Cosmological implications of the Higgs mass measurement

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Abstract. We assume the validity of the standard model up to an arbitrary high-energy scale and discuss what information on the early stages of the Universe can be extracted from a measurement of the Higgs mass. For $M_h \lesssim 130$ GeV, the Higgs potential can develop an instability at large-field values. From the absence of excessive thermal Higgs field fluctuations we derive a bound on the reheat temperature after inflation as a function of the Higgs and top masses. Then we discuss the interplay between the quantum Higgs fluctuations generated during the primordial stage of inflation and the cosmological perturbations, in the context of landscape scenarios in which the inflationary parameters scan. We show that, within the large-field models of inflation, it is highly improbable to obtain the observed cosmological perturbations in a Universe with a light Higgs. Moreover, independently of the inflationary model, the detection of primordial tensor perturbations through the $B$ mode of CMB polarization and the discovery of a light Higgs can simultaneously occur only with exponentially small probability, unless there is new physics beyond the standard model.

Keywords: string theory and cosmology, inflation, physics of the early universe

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1. Introduction

The search for the Higgs boson and the measurement of its properties are one of the primary goals of the LHC. The mere discovery of a Higgs can be viewed as a possible indication of additional new physics not far from the electroweak scale, because of the high sensitivity to short-distance quantum corrections of the mass term associated with a fundamental scalar. However, even in the absence of any new-physics discovery at the LHC, a measurement of the Higgs mass can give us useful hints on the structure of the theory at very high energies. This is because, at large-field values, the Higgs potential can develop an instability or become non-perturbative, depending on the precise value of the Higgs quartic coupling $\lambda$ or, ultimately, on the Higgs mass $M_h$. Because of the logarithmic dependence of $\lambda$ on the energy, such considerations [1]–[5] can test the properties of the theory up to extremely small distances, which are otherwise totally inaccessible to any imaginable collider experiment.

In this paper, we want to use the same considerations for a different purpose. Rather than trying to infer new particle physics properties at small distances, we will assume the validity of the standard model (SM) up to an arbitrary high-energy scale, and find what information on the early stages of the Universe can be extracted from a measurement of the Higgs mass. We will first obtain that, in the Higgs mass range $114 \text{ GeV} \lesssim M_h \lesssim 130 \text{ GeV}$, where the electroweak vacuum is potentially metastable, the absence of excessive thermal Higgs field fluctuations in the early Universe imposes a bound on the reheat temperature after inflation $T_{RH}$.

Then we will discuss the interplay between the quantum Higgs fluctuations generated during the primordial stage of inflation and the cosmological perturbations which are either currently observed in the form of cosmic microwave background (CMB) anisotropies, or might be detected in the near future in the form of tensor (gravity waves) perturbations. The key ingredient is that all these fluctuations depend upon the Hubble rate during
inflation and therefore, under certain assumptions, it is possible to relate the amount of Higgs fluctuations to observable properties of the CMB. However, excessive fluctuations of the Higgs field during inflation can pose a threat to the stability of the present vacuum, if the Higgs mass lies in the metastability window.

We will assume that the various initial inflationary patches of the Universe are characterized by different microphysical parameters. In this sense we take the point of view that the underlying theory has many vacua, realized in different patches of the Universe. This picture, usually referred to as the landscape [6], has been put forward especially in the context of string theory. Under this assumption, from CMB observations and from a measurement of the Higgs mass, we can derive probabilistic conclusions on the properties of our Universe.

In particular, within the class of large-field models of inflation, we will compute the probability to have a Universe at the end of inflation which both survived the quantum Higgs fluctuations and has the right amount of observed cosmological perturbations. Such a probability is extremely (exponentially) small, if the Higgs mass is below 124 GeV (for the present central value of the top mass). Moreover, we find that the discovery of a light Higgs together with the detection of primordial tensor perturbations through the $B$ mode of CMB polarization (at a level quantitatively described in section 6) would imply that we live in a very atypical Universe, whose probability decreases exponentially when the Higgs mass decreases. Such a discovery could be interpreted as indirect evidence for the existence of new physics beyond the standard model at some intermediate energy scale.

This paper is organized as follows. In section 2 we briefly review the Higgs mass instability window. In section 3 the bounds on the reheating temperature after inflation are discussed. In section 4 we compute the survival probability of the electroweak vacuum during inflation and in sections 5 and 6 we relate it to the curvature and tensor perturbations, respectively. Section 7 states our conclusions. The appendix contains technical details relevant to the calculation of the Higgs mass instability window.

2. The Higgs mass instability window

Let us start by reviewing the range of Higgs masses for which the electroweak vacuum is metastable. Since our considerations refer to large-field values, for our purposes it is perfectly adequate to neglect the bilinear term and to approximate the potential of the real Higgs field $h$ as

$$V(h) = \frac{\lambda(h)}{4} h^4.$$  \hfill (1)

We will work in the next-to-leading-order approximation and include the field dependence of the quartic coupling $\lambda$, determined by the two-loop renormalization-group (RG) equations, as well as the one-loop corrections to the effective potential (as in [3]). The couplings $\lambda$ and the top-quark Yukawa coupling $h_t$, entering the RG evolution, are then related to the physical Higgs and top pole masses. The explicit expressions are given in the appendix.

The request that the electroweak vacuum $\langle h \rangle = v$, with $v = (\sqrt{2}G_F)^{-1/2} = 246.22$ GeV, is the true minimum of the potential, up to a cutoff scale $\Lambda$, implies $\lambda(\mu) > 0$ for any $\mu < \Lambda$. This condition is not satisfied at some energy scale whenever $M_h$ is below
Figure 1. The instability scale $\Lambda$ as a function of the Higgs mass $M_h$ for three different values of the top mass $M_t$.

some critical value $M^c_h$ given by

$$M_h < M^c_h = 125.4 \text{ GeV} + 3.8 \text{ GeV} \left( \frac{M_t - 170.9 \text{ GeV}}{1.8 \text{ GeV}} \right) - 1.6 \text{ GeV} \left( \frac{\alpha_s(M_Z) - 0.1176}{0.0020} \right) \pm 2 \text{ GeV}. \quad (2)$$

We have explicitly shown the dependence on the two most important SM parameters, normalizing their effects in units of one standard deviation from their experimental central value, taking $M_t = 170.9 \text{ GeV} \pm 1.8 \text{ GeV}$ [7] and $\alpha_s(M_Z) = 0.1176 \pm 0.0020$ [8]. Besides the uncertainties in the SM parameters, equation (2) has an overall error due to higher-order corrections which, parametrically, are expected to shift the result by an amount $O(\alpha_s^2 M_h/\pi^2)$. However, since the two-loop QCD correction to the top-quark pole mass (included in our calculation and shown in the appendix) has a large coefficient, we conservatively estimate the theoretical error to be 2 GeV. Figure 1 shows the instability scale $\Lambda$, at which the quartic coupling $\lambda$ becomes negative, as a function of the Higgs mass for three different values of the top mass. At scales larger than $\Lambda$, the Higgs potential becomes negative, and then it develops a new minimum. The endpoints of the lines, marked with a square, correspond to $M^c_h$ and to the scale at which the new minimum of the Higgs potential characterized by a large vacuum expectation value becomes degenerate with the electroweak one. For most of the relevant values of the top and Higgs masses, the instability scale $\Lambda$ is sufficiently smaller than the Planck mass, justifying the hypothesis of neglecting effects from unknown Planckian physics.
A lower bound on $M_h$ is obtained by considering the instability of the electroweak vacuum under quantum tunneling. Since the vacuum transition is dominated by late times, this bound is independent of the early history of the Universe. The tunneling probability $p$ is given by

$$p = \max_{h<\Lambda} V_U h^4 \exp \left( -\frac{8\pi^2}{3|\lambda(h)|} \right).$$  \hspace{1cm} (3)

We have also included one-loop corrections to the bounce configuration [2], which, however, have only a small impact on the final bound on $M_h$, contributing less than a GeV. In equation (3), $V_U$ is the space–time volume of the past light cone of the observable Universe and we take $V_U = \tau_U^4$, where $\tau_U$ is the lifetime of the Universe. Taking $\tau_U = 13.7 \pm 0.2$ Gyr from WMAP data [9], the metastability limit $p < 1$ imposes the bound on the Higgs mass [2]

$$M_h > 105.6 \text{ GeV} + 5.2 \text{ GeV} \left( \frac{M_t - 170.9 \text{ GeV}}{1.8 \text{ GeV}} \right) - 2.5 \text{ GeV} \left( \frac{\alpha_s(M_Z) - 0.1176}{0.0020} \right) \pm 3 \text{ GeV}. \hspace{1cm} (4)$$

The error of 3 GeV is estimated by combining uncertainties from higher-order corrections and from the prefactor in $p$.

