Hybrid beamforming in Multiuser mmWave Massive MIMO System

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Abstract. Conventional MIMO systems typically use full digital processing for optimal performance while it requires an independent radio frequency (RF) chain for each antenna, which is not feasible for large-scale antenna arrays due to the high cost, high-power consumption and high complexity of RF chain components in high frequencies. Fortunately, in recent years, the Hybrid beamforming has been proposed as an efficient and auspicious technique for the practical implementation of millimeter-Wave (mmWave) multiple-input multiple-output (MIMO) wireless systems. In this paper we propose a novel HBF (Hybrid beamforming) algorithm and simulate the sum rate of the proposed approach, then compare it with other algorithms in the sum rates. Specifically, the HBF algorithm as well as the maximizing minimum phase difference algorithm are based on the MMSE (Minimum Mean Square Error). For the full-digital beamforming and the beam steering, the ZF (zero forcing) criterion is employed. Simulations demonstrate that the performance of the proposed algorithm is similar to the full digital algorithm, and its robustness is confirmed by comparisons with other existing algorithms under the same channel model.

1. Introduction
The rapid development of wireless communications requires much more spectrum resources that the low frequency bands can provide; therefore, the millimeter-wave (mmWave) frequency band are utilized more and more in the communication systems. The small wavelength of mmWave also enables a large number of antennas to be placed into a limited space, which provides high array gains [1]. However, the mmWave communication usually endures a large path loss [2]. As shown by the Friis’s Formula, path loss is directly proportional to the square of the frequency in free space. As the bands with significantly
high frequency are used in mmWave communications, the pathloss in the mmWave system is much higher than the traditional systems. Fortunately, directional antennas help the mmWave MIMO system to achieve high antenna gain through beamforming, which compensates the high path loss [3].

The conventional MIMO systems use the full digital beamforming, which will yield optimal performances. The digital beamformer is created using the minimum mean square error algorithm (MMSE) [4]. However, each antenna will require an independent radio frequency chain in full digital beamforming, which results in large power consumption and high complexity in the mmWave MIMO systems due to the large array of antennas. Therefore, to overcome this limitation, hybrid beamforming is being developed. Reference [2] proposed a hybrid beamformer design based on maximizing the minimum phase difference. The ABF vector designed by this algorithm makes an optimal compromise between high received signal powers and low inter-user interferences. Reference [5] proposes another hybrid beamforming algorithm in which the ABF vectors are selected by the transmitter from the codebook vectors with the strongest correlation with the channel vectors.

The main problems with the hybrid beamforming algorithms are that when the number of users increases significantly, the problem of interference becomes, which will compromise the performances of these algorithms. This paper explores a modified Gram-Schmidt Algorithm that has robust performances with large number of users. The comparison shows that this modified Gram-Schmidt Algorithm is more capable of handling severe interferences [1].

2. Channel Model

Channel model often helps creating a simplified version of reality. Besides detailing the environment, the simplicity created through mathematical operation further facilitate the process of simulation. In this section, the channel model, which is being represented as a transfer function of the channel, will be deducted.

One of the most generalized mathematical models for channel under three-dimensional condition is being presented in [6] as:

\[
y(t) = \sum_{l=1}^{P} \alpha_l e^{j2\pi v_l t} a_R(\theta_{R,l}, \phi_{R,l}) a_T^*(\theta_{T,l}, \phi_{T,l}) x(t - \tau_l) + n(t) \quad (1)
\]

In equation 1, \(y(t)\) is the vector received at the receiver, and \(x(t)\) is the transmitted signal, while \(n(t)\) is the noise vector that is being created while the signal is being transmitted. The collection that is multiplying \(x(t)\) is called the transfer function, which represents the complete channel, here denoted as \(h\).

\[
h = \sum_{l=1}^{P} \alpha_l e^{j2\pi v_l t} a_R(\theta_{R,l}, \phi_{R,l}) a_T^*(\theta_{T,l}, \phi_{T,l}) \quad (2)
\]

Since the model consist of multiple paths, summing all the vectors is necessary. Here, \(P\) is the total number of path and \(l\) is the index of each path. Inside the transfer function, \(\alpha\) is the complex gain of each path. Here, the index of path is being presented by the subscript. As any velocity user undergo would create Doppler’s effect, the velocity for each path is being denoted as \(v\). Finally, \(a_R\) and \(a_t\) are the steering vector of receiver and transmitter. Inside the parenthesis, \(\theta\) is the horizontal while \(\phi\) is the vertical angle of arrival.

