The FCNC top-squark decay as a probe of squark mixing

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Abstract

In supersymmetry (SUSY) the flavor mixing between top-squark (stop) and charm-squark (scharm) induces the flavor-changing neutral-current (FCNC) stop decay $\tilde{t}_1 \rightarrow c\chi^0_1$. Searching for this decay serves as a probe of soft SUSY breaking parameters. Focusing on the stop pair production followed by the FCNC decay of one stop and the charge-current decay of the other stop, we investigate the potential of detecting this FCNC stop decay at the Fermilab Tevatron, the CERN Large Hadron Collider (LHC) and the next-generation $e^+e^-$ linear collider (LC). We find that this decay may not be accessible at the Tevatron, but could be observable at the LHC and the LC with high sensitivity for some part of parameter space.

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I. INTRODUCTION

Flavor-changing neutral-current (FCNC) interactions are strongly suppressed in the Standard Model (SM) by the GIM mechanism, which is consistent with the current experimental observation. In theories beyond the SM the FCNC interactions are not generally suppressed, and thus are subject to stringent constraints from experiments [1]. On the other hand, the study of FCNC interactions, especially related to the top quark [2], will play an important role in testing the SM and probing new physics.

Weak scale supersymmetry (SUSY) as a leading candidate for new physics beyond the SM provides no further understanding about the origin of flavor. In fact, it extends the mystery of flavor by necessarily adding three families of squarks and sleptons. Without additional assumptions for flavor structure of the soft SUSY breaking, supersymmetric theories often encounter phenomenological difficulties, known as the SUSY flavor problem [3]. This in turn implies that there may exist rich FCNC phenomenology. Some highly suppressed FCNC processes in the SM may be enhanced in supersymmetric models to a level accessible in the future experiments, such as $t \rightarrow cV$ ($V = \gamma, Z, g$) and $t \rightarrow ch$ [4–6]. On the other hand, sfermions may have large flavor mixings via the soft SUSY breaking terms. Even if the flavor-diagonality is assumed for sfermions at the grand unification scale, the flavor mixings at weak scale are naturally generated through renormalization group equations [7]. Therefore, hunting for the exotic FCNC processes predicted by SUSY would be one of the important aspects in SUSY searches at the upcoming colliders.

There have been intensive studies for the FCNC phenomenology in the slepton sector [8]. In the squark sector, some interesting FCNC phenomena may arise from the mixing between the stop and scharm. On the experimental side, we note that despite of the strong constraints on the mixing between first and second generation squarks from $K^0 – \bar{K}^0$ mixing, the mixing between stop and scharm is subject to less low-energy constraints and could be maximal [9] although a recent analysis [10] of electric dipole moment of mercury atom indicated a nontrivial constraint on the $\tilde{t}_L – \tilde{c}_L$ mixing. Such a large mixing would reveal itself in some processes or subject to some constraints in future collider experiments. On the theoretical side, in the Minimal Supersymmetric Standard Model (MSSM) the stop-scharm mixing is likely to be large even if there is no mixing at tree level, as first realized in [11]. The stop-scharm mixing induces the FCNC stop decay $\tilde{t}_1 \rightarrow c\tilde{\chi}_1^0$, where $\tilde{t}_1$ is the lighter one of the stop mass eigenstates and $\tilde{\chi}_1^0$ is the lightest neutralino assumed to be the lightest supersymmetric particle (LSP). Early searches for this channel at the Tevatron experiments have set the bounds $m_{\tilde{t}_1} \gtrsim 120 \text{ GeV}$ for $m_{\tilde{\chi}_1^0} \sim 40 \text{ GeV}$ [12]. However, if kinematically accessible, the tree-level charged-current (CC) decay mode $\tilde{t}_1 \rightarrow b\tilde{\chi}_1^+$, where $\tilde{\chi}_1^+$ is the lighter chargino, will be most likely dominant although it may be via a three-body decay with $\tilde{\chi}_1^{+*} \rightarrow \ell^+\bar{\nu}$ [13]. Experimental searches for this mode have also been performed at the Tevatron [14], and the current bounds are $m_{\tilde{t}_1} \gtrsim 135 \text{ GeV}$ for $m_{\tilde{\nu}} \sim 80 \text{ GeV}$. In their analyses, however, the decay branching fraction of $\tilde{t}_1$ has been simply assumed to be 100% for each channel under their consideration. There have also been studies [15] on the possibility of finding the FCNC stop decay from top quark pair production followed by the decay $t \rightarrow \tilde{t}_1\chi_1^0 \rightarrow c\tilde{\chi}_1^0\tilde{\chi}_1^0$ of one top and the SM decay of the other top. It is shown that such a decay mode, if realized in the $tt$ pair events with a substantial branching fraction, could be observable in some part of the SUSY parameter space at Run 2 of the Tevatron.
In this article, we focus on the direct stop pair production followed by the FCNC decay of one stop \((\tilde{t}_1 \rightarrow c\tilde{\chi}^0_1)\) and the charge-current decay of the other stop \((\tilde{t}_1 \rightarrow b\tilde{\chi}^+_1)\). We allow arbitrary branching fractions for these two channels. By simulating both the signal and the SM backgrounds, we examine to what levels the branching ratio and the stop-scharm mixing parameter can be probed at the Tevatron collider, the CERN Large Hadron Collider (LHC) and the 500 GeV next-generation \(e^+e^-\) linear collider (LC).

