MHD SIMULATIONS OF BONDI ACCRETION TO A STAR IN THE “PROPELLER” REGIME

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ABSTRACT

This work investigates Bondi accretion to a rotating magnetized star in the “propeller” regime using axisymmetric resistive, magnetohydrodynamic simulations. In this regime accreting matter tends to be expelled from the equatorial region of the magnetosphere where the centrifugal force on matter rotating with the star exceeds the gravitational force. The regime is predicted to occur if the magnetospheric radius larger than the corotation radius and less than the light cylinder radius. The simulations show that accreting matters is expelled from the equatorial region of the magnetosphere and that it moves away from the star in a supersonic, disk-shaped outflow. At larger radial distances the outflow slows down and becomes subsonic. The equatorial matter outflow is initially driven by the centrifugal force, but at larger distances the pressure gradient becomes significant. We find that the star is spun-down mainly by the magnetic torques at its surface with the rate of loss of angular momentum \( \dot{L} \) proportional to \(-\Omega_1^{1.3} \mu^{0.8}\), where \( \Omega_1 \) is the star’s rotation rate and \( \mu \) is its magnetic moment. Further, we find that \( \dot{L} \) is approximately independent of the magnetic diffusivity of the plasma \( \eta_m \). The fraction of the Bondi accretion rate which accretes to the surface of the star is found to be \( \propto \Omega_1^{-1.0} \mu^{-1.7} \eta_m^{0.4} \). Predictions of this work are important for the observability of isolated old neutron stars and for wind fed pulsars in X-ray binaries.

Subject headings: accretion, dipole — plasmas — magnetic fields — stars: magnetic fields — X-rays: stars

1. INTRODUCTION

Rotating magnetized neutron stars pass through different stages in their evolution (e.g., Shapiro & Teukolsky 1983; Lipunov 1992). Initially, a rapidly rotating \((P \ll 1\text{ s})\) magnetized neutron star is expected to be active as a radiopulsar. The star spins down owing to the wind of magnetic field and relativistic particles from the region of the light cylinder \( r_L \) (Goldreich and Julian 1969). However, after the neutron star spins-down sufficiently, the light cylinder radius becomes larger than magnetospheric radius \( r_m \) where the ram pressure of external matter equals the magnetic pressure in the neutron star’s dipole field. The relativistic wind is then suppressed by the inflowing matter (Shvartsman 1970). The external matter may come from the wind from a binary companion or from the interstellar medium for an isolated neutron star. The centrifugal force in the equatorial region at \( r_m \) is much larger than gravitational force if \( r_m \) is much larger than the corotation radius \( r_{\text{cor}} \). In this case the incoming matter tends to be flung away from the neutron star by its rotating magnetic field. This is the so called “propeller” stage of evolution (Davidson & Ostriker 1973; Illarionov & Sunyaev 1975).

The “propeller” stage of evolution, though important, is still not well-understood theoretically. Different studies have found different dependences for the spin-down rate of the star (Illarionov & Sunyaev 1975; Davis, Fabian, & Pringle 1979; Davis & Pringle 1981; Wang & Robertson 1985; Lipunov 1992). Observational signs of the propeller stage have been discussed by number of authors (e.g., Stella, White, & Rosner 1986; Treves & Colpi 1991; Cui 1997; Treves et al. 2000).

MHD simulations of disk accretion to a rotating star in the propeller regime were done by Wang and Robertson (1985). However, authors considered only equatorial plane and concentrated on investigation of instabilities at the boundary between the magnetosphere and surrounding medium. Thus they could not investigate accretion along the magnetic poles of the star. An analytical model of disk accretion in the propeller regime was developed by Lovelace, Romanova, and Bisnovatyi-Kogan (1999). Disk accretion at the stage of weak propeller \((r_m \sim r_{\text{cor}})\) investigated numerically by Romanova et al. (2002).

