A Theoretical Model of Non-conservative Mass Transfer with Non-uniform Mass Accretion Rate in Close Binary Stars

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Mass transfer in close binaries is often non-conservative and the modeling of this kind of mass transfer is mathematically challenging as in this case due to the loss of mass as well as angular momentum the governing system gets complicated and uncertain. In the present work a new mathematical model has been prescribed for the non-conservative mass transfer in a close binary system taking into account the gradually decreasing profile of the mass accretion rate by the accreting star with respect to time as well as with respect to the increase in mass of the accreting star. The process of mass transfer is understood to occur up to a critical mass limit of the accreting star beyond which this process may cease to work.

Keywords: Close binary; non-conservative mass transfer; primary star; secondary star

I. INTRODUCTION

A Binary star is a star system consisting of two stars orbiting around their common centre of mass (Walter, 1940; Kuiper, 1941). The brighter star is called the primary and the other is its companion star or secondary. In fact, at least 50%, of the stars are found to be in binary systems. (Abt, 1983; Pinfield et al., 2003).

The orbital periods (Porb) of binary stars range from 11 minutes to $\sim 10^6$ years (Podsiadlowski, 2001). The Roche lobe is the region of space around a star in a binary system within which orbiting material is gravitationally bound to that star (Herbig, 1957; Plavec, 1966). Detached binaries are binary stars where each component is within its Roche lobe and neither star fills its Roche lobe. Mass transfers are unlikely for this category of binaries. Semi-detached binaries (Kopal, 1955) are binary stars where one of the components fills the binary stars Roche lobe and the other does not. A contact binary (Kopal, 1955) is a type of binary in which both components of the binary fill their Roche lobes. In case of semi-detached and contact binaries, mass transfer is quite expected. In fact, the majority of binaries are in fairly wide systems that do not interact strongly and both the stars evolve essentially as single star. But there is a large fraction of systems (for Porb at the left hand neighbourhood of 10 years) that are close enough (Podsiadlowski, 2001) that mass is transferred from one star to another which changes the structures of both the stars and their subsequent evolution. They are termed as close binaries. Although the exact numbers are somewhat uncertain, binary surveys suggest that the close binaries range between 30% to 50% (Duquennoy and Mayor, 1991; Kobulnicky and Fryer, 2007) of the total binary population.

Crawford (1955) and Hoyle (1955) independently proposed that the observed secondaries are originally the more massive stars. At the primary level both the companions in a binary evolve independently. But as the secondary expands beyond its Roche limit, then the material can escape the gravitational pull of the star and eventually the process of mass transfer starts from secondary to primary. The material will fall in through the inner Lagrangian point. It is an approximately tear-drop shaped region bounded by a critical gravitational equipotential. Normally there are two possible mechanisms for mass transfer between stars in a close binary system. The first one is conservative mass transfer in which both the total mass and angular momentum of the binary are conserved and the second one is non-conservative mass transfer where both the total mass and angular momentum decays with time. Observationally, there are evidences for both conservative and non conservative mass transfer in close binaries (Podsiadlowski, 2001; Yakut, 2006; Manzoori, 2011; Onno, 2012).

The modelling of non-conservative mass transfer is a challenging one as in this case governing mechanism becomes very much complicated and uncertain due to the loss of the mass as well as the angular momentum in the binary. Mass transfer in close binaries is often non-conservative and the ejected material moves slowly enough that it can remain available for subsequent star formation (Izzard et al., 2013). Some communications are available which have tried to address some issues in the non-conservative mass transfer. Packet (1981) proposed that accretion and spin up continues until the mass of the primary (accretor) increases by about 10% at which point it rotates so fast that material at its equator is unbounded. Stepień (1995) calculated the relative angular momentum lost from the system for a non-conservative mass transfer with uniform accretion rate.

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Soberman et al. (1997) discussed on different types of modes of mass transfer like Roche lobe overflow, Jeans mode and produced detailed description of orbital evolution for non-conservative mass transfer for different modes as well as for a combination of several modes. Podsiadlowski (2001) discussed briefly the magnetic braking for non-conservative mass transfer in a binary system. Sepinsky et al. (2006) discussed the mass loss in the non-conservative mass transfer and loss of angular momentum in eccentric binaries. Van Rensbergen et al. (2010) discussed on the mass loss out of close binaries. Manzoori (2011) sketched the effects of magnetic fields on the mass loss and mass transfer for non-conservative mass exchange with uniform rate of accretion by the primary. Davis et al. (2013) demonstrated mass transfer in eccentric binary systems using binary evolution code. Interestingly, the above works mainly dealt with uniform mass accretion rate by the primary. But real situations may not always play by this simple rule. In this regard we must mention that Stepien and Kiraga (2013) while detailing on the evolutionary process under the non-conservative mass transfer in close binary system argued for non-uniform rate of mass accretion with respect to time.

