Gravity from Discrete Interactions,
a laboratory of ideas for field theory

Manoelito M de Souza
Universidade Estadual do Sudoeste da Bahia *
Departamento de Ciências Exatas - Vitória da Conquista - BA - Brazil
and
Universidade Federal do Espírito Santo†‡
Departamento de Física - Vitória - ES - Brazil

Abstract

The observed accelerated expansion of the universe is an indication that somehow, in some conditions, gravity may become a repulsive interaction. The very concept of discrete interactions, compatible with General Relativity as an effective theory of averaged fields, can handle this and other pressing problems without the need of searching for ad hoc exotic sources. Semi-quantum models of discrete interactions based on quantum exchanges between point-like masses are a rich laboratory for enlightening field interactions and classical-to-quantum transitions in field theory. A very simple version is presented for exhibiting how naturally it makes contact with inflation, dark matter and dark energy issues.

1 Introduction

In few more years the scientific community will be celebrating the centennial year of the two great pillars of modern theoretical physics, Quantum Mechanics and General Relativity, without being able to make them compatible, harmonize them into a single scheme. Quantum Mechanics carries the idea of interaction discreteness realized through exchanges of a quantum interaction, a packet of energy and momentum, while General Relativity associates the gravitational interaction to the smooth geometry of a curved spacetime. This incompatibility is a serious problem because they are both unbeatable on fitting the experimental data with high accuracy on their respective domain of validity and applicability. General Relativity Overseeds the old Newtonian scheme reproducing its dynamics in its Newtonian limit, a non-relativistic weak field limit. There are great challenges facing General Relativity [1]. The observed accelerated expansion of the universe is an indication that somehow, in some conditions, gravity may become a repulsive interaction. The old question of the flat rotation curves of stars and galaxies is still unsettled. These questions have in common very large scales of distances which lead to the above mentioned weak field Newtonian limit[1]. In the Newtonian scheme there were no room for a repulsive gravity but General Relativity, assuming its validity on such scales of distance, can handle it in various ways but none without controversy. The Einstein field equations are the equality between, in one side, the geometric one representing the observational data, the field, the test mass acceleration, and, in the other side, the sources, the matter content represented by the stress-energy tensor. Feeding this scheme with the observed data for finding the matter content, an exotic fluid that fulfill this data, may be an easy but certainly dangerous procedure as it raises the Popper’s falsifiability question if there is no pre-established

---

*Permanent address
†actual address
‡manoelitosouza@yahoo.com

1The flat rotation curves occurs when the field is smaller than $10^{-9}m/s^2$ which, by the way, is of the same order of the Pioneer anomaly [2]. This may not be a mere coincidence.
criteria on the acceptability of what is found. The same observations are valid for the galactic flat rotation curves. Then we have the dark matter and the dark energy controversies. Inflation in cosmology, its nature, what powers it and what smoothly disconnects it is another controversy, or unsettled question in gravitation theory. These are certainly among the greatest riddles for the modern researcher in gravitation theory.

The objective of this letter is to point to a new remarkable fact that they can altogether be enlightened with just very simple ideas on discrete interactions and no further else assumption. For discrete interaction it is meant the instantaneous emission/absorption of a quantum of interaction triggered by the instantaneous absorption/emission of a quantum of interaction, with free, straight-line propagation between any two consecutive interactions, in a flat spacetime background. As an illustration we present here the simplest discrete interaction model for gravity between two point-like masses in a non-relativistic relative radial motion; it can easily be associated to a cosmological model of a spherically symmetric distribution of dust. Such kind of naive models, with no free parameters and great variety of levels of complexity, are useful laboratories of ideas in the search for a better understanding of field interactions and quantum to classical transitions. The resulting dynamics, strongly dependent on the distance scales and on the initial conditions, oscillates between repulsive and attractive phases. It starts with a repulsive exponential expansion that smoothly changes into a Newtonian phase to return later, again to a repulsive expansion. This rich scenario needs no extra input to develop besides the two point-like masses and discrete interactions. The well known success of General Relativity on high accurately fitting some experimental data does not stand against gravity being a discrete interaction since the very General Relativity \[3\], like Classical Electrodynamics, can be seen as an effective theory of averaged interactions in a discrete interaction context \[4, 5, 6\]. They are both finite discrete interaction theories; singularities and other well known problems are shown to be consequences of the averaging process in the field definition.

