THE POINCARE GROUP IS DEDUCED
FROM
THE LOGIC PROPERTIES OF THE
INFORMATION

Gunn Quznetsov
quznets@geocities.com

March 31, 2022

Abstract

The principal time properties - the one-dimensionality and the irreversibility -, the space metric properties and the spatial-temporal principles of the theory of the relativity are deduced from three natural logic properties of the information, obtained by a physics device. Hence, the transformations of the complete Poincare group are deduced from that.

Contents

1 LOGIC 1
2 TIME 3
3 SPACE 6
4 RELATIVITY 13
5 RESUME 19
6 Acknowledgements 19

1 LOGIC

An information, which is obtained from a physics device, can be expressed by a text.
That is in this article the information is an entity, which can be expressed by a set of any language sentences.

The sentence \( \ll \Theta \gg \) is the true sentence if and only if \( \Theta \).

For example: the sentence \( \ll \text{it rains} \gg \) is the true sentence if and only if it rains.

The sentence \( \ll \Theta \gg \) is the false sentence if and only if it is not that \( \Theta \).

The sentence \( \Theta \) is the conjunction of the sentences \( A \) and \( B \) \( (C = (A \& B)) \), if \( C \) is true if and only if \( A \) is true and \( B \) is true.

The sentence \( C \) is the negation of the sentence \( A \) \( (C = (\neg A)) \), if \( C \) is true if and only if \( A \) is false.

The function \( g(x) \), which has got the domain on the sentences set and has got the range of values in the two-elements set \( \{0; 1\} \), is the Bool’s function, if \( g(x) \) follows to the following conditions:

1. \( g((A \& B)) = g(A) \cdot g(B) \);
2. \( g(\neg A) = 1 - g(A) \).

Therefore, we shall work into the classical propositional logic approach [1].

For every \( A: g(A) \cdot g(A) = g(A) \), since \( g(x) \) has got the range of values in the two-elements set \( \{0; 1\} \).

The sentence \( A \) is the tautology, if for every Bool’s function \( g(x) \): \( g(A) = 1 \).

For example: because
\[
g((\neg (A \& \neg A))) = 1 - g((A \& \neg A)) = 1 - g(A) \cdot g(\neg A) = 1 - g(A) \cdot (1 - g(A)) = 1 - g(A) + g(A) \cdot g(A) = 1,
\]
then \( (\neg (A \& \neg A))) \) is the tautology.

If \( (\neg (A \& \neg B))) \) is the tautology then \( B \) is the logic consequence from \( A \).

Let \( \hat{a} \) be an object, which may to accept, to retain and/or to pass any information [2]. The set \( a \) of the sentences, which expresses the information of \( \hat{a} \), is defined as the recorder of \( \hat{a} \). I.e. the expression \( \ll \hat{a} \gg \) denotes: \( \ll \hat{a} \gg \) has got the information, that the event, which can be expressed by the sentence \( A \), happens., or denotes: \( \ll \hat{a} \gg \) knows, that \( A \). We write down such expression in abridged type as \( \ll a \cdot A \gg \).

The following conditions are fulfilled:

I. For every \( a \) and for every \( A \): it is not, that \( a \cdot (A \& (\neg A)) \), i.e. every recorder does not contain the contradiction.

II. For every \( a \), for every \( B \) and for every \( A \): if \( B \) is the logic consequence from \( A \), and \( a \cdot A \), then \( a \cdot B \).

*III. For every \( a, b \) and for every \( A \): if \( a \cdot b \cdot A \gg \) then \( a \cdot A \).

For example: if the device \( \hat{a} \) has got the information, that the device \( \hat{b} \) has got the information, that the particle mass equals to 7, then the device \( \hat{a} \) has got the information, that the particle mass equals to 7.

1Perhaps, the definition of the truth sentence belongs to A.Tarsky.

2The formalization and the self-consistency see in G.Kuznetsov, Physics Essays, v.4, n.2, (1991), p.157-171.
2 TIME

Let us consider the finite (possible - empty) arrays of the symbols of the type: q*.

The array α is the subarray of the array β (α ≺ β), if α can be obtained from β by the deletion of some (possible - all) elements.

Let us designate: (α)1 is α, and (α)k+1 is α(α)k.

Hence, if k ≤ l, then (α)k ≺ (α)l.

The array α is equivalent to the array β (α ∼ β), if α can be obtained from β by the substitution of the subarray of the type (a•k) by the subarray of the same type ((a•s)k).

In such case:

III. If β ≺ α or β ∼ α, then for every K:
if a•K, then a•(K & (¬((a•A) & (¬(β•A))))).
It is obvious, that III is the refinement of *III.

The number q is the moment, at which a records B by the κ−clock {g0, A, b0} (the designation: q is [a•α B ↑ a, {g0, A, b0}], if:

1. for every K: if a•K, then a•(K & (¬((a•B) & (¬(a•(g0•b0)g0•A))))) and a•(K & (¬((a•(g0•b0)g0•A))));
2. a•((a•B) & (¬(a•(g0•b0)g0•A)))).

In our world the κ−clock {g0, A, b0} accords to the pair (g0, b0) of the physics devices, which dispatch the information, expressed by A, between each other. Here the events, expressed by following sentences, happen:
a•g0•A,
a•b0•g0•A,
a•g0•b0•g0•A,
a•b0•g0•b0•g0•A = a•(g0•b0)g0•A, 
a•(g0•b0)g0•A,
a•b0•(g0•b0)g0•A, 
a•(g0•b0)g0•A, etc.

