On the Soft Supersymmetry Breaking Parameters in Gauge-Mediated Models

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Abstract

Gauge mediation of supersymmetry breaking in the observable sector is an attractive idea, which naturally alleviates the flavour changing neutral current problem of supersymmetric theories. Quite generally, however, the number and quantum number of the messengers are not known; nor is their characteristic mass scale determined by the theory. Using the recently proposed method to extract supersymmetry-breaking parameters from wave-function renormalization, we derived general formulae for the soft supersymmetry-breaking parameters in the observable sector, valid in the small and moderate $\tan\beta$ regimes, for the case of split messengers. The full leading-order effects of top Yukawa and gauge couplings on the soft supersymmetry-breaking parameters are included. We give a simple interpretation of the general formulae in terms of the renormalization group evolution of the soft supersymmetry-breaking parameters. As a by-product of this analysis, the one-loop renormalization group evolution of the soft supersymmetry breaking parameters is obtained for arbitrary boundary conditions of the scalar and gaugino mass parameters at high energies.
1 Introduction

Low energy supersymmetric theories have attracted much attention in the past years, since they can solve the hierarchy problem of the Standard Model and provide an effective low-energy description of grand unified theories, describing all known interactions, including gravity [1]. In spite of these remarkable properties, since supersymmetry is broken at low energies, a complete phenomenological description of the low energy physical phenomena may only be obtained after the exact mechanism leading to the breakdown of supersymmetry in the observable sector is known. It was long ago realized that more important than the nature of this mechanism is the way the supersymmetry-breaking effects are transmitted to the observable sector.

One of the most interesting ideas concerning the origin of supersymmetry breaking is that it is transmitted to the observable sector through superfields which couple to the Standard Model superfields only via gauge interactions [2],[3]. These superfields, usually called messengers, are coupled to the sector in which supersymmetry is originally broken (in a spontaneous or dynamical way). The supersymmetry-breaking effects in the observable sector are obtained by decoupling the messenger states, thus generating an effective theory at low energies, which contain the information about the soft supersymmetry-breaking parameters. In the simplest cases, the coupling of the supersymmetry-breaking sector with the messengers may be parametrized by the introduction of auxiliary chiral superfields $X_I$, coupled to the messengers $\Phi_I$, and with non-vanishing expectation values in their scalar ($X_I$) and auxiliary ($F_{X_I}$) components [2]. For simplicity of notation, in the following, we shall omit the brackets in the definition of the field vacuum expectation values.

One of the phenomenological properties of gauge-mediated supersymmetry-breaking models is that fields with the same quantum numbers under the Standard Model gauge group acquire the same soft supersymmetry breaking masses, leading to a natural suppression of the flavour changing neutral current effects via a super-GIM mechanism. Moreover, in order to preserve the successful unification of couplings in the minimal supersymmetric standard model (MSSM), the messengers are assumed to form complete representations of the group $SU(5)$ and, for these fields to acquire explicit SUSY conserving masses, that they come in vector representations ($5 \bar{5}, 10 \bar{10}$, etc.). In addition, it is usually assumed that the dynamics leading to the non-vanishing $X_I$’s and $F_{X_I}$’s preserves the $SU(5)$ symmetry, and that all messengers acquire a single mass scale, modulo wave-function renormalization and supersymmetry breaking effects. Although these assumptions lead to phenomenologically acceptable models, the way supersymmetry-breaking effects are transmitted to the messenger sector may violate the exact $SU(5)$ relations [4],[5],[6]. Moreover, the messengers may acquire different masses, provided that, apart from the usual doublet-triplet splitting in the Higgs sector, the hierarchy of masses between different $SU(5)$ multiplet states is not extremely large, in order to preserve the unification of couplings (other possibilities are discussed in Ref. [4]).

An example of these generalized scenarios was provided in Ref. [5], where the mess-

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1 In the following, we shall assume the absence of off diagonal $F$-terms in the messenger flavor space [8].
sengers that transmit supersymmetry breaking proceed from two $10 \bar{10}$ multiplets of $SO(10)$, one of which contains the usual Higgs doublets. In this case, the triplets and the heavy doublets act as messengers, which acquire different masses and feel the supersymmetry breaking effects in different ways. The phenomenological consequences of this kind of scenarios may be very different from the minimal one. In the particular case of Ref. [5], light gluinos appear in the spectrum, due to a suppression of the supersymmetry-breaking effects in the colour sector, originated from the Higgs triplet states. Since no such suppression factor appears in the terms originated from the heavy doublet states, particles with electroweak quantum numbers acquire much larger masses than the coloured ones.

Our purpose in this letter is not to concentrate on any particular model, but to present general formulae for an arbitrary messenger sector, independently of their quantum numbers. In section 2, we review the method to extract the supersymmetry-breaking terms from wave-function renormalization effects, presenting a generalization of the method derived in Ref. [7] for the case of many messengers. In section 3 we present an example, with a messenger sector consisting in a $5 \bar{5}$ of SU(5) and with doublet and triplet components whose masses are split in an arbitrary way. In section 4 we present the general formulae. We reserve section 5 for our conclusions. Relevant formulae for the derivation of the expressions presented in sections 3 and 4 are given in the appendix.

2 Method to extract supersymmetry-breaking terms

Let us start with the effective superpotential

$$W = \sum_i X_I \Phi_I \Phi_I^\dagger,$$

where, as mentioned in the introduction, we shall assume that both $X_I \neq 0$ and $F_{X_I} \neq 0$. The fermion components of the messengers $\Phi_I$ acquire a mass equal to $X_I$ in the background of the $X_I$ field, while the scalar components acquire supersymmetry breaking masses $m^2_S = X_I^2 \pm F_{X_I}$. Observe that the sign of the vacuum expectation values is not fixed a priori, what, of course, has no relevance at this stage.

Under these conditions, the values of the soft supersymmetry-breaking parameters in the observable sector may be obtained by a simple generalization of the method developed in Ref. [7]. Let us briefly discuss this method. The gauge superfield kinetic term may be parametrized by

$$L_W = \int d^2\theta S_j W_j^\dagger W_j^\alpha,$$

where $\text{Re}(\langle S_j \rangle) = \alpha_j^{-1}/4\pi$, while $-\langle F_{S_j}/(2S_j) \rangle$ is the renormalized gaugino-mass parameter, with $j$ a group index parameter. Throughout this paper, summation over repeated indices will be implicitly understood. Owing to the presence of the messengers, the vacuum expectation values of $S_j$ and $F_{S_j}$ are a function of $X_I$ and $F_{X_I} \equiv F_I$. Assuming that all $F_I \ll X_I^2$, and at the first order in $F_I/X_I^2$, one can easily show that

$$M_j = -\frac{1}{2} \frac{\partial \ln S_j}{\partial \ln X_I} \frac{F_I}{X_I},$$

where

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for j
In general, the sign and phase of the different contributions to the gaugino masses may be important, since cancellations between them may occur and, in addition, non-vanishing CP-violating phases may be induced in the low-energy theory.

