UNIQUENESS THEOREMS FOR CAUCHY INTEGRALS

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Abstract

If \( \mu \) is a finite complex measure in the complex plane \( \mathbb{C} \) we denote by \( C^\mu \) its Cauchy integral defined in the sense of principal value. The measure \( \mu \) is called reflectionless if it is continuous (has no atoms) and \( C^\mu = 0 \) \( \mu \)-almost every point. We show that if \( \mu \) is reflectionless and its Cauchy maximal function \( C^\mu_* \) is summable with respect to \( |\mu| \) then \( \mu \) is trivial. An example of a reflectionless measure whose maximal function belongs to the “weak” \( L^1 \) is also constructed, proving that the above result is sharp in its scale. We also give a partial geometric description of the set of reflectionless measures on the line and discuss connections of our results with the notion of sets of finite perimeter in the sense of De Giorgi.

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