The semileptonic decays $B_c^+ \to D_{(s)}^*(l^+\nu, l^+l^-, \nu\bar{\nu})$ in the perturbative QCD approach

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Abstract

In this paper we study the semileptonic decays of $B_c^+ \to D_{(s)}^*(l^+\nu, l^+l^-, \nu\bar{\nu})$ (here $l$ stands for $e, \mu$ or $\tau$). After evaluating the $B_c^+ \to (D_{(s)}, D_{(s)}^*)$ transition form factors $F_{0+, T}(q^2)$ and $V(q^2), A_{0,1,2}(q^2), T_{1,2,3}(q^2)$ by employing the perturbative QCD (pQCD) factorization approach, we calculate the branching ratios for all considered semileptonic decays. Based on the numerical results and the phenomenological analysis, we find that: (a) The pQCD predictions for the values of the $B_c^+ \to D_{(s)}$ and $B_c^+ \to D_{(s)}^*$ transition form factors are consistent with those obtained by using other methods; (b) The branching ratios of the decay modes with $\bar{\nu}\nu$ is almost an order larger than the corresponding decays with $l^+l^-$ after the summation over the three neutrino generations. And the branching ratios for the decays with $b \to d$ transitions are much smaller than those decays with the $b \to s$ transitions, due to the CKM suppression. (c) We defined ratios $R_D$ and $R_{D^*}$ among the branching ratios for the charged current process. The pQCD predictions are $R_D \approx 0.7$ and $R_{D^*} \approx 0.6$, which is possible to be measured in the near future.

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I. INTRODUCTION

After the discovery of the standard model (SM) Higgs boson by the ATLAS [1] and CMS [2] collaborations, one of the most important themes in the high energy physics is to find out whether there is new physics beyond the SM. As the only heavy meson consisting of two heavy quarks with different flavors, the $B_c$ meson can provide windows for testing the predictions of the SM and may shed light on new physics signal beyond the SM, which also makes the $B_c$ meson a hot topic on theoretical side especially after a lot of experimental data released by LHCb experiment recently [3].

The $B_c$ meson is a pseudoscalar ground state of quarks $b$ and $c$, which can decay individually. Because it is the ground state, the electromagnetic interaction can not transform the $B_c$ meson into other hadrons containing $b$ and $c$ quarks. The difference of quark flavors forbids its annihilation into gluons and being below the $B-D$ threshold also makes the $B_c$ meson stable for strong interaction. The $B_c$ meson can only decay through weak interactions, so it is an ideal system to study weak decays of heavy quarks. And either the heavy quark $b$ or $c$ can decay individually, which makes it different from the $B_{u,d}$ or $B_s$ meson. The phase space in $c \rightarrow s$ transition is smaller than that in $b \rightarrow c$ transition, but the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{cs}| \sim 1$ is much larger than the CKM matrix element $|V_{cb}| \sim 0.04$. Thus the $c$-quark decays provide the dominant contribution (about 70%) to the decay width of $B_c$ meson [4]. Because the mass of a $B_c$ meson pair has exceeded the threshold of $\Upsilon(4S)$, $B_c$ meson can not be produced on the $B$ factories. So comparing with $B_{u,d}$ or $B_s$ meson, the $B_c$ meson decays received much less experimental considerations in the past decade. But at LHC experiments, around $5 \times 10^{10} B_c$ events per year are expected [4, 5] due to the relatively large production cross section, which provides a very good platform to study various $B_c$ meson decay modes.

Because there is only one hadronic final product, the $B_c$ meson semileptonic decays among the abundant decay modes are relatively clean in the theoretical treatment. These semileptonic decays provide good opportunities to measure not only the CKM matrix elements, such as $|V_{cb}|, |V_{ub}|$ and $|V_{cd}|$, but also the form factors of the $B_c$ to bottom and charmed mesons transitions. The rare semileptonic decays governed by the flavour-changing neutral currents are forbidden at tree level in the SM. Those decays, which are very sensitive to the contributions of new intermediate particles or interactions are especially interesting. There are various approaches working on the semileptonic $B_c$ decays. In Ref. [6], for example, Dhir and Verma presented a detail analysis of the exclusive semileptonic $B_c$ decays in the Bauer-Stech-Wirbel (BSW) framework. While the authors of the Refs. [7–9] studied the semileptonic $B_c$ decays in the relativistic and/or constituent quark model. And also the three point QCD sum rules approach is adopted to investigate the $B_c^+ \rightarrow D_{(s)}^{+} l^+ l^−$ in [10] and $B_c^- \rightarrow D_{(s)}^{+} (l^+ l^−, \bar{\nu} \nu)$ in [11].

In this paper, we will study the semileptonic decays of $B_c^+ \rightarrow D_{(s)}^{+(s)} (l^+ \nu, l^+ l^−, \nu \bar{\nu})$ (here $l$ stands for leptons $e, \mu$, or $\tau$) in the perturbative QCD (pQCD) approach [12]. A form factor, in the pQCD approach, is generally written as the convolution of a hard amplitude with initial-state and final-state hadron distribution amplitudes. While the Sudakov form factors from both the $k_T$ resummation and the threshold resummation can cure the endpoint singularity [13] and make the perturbative calculation of the hard amplitudes (form factors) infrared safe. The pQCD approach is widely adopted to calculate the transition form factors of $B_{u,d}$ and $B_s$ meson [14, 16]. Furthermore, various $B_c$ decay modes have also been studied in Refs. [17, 18] in the pQCD approach.
FIG. 1. The leading order Feynman diagrams for the transition of $B_c^+ \rightarrow (D^{(*)}, D_s^{(*)})$, where $M$ stands for $D^{(*)}$ or $D_s^{(*)}$ meson, and $\otimes$ is the weak vertex.

The structure of this paper is as below: After this introduction, we collect the distribution amplitudes of the $B_c$, $D^{(*)}$ and $D_s^{(*)}$ mesons in Section II. Based on the $k_T$ factorization formalism, we calculate and present the expressions for the $B_c \rightarrow (D^{(*)}, D_s^{(*)})$ transition form factors in the large recoil regions in Section III. The numerical results and relevant discussions are given in Section IV. And Section V contains the conclusions and a short summary.

