The Quest for $\mu \rightarrow e\gamma$ and its Experimental Limiting Factors at Future High Intensity Muon Beams

Francesco Renga\textsuperscript{1}, Gianluca Cavoto\textsuperscript{1,2}, Angela Papa\textsuperscript{3,4}, Emanuele Ripiccini\textsuperscript{5}, and Cecilia Voena\textsuperscript{1}

\textsuperscript{1}Istituto Nazionale di Fisica Nucleare, Sez. di Roma, P.le A. Moro 2, 00185 Roma, Italy
\textsuperscript{2}“Sapienza” Università di Roma, Dipartimento di Fisica, P.le A. Moro 2, 00185 Roma, Italy
\textsuperscript{3}Dipartimento di Fisica, Universit di Pisa, and INFN sez. Pisa, Largo B. Pontecorvo 3, 56127 Pisa, Italy
\textsuperscript{4}Paul Scherrer Institut, 5232 Villigen, Switzerland
\textsuperscript{5}Université de Genève, Département de physique nucléaire et corpusculaire, 24 Quai Ernest-Ansermet, 1211 Genève, Switzerland

March 3, 2022

Abstract

The search for the Lepton Flavor Violating decay $\mu \rightarrow e\gamma$ exploits the most intense continuous muon beams, which can currently deliver $\sim 10^8$ muons per second. In the next decade, accelerator upgrades are expected in various facilities, making it feasible to have continuous beams with an intensity of $10^9$ or even $10^{10}$ muons per second. We investigate the experimental limiting factors that will define the ultimate performances, and hence the sensitivity, in the search for $\mu \rightarrow e\gamma$ with a continuous beam at these extremely high rates. We then consider some conceptual detector designs and evaluate the corresponding sensitivity as a function of the beam intensity.
1 Introduction

In the Standard Model (SM), if the neutrino masses are neglected, three families (or flavors) of leptons exist, and in any process the number of leptons of each family is separately conserved. Lepton flavor conservation is anyway an accidental symmetry: a mere consequence of the particle content of the model, following namely from the absence of right handed neutrinos. Actually, this symmetry is not exact, as already demonstrated by the discovery of neutrino oscillations, but the impact on charged lepton decays is negligible, giving for instance a predicted branching ratio (BR) for the $\mu \to e\gamma$ decay around $10^{-54}$, well below the current experimental limit, $BR(\mu \to e\gamma) < 4.2 \times 10^{-13}$ [1].

On the other hand, the accidental nature of this symmetry makes it very sensitive to new physics (NP) processes beyond the SM. In many models lepton flavor violation (LFV) in the charged lepton sector arises in a measure that is already strongly constrained by the present experimental limits.

These features make the search for LFV very attractive, because negative results are able to strongly constrain the development of NP models, while an observation would be an unambiguous evidence of physics beyond the SM, with no theoretical uncertainty.

In this work we investigate the potential of the next generation of searches for the LFV decay $\mu \to e\gamma$, in the view of the possible availability of high intensity muon beams, delivering a number of muons per second up to two orders of magnitude larger than what is presently possible at the Paul Scherrer Institut (PSI, Switzerland), where the most intense continuous muon beam line in the world is operated, with up to $10^8$ muons per second. Projects to reach a muon beam rate of $10^9$ or even $10^{10}$ muons per second are under considerations at PSI [2] and elsewhere [3, 4]. In this kind of facilities, muons come from the decay of pions produced by a proton beam impinging on a fixed target. At PSI an high intensity muon beam line (HiMB) is studied, that should be able to increase by a factor $>4$ the muon capture efficiency at the production target, thanks to a new design of the solenoid magnets used to convey the muons into the beam line, and by a factor $\sim 7$ the transport efficiency from the production target to the experimental halls, thanks to an improved beam optics. The main limitation at PSI comes from the fact that the proton beam need to be mostly preserved to serve a neutron spallation source downstream of the muon production target. Hence, a thin target is used and only 18% of the original, 2 mA beam is used to produce muons. At RCNP (Japan), the MuSIC project aims to use a thicker target in order to get a similar production rate with a much lower proton beam inten-
sity. The target will be surrounded by an intense solenoidal magnetic field in order to capture pions and muons with a large solid angle acceptance, and a magnetic field adiabatically changing from 3.5 T at the center of the target to 2 T at the exit of the capture solenoid will reduce the angular divergence of the beam and hence increase the acceptance of the solenoidal muon transport beam line. The tests already performed showed that \( \sim 10^6 \) muons per Watt of proton beam power can be produced. Some studies are also going for the production of continuous muon beams in the context of the PIP-II project at Fermilab (USA).

