Robust Physical Layer Security for Power Domain Non-orthogonal Multiple Access-Based HetNets and HUDNs: SIC Avoidance at Eavesdroppers

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Abstract—In this paper, we investigate the physical layer security in downlink of Power Domain Non-Orthogonal Multiple Access (PD-NOMA)-based heterogeneous cellular network (HetNet) in presence of multiple eavesdroppers. Our aim is to maximize the sum secrecy rate of the network. To this end, we formulate joint subcarrier and power allocation optimization problems to increase sum secrecy rate. Moreover, we propose a novel scheme at which the eavesdroppers are prevented from doing Successive Interference Cancellation (SIC), while legitimate users are able to do it. In practical systems, availability of eavesdroppers’ Channel State Information (CSI) is impractical, hence we consider two scenarios: 1) Perfect CSI of the eavesdroppers, 2) imperfect CSI of the eavesdroppers. Since the proposed optimization problems are non-convex, we adopt the well-known iterative algorithm called Alternative Search Method (ASM). In this algorithm, the optimization problems are converted to two subproblems, power allocation and subcarrier allocation. We solve the power allocation problem by the Successive Convex Approximation approach and solve the subcarrier allocation subproblem, by exploiting the Mesh Adaptive Direct Search algorithm (MADS). Moreover, in order to study the optimality gap of the proposed solution method, we apply the monotonic optimization method. Moreover, we evaluate the proposed scheme for secure massive connectivity in 5G networks. Numerical results highlight that the proposed scheme significantly improves the sum secrecy rate compared with the conventional case at which the eavesdroppers are able to apply SIC.

Index Terms—physical layer security, PD-NOMA, Resource allocation, monotonic optimization.

I. INTRODUCTION

A. State of the art and motivation

The increasing demand of high data rates and multimedia applications and scarcity of radio resources encourage operators, research centers, and vendors to devise new methods and products for providing high data rate services for the next-generation 5G network. International Telecommunication Union (ITU) has categorized 5G services into three categories: 1) Ultra-reliable and low latency communication (URLLC), 2) enhanced mobile broadband (eMBB), and 3) massive machine-type communication (mMTC) [1]. In mMTC, massive number of machine-type devices are connected simultaneously. Services like sensing, monitoring, and tagging which are in this category have two main challenges: 1) Scarcity of radio resources which make the deployment massive connections very difficult and 2) broadcast nature of wireless channels which make the massive connections insecure. Massive connectivity is one the main features of future wireless cellular networks which is suitable for IoT [2] and machine-type communications (MTC) services. In massive device connectivity scenarios, a cellular BS connects to large number of devices (in order of $10^4$ to $10^6$ per km$^2$). To achieve high throughput and spectrum efficiency, Heterogeneous Ultra Dense Networks (HUDNs) is a promising solution at which the number of BSs per km$^2$ is very large (about 40-50) [3]. In order to overcome the scarcity of radio resources in this category, a new multiple access method called power domain Non-Orthogonal Multiple Access (PD-NOMA) can be adopted at which users are serviced within a given resource slot (e.g., time/frequency) at different levels of transmit power [4], [5]. In this method, users can remove signals intended for other users which have the worse channel conditions, by employing successive interference cancellation (SIC) [6]–[8]. It is necessary to mention that SIC concept was first proposed by Cover in 1972 [7] which is very useful technique, because it imposes lower complexity than joint decoding techniques [8]. It is worth noting that the PD-NOMA technique has attracted significant attentions in both academia and industry, [9]–[12]. It is necessary to mention that it has been confirmed in theory domain [13] and system-level simulations [14] which PD-NOMA surpasses orthogonal frequency-division multiple access (OFDMA) in different points of view such as device connections and spectrum efficiency. Based on these benefits, PD-NOMA is very appropriate to be employed for meeting the 5G requirements such as massive connectivity [15] which is very vital for mMTC and Internet of Things (IoT). Besides, establishment of security in these networks is a dilemma, because wireless transmission has broadcast nature. Therefore, private information that is exchanged between transmitter and receiver is vulnerable of eavesdropping. During the past years, physical layer security as a promising idea, has been widely investigated since Wyner presented his work in the security domain [16]. Furthermore, as IoT is employed in wide domains such as commercial, military, and governmental application, security plays an important role in IoT appli-
ations [17]. Due to constraints of energy consumption and limited hard-ware in IoT devices, it is very vulnerable with respect to eavesdropping. Physical layer security owing to low computational complexity attracts a lot of attentions and is becoming a suitable solution for secure communications in IoT [18].

B. Related works

In recent years, PD-NOMA has been studied from various perspectives, such as investigating optimal and fair energy efficient resource allocation in downlink of Heterogeneous Network (HetNet) for energy harvesting [10], outage performance analysis [11], and beamforming based 5G millimeter-wave communications [12]. Moreover, power allocation ensuring fairness among users for instantaneous and average CSI, is investigated in [19]. In [20], PD-NOMA based multiple antennas technology is proposed to improve throughput. Multiple input single-output (MISO) in PD-NOMA-based network is investigated in [21]. Additionally, in [3], the authors study usage of PD-NOMA in HUDNs to backup massive connectivity in 5G networks. Cost of active user detection and channel estimation in massive connectivity by employing massive MIMO is evaluated in [22]. The authors in [23], propose an inter-cell interference coordination mechanism in dense small cell networks. Millimeter-Wave PD-NOMA in machine-to-machine (M2M) communications for IoT networks is proposed in [24]. The authors in [25] study dynamic user scheduling and power allocation problem for massive IoT devices based PD-NOMA.

Recently, physical layer security in single-input single-output (SISO) systems based on the PD-NOMA technology is investigated in [27], at which its objective is maximizing the sum secrecy rate. Physical layer security for PD-NOMA-based cognitive radio networks is studied in [28]. The authors' aim is to derive exact and asymptotic expressions of the outage secrecy rate. Comparison secrecy uncasting rate between PD-NOMA and OMA is investigated in [29]. The authors study the achievable secrecy uncasting rate of OMA and PD-NOMA. Moreover, in [30], the authors study physical layer security of PD-NOMA in large-scale networks with employing stochastic geometry, in which a new exact expression of the outage secrecy rate is derived for single-antenna and multiple-antenna cases. In the literature, it is assumed that eavesdroppers know the channel ordering and are able to perform SIC. It is worth noting that eavesdroppers by doing SIC are able to decrease the sum secrecy rate. Hence, to tackle this issue, we propose a novel scheme such that, we do not allow eavesdroppers to be able to perform SIC, even if they know the channel ordering. In this regard, we formulate an optimization problem at which we introduce a new constraint called SIC avoidance at eavesdropper condition and the main aim is to maximize the sum secrecy rate over transmit power and subcarrier allocation variables.

For solving the proposed optimization problems, we adopt the well-known iterative algorithm called Alternative Search Method (ASM) [33]. In this method, the optimization problems are converted to two subproblems which one of them has binary and another has continuous optimization variables, in other words, power and subcarriers are allocated separately. In each iteration of this method, power allocation problem is non-convex and subcarrier allocation is Non-Linear Programing (NLP). We solve the power allocation problem by the Successive Convex Approximation (SCA) approach. To this end, we use Difference of two Concave functions (DC) approximation, to transform the non-convex problem into a canonical form of convex optimization [34]. Also we solve subcarrier allocation problem, by exploiting the Mesh Adaptive Direct Search algorithm (MADS), to this end, we employ Nonlinear Optimization solver with MADS (NOMAD). [35].

In practical systems availability of all eavesdroppers’ channel conditions at legitimate transmitters are impractical, hence, we investigate two scenarios: 1) Perfect Channel State Information (CSI) of eavesdropper, 2) imperfect CSI of eavesdropper, where BSs do not have perfect CSI of eavesdroppers.

We also apply the monotonic optimization method to study the optimality gap of the proposed solution method [36]. For this purpose, we convert the optimization problems to the canonical form of monotonic optimization problem and finally, by exploiting the polyblock algorithm, we solve the monotonic optimization problem, globally.

