Energy Storage Investment and Operation in Efficient Electric Power Systems

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ABSTRACT

We consider welfare-optimal investment in and operation of electric power systems with constant returns to scale in multiple available generation and storage technologies under perfect foresight. We extend a number of classic results on generation, derive conditions for investment and operations of storage technologies described by seven cost/performance parameters, and develop insights on power systems with multiple storage technologies. Simulation of a deeply decarbonized “Texas-like” power system with two available storage technologies shows both the non-existence of simple “merit-order” rules for storage operation and the value of frequency domain analysis to describe efficient operation. Our analysis points to the critical role of the capital cost of energy storage capacity in influencing efficient storage investment and operation.

Keywords: Electricity, Storage, Efficiency, Optimality

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1. INTRODUCTION

Driven importantly by concerns about climate change, variable renewable energy (VRE) resources, mainly wind and solar, are becoming increasingly important sources of electricity in many regions. Because the maximum output of VRE generators is variable and imperfectly predictable, however, increased penetration of VRE generation makes it more difficult for power system operators to match supply and demand at every instant. As the costs of storage, particularly lithium-ion (Li-ion) battery storage, have declined rapidly, storage has emerged as a potentially attractive, carbon-free solution to problems posed by increased VRE penetration (Patel 2018). Policy-makers in the U.S. and the E.U. have accordingly encouraged the deployment of storage. The California Public Utilities Commission has been requiring load-serving entities to procure storage since the promulgation of statutory requirements in 2010 (Petlin et al 2018, California Public Utilities Commission n.d.). As of mid-2021, seven states have established storage targets, and they are under consideration in other states (DSIRE database n.d.). The U.S. Federal Regulatory Commission (2018) Order 841 is intended to open wholesale energy markets (and other wholesale markets) to merchant storage providers. 1 Similarly, The European Union’s Clean Energy Package calls for competitive supply of storage (Glowacki 2020).

1. In addition, at the federal level in the U.S., storage facilities that are charged only by solar generators are eligible for up to a 30% investment tax credit.

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In this essay, we explore what economic theory implies about the general properties of cost-efficient electric power systems in which storage performs energy arbitrage to help balance supply and demand.\textsuperscript{2} We start from an investment planning model ultimately based on the work of Boiteux (1960, 1964) and Turvey (1968).\textsuperscript{3} In models of this sort, constant returns to scale are generally assumed in generation, i.e., costs are assumed to be linear in the capacities and outputs (up to capacity) of each of several types of dispatchable generators. There are no startup costs or ramping constraints, which limit thermal generators’ ability to change output. There are thus no non-convexities or links between time periods on the supply side. Similarly, the demand function in each period is independent of prices charged in other periods. Thus the multiple time periods in these models are linked only by the generation capacities that are chosen at the outset.

It is important to note that these assumptions are not descriptive of systems in which coal or nuclear generation are important supply sources. Both technologies have significant economies of scale, giving rise to nonconvexities. In addition, coal and nuclear plants take time and incur costs to start up and ramp down\textsuperscript{4}, which breaks the independence among time periods. Power systems with these characteristics resist general algebraic analysis, and sophisticated numerical optimization tools have been developed to permit explicit multi-period analysis of particular cases.\textsuperscript{5}

For modern gas generators and VRE facilities, however, neither lumpiness nor startup or ramp down costs are nearly as important. Boiteux-Turvey-style models are thus reasonable approximations for systems without significant coal or nuclear generation.\textsuperscript{6} There are a number of ways that storage has been added to models of this sort. In the earliest formal treatments of storage in this context of which we are aware, Gravelle (1976) and Nguyen (1976) consider two-period—peak and off-peak—models and simply assume that an unlimited amount of the quantity being sold can be transferred between adjacent periods at a constant per-unit cost. Several authors, including Steffen and Weber (2013) and Korpås and Botterud (2020) have added storage to Boiteux-Turvey-style models by assuming that power can be purchased whenever the price of energy is low and resold whenever the price is high. This also amounts to assuming that the amount of energy that can be stored is unbounded, since low-price and high-price periods may be far apart in time. Helm and Mier (2018) consider a dynamic model with a constant demand curve and non-stochastic renewable output that follows a regular cyclic trajectory. Schmalensee (forthcoming) considers a model with stochastic demand and alternating daytime and nighttime periods in which VRE generation is only available in the daytime periods.

Here we follow Crampes and Trochet (2019) and Brown and Reichenberg (2021) and consider an explicitly dynamic Boiteux-Turvey-style model with perfect foresight. We follow most of

\begin{itemize}
  \item Storage can also perform other functions in electric power systems. Depending on the technology employed, storage facilities can provide frequency regulation, deferral of wires investment, and reducing the cost of spinning reserves. For discussions, see Giuletti et al (2018), Balducci et al (2018), and U.S. Government Accountability Office (2018). See Sidhu et al (2018) for a worked example of a storage project that could perform multiple functions.
  \item For an early exposition of models of this sort, see Drèze (1964), and for an excellent recent textbook treatment, see Biggar and Hesamzadeh (2014, esp. ch. 9). Following most of this tradition, we neglect the spatial dispersion of real power systems and assume everything happens at a single point.
  \item Although the existing fleet of nuclear power plants are capable of flexible operation within limits, they are more constrained than competing grid resources like natural gas power generation and energy storage (U.S. Department of Energy, 2015).
  \item See, for instance, Jenkins and Sepulvada (2017) and Johnston et al. (2019).
  \item Modern combined cycle gas turbines (CCGT) and open cycle gas turbine (OCGT) power plants can ramp up or down 100% of their nameplate capacity within an hour. See for example specifications for GE’s 7HA gas turbines (https://www.ge.com/content/dam/gpenergy-pgdpp/global/en_US/documents/product/gas%20turbines/Fact%20Sheet/2017-prod-specs/7ha-power-plants.pdf).
\end{itemize}

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the literature and assume constant returns to scale in storage as well as in generation. The perfect foresight assumption is of course strong and eliminates the precautionary demand for storage. Relaxing that assumption, however, requires explicitly modeling the relevant stochastic processes, as demonstrated by Geske and Green (2019), and it is not clear that general results are available.

Section 2 presents the (linear) capacity planning and operations model employed, and Section 3 provides brief derivations of a number of known results for the sake of completeness. We show explicitly that the problem of maximizing overall social welfare in this model can be decomposed into the problems faced by profit-maximizing, perfectly competitive suppliers of each available technology, even when considering limited energy capacity of energy storage and ramping constraints for dispatchable generation. We demonstrate that marginal-cost-based dispatch for thermal generators is not generally optimal when ramping constraints are binding.

Section 4 provides generalizations of recent results regarding optimal investment in, profitability of, and operation of individual storage technologies. We employ a generalized characterization of storage technologies that uses seven distinct parameters, including independent charging and discharging power capital costs and efficiencies and show that all deployed storage technologies break even at equilibrium under constant returns to scale.

Section 5 presents an analytical framework that yields insight into efficient configurations and operations of systems employing multiple storage technologies and points to the importance of the relative costs of power capacity and energy storage capacity. Finally, Section 6 provides simulation results that illustrate the complexity of operating patterns of storage in systems with multiple storage technologies and supports the insights developed in Section 5. It shows that general analytical results of the “merit-order” variety are not available for storage, and demonstrates the value of frequency domain analysis via Fourier Transforms to characterize the cost-efficient operating regimes of each storage technology. Section 7 provides some concluding observations.

2. OPTIMA AND EQUILIBRIA

We consider a linear $T$-period model with one dispatchable technology (which we will often refer to as gas), one VRE technology, and a single storage technology. The restriction to a single technology of each type in this section is simply to reduce notational clutter. In later sections we consider systems with multiple technologies of each type when appropriate. Throughout we abstract from storage’s ability to supply frequency regulation and other ancillary services and to defer investment in transmission or distribution systems.

Because our focus is on the supply of electricity, we assume perfectly inelastic demand for simplicity. That is, we assume that demand in period $t$ is equal to the exogenous quantity $Q_t$, for prices below $\omega$, the value of lost load. Then total welfare, to be maximized, is given by

$$W = \omega \sum_{t=1}^{T} [C_g G + C_R R + C_P^a P^a + C_P^d P^d + C_E E + \nu \Sigma g_t + o^A \Sigma A_t + o^D \Sigma D_t]$$

where $L_t$ is the non-negative lost load in period $t$. Throughout, sums are over $t$ from 1 to $T$, unless otherwise specified.\(^7\)

7. To allow for price-responsive demand, the first term in (1) would be replaced by $U(Q_t; t)$, with $U$ a concave utility function that is shifted by changes in $t$, and $Q_t$ a non-negative choice variable. With this change, for positive values of $Q_t$ condition (9) below would be replaced by a requirement for marginal cost pricing, and nothing else in the analysis would change. Joskow and Tirole (2007) analyze markets with both price-responsive and unresponsive demand and also consider system collapses and inefficient rationing.

