Heavy baryons in a pion mean-field approach: A brief review

Hyun-Chul Kim

1Department of Physics, Inha University, Incheon 22212, Republic of Korea
2School of Physics, Korea Institute for Advanced Study (KIAS), Seoul 02455, Republic of Korea

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We review in this paper a series of recent works on properties of singly heavy baryons, based on a pion mean-field approach. In the limit of an infinitely heavy-quark mass, the heavy quark inside a heavy baryon can be regarded as a static color source. In this limit, a heavy baryon can be viewed as $N_c - 1$ valence quarks bound by the pion mean fields which are created self-consistently by the presence of the $N_c$ valence quarks. We show that this mean-field approach can successfully describe the masses and the magnetic moments of the lowest-lying singly heavy baryons, using all the parameters fixed in the light-baryon sector except for the hyperfine spin-spin interactions. We also review a recent work on identifying the newly found excited $\Omega_c$ baryons reported by the LHCb Collaboration. We discuss possible scenarios to identify them. Finally, we give a future perspective on this pion mean-field approach.

Keywords: heavy baryons, pion mean fields, chiral quark-soliton model, flavor SU(3) symmetry breaking

* E-mail: hchkim@inha.ac.kr
I. INTRODUCTION

An ordinary heavy baryon constitutes a pair of light quarks and a heavy quark. Since the charm and bottom quarks are very heavy in comparison with the light quarks, it is plausible to take the limit of the infinitely heavy mass of the heavy quark, i.e. $m_Q \to \infty$. In this limit, the physics of heavy baryons become simple. The spin of the heavy quark is conserved, because of its infinitely heavy mass. It results in the conservation of the total spin of light quarks: $J_L \equiv J - J_Q$, where $J_L$, $J_Q$, and $J$ denote the spin of the light-quark pair, that of the heavy quark, and the total spin of the heavy baryon. This is called the heavy-quark spin symmetry that allows $J_L$ to be a good quantum number. Moreover, the physics is kept intact under the placement of heavy quark flavors. This is called the heavy-quark flavor symmetry [1–4]. Then a heavy quark becomes static, so that it can be considered as a static color source. Its importance is only found in making the heavy baryon a color singlet, and in giving higher-order contributions arising from $1/m_Q$ corrections. Consequently, the dynamics inside a heavy baryon is mainly governed by the light quarks.

The flavor structure of the heavy baryon is also determined by them. Since there are two light quarks inside the heavy baryon, we have two different flavor $SU_f(3)$ irreducible representations, i.e. $3 \otimes 3 = 3 \oplus 6$. In the language of a quark model, the spatial part of the heavy-baryon ground state is symmetric due to the zero orbital angular momentum, and the color part is totally antisymmetric. Since the flavor anti-triplet ($3$) is antisymmetric, the spin state corresponding to $3$ should be antisymmetric. Thus, the baryons belonging to the anti-triplet should be $J_L = 0$. Similarly, the flavor-symmetric sextet ($6$) should be symmetric in spin space, i.e. $J_L = 1$. This leads to the fact that the baryon antitriplet has spin $J = 1/2$, while the baryon sextet carries spin $J = 1/2$ or $J = 3/2$, with the spin of the light-quark pair being coupled with the heavy quark spin $J_Q = 1/2$. So, we can classify 15 different lowest-lying heavy baryons as shown in Fig. 1 in the case of charmed baryons.

Recently, there has been a series of new experimental data on the spectra of heavy baryons [5–13], which renewed interest in the physics of the heavy baryons. The lowest-lying singly heavy baryons are now almost classified except for $\Omega^\ast_b$. In the meanwhile, the LHCb Collaboration has announced the first finding of two heavy pentaquarks, $P_c(4380)$ and $P_c(4450)$ [14–17]. Very recently, the five excited $\Omega_c$ baryons were reported [18], among which the four of them was confirmed by the Belle experiment [19]. Interestingly the two of the excited $\Omega_c$s, i.e. $\Omega_c(3050)$ and $\Omega_c(3119)$, have very narrow widths: $\Gamma_{\Omega_c(3050)} = (0.8 \pm 0.2 \pm 0.1)$ MeV and $\Gamma_{\Omega_c(3119)} = (1.1 \pm 0.8 \pm 0.4)$ MeV.

While there is a great deal of theoretical approaches for the description of heavy baryons, we will focus on a pion mean-field approach in the present short review. This mean-field approach was first proposed by E. Witten in this seminal papers [20, 21], where he asserted that in the limit of the large number of colors ($N_c$) the nucleon can be regarded as a bound state of $N_c$ valence quarks in a pion mean field with a hedgehog symmetry [22, 23]. Since a baryon mass is proportional to $N_c$ whereas the quantum fluctuation around the saddle point of the pion field is suppressed by $1/N_c$, the mean-field approach is a rather plausible method for explaining properties of baryons. The presence of $N_c$ valence quarks in this large $N_c$ limit, which consist of the lowest-lying baryons, produce the pion mean fields.
by which they are influenced self-consistently. This picture is very similar to a Hartree approximation in many-body theories. Witten also showed how to construct the mean-field theory for the baryon schematically in two-dimensional quantum chromodynamics (QCD). Though his idea was criticized sometimes ago by S. Coleman because of its technical difficulties, it is worthwhile to pursue it to see how far we can describe the structure of the baryon in the pion mean-field approach.

The chiral quark-soliton model (χQSM) has been constructed based on Witten’s argument. The χQSM starts from the effective chiral action (EχA) that was derived from the instanton vacuum. The EχA respects chiral symmetry and its spontaneous breakdown, in which the essential physics of the lowest-lying hadrons consists. One can derive the classical energy of the nucleon by computing the nucleon correlation function in Euclidean space, taking the Euclidean time to go to infinity. Minimizing the classical energy self-consistently in the large \( N_c \) limit with the \( 1/N_c \) meson quantum fluctuations suppressed, we obtain the classical mass and the self-consistent profile function of the chiral soliton. While we ignore the \( 1/N_c \) quantum fluctuations around the saddle point of the soliton field, we need to take into account the zero modes that do not change the soliton energy. Since the soliton with hedgehog symmetry is not invariant under translational, rotational and isotopic transformations, we impose these symmetry properties on the soliton and obtain a completely new solution with the same classical energy. Because of the hedgehog symmetry, an SU(2) soliton needs to be embedded into the isospin subgroup of the flavor SU(3), which was already utilized by various chiral soliton models. This collective quantization of the chiral soliton leads to the collective Hamiltonian with effects of flavor SU(3) symmetry breaking. The χQSM has one salient feature: the right hypercharge is constrained to be \( Y' = N_c/3 \) imposed by the \( N_c \) valence quarks. This right hypercharge selects allowed representations of light baryons such as the baryon octet, the decuplet, etc. The χQSM was successfully applied to the properties of the lowest-lying light baryons such as the mass splittings, the form factors, the magnetic moments, hyperon semileptonic decays, parton distributions, transversities of the nucleon, generalized parton distributions, and so on.

