Qubit-efficient exponential suppression of errors

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Achieving a practical advantage with near-term quantum computers hinges on having effective methods to suppress errors. Recent breakthroughs have introduced methods capable of exponentially suppressing errors by preparing multiple noisy copies of a state and virtually distilling a more purified version. Here we present an alternative method, the Resource-Efficient Quantum Error Suppression Technique (REQUEST), that adapts this breakthrough to much fewer qubits by making use of active qubit resets, a feature now available on commercial platforms. Our approach exploits a space/time trade-off to achieve a similar error reduction using only \(2N+1\) qubits as opposed to \(MN+1\) qubits, for \(M\) copies of an \(N\) qubit state. Additionally, we propose a method based on near-Clifford circuits to find the optimal number of these copies in the presence of realistic noise, which limits this error suppression. We perform a numerical comparison between the original method and our qubit-efficient version with a realistic trapped-ion noise model. We find that REQUEST can reproduce the exponential suppression of errors of the virtual distillation approach for simple noise models while out-performing virtual distillation when fewer than \(3N+1\) qubits are available.

I. INTRODUCTION

One of the most serious challenges in demonstrating a practical advantage for quantum computing over classical computing in the near term is the hardware noise [1, 2]. While many expect that fault-tolerant quantum computing will eventually become available, this possibility remains distant. Instead, we are approaching the arrival of noisy, intermediate scale quantum (NISQ) devices that employ hundreds or more noisy qubits [3].

A number of error mitigation methods have been proposed for this NISQ era. Perhaps the most prominent method is zero noise extrapolation, whereby the noise is increased in a controlled manner, allowing one to perform a function fit and then extrapolate to the noiseless limit [4–10]. Alternatively, if one knows the noise model of the device being used, gates can be introduced probabilistically in order to cancel the effect of the noise on average [4]. A third approach is to leverage classically simulable quantum circuits to infer properties of the hardware noise by comparing classically simulated outputs with noisy results. With this approach, one can use regression to estimate the noiseless result of interest directly [11], inform the functional form for a zero noise extrapolation [12], or estimate the gate distributions for probabilistic error correction [13]. Circuit optimization by optimal compiling provides another error mitigation method by producing noise-resilient quantum circuits [14–19]. Finally, a number of application specific approaches have been proposed, leveraging symmetries and/or post-selection techniques to mitigate errors [20–23].

In a recent breakthrough, a new approach was proposed that uses additional copies of a quantum state of interest to suppress errors [24, 25]. These methods make use of the fact that, for a given density matrix \(\rho\), the state...
$\rho^M/\text{Tr}[\rho^M]$ approaches a pure state exponentially quickly with increasing $M$. Measurements on this state can be made by preparing $M$ copies of $\rho$, and this protocol has been termed the exponential suppression of quantum errors when $\rho$ represents a noisy state that results from attempting to prepare a given pure state [24].

Nevertheless, the ability of this approach to exponentially suppress errors is subject to limitations. First, in general the pure state this method approaches may not be exactly the intended pure state, which provides a noise floor for the method. Additionally, the protocol includes the action of noisy controlled swaps on the copies. Therefore when one accounts for the increased number of the controlled swaps associated with larger $M$, there is a point where adding copies may increase the impact of noise more than it suppresses it [25].

A different limitation of this error suppression strategy comes from the number of qubits required. If the state $\rho$ requires $N$ qubits to prepare, then $MN + 1$ qubits are required in total. For NISQ devices capable of achieving a quantum advantage, the required number of qubits may prove more limiting to the number of copies that can be used than the noise of the controlled swaps.