The mentioned studies obtained possible trends of the propeller stage of evolution. However, two and three dimensional MHD simulations are needed to obtain more definite answers to the important physical questions. The questions which need to be answered are: (1) What are the physical conditions of the matter flow around the star in the propeller regime of accretion? (2) What is the spin-down rate of the star, and how does it depend on the star’s magnetic moment and rotation rate and on the inflow rate of the external matter? (3) What is the accretion rate to the surface of the star and how does it depend on...
Fig. 1.— Geometry of the MHD simulation region, where \( \dot{M}_B \) is the Bondi accretion rate, \( \mu \) and \( \Omega_* \) are the magnetic moment and angular velocity of the star, \( R_* \) is the radius of the star, \( (R_{\text{max}}, Z_{\text{max}}) \) are the limits of the computational region. In the described calculations \( R_* \ll R_{\text{max}} \).

the star’s rotation rate and magnetic moment? (4) What are the possible observational consequences of this stage of evolution?

This paper discusses results of axisymmetric, two-dimensional, resistive MHD simulations of accretion to a rotating magnetized star in the “propeller” regime. We treat the case when matter accretes spherically with the Bondi accretion rate. Bondi accretion to a non-rotating and a slowly rotating star was investigated by Toropina et al. (1999; hereafter T99). It was shown that the magnetized star accretes matter at a rate less than the Bondi rate. Toropina et al. (2002; hereafter T02) confirmed this result for stronger magnetic fields. In this paper we consider initial conditions similar to those in T02, but investigate the case of rapidly rotating stars. Section 2 gives a rough physical treatment of the propeller regime. Section 3 describes the main results from our simulations, and Section 4 describes the main results from our simulations, and Section 5 gives a numerical application of our results. Section 6 gives the conclusions from this work.

2. PHYSICS OF THE “PROPELLER” REGIME

In the propeller regime the magnetospheric radius \( r_m \) is larger than the corotation radius \( r_{\text{cor}} = (GM/\Omega_*^2)^{1/3} \approx 0.78 \times 10^9 P_{10}^{2/3} \) cm and incoming matter in the equatorial plane is flung away from the star. Here, we assume \( M = 1.4M_\odot \), and \( P_{10} = P/10 \) s with \( P \) the rotation period of the star. The magnetospheric radius is determined by a balance of the ram pressure of the inflowing matter \( p_m + \rho_m v_m^2 \) against the magnetic pressure \( B^2/8\pi \) of the neutron star’s field \( B_m = \mu/4\pi r_m \), where the \( m \)-subscripts indicate that the quantity is evaluated at a distance \( r_m \), and \( \mu \) is the star’s magnetic moment.

Consider first the case of a non-rotating star. The Bondi inflow velocity at \( r_m \) is supersonic for specific heat ratios \( \gamma < 5/3 \). Thus \( \rho_m = M/(4\pi r_m^3\sqrt{GM}) \), where \( M \) is the accretion rate. For this case the ram pressure is approximately \( \rho_m GM/r_m \) so that

\[
r_{\text{m0}} = \left( \frac{\mu^2}{2M\sqrt{GM}} \right)^{2/7},
\]

\[
= 6.1 \times 10^{10} \left( \frac{\mu_{30}}{M_{17}} \right)^{2/7} \text{ cm} \tag{1}
\]

(Davidson & Ostriker 1973; Shapiro & Teukolsky 1983). Here, \( \mu_{30} = \mu/10^{30} \text{ Gcm}^3 \), and \( M_{17} = M/10^{-17} \text{M}_\odot/\text{yr} \).

For a rapidly rotating star, \( r_m \gg r_{\text{cor}} \), the dominant velocity is due to the rotation of the magnetosphere so that \( v_m^2 = (\Omega_* r_m)^2 \). The flow velocity is much larger than the sound speed so that this is a “supersonic propeller” (Davies et al. 1979). Neglecting the complicated two-dimensional nature of the plasma flow, we estimate

\[
\rho_m (\Omega_* r_m)^2 \approx \mu^2/(8\pi r_m^6),
\]

which gives

\[
r_{\text{m0}} \Omega_* \approx \left( \frac{\mu^2 \sqrt{GM}}{2M\Omega_*^2} \right)^{2/13}
\]

\[
\approx 0.81 \times 10^{10} \left( \frac{\mu_{30}^2 P_{10}^2}{M_{17}} \right)^{2/13} \text{ cm} \tag{2}
\]

(Wang & Robertson 1985; Lovelace, Romanova, & Bisnovatyi-Kogan 2002). In this limit, there is to first approximation no accretion to the star; all of the incoming matter is flung away from the star in the equatorial plane by the rotating magnetic field.

