We examine to what extent several recently discovered narrow resonances can be interpreted as conventional $c\bar{c}$ bound states describable using a potential model. In doing so, we use a semirelativistic approach, which includes both the $v^2/c^2$ and QCD one-loop corrections to the short distance potential and a long range linear potential together with its scalar and vector $v^2/c^2$ spin-dependent terms.

1. Introduction

With the recent experimental results for several expected states ($\eta_C$ and $h_C$) in the charmonium spectrum, and the discovery of a state ($X(3872)$), which could be a $^3D_2$ charmonium level, it seems an appropriate time to revisit the potential model interpretation of the $c\bar{c}$ spectrum. For such models, the challenges seem to be:

- Are potential models capable of describing the spin splitting in a quantitatively satisfactory way?
- Including the additional data, is it possible to determine the Lorentz properties of the phenomenological confining potential?
- How well are the leptonic and radiative decays predicted?
- Under what circumstances can the state at 3872 MeV be interpreted as a charmonium level?

Here, we will attempt to answer these questions using a potential model which includes the $v^2/c^2$ and all one-loop corrections to the short distance potential supplemented with a linear phenomenological confining potential and its $v^2/c^2$ corrections.
2. Potential Models

Potential models range from non-relativistic forms such as the Cornell model
\[ \mathcal{H}_C = \frac{p^2}{2m} + Ar - \frac{4\alpha_S}{3r}, \] (1)
recently used to identify new charmonium states observable in B decays, to those including all \( v^2/c^2 \) spin-dependent corrections. One-loop QCD corrections can then be added to complete the non-relativistic treatment.

In our analysis, we utilize a semi-relativistic Hamiltonian of the form
\[ \mathcal{H} = 2\sqrt{p^2 + m^2} + Ar - \frac{4\alpha_S}{3r} F(\mu, r) + V_S + V_L = \mathcal{H}_0 + V_S + V_L. \] (2)
Explicit forms of the short distance potential \( V_S \), the long distance potential \( V_L \) and \( F(\mu, r) \), the one-loop QCD correction to \( 4\alpha_S/3r \), can be found in Ref. The \( c\bar{c} \) mass spectrum and the corresponding wave functions were obtained using a variational approach. The wave functions were expanded as
\[ \psi_n^m(\vec{r}) = \sum_{n=0}^{N} C_n \left( \frac{r}{R} \right)^n e^{-r/R} Y_n^m(\Omega), \] (3)
and the \( C_n \)'s were determined by minimizing \( E = \langle \psi | \mathcal{H} | \psi \rangle / \langle \psi | \psi \rangle \). This procedure results in a linear eigenvalue equation for the \( C_n \)'s and the energies. The wave functions corresponding to different eigenvalues are orthogonal and the \( j \)th eigenvalue \( \lambda_j \) is an upper bound on the exact energy \( E_j \). For \( N = 10 \), the lowest four eigenvalues are stable to a part in \( 10^6 \).

3. Results and Conclusions

The energies and wave functions were obtained by treating \( V_S + V_L \) as a perturbation to \( \mathcal{H}_0 \) and by treating \( \mathcal{H} \) nonperturbatively. We fit the spectrum to ten well-established \( c\bar{c} \) states by adjusting the parameters \( A, \alpha_S, m, \mu \) and the vector fraction \( f_V \) to minimize \( \chi^2 \). Our nonperturbative fit gives: \( A = 0.175 \) GeV, \( \alpha_S = 0.361 \), \( m = 1.49 \) GeV, \( \mu = 1.07 \) GeV and \( f_V = 0.18 \). Interestingly, the perturbative fit yields similar results for \( A, \alpha_S \) and \( m \), but prefers \( \mu = 2.32 \) GeV and \( f_V = 0 \). The results for the levels are given in Table and the predicted \( E_1 \) transitions widths are given in Table.

