The yield of $\Upsilon(1S)$ transverse momentum spectra through dissociation and regeneration in heavy-ion collisions

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We calculate the transition between a quarkonium state and an unbound heavy quark-antiquark pair through gluo-dissociation and inelastic parton scattering using a partonic picture that interpolates between the formal limits based on potential nonrelativistic QCD (pNRQCD) at different temperatures. While the thermal width increases with momentum and temperature, the quarkonium regeneration is affected by the heavy quark distribution function which depends on the diffusion constant. By solving the Boltzmann equation with the dissociation and regeneration terms, we investigate the medium modifications of quarkonium momentum spectra.

Our numerical results indicate that the $\Upsilon(1S)$ $R_{AA}$ at high transverse momentum are influenced by the regeneration effects depending on the heavy quark diffusion. In this picture, the published CMS data that show an almost transverse momentum independence can be explained by the interplay between the suppression by dissociation and enhancement by regeneration at low and high transverse momenta, respectively.

With the same input, we also calculate the transverse momentum dependence of the $\Upsilon(1S) v_2$ and show that it lies within the limits of the available data.

I. INTRODUCTION

As an important signal of the quark-gluon plasma formation \cite{1}, quarkonium suppression has attracted much theoretical and experimental attention in heavy quarkonium physics. Although the yields of heavy quarkonia are reduced by the static color screening and dissociation by in-medium interactions, quarkonia states are known to survive above the phase transition temperature \cite{2} and can even be regenerated by recombination of a heavy quark and antiquark. The quarkonium enhancement by regeneration not only increases the number of bound states but also affects their momentum spectra.

Quarkonium dissociation has been investigated over the years \cite{3,4}, whereas its regeneration is less discussed and often ignored due to the low density of heavy quarks produced in heavy-ion collisions, especially for bottom quarks. The statistical hadronization and coalescence models have been used for $J/\Psi$ production \cite{5,6}. However, quarkonium momentum spectra are more sensitive to the regeneration mechanism than the total yields, and furthermore the centrality and energy-momentum dependences of the enhanced quarkonium distribution are difficult to predict within a statistical model or other models based on hadronic processes \cite{7}.

In addition to the production near the phase transition, we need to consider how the number and momentum distributions of the initially produced quarkonia and heavy quarks change by dissociation and regeneration during the evolution of the quark-gluon plasma. By treating both the thermal medium and quarkonium distributions dynamically, kinetic models with a rate or transport equation have been utilized to describe continuous regeneration of quarkonium through the quark-gluon plasma phase \cite{7-10}.

The yield of $J/\Psi$ by recombination has been shown to be considerable, but the significance of bottomonium regeneration is problematic. This is so because the regeneration by recombining $b$ and $\bar{b}$ seems to be negligible as the density of bottom quarks and cross sections for open bottom are much smaller than those for charm quarks. On the other hand, if the transverse momentum dependent ratio of the bottomonium number to the number of $b$ quarks is much smaller than that of the corresponding values for the $c$ quarks, bottomonium regeneration might be relatively significant \cite{11,12}, especially in high energy heavy-ion collisions at $\sqrt{s_{NN}} = 5.02$ TeV at the LHC.

In a partonic picture, quarkonium dissociation occurs through two scattering processes, gluo-dissociation ($\Upsilon + g \to b + \bar{b}$, where $\Upsilon$ is bottomonium) and inelastic parton scattering ($\Upsilon + p \to b + \bar{b} + p$ with $p = g, q, \bar{q}$). The dipole interaction of color charges with gluon \cite{13} is used at leading order ($\sigma_{LO} \sim g^2 a_0^2$) for the first process and at next-to-leading order ($\sigma_{NLO} \sim g^4 a_0^2$) for the second process. By using the Bethe-Salpeter amplitude and hard thermal loop (HTL) perturbation theory, we have recently rederived the next-to-leading order dissociation cross section \cite{14}. In the relevant kinematical limit, our results reduce to the formal limits obtained by potential nonrelativistic QCD (pNRQCD) at which the thermal width is determined by the imaginary part of the singlet potential \cite{15}.

