Inflated Eccentric Migration of Evolving Gas Giants II – Numerical Methodology and Basic Concepts

Hila Glantz1, Mor Rozner1, Hagai B. Perets1, and Evgeni Grishin1,2

1 Technion—Israel Institute of Technology, Haifa, 3200002, Israel; Glanz@tx.technion.ac.il
2 School of Physics and Astronomy, Monash University, Clayton 3800, VIC, Australia

Received 2021 November 28; revised 2022 April 4; accepted 2022 April 17; published 2022 May 19

Abstract
Hot and warm Jupiters (HJs&WJs) are gas-giant planets orbiting their host stars at short orbital periods, posing a challenge to their efficient in situ formation. Therefore, most HJs&WJs are thought to have migrated from an initially farther-out birth location. Current migration models, i.e., disk migration (gas-dissipation driven) and eccentric migration (tidal evolution driven), fail to produce the occurrence rate and orbital properties of HJs&WJs. Here we study the role of thermal evolution and its coupling to tidal evolution. We use AMUSE, a numerical environment, and MESA, planetary evolution modeling, to model in detail the coupled internal and orbital evolution of gas giants during their eccentric migration. In a companion paper, we use a simple semianalytic model, validated by our numerical model, and run a population-synthesis study. We consider the initially inflated radii of gas giants (expected following their formation), as well study the effects of the potentially slowed contraction and even reinflation of gas giants (due to tidal and radiative heating) on the eccentric migration. Tidal forces that drive eccentric migration are highly sensitive to the planetary structure and radius. Consequently, we find that this form of inflated eccentric migration operates on significantly (up to an order of magnitude) shorter timescales than previously studied eccentric-migration models. Therefore, inflated eccentric migration gives rise to the more rapid formation of HJs&WJs, higher occurrence rates of WJs, and higher rates of tidal disruptions, compared with previous eccentric-migration models that consider constant ~Jupiter radii for HJ and WJ progenitors. Coupled thermal–dynamical evolution of eccentric gas giants can therefore play a key role in their evolution.

Unified Astronomy Thesaurus concepts: Exoplanet formation (492); Hot Jupiters (753); Exoplanet migration (2205); Exoplanet evolution (491)

1. Introduction

Gas-giant planets are thought to have formed from either core accretion, in which runaway gas accretion takes place onto the massive, \(\sim 10M_\oplus\), core (Perri & Cameron 1974; Bodenheimer & Pollack 1986), or from a direct collapse from the gas disk (Mizuno 1980; Boss 1997; Armitage 2010). However, as the efficiencies of both channels are greatly affected by the local environment properties, such as the temperature, density, composition, and velocities, they cannot solely describe the formation of gas giants that have extremely short-period orbits around their host stars. These include the population of Hot Jupiters (HJs), with orbits of a few days (Bodenheimer et al. 2000; Rafikov 2005), as well as some warm Jupiters (WJs) with small pericenters. The remaining nonnegligible fraction of more distant WJs might still be formed in situ, as discussed in Huang et al. (2016) and Anderson et al. (2020). Consequently, HJs&WJs are thought to have formed at larger separations from their stars and migrated inward due to dynamical interactions, either with other bodies leading to high-eccentricity migration or with the gas from the protoplanetary disk, producing a drag force (see Dawson & Johnson 2018 for a review). Nevertheless, past studies on these migration models could not reproduce the observed formation rates and properties of the current population of HJs&WJs (Dawson & Johnson 2018; Zhu & Dong 2021), as the typical migration timescales are potentially too long to produce the inferred numbers of HJs/WJs and their appropriate timescales.

A planet orbiting its host star at close separation experiences significant tidal forces raised by the host star. The gravitational interaction between the star and the bulge raised due to tides on the planet (and to a much lesser degree the tides raised on the star by the planet) eventually gives rise to the dissipation of orbital energy in the planetary atmosphere. This, in turn, leads to the orbital decay of the planet into shorter periods and more circular orbits. Consequently, planets on highly eccentric orbits with a close pericenter approach to the host star may experience tidal migration, generally termed eccentric migration. The strength of the tides strongly depends on the planetary radius, which is typically considered as some constant \(\sim\)Jupiter radius, \(R_p\), in eccentric-migration models. However, gas giants are thought to form with far larger inflated radii and then cool and contract, where external heating by radiation and/or tidal heating may slow down their cooling and possibly even reinflate them. After reaching large radii of up to \(10R_J\) by the end of the core accretion (Ginzburg & Chiang 2019), the gas giants contract to smaller radii, initially in a rapid process to \(4R_J\) (Guillot et al. 1996), followed by a slower thermal contraction within a Kelvin–Helmholtz timescale (\(\sim 10^8\) yr), reaching radii of \(\sim 1.5–2.5R_J\) that continuously shrink in an even slower rate, depending on their mass and external energies. Because tidal migration depends strongly on the planetary radius, inflated planets could give rise to far faster eccentric migration compared with nonevolving constant-Jupiter-radius gas giants typically considered in eccentric-migration models. As the radius of such planets might decrease in a comparable timescale to the high-eccentricity migration
timescale, considering the internal evolution of the planet (initially thermal contraction and cooling) can therefore play a key role in their dynamical evolution.

Here and in a companion paper (Rozner et al. 2021, hereafter Paper I) we explore for the first time a self-consistent thermal–dynamical evolution of migrating planets over a wide parameter space and throughout their evolution beginning at very high eccentricities (but see Wu et al. 2007; Miller et al. 2009; Petrovich 2015, where some of these issues were partially studied). We couple the thermal evolution of gas giants and their dynamical evolution through eccentric tidal migration, as well as consider possible reflation and slow contraction of the planets due to external heating sources. We find that the eccentric migration of such inflated Jovian planets, which we term inflated eccentric migration, significantly alters their dynamical evolution and could play a key role in any type of eccentric migration and, in particular, give rise to much (up to an order of magnitude) faster eccentric migration.