The bound in equation (4) is useful only when it is stronger than the direct experimental limit on the Higgs mass [10]

$$M_h > 114.4 \text{ GeV} \quad \text{at 95\% CL.} \hspace{1cm} (5)$$

In summary, equations (2), (4) and (5) define the Higgs mass window (see figure 2) in which the electroweak vacuum is metastable and therefore potentially sensitive to large-field fluctuations during the early stages of the Universe.

3. Bound on the reheating temperature

At sufficiently large temperatures, thermal fluctuations in the early Universe plasma can trigger the decay of the metastable electroweak vacuum [12, 4, 13] by nucleation of bubbles that probe the Higgs instability region. On the other hand, high-temperature effects also modify the Higgs effective potential, with a tendency of making the origin more stable. The contribution of the different plasma species to the potential (or, rather, free energy) in the non-interacting gas approximation is given by standard one-loop (bosonic/fermionic) thermal integrals. Each particle species, with $h$-dependent mass $M_\alpha(h)$, contributes to the free energy

$$\delta_\alpha V(h) = \frac{T^4}{2\pi^2} N_\alpha \varepsilon_\alpha \int_0^\infty dx \, x^2 \log \left[ 1 - \varepsilon_\alpha e^{-x^2 + M_\alpha^2(h)/T^2} \right]$$

$$+ \frac{T}{12\pi} \varepsilon_\alpha N_\alpha \left\{ M_\alpha^2(h) - \left[ M_\alpha^2(h) + \Pi_\alpha(h, T^2) \right]^{3/2} \right\}, \hspace{1cm} (6)$$

where $N_\alpha$ counts the number of degrees of freedom, $\varepsilon_\alpha = +1(-1)$ for bosons (fermions) and $\Pi_\alpha(h, T^2)$ is the thermally corrected mass of the corresponding species (see, e.g., [14]). The second line in equation (6) takes into account the effect of resumming hard-thermal
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Figure 2. Lower bounds on $M_h$ from absolute stability (upper curves) and $T = 0$ metastability (lower curves). The width corresponds to $\alpha_s(M_Z) = 0.1176 \pm 0.0020$ (with the higher curve corresponding to lower $\alpha_s$) and we do not show the uncertainty from higher-order effects, which we estimate to be below 2-3 GeV. The horizontal line is the LEP mass bound.

loops for Matsubara zero modes. For our numerical work we used a series expansion of these integrals in terms of modified Bessel functions [15], avoiding high $T$ expansions.

The energy $E_c(T)$ of the smallest critical bubble large enough to grow (overcoming the surface tension penalty) controls the false vacuum decay rate through a Boltzmann suppression factor $\exp \left[ -E_c(T)/T \right]$. The quantity $E_c(T)$ is computed by solving for the $O(3)$ bounce solution [12] using the finite $T$ potential described above. It is easy to show [4] that, parametrically, $E_c(T)/T \sim \pi g/|\lambda(T)|$.

The vacuum decay rate per unit volume is

$$\Gamma(T) \simeq T^4 \left[ \frac{E_c(T)}{2\pi T} \right]^{3/2} \exp\left[ -E_c(T)/T \right]. \quad (7)$$

The differential decay probability $dP/d\ln T$ is obtained by multiplying $\Gamma(T)$ above by the volume of the Universe at temperature $T$ and the time spent at that $T$. In a radiation-dominated Universe one has

$$\frac{dP}{d\ln T} \simeq \Gamma(T) \tau_0^3 \frac{M_p}{T^2} \left( \frac{T_0}{T} \right)^3, \quad (8)$$

where $T_0 \simeq 2.73$ K $\simeq 2.35 \times 10^{-4}$ eV and $M_p = 1.2 \times 10^{19}$ GeV is the Planck mass. The previous result assumes $T$ is smaller than the reheating temperature after inflation, $T_{RH}$. 
For temperatures $T > T_{RH}$, the previous result gets modified to

$$\frac{dP}{d \ln T} \simeq \Gamma(T) T^3 \left( \frac{T_{RH}}{T} \right)^{10} \left( \frac{T_0}{T_{RH}} \right)^3,$$

(9)

to take into account the period of expansion during inflaton dominance before reheating is completed. The final decay probability $P$, resulting from integrating in $d \ln T$ the previous expressions, is not a properly normalized probability. Its interpretation is that the fraction of space converted to the true vacuum goes like $e^{-P}$.

For a given value of the Higgs mass in the metastability window of figure 2 the requirement that the false vacuum does not decay during the high $T$ stages of the early Universe (that is, $e^{-P} \lesssim 1$) will set an upper bound on $T_{RH}$. In fact, $T_{RH}$ is not the maximal temperature achieved after inflation. Such maximal temperature occurs after inflation ends and before reheating completes and is given by [16]

$$T_{\text{max}} = \left( \frac{3}{8} \right)^{2/5} \left( \frac{5}{\pi^7} \right)^{1/5} \left( \frac{g_*(T_{RH})}{g_*(T_{\text{max}})} \right)^{1/8} M_p^{1/4} H^{1/4}_{f} T_{RH}^{1/2},$$

(10)

where $g_*(T)$ counts the effective number of degrees of freedom (with a $7/8$ prefactor for fermions) with masses $\ll T$ and $H_f$ is the Hubble parameter at the end of inflation. The metastability bound on $T_{RH}$ therefore depends on the particular value of $H_f$: for a given $T_{RH}$, the value of $T_{\text{max}}$ grows with $H_f$. Therefore the metastability constraint on $T_{RH}$ will be more stringent for larger values of $H_f$.

Figure 3 shows this metastability bound on $T_{RH}$ as a function of the Higgs mass for various values of the top mass and for two choices of the Hubble rate $H_f$ at the end of inflation. The lower curves correspond to $H_f = 10^{13}$ GeV while the upper ones have $H_f = [4\pi^3 g_*(T_{RH})/45]^{1/2}(T_{RH}^2/M_p)$, the lowest value allowed once it is required that the inflaton energy density $\rho = 3M_p^2 H_f^2/(8\pi)$ is larger than the energy density of a thermal bath with temperature $T_{RH}$:

$$T_{RH} < \left[ \frac{45}{4\pi^3 g_*(T_{RH})} \right]^{1/4} M_p^{1/2} H_f^{1/2}.$$

(11)

This condition also corresponds to the requirement that the inflaton lifetime $\Gamma_{\phi}^{-1}$ is larger than the Hubble time $H_f^{-1}$. The bound on $T_{RH}$ gets weaker for smaller values of the top mass (or larger values of the Higgs mass) since the instability scale becomes higher and eventually, for [13]:

$$M_h > 117.4 \text{ GeV} + 4.2 \text{ GeV} \left( \frac{M_t - 170.9 \text{ GeV}}{1.8 \text{ GeV}} \right) - 1.6 \text{ GeV} \left( \frac{\alpha_s(M_Z)}{0.1176} \right) \pm 3 \text{ GeV},$$

(12)

the bound on $T_{RH}$ is lost: the vacuum is sufficiently long-lived even for $T \sim M_p$ (that is, $\int_{T_0}^{M_p} (dP/d \ln T) \ln T < 1$). This is the reason why the lines in figure 3 stop at some value of $M_h$. From this figure it is also clear that the bound on $T_{RH}$ is very sensitive on the value of $M_t$, with the experimental error in $M_t$ being the main source of uncertainty.