We also evaluate the proposed scheme for secure massive connectivity in 5G ultra dense networks. Without loss of generality, for changing our scenario from HetNet to HUDN, we need to extend the dimension of system model. According to [3] and [23], in order to tackle high dimension complexity of resource allocation in HUDNs, it is assumed that the transmit power is uniformly allocated to users. Moreover, we show the performance of uniform power allocation is close to the performance of our proposed solution.

C. Contribution

In the following, we summarize the main contributions of this paper as:

- We consider physical layer security for the downlink of PD-NOMA-based HetNets and HUDNs consisting of one Macro Base Station (MBS), multiple Small Base Stations (SBS), multiple MBS, SBS users, and multiple eavesdroppers.
- We focus on this aspect of PD-NOMA, “how to avoid eavesdroppers from doing SIC and when users are able to perform SIC?”. From the information theory point of view, user A can perform SIC whenever its received Signal-to-Interference-plus-Noise Ratio (SINR) at user B is more than user B’s received SINR at its own signal, [31], [32]. We propose a novel optimization problem such that, we do not allow eavesdroppers to be able to perform SIC, even if they know the channel ordering. In this regard, we formulate an optimization problem at which we introduce a new constraint called SIC avoidance at eavesdropper condition and the main aim is to maximize the sum secrecy rate over transmit power and subcarrier allocation variables.
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- For solving the proposed optimization problems, we adopt the well-known iterative algorithm called Alternative Search Method (ASM) [33]. In this method, the optimization problems are converted to two subproblems which one of them has binary and another has continuous optimization variables, in other words, power and subcarriers are allocated separately. In each iteration of this method, power allocation problem is non-convex and subcarrier allocation is Non-Linear Programing (NLP). We solve the power allocation problem by the Successive Convex Approximation (SCA) approach. To this end, we use Difference of two Concave functions (DC) approximation, to transform the non-convex problem into a canonical form of convex optimization [34]. Also we solve subcarrier allocation problem, by exploiting the Mesh Adaptive Direct Search algorithm (MADS), to this end, we employ Nonlinear Optimization solver with MADS (NOMAD). [35].

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- We also apply the monotonic optimization method to study the optimality gap of the proposed solution method [36]. For this purpose, we convert the optimization problems to the canonical form of monotonic optimization problem and finally, by exploiting the polyblock algorithm, we solve the monotonic optimization problem, globally.
- We also evaluate the proposed scheme for secure massive connectivity in 5G ultra dense networks. Without loss of generality, for changing our scenario from HetNet to HUDN, we need to extend the dimension of system model. According to [3] and [23], in order to tackle high dimension complexity of resource allocation in HUDNs, it is assumed that the transmit power is uniformly allocated to users. Moreover, we show the performance of uniform power allocation is close to the performance of our proposed solution.
D. Organization

The remainder of this paper is organized as follows. In Section II, we present system and signal model, respectively, and explain our novel idea. Section III provides the detailed problem formulation at two scenarios: 1) Perfect CSI, 2) imperfect CSI. In Section IV, the proposed solution is expressed. The proposed scheme for ultra dense network is evaluated in Section V. Our proposed optimal solution is provided in Section VI. Performance evaluation of the proposed resource allocation approach is discussed in Section VII before concluding the paper in Section VIII.

TABLE I

| variables | definition |
|-----------|------------|
| $h_{m,n}$ | Channel coefficient from BS $f$ to the $m^{th}$ user on subcarrier $n$ |
| $h_{e,n}$ | Channel coefficient from BS $f$ to the $e^{th}$ eavesdropper on subcarrier $n$ |
| $p_{m,n}$ | Transmit power of BS $f$ to user $m$ on subcarrier $n$ |
| $\rho_{m,n}$ | Subcarrier allocation to user $m$ on subcarrier $n$ in BS $f$ |
| $\mathcal{F}$ | Set of BSs, $\mathcal{F} = \{1,2,...,F\}$ |
| $N$ | Total number of BSs |
| $\mathcal{M}_f$ | Set of all users in BS $f$, $\mathcal{M}_f = \{1,2,...,M_f\}$ |
| $M$ | Total number of users, $M = \sum_{f \in \mathcal{F}} M_f$ |
| $\mathcal{E}$ | Set of all eavesdroppers, $\mathcal{E} = \{1,2,...,E\}$ |
| $E$ | Total number of eavesdroppers. |

II. SYSTEM AND SIGNAL MODEL

A. System model

In this paper, we focus on secure communication in the downlink of PD-NOMA based HetNet. As illustrated in Fig. 1, our system model consists of one MBS, multiple SBSs, multiple eavesdroppers, multiple MBS, and SBS users. In this system model, we assume that all nodes are equipped with single antenna. For clarity, the main underutilized variables in this paper are listed in Table I. Note that $f = 1$ refers to MBS. When BS $f$ allocates subcarrier $n$ to user $m$, the binary variable $\rho_{m,n,1} \in \{0,1\}$ is equal to one, i.e., $\rho_{m,n,1} = 1$, and otherwise, $\rho_{m,n,0} = 0$. $h_{m,n} = d_{m,n}^{-\alpha} h_{m,n}^f$ is the channel coefficient between BS $f$ and user $m$ on subcarrier $n$, where $h_{m,n}^f$ indicates the Rayleigh fading, $\alpha$ and $d_{m,f}$ are the path loss exponent and the distance between user $m$ and BS $f$, respectively. Moreover, $h_{e,n}^f = d_{e,f}^{-\alpha} h_{e,n}^f$.

B. Signal model

Employing the PD-NOMA technique, a linear combination of $M_f$ signals is diffused by BS $f$ to its users. In other words, BS $f$ transmits $\sum_{j=1}^{M_f} \rho_{j,n}^f p_j f_n^s_j + Z_{m,n}^f + Z_{e,n}^f$ on subcarrier $n$, where $s_{j,n}^f$ denotes the transmitted symbol of the $j^{th}$ user on the $n^{th}$ subcarrier by BS $f$. Without loss of generality, it is assumed $E \left\{ |s_{j,n}^f|^2 \right\} = 1, \forall m \in \mathcal{M}_f, f \in \mathcal{F}, n \in \mathcal{N}$, where $\mathcal{M}_f$, $\mathcal{N}$, and $\mathcal{F}$ are denoted set of all users in BS, set of total subcarriers, and set of BSs, respectively. Moreover, $E \{ x \}$ is the expectation of $x$. The received signals on the $m^{th}$ user and the $e^{th}$ adversary, that are located in the coverage region of BS $f$, on the $n^{th}$ subcarrier are expressed as

$$y_{m,n}^f = h_{m,n}^f \sum_{i \in \mathcal{M}_f} \rho_{i,n}^f p_i f_n^s_i + Z_{m,n}^f$$

and

$$y_{e,n}^f = h_{e,n}^f \sum_{i \in \mathcal{E}} \rho_{i,n}^f p_i f_n^s_i + Z_{e,n}^f$$

respectively, where $Z_{m,n}^f \sim \mathcal{CN}(0,\sigma^2)$ and $Z_{e,n}^f \sim \mathcal{CN}(0,\sigma^2)$ are the complex Additive White Gaussian Noise (AWGN) with zero-mean and variance $\sigma^2$, on subcarrier $n$, over BS $f$, at user $m$ and eavesdropper $e$, respectively.

C. Achievable Rates at the Legitimate Users and the Eavesdroppers

In order to decode signals, in the PD-NOMA-based system, users apply SIC. In these systems, user $m$, firstly detects the $i^{th}$ user’s message when $|h_{m,n}^i|^2 > |h_{m,n}^e|^2$, then removes the detected message from the received signal, in a consecutive way. Note that the $i^{th}$ user’s message for user $m$ behaves as noise if $|h_{m,n}^i|^2 \leq |h_{m,n}^e|^2$. When user $m$ applies SIC, its SINR in BS $f$ on subcarrier $n$ can be expressed as:

$$\gamma_{m,n}^f = \frac{p_{m,n}^f |h_{m,n}^f|^2}{|h_{m,n}^f|^2 + \sum_{i \in \mathcal{M}_f / \{m\}} p_i^f |h_{m,n}^i|^2 + I_{m,n}^f + \sigma^2}.$$
where $I_{m,n}^f = \sum_{f' \in \mathcal{F}/f} |h_{m,n}^{f'}|^2 \sum_{i \in \mathcal{M}_f} \rho_{i,n}^f p_{i,n}^f$ and its achievable rate is given by:

$$r_{m,n}^f = \log(1 + \gamma_{m,n}^f).$$ (4)

In the PD-NOMA-based system, users are able to perform SIC, if the following conditions hold [38], [39]:

- SIC can be applied by user $m$ if user $i$'s received SINR for its signal is less than or equal to user $m$'s received SINR for user $i$'s signal [31], [32].
- $|h_{m,n}^{f_i}|^2 \leq |h_{m,n}^{f_i}|^2, \forall i, m \in \mathcal{M}_f, f \in \mathcal{F}, n \in \mathcal{N}, i \neq m$.

