Cosmological free-free emission from dark matter halos

diffuse free-free (background) radiation from dark matter halos

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We study the diffuse background free-free emission induced in dark matter halos. Since dark matter halos host ionized thermal plasma, they are an important source of the cosmological free-free emission. We evaluate the global background intensity and anisotropy of this free-free emission. We show that the dominant contribution comes from dark matter halos with a mass close to the Jeans mass, $M_{\text{halo}} \sim 10^{10} M_\odot$, around the redshift $z \sim 1$. Therefore, the intensity of the free-free emission is sensitive to the small-scale curvature perturbations that form such small-mass dark matter halos. Considering the blue-tilted curvature perturbations, we find that the free-free emission signal is modified by $\sim 25\%$ even in the parameter set of the spectral index and the running, which is consistent with the recent Planck result. However, our obtained intensity of the global and anisotropic free-free emission is smaller than the ten percent level of the observed free-free emission, which is dominated by the Galactic origin. Therefore, the measurement of the cosmological free-free emission has the potential to provide more stringent constraints on the abundance of small-mass dark matter halos and the curvature perturbations including the spectral index and the running, while carefully removing the Galactic free-free emission is required through the multifrequency radio observation or the cross-correlation study with the galaxy surveys or 21-cm intensity map.

I. INTRODUCTION

The Planck space mission has achieved to measure the cosmic microwave background (CMB) anisotropies with surprising accuracy. The Planck observation result is in favor of the primordial curvature perturbations which are almost scale-invariant, adiabatic, and Gaussian [1]. Besides, combining the galaxy surveys [2, 3], we can confirm that these statistical features arise from the present horizon scale to the 1 Mpc scale [4]. Probing the statistical nature of the primordial curvature perturbations provides the access to the early universe model generating the primordial perturbations, especially the inflation model. Surprisingly, obtained statistical features are consistent with the prediction from a simple inflation model, that is the slow roll inflation model with a single scalar field [5] (for a review, see e.g. Ref. [6]).

Based on this great success, one of the next goals in modern cosmology is to reveal the primordial curvature perturbations on a smaller scale than the Mpc scale. In this context, CMB distortion can be a powerful probe [7–9]. Although the small-scale perturbations are smoothed out due to the Silk damping, the energy flows dissipated in the process of the Silk damping creates the distortions from the blackbody spectrum in the CMB energy spectrum [10–12]. Hence, measurements of the CMB distortion allow us to understand the small-scale perturbations more [11, 13–16]. In fact, from the measurements of CMB distortion by COBE/FIRAS, the constraint of the primordial power spectrum, $P_\zeta \lesssim 10^{-8}$ for the wave number range, $k \approx 1 - 10^4$ Mpc$^{-1}$, was suggested in Ref. [13]. Furthermore, it is suggested that in the next-generation CMB measurements like PIXIE [17], the constraint would improve in order of $P_\zeta \lesssim 10^{-8}$.

Future measurements of redshifted 21-cm signal are also expected to be useful to measure the small-scale perturbations. Since the 21-cm signal is emitted by the transition of the neutral hydrogen, in which the electron flips its spin, the measurements of the spatial fluctuations in the redshifted 21-cm signal can trace the evolutionary history of the matter density fluctuations before the epoch of reionization (for a comprehensive review, see Ref. [18]). Redshifted 21-cm measurements have the potential to explore the density perturbations on smaller scales than the Silk scale [19]. To accomplish these measurements, the Square Kilometre Array (SKA) project, building the largest radio telescope with over a square kilometer of collecting area, is proceeding now.

Observations of the early-formed minihalo signature are also suggested in cosmological observations to constrain the small-scale density fluctuations. If the fluctuations on smaller scales have 100 times larger amplitude than the scale-invariant spectrum confirmed by Planck, a large amount of minihalos forms soon after the matter-radiation equality [20]. Because of such early formation, their core density is higher than one formed in the standard hierarchical structure formation. The measurement of the $\gamma$-ray sky by Fermi can provide the tight constraint on the abundance of minihalos and small scale perturbations, if dark matter (DM) can self-annihilate [21–23]. Redshifted 21-cm observations have been investigated as a probe of these early formed minihalos, which does not require the self-annihilating DM model [24–28]. Recently the free-free emission from these minihalos has been studied and, using the Planck free-free measurement, the author provides the tighter constraint on the small scale structure, [29].

In this work, we focus on the diffuse background free-
free emission as the probe of the small-scale perturbation. The diffuse background free-free emission has been studied in the context of the foreground components of e.g. the CMB in the microwave and radio frequency range. Although most of the observed free-free emission is considered to be composed of Galactic origin, cosmological free-free emission is generated, for example, in the intergalactic medium (IGM) [30], the galaxy groups and clusters after the reionization [31], and the structure formation during the reionization [32]. Among them, the contribution from DM halos could be largest [33].

