Higher Dimensional Reissner-Nordström-FRW metric

Chang Jun Gao\(^1\)\(^*\) Shuang Nan Zhang\(^1,2,3,4\)\(^†\)

\(^1\)Department of Physics and Center for Astrophysics, Tsinghua University, Beijing 100084, China (mail address)

\(^2\)Physics Department, University of Alabama in Huntsville, AL 35899, USA

\(^3\)Space Science Laboratory, NASA Marshall Space Flight Center, SD50, Huntsville, AL 35812, USA

\(^4\)Laboratory for Particle Astrophysics, Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100039, China

(October 18, 2018)

Abstract

By inspecting some known solutions to Einstein equations, we present the metric of higher dimensional Reissner-Nordström black hole in the background of Friedman-Robertson-Walker universe. Then we verify the solution with a perfect fluid. The discussion of the event horizon of the black hole reveals that the scale of the black hole would increase with the expansion of the universe and decrease with the contraction of the universe.

PACS number(s): 04.20.Cv, 04.20.Jb, 97.60.Lf

\(^*\)E-mail: gao cj@mail.tsinghua.edu.cn

\(^†\)E-mail: zhangsn@mail.tsinghua.edu.cn
I. INTRODUCTION

Black holes in non-flat backgrounds are an important topic because astrophysical black holes are not asymptotically flat but embedded in our real universe. In this respect, as early as in 1933, McVittie [1] found his celebrated metric for a mass-particle in the FRW (Friedman-Robertson-Walker) universe. It describes just the Schwarzschild black hole which is embedded in the FRW universe although there was no the notion of black hole at that time. In 1993, the multi-black-hole solution in the background of de Sitter universe was discovered by Kastor and Traschen [2]. In 1999, Shiromizu and Gen extended it to a spinning black hole [3]. In 2000, Nayak etc. [4, 5] studied the solutions for the Schwarzschild and Kerr black holes in the background of the Einstein universe. Recently, we extended the McVittie’s solution to the charged case [6].

On the other hand, with the development of string theory, black holes in higher dimensional spacetimes have come to play a fundamental role in physics. Furthermore, the possibility of black hole production in high energy experiments has recently been suggested in the context of the so-called TeV gravity. To predict some observational and experimental results, we need reliable knowledge about higher dimensional black holes. Exact solutions for higher dimensional black holes have been constructed by many authors [7-21].

Thus the aim of this paper is to extend our recent work in which the McVittie solution is generalized to the four-dimensional charged black hole to higher dimensions. Spherically symmetric, vacuum, asymptotically flat spacetimes and homogeneous, isotropic cosmological ones with fluid matter or cosmological constant can be treated easily in general relativity and give rise, respectively, to the Schwarzschild solution and the FRW or de Sitter spacetimes. However, solutions representing an isolated massive object, especially charged, embedded in an expanding universe are much more difficult to obtain, and fully explicit forms are not usually given (see, for example, the general discussion in [22]). Therefore in this paper, we would present the higher dimensional Reissner-Nordstrom-FRW metric by simply inspecting some known solutions. Then we verify the solution by substituting it into the
Einstein-Maxwell equations with a perfect fluid. Finally we study the evolution of the event horizon of the black hole.

II. HIGHER DIMENSIONAL REISSNER-NORDSTRÖM-FRW METRIC

The well-known four dimensional static Schwarzschild metric in the isotropic spherical coordinates system can be written as

$$ds^2 = -\left(\frac{1 - \frac{r_0}{x}}{1 + \frac{r_0}{x}}\right)^2 du^2 + \left(1 + \frac{r_0}{x}\right)^4 \left(dx^2 + x^2 d\Omega^2_2\right),$$

where the constant $r_0$ is related to the mass of the black hole. For our purpose, we write the McVittie solution which represents the Schwarzschild-FRW metric as follows

$$ds^2 = -\left[\frac{a^\frac{1}{2}}{(1+kx^2/4)^{\frac{1}{2}}} - \frac{r_0}{xa^2}\right]^2 du^2 + \left[\frac{a^\frac{1}{2}}{(1+kx^2/4)^{\frac{1}{2}}} + \frac{r_0}{xa^2}\right]^4 \left(dx^2 + x^2 d\Omega^2_2\right),$$