In other words, user $m$ can successfully decode and remove the $i^{th}$ user's signal on subcarrier $n$ in BS $f$, whenever the following inequality is satisfied, [42]:

$$\gamma_{m,n}^f (i) \geq \gamma_{i,n}^f (i) \forall i, m \in \mathcal{M}_f, f \in \mathcal{F}, n \in \mathcal{N},$$

where $\gamma_{m,n}^f (i)$ is user $m$'s SINR for user $i$'s signal and $\gamma_{i,n}^f (i)$ is user $i$'s SINR for its own signal. Accordingly, (5) can be written as follows:

$$\log_2 \left( 1 + \frac{p_{i,n}^f |h_{i,n}^f|^2}{|h_{m,n}^f|^2 \sum_{l \in \mathcal{M}_f \setminus \{i\}} |h_{l,n}^f|^2 \rho_{l,n}^f p_{l,n}^f + I_{m,n}^f + \sigma^2} \right) \geq 0,$$

for simplicity, we can rewrite (6) to the following inequality:

$$Q_{m,i,n}^f (\rho, p) \triangleq -|h_{m,n}^f|^2 \sigma^2 + |h_{i,n}^f|^2 \sigma^2 + |h_{i,n}^f|^2 I_{m,n}^f - |h_{m,n}^f|^2 I_{i,n}^f - |h_{m,n}^f|^2 |h_{i,n}^f|^2 \sum_{l \in \mathcal{M}_f \setminus \{i\}} |h_{l,n}^f|^2 \rho_{l,n}^f p_{l,n}^f \leq 0,$$

$$|h_{i,n}^f|^2 |h_{i,n}^f|^2 \sum_{l \in \mathcal{M}_f \setminus \{i\}} |h_{l,n}^f|^2 \rho_{l,n}^f p_{l,n}^f \leq 0.$$ (7)

We propose a novel resource allocation algorithm in which eavesdropper is not able to employ SIC to increase its own achievable rate. In this case, eavesdropper $c$ can not apply SIC, hence, all users' messages are treated as interference in the $e^{th}$ eavesdropper. Therefore, SINR of the eavesdropper $e$ in BS $f$ on subcarrier $n$ can be obtained as:

$$\gamma_{e,c}^f = \frac{p_{e,n}^f |h_{e,n}^f|^2}{|h_{e,n}^f|^2 \sum_{i \in \mathcal{M}_f \setminus \{m\}} |h_{i,n}^f|^2 \rho_{i,n}^f p_{i,n}^f + I_{e,n}^f + \sigma^2}.$$ (8)

For SIC avoidance at the eavesdroppers, the following inequality must be satisfied:

$$\gamma_{e,c}^f (i) \leq \gamma_{e,c}^f (i), \forall i, m \in \mathcal{M}_f, e \in \mathcal{E}, f \in \mathcal{F}, n \in \mathcal{N},$$

where $\gamma_{e,c}^f (i)$ is user $e$'s signal on subcarrier $n$ in BS $f$, whenever the following inequality is satisfied:

$$\log_2 \left( 1 + \frac{p_{i,n}^f |h_{i,n}^f|^2}{|h_{e,n}^f|^2 \sum_{l \in \mathcal{M}_f \setminus \{i\}} |h_{l,n}^f|^2 \rho_{l,n}^f p_{l,n}^f + I_{e,n}^f + \sigma^2} \right) \leq 0,$$

by some mathematical manipulation, (11) is equivalent to the following inequality:

$$\Psi_{m,i,n,c}^f (\rho, p) = -|h_{e,n}^f|^2 \sigma^2 + |h_{i,n}^f|^2 \sigma^2 - |h_{i,n}^f|^2 I_{e,n}^f + |h_{i,n}^f|^2 I_{e,n}^f - |h_{m,n}^f|^2 |h_{i,n}^f|^2 \sum_{l \in \mathcal{M}_f \setminus \{i\}} |h_{l,n}^f|^2 \rho_{l,n}^f p_{l,n}^f \geq 0.$$ (12)

In the following, we assume that the eavesdroppers are non-colluding, hence we have

$$r_{e,m,n}^f = \max_{e \in \mathcal{E}} \{ \log(1 + \gamma_{e,m}^f) \},$$

therefore, the secrecy rate at the $m^{th}$ user served by BS $f$ on subcarrier $n$ can be obtained as follows:

$$P_{m,n}^f = \left[ r_{m,n}^f - r_{e,m,n}^f \right]^+, \quad \text{where } [x]^+ = \max\{x, 0\}.$$ (14)

### III. Problem Formulation

In this section, we propose a new policy for resource allocation to maximize the sum secrecy rate. It should be noted, in practical systems, having knowledge of all eavesdroppers’ CSI is impractical, hence we investigate two scenarios:

1) Perfect CSI of the eavesdroppers, 2) imperfect CSI of the eavesdroppers, where BSs do not have perfect CSI of eavesdroppers. We investigate these two scenarios in two Subsections III-A and III-B respectively.
A. Perfect CSI Scenario

In this subsection, we assume the CSI of eavesdroppers are available in the BSs, which is a common assumption in the physical layer security literature. To this end, we propose a policy for resource allocation to maximize the sum secrecy rate. In this policy, unlike the users, the eavesdroppers cannot apply SIC. We formulate the considered optimization problem of sum secrecy rate maximization via the worst-case robust approach as follows:

\[
\begin{align*}
\max_{\mathbf{P}, \mathbf{\rho}} & \sum_{m \in \mathcal{M}_f} \sum_{n \in \mathcal{N}} \sum_{f \in \mathcal{F}} \rho_{m,n}^f \{ r_{m,n}^f - r_{m,n}^{\text{max}} \}, \\
\text{s.t.} & : C_1 : \sum_{m \in \mathcal{M}_f} \sum_{n \in \mathcal{N}} \rho_{m,n}^f \rho_{m,n}^{f,m} \leq \rho_{\text{max}}^f \forall f \in \mathcal{F}, \\
& C_2 : \sum_{m \in \mathcal{M}_f} \rho_{m,n}^f \leq \ell, \forall n \in \mathcal{N}, f \in \mathcal{F}, \\
& C_3 : \rho_{m,n}^f \in \{0,1\}, \forall m \in \mathcal{M}_f, n \in \mathcal{N}, f \in \mathcal{F}, \\
& C_4 : \rho_{m,n}^f \geq 0, \forall m \in \mathcal{M}_f, n \in \mathcal{N}, f \in \mathcal{F}, \\
& C_5 : \rho_{m,n}^f \rho_{i,n}^f Q_{m,i,n}(\mathbf{p}) \leq 0, \forall f \in \mathcal{F}, \\
& n \in \mathcal{N}, m, i \in \mathcal{M}_f, |h_{m,n}^f|^2 \leq |h_{e,n}^f|^2, i \not= m, \\
& C_6 : \rho_{m,n}^f \rho_{e,n}^f |\psi_{m,i,n}(\mathbf{p},\mathbf{\rho})| \geq 0, \forall f \in \mathcal{F}, n \in \mathcal{N}, \\
& m, i \in \mathcal{M}_f, e \in \mathcal{E}, |h_{e,n}^f|^2 \leq |h_{e,n}^f|^2, i \not= m, \\
& C_7 : |v_{h_{e,n}}^f|^2 \leq \epsilon, \forall f \in \mathcal{F}, n \in \mathcal{N}, e \in \mathcal{E}.
\end{align*}
\]

The optimization variables \( \mathbf{P} \) and \( \mathbf{\rho} \) are defined as \( \mathbf{P} = [p_{m,n}^f] \) and \( \mathbf{\rho} = [\rho_{m,n}^f] \) \( \forall m \in \mathcal{M}_f, n \in \mathcal{N}, f \in \mathcal{F} \), moreover \( \rho_{\text{max}}^f \) is the maximum allowable transmit power at BS \( f \). Constraint \( C_1 \) demonstrates the maximum allowable transmit power of BS \( f \). In order to guarantee each subcarrier can be allocated to at most \( \ell \) users, constraint \( C_2 \) is imposed. Constraint \( C_4 \) denotes that the transmit power is non-negative. Constraint \( C_5 \) guarantees user \( m \) can perform SIC successfully on users that \( |h_{m,n}^f|^2 \leq |h_{e,n}^f|^2 \). Constraint \( C_6 \) assures that eavesdropper \( e \) is not able to perform SIC, and other users’ signals are treated as interference.

