| $\mu$ | $N_{\text{max}} = 12$ | $^1_2(6)12$ | $^3_2(6)12$ | $^2_4(6)12$ | $^1_2(6)12$ |
|-------|----------------------|---------------|---------------|---------------|---------------|
| $^1_2$ | 0.839 | 1.866 | 0.970 | 0.383 |  |
| $^3_2$ | 0.839 | 1.866 | 1.014 | 0.383 |  |
| $^2_4$ | 2.110 | 2.044 | 2.180 | 2.290 |  |

For the complete $N_{\text{max}} = 12$ model space, which is $3.95 \times 10^9$ ($J = 1$), $5.88 \times 10^6$ ($J = 2$), and $6.97 \times 10^6$ ($J = 3$). The runtime of the SA-NCSM code exhibits a quadratic dependence on the number of $(\lambda \mu)$ and $(S_p S_q S)$ irreps for a nucleus – there are $1.74 \times 10^9$ irreps for the complete $N_{\text{max}} = 12$ model space of $^6$Li, while only $8.2\%$, $8.3\%$, $8.9\%$, $12.7\%$, and $30.6\%$ of these are retained for $N_{\text{max}}^{\perp} = 2, 4, 6, 8, 10$, respectively. The net result is that calculations in the $10 \geq N_{\text{max}} \geq 2$ range require one to two orders of magnitude less time than SA-NCSM calculations for the complete $N_{\text{max}} = 12$ space.

As illustrated in Table I, the magnetic dipole moments obtained in the (6)12 model space for $^6$Li agree to within $0.3\%$ for odd-$J$ values, and $5\%$ for $\mu(2_1^+)$. Qualitatively similar agreement is achieved for $\mu(2_1^+)$ of $^6$He, as shown in Table II. The results suggest that it may suffice to include all low-lying $\hbar \Omega$ states up to a fixed limit, e.g. $N_{\text{max}}^{\perp} = 6$ for $^6$Li and $N_{\text{max}}^{\perp} = 8$ for $^6$He, to account for the most important correlations that contribute to the magnetic dipole moment.

To explore how close one comes to reproducing the important long-range correlations, we compared observables that are sensitive to the tails of the wavefunctions; specifically, the point-particle root-mean-square (rms) matter radii, the electric quadrupole moments and the reduced electromagnetic $B(E2)$ transition strengths that could hint at rotational features [11]. As Table II shows, the complete-space $N_{\text{max}} = 12$ results for these observables are remarkably well reproduced by the SA-NCSM for $^6$He in the restricted (8)12 space. In addition, the results for the rms matter radii of $^6$Li, listed in Table II, agree to within $1\%$ for the (6)12 model space.

Notably, the (2)12 eigensolutions for $^6$Li yield results for $B(E2)$ strengths and quadrupole moments that track closely with their complete $N_{\text{max}} = 12$ space counterparts (see Fig. 3). It is known that further expansion of the model space beyond $N_{\text{max}} = 12$ is needed to reach convergence [42, 43]. However, the close correlation between the $N_{\text{max}} = 12$ and (2)12 results is strongly suggestive that this convergence can be obtained through the leading SU(3) irreps in a symmetry-adapted space. In addition, the results [Fig. 3 (c)] reproduce the ground-state quadrupole moment $Q^{(1+)}$ that is measured to be $Q^{(1+)} = -0.0818(17) \text{e fm}^2$ [44].

The differences between truncated-space and complete-space results are found to be essentially $\hbar \Omega$ insensitive and appear sufficiently small as to be nearly inconsequential relative to the dependences on $\hbar \Omega$ and on $N_{\text{max}}$ [see Fig. 3 (b) and (d)]. Since the NN interaction dominates contributions from three-nucleon forces (3NFs) in light nuclei, except for selected cases [5-7], we expect our results to be robust and carry forward to planned applications that will include 3NFs.

To summarize, the results reported in this paper demonstrate that observed collective phenomena in light nuclei emerge naturally from first-principle considerations. This is illustrated through detailed calculations in a SA-NCSM framework for $^6$Li, $^6$He, and $^8$Be nuclei using the JISP16 and chiral N$^3$LO NN realistic interactions. The results underscore the strong dominance.
of configurations with large deformation and low spins. The results also suggest a path forward to include higher-lying correlations that are essential to collective features such as enhanced B(E2) transition strengths. The results further anticipate the significance of LS-coupling and SU(3) as well as an underlying symplectic symmetry for an extension of ab initio methods to the heavier, strongly deformed nuclei of the lower ds shell, and, perhaps, even reaching beyond.

We thank David Rowe and Andrey Shirokov for useful discussions. This work was supported in part by the US NSF [OCI-0904874, OCI-0904809, PHY-0904782], the US Department of Energy [DESC0008485, DE-FG02-95ER40934, DE-FG02-87ER40371], the National Energy Research Scientific Computing Center [supported by DOE’s Office of Science under Contract No. DE-AC02-05CH1123], the Southeastern Universities Research Association, and by the Research Corporation for Science Advancement under a Cottrell Scholar Award. This work also benefitted from computing resources provided by the Louisiana Optical Network Initiative and Louisiana State University’s Center for Computation & Technology. T. D. and D. L. acknowledge support from Michal Pajr and CQK Holding.