where $a = a(u)$ is the scale factor and $k$ is the curvature of the universe. When $r_0 = 0$, it recovers the FRW metric. On the other hand, when $a = const, k = 0$, it is just the Schwarzschild metric. For $k = 0, a = e^{Hu}, H$ is a constant, Eq.(2) represents the Schwarzschild-de Sitter metric. This form of Schwarzschild-de Sitter metric can be reduced to the following familiar form via coordinates transformations [6]

$$ds^2 = -\left(1 - \frac{r_0}{r} - H^2r^2\right) dt^2 + \left(1 - \frac{r_0}{r} - H^2r^2\right)^{-1} dr^2 + r^2 d\Omega^2_2.$$

Inspecting Eq.(1) and Eq.(2), we find that in order to obtain the Schwarzschild-FRW metric, we need only to do the following replacements in Eq.(1)

$$1 \rightarrow \frac{a^\frac{1}{2}}{(1+kx^2/4)^{\frac{1}{2}}},$$

$$\frac{r_0}{x} \rightarrow \frac{r_0}{xa^2}. \quad (4)$$
Now we wonder whether the above method of replacement is universal. So in the next let’s look for the higher dimensional version of McVittie solution.

For higher dimensional static Schwarzschild metric

\[
\begin{align*}
ds^2 &= -\left(\frac{1^n - r_0^n}{1^n + r_0^n} \right)^2 dt^2 + \left(1^n + r_0^n \right)^{\frac{4}{n}} \left(dx^2 + x^2d\Omega^2_{n+1}\right). 
\end{align*}
\]  

(5)

Make the above replacements in Eq.(5), then the higher dimensional Schwarzschild-FRW metric is achieved

\[
\begin{align*}
ds^2 &= -\left[\frac{a^\frac{2}{n}}{(1+kx^2/4)^\frac{2}{n}} - \frac{r_0^n}{x^n a^{\frac{2}{n}}}\right]^2 dt^2 + \left[\frac{a^\frac{2}{n}}{(1+kx^2/4)^\frac{2}{n}} + \frac{r_0^n}{x^n a^{\frac{2}{n}}}\right]^\frac{4}{n} \left(dx^2 + x^2d\Omega^2_{n+1}\right). 
\end{align*}
\]  

(6)

It is just the result we have obtained previously [20]. For the de Sitter version, Eq.(6) can also be turned to our familiar form via coordinates transformations

\[
\begin{align*}
\begin{align*}
ds^2 &= -\left(1 - \frac{r_0^n}{r^n} - H^2r^2 \right) dt^2 + \left(1 - \frac{r_0^n}{r^n} - H^2r^2 \right)^{-1} dr^2 + r^2d\Omega^2_{n+1}. 
\end{align*}
\end{align*}
\]  

(7)

So the method of replacement is likely universal. Let’s look for the four dimensional Reissner-Nordström-FRW metric in the following.

For the four dimensional static Reissner-Nordström metric

\[
\begin{align*}
\begin{align*}
ds^2 &= -\left[\frac{1^2 - r_0^2 + \frac{r_1^2}{x^2}}{(1 + \frac{r_0^2}{x^2} - \frac{r_1^2}{x^2})^2} \right] du^2 + \left[\left(1 + \frac{r_0}{x}\right)^2 - \frac{r_1^2}{x^2}\right]^2 \left(dx^2 + x^2d\Omega^2_2\right), 
\end{align*}
\end{align*}
\]  

(8)

where the constant \( r_1 \) is related to the charge of the black hole. Enlightened by Eq.(5), we make the following replacements

\[
\begin{align*}
1 &\rightarrow \frac{a^{\frac{1}{n}}}{(1+kx^2/4)^{\frac{1}{n}}}, \\
r_0 &\rightarrow \frac{r_0}{xa^{\frac{1}{n}}}, \\
x &\rightarrow \frac{x}{xa^{\frac{1}{n}}}, \\
r_1 &\rightarrow \frac{r_1}{xa^{\frac{1}{n}}}, 
\end{align*}
\]  

