THE CHAIN OF ANTICHAINS

Box Protocol: the Dual-Blockchain and a Stablecoin

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October 27, 2018. Version 1.0

Abstract

We are in transition to a cashless society and, for quite some time, blockchain has been around as one of the most thriving technologies with a bright future ahead. Blockchain is a decentralized-distributed system which has revolutionized our perspective of the world. Many forms of digital currency have already been used in a variety of ways and places, e.g., in the online marketplaces and mobile banking systems, where it is now normal to use a phone number or an email address instead of a bank account. Although there are numerous benefits the digital currency has to offer, many cryptocurrencies have very low trading volumes as they failed to build their own ecosystem.

Stabilization is the key as cryptocurrencies are extremely volatile at times. Admittedly, there are a number of active stablecoin projects all over the world, some of those are already playing an important role for more efficient and secure digital ecosystems, and many new stablecoin teams are planning to launch in weeks or months. In this paper we present our new way of thinking – the dual-blockchain algorithm. Our primal space is a directed acyclic graph (DAG). Using the dual-blockchain algorithm a chain of antichains is constructed in real time on another layer, which has a one-to-one correspondence with transactions on a DAG. This chain of antichains has the same structure of blockchain. Using such dual-layer framework, we take desirable aspects from both blockchain and a DAG-based distributed network systems. We consequently came up with a powerful consensus protocol which makes the final confirmation feasible with great efficiency. Our dual approach comes with two distinct cryptocurrencies: Box Dollar (BXD) and Box Coin (BXC). Our stablecoin Box Dollar (BXD) is tied up with a robust fiat currency, anticipating the use as a medium-of-exchange as well as a reliable store-of-value. Another cryptocurrency, called Box Coin (BXC), is a crucial component to keep up our purely distributed peer-to-peer network, with capability to run our unique incentive system at its core.

In this paper we present both deterministic and stochastic aspects of the dual-blockchain. Illustrative and numerical examples are also presented. Our new algorithms were discovered by the use of discrete mathematics as well as probability models. This paper is focused on the mathematical and analytical foundations of a new digital ecosystem, the dual-blockchain of Box protocol.

Keywords: blockchain, antichain, dual-blockchain, stablecoin, cryptocurrency, directed acyclic graph (DAG), consensus protocol, international payment, cross-border e-commerce, peer-to-peer network, Internet-of-Things (IoT), distributed network, decentralization, partially ordered set, optimization, stochastic models, nonhomogeneous Poisson process, compound Poisson distribution, currency conversion, Boolean functions, system of distinct representatives, digital wallet and smart insurance

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1 Introduction and our motivation

Interest in blockchain technology has risen dramatically over the past few years in every industry. The success of a cryptographic currency such as Bitcoin inspired many practitioners and researchers trying to carry out the technology to a variety of real-life applications on a decentralized peer-to-peer network system. Peer-to-peer (P2P) payments are a great fit for such applications due to the similarity of operations and transactions on a decentralized system structure – e.g., they have chains of digital signatures of asset transfers, P2P networking, etc. Yet, most of P2P platforms are not fully utilizing the “purely” distributed system. There are tons of P2P platforms (e.g., PayPal, TransferWise, Google Wallet, Mastercard Send and many others) which made it easier and faster to make payments, send or receive funds. They provide us with clearly better solutions than traditional systems in most aspects, even with lower fees and costs. Still, their pricing and operating models could be more efficient and secure by the blockchain technology, which may lead us to find a more perfect solution.

Most of existing cryptocurrencies are not accepted in regular market places due to their high volatility. Holding some cryptocurrency is extremely risky, so the current forms of cryptocurrencies are not suitable to use in our daily life. It follows, therefore the price stabilization is the key to the application of a cryptocurrency. A cryptocurrency, with guaranteed price stability, will clearly be beneficial. The electronic commerce (or e-commerce) industry is where one can find it suitable to apply the blockchain technology and cryptocurrency to its existing P2P payment system. Most of successful e-commerce marketplaces have flexible and adaptable structure, and more importantly, with less regulation but higher potential. For this reason, the e-commerce is a great fit for incorporating the ideas of cryptocurrency and its related applications.

At this moment the e-commerce market transactions are mostly based on fiat currencies, while there has been a serious need of instantaneous transactions and borderless transfer-of-ownership. Yet, there is a variety of difficulties in the international transfer, which oftentimes is hard to track, and normally takes more time (than domestic) with higher cost to exchange, various fees depending on a sending or receiving financial institution. None of the existing models are well-suited for our daily use, and we consequently started being occupied with thought of a new digital currency. We came up with a new idea, the dual-blockchain algorithm with two distinct currencies – a stablecoin Box Dollar (BXD) with guaranteed reliability and stability; another cryptocurrency Box Coin (BXC) with a small downside risk and a large upside potential. Such dual currency system is essential to keep up a decentralized-distributed system as well as the possibility to use for our daily life. The reason is as follows.

- For BXD: In case of sending and/or receiving international payment caused by any merchandise purchase, first and foremost is the highest reliability and stability – a proper digital currency must be able to be functioning like a robust fiat currency (i.e., as a useful medium of exchange as well as dependable store of value). Therefore, one of straightforward methods is pegging it to a robust fiat currency. Rebalancing of holding currencies in return of BXD pegged to a robust fiat currency (e.g., USD) may help keep its reliability and stability.

- For BXC: We view a decentralized-distributed system as a society. It is not difficult to see that if a society consists of good citizens only, there might be no serious concerns raised. Good citizens are honest and would contrive to help others, not to mention that they act in accordance with the rules (or suggest more suitable regulations and rules). Moreover, a good citizen is committed to do his or her duty such as paying taxes as a citizen for a healthy community.

Proper rewards are essential for the peers to act like a good citizen. In order to encourage rewarding behaviors, a fair opportunity will be given to every peer to play a vital role in the system. In our ecosystem, there are two particular positions called the boxer and box-genesis to which anyone can be appointed multiple times. There are also some small fees charged when transactions are issued, which will provide more security to the system. With no fee at all, one might easily be dumping numerous nonsensical transactions with no reason. The amount of fees will be from tiny to small, depending on the matter. See Section 2.4.3 for more details.
The mentioned elements such as rewards and fees must be realized instantaneously when the events take place – giving prompt incentive and motivation to the peers. When the ecosystem gets larger, stronger and more popular, it is obvious that the value of its currency will be increased if its quantity in circulation is fixed. This possibility of capital appreciation is a great incentive for the peers to act like a great citizen. For this reason, another cryptocurrency for rewards and fees would be necessary to keep the network healthier, more secure and successful.

For this reason we are motivated to build a new distributed peer-to-peer system with two distinct cryptocurrencies: Box Dollar (BXD) and Box Coin (BXC). They have their own roles. The BXD is a stablecoin for our daily use and the BXC is to improve and manage our ecosystem together with all the peers’ active participation.

The organization of this paper is as follows. In Section 2 we introduce our new idea – the dual-blockchain algorithm, which is based on a proper decomposition of a partially ordered set. A simultaneous creation of a dual layer will follow, where a “chain of antichains” will be constructed in real time. A brief comparison with Bitcoin and IOTA can be found from Section 2.4. In Section 3 we present the associated stochastic models, e.g., a nonhomogeneous Poisson process, combination of multiple transaction flows and a compound Poisson distribution. In Section 4 we discuss how to effectively manage our stablecoin Box Dollar (BXD). The value chain of our ecosystem is briefly described in Section 5, followed by our concluding remarks in Section 6. Note that the main idea and contribution of this paper can be found in Section 2, and the other sections present some of its related topics.

2 The Dual-Blockchain Algorithm

2.1 Distributed system on a DAG, some notable differences from the blockchain technology, and previous works on a DAG

An acyclic digraph, or directed acyclic graph (DAG), consists of two entities: vertices and edges, i.e., \( G = (V, E) \), which consists of a set \( V \) of vertices (or nodes) and a set \( E \) of edges (or directed arcs). An edge is an ordered pair \((i, j)\) meaning outgoing from vertex \( i \), incoming to vertex \( j \), a pair \((j, i)\) means from vertex \( j \) to vertex \( i \). A DAG is often called an generalized blockchain because the graph itself is a ledger for storing transactions where each vertex is regarded as a single block. Note that throughout this paper we use “DAG” when we expound in plain English, and employ \( G \) when we make a study of related aspects from the mathematical perspective, or when scientific language is more suitable. We also use “vertex” and “node” in a mixed way – “vertex” will be used when we are more focused on graph itself and “node” when transaction processes and their related topics are discussed. Some of the notable differences from the original blockchain ideas can be written (among others) as follows.

- The main structure of blockchain is a single chain that consists of blocks (a chain of blocks) and each block is a set of multiple transactions. If we look at each transaction as a single block, it is not difficult to understand that such blocks and their connections can be depicted on a DAG. A DAG can also be expressed in terms of trees – note that if there are \( N \) vertices (or nodes), then there can be at least \( N \) binary trees (see Section 2.2).

- Blockchain is based on synchronous time stamps, while a DAG is asynchronously operating. However, our dual-blockchain is updated in a nearly synchronous manner (see Section 2.4).

- On a DAG there is nothing to force to separate participants into different categories. On the other hand, Bitcoin and some other cryptocurrencies justify to maintain two separate types of participants, required either to issue or to approve transactions. There is no “miners” creating blocks and receiving rewards in DAG.

Let us briefly mention how a DAG system works as a distributed ledger. Transactions are issued by nodes and their edges show what previous transactions they approved - users need to approve previous
transactions in order to issue their own transaction. All transactions have its own weight, and in our case they are all equal to one which is adaptable as the system evolves. Speaking of the approval and the weights, the validation comes with cumulative addition of weights and it goes to all the connected previous nodes (i.e., ancestors of the node). Unapproved nodes in a DAG are called “tips.” See Figure 1.

A transaction validation process is simple – a node selects some tips at random and verify their validity in order to issue its own transaction. It is assumed that the approving nodes checks whether or not the two transactions are conflicting in a diligent and honest manner. Note that this validation is different from the final confirmation in a sense of consensus mechanism. A node even before a first transaction is called the “genesis.” The genesis node has all tokens created in the beginning of a DAG (no more tokens will be created) and then send the tokens out to several founder nodes. In order to issue a transaction a node needs to approve the two tips and solve a cryptographic puzzle similar to those in the Bitcoin blockchain. For more technical details we refer the reader to [2], [9], [14], [15], etc.