We present an alternative approach below that uses the same framework to achieve an exponential error suppression (in the same sense) with only $2N + 1$ qubits. To that end we use active qubit resets enabling reusing qubits during a quantum algorithm execution by re-initializing them in a known state [26–29]. In that way, inspired by qubit-efficient algorithms for Renyi entropy computation [30], we replace a circuit implementing the exponential suppression by an equivalent one with increased depth proportional to $M$ and fixed width. We call this new method the resource-efficient quantum error suppression technique (REQUEST). We schematically compare REQUEST with the original exponential suppression methods in Fig. 1. As the qubit resets are enabled by major quantum computing architectures, including superconducting qubit [31] and trapped-ion devices [32], we expect the method to have a wide range of applications.

Below we first briefly review the error suppression formalism presented in Refs. [24, 25]. Next, we introduce our modification, REQUEST, and present a method to estimate the optimal number of copies $M_{\text{opt}}$ using near-Clifford circuits. We then present a numerical comparison of the performance of our method and the original. Finally, we present our conclusions and discuss future directions.

II. EXPONENTIAL SUPPRESSION OF ERRORS

Recently, methods for the suppression of quantum errors by virtual distillation (VD) using $M$ copies of a state have been proposed [24, 25]. These methods are based on the assumption that the desired pure state at the end of a unitary evolution will be close to the eigenvector of the density matrix with the largest eigenvalue. To make this precise, suppose that we have an initial state $|\phi\rangle$ on $N$ qubits that we wish to act on with a unitary $U$, but the device we are working with is noisy. Denoting by $\mathcal{E}_U$ the quantum channel that results from attempting to apply $U$, we end up preparing the state

$$\mathcal{E}_U(|\phi\rangle\langle\phi|) = \rho = \sum_{i=1}^{D} p_i |\psi_i\rangle\langle\psi_i|.$$  

(1)

Here the eigenvalues $p_i$ are ordered in descending order (for convenience) and $D = 2^N$ is the dimension of the Hilbert space. VD then works from the assumption that $U|\phi\rangle \approx |\psi_1\rangle$.  

(2)

Note that if all errors introduced are orthogonal to the state of interest, the approximation in Eq. (2) becomes exact [25].

With the approximation in Eq. (2) in mind, VD considers the quantity [24, 25]

$$\frac{\text{Tr}[X\rho^M]}{\text{Tr}[\rho^M]} = \frac{\langle \psi_1 | X | \psi_1 \rangle}{1 + \sum_{i=2}^{D} (p_i/p_1)^M \langle \psi_i | X | \psi_i \rangle} = \langle X \rangle_{\text{mitigated}}.$$  

(3)

So long as $p_1$ is larger than any other eigenvalue, this ratio approaches $\langle \psi_1 | X | \psi_1 \rangle$ exponentially with $M$ and so is used to calculate the error suppressed expectation value of $X$, which we denote $\langle X \rangle_{\text{mitigated}}$.

VD computes the numerator and denominator of Eq. (3) by preparing $M$ copies of $\rho$ and one ancilla qubit, using a circuit like the one shown in Fig. 2(a). The main idea is to apply a controlled derangement operation commonly used in computing Renyi entropies [33, 34] to the copies (the derangement is a permutation of copies which changes position of each copy). The derangement is implemented with the controlled swap gates. To find $\langle X \rangle_{\text{mitigated}}$ we apply a controlled $\sigma$ gate to the permuted copies (see Fig. 2(a)) and measure the ancilla qubit. Denoting the probability of getting 0 as the result of the measurement with $\sigma = X$ as $p_0$ and with $\sigma = I$ (the identity) as $p_0'$, we then have:

$$\langle X \rangle_{\text{mitigated}} = \frac{\text{Tr}[X \rho^M]}{\text{Tr}[\rho^M]} = \frac{2p_0 - 1}{2p_0' - 1}.$$  

(4)

Even on a quantum device with many qubits there are practical limitations to the error suppression offered by VD. First, many physical error channels will produce errors which are not orthogonal to $U|\phi\rangle$, worsening the approximation in Eq. (2). This effect introduces a floor below which the error cannot be suppressed [25]:

$$\epsilon = |\langle \psi_1 | X | \psi_1 \rangle - \langle \phi | U^\dagger X U | \phi \rangle|.$$  

