Developing a model for simulation of mechanical damages to buckwheat during mechanical grain processing

V V Trotsenko¹, I V Trotsenko¹ and N V Komendantova²

¹Omsk State Agrarian University named after P. A. Stolypin, 1, Institutskaya square, Omsk, 644008, Russia
²Department of the Pension Fund of Russia in Omsk region, 71/1, Chapaeva str., Omsk, 644099, Russia

Email: vv.trotsenko@omgau.org

Abstract. This paper tells about actual problem of the efficiency increasing of buckwheat production by reducing the mechanical damage, which appears as the result of technological processes during harvesting, post-harvesting and pre-sowing. The mechanical processing causes the damages to some of the buckwheat grains during the interaction between the grains and the units of a mechanism. The pressure of the mechanism induces the excess stress and the grains are destroyed. This article shows the buckwheat grain is modeled like a cylindrical trihedron for the estimation of these stresses. The mathematical relationships for the suggested forms let us associate the parameters of the length, width and thickness of grains with the radii of the surfaces of the faces and the location of the axes of the intersecting cylinders. This knowledge of let us use the well-known relationships of the theory of elasticity in determining the values of stresses in various types of contacting grains and units of machines. It has been experimentally established that the main proportion (more than 85%) of the buckwheat grain batches is presented like grains with cylindrical faces, the ratio of the actual grain width to its thickness for different varieties differs from the calculated one by no more than 10%, and the deviation of the actual buckwheat grain volume from the calculated one depending on the variety does not exceed 15%.

1. Introduction
The modern technological process of grain production is impossible without use of machines. However, during the mechanized processing of grain by batches as a result of interaction with the parts of machines individual grains experience excess stress that causes them mechanical damages. The number of mechanically damaged grains in a batch (hereinafter–mechanical damage) according to the data [1–9] reaches 60% or more, which significantly exceeds the limits defined by the standards for the quality of grain material and consequently affects its further use. In this case, there are decrease of the germination rate and yield of the buckwheat seed grain, and degradation of the food processing properties. In addition, the batches of the grains with mechanical damages are less stable in storage. Since it is impossible to completely exclude grain damage during mechanized processing, the search for ways to reduce the amount of mechanical damage by improving the technological scheme of processing and technical means of its implementation is topical and has important economic significance.

The values of mechanical stresses occurring in the grain can be estimated with a known load value
and knowledge of the shape of the grain. Various mathematical dependences were proposed to estimate the contact stresses arising in wheat, pea, soy, and corn grains [10–12]. However, with the same force interaction with the parts of machines, the stresses in the grains of these crops must obviously differ from the stresses arising in the grain of buckwheat, because it has a different form of seeds, and the proposed dependencies are not acceptable for such a case.

So, the main goal is the theoretical approximation of the shape of buckwheat seeds in the form of a cylindrical trihedron with subsequent experimental confirmation.

2. Theory
We suggest the shape of the buckwheat grain as a surface formed by the intersection of three cylinders - a cylindrical trihedron to determine the stresses arising in the buckwheat grains for comparison with the limiting ones. Let us determine the geometric parameters of the proposed shape, for which we consider the intersection of three circular cylinders with axes $O_1 o_1$, $O_2 o_2$ and $O_3 o_3$ and radii equal to $R$ (Figures 1 and 2) [13].

Such a figure has three elliptical edges PAQ, PBQ, PCQ and two vertices P and Q. If we draw the plane $f$ through the middle of the diagonal PQ point O, so that $f$ is perpendicular to PQ, then the result is a flat figure in the form of an equilateral triangle ABC with the point of intersection of the medians O. The Axes of the cylinders $O_1 o_1$, $O_2 o_2$ and $O_3 o_3$ are in the plane $f$ and intersect at points $o_1$, $o_2$, $o_3$.

To determine the necessary geometric dependencies, let's set the Cartesian coordinate system $XYZ$ in such a way that the origin coincides with the point $O$, the X and Y axes belong to the secant plane $f$, while the y axis passes through the point $A$, the z axis coincides with the diagonal of the trihedron $PQ$ and is directed towards the point $P$ (Fig. 2). In the selected coordinate system, the coordinates of the points are: $A (0, y_A, 0)$, $B(x_B, y_B, 0)$, $C(x_C, y_C, 0)$, $P(0, 0, z_P)$, $Q(0, 0, z_Q)$, $O_1 (0, a, 0)$, $O_2 (a \cos 30^\circ, -a \sin 30^\circ, 0)$, $O_3 (-a \cos 30^\circ, -a \sin 30^\circ, 0)$, where $a$ is the distance from the origin to the axes of the cylinders.