2 The model

Let us consider two point-like masses, M and m, in relative non-relativistic radial motion under a discrete gravitational interaction. \(m\) is a test mass (\(M \gg m\)) and its momentum and position (relative to \(M\)) are given by

\[
p_n = p_{n-1} + \Delta p_{n-1} = p_0 + \sum_{j=0}^{n-1} \Delta p_j, \tag{1}
\]

as the momentum only changes at discrete points when there is the emission/absorption of an interaction quantum, a graviton, and

\[
r_n = r_{n-1} + \Delta r_{n-1} = r_0 + \frac{p_0 \Delta t_{n-1}}{m} + \sum_{j=0}^{n-1} \frac{p_j \Delta t_j}{m}, \tag{2}
\]

as there is free straight-line propagation between any two consecutive interactions. The label \(n\) counts the number of discrete interactions starting from \(t_0\) at the initial position \(r_0\) with the initial momentum \(p_0\). With such discrete interactions a trajectory is piecewise-linear with vertices at the interaction events, where velocity, as the trajectory tangent vector, is not defined because the discrete interaction causes a sudden change of velocity. So the notion of acceleration, if not as an approximation, is meaningless since it is null in the free propagation period between two consecutive discrete interaction and is not defined at the vertices.

The kinematics

Among many other alternatives, we take for our model of classical discrete interaction

\[
\Delta t_n = \alpha x_n \tag{3}
\]
where $x_n = \frac{r_n}{r_0}$ and $\alpha$ is the kinematic coefficient that describes the system free propagation between any two consecutive interactions. This choice is justified assuming that, let us say, the graviton, propagates with the speed of light $c$ and, crudely, that the emission of an interaction quantum is triggered without delay by the absorption of an interaction quantum. Then, in the non-relativistic approximation

$$\Delta t_n \approx \frac{2r_n}{c} = \frac{2r_0}{c} x_n, \quad \alpha \approx \frac{2r_0}{c}. \quad (4)$$

**The dynamics**

The dynamics for our model will be chosen with an eye on the above mentioned non-relativistic weak field limit, which, after (3), implies on

$$\Delta p_n = \frac{\beta}{x_n}, \quad (5)$$

as the most immediate choice, since

$$\frac{\Delta p_n}{\Delta t_n} \approx \frac{dp_n}{dt_n} \quad (6)$$

requires

$$\frac{\beta}{\alpha x_n^2} \equiv -\frac{GMm}{r_n^2} = -\frac{GMm}{r_0^2} \frac{1}{x_n^2}, \quad (7)$$

which fixes the dynamic coefficient $\beta$ as

$$\beta = -\frac{2GMm}{cr_0}. \quad (8)$$

There is no free-to-be-adjusted constant in this toy model of discrete classical interactions between two point-like masses. However, we strongly remark that (3) is not the unique possible choice, even after (3), and neither the most appropriate considering that the Newton’s equation is just an approximation to a correct formulation of the problem. It is, nonetheless, interesting to see that discrete interaction can produce repulsive gravity even in such a quasi-Newtonian simple context.

On the other hand, (6) has a price as it requires an approximate continuity on $t_n$ and $p_n$ (grossly $\Delta t_n << 1$ and $\Delta p_n << 1$) which, with (3) and (5), leads to

$$\beta << x_n << \frac{1}{\alpha}, \quad (9)$$

that, for dimensional reasons, must be seen just as a qualitative indication of the existence of such a range of validity for the Newton’s law of gravity.

