Flavor-changing decay $h \to \tau \mu$ at super hadron colliders

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Abstract: We study the flavor-changing decay $h \to \tau \mu$ with $\tau = \tau^- + \tau^+$ and $\mu = \mu^- + \mu^+$ of a Higgs boson at future hadron colliders, namely: a) High Luminosity Large Hadron Collider, b) High Energy Large Hadron Collider and c) Future hadron-hadron Circular Collider. The theoretical framework adopted is the Two-Higgs-Doublet Model type III. The free model parameters involved in the calculation are constrained through Higgs boson data, Lepton Flavor Violating processes and the muon anomalous magnetic dipole moment; later they are used to analyze the branching ratio of the decay $h \to \tau \mu$ and to evaluate the $gg \to h$ production cross section. We find that at the Large Hadron Collider is not possible to claim for evidence of the decay $h \to \tau \mu$ achieving a signal significance about of 1.46$\sigma$ by considering its final integrated luminosity, 300 fb$^{-1}$. More promising results arise at the High Luminosity Large Hadron Collider in which a prediction of 4.6$\sigma$ when an integrated luminosity of 3 ab$^{-1}$ and $\tan \beta = 8$ are achieved. Meanwhile, at the High Energy Large Hadron Collider (Future hadron-hadron Circular Collider) a potential discovery could be claimed with a signal significance around 5.04$\sigma$ (5.43$\sigma$) for an integrated luminosity of 3 ab$^{-1}$ and $\tan \beta = 8$ (5 ab$^{-1}$ and $\tan \beta = 4$).

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1 Introduction

A lepton flavor violation (LFV) is a transition between $e$, $\mu$, $\tau$ sectors that doesn’t conserve lepton family number. Within the Standard Model (SM) with massless neutrinos, individual lepton number is conserved. Even with the addition of non-zero neutrino masses, processes that violate charged lepton number are suppressed by powers of $m_\nu^2/m_W^2$ [1] and they should be extremely sensitive to physics beyond the Standard Model (BSM). Neutrino oscillations are a quantum mechanical consequence of the existence of nonzero neutrino masses and mixings. The experiments with solar, atmospheric, reactor and accelerator neutrinos [2–5] have provided evidences for the existence of this phenomenon [6, 7] giving a clear signal of LFV. On the other hand, the observation of charged lepton flavor-violating (CLFV) processes would be a non-trivial signal of physics BSM. However, no evidence of the LFV in the searches of leptonic decays $\tau^- \to e^- e^+ e^+$, $\tau^- \to \mu^- \mu^- \mu^+$ [8], and $\mu^- \to e^- e^- e^+$ [9], or radiative decays $\mu \to e\gamma$ [10], $\tau \to \mu\gamma$, $\tau \to e\gamma$ [11] which impose very restrictive bounds on the rates of these processes. Particularly interesting is the decay $h \to \tau \mu$, which was studied first by authors of [12], with subsequent analyses on the detectability of the signal.
appearing soon after [13, 14]. This motivated a plethora of calculations in the framework of several SM extensions, such as theories with massive neutrinos, supersymmetric theories, etc., [15–22]. The observation of the SM Higgs boson with a mass close to 125 GeV at the Large Hadron Collider (LHC) [23, 24] opened a great opportunity to search for physics BSM, in particular through the decay $h \rightarrow \tau \mu$. Currently the upper bound on the $BR(h \rightarrow \tau \mu)$ is $2.5 \times 10^{-3}$ at 95% of C.L. [25]. With this value, searches for decay $h \rightarrow \tau \mu$ look promising with luminosities larger than the one reached by the LHC (300 fb$^{-1}$). This could be achieved at the High Luminosity Large Hadron Collider (HL-LHC) [26] which will be a new stage of the LHC starting about 2026 with a center-of-mass energy of 14 TeV. The upgrade aims at increasing the integrated luminosity by a factor of ten (3 ab$^{-1}$, around year 2035) with respect to the final stage of the LHC. In addition, subsequent searches for the decay $h \rightarrow \tau \mu$ could be performed at the High Energy Large Hadron Collider (HE-LHC) [27] and at Future hadron-hadron Circular Collider (FCC-hh) [28], which will reach an integrated luminosity of up to 12 and 30 ab$^{-1}$ with center-of-mass energies of until 27 and 100 TeV, respectively.

On the theoretical side, one of the simplest models reported in the literature is the Two-Higgs-Doublet Model (2HDM) [29, 30]. The versions type I and type II of 2HDM are invariant under a $Z_2$ discrete symmetry that ensure $CP$ conservation ($CP$) in the scalar sector. In the model type I only one of the doublets gives masses to the fermions [31], while in the model type II one doublet is assigned to give mass to the sector up and the other to the sector down, respectively. The Two-Higgs-Doublet Model type III (2HDM-III) without $Z_2$ discrete symmetry both doublet scalar fields give masses to the up and down sectors. This general version generate Flavor Changing Neutral Currents (FCNC) in Higgs-fermions Yukawa couplings and $CP$ violation ($CPV$) in the Higgs potential [31, 32]. The searches for lepton flavor violating decays of the SM Higgs boson in the $\tau \mu$ and $\tau e$ channels by ATLAS and CMS [33, 34] have motivated their study. In this paper, we search for the decay $h \rightarrow \tau \mu$ in the context of the 2HDM-III.

The organization of our work is as follows. In section 2 we discuss generalities of the 2HDM-III including the Yukawa interaction Lagrangian written in terms of mass eigenstates as well as the diagonalization of the mass matrix. Section 3 is devoted to the constraints on the relevant model parameter space whose values will be used in our analysis. The section 4 is focused on the analysis of the production cross section of the SM-like Higgs boson via the gluon fusion mechanism, the decay $h \rightarrow \tau \mu$ and its possible detection at super hadron colliders, namely: HL-LHC, HE-LHC and the FCC-hh. Finally, conclusions and outlook are presented in section 5.