Note that [2], [20] have detailed illustrations of the process together with excellent discussions, so we recommend the reader to see these papers (and references therein) as well. They both investigate many important aspects of the network flows on an acyclic digraph – specifically, [2] studies the network flows from the deterministic perspective while [20] presents how to handle the uncertainty on a DAG and discusses their strategies using probability (or stochastic) models (mainly based on a homogeneous Poison process). We will discuss both aspects in this paper and present what can be done more and how to improve them.

Figure 1: Transactions on an acyclic digraph. Nodes in gray are tips (i.e., unapproved nodes).

In what follows one can find how we develop and modify the existing cryptocurrency models on a DAG, and more importantly, how we make the connection between the blockchain and a DAG. We will begin with introduction of basic notions and develop such concepts using the system of distinct representatives, i.e., decomposition of a partially ordered set (poset). A simultaneous creation of an efficient network layer (called the dual-layer) will follow, and this is the core of our dual-blockchain.

2.2 Acyclic digraph, structure of trees and related set representation

Our space is an acyclic digraph (directed graph) $G$, where vertices (or nodes) designate transactions and edges (or directed arcs) denote how they are connected – the edge $(i, j)$ means that node $i$ approves node $j$. In our network system a node selects exactly “two” tips at random and verify their validity in order to issue its own transaction. More details are presented after this section. There is another notation we need to define together with $G$. Allow us to use $\mathcal{N}$ designating the network associated with $G$, which consists of sub-networks $\mathcal{N} = \{\mathcal{N}_1, \ldots, \mathcal{N}_V\}$, where $V = |V|$. Thus, each network contains a subset of $G$ where a large number of vertices can be included. However complicated these networks look, there are always starting and ending points, and these are the latest vertex (the source node) and the genesis (the sink node), respectively.
An acyclic digraph can be thought of as a combination of trees which consist of child nodes and their parent and ancestor nodes. Speaking of tree, let us list some terminology (well-known) that we will use for this paper. Parent means the predecessor of a node; Child is any successor of a node; Siblings are any pair of nodes that have the same parent; Ancestor is the set of predecessor; Descendant is the set of successors; Subtree denotes a node with its descendants.

**Remark 1 (Binary Trees).** As a node is supposed to approve two previous transactions, it can be seen as a binary tree. It is easy to observe that the whole system can be separated into multiple binary trees. (There at least $V$ binary trees if the total number of vertices is denoted by $V$.)

In Section 2.1 we briefly mentioned the basic mechanics of issuing and approving transactions on a DAG. In connection with trees, it can be said as follows. If a node $v$ approves two previous transactions then such nodes that issued the approved transactions become parent-nodes (or parents) of a node $v$. Note that this validation of the parents automatically approves the parents of that parents, followed by such recursion all the way to the genesis. If we select a single parent node out of two parent nodes then it forms a chain (i.e., a linearly linked set (totally ordered)). This is very efficient and is regarded as one of the main benefits to manage transactions on a DAG.

However, there are a large (typically) number of such chains, which are only partially ordered as there are incomparable ancestors in terms of subset relation (or validation relation). A DAG is a partially ordered set (or poset) itself. The multiple chains of a DAG is the main reason why the system is asynchronous, which makes it almost impossible for all peers to agree to a single version of the truth. This means the final confirmation from a reasonable consensus mechanism would be unreachable if the network system is solely based on a DAG. This was the motivation for this research and our new idea – the dual-blockchain algorithm is all about.

**Remark 2.** A DAG is a partially ordered set (for short, poset) by a subset relation.

Now let us turn our attention to see what happens to a DAG when a node is approved in terms of the cumulative weight.

**Remark 3 (Cumulative weight on nodes, a measure for the integrity and trust, and the counting measure).** As mentioned earlier, we assume all nodes have their own weight equal to one. In addition to the references to the parents (by hashes), the cumulative weight is a good measure for the integrity and trust. By the validation of a single child node, its all ancestors’ weights will be added exactly by one, which is nice for the counting measure as well. That is, the node’s weight provides the system with information of how many child nodes at the moment. The exact cumulative weight of a node is not known to its neighbors due to asynchronous network in a DAG.

**Remark 4 (Inclusion of transaction information on nodes).** Note that a child node includes its parent node by referencing parent’s hash. All information of ancestors (of course, including genesis) can be obtained (reference does not take any memory, only point to the location) in a recursive manner, which means that child node has more information than their parent nodes.

Speaking of the information inclusion by nodes let us use the following notations:

$$T_k = \{\text{transaction information of node } i_k\}.$$  

Suppose that node $i_7$ approved node $i_6$ and node $i_5$, and node $i_6$ approved node $i_4$ and $i_3$, and node $i_5$ approved nodes $i_2$ and $i_1$ as in Figure 2. Then node $i_7$ can get the information of all 6 previous nodes and itself (of course), node $i_6$ includes nodes $i_4$, $i_3$ and itself, and node $i_5$ includes nodes $i_2$, $i_1$ and itself. For $k = 1, \ldots, 7$, let the set $A_k$ designate the union of all ancestor nodes from the point of view of node $i_k$. This example can be written up as:

$$A_7 = T_1 \cup T_2 \cup \cdots \cup T_7,$$

$$A_6 = T_6 \cup T_4 \cup T_3, \quad A_5 = T_5 \cup T_2 \cup T_1$$
and
\[ A_1 = T_1, A_2 = T_2, A_3 = T_3, A_4 = T_4. \]
Thus, we have
\[ A_7 = T_7 \cup A_6 \cup A_5, \]
followed by the subset relation
\[ A_7 \supseteq A_6, A_7 \supseteq A_5, A_6 \supseteq A_4, A_6 \supseteq A_3, A_5 \supseteq A_2, A_5 \supseteq A_1, \tag{1} \]
\( A_5 \not\supseteq A_6 \) and \( A_5 \not\subseteq A_6 \) as they are only partially ordered. This is a pretty simple example, but tells us some important ordering on components of the network. Refer to Figure 3 where child and parent nodes are upside down. In a tree graph, it is typical to have child nodes below, but in our special validation operation child nodes approve the transactions of their parent nodes. Again, the direction of edges are not from the genesis, but towards it.

![Figure 2: Transactions on an acyclic digraph. Nodes in gray are tips (i.e., unapproved nodes). Compare with Figure 3](image)

**Remark 5** (Not a typical binary tree structure). *Again, note that the hierarchical relationship in a typical tree has ancestors above their predecessors. Our case is the opposite due to the selection-approval operation by a child node. This means, the root of a tree is a common child (the youngest “single” one!) of all the others. A child is a “superset” of the parent nodes.*

Also note that the tree in Figure 3 can be divided into three binary trees (i.e., repeated structural form as illustrated), which will help solve a variety of problems in a recursive or iterative manner. And we employ such recursion for our consensus protocol (in Section 2.4.1).

### 2.3 Primal and dual spaces and the dual-layer system

#### 2.3.1 Partially ordered set and its related concepts

In this section we present models and related algorithms making the system more integral, secure and efficient. For the sake of completeness we list some basic definitions and results in connection with partially ordered sets (or *poset*, for short).

We say that two elements \( x \) and \( y \) of \( S \) are *comparable* if \( x \leq y \) or \( y \leq x \), otherwise \( x \) and \( y \) are *incomparable*. If all pairs of elements are comparable then \( S \) is *totally ordered* with respect to \( \leq \). An element \( M \) of \( S \) is called a *maximal* element in \( S \) if there exist no \( x \in S \) such that \( M \leq x \) (i.e., \( M \leq x \rightarrow M = x \)). An element \( m \) of \( S \) is called a *minimal* element in \( S \) if there exist no \( y \in S \) such that \( y \leq m \) (i.e., \( y \leq m \rightarrow m = y \)).
An element $G$ of $S$ is called a greatest element in $S$ if $x \leq G$ for all elements $x$ of $S$, and the term least element is defined dually. A chain (or totally ordered set or linearly ordered set) in a poset $S$ is a subset $C \subseteq S$ such that any two elements in $C$ are comparable (A chain is a sequence). An antichain in a poset $S$ is a subset $A \subseteq S$ such that “no” two elements in $A$ are comparable.

**Remark 6** (Blockchain is linear, so totally ordered and a DAG is only partially ordered.). \textit{The main structure of blockchain is a series of blocks – it is linear, hence a totally ordered set. A DAG is only partially ordered and a mixture of piece-wise linear networks (can be very messy). A DAG can also be restructured in a meaningful way using the idea of the decomposition of a poset into chains and antichains.}

It is well known that if every chain of a poset $S$ has an upper bound in $S$, then $S$ contains at least one maximal element (Zorn’s lemma). If $x, y \in S$, then we say that $y$ covers $x$ or $x$ is covered by $y$, denoted $x < y$ or $y > x$, if $x < y$ and no element $u \in S$ satisfies $x < u < y$. The chain $C$ of $S$ is called maximal if it is not contained in a larger chain of $S$.

**Remark 7.** It is desirable that all maximal chains have the same length from the genesis to the corresponding tips. Typically some of the tips are not in the same antichain.

The size of the largest antichain is known as a poset’s width; The size of the longest chain is known as a poset’s height. This will be used in \((3)\).

**Remark 8.** The height is from the genesis to a tip in the most recent antichain.

A poset can be partitioned using chains, and for its details we refer the reader to the literature, e.g., [4], [6], [5], etc. A poset can also be partitioned using antichains and this is closely related to our formula construction (see, e.g., [7], etc.). The following theorem is well-known, so it is presented without the proof.

**Theorem 1** (Dual of Dilworth). Suppose that the largest chain in a poset $S$ has length $n$. Then $S$ can be partitioned into $n$ antichains.

**Definition 1.** A rank function of a poset $S$ is function $r : S \rightarrow \{0\} \cup \mathbb{N}$ having the following properties:

\begin{itemize}
  \item[(i)] if $s$ is minimal, then $r(s) = 0$.
\end{itemize}
(ii) if \( t \) covers \( s \) (i.e., \( t \trianglerighteq s \)), then \( r(t) = r(s) + 1 \).

