Dissipative instability of converging cylindrical shock wave

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Abstract

The condition of linear instability for the converging cylindrical strong shock wave (SW) in arbitrary viscous medium is obtained in the limit of large stationary SW radius, when it is possible to consider the same Rankine-Hugoniot jump relations as for the plane SW. This condition of instability is substantial different from the condition of instability for the plane SW because cylindrical SW have not chiral symmetry for the direction of the SW velocity (from the left to right or vice versa) as for the case of plane SW. The exponential increment of perturbations for the converging cylindrical SW is positive only for nonzero viscosity in the limit of high, but finite Reynolds numbers as for instability of plane SW.

Introduction

Shock waves (shocks or SW) arise in hydrodynamics, aerodynamics and many fundamental and applied physical problems such as the problems of inertial confinement fusion [1], supernova explosion [2] and underwater electrical explosion of wires and wire arrays [3, 4]. In cylindrical and spherical geometries, a converging shock is strengthened towards the center and it is of high importance to examine its stability [5-9]. For example, in [7-9] is stated that a convergent spherical shock is unstable in linear approximation when the growth rate of the disturbances, which is obtained numerically, is only algebraic and is slowly than exponential growth. Shock is stable in the case of symmetrical perturbations [5, 6, 8] for converging spherical and cylindrical shocks, but a finite region of stability has been found numerically for the case of asymmetrical perturbations. It is also known that plane shock is always stable to
one-dimensional (1D) perturbations in ideal medium [10-12]. As it is shown in [10], this is the result of absence of parameter which has the dimension of inverse time which is necessary, but not sufficient to realize the plane shock instability. Indeed, in [13] for the case of shocks in viscous medium this necessary parameter with dimension of inverse time arise, but stability to 1D perturbations is stated for the weak plane shocks considered in [13] on the base of the Burgers equation. From the other side for strong plane shocks the instability to 1D perturbations is stated in [14] on the base of generalization of the D’yakov theory [10-12] when viscosity is taken into account.

For cylindrical shocks the parameter with dimension of inverse time is always exist due to finite curvature of the shock front and compressibility of medium with finite speed of sound. However, the shock instability in the limit of large stationary radius of converging cylindrical SW velocity may arise only if viscosity is taken into account for strong shocks, as in the case of plane shock front, but at substantially different conditions. Here the instability of the converging cylindrical shocks in viscous medium for the case of symmetric perturbations is stated, when only small acoustic perturbations are considered (without their connection with entropy perturbations, which are also considered in [10-12] in addition to acoustic perturbations).

1. Dispersion equations

Let us consider a converging cylindrical SW of arbitrary intensity, propagating in the directions perpendicular to axis \( z \) in the cylindrical variables \((z, r, \varphi)\).

The radial velocity of the converging shock front is \( D < 0 \) and \( U < 0 \) is the radial velocity of the medium behind the shock wave front. For simplicity, let us consider the case when it is assumed that the SW front is uniform in the direction of the axis \( z \) and there are no perturbations of the velocity field component along this axis. This case allows us to simulate a converging cylindrical shock wave arising from an explosion of a system consisting of a finite
number of long wires bounding a cylindrical region with an axis coinciding with the z axis. In this case, perturbations in the azimuthal direction can be caused by the finite distance between the wires, which determines the wavelength of the corresponding perturbation. Since the wires are assumed to be uniform along the length, this corresponds to the assumption made above that there are no disturbances along the axis z.

Also let us consider quasi stationary limit when \( D \approx \text{const} \); \( U \approx \text{const} \) and it is possible to neglect of the piston influence on shock front propagation and its stability.