Here, we revisit the free-free emission from DM halos in the standard $\Lambda$CDM cosmology from the point of view of the small-scale primordial scalar perturbation. In particular, we investigate the dependency on the spectral index $n_s$ of the primordial power spectrum and the running $r_n$ which appears as the cosmological parameters for the first time. To investigate it, we first study the distributions of redshift and DM halo mass for both the global signal and anisotropy of the free-free emission. After that, we prepare several parameter sets ($n_s, r_n$) which would modify the number density evolution especially for small-mass halos, and test how the emission signal and anisotropy change with the different parameter sets. We also discuss the dependence on the gas profile model within DM halos.

This paper is organized as follows. In Sec.II, we provide the halo model describing the profile of their gas density and temperature. Then, we calculate the intensity of the free-free emission from an individual halo for different mass $M_{\text{halo}}$. In section III, considering the halo formation history, we formulate the diffuse background intensity which is the sum of the free-free emission from individual halos. We also discuss the mass and redshift distribution of the diffuse background intensity. In Sec. IV, we formulate the anisotropies of the free-free emission. We also estimate the mass and redshift distribution of the anisotropies in the same way as the one of the diffuse background intensity. In Sec. V, we discuss the application of our results to the constraint on the primordial curvature perturbations and obtain the limit by comparing the intensity of the observed free-free emission in the CMB frequency range. We conclude in section VI.

In this paper, we apply the flat $\Lambda$CDM cosmology and use the best-fit cosmological parameters of the Planck [1]. We also use the calculation package named $\text{HMFFcalc}$ [34] to estimate the halo mass function, the matter power spectrum, and so on.

II. FREE-FREE EMISSION RATE FROM AN INDIVIDUAL HALO

Thermal plasma can emit free-free radiation. When the number density, temperature, and ionization fraction of the free electrons in a plasma are given by $n_{\text{gas}, x}$, $T_{\text{gas}}$, and $x_e$, the emission coefficient of the free-free radiation at a frequency $\nu$ is given by [35]

$$e_\nu^{\text{ff}} = \frac{2^6 e^6}{3 m_e c^3} \left( \frac{2\pi}{3 m_e k_B T_{\text{gas}}} \right)^{1/2} \times x_e^2 n_{\text{gas}}^2 \exp \left( -\frac{h_p \nu}{k_B T_{\text{gas}}} \right) \bar{g}_\nu ,$$

where $e$ and $m_e$ is the electric charge and mass of electrons, $h_p$ is the Planck constant and $k_B$ is the Boltzman constant. In Eq. (1), $\bar{g}_\nu$ is a velocity-averaged Gaunt factor. We adopt the fitting formula of $\bar{g}_\nu$ in Ref. [36],

$$\bar{g}_\nu = \log \left\{ \exp \left[ 5.960 - \sqrt{3/\pi} \log \left( \nu_\odot T_4^{3/2} \right) \right] + e \right\} ,$$

where $\nu_\odot \equiv \nu/(1 \text{ GHz})$, $T_4 \equiv T_{\text{gas}}/(10^4 \text{ K})$, and $e$ is the Napier’s constant. The free-free emission at a frequency larger than the critical frequency, $\nu_\odot \equiv k_B T_{\text{gas}}/h_p$, suffers the exponential damping as shown in Eq. (1). However, we are interested in the CMB or radio frequency range. Since these frequency range is well below the critical frequency given by the gas temperature in DM halos, we assume $(h_p \nu/k_B T_{\text{halo}}) \approx 1$ hereafter.

To evaluate the free-free emission from DM halos, it is required to model the gas profile for the energy density and temperature in DM halos. In this paper, we adopt the gas profile (KS model) given in Ref. [37, 38].

In the KS model, the gas density and temperature profiles are given by

$$\rho_{\text{gas}}(x) = \rho_{\text{gas}}(0) y_{\text{gas}}(x),$$
$$T_{\text{gas}}(x) = T_{\text{gas}}(0) y_{\text{gas}}^{-1}(x),$$

where $x$ is given by $x = r/r_{\text{vir}}$ with the virial radius, $\rho_{\text{gas}}(0)$ and $T_{\text{gas}}(0)$ are the number density and temperature at $x = 0$, $\gamma$ is the polytropic index, and $y_{\text{gas}}$ is the shape function of the profile which satisfies $y_{\text{gas}}(0) = 1$. Here the shape function, $y_{\text{gas}}$, is found by solving the hydrostatic equilibrium equation [37],

