QUICKEST DETECTION OF CASCADING FAILURE

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ABSTRACT

We consider online detection of cascading failure in the networks using sequential data. Our goal is to detect the failure as quickly as possible after it occurs. To achieve this goal, we propose a temporal diffusion network model to capture the dynamic of the potential change, which will help us to capture the change as quickly as possible since its onset. Under this model, the hazard rate of a node increases as the number of failure neighbors increases. Once the failure affects a node, the distribution of the measurement changes from pre-change distribution to an unknown post-change distribution. We develop a sequential generalized likelihood ratio statistics, which performs joint detection and estimation in detecting the change. Numerical experiments show that our method outperforms the existed methods.

Index Terms—Independent Cascade Model, likelihood ratio test, change-point detection

1. INTRODUCTION

A power system consists of a large number of components (buses and lines). The operations of components are dependent. Failure or anomaly occurs on one component makes the whole system weaken and unstable. The risk of other components being malfunction will increase. Cascading failure is the process that the initial failure of one component propagates and causes the consecutive failure of other components.

On the simulation side, \cite{1,2} propose probabilistic models of how one failure causes later-on failures. With the models, they do the simulation so that they can perform risk assessment, identify the critical components and do mitigation for the failures.

The propagation of failure in the power system can be seen as an example of the problems of information diffusion in a large network \cite{3,4,5}. Use the failure data in power systems to estimate the parameters of the propagation models. Diffusion network has been widely studied in disease spreading \cite{6,7}, viral diffusion through social networks \cite{8}, emergency detection, etc. Most of these works focus on inferring the latent network by observing multiple cascade events. In \cite{9,10}, the propagation duration is modeled as the exponential model, power-law model, and Rayleigh model. The underlying network is inferred by the maximum likelihood approach. \cite{6,7} use the Hawkes process to model the temporal events to capture features of the exciting process and infer the influential paths. \cite{7} assumes that weak signals of epidemic propagates through the network and only occur once on each node. They propose a method to raise an alarm by counting signals in a certain time window.

In online monitoring, it is usual that we can’t observe whether there is a failure on each component. Instead, we use other measures to detect the occurrence of failure. Due to the size of a power system, \cite{12,13} monitor measurements that can reflect the state of an area of the power system. To detect the failure before its occurrence, \cite{14} illustrates that autocorrelation in the frequency signal increase from zero as the system is getting close to the critical slowing down. In \cite{15}, it applies a neural-fuzzy network to detect the change of measurement in the early stage of the cascading failure and identify the type of the anomaly. However, these studies don’t consider the propagation of failure across the network.

Failure in the power system is a change-point when the distribution of measurements change. \cite{16} considers a general setting with multiple change-points in multiple data streams. It does not utilize the graph topology of the data streams, therefore, it doesn’t consider the propagation model of the change-points. Inferring the underlying propagation model is of great use because it improves the resilience, forecasting, and failures localization. Several works consider the propagation of the change-points through the network. \cite{17}, consider the change-point propagate from one end to the other in a line-type network and the duration between two change-points is considered to follow the geometric distribution. \cite{18} assumes prior distribution for the possible propagation paths and given the path, the duration between two change-points is geometrically distributed. In \cite{19}, it does not model the propagation with probability, instead, it partitions the network into several complete graphs to guarantee a result with connected change-points.

In this paper, we aim to detect the cascading failure in the power system with measurements observed by the sensor networks. We model the latent failure propagation with Independent Cascade Model \cite{9,20}, a point process that propagates across the network through the edge. The independent cascade model can model the property that the risk of
a component increase when there are more and more components around it being failed. To the best of our knowledge, we are the first to combine the independent cascade model with change-point detection.

2. PROBLEM SETUP

In this section, we provide a general setting for the detection of cascading failure. Consider a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, which is formed by a set of nodes $\mathcal{V} = \{1, 2, 3, \ldots, N\}$ and a set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. Assume $\mathcal{G}$ is undirected, i.e. $(i, j) \in \mathcal{E} \iff (j, i) \in \mathcal{E}$.

