Neoclassical Light – An Assessment of the Voyage into Hilbert Space

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1. Introduction
I thank Rodger Walser and his colleagues for organizing this conference to celebrate George Sudarshan’s birthday and landmark achievements in science. It is an honor for me to be here and also a great pleasure to be back in Austin to join the chorus of the FOGS.

I would like begin with a brief historical introduction and then turn to topics that we haven’t heard so much about thus far in the symposium, namely experiments in Quantum Optics. Let me remind you that blackbody radiation has had a profound impact on physics since the time of Planck. However, even in more modern times after the development of quantum mechanics and quantum electrodynamics, one really did not know how to describe the underlying field fluctuations as required for diverse interference phenomena. Emil Wolf laid critical foundations for this enterprise with his theory of classical coherence developed in collaboration with Max Born in their classic book *Principles of Optics*.

A great challenge to the physics community was the experiment by R. H. Brown and R. Q. Twiss in 1956 [1], which led to a confusing and controversial time for photons and fields. In 1958 Leonard Mandel helped to resolve the controversy by explaining how light fluctuations are converted to photocurrents in the detection process, albeit in with a classical description of the field [2]. The confusion arose from the effort to understand, on the one hand, interference phenomena that are expressed in terms of the complex amplitude of the electromagnetic field, for which only a classical description was available. On the other hand, quantum electrodynamics had been developed based upon energy eigenstates (i.e., Fock states) with photons as quanta. It was really difficult to understand how the electric and magnetic fields behave in terms of these quantum states. For example, the expectation value of any electromagnetic field is zero in a Fock state. The confusion was compounded by the historic invention of the maser and laser and by the pressing need for quantum theories of the dynamical processes of these and other laboratory advances.

2. The Optical Equivalence Theorem
And then in 1963 the fog lifted. Two landmark papers were published in Physical Review Letters, one by George [3] and one by Roy Glauber [4]. Within a few months, we suddenly had Nature’s language for the quantum theory of coherence, including for generation, propagation, and detection. Certainly, not all phenomena were understood, but Refs. [3, 4] and publications that followed by Glauber and Sudarshan provided a new foundation for the formulation of quantum theories of the interaction of light and matter.
An extremely important aspect of these developments was the Optical Equivalence Theorem, first stated by George in Ref. [3] and elaborated in his essential book with John Klauder [5]. Over the five decades since then, the Optical Equivalence Theorem has provided the central tool in Quantum Optics for distinguishing between classical and manifestly quantum (or nonclassical) regimes for the electromagnetic field.

As reviewed in the presentation by Professor Mehta, the density operator \( \hat{\rho} \) for the electromagnetic field can be expressed in the form [3]

\[
\hat{\rho} = \int d^2 z \phi(z) |z\rangle \langle z|.
\]

Here, the function \( \phi(z) \) is, in modern terms, called the Glauber-Sudarshan phase-space function [6]. The aspect of this function that George emphasized (and formally established) is its universal character for all states of the electromagnetic field, ranging from blackbody radiation, for which \( \phi(z) \) is a Gaussian distribution, to more pathological examples, such as a number (Fock) state \(|n\rangle\) of the field, for which

\[
\phi(z) = \frac{e^{i\delta(z)}}{n!} \frac{\partial^{2n} \delta^{(2)}(z)}{\partial z^n \partial z^{*n}}.
\]

As you can see and as Professor Mehta discussed, this is not an ordinary function. As Dorothy in the Wizard of Oz expressed so appropriately, we’re certainly not in Kansas anymore! For an
experimentalist, this seems like spooky territory, but in fact, once \( \phi(z) \) is clothed by an integral [5], it becomes our best friend.

Indeed as illustrated in Figure 1, the Optical Equivalence Theorem provides a definitive, model independent line of demarcation between the classical and manifestly quantum domains. In this regard, it is critical to know that \( \phi(z) \) is everywhere well-defined (“exists”) but which becomes negative or more singular than the delta function at a particular boundary. George’s Optical Equivalence Theorem provides this foundation. The behavior of \( \phi(z) \) leads to operational consequences in the laboratory and has been essential to advances in the creation of nonclassical light.

