Tire Road Friction Coefficient Estimation: Review and Research Perspectives

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Abstract

Many surveys on vehicle traffic safety have shown that the tire road friction coefficient (TRFC) is correlated with the probability of an accident. The probability of road accidents increases sharply on slippery road surfaces. Therefore, accurate knowledge of TRFC contributes to the optimization of driver maneuvers for further improving the safety of intelligent vehicles. A large number of researchers have employed different tools and proposed different algorithms to obtain TRFC. This work investigates these different methods that have been widely utilized to estimate TRFC. These methods are divided into three main categories: off-board sensors-based, vehicle dynamics-based, and data-driven-based methods. This review provides a comparative analysis of these methods and describes their strengths and weaknesses. Moreover, some future research directions regarding TRFC estimation are presented.

Keywords: Intelligent vehicles, Tire road friction coefficient (TRFC), Off-board sensors-based method, Vehicle dynamics-based method, Data-driven-based method

1 Introduction

Traffic accidents are one of the leading causes of injuries and death in China and abroad. According to the National Bureau of Statistics of China, in 2019, there were 247646 traffic accidents, which resulted in 62763 fatalities, 256101 injuries, and a direct economic loss of 1346.18 million yuan. Therefore, both industry and academia have made great efforts to develop new technologies to reduce or even avoid traffic accidents. Active safety systems are the most representative of these new technologies, which include antilock braking systems [1], electronic stability control systems [2, 3], active collision avoidance systems [4], etc.

The main function of antilock braking systems is to prevent the wheel lock during heavy braking and to maintain the traction between the tires and the road at an optimal value. The magnitude of this optimal traction is usually determined based on the tire road friction coefficient (TRFC). The electronic stability control systems generate a yaw moment based on the desired yaw rate to ensure the lateral stability of the vehicle. The desired yaw rate normally shows a positive correlation with TRFC. The active collision avoidance systems work when the relative distance between the vehicle and the obstacle is lower than the safety distance. This safety distance is negatively correlated with TRFC. The above analysis shows that accurate TRFC information is essential to improve the performance of active safety systems. Unfortunately, TRFC cannot be measured by on-board sensors. To this end, researchers have successively proposed various approaches to address the challenge. Refs. [5–7] also provide a review of the models and methods used for TRFC estimation. However, few types of research have systematically discussed the acquisition of TRFC from the perspective of off-board sensors-based, vehicle dynamics-based, data-driven-based.
This article systematically reviews the recent developments on TRFC estimation from different research directions. It contains a comparative analysis of existing methods and describes their strengths and weaknesses. Also, some future research directions regarding TRFC estimation are presented. Based on this, we believe this work will help the researcher or vehicle engineer to choose a suitable approach for TRFC estimation.

To give some details of the analysis, the rest of this article is organized as follows. In Section 2, the different types of TRFC estimation methods are presented. In addition, the advantages and shortcomings of these existing methods are briefly elaborated. The conclusion and some promising prospects regarding TRFC estimation are given in Section 3.

2 TRFC Estimation Methods

There are three main directions of existing research on TRFC estimation including identification methods based on off-board sensors, vehicle dynamics-based approaches, and data-driven prediction methods. These estimation methods are divided into three groups according to different categories as shown in Figure 1.

2.1 Off-board Sensors-based Methods

Studies have shown that changes in micro/macro-texture of road surfaces affect TRFC [8], which has prompted many scholars to use this knowledge to obtain TRFC. Estimation methods based on this principle usually require the use of a camera to acquire a certain amount of road images and subsequently utilize some algorithms to obtain the TRFC. Baffet et al. [9] used multiple linear regression analysis and the fuzzy logic method to estimate the TRFC. Du et al. [10] proposed a deep neural network based on domain knowledge for estimating TRFC. Leng et al. [11] developed a fusion strategy of a dynamic estimator and visual estimator to identify TRFC. Yu et al. [12] made use of a backpropagation (BP) neural network to predict TRFC. This method usually has better estimation accuracy in high visibility environments while the prediction performance decreases significantly in night driving environments.

To make the estimation algorithm work in a night driving environment, several estimation methods based on the physical deformation of the tire have been proposed successively. Some scholars found that tire deformation and vibration are also related to TRFC. Some sensors such as accelerometers are installed inside the tires (see Figure 2) to measure some key information to obtain TRFC.

