Mechanical Cooper pair transportation as a source of long distance superconducting phase coherence.

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Transportation of Cooper-pairs by a movable single Cooper-pair-box placed between two remote superconductors is shown to establish coherent coupling between them. This coupling is due to entanglement of the movable box with the leads and is manifested in the suppression of quantum fluctuations of the relative phase of the order parameters of the leads. It can be probed by attaching a high resistance Josephson junction between the leads and measuring the current through this junction. The current is suppressed with increasing temperature.

Delocalization of electrons due to tunneling between two bodies constituting a system will, according to the Heisenberg uncertainty principle, lower the total energy of this system and, hence, lead to interbody coupling. If the bodies are superconductors, charge exchange due to Cooper pair tunneling (Josephson tunneling) causes such coupling. Since the Cooper-pair exchange is sensitive to the relative phases of the superconducting order parameters of the bodies this coupling leads to the establishment of phase coherence i.e. a phase ordering of the relative phases occurs. The characteristic energy responsible for this ordering is the Josephson coupling energy $E_J$.

If charge exchange is accompanied by charge accumulation Coulomb energy cannot be neglected. In nanostructures, where the excess charge can be strongly confined, this may result in forces strong enough to cause mechanical deformations of the systems. Coupling between mechanical degrees of freedom and electronic degrees of freedom can then arise in systems having a rigidity comparable to these Coulomb forces.

That such electromechanical coupling can lead to mechanical transportation of charge in a movable nanocluster was suggested in \cite{1} as an unusual mechanism for shuttling of electric charge between two bodies (see also \cite{2,3}). This triggered a number of both experimental \cite{4–6} and theoretical \cite{7,8} activities including \cite{9} where the possibility of coherent transfer of Cooper pairs by a movable superconducting island was proposed.

There, a non stationary quantum mechanical process involving a movable qubit, a coherent superposition of differently charged states on a movable superconducting grain, was shown to be responsible for a finite supercurrent between two remote phase coherent superconductors. However, when confronted with this result one is faced by a most fundamental question: \textit{Could such a non-equilibrium quantum mechanical process serve as a source for the creation of phase coherence (phase ordering) if the two superconductors were initially in states with definite number of particles?} This paper gives a positive answer to the above question.

![FIG. 1. Schematic diagram of the system discussed in the text. A movable nanoscale superconducting grain which may accommodate excess charge carried by either 0 or 1 extra Cooper pairs is placed between two finite isolated superconducting leads. Close to the leads, electrostatic gates ensure that the Coulomb blockade of tunneling is lifted. After a sequence of grain rotations, the grain moving periodically contacting the two leads sequentially, a probe (Josephson) junction is connected through which a current $I$ is measured.]

To demonstrate this, we consider the same system as in \cite{1} but with the low impedance link connecting the two leads removed. This is shown in Fig. 1. Initially the two superconducting leads, labelled left (L) and right (R), are in projected BCS-states $|N_L\rangle$ and $|N_R\rangle$ with $N_L$ and $N_R$ extra Cooper-pairs respectively. In terms of BCS-states $|\phi\rangle$ with definite phase $\phi$ we have

$$|N_{L,R}\rangle = \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} d\phi e^{-iN_{L,R}\phi} |\phi\rangle.$$  

Transportation of Cooper-pairs between the leads is then provided by a movable nanometer-sized grain \cite{10} placed between the two electrodes. When the grain is close to the superconductors electrostatic gates ensure that there exists two nearby charge states, differing by one Cooper...
pair on the grain, for which the difference in charging energy $\Delta E_C$ is much smaller than the Josephson energy $E_J$. Due to the smallness of the grain, Coulomb blockade of tunneling will restrict the available states of the grain to only those two states which we label $|n = 0\rangle$ and $|n = 1\rangle$, i.e., a single-Cooper-pair box situation is realized close to each lead. In this situation, when $E_J \gg \Delta E_C$, a coherent superposition, $\alpha |0\rangle + \beta |1\rangle$, of differently charged states can be created on the grain. This was recently demonstrated experimentally [1].