II. KINEMATICS AND THE WAVE FUNCTIONS

The lowest order diagrams for $B_c^+ \rightarrow (D^{(*)}, D_s^{(*)})$ transitions are displayed in Fig. 1, where $M$ stands for $D^{(*)}$ or $D_s^{(*)}$ meson, the $\otimes$ is the vertex for the leptonic pairs to come out. In the rest frame of $B_c$ meson, with the $m_{B_c}$ standing for the mass of the $B_c$ meson, and $m$ for the $D^{(*)}$ or $D_s^{(*)}$ mesons, the momenta of $B_c$ and $D^{(*)}_s$ mesons are defined in the light-cone coordinate as

$$p_1 = \frac{m_{B_c}}{\sqrt{2}}(1, 1, 0), \quad p_2 = \frac{m_{B_c}}{\sqrt{2}}(r\eta^+, r\eta^-, 0),$$

with $r = m/m_{B_c}$ and $\eta^\pm = \eta \pm \sqrt{\eta^2 - 1}$. As for the $\eta$ in $\eta^\pm$, the result of

$$\eta = \frac{1}{2r} \left[ 1 + r^2 - \frac{q^2}{m_{B_c}^2} \right]$$

(2)

can be evaluated from $q^2 = (p_1 - p_2)^2$ which is the invariant mass of the lepton pairs. The momenta of the spectator quarks in $B_c$ and $D^{(*)}_s$ mesons are parameterized as

$$k_1 = (0, x_1 m_{B_c} \sqrt{2}, k_{1\perp}), \quad k_2 = (x_2 m_{B_c} \sqrt{2} r\eta^+, x_2 m_{B_c} \sqrt{2} r\eta^-, k_{2\perp}).$$

(3)

For the $D^{(*)}_s$ mesons, we define their polarization vector $\epsilon$ as

$$\epsilon_L = \frac{1}{\sqrt{2}}(\eta^+, -\eta^-, 0), \quad \epsilon_T = (0, 0, 1),$$

where $\epsilon_L$ and $\epsilon_T$ denotes the longitudinal and transverse polarization of the $D^{(*)}_s$ mesons, respectively.

In this work, we use the same distribution amplitude for $B_c$ meson as those used in Refs. [18, 20–22]

$$\Phi_{B_c}(x) = \frac{i}{\sqrt{2N_c}}[(\not{p} + m_{B_c})\gamma_5\phi_{B_c}(x)]_{\alpha\beta},$$

(5)
with
\[
\phi_{B_c}(x) = \frac{f_{B_c}}{2\sqrt{2}N_c} \delta(x - m_c/m_{B_c}) \exp\left[-\omega_{B_c}^2 b^2/2\right],
\]

where \(m_c\) is the mass of \(c\)-quark and \(\omega_{B_c} = 0.60\) in the numerical calculations. Because the \(B_c\) meson consists two heavy quarks \(b\) and \(c\), just like a heavy quarkonium, the non-relativistic QCD framework can be applied, which means the leading order wave function should be just the 0-point wave function shown in eq. (6).

As for the \(D_{(s)}^{(*)}\) mesons, up to twist-3 accuracy, the two-particle light-cone distribution amplitudes are defined as [19, 23]

\[
\langle D_{(s)}^{(*)}(p)|q_\alpha(z)\bar{c}_\beta(0)|0\rangle = i \frac{1}{\sqrt{2}N_c} \int_{0}^{1} dx e^{ixp \cdot z} \left[ \gamma_5 (\not{p} + m) \phi_{D_{(s)}}(x, b) \right]_{\alpha\beta},
\]

\[
\langle D_{(s)}^{(*)}(p)|q_\alpha(z)\bar{c}_\beta(0)|0\rangle = -i \frac{1}{\sqrt{2}N_c} \int_{0}^{1} dx e^{ixp \cdot z} \left[ \gamma_5 (\not{p} + m) \phi_{D_{(s)}}^L(x, b) + \phi_{D_{(s)}}^T(x, b) \right]_{\alpha\beta},
\]

with

\[
\int_{0}^{1} dx \phi_{D_{(s)}}(x, 0) = \frac{f_{D_{(s)}}}{2\sqrt{2}N_c},
\]

\[
\int_{0}^{1} dx \phi_{D_{(s)}}^L(x, 0) = \frac{f_{D_{(s)}}^L}{2\sqrt{2}N_c},
\]

\[
\int_{0}^{1} dx \phi_{D_{(s)}}^T(x, 0) = \frac{f_{D_{(s)}}^T}{2\sqrt{2}N_c},
\]

as the normalization conditions. We use \(f_{D_{(s)}}^L = f_{D_{(s)}}^T = f_{D_{(s)}}\) and \(\phi_{D_{(s)}} = \phi_{D_{(s)}}\) in the numerical calculations in this work just like in Ref. [23]. We adopt \(f_{D} = 206.7 \pm 8.9\) MeV and \(f_{D_s} = 260.0 \pm 5.6\) MeV in PDG [24] by experimental average. As for the decay constant of the \(D^{*}\) and \(D_{s}^{*}\) meson, there is no experimental data. We use \(f_{D^{*}} = 270\) MeV and \(f_{D_{s}^{*}} = 310\) MeV for \(D^{*}\) and \(D_{s}^{*}\) meson considering of the results in Refs. [25] and assuming a 10% uncertainty for \(f_{D^{*}}\) and \(f_{D_{s}^{*}}\). The distribution amplitude for the \(D_{(s)}\) mesons is

\[
\phi_{D_{(s)}} = \frac{1}{2\sqrt{2}N_c} f_{D_{(s)}} 6x(1-x) \left[ 1 + C_{D_{(s)}} (1 - 2x) \right] \exp \left[ -\frac{\omega_{D_{(s)}}^2 b^2}{2} \right],
\]

which is a \(k_T\)-dependent form with \(C_D = 0.5, \omega_D = 0.1\) and \(C_{D_s} = 0.4, \omega_{D_s} = 0.2\) for \(D\) and \(D_s\) meson, respectively [23]. In this work, we also adopt the same distribution amplitude for both the vector meson \(D_{(s)}^{*}\) and pseudoscalar meson \(D_{(s)}\) because of the small mass difference between them [23].
III. FORM FACTORS AND SEMILEPTONIC DECAYS

