ON THE FORMATION OF MASSIVE STARS

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ABSTRACT

We calculate numerically the collapse of slowly rotating, nonmagnetic, massive molecular clumps of masses 30, 60, and 120 $M_\odot$, which conceivably could lead to the formation of massive stars. Because radiative acceleration on dust grains plays a critical role in the clump’s dynamical evolution, we have improved the module for continuum radiation transfer in an existing two-dimensional (axial symmetry assumed) radiation hydrodynamic code. In particular, rather than using “gray” dust opacities and “gray” radiation transfer, we calculate the dust’s wavelength-dependent absorption and emission simultaneously with the radiation density at each wavelength and the equilibrium temperatures of three grain components: amorphous carbon particles, silicates, and “dirty ice”–coated silicates. Because our simulations cannot spatially resolve the innermost regions of the molecular clump, however, we cannot distinguish between the formation of a dense central cluster or a single massive object. Furthermore, we cannot exclude significant mass loss from the central object(s) that may interact with the inflow into the central grid cell. Thus, with our basic assumption that all material in the innermost grid cell accretes onto a single object, we are able to provide only an upper limit to the mass of stars that could possibly be formed. We introduce a semianalytical scheme for augmenting existing evolutionary tracks of pre–main-sequence protostars by including the effects of accretion. By considering an open outermost boundary, an arbitrary amount of material could, in principle, be accreted onto this central star. However, for the three cases considered (30, 60, and 120 $M_\odot$ originally within the computation grid), radiation acceleration limited the final masses to 31.6, 33.6, and 42.9 $M_\odot$, respectively, for wavelength-dependent radiation transfer and to 19.1, 20.1, and 22.9 $M_\odot$ for the corresponding simulations with gray radiation transfer. Our calculations demonstrate that massive stars can in principle be formed via accretion through a disk. The accretion rate onto the central source increases rapidly after one initial free-fall time and decreases monotonically afterward. By enhancing the nonisotropic character of the radiation field, the accretion disk reduces the effects of radiative acceleration in the radial direction—a process we call the “flashlight effect.” The flashlight effect is further amplified in our case by including the effects of frequency-dependent radiation transfer. We conclude with the warning that a careful treatment of radiation transfer is a mandatory requirement for realistic simulations of the formation of massive stars.

Subject headings: circumstellar matter — hydrodynamics — radiative transfer — stars: formation — stars: mass loss

1. INTRODUCTION

Although massive stars play a critical role in the production of turbulent energy in the interstellar medium, in the formation and destruction of molecular clouds, and ultimately in the dynamical and chemodynamical evolution of galaxies, our understanding of the sequence of events that leads to their formation is still rather limited. Because of their high luminosities we can expect that (1) radiative acceleration will contribute significantly to the dynamical evolution during the formation process and (2) the thermal evolution timescales of massive pre–main-sequence objects will be extremely short. Thus, we cannot simply “scale up” theories of low mass star formation. Furthermore, OB stars form in clusters and associations; their mutual interactions via gravitational torques, powerful winds, and ionizing radiation contribute further to the complexity of the problem.

Even though no massive disk has yet been directly observed around a main-sequence massive star, it is likely that such disks are the natural consequence of the star formation process even in the high-mass case. In their radio recombination maser studies and CO measurements, Martin-Pintado et al. (1994) do find indirect evidence for both an ionized stellar wind and a neutral disk around MWC 349. Moreover, several other high-luminosity far-IR sources—suspected embedded young OB stars—have powerful bipolar outflows associated with them (e.g., Eiroa et al. 1994; Shepherd et al. 2000). Such massive outflows are probably powered by disk accretion, and, similar to their low-mass counterparts, the flow energetics appear to scale with the luminosity of the source (see Cabrit & Bertout 1992; Shepherd & Churchwell 1996; Richer et al. 2000).

The detailed structure and evolutionary history of massive circumstellar disks has important consequences with regard to the early evolution of these protostars. Disks provide a reservoir of material with specific angular momentum too large to be directly accreted by the central object. Only after angular momentum is transported outward can this material contribute to the final mass of the star. The transition region disk-star will strongly influence the star’s photospheric appearance and how the star interacts with the disk. The relatively high densities in these disks provide the environment for the further growth and evolution of dust grains, affecting the disk’s opacity and consequently its
energetics and appearance. The disk can be expected to interact with stellar outflows and is likely to be directly responsible for the outflows associated with star formation.

Disks surrounding massive stars or disks associated with close companions to massive stars should be short lived compared to their low-mass counterparts. The UV environment within an OB cluster will lead to the photoevaporation of disks on a timescale of a few $10^7$ yr (Hollenbach, Yorke, & Johnstone 2000; Richling & Yorke 1998). Because this process operates on a timescale comparable to the formation of massive stars and is competitive to it, it is important to carefully model the transfer of radiation in the envelopes of accreting massive stars. Numerical tools capable of this task are lacking at present. We consider the present investigation an important step in this direction.

2. THE NUMERICAL MODEL

We consider the simulation of the hydrodynamic collapse of a rotating molecular cloud clump, with wavelength-dependent radiation transfer, under the assumption of symmetry with respect to the rotation axis and the equatorial plane. Our code contains all of the basic features of the two-dimensional code described in detail by Yorke & Bodenheimer (1999, hereafter YB). In the following we shall discuss only the deviations from and enhancements to the YB code.

In addition to artificial viscosity for the treatment of shocks, physical viscosity has been implemented via an $\alpha$-prescription (Shakura & Sunyaev 1973): $\nu = \alpha c_s(r) H(r)$, where we have approximated the local disk scale height by $H(r) = c_s(r)/\Omega(r)$ ($\Omega$ is the angular velocity and $c_s$ is the sound speed). All components of the viscosity tensor are included, as previously implemented in two-dimensional disk models by Różyczka, Bodenheimer, & Bell (1994) and YB. Contrary to YB, who allowed $\alpha$ to vary in time in order to mimic the effects of tidal torques due to the growth of nonaxisymmetric gravitational modes, the value for $\alpha = 10^{-3}$ is kept constant in space and time for all cases considered.

The details of this angular momentum transfer scheme—within certain limits—do not critically affect our results, however, because our central computational zone (where angular momentum transfer is presumably very critical) is so large. Test calculations with $\alpha = 0.03$ yielded essentially the same results. By contrast, for $\alpha \leq 10^{-4}$ ring instabilities developed in the disk, which led to a premature ending of the calculations. As discussed by Yorke, Bodenheimer, & Laughlin (1995), who did not include the effects of angular momentum transfer (i.e., $\alpha = 0$), these ring instabilities are unrealistic. Such a ring would be unstable on a very short (dynamical) timescale, and the resulting clumps would exert tidal torques, resulting in angular momentum transfer.