**Remark 9.** In a DAG, the minimal element is the genesis.

**Definition 2** (by [13]). On a finite poset \( S \) with length \( n \), ordered by inclusion, the reverse rank function \( \rho : S \to \{1, \ldots, n\} \) is defined by

\[
\rho(E) = \sum_i 1_{E \subseteq M_i},
\]

where \( E \) is any element of \( S \) and \( M_i 's \) are incomparable maximal elements of \( S \). The reverse rank function \( \rho(E) \), a counting measure, returns the number of maximal elements containing \( E \).

**Remark 10.** In a DAG, the maximal elements are the tips.

**Theorem 2.** Given a finite poset \((S, \subseteq)\), \( \max_{E \in S} \rho(E) \) is the width (i.e, the size of the largest anti-chain) of a poset and equals to

\[
\min \left\{ m \mid \text{maximal chains } C_1, \ldots, C_m \text{ with } S = \bigcup_{i=1}^m C_i \right\}.
\]

**Proof.** Obvious by [4] and the reverse rank function in Definition 2.

**Definition 3** (system of distinct representatives (SDR)). Suppose that \( A_1, A_2, \ldots, A_N \) are sets. The family of sets \( A_1, A_2, \ldots, A_N \) has a system of distinct representatives (SDR) if and only if there exist distinct elements \( z^{(1)}, z^{(2)}, \ldots, z^{(N)} \) such that \( z^{(i)} \in A_i \) for each \( i = 1, \ldots, N \).

**Theorem 3** (Duality). The minimum number of non-redundant edges from an antichain to its previous one is equal to the maximum number of distinct representatives in that later antichain.

### 2.3.2 A dual layer construction and two essential roles

With the listed notions, we present our main algorithm which constructs a new layer on top of a very complicated partially ordered DAG. We will decompose a DAG using chains and antichains. Note that a DAG will be generated as it normally operates, but there is a dual-layer above, which will be constructed almost simultaneously based on a subset relation among nodes.

![Transaction units connected in a DAG (The genesis is node 1.)](image)

Let us consider a DAG of 20 nodes (with four tips of nodes 15, 18, 19, 20) as in Figure 4. The \( i^{th} \)-created node shows its node number \( i \), i.e, node 1 is the genesis node. Child nodes are the superset of the parent.
nodes. Incomparable nodes form an antichain and the set of incomparable child nodes must be a “cover” of the set of incomparable parent nodes. In Figure 5, for example, the nodes 8, 9, 10, 11 are included in an antichain (the third bluebox from the left) since they are incomparable with each other. The “union” of those four nodes (i.e., antichain) include the “union” of nodes 5, 6, 7 (i.e., antichain) – no single node among the four nodes 8, 9, 10, 11 include all three nodes 5, 6, 7 as a node can only approve two previous nodes.

As described in Figure 5 by the decomposition using antichains, any complicated DAG can be restructured into a linked list (i.e., a chain of blue boxes in Figure 5) along the lines of the blockchain. It follows, therefore the blockchain can be said a special case of a DAG, and this is why some people insist that a DAG is a generalized blockchain.

**Remark 11.** We do not change our space of a DAG to a single chain; Rather, we utilize this counterpart in order to make the system more secure and trusted. This is the staring point of our dual-blockchain algorithm.

To give an account of what we mentioned above, we present a series of descriptions as in Figures 5, 6 and 7. Let us to use some of the set operations and notations, let $B_k, k = 1, \ldots, 6$ denote the bluebox in Figure 5 from the left to right in order. $B_1$ has elements of nodes 2,3,4, $B_2$ has elements of nodes 5,6,7, and so on. Then $B_1 \subseteq B_2 \subseteq B_3 \subseteq B_4 \subseteq B_5 \subseteq B_6$ and $B_1 < B_2 < B_3 < B_4 < B_5 < B_6$ as a bluebox covers its most recent previous one.

![Figure 5: From the view of the series of boxes (SDRs) which have incomparable units, constructed based on the partial order by their subset relations.](image)

![Figure 6: From the view of the series of boxes (SDRs) – a desirable series of boxes after redundant edges (or approvals) are removed.](image)

Speaking of including the most recent previous transaction, it is important that if the same address issue multiple transactions (multiple nodes) then it had better form a single linked list (i.e., a chain, where they are totally ordered). In doing so it would be helpful to track its own series of transactions. Note that a rapid
succession of the multiple transactions using different addresses is a typical way to attack the system. More details are discussed in Section 2.4.1.

Figures 5 and 6 are slightly different – a few number of edges in Figure 5 are missing in Figure 6. One of the two edges are removed from nodes 8, 9, 17, 20 as some of them are redundant and do not ease the job for the consistent integrity of the system. Note that node 9 approved nodes 5 and 2, but node 5 approved node 2 earlier. Node 9’s approval on node 5 means that it automatically approved node 2 as node 5 is the child of node 2. Node 9 is a child of node 5, so the node 9 cannot be a child of node 2 since node 2 is a parent of node 5 – it will hurt the parent-child relationship. (The nodes 17, 13, 11 are of the same case.)

The approval selections of nodes 8 and 20 are OK but not quite desirable. They did not approve redundant transactions and so in this small example nothing is wrong with their selection. What we’re concerned about is the possibility that in a real situation some lazy nodes could select quite unnecessary and old transactions (already approved by many others). This may cause higher complexity in a variety of important operations. It also may put nonsensical and illogical cumulative weights on a DAG. This is because it interferes in the efficient ordering operation of the system. Selection of nonsensical transaction does not take place in our dual-blockchain system, which will be discussed in Section 2.4.1.

The blueboxes of Figure 6 are the antichains as well as the system of distinct representatives (if we see distinct chains from the nodes). From the geometric perspective, the elements of the most recent antichain can be considered as vertices of upper bounded orthants in a multidimensional space, and it follows that the earlier antichains’ elements are inside of such orthants.

Note that there is no link among the elements of the same antichain. Let us pick a single node and appoint it to represent and act for its siblings. The nodes we will choose are the ones that joined their antichains the latest. In this example, such nodes are 4, 7, 11, 14, 18, 20 and they are all over the place on a DAG (see Figure 7). We call them the box-closing-members, or the “boxers” for short.

Remark 12 (The roles of the box-closing-members, the boxers). Note that the roles of the boxers are adaptable as the system evolves. One of the crucial roles is that they communicate with their siblings (i.e., the members of the same antichain) and broadcast to their siblings if there are some notable changes on the system. Moreover, there is a boxers’ own network, on which they can efficiently communicate with each other. The network is just a single chain connected by edges among them, but it is a semi-hidden network since it’s exclusive for the boxers.

We learned that the boxes are the antichains and that the individuals of them may form a chain. Therefore we have a chain of antichains. In keeping with our dual approach we introduce another unique set of vertices in boxes, and we call it a box-genesis group. Individual box-genesis will be denoted by igenesis (or i-genesis) if it’s inside the ith Box (i.e, ith antichain). The i-genesis is the child of (i − 1)-genesis and the parent of i + 1-genesis, and (i + 1)-genesis is the child of i-genesis and the parent of (i + 2)-genesis, and so on.

\[
\text{genesis} \prec \cdots \prec (i - 1)\text{-genesis} \prec i\text{-genesis} \prec (i + 1)\text{-genesis} \prec \cdots \quad (2)
\]

There is also a chain (as in the above covering inequalities in (2)) of box-genesis which is the same height as the whole DAG (not hard to see because the height of a DAG is the size of its longest chain.).

Remark 13 (The roles of the box-genesis). The box-genesis is selected among good citizens with excellent credit history. Once selected, one is responsible to check most recently approved transactions by its siblings, followed by announcing the final confirmation of their validity. Such transactions are in the parent-box (most recent previous box). When the final confirmation is placed, the box will finally be closed in agreement with a boxer and no more validations can be placed. More details are presented in Section 2.4.1.

If the system has a fixed upper bound of the width (i.e., the size of antichain), say M, and if the total number of vertices of a DAG is N, then

the height of the whole DAG is at most \(\lceil N/M \rceil\),

which is easy to see and might seem trivial. However, this is a markedly important measure owing to the fact that the system governs the quantity M. We will be employing this measure for further discussions,
Figure 7: Primal layer: Transaction units on a DAG, our primal space with boxers in blue

Figure 8: Dual layer: Boxers (in gray) and their siblings in the dual layer, constructed in real time.

together with another criterion – the time constraint $\tau$ for each antichain. This is also a vital constraint to control the system. Both $M$ and $\tau$ and their roles are presented in Remark 20, they are both paramount criteria for a boxer selection. The size of antichain will be determined by a boxer, the box-closing member.

This chain of box-genesis is a hidden network to users (from the latest $i$-genesis (a root node) to the genesis (a sink node and the origin)), which helps make the system more secure and compact in a sense of speaking another layer (in Figure 8). The other users do not get informed of such chain, which will come into play whenever the need arises. The detailed roles of the box-genesis are presented in Section 2.4.1.

Let us present our box-making algorithm (which is like, open, examine items and put them in the right package boxes, and then close the boxes). Using the boxes based on the algorithm below we construct a new layer which is a combination of a semi-hidden network and a normal chain. Note that this new layer can be viewed as a separate space but it has one-to-one correspondence with the DAG. This layer will be updated in real time as transactions are released and validated in a DAG.

We present two algorithms – Algorithm 1 at a fixed point of time and Algorithm 2 in real time. The following is to show how to fill a box at a specific time point by the use of all unassigned nodes. Let $B_i$ denote the $i$th antichain (i.e, the $i$th Box) and $V_i'$ the corresponding set of vertices which validated two
previous transactions but not yet approved by others. Put it differently, $V'_i$ is the set of tips, i.e, the vertices which is about to issue new transactions (or already issued one but no validations are given yet).