Equations (1) and (2) give the same values for \( \Omega_* = (2M/\mu^2)^{3/7}(GM)^{5/7} \) which corresponds to a period \( P_2 \approx 7100(\mu_{30}^2/M_{17})^{3/7} \) s. The propeller regime requires periods \( P < P_2 \). At the other extreme, the condition that \( r_{\text{m0}} \Omega_* \) be less than the light cylinder radius gives the condition \( P > P_1 \approx 0.77(\mu_{30}^2/M_{17})^{2/3} \) s.

The spin-down rate of the star can be estimated as \( d\Omega_*/dt \approx 4\pi r_m^2 (P_m B_m^2/8\pi) \), which is the magnetic torque at the radius \( r_m \). This assumes that \( B_\phi \approx B_m \) at \( r_m \). Thus

\[
I \frac{d\Omega_*}{dt} \approx \frac{\mu^2}{2r_{\text{m0}}^3} = \frac{\mu_{30}^{14/13}\Omega_*^{12/13} M_{6/13}}{2^{7/13}(GM)^{3/13}}. \tag{3}
\]

This gives a spin-down time scale

\[
\tau = \frac{\Omega_*}{|\Omega_*|} \approx \frac{2.2 \times 10^7 \text{ yr}}{\mu_{30}^{14/13} M_{6/17}^{1/13} P_{10}^{1/13}},
\]

where the neutron star’s moment of inertia \( I \) is assumed to be \( 10^{45} \text{ gcm}^2 \). The angular momentum lost by the star goes into the outflow of material mainly in the equatorial plane. The rotational power lost by the star \( \Omega_* d\Omega_*/dt \) goes into the kinetic (thermal plus flow) energy of the equatorial outflow.

3. NUMERICAL MODEL

We simulate the plasma flow in the propeller regime using an axisymmetric, resistive MHD code. The code incorporates the methods of local iterations and flux-corrected-transport (Znakov, Zabrodin, & Fedotova 1993). The code is described in our earlier investigation of Bondi accretion to a non-rotating magnetized star (T99). The equations for resistive MHD are

\[
\frac{\partial p}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{4}
\]

\[
\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \frac{1}{c^2} \mathbf{J} \times \mathbf{B} + \mathbf{F}^g, \tag{5}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{c^2}{4\pi \sigma} \nabla^2 \mathbf{B}, \tag{6}
\]

\[
\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -p \nabla \cdot \mathbf{v} + \frac{3\mathbf{J}^2}{\sigma}. \tag{7}
\]

We assume axisymmetry (\( \partial/\partial \phi = 0 \)), but calculate all three components of \( \mathbf{v} \) and \( \mathbf{B} \). The equation of state is
taken to be that for an ideal gas, \( p = (\gamma - 1)\rho\varepsilon \), with specific heat ratio \( \gamma = 7/5 \). A value of \( \gamma \) less than 5/3 is expected in the case of an ionized gas because of the influence of the electron heat conduction. The equations incorporate Ohm’s law \( J = \sigma (E + \mathbf{v} \times \mathbf{B} / c) \), where \( \sigma \) is the electrical conductivity. The associated magnetic diffusivity, \( \eta_m \equiv \sigma^2 / (4\pi\alpha) \), is considered to be a constant within the computational region. In equation (5) the gravitational force, \( \mathbf{F} = -GM\rho\mathbf{r}/R^3 \), is due to the central star.