Lemma 1 If

q is [a•α B ↑ a, {g0, A, b0}],

p is [a•β B ↑ a, {g0, A, b0}],

α ≺ β

then

q ≤ p
Proof: From (3):
\[ a^* \left( (a^* \beta B) \& \left( \neg \left( a^* (g_0^* b_0^*)^{(p+1)} g_0^* A \right) \right) \right). \]

From above and (3) by III:
\[ a^* \left( \left( (a^* \beta B) \& \left( \neg \left( a^* (g_0^* b_0^*)^{(p+1)} g_0^* A \right) \right) \right) \& \left( \neg ((a^* \beta B) \& (\neg (a^* aB))) \right) \). \]

From above by II:
\[ a^* \left( (a^* aB) \& \left( \neg \left( a^* (g_0^* b_0^*)^{(p+1)} g_0^* A \right) \right) \right). \]

From above and (3):
\[ a^* \left( \left( (a^* aB) \& \left( \neg \left( a^* (g_0^* b_0^*)^{(p+1)} g_0^* A \right) \right) \right) \& \left( \neg ((a^* aB) \& (\neg (a^* aB))) \right) \). \]

From above by II:
\[ a^* \left( \left( \neg \left( a^* (g_0^* b_0^*)^{(p+1)} g_0^* A \right) \right) \& (a^* (g_0^* b_0^*)^{q} g_0^* A) \right). \]

(4)

If \( q > p \), that is \( q \geq p \), then from (3) by III:
\[ a^* \left( \left( (\neg \left( a^* (g_0^* b_0^*)^{(p+1)} g_0^* A \right) \right) \& (a^* (g_0^* b_0^*)^{q} g_0^* A) \right) \& \left( (a^* (g_0^* b_0^*)^{q} g_0^* A) \& \left( \neg (a^* (g_0^* b_0^*)^{(p+1)} g_0^* A) \right) \right). \]

From above by II:
\[ a^* \left( \left( \neg \left( a^* (g_0^* b_0^*)^{(p+1)} g_0^* A \right) \right) \& \left( (a^* (g_0^* b_0^*)^{p+1}) g_0^* A \right) \right), \]

This is the contradiction of I. Hence, \( q \leq p \)

The following proposition is the Lemma 1 direct consequence:

if \( q = [a^* B \uparrow a, \{g_0, A, b_0\}] \) and \( p = [a^* B \uparrow a, \{g_0, A, b_0\}] \) then \( q = p \). Hence, the expression \( \ll q = [a^* B \uparrow a, \{g_0, A, b_0\}] \geq \) is equivalent to the expression \( \ll q = [a^* B \uparrow a, \{g_0, A, b_0\}] \gg \).

The \( \kappa \)-clocks \{g_1, B, b_1\} and \{g_2, B, b_2\} have got the identical direction for \( a \), if the following condition fulfills:

if
\[ r = [a^* (g_1^* b_1^*)^{q} g_1^* B \uparrow a, \{g_2, B, b_2\}], \]
\[ s = [a^* (g_1^* b_1^*)^{p} g_1^* B \uparrow a, \{g_2, B, b_2\}], \]
\( q < p \),

then
Theorem 1. All $\kappa$–clocks have got the identical direction.

1. Proof: Let

$$r = [a^*(g_2^*b_2^*)g_2 B \uparrow a, \{g_2, B, b_2\}]$$

$$s = [a^*(g_1^*b_1^*)g_1 B \uparrow a, \{g_2, B, b_2\}]$$

$$q < p.$$ In this case:

$$(g_1^*b_1^*)q \prec (g_1^*b_1^*)p.$$ Hence, by Lemma 1:

$$r \leq s. \blacksquare$$

Therefore, a recorder arranges its own sentences on the moments. And this order is linear and does not depend from which $\kappa$–clock this order is settled.

The $\kappa$–clock $\{g_2, B, b_2\}$ is $k$ times more precise than the $\kappa$–clock $\{g_1, B, b_1\}$ for the recorder $a$, if for every $C$ the following condition fulfills:

if

$$q_1 = [a^* \uparrow a, \{g_1, B, b_1\}],$$

$$q_2 = [a^* \uparrow a, \{g_2, B, b_2\}],$$

then

$$q_1 < q_2 / k < q_1 + 1.$$ The array $\tilde{H}$ of the $\kappa$–clocks:

$$\langle \{g_0, A, b_0\}, \dots, \{g_j, A, b_j\}, \dots \rangle$$

is the utter precise $\kappa$–clock for the recorder $a$, if for every $j$ the natural number $k_j$ exists, for which the $\kappa$–clock $\{g_j, A, b_j\}$ is $k_j$ times more precise than the $\kappa$–clock $\{g_{j-1}, A, b_{j-1}\}$. In this case, if

$$q_j = [a^* \uparrow a, \{g_j, A, b_j\}],$$

$$t = q_0 + \sum_{j=1}^{\infty} (q_j - q_{j-1} \cdot k_j) / (k_1 \cdot k_2 \cdot \cdots \cdot k_j),$$

then
\[ t = [a^*C \uparrow a, \overline{H}] . \]

Hence, "the time" of the \( \kappa \)-clocks of type \( \{g_j, A, b_j\} \) is the natural number, and "the time" of the utter precise \( \kappa \)-clocks is the real number.