Equations of cylinder axes in the $XYZ$ coordinate system.

$$
\begin{align*}
O_1 o_1: & \begin{cases} y = a \\ z = 0 \end{cases} \\
O_2 o_2: & \begin{cases} y = \sqrt{3} \cdot x - 2a \\ z = 0 \end{cases} \\
O_3 o_3: & \begin{cases} y = -\sqrt{3} \cdot x - 2a \\ z = 0 \end{cases}
\end{align*}
$$

(Figure 1. Cylindrical trihedron – the intersection of three cylinders)
The points of intersection of the axes of the cylinders will be determined as a result of the joint solution of these equations and will be respectively:

\[ a_1(\sqrt{3} \cdot a, a, 0), a_2(0, -2a, 0), a_3(\sqrt{3} \cdot a, a, 0). \]

Equations of cylinders with axes

\[ O_1 o_1: (y_1 - a)^2 + z^2 = R^2, \]
\[ O_2 o_2: (y_2 - a)^2 + z^2 = R^2, \]
\[ O_3 o_3: (y_3 - a)^2 + z^2 = R^2. \] (2)

Figure 2. Design scheme

In this equation

\[ y_1 = y; z = z_1 = z_2 = z_3 \]
\[ y_2 = -x \cdot \sin(-120) + y \cdot \sin(-120) = \frac{x \cdot \sqrt{3} - y}{2}, \]
\[ y_3 = -x \cdot \sin(120) + y \cdot \sin(120) = \frac{-x \cdot \sqrt{3} - y}{2}. \] (3)

Substituting the obtained values of \( y_2 \) and \( y_3 \) in (2) we get

\[ (y - a)^2 + z^2 = R^2, \] (4)
\[ \left(\frac{x \cdot \sqrt{3} - y}{2} - a\right)^2 + z^2 = R^2, \] (5)
\[ \left(\frac{-x \cdot \sqrt{3} - y}{2} - a\right)^2 + z^2 = R^2. \] (6)

Solving pairwise equations (4), (5), (6), let's define unknown coordinates of points \( A (0, y_A, 0), B(x_B, y_B, 0), C(x_C, y_C, 0) \) of intersection of cylinders with the plane \( XOY (z=0) \).
Point \( A \)

\[
\begin{cases}
  \left( x \cdot \frac{\sqrt{3}}{2} - y \right)^2 + z^2 = R^2, \\
  \left( -x \cdot \frac{\sqrt{3}}{2} - y - a \right)^2 + z^2 = R^2.
\end{cases}
\]  

(7)

Since the right parts of the equations are equal, we equate the left and after the transformations we get a satisfying root

\[ y_A = 2R - 2a \]

Point \( B \)

\[
\begin{cases}
  (y - a)^2 + z^2 = R^2, \\
  \left( -x \cdot \frac{\sqrt{3}}{2} - y - a \right)^2 + z^2 = R^2.
\end{cases}
\]  

(8)

Solving the first, we get

\[ y_B = a - R. \]

Equating the left parts of the equations and equating them, we come to the square equation

\[- \frac{3}{4} \cdot x^2 + \left( \frac{\sqrt{3}}{2} \cdot R - \frac{3\sqrt{3}}{2} \cdot a \right) \cdot x - \frac{9}{4} \cdot a^2 + \frac{3}{2} \cdot a \cdot R + \frac{3}{4} \cdot R^2 = 0\]

Solving which, we get two roots

\[ x = \frac{-1}{\sqrt{3}} \cdot (3 \cdot a + R), \quad -(a - R) \cdot \sqrt{3} \]

Since in our case \( R > a \), then according to the drawing (Fig. 2) we are satisfied with the positive root

\[ x_B = (R - a) \cdot \sqrt{3} \]

Point \( C \)

\[
\begin{cases}
  (y - a)^2 + z^2 = R^2, \\
  \left( x \cdot \frac{\sqrt{3}}{2} - y - a \right)^2 + z^2 = R^2.
\end{cases}
\]  

(9)

When solving this system, we do the same, as a result we get

\[ y_C = a - R \]

\[ x_C = -(R - a) \cdot \sqrt{3} \]

The solution of the system is the ellipse equation

\[ \left( \frac{y}{2} + a \right)^2 + z^2 = R^2. \]  

(10)

Let us express the length \( l \), width \( b \), and thickness \( h \) of the grain through the values of the radius of curvature of the face \( R \) and the location of the cylinder axis in the accepted coordinate systems \( a \)

\[ l = z_p - z_Q = 2\sqrt{R^2 - a^2}, \]  

(11)

\[ b = x_B - x_C = 2 \cdot (R - a) \cdot \sqrt{3}, \]  

(12)

\[ h = y_A - y_D = y_A - y_B = y_A - y_C = 2(R - a) + (R - a) = 3(R - a), \]  

(13)

Based on the expressions (12) and (13)

\[ b = \frac{2h}{\sqrt{3}} = 1.1547h. \]  

