Electromagnetic duality and light-front coordinates.

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August 30, 2018

Abstract

We review the light-front Hamiltonian approach for the Abelian gauge theory in 3+1 dimensions, and then study electromagnetic duality in this framework.
1 Introduction.

In recent years we have witnessed a resurrection of interest in light-front Hamiltonian physics in two areas. The first one, the new nonperturbative approach to QCD [1], is related to the original application of light-front coordinates [2], i.e. hadron spectroscopy, and the other comes from string theory [3]. The popular M-theory [4] is formulated in light-front coordinates [5]. With the rise of the second application, some rather academic questions became of interest. For example, dualities in string theories are one of the most powerful tools, yet not much is known about how they work on the light front. In this paper, we attempt to study one of the simplest cases of known dualities - the electromagnetic duality in the Abelian gauge field theory in 3+1 dimensions.

Susskind has conjectured [6] that since light-front coordinates are non-local in the longitudinal direction, it might be possible to formulate a light-front theory with both electric and magnetic sources without having to introduce any additional non-localities corresponding to Dirac strings [7]. He observed that the role of electric and magnetic fields reverses in the light-front Hamiltonian (which contains only the physical, transverse fields) when the original fields are replaced by (transverse) fields perpendicular to them. He concluded that the above described transformation of fields is the electromagnetic duality on the light front. Then he suggested that magnetic sources be added into the Hamiltonian by symmetry.

In this paper we investigate this idea. The paper is organized as follows: Since there are some misconceptions in the literature, and since each beginning researcher in this
field has to set up his/her own notes on the light-front conventions, we summarize in the section 2 formalisms of light-front coordinates, free Abelian fields, and we establish the connection between the components of the $F^{\mu\nu}$ tensor and electric and magnetic fields. We show how classical electric sources can be added to the theory. For completeness, we list the surface terms even though they do not enter the calculation presented here, and we list some manipulations with the $(\partial^+)^{-1}$ operator. Further, we wish to mention that there are, in general, problems regarding other than $+\text{-}$components of light-front currents, even though this does not affect our calculation since we restrict ourselves to external classical currents. The section 2 is rather formal; a reader familiar with the light front may want to skip most of it. Section 3 is devoted to Susskind’s idea. The last section contains our conclusions.

## 2 Light-front field theory: Formalities

In a light-front quantum field theory, fields are quantized at an equal light-front time $\tilde{\xi}$. Advantages of a light-front formulation are: The light-front has the largest kinematic subgroup of Lorentz generators, boosts are kinematic $\mathcal{P}$, and the light-front vacuum can be decoupled from the physical states by imposing a longitudinal momentum cutoff $\mathcal{Q}$. The price to pay is more complicated renormalization, rotations involving the z-axis are dynamical, and as a consequence the physical picture is less intuitive for nonrelativistic systems at rest that are naturally described in equal-time coordinates. On the other hand, it is a natural framework for highly relativistic systems (e.g. description of deep inelastic
scattering).

2.1 Light-front coordinates

Dirac showed that it is possible to formulate relativistic dynamics in coordinates other than the usual equal-time form in which everything is expressed in terms of dynamical variables at one instant of time (hence *instant form*). The other forms he found are the *point form* and the *front form* [8].

In the front form (or light-front) $x^+ = t + z$ plays the role of time. The remaining coordinates, $x^- = t - z$ and $x^\perp \equiv (x^1, x^2)$, are spatial. Given a four-vector $a$, its components in light-front coordinates are:

$$
a^- = a^0 - a^3, \quad a^+ = a^0 + a^3, \quad a^\perp = (a^1, a^2).$$

(1)

A scalar product of two four-vectors $a, b$ is:

$$a_\mu b^\mu = \frac{1}{2} a^+ b^- + \frac{1}{2} a^- b^+ - a^\perp \cdot b^\perp.$$  

(2)

There are two slightly different conventions regarding the $+, -$ components. The other one differs from the one used here by a factor of $\sqrt{2}$: $a^\pm = (a^0 \pm a^3)/\sqrt{2}$, so that the metric tensor $g^{+-} = 1$. It is, therefore, a good idea to check the definitions of coordinates before comparing any results.
The metric tensor in light-front coordinates is:

\[
g_{\mu\nu} = \begin{pmatrix}
g_{++} & g_{+-} & g_{+1} & g_{+2} \\
g_{-+} & g_{--} & g_{-1} & g_{-2} \\
g_{1+} & g_{1-} & g_{11} & g_{12} \\
g_{2+} & g_{2-} & g_{21} & g_{22} \\
\end{pmatrix} = \begin{pmatrix}
0 & \frac{1}{2} & 0 & 0 \\
\frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1 \\
\end{pmatrix}
\]  