The inverse reactions of gluo-dissociation and inelastic parton scattering contribute to the quarkonium regeneration. In order to describe the regeneration, the detailed balance condition or a coalescence model has been used in the kinetic approaches mentioned above. The detailed balance allows one to describe the quarkonium regeneration in terms of the dissociation process and the equilibrium distributions. It is valid only if quarkonium and heavy quarks are near equilibrium, which is not the case for the quark-gluon plasma phase. On the other hand, a coalescence model describes instantaneous regeneration regardless of specific reactions. It would be more appropriate to calculate the regeneration continuously by parton-quarkonium interactions in the same approximation used for the dissociation. In this work, to de-
scribe the dissociation and regeneration mechanisms consistently, we use a Boltzmann transport equation with the collision terms which are obtained by the scattering amplitudes of the two processes, convoluting the momentum distributions of incoming and outgoing particles [16, 17]. In this way, the regeneration term will depend on heavy quark distribution functions, and the momentum spectra of quarkonium will reflect the evolution of initial heavy quark spectra with modifications by a thermal medium.

We concentrate on bottomonium because our numerical approach based on the nonrelativistic heavy quark limit is more suitable for bottomonium than charmonium. Especially, Υ(1S) survives up to ~ 600 MeV [18] so that it is crucial to analyze the evolution of the distribution over a wide temperature range within a consistent formalism. This can be accomplished within our approach, as our previously derived partonic formula consistently interpolates the constraints imposed by the pNRQCD formalism at high and low temperature limits. The nuclear modification factor of Υ(1S) has been measured in PbPb collisions at $\sqrt{s_{NN}} = 2.76, 5.02$ TeV by the CMS collaboration [19, 20], and the elliptic flow at $\sqrt{s_{NN}} = 5.02$ TeV by the ALICE and CMS collaborations [21, 22]. The Υ(1S) $R_{AA}$ appears to be independent of the transverse momentum. The experimental data have been compared either with [12] or without [23] the regeneration of Υ(1S) within kinetic models, but the regeneration effects in Ref. [12] are estimated by a coalescence model. The centrality and rapidity dependence of Υ(1S) $R_{AA}$ measured in pPb [24–26] and PbPb collisions have been reproduced in Ref. [27].

Our goal is to understand how the dissociation and regeneration mechanisms influence on quarkonia momentum spectra, and eventually on the nuclear modification factor and the elliptic flow, using a consistent formalism that interpolates between the formal limits provided by the pNRQCD constraints. In Sec. II A, we extend our previous work on the thermal width [14] for quarkonium moving in the plasma. By taking the inverse reactions of gluo-dissociation and inelastic parton scattering, we calculate the regeneration term of the Boltzmann equation in Sec. II B. The regeneration term involves heavy quark distribution functions which can be characterized by a Fokker-Planck equation with a diffusion constant. In Sec. III A, the Boltzmann equation with the dissociation and regeneration terms is solved for a Bjorken expansion at mid-rapidity. The numerical solutions for Υ(1S) are used to determine the nuclear modification factor in Sec. III B and the elliptic flow in Appendix A. Sec. III C is devoted to the discussion about the uncertainties regarding initial conditions when comparing with experimental data. Finally, we summarize our results in Sec. IV.

II. DISSOCIATION AND REGENERATION OF QUARKONIUM MOVING IN THE QUARK-GLUON PLASMA

The dynamic evolution of a quarkonium state can be described by dissociation, regeneration, and elastic scatterings. When the inverse distance between a heavy quark and its antiquark is larger than the temperature scale, elastic collisions are of higher order than dissociation and regeneration [17]. Thus, neglecting this term, the Boltzmann equation for quarkonium is given by [16]

$$ \left( \frac{\partial}{\partial t} + v \cdot \frac{\partial}{\partial x} \right) f_T(t, x, q) = -\Gamma_{\text{diss}}(t, x, q) + \Gamma_{\text{reg}}^{\text{diss}}(t, x, q), $$

(1)

where $v = q/q_0$ is the quarkonium velocity, and the distribution functions $f_T$ and $f_{b,\bar{b}}$ contain the fugacity factors of $\gamma_b^2$ and $\gamma_b$, respectively. The first term on the right hand side corresponds to the dissociation term with the thermal width $\Gamma_{\text{diss}}$, and the second to the regeneration term which depends on the heavy quark and antiquark distribution functions.

A. The thermal width

In Ref. [14], we have calculated the thermal width in the rest frame of quarkonium and integrated the phase space numerically for a phenomenological study. To extend this for quarkonium moving in the plasma, we proceed as follows: First, we calculate the scattering amplitudes (or dissociation cross sections) in the rest frame of quarkonium. Second, medium partons ($g, q, \bar{q}$) are considered to move with respect to quarkonium. Then their thermal distributions are

$$ f(t, x, q, k) = 1/[\gamma(\mathbf{p}^2 - k^2) + T + i \epsilon], $$

where $\gamma = 1/\sqrt{1 - \mathbf{v}^2}$ is the Lorentz factor. Third, the thermal width is obtained by convoluting the momentum distributions of moving partons with the scattering cross sections. Lastly, we divide the thermal width by the Lorentz factor to determine the width in the rest frame of the plasma [28].

