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Cancellation of Leading Divergencies in Left-Right Electroweak Model and Heavy Particles

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Abstract

The fine-tuning principles are analyzed in search for estimations of heavy particle masses in the left-right (LR) symmetric model. The modification of Veltman condition based on the hypothesis of the compensation between fermion and boson vacuum energies within the LR Model multiplets is proposed. The hypothesis is supplied with the requirement of the stability under rescaling. With regard to these requirements the necessity of existence of right-handed Majorana neutrinos with masses of order of right-handed gauge bosons is shown and estimations on the top-quark mass which are in a good agreement with the experimental value are obtained.

1. Introduction

The model with the Left-Right gauge symmetry $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ was proposed [1, 2] as a possible candidate for the generalization of the Standard Model. It is rather attractive both for theoretical and experimental reasons since it treats left and right chiral fermions on equal footing at high energies and the $P$-parity in this theory is broken spontaneously. The possible physical signatures for right-handed currents can be estimated in high-precision low-energy experiments such as $\beta$–decay, muon decay, etc. [3, 4]. As well the search for right-handed interactions in deviations of high-energy experimental data from the SM predictions remains an important part of future collider programmes [4].

This model can be considered as a result of $SO(10)$-GUT symmetry breaking. Its embedding into a grand unified scheme [2] can be implemented consistently when the scale of the discrete symmetry breaking is taken much more than several TeV.

In the present paper we would like to apply the phenomenological principles of the Fine-Tuning (FT) to the Left-Right (LR) Model in order to estimate the heavy mass spectrum of the theory. These principles are based on the assumption that the theory is actually an effective one applicable for low energies. Let us formulate them:

- The strong fine tuning for the scalar field parameters (v.e.v. and their masses) consists in the cancellation of large radiative contributions quadratic in ultraviolet scales which bound the particle spectra in the effective theory (Veltman-type conditions - [6, 7, 8, 9]).
• The strong fine tuning for vacuum energies [7, 8, 9] deals with the cancellation of large divergencies quartic in ultraviolet scales which might affect drastically the formation of the cosmological constant.

• The RG stability of the cancellation mechanism under change of ultraviolet scale of effective theory is provided by the weak fine tuning [7, 8, 9].

Below on we examine the compatibility of these principles for the Left-Right Symmetric model. Their natural motivation for the LR Model can be found in a more fundamental underlying theory which is free from nonlogarithmic divergences (for example, in the SUSY extension of the LR Model).

It will be shown that the FT principles lead to reasonable values for $m_t$ for a wide range of scales $\Lambda$ and require the existence of the right-handed Majorana neutrinos with masses of order of right-handed gauge boson masses [9].

2. Particle Content of the Theory.

We consider LR Model with the $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B−L}$ gauge group. The electromagnetic charge is expressed in terms of quantum numbers of this group by the generalized Gell-Mann–Nishijima formula:

$$Q = T_{3L} + T_{3R} + \frac{Y}{2}$$

This theory contains three generations of Standard Model fermions with the necessary addition of right-handed neutrinos. Fermion assignments for $(T_L, T_R, Y)$ are as follows:

$$\begin{bmatrix} u \\ d \end{bmatrix}_{iL} = \left( \frac{1}{2}, 0, \frac{1}{3} \right); \quad \begin{bmatrix} u \\ d \end{bmatrix}_{iR} = \left( 0, \frac{1}{2}, \frac{1}{3} \right)$$

$$\begin{bmatrix} \nu \\ l \end{bmatrix}_{iL} = \left( \frac{1}{2}, 0, -1 \right); \quad \begin{bmatrix} \nu \\ l \end{bmatrix}_{iR} = \left( 0, \frac{1}{2}, -1 \right)$$

The gauge sector differs from the SM due to presence of right-handed gauge bosons. Besides, the Higgs sector of the model contains more particles than in the SM [3].
In order to generate fermion masses one needs the Higgs bidoublet with
the following quantum numbers:

\[ \Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{2} \end{pmatrix}, (0) \]

This field has to acquire nonzero v.e.v., saving however the electromagnetic
invariance of vacuum:

\[ <\Phi> = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix} \]

The existence of the abovementioned bidoublet is not enough to yield the
spontaneous symmetry breaking of the \( SU(2)_L \otimes SU(2)_R \) gauge group [4].