Here we present our numerical method, where we use MESA and AMUSE to accurately simulate the internal evolution of these planets during their migration. Our numerical results can be used to study the effect of other types of dynamical evolution and external energy sources. In Paper I, we present a semianalytical approach to simulate such a migration, where we use the same equations of motion but simple modeling of the internal/thermal evolution, which therefore requires less computational resources in order to be used. Here we present some comparisons between the results of both methods and find a good agreement. This also validates our use of the semianalytical approach in the study and characterization of a large population of HJ and WJ progenitors, which we present in Paper I.

In the next section, we describe our calculation method. We first discuss the considered external energy sources affecting the evolution of the giant planets (Section 2.1), then we explain the mechanism of high-eccentricity migration with different tide models in Section 2.2. Later (Section 2.3), we describe our numerical simulation method to couple the dynamical and tidal evolution of the planets with their thermal evolution. In Section 3, we present our results and their implications on the formation of HJs&WJs, followed by discussions in Section 4, and finally, we summarize in Section 5.

2. Methods

2.1. External Energy Sources

In the absence of any internal heating sources, following its formation and final runaway accretion stages, a newly born gas giant begins to continuously cool down and contract. However, a variety of external heating sources can affect the planet during its life. These can include heating the planetary surface through irradiation by its host star, tidal heating induced by the star when the planet migrates, or any other potential heating sources resulting from other interactions and dynamical processes (e.g., collisions with other planets, Lin & Ida 1997, which can affect the early stages of planetary evolution and growth). Here, we consider the evolution of fully formed planets after they had been excited to high eccentricity, such that they experience strong tidal interactions with the host star. Besides the initial excitation to high eccentricity, the definition of our initial conditions, and the tidal interaction with the host star, we assume that no further interaction with other stellar or planetary bodies occurs. Figure 2 demonstrates the fast contraction from the initially inflated radii to about 2RJ, in less than a Myr, such that a scattering prior to this stage is less probable, and even in such cases, the binary would more likely be disrupted rather than rapidly migrate to produce an HJ/WJ (see the discussion on flow in parameter space in Paper I). Therefore, we begin our models after a gas giant has already finished the core accretion stages and any planetary-scattering epoch and reached the initial eccentricity for its migration.

Generally, these processes are thought to have been finalized by the first few Myr of evolution. As we describe in Section 2.3, we examine different initial radii at the time of coupling, such that the external energies are included both during the rapid contraction shown in Figure 2 and after all initial models have already converged and continued on the same cooling timescale.

Hereafter we study the effects of two sources of external energy: tidal heating and irradiation flux from the host star, both of them taken into consideration self-consistently together with the migration of the planet toward the host star and the thermal cooling of the migrating planet.

The distribution of the heat from the different sources inside the planet depends on the specific mechanism and the internal structure of the planet. Irradiation flux heats the surface of the planet and dissipates to deeper layers, but tidal heating may cause a deformation of the internal structure and therefore can potentially heat deeper layers more efficiently. We define r_{ext} as the radial distance inside the planet in which most of the external energy source is deposited.

The irradiation luminosity (averaged over an orbital period) is deposited in the photosphere of the planet (i.e., r_{ext} = r_{irr} = R_p, where R_p is the radius of the planet), is given by:

\[ L_{irr} = \frac{1}{T} \int_T L_{irr}(r(t))dt = \left( \frac{R_p}{a} \right)^2 \frac{L_*}{\sqrt{1 - e^2}} \]

where \( T \) is the orbital period and \( r(t) \) is the distance between the planet and its host star.

The energy from tidal heating is given by the tidal model, which determines its internal distribution (i.e., r_{ext} = r_{tides}). We discuss the different heat distributions in Sections 2.2.1 and 2.2.2 for the equilibrium and dynamical tides models. We explain our numerical method of the internal heat distribution in Section 2.3.

We find that due to the planet’s own radiation and cooling of the planet, the effect of deposition of irradiation and/or tidal heating on the dynamical evolution is mostly negligible when the energy is deposited in the planetary photosphere. In this case, most of the deposited energy is quickly irradiated away and does not heat the planetary interior. Consequently, the planetary radius is not affected by the heating processes in this case nor does it affect the migration timescale. However, some processes, such as ohmic dissipation (Batygin & Stevenson 2010), can provide a channel for heat conduction into internal regions. Deeper deposition at the inner layers could lead to a much more significant effect, such that even 1% of the external energy deposited at the center of the planet could induce larger radii than R_J even after Gyr when the planet is already very close to its star (see Bodenheimer et al. 2001; Guillot & Showman 2002; Komacek et al. 2020 and references therein). When using r_{ext} = 0 to deposit the energy around the center of the planet, and multiply the right side of Equation (14) by an efficiency parameter, we find that very high energy deposition
in the core can indeed give rise to planetary inflation, as can be seen in Figures 5, 6, and 11, which in the case of strong inflation can lead to disruption. We further discussed this in Section 4.2; see also the semianalytic study in Paper I.

2.2. High-eccentricity Tidal Migration

Tidal migration occurs when strong tidal forces from the star act to exchange energy and angular momentum between the orbit and the planet, leading to the growth of tidal bulges and thus an orbital decrease. Given the strong dependence of tides on the distance from the host star (Equations (3) and (8)), efficient tidal migration requires a close approach of the planet to the star. If the planet is born far from the star, as expected for gas giants, a close approach can occur only if the planet resides in a highly eccentric orbit, for which the pericenter approach is close to the star for tidal effects to become significant. Therefore, one can divide high-eccentricity tidal migration into two separate stages: reducing the planet’s angular momentum and reducing the planet’s energy. In the first stage, the HJ/WJ progenitor, which is likely formed on a relatively circular orbit, is excited into an eccentric orbit via planet–planet scattering (Rasio & Ford 1996; Chatterjee et al. 2008; Jurić & Tremaine 2008), as we discuss here, or through other channels for eccentricity excitation such as via the Von-Ziepelin–Lidov–Kozai (ZLK) mechanism and secular chaos (e.g., von Zeipel 1910; Kozai 1962; Lidov 1962; Wu & Murray 2003; Fabrycky & Tremaine 2007; Nagasawa et al. 2008; Naoz et al. 2011; Wu & Lithwick 2011; Petrovich 2015; Hamers et al. 2017; Wu 2018). In the second stage, energy extraction via tides leads to migration and circularization of the planet’s orbit. The energy extracted from the orbit during an orbital period is dissipated in the planet, affecting its overall luminosity, which can affect the internal structure as a result. Assuming a complete transfer from the orbital energy to the planet, the injected/incoming luminosity can be described as follows:

\[ L_{\text{tide}} = \frac{E}{a} \frac{da}{dt}, \]  

where \( E \) is the orbital energy and \( a \) is the semimajor axis.