![FIG. 2. Schematic picture of a heavy baryon.](image)

Very recently, Ref. extended a mean-field approach to describe the masses of singly heavy baryons, being motivated by Ref. A singly heavy baryon constitutes a heavy baryon and \( N_c - 1 \) light valence quarks (see Fig. 2). In the limit of \( m_Q \to \infty \), the heavy quark can be considered as a static color source. Thus, the dynamics inside a heavy baryon is governed by the \( N_c - 1 \) valence quarks. The presence of the \( N_c - 1 \) valence quarks will produce the pion mean fields as in the case of the light baryons. However, there is one very significant difference. The constraint right hyper charge is taken to be \( Y' = (N_c - 1)/3 \) and allows the lowest-lying representations: the baryon anti-triplet, the baryon sextet, the baryon anti-decapentaplet. The model reproduced successfully the mass splitting of the baryon anti-triplet and sextet in both the charm and bottom sectors. In addition, the mass of the \( \Omega^*_b \) baryon, which has not yet found, was predicted. The model was further extended by including the second-order perturbative corrections of flavor SU(3) symmetry breaking. The magnetic moments baryons and electromagnetic form factors of the singly heavy baryons were also studied within the same framework. The χQSM was also used to interpret the five \( \Omega_c \) baryons newly found by the LHCb Collaboration. Within the present framework, two of the \( \Omega_c \)’s with the smaller widths are classified as the members of the baryon \( 15 \), whereas all other \( \Omega_c \)’s belong to the excited baryon sextet. The widths were quantitatively well reproduced without any free parameter. In the present
work, we will review briefly these recent investigations on the singly heavy baryons.

We sketch the present work as follows: In Section II, we review the general formalism of the \( \chi \)QSM for singly heavy baryons. In Section III, we examine the mass splittings of the heavy baryons, emphasizing the discussion of the effects of SU(3)\(_c\) breaking. In Section IV, we discuss the recent results of the magnetic moments and electromagnetic form factors of the heavy baryons. In Section V, we briefly introduce a theoretical interpretation of the excited \( \Omega \)\(_c\) baryons found by the LHCb, based on the present mean-field approach. The final Section is devoted to the conclusions and outlook.

II. THE CHIRAL QUARK-SOLITON MODEL FOR SINGLY HEAVY BARYONS

In the present approach, a heavy baryon is considered as a bound state of the \( N_c - 1 \) valence quark in the pion mean field with a heavy quark stripped off from the valence level. Thus, the correlation function of the heavy baryon can be expressed in terms of the \( N_c - 1 \) valence quarks

\[
\Pi_B(0, T) = \langle J_B(0, T/2) J_B^\dagger(0, -T/2) \rangle_0 = \frac{1}{Z} \int D\psi D\bar{\psi} J_B(0, T/2) J_B^\dagger(0, -T/2) e^{\int d^4x \bar{\psi}^\dagger (i\gamma^\mu U^\mu + i\hat{m}) \psi},
\]

where \( J_B \) denotes the light-quark current with the \( N_c - 1 \) light quarks for a heavy baryon \( B \)

\[
J_B(x, t) = \frac{1}{(N_c - 1)!} \varepsilon^{\beta_1 \cdots \beta_{N_c - 1}} \Gamma^{(f)}_{J', J_3, T T_3} \Psi_{\beta_1 \beta_2}(x, t) \cdots \Psi_{\beta_3 \beta_4}(x, t).
\]

\( \beta_i \) stand for color indices and \( \Gamma^{(f)}_{J', J_3, T T_3} \) represents a matrix with both flavor and spin indices. \( J' \) and \( T \) are the spin and isospin of the heavy baryon, respectively. \( J'_3 \) and \( T_3 \) are their third components, respectively. The notation \( \langle \cdots \rangle_0 \) in Eq. (1) is the vacuum expectation value, \( M \) the dynamical quark mass, and the chiral field \( U^{\gamma_5} \) is defined as

\[
U^{\gamma_5} = U^{1 + \gamma_5} + U^\dagger \frac{1 - \gamma_5}{2}
\]

with

\[
U = \exp(i\pi^a \lambda^a).
\]

Here, \( \pi^a \) represents the pseudo-Goldstone boson field and \( \hat{m} \) denotes the flavor matrix of the current quarks, written as \( \hat{m} = \text{diag}(m_u, m_d, m_s) \). We assume isospin symmetry, i.e. \( m_u = m_d \). Since the strange current quark mass is small enough, we will treat it perturbatively.

Integrating over the quark fields, we derive the correlation function as

\[
\Pi_B(0, T) = \frac{1}{Z} \Gamma^{(f)}_{J', J_3, T T_3} \Gamma^{(g)}_{J', J_3, T T_3} \int D\psi D\bar{\psi} \prod_{i=1}^{N_c - 1} \langle 0, T/2 \bigg| \frac{1}{D(U)} \bigg| 0, -T/2 \rangle e^{-S_{\text{eff}}(U)},
\]

where the single-particle Dirac operator \( D(U) \) is defined as

\[
D(U) = i\gamma_k \partial_k + i\gamma^5 + iMU^{\gamma_5} + i\hat{m}
\]

and \( S_{\text{eff}} \) is the effective chiral action written as

\[
S_{\text{eff}} = -N_c \text{Tr} \log D(U).
\]

\( \chi \)QSM can be schematically depicted as Fig. 2. It consists of two different terms: The first and second ones are respectively called the \textit{valence-quark contribution} and \textit{sea-quark contribution} within the \( \chi \)QSM. When the Euclidean time \( T \) is taken from \(-\infty \) to \( \infty \), the correlation function picks up the ground-state energy

\[
\lim_{T \to \infty} \Pi_B(T) \sim \exp \left[ \{- (N_c - 1)E_{\text{val}} + E_{\text{sea}}\} T \right],
\]

where \( E_{\text{val}} \) and \( E_{\text{sea}} \) the valence and sea quark energies. Minimizing self-consistently the energies around the saddle point of the chiral field \( U \)

\[
\frac{\delta}{\delta U} \left[ -(N_c - 1)E_{\text{val}} + E_{\text{sea}} \right]_{U_c} = 0,
\]
we get the classical soliton mass
\[ M_{\text{sol}} = (N_c - 1)E_{\text{val}}(U_c) + E_{\text{sea}}(U_c). \] (10)

Note that a singly heavy baryon has a heavy quark, so its classical is expressed as the sum of the classical and heavy-quark masses
\[ M_{\text{cl}} = M_{\text{sol}} + m_Q. \] (11)

We want to mention that \( m_Q \) is the effective heavy quark mass that is different from that of QCD and will be absorbed in the center mass of each representation.

The rotational excitations of the soliton with \( N_c - 1 \) valence quarks will produce the lowest-lying heavy baryons. To keep the hedgehog symmetry, the SU(2) soliton \( U_c(r) \) will be embedded into SU(3) \[21\]
\[ U(r) = \begin{pmatrix} U_c(r) & 0 \\ 0 & 1 \end{pmatrix}. \] (12)

As mentioned in Introduction, we consider explicitly the rotational zero modes. Assuming that the soliton \( U(r) \) in Eq.(12) rotates slowly, we apply the rotation matrix \( A(t) \) in SU(3) space
\[ U(r, t) = A(t)U(r)A^\dagger(t). \] (13)

