Identification Method of Moment of Inertia with the Observed Angular Velocity Containing Gauss White Noise

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Abstract. The identification of the mass characteristic parameters of the spacecraft plays a vital role in improving the accuracy of the spacecraft control system. Due to the limit of sensor accuracy, there exists so much noise in the measurement angular velocity data, which causes great challenge of parameter identification. In order to solve the problem of identification of the spacecraft's moment of inertia, a new filter-identification method is proposed, which combines the square root volume Kalman filter and the recursive least square. Firstly, the filter method is used to remove the Gaussian white noise in the measurement equation and then the recursive least square method is used to calculate inertia of moment to deal with filtered data. The above steps continue to iterate until the results converge. The simulation results show that, compared with the identification result directly calculated by the recursive least square method, the identification accuracy of filter-identification method is improved by a half, which means this method is of great significance for enhancing the control accuracy and robustness of the spacecraft control system.

1. Introduction
With the development of science and technology and the continuous advancement of aerospace technology, on-orbit tasks have become more complicated. The high-precision pointing capability of the spacecraft will become an inevitable requirement for completing more complex and difficult on-orbit missions. Correspondingly, the control system of the spacecraft also needs to have higher precision and accuracy. In order to achieve high-precision control requirements, it is necessary to master accurate spacecraft control system parameters, including mass, inertia, sensor error, and so on. Therefore, before the satellite is launched, various methods will be carried out on the ground to measure its parameter characteristics. However, due to the existence of model errors and errors caused by measurement methods, the ground measurement results are not completely consistent with reality. In addition, during the process of launching, entering orbit, and long-term on-orbit operation of a spacecraft, vibrations, fuel consumption, etc. may cause changes in its characteristics, especially changes in its quality characteristics. In some cases, changes in quality characteristics cannot be accurately modeled and calculated, such as space robots capturing unknown targets, cleaning up space junk, or operating on non-cooperative targets, etc. [1]. This requires identification of the mass characteristics of the spacecraft with on-orbit data to ensure that the control system can always perform accurate orbit and attitude control. Through the actual data obtained from the orbit operation, using the hidden information in the data, the actual mass characteristics of the spacecraft can be estimated in orbit to ensure that the spacecraft can achieve high-precision control when the spacecraft is on orbit, which is the purpose of this paper.
At present, many domestic and foreign scholars have conducted research on the identification of spacecraft's quality characteristic parameters with changing service parameters on orbit. For the spacecraft with variable mass characteristics, many results have been obtained in the identification algorithm. These identification algorithms are mainly distinguished from different identification objects, different identification modelling principles, and different data processing algorithms.

Wilson E. et al. [2-5] studied the estimation of the mass characteristics of a rotating rigid body. The regression model established is non-linear in measurement, but linear in mass characteristics, and does not involve the linearization of rigid body rotation dynamics. Lampariello R. et al. [6] proposed a method of using accelerometers to directly identify the mass characteristic parameters of the on-orbit robot, which helps to improve the robot’s path planning and tracking capabilities, but the fuel consumption is relatively fast, and the on-orbit life and identification accuracy need to be improved. Bergmann E. et al. [7] used the second-order filter to measure the angular velocity noise based on the viewpoint of random estimation, obtained the inverse inertia matrix elements and the centroid position vector, and introduced the Taylor second-order term into the Kalman filter to improve the identification accuracy. Tanygin S. et al. [8] studied the mass parameter estimation problem of spin-stabilized spacecraft, and used the least square method to achieve identification. Sutter D. W. et al. [9] divided a single nonlinear problem into two or more independent linear problems, and proposed a multi-variable concurrent RLSM for on-orbit service spacecraft parameter identification.

There are some domestic researches on the identification of quality parameters. Wang Shuting et al. [10] used the recursive least squares algorithm to achieve the on-orbit spacecraft by considering structural changes and fuel consumption for the spacecraft whose actuator is a thruster. Parameter identification. Huang He et al. [11] proposed a closed-loop identification method of spacecraft quality characteristic parameters based on a variable structure controller. For the spacecraft with solar arrays, Xu Ying et al. [12] considered the coupling problem of parameters and various constraints in parameter identification, and used the least square algorithm to identify the spacecraft parameters in orbit. Some scholars have made some achievements in the parameter identification of space vehicles with flexible attachments. Lan Congchao et al. [13] aimed at large-scale spacecraft with flexible attachments, considering the impact of attachment vibration on parameter identification, combined the least squares method with the recursive least squares method to identify the spacecraft parameters on-orbit. Zhu Dongfang et al. [14] took the complex flexible satellite with the moment gyroscope as the actuator as the research object, and considered the timeliness and accuracy of parameter identification, and used the extended Kalman filter to identify the quality parameters of the complex flexible satellite in orbit.

