Giant oscillations of the density of states and the conductance in a ferromagnetic conductor coupled to two superconductors

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Giant oscillations of the density of electronic states and the differential conductance of a superconductor-ferromagnet-superconductor structure are predicted for the case that the exchange energy of the interaction between the electron spin and the spontaneous moment of the ferromagnet, $I_0$, is less than the superconductor energy gap, $\Delta$ ($I_0 < \Delta$). The effect is due to an extremely large degeneration of the energy level $\varepsilon \pm I_0$ ($\varepsilon$ is the electron energy measured from the Fermi-energy) if the superconductor phase difference $\varphi$ is close to odd numbers of $\pi$. These quantum interference effects persist even in long ferromagnetic bridges the length of which much exceeds the "magnetic length" $\hbar v_F/\Delta$ for the ballistic case and $\sqrt{\hbar D/I_0}$ for the diffusive one ($D$ is the electron diffusion constant). The predicted effect allows a direct spectroscopy of Andreev levels in the ferromagnet as well as a direct measurement of the exchange energy $I_0$, of the interaction of the electron spin with the spontaneous moment of the ferromagnet.

INTRODUCTION

In recent years much attention has been paid to the conductance of mesoscopic superconductor-normal conductor-superconductor systems (S/N/S heterostructures) which is sensitive to the phase difference between the superconductors (see, e.g., the review paper by Lambert and Rimondi[1] and references there). This effect takes place both in ballistic and diffusive samples due to a quantum interference caused by Andreev reflections of quasi-particles at two (or more) N/S interfaces which imposes the phase of the superconducting condensate on the quasiparticle wave function in the normal conductor.

In normal conductor-superconductor heterostructures, electronic elementary excitations which freely propagate inside the non-superconducting metal can not penetrate into the superconductor if their energy $\varepsilon$ (measured from the Fermi energy) is less than the superconductor energy gap $\Delta$. A correlated transferring of two electrons accompanied by their pairing inside the superconductor is the only mechanism that provides a direct transmission of the charge into the superconducting condensate that is the ground state of the superconductor. This 2-electron transmission may be considered as a scattering process under which electronic excitations inside the normal conductor undergo an electron-hole transformation at the boundary with the superconductor. This scattering (known as the Andreev reflection) couples the incident electron (hole) and the reflected hole (electron) in such a way that their spins are oriented in opposite directions and their energies ($\pm \varepsilon$) are symmetrically positioned with respect to the Fermi energy $\varepsilon_F$ ("Andreev hybrid"), as shown in Fig.1. As a result of the Andreev reflection the electronic excitations reflected at the normal conductor - superconductor interface pick up the phase $\varphi$ of the superconductor order parameter $\Delta = |\Delta| \exp i \varphi$ and keep memory of it inside the normal conductor. Such a 2-electron superconducting correlation persists inside the normal conductor at the distance $c a l L_\varepsilon$ from the superconductor,

$$L_\varepsilon = \min \left( \frac{\hbar}{\Delta \varepsilon}, \sqrt{\frac{\hbar D}{\Delta \varepsilon v_F}} \right)$$

where $\Delta \varepsilon = p_e - p_h \sim \varepsilon/v_F$, $v_F$ is the Fermi velocity and $D$ is the diffusion coefficient. At larger distances from the superconductor the destruction of the phase coherence arises due to the difference between the momenta of the electron ($p_e = p_F + \varepsilon/v_F$) and hole ($p_h = p_F - \varepsilon/v_F$) components in the Andreev hybrid (the typical value of the energy is $\varepsilon \sim k T$, where $T$ is the temperature and $k$ is the Boltzmann constant). The other peculiar feature of the Andreev hybrid is that the electron and hole spins have opposite directions. This spin flip does not change the interference pattern of the non-magnetic metal because all energy levels are twice degenerated with respect to the spin direction. In ferromagnets, however, this degeneracy is lifted due to the interaction of the electron spin with the spontaneous moment of the ferromagnet (below we refer to it as the exchange-interaction energy $I_0$), and electrons with opposite directions of the spins occupy different energy bands (see Fig.2):

$$\varepsilon_\sigma(p) = p^2/2m + \sigma I_0$$

where $p$ is the electron momentum, $m$ is the electron mass, $\sigma = \pm 1$ for the electron spin up and down. In a ferromagnetic metal-superconductor heterostructure, the change of the spin direction of the incident electron (hole)
FIG. 1: Schematic representation of the Andreev reflection: the incident electron with energy $\epsilon$ (measured from the Fermi energy $\epsilon_F$) and the spin up is reflected back as a hole with the energy $-\epsilon$ and the spin down: the incident electron and the reflected hole momenta are $p_e = \sqrt{p_F^2 + 2m\epsilon}$ and $p_h = \sqrt{p_F^2 - 2m\epsilon}$ ($p_F$ is the Fermi momentum, $m$ is the electron mass), respectively.

FIG. 2: Energy bands for electrons with opposite spins under the Andreev reflection shifts the reflected hole (electron) into the other energy band that causes an additional difference $\delta p = I_0/v_F$. The latter drastically decreases the penetration length $L$ by orders of the magnitude that influences the quantum interference affecting many properties of ferromagnet/superconductor heterostructures such as magnetism Refs. [6, 8, 13, 14, 18]. On the other hand, measurements carried out in recent works demonstrate a long-range proximity effect in magnetic materials that is in an obvious contradiction with the above general considerations. As shown in papers such a long-range proximity effect arises in F/S heterostructures in which the ferromagnetic part is magnetically inhomogeneous. In this case a triplet superconducting correlation persists on a length typical for non-magnetic materials. A long-range proximity effect of another type occurs in F/S heterostructures in which the ferromagnetic part is magnetically homogeneous but the exchange-interaction energy $I_0$ is less than the superconductor energy gap $\Delta$. In this case the difference between the electron and hole
momenta of Andreev hybrids with energy \( \varepsilon = I_0 \) do not depend on \( I_0 \) (see the section) and hence their penetration length is equal the one for non-magnetic conductors \( L_c \). It results in giant oscillations of the conductance of an S/F/S Andreev interferometer with a change of the phase difference between the superconductors when the voltage \( V \) applied to the ferromagnet is close to \( 2I_0/e \) \cite{48, 49}; for the case of F/S structures the subgap conductance exhibits a peak below the superconducting energy gap \( \Delta \) at \( eV = I_0 < \Delta \) \cite{12}.

It is important for applications that the both above-mentioned types of the long-range proximity effects allow to create superconducting quantum interference devices with a ferromagnetic normal metal junction of an anomalous large length (a device of such a type was suggested in \cite{50}).