(5)
FIG. 2. Circuit diagrams for the exponential suppression of errors with \( M \) copies. Here each \( p_i \) denotes the circuit to prepare the \( i \)th copy of the state \( \rho \). Also, \( \sigma = I \) or \( \sigma = X \), where \( X \) is the observable whose expectation value will be mitigated. Diagram (a) shows a circuit diagram like the one proposed by [24] to suppress errors. Diagram (b) shows our alternative formulation using active qubit resets, which are represented by a break in a wire followed by \(|0\rangle\).

Additionally, since the application of the controlled derangement is subject to error channels, in practical applications there will usually be a finite optimal number of copies \( M_{\text{opt}} \) that can be used before the additional error introduced outweighs the suppression. (We note that \( M_{\text{opt}} \) may not be finite for highly idealized noise models such as global depolarizing noise, but it will be for realistic noise models based on current hardware.) As determining \( M_{\text{opt}} \) would require detailed knowledge of the noise channels, this value can be difficult to predict in practice. Finally, we note that VD is robust to the copies of \( \rho \) being imperfect (perhaps due to differences in their noise channels) if they still have the same \( |\psi_1\rangle \) corresponding to the largest eigenvalue [24].

III. QUBIT-EFFICIENT ERROR SUPPRESSION

In addition to the accumulation of hardware errors, the number of qubits available (as \( N_{\text{tot}} = MN + 1 \) qubits are required) and the difficulty of entangling them limits the mitigation attainable with VD. Particularly for NISQ devices, the severely limited number of qubits and connectivity may well be more restrictive. Inspired by qubit-efficient methods for computing Renyi entropies [30], we therefore propose a variant VD we call the Resource-efficient Quantum Error Suppression Technique (REQUEST). REQUEST utilizes active qubit resets to reduce \( N_{\text{tot}} \) to \( 2N + 1 \), independent of \( M \).

The prototypical circuit diagram for REQUEST is schematically depicted in Fig. 2(b). We note that the circuit diagrams in Fig. 2 are mathematically equivalent, though the noise channels that result from implementing them will differ. REQUEST therefore reduces \( N_{\text{tot}} \) at the cost of increased circuit depth. This trade-off means that the idling time for the control qubit and one copy of \( \rho \) is greatly increased in REQUEST as compared with VD. However, on devices with limited connectivity, the cost of performing the derangement operation on \( M \) copies may offset this difficulty. In such case, REQUEST may prove a more efficient alternative, even if many qubits are available.

A. Estimating the optimal number of copies

As REQUEST lifts the requirement for more and more qubits, it is especially important to determine the correct number of copies to use. We propose to estimate \( M_{\text{opt}} \) by finding the optimal number of copies for similar but classically simulable systems. To accomplish this we construct a set of near-Clifford circuits that are similar to the state preparation circuit. (See Appendix A for details on how we define similar circuits.)

We will call the states prepared by these near-Clifford circuits \( \{|\Phi_i\rangle\} \). If the observable of interest \( X \) can be efficiently decomposed into a sum of Clifford operators, we are then able to efficiently classically compute (without noise) the exact expectation values of \( X \) for these states. If we attempt to prepare the states \( \{|\Phi_i\rangle\} \) on noisy hardware, we will end up instead preparing corresponding density matrices \( \rho_i \). We expect and further back our claim up with numerical evidence in Section IV that the optimal number of copies for these near-Clifford circuits should be similar to the optimal number of copies for the circuit for which we would like to mitigate errors. Therefore, we approximate:

\[
M_{\text{opt}} \approx \min_M \left\{ \sum_i \left| \frac{\text{Tr}[X \rho_i^M]}{\text{Tr}[\rho_i^M]} - \langle \Phi_i | X | \Phi_i \rangle_{\text{exact}} \right| \right\}.
\] (6)

Note that the traces \( \text{Tr}[X \rho_i^M] \) and \( \text{Tr}[\rho_i^M] \) are computed...
with the noisy quantum device while the expectation values $\langle \Phi_i | X | \Phi_i \rangle_{\text{exact}}$ are computed classically.

