Frontiers in Nuclear Physics

Symposium in honor of Walter Glöckle’s 70th birthday
Bad Honnef, Germany, June 18 - 20, 2009

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ABSTRACT

These are the proceedings of the symposium on “Frontiers in Nuclear Physics” held at the Physikzentrum Bad Honnef from June 18 to 20, 2009. The workshop concentrated on recent advances in the understanding of the nuclear forces, few-nucleon systems and related topics. Included are a short contribution per talk.

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1 Introduction

On January 8 of 2009, Walter Glöckle turned 70. Over many decades, his work had a lasting impact on the theoretical and experimental investigations of few-nucleon systems, on the theory of nuclear forces, relativistic quantum mechanics and most recently, Coulomb effects in few-nucleon systems. We thus found it appropriate to organize a symposium in honor of these achievements and to review the state-of-the-art in these fields and related areas. The meeting was organized at the Physikzentrum Bad Honnef from June 18-20, 2009, with financial support from the Virtual Institute on “Spin and Strong QCD”, the Network WP4 (QCDnet) of the HadronPhysics2 project of the seventh framework of the EU and the young investigator group “Few-Nucleon Systems in Chiral Effective Field Theory” of the Helmholtz Association. The meeting had 35 participants whose names, institutes and email addresses are listed below. 17 of them presented results in half hour presentations. A short description of the contents of each talk and a list of the most relevant references can be found below. We felt that this was more appropriate a framework than full-fledged proceedings. Most results are or will soon be published and available on the archives, so this way we can achieve speedy publication and avoid duplication of results in the archives.

Below follows first the program, then the list of participants followed by the abstracts of the talks. All talks can also be obtained from the workshop web-site

http://www.itkp.uni-bonn.de/~epelbaum/bh09.

We would like to thank the staff of the Physikzentrum, in particular Victor Gomer, for the excellent organization of the workshop and all participants for their valuable contributions.

Evgeny Epelbaum and Ulf-G. Meißen
2 Program

Thursday, June 18, 2009

19:00 Workshop dinner

Friday, June 19, 2009

Morning session, chair: Ulf-G. Meiβner
9:00 U.-G. Meiβner Opening remarks
9:10 A. Kievsky (Pisa) Three-nucleon forces: a comparative study
9:45 J. Golak (Krakow) Two-pion exchange currents in the photodisintegration of the deuteron
10:20 A. Nogga (Jülich) On the action of four-nucleon forces in 4He
10:55 Coffee
11:15 D. Lee (Raleigh) Effective field theory on a lattice
11:50 J. Haidenbauer (Jülich) Aspects of the hyperon-nucleon interaction
11:50 D. Phillips (Athens, Ohio) Compton scattering on Helium-3
13:00 Lunch

Afternoon session, chair: Ulf-G. Meiβner
14:30 N. Kaiser (München) Chiral three-nucleon interaction and 14C beta decay
15:05 H.-W. Hammer (Bonn) Few-body physics with resonant interactions
15:40 F.-K. Guo (Jülich) Heavy meson hadronic molecules
16:15 Coffee
16:45 B. Kubis (Bonn) Cusps in $K \rightarrow 3\pi$ decays
17:20 W. Plessas (Graz) Baryons as relativistic three-quark systems
17:55 N. Kalantar-Nayestanaki (Groningen) What have we learned about three-nucleon systems at intermediate energies?
18:30 R. Beck (Bonn) Recent results in meson photoproduction at ELSA
19:05 End of Session

Saturday, June 20, 2009

Morning Session, chair: Evgeny Epelbaum
09:30 Ch. Elster (Athens, Ohio) Faddeev calculations in three dimensions
10:05 W. Polyzou (Iowa) Euclidean formulation of relativistic quantum mechanics
10:40 Coffee
11:05 A novel approach to include the pp Coulomb force into the 3N Faddeev calculations
11:35 H. Kamada (Kitakyushu) Effects of the $\pi\rho$ exchange three-body force
12:10 W. Glöckle A short look back
12:20 E. Epelbaum Closing remarks
12:25 End of session
12:30 Lunch and end of the Symposium
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Three-nucleon Forces: A Comparative Study

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The use of realistic NN potentials in the description of the three- and four-nucleon systems gives a $\chi^2$ per datum much larger than 1 (see for example Ref.[1]). In order to improve this situation, different three-nucleon force (TNF) models have been introduced as the Tucson-Melbourne (TM) and the Urbana IX (URIX) potentials. More recently, TNF models have been derived based on chiral effective field theory at next-to-next-to-leading order. A local version of this interaction (hereafter referred as N2LO) can be found in Ref. [2]. All these models contain a certain number of parameters that fix the strength of the different terms. It is a common practice to determine these parameters from the three- and four-nucleon binding energies.

In Table 1 we report the triton and $^4$He binding energies, and the doublet $n - d$ scattering length $^2a_{nd}$. These results were obtained using the AV18 or the N3LO-Idaho two-nucleon potentials together with the AV18+URIX, AV18+TM’ and N3LO-Idaho+N2LO TNF models. The results are compared to the experimental values also reported in the table. From the table we observe that only the results obtained using an interaction model that includes a TNF are close to the corresponding experimental values. However the predictions are not in complete agreement with the experimental values. In the following we would like to discuss possible modifications to the TNF models in order to improve the description of these three quantities. As a two body potential we choose the AV18 interaction. The TNF models are summed to this interaction and, in the case of the TM potential, the constants $b$, $d$ and the cutoff $\Lambda$ have been varied. In the case of the URIX potential the constant $A_{2\pi}$ and $U_0$ have been varied. Moreover the constant

| Potential                        | $B(^3\text{H})$ | $B(^4\text{He})$ | $^2a_{nd}$ |
|---------------------------------|-----------------|-----------------|-----------|
| AV18                            | 7.624           | 24.22           | 1.258     |
| N3LO-Idaho                      | 7.854           | 25.38           | 1.100     |
| AV18+TM’                        | 8.440           | 28.31           | 0.623     |
| AV18+URIX                       | 8.479           | 28.48           | 0.578     |
| N3LO-Idaho+N2LO                 | 8.474           | 28.37           | 0.675     |
| Exp.                            | 8.482           | 28.30           | 0.645\pm0.003\pm0.007 |

