Abstract. The Ryder relation between left- and right-spinors has been generalized in my previous works. On this basis Ahluwalia presented a physical content following from this generalization. It is related to non-locality. A similar conclusion can be drawn on the basis of a generalization of Sakurai-Gersten consideration. I correct several calculating and conceptual misunderstandings of the previous works.

*Some parts of this work have also been presented at the seminar at the Brigham Young University, Provo, USA. August 29, 2000.
On the basis of the consideration [1, 3] of the Lorentz transformations of the form:

\[ x' = \frac{x + vt}{\sqrt{1 - v^2/c^2}}, \quad y' = y, \quad z' = z, \quad (1a) \]
\[ t' = \frac{t + vx/c^2}{\sqrt{1 - v^2/c^2}} \quad (1b) \]

and rotations in three-dimensional space

\[ r' = Rr, \quad R^T R = I \quad (2) \]

one can obtain the 4 × 4 matrix representation of the boost and rotation generators. They form the Lorentz algebra:

\[
[K_x, K_y] = -iJ_z \quad \text{and cyclic permutations}, \quad (3a)
\]
\[
[J_x, K_x] = 0 \quad \text{etc.,} \quad (3b)
\]
\[
[J_x, K_y] = iK_z \quad \text{and cyclic permutations}, \quad (3c)
\]
\[
[J_x, J_y] = iJ_z \quad \text{and cyclic permutations}. \quad (3d)
\]

It was shown that the Lorentz transformations are connected with the squeeze transformations [4]. In the (1/2, 0) ⊕ (0, 1/2) representation it was explicitly shown that the Lorentz group is essentially SU(2) ⊗ SU(2), ref [3, p.40].

The Relativity Theory conserves the interval, \( ds^2 = dx^\mu dx_\mu, \mu = 0, 1, 2, 3 \). As a consequence, the second-order differential equation follows:

\[
\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \vec{\nabla}^2\right)\phi + \frac{m^2c^2}{\hbar^2} \phi = 0 \quad (4)
\]

for a field without spin splitting. The Dirac equation hence appears as a relation between 2-spinors of the spinor representations \((1/2, 0)\) and \((0, 1/2)\) of the algebra [5–8]. Of course, its solutions satisfy the momentum-space realization of (4). Under parity the representations interchange \((j, 0) \leftrightarrow (0, j)\), because matrices of the Lorentz transformations (rotations and boosts) for

\[1\] The helicity eigenspinors can be parametrized as follows [5, p.180], [3]:
dotted and undotted spinors are related by the Wigner operator (see the equation (2.75) in [3])

$$\Lambda_L = \zeta \Lambda_R^* \zeta^{-1}, \quad \text{with } \zeta = -i\sigma_2. \quad (6)$$

Ryder writes: “Now when a particle is at rest, one cannot define its spin as either left- or right-handed, so $\phi_R(\vec{0}) = \phi_L(\vec{0})$. It then follows ... that”

$$\phi_R(\vec{p}) = \Lambda_R(\vec{p} \leftarrow \vec{0}) \phi_R(\vec{0}) = \Lambda_R(\vec{p} \leftarrow \vec{0}) \phi_L(\vec{0}) = \Lambda_R(\vec{p} \leftarrow \vec{0}) \Lambda^{-1}_L(\vec{p} \leftarrow \vec{0}) \phi_L(\vec{p}), \quad (7a)$$

$$\phi_L(\vec{p}) = \Lambda_L(\vec{p} \leftarrow \vec{0}) \phi_L(\vec{0}) = \Lambda_L(\vec{p} \leftarrow \vec{0}) \phi_R(\vec{0}) = \Lambda_L(\vec{p} \leftarrow \vec{0}) \Lambda^{-1}_R(\vec{p} \leftarrow \vec{0}) \phi_R(\vec{p}), \quad (7b)$$

where boosts were only used. In the $4 \times 4$ matrix form (after the corresponding substitutions of quantum-mechanical operators $E \to i\hbar \frac{\partial}{\partial t}$, $\vec{p} \to -i\hbar \vec{\nabla}$ and $c = \hbar = 1$) the equations (7a,7b) become to be written

$$[i\gamma^\mu \partial_\mu - m] \psi(x) = 0. \quad (8)$$

This derivation of the Dirac equation has been analyzed in [7]. Different ways of derivations of the Dirac equation (its generalizations and higher spin equations) have been presented in [8] with corresponding citations.