2 Two-Higgs Doublet Model type III

The 2HDM includes two doublet scalar fields with the same hypercharge, $Y = 1$. The classification of the 2HDM types is based on the different ways to introduce Yukawa interactions and scalar potential. The theoretical framework adopted is the 2HDM-III, where both doublets are used to induce interactions between fermions and scalars as described in this section. A characteristic of the 2HDM-III is that the fermion mass matrix is a linear combination of two Yukawa matrices, which is diagonalized by a bi-unitarity transforma-
tion. However, this bi-unitary transformation do not simultaneously diagonalize the two Yukawa matrices. As a result, FCNC can arise at tree level.

2.1 General Higgs potential in the 2HDM-III

The most general $SU(2)_L \times U(1)_Y$ invariant scalar potential is given by [35, 36]:

$$V(\Phi_1, \Phi_2) = \mu_1^2(\Phi_1^\dagger \Phi_1) + \mu_2^2(\Phi_2^\dagger \Phi_2) - \left(\mu_{12}^2(\Phi_1^\dagger \Phi_2) + H.c.\right) + \frac{1}{2} \lambda_1(\Phi_1^\dagger \Phi_1)^2$$

$$+ \frac{1}{2} \lambda_2(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$

$$+ \left(\frac{1}{2} \lambda_5(\Phi_1^\dagger \Phi_2)^2 + (\lambda_6(\Phi_1^\dagger \Phi_1) + \lambda_7(\Phi_2^\dagger \Phi_2))(\Phi_1^\dagger \Phi_2) + H.c.\right),$$

where $\mu_1, \mu_2, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7$ are real parameters while $\mu_{12}$, $\lambda_5, \lambda_6, \lambda_7$ can be complex in general. The doublets can be written as $\Phi_a^T = (\phi_a^+, \phi_a^0)$ for $a = 1, 2$. After the Spontaneous Symmetry Breaking (SSB) the two Higgs doublets acquire non-zero expectation values. The Vacuum Expectation Values (VEV) are selected as

$$\langle \Phi_i \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_i \end{pmatrix}, \text{with } i=1, 2. \quad (2.2)$$

where $v_1$ and $v_2$ satisfy $v_1^2 + v_2^2 = v^2$ for $v = 246$ GeV. Usually, in the 2HDM-I and II the terms proportional to $\lambda_6, \lambda_7$ are removed by imposing the $Z_2$ discrete symmetry in which the doublets are transformed as $\Phi_1 \to \Phi_1$ and $\Phi_2 \to -\Phi_2$. This $Z_2$ discrete symmetry suppresses FCNC in Higgs-fermions Yukawa couplings at tree level. This is the main reason why $Z_2$ discrete symmetry is not introduced.

Once the scalar potential (2.1) is diagonalized, the mass-eigenstates fields are generated. The charged components of $\Phi_a$ lead to a physical charged scalar bosons and the pseudo-Goldstone bosons associated with the $W$ gauge fields are given as follows:

$$G_W^\pm = \phi_1^\pm \cos \beta + \phi_2^\pm \sin \beta,$$

$$H^\pm = -\phi_1^\pm \sin \beta + \phi_2^\pm \cos \beta,$$  

where the mixing angle $\beta$ is defined through $\tan \beta = v_2/v_1 = t_\beta$.

The charged scalar boson mass is given by:

$$m_{H^\pm}^2 = \frac{\mu_{12}^2}{s_\beta c_\beta} - \frac{1}{2} v^2 \left(\lambda_4 + \lambda_5 + t_\beta^{-1} \lambda_6 + t_\beta \lambda_7\right),$$

where we defined $\cos \beta(\sin \beta) = c_\beta(s_\beta)$. Meanwhile, the imaginary part of the neutral component of the $\Phi_a$, i.e., $\text{Im}(\Phi^0)$, defines the $CP$-odd state and the pseudo-Goldstone boson related to the $Z$ gauge boson. The corresponding neutral rotation is given by:

$$G_Z = \text{Im}(\phi_1^0)c_\beta + \text{Im}(\phi_2^0)s_\beta,$$

$$A^0 = -\text{Im}(\phi_1^0)s_\beta + \text{Im}(\phi_2^0)c_\beta,$$ 

$$\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \frac{3}{3}$
where the superscript 0 denotes the neutral part of the doubles. The CP-odd scalar boson mass reads as follows:

$$m_{A^0}^2 = m_{H^0}^2 + \frac{1}{2} v^2 (\lambda_4 - \lambda_5).$$  \hspace{1cm} (2.8)

On the other side, the real part of the neutral component of the $\Phi_a$, i.e., $\text{Re}(\Phi^0)$, defines the CP-even states, namely: the SM-like Higgs boson $h$ and a heavy scalar boson $H$.

The physical CP-even states are written as:

$$H = \text{Re}(\phi_1^0)c_\alpha + \text{Re}(\phi_2^0)s_\alpha,$$  \hspace{1cm} (2.9)

$$h = -\text{Re}(\phi_1^0)s_\alpha + \text{Re}(\phi_2^0)c_\alpha,$$  \hspace{1cm} (2.10)

with

$$\tan 2\alpha = \frac{2m_{12}}{m_{11} - m_{22}},$$  \hspace{1cm} (2.11)

where $m_{11}$, $m_{12}$, $m_{22}$ are elements of the real part of the mass matrix,

$$\text{Re}(M) = \begin{pmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{pmatrix},$$  \hspace{1cm} (2.12)

and are given by:

$$m_{11} = m_A^2 s_\beta^2 + v^2 (\lambda_1 c_\beta^2 + \lambda_5 s_\beta^2 + 2\lambda_6 c_\beta s_\beta),$$  \hspace{1cm} (2.13)

$$m_{12} = -m_A^2 c_\beta s_\beta + v^2 \left[ (\lambda_3 + \lambda_4) c_\beta s_\beta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2 \right],$$  \hspace{1cm} (2.14)

$$m_{22} = m_A^2 c_\beta^2 + v^2 (\lambda_2 s_\beta^2 + \lambda_5 c_\beta^2 + 2\lambda_7 c_\beta s_\beta).$$  \hspace{1cm} (2.15)