For the scalar masses, the kinetic terms arise from

$$\mathcal{L} = \int d^4\theta Z_Q(X_I)\Phi_Q\Phi_Q,$$

where $Q$ denotes any arbitrary superfield in the observable sector. At the lowest order in $F_I/X_I^2$, the renormalized soft supersymmetry-breaking scalar masses are now given by

$$m_Q^2 = -\frac{\partial^2 \ln Z_Q(X_I)}{\partial X_J \partial X^*_I} \frac{F_J F_J^*}{X_I X^*_I}.$$  \hspace{1cm} (5)

Observe that the $X_I$ dependence of $Z_Q$ comes through the gauge-coupling dependence of the wave-function renormalization.

Finally, let us consider the trilinear and bilinear terms in the scalar potential, defined through

$$V = A_Q \Phi_Q \frac{\partial W}{\partial \Phi_Q},$$

where $A_Q$ denotes the soft supersymmetry-breaking parameters and, as before, we have used the same notation for the superfield as for the scalar terms. The expression for $A_Q$ is given by

$$A_Q = \frac{\partial \ln Z_Q}{\partial \ln X_I} \frac{F_I}{X_I X^*_I}.$$  \hspace{1cm} (7)

It is interesting to notice that, in all cases considered above, the supertrace \(\text{Str} \mathcal{M}^2\) vanishes in the messenger sector [9]. The treatment of the case with non-vanishing \(\text{Str} \mathcal{M}^2\) in the messenger sector demands the inclusion of the next-to-leading effects in the wave-function renormalization, an extension that is beyond the scope of this article.

### 3 General Formulae for the Soft Terms. An Example

Let us start with an example, which has all the necessary features to derive the general formulae for the soft terms in gauge-mediated models. Let us imagine that the messengers belong to one set of 5 5 representations of $SU(5)$, but that the doublets and triplets have different masses $X_I$, and, in general, different $F_I$ terms. The theory then has three different energy regimes: above the largest of the two messenger masses, the theory has the MSSM plus the 5 5 with the corresponding $\beta_i$ function coefficients for the couplings $\alpha_i$, which we shall denote by $b_i$. For definiteness, we shall take an example in which $X_2 > X_3$. At scales below $X_2$, but above $X_3$, the SU(2) messenger doublets decouple and we are left with a theory with the MSSM particle content plus two $SU(3)$ colour triplets with the quantum numbers of the right handed down quark and its charge conjugate. The corresponding $\beta_i$ function coefficients will be denoted by $b_i^{33}$. Below $X_3$, we are left with the MSSM, with $\beta_i$ function coefficients equal to...
In this particular example, the numerical values of these coefficients are given by the following table:

| $b_i$ | Full theory | MSSM + 3 3 | MSSM |
|-------|-------------|------------|------|
| $b_3$ | $-2$        | $-2$       | $-3$ |
| $b_2$ | $2$         | $1$        | $1$  |
| $b_1$ | $38/5$      | $7$        | $33/5$ |

More important than the particular values of the $\beta_i$ functions in the three regimes are the contributions of the messenger states to the $\beta_i$ function coefficients, which can be obtained by the information given in the above table. We shall call $B_2^i = 3/5, 1, 0$ the $SU(2)_L$ doublet contribution to these coefficients, and $B_3^i = 2/5, 0, 1$ the $SU(3)$ triplet contribution for $i = 1, 2, 3$, respectively.

Let us begin with the gaugino masses. The gauge couplings dependence on the messenger scales may be simply obtained by renormalization group methods. At the one-loop level, we get

$$\frac{1}{\alpha_j(X_2)} = \frac{1}{\alpha_j(M_G)} + \frac{b_j}{4\pi} \ln \left( \frac{M_G^2}{|X_2|^2} \right),$$

$$\frac{1}{\alpha_j(X_3)} = \frac{1}{\alpha_j(X_2)} + \frac{b_3^j}{4\pi} \ln \left( \frac{|X_2|^2}{|X_3|^2} \right),$$

$$\frac{1}{\alpha_j(\mu)} = \frac{1}{\alpha_j(X_3)} + \frac{b^{MSSM}_j}{4\pi} \ln \left( \frac{|X_3|^2}{|\mu|^2} \right). \quad (8)$$

Applying the above rules, and using the holomorphicity of $S_j(X_I, \mu)$, the gaugino masses $M_j$ are given by

$$M_j(\mu) = \frac{B_2^j \alpha_j(\mu)}{4\pi} \frac{F_2}{X_2} + \frac{B_3^j \alpha_j(\mu)}{4\pi} \frac{F_3}{X_3}. \quad (9)$$

Numerically, we hence obtain,

$$M_3(\mu) = \frac{\alpha_3(\mu)}{4\pi} \frac{F_3}{X_3}, \quad M_2(\mu) = \frac{\alpha_2(\mu)}{4\pi} \frac{F_2}{X_2}, \quad M_1(\mu) = \frac{\alpha_1(\mu)}{4\pi} \left( \frac{3F_2}{5X_2} + \frac{2F_3}{5X_3} \right), \quad (10)$$

a result that agrees with the one obtained, for the same conditions, in Ref. [4]. Observe that the gaugino masses receive contributions from the decoupling of both messenger states, which are proportional to the beta function contribution of the messenger $X_I$ and to $\alpha_i(X_I)$, while their dependence on $\alpha_i(\mu)$ is a reflection of the evolution of the gaugino masses at the one-loop level

$$\frac{dM_i}{dt} = \frac{b_i}{4\pi} \frac{M_i}{\alpha_i}, \quad \frac{d}{dt} \left( \frac{M_i}{\alpha_i} \right) = 0, \quad (11)$$

where $t = \ln \mu^2$. For future purposes, it is useful to define the gaugino masses at the scales $X_2$ and $X_3$, which are given by

$$M_j(X_2) = \frac{B_2^j \alpha_j(X_2)}{4\pi} \frac{F_2}{X_2}, \quad M_j(X_3) = \frac{B_2^j \alpha_j(X_3)}{4\pi} \frac{F_2}{X_2} + \frac{B_3^j \alpha_j(X_3)}{4\pi} \frac{F_3}{X_3}. \quad (12)$$
The gaugino masses at the scale $X_2$ come from the decoupling effects of the doublet messengers, while $M_j(X_3)$ comes in part from the running between $X_2$ and $X_3$ (first term in the second line of Eq. (12)) and in part from the decoupling effects of the colour triplet messengers (last term in the second line of Eq. (13)).