In this limit the equation for the perturbed cylindrical surface of the shock
\[
R_s(\tau = \alpha) + r_{sh}(\varphi, t); \epsilon << 1, \tau \approx \text{const}
\]
depends only on the time and the coordinate \( \varphi \) in the form:
\[
r_{sh} = g(\varphi, t) \tag{1}
\]

Equations for small perturbations of the velocity and pressure fields behind the front of the shock wave in the linear approximation have the form in the reference of frame where shock is in rest (\( w = U - D \approx \text{const} \)), when they are represented in cylindrical coordinates:

\[
\frac{\partial V_{tr}}{\partial t} + \frac{w}{r} \frac{\partial V_{tr}}{\partial r} = - \frac{1}{\rho} \frac{\partial p_1}{\partial r} + V_1 \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial V_{tr}}{\partial r} - \frac{V_{tr}}{r^2} \right) \right] + \frac{(\nu_1 + \nu_2)}{3} \frac{\partial \text{div} \vec{V}_1}{\partial r};
\]

\[
\frac{\partial V_{tp}}{\partial t} + \frac{w}{r} \left( \frac{\partial V_{tp}}{\partial r} + \frac{1}{r} \frac{\partial V_{tp}}{\partial \varphi} \right) = - \frac{1}{\rho} \frac{\partial p_1}{\partial r} + V_1 \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial V_{tp}}{\partial r} - \frac{V_{tp}}{r^2} \right) \right] + \frac{(\nu_1 + \nu_2)}{3} \frac{\partial \text{div} \vec{V}_1}{r \partial \varphi};
\]

\[
\frac{\partial p_1}{\partial t} + w \frac{\partial p_1}{\partial r} + c^2 \rho \left( \frac{1}{r} \frac{\partial r V_{tr}}{\partial r} + \frac{1}{r} \frac{\partial V_{tp}}{\partial \varphi} \right) = 0;
\]

\[
\text{div} \vec{V}_1 = \frac{1}{r} \frac{\partial}{\partial r} \left( r V_{tr} \right) + \frac{1}{r} \frac{\partial V_{tp}}{\partial \varphi} \tag{2}
\]

In (2) \( \nu_1, \nu_2 \) are the first and second (volume) constant coefficients of kinematic viscosity and the perturbation fields are denoted by the subscript 1, where \( c^2 = (\partial p/\partial \rho) \), is the
square of the speed of sound in the region behind the shock wave front where the Mach number $M = \frac{|w|}{c} < 1$. Also in (2) the equation for pressure has the form of the equation of mass conservation when the relationship between the density and pressure perturbations in this region has the form $p_1 = c^2 \rho_1$.

Equations (2) provide a closed description of the evolution of perturbation. Further, system (2) will be considered only for acoustic disturbances without consideration of entropy disturbances, which are taken into account as in the D’yakov theory for plane shock instability in ideal non viscose medium [10-12].

System (2) must be considered together with the boundary conditions on the perturbation (of near stationary cylindrical shock surface with $R_s (\tau = \alpha) \approx R_{s0} = \text{const}; \epsilon << 1$) surface defined by function (1). It is possible to neglect the influence of viscosity in boundary conditions in the limit of small viscosity (or large Reynolds numbers) [14]. Then, for the tangential and normal unit vectors to this surface, we have (see also [11, 12]):

$$\vec{\iota} = (t_r, t_\varphi) = (-g_\varphi / R_{s0}; -1) / \sqrt{1 + g_\varphi^2 / R_{s0}^2};$$

$$\vec{n} = (n_r, n_\varphi) = (1; -g_\varphi / R_{s0}) / \sqrt{1 + g_\varphi^2 / R_{s0}^2};$$

$$g_\varphi = \frac{\partial g(\varphi, t)}{\partial \varphi}$$

(3)

In the linear approximation, one can neglect nonlinear terms in (3). From the condition of continuity of the tangential component of the velocity field on the perturbed front of a shock wave, it follows that scalar products with the vector $\vec{\iota}$ of the velocity vectors on the both sides of the shock front, $(w + V_{r, \varphi} \circ \vec{\iota}) \circ \vec{\iota} = (w_0, 0) \circ \vec{\iota}$; for $w_0 = U_0 - D = -D \approx \text{const}$, that in the linear approximation for $g_\varphi \ll 1$, gives:

$$V_{i\varphi} = g_\varphi (w_0 - w) / R_{s0}$$

(4)
Similarly, from the boundary condition for the normal component of the velocity field (which determines the difference between the scalar products on both sides of the shock front on vector \( \vec{n} \)) as \((w + V_{1r}, V_{1r}) \circ \vec{n} - (w_0, 0) \circ \vec{n} = w - w_0 + w_1 \).

