A new statistical image watermark detector in RHFM domain using beta-exponential distribution

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Abstract
The detection of watermarks can be achieved by statistical approaches. How to select robust modeling object, appropriate statistical model, and decision rules is one of the major issues in statistical image watermark detection. In this paper, we propose a new image watermark detector in robust fast radial harmonic Fourier moments (FRHFMs) magnitudes domain, wherein the Beta-exponential distribution model and locally most powerful (LMP) decision rule are used. We first investigate the statistical modeling of the robust FRHFMs magnitudes by the Beta-exponential distribution. It is shown that the Beta-exponential distribution model fits the empirical data more accurately than the formerly employed statistical distributions, such as the Cauchy, Weibull, BKF, and Exponential, do. Motivated by the statistical modeling results, we design a blind image watermark detector in FRHFMs magnitudes domain by using Beta-exponential distribution and LMP test. Also, we utilize the Beta-exponential model to derive the closed-form expressions for the watermark detector. We provide comparative experimental results to alternative approaches to demonstrate the advantages of the proposed image watermark detector.

Keywords Image watermarking · Beta-exponential distribution · FRHFMs domain magnitudes · Locally most powerful test

1 Introduction

With the rapid development of multimedia and Internet technologies, digital data can be easily acquired, represented, manipulated, and distributed without any quality degradation. As a result, intellectual property right protection has become a major issue worldwide. Owing to its effectiveness and practicality, digital watermarking is one promising solution for copyright protection and integrity authentication in an open network environment. Digital watermarking technology can be used in many applications such as source tracking, secret communication, broadcast monitoring, billing security, and so on. Two basic approaches regarding digital image watermarking include watermark decoding (Amirmazlaghani et al. 2015; Bhinder et al. 2018; Amirmazlaghani 2016) and watermark detection (Amini et al. 2017a, b; Sadreazami et al. 2015a; Amirmazlaghani 2016). In watermark decoding, the problem to be solved is the extraction of watermark information. While in watermark detection, we need to determine if particular watermark information exists in the given data using a binary decision criterion. This paper mainly studies the copyright protection of images, so watermark detection based on a binary decision criterion is sufficient to declare legal ownership. Watermark detection algorithms can be divided into two major categories base on whether the original signal is provided: non-blind detection (Singh 2017) and blind detection (Ahmaderaghi et al. 2018; Liu 2018). When watermark carrier signals obey Gaussian distribution, the correlation-based detection method is optimal. However, the research results have shown that digital signals in both the frequency and spatial domains do not obey Gaussian distribution (Sadreazami et al. 2015a). Hence, the detection method considering statistical properties of the carrier image coefficients can improve the correctness of watermark detection. Image watermark algorithms based on statistical model need to solve three basic problems, they are also important
indicators to measure the pros and cons of a watermarking scheme, namely robustness, imperceptibility, and capacity. Meanwhile, there is a mutually restrictive relationship between the three. Robustness is a core requirement of robust watermarking systems, and it is also a significant sign for judging the resistance of watermarking algorithms. The better a robust watermarking algorithm, the stronger its ability to resist attacks. It is a popular method to evaluate the robustness of detector by using receiver operating characteristic curve. The aim of this work is to enhance the balance between robustness and invisibility while making sure the watermark capacity. Invisibility means that the signal containing the watermark is not different from the original signal visually, and the peak signal-to-noise ratio (PSNR) is the most frequently used method to test imperceptibility in image watermarking. Capacity indicates the quantity of watermarking data that can be hidden. Hence, to maintain the robustness and invisibility of watermark signals in the relational database, we choose the multiplicative embedding method (Etemad and Amirmazlaghani 2017; Amini et al. 2017b; Rabizadeh et al. 2016) to achieve the embedded watermark.

According to different embedding domains, the image watermark is mainly separated into two categories: spatial (Sedighi et al. 2015) and frequency (Amirmazlaghani et al. 2015; Bhinder et al. 2018; Wang et al. 2021a; Amini et al. 2017a; Sadreazami et al. 2015a, 2015b; Amirmazlaghani 2016; Zebbiche et al. 2018; Etemad and Amirmazlaghani 2016). The spatial watermarking algorithm indicates that watermark data is directly inserted into the pixels of the original image. The operation of this method is simple, but the watermark is not robust enough to resist common signal attacks. In the frequency-domain watermark algorithms, watermarking is embedded by altering the image transform domain coefficients. Compared with spatial domain watermark embedding schemes, its robustness is improved to a certain extent. The most widely used transform domains include wavelet transform (Amirmazlaghani 2016; Amini et al. 2017b; Liu 2018), non-subsampled shearlet transform (NSST) (Wang et al. 2016a), non-subsampled contourlet transform (NSCT) (Bi et al. 2016; Wang et al. 2019), discrete cosine transform (DCT) (Dong et al. 2016), contourlet transform (Sadreazami et al. 2015a; Etemad and Amirmazlaghani 2016; Rabizadeh et al. 2016; Amirmazlaghani 2017), discrete shearlet transform (DST) (Ahmaderaghi et al. 2018), and dual tree complex wavelet transform (DTCWT) (Zebbiche et al. 2018; Barazandeh and Amirmazlaghani 2016). In recent years, researchers have proposed a watermarking method considering geometric invariants. Before embedding and detecting the watermark, the geometric invariants of the original image are determined. The geometric attack invariant features are utilized to ensure the optimal watermark embedding position. In 2000, Alghoniemy et al. (Alghoniemy and Tewfik 2000) used 7 Hu moment invariants to apply image moment to image watermarking technology for the first time. However, watermark detection methods that combine statistical model and moment invariant is still of great research significance. Therefore, this paper combines the anti-attack ability of geometric moments with statistical models to propose a robust watermark detector.

The accuracy of watermark detection is affected by many aspects. Besides watermark embedding objects, it also includes statistical model establishment, model parameter estimation and detector construction methods. Some statistical models are often used, mainly including the Bessel K Form (BKF) distribution (Amini et al. 2017a; Rabizadeh et al. 2016), t location-scale distribution (Etemad and Amirmazlaghani 2017, 2016), generalized Gaussian (GG) distribution (Liu 2018; Zebbiche et al. 2018), Cauchy distribution (Amini et al. 2017a; Wang et al. 2019), Gaussian mixture model (GMM) (Amirmazlaghani et al. 2015), Laplacian distribution (Ahmaderaghi et al. 2018), normal inverse Gaussian (NIG) distribution (Sadreazami et al. 2015a), and Weibull distributions (Dong et al. 2016; Barazandeh and Amirmazlaghani 2016). To consider the correlation between coefficients more fully, multivariate Cauchy distribution (Sadreazami et al. 2015b), multivariate generalized Gaussian (MVGG) model (Sedighi et al. 2015), and Hidden Markov Model (HMM) (Amini et al. 2017b, 2019) were proposed. Then, a valid closed expression based on Bayesian log-likelihood ratio test (LLRT) (Sadreazami et al. 2015a, 2015b) is established. In addition, the performance of watermark detection is affected by the accuracy of the parameter estimation algorithm. At present, expectation maximization (EM) and maximum likelihood estimation (MLE) methods are widely used in parameter estimation of statistical models. The function of detector is to detect whether there is hidden binary information from the observed image coefficients. Watermarking detection is often regarded as binary hypothesis testing of signals. In the past studies, the decision rules for constructing detectors include LLRT, the RAO test (Bi et al. 2016), generalized likelihood ratio test (GLRT) (Zebbiche et al. 2018), locally most powerful (LMP) test (Wang et al. 2019), and log-likelihood ratio test (LRT) (Ahmaderaghi et al. 2018; Amirmazlaghani 2017; Barazandeh and Amirmazlaghani 2016).

Although statistical models based digital watermark technology has been generally used in information security, the performance still has many room for improvement. First, embedding the watermark method directly by modifying the transform domain coefficients can not resist the geometric attack well. Second, a single distribution can not well describe the characteristics of coefficients distribution. Thirdly, the traditional parameter estimation methods have
Robust fast radial harmonic Fourier moment (FRHFMs) follows: experiment. The priority of the scheme are proved by the simulation with Beta-exponential distribution. The validity and superiority of the scheme are proved by the simulation experiment.