Modeling tides in giant planets is not trivial, and its strong dependence on the internal structure of the planet, turbulent viscosity processes dissipating energy, and other physical aspects of the problem has induced some long-standing debates on the nature and specific properties of tidal dissipation. Here we adapt the widely used tidal model of weak/equilibrium tides (Darwin 1879; Goldreich & Soter 1966; Alexander 1973; Hut 1981) but also consider more briefly the importance of dynamical tides (e.g., Zahn 1977; Mardling 1995a, 1995b). The latter could be especially important and more efficient during the early migration phases when the planet’s orbit is still highly eccentric, and in that sense, considering only the weak-tides model is potentially conservative in terms of the efficiency of eccentric migration (Lai 1997).

Here we present a general approach, which can account for any tide model and is demonstrated here using both equilibrium tides and dynamical tides. We note that other models, such as chaotic-dynamical tides (Vick & Lai 2018; Vick et al. 2019), are likely to further shorten the migration timescales; these are to be left for future works.

In the next subsections, we explain how we model the migration of a planet due to equilibrium and dynamical tides, where we describe the equations of motion and the corresponding heat that should be transferred to the planet.

2.2.1. Equilibrium Tide Model

In this tidal model, the gravity from the star raises tides on the planet, leading to the formation of an equilibrium bulge on the planet, which is treated as an external point mass along the calculation (Hut 1981). Due to the timescale involved in raising the bulge, and the spin of the planet, the bulge position lags with respect to the position of the star, and the mutual interaction of the stellar gravity and the bulge torques the planet. When the lag time between the objects is much smaller than the spin or orbital period of the planet, one can invoke the weak-tide approximation (Darwin 1879; Goldreich & Soter 1966; Alexander 1973; Hut 1981). Under the assumption of pseudo-synchronization (of the planetary spin and the orbit) and conservation of the angular momentum, the orbital-averaged time evolution of the eccentricity and semimajor axis is given by Hut (1981) and Hamers & Tremaine (2017):

\[ \frac{da}{dt} = -21kAM^2r_p^2M_e\left(\frac{M_p}{M_e}\right)^5 \frac{a}{e^2} \frac{f(e)}{(1-e^2)^{15/2}}, \]  

\[ \frac{de}{dt} = \frac{21}{2}kAM^2r_p^2M_e\left(\frac{M_p}{M_e}\right)^5 \frac{e^3}{(1-e^2)^{15/2}}, \]

where \( M_e \) is the mass of the host star, and \( M_p, R_p, e, a, n, \) and \( \Omega_p \) are the mass, radius, orbital eccentricity, orbital semimajor axis mean motion, and spin frequency of Jupiter correspondingly; \( r_p = 0.66 \) is the planetary tidal-lag time, \( kAM = 0.25 \) is the planetary apsidal motion constant (Hamers & Tremaine 2017), and

\[ f(e) := \frac{1 + \frac{45}{14}e^2 + 8e^4 + \frac{685}{224}e^6 + \frac{255}{448}e^8 + \frac{25}{1792}e^{10}}{1 + 3e^2 + \frac{5}{8}e^4}. \]

Here we ignore the influence of the tides on the host star, as these are typically negligible in comparison with the tides on the planet. The energy associated with the tides according to Equations (2) and (3) scales as \( R_p^5 \), leading to a very strong dependence of the migration timescale on the planet’s radius. Consequently, the migration timescales of initially inflated gas giants should be shorter than the timescales of non-inflated gas giants with a constant \( R_p \) radius. We note that the contraction timescales are sufficiently long to maintain inflated gas giants throughout a significant part of their dynamical evolution, such that the initial radius of an HJ/WJ will leave a signature on its expected final parameters, which could be also observed.

We consider the location of the tidal bulge, given by Murray & Dermott (1999),

\[ h_{\text{weak}} = M_e \frac{R_p}{M_p} \left(\frac{R_p}{a}\right)^3, \]

implying a peak of the external heat from tides at \( r_{\text{tides}} = R_p - h_{\text{weak}} \) from the center of the planet.

2.2.2. Dynamical Tides

At very large eccentricities, tidal energy mostly dissipates near periastron, raising a large tidal bulge on the primary (the giant planet in our case). Consequently, such tidal evolution cannot be parameterized by its average over the entire orbit, as
done in the equilibrium tide model \cite{Moe2018}. The energy associated with this tidal deformation might excite internal energy modes of the planet (mainly the fundamental f-mode), which might induce an enhanced response \cite{Mardling1995a, Mardling1995b, Lai1997, Ogilvie2014}, potentially leading to even more rapid circularization and migration of the planet. The eccentricity decay is accompanied by pseudo-synchronization with the angular frequency of the host star and the excitation of oscillations in the planet becomes less pronounced as the orbital eccentricity decreases. As a result, the energy dissipation by the various modes is gradually suppressed, until a transition to the regime in which equilibrium tides are more dominant \cite{Mardling1995b}. The quadrupole order of the energy dissipation can be written as follows \cite{Press1977, Moe2018}:

\[
\Delta E = f_{\text{dyn}} \frac{M_* + M_p GM_p^2}{M_p} \left[ \frac{a(1 - e)}{R_p} \right]^{-9},
\]

with \( f_{\text{dyn}} = 0.1 \), as \cite{Moe2018} \cite{Mardling1995a} \cite{Moe2018} \cite{McMillan1986} \cite{Press1977} \cite{Moe2018} \cite{Press1977} \cite{Moe2018}. The ratio of the migration rate due to dynamical tides with a very small pericenter gets its minimum value at \( \beta \sim 1 \), and we set a lower artificial cutoff at \( e = 0.2 \), at approximately the point where \( \tilde{A}(e) \) gets its minimum value \cite{Glanz2022}. In this way, we avoid the divergence of dynamical tides at \( e = 0 \), and the transition occurs at max \( \{0.2, e_{\text{trans}} \} \). We note that considering migration due to dynamical tides with a very small pericenter such that \( B \) \cite{Glanz2022} is greater than the maximum value of \( \frac{1}{\tilde{A}(e)} \), leads to \( \beta > 1 \) for the entire migration until \( e = e_{\text{trans}} \).