In this case, the value of the disturbance \( w_1 \) is determined from the well-known representation [10, 12] for \( w - w_0 = \sqrt{(p - p_0)(1/ \rho_0 - 1/ \rho)} ; \delta = \rho/ \rho_0 > 1 \) when replacing in it:

\[ p \to p + p_1 ; \rho \to \rho + \rho_1 \] and decomposing in the Taylor serious in the limit \( p_1 << p ; \rho_1 << \rho \) in which it is necessary to drop the terms quadratic in the perturbation of density and pressure.

As a result, the boundary condition for the normal component of the velocity field leads to the equation (see the same equation in [11, 12] and [14]):

\[
V_{1r} = \frac{(w - w_0)}{2} \left( \frac{p_1}{p - p_0} + \frac{\rho_1}{\rho^2(1/ \rho_0 - 1/ \rho)} \right)
\]  (5)

To determine the relationship of perturbation of density and pressure in (5), it is already unacceptable to use a relation \( p_1 = c^2 \rho_1 \) applied to obtain the equation for the perturbation of the pressure field in a compressed medium from the continuity equation, as it was done to obtain the system (2). This is due to the fact that (5) is considered on the very boundary of the discontinuity of the shock front, where the assumption of the constancy of the entropy does not apply. In this connection, in [10-12], it is proposed to use of the relation between density and pressure perturbations on the Huganiot shock curve in the form of \( p_1 = (\frac{dp}{d(1/ \rho)})_H \left( - \frac{\rho_1}{\rho^2} \right) \) or:

\[ p_1 = \left( \frac{dp}{d\rho} \right)_H \rho_1 \] (6)
To find the equation that determines the function (1) depending on time, we use the equality
determining the perturbation of the velocity of the shock wave in the form \( D_1 = \partial g / \partial t \) and the
well-known expression [10, 12] \( w_0^2 = D^2 = \frac{(p - p_0)\delta}{\rho_0(\delta - 1)} \).

At the same time, to get the expression \( D_1 \), it is necessary to make a replacement on the left
side of the equality \( D \to D + D_1 \), and \( p \to p + p_1; \rho \to \rho + \rho_1 \) in the right side, and carry out
the decomposition in a Taylor series, leaving only terms not higher than the first order of
smallness of the perturbations of the density, pressure, and velocity of the shock wave. This
gives the equation (see also [12, 14]):

\[
\frac{\partial g}{\partial t} = -\frac{w_0}{2} \left( \frac{p_1}{p - p_0} - \frac{\rho_1}{\rho^2(1/\rho_0 - 1/\rho)} \right) \quad (7)
\]

As in (5), in (7), the relation between density and pressure perturbations gives by relation (6).

Let us look for a solution of equations (2) with boundary conditions (4), (5) and relations (6),
(7) as

\[
g(\varphi, t) = \bar{g} \exp(i(k\varphi - \omega t)); \\
(V_{1r}, V_{1\varphi}, p_1) = (\bar{V}_{1r}, \bar{V}_{1\varphi}, \bar{p}_1) K_1(1r) \exp(i(k\varphi - \omega t)) \quad (8)
\]

In (8) \( K_1(1r) \) is the MacDonald function (Bessel’s function of second kind) of first order that
gives zero boundary conditions for perturbations at infinity \( r \to \infty \). In (8) the value of
longitudinal wave number \( l \) is always positive \( l > 0 \) and this is the result of the cylindrical
symmetry, when negative value of \( l \) is impossible in principle as negative value of radial
coordinate \( r \). From the other side, cylindrical symmetry is breaking for the plane SW where
arising new- chiral- symmetry (independence of problem from the direction of the SW
velocity – from the left to right or vice versa). Indeed, when the front of plane SW move, as
considered in [12, 14], from left to right in the positive direction of axis \( x > 0 \) the perturbations existing only after the front of the plane SW (in the system where the shock front is located at \( x = 0 \)) in the region \( x < 0 \) and for zero boundary condition at \( x \to -\infty \) it is necessary to consider only perturbations which are proportional to \( \exp(\text{i}x\ell); l = il_i; l_i < 0 \) [12, 14]. Thus, for the plane SW condition of negative longitudinal wave number \( l_i < 0 \) is the necessary condition, when positive values are possible in principle, but they must be excluded by zero boundary condition at infinite \( x \to -\infty \). In this connection it is also important to note the difference of the converging SW from the case of the diverging cylindrical SW where the compression region after the front of SW is always finite and so the representation (8) for the case of diverging SW must be change on the condition at \( r = 0 \), or at the moving piston boundary.