**A word of caution**

A word of caution, with respect to (4), is however necessary. We are dealing here with the gravitational interaction between two point-like mass. In practice this would imply on something like the gravitational interaction between two electrons, or neutrinos or some other elementary particle. For a long time still to come this would be a purely academic business! If $M$ is due to a number $N_M$ of point-like masses distributed on a region of radius $r_M$, with $r_M << r_0$, and uncorrelatedly interacting with $m$, then $\alpha = \frac{2c}{r_0}$ must be replaced by

$$\alpha = \frac{2r_0}{cN_M}. \quad (10)$$

Just for an order of magnitude, we remind that light takes about 8 minutes to reach Earth from the Sun which cannot, obviously, be the order of the $\Delta t_n$. $N_M$ for the Sun is such a huge number that most certainly we will never be able to measure such a tiny lapse of time $\Delta t_n$. On the other hand, in this non-relativistic approach $\alpha$ is the only place where $c$, the speed of light, will appear. We will
keep $N_M = 1$ everywhere and in the case we need to consider a real physical observation involving a composed object we just have to replace $c$ by $c N_M$:

$$c \Rightarrow c N_M$$  \hspace{1cm} (11)

In general we do not know $N_M$ but we can assume, in a gross approximation, that

$$N_M \propto M.$$  

A similar reasoning [9] must be considered at the moment that $m$ absorbs an interaction quantum and if $m$ is composed of $N_m$ point-like objects then the dynamic coefficient $\beta$ (or the probability that this absorption occurs) is proportional to $N_m$ and then to $m$. This justifies part of the Newton’s statement on the gravitational force (being proportional to the product of the masses).

3 Solving for the equation of movement

After (3) and (5), with $a \equiv \frac{p_0}{m r_0} = \frac{v_0}{r_0}$ and $b \equiv \frac{\beta}{\alpha m r_0} = -\frac{G M}{r_0^3} = -\frac{v_0^2}{2 r_0^2}$, (2) may be written as

$$x_n = 1 + \frac{\alpha}{m r_0} \sum_{j=0}^{n-1} (p_0 + \sum_{j_1=0}^{j-1} x_{j_1}) x_j = 1 + \sum_{j=0}^{n-1} (\alpha a + \sum_{j_1=0}^{j-1} b a^2) x_j,$$  \hspace{1cm} (12)

which can be solved by re-iterations but it is more convenient to use an expansion in series powers of $\alpha$

$$x_n = \sum_{s \geq 0} \alpha^s x_{n,s}, \quad x_{n,0} = 1,$$  \hspace{1cm} (13)

where $x_{n,s}$ are coefficients to be determined in terms of the initial conditions $r_0, p_0$ and of $n$. We find [9]

$$x_{n,0} = 1, \quad x_{n,1} = a {n \choose 1}, \quad x_{n,k} = 0 \quad \text{for} \quad k > \lfloor n/2 \rfloor$$

and for $k < \lfloor n/2 \rfloor$:

$$x_{n,2k} = B A^{k-1} {n \choose 2k} + 2(k-1)b BA^{k-2} {n \choose 2k-1} + \ldots$$  \hspace{1cm} (14)

$$x_{n,2k+1} = a A^{k} {n \choose 2k+1} + a b A^{k-1} (k A + B(k-1)) \left( \frac{n}{2k+1} \right) + \ldots,$$  \hspace{1cm} (15)

where the dots indicate decreasing sequences of $n$-binomial numbers. We have introduced, for convenience, the two constants $A$ and $B$, defined as

$$A \equiv a^2 + 2b = \frac{2}{m r_0^2} \left( \frac{p_0^2}{2m} - \frac{G M m}{r_0} \right) = \frac{v_0^2 - v_{E,0}^2}{v_0^2} = \frac{w_0^2}{v_0^2},$$  \hspace{1cm} (16)

$$B = A - b = \frac{w_0^2 + v_{E,0}^2}{2v_0^2},$$

where $v_0$ and $v_{E,0}$ are, respectively, the velocity and the escape velocity at the initial position $r_0$ in the standard Newtonian potential; $w_0^2 \equiv v_0^2 - v_{E,0}^2$. $A$ is clearly related to the system initial energy.
4 The \( n \)-dominant order interaction.