However, this model is not the final channel model that has been utilized in our analysis. Before presenting the simplification, a Fourier transform is being applied to the original transfer function:

\[
H = \sum_{l=1}^{P} \alpha_l e^{j2\pi (\nu_l t - \tau_l)} a_R(\theta_{R,l}, \phi_{R,l}) a_T^*(\theta_{T,l}, \phi_{T,l}) \quad (3)
\]

Here, one more term is being added to the exponential of e, which is being caused by the time shift. In our analysis, several assumptions are being made to simplify this model. First, we assumed user are moving in a slow enough way such that the Doppler’s effect can be ignored:

\[
H = \sum_{l=1}^{P} \alpha_l e^{j2\pi (\nu_l t - \tau_l)} a_R(\theta_{R,l}, \phi_{R,l}) a_T^*(\theta_{T,l}, \phi_{T,l}) \quad (4)
\]

Next, as being introduced in [4], we assumed the bandwidth to be small enough so that \(e\) term would be eliminated completely:
Lastly, we also assumed that we are only dealing with receivers, or users, who have only one antenna and in two-dimensional space, the term is being further simplified as:

$$H = \sum_{i=1}^{p} \alpha_i a_T(\theta_i)$$  \hspace{1cm} (6)

In equation 6, $a_T$ can be represented as a matrix:

$$a_T(\theta) = \begin{bmatrix} 1, e^{-j2\pi d \cos \theta}, \ldots, e^{-j2\pi d(N-1) \cos \theta} \end{bmatrix}^T$$  \hspace{1cm} (7)

Where $d$ is the distance between adjacent antennas at the base station, which is half of the wavelength of the transmitted signal, each exponential term represents the phase change and the delay due to different angle of arrival.

As for $\alpha_i$, which would be the gain of each path, to aid the computation simplicity, the strategy in [1] is employed such that would make the complex gain to be generated easier. In our channel model, the complex gain is expressed as: $\alpha_i \sim CN(0, g_k)$ Where $1/P \sum_{i=1}^{p} \alpha_i = 1$. In such way, the final channel model would become:

$$H = \frac{1}{\sqrt{P}} \sum_{i=1}^{p} \alpha_i a_T(\theta_i)$$  \hspace{1cm} (8)

3. Calculation

In the ABF stage, the Gram-Schmidt Orthogonalization [7] is applied to obtain the orthogonalized channel vector.

Gram-Schmidt Orthogonalization is a method to transfer a matrix $A$ into a standard orthogonal basis, for which $A$ is composed of linear independent column vector, denoted as $A = [a_1, a_2, \ldots, a_n]$ and the transfer relationship is denoted as $A = QR$ where $Q = [q_1, q_2, \ldots, q_n]$ is the obtained orthogonal matrix and $R$ is the upper triangle matrix. Based on the difference of obtaining the relationship of $A$ and $Q$, there are two types of orthogonalization called Classical Gram-Schmidt (CGS) orthogonalization and Modified Gram-Schmidt (MGS) orthogonalization.

Classical Gram-Schmidt (CGS)

$$q_1 = a_1$$
$$q_2 = a_2 - \frac{(a_2, a_1)}{(a_1, a_1)} q_1$$
$$q_n = a_n - \sum_{i=1}^{n-1} \left( \frac{(a_n, q_i)}{(q_i, q_i)} \right) q_i$$  \hspace{1cm} (9)

Modified Gram-Schmidt (MGS)

$$q_1 = a_1$$
$$q_2 = a_2$$
$$\ldots$$
$$q_n = a_n$$
$$e_1 = \frac{a_1}{|q_1|}; b_2 = b_2 - b_2^T e_1 e_1; b_3 = b_3 - b_3^T e_1 e_1; \ldots; b_n = b_n - b_n^T e_1 e_1;$$
$$e_2 = \frac{q_2}{|q_2|}; b_3 = b_3 - b_3^T e_2 e_2; \ldots; b_n = b_n - b_n^T e_2 e_2;$$
$$\ldots$$
$$e_n = \frac{q_n}{|q_n|}$$  \hspace{1cm} (10)

In the DBF stage, we use the MMSE algorithm. MMSE is essentially a method of getting the received data as close as possible to the sender data. After using Schmidt orthogonalization to normalize the channel matrix, the processed channel matrix is $\tilde{H} = w_{RF} \ast H$. If the input signal is $x$ and the output signal is $y$, then $y = \tilde{H}x + n$. 
Assume that input $x$ is the normalized input signal and $n$ is the normalized Gaussian noise. Therefore, the purpose of MMSE is to find the $w_{BB}$ matrix, make $w_{BB}$ as close as possible to matrix $X$, and reduce signal distortion.

Let $w_{BB} \ast y - x = e, E\{e^H e\}$ is the mean square error (MSE). We assume that the data received is irrelevant to $e$, then $E\{e^H e\} = 0$.