We have used the hard thermal loop (HTL) perturbation theory with an effective vertex derived from the Bethe-Salpeter amplitude to calculate the dissociation cross sections [14]. The effective vertex describes the dipole interaction of color charge with gluon in the large $N_c$ limit (where the interaction between heavy quark and antiquark after dissociation can be neglected). For gluo-dissociation and inelastic parton scattering, the matrix elements have been obtained as

$$ |M|_{\text{diss}}^2 = \frac{8(N_c^2 - 1)}{N_c} g^2 m_T k_0^2 |\nabla \psi(p)|^2, $$$$ |M|_{\text{inel}}^2 = 16(N_c^2 - 1) g^4 m_T |\nabla \psi(p)|^2 \times \frac{(k_1 - k_2)^2 k_0^2}{[(k_1 - k_2)^2 + m_D^2]^2} \left[ 1 + (k_1 + k_2)^2 \right] (g, \bar{g}), $$$$ (2)
for a Coulombic bound state, $|\nabla \psi(p)|^2 = 2^{10} \pi a_s^2 p^2 /[m^2 + 1]^6$, with the relative momentum $p = (p_1 - p_2)/2$. The Debye screening mass depends on temperature as $m_D^2 = \frac{2^{2/3} T}{\pi^2} (N_c + N_f)$. Following the above procedure, the thermal widths in the rest frame of the quark-gluon plasma are then given by

$$\Gamma_{\text{diss}}^{\text{inel}}(t, x, q) = \frac{1}{2dT^0} \int \frac{d^3 k}{(2\pi)^3 2k^0} \int \frac{d^3 p_1}{(2\pi)^3 2p_1^0} \int \frac{d^3 p_2}{(2\pi)^3 2p_2^0} |M|_{\text{inel}}^2 \delta^4(Q + K_1 - K_2 - P_1 - P_2) f(t, x, k_1) \left[ 1 \pm f(t, x, k_2) \right],$$

where $Q, K, P_1, P_2$ denote the momenta of quarkonium, medium partons, heavy quark and heavy antiquark, respectively ($q, p_1, p_2 \gg k$). In inelastic parton scattering, the thermal distribution of an outgoing parton has the momentum $k_2 \simeq k_1$ for small energy transfer, where 1, 2 refer to incoming and outgoing momenta, respectively.

Figure 1 shows the numerical results for the $\Upsilon(1S)$ thermal widths which depend on the momentum. We have used the same values of the parameters as Ref. [14]: $\alpha_s = 0.4, a_0 = 0.14$ fm, $m = 4.8$ GeV, and the binding energy $E$ estimated in lattice QCD [29]. The thermal width of inelastic parton scattering increases with momentum but the width of gluo-dissociation decreases. This behavior comes from the Lorentz factor and the thermal distributions of partons involved in the width. Specifically, the factor of $f(t, x, k_1) [1 \pm f(t, x, k_2)]/\gamma$ increases with the quarkonium momentum especially for high $k_1$ where inelastic parton scattering is dominant. Furthermore, the inelastic scattering cross section increases with $k_1$. For low $k$, gluo-dissociation becomes effective and the factor of $f(t, x, k)/\gamma$ decreases as the $\Upsilon(1S)$ momentum grows. These behaviors toward momentum are reflected in the thermal width as seen in Fig. 1 (a). For low temperature at which the binding energy is larger than the Debye mass, gluo-dissociation is effective especially for quarkonium at rest. On the other hand, inelastic parton scattering is dominant over gluo-dissociation at high temperature where $m_D \gg E$, as in pNRQCD [15]. Since the momentum dependence of inelastic parton scattering is stronger than that of gluo-dissociation, the sum of two contributions grows with the momentum (see Fig. 1 (b)). Our numerical results of the thermal width qualitatively agree with those of Ref. [28].