2 Materials and Methods

Our discussion of the sensitivity reach of future experiments looking for \( \mu \rightarrow e\gamma \) considers the typical features of this kind of searches. First of all, positive muons are used in these experiments, in order to avoid their capture in the target nuclei, which would distort the energy spectrum of the decay products. In order to get an intense muon beam with low contamination of pions and electrons, the beam lines are tuned to transport particles of about 28 MeV/c of momentum, corresponding to muons produced by pions decaying at rest just at the surface of the production target. Muons of such a low momentum can be stopped on thin targets, and typically a few hundred microns of plastic material are sufficient. The muons decay at rest and the two-body \( \mu \rightarrow e\gamma \) kinematics is exploited, by looking for a positron and a photon emitted back to back, with equal energies (neglecting the electron mass), \( E_e = E_\gamma = m_\mu c^2/2 \sim 52.8 \, \text{MeV} \).

There is a physics background coming from the radiative muon decay (RMD), \( \mu^+ \rightarrow e\gamma \nu_e \bar{\nu}_\mu \), when the two neutrinos carry off little energy. The other is due to the accidental coincidence of a positron from a Michel muon decay, \( \mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu \), with a high energy photon, whose source might be either a RMD, the annihilation-in-flight (AIF) of a positron in a Michel decay or the bremsstrahlung from a positron. The rate of accidental coincidences, goes with the square of the muon beam rate \( \Gamma_\mu \), and hence it becomes dominant over RMDs at very high muon beam rates. In order to discriminate against accidental coincidences, the difference between the positron and photon emission time, \( T_{e\gamma} \), is also required to be zero.

If a signal region is defined in the parameter space given by the photon and positron energies, the relative stereo angle \( \Theta_{e\gamma} \) and the relative time, with dimensions proportional to the detector resolutions
\( \delta E_e, \delta E_\gamma, \delta \Theta_{e\gamma} \) and \( T_{e\gamma} \), the accidental rate is found to be [5]:

\[
\Gamma_{\text{acc}} \propto \Gamma_\mu^2 \cdot \delta E_e \cdot (\delta E_\gamma)^2 \cdot \delta T_{e\gamma} \cdot (\delta \Theta_{e\gamma})^2
\]

(1)

It indicates that, if a significative background yield is expected during the lifetime of the experiment, a further increase of the beam intensity is useless, because the sensitivity, depending on the ratio of the signal yield over the square root of the background yield remains constant, unless the resolutions are improved in such a way that the background yield becomes negligible. Equivalently, given an experimental setup running for a given time, there is an ideal muon beam rate: the one giving only very few expected background events.

We analyzed in detail the experimental factor that will limit the sensitivity of the future searches for \( \mu \rightarrow e\gamma \). We considered different detector options, both for the positron and the photon, and performed simulations to determine the performances that could be reasonably reached. We made this exercise considering different experimental approaches.