As in YB, we utilize a series of hierarchically nested grids (Berger & Colella 1989; Yorke & Kaisig 1995). Whereas YB considered nesting levels of 6 and 60 $\times$ 60 grids, allowing the innermost grid to be $\sim 1/2000$ of the cloud radius, here we have considered nesting levels of 3 only and slightly larger grids (64 $\times$ 64). We assume constant density $\rho = \rho_0$ along the outermost radius $r_{\text{max}}$ and allow material to enter into or exit from our computational grid $R^2 + Z^2 \leq r_{\text{max}}^2$ ($R$ and $Z$ are the cylindrical coordinates), based on the sign of the radial component of the velocity. By contrast, YB assume a semipermeable outer boundary at $r_{\text{max}}$: material with positive radial velocity can leave the computational grid, but no mass is allowed to enter.

As in YB, the boundary of the innermost cell of our innermost grid is considered to be semipermeable: material can flow into this cell but cannot flow out of it. Material entering this cell is assumed to accrete onto a single central object. We realize that this “sink cell” procedure is a gross simplification of the physics in the innermost regions of our computational domain and a number of possibly important effects are being ignored, e.g., fragmentation and accretion of material onto multiple objects and the interaction of the accretion flow with powerful outflows. For the cases F30 and G30 (see § 3), for instance, our innermost cell is a cylinder of radius 40 AU and height 80 AU, whereas for the cases F120 and G120, the cell is larger by a factor of 4. Although we cannot follow the mass flow within and possibly out of this cell, our simulations do provide upper limits to the amount of material that is available to be accreted by the central object: If radiative acceleration prevents the flow of material into the central sink cell, then the central object cannot accrete it.

2.1. Modeling the Central Star

In contrast to YB, we consider a slightly more sophisticated treatment of the radius $R_*$, luminosity $L_*$, and effective temperature $T_{\text{eff}}$ of the central object. Its mass $M_*$ is uniquely determined by integrating the mass flux into the center sink cell:

$$M_* = \int \dot{M}_* \, dt .$$

We can express the total energy of the central core in terms of a “structure parameter” $\eta$:

$$E_{\text{tot}} = -\eta \frac{GM^2_*}{R_*} .$$

In principal, $\eta$, a parameter describing the compactness of the hydrostatic core, must be calculated by solving for the stellar structure of the accreting hydrostatic core. For polytropes of degree $n$, $\eta$ can be derived analytically (e.g., Kippenhahn & Weigert 1990):

$$\eta = \frac{3}{10 - 2n} .$$

A fully convective pre-main-sequence protostar can be approximated by an $n = 3/2$ polytrope and $\eta = 3/7$. As the star approaches the main sequence, a greater proportion of it becomes radiative, its core becomes more compact, and $\eta$ increases.

For purposes of discussion we shall assume for the moment that $\eta = \eta(M_*, R_*)$ is a known function. In this case the intrinsic core luminosity $L_*$ (which does not include the contribution to the total luminosity emitted in the accretion shock front and dissipated within the disk) is given by

$$L_* = L_{\text{nuc}} - E_{\text{tot}} - \beta \frac{GM_* M_*}{R_*}$$

$$= L_{\text{nuc}} - E_{\text{tot}} \left[ \frac{\eta}{\eta} + \left( 2 - \frac{\beta}{\eta} \right) \frac{M_*}{M_*} \frac{R_*}{R_*} \right] ,$$

where $L_{\text{nuc}}$ is the contribution from nuclear burning.
In the following we will assume that \( \beta = 1 \): the material accreted by the star is being added ever so gently at its current radius \( R_* \), and that this material adds negligible entropy to the star.\(^1\) For the total bolometric luminosity of a spherically accreting star, however, we must include the contribution of the potential energy of infalling material as it is dissipated on its way to the stellar surface:

\[
L_{\text{bol}} = L_* + L_{\text{acc}} = L_* + \beta \frac{GM_* \dot{M}_*}{R_*} .
\] (5)

Equation (4) shows that for constant \( L_* - L_{\text{acc}} \) the star should increase or decrease its radius because of mass accretion, depending on the sign of the coefficient of \( \dot{M}_*/M_* \). Because \( \beta \approx 1 \) and \( \eta \approx 3/7 \) during the fully convective Hayashi phase, this coefficient is negative (\( \approx -\frac{1}{3} \)). Thus, mass accretion onto a fully convective star has a tendency to decrease the star’s radius, whereas close to the main sequence, where \( \eta \) is closer to unity, mass accretion has a tendency to cause the star to bloat up (see Kippenhahn & Meyer-Hofmeister 1977 for a more detailed discussion). In reality, the internal readjustment of the star after it has gained mass also affects the nuclear burning rate and thus has an effect on both radius and luminosity, depending on the magnitude of the mass accretion rate and the star’s current position in the Hertzsprung-Russell (H-R) diagram.

A classical problem of the mathematical theory of stellar structure is the question of whether for stars of given fixed parameters, say, chemical composition, mass, and radius, there exists one and only one solution of the basic structure equations. There is actually no mathematical basis for the so-called Vogt-Russell conjecture of uniqueness, and, indeed, multiple solutions for the same set of parameters have been found numerically in some cases (see discussion by Kippenhahn & Weigert 1990). However, for the rather simple cases considered here, spherically symmetric, quasi-hydrostatic homogeneous pre-main-sequence and young main-sequence stars, knowledge of the star’s mass and age at any given time does allow us to uniquely fix its position in the H-R diagram, from which we can determine \( L_* \), \( T_{\text{eff}} \), and \( L_{\text{acc}} \).

For the pre-main-sequence phase we shall account for deuterium burning only and use the following approximate expression:

\[
L_{\text{acc}} = L_D \approx L_0(M_*) \left( \frac{\chi_D}{\chi_D,0} \right) \left( \frac{R_0(M_*)}{R} \right)^p ,
\] (6)

where \( L_0 \) and \( R_0 \) are the equilibrium deuterium burning rate and equilibrium radius for a star of mass \( M_* \) at its “birth line,”\(^2\) \( \chi_D,0 \) is the cosmic mass abundance of deuterium, and \( \chi_D \) is the star’s net deuterium abundance. We have assumed \( p = 21 \), which—because the star’s central density \( \rho_c \propto R^{-3} \)—corresponds to \( L_D \propto \rho_c^7 \). This insures that a non-accreting star remains close to its birth line until a significant fraction of its deuterium is consumed.

Assuming instantaneous mixing during accretion, the deuterium mass fraction \( \chi_D \) can be calculated from the following equation:

\[
\frac{d\chi_D M_*}{dt} = \chi_D,0 M_* - \epsilon_D L_D ,
\] (7)

where \( \epsilon_D L_D \) is the rate of deuterium consumption due to deuterium burning (\( \epsilon_D = 1.76 \times 10^{-19} \) s\(^{-2} \) cm\(^{-2} \) is a constant).

