CP violation including universal one-loop corrections and heterotic M-theory

D.BAILIN♣, G. V. KRANIOTIS♠ and A. LOVE♣

♣ Theory Division, CERN,
CH1211 Geneva 23, Switzerland.
♠ Centre for Particle Physics,
Royal Holloway and Bedford New College,
University of London, Egham,
Surrey TW20-0EX, U.K.

ABSTRACT

CP violation by soft supersymmetry-breaking terms in orbifold compactifications is investigated. We include the universal part of the moduli-dependent threshold corrections in the construction of the non-perturbative effective potential due to gaugino-condensation. This allows interpolation of the magnitude of CP violating phases between the weakly and strongly coupled regimes. We find that the universal threshold corrections have a large effect on the CP violating phases in the weakly coupled regime.
Too large CP violation by soft supersymmetry-breaking terms is a generic problem in supergravity and superstring theories. This can result in a neutron electric dipole moment much larger than the experimental upper bound. We found elsewhere \cite{1} that the modular properties of orbifold compactifications of the weakly coupled heterotic string can lead to very small or zero CP violating phases in the soft supersymmetry-breaking terms even when the moduli have large phases at the minimum of the effective potential.

Recently, many new facts about the strong coupling limit of string theory have been accumulated with the advent of M-theory \cite{2}-\cite{19}. One would like to interpolate between the weak and strong coupling limits of this unique theory and study differences (if any) in the resulting phenomenology. As was emphasized first by Nilles and Stieberger \cite{5}, the calculated gauge-group-independent universal threshold effects in the gauge kinetic function of weakly coupled orbifold theories allow such an interpolation to be made for some purposes.

As we shall see in this paper, the magnitude of CP violation due to soft supersymmetry-breaking terms varies as one goes between regions corresponding to weakly and strongly coupled regimes of the moduli space. We shall also see that the universal threshold corrections have a large effect on the CP violating phases in the weakly coupled regime.

The gauge kinetic function including universal threshold effects is given by

\[ f_a = S - \frac{1}{16\pi^2} \sum_i \frac{|G_i|}{|G|} \left[ (b_a^{N=2}), \ln(\eta(T_i))^4 - \sigma_1(T_i, U_i) \right] \]  

where \( |G_i| \) are the orders of the subgroups of the point group \( G \) which leave the \( i \)-th complex plane unrotated in the six compact dimensions. Also, for \( \text{Re}T > \text{Re}U \) \cite{3}

\[ \sigma_1(T, U) = -2 \ln[j(T) - j(U)] - 2 \sum_{(k,l)>0} c(kl) kl \ln[1 - e^{-2\pi(kT+lu)}] \]  

The notation \((k, l) > 0\) means that we sum over the orbits: (i) \( k > 0, l = 0 \), (ii) \( k = 0, l > 0 \), (iii) \( k, l > 0 \), (iv) \( k = 1, l = -1 \). The coefficients \( c(n) \) are defined by \( F(q) = \sum_{n=-1}^{\infty} c(n) q^n = \frac{E_4 E_6}{\eta^4} \), where \( E_4, E_6 \) are the Eisenstein series with modular weight 4 and 6 respectively. The term \( j(T) - j(U) \) is the denominator formula of the Monster Lie Algebra \cite{7}.

The non-perturbative superpotential due to a single gaugino condensate is taken to be \( W_{np} \sim e^{\frac{3\pi^2}{\sigma_1}} f_a \), and substituting \cite{4} gives

\[ W_{np} \sim e^{\frac{3\pi^2}{\sigma_1}} S \prod_i e^{\frac{3\pi^2}{2\sigma_1}(T_i, U_i)} \frac{|G_i|}{|G|} \prod_i \left( \frac{\eta(T_i)^{-6(b_a^{N=2})}}{\eta(t)^{(b_a^{N=2})}} \right)^{\frac{|G_i|}{|G|}} \]  