However, the declaration of impossibility to distinguish spin of a particle as either left- or right-handed assumes that it is also possible to set

$$\phi_R(\vec{0}) = e^{i\alpha} \phi_L(\vec{0}), \quad (9)$$

$$\xi_\uparrow = Ne^{i\vartheta_1} \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)e^{i\phi} \end{pmatrix}, \quad (5a)$$

$$\xi_\downarrow = Ne^{i\vartheta_2} \begin{pmatrix} \sin(\theta/2) \\ -\cos(\theta/2)e^{i\phi} \end{pmatrix}, \quad (5b)$$

with $\theta$ and $\phi$, the angles of $\vec{p}$ in the spherical coordinate system; $N$ and $\vartheta_{1,2}$ are arbitrary parameters.
with arbitrary parameter $\alpha$. This has been studied in [11, 12], see also [13, 14] and the commented paper [15]. Furthermore, the relation of the Ryder book can be generalized in different ways, see the generalized formulas (8) and (10), e.g., in ref. [11a] and the formulae (5) in [12]:

$$
\phi^h_L(\vec{p}^{\mu}) = a(-1)^{\frac{1}{2}} e^{i(\vartheta_1 + \vartheta_2)} \Theta_{[1/2]}[\phi^{-h}_L(\vec{p}^{\mu})]^* + b e^{2i\vartheta_h} \Xi_{[1/2]}^{1}[\phi^h_L(\vec{p}^{\mu})]^* 
$$

(10)

(the notation is explained in the cited papers). Our intention of modification of the Dirac formalism originates from the classical works of Markov, Gelfand and Tsetlin, and Sokolik [16]. Recently, by using the coordinate-dependent phase Ahluwalia derived the ‘CP-violating Dirac equation’, ref. [17], and suggested to interpret the resulting non-locality as that which “manifests in the spinorial space (i.e., the spinorial indices) and not in the configurational space (i.e., the $\vec{x}$ space)” [15]. Unfortunately, I have to correct several Ahluwalia’s claims. I show that this interpretation is incorrect: the resulting non-locality manifests itself in the coordinate space in the sense that the fermionic anticommutator does not vanish outside the light cone $(x - x')^2 < 0$ (cf. the discussion in ref. [18, p.150]). This Ahluwalia’s misconception originates from a calculating error in the derivation of the formulas (23,24) of ref. [15]. I also give several remarks on the Ahluwalia

2I am grateful to an anonymous referee of Physical Review D on the paper [17d]

“Additional Equations Derived from the Ryder Postulates in the $(1/2, 0) \oplus (0, 1/2)$ Representation of the Lorentz Group.’ for independent confirmation of this obvious fact. I acknowledge discussions of the first version of the AFDB paper with Dr. Ahluwalia in the beginning of 1998 during his second visit in Zacatecas, México.

3It is not very convenient for a reader to have different physical quantities denoted by the same symbol (as in [15, Eq. (4)] for 2-spinors and phases). But, in order not to mislead those who have read the work [15] and is reading this
misunderstandings, provide several insights into the problem of the “kinematical” CP violation and present relations with research of other authors.