From equations (4) and (6) we can derive an expression for \( R_* \):

\[
\frac{\dot{R}_*}{R_*} = (1 - \eta R)^{-1} \times \left[ \left( 2 - \frac{\beta}{\eta} + \eta M \right) \frac{M_0}{M_*} + \frac{L_* - L_{\text{acc}}}{E_{\text{tot}}} \right] ,
\] (8)

where

\[
\eta R = \left( \frac{\partial \ln \eta}{\partial \ln R} \right) , \quad \eta M = \left( \frac{\partial \ln \eta}{\partial \ln M} \right) .
\] (9)

From knowledge of \( \eta(M_*, R_*) \), \( L_0(M_*, R_*) \), \( L_0(M_*) \), and \( R_0(M_*) \), we approximate the pre-main-sequence evolution of an accreting protostar by integrating equations (7) and (8) simultaneously.

How does one actually determine \( \eta \) and \( L_* \) from knowledge of \( M_* \) and \( R_* \)? For the Hayashi phase we have used published pre-main-sequence tracks\(^3\) for \( L_0(M_*, \tau) \), \( R_0(M_*, \tau) \) and set \( \eta = 3/7 \). For the mass range \( 0.1 M_\odot \leq M_* \leq 2.5 M_\odot \), we use the evolutionary tracks of D’Antona & Mazzitelli (1994), which assume “CM convection” (\( \alpha_{\text{ML}} = 2 \)) and Alexander + RI opacities (\( Y = 0.28 \), \( Z = 0.019 \)). For \( 3 M_\odot \leq M_* \leq 15 M_\odot \), we use tracks published by Iben (1965). Both sets of tracks represent a series of stellar models that incorporate the detailed microphysics of convection and stellar atmospheres. For masses \( M_* > 15 M_\odot \), we have assumed that the evolutionary tracks for non-accreting stars are horizontal lines at the main-sequence luminosity, similar to the \( 15 M_\odot \) track (see Fig. 1), and the main-sequence values of \( R_0(M_*) \) and \( L_0(M_*) \) were taken from Allen (1973).

As the contracting protostar becomes radiative, \( \eta \) increases. Once the star reaches the main sequence, hydrogen burning commences and \( \eta \approx \text{const} \). We approximate \( \eta \) during this phase by approximating the main sequence by polytropic models with the restriction that the main-sequence value \( \eta_{MS} \leq 1 \). Obviously, this is a very rough approximation, but because we expect \( \eta \approx 0 \) and \( L_* \approx L_{\text{acc}} \) on the main sequence, we are not making a significant error. Our greatest error arises during the transition from \( \eta = 3/7 \) (fully convective) to \( \eta_{MS} \), where we have used interpolated values.

Of course, we could obtain a much better approximation to these physical parameters by solving the full set of radiation hydrodynamic equations for an accreting hydrostatic object. This, however, goes far beyond the scope of the

\(^1\) For the newly accreted material, we must subtract the difference of potential energy from infinity to the stellar surface when considering the star’s net change of total energy. For \( \beta = 1 \), the energy gained by the star because of heating from the accretion shock “backwarming” or by dissipating rotational energy within the star is negligible.

\(^2\) There are alternate definitions of the concept of “birth line.” Here we use the word to describe the equilibrium position of a homogeneous, deuterium-burning, pre-main-sequence star with a cosmic abundance of deuterium.

\(^3\) We use the evolutionary time \( \tau \) given for the published tracks as a parameterization of the curves. Our evolutionary time \( t \) results from integrating eq. (8).
present investigation. We merely wish to obtain more realistic approximations for $L_*$ and $R_*$ than those used by YB, which were based on the pre–main-sequence evolution of nonaccreting low-mass protostars. Because massive stars require at least one phase of high accretion rates, typically $M_\dot{\nu} \gtrsim 10^{-4} M_{\odot}$ yr$^{-1}$, the effects of accretion on the evolution of $L_*$ and $R_*$ cannot be completely neglected. To illustrate this quantitatively, we display the evolution of (proto)stars accreting at given constant rates in the H-R diagram (Fig. 1).

These tracks compare very well with published, more detailed calculations by Behrend $&$ Maeder (2001) and by Meynet $&$ Maeder (2000). Not only do the tracks lie slightly below the equilibrium deuterium burning birth line in all cases, but the qualitative effect of rapid accretion—namely, to shift the tracks to even smaller radii below the birth line—occurs both in our simplified model and the above cited calculations. The reason for this is the negative sign of $2 - \beta/\eta$ for fully convective stars. The tracks of our simplified model converge to the main sequence in a manner similar to those of the detailed calculations.

There are some differences, however. Behrend $&$ Maeder and Meynet $&$ Maeder begin their tracks assuming a fully convective $0.7 M_{\odot}$ star taken to be $7 \times 10^5$ yr old, whereas we show here that the equilibrium deuterium burning position at cosmic deuterium abundance is never reached via accretion for masses $M \lesssim 1 M_{\odot}$. If you add mass too quickly, the star’s radius is reduced as discussed above. If you add it too slowly (see $10^{-5} M_{\odot}$ yr$^{-1}$ track in Fig. 1), a significant amount of the deuterium is consumed even before $0.7 M_{\odot}$ has been accreted. Other differences can be accounted for by the different time-dependent accretion rates.

We remind the reader that these tracks in the H-R diagram do not reflect the actual observable bolometric luminosities of accreting protostars. Much of the accretion luminosity will be indistinguishable from the intrinsic luminosity of the star. For our hydrodynamic simulations we will include the effects of the accretion luminosity when discussing the star’s evolution within the H-R diagram.

As in YB, we add the accretion luminosity $L_{\text{acc}}$ (Adams $&$ Shu 1986) to the core’s intrinsic luminosity to obtain the total luminosity

$$L_{\text{tot}} = L_* + \frac{3}{4} \frac{G M_* \dot{M}_*}{R_*}.$$  

Equation (10) differs from equation (5) (for $\beta = 1$), because approximately one-fourth of the total potential energy of the accreted material is dissipated within the disk and is already accounted for by our treatment of viscosity.

Finally, knowing $L_{\text{tot}}$ and $R_*$ allows us to determine $T_{\text{eff}}$ from

$$L_{\text{tot}} = 4\pi\sigma_{\text{SB}} R_*^2 T_{\text{eff}}^4.$$  

The term $\sigma_{\text{SB}}$ is the Stefan-Boltzmann radiation constant. When discussing the evolution of the central star in the H-R diagram, we use $L_*$ rather than $L_{\text{tot}}$ in equation (11).

### 2.2. The Opacity Model

The principal source of opacity is due to absorption and scattering by dust (see Yorke $&$ Henning 1994). We have adopted the detailed frequency-dependent grain model of Preibisch et al. (1993), which assumes a mixture of small amorphous carbon particles (for grain temperatures $T_{\text{gr}} \leq 2000$ K) and “astrophysical silicate” grains (for temperatures $T_{\text{gr}} \leq 1500$ K; see Draine $&$ Lee 1984). At grain temperatures $T_{\text{gr}} < 125$ K the silicates are coated with a layer of “dirty” NH$_3$/H$_2$O ice, contaminated with 10% of the amorphous carbon particles. The grain sizes are assumed to follow an MRN power law $n(a) \sim a^{-3.5}$ (Mathis, Rumpl, $&$ Nordsieck 1977) in the size ranges 7 nm $\leq a_{\text{gr}}$ $\leq 30$ nm (carbon particles) and 40 nm $\leq a_{\text{gr}}$ $\leq 1$ $\mu$m (silicates). As evident in Figure 2, the specific extinction is strongly wavelength dependent.