There may be an alternative choice of additional Higgs fields: a) Higgs
doublets:

\[ \delta_L = \begin{pmatrix} \delta_L^+ \\ \delta_L^0 \\ \delta_L^- \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 0 \\ 1 \end{pmatrix}; \delta_R = \begin{pmatrix} \delta_R^+ \\ \delta_R^0 \\ \delta_R^- \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ 1 \end{pmatrix} \]

(3)

They can generate heavy right-handed gauge bosons, but cannot interact
with fermions.

b) or Higgs triplets:

\[ \Delta_L = \begin{pmatrix} \Delta_L^{++} \\ \Delta_L^+ \\ \Delta_L^0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}; \Delta_R = \begin{pmatrix} \Delta_R^{++} \\ \Delta_R^+ \\ \Delta_R^0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \]

(4)

They can produce large \( M_{W_R} \) as well as Majorana masses for neutrinos.

We remind that the cancellation of all quadratic divergencies in the scalar
sector can be produced only by the compensation between the bosonic and
fermionic loops. As a result, in the case a) the fine tuning cannot be im-
plemented, since no fermions with usual quantum numbers can have Yukawa
couplings to these fields. In the case b) (with the triplet Majorana-Higgs
representation \( \Delta_R \)) Majorana masses for right-handed neutrinos should be
generated, having the same order of magnitude as right-handed gauge boson
masses. Then one can expect the cancellation of the quadratic divergences in
the scalar sector. The presence of the left-handed Higgs-Majoron fields \( \Delta_L \)
in general case is not necessary. But if these fields exist – for example in the
case of the manifest LR symmetry (see [4]), the vacuum expectation of the
left-handed Higgs-Majoron should be extremely small because of the upper bound $\sim 1\text{eV}$ on the left-handed Majorana neutrino masses \[1\].

Thus the FT in the LR Model leads to the unambiguous determination of the symmetry breaking sector of the theory.

3. Vacuum-Energy Cancellation Condition

It is well known that in the Left-Right symmetric model, as well as in all non-supersymmetric models the vacuum energy diverges like the forth power of the ultraviolet cutoff scale. This divergence contains contributions with opposite signs from bosons and fermions which can compensate each other. As far as we treat the model as a low-energy effective theory, the cutoff scales for bosons and fermions may differ. In a SUSY underlying theory these scales are related to heavy superparticle masses and their difference reflects the soft SUSY breaking. In this case it happens to be possible to cancel vacuum-energy contributions by the fine-tuning of the cutoff scales (fine-tuning for vacuum energies \[7\]).

We consider the universal cutoff $\Lambda_F$ for all fermions in order to preserve the horizontal symmetry and the universal boson cutoff $\Lambda_B$ implying a Grand Unification at high energies. Then the fine-tuning for vacuum energies at one-loop level reads:

$$\frac{\Lambda_B^4}{\Lambda_F^4} = \alpha^2 = \frac{4N_F}{2N_B + N_S} = \begin{cases} 
96/50 & \Delta_L \text{ exist} \\
96/44 & \text{without} \Delta_L 
\end{cases}$$

(Here $N_F = 24$ is the number of fermionic degrees of freedom, including quark colors (left- and right-handed neutrinos are considered here as one Dirac fermion), $N_B = 15$ is the number of the gauge bosons, $N_S = 20$; 14 –number of scalar fields, including Goldstone modes, with and without left Higgs-Majoron fields $\Delta_L$ respectively.