The form factors \( F_+(q^2) \), \( F_0(q^2) \) for \( B_c \) to the pseudoscalar meson \( D_{(s)} \) transition induced by the vector current can be defined as \[26, 27\]

\[
\langle D_{(s)}(p_2)|\bar{q}(0)\gamma_\mu b(0)|B_c(p_1)\rangle = \left[(p_1 + p_2)_\mu - \frac{m_{B_c}^2 - m^2}{q^2}q_\mu\right]F_+(q^2) \\
+ \frac{m_{B_c}^2 - m^2}{q^2}q_\mu F_0(q^2), \quad (10)
\]

where \( q = p_1 - p_2 \) is the momentum of the lepton pairs, and \( m \) is the mass of \( D_{(s)} \) meson. In order to cancel the poles at \( q^2 = 0 \), \( F_+(0) \) should be equal to \( F_0(0) \). For the sake of convenience, we define the auxiliary form factors \( f_1(q^2) \) and \( f_2(q^2) \)

\[
\langle D_{(s)}(p_2)|\bar{q}(0)\gamma_\mu b(0)|B_c(p_1)\rangle = f_1(q^2)p_{1\mu} + f_2(q^2)p_{2\mu}. \quad (11)
\]

In terms of \( f_1(q^2) \) and \( f_2(q^2) \) the form factor \( F_+(q^2) \) and \( F_0(q^2) \) read as

\[
F_+(q^2) = \frac{1}{2} \left[f_1(q^2) + f_2(q^2)\right], \\
F_0(q^2) = \frac{1}{2} f_1(q^2) \left[1 + \frac{q^2}{m_{B_c}^2 - m^2}\right] + \frac{1}{2} f_2(q^2) \left[1 - \frac{q^2}{m_{B_c}^2 - m^2}\right]. \quad (12)
\]

The form factor \( F_T(q^2) \) for \( B_c \to D_{(s)} \) transition induced by the tensor current can be defined as \[27\]

\[
\langle D_{(s)}(p_2)|\bar{q}(0)\sigma_{\mu\nu}b(0)|B_c(p_1)\rangle = i \left[p_{2\mu}q_\nu - q_{\mu}p_{2\nu}\right] \frac{2F_T(q^2)}{m_{B_c} + m}. \quad (13)
\]

There are totally seven form factors \( V(q^2), A_{0,1,2}(q^2) \) and \( T_{1,2,3}(q^2) \) needed for the transition of \( B_c \to D_{(s)}^* \) in this work. The form factors \( V(q^2) \) and \( A_{0,1,2}(q^2) \) are defined by \[27, 26\]

\[
\langle D_{(s)}^*(p_2)|\bar{q}(0)\gamma_\mu b(0)|B_c(p_1)\rangle = \epsilon_{\mu\rho\alpha\delta}^* p_1^\alpha p_2^\rho \frac{2V(q^2)}{m_{B_c} + m}, \quad (14)
\]

\[
\langle D_{(s)}^*(p_2)|\bar{q}(0)\gamma_\mu\gamma_5 b(0)|B_c(p_1)\rangle = i \left[\frac{\epsilon_{\mu}^* \cdot q}{q^2}q_\mu\right] (m_{B_c} + m)A_1(q^2) \\
- i \left[(p_1 + p_2)_\mu - \frac{m_{B_c}^2 - m^2}{q^2}q_\mu\right] (\epsilon^* \cdot q) A_2(q^2) \\
+ \frac{2m(\epsilon^* \cdot q)}{q^2}q_\mu A_0(q^2), \quad (15)
\]

where \( \epsilon^* \) is the polarization vector of \( D_{(s)}^* \) meson and \( m \) is its mass. The form factors \( T_{1,2,3} \) are defined by \[27, 30\]

\[
\langle D_{(s)}^*(p_2)|\bar{q}(0)\sigma_{\mu\nu}q^\rho(1 + \gamma_5)b(0)|B_c(p_1)\rangle = i\epsilon_{\mu\rho\alpha\beta}^* p_1^\alpha p_2^\beta 2T_1(q^2) \\
+ \left[\epsilon_{\mu}^* (m_{B_c}^2 - m^2) - (\epsilon^* \cdot q)(p_1 + p_2)_\mu\right] T_2(q^2) \\
+ (\epsilon^* \cdot q) \left[q_\mu - \frac{q^2}{m_{B_c}^2 - m^2}(p_1 + p_2)_\mu\right] T_3(q^2), \quad (16)
\]
with $T_1(0) = T_2(0)$ implied by the identity

$$\sigma_{\mu\nu}\gamma_5 = -\frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}.$$  \hfill (17)

We stress here that maybe those form factors have different form of definition in different literature, but they are equivalent. In the transverse configuration b-space and by including the Sudakov form factors and the threshold resummation effects, we obtain the $B_c \to D_{(s)}$ form factors $f_1(q^2), f_2(q^2)$ and $F_T(q^2)$ as follows

$$f_1(q^2) = 8\pi m_{B_c}^2 r C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_c}(x_1)\phi_{D_{(s)}}(x_2, b_2) \times \left\{2 \left[1 - r x_2\right] \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \exp \left[-S_{ab}(t_1)\right] + \frac{x_1\eta^+ (\eta^+ - 2)}{\sqrt{\eta^2 - 1}} - 2r \right\} \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \exp \left[-S_{ab}(t_2)\right]\right\}, \hfill (18)$$

$$f_2(q^2) = 8\pi m_{B_c}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_c}(x_1)\phi_{D_{(s)}}(x_2, b_2) \times \left\{2 \left[1 - 2r x_2 (1 - \eta)\right] \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \exp \left[-S_{ab}(t_1)\right] + 4r - \frac{x_1(\eta^+ - 2)}{\sqrt{\eta^2 - 1}} \right\} \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \exp \left[-S_{ab}(t_2)\right]\right\}, \hfill (19)$$

$$F_T(q^2) = 8\pi m_{B_c}^2 C_F (1 + r) \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_c}(x_1)\phi_{D_{(s)}}(x_2, b_2) \times \left\{\left[1 - r x_2\right] \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \exp \left[-S_{ab}(t_1)\right] + 2r + \frac{x_1(2 - \eta^+)}{2\sqrt{\eta^2 - 1}} \right\} \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \exp \left[-S_{ab}(t_2)\right]\right\}, \hfill (20)$$

where $C_F = 4/3$ is the color factor, $r$ and $\eta^+$ are the same as in Eqs. (13). The functions $h_1$ and $h_2$, the scales $t_1, t_2$ and the Sudakov factors $S_{ab}$ are given in ref. [14, 19].