8. With a single investment period, allowing for uncertainty that is resolved after investment would mainly complicate formulas and change the focus of break-even analysis from total net revenue to total expected net revenue. Similarly, allowing...
We assume constant returns to scale, so that we can work with the aggregate capacities and outputs of all facilities using the same technology. From left to right the Cs in equation (1) are the T-period per-MW capital costs of dispatchable capacity, $G$, of renewable capacity, $R$, of charge power capacity of storage, $P_A$, and of discharge power capacity of storage, $P_D$, respectively. $C_E$ is the T-period per-unit capital cost of energy storage capacity, $E$.

For a pumped hydro storage facility, for instance, $P_A$ would be the maximum rate at which water can be pumped into the uphill reservoir, measured by the instantaneous power consumption of the pumping system in MW, $P_D$ would be the maximum rate at which the facility can generate electricity, again in MW, and $E$ would be the capacity of the reservoir. For convenience we assume that $P_A$, $P_D$, and $E$ can be chosen independently, though for some storage technologies this may not be possible. For pumped hydro for instance, the same turbine is often used to pump water into the reservoir and to generate electricity when the water is released.

Also in equation (1), $v$ is the (constant) marginal cost of dispatchable generation, $g_t$ is dispatchable generation in period $t$, $o^A$ is the variable operation and maintenance (O&M) cost per MWh used to charge storage, $A_t$ is MWh used to charge storage in period $t$, $o^D$ is the variable O&M per MWh discharged, and $D_t$ is MWh discharged from storage in period $t$. In the context of pumped hydro storage, one can think of $o^A$ and $o^D$ reflecting the marginal wear and tear caused by pumping water into the uphill reservoir and using water from that reservoir to generate electricity, respectively. In the case of battery storage systems, these parameters are best thought of as providing an approximation to the degradation caused by charging and discharging.

We consider maximization of $W$ subject to a set of linear constraints. The Karush-Kuhn-Tucker (KKT) stationary conditions can thus be employed to characterize maxima. The constraints that supply (including lost load) is equal to demand in each period is

$$L_t + g_t + (\theta R - R^C_t) + D_t - A_t - Q_t = 0 \quad (\lambda_t), \quad t = 1, \ldots, T. \quad (2)$$

Here $\theta_t$ is the fractional capacity factor of renewables, $R^C_t$ is the non-negative amount of renewable output that is curtailed, both in period $t$. The KKT multipliers $\lambda_t$ are the marginal values of electric energy in period $t$ in the planner’s problem. In all that follows, we show the multipliers corresponding to each set of constraints in parentheses after the constraints. Multipliers corresponding to non-negativity constraints are themselves non-negative. The constraints that renewable curtailment not exceed renewable output are

$$\theta_t R - R^C_t \geq 0 \quad (\vartheta_t), \quad t = 1, \ldots, T. \quad (3)$$

In more general models (like the simulation model used to generate the results in Section 6) that enforce minimum stable outputs of thermal generators and costs of startup and shutdown of for discounting would complicate formulas and change the focus of break-even analysis from total net revenue to total discounted net revenue. The only difference in optimal operation would be in the evolution of the value of stored energy, which is discussed below Proposition 7.

9. For gas generation in moderately large systems, constant returns is a good approximation. Some storage technologies may exhibit increasing returns, however: the surface area of a tank rises less than proportionately with its volume, for instance.

10. If for technological reasons $P^A = P^D = P$ for some technology, then if some facilities using that technology are charging and others are simultaneously discharging, total charging plus total discharging cannot exceed $P$. Since, as discussed below, simultaneous charge and discharge for the same technology can occur only in very special cases, this constraint is not explicitly imposed here.

11. The degradation of Lithium-ion batteries with both time and usage has been much studied; see Gailani et al (2020) for a recent contribution and references to that literature. Implications for scheduling generators have been explored by Duggal and Venkatesh (2015).
such generators, the marginal value of energy can be negative. If thermal generation is needed in period \( t+1 \) but optimally shut down in period \( t \), for instance, an increase in demand in period \( t \) that would enable avoidance of startup costs in period \( t+1 \) could lower system cost.\(^\text{12}\) The competitive energy price in period \( t \) would accordingly be negative. Even though we do not consider startup costs here, for the sake of generality, we do not constrain \( \lambda_t \) to be non-negative.

Let \( S_t \) be the amount of energy in storage at the end of period \( t \), with value at the start of period one (end of period zero) equal to \( S_0 \). We impose the constraint that storage not accumulate or dissipate energy over the \( T \) periods:

\[
S_0 - S_T = 0 \quad (\mu_0).
\]

If for every MWh used to charge storage, \( r_A \) of energy is actually stored, and in order to discharge one MWh from storage, \( (1/r_D) \) of stored energy must be used,\(^\text{13}\) then the equations of motion for end-of-period energy in storage are

\[
\chi S_{t+1} + r_A D_t - (1/r_D) S_t = 0, \quad t = 1, \ldots, T.
\]

Here \( \chi \leq 1 \) is a constant reflecting self-discharge in some storage systems (e.g., evaporation from pumped hydro reservoirs, battery self-discharge). In general, the values of \( r_A \) and \( r_D \) will depend on the units in which \( S \) and \( E \) are measured. From the point of the power system, however, all that matters is round-trip efficiency, the incremental MWh discharge made possible by an incremental use of one MWh in charging, \( r \equiv r_A r_D \).\(^\text{14}\) This quantity is independent of how \( E \) and \( S \) are measured and is strictly less than one in real storage systems. It is convenient then to define \( S \) as the energy that can be delivered to the power system, with \( E \) its upper limit. This is equivalent to assuming that all energy loss occurs during charging, so that \( r_A = r \) and \( r_D = 1 \), and the equation of motion of \( S \) becomes

\[
\chi S_{t+1} + r_A D_t - S_t = 0 \quad (\mu_t), \quad t = 1, \ldots, T
\]

The multiplier \( \mu_t \) is the marginal value of energy in storage at the end of period \( t \). (Note that unlike \( \lambda_t \), \( \mu_t \) does not correspond to an observable market price.) Because energy is lost in the process of charging and discharging storage, one would expect the value of stored energy to be non-negative, but, for the sake of generality, we do not impose this constraint. A storage technology in this model is thus described by seven parameters: three capital cost parameters \((C_P^A, C_P^D, \text{ and } C_E)\) and four flow parameters \((\sigma_A, \sigma_D, r, \text{ and } \chi)\).

\(^{12}\) The idea that increasing demand can sometimes lower system cost is not just a theoretical possibility (Hawai‘i Natural Energy Institute 2019). A diesel generator is used to follow load on the Hawaiian island of Moloka‘i, population 7,345. In 2015 the local utility found that if it granted all pending applications for rooftop solar generation, the difference between demand and solar output would occasionally fall below the diesel’s minimum output level, causing the generator to trip off and the island to black out. To avoid the high cost of blackouts when those applications had been granted, the utility installed a “load bank”, a dispatchable resistive load that could be used when necessary to transform electric energy into waste heat.

\(^{13}\) This model of storage generalizes that of Crampes and Trochet (2019). They assume that \( \chi = 1, \sigma_A = \sigma_D = 0, \text{ and } r^D = 1 \). As the discussion below indicates, this last restriction does not entail a loss of generality. Brown and Reichenberg (2021, Appendix B.3) assume \( \sigma_A = \sigma_D = 0 \).

\(^{14}\) Charge and discharge efficiency may have implications for the cost of storage facilities, however. Consider two storage devices, both with \( r = 0.4 \), and both capable of providing 2 MWh to the grid for one hour. Using the same units of measurement for \( S \) and \( E \), device A has \( r_A = 0.8 \) and \( r_D = 0.5 \), while device B has \( r_A = 0.5 \) and \( r_D = 0.8 \). To be able to provide 2 MWh for one hour, device A needs \( E_A = 4.0 \), while device B needs only \( E_B = 2.5 \). The costs of these two devices are unlikely to be the same. This issue is explored in Sepulvada et al (2021).
We assume that limitations on per-period changes in the outputs of dispatchable generators relative to their capacities, so-called ramping constraints, may sometimes be binding:

\[ \beta^U G + g_{t-1} - g_t \geq 0, \quad (\rho^U_t), \quad t = 2, \ldots, T, \quad (6a) \]

\[ \beta^D G + g_t - g_{t-1} \geq 0, \quad (\rho^D_t), \quad t = 2, \ldots, T. \quad (6b) \]

Here \( \beta^U \) and \( \beta^D \) are exogenous, positive constants strictly less than one. Note that there are no ramping constraints on first-period output. Biggar and Hesamzadeh (2014, Section 4.9) provide a formal analysis of a ramping constraint in a model of this sort, though they constrain only the absolute increase in generation.