For all our calculations we used 64 nonuniformly distributed frequency points between 5000 and 0.1 $\mu$m.

### 2.3. Modeling Radiation Transfer

A complete description of the radiation field requires knowledge of the radiation intensity $I_\nu(x, \hat{n}, \mathbf{T})$, where $x$ indicates the position and $\hat{n}$ the direction under consideration. Even assuming an axially symmetric configuration and plane symmetry with respect to the equatorial plane, $I_\nu$ is a function of six independent variables: frequency $\nu$, two spatial variables ($R, Z$), two direction variables, say, ($\theta, \phi$), and time $t$. Solving the rather innocuous looking equation for radiation transfer

$$\frac{1}{c} \frac{dI_\nu}{dt} + \nabla \cdot I_\nu = \kappa_{\text{ext}}^i (S_\nu - I_\nu)$$  

(12)

($S_\nu$ is the source function and $\kappa_{\text{ext}}^i = \sum_i \kappa_{\text{ext}}^i$ is the net contribution to the extinction coefficient from all components $i$) together with the equations for hydrodynamics, energy bal-
accuracy, radiation equilibrium, and the Poisson equation for
the gravitational potential becomes a formidable task with
present-day computers because of the vast number of com-
putations that are necessary to obtain reasonable numerical
resolution.

2.3.1. Flux-limited Diffusion

The numerical problem can be greatly simplified by
resorting to the flux limited diffusion (FLD) approxima-
tion (see Yorke & Kaisig 1995; Levermore & Pomraning 1981).
If we indicate by $J_\nu$ and $H_\nu$ the zeroth and first moments of
the radiation field, respectively, where

$$
J_\nu = \frac{1}{4\pi} \int I_\nu d\Omega, \quad H_\nu = \frac{1}{4\pi} \int I_\nu \hat{n} d\Omega, \tag{13}
$$

then the time-independent form of the zeroth moment equation—obtained by integrating equation (12) over all direc-
tions—can be written

$$
\nabla \cdot H_\nu = j_\nu + \sum_i \kappa_{\nu,i}^{abs}[B_i(T_i) - B_\nu(T_\nu)]. \tag{14}
$$
The term $\kappa_{\nu,i}^{abs}$ is the absorption coefficient of grain type $i$, $T_i$ is its temperature, $B_i(T)$ is the Planck function, and $j_\nu$ is the contri-
bution to the emissivity from internal sources other than dust emission. The next higher order moment equation, obtained by multiplying the time independent form of equation (12) by $\hat{n}$ and then integrating over all directions, would contain terms involving the second moment of the radiation field.

The FLD approximation is a procedure for closing this series of moment equations by approximating the second moment in terms of the zeroth and first moments, utilizing knowledge of the opacity distribution:

$$
H_\nu = -\frac{\lambda_\nu}{\omega_\nu \kappa_{\nu,i}^{ext}} V J_\nu. \tag{15}
$$
The effective albedo $\omega_\nu$ and the flux limiter $\lambda_\nu$ are defined as

$$(\kappa_{\nu,i}^{sca} \text{ is the scattering coefficient for grain type } i)$$

$$
\omega_\nu = \sum_i \frac{\kappa_{\nu,i}^{abs}}{\kappa_{\nu,i}^{sca}} B_i(T_i) + \kappa_{\nu,i}^{sca} J_\nu, \tag{16}
$$

$$
\lambda_\nu = \frac{1}{d} \left( \coth d - 1 \right) \quad \text{with} \quad d = \frac{|V J_\nu|}{\kappa_{\nu,i}^{sca} \omega_\nu J_\nu}. \tag{17}
$$

In the limits of high opacity or low opacity, the FLD approxima-
tion asymptotically approaches the diffusion limit or the free-streaming limit, respectively, as expected.

Equations (14) and (15) have to be solved for all frequen-
cies simultaneously with the condition for radiative equili-
rium of each of the absorbing species:

$$
\int \kappa_{\nu,i}^{abs}[B_i(T_i) - B_\nu(T_\nu)] d\nu = 0. \tag{18}
$$

When considering “gray” radiation transfer as YB did, this
condition can be put into tabular form (for rapid look-up):

$$
T_i = \mathcal{F}_i(J) \quad \text{with} \quad J = \int J_\nu d\nu. \tag{19}
$$

2.3.2. Finite Difference Equations

Following a “staggered mesh” discretization philosophy for
converting partial derivatives into finite differences, we
define $J_\nu$ at the cell centers of an $(R_j, Z_k)$ cylindrical grid and the vector $H_\nu = (H_{\nu,R}, H_{\nu,Z})$ at the centers of cell bound-
daries. Combining equations (14) and (15), we obtain a dif-
fusion equation:

$$
\nabla \cdot (\mathcal{A}_\nu V J_\nu) = j_\nu + \sum_i \kappa_{\nu,i}^{abs}[B_i(T_i) - B_\nu(T_\nu)]. \tag{20}
$$

The discretized form of this FLD equation can be expressed as a matrix equation:

$$
\mathcal{A}_\nu J_\nu = \epsilon_\nu(T), \tag{21}
$$

where the vector $J_\nu$ represents the solution for the zeroth
moment and $\epsilon_\nu(T)$ the dust temperature-dependent
components for the source terms at all grid centers. We employ an equidistant $(R_j, Z_k)$ grid at each level of nesting and, while
utilizing a five-point discretization scheme, $\mathcal{A}_\nu J_\nu = \epsilon_\nu(T)$, (21)

$$
a_{ij,k}^1 J_{ij-1,k} + a_{ij,k}^{10} J_{ij,k-1} + a_{ij,k}^{11} J_{ij,k} + a_{ij,k}^{12} J_{ij,k+1} + a_{ij,k}^{13} J_{ij+1,k} = j_{i,j,k} - \sum_i \kappa_{\nu,i}^{abs} B_i(T_{ij,k}), \tag{22}
$$

insure by proper centering that our finite difference equation
(21) is accurate to second order $\mathcal{O}(h^2)$ of the grid spacing
$h = R_{i+1} - R_i = Z_{k+1} - Z_k$. In equation (22) we have used
the subscript $l$ for frequency and $i$ for grain type.

