Local Actions with Electric and Magnetic Sources

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Superstring field theory was recently used to derive a covariant action for a self-dual five-form field strength. This action is shown to be a ten-dimensional version of the McClain-Wu-Yu action. By coupling to D-branes, it can be generalized in the presence of sources. In four dimensions, this gives a local Maxwell action with electric and magnetic sources.
1. Introduction

Because duality symmetry relates theories with strong and weak coupling, it is interesting to look for actions where this symmetry is manifest. This problem is closely related to looking for actions of self-dual $p$-form field strengths in $2p$ dimensions where $p$ is odd. If manifest Lorentz invariance is sacrificed, it is straightforward to construct quadratic actions for these systems.\textsuperscript{1} Although manifest Lorentz invariance can be recovered by introducing “harmonic-like” variables, the resulting action is non-polynomial and may be difficult to quantize.\textsuperscript{2}

An alternative approach is to use the Hamiltonian formalism. After introducing an infinite set of fields, it is possible to construct a set of first-class covariant constraints which impose the appropriate duality conditions. This approach was first developed by McClain, Wu and Yu for two-dimensional chiral bosons\textsuperscript{3}, and later generalized to four-dimensional Maxwell \textsuperscript{4} and self-dual $p$-forms\textsuperscript{5}. Recently, it was shown by Bengtsson and Kleppe that by performing a Legendre transformation, the McClain-Wu-Yu Hamiltonian can be converted into a manifestly Lorentz-invariant action.\textsuperscript{6}

Since the massless Ramond-Ramond sector of the ten-dimensional Type IIB superstring contains a self-dual five-form field strength, it is natural to ask how superstring field theory solves the problem. Due to picture-changing difficulties, the massless Ramond-Ramond contribution to the field theory action has only recently been computed in \textsuperscript{7}, using the techniques of \textsuperscript{8} and \textsuperscript{9}. Because of bosonic ghost zero modes, there are an infinite number of fields in the superstring action, so one suspects a relationship with the McClain-Wu-Yu action. Indeed, as will be shown in this paper, the two actions coincide after gauge-fixing certain fields.\textsuperscript{10}

By coupling to D-branes, it is possible to generalize this action in the presence of sources. Because of manifest duality, electric and magnetic D-branes couple symmetrically. After dimensional reduction to four dimensions, this gives for the first time a local Maxwell

\begin{equation}
\frac{1}{2} \epsilon_{mn} \int d^4 y g_{m} (y) \int \partial^4 \delta^4 (x - \xi) d\xi \text{ and } (j_m(x),g_m(x)) \text{ are the electric and magnetic sources.}
\end{equation}

\textsuperscript{1} The fact that these two actions coincide was also noticed independently by Dmitri Sorokin.\textsuperscript{11}
action with electric and magnetic sources. Dirac charge quantization is implied by the existence of solutions to the classical equations of motion.

In section II of this paper, the free superstring field theory action of reference [9] is reviewed. In section III, it is shown how this action reduces to the McClain-Wu-Yu action after algebraically gauge-fixing certain fields. In section IV, the D-brane boundary state is constructed, and its contribution to the action is computed. In section V, the action is generalized to four-dimensional Maxwell in the presence of electric and magnetic sources. In section VI, some concluding remarks are made, including a conjecture that the eleventh dimension of M-theory might be related to ghost degrees of freedom in superstring theory.

2. Review of Action without Sources

In reference [9], it was shown that by adding a non-minimal set of variables to the usual RNS variables, the free action for the Ramond-Ramond sector can be computed from a \( \langle \Phi | Q | \Phi \rangle \) action. These non-minimal variables were first introduced in [10] and consist of a left-moving pair of conjugate bosons \((\tilde{\gamma}_L, \tilde{\beta}_L)\) of weight \((-\frac{1}{2}, \frac{3}{2})\), a left-moving pair of conjugate fermions \((\chi_L, u_L)\) of weight \((-\frac{1}{2}, \frac{3}{2})\), and their right-moving counterparts, \((\tilde{\gamma}_R, \tilde{\beta}_R)\) and \((\chi_R, u_R)\). The BRST operator is modified to \(Q_{\text{new}} = Q_{RNS} + \int d\sigma (u_L \tilde{\gamma}_L + u_R \tilde{\gamma}_R)\), so using the standard “quartet” argument, the new non-minimal fields do not affect the physical cohomology. Like the \(\psi^\mu\) matter fields and \((\gamma, \beta)\) ghost fields, \((\tilde{\gamma}, \tilde{\beta})\) and \((\chi, u)\) are defined to be odd under \(G\)-parity.