First of all, let me check the correctness of the derivation of the formula (23). It is very strange from the first sight that in the formula (23) the left-hand side depends only on $x^\mu$ and $x'^\mu$, but the right-hand side depends on the $k^0 = \sqrt{\vec{k}^2 + m^2}$, see the formula (24) for $O_{ij}$. Even if one assumes that one can neglect the difference of coordinate-dependent phases in $u$ (and $v$) 4-spinors in points $\vec{x}$ and $\vec{x}'$ for which the anticommutator (23) has been calculated (see the footnote $d$ in [15]), we note that Ahluwalia put the term $\frac{1}{k_0}$ outside the integration on $d^3k$. We give here corrected calculations in detail. The equal-time anticommutator is:

$$\{\Psi_i(\vec{x}, t), \Psi_j(\vec{x}', t)\}_+ = \sum_{\sigma \sigma'} \int \int \frac{d^3\vec{k} \, d^3\vec{k}'}{(2\pi)^6} \frac{m^2}{k_0 k_0'} \left[ u^\sigma_i(\vec{k}) \bar{\pi}^\sigma'_{\vec{k}'} (\vec{k}') \gamma^0_{kj} \left\{ b_\sigma, b^\dagger_{\sigma'} \right\} e^{-i\vec{k} \cdot \vec{x} + i\vec{k}' \cdot \vec{x}'} +$$

$$v^\sigma_i(\vec{k}) \bar{\pi}^\sigma'_{\vec{k}'} (\vec{k}') \gamma^0_{kj} \left\{ d_\sigma, d^\dagger_{\sigma'} \right\} e^{i\vec{k} \cdot \vec{x} - i\vec{k}' \cdot \vec{x}'} \right]$$

$$= \frac{1}{\cos(\phi)} \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{m}{k_0} \left[ (\vec{k} + \zeta - 1 m) \gamma^0_{kj} (\vec{x} - \vec{x}') + (\vec{k} - \zeta - 1 m) \gamma^0_{kj} e^{-i\vec{k} \cdot (\vec{x} - \vec{x}')} \right] =$$

$$= \frac{1}{\cos(\phi)} \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{m}{k_0} \left[ 2k_0 \delta_{ij} + m(\zeta - 1 + \zeta - 1) \gamma^0_{kj} \right] e^{i\vec{k} \cdot (\vec{x} - \vec{x}')} =$$

$$= \frac{2m}{\cos(\phi)} \left[ \delta_{ij} \delta(\vec{x} - \vec{x}') + im \sin(\phi)(\gamma^5 \gamma^0)_{ij} D^1(0, \vec{x} - \vec{x}') \right], \quad (11)$$

comment, we shall follow the notation of the commented paper.

4One should remember that the transformation $d^4k \delta(k^2 - m^2) \sim \frac{d^3\vec{k}}{2k_0}$ is used when transferred to the 3-dimensional momentum-space integrals in the field operators. Furthermore, one can also show that the connection between $k_0$ and $\vec{k}$ is used when deriving the Dirac-(like) equations by the Ryder method, see [15, Eq.(10)].

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where \( D^1(x^\mu - x'^\mu) \) is the even solution of the homogeneous Klein-Gordon equation. Its explicit form was given in ref. \[19, \text{formula (A2B.6)}\] (see also p. 150 of \[18\]):

\[
D^1(x) = i(D^+(x) - D^-(x)) = \frac{m}{4\pi \sqrt{\lambda}} \theta(\lambda) N_1(m \sqrt{\lambda}) + \frac{m}{2\pi^2 \sqrt{-\lambda}} \theta(-\lambda) K_1(m \sqrt{\lambda}) \approx \frac{1}{2\pi^2 \lambda} + \frac{m^2}{4\pi^2} \ln \frac{m \sqrt{\lambda}}{2},
\]

with \( \lambda = (x^0)^2 - \vec{x}^2 \), and \( N_1, K_1 \) are the first-order cylinder functions (the Neumann function and the MacDonald-Hankel function, respectively). As it is readily seen, the formula (11) does not coincide with the formula (23) of \[15\].