Then, we can derive the collective Hamiltonian for heavy baryons
\[ H = H_{\text{sym}} + H_{\text{sb}}^{(1)} + H_{\text{sb}}^{(2)}, \] (14)
where \( H_{\text{sym}} \) represents the flavor SU(3) symmetric part, \( H_{\text{sb}}^{(1)} \) and \( H_{\text{sb}}^{(2)} \) the SU(3) symmetry-breaking parts respectively to the first and second orders. \( H_{\text{sym}} \) is expressed as
\[ H_{\text{sym}} = M_{\text{cl}} + \frac{1}{2I_1} \sum_{i=1}^{3} \hat{J}_i^2 + \frac{1}{2I_2} \sum_{a=4}^{7} \hat{J}_a^2, \] (15)
where \( I_1 \) and \( I_2 \) are the moments of inertia of the soliton and the operators \( \hat{J}_i \) denote the SU(3) generators. We get the eigenvalue of the quadratic Casimir operator \( \sum_{i=1}^{8} J_i^2 \) in the \((p, q)\) representation, given as
\[ C_2(p, q) = \frac{1}{3} \left[ p^2 + q^2 + pq + 3(p + q) \right], \] (16)
which leads to the eigenvalues of \( H_{\text{sym}} \)
\[ E_{\text{sym}}(p, q) = M_{\text{cl}} + \frac{1}{2I_1} J_L(J_L + 1) + \frac{1}{2I_2} \left[ C_2(p, q) - J_L(J_L + 1) \right] - \frac{3}{8I_2} Y^{r^2}. \] (17)
The right hypercharge $Y'$ is constrained by the $N_c - 1$ valence quarks inside a singly heavy baryon, i.e. $Y' = (N_c - 1)/3$. The corresponding collective wave functions of the singly heavy baryon is then obtained as

$$\psi_B^{(R)}(J J_3, J_L; A) = \sum_{m_3 = \pm 1/2} C_{J q m_3 J_L} \chi_{m_3} \sqrt{\text{dim}(p, q)} (-1)^{-J K_3 + J L_3} D^{(R)\ast}_{Y', T, T_3}(Y', J_L, -J L_3)(A),$$

(18)

where

$$\text{dim}(p, q) = (p + 1)(q + 1) \left(1 + \frac{p + q}{2}\right).$$

(19)

$J$ and $J_3$ in Eq. (18) are the spin angular momentum and its third component of the heavy baryon, respectively. $J_L$ and $J_Q$ represent the soliton spin and heavy-quark spin, respectively. $J_{L3}$ and $m_3$ are the corresponding third components, respectively. Since the spin operator for the heavy baryon is given as

$$\mathbf{J} = \mathbf{J}_Q + \mathbf{J}_L,$$

(20)

the relevant Clebsch-Gordan coefficients appear in Eq. (18). The SU(3) Wigner $D$ function in Eq. (18) means just the wave-function for the quantized soliton with the $N_c - 1$ valence quarks, and $\chi_{m_3}$ is the Pauli spinor for the heavy quark. $R$ designates a SU(3) irreducible representation corresponding to $(p, q)$. Since the soliton is coupled to the heavy quark, we finally obtain the three lowest-lying representations illustrated in Fig. 1. In the limit of $m_Q \to \infty$, the two sextet representations are degenerate. One needs to introduce a hyperfine spin-spin interaction to lift this degeneracy. As will be discussed soon, this hyperfine interaction will be determined by using the experimental data on the masses of heavy baryons.

In the present zero-mode quantization scheme, we find the following the two important selection rule. The allowed SU(3) representations must contain states with $Y' = (N_c - 1)/3$ and the isospin $T$ of the states with $Y' = (N_c - 1)/3$ are coupled with the soliton so that we have a singlet $\mathbf{K} = T + \mathbf{J}_L = \mathbf{0}$, where $\mathbf{K}$ is called the grand spin. The lowest-lying heavy baryons have the grand spin $\mathbf{K} = 0$, that is, we must have always $J_L = T$ with $Y' = (N_c - 1)/3$ for the ground-state heavy baryons as shown in Fig. 1.

**FIG. 4.** The baryon anti-triplet has the $J_L = T = 0$ state with $Y' = 2/3$ whereas the baryon sextet contains the $J_L = T = 1$ state with $Y' = 2/3$.

An observable of the heavy baryon can be expressed in general as a three-point correlation function

$$\langle B, p' | J_\mu(0) | B, p \rangle = \frac{1}{2} \lim_{T' \to \infty} \exp \left(i p' \frac{T'}{2} - i p \frac{T'}{2}\right) \int d^3 x d^3 y \exp(-i p' \cdot y + i p \cdot x)$$

$$\times \int D U \int D \psi \int D \psi' J_B(y, T/2) \psi(0) \Gamma \psi(0) J_B^\dagger(x, -T/2) \exp \left[-\int d^4 z \psi^\dagger i D(U) \psi\right],$$

(21)

where $\Gamma$ and $\mathcal{O}$ represent respectively generic Dirac spin and flavor matrices. Computing Eq. (21), one can study heavy baryonic observables such as form factors, magnetic moments, axial-vector constants, etc. For the detailed formalism, we refer to Refs. [26, 35].

### III. MASS SPLITTINGS OF THE SINGLY HEAVY BARYONS

We first discuss the mass splittings of the singly heavy baryons. In order to obtain the mass splittings, one should include the symmetry-breaking part of the collective Hamiltonian [26, 33]

$$H_{sb}^{(1)} = \alpha D_{ss}^{(8)} + \beta \hat{Y} + \gamma \sqrt{\beta} \sum_{i=1}^{3} D_{si}^{(8)} \hat{J}_i,$$

(22)
where
\[ \bar{\alpha} = \left(-\frac{\Sigma_{\pi N}}{3m_0} - \frac{K_2}{I_2} Y' \right) m_s, \quad \beta = \frac{K_2}{I_2} m_s, \quad \gamma = 2 \left( \frac{K_1}{I_1} - \frac{K_2}{I_2} \right) m_s. \] (23)

The parameters \( \bar{\alpha}, \beta, \) and \( \gamma \) are the essential ones in determining the masses of the lowest-lying singly heavy baryons, which are expressed in terms of the moments of inertia \( I_{1,2} \) and \( K_{1,2} \). However, we do not need to fit them, since they are related to \( \alpha, \beta, \) and \( \gamma \) in the light-baryon sector. The valence parts are only different from those in the light baryon sector by the color factor \( N_c - 1 \). So, we need to replace \( N_c \) by \( N_c - 1 \) in the valence parts of all the relevant dynamical parameters determined in the light-baryon sector. The valence part of \( \Sigma_{\pi N} \) is just the \( \pi N \) sigma term with different \( N_c \) factor: \( \Sigma_{\pi N} = (N_c - 1)N_c^{-1}\Sigma_{\pi N} \), where \( \Sigma_{\pi N} = (m_u + m_d)(\bar{u}u + \bar{d}d|N) = (m_u + m_d)\sigma \). On the other hand, the sea parts should be kept intact as in the light baryon sector.

The dynamical parameters \( \alpha, \beta \) and \( \gamma \) have been fixed by using the experimental data on the baryon octet masses and a part of the baryon decuplet and anti-decuplet masses with isospin symmetry breaking effects \[53\]. The values of \( \alpha, \beta, \) and \( \gamma \) have been obtained by the \( \chi^2 \) fit \[34\]

\[ \alpha = -255.03 \pm 5.82 \text{ MeV}, \quad \beta = -140.04 \pm 3.20 \text{ MeV}, \quad \gamma = -101.08 \pm 2.33 \text{ MeV}, \] (24)

While \( \beta \) and \( \gamma \) are not required to be changed in the heavy-baryon sector, \( \alpha \) should be modified by

\[ \bar{\alpha} = \rho \alpha, \] (25)

where \( \rho = (N_c - 1)/N_c \). However, there is a caveat when one uses the values of Eq. (24). As mentioned above, only the valence parts should be modified, while the scaling in Eq. (25) changes the sea part too. To compensate this we choose \( \rho \approx 0.9 \). If one computes the parameters \( \bar{\alpha}, \beta, \) and \( \gamma \) in a self-consistent way, we do not have this problem \[52\].