Remark 2.1 The graph $\mathcal{G}$ can be the same as the physical graph, i.e. $(i, j) \in \mathcal{E}$ if the $i$th and $j$th components are connected physically. However, it is known that the failure does not necessarily propagate between the physically connected neighbor. Instead, the failure propagates through the components which are electrically related to each other by Kirchhoff’s and Ohm’s laws [21]. Therefore it is more reasonable to consider $\mathcal{G}$ as the interaction graph [12,21].

Model of measurement. On each node, we observe some measurement at each time stamp $t$, $X_i(t) \in \mathbb{R}^d$. Any measurements that depends on the state of power system can be used. There are some examples in [12,14]. Besides, there is a latent (unobservable) state for each node $Z_i(t) \in \{0, 1\}$, which indicates whether the failure affects $i$th node. Define a vector $\vec{\tau} = (\tau_1, \tau_2, \ldots, \tau_N) \in \mathbb{R}^N \cup \{\infty\}$, which is the time of failure for each node. Then, $Z_i(t) = 1_{\{\tau_i < t\}}$.

The distribution of $X_i(t)$’s depends on the state $Z_i(t)$’s, i.e. $\forall i = 1, \ldots, N, t = 1, \ldots, T$

$$X_i(t) \sim \prod_{i=1}^T f(X_1(t), \ldots, X_N(t)|Z(t)),$$

where $f(\cdot|Z(t))$ is the joint probability density of measurements at time $t$ given the vector of latent state, $Z(t) \triangleq (Z_1(t), \ldots, Z_N(t))$. We can see $\tau_i$’s are change-points of the distribution, since $Z(t)$ changes at the $\tau_i$’s.

Failure. Let $T_f = (t_f, v_f)$ denote the first failure, where $t_f$ is the time of the failure, and $v_f$ is the location of the failure. If there is failure, we have $\tau_1 = \tau_{v_f} = t_f$, other wise let $T_f = (\infty, \emptyset)$.

Pairwise transmission hazard rate. After the first failure, the failure propagates through the edges. Let $\lambda_{i,j}(t)$ denote the pairwise transmission hazard rate, which determines the likelihood that the failure propagates from $i$ to $j$.

Model of failure diffusion. Given $T_f$ and $\lambda_{i,j}(t)$, the distribution of $\vec{\tau}$ is determined by the hazard rate of each node, $\lambda^i(t)$, which is the sum of pairwise transmission hazard rate, this is the independent cascade model [9]:

$$\lambda^i(t) = \begin{cases} \sum_{j: (i,j) \in \mathcal{E}, \tau_j < t} \lambda_{i,j}(t - \tau_j), & t_f < t \leq \tau_i, \\ 0, & t > \tau_i. \end{cases} \quad (1)$$

Fig. 1: Example of failure diffusion. Red circles with the black bold outline: failure nodes. Yellow solid lines: possible paths for failure diffusion. Yellow dash lines: paths with two failure nodes at both ends. Yellow circles: nodes being affected by failure neighbors. In this example, we can see the failure first occurs at A. Then all the neighbors of A are affected. Second and Third failures are at B and D respectively. As the failure is diffusing, C is surrounded by more and more failure nodes and hence the hazard rate of it keeps increasing.

Remark 2.2 Let’s define the history (filtration) up to time $t$:

$$\mathcal{H}(t) = \{\tau_i \leq t, \forall i = 1, \ldots, N\}.$$

Then give the $\mathcal{H}(t)$, the intensity function of $i$th node is defined as:

$$\lambda^i(t) \triangleq \lim_{\Delta t \to 0} \mathbb{P}\{\tau_i \in [t + \Delta t]|\mathcal{H}(t), \tau_i > t\}/\Delta t$$

$$= \frac{f^i(t)}{1 - F^i(t)},$$

where $f^i(t)$ and $F^i(t)$ are the probability density function and cumulative distribution function conditional on $\mathcal{H}(t)$. Equivalently, we have

$$F^i(t) = 1 - \exp\left(-\int_{\tau_i}^t \lambda^i(u)du\right),$$

$$f^i(t) = \lambda^i(t) \exp\left(-\int_{\tau_i}^t \lambda^i(u)du\right),$$

where $\tau_i = \max\{\tau_i \leq t, \forall i = 1, \ldots, N\}$, the last failure up to time $t$.

