Formation Of Emergent Universe in Brane Scenario as a Consequence of Particle Creation

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Here we formulate scenario of emergent universe from particle creation mechanism in spatially flat braneworld models. We consider an isotropic and homogeneous universe in Braneworld cosmology and universe is considered as a non-equilibrium thermodynamical system with dissipation due to particle creation mechanism. Assuming the particle creation rate as a function of the Hubble parameter, we formulate emergent scenario in RS2 and DGP models of Braneworld.

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I. INTRODUCTION

The existence of big bang singularity in the very early universe remains an open issue although most of the mysteries of standard hot big bang cosmological model have been addressed by inflationary epoch [1]. To overcome this initial singularity, Ellis et al proposed a scenario called Emergent Universe, which is ever existing without singularity and has an almost static behaviour in the infinite past [2, 3]. Eventually the model evolves into an inflationary phase.

Recently there have been lot of interest in emergent universe models based on standard as well as modified gravity [4–7]. This resurgence of interest is because of CMB observations favouring early inflationary universe [8, 9] and probably this is the possible mechanism for present acceleration [10]. Historically the credit goes to Harrison [11] for obtaining a model of closed universe with radiation and he showed that asymptotically it approached Einstein static universe (ESU). Then after a long gap Ellis and Maartens [2, 3] in recent past were able to formulate closed universe with a minimally coupled scalar field \( \phi \). However, exact analytic solutions were not presented in their work and only asymptotic behaviour agreed with emergent universe. Subsequently, Mukherjee et al [12] obtained solution for Starobinsky model with features of an emergent universe. Also Mukherjee et al [13] formulated a general framework for an emergent universe using an adhoc equation of state which has exotic behavior in some cases. Very recently the idea of quantum tunnelling has been used to model emergent universe [14].

In most of the General Relativity (GR) based models, Emergent universe is generally obtained for spatially closed past ESU which is not stable. Contrary to this in modified gravity, emergent scenario can be obtained in spatially flat as well as open/closed model of universe [4, 5]. Also in the context of modified gravity, including Braneworld models it has been found, a stable ESU can be obtained [15]. It may be noted that a stable ESU also exists in an open universe in \( f(T) \) gravity, loop quantum cosmology and Horova Lifshitz gravity [16–18]. Since astronomical observations including very recent Planck results favour spatially flat universe it is imperative to examine Emergent scenario in spatially flat universe. In this paper, we look for a possibility of emergent scenario from particle creation mechanism in spatially flat braneworld models. The modern era of brane cosmology began in the context of large extra dimensions, made possible by the hypothesis that the standard model
of particle physics is localized on a D-brane. Braneworld models of the universe have been the focus of attention in the current decade for the search of many outstanding problems in Cosmology. These kind of models are inspired by String Theory. In braneworld scenario, our four dimensional universe (a brane) is a hypersurface embedded in higher dimensional bulk spacetime. In this brane-bulk scenario, all matter and gauge interactions (described by open strings) are localised on a brane while gravity (described by closed strings) may propagate into whole space time. This means that gravity is fundamentally a higher dimensional interaction and we only see the effective 4D theory on brane. Among the different proposals for brane models, the two prominent are RS (proposed by Randall and Sundrum) and DGP(proposed by Dvali,Gabadaze and Porrati) models. In RS models, the hierarchy problem could be solved by a warped or curved extra dimension showing that fundamental scale could be brought down from the Planck scale to 100 GeV.

While RS2 model have only \textit{one} \((1 + 3)\)-brane [20], the RS1 model have two \textit{two} \((1 + 3)\)-branes at the ends of the orbifold \(S^1/Z_2\) [19]. Due to its simple and rich conceptual base, RS2 model is very popular and got much attention [21–24] and in particular, on the inflationary scenario.

The main idea of the DGP model is the inclusion of a four dimensional Ricci-Scalar into the action. On the 4-dimensional brane the action of gravity is proportional to \(M_P^2\) whereas in the bulk it is proportional to the corresponding quantity in 5-dimensions. The model is then characterized by a cross over length scale

\[r_c = \frac{M_P^2}{2M_5^2}\]

such that gravity is 4-dimensional theory at scales \(a \ll r_c\) where matter behaves as pressure less dust but gravity \textit{leaks out} into the bulk at scales \(a \gg r_c\) and matter approaches the behaviour of a cosmological constant [25–27].