Apply $w_{BB} \ast y - x$ to the equation above, we can obtain:

$w_{BB} \ast E\{y \ast y^H\} - E\{x \ast y^H\} = 0$,

$w_{BB} = E\{x \ast y^H\}E\{y \ast y^H\}^{-1}$.

Then we will calculate $E\{x \ast y^H\}$ and $E\{y \ast y^H\}$. We assume that there is no correlation between input and noise, then $x n^H = 0, n x^H = 0$. So:

\[
E\{y \ast y^H\} = E\{(\tilde{H}x + n) \ast (\tilde{H}x + n)^H\} \\
= E\{(\tilde{H}x + n) \ast (x^H \tilde{H}^H + n^H)\} \\
= \tilde{H} \ast I_K \ast \tilde{H}^H + I_K, \\
= I_K \ast \tilde{H}^H
\]  

(11)

Where, $I_K$ is the identity matrix.

\[
E\{x \ast y^H\} = E\{x \ast (\tilde{H}x + n)^H\} \\
= E\{x \ast (x^H \tilde{H}^H + n^H)\} \\
= E\{x x^H\} \tilde{H}^H \\
= I_K \ast \tilde{H}^H
\]  

(12)

Thus,

$w_{BB} = E\{x \ast y^H\}E\{y \ast y^H\}^{-1} = \tilde{H}^H (\tilde{H} \tilde{H}^H + I_K)^{-1}$.

Through the above two steps, we can get the final HBF matrix $w_{HBF} = w_{RF} \ast w_{BB}$.

4. Result and Discussion

The base station (BS) is equipped with 128-antenna uniform linear array (ULA). The space between each antenna is half of the wavelength in the array. In order to serve five single-antenna users at the same time, BS is equipped with 5 (radio frequency) RF chains. Each user has five propagation paths, and each path’s average power is generated randomly from a uniform random variable in $[0, 1]$. Each user’s total received power need to be normalize which is required by power constraint. The angles of arrival (AoA) of each path is distributed uniformly in $[0, 2\pi]$.

We evaluate the uplink sum rate of the proposed hybrid beamforming (HBF) algorithm in different situations. We also provide several algorithms for comparison, including the full digital algorithm in [4], the maximizing minimum phase difference algorithm in [2] and the beam steering algorithm in [5].

In order to compare the advantages and disadvantages of each algorithm, we change the SNR, the number of users and the number of paths separately.

In SNR changing part, we give the different algorithm’s uplink sum rate when the SNR is - 20, - 10, 0, 10 and 20. In number of paths changing part, we select the number of paths to be 5, 6, 7, 8 and 9. In number of users changing part, we select the number of use to be 2, 4, 6, 8 and 10. For every different situation, the complex propagation gains are generated independently for 10000 times, then average the sum of 10000 times results to obtain a stable result.
Figure 1. Sum rates versus SNR.

The Figure 1 above shows the performance of our proposed algorithm and performance of other algorithms with different SNR. We assume that the number of the users is 5, and the number of paths is 5. We select the performance when the SNR is -20dB, -10dB, 0, 10dB, and 20dB. Our proposed algorithm has a higher sum rate than the beam steering algorithm and the maximizing minimum phase difference algorithm in every SNR. Compare to full digital algorithm, our proposed algorithm gets very close to it, however, there are still something can improve.

Figure 2. Sum rates versus number of paths.

In figure 2, We assume the SNR is 10dB and the number of users is 5. Our proposed algorithm still keeps good performance, which is only slightly worse than the full digital algorithm, but much better than other two.
Figure 3. Sum rates versus number of users.

In figure 3, we show the sum rates in different numbers of users. We assume the SNR is 10dB and the number of paths is 5. Obviously, compare to other hybrid beamforming algorithms, our proposed algorithm performs better. When the number of users is 2 and 4, our proposed algorithm’s performance is very close to fully digital. But as the number increasing, the gap between the performance of our recommended algorithm and all digital algorithm becomes more and more obvious. When the number of users increase, the problem of interference become harder, and that is the main point why our proposed algorithm cannot perform as good as full digital algorithm do.

5. Conclusion
In this work, the proposed hybrid beamforming is applied to multiuser mmWave MIMO systems with the fully connected HBF structure. The HBF matrix consists of ABF matrix and DBF matrix; The ABF matrix can be obtained by using the Gram-Schmidt Orthogonalization; The DBF matrix can be obtained by using MMSE. Finally, we have compared the performance of full-digital beamforming, hybrid beamforming and beam steering in mmWave Massive MIMO systems. By analyzing the sum rate of three different types of beamforming, our proposed algorithm can achieve a performance very close to the full digital algorithm, which proves the effectiveness of our proposed HBF algorithm.

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