IV. NUMERICAL IMPLEMENTATION

We investigate the performance of both VD and REQUEST with a realistic noise model of trapped ion quantum computers [15, 35]. This architecture is favorable for REQUEST’s applications as the qubits have long decoherence times which limit the effects of idling noise during the derangement application. Furthermore, such devices typically enable all-to-all qubit connectivity reducing circuit depth of the controlled derangements and observables. We assume all-to-all qubit connectivity in our implementation.

Our goal here is to introduce REQUEST method and gain understanding of the effects of a realistic noise model on the exponential error suppression methods. Therefore, for simplicity we choose to not combine the methods with other error mitigation methods. It seems probable that such a combination will further enhance the power of both VD and REQUEST.

To test the performance of the method we use random quantum circuits (RQC) obtained with a trapped-ion hardware efficient ansatz. The ansatz is built from layers of nearest-neighbor two-qubit $XX(\delta)$ gates that are parametrized by random angles $\delta$ and decorated with general random single-qubit unitaries. See Fig. 3 for details. We test the method for a range of system sizes $N$ and numbers of ansatz layers $p$.

First, to clearly demonstrate the exponential suppression of errors, we consider a special case when the noise acts only during the preparation of the various copies.

![FIG. 4. Exponential suppression of errors. Mitigating $\langle \sigma_Z^2 \rangle$ for RQC. Here, to clearly demonstrate the exponential suppression, we consider the noise acting only during the state preparation for the various copies. Furthermore, we plot the error with respect to $\langle \sigma_Z^2 \rangle$ for $|\psi_1\rangle$. The error is averaged over 44 instances of RQC and plotted versus $M$. $M = 1$ corresponds to noisy, single-copy results.](image)

![FIG. 5. Error mitigation for random quantum circuits. Error mitigation of $\langle \sigma_Z^2 \rangle$ for the random quantum circuits with different $p$, $N$ and a realistic case when all gates are noisy. The error is averaged over 44 instances of the random circuits. In (a) and (c) we show the results obtained with the REQUEST method. The VD method is presented in panels (b) and (d). Note that in our implementation both methods are equivalent for $M = 2$. REQUEST allows us to obtain improvement over $M = 2$ without extra qubits, providing the method’s proof of principle. Furthermore, for small enough $p$ the best results obtained with REQUEST and the original method have similar quality.](image)

Furthermore, we consider an error with respect to the leading eigenvector of the noisy state $|\psi_1\rangle$. We mitigate $\langle \sigma_Z^2 \rangle$ (the Pauli Z operator on qubit 1) for RQC with $N = 4$, $p = 5, 15, 25, 35$ averaging the error over 44 instances of RQC. Indeed for such a setup we find exponential suppression of errors, see Fig. 4. This is similar to the suppression observed in Ref. [24].

Next we consider REQUEST mitigation for a realistic case when all gates are noisy. We mitigate $\langle \sigma_Z^2 \rangle$ for RQC with $N = 4$ and $p = 5, 15, 25, 35$, and with $N = 2, 3, 4, 5$ and $p = 25$. We consider the mean absolute error of $\langle \sigma_Z^2 \rangle$ (computed with respect to the exact expectation value $\langle \phi | \sigma_Z^2 | \phi \rangle$) obtained by averaging over 44 instances of RQC plotted versus $M = 1 - 8$. We gather the results in Fig. 5(a,c). In most cases we find a clearly visible $M_{\text{opt}} = 3, 4$ and monotonic increase of the error for $M > M_{\text{opt}}$. As $M_{\text{opt}} > 2$, the results demonstrate advantage of the REQUEST method in the case of limited qubit counts.