Thus we have $\alpha \approx 1.3856$ –if $\Delta_L$ exist or $\alpha \approx 1.4771$ without $\Delta_L$. One can see that $\alpha$ in the LR-model is more close to unity than in the Standard Model or in the Two-Higgs Standard Model \[7, 8, 9\].
4. Quadratic Divergencies Cancellation and Right-Handed Neutrino Masses.

Let us remind that the scalar sector in the Weinberg-Salam electroweak theory contains quadratic divergences in the tadpole diagrams and in the scalar particle self-energy. In order to fit the finite masses the cancellation for quadratic divergences, the fine-tuning \[6\] is required. This cancellation holds only if fermion and boson loops are tuned due to specific values of coupling constants. At the one loop level the condition:

\[
(2M_W^2 + M_Z^2 + m_H^2) = \frac{4}{3} \sum_{\text{flavors, colors}} m_f^2
\]  

removes quadratic divergences both from the Higgs-field v.e.v. and the Higgs boson self-energy if the universal momentum cutoff is implemented for all the fields.

The original Veltman condition is however not stable under rescaling since its renorm-derivative cannot vanish simultaneously for any choice of the cutoff \(\Lambda\) below the Plank scale \([10]\), i.e. the required cancellation of quadratic divergences can be provided only at a selected scale.

On the other hand, the usage of the universal scale for all bosons and fermions roughly implies the existence of large symmetry involving all the observable particles in one multiplet and therefore is not well motivated within the framework of effective theory.

We shall implement the Veltman’s idea in the LR model for the case with different cutoff scales for bosons and fermions. Let us examine the form of the Higgs-Yukawa and Higgs self-coupling lagrangian in the LR-model with different choice of particle content.

The general Higgs-Yukawa lagrangian for the bidublet field \(\Phi\) and the three fermion generations reads:

\[
L_{Yuk} = \overline{\psi}_L^i(F_{ij}\Phi + G_{ij}\tilde{\Phi})\psi_R^i + \overline{\psi}_R^i(F_{ij}^*\Phi^+ + G_{ij}^*\tilde{\Phi}^+)\psi_L^i
\]

(Here \(i, j = 1, 2, 3\))

Since in the LR-model one has more scalar fields than in the Standard Model, different ways of producing fermion mass hierarchy are available.
For our purposes we shall neglect the fermion masses of the first two generations, i.e. set $F_{ij} = G_{ij} = 0$ for $i, j = 1, 2$. In order to avoid problems, connected with the flavour-changing neutral currents, we shall set the constant $G_{33} = 0$. Hence only $F_{33} \equiv F \neq 0$ and the top-bottom mass difference is produced by the hierarchy of v.e.v’s: $m_t / m_b = v_1 / v_2$.

The Yukawa-type lagrangian for the Higgs-Majoron fields may be written only for the lepton sector, because only neutrinos can have Majorana masses.

It has the form (for right-handed fields):

$$L_{MYu} \sim -\frac{h_M}{2} \bar{\omega}(i\tau_2\Delta_R \frac{(1 - \gamma_5)}{2} - \Delta_R^+ i\tau_2 \frac{(1 + \gamma_5)}{2})\omega$$

Here: $\omega \equiv \psi_R + \bar{C}\psi_R^T$; and $\psi_R$ is the right-handed component of Dirac spinor for the weak isodoublet lepton fields;

$$\Delta_R = \begin{pmatrix} \delta^+_+ & \delta^+_- \\ \sqrt{2} & \delta^0 \\ \delta^0_+ & -\delta^+_- \sqrt{2} \end{pmatrix}$$

For the theory with $\Delta_L$ fields one has the similar lagrangian for the left Higgs-Majoron and left-handed leptons, but we shall not write it down, since the corresponding v.e.v. is very small and does not influence on the heavy particle spectrum.