The star rotates with angular velocity \( \Omega = \Omega_\ast \hat{z} \). It is useful to introduce the dimensionless quantity \( \omega_* \equiv \Omega_\ast / \Omega_\ast \leq 1 \), where \( \Omega_\ast = (GM/R_\ast^2)^{1/2} \) is the Keplerian angular velocity at the stellar radius \( R_\ast \). Simulations have been done for different values of \( \omega_* = 0 - 0.7 \). The intrinsic magnetic field of the star is taken to be an aligned dipole, \( \mathbf{B} = [3\mathbf{R} (\mu \cdot \mathbf{R}) - R^2 \mu / R^5 \) with \( \mu = \mu \hat{z} \) and vector-potential \( \mathbf{A} = \mu \times \mathbf{R} / R^3 \).

We use a cylindrical, inertial coordinate system \((r, \phi, z)\) with the \( z-\)axis parallel to the star’s dipole moment \( \mu \) and rotation axis \( \Omega \). The equatorial plane is treated as a symmetry plane. The vector potential \( \mathbf{A} \) is calculated so that \( \nabla \cdot \mathbf{B} = 0 \) at all times. The full set of equations is given in T99.

To convert our equations to dimensionless form we measure length in units of the Bondi radius \( R_B \equiv GM/c_s^2 \), with \( c_s \) the sound speed at infinity, density in units of the density at infinity \( \rho_\infty \), and magnetic field strength in units of \( B_0 \) which is the field at the pole of the numerical star \((r = 0, z = Z_\ast) \). Pressure is measured in units of \( B_0^2 / 8\pi \). The magnetic moment is measured in units of \( \mu_0 = B_0 R_B / 2 \). We measure velocity in units of the Alfvén speed \( v_\alpha = B_0 / \sqrt{4\pi\rho_\infty} \). However, in the plots we give the velocities in units of \( c_s \).

After putting equations (4)-(7) in dimensionless form, one finds three dimensionless parameters, two of which are

\[
\beta \equiv \frac{8\pi \rho_\infty}{B_0^2}, \quad \eta_m \equiv \frac{\eta_m}{B_0 v_\alpha} = \frac{1}{S}. \tag{8}
\]

Here, \( \eta_m \) is the dimensionless magnetic diffusivity which may be interpreted as the reciprocal of the Lundquist number \( S \); and \( \rho_\infty \equiv \rho c_s^2 / \gamma \) is the pressure at infinity. The third parameter is \( g = GM_\ast / (\Omega_\ast^2 Z_\ast^2) = \gamma/2 \) is of the order of \( \beta \). We also use a parameter \( b_0 \) to control the field strength of the star. It is defined by the equation

\[ \mathbf{A} = b_0 \mathbf{A}_0, \]

where \( b_0 \) has been varied in the range \( 1 - 25 \).

Simulations were done in a cylindrical “can” \((0 \leq z \leq Z_{\text{max}}, 0 \leq r \leq R_{\text{max}}) \). The size of the region was taken to be \( R_{\text{max}} = Z_{\text{max}} = R_\ast / \sqrt{2} = R_B / 5 \sqrt{2} \approx 0.141R_B \), which is less than the sonic radius of the Bondi flow \( R_s = (5 - 3\gamma)/2\sqrt{2} = 0.2R_B \). Thus matter inflows supersonically to the computational region. The inflow rate is taken to be the Bondi (1952) accretion rate, \( \dot{M}_B = 4\pi \lambda GM_\ast^2 \rho_\infty / c_s^3 \), where \( \lambda = 0.625 \) for \( \gamma = 7/5 \). The incoming matter is assumed to be unmagnetized. A uniform \((r, z)\) grid with \( N_R \times N_Z \) cells was used. For the results discussed here \( N_R \times N_Z = 513 \times 513 \).