As expressed qualitatively in Figure 1, the Optical Equivalence Theorem provides a definitive boundary between the classical and quantum domains. But of course, everything is quantum, right? Well, for many phenomena, a manifestly quantum description is not required - such as for the fluctuations of blackbody radiation, radio waves, cell phones, lasers, and so on. It is perfectly fine to treat the field in such cases as a stochastic mixture of appropriate classical electromagnetic fields with \( \phi(z) \) serving as the underlying probability. This is what is indicated as the classical domain in the figure.

By contrast, the manifestly quantum or nonclassical domain encompasses the region for which a quantum description is essential and includes number states, squeezed states, photon twins, and even more bizarre states. This is a domain where \( \phi(z) \) does not exist in the usual sense of an ordinary function and for which our classical intuition about coherence phenomena fails. The question that has been central to the development of Quantum Optics since the early 1960s is how to move from the classical into the quantum domain depicted in Fig. 1. This is the voyage into Hilbert space that is expressed in the title of my presentation. It is a journey that has taken many decades to travel from the land of classical random processes to manifestly quantum phenomena. Our compass for this journey has been the Optical Equivalence Theorem for providing unambiguous direction into the nonclassical realm.

Certainly, I think that George appreciated much of this in 1963. His paper has the following footnote [3]:

> The only case where all correlation functions are known is for the important but familiar example of the blackbody radiation. We hope that this circumstance is not time independent!

I really like this note, which conveys the sense that light wasn’t very interesting in 1963, and that it should be – that we should try to do other, less “familiar” things.

3. The Creation of Nonclassical Light

In the ensuing years after the papers by Glauber and Sudarshan, there were many theoretical papers written in the search for physical processes that could generate quantum effects for light. Some of the theories were quite intimidating, the proposed experiments complicated, and the predicted effects small. However, it turns out there is a really simple way to generate nonclassical light, namely by way of the fluorescence from one atom [7,8]. The idea is to illuminate an atom, collect the scattered field, and direct it to two photodetectors by way of a beam splitter. If a detection event is recorded by one detector, surely a second simultaneous event cannot occur at the other detector since the atom can only scatter photons one by one. More formally, the conditional state of the atom following a detection event is the atomic ground state from which no emission can occur until after a finite interval for re-excitation of the atom.

Simple enough, but it took many years in the 1970s to build this experiment in the laboratory of Professor Leonard Mandel at the University of Rochester. I will not go into the technical
problems, which required the marriage of traditional laser spectroscopy with the new field of Quantum Optics.

Results from this effort are described in Ref. [9]. In particular, Figure 2 in Ref. [9] displays the number of coincidences $n(\tau)$ for photoelectric counts as a function of time delay $\tau$ between the counts. The tools to record time-resolved detection events from fast photoelectric detectors came from the high energy physics community, with some electronic components borrowed from colleagues at the University of Rochester.

What’s interesting about our results in Ref.9? Well, after calibrations and checks and cross-checks, we observed the simple feature $n(0) < n(\tau)$, which is to say that there was a rising slope in coincidences from $\tau = 0$. Anybody who knows Quantum Optics knows that then we’re done.

Based upon the Optical Equivalence Theorem, if the number of recorded pulse pairs $n(\tau)$ rises from $\tau = 0$, then the field that generated these photoelectric statistics must be nonclassical [6]. This is an absolutely unambiguous signature of a manifestly quantum state of light independent of one’s favorite theoretical model.

The experiment in Ref. [9] and subsequent work by Short and Mandel [10] are examples of nonclassical behavior associated with lumps of light - photons. There is another “flavor” of non-classical light that relates more directly to the complex amplitude of the field. Maxwell’s equations can be expressed in canonical form to resemble a collection of harmonic oscillators. For each oscillator, the two canonical variables $E_1$ and $E_2$ that describe the field are completely analogous to the position $x$ and momentum $p$ for a mechanical oscillator. The qualitative “rules” for states in the phase space are illustrated in Fig. 2 in terms of the Wigner phase-space function $W(x, p)$ for several states of the electromagnetic field.