In the present work we have tried to introduce a new mathematical model for the non-conservative mass transfer in the close binary which can address the issues like the mode of mass transfer and mass loss under the consideration of the critical mass limit (for the accretor) (Packet, 1981) of the transfer. The present model has been prescribed taking in to account the gradually decreasing profile of the mass accretion rate by the accreting star with respect to time as well as with respect to the increase in mass of the accreting star and consequent time dependent mass profiles of the component stars and the orbital angular momentum of the close binary system have been demonstrated. We have also provided a numerical model in view of our present consideration.

II. THEORY: NON-CONSERVATIVE MASS TRANSFER:

We first assume that $M_1$ is the mass of the primary star (accretor) and $M_2$ is the mass of the secondary star (donor) at time $t$. Initially $M_2 > M_1$ i.e. greater mass is discharging mass and lower mass is in taking mass.

We here consider that the process of this mass exchange continues within the range $M_1 < M^*_1$, where $M^*_1$ is supposed to be the critical mass of the accretor beyond which the process completely stops. This assumption is made following the argument made by Packet (1981) that the process of mass transfer continues till the accretor gets 10% increase in its weight as beyond this the accretor rotates so fast that material at the equator gets unbounded.

We consider the following model of non-conservative mass transfer:

\[ M_1 = \beta e^{-\alpha t} \left( 1 - \frac{M_1}{M^*_1} \right) M_2 \]  \hspace{1cm} (1)

\[ (\alpha > 0, \, \beta > 0) \]

The above model is proposed following the argument of Stepien and Kiraga (2013) that for non-conservative mass transfer this mass exchange rate decreases gradually with time and also according to Izzard et al. (2013) this mass exchange rate is very slow and decreases as the accretor captures mass. In view of this we introduced two factors in the above model: the first factor $e^{-\alpha t}$ is to trace the gradually decreasing profile of mass exchange rate with time and in this connection the parameter $\alpha > 0$ must be small enough so that this rate of exchange remains feeble altogether. The second factor $\left( 1 - \frac{M_1}{M^*_1} \right)$ is to capture the gradually decreasing profile of this exchange rate with the increase in $M_1$ within the limit $M_1 < M^*_1$ as mentioned earlier. Here the parameter $\beta$ is dimensionless but the parameter $\alpha$ has the unit ‘per unit time’.

Now applying Bernoulli’s law to the gas flow through the inner Lagrangian point, we get

\[ M_2 = -A \frac{M_2}{P} \left( \frac{\Delta R}{R} \right) \]  \hspace{1cm} (Onno, 2012) \hspace{1cm} (2)

where $A$ is a numerical constant likely to be between 1 and 2.

Since, \( \left( \frac{\Delta R}{R} \right) \approx \left( \frac{\dot{M}}{\tau} \right)^{\frac{1}{2}} \) (Onno, 2012) where $\tau$ is the total timescale of mass transfer and $P$ is the orbital time period we therefore have,

\[ \dot{M}_2 = -AP \frac{\dot{M}_2}{\tau} \frac{1}{\left( \frac{\dot{M}_1}{\tau} \right)^{\frac{1}{2}}} M_2 \]  \hspace{1cm} (3)

For stars with convective envelopes, i.e. for red-giants or low-mass main sequence stars, $\gamma = \frac{5}{3}$ and this gives us from (3),

\[ \dot{M}_2 = -A \frac{\dot{M}_2}{\tau} M_2 \]  \hspace{1cm} (4)

Using (4) in (1),

\[ \dot{M}_1 = -A \frac{\beta}{\tau} e^{-\alpha t} \left( 1 - \frac{M_1}{M^*_1} \right) M_2 \]  \hspace{1cm} (5)