In particular, we studied the two different techniques that can be used to detect the photon. In a calorimetric approach, a fast and luminous inorganic scintillating material is used to measure the photon energy, time and position at the detector. Alternatively, thin layers of dense material can be used to produce a photon conversion to \( e^+e^- \), and the two charged particles are tracked in a magnetic field to determine the photon energy and conversion point. The first approach, used in the last decades by the CrystalBox [6] and MEG [7] experiments, gives a relatively high efficiency (> 60% in MEG), while the resolutions are determined by the physical properties of the scintillating material. The MEG experiment, and its upgrade MEG-II (currently under construction) have a LXe calorimeter, with an homogeneous volume of 900 \( \ell \), giving very good energy (\( \sim 2\% \)) and time (\( \sim 60 \) ps) resolutions. The second approach, adopted for the MEGA experiment [8], suffers from a very low efficiency (a few percent per conversion layer) due to the low conversion probability, but allows to reach extremely good resolutions, which according to the discussion above can allow to exploit a higher beam rate to recover the loss of efficiency. Neither technique provides a precise determination of the photon direction. It is more precisely determined by tracking the positron back to the target, assuming that the photon comes from the same place, and taking the line joining this point to the photon conversion point as the photon direction. Nonetheless, the conversion technique gives some information about the photon direction, as the combination of the directions of the \( e^+e^- \) pair. This supplementary information can be used to require the photon and the positron to
come approximately from the same point, and it helps to reduce the accidental background.

Second, we considered the best performances that could be reached in the positron reconstruction, which is typically carried on with a magnetic spectrometer, which provides high efficiency and the best resolutions in momentum and direction.

We also considered the impact of the target and other detector materials. Due to the low positron momentum, the multiple Coulomb scattering (MS) plays a dominant role in the determination of the positron kinematics, and the target itself, as thin as it can be, still gives non negligible contributions. Moreover, materials on the positron trajectory increase the probability of producing AIF photons and hence the accidental background.

In Figure 1, the conceptual design of a detector searching for $\mu \rightarrow e\gamma$ is shown, for the calorimetric and the conversion techniques.

![Figure 1](image)

Figure 1: Conceptual detector designs exploiting the calorimetric (left) or conversion (right) technique for the photon detection, and a tracking approach in a magnetic field for the positron reconstruction. Muons are stopped in a target (dark red ellipse) at the center of the magnet. Positron tracks from the muon decays (in black) are reconstructed in a tracking detector (dark blue), photons (in green) either produce a shower in a calorimeter (light blue) or are converted by a thin layer of high-Z material (in gray) into an electron-positron pair (in red and black, respectively) which is then reconstructed by an outer tracking detector. The magnet coil (hatched area) surrounds the tracking detectors.
3 Results

3.1 Experimental limiting factors

3.1.1 Efficiency

The signal efficiency is determined by the positron and photon reconstruction efficiencies, $\epsilon_e$ and $\epsilon_\gamma$. First of all, the geometrical acceptance has to be considered. It is typically constrained by the cost of the photon detector. The MEG experiment, for instance, only had a 10% acceptance, limited by the angular coverage of the (very expensive) LXe calorimeter. Though mitigated, this point could be relevant also for the innovative crystals we will discuss in Sec. 3.1.2.

If the calorimetric technique is used, the efficiency is limited by the number of photons converting before reaching the calorimeter, typically in the material of the magnet of the positron spectrometer. A reconstruction efficiency of $\sim 60\%$ was obtained in MEG.

If the photon conversion technique is adopted, thin converters are needed in order to preserve very good resolutions. It implies in turn a few percent $\epsilon_\gamma$. In Fig. 2 the conversion probability for 52.8 MeV photons in lead and tungsten for different thicknesses are shown.

![Figure 2: The conversion efficiency (black, left axis) and the contribution to the energy resolution from the energy loss in the converter (red, right axis), for Lead (full lines) and Tungsten (dashed lines), as a function of the converter thickness (in units of radiation length). The dash-dotted line shows the asymptotic conversion probability, $7/9$ times the thickness in units of radiation length.](image)

Concerning the positron from the muon decay, tracking in a spectrometer usually provides very large $\epsilon_e$. 