The scalar sector is more interesting. If all Yukawas are neglected, the wave-function renormalization is given by

$$
\ln \left( \frac{Z_Q(X_I, X_I^I, \mu)}{Z_Q(M_G)} \right) = - \frac{2c_Q^i}{b_i} \ln \left( \frac{\alpha_i(M_G)}{\alpha_i(X_2)} \right) - \frac{2c_Q^i}{b_i^{33}} \ln \left( \frac{\alpha_i(X_2)}{\alpha_i(X_3)} \right) - \frac{2c_Q^i}{b_i^{MSSM}} \ln \left( \frac{\alpha_i(\mu)}{\alpha_i(X_3)} \right),
$$

(13)

where $c_Q^i$ is the quadratic Casimir of the superfield $Q$ under the $i$ gauge group, which, for a fundamental representation of $SU(N)$ takes the value $c_Q = (N^2 - 1)/2N$, while $c_Q = Y^2/4$ for $U(1)$. Observe that we are implicitly working with a normalization of the gauge couplings consistent with their unification at high energies, so $\alpha_1 = 5/3 \alpha_1^{SM}$, and $Y^2/4 \equiv 3/5 \times (Q - T_3)^2$. Applying the above rules, one obtains

$$
- \frac{\partial^2 \ln Z_Q}{\partial \ln X_2 \partial \ln X_2^+} = - \frac{2c_Q^i}{16\pi^2} \left[ -B_2^i \alpha_i^2(X_2) + \frac{(B_2^i)^2}{b_i^{33}} \left( \alpha_i^2(X_3) - \alpha_i^2(X_2) \right) \right] + \frac{(B_2^i)^2}{b_i^{MSSM}} \left( \alpha_i^2(\mu) - \alpha_i^2(X_3) \right),
$$

$$
- \frac{\partial^2 \ln Z_Q}{\partial \ln X_2 \partial \ln X_3^+} = - \frac{2c_Q^i}{16\pi^2} \left[ B_2 B_3 \alpha_i^2(X_3) \right] + \frac{(B_3^i)^2}{b_i^{MSSM}} \left( \alpha_i^2(\mu) - \alpha_i^2(X_3) \right),
$$

$$
- \frac{\partial^2 \ln Z_Q}{\partial \ln X_3 \partial \ln X_3^+} = - \frac{2c_Q^i}{16\pi^2} \left[ -B_3^i \alpha_i^2(X_3) + \frac{(B_3^i)^2}{b_i^{MSSM}} \left( \alpha_i^2(X_3) - \alpha_i^2(X_3) \right) \right].
$$

(14)

Taking into account Eqs. (3) and (12), the scalar masses at the scale $\mu$, can be defined

$$
m_Q^2(\mu) = \frac{2c_Q^i}{16\pi^2} B_2 \alpha_i^2(X_2) \left( \frac{F_2}{X_2} \right)^2 - \frac{2c_Q^i}{16\pi^2} M_2^2(X_2) \alpha_i^2(X_2) \left[ \frac{\alpha_i^2(X_3) - \alpha_i^2(X_2)}{b_i^{33}} \right] + \frac{2c_Q^i}{16\pi^2} B_3 \alpha_i^2(X_3) \left( \frac{F_3}{X_3} \right)^2 - \frac{2c_Q^i}{16\pi^2} M_3^2(X_3) \alpha_i^2(X_3) \left[ \frac{\alpha_i^2(\mu) - \alpha_i^2(X_3)}{b_i^{MSSM}} \right].
$$

(15)

The above expression has a simple interpretation. The first terms in the first and second lines of Eq. (13) are the threshold contributions of the messenger of mass $X_2$ and $X_3$ to the scalar soft supersymmetry-breaking mass parameters, respectively. The second terms in these lines are the terms resulting from the evolution of the scalar mass parameters from $X_2$ to $X_3$ and from $X_3$ to $\mu$, respectively. Indeed, the renormalization group equations for the scalar mass parameters are given by

$$
\frac{dm_Q^2}{dt} = -4c_Q^i \alpha_i^2 M_2^2.
$$

(16)

Using this equation, plus the evolution of the gaugino mass parameters, Eq. (11), it is easy to derive the last terms in the first and second lines of Eq. (13).

The scalar masses at the scale $X_2$, $X_3$ are given by

$$
m_Q^2(X_2) = \frac{2c_Q^i}{16\pi^2} B_2 \alpha_i^2(X_2) \left( \frac{F_2}{X_2} \right)^2
$$

5
\[
m^2_{Q}(X_3) = m^2_{Q}(X_2) - 2c^i_Q M^2_{i}(X_2) \left[ \frac{\alpha^2_{i}(X_3) - \alpha^2_{i}(X_2)}{b^3_i} \right] + \frac{2c^i_Q}{16\pi^2} B^i_3 \alpha^2_{i}(X_3) \left( \frac{F_3}{X_3} \right)^2. \tag{17}
\]

### 3.1 Yukawa Coupling Effects

The most interesting contributions to the evolution of the scalar masses come from the Yukawa terms. The top quark Yukawa effects in the renormalization group evolution of the Higgs scalar masses is indeed essential to induce the radiative breaking of the electroweak symmetry \([10]–[21]\). The top Yukawa effects on the supersymmetry-breaking parameters may be obtained by studying the evolution of the wave-function renormalization

\[
\frac{d \ln Z_Q}{d t} = \frac{2c^i_Q \alpha_i}{4\pi} - \frac{d^i_2 \alpha_t}{4\pi},
\tag{18}
\]

where \(\alpha_t = h_t^2/4\pi\), \(d^i_{Q_L} = 1\), \(d^i_{U_R} = 2\), and \(d^i_{H_2} = 3\). \(Q_L\) and \(U_R\) are the third generation left-handed quark doublet and right-handed up quark, respectively, and \(H_2\) is the Higgs that couples to up-type quarks superfields at tree level. In the above, we have ignored the bottom Yukawa coupling, considering it to be small compared with the top Yukawa coupling. The bottom Yukawa effects on the evolution of the soft supersymmetry breaking parameters may be quite relevant if tan \(\beta\) is large, tan \(\beta \approx m_t/m_b\) \([28]–[33]\). Hence, our analysis is valid in the small and moderate tan \(\beta\) regimes. To obtain the form of the wave-function renormalization, Eq. (18) must be complemented with the evolution of the Yukawa coupling

\[
\frac{d \alpha_t}{d t} = -\frac{\alpha_t}{4\pi} \left[ c^i_t \alpha_i - \left( d^i_{Q_L} + d^i_{U_R} + d^i_{H_2} \right) \alpha_t \right],
\tag{19}
\]

where \(c^i_t = 2(c^i_{Q_L} + c^i_{U_R} + c^i_{H_2})\), while \(\sum_{Q=U_R,Q_L,H_2} d^i_Q = 6\).