\[
\beta(R_p, a, e) = \tilde{A}(e) \cdot R_p^3 \cdot B(m_p, \ldots),
\]

where because the pericenter is assumed to remain constant during the migration with dynamical tides, one can write

\[
\frac{da}{dt} = -\frac{a}{T} \Delta E, \quad \frac{de}{dt} = \frac{1 - e}{a} \frac{da}{dt},
\]

where \( T = 2\pi \sqrt{\frac{a^3}{GM_* + M_p}} \) is the orbital period of the planet around its host star.

While dynamical tides dominate for large eccentricities, weak tides will be a more physical description for low ones \cite{Mardling1995b}. The ratio of the migration rate due to dynamical tides to the migration rate due to weak tides is given by

\[
\beta(R_p, a, e) \equiv \frac{\frac{da}{dt} \text{dyn}}{\frac{da}{dt} \text{weak}} = \frac{2f_{\text{dyn}} R_p^3 \tilde{A}(e)}{21GM_p k_{AM} \tau_p T},
\]

where

\[
\tilde{A}(e) \equiv \frac{(1 - e^2)^{15/2}}{(1 - e)^9 e^2 f(e)}.
\]

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{Figure1.png}
\caption{\( \tilde{A}(e) \) from Equation (12) showing the dependence of the dynamical-to-weak-tide migration strengths ratio \( \beta \) on the eccentricity.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{Figure2.png}
\caption{Contraction of a gas giant, modeled with MESA version 15140 \cite{Paxton2013, Paxton2018, Paxton2019}, with different initial radii, without deposition of external heat.}
\end{figure}

We further discuss the transition point between the different tide models in Section 3.2.

We note that at a sufficiently large eccentricity and low pericenter, the oscillatory modes inside the planet due to tides could grow chaotically \cite{Ivanov2004, Ivanov2007, Vick2018, Wu2018} and can potentially increase the energy exchange and hence lead to faster migration and circularization. However, modeling such a scenario will be left for future work.

In our modeling, the energy from the dynamical tides is deposited into the planet’s photosphere, i.e., \( \tau_{\text{tides}} = R_p \).

Because of deposition of internal distributions, as the deposition heat from dynamical tides inside a planet is not yet understood \cite{Sun2018} and is beyond the scope of this work. In Section 4.2, we briefly
discuss the effect of a deeper deposition of the tides, which may arise from efficient ohmic dissipation.

We compare the evolution of inflated eccentric migrating planets due to two different tide models—weak and dynamical tides in Section 3.2—where one can notice the stronger effect of the dynamical tides not only on the migration process but on the planet’s structure as well. In the same subsection, one can find a further discussion on the differences between the different tide models, in addition to the influence of different parameter choices for the dynamical tides model.

2.3. Numerical Coupling of the Thermal–Dynamical Evolution

In our numerical approach, we couple the orbital-averaged equations, derived from the tidal migration model, with numerical modeling of the internal evolution of the planet, affected by the deposition of heat and the cooling due to its own irradiation. We use the AMUSE framework (Portegies Zwart et al. 2009) (version 13.2.1 including self-contributions that should be available in future versions) to combine between the different codes. The internal evolution of our planets is modeled with the stellar evolution code MESA (Paxton et al. 2011, 2013) version 2208, which is a one-dimensional code that solves the stellar equations (Kippenhahn et al. 2012) assuming hydrostatic equilibrium and spherical symmetry. We use the OPAL/SCVH equations of state (Rogers & Nayfonov 2002) and opacity tables corresponding to the existence of molecules at low temperatures at the outer layers of the planets (Grevesse & Sauval 1998; Freedman et al. 2008). In order to simulate such planets, our code can combine any given opacity table by specifying the transition temperatures. In this way, one can consider available dust opacities as well as any future opacity tables that are relevant for planets and were not available in the original version of MESA within AMUSE. We begin by creating the initial planet model as a pre-main-sequence low-mass star, which has no nuclear burning and hence mimics the evolution of a planet. The planet model then contracts according to the equilibrium between its own gravity and thermal cooling, as can be seen in Figure 2. We thus evolve this model in isolation until it reaches the initial radius at which the migration process is assumed to begin, and from this point, we couple its further internal evolution (i.e., thermal cooling) to the dynamical evolution. When simulating the coupled migration process, after each orbital evolution step, we calculate the corresponding external heat source (extra heat as termed in MESA Paxton et al. 2013, $L_{\text{ext}}$, such that the energy equation of the planet at each radial distance from its core becomes

$$\frac{dL}{dm} = -T \frac{ds}{dt} + \frac{dL_{\text{ext}}}{dm},$$

where $T$ is the temperature of the model and $s$ is the specific entropy, and $L_{\text{ext}}$ is calculated according to the orbital parameters, through the dependency of tides and irradiation, described in the previous sections. In order to include the external heat term via AMUSE, we updated the current interface to support the inclusion of any external heat distribution during the evolution with MESA.