Finally, the neutral CP-even scalar masses are written as follows:

$$m_{H,h}^2 = \frac{1}{2} \left( m_{11} + m_{22} \pm \sqrt{(m_{11} - m_{22})^2 + 4m_{12}^2} \right).$$  \hspace{1cm} (2.16)

### 2.2 Yukawa Lagrangian and flavor-changing neutral scalar interactions

In the most general case both doublets can participate in the interactions with the fermion fields. The Yukawa Lagrangian is written as

$$\mathcal{L}_Y = Y^u_1 \Bar{Q}_1^0 \bar{U}_1 u_R + Y^u_2 \Bar{Q}_2^0 \bar{U}_2 u_R + Y^d_1 \Bar{Q}_1^0 \bar{d}_1 d_R + Y^d_2 \Bar{Q}_2^0 \bar{d}_2 d_R + Y^\ell_1 \Bar{L}_1^0 \bar{\ell}_1 \ell_R + Y^\ell_2 \Bar{L}_2^0 \bar{\ell}_2 \ell_R + H.c.,$$  \hspace{1cm} (2.17)

with

$$Q^0_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad L^0_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix},$$

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix},$$  \hspace{1cm} (2.18)

$$\tilde{\Phi}_j = i\sigma_2 \Phi^*_j.$$
The zero superscript in fermion fields and Yukawa matrix elements stands for the interaction basis. The left-handed doublets and right-handed singlets are denoted with the subscripts $L$ and $R$, respectively.

After of the SSB and a little bit of algebra, the fermion mass matrices are defined by:

$$M^0_f = \frac{v_1}{\sqrt{2}} Y^0_1 + \frac{v_2}{\sqrt{2}} Y^0_2. \quad (2.19)$$

In order to diagonalize the fermion mass matrices a bi-unitary transformation given through the $O_f$ matrix is required:

$$O_f = \begin{pmatrix}
\frac{m_{f_3} m_{f_3} (A - m_{f_1})}{A(m_{f_2} - m_{f_1}) (m_{f_3} - m_{f_1})} & \frac{m_{f_1} m_{f_3} (m_{f_2} - A)}{A(m_{f_2} - m_{f_1}) (m_{f_3} - m_{f_2})} & \frac{m_{f_1} m_{f_3} (A - m_{f_3})}{A(m_{f_3} - m_{f_1}) (m_{f_3} - m_{f_2})} \\
- \frac{m_{f_1} (m_{f_1} - A)}{(m_{f_2} - m_{f_1}) (m_{f_3} - m_{f_1})} & \frac{m_{f_2} (A - m_{f_2})}{(m_{f_2} - m_{f_1}) (m_{f_3} - m_{f_2})} & \frac{m_{f_2} (A - m_{f_2})}{(m_{f_3} - m_{f_1}) (m_{f_3} - m_{f_2})} \\
\frac{m_{f_1} (A - m_{f_3}) (m_{f_3} - m_{f_1})}{A(m_{f_2} - m_{f_1}) (m_{f_3} - m_{f_1})} & - \frac{m_{f_2} (A - m_{f_3}) (m_{f_3} - m_{f_2})}{A(m_{f_2} - m_{f_1}) (m_{f_3} - m_{f_2})} & \frac{m_{f_2} (A - m_{f_3}) (A - m_{f_2})}{A(m_{f_3} - m_{f_1}) (m_{f_3} - m_{f_2})}
\end{pmatrix} \quad (2.20)$$

where $m_{f_i}$ ($i = 1, 2, 3$) are the physical fermion masses. We assume the hierarchy $m_3 > A > m_2 > m_1$, with $A = m_3 - m_2$.

Once the bi-unitary transformation is applied, the charged lepton mass matrix is

$$\bar{M}_f = \frac{v_1}{\sqrt{2}} \tilde{Y}_1 + \frac{v_2}{\sqrt{2}} \tilde{Y}_2, \quad (2.21)$$

where $\bar{M}_f = O M^0_f O^\dagger = \text{Diagonal} \{m_{f_1}, m_{f_2}, m_{f_3}\}$ and $\tilde{Y}_{1,2} = O Y^0_{1,2} O^\dagger$. Unitary matrices only diagonalize to the mass matrices $M_f$, leaving Yukawa matrices, in general, as non-diagonal. Then, FCNCs are induced at tree level.

The equation (2.21) not only defines the mass matrix but also provide relations to eliminate one of the Yukawa matrices in the physical states. In order to obtain the interactions in terms of only one Yukawa matrix, the equations (2.21) can be written in two possible forms $Y^f_1 = \sqrt{2} M_f - \tan \beta Y^f_2$ or $Y^f_2 = \sqrt{2} M_f - \cot \beta Y^f_1$. Thus, from Eq. (2.17), the 2HDM-III can be written in four different versions, however, we choose to write the interactions as a function of $Y_2$. From now on, in order to simplify the notation, the subscript 2 in the Yukawa couplings will be omitted. After SSB and doing the proper rotations, the interactions between charged leptons and Higgs boson is explicitly written as

$$\mathcal{L}_Y^l = \frac{g}{2} \left( \frac{m_{\bar{h}}}{m_W} \right) \ell_i \left[ - \sin \alpha \cos \beta \delta_{ij} + \sqrt{2} \cos (\alpha - \beta) \left( \frac{m_W}{m_{\bar{h}}} \right) \frac{Y^f_{ij} \bar{Y^f_{ij}}}{m_{\bar{h}}} \right] \ell_j h. \quad (2.22)$$

The complete Yukawa Lagrangian is shown in the appendix A.