The top-quark Yukawa coupling at the different scales is given by

\[
\begin{align*}
\alpha_t(X_2) &= \frac{\alpha_t(M_G) E(X_2)}{1 - \frac{6\alpha_t(M_G)}{4\pi} F(X_2)}, \\
\alpha_t(X_3) &= \frac{\alpha_t(X_2) E'(X_3)}{1 - \frac{6\alpha_t(X_2)}{4\pi} F'(X_3)}, \\
\alpha_t(\mu) &= \frac{\alpha_t(X_3) E''(\mu)}{1 - \frac{6\alpha_t(X_3)}{4\pi} F''(\mu)}. 
\end{align*}
\tag{20}
\]

where \(dF/\ln t = E\), \(dF'/dt' = E'\), \(dF''/dt'' = E''\), with \(X_2 \leq t \leq M_G\), \(X_3 \leq t' \leq X_2\), \(\mu \leq t'' \leq X_3\), respectively. The functions \(E, E'\) and \(E''\) are defined by

\[
\begin{align*}
E &= \prod_i \left[ \frac{\alpha_t(t)}{\alpha_t(M_G)} \right]^{-c^i_t/b^i_t}, & E' &= \prod_i \left[ \frac{\alpha_t(t)}{\alpha_t(X_2)} \right]^{-c^i_t/b^3_t}, & E'' &= \prod_i \left[ \frac{\alpha_t(t)}{\alpha_t(X_3)} \right]^{-c^i_t/b^{MSSM}_t}. 
\end{align*}
\tag{21}
\]

The expression of the wave-function renormalization may be easily obtained by solving Eq. (18), making use of the above expressions. The derivatives of ln \(Z_Q\) with respect to \(X_t\) may be obtained from the formulae given in the appendix. For instance,
\frac{\partial \ln Z_Q}{\partial \ln X_2} = \frac{c_i B_3^2 d_i^Q}{(4\pi)^3} \alpha_t(X_2) F'(X_3) H'(\alpha_t^2) \left[ \frac{1}{1 - \frac{6\alpha_t(X_2) F'(X_2)}{4\pi}} \right] + 2c_i B_2^j \int_{X_2}^{X_3} \frac{\alpha_t^2}{(4\pi)^2} dt \\
+ \frac{\alpha_t(X_3) F''(\mu) d_i^Q}{(4\pi)^3} \left[ \frac{1}{1 - \frac{6\alpha_t(X_3) F'(X_3)}{4\pi}} \right] \int_{X_2}^{X_3} \alpha_t^2 + \frac{6\alpha_t(X_2) F'(X_3) H'(\alpha_t^2)}{4\pi \left( 1 - \frac{6\alpha_t(X_3) F'(X_3)}{4\pi} \right)} \\
+ \frac{c_i B_2^j}{(4\pi)^3} \frac{1}{1 - \frac{6\alpha_t(X_3) F'(X_3)}{4\pi}} \int_{X_2}^{X_3} \frac{\alpha_t^2}{(4\pi)^2} dt. \quad (22)

The form of the linear integral functions \(H'(f(t))\) and \(H''(f(t))\) are given in the appendix. In order to extract the trilinear mass parameter governing the stop mixing, it is more useful to write

\[
\sum_{Q,U,H_2} \frac{\partial \ln Z_Q}{\partial \ln X_2} = \frac{c_i B_3^2}{(4\pi)^2} \left[ \frac{6\alpha_t(X_2) F'(X_3) H'(\alpha_t^2)}{4\pi} \right] + \int_{X_2}^{X_3} \frac{\alpha_t^2}{(4\pi)^2} dt \\
\times \left[ 1 + \frac{6\alpha_t(X_3) F''(\mu)}{(4\pi) \left( 1 - \frac{6\alpha_t(X_3) F'(X_3)}{4\pi} \right)} \right] \\
+ \frac{c_i B_2^j}{(4\pi)^2} \left[ \frac{6\alpha_t(X_3) F''(\mu) H''(\alpha_t^2)}{4\pi} \right] + \int_{X_2}^{X_3} \frac{\alpha_t^2}{(4\pi)^2} dt. \quad (23)
\]

Analogously,

\[
\sum_{Q_L,U_R,H_2} \frac{\partial \ln Z_Q}{\partial \ln X_3} = \frac{c_i B_3^j}{(4\pi)^2} \left[ \frac{6\alpha_t(X_3) F''(\mu) H''(\alpha_t^2)}{4\pi} \right] + \int_{X_2}^{X_3} \frac{\alpha_t^2}{(4\pi)^2} dt. \quad (24)
\]

Let us rewrite the formulae derived above in a clearer way, by introducing the quantities

\[
y' = -\frac{6\alpha_t(X_2) F'(X_3)}{(4\pi) \left( 1 + \frac{6\alpha_t(X_2) F'(X_2)}{4\pi} \right)}, \\
y'' = -\frac{6\alpha_t(X_3) F'(\mu)}{(4\pi) \left( 1 + \frac{6\alpha_t(X_3) F'(X_3)}{4\pi} \right)}. \quad (25)
\]

The function \(y'\) denotes the square of the ratio of the Yukawa coupling at the scale \(X_3\) to the value that it would acquire at the scale \(X_2\) if the coupling at \(X_2\) were strong. Similarly, \(y''\) is the square of the ratio of the Yukawa coupling at the scale \(\mu\) to the value that it would acquire at the scale \(\mu\) if the coupling at the scale \(X_3\) were strong. The functions \(y'\) and \(y''\) are analogous to the function \(y\), equal to the ratio of the top Yukawa coupling to its fixed point value, which plays a relevant role in the solution of the renormalization group equations in models in which supersymmetry breaking is mediated by gravity effects \[14, 15, 16\].
The expression for the parameter $A_t = \sum_{Q=Q, U, H_2} \partial \ln Z_Q / \ln X_1 \times F_t / X_1$ becomes

$$A_t(\mu) = \frac{c_i^t M_i(X_2)}{4\pi \alpha_i(X_2)} \left[ \int_{X_2}^{X_3} \alpha_i^2(t) dt - y' H'(\alpha_i^2) \right] \times (1 - y'')$$

$$+ \frac{c_i^t M_i(X_3)}{4\pi \alpha_i(X_3)} \left[ \int_{X_3}^{X_2} \alpha_i^2(t) dt - y'' H''(\alpha_i^2) \right]. \quad (26)$$

The above expression has a clear interpretation: at the scales $X_2, X_3$, the trilinear coupling takes values

$$A_t(X_2) = 0,$$

$$A_t(X_3) = \frac{c_i^t M_i(X_2)}{4\pi \alpha_i(X_2)} \left[ \int_{X_2}^{X_3} \alpha_i^2(t) dt - y' H'(\alpha_i^2) \right]. \quad (27)$$

There is no one-loop threshold contribution to the couplings $A_t$ and the value at $X_3$ is generated from the running from the scales $X_2$ and $X_3$. The value of $A_t(X_3)$ then serves as a boundary condition for the running between $X_3$ and $\mu$ which generate both the $(-y'')$ factor in the first line of Eq. (26), as well as the whole second line.