In this paper, we propose a locally optimal (LO) image watermark detector by modeling FRHFMs magnitudes with Beta-exponential distribution. The validity and superiority of the scheme are proved by the simulation experiment.

In conclusion, the characteristics of this method are as follows:

- Robust fast radial harmonic Fourier moment (FRHFMs) magnitudes is introduced to digital image watermarking domain, and is applied for embedding watermark message and developing watermark detector.
- We use Beta-exponential distribution to model the FRHFMs magnitudes, whose peak and heavy tail statistical characterization can be described accurately.
- The modified ML estimation algorithm is employed to calculate the parameters of the Beta-exponential distribution.
- Based on Beta-exponential distribution and LMP test, a locally optimal image watermark detector is developed. In addition, the closed-form expressions of the detection statistics are derived from the Beta-exponential model to verify the performance of the detector.
- The excellent performance of the proposed watermark detector is proved by a lot of experiments.

The remaining chapters are organized as follows. Section 2 mainly introduces the digital image watermarking technology based on a statistical model in recent years. The concept of the FRHFMs is briefly introduced, and the robustness of the FRHFMs magnitudes is studied in Sect. 3. Section 4 mainly studies the statistical characteristics of FRHFMs magnitudes and then uses Beta-exponential model to fit the moment magnitude coefficients. MMLE parameter estimation method is also given to improve the accuracy and reliability of the model. In Sect. 5, we detail the watermark embedding process of this scheme. Section 6 deduces the LO watermarking detector based on Beta-exponential model, and discusses the performance of the constructed detection method. In Sect. 7, we analyze the detection probability of the suggested watermarking detection method and contrast to other excellent detectors through simulation experiments. Section 8 draws conclusions.

2 Related work

The primary target of image watermark technology is to settle the balance problems among the transparency, payload, and robustness of watermark information. Therefore, watermark methods based on statistical models have been widely studied.

In paper (Etemad and Amirmazlaghani 2016), an additive watermark detector has been proposed according to Neyman–Pearson (N–P) criterion in contourlet domain. This method has modeled the contourlet transform coefficients using t location-scale distribution. In (Amirmazlaghani 2016; Singh 2017; Etemad and Amirmazlaghani 2017; Sedighi et al. 2015) (Etemad and Amirmazlaghani 2016; Bi et al. 2016; Dong et al. 2016; Barazandeh and Amirmazlaghani 2016; Alghoniemy and Tewfik 2000) new detection method based on 2D-GARCH model is proposed, which fully considering the dependencies between wavelet coefficients. Khalil et al. (Zebbiche et al. 2018) used GG distribution for modeling in DT-CWT domain and developed an additive watermark detector using a generalized likelihood ratio test. However, the additive watermark algorithm is not strong in robustness and invisibility. And multiplication embedding rule can improve the detection rate of watermarking. Therefore, the multiplicative watermarking method is more popular in watermarking embedding. Wang et al. (2019) developed a multiplicative watermark detector based on Cauchy distribution by using the LMP test criterion, which can well describe the local correlation of NSCT difference coefficients and improve the detection probability. Meanwhile, the robustness of the detector has been improved by using the multiplicative watermark embedding method. However, since Cauchy distribution is only suitable for symmetric data types, many coefficient features cannot be fully fitted. Therefore, it is necessary to select a more fit distribution to build the model. Sadegh et al. (Etemad and Amirmazlaghani 2017) developed an optimal detector using t location-scale distribution to model the contourlet coefficients, wherein the receiver operating characteristics (ROC) curve has been obtained for testing the detection probability of the suggested detector in the contourlet domain. In Barazandeh and Amirmazlaghani (2016), the digital image watermarking technology based on a binary hypothesis test was introduced. The statistical models such as Gamma, Rayleigh, and Weibull have been used to fit the DT-CWT coefficients. But when images are subjected to geometric attacks, the detector is less robust.

Sadreazami et al. (2015a) developed a blind watermark detector modeling the contourlet coefficients, and in which the transform coefficients obeying non-Gaussian distribution were modeled by normal inverse Gaussian distribution.
Ahmaderaghi et al. (2018) introduced a watermark inaccuracy of the single model to the coefficients. In introduced, which can accurately fit the distribution characteristic of contourlet coefficients, a multivariate Cauchy distribution was designed. To consider the inter-scale correlations of contourlet coefficients, a multiplicative watermarking scheme based on Bayesian log-likelihood ratio has been proposed in color images, which was detected according to the principle of likelihood ratio test. The inter-channel correlations in RGB color channels and the inter-scale correlations of image coefficients have been considered by adopting the hidden Markov model.

In Amini et al. (2017b), proposed a multi-channel blind watermark detector based on color images, which used a log-likelihood ratio decision criterion to obtain a valid closed expression for the test statistics. In paper Sadreazami et al. (2015b), a multiplicative watermarking scheme based on Bayesian log-likelihood ratio has been designed. To consider the inter-scale correlations of contourlet coefficients, a multivariate Cauchy distribution was introduced, which can accurately fit the distribution characteristics of the transform coefficients and eliminate the inaccuracy of the single model to the coefficients. In Ahmaderaghi et al. (2018), introduced a watermark detector based on Laplace distribution to model DST coefficients, which was detected according to the principle of likelihood ratio test. In Rabizadeh et al. (2016), a watermark detection method is designed according to maximum likelihood (ML) criterion, which used BKF distribution to model contourlet coefficients, and they analyzed its receiver operating characteristics by Monte Carlo simulations. However, transformation domain coefficients have a weak ability to resist various attacks.

Hosny et al. (2021) derived the new fractional-order multi-channel orthogonal exponent moments (MFrEMs), and proposed MFrEMs based color image watermarking algorithm. Zhou et al. (2021) proposed a novel robust reversible watermarking (RRW) scheme based on the discrete wavelet transform (DWT), in which the Zernike moments based geometric correction is utilized to predict attack parameters. Xia et al. (2019a) proposed a geometrically invariant color medical image null-watermarking scheme based on quaternion polar harmonic Fourier moments (QPHFM). Based on quantization technique and the distribution of moment magnitude, Hosny and Darwish (2019) inserted the watermark information into host color images by modifying the quaternion radial substituted Chebyshev moments (QRCMs) magnitudes. Hosny and Darwish (2018) presented a geometrically invariant color image watermarking method using Quaternion Legendre-Fourier moments (QLFM). These moments based image watermarking schemes generally have better robustness, but they all ignore the tradeoff among imperceptibility, robustness, and watermark capacity.

Liu et al. (2017) proposed a novel fractional transform, named the fractional Krawtchouk transform (FrKT), and developed an FrKT-based robust image watermarking scheme for copyright protection. Xia et al. (2019b) proposed the quaternion polar harmonic transforms (QPHTs) using quaternion theory and PHTs, and based on this proposal and chaotic mapping, they proposed a novel robust zero-watermarking scheme for color medical images. In Ma et al. (2020), Ma et al. proposed an accurate Polar harmonic Fourier moments (PHFMs) computation method based on Gaussian numerical integration (GNI), and designed a novel watermarking algorithm using accurate PHFMs and chaotic mapping. Prabha et al. (2019) presented a robust watermarking method depending on Quaternion Hadamard transform (QHT) and QR decomposition. Here, the QHT can preserve the relationship among the RGB component and color information. In Liu et al. (2021), the discrete fractional Krawtchouk was generalized with quaternion algebra to obtain the quaternion discrete fractional Krawtchouk transform (QDFrKT). Also, a color image watermarking scheme was designed to verify the effectiveness of the QDFrKT. Liu et al. (2020) proposed a novel color image watermarking algorithm based on quaternion polar harmonic transform (QPHT) and Bessel K form (BKF) distribution. Wang et al. (2021b) combined the octonion theory with continuous orthogonal moments (COMs) to propose the octonion continuous orthogonal moments (OCOMs), and applied the constructed OCOMs to the zero-watermarking algorithm for color stereoscopic images. Yami et al. (2020) proposed a new set of discrete orthogonal polynomials called fractional Charlier polynomials (FrCPs). Also, they defined the fractional discrete orthogonal Charlier moments (FrCMs) and applied FrCMs for image reconstruction and watermarking. In Xiao et al. (2020), a novel framework for deriving fractional-order DTM (FrDTMs) by the eigen-decomposition of kernel matrices was proposed, and some properties of the proposed FrDTMs were analyzed. Also,
image encryption and image watermarking were investigated to validate the superiorities of the proposed FrDTMs.