Table 1: The triton and $^4$He binding energies (in MeV), and doublet scattering length $^2a_{nd}$ (in fm) calculated using the AV18 and the N3LO-Idaho two-nucleon potentials, and the AV18+URIX, AV18+TM’ and N3LO-Idaho+N2LO two- and three-nucleon interactions.
Figure 1: Values of $D_{2\pi}$, the doublet scattering length and the $^4$He binding energy, as a function of $A_{2\pi}$ for the URIX potential. The crosses correspond to the original values

$D_{2\pi}$ in front of the anticommutator term and originally fixed to $D_{2\pi} = (1/4)A_{2\pi}$, was allowed to vary independently of the value of $A_{2\pi}$. Finally, for the N2LO potential the constants $c_3$, $c_4$, $c_D$ and $c_E$ have been varied. As an example, in Fig.1 the values of $D_{2\pi}$ in function of $A_{2\pi}$ are shown as well as the doublet scattering length and the $^4$He binding energy for the AV18+URIX potential. In each point the value of $U_0$, which fixes the strength of the repulsive term, has been fixed to describe the triton binding energy. The values corresponding to the original potential are indicated by crosses in the figure.

Similar modifications were performed for the TM and N2LO. A detailed description of this study for the three TNF potentials under consideration is in progress [3]. The main conclusions are the following. The TM potential, as originally defined, does not include a repulsive term. Accordingly, a simultaneous description of $B(^3\text{H})$, $B(^4\text{He})$ and $^2a_{nd}$ can be obtained with unrealistic values of the parameters $b$, $d$ and $\Lambda$. The introduction of a $c_E$--term helps to obtain a much better agreement. In the case of the N2LO potential it is possible to obtain a good agreement with a small change of the original values. The next step is to analyze the new parametrizations of the potentials in the description of $N-d$ and $N-^3\text{He}$ scattering.

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Two-Pion Exchange Currents in the Photodisintegration of the Deuteron

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Chiral effective field theory (ChEFT) is a modern framework to analyze properties of few-nucleon systems at low energies [1]. It is based on the most general effective Lagrangian for pions and nucleons consistent with the chiral symmetry of QCD. For energies below the pion-production threshold it is possible to eliminate the pionic degrees of freedom and derive nuclear potentials and nuclear current operators. This is very important because, despite a lot of experience gained in the past, the consistence between two-nucleon forces, many-nucleon forces and corresponding current operators has been not yet achieved.

In this presentation we consider recently derived two-pion exchange (TPE) contributions to the nuclear current operator [2]. These operators do not contain any free parameters. Due to their isospin structures they do not contribute to elastic electron scattering off the deuteron. We thus study their role in the deuteron photodisintegration reaction. We show how partial wave decomposition for these operator is performed using the Mathematica software and parallel computing techniques. We address also a problem of numerical stability of some scalar functions which appear in TPE contributions. Finally, using a two-nucleon (2N) current operator containing the single-nucleon, lowest order one-pion exchange and TPE contributions, we show predictions for a number of observables at photon energy $E_\gamma = 60$ and $120$ MeV. The bound and scattering states are calculated with five different chiral N2LO 2N potentials which results in the so-called bands for the predicted results.

For some observables the widths of the bands and their vicinity to the reference predictions based on the AV18 2N potential and the current operator (partly) consisted with this force indicate that the missing contributions in the chiral framework (like contact currents) do not play a big role.

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On the action of four-nucleon forces in $^4$He

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Due to the rather good description of *ab-initio* calculations for light nuclei, it is generally expected that the contribution of four-nucleon forces (4NF) is insignificant. However, since small deviation of the predictions remain, it is timely to study the possible impact of the 4NF more stringently. In this talk, we present the results of such a study for $^4$He within the framework of chiral perturbation theory, where NN, 3N and 4N forces are formulated consistently on the same footing.

It was recently shown that the leading chiral 4NF is completely determined by parameters of the leading NN interaction [1][2]. Based on this formulation, we have performed a first study of the 4NF contribution to the binding energy of $^4$He [3], where we could not take into account the full complexity of the $^4$He wave function. Due to this, our results especially for the short range 4NF’s were not reliably enough for final conclusions on the size of 4NF’s.

In this talk, we present for the first time the results of complete calculations of the 4NF contribution to the $^4$He binding energy. We estimate the 4NF perturbatively. For the pertinent expectation values, we devised a Monte Carlo method suitable for calculations in momentum space. We found that the 4NF contributions depend on the cutoff and order of the chiral interactions. They are however in line with power counting estimates. Individual classes of diagrams of the 4NF contribute of the order of 1 MeV to the binding energy of $^4$He. We also found that attractive and repulsive classes of diagrams cancel each other in parts, so that the net contribution mostly is below 500 MeV for $^4$He. Since such a cancellation could be less prominent in nuclei other $^4$He, we expect a natural contribution of the 4NF of 200-300 keV per nucleon. Based on these results, the 4NF will not be negligible in high precision nuclear structure calculations. The results presented will be published in [4].

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Effective field theory on a lattice

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Lattice effective field theory combines the theoretical framework of effective field theory with numerical lattice methods. This approach has been used to simulate nuclear matter \cite{1} and neutron matter \cite{2}--\cite{4}. It has also been used to study light nuclei in pionless effective field theory \cite{5} and chiral effective field theory at leading order \cite{6}. A review of the literature in lattice effective field theory can be found in Ref. \cite{7}.

Recently calculations at next-to-leading order in chiral effective field theory have been carried out for the ground state of dilute neutron matter \cite{8}--\cite{10}. With auxiliary fields and Euclidean-time projection Monte Carlo, the ground state energy for 8, 12, and 16 neutrons in a periodic cube was calculated for a density range from 2\% to 10\% of normal nuclear density.

Another recent study considered low-energy protons and neutrons on the lattice at next-to-next-to-leading order in chiral effective field theory \cite{11}. Three-body interactions first appear at this order, and several methods were considered for determining three-body interaction coefficients on the lattice. The energy of the triton was calculated as well as low-energy neutron-deuteron scattering in the spin-doublet and spin-quartet channels using Lüscher’s finite volume method \cite{12}. In the four-nucleon system the energy of the $\alpha$-particle was computed using auxiliary fields and Euclidean-time projection Monte Carlo.