The advantage of adding the non-minimal fields is that they allow consistent boundary conditions for the bosonic zero modes, since the zero modes of \(\gamma - i\tilde{\gamma}\) and \(\beta - i\tilde{\beta}\) can be defined to annihilate the incoming ground state while the zero modes of \(\gamma + i\tilde{\gamma}\) and \(\beta + i\tilde{\beta}\) annihilate the outgoing ground state. As shown in reference [11], this solves the picture-changing problem associated with the Ramond sector of superstring field theory.

Using the methods of [11], it was shown in [9] that the massless components of the incoming Ramond-Ramond string field can be described by

\[
| \varphi \rangle = f^{\alpha\beta}(x^\mu, \psi_L^\mu, \psi_R^\mu, u_L, u_R, y)|L_\alpha\rangle|R_\beta\rangle
\]

(2.1)

where \(\alpha\) is a Weyl or anti-Weyl SO(9,1) spinor index depending if it is a superscript or subscript,

\[
y = \frac{i}{2}[(\gamma_L + i\tilde{\gamma}_L)(\beta_R + i\tilde{\beta}_R) - (\gamma_R + i\tilde{\gamma}_R)(\beta_L + i\tilde{\beta}_L)],
\]

(2.2)
sector of the Type II superstring. (For the Type IIA superstring, the only difference in So at the massless level, an infinite number of fields are present in the Ramond-Ramond action and energy of the system is finite and well-defined.

In order to remove subtleties associated with an infinite number of fields, it will be assumed that at each point in spacetime, there exists an \( N \) such that for all \( n > N, |F^\alpha_{(n)}| < \frac{1}{n}, |E^\beta_{(n)}| < \frac{1}{n}, |D^\alpha_{(n)}| < \frac{1}{n}, \) and \( |C_{(n)\alpha\beta}| < \frac{1}{n} \). This limiting procedure differs from that of reference [9], and is similar to the restriction of reference [8]. It implies that the action and energy of the system is finite and well-defined.

The action for these fields was calculated in [9] from a \( \langle \Phi|Q|\Phi \rangle \) action and is

\[
S = \sum_{n=0}^{\infty} \left[ 2C_{(n)\alpha\beta} \partial_\mu \partial^\mu (\partial^\alpha \gamma E^\beta_{(n)} + (-1)^n \partial^\beta \gamma D^\alpha_{(n)} \gamma) \right. \\
-2F^\alpha_{(n)} \left( (-1)^n \partial^{\gamma}_{(n)\alpha} \gamma + \partial_{\alpha} \gamma D^\alpha_{(n)\beta} \right) \\
- \left( F^\alpha_{(n)} + \partial^{\alpha \gamma} E^\beta_{(n)} \gamma + (-1)^n \partial^{\beta \gamma} D^\alpha_{(n)} \gamma - (-1)^n \partial^{\alpha \gamma} \partial^{\beta \delta} C_{(n)\gamma \delta} \right) \\
\left. (F_{(n+1)\alpha \beta} + (-1)^n \partial_{\beta \kappa} D^\kappa_{(n+1)\alpha} - \partial_{\alpha \kappa} E^\kappa_{(n+1)\beta} + (-1)^n \partial_{\alpha \kappa} \partial_{\beta \kappa} C^\kappa_{(n+1)}) \right] 
\]

where it is understood that for odd \( n \), the positions of all spinor indices are reversed (e.g. the first term for \( n = 1 \) is \( 2C^\alpha_{(1)} \partial_\mu \partial^{\alpha \gamma} E^\gamma_{(1)\beta} \)).
3. Equivalence with McClain-Wu-Yu action