It is natural to ask what implications the bound on $T_{RH}$ shown in figure 3 has for leptogenesis. SM thermal leptogenesis, with hierarchical right-handed neutrinos, sets a
Figure 3. Upper bounds on $T_{RH}$, as functions of $M_h$, from sufficient stability of the electroweak vacuum against thermal fluctuations in the hot early Universe for three different values of the top mass. The lower curves are for $H_f = 10^{13}$ GeV, the upper ones for $H_f$ deduced from equation (11), $H_f = [4\pi^3 g_*(T_{RH})/45]^{1/2}(T_{RH}/M_h)$, which corresponds to the case of instant reheating. We take $\alpha_S(M_Z) = 0.1176$. Lowering (increasing) $\alpha_S(M_Z)$ by one standard deviation lowers (increases) the bound on $T_{RH}$ by up to one order of magnitude.

The Yukawa couplings $h_{\nu}$ of the heavy right-handed neutrinos could in principle affect the bound on $T_{RH}$, since $h_{\nu}$ can modify the instability scale of the Higgs potential [19] with its effect on the evolution of $\lambda$ above the $M_1$ threshold. Because $h_{\nu}^2 = m_\nu M_1/v^2$, such effects turn out to be important only if the mass of the right-handed neutrinos is sufficiently large, $M_1 \gtrsim (10^{13}-10^{14})$ GeV [19]. Therefore, the existence of heavy right-handed neutrinos can modify the bounds on $T_{RH}$ we have obtained only at such large energy scales, i.e. for $T_{RH} > M_1 \gtrsim (10^{13}-10^{14})$ GeV.

4. Survival probability of the electroweak vacuum during inflation

In the previous section we have discussed the stability of the electroweak vacuum against thermal fluctuations. These are expected to drive the Higgs field towards the instability lower bound on $T_{RH}$ as a function of $M_1$, the mass of the lightest right-handed neutrino [17]. This bound reaches its minimum for $M_1 \sim T_{RH}$, when $T_{RH} > 3 \times 10^9$ GeV [18]. This condition could be in conflict with the upper bound on $T_{RH}$ shown in figure 3, if the Higgs mass turns out to be very close to the LEP lower limit and if the top mass is on the high side of the allowed experimental range. However we stress that these considerations apply only to the case of hierarchical thermal leptogenesis in the SM, with no new physics present below the scale $M_1$. The Yukawa couplings $h_{\nu}$ of the heavy right-handed neutrinos could in principle affect the bound on $T_{RH}$, since $h_{\nu}$ can modify the instability scale of the Higgs potential [19] with its effect on the evolution of $\lambda$ above the $M_1$ threshold. Because $h_{\nu}^2 = m_\nu M_1/v^2$, such effects turn out to be important only if the mass of the right-handed neutrinos is sufficiently large, $M_1 \gtrsim (10^{13}-10^{14})$ GeV [19]. Therefore, the existence of heavy right-handed neutrinos can modify the bounds on $T_{RH}$ we have obtained only at such large energy scales, i.e. for $T_{RH} > M_1 \gtrsim (10^{13}-10^{14})$ GeV.
region if \( T_{\text{RH}} \) is larger than the critical temperature shown in figure 3. If reheating is an instantaneous process, so that \( H_{f} \approx g_{1}^{1/2} (T_{\text{RH}}^{2}/M_{p}) \), then the upper bound on \( T_{\text{RH}} \) (which in this case coincides with \( T_{\text{max}} \)) can be translated into an upper bound on \( H_{f} \).

We obtain that (for \( M_{t} = 170.9 \text{ GeV} \)) \( H_{f}/\Lambda \approx 0.1 \) for \( M_{h} = 115 \text{ GeV} \) and \( H_{f}/\Lambda \approx 10^{4} \) for \( M_{h} = 117 \text{ GeV} \). Not only does this limit on \( H_{f} \) become quickly weaker as \( M_{h} \) is increased inside the metastability window, but it also completely evaporates if reheating is a prolonged process with \( T_{\text{max}} \gg T_{\text{RH}} \). Moreover, during inflation the Hubble rate could be (much) larger than its value at the end of inflation. Therefore, even if thermal fluctuations do not destabilize the electroweak vacuum, the ratio \( H/\Lambda \) during inflation can be much larger than one. It is thus interesting to address the issue of what is the fate of the electroweak vacuum when we account for the Higgs fluctuations generated during inflation, whose amplitude is directly proportional to the Hubble rate and can probe the instability region.

In the inflationary cosmology picture, the present structure in the Universe is supposed to originate from vacuum fluctuations which were quantum mechanically excited during an inflationary stage [20, 21]. Indeed, any scalar field whose mass is lighter than \( H \), where \( H \) denotes the Hubble rate during inflation, gives rise to an almost scale-invariant spectrum of perturbations on superhorizon scales [22, 23]. The Higgs field, if light enough during inflation, does not represent an exception. However, this requires nonvanishing couplings between the inflaton and the Higgs and a minimal coupling to gravity, or else a Higgs mass of the order of the Hubble rate is induced. We will start by considering the case in which the Higgs field is minimally coupled to gravity and therefore it can be considered nearly massless during inflation, as long as \( H \gg M_{h} \). At the end of section 5 we will discuss the interesting case in which there is a direct coupling between the Higgs bilinear and the Ricci scalar.

If the Higgs mass lies in the metastable window (114 GeV < \( M_{h} \approx 130 \text{ GeV} \)), excessive fluctuations of the Higgs field may pose a threat to the stability of the electroweak vacuum. In the inflationary picture, long wavelength perturbations of the Higgs field may be generated and they behave in the same way as a homogeneous classical field. We suppose that, in the beginning of inflation, the whole inflationary domains are characterized by a very homogeneous Higgs field with vanishing vacuum expectation value. Then the domains become exponentially large. The Higgs quantum fluctuations divide the Universe into exponentially large regions with different values of the Higgs field: the Universe becomes quickly filled with domains in which \( h \) typically changes from \(-\langle h^{2} \rangle^{1/2} \) to \( \langle h^{2} \rangle^{1/2} \). Here \( \langle h^{2} \rangle \) is the variance of the Higgs field which we will estimate in the following.

The process of generating a classical Higgs field configuration in the inflationary Universe can be interpreted as the result of the Brownian motion of the Higgs scalar field under the action of its quantum fluctuations which are converted into the classical field when their wavelengths overcome the horizon length. The best way to describe the structure of the Higgs fluctuations is provided by the stochastic approach in which one defines the distribution of probability \( P_{c}(h, t) \) to find the Higgs field value \( h \) at a given time at a given point [24]. Perhaps a more adequate approach might be based on the distribution of probability \( P_{p}(h, t) \) to find the Higgs field value \( h \) at a given time in a given physical volume [25] which takes into account the exponential growth of the volume of the various domains. Both distributions of probabilities are plagued by various problems (for a recent discussion on this issue related to the idea of eternal inflation, see [26]).
comoving probability $P_c$ at a given point ignores the possible creation of new volumes (new points) during inflation; the physical probability suffers from normalizability problems. In this paper we infer our results by making use of the comoving probability $P_c$ which is technically more feasible. The subscript $c$ serves to indicate that $P_c$ corresponds to the fraction of original comoving volume filled by the Higgs field $h$ at time $t$. The comoving probability satisfies the Fokker–Planck equation

$$\frac{\partial P_c}{\partial t} = \frac{\partial}{\partial h} \left[ \frac{H^3}{8\pi^2} \frac{\partial P_c}{\partial h} + \frac{V'(h)}{3H} P_c \right].$$

(13)

In writing this equation we have supposed that the Higgs field gives a negligible contribution to the energy density of inflation and that the probability does not sensitively depend upon the value of the inflaton field. The first assumption always holds in the relevant region of Higgs field configurations and we will come back to the second point later on.