We consider a heating source that deposits its energy at some typical region inside the planet, at a distance of $r_{\text{ext}}$ from the planet’s center. Owing to the lack of known distribution of the heat, we adapt a Gaussian heat distribution similar to Spiegel & Burrows (2013) and Komacek & Youdin (2017):

$$\frac{dL_{\text{ext}}}{dm} = \frac{L_{\text{ext}}}{\sqrt{2\pi \sigma_{\text{ext}}^2}} \exp\left[\frac{-1}{2} \left( \frac{r - r_{\text{ext}}}{\sigma_{\text{ext}}} \right)^2 \right] \frac{dr}{dm},$$

where $dr/dm = (4\pi r^3)^{-1}$ and $\sigma_{\text{ext}} = 0.5H_{p,\text{ext}}$ is half the scale height, computed at $r_{\text{ext}}$ according to Paxton et al. (2011):

$$H_p(r) = \begin{cases} \min \left\{ \frac{P}{pg}, \frac{PL}{Gp^2} \right\}, & r \neq 0, \\ R_p, & r = 0 \end{cases},$$

$$\frac{dL_{\text{ext}}}{dm} = \frac{L_{\text{ext}}}{\sqrt{2\pi \sigma_{\text{ext}}^2}} \exp\left[\frac{-1}{2} \left( \frac{r - r_{\text{ext}}}{\sigma_{\text{ext}}} \right)^2 \right] \frac{dr}{dm},$$

where $dr/dm = (4\pi r^3)^{-1}$ and $\sigma_{\text{ext}} = 0.5H_{p,\text{ext}}$ is half the scale height, computed at $r_{\text{ext}}$ according to Paxton et al. (2011):

$$H_p(r) = \begin{cases} \min \left\{ \frac{P}{pg}, \frac{P}{Gp^2} \right\}, & r \neq 0, \\ R_p, & r = 0 \end{cases},$$

$$H_p(r) = \begin{cases} \min \left\{ \frac{P}{pg}, \frac{P}{Gp^2} \right\}, & r \neq 0, \\ R_p, & r = 0 \end{cases},$$

Figure 3. The numerical models’ results of the thermal and orbital evolution of an HJ progenitor with a different initial mass, migrating due to weak tides, including irradiation and tidal heating. The initial radius is $1.5 R_p$, the initial semimajor axis is 1 au, and the initial eccentricity is 0.975.
where $P$ and $\rho$ are the pressure and density distributions of the planet. Because this distribution depends on the pressure and density profiles, a deposition at a higher location in the atmosphere, at low-pressure regions, is likely to have only a little influence on the interior of the planet, whereas deposition of energy directly into the core significantly affects the evolution of the planetary structure. Integrating over Equation (14) gives approximately the same amount of heat as $L_{\text{ext}}$. We note that the sum of this discrete numerical distribution over all shells might be different (lower) than the total heat calculated from Equations (1) and (2) when using a Gaussian distribution around some radial distance inside a spherical model with a discrete mass distribution. The change we find is by a factor of 2 at most, i.e., this can effectively be
translated to a lower efficiency of the heat conductance to the central parts. Other distributions can be chosen and easily used by changing the heat distribution.

The time of each step in our simulation is chosen to be much shorter than the typical timescales for the orbital/thermal changes. At each step, we use the current properties of the planet and evolve the orbital parameters according to the specific tides model (i.e., Equation (3) or (8)).

The amount of deposited energy by tides is derived from the tides model and is deposited in the planet; see Equation (2). We assume the energy is deposited at a typical radius of the planet, as discussed above, and smooth it as a Gaussian distribution of the corresponding heat (Komacek & Youdin 2017) $L_{ext} = L_{tides}$, with a pick at $r_{ext} = r_{tides}$. Inside the MESA model, as described in Equation (14), where the exact $r_{tides}$ depends on the tide model, as described in Section 2.2. The irradiation flux from the host star, which changes due to the orbital evolution throughout the migration process, is described in Equation (1). The corresponding heat is distributed in the photosphere of the planet, using $r_{ext} = R_p$ in Equation (14).

After injecting both tidal and radiation energies, i.e., $L_{ext} = L_{tides} + L_{irr}$, we evolve the planet model with MESA for the same duration as was done for the orbital evolution. Our simulations terminate when one of the following conditions is fulfilled: (1) the planet has passed its Roche limit, defined as $r = R_p \eta (M_2/M_1)^{1/3}$ (Guillochon et al. 2011), and cannot survive in its current condition; (2) the evolution time has passed the Hubble time; (3) the planet has cooled and contracted to levels such that currently used finite opacity and equations of states (EOS) tables, as well as other parameters in the version of MESA used, are no longer adequate. Satisfying the last criterion (3) means that the contraction timescale of the current model (including external heating) is much shorter than the migration timescale. During the migration, this termination condition was achieved only in some of our simulations that produced WJs. In such cases, the numerical model cannot be compared with the semianalytic model throughout the evolution. Modeling of these regimes can, however, be followed in the semianalytical model (see Paper I).

Our numerical model has been developed such that one can choose different orbital evolution models and different internal evolution codes. Using the current MESA module, one can follow the evolution of an externally built MESA model and include any external heat sources distributed around any desired location according to Equation (14).

### 3. Results

In the next section we present the results of our numerical simulations of the inflated eccentric migration of gas giants. We simulated different candidate models producing HJs&WJs, where we tested the effect of the different heat sources on their migration under the different tide models.

In Paper I we present a simpler semianalytical model that uses the same equations of motion for the orbital change but is coupled with equations approximating the thermal/radius change, instead of coupled to the much more detailed, yet computationally expensive, numerical model of the internal evolution as done here. We compare the results of the semianalytical model to those of our numerical model described here and find excellent agreement (see also Paper I).

#### 3.1. Hot and Warm Jupiter Candidates with Different Initial Properties

The thermal–dynamical evolution is affected by the different properties of the migrating planet: its mass, radius, and the internal distribution of heat from external energy sources.

Observations show that the majority of the giant-planet population have mass in the range of $0.1–10 \, M_J$ (Butler et al. 2006).
In Figure 3 we present a comparison between the evolution of planets of different masses according to the equilibrium tides model, showing that planets with lower masses migrate faster. The slower contraction of the radii shows inflated eccentric migration to have an even larger impact in these cases when considering the formation of low-mass HJs&WJs. This can be explained by the opposite dependence of the weak-tides EOS on the planetary mass (Equation (3)). In this case, the resulting giants have migrated to become HJs with a final orbital period of ~4 days.

The strong dependence of tides on the radius of the migrating object implies a faster migration for larger initial radii. In Figure 4 we compare the dynamical and thermal evolution of gas planets, considering weak tides, which are initialized with different radii. We notice that an initially more inflated planet can migrate an order of magnitude faster, or even more, than a planet with a constant 1 $R_J$. We discuss the possible implications of the different assumptions regarding the initial radii on the HJ and WJ population in Paper I.