Note that, with accuracy up to the values of the first order of smallness, when substituting (8) into the boundary conditions (4), (5), relations (6), (7) also should be considered on the unperturbed surface of the shock front.

Unknown value \( g \) can be excluded from (4) and (7). After substitution of (8) in the resulting system of equations, from (5) - (6), it follows a homogeneous system of equations:

\[
\begin{align*}
\vec{V}_{tr} &= \frac{(1 - h)}{2\rho_0|\omega_0|} \vec{P}_1; \\
\vec{V}_{t\varphi} &= \frac{(1 + h)k}{2R_{r0}\omega_0 \rho_0} \vec{P}_1 \\
\vec{P}_1 &= \frac{c^2}{(\omega - w_1)} \left( \frac{k}{R_{r0}} \vec{V}_{t\varphi} + \vec{V}_{tr} l_2 \right) \\
l_1 &= il \left( \frac{K_0(\text{i}R_{r0})}{K_1(\text{i}R_{r0})} \right) + \frac{1}{lR_{r0}}; l_2 = il \left( \frac{K_0(\text{i}R_{r0})}{K_1(\text{i}R_{r0})} \right)
\end{align*}
\]
In (9) $K_1(\text{IR}_{s0})$ is the MacDonald function of first order and equation for pressure is obtained from (2) for the boundary value $r = R_{s0}$.

From the condition of solvability of the system (9) we obtain the following dispersion equation which gives the generalization of equation (2.15) in [14] on the cylindrical case in the limit of zero viscosity):

$$\omega + Mcl_1(1 - \frac{(1 - h)l_2^2}{2M^2l_1}) = \frac{(1 + h)k^2c^2\delta}{2R_{s0}^2\omega} \tag{10}$$

In the limit $\text{IR}_{s0} \rightarrow \infty$ equation (10) has exactly the same form as in the case of plane SW in the limit of high Reynolds numbers (see (2.15) in [14] where $a_2 \rightarrow 1 - h; a_1 \rightarrow 1 + h$ in the limit $\text{Re}_a = |\omega_0|\ell / \nu >> 1$; $\ell$ - is the width of plane SW which is connected with viscosity $\nu$), because in this limit $K_0(\text{IR}_{s0}) / K_1(\text{IR}_{s0}) \rightarrow 1$. Only it is need to replace $k / R_{s0}$ in (10) on wave number $k$ - of 2D corrugation perturbations, which is not dimensionless as $k$ in (10). But even in this limit it is substantial difference in the determination of longitudinal wave number $l$ between the converging cylindrical SW in (10) and for the plane SW, as it is mentioned above. Indeed, in (10) the value $l$ is always positive.

Dispersion equation (10) must be considered together with dispersion equation which is defined after substitution of (8) in (2) and integration of all equations $\int_0^\infty drr^3$ (when relations

$$\int_0^\infty drr^{\mu-1} K_\nu (lr) = \frac{2^{\mu-2}}{l^{\mu}}\Gamma(\frac{\mu + \nu}{2})\Gamma(\frac{\mu - \nu}{2})$$

are taken into account, where $\Gamma$ - gamma function):
\[ 3A_1A_2 = k^2A_3; \]
\[ A_1 = \Omega + 2Mcl + \frac{\nu_1 l^2}{2} (3 + \frac{2k^2}{3}); \Omega = i\omega \frac{3\pi}{2} - 6Mcl; \tag{11} \]
\[ A_2 = -4 + \Omega (\frac{3\pi}{2} l^2 (\frac{4}{3} V_1 + V_2)) \frac{6l^2 c^2}{2}; A_3 = \Omega + \frac{9\pi}{2} l^2 V_1 \]

Dispersion equation (11) is derived from (2) and (8) as the weak solution of (2) while dispersion equation (10) is the result of strong solution which is obtained exactly from boundary conditions (4), (5) and (7) for representations (6) and (8).