3 Robustness analysis of FRHFMs domain magnitudes

3.1 An introduction to FRHFMs

The gray image in polar coordinates is \( f(r, \theta) \), then traditional radial harmonic Fourier moments (RHFMs) on the unit circle can be expressed as

\[
\psi_{nm} = \frac{1}{2\pi} \int_0^{2\pi} \int_0^1 f(r, \theta) T_n(r) e^{-jm\theta} \, rdrd\theta
\]

where \( 0 \leq r < 1, 0 \leq \theta \leq 2\pi \), the RHFMs are denoted by \( \psi_{nm} \), \( m | m \geq 0 \) represents repetition and \( n | n \geq 0 \) is order. The angular function \( e^{-jm\theta} \) represents the Fourier exponential factor and \( T_n(r) \) represents the radial function

\[
T_n(r) = \begin{cases} 
\sqrt{\frac{2}{\pi}} r^n, & \text{while } n = 0 \\
\sqrt{\frac{2}{\pi}} \cos(n\pi r), & \text{while } n \text{ is even} \\
\sqrt{\frac{2}{\pi}} \sin((n+1)\pi r), & \text{while } n \text{ is odd}
\end{cases}
\]

The function set \( P_{nm}(r, \theta) = T_n(r) e^{-jm\theta} \) is orthogonal inside unit circular satisfies

\[
\int_0^{2\pi} \int_0^1 P_{nm}(r, \theta) P_{n'}(r, \theta) rdrd\theta = 2\pi \delta_{nn'} \delta_{mm'}
\]

where \( \delta \) is Kronecker delta and \( 2\pi \) indicates normalization factor. \( \delta_{nn'} \) and \( \delta_{mm'} \) represent Kronecker symbols, \( P_{n'}(r, \theta) \) is the conjugate of \( P_{n}(r, \theta) \).

The original grayscale image \( f(r, \theta) \) is reconstructed as

\[
f(r, \theta) = \sum_{n=-N}^{N} \sum_{m=-M}^{M} \psi_{nm} T(r) e^{jm\theta}
\]

where \( \psi_{nm} \) denotes the RHFM, and \( T(r) \) is the radial function.

To obtain better performance, we adopted a fast radial harmonic Fourier moments (FRHFMs) algorithm (Wang et al. 2018) based on FFT. The traditional RHFMS for calculating the inscribed circle mapping has rotation invariance. Therefore, the inscribed circle mapping is still used in the image watermark for FRHFMs (Wang et al. 2018, 2016b). In addition, FRHFMs provide higher image reconstruction quality, lower computational complexity, lower noise sensitivity, and magnitude invariance. The following describes the specific calculation method in the polar coordinate system.

In the unit circle, radial \( r_x \) and angular \( \theta_y \) are first divided into \( H \) equal parts, and the unit circle is nearly segmented into \( H^2 \) small regions. So converting Cartesian coordinate system of the image with pixel \( A \times A \) to \( f_p(r_x, \theta_y) \) (Wang et al. 2016b) in polar coordinates is

\[
f_p(r_x, \theta_y) = \left\{ \begin{array}{ll}
-\left[ r_x \times A/2 \times \sin \theta_y \right] + A/2 \\
+1, & \left[ r_x \times A/2 \times \cos \theta_y \right] + A/2
\end{array} \right.
\]

where \( r_x = \frac{x}{H}, \theta_y = \frac{2\pi y}{H}, x, y = 0, 1, \cdots, H - 1 \) and \( H = 4A \).

In addition, the FRHFMS \( \phi_{nm} \) obtained by FFT can be expressed as:

\[
\phi_{0,0} = \sqrt{2T}(H/2 + 1, H/2 + 1 + m), \quad k = n = 0
\]

\[
\phi_{n\pm 2k, m} = T(H/2 + 1 + k, H/2 + 1 + m) + T(H/2 + 1 + k, H/2 + 1 + m), \quad n = 2k, k = 1, 2, \cdots
\]

\[
\phi_{n\pm 2k-1, m} = j(T(H/2 + 1 + k, H/2 + 1 + m) - T(H/2 + 1 + k, H/2 + 1 + m)), \quad n = 2k - 1, k = 1, 2, \cdots
\]

where \( T \) is the FFT of function \( G_p(r_u, \theta_v) = f_p(r_u, \theta_v) \sqrt{pa/2} \) that moves zero-frequency component to the center of the spectrum.

3.2 Robust FRHFMs domain magnitudes

Because FRHFMS has geometric invariance, low time complexity, and strong anti-noise ability (Xia et al. 2019a), the FRHFMS amplitude of the image block is selected as the embedding position of the watermark in this paper. Firstly, the 512 × 512 pixel carrier image is split into 4×4 non-overlapping sub-blocks. Next, a second-order FRHFMs transform is performed in each block to get the FRHFMS magnitude of the host image. Figure 1 shows second-order magnitude domains of different images with the size of 640 × 384.

In this paper, to verify that the FRHFMs magnitudes possess better robustness than spatial domain, and is more suitable for watermark watermarking, the normalized error is introduced. In order to facilitate the comprehensive comparison and evaluation, the paper needs to standardize the initial information to ensure that the error values in space and magnitude domains have the same order of magnitude. Data Z-score normalization is the most classic normalization way, which maps the data uniformly to the interval \([0, 1]\). Then the normalized error is expressed as

\[
P = |I_0 - I_{\text{attack}}|
\]

\[
E = \frac{1}{n} \sum_{i=1}^{n} \frac{|P_i - \mu|}{\sigma}
\]
Here, \( I_{\text{attack}} \) denotes the attacked signal, and \( I_0 \) denotes the original unattacked signal. The mean of \( P \) is \( \mu \) and the standard deviation of \( P \) is \( \sigma \). The amount of signals is \( n \) and \( \sum \) is the cumulative sum. Table 1 records the normalized error values of the second-order FRHFMs magnitudes under different attacks. And the experimental images are three grayscale images of Lena, Barbara and Peppers.

It is well known that the smaller normalized error value means the stronger robustness. According to the normalized error formula, we test the robustness of four grayscale images: Lena, Barbara, Peppers, and Baboon. Figure 2 shows the error images of the spatial pixels and moment magnitudes.

Table 1 shows the normalized error results of FRHFMs magnitudes coefficients are smaller than that of image pixels. And Fig. 2 shows that the normalized error images of FRHFMs magnitudes are darker than that of image pixels. Both the subjective and objective results indicate

![Fig. 1 Original images and FRHFMs magnitudes images](image)

**Table 1** Normalize error between the original signal and the attacked signal

| Attack types                  | Lena                  | Barbara               | Peppers               |
|------------------------------|-----------------------|-----------------------|-----------------------|
|                              | FRHFMs magnitudes     | Host image            | FRHFMs magnitudes     | Host image            | FRHFMs magnitudes     | Host image            |
| JPEG Compression (QF = 90)   | 0.0244                | 0.0651                | 0.0252                | 0.0518                | 0.0280                | 0.0668                |
| JPEG Compression (QF = 30)   | 0.0339                | 0.0838                | 0.0314                | 0.1087                | 0.0371                | 0.1137                |
| Median filtering (9 x 9)     | 0.0119                | 0.0377                | 0.0375                | 0.0673                | 0.0136                | 0.0250                |
| Median filtering (5 x 5)     | 0.0111                | 0.0271                | 0.0260                | 0.0611                | 0.0076                | 0.0184                |
| Gaussian filtering (9 x 9)   | 0.0216                | 0.0518                | 0.0412                | 0.0700                | 0.0219                | 0.0383                |
| Gaussian filtering (5 x 5)   | 0.0207                | 0.0511                | 0.0389                | 0.0698                | 0.0212                | 0.0380                |
| Gamma correction \( \gamma = 2 \) | 0.0667                | 0.8482                | 0.0654                | 0.7824                | 0.0632                | 0.7892                |
| Gamma correction \( \gamma = 0.9 \) | 0.0341                | 0.8882                | 0.0343                | 0.8504                | 0.0325                | 0.8346                |

Best results are shown in bold
that the FRHFMs magnitudes are more robust than the spatial domain. Hence, this scheme selects local FRHFMs domain magnitudes to insert and detect watermarking information.