In addition to conditions (2)–(6), the following inequalities must also be satisfied, with \( R^U \) and \( E^U \) positive constants:

\[ R \geq 0 \quad (R_0), \quad R^C - R \geq 0 \quad (R_u), \quad G \geq 0 \quad (G_0). \quad (7a) \]

\[ P^A \geq 0 \quad (P^A_0), \quad P^D \geq 0 \quad (P^D_0), \quad E \geq 0 \quad (E_0), \quad E^U - E \geq 0 \quad (E_u). \quad (7b) \]

The upper bound constraint on renewable capacity in (7a) can arise if, for instance, there is a limit on the number of turbines that can be installed on sites that are good for wind generation. Similarly, the upper bound constraint on energy storage capacity could reflect, for instance, limits on the size of the uphill reservoir in a pumped hydro system. Adding these two constraints entails a slight relaxation of the assumption of constant returns to scale.

In addition, the following inequalities must hold for all \( t \):

\[ L_t \geq 0 \quad (\eta_t), \quad R^C_t \geq 0 \quad (r^C_t), \quad g_t \geq 0 \quad (\gamma^U_t), \quad G - g_t \geq 0 \quad (\gamma^U_t). \quad (7c) \]

\[ D_t \geq 0 \quad (\delta_t^D), \quad P^D - D_t \geq 0 \quad (\delta^U_t), \quad A_t \geq 0 \quad (\alpha^D_t), \quad P^A - A_t \geq 0 \quad (\alpha^U_t). \quad (7d) \]

\[ S_t \geq 0 \quad (\varepsilon^U_t), \quad E - S_t \geq 0 \quad (\varepsilon^U_t). \quad (7e) \]

Note that in (7e), the non-negativity constraint is also enforced for \( S_0 \) with a corresponding multiplier, \( \varepsilon^U_0 \). The planner’s problem is to choose capacities \( (G, R, P^A, P^D, \text{ and } E) \) and flow variables \( (L_t, g_t, R^C_t, D_t, A_t, \text{ and } S_t \text{ for } t = 1, \ldots, T) \) to maximize \( W \) subject to constraints (2)–(7). The Lagrangian for this problem can be written as follows:

\[ \Lambda = \Lambda_L + \Lambda_R + \Lambda_G + \Lambda_S, \quad (8a) \]

where

\[ \Lambda_L = \Sigma(\lambda_t - \omega)L_t + \Sigma(\omega - \lambda_t)Q_t + \Sigma \eta_t L_t \quad (8b) \]

\[ \Lambda_R = \Sigma(\lambda_t + \beta_t)(\theta_t R - R^C_t) - C_k R + R_v R + R_v [R^U_t - R] + \Sigma r^U_t R^C_t \quad (8c) \]

\[ \Lambda_G = \Sigma(\lambda_t - \nu)g_t - C_g G + \Sigma \rho^U_t [\beta^U G + g_{t-1} - g_t] + \Sigma \rho^D_t [\beta^D G + g_t - g_{t-1}] + G_0 G + \Sigma \gamma^U_t g_t + \Sigma \gamma^D_t [G - g_t], \quad (8d) \]
\[
\Lambda_s = \sum_{t} \lambda_t (D_t - A_t) - \left[ C^A_p A^A + C^0_p P^D + C_e E + \delta A_t + \delta D_t \right] \\
+ \left[ \mu_t [\chi S_{t-1} + r A_t - D_t - S_t] + \mu_0 [S_0 - S_T] \right] \\
+ P^A_0 P^A + P^D_0 P^D + E_0 E_i [E^U - E] + \sum \delta_i^0 D_t + \sum \delta_i^U [P^D - D_t] \\
+ \sum \alpha_i^0 A_t + \sum \alpha_i^U [P^A - A_t] + \sum_{i=0}^T \epsilon_i^0 S_t + \sum \epsilon_i^U [E - S_t].
\]

(8e)

By inspection, each of expressions (8b)-(8e) is the Lagrangian for the problem of choosing associated stock and flow variables to maximize the profit of a particular technology (respectively, lost load, renewable generation, gas generation, and storage) subject to the inequality constraints relevant to that technology, treating the (shadow) price of energy, \( \lambda_t \), as exogenous. Given our assumption of constant returns to scale, it is not surprising that this is exactly the problem that would be solved by a perfectly competitive industry supplying that technology and treating energy prices as given. We have thus established

**Proposition 1: Equilibria and Optima.** Under constant returns to scale, the necessary conditions for maximizing social welfare are identical to the necessary conditions for maximizing the profits of competitive industries supplying each of the available technologies.

In what follows we consider operation of and (except for loss of load) investment in each of the available technologies in turn. We will refer to a point at which all the KKT conditions for constrained welfare maximization are satisfied as “an optimum,” understanding that such a point is also a constrained maximum of technology-specific profits under competition with the \( \lambda_t \) as energy prices. Under perfect competition and constant returns to scale, one might expect that the suppliers of each technology would just break even at an optimum. We verify this expectation below.

The KKT necessary conditions for constrained maxima include both that the derivatives of the corresponding Lagrangian with respect to each decision variable be zero and the complementary slackness conditions corresponding to the inequality constraints in (3), (6) and (7) be satisfied. These require that the products of the non-negative multipliers and the corresponding constrained quantities be zero. Thus, for instance, at the optimum \( R^0 R = 0 \), so that if \( R > 0 \), then \( R^0 = 0 \), and if \( R^0 > 0 \), then \( R = 0 \).

### 3. GENERATION AND LOAD

This brief section provides a reasonably complete presentation of general results relating to investment in and operation of renewable and dispatchable generation, as well as the conditions for loss of load.

Differentiating (8b), at an optimum we must have

\[
\frac{\partial \Lambda_L}{\partial L_t} = \lambda_t - \omega + \eta_t = 0.
\]

(9)

From (7c), if lost load is positive the shadow price on the corresponding non-negativity constraint, \( \eta_t \), must be zero. Then condition (9) implies that \( \lambda_t = \omega \), the price of energy must equal the value of lost load, establishing

15. Purely to simplify the presentation, we will generally not deal explicitly with knife-edge cases in which both the multiplier and the constrained quantity are zero.
Proposition 2: Lost Load. At an optimum, if lost load is positive in any period, the energy price equals the value of lost load, $\omega$.

Differentiating (8c), at an optimum the following first-order conditions related to VRE technologies must be satisfied:

\[
\frac{\partial \Lambda_R}{\partial C_t} = - (\lambda_t + \vartheta_t) + r_t^0 = 0, \quad t = 1, \ldots, T, \quad (10a)
\]

\[
\frac{\partial \Lambda_R}{\partial R} = \Sigma (\lambda_t + \vartheta_t) \vartheta_t - C_R + R_0 - R_u = 0. \quad (10b)
\]

Condition (10a) establishes that if curtailment is optimal in some period, so that the shadow price on the corresponding non-negativity constraint, $r_t^0$, must be zero, then $(\lambda_t + \vartheta_t)$ must also equal zero in that period. If curtailment is partial, so that constraint (3) is not binding, then $\vartheta_t = 0$, and the energy price must also be zero. If VRE output is completely curtailed, and the constraint (3) that curtailment not exceed generation is strictly binding, so $\vartheta_t > 0$, the energy price must be negative. In a razor’s-edge case, complete curtailment is also optimal when $\lambda_t = \vartheta_t = 0$.

If there is no curtailment in period $t$, $\vartheta_t = 0$, and revenue per unit of VRE capacity is just $\lambda_t \vartheta_t$. If there is curtailment, $(\lambda_t + \vartheta_t) = 0$, as above, and revenue is zero either because the energy price is zero or because VRE output is completely curtailed or (the razor’s-edge case) both. Thus the first two terms on the right of (10b) are total VRE generator profit per unit of capacity minus the per-unit capacity cost. Now note that from (7a), at most one of the multipliers $R_u$ and $R_0$ can be positive. If both are zero, so is total per-unit (supra-normal) profit, consistent with competitive investment behavior. If the lower-bound constraint on renewable capacity binds, so that $R_0 > 0$ and the socially optimal investment is zero, it follows that the derivative of profit with respect to capacity is negative at zero capacity, so that profit would be reduced below zero if capacity were increased above zero. Finally, if the upper-bound constraint binds and $R_u > 0$, profit is positive in competitive equilibrium and at a social optimum. As noted above, a binding upper-bound constraint most plausibly reflects the limited capacity of suitable sites for VRE generation, in which case the value of the multiplier on that constraint corresponds to the rental value of the corresponding sites under perfect competition.

We have thus established.

Proposition 3: VRE. At an optimum, (a) since the marginal cost of VRE supply is zero, VRE generation is curtailed only when the energy price is non-positive, (b) any VRE technology for which investment is positive earns a positive profit if and only the upper-bound constraint on capacity is binding (most plausibly reflecting site-specific rents), otherwise its profit is zero.