B. Imperfect CSI Scenario

In this subsection, we assume imperfect CSI of eavesdroppers is available in the BSs. In particular, the BSs have the knowledge of an estimated version of channel i.e., \( h_{e,n}^f \), and the channel error is defined as \( e_{h_{e,n}}^f = h_{e,n}^f - h_{e,n} \). We assume that the channel mismatch lies in the bounded set, i.e., \( \mathbb{E}_{h_{e}} = \{ e_{h_{e,n}}^f : |e_{h_{e,n}}^f|^2 \leq \epsilon \} \) \( \forall m \in \mathcal{M}_f, n \in \mathcal{N}, f \in \mathcal{F} \), where \( \epsilon \) is a known constant. Therefore, we model the channel coefficient from BS \( f \) to the \( e^{th} \) eavesdropper on subcarrier \( n \) as follows:

\[
|h_{e,n}^f|^2 = |h_{e,n}^f| + |e_{h_{e,n}}^f|^2.
\]

We focus on optimizing the worst-case performance, where we maximize the worst case sum secrecy rate for the worst channel mismatch \( e_{h_{e,n}}^f \) in the bounded set \( \mathbb{E}_{h_{e}} \). Hence, the imperfect CSI optimization problem can be formulated as follows:

\[
\begin{align*}
\max_{\mathbf{P}, \mathbf{\rho}, \mathbf{\nu}} & \sum_{f \in \mathcal{F}} \sum_{m \in \mathcal{M}_f} \sum_{n \in \mathcal{N}} \rho_{m,n}^f \{ r_{m,n}^f - r_{m,n}^{f,m} \}, \\
\text{s.t.} & : C_1 : \sum_{m \in \mathcal{M}_f} \sum_{n \in \mathcal{N}} \rho_{m,n}^f \rho_{m,n}^{f,m} \leq \rho_{\text{max}}^f \forall f \in \mathcal{F}, \\
& C_2 : \rho_{m,n}^f \leq \ell, \forall n \in \mathcal{N}, f \in \mathcal{F}, \\
& C_3 : \rho_{m,n}^f \in \{0,1\}, \forall m \in \mathcal{M}_f, n \in \mathcal{N}, f \in \mathcal{F}, \\
& C_4 : \rho_{m,n}^f \geq 0, \forall m \in \mathcal{M}_f, n \in \mathcal{N}, f \in \mathcal{F}, \\
& C_5 : \rho_{m,n}^f \rho_{i,n}^f Q_{m,i,n}(\mathbf{p}) \leq 0, \forall f \in \mathcal{F}, \\
& n \in \mathcal{N}, m, i \in \mathcal{M}_f, |h_{m,n}^f|^2 \leq |h_{e,n}^f|^2, i \not= m, \\
& C_6 : \rho_{m,n}^f \rho_{e,n}^f |\psi_{m,i,n}(\mathbf{p},\mathbf{\rho})| \geq 0, \forall f \in \mathcal{F}, n \in \mathcal{N}, \\
& m, i \in \mathcal{M}_f, e \in \mathcal{E}, |h_{e,n}^f|^2 \leq |h_{e,n}^f|^2, i \not= m, \\
& C_7 : |v_{h_{e,n}}^f|^2 \leq \epsilon, \forall f \in \mathcal{F}, n \in \mathcal{N}, e \in \mathcal{E}.
\end{align*}
\]

IV. SOLUTIONS OF THE OPTIMIZATION PROBLEM

The optimization problems \([15]\) and \([17]\) are non-convex because they have binary and continuous variables for subcarrier and power allocation, respectively. Besides, the objective functions are non-convex. Hence, we can not employ existing convex optimization methods straightforwardly. Hence, to tackle this issue, we adopt the well-known alternative method \([42]\), to solve the optimization problems.

A. Solution of the optimization problem in perfect CSI Scenario

As there is the \( \max \) operator in the objective function, we use a slack variable \( v_{h_{e,n}}^f \) which is defined as

\[
\max_{\mathcal{E}} \{ \log(1 + \gamma_{e,n}^{f,m}) \} = v_{m,n}^f,
\]

by applying the epigraph method, the optimization problem \([15]\) is rewritten as

\[
\begin{align*}
\max & \sum_{f \in \mathcal{F}} \sum_{m \in \mathcal{M}_f} \sum_{n \in \mathcal{N}} \rho_{m,n}^f \{ r_{m,n}^f - v_{m,n}^f \}, \\
\text{s.t.} & : C_1 - C_6, \\
& C_7 : \log \left(1 + \gamma_{e,n}^{f,m} \right) \leq v_{m,n}^f \forall e \in \mathcal{E}, n \in \mathcal{N}, \forall f \in \mathcal{F}, \forall m \in \mathcal{M}_f.
\end{align*}
\]

where \( \mathbf{\nu} \) is defined as \( \mathbf{\nu} = [v_{m,n}^f, \forall m \in \mathcal{M}_f, n \in \mathcal{N}, f \in \mathcal{F}] \). For solving \([19]\), we adopt the well-known iterative algorithm called ASM. In this method, the optimization problems are converted to two subproblems which one of them has binary and another has continuous optimization variables, in other words, power and subcarrier are allocated alternatively \([33]\). In this method, in each iteration, we allocate transmit power and
Algorithm 1: ITERATIVE RESOURCE ALLOCATION ALGORITHM For PERFECT CSI

1: Reformulate the optimization problem via the epigraph method
2: Initialization: Set \( \mu = 0 \) (\( \mu \) is the iteration number) and initialize to \( \rho (0) \) and \( \textbf{P} (0) \).
3: Set \( \rho = \rho (\mu) \),
4: Solve \( (20) \) and set the result to \( \textbf{P} (\mu + 1) \),
5: Solve \( (20) \) and set the result to \( \rho (\mu + 1) \),
6: If \( ||\textbf{P} (\mu + 1) - \textbf{P} (\mu)|| \leq \Theta \), stop,
7: else
8: set \( \mu = \mu + 1 \) and go back to step 3.

Subcarriers, separately. In other words, in this iterative method, in each iteration we consider fixed subcarrier and allocate power, then, for subcarrier allocation we consider fixed power. We summarize the explained algorithm in Algorithm 1. As seen in this algorithm, it is ended when the stopping condition is satisfied i.e., \( ||\textbf{P} (\mu + 1) - \textbf{P} (\mu)|| \leq \Theta \), where \( \mu \) and \( \Theta \) are the iteration number and stopping threshold, respectively.

1) Initialization Method: In order to begin the algorithm, we need to initial vectors \( \textbf{P} \) and \( \rho \). For initialization, it is supposed that the SBSs do not transmit data, i.e., SBSs at initialization do not serve any users \([43], [46]\). In other words, \( p_{m,n}^f = 0 \) if \( f \in \mathcal{F}/\{0\} \). Moreover, the subcarriers are allocated to one MBS user that has the highest secrecy rate.