To solve equation (13) we assume that the Hubble rate is approximately constant during inflation. We denote from now on by $\Lambda$ the value of the Higgs field at which the maximum of the potential $V(h)$ occurs. It is very close to the value of the energy at which the Higgs potential vanishes, shown in figure 1. In those patches where the Higgs field takes any value larger than $\Lambda$, the Higgs field rapidly rolls down towards the region where the potential is negative, $V(h) = -|\lambda(h)|h^4/4$. Therefore, the domain where the barrier has been surmounted will correspond to bubbles of AdS space which will rapidly collapse and disappear. To account for this process we impose that the solution of equation (13) satisfies the boundary condition

$$P_c(\Lambda, t) = 0.$$ 

(14)

Physically, this condition dictates that, once the unstable region of the Higgs potential is probed, the probability to jump back to the stable region vanishes. As $\partial P_c/\partial h$ does not vanish at $h = \Lambda$, the probability that the Higgs field is in the stable region $h < \Lambda$ will decrease with time. Since $P_c(h, t)$ gives the fraction of the comoving volume occupied by regions inside which the Higgs field takes the value $h$ at a given time $t$, we expect that this fraction will become smaller and smaller as time goes by during the inflationary stage.

To investigate the amount of Higgs fluctuations during inflation, we can also parameterize them in terms of the Higgs correlations determined with the help of the comoving probability given by equation (13):

$$\frac{d}{dt} \langle h^m \rangle = \frac{H^3}{8\pi^2} m(m - 1) \langle h^{m-2} \rangle - \frac{m}{3H} \langle h^{m-1} V'(h) \rangle + \frac{H^3 \Lambda^m}{4\pi^2} P'_c(\Lambda),$$

(15)

where $m$ is an even integer, the prime denotes derivatives with respect to $h$ and

$$\langle h^m \rangle = \int_{-\Lambda}^{\Lambda} dh h^m P_c(h, t).$$

(16)

We consider the case in which $P_c(h, t)$ is an even function of $h$, so that any Higgs correlation functions with odd $m$ identically vanish.

For constant $H$, equation (13) can be solved by the separation of variables:

$$P_c(h, t) = \sum_{n=0}^{\infty} c_n e^{-(\alpha V + H^3 a_n t)/8\pi^2} \Phi_n(h), \quad \alpha = \frac{8\pi^2}{3H^2},$$

(17)
where $\Phi_n$ and $a_n$ are the eigenfunctions and the eigenvalues of the equation

$$\Phi''_n - \alpha V'\Phi'_n = -a_n \Phi_n.$$

(18)

Since $V(h)$ is an even function of $h$ and equation (18) does not mix even and odd eigenfunctions, we look for solutions which are even, in the range $-\Lambda < h < \Lambda$. Moreover, since the condition in equation (14) has to be satisfied at any time $t$, we have to impose $\Phi_n(\Lambda) = 0$, for any $n$.

We assume that the Higgs field is initially localized at $h = 0$, $P_c(h, 0) = \delta(h)$, and study its evolution. We have solved the Fokker–Planck equation numerically, but it is useful to give here approximate analytic solutions in order to describe our results.

(i) Case $H \gg \Lambda$. Let us first consider the most interesting case in which the Hubble rate is much larger than $\Lambda$. In this case the potential term in equation (18) can be neglected since $\alpha V \ll \lambda (\Lambda) H^4 \ll 1$, once we use the condition $|h| < \Lambda$. This means that the loss of probability caused by the AdS instability dominates the dynamics, while the effects from the Higgs potential are negligible. Then the solution of the Fokker–Planck equation satisfying the appropriate boundary conditions is

$$P_c(h, t) = \frac{1}{L} \sum_{n=0}^{\infty} e^{-\left(\frac{n+1/2}{\Lambda}\right)^2 \left(\frac{H^3t}{8\Lambda^2}\right)} \cos \left(\frac{n + \frac{1}{2}}{\Lambda}\right).$$

(19)

Notice that, in the limit of large $\Lambda$, the probability distribution in equation (19) reduces to a Gaussian

$$\lim_{\Lambda \to \infty} P_c(h, t) = \sqrt{\frac{2\pi}{H^3 t}} e^{-2\pi^2 h^2 / H^3 t},$$

(20)

as can be easily obtained by switching from discrete to continuous variables $(n + 1/2)\pi/\Lambda \to k$ and by integrating over $k$.

The survival probability $P_\Lambda$ for the Higgs to remain in the region $|h| < \Lambda$ and the variance of the Higgs field are given by

$$P_\Lambda(t) \equiv \int_{-\Lambda}^{\Lambda} dh P_c(h, t) = \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{(-)^n}{n + 1/2} e^{-\left(\frac{n+1/2}{\Lambda}\right)^2 \left(\frac{H^3t}{8\Lambda^2}\right)},$$

(21)

$$\langle h^2 \rangle \equiv \int_{-\Lambda}^{\Lambda} dh h^2 P_c(h, t) = \frac{2\Lambda^2}{\pi} \sum_{n=0}^{\infty} \frac{(-)^n}{n + 1/2} \left[1 - \frac{2}{(n + 1/2)^2 \pi^2}\right] e^{-\left(\frac{n+1/2}{\Lambda}\right)^2 \left(\frac{H^3t}{8\Lambda^2}\right)}.$$

(22)

At small $t$, equations (21) and (22) become

$$P_\Lambda(t) \simeq 1 - \sqrt{\frac{H^3 t}{2\pi^3 \Lambda^2}} e^{-2\pi^2 \Lambda^2 / H^3 t} \quad \text{(small $t$)},$$

(23)

$$\langle h^2 \rangle \simeq \frac{H^3 t}{4\pi^2} \quad \text{(small $t$)}.$$

(24)

At the initial stages, the loss of probability is very small and $P_\Lambda$ is exponentially close to one, while $\langle h^2 \rangle$ starts growing linearly with time. To study how fast the probability

4 One can show that $P_c(h, 0) = \delta(h)$ using $\sum_{n=0}^{\infty} \cos[(n+1/2)x] = \pi \delta(x)$. 
decays with time, we can consider equations (21) and (22) in the limit of large $t$:

$$P_\Lambda(t) \approx \frac{4}{\pi} e^{-H^3 t/32\Lambda^2} \quad (H \gg \Lambda, \text{ large } t), \quad (25)$$

$$\langle h^2 \rangle \approx \frac{4(\pi^2 - 8)\Lambda^2}{\pi^3} e^{-H^3 t/32\Lambda^2} \quad (H \gg \Lambda, \text{ large } t). \quad (26)$$

The survival probability $P_\Lambda$ and the Higgs correlation function exponentially decay with time. The normalized Higgs variance, which measures the correlation function within the surviving domains, tends asymptotically to a constant,

$$\langle h^2 \rangle \rightarrow \left(1 - \frac{8}{\pi^2}\right)\Lambda^2 \approx (0.44 \, \Lambda)^2. \quad (27)$$

The asymptotic behavior shown in equations (21) and (22) is valid for $t \gg \frac{\Lambda^2}{H^3}$ and therefore it is reached very rapidly, justifying the assumption of neglecting the time dependence of the Hubble constant.

From equation (22) we can also obtain the maximum value of the Higgs variance:

$$\langle h^2 \rangle_{\text{max}} \approx (0.39 \, \Lambda)^2, \quad (29)$$

which is obtained at the time $t \approx 10.6 \, \Lambda^2/H^3$.