If we consider in each \( x_{n,s} \) only the contributions from the term with the highest power in \( n \), \( (n >> 1) \),

\[
\binom{n}{k} = \frac{n^k}{(k)!} + 0(n^{k-1})
\]

we have

\[
x_{n,2k} \approx A^{k-1}B \frac{n^{2k}}{(2k)!} \quad \text{for} \quad k > 0
\]

\[
x_{n,2k+1} \approx aA^k \frac{n^{2k+1}}{(2k+1)!} \quad \text{for} \quad k \geq 0,
\]

and then, from (13), assuming \( A \neq 0 \),

\[
x_n \approx 1 + \sum_{k \geq 1} \alpha^k A^{-1}B \frac{n^{2k}}{(2k)!} + \sum_{k \geq 0} \alpha^{2k+1} aA^k \frac{n^{2k+1}}{(2k+1)!} =
\]

\[
= 1 + \sum_{k \geq 1} B \frac{X_n^{2k}}{A (2k)!} + \sum_{k \geq 0} a \frac{X_n^{2k+1}}{\sqrt{A} (2k+1)!},
\]

with \( X_n = \alpha \sqrt{An} = \frac{2w_0 n}{c}, n >> 1 \), and \( \alpha \sqrt{A} = \frac{2w_0}{c} << 1 \). It is a good approximation to replace the partial sums by their respective asymptotic series,

\[
x_n \approx \frac{b}{A} + \frac{B}{A} \cosh X_n + \frac{a}{\sqrt{A}} \sinh X_n
\]

\[
r_n \approx \frac{GM}{w_0^2} (\cosh \phi_n - 1),
\]

with \( \phi_n \equiv X_n + \phi_0 = \frac{2w_0 n}{c} + \phi_0 \) and

\[
tanh \phi_0 = \frac{a \sqrt{A}}{B} = \frac{2v_0 w_0}{w_0^2 + v_0^2}.
\]

Replacing in (11) the sum by an integral, which is proved to be valid \( [9] \) at this order of approximation, leads to

\[
q \equiv p_n - k_n = p_0 - k_0, \quad \text{with} \quad k_n = 2mw_0(e^{\phi_n} - 1)^{-1}.
\]

\( q \), which has the physical dimension of linear momentum, is a conserved quantity as it does not change with \( n \). It would be kind of peculiar to have linear momentum conservation under gravitational interaction but actually it boils down \( [9] \) to energy conservation

\[
q = mw = m \sqrt{v_n^2 - \frac{2GM}{r_n}} = m \sqrt{v_0^2 - \frac{2GM}{r_0}} = \sqrt{2mE}.
\]

The dominant-order contributions reproduce the usual Newtonian picture of continuous interactions with its known interaction potential.
The “potential energy”

The “potential energy”, retrieved from the $q$ conservation

\[
U_n = \frac{q^2 - p_n^2}{2m} = \frac{(q - p_n)(q + p_n)}{2m} = \frac{k_n(2q + k_n)}{2m}
\]

, can be seen from a new perspective: we can be interested on knowing the zeroes of the potential, $2q + k_n = 0$ and $k_n = 0$. In this dominant order contribution approximation each condition equally leads, with (23), to the same point at infinity

\[
v_{E,0} = 0, \quad \text{or, equivalently} \quad 1/r_0 = 0,
\]

as expected once the potential is the old Newtonian one,

\[
U_n = \frac{m^2(v_n^2 - v_{E,n}^2) - p_n^2}{2m} = -\frac{GMm}{r_n^2}.
\]

But it is remarkable the possibility of having two distinct zeroes (this can happen with the inclusion of sub-dominant contributions) and then there is a place for a repulsive gravity too, as we will see next.