(3)

and

\[
g^{\mu\nu} = \begin{pmatrix}
g^{++} & g^{+-} & g^{+1} & g^{+2} \\
g^{-+} & g^{--} & g^{-1} & g^{-2} \\
g^{1+} & g^{1-} & g^{11} & g^{12} \\
g^{2+} & g^{2-} & g^{21} & g^{22} \\
\end{pmatrix} = \begin{pmatrix}
0 & 2 & 0 & 0 \\
2 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1 \\
\end{pmatrix}
\]  

(4)

We can now write down the derivatives with respect to coordinates:

\[
\partial^- = \frac{\partial}{\partial x_-} = 2 \frac{\partial}{\partial x^+} = \partial^0 - \partial^3; \\
\partial^+ = \frac{\partial}{\partial x_+} = 2 \frac{\partial}{\partial x^-} = \partial^0 + \partial^3; \\
\partial^\perp = (\partial^1, \partial^2).
\]  

(5)

\(\partial^-\) is the time derivative, the remaining derivatives are spatial.

The four-dimensional volume element is:

\[
[d^4x] = \frac{1}{2} dx^+ dx^- d^2x^\perp.
\]  

(6)

Let \(p = (p^-, p^+, p^1, p^2)\) be the four-momentum of a free particle with mass \(m\) in light-front coordinates. Then

\[
p_\mu x^\mu = \frac{1}{2} p^+ x^- + \frac{1}{2} p^- x^+ - p^\perp \cdot x^\perp.
\]  

(7)
$p^+$ is the *longitudinal* momentum, $p^1$ and $p^2$ are the *transverse* momenta, and $p^-$ is the *light-front energy*:

$$p^- = \frac{p^1^2 + m^2}{p^+}.$$  \hspace{1cm} (8)

The Lorentz invariant momentum integration element is obtained as follows:

$$[d^4 q] 2 \pi \delta(m^2 - q^2) = \frac{1}{2} \frac{d q^- d q^+ d^2 q^\perp}{(2\pi)^3} \delta(m^2 - q^+ q^- + q^\perp^2) = \frac{d q^+ d^2 q^\perp}{2(2\pi)^3 q^+}. \hspace{1cm} (9)$$

The light-front energy is well defined apart from peculiar modes which have zero longitudinal momentum (so-called *zero modes*). $p^+$, which is equal to $p^0 + p^3$, satisfies $p^+ \geq 0$. This means that in the vacuum all particles must have precisely zero longitudinal momentum. From the expression for the light-front energy we can see that the energy diverges as $p^+ \rightarrow 0$ for massive particles. For massless particles, the light-front energy can be finite even at $p^+ = 0$, but the vacuum can be made trivial by imposing a small longitudinal momentum cutoff, e.g. requiring that all longitudinal momenta satisfy $p_1^+ > \epsilon$. Another frequently used method of regularization is a discretized light-cone quantization (DLCQ) \[11\] which removes both ultraviolet and infrared divergences. The physics of $p^+ = 0$ cannot be recovered by renormalization with respect to high energy states, and has to be added by hand using counterterms. The so-called “constraint zero mode” \[12\] is a specific counterterm consequent of DLCQ, and it does not require a nontrivial vacuum structure.
2.2 Free Abelian gauge fields

Let us start with pure electromagnetism. The Lagrangian density is:

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \]  

where

\[ F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu. \]

In a light-front formulation, indices \( \mu, \nu \) run through \( +, -, \) and \( \perp = (1, 2) \).