**Algorithm 1** Constructing the $i$th Box ($B_i$) at a point of time for $i = 1, \ldots, n$

Suppose $B_{i-1}$ is full, and $V'_i = \{v_1, \ldots, v_{n_i}\}$

\[
\text{while there exists } v \notin B_i \text{ for every } v \in V' \cup \{v\} \text{ do}
\]

if $v$ is a cover of the element(s) of $B_{i-1}$ then

$B_i \leftarrow B_i \cup \{v\}$

$V'_i \leftarrow V'_i \setminus \{v\}$

else

$V'_{i+1} \leftarrow V'_{i+1} \cup \{v\}$

end if

end while

Stop: The Box $B_i$ has been filled up, let the last $v$ be the boxer. Then the $i$th genesis is assigned, and we move on to the next Box $B_{i+1}$ with $V'_{i+1}$

**Remark 14.** The box-genesis will be selected from the peers (among good citizens) only after a boxer is determined. This is a very important rule to keep the system secure. (see Remarks 22 and 32.)

**Remark 15 (Timestamp server).** Timestamps form a chain, note that each timestamp includes is previous ones in its hash as in Figure 8.

**Remark 16.** Each box is identified by a hash, and is linked to its previous box by referencing the previous box’s hash.

The following is how to build a dual-layer in real time. See Remark 20 for the constraints $M$ and $\tau$ – useful tools for the antichain size control. Note that a node will be assigned to the corresponding antichain (based on the partial order) right after the node approves two previous transactions.

**Algorithm 2** Constructing the antichains in real time (i.e, a single node at a time)

Suppose $|B_i| = k_i$ for all $i$

if a node $v$ just approved two previous transactions, and is a cover of the element(s) of $B_j$ then

$B_j \leftarrow B_j \cup \{v\}$

Update $B_j = \{v^{(1)}_j, \ldots, v^{(k_j)}_j, v^{(k_j+1)}_j\}$

end if

**Remark 17 (Blockchain on our dual-layer).** This dual-layer has a chain of antichains, which is actually a blockchain (or a box-chain) since each block consists of transactions and such blocks are linearly linked.

Similar to Bitcoin transactions, in our dual-blockchain system most of the conflicts are easily determined if they are in different antichains (or boxes, or blocks). If they are in different antichains (i.e, totally ordered), the latter one will be rejected. If suspicious transactions are in the same antichain, their cumulative weights can be compared. If such transactions happen to have the same weight (i.e, same number of descendants), one can think of some tie-break rules coming into play, including a fixed maximum width (or the size of the largest antichain), a boxers role, etc. These mentioned ideas seem all useful, however, we prefer robust criteria for the final confirmation. For this reason, we developed a powerful consensus mechanism to reach a final stage of agreement. It became feasible due to the dual-blockchain which has both a blockchain and a DAG.

**Remark 18 (The dual-blockchain).** The dual-blockchain system consists of a “blockchain” in dual layer and a “DAG” in primal layer.
2.4 The Chain of Antichains

2.4.1 Consensus protocol for the final confirmation with box-closing, and a subtree problem

As many practitioners and researchers have already pointed out, there can be plenty of subtrees, and as a result, a DAG can be extremely wide and inefficient (in terms of the final confirmation possibility). Therefore, one of the most serious concerns on a DAG would be a subtree problem. As in Figure 9, there is no path from some later blue nodes (say, good citizens) to red nodes (say, dishonest ones). With subtree(s) there is a possibility that nonsensical transactions can be “issued and approved” by a group of red nodes without any proper investigation to see if the transactions are true and valid. This is a major obstacle to reaching a “consensus” on validity of previous transactions, which is critical in any distributed peer-to-peer system.

![Figure 9: Main subtree in blue vs. dishonest subtree in red. (tips are in gray.)](image)

A subtree can take place anytime and a subtree always exists in a DAG, which renders negative behaviors possible. Whether it’s formed by a good-will or not (by a good citizen or by a malicious attacker), there must be a way to check transactions’ validity from subtrees. Allow us to present what can be done by our particular algorithm regarding a consensus mechanism. This is about reaching a final state of agreement from the peers – the final confirmation of already-approved-transactions.

The main process of our consensus protocol is as follows. For the final confirmation of transactions of $B_{i-1}$, it is required to reach agreement among the peers in the next antichain $B_i$. This means, a final confirmation process for $B_{i-1}$ gets started when a boxer of $B_i$ is selected. To join $B_i$ a node must approve at least one node from $B_{i-1}$. Thus, antichain is totally based on what transactions the node approved.

**Remark 19 (The rule for the tip selection (or transaction selection and validation)).** The users are recommended to select most recent transactions. We use the rank function (in Definition 7) for this rule as the following.

$$r(v_k) \leq r(v_{k+1}) \leq r(v_k) + 1,$$

where $v_k$ denotes a node that just finished its transaction validation right before $v_{k+1}$’s completion of validation. The inequality (4) is equivalent to:

$$v_k \in B_i \rightarrow v_{k+1} \in B_i \text{ or } B_{i+1}.$$

Once the final confirmation process begins, a later node can no longer select and approve transactions from the corresponding antichain whose transactions are being checked.
The final confirmation will be placed on each box by its child box (the next neighboring box, or cover). Again, the final confirmation process for \( B_{i-1} \) gets started when a boxer of \( B_i \) is selected. Hence we list below the following dual-criteria for a boxer selection problem.

**Remark 20** (Dual criteria for a boxer selection).

- **By the use of cardinality.** As mentioned in [3], the size of antichains can be determined by the number of nodes. Let \( M \) denote the upper limit of the number of elements of \( i \)-th Box (i.e., a set \( B_i \)). Then we write \( |B_i| \leq M \). If we only take this into account for a boxer selection, then the \( M \)-th node in \( B_i \) will be designated as a boxer.

- **By the use of time.** Let the time \( \tau \) denote the acceptable and agreeable time-limit (e.g., 20 seconds) for a user to wait until the final confirmation. Note that the average time until the final confirmation would be about \( 1.5\tau \). (See Remark 26 for details.) Suppose that the first node \( v_{i_1} \) of \( B_i \) comes in at time \( t = s \) in other words, at time \( t = s \) a node \( v_{i_1} \) just finished its validation of two previous transactions in \( B_{i-1} \) (or one in \( B_{i-1} \) and another one in \( B_{i-2} \)). Suppose that a node \( v_{i_k} \) finishes validation of two previous transactions at time \( t \leq s + \tau \), with the next one \( v_{i_{k+1}} \) completing its approvals at time \( t > s + \tau \). Then the node \( v_{i_k} \) will become a boxer and \( v_{i_{k+1}} \) will be the first member of \( B_{i+1} \). In fact the node \( v_{i_{k+1}} \) cannot select any transactions from \( B_{i-1} \) by (5).

- **Exception:** If the size \( M \) is reached at beyond allowable speed (i.e., in case that incoming transactions are extremely frequent than normal), the time constraint will be used. This is to keep the system safe from possible attacks. (see Section 2.4.2).

Note that the size of antichain depends on a boxer selection problem. Transactions issued by the members of \( B_{i-1} \) are approved by the members \( B_i \), but they are just individual validations. We need a strong consensus protocol in order to place the final confirmation of entire set of transactions of a previous block (or antichain).

**Algorithm 3** The final confirmation of the transactions in \( B_{i-1} \), conducted by \( B_i \)

**Step 1.** Once the antichain \( B_i \)'s size becomes \( M \) or time is up \((\geq \tau)\), i.e., \( B_i \) is full, the \( i \)-genesis starts to check its siblings’ validations (i.e., validity of transactions of \( B_{i-1} \)).

**Step 2.** If all transactions of \( B_{i-1} \) are legitimate based on the consensus protocol (below), go to Step 4. Otherwise, go to Step 3.

**Step 3.** Disable the nodes with illegal transactions in \( B_{i-1} \), and report it to the genesis group. Disable all nodes in \( B_i \) that approved such illegal nodes.

**Step 4.** The final confirmation is placed by the \( i \)-genesis. \( B_{i-1} \) is closed and marked “true,” and every transaction issued in \( B_{i-1} \) becomes final.

Therefore, the whole chain of antichains can get marked “true” in a recursive way. Note that there is another recursion inside each box as in the following consensus mechanism.

**Consensus protocol in a dual recursion**

- **The 2+2 process in recursion:** When a node approved two previous transactions it will be added to the box as in the Algorithm 2. As a node joins a box, it is automatically assigned to its neighboring node and is required to check the validity of transactions that its neighboring node approved. It is easy to see that there is a single
neighbor when joined (i.e., a node joined right ahead) and this process continues recursively from the second to the last (i.e., a boxer). Simply put, each node checks its prior neighbor’s approvals as they join. This is a dual validation process for every node conducted both in primal and dual layers.

- Final confirmation by the box-genesis:
  The final confirmation of recent transactions will be made by a box-genesis as presented in Algorithm 3. This role is similar to that of miners’ of Bitcoin in a sense, and such validation process is conducted by a box-genesis in a dual layer (a box-genesis exists only in a dual layer). Rewards are given to a box-genesis when the final confirmation is placed.

![Figure 10: The dual-blockchain. Boxes are in blue in both layers; box-genesis’ are in pink in the dual layer.](image)

Final confirmation is placed when the consensus protocol is completed. We understand the box-genesis is the one who broadcasts the termination of validation process to all other members. In our peer-to-peer system anyone can take such role (there is no two distinct types of participants!) as long as one shows a record of positive behaviors.

**Remark 21** (The box-genesis group membership (only for good citizens)). The roles of box-genesis is crucial to keep our ecosystem healthy and secure. Excellent history of positive behavior is required for peers to renew this membership and new members with outstanding history of positive behaviors can also be selected. The box-genesis is the leader of consensus protocol, and this position is open to everyone but follows a random assignment process.

**Remark 22** (Random assignment process). The members of a box-genesis group are not totally trusted even though they qualify – its box assignment is completely random with some conditions, and not known to a box-genesis until the last node of box (i.e., a boxer) is determined. After a boxer is selected the box genesis will be appointed by a random selection with a condition being a different user from the selected boxer.

**Remark 23** (Boxer’s role for dual safety). Boxer is the one communicating with its siblings (the members of an antichain), together with other boxers. However, the final confirmation of validations of previous transactions must be determined solely by a box-genesis. For dual safety, a boxer keeps a final confirmation from a box-genesis in check. Some rewards are given to a boxer.