(9)

then we obtain the four dimensional Reissner-Nordström-FRW metric
\[ ds^2 = -\frac{\left[ 1 + \frac{a^2}{(1+kx^2/4)^2} - \frac{r_0^2}{x^2a^2} + \frac{r_1^2}{x^2a^2} \right]^2}{\left[ \frac{a^2}{(1+kx^2/4)^2} + \frac{r_0^2}{x^2a^2} \right]^2 - \frac{r_1^2}{x^2a^2}} du^2 \]

\[ + \left[ \frac{a^2}{(1+kx^2/4)^2} + \frac{r_0^2}{x^2a^2} \right]^2 - \frac{r_1^2}{x^2a^2} \left( dx^2 + x^2d\Omega_2^2 \right). \]

It is just the result we have obtained [6]. For de Sitter version, Eq. (10) can also be written in the Schwarzschild coordinates

\[ ds^2 = -\left( 1 - \frac{r_0}{r} + \frac{r_1^2}{r^2} - H^2 r^2 \right) dt^2 + \left( 1 - \frac{r_0}{r} + \frac{r_1^2}{r^2} - H^2 r^2 \right)^{-1} dr^2 + r^2 d\Omega_2^2. \]

Now it seems that the method of replacement is highly likely universal. We would admit it and conclude the higher dimensional Reissner-Nordström-FRW metric with it.

The higher dimensional static Reissner-Nordström metric can be written as

\[ ds^2 = -\frac{\left[ 1 + \frac{a^2}{(1+kx^2/4)^2} - \frac{r_0^2}{x^2a^2} + \frac{r_1^2}{x^2a^2} \right]^2}{\left[ \frac{a^2}{(1+kx^2/4)^2} + \frac{r_0^2}{x^2a^2} \right]^2 - \frac{r_1^2}{x^2a^2}} du^2 + \left[ \frac{a^2}{(1+kx^2/4)^2} + \frac{r_0^2}{x^2a^2} \right]^2 - \frac{r_1^2}{x^2a^2} \left( dx^2 + x^2d\Omega_2^2 \right). \]

Make the replacements in Eq. (9), then the higher dimensional Reissner-Nordström-FRW metric is obtained

\[ ds^2 = -\frac{\left[ 1 + \frac{a^2}{(1+kx^2/4)^2} - \frac{r_0^2}{x^2a^2} + \frac{r_1^2}{x^2a^2} \right]^2}{\left[ \frac{a^2}{(1+kx^2/4)^2} + \frac{r_0^2}{x^2a^2} \right]^2 - \frac{r_1^2}{x^2a^2}} du^2 \]

\[ + \left[ \frac{a^2}{(1+kx^2/4)^2} + \frac{r_0^2}{x^2a^2} \right]^2 - \frac{r_1^2}{x^2a^2} \left( dx^2 + x^2d\Omega_2^2 \right), \]

namely

\[ ds^2 = -\frac{\left[ 1 + \frac{a^2}{(1+kx^2/4)^2} - \frac{r_0^2}{x^2a^2} + \frac{r_1^2}{x^2a^2} \right]^2}{\left[ \frac{a^2}{(1+kx^2/4)^2} + \frac{r_0^2}{x^2a^2} \right]^2 - \frac{r_1^2}{x^2a^2}} du^2 \]

\[ + \frac{a^2}{(1+kx^2/4)^2} \left[ \frac{a^n}{x^n a^n} \right]_0^0 \left[ \frac{a^n}{x^n a^n} \right]_0^0 \left( dx^2 + x^2d\Omega_2^2 \right). \]

When \( r_1 = 0 \), the metric restores to the higher dimensional McVittie solution. When \( r_1 = r_2 = 0 \), it restores to the higher dimensional FRW metric. When \( a = const, k = 0 \), it
restores to the higher dimensional static Reissner-Nordström metric. When \( a = \text{const}, k = 1 \), the metric restores to the higher dimensional static Reissner-Nordström black hole in the Einstein universe. In one word, it covers all the known solutions with the background of FRW universe. For the de Sitter version, we show in the following it can be reduced to our familiar form. To this end, make variable transformation

\[
\begin{align*}
    r &= ax \left[ \left( 1 + \frac{r_0^n}{a^n x^n} \right)^2 - \frac{r_1^{2n}}{a^{2n} x^{2n}} \right]^{1/n},
\end{align*}
\]