2. Instability of shocks (1D case). The instability of shock is possible when in (10) and (11) for the always positive real value of radial wave number \( l > 0 \) the solution with \( \omega_t = \text{Im} \omega > 0; \omega = i\omega_t \) may be obtained.

Let us consider the solutions of dispersion equations (10) and (11) in the limit \( k \to 0 \) which is corresponded to the case one-dimensional (1D) perturbations. From (11) in this case it is necessary to be valid equation \( A_1 A_2 = 0 \). For the case, when \( A_1 = 0 \) from (11) it is possible to obtain representation:

\[ \omega_t = \nu_1 l^2 - \frac{2Mcl}{3\pi} \tag{12} \]

Equation (10) in the limit \( k \to 0 \) may be represented in the form:

\[ \omega_1 = Mcl \left( \frac{K_0(lR_{s0})}{K_1(lR_{s0})} + \frac{1}{lR_{s0}} \right) \left[ 2M^2 \left( 1 + \frac{K_1(lR_{s0})}{lR_{s0}K_0(lR_{s0})} \right) - 1 \right] = Q(lR_{s0}) \tag{13} \]

From (12) and (13) it is possible to obtain equation for the wave number:

\[ l(\nu_1 l - \frac{2Mc}{3\pi}) = Q(lR_{s0}) \tag{14} \]
Let us consider equation (14) in the limit $lR_{s0} >> 1$ when is valid

$$
\frac{K_1(lR_{s0})}{K_0(lR_{s0})} \approx 1 + \frac{1}{2lR_{s0}} + O(1/l^2 R_{s0}^2). \quad \text{In this limit from (14) the equation for } l \text{ has representation:}
$$

$$
l^2 - l_s l + \frac{c(1-h)}{4Mv_1R_{s0}} = 0;
$$

$$
l_s = \frac{Mc}{v_1}\left(\frac{1-h}{2M^2} - 1 + \frac{2}{3\pi}\right) \quad \text{(15)}
$$

For the solution of (15) it is possible to obtain representation:

$$
l = l_s = \frac{l_s}{2}\left[1 \pm \sqrt{1 - \frac{2(1-h)}{M\Re(1-h - 2M^2(1-2/3\pi))}}\right]; \quad \Re = \frac{R_{s0}c}{v_1} \quad \text{(16)}
$$

For the limit of large Reynolds numbers $\Re \gg 1/M$ in (16) one of the two solutions (16) is equal to the value:

$$
l = l_s \approx l_s (1 + O(1/M \Re)) \quad \text{(17)}
$$

From (17) and (12) it is possible to obtain the representation to the increment of exponential evolution of perturbations in the form:

$$
\omega_1 = \frac{c^2}{2v_1}\left(1 - 2M^2 - h\right)\left(\frac{1-h}{2M^2} - 1 + \frac{2}{3\pi}\right)(1 + O(1/M \Re)) \quad \text{(18)}
$$

The necessary and the sufficient condition for instability of the converging cylindrical SW with $\omega_1 = \Im \omega > 0$ in (18) (when also $l = l_s > 0$ in (17), see (15)) is possible only for condition (because $M < 1$):

$$
-1 < h < h_1 \equiv 1 - 2M^2 \quad \text{(19)}
$$

It is important that even in the limit $lR_{s0} >> 1$ the condition (19) has substantial difference from conditions of instability for plane SW in ideal medium in the D’yakov theory [10-12]
(where it is necessary $h > 1 + 2M$ or $h < -1$ for 2D perturbations) and condition of instability to 1D and 2D perturbations for the plane SW in viscose medium (where it is need to hold inequality $1 - 2M^2 < h < 1$ [14]).

