Modelling and simulation of narrow car dynamic

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Abstract. Torque vectoring has huge impact on vehicles steering characteristics. Most of the literature refer to the vehicles with a wheel track typical for passenger cars. This paper examines whether for the narrow vehicle (with a very small wheel track) torque vectoring is possible to be used in an active safety systems, securing it from roll over (roll mitigation). This paper deals with the description of current vehicle technology related to traction control through the employment of effective torque vectoring strategies for electric vehicles. The work contains a description of simulation models. The bicycle model of a vehicle is arranged and suitable adapted for the test subject. The paper contains a set of simulation compared with road tests of the narrow vehicle. The usefulness of a simple linear single track vehicle model to describe the movement was tested. The vehicle understeer characteristic as a function of a power distribution is presented.

1. Introduction

1.1. Narrow car
One of the most serious problems of modern cities are a traffic jams, congestion and lack of parking spaces. The solution to these problems seems to be a narrow car, a the special kind of microcar. Narrow cars are small-sized vehicles designed to run short routes by one or two people with little luggage. Due to their small dimensions, the narrow cars perform perfectly in the urban agglomeration spaces. Majority of residents drive short distances between the place of residence, place of work and service premises (often collected in one shopping and service centre). Common in European cities are setting a special zones with limited or total ban entry of cars equipped with combustion engines (zero emission zone), where electric vehicles can freely drive. Often the legal system is favouring microcars in relation to full size vehicles (often by lower taxes and lower insurance rates). Because of the above the electric motors are often used to drive microcars. It reduces the traveling costs, but on the other hand electric vehicles have a small range compared to vehicles powered by internal combustion engines. Full electric vehicle have also reduced the negative impact of the vehicle on the natural environment. Societies are becoming more and more eco-friendly, therefore environmental protection is one of the reasons determining the purchase of a electric car. As a tested narrow car a MIST car was chosen. The microcar MIST was constructed and fabricated on Cracow University of Technology. It is presented in Figure 1.
1.2. Electric drivetrain

Electric motors (e.g. BLDC or PMSM) have torque and power characteristics similar to the desired one. The electric drive enables work with constant torque from speed 0 rpm to rated rpm (I regulation zone). Above the rated rpm, the motor operates at a constant power - decreasing torque characteristic (II control zone). Such properties of the electric motor simplify the construction of drive systems. Due to the generation of torque at a speed of 0, the clutch becomes unnecessary. The gearbox is also not needed. The value of the maximum speed and the driving torque is determined by selecting a fixed ratio and the dynamic radius of the drive wheels. Two groups of drive systems for electric vehicles can be distinguished [1]:

1.2.1. 1st generation

In place of the internal combustion engine, clutch and gearbox, an electric motor is installed. One electric motor cooperating with a differential gear drives the vehicle's axle. The choice of a higher main gear ratio allows the engine to be run at a higher rotational speed. Examples of vehicles available on the market with the above-mentioned drivetrain: Mitsubishi i-Miev, Nissan Leaf, Renault Fluence, Renault ZOE, Smart ED, VW e-up!, Tesla Model S, Ford Focus Electric, Volvo C30 Electric, BMW i3, Toyota RAV4 EV, Chevrolet Spark EV, Honda Fit EV.

1.2.2. 2nd generation

There is no differential mechanism in this construction. Electric motors (two or four with a 4WD drive) drive separate vehicle wheels. Such a drive system gives the possibility of individual control of the vehicle's engines. A market available example is the Mercedes-Benz AMG SLS C 197 Electric Drive. This vehicle is equipped with four electric motors driving the wheels. Most manufacturers have prototypes built with the 2nd generation of the electric drive system. They are min. Audi R8 E-tron 2.0, Mitsubishi iMIEV Sport Concept. Vehicles in this generation are constructed in the version with rear-drive (Audi), front (Mitsubishi), or all (Mercedes) wheels.

2. Torque vectoring

The vehicle motion occurs by supplying the driving torque to the vehicle's wheels. The driving torque of the wheels generates the longitudinal force that sets the vehicle in motion. This force is used for acceleration (resistance of inertia) or constant speed motion (rolling resistance and aerodynamic resistance of the vehicle). Balance of forces acting on the car in rectilinear motion (1):

![Figure. 1. MIST microcar constructed on Cracow University of Technology](image)
\[ \vec{F}_d + \vec{F}_a + \vec{F}_r + \vec{F}_h + \vec{F}_i = 0 \]  

(1)

where:
- \( F_d \) - driving force
- \( F_a \) - air resistance force
- \( F_r \) - rolling resistance force
- \( F_h \) - hill resistance force
- \( F_i \) - inertial force

Normally, on a vehicle equipped with differential mechanism or with single power distribution line the driving torque is equal on both wheels of the driving axle. But unbalanced dividing of driving torque can influence the steering characteristic of a vehicle by providing an additional deflecting moment [2].