The expressions of form factors $V(q^2), A_{0,1,2}(q^2)$ and $T_{1,2,3}(q^2)$ for $B_c \to D_{(s)}^*$ transition in pQCD approach are:

$$V(q^2) = 4\pi m_{B_c}^2 C_F (1 + r) \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_c}(x_1)\phi_{D_{(s)}^*}(x_2, b_2) \times \left\{2 \left[1 - r x_2\right] \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \exp \left[-S_{ab}(t_1)\right] + 2r + \frac{x_1}{\sqrt{\eta^2 - 1}} \right\} \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \exp \left[-S_{ab}(t_2)\right]\right\}, \hfill (21)$$
\[ A_0(q^2) = 8\pi m_B^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1)\phi_{D_t}^T(x_2, b_2) \]
\[ \times \left\{ [1 - r x_2(r - 2\eta) + r (1 - 2x_2)] \right. \]
\[ \times h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \exp [-S_{ab}(t_1)] \]
\[ + \left. \left[ r^2 + \frac{x_1}{2} - rx_1\eta + \frac{x_1(\eta + r(1 - 2\eta^2))}{2\sqrt{\eta^2 - 1}} \right] \right. \]
\[ \times h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \exp [-S_{ab}(t_2)] \right\}, \] (22)

\[ A_1(q^2) = 8\pi m_B^2 C_F \frac{r}{1 + r} \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1)\phi_{D_t}^T(x_2, b_2) \]
\[ \times \left\{ 2 [1 + r x_2\eta - 2rx_2 + \eta] \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \exp [-S_{ab}(t_1)] \right. \]
\[ + \left. [2r\eta - x_1] \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \exp [-S_{ab}(t_2)] \right\}, \] (23)

\[ A_2(q^2) = \frac{(1 + r)^2(\eta - r)}{2r(\eta^2 - 1)} \cdot A_1(q^2) - 8\pi m_B^2 C_F \frac{1 + r}{\eta^2 - 1} \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1) \]
\[ \times \phi_{D_t}^T(x_2, b_2) \cdot \left\{ \left[ (\eta(1 - 2x_2^2) - 2x_2(1 - 2\eta^2 - 2r) + (1 - r) - r\eta(1 + 2x_2) \right] \right. \]
\[ \times h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \exp [-S_{ab}(t_1)] \]
\[ + \left. \left[ x_1 \left( \eta - \frac{1}{2} \right) \sqrt{\eta^2 - 1} + \left( r^2 - \frac{x_1}{1} \right) \eta + r \left( 1 - \frac{x_1}{2} + x_1\eta^2 \right) \right] \right. \]
\[ \times h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \exp [-S_{ab}(t_2)] \right\}, \] (24)

\[ T_1(q^2) = 8\pi m_B^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1)\phi_{D_t}^T(x_2, b_2) \]
\[ \times \left\{ [1 + r(1 - x_2(2 + r - 2\eta))] \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \exp [-S_{ab}(t_1)] \right. \]
\[ + \left. r \left[ 1 - \frac{x_1}{2} + \frac{x_1}{2\sqrt{\eta^2 - 1}} (1 - \eta) \right] \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \exp [-S_{ab}(t_2)] \right\}, \] (25)

\[ T_2(q^2) = 8\pi m_B^2 C_F \frac{r}{1 - r^2} \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_B(x_1)\phi_{D_t}^T(x_2, b_2) \]
\[ \times \left\{ 2 [(1 - r)(1 + \eta) + 2rx_2(r - \eta) + rx_2(2\eta^2 - r\eta - 1)] \right. \]
\[ \times h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \exp [-S_{ab}(t_1)] \]
\[ + \left. \left[ r \left( 2(\eta - r) + x_1\eta \right) \right] \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \exp [-S_{ab}(t_2)] \right\}, \] (26)
\[ T_3(q^2) = \frac{r + \eta}{r} \cdot \frac{1 - r^2}{2(q^2 - 1)} \cdot T_2(q^2) - \frac{1 - r^2}{2(q^2 - 1)} \]
\[ \times 8\pi m_{B_c}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_c}(x_1) \phi_{D^{(*)}}^F(x_2, b_2) \]
\[ \times \left\{ 2 \left[ 1 + r x_2(\eta - 2) + \eta \right] \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \exp \left[-S_{ab}(t_1)\right] \right. \]
\[ + \left[ 2r - x_1 \eta^+ \right] \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \exp \left[-S_{ab}(t_2)\right] \right\}, \quad (27) \]

where \( r = m_{D^{(*)}}/m_{B_c} \). One should note that the expressions of the form factors \( f_{1,2}(q^2) \), \( F_T(q^2), V(q^2) \), \( A_{0,1,2}(q^2) \) and \( T_{1,2,3}(q^2) \) given in Eqs. (18-27) are the results at leading order of the pQCD approach. The next-to-leading-order contributions to the form factors of \( B \to (\pi, K, \eta^{(0)}) \) transitions given in Refs. [14,15,31] are not available here because of the large mass of \( c \)-quark and \( (D_s, D^{(*)}_s) \) mesons.

The \( B_c^- \to \bar{D}^0 l^- \bar{\nu} \) and \( B_c^- \to \bar{D}^* l^- \bar{\nu} \) decays are from the quark level \( b \to ul^- \bar{\nu} \) charged current transition. The effective Hamiltonian for such transition is [32]

\[ \mathcal{H}_{\text{eff}}(b \to ul^- \bar{\nu}_l) = \frac{G_F}{\sqrt{2}} V_{ub} \bar{u}_l \gamma_\mu(1 - \gamma_5)b \cdot \bar{l}_\mu(1 - \gamma_5)\nu_l, \quad (28) \]

where \( G_F = 1.16637 \times 10^{-5}\text{GeV}^{-2} \) is the Fermi-coupling constant and \( V_{ub} \) is one of the CKM matrix elements. The corresponding differential decay width expression for \( B_c^- \to \bar{D}^0 l^- \bar{\nu} \) is the same of Eq.(23) in Ref. [14] except the changes of \( m_B \to m_{B_c} \) and \( m_F \to m \). While the corresponding differential decay width for the decay of \( B_c^- \to \bar{D}^* l^- \bar{\nu} \) can be found in Ref. [33].