Although (2.4) looks complicated, it is easy to analyze because of the gauge invariances:

\[ \delta C_{(n)\alpha \beta} = \Theta_{(n)\alpha \beta} + \Xi_{(n)\alpha \beta}, \quad \delta D_{(n)\beta} = \Omega_{(n)\beta} - \partial^{\alpha \gamma} (\Theta_{(n)\gamma \beta} - \Xi_{(n)\gamma \beta}) , \quad (3.1) \]

\[ \delta E_{(n)\alpha} = \Lambda_{(n)\alpha} + (\alpha) \partial^{\beta \gamma} (\Theta_{(n)\alpha \gamma} - \Xi_{(n)\alpha \gamma}) , \]

\[ \delta F_{(n)\alpha \beta} = \partial^{\alpha \gamma} \Lambda_{(n)\gamma} + (\alpha) \partial^{\alpha \gamma} (\Theta_{(n)\gamma} + \Xi_{(n)\gamma}), \quad \delta D_{(n+1)\alpha} = -\Lambda_{(n+1)\alpha} , \quad \delta E_{(n+1)\beta} = \Omega_{(n+1)\beta} , \quad \delta F_{(n+1)\alpha \beta} = -(\alpha) \partial^{\beta \gamma} \Lambda_{(n)\alpha} - \partial^{\alpha \gamma} \Omega_{(n)\alpha} \beta. \]

Since the gauge transformations parameterized by \( \Theta_{(n)\alpha \beta} + \Xi_{(n)\alpha \beta} , \Lambda_{(n)\beta} \) and \( \Omega_{(n)\alpha} \) act algebraically on \( C_{(n)\alpha \beta} , D_{(n+1)\alpha} \) and \( E_{(n+1)\beta} \), they can be used to gauge

\[ C_{(n)\alpha \beta} = D_{(n+1)\alpha} = E_{(n+1)\beta} = 0. \quad (3.2) \]

In this gauge, the only non-zero fields are \( D_{(0)\beta} \), \( E_{(0)\alpha} \) and \( F_{(n)\alpha \beta} \), and the action of (2.4) simplifies to

\[ S = \int d^{10} x \left[ -2 F_{(0)\beta} (\partial^{\beta \gamma} E_{(0)\alpha} + \partial_{\alpha \gamma} D_{(0)\beta} ) \right] \quad (3.3) \]

\[ - F_{(1)\alpha \beta} (\partial^{\alpha \gamma} E_{(0)\gamma} + \partial_{\gamma \beta} D_{(0)\alpha} ) - \sum_{n=0}^{\infty} (F_{(2n+2)\beta} + F_{(2n+2)\alpha \beta}) F_{(2n+1)\alpha \beta} \].

Note that (3.3) is gauge invariant under

\[ \delta D_{(0)\beta} = -\partial^{\alpha \gamma} \lambda_{(0)\alpha \gamma}, \quad \delta E_{(0)\alpha} = \partial^{\beta \gamma} \lambda_{(0)\alpha \gamma}, \]

3 These invariances are slightly different from those of reference [9]. In the language of [9], these invariances are obtained from

\[ \delta |\varphi\rangle = Q^{j} \sum_{n=0}^{\infty} \frac{(it_{L} t_{R})^{n}}{(n+2)!} \]

\[ (-2 t_{j L} (\Lambda_{(n)\alpha} + (\alpha) \partial^{\beta \gamma} (\Theta_{(n)\alpha \gamma}) |L_{\alpha}\rangle |R_{\beta}\rangle - 2 t_{j R} (\Omega_{(n)\beta} + \partial^{\alpha \gamma} \Xi_{(n)\gamma \beta}) |L_{\alpha}\rangle |R_{\beta}\rangle \]

\[ -2 t_{j L} u_{R} \Theta_{(n)\alpha \beta} |L_{\alpha}\rangle |R_{\beta}\rangle - 2 t_{j R} u_{L} \Xi_{(n)\alpha \beta} |L_{\alpha}\rangle |R_{\beta}\rangle). \]
which comes from $\Theta_{(0)}^{\alpha\beta} - \Xi_{(0)}^{\alpha\beta}$ in (3.4).

