The Localization of the Single Pulse in VLBI Observation

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Abstract

In our previous work, we proposed a cross spectrum–based method to extract single-pulse signals from RFI-contaminated data, which is originated from geodetic VLBI postprocessing. This method fully utilizes fringe-phase information of the cross spectrum and hence maximizes the signal power. However, the localization was not discussed in that work. As the continuation of that work, in this paper, we further study how to localize single pulses using an astrometric solving method. Assuming that the burst is a point source, we derive the burst position by solving a set of linear equations given the relation between the residual delay and the offset to a priori position. We find that the single-pulse localization results given by both astrometric solving and radio imaging are consistent within the 3σ level. Therefore, we claim that it is possible to derive the position of a single pulse with reasonable precision based on only three or even two baselines with 4 ms integration. The combination of cross spectrum–based detection and the localization proposed in this work then provide a thorough solution for searching for single pulses in VLBI observation. According to our calculation, our pipeline gives comparable accuracy to the radio imaging pipeline. Moreover, the computational cost of our pipeline is much smaller, which makes it more practical for a fast radio burst (FRB) search in regular VLBI observation. The pipeline is now publicly available and named the VLBI Observation for FRB Localization Keen Searcher (VOLKS).

Key words: methods: data analysis – pulsars: general – radio continuum: general – techniques: interferometric

1. Introduction

The search for fast radio bursts (FRBs; Lorimer et al. 2007) is now becoming an important topic in time-domain astronomy. Their high-precision localization is crucial in finding the possible background counterpart and finally explaining the burst mechanism. Since the first discovery of FRBs, only about 65 FRBs have been found (Petroff et al. 2016). In FRB searching, large single-dish telescopes first play an important role (Lorimer et al. 2007; Thornton et al. 2013; Ravi et al. 2015; Petroff et al. 2017; Bhandari et al. 2018). However, the resolution of a single-dish telescope is of arcminute level, which is too large to isolate the transients from background sources or associate them with possible counterparts (Chatterjee et al. 2017). In this case, interferometers with higher angular resolution provide another choice. To fully explore the performance of different types of interferometric instruments, several single-pulse search methods are proposed (Law et al. 2011), including beam forming, radio imaging, etc.

Aperture arrays such as UTMOST (Caleb et al. 2016) are dedicated to FRB searches, and ASKAP and CHIME (Ng et al. 2017; the CHIME/FRB Collaboration et al. 2018) take FRB searches as one of their main scientific goals. These arrays take the beam-forming approach,5 in which radio signals from multiple receivers are aligned in both time and frequency domain and then combined together to form multiple data beams to cover a large searching area. After that, these beams are searched for single pulses using similar methods as for the data from large single-dish telescopes. Until now, UTMOST has successfully detected four FRB events (Caleb et al. 2017; Farah et al. 2018). CHIME reported detections of 13 FRBs at radio frequencies as low as 400 MHz, including one repeating burst (the CHIME/FRB Collaboration et al. 2019a, 2019b). Shannon et al. (2018) reported the discovery of 23 FRBs in a fly’s-eye survey with ASKAP, which almost doubles the number of known events. Based on this sample, Macquart et al. (2019) derived a mean spectral index of −1.6.

As the astronomical technique with the highest angular resolution (Thompson et al. 2001), very long baseline interferometry (VLBI) is expected to provide high-precision FRB localization. However, due to the relatively small field of view (FoV), the search is usually carried out as a commensal task in regular VLBI observations, e.g., V-FASTR (Thompson et al. 2011; Wayth et al. 2011) for VLBA and LOCATe for EVN (Paragi 2016). In these projects, the station auto spectrum is first dispersed and searched for single pulses, then candidates from multiple stations are cross-matched. This method is fast and easy to implement. However, it does not utilize the cross-spectrum fringe-phase information. According to our study in Liu et al. (2018b), this potentially reduces its single-pulse detection capability with RFI-contaminated data. In this case, cross spectrum–based search methods are more suitable for VLBI observation. Among these methods, the most successful one is radio imaging, which detects single pulses in fast dumped images (Law et al. 2015).