2) Subcarrier Allocation: The subproblem for subcarrier allocation with fixed transmit power (which are computed in the previous iteration) is expressed as:

\[
\max_{\rho} \sum_{f \in \mathcal{F}} \sum_{m \in \mathcal{M}_f} \sum_{n \in \mathcal{N}} \rho_{m,n}^f \left( r_{m,n}^f - v_{m,n}^f \right),
\]

s.t. : \( C_1 - C_3 \),
\( C_5 - C_7 \). \hspace{1cm} (20)

Since this optimization problem is Integer Nonlinear Programming (INLP), we can solve it by exploiting MADS, to this end, we employ the NOMAD solver \([35]\).

3) Power Allocation: The power allocation subproblem at each iteration when the subcarriers allocation variables are fixed, is expressed as:

\[
\max_{\textbf{P}, \rho} \sum_{f \in \mathcal{F}} \sum_{m \in \mathcal{M}_f} \sum_{n \in \mathcal{N}} \rho_{m,n}^f \left( r_{m,n}^f - v_{m,n}^f \right),
\]

s.t. : \( C_1, C_4 - C_7 \). \hspace{1cm} (21)

This optimization subproblem is non-convex because constraints \( C_7 \) is non-convex and the objective function is non-concave. To tackle this difficulty, we utilize the SCA approach to approximate constraints \( C_7 \) and the objective function.

First, we investigate \( C_7 \):

\[
C_7^\prime: \log \left( 1 + \gamma_{e,n}^f \right) \leq v_{m,n}^f \hspace{1cm} (22)
\]

by substitution (8) into (22), we have

\[
\log \left( 1 + \frac{p_{m,n}^f |h_{e,n}^f|^2}{|h_{e,n}^f|^2} \sum_{i \in \mathcal{M}_f \setminus \{m\}} p_{i,n}^f \rho_{i,n}^f + I_{e,n}^f + \sigma^2 \right) - v_{m,n}^f \leq 0,
\]

the left hand side of (23) is written as follows:

\[
\log \left( |h_{e,n}^f|^2 \sum_{i \in \mathcal{M}_f \setminus \{m\}} p_{i,n}^f \rho_{i,n}^f + I_{e,n}^f + \sigma^2 + p_{m,n}^f |h_{e,n}^f|^2 \right) - \log \left( |h_{e,n}^f|^2 \sum_{i \in \mathcal{M}_f \setminus \{m\}} p_{i,n}^f \rho_{i,n}^f + I_{e,n}^f + \sigma^2 \right) - v_{m,n}^f,
\]

As seen, (24) is the difference between two concave functions. Hence, we can employ the DC method to approximate (24) to a convex constraint. To this end, we write (24) as follows:

\[
\Xi_{e,n}^f (\textbf{P}) = \Xi_{e,n}^f (\textbf{P}) - \Phi_{e,n}^f (\textbf{P}), \hspace{1cm} (25)
\]

where

\[
\Xi_{e,n}^f (\textbf{P}) = - \log \left( |h_{e,n}^f|^2 \sum_{i \in \mathcal{M}_f \setminus \{m\}} p_{i,n}^f \rho_{i,n}^f + I_{e,n}^f + \sigma^2 \right) - v_{m,n}^f,
\]

and

\[
\Phi_{e,n}^f (\textbf{P}) = - \log \left( |h_{e,n}^f|^2 \sum_{i \in \mathcal{M}_f \setminus \{m\}} p_{i,n}^f \rho_{i,n}^f + I_{e,n}^f + \sigma^2 + p_{m,n}^f |h_{e,n}^f|^2 \right) - v_{m,n}^f.
\]

\( \Phi_{e,n}^f (\textbf{P}) \) and \( \Phi_{e,n}^f (\textbf{P}) \) are convex, by utilizing a linear approximation, \( \Phi_{e,n}^f (\textbf{P}) \) can be written as follows:

\[
\Phi_{e,n}^f (\textbf{P}) \simeq \hat{\Phi}_{e,n}^f (\textbf{P}) = \Phi_{e,n}^f (\textbf{P} (\mu - 1)) + \left[ \nabla^T \Phi_{e,n}^f (\textbf{P} (\mu - 1)) (\textbf{P} - \textbf{P} (\mu - 1)) \right],
\]

where \( \nabla \Phi_{e,n}^f (\textbf{P}) \) is the gradient of \( \Phi_{e,n}^f (\textbf{P}) \) which is defined as:

\[
\nabla \Phi_{e,n}^f (\textbf{P}) = \frac{\partial}{\partial \textbf{P}} \Phi_{e,n}^f (\textbf{P}) = \begin{bmatrix} \frac{\partial \Phi_{e,n}^f (\textbf{P})}{\partial p_{m,n}^f} \end{bmatrix}, \forall m \in \mathcal{M}_f, \forall n \in \mathcal{N}, \forall f \in \mathcal{F}, \forall e \in \mathcal{E},
\]

and

\[
\frac{\partial \Phi_{e,n}^f (\textbf{P})}{\partial p_{a,b}^c} = \begin{cases} X, & a = m, b = n, c = f, \\
Y, & \forall a \in \mathcal{M}_f \setminus \{m\}, b = n, c = f, \\
B, & \forall a \in \mathcal{M}_f, b = n, c = f', e \in \mathcal{F}/f, \\
0, & O.W.
\end{cases}
\]

(30)
where, after approximation

\[
B = - \frac{|h_{c,n}^f|^2 \rho_{a,n}}{|h_{c,n}^f|^2 \sum_{i \in \mathcal{M}_f \setminus \{m\}} p_{i,n}^f p_{i,n}^f + p_{m,n}^f |h_{c,n}^f|^2 + I_{m,n}^f + \sigma^2},
\]

(32)

Therefore, \( \nabla^T \Phi_{e,m} (P) \) is a vector that its length is \( N \times M \times F \). After approximation \( C_{k_b}^f \) to the convex constraint, we convert the objective function to a concave function by exploiting the DC method, hence we have:

\[
\log (1 + \gamma_{m,n}^f) - \nu_{m,n}^f,
\]

(34)

By substitution (3) into (44), we have

\[
\log (1 + \frac{p_{m,n}^f |h_{m,n}^f|^2}{|h_{m,n}^f|^2 \sum_{i \in \mathcal{M}_f \setminus \{m\}} p_{i,n}^f p_{i,n}^f + I_{m,n}^f + \sigma^2}) - \nu_{m,n}^f,
\]

(35)

Where can be written as:

\[
\log (|h_{m,n}^f|^2 \sum_{|h_{m,n}^f|^2 \leq |h_{m,n}^f|^2} p_{i,n}^f p_{i,n}^f + I_{m,n}^f + \sigma^2) - \log (|h_{m,n}^f|^2 \sum_{|h_{m,n}^f|^2 \leq |h_{m,n}^f|^2} p_{i,n}^f p_{i,n}^f + I_{m,n}^f + \sigma^2) - \nu_{m,n}^f,
\]

(36)

We can write (36) as follows:

\[
U_{m,n}^f (P) = G_{m,n}^f (P) - H_{m,n}^f (P),
\]

(37)

Where \( H_{m,n}^f (P) \) and \( G_{m,n}^f (P) \) are defined as

\[
G_{m,n}^f (P) = -\nu_{m,n}^f + \log (|h_{m,n}^f|^2 \sum_{|h_{m,n}^f|^2 \leq |h_{m,n}^f|^2} p_{i,n}^f p_{i,n}^f + I_{m,n}^f + \sigma^2),
\]

(38)

And

\[
H_{m,n}^f (P) = \log (|h_{m,n}^f|^2 \sum_{|h_{m,n}^f|^2 \leq |h_{m,n}^f|^2} p_{i,n}^f p_{i,n}^f + I_{m,n}^f + \sigma^2),
\]

(39)

Respectively, \( G_{m,n}^f (P) \) and \( H_{m,n}^f (P) \) are concave, by utilizing a linear approximation we can write \( H_{m,n}^f (P) \) as follows:

\[
H_{m,n}^f (P) \simeq \tilde{H}_{m,n}^f (P) = H_{m,n}^f (P (\mu - 1)) + \nabla^T H_{m,n}^f (P (\mu - 1)) (P - P (\mu - 1)),
\]

(40)