In this paper we consider the density of states, the differential conductance and the current-voltage characteristic (CVC) of an superconductor-ferromagnet-superconductor heterostructure (S/F/S) of the Andreev interferometer type in which the ferromagnetic part is separated from the reservoir of normal electrons with a potential barrier of low transparency \( t_r \ll 1 \), (see Fig.3). For the case that the exchange-interaction energy \( I_0 \) is less than the superconductor energy gap \( \Delta \) we predict a sharp peak in the density of states at energies close to \( |I_0| \) and sharp dependence of the differential conductance \( G(V, \varphi) \) on the applied voltage \( V \) that is accompanied with giant oscillations of the density of states and the conductance with a change of the phase difference \( \varphi \) between the superconductors. For the geometry under consideration the differential conductance is proportional to the density of Andreev states that allows their direct spectroscopy with electrical measurements. On the other hand, the predicted effect allows to find the electron spin - ferromagnet moment exchange interaction energy \( I_0 \) with a direct electric measurement because a sharp and high peak in the differential conductance takes place exactly at \( V = 2I_0 \).

In section we qualitatively explain the phenomenon under consideration. In sections and we present analytical and numerical calculations for the density of states and for the current and the differential conductance in ballistic S/F/S structures, respectively. In section we calculate the density of states in a disordered S/F/S structure using the Gutzwiller path-integral approach.

GIANT CONDUCTANCE OSCILLATIONS; QUALITATIVE CONSIDERATIONS

In this section we qualitatively show that a change of the superconductor phase difference \( \varphi \) can result in giant oscillations of the conductance of a superconductor-ferromagnetic conductor - superconductor structure if the exchange-interaction energy \( I_0 \) of the ferromagnet is less than energy gap \( \Delta \) of the superconductor. The effect arises due to a resonant transmission of quasi-particles from the normal reservoir through Andreev states in the ferromagnet which macroscopically concentrate near the exchange-interaction energy \( I_0 \) if the superconductor phase difference \( \varphi \) is close to an odd number of \( \pi \) and the voltage \( V \) applied to structure (see Fig.3) is close to \( 2I_0/e \) \((I_0 < |\Delta|)\). A qualitative explanation of the effect is as follows.

Under Andreev reflection at an F/S interface the spin directions of the incident and reflected quasi-particles are opposite and hence the longitudinal momenta (parallel to the \( x \)-axis which is perpendicular to the F/S interfaces) of the electron and the hole are, respectively, (de Jong and Beenakker \cite{52})

\[
p_{n,s}^{(c)}(\varepsilon) = \sqrt{p_F^2 - p_{\perp}^2(n)} + 2m(\varepsilon - \sigma I_0)
\]

\[
p_{n,\sigma}^{(h)}(\varepsilon) = \sqrt{p_F^2 - p_{\perp}^2(n)} - 2m(\varepsilon - \sigma I_0)
\]

where \( p_F \) is the Fermi momentum, \( p_{\perp}(n) = (0, h n_y/d_y, h n_z/d_z) \) is the quantized transverse quasi-particle momentum parallel to the F/S interfaces, \( (d_y \) and \( d_z \) are the transverse sizes of the ferromagnetic section, \( n = (0, n_y, n_z) \); \( n_y, n_z = 0, 1, 2, ..., |n| \leq N_{\perp} \) are the transverse mode quantum numbers) assuming a hard wall confining potential with the number of transverse modes \( N_{\perp} \approx S/\lambda_F^2 \) inside it, \( S = d_y d_z \) is the cross section area of the ferromagnet (equal to the area of the F/S interfaces), \( \lambda_F = 2\pi\hbar/p_F \) is the Fermi wave length. From Eq. it follows that in contrast to the non-magnetic case, near the Fermi level \( (\varepsilon \approx 0) \) the electron and the hole momenta in the ferromagnet are different, and for a large enough \( I_0 \) (usually \( I_0 \) is greater than the Thouless energy ) the interference effects are absent due to the destructive interference. This fact demonstrates the conflict between superconductivity and magnetic ordering in S/F/S structures.

However, interference effects in the ferromagnet can exist albeit at some finite voltage \( V \) applied between the reservoir and the superconductor. Our argument starts from a description of the electron transport in terms of resonant tunneling through quantized energy levels of the ferromagnet mesoscopic part of the system shown in Fig. 3.
FIG. 3: Superconductor-ferromagnet-superconductor heterostructure of the Andreev interferometer type. Thick lines indicate potential barriers of low transparency, \( t_r \ll 1 \), separating the ferromagnet from the reservoir of normal electrons; voltage \( V \) is applied between the reservoir and the superconductor.

Taking into account the amplitude of the Andreev reflection at a normal conductor - superconductor interface, \( r_A = \exp(i\pi/2 + \phi) \)

one easily finds the semiclassical quantization condition for an S/F/S system in the absence of potential barriers at F/S interfaces to be

\[
\left( p_{n,\sigma}^{(e)}(\varepsilon) - p_{n,\sigma}^{(h)}(\varepsilon) \right) L/\hbar + \pi \pm \phi = 2\pi l
\]