In the rarest occasion of repeating FRBs, localization can, in principle, be measured accurately. The position of the first discovered repeating burst, FRB 121102 (Spitler et al. 2016), is measured (Chatterjee et al. 2017; Marcote et al. 2017) with VLBI observation, and even the possible counterpart is identified in other bands (Bassa et al. 2017; Scholz et al. 2017; Tendulkar et al. 2017). In that work, the radio imaging

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5 ASKAP antennas are equipped with a phased array feed (PAF): the whole focal plane is sampled, and beams are formed computationally by combining signals from multiple PAF elements with complex coefficients (weights; Johnston et al. 2008). In this way, ASKAP obtains good angular resolution and increases the FoV simultaneously.
search pipeline “realfast” plays a key role in detecting and localizing the burst in VLA data (Chatterjee et al. 2017). Other nonimaging methods also exist, e.g., the $uv$-fitting method and the bispectrum method, which have been proposed and tested with the “PoCo” data (Law et al. 2011; Law & Bower 2012). However, these methods are not widely deployed in current search projects.

Although “realfast” has achieved great success for its detection and localization of the repeating bursts of FRB 121102, our calculation in Section 4 suggests that when it comes to VLBI observation with a much longer baseline and therefore a much higher angular resolution, to cover a similar searching area as the VLA antenna, the map size (pixel number along one side) becomes several orders of magnitude larger. The corresponding computational cost increases respectively, which makes it difficult to carry out an FRB search in a real VLBI observation.

In Liu et al. (2018a), we propose a geodetic VLBI-based single-pulse detection method. It takes the idea of geodetic VLBI fringe fitting that utilizes cross-spectrum fringe-phase information to maximize the signal power. Compared with the auto spectrum–based method, it is able to extract single pulses from highly RFI-contaminated data (Liu et al. 2018b). As a continuation of that work, and to construct the whole single-pulse search and localization pipeline, in this paper, we further propose to localize single pulses in an astrometric solving approach: by assuming the burst is a point source, we may derive its accurate position by solving a set of linear equations based on the relation between the residual delay and the correction to an a priori position. Compared with radio imaging, astrometric solving is much faster and gives comparable accuracy.

Our final goal for developing a complete single-pulse search and localization pipeline is to carry out an FRB search in regular VLBI observations. According to our calculation (Liu et al. 2018a), by assuming a reasonable event rate (Keane & Petroff 2015) and appropriate spectral index and fluence index (Caleb et al. 2017), the FRB detection rate for the VLBI2010 Global Observation System (VGOS; Petrenko et al. 2013) antenna is 0.0076 events per sky per day. By assuming a 50% observation efficiency, the expected detection rate is 1.387 events per year. Moreover, the number could double if we take a higher FRB event rate (Champion et al. 2016). Therefore, searching for FRB and other transient events in regular VLBI observations is technically feasible and scientifically promising.

For the real deployment of the FRB search pipeline, we have to admit that there are still some problems that must be solved. One question that is often asked is whether the burst is detectable when it appears far from the phase center. Besides that, the performance of a dispersion measure (DM) search with our cross spectrum–based method has not been tested, since the DM of the current pulsar data set is too low. We present detailed discussions of these two issues in Sections 5.2 and 5.4, respectively.

In this paper, we introduce the astrometric solving–based single-pulse localization method, compare its localization accuracy with radio imaging, and analyze the computational cost of our geodetic VLBI-based pipeline and radio imaging pipeline. This paper is organized as follows. In Section 2, we introduce the astrometric solving–based localization method. In Section 3, we present the single-pulse localization result with a VLBI pulsar data set. In Section 4, we analyze the computational cost of our pipeline and radio imaging. In Section 5, we discuss unresolved issues in the current pipeline. In Section 6, we give our conclusion.

2. The Astrometric Solving Method for Single-pulse Localization

In this section, we introduce the astrometric solving method for single-pulse localization. The localization part, together with the search part in Liu et al. (2018a), builds up the complete pipeline for single-pulse search and localization in a geodetic VLBI approach.