**Remark 24.** As we discussed, our recursive consensus mechanism can perform well in terms of both efficiency and scalability. Our consensus mechanism is unique – in our dual-blockchain ecosystem there are no heterogeneous peer groups. Anyone can play a vital role in the system, e.g., as a box-genesis or as a boxer. It is easy to see that more active peers will have higher chance to get more rewarding opportunities.

### 2.4.2 Possible attack to the system and its success probability

A double spending attack can cause a serious integrity violation in any distributed peer-to-peer system. Although our recursive validation and final confirmation systems are powerful, we should consider every
possible attack in order to keep the network safe. A possible attack scenario we came up with is as follows. (It is probably the worst case scenario). Suppose that a malicious and rich person (or group) will try to dominate the system over some time-window (this guy will try to dominate with all different addresses!). This means, that guy can issue a huge number of transactions almost simultaneously, which makes it possible to take all the nodes including boxers in two consecutive boxes. Note that a takeover of a single antichain won’t be good enough for a successful attack as our consensus protocol is always conducted over two boxes. Therefore, a complete dominance of back-to-back antichains would be necessary for a successful attack.

It is not difficult to see that the above mentioned attack would fail if there is at least one node in such antichains. From the attacker’s perspective, the favorable event is that there are no other transactions issued while two consecutive boxes are formed. In order to calculate such probability, let us suppose that a time constraint $\tau$ (in Remark 20) is used for a single box-closing. For simplicity we also assume that the transaction arrivals follow a homogeneous Poisson process. Note that in Section 3 more detailed and realistic stochastic processes are studied (e.g., a nonhomogeneous compound Poisson process, etc.). It is well known that the corresponding inter-arrival times are i.i.d. exponential random variables with the same parameter $\lambda$ of the Poisson process which means the average number of transactions in a given unit time.

Let a random variable $W$ denote the time until the next transaction, which follows an exponential distribution with a p.d.f. $f(x) = \lambda e^{-\lambda x}, x \geq 0$. Then the following can be written:

$$p = P(\text{no transactions while two boxes are formed}) = P(W > 2\tau) = e^{-2\lambda \tau}. \quad (6)$$

Suppose that the system had set a time limit $\tau$ of 20 seconds (1/3 minute) for the final confirmation process. Assuming the average number of transactions per minute is 30 (i.e., $\lambda = 30$), we have

$$p = P(W > 2(1/3)) = e^{-2(30)(1/3)} \approx 0.00000002061. \quad (7)$$

For a more speedy final confirmation process, let $\tau = 10$ seconds with the same $\lambda = 30$.

$$p = P(W > 2(1/6)) = e^{-2(30)(1/6)} \approx 0.00004539, \quad (8)$$

which is still quite small. The results clearly demonstrate the following.

- More transactions will bring down the chance of a successful attack, i.e., $\lambda \uparrow \implies p \downarrow$.
- Greater patience of a user (waiting for the final confirmation) also brings down the chance of a successful attack, i.e., $\tau \uparrow \implies p \downarrow$.

Note that the number of transactions will be much higher in any well known e-commerce marketplace, which makes it possible to have a very speedy final confirmation process. With this reasoning, let us see another case, e.g., say 100 transactions per minute on average (i.e., $\lambda = 100$). Then our time constraint $\tau$ can be selected so that the probability of an attacker’s favorable event is kept negligible. Let us put an upper bound of $p = 0.000001$ (one out of a million) on a successful attack chance. Then we can find $\tau$ using

$$P(W > 2\tau) = e^{-2(100)\tau} \leq 0.000001, \quad (9)$$

followed by

$$\tau \geq \frac{\ln(0.000001)}{-200} \approx 0.06907755 \text{ minutes}, \quad (10)$$

which is about 4.14465316 seconds. This means that if we put on a 5 second rule for the time constraint $\tau$ then the attacker’s success probability would be less than 0.000001 (again, $\tau \uparrow \implies p \downarrow$). If it still seems possible, note that we have a series of box-genesis (two different ones) in the associated back-to-back boxes and they are independent of transactions.

Remark 25. Together with a random selection of box-genesis, the probability of successful attack would be almost zero in any case.
Detailed stochastic aspects of transaction flows will be discussed in Section 3 where one can find more general and useful ideas about how to analyze related random processes.

**Remark 26** (Average time until the final confirmation). *Roughly, it is about $1.5\tau$ which is the average of $\tau < T < 2\tau$.*

The actual amount of time until the final confirmation for a single transaction (or a node) is varied depending on the time point when joined a corresponding antichain. Such waiting time until the final confirmation is $\tau < T < 2\tau$ since the final confirmation is made by the next neighboring box (child box). Note that it takes $\tau$ to form a single box. It is not hard to see that if a node is the first member of a box, its transaction’s final confirmation time would be approximately $2\tau$ since two more boxes need to be closed. In case of a boxer, the confirmation time will be about $\tau$. Recall that the transactions are already approved by their child nodes on a DAG, and therefore our consensus protocol on the dual layer can place a powerful final-confirmation.

**Remark 27.** *On a dual layer, we have a chain of antichains and each antichain consists of a single chain. This means we have a total order in the dual layer and obviously any conflicting nodes can easily be checked.*

Let us finalize this section by supporting Remarks 19 and 20 which enable the following.

- No purely random approvals of tips - a lazy user can approve a fixed pair of old transactions
- Inclusion in a more recent antichain, followed by a 2+2 validation process.
- In case that a new legitimate transaction does not get approved and waits longer than two new boxes formation, an empty transaction will be issued as a series. But it still needs to select and approve another node to issue the same transaction in order to get a validation by others. Note that such empty transaction contributes the network’s security.
- $M$ - the width constraint for antichain: This will be determined by the frequency of transaction flows (see Section 3 for more details about a random transaction flow).
- $\tau$ - the time constraint for antichain: We do not wait until the box is filled up to the given limit $M$. Recall: If the size $M$ is reached at beyond allowable speed (incoming transactions are extremely frequent than normal), the time constraint had better be used. This is to keep it safe from the worst case scenario we just studied.

### 2.4.3 Incentive system with proper rewards and fees

Proper rewards and fees are essential to keep a distributed peer-to-peer network at a desirable level. Note that in Bitcoin “mining” is the incentive system and the mechanism for decentralized security. Our incentive system is for everyone (not for a particular group) by the use of a mixture of rewards and fees – suitable returns for good behaviors. Note that our fees are not about a bad behavior, the fees in our ecosystem are tiny but functioning as a very important obstacle to putting any nonsensical transactions. We believe that rewards and fees should not be mutually exclusive; Rather they need to be given as a combination depending on an attribute of cases. Below is the possible cases for rewards. Rewards are given to such user(s) who

- put legitimate validations in the primal layer, i.e., in a DAG (or a generalized blockchain).
- put legitimate validations in the dual layer, i.e., in a chain of antichains (or a blockchain).
- complete the job as a box-genesis
- complete the job as a boxer
- report abnormal event(s)
Recall that the appointment as a boxer or a box-geneis is totally random – any user can play these roles multiple times (as mentioned previously, e.g., see Remarks [12][13][21][22]). It is not difficult to see that more active and honest participation will increase the chance to take such positions and receive more rewards, which could end up with more rewards than fees. Fees are paid whenever a transaction is issued. This incentive system is certainly beneficial to both users and the whole ecosystem. Good citizenship is crucial for our purely distributed peer-to-peer system as much as any type of society is in need of.

### 2.4.4 Dual-blockchain compared with others

The main structure of blockchain is a single linear link of blocks (hence the name) and each block consists of transactions, therefore the blockchain is a totally ordered set. If we look at each transaction as a single block, there is no longer a single chain; Rather, we see very complicated, tons of blockchains which look tangled up in a sense. This structure is called a generalized blockchain and can be depicted on a directed acyclic graph (for short, DAG), consisting of a set of vertices (transactions) and a set of edges (issue and validation of transactions). The transactions on a DAG are only partially ordered. Indeed, the validation process on a DAG is very efficient and relatively secure (not as secure as the original blockchain), and this is why there are already a number of groups making their cryptocurrencies and payment systems on a DAG (or modified graphs), headed by IOTA and many others. A DAG is regarded as a third generation of the blockchain technology. (Bitcoin, followed by Ethereum, and then there are new protocols on a DAG.) Between the blockchain (a block = a set of transactions) and a complicated and messy mixture of numerous blockchains (a block = an individual transaction), we cannot say which one offers a more perfect solution to a purely distributed peer-to-peer system – there are pros and cons in both systems (see Table 1).

Upon the structural basis of a DAG, one could summon up the images of connected antichains since a DAG is only partially ordered. Such messy tangle can be reshaped into a more compact form using chains and antichains, followed by the generation of a series of systems of distinct representatives, i.e., a chain of antichains. This idea, together with suitable validation and selection processes, inspired the author to write an algorithm, called the dual-blockchain. Note that this algorithm is partially based on the scheme of the author’s recent publications [11], [12], [13]. As we have seen from the previous sections, this reshaping does not mean to replace a DAG – we create its efficient counterpart on a dual layer in a more compact form in real time, together with effective consensus protocol which enables a final confirmation. And this is the core of our dual-blockchain algorithm.

|                | Bitcoin | IOTA          | Box Protocol |
|----------------|---------|---------------|--------------|
| Usage          | Payment | Used for IoT applications | Payment     |
| Finality       | Yes     | No            | Yes          |
| Confirmation time | 10 minutes at least | 2 minutes (validation) | 30 seconds on average if \( \tau = 20 \) seconds (depending on the values of \( \tau \) and \( \lambda \)) |
| Transaction fees | 0.001 BTC | None | Tiny or none (if rewards are given) |
| TPS            | 7       | 1000          | 5000         |
| Decentralization | Yes     | No            | Yes          |
| Security       | high    | low           | high         |

Table 1: Brief Comparison of Bitcoin, IOTA and Box Protocol
3 Stochastic models of the dual-blockchain

3.1 A nonhomogeneous Poisson process - practical assumptions on the frequency of transactions

Unlike some of the assumptions made in [20], we suppose that \( X(t) \) is a nonhomogeneous P.P. (Poisson Process), where \( \{\lambda(t), t \geq 0\} \) is a stochastic process itself. This is because we believe that transactions’ frequency can be varied over certain time periods (e.g., there might be more transactions in the middle of night). For the sake of completeness, allow us to present some basic concepts first. Stochastic process can simply be called “one-parametric family” of random variables \( X(t), t \in T \). For example, in case of Markov Chains we have \( T = \{0, 1, 2, \ldots\} \). In our case we suppose \( T = \{t | t \geq 0\} \), which means “time” (mathematically it is nothing but nonnegative half of the real line). The following is well known.