3 SPACE

Let us denote:
\[ j(\overline{aH})(a^*oa^*C) = [a^*oa^*C \uparrow a, \overline{H}] - [a^*C \uparrow a, \overline{H}] . \]

In our world \( j(\overline{aH})(a^*oa^*C) \) is the time, at which the information about the event, expressed by sentence \( C \), runs by the path \( \hat{a}, \hat{a}_1, \hat{a}_2, \hat{a} \).

Let us denote:
1) for every recorder \( a \): \( (a)^\dagger = (a) \);
2) for all recorders series \( \alpha \) and \( \beta \): \( (\alpha\beta)^\dagger = (\beta)^\dagger (\alpha)^\dagger \).

The set \( \mathbb{R} \) of the recorders is the internally stable system for the recorder \( a \) with the \( \kappa \)-clock \( \overline{H} \) (denote: \( \mathbb{R} \) is \( ISS(\overline{aH}) \)), if for all sentences \( B \) and \( C \), for all elements \( a_1 \) and \( a_2 \) of \( \mathbb{R} \) and for all series \( \alpha \), formed by elements of \( \mathbb{R} \), the following conditions are fulfilled:
1) \[ [a^*a_2^*a_1^*C \uparrow a, \overline{H}] - [a^*a_1^*C \uparrow a, \overline{H}] = [a^*a_1^*B \uparrow a, \overline{H}] - [a^*a_2^*B \uparrow a, \overline{H}] ; \]
2) \[ j(\overline{aH})(a^*oa^*C) = j(\overline{aH})(a^*a_1^*a^*C) . \]

In our world \( ISS(\overline{aH}) \) accords to the set of the physics devices, which all are immovable between each other, and this set does not birl (does not circumvolve).

**Lemma 2.** If \( \{a, a_1, a_2\} \) is \( ISS(\overline{aH}) \), then
\[ [a^*a_2^*a_1^*C \uparrow a, \overline{H}] - [a^*a_1^*C \uparrow a, \overline{H}] = [a^*a_1^*a_2^*B \uparrow a, \overline{H}] - [a^*a_1^*B \uparrow a, \overline{H}] \]

**Proof:** Let us denote:
\[ p = [a^*a_1^*B \uparrow a, \overline{H}] , \]
\[ q = [a^*a_1^*a_2^*a_1^*B \uparrow a, \overline{H}] , \]
\[ r = [a^*a_2^*C \uparrow a, \overline{H}] , \]
\[ s = [a^*a_2^*a_1^*a_2^*C \uparrow a, \overline{H}] . \]
\[ u = \left[ a^* a_1^* a_1^* B \uparrow a, \tilde{H} \right], \]
\[ w = \left[ a^* a_1^* a_2^* C \uparrow a, \tilde{H} \right]. \]

From above by the definition of ISS:
\[ u - p = s - w, w - r = q - u. \]

From above:
\[ s - r = q - p. \]

Let us denote:
\[ p = \left[ a^* a_1^* B \uparrow a, \tilde{H} \right], \]
\[ q = \left[ a^* a_1^* a_2^* a_1^* B \uparrow a, \tilde{H} \right], \]
\[ r = \left[ a^* a_1^* C \uparrow a, \tilde{H} \right], \]
\[ s = \left[ a^* a_1^* a_2^* a_1^* C \uparrow a, \tilde{H} \right], \]
\[ u = \left[ a^* a_1^* B \uparrow a, \tilde{H} \right], \]
\[ w = \left[ a^* a_1^* a_2^* C \uparrow a, \tilde{H} \right]. \]

From above by the definition of ISS:
\[ u - p = w - r, q - u = s - w. \]

From above:
\[ q - p = s - r. \]

Hence, in this case:
\[ \ell \left( a, \tilde{H}, B \right) (a_1, a_2) = \ell \left( a, \tilde{H}, C \right) (a_1, a_2). \]
Theorem 2: If \( \{a, a_1, a_2, a_3\} \) is \( ISS(a, \bar{H}) \), then:

1) \( \ell(a, \bar{H})(a_1, a_2) \geq 0; \)
2) \( \ell(a, \bar{H})(a_1, a_1) = 0; \)
3) \( \ell(a, \bar{H})(a_1, a_2) = \ell(a, \bar{H})(a_2, a_1); \)
4) \( \ell(a, \bar{H})(a_1, a_2) + \ell(a, \bar{H})(a_2, a_3) \geq \ell(a, \bar{H})(a_1, a_3). \)

Proof: 1) and 2) are the direct consequence from Lemma 1, and 3) - from Lemma 2.

Let us denote:

\[
\begin{align*}
p &= [a^*a_1^*a_2^*C \uparrow a, \bar{H}], \\
q &= [a^*a_1^*a_2^*a_3^*C \uparrow a, \bar{H}], \\
r &= [a^*a_1^*a_2^*a_3^*a_1^*C \uparrow a, \bar{H}], \\
s &= [a^*a_2^*a_3^*C \uparrow a, \bar{H}], \\
u &= [a^*a_2^*a_3^*a_2^*B \uparrow a, \bar{H}], \\
w &= [a^*a_1^*a_2^*a_3^*a_1^*a_3^*C \uparrow a, \bar{H}].
\end{align*}
\]

From above by the definition of \( ISS \):

\[
w - u = q - s,
\]

Hence,

\[
w - p = (q - p) + (u - s).
\]

And by Lemma 1:

\[
w \geq r.
\]

Therefore:

\[
(q - p) + (u - s) \geq r - p. \]

Thus, all metric space \( \square \) axioms are fulfilled for \( \ell(a, \bar{H}) \) in the internal stable system.

