Effect of confining pressure on pendulum wave spectra recorded in block media under seismic loading

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Abstract. Effect of external compression on seismic waves in rock mass was tested on one-dimensional models of block media subjected to impulsive loading. It is shown that a low-frequency pendulum wave appears at the shock wave head as a result of dispersion. The pendulum wave parameters are governed by the mass of blocks and by the yielding properties of block interfaces. External compression increases stiffness of block interfaces, which leads to an increase of the wave velocities and to a change of the wave spectra. The model experiments tested the possibility of external compression assessment from the parameters of pendulum wave spectra.

1. Introduction
The known concept represents a rock mass as a nested system of various-scale blocks parted by often soft and jointed interlayers [1]. Either static or dynamic, the block rock mass deforms owing to deformation of these yielding interlayers, which allows detecting low-frequency pendulum waves in the seismic response of rocks to impulsive loading [2, 3].

The waveguard properties of one-dimensional block models in the form of a chain of elastic beams with elastic interlayers were tested theoretically and experimentally [4–6]. A good approximation for the description of wave propagation in such media is the assumption that the blocks move as unyielding bodies. In this case, the low-frequency wave induced by impulsive loading can be sufficiently accurately described.

From the comparison of the modeling data and experimental results, the pendulum wave velocities and spectra are governed by the mass of the blocks and by the rheological properties of interlayers, which depend on the confining pressure. The interrelation between the confining pressure and seismic wave velocity offers an opportunity of ground pressure control using the well shooting data [7, 8].

This paper presents the studies on the effect of external compression of a block medium on the spectrum of seismic waves induce in the block medium by shock loading.

2. Mathematical model and the results of numerical study
It is known that propagation of waves in block media is possible in certain frequency ranges [9–11]. In the simplest model of a block medium—a one-dimensional chain of masses \( m \) connected by linear springs having stiffness \( k \) at intervals \( l \), the wave pass band is limited from above by a resonant frequency. Due to dispersion, the wave velocity depends on the wave length. Infinitely long waves
propagate at a maximal velocity. The resonant frequency of a chain of masses with springs, \( c \), and the maximal wave velocity in this chain are given by:
\[
\omega_0^c = 2 \sqrt{\frac{k}{m}}, \quad c^c_g = l \sqrt{\frac{k}{m}}.
\]  
(1)

The pass bands in the block medium model of a chain of elastic beams connected by springs were determined in [4]. It appeared that alongside with low-frequency zone of long waves in the frequency range \([0–\omega_0]\), there were an infinite number of shorter waves inside the beams.

Regarding the chain of elastic beams having the length \( l \), cross-sections \( F \) and mass \( m \), connected by springs with a stiffness \( k \):
\[
\omega_0 = 2 \sqrt{\frac{k}{m(1 + \beta/3)}}, \quad \omega_1 = \frac{\pi c}{l}, \quad \omega_2 = \frac{\pi c}{l} \left( 1 + \frac{4 \beta}{\pi^2 + 4 \beta} \right).
\]
Here, \( \beta = k l / (F E) \); \( E \) is Young’s modulus; \( c \) is the sound speed in the beams.

The maximum group velocities of the low-frequency waves are:
\[
c^g = l \sqrt{\frac{k}{m(1 + \beta)}}. \quad (2)
\]

The found resonant frequency and group waves of the low-frequency waves have similar values as in the chain of masses at small \( \beta \) fitting the interlayers of low stiffness.

The calculated spectra of shock waves in a chain of steel beams connected by springs under shock loading [4] are illustrated by the curve of spectral density of beam strain in Figure 1. The bolder curve depicts the numerical calculations for wave propagation in the chain of elastic beams, while the finer curve in the low frequency range \([0–\omega_0]\) describes the analytical solution. As is seen, the shock wave in the block model of elastic beams–spring consists of high-frequency resonance vibrations of individual blocks and low-frequency vibrations of their collective motion.

![Figure 1. Spectral strain density in the center of the 10th beam interrelated with amplitude of loading pressure \( P_0 \).](image)

The experimental studies described in the paper were aimed to record propagation of waves in a block model under shock loading, and to find low-frequency wave velocity and spectrum at different values of external compression.

The model of the block medium represented the vertical on-dimensional set of five marble blocks with mass \( m = 9.5 \) kg and dimension \( 150 \times 150 \times 150 \) mm arranged in hydraulic press. Accelerometers KD91 were attached to the blocks using plasticine. The hydraulic press created compression of the model to 60 or 2670 kPa as per the block face area. The pressure to the top block was transferred via a coupling with hammer arranged inside it. Brüel&Kjær type accelerometer 8309 was attached to the hammer to register the shock wave strength. All accelerometers were connected via Brüel&Kjær type amplifiers 2645 to AD converter E-1440 and to a computer for data download and storage.
The contacts of the test blocks were natural, produced by diamond sawing, without additional interlayers. The illustration of the vibration accelerations of marble blocks 1, 3 and 5 in the block assembly subjected to varied longitudinal compression is given in Figure 2.

In the blocks nearby the shock load application point, free vibrations arise. They attenuate as the shock wave travels, and the attenuation rate is directly proportional to the vibration frequency. The propagation length of high-frequency vibrations grows with increasing compression of the test assembly (Figure 2). Far from the shock point, a low-frequency pendulum wave caused by interaction of the beam through the yielding interlayers is detected at the wave head. From interpretation of the acceleration oscillograms, the wave velocities along the block assembly were determined.