For benchmark purposes, we compare the results with the ones obtained by the VD method, shown in Fig. 5(b,d). It is expected that VD will perform better for large $M$ as the REQUEST circuit is much deeper for large $M$ for a device with full connectivity, such as the ion...
FIG. 6. Determining \( M_{\text{opt}} \) with near-Clifford circuits. A Comparison of the mean \( \langle \sigma_z^2 \rangle \) absolute error for REQUEST mitigation of RQC and near-Clifford circuits. RQC circuits are the ones for which results from Fig. 5 were obtained while the near-Clifford ones are obtained by projecting these RQC as described in Section IIIA and Appendix A. Here the error is averaged over 44 instances of the random circuits and 440 instances of near-Clifford circuits. The behavior is similar enough in both cases to use the near-Clifford circuits in order to find a good approximation of \( M_{\text{opt}} \) for the random circuits. We show here the behavior for \( N \) and \( p \) for which using the approximate \( M_{\text{opt}} \) instead of the exact one results in the largest increase of the error (which is smaller than 1.5\%).

FIG. 7. Summary of results obtained for RQC mitigation. Scaling of the mean and maximal values of \( \langle \sigma_z^2 \rangle \) error for REQUEST RQC mitigation from Fig. 5 and selected \( M \) values. In (a) results for \( N = 4 \), and in (b) results for \( p = 25 \). \( M_{\text{opt}} \) was estimated here with the near-Clifford circuits. The results demonstrate systematic improvement obtained with the REQUEST method with respect to VD \( M = 2 \) exponential mitigation and unmitigated results.

V. CONCLUSION

The next major milestone for quantum computing is demonstrating an advantage over classical computing for some task that is of practical use. Achieving such a quantum advantage with NISQ devices will likely require algorithms that are qubit efficient. Additionally, as NISQ devices cannot support full error correction, quantum advantage will also require robust error mitigation strategies.

Building upon recent proposals to use multiple copies of a noisy state \( \rho \) to distill the pure state of interest \([24, 25]\), so-called Virtual Distillation (VD), we have presented a variation that achieves qubit efficiency by resetting and reusing qubits. Specifically, for \( N \) qubit states, the total qubit requirement of our method, REQUEST, is only \( 2N + 1 \) for any number of copies while the previous approach required \( MN + 1 \) qubits to use \( M \) copies.

As the number of copies used by REQUEST is not limited by the size of the physical device, we also address how to estimate the optimal number of copies, \( M_{\text{opt}} \). We propose to find \( M_{\text{opt}} \) by using (classically simulable) near-Clifford circuits. Choosing these near-Clifford circuits to be similar to the circuit that prepares \( \rho \), one can compare results mitigated with different values of \( M \) to exact quantities.

While REQUEST achieves this reduction in qubit resources by increasing the overall depth of the quantum circuit, this trade-off can still be worthwhile. Using a re-
alis tic trapped-ion noise model and random quantum cir-

cuits, we compare REQUEST and VD. For this test case, 

we find that the REQUEST method with $M_{\text{opt}}$ copies 

provides a clear advantage over VD with $M = 2$ copies.

We note that when enough qubits are available on 

a fully connected device the qubit-hungry VD method 

tends to outperform REQUEST on $2N + 1$ qubits when 

considering the same number of copies. However, while 

we have focused on the likely near-term case where few 

qubits are available, the active reset approach of RE-

QUEST can be generalized further. If sufficient qubits 

and connectivity are available, REQUEST’s active resets 

can be applied to more than one subsystem. Doing so 

would potentially increase the optimal number of copies 

$M_{\text{opt}}$ and with it the error suppression. REQUEST will 

therefore be relevant even when larger devices with good 

connectivity are available.

Beyond the NISQ regime, Ref. [25] considers whether 

this kind of error suppression with surface codes might 

be useful. They note that if one needs to get the most 

physical qubits possible out of a device, low code dis-


tances would have to be considered. In such a situation, 

they argue that the exponential suppression technique 

with $M = 2$ may offer a better improvement than using 

the extra qubits to increase the code distance. (This im-


provement vanishes for sufficiently large code distances.) 