Singh et al. [14] proposed a method to predict TRFC using the frequency response of tire vibration. A method of using acceleration information from intelligent tires was presented in Refs. [13, 15, 16], and experimental results proved the effectiveness of this approach. However, due to the complex working conditions of tires, the sensors inside the tires are easily dislodged or damaged, which limits the further practical application of
such methods. Furthermore, some estimation methods based on ultrasonic sensors\[17\], laser profilometer \[18\], wireless piezoelectric tire sensor \[19\], and magnetometer \[20\] have also been reported. The advantages of these off-board sensor-based methods are that they require fewer measurement variables, are insensitive to the vehicle’s dynamic response, and do not require specific excitation inputs. Obviously, these off-board sensors need to be additionally assembled on series production cars. Also, the off-board sensor-based method can only obtain the approximate range of the TRFC and the measurement accuracy is sensitive to the interference of the external environment. For these reasons, vehicle dynamics-based methods have received increasing attention in recent years.

2.2 Vehicle Dynamics-based Approaches

The vehicle dynamics-based method identifies TRFC according to the dynamic response of the vehicle on different road surfaces. The vehicle dynamics-based approaches can be divided into three categories: longitudinal dynamics-based, lateral dynamics-based, and coupled dynamics-based methods.

2.2.1 Longitudinal Dynamics-based Methods

Longitudinal dynamics-based estimation methods typically have high estimation accuracy for acceleration and braking conditions. The principle underlying most of these estimation methods is the relationship between longitudinal slip and TRFC.

Figure 3 shows the relationship between the TRFC and the longitudinal slip for a variety of road surfaces. From Figure 3, we can see that TRFC is an increasing function of slip. As the slip increases, TRFC will reach a maximum value and then decrease slightly. Gustafsson et al. \[21\] first proposed a classical approach to estimate TRFC utilizing the longitudinal slip slope. In Refs. \[21, 22\], an adaptive filter based on Kalman filter theory by using a magic formula (MF) tire model is proposed to estimate TRFC. Similarly, Yi et al. \[23, 24\] supplied more experimental data to sustain Gustafsson’s view. Based on Gustafsson’s idea, Rajamani et al. \[25, 26\] further extended the applicability of the method by adding global positioning system (GPS) signals. In addition, a rule-based TRFC estimation method was also proposed in Ref. \[27\], which can still have a good estimation accuracy when the wheels reach the limit of adhesion. Since TRFC varies not only with road conditions but also with tires, the experiment-based real-time TRFC estimation method can further improve the accuracy of TRFC estimation under different driving conditions \[28\]. Although the slip-based TRFC estimation method requires few sensors and shows promising results, it has major problems in terms of robustness and calibration. Accordingly, nonlinear curve fitting techniques were presented to address this problem \[29–33\]. In addition, another mainstream estimation method is to determine the TRFC from the obtained tire force. Some multiple analytical models were used to identify the TRFC in Refs. \[34–40\]. State observers and recursive least squares (RLS) are also used for TRFC estimation. For example, state observers based on the LuGre dynamic model \[41–43\], the Burckhardt model \[44–47\], the Magic Formula model \[48, 49\], the quarter wheel model \[50\], and the tire torsion model \[51\] were designed to identify the TRFC. RLS methods, which are mainly based on the brush model \[37\], the Burckhardt model \[52, 53\], longitudinal vehicle model \[54–58\], single-wheel model \[59\] and six-degree-of-freedom (DOF) vehicle model \[60\], have been extensively studied in recent years. It is well known that the RLS usually has only one degree of freedom to adjust the adaptivity of the filter, which may limit its application.

To fill the gap, Kalman filter-based methods are attracting more and more attention. Krisztian et al. \[61\] made use of extended Kalman filter for TRFC estimation. Castillo et al. \[62\] developed a TRFC estimation method incorporating fuzzy logic and a Kalman filter. In addition to the common methods mentioned above, some interesting estimation methods have been proposed by scholars. TRFC prediction method based on tire force information using conditional probability theory was presented in Ref. \[63\]. Resonance frequency-based TRFC estimation methods were developed in Refs. \[64, 65\]. A fuzzy logic-based TRFC identification method was presented in Ref. \[66\]. Also, the transformation of the TRFC estimation problem into an optimization problem is an interesting research direction \[67\]. To compare the various methods more clearly, the estimation methods based on vehicle longitudinal dynamics are presented in Table 1.
2.2.2 Lateral Dynamics-based Methods

Estimation approaches that consider vehicle longitudinal dynamics commonly require larger excitation, while methods based on vehicle lateral dynamics may also yield better estimation results when the excitation is relatively small. The general framework of TRFC estimation based on vehicle lateral dynamics is shown in Figure 4. When the driver applies a steering maneuver to the vehicle, several key state variables are first measured by on-board sensors. Then, after obtaining measurement signals, the lateral force of the tire is estimated by some advanced algorithms. Finally, the tire model and tire lateral force are used to predict TRFC.