5 Sub-dominant contributions.

It is convenient now, for $w_0 \neq 0$, to combine the initial condition parameters $w_0$, $v_0$ ($v_0 > 0$) and $r_0$ (or $v_{E,0}$) into a single one $h$, $h \equiv \frac{v_0}{|w_0|}$

\[
h \equiv \begin{cases} 
\frac{v_0}{\sqrt{v_0^2 - v_{E,0}^2}} = \frac{1}{\sqrt{1-(\frac{v_{E,0}}{v_0})^2}} \geq 1, & \text{if } v_0 > v_{E,0} \\
\frac{v_0}{\sqrt{v_{E,0}^2 - v_0^2}} = \frac{1}{\sqrt{1-(\frac{v_0}{v_{E,0}})^2}} \geq 0, & \text{if } v_0 < v_{E,0}
\end{cases}
\]

(25)

As $h \geq 1$ is finite with the equality to 1 being only asymptotically reached for

\[
v_{E,0} \to 0 \quad \text{or} \quad r_0 \to \infty,
\]

we will be assuming from now on $h >> 1$ except when the opposite is explicitly stated.

Including now the next $n$-order contributions i.e. the terms with the second highest power of $n$ in each $x_{n,s}$,

\[
\binom{n}{k} = \frac{n^k - \binom{k}{2} n^{k-1}}{(k)!} + 0(n^{k-2})
\]

(26)

we have, instead of (13) and (15),

\[
x_{n,2k} = BA^{k-1} \frac{n^{2k} - k(2k-1)n^{2k-1}}{(2k)!} + 2(k-1)bBA^{k-2} \frac{n^{2k-1}}{(2k-1)!} + \ldots
\]

\[
= B \left( \frac{\sqrt{An}^{2k}}{(2k)!} - \frac{b}{\sqrt{A}} \left( \frac{\sqrt{An}^{2k-1}}{(2k-1)!} - \frac{na^2}{2} \frac{(\sqrt{An})^{2k-2}}{(2k-2)!} \right) \right) + \ldots
\]

and

\[
x_{n,2k+1} = aA^{k} \left( \frac{n^{2k+1} - k(2k+1)n^{2k}}{(2k+1)!} \right) + abA^{k-2} (kA + (k-1)B) \frac{n^{2k}}{(2k)!}
\]

\[
= a \left( \frac{(n\sqrt{A})^{2k+1}}{(2k+1)!} - \frac{bB}{A^2} \frac{(n\sqrt{A})^{2k}}{(2k)!} + \frac{nB^2}{2A} \frac{(n\sqrt{A})^{2k-1}}{(2k-1)!} \right) + \ldots
\]
and they lead, with (13), to

\[
x_n = T_0 + T_1 \cosh X_n + T_2 \sinh X_n + \frac{1}{2} X_n W (T_3 \cosh X_n + T_4 \sinh X_n)
\]  

(27)

where

\[
X_n = \alpha n \sqrt{A} = \frac{\nu_0}{\hbar c} n, \quad W = \alpha \frac{bB}{A \sqrt{A}} = \frac{w_0}{2c} \left(1 - h^4\right) < 0
\]

\[
T_0 = \frac{b}{A} + \frac{a}{\sqrt{A}} W = \frac{1}{2} (1 - h^2)(1 + \frac{\nu_0}{c} (1 + h^2)) < 0
\]

\[
T_1 = \frac{B}{A} - \frac{a}{\sqrt{A}} W = \frac{1}{2} (1 - h^2)(1 - \frac{\nu_0}{c} (1 + h^2)) > 0
\]

\[
T_2 = \frac{a}{\sqrt{A}} - W = h - \frac{\nu_0}{2hc} (1 - h^4) > 0, \quad T_3 = -\frac{a^2}{b} = -\frac{2h^2}{1 - h^2} > 0
\]

\[
T_4 = -\frac{Ba}{b\sqrt{A}} = -h \frac{1 + h^2}{1 - h^2} \approx h > 0
\]

The term \(\frac{aW}{\sqrt{A}} = -\frac{\nu_0}{c} \frac{(\nu_0^2 - w_0^2)}{2c^2} = \frac{\nu_0}{c} (1 - h^4)\) marks the sub dominant contributions; being proportional to \(\frac{\nu_0}{c}\) it shows the importance of the velocity of propagation of the signal (quanta) and suggests that the next order of interaction may require a proper relativistic approach. On the other hand \(\frac{\nu_0}{c} << 1\) does not assure \(|\frac{aW}{\sqrt{A}}| < 1\), as long as there is no a priori constraint on the size of \(h\).