In light-front gauge, \( A^+ = 0 \), and the Lagrangian density reduces to:

\[ \mathcal{L} = \frac{1}{8} (\partial^+ A^-)^2 + \frac{1}{2} \partial^+ A^i \partial^- A^i - \frac{1}{2} \partial^+ A^i \partial^i A^- - \frac{1}{4} (\partial^i A^j - \partial^j A^i)^2. \]

The Lagrangian density does not contain a time-derivative of \( A^- \) so it is immediately obvious that this component of \( A^\mu \) is not dynamical. Indeed, conjugate momenta, \( \Pi \), to the fields are:

\[ \Pi_{A^i} = \frac{\partial \mathcal{L}}{\partial \partial^- A^i} = \frac{1}{2} \partial^+ A^i, \]
\[ \Pi_{A^-} = \frac{\partial \mathcal{L}}{\partial \partial^- A^-} = 0. \]

\( A^- \) can be eliminated using the equations of motion:

\[ \frac{\partial \mathcal{L}}{\partial A^-} = \partial^\mu \frac{\partial \mathcal{L}}{\partial \partial^\mu A^-}, \]

leading to

\[ (\partial^+)^2 A^- = 2 \partial^+ \partial^i A^i. \]
Apart from zero modes (i.e. $p^+ = 0$ states introduced above), $\partial^+$ can be inverted, and $A^-$ is then given by:

$$A^- = \frac{2}{\partial^+} \partial^i A^i.$$ (16)

To proceed further, some manipulations with $(\partial^+)^{-1}$ are needed. Up to a constant in $x^-$ which can depend on remaining coordinates, the operator $(\partial^+)^{-1}$ is defined as follows:

$$\frac{1}{\partial^+} f(x^-) \equiv \int dy^- \epsilon(x^- - y^-) f(y^-),$$

where $\epsilon(x) = 1$, if $x > 0$, and $\epsilon(-x) = -\epsilon(x)$, and $f(x^-)$ is an arbitrary function. Using the properties of $\epsilon(x)$ it is straightforward to find:

$$\int d^3 x \left( \frac{1}{\partial^+} f(x^-) \right)^2 = -\int d^3 x f(x^-) \left( \frac{1}{\partial^+} \right)^2 f(x),$$

$$\int d^3 x f(x^-) \left( \frac{1}{\partial^+} g(x^-) \right) = -\int d^3 x \left( \frac{1}{\partial^+} f(x^-) \right) g(x).$$

Substituting for $A^-$, and using the properties of the operator $(\partial^+)^{-1}$ shown above, gives the Lagrangian in terms of physical degrees of freedom:

$$\mathcal{L} = \frac{1}{2} \partial^+ A^i \partial^- A^i - \frac{1}{2} \left( \partial^i A^i \right)^2 - \frac{1}{4} \left( \partial^i A^j - \partial^j A^i \right)^2 + \text{surface terms},$$ (17)

where the surface terms,

$$- \left\{ \partial^+ \left( A^i \partial^j \frac{1}{\partial^+} \partial^i A^i \right) - \partial^i \left( A^i \partial^j A^i \right) \right\},$$ (18)

are traditionally dropped. $F^{\mu\nu}$ in terms of $A^i$ is:

$$F^{+\nu} = 2 \partial^i A^i, \quad F^{+i} = \partial^+ A^i,$$

$$F^{-i} = \partial^- A^i - \partial^i \frac{2}{\partial^+} \partial^j A^j, \quad F^{ij} = \partial^i A^j - \partial^j A^i.$$ (19)
Let us also introduce the dual tensor $\tilde{F}^{\mu\nu} = 1/2\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}$, where $\epsilon^{\mu\nu\lambda\rho}$ is totally antisymmetric, 

$$\epsilon^{+\cdot-12} \equiv 2,$$  

so that it satisfies 

$$\epsilon^{\mu\nu\alpha\beta}\epsilon^{\mu'}_{\nu'\alpha'\beta'} = g^{\nu\alpha'}g^{\alpha\beta'} + g^{\mu\nu'}g^{\alpha\beta'} + g^{\mu'\beta'}g^{\alpha\nu'}g^{\beta\nu'} - g^{\nu'}g^{\nu\alpha'}g^{\alpha\beta'} - g^{\nu\beta'}g^{\nu\alpha'}g^{\alpha\beta'} - g^{\nu'\nu}\lambda^{\alpha\beta}g^{\alpha\beta'},$$  

(21)

Then, 

$$\tilde{F}^{++} = \epsilon_{ij}F^{ij}, \quad \tilde{F}^{+i} = \epsilon_{ij}F^{+j},$$

$$\tilde{F}^{ij} = -\frac{1}{2}\epsilon_{ij}F^{++}, \quad \tilde{F}^{-i} = -\epsilon_{ij}F^{-j}. \quad (22)$$

where $\epsilon_{12} = 1$ is antisymmetric. Let us note that $F$ and $\tilde{F}$ are related by electromagnetic duality $\vec{B} \rightarrow \vec{E}, \vec{E} \rightarrow -\vec{B}$. The connection between electric and magnetic fields $\vec{E}$ and $\vec{B}$ and the tensor $F^{\mu\nu}$ given in light-front coordinates is shown in the next section.