In the standard cosmology RS2 model modifies the early universe whereas DGP gravity modifies the late universe. So in the present universe brane corrections are not effective in RS2 model. But as phantom energy increases with the expansion, it brings drastic changes in the RS2 model [28]. This interesting and recent feature of RS2 model which can modify late time cosmic expansion (if the energy density of the matter content increases at late time e.g., phantom scalar field) has also been beautifully shown by using dynamical system tools in ref [29]. But this kind of modification does not appear if the energy density of the matter content decreases with cosmic expansion e.g, quintessence scalar field, radiation, dust etc. Thus RS brane model can also modify late time cosmic expansion in addition to its appreciable impact on early universe cosmology.

Very recently Chakraborty in ref [6] formulated a emergent universe model in GR using the mechanism of particle creation. The aim of this paper is to formulate an Emergent Scenario in Braneworld
as consequence a of particle mechanism. In a sense this paper is a generalisation of the one reported in [6] to include higher dimensional behaviour. Often using BG one obtains cosmological surprises. It may be noted that from thermodynamical aspect it has been proposed that entropy consideration favors the Einstein static phase as the initial state of our universe [30, 31]. The paper is organized as follows: Section 2 deals with non-equilibrium thermodynamics from the perspective of particle creation in cosmology while in section 3 we present Emergent scenario in Braneworld models. The paper ends with a short discussion and Concluding Remarks 4.

II. NON EQUILIBRIUM THERMODYNAMICS AND PARTICLE CREATION IN COSMOLOGY

Suppose $E$ be the internal energy of a closed thermodynamical system having $N$ particles. The first law of thermodynamics which is essentially the conservation of internal energy is given by [32]

$$dE = dQ - pdV$$

(1)

where as usual $p$ is thermodynamic pressure, $V$ any co-moving volume and $dQ$ represents the heat received by the system in time $dt$.

From the above conservation equation, we get the Gibb’s equation given by

$$Tds = dq = d\left(\frac{u}{n}\right) + p d\left(\frac{1}{n}\right)$$

(2)

$n = \frac{N}{V}, \ dq = \frac{dQ}{N}$ and $\rho = \frac{E}{V}$ being the particle number density, heat per particle and energy density of the system respectively. It may be noted that this equation is also true when the particle number is not conserved i.e., the system is not a closed system [33]. Here we shall consider an open thermodynamical system where the number of fluid particles is not conserved and the non conservation of fluid particles is expressed as ($N^\mu_{\mu} \neq 0$)

$$\dot{n} + \Theta n = n\Gamma$$

(3)

where $N^\mu = nu^\mu$ is the particle flow vector, $u^\mu$ is the particle four velocity, $\Theta = u^\mu_{\mu}$ is the fluid expansion, $\Gamma$ stands for the rate of change of the number of particles ($N = na^3$) in a co-moving volume $V = a^3$ and by notation $\dot{n} = n_{\mu}u^\mu$.

The positivity of $\Gamma$ indicates creation of particles while there is annihilation of particles for negative $\Gamma$. Any non zero $\Gamma$ will behave as an effective bulk pressure of thermodynamical fluid and one can use non equilibrium thermodynamics [34].
In this work, we shall consider a flat, homogeneous and isotropic model of the Universe, given by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric in comoving coordinates \((t, r, \theta, \phi)\) as

\[
ds^2 = dt^2 - a^2(t) \left[dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)\right],
\]

(4)

where \(a(t)\) is the scale factor of the Universe. Here we consider universe as an open thermodynamical system which is non equilibrium in nature due to particle creation.