4 Modeling of robust FRHFMs domain magnitudes

4.1 Statistical analysis of robust FRHFMs domain magnitudes

The model effectiveness in the FRHFMs magnitudes affects the performance of the suggested watermark method, and one of the key steps in accurate modeling is to study the distribution characteristics of FRHFMs magnitude coefficients. First, each test image with a size of $512 \times 512$ is divided into $N_{\text{block}}$ non-overlapping sub-blocks of $n \times n$ in size, and the second-order FRHFMs of the image blocks is calculated to obtain the moment magnitudes of each block. The most stable moment magnitude in each block is selected to form $N_{\text{block}}$ magnitude coefficients.

Here, we take four typical grayscale images as examples and use distribution histogram and kurtosis value to analyze the edge statistical character of FRHFMs magnitudes. In the experiment, the carrier images with 512*512 pixels are chosen, and each image is split into 16,384 non-overlapping sub-blocks, and FRHFMs of each image block are calculated. The position (2, 2) of each block is chosen to form a total of 16,384-moment magnitudes and Fig. 3 provides the histograms of these moment magnitudes. We can clearly see from the histogram that the FRHFMs magnitude coefficients have the feature of sharp peak and heavy tail. The kurtosis values are 16.7605, 11.3751, 21.5277, and 15.1866, respectively, which are far greater than 3, indicating that they have non-Gaussian distribution characteristics. Therefore, a reasonable model is needed to accurately describe the characteristics of the magnitude coefficients.

4.2 Statistical modeling of robust FRHFMs domain magnitudes

The performance of the watermark detector largely determined by the modeling accuracy of the FRHFMs magnitudes, so Beta-exponential distribution (Wang et al. 2021a; Nadarajah and Kotz 2006) suitable for the magnitudes coefficients is selected to describe the statistical characteristics of the FRHFMs magnitudes in this scheme.

The exponential distribution is a very simple distribution function. However, the three-parameter Beta-exponential distribution appears, which effectively makes up for the deficiency of two-parameter exponential distribution. Then the probability density function (PDF) $f(x; k, m, n)$ of Beta-exponential distribution is

$$f(x; k, \alpha, \beta) = \frac{k}{C(\alpha, \beta)} e^{-\beta k x} (1 - \exp^{-k x})^{\alpha - 1} \quad (9)$$

where $k$ is the scale parameter, $\alpha$ and $\beta$ denote the shape parameters of Beta-exponential distribution function. Here, $C(\alpha, \beta) = \Gamma(\alpha)\Gamma(\beta)/\Gamma(\alpha + \beta)$ and $x_i > 0$ denotes the $ith$ random variable. And the distribution has the characteristics of high peak and heavy tail.
We next study how to precisely select the appropriate model to fit the FRHFM magnitudes. In Fig. 4, the modeling results of the magnitude coefficients, in which four images are fitted by different distribution functions. It is observed that the Beta-exponential model has a higher fitting degree than other distributions. Hence, the Beta-exponential distribution can delineate the FRHFM magnitudes more accurately.

In addition to the subjective tests above, we also compare the results based on the Kolmogorov–Smirnov (KS) metric. The KS test value is calculated as

$$Q_{ks} = \max |C_e(x) - C_r(x)|$$

(10)

Among them, $C_e(x)$ and $C_r(x)$ denote the referenced and empirical cumulative distribution function, respectively. The smaller the KS value, the better the fitting effect of the
distribution function used. In this paper, the FRHFMs magnitudes of Lena, Barbara, Peppers, and Boat images are fitted using different distributions. Then, the KS test values of each distribution are calculated separately. Table 2 shows that the KS test values of Beta-exponential distribution is the smallest. It is reasonable to believe that the Beta-exponential distribution can better describe the FRHFMs magnitudes.

### 4.3 Modified maximum likelihood parameter estimation

Parameter estimation is an important work in statistical model watermarking technology, and accurate parameters can ensure the performance of the watermarking detector, so an appropriate parameter estimation algorithm is very important. As optimization of the maximum likelihood estimation (MLE), the modified maximum likelihood estimation (MMLE) method has more generality. Moreover, when the sample data is small, the estimator obtained by MMLE convergence still has the advantages of consistency and unbiased. Since direct iteration may lead to some problems such as local optimality, the likelihood function is linearized by Taylor expansion method to accurately solve the estimate values. Hence, the MMLE method not only reduces the computational complexity but also improves the numerical accuracy. Kumar et al. (2016) demonstrated that MMLE is stable for the results of estimating finite populations, and the estimators of this method can be obtained easily with the use of computational tools.

Then, the parameter values of Beta-exponential model are estimated by MMLE method. Let random samples be $x_1, x_2, \cdots, x_h$, then according to the function $\Phi(x) = d\ln \Gamma(x)/dx$, the logarithmic likelihood function in Eq. (9) can be

$$
\ln L(x, \beta, k) = h \ln k - \ln C(x, \beta) + (x - 1) \sum_{i=1}^{h} \ln(1 - \exp(-kx_i)) - \beta k \sum_{i=1}^{h} x_i
$$

(11)

The specific process of the robust MMLE method is as follows (Wang et al. 2021a; Kumar et al. 2016):

**Step 1:** The likelihood equation is denoted by ordinal variables: $x_1 \leq x_2 \leq \cdots \leq x_h$.

**Step 2:** The linearized awkward function $e^{-kv}$ is derived from Taylor expansion method around the population quantile. Let function $g(x_i) = e^{-kx_i}$ expanded at point $x_i = t_i(t_i = E(x_i))$, and we can obtain

$$
g(x_i) = g(t_i) + \left(\frac{dg(x_i)}{dx_i}\right)_{x_i = t_i} \cdot (x_i - t_i) \approx \delta_i + \gamma_i x_i
$$

(12)

$$
\delta_i > 0, \quad i = 1, 2, \ldots, h
$$

Among them, $\delta_i = e^{-kx_i} + kt_i e^{-kx_i}$, $\gamma_i = -ke^{-kx_i}$. In order to calculate $x_i = t_i(t_i = E(x_i))$, the PDF of a virtual random variable $\nu$ is defined as $g(v) = e^\nu$ (Oral 2017), and $E(X) = \mu_i = F(x_i)$, where $F(u) = e^u$. When $h \geq 20$, $t_i$ can be obtained from the following formula (Vaughan and Tiku 2000): $\int_{0}^{\nu_i} g(v)dv = i/(h + 1)$. Therefore, $t_i$ can be derived as $t_i = \ln(i/(h + 1))$.

**Step 3** In the end, the likelihood equation is solved and the unique solution is obtained. The modified likelihood function of parameters $\alpha$, $\beta$, and $k$ can be obtained as follows

$$
\frac{\partial \ln L}{\partial \alpha} \approx -h \Phi(x) + h \Phi(x + \beta) + \sum_{i=1}^{h} \ln(1 - (\delta_i + \gamma_i x_i))
$$

$$
\frac{\partial \ln L}{\partial \beta} \approx -h \Phi(x) + h \Phi(x + \beta) - k \sum_{i=1}^{h} x_i
$$

$$
\frac{\partial \ln L}{\partial k} \approx \frac{h}{k} - \beta \sum_{i=1}^{h} x_i + (x - 1) \sum_{i=1}^{h} x_i (\delta_i + \gamma_i x_i) \frac{1 - (\delta_i + \gamma_i x_i)}{1 - (\delta_i + \gamma_i x_i)}
$$

(13)

Let Eq. (13) equals to 0, the simultaneous three equations and use the function $\Gamma(x) = \int_{0}^{\infty} t^{x-1}e^{-t}dt \approx \sqrt{2\pi} x^{x-1/2}e^{-x}$ to get the specific parameter values.