For those flavor-changing neutral current one-loop decay modes, such as \( B_c \to Dl^+l^- \) and \( B_c \to Ds l^+l^- \), they are transitions of \( b \to dl^+l^- \) and \( b \to sl^+l^- \) at quark level, respectively. For the detailed discussion of their effective Hamiltonians and the corresponding differential decay widths please see Ref. [14]. For the semileptonic decay modes of \( B_c \to (D^*, D^*_s) l^+l^- \), their effective Hamiltonians are the same as \( B_c \to (D, D_s) l^+l^- \). But the expressions of the differential decay widths are very different. The differential decay widths of \( B(B_c) \to Vl^+l^- \) (\( V \) means a vector meson here) have been discussed in literature [30,34,36], and we adopt the result given by Ali et al. in Ref. [36].

For the decay modes of \( B_c \to D^{(*)}_s l^- \nu \), the effective Hamiltonian is [32]

\[ \mathcal{H}_{\text{eff}}(b \to sl^- \nu) = \frac{G_F}{\sqrt{2}} \frac{\alpha_{em}}{2\pi \sin^2(\theta_W)} V_{ib} V_{ts}^* \eta_X X(xt) \left[ s^\mu(1 - \gamma_5)b \right] \left[ \bar{\nu} \gamma_\mu(1 - \gamma_5)\nu \right] \quad (29) \]

where \( \theta_W \) is the Weinberg angle with \( \sin^2(\theta_W) = 0.231 \) [24], \( V_{ib} \) and \( V_{ts} \) are CKM matrix elements and \( \alpha_{em} \approx 1/137 \) is the fine structure constant. The function \( X(xt) \) can be found in Ref. [32], while \( \eta_X \approx 1 \) is the QCD radiative correction factor [32]. As for the decay modes of \( B_c \to D^{(*)}_s l^- \nu \), their effective Hamiltonian can be get by a simple replacement of \( s \to d \) in Eq. (29). The corresponding differential decay widths for \( B_c \to D^{(*)}_s l^- \nu \) is the same as \( B \to \pi(K)\nu \bar{\nu} \) in Ref. [14] except the replacements \( m_B \to m_{B_c} \) and \( m_F \to m \). While for
the decay modes of $B_c \to D_s^* \nu \bar{\nu}$, the differential decay width is \[37\]

\[
\frac{d\Gamma(B_c \to D_s^* \nu \bar{\nu})}{dq^2} = \frac{G_F^2 \alpha_{em}^2}{2^{10} \pi^{5} m_{B_c}^3} \left| X(x_t) \right|^2 \frac{\eta_X^2 \cdot |V_{tb} V_{ts}|^2}{(m_{B_c} + m)^2} \lambda^2 \left\{ 8 \lambda q^2 (m_{B_c} + m)^2 ight. \\
+ \frac{1}{m^2} \left[ \lambda^2 \left( \frac{A_2}{(m_{B_c} + m)^2} + (m_{B_c} + m)^2(\lambda + 12m^2 q^2) \cdot A_1 \right) \right. \\
- 2\lambda(m_{B_c}^2 - m^2 - q^2) \cdot Re[A_1 A_2] \right\}, \tag{30}
\]

where $V, A_1$ and $A_2$ are the form factors of $B_c \to D_s^*$ transition, and the phase space factor

\[
\lambda = (m_{B_c}^2 + m^2 - q^2)^2 - 4m_{B_c}^2 m^2. \tag{31}
\]

The differential decay width for $B_c \to D_s^* \nu \bar{\nu}$ can be get from Eq. \[30\] by the replacement of $V_{ts} \to V_{td}$.

**IV. NUMERICAL RESULTS AND DISCUSSIONS**

In the numerical calculations we adopt the following input parameters (masses and decay constants in unit of GeV) \[18, 24, 38\]

\[
\Lambda^{(f=4)} = 0.287, \quad m_{B_c} = 6.277, \quad m_{D_s} = 1.865, \quad m_{D_s} = 1.870, \\
m_{D_{s0}} = 2.007, \quad m_{D_{s-}} = 2.010, \quad m_{D_{s-}} = 1.969, \quad m_{D_{s-}} = 2.112, \\
m_c = 1.777, \quad m_c = 1.275 \pm 0.025, \quad \tau_{B_c} = (0.45 \pm 0.04) ps, \\
|V_{tb}| = 0.999, \quad |V_{ts}| = 0.0404, \quad |V_{td}/V_{ts}| = 0.211, \quad f_{B_c} = 0.489. \tag{32}
\]

As for the CKM matrix element $V_{ub}$, we adopt $|V_{ub}| = 0.0038 \ [14, 39]$ in this work.

By using the expressions in Eqs. \[18, 27\] and the definitions in Eq. \[12\], we calculate the values of the form factors $F_{0,+}(q^2), V(q^2), A_{0,1,2}(q^2)$ and $T_{1,2,3}(q^2)$ of $B_c \to D_s^{(*)}$ transitions for given value of $q^2$ in the region of $0 \leq q^2 \leq (m_{B_c} - m)^2$. But one should note that the pQCD predictions for the considered form factors are reliable only for the small value of $q^2$. For the form factors in the large $q^2$ region, one has to make an extrapolation for them from the low $q^2$ region to large $q^2$ region. In this work we make the extrapolation by using the formula in Refs. \[18, 33\]

\[
F(q^2) = F(0) \cdot \exp \left[ a \cdot q^2 + b \cdot (q^2)^2 \right], \tag{33}
\]

where $F$ stands for the form factors $F_{0,+}, V, A_{0,1,2}$ and $T_{1,2,3}$, and $a, b$ are the constants to be determined by the fitting procedure.

