Many-body synchronization of interacting qubits by engineered ac-driving

Sergey V. Remizov\textsuperscript{1,2}, Dmitry S. Shapiro\textsuperscript{1,2,3,*} and Alexey N. Rubtsov\textsuperscript{1,4,5}

\textsuperscript{1}Dukhov Research Institute of Automation (VIKTA), Moscow 127055, Russia
\textsuperscript{2}V. A. Kotelnikov Institute of Radio Engineering and Electronics, Russian Academy of Sciences, Moscow 125009, Russia
\textsuperscript{3}National University of Science and Technology MISIS, Leninsky prosp. 4, Moscow, 119049, Russia
\textsuperscript{4}Russian Quantum Center, Skolkovo, 143025 Moscow Region, Russia and
\textsuperscript{5}Department of Physics, Moscow State University, 119991 Moscow, Russia

In this work we introduce the many-body synchronization of an interacting qubit ensemble which allows one to switch dynamically from many-body-localized (MBL) to an ergodic state. We show that applying of $\pi$-pulses with altering phases, one can effectively suppress the MBL phase and, hence, eliminate qubits disorder. The findings are based on the analysis of the Loschmidt echo dynamics which shows a transition from a power-law decay to more rapid one indicating the dynamical MBL-to-ergodic transition. The technique does not require to know the microscopic details of the disorder.

\section{I. INTRODUCTION}

Contemporary view of generic properties of quantum ensembles where the interactions and disorder are simultaneously present is that those systems reveal the two states: the ergodic state and many-body localized (MBL) phase. The general statement is that the ergodic, or delocalized, phase of a closed many-body quantum system is characterized by the eigenstate thermalization hypothesis \cite{1}. It says that the wave function evolution of excitations from a narrow energy domain is nothing but a relaxation of observables to the Gibbs distribution \cite{2}. In contrast, the MBL phase reveals the non-ergodicity with an absence of an eigenstate thermalization \cite{3}. It is described by a class of Hamiltonians where the eigenvalues do not repel and have Poisson statistics. The notion of MBL was initially introduced in the context of finite temperature metal-insulator transition in a system of interacting electrons with a static random potential \cite{4–6}. Further, the MBL was intensively explored for disordered spin-1/2 systems, see \cite{7} for a review. One of the remarkable features of MBL phase is that von Neumann entropy of entanglement \cite{8,9} does not show a volume-law scaling for finite temperatures. Instead of that there is an area-law behavior of the entropy \cite{10–12} which is similar to the low temperature results obtained for the ground state of gapped spin liquids \cite{9}. The recent experimental observations of MBL phase have been reported for fermions in an optical lattice \cite{13}, driven dipolar spin impurities in diamond \cite{14} and in a gas of ultracold fermionic potassium atoms \cite{15}. Probing of a many-body dynamics on a quantum simulator with controllable 51\textsuperscript{87}Rb atoms was reported in \cite{16}. Most of the theoretical and numerical realizations of the MBL phase deal with arrays of qubits \cite{3,17–22}. Their theoretical models are of Heisenberg type with exchange interaction \cite{23,24} and random qubit frequencies.

Collective properties of the MBL phase can be probed by a temporal dynamics of the quantum information \cite{18,21,22} stored in a system during its evolution from a particular initial state. Namely, the break of ergodicity with the transition into MBL is seen by a change of the entropy time dependence from the linear to logarithmic one. Another technique is the Loschmidt echo \cite{25}, being a quantum mechanical counterpart of the Lyapunov exponent in classical chaotic systems. The Loschmidt echo is defined through an overlap of wave functions which were identical for $t = 0$ and start to evolve with two slightly different Hamiltonians. If the system is in MBL phase then the Loschmidt echo has a power-law decay, see \cite{26} and references therein for details. The regime of non-interacting Anderson insulating phase is expected to show a non-vanishing echo while the ergodicity is manifested by a fast exponential decay.

The external periodic driving and parametric pumping \cite{27–34} allow to introduce an additional control over the properties of the superconducting metamaterials realized as qubit ensembles \cite{35–37} and Josephson chains \cite{38}. For non-interacting qubits a suppression of an inhomogeneous broadening was demonstrated for the nitrogen-vacancies in diamonds \cite{39–41}. We later proposed an optimized shape of the driving pulses for the broadening suppression and studied the effect of strong driving on an entanglement in the hybrid qubit-cavity system \cite{42,43}. In the context of disordered interacting qubits it was shown that an external driving can enhance the localization effects, namely, a driven transition to the dynamical MBL phase was discovered for an engineered Floquet Hamiltonian \cite{44}. The main idea of \cite{44} is to impose a phase rotation on each even site of the qubit chain. It allows to effectively suppress the coupling between qubits while preserving the disorder in the qubit frequencies. However this technique does not seem to allow for an inverse switching from the MBL to an ergodic state, because phase rotations cannot lead to an effective increase of the qubit-qubit coupling.

In this Letter we introduce the many-body synchronization technique allowing for a switching from the MBL
to an ergodic state by an effective suppression of the disorder. We demonstrate that the sequence of \( \pi \)-pulses of an alternating phase, similar to that used in [41], leads to the synchronization of the inhomogeneously broadened qubits even in the presence of qubit-qubit interaction. Importantly, this technique requires that all qubits are subjected to the same engineered driving, irrespective of a particular disorder realization.

The paper is organized as follows. In the Section II we present the model of the driven qubits and their non-stationary Hamiltonian. In Section III we show the results for a transition from the MBL to ergodic phase and in IV we conclude.