$$y_{\text{gas}} \equiv \left\{ 1 - B \left[ 1 - \frac{\ln(1 + x)}{x} \right] \right\}^{1/(\gamma-1)} ,$$

with

$$B \equiv 3 \eta_0^{-1} \gamma - 1 \frac{\ln(1 + c_s)}{c_s} - \frac{T}{1 + c_s} ,$$

where $\eta_0$ is the mass-temperature normalisation factor which is defined by

$$\eta_0 \equiv \frac{3 k_B r_{\text{vir}} T_{\text{gas}}(0)}{G \mu m_p M} ,$$

and $c_s$ is the concentration parameter. For the values of $\eta_0$ and $\gamma$, Ref. [38] provides the useful fitting formula as functions of $c_s$,

$$\eta_0 \approx 2.235 + 0.202(c_s - 5) - 1.16 \times 10^{-3}(c_s - 5)^2 ,$$

and $\gamma$ is given by

$$\gamma \approx 5.265 - 1.37(c_s - 5) + 1.15 \times 10^{-3}(c_s - 5)^2 .$$
and
\[ \gamma = 1.137 + 8.94 \times 10^{-2} \ln(c_s/5) - 3.68 \times 10^{-3}(c_s - 5). \]  
(9)

These functions are valid in range of \( 0 < c_s < 25 \).

Ref. [39] has investigated the concentration parameter \( c_s \) by performing N-body simulations from the high redshift to the present. Based on the simulation results, they provide the analytic fitting formulae of \( c_s \) for the DM halos with mass \( M_{\text{halo}} \gtrsim 10^9 M_{\odot} \) in the redshift range of \( 0 < z < 14 \). In this paper, we employ their result for the concentration parameter \( c_s \). We confirmed that \( c_s \) is less than \( c_s < 25 \) for the DM halo mass range and the redshifts investigated in this paper. Therefore, the fitting formulae of Eqs. (8) and (9) are valid throughout the paper.

With \( \eta_0 \) in Eq. (7), we can estimate \( T_{\text{gas}}(0) \) as
\[ T_{\text{gas}}(0) = 2eV \left( \frac{\mu}{0.6} \right) \left( \frac{M_{\text{halo}}}{10^{10}h^{-1} M_{\odot}} \right) \left( \frac{R_{\text{vir}}}{67h^{-1} \text{kpc}} \right)^{-1}. \]
(10)

The integration of the density profile provides \( \rho_{\text{gas}}(0) \) as
\[ \rho_{\text{gas}}(0) = M_{\text{gas}} \left[ 4\pi \right] \int_{0}^{\epsilon} y_{\text{gas}}(u) u^2 du \left( \frac{M_{\text{halo}}}{10^{10}h^{-1} M_{\odot}} \right) \left( \frac{R_{\text{vir}}}{67h^{-1} \text{kpc}} \right)^{-3} \times \frac{\Omega_{b}h^2}{\Omega_{m}h^2} \left[ \ln(1 + c_s) - \frac{c_s}{1 + c_s} \right]^{-1}, \]
(11)
where \( M_{\text{gas}} \) is the total baryonic mass contained in the halo with mass \( M_{\text{halo}} \). Here we assume that the dark matter halos can host the baryon gas whose mass is given by
\[ M_{\text{gas}} = \frac{\Omega_{b}}{\Omega_{m}} M_{\text{halo}}. \]
(12)

The ionization fraction is determined by the balance between the recombination and the ionization by thermal collision or UV photons from galaxies and stars. Most DM halos that contribute the free-free radiation form after the Epoch of reionization. Therefore, we simply assume that UV photons from galaxies and stars are enough to keep the ionization in DM halos and we set \( x_e = 1 \).

### III. Diffuse Background Free-Free Emission Induce by \( \Lambda CDM \) Halos

The free-free emission from an individual DM halo can be evaluated by Eq. (1) with the gas profile discussed in the previous section. Now we calculate the global intensity of the diffuse free-free background emission which is the sum of the emission from DM halos in the universe, following the calculation in Ref. [40].

To calculate the global intensity, it is useful to evaluate the mean intensity of the free-free emission from an individual halo with mass \( M \) at a redshift \( z \),
\[ I_{\nu}^\text{ind}(z, M_{\text{halo}}) = \frac{\int_{V_{\text{halo}}} \epsilon_{\text{ff}} dV}{S_{\text{halo}}}, \]
(13)
where \( V_{\text{halo}} \) and \( S_{\text{halo}} \) is the physical volume and cross section on the sky for the DM halo, \( V_{\text{halo}} = 4\pi R_{\text{vir}}^3/3 \) and \( S_{\text{halo}} = \pi R_{\text{vir}}^2 \), respectively. Note that, to obtain Eq. (13), we apply the optically thin approximation, because the free-free absorption is negligible in our case.