\( B \) happens in the same place with \( a_1 \) for \( a \) (denote: \( z(a)(a_1, B) \)), if for every array \( \alpha \) and for every sentence \( K \) the following condition fulfills:

- if \( a^*K \), then \( a^* (K \& \neg((\alpha B) \& \neg(\alpha a_1^*B))) \).

Theorem 3 \( z(a)(a_1, a_1^*B) \).

Proof: Since \( \alpha a_1^* \sim \alpha a_1^* a_1^* \), then by III: if \( a_1^* K \), then
Theorem 4. If

\[ \uparrow (a_1, B), \quad (5) \]

\[ \uparrow (a, B), \quad (6) \]

then

\[ \uparrow (a_2, a^* B). \]

**Proof:** Let \( a^* K \). In this case from (6):

\[ a^* (K & (-((aa^*_1 B) & (-aa^*_2 B))))). \]

From (5):

\[ a^* \left( \left( K & (-((aa^*_1 B) & (-aa^*_2 B)))) \right) \& \left( -((aa^*_1 a^*_2 B) & (-aa^*_1 a^*_2 B))) \right) \right). \]

From above by II:

\[ a^* (K & (-((aa^*_1 B) & (-aa^*_1 a^*_2 B))))). \]

From above by III:

\[ a^* \left( \left( K & (-((aa^*_1 B) & (-aa^*_1 a^*_2 B)))) \right) \& \left( -((aa^*_1 a^*_2 B) & (-aa^*_2 B))) \right) \right). \]

From above by II:

\[ a^* (K & (-((aa^*_1 B) & (-aa^*_2 B))))). \]

Lemma 4. If

\[ \uparrow (a_1, B), \quad (7) \]

\[ t = \left[ a^* \alpha B \uparrow a, \bar{H} \right], \quad (8) \]

then

\[ t = \left[ a^* a^*_1 B \uparrow a, \bar{H} \right]. \]

**Proof:** Let us denote:
\[ t_j = [a^\cdot \alpha_B \uparrow a, \{g_j, A, b_j\}] . \]

Hence:
\[
a^\cdot \left((a^\cdot \alpha_B) \& \neg \left(a^\cdot \left(g_j^\ast b_j^{t_j}g_j^\ast A\right)\right)\right),
\]
and from (8):
\[
a^\cdot \left((a^\cdot \alpha_B) \& \neg \left(a^\cdot \left(g_j^\ast b_j^{t_j}g_j^\ast A\right)\right)\right) \& \neg ((a^\cdot \alpha_B) & (\neg (a^\cdot \alpha a_1^B)))).
\]

From above by II:
\[
a^\cdot \left((a^\cdot \alpha a_1^B) \& (\neg \left(a^\cdot \left(g_j^\ast b_j^{t_j}g_j^\ast A\right)\right)\right),
\]
Let \(a^\cdot K\). In this case from (8):
\[
a^\cdot \left(K \& (\neg ((a^\cdot \alpha_B) & (\neg (a^\cdot \alpha a_1^B)))).
\]

Hence, by III:
\[
a^\cdot \left((K \& (\neg ((a^\cdot \alpha_B) & (\neg (a^\cdot \alpha a_1^B)))).
\]

From above by II:
\[
a^\cdot \left(K \& (\neg ((a^\cdot \alpha a_1^B) \& (\neg \left(a^\cdot \left(g_j^\ast b_j^{t_j}g_j^\ast A\right)\right)\right)\right) \& \neg ((a^\cdot \alpha_B) & (\neg (a^\cdot \alpha a_1^B)))).
\]

From above and from (8):
\[
a^\cdot \left(K \& (\neg ((a^\cdot \alpha a_1^B) \& (\neg \left(a^\cdot \left(g_j^\ast b_j^{t_j}g_j^\ast A\right)\right)\right)\right) \& \neg ((a^\cdot \alpha_B) & (\neg (a^\cdot \alpha a_1^B)))).
\]

From above by II:
\[
a^\cdot \left(K \& (\neg ((a^\cdot \alpha a_1^B) \& (\neg \left(a^\cdot \left(g_j^\ast b_j^{t_j}g_j^\ast A\right)\right)\right)\right) \& \neg ((a^\cdot \alpha_B) & (\neg (a^\cdot \alpha a_1^B)))).
\]

From above and from (8), (9) for all \(j\):
\[ t_j = [a^\cdot \alpha a_1^B \uparrow a, \{g_j, A, b_j\}] . \]

Therefore,
Theorem 5. If \( \{a, a_1, a_2\} \) is ISS \( (a, \tilde{H}) \),

\[
\sharp (a) (a_1, B), \quad (11)
\]

\[
\sharp (a) (a_2, B), \quad (12)
\]

then

\[
\ell (a, \tilde{H}) (a_1, a_2) = 0.
\]

Proof: Let us denote:

\[
t = \left[ a^\bullet a_1^\bullet B \uparrow a, \tilde{H} \right].
\]

From above by Lemma 4:

from (11):

\[
t = \left[ a^\bullet a_1^\bullet B \uparrow a, \tilde{H} \right],
\]

from (12):

\[
t = \left[ a^\bullet a_1^\bullet a_2^\bullet B \uparrow a, \tilde{H} \right],
\]

again from (11):

\[
t = \left[ a^\bullet a_1^\bullet a_2^\bullet a_3^\bullet B \uparrow a, \tilde{H} \right].
\]