(ii) Case $\Lambda \gg H$. At the first stages of inflation, the first term in equation (18) dominates and the results given in equations (23) and (24) for small $t$ are valid also in the case $\Lambda \gg H$. At later times the shape of the probability distribution changes because the second term in equation (18) becomes relevant. To estimate when the effect of the Higgs potential is important, we study equation (15) for the variance of the Higgs field ($m = 2$):

$$\frac{d}{dt} \langle h^2 \rangle = \frac{H^3}{4\pi^2} \left[ P_\Lambda + \Lambda^2 P'_c(\Lambda) \right] - \frac{2\lambda}{H} \langle h^2 \rangle^2. \quad (28)$$

Here we have neglected the contribution to the correlators from the Higgs dependence in the Higgs quartic coupling and we have adopted a Hartree–Fock approximation for the nonlinear term by taking $\langle h V'(h) \rangle \approx 3\lambda \langle h^2 \rangle^2$, where $\lambda$ is evaluated at the scale $\sqrt{\langle h^2 \rangle}$. From equation (28) we deduce that the Higgs variance starts growing linearly with time, $\langle h^2 \rangle = H^3 t / (4\pi^2)$, and then it reaches a maximum value which can be estimated by equating the right-hand side of equation (28) to zero:

$$\langle h^2 \rangle_{\text{max}} \approx \frac{H^2}{2\pi \sqrt{2\lambda}}. \quad (29)$$

This value is reached at a number of e-foldings approximately given by $N \sim H t \sim \pi \sqrt{2/\lambda}$. Therefore this maximum is reached very promptly which, also in this case, justifies neglecting the time dependence of the Hubble rate. At times $t \gtrsim 1/(\sqrt{\Lambda H})$, the friction term starts being relevant in equation (13). In the absence of the AdS instability region, the comoving probability would reach a stationary form $P_c(h) \propto \exp[-8\pi^2 V(h)/3H^4]$, i.e. $a_0 = 0$ and $\Phi_0 = 1$ in equation (17). However, in the presence of the AdS region, this solution has to be modified to account for the loss of the probability. The stationary solution does not satisfy the boundary condition in equation (14) but, for $\Lambda \gg H$, $P_c(\Lambda)$ is exponentially close to zero. Therefore we can obtain the correct solution by perturbing around the stationary solution valid for $\Lambda \rightarrow \infty$, and we find $\Phi_0(h) \approx 1 - I(h)/I(\Lambda)$ and
\[a_0 \simeq 1/I(\Lambda),\]

where

\[I(h) = \int_0^h dh' e^{-\alpha V(h')} \int_{h'}^h dh'' e^{\alpha V(h'')} \tag{30}\]

At late times, the survival probability decays with time as \(P_\Lambda(t) \propto e^{-\gamma t}\), with

\[\gamma = \frac{H^3 a_0}{8\pi^2} \simeq \frac{2\sqrt{\pi}}{15/4} \left( \frac{2\lambda}{3} \right)^{5/4} \Lambda^3 \frac{H^2 e^{-8\pi^2 V(\Lambda)/3H^4}}{\Lambda^3} \tag{31}\]

Here we have integrated equation (30) neglecting the field dependence of the quartic Higgs coupling \(\lambda\) and we have taken the limit \(\Lambda \gg H\). In the more relevant case in which the Hubble rate is changing with time, we have numerically checked that the survival probability decays as \(e^{-\int t \, dt' \gamma(t')}\), where one can approximately use the expression of \(\gamma\) in equation (31) simply plugging in the appropriate time dependence of the Hubble rate. This is also true in the case \(H \gg \Lambda\), using the time dependence shown in equation (25).

5. The survival probability and the comoving curvature perturbation

From the results of the previous section we deduce that, if \(H \gg \Lambda\), an exponentially small fraction of the initial comoving volume survives the Higgs quantum fluctuations during inflation. The number of surviving domains will be suppressed by \(\exp(-H^2 N/32\Lambda^2)\), where \(N\) is the total number of e-folds. May we conclude that our Universe is very unlikely? The answer is no. Suppose, indeed, that to quantify the probability of ending up in a ‘well-behaving’ domain (the vacuum has the correct Higgs vacuum expectation value) we simply count the fraction of domains where the value of the Higgs field is less than \(\Lambda\). This fraction is given by the comoving survival probability. However, those regions where the value of the Higgs field has become larger than the critical value during the inflationary stage simply do not exist. As soon as the instability point is reached, those regions collapse and disappear. Only the ‘well-behaving’ regions remain and their corresponding fraction is unity.

Can we then conclude that the parameter space where \(P_\Lambda\) is exponentially small is acceptable? As we will show, the answer is again no, at least in a probabilistic sense. Indeed, in the class of inflationary models dubbed large-field models, the ‘well-behaving’ domains, where the vacuum expectation value of the Higgs is correct, typically have an insufficient amount of curvature perturbations generated during inflation. In the inflating domains that are characterized by the correct amount of curvature perturbations, the Hubble rate, and therefore the square root of the Higgs variance, turns out to be much larger than the scale \(\Lambda\). Therefore, among the surviving regions, the ones having the right amount of curvature perturbations will be exponentially rare.

Observations of the CMB anisotropies are consistent with a smooth and nearly Gaussian power spectrum of curvature perturbations with an amplitude

\[\zeta_{\text{obs}} \simeq 5 \times 10^{-5}. \tag{32}\]

Inflation predicts such a spectrum with amplitude

\[\zeta = \frac{H^2}{2\pi \phi_e}, \tag{33}\]
where $H_*$ and $\phi_*$ are the value of the Hubble rate and the inflaton field, respectively, when there are about 60 e-folds to go till the end of inflation. The dot stands for differentiation with respect to time.

Let us focus on the large-field models of inflation. We parameterize their potential by

$$V(\phi) = \frac{\mu^{4-p}}{p} \phi^p,$$

with $p$ a positive integer. This determines the amplitude of the curvature perturbations

$$\zeta = \left( \frac{4N_*}{\pi p} \right)^{1/2} \frac{H_*}{M_p},$$

where $N_*$ is the number of e-folds till the end of inflation when the observable scales leave the horizon\(^5\). Notice, in particular, that the perturbation is directly proportional to the Hubble rate. This will play a crucial role in the following because larger values of the Hubble rate, and therefore larger values of $\zeta$, tend to destabilize the electroweak vacuum.

In the case in which $H \gg \Lambda$, the survival probability becomes

$$P_\Lambda \propto \exp \left[ -\int_0^t dt' \frac{H^3(t')}{32\Lambda^2} \right] \simeq \exp \left[ -\frac{H_i^2 N}{16(p+2)\Lambda^2} \right],$$

where $H_i$ is the Hubble rate at the beginning of inflation and $N$ is the total number of e-folds. Using equation (35) and $(H_i/H_*) = (N/N_*)^{p/4}$, the survival probability can be expressed in terms of the total number of e-folds and the comoving curvature perturbation:

$$P_\Lambda \propto \exp \left[ -\frac{\pi p}{p+2} \left( \zeta M_p \right)^2 \left( \frac{N}{N_*} \right)^{(p+2)/2} \right].$$

We now take a crucial step and assume that initially the Universe is characterized by local domains where the inflationary parameters may assume any value. As mentioned in the introduction, this assumption is inspired by the string landscape picture [6], in which the theory has an enormous number of vacua, each determining different values for the underlying parameters. This hypothesis agrees especially well with the point of view followed in this paper of ignoring the hierarchy problem associated with the Higgs mass and extending the validity of the SM up to very high-energy scales. Indeed, this approach could be justified in the context of a landscape scenario in which the Higgs mass parameter scans among the different vacua. In practice, in the case under consideration, we allow for the possibility that the various initial comoving patches of the Universe are characterized by different microphysical parameters, such as the parameter $\mu$ in equation (34) and the initial value of the inflaton field $\phi_i$. These two parameters determine the other cosmological variables

$$N = \frac{4\pi \phi_i^2}{p M_p^2}, \quad H_i^2 = \frac{8\pi \mu^{4-p} \phi_i^p}{3p M_p^2}, \quad \zeta = \sqrt{\frac{2}{3}} \frac{N_*^{(p+2)/4}}{\pi^{p/4}} \left( \frac{2\mu}{\sqrt{p M_p}} \right)^{(4-p)/2}.$$ 

\(^5\) The number of e-folds to go till the end of inflation for those scales which exit the horizon and are today relevant for observation depends on the reheating temperature $T_{RH}$ through the relation $N_* \approx 60 + 1/6 \ln(-n_T) + 1/3 \ln(T_{RH}/10^{16} \text{ GeV}) - 1/3 \ln \gamma$ [21], where $n_T$ is the spectral index of tensor perturbations and $\gamma$ is the ratio of the entropy per comoving volume today to that after reheating.
Using equation (38), we trade the parameters $\phi_i$ and $\mu$ for the two more physical quantities $N$ and $\zeta$. The way the inflationary parameters are distributed at the beginning of inflation among these different regions is unknown and we take it to be simply flat in $N$ and $\zeta$. Our results are not very sensitive to this assumption (as long as the parameter distributions are not sharply peaked), because the probability function we are interested in has an exponential dependence on our variables. Among the comoving patches which have survived the Higgs instability, the probability that the total number of e-folds is between $N$ and $N + dN$ and that the comoving curvature perturbation is between $\zeta$ and $\zeta + d\zeta$ is $P_\Lambda(\zeta, N) d\zeta dN$, with $P_\Lambda(\zeta, N)$ given in equation (37).