The FC neutral scalar interactions can be suppressed through assuming a structure in the non-diagonal mass matrix, which is in the interaction base; this assumption is known as textures. One of the first proposals was based on the texture formalism [37–41] in which the mass matrix has the following structure:

$$M_f = \begin{pmatrix}
0 & D_f & 0 \\
D_f^* & C_f & B_f \\
0 & B_f^* & A_f
\end{pmatrix}, \quad (2.23)$$
whose elements are related to eigenvalues $m_i, (i = 1, 2, 3)$, through the following invariants:

$$
\begin{align*}
det(M) &= -D^2 A = m_1 m_2 m_3, \\
Tr(M) &= C + A = m_1 + m_2 + m_3, \\
\lambda(M) &= CA - D^2 - B = m_1 m_2 + m_1 m_3 + m_2 m_3,
\end{align*}
$$

(2.24)

where we have omitted the subscript $f$ to not overload the notation. From these expressions we find a relation between the components of the four-zero matrix mass and the mass eigenstates, namely:

$$
\begin{align*}
A &= m_3 - m_2, \\
B &= m_3 \sqrt{\frac{r_2(r_2 + r_1 - 1)(r_2 + r_2 - 1)}{1 - r_2}}, \\
C &= m_3(r_2 + r_1 + r_2), \\
D &= \sqrt{\frac{m_1 m_2}{1 - r_2}},
\end{align*}
$$

(2.25)

with $r_i = m_i/m_3$.

Under the structure of the mass matrix Eq. (2.23), the elements of the Yukawa matrix will have the form of so called Cheng−Sher ansatz [42]:

$$
Y_{ij} = \sqrt{\frac{m_i m_j}{v}} \chi_{ij},
$$

(2.26)

where $m_{i,j}$ are the fermion masses. In particular, we have:

$$
\bar{Y}_{\tau \mu} = \sqrt{\frac{m_{\tau} m_{\mu}}{v}} \chi_{\tau \mu},
$$

(2.27)

$$
\chi_{\tau \mu} = \sqrt{\frac{m_{\tau}}{m_{\mu}}} \left( \frac{b_1 v}{m_{\tau} \tan \beta} - \frac{F_1}{\sin \beta} \right) + \left( \frac{a_1 - c_1}{m_{\tau} \tan \beta} - \frac{\sqrt{2}}{\sin \beta} F_2 \right),
$$

(2.28)

where we define $F_1 = \sqrt{2} G$, $F_2 = Q - 2 G$, $G = r_{\mu}$, $Q = 1 - r_{\mu}$ and $r_{\mu} = m_{\mu}/m_{\tau}$. Here, $a_1$, $b_1$ and $c_1$ are elements of the Yukawa matrix $Y_{1}^\ell$. In this work, instead of constraining the parameters that come from the explicit form of Yukawa matrices, we restrict the $\chi_{\tau \mu}$ parameter as a whole.

3 Model parameter space

In order to evaluate the branching ratio of the $h \to \tau \mu$ decay and the production cross section of the SM-like Higgs boson by the gluon fusion mechanism, we need to analyze the 2HDM-III free model parameters. The most relevant 2HDM-III parameters involved in this work are the $\cos(\alpha - \beta) = c_{\alpha \beta}$ and $\tan \beta = t_{\beta}$ because $g_{h\tau\mu}$ and $g_{h\ell\ell}$ couplings are proportional to them. Figure 1 illustrates this.

To constrain the above mentioned parameters, we consider LHC Higgs boson data, $B_s^0 \to \mu^- \mu^+$, the tau lepton decays $\tau \to \bar{\ell}_i \ell_j \ell_j$ and $\tau \to \ell_i \gamma$ as well as the experimental
constraint on the $h \to \tau \mu$ and the muon anomalous magnetic dipole moment $\delta a_\mu$. Direct searches for additional heavy neutral $C\bar{P}$-even and $C\bar{P}$-odd scalars through $gb \to \phi \to \tau \tau$ [43, 44], with $\phi = H, A$ are also used in order to constrain their masses, we denote them as $m_H, m_A$. Finally, the charged scalar boson mass $m_{H^\pm}$ is constrained with the upper limit on $\sigma(pp \to tbH^\pm) \times BR(H^\pm \to \tau^\pm \nu)$ [45] and the decay $b \to s\gamma$ [46–51].

3.1 Constraint on $c_{\alpha\beta}$ and $t_\beta$

In order to have values of $c_{\alpha\beta}$ in accordance with current experimental results, we use the coupling modifiers $\kappa$-factors reported by ATLAS and CMS collaborations [52, 53]. They are defined as following:

$$\kappa_{pp}^2 = \frac{\sigma(pp \to H^{2\text{HDM-III}})}{\sigma(pp \to h^{\text{SM}})} \text{ or } \kappa_{x\bar{x}}^2 = \frac{\Gamma(H^{2\text{HDM-III}} \to x\bar{x})}{\Gamma(h^{\text{SM}} \to x\bar{x})}. \tag{3.1}$$

where $\Gamma(H_i \to x\bar{x})$ is the decay width of $H_i$ into $x\bar{x} = b\bar{b}, \tau^-\tau^+, ZZ, WW, \gamma\gamma$ and $gg$; with $H_i = h^{2\text{HDM-III}}$ and $h^{\text{SM}}$. Here $h^{2\text{HDM-III}}$ is the SM-like Higgs boson coming from 2HDM-III and $h^{\text{SM}}$ is the SM Higgs boson; $\sigma(pp \to H_i)$ is the Higgs boson production cross section via proton-proton collisions. In addition, we also consider the current experimental limits on the tau decays $\tau \to \mu\gamma$, $\tau \to \ell_i\ell_j\ell_i\ell_j$, $\delta a_\mu$, $B_0^0 \to \mu^+\mu^-$ [25] and the direct upper bound on the branching ratio of the Higgs boson into $\tau\mu$ pair [54, 55]. All the necessary formulas to perform our analysis of the model parameter space are presented in Appendix B.

In figure 2 we present the $c_{\alpha\beta} - t_\beta$ plane in which the colored areas represent the consistent regions with results reported for $\kappa_{x\bar{x}}$’s, LFV processes and $B_0^0 \to \mu^+\mu^-$, namely, green: $\delta a_\mu$, cyan: $\tau \to \mu\gamma$, magenta: $\kappa_V$ with $V = W, Z$, pink: $\tau \to 3\mu$, orange: $\kappa_b$, yellow: $\kappa_\tau$, purple: $h \to \tau\mu$ and the dotted region corresponds to $B_0^0 \to \mu^+\mu^-$. As a particular case, we observe that the 2HDM-III is able to accommodate the current discrepancy between the theoretical SM prediction and the experimental measurement of the muon anomalous magnetic dipole moment $\delta a_\mu$. However, from figure 2, we note that the allowed region by $\delta a_\mu$ is out of the intersection of the additional observables. This happens by choosing the parameters shown in table 1. We find that $\delta a_\mu$ is sensitive to $\chi_{\tau\mu}$ which is set to the unit in order to obtain the best fit of the model parameter space. Under this choice, $\delta a_\mu$ is explained with high values of $t_\beta$.