II. STOP-SCHARM MIXING AND FCNC STOP DECAY

Following Ref. [11], we start in the framework of the MSSM and assume that the tree level interactions are flavor diagonal for stops and scharms. The flavor mixing between stops and scharms is then induced via loops. The dominant effects are from the logarithmic divergences caused by soft breaking terms. Such divergences must be subtracted using a soft counter-term at the SUSY breaking scale, such as the Plank scale \(M_p\) in supergravity (SUGRA) models. Thus a large logarithmic factor \((1/16\pi^2)\ln(M_p^2/m_W^2)\approx 0.5\) remains after renormalization. In the approximation of neglecting the charm quark mass, \(\tilde{c}_R\) does not mix with stops. The mixing of \(\tilde{c}_L\) with stops result in the physical states given approximately by

\[
\begin{pmatrix}
\tilde{t}_1 \\
\tilde{t}_2 \\
\tilde{c}_L
\end{pmatrix}_{\text{phys}} =
\begin{pmatrix}
1 & 0 & \epsilon \\
0 & 1 & \epsilon' \\
-\epsilon & -\epsilon' & 1
\end{pmatrix}
\begin{pmatrix}
\tilde{t}_1 \\
\tilde{t}_2 \\
\tilde{c}_L
\end{pmatrix},
\]

where

\[
\epsilon = \frac{\Delta_L \cos \theta_t + \Delta_R \sin \theta_t}{m_{\tilde{t}_1}^2 - m_{\tilde{c}_L}^2}, \quad \epsilon' = \frac{\Delta_R \cos \theta_t - \Delta_L \sin \theta_t}{m_{\tilde{t}_2}^2 - m_{\tilde{c}_L}^2}.
\]

with \(\Delta_{L,R}\) given by

\[
\Delta_L = -\frac{\alpha}{4\pi} \ln \frac{M_{\tilde{t}}^2 V_{tb}^* V_{td} m_b^2}{2 m_W^2 s_W^2} (1 + \tan^2 \beta) \left( \tilde{M}_Q + \tilde{M}_D + \tilde{M}_{H_2} + |A_d|^2 \right),
\]

\[
\Delta_R = -\frac{\alpha}{4\pi} \ln \frac{M_{\tilde{t}}^2 V_{tb}^* V_{td} m_b^2}{2 m_W^2 s_W^2} (1 + \tan^2 \beta) m_t A_d^*.
\]

\(\theta_t\) is the mixing angle\(^2\) between left- and right-handed stops, defined by

\[
\begin{pmatrix}
\tilde{t}_1 \\
\tilde{t}_2
\end{pmatrix} =
\begin{pmatrix}
\cos \theta_t & \sin \theta_t \\
-\sin \theta_t & \cos \theta_t
\end{pmatrix}
\begin{pmatrix}
\tilde{t}_L \\
\tilde{t}_R
\end{pmatrix}.
\]

---

\(^1\)For some mechanisms of SUSY breaking other than gravity mediation, such as gauge mediation, the SUSY breaking scale can be much lower than the Plank scale and thus this factor may be smaller.