(15)

where \( a = e^{Hu} \). Then Eq.(14) becomes

\[
\begin{align*}
    ds^2 &= - \left( 1 - \frac{4r_0^n}{r^n} + \frac{4r_1^{2n}}{r^{2n}} - H^2 r^2 \right) du^2 - \frac{2Hr}{\sqrt{1 - \frac{4r_0^n}{r^n} + \frac{4r_1^{2n}}{r^{2n}}}} dudr \\
    &\quad + \left( 1 - \frac{4r_0^n}{r^n} + \frac{4r_1^{2n}}{r^{2n}} \right)^{-1} dr^2 + r^2 d\Omega^2_{n+1}.
\end{align*}
\]

(16)

In order to eliminate the \( dudr \) term, we introduce a new time variable \( t \), namely, \( u \to t \)

\[
\begin{align*}
    u &= t - \int \frac{Hr}{\left( 1 - \frac{4r_0^n}{r^n} + \frac{4r_1^{2n}}{r^{2n}} - H^2 r^2 \right) \sqrt{1 - \frac{4r_0^n}{r^n} + \frac{4r_1^{2n}}{r^{2n}}}} dr,
\end{align*}
\]

(17)

Finally in the new coordinates system \((t, r)\), Eq.(16) is reduced to

\[
\begin{align*}
    ds^2 &= - \left( 1 - \frac{4r_0^n}{r^n} + \frac{4r_1^{2n}}{r^{2n}} - H^2 r^2 \right) dt^2 \\
    &\quad + \left( 1 - \frac{4r_0^n}{r^n} + \frac{4r_1^{2n}}{r^{2n}} - H^2 r^2 \right)^{-1} dr^2 + r^2 d\Omega^2_{n+1}.
\end{align*}
\]

(18)

Absorb the constant 4 by \( r_0 \) and \( r_1 \), we obtain the higher dimensional Reissner-Nordström-de Sitter metric in the Schwarzschild coordinates system

\[
\begin{align*}
    ds^2 &= - \left( 1 - \frac{r_0^n}{r^n} + \frac{r_1^{2n}}{r^{2n}} - H^2 r^2 \right) dt^2 \\
    &\quad + \left( 1 - \frac{r_0^n}{r^n} + \frac{r_1^{2n}}{r^{2n}} - H^2 r^2 \right)^{-1} dr^2 + r^2 d\Omega^2_{n+1}.
\end{align*}
\]

(19)

In the next section, we will show our solution Eq.(14) satisfies the Einstein-Maxwell equations.
III. VERIFICATION OF THE METRIC

In the last section, we deduced the higher dimensional Reissner-Nordström-FRW metric. In this section we will verify that it satisfies the Einstein-Maxwell equations. The Einstein-Maxwell equations can be written as [23]

\[ G_{\mu\nu} = 8\pi (T_{\mu\nu} + E_{\mu\nu}), \]
\[ F_{\mu\nu} = A_{\mu;\nu} - A_{\nu;\mu}, \]
\[ F_{\mu\nu}^\prime = 4\pi J^\mu. \]  

(20)

Here \( J^\mu \) is the current density of the charge. \( T_{\mu\nu} \) and \( E_{\mu\nu} \) are the energy momentum for the perfect fluid and electromagnetic fields, respectively, which are defined by

\[ T_{\mu\nu} = (\rho + p) U_\mu U_\nu + pg_{\mu\nu}, \]
\[ E_{\mu\nu} = \frac{1}{4\pi} \left( F_{\alpha\beta} F^{\alpha\beta}_{\nu} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right). \]  

(21)

where \( \rho \) and \( p \) are the energy density and pressure. \( U_\mu \) is the \((n+3)\)-velocity of the particles. \( F_{\mu\nu} \) and \( A_\mu \) are the tensor and the potential for electromagnetic fields.