The critical question is how to quantify the manifestly quantum character of the fluctuations of $E_1, E_2$. The Heisenberg uncertainty principle for light is analogous to that for a harmonic oscillator. The uncertainty product for $E_1, E_2$ in appropriate units is “1” for each oscillator (i.e., each mode of the E-M field), precisely as for $(x,p)$, which I will use in place of $(E_1, E_2)$. For nonclassical states, it is not possible to display the Glauber-Sudarshan phase-space function, which is highly singular. Hence we plot $W(x, p)$ in Fig. 2. The vacuum state has equal fluctuations for $x$ and $p$ set by Planck’s constant. A thermal state, which is a particularly important state historically, is a Gaussian just as for the vacuum state, but now with finite average photon number.
Of more interest is a squeezed state, whose nonclassical character follows directly from the Optical Equivalence Theorem [6]. In qualitative terms, a state whose Wigner distribution varies on a scale less than “1” (the dimension of the cells of the quantum phase space) is nonclassical, with the example of a squeezed state shown.

How do we translate these concepts to the laboratory? In 1984 when I was at UT Austin, there was a big race to observe “squeezing,” which was won by Richart Slusher and his colleagues at Bell Labs [11]. Subsequently, my group observed squeezing by what has become the world’s prototypical system [12]. As before, the key question is: how do we tell if the light coming out of one’s favorite black box exhibits quantum squeezing? And again, a model independent criterion is provided by the Optical Equivalence Theorem.

The crux of the matter is to define operationally the line shown in Fig. 2 that sets the level for vacuum fluctuations of light. And believe me, this was a hard part of the experiment in which my group invested considerable effort [Figs. 1 and 3 in Ref. [12]]. Determining this level has nothing to do directly with the physics of squeezing. Rather, it involves great care in understanding whether a particular level of noise current from a specific photodiode and electronic amplifier actually corresponds to the level associated with a vacuum input to a homodyne detector. Different groups accomplished this determination with various levels of “believability”. The best that my colleagues and I have done over the years is to define the “1” associated with vacuum fluctuations to an accuracy ~ 0.002.

Once we have that “1” nailed down, we have translated the abstract mathematics of the Optical Equivalence Theorem into a definitive laboratory “meter” for detecting nonclassical light. Based upon this razor’s edge for the determination of squeezing, my group at UT Austin carried out a series of experiments related to squeezed light. Two experiments of which I am particularly proud are (1) the empirical mapping of the Heisenberg Uncertainty Principle for light [12,13] and (2) the first measurement with sensitivity beyond the Standard Quantum Limit (in our case, in optical interferometry) [14].

4. Dual Beam Correlation
The discussion thus far has centered on quantum correlation for a single beam of light, whether in terms of “photons” as for photon antibunching or for fluctuations of the quantum amplitudes as for squeezed light. However, another area of great historic significance relates to the quantum character of correlation between two beams of light. There are a number of nonclassical inequalities that we could discuss, which fall broadly into two classes. The first relates to investigations of Bell inequalities for discrete variables (i.e., polarization correlation for beams of light) that are of profound importance but not so directly related to our story [15]. The second involves correlation in the infinite dimensional Hilbert space of light in the spirit of our previous discussion, about which I’ll say more.

An important experiment that is often overlooked was reported by John Clauser in 1974 [16]. As part of his investigations of the inequalities of John Bell, Clauser observed photon correlation for light from an atomic cascade [16], which is roughly the time domain version of the energy correlation studied by Weisskopf and Wigner in 1930 [17]. Clauser’s measurements of intensity-intensity correlation for light emitted in the successive steps of an atomic cascade demonstrated a clear violation of a classical Cauchy-Schwarz inequality (roughly, that the magnitude of cross correlation is bounded by the product of autocorrelation for two beams) and hence established the nonclassical character of the beams by way of the Optical Equivalence Theorem.