To prove the powerful performance of this method, we use Beta-exponential distribution to conduct Monte Carlo simulation experiments for MMLE and MLE methods. For

| Images       | Gamma distribution | BKF distribution | Exponential Distribution | Cauchy distribution | Weibull distribution | Beta-Exponential distribution |
|--------------|--------------------|------------------|--------------------------|--------------------|----------------------|-------------------------------|
| Barbara      | 0.1427             | 0.2027           | 0.1910                   | 0.2126             | 0.1100               | 0.0244                        |
| Peppers      | 0.0806             | 0.3355           | 0.1106                   | 0.1463             | 0.0840               | 0.0128                        |
| Boat         | 0.0891             | 0.1893           | 0.0707                   | 0.2031             | 0.0727               | 0.0206                        |
| Lena         | 0.0868             | 0.2862           | 0.1041                   | 0.1548             | 0.0859               | 0.0135                        |
| Average      | 0.0998             | 0.2535           | 0.1191                   | 0.1792             | 0.0882               | 0.0178                        |

Best results are shown in bold

| Images       | Gamma distribution | BKF distribution | Exponential Distribution | Cauchy distribution | Weibull distribution | Beta-Exponential distribution |
|--------------|--------------------|------------------|--------------------------|--------------------|----------------------|-------------------------------|
| Barbara      | 0.1427             | 0.2027           | 0.1910                   | 0.2126             | 0.1100               | 0.0244                        |
| Peppers      | 0.0806             | 0.3355           | 0.1106                   | 0.1463             | 0.0840               | 0.0128                        |
| Boat         | 0.0891             | 0.1893           | 0.0707                   | 0.2031             | 0.0727               | 0.0206                        |
| Lena         | 0.0868             | 0.2862           | 0.1041                   | 0.1548             | 0.0859               | 0.0135                        |
| Average      | 0.0998             | 0.2535           | 0.1191                   | 0.1792             | 0.0882               | 0.0178                        |

Best results are shown in bold
the convenience of comparison, the shape parameter \( \beta = 1 \) and scale parameter \( k = 1 \) are first fixed and the discrete random variables are generated, and then the shape parameter \( z \) is estimated. We set the sample size of 5000 and randomly generate 1000 groups of samples in the experiment. Estimates of different parameter \( z \) are generated for each group of sampling experiment and run independently for 1000 times. Table 3 reports the parameter estimation results using different approaches. It implies that MMLE method is better than traditional MLE in terms of estimation accuracy and computational complexity.

To compare the two algorithms more intuitively, Fig. 5 shows the average running time and average errors of the two algorithms in different sample sizes. It can be concluded that MMLE method is superior to MLE method in both estimation accuracy and computational complexity under different amount of sample sets. As the sample size increases, the parameter estimation error decreases and the average calculation time becomes longer. The main reason for such a change rule is that the small sample sizes cannot adequately represent the overall trend, resulting in the lack of accuracy of the estimation values. Furthermore, more sample data will inevitably lead to more calculation time. Hence, MMLE algorithm is used to calculate the parameters of Beta-Exponential distribution in this scheme.

### 5 Digital watermark embedding

This section describes the embedding part of digital image watermarking technology in detail. This embedding method selects the multiplicative method to hide watermark information in FRHFMs magnitudes with local geometric invariance. Figure 6 shows the embedding process of watermarking information. Where \( I = \{ f(x, y), 0 \leq x \leq N, 0 \leq Y \leq N \} \) is the original carrier image, \( f(x, y) \) denotes the image element, \( w = \{ w_l \in \{-1, 1\}, 1 \leq l \leq L \} \) are binary watermark bits with the same probability of \(+1\) and \(-1\). (i.e., \( \sum_{l=1}^{L} w_l = 0 \)), and \( I' \) is a watermarked image.

**Step 1** Divide the original image into \( 4 \times 4 \) blocks. The original carrier image \( I \) is divided into \( N \) non-overlapping, equally sized sub-blocks, each of which has a size of \( 4 \times 4 \). Then the \( N \) sub-blocks are sorted by high entropy values.

An imperceptible watermarked image can be effectually acquired by using the entropy masking model. The model shows that the high entropy regions of an image are highly complex. The larger the entropy value, the greater the uncertainty of image information sources. In addition, high entropy regions contain more image texture features, which is beneficial to resist noise and information hiding. Then the entropy (H) (Bhinder et al. 2018) formula is expressed as

\[
H = - \sum_{i=1}^{R} p(a_i) \cdot \log p(a_i)
\]

where \( a_i \) indicates an array of discrete possible events, and its probability is indicated by \( p(a_i) \). \( R \) represents the number of possible events.

**Step 2: High entropy blocks choice.** The first \( L \) high entropy blocks \( B_l(l = 1, 2, 3, \ldots L) \) are selected and the magnitude coefficients based on the second-order FRHFMs are calculated for each image block. The target point \((2, 2)\) of the magnitude coefficient blocks is changed by multiplicative rule to achieve the embedding of the watermark sequence \( w_l \). What is particularly noted here is that each amplitude is embedded with the same watermark bit. The embedded expression is written as

\[
y_i = \begin{cases} 
  x_i \cdot (1 + \lambda) & \text{if watermark bit } w_l = 1 \\
  x_i \cdot (1 - \lambda) & \text{if watermark bit } w_l = -1
\end{cases}, \quad i \in B_l
\]

Among them, \( y_i \) indicates containing watermark moment magnitudes, \( x_i \) indicates original moment magnitudes, and \( \lambda \) denotes an embedding strength (positive weighting factor), which can well adjust the balance between imperceptibility and robustness of watermarking.

### Table 3 The average estimated result of shape parameter \( z \)

| Actual shape parameters \( z \) | MMLE | | MLE |
|-------------------------------|------|------------------|------|
|                              | Average errors | Average estimated values | Average errors | Average estimated values |
| 5.0                           | 0.0220 | 5.0220           | 0.0246 | 5.0246 |
| 4.0                           | 0.0178 | 4.0178           | 0.0173 | 3.9827 |
| 3.0                           | 0.0115 | 3.0115           | 0.0151 | 3.0151 |
| 2.0                           | 0.0164 | 1.9836           | 0.0167 | 2.0167 |
| 1.0                           | 0.0098 | 0.9902           | 0.0116 | 0.9884 |
In order to keep watermark invisible, the watermarking strength is usually \( \lambda < 1 \).

The embedding strength \( \lambda \) is determined by the formula of watermark document ratio (WDR) (Etemad and Amir-mazlaghani 2017)

\[
WDR = 10 \log_{10} \left( \frac{\lambda^2 \sigma_w^2}{\sigma^2_{\xi}} \right) \tag{16}
\]

where the variance of the original image moment magnitudes is \( \sigma^2_{\xi} = \frac{1}{q} \sum_i x_i^2 \) and \( q \) represents the count of magnitude coefficients. \( \sigma^2_w \) represents the variance of the watermarking data, which is equal to 1 in this scheme. The image may be slightly distorted during the watermarking process, so WDRs are negatively related to the quality of images. According to Eq. (16), the embedding strength \( \lambda \) is given by

\[
\lambda = \sqrt{10^{\frac{WDR}{10}} \times \sigma^2_{\xi}} \tag{17}
\]

**Step 3:** Obtain watermarked image blocks. The formula for obtaining image blocks containing watermarking information is

\[
f_r(x, y) = f(x, y) - f_e(x, y) + f_e^0(x, y) \tag{18}
\]

Among them, \( f_r(x, y) \) and \( f_e^0(x, y) \) represents the image block reconstructed by original FRHFMs and modified FRHFMs, respectively, and \( f(x, y) \) is defined the original image block.