The energy momentum tensor of the cosmic fluid is characterized by the energy momentum tensor

\[
T_{\mu\nu} = (\rho + p + \Pi)u_\mu u_\nu + (p + \Pi)g_{\mu\nu}
\]

(5)

The energy conservation relation of \(T^{\mu\nu}_{;\nu} = 0\) takes the following form

\[
\dot{\rho} + 3H(\rho + p + \Pi) = 0
\]

(6)

where \(H = \frac{\dot{a}}{a}\) is the Hubble parameter. Here the pressure term \(\Pi\) is related to some dissipative phenomena (say bulk viscosity). In the present context, however the cosmic fluid may be considered as perfect fluid where dissipative term \(\Pi\) is the effective bulk pressure due to particle creation or equivalently the conventional dissipative fluid is not taken as cosmic substratum, rather a perfect fluid with varying particle number is considered. For an isentropic (or adiabatic) particle production this equivalence can be nicely described as follows [34, 35]:

Using the conservation eqs. (3) and (6) the entropy variation can be obtained from Gibb’s equation (2) as

\[
nT\dot{s} = -3H\Pi - \Gamma(\rho + p)
\]

(7)

where \(T\) is the temperature of the fluid.

But due to isentropic (or adiabatic) process, the equilibrium entropy per particle does not change (as it does in dissipative process), i.e, \(\dot{s} = 0\), and from eq.(7) the effective bulk pressure is determined by particle creation rate as

\[
\Pi = -\frac{\Gamma}{3H}(\rho + p)
\]

(8)

Hence, the bulk viscous pressure is entirely determined by particle production rate. So we may say that a dissipative fluid is equivalent to a perfect fluid having varying particle number. It may be
noted that, although $\dot{s} = 0$, still there is entropy production due to enlargement of the phase space of the system. In the present context the same effect is due to the expansion of the universe. This effective bulk pressure does not correspond to conventional non-equilibrium phase, rather a state having equilibrium properties as well. We note that it is not the equilibrium era with $\Gamma = 0$.

From eq. (8), we see that if we know the Hubble parameter $H$, then the particle creation rate can be determined. Further using Friedman equations one can relate $\Gamma$ to the evolution of universe. In what follows as in [34], assuming the particle creation rate as a function of the Hubble parameter, we can study the cosmological evolution in Braneworld.

III. EMERGENT UNIVERSE IN BRANEWORLD SCENARIO

A. RS2 model

In a homogeneous and isotropic model of universe given by FRLW metric (4), the modified Friedmann equation of RS2 model is given by

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3} \rho \left[1 + \epsilon \frac{\rho}{2\lambda}\right],$$

(9)

where $\epsilon = \pm 1$ and the sign of $\epsilon$ are related to positive and negative brane tension respectively. (For simplicity we are using $8\pi G = 1$)

In the absence of bulk pressure, the matter content sector satisfy

$$\dot{\rho} + 3H(\rho + p) = 0,$$

(10)

In order to highlight some characteristics of this model, we review our paper [28] for the case $\epsilon = -1$ with $p = \rho \omega$, $\Pi = 0$ and $\omega < -1$. Using eqs. (9) and (10), we see that exact solution of energy density is given by

$$\rho(t) = \left[-\epsilon \frac{1}{2\lambda} + \left\{\frac{1}{\rho_0} + \frac{1}{2\lambda} + \sqrt{\frac{3}{4}\omega(t - t_0)}\right\}^{-2}\right]^{-1}$$

(11)

where $\rho_0 < 2\lambda$ is the current energy density of dark energy and $t_0$ is the present time. From above equation we see that the braneworld model with $\epsilon = 1$ ($\lambda > 0$) that describes an accelerated expansion ends in big-rip singularity in time.
\[ t_s = t_0 - \frac{2}{\sqrt{3}|1 + w|} \left[ \sqrt{\frac{1}{2\lambda}} + \sqrt{\frac{1}{\rho_0} + \frac{1}{2\lambda}} \right] \]  

(12)

as obtained in most of the GR based models. Brane gravity corrections make changes in the behaviour of phantom fluid in case phantom fluid dominates RS2 model of the universe with \( \epsilon = -1 \) \((\lambda < 0)\). In this case it is found that it explains the present cosmic acceleration and exhibits deceleration after a finite time, when energy density grows sufficiently [28].