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Aspects of the hyperon-nucleon interaction

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The derivation of nuclear forces within chiral effective field theory (EFT) has been pursued extensively since the work of Weinberg \cite{Weinberg1967}. The most recent publications in this direction demonstrate that the nucleon-nucleon ($NN$) interaction can be described to a high precision in chiral EFT \cite{Epelbaum2006}\cite{Meissner2008}. For reviews we refer the reader to \cite{Epelbaum2010}\cite{Epelbaum2018}\cite{Epelbaum2019}.

The situation is different for baryon-baryon systems with strangeness. Only very few studies exist for the strangeness $S = -1$ sector, i.e. for the hyperon-nucleon ($YN$) interaction ($Y = \Lambda, \Sigma$) \cite{Haidenbauer2003}\cite{Haidenbauer2004}\cite{Haidenbauer2005}. The strangeness $S = -2, -3,$ and $-4$ channels have not been considered within chiral EFT at all.

In my talk I present results from our ongoing investigation of the baryon-baryon ($BB$) interaction in the strangeness $S = -1, -2, -3,$ and $-4$ channels, performed within the framework of chiral EFT \cite{Haidenbauer2010}\cite{Haidenbauer2011}\cite{Haidenbauer2012}\cite{Haidenbauer2013}. At leading order (LO) in the power counting the $BB$ interactions consist of four-baryon contact terms without derivatives and of one-pseudoscalar-meson exchanges, analogous to the $NN$ potential of \cite{Meissner2008}. The potentials are derived using $SU(3)$ flavor symmetry constraints. Then there are in total only six independent contact terms whose parameters, the low-energy constants (LECs), need to be determined by a fit to data. $SU(3)$ symmetry also interrelates the coupling constants at the various (pseudoscalar) meson-baryon-baryon vertices \cite{Haidenbauer2010}. The reaction amplitudes are obtained by solving a (single or coupled channels) Lippmann-Schwinger equation for the LO potential. We use an exponential regulator function to regularize the potential and apply cutoffs in the range between 550 and 700 MeV, cf. Ref. \cite{Haidenbauer2010}\cite{Haidenbauer2012} for details.

Five of the six contact terms appear in the strangeness $S = -1$ $BB$ interaction and can be fixed by a fit to low-energy $\Lambda N$ and $\Sigma N$ scattering data. It turned out that already at LO in chiral EFT a description of the available 35 $YN$ data can be achieved that is as good as the one for conventional meson-exchange models \cite{Haidenbauer2010}. Furthermore, also the binding energies of the light hypernuclei are predicted well within chiral EFT \cite{Haidenbauer2014}\cite{Haidenbauer2012}.

The additional sixth contact term appears only in the $\Xi N$ and $YY$ systems and can only be determined in the $S = -2$ sector. Adopting natural values for this additional contact term, a moderately attractive $\Lambda \Lambda$ interaction is obtained - in line with recent empirical information on doubly strange hypernuclei \cite{Haidenbauer2011}. Furthermore, we could show that the chiral EFT predictions are consistent with
the recently deduced doubly strange scattering cross sections.

With regard to the strangeness $S = -3$ and $-4$ sectors the occurring five contact terms are the same as those that were already fixed from our study of the $YN$ interaction. Thus genuine predictions can be made for the baryon-baryon interactions in those channels based on chiral EFT and the assumed $SU(3)_f$ symmetry [13].

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Compton scattering on Helium-3

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There has been much recent experimental and theoretical work regarding neutron electromagnetic polarizabilities. These quantities are fundamental properties of the neutron, and contain interesting information on the different mechanisms that contribute to neutron structure. (See Ref. [1] for a recent overview.) Until a couple of years ago most attention was focused on experiments that used Compton scattering on the deuterium nucleus (elastic or breakup) to obtain constraints on neutron polarizabilities. A series of experiments (see Ref. [1] for details) has allowed the extraction of useful information on the spin-independent polarizabilities, \( \alpha(n) \) and \( \beta(n) \). The classic chiral-perturbation-theory predictions for these quantities [2] seem to be in fairly good agreement with these extractions.

The four neutron spin polarizabilities (here denoted \( \gamma_1^{(n)} \)–\( \gamma_4^{(n)} \)) parameterize the \( O(\omega^3) \) response of the neutron to applied electric and magnetic fields, and are also calculable in \( \chi \)PT. As with \( \alpha(n) \) and \( \beta(n) \), their leading behaviour is given by constants of QCD: \( g_A, f_\pi, \) and \( m_\pi \). But higher-order effects in the spin polarizabilities are known to be sizeable [3]. The extraction of information on \( \gamma_1^{(n)} \)–\( \gamma_4^{(n)} \) will therefore teach us about the interplay of different effects in neutron electromagnetic structure.

Recently we completed the first calculation of Compton scattering from the Helium-3 nucleus [4]. It showed that polarized Helium-3 targets provide an excellent opportunity to obtain information on the \( \gamma_i^{(n)} \). There are, at present, only two constraints on combinations of these quantities, so such experiments—which are within the projected capabilities of the HI\( \gamma \)S facility [5]—will significantly advance our knowledge of neutron structure.

In the low-photon-energy region, \( \omega \sim m_\pi^2/M \), with \( M \) the nucleon mass, a correct treatment of \( \gamma^3\text{He} \) scattering requires as input (1) a three-nucleon current operator that is consistent with the potential employed to bind the nucleus and (2) the fully-interacting Green’s function of the three-nucleon state. In the case of the \( A = 2 \) system it has been shown that such a treatment produces the correct Thomson limit for Compton scattering from the nucleus [6]. Carrying out an analogous treatment in the \( A = 3 \) case is an important future step.