\( X(t), t \geq 0 \) is a homogeneous Poisson process if it has two properties: (i) \( X(t) \) has independent increments, i.e., for \( 0 \leq t_0 < t_1 < t_2 < \cdots < t_{2n-1} < t_{2n}, X(t_1) - X(t_0), X(t_3) - X(t_2), \ldots, X(t_{2n}) - X(t_{2n-1}) \) are independent random variables; (ii) For \( s \geq 0, t > 0 \), the random variable has Poisson distribution with parameter \( \lambda t \),

\[
P(X(s + t) - X(s) = k) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}, k = 0, 1, \ldots,
\]

which follows that \( E(X(s + t) - X(s)) = \lambda t \) and \( \text{var}(X(s + t) - X(s)) = \lambda t \). Poisson process is a random event process, e.g., transactions occur in the system (our case), customers arrive to a store, cars arrive to a gas station, etc. \( \lambda \) is the expected number of such random events in unit time.

For nonhomogeneous Poisson process, Condition (ii) only needs to be changed. Instead of a constant rate lambda we assume that there exists a nonnegative function \( \lambda(u), u \geq 0 \) such that for \( s < t \) \( X(t) - X(s) \) has Poisson distribution with parameter \( \int_s^t \lambda(u) du \), which means the parameter depends on time interval. This assumption is crucial in our business because our data set shows some clear trends in frequency of transactions (and their sizes as well). \( \lambda(u) \) is called “event density” (in our case, transaction density) because in the interval \((t, t + \Delta t)\)

\[
\int_t^{t+\Delta t} \lambda(u) du \approx \lambda(t) \Delta t
\]

is the expected number of events.

**Example 1** (Transaction Density from Data Set). Suppose that the following transaction density (or event density) are found from recent data set (unit time = 1 hour).

\[
\lambda(u) = \begin{cases} 
2u, & 0 \leq u \leq 1 \\
2, & 1 \leq u \leq 2 \\
4 - u, & 2 \leq u \leq 4.
\end{cases}
\]

For simplicity, we are interested only in the time interval \([0, 4]\). We want to calculate

(i) the probability that two transactions occurred during the first two hours,

(ii) the probability that two transactions occurred during the second two hours.

For (i), we need to calculate the event density, say \( \mu \), of the first two hours:

\[
\mu = \int_0^2 \lambda(u) du = \int_0^1 \lambda(u) du + \int_1^2 \lambda(u) du = \int_0^1 2udu + \int_1^2 2du = 1 + 2 = 3.
\]
Thus, \( P(X(2) = 2) = \frac{3^2}{2!}e^{-3} = 0.2240 \). For (ii), we have:

\[
P(X(4) - X(2) = 2) = \frac{\left( \int_2^4 \lambda(u)du \right)^2}{2!}e^{-\int_2^4 \lambda(u)du} = \frac{\left( \int_2^4 (4 - u)du \right)^2}{2!}e^{-\int_2^4 (4 - u)du} = \frac{2^2}{2!}e^{-2} = 0.2707.
\]

**Remark 28** (Doubly stochastic Poisson processes). A nonhomogeneous Poisson process with the rate function \( \{\lambda(t), t \geq 0\} \) – it is a stochastic process itself – is called a “doubly stochastic Poisson process.” This is a great fit for applications which have “dependent” process increments, e.g., some seasonal products, and new IT products (with business cycles, replaced by newer products), etc.

The simplest doubly stochastic process (sometimes called a mixed Poisson process) has a single random variable \( \theta \) with \( X'(t) = X(\theta t) \), where \( \{X(t), t \geq 0\} \) is a Poisson process with \( \lambda = 1 \). Given \( \theta \), \( X' \)’s a Poisson process of constant rate \( \lambda = \theta \), but \( \theta \) is random (unobservable, typically). If \( \theta \) is a continuous random variable with p.d.f. \( f(\theta) \), the marginal distribution is as follows.

\[
P(X'(t) = k) = \int_0^\infty \frac{(\theta t)^k e^{-\theta t}}{k!}f(\theta)d\theta.
\]

### 3.2 Some important random behaviors regarding transaction interarrival times in a Poisson process

It is well known that inter-arrival times, let’s say \( S_0, S_1, \ldots \), are i.i.d. exponential random variables with parameter \( \lambda \), where \( \lambda \) is the parameter of the Poisson process. Let \( W_n \) denote the time of occurrence of the \( n \)th event (setting \( W_0 = 0 \)). The differences \( S_n = W_{n+1} - W_n \) are the duration that the Poisson process sojourns in state \( n \). See Figure 11 for description, where \( S_0, S_1, \ldots \) denote the inter-arrival times in a Poisson process and \( W_1 = S_0, W_2 = S_0 + S_1, \ldots \) represent the occurrence times of the random events in a Poisson process.

![Figure 11: Interarrival times of transactions - exponential random variables in the Poisson process.](image)

We present the following theorems without proofs (see some stochastic models literature for more details).

**Theorem 4.** Conditioned on \( N(t) = n \), i.e., \( N(t) = N((0, t)) = \text{number of events in} (0, t) \), the random variables \( W_1, \ldots, W_n \) have joint p.d.f.:

\[
g(t_1, \ldots, t_n) = \begin{cases} n!t^n & \text{if } 0 \leq t_1 \leq \cdots \leq t_n \\ 0 & \text{elsewhere} \end{cases}
\]

The above theorem tells us that if conditioned on the number of events up to \( t \), i.e., \( N(t) = n \), then the \( n \) points behave as \( n \) independent random points, each chosen from \((0, t)\), according to uniform distribution.

**Theorem 5.** Probability density function of sum of \( n \) independent, exponentially distributed random variables with the same parameter \( \lambda \): \( X = X_1 + \cdots + X_n \), where \( X_i, i = 1, \ldots, n \) have the p.d.f. \( f(x) = \lambda e^{-\lambda x}, x \geq 0 \). Let \( f_n(x) \) be the p.d.f. of \( X \). Then, for \( n = 1, 2, \ldots \), we have

\[
f_n(x) = \frac{\lambda^n x^{n-1}e^{-\lambda x}}{(n-1)!}, x \geq 0.
\]
It follows that another derivation of probability distribution for interarrival times of transactions. Let $F_n(t) = P(S_0 + \cdots + S_{n-1} \leq t)$. Then we have, using Theorem [5]

$$F_n(t) = \int_0^t f_n(u)du = \int_0^t \frac{n^n e^{-nu}}{(n-1)!} du.$$  

Furthermore, for $n \geq 1$,

$$P_n(t) = P(N((0, t)) = n) = F_n(t) - F_{n+1}(t) = \int_0^t \frac{n^n e^{-nu}}{(n-1)!} du - \int_0^t \frac{(n+1)^{n+1} e^{-u(n+1)}}{n!} du = \left[\frac{n^n u^n e^{-nu}}{n!}\right]_0^t + \int_0^t \frac{n^n e^{-nu}}{n!} du - \int_0^t \frac{(n+1)^{n+1} e^{-u(n+1)}}{n!} du$$  

$$= \frac{(\lambda t)^n}{n!} e^{-\lambda t}.$$  

Note that $F_k(t)$ means the probability that there are at least $k$ transactions. It follows that $P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}$ for $n = 1, 2, \ldots$ and this implies $P_0(t) = e^{-\lambda t}$. See our related application below.

**Example 2** (Expected total BXC in time $(0, t)$). Suppose that transactions occur in the system according to a Poisson process. Upon making a transaction each user pay 1 BXC. Find the expected total sum collected in $(0, t)$, discounted back to time 0. Let $\beta$ be the discount rate. Note that the reason of discounting is to perform more accurate comparison, especially for different time periods.

In case of continuous compounding $\beta$ is divided by $n$ and then $n \to \infty$. The result is

$$\lim_{n \to \infty} \left(1 + \frac{\beta}{n}\right)^n = e^\beta.$$  

The discounting factor is its reciprocal value $e^{-\beta}$. Discounting from time $W_k$ back to initial time 0, the result is $e^{-\beta W_k}$. We want to calculate the expected total BXC in a given time as follows.

$$E\left(\sum_{k=1}^{X(t)} e^{-\beta W_k}\right) = \sum_{n=1}^\infty E\left(\sum_{k=1}^{X(t)} e^{-\beta W_k} | X(t) = n\right) P(X(t) = n)$$  

$$= \sum_{n=1}^\infty \sum_{k=1}^\infty n \int_0^t e^{-\beta u} du P(X(t) = n)$$  

$$= \sum_{n=1}^\infty \int_0^t 1 - e^{-\beta u} du P(X(t) = n)$$  

$$= \frac{1}{\beta t} (1 - e^{-\beta t}) \sum_{n=1}^\infty n P(X(t) = n)$$  

$$= \frac{\lambda}{\beta} (1 - e^{-\beta t}).$$  

It is well known that if $X_1, \ldots, X_n$ are i.i.d. exponential random variables, then $X_1 + \cdots + X_n \sim \text{Gamma}(n, \mu)$, which has the p.d.f.

$$f(x) = \mu e^{-\mu x} (\mu x)^{n-1} / (n-1)!,$$  

where $(n-1)! = \Gamma(n)$ because $\Gamma(n) = \int_0^\infty e^{-y} y^{n-1} dy$ and $\Gamma(n) = (n-1)\Gamma(n-1)$. Its expectation and variance are $E(X) = n/\mu, \text{Var}(X) = n/\mu^2$, respectively. Note that the gamma distribution is log-concave,
so we can formulate a convex optimization problem using some stochastic optimization techniques.