Where \( \nabla^T H_{m,n}^f (P) \) is calculated as follows:

\[
\nabla^T H_{m,n}^f (P) = \frac{\partial}{\partial p_{a,n}} H_{m,n}^f (P) = \frac{\partial H_{m,n}^f (P)}{\partial p_{a,n}}, \forall m \in \mathcal{M}_f, \forall n \in \mathcal{N}, \forall f \in \mathcal{F},
\]

We take derivative of \( H_{m,n}^f (P) \) with respect to \( p_{a,n}^c \) as follows:

\[
\frac{\partial H_{m,n}^f (P)}{\partial p_{a,n}^c} = \begin{cases} Z & \forall a \in \mathcal{M}_f \setminus \{m\}, b = n, c = f, |h_{m,n}^f|^2 \leq |h_{a,n}^f|^2, \\
T & \forall a \in \mathcal{M}_f, b = n, c = f \in \mathcal{F} \setminus \{f\} \\
0 & \text{O.W}
\end{cases}
\]

(41)

Where \( Z \) and \( T \) are calculated as follows:

\[
Z = \frac{|h_{m,n}^f|^2 \rho_{a,n}^f}{|h_{m,n}^f|^2 \sum_{|h_{m,n}^f|^2 \leq |h_{m,n}^f|^2} p_{i,n}^f p_{i,n}^f + I_{m,n}^f + \sigma^2},
\]

(43)

\[
T = \frac{|h_{m,n}^f|^2 \rho_{a,n}^c}{|h_{m,n}^f|^2 \sum_{|h_{m,n}^f|^2 \leq |h_{m,n}^f|^2} p_{i,n}^f p_{i,n}^f + I_{m,n}^f + \sigma^2}.
\]

(44)

Consequently, we have a convex optimization problem in the canonical form, by exploiting the DC approximation, which is formulated as:

\[
\max_{P, \nu} \sum_{\forall f \in \mathcal{F}} \sum_{\forall m \in \mathcal{M}_f} \sum_{\forall n \in \mathcal{N}} \rho_{m,n}^f \left\{ G_{m,n}^f (P) - H_{m,n}^f (P) \right\},
\]

s.t. : \( C_1 - C_7 \),

(45a)

\( C_7^f : \Phi_{e,m}^f (P) - \Phi_{e,m}^f (P) \leq 0 \ \forall e \in \mathcal{E}, \)

\( n \in \mathcal{N}, f \in \mathcal{F}, m \in \mathcal{M}_f \),

(45c)

For solving the convex optimization problem (45), we can use available softwares, such as CVX solver [43].

4) Convergence of the algorithm: In this subsection, we prove the convergence of the algorithm and illustrate that after each iteration the value of objective function \( f (P, p) = \rho_{m,n}^f \left\{ r_{m,n}^f - \nu_{m,n}^f \right\} \), is improved and converged.

Proof: In this algorithm, after applying the third step, with a given \( p = p (\mu) \), the power allocation of iteration \( \mu + 1 \) is obtained. According to Appendix I, we will have \( f (p (\mu), p (\mu)) \leq f (p (\mu), p (\mu + 1)) \). Moreover, in the fourth step, with a given \( p = p (\mu + 1) \), the subcarrier
allocation of this iteration is obtained. According to this fact that, after each iteration, subcarrier allocation with feasible power solution improves the objective function, hence we have:

\[ ... \leq f(\rho(\mu), p(\mu)) \leq f(\rho(\mu), p(\mu + 1)) \leq f(\rho(\mu + 1), p(\mu + 1)) \leq ... \leq f(\rho^*, p^*) \]

where \( \rho^* \) and \( p^* \) are obtained at the last iteration. [44]. Convergence behavior of the proposed algorithm is shown in Fig. [3]. It should be noted, globally optimal solution is not guaranteed by this solution even after convergence. Hence, for finding the globally optimal solution, we utilize the monotonic optimization method which is explained, in Section [VI].

5) Computational complexity: Solution of the optimization problem (15) consists of two stages) Calculation of power allocation from problem (45), 2) calculation of subcarrier allocation from problem (20). As we know, CVX software employs geometric programming with the Interior Point Method (IPM) [43], hence, the order of computational complexity can be obtained as:

\[ O\left(\frac{\log(NOC)}{\log(\xi)}\right) \],

where NOC is the total number of constraints, \( \xi \) and \( t \) are parameters of IPM. \( 0 \leq \xi \ll 1 \) is the stopping criterion of IPM, \( \xi \) is used for the accuracy IPM and \( t \) is initial point for approximated the accuracy of IPM. [45], [47]. Hence, the complexity order is given by:

\[ O\left(\frac{\log\left(F(1+N(1+M+M(M-1)+ME(M-1)+ME))\right)}{\log(\xi)}\right) \] (48)

B. Solution of the optimization problem in Imperfect CSI Scenario

For solving (17), first we solve the inner minimization and obtain \( \epsilon \), then solve the maximization problem according to Section IV-A. The inner minimization can be written as follows:

\[ \min_\epsilon \sum_{\forall f \in F} \sum_{\forall m \in M_f} \sum_{\forall \epsilon_1 \in N_f} \rho_{f,m} \bigl( r_{f,m} - v_{f,m} \bigr), \] (49a)

s.t. : \( C_6, C_7, C_9. \) (49b)

Our aim is to minimize the objective function, to this end, we should maximize \( v_{f,m} \). Hence, according to \( C_9 \), we maximize lower bound of \( v_{f,m} \), i.e., \( \log (1 + \gamma_{f,m}) \). Since the logarithmic function is increasing, we can maximize \( \gamma_{f,m} \) instead of \( \log (1 + \gamma_{f,m}) \). As \( \gamma_{f,m} \) is fractional, we should maximize numerator and minimize denominator. To this end, we use the triangle inequality which is defined as follows:

\[ \left| \hat{h}_{f,m}^2 - \epsilon \right| \leq \left| \hat{h}_{f,m}^2 - e_{h_{f,m}}^2 \right| \leq \left| \hat{h}_{f,m}^2 + e_{h_{f,m}}^2 \right| \leq \left| \hat{h}_{f,m}^2 + e_{h_{f,m}}^2 \right| \leq \hat{h}_{f,m}^2 + \epsilon. \] (50)

By using (50), we can write the upper bound of \( \gamma_{f,m}^2 \) as follows:

\[ \hat{h}_{f,m}^2 \leq \sum_{i \in M_f} \rho_{f,m}^i I_{f,m}^i + \sigma^2 \]

\[ \hat{h}_{f,m}^2 \leq \frac{p_{m,n} \left( \hat{h}_{f,m}^2 + \epsilon \right)}{\left( \hat{h}_{f,m}^2 - \epsilon \right)} \sum_{i \in M_f} \rho_{i,m}^i I_{i,m}^i + \sigma^2 \] (51)

where \( I_{f,m}^i \) = \( \frac{\hat{h}_{f,m}^2 - \epsilon}{\hat{h}_{f,m}^2 + \epsilon} \sum_{i \in M_f} \rho_{i,m}^i p_{i,m}^i \). Also, we consider the worst case for constraint \( C_6 \). According to (50), we can rewrite the worst case \( C_6 \) as follows:

\[ \Psi_{m,i,n,e}(\rho, p) \geq \hat{\Psi}_{m,i,n,e}(\rho, p) = \left| h_{f,m}^i \right|^2 \sigma^2 + \left| h_{f,m}^i \right|^2 I_{f,m}^i - \left( \left| \hat{h}_{f,m}^2 - \epsilon \right| \sum_{i \in M_f} \rho_{i,m}^i \right) + \left( \left| \hat{h}_{f,m}^2 - \epsilon \right| \sum_{i \in M_f} \rho_{i,m}^i I_{f,m}^i + \sigma^2 \right) \] (52)

In the following, we should solve the outer maximization, which is written as follows:

\[ \max_{\rho, p} \sum_{\forall f \in F} \sum_{\forall m \in M_f} \rho_{f,m} \left( r_{f,m} - v_{f,m} \right), \] (53a)

s.t. : \( C_1 - C_5, \) (53b)

\[ C_6 : \rho_{m,n} \rho_{m,n} \hat{\psi}_{m,n,e}(\rho, p) \geq 0, \forall f \in F, n \in N, \]

\[ m, i \in M_f, \epsilon \in E, \left| h_{f,m}^i \right|^2 \leq \left| h_{f,m}^i \right|^2, i \neq m, \] (53c)

\[ C_7 : \]

\[ \log \left( 1 + \frac{\rho_{m,n} \hat{h}_{f,m}^2 + \epsilon}{\sum_{i \in M_f} \rho_{i,m}^i I_{f,m}^i + \sigma^2} \right) \]

\[ - v_{m,n} \leq 0, \forall \epsilon \in E, n \in N, f \in F, \forall m \in M_f. \] (53e)

This optimization problem (53) can be solved similar to the proposed approach in Section IV-A.