### 3.2. Different Tide Models

The equations of motion of a migrating planet due to equilibrium tides are derived with the assumption of a small eccentricity. As was described in Section 2.2.2, high eccentricities are likely to excite additional modes inside the planet that can lead to even larger effect on the migration process. In Figure 5 we show the migration of gas-giant planets with different initial radii but now affected by dynamical tides, compared with the migration of a constant 1 $R_J$ radius gas giant. The migration of the initially inflated planets is indeed shorter by more than an order of magnitude compared with the migration of the constant radius planet.

In Figure 6 we compare the evolution of migrating planets when considering the two different tidal models, where one can notice the larger effect of the dynamical tides, with a greater dependence on the planetary radius.

As described in Section 2.2.2, the dynamical tides model still has many uncertainties, among them are the value of $f_{\text{dyn}}$ in Equation (8) (McMillan 1986; Mardling 1995b; Lai 1997) and the transition point to equilibrium tides (Moe & Kratter 2018; Grishin & Perets 2022). When using a value of $f_{\text{dyn}} = 0.1$, and if $\beta > 1$ all the way to $e_{\text{trans}}$, the energy associated with the dynamical tides increases rapidly as the semimajor axis decreases. In this case, even when the heat is deposited only at the planet’s photosphere, sufficient heat is transported to the central part as to give rise to a radial expansion (reinflation) of the gas giant (see Figures 5, 6, and 7). This can be seen in Figure 1, showing the dependence of $\beta$ on the eccentricity, which has a minimum at 0.2, and goes to infinity for $e \rightarrow 0$ and $e \rightarrow 1$. Because Equations (7) and (8) are no longer valid at $e = 0$, one must use another condition to cease the dynamical tides prior to this point; in correspondence with Figure 1, we choose a lower limit of the eccentricity with dynamical tides between 0.2 and 0.3. However, as there must be a smooth transition into equilibrium tides prior to circularization, the large jump in the tidal energy during the transition is probably not physical. In our model, we consider the heat associated with the equilibrium tides to be deposited at a depth of the bulge height, while the heat from the dynamical tides, which might be larger, is deposited around the surface, such that the depth of energy deposition changes after the transition to a slightly deeper layer and a lower luminosity for the weak tides, as can be seen in Figure 10. In Figure 6 we also examine the evolution when $r_{\text{eq}}$ always equals the difference between $R_p$ and the height of the bulge, also during the influence of the dynamical tides (light-blue dashed line), and because the bulge is much smaller than the size of the planet, and therefore very close to the surface, the difference, compared with the evolution when dynamical tides are deposited around the surface (purple line), is relatively negligible. The same behavior can be seen in Figure 11, when the external heat is deposited around the core for the entire evolution, which does not affect the dynamical evolution. In addition, as can be seen in Figure 7, a lower value of $f_{\text{dyn}}$ leads to a smoother transition, with a lower impact of the dynamical tides on the migration. Figure 7 shows only a minor difference in the eccentricity evolution between $e_{\text{trans}} = 0.2$ and $e_{\text{trans}} = 0.3$, but a significant difference in the migrations with the different efficiency parameters ($f_{\text{dyn}}$). We compare the effect of the efficiency parameter on the overall formation rate of the HJ and WJ population in Paper I, showing an increase in both populations when using larger values of $f_{\text{dyn}}$. We note that very large dynamical tides can lead to planet disruption, even when still not reaching the tidal radius.
3.3. The Effect of Different Energy Sources

Here and in Paper I we considered the influence of two different external heat sources on the migration of HJ and WJ candidates—irradiation from the host star and the energy from the tides acting on the planet. Our approach allows the inclusion of any external energy source and couples its effects to both the thermal and dynamical processes. In Figure 8 and Figure 9, we demonstrate the differences in the migration of a gas giant with both tide models described in Section 2.2 when including the different combinations of heat sources in the thermal evolution of the planet. Both figures show the importance of the irradiation energy to achieve the observed effective temperature range of such planets. The effect of tidal heating in these two cases is very minor in terms of the final properties of the planet, when the migration terminates. However, as was stated in Section 2.2.2, the exact deposition of the dynamical tides inside the planet is unknown, in edition...
Figure 10. The calculated external heat sources along the migration of a 1 M_j planet with an initial radius of 1.5 R_j, initial semimajor axis of 1 au, and initial eccentricity of 0.97, under the influence of equilibrium tides (upper panel) and dynamical tides (lower panel); simulations are presented in Figure 6.

to its efficiency (i.e., f_dyn) and the exact transition to equilibrium tides (e_trans). Therefore, the effect of the dynamical tides can be greater if the efficiency parameter is larger, as well as when the transition to weak tides occurs at a lower eccentricity. In the upper panel of Figure 10 we see that the radiation is indeed the most dominant along the entire evolution. On the other hand, the lower panel of the same Figure 10 shows an example where the dynamical tides play the dominant role down to the transition point at e_trans = 0.2. In this case, we used a value of f_dyn = 0.1, while a larger value would increase this effect, as can be seen in Figure 7.

We emphasize that there are many uncertainties regarding the deposition of heat by dynamical tides, which generally affect any eccentric-migration model and the structure of tidal-migrating planets. We discuss the possible implications of the different assumptions on the HJ&WJ population in Paper I. However, an in-depth study of the exact behavior of dynamical tides and their workings is beyond the scope of this paper, while here we consider several models and bracket their general implications.

4. Discussion

4.1. Inflated Hot Jupiters and Heat Transfer

Although our study focuses on the early evolution of migrating Jupiters, at which time they still retain large inflated radii following their initial formation, observations show the existence of at least some older inflated HJs, even at Gyr timescales. The abundance of such inflated HJs was suggested to indicate that an external source of deposited energy is required in order to keep HJs at an inflated phase or to reinflate them after they already contracted (Guillot & Showman 2002; Baraffe et al. 2010; Thorngren & Fortney 2018). Several external energy sources and/or processes that conduct heat from the outer layers of the planet to the interior part (hence keeping the planets hotter) were suggested as a solution to the inflation (Ginzburg & Sari 2015). These include tidal heating (Bodenheimer et al. 2001), ohmic heating (Batygin & Stevenson 2010), and irradiation from the star (Burrows et al. 2007). However, there is still no consensus on the origin of the population of such old inflated HJs. Nonetheless, because observed WJs are usually not inflated (Miller & Fortney 2011), this energy should potentially relate to the orbital separation from the host star or to the migration timescales, which are correlated with the orbital energy and angular momenta of the migrating planet. Depending on the energy source, its duration, and its strength, it could potentially affect the migration process and shorten it. More generally, if other processes exist that keep planets inflated, i.e., leading to even longer contraction timescales, our suggested inflated eccentric migration should be even more efficient than already suggested by our results.