### 2.3 Connection between electric and magnetic fields $\vec{E}$ and $\vec{B}$ and the tensor $F^{\mu\nu}$ in light-front coordinates.

The connection between $\vec{B}, \vec{E}$ and $F^{\mu\nu}$ can be established using the definition of the potential:

$$\vec{E} = -\vec{\nabla}A^0 - \frac{\partial}{\partial t}\vec{A},$$

$$\vec{B} = \vec{\nabla} \times \vec{A}. \quad (23)$$
Substituting:
\[ A^0 = \frac{1}{2} (A^+ + A^-), \quad A^3 = \frac{1}{2} (A^+ - A^-), \]
\[ \partial \frac{\partial}{\partial t} = \partial^0 = \frac{1}{2} (\partial^+ + \partial^-), \quad \partial \frac{\partial}{\partial z} = -\partial^3 = -\frac{1}{2} (\partial^+ - \partial^-), \]  \( \quad \text{(24)} \)

we obtain:
\[ E^i = -\frac{1}{2} \left( F^{+i} + F^{-i} \right), \quad E^z = \frac{1}{2} F^{+-}, \]
\[ B^i = \epsilon_{ij} \frac{1}{2} \left( F^{+j} - F^{-j} \right), \quad B^z = -\frac{1}{2} \epsilon_{ij} F^{ij}, \]  \( \quad \text{(25)} \)

or,
\[ F^{+-} = 2E^z, \quad \tilde{F}^{+-} = -2B^z \]
\[ F^{+i} = - \left( E^i + \epsilon_{ij} B^j \right), \quad \tilde{F}^{+i} = \left( B^i - \epsilon_{ij} E^j \right), \]
\[ F^{ij} = -\epsilon_{ij} B^z, \quad \tilde{F}^{ij} = -\epsilon_{ij} E^z, \]
\[ F^{-i} = - \left( E^i - \epsilon_{ij} B^j \right), \quad \tilde{F}^{-i} = \left( B^i + \epsilon_{ij} E^j \right), \]  \( \quad \text{(26)} \)

where \( i, j \) are transverse indices \( (i, j = (1, 2)) \). These definitions ensure that the Lagrangian equations of motion give the correct set of Maxwell’s equations. In ref. 10 the magnetic and electric fields are defined differently, in particular, in analogy with equal time, \( E^\mu = 1/2 F^{+\mu} \) and \( B^- = F^{12} \), but this is misleading, because these definitions do not lead to Maxwell’s equations. Moreover, \( \vec{E} \) and \( \vec{B} \) are not four-vectors, \( E^0 \) and \( B^0 \) are not defined, so there is no natural way to form the minus and plus components.
2.4 Adding classical electric sources

In this section we add classical electric sources. The Lagrangian density in the presence of classical sources $j_\mu$ is:

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j_\mu A^\mu$$  \hspace{1cm} (27)

As before, $A^-$ is not dynamical and it can be eliminated using the equations of motion, leading to:

$$A^- = \frac{2}{\partial^+} \left( \partial^i A^i - \frac{1}{\partial^+} j^+ \right) = A^-_{\text{free}} - \frac{2}{(\partial^+)^2} j^+. \hspace{1cm} (28)$$