We now turn to dispatchable generation. In addition to complementary slackness conditions on $G$, the $g_t$, and the percentage increases in gas generation, the following first-order conditions must hold at an optimum:

\[
\frac{\partial \Lambda_G}{\partial g_t} = 0, \quad t = 1 \ldots T, \quad \text{or}
\]

\[
\lambda_t - v = (\gamma_t^u - \gamma_t^o) - (\rho_t^u - \rho_t^o), \quad t = 1, \quad (11a)
\]

\[
\lambda_t - v = (\gamma_t^u - \gamma_t^o) + (\rho_t^u - \rho_t^o) - (\rho_{t+1}^u - \rho_{t+1}^o), \quad t = 2, \ldots, T - 1,
\]

\[
\lambda_T - v = (\gamma_T^u - \gamma_T^o) + (\rho_T^u - \rho_T^o) \quad t = T.
\]

\[
\frac{\partial \Lambda_G}{\partial G} = -C_G + \Sigma_{t=2}^T \rho_t^u \beta_t^u + \Sigma_{t=2}^T \rho_t^o \beta_t^o + G_0 + \Sigma_{t=1}^T \gamma_t^u = 0. \quad (11b)
\]
Suppose initially that for some dispatchable technology both the upward and downward ramping constraints are either absent or never binding so that $\rho^u_t = \rho^d_t = 0$ for all $t$. Then condition (11a) becomes

$$\lambda_t - v = \gamma^u_t - \gamma^0_t, \quad t = 1, \ldots, T.$$  

(11a’)

Whenever this technology has positive output, $\gamma^0_t = 0$, and if generation is at capacity, $\gamma^u_t > 0$, and we have established

**Proposition 4: Operation of Dispatchable Generation Without Ramping Constraints.** At an optimum, for any dispatchable generation technology for which optimal capacity is positive, if ramping constraints are absent or never binding, then in any period (a) generation is positive only if the market price of energy is greater than or equal to marginal cost, (b) if the inequality is strict, generation is at capacity, and (c) if two dispatchable technologies have positive capacities and different marginal costs, if the one with the higher marginal cost has positive generation, so does the one with lower marginal cost.

Parts (a) and (c) describe classic merit-order dispatch, in which plants with lower marginal costs are dispatched before those with higher marginal costs.

To understand condition (11a) in the general case, note from conditions (6) that beginning in period 1, incremental dispatchable generation in period $t$ serves to relax the upward ramping constraint in period $t+1$ and tighten the downward ramping constraint in that period, but generation in period $T$ has neither effect. Suppose that only period $t$’s upward ramping constraint is binding (i.e., $\rho^u_{t+1} = 0, \rho^d_{t+1} = 0, \rho^u_t = 0, \rho^v_t > 0$), then $g_t$ must be positive, so that $\gamma^0_t = 0$, and from (11a), price is strictly greater than marginal cost. Alternatively, if only period $t$’s downward ramping constraint is binding (i.e., $\rho^u_{t+1} = 0, \rho^d_{t+1} = 0, \rho^d_t > 0, \rho^v_t = 0$), then $g_t$ must be less than capacity, so that $\gamma^v_t = 0$, and price is strictly less than marginal cost. Similarly, if only period $t+1$’s upward ramping constraint is binding (i.e., $\rho^v_{t+1} > 0, \rho^d_{t+1} = 0, \rho^d_t = 0, \rho^u_t = 0$), $g_t$ must be less than capacity and $\gamma^v_t = 0$. In this case, the marginal benefit from current generation exceeds the energy price, and generation may be positive even if marginal cost exceeds that price. Finally, if only period $t+1$’s downward ramping constraint binds (i.e., $\rho^v_{t+1} = 0, \rho^u_{t+1} > 0, \rho^d_t = 0, \rho^u_t = 0$), it must be that $g_t$ is positive and $\gamma^0_t = 0$, so price must strictly exceed marginal cost. This establishes

**Proposition 5: Operation of Dispatchable Generation with Ramping Constraints.** At an optimum, for any gas generation technology for which capacity is positive, then in any period (a) if only the current period’s upward (downward) ramping constraint is binding, the energy price is strictly greater than (less than) marginal cost, (b) if only the next period’s upward ramping constraint is binding, positive generation may be optimal even when the energy price is less than marginal cost, (c) if only the next period’s downward ramping constraint is binding, price is strictly greater than marginal cost and (d) if ramping constraints are sometimes binding, dispatch based on marginal cost may not always be optimal.

16. Biggar and Hesamzadeh (2014, Section 4.9) provide a formal discussion of operation with ramping constraints in this basic setup. They consider a constraint on the absolute increase in generation, independent of the level of capacity, and they derive versions of (a) and (b) of Proposition 5. Wang and Shahidepour (1993) analyze the effects of ramping constraints on unit commitment and economic dispatch in practice.
Part (d) follows because (b) implies that in some period a high-\( v \) technology may generate at a price below its marginal cost if only its next-period upward ramping constraint is binding.

Even though classic merit-order dispatch is not always optimal in the presence of ramping constraints, one would expect the predictions of Proposition 4 to hold most of the time. As we discuss in Section 5, one would expect a similar (though less general) set of predictions regarding the optimal dispatch of different storage technologies to hold most of the time, and we present simulation evidence using Texas data to support that expectation.

Online Appendix A contains a proof that any dispatchable generation with positive capacity at an optimum breaks even, even with ramping constraints:

**Proposition 6: Investment in Dispatchable Generation.** At an optimum, any dispatchable generation technology for which investment is positive earns zero profit.

### 4. STORAGE: GENERAL RESULTS

Proceeding as above, in addition to the complementary slackness conditions corresponding to the storage-related inequality constraints in (7), the necessary conditions for operating and investing in any storage technology to maximize welfare or for a competitive equilibrium in storage supply using that technology are

\[
\frac{\partial \Lambda}{\partial D_i} = \lambda_i - o_i^d - \mu_i + \delta_i^d - \delta_i^u = 0. \quad (12a)
\]

\[
\frac{\partial \Lambda}{\partial A_t} = -\lambda_t - o_t^A + r \mu_t + \alpha_t^A - \alpha_t^U = 0. \quad (12b)
\]

\[
\frac{\partial \Lambda}{\partial S_t} = \mu_0 + \chi \mu_t + e_0^0, \quad t = 0,
\]

\[
= -\mu_t + e_t^0 - e_t^U + \chi \mu_{t+1} = 0, \quad t = 1, \ldots, T - 1,
\]

\[
= -\mu_T + e_T^0 - e_T^U - \mu_0 = 0, \quad t = T. \quad (12c)
\]

\[
\frac{\partial \Lambda}{\partial P^A} = -C_p^A + P_0^A + \Sigma \alpha_t^U = 0. \quad (12d)
\]

\[
\frac{\partial \Lambda}{\partial P^D} = -C_p^D + P_0^D + \Sigma \delta_t^U = 0. \quad (12e)
\]

\[
\frac{\partial \Lambda}{\partial E} = -C_E + E_0 - E_u + \Sigma \epsilon_t^U = 0. \quad (12f)
\]

Conditions (12a)-(12c) show the key role of the (technology-specific, unobservable) shadow price of stored energy, \( \mu_t \), as well as the non-negativity and upper bound constraints on \( A_t, D_t, \) and \( S_t \). The multipliers \( \alpha_t^A, \delta_t^D, \) and \( e_t^0 \) correspond to the non-negativity constraints, and \( \alpha_t^U, \delta_t^U, \) and \( e_t^U \) correspond to the upper-bound constraints. Similarly, \( P^A, P^D, \) and \( E \) must satisfy non-negativity constraints, with multipliers \( P_0^A, P_0^D, \) and \( E_0, \) and \( E \) may have to satisfy an upper-bound constraint with multiplier \( E_u \).

---

17. We have not seen a zero-profit proof for dispatchable generation with ramping constraints elsewhere. Without those constraints, the zero profit result seems to have first been asserted, but not proven, in Crew and Kleindorfer (1986, Section 3.3). For recent zero-profit proofs for dispatchable generation without ramping constraints in a linear model of the sort considered here, see Biggar and Hesamzadeh (2014, chs. 9–10) for a timeless model and Brown and Reichenberg (2021) for a dynamic model with perfect foresight.

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Inspection of (12a) and (12b) and complementary slackness conditions (7d) imply that if 
storage is discharging (charging) then $\delta_t^0 = 0$ ($\alpha_t^1 = 0$) and if the charging (discharging) rate is below 
capacity, then $\delta_t^U = 0$ ($\alpha_t^U > 0$); if not, then $\delta_t^U > 0$ ($\alpha_t^U > 0$). This serves to establish

**Proposition 7: Operation of Storage.** At an optimum, in every period (a) if 
$\lambda_t \geq o^D + \mu_t$, storage is discharging, (b) it is discharging at capacity if the inequality is 
strict, (c) if $(\lambda_t + o^D)(1/r) \leq \mu_t$, storage is charging, (d) it is charging at capacity if the 
inequality is strict, and (e) otherwise, it is idle.

These are simple arbitrage conditions. Part (a) reflects the fact that energy in storage can be 
delivered to the grid at a marginal cost of $(\mu_t + o^D)$. If the value of energy is at least equal to that cost, 
discharge may be optimal. Similarly, the marginal cost per MWh used to charge storage is $(\lambda_t + o^A)$, 
and it takes $(1/r)$ MWh from the grid to increase energy in storage by one MWh.