4.2. Internal Distribution of Energy

When considering tidal heating and irradiation flux around the migrating planet’s photosphere, we find that due to the efficient radiation of this energy, the effect on the migration is mostly negligible (though it does determine the planet’s effective surface temperature).

However, as can be seen in Figure 11, if the energy is deposited at a deeper region (when using a smaller r_ext in Equation (14)), the planet may slow its contraction or even reinflate, thus its migration will be further accelerated. One can see that even a low efficiency of heat conductance to the center of the planet of only 1% of the energy is distributed around the center of the planet. When multiplying the expression of dL/m in Equation (14) by 0.01, the planet’s radius can increase and affect the migration time. We note that the strength of the dissipation of irradiation energy on the planet slightly varies due to the change in the mass distribution of the model and therefore the change in σ_ext and the result of the integral in Equation (14).

4.3. Formation of Warm Jupiters

Inflated eccentric migration enhances the migration rate such that planets that could not become HJs/WJs when considering an initial and constant 1 R_j radii, migrate more efficiently and now become WJs. Furthermore, inflated WJs, given the same initial conditions, would be less eccentric because they proceed faster in their migration; some of the expected WJs from the 1 R_j case will turn out to be HJs because their inflated migration sped up to enable that. Figure 12 shows the evolution of two models of migrating WJ candidates during a Hubble time, where one outcome can be considered as a WJ (initialized with R = 1R_j) and the other already as an HJ (initialized with R = 3R_j). As both migration cases have not yet terminated at a Hubble time, one can deduce that ongoing star formation will enlarge the fraction of eccentric WJs, which are effectively on their way to become HJs on longer timescales. The fractions of WJs decay with time and the fraction of HJs increases, as WJ candidates end as HJs if their migration is efficient enough. Therefore, star formation gives rise to an increment in the fraction of WJs on account of the fraction of HJs.

5. Summary

In this paper, together with Paper I, we proposed a new efficient model for the formation of HJs and WJs by considering the radial/thermal evolution of the originally inflated planet along its migration. Here we used AMUSE (Portegies Zwart et al. 2009) to couple numerically the dynamical evolution of such planets according to different
tidal models with their internal evolution along the migration process (using MESA; Paxton et al. 2011).

Here and in Paper I, we showed that the inflated eccentric-migration process efficiently accelerates the migration of such gas planets, compared with eccentric-migration models where the thermal evolution of the planets is not considered. Initially inflated planets and planets that reinflated due to tidal and/or radiative heating experience stronger tides, allowing for planets initialized at larger separations to migrate inwards and inducing higher rates of tidal disruptions of gas giants.

We find that the energy deposited by tides is mostly negligible in the equilibrium tides regime (weak tides) when deposited close to the planet’s surface. Tidal heating can be important and even lead to planetary inflation if highly efficient dynamical tides are considered ($f_{\text{dyn}} > 0.1$). In addition, efficient heat transfer from the outer regions of the planets

Figure 11. Comparison of the migration of a constant ($R_0 = 1 R_J$) radius Jupiter-mass planet (blue) and the migration of a similar planet that is also affected by dynamical tides and irradiation (purple) with a constant $R_0 = 1 R_J$ (blue). The injection of external heat is distributed according to Equation (14). The yellow line corresponds to the case where only 1% of the external heat is injected, but it is now distributed around the center of the planet, i.e., at $r_{\text{ext}} = 0$. In all simulations, $a_0 = 1$ au and $e_0 = 0.955$.

Figure 12. The thermal and orbital evolution of HJ and WJ candidates migrating due to dynamical tides, initialized with a semimajor axis of 1.5 au and initial eccentricity of 0.963. The blue line shows the dynamical evolution of a constant $R_0 = 1 R_J$, whereas in orange is the evolution (thermal and dynamical) of $R_0 = 3 R_J$ affected by both irradiation and tidal energies. The initial $3 R_J$ model reached an orbital period of $\sim 6.4$ days after a Hubble time and became an HJ, and the constant $1 R_J$ finalized at an orbit of 24.2 days, in the WJ regime.
where radiative and/or tidal heating is deposited to the central parts also gives rise to significant thermal evolution and possible inflation of planets during their migration, even when only weak tides or less efficient dynamical tides are considered. As the planets reinflate, the radii of HJs may become larger, but the number of disruptions may increase (see Paper I for further discussion). Identifying the exact processes and efficiencies of heat transfer in gas giants is therefore critical for our understanding of their dynamical evolution and the formation of HJs and WJs. However, this is out of the scope of this paper, and we leave it to future works.

Our numerical and analytical approaches complement each other, and both can account for additional types of dynamical processes and other types of external energies. Our numerical model can be used to simulate the detailed evolution of stellar multiples where one can use the coupled internal evolution part on more than one component. The good agreement between the numerical model presented here and the semianalytic models presented in Paper I support the use of the latter, and the analytical model can well reproduce all of the Juri Ivankov, P. B., & Papaloizou, J. C. B. 2007, MNRAS, 376, 682

The more computationally efficient semianalytical could then be used to study the large parameter space of the HJ/WJ populations, as described in Paper I.

6. Software and Third-party Data Repository Citations

Our numerical code can be found under the public repository https://github.com/hilaglanz/InflatedEccentricMigration. Here we used AMUSE version 13.2.1 with self-contribution as described, combined with MESA version 2208.