Considering the first-order perturbative corrections of \( m_s \), one can express the masses of the singly heavy baryons in representation \( \mathcal{R} \) as

\[ M_{B,\mathcal{R}}^Q = M_{\mathcal{R}}^Q + M_{B,\mathcal{R}}^{(1)} \] (26)

with

\[ M_{\mathcal{R}}^Q = m_Q + E_{\text{sym}}(p,q). \] (27)

Here, \( M_{\mathcal{R}}^Q \) is the center mass of a heavy baryon in representation \( \mathcal{R} \). \( E_{\text{sym}}(p,q) \) is the eigenvalue energy of the symmetric part of the collective Hamiltonian defined in Eq. (17). Note that the lower index \( B \) designates a certain baryon in a specific representation \( \mathcal{R} \). The upper index \( Q \) denotes either the charm sector (\( Q = c \)) or the bottom sector (\( Q = b \)). Then the center masses for the anti-triplet and sextet representations are obtained as

\[ M_{3}^{(1)} = M_{3}^{(1)} = \frac{1}{2I_2}, \quad M_{6}^{(1)} = M_{6}^{(1)} + \frac{1}{I_1}, \] (28)

where \( M_{\mathcal{R}}^{(1)} \) was defined in Eq. (11). The second term in Eq. (26), which arises from the linear-order \( m_s \) corrections, is proportional to the hypercharge of the soliton with the light-quark pair

\[ M_{B,\mathcal{R}}^{(1)} = \langle B, \mathcal{R} | H_{sb}^{(1)} | B, \mathcal{R} \rangle = Y \delta_{\mathcal{R}}, \] (29)

where

\[ \delta_{3} = \frac{3}{8} \bar{\alpha} + \beta, \quad \delta_{6} = \frac{3}{20} \bar{\alpha} + \beta - \frac{3}{10} \gamma. \] (30)

Finally, we arrive at the expressions for the masses of the lowest-lying baryon anti-triplet and sextet as follows

\[ M_{B,\mathcal{R}}^{3} = M_{\mathcal{R}}^{3} + Y \delta_{3}, \quad M_{B,\mathcal{R}}^{6} = M_{\mathcal{R}}^{6} + Y \delta_{6}. \] (31)

with the linear-order \( m_s \) corrections taken into account.

Since the baryon sextet with spin 1/2 and 3/2 are degenerate, we need to remove the degeneracy by introducing the hyperfine spin-spin interaction Hamiltonian \[54\]. Typically, the hyperfine Hamiltonian is written as

\[ H_{LQ} = \frac{2}{3} \kappa \frac{\kappa}{m_Q} M_{\text{sol}} \cdot J_L \cdot J_Q = \frac{2}{3} \kappa \frac{\kappa}{m_Q} J_L \cdot J_Q, \] (32)
where $\kappa$ stands for the flavor-independent hyperfine coupling. $M_{\text{tot}}$ has been incorporated into an unknown coefficient $\kappa$ that will be fixed by using the experimental data. The Hamiltonian $H_{LQ}$ does not affect the $\overline{3}$ states with $J_L = 0$. On the other hand, the baryon sextet acquire additional contribution from $H_{LQ}$ which bring about the splitting between different spin states

$$M^Q_{B,6} = M^Q_{B,0} - \frac{2}{3} \frac{\kappa}{m_Q}, \quad M^Q_{B,6,1/2} = M^Q_{B,0} + \frac{1}{3} \frac{\kappa}{m_Q}, \quad (33)$$

which leads to the splitting

$$M^Q_{B,6} - M^Q_{B,6,1/2} = \frac{\kappa}{m_Q}. \quad (34)$$

The numerical values of $\kappa/m_Q$ were determined by using the center values of the masses of the baryon sextet $[50]$

$$\frac{\kappa}{m_c} = (68.1 \pm 1.1) \text{ MeV}, \quad \frac{\kappa}{m_b} = (20.3 \pm 1.0) \text{ MeV}. \quad (35)$$

Note that $\kappa$ is flavor-independent. So, knowing the ratio $m_c/m_b$, one can extract the value of $\kappa$ from Eq. $(35)$.

We now present the numerical results of the masses of the heavy baryons $[50]$. Using the values of $\kappa$, $\beta$, and $\gamma$, we can immediately determine the values of $\delta_\pi$ and $\delta_0$ defined in Eq. $(30)$

$$\delta_\pi = (-203.8 \pm 3.5) \text{ MeV}, \quad \delta_0 = (-135.2 \pm 3.3) \text{ MeV}. \quad (36)$$

Including the results of $\kappa/m_c$ and $\kappa/m_b$, we can obtain the numerical results of the heavy baryon masses. In Table I and Table II the numerical results of the charmed and bottom baryon masses are presented, respectively. They are in good agreement with the experimental data taken from Ref. $[60]$. The mass of $\Omega^*_b$ is still experimentally unknown. Thus, the prediction of its mass is given as

$$M_{\Omega^*_b} = (6095.0 \pm 4.4) \text{ MeV}. \quad (37)$$

The uncertainties in Tables I and II are due to those in $\kappa$, $\beta$, $\gamma$, and $\kappa/m_Q$.

### Table I: The numerical results of the charmed baryon masses in comparison with the experimental data $[60]$.

| $^Q \overline{3}_{1/2}$ | $^Q \overline{6}_{1/2}$ | $^Q \overline{6}_{3/2}$ |
|-------------------------|--------------------------|--------------------------|
| $\Lambda_c^+$ | $2272.5 \pm 2.3$ | $2286.5 \pm 0.1$ |
| $\Xi_c^-$ | $2476.3 \pm 1.2$ | $2469.4 \pm 0.3$ |
| $\Sigma_c^+$ | $2445.3 \pm 2.5$ | $2453.5 \pm 0.1$ |
| $\Xi_c^+$ | $2580.5 \pm 1.6$ | $2576.8 \pm 2.1$ |
| $\Omega_c^0$ | $2715.7 \pm 4.5$ | $2695.2 \pm 1.7$ |
| $\Sigma_b^*$ | $2513.4 \pm 2.3$ | $2518.1 \pm 0.8$ |
| $\Xi_b^*$ | $2648.6 \pm 1.3$ | $2645.9 \pm 0.4$ |
| $\Omega^*_b$ | $2783.8 \pm 4.5$ | $2765.9 \pm 2.0$ |

### Table II: The results of the masses of the bottom baryons in comparison with the experimental data $[60]$.

| $^Q \overline{3}_{1/2}$ | $^Q \overline{6}_{1/2}$ | $^Q \overline{6}_{3/2}$ |
|-------------------------|--------------------------|--------------------------|
| $\Lambda_b^0$ | $5599.3 \pm 2.4$ | $5619.5 \pm 0.2$ |
| $\Xi_b^0$ | $5803.1 \pm 1.2$ | $5793.1 \pm 0.7$ |
| $\Sigma_b^0$ | $5804.3 \pm 2.4$ | $5813.4 \pm 1.3$ |
| $\Xi_b^-$ | $5939.5 \pm 1.5$ | $5935.0 \pm 0.05$ |
| $\Omega_b^0$ | $6074.7 \pm 4.5$ | $6048.0 \pm 1.9$ |
| $\Sigma_b^*$ | $5824.6 \pm 2.3$ | $5833.6 \pm 1.3$ |
| $\Xi_b^*$ | $5959.8 \pm 1.2$ | $5955.3 \pm 0.1$ |
| $\Omega^*_b$ | $6095.0 \pm 4.4$ | $-$ |
IV. MAGNETIC MOMENTS OF HEAVY BARYONS

