The critical properties of the agent-based model with environmental-economic interactions

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Abstract

The steady-state and nonequilibrium properties of the model of environmental-economic interactions are studied. The interacting heterogeneous agents are simulated on the platform of the emission dynamics of cellular automaton. The model possess the discontinuous transition between the safe and catastrophic ecology. Right at the critical line, the broad-scale power-law distributions of emission rates have been identified. Their relationship to Zipf’s law and models of self-organized criticality is discussed.

Key words: agent-based distributed feedback, discontinuous transition, environmental-economic interactions
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1 Introduction

On the onset of global ecological crisis questions related to regulation technologies and safe handling of environmental resources become urgent. The equilibrium between the economic growth and environmental variables is an actual problem of every government. The environmental science must deal with the task how to design robust sensorial nets and control systems to manage the balance between the immediate economic needs and the long-time perspectives of mankind. There are several instruments for control and decrease the local and global pollution rate: energy taxation, pollution rights and introduction of cleaner technologies. Unfortunately, direct and naive application of these approaches does not result relevant emission reductions. Enforcing pollution rights is difficult and has high administrative costs [1,2]. As an example can serve the multi-country control model of international coordination of emission charges and pollution trading [3]. As shown in Ref. [3] pollution trading
may result a non-cooperative equilibrium with high pollution rates. It is often argued that if the only tool of policy is the energy taxation it may cause the loss of the competitiveness between firms. Moreover, the tax rate which is be able to stabilize the emissions would be too high to be economically and politically acceptable. Because of energy taxation and network effects in society, the firms tend to delay adoption to cleaner technologies [4,5]. All these unexpected emergent regulatory effects stimulated our interest about the problem of dynamics of emissions that can be seen as a consequence of the competitive interaction of the environmental and economical (acquisition) agents.

A key principle we applied to define a robust environmental policy is the feedback mechanism. In general, the nonlinear feedback is one of the key up-bottom mechanism that enables to keep control over the dynamics of complex systems. The mechanism has been exploited in many different systems. The problem discussed in Ref.[6] is focused to the feedback localization of the parametric boundary between periodicity and chaos generated by a logistic map. Stabilization of dynamical regimes via feedback appears in the simulations of investments Ref.[7]. In Ref.[8,9] model has been suggested, where the speculative market expectations are linked via the positive feedback to the prevailing price trend. As stated in Ref.[10], the steady self-organized critical (SOC) regime Ref.[11] implies the operation of inherent feedback mechanism that ensures a steady state marginally stable against disturbances. It is important here to note that criticality is accompanied by the absence of characteristic scale of the power-law distributions of avalanches. According Ref.[12] the nonlinear feedback allows to transform the models of the ”unstable” phase transitions (i.e. classic models) to SOC models. In the works devoted to self-adjusted Monte Carlo methods Ref.[13,14], the feedback which drives the physical system to a critical regime is expressed in terms of estimated statistical averages. Additionally, catastrophic environmental collapses in consumption/pollution feedback models were described in [15,16]. In this paper we present feedback control distributed over the population of autonomous sensing mobile agents that control the behavior of factory agents.

As usual, the thinking about distributed control is motivated by the expected robustness against accidental failures. Nevertheless, most of agent systems face to the general problem of agency that is the imperfect incentives [17] to satisfy the global goal function of system. Our motivation is to show, that this property of distributed agent system may drive the environment to a critical state with extremal emission ratio. The idea of walking sensorial agents in our model is very similar to the idea of distributed pollution sensing by pigeon bloggers proposed by de Costa Ref.[18], where pigeon agents are equipped by communication chips and sensors for carbon monoxide. Similar idea had been reported as a concept of randomly walking sensors for detection of dynamically changing gradient sources [19].
The plan of the paper is as it follows. In the next section the multi-agent ecological model is introduced. In section sec.3 the results of numerical study are discussed. Finally the conclusions are presented.