Therefore:

\[
\ell (a, \tilde{H}) (a_1, a_2) = 0.5 \cdot (t - t) = 0. \]

Theorem 6. If \( \{a_1, a_2, a_3\} \) is ISS \( (a, \tilde{H}) \) and the sentence \( B \) exists, for which:

\[
\sharp (a) (a_1, B), \quad (13)
\]

\[
\sharp (a) (a_2, B), \quad (14)
\]

then
\[
\ell (a, \vec{H}) (a_3, a_2) = \ell (a, \vec{H}) (a_3, a_1).
\]

**Proof:** By Theorem 5 from (13) and (14):
\[
\ell (a, \vec{H}) (a_1, a_2) = 0; \tag{15}
\]
By Theorem 2:
\[
\ell (a, \vec{H}) (a_1, a_2) + \ell (a, \vec{H}) (a_2, a_3) \geq \ell (a, \vec{H}) (a_1, a_3),
\]

hence, from (15):
\[
\ell (a, \vec{H}) (a_2, a_3) \geq \ell (a, \vec{H}) (a_1, a_3),
\]

hence, by Theorem 2:
\[
\ell (a, \vec{H}) (a_3, a_2) \geq \ell (a, \vec{H}) (a_1, a_3). \tag{16}
\]

From
\[
\ell (a, \vec{H}) (a_3, a_1) + \ell (a, \vec{H}) (a_1, a_2) \geq \ell (a, \vec{H}) (a_3, a_2):
\]
\[
\ell (a, \vec{H}) (a_3, a_1) \geq \ell (a, \vec{H}) (a_3, a_2).
\]

From above and from (16):
\[
\ell (a, \vec{H}) (a_3, a_1) = \ell (a, \vec{H}) (a_3, a_2). \blacksquare
\]

The real number \( t \) is the moment of \( B \) for the frame of reference \( (\mathbb{R}a\vec{H}) \)
(denote: \( t = \left[ B \mid \mathbb{R}a\vec{H} \right] \), if
1) \( \mathbb{R} \) is ISS \( (a, \vec{H}) \),
2) the recorder \( b \) exists, for which: \( b \in \mathbb{R} \) and \( \zeta (a) (b, B) \),
3) \( t = \left[ a^* B \uparrow a, \vec{H} \right] - \ell (a, \vec{H}) (a, b). \)

**Lemma 5.**
\[
\left[ a^* B \uparrow a, \vec{H} \right] = \left[ a^* B \mid \mathbb{R}a\vec{H} \right].
\]

**Proof:** Let \( \mathbb{R} \) be ISS \( (a, \vec{H}) \), \( a_1 \in \mathbb{R} \) and
\(\hat{z}(a) (a_1, a \cdot B)\).

By Theorem 3:
\(\hat{z}(a) (a, a \cdot B)\).

From above and from (17) by Theorem 5:
\[\ell \left( a, \tilde{H} \right) (a, a_1) = 0,\]

hence,
\[
\left[ a \cdot B \mid \Re a \tilde{H} \right] = \left[ a \cdot B \uparrow a, \tilde{H} \right] - \ell \left( a, \tilde{H} \right) (a, a_1) = \left[ a \cdot B \uparrow a, \tilde{H} \right].
\]

The real number \(z\) is the distance between \(B\) and \(C\) for the frame reference \(\left( \Re a \tilde{H} \right)\) (denote: \(z = \ell \left( \Re a \tilde{H} \right) (B, C)\), if
1) \(\Re\) is ISS \(\left( a, \tilde{H} \right)\),
2) the recorders \(a_1\) and \(a_2\) exist, for which: \(a_1 \in \Re, a_2 \in \Re, \hat{z}(a) (a_1, B)\) and \(\hat{z}(a) (a_2, C)\),
3) \(z = \ell \left( a, \tilde{H} \right) (a_2, a_1)\).

From Theorem 2: all axioms of the metric space fulfill for such distance.

4 RELATIVITY

The recorders \(a_1\) and \(a_2\) receive the information, expressed by \(B\), identically for the recorder \(a\), if \(< \hat{z}(a) (a_2, a \cdot B) \gg\) is consequence of \(< \hat{z}(a) (a_1, a \cdot B) \gg\) and vice versa.

The recorders set is the homogeneous space, if all elements of this set receive every information identically.

The real number \(c\) is the propagation velocity of the information, expressed by \(B\), to the recorder \(a_1\) for the frame reference \(\left( \Re a \tilde{H} \right)\), if
\[
c = \frac{\ell \left( \Re a \tilde{H} \right) (B, a_1 \cdot B)}{\left[ a_1 \cdot B \mid \Re a \tilde{H} \right] - \left[ B \mid \Re a \tilde{H} \right]}.
\]