We gratefully acknowledge helpful discussions with Sivan Ginzburg, Thaddeus D. Komacek, Michelle Vick, Nicholas C. Stone, and Eden Saig. M.R. acknowledges the generous support of the Azrieli fellowship. H.B.P. acknowledges support for this project from the Minerva center for life under extreme planetary conditions.

Software: AMUSE (Portegies Zwart et al. 2009), MESA (Paxton et al. 2011, 2013).

ORCID iDs

Hila Glanz @ https://orcid.org/0000-0002-6012-2136
Mor Rozner @ https://orcid.org/0000-0002-2728-0132
Hagai B. Perets @ https://orcid.org/0000-0002-5004-199X
Evgeni Grishin @ https://orcid.org/0000-0001-7113-723X

References

Alexander, M. E. 1973, Ap&SS, 23, 459
Anderson, K. R., Lai, D., & Pu, B. 2020, MNRAS, 491, 1369
Armitage, P. J. 2010, Astrophysics of Planet Formation (Cambridge, UK: Cambridge Univ. Press)
Baraffe, I., Chabrier, G., & Barman, T. 2010, RPPh, 73, 016901
Batygin, K., & Stevenson, D. J. 2010, ApJ, 714, L238
Bodenheimer, P., Hubbard, W. B. 2007, ApJ, 661, 502

Butler, R. P., Wright, J. T., Marcy, G. W., et al. 2006, ApJ, 646, 505
Chatterjee, S., Ford, E. B., Matsumura, S., & Rasio, F. A. 2008, ApJ, 686, 580
Darwin, G. H. 1879, RSPT, 170, 1
Dawson, R. I., & Johnson, J. A. 2018, ARA&A, 56, 175
Fabrycky, D., & Tremaine, S. 2007, ApJ, 669, 1298
Freedman, R. S., Marley, M. S., & Lodders, K. 2008, ApJS, 174, 504
Ginzburg, S., & Chiang, E. 2019, MNRAS, 490, 4334
Ginzburg, S., & Sari, R. 2015, ApJ, 803, 111
Goldreich, P., & Soter, S. 1966, Icar, 5, 375
Grevesse, N., & Sauval, A. J. 1999, SSRv, 85, 161
Grishin, E., & Perets, H. B. 2022, MNRAS, 512, 4993
Guillochon, J., Ramirez-Ruiz, E., & Lin, D. N. 2011, ApJ, 732, 74
Gullott, T., Burnrows, A., Hubbard, W. B., Lunine, J. I., & Saumon, D. 1996, ApJ, 459, L35
Gullott, T., & Showman, A. P. 2002, A&A, 385, 156
Hamers, A. S., Antonini, F., Lithwick, Y., Perets, H. B., & Portegies Zwart, S. F. 2017, MNRAS, 464, 688
Hamers, A. S., & Tremaine, S. 2017, AJ, 154, 272
Huang, C., Wu, Y., & Triaud, A. H. M. J. 2016, ApJ, 825, 98
Hut, P. 1981, A&A, 99, 126
Ivanov, P. B., & Papaloizou, J. C. B. 2004, MNRAS, 347, 437
Ivanov, P. B., & Papaloizou, J. C. B. 2007, MNRAS, 376, 682
Jurić, M., & Tremaine, S. 2008, ApJ, 686, 603
Kippenhahn, R., Weigert, A., & Weiss, A. 2012, Stellar Structure and Evolution (Berlin: Springer), 2012
Komacek, T. D., Thomgren, D. P., Lopez, E. D., & Ginzburg, S. 2020, ApJ, 893, 36
Kozai, Y. 1962, AJ, 67, 591
Lai, D. 1997, ApJ, 490, 847
Lidov, M. L. 1962, P&S, 9, 719
Lin, D. N. C., & Ida, S. 1997, ApJ, 477, 781
Mardling, R. A. 1995a, ApJ, 450, 722
Mardling, R. A. 1995b, ApJ, 450, 732
McMillan, S. L. W. 1986, ApJ, 306, 552
Miller, N., & Fortney, J. J. 2011, ApJL, 736, 129
Miller, N., Fortney, J. J., & Jackson, B. 2009, ApJ, 702, 1413
Mizuno, H. 1980, PThPh, 64, 544
Moe, M., & Kratter, K. M. 2018, ApJ, 854, 44
Murray, C. D., & Dermott, S. F. 1999, in Solar System Dynamics, ed. C. D. Murray & S. F. Dermott (Cambridge: Cambridge Univ. Press)
Nagasawa, M., Ida, S., & Bessho, T. 2008, ApJ, 678, 498
Naoz, S., Farr, W. M., Lithwick, Y., Rasio, F. A., & Tremssander, J. 2011, Natur, 473, 187
Ogilvie, G. I. 2014, ARA&A, 52, 171
Portegies Zwart, S. F., Hergenrother, M. S., & Maehlen, J. H. 2006, ApJ, 646, 1064
Press, W. H., & Teukolsky, S. A. 1977, ApJ, 213, 183
Rafikov, R. 2005, ApJL, 621, L69
Rasio, F. A., & Ford, E. B. 1996, Sci, 274, 954
Rogers, F. J., & Nayfonov, A. 2002, ApJ, 576, 1064
Rozner, M., Glanz, H., Perets, H. B., & Grishin, E. 2021, arXiv:2111.12718
Spiegel, D. S., & Burrows, A. 2013, ApJ, 772, 76
Sun, M., Arras, P., Weinberg, N. N., Troup, N. W., & Majewski, S. R. 2018, MNRAS, 481, 4077
Thorngren, D. P., & Fortney, J. J. 2018, AJ, 155, 214
Vick, M., & Lai, D. 2018, MNRAS, 476, 482
Vick, M., Lai, D., & Anderson, K. R. 2019, MNRAS, 484, 5645
von Zeipel, H. 1910, A&A, 99, 126
Wu, Y. 2018, AJ, 155, 118
Wu, Y., & Lithwick, Y. 2011, ApJ, 735, 109
Wu, Y., & Murray, N. 2003, ApJ, 589, 605
Wu, Y., Murray, N. W., & Ramasahai, J. M. 2007, ApJ, 670, 820
Zahn, J. P. 1977, A&A, 500, 121
Zhu, W., & Dong, S. 2021, ARA&A, 59, 42