where \( p_{n,\sigma}^{(e)}(\varepsilon) \) and \( p_{n,\sigma}^{(h)}(\varepsilon) \) are the longitudinal momenta determined by Eq. (3); \( L \) is the distance between the F/S interfaces; \( \phi = \phi_1 - \phi_2 \) is the phase difference between the superconductors 1 and 2; \( l = 0, \pm 1, \pm 2, \ldots \) is the longitudinal quantum number; while writing equations Eq.(4) and Eq.(5) we assumed \( \varepsilon \ll |\Delta| \) for simplicity’s sake. Dispersion equation Eq.(5) determines Andreev levels \( \varepsilon_{l,n} \) which can be shifted with a change of \( \phi \) by means, for instance, of an external magnetic field. When an Andreev level is lined with the energy of electrons injected from the reservoir of normal electrons, the resonant transmission of electrons through the ferromagnetic section takes place that causes an increase of the system conductance. Typically each Andreev level is only 2-fold (spin up and down) degenerated, and hence simultaneously only 2 electrons can be resonantly transmitted through this energy level. It means that the amplitude of oscillations of the differential conductance \( G(V) = dJ/dV \) \( (J \) is the dissipative current current, see Fig. 3) with a change of the superconductor phase difference \( \phi \) is of order of \( e^2/h \). The situation drastically changes when the superconductor phase difference is equal to an odd number of \( \pi \). In this case the energy level \( \varepsilon = \varepsilon_0 \) is highly degenerated since all \( N_\perp \) transverse modes are simultaneously at this level (if \( \phi = \pi(2l + 1) \) the dispersion equation Eq.(5) is satisfied with \( \varepsilon = \varepsilon_0 \) for all transverse modes because \( p^{(e)} = p^{(h)} \) for any \( n \), see Eq. (3) and Eq. (5)). Therefore the resonant transmission occurs simultaneously in all transverse modes when the superconductor phase difference is equal to odd numbers of \( \pi \) and the applied voltage takes such a value that \( eV/2 \) is equal to the exchange energy \( \varepsilon_0 \), thus producing a giant conductance peak. The width of the peak in the dependence of the conductance on \( \phi \) at \( V = 2\varepsilon_0/e \) is of the order of the single-electron transparency \( t_r \) of the potential barriers between the ferromagnet and the electron reservoir, and the width of the peak in the \( V \)-dependence of the conductance at \( \phi = \pi(2l + 1) \) is \( t_r \varepsilon_0 \) \( (\varepsilon_0 = \hbar v_F/L \) is the distance between neighboring energy levels); the amplitude of the peak is of the order of \( N_\perp e^2/h \) reflecting the above-mentioned \( N_\perp \)-fold degeneracy of the resonant level. When this result is compared with that for a non-magnetic normal part (see [53, 54]) it is apparent that the exchange interaction of the electron spin with the ferromagnet spontaneous momenta destroys the giant oscillations of the conductance on the Fermi level (that is at \( V \) close to zero) shifting them to the energy range \( eV/2 \approx \varepsilon_0 \) where the giant conductance oscillations are restored even if \( \varepsilon_0 \geq \varepsilon_0 \).

Giant conductance oscillations for a diffusive ferromagnet in the case that the temperature \( T \) satisfies the inequality \( t_r E_{Th}^{(D)} \ll kT \ll E_{Th}^{(D)} = \hbar D/L^2 \) were considered in paper [55]. In section 4, we show that the high degeneracy of the
energy level $\varepsilon = I_0$ at $\varphi = \pi$ is not lifted in diffusive S/F/S structures that results in a sharp peak in the density of states and hence in giant conductance oscillations.

**DENSITY OF STATES IN BALLISTIC S/F/S STRUCTURES WITH $I_0 < |\Delta|$**

In this Section we find the dispersion equation and the density of Andreev states in a ferromagnetic conductor-superconductor-ferromagnetic conductor structure in which potential barriers are present at the ferromagnet-superconductor interfaces.

In order to find Andreev energy levels $\varepsilon_n^\sigma$ inside the ferromagnetic conductor we use the Stoner model with the exchange interaction energy $I_0$ so that the Bogolubov-de Gennes equations are written as follows [52]:

$$
\begin{align*}
\left\{ 
\begin{array}{ll}
\left( \hat{H}_0 + \sigma I(r) - \varepsilon \right) u_{+\sigma} + \Delta v_{-\sigma} = 0 \\
\Delta^* u_{+\sigma} - \left( \hat{H}_0 - \sigma I(r) + \varepsilon \right) v_{-\sigma} = 0
\end{array}
\right.
\end{align*}
$$

(6)

where $\hat{H}_0 = \mathbf{p}^2 / 2m - \varepsilon_F$; the superconducting energy gap $\Delta(r)$ and the ferromagnet exchange energy $I(r)$ have non-zero values in complementary space regions, $\Delta = const \neq 0$; $I = 0$ in the superconductor and $\Delta = 0$, $I = I_0 = const \neq 0$ in the ferromagnet. We assume Andreev reflections of quasi-particles at the F/S interfaces to be accompanied by normal reflections, that is the scattering process at an F/S interface is described by a $2 \times 2$ scattering matrix [58, 59] as follows [68]:

$$
\hat{T} = e^{i\chi} \begin{pmatrix} r_N \exp(i\eta) & r_A \\ -r_A^* & r_N^* \exp(-i\eta) \end{pmatrix}
$$

(7)

where

$$
e^{i\chi} = -i \frac{\sqrt{|t_0|^2 + 4|r_0|^2 \sin^2 \psi_e}}{\exp(-i\psi_e) - |r_0|^2 \exp(i\psi_e)};
$$

$$
e^{i\psi_e} = \frac{|\Delta|}{\varepsilon + i\sqrt{|\Delta|^2 - \varepsilon}};
$$

(8)

In Eq. 8 the probability amplitudes of Andreev ($r_A$) and normal ($r_N$) reflections are written as follows:

$$
r_A = i|t_0|^2 \exp(i\phi) \sqrt{|t_0|^2 + 4|r_0|^2 \sin^2 \psi_e} / |r_0|^2 \exp(i\psi_e);
$$

$$
r_N = 2r_0 \sin \psi_e \sqrt{|t_0|^2 + 4|r_0|^2 \sin^2 \psi_e};
$$

where $\phi$ is the phase of the superconductor energy gap $\Delta = |\Delta| \exp(i\phi)$; $r_0$ and $t_0$ are the probability amplitudes for an incident electron to be reflected back and to be transmitted through the interface in case that the conductors on the both sides of the interface are in the normal state; this scattering arises due to a potential barrier at the interface, mismatch between the Fermi velocities of electrons of the conductors or of their effective masses, and so on. In general case this scattering is described by the scattering matrix

$$
\hat{\rho} = e^{i\eta} \begin{pmatrix} \left|t_0\right| \exp(i\phi) & i|t_0| \\ i|t_0| & \left|r_0\right| \exp(-i\phi) \end{pmatrix}; \quad |r_0|^2 + |t_0|^2 = 1;
$$

(9)

According to Eq. 9 the electron- and hole-like components of the wave-function in the $n$-th transverse mode inside the ferromagnet is

$$
u_{\alpha}(x, y) = \sum_{n} \left( a_n^{(e)} e^{ik_n^{(e)} x} + b_n^{(e)} e^{-ik_n^{(e)} x} \right) \sin k_{\perp}(n) y
$$

$$
u_{\alpha}(x, y) = \sum_{n} \left( a_n^{(h)} e^{-ik_n^{(h)} x} + b_n^{(h)} e^{ik_n^{(h)} x} \right) \sin k_{\perp}(n) y
$$

(9)

Here $a_n^{(e)}$ and $b_n^{(e)}$ [$a_n^{(h)}$ and $b_n^{(h)}$] are the probability amplitudes for free motion of electrons [holes] forward and backward, respectively, in channel $n$; $k_{\perp}^{(e,h)} = p_{n,\alpha}/\hbar$ (see Eq. 3) is the electron (hole) longitudinal momentum for
an electron (hole) in the $n$-th transverse mode and its spin direction $\sigma$ ($\sigma = \pm 1$), while $x$ and $(y, z)$ are longitudinal and transverse coordinates in the sample, respectively; here and below $\{n\}$ stands for the summation over all the open modes $|n| \leq N_L$.