We consider that at $t=0$ (when this mass exchange started) the initial masses of accretor and donor were $M_{1,0}$ and $M_{2,0}$ respectively. As $\tau$ is taken to be the total timescale of mass transfer we can believe that $\tau$ is the hypothetical time taken by the accretor to reach the mass
starting from $M_{1,0}$, provided there is no reverse mass transfer or any other issue preventing this mass exchange. On integration, (4) gives,

$$M_2 = M_{2,0}e^{-\frac{\Delta t}{\tau}}$$

(6)

This gives,

$$M_{2,\tau} = M_{2,0}e^{-A}$$

(7)

Again integrating (5) we get,

$$\int_{M'_{1}=M_{1,0}}^{M_1} \frac{dM'}{1 - \frac{M'}{M_1}} = -\frac{A\beta}{\tau} \int_{t'=0}^{t} e^{-\alpha t'} M'_2 dt'$$

(8)

Using (6) in (8),

$$\int_{M'_{1}=M_{1,0}}^{M_1} \frac{dM'}{1 - \frac{M'}{M_1}} = -\frac{A\beta}{\tau} \int_{t'=0}^{t} e^{-\alpha t'} M_{2,0}e^{-\frac{\Delta t}{\tau}} dt'$$

This gives,

$$M_1 = M'_1 \left[ 1 - \left(1 - \frac{M_{1,0}}{M_1} \right)e^{-\frac{A\beta}{\tau} \frac{M_{2,0}}{M_1^{(\alpha + \beta)^{\tau}}} \left(1 - e^{-\left(\alpha + \beta\right)\tau}\right)} \right]$$

(10)

for $t \leq \tau$.

Now as the process of mass transfer continues till the accretor gets 10% increase in its weight (Packet, 1981) we have,

$$M'_1 = \frac{11}{10} M_{1,0}$$

(11)

Thus from (10) we get,

$$M_1 = \frac{11}{10} M_{1,0} \left[ 1 - \frac{1}{11} e^{-\frac{10 A \beta}{11 M_{1,0}^{(\alpha + \beta)^{\tau}}} \left(1 - e^{-\left(\alpha + \beta\right)\tau}\right)} \right]$$

(12)

for $t \leq \tau$.

We expect that at $t = \tau$, $M_1$ appreciably nears to the critical value $M_1''$ and this suggests us to take

$$\frac{1}{11} e^{-\frac{10 A \beta}{11 M_{1,0}^{(\alpha + \beta)^{\tau}}} \left(1 - e^{-\left(\alpha + \beta\right)\tau}\right)}$$

at the right hand neighbourhood of zero. The parameters $\alpha$ and $\beta$ must assume their magnitudes in accordance with this.

As we know that in non-conservative mass transfer, less than 25% of the mass ejected from the secondary reaches the primary i.e. as the accretion efficiency is less than about 0.25 (de Mink et al., 2009) we have $\forall t \leq \tau$,

$$\beta e^{-\alpha t} \left(1 - \frac{M_1}{M_1'}\right) < \frac{1}{4}$$

This gives,

$$\beta < \text{Min}_t \left[ \frac{e^{\alpha t} \left(1 - \frac{M_1}{M_1'}\right)}{4} \right]$$

(13)

We have, $\Delta R \approx \left(\frac{2}{R}\right)^{1/2} \text{(Onno, 2012)}$ where $\Delta R = R - R_L$, $R$ being the radius of the donor and $R_L$ being the Roche lobe radius. We also have $\Delta R < 0.01$ (Onno, 2012). So we can take,

$$\left(\frac{P}{\tau}\right)^{1/2} < 0.01$$

(14)

The orbital period should be less than 100 days so that the star can fill its Roche lobe during its expansion through main sequence phase till to a red giant (Mon- zooi, 2011). This gives us $P \sim 10$ (days) and from (14) we can consider $\tau > 10^7$ days i.e. $\tau > 10^4$ years. On the other hand, we have $\tau < 10^6$ years (Postonov and Yungelson, 2014). This gives finally,

$$10^4 < \tau < 10^6 \text{ (years)}$$

(15)