V. MASSIVE CONNECTIVITY SCENARIO

In this section, we aim to evaluate the PD-NOMA technique in ultra dense network for secure massive connectivity in 5G networks. Without loss of generality, for changing our scenario from HetNet to HUDN, we need to extend the dimension of system model. According to [3], [23], and [48], in order to tackle high dimension complexity of resource allocation in HUDNs and overcome hardware computation limitations, it is assumed that the transmit power is uniformly allocated to devices/users and subcarriers are dynamically allocated.

To know the performance degradation due to the uniform power allocation, we compare the uniform power allocation method for a small network dimension with our proposed
solution i.e., joint power and subcarrier allocation in section of simulation result. Based on simulation results, we show the performance of uniform power allocation is close to the performance of our proposed solution i.e., joint power and subcarrier allocation in the small network dimension.

VI. OPTIMAL SOLUTION

In this section, our aim is to solve problem (19) by utilizing the monotonic optimization method. Hence, in this section, three main steps are performed as follows:

1) Problem (19) is transformed into an optimization problem at which its optimization variables are only transmit power.
2) The new optimization problem is converted to a canonical form of monotonic optimization problem.
3) Finally, by exploiting the polyblock algorithm, we solve the monotonic optimization problem globally.

A. Problem Transformation

For the first step, we assume each subcarrier can be allocated to at most two users, i.e., \( \ell = 2 \). Hence, based on constraints (15c) and (15d), if \( p_{m,n}^f \neq 0 \) and \( p_{m,n}^{f'} \neq 0 \), we have \( p_{w,n}^f = 0 \) \( \forall m, i, w \in M_f, m \neq i \neq w \). Therefore, constraint (15c) is equivalent to

\[
p_{m,n}^f p_{n,w}^f \leq 0, \quad \forall n \in \mathcal{N}, \quad f \in \mathcal{F}, \quad m, i, w \in M_f,
\]

\[
m \neq i \neq w
\]

Therefore, the optimization problem (19) can be transformed into a new optimization problem with only transmit power variables as follows:

\[
\begin{align}
\max_{P} & \sum_{f \in \mathcal{F}} \sum_{n \in \mathcal{N}} \{ \tilde{r}_{m,n}^f - v_{m,n}^f \}, \\
\text{s.t.} : & \sum_{m \in \mathcal{M}_f} p_{m,n}^f \leq p_{\text{max}}^f \forall f \in \mathcal{F}, \\
& p_{m,n}^f, p_{n,w}^f \geq 0, \forall m \in \mathcal{M}_f, n \in \mathcal{N}, m \in \mathcal{M}_f, \\
& p_{m,n}^f p_{n,i}^f \tilde{Q}_{m,i,n}^f \leq 0, \forall f \in \mathcal{F}, \\
& n \in \mathcal{N}, f, i \in \mathcal{M}_f, |h_{m,n}^f|^2 \leq |h_{m,i}^f|^2, i \neq m, \\
& -p_{m,n}^f p_{n,w}^f \tilde{Q}_{m,w,n}^f \leq 0, \forall f \in \mathcal{F}, n \in \mathcal{N}, \\
& m, i \in \mathcal{M}_f, e \in \mathcal{E}, |h_{e,n}^f|^2 \leq |h_{e,m}^f|^2, i \neq m, \\
& \log \left( |h_{e,n}^f|^2 \sum_{i \in \mathcal{M}_f / \{m\}} p_{i,n}^f + \tilde{I}_{m,n}^f + \sigma^2 + p_{m,n}^f |h_{e,n}^f|^2 \right) - v_{m,n}^f \leq 0, \\
& \forall e \in \mathcal{E}, n \in \mathcal{N}, f \in \mathcal{F}, \forall m \in \mathcal{M}_f, \\
& p_{m,n}^f p_{n,i}^f p_{n,w}^f \leq 0, \forall n \in \mathcal{N}, f \in \mathcal{F}, m, i, w \in \mathcal{M}_f, \\
& m \neq i \neq w,
\end{align}
\]

where

\[
Q_{m,i,n}^f = -|h_{m,n}^f|^2 \sigma^2 + |h_{m,i}^f|^2 \tilde{I}_{m,n}^f + -|h_{m,n}^f|^2 |h_{i,n}^f|^2 + \sum_{i \in \mathcal{M}_f / \{i\}} p_{i,n}^f + |h_{i,n}^f|^4 \sum_{i \in \mathcal{M}_f / \{i\}} p_{i,n}^f,
\]

\[
\hat{\psi}_{m,i,n,e} = -|h_{e,n}^f|^2 \sigma^2 + |h_{e,i}^f|^2 \tilde{I}_{m,n}^f + |h_{e,n}^f|^2 \tilde{I}_{m,i}^f - |h_{e,i}^f|^2 |h_{e,n}^f|^2 \sum_{i \in \mathcal{M}_f / \{i\}} p_{i,n}^f + |h_{e,i}^f|^4 \sum_{i \in \mathcal{M}_f / \{i\}} p_{i,n}^f,
\]

\[
\hat{I}_{m,n}^f = \sum_{f \in \mathcal{F}} |h_{m,n}^f|^2 \sum_{i \in \mathcal{M}_f} p_{i,n}^f \text{ and } \hat{I}_{m,i}^f = \sum_{f \in \mathcal{F}} |h_{m,i}^f|^2 \sum_{i \in \mathcal{M}_f} p_{i,n}^f.
\]

B. Monotonic Optimization

In the second step, our aim is to formulate the optimization problem (55) as a monotonic optimization problem in the canonical form. As we know, the optimization problem (55) is a non-monotonic problem because the objective function is not increasing function and constraints (55d), (55e), and (55f) are not inside normal or conormal sets. Since the optimization problem is a problem with hidden monotonicity [59], we can rewrite the objective function and non-monotonic constraints to a differences of increasing function form. Hence, let us reformulate the objective function as

\[
\hat{r}_{m,n}^f = \log \left( 1 + \frac{p_{m,n}^f |h_{m,n}^f|^2}{|h_{m,n}^f|^2 \sum_{i \in \mathcal{M}_f / \{m\}} p_{i,n}^f + \hat{I}_{m,n}^f + \sigma^2} \right)
\]

and

\[
\nu_{m,n}^f = \log \left( 1 + \frac{p_{m,n}^f |h_{m,n}^f|^2}{|h_{m,n}^f|^2 \sum_{i \in \mathcal{M}_f / \{m\}} p_{i,n}^f + \hat{I}_{m,n}^f + \sigma^2} \right)
\]

where \( t_1, t_2, t_3, \) and \( t_4 \) to reformulate (55) as (50), (51).
According to problem (59), we define two sets as follows:
\[ N_1 = \left\{ (\mathbf{P}, \mathbf{v}, \mathbf{T}_1, \mathbf{T}_2, \mathbf{T}_3) : \mathbf{P} \preceq \mathbf{P}^{\text{mask}}, (55b), (55c), (55d), (55g) \right\}, \]
and
\[ N_2 = \left\{ (\mathbf{P}, \mathbf{v}, \mathbf{T}_1, \mathbf{T}_2, \mathbf{T}_3) : \mathbf{P} \succeq 0, (55c), (55d), (55g), (59d), (59m) \right\}. \]

Finally, problem (59) is a monotonic problem in a canonical form, based on Definition 5 in [51]. Hence, the optimization problem (59) can be solved by using the polyblock algorithm. As mentioned way, at which we convert (19) to a canonical form of monotonic optimization, we can convert (53) to a canonical form.