Consider now the situation when the system is initially in a pure state with charge neutral components $|\Psi\rangle = |n = 0\rangle |N_L = 0\rangle |N_R = 0\rangle$. If the grain executes periodic motion, performing repeated alternating contacts with the two leads, Cooper-pair exchange between the leads will result. Thus the initial state will evolve into a superposition of states with different numbers of particles on the leads,

$$|n = 0\rangle |N_L = 0\rangle |N_R = 0\rangle \rightarrow \sum_{n=0}^{\infty} \sum_{N_L}^{\infty} \sum_{N_R}^{\infty} c_{N_L, n} c_{N_R, n} |n\rangle |N_L\rangle |N_R\rangle.$$  

The Kroenecker delta makes sure that the total number of particles is conserved. To investigate whether this corresponds to a phase ordered state with a well defined phase difference, $\Delta \Phi \equiv \Phi_R - \Phi_L$, between the leads a small probe junction, a Josephson weak link, is connected as shown in Fig. 1. This provides the means for measuring such order since the average current $I$ through this link is related to $\Delta \Phi$,

$$I = \text{Tr}_p \rho \hat{I}, \quad \hat{I} \equiv I_c \sin(\hat{\Phi}_R - \hat{\Phi}_L).$$

Here $\rho$ is the system density matrix, $I_c$ is the critical current of the probe junction and $\hat{\Phi}_L, R$ are the phase operators of the respective lead.

To be more concrete, consider the following Hamiltonian describing the system dynamics

$$H_0(x(t)) = -\sum_{s=L,R} E_s^{J}(x(t)) \cos(\hat{\Phi}_s - \hat{\phi}) + \Delta E_C(x(t)) \hat{n}.$$  

$H_0$ depends on time through the grain position $x(t)$. The first term represents the coupling between the grain and the leads in terms of the phase operators of the leads, $\hat{\Phi}_L, R$, and the grain, $\hat{\phi}$, and time varying Josephson coupling energies $E_s^{J}(x(t))$. The second term, containing the operator $\hat{n}$ for the grain, accounts for the difference in charging energy, $\Delta E_C$, associated with different charge states when the grain is moving between the leads. In order for a phase ordering to occur two additional ingredients are required: First, in its present form $[H_0, \hat{I}] = 0$ which means that no sign of any ordering can be detected. Second, a term describing dissipation decoherence is needed in order for the system to reach a well defined final state irrespective of initial conditions. If one takes into account that any real isolated superconductor has a finite capacitance the first point no longer holds. To mimic the behavior when the leads have finite capacitance we restrict the Hilbert space of the leads to states $|N_{L,R} = m\rangle$ with $|m| \leq N$. To include the effect of the environment we consider the effect of the coupling between the charge on the grain and the fluctuations of the gate voltage through the term

$$V = \epsilon(2e\tilde{n}) \sum_{\alpha=1}^{M} C_\alpha \hat{x}_\alpha, \quad 0 \leq \epsilon \leq 1,$$

where $\epsilon$ is a dimensionless coupling constant [12]. The $\hat{x}_\alpha$ are the coordinates of a harmonic oscillator bath

$$H_E = \sum_{\alpha} \left( E_{\alpha} \hat{\rho}_{\alpha} + \frac{1}{2} m_\alpha \omega_{\alpha}^2 \hat{x}_{\alpha}^2 \right)$$

characterized by a spectral density

$$J(\omega) \equiv \frac{\pi}{2} \sum_{\alpha} \frac{C_\alpha^2}{m_\alpha \omega_{\alpha}} \delta(\omega - \omega_{\alpha}) = \frac{\hbar \omega R}{1 + \omega^2 R^2 C^2}$$

where $R$ is the resistance and $C$ the capacitance as seen from the gate. Hence, the Hamiltonian describing system, environment and interaction between the two is

$$H(t) = H_0(t) + H_E + V.$$

Writing the Liouville-von-Neumann equation in the interaction representation,

$$\frac{d\hat{\rho}(t)}{dt} = -\frac{i}{\hbar} \left[ \hat{V}(t), \hat{\rho}(t) \right],$$