**Step 4:** Acquire watermarked image. The high entropy image blocks with the watermark are swapped with the original image blocks to get a watermarked image \( I' \).

### 6 Digital watermark detection

This section describes the construction of the watermark detector in detail. Actually, the purpose of watermarking detection is to detect whether the host image contains watermark information. For watermark detection, a detector based on the statistical property of moment magnitude coefficients helps to obtain accurate and reliable results. If \( I' \) is a watermarked image with \( n \times n \) pixels, then the specific process of watermarking detection is shown in Fig. 7.

The image \( I' \) with watermark is split into \( N \) non-overlapping sub-blocks of size 4 \( \times \) 4. After calculating the entropy value of each block, these blocks are sorted in descending order. The second-order FRHFMs of the first \( L \) high entropy blocks with the same number of watermarks are calculated to be the magnitude coefficients. Then, using the FRHFMs magnitudes in the same area as the embedding watermark position to form the target domain, and then \( L \) accurate moment magnitudes are obtained. A basic assumption is that after the watermark is embedded, the statistical distribution of FRHFMs magnitudes will not change. The watermark bit is defined as equal probability,
and the moment magnitudes are assumed to be isolated and uniformly distributed. To detect watermark information hidden in the FRHFMs magnitudes, a statistical watermark detector based on the Beta-exponential model is constructed through the LMP test.

6.1 Locally optimal watermark detector

When watermark embedding intensity is weak, the watermark detection can be distinctly expressed as a weak signal detection problem. Otherwise, it can be regarded as a strong signal detection problem. Since the strength of the watermark is suppressed by some attacks such as noise, rotation, and filtering, watermark detection under strong signals can also be considered as a smaller signal detection problem. The LO detector is specifically designed to detect weak signals, so it is very significant to introduce this detector in watermark detection scheme.

The watermarking detection problem is customarily regarded as a binary hypothesis testing problem, then the multiplicative watermark detection method is

\[
H_0 : y = x \\
H_1 : y = x \cdot (1 + \lambda w)
\]

(19)

Among them, \(H_1\) represents the alternative hypothesis that there is hidden information, and \(H_0\) is the null hypothesis. \(y = \{y_1, y_2, \cdots, y_L\}\) and \(x = \{x_1, x_2, \cdots, x_L\}\) are the original FRHFMs magnitude coefficients and the FRHFMs magnitude coefficients with watermark, respectively. \(w = w_i \in \{+1, -1\}, 1 \leq i \leq L\) is watermark information and \(\lambda\) is watermark strength. A watermark detector based on Neyman–Pearson lemma is presented

\[
\Lambda(y) = \frac{f_y(y|H_1)}{f_y(y|H_0)} \xrightarrow{H_1} \eta
\]

(20)

where \(\Lambda(y)\) represents the likelihood ratio and \(\eta\) denotes the threshold. Assuming the FRHFMs magnitudes of the watermarked image follows Beta-exponential distribution (optimal distribution). The conditional probability density functions under both assumptions are expressed as

\[
f_Y(y_i|H_0) = f_X(y_i) \\
f_Y(y_i|H_1) = \frac{1}{1 + \lambda w_i} f_X \left( \frac{y_i}{1 + \lambda w_i} \right)
\]

(21)

Watermarks can be considered as weak signals added to a strong background (the original image), so the statistical properties of original magnitude coefficients do not be changed by the embedded watermark. Then the logarithmic likelihood ratio is determined by

\[
\ln(l_{LOD}(y_i)) = \ln \frac{f_Y(y_i|H_1)}{f_Y(y_i|H_0)} = \sum_{i=1}^{L} \left[ \ln \frac{1}{1 + \lambda w_i} + \ln \frac{f_X(y_i)}{f_X(y_i)} \right]_{H_1} \leq \tau
\]

(22)

where \(\tau = \ln(\eta)\). Equation (22) is expanded to Taylor series at \(\lambda = 0\) based on the approximation of the likelihood ratio test, and the LO detector is obtained after ignoring the second and higher orders by

\[
l_{LOD}(y_i) \mid_{\lambda=0} = l(y_i) \mid_{\lambda=0} + \frac{\partial l(y_i)}{\partial \lambda} \mid_{\lambda=0} \cdot \lambda + o(\lambda) \\
\cong -\lambda w_i - \frac{\delta f_Y(y_i)}{f_X(y_i)} \cdot \lambda y_i w_i = -\lambda w_i + g_{LO}(y_i) \cdot \lambda y_i w_i
\]

(23)

where \(g_{LO}(y)\) denotes “locally optimal nonlinearity”. The derivation process of applying the PDF of Beta-exponential model to this formula is given by

\[
g_{LO}(y) = -\frac{\delta f_X(y_i)}{f_X(y_i)} = -\frac{f'_X(y_i)}{f_X(y_i)} \equiv \beta \cdot \frac{(x - 1) \cdot ke^{-\beta y}}{1 - e^{-\beta y}}
\]

(24)

Now using (23) and (24) in (22), the final statistical decision formula of the LO detector is expressed as
\[ l_{LOD}(y) = \sum_{i=1}^{L} -\lambda w_i + g_{LO}(y_i) \cdot \lambda y_i w_i \]

\[ = -\lambda \sum_{i=1}^{L} w_i + \sum_{i=1}^{L} \left( \beta k - \frac{(x - 1) \cdot ke^{-\lambda x}}{1 - e^{-\lambda x}} \right) \cdot \lambda y_i w_i \]

\[ = \sum_{i=1}^{L} \left( \beta k - \frac{(x - 1) \cdot ke^{-\lambda x}}{1 - e^{-\lambda x}} \right) \cdot \lambda y_i w_i \]

where the embedding strength \( \lambda \) can be obtained from Eq. (17), if \( l_{LOD} \) is more than the decision threshold \( \tau \), \( H_1 \) is accepted; otherwise, \( H_0 \) is accepted.

### 6.2 Performance analysis of the proposed watermark detector

Performance of watermark detection methods must be analyzed before they are applied in practice. Next, we test the performance of the watermark detector for the given image based on the detection probability \( P_{det} \) and the false alarm probability \( P_{fa} \). Generally, the false alarm probability \( P_{fa} \) is fixed. The optimal detector should minimize the probability of miss \( (P_m) \), that is, maximize the detection probability \( P_{det} = 1 - P_m \). The determination threshold \( \tau \) is gained by Naiman–Pearson criterion. This threshold minimizes the watermark missing probability \( P_m \) under the condition that the false alarm probability \( P_{fa} \) is bounded. We can get the following expression

\[ P_{fa} = P(T_{LOD}(y) > \tau | H_0) \]

where \( Q(x) = \frac{1}{2} \text{erfc}(\frac{x}{\sqrt{2}}) \), \( \mu_0 \) is the mean and \( \sigma_0 \) is the variance under the \( H_0 \) assumption, and the specific process of the calculation is shown in the Appendix A. \( \text{erfc}(\cdot) = 1 - \text{erf}(\cdot) \) represents the complementary error function. For a given \( P_{fa} \), the threshold expression is derived by

\[ \tau = \mu_0 + \sigma_0 Q^{-1}(P_{fa}) \]

If \( Q(x) = P_{fa} \), then \( Q^{-1}(P_{fa}) = x \). According to the above formulas, the relationship between \( P_{fa} \) and \( P_{det} \) (that is the ROC curve of the proposed detector) is expressed as

\[ P_{det} = Q \left( \frac{\tau - \mu_1}{\sigma_1} \right) \]

\[ = Q \left( \frac{\sigma_0 Q^{-1}(P_{fa}) - m_1}{\sigma_1} \right) \]

### 7 Experimental results

In this section, we first evaluate the performance of the proposed watermark detector on some standard grayscale images with different sizes from Computer Vision Group Test Images database s (http://decsai.ugr.es/cvg/dbimages/index.php), and various length pseudorandom watermark sequences. Then, we compare our approach with the state-of-the-art methods such as Etemad’s t LS(Etemad and Amirmazlaghani 2017), Rabizadeh’s BKF(Rabizadeh et al. 2016), Sadreazami’s Cauchy(Sadreazami et al. 2015b), Amirmazlaghani’s CT-GARCH(Amirmazlaghani 2019), Sadreazami’s NIG(Sadreazami et al. 2015a), Amini’s CHMM(Amini et al. 2019), Amirmazlaghani’s WT-GARCH (Qu and Peng 2008), and Amini’s WHMM(Amini et al. 2017b) based approaches.

