The quark inside hadron.

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We study and solved Schwinger-Dayson equation for massive quarks in the quark model. Interaction potential is choose as sum oscillator and Coulomb terms. The calculation show us that negative energy of quark exist under $m_o \ll \left(\frac{4V_o^3}{3}\right)^{1/3}$ i.e. when mass quark is less that parameters of scale. Us are offered to use asymptotical behaviour at the solutions Schwinger-Dayson equation selection physical and non-physical conditions. The quark condensate under different values mass quarks are calculated. The scale parameters $\left(\frac{4V_o^3}{3}\right)^{1/3} = 520$ MeV.

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I. INTRODUCTION.

For description of decays of particles to us needed will know a behaviour of wave functions and the spectra of masses of mesons and baryons. In literature there are several general investigation ways: quark model, lattice QCD, bag model, QCD sum rules, quenched QCD and other phenomenological models and approaches. They have their own advantages and defect.

The most real approach is quark model based on Schwinger-Dyson (SD) and Bethe-Salpeter (BS) equations. This quark model proposed by the Orsay group [1], perfection by Pervushin [2] and Bicudo-Ribeiro [3]. The main purpose the works [1–3] was offer a method define wave functions and obtain spectrum masses of mesons and hadrons. Next step development of quark model is calculate meson spectra of masses of mesons and baryons. In literature there are several general investigation ways: quark gap equation and the energy of quarks are calculated in this approach. Using asymptotic behavior when hadron with interaction potential chosen as sum Coulomb and oscillator terms. We has solve SD mass form of quark-antiquark potential is one of the actual problem of the meson spectroscopy.

Thereby we have quark potential model which based on SD and BS equations. The definition of the form of quark-antiquark potential is one of the actual problem of the meson spectroscopy.

In the present paper we investigate quark mass function based on SD equation. We consider quark in hadron with interaction potential chosen as sum Coulomb and oscillator terms. We has solve SD mass gap equation and the energy of quarks are calculated in this approach. Using asymptotic behavior when $p \to \infty$ we can select solution on physical and non-physical stages of quarks. By means of solutions SD equation we have calculate $< \bar{q}q >$ quark condensate under different masses of quarks.

Let us consider the effective covariant relativistic action for singlet channel [2]:

$$S_{eff} = \int d^4x \bar{q}(x)(i\gamma \cdot \nabla - m_o)q(x) - \frac{1}{2} \int d^4xd^4y \bar{q}\gamma_\mu q \eta_{\alpha \beta} \eta_{\alpha \beta}$$

where $\bar{q}(x) = (m_o, \ldots, m_o)$ are the bare masses, $\alpha$ or $\beta$ are the short notation for the Dirac and flavour index. Quantization axis is chosen

$$\eta_{\mu} \sim \frac{1}{i} \frac{\partial}{\partial X_{\mu}},$$

where $\eta_{\mu}$ - single time-axis. From the covariant effective action [2] we can obtain SD equation for mass operator $(\Sigma(p^\perp))$ depends on transversal momentum $p^\perp$ with nonzero mass current quark $(m_o)$ [4].

$$\Sigma(p^\perp) = m_o + i \int \frac{d^4q}{(2\pi)^4} V(p^\perp - \bar{q}) \eta G_{\Sigma}(q) \eta$$

where $\bar{q} = \eta^\mu \gamma_\mu, \eta_{\mu}$ is satisfy Markov-Yukava principle [3], $\bar{q}^\perp = \bar{q} - \eta \bar{q}$ - momentum, which orthogonal on time axes ($\bar{q}^\perp, \eta = 0$), and $G_{\Sigma}$ - Green’s function of quark

$$G_{\Sigma}(q) = [\bar{q} - \Sigma(q)]^{-1}$$

Let us note that after integration [3] over "time" component $(q^\mu \equiv q \cdot \eta)$ we obtained the mass operator with perpendicular component only. We can write in Lorentz covariant presentation :

$$\Sigma(\bar{p}^\perp) = \bar{p}^\perp + E(\bar{p}^\perp)S^{-1}(\bar{p}^\perp)$$

where $E$ - energy of quark, and $S$ - unitary operator which can to present(parametrize) through $v$ function as

$$S(\bar{p}^\perp) = \exp\{-\bar{p} \cdot v(\bar{p}^\perp)\}$$

where $\bar{p} = |\bar{p}^\perp|$, $|\bar{p}^\perp| = \sqrt{(\bar{p}^\perp)^2}$. The one-particle Green function [1] can be represented in the form of the expansion over the states with positive and negative energies
\[ G_\Sigma(q) = \left[ \frac{\Lambda^\eta_{(+)}(q_\perp)}{q^\eta - E(q_\perp) + i\epsilon} + \frac{\Lambda^\eta_{(-)}(q_\perp)}{q^\eta + E(q_\perp) + i\epsilon} \right] \eta \] (7)