\(^2\)Note that our definition of \(\theta_t\) differs from that in Ref. [11] by a minus sign.
In the above, we have adopted the notation in Ref. [17], with \( m_{\tilde{t}_i} < m_{\tilde{t}_i} \). \( \tilde{M}_Q^2, \tilde{M}_D^2 \) and \( \tilde{M}_{H_1}^2 \) are soft-breaking mass terms for left-handed squark doublet \( \tilde{Q} \), right-handed down squark \( \tilde{D} \) and Higgs doublet \( H_1 \), respectively. \( A_d \) is the coefficient of the trilinear term \( H_1 \tilde{Q} \tilde{D} \) in soft-breaking terms and \( \tan \beta = v_2/v_1 \) is ratio of the vacuum expectation values of the two Higgs doublets. Note that \( \epsilon \) is a mass eigenstate in our analysis since we do not consider the mixing between \( \tilde{c}_L \) and \( \tilde{c}_R \), which is proportional to the charm quark mass.

From the above equations, we note that besides the large logarithmic factor \( \ln(M_p^2/m_W^2) \), the mixings are proportional to \( \tan^2 \beta \) and thus can be further enhanced at large \( \tan \beta \). If we assume that all soft SUSY-breaking parameters are of the same orders in magnitude, we then have typically \( \epsilon \approx 0.01(\tan \beta/10)^2 \) and thus \( \epsilon \) is much smaller than unity. (Note that to make the approximate expansion of Eq. (1) valid, \( \epsilon \) should be much smaller than unity.) Without such an assumption, \( \epsilon \) can be larger because in the sum \( \tilde{M}_Q^2 + \tilde{M}_D^2 + \tilde{M}_{H_1}^2 + |A_d|^2 \) only \( \tilde{M}_Q \) is related to stop and scharm masses while other parameters are independently free in the MSSM.

The stop mass \( m_{\tilde{t}_i} \) is particularly important for our study and will be retained as a free parameter in our numerical calculations. The lightness of the stop is quite well motivated in some SUSY models like SUGRA and is also preferred by electroweak baryogenesis [18]. On the other hand, the current lower bound on its mass is about 135 GeV [14], albeit under some assumptions. We will thus explore the mass range

\[
150 \text{ GeV} < m_{\tilde{t}_i} < 250 \text{ GeV} \tag{6}
\]

where the upper end is the kinematic limit for a 500 GeV linear collider. So we assume an upper bound of about 250 GeV in our numerical analysis.

The flavor mixing between stop and scharm will induce the FCNC stop decay \( \tilde{t}_1 \rightarrow c\chi_1^0 \). Since the charge-current decay \( \tilde{t}_1 \rightarrow b\chi_1^+ \) can be the other important decay mode, the branching ratio of the FCNC decay is obtained by

\[
BF = \frac{\Gamma(\tilde{t}_1 \rightarrow c\chi_1^0)}{\Gamma(\tilde{t}_1 \rightarrow c\chi_1^0) + \Gamma(\tilde{t}_1 \rightarrow b\chi_1^+)} \tag{7}
\]

with

\[
\Gamma(\tilde{t}_1 \rightarrow c\chi_1^0) = \frac{\alpha}{2} |\epsilon|^2 m_{\tilde{t}_1} \left( 1 - \frac{m_{\chi_1^0}^2}{m_{\tilde{t}_1}^2} \right)^2 \frac{1}{s_W c_W} \left( 1 - e_c s_W^2 \right) \sin \theta_t \tag{8}
\]

\[
\Gamma(\tilde{t}_1 \rightarrow b\chi_j^+) = \frac{\alpha}{4} m_{\tilde{t}_1} \left( 1 - \frac{m_{\chi_j^+}^2}{m_{\tilde{t}_1}^2} \right)^2 \left| \frac{V_{ij}^*}{s_W} \right|^2 \cos \theta_t + \frac{m_{\tilde{t}_1} V_{ij}^*}{\sqrt{2} m_W s_W \sin \beta} \sin \theta_t \tag{9}
\]