In order to measure a vehicle steering the gradient of steering wheel angle related to the lateral acceleration, normalized by steering system transmission ratio has been adopted. It is called the steering gradient, which is defined as (2) [3]:

\[ GS = \frac{\partial \delta_H}{\partial a_y} \cdot \frac{1}{i_s} - \frac{\partial \delta_D}{\partial a_y} \]  

(2)

where:
- \( \delta_H \) – steering angle,
- \( i_s \) – the kinematic ratio of the steering system,
- \( \delta_D \) – dynamic steering angle of the wheel (3),
- \( a_y \) – lateral acceleration.

\[ \delta_D = \frac{L}{R} \]  

(3)

where:
- \( L \) – wheel base,
- \( R \) – radius of turn

The trend of lateral acceleration at understeer vehicles is at the small steering angle linear up to certain value of lateral acceleration and then increases non-linearly to a maximum value. It is the maximum possible lateral acceleration to obtain in a steady state cornering. Potential modifications of the vehicle understeer characteristic achievable through torque vectoring with individually controlled powertrains is presented in Figure 2.

![Figure 2](image-url)  

**Figure 2.** modifications of the vehicle understeer characteristic achievable through torque vectoring [4]

In the transient state the increasing of the longitudinal acceleration reduces the linear response region of the vehicle to the steering signal. Improve the vehicle performance at the transient state is widely described in the academic papers, but generally the two different torque vectoring strategies can be implemented: i) driving torque proportional to the each wheel normal (vertical) load; ii) driving torque distribution allowing achieving the same longitudinal slip ratio on each wheel of the vehicle
Both strategies allow to reduce the understeer gradient difference and expand the linear respond region. Torque vectoring can also be used to reduce the vehicle power consumption while cornering. Simulations carried out by [5] and [4] point to a few percent reduction of the vehicle energy consumption while cornering in relation to the vehicle with the same parameters without differentiation of torque. This result shows that the torque vectoring strategies not only increases the vehicle dynamic performance, but also optimizes the usage of the battery energy. In addition, torque vectoring has an effect on the improvement of the drivability [6], [7], improvement of the vehicle's off-road properties and driving in poor road conditions [8], as well as the driver's "feeling" of the vehicle [9].

Most papers referred to vehicles with wheelbases and track width typical for passenger cars. In this paper a narrow car motion was tested and simulated. The narrow cars have greater height in relation to the track width to ensure good visibility and adequate passengers space. Because of that they are susceptible to rolling over. Occasionally at the narrow cars, rolling protection is provided by tilting the vehicle. However, it is necessary to build additional devices such as an actuator and it requires the development of a tilt control method. Other commonly used track stabilization and anti-rollover systems use vehicle brakes to create the deflection or tilting moment of the vehicle. Each activation of the vehicle's brakes dissipates the kinetic energy of the vehicle in the form of heat. It is a loss of kinetic energy and it reduces the overall efficiency of the vehicle. Therefore, each operation of the vehicle's stabilization system contributes to reducing the maximum range of the vehicle. This is especially important in electric vehicles, because the energy density in traction batteries is lower than in liquid fuels and the range of electric vehicles is much lower. The distribution of driving forces on individual wheels of the driven axle can be used to prevent rolling over and to stabilized the vehicle motion. Such a solution can be used in vehicles with independent power supply for electric motors.

Currently, there are no stabilization systems using the distribution of drive forces to produce a stabilizing torque [10]. The operating range of the stabilization system is narrower than systems based on vehicle brakes, however the described advantages are the basis for developing the system based on the torque vectoring principals.

3. Vehicle models
A mathematical model of a vehicle allows to simulate vehicle motion. Thanks to them, you can predict the behaviour of the vehicle under certain conditions and calculate the vehicle's response and the path of its movement.

3.1. Multipart models
Models built as a set of connected rigid bodies. The whole vehicle is a very complicated device, but the number of vehicle components in simulation is limited. Simplification of the model must be carried out. In the simulations carried out to examine the effect of torque differentiation on the dynamic behaviour of a multi-body vehicle, the model can be simplified to one rigid body describing the vehicle body and four vehicle wheels. Such a number of elements allows modelling of wheel turns, drive forces taking into account the differentiation of the drive torque and suspension movements. The large number of equations that require calculation means that significant computing power is needed for this type of modelling. Simulations using multipart models require a lot of time.