Similar methods may be used to extract the soft supersymmetry breaking scalar masses from the second derivatives with respect to the $X_I$’s of the chiral superfield wave-function renormalization. Keeping only the terms depending on the top Yukawa coupling, we obtain

$$- \frac{\partial \ln Z_Q}{\partial \ln X_2} \bigg|_{\alpha_t} = \frac{d_Q^t}{4\pi} \left\{ \frac{c_i^t \alpha_i(X_2)^2 B_i^2 \alpha_i(X_2) F'(X_3)}{(4\pi)^2 \left[ 1 - \frac{6\alpha_i(X_2) F'(X_3)}{4\pi} \right]} \right.$$ 

$$+ \frac{2\alpha_i(X_2) F'(X_3) c_i^t B_i^2 \dot{c}_i B_i^2 H'(\alpha_i^2(t)) \left( \alpha_i^2(t) \int_{X_2}^{X_3} \alpha_i^2(t') dt' \right)}{(4\pi)^3 \left[ 1 - \frac{6\alpha_i(X_2) F'(X_3)}{4\pi} \right]}$$

$$- \frac{2\alpha_i(X_2) F'(X_3) c_i^t (B_i^3)^2 H''(\alpha_i^2(t))}{(4\pi)^4 \left[ 1 - \frac{6\alpha_i(X_2) F'(X_3)}{4\pi} \right]}$$

$$+ \frac{6\alpha_i^2(X_2) (F'(X_3))^2 \left[ c_i^t B_i^2 H'(\alpha_i^2(t)) \right]^2}{(4\pi)^5 \left[ 1 - \frac{6\alpha_i(X_2) F'(X_3)}{4\pi} \right]^2} \right\}$$

$$+ \text{running between scales } X_3 \text{ and } \mu, \quad (28)$$

where the last line represents the running effects between the scale $X_3$ and $\mu$, which, because of their lengthy analytical expression, we shall not write it down explicitly. Its form will become clear after giving the expression for $m_Q^2(\mu)$.

Assuming real gaugino mass parameters, additional contributions to the scalar mass parameters are obtained through,

$$- \frac{\partial \ln Z_Q}{\partial \ln X_2} \bigg|_{\alpha_t} = \frac{d_Q^t}{4\pi} \left\{ \frac{\alpha_t(X_3) F''(\mu) c_i^t B_i^2 \dot{c}_i B_i^2 H''(\alpha_i^2(t))}{(4\pi)^3 \left[ 1 - \frac{6\alpha_t(X_3) F''(\mu)}{4\pi} \right]^2} \right.$$ 

$$\times \left[ \int_{X_2}^{X_3} \frac{\alpha_i^2(t) dt}{4\pi} + \frac{6\alpha_t(X_2) F'(X_3) H'(\alpha_i^2(t))}{(4\pi)^2 \left[ 1 - \frac{6\alpha_t(X_2) F'(X_3)}{4\pi} \right]} \right]$$

$$\left. \right\}$$

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\[ + \frac{2\alpha_t(X_3) F''(\mu) c_i B_i^3 B_i^3 H'' \left( \alpha_i^2(t) \right) f_{X_3}^t}{(4\pi)^4 \left( 1 - \frac{6\alpha_t(X_3) F''(\mu)}{4\pi} \right)} \]

\[ \left( \frac{2\alpha_t(X_3) F''(\mu) c_i B_i^3 B_i^3 H''(\alpha_i^2(t))}{(4\pi)^3 \left( 1 - \frac{6\alpha_t(X_3) F''(\mu)}{4\pi} \right)} \right) \]

\[ + \frac{6\alpha_t^2(X_3) (F''(\mu))^2 [c_i B_i^3 H''(\alpha_i^2(t))] [c_i B_i^3 H''(\alpha_i^2(t))]}{(4\pi)^5 \left( 1 - \frac{6\alpha_t(X_3) F''(\mu)}{4\pi} \right)^2} \]

\[ + (X_2 \leftrightarrow X_3). \] (29)

Finally, the expression of $\partial \ln Z_Q / \ln X_3 \ln X_3^\dagger$ can be obtained by changing in an obvious way the relevant scales and $\beta_i$ function coefficients in the expression given between curly brackets in Eq. (28). The mass parameter $m_Q^2(\mu)$ can be obtained by using the above expressions. At the scale $X_3$, for instance, taking into account Eqs. (15), (17) and (28), one obtains

\[ m_Q^2(X_3) = m_Q^2(X_2) - 2c_i Q M_i^2(X_2) \left[ \frac{\alpha_i^2(X_3) - \alpha_i^2(X_2)}{b_i^3} \right] \]

\[ - \frac{y'd_Q}{6} \left( m_{Q_L}(X_2) + m_{U_R}(X_2) + m_{H_2}(X_2) \right) \]

\[ - 2d_Q y' \frac{c_i M_j(X_2) c_i M_j(X_2)}{\alpha_i(X_2) \alpha_j(X_2)} H' \left( \frac{\alpha_i^2}{4\pi} \right) \int_{X_2}^{\mu} \alpha_i^2(t') dt' \]

\[ + 2d_Q y' \frac{c_i M_i^2(X_2)}{\alpha_i^2(X_2)} H' \left( \frac{\alpha_i^3}{4\pi} \right) + \frac{d_Q}{6} \left( \frac{y' c_i M_i(X_3)}{\alpha_i(X_2)} \right) H' \left( \frac{\alpha_i^2}{4\pi} \right)^2 \]

\[ + \frac{2c_i Q B_i^3 \alpha_i^2(X_3) (F_3^\dagger)^2}{16\pi^2} \]

(30)

where the last term is, as said before, the threshold contribution at the scale $X_3$ and the other terms, apart from $m_Q^2(X_2)$, represent the running between the scales $X_2$ and $X_3$.