**Higher n-order contributions**

The coefficients \(T_0, T_1\) and \(T_2\) carry contributions from both the dominant and the sub dominant \(n\)-orders and they will receive contributions from any other lower \(n\)-order that may be added. Considering the possibility of including the \(N\) first \(n\)-orders, (27) can be generalized to

\[
x_n = T_0(N) + \sum_{k=0}^{N} \frac{(WX_n)^k}{2k!} (T_{2k+1}(N) \cosh X_n + T_{2k+2}(N) \sinh X_n).
\]  

(28)

Every new \(n\)-order added adds contributions to every \(T\)-coefficient, including the \(T_0\) one. It is a peculiar aspect of these \(n\)-order expansions that the relevance of lower \(n\)-order terms increases as \(n\) grows in contraposition to the usual approximation expansions in terms of small coupling constants where each order of contribution is fixed. This is unexpected as in general the gravitational effects decrease with the distance; these non-Newtonian effects increase. As the distance keeps increasing, contributions from lower \(n\)-orders become more and more relevant. Probably there is a closed form, still to be found, for each \(T\)-coefficient as well as for the entire equation of which the expansion (28) would be just a partial sum. On the other hand there is no point on adding new contributions if they only become relevant with distances that go beyond our observation horizon.

### 6 Discrete versus continuous

Let us compare (21) and (27). They represent two distinct perspectives, two different approaches to the dynamics of gravity. (21) stands for the non-relativistic weak-field limit of General Relativity, for the vision of an interaction field as continuous as a smooth surface while (27) shows additional consequences of accumulated effects of successive discrete interactions. Let us now point to some striking differences between them.

While (21) describes just one phase evolution, (27) describes a multi-phase one. The dynamics is very sensitive to the initial conditions. \(|W| << 1\) reduces (27) to (21) as it turns the sub dominant
\( n \)-order contributions negligible. On the other hand as \( n \) (or equivalently the distance or the time) increases, the last two terms on the right-hand side of (27), proportional to \( WX_n \), and that we are calling the sub dominant contributions, increases \((WX_n \text{ times})\) faster and may compete or even dominate, radically changing the dynamics. Let

\[
|W| X_n \approx 1
\]  

(29)

mark the point where these two contributions grow even; for \(|W| X_n > 1\) the non-Newtonian effects increase faster than the Newtonian ones but for \(|W| X_n << 1\) (27) reduces to

\[
x_n \approx T_0 + T_1 \cosh X_n + T_2 \sinh X_n \quad \text{for} \quad |W| X_n << 1
\]

(30)

which is just (21) with different constants. The dynamics is strongly dependent on the initial conditions and, even after this simplification, it still encompass a great variety of physical situations of which we mention here some few. The same procedure applied to (21) leads now to a distinct conservation law

\[
q = p_n - \frac{\beta}{\alpha \sqrt{A W}} \ln \left( \frac{x_n - Z_+}{x_n - Z_-} \right) = p_0 - \frac{\beta}{\alpha \sqrt{A W}} \ln \left( \frac{1 - Z_+}{1 - Z_-} \right)
\]

with

\[
Z_\pm(T_1 + T_2) = -T_0 \pm \sqrt{T_0^2 + T_2^2 - T_1^2} = -T_0 \pm W
\]

(31)

and leads to a dominant logarithmic contribution to the potential

\[
U_n \approx \frac{q^3}{m W \sqrt{A}} \ln \left( \frac{x_n + \sqrt{x_n^2 - 2x_n T_0 + W^2 - W}}{x_n + \sqrt{x_n^2 - 2x_n T_0 + W^2 + W}} \right).
\]

(32)

For

\[
|W| << x_n
\]

(33)

both \( q \) and \( U_n \) reduce, in a first order approximation, to the Newtonian results, as expected.