The Kähler potential, including non-perturbative corrections to the dilaton part of the Kähler potential parametrized by the function \( P(y) \), is given by

\[ K = P(y) + \dot{K} \]
where $\dot{K} = - \sum_i \ln(T_i + T_i)$. The modular invariant function $y$ includes loop-corrections due to universal threshold corrections and is given by the equation

$$y = S + \bar{S} - \Delta$$

where

$$\Delta = \sum_i \frac{\delta^{i}_{GS} \ln(T_i + \bar{T}_i)}{8\pi^2} + \frac{1}{8\pi^2} \sum_i \frac{|G_i|}{|G|} G^{(1)}(T_i, U_i, \bar{T}_i, \bar{U}_i)$$

with $\delta^{i}_{GS}$ the Green-Schwarz anomaly cancelling coefficients, and

$$G^{(1)}(T, U, \bar{T}, \bar{U}) = -\frac{4\pi (\text{Re}U)^2}{3 \text{Re}T} \Theta(\text{Re}T - \text{Re}U) - \frac{4\pi (\text{Re}T)^2}{3 \text{Re}U} \Theta(\text{Re}U - \text{Re}T) + \frac{1}{\pi \text{Re}T \text{Re}U} \text{Re}\tilde{P}(T - U) - \frac{60}{\pi^2 \text{Re}T \text{Re}U} (\zeta(3) + 4\pi \text{Re} \sum_{k>0} \tilde{P}(kT) + 4\pi \text{Re} \sum_{l>0} \tilde{P}(lU)) + \text{Re} \sum_{k,l>0} \frac{c(kl)}{\pi \text{Re}T \text{Re}U} \tilde{P}(kT + lU)$$

The function $\tilde{P}(x)$ is defined by $\tilde{P}(x) = \text{Re} x Li_2(e^{-2\pi x}) + \frac{1}{2\pi} Li_3(e^{-2\pi x})$ where $Li_j$ are the polylogarithms. The polylogarithms are given in the unit disk $|z| < 1$ by the expression

$$Li_1(z) = \sum_{n=1}^{\infty} \frac{z^n}{n}$$
$$Li_2(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^2}$$
$$Li_3(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^3}$$

The dilogarithm $Li_2$ and the trilogarithm $Li_3$ can be continued analytically outside the unit circle, i.e.

$$Li_3(e^x) = Li_3(e^{-x}) + \frac{\pi^2}{3} x - \frac{i\pi x^2}{2} - \frac{1}{6} x^3$$
$$Li_2(e^x) = -Li_2(e^{-x}) + \frac{\pi^2}{3} - i\pi x - \frac{1}{2} x^2$$

The $L$-functions obtained by constructing series of polylogarithms with Fourier coefficients the Fourier expansion coefficients of the modular form $E_4 E_6 \eta^{24}$ have highly non-trivial modular properties. For instance under $U \to 1/U, T \to T$
The prepotential satisfies the identity (4.36) and (4.37) of Harvey and Moore [21]. We have verified this numerically as a check on the accuracy of our codes. We have also verified numerically that the prepotential satisfies the identity (4.36) and (4.37) of Harvey and Moore [21].

The effective potential in the case of a single modulus $T_3$, and $U$-modulus fixed at a constant value, is given by the expression

$$e^{-G} V_{eff} = -3 + \left( \frac{d^2 P}{dy^2} \right)^{-1} \left| \frac{dP}{dy} + \frac{\partial \ln W_{np}}{\partial S} \right|^2$$

$$+ \left[ 1 + \delta G_3 \frac{dP}{dy} - \frac{1}{24\pi^2} \frac{dP}{dy} (T_3 + \bar{T}_3)^2 \right]^{-1}$$

$$\times \left| (T_3 + \bar{T}_3) \frac{\partial \ln W_{np}}{\partial T_3} - 1 + \frac{\partial \ln W_{np}}{\partial S} \left( \delta G_3 + (T_3 + \bar{T}_3) \frac{1}{24\pi^2} G^{(1)}_{ij} \right) \right|^2$$

where $e^G = |W_{np}|^2 (T_3 + \bar{T}_3)^{-1} e^{P(y)}$, $\delta G_3 = \frac{G_{ij}}{G_{ij}}$, and for a pure gauge hidden sector $\delta G_3 = \frac{1}{2} \left( 1 - 2 \frac{|G_{ij}|}{|G_{ij}|} \right)$; the suffices 3 and $\bar{3}$ on $G^{(1)}$ denote differentiation with respect to $T_3$ and $\bar{T}_3$. To ensure physical dilaton kinetic energy terms we require $P'' \equiv \frac{\partial^2 P}{\partial y^2} > 0$. We have checked that $V_{eff}$ is invariant under the modular transformations $T \rightarrow T' \equiv \frac{T}{T}$ and $T \rightarrow T + i$ for test values of $T$ and $U$ satisfying $\Re T$, $\Re T' < \Re U$. This requires the analytic continuation (3) of the polylogarithms to the region of moduli space reached by the above modular transformation. However, we were unable to obtain the modular repeats of the minima of $V_{eff}$ obtained later, with $U = e^{\pi/6}$, because of the slow rate of convergence of the $L$-functions for these values.