**Theorem 7.** In all homogenous spaces:
\[c = 1.\]
**Proof:** Let $c$ be the propagation velocity of the information, expressed by $B$, to the recorder $a_1$ for the frame reference $(RaH)$. That is: if $\mathcal{R}$ is ISS $(a, \tilde{H})$, then

$$z = \ell \left( RaH \middle| B, a_1^*B \right),$$  \tag{18}

$$t_1 = \left[ B \mid RaH \right],$$  \tag{19}

$$t_2 = \left[ a_1^*B \mid RaH \right],$$  \tag{20}

then

$$c = z / (t_2 - t_1).$$  \tag{21}

From (18): the elements $b_1$ and $b_2$ of $\mathcal{R}$ exist, for which:

$$\natural (a) (b_1, B),$$  \tag{22}

$$\natural (a) (b_2, a_2^*B),$$  \tag{23}

$$z = \ell \left( a, \tilde{H} \right) (b_1, b_2).$$  \tag{24}

From (19) and (20): the elements $b_1'$ and $b_2'$ of $\mathcal{R}$ exist, for which:

$$\natural (a) (b_1', B),$$  \tag{25}

$$\natural (a) (b_2', a_2^*B),$$  \tag{26}

$$t_1 = \left[ a_1^*B \uparrow a, \tilde{H} \right] - \ell \left( a, \tilde{H} \right) (a, b_1'),$$  \tag{27}

$$t_2 = \left[ a_1^*a_2^*B \uparrow a, \tilde{H} \right] - \ell \left( a, \tilde{H} \right) (a, b_2').$$  \tag{28}

From (18), (22), (25) by Theorem 6:

$$\ell \left( a, \tilde{H} \right) (a, b_1) = \ell \left( a, \tilde{H} \right) (a, b_1').$$  \tag{29}

Likewise from (18), (23), (26):

$$\ell \left( a, \tilde{H} \right) (a, b_2) = \ell \left( a, \tilde{H} \right) (a, b_2').$$  \tag{30}
From (27), (22), (29) by Lemma 4:
\[ t_1 = \left[ a^* b_1^* B \uparrow a, \bar{H} \right] - \ell \left( a, \bar{H} \right) (a, b_1). \] (31)

From (23) by Lemma 4:
\[ \left[ a^* a_2^* B \uparrow a, \bar{H} \right] = \left[ a^* b_2^* a_2^* B \uparrow a, \bar{H} \right]. \] (32)

By Lemma 1:
\[ \left[ a^* b_2^* a_2^* B \uparrow a, \bar{H} \right] \geq \left[ a^* b_2^* B \uparrow a, \bar{H} \right]. \] (33)

From (23):
\[ \natural (a) (a_2, b_2^* B). \]

From above by Lemma 4:
\[ \left[ a^* a_2^* b_2^* B \uparrow a, \bar{H} \right] = \left[ a^* b_2^* B \uparrow a, \bar{H} \right]. \] (34)

Again by Lemma 1:
\[ \left[ a^* a_2^* b_2^* B \uparrow a, \bar{H} \right] \geq \left[ a^* a_2^* B \uparrow a, \bar{H} \right]. \]

From above and from (34), (32), (33):
\[ \left[ a^* a_2^* B \uparrow a, \bar{H} \right] \geq \left[ a^* b_2^* B \uparrow a, \bar{H} \right] \geq \left[ a^* a_2^* B \uparrow a, \bar{H} \right], \]

hence,
\[ \left[ a^* a_2^* B \uparrow a, \bar{H} \right] = \left[ a^* b_2^* B \uparrow a, \bar{H} \right]. \]

From above and from (28), (30):
\[ t_2 = \left[ a^* b_2^* B \uparrow a, \bar{H} \right] - \ell \left( a, \bar{H} \right) (a, b_2). \]

From above and from (22) by Lemma 4:
\[ t_2 = \left[ a^* b_2^* b_1^* B \uparrow a, \bar{H} \right] - \ell \left( a, \bar{H} \right) (a, b_2). \] (35)

Let us denote:
\[ u = \left[ a^* C \uparrow a, \bar{H} \right], \] (36)
\[ d = \left[ a^* b_1^* a^* C \uparrow a, \bar{H} \right]. \] (37)
\[ w = \left[ a^* b_2^* a^* C \uparrow a, \bar{H} \right], \quad (38) \]

\[ j = \left[ a^* b_2^* b_1^* a^* C \uparrow a, \bar{H} \right], \quad (39) \]

\[ q = \left[ a^* b_1^* b_2^* a^* C \uparrow a, \bar{H} \right], \quad (37) \]

\[ p = \left[ a^* b_1^* b_2^* b_1^* a^* C \uparrow a, \bar{H} \right], \quad (40) \]

\[ r = \left[ a^* b_2^* b_1^* b_2^* a^* C \uparrow a, \bar{H} \right]. \]

Since \( \Re \) is ISS \((a, \bar{H})\), then
\[ q - w = p - j, \quad (41) \]
\[ j = q. \quad (42) \]

And from \((35), (31), (37), (39)\):
\[
(t_2 + \ell \left( a, \bar{H} \right) (a, b_2)) - (t_1 + \ell \left( a, \bar{H} \right) (a, b_1)) = j - d,
\]
hence,
\[
t_2 - t_1 = j - d - \ell \left( a, \bar{H} \right) (a, b_2) + \ell \left( a, \bar{H} \right) (a, b_1). \quad (43)
\]

From \((36), (37), (38)\) by Lemma 1:
\[
\ell \left( a, \bar{H} \right) (a, b_2) = 0.5 \cdot (w - u), \quad \ell \left( a, \bar{H} \right) (a, b_1) = 0.5 \cdot (d - u).
\]

From above and from \((41), (42), (43)\):
\[
t_2 - t_1 = 0.5 \cdot ((j - d) + (j - w)) = 0.5 \cdot (j - d + p - j) = 0.5 \cdot (p - d).
\]

From \((40), (37), (24)\):
\[
z = 0.5 \cdot (p - d).
\]

Therefore,
\[
z = t_2 - t_1.
\]

That is, in every homogenous space the propagation velocity of every information to every recorder for every frame reference equals to 1.