Let me also present the result of calculation of the anticommutator of two free fields at arbitrary separations, the analogue of the Pauli-Jordan function for this kind of \( (1/2, 0) \oplus (0, 1/2) \) fields:

\[
\{ \Psi(t, \vec{x}), \Psi(t', \vec{x}') \}_+ = \frac{2m}{i \cos(\phi)} [i \partial_x + m \cos(\phi)] D(x - x') + 2im^2 \gamma^5 \tan(\phi) D^1(x - x').
\]

(13)

Since the function \( D^1(x - x') \neq 0 \) for \( (x - x')^2 < 0 \), the ‘local’ observables would not commute at equal times (cf. with the condition (1.5) of \[20\]).\[5\] In

\[5\] One can still write the equation (13) in a symbolic form with the operator \( \hat{\epsilon} = i \partial_t / |i \partial_t| \) (introduced by Weaver, Hammer and Good, Jr. \[21\]):

\[
\{ \Psi(t, \vec{x}), \Psi(t', \vec{x}') \}_+ = \frac{2m}{i \cos(\phi)} [i \partial_x + m \cos(\phi) + im \gamma^5 \sin(\phi) \hat{\epsilon}] D(x - x').
\]

(14)

It is relevant to the dynamical equation obtained \[13, 17, 13\] in the approximation \( \phi(x) \approx \text{const} \):

\[
[i \gamma^\mu \partial_\mu - m \cos(\phi) \pm im \gamma^5 \sin(\phi)] \Psi_\pm (x^\mu) = 0.
\]

(15)

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conclusion, the derived non-locality is not the non-locality, which “manifests in the spinorial space”; the anticommutators explicitly contains the even solution $D^1(x - x')$ of the Klein-Gordon equation in the formulas (11,13), which does not vanish for space-like intervals $(x - x')^2 < 0$. The non-locality is obviously in the $\vec{x}$-space. The contribution of the even $D^1(x - x')$ function is larger for small intervals, and if $t - t' = 0$ we have the inverse proportionality to $|\Delta \vec{x}|^2$. In a more detailed version of this paper (submitted to Mod. Phys. Lett. A) we took into account the dependence of phase factors on the space points and instead of Eqs. (21) and (22) of the commented paper, the analogues of the projection operators, we obtained generalized expressions, as well as those instead of Eqs. (23) and (24) of [15].

I want to indicate that the concept of a variable coordinate-dependent mass is not a new one, e. g. [24] (see also old related papers [25]). Moreover, these results can be obtained without the use of the Ryder procedure, but instead, one can use the Sakurai-Gersten method [26]. I start from

$$ (E^2 - c^2 \vec{p}^2) I^{(2)} \Psi = \left[ EI^{(2)} + c\vec{p} \cdot \vec{\sigma} \right] \left[ EI^{(2)} - c\vec{p} \cdot \vec{\sigma} \right] \Psi = M^2 c^4 \Psi \quad (16) $$

(cf. Eq. (4) of [26b]). Then, its solutions can be found from

$$ (ih \frac{\partial}{\partial x^0} + i h \vec{\sigma} \cdot \vec{\nabla}) \Psi(x) = m(x) c \Phi(x) , \quad (17a) $$
$$ (ih \frac{\partial}{\partial x^0} - i h \vec{\sigma} \cdot \vec{\nabla}) m(x) c \Phi(x) = M^2 c^2 \Psi(x) . \quad (17b) $$
or

But, in my opinion, such a formulation only put cover on the intrinsic non-locality of the theory in the $\vec{x}$-space. Of course, the last term in (15) can be considered as a Higgs-like ‘interaction’. We noted that equations with $\gamma^5$ ‘interaction’ term have also been introduced (apart of our previous works) in the context of the Dirac oscillator [22] and of the Dirac supersymmetry and pseudoscalar-Higgs mass generation [23].
\[
(i\hbar \frac{\partial}{\partial x^0} + i\hbar \vec{\sigma} \cdot \vec{\nabla})\Psi(x) = Mc\phi(x)\Phi(x),
\]
(18a)

\[
Mc\{\phi(x)(i\hbar \frac{\partial}{\partial x^0} - i\hbar \vec{\sigma} \cdot \vec{\nabla})\Phi(x) + (i\hbar \frac{\partial \phi(x)}{\partial x^0} - i\hbar (\vec{\sigma} \cdot \vec{\nabla}\phi(x)\Phi(x))\} = M^2c^2\Psi.
\]
(18b)

