A damage sensitive feature: Beam case

Fergyanto E Gunawan¹ and Nobumasa Sekishita²

¹Industrial Engineering Department, BINUS Graduate Program - Master of Industrial Engineering, Bina Nusantara University, Jakarta, Indonesia 11480
²Department of Mechanical Engineering, Toyohashi University of Technology, Toyohashi, Aichi 441-8580, Japan

E-mail: fgunawan@binus.edu

Abstract. With the adoption of damage tolerance design principle, the health monitoring system has become an integral part of the engineering structure in operation. For the system to work, damage indicator that describes the structural integrity level should be established and monitored. Damage indicator is usually derived from structural responses. Many quantities have been proposed for damage indicator. Some popular damage indicators are natural frequency, mode shape, curvature, strain energy, and \( t \)-, \( F \)-, and \( z \)-statistics. In this paper, we propose a new damage indicator derived from the theory of the strength of materials. We evaluate the proposal for a case involving damage on a simply-supported beam. The required data are established numerically by the finite element method for structure in healthy and damaged conditions. The results suggest that the current proposal is more sensitive to damage than the natural frequency.

1. Introduction

The system of Structural Health Monitoring (SHM) has been implemented in various engineering structures, such as bridges [1–3] and airplanes [4–6]. Damage indicator is required in order to build an SHM system. Natural frequency and mode shape are two most widely used damage indicators [7–13]. The power spectral density was deployed by Ref. [14]. Lately, the SHM system is also built by deploying machine learning techniques [15–17, 4].

The machine learning approach generally requires a large amount of data and intensive computational resources. They often are unavailable on the remote sites where SHM is required.

This paper intends to propose a new indicator that is easy to compute and is better in detecting damage. The indicator is derived by applying the theory of the strength of materials, which is well understood.

2. Research method

This paper proposes a damage indicator derived from the theory of the strength of materials. Notably, it focuses on the Euler-Bernoulli beam theory. Thus, the derived indicator is useful for the SHM of beam-like structures.

The Euler-Bernoulli beam theory is accurate when the beam deflection is small such that the beam cross-section remains flat upon deformation. Also, the section is still perpendicular to the beam neutral plane.

With those assumptions, the theory links the applied distributed loads onto the beam \( q(x) \) to deformation \( w(t, x) \) with the second-order differential equation of
\[ EI \frac{\partial^4 w(t,x)}{\partial x^4} = -\rho \frac{\partial^2 w(t,x)}{dt^2} + q(x) \] (1)

The symbol \( E \) denotes the beam elastic modulus, \( I \) denotes the second moment of area of the beam cross-section, and \( \rho \) is the material density.

The dynamic equilibrium equation is applicable for any point at the beam at any time. We hypothesize that a deviation from the condition may signify a change in either the material properties or the beam geometry or both. Corrosion or crack may alter beam cross-section. Thus, the change may reflect the deterioration in material integrity.

In this work, we use the Euler-Bernoulli theory not to predict a beam deformation, nor to predict the exerted force, but to estimate the beam integrity. For the purpose, we propose:

\[ d(t,x) = \left| EI \frac{\partial^4 w(t,x)}{\partial x^4} + \rho \frac{\partial^2 w(t,x)}{dt^2} - q(x) \right| \] (2)

We hypothesize when the beam is intact, and \( E, I, \) and \( \rho \) are assigned with values on the intact condition, the damage indicator \( d(t,x) \) should be zero or very small. The indicator may shift from zero when the beam contains damages. We assume that damages alter the beam deformation \( w(t,x) \).

For SHM purpose, we have the freedom to choose the observation or measurement point. We may select the point where the external load is absent. Thus, it simplifies the computation of the damage indicator.

In this paper, we evaluate the above proposed indicator with a case of damage on a simply-supported beam. The beam has 300-mm length, 20-mm width, and 20-mm height. The beam and its discretization are shown in Fig. 2. It is made of steel material with the elastic modulus of 207 GPa, the Poisson ratio of 0.3, and the material density of \( 8050 \times 10^{-9} \) kg/mm\(^3\). The beam is subjected to a harmonic load \( f(t) = 100 \cdot \sin(2\pi f_p t) \) at the location \( x = 100 \) mm with \( f_p = 2 \) kHz.

3. Results

In this paper, we propose the formula in Eq. (2) for SHM. Our early work suggested the formula was able to detect damage when the measurement was performed near the location of damage [18]. It was evaluated on simple cases involving lumped-mass systems. In this paper, we evaluate the theory for detecting the damages in a beam.