It does not seem possible to completely rule out the optimality of charging some facilities 
using a particular storage technology while simultaneously discharging other facilities using the 
same technology, even though this model does not allow for thermal generator startup costs. If both 
$A_t$ and $D_t$ are positive, then $\delta_t^0 = \alpha_t^0 = 0$, and both $\delta_t^U$ and $\alpha_t^U$ are non-negative. Conditions (12a) and 
(12b) then imply

\[
\lambda_t = o^D + \mu_t + \delta_t^U \geq o^D + \mu_t, \tag{13a}
\]

\[
\lambda_t = -o^A + r(\mu_t - \alpha_t^U) \leq -o^A + r \mu_t. \tag{13b}
\]

Combining and re-arranging (13a) and (13b) yields a necessary condition for simultaneous charge 
and discharge to be optimal:

\[
\mu_t \leq -(o^A + o^D) / (1 - r). \tag{13c}
\]

If $\mu_t$ is positive, this condition cannot be satisfied. Condition (13a) then implies that $\lambda_t$ is also posi-
tive, and system cost cannot be reduced by increasing demand.

If $(o^A + o^D) = 0$, condition (13c) can be satisfied with $\mu_t \leq 0$. If $\mu_t = 0$, conditions (13a) and 
(13b) require $\lambda_t = 0$. If condition (13c) is satisfied with $\mu_t < 0$, which is the only way it can be satisfied 
if variable O&M cost is positive, condition (13b) requires $\lambda_t < 0$. Summarizing this discussion, we have

**Proposition 8: Simultaneous Charge and Discharge.** If for any type of storage in any 
period (a) if $\mu_t > 0$, simultaneous charge and discharge is not optimal, (b) if $(o^A + o^D) = 0$, 
simultaneous charge and discharge may be optimal if $\mu_t = \lambda_t = 0$ or if both quantities are 
negative, and (c) if $(o^A + o^D) > 0$, simultaneous charge and discharge may be optimal only 
if $\mu_t < 0$ and $\lambda_t < 0$.

It is not clear whether cases (b) and (c) are more than mathematical curiosities. If renewables are be-
ing curtailed, $\lambda_t$ will equal zero, and a binding downward-ramping constraint can in principle force 
the energy price below zero, but since energy in storage can be held until it becomes valuable, it is 
not apparent how its marginal value, $\mu_t$, could ever be non-positive.

---

18. This is a generalization of Proposition 1 in Crampes and Trochet (2019) to allow for more general storage technol-
ogies. They begin with the problem of maximizing the profit of a price-taking storage supplier and do not embed it in the 
problem of welfare maximization as we do here. In addition, they do not allow for variable O&M.
If two or more types of storage are optimally employed, the numerical analysis discussed in Section 6 has revealed that it is occasionally optimal to charge units of one type while discharging units of another type even if the conditions of Proposition 8 are not satisfied for either type.

Inspection of condition (12c) immediately establishes

**Proposition 9: Value of Stored Energy.** At an optimum for any storage technology, for \( t = 1, \ldots, T - 1 \), (a) when storage is neither full nor empty, \( \mu_{t+1} = (1/\chi)\mu_t \geq \mu_t \), (b) if storage is full \( \mu \) increases more rapidly, and (c) if storage is empty \( \mu \) decreases if \( \chi = 1 \) but may increase if \( \chi < 1 \).

Crampes and Trochet (2019) note that when \( \chi = 1 \), the behavior of the \( \mu \), when storage is neither empty nor full is consistent with Hotelling’s (1931) rule: under competition and perfect foresight, the value of a durable asset must rise at the rate of interest, which is zero here. When \( \chi < 1 \), so the physical quantity of \( S \) declines when storage is neither empty nor full, the per-unit shadow value \( \mu \) increases so keep the aggregate value of \( S \) constant.\(^{19}\)

Online Appendix A provides the proof of

**Proposition 10: Investment in Storage.**\(^{20}\) Any storage technology for which optimal capacity is positive earns a positive profit only if the upper-bound constraint on energy storage capacity is binding (again, most plausibly reflecting site-specific rents). Otherwise, profit is zero.

5. MULTIPLE STORAGE TECHNOLOGIES: GENERAL ANALYSIS

In Section 3, we considered situations in which it was optimal to have both baseload (e.g. combined-cycle gas plant) and peaker (e.g. simple cycle gas plant) gas generation capacity. In the absence of ramping constraints, it was easy to establish in this multi-period framework the classic result that peaking gas plants, which have higher variable cost, are used only when demand is particularly high and baseload capacity is fully utilized. We also showed, however that the intertemporal linkages that follow from ramping constraints add a level of operational complexity and destroy the universal validity of that classic result.

Because storage technologies with constant returns to scale are characterized in the most general case in this model by the values of seven parameters (\( C^A_p, C^D_p, C^A_e, C^D_e, o^A, o^D, \chi \)), it is not as simple to compare storage technologies as to compare constant-returns generation technologies that are completely described by their levels of per-unit fixed and variable cost. Moreover, one might expect the intertemporal linkages inherent in storage operation to invalidate any general rules as to which storage technologies would be used under what conditions. A natural, if informal, division is between short-term storage, in which intervals of charging and of discharging are close in time and long-term storage, in which energy remains in storage for longer periods before it is discharged. An example of short-term storage would be charging batteries in mid-day using excess solar generation and then discharging them as the sun goes down. In contrast, some have argued that it could be valu-

\(^{19}\) Suppose \( B \) equals one plus the positive rate of interest. Since \( \mu \), is the period-\( t \) value of stored energy, it needs to be discounted by \( B^t \) to obtain the value as of period zero. Equation (12c) then implies that when \( S \) is away from its bounds, \( \mu_{t+1} = B(1/\chi)\mu_t \). The current-period value of stored energy rises at the rate of interest when \( \chi = 1 \) (the discounted value as of period zero is constant) and more rapidly when \( \chi < 1 \).

\(^{20}\) The only prior zero-profit proof for storage of which we are aware is in Appendix B.3 of Brown and Reichenberg (2021). As noted above, they assume \( \sigma^D = o^D = 0 \).
able to have long-term storage that would enable energy provided by solar generators in the summer to be used to make up for lower solar output in the winter.

In the rest of this section we first present a simple cost analysis that suggests which sorts of storage technologies would be more suitable for short-term storage and which would be more suitable for long-term storage. We then use the KKT conditions developed above to provide additional support for this suggestion, which is further substantiated via numerical experiments in Section 6.

It is useful to begin by considering a symmetric charge-discharge cycle for a storage facility with no variable O&M cost and no self-discharge. Suppose the facility is charged for a time \( t^A \) at average power \( p \) and then discharged at the same average power for a time \( t^D \) until the original state of charge is reached. Continuing to use the “maximum electric energy recoverable” definition of capacity, the total amount of energy stored in this cycle, \( e \), is just \( pt^A \). Letting \( Z = 1/r \), a measure of round-trip inefficiency, the total amount of energy taken from the grid during the charging phase is \( eZ = pt^A \). The total length of this cycle, \( t' \), is thus given by

\[
t' = t^A + t^D = \frac{eZ}{p} e + \frac{e(1+Z)}{p} = \frac{e(1+Z)}{p}.
\]

(14)

Longer cycles involve higher ratios of energy stored to average power employed in charging and discharging. This suggests that technologies with low ratios of energy storage capacity cost to charge and discharge power capacity cost are best suited to providing long-duration storage, all else (including \( Z \)) equal.

To refine this suggestion, it is necessary to consider a specific charge/discharge cycle. As we discuss further below, if it is optimal to employ a particular storage technology, it will be optimal for that technology to be fully charged during some periods. Similarly, if it were not fully discharged during some (other) periods, costs could have been saved by reducing energy storage capacity. Let us therefore consider a facility that is initially fully discharged, then charges at power \( P^A \) until it is fully charged, then completes the cycle by discharging at power \( P^D \) until it is fully discharged.

Putting aside the cost of energy to charge the facility and the revenue from discharging and selling energy from storage, the total capital and operating cost of such a maximal cycle is given by

\[
TC = t'[c_p^A P^A + c_p^D P^D + c_E E] + o^A \Sigma A_r + o^D \Sigma D_r,
\]

(15)

where \( t' \) is the total time the cycle takes in hours, and the \( c \)’s are per-hour costs of the various capacities. To simplify formulas, let \( k = P^D/P^A \), where \( k \) is a positive constant. For some technologies \( k \) is fixed (e.g., \( k = 1 \) for electrochemical storage), while for others it is an outcome of an optimization that we suppress here. In addition, it is convenient to define \( x \equiv 1 - \chi \), the rate of self-discharge.