In the present work we take \( \Pi \neq 0 \), the eq. (9) can be rewritten by introduction of dimensional variable \( x = \rho/2\lambda \)

\[ 3H^2 = \rho_{\text{eff}} \]  

(13)

where the effective density is given by

\[ \rho_{\text{eff}} = 2\lambda x(1 + \epsilon x) \]  

(14)

It is worth noting that when \( x << 1 \), the GR based cosmology is recovered and brane effects give new kind of behaviour when \( x >> 1 \). Acceleration and deceleration of this model without bulk pressure have been studied in [28, 37]. The case of \( \epsilon = -1 \) is (also known as bouncing universe) have been extensively studied in [38, 39]. Also the effect of bulk brane viscosity in this model in the framework of Eckart’s theory has been studied in [40].

Taking derivative of eq. (13) w.r.t cosmic time \( t \) and using (6), we get

\[ \dot{H} = -\frac{1}{2}[\rho_{\text{eff}} + p_{\text{eff}} + \Pi_{\text{eff}}] \]  

(15)

where effective pressures are given by

\[ p_{\text{eff}} = 2\lambda[\omega x(1 + \epsilon 2x) + \epsilon x^2] \]  

(16)

\[ \Pi_{\text{eff}} = \Pi(1 + \epsilon 2x) \]  

(17)

and the effective equation of state is given by

\[ \omega_{\text{eff}} = \frac{p_{\text{eff}} + \Pi_{\text{eff}}}{\rho_{\text{eff}}} = \frac{1}{1 + \epsilon x} \left[ (1 + 2\epsilon x) \left( \omega + \frac{\Pi}{2\lambda x} \right) + \epsilon x \right] \]  

(18)
FIG. 1. Variation of $\Gamma$ with $H$ in RS-II Model, considering $e = 0.787$ and $f = -6.804$.

Adding eqs. (14), (16) and (17) and using (8), we can rewrite eq (15) as

$$2\dot{H} = -(1 + \epsilon 2x)(1 + \omega)2\lambda x\left(1 - \frac{\Gamma}{3H}\right)$$  \hspace{1cm} \text{(19)}$$

Again rewriting modified Friedmann equation (13) in terms of $x$, we get

$$2\epsilon \lambda x = \epsilon \lambda [B_{RS}(\epsilon, H) - 1] \text{ where } B_{RS}(\epsilon, H) = \sqrt{\frac{6\epsilon H^2}{\lambda} + 1}$$  \hspace{1cm} \text{(20)}$$

Substituting the value of $2\lambda x$ in eq. (19), we get

$$2\dot{H} = -\frac{6(1 + \omega)H^2}{[1 + 1/B_{RS}(\epsilon, H)]}\left(1 - \frac{\Gamma}{3H}\right)$$  \hspace{1cm} \text{(21)}$$

As the particle creation rate is a function of Hubble parameter $H$ [34], we choose

$$\Gamma = 3H\left[(1 - e) + \frac{f}{H} - \frac{1}{B_{RS}(e, H)}(e - \frac{f}{H})\right]$$  \hspace{1cm} \text{(22)}$$

where $e$ and $f$ are constants.

Thus using eqs. (21) and (22), we get the differential equation of $H$ describing cosmic evolution as

$$2\dot{H} = -6(1 + \omega)(eH^2 - fH)$$  \hspace{1cm} \text{(23)}$$

So the Hubble parameter can be obtained by solving eqs. (23), as $(\omega \neq -1)$,

$$\frac{H_0}{H} = H_1 + \exp\left[-\frac{H_0}{2}(t-t_0)\right]$$  \hspace{1cm} \text{(24)}$$
where \( H_0 = 6(1 + \omega)f \), \( H_1 = 6(1 + \omega)e \) and \( t_0 \) is the constant of integration. Hence the scale factor can be obtained from the above expression for Hubble parameter as
\[
\frac{a}{a_0} e^{\frac{H_1 t}{2}} = 1 + H_1 \exp \left[ \frac{H_0(t - t_0)}{2} \right]
\]
(25)
where \( a_0 = a(t_0) \).

The above cosmological solution shows the following asymptotic behavior for \( f < 0 \):

(i) \( a \to a_0 \), \( H \to 0 \) as \( t \to -\infty \)

(ii) \( a \approx a_0 \), \( H \approx 0 \) for \( t << t_0 \)

(iii) \( a \approx \exp \left[ \frac{H_0}{H_1}(t - t_0) \right], \ H \approx \frac{H_0}{H_1} \) for \( t >> t_0 \).