Let us consider the redshift shell at a redshift \( z \) with the width \( dz \). The free-free emission contribution from DM halos in this redshift shell is
\[ dI_{\nu}^\text{ff}(z) = dz \frac{dV_{\text{com}}}{dz} \int_{M_{\text{min}}}^{M_{\text{halo}}} dM_{\text{halo}} \frac{\Omega_{\text{halo}}}{4\pi} I_{\nu}^\text{ind} \frac{dn_{\text{halo}}^\text{com}}{dM_{\text{halo}}}, \]
(14)
where \( dn_{\text{halo}}^\text{com}/dM \) is the comoving mass function of DM halos and \( V_{\text{com}} \) is the comoving volume and \( \Omega_{\text{halo}}(z, zf, M_{\text{halo}}) \) is the solid angle of a DM halo given by \( \Omega_{\text{halo}} = \pi((1 + z)R_{\text{vir}})^2/\chi^2 \) with the comoving distance \( \chi \) to \( z \). In Eq. (14), we set the DM minimum mass \( M_{\text{min}} \) to the Jeans mass with the background baryon temperature \( T_{b, \text{bg}} \). Since we are interested in the redshifts after the epoch of reionization, we take \( T_{b, \text{bg}} \sim 10^4 \) K [41–45]. Below \( M_{\text{min}} \), the baryon gas cannot collapse at the formation of DM halos and DM halos can retain baryon gas through the accretion, which is a relatively smaller amount than through collapsing. Therefore, we neglect the contribution from such small-mass DM halos.

Finally, the redshift integration yields the total global intensity at an observed frequency \( \nu_{\text{obs}} \) from DM halos,
\[ I_{\nu_{\text{obs}}} = \int_{0}^{\infty} dz \frac{dV_{\text{com}}}{dz} \frac{dI_{\nu}^\text{ff} \epsilon_{\text{em}}(z, M_{\text{halo}})}{dz}, \]
(15)
where \( \epsilon_{\text{em}} \) is \( \epsilon_{\text{em}} = (1 + z)\nu_{\text{obs}} \) and the factor \( (1 + z)^{-3} \) is the redshift effect for the intensity.

We plot the result of Eq. (15) in Fig. 1. To obtain this result, we adopt the Press-Schechter mass function with the Planck best-fit cosmological parameter set. It might be helpful to represent the intensity in terms of the brightness temperature, \( T_{b, \nu} \). In the Rayleigh-Jeans limit, the brightness temperature is related to the intensity by
\[ T_{b, \nu} = \frac{c^2}{2\nu_{\text{obs}}^2} I_{\text{obs}} \approx 0.32 \left( \frac{I_{\text{obs}}}{1 \text{ Jy/str}} \right) \left( \frac{\nu_{\text{obs}}}{10 \text{ GHz}} \right)^{-2} [\mu\text{K}]. \]
(16)
Therefore, while the intensity of the free-free signals is almost frequency independent, the brightness temperature of the free-free signal is proportional to \( \nu^2 \).
FIG. 1. The intensity of the global free-free emission from DM halos. The horizontal axis is for the frequency in a unit of GHz. The red solid line shows the global intensity for the KS model, and the blue dashed line is the one for the homogeneous gas model. The dark grey shaded region is excluded by the global mean intensity observed by Planck. The intensity represented in the light grey region is larger than the mean intensity of the free-free signals at the high galactic latitude.

Since the free-free intensity is proportional to the square of the gas number density, the strength of the intensity highly depends on the gas profile model in DM halos, for which we take the analytical model based on the hydrostatic equilibrium. In order to show the impact of the gas profile, we plot the intensity in the homogeneous gas model, where $n_{\text{gas}} = 200n_{\text{bg}}$, and $T_{\text{halo}} = T_{\text{vir}}(M_{\text{halo}})$, as a blue dashed line in Fig. 1. We plot the intensity with the homogeneous gas model in the blue dashed line in Fig. 1.

The clumpiness is a good indicator representing the impact of the gas profile model on the free-free emission and is defined as

$$C(z) \equiv \frac{\int dM_{\text{halo}} \int V_{\text{halo}} dV n_e^2(r, M_{\text{halo}}) \frac{dn_{\text{com}}}{dM_{\text{halo}}}}{\bar{n}_{e,\text{IGM}}},$$

where $n_e(r, M_{\text{halo}}) = x_e n_{\text{gas}}$ is the electron number density in an individual halo, and $\bar{n}_{e,\text{IGM}}$ represents the mean value of the free electron number density in the IGM. We show the clumpiness for both models in Fig. 2. The clumpiness in the KS model is roughly 10 times larger than in the homogeneous model. However, the free-free intensity in the KS model is not 10 times more amplified than in the homogeneous model as expected from the clumping factor. This is because the gas temperature in DM halos is also large in the KS model and the increase of the gas temperature weakens the amplification due to the enhancement of the clumpiness. Note that the homogeneous model can provide the minimum intensity of the free-free emission from DM halos, compared with other gas models. Therefore, Fig. 1 tells us that the difference in the gas profile modifies the intensity by, at least, a factor of a few.