Let \( t^A(x) \) be the time taken to charge storage fully as a function of the self-discharge rate, let \( t^D(x) \) similarly be the time taken to discharge storage completely, so that the total time for a charge/discharge cycle, \( t'(x) \), is just the sum of \( t^A \) and \( t^D \). The total amounts of energy delivered to and taken from the grid in one cycle are just \( t^D P^D \) and \( t^A P^A \), respectively. Dividing equation (15) by total energy delivered to the grid, \( t^D P^D \), yields the average cost per MWh:

\[
AC(x) = \left[ 1 + \left( \frac{t^A}{t^D} \right) \left[ c_p + c_E E \right] + \left( \frac{t^A}{t^D} \right) \frac{o^A}{k} + o^D \right],
\]

(16a)

21. As a practical matter, storage facilities may be degraded by either being fully charged or fully discharged, so that the normal range of operation is somewhat smaller than the nameplate level of capacity would indicate.
where
\[ c_p \equiv (c_p^A / k) + c_p^D. \] (16b)

When \( x = 0 \), so there is no self-discharge, \( t^A = E/rP^A = ZkE/P^D, t^D = E/P^D \), and equation (16a) becomes
\[ AC(0) = (1 + Zk)c_p + t'c_k + (Zo^A + o^D). \] (17)

This equation implies that for values of \( x \) sufficiently close to zero, if a storage unit is continuously charged at maximum charging power capacity until it is fully charged and then discharged at maximum discharge power until it is empty, the average cost per discharged MWh over that cycle has three components. The first reflects the average capital cost of power charge and discharge capacity. Low round-trip efficiency (high \( Z \)) in effect raises power capacity cost, because each unit of power capacity is less effective at producing deliverable energy. The third component measures effective round-trip O&M cost. Low round-trip efficiency (high \( Z \)) increases round-trip O&M cost because more energy must be taken from the grid for each MWh later returned to it. The second component is the only one that depends on the total duration of the charge/discharge cycles, \( t' \). The derivative of overall cost with respect to duration here is exactly equal to the per-period energy storage capacity cost (\( c_e \)).

If self-discharge is positive, it takes longer to charge the storage fully because energy is lost during the charging process through self-discharge, and it takes less time to completely empty the storage for the same reason. It follows that more energy is taken from the grid during charging, and less energy is delivered to the grid during the discharge phase of the cycle. Online Appendix A evaluates the charge and discharge times for positive values of \( x \) and obtains an approximate value of \( AC \) for small but non-zero values of \( x \), equation (A.11). \( AC \) is increasing in \( x \) for \( x \) near zero, as one would expect, but the main implications of equation (17) are preserved.

Consider two storage technologies, 1 and 2, with technology 1 having higher ratio of energy storage capacity costs to average power capacity costs compared to technology 2. For the same flow cost parameters (\( Z, o^A, o^D \)), then the average cost per discharged MWh of technology 2 can only be equal to average cost per discharged MWh of technology 1 so long as the duration of charge/discharge cycles of technology 2 is greater than the duration of cycles for technology 1. This is a (very) rough analog to the usage implications of dispatchable generators with different levels of fixed and variable costs.

In the case of gas generation, it is a familiar result that if it is optimal to have positive capacities of two different technologies, the one with the higher variable cost must have lower fixed costs or it would have been dominated and not part of an efficient mix. In the case of storage, one expects that if it is optimal to have positive capacities of two storage technologies, the one with the lower cost of storage capacity must have higher charging/discharging costs. Equation (17) suggests that the technology with the lower energy storage capacity cost will tend to be used for longer duration storage, generally involving in effect higher values of \( t' \), than the one with the higher energy storage cost.\(^{22}\) This suggestion has implications for the focus of R&D efforts concerned with long-term storage.

Additional support for this suggestion can be derived from the KKT necessary conditions, equations (12), and the relevant complementary slackness conditions. If a particular energy storage...

---

\(^{22}\) Crampes and Trochet (2019, section 3.2) provide a less formal discussion that reaches the same general conclusion. Sepulveda et al (2021) argue that energy discharge efficiency (\( 1/r^D \)) is also an important performance parameter.
technology is deployed, then by complementary slackness conditions applied to storage related constraints in (7), \( E_0 = 0 \) and, if the upper bound on energy storage capacity constraint is not binding, \( E_u = 0 \). Condition (12f) then reduces to

\[
C_E = \sum e_i^U
\]

(18)

Additionally, by complementary slackness conditions in (7) related to storage energy capacity, \( e_i^U = 0 \) for all periods when energy in storage is below the installed storage capacity. Thus the summation on the right hand side of equation (18) can be reduced to periods when energy storage is at capacity. Letting the set of such periods be \( F \), we have

\[
C_E = \sum_{t \in F} e_i^U
\]

(19)

The right hand side of (19) can be written terms of the stored value of energy using condition (14c), with the understanding that \( e_i^0 = 0 \ \forall t \in F \). For simplicity, we assume that storage is not full at the end of the last period:

\[
C_E = \sum_{t \in F} (\chi \mu_{t+1} - \mu_t)
\]

(20)

If two storage technologies, 1 and 2, are deployed with non-zero energy storage capacities such that \( C_{E1} > C_{E2} \), equation (20) implies

\[
\sum_{t \in F} \left( \chi^1 \mu_{t+1}^1 - \mu_t^1 \right) > \sum_{t \in F} \left( \chi^2 \mu_{t+1}^2 - \mu_t^2 \right)
\]

(21)

Here the superscripts 1 and 2 correspond to technologies 1 and 2, respectively. In Online Appendix A we demonstrate that for \( r = 1 \), all of the terms in parentheses in (21) are bounded above by \((\chi \hat{\lambda}_{t+1} - \hat{\lambda}_t)\). For \( r < 1 \), the bounds depend on the charge/discharge patterns for energy storage in periods \( t \) and \( t+1 \).

The most natural way for condition (21) to be satisfied is for storage technology 1 with higher capital costs of energy storage to spend more periods fully charged than storage technology 2. This is consistent with storage technology 1 following something like the fast-cycling pattern seen for Li-ion storage in the numerical results from the optimization model discussed in Section 6.

The analysis in this section can only be suggestive. Real storage technologies generally have different round-trip efficiencies and self-discharge rates, and O&M costs may not be negligible. For arbitrary time-paths of renewable generation and load, the optimal pattern of charging and discharging will never be as regular as the cycles analyzed above. Similarly, in the analysis immediately above, there is no guarantee that the two technologies will be fully charged under comparable conditions. To shed more light on how different storage technologies are optimally employed together in practice, we turn to a numerical optimization exercise.

### 6. MULTIPLE STORAGE TECHNOLOGIES: SIMULATION

To illustrate optimal investment in and operation of a power system with multiple storage technologies, we simulated a simplified representation of a future “Texas-like” grid under greenfield

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23. If storage is full at the end of the last period, then \( C_E = \sum_{t \in F} (\chi \mu_{t+1} - \mu_t) \) based on applying condition 14c and the fact that \( e_i^0 = 0 \) (because of (4) and given that storage is full at period \( T \)). This is equivalent to \( C_E = \sum_{t \in F} (\chi \mu_{t+1} - \mu_t) \) with the condition that period \( T+1 \) is identical to period 1.
conditions and different combinations of low-carbon emissions constraints and storage technology availability scenarios. This model was developed as part of the MIT Energy Initiative’s *Future of Storage* study. As in earlier sections, the ability of storage to provide ancillary services and to enable deferral of investment in transmission and distribution systems was not modeled. We employed a capacity expansion model, GenX, to determine the optimal generation and energy storage investments needed to meet exogenous demand over time, while satisfying various grid operation constraints, resource availability limits, and other policy/environmental constraints at an hourly temporal resolution. GenX implements the optimization problem described in Section 2, while adhering to various additional technology-specific constraints, such as linearized representation of unit commitment (with startup costs) and minimum up/down time constraints of thermal generators, VRE resource availability limits and other imposed policy/environmental constraints. Notably, the model considers a high temporal resolution, in this case seven years of grid operations with hourly time steps, which allows for assessing the role for both short-duration and long-duration storage technologies. Main model features are listed in Table 1, while data sources, assumptions on capital costs and technological parameters used for generation and storage technologies are reported in Tables B.1–B.3 in Online Appendix B. The assumptions employed here for illustrative purposes may differ from those finally adopted in the *Future of Storage* study.

**Table 1: Model Main Features**

| Feature                              | Description                                                                 |
|--------------------------------------|-----------------------------------------------------------------------------|
| Available dispatchable generation    | Combined cycle gas turbine (CCGT); Combined cycle gas turbine with Carbon   |
|                                      | Capture and Storage (CCGT-CCS); open cycle gas turbine (OCGT)               |
| Variable renewables                  | Onshore Wind and utility-scale PV, with 7 resource bins per technology used  |
|                                      | to characterize different types of wind and PV sites. Each resource bin has  |
|                                      | a unique hourly capacity factor profile. Interconnection cost is added to the |
|                                      | baseline VRE capital cost, and maximum capacity in MW. See Brown and Botterud |
|                                      | (2021) for further details. Wind data are from Draxl et al (2015); PV data   |
|                                      | are from the National Solar Radiation Database (NSRDB n.d.)                 |
| Available storage technologies       | Li-ion (P^A = P^D); power to hydrogen to power (“H2”) (P^A≠P^D)             |
| Demand                               | Peak demand = 151 GW, Annual demand = 715 TWh; value of lost load = $50,000/ |
|                                      | MWh                                                                         |
| Spatial resolution                   | Single zone, no transmission constraints                                     |
| Temporal resolution                  | 2007–2013 weather years (61,314 hours)                                      |
| Carbon emission constraints          | Two constraints: 10 and 1 gCO\textsubscript{2}/ kWh                        |
| Thermal plant operating constraints  | Linearized unit commitment with ramping constraints and minimum up and down |
|                                      | time constraints                                                            |

We consider Lithium-ion batteries and power-to-hydrogen-to-power (“Li-ion” and “H\textsubscript{2}” for short) as the available storage technologies, with the estimated energy storage capacity cost much lower for H\textsubscript{2} than for Li-ion (Table B.3). We focus our numerical analysis on scenarios with stringent carbon emission intensity constraints in which storage is important: 10 grams and 1 gram of CO\textsubscript{2} per kWh. Model-optimal investment results for the two emissions constraint scenarios are summarized in Table 2.

