Design of bevel gears using accelerated particle swarm optimization technique

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Abstract. Straight bevel gears are simple type of bevel gears with their teeth being straight and tapered. They transmit power between shafts which are perpendicular with each other. In various applications of bevel gears, the issue of minimization of volume of gears is of immense importance for the designers, since it leads to reduced power consumption and material requirement for manufacturing. In this study, Accelerated Particle Swarm Optimization (APSO) is used to minimize the volume of straight bevel gears and optimized design variables are obtained. The design variables employed in the optimization procedure are module, face width and number of teeth. A design problem is considered for design optimization of bevel gear using APSO and obtained results are compared with the results produced by genetic algorithm (GA) and simulated annealing (SA) from previous literature. Obtained results show that there is improvement in the volume attained using APSO.

1. Introduction
Designing a gear drive is an intricate task which involves various factors such as gear geometry, heat treatment, manufacturing etc. in order to satisfy different practical considerations of high strength, high accuracy, low noise and small size. Several approaches are used by the researchers and designers in order to improve the gear design with respect to weight, volume, life, noise etc.

To obtain optimum gear design, numerous optimization techniques starting from traditional one to meta-heuristic have been used by the researchers in recent times. Golabi et al. [1] used non-linear programming technique to minimize the volume of one-, two- and three- stage gear train. Moving towards application of meta-heuristic optimization technique, Gologlu and Zeyveli [2] illustrated the use of Genetic Algorithm (GA) for finding minimum volume of two stage helical gearbox. Zhang et al. [3] proposed design optimization software based on genetic algorithm for obtaining optimized bevel gear design. Tudose et al. [4] used genetic optimization for performing optimization on two-stage co-axial helical gearbox. To obtain low weight and energy efficient helical gear pair, Tamboli et al. [5] used particle swarm optimization to minimize volume of a helical gear pair. In recent times, Zolfaghari et al. [6] designed a straight bevel gear using genetic algorithm (GA) and simulated annealing (SA). The results obtained from these two algorithms are then compared with traditional algorithm (TA). Panda et al. [7] introduced differential evolution algorithm for minimizing the weight of spur gear. Using obtained gear geometry parameters, a CAD model has been developed and stress analysis has been done using ANSYS. Straight bevel gears are utilized in transmission system where the axes intersect at 90°. Thus, they find applications where low velocity and production cost are desired [8]. Despite of this, availability of literature related to bevel gear volume optimization is
limited. There are various PSO variants and hybrid algorithms, developed by combining other existing algorithms, which are increasingly popular and able to solve many design problems. But APSO is much simpler as it utilizes only two parameters and has faster convergence [9]. In this paper, volume of bevel gear is optimized using APSO and optimum design values are obtained. All the formulas and instructions are as per AGMA standards. A design example has been taken from literature [6] for optimization. The obtained results are validated by comparing it with previously published results.

Rest of the paper is organized in following manner. The next section consists of explanation of optimization algorithm i.e. APSO. In section three, mathematical modelling is done. A design example has been presented in section four. Simulation results of above problem formulated are reported in Section 5. Finally, conclusion is drawn in Section 6.

2. Accelerated Particle Swarm Optimization

Global best and individual best position of particles both are used in standard particle swarm optimization problem [10]. When individual best are used, found to be diversity in the quality solution; however this diversity can be simulated using randomness. Consequently, there is no compelling condition to use individual best, until the objective function is very complex and non-linear in nature. By using global best, the algorithm can be accelerated to converge. A simple formula is employed to generate velocity vector, with $x_i^t$ and $v_i^t$ as the current position and velocity vector respectively, and is given by

$$v_i^{t+1} = v_i^t + \alpha \epsilon_n + \beta (G^* - x_i^t)$$

(1)

where $\epsilon_1$ and $\epsilon_2$ are the random numbers between 0 and 1. Here, $\alpha$ and $\beta$ are the acceleration constant. The expression to update the position is same as used in standard PSO. To enhance the convergence even more, the update of position can be written as

$$x_i^{t+1} = (1 - \beta)x_i^t + \beta G^* + \alpha \epsilon_n$$

(2)

Thus APSO based optimization technique is quiet simple and results in quick convergence [9].

3. Mathematical Modelling

In this study, minimizing the volume of bevel gears is treated as objective function for the design optimization problem. For formulating the mathematical model, objective function, design variables and design constraints are required. In the following section, brief discussion has been made on problem formulation.