The semi-relativistic model provides a quantitatively good description of the charmonium spectrum. Of the states included in the fit, only the \( ^3D_1(3770) \) is poorly described. The \( E_1 \) widths agree reasonably well with experiment. However, based on the model considered here, the \( X(3872) \) cannot be explained solely in terms of a charmonium \( ^3D_2 \) state described by a potential. Spin effects alone can only separate the \( ^3D_2 \) from the \( ^3D_1 \) by 40 MeV or so, which suggests that the inclusion of open channel effects is essential if this identification is to be established.

\[^1S_0, ^3S_1, ^3P_J, ^2S_0, ^2S_1, ^1D_1, ^3S_1 \text{ and } ^2D_1.\]
Describing Recently Discovered Narrow States as Quarkonia Using a Potential Model

Table 1. Perturbative and nonperturbative results for the $c\bar{c}$ spectrum.

|       | Pert       | Non-pert   | Expt       |       | Pert       | Non-pert   | Expt       |
|-------|------------|------------|------------|-------|------------|------------|------------|
| $\eta_c$ | 2985        | 2981       | 2979.7 ± 1.5 | $\eta'_c$ | 3599       | 3624       | (3637.7 ± 4.4) |
| $J/\psi$ | 3096.9      | 3096.9     | 3096.87 ± 0.04 | $\psi'$ | 3686       | 3686       | 3686.0 ± 0.1    |
| $\chi_0$ | 3418.4      | 3415.8     | 3415.1 ± 0.8  | $\chi'_0$ | 3849       | 3872       |            |
| $\chi_1$ | 3510.2      | 3510.4     | 3510.51 ± 0.12 | $\chi'_1$ | 3946       | 3951       |            |
| $\chi_2$ | 3556.5      | 3556.3     | 3556.18 ± 0.17 | $\chi'_2$ | 3999       | 3996       |            |
| $h_c$    | 3527        | 3524       | (3526.21 ± 0.25) | $h'_c$ | 3966       | 3966       |            |
| $1^3D_1$ | 3809        | 3790       | 3770 ± 2.5   | $2^3D_1$ | 4174       | 4157       | 4160 ± 20  |
| $1^3D_2$ | 3827        | 3826       | 3872 ± 1.0   | $2^3D_2$ | 4198       | 4201       |            |
| $1^3D_3$ | 3845        | 3845       | $2^3D_3$    | 4209       | 4223       |            |
| $1^3D_2$ | 3824        | 3825       | 3836 ± 13.0  | $2^1D_2$ | 4199       | 4202       |            |

Table 2. $E_1$ transition widths.

| $\Gamma(E_1)$ (keV) | TH | EX | $\Gamma(E_1)$ (keV) | TH | EX |
|---------------------|----|----|---------------------|----|----|
| $\chi_0 \rightarrow \gamma J/\psi$ | 169 | 119 ± 17 | $1^3D_2(3826) \rightarrow \gamma \chi_1$ | 314 |  |
| $\chi_1 \rightarrow \gamma J/\psi$ | 357 | 288 ± 51 | $1^3D_2(3826) \rightarrow \gamma \chi_2$ | 76.3 |  |
| $\chi_2 \rightarrow \gamma J/\psi$ | 468 | 426 ± 48 | $1^3D_2(3872) \rightarrow \gamma \chi_1$ | 459 |  |
| $h_c \rightarrow \gamma h_c$ | 670 |  | $1^3D_2(3872) \rightarrow \gamma \chi_2$ | 119 |  |
| $\psi' \rightarrow \gamma \chi_0$ | 22 | 24.2 ± 2.5 | $\psi(3770) \rightarrow \gamma \chi_0$ | 291 | 320 ± 100 |
| $\psi' \rightarrow \gamma \chi_1$ | 33 | 23.6 ± 2.7 | $\psi(3770) \rightarrow \gamma \chi_1$ | 125 | 280 ± 100 |
| $\psi' \rightarrow \gamma \chi_2$ | 29 | 24.2 ± 2.5 | $\psi(3770) \rightarrow \gamma \chi_2$ | 5.6 | ≤ 330 |

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