Input the components of the metric Eq.(14) to the Maple software package, we obtain the Einstein tensor \( G_{\mu\nu} \) and then the energy momentum tensor \( T_{\mu\nu} \) and \( E_{\mu\nu} \), respectively, for the perfect fluid and the electromagnetic fields

\[ T_0^0 = -\rho, \quad T_1^1 = T_2^2 = \cdots = T_{n+2}^{n+2} = p, \]
\[ 8\pi E_0^0 = 8\pi E_1^1 = -8\pi E_2^2 = -8\pi E_3^3 = \cdots = -8\pi E_{n+2}^{n+2} = \]
\[ = - \frac{2n (n+1) r_1^{2n} [1 + kx^2/4]^n}{x^{2n+2} a^{2n+2} \left\{ \left[ 1 + \frac{r_0^2}{a^2 x^n} (1 + kx^2/4)^{\frac{n}{2}} \right]^2 - \frac{r_2^{2n}}{a^{2n+2} x^n} (1 + kx^2/4)^n \right\}^{\frac{2n}{n}}} \]  

(22)

Substituting the above components of electromagnetic tensor in the second equation of Eqs.(21), we obtain the non-vanishing components of electromagnetic tensor \( F_{\mu\nu} \)

\[ F^{01} = \frac{\sqrt{2n (n+1) r_1^n (1 + kx^2/4)^{1+\frac{n}{2}}}}{x^{n+1} a^{n+2} \left[ 1 - \frac{r_0^{2n}}{a^{2n} x^{2n}} (1 + kx^2/4)^n + \frac{r_2^{2n}}{a^{2n+2} x^{2n}} (1 + kx^2/4)^n \right]^{\frac{n}{2}}} \cdot \]
\[ \left\{ \left[ 1 + \frac{r_0^2}{a^2 x^n} (1 + kx^2/4)^{\frac{n}{2}} \right]^2 - \frac{r_2^{2n}}{a^{2n+2} x^n} (1 + kx^2/4)^n \right\}^{\frac{2n}{n}}. \]  

(23)
Then substituting Eq.(23) in the second equation of Eqs.(20), we obtain the non-vanishing components of the potential $A_{\mu}$

$$A_0 = \int F^{01} g_{00} g_{11} d x.$$  \hspace{1cm} (24)

In the end, from the last equation of Eqs.(20), we obtain the non-vanishing component of the flux density

$$J^0 = \frac{1}{4\pi \sqrt{-g}} \frac{\partial}{\partial x} \left( \sqrt{-g} F^{10} \right)$$

$$= -\frac{1}{16\pi} k x^{-n} r_0^n a^{-n-2} (n + 2) \sqrt{2n (n + 1)} \left( 1 + k x^2 / 4 \right)^{n-n/2}$$

$$\cdot \left[ 1 - \frac{r_0^n}{a^{2n} x^{2n}} \left( 1 + k x^2 / 4 \right)^n + \frac{r_1^{2n}}{a^{2n} x^{2n}} \left( 1 + k x^2 / 4 \right)^n \right]^{-1}$$

$$\cdot \left\{ \left[ 1 + \frac{r_0^n}{a^n x^n} \left( 1 + k x^2 / 4 \right)^{n/2} \right] - \frac{r_1^{2n}}{a^{2n} x^{2n}} \left( 1 + k x^2 / 4 \right)^n \right\}^{-2/n},$$  \hspace{1cm} (25)

where $g$ is the determinant of the metric tensor. We note that $\sqrt{-g} F^{10}$ does not depend on the variable $u$. It is the function of only variable $x$. We also note that for the space-flat universe, i.e., $k = 0$, the flux density vanishes. For $k \neq 0$, there is a charge density in the universe. The universe of $k = 0$ has the topology of $R^3$ and it is infinite both in space and in radial variable $x$. The field lines of the charge inside the black hole end in the infinity of the universe. So the charge density is zero outside the black hole. On the other hand, the universe of $k = 1$ has the topology of $S^3$ and it is finite in space. So there must be charge density in this universe to end the field lines. For $k = -1$, the background universe is infinite in space but the radial variable is finite. This is indicated by the constraint of $1 - k x^2 / 4 \geq 0$ in Eq.(14). The field lines can not end in infinity. So charge density should also exist in this universe.

