Sensitivity Analysis in Data Envelopment Analysis for Interval Data

N H Nasution 1, S Efendi 2, and Tulus 3

1, 3 Department of Mathematics, Universitas Sumatera Utara, Indonesia
2 Department of Computer Science and Information Technology, Universitas Sumatera Utara, Indonesia

* nurhasanahnasution35@yahoo.com

Abstract. Data Envelopment Analysis (DEA) analysis model can evaluate the relative efficiency of a set of DMU decision-making units with the right values but cannot handle data that is not correct, for example data in the form of intervals. In this paper, the DEA model is able to calculate the data efficiency of interval values. Data Envelopment Analysis interval or DEA interval is an approach that can make input and output data from Data Envelopment Analysis which is used to measure the relative efficiency of Decision Making Unit (DMU) in multi input and multi output processes which will be examined in this study. This model is based on arithmetic operations with interval numbers not the same as the DEA model in general that uses single data. Measurement of the relative efficiency of DMUs with inputs and outputs that have interval data using a constant interval of interval, depending on the application, can also be estimated using statistical techniques or experience. Because in the world most data is interval data, we present a model to calculate the lower and upper limits for each DMU and prove that this model provides higher and lower efficiency limits. In this paper the number of DMUs is increased where if one DMU is added then all highly efficient units will remain.

1. Introducing

Data Envelopment Analysis (DEA) was first introduced by Charnes, et al. in 1978 [1, 2]. The DEA is basically a development of Linear Programming (LP) [2]. Envelopment Analysis Data serves to assess efficiency in the use of resources (input) to achieve results (output) whose purpose is to maximize efficiency. Envelopment Analysis data is a nonparametric method in operations and economic research to estimate production limits. This is used to empirically measure the Decision Making Unit (DMU) production efficiency.

The term DMU in DEA can be in the form of various units such as banks, hospitals, units from factories, departments, universities, schools, electric generators, police stations, samsat offices, tax offices, prisons, and anything that has similar operational characteristics [2]. There are two factors that influence the selection of DMU, namely: DMU must be homogeneous units. These units perform the same task, and have the same objective. The inputs and outputs that characterize the performance of the DMU must be identical, except that differ only in intensity and number/size (magnitude).

DEA serves to assess efficiency in the use of resources (input) to achieve results (output) whose purpose is to maximize efficiency. To find out what effect the data modification has on the optimal solution. DEA sensitivity analysis that is based on linear programming theoretically and practically is equally important [2]. To date, because the DEA has been used in a variety of problems, many studies...
have been carried out on this issue [3][4][5][6]. To date, studies have considered modifying input and output data.

DEA which is a useful method for measuring performance can produce errors because of its sensitivity to data uncertainty [7]. Extreme points in the data greatly affect the efficiency measurement of most DMUs. With data disturbance (perturbed data), it is necessary to know how sensitive the interval data in data envelopment analysis is to efficiency. This study aims to see how far the change with sensitivity analysis is so that the efficiency in data envelopment analysis for interval data remains guaranteed and to improve the efficiency of data envelopment analysis modeling in order to solve the problem of sensitivity analysis of decision making units for interval data [8].

2. Method

Data Envelopment Analysis (DEA) is basically a linear programming technique to measure the relative efficiency of a Decision Making Unit (DMU) on the basis of several (multiple) inputs and outputs [9]. We can find out the efficiency of the performance of a DMU compared to other DMUs through the DEA [2].

The advantage of DEA is that the results of its analysis can be used to set targets that must be achieved by a DMU to produce efficient performance, besides that we can know the value of inputs and outputs that must be raised or lowered to achieve the potential improvement target values and attributes that must be corrected.

DEA measures the efficiency of the Decision Making Unit by maximizing the ratio of output to input or vice versa. Efficiency is assessed based on intervals between 0-1, where value 1 represents the unit of efficiency [8]. An efficient DMU will form and be in the frontier line.

DEA efficiency is measured by the ratio of the output to input ratio. Measurement of relative efficiency stems from a concept developed by Farrel (Bowlin) which explains that a production frontier is a technology relationship that describes the maximum output produced by an efficient company from a combination of inputs in several periods. Farrel's efficiency ratio formulation is:

Efficiency of DMU_o = \frac{\sum_{i=1}^{m} u_{io} y_{io}}{\sum_{j=1}^{n} v_{jo} x_{jo}} (1)

Where:
O = DMU to be evaluated (o = 1,2, ..., N)
m = DMU_o output observed
n = DMU_o input observed
y_{io} = the amount of output i produced by DMU_o (i = 1,2, ..., m)
x_{jo} = number of inputs j used by DMU_o (j = 1,2, ..., n)
u_{io} = output weight i produced by DMU_o
v_{jo} = input weight j used by DMU_o

Note: u_{io} and v_{jo} are decision variables whose values will be determined through linear programming iterations.

After DEA is transformed in the form of linear programming, to get the optimal solution the algorithm in linear programming can be applied

Sensitivity Analysis

An important part of linear programming applications is sensitivity analysis. The aim is to determine the level of sensitivity of the optimal solutions that have been obtained from linear programming problems to changes in coefficients in the linear programming model [6]. Sensitivity analysis will produce output in the form of parameter ranges (upper and lower limits of parameters) that do not interfere with the optimal solution.