2 The model

The first point we consider is the dynamics of diffusive emissions of factory agents. The base substance of model is a square lattice with dimension $L \times L$ which represents the physical space for emission spreading. The factories are located on random lattice sites and the spatial distribution of their emissions is directed by discrete rule of diffusion. For this purpose, we implement a cellular automaton (CA) based on integer rules. Our suggestion for computation of diffusion is partially inspired by the lattice gas models of pollutants Ref.[20]. However, we preferred formulation where the concentration field on lattice sites varies via the recursive rule

$$\forall \mathbf{R} \in \{L \times L\}, \quad m(t+1, \mathbf{R}) = m(t, \mathbf{R}) + I_{nn}(t, \mathbf{R}) + I_{F}(t, \mathbf{R}) \quad (1)$$

that replaces in one time step $t$ the $m(t, \mathbf{R})$ ”integer pieces” of emission at the position $\mathbf{R} \equiv [R_x, R_y]$. The emission inflow caused by the factory agents $I_{F}$ is discussed in below Eq.(6) The emission current between the nearest neighbor cells

$$I_{nn}(t, \mathbf{R}) \equiv -4[m(t, \mathbf{R})/5]_{\text{int}} + \sum_{\mathbf{r} \in \text{nn}(\mathbf{R})} [m(t, \mathbf{r})/5]_{\text{int}}, \quad (2)$$

where the brackets $[\ldots]_{\text{int}}$ denote the integer part of argument and $\sum_{\text{nn}(\mathbf{R})}$ refers to the four nearest neighboring sites of $\mathbf{R}$. Eq.(2) preserves the total emission for transitions between the lattice sites . Other words, the natural purification of emission is absent, emissions are exclusively depleted by the outflow through opened lattice bounds. For $m(t, \mathbf{R}) \gg 1$ the mass equidistribution of emissions becomes satisfied with $1/\sqrt{m}$ order. Under these conditions, $I_{nn}(t, \mathbf{R})$ acts as 2d Laplace of $m$, and thus CA rules describe the stylized diffusive transport.

The local concentration of emissions is permanently monitored by $N_S$ number of sensing agents. The sensing agents are random walkers with the lattice positions $\mathbf{S}(t, j) \equiv [S_x(t, j), S_y(t, j)], j = 1, 2, \ldots, N_S$. The lattice position of the $j$th walker is given by

$$S_x(t+1, j) = [S_x(t, j) + s_x(t, j)] \mod L, \quad (3)$$
$$S_y(t+1, j) = [S_y(t, j) + s_y(t, j)] \mod L;$$

3
where the shift \( \mathbf{s}(t, j) \equiv [s_x(t, j), s_y(t, j)] \) is chosen randomly from the set of four unit vectors \( \{(0, 1), (0, -1), (1, 0), (-1, 0)\} \). Eq.(3) preserves the periodic boundary conditions for \( \mathbf{S}(t, j) \). Moreover, we checked that the statistical dependencies studied in this paper do not change remarkably for closed boundary conditions.

The source of emissions are the factory agents localized at random positions \( \mathbf{F}(t, k) \in \{L \times L\}, k = 1, 2, \ldots, N_F \). They are characterized by an integer value of emission production \( n(t, k) \). The condition of sensorial feedback system is that over a predetermined local pollution threshold \( m_c \) every sensorial agent with \( m(t, \mathbf{S}(t, j)) > m_c \) sends a feedback request to the nearest (in the Euclidean sense) factory \( \mathbf{F}(t, k^{(j)}_{\text{near}}) \)

\[
\min_{k=1,2,...,N_F} \| \mathbf{S}(t, j) - \mathbf{F}(t, k) \| = \| \mathbf{S}(t, j) - \mathbf{F}(t, k^{(j)}_{\text{near}}) \|. \tag{4}
\]

The concentration threshold \( m_c \) is analogous to that in Ref.[21] where the feedback operation is mediated by the chemical sensors with specific thresholds for different pollutants. Any request of sensorial agent instantly decreases the local emission production \( n(t, k) \) of the \( k \)th factory by a constant additive factor \( \Delta > 0 \). In the absence of feedback signal, to satisfy economic needs, the factory increases its production automatically. Both alternatives are described by the equation

\[
\forall k, \quad n(t + 1, k) = \begin{cases} 
  n(t, k) + \Delta & \text{when no signal from any } \mathbf{S}(t, j) \text{ agent is received} \\
  \max\{n(t, k) - \Delta; 0\} & \text{feedback signal received}
\end{cases} \tag{5}
\]