From this theorem: in all homogenous spaces: (the time irreversibility)
Therefore, in every homogenous space: **nobody can learn about that, what any event occurred, before, than it occurred.**

By the Urysohn theorem \(\mathbb{3}\): every homogenous space is homeomorphic to some set into the real Hilbert space. If this homomorphism is not the identity transformation, then \(\mathbb{R}\) is the noneuclidean space. In this case some variant of General Relativity Theory can be constructed into this ”space-time”. If this homomorphism is some identity transformation, then \(\mathbb{R}\) is the Euclidean space. In this case some coordinates system \(\mathbb{R}^\mu\) exists, for which the following condition fulfills:

for all elements \(a_1\) and \(a_2\) of \(\mathbb{R}\) the points \(-\vec{x}_1\) and \(-\vec{x}_2\) of \(\mathbb{R}^\mu\) exist, for which:

\[
\ell\left(\vec{a}, \vec{H}\right) (a_k, a_s) = \left(\sum_{j=1}^{\mu} (x_{s,j} - x_{k,j})^2\right)^{0.5}.
\]

In this case \(\mathbb{R}^\mu\) is denoted as the coordinates system of the frame reference \((\mathbb{R}a\vec{H})\), and the numbers \(\langle x_{k,1}, x_{k,2}, \ldots, x_{k,\mu}\rangle\) - as the coordinates of the recorder \(a_k\) in \(\mathbb{R}^\mu\).

The coordinate system of the frame reference can be determined accurate to the transformations of the replacement, the rotating and the inversion.

\(B\) has got the coordinates \(\langle x_1, x_2, \ldots, x_\mu\rangle\) in the coordinate system \(\mathbb{R}^\mu\) of the frame reference \((\mathbb{R}a\vec{H})\), if the recorder \(b\) exists, for which: \(b \in \mathbb{R}, \xi(a) (b, B)\) and the coordinates of \(b\) in \(\mathbb{R}^\mu\) are \(\langle x_1, x_2, \ldots, x_\mu\rangle\).

The recorder \(b\) has got the coordinates \(\langle x_1, x_2, \ldots, x_\mu\rangle\) in the coordinate system \(\mathbb{R}^\mu\) in the moment \(t\) of the frame reference \((\mathbb{R}a\vec{H})\), if for every \(B\) the following condition fulfills:

if \(t = \left\lceil b^* B \mid \mathbb{R}a\vec{H}\right\rceil\), then \(\ll b^* B \gg\) has got the coordinates \(\langle x_1, x_2, \ldots, x_\mu\rangle\) in the coordinate system \(\mathbb{R}^\mu\) of the frame reference \((\mathbb{R}a\vec{H})\).

From Theorem 9: For all real numbers \(v (|v| < 1)\) and \(l\), for the coordinate system \(\mathbb{R}^\mu\) of the frame reference \((\mathbb{R}a\vec{H})\), if at every moment \(t\) the coordinates of:

- \(b\) are: \(\langle x_{b,1} + v \cdot t, x_{b,2}, x_{b,3}, \ldots, x_{b,\mu}\rangle\);
- \(a_0\) are: \(\langle x_{0,1} + v \cdot t, x_{0,2}, x_{0,3}, \ldots, x_{0,\mu}\rangle\);
- \(b_0\) are: \(\langle x_{0,1} + v \cdot l, x_{0,2} + l, x_{0,3}, \ldots, x_{0,\mu}\rangle\); and

\[t_C = \left\lceil b^* C \mid \mathbb{R}a\vec{H}\right\rceil;\]
\[t_D = \left\lceil b^* D \mid \mathbb{R}a\vec{H}\right\rceil;\]
\[q_C = \left\lceil b^* C \uparrow b, \{g_0, A, b_0\}\right\rceil;\]
\[q_D = \left\lceil b^* D \uparrow b, \{g_0, A, b_0\}\right\rceil;\]

then
\[
\lim_{t \to 0} 2 \cdot l/\sqrt{(1-v^2)} \cdot ((q_D - q_C) / (t_D - t_C)) = 1.
\]

If denote: \( q_D^0 = q_D \) and \( q_C^0 = q_C \) for \( v = 0 \), then
\[
\lim_{t \to 0} 2 \cdot l \cdot ((q_D^0 - q_C^0) / (t_D - t_C)) = 1.
\]

Therefore:
\[
q_D - q_C = (q_D^0 - q_C^0) \cdot \sqrt{(1-v^2)}.
\]

Hence, a \( \kappa \)-clock, which moves with velocity \( v \), runs \((1-v^2)^{-0.5}\) times slower than a static \( \kappa \)-clock.

Let \( v \) \((|v| < 1)\) and \( l \) be a real numbers, and \( k_i \) be a natural.