3.1 Design Variables

Design variables considered for optimization procedure are module, pinion teeth number and face width of gear. Description of these variables with their minimum and maximum bounds is given in Table 1.

| Table 1. Description of Design variables |
|------------------------------------------|
| Design Variables | Min. Value | Max. value |
|------------------|------------|------------|
| Module (m)       | 1          | 8          |
| Pinion teeth number ($z_1$) | 14         | 60         |
| Face width (b)   | 10         | 60         |

3.2 Formulation of Objective function

Volume is regarded as one the significant criteria for a gear reducer. Therefore, minimization of volume of bevel gear is considered as the objective function and expressed as.

$$V = \frac{\pi}{3} b \cos \delta_1 \left[ \left( \frac{mz_1}{2} \right)^2 + \left( \frac{mz_1}{2} \left( \frac{r-b}{2} \right) \right)^2 + \left( \frac{mz_1}{2} \left( \frac{r-b}{2} \right) \right)^2 \right]$$
\[ + \frac{z}{2} b \cos \delta_2 \left[ (\frac{m_{xz}}{2})^2 + \left( \frac{m_{xz}}{2} \right) \left( \frac{r-b}{2} \right) \left( \frac{m_{yz}}{2} \right) + \left( \frac{m_{yz}}{2} \right) \left( \frac{r-b}{2} \right)^2 \right] \] (3)

where \( \delta_1 \) and \( \delta_2 \) the cone angle for pinion and gear respectively and \( r \) is called cone pitch.

3.3 Constraints

Various design constraints are required for satisfying the practical demands such as bending strength, pitting resistance and linear velocity of gear at outer diameter of the design problem respectively. These established design constraints are expressed as shown below

\[ g_1(x) = Z_{y} W_{i} K_{r} K_{h} K_{\phi} Z_{x}^2 S_{w} Z_{x} c - (\sigma_{allow} Z_{NT} Z_{w})^2 bdZ_i \leq 0 \] (4)

\[ g_2(x) = K_{r} W_{i} K_{h} K_{\phi} Y_{S} S_{F} - \sigma_{allow} Y_{NT} Z_{i} b m_{et} Y_{P} \leq 0 \] (5)

\[ g_3(x) = 1047.2 \left(10^{-6}\right)m_{z} Z_{i} n_{i} - \left(50 + 56(1 - 0.25(12 - Q_{v})^{2/3}) + (Q_{v} - 3)\right)^2 \leq 0 \] (6)

\[ g_4(x) = b \leq \left(0.3A_{m}, 10m_{et}\right) \] (7)

where \( K_{r}, K_{h}, K_{\phi}, K_{G} \) is the dynamic, load distribution, temperature and overload factor respectively. \( W_{i} \) is the transmitted load. Here \( S_{F} \) and \( S_{P} \) is safety factor for bending and contact and \( m_{et} \) is the outer transverse module. Stress cycle factor for bending and pitting is denoted by \( Y_{NT} \) and \( Z_{NT} \) respectively and \( Y_{S} \) and \( Z_{S} \) is the size factor bending and pitting resistance respectively. \( Y_{P}, Z_{w}, Z_{xc} \) and \( Z_{x} \) length-wise curvature, hardness, crown and reliability factor respectively for bevel gears. \( Z_{i} \) and \( Z_{g} \) are bending geometry factor and elastic coefficient for pitting respectively. All the constants used in above inequalities are explained and illustrated in [6] and [8]. To prevent undercut in straight bevel gears, minimum number of teeth required are 14. Furthermore, based on literature [6], following equalities are considered in order to avoid undercutting.

\[ g_{eq1}(x) = A_{ig} = A_{m} - 0.5b \] (8)

\[ g_{eq2}(x) = \psi_{ig} = 0 \] (9)

\[ g_{eq3}(x) = \delta_{p} = \delta_{G} \] (10)

\[ g_{eq4}(x) = \tan \varphi_{ti} = \tan \varphi \] (11)

\[ g_{eq5}(x) = b_{lip} = A_{ig} \tan \gamma \sin^{2} \varphi \] (12)

\[ g_{eq6}(x) = b_{p} = b_{p} \] (13)

Here \( A_{ig} \) and \( A_{m} \) are gear cone distance and mean cone distance (mm) respectively. \( \psi_{ig} \) represents the inner gear spiral angle. \( \varphi_{ti} \) and \( \varphi \) denote the transverse and normal pressure angle respectively. \( b_{lip}, b_{p} \) and \( b_{p} \) is the pinion limit inner dedendum, pinion mean inner dedendum and pinion inner dedendum respectively. \( \delta_{p} \) and \( \delta_{G} \) is pinion and gear dedendum angle respectively.