By assuming the burst is a point source, we are able to derive the single-pulse positions by solving a set of linear equations based on the relation between the residual delay and the correction to an a priori position. The idea is taken directly from solutions for Earth orientation parameters (usually including precession and nutation, polar motion, and universal time), baseline and source position vectors, etc. in standard geodetic and astrometric VLBI measurements (Takahashi et al. 2000; Thompson et al. 2001). Actually, there is already work that tries to obtain the high-precision pulsar position in a geodetic VLBI approach: Sekido et al. (1999) obtained the position of B0329+54 and further derived its proper motion by first deriving the group delay with the Japanese bandwidth synthesis software “KOMB” (Takahashi et al. 1991), then carrying out analysis with CALC/SOLVE. However, their approach is still quite different from our work: they treat the pulsar as a normal radio source. Since B0329+54 is strong, they do not even use pulse gating. The total duration of the pulsar observation that is used to derive the position is as long as several hours. In contrast, in our work, we try to resolve and localize every individual single pulse with durations as short as 4 ms. Actually, we are the first to try to localize the single pulse in an astrometric solving approach.

Concerning this work, since the pulsar is observed in phase reference mode, most of the geodetic and atmospheric effects can be removed by phase reference calibration. What we need to estimate is just the correction to an a priori position for every individual single pulse. Assuming the single-pulse cross spectrum has been phase reference–calibrated, for each baseline, the linear relation between the residual delay and the correction to an a priori position can be expressed as

$$\tau = \frac{\partial \tau}{\partial \alpha} \Delta \alpha + \frac{\partial \tau}{\partial \delta} \Delta \delta,$$

where $\tau$ is the residual delay of this baseline, $\frac{\partial \tau}{\partial \alpha}$ and $\frac{\partial \tau}{\partial \delta}$ are partial derivatives of the delay by R.A. and decl., and $\Delta \alpha$ and $\Delta \delta$ are corrections to an a priori position. The residual delay can be derived by fitting the fringe phase. Two partial derivatives of delay by the source position are given by the VLBI delay model and can be obtained from popular model calculation programs, e.g., CALC. The above equations are solvable with two or more baselines. The least-squares solutions that take the uncertainties of residual delay into account are described in the Appendix.

Figure 1 demonstrates the whole geodetic VLBI-based single-pulse search pipeline. In this pipeline, the search and localization are two independent steps. In the first step, single-pulse candidates are extracted by cross-spectrum fringe fitting. A cross spectrum that takes the single-pulse information is
extracted for further localization. In the second step, the single-pulse cross spectrum is calibrated with a phase reference source and then fitted to derive the residual delay.

The single-pulse search and localization scheme described in Figure 1 has been implemented as the VLBI Observation for single-pulse Localization Keen Searcher (VOLKS) pipeline. At present, this pipeline supports dedispersion, fringe fitting of the fast dump cross spectrum, filtering of single-pulse candidates from multiple resampling times (window length), and multiple baselines cross matching. For localization, it supports both radio imaging and astrometric solving methods. The pipeline is still being improved so as to support more features, e.g., cross spectrum–based DM search, GPU acceleration of fringe fitting, etc. We have made this pipeline publicly available (Liu 2018; Codebase: https://github.com/liulei/volks).

3. Localization Result

We carry out single-pulse search and localization in a VLBI pulsar data set that was used in Liu et al. (2018a). All works except for the AIPS calibration part are carried out with the VOLKS pipeline described in Section 2. We present the localization results using both radio imaging and astrometric solving methods and compare their accuracies.

3.1. Data Set

Data are taken from the Chinese VLBI Network (CVN) VLBI pulsar observation psrf02. The 96 MHz bandwidth data in the S band are recorded in six frequency channels, two bits sampling. Three CVN stations, Sh, Km, and Ur, participate the observation. The details of the observation are presented in Liu et al. (2018a). Among the 293 scans in the 24 hr observation, single pulses of PSR J0332+5434 in scans 69, 71, and 73 are extracted for localization. In scans 68, 70, 72, and 74, J0347+5557 is used as a phase reference source; 3C 273 in scan 293 is used for PCAL, clock, and channel delay calibration.