Here, \( N'_{ij} \) denotes the matrix element projecting the \( i \)-th neutralino into photino \( (j = 1) \), zino \( (j = 2) \), and two neutral Higgsinos \( (j = 3, 4) \). \( V_{ij} \) is the matrix element projecting the \( i \)-th left-handed chargino into wino \( (j = 1) \) and the charged Higgsino \( (j = 2) \). The gaugino masses and mixing are determined by the soft SUSY-breaking parameters \( M_1, M_2 \), as well as \( \mu, \tan \beta \). There are strong theoretical motivations to further constrain these parameters [19]. First of all, the supergravity models predict the unification relation \( M_1 = \frac{2}{3} M_2 \tan^2 \theta_W \simeq 0.5 M_2 \). Radiative electroweak symmetry breaking generally yields a large \( \mu \)
parameter, although the naturalness arguments prefer a lower value of \( \mu \). This scenario leads to the LSP \( \tilde{\chi}_0^0 \) bino-like, and \( \tilde{\chi}_1^\pm \) wino-like, which is also favored for a SUSY dark matter interpretation. Regarding the other parameter \( \tan \beta \), the LEP experiments excluded small values \( \tan \beta < 2 \) \cite{20}. For the sake of illustration, we thus choose the following representative set of parameters

\[
M_2 = 150 \text{ GeV}, \quad \mu = 300 \text{ GeV}, \quad \tan \beta = 10.
\]  

(10)

The chargino and neutralino masses in units of GeV are then given by

\[
egin{align*}
m_{\tilde{\chi}_1^\pm} &= 133, & m_{\tilde{\chi}_2^\pm} &= 328, \\
m_{\tilde{\chi}_1^0} &= 72, & m_{\tilde{\chi}_2^0} &= 134, & m_{\tilde{\chi}_3^0} &= 308, & m_{\tilde{\chi}_4^0} &= 327.
\end{align*}
\]

(11)

In our analysis the chargino \( \tilde{\chi}_1^\pm \) must be lighter than stop \( \tilde{t}_1 \). Such a light chargino decays into \( ff'\tilde{\chi}_1^0 \) (\( f \) is a quark or lepton) through exchanging a \( W \)-boson, or a charged Higgs boson, a slepton, a squark \cite{21}. Since, typically, the charged Higgs, sleptons and squarks are much heavier than the \( W \)-boson, such decays occur dominantly through the \( W \)-exchange diagram and the branching ratio for the clean channels \( \tilde{\chi}_1^\pm \to \ell^+ \nu \chi_0^0 \) (\( \ell = e \) and \( \mu \)) is thus approximately \( 2/9 \).

With the parameters in Eq. (10), the branching fraction \( B(\tilde{t}_1 \to c\tilde{\chi}_1^0) \) in the no mixing limit is approximately given by

\[
BF \approx \begin{cases} 
1.3|\ell|^2, & \text{for } m_{\tilde{t}_1} = 150 \text{ GeV}, \\
0.16|\ell|^2, & \text{for } m_{\tilde{t}_1} = 250 \text{ GeV}.
\end{cases}
\]  

(12)

For a lighter \( m_{\tilde{t}_1} \), the decay \( \tilde{t}_1 \to b\tilde{\chi}_1^+ \) is kinematically suppressed; and for a heavier \( m_{\tilde{t}_1} \), this charged-current channel becomes dominant.

Note that our choice of parameters in Eq. (10) is rather representative for which the decay modes \( \tilde{t}_1 \to c\tilde{\chi}_1^0 \) and \( b\tilde{\chi}_1^+ \) are both kinematically accessible. The exception is in the Higgsino-like region \( (M_2 > |\mu|) \). In this case, both the LSP and \( \tilde{\chi}_1^\pm \) are mainly Higgsino-like, and are about degenerate in mass close to \( \mu \). The lepton produced in the decay \( \tilde{\chi}_1^\pm \to \tilde{\chi}_1^0 \ell^+ \nu \) will be too soft to be experimentally identifiable, making the signal difficult to observe. As we indicated earlier, this situation is disfavored by the arguments of SUSY-GUT and dark matter. We will thus not pursue this special case further.