By omitting $\delta a_\mu$, the yellow dotted area filled with segmented lines shows the intersection of all individual allowed regions. We observe that for the particular value of $c_{\alpha\beta} = 0.05$...
corresponds to \( t_\beta \approx 8 \), while for \( c_{\alpha \beta} = 0 \) corresponds to \( t_\beta \approx 11 \). The graph was generated with the package \text{SpaceMath} [56].

Figure 2. Allowed regions by LFV processes; \( \kappa_i \), with \( i = \tau, b, W, Z \) and \( B_0 \to \mu^- \mu^+ \) in the plane \( c_{\alpha \beta}-t_\beta \).

3.2 Constraint on \( m_H, m_A \) and \( m_{H^\pm} \)

3.2.1 \( m_H \) and \( m_A \)

The ATLAS and CMS collaborations presented results of a search for additional neutral Higgs bosons in the ditau decay channel [43, 44]. The former of them searched through the process \( gb \to \phi \to \tau\tau \), with \( \phi = A, H \); figure 3 shows the Feynman diagram of this reaction. However, no evidence of any additional Higgs boson was observed. Nevertheless, upper limits on the production cross section \( \sigma(gb \to \phi) \times \text{BR}(\phi \to \tau\tau) \) were imposed. In this work we focus on the particular case of the search carried out by the ATLAS collaboration.

In figure 4(a), we present the \( \sigma(gb \to Hb) \times \text{BR}(H \to \tau\tau) \) as a function of the \( \mathcal{CP} \)-even neutral scalar mass \( m_H \) for different values of \( t_\beta = 1, 5, 10, 40 \) and \( c_{\alpha \beta} = 0.05 \). Figure 4(b) shows the same but for the \( \mathcal{CP} \)-odd neutral scalar mass \( m_A \) and \( t_\beta = 10, 30, 40 \). In both plots, the black points and red crosses represent the expected and observed values at 95% CL upper limits, respectively; while the green (yellow) band indicates the interval at \( \pm 1\sigma \)
(±2σ) with respect to the expected value. We implement the Feynman rules in CalcHEP [57] in order to evaluate \( \sigma(gb \to \phi b) \times BR(\phi \to \tau\tau) \).

From figure 4(a) we note that \( \mathcal{C}P \)-even scalar masses \( m_H \lesssim 210 \text{ GeV} \) (\( m_H \lesssim 520 \text{ GeV} \)) are excluded at 2σ (1σ) for \( t_\beta = 10 \), while for \( t_\beta = 1, 5 \), the upper limit on \( \sigma(gb \to \phi b) \times BR(\phi \to \tau\tau) \) is easily accomplished. Although \( t_\beta = 40 \) is discarded, as shown in figure 2, we include it to have an overview of the behavior of the model. On the other side, from figure 4(b), we find that \( \mathcal{C}P \)-odd scalar masses \( m_A \lesssim 310 \text{ GeV} \) (\( m_H \lesssim 360 \text{ GeV} \)) are excluded at 2σ (1σ) for \( t_\beta = 10 \).

### 3.2.2 Constraint on the charged scalar mass \( m_{H^\pm} \)

The discovery of a charged scalar \( H^\pm \) would constitute unambiguous evidence of new physics. Direct constraints can be obtained from collider searches for the production and decay of on-shell charged Higgs bosons. These limits are very robust and model-independent if the basic assumptions on the production and decay modes are satisfied [58–61].
recently the ATLAS collaboration reported a study on the charged Higgs boson produced either in top-quark decays or in association with a top quark. Subsequently the charged Higgs boson decays via $H^\pm \to \tau^\pm \nu_\tau$ with a center-of-mass energy of 13 TeV [45]. We analyze this process through the CalcHEP package, however, we find that this process is not a good way to impose a stringent bound on $m_{H^\pm}$.

Conversely, the decay $b \to s\gamma$ imposes stringent limits on $m_{H^\pm}$ because a new ingredient with respect to the SM contribution [46–49] is the presence of the charged scalar boson coming from 2HDM-III which gives contributions to the Wilson coefficients of the effective theory as is shown in the Refs. [50, 51].

In figure 5 we show $R_{\text{quark}}$ at NLO in QCD as a function of the charged scalar boson mass for $t_\beta = 2, 5, 10$, where $R_{\text{quark}}$ is defined as following:

$$R_{\text{quark}} = \frac{\Gamma(b \to Xs\gamma)}{\Gamma(b \to Xc\nu_\tau)}.$$  \hspace{1cm} (3.2)

We observe that for $t_\beta = 2$, the charged scalar boson mass $100 \text{ GeV} \lesssim m_{H^\pm}$ ($700 \text{ GeV} \lesssim m_{H^\pm}$) is excluded, at $2\sigma$ ($1\sigma$); while $t_\beta = 10$ imposes a more restrictive lower bound $1.6 \text{ TeV} \lesssim m_{H^\pm}$ ($3.2 \text{ TeV} \lesssim m_{H^\pm}$) at $2\sigma$ ($1\sigma$).

![Figure 5](image)

**Figure 5.** $R_{\text{quark}}$ at NLO in QCD as a function of the charged scalar boson mass for $t_\beta = 2, 5, 10$. Solid line represents the experimental central value while red crosses indicate the theoretical SM central value. Green and yellow bands stand for $1\sigma$ and $2\sigma$, respectively. $R_{\text{quark}}$ is defined in the main text.

In summary, table 1 shows the values of the 2HDM-III parameters involved in the subsequent calculations.