The range in which the parameters scan is, in principle, arbitrary. We can restrict it using anthropic priors, although this is not essential for our conclusions. If perturbations are too large, $\zeta \gtrsim 10^{-4}$, they become nonlinear when the average density of the Universe is too high to allow the resulting structures to guarantee a stable environment for life [27]. On the opposite side, if the perturbations are too small, $\zeta \lesssim 10^{-6}$, the majority of the overdense regions are not capable of cooling quickly enough to form structures [27]. Therefore, we scan the comoving curvature perturbations only over the anthropically allowed region $10^{-6} \lesssim \zeta \lesssim 10^{-4}$.

The probability in equation (37) is dominated by the smallest value of $\zeta$. Integrating over the total number of e-folds, the probability becomes

$$P_\Lambda(\zeta) \propto E_{p/(p+2)}(\alpha) \simeq \frac{e^{-\alpha}}{\alpha}, \quad \alpha \equiv \frac{\pi p}{p + 2} \left( \frac{\zeta M_p}{8\Lambda} \right)^2. \quad (39)$$

Here $E_n(x)$ is the exponential integral function and we have expanded for large values of the parameter $\alpha$, which is appropriate since equation (37) is valid only for $H \gg \Lambda$ (i.e. $\alpha \gg 1$). When $\alpha < 1$, $P_\Lambda(\zeta)$ should be replaced by a constant. The probability $P_\Lambda(\zeta)$ is peaked at the smallest value of the cosmological perturbation $\zeta$. Therefore, the fraction of domains whose value of the cosmological perturbation is as large as the measured $\zeta_{\text{obs}}$ is extremely small, even if we impose the prior of restricting our considerations only to values of $\zeta$ within the narrow anthropic range:

$$\frac{\int_{\zeta > \zeta_{\text{obs}}} d\zeta P_\Lambda(\zeta)}{\int_{\zeta > 10^{-6}} d\zeta P_\Lambda(\zeta)} \simeq \left( \frac{10^{-6}}{\zeta_{\text{obs}}} \right)^3 \exp \left[ -\frac{\pi p}{p + 2} \left( \frac{\zeta_{\text{obs}} M_p}{8\Lambda} \right)^2 \right]. \quad (40)$$

We recall that equation (40) is valid for $H \gg \Lambda$ which holds in the anthropic window for $\zeta$ as long as $\Lambda < 10^{12}$ GeV. In figure 4 we plot this ratio as a function of the Higgs mass and for different values of the top mass and for $p = 2$. As expected, if the Hubble rate $H_\star$ is large enough to account for $\zeta$, see equation (35), the fraction of volumes which survived the quantum Higgs fluctuations is extremely small, unless the mass of the Higgs is close to (or larger than) the critical $M_\chi^c$ of equation (2) shown by the upper curves in figure 2.

Let us now discuss how our results change if we adopt a physical distribution $P_p(h, t)$ instead of the comoving one $P_c(h, t)$. As we already mentioned, $P_p(h, t)$ describes the probability of finding the Higgs field value $h$ at a given time in a physical volume and takes into account the exponential growth of the volume of the various inflationary patches. This amounts to multiplying the comoving probability by $\exp(3N)$, where $N$ is the total number of e-folds. Putting aside the technical problems related to the normalizability of
The quantity $\log_{10}\left[ \int_{\zeta > \zeta_{\text{obs}}} d\zeta P_\Lambda / \int_{\zeta > 10^{-6}} d\zeta P_\Lambda \right]$ as a function of the Higgs mass for three values of $M_t$ and $p = 2$.

From this expression it is clear that, when $H_i^2 \gg 48(p + 2)\Lambda^2$, the term in the exponential accounting for the dynamics of the Higgs fluctuations dominates over the one due to the physical volume expansion. Also, when $\zeta$ and $N$ are used as independent scanning variables, see equation (37), the Higgs term $\exp[-\alpha(N/N_*)^{1+p/2}]$ has a steeper dependence on $N$ than the volume term $\exp(3N)$, when $N$ grows. This implies that even the physical survival probability is pushed towards small values of the number of e-folds $N$ and our results are not modified by using $P_p$ rather than $P_c$. Because we have found that statistics prefer small $N$ (or equivalently small $\phi_i$), we can avoid problems related to the computation of the physical probability when this is maximized for large values of the inflaton field. Indeed, for values of the inflaton field $\phi \gtrsim \mu(M_p/\mu)^{6/(p+2)}$, the inflaton evolution is quantum rather than classical, giving rise to eternal inflation [25]. The computation of the physical probability to get a certain value of the cosmological perturbation $\zeta$ in a physical volume would become therefore a complicated task as the Brownian motion of the inflaton field described by the Fokker–Planck equation has to be taken into account. In [28] it is argued that the volume factor $\exp(3N)$ statistically favors small values of $\zeta$. This conclusion is based on the requirement that the inflaton evolution remains classical and this determines a maximum value of the number of e-folds $N_{\text{max}} \sim (M_p/\mu)^{2(4-p)/(p+2)} \sim \zeta^{-4/(p+2)}$, which selects small $\zeta$. This requirement is not well
motivated on physical grounds and the quantum regime might modify the parametric dependence of the probability on the perturbation $\zeta$. In contrast, in our case the Higgs fluctuations enhance the probability of domains with small values of $\phi_i$, away from the region dominated by quantum evolution.

So far, we have discussed the case in which inflation is driven by an inflaton field with a polynomial potential. There are two other classes of inflationary models, called small-field and hybrid models [21], where the potential is made of a constant vacuum energy plus a field-dependent term responsible for the slow roll. In such a case the connection between the Hubble rate and the cosmological perturbation is lost. For instance, in a model with potential $V(\phi) = V_0 + m^2 \phi^2/2$, the Hubble rate during inflation is roughly $H_* \approx (\zeta \phi_f/N_*)$, where $\phi_f$ is the value of the inflaton field at which inflation stops. Therefore, one can have a large probability of having domains with the correct amount of perturbation, but values of the Hubble rate are easily smaller than the instability point $\Lambda$ by simply choosing $\phi_f$ small enough. However, as will be discussed in the next section, if a future measurement of gravity waves through the $B$ mode of the CMB polarization indicates that the Hubble rate is sizable, even within these classes of models, one would have to conclude that we live in an unlikely Universe.

Finally, we study the case in which we relax the assumption that the Higgs is minimally coupled to gravity. Indeed, the Higgs field might be coupled nonminimally to gravity through a term in the Lagrangian $|\xi| R h^2/2$, where $\xi$ is a numerical coefficient and $R$ is the Ricci scalar. During inflation this term gives a contribution to the Higgs mass equal to $12|\xi| H^2$ (since $R = -12 H^2$). If $0 < \xi \ll 10^{-1}$, the Higgs field is practically massless during inflation and our conclusions do not change. However, when $|\xi|$ is sizable, our results are drastically modified. If $\xi \gg 10^{-1}$, the electroweak vacuum is stable and the Higgs fluctuations are damped exponentially because of the large positive effective mass. However, if $\xi < 0$ (as in the case of conformal coupling, where $\xi = -1/6$), the electroweak vacuum is destabilized by a large tachyonic mass even without inflationary fluctuations. In figure 5 we show the upper bound on $\sqrt{|\xi|} H$, in the case of negative $\xi$, obtained by imposing that the electroweak vacuum is not destabilized by the large negative mass squared induced by the $|\xi| R h^2/2$ term. Here $H$ has to be understood as the maximal Hubble rate during inflation. Note that this bound does not depend on the statistical considerations developed in this section, since it follows directly from the stability of the electroweak vacuum in our patch of the Universe.