**Gamma distribution, a family member of logconcave distributions.** If $\mu = 1$ of (13), then the distribution is said to be standard. If $\xi$ has gamma distribution, then $n\xi$ has standard gamma distribution. Both the expectation and the variance of a standard gamma distribution are equal to $n$. An $m$-variate gamma distribution can be defined in the following way. Let $A$ be the $m \times (2^m - 1)$ matrix the columns of which are all 0-1 component vectors of size $m$ except for the 0 vector. Let $\eta_1, \ldots, \eta_s$, $s = 2^m - 1$ be independent standard gamma distributed random variables and designate by $\eta$ the vector of these components. Then we say that the random vector

$$\xi = A\eta$$

has an $m$-variate standard gamma distribution.

### 3.3 Combining nonhomogeneous Poisson processes of transactions

Multiple transaction inflows into the system must be well analyzed in order to help monitor overall activity of the system. Let us begin with the simplest case and extend to the generality. Let $X_1(t)$ and $X_2(t)$ be the two Poisson flows with parameter $\lambda_1$ and $\lambda_2$, respectively. We also assume that they are independent. Consider nonoverlapping intervals: $(t_0, t_1)$ and $(t_2, t_3)$. Then $X_1(t_1) - X_1(t_0)$ and $X_1(t_2) - X_1(t_1)$ are independent, and $X_2(t_1) - X_2(t_0)$ and $X_2(t_2) - X_2(t_1)$ are also independent. Since the two processes are also independent, the four random variables are independent. It follows that $X_1(t_1) - X_1(t_0) + X_1(t_3) - X_1(t_2)$ and $X_2(t_1) - X_2(t_0) + X_2(t_3) - X_2(t_2)$ are independent. Thus, we can say that $X(t) = X_1(t) + X_2(t)$ has independent increments. Note that

$$X(s + t) - X(s) = X_1(s + t) - X_1(s) + X_2(s + t) - X_2(s),$$

where $X_i(s + t) - X_i(s)$ has Poisson parameter $\lambda_i t$, $i = 1, 2$. Hence $X(s + t) - X(s)$ has Poisson distribution with parameter $(\lambda_1 + \lambda_2)t$.

The procedure is the same in the general case. We can unite arbitrary finite number of independent Poisson process: if $X_1(t), \ldots, X_n(t)$ are independent Poisson processes with parameters $\lambda_1, \ldots, \lambda_n$, respectively, then $X(t) = X_1(t) + \cdots + X_n(t)$ is a Poisson process with parameter $\lambda_1 + \cdots + \lambda_n$.

### 3.4 Transaction “size”, and the recursive formulas for the probability mass function of compound random variables

Transactions are randomly occurring events. Since a number of events occurring in a fixed period of time, say $N$, is uncertain, it would be suitable to use a compound distribution modelling for a random sum $S = X_1 + X_2 + \cdots + X_N$, where $N$ is a nonnegative integer-valued random variable. For the frequency of transactions, let $N(t)$ be a Poisson process with a fixed rate $\lambda$, for simplicity. It is not a problem, however, if the Poisson parameter $\lambda$ is also uncertain as discussed in the previous sections.

If we need to consider the magnitude (or size) of transactions but their sizes are random (or unknown), it is suitable to use a compound Poisson process. The following is well-known (S):

$$N \sim \text{Poisson}(\lambda), \text{ where } \lambda \sim \text{Gamma}(r, p/(1 - p)) \Rightarrow N \sim \text{NegBinomial}(r, p),$$

which means that if the Poisson parameter $\lambda$ is uncertain due to the behavior of heterogeneous users, a gamma distribution may be suitable for capturing the $\lambda$ information. Then $N$ will have a negative binomial distribution. Negative binomial distributions can be used for our model with no problem, but we restrict ourselves to the compound Poisson distribution for this paper.

The frequency of transactions plays a central role and in our view, the size of transactions would also be very important to proper management of a DAG. For the transaction size, we let $X_1, X_2, \ldots$ denote independent, identically distributed random variables (meaning the transaction sizes at the corresponding events), which are also independent of the Poisson process. (i.e., the random variables $N, X_1, X_2, \ldots$ are
mutually independent.) Then \( S(t) = \sum_{k=1}^{N(t)} X_k, \forall t \geq 0 \) (i.e., \( S(t) \) is a compound random variable) means the aggregate transaction weight in time interval \((0, t]\). It is well known that \( E(S(t)) = \lambda t \mu, \text{var}(S(t)) = \lambda t(\sigma^2 + \mu^2) \) if \( \mu = E(X_k), \sigma^2 = \text{var}(X_k), k = 1, 2, \ldots \).

In order to calculate the distribution of aggregate transaction weights we use Panjer’s recursion formula (see for details, e.g., [16], [18], [3], [1], etc.). For completeness, allow us to present here some basic notions and related formulas. The size \( X_k \) can be fitted with continuous or discrete distributions, but the continuous distribution needs to be discretized for the use of Panjer’s recursion formula. This is a great fit for our model because every transaction has a positive integer (its weight). For practical discretization methods we refer the reader to the literature, e.g., [17] and [3]. We restrict our attention to the case that the transaction size \( X_k \) follows a discrete distribution on the positive integers because a continuous variable needs to be discretized to use the recursion formula and also because any positive integer-valued variables can easily be scaled to the suitable size. Then \( S \) is also distributed on the nonnegative integers, and the probability mass function of the compound process \( S(t) \) can be calculated recursively by the following well-known recursion:

\[
P(S(t) = k) = \frac{1}{k} \lambda \sum_{i=1}^{k} iP(X_1 = i)P(S(t) = k - i).
\]  

(14)

Figure 12: Illustration of Compound Poisson Distributed Transactions. \( H_j \) is the number of events incurred over the period \( j \) and the unit transaction size is one \( BXC \).

Given a Poisson process \( N_j(t) \) and nonnegative integer-valued random transaction size \( X_{jk} \)'s for the upcoming time periods \( j, j = 1, \ldots, M \), we can write:

\[
N_j(t) = x \Rightarrow P(N_j(t) = x) = \frac{(\lambda_j t)^x}{x!} e^{-\lambda_j t}, \ x = 0, 1, \ldots; \ j = 1, \ldots, M
\]

\[
S_j(t) = X_{j1} + X_{j2} + \cdots + X_{jN_j(t)}, \ j = 1, \ldots, M,
\]

(15)

where \( X_{jk} \) is the \( k \)th transaction size in the \( j \)th process. See Figure 12 for description.

Let \( f_j(x) = P(S_j(t) = x) \), where \( x \) is a positive integer. Then, the recursion formula for the p.m.f \( f_j(x), j = 1, \ldots, M \) can be written as:

\[
f_j(x) = \frac{\lambda_j x}{x} \sum_{k=1}^{x} kp_j(k)f(x - k), \ x = 1, 2, \ldots,
\]

\[
f_j(0) = e^{-\lambda_j}, \ j = 1, \ldots, M,
\]

(16)

where \( p_j(k) = P(X_{j1} = k) \).
4 Models for Box Dollar (BXD), our stablecoin

The Box Dollar (BXD), a stablecoin will be used primarily as a useful medium of exchange as well as a dependable store of value. The BXD value is at a fixed exchange rate one to one to the USD, hence the BXD is a USD backed asset and keeps its stability. The goal of analytical models of BXD is twofold – currency conversion and currency rebalancing problems. These models are well known, and therefore we list up a few models in this section. Note that there is a variety of useful optimization models for financial applications, which can be found from operations research literature (see, e.g., [21]).

4.1 Currency Conversion Problem

We may encounter situations of needing to exchange Box Dollar (BXD) to some other fiat currency (possibly to multiple currencies) – observation of nontrivial movement in exchange rates, our market participants’ requests, and/or regular portfolio (multicurrency accounts) rebalancing, etc. When it comes to currency exchange, the loss of stabilization is what we’re most concerned about. This means we should always maximize the USD value (i.e., the amount of BXD) of desired positions in converting to other currencies. It is very important that we obtain the best currency conversion strategy to keep up with our stable Box Dollar models. Such currency conversion problems (with possibility of arbitrage detection) are well known in operations research literature.

Figure 13: Stable Coin System Topology - BXD matching with fiat currencies: CNY (red), JPY (yellow), KRW (blue), HK$ (green), NT$ (purple)

In what follows, we present optimization problems suitable to rebalance the combination of multinational currencies. Their optimal solutions (i.e., optimal weights on currencies) will be employed for a conversion problem. Note that currency rebalancing will take place only if there is a guaranteed capital gain.

4.2 Currency Rebalancing – Kataoka’s Problem: the Safety First Model

Typically random variables appear only on the right hand sides of constraints of stochastic program. The following problem was formulated by [10] and it has stochastic constraints where their technology matrix has random variables. Many real life applications can be solved, but we restrict ourselves to our problem in this paper. Let $x_i$ denote the weight of the $i$th currency, $i = 1, \ldots, n$ and the random vector $\xi$ consists of
components meaning the return on holding the corresponding currency. Our model is the following:

\[
\begin{align*}
\max \ d \\
\text{subject to} \\
\quad P \left( \sum_{i=1}^{n} \xi_i x_i \geq d \right) \geq p \\
\quad \sum_{i=1}^{n} x_i = 1, \ x \geq 0
\end{align*}
\]

(17)

where we assume that \( \xi = (\xi_1, \ldots, \xi_n)^T \) has an \( n \)-variate normal distribution with

\[
\mu_i = E(\xi_i), \ i = 1, \ldots, n, \mu = (\mu_1, \ldots, \mu_n)^T,
\]

\[
C = E(\xi - \mu)(\xi - \mu)^T.
\]

Note that \( p \) and \( M \) are constants and the decision variables are \( x_1, \ldots, x_n, d \). Using some mathematical steps (see, e.g., [22]) the formulation (17) can be written up as

\[
\begin{align*}
\max \ \{ \mu^T x + \Phi^{-1}(1-p) \sqrt{x^T C x} \} \\
\text{subject to} \\
\quad \sum_{i=1}^{n} x_i = 1, \ x \geq 0
\end{align*}
\]

(18)

Since \( C \) is a positive semidefinite matrix, the function \( \sqrt{x^T C x} \) is convex. When the probability level is set \( p \geq 0.5 \) (i.e., \( \Phi^{-1}(1-p) \leq 0 \)), the objective function is concave so (18) turns out to be a convex programming problem.