The free-free emission in the CMB frequency has been studied well as the foreground of the CMB. In Fig. 1, we plot the (all-sky) mean free-free intensity provided by Planck collaboration and the free-free component at the high galactic latitude given in Ref. [40]. The current observed signal even at high galactic latitude is 10 times larger than our prediction with the Planck best-fit parameters. In the observed free-free signals, the cosmological contribution has not been identified yet, most of the observed free-free signals are considered to be of Galactic origin. To identify the cosmological contribution as we predict here, further investigation including removing Galactic signals, analyzing the statistical anisotropy of the signal, and taking the cross-correlation with cosmological observations is required.

A. Mass and redshift distribution

The free-free signals calculated above are the sum of the contribution from all DM halos spread in the wide range of the mass and redshift. It is worth investigating the mass and redshift contribution of the global intensity. The halo mass distribution of $I_{\text{obs}}$ at $\nu_{\text{obs}} = 70$ GHz is represented in Fig. 3. The figure clearly shows that the dominant contribution comes from relatively small-mass halos around $M_{\text{halo}} \sim 10^{10}M_\odot$. The monotonic decrease in a large mass side is due to the shape of the halo mass function. As the DM mass increases, the number density of DM halos becomes small and, as a result, the intensity...
FIG. 3. Mass distribution of the global free-free emission from DM halos. The red solid and blue dashed lines are for the KS model and the homogeneous gas model, respectively. These lines are identical because the dependency of the mass function employed for both models is stronger than other terms depending on the halo mass.

contribution decreases. The cutoff on the lower mass side is due to the Jeans mass. Therefore, most the free-free emission signals come from DM halos around the Jeans mass scale. The concentration parameter also depends on the mass. As DM halos have a small mass, the concentration parameter increases, and the resultant signal is enhanced. However, we found that the mass distributions of the signals for both the KS and the homogeneous gas models are almost identical. This is because the contribution coming from the halo mass function is much larger than the one of the concentration parameter.

We also plot the redshift distribution for these two models in the same way of Fig. 3. Basically, the redshift distribution reflects the history of the formation efficiency of DM halos, especially with $M_{\text{peak}} \sim 10^{10} M_\odot$ which is corresponding the peak in Fig. 3. That is, the peak feature represents in the redshift $z_{\text{peak}} \sim 4$ when $\sigma(M_{\text{peak}}, z_{\text{peak}})/\delta_c \sim 1$. Unlike the mass distribution, the evolution of the concentration parameter significantly contribute on this redshift distribution. As shown in Fig.10 of Ref. [39], for the halo with a mass around $M_{\text{halo}} \sim M_{\text{peak}}$, the concentration parameter increases as the redshift becomes small. Therefore, the redshift distribution for the KS model shifts to the smaller redshift side than in the homogeneous model.

Because of this redshift dependence, we suggest that if we obtained the redshift tomographic information on the global signals of free-free emission through e.g., the cross-correlation analysis with 21cm line intensity, we could know about the small-scale structure formation in Universe and the gas profile in small-mass halos.

FIG. 4. Redshift distribution of the cosmological free-free emission from DM halos. The red solid line is for the KS model and the blue dashed line is for the homogeneous gas model.

IV. STATISTICAL ANISOTROPY OF THE FREE-FREE EMISSION FROM $\Lambda$CDM HALOS

In this section, we investigate the anisotropy of the free-free emission induced by DM halos. The anisotropy of the cosmological free-free signals is created by both the clustering of DM halos and the Poisson contribution in the number density of DM halos. The statistical value of the anisotropy is evaluated in term of the angular power spectrum. To compute the angular power spectrum, we adopt the halo formalism of Ref. [46]. Accordingly, the power spectrum can be divided into two components,

$$C^{\text{eff}}_\ell = C^{\text{1h}}_\ell + C^{\text{2h}}_\ell. \tag{18}$$

Here $C^{\text{1h}}_\ell$ is the “one-halo” term describing the Poisson contribution given in

$$C^{\text{1h}}_\ell = \int_0^\infty dz \frac{d^3V}{dzd\Omega} \int dM \frac{dn_{\text{halo}}}{dM} |\tilde{I}_\ell(z)|^2, \tag{19}$$

and $C^{\text{2h}}_\ell$ is the “two-halo” term describing the clustering contribution written in

$$C^{\text{2h}}_\ell = \int_0^\infty dz \frac{d^3V}{dzd\Omega} P \left( \frac{\ell}{\chi(z)} \right) \left| \int dM \tilde{\Psi}(M, z) \right|^2. \tag{20}$$