In this model, we don’t assume any distribution for the first failure. In other words, we assume the time of the first failure is adversarial.

Hypothesis testing. To detect the failure, we consider the following hypothesis test:

$$H_{0,T} : t_f > T, \quad H_{1,T} : 0 \leq t_f \leq T.$$

3. SPECIFIC MODEL

In this section, we provide a specific model for this problem. We derive the likelihood functions, the test statistics, and the corresponding detection procedure.

Let’s assume the pre-change distributions of measurements are independent standard Normal distributions. Notice that we can always use a certain length of data as a warm start to compute the sample mean and variance of the pre-change
distribution. Therefore we assume that the pre-change distribution is known and can be standardized. Then, under $H_0$:

$$
\mathcal{L}_0 = \prod_{i=1}^{N} \prod_{t=1}^{T} f(X_i(t)|\mu = 0, \sigma = 1) = \left(\frac{1}{\sqrt{2\pi}}\right)^{NT} \exp\left\{ - \sum_{i=1}^{N} \sum_{t=1}^{T} \frac{X_i(t)^2}{2} \right\}. \tag{2}
$$

After failure occurs, the measurements follow Normal distribution with unknown means and variances, independently. Given a $\bar{F}$, we can estimate $\mu_i$’s and $\sigma_i$’s by MLE, $\hat{\mu}_i = (T - \tau_i + 1)^{-1} \sum_{t=\tau_i}^{T} X_i(t)$, $\hat{\sigma}_i^2 = (T - \tau_i + 1)^{-1} \sum_{t=\tau_i}^{T} (X_i(t) - \hat{\mu}_i)^2$. Hence:

$$
\mathcal{L}_1^M = \prod_{i=1}^{N} \mathcal{L}_{1,i}^M = \prod_{i=1}^{N} \exp\left\{ \frac{1}{2} \sum_{t=\tau_i}^{T} X_i(t)^2 - \frac{1}{2} \sum_{t=\tau_i}^{T} (T - \tau_i + 1) \cdot \left[ 1 + \log\left(\frac{\sum_{t=\tau_i}^{T} (X_i(t) - \hat{\mu}_i)^2}{T - \tau_i + 1}\right) \right] \right\}. \tag{3}
$$

For the model of failure diffusion, we assume the hazard rate of each node (except for the first failure) follows the exponential model $\{\mathcal{H}\}$, i.e. eq.1 with $\lambda_{j,i}(t) = \alpha_{j,i}$. In [3], there are several other choices of $\lambda_{j,i}(t)$. We assume $\alpha_{j,i}$’s are known, since it can be estimated depend on the topology of the power grid and power flow. Hence, for a given $\bar{F}$ we have the following likelihood for diffusion in [1, $T$]:

$$
\mathcal{L}_1^D = \mathcal{g}(\tau_1, \tau_2, \ldots, \tau_N|\alpha_{i,j}, \forall i, j = 1, \ldots, N) = \prod_{i : \tau_i \leq T} \sum_{j \in C(i)} \alpha_{j,i} I(\tau_j < \tau_i) \cdot \exp\left\{- \sum_{j \in C(i)} \alpha_{j,i} (\tau_i - \tau_j)^+ \right\} \prod_{i : \tau_i > T} \exp\left\{- \sum_{j \in C(i)} \alpha_{j,i} (T - \tau_j)^+ \right\}. \tag{4}
$$

where $C(i) = \{ j \in \mathcal{V} | (j,i) \in \mathcal{E} \}$ is the set of $i$th node’s neighbors, $(\cdot)^+ = \max(\cdot, 0)$ and $I(\cdot)$ is the indicator function.

**Remark 3.1** In [19], to reduce the false alarm such that the failure nodes are not connected they partition the graph into several fully connected subgraph. Here we reduce the false alarm by considering the diffusion pattern.