### III. OBSERVABILITY OF FCNC STOP DECAY AT COLLIDERS

Since the stop \( \tilde{t}_1 \) is likely to be significantly lighter than any other squark and thus the production rate of \( \tilde{t}_1\tilde{t}_1 \) is larger than other squark pairs, as well as than \( \tilde{t}_1\tilde{t}_2 \) or \( \tilde{t}_2\tilde{t}_2 \), we only consider the production of \( \tilde{t}_1\tilde{t}_1 \) in our analysis. Inclusion of the channels \( \tilde{t}_1\tilde{t}_2 \) and \( \tilde{t}_2\tilde{t}_2 \) would enhance the signal observability although the kinematics of the final states would be more involved to study. For a light stop with a mass close to the top quark, the QCD corrections enhance the total cross section of stop pair by a factor of about 1.2 at the Tevatron energy and 1.4 at the LHC energy \cite{22}. This enhancement (the so-called \( K \) factor) will be taken into account in our calculation. The one-loop corrections to stop pair production in a 500 GeV \( e^+e^- \) collider were found to increase the cross section by 10–20% \cite{23} and we assume
an enhancement factor $K = 1.1$ in our analyses. Going beyond the crude assumption on the branching fractions of the $\tilde{t}_1$ decay in the Tevatron studies [12,14], we consider the FCNC decay of one stop $\tilde{t}_1 \to \tilde{c}\tilde{\chi}_1^0$, and the charge-current decay of the other one, $\tilde{t}_1 \to b\tilde{\chi}_1^+ \to b\ell^+\nu\tilde{\chi}_1^0$. The signal we are proposing to look for is a $\tilde{t}_1\tilde{t}_1$ event giving rise to an energetic isolated charged lepton ($e$ or $\mu$), a $b$-quark jet, a (charm) jet and missing transverse energy, denoted by $j\ell + E_T$.

First, we consider the search at hadron colliders. To simulate the acceptance of the detectors, we impose some kinematical cuts on the transverse momentum ($p_T$), the pseudo-rapidity ($\eta$), and the separation in the azimuthal angle-pseudo rapidity plane ($\Delta R = \sqrt{(\Delta \phi)^2 + (\Delta \eta)^2}$) between a jet and a lepton or between two jets. We choose the basic acceptance cuts for the Tevatron

$$
P_T^\ell, P_T^{\text{jet}}, E_T \geq 20 \text{ GeV}, \eta^{\text{jet}}, \eta^{\ell} \leq 2.5, \Delta R_{jj}, \Delta R_{j\ell} \geq 0.5. \tag{13}
$$

We increase the threshold for the LHC as

$$
P_T^\ell \geq 20 \text{ GeV}, P_T^{\text{jet}} \geq 35 \text{ GeV}, E_T \geq 30 \text{ GeV}, \eta^{\text{jet}}, \eta^{\ell} \leq 3, \Delta R_{jj}, \Delta R_{j\ell} \geq 0.4. \tag{14}
$$

Furthermore, we assume the tagging of a $b$-quark jet with 50% efficiency and the probability of 0.4% (15%) for a light quark ($c$-quark) jet to be mis-identified as a $b$-jet.

To make the analyses more realistic, we simulate the energy resolution of the calorimeters by assuming a Gaussian smearing on the energy of the final state particles, given by

$$
\frac{\Delta E}{E} = 30\% \oplus 1\% \quad \text{for leptons}, \tag{15}
$$

$$
\frac{\Delta E}{E} = 80\% \oplus 5\% \quad \text{for hadrons}, \tag{16}
$$

where $E$ is in GeV.

The potential SM backgrounds at hadron colliders are

1) $bq(\bar{q}) \to tq'(\bar{q}')$;
2) $qq' \to W^* \to t\bar{b}$;
3) $Wb\bar{b}; Wc\bar{c}; Wc\bar{j}; Wjj$;
4) $t\bar{t} \to W^-W^+b\bar{b}$;
5) $gb \to tW$;
6) $qg \to q't\bar{b}$.
 TABLE I. Signal $\tilde{t}_1 \tilde{t}'_1 \to \ell bc \not{E}_T$ and background cross sections in units of fb. The signal results were calculated by assuming $m_{\tilde{t}_1} = 150$ GeV and other parameters are in Eq. (10). The charge conjugate channels have been included. The signal results do not include the branching fraction factor $2BF(1 - BF)$, which should be multiplied to obtain the actual signal rate for a given value of $BF$.