![FIG. 1. The momentum dependence of the $\Upsilon(1S)$ thermal width as a function of the plasma temperature. (a) The comparison between gluo-dissociation and inelastic parton scattering. (b) The total thermal width as the sum of two processes.](image-url)

**B. The regeneration term**

The regeneration terms in the Boltzmann equation are obtained by the inverse reactions of gluo-dissociation and inelastic parton scattering,

$$C_{\text{reg}}^{\text{gluo}}(t, x, q) = \frac{1}{2dT^0} \int \frac{d^3 k}{(2\pi)^3 2k^0} \int \frac{d^3 p_1}{(2\pi)^3 2p_1^0} \int \frac{d^3 p_2}{(2\pi)^3 2p_2^0} |M|_{\text{gluo}}^2 \delta^4(Q + K_1 - K_2 - P_1 - P_2) f_b(t, x, p_1) f_b(t, x, p_2) [1 + f(t, x, k)],$$

$$C_{\text{reg}}^{\text{inel}}(t, x, q) = \frac{1}{2dT^0} \int \frac{d^3 k_1}{(2\pi)^3 2k_1^0} \int \frac{d^3 k_2}{(2\pi)^3 2k_2^0} |M|_{\text{inel}}^2 \delta^4(Q + K_1 - K_2 - P_1 - P_2) f_b(t, x, p_1) f_b(t, x, p_2) [1 + f(t, x, k)],$$
\[ (2\pi)^4 \delta^4(Q + K_1 - K_2 - P_1 - P_2) \times f_b(t, x, p_1) f_b(t, x, p_2) f(t, x, k_2) [1 \pm f(t, x, k_1)] \], \quad (4) 

where the phase space integrations are numerically performed as done in the thermal width calculation [14]. The momenta of partons and heavy quarks are approximated as \( k_1 \approx k_2 \) and \( p_1 = -p_2 \approx p \) in the rest frame of quarkonium, respectively.

Through the regeneration term, the momentum spectrum of bottomonium reflects the initial \( b, \bar{b} \) spectra which themselves evolve by interacting with partons in a thermal medium. Bottom quarks are produced with a power-law transverse momentum spectrum which is harder than the thermal one. The approach to the thermal equilibrium is slower than that of light quarks by a factor \(~\sim m/T\) [30], and thus they are not likely to be thermalized in the quark-gluon plasma phase. In principle, the \( b \) distribution function should be obtained by solving a Boltzmann equation, but we use a simpler Fokker-Planck equation which characterizes the time evolution of \( b \) quarks.

To determine the \( b \) distribution function, we consider elastic scatterings, \( p + b \rightarrow p + b \). In the limit of small energy transfer, the \( t \)-channel gluon exchange is dominant, and the collision term at leading-log in \( T/m_D \) can be approximated as a Fokker-Planck operator [30–32]. For a uniform plasma, we then have

\[
\frac{\partial f_b(t, p)}{\partial t} = \left[ \frac{\partial}{\partial p} \cdot \eta(t) p + \frac{1}{2} \frac{\partial^2}{\partial p^2} \kappa(t) \right] f_b(t, p), \quad (5)
\]

where \( \eta(t) \) is the drag coefficient and \( \kappa(t) \) is the momentum diffusion constant. By ignoring the momentum dependences of drag and diffusion, (when the heavy quark velocity vanishes) \( \kappa(t) = 2mT\eta(t) \). The momentum diffusion constant is related to the diffusion constant in space, \( D(t) = 2T^2/\kappa(t) \). Then \( f_b(t, p) \) can be obtained from the initial condition \( f_b(t_0, p_0) \) by convolving the Green’s function as follows [30, 32]:

\[
f_b(t, p) = \int d^3 p_0 G(t, p; t_0, p_0) f_b(t_0, p_0), \quad (6)
\]

where

\[
G(t, p; t_0, p_0) = \frac{1}{[4\pi K(t)]^{3/2}} \exp \left[ -\frac{(p - p_0 e^{-H(t)})^2}{4K(t)} \right], \quad (7)
\]

with \( H(t) = \int_{t_0}^{t} dt' \eta(t') \) and \( K(t) = \frac{1}{2} e^{-2H(t)} \int_{t_0}^{t} dt' \kappa(t') e^{2H(t')} \).

All the \( b \) quarks are expected to be produced in initial hard collisions. The fugacity factor \( \gamma_b(t) \) is determined by requiring that the total number of \( b \) quarks is constant throughout the quark-gluon plasma phase,

\[
N_b(t) = d_b \gamma_b(t) V(t) \int d^3 p (2\pi)^3 f_b(t, p), \quad (8)
\]

where \( d_b \) is the degeneracy factor of \( b \) quarks. Because the production cross section of a hidden bottom state is much smaller than that of \( \bar{b}b \) pairs (\( \sigma_{b\bar{b}} \sim 10^{-3} \) [12], we ignore the \( \Upsilon \) contribution to \( N_b(t) \). At the LHC and RHIC, at most a few \( b, \bar{b} \) pairs are produced so we present the numerical results for the case \( N_b(t) = 1 \) in the following.