3.2. Double-track vehicle model
It is a vehicle simplified into two dimensions. It describes the movement of the vehicle's projection to the X-Y plane. Double-track model describes the flat motion and rotation of the vehicle's centre of gravity. Simplification includes the omission of the vertical movement of the vehicle and the tilt and yaw rotation. The dynamics of the wheels depend on the static normal forces resulting from the distance of the centre of mass from the axis of the vehicle. Some models include accelerations
(longitudinal and transverse) acting on the centre of mass to model the effects of loading and unloading of individual vehicle wheels while driving on a curve.

3.3. Single-track model

The simplest mathematical model which describes vehicles dynamics and is used in simulations [11]. It is a flat, three degrees of freedom model. It assumes that the centre of mass of the vehicle is its projection on the road surface. The vehicle itself is symmetrical. Axes have been reduced to individual wheels. The model assumes that the left and right wheels of the vehicle generate equal lateral forces and depend only on the slip angle of axles. This means that the description of the vehicle is identical to the description of a single-track vehicle. The model assumes also that the left and right wheel of the vehicle generate equal longitudinal driving forces. A serious limitation is the omission of the overturning of the vehicle.

The schematic of the model is shown in Fig. In the description of the model the following coordinate systems are used:
- stationary CS Ox/y0
- local CS associated with the vehicle Sxy, beginning in the middle of vehicle mass S

![Figure 3. Single-track vehicle model](image)

Because the translational motion acts at a constant speed \( v_x = \text{const} \), the equations of forces and momentous acting on the vehicle takes the form:

\[
-F_{int_y} + Y_1 \cos \delta_1 + Y_2 \cos \delta_2 + F_{ext_y} = 0
\]

(3)

\[
-M_{int} + Y_1 \cos \delta_1 l_f - Y_2 \cos \delta_2 l_r + M_{ext} = 0
\]

(4)

Vehicle slip angle is calculated as:

\[
\beta = \arctan \left( \frac{v_y}{v_x} \right)
\]

(5)

Equations of motion of a single-track vehicle model:

\[
\dot{v}_x = v_y r + \frac{1}{m} F_{ext_x}
\]

(6)

\[
\dot{v}_y = -\frac{K_1 + K_2}{mv_x} v_y + \left( -\frac{l_y K_1 + l_r K_2}{mv_x} - v_x \right) r + \frac{K_1}{m} \delta_1
\]

(7)

\[
\dot{r} = \frac{-l_y K_1 + l_r K_2}{l_x v_x} v_y - \frac{l_y^2 K_1 + l_r^2 K_2}{l_x v_x} r + \frac{l_y K_1}{l_x} \delta_1 + \frac{1}{l_x} M_{ext}
\]

(8)
where:
\( m \) – mass of the vehicle [kg]
\( I_z \) – moment of inertia around vertical axis [kg m\(^2\)]
\( \delta_1 \) – steering angle of the front wheels [rad]
\( \delta_2 \) – steering angle of the rear wheels [rad]
\( l_f \) – distance from front axle to centre of gravity [m]
\( l_r \) – distance from rear axle to centre of gravity [m]
\( Y_1 \) – horizontal force on the front axle [N]
\( Y_2 \) – horizontal force on the rear axle [N]
\( M_{ext} \) – additional yaw moment [Nm]
\( F_{ext} \) – additional horizontal force [N]
\( K_1 \) – cornering stiffness of the front axle [N/rad]
\( K_2 \) – cornering stiffness of the front axle [N/rad]
\( F_{int} \) – force of inertia [N]
\( M_{int} \) – moment of inertia [Nm]

Due to significant simplifications, this model does not require much computing power. Predictions of vehicle behavior can be calculated in real time and provide a basis for making corrections by the vehicle stabilization systems.

The \( K_i \) values are substitute stiffness calculated for whole vehicle axle. These data can be directly used in the bicycle mathematical model of the vehicle and are necessary to simulate vehicle motion.

4. Simulations

The preliminary test allowed to determine the forces acting in the axes of the vehicle and the slip angles of individual axles. Using these data the substitute cornering stiffness have been calculated.

The lateral forces of the front and rear axles can be determined as:

\[
\begin{align*}
Y_1 &= K_1 \alpha_1 \\
Y_2 &= K_2 \alpha_2
\end{align*}
\]

(9) (10)

The results of the preliminary tests carried out indicate the possibility of changing the understeer characteristics of narrow vehicles by varying the torque distribution. The results of tests carried out on the narrow vehicle showed the scope of possibilities of changes in the steering characteristics from the understeer to even towards oversteer [12] (see Figure 4.).