But, observables are much more sensitive to polarizabilities, and especially to spin polarizabilities, at photon energies of order 100 MeV. In this regime the three-nucleon Green’s function can be evaluated in perturbation theory. This statement is contained in Weinberg’s power counting for the \( \gamma NNN \rightarrow \gamma NNN \) operator. This operator has been constructed up to \( O(e^2Q) \) [NLO] in Ref. [4],
and includes the same mechanisms that were demonstrated to be important in describing Compton scattering from deuterium in this energy range [7].

The resulting calculations of $\gamma^3\text{He}$ scattering matrix elements predict cross sections that are appreciably larger than those for deuterium in the same energy range. There is also a larger absolute sensitivity to $\alpha^{(n)}$ and $\beta^{(n)}$. Meanwhile the results for the double-polarization observables $\Sigma_z$ and $\Sigma_x$ (asymmetries for circularly-polarized photons on a longitudinally and transversely polarized target) are quite close to those predicted for a free neutron. Our calculation, which includes the full three-body wave function, as well as two-body currents, shows that the dominant effect in these asymmetries is Compton scattering from the unpaired neutron in the polarized $^3\text{He}$ ground state. In consequence, $\Sigma_x$ and $\Sigma_z$ show significant sensitivity to novel combinations of the $\gamma_i^{(n)}$.

The calculation of Ref. [4] is a first step towards understanding $\gamma^3\text{He}$ scattering. Several improvements are necessary if a precision extraction of neutron spin polarizabilities from data is sought. This work does, though, open up a new avenue through which information on neutron spin-structure can be obtained. The Bochum-Cracow group, led by Walter Glöckle, played a key role in the use of $^3\text{He}$ targets to measure neutron form factors (see, e.g., Ref. [8]). This expertise, used in concert with consistent chiral expansions for interactions and current operators, can, when applied to precise experimental data, lead to similar success in determining the neutron Compton amplitude.

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Chiral three-nucleon interaction and $^{14}$C beta decay

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The anomalously long lifetime of $^{14}$C, which makes possible the radiocarbon dating method, has long been a challenge to nuclear structure theory. The transition from the $J^P_i = 0^+, T_i = 1$ ground state of $^{14}$C to the $J^P_f = 1^+, T_f = 0$ ground state of $^{14}$N is of the allowed Gamow-Teller type, yet the known lifetime of $\sim 5730 \pm 30$ years is nearly six orders of magnitude longer than would be expected from typical allowed transitions in $p$-shell nuclei. The associated Gamow-Teller transition matrix element must therefore be accidently small, $M_{GT} \simeq 2 \times 10^{-3}$. This feature makes it a sensitive test for both nuclear interactions and nuclear many-body methods. Recently, it has been suggested [1] that the $^{14}$C beta decay transition matrix element should be particularly sensitive to the density-dependence of the nuclear interaction. The study in ref.[1] used a medium-dependent one-boson-exchange interaction modeled with Brown-Rho scaling, where the masses of the vector mesons decrease due to partial restoration of chiral symmetry at finite density.

In the present work [2], we have investigated in detail the role of density-dependent corrections to the nuclear interaction generated by the leading-order chiral three-nucleon force. Pauli-blocking effects by the nuclear medium introduce a particular density-dependence into the long-range one-pion and two-pion exchange interactions and also into the short-range NN-interaction. With the residual nuclear interaction $V_{low-k} + V_{NN}^{med}$ we perform a highly constrained shell-model calculation (including second order terms) for the ground state wavefunctions of the $^{14}$C and $^{14}$N nuclei. After examining the different contributions to the in-medium NN-interaction $V_{NN}^{med}$ we find that the large suppression of the $^{14}$C beta decay matrix element comes almost entirely from to the short-range component of the chiral three-nucleon interaction $\sim c_E$.

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14
Few-Body Physics with Resonant Interactions

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Particles with a large scattering length have universal low-energy properties that do not depend on the details of their interactions at short distances [1]. These properties include the existence of a geometric spectrum of three-body bound states (so-called Efimov trimers) and a discrete scale invariance. In the four-body sector, a new class of universal tetramer states connected to the trimer states was predicted and recently observed in few-body loss processes in an ultracold gas of Cs atoms [2]. Other applications of this approach include halo nuclei and shallow hadronic molecules.

First, we discuss the possibility to observe excited Efimov states in $^{20}$ halo nuclei [3]. Based on the experimental data, $^{20}$C is the only halo nucleus candidate to possibly have an Efimov excited state, with an energy less than 7 keV below the scattering threshold. Second, we investigate the structure of $^{20}$C and other $^{2n}$ halo nuclei. In particular, we calculate their matter form factors, radii, and two-neutron opening angles.

The mass and the likely quantum numbers ($J^{PC} = 1^{++}$) of the $X(3872)$ suggest that it is either a weakly-bound hadronic “molecule” or a virtual state of neutral charm mesons. Assuming the $X(3872)$ is a weakly-bound molecule, we calculate the phase shifts and cross section for scattering of $D^0$ and $D^{*0}$ mesons and their antiparticles off the $X(3872)$ in an effective field theory for short-range interactions [4]. It may be possible to extract the scattering within the final state interactions of $B_c$ decays and/or other LHC events.

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Heavy meson hadronic molecules

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Some hidden or open charmed mesons discovered in recent years were suggested to have exotic nature other than $c\bar{c}$ or $cq$. Among various exotics, hadronic molecules have specific features. In this talk, we discuss how to identify heavy meson hadronic molecules. 1) For a loosely $S$-wave bound state, the effective coupling constant can be related to the binding energy \cite{1}, which means the structure information of such a bound state is hidden in the coupling to its components. 2) A heavy meson hadronic molecule consisting of a light hadron and a heavy meson should have spin-multiplet partner, and the mass splitting should be almost the same as the heavy meson hyperfine splitting \cite{2}. 3) If there is an $S$-wave bound state, one would observe a repulsive-like sign of the $S$-wave scattering length in the attractive channel using lattice simulations \cite{3}. We give two examples, i.e., the $Y(4660)$ and the $D_s^*(2317)$.