Leonard Mandel at the University of Rochester was searching for a more powerful means to carry forward his investigations of nonclassical light beyond his early experiments in resonance fluorescence. To circumvent the limitations of atomic radiative processes in free space, in the early 1980s Mandel undertook experiments to utilize parametric down conversion as a source to generate quantum fields. Mandel thus began a landmark set of experiments that
continued until his death in 2001 and that opened new vistas into the nature of the quantum world [6,18]. E. Wolf and I summarized the impact of these experiments in 2001 as follows:

> Beyond what had by then become “the standard model” of manifestly quantum effects, Mandel’s genius for probing the quantum realm led to advances throughout the 1990s that challenged and ultimately redefined our understanding of the quantum character of light and, more generally, of quantum measurement.

There is now a flourishing world community whose work is built upon the foundations laid by Mandel, which in turn, rest upon George’s Optical Equivalence Theorem.

Perhaps you will wonder that I have said very little about the “black boxes” employed in these various experiments to generate nonclassical light. It is really important from my perspective that we need not delve into such details to support the case for nonclassical light. Certainly we must understand the characteristics of our detectors [19]. But with this done, the Optical Equivalence Theorem provides an unambiguous means for validating the quantum character of the electromagnetic field for an incredible diversity of experiments.

5. The Paradox of EPR

Another important non-classical phenomenon is related to the well-known paper by Einstein, Podolsky and Rosen [20]. These authors considered the collision of two massive particles that led to entanglement for the positions \((x_1, x_2)\) and momenta \((p_1, p_2)\) for the final state of the particles. Although it has not been possible to realize this gedanken experiment with massive particles, there is an optical implementation, as was first analyzed by Margaret Reid in 1989 [21].

The idea is not to collide two particles, but rather two beams of light at a beam splitter. If each incident beam is in a squeezed state and if the relative phase for these two beams is precisely controlled, then the output state is identical to the EPR state in the limit of large squeezing for the inputs. The crux of the EPR paper is that if one measures \(x_1\) then \(x_2\) can be inferred perfectly, and likewise for \(p_1\) and \(p_2\). Hence, the product of uncertainties for the inferences of the variables \((x_2, p_2)\) given measurements of \((x_1, p_1)\) is not constrained by an uncertainty principle with lower bound of “1”. What might all this have to do with the Optical Equivalence Theorem? My group at Caltech carried out the initial experiment to realize the EPR paradox in 1992 [22]. We demonstrated that the product of uncertainties for inference of \((x_2, p_2)\) given measurements \((x_1, p_1)\) does indeed lie below the limit set by the uncertainty principle for \((x_2, p_2)\), where the relevant constraint was set by the Optical Equivalence Theorem.

However, it was not until 2000 that Professor Simon [23] and Duan et al. [24] were able to show that the line of demarcation set by the Optical Equivalence Theorem also determines the boundary between quantum entanglement and classical correlation for the EPR paradox, including for finite degrees of correlation. I consider this to be quite a remarkable result. Not until the work in Refs. [23,24] did we have the tools to quantify entanglement for states of continuous quantum variables.

6. Angels on a Pinhead?

At this point, one might well wonder “This business about the Optical Equivalence Theorem is all very interesting, but seems somewhat arbitrary in drawing a line of demarcation - so what?” It turns out that there really is an important “so what”. Nonclassical states as quantified by the Optical Equivalence Theorem are essential in diverse applications, ranging from the quantum limits to measurement to the teleportation of quantum states.

In fact, in 1987 my group at UT Austin carried out an experiment that first achieved measurement sensitivity beyond the so-called Standard Quantum Limit (SQL) [14]. I will not go into the details except to say that we followed a theoretical proposal by C. Caves [25] and used squeezed states of light to surpass the SQL for optical interferometry. The boundary associated with the SQL was set by the Optical Equivalence Theorem.
Another application of these ideas is the phenomenon called quantum teleportation for
transporting quantum states from A to B – Alice to Bob. This protocol was invented by C.
Bennett and his colleagues in 1993 [26]. It is a remarkable protocol with diverse applications in
Quantum Information Science, including for the realization of quantum networks.
In 1998, my group at Caltech performed an experiment to teleport a beam of light from one side
of an optical table to the other [27]. Success in such an experiment is determined by surpassing
the limit for the transport of the state without the use of quantum entanglement. Again, it turns
out that the demarcation between the classical and quantum domains for teleportation of light
beams is based upon the Optical Equivalence Theorem.