Based on this general framework, some interesting estimation algorithms have been proposed to estimate TRFC. An analytical model based on lateral acceleration to estimate TRFC was presented in Refs. [68, 69], and the method has good estimation results in the linear range of the tire. To further extend the scope of application of this...
method, an analytical model based on cornering stiffness coefficient to identify TRFC at large slip angles was developed in Ref. [70]. The use of yaw rate and aligning torque information to estimate TRFC was also presented in Refs. [71–73] respectively.

To reduce reliance on in-vehicle sensor information, Rajamani et al. [74] only used a differential global positioning system (DGPS) to identify TRFC. Erdogan et al. [75] utilized a new tire force measurement device to estimate TRFC without using information from the braking system. Although using less sensor information to estimate TRFC can help reduce costs in some normal driving conditions, the accuracy of TRFC estimation may be decreased in some complex driving situations. An integrated TRFC estimation strategy [76–78] was proposed using sensor fusion techniques to address this problem.

In addition, due to the coupling relationship between the vehicle state and TRFC, some scholars usually carry out the identification of sideslip angle and TRFC simultaneously to improve the estimation accuracy. An RLS algorithm for estimating the vehicle sideslip angle and TRFC was developed in Refs. [79, 80]. The Kalman filter as a special form of RLS has obvious advantages in dealing with estimation problems with measurement noise. Considering the sensor measurement noise interference and the nonlinearity of vehicle dynamics, an estimation method based on the double extended Kalman filter (EKF) was presented in Refs. [81, 82]. The unscented Kalman filter avoids solving the Jacobian matrix and can obtain higher estimation accuracy than EKF. Hence, the UKF was used for the estimation of TRFC [83]. However, these Kalman-based methods are only valid for Gaussian-distributed noise. For non-Gaussian and nonlinear systems, particle filters have high estimation accuracy. Liu et al. [84] proposed a prediction method combining auxiliary particle filter and iterative estimator and verified the effectiveness of the algorithm by real vehicle test.

On the other hand, methods using state observers are often reported such as extended Luenberger observer [85], online gradient descent algorithm [86], nonlinear observer [87, 88], high-order sliding mode observer [89], etc. The observer-based approach usually has a certain range of applicability, and an adaptive observer [90, 91] for all road conditions was proposed to solve this problem. To compare the various methods more clearly, the estimation methods based on vehicle lateral dynamics are presented in Table 2.

### 2.2.3 Coupled Dynamics-based Methods

The above studies only considered vehicle longitudinal or lateral dynamics, which may result in a serious underestimation of the TRFC [92]. To increase the estimator working range, hybrid estimators began to be proposed one after another. The general framework of hybrid estimators is shown in Figure 5. This hybrid estimator improves the estimation accuracy by designing some hybrid algorithms to weigh the estimation results from different modules. Shim et al. [93] proposed a fusion method

| Number | Models                     | Methodology                        | References |
|--------|----------------------------|-----------------------------------|------------|
| 1      | Single-track vehicle model | Analytical model                  | [71]       |
| 2      | Brush model                | Analytical model                  | [68]       |
| 3      | Single-track vehicle model | Parameter identification algorithm | [74]       |
| 4      | Lateral tire forces        | Analytical model                  | [75]       |
| 5      | Seven DOF vehicle model    | Multi-sensor signal fusion method  | [76–78]    |
| 6      | Single-track vehicle model | RLS                               | [79, 80]   |
| 7      | Single-track vehicle model | Analytical model                  | [69]       |
| 8      | Brush model                | Analytical model                  | [72]       |
| 9      | Three DOF vehicle model    | Dual extended Kalman filter        | [81, 82]   |
| 10     | Nonlinear vehicle model    | Switched multiple nonlinear observer | [90]     |
| 11     | Single-track vehicle model | Analytical model                  | [70]       |
| 12     | Random-walk model          | Extended Luenberger observer      | [85]       |
| 13     | Single-track vehicle model | Iteration estimator               | [84]       |
| 14     | Seven DOF vehicle model    | Online gradient descent algorithm  | [86]       |
| 15     | Single-track vehicle model | Nonlinear observer                | [87, 88]   |
| 16     | Single-track vehicle model | Nonlinear adaptive observer       | [91]       |
| 17     | Hypothetical brush model   | Direct model inversion            | [73]       |
| 18     | Single-track vehicle model | High-order sliding mode differentiator. | [89] |
| 19     | Single-track vehicle model | Unscented Kalman filter           | [83]       |
based on steering angle information to estimate TRFC. Villagra et al. [94] used new algebraic filtering techniques to consecutively estimate tire forces and TRFC. Ahn et al. [95] developed an integration logic to switch among the developed algorithms based on the nature and level of excitations. Ren et al. [96] designed an integrated estimator to predict TRFC on basis of information about longitudinal, lateral, and yaw motions.