The general form of Higgs potential of the model contains 15 (!) self-coupling vertices \[4\] and it is presented in the Appendix. For the simplicity we shall consider the situation, when ”non-diagonal” interactions (i.e. mixing fields $\Phi$ and $\Delta_{L,R}$ are suppressed. This condition is one-loop renorm-invariant for all “non-diagonal” couplings except for $a_1$. The latter one is assumed to be zero only at the scale $\Lambda$, and its non-zero renorm-group flow is to be taken into account.

Then the scalar potential is divided into two parts: the bidoublet potential and the triplet potential. The potential for the $\Phi$ fields contains 5 self-coupling vertices:

$$V_\Phi \sim l_1 Tr^2(\Phi\Phi^+) + l_2 [Tr^2(\tilde{\Phi}\Phi^+) + Tr^2(\tilde{\Phi}^+\Phi)] + l_3 Tr(\tilde{\Phi}\Phi^+) Tr(\tilde{\Phi}^+\Phi) +$$

$$+ l_4 Tr(\Phi\Phi^+) Tr[(\tilde{\Phi}\Phi^+) + (\tilde{\Phi}^+\Phi)] + a_1 [Tr(\Phi\Phi^+) \cdot Tr [(\Delta_L \Delta^+_L) + (\Delta_R \Delta^+_R)]]$$
Here \( a_1(\Lambda) = 0 \). The potential for the right Higgs-Majoron fields then reads (at the scale \( \Lambda \)):

\[
V_\Delta \sim \rho_1 tr^2(\Delta_R^\dagger \Delta_R^\dagger) + \rho_2 tr(\Delta_R^\dagger)^2 tr(\Delta_R^\dagger)^2.
\]

Using this lagrangian one can derive the modified Veltman equations cancelling quadratic divergences for the scalar tadpoles of the theory. These equations for the bidoublet fields \( \Phi \) are – for the case without \( \Delta_L \):

\[
\begin{align*}
f_1 &\equiv 4F^2 - \alpha[\frac{3}{2}g_L^2 + \frac{3}{2}g_R^2 + \frac{20}{3}l_1 + \frac{8}{3}l_3 + 4l_4 \frac{v_1}{v_2} + 2a_1] \\
f_2 &\equiv 4F^2 - \alpha[\frac{3}{2}g_L^2 + \frac{3}{2}g_R^2 + \frac{20}{3}l_1 + \frac{8}{3}l_3 + 4l_4 \frac{v_1}{v_2} + 2a_1]
\end{align*}
\]

and for the case with \( \Delta_L \):

\[
\begin{align*}
f_1 &\equiv 4F^2 - \alpha[\frac{3}{2}g_L^2 + \frac{3}{2}g_R^2 + \frac{20}{3}l_1 + \frac{8}{3}l_3 + 4l_4 \frac{v_1}{v_2} + 4a_1] \\
f_2 &\equiv 4F^2 - \alpha[\frac{3}{2}g_L^2 + \frac{3}{2}g_R^2 + \frac{20}{3}l_1 + \frac{8}{3}l_3 + 4l_4 \frac{v_1}{v_2} + 4a_1]
\end{align*}
\]

Here \( a_1 \approx 0 \). Hence if \( m_t \neq m_b \Rightarrow l_4 = 0 \). This condition is renorm-invariant. The requirement \( l_4 = 0 \) follows also from the independent cancellation of quadratic divergences in the self-mass diagrams for fields \( \Phi \).

The modified Veltman condition for the right-handed Higgs-Majoron fields \( \Delta_R \) takes the form:

\[
2h_M^2 = 3\alpha[2(2g^2 + g'^2) + 32\rho_1 + 16\rho_2].
\]

Analyzing the modified Veltman conditions for the LR model, one can find that in contrast to the Standard Model, equations (7), (8) do not yield any bounds from below for the \( t \)-quark mass, because not all \( l_i \) constants must have positive values (for example \( l_3 \) may be negative). However such bounds can be obtained for the right-handed Majorana neutrinos.