As REQUEST can provide better results without requir-

ing additional physical qubits, it would prove even more 

useful in that regime. We therefore expect that, even 

once fault tolerance is achieved, REQUEST will be use-

ful for problems that are qubit limited.

To further establish the method it will be important 

to benchmark its hardware implementation. Finally, the 

best way to combine these error suppression techniques 

with the established error mitigation strategies (such as 

zero noise extrapolation [4–10]. Clifford data regres-

[11, 12], etc.) remains a question for future research.

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FIG. 8. Classical simulation of VD mitigation. In (a) a schematic representation of qubit-efficient classical simulation of VD. The circuit shown in (a) is simulated classically with $2N + 1$ qubits while switching off an idling noise channel [15] at the ancillary qubit and the first register of qubits (corresponding to $\rho_1$) during preparation of $\rho_2, \ldots, \rho_M$. The switched off idling gates correspond to the blue areas in (a). The computation shown in (a) is equivalent to a noisy circuit (b) implementing VD under assumptions (i) that there is no cross-talk noise associated with preparation of $\rho_2, \ldots, \rho_M$ and (ii) that depth of circuits preparing $\rho_2, \ldots, \rho_M$ is the same as depth of circuits implementing CTRL-SWAPs. Assumption (ii) is necessary to ensure equivalent distribution of idling gates [15] in both cases. Assumption (i) is true for our noise model [15]. Assumption (ii) is not true in general. In the case when (ii) is not true the equivalent circuit is similar to (b) although it can not be correctly visualized without considering structure of native gates in the circuits implementing CTRL-SWAPs and state preparation. Finally, note that we apply the controlled-$\sigma$ operation to $\rho_M$. While in the idealized case of noise acting only during state preparation and $\rho_1 = \rho_2 = \cdots = \rho_M$ all choices of the measured copy are equivalent that’s no longer true for realistic noise. We find that for our implementation choosing $\rho_M$ minimizes the error.

with the weights

$$w_{jk} = e^{-d^2/\sigma^2}, \quad d = \frac{||e^{i\delta_j XX(\delta_j)} - e^{ik\pi/4 XX(k\pi/4)}||}{||XX(\delta_j)||}.$$  

We choose $\sigma = 0.5$.

4. We replace each $R_Y(\beta_j)(XX(\delta_j))$ by Clifford $R_Y(k\pi/2)(XX(k\pi/4))$ with probability

$$p_{jk} = \frac{w_{jk}}{\sum_k w_{jk}}.$$  

5. We replace one of $R_Z(\alpha_j)$ by $R_Z(k\pi/2)$ with probability

$$p_{jk} = \frac{w_{jk}}{\sum_{jk} w_{jk}}.$$  

6. If after the replacement the number of non-Clifford $R_Z(\alpha_j)$ is larger than $N_{nC}$, we repeat step 5 until only $N_{nC}$ non-Clifford $R_Z(\alpha_j)$ remains.

To determine $M_{opt}$ we generate $4 - 10$ near-Clifford circuits per each random quantum circuit.

Appendix B: Implementation details

1. Quantum gates decomposition to native trapped-ion gates.

To implement CTRL-SWAP gate we decompose it to two CNOTs and a Toffoli gate as described in [36]. We decompose Toffoli gates, CNOTs and CTRL-Z gates to the native gates using a decomposition from [37] and assuming full connectivity of the device.

2. Classical simulation of the VD method

Noisy classical simulation of VD is challenging for large $M$ as the number of the required qubits grows linearly with $M$. Nevertheless in absence of cross-talk noise (as in the case of our noise model) an equivalent classical simulation can be performed with $2N + 1$ qubits, see Fig. 8. We use a method from Fig. 8 to simulate VD.