Therefore, under $H_1$:

$$
\mathcal{L}_1 = \mathcal{L}_1^M \cdot \mathcal{L}_1^D.
$$

Then, we have the test statistics, log likelihood ratio:

$$
S_1(T) = \max_{\bar{F}} \log \mathcal{L}_1 - \log \mathcal{L}_0 = \max_{\bar{F}} \log \mathcal{L}_1^M + \log \mathcal{L}_1^D - \log \mathcal{L}_0
$$

In [19], it proposed a detection algorithm to raise alarm when there are at least $\eta$ change-points given constraints of average run length when the number of change-points is less than $\eta$. To cope with this problem, we can alter our test statistics as follows:

$$
S_2(T) = \max_{\bar{F}} \log \mathcal{L}_1 (\bar{F}) - \max_{\bar{F}} \log \mathcal{L}_1 (\bar{F})
$$

Then the corresponding stopping times are:

$$
\Gamma_i = \inf\{ t > 0 : S_i(t) > b_i, i = 1, 2 \}.
$$

4. ALGORITHM

In practice, we hope to detect the failure as soon as possible, therefore we can only search for the propagation paths with at most $m$ nodes affected by the failure. However, the computation cost of the maximum likelihood under $H_1$ is still large. For example for a fully connected graph with $N$ nodes and a time $T$, the computation cost is $O(T^m)$. Therefore, a fast algorithm is needed for applying our detection in practice.

**Sliding window**: Instead of using all the data up to time $T$, we adopt a sliding window approach to reduce the computation cost. For a window length $L$, we compute the test statistics with the data from $T - L + 1$ to $T$.

**Random sampling propagation paths**: As stated above, given the maximum number of failure $m$, the number of possible propagation paths in a fully connected network grows exponentially as the number of nodes increases. Therefore to reduce the computation cost, we propose a random sampling method to sample the propagation paths. Denote the failure set as $\mathcal{F}$ which contains the failure nodes, and risk set as $\mathcal{R} = \{ j \notin \mathcal{F} : \exists i \in \mathcal{F}, \lambda_{i,j} > 0 \}$. Then, we generate next possible failure points by the way that we randomly pick $P$ points in $\mathcal{R}$ without replacement with probability vector $\mathbf{p} = (p_i)_{i \in \mathcal{R}}$, where $\forall i \in \mathcal{R}$, $p_i = \tilde{p}_i (\sum_{j \in \mathcal{R}} \tilde{p}_j)^{-1}$ and $\tilde{p}_i = \sum_{j \in \mathcal{R}} \lambda_{i,j}$. With this scheme, we reduce the number of paths to $O((NP)^m)$, which grows linearly.

**Thinning**: Given a propagation path, we need to compute maximum of $\mathcal{L}_1 (\bar{F})$, which is the product of $\mathcal{L}_1^M$ and $\mathcal{L}_1^D$. We define the $q$th percentile of $i$th node as $L_{i,q} \triangleq \inf\{ x \in Q_{i,0} : |Q_{i,x}|/L \geq q \}$, where $|A|$ is the cardinality of set $A$ and

$$
Q_{i,x} = \{ \mathcal{L}_{1,i}^M (\tau_i) \geq x : \tau_i = T - L + 1, \ldots, T \}.
$$

Moreover, we also define a lower bound $l_D$ for $\mathcal{L}_1^D$. 
Hence, instead of maximizing $\mathcal{L}_{1}(\bar{\tau})$ over all the possible choice, we maximize it only in a thinning set $\{\bar{\tau} : \mathcal{L}_{1}^{M}(\bar{\tau}) \geq l_{i,Q}, \forall i = 1, \ldots, N\}$ \& $\{\bar{\tau} : \mathcal{L}_{1}^{D}(\bar{\tau}) \geq l_{D}\}$. The computation cost of this step is $O(h)$, where $h$ depends on the topology of $G, \lambda_{i,j}$’s, $l_{D}$ and $q$. Moreover, we know that $h \leq [L(1-q)]^{m}$.