C. Computational complexity

In this section, we discuss about the computational complexity of the polyblock algorithm. As we know, the computational complexity of this algorithm depends on the number of variables and form of the functions in the optimization problem. In the polyblock algorithm four main steps are performed. In the first step, the best vertex should be found, in the second step we find projection of the selected vertex, improper vertexes are removed in the third step, and new vertext set is found in the fourth step. The dimension of our optimization problem is \( 3_0 = NF + NFME (3M + 1) + (M - 1) (M (M - 2) + 3 NFME + 3 NFME) + 5 NFME \), the convergence of algorithm for stopping threshold is \( 10^{-3} \), occurs approximately after \( 31 = 10^4 \) iterations, the bisection algorithm which gives projection of vertex, with stopping threshold \( 10^{-3} \), has \( 32 = 10^3 \) iterations, approximately. Hence, the complexity order can be written as [10]:
\[ O(3_1 \times 3_0 + 3_2) \]

VII. SIMULATION RESULTS

In this section, we provide numerical results to evaluate the performance of the proposed scheme. The simulation parameters are considered as: \( p_{\text{max}} = 16 \text{ dB} \) (maximum allowable transmit power of MBS), \( p_{\text{max}} = 6 \text{ dB} \), \( \forall m \in \mathcal{M}/\{1\} \) (maximum allowable transmit power of SBS), Power Spectral Density (PSD) of noise is \(-130 \text{ dBm/Hz} \), \( \alpha = 4 \). Maximum coverage MBS and SBS are supposed 1500 m and 15 m, respectively.

In Fig. 2, the sum secrecy rate versus the number of subcarriers is shown. Also this figure compares our proposed
scheme at which the eavesdroppers are not able to perform SIC with the case they can perform SIC. As seen in this figure, the sum secrecy rate in our proposed scheme has \%82.52 gap with the conventional type, because in the proposed scheme we do not allow eavesdroppers to perform SIC, even if they know the channel ordering, but in the conventional type the eavesdroppers can perform SIC. This issue is shown for two different number of eavesdroppers, i.e., $E = 4, E = 6$.

In Fig. 3, we compare the performance of the proposed scheme for the perfect and imperfect CSI scenarios. Moreover, this figure shows imperfect CSI sensitivity with respect to the upper bound of error. As seen, when $\epsilon = 0.1$, the imperfect SCI scenario has \%14.34 gap with respect to perfect CSI. By increasing $\epsilon$ to 0.3 and 0.5, this gap is increased to \%17.37 and \%31.45, respectively.

Fig. 4 shows the sum secrecy rate versus the number of eavesdroppers. As seen, with increasing the number of eavesdroppers in our system model, the sum secrecy rate is decreased. This is because, as we know, it is assumed the eavesdroppers are non-clusters, therefore, when an eavesdropper is added to the system, its channel maybe better than others, hence the secrecy rate changes (decreases). In the simulation, we use the Monte Carlo method, therefore when the number of eavesdroppers is increased, the secrecy rate decreases on average.

Fig. 5 presents the convergence behavior of the proposed algorithm. We observe that the algorithm converges in iteration 8, in ASM, approximately. In this figure, we assume $E = 2$ per km$^2$, $F = 2$ per km$^2$, $M = 3$ per km$^2$, and $N = 2$.

In addition, we show optimal solution in all of these figures. As seen, the proposed suboptimal solution which has low complexity with respect to the monotonic optimization problem, is closed to the optimal solution, for example in Fig. 8, in ASM, approximately.
dimension with our proposed solution i.e., joint power and subcarrier allocation in Fig. 6. As shown in this figure, there is approximately 9% performance gap between these methods.

In Fig. 7, secrecy sum rate versus the number of BSs in HUDN for massive connectivity is evaluated. Besides, this figure investigates effect of number of eavesdroppers in massive connectivity. As seen, by increasing one BS per Km², the sum secrecy rate 2.5 unit increases, approximately.

VIII. CONCLUSION

In this paper, we investigated physical layer security for power domain non-orthogonal multiple access based HetNet. We proposed a novel resource allocation to maximize the sum secrecy rate in PD-NOMA based HetNet. In the proposed scenario, the eavesdroppers are not allowed to perform successive interference cancellation, but the legitimate users are able to perform it. Hence, all users’ signals in the eavesdroppers are treated as interference, while some users’ signals can be canceled in the desired users, therefore, less interference is experienced by users. In order to solve the optimization problem, we adopted the iterative algorithm called ASM, i.e., to convert the optimization problems to two subproblems power and subcarrier allocation and solve them alternatively. In each iteration of ASM, we consider fixed subcarrier and allocated power, then, for subcarrier allocation we consider fixed power. In each iteration of this method, power allocation problem, is non-convex and subcarrier allocation is NLP. Hence, we solve the power allocation by the SCA approach. To this end, we use the DC approximation, to transform the non-convex problem into canonical form of convex optimization. Also, we solve subcarrier allocation, by exploiting MADS, hence we employ the existing solver called NOMAD. Moreover, we obtained optimal solution and the optimality gap of the proposed solution method, by converting the optimization problems to a canonical form of the monotonic optimization problem and exploiting the polyblock algorithm. Besides, we evaluated the proposed scheme for secure massive connectivity in the massive machine-type communication (mMTC) cases in 5G networks. As resource allocation in this networks has high dimension complexity, we allocated the transmit power uniformly to users and showed the performance of uniform power allocation is close to the performance of our proposed solution. Numerical results show the sum secrecy rate in our novel resource allocation has %82.52 gap with the conventional type at which the eavesdroppers are able to perform SIC. Moreover, we investigated imperfect CSI of the eavesdroppers scenario and compared it with the perfect CSI case.

Appendix I

In the SCA approach with the DC approximation, a sequence of improved feasible solutions is generated and is converged to a local optimum. \[\text{[2]}\]

Proof: As mentioned, we approximate \[\text{[59]}\] with function \[\text{[40]}\]. Gradient of function \(H_{m,n}^f(P)\) is its super gradient, because \(H_{m,n}^f(P)\) is a concave function \[\text{[49]}\]. Hence, we have

\[
H_{m,n}^f(P(\mu)) \leq H_{m,n}^f(P(\mu - 1)) + \nabla^T H_{m,n}^f(P(\mu)) (P(\mu) - P(\mu - 1)),
\]

Therefore, in the objective function, we have:

\[
\sum_{f \in F} \sum_{m \in M_f} \sum_{n \in N_f} \rho_{m,n} \{G_{m,n}^f(P(\mu)) - H_{m,n}^f(P(\mu))\} \geq \sum_{f \in F} \sum_{m \in M_f} \sum_{n \in N_f} \rho_{m,n} \{G_{m,n}^f(P(\mu)) - H_{m,n}^f(P(\mu - 1)) - \nabla^T H_{m,n}^f(P(\mu - 1)) (P(\mu) - P(\mu - 1))\} = \max_P \sum_{f \in F} \sum_{m \in M_f} \sum_{n \in N_f} \rho_{m,n} \{G_{m,n}^f(P(\mu)) - H_{m,n}^f(P(\mu - 1)) - \nabla^T H_{m,n}^f(P(\mu - 1)) (P(\mu) - P(\mu - 1))\} \geq \sum_{f \in F} \sum_{m \in M_f} \sum_{n \in N_f} \rho_{m,n} \{G_{m,n}^f(P(\mu - 1)) - H_{m,n}^f(P(\mu - 1)) - \nabla^T H_{m,n}^f(P(\mu - 1)) (P(\mu - 1) - P(\mu - 1))\}.
\]
\[
\sum_{f \in F} \sum_{m \in M_f} \sum_{n \in N_f} \mu_{m,n}^f \{ G_{m,n}^f (\mathbf{P} (\mu - 1)) - H_{m,n}^f (\mathbf{P} (\mu - 1)) \}.
\]

Therefore, after iteration \(\mu\), the objective function value either increases or stays unchanged with respect to iteration \(\mu - 1\).

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