Giant enhancement of the thermal Hall conductivity $\kappa_{xy}$ in the superconductor YBa$_2$Cu$_3$O$_7$

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In high-purity crystals of YBa$_2$Cu$_3$O$_7$, the quasiparticle (qp) lifetime $\tau$ and the (weak-field) thermal Hall conductivity $\kappa_{xy}$ undergo dramatic increases below 90 K. We present a detailed picture of the behavior of $\kappa_{xy}$ at low temperature, in particular its scaling properties, which are directly relevant to the issue of whether Landau quantization of the qp states occurs.

The problem of excitations of the superconducting condensate in the cuprates at low temperatures is of strong current interest. In a $d$-wave superconductor, the energy-momentum dispersion of quasiparticles near a node is Dirac-like. The effect of an intense magnetic field on the quasiparticle (qp) states is an interesting open question (see below). Landau quantization of the qp states, first proposed by Schrieffer and Gorkov [1], has been recently re-derived using different arguments [2–4]. However, the case against Landau-level formation has also been argued [5–7].

A second problem is the temperature dependence of the qp mean-free-path $\ell$ (in zero field) close to $T_c$. Transport evidence from thermal conductivity [6], microwave and teraHertz experiments [8–10], and thermal Hall conductivity [11,12] point to a sharp increase in the qp lifetime just below $T_c$. Recent high-resolution angle-resolved photoemission (ARPES) experiments [11,12] have started to address the lifetime issue as well, but with conflicting results (see below).

These issues reflect the strong interest in the low-lying excitations of the $d$-wave superconductor. While microwave absorption and ARPES experiments provide valuable information on the quasiparticles, they are less effective in a field. For in-field experiments, teraHertz techniques [10] and the thermal Hall effect [11,12], in particular, have emerged as powerful probes of qp transport. In a field, the qp heat current develops a transverse component that is observed as a thermal Hall conductivity $\kappa_{xy}$ (by contrast, phonons do not display a Hall effect since they are charge-neutral). Hence, $\kappa_{xy}$ selectively senses the qp current alone [13]. To fully exploit this technique at low temperatures, however, samples with a very long $\ell$ are needed.

A recent innovation is the growth, using BaZrCO$_3$ (BZO) crucibles, of crystals of YBa$_2$Cu$_3$O$_y$ (YBCO) with nearly perfect crystalline order (from X-ray rocking curves [10]) and very low impurity concentration. The step-wise improvement in crystal quality results in strong enhancements of the qp lifetime $\tau$. A number of novel features of qp heat transport become apparent in these crystals. The weak-field $\kappa_{xy}$ undergoes a remarkable thousand-fold increase between $T_c$ and 30 K. Below 30 K, the curves of $\kappa_{xy}$ vs $H$ provide new, specific information on scaling behavior at low $T$ [21]. Both features are directly relevant to the two issues mentioned above.

![FIG. 1. (Main Panel) The thermal Hall conductivity $\kappa_{xy}$ vs. $H$ in BZO-grown YBa$_2$Cu$_3$O$_{6.99}$ ($T_c = 89$ K) at temperature from 85 to 40 K. As $T$ decreases below $T_c$, the initial slope $\kappa_{xy}^0/B$ increases sharply. The prominent peak in $\kappa_{xy}$ below 55 K is a new feature in BZO-grown YBCO. The inset compares the zero-field $\kappa_{xx} \equiv \kappa_a$ in the BZO-grown crystal (solid circles) with a detwinned non-BZO grown crystal (open).](image-url)
tial slope $\kappa_{xy}^0/B \equiv \lim_{B \to 0} \kappa_{xy}/B$ increases very rapidly as the temperature $T$ falls below $T_c$. Further, the curves are strongly non-linear in $H$. Both features reflect a $\tau$ that increases rapidly with decreasing $T$. An important new feature, absent in previous studies, is the prominent ‘overshoot’ that produces a maximum in $\kappa_{xy}$ at the field scale $H_{\text{max}}$. As $T$ falls below 40 K (see Fig. 2), the peak continues to narrow. For later reference, we note that, over a broad range of temperatures ($10 < T < 70$ K), $H_{\text{max}}$ varies as $T^2$. Moreover, at low temperatures ($T < 28$ K), the peak magnitude $\kappa_{xy}^{\text{max}}$ also scales as $T^2$.

The initial slope $\kappa_{xy}^0/B$, plotted as solid circles in Fig. 2, undergoes a thousand-fold increase between $T_c$ and 30 K (the $T$-linear variation of $\kappa_{xy}$ above $T_c$ are displayed as open circles [18]). We now show that this giant enhancement is driven by a 100-fold increase in the qp lifetime.