For the reason, we focus on the six measurement points depicted in Fig. 2. The points position around the damage. The damage spans the segment between \( x = 5 \) mm and \( x = 60 \) mm. The six points are located at \( x = 51 \) mm, 54 mm, \( \cdots \), 63 mm. Thus, two points are at the ends of the damaged segment, two points on the left of the damaged element, and two points on the right.

For the first result, the beam displacements in the lateral direction at the six measurement points are shown in Fig. 1 for healthy and damaged conditions.

The profiles look rather similar because the six measurement points are separated by the small distance of 3 mm only. It is interesting to witness that the profiles for healthy and damaged conditions look rather identical. The displacements are practically not affected by the five-percent stiffness degradation on Element 20. The fact suggests the quantity is not suitable for damage detection.

For the second result, we focus on the natural frequency. The quantity is computed by solving \( \det(K - \omega^2 M) = 0 \), where \( K \) is the stiffness matrix, \( M \) is the mass matrix, and \( \omega \) is the natural frequency. The computed natural frequency is precise. The approach avoids errors that may occur when the quantity is reduced from the frequency analysis. The computed natural frequency is shown in Table 1. Only the first nine modes are provided. The table presents the quantity for healthy and damaged conditions. On the last column, the change of the frequency due to the damage is provided. The results
suggest that the change of the natural frequency is too small. The damage does not lead to a significant change in the natural frequency.

\[ x = 51 \text{ mm} \]

\[ x = 54 \text{ mm} \]

\[ x = 57 \text{ mm} \]

\[ x = 60 \text{ mm} \]

\[ x = 63 \text{ mm} \]

\[ x = 66 \text{ mm} \]

**Figure 1.** The lateral displacements at the six observation points for the healthy and damaged conditions.
Table 1. The natural frequencies for the healthy and damaged conditions

| Modes | Healthy Frequency (kHz) | Damaged Frequency (kHz) | Change (%) |
|-------|-------------------------|-------------------------|------------|
| 1     | 0.507                   | 0.507                   | 0.037      |
| 2     | 1.987                   | 1.982                   | 0.095      |
| 3     | 4.226                   | 4.222                   | 0.101      |
| 4     | 4.327                   | 4.323                   | 0.095      |
| 5     | 4.849                   | 4.847                   | 0.037      |
| 6     | 7.382                   | 7.379                   | 0.048      |
| 7     | 9.699                   | 9.690                   | 0.098      |
| 8     | 11.003                  | 11.000                  | 0.022      |
| 9     | 12.678                  | 12.673                  | 0.041      |

Figure 2. The indicator $d$ computed at the observation points of 51 mm, 54 mm, 57 mm, 60 mm, 63 mm, and 66 mm for the healthy and damaged conditions. The damaged element spans between $x = 57$ mm and $x = 60$ mm. The duration of analysis is 10 ms. The lateral displacements at the six observation points for the healthy and damaged conditions.

The natural frequency is well known to be sensitive to damage. Theoretically, damage at any point at the beam should affect the quantity. With this nature, the indicator is said to be global in detecting damage. However, for the present case, it becomes apparent that sufficiently large damage is required to be detectable by the approach. As the natural frequency is proportional to the square root of the stiffness, a 75-percent reduction in the stiffness only changes the natural frequency by half.

For the third result, we present the current proposal of the damage indicator. We compute the indicator at the six observation points. The results are shown in Fig. 2. We observe the indicator closely and obtain the following conclusions. The values of the indicator computed at the six observation points are always zero for the healthy condition. As for the damaged condition, the indicator computed at $x = 60$ mm is the most sensitive to the damage. The point is on the right end of the damaged segment. The sensitivity to the damage quickly drops by increasing the distance from the damaged element.
4. Conclusion
In this paper, we propose a damage indicator for assessing the change in structural condition. In general, the indicator is derived from our understanding of mechanics of materials and theory of elasticity. With those theories in mind, we understand and can predict the dynamical behavior of engineering structure upon receiving external loads. The deviation of responses from our prediction may signify alteration of the structural integrity. We evaluate our theory with data collected from a vibration analysis of a simply supported beam. The vibration is simulated numerically by the finite element method. To simplify the issue, we alter the elastic modulus of an assumed damaged element by a small amount. We analyze the structural responses on the healthy and damaged conditions. Then, we compute the proposed damage indicator. We witness the sensitivity of the indicator to the altered element. We also evaluate the change of the natural frequencies due to the damaged element and witness that the change is too small for the frequencies to be affected. With that example, we conclude that the proposed indicator can potentially detect structural changes better than the tradition method based on the natural frequency.

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