Replacing $A^-$ modifies $F^{\mu\nu}$, in particular, the +-component of the current is absorbed into $F^{+ -}$ and $F^{- i}$:

$$F^{+ -} = F^{+ -}_{\text{free}} - \frac{2}{\partial^+} j^+, \quad F^{- i} = F^{- i}_{\text{free}} - \partial^i \frac{2}{(\partial^+)^2} j^+, \hspace{1cm} (29)$$

where $F^{\mu\nu}_{\text{free}}$ is given in previous sections. The remaining two components of $F^{\mu\nu}$ are unchanged. The Lagrangian density then reads:

$$\mathcal{L} = \frac{1}{2} \partial^+ A^i \partial^- A^i - \frac{1}{2} \left( \partial^i A^i - \frac{1}{\partial^+} j^+ \right)^2 - \frac{1}{4} \left( \partial^i A^j - \partial^j A^i \right)^2 + j^+ A^+ + \text{surface terms}, \hspace{1cm} (30)$$

and the surface terms are modified also:

$$\text{surface terms} =$$
\[ - \partial^+ \left[ A^i \partial^j \left( \partial^j A^i - \frac{1}{\partial^+} j^+ \right) \right] - \partial^j \left[ A^i \left( \partial^j A^i - \frac{1}{\partial^+} \right) \right] \\
+ \frac{1}{\partial^+} \left[ j^+ \left( \partial^j A^i - \frac{1}{\partial^+} j^+ \right) \right]. \] (31)

The Lagrangian equations of motion are:

\[ \partial_\mu F^{\mu i} = j^i, \] (32)

\( \partial_\mu F^{\mu +} = j^+ \) is satisfied identically, and using equations of motion it can be shown that

\[ \partial_\mu F^{\mu -} = -\frac{2}{\partial^+} \left[ \frac{1}{2} \partial^- j^+ - \partial^j j^i \right], \]

which implies a continuity equation for \( j^\mu \).

The Hamiltonian density in the presence of classical sources is:

\[ \mathcal{H} = \frac{1}{2} \left( \partial^j A^i - \frac{1}{\partial^+} j^+ \right)^2 + \frac{1}{4} \left( \partial^j A^i - \partial^i A^j \right)^2 - j^\perp A^\perp, \] (33)

and the fields \( A^i \) can be quantized as if they were free.

### 3 Electromagnetic duality

In this section we investigate the question of whether it is possible to formulate electromagnetic duality as a transformation of the potential \( A^\perp \) itself rather than the field strength tensor and its dual. Given the Hamiltonian in the transverse degrees of freedom, a natural starting point is a transformation

\[ A^i \rightarrow \tilde{A}^i \equiv -\epsilon_{ij} A^j. \] (34)
Indeed, under this transformation the first and second term in the free Hamiltonian “interchange”:

\[ \mathcal{H} = \frac{1}{4}(\partial^i \tilde{A}^j - \partial^j \tilde{A}^i)^2 + \frac{1}{2}(\partial^i \tilde{A}^i)^2. \] (35)

By comparison with the Hamiltonian including electric sources (see eqn. (33)), it appears that one can by symmetry add magnetic sources, as well as the electric sources. In complete analogy one could then expect that the +component of the magnetic current \( \tilde{j}^\mu \) was absorbed into the definition of the field strength tensor and/or the dual tensor \( \tilde{\mathcal{F}}_{\mu\nu} \).

Taking advantage of the kinematical boost invariance (for a review see [6]), it would be sufficient to consider the simple case of a magnetic current with only the +component (the so-called good component) being non-zero, viz.

\[ \mathcal{H} = \frac{1}{2}(\partial^i A^i - \frac{1}{\partial^+ j^+})^2 + \frac{1}{2}(\partial^i \tilde{A}^i - \frac{1}{\partial^+ \tilde{j}^+})^2 - j^\perp A^\perp. \]

It is straightforward to show that if one proceeds as described, the Hamiltonian leads to the desired equations of motion, including the continuity equation for the \( -\)component of \( \tilde{j} \).

The catch is that the Hamiltonian itself is not equivalent to the complete set of Maxwell equations. It is, rather, the Hamiltonian and the gauge conditions [13]. In particular, only with the gauge conditions \( A^+ = 0 \) and \( A^- = 2(\partial^+)^{-1} \partial^i A^i \) are all components of the field

\[ \partial_\mu \tilde{F}^{\mu\nu} = 0. \] However, absorbing the \( \tilde{j}^+ \) appropriately into the definition of \( \tilde{F} \) produces a non-zero right-hand side. It is somewhat reminiscent of introducing a Dirac string. Note that in our case this trick does not work for \( \tilde{j}^i \neq 0 \).
Let us look at what happens with the field strength tensor and its dual under the transformation (34). In order for the transformation (34) to be the operation of electromagnetic duality, it has to lead to

\[- \tilde{F}^{\mu\nu}(A^\perp) = F^{\mu\nu}(\tilde{A}^\perp),\]

\[F^{\mu\nu}(A^\perp) = \tilde{F}^{\mu\nu}(\tilde{A}^\perp).\]  