The eventuality \( n(t, k) - \Delta \leq 0 \) (zero emission means no production) yields the bankruptcy of \( \mathbf{F}(t, k) \) and the appearance of a new factory at random position with initial production \( n(t + 1, k) = 0 \). The zero initial emission assure that new factory may start its production only when it survives at least one time period without feedback request. The total number of factories is conserved by the death-birth rules. Since the production strategies of new generation of factories does not change, the implicit assumption here is that system is locked to the inferior technology [22].
What remains unspecified in Eq.(1) is the emission inflow caused by the \( F \) type of agents

\[
I_F(t, R) = n(t, k) \delta_{R,F(t,k)} .
\]  

It is worth nothing that all the agent systems in common with CA subsystem are updated synchronously. The asynchronous update rules can be also of interest, but we dismiss them from our considerations.

After the completion of the agent-based model, its relationship to the standard sandpile model of SOC [11] should be noted. The level of emissions is confronted with a threshold which is similar to the height (slope) threshold in SOC models. In sandpile models if the height of a pile exceeds a given threshold, a "signal" is transmitted to the central site and to its neighboring sites. The abundant grains are then toppled from the center. Spatial expansion of a sandpile avalanche corresponds to the emission spreading around a factory agent. The power-law distribution of avalanches in sandpile model is a consequence of the local conservation of sand grains. The property coincide with the conservation of emissions, but contradicts to the non-local conservation of factory agents. The opened boundary conditions for emissions are also borrowed from the sandpile model.

3 Simulation results

The simulations have been performed for lattices \( L = 20, 30, 40, 50 \). To avoid the influence of transients, the initial \( 10^5 \) updates per site were discarded; about \( \tau = 5 \times 10^7 \) updates have been used to collect information about the steady-state for the fixed parameters \( m_c = 5 \gg \Delta = 1 \). In further we limit ourselves to the systems with \( \rho_F = N_F/L^2 = 1/100 \). In the world with large number of sensors \( N_S \) the emissions fluctuate around \( m_c \). This oversaturated population of the sensors prevents from unbearable contamination but yields a non-efficient economy. Much different is the situation for \( N_S \) small, where the steady-state regime become unstable since the rate of emissions diverges in time. The dynamics corresponds to a catastrophic regime. The preliminary numerical work on steady-state statistics yields the conclusion that \( m(t, R) \) is remarkably lowered at the corners of \( L \times L \) lattice world with opened bounds. Such inhomogeneity induces secondary effect of the accumulation of factories at boundary regions. As \( L \) increases, the overall contribution of corners decays faster than the contribution of edges. In this context, the important is the aspect of finite-size scaling carried out in the next section.

The outcome of our simulation experience is the decision to accumulate the
statistics of those factories that have received even one feedback request during
the step $t$. Thus, we define the following order parameter

$$\langle \phi \rangle = \frac{1}{\tau} \sum_{t=0}^{\tau-1} \phi(t) , \quad \phi(t) = \frac{1}{N_F} \sum_{k=1}^{N_F} \sigma(t, k) , \quad (7)$$

where $\langle \phi \rangle$ is the temporal average calculated over $\tau$ epochs. If source $F(t, k)$ received a feedback signal $\sigma(t, k) = 1$ and $\sigma(t, k) = 0$ otherwise. Fig.1 shows the $N_S$ dependencies of $\langle \phi \rangle$ that attain abrupt jump interpreted as a discontinuous transition from the safe (nondestructive) to the destructive economy. Moreover, we found that $N_S$ dependencies of $\langle \phi \rangle$ collapse onto the single finite-size scaling form

$$\langle \phi \rangle = L^{-a} f(\theta) , \quad \theta = N_S/L^{2+b} \quad (8)$$

with exponents $a = 0.38$, $b = 1.39$. The algebraic combination $\theta_{\text{crit}} = N_S/L^{2+b}$, where the scaling function $f(\ldots)$ attains the jump $f(\theta_{\text{crit}})$ and $\langle \phi \rangle$ tends to the $L^{-a}$ asymptotics, is interpreted as a critical point. For given $\rho_F$ we have estimated $\theta_{\text{crit}} \simeq 2.8 \times 10^{-4}$. For sufficiently small lattices ($L \leq 20$) the scaling violation is indicated.