4. Design Problem

In this study, the optimal design values of bevel gear have been achieved using APSO in MATLAB. For designing bevel gear pair, various design parameters and working condition required for optimization of bevel gears are shown in Table 2.

| Table 2. Various design parameters used in design optimization problem [6] |
|----------------------------------|------------------|
| Design Parameters               | Values           |
| Power transmitted (kW)          | 5                |
| Reduction ratio                 | 1:1              |
| Input speed (rpm)               | 900              |
| Load on prime mover             | Medium shock     |
| Contact safety factor           | 1.2              |
| Bending safety factor, \( \sigma_{allow} \) | 1.2       |
| Pressure angle, \( \varphi \)   | 20°              |
| Material                         | Steel (Grade II) |
| Hardness (HB) (Gear/Pinion)     | 350              |
| Transmission Accuracy number, \( Q_{v} \) | 11        |
| Life (No. of cycles)            | 10^7             |
Load of driven machine | Medium shock
--- | ---
Reliability | 0.99
Installation condition | Both members straddle mounted
Gear/pinion shape | Properly crowned teeth
Tooth form | Full depth

5. Results and Discussion
To produce optimum results, APSO algorithm has been coded in MATLAB. Maximum number of iterations is taken as stopping criteria for optimization. The typical values of acceleration constants are assumed to lie between: \( \alpha \approx 0.1 \) to 0.4 and \( \beta \approx 0.1 \) to 0.7. Description of various parameter used in APSO are reported in Table 3.

Table 3. Various APSO parameters used for optimization of bevel gears

| APSO parameters | Values |
|-----------------|--------|
| Population size | 20     |
| Iterations      | 150    |
| Time steps      | 250    |
| \( \alpha \)     | 0.2    |
| \( \beta \)      | 0.5    |

A comparison of results has been shown in Table 4. The effectiveness of transmitted power on the volume of straight bevel gear is highlighted in the table. Convergence curve for volume vs iterations is shown in Figure 1. As demonstrated in Table 4, APSO led to improvement in gear volume from 1.12\% to 29.9\% for all transmitted power (Figure 2). The face width values are increasing from 5 kW power to 30.65 kW power, but the obtained values are smaller in comparison with the earlier values obtained from other algorithms. This will help in quiet, smooth and reliable gear mesh action. Module of bevel gear increases with increase in transmitted power value. Finally, the numbers of teeth increases and then decreases as power transmitted varies from 5 kW to 30.65 kW (Figure 3).

Table 4. Comparison of results for APSO with TA, GA and SA [6]

| Power(kW) | 5 | 10.65 | 30.65 |
|-----------|---|-------|-------|
| Design Variables | TA | GA | SA | APSO | TA | GA | SA | APSO | TA | GA | SA | APSO |
| Module, m | 3.5 | 2.75 | 4.0 | | 4.5 | 5.5 | 4.5 | 4.5 | 5.5 | 5.5 | 5.5 | 5.5 |
| No. of teeth, \((z_1/z_2)\) | 25 | 20 | 25 | 25 | | 30 | 15 | 20 | 27 | 35 | 20 | 30 | 20 |
| Face width, (mm) | 19.2 | 27.3 | 18.5 | 15.86 | | 23.1 | 26.6 | 22.8 | 17.76 | 26.9 | 46.02 | 38.2 | 22.47 |
| Volume, dm³ | 69.8 | 63.3 | 68.7 | 51.06 | | 206.8 | 158.9 | 170.8 | 144.9 | 502 | 453.6 | 453.8 | 448.59 |
| % Improve. by APSO | 26.8 | 19.3 | 25.6 | _ | 29.9 | 8.8 | 15.1 | _ | 10.6 | 1.12 | 1.16 | _ |

Figure 1. Convergence curves for Fitness Function vs iterations
Figure 2. Optimized volume of bevel gear compared for TA, GA, SA and APSO

Figure 3. Variation of No. of teeth vs transmitted power for TA, GA, SA and APSO

6. Conclusion
In this study, accelerated particle swarm optimization is used for volume optimization of bevel gears. The optimal gear volume achieved using APSO is found to be better than earlier used optimization techniques (TA, GA and SA). It has been found that the values of face width are smaller in comparison to previously published work. This will make gears less sensitive to errors of manufacturing and misalignment. There is slight change in value of gear module using APSO. Number of teeth obtained from APSO is greater than that obtained from GA, but less for SA. As the value of transmitted power changes, the percentage of reduction in volume of bevel gear when compared to other algorithms lie in between 1.12% to 29.9%. Hence the achieved results show that proposed APSO approach surpasses the other approaches in minimizing volume of bevel gears. This will reduce material requirement for manufacturing which will further reduce the manufacturing cost.

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