In the 4-component form after algebraic transformations one has
\[
\begin{pmatrix}
-Mc\phi(x) & i\hbar \frac{\partial}{\partial x^0} - i\hbar \vec{\sigma} \cdot \vec{\nabla} \\
(i\hbar \frac{\partial}{\partial x^0} + i\hbar \vec{\sigma} \cdot \vec{\nabla}) & -Mc\phi(x)
\end{pmatrix}
\begin{pmatrix}
\Psi \\
\Phi
\end{pmatrix}
\begin{pmatrix}
1 + \frac{\gamma_5}{2} \\
\frac{1}{\phi(x)}(i\hbar \frac{\partial \phi(x)}{\partial x^0} - i\hbar \vec{\sigma} \cdot \vec{\nabla}\phi(x))
\end{pmatrix}
\begin{pmatrix}
\Psi \\
\Phi
\end{pmatrix} = 0.
\]
(19)

So, we have an equation
\[
[ih\gamma^\mu \partial_\mu - \frac{Mc}{\phi(x)} \left(\frac{1}{2} + \gamma_5\right) - Mc\phi(x) \frac{1 - \gamma_5}{2} + i\hbar \gamma^\mu \frac{1}{\phi(x)} \partial_\mu \phi(x)]\psi^D(x) = 0.
\]
(20)

When presenting \(\phi(x) = \exp(\eta + i\chi(x))\) we obtain:
\[
[ih\gamma^\mu \partial_\mu - Mc(\eta - i\chi(x)) \frac{1 + \gamma_5}{2} - Mc(\eta + i\chi(x)) \frac{1 - \gamma_5}{2} - h\gamma^\mu \partial_\mu \chi(x)]\psi^D(x) = 0.
\]
(21)

The charge-conjugate Dirac field function satisfies the equation with the opposite sign before some terms (according to the formulas in [30, footnote 3]). You may see that these equations lead to a theory, which is similar to that based on Eq. (15). As opposed to that, the formulation based on the Sakurai-Gersten procedure is a manifestly relativistic covariant one. In general, one can consider the right-hand side of (18a) to be \(Mc(\sigma^\mu c_\mu)\Phi\) or \(Mc\sigma^2\sigma^\mu \tilde{c}_\mu \Phi^*\), or even in more general form (like in [3]). Additional terms in the Dirac-like equation will answer to some specific forms of interaction.

Finally, I present several remarks.

**Remark 1** to the Section 2.1 of the paper [15] follows. That author introduces the \(\lambda\) and \(\rho\) spinors

\[\text{Below the formulas numeration refers to the Ahluwalia paper.} \]
\[ \lambda(p^\mu) \equiv \begin{pmatrix} (\zeta_\lambda \Theta_{[j]}) \phi_L^*(p^\mu) \\ \phi_L(p^\mu) \end{pmatrix}, \quad \rho(p^\mu) \equiv \begin{pmatrix} \phi_R(p^\mu) \\ (\zeta_\rho \Theta_{[j]})^* \phi_R^*(p^\mu) \end{pmatrix}. \tag{22} \]

with \( \Theta_{[j]} \) being the Wigner time-reversal operator and he writes: “... for fermion fields these phases must take on the values \( \pm i \) to ensure that the spinors of the \((j, 0) \oplus (0, j)\) representation are self/anti-self charge conjugate, i.e. they are of the extended Majorana type.”

The spin-1/2 charge-conjugate operator, which was defined in old papers, is:

\[ S_{[1/2]}^c = e^{i\theta_{[1/2]}} \begin{pmatrix} 0 & i\Theta_{[1/2]} \\ -i\Theta_{[1/2]} & 0 \end{pmatrix} \mathcal{K}. \tag{23} \]

As readily seen from the condition of self/anti-self conjugacy, the phases \( \zeta_{\lambda,\rho} \) (which Ahluwalia refers to) depend on the phase factor in the definition (23):

\[ \zeta_\lambda = \pm ie^{i\theta_{[1/2]}}, \quad \zeta_\rho = \pm ie^{-i\theta_{[1/2]}}. \tag{24} \]

For instance, if \( \theta_{[1/2]} = \pi/2 \) then one has \( \zeta_\lambda = \mp 1 \) and \( \zeta_\rho = \pm 1 \).