The “numerical star”, taken to be a cylinder of radius \( R_\ast \) and half axial length \( Z_\ast \) with \( R_\ast = Z_\ast << R_{\text{max}}, Z_{\text{max}} \). In the results presented here, \( R_\ast = Z_\ast = 0.0044R_B \). Clearly, the numerical star is very large: the value of \( R_\ast / R_B \), although much less than unity, is much larger than the astrophysical ratio for a neutron star \( (\sim 10^{-4}) \). The grid size, \( DR = R_{\text{max}} / N_R = 2.76 \times 10^{-4} R_B \), is about 16 times smaller than the star’s radius. The vector-potential \( \mathbf{A} \) of the intrinsic dipole field of the star is determined on the surface of the numerical star. The value of the vector potential on this surface is fixed corresponding to the numerical star being a perfect conductor. Equivalently, the component of the \( \mathbf{B} \) field normal to the surface is fixed, but the other two components vary.

The density and internal energy at the inner boundary (the numerical star) are set equal to small numerical values. Thus, incoming matter with higher values of these parameters, is absorbed by the star. This treatment of the star is analogous to that used by Ruffert (1994). Typically, the density of incoming matter is \( \gtrsim 10^3 \) times larger than density inside the star \( \rho = 1 \). For a purely hydrodynamic spherical accretion flow, we verified that this accretor absorbs all incoming matter at the Bondi rate.

4. Matter Flow in the Propeller Regime

Here, we first discuss simulations for the case \( \omega_* = 0.5 \) \( \beta = 10^{-7} \), magnetic diffusivity \( \eta_m = 10^{-5} \) (equation 8), and for field strength \( b_0 = 10 \). At the end of this section we discuss the dependence of the results on \( \omega_* \), \( \mu \) (and \( b_0 \)), and \( \eta_m \).

Time is measured in units of the free-fall time from the top or the side of the simulation region, \( t_{\text{ff}} = Z_{\text{max}} / v_{\text{ff}} \), with \( v_{\text{ff}} = \sqrt{2GM/Z_{\text{max}}} \). For the rotating star, time is also given in terms of rotation periods of the star, \( P_\ast = 2\pi/\Omega_\ast \).

For the mentioned values of \( \beta, b_0 \), and \( \eta_m \), the magnetospheric radius \( r_{\text{m}} \) and corotation radius \( r_{\text{cor}} = (GM_\ast \Omega_\ast^2)^{1/3} / \Omega_\ast \) are equal for \( \omega_* \approx 0.16 \). For smaller angular velocities, \( \omega_* < 0.16 \), the matter flow around the star is close to that in the non-rotating case. For \( \omega_* > 0.16 \) the flow exhibits the features expected in the “propeller” regime.

Figures 2 and 3 show the general nature of the flow in the propeller regime. Two distinct regions separated by a shock wave are observed: One is the external region where matter inflows with the Bondi rate and the density and velocity agree well with the Bondi (1952) solution. The second is the internal region, where the flow is strongly influenced by the stellar magnetic field and rotation. The shock wave, which divides these regions, propagates outward as in the non-rotating case (T99). For a rotating star in the propeller regime the shock wave has the shape of an ellipsoid flattened along the rotation axis of the star.

The region of the flow well within the shock wave is approximately time-independent. The accretion rate to the star becomes constant after about 1 – 2 rotation periods of the star.

A new regime of matter flow forms inside the expanding shock wave. The rapidly rotating magnetosphere expels matter outward in the equatorial region. This matter flows radially outward forming a low-density rotating torus. The outflowing matter is decelerated when it reaches the shock wave. There, the flow changes direction and moves towards the rotation axis of the star. However, only a small fraction of this matter accretes to the surface of the star. Most of the matter is expelled again in the equatorial direction by the rotating field of the star. Thus, most of the matter circulates inside this inner region driven by the
rapidly rotating magnetosphere. Large-scale vortices form above and below the equatorial torus (see Figures 2 and 3). In three dimensions vortices of smaller scale may also form. Thus, at the propeller stage the significant part of the rotational energy of the star may go into the directed form. Thus, at the propeller stage the significant part of the rotational energy of the star may go into the directed form. Thus, at the propeller stage the significant part of the rotational energy of the star may go into the directed form. Thus, at the propeller stage the significant part of the rotational energy of the star may go into the directed form.

