The most correct $\rho^0(770)$ meson mass and width values

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Abstract. By application of the pion electromagnetic form factor and also the P-wave isovector $\pi\pi$ scattering phase shift parametrizations, following from the first principles, like unitarity and analyticity, for a description of the corresponding accurate data, it is clearly demonstrated that the $\rho^0(770)$ meson mass and width values obtained by the Gounaris-Sakurai model in a description of the same data can not be accepted as correct ones.

1 Introduction

The initial state radiation method gives very precise information on the pion electromagnetic (EM) form factor ($F_{\pi}^{EM,I=1}$) [1, 2], measuring total cross-section

$$\sigma_{tot}(e^+e^- \rightarrow \pi^+\pi^-) = \frac{\pi\alpha^2(0)}{3t} \beta_\rho^2(t) \left| F_{\pi}^{EM,I=1}(t) + Re^{i\phi} \frac{m_\omega^2}{m_\rho^2 - t - im_\omega\Gamma_\omega} \right|^2$$

with the pion velocity $\beta_\rho(t) = \sqrt{1 - \frac{4m_\rho^2}{t}}$, the $\rho - \omega$ interference phase $\phi$ to be expressed as $\phi = arctg \frac{m_\omega\Gamma_\rho}{m_\rho^2 - m_\omega^2}$ and the amplitude $R$ as a free parameter.

On the other hand, the most accurate up to now P-wave isovector $\pi\pi$ scattering phase shift $\delta_1(t)$ data at the elastic region with theoretical errors, to be generated from the existing inaccurate experimental information by the Garcia-Martín-Kamiński-Peláez-Yndurain (GKPY) Roy-like equations with an imposed crossing symmetry condition [3], appeared.

Further we demonstrated that the the mass and width values of the $\rho^0(770)$ meson following from the application of the Gounaris-Sakurai model to a description of these data are not trustworthy.

2 Analysis

The extensively quoted pion EM FF G.-S. model [4] has been constructed by assuming that for a wide energy of the elastic region up to $t = 1$ GeV² the P-wave isovector $\pi\pi$ scattering
Figure 1: Optimal description of the unified BESIII-BaBar data on $\sigma_{tot}(e^+e^- \rightarrow \pi^+\pi^-)$ at the elastic region: (a) by pion EM FF G.-S. model (b) by pion EM FF U&A model.

The phase shift $\delta_1^1(t)$ satisfies a two parametric effective-range formula of the Chew-Mandelstam type [5]

$$\frac{q^2}{\sqrt{t}} \cot g \delta_1^1(t) = a + bq^2 + q^2h(t); \quad h(t) = \frac{2}{\pi} \frac{q}{\sqrt{t}} \ln(\frac{\sqrt{t} + 2q}{2m_\pi})$$

(2)

and takes the following form

$$F_{\pi}^{GS}(t) = \frac{m_\rho^2 + m_\rho \Gamma_\rho(\frac{3}{2}m_\pi^22q_{\rho} + \frac{m_\rho}{2q_{\rho}} - \frac{m_\rho m_\rho}{q_{\rho}})}{(m_\rho^2 - t + \Gamma_\rho(\frac{m_\rho^2}{q_{\rho}})(q^2(h(t) - h(m_\rho^2)) + q^2h'(m_\rho^2)(m_\rho^2 - t)) - im_\rho \Gamma_\rho(\frac{q}{q_{\rho}})^3m_\rho^2)}.$$  

(3)

Its application to an optimal description of the unified BESIII-BaBar data [1, 2] at the elastic region (see Fig.1a) with $\chi^2/ndf = 40.6341$ gives parameters

$$m_\rho = (775.73 \pm 0.10) MeV$$

$$\Gamma_\rho = (126.51 \pm 0.13) MeV$$

(4)

not in a coincidence with values presented by Review of Particle Physics [6].

On the other hand an application of the pion EM FF model (see Fig.1b)

$$F_{\pi}^{EM,J=1}(q) = \frac{(q - q_{\rho})(q_{N} - q_{\rho})(q_{N} - q_{P})}{(q - q_{P})(q_{N} - q_{\rho})}(f_{\pi}^P/f_{\rho}).$$

(5)

following from the first principles [7] to a description of the same data gives slightly lower values of parameters

$$m_\rho = (763.026 \pm 0.10) MeV$$

$$\Gamma_\rho = (144.233 \pm 0.13) MeV$$

(6)

than in [6].

Which of these three sets of $\rho^0$ meson parameters can be considered to be correct ones? For a solution of this problem the most accurate up to now P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ data [3] have been exploited.
If the effective-range formula of the Chew-Mandelstam type (2) is used to a description of $\delta_1^1(t)$ data (see Fig.2a) the following $\rho^0$ meson parameter values

$$m_\rho = (772.42 \pm 0.03)\text{MeV}$$
$$\Gamma_\rho = (153.85 \pm 0.11)\text{MeV}.$$  (7)

are found [8], which do not coincide, neither with the values obtained by description of the BESIII-BaBaR data on $\sigma_{tot}(e^+e^- \rightarrow \pi^+\pi^-)$ by G.-S. charged pion EM FF model, nor with values of PDG.

On the other hand, if fully solvable mathematical scheme through the phase representation of the pion EM FF

$$F_x^{EM,I=1}(t) = P_n(t)\exp\left[\frac{t}{\pi} \int_4^{\infty} \frac{\delta_1^1(t')}{t'(t'-t)} dt'\right],$$  (8)

elaborated in [9, 10], is applied, a perfect description of $\delta_1^1(t)$ data (see Fig.2b), by a model independent phase shift $\delta_1^1(t)$ representation [8]

$$\delta_1^1(q) = \arctg\frac{A_3q^3 + A_5q^5 + \ldots}{1 + A_2q^2 + A_4q^4 + \ldots}.$$  (9)

with $\chi^2/ndf = 0.0244$ is achieved and the $\rho^0$ meson mass and width

$$m_\rho = (763.56 \pm 0.51)\text{MeV}$$
$$\Gamma_\rho = (143.09 \pm 0.82)\text{MeV}.$$  (10)

are found to be consistent with (6).

The same values of $\rho^0$ meson parameters are found also by a generalization of formula (5) to the region of the excited states of the $\rho^0$ meson [8], what can not be said about results obtained by a generalization of the pion EM FF G.-S. model [4] to the same region.

This all convinces us that the $\rho^0(770)$ meson mass and width values obtained by the Gounaris-Sakurai model can not be accepted as correct ones.

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