7. My First Colloquium

One of my first interactions with George was at a colloquium that I gave at UT Austin in 1979
entitled “Light work with optical forces”. George was in the front row (just like he was at the
symposium in his honor), and he interrupted me by saying “… that’s not right”. I remember this
very distinctly; I was a new assistant professor just starting my career and “ka boom”! I asked
George “why not?” and he said something that I did not understand, but he let me off the hook.
Later, I went to his office to understand his concern. Indeed, the formula that I had
showed assumed a classical random walk for an atom’s momentum driven by light scattering. In
some approximation this is indeed the case, but the basic equations have much more complex and
interesting possibilities, including entanglement between atomic momentum steps and the state of
the field. George appreciated this in my talk about a subject that he had not previously studied.
He encouraged me to go back to the basics and find out where things really come from. This and
other interactions with George over the years had a profound impact on me and on the design of
my experiments. Although often without success, my goal inspired by George has been always to
drill to the core of the matter.

One direction that my research took was that of cavity quantum electrodynamics (cQED).
My group and I tried to push not towards conventional lasers and optical processes, but into a
new regime to access fundamental radiative processes for single atoms and photons.

It took a couple of decades, but one of the things that we have been able to achieve is the
generation of single photons on demand within the setting of cQED [28]. The photons are
generated by one atom trapped inside an optical cavity in a deterministic fashion with a Gaussian
spatial mode and a “user” controlled wave packet. This was a long voyage from my work in
graduate school with photons randomly emitted into 4π.

And how do we know they are single photons? By now the story line is clear; we go back to the
Optical Equivalence Theorem, which leads to operational criteria for a nonclassical field.

Further Destinations into Hilbert Space - By now there is a large, worldwide
community pursuing experiments in Quantum Optics that rest upon the foundation of the Optical
Equivalence Theorem. One example in my group is an experiment that generates the following
entangled state [29]:

\[
|\Psi_{L,R}\rangle = |0\rangle_L |1\rangle_R \pm e^{i\phi} |1\rangle_L |0\rangle_R .
\]

This state has one quantum of excitation shared collectively between two ensembles of atoms, a
left ensemble (L) and a right ensemble (R) which are separated by 3 meters. Certainly, as written,
this is an entangled state, but how do we quantify entanglement in the laboratory? First, we map
the purported entangled state of the two material systems (L,R) to two beams of light (1_L,1_R), and
then employ the powerful tools of Quantum Optics to analyze the light. However, for the state
|\Psi_{L,R}\rangle and the resulting optical fields (1_L,1_R), the Optical Equivalence Theorem provides a
necessary but not sufficient condition to establish entanglement.

To demonstrate entanglement, we must turn to criteria that go beyond the Optical
Equivalence Theorem [29,30]. The tool that has served us so well for five decades now proves
insufficient for the general task of the classification and quantification of entanglement. However,
the Optical Equivalence Theorem does provide the standard for our quest, namely to find model-independent criteria for the classification and understanding of the zoology of states as we journey further into Hilbert space.

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The work that I have described has been carried out by a remarkable group of young people over the years -- graduate students, postdocs, and visiting scientists, as well undergraduates. We all owe a great debt to Professor Sudarshan for the future that he has so powerfully enabled with his historic contributions to Quantum Optics. We call ourselves DOGS – Disciples of George Sudarshan. And how can we best express our gratitude and honor George’s lifetime of landmark contributions? In the best California tradition, all great things are immortalized by a T-shirt, here presented to George with a photograph of two atomic ensembles entangled by his DOGS.

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