3.1.2 Photon energy

In a calorimetric approach, the $E_\gamma$ resolution is dominated by the photon statistics. Hence, the light yield determines the choice of the scintillator to be used, along with the fast response that is needed in order to reach a very good time resolution. Table 1 summarizes the relevant properties of some state-of-the-art scintillating materials. LaBr$_3$(Ce) crystals are a good candidate for future experiments, thanks to the high light yield, which should guarantee a good energy resolution and the low decay time, which is necessary to get a very good time resolution.

Table 1: Properties of state-of-the-art scintillators relevant for the application on $\mu^+ \rightarrow e^+\gamma$ searches.

| Scintillator | Density [g/cm$^3$] | Light Yield [ph/keV] | Decay Time [ns] |
|--------------|---------------------|----------------------|----------------|
| LaBr$_3$(Ce) | 5.08                | 63                   | 16             |
| LYSO         | 7.1                 | 27                   | 41             |
| YAP          | 5.35                | 22                   | 26             |
| LXe          | 2.89                | 40                   | 45             |
| NaI(Tl)      | 3.67                | 38                   | 250            |
| BGO          | 7.13                | 9                    | 300            |

I pair conversion is used, the limiting factor of the $E_\gamma$ resolution is the interaction of the $e^+e^-$ pair within the material of the photon converter itself. The energy loss fluctuation predominantly contributes to the resolution, since $E_\gamma$ is estimated as the sum of the $e^+$ and $e^-$ energies. We performed simulations with GEANT4 [9] showing that a 280 $\mu$m Pb layer ($\sim$ 5% $X_0$) would give a resolution of $\sim$ 240 keV in the limit of perfect tracking of the $e^+e^-$ pair. Figure 2 also show the contribution of the material effects to the resolution.

3.1.3 Positron energy

The positron energy and positron angular resolutions in a spectrometer are ultimately determined by MS and energy loss fluctuations. For this reason, gaseous detectors give the best performances and have been used in the latest experiments. A silicon vertex tracker is used for the search of $\mu^+ \rightarrow e^+e^+e^-$ by the Mu3e Collaboration [10], and a similar design has been suggested for future $\mu^+ \rightarrow e^+\gamma$ searches [11], considering that very thin sensors are now available, with a thickness of 50 $\mu$m Si + 25 $\mu$m Kapton per layer [12], corresponding to $\sim 10^{-3}$ radiation lengths per layer. On the other hand, the complete
drift-chamber spectrometers of MEG or MEG-II amount to less than $3 \times 10^{-3}$ radiation lengths over the whole track length within the tracking volume, nonetheless material effects gave a significant contribution in MEG and will almost be dominant in MEG-II. It clearly indicates that more than a few silicon layers cannot be used: indeed, simulations [11] point toward $E_e$ resolutions of $\sim 200$ keV, which are not competitive with what can be obtained with gaseous detectors [13].

3.1.4 Relative angle $\Theta_{e\gamma}$

The relative angle $\Theta_{e\gamma}$ is measured by combining the positron angle, the photon conversion point and the positron vertex on the target, which has to be as thin as possible to reduce the MS affecting the positron angle measurement. On the other hand it has to be thick enough to provide a good stopping power for muons. A good compromise has been obtained by slanting the target with respect to the beam axis (in MEG the target normal vector makes an angle $\alpha \sim 70^\circ$ with the beam axis, which will be increased to $76^\circ$ in MEG-II). In this configuration, the effective thickness seen by muons is magnified by a factor $1/\cos(\alpha) \sim 3$, while positrons emitted at the center of the detector acceptance ($90^\circ$ with respect to the beam axis) see a thickness magnified only by a factor $1/\sin(\alpha) \sim 1.06$. Nonetheless, if a 90% stopping efficiency is required, simulations suggest that the contribution to the angular resolutions is always larger than 3 mrad, even with the material (Beryllium) giving the best performances. Also, reducing the thickness by accepting a loos off stopping efficiency is not feasible: the target has to be placed at the Bragg peak to have a reasonable stopping efficiency, and survived muons would decay in the gas just after the target, giving a contribution to the background without increasing the signal rate. Hence, a positron angle resolution better than a few mrad cannot be obtained with conventional techniques.