(14)

Let’s express the parameters \( R \) and \( a \) through \( l, b, h \). to do this, we solve together the equations (11) and (13)

\[ \begin{aligned}
  2\sqrt{R^2 - a^2} - l &= 0, \\
  3(R - a) - h &= 0.
\end{aligned} \]  

(15)
Where from

\[ R = \frac{4h^2 + 9l^2}{24h}, \]
\[ a = \frac{9l^2 - 4h^2}{24l}. \]

To write the equation of tangent planes to the surface of the grain at point \( A \) we use the expression

\[ \frac{\partial F(x_1, y_1, z_1)}{\partial x}(x - x_1) + \frac{\partial F(x_1, y_1, z_1)}{\partial y}(y - y_1) + \frac{\partial F(x_1, y_1, z_1)}{\partial z}(z - z_1) = 0 \]

Having differentiated expressions (5) and (6), having found values of a derivative at the specified point and having substituted them in (18), we obtain equations of tangents to cylinders with axes \( O_2O_3 \) and \( O_3O_3 \) respectively

\[ y = -\sqrt[3]{3} \cdot x + 2(R - a), \]
\[ y = \sqrt[3]{3} \cdot x + 2(R - a) \]

The angle between the tangent plane at point \( A \) and the \( XOZ \) plane

\[ \alpha = \arctg \sqrt[3]{3} = 60^\circ \]

The volume of the considered figure can be determined from the expression

\[ V = 3 \int_{-l/2}^{l/2} S_{OBC} dz, \]

where \( S_{OBC} \) is the cross-sectional area of a cylindrical trihedron by a plane perpendicular to \( PQ \) at \( z \), belonging to the interval from \(-l/2\) to \( l/2\).

The area of the OBC triangle is defined by the formula

\[ S_{OBC} = \frac{bh}{6}, \]

which, taking into account (13) and (14) has the form

\[ S_{OBC} = \frac{h^2}{3\sqrt[3]{3}} = \sqrt[3]{3} \cdot (r - a)^2, \]

where \( r \) is the current value of the parameter defined according to (Fig. 2) according to the formula

\[ r = \sqrt[3]{R^2 - z^2}, \]

As a result

\[ V = 3\sqrt[3]{3} \left[ R^2 l + a^2 l - \frac{l^2}{12} - a l \left( R^2 - \frac{l^2}{4} - 2aR^2 \cdot \arcsin \frac{l}{2R} \right) \right]. \]

Thus, knowing the overall dimensions, we can determine the radius of curvature of the grain face \( R \), the location of the cylinder axis in the accepted coordinate systems (Fig. 2), as well as the volume of the resulting figure.

3. Experimental result

Grain lots of buckwheat in its composition contain three types of grains in shape: first – these are grains with hollow faces, second – grains with straight faces and, third – grains with convex faces. The quantitative ratio between the individual types of grains depends on the period of their maturation. For buckwheat, as it is known, maturation is a long period of 25 – 30 days. The cleaning of such culture is made in a separate way and begins at browning of 65-75% of fruit.

We carried out visual control of the shape and measurement of the overall dimensions of the grain harvested in compliance with the appropriate technology. Moreover, the average samples were taken from three batches of different varieties and 500 pieces from each average sample [14, 15]. The results are summarized in Table 1.
Table 1. Buckwheat grain shape and parameters.

| Variety        | Weight of 1000 grains, g. | The content of grains of different shapes, % | Overall dimensions, mm | Design parameters for a cylindrical trihedron, mm |
|----------------|---------------------------|---------------------------------------------|------------------------|-----------------------------------------------|
|                |                           | with sunken faces | with straight faces | with a bulge | length l | width b | thickness h | radius of curvature of the face |
| Chishminskaya  | 27.64±0.41               | 3.26             | 87.50             | 9.24        | 6.34±0.13 | 4.2±0.09 | 3.55±0.05 | 4.84 | 3.65 |
| Shatilovskaya 5 | 28.16±0.51               | 6.28             | 85.31             | 8.41        | 6.64±0.15 | 4.58±0.08 | 3.71±0.06 | 5.08 | 3.84 |
| Idel           | 31.16±0.51               | 8.14             | 86.22             | 5.64        | 7.30±0.15 | 5.02±0.10 | 3.95±0.06 | 5.72 | 4.40 |

Based on this table, we can say that the main share in grain batches of buckwheat are grains that have straight faces. The ratio (14) will actually be:

for grain class "Chishminskaya" \( \frac{b}{h} = \frac{4.2}{3.55} = 1.1831 \), the deviation from the design of 2.46%;

for grain of the Shatilovskaya 5 variety \( \frac{b}{h} = \frac{4.48}{3.71} = 1.2075 \), the deviation from the calculated 4.57%;

for “Idel” grain \( \frac{b}{h} = \frac{5.02}{3.95} = 1.2709 \), the deviation from the calculated 9.53%.