Based on the above investigation, it can be seen that there is still much room for improvement in the timeliness and accuracy of the identification of spacecraft parameters. For the combination of space target rendezvous and docking, non-cooperative target capture and other tasks, the moment of inertia is usually unknown. How to effectively and accurately complete the estimation of the quality characteristics is the focus of such research. This article will focus on the identification of the spacecraft's moment of inertia. The least square method is used to identify its moment of inertia. In order to reduce the influence from the measurement noise, square root volume Kalman filter is combined with the least square method. First, the measurement matrix is filtered by the volume Kalman filter, and then the least square method is used for parameter identification.

The outline of the whole paper is organized as follows. Section 2 presents the rigid spacecraft attitude dynamics and defines the notation used in this paper. The recursive least square method is derived in Section 3. Then in the Section 4, a filtering method is first introduced for noise, and then a filtering-identification strategy is proposed. The Section 5 demonstrates the filter-identification’s performance by simulations, and Section 6 captures conclusions.

2. Model

The dynamic equation of the spacecraft can be written as:

$$ J\ddot{\omega} + [\omega]^T J\omega = u $$

(1)
Taking into account the characteristics of the moment of inertia matrix, there are
\[ J_{xy} = J_{yx}, \quad J_{xz} = J_{zx}, \quad J_{yz} = J_{zy} \],
so the moment of inertia matrix can be written in the form of a column vector
\[ \theta = \begin{bmatrix} J_{xx} & J_{xy} & J_{xz} & J_{yx} & J_{yz} & J_{zz} \end{bmatrix}^T \],
thus, the observable on the left side of the equal sign of the equation is separated from the quantity to be identified, and the formula (1) is transformed into the following form of linear equation set:
\[ u = H \cdot \theta \]

\[
\begin{bmatrix}
\dot{\omega}_1 \\
\dot{\omega}_2 \\
\dot{\omega}_3 \\
-\omega_1 \omega_3 \\
\omega_1 \omega_2 \\
-\omega_2 \omega_3 \\
\omega_1^2 - \omega_2^2 \\
-\dot{\omega}_1 - \dot{\omega}_2 \\
\omega_1 \omega_3 \\
\omega_1 \omega_2 \\
\dot{\omega}_2 - \dot{\omega}_3 \\
\omega_2 \omega_3 \\
\dot{\omega}_3 \\
-\omega_2 \omega_3 \\
\omega_2 \omega_1 \\
-\dot{\omega}_3 \\
\omega_3 \\
\dot{\omega}_3 \\
\end{bmatrix}
\begin{bmatrix}
J_{xx} \\
J_{xy} \\
J_{xz} \\
J_{yx} \\
J_{yz} \\
J_{zz} \\
\end{bmatrix} = \begin{bmatrix}
\omega_1 \omega_2 \\
\omega_1 \omega_3 \\
-\omega_1^2 + \omega_2^2 \\
-\omega_1^2 - \omega_2^2 \\
\omega_1 \omega_2 \\
\omega_1 \omega_3 \\
\omega_2 \omega_3 \\
\omega_2 \omega_1 \\
-\omega_2^2 + \omega_3^2 \\
-\omega_2^2 - \omega_3^2 \\
\omega_2 \omega_3 \\
\omega_3 \\
\end{bmatrix}
\begin{bmatrix}
J_{xx} \\
J_{xy} \\
J_{xz} \\
J_{yx} \\
J_{yz} \\
J_{zz} \\
\end{bmatrix}
\]

According to formula (2), the discrete observation equation at moment \( k \) is:
\[ u(k) = h(k) \cdot \theta \]
where, \( u(k) \in \mathbb{R}^{3 \times 1} \) is the given control moment, \( h(k) \in \mathbb{R}^{3 \times 6} \) is the observation matrix, and \( \theta \in \mathbb{R}^{6 \times 1} \) is the moment of inertia to be identified.