$V$ can be treated perturbatively to second order. Tracing out the environmental states yields for the reduced density matrix of the system, $\hat{\rho}_S(t) \equiv \text{Tr}_E \hat{\rho}(t)$,

$$\hat{\rho}_S(t) = \hat{\rho}_S(t_0) - \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 K_1(t_1 - t_2) [\hat{n}(t_2), \hat{\rho}_S(t_0)]$$

$$+ \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 K_2(t_1 - t_2) [\hat{n}(t_2), \hat{\rho}_S(t_0)]$$

$$= \hat{\rho}_S(t_0) + \hat{\mathcal{L}}[\rho(t_0)](t). \tag{1}$$

The kernels $K_{1,2}$ appearing here are related to the spectral density and the inverse temperature $\beta = (k_B T)^{-1}$ through

$$K_1(t) = \frac{4e^2 \bar{e}^2}{\pi \hbar^2} \int_{0}^{+\infty} d\omega J(\omega) \cos \omega t \coth \frac{\beta \hbar \omega}{2}.$$  


and

\[ K_2(t) = \frac{4e^2C^2}{\pi \hbar^2} \int_0^{+\infty} d\omega J(\omega) \sin \omega t. \]

To find the (quasi) stationary state of the system eq. (1) is iterated. To be more precise, suppose the grain performs rotations with a period 4T_0. Introduce the following time labels

- t_0: The grain leaves the left electrode.
- t_1: The grain arrives at the right electrode.
- t_2: The grain leaves the right electrode.
- t_3: The grain arrives at the left electrode.
- t_4 = t_0 + 4T_0 the grain leaves the left electrode.

The reduced density matrix after a complete rotation is then found by (numerical) iteration, i.e.

\[ \hat{\rho}_S(t_{i+1}) = \hat{\rho}_S(t_i) + \hat{\mathcal{L}}[\hat{\rho}_S(t_i)](t_{i+1}). \]

For the algorithm to converge the coupling to the bath must be weak enough. The relevant strength of the perturbation is characterized by the factor \( e^2 \eta \) where \( \eta \equiv (4e^2T_0)/(\pi \hbar C) = 4T_0/R_QC. \)

Inspired by [4] we consider the grain oscillating in such a way that it contacts each of the leads for \( T_0 = 1 \) ns and that it takes the same time for it to cross the gap between the leads. Further, a reasonable value of the Josephson energy during the contacts is \( E_J = 7 \mu eV \) and we will assume that \( \Delta E_C \) is of the same order. For simplicity we take the time dependence of \( E_J \) and \( \Delta E_C \) to be step like. Thus close to the leads \( E_J \) takes a constant value while \( \Delta E_C \) is zero and vice versa. More relevant than the exact value of \( \Delta E_C \) are the dynamical phase differences \( \chi^\pm \) picked up by the grain during the motion between the leads. More precisely we define

\[ \chi_+ = \hbar^{-1} \int_{t_0}^{t_1} dt \Delta E_C(t), \quad \chi_- = \hbar^{-1} \int_{t_2}^{t_3} dt \Delta E_C(t). \]

The dependence of the probe junction current on the dynamical phase difference is shown in figure 2. Here the maximum capacity of the leads is \( N = 8 \) and the temperature set to 1 mK. Starting from an initially pure state the current stabilizes to a fixed value after a large number of rotations. This current shows a distinct oscillatory dependence on the dynamical phases \( \chi^\pm \). The number of rotations required before the system stabilizes depends on the strength of the coupling to the bath. Since our approach is perturbative a very weak coupling has been used in order to obtain the numerical results. This implies in our case, as is evident from Fig. 3, that approximately \( 10^4 \) rotations are neccessary for the value of the probe junction current to stabilize. In a real situation this time may be considerably shorter.