At early times of heavy-ion collisions, the transverse dimension of a central collision system is so large that the dynamics is dominated by a longitudinal motion. We suppose a thermal medium to be in a local equilibrium at time \( t_0 \) with temperature \( T_0 \). For a Bjorken expansion [33], the time dependences of drag, temperature and volume are given by \( \eta(t) = \eta_0(t_0/t)^2/3 \), \( T(t) = T_0(t_0/t)^1/3 \), and \( V(t) = V_0 t/t_0 \), respectively, where we set \( T_0 = 550 \) MeV and \( V_0 = 60 \) fm$^3$ at \( t_0 = 0.3 \) fm/c. Since there are neither much data nor theoretical calculations for \( b \) quarks, for an initial distribution we use the \( B \) meson differential cross section measured in pp collisions [34] as a starting point (the uncertainties related to the ini-

FIG. 2. (a) The time evolution of the \( b \) quark distribution in momentum space. (b) The dependence on the diffusion constant.
FIG. 3. The momentum dependence of the $Y(1S)$ regeneration term in the Boltzmann equation, Eq. (1). (a) The comparison of the inverse gluo-dissociation with inelastic parton scattering. (b) The sum of two contributions.

![Graph 3a](image3a.png)

![Graph 3b](image3b.png)

FIG. 4. The regeneration term depending on the $b$ quark diffusion constant.

![Graph 4](image4.png)

The regeneration term exhibits a similar momentum dependence to the thermal width case, because the factor $f(t, x, k_2)[1 \pm f(t, x, k_1)]/\gamma$ or $[1 + f(t, x, k)]/\gamma$ acts in the same way as discussed in Sec. II A. However, the regeneration effects by the inverse gluo-dissociation can be rather strong at low temperature, especially when the heavy quark diffusion is small. Figure 4 shows the $D$ dependence of the regeneration term by two mechanisms. A less diffuse $f_b(t, p)$ makes the $Y(1S)$ regeneration more probable, and the regeneration through the inverse gluo-dissociation is more influenced by the diffusion constant. The heavy quark diffusion has scanty effect on the regeneration by inelastic parton scattering, because the process is important only at high temperature where the distribution of $b, \bar{b}$ is close to the initial input.

Strong drag can lead to the enhanced production of a bound state, whereas fast diffusion should decrease quarkonium yields [31]. The drag and diffusion constants are related to the equilibration rate of heavy quarks and affect the regeneration process. For smaller $\eta$ and larger $D$, the equilibration is slower and the regeneration effects are weaker. Because the numerically calculated distribution of $b, \bar{b}$ are harder than the thermal distribution, the regeneration term presented here is smaller than that calculated with the thermal distributions.

III. MEDIUM MODIFICATIONS OF QUARKONIUM TRANSVERSE MOMENTUM SPECTRA

In the previous section, we have obtained the thermal width and the regeneration term which depend only on time and momentum. Now, we apply it in the central rapidity region with the Lorentz invariance under a longitudinal boost. Since the right hand side of Eq. (1) is independent of space, taking the average over $x_T$ yields
the transverse momentum spectrum, $f_T(t, q_T)$:
\[
\frac{\partial f_T(t, q_T)}{\partial t} = -\Gamma_{\text{diss}}(t, q_T) f_T(t, q_T) + C_{\text{reg}}(t, q_T) \, . \tag{9}
\]
We are interested in the medium modifications of $R_{AA}(q_T)$ and the elliptic flow $v_2(q_T)$ (see Appendix A) which are induced by the momentum-dependent dissociation and regeneration. Because the quarkonium regeneration takes place only below the dissociation temperature $T_{\text{diss}}$ at which the thermal width becomes comparable to or exceeds the reduced binding energy, we consider the quark-gluon plasma phase within the temperature range $T_c \lesssim T \lesssim T_{\text{diss}}$.

A. The $\Upsilon(1S)$ distribution function

The quarkonium distribution function is obtained by solving Eq. (9) numerically for each momentum. As with $b$ quarks, the initial number and momentum distribution of $\Upsilon(1S)$ are not well known. For an initial condition, we use the differential cross section for $\Upsilon(1S)$ as a function of its transverse momentum and per unit rapidity measured in pp collisions [20], normalized by $N_\Upsilon(0) / N_\Upsilon(0) = 1.76 \times 10^{-3}$ [12].