The total observation equation at moment \( k \) and before, denoted as:
\[ U(k) = H(k) \cdot \theta \]
where the given control torque \( U(k) \in \mathbb{R}^{3 \times 1} \), the observation matrix \( H(k) \in \mathbb{R}^{3 \times 6} \), and the moment of inertia to be identified \( \theta \in \mathbb{R}^{6 \times 1} \) are the data at moment \( k \) and before.\[15\]

### 3. Identification of Moment of Inertia by Least Square Method

According to the least square estimation criterion, the estimated value of the moment of inertia at the moment \( k \) \( \hat{\theta}_k \) can be obtained by calculation based on the data at time \( k \) and before
\[ \hat{\theta}_k = \left[H^T(k) \cdot H(k)\right]^{-1} H^T(k) \cdot U(k) \]

Add a new observation equation at moment \( k+1 \)
\[ u(k+1) = h(k+1) \cdot \theta + v(k+1) \]
Then the total observation equation at moment \( k+1 \) and before is denoted as
\[ U(k+1) = H(k+1) \cdot \theta + V(k+1) \]
Similarly, you can use the least square method to calculate the new estimated value of the moment of inertia with reference to equation (5)
\[ \hat{\theta}_{k+1} = \left[H^T(k+1) \cdot H(k+1)\right]^{-1} H^T(k+1) \cdot U(k+1) \]

However, through observation, it can be found that equation (8) includes the workload of repeated calculation in equation (5), and as the number of observations increases, the observation matrix becomes more and more complex, and the calculation results at the latter moment are greatly affected by the previous data, the calculation efficiency is also reduced. In order to overcome the above shortcomings, the recursive least squares algorithm is adopted—using the estimated moment of inertia calculated at the previous moment \( \hat{\theta}_k \), combined with the observation data at the next moment to obtain a new estimated moment of inertia \( \hat{\theta}_{k+1} \).
Expand the first term of equation (8)

\[ H^T (k+1) \cdot H(k+1) = \begin{bmatrix} H(k) \\ h(k+1) \end{bmatrix}^T \begin{bmatrix} H(k) \\ h(k+1) \end{bmatrix} \]

\[ = H(k)^T H(k) + h(k+1)^T h(k+1) \]

Equation (8) can be written as

\[ \hat{\theta}_{k+1} = \left[ H^T (k) H(k) + h^T (k+1) h(k+1) \right]^{-1} \left[ H^T (k) U(k) + h^T (k+1) u(k+1) \right] \]

Introduce the matrix inversion formula

\[ (A + B \cdot B^T)^{-1} = A^{-1} - A^{-1} B \cdot (I + B^T A^{-1} B)^{-1} B^T A^{-1} \]

Note \( P(k) = \left[ H^T (k) H(k) \right]^{-1} \), then \( P(k+1) = \left[ P(k)^{-1} + h^T (k+1) h(k+1) \right]^{-1} \) may be expanded to

\[ P(k+1) = P(k) - P(k) h^T (k+1) \cdot \left[ I + h(k+1) P(k) h^T (k+1) \right]^{-1} h(k+1) P(k) \]

Record the gain matrix \( K_k \) as

\[ K_k = P(k) \cdot h^T (k+1) \cdot \left[ I + h(k+1) P(k) h^T (k+1) \right]^{-1} \]

Equation (12) can be written as

\[ P(k+1) = P(k) - K_k h(k+1) P(k) \]

Substituting formula (14) into formula (10), we can get

\[ \hat{\theta}_{k+1} = \hat{\theta}_k - K_k h(k+1) \hat{\theta}_k + \left[ P(k) h^T (k+1) - K_k h(k+1) P(k) h^T (k+1) \right] u(k+1) \]

According to formula (13), we can get

\[ P(k) \cdot h^T (k+1) = K_k \left[ I + h(k+1) P(k) h^T (k+1) \right] \]

Replace the first product term in the brackets in (15) with the above formula, we can get

\[ \hat{\theta}_{k+1} = \hat{\theta}_k - K_k h(k+1) \hat{\theta}_k + K_k \left[ I + h(k+1) P(k) h^T (k+1) \right] y(k+1) \]

In summary, the recursive calculation process can be written as

1) Calculate the estimated value of the moment of inertia at moment \( k \) \( \hat{\theta}_k \) according to formula (5);

2) Use the formula \( P(k) = \left[ H^T (k) H(k) \right]^{-1} \) to calculate the information inverse matrix \( P(k) \) at moment \( k \);