In the limit $\text{Re} >> 1/M$ it is also existed an additional solution of (16) and (12) in the form:

$$l = l_0 = \frac{(1 - h)}{4R_{x0}M^2}$$

$$\omega_0 = \frac{\nu_1 (1 - h)}{16M^2R_{x0}^2} \left(1 - h - \frac{8M^3 \text{Re}}{3\pi}\right)$$

From (21) it is possible to obtain the instability condition in the form:

$$h < h_2 \equiv 1 - 8M^3 \text{Re} / 3\pi$$

For the case when $M \text{Re} > 3\pi$ from (22) (which is valid only in the limit $M \text{Re} >> 1$) it is possible to obtain that $h_2 < h_1$ and from (22) always is also valid condition (19). From the other side, in the limit $\text{Re} >> 1/M$ the value of increment in (21) $\omega_0 \equiv \nu_1 / 4R_{x0}^2$ is much smaller than in (18). So, the dominative condition of instability is condition (19), but estimation $\omega_0 \equiv \nu_1 / 4R_{x0}^2$ gives the tendency for the increasing of exponential instability increment $\omega_0$ when the radius of converging shock is decreasing.

It must be also note that in the limit $lR_{x0} << 1$ when $K_0(lR_{x0}) / K_1(lR_{x0}) \approx lR_{x0} \ln(2/lR_{x0})$ in (13) instability is impossible because $l = \frac{Mc}{3\pi \nu_1} \left(l \pm \sqrt{1 - 9\pi^2 / M \text{Re}}\right) \omega_0 = -Mc / R_{x0} < 0$ in this case from (12) and (14).
For the case with $A_2 = 0$ instability is possible only in the limit $lR_{s0} >> 1$ when condition of instability is the same as (19) because for the case $Re_2/lR_{s0} << 1; Re_2 = R_{s0}c/4(4v_1 + v_2)$ it is possible to obtain representations:

$$l = \frac{Mc}{4}\left(3v_1 + v_2\right)\left(\frac{1 - h}{2M^2} + \frac{4}{\pi} - 1\right);$$

$$\omega_1 = \frac{c^2(1 - 2M^2 - h)}{2(3v_1 + v_2)}\left(\frac{1 - h}{2M^2} + \frac{4}{\pi} - 1\right)$$

(23)

From (23) it is possible to obtain also the condition of instability in the form (19).

On the other side in the limit $lR_{s0} << 1; k >> 1$ instability is impossible, as for the case $A_1 = 0$.

3. Instability of shocks (2D case)

Let us consider 2D perturbation with $k \neq 0$ in the limit $k >> 1$ in (10), (11), when from (11) it is possible to obtain representation of (11) in the form:

$$\Omega^2 + 3\Omega\left(\frac{\pi^2}{2}\left(3v_1 + v_2\right) - \frac{2c^2}{\pi v_1}\right) - 51c^2l^2 = 0$$

(24)

The solution of (24) may be represented in the form:

$$\omega_1 = \text{Im} \omega = \frac{2}{3\pi}\left[-6Mc + 3a \pm \sqrt{9a^2 + 51c^2l^2}\right]$$

$$a = \frac{\pi^2}{4}(\frac{4}{3}v_1 + v_2) - \frac{c^2}{\pi v_1}$$

(25)

For the case of instability in (25) it is interesting only the solution with sign plus. From (25) in the limit $Re_{12} = R_{s0}c/\sqrt{v_1(v_2 + 4v_1/3)} >> \pi R_{s0} / 2$ it is possible to obtain representation:

$$\omega_1 = \frac{2}{3\pi}\left[-6Mc + \frac{17\pi}{2}v_1l\right]$$

(26)
From (10) in the limit \( IR_{s0} \gg 1; k \gg 1 \) it is possible to obtain representation:

\[
\omega_1 = \omega_{1s} = \frac{Mcq}{2} \left( 1 \pm \sqrt{1 - \frac{2(1 + h)k^2\delta}{l^2R_{s0}^2M^2q^2}} \right);
\]

\[
q = \frac{1 - h}{2M^2} - 1
\]