As a first step, we can ignore the regeneration term because bottomonium regeneration depends on the densities of $b, \bar{b}$ quarks which are fairly small. Then the distribution function has an exponential form,
\[
f_{\Upsilon}^{\text{diss}}(t, q_T) = f_{\Upsilon}^{\text{diss}}(t_0, q_T) e^{-\int_{t_0}^{t} \Gamma_{\text{diss}}(t', q_T) dt'}, \tag{10}
\]
as denoted by the dashed lines in Fig. 5 (a). If we include the regeneration term, the suppression is slightly reduced (see the solid lines). At higher momentum where the initial number is smaller, the regeneration effects are more prominent. From Figs. 1 (b), 3 (b), and 5 (a), we note that $C_{\text{reg}} / f_T \sim 10$ MeV at $q_T = 15$ GeV but $\Gamma_{\text{diss}} \sim 100$ MeV. Thus, using these numbers in the first and second terms of Eq. (9), one expects that the regeneration effects are at most $\sim 10\%$ compared to the dissociation contribution. In Fig. 5 (b), we exhibit the time evolution of the $\Upsilon(1S)$ momentum distribution. The accumulated dissociation effect increases with time but its change decreases, so that the numerical solution freezes around $t \approx 7$ fm/c at the end of the quark-gluon plasma phase. Since the bottomonium dissociation is dominant over regeneration, the number of $\Upsilon(1S)$ is reduced to approximately 40%.

B. The nuclear modification factor

The nuclear modification factor can be estimated by the ratio of the final spectrum of $Y$ to the initial one,
\[
R_{AA}(q_T) = \frac{\frac{dN_\Upsilon}{dq_T dq_T} |_{t=t_f}}{\frac{dN_\Upsilon}{dq_T dq_T} |_{t=t_0}} , \tag{11}
\]
where $t_f \approx 7$ fm/c is the time when the dissociation and regeneration mechanisms stop working at $T_f \approx T_c$. After the phase transition, hadronic effects are expected to be insignificant because of small reaction cross sections.

Figure 6 (a) shows the time evolution of the medium modifications. In the absence of regeneration, $R_{AA}$ corresponds to the exponentially decaying factor of Eq. (10) and is denoted by the dashed lines. As a collision system evolves, the temperature decreases and so does the thermal width (see Fig. 1 (b)). Thus, the suppression increases ($R_{AA}$ decreases) with time but eventually freezes at $t \approx 7$ fm/c. Furthermore, the thermal width grows with the transverse momentum, and the $R_{AA}$ decreases as the momentum increases. However, as seen in the solid lines of Fig. 6 (a), at high $q_T$ the regeneration effects are substantial because the dissociation term comes with a

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{(a) The numerical solution of Eq. (9). The regeneration effects are included in the solid lines, in comparison with the dashed lines which are obtained by the dissociation term only. (b) The time evolution of the $\Upsilon(1S)$ momentum distribution.}
\end{figure}
The regeneration constant of $b$ quark also affects the nuclear modification factor. From Fig. 6 (b), we see that the suppression at high $q_T$ changes for the different diffusion constant as in the case for the regeneration term. As the number of $\Upsilon(1S)$ is reduced to 40% in Fig. 5 (b), $R_{AA} \approx 0.4$ at low momentum which is independent of the diffusion constant. On the other hand, $R_{AA}$ at high momentum depends considerably on the diffusion of $b, \bar{b}$. The $D$ dependence of $R_{AA}$ is agreeable with the findings in Ref. [35] which is based on the Langevin equation and Wigner function.

C. The dependence on initial conditions

There are significant uncertainties in the initial stage of heavy-ion collisions. Although we have assumed a locally equilibrated medium at $t_0$ with $T_0 < T_{\text{diss}}$, there is a pre-equilibrium stage where quarkonia formation is in progress and the dissociation mechanism begins to work before regeneration [9]. Since the pre-equilibrium effects can lead to different initial conditions, we discuss the possible consequences in this section.

For initial distributions of $b, \bar{b}$ and $\Upsilon(1S)$, we have used the differential cross section measured in pp collisions, fit to the following form:

$$f(t_0, p_T) \propto \frac{1}{[\left(\frac{p_T}{\Lambda} \right)^2 + 1]^{-\alpha}}.$$  \hspace{1cm} (12)

For $f_b(t_0, p_T)$ and $f_{\Upsilon}(t_0, q_T)$, $\alpha = 2.85$, $2.44$ and $\Lambda = 6.07$, $6.05$ GeV, respectively. The fitted spectra are questionable as the current experimental data have large uncertainties especially at high momentum [20, 34].