3) Calculate the gain matrix \( K_k \) according to formula (13);
4) Combining $K_k$ and the observed value at moment $k + 1$, use equation (17) to update the estimated value of the moment of inertia at moment $k + 1$;

5) Update the information inverse matrix $P(k + 1)$ at moment $k + 1$ according to formula (14);

6) Judge whether it meets the accuracy requirements $\max_i \left| \frac{\dot{\theta}_{k+1}(i) - \hat{\theta}_k(i)}{\dot{\theta}_{k+1}(i)} \right| < \varepsilon$, stop the loop if it meets the requirements, and skip to step 3 to continue iterating if it doesn't.[16]

4. Identification of Moment of Inertia with Measurement Noise

Since for the identification equation $H = U$, the measurement data containing Gaussian white noise mainly affects the accuracy of the measurement matrix, it is necessary to filter the angular velocity information before performing the moment of inertia identification. The common extended Kalman filter uses Taylor expansion method to convert nonlinear equations into linear equations. Although this method has many engineering applications in weakly nonlinear systems, the filtering effect is not very good for strongly nonlinear systems. Unscented Kalman filtering is only suitable for low-order nonlinear systems, so the volumetric Kalman filtering method is used to filter the angular velocity data with Gaussian white noise, and based on this online identification of the moment of inertia is performed. Taking into account the possible non-negative situation of the covariance matrix of the nonlinear system, the square root volume Kalman filter is used, and the QR decomposition is used to replace the Cholesky decomposition of the state covariance matrix except the initial time.

4.1. Square root volume Kalman filter

The state space of the spacecraft considering the observation noise is described as

$$
\begin{align*}
\dot{x}(k + 1) &= x(k) - J^{-1} x(k) \cdot (x(k)) \cdot \Delta t + J^{-1} u(k) \cdot \Delta t + \xi_k \\
v(k) &= x(k) + \eta_k
\end{align*}
$$

(18)

where $x$ is the angular velocity, $\Delta t$ is the time interval, $\{\xi_k\}$ and $\{\eta_k\}$ are the system noise sequence and the observation noise sequence (unknown) with statistical information such as known mean, variance and covariance, respectively. Here $\xi_k = 0$.[17]

**Assumption 4.1** Let $\{\xi_k\}$ and $\{\eta_k\}$ be zero-mean Gaussian white noise sequences, then for all $k$ and $l$, $\text{Var}(\xi_k) = Q_k$ and $\text{Var}(\eta_k) = R_k$ are positive definite matrices, and $E(\xi_k \eta_l) = 0$.

Knowing the state estimate $\hat{x}_0$ and the state covariance $P_0$ at moment 0, find the square root of $P_0$

$$
S_0 = \text{chol}(P_0)
$$

(19)

1) Time update

Generate $2n$ equal weighted volume points $\zeta_i$, namely

$$
\zeta_i = \begin{bmatrix} 1 \\ \sqrt{n} \end{bmatrix}
$$

(20)

where $[1]$ represents an n-dimensional column vector with the i-th element being 1 and the remaining elements being 0.

Calculate volume point

$$
x'_i = S_k \zeta_i + \hat{x}_k
$$

(21)

After the volume point is transferred through the kinematic equation, the volume point estimate at time $k + 1$ can be obtained based on the volume point at time $k$.
\[ x_{k+1|k}^{i} = f\left( x_{k}^{i} \right) \]  

(22)

Taking the average of the above 2n volume points, the estimated value of state quantity at moment \( k+1 \) can be obtained

\[ \hat{x}_{k+1|k} = \frac{\sum_{i=1}^{2n} x_{k+1|k}^{i}}{2n} \]  

(23)

2) Measurement update

Calculate the square root of the state prediction covariance matrix

\[ S_{k+1|k} = \text{Tria}\left( \begin{bmatrix} A_{k+1|k} & S_{0k} \end{bmatrix}^{T} \right) \]  

(24)

where

\[ A_{k+1|k} = \frac{1}{\sqrt{2n}} \left[ x_{k+1|k}^{1} - \hat{x}_{k+1|k}, x_{k+1|k}^{2} - \hat{x}_{k+1|k}, \ldots, x_{k+1|k}^{2n} - \hat{x}_{k+1|k} \right] \]  

(25)

\( \text{Tria}(\cdot) \) means QR decomposition first, and then the transpose of the upper triangular matrix R.