\[ \Lambda^\eta_{(+)} = S(q^+)\Lambda^\eta_{(+)}(0)S^{-1}(q_\perp) \] (8)

\[ \Lambda^\eta_{(-)} = S(q^+)\Lambda^\eta_{(-)}(0)S^{-1}(q_\perp) \] (9)

where \( \Lambda^\eta_{(\pm)}(0) \) are projection operators of positive and negative value,

\[ \Lambda^\eta_{(+)}(0) = \frac{1}{2}(I + \frac{\sigma^\eta}{p_\|}) \] (10)

\[ \Lambda^\eta_{(-)}(0) = \frac{1}{2}(I - \frac{\sigma^\eta}{p_\|}) \] (11)

here \( I \) – one-dimensional 4-matrix.

Since energy of quark \( E(0) \sim m_\circ \), \( E(\infty) \sim p_\perp \) and considering that \( S^2(0) = 1, S^2(\infty) = -1 \), so propagator of quark in limit \( p_\perp \to 0 \) coincide with propagator free quark in rest system

\[ G(0) = \frac{1}{2} \left[ \frac{1 + \gamma^\eta}{p_\| - m - i\epsilon} - \frac{1 - \gamma^\eta}{p_\| + m - i\epsilon} \right] \]

In limit \( p_\perp \to \infty \) propagator \( G(\infty) \) coincide with propagator massless quark

\[ G(\infty) = \frac{1}{2} \left( \frac{1 + \gamma^\eta p_\perp}{p_\| - p_\perp - i\epsilon} - \frac{1 - \gamma^\eta p_\perp}{p_\| + p_\perp - i\epsilon} \right) \]

Let us consider SD equation (3) in rest system \( \eta_\mu = (1,0,0,0) \), of bound state, one of the particles is quark. Inserting (5)-(11) into (3) and after integration over \( q_\eta = q^0 \) we have get next equation for \( \nu \) function

\[ m_\circ \sin 2\nu(p) - p \cos 2\nu(p) = \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} V(|\vec{p} - \vec{q}|) [\cos 2\nu(q) \sin 2\nu(p) - \frac{\vec{p} \cdot \vec{q}}{pq} \sin 2\nu(q) \cos 2\nu(p)] \] (12)

and expression for energy of quark

\[ E(p) = m_\circ \cos 2\nu(p) + p \sin 2\nu(p) + \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} V(|\vec{p} - \vec{q}|) [\cos 2\nu(q) \cos 2\nu(p) - \frac{\vec{p} \cdot \vec{q}}{pq} \sin 2\nu(q) \sin 2\nu(p)] \] (13)

II. SCHWINGER-DYSON MASS GAP EQUATION.

In the literature the equations (12),(13) usually called as mass gap equation. Let us make \( \nu(p) = \frac{1}{2} [\frac{1}{2} - \phi(p)] \) variable change in (12),(13). So we obtain the following equations

\[ E(p) \sin \phi(p) = m_\circ + \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} V(|p - q|) \sin \phi(q) \]

\[ E(p) \cos \phi(p) = p + \frac{1}{2} \int \frac{d^3q}{(2\pi)^3} V(|p - q|) \hat{p} \hat{q} \cos \phi(q) \] (14)

where \( p = |\vec{p}|, \hat{p} = \frac{\vec{p}}{p} \).
The SD equation define one-particle energy of flavour quark. The potential phenomenology of spectroscopy of quarkonia \[4\] in the meaning of the real interaction, uses in general, the sum Coulomb and the increasing potential. In the increasing potential is used the lattice calculation for description of the heavy quarkonia. The definition of the form \[\bar{q}q\] – potential is one of the actual problem of the meson spectroscopy. We chose the potential in the form as sum of oscillator and Coulomb terms.

\[
V(p) = \frac{4}{3} \frac{4\pi \alpha_s}{p^2} + (2\pi)^3 V_o^3 \Delta p_\delta^3(p)
\]  \hspace{1cm} (15)

One particle SD equation \[14\] for quark in dimensionless units takes a form

\[
\frac{d^2 \varphi(p)}{dp^2} + \frac{2}{p} \frac{d \varphi(p)}{dp} + \frac{\sin 2 \varphi(p)}{p^2} - 2p \sin \varphi(p) + 2m_o \cos \varphi(p) + \\
\frac{2\alpha_s}{3\pi} \int dq \frac{q}{p} \{ \mathcal{R}_1(p, q) \cos \varphi(p) - \mathcal{R}_2(p, q) \sin \varphi(p) \} = 0
\]  \hspace{1cm} (16)