The soft supersymmetry-breaking $A$ terms are in general given by

$$A_{\alpha \beta \gamma} = \left( \frac{d^2 P}{dy^2} \right)^{-1} \left( \frac{dP}{dy} + \frac{24\pi^2}{b} \right) \frac{dP}{dy}$$

$$+ C_{ij}^{-1} \left( \Delta_i \frac{\partial \ln W_{np}}{\partial S} + \frac{\partial \ln W_{np}}{\partial T_i} - (T_i + \bar{T}_i)^{-1} \right)$$

$$\times \left( \frac{\partial \log h_{\alpha \beta \gamma}}{\partial T_j} - (T_j + \bar{T}_j)^{-1} \left[ 1 + n_{\alpha}^j + n_{\beta}^j + n_{\gamma}^j \right] \right)$$

where the superpotential term for the Yukawa couplings of $\phi_\alpha$, $\phi_\beta$ and $\phi_\gamma$ is $h_{\alpha \beta \gamma} \phi_\alpha \phi_\beta \phi_\gamma$, the modular weights of these states are $n_{\alpha}^j$, $n_{\beta}^j$ and $n_{\gamma}^j$, and the usual rescaling by a
factor \( \frac{W_{np}}{|W_{np}|} \) required to get from the supergravity theory derived from the orbifold compactification of the superstring theory to the spontaneously broken globally supersymmetric theory has been carried out. In the case of a single modulus \( T_3 \)

\[
C_{33} = (T_3 + \bar{T}_3)^{-2} \left[ 1 + \delta_3 \frac{dP}{dy} - \frac{1}{24\pi^2} \frac{dP}{dy} (T_3 + \bar{T}_3)^2 G_{33}^{(1)} \right]
\]

and

\[
\Delta_3 = \delta_3 (T_3 + \bar{T}_3)^{-1} + \frac{1}{8\pi^2} \frac{|G_3|}{|G|} G_{33}^{(1)}
\]

In particular, for the \( Z_6 \)' orbifold with standard embedding, the hidden sector is a pure \( E_8 \) sector, so \( b_a = -90 \); also \( \frac{|G_3|}{|G|} = \frac{1}{3} \), so \( \delta_3^{GS} = -10 \); the U modulus is fixed at \( e^{i\pi/6} \) and \( W_{np} \) is given by

\[
W_{np} \sim e^{-\frac{4\pi^2}{12} S} e^{-\frac{\sigma_1 (T_3 + \bar{T}_3)}{24\pi^2}} \eta(T_3)^{-4/3}
\]

The case without inclusion of the universal terms is obtained by setting \( \sigma_1 = G^{(1)} = 0 \). The Yukawa couplings with a non-trivial moduli dependence are given by

\[
h(T_3, k) \sim e^{-\frac{2\pi k^2 T_3}{3}} \times \left[ \theta_3(i k T_3, 2i T_3) \theta_3(i k T_3, 6i T_3) \right. \\
\left. + \theta_2(i k T_3, 2i T_3) \theta_2(i k T_3, 6i T_3) \right]
\]

where \( k = 0, \pm 1 \) is related to the fixed points associated with the three (twisted-sector) states \( \phi_\alpha, \phi_\beta \) and \( \phi_\gamma \). These couplings are invariant under the modular transformation \( T \to T + i \), and the modular weights are \( n_\alpha^3 = n_\beta^3 = n_\gamma^3 = -2/3 \). We do not consider the more model-dependent \( B \) soft supersymmetry-breaking term.