In Eq. (20), $\tilde{\Psi}$ is defined as

$$\tilde{\Psi} = \frac{dn_{\text{halo}}}{dM} \tilde{I}_\ell(M, z)b(M, z), \tag{21}$$

where $\tilde{I}_\ell(M, z)$ is the 2D Fourier modes of the intensity, and $b(M, z)$ is the halo bias which we employ in Ref. [47].
location of the contribution shift to large-mass halos. As discussed above, small-mass halos produce the dominant contribution. However, in the anisotropy, the contribution of small mass halos on small $\ell$ is suppressed proportionally to $\ell^2$ because of the Poisson contribution. As a result, the profile structure of large mass halo cannot be relatively neglected on large scales, compared with small-mass halo contribution.

Next, we study the redshift contribution. The redshift distributions is obtained through

$$\frac{d \ln C_{\ell}^{1h}}{d \ln z} = \frac{z^2}{\ell^2} \int \frac{dV}{dz} \int \frac{dM}{dM} \frac{d\rho_{\text{halo}}^\text{com}(M,z)}{dM} |\tilde{I}_\ell(M,z)|^2, \quad (24)$$

and

$$\frac{d \ln C_{\ell}^{2h}}{d \ln z} = \frac{z^2}{\ell^2} P \left( \frac{\ell}{\ell_{\text{halo}}} \right) \int dV \frac{d\Psi(M,z)}{dz} |\tilde{I}_\ell(M,z)|^2. \quad (25)$$

Fig. 7 and 8 show the redshift distribution for the one- and two-halo terms respectively. The dependence of the peak location on $\ell$ modes is different between the one- and two-halo terms.

In the one-halo term, the peak location depends on $\ell$ modes. It moves toward lower redshifts as the $\ell$ mode decreases. This is because the apparent size of DM halos is important through $I_\ell$. As shown in the mass contribution of the one-halo term in Fig. 6, most of the contributions come from small-mass halos as the Poisson contributions. On small $\ell$ modes, the larger apparent angle size the DM halos have, the smaller the suppression of the Poisson noise is, because the suppression is proportional to $\ell/\ell_{\text{halo}}^3$, where $\ell_{\text{halo}}$ is the $\ell$ mode corresponding to the apparent angular size of halos. Therefore, since DM halos at lower redshift have large apparent angular scales, the peak locates on lower redshifts as the $\ell$ mode becomes small.

On the other hand, in the two-halo terms, all $\ell$ modes have a sharp peak at the same redshift, $z \gtrsim 3$. The two-halo term has a strong dependence on the abundance of DM halos with $M \sim 10^{10} M_\odot$ because of $C_{\ell}^{2h} \propto (d\ln_{\text{halo}}/dM)^2$. DM halos with $M \sim 10^{10} M_\odot$ actively form at $z \gtrsim 3$. As a result, the peak of the redshift contribution locates on $z \gtrsim 3$.

V. COSMOLOGICAL APPLICATION: THE SPECTRAL INDEX AND RUNNING OF THE PRIMORDIAL CURVATURE PERTURBATIONS

As discussed in the previous sections with Figs. 3 and 6, most contribution of the global signal and the anisotropy of free-free emission comes from small-mass DM halos whose mass is corresponding to around the Jeans scale. This suggests that the observations of cosmological free-free signal might be a good tool to probe the abundance
of such DM halos. The probing the abundance of small-mass halos is important to study the statistics of the primordial curvature perturbations, in particular, on small scales. In this section, we investigate the dependence of the free-free signals on the statistic of the primordial curvature perturbations.

The statistical nature of the primordial curvature perturbations can be provided in the non-dimensional primordial curvature power spectrum $P_\zeta$. To describe the $k$-dependence, we often introduce the spectral index, $n_s$, and the running, $r_s$ as

$$P_\zeta = \left( \frac{k}{k_{\text{pivot}}} \right)^{n(k)},$$

$$n(k) \equiv n_s - 1 + \frac{1}{2} r_s \ln \left( \frac{k}{k_{\text{pivot}}} \right),$$

where $k_{\text{pivot}}$ is the pivot scale given in $k_{\text{pivot}} = 0.05\text{Mpc}^{-1}$.