II. MODEL

The quantum gates are the pulse techniques which allow to rotate a vector of the qubit state on the Bloch sphere on a particular angles. Superconducting technologies offer quantum gates via the coupling between a Josephson qubit and GHz transmission line [45–47]. The pulse duration time \( T/2 \) between the phase switch and its amplitude \( f \) are tuned such that it acts as \( \pi \)-pulse for a qubit of the median frequency \( h \). It follows from Rabi physics that rectangular \( \pi \)-pulse is realized if the qubits’ and carrying frequencies are brought into the resonance and if the following relation holds:

\[
T = \frac{2\pi}{\bar{f}}.
\]

The further investigations involve dynamics of the Neel’s antiferromagnetic ordered state in a view of the Loschmidt echo. Also, the qubit-qubit correlations, spin-density wave decay and fidelity, which is an overlap between initial and final states, are analyzed.

The quantum mechanical definition of the Loschmidt echo reads as

\[
S(t) = \frac{\left| \langle \psi(0) | T e^{i \int_0^t H(t') dt'} + \delta H t e^{-i \int_0^t H(t') dt'} | \psi(0) \rangle \right|}{T}.
\]

We follow the notation of the Ref. [26] where the initial state describes the spin-density wave

\[
|\psi(0)\rangle = |\uparrow\downarrow\uparrow\downarrow\ldots\downarrow\rangle
\]

and \( \delta H = \Delta \sigma^z_i \) is the perturbation term with \( g = \Delta \). The brackets \( \langle \rangle \) also assumes the averaging over the disorder realizations. In order to analyze the entanglement we also calculate a correlation function of qubits polarization, similar to [3], in a translation invariant form

\[
C(d) = \sum_i \left( (\psi(t) | \sigma^z_{i+d} \sigma^z_i | \psi(t) ) - (\psi(t) | \sigma^z_i | \psi(t) ) (\psi(t) | \sigma^z_{i+d} | \psi(t) ) \right),
\]

where the periodic boundary conditions for \( i = 1, L \) are applied.

We calculate the fidelity function \( \alpha(t) \) and the spin-density-wave amplitude \( A(t) \) defined as

\[
\alpha(t) = |\langle \psi(0) | \psi(t) \rangle|,
\]

and

\[
A(t) = \left| \sum_i (-1)^i \langle \psi(t) | \sigma^z_i | \psi(t) \rangle \right|.
\]
III. RESULTS FOR LOSCHMIDT ECHO AND QUBIT-QUBIT CORRELATIONS

In this part we analyze numerically the effect of the transversal driving on MBL phase by means of the Loschmidt echo $S(t)$ averaged by disorder realizations. In Fig. 1(a) the echo is shown for the steady state Hamiltonian (1) without the driving. The exchange constant in simulations is $J = 1/2$ and the number of qubits is $L = 8$. Increasing the disorder parameter $W$ from $W = 1$ (lower red curve) to $W = 8$ (upper magenta curve) we observe the decreasing of the decay rate (this data is in correspondence with the results of Ref. [26]). In the double logarithmic scale all of the curves in Fig. 1(a) have linear sectors. This stands for a power-law decay of the Loschmidt echo and indicates the MBL phase for all of the $W$. There are no curves with sub-power-law decay because the initial Neel state $|\psi(t)\rangle$ introduced in (7) is a low lying state on energy scale which does not show ergodicity even with low disorder. Appearing of the driving, described by the Hamiltonian (2), brings ergodicity. The data is shown in Fig. 1(b) where the driving strength in simulations is set to $f = 1$. The dynamic transition into the ergodic phase is seen by the disappearing of the linear sectors in the curves. We associate this phenomena with the many-body synchronization of the interacting qubits. The value of the entropy saturation for long times in the driven system, Fig. 1(b), is two orders less than for $f = 0$, Fig. 1(a). This follows from the increase of an effective temperature induced by the driving.

The second part of the results is related to the evolution dynamics of the Neel state in terms of the fidelity and antiferromagnetic order parameter, $\alpha(t)$ and $A(t)$. As it is shown in Figs. 2(a) and 3(a), if the driving is zero, $f = 0$, then the Neel state evolution has a saturation to the finite values of $\alpha(t)$ and $A(t)$. The local momenta are formed in the system and do not decay in this case. Indeed, the switching on of the driving $f = 1$, see Figs. 2(b) and 3(b), reveals their decay and means that system is in ergodic phase. In contrast to 'single-particle' characteristics $\alpha(t)$ and $A(t)$, the correlator $C(d)$ shows more complicated behavior. It does not decay for the driven system, as presented in the right column of Fig. 4. The correlator $C(d)$ has finite value for low distances $d = 1, 2$ and zero driving, see Figs. 4 in left column. In the non-zero driving regime, Figs. 4 in right column, all correlations are actually absent for long times.

IV. DISCUSSION

Being motivated by the work [44], where the dynamical transition from the ergodic state into the MBL phase has been predicted for qubits with switchable interaction parameters, we searched for a dual regime with a controllable transition out of the MBL phase. Implementation of this transition is relevant for state-of-the-art quantum interfaces where a large amount of controllable qubits are integrated with each other. If a dynamical transition is possible then it would provide a method of the disorder suppression. The unavoidable disorder in qubit frequencies, along with the decoherence, is always present and limits their collective performance. In this Letter we proposed the scenario of a controllable transition out of the MBL phase. It is realized as the driving of a transversal type (as Zeeman field in $xy$-plane in terms...
FIG. 3: Results for the antiferromagnetic order parameter $A(t)$: (a) in steady state system without the driving, $f = 0$, and (b) in driven system, $f = 1$.

V. ACKNOWLEDGMENTS

The research was funded by the Russian Science Foundation under Grant No. 16-12-00095.
FIG. 4: Results for the correlation function of qubits polarization $C(d)$ for different inter-qubit distances $d$: (left column) in steady state system without the driving, $f = 0$, and (right column) in driven system $f = 1$. 

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