Up to now, we have verified the solution satisfies the Einstein-Maxwell equations. Of course, any solution solves the Einstein-Maxwell equations. However, not all the solutions are physically meaningful. A solution is physically meaningful if and only if it satisfies both the equations of fields and the conditions of energy-momentum tensor. Fortunately, Eq.(14) meets both the field equations Eqs.(20) and the energy-momentum conditions Eqs.(21).
IV. EVENT HORIZON OF THE BLACK HOLE

In this section, we make a discussion on the evolution of the event horizon of the black hole. For simplicity in mathematics, we consider the black holes in space-flat universe. Set \( k = 0 \) in the metric Eq.(14), we have

\[
\begin{align*}
    ds^2 &= -\left(1 - \frac{r_0^{2n}}{a^n x^n} + \frac{r_1^{2n}}{a^n x^n}\right)^2 du^2 + a^2 \left[\left(1 + \frac{r_0^n}{a^n x^n}\right)^2 - \frac{r_1^{2n}}{a^{2n} x^{2n}}\right] \frac{2}{n} \left(dx^2 + x^2 d\Omega_{n+1}^2\right). \tag{26}
\end{align*}
\]

In order to study the event horizons of the black holes, we should rewrite the metric in the Schwarzschild coordinates. So make variable transformation

\[
    r = ax \left[\left(1 + \frac{r_0^n}{a^n x^n}\right)^2 - \frac{r_1^{2n}}{a^{2n} x^{2n}}\right]^{1/n}, \tag{27}
\]

Then Eq.(26) becomes

\[
\begin{align*}
    ds^2 &= -\left(1 - \frac{r_0^n}{r^n} + \frac{r_1^{2n}}{r^{2n}} - H^2 r^2\right) du^2 - \frac{2Hr}{\sqrt{1 - \frac{r_0^n}{r^n} + \frac{r_1^{2n}}{r^{2n}}}} dudr \nonumber \\
    &\quad + \left(1 - \frac{r_0^n}{r^n} + \frac{r_1^{2n}}{r^{2n}}\right)^{-1} dr^2 + r^2 d\Omega_{n+1}^2. \tag{28}
\end{align*}
\]

where \( H \equiv \dot{a}/a \) which has the meaning of Hubble parameter. Some constants have been absorbed by \( r_0 \) and \( r_1 \) in Eq.(28). From the null surface equation,

\[
    g^{\mu\nu} \frac{\partial f}{\partial x^\mu} \frac{\partial f}{\partial x^\nu} = 0, \tag{29}
\]

where \( f \equiv f (x^\mu) = 0 \) is the location of the event horizon, we obtain the differential equation of evolution of the event horizon \( r_{EH} \)

\[
    \frac{dr_{EH}}{du} = \left(-Hr_{EH} \pm \sqrt{1 - \frac{r_0^n}{r_{EH}^{n}} + \frac{r_1^{2n}}{r_{EH}^{2n}}} \sqrt{1 - \frac{r_0^n}{r_{EH}^{n}} + \frac{r_1^{2n}}{r_{EH}^{2n}}} \right). \tag{30}
\]

We note that the two signs ”+” and ”−” in ”±” are for \( H > 0 \) and \( H < 0 \), respectively.

Otherwise the cosmic event horizon which is far away from the black hole, i.e., \( r_{EH} \gg r_0 \) and...
$r_{EH} \gg r_2$, will expand or contract with the superluminal motion, $|\dot{r}_{EH}| = |-Hr_{EH} \pm 1| > 1$. This is physically forbidden. Since the $g_{00}$ term in Eq.(28) is always negative outside the black hole, we conclude $\sqrt{1 - \frac{r_0^2}{r_{EH}^2} + \frac{r_1^{2n}}{r_{EH}^{2n}}} > |Hr|$. So Eq.(30) tells us the scale of the black hole would increase with the expansion of the universe ($H > 0$) and decrease with the contraction of the universe ($H < 0$). Compared with the four dimensional black hole, it is easy to find that the higher dimensional back hole would increase or decrease even faster.