### 4 Search for the $h \to \tau\mu$ decay at future hadron colliders

We are interested in a possible evidence for the $h \to \tau\mu$ decay at future hadron collider. Thus, in this section we analyze the LFV process of the Higgs boson decaying into a $\tau\mu$ pair and its production at future hadron colliders via the gluon fusion mechanism.
Table 1. Values of the parameters used in the calculations.

| Parameter | Values            |
|-----------|-------------------|
| $c_{\alpha\beta}$ | 0.05             |
| $t_\beta$   | 0.1-8             |
| $\chi_{\tau\mu}$ | 0.1, 0.5, 1      |
| $m_H = m_A$ | 800 GeV          |

We analyze three scenarios that correspond to each of the future hadron colliders, namely:

- **Scenario A (SA)**: HL-LHC at a center-of-mass energy of 14 TeV and integrated luminosities in the interval 0.3-3 ab$^{-1}$,
- **Scenario B (SB)**: HE-LHC at a center-of-mass energy of 27 TeV and integrated luminosities in the range 0.3-12 ab$^{-1}$,
- **Scenario C (SC)**: FCC-hh at a center-of-mass energy of 100 TeV and integrated luminosities from 10 to 30 ab$^{-1}$.

4.1 Number of signal and background events

Once the free model parameters were constrained in section 3, we now turn to evaluate the number of events produced of the signature $gg \rightarrow h \rightarrow \tau\mu$.

In figure 6 we present the $\sigma(gg \rightarrow h)BR(h \rightarrow \tau\mu)$ as a function of $t_\beta$ (left axis) and the Events-$t_\beta$ plane (right axis) for scenarios SA, SB and SC. In all figures, the dark area represents the consistent region with allowed parameter space found in section 3 (see table 1). We observe that the maximum signal number of events ($N^{SA}$) produced are of the order of $N^{SA} = \mathcal{O}(10^5)$, $N^{SB} = \mathcal{O}(10^6)$, $N^{SC} = \mathcal{O}(10^7)$, by considering $t_\beta = 8$ and $\chi_{\tau\mu} = 1$.

![Figure 6](image_url)

**Figure 6.** (a) Scenario SA, (b) Scenario SB, (c) Scenario SC. Left axis: $\sigma(gg \rightarrow h)BR(h \rightarrow \tau\mu)$ as a function of $t_\beta$ for $\chi_{\tau\mu}=0.1$, 0.5, 1. Right axis: Events-$t_\beta$ plane. The dark area corresponds to the allowed region. See table 1.
4.2 Monte Carlo analysis

We will now analyze the signature of the decay $h \rightarrow \tau \mu$, with $\tau \mu = \tau^{-}\mu^{+} + \tau^{+}\mu^{-}$ and its potential SM background. The ATLAS and CMS collaborations [62, 63] searched two $\tau$ decay channels: electron decay $\tau \rightarrow e\nu_{e}\tau\nu_{e}$ and hadron decay $\tau_{h}\mu$. In our analysis, we will concentrate on the electron decay. As far as our computation scheme is concerned, we first implement the relevant Feynman rules via LanHEP [64] for MadGraph5 [65], later it is interfaced with Pythia8 [66] and Delphes 3 [67] for detector simulations. Subsequently, we generate $10^5$ signal and background events, the last ones at NLO in QCD. We used CT10 parton distribution functions [68].

Signal and SM background processes

The signal and background processes are as following:

- **SIGNAL**: The signal is $gg \rightarrow h \rightarrow \tau \mu \rightarrow e\nu_{e}\tau\nu_{e}\mu$. The electron channel must contain exactly two opposite-charged leptons, namely, one electron and one muon. Therefore, we search for the final state $e\mu$ plus missing energy due to neutrinos not detected.

- **BACKGROUND**: The main SM background arises from:
  1. Drell-Yan process, followed by the decay $Z \rightarrow \tau\tau \rightarrow e\nu_{e}\mu\nu_{\mu}$.
  2. $WW$ production with subsequent decays $W \rightarrow e\nu_{e}$ and $W \rightarrow \mu\nu_{\mu}$.
  3. $ZZ$ production, later decaying into $Z \rightarrow \tau\tau \rightarrow e\nu_{e}\mu\nu_{\tau}\nu_{\mu}$ and $Z \rightarrow \nu\nu$.

Signal significance

The main kinematic cuts to isolate the signal are the collinear and transverse mass defined as following:

$$m_{\text{col}}(e\mu) = \frac{m_{\text{inv}}(e\mu)}{\sqrt{x}}, \text{ with } x = \frac{|\vec{P}_{T}|}{|\vec{P}_{T}| + E_{\text{miss}}^{T}.\vec{P}_{T}}, \quad \text{(4.1)}$$

and

$$M_{T}^{T} = \sqrt{2|\vec{P}_{T}||E_{\text{miss}}^{T}|(1 - \cos \Delta\phi_{\vec{P}_{T},E_{\text{miss}}^{T}})}, \quad \text{(4.2)}$$

In figure 7 we show the distribution of collinear mass versus number of signal events for the scenarios (a) SA, (b) SB and (c) SC with integrated luminosities of 3, 12 and 30 ab$^{-1}$, respectively. In all scenarios we consider $t_{\beta} = 5, 8$. We use the package MadAnalysis5 [69] to analyze the kinematic distributions. Additional cuts applied both signal and background are shown in Table 2 for scenario SA. We also display the event number of the signal ($N_{S}$) and background ($N_{B}$) once the kinematic cuts were applied. The signal significance considered is defined as the ratio $N_{S}/\sqrt{N_{S} + N_{B}}$. The efficiency of the cuts for the signal and background are: $\epsilon_{S} \approx 0.13$ and $\epsilon_{B} \approx 0.014$, respectively.

We find that at the LHC is not possible to claim for evidence of the decay $h \rightarrow \tau \mu$ achieving a signal significance about $1.46\sigma$ by considering its final integrated luminosity, $300$ fb$^{-1}$. More promising results arise at HL-LHC in which a prediction of about $4.6\sigma$ when an integrated luminosity of $3$ ab$^{-1}$ and $t_{\beta} = 8$ are achieved. Meanwhile, at HE-LHC
Figure 7. Distribution of the collinear mass versus number of signal events for scenarios (a) SA, (b) SB and (c) SC.