\[
E(p) = p \cos \varphi(p) + m_o \sin \varphi(p) - \frac{1}{2} [\varphi'(p)]^2 - \frac{\cos^2 \varphi(p)}{p^2} + \\
\frac{2\alpha_s}{3\pi} \int dq \frac{q}{p} \{ \mathcal{R}_1(p, q) \sin \varphi(p) - \mathcal{R}_2(p, q) \cos \varphi(p) \}
\]  \hspace{1cm} (17)

where

\[
\mathcal{R}_1(p, q) = \ln \left| \frac{p + q}{p - q} \right| \left[ \frac{\sin \varphi(q) - \frac{m_o}{r(q)}}{r(q)} \right],
\]

\[
\mathcal{R}_2(p, q) = \left( \frac{p^2 + q^2}{2pq} \ln \left| \frac{p + q}{p - q} \right| - 1 \right) \left[ \cos \varphi(q) - \frac{q}{r(q)} \right],
\]

\[
r(q) = \sqrt{q^2 + (m_o)^2}.
\]

The equation \[13\] and expression for self-energy of quarks \[17\] is present in dimensionless unit \((4V_o^3/3)^{1/3} = 1\). We can define this parameter in physical region after solve BS equation, from spectroscopy mesons and baryons. In literature \[1, 3, 4\] usually use as: \((4V_o^3/3)^{1/3} \approx 280 \div 480\) MeV.

**III. NUMERICAL METHOD.**

To solve the equation \[14\] by analytical method is difficult that is way we’ll find the solution it by use numerical method. Rewrite equation \[14\] in following type:

\[
\mathcal{F}_1[\varphi(p)] + \mathcal{F}_2[\varphi(p)] = 0,
\]  \hspace{1cm} (18)

where

\[
\mathcal{F}_1[\varphi(p)] = \frac{d^2 \varphi(p)}{dp^2} + \frac{2}{p} \frac{d \varphi(p)}{dp} + \frac{\sin 2 \varphi(p)}{p^2} - 2p \sin \varphi(p) + 2m_o \cos \varphi(p),
\]

\[
\mathcal{F}_2[\varphi(p)] = \frac{2\alpha_s}{3\pi} \int dq \frac{q}{p} \{ \mathcal{R}_1(p, q) \cos \varphi(p) - \mathcal{R}_2(p, q) \sin \varphi(p) \}.
\]

The problem can be solved step by step scheme:

i) \(\alpha_s = 0, m_o = 0, (4V_o^3/3)^{1/3} = 1\). The current quark mass equal null and equation \[18\] disregarding Coulomb interaction potential is considered. We have chosen interaction potential as increasing oscillator potential.

ii) \(\alpha_s = 0, m_o \neq 0, (4V_o^3/3)^{1/3} = 1\). The equation \[18\] with provision for current quark mass on the interval \(0 \leq m_o \leq M_{\text{Largest mass quark}}\).

iii) \(\alpha_s \neq 0, m_o \neq 0, (4V_o^3/3)^{1/3} = 1\) The equation \[18\] oscillator plus Coulomb term of interaction potential and current quark mass not equal zero.
Unfortunately to consider the equation $[18]$ for case $iii$) without oscillator terms of interaction potential (i.e. $(4V_o^3/3)^{1/3} = 0$) to us it was not possible.

We have derived numerical solutions to the integr-differential equation $[18]$ with boundary conditions,

$$\varphi(0) = \frac{\pi}{2}, \quad \varphi(\infty) = 0$$

using the computational scheme developed in ref. [3]. This scheme consist the modification of the Newton iterations by combining the continuation over a parameter ($m_o$ or $\alpha_s$) method. This scheme has a convenient algorithm for assignment of the initial conditions as functions of the external parameters ($m_o$ or $\alpha_s$) due to a special choice of the iteration parameter as a step of the modified Newton iteration. So, in this scheme the solutions to problem $[18],[19]$ are used as an input.

The numerical solutions to boundary problem $[18],[19]$ for several values of $m_o$ are shown in Figure 1, under $\alpha_s = 0.2$. The values of free parameters $m_o,(4V_o^3/3)^{1/3}$ and $\alpha_s$ belonging to the physical region will be defined from the solutions to the BS equation for mesons.

\section*{IV. RESULTS AND CONCLUSION.}

We solve SD equation by using numerical method with choose potential $[3]$ for massive quarks. In the event of massless quarks ($m_o = 0$) and without Coulomb term of potential our results coincide with $[1]$. The SD equation have solution with some nodes. The asymptotical behaviour $\varphi$ when $p \to \infty$ is positive value because $\varphi(p \to \infty) > 0$ for masses quark and $\varphi(p \to \infty) = \frac{m_o}{p}$. The authors $[1]$ study asymptotic behaviour when $p \to \infty$ for masses quarks and without Coulomb term of interaction potential.