Matching the wave-functions Eq. (9) at two nonequivalent F/S boundaries with the use of the scattering matrix Eq. (10) results in a spectral function of the form

$$
D_n^{(\sigma)}(\varepsilon) = \cos \varphi - |r_N^{(1)}||r_N^{(2)}| \cos \varphi_+ + |r_A^{(1)}||r_A^{(2)}| \cos \varphi
$$

where $\varphi = \cos \left(\frac{k_n^{(\sigma)} - k_n^{(h)}}{k_n^{(h)}}L + \chi_+\right)$ and $\varphi_+ = \cos \left(\frac{k_n^{(\sigma)} + k_n^{(h)}}{k_n^{(h)}}L + \mu_+\right)$ where $L$ is the length of the ferromagnet, $\chi_+ = \chi_1 + \chi_2$; $\mu_+ = \eta_1 + \eta_2 + \vartheta_1$ and $\varphi = \phi_1 - \phi_2$ is the phase difference between the superconductors; the indices 1 and 2 indicate the F/S boundaries. The Andreev discrete energy levels $\varepsilon_{n,l}^\sigma$ of the system are determined by solutions of the equation $D_n^{(\sigma)}(\varepsilon) = 0$, that is

$$
\cos \left(\frac{k_n^{(\sigma)} - k_n^{(h)}}{k_n^{(h)}}L + \chi_+\right)
$$

and

$$
|r_N^{(1)}||r_N^{(2)}| \cos \left(\frac{k_n^{(\sigma)} + k_n^{(h)}}{k_n^{(h)}}L + \mu_+\right) - |r_A^{(1)}||r_A^{(2)}| \cos \varphi
$$

In the case that the ferromagnetic part of the structure is coupled to a reservoir of normal electrons through a potential barrier of a low transparency $t_r \ll 1$ (see Fig. 3), the density of Andreev states inside the ferromagnetic conductor can be written as follows:

$$
\nu(\varepsilon) = \frac{1}{\mathcal{V}} \sum_{\sigma = -1}^{+1} \sum_{\{n\}} \nu_n^{(\sigma)}(\varepsilon)
$$

where $\mathcal{V}$ is the volume of the ferromagnetic part, $\nu_n^{(\sigma)}$ is the density of Andreev states at a fixed transverse mode quantum number $n$ and the spin projection $\sigma$.

$$
\nu_n^{(\sigma)}(\varepsilon) = \frac{1}{\pi} \sum_l \frac{t_r E_0}{(\varepsilon - \varepsilon_{n,l}^{\sigma})^2 + (t_r E_0)^2} = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{t_r E_0}{(\varepsilon - \varepsilon')^2 + (t_r E_0)^2} \nu_n^{(\sigma)}(\varepsilon')d\varepsilon'
$$

and

$$
\nu_n^{(\sigma)}(\varepsilon) = \sum_l \delta(\varepsilon - \varepsilon_{n,l}^{\sigma})
$$

For convenience sake of further analytical calculations we write the density of Andreev states in the following form

$$
\nu_n^{(\sigma)}(\varepsilon) = \sum_l \delta(\varepsilon - \varepsilon_{n,l}^{\sigma}) = \left| \frac{\partial D_n^{(\sigma)}(\varepsilon)}{\partial \varepsilon} \right| \delta \left( D_n^{(\sigma)}(\varepsilon) \right)
$$

where $D_n^{(\sigma)}(\varepsilon)$ is defined by Eq. (10).

To find the density function $\nu_n^{(\sigma)}(\varepsilon)$ we use the method developed in Ref. [61]. As $\partial \varphi_\pm / \partial \varepsilon \approx \pm 1/E_n$ (here $E_n = h\nu_n/L$ and $\nu_n = \sqrt{P_x^2 + \frac{P_y^2 + P_z^2}{m}}$) the factor $\partial D_n^{(\sigma)}(\varepsilon)/\partial \varepsilon$ is a trigonometrical function of $\varphi_\pm$, as well as $D_n^{(\sigma)}$ (see Eq. (10)), and it is productive to expand $\nu_n^{(\sigma)}$ into Fourier series in $\varphi_\pm$ and write it as follows:

$$
\nu_n^{(\sigma)}(\varepsilon) = \sum_{k=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} A_{s,k}^{n,\sigma} \exp(is\varphi_-(\varepsilon) + ik\varphi_+(\varepsilon));
$$

$$
A_{s,k}^{n,\sigma} = \frac{1}{(2\pi)^2} \int_0^{2\pi} d\varphi_+ \int_0^{2\pi} d\varphi_- \left| \frac{\partial D_n^{(\sigma)}(\varepsilon)}{\partial \varepsilon} \right| \delta \left( D_n^{(\sigma)}(\varphi_+,\varphi_-) \right) \exp(-is\varphi_- - ik\varphi_+)
$$
As shown in Ref. [56] the main contribution to the state density function Eq. (16) is of the terms \( A^{n,\sigma}_{s,0} \), which, after integrating Eq. (18) with respect to \( \varphi' \), is written as

\[
A^{n,\sigma}_{s,0} = \left( \frac{2}{E_n} + \frac{\partial \chi_+}{\partial \varepsilon} \right) \frac{1}{(2\pi)^2} \int_0^\pi d\varphi_+ \left\{ \exp\left[i\varphi_1(\varphi_+)\right] + \exp\left[-i\varphi_1(\varphi_+)\right] \right\}
\]  

(18)

where

\[
\varphi_1(\varphi_+) = \arccos(|r_N^{(1)}||r_N^{(2)}| \cos \varphi_+ - |r_A^{(1)}||r_A^{(2)}| \cos \varphi), \quad 0 \leq \varphi_1 \leq \pi
\]