From the positive definiteness of the Higgs-Majoron potential one can get that \( \rho_1 > 0, \rho_1 + \rho_2 > 0 \). Then from the modified Veltman condition it comes out that

\[
2h_M^2 \geq 6\alpha(2g^2 + g'^2)
\]

Thus right-handed Majorana neutrinos must have masses of the same order of magnitude as right-handed gauge bosons. Taking \( M_R \) evaluations from
different experiments one obtains the corresponding lower bounds on $m_{\nu_R}$ (see the Table 1).

Table 1

| Exp. data | $M_{W_R}$ | $m_{\nu_R}(\Delta_L)$ | $m_{\nu_R}$ | $m_{\nu_R}^3(\Delta_L)$ | $m_{\nu_R}^3$ |
|-----------|-----------|-----------------------|-------------|-----------------------|-------------|
| $\Delta m_K^+$ | 800 | 2.5 | 2.3 | 1.44 | 1.33 |
| $+B\bar{B}_d^+$ | 670 | 2.1 | 1.9 | 1.21 | 1.10 |
| $+b + \beta\beta$ | 740 | 2.31 | 2.2 | 1.33 | 1.26 |
| man. LR | 1400 | 4.4 | 4.0 | 2.52 | 2.3 |
| $\Delta m_K^+$ | 500 | 1.56 | 1.43 |
| $+B\bar{B}_d^+$ | 500 | 1.56 | 1.43 |
| $+\mu$ | 740 | 2.30 | 2.2 |
| man. LR | 1300 | 4.1 | 3.8 |
| $m_{1R} < 10MeV$, | 720 | 2.2 | 2.0 |
| Supernova | 16200 | 50 | 46 |
| Dir. search | 520; 610 | 1.6;1.9 | 1.59 |
| Rad.corr. | 439 | 1.36 | 1.25 | 0.80 | 0.74 |

The case of single heavy neutrino is represented in two central columns of the Table, the case of equal neutrino masses for all 3 generations - in two right columns. The left column of each pair corresponds to the theory with $\Delta_L$ field ($\alpha = 1.215$) while the right one - to the theory without $\Delta_L$ ($\alpha = 1.276$). The left two columns are taken from [1, 4, 12, 13].

One should bear in mind that all the bounds on right-handed boson masses are model dependent. All the data from experiments with strange mesons depend essentially on the right-handed Kobayashi-Maskawa matrix elements. For example the double $\beta$-decay data can be eliminated after fine-tuning in the corresponding leptonic mass matrix. However, all these bounds are valid under rather reasonable assumptions and may be used for estimations of the...
right-handed neutrino masses.

For the theory without left Higgs-Majoron ($\Delta_L$) the equality $g_L = g_R$ holds only at the GUT scale. Then $\alpha_{2L}(m_Z) = 0.0354$ implies $\alpha_{2R} = 0.0265$. It is taken into account in this table.

The main result of the quadratic fine-tuning in the Higgs-Majoron sector is that the absence of quadratic divergences leads to rather heavy right-handed Majorana neutrinos.

5. RG-Stability of Fine-Tuning and Top-quark Mass.

Let us apply the third of the Fine-Tuning principles (Weak Fine-Tuning) – the RG-stability of of quadratic divergencies cancellation– in order to obtain estimations on the top-quark mass. As it was shown in [7] the RG stability of the modified Veltman condition in the Standard Model can be achieved only due to different cutoff scales for bosons and fermions. The abovementioned three Fine-Tuning principles lead in the Standard Model to the following top- and Higgs-mass predictions: $m_t = 175 \pm 5$ GeV, $m_H = 210 \pm 10$ GeV. In contrast ot the Standard Model, the LR model contains more scalar degrees of freedom and its scalar self-couplings may have different signs. However, the providing of the RG-stability of leading divergencies cancellation leads to a rather narrow range for the $m_t$ values, which is compatible with the modern experimental data.