With the photon conversion technique, the photon conversion point can be measured very precisely, essentially with the single hit resolution of the $e^+e^-$ tracker. As a consequence, the photon angle resolution is completely dominated by the positron vertex resolution. With calorimetry, the granularity of the detector determines the resolution, but it is reasonable to assume a resolution below 1 cm. In both cases, the positron angle resolution is dominant and defines the ultimate $\Theta_{e\gamma}$ resolution.

3.1.5 Relative time $T_{e\gamma}$

A good $T_{e\gamma}$ resolution has been guaranteed in the MEG experiments by the use of scintillation detectors placed at the end of the positron
trajectory, in combination with the good time resolution of the LXe calorimeter. Replicating these performances in future experiments will require either the use of a calorimetric approach with very fast crystals or the inclusion of scintillators on the $e^+e^-$ trajectory if the photon conversion is used.

3.1.6 Summary

Tab. 2 shows a summary of the limiting factors for the efficiency and resolutions of future $\mu^+ \rightarrow e^+\gamma$ searches, as derived from simulations and analysis of past experiments. More details can be found in [14].

3.2 Sensitivity reach

We considered a conceptual $\mu^+ \rightarrow e^+\gamma$ detector based on the photon conversion technique. In this design, a target identical to the one of MEG-II is surrounded by a cylindrical gaseous positron tracker. Externally, a Lead conversion layer is placed, with a $0.1X_0$ thickness. Behind it, another gaseous detector is used as an $e^+e^-$ pair spectrometer.

Optionally, a small gaseous or two-layer solid state detector can be considered as a vertex tracker to improve the determination of the positron angles and the muon decay point.

Everything is immersed in a magnetic field. The signal positron curls before reaching the converter layer and finally reaches a set of scintillators for positron timing, while at least one of the tracks from the photon conversion goes through the whole $e^+e^-$ pair spectrometer and reaches another set of scintillators for the photon timing.

We estimated the expected performances of such a detector, assuming that the ultimate resolutions of Table 2 can be reached with an incremental improvement of the present experimental techniques. We consider two scenarios for the inner vertex detector. In the first, conservative one, the only improvement comes from having the first measured point which is closer to the target, while the momentum and angular resolutions are still dominated by the extended tracker, and the angular resolution is deteriorated by the MS in the inner wall of the TPC or the inner layer of the silicon vertex tracker. In the second, optimistic one, the vertex detector makes also the tracking contribution to the angular resolution negligible. This resolution is then completely determined by material effects before and inside the first layer of the inner vertex detector. A summary of the expected performances can be found in Tab. 3 and 4. It is evident that a silicon vertex detector cannot help, because the MS in the first layer of such
Table 2: Limiting factors for the efficiency and resolutions of future $\mu^+ \rightarrow e^+\gamma$ searches.

| Efficiency                  | Typical figure | Comments                      |
|-----------------------------|----------------|-------------------------------|
|                             | **Calorimetry** | **$\gamma$ Conversion**       |
| Material budget             | 0.5 $\sim$ 0.9 | $-$                           | magnet coil |
| Pair production             | $-$            | 0.02 $\sim$ 0.04              | 0.05 $\sim$ 0.1 $X_0$ |
| Minimum $e^+e^-$ energies   | $-$            | 0.8                           | $E_{e^+}, E_{e^-} > 5$ MeV |

**Photon Energy Resolution**

| Photon Statistics & segmentation | 800 keV | $-$ |
| Energy loss                     | $-$     | 250 $\sim$ 800 keV           | 0.05 $\sim$ 0.1 $X_0$ |