Let for the coordinate system \( \mathbb{R}^n \) of the frame reference \((\mathbb{R}a\overrightarrow{H})\) at every moment \( t \) the coordinates of:
- \( b \) be: \( \langle x_{b,1} + v \cdot t, x_{b,2}, x_{b,3}, \ldots, x_{b,n} \rangle \),
- \( g_j \) be: \( \langle x_{j,1} + v \cdot t, x_{j,2}, x_{j,3}, \ldots, x_{j,n} \rangle \),
- \( b_j \) be: \( \langle x_{j,1} + v \cdot t, x_{j,2}, x_{j,3} + l / (k_1 \cdot \ldots \cdot k_j), x_{0,3}, \ldots, x_{0,n} \rangle \),

for all \( q_i \) if \( q_i \in \mathbb{H} \) then the coordinates of
- \( q_i \) be \( x_{i,1} + v \cdot t, x_{i,2}, x_{i,3}, \ldots, x_{i,n} \),
- \( \mathbb{T} \) be \( \{ \langle g_1, A, b_1 \rangle, \{ g_2, A, b_2 \}, \ldots, \{ g_j, A, b_j \}, \ldots \} \).

In this case from Theorem 9:
\( \mathbb{H} \) is ISS \( \left( b, \mathbb{T} \right) \).

Hence, the internal stableness survives for the uniform in-line motion.

Let:
1) for the coordinate system \( \mathbb{R}^n \) of the frame reference \((\mathbb{R}a\overrightarrow{H})\) at every moment \( t \):
- \( b \) : \( \langle x_{b,1} + v \cdot t, x_{b,2}, x_{b,3}, \ldots, x_{b,n} \rangle \),
- \( g_j \) : \( \langle x_{j,1} + v \cdot t, x_{j,2}, x_{j,3}, \ldots, x_{j,n} \rangle \),
- \( b_j \) : \( \langle x_{j,1} + v \cdot t, x_{j,2}, x_{j,3} + l / (k_1 \cdot \ldots \cdot k_j), x_{0,3}, \ldots, x_{0,n} \rangle \),

for all \( q_i \) if \( q_i \in \mathbb{H} \) then the coordinates of
- \( q_i \) : \( x_{i,1} + v \cdot t, x_{i,2}, x_{i,3}, \ldots, x_{i,n} \),
- \( \mathbb{T} \) be \( \{ \langle g_1, A, b_1 \rangle, \{ g_2, A, b_2 \}, \ldots, \{ g_j, A, b_j \}, \ldots \} \),
- \( C \) : \( \langle C_1, C_2, C_3, \ldots, C_n \rangle \),
- \( D \) : \( \langle D_1, D_2, D_3, \ldots, D_n \rangle \),
- \( t_C = \left[ b^* C \mid \mathbb{R}a\overrightarrow{H} \right] \),
- \( t_D = \left[ b^* D \mid \mathbb{R}a\overrightarrow{H} \right] \);
2) for the coordinate system \( \mathbb{R}^n \) of the frame reference \( \mathbb{H}b\overrightarrow{T} \):
- \( C \) : \( \langle C_1, C_2', C_3', \ldots, C_n' \rangle \),
- \( D \) : \( \langle D_1', D_2', D_3', \ldots, D_n' \rangle \).
\[ t'_C = \begin{bmatrix} b^* C \mid 3b^T \end{bmatrix}, \]
\[ t'_D = \begin{bmatrix} b^* D \mid 3b^T \end{bmatrix}. \]

In this case from I, II, III:

\[
(t'_D - t'_C)^2 - (D'_1 - C'_1)^2 - (D'_2 - C'_2)^2 - \ldots - (D'_\mu - C'_\mu)^2 =

(t_D - t_C)^2 - (D_1 - C_1)^2 - (D_2 - C_2)^2 - \ldots - (D_\mu - C_\mu)^2.
\]

From above the Lorentz transformations are obtained.

5 RESUME

The clock-like structure can be constructed from the recorders, and the following results are obtained from I, II and III:

First, all such clocks have got the same direction, i.e. if the event, expressed by the sentence \( A \), precedes to the event, expressed by the sentence \( B \), with respect to any such clock, then it is the same for all other such clocks.

Second, the Time, defined by such clocks, proves irreversible, i.e. no the recorder can obtain the information, that a certain event has taken place, before it has actually taken place. Thus, nobody can return back into the Past Times or obtain the information from the Future Times.

Third, the set of recorders has been embedded in the metric space by some natural method; i.e. all metric space axioms are obtained from I, II and III.

Fourth, if this metric space proves to be the Euclidean space, then the corresponding recorders "space-time" obeys the Poincare complete group transformations. I.e. in this case the Special Theory Relativity follows from the logical properties of the information. If this metric space is not Euclidean, then any non-linear geometry exists on the space of the recorders, and any variant of the General Relativity Theory can be realized on this space.

Therefore, the principal time properties - the one-dimensionality and the irreversibility -, the space metric properties and the spatial-temporal principles of the theory of the relativity are deduced from I, II, and III. Hence, if you have got any set of the objects, which able to get, to keep or to give any information, then "the time" and "the space" are inevitable on this set. And it is all the same: or this set is in our world or this set is in any other worlds, in which the spatial-temporal structure does not exist initially. Hence the spatial-temporal structure arises from the logic properties of the information.

The transformations of the complete Poincare group are obtained from the logic properties of a information.

6 Acknowledgements

Special thanks to Prof. Emilio Panarella.
References

[1] Elliot Mendelson, "Introduction to Mathematical Logic", D.VAN NOS-TRAND COMPANY, INC., (1964)

[2] P.S.Alexandrov, "Introduction to Set Theory and General Topology", NAUKA, Moscow, (1977), p.96

[3] Item, p.174