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24. Jenkins and Sepulvada (2017). See Mallapragada et al. (2020) and the available code on the github repository (https://github.com/GenXProject/GenX) for detailed discussion of most current storage operational constraints in GenX.

25. Without a carbon emissions constraint, the least-cost model solution yields an emissions intensity of 82.9 gCO\textsubscript{2}/ kWh and a system average electricity cost of $39.5/MWh.
Table 2: Model Results

| Result                        | Technology | Emissions Constraint | 10 gCO₂/kWh | 1 gCO₂/kWh |
|-------------------------------|------------|----------------------|-------------|------------|
| Installed Power Capacity (GW)¹| CCGT       | 33.9                 | 9.8         |            |
|                               | CCGT-CCS   | 18.4                 | 23.6        |            |
|                               | OCGT       | 3.1                  | 0.0         |            |
|                               | PV         | 103.9                | 128.3       |            |
|                               | Wind       | 121.9                | 136.8       |            |
|                               | H₂ (discharge) | 5.5           | 20.6        |            |
|                               | H₂ (charge) | 3.6                  | 12.4        |            |
|                               | Li-ion     | 38.8                 | 42.8        |            |
| Total Discharge               |            | 325.5                | 361.9       |            |
| Installed Energy Storage Capacity (GWh) | H₂ | 199.5 | 1279.9 | |
|                               | Li-ion     | 130.9                | 168.8       |            |
|                               | Total      | 330.4                | 1448.8      |            |
| Average Discharged Energy (TWh/year)² | H₂ | 7.5 | 25.2 | |
|                               | Li-ion     | 28.9                 | 22.3        |            |
| Equivalent discharges / year³ | H₂         | 37.5                 | 19.7        |            |
|                               | Li-ion     | 220.5                | 132.3       |            |
| Number of periods when storage is fully charged (hours/year) | H₂ | 232.0 | 68.0 | |
|                               | Li-ion     | 2942.0               | 3911.0      |            |
| Average system electricity cost ($/MWh) | — | 45.4 | 50.7 | |
| Average load shedding per year (GWh/year) | — | 0.1 | 0.1 | |

¹In the case of hydrogen, installed power capacity has two components, one each for charging and discharging power.
²Average annual energy discharged is calculated as total energy discharged over the seven-year period divided by seven.
³Equivalent discharges per year for each storage technology is calculated as the ratio of average annual energy discharged divided by the maximum electrical energy recoverable measure of storage capacity.

Not surprisingly, increasing the stringency of carbon emissions constraints leads to increased roles for VRE generation and for storage technologies and a reduced role for thermal generation.²⁶ Notably, going from the looser to the tighter emissions constraint leads to a 5-fold increase in the optimal energy storage capacity of H₂. Overall system average electricity cost increases by 12% as the CO₂ emissions constraint is tightened from 10 gCO₂/kWh to 1 gCO₂/kWh. Table 2 highlights that overall lost load is generally small compared to total demand, owing to the relatively high value of lost load (i.e., the maximum wholesale price) assumed in the scenarios ($50,000/MWh, see Table 1).

Figure 1 illustrates how the stored energy for the two technologies changes over time for the two CO₂ emissions constraint cases during three illustrative months of operation.²⁷ Storage operation is not described by regular charge-discharge cycles. On the contrary, the pattern of operation changes over time and between emission constraints, and most charge-discharge cycles are not complete. In the 1 g CO₂/kWh case, H₂ mainly (but not exclusively) displays long-term storage behavior, while in the 10 g CO₂/kWh cases H₂ cycles more frequently. In both cases, the frequency of storage discharge varies from month to month. Li-ion, on the other hand is primarily used for shorter cycles across both CO₂ emissions constraint cases, but it cycles more frequently in some periods than in others.

As noted above, the model setup does allow for the possible optimality of simultaneous charging and discharging of an individual storage technology, thus wastefully increasing demand,
in order to avoid the startup and shutdown costs of thermal generators. In the runs reported here, however, there were no instances of such operation.

The numerical results also indicate that it may be optimal to simultaneously charge one storage technology and discharge the other, with such instances occurring 1.2% and 0.4% of the time for the 10 gCO$_2$/kWh and 1 gCO$_2$/kWh cases, respectively. In most of these instances, Li-ion is discharging and H$_2$ is charging. This behavior involves a loss of energy, but it must be that stored

28. This behavior has also been observed by Dowling et al (2020).

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energy is sufficiently more valuable on the margin in H₂ than in Li-ion to make up for the loss, perhaps because stored energy will be required for a longer period in the future than Li-ion’s limited energy capacity can handle.

In both emission intensity cases, Table 2 highlights that Li-ion spends more hours per year in a fully charged state compared to H₂, which is consistent with the discussion of equation (21) above: the technology with the higher energy capital cost spends more periods in a fully charged state. Overall, these numerical observations of the optimal use of these technologies are broadly consistent with the intuition developed in Section 5, which suggested that the lower energy capital cost storage technology generally is deployed for longer-duration charge-discharge cycles.

Since the metric of equivalent discharges/year does not fully capture and reveal the complex nature of the operation of these systems, we use frequency domain analysis of the state of charge to produce a quantitative picture of the dominant modes of storage operation. First, we applied the Fast Fourier Transform (FFT) to each state-of-charge time series. Next, we selected frequency bands of interest that contribute to different cycling periods and computed each band’s contribution to the signal root mean square (RMS) value. As noted before, the cycling of storage technologies is complex and therefore it is not only described by single frequency components. Consequently, the frequency bands analysis allows us to aggregate multiple frequency components in a simple metric that provides an instructive summary of the relative importance of different modes of storage operation. We present results using the following indicative frequency bands:

- 0 to 12 cycles/year: Long-term or seasonal cycling
- 12 to 52 cycles/year: Intra-month cycling
- 52 to 365 cycles/year: Intra-week cycles
- Above 365 cycles/year: Intra-day cycles.

Table 3 displays the results of this analysis, using the full seven years of simulated data. It indicates that whereas in the 10g CO₂/kWh case H₂ storage mainly oscillates at monthly frequencies, in the 1g CO₂/kWh case seasonal oscillations become more relevant, and the contribution from the weekly and monthly cycling bands decreases. With a very tight emissions constraint, natural gas cannot be used to provide energy for appreciable seasonal storage, and H₂ is the cheaper long-term storage technology as compared to Li-ion. Similarly, Li-ion displays a change in operation towards lower frequency cycling (less daily and weekly cycling) as the carbon constraint is tightened.

Finally, it is instructive to examine how the presence or absence of one storage technology influences the operating pattern of the other storage technology. Table 4 summarizes the model investment results for the scenario in which Li-ion is the only available energy storage technology, with all else equal. Comparing Tables 2 and 4 indicates that when H₂ is not available, VRE capacity is increased along with gas generation capacity, as well as Li-ion power and energy storage capacity, most noticeably in the tightest emissions constraint scenario (1 gCO₂/kWh). Total storage capacity is decreased substantially, however, since the relatively cheap storage provided by H₂ is not available.

29. Fourier analysis has been addressed in an extensive literature. Brigham (1988, chapters 1 and 2) describes the FFT in a succinct way and shows the ubiquitous use of the method in different fields. In the context of storage integration in power systems, Victoria et al. (2019) have previously used Fourier-spectra analysis to illustrate the different operational behavior of storage under various carbon emissions constraint scenarios.

30. Since the ratio of Li-ion energy storage capacity to power capacity is less than four hours in both cases, the importance of cycles longer than intra-day may be surprising at first glance. Storage operation does not only involve regular intra-day charge/discharge cycles, however, and the pattern of operation varies over time, as Figure 1 shows. In some periods there is frequent cycling, while in other periods storage is kept fully charged most of the time and only operates with small discharges. Variations of this sort are captured in the frequency domain analysis as low frequency components.
Generating capacity is a substitute for storage, so in the absence of H₂ storage VRE curtailments increase from 14.7% to 16.3% of generation with the 10g CO₂/kWh constraint and from 19.8% to 31.2% with the 1g CO₂/kWh constraint. The unavailability of H₂ results in a negligibly higher average electricity cost under the looser emissions constraint and a 5.2% higher average cost under the tighter constraint. Table 5 shows that when Li-ion is operating as the only storage technology, its total contribution in weekly, seasonal and monthly frequency bands are larger (87% vs. 81%) for the tightest emissions constraint 1g CO₂/kWh cases respectively. This change in operating behavior is consistent with the fact that (per Tables 2 and 4) Li-ion spends fewer periods fully charged when H₂ is not available to supply emissions from CCGT-CCS. At the 1g CO₂/kWh emissions constraint, emissions from CCGT-CCS limits its adoption and leads to greater reliance on VRE generation and storage to meet system demand.