Table 2. The amount of grains of buckwheat.

| Variety      | Grain volume, mm³ | Deviation, % |
|--------------|-------------------|--------------|
|              | Calculated        | measured     |              |
| Chishminskaya| 25.56             | 27.81        | 10.09        |
| Shatilovskaya 5 | 29.23            | 24.95        | 14.64        |
| Idel         | 36.35             | 31.71        | 12.52        |

The measurement of the volume of the grain was carried out in a five-fold repetition by immersing five hundred grains in a measuring vessel with water. In accordance with the table, 2 the deviation of the actual volume of grain from the calculated one, depending on the variety, is approximately 10-15%.

Figure 3 shows a grain of buckwheat varieties "Chishminskaya" in three axonometric projections.
4. Conclusion
1. The mathematical relationships for the proposed forms let us associate parameters of length, width, thickness of grains with radii of the surfaces of faces and location of the axes of the intersecting cylinders, the knowledge of which will let use the known equations of the theory of elasticity in different types of grain contact and working bodies of machines.
2. The main share of more than 85% in grain lots of buckwheat is made up of grains having cylindrical faces.
3. The ratio of the actual width of the grain to its thickness for different varieties differs from the calculated one by no more than 10%, which is quite acceptable for biological objects.
4. The deviation of the actual volume of buckwheat grain from the calculated one, depending on the variety, does not exceed 15%.

References
[1] Shevchenko A P, Bankrutenko A V, Koval V S, Begunov M A and Demchuk E V 2018 Scarification of seeds as an increasing element of perennial legume grasses productivity Journal of Physics: Conference Series 1059(1) 012011
[2] Koval V, Chernyakov A, Shevchenko A, Begunov M and Demchuk E 2019 Determination of separator constructive parameters IOP Conference Series: Materials Science and Engineering 582 012040 Available at: https://doi.org/10.1088/1757-899X/582/1/012040
[3] Ng H F, Wilcke W F, Morey R V, Meronuck R A and Lang J P 1998 Mechanical damage and corn storability Transactions of the American Society of Agricultural Engineers 41 1095-1100 Available at: https://doi.org/10.13031/2013.17239
[4] Resende O, Correa P C, de Oliveira G H H, Goneli A L D and Jaren C 2013 Mechanical properties of rough and dehulled rice during drying International Journal of Food Studies 2 158–166 Available at: https://doi.org/10.7455/ijfs.2.2.2013.a3
[5] Tarasenko A P, Orobinskii V I, Merchalova M E and Buravlev N E 2015 Innovative ways of improving mechanization of high-quality seeds Revista Ciencias Técnicas Agropecuarias 24(2) 49–52
[6] Henry Z A, Su B and Zhang H 2000 Resistance of soya beans to compression Journal of Agricultural Engineering Research 76 175–181 Available at: https://doi.org/10.1006/jaer.2000.0546
[7] Altuntas E and Yildiz M 2007 Effect of moisture content on some physical and mechanical properties of faba bean (Vicia faba L.) grains Journal of Food Engineering 78(1) 174–183 Available at: doi:10.1016/j.jfoodeng.2005.09.013
[8] Corrêa P C, Schwanz da Silva F, Jaren C, Afonso Júnior P C and Arana I 2007 Physical and mechanical properties in rice processing Journal of Food Engineering 79(1) 137–142 https://doi.org/10.1016/j.jfoodeng.2006.01.037
[9] Shabbazi F, Valizade S and Dowlatshah A 2017 Mechanical damage to green and red lentil...
seeds Food Science & Nutrition 5(4) 943–947 doi: 10.1002/fsn3.480
[10] Jonson K L 1985 Contact mechanics (London: Cambridge University Press)
[11] Pisarenko G S, Yakovlev A P and Matveev V V 1988 Handbook on resistance of materials (Kiev: Naukova dumka)
[12] Tymoshenko S P and Gudier J 1979 Theory of elasticity (Moscow: Nauka)
[13] Korn G A and Korn T M 2000 Mathematical Handbook for Scientists and Engineers: Definitions, Theorems, and Formulas for Reference and Review (New York: Dover Publications Inc.)
[14] Trotsenko V V and Trotsenko I V 2019 Ways to reduce mechanical damage of barley for mechanical processing Journal of Physics: Conference Series 1260 112030 DOI: 10.1088/1742-6596/1260/2/022003
[15] Trotsenko V V, Zabudsky A I and Komendantov V V 2017 Damage of barley grain machines for mechanical processing Electronic scientific and methodical journal of Omsk State Agrarian University 1(8) Available at: http://e-journal.omgau.ru/index.php/2017/1/35-statya-2017-1/783-00310.