Since $\Omega_{(n)}^{\alpha\beta}$ and $\Lambda_{(n)}^{\beta\alpha}$ also transform $D_{(n)}^{\alpha}$ and $E_{(n)}^{\beta\alpha}$ algebraically, one could have used them to gauge $D_{(n)}^{\alpha} = E_{(n)}^{\beta\alpha} = 0$ (as opposed to gauging $E_{(n+1)}^{\alpha\beta} = D_{(n+1)}^{\beta\alpha} = 0$). In this way, it would naively seem that one could gauge $D_{(n)}^{\alpha} = E_{(n)}^{\beta\alpha} = 0$ for all $n$, including $n = 0$. However, for large $n$, there is a problem with this gauge since it does not preserve the property that $|E_{(n)}^{\beta\alpha}| < \frac{1}{n}$ for all $n > N$. For example, suppose $D_{(N)}^{\alpha} = 1$, and $D_{(N+1)}^{\beta\alpha} = E_{(N+1)}^{\alpha\beta} = 0$. After using $\Omega_{(N)}^{\alpha\beta}$ to gauge $D_{(N)}^{\alpha} = 0$, $E_{(N+1)}^{\beta\alpha}$ will equal $-1$, which does not satify $|E_{(N+1)}^{\beta\alpha}| < \frac{1}{N+1}$. This problem does not occur if one instead uses $\Lambda_{(N-1)}^{\alpha\beta}$ to gauge $D_{(N)}^{\alpha}$ to zero.

The equations of motion of (3.3)are easily calculated to be

$$
F_{(0)}^{\alpha\beta} + \partial^\alpha E_{(0)}^{\beta\gamma} + \partial^\beta E_{(0)}^{\alpha\gamma} = -F_{(2)}^{\alpha\beta} = F_{(4)}^{\alpha\beta} = -F_{(6)}^{\alpha\beta} = \ldots,
$$

$$
2(\partial_{\beta\gamma} E_{(0)}^{\gamma\alpha} + \partial_{\alpha\gamma} D_{(0)}^{\gamma \beta}) = -F_{(1)}^{\alpha\beta} = F_{(3)}^{\alpha\beta} = -F_{(5)}^{\alpha\beta} = \ldots,
$$

$$
2\partial_{\beta\gamma} F_{(0)}^{\alpha\beta} + \partial^\alpha F_{(1)}^{\delta\beta\gamma} = 2\partial_{\beta\gamma} F_{(0)}^{\beta\alpha} + \partial^\alpha F_{(1)}^{\gamma\delta\beta} = 0.
$$

If we assume that there exists an $N$ such that $|F_{(n)}^{\alpha\beta}| < \frac{1}{n}$ for $n > N$, then the only solutions to (3.4) satisfy

$$
F_{(0)}^{\alpha\beta} + \partial^{\alpha\gamma} E_{(0)}^{\beta\gamma} + \partial^{\beta\gamma} D_{(0)}^{\alpha\gamma} = 0,
$$

$$
\partial_{\beta\gamma} E_{(0)}^{\gamma\alpha} + \partial_{\alpha\gamma} D_{(0)}^{\gamma \beta} = 0,
$$

$$
\partial_{\beta\gamma} F_{(0)}^{\alpha\beta} = \partial_{\beta\gamma} F_{(0)}^{\beta\alpha} = 0,
$$

$$
F_{(2n+2)}^{\alpha\beta} = F_{(2n+1)}^{\alpha\beta} = 0.
$$

Expanding $F_{(0)}^{\alpha\beta}$ in vector notation, these equations are easily seen to imply the Bianchi identities and equations of motion for a 1-form, 3-form, and self-dual 5-form field strength. \[9\]

The action for a self-dual 5-form field strength can be extracted from (3.3) as

$$
S = \int d^{10}x \left[ (-2F_{(0)pqrst} - F_{(1)pqrst}) \partial^p A^{qrst} - \sum_{n=0}^{\infty} (F_{(2n)pqrst} + F_{(2n+2)pqrst}) F_{(2n+1)}^{pqrst} \right]
$$

where $A^{qrst} = (\gamma^{qrst})^{\beta\alpha} (D_{(0)}^{\alpha\beta} + E_{(0)}^{\beta\alpha})$ and

$$
F_{(2n+2)pqrst} = \frac{1}{120} \epsilon_{pqrstuvwx} F_{(2n)}^{uvwxy} = F_{(2n)}^{\alpha\beta} (\gamma^{pqrst})^{\alpha\beta},
$$