Now, inspection of the result for $\partial Q^2 / \ln X_2 \ln X_3^\dagger$ shows that there is now a new contribution absent in the running between $X_2$ and $X_3$. This is due to the appearance of a non-vanishing value of the trilinear soft supersymmetry-breaking parameter at the scale $X_3$, $A_t(X_3) \neq 0$ (see Eq. (27)). Actually,

\[ m_Q^2(\mu) = m_Q^2(X_3) - 2c_i Q M_i^2(X_3) \left[ \frac{\alpha_i^2(\mu) - \alpha_i^2(X_3)}{b_i^{MSSM}} \right] \]

\[ - \frac{y''d_Q}{6} \left( m_{Q_L}(X_3) + m_{U_R}(X_3) + m_{H_2}(X_3) \right) \]

\[ - 2d_Q y'' \frac{c_i M_j(X_3) c_i M_j(X_3)}{\alpha_i(X_3) \alpha_j(X_3)} H'' \left( \frac{\alpha_i^2}{4\pi} \right) \int_{X_3}^{\mu} \alpha_i^2(t') dt' \]

\[ + 2d_Q y'' \frac{c_i M_i^2(X_3)}{\alpha_i^2(X_3)} H'' \left( \frac{\alpha_i^3}{4\pi} \right) + \frac{d_Q}{6} \left( \frac{y'' c_i M_i(X_3)}{\alpha_i(X_3)} \right) H'' \left( \frac{\alpha_i^2}{4\pi} \right)^2 \]

\[ - \frac{d_Q}{6} \left( 1 - y'' \right) 2c_i M_i(X_3) \frac{d_Q}{\alpha_i(X_3)} H'' \left( \frac{\alpha_i^2}{4\pi} \right) A_t(X_3) \]
\[
- \frac{d_Q y''(1 - y'')}{6} A_t(X_3)^2.
\] (31)

Observe that the last term in Eq. (31) appears only through the contribution to \(\partial \ln Z_Q / \partial \ln X_2 \partial \ln X_2^\dagger\) omitted in Eq. (28). In order to understand the origin of this term it is easier to study the contributions to the soft supersymmetry breaking parameter \(\bar{m}^2 = m^2_{Q_L} + m^2_{H_2} + m^2_{U_2}\). It is easy to see that the expression given between curly brackets in Eq. (28) (suming over \(Q = H_2, U_2\) and \(Q_L\)) comes from the derivative of the first line of Eq. (23) with respect to \(X_2^\dagger\). The derivative with respect to \(X_2^\dagger\) of the last line of Eq. (23) may be obtained by using Eq. (29) and observing that the last line in Eq. (23) differs from Eq. (24) only in the replacement of \(B_i^3\) by \(B_i^2\). Finally, the contribution from the second line may be easily obtained by using the following relation

\[
\frac{\partial y''}{\partial \ln X_2} = -y''(1 - y'') \times \left[ \frac{c^2_i B_i^2 H''(\alpha_i^2)}{(4\pi)^2} \right]
\]

\[
+ \frac{c^2_i B_i^2}{(4\pi)^2} \left( \int_{X_2}^{X_3} \alpha_i^2 dt - y' H'(\alpha_i^2) \right). \tag{32}
\]

Using Eq. (27) and the fact that the top Yukawa contributions to \(m^2_Q\) are weighted by the factors \(d_Q\), the appearance of the last term in Eq. (31) becomes clear.

### 4 General Formulae

After the above example, we can give general formulae for the extraction of soft supersymmetry-breaking parameters in the case of split messengers.

In general, in a theory with \(N\) thresholds \(y^{(n)}\) can be defined as the ratio of the square of the Yukawas at one threshold with respect to the value that would be obtained if the Yukawa coupling at the immediately upper threshold were large,

\[
y^{(n)} = -\frac{6\alpha_t(X_N) F^{(n)}(X_{(N+1)})}{(4\pi) \left[ 1 - \frac{6\alpha_t(X_N) F^{(n)}(X_{(N+1)})}{4\pi} \right]}, \tag{33}
\]

where \(X_N\) is the n-th mass threshold scale and \(X_N > X_{(N+1)}\). The function \(F^{(n)}\) is defined by

\[
F^{(n)}(X_{(N+1)}) = \int_{X_N}^{X_{(N+1)}} E^{(n)}(t) dt, \quad E^{(n)}(t) = \prod_i \left( \frac{\alpha_i(t)}{\alpha_i(X_N)} \right)^{-c_i/b_i^n}, \tag{34}
\]

where \(b_i^n\) is the \(\beta_i\) function coefficient of the effective theory defined in the energy range between the scales \(X_N\) and \(X_{(N+1)}\). We shall also define the linear integral function \(H^{(n)}\) by

\[
H^{(n)}(f) = \int_{X_N}^{X_{(N+1)}} f(t) dt - \frac{1}{F^{(n)}(X_{(N+1)})} \int_{X_N}^{X_{(N+1)}} F^{(n)}(t) f(t) dt, \tag{35}
\]

which is most useful to express the soft supersymmetry-breaking parameters at the weak scale in a compact form.
Generalizing the formulae presented in the last section, the value of the gaugino masses at the threshold \( X_I \) is given by,

\[
M_j(X_I) = \frac{B^i_j (X_I)}{4\pi} \frac{F_1}{X_i^2}
\]

\[
M_j(X_{(N+1)}) = M_j(X_N) \frac{\alpha_j(X_{(N+1)})}{\alpha_j(X_N)} + \frac{B^j_{(N+1)} \alpha_j(X_{(N+1)})}{4\pi} \frac{F_{(N+1)}}{X_{(N+1)}}
\]

\[
M_j(\mu) = M_j(X_N) \frac{\alpha_j(\mu)}{\alpha_j(X_N)},
\]

where \( B^i_N \) represents the contributions to the coefficient of the function \( \beta_i \) of the messenger with mass \( X_N \). As in the above examples, the gaugino masses receive threshold contributions at each messenger mass scale and their simple renormalization group evolution makes the general expression for their values at low energies rather simple. In particular, the gaugino masses are not affected by the Yukawa coupling of the top quark at this order.