The numerical values of the $B_c \to D$ and $B_c \to D_s$ transitions form factors $F_{0,+}, V, T_{1,2,3}$ at $q^2 = 0$ and their fitted parameters $a, b$ are listed in Table\[I\]. The numerical values of the form factors $V, A_{0,1,2}$ and $T_{1,2,3}$ at $q^2 = 0$ for the $B_c \to D_s^*$ and $B_c \to D_s^*$ transitions are listed in Table\[II\]. The first error of the pQCD predictions for the form factors in Table\[I\] and Table\[II\] comes from the uncertainty of decay constants of the $D_s^{(*)}$ mesons, the second one is induced by $\omega_{B_c} = 0.60 \pm 0.06$, the third one of these form factors in Table\[I\] and Table\[II\] comes from the uncertainty of $C_{D_s^{(*)}} = 0.5 \pm 0.1$ or $C_{D_s^{(*)}} = 0.4 \pm 0.1$ and the forth error comes from $m_c = 1.275 \pm 0.025$ GeV. While the errors from the uncertainty of $\omega_{D_s^{(*)}} = 0.10 \pm 0.02$ or
\( \omega_{D^{(*)}} = 0.20 \pm 0.04 \) and \( f_{B_c} = 0.489 \pm 0.004 \pm 0.003 \) GeV are very small that have been neglected. The error of the parameters \( a, b \) in Table II comes from \( m_c = 1.275 \pm 0.025 \) GeV. The form factors are proportional to the decay constants \( f_{B_c} \) and \( f_{D^{(*)}} \), which makes the parameters \( a, b \) unchanged when they suffering the uncertainty of the decay constants \( f_{B_c} \) and \( f_{D^{(*)}} \). Those errors from the uncertainty of \( \omega_{B_c}, C_{D^{(*)}} \) and \( \omega_{D^{(*)}} \) are also very small for the parameters \( a, b \) and have been neglected in Table II and Table III. As a comparison, we also present some results obtained by other authors based on different methods in Table III. And from Table III we find that our results are consistent with the results in other literatures.

### TABLE I. The pQCD predictions for form factors \( F_0, F_+ \) and \( F_T \) at \( q^2 = 0 \) and the parametrization constants \( a \) and \( b \) for \( B_c \to D \) and \( B_c \to D_s \) transitions.

| \( F(0) \) | \( a \) | \( b \) |
|---|---|---|
| \( F_{0}^{B_c\to D} \) | 0.19 \( \pm 0.01 \) \( \pm 0.01 \) \( \pm 0.01 \) \( \pm 0.01 \) | 0.040 | 0.0013 |
| \( F_{0}^{B_c\to D} \) | 0.19 \( \pm 0.01 \) \( \pm 0.01 \) \( \pm 0.01 \) \( \pm 0.01 \) | 0.061 | 0.0021 |
| \( F_{0}^{B_c\to D} \) | 0.20 \( \pm 0.01 \) \( \pm 0.01 \) \( \pm 0.01 \) \( \pm 0.01 \) | 0.072 | 0.0023 |
| \( F_{0}^{B_c\to D_s} \) | 0.27 \( \pm 0.02 \) \( \pm 0.01 \) \( \pm 0.01 \) \( \pm 0.01 \) | 0.042 | 0.0015 |
| \( F_{0}^{B_c\to D_s} \) | 0.27 \( \pm 0.02 \) \( \pm 0.01 \) \( \pm 0.01 \) \( \pm 0.01 \) | 0.064 | 0.0024 |
| \( F_{0}^{B_c\to D_s} \) | 0.29 \( \pm 0.02 \) \( \pm 0.01 \) \( \pm 0.01 \) \( \pm 0.01 \) | 0.075 | 0.0028 |

### TABLE II. The pQCD predictions for form factors \( A_{0,1,2}, V \) and \( T_{1,2,3} \) at \( q^2 = 0 \) and the parametrization constants \( a \) and \( b \) for \( B_c \to D^* \) and \( B_c \to D_s^* \) transitions.

| \( F(0) \) | \( a \) | \( b \) |
|---|---|---|
| \( A_{0}^{B_c\to D^*} \) | 0.17 \( \pm 0.02 \) \( \pm 0.01 \) \( \pm 0.01 \) \( \pm 0.00 \) | 0.064 \( \pm 0.001 \) | 0.0027 \( \pm 0.0000 \) |
| \( A_{0}^{B_c\to D^*} \) | 0.19 \( \pm 0.02 \) \( \pm 0.01 \) \( \pm 0.01 \) \( \pm 0.01 \) | 0.045 \( \pm 0.001 \) | 0.0019 \( \pm 0.0001 \) |
| \( A_{0}^{B_c\to D^*} \) | 0.20 \( \pm 0.02 \) \( \pm 0.01 \) \( \pm 0.01 \) \( \pm 0.01 \) | 0.071 \( \pm 0.001 \) | 0.0028 \( \pm 0.0001 \) |
| \( V^{B_c\to D^*} \) | 0.26 \( \pm 0.03 \) \( \pm 0.02 \) \( \pm 0.01 \) \( \pm 0.01 \) | 0.073 \( \pm 0.001 \) | 0.0035 \( \pm 0.0001 \) |
| \( T_{1}^{B_c\to D^*} \) | 0.23 \( \pm 0.02 \) \( \pm 0.01 \) \( \pm 0.01 \) \( \pm 0.00 \) | 0.065 \( \pm 0.001 \) | 0.0029 \( \pm 0.0001 \) |
| \( T_{2}^{B_c\to D^*} \) | 0.23 \( \pm 0.02 \) \( \pm 0.01 \) \( \pm 0.01 \) \( \pm 0.00 \) | 0.041 \( \pm 0.001 \) | 0.0020 \( \pm 0.0001 \) |
| \( T_{3}^{B_c\to D^*} \) | 0.20 \( \pm 0.02 \) \( \pm 0.01 \) \( \pm 0.01 \) \( \pm 0.00 \) | 0.074 \( \pm 0.001 \) | 0.0037 \( \pm 0.0001 \) |
| \( A_{0}^{B_c\to D_s^*} \) | 0.21 \( \pm 0.02 \) \( \pm 0.01 \) \( \pm 0.01 \) \( \pm 0.00 \) | 0.065 \( \pm 0.001 \) | 0.0037 \( \pm 0.0002 \) |
| \( A_{0}^{B_c\to D_s^*} \) | 0.24 \( \pm 0.02 \) \( \pm 0.01 \) \( \pm 0.01 \) \( \pm 0.00 \) | 0.046 \( \pm 0.001 \) | 0.0027 \( \pm 0.0002 \) |
| \( A_{0}^{B_c\to D_s^*} \) | 0.27 \( \pm 0.03 \) \( \pm 0.02 \) \( \pm 0.01 \) \( \pm 0.00 \) | 0.073 \( \pm 0.001 \) | 0.0040 \( ^{+0.0003}_{-0.0002} \) |
| \( V^{B_c\to D_s^*} \) | 0.34 \( \pm 0.03 \) \( \pm 0.02 \) \( \pm 0.02 \) \( \pm 0.01 \) | 0.076 \( \pm 0.001 \) | 0.0048 \( \pm 0.0002 \) |
| \( T_{1}^{B_c\to D_s^*} \) | 0.29 \( \pm 0.03 \) \( \pm 0.02 \) \( \pm 0.02 \) \( \pm 0.00 \) | 0.067 \( \pm 0.001 \) | 0.0041 \( \pm 0.0002 \) |
| \( T_{2}^{B_c\to D_s^*} \) | 0.29 \( \pm 0.03 \) \( \pm 0.02 \) \( \pm 0.02 \) \( \pm 0.00 \) | 0.043 \( \pm 0.001 \) | 0.0028 \( \pm 0.0002 \) |
| \( T_{3}^{B_c\to D_s^*} \) | 0.26 \( \pm 0.03 \) \( \pm 0.02 \) \( \pm 0.02 \) \( \pm 0.00 \) | 0.078 \( \pm 0.001 \) | 0.0052 \( \pm 0.0002 \) |