In this Section, we briefly summarize a recent work on the magnetic moments of the heavy baryons [55]. Starting from Eq. (21), one can derive the general expressions of the collective operator for the magnetic moments

\[ \hat{\mu} = \hat{\mu}^{(0)} + \hat{\mu}^{(1)}, \]

(38)

where \( \hat{\mu}^{(0)} \) and \( \hat{\mu}^{(1)} \) denote the leading and rotational \( 1/N_c \) contributions, and the linear \( m_s \) corrections respectively

\[ \begin{align*}
\hat{\mu}^{(0)} &= w_1 D^{(8)}_{Q3} + w_2 d_{pq3} D^{(8)}_{Qp} \cdot \hat{J}_q + \frac{w_3}{\sqrt{3}} D^{(8)}_{Q8} J_3, \\
\hat{\mu}^{(1)} &= \frac{w_4}{\sqrt{3}} d_{pq3} D^{(8)}_{Qp} D^{(8)}_{q4} + w_5 \left(D^{(8)}_{Q3} D^{(8)}_{88} + D^{(8)}_{Q8} D^{(8)}_{83} \right) + w_6 \left(D^{(8)}_{Q3} D^{(8)}_{88} - D^{(8)}_{Q8} D^{(8)}_{83} \right).
\end{align*} \]

(39)

d_{pq3} is the SU(3) symmetric tensor of which the indices run over \( p = 4, \cdots, 7 \). \( \hat{J}_3 \) and \( \hat{J}_p \) denote the third and the \( p \)th components of the spin operator acting on the soliton with the light-quark pair. \( D^{(8)}_{Q3} \) arises from the rotation of the electromagnetic current

\[ D^{(8)}_{Q3} = \frac{1}{2} \left( D^{(8)}_{33} + \frac{1}{\sqrt{3}} D^{(8)}_{83} \right). \]

(40)

The coefficients \( w_i \) in Eq. (39) are independent of baryons involved, which encode the interaction of light quarks with the electromagnetic current. Each term has a physical meaning: \( w_1 \) represents the leading-order contribution, a part of the rotational \( 1/N_c \) corrections, and linear \( m_s \) corrections whereas \( w_2 \) and \( w_3 \) describe the rest of the rotational \( 1/N_c \) corrections. \( w_4 \) includes the \( m_s \)-dependent term, which is not explicitly involved in the breaking of flavor SU(3) symmetry. So, we need to treat \( w_1 \) as if it had contained the SU(3) symmetric part. On the other hand, \( w_4, w_5, \) and \( w_6 \) are the SU(3) symmetry breaking terms. There are yet another \( m_s \) corrections, which arise from the collective wave functions. Though \( w_1 \) can be determined within a specific chiral solitonic model such as the \( \chi \) QSM [55] [58], we will use the values of \( w_1 \), which have been already fixed from the experimental data on the magnetic moments of the baryon octet.

The baryon wave function given in Eq. (18) is not enough to compute the magnetic moments, because the collective wave functions should be revised when the perturbation coming from the strange current quark mass is considered. In this case, the baryon is no more in a pure state but is mixed with higher representations. In Ref. [56], the collective baryon wave functions for the heavy baryons have been already derived. Those for the baryon anti-triplet \( (J_L = 0) \) and the sextet \( (J_L = 1) \) are expressed respectively as [56]

\[ \begin{align*}
|B_3\rangle &= (\bar{3}, B) + p_{15} \left[ \frac{4}{15} |\bar{15}, B\rangle, \\
|B_0\rangle &= (6, B) + q_{15} \left[ \frac{3}{10} |\bar{15}, B\rangle, \right] + \phi_{15} |\bar{15}, B\rangle, \end{align*} \]

(41)

with the mixing coefficients

\[ \begin{align*}
p_{15} &= \frac{3}{4\sqrt{3}} \mathbf{I}_2, & q_{15} &= -\frac{1}{\sqrt{2}} \left( \bar{\alpha} + \frac{2}{3} \right) \mathbf{I}_2, & \phi_{15} &= -\frac{4}{5\sqrt{10}} \left( \bar{\alpha} - \frac{3}{5} \right) \mathbf{I}_2.
\end{align*} \]

(42)

respectively, in the basis \( [\Lambda_Q, \Xi_Q] \) for the anti-triplet and \( [\Sigma_Q (\Sigma_Q^*), \Xi_Q (\Xi_Q)^*], \Omega_Q (\Omega_Q^*) \) for the sextets. The parameters \( p_{15}, q_{15}, \) and \( \phi_{15} \) are written by

\[ \begin{align*}
p_{15} &= \frac{3}{4\sqrt{3}} \mathbf{I}_2, & q_{15} &= \frac{1}{\sqrt{2}} \left( \bar{\alpha} + \frac{2}{3} \right) \mathbf{I}_2, & \phi_{15} &= \frac{4}{5\sqrt{10}} \left( \bar{\alpha} - \frac{3}{5} \right) \mathbf{I}_2.
\end{align*} \]

(43)

Combining Eq. (41) with the heavy-quark spinor as in Eq. (18), one can construct the collective wave functions for the heavy baryon states [55].

Computing the baryon matrix elements of \( \hat{\mu} \) in Eq. (38), we get the magnetic moments of the heavy baryons

\[ \mu_B = \mu_B^{(0)} + \mu_B^{(op)} + \mu_B^{(wf)} \]

(44)

where \( \mu_B^{(0)} \) is the part of the magnetic moment in the chiral limit and \( \mu_B^{(op)} \) comes from \( \hat{\mu}^{(1)} \) in Eq. (38), which include \( w_4, w_5, \) and \( w_6 \). \( \mu_B^{(wf)} \) is derived from the interference between the \( \mathcal{O}(m_s) \) and \( \mathcal{O}(1) \) parts of the collective wave functions in Eq. (41).
Since the soliton with the light-quark pair for the baryon anti-triplet has spin $J_L = 0$, the magnetic moments of the baryon anti-triplet vanish. In this case $1/m_Q$ contributions are the leading ones. However, we will not include them, since we need to go beyond the mean-field approximation to consider the $1/m_Q$ contributions within the present framework.

Since $w_1$ contains both the leading-order contributions and the $1/N_c$ rotational corrections, we have to decompose them. Following the argument of Ref. [55], we can separately consider each contribution. The coefficients $w_1, w_2,$ and $w_3$ are expressed in terms of the model dynamical parameters

$$ w_1 = M_0 - \frac{M_1(-)}{I_1^{(+)}}; \quad w_2 = -2 \frac{M_2(-)}{I_2^{(+)}}; \quad w_3 = -2 \frac{M_1^{(+)} - M_2^{(+)}}{I_1^{(+)}}; \quad (45) $$

where the explicit forms of $M_0, M_1^{(+)}, M_2^{(-)}$ are given in Refs. [35, 61]. $I_1^{(+)}, I_2^{(+)}$ are the moments of inertia with the notation of Ref. [61] taken. In the limit of the small soliton size, the parameters in Eq. (45) can be simplified as

$$ M_0 \rightarrow -2N_cK, \quad \frac{M_1(-)}{I_1^{(+)}} \rightarrow \frac{4}{3}K, \quad \frac{M_1^{(+)} - M_2^{(+)}}{I_1^{(+)}} \rightarrow \frac{2}{3}K, \quad \frac{M_2^{(-)}}{I_2^{(+)}} \rightarrow -\frac{4}{3}K. \quad (46) $$