Inserting Eq. (18) into Eq. (16) (other terms in Eq. (16) with \( k \neq 0 \) are negligibly small, see Ref. [56]) one find \( \nu^{\sigma}_n \) to be

\[
\nu^{\sigma}_n(\varepsilon) = \left( \frac{2}{E_n} + \frac{\partial \chi_+}{\partial \varepsilon} \right) \sum_{s=-\infty}^{\infty} \frac{1}{(2\pi)^2} \int_0^\pi d\varphi' \left\{ \exp\left[i\varphi_1(\varphi') + \varphi_-(\varepsilon)\right] + \exp\left[-i\varphi_1(\varphi') + \varphi_-(\varepsilon)\right] \right\}
\]

(19)

Using Eq. (19), Eq. (13) and Eq. (12) one finds the the density of electronic states in the ferromagnet to be equal to

\[
\nu(\varepsilon) = \frac{4N_{\perp}}{\pi E_0} \sum_{\sigma=-1}^{1} \sum_{\sigma'=-1}^{1} \int_1^\infty dy \left( 2 + \frac{E_0}{y} \frac{d\chi_+(\varepsilon)}{d\varepsilon} \right) \times
\]

\[
\int_0^\pi d\varphi_+ \frac{t_r}{2\pi} \frac{t_r}{1 - \cos(2(\varepsilon + \sigma I_0)y/E_0 + \chi_+(\varepsilon) + \sigma' \varphi_1(\varphi_+))}
\]

(20)

Numerical results for the density of states based on Eq. (20) are shown in Fig. 4. The sharp peak in the dependence of the density of states on energy corresponds to the \( N_{\perp} \)-fold degeneracy of the energy level \( \varepsilon = I_0 \) in the case that the superconductor phase difference \( \varphi \) is equal to odd numbers of \( \pi \), as was explained in the previous section.

**NORMAL CURRENT AND CONDUCTANCE OF S/F/S STRUCTURES WITH \( I_0 < |\Delta| \)**

In this Section we find the differential conductance and the current through the ferromagnet in the geometry of Fig. 3 for the case that the ballistic ferromagnet is weakly coupled to a normal electron reservoir (that is \( t_r \ll 1 \)) with a voltage drop \( V \) to be applied between the reservoir and the superconductor. We assume that there are
Andreev and normal reflections at F/S interfaces 1 and 2 with the probability amplitudes \( r^{(1,2)}_A \) and \( r^{(1,2)}_N \), respectively \((|r^{(1,2)}_A|^2 + |r^{(1,2)}_N|^2 = 1)\).

According to the Landauer-Lambert formula [57], the current from the normal electron reservoir into the superconductor is written as follows:

\[
J = \frac{e}{h} \int_{-\infty}^{\infty} \left( f_0(\varepsilon - \frac{eV}{2}) - f_0(\varepsilon + \frac{eV}{2}) \right) R_A(\varepsilon);
\]

\[
R_A = \sum_{\{n\}} \rho_A^{(n)}
\]

where \( f_0(\varepsilon) \) is the Fermi distribution function, \( \rho_A^{(n)} \) is the probability for an electron approaching the ferromagnet along the lead to be reflected back into the reservoir as a hole.

As we are interested in the resonant transmission of quasi-particles through Andreev levels \( \varepsilon^\sigma_{n,l} \) inside the ferromagnet we use the resonant Breight-Wigner formula to write \( R_A(\varepsilon) \) in the form

\[
R_A(\varepsilon) = \frac{1}{\pi} \sum_{\sigma = \pm 1} \frac{1}{\int_{1}^{\infty} \frac{t_r^2}{y^4} \int_{0}^{\pi} d\varphi_+ \frac{d\varphi_+}{2\pi} }
\]

\[
2t_r^2 \int_{0}^{\pi} \frac{d\varphi_+}{2\pi} \left[ \arctan \left( \frac{\sqrt{1 + t_r^2}}{t_r} \tan \left( \frac{eV/2 + I_0}{E_0} + \sigma \varphi_1(\varphi_+) \right) \right) \right] - \arctan \left( \frac{\sqrt{1 + t_r^2}}{t_r} \tan \left( \frac{-eV/2 + I_0}{E_0} + \sigma \varphi_1(\varphi_+) \right) \right)
\]

Inserting Eq.(19) into Eq.(23) one finds the electron-hole transmission coefficient \( R_A(\varepsilon) \) to be equal to

\[
R_A(\varepsilon) = \frac{2N_0}{2\pi} \int_{\sigma = \pm 1} \int_{\sigma' = \pm 1} \int_{0}^{\infty} \frac{d\varphi_+}{2\pi} \left[ \arctan \left( \frac{\sqrt{1 + t_r^2}}{t_r} \tan \left( \frac{eV/2 + I_0}{E_0} + \sigma \varphi_1(\varphi_+) \right) \right) \right] - \arctan \left( \frac{\sqrt{1 + t_r^2}}{t_r} \tan \left( \frac{-eV/2 + I_0}{E_0} + \sigma \varphi_1(\varphi_+) \right) \right)
\]
Numerical results for the differential conductance and the current based on Eq. (25) and Eq. (26) are shown in Fig. 5 and Fig. 6. They demonstrate a high sensitivity of the differential conductance and the non-linear current-voltage characteristics to both the superconductor phase difference $\varphi$ and the applied voltage $V$.

At low voltages, far from $2I_0/e$, we have a resonant tunneling of quasi-particles through separate Andreev levels, and the conductance and the current are low. When $eV/2 \approx I_0$ and $\varphi = \pi(2l + 1)$ ($l = 0, \pm 1, \pm 2, \ldots$) Andreev levels concentrate near $I_0$ as can be readily seen from Eq. (10), and we have simultaneous resonant transmission of quasi-particles through the whole number of $N_\perp$ states resulting in a high peak in the conductance and a sharp jump in the current. When $\varphi$ deviates from an odd number of $\pi$, the number of Andreev levels concentrated near $I_0$ is decreasing that results in a decrease of the sensitivity of the conductance and the current to the voltage.

As is evident from Eq. (11), in case $r_N^{(1)} \neq r_N^{(2)}$ Andreev levels are repelled from the level $I_0$. It results in a splitting (proportional to $\delta r_N = |r_N^{(1)} - r_N^{(2)}|$), see [56]) of the conductance peak if this splitting is larger than the Andreev level broadening (proportional to $t_r$) caused by the coupling of the ferromagnet to the reservoir through a potential barrier.
of the transparency \( t_r \ll 1 \). In Fig. 5 this splitting is absent at \( \varphi = \pi \) because in this case \( \delta r_N = 0.05 < t_r = 0.1 \). However, with a deviation of the superconductor phase difference \( \varphi \) from \( \pi \) the splitting sharply increases, and a dip in the conductance peak appears as is seen in Fig. 4, at \( \varphi = 1.1\pi \) and \( \varphi = 1.2\pi \).