### 4.3 Currency Rebalancing – Conditional Value-at-Risk: Minimization of risk

Optimization problems using Conditional Value-at-Risk (CVaR) have been researched and used in practice, so allow us to present the formulation of CVaR. We recommend the readers to the literature, e.g., [19], [23], [24] and the references therein. In order to find the optimal portfolio using the classical CVaR, let \( \xi \) denote the loss vector and \( x \) the currency weights (the decision vector) for a portfolio of \( n \) currencies. Then \( \xi^T x \) means the loss of the currency holding, and the following model can be written:

\[
\begin{align*}
\min_{x, a} \ a + \frac{1}{1-p} E \left( [\xi^T x - a]_+ \right) \\
\text{subject to} \\
\quad \sum_{i=1}^{n} x_i \leq 1 \\
\quad x \geq 0, \ i = 1, \ldots, n
\end{align*}
\]

(19)

where \( \mu = E(\xi) \) and \( \mu_0 \) is some constant. Note that \( a = \text{VaR}_p(X) \) at optimality and the optimal objective value is the smallest among all values of \( E \left( (\xi^T x \mid \xi^T x \geq \text{VaR}_p(\xi^T x)) \right) \) with \( x \) in the feasible set of the constraints. Also note that we can write \( \mu_0 = -R \) where \( R \) means the minimum required return for the portfolio.

It is well known that the following LP is a discrete version of (19) with \( \mu_0 = -R \),

\[
\begin{align*}
\min a + \frac{1}{K(1-p)} \sum_{k=1}^{K} u_k \\
\text{subject to} \\
\quad a - x^T y_k + u_k \geq 0, \ k = 1, \ldots, K \\
\quad u_k \geq 0, \ k = 1, \ldots, K \\
\quad -x^T \mu \geq R \\
\quad \sum_{i=1}^{n} x_i \leq 1 \\
\quad x_i \geq 0, \ i = 1, \ldots, n,
\end{align*}
\]

(20)
where \( \{y_1, \ldots, y_K\} \) denotes \( K \) i.i.d. samples of the loss random vector \( \xi \in \mathbb{R}^n \). (20) can equivalently be written in the following matrix notation:

\[
\min a + \frac{1}{K(1-p)} \sum_{k=1}^{K} u_k \\
\text{subject to} \\
\begin{pmatrix}
1_K & -Y & I_K \\
0 & -\mu^T & 0 \ldots 0 \\
0 & -1_n^T & 0 \ldots 0 \\
\end{pmatrix}
\begin{pmatrix}
a \\
x \\
u \\
\end{pmatrix} \geq 
\begin{pmatrix}
0 \\
R \\
-1 \\
\end{pmatrix}
\]

(21)

where \( I_K \) means \( K \times K \) identity matrix, \( Y \) is a \( K \times n \) matrix with samples of the loss random vector \( \xi \in \mathbb{R}^n \), and \( 1_K \) is a \( K \times 1 \) all ones vector.

### 4.4 More general formulations

Let \( x \) denote the vector of currency weights with its related cost vector \( c \), and \( \xi \) the random vector with an estimated distribution; the matrices \( A \) and \( T \) are present and future constraints, respectively. Then a more general stochastic programming model is formulated in the following way:

\[
\min c^T x \\
\text{subject to} \\
Ax = b, x \geq 0 \\
P(Tx \geq \xi) \geq p, 
\]

(22)

where \( p \) is a fixed probability chosen by ourselves. In practice \( p \) is near 1, for example we may choose \( p \) as 0.8, 0.9, 0.95, 0.99, depending on our reliability requirement, i.e., in what proportion of the cases do we want the inequality \( Tx \geq \xi \) to be satisfied. Let \( T_i \) denote the \( i \)th row of matrix \( T \) and \( \xi_i \) the \( i \)th component of random vector \( \xi \).

There is a simplification possibility for the problem (22). Instead of \( P(Tx \geq \xi) \geq p \) we take

\[
E(\xi_i - T_i x \mid \xi_i - T_i x > 0) \leq d_i, i = 1, \ldots, r. 
\]

If the function \( g_i(z) = E(\xi_i - z \mid \xi_i - z > 0) \) is decreasing, then \( g_i(T_i x) = E(\xi_i - T_i x \mid \xi_i - T_i x > 0) \leq d_i \) is equivalent to \( T_i x \geq g_i^{-1}(d_i), i = 1, \ldots, r \) and the whole problem becomes:

\[
\min c^T x \\
\text{subject to} \\
Ax = b, x \geq 0 \\
T_i x \geq g_i^{-1}(d_i), i = 1, \ldots, r, 
\]

(23)

which is an LP.

**Remark 29** (Related measures of violation). In reliability theory and insurance problems \( E(\xi - t \mid \xi - t > 0) \) is called “Expected Residual Lifetime.” It is natural that it is a decreasing function of \( t \), but it is not always decreasing. (i.e., there are probability distributions for which it is not true, e.g., lognormal, Pareto if \( t \geq 1 \), etc.) If \( \xi \) has a logconcave p.d.f., then \( E(\xi - t \mid \xi - t > 0) \) is a decreasing function of \( t \).
where \( x \) is the decision vector, \( \xi \) is a random vector, \( h(x), h_1(x), \ldots, h_m(x) \) are given functions, \( 0 < p_0 \leq 1, p_1, \ldots, p_m \) are given numbers.

**Remark 30** (Convexity of the problem (24)). Any logconcave function is quasi-concave, hence if \( \xi \in \mathbb{R}^r \) has a continuous distribution and logconcave density then \( h_0(x) \) in problem (24) is quasi-concave. Hence, \( h_0(x) \) allows for the convex programming property. If the objective function \( h \) is convex and we assume that \( h_1, \ldots, h_m \) are quasi-concave, then the problem is indeed convex.

**Remark 31.** If the random vector \( \xi \) has independent components \( \xi_1, \ldots, \xi_r \), then

\[
P(Tx \geq \xi) = \prod_{i=1}^{r} P(T_i x \geq \xi_i) = \prod_{i=1}^{r} F_i(T_i x),
\]

where \( F_i \) is the c.d.f. of \( \xi_i \) for \( i = 1, \ldots, r \). The probabilistic constraint takes the form:

\[
\prod_{i=1}^{r} F_i(T_i x) \geq p.
\]

\( x \) is feasible if this inequality, in addition to \( Ax = b, x \geq 0 \), is satisfied.

Note that a variety of optimization applications can be found in [21] and the author’s numerous excellent scientific papers.

### 5 The Dual: Box Coin (BXC) and Box Dollar (BXD) in the Value Chain

The Box Dollar (BXD), a stablecoin will be used primarily as a medium of exchange. The participants will exchange their local fiat currencies to BXD based on a real time conversion rate to USD. This is quite simple but strong enough to obtain desirable level of trust and security – one can put down a deposit with a US Dollar for every Box Dollar issued (i.e., 1 to 1) so that the BXD is asset backed and keeps its stability. There will be a unique and efficient digital wallet, called the boxpay-wallet. Using the boxpay-wallet, the market participants will see all previous transactions, conversion records, and current balances of BXD and BXC. If registered, fiat currencies in his or her bank accounts can be seen as well. The real time exchange rates of BXD to BXC, BXC to BXD and ones among all related fiat currencies are also presented on the boxpay-wallet.

The boxpay protocol consists of the most efficient dual currencies – Box Coin (BXC) and Box Dollar (BXD) on public and private networks, respectively. In what follows we list up some of the notable functions of Box Dollar (BXD) and Box Coin (BXC). The Box Dollar has the following desirable features:

- Reliability and security
- Medium of exchange
- Store of value
- Diversification of currency holdings
- Transferability (e.g., efficient transfer (cross-border) in P2P transactions, among merchants and customers, etc.)
- Tracking all previous transactions
The BXC is mainly for making micropayment on public network (see Section 2 for details). There are some specifics for the use of Box Coin (BXC), the key functions and benefits of Box Coin (BXC) include the following:

- Rewards and Fees (as presented in Section 2.4.3)
- Possibility of capital appreciation

Let’s see from a standpoint of a buyer, say Alice. After she exchanged her local currency to BXD (based on a conversion rate to a USD), the calculated amount of BXDs will be stored in her boxpay-wallet and be ready to use. If Alice wants to use her local currency CNY to buy an item priced at BXD 500, first thing to do is to spend CNY 3,440 to receive BXD 500 into her boxpay-wallet. (Buying 1 US dollar for Chinese Yuan requires CNY 6.88 using the conversion rates as of 1 PM (UTC -4), 8/14/18.) Note that it might be a good idea to put down more CNY to get more BXD if the USD appreciation is expected. (For example, the rate 1 USD = CNY 6.88 at the moment may later be changed to 1 USD > CNY 6.88. It’s been a while the USD gets more valuable compared to the other currencies.) She may have multiple items in her to-buy list even if she does not want to buy them at the moment. After transaction completed the seller will be able to see the BXDs from the boxpay-wallet, ready to use in the e-marketplace (e.g., for shipping cost) or get an exchange for some local fiat currency (or multiple currencies) as needed.

Remark 32 (The boxpay wallet and smart insurance). The boxpay wallet is now being developed, and is a file (a simple database) of the digital keys, which are completely independent of the protocol. This will come with a smart insurance capability, systematically identifying claims to report.

6 Concluding remark: The dual approach of Box Protocol

Our unique way of thinking – the dual blockchain algorithm – has taken desirable aspects both from the blockchain and a DAG. The chain of antichains will certainly provide us with another dimension for a purely distributed peer-to-peer network system. Our dual approaches throughout the paper carry out many useful algorithms, e.g., the 2+2 dual recursion of validation, the duo: box-genesis and boxer groups, the dual-layer system: primal space (a DAG) and dual space (blockchain). We hope our new ideas will be beneficial to the readers and be helpful to improve decentralized-distributed network systems.

7 Acknowledgements

The author would like to express sincere gratitude to Professor Endre Boros for his kind critics, which inspired the author to come up with a consensus algorithm. The author would also like to thank the Box Protocol team for their comments and suggestions. The author dearly misses his academic father Professor András Prékopa (1929-2016)

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