These results yield the expressions of the magnetic moments in the nonrelativistic (NR) quark model. For example, the ratio of the proton and magnetic moments can be correctly obtained as $\mu_p/\mu_n = -3/2$. In the NR limit, we also derive the relation $M_1^{(-)} = -2M_1^{(+)}$. Furthermore, we have to assume that this relation can be also applied to the case of the realistic soliton size. Then, we can write the leading-order contribution $M_0$ in terms of $w_1$ and $w_3$

$$ M_0 = w_1 + w_3. \quad (47) $$

Since a heavy baryon constitutes $N_c - 1$ valence quarks, the original $M_0$ is modified by introducing $(N_c - 1)/N_c$. As mentioned previously, only the valence part of $M_0$ should be changed by this scaling factor. Since, however, we have determined the values of $w_1$ using the experimental data, we cannot fix separately the valence and sea parts. Thus, we introduce an additional scaling factor $\sigma$ to express a new coefficient $\tilde{w}_1$

$$ \tilde{w}_1 = \left[ \frac{N_c - 1}{N_c} (w_1 + w_3) - w_3 \right] \sigma. \quad (48) $$

$\sigma$ compensates also possible deviations from the NR relation $M_1^{(-)} = -2M_1^{(+)}$ assumed to be valid in the realistic soliton case. The value of $\sigma$ is taken to be $\sigma \sim 0.85$.

Considering the scaling parameters, we are able to determine the following values for $w_1$

$$ \tilde{w}_1 = -10.08 \pm 0.24, \quad w_2 = 4.15 \pm 0.93, \quad w_3 = 8.54 \pm 0.86, \quad \tilde{w}_4 = -2.53 \pm 0.14, \quad \tilde{w}_5 = -3.29 \pm 0.57, \quad \tilde{w}_6 = -1.34 \pm 0.56. \quad (49) $$

Before we carry on the calculation of the magnetic moments, we examine the general relations between them. First, we find the generalized Coleman and Glashow relations [63], which arise from the isospin invariance

$$ \mu(\Sigma^{++}_c) - \mu(\Sigma^+_c) = \mu(\Sigma^+_c) - \mu(\Sigma^-_c), $$

$$ \mu(\Sigma^+_c) - \mu(\Xi^0_c) = \mu(\Xi^0_c) - \mu(\Omega^0_c), $$

$$ 2[\mu(\Sigma^+_c) - \mu(\Xi^0_c)] = \mu(\Sigma^{++}_c) - \mu(\Omega^0_c). \quad (50) $$

Similar relations were also found in Ref. [62]. However, there is one very important difference. While the Coleman-Glashow relations are known to be valid in the chiral limit, the relations in Eq. (50) are justified even when the effects of SU(3) flavor symmetry breaking are considered. We also find the relation according to the $U$-spin symmetry

$$ \mu(\Sigma^0_c) = \mu(\Xi^0_c) = \mu(\Omega^0_c) = -2\mu(\Sigma^+_c) = -2\mu(\Xi^+_c) = -\frac{1}{2} \mu(\Omega^0_c). \quad (51) $$
Belle data unambiguously confirmed the existence of the $\Omega^0_c$ Collaboration, are listed in Table V. The Belle Collaboration has confirmed the four of them [19] (see Table VI). The masses and decay widths of the $\Omega^0_c$ by the LHCb Collaboration [18]. The masses and decay widths of the $\Omega^0_c$ models.

We obtain exactly the same results for the bottom baryons because of the heavy-quark symmetry in the $m \to \infty$. In the SU(3) symmetric case.

In Tables III and IV, we list the numerical results of the charmed baryon sextet with spin 1/2 and 3/2, respectively.

$$\sum_{B_c \in \text{sextet}} \mu(B_c) = 0 \quad (52)$$

which are only valid in the SU(3) symmetric case. We derive also the sum rule given as

in the SU(3) symmetric case.

In Tables III and IV we list the numerical results of the charmed baryon sextet with spin 1/2 and 3/2, respectively. We obtain exactly the same results for the bottom baryons because of the heavy-quark symmetry in the $m_Q \to \infty$ limit. In Ref. [55], a detailed discussion can be found, the present results being compared with those from many other models.

V. EXCITED $\Omega_c$ BARYONS

The present mean-field approach was applied to the classification of the excited $\Omega^0_c$'s that were recently reported by the LHCb Collaboration [18]. The masses and decay widths of the $\Omega^0_c$'s, which were reported by the LHCb Collaboration, are listed in Table V. The Belle Collaboration has confirmed the four of them [19] (see Table VI). The Belle data unambiguously confirmed the existence of the $\Omega_c(3066)$ and $\Omega_c(3090)$, and $\Omega_c(3000)$ and $\Omega_c(3050)$ are also confirmed with reasonable significance. On the other hand the narrow resonance $\Omega_c(3119)$ was not seen in the Belle experiment but the nonobservation of $\Omega_c(3119)$ is not in disagreement because it is due to the small yield.

When one examines the excited heavy baryons in the present work, we need to consider states with the grand spin $K = 1$. Since we have the quantization rule $K = J_L + T$, the possible values of the spin are determined by

$$J_L = |k - T|, \ldots, K + T. \quad (53)$$

Thus, In the case of $T = 0$ which corresponds to the anti-triplet with $Y' = 2/3$, we must have $J_L = 1$ because of $K = 1$. Combining it with the heavy-quark spin 1/2, we have two excited baryon anti-triplet. Similarly, $T = 1$ corresponds to the sextet. In this case $J_L$ can have the values of 0, 1, and 2. Being coupled with the heavy-quark spin 1/2, we get five excited baryon sextets: (1/2), (1/2, 3/2), and (3/2, 5/2), corresponding to $J_L = 0$, and $J_L = 1$, and $J_L = 2$. In each sextet representation, we have a isosinglet $\Omega^0_c$. Thus, is is natural to think that the newly found five $\Omega^0_c$'s are those in the excited baryon sextets. Note that the representations for each value of $J$ are degenerate in the limit of $m_Q \to \infty$. So, we need to introduce an additional hyperfine spin-spin interaction as done for the ground-state
of the excited heavy baryons

The SU(3) symmetry-breaking Hamiltonian in Eq. (22) also needs to be extended to describe the mass splittings of the excited heavy baryons

\[
H_{\text{sb}}^{(K)} = \bar{\sigma} D_{88}^{(8)} + \beta \bar{Y} + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^{3} D_{81i}^{(8)} \bar{T}_i + \frac{\delta}{\sqrt{3}} \sum_{i=1}^{3} D_{81i}^{(8)} \bar{K}_i.
\]

The additional parameter \(\delta\) can be determined by using the mass spectrum of excited baryons.