We note here that the differential conductance \( G \) as a function of \( eV \) is proportional to the density of Andreev states in the ferromagnet permitting a direct spectroscopy of Andreev levels by conductance and current measurements.

**DENSITY OF STATES IN DIFFUSIVE S/F/S STRUCTURES WITH \( I_0 < |\Delta| \)**

As is shown in Section 4 for ballistic S-F-S structures, the energy level \( \varepsilon = I_0 < |\Delta| \) is \( N_i \)-fold degenerated and the density of states \( \nu(\varepsilon) \) at \( \varepsilon = I_0 \) and \( \varphi = \pi \) is proportional to \( N_i \sim S/\lambda_F^2 \gg 1 \) (see Fig. 4) in the same manner as it takes place in S-N-S (non-magnetic) structures at \( \varepsilon = 0 \) (see [53, 56]). In diffusive non-magnetic S-N-S structures there is an energy gap in the density of states around \( \varepsilon = 0 \) which is maximal at \( \varphi = 0 \) and shrinking to zero together with a sharp increase of the density of states at \( \varepsilon = 0 \) as \( \varphi \) approaches \( \pi \) [53]. In order to see whether the above-mentioned degeneracy of the level \( \varepsilon = I_0 < |\Delta| \) survives in diffusive S/F/S structures, we consider the density of states in a diffusive S/F/S structure in the vicinity of \( \varphi = \pi(2k + 1), k = 0, \pm 1, \pm 2, \ldots \) assuming the motion of quasiparticles inside the ferromagnet to be semiclassical. It allows us to use Gutzwiller’s approach [64] which has the advantage of the clarity of physical presentation. On the other hand, the new class of two-dimensional (2D) magnetic semiconductors with large dielectric constants and small effective masses [62] very well satisfies the condition of semiclassical motion \( \alpha = r_s p_F / \hbar \gg 1 \) (\( r_s \) is the screening length in the ferromagnet) with \( \alpha = 10 \div 10^2 \). We show, that in much the same way as in the ballistic case (see Section 4 and Section 5), in a diffusive S/F/S structure the energy level \( \varepsilon = I_0 \) \( (I_0 < |\Delta|) \) is also \( N_0 \sim S/\lambda_F^2 \)-fold degenerated and the height of the peak in the density of states at \( \varepsilon = I_0 \) is proportional to \( N_0 \), if the phase difference between the superconductors is in the vicinity of an odd number of \( \pi \). Below we consider the case that the Andreev reflection of a quasiparticle at the F/S interfaces is accompanied by a normal specular reflection in the same manner as for the ballistic case, that is the probability amplitudes \( |r_N^{(1,2)}| \ll 1 \).

For calculations of the density of states \( \nu_{SFS}(\varepsilon) \) at \( \varphi = \pi(2k + 1), k = 0, \pm 1, \ldots \) we use the Gutzwiller semiclassical trace formula which shows that \( \nu_{SFS}(\varepsilon) \) can be presented as a sum over all periodic classical trajectories [64]:

\[
\nu_{SFS}(\varepsilon) = \sum_j A_j(\varepsilon)e^{iS_j(\varepsilon)}
\]  

(27)

(here \( j \) is the trajectory index, \( S_j = \oint pdq \) is the classical action integral (the integral is taken along the periodic \( j \)-trajectory), \( A_j \) is the trajectory amplitude which is equal to the Gaussian path integral around the stationary phase path (which is the classical trajectory)).

If there are Andreev and normal reflections at the F/S boundaries, the semiclassical trajectories contributing to the density of states Eq. (27) are of the type shown in Fig. 4. In this case the problem of finding the density of states is reduced to a quantum scattering problem for the configuration shown in Fig. 4B. The points of reflections at the F/S interfaces are shown with black dots. Propagation between these points is coherent in both electron and hole channels, which is illustrated by dashed and solid lines of equal lengths. For a quasiparticle with a non-zero energy \( \varepsilon \) the phase gains along electron and hole trajectories do not completely cancel since the momenta are now different. The resulting decompensation effect is of order of \( ||\varepsilon|| - I_0||/E_{Th} \) [63].

If \( r_N^{(1,2)} \neq 0 \), the motion of a quasiparticle along such a trajectory is of a quantum character despite the quasiclassical parameter is small because quasi-particle waves split at the scattering points. The semiclassical wave-functions in the adjacent sections of Fig. 4B are connected by the 2-channel scattering matrix 7 describing Andreev and normal reflections at the F/S boundaries. The problem of finding the spectrum and the wave-function in such a one-dimensional chain is reduced to solving a set of matching equations for amplitudes of electron \( a_n^{(e)} \) and hole \( a_n^{(h)} \) excitations in every section of propagation between scattering points. If \( r_N^{(1,2)} \ll 1 \), the main contribution to the density of states Eq. (27) at \( \varphi = \pi(2k + 1), k = 0, \pm 1, \pm 2, \ldots \) comes from trajectories which do not include successive reflections at the same F/S boundary. Taking these observation into account the following set of algebraic matching
The amount of the phase gain after propagation along the trajectories in section \( n \) is
\[
\Phi_{n}^{(e,h)} = \int_{L_n} p_n^{(e,h)} dl / \hbar \approx \Phi_{n}^{(0)} \pm \tau_n (\epsilon - \sigma I_0) / \hbar
\]
where \( \Phi_{n}^{(0)} = p_F L_n / \hbar \) and \( \tau_n = L_n / v_F \) is the propagation time in section \( n \). The quantity \( r_A^{(1,2)} \) and \( r_N^{(1,2)} \) are, respectively, the probability amplitudes for Andreev and normal reflections at F/S boundaries 1 and 2. Phases and amplitudes of electrons or holes along semiclassical paths are defined in such a way that no phase has been gained at the beginning of a particular electron or hole section \( n \). Hence the amplitude is \( a_{n}^{(e,h)} \) at the beginning of the section, and \( a_{n}^{(e,h)} \exp \pm \Phi_{n}^{(e,h)} \) at its end. We note that one can show that the large phases \( \Phi_{n}^{(0)} \) can be removed from the set of eqns. \( 28 \). This is a manifestation of the fact that the electron and hole phase gains compensate each other at \( \epsilon = \sigma I_0 \).