|                | Tevatron 2 TeV | LHC 14 TeV |
|----------------|---------------|------------|
|                | basic cuts    | basic + $m_T$ | basic cuts | basic + $m_T$ |
| signal        | 23            | 6.6        | 720        | 220          |
| $qb \to q't$  | 120           | 5.0        | 7400       | 400          |
| $q\bar{q}' \to tb$ | 39            | 2.3        | 280        | 17           |
| $Wbb$         | 130           | 2.5        | 570        | 45           |
| $Wcc$         | 80            | 1.5        | 450        | 36           |
| $Wcj$         | 670           | 3.8        | 7600       | 650          |
| $Wjj$         | 500           | 2.9        | 1700       | 150          |
| $t\bar{t}$    | 7.9           | 3.8        | 600        | 300          |

The backgrounds 5) and 6) are of modest rates and can mimic our signal only if the extra jet is missed in detection. After vetoing the extra central jet, these backgrounds are effectively suppressed. The $t\bar{t}$ background 4) is of a large production cross section, especially at the LHC. It can mimic our signal if both $W$’s decay leptonically and one of the charged leptons is not detected, which is assumed to occur if the lepton pseudo-rapidity and transverse momentum are in the range $|\eta(\ell)| > 3$ and $p_T(\ell) < 10$ GeV. In addition, we also have some SUSY backgrounds. In case of $\tilde{t}_1$ being significantly lighter than other squarks, the dominant SUSY background is the pure charged-current decay $\tilde{t}_1 \tilde{t}_1 \to \tilde{\chi}_1^+ \tilde{\chi}_1^- bb$. Also, at the high end of the top squark mass range considered in our analysis, $\tilde{t}_1$ can decay to $t^* + \tilde{\chi}_1^0$ or even $t + \tilde{\chi}_1^0$. All these processes give a $t\bar{t}$-like signature [24] and can mimic our signal just like the SM production of $t\bar{t}$. However, compared with $t\bar{t}$ background 4), such backgrounds are much smaller since their production rates are much lower than the $t\bar{t}$ background.

We notice that for most of the background events the missing energy comes only from neutrinos in $W$ decay, while for the signal the missing energy contains the extra contribution from the neutralinos. From the transverse momentum of the lepton $\vec{p}_T^\ell$ and the missing transverse momentum $\vec{p}_T$, we construct the transverse mass as

$$m_T = \sqrt{(|\vec{p}_T^\ell| + |\vec{p}_T|)^2 - (\vec{p}_T^\ell \cdot \vec{p}_T)^2}.$$  

(17)

For the background events where the only missing energy comes from a neutrino from $W$ decay, $m_T$ is always less than $M_W$ (and peaks just below $M_W$) without energy smearing. Smearing pushes some of the events above $M_W$. For the signal $m_T$ is spread out widely above and below $M_W$, due to the extra missing energy of the neutralinos. In order to substantially enhance the signal-to-background ratio $(S/B)$, we apply a cut

$$m_T > 90 \text{ GeV}.$$  

(18)
We first present the signal and background cross sections at the Tevatron (2 TeV) and LHC (14 TeV) under various cuts in Table I. One sees that with only the basic acceptance cuts, the various SM backgrounds can overwhelm the signal. The implementation of the $m_T$ cut reduces the backgrounds $Wb\bar{b}$, $Wc\bar{c}$, $Wjj$ and $Wcj$ efficiently. The production rate of the signal $b+$jet+\ell+\not{E}_T$ can be obtained by multiplying the $\tilde{t}_1\tilde{t}_1$ cross section by the branching fraction factor $(2/9)2BF(1-BF)$ for a given value of $BF$. Our results for the $2\sigma$ sensitivity on $BF$ at the Tevatron versus the stop mass are shown in Fig. 1 (the top curve). We find that due to limited statistics, Tevatron Run 2 with a luminosity of 2 fb$^{-1}$ is not able to discover the signal nor even set any significant bounds on the branching fraction $BF$. A bound at $2\sigma$ level could be reached at the Tevatron energy with a luminosity of 20 fb$^{-1}$ for $m_{\tilde{t}_1} < 180$ GeV, corresponding to $BF \sim 20\%$. At the LHC the $5\sigma$-discovery is accessible, reaching the branching fraction below 1% even for a low luminosity 100 fb$^{-1}$. From Fig. 1 one sees that the detection sensitivity for hadron colliders does not monotonously increase as the stop mass decreases. Instead, when the stop becomes too light, the detection sensitivity decreases. This is the effect of the cuts applied in our simulation and can be understood as follows. As the stop mass decreases, the stop pair production rate increases. However, when the stop becomes too light, the $b$-jet from $\tilde{t}_1 \rightarrow \tilde{\chi}_1^+ b$ becomes very soft and thus failed to pass the selection cuts so that it decreases the detection sensitivity.