Let us start our discussion with the moduli-dominated limit, i.e. we neglect the dilaton F-term \( F_S \equiv P' + W^{-1} W_S \); (then \( P' = -\frac{24\pi^2}{b} \), and \( P'' \) is arbitrary.) In this case, minimising \( V_{eff} \) (see Figs.\[3\] and \[4\]) with respect to \( T_3 \) leads to

\[
T_3 = 1.1239 \pm 0.0830 \ i
\]

which is in the interior of the standard fundamental domain of \( PSL(2, Z) \). \( V_{eff} \) is negative at this point, and the phase of the soft supersymmetry-breaking \( A \) term is \( \phi(A) = 3.8 \times 10^{-2} \). This should be contrasted with previous results \[4\], obtained without inclusion of the universal threshold corrections, the only modular function present being the Dedekind \( \eta \) function, in which real values of \( T \) at the minimum were obtained. In the present case, omission of the universal terms gives a minimum at \( T = 1.33 \). However, the picture is consistent with previous results \[4\] in which, when the absolute modular invariant \( j(T) \) was present in \( W_{np} \), larger phases occurred when the minima of \( V_{eff} \) were at values of \( T \) in the interior of the standard fundamental domain of \( PSL(2, Z) \) than for values of \( T \) on the boundary.
For $P' = 1/3$ and $P'' = 1/4$ we also find the minimum in the interior of the fundamental domain, this time at

$$T_3 = 1.070 + 0.44986 i$$

(19)

which gives $\phi(A) = 0.05$. Again, this differs considerably from the real minimum which occurs when the universal terms are absent. If instead we consider $P'(y) = 3.4, P''(y) = 2$, the minimum of $V_{eff}$ is again in the interior of the fundamental domain, at

$$T_3 = 1.10046 + 0.23770 i$$

(20)

and the phase of the trilinear soft-terms in this case is $\phi(A) = O(10^{-6})$. Other values of the non-perturbative Kähler potential parameters produce minima at real $T$, but with significantly different values from the case when the universal terms are absent. For example, for $P' = 2.4, P'' = 4$ (see Fig. 3), we find the minimum at

$$T_3 = 1.29459 + n i$$

(21)

Without universal threshold effects the minimum is located at $T_3 = 1.383$.

In conclusion, the introduction of universal threshold corrections in the construction of the gaugino condensate superpotential $W_{np}$ and the Kähler potential $K$ leads to CP-violating phases in soft supersymmetry-breaking terms of order $10^{-4} - 10^{-2}$ in some regions of the parameter space close to the current experimental limit from the neutron dipole moment. This is the case for the particular orbifold model studied in this paper, namely the $Z'_6$ orbifold with single gaugino condensate and an $E'_8$ pure gauge hidden sector. In other regions of the parameter space CP violation is zero or negligible. Both the absolute modular invariant $j(T)$ and modular forms formed from the polylogarithms $Li_m$ were present in the effective Lagrangian besides the Dedekind eta function. This is contrast to the case without universal corrections where the standard form of $W_{np}$ contains only the Dedekind eta function. Then the CP violating phases were always zero or extremely small (of $O(10^{-15})$).

Although minima with large values of $T$ were not obtained in our calculations, if such complex minima had been obtained (corresponding to strongly coupled M-theory), it is clear that the phases in the soft supersymmetry-breaking $A$ terms would have been negligible owing to the properties of the modular functions. We studied the variation of $|\text{Im}A|$ as a function of $\text{Re}T$ for various values of $\text{Im}T$. One such case is displayed in Fig. 4 which shows that $|\text{Im}A| \to 0$ rapidly for large $\text{Re}T$.

This behaviour originates from the generalised Eisenstein functions and Jacobi theta functions $\theta_i$ involved in the soft supersymmetry-breaking $A$ term. In Fig. 5 we plot the imaginary part of the derivative of the generalised Eisenstein function, $Eisen \equiv -\frac{2}{3}\hat{G}(T, \bar{T}) - \frac{1}{9g_0}\partial_T(G^{(i)} + \frac{1}{2}\sigma_1)$, as a function of $\text{Re}T$; (As usual, $\hat{G} \equiv (T + \bar{T})^{-1} + 2\partial_T\text{Im}\eta.$) Like $\hat{G}$, we again see that $\text{Im}Eisen \to 0$ rapidly as $\text{Re}T \to \infty$. The Jacobi theta functions have a similar behaviour as $\text{Re}T \to \infty$.

Thus the following physical picture seems to emerge. At weak coupling, we find minima in the interior of the fundamental domain, and the theory breaks CP.
As we go to strong-coupling, i.e. large \( T \)-minimum, CP violation becomes negligible owing to properties of the modular functions.