From (26) and (27) for the case \( \omega_1 = \omega_{1s} \) in (27) in the limit \( 1 << k^2 << l^2R_{s0}^2 \) it is possible to obtain representations:

\[
l = \frac{3Mc}{17
\nu_1 \left( \frac{q + \frac{4}{\pi}}{q} \right)} \quad (28)
\]

\[
\omega_1 = \frac{3c^2}{34\nu_1} (1 - 2M^2 - h)(q + \frac{4}{\pi}) \quad (29)
\]

For (29) the instability condition is exactly the same as condition (19). From (26) and (27) in the case \( \omega_1 = \omega_{1s} \) in (27) in the limit \( 1 << k^2 << Re = R_{s0}c/\nu_1 \) it is possible to obtain representation:

\[
l = \frac{12Mc}{17\pi\nu_1} \left( 1 + \frac{k^2\pi^317^2\delta(1 + h)}{4Re^212^2q} + o(k^2 / Re^2) \right) \quad (30)
\]

\[
\omega_1 = \frac{17k^2M^2\nu_1\pi\delta(1 + h)}{6R_{s0}^2(1 - 2M^2 - h)} \quad (31)
\]

In (31) and (29) the condition \( \omega_1 > 0 \) is the same as the instability condition (19).

4. Discussion and comparison with experiment

From (31) and (29) for the 2D perturbations and from (18) and (23) for the 1D perturbations it is easy to see that the value of increment for 1D perturbation is larger than for the case of 2D perturbations of the cylindrical converging SW.
From the other side, the condition of instability (19) is the same for 1D and 2D cases.

Thus, let us consider condition (19) on the base of the Huganiot curve, which is obtain from experimental date on the SW in water in the form of dependence between the SW velocity $D$ and velocity of medium $U$ [15-17]:

$$D = A + BU$$

(32)

For example, in (32) $A = 2.393km/sec; B = 1.333; c_0 = 1.483; 1.5km/sec < U < 7.1km/sec$ for date [16].

From (32) it is possible to obtain representations [14]:

$$h = -\frac{B - \delta(B - 1)}{(B + 1)\delta - B}$$

(33)

$$M = \frac{A}{c(B - \delta(B - 1))};$$

$$1 < \delta < \delta_{\text{max}} = B/(B - 1)$$

(34)

In (33) $h > -1$ for all $\delta > 1$ and $B > 0$.

From (33), (34) the condition of instability (19) may be represented in the form:

$$\frac{c^2}{A^2} > \frac{(B + 1)\delta - B}{B(\delta - 1)(B - \delta(B - 1))} > \frac{1}{(B - \delta(B - 1))^2}$$

(35)

The second inequality in (35) is obtained from (34) and condition $M < 1$. For $B = 1.333$ the second inequality is valid only for $1 < \delta < \delta_M \approx 2.642$ when $\delta_M < \delta_{\text{max}}$.

Let us take into account the adiabatic equation of state for water in the form [18, 19]:

$$p - p_0 = \frac{\rho_0 c_0^2}{n}(\delta^n - 1)$$

(36)
For (36) the SW velocity has representation $D = c_0 \left( \frac{\delta (\delta^n - 1)}{n (\delta - 1)} \right)^{1/2}$ which is based on Renkin-Hugoniot jump condition on the front of the SW.

For the pressure up to 25 kbar in (36) is useful value $n = 7.15 [19]$. If the representation

$$c^2 = \frac{dp}{d\rho} = c_o^2 \delta^{n-1}; \delta = \rho / \rho_0$$

is used on the base of (36) the condition of instability (35) may be represented in the new form:

$$F(\delta) \equiv \delta^{n-1} (\delta - 1) (B - \delta (B - 1))^2 - \frac{A^2}{Bc_0^2} ((B + 1) \delta - B) > 0 \quad (37)$$

In the case with $n = 7.15$ in (37) and $c_0 = 1.483 km/sec$; $A = 2.393 km/sec; B = 1.333 [16]$ from (37) it is possible to obtain that instability of converging cylindrical SW may arise only for compressions and SW velocities from intervals:

$$1.503 \leq \delta \leq \delta_M \approx 2.642 \quad (38)$$

$$2.7 c_0 < D < D_{max} \approx 7.995 c_0 \quad (39)$$

In (39) the upper limit is represented by taken into account (32) because

$$D = D_{max} = A + DU_{max} \approx 11.857 km/sec \text{ when } U = U_{max} \approx 7.1 km/sec \text{ in (32).}$$