The results at the LHC are obtained by applying the basic and the $m_T$ cuts. The signal significance is obtained in terms of Gaussian statistics, given by the signal and background events $S/\sqrt{B}$. Although the sensitivity reach at the LHC is impressive as shown in Fig. 1,
FIG. 2. 5σ discovery limits of the stop-scharm mixing parameter $\epsilon$ versus the stop mass with the stop mixing angle $\theta_t = \pi/10$. The region above each curve is the corresponding observable region.

the signal-to-background ratio $S/B$ becomes rather low when reaching the small branching fraction. Thus the sensitivity relies on the successful control of the systematics in the experiments.

It is known that the experimental environment is much cleaner at an $e^+e^-$ collider. Now we recapitulate our analyses for an $e^+e^-$ linear collider with C. M. energy of 500 GeV. Since the environment of $e^+e^-$ colliders is much cleaner, we will evaluate the production rate of the signal $b+$jet+$\ell$ $E_T$ simply by multiplying the cross section $\sigma(e^+e^- \rightarrow \tilde{t}_1\tilde{t}_1)$, the branching ratio $(4/9)BF(1 - BF)$, the $b$-tagging efficiency assumed to be 50%, and the detection efficiency of kinematics assumed to be 80%. The possible SM backgrounds are

$$e^+e^- \rightarrow W^+W^- \rightarrow jj'\ell\nu,$$

$$e^+e^- \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^- \rightarrow bbjj'\ell\nu.$$  \hspace{1cm} (19)

However, these backgrounds can be effectively separated due to the rather different kinematical features from the signal. If we define the recoil mass as

$$m_r^2 = (P_{e^+} + P_{e^-} - \sum P_{\text{obs}})^2,$$  \hspace{1cm} (21)

where the sum is over all momenta of the observed final state particles, then we notice that the backgrounds have rather small recoil mass from the single missing neutrino. The recoil mass for the signal on the other hand is quite large since the neutralino is very massive. Similar to the case of hadron colliders, the dominant SUSY background is $\tilde{t}_1\tilde{t}_1 \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^- bb$, which is small and neglected in our numerical analysis. For a stop $m_{\tilde{t}_1} \lesssim 230$ GeV when the threshold is sufficiently open for $\sqrt{s} = 500$ GeV, one can reach a 5σ observation at the
FIG. 3. 5σ discovery limits of the stop-scharm mixing parameter $\epsilon$ versus the stop mixing angle $\theta_t$ for $m_{\tilde{t}_1} = 170$ GeV. The region above each curve is the corresponding observable region.

LC with a branching fraction of 1–5% even for a luminosity of only 100 fb$^{-1}$, as shown in Fig. 1.

The exclusion and discovery limits of the branching fraction can be translated into the limits on the stop-scharm mixing parameter $\epsilon$, which can be predicted for a specific SUSY model. At this stage, the stop mixing angle $\theta_t$ needs to be specified. For illustration, we first fix $\theta_t = \pi/10$ and the resulting limits on $\epsilon$ are shown in Fig. 2, corresponding to the results of Fig. 1. For stop mass of 150 GeV the 5σ discovery limit with a luminosity of 100 fb$^{-1}$ is $\epsilon \gtrsim 0.09$ at the LC, and $\epsilon \gtrsim 0.20$ at the LHC. For a heavier stop, the detection sensitivity at the LC drops much more rapidly than that at the LHC due to the limited C.M. energy of the LC. Note that although Run 2 with 20 fb$^{-1}$ has a 2σ sensitivity (as shown in Fig. 1) to the decay branching ratio, it has no sensitivity to $\epsilon < 0.3$. When the 2σ sensitivity limits of Run 2 in Fig. 1 are translated to the mixing parameter $\epsilon$, rather large $\epsilon$ values ($\gtrsim 0.5$) are obtained. A large $\epsilon$ is not theoretically favored, as implied in Eqs. (1–4).