To extract the zero-field mean-free-path (mfp) $\ell$ from $\kappa_{xy}^0/B$, we apply the Boltzmann-equation approach [16], which should be valid in the weak-field regime $\omega_c \tau \ll 1$ ($\omega_c$ is the cyclotron frequency). In terms of the ‘qp heat capacity’ $c_c = T^{-1} \sum_q (\partial f/\partial E_k)E_k^2$ where $E_k$ is the qp energy, the zero-$H$ thermal conductivity may be written as $\kappa_c = c_c (v \ell)/2$, with the group velocity $v_k = \nabla E_k/h$. (Close to a node $k^*$, the qp energy may be approximated as $E_q = h\sqrt{(v_f q_1)^2 + (v_\Delta q_2)^2}$, where $v_f$ and $v_\Delta$ are velocity parameters normal and parallel to the FS, and $q = k - k^*$.)

The thermal Hall conductivity is related to $\kappa_c$ by $\kappa_{xy} = \kappa_c \tan \theta$. We assume that, in the weak-field limit, the thermal Hall angle $\tan \theta$ is proportional to $\omega_c \tau$, viz.

$$\tan \theta = \eta \omega_c \tau = \eta \ell/k_f \ell_B^2, \quad (B \to 0) \quad (1)$$

where the magnetic length $\ell_B = \sqrt{\hbar/eB}$. The parameter $\eta$ is less than 1 if $\ell$ is anisotropic around the FS. To obtain $\tan \theta$ [15], we first fit the profile of $\kappa_{xx}$ vs. $H$ to the empirical expression $\kappa_{xx}(B,T) = \kappa_{xx}^0(T)/(1 + p|B|^q) + \kappa_{bg}(T)$, where the background term $\kappa_{bg}(T)$ is $H$-independent and identified with the phonon contribution. The initial Hall angle is then obtained as $\tan \theta = \lim_{B \to 0} \kappa_{xy}(B)/|\kappa_{xx}(B) - \kappa_{bg}|$. This procedure allows us to extract $\tan \theta$ (hence, $\ell$ using Eq. 1).

As a consistency check, we adopt a second way to obtain $\ell$ from $\kappa_{xy}^0$ that relies on measurements of the electronic heat capacity $c_e$. Using Eq. 1, we may write

$$\kappa_{xy}^0 = \frac{c_e v_f \ell^2 \eta}{4k_f \ell_B^2} \quad (2)$$

FIG. 3. The $T$ dependence of the initial Hall slope $\kappa_{xy}^0/B$ in BZO-grown YBCO (solid circles). Between $T_c$ and 30 K, $\kappa_{xy}^0/B$ increases by $10^3$. The $1/T$ dependence of $\kappa_{xy}^0/B$ above $T_c$ (measured in a non-BZO grown YCBO) is shown as open circles. The inset shows a qp energy contour on the Dirac cone. Group velocities on the particle- ($p$) and hole-like ($h$) branches are indicated.

In a $d$-wave superconductor, $c_e = \alpha_c T^2$ for $T < T_c$. Using the measured value $\alpha_c \approx 0.064$ mJK$^{-3}$mol$^{-1}$ [20], we may invert Eq. 3 to find $\ell$. We find that the values of $\ell$ obtained from the two methods share the same $T$ dependence, but differ by a fixed factor of 1.5 if $\eta = 1$. By adjusting $\eta$ to 0.6, we obtain numerical agreement between the two methods.

Figure 4 shows the $T$ dependence of $\ell$ derived from the two methods. The agreement between the two sets of data is evidence that our assumption Eq. 3 is physically reasonable. Remarkably, between $T_c$ and 20 K, the mfp increases by a factor of ~120 from 80 A to 1 micron. In the expanded scale, we show that this increase is abrupt,
of $\kappa_{xy}/T^2$ versus $\sqrt{H}/T$ should collapse to the universal curve $F_{xy}(u)$.

We proceed to plot our results in this way in Fig. 5. While the curves above 28 K are spread out, the ones below collapse onto a common curve for $H < H_{\text{max}}$. The data taken at 25 K (and below) collectively determine the form of $F_{xy}(u)$. Its most notable feature is the nominally straight segment that extends from $u \sim 0$ to just below $u_0 \equiv \sqrt{H_{\text{max}}}/\alpha T$, i.e. $F_{xy}(u) \sim u$ for $0 < u < u_0$. This simple form for $F_{xy}$ implies that, below 25 K and for $H < H_{\text{max}}$, $\kappa_{xy}$ reduces to the form

$$\kappa_{xy}(H, T) = C_0 T \sqrt{H},$$

where the constant $C_0 = 1.51 \times 10^{-2}$ in SI units. Remarkably, when Eq. \(3\) applies, the magnitude of $\kappa_{xy}$ is just proportional to $T \sqrt{H}$, and is insensitive to all transport quantities such as $\ell$ and $\theta$. This interesting result has not been anticipated theoretically.

At larger values of $u$, $F_{xy}$ attains a maximum value $F_{xy}^0$ before falling slowly. The $T^2$ dependence of the peak value $\kappa_{xy}^{\text{max}}$ noted earlier in Fig. 4 is now seen to be a simple consequence of scaling behavior (i.e. $\kappa_{xy}^{\text{max}} \sim T^2 F_{xy}^0$).