As shown in Fig. 3, the transition from a \(K^p\) = 1 Dirac-sea level to an unoccupied \(K^p = 0^+\) state may correspond to the first excited heavy baryons. Note that such a transition is only allowed in the heavy-baryon sector, not

### Table V. Experimental data on the five \(\Omega^c_b\) baryons reported by the LHCb Collaboration [18].

| Resonance     | Mass (MeV) | Decay width (MeV) |
|---------------|------------|-------------------|
| \(\Omega_c(3000)^0\) | 3000.4 ± 0.2 ± 0.1 ±0.3 | 4.5 ± 0.6 ± 0.3 |
| \(\Omega_c(3050)^0\) | 3050.2 ± 0.1 ± 0.1 ±0.3 | 0.8 ± 0.2 ± 0.1 |
| \(\Omega_c(3066)^0\) | 3065.6 ± 0.1 ± 0.3 ±0.3 | 3.5 ± 0.4 ± 0.2 |
| \(\Omega_c(3090)^0\) | 3090.2 ± 0.3 ± 0.5 ±0.3 | 8.7 ± 1.0 ± 0.8 |
| \(\Omega_c(3119)^0\) | 3119.1 ± 0.3 ± 0.9 ±0.3 | 1.1 ± 0.8 ± 0.4 |
| \(\Omega_c(3188)\) | 3188 ± 5 ± 13 | 60 ± 15 ± 11 |

### Table VI. Experimental data on the four \(\Omega^0_b\) baryons reported by the Belle Collaboration [19].

| Resonance     | Mass (MeV) |
|---------------|------------|
| \(\Omega_c(3000)^0\) | 3000.7 ± 1.0 ± 0.2 |
| \(\Omega_c(3050)^0\) | 3050.2 ± 0.4 ± 0.2 |
| \(\Omega_c(3066)^0\) | 3064.9 ± 0.6 ± 0.2 |
| \(\Omega_c(3090)^0\) | 3089.3 ± 1.2 ± 0.2 |
| \(\Omega_c(3119)^0\) | – |
| \(\Omega_c(3188)\) | 3199 ± 9 ± 4 |

baryon sextet

\[
H_{LQ} = \frac{2}{3} \frac{\kappa'}{m_Q} J_L \cdot J_Q,
\]

which is very similar to Eq. (32). \(\kappa'\) can be fixed by using the experimental data on the masses of the excited baryon anti-triplet.

Following Refs. [64, 65], we revise the eigenvalues of the symmetric Hamiltonian for the excited baryons \((K \neq 0)\) as follows

\[
M^{(K)}_R = \frac{1}{2T_2} \left[ C_2(R) - T(T+1) - \frac{3}{4} Y'^2 \right] + \frac{1}{2T_1} \left[ (1-a_K)T(T+1) + a_K L_L(J_L+1) - a_K(1-a_K)K(K+1) \right],
\]

where \(C_2(R)\) is the eigenvalue of the SU(3) Casimir operator, which was already defined in Eq. (16). The parameter \(a_K\) is related to one-quark excitation. The collective wave functions for the soliton are derived as

\[
\Phi_{B,L,J,L,T,K}^{(R)\ast} = \sqrt{\frac{2J_L+1}{2K+1}} \sum_{T_3,J_{K3}L_3} C^{KK_3J_{K3}L_3}_{TT_3J_LJ_{L3}} (-1)^{T+T_3} \Psi_{B,R;-.Y'TT_3}^{(R;B)} D^{(J_L)}_{J_{L3}J_{L3}} (S) \chi_{K3},
\]

where index \((R;Y'TT_3)\) denotes the SU(3) quantum numbers of a corresponding baryon in representation \(R\), and \((R;-.Y'TT_3)\) is attached to a fixed value of \(Y'\) and is formally given in a conjugate representation to \(R\). The function \(D^{(J_L)}\) represents the SU(2) Wigner \(D\) function and \(\chi_{K3}\) is the spinor corresponding to \(K\) and \(K_3\). The wave function for the excited baryons can be constructed by coupling \(\Phi_{B,L,J,L,T,K}^{(R)\ast} \) with the heavy-quark spinor.
FIG. 5. Schematic picture of the first excited heavy baryons. A possible excitation of a quark from the Dirac sea to the valence level might have $K^P = 1^-$. 

in the light-baryon sector. As discussed already, there are two baryon anti-triplets and five baryon sextets. From Eq. (55), we can derive the following expressions

$$M_3' = M_3'_{cl} + \frac{1}{2I_2} + \frac{1}{I_1}(a_1^2),$$

$$M_6' = M_6'_{cl} + \frac{a_1}{I_1} + \frac{a_1}{I_1} \times \begin{cases} -1 & \text{for } J_L = 0 \\ 0 & \text{for } J_L = 1 \\ 2 & \text{for } J_L = 2 \end{cases}. \quad (58)$$

Considering the SU(3) symmetry breaking from Eq. (57), we find the splitting parameters for the $\mathbf{3}$ and $\mathbf{6}$

$$\delta_{\mathbf{3}}' = \frac{3}{8} \pi + \beta = \delta_{\mathbf{3}} = -180 \text{ MeV},$$

$$\delta_{\mathbf{6},J_L} = \delta_{\mathbf{6}} - \frac{3}{20} \delta \times \begin{cases} -1 & \text{for } J_L = 0 \\ 0 & \text{for } J_L = 1 \\ 2 & \text{for } J_L = 2 \end{cases}. \quad (59)$$

where we see that $\delta_{\mathbf{3}}'$ is just the same as $\delta_{\mathbf{3}}$ given in Eq. (30). $\delta_{\mathbf{6}}$ is given as $-120 \text{ MeV}$. Though we do not know the numerical value of the new parameter $\delta$, we still can analyze the mass splittings of the newly found $\Omega_c$'s, using the splittings between the states with different values of $J_L$.

We now turn to the hyperfine splittings. The two anti-triplets of spin $1/2$ and $3/2$ and the two sextets of spin $1/2$ and $3/2$ are split by

$$\Delta_{\mathbf{3}}^{hf} = \Delta_{\mathbf{6},J_L=1}^{hf} = \frac{\kappa'}{m_c},$$

whereas another two sextets of spin $3/2$ and $5/2$ are split by

$$\Delta_{\mathbf{6},J_L=2}^{hf} = \frac{\kappa'}{3m_c}. \quad (61)$$

One sextet of spin $1/2$ from the $J_L = 0$ case has no hyperfine splitting. The results are depicted in Fig. 6. Note that the $\Delta_1$ represent the splittings between the $J_L = 0$ state and the degenerate $J_L = 1$ state, whereas $\Delta_2$ denote those between degenerate $J_L = 1$ and $J_L = 2$ states

$$\Delta_1 = \frac{a_1}{I_1} + \frac{3}{20} \delta, \quad \Delta_2 = 2 \Delta_1. \quad (62)$$

We will soon see that the relation $\Delta_1 = 2\Delta_2$ will play an critical role in identifying the excited $\Omega_c$'s within the $\chi$QSM.

If one identifies $\Lambda_c(2592)$ and $\Xi_c(2790)$ as the members of the excited baryon anti-triplet of spin $(1/2)^-$ with negative parity, and $\Lambda_c(2592)$ and $\Xi_c(2790)$ as those of the excited baryon anti-triplet of spin $(3/2)^-$, then we find
\[ \delta_\pi = -198 \text{ and } -190 \text{ MeV, which are more or less in agreement with the value given in Eq. (59). The } \kappa'/m_c \text{ can be also determined as} \]
\[ \frac{\kappa'}{m_c} = \frac{1}{3} (M_{\Lambda_c(2628)} + 2M_{\Xi_c(2818)}) - \frac{1}{3} (M_{\Lambda_c(2592)} + 2M_{\Xi_c(2790)}) = 30 \text{ MeV}, \] (63)

and \( M_\pi \) is also fixed by
\[ M_\pi = \frac{2}{9} (M_{\Lambda_c(2628)} + 2M_{\Xi_c(2818)}) + \frac{1}{9} (M_{\Lambda_c(2592)} + 2M_{\Xi_c(2790)}) = 2744 \text{ MeV}. \] (64)

We now assert that as a minimal scenario the newly found \( \Omega_c \) baryons by the LHCb Collaboration belong to the five excited sextets. Then \( \Omega_c(3000) \) can be identified as the state with \( (J_L = 0, \frac{1}{2}^-) \), which corresponds to the lightest state in Fig. 6. All other four states can consequently be identified as depicted in Fig. 6. Including the hyperfine interactions, we get the results as summarized in Table VII. We find at least three different contradictions