The set of equations Eq. \( 28 \) can be presented in a matrix form:
\[
|a⟩ = Û|a⟩
\]
where components of the vector \(|a>\) are the coefficients \(a_{k}^{(e,h)}\) of Eq(28); the definition of the matrix \(\hat{U}\) is obvious from Eq (28); it is a unitary matrix written as follows:

\[
U_{mk}(\Phi^{(e,h)}) = V_{mk} \exp \{i\Phi_{k}^{(e,h)}\}
\]

(30)

where the \(m\)-th row of the matrix \(\hat{V}\) has only two non-zero elements which are elements of the matrices \(\hat{T}^{(1,2)}\) (see Eq(17)); \(\Phi^{(e,h)} = (\ldots, \Phi_{1}^{(e,h)}, \Phi_{2}^{(e,h)}, \ldots)\).

According to general principles of quantum mechanics [60], the spectrum of the system is defined by zeros of the determinant

\[
Det \left[ \hat{I} - \hat{U}(\Phi) \right] = 0
\]

(31)

where \(\hat{I}\) is the unit matrix.

Summation in Eq(27) is over different trajectories and one can think of it as averaging over various distributions of \(\tau_{n}\). Therefore, the density of states Eq (27) for the S/F/S case under consideration can be re-written as follows:

\[
\nu_{SFS}(\varepsilon) \approx N_{0} \sum_{\sigma} \langle \nu_{\sigma \text{ random}}^{\varepsilon} \rangle
\]

(32)

where \(N_{0} = S/\lambda_{p}^{2}\) and \(\langle \ldots \rangle\) implies an averaging over \(\tau_{n}\); \(\nu_{\sigma \text{ random}}^{\varepsilon}\) is the density of Andreev states for a fixed spin projection \(\sigma\) generated by a given semiclassical trajectory of the type shown in Fig. 7B. The distribution of propagation times \(\tau_{n}\) depends on details of the disordered potential in the mesoscopic ferromagnetic region. These are not known, but it is natural to assume that the propagation times along different sections of the semiclassical trajectories (see Fig. 7A) are uncorrelated.

In order to find the averaged density of states \(\langle \nu_{\sigma \text{ random}}^{\varepsilon} \rangle\) we use the presentation suggested by Slutskin [61] for the density of states of a one-dimensional chain of the type in Fig. 7B. In the general case it can be written as follows:

\[
\nu(\varepsilon) = \frac{1}{\hbar N} \sum_{n=1}^{N} \tau_{n} (F_{nn} + F_{nn}^{*} + 1)
\]

(33)

where matrix \(\hat{F}\) satisfies the matrix equation \(\hat{F} = \hat{F} \hat{U} + \hat{U}\) (in our case \(\hat{U}\) is defined by Eq(31)) and

\[
F_{nn} = \sum_{l} A_{n}(l) \exp \{il\}
\]

(34)

Here \(l = \{l_{1}, \ldots, l_{N}\}\) (\(N\) is the number of sections in the chain), \(l_{n}\) are integers, either positive or equal to zero, \(lf = \sum_{k=1}^{N} l_{k} f_{k}\), \(f = \{f_{1}(\varepsilon), \ldots, f_{N}(\varepsilon)\}\), \(f_{n}(\varepsilon)\) is the phase gain along section \(n\) of the chain, Fourier coefficients \(A_{n}\) depends on "hopping integrals" \(\phi_{N}^{(1,2)}\) and are independent of \(\tau_{k}, k = 1, 2, \ldots N\); summation is over integer \(l_{k}\), either positive or equal to zero. Therefore, the averaged density of states is

\[
\langle \nu_{\sigma \text{ random}}^{\varepsilon} \rangle = \frac{1}{\hbar N} \sum_{n=1}^{N} \langle \tau_{n} (F_{nn} + F_{nn}^{*} + 1) \rangle
\]

(35)

where \(F_{n} = F_{nn}\).

For the case under consideration \(f_{k} = \tau_{k}(\varepsilon - \sigma I_{0})/\hbar\) and for the convenience sake we re-write Eq(33) as follows:

\[
F_{n} = \sum_{l} A_{n}(l) \prod_{k} \exp \{il\tau_{k}(\varepsilon - \sigma I_{0})/\hbar\}
\]

(36)

Since amplitudes \(A_{n}\) do not depend on \(\tau_{k}\) one only has to average the product \(\tau_{n} \prod_{k} \exp il\tau_{k}(\varepsilon - \sigma I_{0})/\hbar\) while calculating the average density of states Eq (35).

For the configuration of Fig. 7B the averaged density of states can be found exactly if one chooses a Lorentz form for the distribution function \(P(\tau)\),

\[
P(\tau) = \frac{1}{\pi} \frac{\gamma}{(\tau - \tilde{\tau})^{2} + \gamma^{2}}\]

(37)
Using the Lorenzian distribution Eq. (37) one finds the result
\[
\langle \tau_k \prod_k \exp{i k \tau_k (\varepsilon - \sigma I_0) / \hbar} \rangle = \frac{\gamma}{\pi \tau} \int_{-\infty}^{\infty} \frac{\varepsilon_1}{(\varepsilon_1 - (\varepsilon - \sigma I_0))^{3/2} + (\gamma/\tau)^2 (\varepsilon - \sigma I_0)^2} \left( \tau \prod_k \exp{i k \tau \varepsilon_1 / \hbar} \right)
\]
Inserting Eq. (38) in Eq. (39) one finds the averaged density of states:
\[
\langle (\nu_{\text{random}}^\sigma (\varepsilon)) \rangle = \frac{2 \gamma |\varepsilon - \sigma I_0|}{\pi \tau} \int_{-\infty}^{\infty} \frac{\varepsilon_1^2 \nu_p (\varepsilon_1) d\varepsilon_1}{\varepsilon_1^4 + 4 (\gamma/\tau)^4 (\varepsilon - \sigma I_0)^4}
\]
where
\[
\nu_p (\varepsilon) = \frac{1}{\hbar N} \sum_{n=1}^{N} \tau \left( F_n^{(0)} + F_n^{(0)*} + 1 \right)
\]
and
\[
F_n = \sum_{l=1} A_n (l) \prod_k \exp \{ i k \tau \varepsilon / \hbar \}
\]
While writing Eq. (39) we took into account the fact that \( \nu_p (\varepsilon_1) = \nu_p (-\varepsilon_1) \).

From Eq. (40) and Eq. (41) one sees \( \nu_p (\varepsilon) \) to be the density of states for the case that the system in Fig. 8B is non-magnetic \( (I_0 = 0) \) and periodic with all \( \tau_n = \tau \) (that is all \( L_n = L \) where \( L = < < L_n > > \equiv \tau / v_F \sim (v_F / D) L^2 \) where \( L \) is the distance between the S/F interfaces).