5
This action will now be shown to be equivalent to the McClain-Wu-Yu action for a self-dual 5-form field strength.

After performing a Legendre transformation, the McClain-Wu-Yu action is:

\[
S_{MWY} = \int d^{10} x \sum_{n=0}^{\infty} (-1)^n \left[ \frac{1}{4} G^{(n)}_{pqurst} G^{pqrst}_{(n)} + \Lambda^{pqrst}_{(n+1)} (G^{(n)}_{pqurst} - G^{(n+1)}_{pqrst}) - \Lambda^{pqrst}_{(n+1)} \Lambda^{(n+2)}_{pqrst} \right]
\]

where \( G^{pqrst}_{(n)} = \frac{1}{120} \partial^p B^{qrst}_{(n)} \) and \( \Lambda^{(n+1)}_{pqrst} = \frac{(-1)^n}{120} \epsilon_{pqrstuvwx} \Lambda_{(n+1)}^{uvwxy} \). Since (3.7) is invariant under the gauge transformations

\[
\delta B^{pqrs}_{(n)} = \Theta^{pqrs}_{(n)}, \quad \delta B^{pqrs}_{(n+1)} = \Theta^{pqrs}_{(n)},
\]

\[
\delta \Lambda^{pqrst}_{(n+1)} = -\frac{1}{240} (\partial^p G^{qrst}_{(n)} + (-1)^n \epsilon_{pqrstuvwx} \partial_u \Theta^{(n)}_{uvwxy}),
\]

one can algebraically gauge \( B^{pqrs}_{(n+1)} = 0 \) for all \( n \). In this gauge, (3.7) becomes

\[
S_{MWY} = \int d^{10} x \sum_{n=0}^{\infty} (-1)^n \left[ \frac{1}{4} G^{(0)}_{pqrst} G^{pqrst}_{(0)} + \Lambda^{pqrst}_{(1)} G^{(0)}_{pqrst} - \sum_{n=0}^{\infty} (-1)^n \Lambda^{pqrst}_{(n+1)} \Lambda^{(n+2)}_{pqrst} \right].
\]

So \( S_{MWY} \) is equal to (3.6) after identifying

\[
A_{pqrs} = \frac{1}{2} B^{(0)}_{pqrs}, \quad F^{pqrst}_{(0)} = -\Lambda^{pqrst}_{(1)} - \frac{1}{4} G^{pqrst}_{(0)} - \frac{1}{120} \epsilon^{pqrstuvwx}_{pqrstuvwx} G^{(0)}_{uvwxy},
\]

\[
F^{pqrst}_{(2n+1)} = -(-1)^n \Lambda^{pqrst}_{(2n+2)}, \quad F^{pqrst}_{(2n+2)} = (-1)^n \Lambda^{pqrst}_{(2n+3)}.
\]

\[\text{This action differs by an overall sign from equation 27 of reference [8] since we use a metric of signature (+, -, ..., -).}\]
4. D-Brane Coupling

In superstring field theory, there are no perturbative states which act as sources for Ramond-Ramond fields. However, it was recently shown that D-branes act as non-perturbative sources for Ramond-Ramond fields.\[13\] At linearized level, the coupling is simply

$$\langle \varphi | D \rangle$$

where $$\langle \varphi |$$ is the Ramond-Ramond string field and $$| D \rangle$$ is the boundary state of the D-brane.\[14\] [15\]

So to add source terms to the action of (2.4), one simply needs to construct the D-brane boundary state and compute its contribution. For a $$(P + 1)$$-dimensional D-brane, one defines $$| D_P \rangle$$ by requiring that

$$\int d\sigma (u_L \tilde{\gamma}_L + u_R \tilde{\gamma}_R).$$

where $$\gamma$$ is defined in (2.2) and $$\mu_P$$ is a constant which can be determined from a one-loop calculation. (We are considering the Type IIB superstring, so $$P$$ is odd. For the Type IIA superstring, $$P$$ is even and the last line of (1.2) is replaced with $$(\gamma^0\ldots P)_{\alpha\beta} | L_\alpha \rangle | R_\beta \rangle + i\eta (\gamma^0\ldots P)_{\alpha\beta} | L_\alpha \rangle | R_\beta \rangle.$$ Note that no sum over pictures is necessary in the definition of $$| D \rangle.$$\[15\]