Relating the effective coupling to the binding energy, we show that the data of the $Y(4660)$ \cite{4} support a $\psi'f_0(980)$ bound state interpretation \cite{5}. The enhancement at about 4630 MeV observed in the $\Lambda_c^+\Lambda_c^-$ \cite{6} can also be understood as the same state once taking into account the $\Lambda_c^+\Lambda_c^-$ $S$-wave final state interaction \cite{7}. Heavy quark spin symmetry predicts a pseudoscalar $\eta_c'f_0(980)$ bound state with a mass of $4616^{+5}_{-6}$ MeV and width of $\Gamma(Y_{\eta'} \rightarrow \eta_c'\pi\pi) = 60 \pm 30$ MeV. It can be searched for in the $B \rightarrow K\eta_c'\pi\pi$ and $B \rightarrow K\Lambda_c^+\Lambda_c^-$. There might already have been a signal of such a state in the BABAR data of the latter channel \cite{8}, but the bad statistics prevents us from making any decisive conclusion \cite{7}.

The scattering between charmed mesons and light pseudoscalar mesons are studied using unitarized chiral perturbation theory \cite{9}\cite{10}. The scalar and isoscalar $D_{s0}^*(2317)$ is generated as a bound state pole in the isospin limit, and as a pole in the second Riemann sheet when the isospin breaking is allowed. By studying its pole position and the $S$-wave $DK$ scattering length in the isoscalar channel, we propose two ways towards identifying the nature of the $D_{s0}^*(2317)$. Experimentally, we suggest to measure directly its decay width into the $D_s^+\pi^0$, which is of order of about 100 keV in the hadronic molecule picture \cite{9}\cite{11}, much larger than that obtained assuming a $c\bar{s}$ or a tetraquark structure. We also suggest to calculate the $DK$ scattering length using lattice simulations, the result for its
absolute value would turn out to be about 1 fm if there is an S-wave $DK$ bound state, and smaller if it has an elementary $c\bar{s}$ component. Thus the nature of the $D_{s0}^*(2317)$ could be measured directly in the lattice.

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Cusps in \( K \to 3\pi \) decays

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It has been pointed out by Cabibbo and Isidori [1] that the pion mass difference generates a pronounced cusp in \( K^+ \to \pi^0\pi^0\pi^+ \) decays, an accurate measurement of which may allow one to determine the combination \( a_0 - a_2 \) of S-wave \( \pi\pi \) scattering lengths to high precision. A first analysis of the data taken by the NA48/2 collaboration was performed in Ref. [2]. In order for this program to be carried out successfully, one needs to determine the structure of the cusp with a precision that matches the experimental accuracy.

Non-relativistic effective field theory is the appropriate systematic framework to analyze the structure of \( K \to 3\pi \) amplitudes and their dependence on the \( \pi\pi \) scattering lengths [3], as the non-relativistic Lagrangian directly contains the parameters of the effective range expansion of the \( \pi\pi \) scattering amplitude. This is in contrast to, e.g., chiral perturbation theory, where scattering lengths etc. are expanded in powers of the pion mass [4]; see also Ref. [5] for an alternative approach. As the \( \pi\pi \) scattering lengths are small, it is useful to perform a combined expansion in powers of the scattering lengths and a non-relativistic small parameter \( \epsilon \).

The power counting is set up such that pion three-momenta are counted as \( \mathcal{O}(\epsilon) \); the kinetic energies are therefore of \( \mathcal{O}(\epsilon^2) \), and consequently so is the mass difference \( M_K - 3M_\pi \). In this way, the non-relativistic region covers the whole decay region. It is convenient to formulate the non-relativistic approach in a manifestly Lorentz-invariant/frame-independent manner, which can be achieved by employing a non-local kinetic-energy Lagrangian for the pion fields of the form

\[
\mathcal{L}_{\text{kin}} = \hat{\Phi}^\dagger (2W)(i\partial_t - W)\Phi , \quad W = \sqrt{M_\pi^2 - \Delta} .
\]

(1)

Non-relativistic Lagrangians for \( K \to 3\pi \) as well as for the \( \pi\pi \) interaction are given by

\[
\begin{align*}
\mathcal{L}_K &= \frac{G_0}{2}(K^+_\pi \Phi_+ (\Phi_0)^2 + \text{h.c.}) + \frac{H_0}{2}(K^+_\pi \Phi_- (\Phi_0)^2 + \text{h.c.}) + \ldots , \\
\mathcal{L}_{\pi\pi} &= C_x (\hat{\Phi}^\dagger \Phi_+ (\Phi_0)^2 + \text{h.c.}) + \ldots ,
\end{align*}
\]

(2)

where the ellipses include higher-order derivative terms. The parameters \( G_0, H_0, C_x \), etc. have to be determined from a simultaneous fit of \( K^+ \to \pi^0\pi^0\pi^+ \) and \( K^+ \to \pi^+\pi^+\pi^- \) amplitudes to experimental data. \( C_x \) in particular is proportional to \( a_0 - a_2 \) up to isospin breaking corrections.

The non-relativistic representation of the \( K \to 3\pi \) amplitudes has been worked out up to \( \mathcal{O}(\epsilon^4, a\epsilon^5, a^2\epsilon^4) \). This representation is valid to arbitrary orders in the quark masses. As the approach is based on a Lagrangian framework,
constraints from analyticity and unitarity are automatically obeyed. Special care has to be taken for the representation of overlapping two-loop graphs, which have a particularly complicated analytical structure and which, for certain combinations of pion masses running in the loops, develop anomalous thresholds in the decay region.

The non-relativistic Lagrangian approach has been extended to include radiative corrections due to real and virtual photons [6]. Photons modify the singularity structure at threshold at $\mathcal{O}(\alpha)$, and therefore can have a significant effect on the scattering length extraction.

Similar cusp effects also occur in other decay channels, such as $K_L$, $\eta \to 3\pi^0$ [7] or $\eta' \to \eta\pi^0\pi^0$ [8]. The extent to which they affect the decay spectra however critically depends on the relative decay strength into the charged vs. the neutral final states, which turns out to be rather small for the $K_L$ and $\eta$ decays, but more promising in the case of the $\eta'$. Non-analytic effects due to $\pi\eta$ rescattering at the border of the Dalitz plot in the latter channel cancel and cannot be observed.

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Baryons as Relativistic Three-Quark Systems

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A selective review of the performance of relativistic constituent-quark models for low-energy baryon physics is given.