The solid line in Figure 3 shows the Alfvén surface, where the magnetic field lines are approximately that of a dipole. At larger distances the field is stretched by the outflowing plasma. Matter inside the magnetopause (the region of closed magnetic field lines inside the Alfvén surface) rotates with the angular velocity of the star (see Figure 4). For $r > r_A$, matter continues to rotate in the equatorial region but the angular velocity decreases with $r$. Figure 5a shows the radial variation of the velocities in the equatorial plane. Matter is accelerated by the rotating magnetosphere for $r > 4R_*$ up to $r \sim 10R_*$, but at larger distances the radial velocity decreases. The azimuthal velocity $v_\phi$ is significantly larger than the radial velocity at $r \sim 4R_*$, but they become equal at larger distances. The sound speed $c_s$ is high inside the shock wave owing to the Joule heating $\mathbf{B}^2/\sigma$. Nevertheless, the outflow is supersonic in the region $r \sim (4 - 7)R_*$. Note, that above and below the equatorial plane the region of the supersonic outflow is larger (see sonic surface line in the Figures 3 and 4). Figure 5b shows the variation of the velocities along the $z$–axis.

Figure 6 shows the streamlines of the matter flow. Matter free falls along the field lines going into the poles of the star. Some matter which flows close to the $z$–axis accretes to the surface of the star. However, matter more removed from the $z$–axis comes close to the star, it is deflected by the rotating magnetic field, and it then moves outward in the equatorial plane.

Figure 7 shows the radial dependences of the radial forces in the equatorial plane. One can see that for $r > 3.5R_*$, the centrifugal force becomes dominant in accelerating the matter outward. However, at larger distances, $r > 5.5R_*$, the pressure gradient force becomes larger and determines acceleration of matter. Thus, centrifugal and pressure gradient forces accelerate matter in the radial direction. Note, that in most of the region ($r > 4.3R_*$) the magnetic force is negative so that it opposes the matter outflow.

The results we have shown are for a relatively strong propeller. If the rotation rate is smaller, then the matter outflow become less intense. For $\omega_\ast < 0.16$, no equatorial outflow is observed and the shock wave becomes more nearly spherical as it is in the non-rotating case. On the other hand the shock wave becomes more a more flattened ellipsoid for larger $\omega_\ast$. Matter rotates rigidly inside the Alfvén radius $r_A$ while at large distances $\Omega = v_\phi/r$ decreases. Figure 8 shows the radial dependences of the angular velocity in the equatorial plane for different values of $\omega_\ast$.

Figure 9 shows that only a small fraction of the Bondi accretion rate accretes to the surface of the star. Figure 9a shows that the accretion rate to the star decreases as the angular velocity of the star increases, $\dot{M}/M_B \approx 0.0075\omega_\ast^{-1.0}$. For a non-rotating star, $\dot{M}/M_B \approx 0.07$ in agreement with our earlier work (T99). Figure 9b shows that the accretion rate to the star decreases as the star’s magnetic moment increases, $\dot{M}/M_B \sim \mu^{-1.7}$. We have also done a number of simulation runs for different magnetic diffusivities in the range $\eta_m = 10^{-6} - 10^{-4.5}$, and from this we conclude that $\dot{M}/M_B \propto (\eta_m)^{-0.4}$. Accretion along the rotation axis of stars in the propeller regime was discussed by Nelson, Salpeter, and Wasserman (1993). Three-dimensional instabilities (e.g., Arons & Lea 1976) not included in the present two-dimensional simulations may act to increase the accretion rate to the the star’s surfaces.