**Gravitation between elementary particles**

The interesting result comes from the other extreme condition

\[
|W| >> x_n
\]

(34)

that reduces (22) to

\[
U_n \approx -\frac{q^3}{m a W \sqrt{A}} \ln \left( \frac{1 + 2W}{x_n} \right)
\]

(35)

whose gradient produces an effective “acceleration”

\[
a_n \approx -\frac{r_0}{m} \frac{dU_n}{dx_n} = -\frac{qGM}{W w_0 r_n}.
\]

An \( r_n \)-dependence like that could explain the flat rotation curves of galaxies. Besides, considering the conservative potential we can imagine, just to simplify, that a given star was carried to its actual state from an energetically equivalent initial position \( r_0^* \) and an initial velocity \( v_0 \approx 0 \), then the condition (34) reduces to

\[
\frac{GM}{2c^2 r_0^*} >> \left( \frac{r_n}{r_0^*} \right)^2
\]

or

\[
a_{n,N} >> \frac{2c^2}{r_0^*}
\]
where $a_{n,N} = GM/r_n^2$ is the Newtonian value for the star acceleration at $r_n$ which must be bigger than a reference value in a kind of twisted Milgron’s condition [8], in reference to an empirically found condition satisfied by most of flat rotation curves of galaxies. The nagging point is that plugging back the $N_M$ factor of (11) turns the condition (34) into

$$\left(\frac{v_0 h^3}{N_M c}\right)^2 \geq x^2_n$$

which can be realized only with few-components objects like elementary particles as they have small values of $N_M$ ($N_M = 1$), $x_n \approx 1$, excluding any macroscopic object, not to mention a star or galaxy. It is saying that the gravitational interaction between two elementary particles could, in principle (it depends on $h$, the initial conditions as the dynamics is very sensitive to them) radically differ from the one we know between macroscopic objects in usual conditions! Had we only a dream technology capable of measuring such discrepancies!

7 Discrete interaction and cosmology

The discrete interaction toy model considered in this paper can easily be associated to a Newtonian cosmological model of a spherically symmetrical distribution of dust of which $m$ is the mass of the point dust at $r_n$ and $M$ is the total mass of the dust spherically distributed inside the sphere of radius $r_n$. It is mostly a consensus that inflation, an early exponential expansion of the universe, is a necessary ingredient of big-bang cosmologies despite the many questions lingering on, waiting for consistent answers. Questions like, how did inflation started? powered by what? and how did it smoothly ended into the present known state of the universe? Similar questions can also be made with respect to the observed accelerated expansion of the universe, that seems to require an evolution system with a repulsive gravitation phase. Discrete interaction, we believe, have something to add on these issues.

Exponential expansion phases

In the dynamics described by (27) there are two main exponential expansion phases, an early and a late one. Let us start re-writing it as

$$x_n = T_0 + \left( T_1 + T_2 + X_n W \frac{T_3 + T_4}{2} \right) \frac{e^{X_n}}{2} +$$

$$+ \left( T_1 - T_2 + X_n W \frac{T_3 - T_4}{2} \right) \frac{e^{-X_n}}{2}$$

from which we can see that it describes an exponential expansion as long as

$$T_1 + T_2 + X_n W \frac{T_3 + T_4}{2} >> T_1 - T_2 + X_n W \frac{T_3 - T_4}{2}$$

or

$$X_n << \frac{2T_2}{|W|T_4}$$

and also when

$$X_n >> \frac{2(T_1 + T_2)}{|W|(T_3 + T_4)} \approx \frac{2T_1}{|W|T_4} >> \frac{2T_2}{|W|T_4}$$

as $T_1 >> T_2$ and $T_4 >> T_3$. It is natural that if we want to use this discrete interaction model for pursuing a more comprehensive understanding of gravity we must consider the whole (27) and not just (30). To repeat with (27) the same treatment given to (21) and to (30) is not as easy but we can do, for now, with numerical computation for a glance on its dynamics. See the figures below where we have used the command
Figure 1: 3 phases from an infinite periodical series of repulsion, attraction and repulsion.