The trilinear soft supersymmetry-breaking mass terms, instead, not only depend on the gauge sector, but also are affected by top quark Yukawa dependent effects, that modify their renormalization group evolution. Following the example given in the last section we find for the general case, at this order:

\[
A_t(X_I) = 0
\]

\[
A_t(X_{(N+1)}) = \frac{\epsilon^i_t M_i(X_N)}{4\pi \alpha_i(X_N)} \left[ \int_{X_N}^{X_{(N+1)}} \alpha_i^2(t) \, dt - y'^{(n)}(n) H'^{(n)}(\alpha_i^2) \right] + A_t(X_N) \left( 1 - y'^{(n)} \right). \tag{37}
\]

Since there are no threshold contributions to the trilinear soft supersymmetry-breaking parameters, the relation between the parameters \( A_t(X_N) \) and \( A_t(\mu) \) is governed by the same expression as governs the evolution between two different threshold scales, Eq. (37).

The scalar mass parameters are also affected by gauge and top-quark Yukawa contributions. The Yukawa dependence enters only through the renormalization group evolution of these parameters. In the case of real gaugino masses, the general expression is given by

\[
m^2_Q(X_1) = \frac{2\epsilon^i_Q}{16\pi^2} B^i_Q \alpha^2_i(X_1) \left( \frac{F_1}{X_1} \right)^2,
\]

\[
m^2_Q(X_{(N+1)}) = m^2_Q(X_N) - 4\epsilon^i_Q M^2_i(X_N) \times \int_{X_N}^{X_{(N+1)}} \frac{\alpha_i^2(t)}{4\pi} \, dt
\]

\[
- \frac{y'^{(n)} d_Q^0}{6} \left( m^2_{Q_L}(X_N) + m^2_{U_R}(X_N) + m^2_{H_2}(X_N) \right)
\]

\[
- \frac{2d_Q^0 y'^{(n)} c_i^i M_j(X_N) c_j^i M_i(X_N)}{6 \alpha_i(X_N) \alpha_j(X_N)} H'^{(n)} \left( \frac{\alpha_i^2(t)}{4\pi} \right)^2 \int_{X_N}^{X_{(N+1)}} \alpha_i^2(t') \, dt'
\]

\[
+ \frac{2d_Q^0 y'^{(n)} c_i^i M^2_i(X_N) H'^{(n)}}{6 \alpha_i^2(X_N)} \left( \frac{\alpha_i^3}{4\pi} \right) + \frac{d_Q^0}{6} \left( \frac{y'^{(n)} c_i^i M_i(X_N) H'^{(n)}}{\alpha_i(X_N)} \right) \left( \frac{\alpha_i^2}{4\pi} \right)^2
\]
while the relation between $m^2_Q(X_N)$ and $m^2_Q(\mu)$ is analogous to the one between $m^2_Q(X_3)$ and $m^2_Q(\mu)$ given in Eq. (31), by changing $X_3$ by $X_N$ and the corresponding $\beta_i$ functions in an obvious way.

Finally, let us write the expression for the $B_H$ parameter, which governs the relation between the bilinear terms in the superpotential and the bilinear soft supersymmetry-breaking parameters in the scalar potential. This is given by

$$B_H(X_1) = 0,$$
$$B_H(X_{(N+1)}) = B_H(X_N) - \frac{y(n)d^i H_2}{6} A_i(X_N)$$
$$+ \frac{4\epsilon^i H_2 M_i(X_N)}{4\pi\alpha_i(X_N)} \times \int_{X_N}^{X_{(N+1)}} \alpha_i^2(t) \, dt - \frac{c^i M_i(X_N) y(n)d^i H_2}{4\pi\alpha_i(X_N)} \frac{H(n)}{6} \alpha_i^2,$$

with an analogous relation holding for the evolution between the scales $X_N$ and $\mu$. In general, however, we expect $B_H(X_1)$ to be modified by the dynamics leading to the supersymmetric mass parameter $\mu$ in the superpotential, and hence in phenomenological analysis, $B_H(X_1)$ (or $B_H$ in general) may be taken as a free parameter to be fixed by the requirement of consistent radiative electroweak symmetry breaking. Observe that the same dynamics might modify the Higgs mass parameters $m^2_{H_1}$. These corrections, if present, will change the boundary conditions of the parameters $m^2_{H_1}$ at the relevant scale, but will preserve the form of the evolution of the soft supersymmetry breaking parameters to low energies.

5 Conclusions

In this work, we presented formulae for the soft supersymmetry-breaking parameters in the observable sector in gauge-mediated supersymmetry-breaking scenarios, for the case of arbitrary quantum numbers of the messenger superfields. The expressions for the low-energy soft supersymmetry-breaking parameters were obtained by using the recently proposed method to extract supersymmetry-breaking parameters from wave-function renormalization. The full dependence on the gauge and top-quark Yukawa coupling effects was presented. These formulae are particularly useful to study the radiative breaking of the electroweak symmetry in the regime of low and moderate values of $\tan \beta$.\(^2\)

\(^2\)The Higgs soft supersymmetry-breaking parameters at the messenger mass scales should be corrected by the finite next-to-leading order contributions, in case they are sizeable. Moreover, in order to cancel the scale and scheme dependence of these parameters, the full one-loop effective potential must be considered.\(^3\)
In the most general case, the gaugino masses are very different from those that would be obtained starting from universal values at the unification scale. The generalized expressions for the low energy soft supersymmetry breaking parameters hence make contact with similar expressions, presented recently in the literature, for the evolution of the supersymmetry breaking parameters for arbitrary boundary conditions for the soft supersymmetry breaking parameters at a given scale $\Lambda$ \cite{21}. In fact, the expression for the soft supersymmetry-breaking parameters at low energies may be interpreted as resulting from the successive running between different threshold scales, defined by the masses of the fermion messengers, plus the matching contributions coming from the decoupling of each messenger superfield. The advantage of the method used in this article is that it provides both the matching and the running contributions without the risk of double-counting, or the need of explicit evaluation of the Feynman diagrams. The expressions presented in this article may be generalized to include the next-to-leading order effects or off-diagonal $F$-terms. We reserve these improvements for further analysis.