For the case of a periodic chain, Eq. (28) with all \( \tau_n = \tau \) can be easily solved, and for the phase difference \( \varphi \) in the vicinity of an odd number of \( \pi \) one gets the dispersion low for the quasiparticle moving along the chain as follows:
\[
\epsilon_{\pm} (k) = \pm E_{Th} \sqrt{(\delta \varphi)^2 + |\nu_N^{(1)}|^2 + |\nu_N^{(2)}|^2 - 2 |\nu_N^{(1)}| |\nu_N^{(2)}| |\cos k|}
\]
where \( E_{Th} = \hbar D / L^2 \) is the Thouless energy, \( \delta \varphi = \varphi - l \pi \) \( (l \) is an odd number, \( |\delta \varphi| \ll 1 \), \( k \) is a continuous quantum number - the "quasi-momentum" in the periodic one-dimensional chain.

Using Eq. (42) one finds the density of states \( \nu_p \) of the periodic chain to be
\[
\nu_p (\varepsilon) = \frac{2 |\varepsilon|}{\pi \sqrt{(\varepsilon^2 - \varepsilon_{\min}^2) (\varepsilon_{\max}^2 - \varepsilon^2)}}
\]
where the minimal and the maximal energies of the energy band Eq. (12) are
\[
\varepsilon_{\min} = E_{Th} \sqrt{(\delta \varphi)^2 + \left( |\nu_N^{(1)}| - |\nu_N^{(2)}| \right)^2}
\]
\[
\varepsilon_{\max} = E_{Th} \sqrt{(\delta \varphi)^2 + \left( |\nu_N^{(1)}| + |\nu_N^{(2)}| \right)^2}
\]
Inserting Eq. (11) in Eq. (39) and integrating the resulting expression, we find an exact formula for the averaged density of states of the random chain in Fig. 7
\[
\nu_{\text{random}}^\sigma (\varepsilon) = \frac{2 \kappa |\varepsilon - \sigma I_0|}{\pi} \left\{ \sqrt{(\varepsilon_{\min}^4 + 4 (\kappa |\varepsilon - \sigma I_0|)^4) (\varepsilon_{\max}^4 + 4 (\kappa |\varepsilon - \sigma I_0|)^4) + \varepsilon_{\min}^2 \varepsilon_{\max}^2 - 4 (\kappa |\varepsilon - \sigma I_0|)^4} \sqrt{2 (\varepsilon_{\min}^4 + 4 (\kappa |\varepsilon - \sigma I_0|)^4) (\varepsilon_{\max}^4 + 4 (\kappa |\varepsilon - \sigma I_0|)^4)} \right\}^{1/2}
\]
where \( \kappa = \gamma / \tau \).

Multiplying Eq. (43) by \( N_0 \) and summing over the spin projections (see Eq. (12)) we get the average density of states of a diffusive S-F-S structure as follows:
\[
\nu_{\text{SFS}} (\varepsilon) = N_0 \sum_{\sigma} \frac{2 |\varepsilon - \sigma I_0|}{\pi} \left\{ \sqrt{(\varepsilon_{\min}^4 + 4 (\varepsilon - \sigma I_0)^4) (\varepsilon_{\max}^4 + 4 (\varepsilon - \sigma I_0)^4) + \varepsilon_{\min}^2 \varepsilon_{\max}^2 - 4 (\varepsilon - \sigma I_0)^4} \sqrt{2 (\varepsilon_{\min}^4 + 4 (\varepsilon - \sigma I_0)^4) (\varepsilon_{\max}^4 + 4 (\varepsilon - \sigma I_0)^4)} \right\}^{1/2}
\]
(we have used \( \gamma = \tau \), see Ref. 56.)

Numerical results for the averaged density of states for a diffusive S/F/S structure based on Eq. (46) are presented in Fig. 8. The sharp peak in the dependence of the averaged density of states on energy in Fig. 8B corresponds to the \( N_0 \)-fold degeneracy of the energy level \( \varepsilon = I_0 \) at \( \varphi \) equal to odd numbers of \( \pi \); the splitting of the peak in Fig. 8A is proportional to \(|\nu_N^{(1)}| - |\nu_N^{(2)}|\).
CONCLUSION

In conclusion, for S/F/S structures with $I_0 < \Delta$ ($I_0$ is the ferromagnet exchange energy, $\Delta$ is the superconductor energy gap) we have demonstrated that an extremely high degeneracy of the energy level $\varepsilon = I_0$ proportional to $N_0 = S/\lambda_F^2$ at $\varphi$ equal to odd numbers of $\pi$ ($S$ is the cross-section area of the ferromagnet, $\lambda_F$ is the electron wave length) results in giant oscillations of the density of Andreev states and the conductance of the ferromagnet with a change of the superconductor phase difference $\varphi$. This phenomena is a convenient tool for the Andreev level spectroscopy (the differential conductance $G$ is proportional to the density of Andreev states) and enables applications, e.g. as a double-gate ferromagnet transistor analogous to the one described in [50]. On the other hand, this effect permits to find the exchange energy, $I_0$, of the interaction between the electron spin and the spontaneous moment of the ferromagnetic conductor by an electric measurement of the differential conductance because a sharp and giant peak in the dependence of the conductance on the applied voltage $V$ arises at $V = 2I_0/e$ (the corresponding peak in the density of Andreev states takes place at energy $\varepsilon = I_0$; see Fig.4, Fig. 5 and Fig. 8).

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For $I_0 \gg kT$, one should change $\Delta p$ in Eq. (1) to $\delta = I_0/vF$ and the penetration length inside a diffusive ferromagnet turned out to be $L_{lo} = \sqrt{\hbar D/I_0}$; the superconducting correlator inside a ballistic ferromagnet oscillates decreasing linearly with
an increase of the distance from the ferromagnet-superconductor interface \[2\].

[67] The conductance of an S/F/S structure for a diffusive case in the absence of such a barrier was considered in Ref. \[51\].

[68] As was shown by M.J.M. de Jong and C.W.J. Beenakker \[52\], the scattering matrix at an F/S interface differs from the one at an N/F interface, but estimations show Eq. \(7,8\) to be correct to the first order of \(I_0/\varepsilon_F \ll 1\).

[69] The semiclassical trajectories for an electron of energy \(\varepsilon\) and the reflected hole of energy \(-\varepsilon\) are to be considered identical since they separate by less than \(\lambda_F\) while diffusing a distance \(L\) if \(||\varepsilon| - I_0| \ll E_{Th}\).