Figure 7 shows the $R_{AA}$ dependence on the initial distributions of $b, \bar{b}$ and $\Upsilon(1S)$. If we use a harder quarkonium spectrum with larger $\Lambda = m_{\Upsilon}$ and smaller $\alpha = 2$ [23], the nuclear modification factor decreases at high momentum (compare the red dotted line with the solid line). On the other hand, when $b, \bar{b}$ spectra become softer with smaller $\Lambda = m$ (within the experimental uncertainties), $R_{AA}$ increases as shown in the green dashed-dotted line compared to the dashed line. As with a smaller diffusion constant, a softer initial spectrum leads to a larger nuclear modification factor at high momentum. We notice that the $D$ dependence of $R_{AA}$ shown in Fig. 6 (b) might not be satisfied when different initial distributions for $b, \bar{b}$ and $\Upsilon(1S)$ are used for each $D$ value.

The $\Upsilon(1S) R_{AA}$ has been measured in PbPb collisions at $\sqrt{s_{\text{NN}}} = 2.76, 5.02$ TeV by the CMS Collabora-

FIG. 6. (a) The regeneration effects on the nuclear modification factor for $\Upsilon(1S)$. (b) The dependence on the $b$ quark diffusion constant.

FIG. 7. The $R_{AA}$ depending on the initial distributions of $f_b(t_0, p_T)$ and $f_{\Upsilon}(t_0, q_T)$. The default $\Lambda$ and $\alpha$ values given in the text are used for those not specified in the figure.
The centrality-integrated $R_{AA} = 0.453 \pm 0.014 \text{(stat)} \pm 0.046 \text{(syst)}$, $0.376 \pm 0.013 \text{(stat)} \pm 0.035 \text{(syst)}$ at $\sqrt{s_{NN}} = 2.76, 5.02 \text{ TeV}$, respectively, and $R_{AA}$ appears to be independent of the transverse momentum. Figure 8 compares our calculations with the experimental data. The initial distributions $f_b(t_0, p_T)$ with $\alpha = 2.85$, $\Lambda = m$ and $f_T(t_0, q_T)$ with $\alpha = 2$, $\Lambda = m_T$ have been used. Using the same initial distributions, the $R_{AA}$ with smaller diffusion is larger as discussed before. The initial temperature $T_0 = 525, 550 \text{ MeV}$ has been assumed for $\sqrt{s_{NN}} = 2.76, 5.02 \text{ TeV}$, respectively. When the initial temperature is larger at higher energy collisions, the phase transition takes place later and the $R_{AA}$ is smaller at low momentum where dissociation is dominant (compare Fig. 8 (a) with (b)). The experimental data are comparable to our results with $D(2\pi T) \approx 5 - 8$. We note that both gluo-dissociation and inelastic parton scattering need to be taken into account to describe the nuclear modification factor $= 0.4$. Since the medium suppression exponentially increases with momentum due to dissociation (as denoted by the violet dashed line), the regeneration effects might account for the seemingly momentum independence of the measured data.

To estimate the feed-down effects, we can apply our numerical approach to the excited states of bottomonium (for which the gluo-dissociation can be more important than inelastic parton scattering because $T_{\text{diss}} < 300 \text{ MeV}$ [18]). The direct production of $\Upsilon(1S)$ is approximately 67% and the remainder is mostly from $1P$ and $2S$ states [12, 23]. However, the binding energies of the excited states are smaller at least by a factor of $3 - 5$ than the ground state energy [29], so their thermal widths are much larger than the $\Upsilon(1S)$ width. As a result, a major part of the inclusive $R_{AA}$ is expected to come from the direct $\Upsilon(1S)$ suppression. For instance, the $R_{AA}^{\Upsilon(2S)} \sim 0.1$ [19, 20] is roughly four times smaller than $R_{AA}^{\Upsilon(1S)}$. If the other excited states have nuclear modification factors of the same order as $R_{AA}^{\Upsilon(2S)}$, then the feed-down effects amount to $\sim 10\%$ of the direct $\Upsilon(1S)$ contribution. In principle, the feed-down can reduce the inclusive $R_{AA}$, especially at high $q_T$ in Fig. 8, but the spectrum would be still enhanced by regeneration in comparison to the suppressed one by dissociation only.