We consider without nodes solution of SD equation, though needed to note that equation $[13]$ has an infinite number of solutions. From asymptotic behavior $\varphi$ we are to select on physical and non-physical solutions. Behavior $\varphi$ under $p \to \infty$ following:

$$\varphi(p \to \infty) \to \frac{m_o}{p}$$

and it has positive sign, this fact indicates us that the nodes solution of number $n = 0, 2, 4, \ldots$ (i.e. even number) they are physical. The nodes solution of number $n = 1, 3, 5, \ldots$ makes no physical sense because asymptotic condition $[20]$ is violated.

We have calculate self-energy of quarks under different parameters mass, see Figure 2. Studies have reveal us behavior $E(p)$ under $m_o > (4V_o^3/3)^{1/3}$ with certain accuracy comply with nonrelativistic approximation $\sqrt{p^2 + m_o^2}$, authors $[3]$ were calculated such results. Presence of negative energy (when $m_o = 0$) note on $[1]$. Authors $[3]$ researched a negative energy and adding in the potential of a constant energy shift they verified BS equation for mesons and baryons. In fact $[6]$ adding to the potential in space coordinates a constant energy shift then the quark energy is shifted by exactly half of that constant opposite sign.

We consider a region when $m_o < (4V_o^3/3)^{1/3}$. On the figure 2 we see that negative energy has an end value (under $p = 0, m_o = 0$) $E(0) = -6(4V_o^3/3)^{1/3}$, reference to $[1]$. With increasing masses an energy of quark grows in the negative area and for $m_o > (4V_o^3/3)^{1/3}$ continues to grow in the positive area. On the Figure 2 (line (a)) have plot a curved line $E(p) = p - \frac{1}{p^2}$ in the neighborhood $p = 0$.

In our approximation the quark condensate define through the part nonperturbative Green’s function $[2]$ \\

\< \bar{q}q > = i2N_c \text{tr} \int \frac{d^3p}{(2\pi)^3} [G_\Sigma(p) - G_m(p)] = \\\n-2N_c \int \frac{d^3p}{(2\pi)^3} [\sin \varphi(p) - \frac{m_o}{\sqrt{p^2 + m_o^2}}], \quad (21)\\

where $G_\Sigma(p) = [\hat{p} - \Sigma(p)]^{-1}$ and $G_m(p) = [\hat{p} - m]^{-1}$ are Green’s functions of “dressed” and “bare” quarks respectively, and $\varphi(p)$ - the solution of SD equation of quark $q$.

In the paper $[1]$, as we already mentioned earlier, SD equation is considered in the case $m_o = 0$ and they calculated $\< \bar{q}q > = -140 MeV^3$ in units $(4V_o^3/3)^{1/3} = 289 MeV$, we have repeat this result. Moreover have calculate behaviour $\< \bar{q}q >$ under different values of masses $m_o$ and unit parameter $(4V_o^3/3)^{1/3}$, see Figure 3. In the event $(4V_o^3/3)^{1/3} = 520 Mev$ (line (c) in the Figure 3) our results compare excellent with commonly value of quark condensate (-250 Mev)$^3$ $[3]$. Completely proper value of scale $(4V_o^3/3)^{1/3}$ will be calculate after solving the equation BS in our approach.
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FIG. 1. General solution of the SD equation (16) for potential (15) in dimensionless units and \( \alpha_s = 0.2 \). Solid line – for current quark mass \( m_o = 0.021(4V_o^3/3)^{1/3} \); Dashed line - for \( m_o = 0.2(4V_o^3/3)^{1/3} \); dashed point line – for \( m_o = 5(4V_o^3/3)^{1/3} \).
FIG. 2. Quark self-energy $E(p)$ is calculated from $\varphi(p)$ under different quark masses in dimensionless unit. (a) In the neighborhood $p=0$ and $\alpha_s = 0$ behaviour $E(p)$ from expression (17) is $E(p) = -\frac{1}{2p^2}$; (b) under $m_0 = 0, \alpha_s = 0$ repeat results Orsay group; (c) massive quark $m_0 = 0.2(4V_o^3/3)^{1/3}$, without Coulomb term potential $\alpha_s = 0$; (d) massive quark $m_0 = 5(4V_o^3/3)^{1/3}$, potential as sum oscillator and Coulomb term $\alpha_s = 0.2$; (e) massive quark $m_0 = 5(4V_o^3/3)^{1/3}$, without Coulomb term $\alpha_s = 0$
FIG. 3. The quark condensate under different masses quark $m = m_o + m_{o2}$ (a) under $(4V_o^3/3)^{1/3} = 289$ Mev in point $m_o = 0$ this results complies with Orsay group; (b) under $(4V_o^3/3)^{1/3} = 368$ Mev; (c) under $(4V_o^3/3)^{1/3} = 520$ Mev.