There has been a vivid development of constituent-quark models with regard to baryon spectroscopy over the past decade. At present one of the most realistic descriptions of the (low-lying) spectra of all light, strange, and charmed baryons in a unified framework is provided by the relativistic constituent-quark model (RCQM) relying on Goldstone-boson exchange (GBE) [1][2][3]. A similar attempt is, e.g., undertaken by the Bonn group [4]. The problem of level orderings in the \( N^* \) and \( \Lambda^* \) spectra appears to be resolved, while, most prominently, the \( \Lambda(1405) \) remains as a notorious problem [5].

What concerns the nucleon ground states, many phenomenological evidences are available from experiments. Certainly, a realistic quark model should be able to describe them. The electromagnetic structures of both the proton and the neutron are well predicted by the RCQM at low momentum transfers up to \( Q^2 \sim 4 \text{ GeV}^2 \) [6][7], i.e. within a range where the validity of a constituent-quark model can be expected. In this context it is essential to look at covariant results, and relativistic effects are important in any respects, even in low-momentum-transfer observables such as the electric radii and magnetic moments of the ground-state baryons [7]. While the Graz group produced these covariant results in point-form relativistic quantum mechanics, the Bonn group arrived at qualitatively similar predictions through a Bethe-Salpeter approach [8]. Essentially the same is true with regard to the axial and induced pseudoscalar form factors, \( G_A(Q^2) \) and \( G_P(Q^2) \), of the nucleons, which are naturally described by the GBE RCQM in agreement with phenomenology [9].

The situation is not yet so clear-cut with regard to baryon resonances. Only in recent years, first covariant results have become available from RCQMs for the various strong decay modes of light and strange resonances [10][11]. In general, the decay widths contain considerable relativistic effects but turn out to be too small, hinting to defects in the assumed decay dynamics. Nevertheless, one has obtained interesting insights into the resonance structures, leading to a partially new classification into flavor multiplets [12].

Most recent results from the RCQM concern the structures of strong meson-baryon interaction vertices [13] and the axial charges of the nucleon and \( N^* \) resonances [14].

In summary, relativistic constituent-quark models, if well done, appear to be capable to serve as an effective tool for a comprehensive description of the wealth of low-energy hadronic phenomena on a uniform basis.
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What have we learned about three-nucleon systems at intermediate energies?

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Three and four-body systems have been studied in detail at KVI and other laboratories around the world in the last few years. Two categories of reactions have been chosen to investigate these systems: i) elastic, transfer and break-up reactions in proton-deuteron and deuteron-deuteron scattering in which only hadrons are involved, and ii) proton-deuteron capture reaction involving real and virtual photons in the final state. In this presentation, I focus mainly on the reactions where only hadrons were involved.

Hadronic reactions excluding photons give a handle on effects such as those from three-body forces. Three-body forces are, though small, very important in nature. The effect of these forces have far-reaching consequences in many fields of physics. Even though a relatively good understanding of most phenomena in nuclear physics has been arrived at by only considering two-nucleon forces, high precision three-nucleon data have revealed the shortcomings of these forces. In the last few decades, the two-nucleon system has been thoroughly investigated both experimentally and theoretically. These studies have resulted in modern potentials which describe the bulk of the data in a large range of energy. This knowledge can be employed in a Faddeev-like framework to calculate scattering observables in three-body systems [1]. In regions and for the reactions in which the effects of Coulomb force are expected to be small or can be calculated accurately, and energies are low enough to avoid sizable relativistic effects, deviations from experimental data must then be a signature of, for instance, three-body force effects.

At KVI and other laboratories, various combinations of high-precision cross sections, analyzing powers and spin-transfer coefficients have been measured at different incident proton or deuteron beam energies between 100 and 200 MeV for a large range of scattering angles and for all the reactions mentioned above. Calculations based on two-body forces only do not describe the data sufficiently. The inclusion of three-body forces improve these discrepancies with data significantly. However, there are still clear deficiencies in the calculations. Due to the extended data set which has been made available, one should now be able to develop a similar approach like the partial-wave analysis in the two-nucleon sector to see where the possible problems might lie. Results of some recent measurements for a number of observables at intermediate energies performed at KVI and RIKEN for the elastic scattering [2]–[10] as well as for the break-up channel [11]–[16] in proton-deuteron system were presented. In addition, preliminary results were shown for the deuteron-deuteron scattering where all possible outgoing
channels were studied. For the four-body system, the theoretical developments are in their infancy. This makes this field of research very exciting as experimental results which have been obtained recently will be available to put to test the results of any ab-initio calculations for the four-body system.

The three-nucleon results presented in the talk have been obtained in a long-term collaboration between KVI and the Cracow and Katowice groups. The four-body systems were recently initiated at KVI with the addition of the IUCF group in the collaboration.

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Recent Results in Meson Photoproduction at ELSA

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The leading goal of the experimental program within the CBELSA/TAPS collaboration is to perform precise measurements on photoproduction of mesons on the nucleon in the mass region up to 2.5 GeV with the explicit inclusion of polarization degrees of freedom to extract information on the dynamics of the production process and on the baryon spectrum. Of crucial importance for a detailed understanding of the baryon spectrum is the measurement of single and double polarization observables to reduce the existing ambiguities in the partial wave analysis and to increase the sensitivity on small resonance contributions. The final goal is of course to get as close as possible to a complete data base, which would allow for a model-independent partial wave analysis. For a complete data base, allowing for this model-independent partial wave analysis, eight carefully chosen observables need to be measured [1].