Select volume point

\[ x_{k+1|k}^{i} = S_{k+1|k} z_{i} + \hat{x}_{k+1|k} \]  

(26)

The volume point can be obtained after passing the measurement equation

\[ z_{k+1}^{i} = h\left( x_{k+1|k}^{i} \right) \]  

(27)

Take the average of the above 2n volume points to obtain the predicted value of the measurement

\[ \hat{z}_{k+1|k} = \frac{\sum_{i=1}^{2n} z_{k+1|k}^{i}}{2n} \]  

(28)

Calculate the square root of the measurement error covariance matrix is

\[ S_{k+1|k}^{zz} = \text{Tria}\left( \begin{bmatrix} B_{k+1|k} & S_{Rk} \end{bmatrix}^{T} \right) \]  

(29)

where

\[ B_{k+1|k} = \frac{1}{\sqrt{2n}} \left[ z_{k+1}^{1} - \hat{z}_{k+1}, z_{k+1}^{2} - \hat{z}_{k+1}, \ldots, z_{k+1}^{2n} - \hat{z}_{k+1} \right] \]  

(30)

Measuring cross-correlation covariance matrix is

\[ P_{k+1|k}^{xz} = C_{k+1|k} B_{k+1|k}^{T} \]  

(31)

where

\[ C_{k+1|k} = \frac{1}{\sqrt{2n}} \left[ x_{k+1|k}^{1} - \hat{x}_{k+1|k}, x_{k+1|k}^{2} - \hat{x}_{k+1|k}, \ldots, x_{k+1|k}^{2n} - \hat{x}_{k+1|k} \right] \]  

(32)

The gain matrix \( K_k \) can be written as

\[ K_k = P_{k+1|k}^{xz} \left( S_{k+1|k}^{zz} \right)^{-1} \]  

(33)

State estimate is

\[ \hat{x}_{k+1} = \hat{x}_{k+1|k} + K_k \left( z_k - \hat{z}_k \right) \]  

(34)

Square root of covariance matrix is

\[ S_{k+1} = \text{Tria}\left( \begin{bmatrix} C_{k+1|k} - K_k B_{k+1|k} & K_k S_{Rk} \end{bmatrix}^{T} \right) \]  

(35)
4.2. Identification Strategy of Moment of Inertia by Introducing Measurement Noise

This section proposes the filtering-least squares method. After collecting the data, perform least-squares identification on all the collected data, and then substitute the relatively stable moment of inertia into the original data again for filtering, and iterate repeatedly until the moment of inertia identification value converges.

The filtering-least squares calculation process can be sorted into:

1) Assuming that the moment of inertia in the dynamic equation of the spacecraft is $J_k$, the square root volume Kalman filter method is used to filter the angular velocity signal to obtain a set of angular velocity estimated values $\hat{\omega}$.

2) Use the recursive least square method to identify the angular velocity estimated value $\hat{\omega}$ obtained in step 1) to obtain the moment of inertia $J_{k+1}$.

3) Judge whether the moment of inertia satisfies the accuracy condition. If the condition is not satisfied, substitute the identified value of moment of inertia $J_{k+1}$ obtained in step 2) as the updated moment of inertia $J_k$ of step 1), and continue iterating; Condition, stop the iteration, and output the moment of inertia.

5. Simulation

Before verifying the algorithm, the angular velocity measurement value needs to be obtained first. Give the system a continuously changing excitation, and according to the dynamic equation (36) of the spacecraft, a series of data sets of angular velocity varying with time are obtained. The simulation parameters are shown in the table, where the moment of inertia of spacecraft $J$ is the true value of the subsequent moment of inertia to be identified, and the control moment $u$ is the given and known continuous excitation of the system. The measured angular velocity versus time curve is shown in Figure 1.

$$\ddot{\omega} = -J^{-1}[\omega]^T J \omega + J^{-1}u$$  \hspace{1cm} (36)