The problem with sensorial system manifests itself when $\theta < \theta_{\text{crit}}$. In this regime a sensor multiply affects the same nearest factory and lowers its emission production, while other hidden sources, may be more influential and dangerous polluters, could left without feedback requests. Hereafter, this process is referred as spatial screening. The screening emerges as a result of co-evolution and self-organization in multi-agent system. The statistical significance of the screening is clarified in Fig.2 where the auxiliary order parameter

$$\langle \phi' \rangle = \frac{1}{\tau} \sum_{t=0}^{\tau-1} \phi'(t) , \quad \phi'(t) = \frac{1}{N_S} \sum_{j=1}^{N_S} \sigma'(t, j) , \quad (9)$$

is plotted against $\theta$. The $\sigma'(t, j) = 1$ when the request comes from the $j$th sensorial agent, $\sigma'(t, j) = 0$ otherwise. At the critical point $\theta = \theta_{\text{crit}}$ both the order parameters attain discontinuous jump. The ratio $N_S/N_F$ reflects the screening efficiency which at the critical point reads $N_S/N_F = \theta_{\text{crit}} L^b / \rho_F = \theta_{\text{crit}} \rho_F^{-b/2} N_F^{b/2}$. The proportionality $N_S/N_F \sim N_F^{b/2}$ reveals the interpretation of the index $b$. The difference $\langle \phi' \rangle - \langle \phi \rangle$ reflects the impact of the screening as well as the most pronounced situation at $\theta \leq \theta_{\text{crit}}$.

The vicinity of criticality is corroborated by the broad-scale statistics of production rates (see Fig.3). We see that the most pronounced power-law dependence belongs to $\theta \simeq \theta_{\text{crit}}$. In the economic models Ref.[3], the emission rates
are usually related to the overall production of goods with a proportionality factor called emission-output ratio. Standard and widely accepted assumption is that this ratio is fixed and irrespective to the investments in clean technology. With such idealization, the critical fluctuations of $n(t,k)$ can be assigned to the empirical Zipf’s law. As we see from Fig.4, the cumulative probability distribution function pdf$_>(n)$ corresponding to $(N_S,L)$-parametric domain have been characterized by the effective exponents within the broad range $\beta \simeq 0.5-6.8$. It means that feedback mechanism we proposed is not sufficient to fix the realistic exponents of the Zipf’s law (according Ref.[23] the empirical exponents are $\beta = 0.84; 0.995$).

Since the place for new factory is chosen randomly, the snapshots showing the spatial distributions of $F$ agents evoke the images of the short-range ordered molecular patterns. From this perspective the system of factories have features of 2d molecular liquid or "vapor". Within such viewpoint the "vapor" of $F(t,k)$ agents prepared for $\theta > \theta_{\text{crit}}$ belongs to the safe economy, whereas stronger correlated "liquid condensate" ($\theta < \theta_{\text{crit}}$) corresponds to the catastrophic regime. In further, these arguments are applied in favor of scenario of discontinuous transition. The transition concept is examined in further (in a manner that is independent of $\langle \phi \rangle$) via damage spreading (DS) method Ref.[24]. The DS is based on the comparative analysis of simultaneously evolving replicas. Let us consider the pair of the systems I,II with equal number of agents. At certain time $t$ the copy II is replicated from I and, consequently, single grain of the emission (damage) is added to its randomly chosen cell $R'$:

$$\forall R, t' = 0, \quad m^{(ii)}(0,R) = m^{(i)}(t,R) + \delta_{R,R'}. \quad (10)$$
Fig. 2. The order parameters as function of sensorial agent density $N_S/L^2$ scaled by $L^{-b}$ factor for $L = 40$. The other parameters as in Fig.1.