One could also envision the case in which there is a direct coupling between the inflaton and the Higgs fields, say of the form $\phi^2 h^2$. During inflation, this can play the role of a Higgs mass term, which can stabilize or destabilize the electroweak vacuum, depending on the sign of the corresponding coupling constant, in analogy with the case of the Higgs–curvature interaction.

6. Implications for gravity wave signals

The considerations presented in the previous sections can have direct observational consequences once we realize that the Hubble rate parameterizes the amount of tensor perturbations during inflation. During the inflationary epoch, tensor perturbations, as for any other massless scalar field, are quantum mechanically generated. They can give rise to $B$ modes of polarization of the CMB radiation through Thomson scatterings of
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Figure 5. Upper bound on $\sqrt{|\xi|H}$ as a function of $M_h$ obtained by requiring that the electroweak vacuum is stable. Here $\xi$ is the negative coupling between the Higgs bilinear and the scalar curvature, and $H$ is the maximal Hubble rate during inflation.

the CMB photons off free electrons at last scattering \cite{29}. The amplitude of the $B$ modes depends on the amplitude of the gravity waves generated during inflation, which in turn depends on the energy scale at which inflation occurred. The tensor-to-scalar power ratio is often defined as $T/S$ and is given by $T/S \simeq (H_s/6.6 \times 10^{13} \text{ GeV})^2$. Current CMB anisotropy data impose the upper bound $T/S \lesssim 0.6$ \cite{9,30}.

The possibility of detecting gravity waves from inflation via $B$ modes is currently being considered by a number of ground-, balloon- and space-based experiments. The decomposition of the CMB polarization into $E$ and $B$ modes requires a full sky data coverage and, as such, is limited by the foreground contamination. The latter introduces a mixing of the $E$ polarization into $B$ with the corresponding cosmic variance limitation. Achieving levels below $T/S = 10^{-4}$, or $H_s \lesssim 7 \times 10^{11} \text{ GeV}$, requires observing 70% of the sky with dust emission at 0.01% level \cite{31}.$^6$

Detection of the tensor mode would therefore imply that the value of the Hubble rate during inflation is larger than about $10^{12} \text{ GeV}$. This will have important implications for the Higgs mass within all inflationary models. In the vast class of models where the potential is dominated by a constant vacuum energy and the Hubble rate $H_s$ during inflation is roughly constant (including the small-field and the hybrid models of inflation) the regions which survived the Higgs fluctuations are simply distributed as

$$P_\Lambda \propto e^{-H_s^2N/32\Lambda^2}.$$  \hspace{3.5cm} (42)

$^6$ Gravitational lensing also contaminates the tensor signal by converting the dominant $E$ polarization into $B$ polarization. Cleaning this contamination by reconstructing the lensing potential from CMB itself, one can achieve values of $T/S$ as small as $10^{-6}$ \cite{32}.

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Figure 6. The probability $P_\Lambda$ versus the Higgs mass normalized with its value at $M_h'$ for the Hubble rate $H_* = 10^{13}$ GeV (upper curves) or $H_* = 5 \times 10^{13}$ GeV (lower curves), $N = N_\star = 60$ and for three different values of the top mass. This probability is dominated by the smallest value of the number of e-folds and by the smallest value of the Hubble rate. Along the lines of the previous section, we conclude that a successful detection of gravity waves with a value of $H_*$ larger than about $10^{12}$ GeV, in conjunction with the discovery of a light Higgs, will indicate that we live in a highly improbable Universe. In figure 6 we plot the probability $P_\Lambda$ versus the Higgs mass normalized with its value at $M_h = M_h'$ for the Hubble rate $H_* = 10^{13}$ GeV (upper curves), $N = N_\star = 60$ and for three different values of the top mass. We remind the reader that $M_h'$ is the upper metastability limit of the Higgs mass given by equation (2) and shown by the upper curves in figure 2.

Similar results are also valid in the case of large-field models of inflation, with the only modification that $P_\Lambda$ is given by equation (36). Therefore, in figure 6 we also plot the probability $P_\Lambda$ versus the Higgs mass normalized with its value at $M_h = M_h'$ for the Hubble rate $H_* = 5 \times 10^{13}$ GeV (lower curves), $N = N_\star = 60$ and for three different values of the top mass. This value of $H_*$ corresponds to the Hubble rate necessary to reproduce the observed curvature perturbations for $p = 2$, see equation (35).

In conclusion, our Universe becomes exponentially unlikely if measurements of the Higgs mass and of the tensor-to-scalar power ratio $T/S$ violate the condition

$$\frac{T}{S} < \left( \frac{\Lambda}{10^{13} \text{ GeV}} \right)^2,$$

where the value of $\Lambda$ has to be inferred from the measured value of the Higgs mass from figure 1. The violation of equation (43) could then be interpreted as an indication for
the existence of new physics beyond the SM, which affects the extrapolation of the Higgs potential to large energy scales.

7. Conclusions

In this paper we have investigated some possible cosmological implications of the Higgs mass measurement. If the LHC discovers a Higgs with mass in the metastability window shown in figure 2, and does not find direct evidence for other new physics, then there is a concrete possibility that we live in a metastable state. If we assume that the pure SM is valid up to very large energy scales, then stability against field fluctuations tests properties of the early Universe. We have revisited the known considerations about thermal fluctuations, interpreting the result as an upper limit on the reheating temperature \( T_{RH} \) after inflation. The bound is summarized in figure 3.

We have also discussed the possibility that the inflationary vacuum fluctuations destabilize the electroweak vacuum. If, during inflation, the Hubble rate \( H \) is large enough and the Higgs mass remains smaller than \( H \) (which requires the assumption that the Higgs is not directly coupled to the inflaton and is only minimally coupled to gravity), then the danger exists that the classical value of the Higgs field is pushed above its instability point, causing the collapse of the corresponding inflating domain. This does not necessarily pose a problem, since all the surviving domains will be characterized by the correct electroweak vacuum. However, interesting probabilistic conclusions can be reached under the assumption of a ‘landscape’ scenario in which the inflationary parameters take different values in different patches of the Universe. In the context of large-field models of inflation we have argued that, among the surviving regions, only an exponentially tiny fraction of them will be characterized by the correct amount of cosmological perturbations. If the Higgs mass is found below (120–130) GeV, either we live in a very special, and exponentially unlikely, domain or new physics must exist below the scale \( \Lambda \).

Moreover, our considerations can be directly related to the amount of primordial gravity waves which can be measured through their imprint on the CMB. The discoveries of a light Higgs boson and of tensor modes would be mutually conflicting (at least in a probabilistic sense) if the condition in equation (43) is not satisfied. Again, this evidence could be interpreted as an indication of new physics modifying the extrapolation of the Higgs potential to large-field values. This result is valid, independently of the particular inflationary model considered.

Finally, a measurement of the Higgs mass below about 130 GeV provides a direct bound, shown in figure 5, on the quantity \( \xi H^2 \), where \( \xi \) is the (negative) coupling between the Higgs bilinear and the curvature, and \( H \) is the maximal Hubble rate during inflation. This bound becomes particularly interesting in the case of detection of primordial gravity waves, which provide a measurement of \( H \). Note that the bound in figure 5 does not depend on any statistical consideration of parameter scanning.