[1] P. Sternberg, E. G. Ng, C. Yang, P. Maris, J. P. Vary, M. Sosonkina and H. V. Le, In Proc. 2008 ACM/IEEE Conf. on Supercomputing, Austin, November 2008, IEEE Press, Piscataway, NJ, 15:1 (2008).
[2] P. Maris, M. Sosonkina, J. P. Vary, E. G. Ng and C. Yang, ICCS 2010, Procedia Computer Science 1, 97 (2010).
[3] H. M. Aktulga, C. Yang, E. N. Ng, P. Maris and J. P. Vary, Euro-par 2012, Lecture Notes on Computer Science 7849, 830 (2012).
[4] P. Navrátíl, J. P. Vary and B. R. Barrett, Phys. Rev. Lett. 84, 5728 (2000); Phys. Rev. C 62, 054311 (2000).
[5] B.R. Barrett, P. Navrátíl and J.P. Vary, Prog. Part. Nucl. Phys. 69, 131 (2013).
[6] P. Maris, J. P. Vary, P. Navrátíl, W. E. Ormand, H. Nam and D. J. Dean, Phys. Rev. Lett. 106, 202502 (2011).
[7] P. Maris, J. P. Vary and P. Navrátíl, Phys. Rev. C 87, 014327 (2013).
[8] R. B. Wiringa and S. C. Pieper, Phys. Rev. Lett. 89, 182501 (2002).
[9] G. Hagen, T. Papenbrock, D. J. Dean and M. Hjorth-Jensen, Phys. Rev. Lett. 101, 092502 (2008).
[10] T. Neff and H. Feldmeier, Nucl. Phys. A 738 (2004) 357.
[11] S. Quaglioni and P. Navrátíl, Phys. Rev. Lett. 101, 092501 (2008).
[12] S. K. Bogner, R. J. Furnstahl, P. Maris, R. J. Perry, A. Schwenk and J. P. Vary, Nucl. Phys. A 801, 21 (2008).
[13] R. Roth, J. Langhammer, A. Calci, S. Binder and P. Navrátíl, Phys. Rev. Lett. 107, 072501 (2011).
[14] E. Epelbaum, H. Krebs, D. Lee and U.-G. Meissner, Phys. Rev. Lett. 106, 192501 (2011); E. Epelbaum, H. Krebs, T. A. Lähde, D. Lee, U.-G. Meissner, Phys. Rev. Lett. 109, 252501 (2012).
[15] P. Maris, A. M. Shirokov and J. P. Vary, Phys. Rev. C 81, 021301(R) (2010).
[16] V.Z. Goldberg et al., Phys. Lett. B 692, 307 (2010).
[17] B. Davids, R. H. Cyburt, J. Jose and S. Mythili, Astrophys. J. 735, 40 (2011).
[18] A. M. Laird et al., Phys. Rev. Lett. 110, 032502 (2013).
[19] A. M. Shirokov, J. P. Vary, A. I. Mazur and T. A. Weber, Phys. Letts. B 644, 33 (2007).
[20] D. R. Entem and R. Machleidt, Phys. Rev. C 68, 041001 (2003).
[21] J. P. Elliott, Proc. Roy. Soc. A 245, 128 (1958).
[22] J. P. Draayer, T. Dytrych, K. D. Launey and D. Langr, Prog. Part. Nucl. Phys. 67, 516 (2012).
[23] O. Castaños, J. P. Draayer and Y. Leschber, Z. Phys. 329, 33 (1988).
[24] G. Rosensteel and D. J. Rowe, Ann. Phys. N.Y. 104, 134 (1977).
[25] Y. Leschber and J. P. Draayer, Phys. Letts. B 190, 1 (1987).
[26] J. P. Draayer, Nucl. Phys. A216, 457 (1973).
[27] N. Anantaraman et al., Phys. Rev. Lett. 35, 1131 (1974).
[28] J. P. Draayer, Nucl. Phys. A237, 157 (1975).
[29] J. P. Draayer, K. J. Weeks and G. Rosensteel, Nucl. Phys. A413, 215 (1984).
[30] G. Rosensteel, J. P. Draayer and K. J. Weeks, Nucl. Phys. A419, 1 (1984).
[31] J. P. Draayer and K. J. Weeks, Phys. Rev. Lett. 51, 1422 (1983).
[32] G. Rosensteel and D. J. Rowe, Phys. Rev. Lett. 38, 10 (1977).
[33] G. Rosensteel and D. J. Rowe, Ann. Phys. N.Y. 126, 343 (1980).
[34] D. J. Rowe, Rep. Prog. Phys. 48, 1419 (1985).
[35] D. R. Peterson and K.T. Hecht, Nucl. Phys. 344, 361 (1980).
[36] T. Dytrych, K. D. Sviratecheva, C. Bahri, J. P. Draayer and J. P. Vary, Phys. Rev. Lett. 98, 162503 (2007).
[37] B. J. Verhaar, Nucl. Phys. A 21, 508 (1960).
[38] D. H. Gloeckner and R. D. Lawson, Phys. Letts. B 53, 313 (1974).
[39] The near vanishing of the quadrupole moment, an L=2 object, in the ground-state of 6Li, Q(17), can be attributed to a very strong dominance (∼87%) of L=0 configurations in the ground state.
[40] D. R. Tilley et al., Nucl. Phys. A 708, 3 (2002).
[41] M. A. Caprio, P. Maris and J. P. Vary, Phys. Letts. B 719, 179 (2013).
[42] C. Cockrell, J. P. Vary and P. Maris, Phys. Rev. C 86, 034325 (2012).
[43] P. Maris and J. P. Vary, Int. J. Mod. Phys. E 22, 1330016 (2013).