According to the latest Planck paper, the spectral index and the running are evaluated as

$$n_s = 0.9659 \pm 0.0040 \ (69\%\text{C.L.}),$$

$$r_s = -0.0041 \pm 0.0067 \ (69\%\text{C.L.}),$$

In order to determine $n_s$ and $r_s$ with smaller error bars, it is important to measure the amplitude of the perturbations in the wide range of the scale, in particular, on small scales which we can probe through the measurement of the free-free signals.

In this work, we set six parameter combinations, $(n_s, r_s)$ based on Eq. (27) to investigate the dependency of the cosmological free-free signal on the spectral index summarized in the Table I. The first three models (I-III) are no-running models but with the best-fit value of the spectral index and the maximum values in the $1\sigma$- and $2\sigma$-region. The latter three models (IV-VI) have not only the blue tilts but also the running of the best-fit value and the $1\sigma$ and the $2\sigma$-maximum values for the running, respectively.\footnote{It is noted that in the Planck analysis, the estimated value from...}

Our parameter set is consistent with the...
CMB anisotropy measured by Planck.

|   | I  | II | III | IV | V  | VI |
|---|----|----|-----|----|----|----|
| $n_s$ | 0.9659 | 0.9699 | 0.9739 | 0.9659 | 0.9699 | 0.9739 |
| $r_{ns}$ | -0.0041 | 0.0026 | 0.0093 | |

**TABLE I. parameter sets**

Fig. 9 shows the global signals with six parameter sets. Although the frequency dependence is the same as in the six parameter sets, the amplitude is different, depending on the parameter set. As the spectrum becomes blue, the signal amplitude increases. This is because the abundance of small-mass DM halos, which contribute to the free-free signals, is enhanced. The impact of the running is more prominent. Even in the parameter set consistent with the Planck data, the amplitude of the free-free signal is enhanced at most by $\sim 12\%$.

The anisotropy of the free-free emission is plotted in Fig. 10 for six parameter sets. For comparison, we provide the ratio of the anisotropy of free-free emission to the parameter set I for the other parameter sets in Fig. 11. Similarly to the global signal, the anisotropy signal is enhanced in the models which have a large amplitude of the primordial perturbations. In particular, the enhancement becomes large on small scales and reach 20% amplification for the parameter set VI.

It is worth comparing this anisotropy dependence on the parameter set free-free anisotropy with CMB signals; the CMB primordial temperature anisotropy and the thermal Sunyaev-Zel’dovich (SZ) effect.

The CMB primordial temperature anisotropy is one of the great probes to know the primordial curvature perturbations. We know its powerfulness by the Planck analysis on cosmological parameter sets mentioned above. Similarly to Fig. 11, we plot the ratio of the CMB temperature anisotropy for the different parameter sets in Fig. 12. In the observational range of the scales, the...
temperature anisotropy depends on the curvature perturbations linearly. At a given scale (multipole $\ell$), the ratio of the signal is proportional to the primordial curvature perturbations. As the scale deviates from the pivot scale corresponding to $\ell \sim 100$, the difference of the ratio from the unity increase. For example, on small scale, $\ell \sim 1000$ the signal enhancement is, at most, $\sim 6\%$ among six parameter sets. This deviation is within the error bar of the Planck measurement. It is expected that the difference becomes large on smaller scales. However, on such smaller scales, the primordial temperature anisotropies are significantly damped by the Silk damping and it is challenging to measure them. Therefore, it is difficult to probe the small-scale primordial curvature perturbations through the primordial temperature perturbations.

![FIG. 12. The difference of CMB temperature anisotropies with the six parameter sets summarized in Table I. We normalized them by the one of the parameter set, $(n_s, r_{ns}) = (0.9659, 0)$.](image)

Next, we consider the thermal SZ effect. The thermal SZ effect is the inverse Compton scattering of CMB photons caused by hot electrons inside the dark matter halos. The brightness temperature of the CMB photons is lifted up by this thermal SZ effect and, as a result, CMB temperature anisotropy is produced, following the distribution of dark matter halos in the sky. Therefore, the SZ effect temperature anisotropy is sensitive to the dark matter halo abundance. Using the same gas model, we calculate the thermal SZ effect temperature anisotropy and investigate the signal dependence on the six parameter set models. In Fig. 13, we plot the dependence on the parameter set models as the ratio of the thermal SZ signals to the one of the model I in Table I.