TABLE VII. Scenario 1: All five LHCb \( \Omega_c \) states are assigned to the excited baryon sextets.

| \( J_L \) | \( S' \) | \( M \) [MeV] | \( \kappa'/m_c \) [MeV] | \( \Delta J_L \) [MeV] |
|---|---|---|---|---|
| 0 | \( \frac{1}{2}^- \) | 3000 | not applicable | not applicable |
| 1 | \( \frac{1}{2}^- \) | 3050 | 16 | 61 |
| 2 | \( \frac{3}{2}^- \) | 3066 | 17 | 47 |
| 5 | \( \frac{5}{2}^- \) | 3119 | 17 | 47 |

arising from the assignment of these \( \Omega_c \) states as the members of the excited sextets within the \( \chi \)QSM. Firstly, this assignment requires that the hyperfine splitting should be almost as twice as smaller than in the \( \mathbf{3} \) case. Secondly, the robust relation \( \Delta_2 = 2\Delta_1 \) given in Eq. (62) is badly broken. Finally, there are two orthogonal sum rules \( \sigma_1 = \sigma_2 = 0 \) derived from the \( \chi \)QSM
\[
\sigma_1 = 6 \Omega_c(J_L = 0, 1/2^-) - \Omega_c(J_L = 1, 1/2^-) - 8 \Omega_c(J_L = 1, 3/2^-) + 3 \Omega_c(J_L = 2, 5/2^-),
\] (65)
\[
\sigma_2 = -4 \Omega_c(J_L = 0, 1/2^-) + 9 \Omega_c(J_L = 1, 1/2^-) - 3 \Omega_c(J_L = 1, 3/2^-) - 5 \Omega_c(J_L = 2, 3/2^-) + 3 \Omega_c(J_L = 2, 5/2^-),
\]

which are also badly broken. Thus, we come to the conclusion that the five \( \Omega_c \) baryons is unlikely to belong to the excited sextets. A similar conclusion was drawn by Ref. [66] in a different theoretical framework. Moreover, the computed decay widths of the excited \( \Omega_c \)'s do not match with the experimental data. Therefore, the first scenario is unrealistic in the present mean-field approach.

Since the first scenario is not suitable for identifying the five excited \( \Omega_c \) baryons, we have to come up with another scenario. Observing that two of them have rather narrower decay widths than other three \( \Omega_c \)'s, we assert that these narrow \( \Omega_c(3050) \) and \( \Omega_c(3119) \) belong to the possible exotic anti-decapentaplet (\( \overline{15} \)) which is yet another lowest-lying allowed representation, whereas three of them belong to the excited sextet. We find in this scenario that two other members of the excited baryon sextet with \( J_L = 2 \) have masses above the \( \Xi D \) threshold at 3185 MeV. Since they have
TABLE VIII. Scenario 2. Only three LHCb states are assigned to the sextets.

| $J_L$ | $S^P$ | $M$ [MeV] | $\kappa'/m_c$ [MeV] | $\Delta J_L$ [MeV] |
|-------|-------|-----------|---------------------|-------------------|
| 0     | $\frac{1}{2}^+$ | 3000      | not applicable      | not applicable    |
| 1     | $\frac{1}{2}^-$ | 3066      | 24                  | 82                |
| 2     | $\frac{3}{2}^-$ | 3222      | input               | input             |
| 5     | $\frac{5}{2}^-  $ | 3262      | 24                  | 164               |

rather broad widths, they are not clearly seen in the LHCb data and may fall into the bump structures appearing in the LHCb data.

The results of the second scenario are summarized in Table VIII except for the $\Omega_c(3050)$ and $\Omega_c(3119)$ which will be discussed separately. The italic numbers correspond to the bump structures from which $\Omega_c(3222)$ used as input. Scenario 2 provides a much more plausible prediction than scenario 1 does. Interestingly, the value of $\kappa'/m_c \approx 24$ MeV is closer to that determined from the excited baryon anti-triplets, given in Eq. (63). Moreover, the relation $\Delta_1 = 2\Delta_2$ is nicely satisfied in this scenario.

The anti-decapentaplet ($\bar{15}$) was first suggested by Diakonov [51]. Figure 7 illustrates the representation of the $\bar{15}$. Since the $\bar{15}$ belongs to the allowed representations for the ground-state heavy baryons, it satisfies the quantization rule $J_L + T = 0$, so $T = J_L = 1$ (see Fig. 8). When the light-quark pair with $J_L = 1$ is coupled to the heavy-quark spin, there are two possible $\bar{15}$ representations that are degenerate in the limit of $m_Q \to \infty$. It means that one needs to consider the hyperfine interaction defined in Eq. (32). As given in Eq. (35), the value of $\kappa'/m_c$ is around 68 MeV.
Surprisingly, the mass difference between the $\Omega_c(3050)$ and the $\Omega_c(3119)$ is

$$M_{\Omega_c(3/2^+)(3119)} - M_{\Omega_c(1/2^+)(3050)} = \frac{\kappa}{m_c} \approx 69 \text{ MeV}$$

(66)

which is almost the same as what was determined from the lowest-lying sextet baryons. The decay widths of the excited $\Omega_c$ baryons predicted within the present framework further support the plausibility of scenario 2 \cite{55}. The decay widths for the $\Omega_c(3050)$ and $\Omega_c(3119)$ are predicted to be

$$\Gamma_{\Omega_c(3050)|^{1S}_0(1/2^+)} = 0.48 \text{ MeV}, \quad \Gamma_{\Omega_c(3119)|^{1S}_0(3/2^+)} = 1.12 \text{ MeV},$$

(67)

which are in good agreement with the LHCb data $\Gamma_{\Omega_c(3050)} = (0.8 \pm 0.2 \pm 0.1) \text{ MeV}$ and $\Gamma_{\Omega_c(3119)} = (1.1 \pm 0.8 \pm 0.4) \text{ MeV}$. For detailed discussion related to the decay widths of $\Omega_c$, we refer to Ref. \cite{55}.

In addition to scenarios 1 and 2, we also tried to examine several other scenarios but find that they all turned out to be inconsistent with the experimental data. Finally, we want to emphasize that the $\Omega_c(3050)$ and $\Omega_c(3119)$ assigned to the members of the $^{15}_0$ channel are isotriplets. It implies that if they indeed belong to the $^{15}_0$, charged $\Omega_c^\pm$ should exist. Knowing that the excited $\Omega_c^0$'s have been measured in the $\Xi_c^0 K^-$ channel, we propose that the $\Xi_c^0 K^0$ and $\Xi_c^0 K^-$ channels need to be scanned in the range of the invariant mass between 3000 MeV and 3200 MeV to find an isovector $\Omega_c$'s. If they do not exist, this will falsify the present predictions.

VI. CONCLUSION AND OUTLOOK

In the present short review, we briefly summarized a series of recent works on the properties of the singly heavy baryons within a pion mean-field approach, also known as the chiral quark-soliton model. In the limit of the infinitely heavy quark mass ($m_Q \to \infty$), the heavy quark inside a heavy baryon can be treated as a mere static color source. Then a heavy baryon is portrayed as a state of $N_c - 1$ valence quarks bound by the pion mean field with a heavy quark stripped off from the valence level. This mean-field approach has a certain virtue since both the light and heavy baryons can be dealt with on an equal footing. It means that we can bring all dynamical parameters which have been stripped off from the valence level. This mean-field approach has a certain virtue since both the light and heavy baryons are under way.

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