As $A$ is expected to lie in the range between 1 and 2 and further $\beta < \frac{1}{4}$, the expression

$$\frac{10 A \beta}{11 M_{1,0}^{(\alpha + \beta)^{\tau}}} \frac{M_{2,0}}{M_1''}$$

is expected to be in the range of order between 1 and $10^2$ depending on the ratio $\frac{M_{2,0}}{M_1''}$. Thus in order to bring the expression

$$\frac{1}{11} e^{-\frac{10 A \beta}{11 M_{1,0}^{(\alpha + \beta)^{\tau}}} \left(1 - e^{-\left(\alpha + \beta\right)\tau}\right)}$$

at the vicinity of zero as argued previously we must take $(\alpha \tau + A)$ in the order of 10 which gives also, $\alpha \tau \sim 10$ as
A is expected to be in the range between 1 and 2. This gives,

\[10^{-5} < \alpha < 10^{-3} \quad \text{(per years)} \quad \text{[using (15)]} \quad (16)\]

The orbital angular momentum (OAM) of a circularized close binary system at time \( t \) is given by, (Demircan et al., 2006)

\[J = M_1M_2 \sqrt{\frac{Ga}{M}} \quad (17)\]

where, \( M = M_1 + M_2 \) is the total mass of the system at time \( t \), \( a \) is the separation between \( M_1 \) and \( M_2 \), and \( G \) is the gravitational constant. (17) can be written as

\[J = M_1M_2 \left( \frac{G}{\omega M} \right) \quad (18)\]

where

\[\omega^2 = \frac{GM}{a^3}, \quad (19)\]

\( \omega \) being the binary angular velocity.

We have from (18),

\[J = M_1M_2 \left( \frac{PG^2}{2\pi M} \right)^{\frac{1}{3}} \quad (20)\]

where the orbital period is \( \frac{2\pi}{\omega} \).

From (20) combining the results of (6) and (12), we can obtain the time dependent profile of the orbital angular momentum \( J \).

**III. RESULTS:**

We here produce a numerical example for the non-conservative mass transfer in close binary system taking the initial masses of the primary and secondary as \( M_{1,0} = 2.5 \times 10^{32} \) (gm) and \( M_{2,0} = 5 \times 10^{33} \) (gm). For the present calculation, we consider the values of the parameters as \( A = 1.5, \alpha = 10^{-4} \) (per year) and \( \beta = 2 \). The profiles of the change in masses for both primary and secondary as well as the changing profile of the orbital angular velocity with time are observed within the pre-assigned time scale of mass exchange as \( \tau = 10^5 \) (years) and orbital period \( P = 90 \) (days). In this regard the graphs for \( \frac{M_1}{M_{1,0}} \) against \( t \) (Fig. 1), \( \frac{M_2}{M_{2,0}} \) against \( t \) (Fig. 2) and \( J \) against \( t \) (Fig. 3) are demonstrated to visualize this changing profile in the binary. Interestingly in the present calculation we can observe some increase in the magnitude of \( J \) at the very initial phase (around up to 3000 years). But after that phase we can see a steady decrease in \( J \) (see Fig. 3). This might have occurred due to the reason that at the initial level \( M_1 \) increases sharply while \( M_2 \) experiences steady decrease. This altogether may have generated this kind of convex shape in the graph of \( J \) at the initial phase. But afterwards as the mass accretion rate by the primary varies slowly and consequently \( M_1 \) shows a very weak variation with time, \( J \) continues to show a steady decreasing profile with the decrease in \( M_2 \) [in view of (20)].

**IV. DISCUSSION:**

In this paper we have made an effort to establish a mathematical model of non-conservative mass transfer in close binary systems which can address the gradually reducing rate of the mass accretion by the accretor from the donor with respect to time (Stepien and Kiraga, 2013) as well as with respect to the increase in mass of the accretor (Izzard et al., 2013). In the present model we have taken into account a critical mass limit of the accretor for this mass exchange (Packet, 1981) beyond which this process may not be continued. In view of this we have presented the time dependent profiles of the masses of both the component stars in a close binary as well as...
of the orbital angular momentum of the corresponding binary system. We have presented a numerical example in this regard to demonstrate the changing profiles of masses of the component stars in a close binary as well as the orbital angular momentum of the system. Magnetic field can play a significant role in this mass transfer and mass loss for some close binary systems (Podsiadlowski, 2001; Manzoori, 2011). This can be an interesting future study to incorporate the effect of the magnetic field in the present model. On the other hand there have been claims on the possibility of reverse mass transfer in binary systems (Crawford, 1955; Hoyle, 1955). There is also a suggestion to include the effect of neutrino emission according to the photon-neutrino coupling theory for better understanding of the evolution of binary systems (Raychaudhuri, 2013). Future study in this context may focus also on these issues.

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