Let us consider the case without $\Delta_L$. The vacuum energy cancellation yields: $\alpha = 1.4771$. Using RG-equations (see Appendix 1) one can get (assuming at the scale $\Lambda$ that $g_L = g_R \equiv g$):

\[
Df_\Phi = 40F^4 - F^2(64g_3^2 + 36g^2 + \frac{4}{3}g'^2) - \frac{\alpha}{3}(640l_1^2 + 512l_1l_3 + 448l_3^2 + 1304l_2^2 + \\
+ 240l_1(F^2 - \frac{3}{2}g^2) + 96l_3(F^2 - \frac{3}{2}g^2) + 28.5g^4 - 96F^4) = 0
\] (11)

Excluding $l_1$ by using $f_\Phi = 0$:

\[
l_1 = \frac{3}{5\alpha} F^2 - \frac{9}{20} g^2 - \frac{2l_3}{5}
\]

one has quadratic equation for $F^2$:

\[
c_1F^4 + c_2F^2 + c_3 = 0;
\]
here:

\[ c_1 = 32\alpha - 8 - \frac{384}{5\alpha} \approx -12.727 \]

\[ c_2 = -64g_3^2 - 36g^2 - \frac{4}{3}g'^2 + \frac{576}{5}g^2 + 72g^2 + 36\alpha g^2 \]

\[ c_3 = -\alpha\left(\frac{144}{5}l_3 + 96l_2 + 110.7g^4\right) \]

It can be easily checked up that this equation can have positive solutions only for such values of gauge couplings which they have at energies much more than 100 GeV. Let us assume that at the scale \( \Lambda \) the left-handed and the right-handed couplings are equal: \( g_L = g_R \) (in the absence of \( \Delta L \) fields they have different RG-flows). Then the solutions of the above equations result in rather narrow range of possible values for \( m_t \) for different \( \Lambda \) (see two left columns of the Table 2).

In the case with \( \Delta L \) the vacuum energy cancellation yields \( \alpha \approx 1.3856 \) and the RG-flow for \( g_L \) is the same as for \( g_R \) (see Appendix 1). Then the equation for \( F^2 \)

\[ c_1F^4 + c_2F^2 + c_3 = 0; \]

has the coefficients:

\[ c_1 = 32\alpha - 8 - \frac{384}{5\alpha} \approx -19.085 \]

\[ c_2 = -64g_3^2 - 36g^2 - \frac{4}{3}g'^2 + \frac{576}{5}g^2 + 72g^2 + 36\alpha g^2 \]

\[ c_3 = -\alpha\left(\frac{144}{5}l_3 + 96l_2 + 110.2g^4\right) \]

Possible solutions are displayed in two right columns of the Table. They lead to more strict bounds for possible \( \Lambda \) scale and allowed values for \( m_t \).
Table 2. Masses of the $t$-quark in the LR Model.

| $\Lambda$ GeV | “$m_t(\Lambda)$” | $m_t(100 \text{ GeV})$ | “$m_t(\Lambda)$” | $m_t(100 \text{ GeV})$ |
|---------------|-------------------|------------------------|-------------------|------------------------|
| $10^{15}$     | 107–287           | 166–202                | 110–225           | 167–197                |
| $10^{14}$     | 110–287           | 167–205                | 113–224           | 169–199                |
| $10^{13}$     | 113–287           | 168–208                | 116–223           | 170–201                |
| $10^{12}$     | 116–285           | 169–211                | 121–221           | 172–203                |
| $10^{11}$     | 120–283           | 171–214                | 126–217           | 175–205                |
| $10^{10}$     | 125–279           | 173–218                | 133–212           | 178–207                |
| $10^{9}$      | 132–273           | 177–222                | 144–202           | 184–207                |
| $10^{8}$      | 141–264           | 181–225                | —                 | —                      |
| $10^{7}$      | 155–247           | 189–227                | —                 | —                      |
| $10^{6}$      | —                 | —                      | —                 | —                      |

The denotation “$m_t(\Lambda)$” means $g_t(\Lambda) \cdot 175 \text{ GeV}$.