Since CALC is easy to integrate into the localization pipeline, in this work, we use CALC 9.1 for partial derivative, $uv$, and delay model calculation. In order to keep the consistency, we reprocess the raw data using the DiFX correlator and carry out a single-pulse search with exactly the same procedure described in Liu et al. (2018a).

Figure 2 presents the single-pulse detection result using the DiFX correlator (Deller et al. 2007, 2011). We expect it to show an identical result to Figure 5 in Liu et al. (2018a) using the CVN software correlator (Zheng et al. 2010). However, at first glance, they are not consistent with each other. According to our analysis, the main reason is that the two correlators behave differently when the signal-to-noise ratio (S/N) is low. In this case, when the normalized power is less than 5, the results are different. Since the two correlators use totally different delay models, it is not surprising to have such a discrepancy. Besides that, the implementations of the algorithm in the two correlators are different. The good thing is, when it comes to strong signals, the results given by the two correlators are quite consistent: singles pulse at 17.5 s of scan 71 and at 49.1 and 113.4 s of scan 73 are detected on all three baselines by both correlators. However, the low sensitivity of the Sh–Ur baseline still makes the result different: based on DiFX output, two single pulses at 116.2 and 116.9 s of scan 71 are detected on all three baselines, while the single pulse at 21.0 s of scan 69 is missed on the Sh–Ur baseline, although it is detected on all three baselines in Liu et al. (2018a). In summary, 17 single pulses are detected on at least two baselines. According to their pulsar phases, we may know that the one enclosed by a dotted rectangle is a false detection.

We extract the visibility records that contain single-pulse information from the original visibility files and convert them, together with the visibilities of the phase reference source (J0347+5557) and calibration source (3C 273), to FITS-IDI format for further calibration and localization.

3.2. Localization

We use AIPS (31DEC18) for calibration. According to the standard recipe for phase reference observations, the whole process consists of three steps. (a) Calibrate the delay and phase in every individual frequency channel (IF) for single pulses and the phase reference source (J0347+5557) using the solution derived from the calibration source (3C 273). (b) Derive solutions for the phase reference source, including delays (combining all IFs), phases, and delay rates, then interpolate them to the pulsar scans. (c) Calibrate every single pulse using the solutions interpolated from the phase reference source and output them with FITS-IDI formats for further localization. One thing we want to point out is, the above calibration procedure is only intended for our testing pulsar data set. In a real FRB search, the burst and target source appear in the same FoV. The corresponding calibration is somewhat similar to phase reference observation, but more
Figure 2. Single-pulse detection result of VLBI pulsar data set psrf02. Single pulses detected on multiple baselines are enclosed by rectangular boxes. Thin and thick solid rectangles correspond to those detected on two and three baselines, respectively. The one enclosed by a dotted rectangle is a false detection, according to its pulsar phase. Note that the actual width of the single pulse is much narrower than the width of the rectangular box. In total, 17 single pulses (including one false detection) are detected on two or three baselines. The detailed parameters of these single pulses, together with their localization results, are presented in Table 1.