The value of exponential increment for SW instability may be evaluated as

$$c^2 / \nu \approx 10^{-13} \text{ sec for water with } \nu \approx 10^{-6} m^2 / \text{sec}.$$ Thus the characteristic time of the perturbation arising is very small and it is possible to consider the approximation when it is possible to neglect the changing in time of the SW radius and values of velocities $D; U$, as it is suggested in this paper.
5. Conclusions

The condition of exponential instability for the converging cylindrical SW is obtained only when viscosity is taken into account. This condition is more wide realizable than the condition of the plane SW dissipative instability which is obtained in [14]. Here only the case of small perturbation of acoustic type is considered in the quasi stationary limit, when the characteristic time of the perturbation growth is much smaller than the characteristic time of the SW radius changing in time. In this case, the possibility of dissipative instability with respect to the one-dimensional perturbations that do not violate the cylindrical symmetry of the shock wave is shown, in contrast to previous studies that do not take into account the viscosity (see [6]). Moreover, due to the dissipative instability of the front of a converging cylindrical shock wave, a vortex flow may occur that has not only a radial, but also a tangential component of the velocity field of the substance behind the shock wave front in the case of two-dimensional perturbations. As a result, it is unattainable to obtain an ultra-high pressure mode when the shock wave converges to the symmetry axis under the conditions of dissipative instability obtained in this paper.

Acknowledgments

I would like to thank Ya. E. Krasik for his attention to the work and discussions.

The work was supported by Israel Science Foundation, Grant number: 492/18
Literature

1. J. D. Lindl, E. M. Campbell, and R. L. McCrory, “Progress toward ignition and burn propagation in inertial confinement fusion”, Phys. Today 45(9), 32 (1992).

2. W. D. Arnett, J. N. Bahcall, R. P. Kishner, and S. E. Woosley, “Supernova 1987a”, Annu. Rev. Astron. Astrophys. 27, 629 (1989)

3. V. E. Fortov and I. T. Iakubov, The Physics of Non-Ideal Plasma, Singapur: World Scientific, 2000

4. Ya. E. Krasik, S. Efimov, D. Sheftmann, et.al., “Underwater electrical explosion of wires and wire arrays and generation of converging shock waves”, IEEE Transactions on Plasma Science, 44, 412 (2016)

5. M. Murakami, J. Sanz, and Y. Iwamoto, “Stability of spherical converging shock wave”, Phys. Plasmas, 22, 072703 (2015)

6. K. Fong and B. Ahlborn, Phys. Fluids, 22, 416 (1979)

7. J. Gardner, D. Book, and I. Bernstein, J. Fluid Mech. 114, 41 (1982)

8. C.C. Wu and P. H. Roberts, Q. J. Mech. Appl. Math., 49, 501 (1996)

9. K. V. Brushlinskii, USSR Comput. Math. Math. Phys. 22, 193 (1982)

10. L.D. Landau, E.M. Lifshitz, Theoretical physics. Hydrodynamics, Pergamon Press, Oxford, 1987

11. S.P.D'yakov,Zh.Exp.Teor.Fiz.,27, 288, 1954

12. G.W. Swan, G.R. Fowles, Phys.Fluids,18, 28, 1975

13. E. A. Kuznetsov, M. D. Spector, G. E. Fal’kovich, JETP Lett., 30, 328, 1979

14. S. G. Chefranov, JETP, 157(3), 2020; https://doi.org/10.1134/S0044451020030000

15. M. H. Rice and J. M. Walsh, J. Chem. Phys., 26, 824 (1957)
16. A. C. Mitchell and W. J. Nellis, J. Chem. Phys., 76, 6273 (1982)
17. K. Nagayama, Y. Mori, K. Shimada, and M. Nakahara, J. Appl. Phys., 91, 476 (2002)
18. J. M. Richardson, A. B. Arons, and R. R. Halverson, J. Chem. Phys., 15, 785 (1947)
19. S. Ridah, J. Appl. Phys., 64, 152 (1988)