Fig. 3. The log-log plot of the cumulative distribution of emissions $n(t, k)$ for $L = 50$. The boundary power-law regime ($N_S = 160$) is the most pronounced. Thereupon for $N_S < 160$ the dynamics has no steady state and steady-state distributions do not exist.

The subsequent $t'$ time steps are executed with the identical sequences of random numbers for updating of the both of replicas I and II. The whole process starting from the configuration given by Eq.(10) is repeated many
Fig. 4. The cumulative pdf's of the production rates $n(t, k)$ obtained for different $L$ and $N_S(L) = \theta_{\text{crit}}L^b$. The inset table lists the effective exponents $\beta$.

Fig. 5. The application of DS technique for $L = 40, t' = 250$. The averaged inter-replica distance $D(t')$ [see Eq.(11)] is plotted versus $\theta = N_S/L^{2+b}$. The position of the critical point coincides with Fig.1 within the statistical error.

times. The idea of DS is to look for averages $\langle D(t') \rangle$ of the Hamming distance

$$D(t') = \frac{1}{L^2} \sum_{R}^{L \times L} (1 - \delta_{m^{(1)}(t+t', R), m^{(1)}(t', R)}) ,$$

where $D(0) = 1/L^2$. In Fig.5 we show $\langle D(t') \rangle$ calculated as an average taken over the 1000 steps of the steady-state. It is seen that DS technique confirms $\theta$-driven transition $\theta = \theta_{\text{crit}}$ maintained previously by the direct inspection of
Fig. 6. The differences between nonequilibrium relaxation below and above the critical number (i.e. minimum number) of sensors sufficient to prevent from dangerous pollution is $N_s \simeq 80$ (for $L = 40$). The interval $(2000 < t < 4000)$ approximately delimits the fast CA purification through the opened bounds. The steady-state becomes formed above $t \simeq 7000$ (above the crossing point).

$\langle \phi \rangle (\theta)$ dependence.

Next, the system is studied via nonequilibrium relaxation method[25] that is an efficient tool of analysis of equilibrium phase transitions. The numerical results are shown in Fig.6. The initial configuration has been prepared in a such way that at the beginning, the factories are generated at random positions. Subsequently, the emissions have been left to increase freely without control. After the level $\sum_{\mathbf{r}} m(t, \mathbf{r}) = 5000L^2$ is reached, the sensors become active. The resulting relaxation dependence is then constructed by averaging over 1000 repetitions of the whole relaxation process. We found that relaxation strongly depends on how far $\theta$ is from $\theta_{\text{crit}}$. Three distinct dynamical regimes have been identified when inspecting averaged relaxation process. In early stage of development practically all sensors are active since $\langle \phi \rangle \sim 1$. Later, when content of the most of cells are still above $m_c$, relatively fast exponential relaxation dominates. It corresponds to the purification through bounds. As it can be clearly seen, the processes at large times are well separated from the initial relaxation. The long-time process is associated with the positional adjustment (or self-organization) of the factories.

4 Conclusions

The statistical properties of the multi-agent system including the environmental-economic interactions have been studied. It should be noted that at early stages of the work we were motivated by the particular hypothesis about implementation of feedback and direct generation of SOC regime. However, even
preliminary runs have uncovered that not all the regimes conducted by feedback acquire SOC signatures as are the power-law distributions and power-law autocorrelation functions. With these facts in mind, we shift our focus to give an interpretation in terms of phase transitions ("unstable" in sense of Ref.[12]). Our simulation confirmed that only optimized feedback induces attractiveness of the critical state. In other words, the distributed feedback (as many other approaches) does not automatically guarantees the attractiveness of critical point. The agents proposed here hold only rudimentary thinking and thus more intelligent entities should be considered in future work. A variety of problems can be formulated where the establishment of new factory is subjected to a place-search optimization strategy. With such modification the formation of highly correlated factory - sensorial agent structures are expected. One can extend the idea to models with crystal-like ordering and formation of the compact aggregates of factories - industrial complexes enhancing the effect of screening. An alternative representation of distributed feedback mechanism may be the notion of individual energy taxation of factories which is proportional to the local contamination level. Contrary to plethora of possible additional realistic entries omitted, the hypothesis posed by simulation is that the phase transition appears to be a generic statistical aspect of the groups of autonomous agents with conflicting interests. The main implication from the present model is the identification of power-law (nearly catastrophic) pdf’s of fluctuations of emission rates. We believe that focus to power-law distributions will open new perspectives of the environmental diagnostics.