The above asymptotic features show that the above cosmological solution (described by equations (24) and (25)) describes a scenario of emergent universe.

B. DGP model

The modified Friedmann equation in this case is given by
\[
H^2 - \epsilon \frac{H}{r_c} = \rho
\]
(26)
where \( r_c = \frac{M^2}{2M_5^2} \) is the crossover scale which determines the transition from 4D to 5D behavior and \( \epsilon = \pm 1 \). For \( \epsilon = 1 \), we have standard DGP(+) model which is self accelerating model without any form of dark energy, and effective \( \omega \) is always non phantom. However for \( \epsilon = -1 \), we have DGP(-) model which does not self accelerate but requires dark energy on the brane. It experiences 5D gravitational modifications to its dynamics which effectively screen dark energy.

The above equation can be rewritten as
\[
3H^2 = \rho_{\text{eff}} \quad \text{where effective density is} \quad \rho_{\text{eff}} = \rho + \epsilon \frac{3H}{r_c}
\]
(27)

Taking derivative of eq. (26) w.r.t cosmic time \( t \) and using eq. (6), we get
\[
(2H - \epsilon/r_c)\dot{H} = -H(\rho + p + \Pi)
\]
(28)

Now using eq. (8) in above equation we get
\[
2\dot{H} = -6H^2(1 + \omega)\left(1 - \frac{\Gamma}{3H}\right)B_{\text{DGP}}(\epsilon, H)
\]
(29)
where
\[
B_{\text{DGP}}(\epsilon, H) = \frac{H - \epsilon/r_c}{2H - \epsilon/r_c} \quad \text{and it gives brane effects}
\]
(30)
FIG. 2. Variation of $\Gamma$ with $H$ in DGP Model, considering $\epsilon = -1$ and $r_c = 4$.

In this case, we choose the particle creation rate as

$$\Gamma = 3H[1 - (c - d/H)/B_{DGP}(\epsilon, H)], \quad c \text{ and } d \text{ are constants} \quad (31)$$

As before using modified Friedmann eqs. (29) and (31) we get the differential equation of $H$ describing cosmic evolution of DGP model as

$$2\dot{H} = -6(1 + \omega)cH^2 + 6d(1 + \omega)H \quad (32)$$

This evolution equation is identical in form to that for RS-II brane gravity (i.e. eq. (23)) and hence it is also possible to have emergent scenario in DGP brane model in the framework of particle creation mechanism.

**IV. SHORT DISCUSSION AND CONCLUDING REMARKS**

The present work deals with spatially flat braneworld models in non-equilibrium thermodynamics, based on particle creation mechanism. For simplicity, we assume the thermal process to be adiabatic in nature and as a result the particle creation rate is related to the cosmic evolution. In both RS-II and DGP brane model, cosmic fluid with constant equation of state ($p = \omega \rho$, $\omega \neq -1$) is chosen. In both the brane models the particle creation rate is chosen as different functions of the Hubble parameter and the brane effect is incorporated through the functions $B_{RS}$ and $B_{DGP}$ respectively. However the
cosmic evolution equation is similar in both the models and we have emergent scenario in the early phase of the evolution. Further it should be noted that the present model of emergent scenario due to particle creation is on ever expanding model of the universe. As the particle creation rate is not constant, so the present model has the basic difference with the steady state theory of Fred Hoyle et al [41, 42] in contrast to the emergent scenario in Einstein gravity with constant particle creation rate [6]. Moreover, one can easily check that if we remove the brane effect (i.e. $\lambda \to \infty$ in RS2 or $r_c \to \infty$ in DGP model) then the particle creation rate may be taken to be constant (by choosing $e = 1/2$ in RS2 model and $c = 1/2$ in DGP brane model). The variation of $\Gamma$ with respect to the Hubble parameter has been presented in figs. (1) and (2) for RS2 and DGP brane model respectively for various choices of the parameters involved. Finally, we conclude that it is possible to have emergence scenario in brane gravity models for non-equilibrium thermodynamics in the framework of particle creation mechanism.

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