To ensure that the estimator can adapt to complex and variable working conditions, a moving horizon estimation strategy [97] was proposed to estimate TRFC. In addition, time-domain-based signal fusion methods [98, 99] have also proven to be effective in dealing with the TRFC estimation problem. However, time-domain-based signal fusion methods usually have degraded estimation performance at small acceleration conditions. A frequency-domain data fusion was proposed to estimate the TRFC based on the natural frequencies of the steering system and the in-wheel motor driving system [100]. RLS-based methods were also proposed to dynamically predict TRFC based on longitudinal and lateral tire forces in Refs. [92, 101, 102]. Since Kalman filtering has a significant advantage over RLS in dealing with the estimation problem with measurement noise. Considering the nonlinearity of vehicle dynamics, an identification method based on the EKF was presented in Refs. [103, 104]. To reduce the influence of old measurement data on the filtering in the EKF algorithm, a limited-memory adaptive extended Kalman Filter [105] was proposed to solve the problem. Also, UKF [106] can obtain higher accuracy when dealing with nonlinear system state estimation, and it has also been used for TRFC estimation in recent years. Furthermore, due to the bad adaptability of traditional Kalman filters to variable system structure, an improved Strong Tracking UKF [107] was constructed to identify the TRFC. To reduce the workload required for mathematical derivations of the Kalman filtering method, a nonlinear state observer was proposed to estimate TRFC [108, 109]. In addition to some of the improved fusion strategies discussed above, neural network-based fusion methods [110] were also an interesting research direction. To compare the various methods more clearly, the estimation methods based on vehicle coupled dynamics are presented in Table 3. The advantage of the vehicle dynamics-based method is that the TRFC can be estimated using on-board sensors; the estimation cost is low, and the real-time performance is effective. The estimated

![Image](image_url)

**Table 3** Summary of estimation methods based on vehicle coupled dynamics

| Number | Models               | Methodology                          | References |
|--------|----------------------|--------------------------------------|------------|
| 1      | Four wheel vehicle model | Analytical model                      | [93]       |
| 2      | Four wheel vehicle model | Unscented Kalman filter               | [106]      |
| 3      | Kinematic model       | Analytical model                      | [94]       |
| 4      | Single-track vehicle model | Analytical model                     | [95]       |
| 5      | Brushed tire model    | Linearized RLS                        | [92]       |
| 6      | Three DOF vehicle model   | Extended Kalman filter               | [103]      |
| 7      | Three DOF vehicle model   | Integrated estimator                | [96]       |
| 8      | Planar vehicle model   | RLS                                  | [102]      |
| 9      | Three DOF vehicle model   | Signal fusion method                | [98]       |
| 10     | Four-DOF vehicle model  | Extended Kalman filter               | [104]      |
| 11     | Planar vehicle model   | MSE-weighted fusion method           | [99]       |
| 12     | Three DOF vehicle model   | RLS                                  | [101]      |
| 13     | Fourteen DOF vehicle model   | Multilayer perceptron neural network | [110]      |
| 14     | Active front steering model   | Frequency domain data fusion         | [100]      |
| 15     | Planar vehicle model   | Nonlinear observer                  | [108]      |
| 16     | Three DOF vehicle model   | Limited-memory adaptive EKF          | [105]      |
| 17     | Kinematic model        | Nonlinear observer                   | [109]      |
| 18     | Three DOF vehicle model   | Moving horizon estimation strategy   | [97]       |
| 19     | Seven DOF vehicle model  | Improved strong tracking UKF         | [107]      |
TRFC can meet the needs of advanced chassis control with guaranteed vehicle model accuracy. It is well known that many assumptions and corresponding mathematical simplifications need to be made before building a vehicle dynamics model. Some idealized assumptions will increase the inaccuracy of the vehicle model and thus affect the accuracy of TRFC estimation, especially for some extreme driving conditions.