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References

[1] M. Pearson, International Tax and Public Finance 2, (1995) 357.
[2] S. Speck, Energy Policy 27 (1999) 659.
[3] F. Ploeg, A. Zeeuw, System & Control letters 17 (1991) 409.
[4] C. Carraro, M. Galeotti, Energy Economics 19 (1997) 2.
[5] E.S. Sartzetakis, P. Tsigaris, J. of Regulatory Economics 28 (2005) 309.
[6] P. Melby, J. Kaidel, N. Weber, A. Hübner, Phys. Rev. Lett. 84 (2000) 5991.
[7] J.A. Holyst, K. Urbanowicz, Physica A 287 (2000) 587.
[8] M. Youssefmir, B.A. Huberman, T. Hogg, Computational Economics 12 (1998) 97;

[9] J. Bradford De Long, A. Shleifer, L. H. Summers, R. J. Waldmann, Journal of Finance 45 (1990) 379.

[10] L.P. Kadanoff, Physics Today (March 1991) p. 9

[11] P. Bak, C. Tang, K. Wiesenfeld, Phys. Rev. A 38 (1988) 364;
    P. Bak, C. Tang, K. Wiesenfeld, Phys. Rev. Lett. 59 (1987) 381;
    C. Tang, P. Bak, Phys. Rev. Lett. 60 (1988) 2347.

[12] D. Sornette, J. Phys. I France 2 (1992) 2065;
    D. Sornette, A. Johansen, I. Dornic, J. Phys. I France 5 (1995) 325.

[13] Y. Tomita, Y. Okabe, Phys. Rev. Lett. 86 (2001) 572.

[14] D. Horváth, M. Gmitra, Z. Kuscsík, Czech. J. Phys. 54 (2004) 921;
    D. Horváth, M. Gmitra, Int. J. Mod. Phys. C 15 (2004) 1269;
    M. Gmitra, D. Horváth, Int. J. Mod. Phys. C 16 (2005) 1943;
    D. Horváth, M. Gmitra, Z. Kuscsik, Physica A, 361 (2006) 589.

[15] H.R. Clarke, W.J. Reed, J. of Economic Dynamics and Control 18 (1994) 991.

[16] Y. Tsur, A. Zemel, J. of Economic Dynamics and Control 22 (1998) 967.

[17] R.L. Axtell, C.J. Andrews, M.J. Small J. of Industrial Ecology 5 (2006) 10.

[18] New Scientist magazine, (2. Febr. 2006), 29.

[19] A. Dhariwal, G.S. Sukhatme, A. Requicha, IEEE International Conference on Robotics and Automation, New Orleans, Louisiana, Apr.2004, 1436-1443.

[20] M. Marín, V. Rauch, A. Rojas-Molina, C.S. Lopez-Cajun, A. Herrera, V.M. Castano, Computational Materials Science 18 (2000) 132.

[21] K. Cammann, Sensors and Actuators B 6 (1992) 19.

[22] W.B. Arthur, The Economic Journal, 99 (1989) 116.

[23] Y. Fujiwara, H. Aoyama, C. Di Guilmi, W. Souma, M. Gallegati, Physica A, 334 (2004) 112.

[24] S. Kauffman, J.Theor.Biol. 22 (1969) 437.
    H.E. Stanley, D. Stauffer, J. Kertesz, H.J. Herrmann, Phys. Rev. Lett. 59 (1987) 2376;
    E.M. Sousa Luz, M.P. Almeida, U.M.S. Costa, M.L. Lyra, Physica A 282 (2000) 176.

[25] Y. Ozeki, K. Kasono, N. Ito, S. Miyashita, Physica A, 321 (2003) 271.