**Positron Energy Resolution**

| Tracking & MS                  | 15 keV |
| Energy loss                    | 100 keV |

**Relative Angle Resolution**

| MS on target                   | 2.6 / 2.8 mrad ($\theta_{e\gamma}$ / $\phi_{e\gamma}$) |
| MS on gas & walls              | 3.3 / 3.3 mrad ($\theta_{e\gamma}$ / $\phi_{e\gamma}$) |
| Tracking                       | 6.0 / 4.5 mrad ($\theta_{e\gamma}$ / $\phi_{e\gamma}$) |
| Alignment                      | $< 1$ mrad                                     |
|                               | $< 100$ $\mu$m target alignment              |

$R_e = 20$ cm, $R_{\gamma} = 30$ cm, $B = 1$ T
a detector negates the advantage of having a very good determination of the track angle between the first and the subsequent layers.

Table 3: Expected performances (efficiency and resolutions) for a basic design with different options as discussed in the text.

| Observable            | one photon conversion layer | photon calorimeter |
|-----------------------|-----------------------------|--------------------|
| $T_{e\gamma}$ [ps]    | 60                          | 50                 |
| $E_e$ [keV]           | 100                         | 100                |
| $E_{\gamma}$ [keV]    | 320                         | 850                |
| Efficiency [%]        | 1.2                         | 42                 |

Table 4: Angular resolutions for different types of a vertex detector. A conservative estimate is given in parenthesis.

|                  | $\theta_{e\gamma}$ [mrad] | $\phi_{e\gamma}$ [mrad] |
|------------------|----------------------------|--------------------------|
| None             | 7.3                        | 6.2                      |
| TPC              | 3.5 (6.1)                  | 3.8 (4.8)                |
| Silicon          | 8.0 (6.3)                  | 7.4 (6.9)                |

A conceptual $\mu^+ \rightarrow e^+\gamma$ experiment based on calorimetry could have a design very similar to the one above for the central part of the detector, but the external $e^+e^-$ pair tracker would be replaced by a scintillation detector placed outside of the magnet. With LaBr$_3$(Ce) crystals, the calorimeter could be about 20 cm deep and the performance summarized in Tab. 3 and 4 could be reached. Here we assume that the photon conversion point can be still determined with a negligible resolution compared to the positron vertex resolution.

With these performances and 100 weeks of data taking (3 to 4 years at PSI), with muon rates from $10^8$ to $10^{10}$ muons per second, and assuming the same photon background rate of the MEG experiment (scaled linearly with the muon beam intensity), we could estimate the the expected sensitivity of the experiment according to a frequentistic approach [15].

Figures 3 and 4 show the expected sensitivity to the $\mu^+ \rightarrow e^+\gamma$ decay as a function of the beam intensity in different scenarios. We also considered the possibility of having multiple conversion layers. In this case, the preservation of a good time resolution requires the inclusion of thin and fast detectors in the conversion layer itself [14].
Figure 3: Expected 90% C.L. upper limit on the Branching Ratio of $\mu^+ \rightarrow e^+\gamma$ in different scenarios for a 3-year run. A few different designs based on the photon conversion technique are compared, including the TPC vertex detector option in the conservative and optimistic hypotheses. The lines turn from continuous to dashed when the number of background events exceeds 10. The horizontal dashed and dotted lines show the current MEG limit and the expected MEG-II sensitivity, respectively.