Table 2. Kinematic cuts applied to the signal and main SM background for scenario SA, i.e, at HL-LHC with a center-of-mass energy $\sqrt{s} = 14$ TeV and $L_{\text{int}} = 3$ ab$^{-1}$ for $t_\beta = 8$.

| Cut number | Cut                        | $N_S$   | $N_B$   | $N_S/\sqrt{N_S + N_B}$ |
|------------|----------------------------|---------|---------|-------------------------|
| Initial    | (no cuts)                  | 57665   | 200089020 | 4.08                    |
| 1          | $|\eta^e| < 2.3$            | 25282   | 132346436 | 2.1975                  |
| 2          | $|\eta^\mu| < 2.1$         | 16378   | 106936728 | 1.5837                  |
| 3          | $0.1 < \Delta R(e, \mu)$  | 16355   | 106801230 | 1.5825                  |
| 4          | $20 < p_T(e)$              | 15533   | 38846174  | 2.4817                  |
| 5          | $30 < p_T(\mu)$           | 12119   | 20357367  | 2.6852                  |
| 6          | $10 < \text{MET}$         | 11185.9 | 20086662  | 2.4952                  |
| 7          | $100 < m_{\text{cut}}(e, \mu) < 150$ | 9645.1 | 9330510 | 3.1560 |
| 8          | $25 < M_T(e)$             | 8669.4  | 4827617   | 3.942                   |
| 9          | $15 < M_T(\mu)$          | 7869    | 2867711   | 4.6404                  |

(FCC-hh) a potential discovery could be claimed with a signal significance of around 5.04σ ($\sim 5.43\sigma$) for an integrated luminosity of 9 ab$^{-1}$ and $t_\beta = 6$ (15 ab$^{-1}$ and $t_\beta = 3$). To illustrate the above, in Figure (8) we present the signal significance as a function of $t_\beta$ for integrated luminosities associated with each scenario, namely:

- **SA**: from 0.3 ab$^{-1}$ at 3 ab$^{-1}$ for the HL-LHC,
- **SB**: from 3 ab$^{-1}$ at 12 ab$^{-1}$ for the HE-LHC,
- **SC**: from 10 ab$^{-1}$ at 30 ab$^{-1}$ for the FCC-hh.

Finally, we present in Figure (9) an overview of the signal significance as a function of the integrated luminosity for representative values of $t_\beta$. 

− 13 −
5 Conclusions

In this article we have studied the LFV decay $h \to \tau\mu$ within the context of the 2HDM type III and we analyze its possible detectability at future super hadron colliders, namely, HL-LHC, HE-LHC and the FCC-hh.

We find the allowed model parameter space by considering the most up-to-date experimental measurements and later is used to evaluate the Higgs boson production cross section via the gluon fusion mechanism and the branching ratio of the $h \to \tau\mu$ decay.
A Monte Carlo analysis of the signal and its potential SM background was realized. We find that the closest evidence could arise at the HL-LHC with a prediction of the order of $4.66\sigma$ for an integrated luminosity of $3\text{ ab}^{-1}$ and $\tan\beta = 8$. On the other hand, a potential discovery could be claimed at the HE-LHC (FCC-hh) with a signal significance about $5.046\sigma$ ($5.43\sigma$) for an integrated luminosity of $3\text{ ab}^{-1}$ and $\tan\beta = 8$ ($5\text{ ab}^{-1}$ and $\tan\beta = 4$).

If the decay considered in this research is observed in a future super hadron collider, then it will be a clear signal of physics BSM.

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A Yukawa Lagrangian

The Yukawa Lagrangian of the Type-III Two-Higgs Doublet Model in terms of the physical fields are given by:

\[
\mathcal{L}_Y^{\ell} = \frac{g}{2} \left( \frac{m_{\ell_i}}{m_W} \right) \bar{\ell}_i \left[ -\sin \alpha \cos \beta \delta_{ij} + \frac{\sqrt{2} \cos(\alpha - \beta)}{g \cos \beta} \left( \frac{m_W}{m_{\ell_i}} \right) \tilde{Y}^\ell_{ij} \right] \ell_j h \\
+ \frac{g}{2} \left( \frac{m_{\ell_i}}{m_W} \right) \bar{\ell}_i \left[ \cos \alpha \cos \beta \delta_{ij} + \frac{\sqrt{2} \sin(\alpha - \beta)}{g \cos \beta} \left( \frac{m_W}{m_{\ell_i}} \right) \tilde{Y}^\ell_{ij} \right] \ell_j H \\
+ ig \left( \frac{m_{\ell_i}}{m_W} \right) \bar{\ell}_i \left[ -\tan \beta \delta_{ij} + \frac{\sqrt{2}}{g \cos \beta} \left( \frac{m_W}{m_{\ell_i}} \right) \tilde{Y}^\ell_{ij} \right] \gamma^5 \ell_j A \\
+ \frac{g}{2} \left( \frac{m_{d_i}}{m_W} \right) \bar{d}_i \left[ -\sin \alpha \cos \beta \delta_{ij} + \frac{\sqrt{2} \cos(\alpha - \beta)}{g \cos \beta} \left( \frac{m_W}{m_{d_i}} \right) \tilde{Y}^d_{ij} \right] d_j h \\
+ \frac{g}{2} \left( \frac{m_{d_i}}{m_W} \right) \bar{d}_i \left[ \cos \alpha \cos \beta \delta_{ij} + \frac{\sqrt{2} \sin(\alpha - \beta)}{g \cos \beta} \left( \frac{m_W}{m_{d_i}} \right) \tilde{Y}^d_{ij} \right] d_j H \\
+ ig \left( \frac{m_{d_i}}{m_W} \right) \bar{d}_i \left[ -\tan \beta \delta_{ij} + \frac{\sqrt{2}}{g \cos \beta} \left( \frac{m_W}{m_{d_i}} \right) \tilde{Y}^d_{ij} \right] \gamma^5 d_j A \\
+ \frac{g}{2} \left( \frac{m_u}{m_W} \right) \bar{u}_i \left[ \sin \alpha \cos \beta \delta_{ij} + \frac{\sqrt{2} \sin(\alpha - \beta)}{g \sin \beta} \left( \frac{m_W}{m_u} \right) \tilde{Y}^u_{ij} \right] u_j H \\
+ \frac{g}{2} \left( \frac{m_u}{m_W} \right) \bar{u}_i \left[ -\cos \alpha \cos \beta \delta_{ij} + \frac{\sqrt{2} \cos(\alpha - \beta)}{g \sin \beta} \left( \frac{m_W}{m_u} \right) \tilde{Y}^u_{ij} \right] u_j h \\
+ ig \left( \frac{m_u}{m_W} \right) \bar{u}_i \left[ -\cot \beta \delta_{ij} + \frac{\sqrt{2}}{g \sin \beta} \left( \frac{m_W}{m_u} \right) \tilde{Y}^u_{ij} \right] \gamma^5 u_j A, \tag{A.1}
\]