To predict the top-quark mass $m_t(100 \text{ GeV})$ we need to use the RG flow:

$$F^2(\mu) = \left(\frac{g_3^2(\mu)}{g_3^2(\Lambda)}\right)^{8/7} \left(\frac{g_2^2(\mu)}{g_2^2(\Lambda)}\right)^{3/4} \left(\frac{g_{\prime}^2(\mu)}{g_{\prime}^2(\Lambda)}\right)^{17/84} \cdot$$

$$\cdot \left\{1 + \frac{5F^2(\Lambda)}{8\pi^2} \int^\Lambda \! dt \left(\frac{g_3^2(t)}{g_3^2(\Lambda)}\right)^{8/7} \left(\frac{g_2^2(t)}{g_2^2(\Lambda)}\right)^{3/4} \left(\frac{g_{\prime}^2(t)}{g_{\prime}^2(\Lambda)}\right)^{17/84}\right\}^{-1}.$$

In the Table 2 the largest and the smallest values of $m_t$ are shown. They correspond to the choice $l_3 = l_2 = 0$, while nonzero values of these self-couplings push two possible values of the top mass inside the interval. One can see that the above equation contains restrictions on the maximal possible values of the $l_3$ and $l_2$. For the gauge couplings the experimental input was taken as follows [11]:

$$\alpha_3(M_Z) \equiv \frac{g_3^2}{4\pi} = 0.118 \pm 0.007$$

$$\alpha_{em} \equiv \sin^2 \theta_W \frac{g_{\prime}^2(M_Z)}{4\pi} = (127.8 \pm 0.1)^{-1}$$

$$\sin^2 \theta_W(M_Z) = 0.2333 \pm 0.0002$$
One can notice that the experimental value \( m_t \approx 175 \) GeV is compatible with the range given in the Table 2. The presented estimations are rather sensitive to the input values of the gauge couplings, especially to \( \alpha_3 \).

6. Conclusion

We have shown that in the Left-Right symmetric Model as well as in the Standard Model with one and two Higgs doublets \([4, 5]\) the selection rule based on the vacuum fine-tuning can be implemented for the parameters of \( t \)-quark and Higgs-boson potentials. This rule requires the existence of the right-handed Majorana neutrinos and yields lower bounds on their masses. The FT conditions for the \( t \)-quark parameters lead to predictions of the \( t \)-mass in a good agreement with the experimental value. The approximate RG invariance, which is used for these predictions, is an important property of the fine-tuning conditions which otherwise do not acquire the universal meaning. We notice that in the LR model it could be very interesting to analyze a more general form of the scalar potential, using the available experimental bounds on its constants.

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APPENDIX 1. Scalar Potential in the Left-Right Symmetric Model.

This is the general form of the scalar potential in the LR model with bidublet and left and right Higgs-Majoron fields:

\[
V = -\mu_1^2 \left[ Tr(\Phi^+ \Phi) \right] - \mu_2^2 \left[ Tr(\Phi \Phi^+) + Tr(\bar{\Phi}^+ \bar{\Phi}) \right] - \\
- \mu_3^2 \left[ Tr(\Delta_L \Delta_L^+) + Tr(\Delta_R \Delta_R^+) \right] + \\
+ l_1 Tr^2(\Phi \Phi^+) + l_2 [Tr^2(\bar{\Phi} \Phi^+) + Tr^2(\bar{\Phi}^+ \bar{\Phi})] + \\
+ l_3 Tr(\bar{\Phi} \Phi^+) Tr(\bar{\Phi}^+ \Phi) + l_4 Tr(\Phi \Phi^+) [Tr(\bar{\Phi} \Phi^+) + Tr(\Phi^+ \bar{\Phi})] + \\
+ \rho_1 \left[ Tr^2(\Delta_L \Delta_L^+) + Tr^2(\Delta_R \Delta_R^+) \right] + \\
+ \rho_2 \left[ Tr(\Delta_L \Delta_L) Tr(\Delta_R^+ \Delta_L^+) + Tr(\Delta_R \Delta_R) Tr(\Delta_R^+ \Delta_R^+) \right] +
\]
\[
+ \rho_3 [Tr(\Delta_L \Delta_L^+) Tr(\Delta_R \Delta_R^+)] + \\
+ \rho_4 [Tr(\Delta_L \Delta_L) Tr(\Delta_R^+ \Delta_R^+) + Tr(\Delta_L^+ \Delta_R) Tr(\Delta_R \Delta_R)] + \\
a_1 [Tr(\Phi \Phi^+) \cdot [Tr(\Delta_L \Delta_L^+) + Tr(\Delta_R \Delta_R^+)]] + \\
a_2 \{ [Tr(\Phi \Phi^+) \cdot Tr(\Delta_R \Delta_R^+) + [Tr(\Phi^+ \Phi) Tr(\Delta_L \Delta_L^+)]] + \\
a_2^* \{ [Tr(\Phi^+ \Phi) \cdot Tr(\Delta_R \Delta_R^+) + [Tr(\bar{\Phi}^+ \Phi) Tr(\Delta_L \Delta_L^+)]] + \\
a_3 [Tr(\Phi \Phi^+ \Delta_L \Delta_L^+) + Tr(\Phi^+ \Phi \Delta_R \Delta_R^+)]] + \\
+ \beta_1 [Tr(\Phi \Delta_R \Phi^+ \Delta_L^+) + Tr(\Phi^+ \Delta_L \Phi \Delta_R^+)] + \\
+ \beta_2 [Tr(\bar{\Phi} \Delta_R \Phi^+ \Delta_L^+) + Tr(\bar{\Phi}^+ \Delta_L \Phi \Delta_R^+)] + \\
+ \beta_3 [Tr(\Phi \Delta_R \bar{\Phi}^+ \Delta_L^+) + Tr(\Phi^+ \Delta_L \bar{\Phi} \Delta_R^+)].
\]

APPENDIX 2. Renorm-Group Equations for the Left-Right Symmetric Model.

Case without left Higgs-Majoron fields:

\[
Dg' = \frac{10}{3} g^3 \\
Dg_L = -3g_L^3 \\
Dg_R = -\frac{7}{3} g_R^3 \\
Dg_3 = -7g_3^3 \\
DF = F \left( 5F^2 - 8g_3^2 - \frac{9}{4} (g_L^2 + g_R^2) - \frac{1}{6} g'^2 \right) \\
Dl_1 = 32l_1^2 + 16l_1 \cdot l_3 + 16l_3^2 + 64l_2^2 \\
-3l_1(3g_L^2 + 3g_R^2) + 12l_1 F^2 + \frac{9}{8} (g_L^4 + \frac{2}{3} g_L^2 g_R^2 + g_R^4) - 6F^4 \\
Dl_3 = 8l_1 \cdot l_3 + 16(l_1 + l_3) \cdot l_3 + 16l_2^2 - 3l_3(3g_L^2 + 3g_R^2) + \\
+ 12l_3 \cdot F^2 + 12F^4 + 6g_L^2 g_R^2
\]
\[
D a_1/a_1=0 = \frac{3}{4} g^4
\]

For the case with left Higgs-Majoron fields

\[
D g_{L,R} = -\frac{7}{3} g_{L,R}^3; \quad D g' = 4 g'^3
\]

(So the equality \( g_L = g_R \) holds for any energy.) All other equations are the same.

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