Figure 4: Expected 90% C.L. upper limit on the Branching Ratio of $\mu^+ \rightarrow e^+\gamma$ in different scenarios for a 3-year run. Calorimetry and the photon conversion technique are compared. The lines turn from continuous to dashed when the number of background events exceeds 10. The horizontal dashed and dotted lines show the current MEG limit and the expected MEG-II sensitivity.
4 Discussion

The search for LFV is one of the most promising field in the quest for NP. The present limit on $\mu \rightarrow e\gamma$ by the MEG collaboration already strongly constrains the NP models and an improvement of one order of magnitude is expected with MEG-II. We investigated some long term prospects for the $\mu \rightarrow e\gamma$ search. Our estimates show that a 3-year run with an accelerator delivering around $10^{9}$ muons per second could allow to reach a sensitivity of a few $10^{-15}$ (expected 90% upper limit on the $\mu \rightarrow e\gamma$ BR), with poor perspectives of going below $10^{-15}$ even with $10^{10}$ muons per second. Below $5 \times 10^{8}$ muons per second, the calorimetric approach needs to be used in order to reach this target. If a muon beam rate exceeding $10^{9}$ muons per second is available, the much cheaper photon conversion option would be recommended and would provide similar sensitivities.

The sensitivity would be eventually limited by the fluctuations of the interaction of the particles with the detector materials: this indicates that a further step forward in the search for $\mu \rightarrow e\gamma$ would require a radical rethinking of the experimental concept.

References

[1] A. M. Baldini et al. [MEG Collaboration], Search for the lepton flavour violating decay $\mu^+ \rightarrow e^+\gamma$ with the full dataset of the MEG experiment, Eur. Phys. J. C 76 (2016) no.8, 434.

[2] P. R. Kettle, contribution to Future Muon Sources 2015, University of Huddersfield, United Kingdom; A. Knecht, contribution to SWHEPPS2016, Unterägeri, Switzerland;

[3] S. Cook et al., Delivering the world’s most intense muon beam, Phys. Rev. Accel. Beams 20 (2017) no.3, 030101.

[4] V. Lebedev, ed. [PIP-II Collaboration], The PIP-II Reference Design Report, FERMILAB-DESIGN-2015-01.

[5] Y. Kuno and Y. Okada, Muon decay and physics beyond the standard model, Rev. Mod. Phys. 73 (2001) 151.

[6] R. D. Bolton et al. [Crystal Box Collaboration], Search for Rare Muon Decays with the Crystal Box Detector, Phys. Rev. D 38 (1988) 2077.

[7] J. Adam et al. [MEG Collaboration], The MEG detector for $\mu^+ \rightarrow e^+\gamma$ decay search, Eur. Phys. J. C 73 (2013) no.4, 2365.
[8] M. L. Brooks et al. [MEGA Collaboration], New limit for the family number nonconserving decay $\mu^+ \rightarrow \mu^+ e^- \gamma$, Phys. Rev. Lett. 83 (1999) 1521.

[9] S. Agostinelli et al. [GEANT4 Collaboration], GEANT4: A Simulation toolkit, Nucl. Instrum. Meth. A 506 (2003) 250.

[10] N. Berger et al., A Tracker for the Mu3e Experiment based on High-Voltage Monolithic Active Pixel Sensors, Nucl. Instrum. Meth. A 732 (2013) 61.

[11] C. h. Cheng, B. Echenard and D. G. Hitlin, The next generation of $\mu^- \rightarrow e^- \gamma$ and $\mu^- \rightarrow 3e$ CLFV search experiments, arXiv:1309.7679 [physics.ins-det].

[12] I. Peric et al., High-voltage pixel detectors in commercial CMOS technologies for ATLAS, CLIC and Mu3e experiments, Nucl. Instrum. Meth. A 731 (2013) 131.

[13] A. M. Baldini et al. [MEG II Collaboration], The design of the MEG II experiment, Eur. Phys. J. C 78, no. 5, 380 (2018).

[14] G. Cavoto, A. Papa, F. Renga, E. Ripiccini and C. Voena, The quest for $\mu \rightarrow e\gamma$ and its experimental limiting factors at future high intensity muon beams, Eur. Phys. J. C 78, no. 1, 37 (2018).

[15] G. J. Feldman and R. D. Cousins, A Unified approach to the classical statistical analysis of small signals, Phys. Rev. D 57 3873 (1998).