Note that $\mathcal{A}_\nu$ is an implicit function of $J_\nu$. Each element
of $\mathcal{A}_\nu$ for frequency $\nu$ at a grid cell $(j,k)$ contains, with
proper centering, $J_\nu$ dependencies for all frequencies $\nu'$ and
extending beyond the five-point discretization star of grid
cells $(j,k)$ and $(j \pm 1, k \pm 1)$. As evidenced by equation
(18), the right-hand side of equation (22) is also an implicit
function of $J_\nu$. The fact that we must find a new solution
iteratively on each nested grid for each hydrodynamic time
step places strong demands on our solution algorithm.
2.3.3. Boundary Conditions

Boundary conditions are required for each level of nested grids. For each grid cell \((R_j, Z_k)\) on the outermost grid that satisfies the condition \(R_j^2 + Z_k^2 \geq r_{\text{max}}^2\), we specify \(J_v = w B_v (T_{\text{out}})\), where \(T_{\text{out}}\) is the color temperature of an external isotropic radiation field and \(w\) is the radiation dilution factor. For all cases considered here, we choose \(w = 1\) and \(T_{\text{out}} = 20\) K. Along the outer edges of interior (fine) grids, we use the interpolated value of \(J_v\) from the next (coarser) grid level. Additional boundary conditions along the rotation axis and equatorial plane result from the assumed symmetry.

Within the innermost central grid cell of each level of nested grids, we treat the central star as an additional internal source of emissivity and define \(j_v\) (see eq. [14]) accordingly:

\[
j_v = \frac{\pi R_1^2}{2\pi R_1^2 Z_1} B_v (T_{\text{eff}}),
\]

where \(R_1\) and \(Z_1\) are the radial extent and height of the central cell.

2.4. Solution Algorithms

Because of the necessity of solving equations (18) and (20) repeatedly on several grids during the course of hydrodynamic evolution, it was imperative to make this module fast and robust. Several promising algorithms that rely on fine-tuning adjustable parameters had to be abandoned, because a wide range of problem classes (optically thin, optically thick, strong density gradients, emission dominated, scattering dominated, etc.) occurred, which were not well suited to a single set of parameters. We were able to make vast improvements with respect to the iterative procedures used by Sonnhalter, Preibisch, & Yorke (1995), who solved these same frequency-dependent equations for a few selected density configurations. We sometimes sacrificed robustness (but never accuracy) for speed but automatically fell back to slower and more robust iterative schemes when the “faster” algorithms failed to converge. We feel it is useful to discuss our “failures” as well as our “successes” in our endeavor to improve the speed of our frequency-dependent radiation transfer module.

2.4.1. Temperature Determination

In analogy to the transfer of line radiation in stellar atmospheres, we consider the method of approximate \(\Lambda\) operators (see, e.g., Cannon 1973a, 1973b and Scharmer 1981). Rewriting equation (21) with the operator \(\Lambda = \mathcal{A}^{-1}\), we find the formal solution:

\[
J_v = \Lambda^t \epsilon_v (T).
\]

A simple \(\Lambda\)-iteration would entail calculating \(T^{\text{old}}\) and thus \(\epsilon_v (T^{\text{old}})\) from \(J_v^{\text{old}}\) using equation (18). From this, a new, improved estimate for \(J_v\) can be calculated: \(J_v^{\text{new}} = \Lambda^t \epsilon_v (T^{\text{old}})\). Replacing \(J_v^{\text{old}}\) by \(J_v^{\text{new}}\) and repeating this procedure several times may or may not (usually not) quickly converge to the equilibrium values for \(T\) and \(J_v\), satisfying equation (24). This procedure is labeled “\(\Lambda\)” in Table 1.

In order to improve convergence we consider an appropriate approximation \(\Lambda^t\) to our operator \(\Lambda\). Rather than using \(J_v^{\text{old}}\) in equation (18), we substitute the expression

\[
J_v^{\text{new}} = J_v^{\text{old}} + \Lambda^t [\epsilon_v (T^{\text{new}}) - \epsilon_v (T^{\text{old}})]
\]

to obtain the equilibrium conditions for each grain component \(i\):

\[
\int \epsilon_{v,j}^{\text{abs}} [B_v (T^{\text{new}}) - \Lambda^t \epsilon_v (T^{\text{new}})] d\nu = \int \epsilon_{v,j}^{\text{abs}} [J_v^{\text{old}} - \Lambda^t \epsilon_v (T^{\text{old}})] d\nu,
\]

which can be rewritten in the form \(G(T^{\text{new}}) = 0\). The solution vector \(T^{\text{new}}\) can be obtained by solving equation (26) using a multidimensional Newton-Raphson procedure.

Our approximate \(\Lambda\)-iteration procedure entails alternatively solving equation (26) for \(T^{\text{new}}\) and equation (21) for \(J_v^{\text{new}}\).

There are many possibilities for \(\Lambda^t\), and much effort has been invested in order to optimize its construction. Generally speaking, the choice of a particular \(\Lambda^t\) is based on performance during numerical experimentation and varies from problem to problem. Following this heuristic approach, we considered several different approximate operators and compared their convergence properties with the simple \(\Lambda\)-iteration. Our test cases were the first 20 time steps of two single grid collapse calculations with (1) \(13 \times 13\) and (2) \(128 \times 128\) grid cells and (3) the results of time step 2900 of the 30 \(M_\odot\) case discussed in § 5. For these test calculations we utilized the most efficient procedure for solving equation (21) for \(J_v^{\text{new}}\) (discussed below). The average number of \(\Lambda\)-iterations necessary for each approximate operator and for each test case is given in Table 1.

Our approximate “core-wing” operator is constructed in analogy to the “core-wing” \(\Lambda\)-operator of Scharmer (1981), which takes into account that lines are often optically thin in the wings and optically thick in the line’s core. The analogy with a more or less monotonously decreasing (or increasing) continuum absorption coefficient is to assume a threshold value for which the dusty material becomes optically thin. We conducted several numerical experiments with different threshold values but found little improvement in comparison to the simple \(\Lambda\)-iteration procedure.

Defining \(\Lambda^t_{\text{diag}} = \text{diag}(\mathcal{A})^{-1}\) resulted in improved convergence behavior. Use of additional accelerator terms as outlined by Auer (1987), however, did not always lead to further improvement. Our best convergence results were attained with the \(\Lambda^t_{\text{diag}}\)-operator constructed as follows. We first approximate \(J_{v,j,k}\) by using the diagonal operator \(\Lambda^t_{\text{diag}}\).
in equation (24):

\[
J_{i,j,k}^\nu (T) = \frac{j_{i,j,k} - \sum_{\nu} T_{i,j,k}}{a_{11}^{\nu}} = \sum_{\nu} \rho_{i,j,k}^{\nu} b_i (T_{i,j,k}). \tag{27}
\]

This approximation is used for the off-diagonal terms in equation (22) to obtain

\[
J_{i,j,k} = (a_{11}^{\nu})^{-1} \left[ j_{i,j,k} - \sum_{\nu} \rho_{i,j,k}^{\nu} b_i (T_{i,j,k}) \right] = \sum_{\nu} \rho_{i,j,k}^{\nu} \left[ J_{i,j,k}^\nu (T) - a_{12}^{\nu} J_{i,j,k}^\nu (T) \right] \tag{28}
\]

Because the right-hand side depends only on known quantities and the temperatures \( T \), this corresponds to the operator equation

\[
J_\nu = \Lambda_\nu^{\nu} (T). \tag{29}
\]