By using the relevant formulas and the input parameters as defined or given in previous sections, it is straightforward to calculate the branching ratios for all the considered decays.
and T decay rates are the following: in Table IV come from the uncertainties of literature.
By making the numerical integration over the whole range of $q^2$, we find the pQCD predictions for the branching ratios. For the $b \to u$ charged current process, with $l = (e, \mu)$, the decay rates are the following:

\[
Br(B_c^- \to \bar{D}^0 l^- \bar{\nu}_l) = (3.29^{+0.53}_{-0.47}(\omega_{B_c})^{+0.30}_{-0.28}(m_c)^{+0.32}_{-0.30}(C_D)^{+0.29}_{-0.28}(f_D) \pm 0.28(\tau_{B_c})) \cdot 10^{-5},
\]

\[
Br(B_c^- \to \bar{D}^0 \tau^- \bar{\nu}_\tau) = (2.27^{+0.38}_{-0.34}(\omega_{B_c})^{+0.29}_{-0.20}(m_c)^{+0.21}_{-0.20}(C_D)^{+0.20}_{-0.19}(f_D) \pm 0.20(\tau_{B_c})) \cdot 10^{-5},
\]

\[
Br(B_c^- \to \bar{D}^0 l^- \bar{\nu}_l) = (1.31^{+0.23}_{-0.20}(\omega_{B_c})^{+0.16}_{-0.13}(m_c)^{+0.12}_{-0.11}(C_D)^{+0.27}_{-0.25}(f_D^*) \pm 0.12(\tau_{B_c})) \cdot 10^{-4},
\]

\[
Br(B_c^- \to \bar{D}^0 \tau^- \bar{\nu}_\tau) = (0.79^{+0.14}_{-0.12}(\omega_{B_c})^{+0.10}_{-0.08}(m_c)^{+0.07}_{-0.06}(C_D)^{+0.16}_{-0.15}(f_D^*) \pm 0.07(\tau_{B_c})) \cdot 10^{-4},
\]

where the errors come from the uncertainties of $\omega_{B_c} = 0.60 \pm 0.06$, $m_c = (1.275 \pm 0.025)$ GeV, $C_D = 0.5 \pm 0.1$, $f_D = (206.7 \pm 8.9)$ MeV or $f_D^* = (270 \pm 27)$ MeV and $\tau_{B_c} = (0.45 \pm 0.04)\,ps$, respectively.

For the neutral current processes, after making the numerical integration over the whole range of $4m_l^2 \leq q^2 \leq (m_{B_c} - m)^2$, we get the pQCD predictions for the branching ratios of considered decay modes which are listed in Table IV. The errors of the pQCD predictions in Table IV come from the uncertainties of $\omega_{B_c}$, $m_c$, $C_{D(*)}$ or $C_{D_s(*)}$, $f_{D(*)}$ or $f_{D_s(*)}$ and $\tau_{B_c}$, respectively.

From the pQCD predictions for the form factors $F_{0,+,T}$ in Table III, the form factors $V, A_{0,1,2}$ and $T_{1,2,3}$ in Table I and the pQCD predictions for the branching ratios as listed in Eq. (34) and in Table IV we have the following points:

(i) All the form factors for the transitions $B_c \to D_s^{(*)}$ are some larger than the corresponding values for the transitions $B_c \to D^{(*)}$ at $q^2 = 0$, which characterizes the SU(3) breaking effect.

(ii) $F_0(0)$ equals to $F_c(0)$ by definition for the $B_c \to D$ or $B_c \to D_s$ transition, but they have different $q^2$ dependence by the different parameters $(a, b)$. $T_1(0)$ equals to $T_2(0)$ for the $B_c \to D^*$ or $B_c \to D_s^*$ transition claimed by the Eq. (17) as they are given in Table II although their expressions are some different in Eqs. (23) and (24).