Table 2: Observables in single pseudoscalar meson photoproduction. The observables in green have been measured in the past, the ones in blue are presently measured using a longitudinally polarized or unpolarized target and a linear, circular or unpolarized photon beam. Red: Single and double polarization observables accessible with a transversally polarized target and a polarized or unpolarized beam, not measuring the recoil polarization.

| Photon | Target | Recoil | Target–Recoil |
|--------|--------|--------|---------------|
|        |        |        |               |
| x      | y      | z      | x’ y’ z’      |
| x      | z      |        |               |

|            | Target–Recoil |
|------------|---------------|
| x’         |               |
| y’         |               |
| z’         |               |

Table 2 shows the observables accessible in single pseudoscalar photoproduction, which have been already measured (green) at ELSA for $p\pi^0$ [2] and $p\eta$ [3]. In blue shown are the observables that are presently measured with a longitudinally polarized target and linearly as well as circularly polarized photons. Preliminary results for the observables $G$ and $E$ in the channels $\gamma\vec{p} \rightarrow p\pi^0$ and $\gamma\vec{p} \rightarrow p\eta$ have been presented at the NSTAR2009 conference [4]. In red the observables are shown that become accessible using a transversally polarized target.

A complete data base for pseudoscalar meson photoproduction (the “complete” experiment) requires at least eight independent observables to be measured. Such complete information is not available at present, however, close to
thresholds or in the $P_{33}(1232)$ resonance region, where only a few partial waves contribute, an almost model-independent analysis can be performed. This has been demonstrated in the s- and p-wave determination at the $\pi^0$-threshold [5], where information on the differential cross section ($\frac{d\sigma}{d\Omega}$) and the photon beam asymmetry ($\Sigma$) were sufficient to determine the four s- and p-wave amplitudes ($E_{0+}, M_{1+}, M_{1-}$ and $E_{1+}$). Another nice example is the determination of the $E_2/M_1$-ratio of the $P_{33}(1232)$-resonance. Again, precise data on $\frac{d\sigma}{d\Omega}$ and $\Sigma$ for the reactions $\vec{\gamma}p \to p\pi^0$ and $\vec{\gamma}p \to n\pi^+$ have been used to determine the isospin 1/2 and 3/2 contributions of the four s- and p-waves [6]. One important constraint in this analysis is the Fermi-Watson theorem. The multipole amplitudes $M_{I+}^l$ are complex functions of the c.m. energy $W$. Below the two-pion production threshold, the Fermi-Watson theorem allows one to express the phases of the complex multipole amplitudes by the corresponding pion-nucleon scattering phase shifts. Above the two-pion production threshold a nearly complete data base is necessary to constrain the PWA-solutions. The new Crystal Barrel experiment at ELSA will make major contributions to the new meson-photoproduction data base.

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Faddeev Calculations in Three Dimensions

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An enormous effort has been made to understand the scattering of three nucleons in the energy regime below the pion-production threshold[1]. However, if one wants to understand the same reactions in the intermediate energy regime, the standard partial wave description, successfully applied at lower energies, is no longer an adequate numerical scheme due to the proliferation of the number of partial waves. In addition, a consistent treatment of intermediate energy reactions requires a Poincaré symmetric quantum theory [2]. Thus, the intermediate energy regime is a new territory for few-body calculations, which waits to be explored.

In this work two aspects in this list of challenges are addressed: exact Poincaré invariance and calculations using vector variables instead of partial waves. In order to carry out this work a simplification in the underlying force has been made, namely scalar nuclear forces are employed consisting of a superposition of an attractive and repulsive Yukawa force such that a two-body bound state at $E_d = -2.23$ MeV is supported. The three-body bound state (including scalar three-body forces) was computed in Ref. [3]. In Ref. [4] the non-relativistic Faddeev equations were solved directly as function of vector variables for scattering up to 1 GeV, and the feasibility as well as numerical reliability of the approach were established.

The Faddeev equation, based on a Poincaré invariant mass operator has been formulated in detail in [5] and has both kinematical and dynamical differences with respect to the corresponding non-relativistic equation. The formulation of the theory is given in a representation of Poincaré invariant quantum mechanics where the interactions are invariant with respect to kinematic translations and rotations [6]. The model Hilbert space is a three-nucleon Hilbert space (thus not allowing for absorptive processes). The method introduces the NN interactions in the unitary representation of the Poincaré group and allows to input e.g. high-precision NN interactions in a way that reproduces the measured two-body observables. Poincaré invariance and $S$-matrix cluster properties dictate how the two-body interactions must be embedded in the three-body dynamical generators. Scattering observables are calculated using Faddeev equations formulated with the mass Casimir operator (rest Hamiltonian) constructed from these generators.

To obtain a valid estimate of the size of relativistic effects, it is important that the interactions employed in the relativistic and non-relativistic calculations are phase-shift equivalent. We follow the suggestion by Coester, Piper, and Serduke (CPS) and construct a phase equivalent interaction from a non-relativistic 2N...
interaction [7][8] by adding the interaction to the square of the mass operator. Thus, differences in the relativistic and non-relativistic calculations first appear in the three-body calculations. Those differences are in the choice of kinematic variables (Jacobi momenta are constructed using Lorentz boosts rather then Galilean boosts) and in the embedding of the two-body interactions in the three-body problem, which is a consequence of the non-linear relation between the two and three-body mass operators. These differences modify the permutation operators and the off-shell properties of the kernel of the Faddeev equations. We studied three-body elastic scattering as well as breakup reactions [9][10], and found that especially in breakup reactions relativistic effects can be quite large (depending on the configuration) already at energies as low as 500 MeV. We also found that the Faddeev multiple scattering series converges rather rapidly once the projectile laboratory energy exceeds 1 GeV.

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Euclidean formulation of relativistic quantum mechanics

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We discuss preliminary work on a formulation of relativistic quantum mechanics that uses reflection-positive Euclidean Green functions or generating functionals as phenomenological input. This work is motivated by the Euclidean axioms of quantum field theory [1][2]. The key observations are (1) locality is not used to reconstruct the quantum theory and (2) it is possible to construct a fully relativistic quantum theory without performing an explicit analytic continuation.