We do not expect that our results are considerably affected by varying other parameters involved in our calculations: (1) $T_0$ and $t_0$ can alter the lifetime of the quark-gluon plasma and the shapes of the $R_{AA}$ curves. (2) With the larger $N_b$ and the smaller $N_T/N_b$, the regeneration effects become more important. These parameters might change the numerical results quantitatively, but the tendency of dissociation and regeneration to the quarkonium momentum remains the same.

### IV. SUMMARY

We have discussed the dissociation and regeneration effects on the quarkonium momentum distributions. By taking into account the gluo-dissociation, inelastic parton scattering, and their inverse reactions through a partonic cross section formula that interpolates the formal limits at different temperature region, we have calculated the thermal width and the regeneration term of the Boltzmann equation for quarkonium moving in the quark-gluon plasma. For a Bjorken expansion geometry, the nuclear modification factor of $\Upsilon(1S)$ has been determined and compared with the experimental data. Due to the dominant dissociation, the $R_{AA}$ is exponentially decaying but the regeneration effect tends to reduce the medium suppression at high transverse momentum. The quarkonium regeneration depends on the heavy quark distribution which has been determined by a Fokker-Planck equa-
tion with a heavy quark diffusion constant. For smaller diffusion, the heavy quark distribution is more localized and the regeneration effects are more significant. With the heavy quark diffusion constant \( D(2\pi T) \approx 5 - 8 \) and a rather hard initial distribution of \( \Upsilon(1S) \), our numerical results seem to agree with the experimental measurements of the \( R_{\AA} \). Our analysis implies that the bottomonium spectra at high momentum are influenced by the regeneration effects, whereas those at low momentum are controlled by dissociation.

The \( \Upsilon(1S) \) spectrum has been described by the Boltzmann equation with the dissociation and regeneration terms, given initial conditions. The two collision terms have been obtained consistently based on the Bethe-Salpeter amplitude and hard thermal loop (HTL) resummation. As our thermal widths reduce to the effective field theory results in the relevant kinematical limit, which should be more applicable for \( b \) quarks than \( c \) quarks, the uncertainties involved in our calculations will be mainly from the initial conditions and the unknown heavy quark diffusion constant.

The upcoming experimental data and the lattice QCD computations on the diffusion constant can be useful to reduce the uncertainties in our approach. Furthermore, we need to include the nuclear geometric evolution and other mechanisms such as feed-down and quarkonium diffusion (and even the quantum decoherence effects caused by the noise correlations [37, 38]) for a more sophisticated phenomenological study.

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**Appendix A: The elliptic flow**

In the main text, we have assumed that the (transverse) reaction plane is isotropic. Here, we consider the azimuthal angular anisotropy \( v_2^b \) of \( b, \bar{b} \) distributions

\[
\frac{dN_b}{d\phi} \propto 1 + 2v_2^b(p_T)\cos(2\phi),
\]

and study its contribution to the \( \Upsilon(1S) \) \( v_2 \) [16]. For the momentum dependent anisotropy of \( b \), we use the \( b \) quark \( v_2(p_T) \) calculated in Ref. [36]. Then, the elliptic flow induced by the regeneration term is given by

\[
v_2(q_T) = \left. \frac{\int d\phi \frac{dN_T}{dq_T dq_T d\phi} \cos(2\phi)}{\int d\phi \frac{dN_T}{dq_T dq_T d\phi}} \right|_{t=t_f}.
\]

Figure 9 shows the regeneration contribution to the elliptic flow of \( \Upsilon(1S) \), supposing the \( v_2^b(p_T)/3 \) is given by the violet dashed line. When \( v_2^b(p_T) \) increases with momentum up to \( p_T \sim m \) and then decreases, \( v_2^b(q_T) \) grows with \( q_T \) and seems to level out at high \( q_T \). As the heavy quark diffusion decreases, the regeneration effects become stronger so \( v_2 \) increases. The calculated \( v_2 \) is comparable with the ALICE data at forward rapidity measured in PbPb collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV [21]. In addition to the regeneration, there are other contributions from the dissociation term, such as the path-length difference, on the elliptic flow, but such effects will be left for a future investigation.

**FIG. 9.** The regeneration effects on the elliptic flow anisotropy for \( \Upsilon(1S) \), comparing with the ALICE data at forward rapidity measured in PbPb collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV [21]. The same initial conditions as Fig. 8 have been used. The violet dashed line indicates the azimuthal angular anisotropy of \( b, \bar{b} \) quarks (divided by 3) [36] used in the calculations.

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