Figure 10 shows the total angular momentum loss rate from the star as a function of $\omega_\ast$. This was obtained by evaluating the integral

$$L = -\int d\mathbf{s} \cdot \left( \rho v_\phi v_\phi - \frac{\mathbf{B} \times \mathbf{B}}{4\pi} \right),$$

over the surface of the numerical star. Here, $d\mathbf{s}$ is the outward pointing surface area element and the $p$– subscript indicates the poloidal component. The dominant contribution to equation (9) is the magnetic field term, and this term increases relative to the matter term as $\Omega_\ast$ increases. The dependence predicted by equation (3), $\dot{L} \propto -\Omega_\ast^{12/13}$ is shown by the dashed line. The fact that the observed dependence, $\dot{L} \propto -\Omega_\ast^{3}$, is steeper may be due to the fact that the magnetospheric radius is not much larger than the corotation radius. Also, we find that $\dot{L} \propto -\mu^{0.8}$ for a factor of four range of $\mu$ about our main case. From simulation runs with magnetic diffusivities in the range $\eta_m = 10^{-6} - 10^{-4.5}$, we conclude that $\dot{L}$ is approximately independent of $\eta_m$.

5. ASTROPHYSICAL EXAMPLE

Here, we give the conversions of the dimensionless variables to physical values for the case $\beta = 10^{-7}$, $\nu_0 = 10$, 

FIG. 2.— Matter flow in the “propeller” regime for a star rotating at $\Omega_\ast = 0.5\Omega_K$, after 6.9 rotation periods of the star. The axes are measured in units of the star’s radius. The background represents the density and the length of the arrows is proportional to the poloidal velocity. The thin solid lines are magnetic field lines.

FIG. 3.— Enlarged view of Figure 2. The bold line represents the Alfvén surface. Dotted line shows sonic surface. The axes are measured in units of stellar radii $R_\ast$.

FIG. 4.— Same as on Figure 3, but the background represents angular velocity $\Omega = v_\phi(r,z)/r$. The axes are measured in units of stellar radii $R_\ast$. 

FIG. 4.—— Matter flow in the “propeller” regime for a star rotating at $\Omega_\ast$.
and $\tilde{\eta}_m = 10^{-5}$ (equation 8). We consider accretion to a magnetized star with mass $M = 1.4 \, M_\odot = 2.8 \times 10^{33}$ g. The density of the ambient interstellar matter is taken to be $\rho_\infty = 1.7 \times 10^{-24} \, g/cm^3$ ($n_\infty = 1 / cm^3$). We consider a higher than typical sound speed in the ISM, $c_\infty > 0.12$. Using the definitions $\beta = 8 \pi \rho_\infty / B_0^2$, $p_\infty = \rho_\infty c_\infty^2 / \gamma$, and the fact that $b_0 = 10$, we obtain the magnetic field at the surface of the “numerical” star $B_\ast \approx 1.7$ G.

Recall that the size of the numerical star is $R_\ast = 0.0044 R_\odot \approx 0.82 \times 10^{10}$ cm in all simulations. The magnetic moment of the numerical star is $\mu = B_\ast r_\ast^2/2 \approx 0.48 \times 10^{30}$ G cm$^3$. Note that this is order of the magnetic moment of a neutron star with surface field $10^{24}$ G and radius $10^{8}$ cm.

The numerical star rotates with angular velocity $\Omega_\ast = \omega_0 \Omega_K$, where $\Omega_K = \sqrt{GM/ R_\ast^3}$. For most of the figures $\omega_0 = 0.5$, and this corresponds to a period of $P_\ast = 2\pi/ \Omega_\ast \approx 685$ s.

Using the mass accretion rate of equation (11) and $\mu_{30} = 1$, the magnetospheric radius of the non-rotating star, from equation (1), is $r_{m0} \approx 6.6 \times 10^{10}$ cm $\approx 8 R_\ast$. The magnetospheric radius of the rotating star with $\omega_\ast = 0.5$, from equation (2), is $r_{m0} \approx 3.1 \times 10^{10}$ cm $\approx 3.8 R_\ast$. From Figure 8 note that the equatorial profile of the angular rotation rate $\Omega(r, 0)$ for $\omega_\ast = 0.5$ is approximately flat out to $r_{flat} \approx 4.2 R_\ast$ which is roughly the same as $r_{m0}$. As required, the corotation radius, $r_{cor} = R_\ast / \omega_0^{2/3} \approx 1.6 R_\ast$, is appreciably less than $r_{m0}$. For smaller values of $\omega_\ast$, the dependence on $\omega_\ast$ differs from that of equation (2) which may be due to the fact that $r_{m0}$ is not very large compared with $r_{cor}$. As $\omega_\ast$ decreases, $r_{cor}$ increases rapidly. Thus the case $\omega_\ast = 0.1$, where $r_{cor} > r_{flat}$, is not in the propeller regime.