Table 3: Relative RMS Contribution of Different Frequency Bands to State of Charge Variation

| Mode of operation | 10 gCO₂/kWh | 1 gCO₂/kWh |
|-------------------|-------------|------------|
|                   | Li-ion      | H₂         | Li-ion      | H₂         |
| Daily             | 31.4%       | 0.5%       | 18.2%       | 0.2%       |
| Weekly            | 37.8%       | 13.0%      | 32.1%       | 3.5%       |
| Monthly           | 13.6%       | 52.9%      | 15.5%       | 28.4%      |
| Seasonal          | 17.2%       | 33.6%      | 34.2%       | 67.9%      |

Table 4: Model Results – System with Li-ion as the Only Storage Technology

| Emissions Constraint | Installed Power Capacity (GW) | Installed Energy Storage Capacity (GWh) | Average Discharged Energy (TWh/year) | Equivalent discharges / year | Number of periods fully charged (hours/year) | Average system electricity cost ($/MWh) | Average Load Shedding per year (GWh/year) |
|----------------------|--------------------------------|----------------------------------------|-------------------------------------|------------------------------|--------------------------------------------|----------------------------------------|--------------------------------------------|
| 10 gCO₂/kWh          | CCGT 33.6                      | Li-ion 129.1                           | Li-ion 29.8                         | Li-ion 231.0                 | Li-ion 3110.0                              | Li-ion 45.5                             | Li-ion 0.1                                |
| 1 gCO₂/kWh           | CCGT-CCS 22.0                  | Li-ion 249.5                           | Li-ion 25.3                         | Li-ion 101.5                 | Li-ion 5397.0                              | Li-ion 53.3                             | Li-ion 0.0                                |
|                      | OCGT 5.5                       |                                        |                                     |                              |                                            |                                        |                                            |
|                      | Wind 102.9                     |                                        |                                     |                              |                                            |                                        |                                            |
|                      | Li-ion 122.0                   |                                        |                                     |                              |                                            |                                        |                                            |
|                      | Total 324.5                    |                                        |                                     |                              |                                            |                                        |                                            |

Energy storage capacity is based on the maximum electrical energy recoverable definition discussed above.

Table 5: Frequency RMS Analysis for Li-Ion—Comparison Between Scenarios

| Mode of operation of Li-ion | 10 gCO₂/kWh | 1 gCO₂/kWh |
|-----------------------------|-------------|------------|
| Daily                       | 31.4%       | 18.2%      |
| Weekly                      | 37.8%       | 32.1%      |
| Monthly                     | 13.6%       | 15.5%      |
| Seasonal                    | 17.2%       | 34.2%      |

Emissions constraint 10 gCO₂/kWh Emissions constraint 1 gCO₂/kWh

| Mode of operation | Li-ion + H₂ scenario | Li-ion only scenario |
|-------------------|----------------------|----------------------|
|                   | 10 gCO₂/kWh          | 1 gCO₂/kWh           |
| Daily             | 31.4%                 | 18.2%                 |
| Weekly            | 37.8%                 | 32.1%                 |
| Monthly           | 13.6%                 | 15.5%                 |
| Seasonal          | 17.2%                 | 34.2%                 |

31. We don’t see this trend with the 10g CO₂/kWh emissions constraint, in part due to the availability of CCGT-CCS. At the 1 gCO₂/kWh emissions constraint, emissions from CCGT-CCS limits its adoption and leads to greater reliance on VRE generation and storage to meet system demand.
longer-term storage. This comparison indicates that when a new storage technology (H₂ here) becomes economic, the efficient operating pattern of the pre-existing technology (Li-ion here) is likely to change.

7. CONCLUDING OBSERVATIONS

In the classic Boiteux (1960, 1964)-Turvey (1968) framework for describing investment and operations of electric power systems, there are no links between supply or demand conditions in different periods. In order to permit an analysis of energy storage in which energy storage capacity has positive costs, we modified that framework to allow for sequences of periods linked by the operation of storage facilities (and, possibly, ramping constraints on thermal generators), with no restrictions on period-to-period changes in demand or in the output of VRE generators.

Making the standard assumption that energy prices are allowed to rise to the value of lost load in shortage conditions,³² the classic results for generation hold in this setting. At a welfare optimum or competitive equilibrium, all thermal and renewable generation technologies employed just break even, and the classic merit-order results for thermal generation hold. (Though the latter results are modified when ramping constraints bind.)

Our analysis reveals the greater complexity of efficient investment in and operation of storage facilities. In general, even under constant returns to scale as assumed here, storage technologies are described by the values of seven cost and performance parameters. Like reservoir hydroelectric facilities, optimal energy storage discharge depends on expectations about future demand and supply conditions, encapsulated in the shadow value of stored energy. Unlike reservoir hydro facilities, charging energy storage facilities (including pumped hydro facilities) is a decision, not something determined by nature, and the choice of storage capacity is generally less constrained than the choice of reservoir capacity.

We have nonetheless proven that all storage technologies employed just break even at a social optimum. Since social optima and competitive equilibria coincide in this model, this break-even result provides some support for general reliance on markets to drive investments in energy storage. We have also shown how optimal storage operation depends on the shadow value of stored energy, though that unobservable shadow value depends on conditions in future periods. It is not possible to establish fully general results regarding investment in and operation of multiple storage technologies, however; there is no simple merit-order analog even under perfect foresight.

We have shown that if it is optimal to employ multiple storage technologies, the ones with the lowest capital cost of energy storage capacity are generally best suited to providing long-term storage.³³ But we have also shown by example that storage technologies optimally play multiple roles in grid operations, providing charge-discharge cycles of various durations. Our simulation exercises show that when multiple storage technologies are employed, frequency domain analysis can be used to characterize the relative importance of the different cycle durations that each provides and that these relative weights depend on the mix of generation and storage technologies employed.

We see three important directions for future work. First, as noted above, we have assumed that the market price of energy can rise to the value of lost load under shortage conditions, and in

³². It should be noted that this assumption, while standard, is not generally descriptive of the behavior of system operators in the U.S.

³³. If it is also optimal to employ a storage technology with a higher energy storage capacity cost, that technology must be superior on some other dimension. In our simulation exercise, Table B3 reveals that Li-Ion has lower charge and discharge power capacity costs, as well as higher round-trip efficiency, than H₂.
our simulation exercises non-served energy events do sometimes occur. In the model analyzed here the quantity $\omega$, which we have called the value of lost load, simply serves as an exogenous cap on the price of energy. If, as in many organized markets, the cap on energy prices is set below the true value of lost load, the competitive market will exhibit a “missing money” problem (Joskow 2008): the equilibrium level of reliability provided will be too low because it will reflect the price cap and not the true value of lost load. This means that non-served energy events will be more important than would be socially optimal.

In systems dominated by dispatchable generation, non-served energy events generally occur at demand peaks, and the prescription for solving the missing money problem has been to provide incentives for investment in generation to bring the nameplate capacity level to approximately that implied by the true value of lost load. Capacity mechanisms intended to implement that prescription have been controversial and have been frequently re-designed, however. The missing money problem is more complex when VRE generation is important, so that troughs in supply may be more important than peaks in demand, and the availability of VRE generation is weather-dependent. Storage poses even more difficult problems. The ability of storage to relieve system stress depends on its state of charge, which depends on prior operator decisions. It seems plausible that the second-best response to energy price caps set below the true value of lost load includes subsidies to investment in storage, but this has not been proven. Moreover, even if such subsidies are second-best optimal, they surely vary with the characteristics of storage technologies in ways that are not yet understood.

Second, our use of frequency domain analysis here to describe the optimal operation of storage systems seems to us likely to have merely scratched the surface of what that approach can contribute. While no simple merit-order result for storage operations exists, even under perfect foresight, examining how the power spectra of optimally employed alternative storage technologies respond to changes in cost parameters and system conditions may yield broadly useful insights.

Finally, there is clearly a need for computational models that can be used to optimize the operation of real storage systems under realistic stochastic processes of demand and VRE generation, with realistically imperfect foresight. Those models seem likely broadly to resemble the complex stochastic models that have been constructed for reservoir hydro systems, but, as noted above, the storage optimization problem involves deciding on both charging and discharging and is thus more complex than the reservoir hydro problem. Cruise et al (2019) and Geske and Green (2019) develop alternative approaches to the problem of optimizing storage operations; choosing optimal capacity is even more complex.

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34. See DeLadurantaye et al (2009) and the sizeable literature there cited.

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