Presumably, the same result will hold for higher fermion spins.

**Remark 2.** The 8-component Dirac-like equations (i.e. obtained on the basis of the different choice of phase factors between left- and right-momentum-space 2-spinors) have been given (apart from [10] and references therein) in [16a] and references therein. They naturally lead to the idea of opposite gravitational masses of particle and its antiparticle (cf. the Introduction in ref. [15] and the Santilli results [27] obtained in different frameworks).

**Remark 3.** It was recently shown [28] that additional phase factors in the definition of parts of the field operator may lead (apart of difficulties in constructing a scalar Lagrangian in the case under consideration\(^7\))

\[ \mathcal{L} \sim \overline{\Psi} \hat{\epsilon} \Psi \quad \text{\( \hat{\epsilon} \) is the Weaver-Hammer-Good sign operator [21] would be a scalar in any case of definition of the field operator.} \]
to unusual relativistic transformation laws of the corresponding Noether currents.

**Remark 4.** The dimension of spin-1/2 fermion field operator is usually chosen to be equal to \( [\text{energy}]^{3/2} \) in the system of units \( c = \hbar = 1 \). This is the consequence of the convention that the action used in the variation procedure must be **dimensionless** and, hence, the Lagrangian density must have the dimension \( [\text{energy}]^4 \). Thus, taking into account the definitions of classical 4-spinors (16,17) we conclude that the creation/annihilation operators used in Eq. (19) of the cited paper should have the dimension \( [\text{energy}]^{-2} \). In the mean time the Ahluwalia equation (20), the anticommutation relations, manifests that the creation/annihilation operators have the dimension \( [\text{energy}]^{-3/2} \); let us not forget that the dimension of the delta-function is inverse to its argument. Therefore, we either have to add some mass factor to the denominator of the right-hand side of (20) or to substitute the mass factor \( m \) by \( \sqrt{m} \) in the definition of the field operator Eq. (19) of [13]. Finally, one can also present the mass term in the Lagrangian for spin-1/2 field in a rather unusual form, \( \mathcal{L} \sim \overline{\Psi} \Psi \). It is the latter case which was implicitly implied in Ahluwalia’s paper (we learnt this looking at the dimension of the right-hand side of his equation (23)).

**Remark 5.** The basis of (16,17) is not much convenient because in the case of \( \phi(x) = \frac{\pi}{2} (2n + 1), \ n = 0, 1, 2 \ldots \) we would have divergent behavior of the corresponding 4-spinors. Reasons of the choice of this basis have not been given therein.

**Remark 6.** Another possibility for construction of the corresponding 4-spinors exists: to use the first \( \pm \) sign in [13, Eq.(4)] for spin-up and spin-down spinors (and not for \( \phi_{R,L} \) spinors), respectively.

**Remark 7.** With respect to the Ahluwalia’s footnote \( f \), C. DeWitt-Morette *et al.* have already argued that the sign of metric may have physical

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8I am very grateful to an anonymous referee of *Foundation of Physics* for the discussion on how is the dimension of field operators to be fixed. If we would not be careful in this question some paradoxes related to the Noether currents eigenvalues may arise.
significance [29].

On the basis of the above, our conclusion is the following: though the Ahluwalia physical idea is interesting, its presentation suffers from several misunderstandings and it contains many encrypted statements. Mathematical and physical foundations of this theory have been presented in several papers before. In the present article the non-locality in $\vec{x}$-space was explicitly shown, which may lead to the violation of the Causality Principle (or even to the energy-momentum non-conservation). However, I agree that this formalism deserves further elaboration, particularly in the way, if the usual momentum-space commutation relations (used in [15, Eq.(20)]) may be considered to be valid in these frameworks.

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