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\begin{align*}
\text{FIG. 2. Average current } I & \text{ through a probe junction attached as in Fig. 1 after many grain rotations. The magnitude of the current is shown as a function of the phases } \chi^\pm \text{ (in units of } \pi). \text{ Bright areas correspond to large current while + and } - \text{ indicate its direction.}
\end{align*}
\]

Although a finite current will flow through the weak link in the probe junction it is not immediately clear that the state built up by the rotations is a BCS-type state. To see that this is indeed the case consider the BCS-type phase-states

\[ |\phi\rangle = \frac{1}{\sqrt{2\pi}} \sum_n e^{i n \phi} |n\rangle. \]

Any state of the system can then be represented in the basis of the phases of the leads \( |\Phi_{L,R}\rangle \) and the grain \( |\phi\rangle \) but also in terms of the relative phases \( \Delta \Phi = \Phi_R - \Phi_L \) and \( \Delta \phi = \Phi_R - \phi \)

\[ |\Delta \Phi, \Delta \phi\rangle = \frac{1}{2\pi} \sum_{n=0,1}^{N_n} \sum_{N_L=-N}^{N_L} e^{-i N_L \Delta \Phi} e^{-i n \Delta \phi} |n\rangle |N_L\rangle |N_L - n\rangle. \]

Since the relative phase \( \Delta \phi \) between the grain and the right lead necessarily suffers from large quantum fluctuations we define the 'phase difference probability density' \( f(\Delta \Phi) \) as the average

\[ f(\Delta \Phi) \equiv \frac{2\pi}{2\pi} \int_0^{2\pi} d(\Delta \phi) \langle \Delta \Phi, \Delta \phi | \rho | \Delta \Phi, \Delta \phi \rangle \]

In Fig. 3, \( f(\Delta \Phi) \) has been calculated as a function of the number of rotations performed for a system with \( N = 20 \) instead of \( N = 8 \). In this simulation the fact that a definite phase relation is built up is numerically verified. In the beginning of the simulation when the system is in a pure state with definite number of particles on each of
the individual components the phase uncertainty is maximal, i.e. $f(∆Φ) = (2π)^{-1}$. As the system approaches a stable regime a sharp peak appears around a well defined value of $∆Φ$. The location of the peak then defines the phase difference between the two leads.

The exact value of the built up phase difference depends on the parameters of the system. Especially it depends on the phases $χ_\pm$ but also on the time spent by the grain in contact with the leads. In general, the final distribution $f(∆Φ)$ has two peaks separated by a distance $π$. The relative magnitude of the peaks vary. For instance, the black lines, representing zero current, in Fig. 2 corresponding to the lines $χ_+ + χ_- = constant$ are due to a superposition of two equally strong built up phases differing by $π$ and hence carrying an equal amount of current in opposite directions. A remnant of such a superposition can bee seen in the distribution in Fig. 3. Here, however, only one of the two states is stable while the other one rapidly decays. The largest magnitude of the current is obtained when one of the two peaks is completely suppressed. We note further, that it is not possible to obtain the value of the built up phase difference by equating the final expression for the current in [9] to zero since that expression was derived under the explicit assumption that the phases were fixed.

By varying the temperature, $T$, in the simulations the current $I(T)$ through the probe junction was calculated. In Fig. 4 $I(T)/I_c$ is shown as a function of temperature. The decrease in current with increasing temperature is due to an increased width of the phase distribution. At very low temperatures the current saturates due to the fact that quantum fluctuations dominate over the thermal fluctuations. Then, with increasing temperature, the current decays exponentially at first but with a crossover to algebraic decay for high temperatures.

In conclusion we have shown that mechanically assisted transportation of Cooper pairs between initially uncoupled, remote superconductors leads to a suppression of quantum fluctuations of the relative phase of their order parameters. The movable single Cooper-pair-box, (which may be in a superposition of two charge states (qubit)) executing sequential tunneling contacts with each of the superconductors, is responsible for creating the strong entanglement causing coherent coupling. This coupling is suppressed with increasing temperatures.

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