![Figure 4. Steering characteristics. Test with constant steering angle; inside motor drive (black), both motor drive (dark grey), outside motor drive (light grey).](image-url)
The data obtained in the preliminary tests allowed to derive the following system of equations describing the behaviour of the vehicle [13]:

\[
m v_x \ddot{r} = \left( K_1 + K_2 \right) \frac{v_y}{v_x} + \left( l_f K_1 - l_r K_2 \right) \frac{\dot{r}}{v_x} - K_1 \delta_1
\]

\[
I_z \ddot{r} = \left( l_f K_1 - l_r K_2 \right) \frac{v_y}{v_x} + \left( l_f^2 K_1 + l_r^2 K_2 \right) \frac{\dot{r}}{v_x} - l_f K_1 \delta_1 + M_z
\]

where:

- \( v_x \) – longitudinal velocity [m/s]
- \( v_y \) – lateral velocity [m/s]
- \( m \) – mass of the vehicle [kg]
- \( I_z \) – moment of inertia around vertical axis [kg m²]
- \( \delta_1 \) – steering angle of the front wheels [rad]
- \( l_f \) – distance from front axle to centre of gravity [m]
- \( l_r \) – distance from rear axle to centre of gravity [m]
- \( M_z \) – additional yaw moment from torque vectoring [Nm]
- \( K_1 \) – cornering stiffness of the front axle [N/rad]
- \( K_2 \) – cornering stiffness of the front axle [N/rad]
- \( r \) – yaw angle [rad]

The schematic of the model is shown in Fig 5.

Figure 5. Simplified single-track model used for simulation

In the conducted simulations, the influence of the torque vectoring was also studied. For simulating the torque vectoring, an additional yaw torque was taken into account. The additional torque depends on the distribution of drive forces between the left and right wheel of the vehicle. It is generated by the action of the unbalanced driving forces each on the arm of the half-track width.

Matlab 2014 software has been selected for the motion simulation. The geometric and mass parameters of the vehicle, the previously determined [14] cornering stiffness for the front and rear axles were used in the bicycle model. The input to the model was the steering angle as a function of time. Steering angle function obtained from real road tests with the vehicle.

The following test apparatus was installed in the vehicle during tests:
- Datron Correvit apparatus measuring the longitudinal and lateral velocities of the vehicle
- Crossbow measuring velocities and lateral, longitudinal and rolling accelerations
- Kübler cable sensor - the steering wheel.

The test object is equipped with apparatus (Fig. 6).
The simulations included a single turn and single lane change performed at different driving velocities and various drive torque distribution between the wheels. Three states of torque distribution were taken into account:

- Equal torque distribution between drive wheels
- Drive only on the outer wheel in relation to the travelled arc
- Drive only on the inner wheel in relation to the travelled arch

The result of the simulation is the lateral movement of a vehicle in local CS, yaw angle and yaw rate as a function of time. The obtained data of the yaw rate was compared with the values measured during the tests. In order to determine the suitability of the model, a correlation coefficient was determined (PCC - Pearson’s Correlation Coefficient) to describe the vehicle movement. The Figures 7., 8. and 9. show simulation result examples.

**Figure. 6.** Tested microcar equipped with apparatus during tests.

**Figure. 7.** Single lane change with an equal torque distribution. Some perturbations at measured value around 1.2 s caused by steering angle sensor failure.
Figure 8. Single lane change with only outer wheel driven. Additional yaw torque included

Figure 9. Single lane change with only inner wheel driven. Additional yaw torque included

The above simulations coincide with the results obtained during the tests. The comparable vehicle response during simulation and road tests are the basis for considering the usage of a bicycle model to predict the behaviour of the vehicle during the movement. The anticipated behaviour of the vehicle can be used for activating the vehicle's active safety systems.

5. Conclusions
A. The single-track model can be used to simulate atypical vehicle - a narrow car. The narrow car dimensions differ significantly from passenger vehicles with the usual wheelbase and track width, where the bicycle model is successfully used to control active safety systems. The simple, linear model can be used to simulate narrow car motion. In a vehicle of this type there is a significant change in normal load on specific wheels during turning. Changing the normal load causes a change in the ability of tangential forces transfer.
B. The results of the simulation carried out indicate the possibility of changing the understeer characteristics of narrow vehicles by varying the torque distribution

C. The narrow cars are very sensitive to torque vectoring. Conducted tests have shown that torque vectoring has a huge influence on the characteristics of steering. Change from understeer to even oversteering. It is absolutely necessary to use a complex drive force distribution system between the vehicle wheels. Changing the characteristics in critical situations can prevent an accident.

The torque vectoring impact can be simulated by an additional torque that acts on the vehicle in the bicycle model.

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