(36)

However,

\[- \tilde{F}^{+-} = 2\partial^i \tilde{A}^i,\]

\[- \tilde{F}^{+i} = \partial^+ \tilde{A}^i,\]

\[- \tilde{F}^{-i} = \partial^- \tilde{A}^i - \partial^i \left( \frac{2}{\partial^+} \partial^k \tilde{A}^k \right) - \frac{2}{\partial^+} \Box \tilde{A}^i,\]

\[- \tilde{F}^{ij} = \partial^i \tilde{A}^j - \partial^j \tilde{A}^i.\]  

(37)

shows that the transformation (34) is not quite electromagnetic duality: It works for all components except \(F^{-i}\). The \(\tilde{F}^{-i}\) contains an additional term \(-2(\partial^+)^{-1} \Box \tilde{A}^i\).

For free fields, the additional term is zero, and (34) is therefore electromagnetic duality. Is it possible to remove the additional term in general, realizing the electromagnetic duality as a generalization of the original Susskind’s suggestion, e.g. (34) plus a gauge transformation?

After fixing the gauge, there is still a residual gauge freedom. In order not to disturb the gauge conditions used to derive the Hamiltonian, the residual gauge function \(\Lambda\) has...
to satisfy

\[ \partial^+ \Lambda = 0, \quad \partial^- \Lambda = \frac{2}{\partial^+} (\partial^\lambda)^2 \Lambda. \]  \hspace{1cm} (38)

Ignoring for a moment the question of zero modes, this implies that

\[ \frac{2}{\partial^+} \Box \Lambda = 0 \]

and thus cannot cancel the unwanted term \(-2(\partial^+)^{-1} \Box \vec{A}_i\).

We now return to the question of zero modes. Since they correspond to a constant in \(x^-\), they cannot cancel the \(-2(\partial^+)^{-1} \Box \vec{A}_i\) term which, in general, does depend on \(x^-\).

4 Conclusion and summary

We reviewed the formalism of Abelian gauge theory in light-front coordinates. We argued that while the potential \(A^\mu\) can be described in light-front coordinates, there is no light-front analogue to electric and magnetic fields \(\vec{E}, \vec{B}\) in the sense that if one defines electric and magnetic fields as components of the light-front field strength tensor, the definitions do not lead to Maxwell equations, and electromagnetic duality is not realized as \(\vec{B} \rightarrow \vec{E}, \vec{E} \rightarrow -\vec{B}\).

We then studied electromagnetic duality on the level of fields \(A^\mu\) (in light-front gauge \(A^+ = 0\)). Our study was motivated by the fact that the light-front Hamiltonian is in this case expressed in terms of transverse fields only, and that by under a specific transformation of transverse fields the electric and magnetic terms in the Hamiltonian interchange.
However, the electromagnetic duality in light-front coordinates cannot be realized by a transformation of transverse fields only. Neither can it be written as a transformation of transverse fields plus a gauge transformation, not even when the gauge transformation has a zero mode. Altering $A^{-}$ in addition to $A^{\perp}$ is not likely to fix the problem either, because it is only one of the two components of the dual tensor involving $A^{-}$ (i.e. $\tilde{F}^{-i}$) that does not transform as desired; fixing $\tilde{F}^{-i}$ would spoil the transformation of $\tilde{F}^{-+}$.

To include magnetic monopoles one would have to allow for additional non-localities (in the gauge function), most likely equivalent to Dirac strings in an equal-time theory [7].

5 Acknowledgments

My work has been supported by the United States Department of Energy. I would like to acknowledge L. Susskind for bringing this problem into my attention. I am grateful to G. t’Hooft for useful discussions during the NATO ASI Workshop on Confinement, Duality, and Non-Perturbative Aspects of QCD held June 23 - July 4, 1997, and to the organizers of the workshop, particularly P. van Baal for providing such a stimulating research environment. I am also grateful to R. Furnstahl, T. Goldman and R. Perry for reading the manuscript. Last but not least, I want to thank Mária Barnášová for making this work possible.
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