where \(i\) and \(j\) stand for the fermion flavors, in general \(i \neq j\).

B Complementary formulas used in the analysis of the model parameter space

In this Appendix we present the analytical expressions in order to obtain the constraints on the LFV couplings as is shown in Figure 2.

We first start with the expression for the width decay of SM-like Higgs boson into fermion pair, which is given by:

\[
\Gamma(h \to f_if_j) = \frac{g_{h,fi}^2 N_c m_h}{128\pi} \left( 4 - \left( \sqrt{\lambda_f} + \sqrt{\lambda_f} \right)^2 \right)^{3/2} \left( 4 - \left( \sqrt{\lambda_f} - \sqrt{\lambda_f} \right)^2 \right)^{1/2}, \tag{B.1}
\]

where \(\lambda_f = 4m_{f_i}^2/m_h^2\), with \(k = i, j\); \(N_c\) is the color number. In our case \(g_{h,\tau\mu} = \frac{c_{\alpha\beta\tau\mu}}{\sqrt{2} s_{\alpha\beta}} \tilde{Y}_{\tau\mu}\).

As far as the \(\tau \to \mu\gamma\) decay is concerned, it arises at the one-loop level and receives contributions of the SM Higgs boson and the \(CP\)-even and \(CP\)-odd neutral heavy scalar.
bosons ($\phi = h, H, A$). Feynman diagrams for this process are displayed in Figure 10(a). The decay width is given by:

$$\Gamma(\tau \to \mu \gamma) = \frac{\alpha m_{\tau}^5}{64\pi^2} \left( |A_S|^2 + |A_P|^2 \right), \quad (B.2)$$

where the $A_S$ and $A_P$ coefficients indicate the contribution of $\mathcal{CP}$-even and $\mathcal{CP}$-odd scalar bosons, respectively. In the limit of $g_{\phi \tau \tau} \gg g_{\phi \mu \mu} \gg g_{\phi ee}$ and $m_\tau \gg m_\mu \gg m_e$, they can be approximated as [70]

$$A_S = A_P \simeq \sum_{\phi=h,H,F} \frac{g_{\phi \tau \tau} g_{\phi \mu \tau}}{12m_\phi^2} \left( 3 \ln \left( \frac{m_\phi^2}{m_\tau^2} \right) - 4 \right). \quad (B.3)$$

Two-loop contributions can be relevant, their expressions are reported in [70]. The current experimental limit on the branching ratio is $BR(\tau \to \mu \gamma) < 4.4 \times 10^{-8}$.

---

**Figure 10.** Feynman diagrams that contribute to (a) $\tau \to \mu \gamma$ and (b) $\tau \to \mu \bar{\mu}$ decays with exchange of a scalar boson $\phi$. We omit both the bubble diagrams for the LFV decay $\tau \to \mu \gamma$, because only serve to cancel the ultraviolet divergences.

As for the $\tau \to \mu \bar{\mu}$ decay, it receives contributions from $\phi$ as depicted in the Feynman diagram of Figure 10(b). The tree-level decay width can be approximated as

$$\Gamma(\tau \to \mu \bar{\mu}) \simeq \frac{m_\tau^5}{256\pi^3} \left( \frac{S_h^2}{m_h^4} + \frac{S_H^2}{m_H^4} + \frac{S_{HF}^2}{m_{HF}^4} + 2S_hS_H + \frac{2S_A}{3m_A^2} \left( \frac{S_h + S_H}{m_h^2} + \frac{S_{HF}}{m_{HF}^2} \right) \right), \quad (B.4)$$

where $S_\phi = g_{\phi \mu \mu}g_{\phi \mu \tau}$. The upper bound on the branching ratio is $BR(\tau \to \mu \bar{\mu}) < 2.1 \times 10^{-8}$ [25].

Finally, the muon AMDM also receives contributions from $\phi$, which are induced by a triangle diagram similar to the diagram of Figure 10(a) but with two external muons. The corresponding contribution can be approximated for $m_\phi \gg m_l$ as [70]

$$\delta a_\mu \sim \frac{m_\mu}{16\pi^2} \sum_{\phi=h,H,A} \sum_{l=\mu,\tau} \frac{mg_{\phi l}^2}{m_\phi^2} \left( 2 \ln \left( \frac{m_\phi^2}{m_l^2} \right) - 3 \right), \quad (B.5)$$

where one must take into account the NP corrections to the $g_{h\mu \mu}$ coupling only. If $H$ and $A$ are too heavy, the dominant NP contribution would arise from the SM Higgs boson.
The discrepancy between the experimental value and the SM theoretical prediction is

\[ \Delta a_\mu = a_\mu^{\exp} - a_\mu^{SM} = (2.88 \pm 0.63 \pm 0.49) \times 10^{-9}. \]  

(B.6)

Thus, the requirement that this discrepancy is accounted for by Eq. (B.5) leads to the bound \[1.32 \times 10^{-9} \leq \Delta a_\mu \leq 4.44 \times 10^{-9}\] with 95% C.L.

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