The efficiency of DEA is very sensitive to even small data, because DEA is a non-parametric technique, testing hypotheses becomes very difficult. Therefore, as with other modeling techniques,
the output produced by the DEA must be viewed carefully, and must be used only after appropriate sensitivity analysis.

3. Results and Discussion

Using the right non-data intervals, the DEA model is oriented to non-linear VRS outputs, separate from the w0 parameters and ur output weights \((r = 1, \ldots, s)\) and input weights \(vi (i = 1, \ldots, m)\). The level of input \(x_{ij}\) and output is also a variable whose exact value must be expected to support the \(j_0\) unit evaluated.

Minimum

\[
h_{j_0} = \sum_{i=1}^{m} v_i x_{ij_0} - w_0
\]

with constraints

\[
\sum_{r=1}^{s} u_r y_{rj_0} = 1
\]

\[
\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} - w_0 \geq 0, j = 1, \ldots, n
\]

\(Ur, Vi \geq \epsilon \forall r, i\)

In the model (2), variables that are not known under estimation are \(u_r\) weights, \(v_i\), and \(q_{ij}, p_{rj}\) variables which indicate the level of input and output values in a limited interval \([x_{ij}, x_{ij}^U]\) and \([y_{rj}, y_{rj}^U]\).

Say \(h_{j_0}^L\) is the efficiency score for a unit that comes from the most profitable position (the input is set lower and the output is to the upper limit) while all remaining units are set to the least profitable position (the input is set to the upper limit and the output is to the lower limit). Let \(h_{j_0}\) be the efficiency score obtained by such an arrangement. This efficiency score is obtained by the following model with the right data.

Minimum

\[
h_{j_0} = \sum_{i=1}^{m} v_i x_{ij_0}^L - w_0
\]

with constraints

\[
\sum_{r=1}^{s} u_r y_{rj_0}^U = 1
\]

\[
\sum_{i=1}^{m} v_i x_{ij_0}^L - \sum_{r=1}^{s} u_r y_{rj_0}^U - w_0 \leq 0, j = 1, \ldots, n, j \neq j_0
\]

\[
ur, v_i \geq \epsilon \forall r, i
\]

Based on input-output data considered in model (5), unit \(j_0\) is in the best position vis-à-vis other units and thus \(h_{j_0}\) is the lowest possible efficiency score that can be achieved by this unit. So \(h_{j_0} \leq h_{j_0}^L\) we notice that the solution \((u, v, Q, P)\) with \(u = (u_r, r = 1, \ldots, s), v = (v_i, i = 1, \ldots, m)\), \(Q = (q_{ij}, i = 1, \ldots, m; j = 1, \ldots, n), P = (p_{rj}, r = 1, \ldots, s; j = 1, \ldots, n)\) and \(q_{ij} = 0, j = j_0\) are feasible model solutions. By applying \(Q, P\) on it means that the solution that matches the efficiency score \(h_{j_0}\), cannot be lower than the value \(h_{j_0}^L\) thus \(h_{j_0} \leq h_{j_0}^L\) becomes a viable solution.
4. Conclusion

Based on the results of the analysis, a conclusion is obtained that the sensitivity analysis shows the level of sensitivity to each weight of activity in achieving the level of efficiency. In addition, sensitivity analysis can illustrate the level of change that does not interfere with the achievement of the level of data interval efficiency in Data Envelopment Analysis.

References

[1] Banker, R. D., Charnes, A., & Cooper, W. W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management science*, 30(9), 1078-1092.

[2] Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European journal of operational research*, 2(6), 429-444.

[3] Cooper, W. W., Park, K. S., & Ciurana, J. T. P. (2000). Marginal rates and elasticities of substitution with additive models in DEA. *Journal of Productivity Analysis*, 13(2), 105-123.

[4] A. Wanto, M. Zarlis, Sawaluddin, and D. Hartama, “Analysis of Artificial Neural Network Backpropagation Using Conjugate Gradient Fletcher Reeves in the Predicting Process,” *Journal of Physics: Conference Series*, 930(1), 1–7, 2017

[5] Farrell, M. J. (1957). The measurement of productive efficiency. *Journal of the Royal Statistical Society: Series A (General)*, 120(3), 253-281.

[6] Hadi, P. 2006. *Dasar-dasar Pemrograman Linear*. Jember: Jember University Press.

[7] N. Nasution, A. Zamsuri, L. Lisnawita, and A. Wanto, “Polak-Ribiere updates analysis with binary and linear function in determining coffee exports in Indonesia,” *IOP Conference Series: Materials Science and Engineering*, 420(12089), 1–9, 2018.

[8] Eken, M. H., & Kale, S. (2011). Measuring bank branch performance using Data Envelopment Analysis (DEA): The case of Turkish bank branches. *African Journal of Business Management*, 5(3), 889-901.

[9] Smirlis, Y. G., Maragos, E. K., & Despotis, D. K. (2006). Data envelopment analysis with missing values: An interval DEA approach. *Applied Mathematics and Computation*, 177(1), 1-10.