Hilbert space vectors are represented by wave functionals $A[\phi]$ with inner product

$$\langle A|B \rangle := \sum_{j,k} a_j^* b_k Z[g_j - \Theta f_j]$$

where $a_j$ are complex constants, $f_j$ are real Schwartz functions on 4 dimensional Euclidean space with positive-time support, $\Theta$ is the Euclidean time-reflection operator, and $Z[f]$ is the Euclidean generating functional. Reflection positivity is the condition that $\langle A|A \rangle \geq 0$. For $\beta \geq 0$ and $a \in \mathbb{R}^3$ we define

$$T(\beta, a)A[\phi] := \sum_{j=1}^{n_a} a_j e^{i\phi(f_j, a)} \quad f_{j,\beta,a}(\tau, x) := f_j(\tau - \beta, x - a).$$

The square of the mass operator operating on a wave functional $A[\phi]$ is

$$M^2 A[\phi] = \left( \frac{\partial^2}{\partial \beta^2} + \frac{\partial^2}{\partial a^2} \right) T(\beta, a)A[\phi]|_{\beta=a=0}.$$

Solutions of the mass eigenvalue problem with eigenvalue $\lambda$ can be expanded in terms of an orthonormal set of wave functionals $A_n[\phi], \langle A_n|A_m \rangle = \delta_{nm}$:

$$\Psi_\lambda[\phi] = \sum_n \alpha_n A_n[\phi].$$

Simultaneous eigenstates of mass, linear momentum, spin, and z component of spin can be constructed from $\Psi_\lambda[\phi]$ using

$$\Psi_{\lambda,j,p,\mu}[\phi] = \int_{SU(2)} dR \int_{\mathbb{R}^3} \frac{da}{(2\pi)^3} e^{-ip \cdot Ra} U(R)T(0, a)\Psi_\lambda[\phi] D_{\mu j}^R[R]$$

where $U(R)$ rotates the vector arguments of $f_j(\tau, x)$ in $A[\phi]$. When $\lambda$ is in the discrete spectrum of $M$, $\Psi_{\lambda,j,p,\mu}[\phi]$ is a wave functional for a single-particle state that necessarily transforms as a mass $\lambda$ spin $j$ irreducible representation.
Products of suitably normalized single-particle wave functionals define mappings from the product of single-particle irreducible representation spaces of the Poincaré group to the model Hilbert space. Because these wave functionals create only single particle states out of the vacuum, their products are Haag-Ruelle injection operators \[ \text{[3][4]} \] for the two-Hilbert-space formulation \[ \text{[4]} \] of scattering theory. If we define \( \Phi(\phi) := \prod_k \Psi_{\lambda_k, j_k, \mu_k}^{p_k} \), \( \otimes g_k = \prod g_k(p_k, \mu_k) \), and \( H_f = \sum_k \sqrt{\lambda_k^2 + p_k^2} \), then scattering wave operator can be defined by the limit

\[
\Omega_{\pm} | \otimes g_k \rangle := \lim_{t \to \pm \infty} e^{iHt} \Phi e^{-iH_f t} | \otimes g_k \rangle.
\]

Using the Kato-Birman invariance principle \[ \text{[4]} \] to replace \( H \) by \( -e^{-\beta H} \) gives

\[
\Omega_{\pm} | \otimes g_k \rangle := \lim_{n \to \pm \infty} e^{-ine^{-\beta H}} \Phi e^{ine^{-\beta H}} | \otimes g_k \rangle.
\]

Since the spectrum of \( e^{-\beta H} \) is compact, for large fixed \( n \) \( e^{-ine^{-\beta H}} \) can be uniformly approximated by a polynomial in \( e^{-\beta H} \), which is easy to calculate in this framework. These steps provide a means to construct all single-particle states, all scattering states, and compute the action of the Poincaré group on all single-particle states and \( S \)-matrix elements, using only the Euclidean generating functional as input. The advantages of this framework are the relative ease with which cluster properties can be satisfied, the close relation to the quantum mechanical interpretation of quantum field theory, and the ability to perform calculations directly in Euclidean space without analytic continuation.

We tested the general method for calculating scattering observables using a solvable quantum mechanical model of the two-nucleon system. These test calculations, which used narrow wave packets, a large finite \( n \) and a Chebyshev polynomial expansion of \( e^{inx} \), exhibited convergence to the exact transition matrix elements for a range of relative momenta between about 100 MeV up to 2 GeV. This success warrants further investigation of this framework.

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A novel approach to include the pp Coulomb force into the 3N Faddeev calculations

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The long-range nature of the Coulomb force prevents the application of the standard techniques developed for short-range interactions in the analysis of nuclear reactions involving two protons. In [1] we developed a novel approach to include the pp Coulomb force into the momentum space 3N Faddeev calculations. It is based on a standard formulation for short range forces and relies on the screening of the long-range Coulomb interaction. In order to avoid all uncertainties connected with the application of the partial wave expansion, inadequate when working with long-range forces, we used directly the 3-dimensional pp screened Coulomb t-matrix [2]. We demonstrated the feasibility of that approach in case of elastic pd scattering using a simple dynamical model for the nuclear part of the interaction. It turned out that the screening limit exists without the need of renormalization not only for pd elastic scattering observables but for the elastic pd amplitude itself.

In [1] we extended that approach to the pd breakup. Again we apply directly the 3-dimensional screened pp Coulomb t-matrix without relying on a partial wave decomposition. In [1] we demonstrated that the physical pd elastic scattering amplitude can be obtained from the off-shell solutions of the Faddeev equation and has a well defined screening limit. In contrast to elastic scattering, where the amplitude itself does not require renormalization, in case of the pd breakup the on-shell solutions of the Faddeev equation are required. They demand renormalization in the screening limit which can be achieved through renormalization of the pp t-matrices.

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Effects of the $\pi\rho$ exchange three-body force in proton-deuteron scattering

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The Tucson-Melbourne three-body force (3BF) has been investigated by calculating triton binding energy [1] and three-nucleon continuum [2]. Using the recent partial wave decomposition scheme PWD [3] of the $2\pi$ exchange type of 3BF we calculated the elastic observables of the pd elastic scattering in intermediate energies to show [4] the significant effects. We apply the same scheme not only to the $2\pi$ exchange 3BF but also to the $\pi\rho$ exchange one. In the pd elastic scattering at 135MeV/u we compare these theoretical predictions (cases without 3BF, with $2\pi$ 3BF and with $2\pi+\pi\rho$ 3BF) to recent data[5]. Although the effects of $\pi\rho$ exchange 3BF are almost invisible except for some observables ($A_{zz}$ etc.). The PWD technique will be used [6] to the 3BF of N$^3$LO version in the chiral field theory.

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