On the other hand, Noerdlinger and Petrosion [24] found that clusters or super-clusters would expand with the expansion of the universe. Gautreau [25] also concluded that the planetary orbits would expand by considering a model of a particle embedded in an inhomogeneous, pressure free expanding universe. Bonner [26] showed that a local system of electrically counterpoised dust expands with the expansions of universe. Thus our conclusion is consistent with their discussions.

V. CONCLUSION AND DISCUSSION

In conclusion, we have extended the four dimensional Reissner-Nordström-FRW metric to higher dimensions. The solution covers all of the known metrics, such as the higher dimensional static Reissner-Nordström metric, the higher dimensional static Reissner-Nordström-de Sitter metric, the McVittie metric and the four dimensional Reissner-Nordström-FRW metric. Then we verified the solution by substituting it into the Einstein-Maxwell equations. We find that there exists a charge density in the universe of $k \neq 0$. It is due to the fact the field lines of charge inside the black hole can not end in infinity in these two kinds of universes. In the end, we make a discussion on the evolution of the event horizon of the black hole. It is found that the scale of the black hole would increase with the expansion of the universe and decrease with the contraction of the universe. This is consistent with the previous discussions.
ACKNOWLEDGMENTS

We thank the anonymous referee for the expert and insightful comments, which have certainly improved the paper significantly. This study is supported in part by the Special Funds for Major State Basic Research Projects and by the National Natural Science Foundation of China. SNZ also acknowledges supports by NASA’s Marshall Space Flight Center and through NASA’s Long Term Space Astrophysics Program.
REFERENCES

[1] G. C. McVittie, Mon. Not. R. Astron. Soc. 93, 325 (1933).

[2] D. Kastor and J. Traschen J, Phys. Rev. D47, 5401 (1993).

[3] T. Shiromizu and U. Gen, Class. Quntum. Grav. 17, 1361 (2000).

[4] K. R. Nayak, M. A. H. MacCallum and C. V. Vishveshvara, Phys. Rev. D63, 024020 (2000).

[5] K. R. Nayak and C. V. Vishveshvara, "Geometry of the Kerr Black Hole in the Einstein Cosmological Background,” report (2000).

[6] C. J. Gao and S. N. Zhang, Phys. Lett. B595, 28 (2004).

[7] F. R. Tangherlini, Nuovo. Cimento. 27, 636 (1963).

[8] P. F. Gonzales-Diaz, Lett. Nuovo. Cimento. 32, 161 (1981).

[9] P. C. Vaidya and R. Tikekar, J. Astrophys. Astron. 3, 325 (1982).

[10] R. C. Myers and M. T. Perry, Ann. Phys. 172, 304 (1986).

[11] D. Y. Xu, Class. Quantum. Grav. 5, 871 (1988).

[12] K. D. Krori, et al., Phys. Lett. A132, 321 (1988).

[13] Y. G. Shen and Z. Q. Tan, Phys. Lett. A137, 96 (1989).

[14] Y. G. Shen and Z. Q. Tan, Phys. Lett. A142, 341 (1989).

[15] R. Tikekar, Indian Math. Soc. 61, 37 (1995).

[16] L. K. Patel, et al., Nuovo. Cimento. 112, 7 (1997).

[17] S. W. Hawking, C. J. Hunter and M. M. Taylor-Robinson, Phys. Rev. D59, 064005 (1999).

[18] A. Awad, Class. Quant.Grav. 20, 2827 (2003).
[19] H. S. Reall, Phys. Rev. D68 024024, 024024 (2003).

[20] C. J. Gao, Class. Quantum. Grav. 21, 4805 (2004).

[21] R. G. Cai, A. Z. Wang, hep-th/0406040.

[22] D. Kramer, H. Stephani, et al., Exact solutions of Einstein field’s equations (Cambridge: Cambridge University Press, 1980).

[23] R. M. Wald, General relativity (The University of Chicago Press, 1984).

[24] P. D. Noerdlinger and V. Petrosion, Astrophys. J. 168, 1 (1971).

[25] R. Gautreau, Phys. Rev. 29, 198 (1984).

[26] W. B. Bonner, Mon. Not. Astro. Soc. 282, 1467 (1996).