The obtained limits on $\epsilon$ are sensitive to the mixing angle $\theta_t$, which controls the partial width for the CC decay as seen in Eq. (9). The dependence of $\epsilon$ limits on $\theta_t$ is shown in Fig. 3 for a fixed value of stop mass $m_{\tilde{t}_1} = 170$ GeV. For the mixing angle to be near a certain value, $\tan \theta_t \approx (\sqrt{2}m_W \sin \beta/m_t)(V_{11}/V_{12})$, the CC mode is suppressed and the sensitivity to the FCNC mode is greatly enhanced. For our choice of the SUSY parameters, this occurs near $\theta_t \approx 9^\circ$. The 5σ sensitivity with 500 fb$^{-1}$ for the LHC and LC could reach as far as $\epsilon \approx 0.01$. For nearly maximal mixing $\theta_t \approx 45^\circ$, on the other hand, the sensitivity could be reduced to about $\epsilon \approx 0.4$.

Furthermore, the limits or observation of the mixing parameter $\epsilon$ can be translated into some knowledge on certain soft SUSY breaking parameters. From Eqs. (2) and (3),
we see that the mixings are proportional to a sum of certain parameters, typically like
\((\tilde{M}_Q^2 + \tilde{M}_D^2 + \tilde{M}_{H_1}^2 + |A_d|^2)/(m_t^2 - m_{\tilde{c}_L}^2)\). This can vary independently of \(m_t\) and \(m_{\tilde{c}_L}\) since only \(\tilde{M}_Q\) in this sum is related to \(m_t\) and \(m_{\tilde{c}_L}\). For the purpose of illustration, taking \(m_{\tilde{t}_1} = 150\) GeV, \(\theta_t = \pi/10\), \(m_{\tilde{c}_L} = 200\) GeV and other SUSY parameters given in Eq. (10), we obtain the LHC discovery (5\(\sigma\) limit) with a luminosity of 100 fb\(^{-1}\)

\[\sqrt{\tilde{M}_Q^2 + \tilde{M}_D^2 + \tilde{M}_{H_1}^2 + |A_d|^2 + 0.3m_tA_d^*} \gtrsim 1.4\text{ TeV},\] (22)

or, in case of non-observation, the 2\(\sigma\) bound given by

\[\sqrt{\tilde{M}_Q^2 + \tilde{M}_D^2 + \tilde{M}_{H_1}^2 + |A_d|^2 + 0.3m_tA_d^*} \lesssim 1.0\text{ TeV}.\] (23)

Due to the nature of the multiple parameters as a combination involved in the expression, more comprehensive analyses would be needed, possibly to combine with other experimental knowledge on the SUSY parameters, in order to extract the information for the theory parameter space.

IV. CONCLUSIONS

In summary, we studied the potential of detecting the FCNC stop decay \(\tilde{t}_1 \rightarrow c\tilde{\chi}_1^0\), as a probe of stop-scharm mixing, at the upgraded Tevatron, the LHC and the LC. Rather than performing an exhaustive scan of the SUSY parameters, we chose a representative set of the relevant parameters to demonstrate the possibility of observation. Through Monte Carlo simulation, we found that the signal at the Tevatron is too weak to be observable for the choice of well-motivated SUSY parameters. At the LHC on the other hand, with judicial kinematical cuts, it is quite possible to observe a 5\(\sigma\) signal with a branching fraction as low as 1% even for a luminosity of 100 fb\(^{-1}\). However, it should be noted that systematic effects in the experiments must be under control. At an LC of \(\sqrt{s} = 500\) GeV, one can reach a 5\(\sigma\) observation with a branching fraction of 1–5% for a luminosity of 100 fb\(^{-1}\). The limits or observation of this important decay mode can be translated into some knowledge on certain soft SUSY breaking parameters. We finally note that in our study we have chosen a representative scenario for relevant SUSY parameters in which the lightest neutralino and chargino are gaugino-like. In the region of SUSY parameters where the lightest neutralino and chargino are Higgsino-like, the signal would be more difficult to observe.

Acknowledgments

The work of T.H. is supported in part by the US Department of Energy under grant DE-FG02-95ER40896, in part by the Wisconsin Alumni Research Foundation, and in part by National Natural Science Foundation of China (NNSFC). The work of K.H. is supported in part by the Grant-in-Aid for Scientific Research (No. 12640248 and 14046201) from the Japan Ministry of Education, Culture, Sports, Science, and Technology. X.Z. is supported by NNSFC.
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