In this work, all experiments are implemented in MATLAB R2016a, where the personal computer configuration is Windows 10 system and Intel(R) Xeon(R) CPU i5-3470 @ 3.20 GHz 8 GB memory.

### 7.1 Performance evaluation of the proposed watermark detector

#### 7.1.1 Accuracy

For the purpose of validating the theoretical expressions of the suggested detection method, the theoretical and experimental ROC curves are compared through simulation experiments. Figure 8 exhibits the averaged ROC curves for 96 test images, in which WDR varies from \(-30 \text{ dB} \) to \(-36 \text{ dB} \) in the range of \( 10^{-12} \leq P_{fa} \leq 10^{-4} \). In Monte Carlo simulation experiments, 100 binary watermark sequences with length of 4000 bits are generated randomly. As can be observed from the figure, the two ROC curves are basically coincident, indicating the availability of the closed-form theoretical expressions of statistical properties.

![Fig. 8 The experimental (solid) and theoretical (dashed) ROC curves](image-url)
7.1.2 Imperceptibility

Imperceptibility is one of the main requirements of watermark algorithms and the objective measure (Wang et al. 2021a) for assessing this feature is the PSNR between original and the watermarked image. Figure 9 shows the test results of the imperceptibility of watermarked images using our proposed watermarking method. We choose host images with $512 \times 512$ pixels as the test images. At the same time, the WDR is defined as $-40$ dB and a set of 1000-bit pseudorandom sequence is used. Figure 9c shows that the naked eye cannot notice the distinction between the watermarked and no-watermark image without the help of image processing technology. Moreover, PSNR values are all over 38, which is enough to show that our watermarking scheme has good imperceptibility.

7.1.3 Robustness

Next, we discuss the robustness of the presented LO detection method under different attacks, including JPEG compression, Gaussian filtering, cropping, and AWGN. For a given image, we compare the statistical decision formula $l_{LOD}$ with the decision threshold $\tau$ to get the detector response under a given false alarm probability ($P_{fa} = 10^{-8}$). And we give the average detection responses of the Lena image in 100 randomly generated binary watermark sequences with length of 6000 bits. Figure 10a demonstrates detection responses under JPEG compression attacks, in which the quality factor increases from 10 to 100. Figure 10b shows detection responses based on Gaussian filtering, where the window sizes are $3 \times 3$, $5 \times 5$, and $7 \times 7$. Figure 10c, d shows detection responses under cropping (cropping ranges from 2 to 20%) and AWGN attacks ($\sigma_n$ varies from 5 to 35), respectively. The results show that the Beta-exponential detector based on the LMP test can provide higher detection rates under

![Imperceptibility Analysis](image-url)
different attacks. Therefore, the proposed detector has strong robustness.

7.1.4 Capacity and time

In the simulations, we embed watermark sequences of different message lengths into twenty standard grayscale images (512 × 512 × 8 bits). Table 4 shows the relationship between average PSNR, average time of watermark embedding/detection, and watermark capacity. As shown in this table, the proposed watermark scheme provides lower time complexity, larger watermark capacity, and stronger imperceptibility.

7.2 Comparisons with state-of-the-art methods

In this section, we compare the proposed approach with eight state-of-the-art statistical image watermarking methods, including Etemad’s t LS (Etemad and Amirmazlaghani 2017), Rabizadeh’s BKF (Rabizadeh et al. 2016), Sadreazami’s Cauchy (Sadreazami et al. 2015b), Amirmazlaghani’s CT-GARCH (Amirmazlaghani 2019), Sadreazami’s NIG (Sadreazami et al. 2015a), Amini’s CHMM (Amini et al. 2019), Amirmazlaghani’s WT-GARCH (Qu and Peng 2008), and Amini’s WHMM (Amini et al. 2017b)-based methods. We selected these eight methods based on their similarities to the proposed approach, and based on the presence of sufficient algorithm descriptions (including implementation details and parameter settings) provided in the respective publications.

7.2.1 Probability of detection for varying watermark strengths

For investigating the proposed detector performance under various watermark powers, we take five different WDRs into account, ranging from −60 (dB) to −40 (dB). In Fig. 11, we plot the detection probabilities for different WDRs with the false alarm probability of 0.01, and test 6 Gy images of 512 by 512, including Barbara, Airplane, Boat, Couple, Lena, and Peppers. It can be noticed that as the watermark powers enhances, the detection probabilities of four detectors increase. As the same time, we can observe that the detection performance of the Beta-

| Watermark length (bits) | The average embedding time (seconds) | The average PSNR (dB) | The average detecting time (seconds) |
|------------------------|-----------------------------------|----------------------|----------------------------------|
| 10000bit               | 2.5189                            | 48.7496              | 2.5189                           |
| 5000bit                | 3.6118                            | 42.6870              | 3.2358                           |
| 10000bit               | 4.9220                            | 40.5230              | 4.0671                           |
exponential detector appears more powerful than other contourlet domain detectors (t-LS (Etemad and Amirmazlaghani 2017), BKF (Rabizadeh et al. 2016) and NIG (Sadreazami et al. 2015a) for different watermark strengths.

### 7.2.2 AUROC values under various attacks

In this part, we compare the suggested digital image watermark detector with other detectors based on multivariate Cauchy distribution (Sadreazami et al. 2015b), BKF distribution (Rabizadeh et al. 2016), and HMM (Amini et al. 2017b, 2019). The 100 experiments on 24 grayscale images with size of 256 × 256 are tested under pseudo-random watermark sequences of the same size as the compared algorithm. In Table 5, we give the average AUROC values of 24 experimental images in the range of $0 \leq P_{fa} \leq 10^{-4}$. As can be observed from this table, the suggested multiplicative Beta-exponential detector provides the highest AUROC value, indicating the detector has superior performance to that of other existing detectors.

| Methods                  | AUROC  |
|--------------------------|--------|
| WHMM (Amini et al. 2017b)| 0.9117 |
| BKF (Rabizadeh et al. 2016)| 0.7286 |
| Cauchy (Sadreazami et al. 2015b)| 0.8362 |
| CHMM (Amini et al. 2019) | 0.9934 |
| Proposed                | **0.9948** |

Table 5 AUROC values ($\times 10^{-4}$) obtained using different watermark detectors in the area $[0, 10^{-4}]$ (WDR = -42 dB)

Best result is shown in bold

In Table 6, the average AUROC values of multiple test images are obtained under Gaussian filtering, cropping, rotation, scaling, and gamma correction attacks. As can be observed from the table, although the performance of the detector in this paper is similar to that of HMM-based detector (Amini et al. 2019) under strong gamma correction and cropping attacks, on the whole, the proposed detector provides larger AUROC values than other detectors under both conventional attacks and geometric attacks.
To clearly prove the robustness of the suggested algorithm, Figs. 12–15 provide the average AUROC test results of this detector and other existing advanced detectors under various attacks, where the watermarked images undergo different types of attacks including JPEG compression, salt-and-pepper noise, median filtering, and AWGN. In Fig. 12, we can see that the suggested LO detector under JPEG compression attacks provides more robust properties than that of other detectors. Especially compared with the optimal algorithm (Amini et al. 2019) in existing detectors, the proposed detector is more likely to detect watermarks under strong attack condition with QF = 5.