One can now use the fact that

$$\langle R_\alpha | (L_\beta | \tilde{g}^m y^n (c_L + c_R) u_0 L u_0 R h(x) | L^\gamma \rangle | R^\delta \rangle$$
to compute $\langle \varphi | D \rangle$. Plugging in the expression of (2.3) for $\langle \varphi |$, one finds

$$S_P = \langle \varphi | D_P \rangle = 2\eta_P \int_{x^\mu = f^\mu} d^{P+1} x^i (\gamma^0 \cdots P)^{\alpha \beta} \sum_{n=0}^{\infty} (-1)^n (E^{\alpha}_{(2n)\beta} - D^{\alpha}_{(2n)\beta} + E^{\alpha}_{(2n+1)\beta} + D^{\alpha}_{(2n+1)\beta}).$$

So in the presence of D-branes, the massless Ramond-Ramond contribution to the Type IIB superstring field theory action is

$$S = S_{\text{free}} + \sum_{i=0}^{5} S_{2i-1}$$

where $S_{\text{free}}$ is defined in (2.4) and $S_{2i-1}$ is defined in (4.4).

This action is still invariant under the gauge transformations of (3.1), and after gauge-fixing $C_{(n)\alpha \beta} = D^{\beta}_{(n+1)\alpha} = E^{\beta}_{(n+1)\alpha} = 0$, it simplifies to

$$S = \int d^{10} x \left[ -2F^{\alpha \beta}_{(0)} (\partial_{\beta \gamma} E^{\gamma}_{(0)\alpha} + \partial_{\alpha \gamma} D^{\gamma}_{(0)\beta}) - F^{\alpha \beta}_{(1)\alpha \beta} (\partial_{\alpha \gamma} E^{\gamma}_{(0)\beta} - D^{\alpha}_{(0)\beta}) \right].$$

The equations of motion of (3.5) are now modified to

$$F^{\alpha \beta}_{(0)} + \partial^{\alpha \gamma} E^{\beta}_{(0)\gamma} + \partial^{\beta \gamma} D^{\alpha}_{(0)\gamma} = 0,$$

$$\partial_{\beta \gamma} E^{\gamma}_{(0)\alpha} + \partial_{\alpha \gamma} D^{\gamma}_{(0)\beta} = 0,$$

$$\partial_{\beta \gamma} F^{\alpha \beta}_{(0)} = \sum_P \eta_P (\gamma^0 \cdots P)^{\alpha \beta} \delta^{9-P} (x^\mu - f^\mu),$$

$$\partial_{\beta \gamma} F^{\alpha \beta}_{(0)} = -\sum_P \eta_P (\gamma^0 \cdots P)^{\alpha \beta} \delta^{9-P} (x^\mu - f^\mu),$$

$$F^{\alpha \beta}_{(2n+2)} = F^{\alpha \beta}_{(2n+1)\alpha \beta} = 0.$$

By plugging equations (4.6) into equations (4.7), it naively appears that the charges $\mu_P$ must vanish. However, this is not true since $D^{\alpha}_{(0)\beta}$ and $E^{\beta}_{(0)\alpha}$ are only single-valued up to the gauge transformation

$$\delta D^{\alpha}_{(0)\beta} = -\partial^{\alpha \gamma} \lambda^{(0)\gamma} \lambda^{(0)\beta}, \quad \delta E^{\beta}_{(0)\alpha} = \partial^{\beta \gamma} \lambda^{(0)\alpha \gamma}.$$

The equations of motion, when combined with single-valuedness of gauge-invariant objects, imply (using the usual arguments [19]) that the charges satisfy the quantization condition that $\frac{1}{2\pi} \mu_P \mu_6 - P$ is an integer and $\mu_9 = 0$. In other words, if $\frac{1}{2\pi} \mu_P \mu_6 - P$ is not an integer or $\mu_9$ is non-zero, the equations of motion have no solution.
5. Generalization to Four-Dimensional Maxwell