Use of \( \Lambda_\nu^{\nu} \) sometimes leads to oscillations during the iterations, which we are able to damp by interjecting simple \( \Lambda \)-iterations. Occasionally, the Newton-Raphson iteration procedure used for solving equation (26) did not converge. For those special cases we had to abandon our approximate \( \Lambda \)-iterations in favor of the following more robust but much more CPU-intensive procedure. If we indicate by \( T_i \) our (inaccurate) estimate of \( T_i \), then a temperature correction \( \Delta T_i \) can be determined from

\[
\Delta T_i = \frac{\sum_{\nu} e_{ij} \rho_{i,j,k}^{\nu} \left[ J_{i,j,k} - B_{\nu} (T_i) \right]}{\sum_{\nu} e_{ij} \rho_{i,j,k}^{\nu} d b_{\nu} / d T |_{T=T_i}} , \tag{30}
\]

where we have replaced the frequency integration by a weighted sum with the weights \( e_{ij} \). By substituting \( B_{\nu} (T_i) + d B_{\nu} / d T |_{T=T_i} \Delta T_i \) for \( B_{\nu} (T_i) \) on the right-hand side of equation (20) and replacing \( \Delta T_i \) by the expression given in equation (30), we find a modified FLD equation:

\[
\mathbf{v} \cdot (\mathbf{\nabla} T_{\nu} + \mathbf{v} J_\nu) = j_\nu + \sum_{\nu} \rho_{i,j,k}^{\nu} \left[ J_{i,j,k} - B_{\nu} (T_i) \right] - d b_{\nu} / d T |_{T=T_i} \left[ \sum_{\nu} e_{ij} \rho_{i,j,k}^{\nu} d b_{\nu} / d T |_{T=T_i} \right] \tag{31}
\]

which can be expressed in matrix form as

\[
\mathbf{\nabla} J_\nu + \sum_{\nu} e_{ij} \rho_{ij}^{\nu} \mathbf{J}_\nu = \chi_{\nu} (T). \tag{32}
\]

The relative importance of individual terms can vary strongly from position to position and with time: i.e., both

\[
| \mathbf{v} \cdot (\mathbf{\nabla} v)_{\nu} | \geq \left| j_\nu + \sum_{\nu} \rho_{i,j,k}^{\nu} \left[ J_{i,j,k} - B_{\nu} (T_i) \right] \right|
\]

and

\[
| \mathbf{v} \cdot (\mathbf{\nabla} v) J_\nu | \leq \left| j_\nu + \sum_{\nu} \rho_{i,j,k}^{\nu} \left[ J_{i,j,k} - B_{\nu} (T_i) \right] \right|
\]

occur.

In addition to the "alternating direction–implicit" procedure (ADI), we considered the multigrid scheme "full approximation storage" (FAS), "successive overrelaxation" (SOR) and its special case by Gauss-Seidel (GS), the method of "quasi-minimal residues" (QMR), and bicgstab(2), an improvement on the Bi-CGSTAB algorithm (Bi-CGSTAB combines the advantages of GMRES and Bi-CG). More information on the ADI, FAS, SOR, and GS procedures can be found in Press et al. (1992). Our variant of the QMR procedure is described by Bücker & Sauren (1996), and the bicgstab(2) method is described in full detail by Sleijpen & Fokkema (1993). 4

Our standard test problem was constructed from the density distribution calculated by Yorke et al. (1995) for the case of a slowly rotating 10 \( M_\odot \) molecular clump, 7074 yr after the beginning of collapse (see their Fig. 4). The density was remapped onto a single 66 \( \times \) 66 grid. Assuming a central luminosity of 100 \( L_\odot \) and constant dust temperature (\( T_\mathrm{dust} = 20 \mathrm{K} \) as starting values, we constructed an initial model by iterating equation (20) until convergence using SOR, subsequently solving for the corrected temperatures using equation (18). The time required for this first iterative step was not included in our comparison. Each procedure was coded in FORTRAN 90 and optimizes for vector processing; a single processor of a Cray T90 was used for the comparison calculations, the results of which are shown in Figure 3 and Table 2 for ADI, QMR, bicgstab(2), and GS.

We first note that the ADI procedure used by Sonnhalter et al. (1995) did not converge for several frequencies for our test problem; the residual in the central cell remained constant after a few iterations. This problem was alleviated by resorting to SOR; the most robust variant used an overrelaxation parameter \( \omega = 1 \), reducing SOR to the GS procedure. GS iterations resulted in steadily (but slowly) decreasing residuals. QMR and bicgstab(2) required the fewest number of iterations to reach convergence, but because of their complexity, it is not immediately apparent that these procedures would also require the least amount of CPU time. As shown in Table 2, bicgstab(2) did indeed prove to be the most efficient partial differential equation solver. It was used for our numerical simulations discussed in § 4. Every variant of FAS we attempted diverged for our test problem and the method was quickly abandoned.

3. INITIAL CONDITIONS

In spite of recent observational and theoretical progress, the initial conditions for protostellar collapse are still poorly known. Whereas it is clear that the formation of massive stars requires high masses, neither the average temperatures

4 Available at http://etna.mcs.kent.edu.
nor the sizes and density distributions of those molecular clumps on the brink of forming massive stars are especially well known (Stahler, Palla, & Ho 2000).

Because we are investigating whether massive stars can form by accretion, comparable to our current understanding of low-mass star formation rather than by coalescence of lower mass hydrostatic components or stellar coalescence (see, e.g., Bonnell, Bate, & Zinnecker 1998), we will adopt for our initial conditions a scaled-up version of the initial configuration expected for the formation of low-mass stars (see, e.g., Williams, Blitz, & McKee 2000 or André, Ward-Thompson, & Barsony 2000 for extensive reviews): clump sizes are a fraction of a parsec, temperatures lie in the range $10$–$30$ K (we adopt $T_0 = 20$ K), and the clumps are density peaked toward the center (typically, $n = \rho \rho_0 / r^p$, where $p \approx 1$–2). Because high-mass star formation is an extremely rare event, we do not restrict ourselves to clump masses that are 1 or 2 Jeans masses only. We adopt a thermal-to-gravitational binding energy ratio of $E_T / E_G = 0.05$, corresponding to about 10 Jeans masses. However, because gravity always dominated thermal pressure forces in our domain of integration, we expect no significant evolutionary differences as long as $T_0 \leq 100$ K ($M \gtrsim 2$ Jeans masses).

The initial configuration is summarized in Table 3. We begin with a rotating ($\Omega = 5 \times 10^{-13}$ s$^{-1}$) density configuration $\rho \propto r^{-p}$, with $p = 1$–2. Because high-mass star formation is an extremely rare event, we do not restrict ourselves to clump masses that are 1 or 2 Jeans masses only. We adopt a thermal-to-gravitational binding energy ratio of $E_T / E_G = 0.05$, corresponding to about 10 Jeans masses. However, because gravity always dominated thermal pressure forces in our domain of integration, we expect no significant evolutionary differences as long as $T_0 \leq 100$ K ($M \gtrsim 2$ Jeans masses).