Figure 13 shows that under AWGN attacks, the suggested LO detector provides the largest AUROC values among all the detection algorithms compared. It should be observed that when $\sigma_n = 40$, the proposed detector still outperforms to other methods. Salt-and-pepper noise and median filtering attacks are considered as common attacks when assessing the performance of any watermarking method. Figures 14 and 15 indicate that the suggested watermark detector has greater advantages than competing techniques when the watermarked image undergoes common signal attacks.

Next, we also compare the average performance of the presented watermark detector and other existing watermark detectors, including CT-GARCH (Amirmazlaghani 2019) and WT-GARCH (Qu and Peng 2008), as shown in Table 7. Here, we used 24 images with size of $512 \times 512 \times 8$ bits as host images, including Peppers, Living room, Lake, Pirate, Bridge, and Gold hill. Meanwhile, we randomly generated 24 different watermark

| Table 6 | The AUROC values ($\times 10^{-4}$) obtained under different attacks |
|---------|---------------------------------------------------------------|
| WHMM (Amini et al. 2017b) | BKF (Rabizadeh et al. 2016) | Cauchy (Sadreazami et al. 2015b) | CHMM (Amini et al. 2019) | Proposed |
| Cropping | | | | |
| 5% | 0.8567 | 0.69983 | 0.8310 | 0.9104 | **0.9267** |
| 10% | 0.7517 | 0.6118 | 0.7369 | **0.8045** | 0.7928 |
| Gaussian filtering | | | | |
| 3 $\times$ 3 | 0.8854 | 0.7009 | 0.7893 | 0.9007 | **0.9315** |
| 5 $\times$ 5 | 0.8032 | 0.6875 | 0.7245 | 0.8865 | **0.9047** |
| 7 $\times$ 7 | 0.7769 | 0.4765 | 0.6879 | 0.8644 | **0.8830** |
| Gamma correction | | | | |
| 0.9 | 0.8876 | 0.6998 | 0.8004 | 0.9032 | **0.9185** |
| 1.1 | 0.8132 | 0.6435 | 0.7993 | **0.9007** | 0.8974 |
| Rotation | | | | |
| 0.5$^\circ$ | 0.8921 | 0.8821 | 0.7832 | 0.9121 | **0.9370** |
| 1$^\circ$ | 0.8764 | 0.8054 | 0.7251 | 0.9003 | **0.9243** |
| 2$^\circ$ | 0.8021 | 0.7994 | 0.6673 | 0.8732 | **0.8821** |
| Scaling | | | | |
| 0.8 | 0.7591 | 0.5889 | 0.6554 | 0.8548 | **0.8940** |
| 1.2 | 0.7254 | 0.5982 | 0.6118 | 0.8003 | **0.8335** |

*Best result is shown in bold.*
sequences (128 × 256 bits), the range of the given $P_{\text{FA}}$ is [0,1]. Thus, 576 different combinations of watermark messages and host images (24 watermarks × 24 hosts) were used in the evaluation. The comparative experiments are carried out under the same experimental conditions. According to the above experimental results, the proposed detector demonstrates excellent performance against various attacks under different WDR values.

According to the above comparison results, we can clearly conclude that our proposed Beta-exponential distribution-based watermark detector achieves high work performance compared with some state-of-the-art methods. This improvement mainly comes from four aspects: first, we introduced FRHFMs to statistical image watermarking, and apply robust local FRHFMs magnitudes for inserting watermark signal and developing watermark detector. Second, we modeled the robust local FRHFMs magnitudes with Beta-exponential distribution, which can capture accurately the non-Gaussian and heavy-tailed statistical characterization of local FRHFMs magnitudes. Also, we estimate effectively the statistical model parameters of the Beta-exponential PDF by modified ML estimation approach. Third, we developed the blind statistical watermark detector using Beta-exponential distribution and locally most powerful test.

### 8 Conclusion

In this algorithm, we have used Beta-exponential distribution to fit the FRHFMs magnitude coefficients and designed an optimal watermark detector. The multiplicative method has been used to insert the watermarking information into the magnitude coefficients. To enhance the detection probability, we also used MMLE algorithm to calculate the model parameters. Further, the optimal detector is derived by the LMP test and assessed its performance. A theoretical expression of the detection has been verified through Monte Carlo simulation experiments in detail. Then the AUROC curves and the ROC curves of this algorithm are compared with other advanced detection algorithms. It has been observed that this detector presents a higher detection probability than other detection methods.

| Attack Types                  | WT-GARCH (Qu and Peng 2008) | CT-GARCH (Amirmazlaghani 2019) | Proposed |
|-------------------------------|-----------------------------|---------------------------------|----------|
| JPEG Compression (QF = 60)    | WDR = − 50 dB               | 0.8591                          | 0.8994   | 0.9282   |
|                               | WDR = − 45 dB               | 0.9413                          |          | 0.9853   |
| Gaussian Filtering (5 × 5)    | WDR = − 60 dB               | 0.6401                          | 0.9038   | 0.9539   |
| Median Filtering (5 × 5)      | WDR = − 50 dB               | 0.8504                          | 0.9688   | 0.9993   |
| Gaussian Filtering (5 × 5)    | WDR = − 50 dB               | 0.7582                          | 0.9780   | 0.9931   |
| and AWGN (σ = 10)             | WDR = − 50 dB               |                                  |          |          |
| Median Filtering (5 × 5) and AWGN (σ = 10) | WDR = − 50 dB   | 0.8149                          | 0.9334   | 0.9803   |
| Scaling with WDR = − 50 dB    | SF = 0.75                   | 0.8063                          | 0.9926   | 0.9986   |
|                               | SF = 2                      | 0.7814                          | 0.9157   | 0.9578   |
| Rotation with WDR = − 45 dB   | $\theta = 3$               | 0.8561                          | 0.9529   | 0.9633   |
|                               | $\theta = − 3$              | 0.8774                          | 0.9454   | 0.9768   |

Best result is shown in bold
Appendix A: Variance and mean of log-likelihood ratio under hypotheses $H_0$ and $H_1$.

In this section, the likelihood ratio provided can be regarded to obey the Beta-exponential distribution conditioned on each of the $H_0$ and $H_1$ hypotheses. We can calculate the variance and mean under the two hypotheses, i.e., $\sigma_0$, $\sigma_1$, $\mu_0$, $\mu_1$. An expression for the mean $\mu_0$ under the $H_0$ hypothesis is derived by

$$
\mu_0 = E(T_{LOD}(y)|H_0) = E(T_{LOD}|x) = E\left[\sum_{i=1}^{L} \left( \frac{\lambda x_i}{2} \right) (\beta k - \frac{(x-1) \cdot ke^{-\frac{\lambda x_i}{2}}}{1 - e^{-\frac{\lambda x_i}{2}}} \right) \right]
$$

$$
= \sum_{i=1}^{L} \left( \frac{\lambda x_i}{2} \right) \left( \beta k - \frac{(x-1) \cdot ke^{-\frac{\lambda x_i}{2}}}{1 - e^{-\frac{\lambda x_i}{2}}} \right) = 0
$$

where $\omega_i = \left( \frac{(x-1) \cdot ke^{-\frac{\lambda x_i}{2}}}{1 - e^{-\frac{\lambda x_i}{2}}} \right) \cdot \lambda x_i w_i$.

The variance under hypothesis $H_1$ is given by

$$
\sigma_1^2 = Var(T_{LOD}(y)|H_1) = E[(E(T_{LOD}(y)|H_1) - \mu_1)^2]
$$

$$
= \sum_{i=1}^{L} E \left[ \left( \frac{\lambda x_i}{2} \right) \left( \beta k - \frac{(x-1) \cdot ke^{-\frac{\lambda x_i}{2}}}{1 - e^{-\frac{\lambda x_i}{2}}} \right) \right]^2
$$

$$
= \sum_{i=1}^{L} \left( \frac{\lambda x_i}{2} \right)^2 \left( \frac{\lambda x_i w_i}{1} \right)^2 \cdot \lambda x_i w_i - \omega_i - v_i
$$

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Data availability
Enquiries about data availability should be directed to the authors.

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