In the absence of sources, the four-dimensional Maxwell action was found in reference [9] by performing dimensional reduction on (2.4). After algebraically gauge-fixing \( C(n)_{ab} = D_{(n+1)ab} = E_{(n+1)ab} = 0 \) and separating out the Maxwell field, the action is

\[
S_{free} = \int d^4x \left[ -F_{(0)}^{pq}(\partial_p A_q - \frac{1}{2} \epsilon_{pqrs} \partial^r B^s) \right] + \frac{1}{2} F_{(1)}^{pq} (\partial_p A_q + \frac{1}{2} \epsilon_{pqrs} \partial^r B^s) \right]
\]

where \( F_{(n)}^{pq} = 2 \text{Re}(\sigma_{ab}^{pq} F_{(n)}^{ab}) \), \( A^p = 4 \text{Re}(D_{(0)}^{ab} + E_{(0)}^{ba}) \), and \( B^p = 4 \text{Re}(D_{(0)}^{ab} + E_{(0)}^{ba}) \). (\( a \) and \( \dot{a} \) are two-component Weyl indices which come from dimensionally reducing a sixteen-component \( \text{SO}(9,1) \) spinor.) Note that (5.1) is manifestly invariant under the duality rotation

\[
\delta F_{(n)pq} = \frac{1}{2} (-1)^n \epsilon_{pqrs} F_{(n)}^{rs}, \quad \delta A_p = B_p, \quad \delta B_p = -A_p.
\]

To couple to \( M \) dyons, one simply adds to (5.1) the source term

\[
S_{source} = \sum_{I=1}^{M} \int d\tau [m_I \sqrt{\dot{y}_I^p \dot{y}_I^p} + \dot{y}_I^p (e_I A_p(y_I(\tau)) + g_I B_p(y_I(\tau)))]
\]

where \( y_I^p(\tau) \) is the worldline of the \( I^{th} \) dyon, \( \dot{y}_I^p = \frac{dy_I^p}{d\tau} \), and \((e_I, g_I)\) is its electric and magnetic charge.

Using the requirement that \( |F_{(n)}^{pq}| < \frac{1}{n} \) for \( n > N \), it is easily verified that the equations of motion for \( S_{free} + S_{source} \) are

\[
F_{(0)}^{pq} = \partial^p A^q - \frac{1}{2} \epsilon_{pqrs} \partial^r B^s, \quad F_{(2n+1)}^{pq} = F_{(2n+2)}^{pq} = 0,
\]

\[
\partial_q F_{(0)}^{pq} = \sum_{I=1}^{M} e_I \int d\tau \dot{y}_I^p \delta^4(x - y_I(\tau)),
\]

\[
\frac{1}{2} \epsilon_{pqrs} \partial_q F_{(0)rs} = \sum_{I=1}^{M} g_I \int d\tau \dot{y}_I^p \delta^4(x - y_I(\tau)),
\]

\[
m_I \frac{d^2 y_p}{d\tau^2} = e_I F_{(0)}^{pq} \frac{dy_q}{d\tau} + \frac{1}{2} g_I \epsilon_{pqrs} F_{(0)rs} \frac{dy_q}{d\tau},
\]

9
where the scale gauge \( \dot{y}_I \dot{y}_I = 1 \) has been chosen. Because \( \exp[i \oint_C dx^p (e_I A_p + g_I B_p)] \) must be single-valued for any closed contour \( C \), these equations of motion only contain solutions if \( (e_I, g_I) \) satisfy the Dirac-Zwanziger quantization condition \( e_I g_J - g_I e_J = 2\pi n_{IJ} \) for some integers \( n_{IJ} \).

Note that one can replace the particle sources with fields, e.g.

\[
S_{\text{source}} = i \sum_{I=1}^{M} \int d^4 x \bar{\psi}_I^a (\partial_x - ie_I A_p - ig_I B_p) \sigma_{a\dot{a}}^p \psi^a_I \tag{5.4}
\]

where \( \psi^a_I \) are dyonic spinors. This gives the appropriate Maxwell equations of motion

\[
F^{pq}_{(0)} = \partial^p A^q - e^{pqrs} \partial_r B_s, \quad F^{pq}_{(2n+1)} = F^{pq}_{(2n+2)} = 0, \tag{5.5}
\]

\[
\partial_q F^{pq}_{(0)} = \sum_{I=1}^{M} e_I \bar{\psi}_I^a \sigma_{a\dot{a}}^p \psi^a_I, \quad \frac{1}{2} \epsilon_{pqrs} \partial^q F^{rs}_{(0)} = \sum_{I=1}^{M} g_I \bar{\psi}_I^a \sigma_{a\dot{a}}^p \psi^a_I,
\]

\[
(\partial_x - ie_I A_p - ig_I B_p) \sigma_{a\dot{a}}^p \psi^a_I = 0.
\]