\[ \text{Manipulate[Plot[f(x,h),\{x,x_{min},x_{max}\},\{h,h_{min},h_{max}\}]}] \]

of the software Mathematica 8.0 which is very useful as it allows the observation of the dynamics dependence on the initial conditions through the explicit variation of \( h \). For the Figure 1, showing the repulsive, attractive and again repulsive sequence of phases, we have plotted the "acceleration" \( v_n dv_n/dx_n \) against \( x_n \) for \( A < 0 \) and with the initial conditions partially fixed by taking \( v_0 = 1000 \text{m/s} \) and leaving \( r_0 \) free, hidden in the parameter \( h \) which was allowed to vary in the range from \( 10^{-3} \) to \( 10^5 \) and \( x \) (that stands here for \( X_n \)) from 0 to \( 2\pi \). Increasing the range of \( x \) would mean including distances beyond, perhaps, our observation horizon; for consistency this would require including more \( n \)-order contributions. A smaller \( h \) implies, for a fixed initial velocity, a larger escape velocity which means, for a fixed \( M \), a smaller initial position, while a larger \( h \) implies \( v_{E,0} \) closer to \( v_0 \). For comparison, using (21) instead of (27), for the same initial conditions, produces Figure 2 and an always negative Newtonian "acceleration"

\[
v_n dv_n/dx_n = -4.5 \times 10^6(1 + \frac{1}{h^2})(\cos 0.5X_n + h \sin 0.5X_n)^{-4} \approx -4.5 \times 10^6 x_n^{-2} < 0
\]

in its unique attractive phase.

We should observe that in both figures the scales of \( X_n \) in the horizontal axis are not linear with the scale of distance \( x_n \); they are related by (21) and (27), respectively. In the notation of Mathematica, with the chosen initial condition, we write (27) as

\[
x_n = ((1 + h^2)/2 + 10^{-5}(1 - h^4)/2) + \\
((1 - h^2)/2 - 10^{-5}(1 - h^4)/2 - h10^{-5}x(1 - h^2)/2) \cos[x] + \\
(h - 10^{-5}(1 - h^4)/2 + x10^{-5}(1 - h^2)^2/4) \sin[x].
\]

Changing \( h \) changes the overall aspect of the plot but not the existence of changes of sign in the "acceleration" as it is naturally dependent on the initial conditions. The choice of the snapshot taken at \( h = 360 \) just reflects the author’s esthetic bias without any further deep physical concern.
8 Conclusions

The objective of this paper of pointing to the relevance of discrete interaction models as laboratory of ideas in the search for a better comprehension of field interaction and of its discrete to continuous or quantum to classical transitions has been accomplished. Although being based on a very simple classical model of discrete interactions for gravity and being mirrored on the non-relativistic weak field limit of General Relativity which is just an approximation to a correct formulation of the problem it makes contact with actual major issues in the physics of gravity. Most remarkable, of course, is the natural appearing of repulsive gravity in a simple quasi-Newtonian context. The next question is how far can it go.

References

[1] Perlmutter, S., et al 1999 Ap. J 517,565
Komatsu, E. et al. 2009 Astrophys. J. Suppl. 180 330
Riess, A. G. et al.1998 Astron. J 116,1009

[2] Anderson J. D., Laing P. A., Lau E. L. Liu A. S., Nieto M.M., Turyshev S. G., Study of the anomalous acceleration of Pioneer 10 and 11, gr-qc/0104064.

[3] Souza M M and Silveira R N 1999 Class. & and Quantum Gravity 16 619. Preprint gr-qc/9801040

[4] Souza M M 1997 J. of Phys. A: Math.and Gen. 30 6565-6585. Preprint hep-th/9610028

[5] Souza 1999 Braz. J. of Phys. 28 n. 3, 250. Preprint hep-th/9505159

[6] Souza M M 2002 Brazilian Journal of Physics 32 no. 2B. Preprint hep-th/0006214

[7] Segatto B R, Azevedo J C S and Souza M M 2003 J. Phys. A: Math.and Gen. 36, 5135 Preprint gr-qc/0301081

[8] Milgrom M.Ap. J.270, 365(1983).

[9] Souza M M. To be published.