![Oscillograms of vibration accelerations in marble blocks 1, 3 and 5 in test assembly subjected to compression by pressure](image1)

**Figure 2.** Oscillograms of vibration accelerations in marble blocks 1, 3 and 5 in test assembly subjected to compression by pressure (a) 78 kPa and (b) 2600 kPa under shock loading.

The low-frequency pendulum wave velocity, which is the maximum group velocity $c_g$ of waves in the block medium, was calculated by the time moments of maximum values of the first acceleration peak. This procedure follows from the calculations from the simple block model, i.e. a chain of masses connected by springs, according to which the pendulum wave propagation at high times is described by the Airy stress function. For this function, it is shown that the first acceleration peak travels at the velocity of the pendulum wave quasi-front. The pendulum wave velocity–compression $p$ curve in the test block model is presented in Figure 3 (curve $I$).

![Pendulum wave velocity in assembly of marble blocks versus compression under shock loading](image2)

**Figure 3.** Pendulum wave velocity in assembly of marble blocks versus compression under shock loading (curve $I$); curve $2$—interpolating relation (4).
It is seen from the experiment that the low-frequency pendulum wave velocity quickly grows first at the external compression increase to 1000 kPa. As compression is increased further, the wave velocity rises slower. An explanation to this phenomenon is given in [12]. After some tests of blocks with differently manufactured faces, the authors arrived to a conclusion that the increase in compression in the early stages led to a fast enlargement of contact areas between the neighbor blocks, which, in its turn, escalated stiffness of the contact zones and resulted in higher velocity of the low-frequency pendulum wave in accordance with (1). The compression dependence of pendulum wave velocities can be used in control of external compression change due to confining pressure.

It follows from (1) for the mass–spring block model that the velocity of the low-frequency waves in such medium is connected with the boundary pass frequency $v_g$ as:

$$c_g^c = l \omega_g^c / 2 = \pi l v_g^c .$$  

According to (3), the external compression control, alongside the low-frequency wave velocity, can use the data on the boundary frequency of their spectra. The spectrum analysis of the test data has proved this inference. By way of example, Figure 4a shows the spectral density $S$ of acceleration versus the frequency $v$ of block 3 under compression by 520 kPa. The shock wave travelling in the set of marble block is composed of low-frequency (to 8 kHz) pendulum waves and high-frequency (starting from 10 kHz) corresponding to free vibrations of these blocks; these vibrations are represented by many peaks.

The change in the boundary frequency $v_g$ of the pendulum wave spectrum with increasing external compression is illustrated in Figure 4b (curve 1). The shape of this curve is very similar to the pendulum wave velocity–compression curve in Figure 3. The experimental values of $v_g$ can also be used in ground pressure control.

![Figure 4. (a) Changing spectral density of acceleration of block 3 in model set under compression of 520 kPa; (b) boundary frequency of pendulum wave spectrum versus external compression (curve 1). Curve 2—interpolating relation (5).](image)

The experimental simulation [12] shows that in the test assemblies of blocks made of different materials, stiffness of interlayers grows with increasing compression proportionally to the square root of the compressive stress. Let $k = k_o \sqrt{\sigma}$, where $\sigma$ is the external compression stress. The value of $k_o$ can be found from expression (1) with an assumption that is hold true at the maximal compression stress $\sigma_{\text{max}} = 2.6$ MPa in a set of marble blocks. In this case, the pendulum wave velocity $c_g(\sigma_{\text{max}}) = 2400 \text{ m/s}$ and $k_o = 1.49 \cdot 10^6 \text{ N}^{1/2}$. Placement of this value in (1) produces an expression for the pendulum wave velocity at various compression of the experimental assembly:

$$c_g = l \sqrt{k_o / m} \sigma^{1/4} = 59.3 \sigma^{1/4} \text{ m/s}. \quad (4)$$
Here, \( l = 0.15 \) m and \( m = 9.5 \). The values calculated in this fashion are marked by rhombs in curve 2 in Figure 3. They agree with the pendulum wave velocities determined from the oscillograms of accelerations of blocks.

Similarly, from (1), the interpolating relation is constructed for the boundary frequency of the vibration spectrum of pendulum waves:

\[
v_g = \frac{\omega}{2\pi} = \frac{1.57}{\pi} \sqrt{\frac{k_0}{m}} \sigma^{1/4} = 197.8 \sigma^{1/4}
\]

depicted by curve 2 in Figure 4b. The experimental relationships of the pendulum wave velocities, boundary frequencies of spectra and the compression level of a block assembly agree quantitatively well with interpolating relations (4) and (5).

3. Conclusions
The spectrum of vibrations induced in a block medium by shock loading is governed by the frequencies of free vibrations of individual blocks and by the frequency spectrum of the collective motion of the blocks. As waves propagate along a block assembly, free vibrations die out quicker with higher frequency. In the distance from the shock point, a low-frequency pendulum wave is identified in vibrations of the blocks. It originates from interactions of the block through their yielding interlayers.

External compression has a great influence on the wave velocities and spectra, which is explained by increased compression of the active contacts between the block and by enlargement of total area of these contacts. The data on velocities and boundary frequencies of spectra of pendulum waves can be used to control the change of external compression.

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