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Appendix

In this appendix we present the relations between running couplings and pole masses for the top quark and the Higgs, and the RG equations used in our numerical calculation. We work at the next-to-leading order and use the \( \overline{\text{MS}} \) renormalization scheme. We define

\[
\lambda(M_t) = \frac{M_t^2}{2v^2} (1 + 2 \Delta_h),
\]

\[
h_t(M_t) = \frac{\sqrt{\gamma}}{v} M_t (1 + \Delta_t).
\]

The corrections to the top-quark mass are given by

\[
\Delta_t = \delta^\text{QCD}_t + \delta^W_t + \delta^\text{QED}_t.
\]

In the QCD part we include up to two-loop corrections using \([33,34]\)

\[
\delta^\text{QCD}_t = -\frac{4\alpha_s(M_t)}{9\pi} + \left(1.0414N_f - 14.3323\right) \left[\frac{\alpha_s(M_t)}{\pi}\right]^2,
\]

where \(N_f = 5\). Although the two-loop effect is beyond the precision of our computation, we include it because it is known to be large. The two-loop mixed electroweak-QCD \([35]\) and the three-loop QCD corrections are also known \([36]\), but give only a small effect.

The electroweak part, including subleading corrections, is well approximated by \([37]\)

\[
\delta^\text{QED}_t + \delta^W_t = -\frac{4\alpha_s(M_t)}{9\pi} + \frac{h_t^2}{32\pi^2} \left[\frac{11}{2} - r + 2r(2r - 3) \ln(4r) - 8r^2 \left(\frac{3}{r} - 1\right)^{3/2} \arccos r\right]

- 6.90 \times 10^{-3} + 1.73 \times 10^{-3} \ln \frac{M_t}{175 \text{ GeV}} - 5.82 \times 10^{-3} \ln \frac{M_t}{175 \text{ GeV}},
\]

where \(r \equiv M_t^2/(4M_Z^2)\). The formula above is valid for \(r < 1\), the case of interest for us. For \(r \geq 1\) one has to replace \((r - 1)^{3/2}\) \(\arccos r\) by \((1 - r)^{3/2}\) \(\arccosh r\). The sign of 6.90 \(\times 10^{-3}\) corrects \([38]\) the misprint of \([37]\).

For the relation \((A.1)\) between the Higgs pole mass and the quartic coupling \(\lambda\) we use \([39]\)

\[
\Delta_h = \frac{G_F}{\sqrt{2}} \frac{M_Z^2}{16\pi^2} \left[\xi f_1(\xi) + f_0(\xi) + \xi^{-1} f_{-1}(\xi)\right],
\]

where

\[
f_1(\xi) = 6 \ln \frac{M_t^2}{M_h^2} + \frac{3}{2} \ln \xi - \frac{1}{2} Z \left(\frac{1}{\xi}\right) - Z \left(\frac{c_w^2}{\xi}\right) - \ln c_w^2 + 9 \left(\frac{25}{9} \ln \frac{\sqrt{5}}{3}\right),
\]

\[
f_0(\xi) = -6 \ln \frac{M_t^2}{M_Z^2} \left[1 + 2c_w^2 - 2 \frac{M_t^2}{M_Z^2}\right] + \frac{3c_w^2}{\xi} \ln \frac{\xi}{c_w^2} + 2Z \left(\frac{1}{\xi}\right)

+ 4c_w^2 Z \left(\frac{\xi}{c_w^2}\right) + 3c_w^2 \left(\frac{2}{s_w^2 + 12c_w^2}\right) \ln c_w^2 - \frac{15}{2} \left(1 + 2c_w^2\right)

- 3 \frac{M_t^2}{M_Z^2} \left[2Z \left(\frac{M_t^2}{M_Z^2}\right) + 4 \ln \frac{M_t^2}{M_Z^2} - 5\right],
\]
f_1(\xi) = 6 \ln \frac{M_Z^2}{M_Z^2} \left[ 1 + 2c_w^4 - 4 \frac{M_Z^4}{M_Z^2} \right] - 6Z \left( \frac{1}{\xi} \right) - 12c_w^4 \left( \frac{c_w^2}{\xi} \right) - 12c_w^4 \ln c_w^2
+ 8 \left( 1 + 2c_w^4 \right) + 24 \frac{M_Z^4}{M_Z^2} \left[ \ln \frac{M_Z^2}{M_Z^2} - 2 + Z \left( \frac{M_Z^4}{M_Z^2} \right) \right],
\tag{A.9}

with \( \xi \equiv M_h^2/M_Z^2, s_w^2 = \sin^2 \theta_W, c_w^2 = \cos^2 \theta_W \) and \( \theta_W \) the Weinberg angle. In addition

\[ Z(z) = \begin{cases} 2A \arctan(1/A) & (z > 1/4) \\ A \ln \left[(1 + A)/(1 - A)\right] & (z < 1/4), \end{cases} \quad A = \sqrt{1 - 4z}. \tag{A.10} \]

We also collect here the two-loop renormalization-group equations that describe the evolution of the SM couplings and Higgs wavefunction renormalization at scales beyond \( M_t \) \cite{[40]}). We keep only the gauge couplings, the top Yukawa coupling and the scalar quartic coupling. The two-loop RG equations for the gauge couplings \( g_i = \{g', g, s\} \) are

\[ \frac{d g_i}{dt} = \kappa \left[ g_i^2 h_i + \kappa^2 g_i^3 \left( \sum_{j=1}^{3} B_{ij} g_j^2 - d_i h_i \right) \right], \tag{A.11} \]

where \( t = \ln Q, Q \) is the renormalization scale, \( \kappa = 1/(16\pi^2) \) and

\[ b = (41/6, -19/6, -7), \quad B = \begin{pmatrix} 199/18 & 9/2 & 44/3 \\ 3/2 & 35/6 & 12 \\ 11/6 & 9/2 & -26 \end{pmatrix}, \quad d' = (17/6, 3/2, 2). \tag{A.12} \]

For the top Yukawa coupling

\[ \frac{d h_t}{dt} = \kappa h_t \left( \frac{9}{2} h_t^2 - \sum_{i=1}^{3} c'_i g_i^2 \right) + \kappa^2 h_t \left[ \sum_{ij} D_{ij} g_i^2 g_j^2 + \sum_{i} E_i g_i^2 h_t^2 + 6(\lambda^2 - 2h_t^4 - 2\lambda h_t^2) \right], \tag{A.13} \]

with

\[ c' = (17/12, 9/4, 8), \quad D = \begin{pmatrix} 1187/216 & 0 & 0 \\ -3/4 & -23/4 & 0 \\ 19/9 & 9 & -108 \end{pmatrix}, \tag{A.14} \]

\[ E = (131/16, 225/16, 36). \]

The RG equation for the Higgs quartic coupling is

\[ \frac{d \lambda}{dt} = \kappa \left\{ -6h_t^4 + 12h_t^2 \lambda + \frac{3}{8} \left[ 2g'^4 + (g^2 + g'^2)^2 \right] - 3\lambda(3g^2 + g'^2) + 24\lambda^2 \right\}
+ \kappa^2 \left\{ 30h_t^6 - h_t^4 \left( 32g_s^2 + \frac{8}{3} g'^2 + 3\lambda \right) + h_t^2 \left[ -\frac{9}{7} g'^4 + \frac{21}{2} g^2 g'^2 - \frac{19}{4} g'^4 \right]
+ \lambda \left( 80g_s^2 + \frac{45}{2} g^2 + \frac{85}{6} g'^2 - 144\lambda \right) \right\}
+ \frac{1}{18} \left( 915g_s^6 - 289g_s^4 g'^2 - 559g_s^2 g'^4 - 379g_s^6 \right)
+ \lambda \left\{ -\frac{73}{8} g'^4 + \frac{38}{7} g^2 g'^2 + \frac{629}{20} g^4 - 108g_s^2 + 36\lambda g^2 - 312\lambda^2 \right\}. \tag{A.15} \]
Finally, the RG equation for the Higgs field wavefunction renormalization is
\[
\frac{1}{h} \frac{dh}{dt} = \kappa \left[ -3h_t^2 + \frac{3}{4} \left( 3g^2 + g'^2 \right) \right] + \kappa^2 \left[ \frac{27}{8} h_t^4 - \frac{5}{2} h_t^2 \left( 8g_2^2 + \frac{9}{4} g'^2 + \frac{17}{12} g^2 \right) + \frac{271}{32} g^4 - \frac{9}{16} g^2 g'^2 - \frac{431}{96} g'^4 - 6\lambda^2 \right].
\]

\[\text{(A.16)}\]

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