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TABLE 2

| Procedure       | 1100 µm | 435 nm |
|-----------------|--------|--------|
| ADI             | 2.12   | >600   |
| GS              | 3.98   | 14.30  |
| QMR             | 2.17   | 2.03   |
| biegstab(2)     | 1.15   | 1.38   |

Notes.—Average over 10 tests. Calculations were run until the residual fell below $10^{-30}$ ergs s$^{-1}$ cm$^{-3}$ Hz$^{-1}$.

TABLE 3

| Parameter                | F30, G30 | F60, G60 | F120, G120 |
|-------------------------|---------|---------|-----------|
| Mass ($M_\odot$)        | 30      | 60      | 120       |
| Radius (pc)             | 0.05    | 0.1     | 0.2       |
| $T_0$ (K)               | 20      | 20      | 20        |
| $\Omega_0$ ($10^{-13}$ s$^{-1}$) | 5      | 5      | 5         |
| $\rho_0$ ($10^{-20}$ g cm$^{-3}$) | 1      | 1      | 1         |
| $(n_0)$ ($10^8$ cm$^{-3}$) | 23     | 5.8    | 1.5       |
| $E_T / E_G$             | 0.05    | 0.05    | 0.05      |
| $E_{rot} / E_G$         | 0.023   | 0.094   | 0.374     |
| $t_{ff}$ (10$^3$ yr)    | 32      | 65      | 129       |

Notes.—F = frequency-dependent cases. G = gray cases. The initial clump temperature $T_0$ was also the outer temperature boundary condition $T_{out}$. 
Lizano & Shu 1989; Crutcher et al. 1994). Probably the best measured precollapse core, L1689B, has a somewhat flatter central region \((R < 4000 \text{ AU})\) with \(\rho \propto r^{-0.4}\) or \(\rho \propto r^{-1.2}\), depending on geometric and temperature assumptions (André, Ward-Thompson, & Motte 1996). Outside 4000 AU the density decreases as \(\rho \propto r^{-2}\). By contrast, Motte, André, & Neri (1998) find \(\rho \propto r^{-2}\) density profiles in 10 compact precollapse cores in the \(\rho\) Oph region, which they were able to resolve down to \(~140\) AU.

In spite of the fact that the radial dependence of density in precollapse cores is observationally ill constrained, especially for the high-mass case, we have restricted ourselves to an initial \(\rho \propto r^{-2}\) density configuration, corresponding to that of a “singular isothermal sphere,” which has been used for many years to model the formation of low-mass stars. This allows direct comparison with the results of analogous collapse calculations. During the course of evolution, however, rotation, thermal forces at the outermost radius, and radiation forces quickly modify this initial density distribution. Moreover, in contrast to similarity solutions that rely on the assumption of a singular isothermal sphere, our clumps are initially at rest. The mass accretion rates are thus not fixed but rather a result of the radiation hydrodynamic calculations.

For the cases F120 and G120, the rotational velocity at the equator and \(r_{\text{max}}\) exceeded the escape velocity. Thus, about \(7.3 \, M_{\odot}\) of the original 120 \(M_{\odot}\) in the cloud was not bound initially.

4. RESULTS

4.1. Evolution of the Central (Proto)star

We have implicitly assumed that all material flowing into the central zone is accreted onto a single object. Its mass is known from integrating the mass accretion rate \(\dot{M}\) over time; its luminosity and effective temperature are modeled according to the procedure described in \(\S\) 2.1. In Figure 4 we display the evolution of the total and accretion luminosities for the frequency-dependent “F” sequences.

Considering the fact that the initial free-fall times double from case F30 to F60 and from F60 to F120, the initial rapid rise of luminosity—due to the emission from an accretion shock surrounding the central star—appears similar for all three cases. This is not too surprising, because neither rotation nor radiation has an important influence during these early phases; the flow is dominated by gravity. After several thousand years, the energy released within the accretion shock front is not the dominant source of luminance. Adding material onto the central star at the very high rates considered here causes it to bloat up to radii exceeding the deuterium-burning limit (see Fig. 5). Note that there is very little difference of the evolution within the H-R diagram of these three cases. Indeed, except for a “shift” in time and the maximum mass attained in each simulation, the growth of mass within the central computational zone was remarkably similar (see Fig. 6). The rate of mass accretion rises sharply within a few thousand years and reaches a maximum value of \(~2 \times 10^{-3}\) \(M_{\odot}\) yr\(^{-1}\).

Comparing these tracks with those published by Behrend & Maeder (2001) and by Meynet & Maeder (2000), we note that our tracks lie significantly higher in the H-R diagram (larger radii, larger luminosity, but slightly lower \(T_{\text{eff}}\) for equivalent masses) and cross the main sequence at a somewhat higher luminosity by about 0.5–0.8 dex. This is not too surprising, because our luminosity includes three-fourths of the accretion luminosity (see eq. [10]) and our accretion rate varies in time according to the results of hydrodynamic calculations (see Figs. 6 and 7). By contrast, Behrend & Maeder assumed a given accretion rate onto the star based on observations of outflows \(\dot{M} = \max\{10^{-5}\, M_{\odot} \text{ yr}^{-1}, f \dot{M}_{\text{out}}\}\), where \(f\) was chosen to lie between 0.3 and 0.5. For outflow mass loss \(\dot{M}_{\text{out}}\), the authors used the observed relation between the outflow mass rates and the stellar bolometric luminosities in ultracompact H \(\text{II}\) regions found by Churchwell (1998) and con-
The formula used by Meynet & Maeder is

\[ \frac{M}{M_\odot} \approx \frac{1}{10} \left( \frac{M_\odot}{M_\odot} \right)^{1/2} \]

The formulae assumed by Behrend & Maeder and by Meynet & Maeder result in higher accretion times (lower average accretion rates) and a shift of the phase of extremely high accretion rates to later evolutionary times, compared to what we find in our collapse calculations. At the high-mass end, however, the evolution of the star is similar; the tracks follow closely along the main sequence in spite of high mass accretion rates.

The growth of mass in the central computational zone was governed by the relative importance of centrifugal, radiative, and gravitative forces in the molecular clump. For the six cases calculated we found that the detailed treatment of radiation transfer strongly influenced the clump’s evolution. In general, the accretion rate increased sharply after about one free-fall time and then decreases. To exemplify this we display the growth of central mass and the time dependence of the accretion rate for the frequency-dependent case F60 and for the gray case G60 in Figure 7. Assuming gray radiation transfer, infall of material into the central regions of the molecular clump is strongly hampered after about 14,000 yr. The central star is a 20.7 \( M_\odot \) main-sequence star with a luminosity of \( 5.2 \times 10^4 \) \( L_\odot \). Assuming frequency-dependent radiation transfer, more than 60% more mass could accrete onto the central object. For case F60, the final mass was 33.6 \( M_\odot \) and the final main-sequence luminosity was \( 2.2 \times 10^5 \) \( L_\odot \).