The considered case of $\beta = 10^{-7}$, $b_0 = 10$, $\tilde{\eta}_m = 10^{-5}$, and $\omega_0 = 0.5$, we have evaluated equation (9) numerically with the result that $\dot{L} \approx -4.7 \times 10^{28}$ g(cm/s)$^2$. The theoretical estimate of $\dot{L}$ from equation (3) for the mentioned values of $\mu$ and $r_{m0}$ is $\dot{L}_{th} = 1.5 \times 10^{28}$ g(cm/s)$^2$. A small, 70% reduction of $r_{m0}$ brings $\dot{L}_{th}$ into coincidence with the $\dot{L}$ from the simulations.

6. CONCLUSIONS

Axisymmetric magnetohydrodynamic simulations of Bondi accretion to a rotating magnetized star in the propeller regime of accretion have shown that: (1) A new regime of matter flow forms around a rotating star. Matter falls down along the axis, but only a small fraction of the incoming matter accretes to the surface of the star. Most of the matter is expelled radially in the equatorial plane by the rotating magnetosphere of the star. A low-density torus forms in the equatorial region which rotates with velocity significantly larger than the radial velocity. Large scale vortices form above and below the equatorial plane. (2) The star is spin-down by the magnetic torque and to a lesser extent the matter torque at its surface. The rate of loss of angular momentum $\dot{L}$ is proportional to $-\Omega_\ast^3 \mu^{0.8}$, and it is approximately independent of $\tilde{\eta}_m$. This dependence differs from the predicted dependence of equation (3) probably because the corotation radius is not much smaller than the magnetospheric radius $r_m$. The rotational energy lost by the star goes into the directed and thermal energy of plasma. (3) The accretion rate to the star is much less than the Bondi accretion rate and decreases as (a) the star’s rotation rate increases ($\propto \Omega_\ast^{-1.0}$), (b) as the star’s magnetic moment increases ($\propto \mu^{-1.7}$), and as the magnetic diffusivity decreases ($\propto (\eta_m)^{0.4}$. (4) Because the accretion rate to the star is less than the Bondi rate, a shock wave forms in our simulations and propagates outward. It has the shape of an ellipsoid flattened along the rotation axis of the star.

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**Fig. 6.**—Same as on Figure 3, but bold solid lines are added which show the streamlines of the matter flow.

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**Fig. 7.**—The figure shows the radial dependence of the radial forces acting to matter in the equatorial plane. The vertical scale is arbitrary. The centrifugal force is the dominant one which drives matter to the centrifugal equatorial wind.
Fig. 8.— Angular velocity of matter \( \Omega = v_{\phi}/r \) versus \( r \) in the equatorial region for different angular velocities of the star. The dashed line represents the Keplerian angular velocity \( \Omega_K = \sqrt{GM/r^3} \).

Fig. 9.— The left-hand panel (a) shows the fraction of Bondi accretion rate reaching the star as a function of the star’s angular velocity \( \omega_* = \Omega_*/\Omega_K* \). For a non-rotating star, \( \dot{M}/\dot{M}_B = 0.07 \) or \( \log(\dot{M}/\dot{M}_B) = -1.1 \). The right-hand panel (b) shows the dependence on the magnetic moment \( \mu \). The straight lines show the approximate power law dependences.

Fig. 10.— Total angular momentum outflow from the star as a function of \( \omega_* = \Omega_*/\Omega_K* \). The straight line is a power law fit \( \dot{L} \propto -(\Omega_*)^{1.3} \). This is somewhat steeper than the predicted dependence of equation (3), \( \dot{L} \propto -(\Omega_*)^{12/13} \), which is shown by the dashed line in the figure. The vertical scale is arbitrary.
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