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Appendix

To obtain the dependence of the wave-function renormalization on the different mass scales, the following formulae are very useful:

\[
\frac{\partial \alpha_i(X_2)}{\partial \ln X_2} = \frac{b_i}{4\pi} \alpha_i^2(X_2), \quad \frac{\partial \alpha_i(X_3)}{\partial \ln X_2} = \frac{b_i - b_i^{33}}{4\pi} \alpha_i^2(X_3), \quad \frac{\partial \alpha_i(\mu)}{\partial \ln X_2} = \frac{b_i - b_i^{33}}{4\pi} \alpha_i^2(\mu),
\]

\[
\frac{\partial F(X_2)}{\partial \ln X_2} = E(X_2), \quad \frac{\partial \alpha_i(X_2)}{\partial \ln X_2} = -\frac{\alpha_i(X_2)}{4\pi} \left( c_i^i \alpha_i(X_2) - 6 \alpha_i(X_2) \right),
\]

\[
\frac{\partial E(X_2)}{\partial \ln X_2} = -\frac{c_i^i \alpha_i(X_2)}{4\pi} E(X_2),
\]

\[
\frac{\partial F'(X_3)}{\partial \ln X_2} = -1 + \frac{c_i^i b_i \alpha_i(X_2)}{4\pi b_i^{33}} F'(X_3) - \frac{c_i^i (b_i - b_i^{33})}{4\pi b_i^{33}} H_i'(X_2, X_3),
\]

\[
\frac{\partial E'(X_3)}{\partial \ln X_2} = -\frac{c_i^i}{4\pi b_i^{33}} \left[ (b_i - b_i^{33}) \alpha_i(X_3) - b_i \alpha_i(X_2) \right] E'(X_3),
\]

\[
\frac{\partial \alpha_i(X_3)}{\partial \ln X_2} = \frac{c_i^i (\alpha_i(X_3) - \alpha_i(X_2))}{4\pi} \left( 1 - \frac{b_i}{b_i^{33}} \right) \alpha_i(X_3) - \frac{6 \alpha_i(X_3) \alpha_i(X_2) c_i^i (1 - b_i/b_i^{33})}{4\pi [4\pi + 6 \alpha_i(X_2) F'(X_3)]}
\]

\[
\times \left[ \alpha_i(X_2) F'(X_3) - H_i'(X_2, X_3) \right],
\]

\[
\frac{\partial E''(\mu)}{\partial \ln X_2} = -\frac{c_i^i}{4\pi b_i^{MSSM}} (b_i - b_i^{33}) (\alpha_i(\mu) - \alpha_i(X_3)) E''(\mu),
\]

\[
\frac{\partial F''(\mu)}{\partial \ln X_2} = -\frac{c_i^i (b_i - b_i^{33})}{4\pi b_i^{MSSM}} \left[ H''(\mu) - \alpha_i(X_3) F''(\mu) \right],
\]

\[
\frac{\partial H_i'(X_3)}{\partial \ln X_2} = -\alpha_i(X_2) + \frac{b_i - b_i^{33}}{4\pi} H_i'(X_2, X_3) + \frac{c_i^i b_j \alpha_j(X_2)}{4\pi b_j^{33}} H_i'(X_2, X_3)
\]

\[
-\frac{c_i^j (b_j - b_j^{33})}{4\pi b_j^{33}} H_{ij}'(X_2, X_3), \tag{40}
\]

where

\[
H_i' = \int_{X_2}^{X_3} \alpha_i(t) E'(t) \, dt, \quad H_{ij}' = \int_{X_2}^{X_3} \alpha_i(t) \alpha_j(t) E'(t) \, dt. \tag{41}
\]

It is convenient to reexpress the above equations as a function of the linear integral function

\[
H'(f) = \int_{X_2}^{X_3} f(t) \, dt - \frac{1}{F'(X_3)} \int_{X_2}^{X_3} F'(t) f(t) \, dt. \tag{42}
\]

Hence,

\[
\int_{X_2}^{X_3} f(t) E'(t) \, dt = F'(X_3) \left[ H'(d f/dt) + f(X_2) \right]. \tag{43}
\]

Similar equations are found for the derivatives of the different functions with respect to \( \ln X_3 \), with the simple facts that \( \partial f(X_2)/\partial \ln X_3 = 0 \), and

\[
\frac{\partial \alpha_i(X_3)}{\partial \ln X_3} = \frac{b_i^{33}}{4\pi} \alpha_i^2(X_3), \quad \frac{\partial \alpha_i(\mu)}{\partial \ln X_3} = \frac{b_i^{33} - b_i^{MSSM}}{4\pi} \alpha_i^2(\mu),
\]

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\[
\frac{\partial F(X_3)}{\partial \ln X_3} = E(X_3), \quad \frac{\partial \alpha_t(X_3)}{\partial \ln X_3} = -\frac{\alpha_t(X_3)}{4\pi} \left( c^i \alpha_i(X_3) - 6 \alpha_t(X_3) \right),
\]
\[
\frac{\partial E(X_3)}{\partial \ln X_3} = -\frac{c^i \alpha_i(X_3)}{4\pi} E(X_3),
\]
\[
\frac{\partial F''(\mu)}{\partial \ln X_3} = -1 + \frac{c^i j^3 \alpha_i(X_3)}{4\pi b_{iMSSM}} F''(\mu) - \frac{c^i (b_{iMSSM}^3 - b_{iMSSM}^3)}{4\pi b_{iMSSM}} H''_i(X_3, \mu),
\]
\[
\frac{\partial E''(\mu)}{\partial \ln X_3} = -\frac{c^i}{4\pi b_{iMSSM}} \left[ (b_{iMSSM}^3 - b_{iMSSM}^3) \alpha_i(\mu) - b_i \alpha_i(X_3) \right] E''(\mu),
\]
\[
\frac{\partial \alpha_t(\mu)}{\partial \ln X_3} = \frac{c^i (\alpha_i(\mu) - \alpha_i(X_3))}{4\pi} \left( 1 - \frac{b_{iMSSM}^3}{b_{iMSSM}^3} \right) \alpha_t(\mu) - \frac{6 \alpha_t(\mu) \alpha_t(X_3) c^i j^3 (1 - b_{iMSSM}^3 / b_{iMSSM}^3)}{4\pi [4\pi + 6 \alpha_t(X_3) F''(\mu)]}
\times \left[ \alpha_t(X_3) F''(\mu) - H''_i(X_3, \mu) \right],
\]
\[
\frac{\partial H''_i(\mu)}{\partial \ln X_3} = -\alpha_t(X_3) + \frac{b_{iMSSM}^3}{4\pi} H''_i(X_3, \mu) + \frac{c^i j^3 \alpha_i(X_3)}{4\pi b_{iMSSM}^3} H''_i(X_3, \mu)
\]
\[
- \frac{c^i (b_{iMSSM}^3 - b_{iMSSM}^3)}{4\pi b_{jMSSM}^3} H''_i(X_3, \mu),
\]

where, much as in the previous case, we have defined the functions
\[
H''_i(X_3, \mu) = \int_{X_3}^{\mu} \alpha_t(t) E''(t) dt, \quad H''_{ij}(X_3, \mu) = \int_{X_3}^{\mu} \alpha_t(t) \alpha_j(t) E''(t) dt.
\]

Once again, the equations take a simplified form if they are expressed as a function of the integral function
\[
H''(f(t)) = \int_{X_3}^{\mu} f(t) dt - \frac{1}{F''(\mu)} \int_{X_3}^{\mu} F''(t) f(t) dt.
\]
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