### 4.2. Evolution of Molecular Cloud Clumps

In the following four figures the distributions of the gas density, of the temperatures of amorphous carbon and silicate grains, and of the gas velocity are shown for selected cases (F60, G60, F30, and F120) at six selected times. The evolutionary age, total mass within the computational grid, total luminosity (including accretion luminosity), and mass of the central star that correspond to each frame are given in Table 4.
sition velocity has also grown (Fig. 8). Outside the inner region there has been a significant amount of material accreted (see Fig. 7). Contrary to case F60, there is no indication of the formation of a polar cavity, evacuated by radiative forces, at the most advanced evolutionary time considered, 110,000 yr (Fig. 9). Instead, the material flows onto a thin, disklike structure, supported in the radial direction by both centrifugal and radiative forces.

4.2.3. Case F30

As for case F60, the initial collapse of case F30 is nearly spherically symmetric until an evacuated polar cavity is formed, encased in a system of expanding shock fronts. The infalling material collides with the radiatively accelerated outflow. In the condensed cylindrical shell bounded by the shock fronts (see Fig. 10), the hard radiation from the central source is absorbed and reemitted at longer wavelengths. Because of this degrading of the stellar radiation “hardness,” the material outside the cylindrical shell is able to flow radially inward more or less parallel to the equator for |z| < 10^{17} cm. Material continues to flow into the central zone via the equatorial plane, and the central stellar mass ultimately grows to 31.6 M_☉, more than originally present within the computational grid. The cylindrical shell depicted in Figs. 10a and 10b is a short-lived phenomenon, however. Within a few thousand years it collapses into a long, narrow, filamentary structure (Fig. 10f) containing about 13 M_☉.

4.2.4. Case F120

Again, as in cases F60 and F30 discussed above, the initial collapse is spherically symmetric (Fig. 11a), followed by the formation of a polar cavity evacuated by radiative forces (Fig. 11b), after a significant amount of material has accumulated within the central zone (M_☉ = 32.9 M_☉ at t = 28,000 yr). The central star continues to accrete an additional 10 M_☉ via an equatorial flow through a disklike structure over the next 30,000 yr albeit at an ever decreasing rate. This “disk” is short lived, however. At an evolutionary age of 60,000 yr (Fig. 11d), the accretion process has stopped and a cylindrical shell first forms, contracts to a smaller cylindrical radius with a more focused polar outflow (Fig. 11c), and then reexpands (Fig. 11f). Whereas for earlier evolutionary times the gas density was higher in the equatorial regions than in the polar outflow regions, the final frame strongly resembles an expanding “cocoon” shell, punctured and elongated by a polar outflow. No disklike structure is visible.

4.3. Appearance of Molecular Clumps

With the known density and equilibrium temperature distributions of each dust component calculated for each time...
Fig. 8.—Distribution of density (gray scale and white contour lines), velocity (arrows), temperature of amorphous carbon grains (solid black contour lines), and temperature of silicate grains (dotted contour lines) for case F60 (see Table 3) at evolutionary times as indicated in Table 4.
step, we can perform ray-tracing radiation transfer calculations analogous to those of YB, who—by contrast—used the single dust temperature obtained in a gray radiation transfer code. From these ray-tracing calculations, we can extract both spectral energy distributions (SEDs; see Figs. 12 and 13) and isophote maps at selected wavelengths for any given evolutionary age.

Because, however, our spatial resolution in the innermost regions is much worse than that of YB and, furthermore, we have made no attempt to model the emission from a hypothetical disk within the central cell (see YB’s “central zone disk model”), we will not be able to accurately model the emission from dust warmer than about 400 K. Even in the rather late formation stages considered here—the central

Fig. 9.—Distribution of density, velocity, and grain temperature for case G60 at evolutionary times as indicated in Table 4. Symbols and lines are as in Fig. 8.
Fig. 10.—Distribution of density, velocity, and grain temperatures for case F30 at evolutionary times as indicated in Table 4. Symbols and lines are as in Fig. 8.
Fig. 11.—Distribution of density, velocity, and grain temperatures for case F120 at evolutionary times as indicated in Table 4. Symbols and lines are as in Fig. 8.
stars have evolved to main sequence O-stars—very little near-infrared and no optical/ultraviolet radiation escapes the remnant molecular cloud. It is reasonable to assume that a nonhomogeneous distribution of material within the central zone (i.e., clumpiness or a flattened disk) would have allowed at least some of the hard photons from the central source to escape. Thus, the examples given here can be illustrative only of the basic method rather than an accurate model of the expected near-infrared to ultraviolet flux. Unfortunately, our ignorance of the goings-on within the central zone preclude a more definitive treatment.

5. DISCUSSION AND CONCLUSIONS

Our improved frequency-dependent radiation hydrodynamics code is able to track the infall of material within a molecular clump against radiative forces. We find that the “flashlight effect” first discussed by Yorke & Bodenheimer (1999), i.e., the nonisotropic distribution of radiative flux that occurs when a circumstellar disk forms, is strongly compounded by the frequency-dependent radiation transfer. The shortest wavelength radiation (which is also the most effective for radiative acceleration) is most strongly concentrated toward the polar directions, whereas the longer wavelength radiation (less effective radiative acceleration) is more or less isotropic. We conclude that massive stars can in principle be formed via accretion through a disk, in a manner analogous to the formation of lower mass stars. A powerful radiation-driven outflow in the polar directions and a “puffed-up” (thick) disk result from the high luminosity of the central source.

We have developed a simplified model for following the evolution of accreting (proto)stars, using existing tracks for nonaccreting stars. With this model we have shown that in the case of massive star formation the energy released within the accretion shock front, the “accretion luminosity,” is not the dominant source of luminosity after a few thousand years of evolution.

The accretion rate onto the central source is time dependent. It rises sharply after one free-fall time to a maximum value and falls off gradually (in the frequency-dependent cases). This is in contrast to the expectations of Meynet & Maeder (2000) and Behrend & Maeder (2001), who have assumed mass accretion rates $\dot{M}_* = M$ that increase monotonically in time up to a maximum value.

We have also shown that the concept of the “birth line,” the equilibrium position of fully convective, deuterium-burning stars in the H-R diagram with cosmic deuterium abundance, is—strictly speaking—unattainable for stars more massive than $1 \ M_\odot$. Beginning with a protostar of a fraction of a solar mass and building up via accretion to $1 \ M_\odot$ and higher masses, it either accretes too rapidly (shifting the H-R position to smaller radii) or it accretes too slowly (significant amounts of previously accreted deuterium are consumed). For masses $M < 10 \ M_\odot$, however, the contribution of the accretion luminosity may make the star appear to lie on or above the birth line.

In this investigation we have not addressed the issues of the longevity of the circumstellar disk or the possible formation of a dense stellar cluster within our central computational zone rather than a single star. However, even without the assumption of ionizing radiation, we find that these disks are not long-lived phenomena. In the most massive cases the effects of radiative acceleration eventually disperse the remnant disks. Future studies will have to address the issues of ionization and the interactions of the disk with powerful stellar winds. The effects of nearby companions in a dense stellar cluster will also have to be considered in future work.

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