2.3 Data Driven-based Approaches
To compensate for the shortcomings of the vehicle dynamics-based method, neural networks were used to describe the tire and wheel suspension behavior [111]. A genetic algorithm optimized neural network is then employed to identify the TRFC. A similar idea was also reported in Ref. [112]. Zhang et al. [113] developed a mapping from input parameters to TRFC using general regression neural network, which effectively avoids storing complex tire models. Ribeiro et al. [114] employed the time-delay neural network to estimate TRFC can avoid using the standard tire mathematical model which makes the estimation method more robust. In addition, other neural networks have also been applied to the estimation of TRFC such as gated recurrent unit (GRU) network [115], deep neural network [116], BP neural network [12], deep convolutional neural network [17], etc. Furthermore, the time series characteristics of TRFC are not considered in the above data-driven estimation method. A long-short term memory (LSTM) neural network [117] was developed to address this problem. In general, with the enhancement of the computing power of the central processing unit, data-driven methods have received more and more attention. However, it should be noted that the prediction accuracy of such methods relies on the completeness of the dataset, and a comprehensive dataset is usually more difficult to obtain in practice. In addition, the generalization capability of the data-driven approach may further affect its applicability.

3 Summary and Perspectives
In this article, we review and compare typical TRFC estimation approaches. Three types of TRFC estimation methods have been systematically assessed and summarized. Although many outcomes have been achieved in TRFC estimation, some interesting points should be noted for future research. The first and foremost idea is to combine the advantages of various methods to improve the estimation accuracy of TRFC. Secondly, with the advent of the Internet of Things era, intelligent connected vehicles are gradually moving from the laboratory to public roads. Vehicles can exchange information with various traffic elements to obtain the friction coefficient of surrounding roads. In addition, with the development of prediction theory, predicting road friction in the future period is becoming a reality. Based on the above discussion, future studies on TRFC estimation are shown in Figure 6.

The development of technology and theory allows us to use advanced sensors and algorithms to estimate TRFC and gradually improve the estimation accuracy from various aspects. Some future research directions on TRFC estimation are as follows.

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**Figure 6** Future research directions on TRFC estimation
(1) Off-board sensors-based estimation methods can obtain road friction information without any excitation, the disadvantage is that it is easily disturbed by the environment. Although the dynamics-based estimation method has strong robustness to environmental interference, it needs suitable excitation to obtain the ideal estimation performance. Furthermore, in the vehicle dynamics-based approaches, the simplification of the model often leads to a decrease in the accuracy of TRFC estimation. A data-driven approach can effectively solve the problem but its performance is heavily dependent on the integrity of the dataset. Therefore, it is necessary to combine different types of estimation methods with well-designed fusion rules to obtain good prediction performance. For example, the effective integration of vehicle dynamics-based approaches with sensing approaches to improve estimation accuracy is one of the popular research directions.

(2) With the development of mobile communication technology, vehicles can exchange data with various elements in the intelligent transportation system, including other vehicles, Internet gateways, and transport infrastructure. When some sensors on the vehicle fail, it can exchange information with other vehicles to obtain TRFC. In addition, the vehicle can also modify the estimated value of TRFC by exchanging data with the camera on the signal light. This means that the road friction of the adjacent road of the vehicle can also be obtained, which will help the driver to plan the driving route reasonably. Therefore, it is a promising research direction to integrate multiple information to predict the TRFC of local roads in a networked environment.

(3) Existing studies on TRFC estimation can only estimate the current road conditions based on the current sensor measurements and they cannot predict the future road conditions. Accurate prediction of TRFC in the next few days allows travelers and road managers to rationalize their trips and road maintenance activities, which contributes to the safety and efficiency of traffic. Combining historical road condition data with weather forecast data and using a data-driven method to estimate long-term TRFC is also an interesting research direction.

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Authors’ Contributions
GY was in charge of the whole trial; YW wrote the manuscript; JH and FW assisted the trial; HD, YY, YR, and CZ conducted proofreading and made some critical revisions. All authors read and approved the final manuscript.

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Competing Interests
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