Remark 4.1 We know that $\mathcal{L}_{1} = \mathcal{L}_{1}^{M} \cdot \mathcal{L}_{1}^{D}$. $\mathcal{L}_{1}^{M}$ can be updated fast by keeping tracking the mean and the sum of square of the data since the computation for each node is not related to each other. However the computation of $\mathcal{L}_{1}^{D}$ is very costly. In this algorithm, given a propagation path, we enumerate $\bar{\tau} \in \{\bar{\tau} : \mathcal{L}_{1}^{M}(\bar{\tau}) \geq l_{i,Q}, \forall i = 1, \ldots, N\}$ from the small to the large. Specifically, given $\tau_{i}$, we try $\tau_{j} \in \{\tau_{i} + 1, \tau_{i} + 2 \ldots\} \cup \{\tau_{j} : \mathcal{L}_{1}^{M}(\tau_{j}) \geq l_{i,Q}\}$ until $\mathcal{L}_{1}^{D} < l_{D}$ as shown in figure 2. Here, we accelerate our computations by exploiting the monotonicity of $\exp(-x)$. In [22], they accelerate the search of change-points by binary search.

![Fig. 2: Illustration of searching the $\tau_{i}$ given last failure point $\tau_{i}$. We search $\tau_{j}$ from $\tau_{i} + 1$. By monotonicity, we stop the search at $\tau_{i} + 5$.](image)

With the above three strategies, we can reduce the computation cost to $O(NP^{m}h)$. The computation cost of our algorithm is linear to $N$ and independent of $T$. As shown in the following numerical section, we can compute the test statistics for a 300-bus power system at a speed that is fast enough to have our algorithm applied in the practice.

5. NUMERICAL EXPERIMENTS

In this section, we are going to use numerical examples to compare the performance of our methods with some existed methods.

Performance metrics: Two commonly used performance metrics in change-point detection are average run length (ARL) and expected detection delay (EDD). For a stopping time $\Gamma$, we define ARL as $\mathbb{E}_{0}[\Gamma]$ and EDD as $\mathbb{E}_{1}[\Gamma - t_{j}^{+}]$, where $\mathbb{E}_{i}$ is the expectation with the probability measure under hypothesis $H_{i}$. It is easy to see that a good procedure should have a large ARL with a small EDD.

Experiments: In the experiments of this section, the pre-change distribution is $\mathcal{N}(0, 1)$ and post-change distribution is $\mathcal{N}(1, 1)$.

Case I: Detect when the first change-point exists. In Figure 3, we show the result in a 300-bus power system (from MATPOWER [23]). In this large system, we apply our fast algorithm mentioned in previous section, with $L = 100$, $q = 0.8$, $P = 1$, $l_{D} = e^{-5}$ and $m = 5$. The average computation time for each time step is less than 3 seconds. When parameters are unknown, we compare our method with the generalized log-likelihood ratio. When parameters are known, we compare our method with CuSum. Also, we show the results of CuSum when $\mu$ is misspecified ($\mu = 2, 2.5$). Overall, our method shows the best performance.

Case II: Detect when there are at least $\eta$ change-points. In figure 4, it shows the comparison between Generalized Multi-chart CuSum, S-CuSum [19] and proposed method. In this experiment, the graph is fully connected with 15 nodes. The parameter for the algorithm is $L = 100$, $q = 0.8$, $P = 1$, $l_{D} = e^{-7}$ and $m = 5$. We set $\eta = 3$. To compute the ARL, we generate data with $\eta - 1$ affected nodes. The result shows our method outperforms both Generalized Multi-chart CuSum and S-CuSum.

![Fig. 3: Comparison of CuSum, Generalized log likelihood ratio and Proposed method.](image)

![Fig. 4: Comparison between General Multi-chart CuSum, S-CuSum and Our proposed method.](image)

6. CONCLUSION

In this paper, we model the cascading failure as a temporal diffusion process in a network. Under this model, we also propose a fast algorithm to perform the change-point detection. Numerical experiments show our proposed method outperforms the existed methods.

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