There remains the following question. Is it possible to obtain strong-coupling \( M \)-theory minima in modular invariant theories? With a single gaugino condensate, the answer is negative. In Fig. 5 we plot the potential \( V_{\text{eff}} \) (for \( F_S = 0 \)) as a function of \( \text{Re}T \) and \( \text{Im}T \), and we observe that the potential does not have a minimum for large values of \( \text{Re}T \). It might be possible to obtain an \( M \)-theory minimum in modular theories using more than one gaugino condensate and/or using five-branes in the effective action. However, in the case of five-branes it is unclear how modular invariance can be incorporated. On the other hand, even if one manages to obtain a minimum at large \( \text{Re}T \), it seems that CP violation in the soft supersymmetry-breaking terms will be negligible, in the strong coupling regime, due to properties of the Eisenstein and Jacobi modular functions.

**Acknowledgements**

This research is supported in part by PPARC. DB and GVK thank the Theory Division at CERN for hospitality received during the completion of this work.

**References**

[1] D. Bailin, G.V. Kraniotis and A. Love, Phys. Lett. B 414 (1997) 269; Nucl. Phys.B 518 (1998) 92; Phys.Lett. B 432 (1998) 343; Phys. Lett. B435 (1998) 323

[2] P. Hořava and E. Witten, Nucl. Phys. B460(1996)506; Nucl. Phys. B475 (1996) 94

[3] E. Witten, Nucl. Phys. B471 (1996) 135

[4] T. Banks and M. Dine, Nucl. Phys. B505 (1997) 445

[5] H. P. Nilles and S. Stieberger, Nucl.Phys. B 499 (1997) 3

[6] E. Kiritsis, C. Kounnas, P.M. Petropoulos and J. Rizos, Nucl. Phys. B 483 (1997) 141

[7] R. E. Borcherds, Invent. Math. 109 (1992) 405

[8] I. Antoniadis and M. Quirós, Phys. Lett. B392 (1997) 61; Nucl. Phys. B 505 (1997) 109

[9] T. Li, J.L. Lopez and D.V. Nanopoulos, Phys. Rev. D 56 (1997) 2602; Mod. Phys. Lett. A 12 (1997) 2647; D.V. Nanopoulos, [hep-th/9711080](http://arxiv.org/abs/hep-th/9711080)

[10] T. Li, Phys. Rev. D 57 (1998) 7539
[11] H.P. Nilles, M. Olechowski and M. Yamaguchi, Phys. Lett. B 415 (1997) 24; Nucl. Phys. B 530 (1998) 43

[12] E. Dudas and C. Grojean, Nucl. Phys. B 507 (1997) 553; E. Dudas, Phys. Lett. B 416 (1998) 309

[13] Z. Lalak and S. Thomas, Nucl. Phys. B 515 (1998) 55

[14] K. Choi, H.B. Kim and C. Muñoz, Phys. Rev. D 57 (1998) 7521

[15] A. Lukas, B.A. Ovrut and D. Waldram, Phys. Rev. D 57 (1998) 7529; Nucl. Phys. B 532 (1998) 43

[16] A. Lukas, B.A. Ovrut and D. Waldram, Phys. Rev. D 59 (1999) 106005; JHEP 9904 (1999) 009; R. Donagi, A. Lukas, B.A. Ovrut and D. Waldram, JHEP 9906 (1999) 034

[17] D. Bailin, G.V. Kraniotis and A. Love, Phys. Lett. B 432 (1998) 90; Nucl. Phys. B 556 (1999) 23

[18] T. Kobayashi, J. Kubo and H. Shimabukuro, HIP-1999-14/TH, hep-ph/9904201

[19] D.C. Cerdeño and C. Muñoz, Phys. Rev. D 61 (2000) 016001

[20] L. Lewin, Polylogarithms and Associated functions, North Holland

[21] J. A. Harvey and G. Moore, Nucl. Phys. B 463 (1996) 315

7
Figure 1: Moduli potential in the limit $F_S = 0$
Figure 2: Moduli potential in the limit $F_S = 0$ for large $ReT$
Figure 3: Moduli potential for $P' = 2.4$ and $P'' = 4$. 
Figure 4: Imaginary part of $A$ soft term as a function of Re$T$ for Im$T=0.1$. 
Figure 5: Imaginary part of $Im\,Eisen$ as a function of $Re\,T$ for $Im\,T=0.3$. 