Above 28 K, Simon-Lee scaling no longer holds. Three field regimes are now apparent. In weak fields ($0 < H < H_x$), $\kappa_{xy}$ is strictly linear in $H$. Above $H_x$, we enter a regime reminiscent of the $\sqrt{H}$ behavior at low-$T$ (the $H$-linear regime is too small to resolve below 28 K). This

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**FIG. 4.** The zero-field mean-free-path $\ell$ extracted from the weak-field Hall angle $\tan \theta$ (open circles), and from Eq. \(2\) (closed). The symbols ($\times$) represent $\tan \theta$ measured in a non-BZO detwinned YBCO crystal (Krishna et al. [14]). The expanded scale (dashed lines) highlights the steep increase below $T_c$. To extract $\ell$, we used the values $\eta = 0.60$, $v_f = 1.78 \times 10^7$ cm/s, and $k_f = 0.8 A^{-1}$.

[For comparison, $\tan \theta$ measured previously in a non-BZO crystal \(13\) is shown as $\times$. Based on the higher sensitivity and broader range in $T$ in the present experiment, we now conclude that $\tan \theta$ does not lie on the extrapolated curve for the electrical Hall angle $\tan \theta_e$.]

Beyond the weak-field regime, we need a fully microscopic description of the qp thermal Hall current to properly analyze $\kappa_{xy}$ vs. $H$. As the theoretical situation is unsettled, we adopt instead scaling arguments \(21\). This approach reveals some rather striking features in the data.

For states close to the node $k^*$, the linear energy dispersion $E = \hbar \tilde{v} \tilde{q}$ ($\tilde{v}$ is an average velocity) implies a general relation between $k_B T$ and the magnetic length $\ell_B$ at a characteristic field scale $B_x$, viz.

$$k_B T = \hbar \tilde{v} \sqrt{\frac{e B_x}{\hbar}}.$$

In addition to this general relation, Simon and Lee \(21\) have proposed that, at low $T$ ($< 30$ K for YBCO), the magnitude of $\kappa_{xy}$ should scale as

$$\kappa_{xy}(H, T) \sim T^2 F_{xy}(\sqrt{H}/\alpha T),$$

where $\alpha \equiv k_B \tilde{v} \sqrt{\hbar}$, and $F_{xy}(u)$ is a scaling function of the dimensionless parameter $u = \sqrt{H}/\alpha T$. Hence, plots starting slightly below $T_c$. 

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**FIG. 5.** Simon-Lee scaling plot of $\kappa_{xy}/T^2$ versus $\sqrt{H}/T$ (Eq. \(5\)). Below 28 K, the curves collapse onto a ‘universal’ curve $F_{xy}(u)$. Above 28 K, scaling is violated. However, the peaks still occur at the same $x$-coordinate ($\sqrt{H_{\text{max}}}/T = 0.042$). The arrows indicate the field scale $H_{\text{arc}}$. 

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intermediate regime appears as straight-line segments in Fig. 3. Finally, closer to $H_{\text{max}}$, $\kappa_{xy}$ deviates from $\sqrt{H}$ behavior, and goes through a broad maximum. Surprisingly, as noted earlier, the weaker scaling relation in Eq. 3 continues to hold: Between 15 and 70 K, the maximum in $\kappa_{xy}$ occurs at the same $x$-coordinate in Fig. 3, i.e. $\sqrt{H_{\text{max}}} = 0.042$ T. Substituting $H_{\text{max}}$ for $B_{x}$ in Eq. 3, we find that $v \sim 8.0 \times 10^{6}$ cm/s, which is close to the geometric-mean velocity $\sqrt{v_{f}\Delta} \sim 6.8 \times 10^{6}$ cm/s (with $v_{f} = 1.78 \times 10^{7}$ cm/s and $v_{f}/v_{\Delta} \sim 7$).

We may estimate semiclassically the time that an excitation, in a field, takes to move from 1 to 2 along the arc (inset, Fig. 5) by $\Delta t = (\hbar/eH) \int_{1}^{2} ds_k \left| \mathbf{v}_k \right|^{-1}$, where $s_k$ is the arc-length. For this time to equal $\tau$, the field required is $H_{\text{arc}} = \pi E/(e\Delta v_{f}\tau)$. Using the measured $\ell \sim v_{f}\tau$ at each $T$ and setting $E = k_B T$, we indicate $H_{\text{arc}}$ as arrows in Fig. 5. The rough estimate shows that the peak is related to the maximum arc length of the dominant energy contour on the Dirac cone. Hence, the abrupt appearance of the qp state below $T_c$ from transport undergoes a steep increase just below $200$ times narrower than the peaks resolved in the current ARPES studies). Hence, in high-purity YBCO, there are exceedingly sharp qp peaks in the spectral function that remain to be resolved and investigated. Understanding the abrupt appearance of the qp state below $T_c$, as implied by the steep increase in $\ell$ and $\kappa_{xy}^0/B$ near $T_c$, seems a key problem in the cuprates.

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