---

**TABLE III.** $B_c \to D_s^{(*)}$ transition form factors at $q^2 = 0$ evaluated in this paper and in other literature.

| $B_c \to D_s^{(*)}$ | $F_+(0) = F_0(0)$ | $F_T(0)$ | $A_0(0)$ | $A_1(0)$ | $A_2(0)$ | $V(0)$ | $T_1(0) = T_2(0)$ | $T_3(0)$ |
|----------------------|-----------------|-----------|-----------|-----------|-----------|--------|-----------------|--------|
| pQCD                 | 0.19            | 0.20      | 0.17      | 0.19      | 0.20      | 0.26   | 0.23            | 0.20   |
| Ref.[33]             | 0.16            | –         | 0.09      | 0.08      | 0.07      | 0.13   | –               | –      |
| Ref.[40]             | 0.14            | –         | 0.14      | 0.17      | 0.19      | 0.18   | –               | –      |
| Ref.[41]             | 0.69            | –         | 0.47      | 0.56      | 0.64      | 0.98   | –               | –      |
| Ref.[42]             | 0.35            | –         | 0.05      | 0.32      | 0.57      | 0.57   | –               | –      |
| Ref.[43]             | 0.32            | –         | 0.35      | 0.43      | 0.51      | 1.66   | –               | –      |
| Ref.[44]             | 0.075           | –         | 0.081     | 0.095     | 0.11      | 0.16   | –               | –      |

---
The pQCD predictions for the branching ratios of the considered decays ($l = e, \mu$).

| Decay modes | pQCD predictions |
|-------------|------------------|
| $Br(B_c^- \rightarrow D^{-} l^+ l^-)$ | $(3.94^{+0.34}_{-0.26}(m_c) + 0.20 (m_c) + 0.39(C_D) + 0.27(f_D) \pm 0.35(\tau_{B_c}) \cdot 10^{-9}$ |
| $Br(B_c^- \rightarrow D^{-} \tau^+ \tau^-$ | $(1.09^{+0.20}_{-0.19}(m_c) + 0.10 (m_c) + 0.20(C_D) + 0.09(f_D) \pm 0.10(\tau_{B_c}) \cdot 10^{-9}$ |
| $Br(B_c^- \rightarrow D^{-} \bar{\nu}\nu)$ | $(3.27^{+0.34}_{-0.40}(m_c) + 0.30(C_D) + 0.32(f_D) \pm 0.28(\tau_{B_c}) \cdot 10^{-7}$ |
| $Br(B_c^- \rightarrow D^{*-} l^- l^+)$ | $(1.67^{+0.16}_{-0.20}(m_c) + 0.13(C_D) \pm 0.07(f_D) \pm 0.15(\tau_{B_c}) \cdot 10^{-7}$ |
| $Br(B_c^- \rightarrow D_-^{*} \tau^+ \tau^-$ | $(0.41^{+0.08}_{-0.07}(m_c) + 0.06(C_D) \pm 0.03(f_D) \pm 0.02(\tau_{B_c}) \cdot 10^{-7}$ |
| $Br(B_c^- \rightarrow D_-^{*} \bar{\nu}\nu)$ | $(1.39^{+0.20}_{-0.13}(m_c) + 0.12(C_D) \pm 0.06(f_D) \pm 0.12(\tau_{B_c}) \cdot 10^{-6}$ |

(iii) Because of the phase space suppression, the branching ratios of the decay modes with a $\tau$ in the final product are smaller than those decay modes with electron or muon in the final product for the the charged current process. And for the flavor changing neutral current processes, with two $\tau$'s in the final product, the branching ratios are much smaller than the corresponding decays with electron or muon pairs in the final product.

(iv) The branching ratios of the decay modes with $\bar{\nu}\nu$ are almost an order magnitude larger than the corresponding decays with $l^- l^+$ after the summation over the three neutrino generations. Because of the strong suppression of the CKM factor $|V_{td}/V_{ts}|^2 = 0.211^2$, the branching ratios for the decay modes with $b \rightarrow d$ transitions are much smaller than those decay modes with the $b \rightarrow s$ transitions.

(v) In order to reduce the theoretical uncertainty of the pQCD predictions, we defined two ratios $R_D$ and $R_{D^*}$ among the branching ratios for the the charged current processes

$$R_D = \frac{Br(B_c^- \rightarrow \bar{D^0} \tau^- \bar{\nu}_\tau)}{Br(B_c^- \rightarrow \bar{D^0} l^- \nu_l)} \approx 0.7,$$

$$R_{D^*} = \frac{Br(B_c^- \rightarrow \bar{D^{*0}} \tau^- \bar{\nu}_\tau)}{Br(B_c^- \rightarrow \bar{D^{*0}} l^- \nu_l)} \approx 0.6,$$

with $l = (e, \mu)$. These two relations will be tested by experiments.

V. SUMMARY AND CONCLUSIONS

In this paper we study the $B_c \rightarrow (D(s), D^{*}(s))$ transition form factors $F_{0,+,-}(q^2)$ and $V(q^2), A_{0,1,2}(q^2), T_{1,2,3}(q^2)$ in the pQCD factorization approach based on $k_T$ factorization. The pQCD predictions for the values of the $B_c \rightarrow D(s)$ and $B_c \rightarrow D^{*}(s)$ transition form
factors agree with those obtained by using other methods. Utilizing these form factors, we calculate the branching ratios for all the semileptonic decays of $B^+_c \rightarrow D^{(+)}_s(l^+\nu, l^+l^-, \nu\bar{\nu})$. Because of phase space suppression, the production ratios of the decay modes with lepton $\tau$ in the final product are smaller than the corresponding decays which is electron or muon in the final product. The branching ratios of the decay modes with $\bar{\nu}\nu$ is almost an order magnitude larger than the corresponding decays with $l^+l^-$ after the summation over the three neutrino generations. And the branching ratios for the decays with $b \rightarrow d$ transitions are much smaller than those decays with the $b \rightarrow s$ transitions.

In order to reduce the theoretical uncertainty of the pQCD predictions, we defined two ratios $R_D$ and $R_{D^*}$ among the branching ratios for the the charged current processes. The pQCD predictions are

$$R_D = \frac{Br(B^+_c \rightarrow \bar{D}^0\tau^-\bar{\nu}_\tau)}{Br(B^+_c \rightarrow \bar{D}^0l^-\bar{\nu}_l)} \approx 0.7,$$

(37)

$$R_{D^*} = \frac{Br(B^+_c \rightarrow \bar{D}^{*0}\tau^-\bar{\nu}_\tau)}{Br(B^+_c \rightarrow \bar{D}^{*0}l^-\bar{\nu}_l)} \approx 0.6,$$

(38)

with $l = (e, \mu)$. This is possible be tested by LHCb and the forthcoming Super-B experiments.

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