| Parameter name                        | Numerical value                  |
|---------------------------------------|----------------------------------|
| Sampling interval $T$ (s)             | 1                                |
| Initial angular velocity $\omega$ (rad/s) | $[0.051 \ 0.046 \ 0.048]^T$     |
| Moment of inertia of spacecraft $J$ (kgm$^2$) | $\begin{bmatrix} 106.8 & 20.6 & 28.1 \\ 20.6 & 108.5 & 22.7 \\ 28.1 & 22.7 & 109.5 \end{bmatrix}$ |
| Control torque $u$ (Nm)               | $\begin{bmatrix} \sin \left(\frac{\pi t}{50} \right) \\ \sin \left(\frac{\pi t}{50} \right) \\ \cos \left(\frac{\pi t}{50} \right) \end{bmatrix} \times 0.1$ |
| Expectation of Gauss White Noise      | 0                                |
| Variance of Gauss White Noise         | $10^{-8}$                        |
The simulation results of the recursive least squares method are shown in the Figure 2. It is obvious that when $t=2203s$, the identification results are considered as convergent. The maximum error ratio is $\Delta J_{\text{max}}$, which can reach 12.8641%, and the minimum error ratio is $\Delta J_{\text{min}}$, which is 3.2375%. This method has poor recognition accuracy. It can be summarized into two reasons: 1) the precision of angular velocity is greatly affected by the Gaussian white noise; 2) the angular acceleration is calculated by the angular velocity differential at adjacent moments which makes angular acceleration imprecise. The angular velocity observation error leads to a large error in the observation matrix. In order to solve the first problem, filtering algorithm is introduced to preserve the effective angular velocity information and remove the useless noise. When it comes to the second problem, for the true angular velocity the sampling time is less and the angular acceleration is more accurate, and the simulation accuracy can be less than 1% as $T=0.1s$. However, if $T<1$, the angular acceleration is amplified, the simulation accuracy of the recursive least squares method is greatly dropped. The simplest method is to add angular accelerometer, which is not considered in this paper.

The simulation result of filtering-least squares method is shown in the Figure 3, where $[100,20,20,100,20,100]^T$ is selected as the initial value of $J_k$. The comparison of the two methods is shown in the table 2. In order to satisfy the convergent requirements, filtering-identification process is looped 8 times. It can be seen from Figure 3 that at the last iteration when $t=2005s$, the identification...
result can be regarded as convergent. The maximum error rate is $\Delta J_{xx}$, which can reach 6.5506%, and the minimum error rate is $\Delta J_{zz}$, which is 1.1045%. Compared with the recursive least squares method, the convergence time is shortened, and the maximum and minimum error rates are reduced in half, that is, the recognition accuracy is greatly improved. The square root volume Kalman filter effectively improves the angular velocity accuracy and removes the influence of Gaussian white noise. Moreover, the improvement of the angular velocity accuracy also effectively improves the accuracy of the angular acceleration, making the angular acceleration more accurate. During simulation, it can be found that this method has little dependence on the initial value. In addition, the filter-identification method has a strong robustness and the anti-interference ability.

![Identification value of moment of inertia](image1.png)

(a) Identification value of moment of inertia

![The difference between the identified value of the moment of inertia and the true value](image2.png)

(b) The difference between the identified value of the moment of inertia and the true value

Figure 3 Filter-least squares method of moment of inertia identification result

| Identification method                      | $\Delta J_{xx}$ | $\Delta J_{xy}$ | $\Delta J_{xz}$ | $\Delta J_{yy}$ | $\Delta J_{yz}$ | $\Delta J_{zz}$ |
|-------------------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| SCKF filtering-least squares method       | -4.5056         | 0.9845          | -1.2011         | -3.5228         | -1.4870         | -1.2094         |
|                                           | 4.2148%         | 4.7790%         | 4.2744%         | 3.2468%         | 6.5506%         | 1.1045%         |
| Recursive least squares                   | -10.8069        | -0.6669         | -2.4867         | -9.7919         | -2.9202         | -8.7426         |
|                                           | 10.1093%        | 3.2375%         | 8.8496%         | 9.0248%         | 12.8641%        | 7.9841%         |

6. Conclusion
The identification accuracy of the least squares method based on square root volume Kalman filter is obviously improved compared with the recursive least square method, which is of great significance for improving the accuracy of the control system. The maximum identification error of the recursive least square method is 12.8641%, while the maximum error is reduced to 6.5506%, which is almost a half of previous value, after introducing square root volume Kalman filter to remove Gaussian white noise. This shows that for the observed angular velocity data containing Gaussian white noise, using the square root volume Kalman filter and the least square method for parameter identification will effectively improve the accuracy of the algorithm and greatly reduce the error. This conclusion is of great significance to practical applications. The algorithm can be applied to the identification of the spacecraft's mass parameters with the existence of measurement noise, thereby achieving the enhancement of the accuracy of the spacecraft control system.
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