These equations (when combined with the fact that \( \exp[i \oint_C dx^p (e_I A_p + g_I B_p)] \) is single-valued for any closed contour \( C \)) imply that

\[
\frac{1}{2\pi} \sum_{I=1}^{M} (e_I g_J - g_I e_J) \int_V d^3 x \bar{\psi}_J^\dot{a} \sigma_{a\dot{a}}^0 \psi^a_J
\]

is an integer for any volume \( V \). Although this is untrue at the classical level even when \( (e_I, g_I) \) satisfy the Dirac-Zwanziger quantization condition, it is true at the quantum level if \( \int_V d^3 x \bar{\psi}_J^\dot{a} \sigma_{a\dot{a}}^0 \psi^a_J \) is interpreted as the expectation value of

\[
\frac{\langle \Phi | \int_V d^3 x \bar{\psi}_J^\dot{a} \sigma_{a\dot{a}}^0 \psi^a_J | \Phi \rangle}{\langle \Phi | \Phi \rangle}
\]

where \( | \Phi \rangle \) is any normalizable state. This is because \( \bar{\psi}_J^\dot{a}(x) \sigma_{a\dot{a}}^0 \psi^a_J(x) \) is an operator with eigenvalues \( \sum_{i=1}^{N} \delta^3(x - y_i) \), where \( \Phi \) is a state constructed with \( N \) creation operators.\(^5\)

\(^5\) I thank Warren Siegel for pointing this out to me.
6. Conclusions

In this paper, D-branes were used to construct local actions with electric and magnetic sources. These actions contain an infinite number of fields and are closely related to the McClain-Wu-Yu action. It should be straightforward to quantize them using the methods of [5][6][7][8].

Since these action appear in superstring field theory, it is natural to try to interpret the infinite set of fields as coming from some stringy degree of freedom. In fact, by looking at equation (2.3), it is clear that this new degree of freedom is simply the $y$ variable, which is constructed from an SU(1,1)-invariant combination of worldsheet ghosts. In this sense, the massless Ramond-Ramond string field $\varphi$ can be interpreted as living in eleven dimensions, where the eleventh dimension is parameterized by $y$.

Recently, it was conjectured that there exists an eleven-dimensional theory called $M$-theory which, after compactification on $S_1$, is dual to Type IIA superstring theory.[17] $D_0$-branes are associated with Kaluza-Klein states in $M$-theory and their Ramond-Ramond charge comes from momentum in the compactified eleventh direction. Furthermore, the radius of compactification of the eleventh dimension is related to the expectation value of the dilaton in superstring theory. By compactifying on $S_1/Z_2$ instead of $S_1$, $M$-theory can be related to the heterotic superstring.[18]

Is there a connection between $y$ and the eleventh dimension of $M$-theory? Although this question will not be answered here, three pieces of favorable evidence will be presented.

Firstly, the $y$ dependence of the $D_0$-brane boundary state is simply $e^{\pm iy}$, where the $\pm$ depends if it is a $D_0$-brane (carrying momentum $P_{11} = +1$) or an anti-$D_0$-brane (carrying $P_{11} = -1$). At least naively, this suggests that $y = x_{11}$.

Secondly, $y$ is constructed from worldsheet ghosts, which couple to worldsheet curvature through their background charge. Therefore, it is possible that $y$ might be related to the expectation value of the dilaton, which also couples to worldsheet curvature.

Thirdly, if the eleventh dimension of $M$-theory were related to worldsheet ghost degrees of freedom, it might explain in a stringy manner why different superstring theories can be unified. As was shown in [19], by mixing matter and ghost degrees of freedom, it is possible to relate string theories with different numbers of worldsheet supersymmetries. Perhaps duality symmetry can be understood as rotations which mix matter and ghost degrees of freedom.

Clearly, more evidence needs to be gathered before this question can be answered. Hopefully, the three pieces of evidence suggested in this conclusion will motivate others...
to investigate if the eleventh dimension of $M$-theory can be related to worldsheet ghost degrees of freedom.

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