![FIG. 13. The difference of anisotropies of CMB temperature induced by Compton scattering with the six parameter sets summarized in Table I. We normalized them by the one of the parameter set, $(n_s, r_{ns}) = (0.9659, 0)$.](image)

In the case of the thermal SZ effect, the angular power spectrum is sensitive to the abundance of DM halos with mass $M_{\text{halo}} \gtrsim 10^{13} M_\odot$ as shown in Ref. [38]. The mass variance of such DM halos is not sensitive to the spectral index and running because the scale corresponding to these mass scales is close to the pivot scale in Eq. (26). Therefore, the dependence of the signal on the parameter set is not strong. However, in smaller scales ($\ell \gtrsim 10^3$), the inner structure of such large DM halos effect on the thermal SZ anisotropy. The concentration parameter which determines the inner structure depends on the spectral index and running in our model. Therefore, the dependence on the parameter set becomes stronger. However, even so, the difference is only by $\sim 15\%$ at best in the very small scale, $\ell \sim 10^5$.

Compared with these CMB observations accessing the primordial curvature perturbations, the free-free signals have the stronger dependence on the spectral index and the running, because the most contribution of the free-free signal comes from DM halos with smaller mass with which other observations cannot reach DM halos. Consequently, these results suggest that the observation of free-free emission could be a powerful probe of the small-scale primordial perturbations, in particular, the spectral index and the running.

VI. CONCLUSION

In this paper, we have investigated cosmological free-free emission originating dark matter halos in the $\Lambda$CDM model. We adopt the analytic gas model which follows the hydrostatic equilibrium in the DM NFW profile to estimate the free-free emission from individual halos. Using this result, we have calculated the global signal and the
anisotropy, considering the cosmological structure formation based on the Press-Schechter formalism.

The evaluated amplitude of the global free-free emission spectrum is roughly 10 Jy/str and is nearly frequency-independent in the CMB and radio frequency range. With regard to the anisotropy, the Poisson contribution basically dominates, and the resultant angular power spectrum is proportional to the square of the multipole. However, the clustering contribution would be comparable or larger than the Poisson one on large scales, $\ell < 1000$. Therefore, the anisotropy of cosmological free-free emission on large scales traces matter density fluctuations. We have also done the above calculations for the constant gas profile model to confirm the dependence on the gas model. This constant gas profile model could provide the lowest signal. We have found that the amplitude of the free-free signal becomes 5 Jy/str.

In addition, we have investigated the mass distribution of the free-free signals. We have shown that most of the contribution comes from DM halos with mass $\sim 10^{11}M_\odot$ for both the global signal and the anisotropy. This tells us that the measurement of cosmological free-free signals have the potential to probe the abundance of such small-mass DM halos, that is, the primordial curvature perturbations on small scales which is the origin of these halos.

As an application of probing the small-scale perturbations in the cosmological context, we examine the sensitivity of the cosmological free-free signal on the spectral index and the running. The difference of these parameters modifies the amplitude of the global signal and creates the deviation from the scale dependence with $\propto \ell^2$ in the angular power spectrum. Our result shows the free-free signals are altered by 25% even in the parameter set which is consistent with the Planck result. This modification is larger than the one of the CMB temperature anisotropy induced by the halo thermal SZ effect, which changes the signal by 15% for the same parameter set. Therefore, the measurement of the cosmological free-free signals could provide a more stringent constraint on the spectral index and the running.

However, the measurement of the cosmological free-free signal is challenging. Although the free-free emission on the sky has been investigated in the CMB and radio frequency range, the observed free-free emission sky is dominated by the signals from the Milky way galaxy. They are roughly 100 times larger in the all-sky average and 10 times larger in the high galactic latitude than the cosmological free-free emission obtained in this paper. Therefore, the cosmological free-free signal cannot be measured without removing the Milky way contribution with high accuracy. The future radio observation, SKA, has enough sensitivity to measure the cosmological free-free signals. The cross-correlation study is useful to reduce foreground contamination. The clustering effect of DM halos due to the underlying matter density fluctuations can make a non-negligible contribution of the cosmological free-free emission anisotropy on large scale. Therefore, the cross-correlation with other cosmological probes of the matter density fluctuations including galaxy survey or future 21 cm intensity map around $z \sim 1$ could be useful to reveal the cosmological free-free anisotropy on large scales. We have addressed this issue in the next paper.

The theoretical uncertainty of the gas profile in DM halos is also one of the difficulties to probe the dark matter halo abundance and the primordial curvature perturbations. The calibration by other cosmological observations, e.g., X-ray and SZ cluster observations might be useful, although the free-free emission signals come from smaller DM halos than in other observations. In this study, we show that the difference of the gas profile model arises on the overall amplitude of the anisotropy and they do not modify its scale dependence, $D_\ell \propto \ell^2$. On the other hand, the scale dependence of the primordial curvature perturbations, the spectral index, and the running change its scale dependence. Therefore, the detailed measurement of the scale dependence in the anisotropy can help to solve the degeneracy of the uncertainties between the gas profile model and the scale dependence of the primordial curvature perturbations.

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