| No. | Scan | Time (s) | Baseline | Solving | Imaging |
|-----|------|---------|----------|---------|---------|
|     |      |         | Km–Sh    | Δα*     | Δδ      | Δα*     | Δδ      | S/N     |
| 1   | 69   | 14.565  | ✓ ✓ ✓    | 421.9 ± 17.9 | −313.3 ± 15.6 | 392.0 | −294.0 | 9.0     |
| 2   | 69   | 21.000  | ✓ ✓ ✓    | 392.6 ± 14.6 | −223.3 ± 13.2 | 358.0 | −248.0 | 9.6     |
| 3   | 69   | 95.306  | ✓ ✓ ✓    | 498.7 ± 21.0 | −203.7 ± 25.5 | 468.0 | −252.0 | 9.1     |
| 4   | 69   | 107.463 | ✓ ✓ ✓    | 481.5 ± 15.6 | −264.3 ± 13.7 | 460.0 | −270.0 | 9.4     |
| 5   | 69   | 136.765 | ✓ ✓ ✓    | 401.0 ± 17.4 | −262.3 ± 16.1 | 392.0 | −256.0 | 8.3     |
| 6   | 71   | 17.547  | ✓ ✓ ✓    | 377.9 ± 10.2 | −327.2 ± 13.4 | 350.0 | −306.0 | 11.1    |
| 7   | 71   | 26.122  | ✓ ✓ ✓    | 579.7 ± 18.1 | −187.6 ± 15.9 | 518.0 | −222.0 | 8.9     |
| 8   | 71   | 75.431  | ✓ ✓ ✓    | 444.9 ± 16.0 | −216.3 ± 15.1 | 410.0 | −192.0 | 8.4     |
| 9   | 71   | 86.636  | ✓ ✓ ✓    | 7673.9 ± 16.8 | −2873.4 ± 15.4 | −26.0 | 540.0 | 6.5     |
| 10  | 71   | 116.158 | ✓ ✓ ✓ ✓  | 417.2 ± 11.7 | −225.2 ± 13.8 | 396.0 | −266.0 | 11.8    |
| 11  | 71   | 116.872 | ✓ ✓ ✓    | 389.6 ± 11.7 | −290.3 ± 15.8 | 358.0 | −288.0 | 10.1    |
| 12  | 71   | 131.879 | ✓ ✓ ✓    | 416.5 ± 16.4 | −228.2 ± 15.1 | 382.0 | −234.0 | 9.1     |
| 13  | 71   | 142.605 | ✓ ✓ ✓    | 414.4 ± 17.5 | −301.3 ± 16.3 | 422.0 | −290.0 | 8.8     |
| 14  | 73   | 49.109  | ✓ ✓ ✓ ✓  | 386.2 ± 8.2  | −280.1 ± 9.4  | 388.0 | −284.0 | 10.7    |
| 15  | 73   | 100.557 | ✓ ✓ ✓ ✓  | 447.5 ± 17.3 | −190.4 ± 15.5 | 432.0 | −204.0 | 7.7     |
| 16  | 73   | 112.702 | ✓ ✓ ✓ ✓  | 442.7 ± 15.1 | −245.5 ± 14.6 | 416.0 | −240.0 | 8.6     |
| 17  | 73   | 113.418 | ✓ ✓ ✓ ✓  | 433.0 ± 10.5 | −272.4 ± 14.1 | 408.0 | −258.0 | 9.5     |

Table 1

Parameters of 17 Single Pulses Detected in Scans 69, 71, and 73 of CVN Observation psrf02

Note. All positions are given as offsets to an a priori position. The offset in the R.A. direction is the tangent plane projection: Δα* = Δα cos δ. For radio imaging, we take a pixel size of 2.0 mas × 2.0 mas. The S/N is calculated as the peak flux subtracted by the average flux and then normalized with noise (standard deviation).

simplified. Please refer to Section 5.3 for a detailed explanation.

Our implementation of radio imaging has a similar procedure as in DIFMAP (Shepherd 1997). The main difference is that DIFMAP only deals with data that all frequency points (channels) in one IF are averaged together as one point, which greatly reduces the time consumption of the imaging process at the expense of a small imaging area. However, this is based on the assumption that the target source is close to its a priori position, and therefore no fringe-phase ambiguity exists inside one IF. For a single-pulse search, large ambiguities might still exist after phase calibration if the burst is far from an a priori position. To keep the full fringe-phase ambiguity information, in this work, the frequency points (channels) in one IF are not averaged together before they are gridded in the uv plane.

To calculate the delay model of PSR J0332+5434 for VLBI correlation, we use the a priori position (R.A.: $3^h32^m59^s$, decl.: $54^d34^m37^s$, in J2000.0) given by the ATNF pulsar database8 (Manchester et al. 2005) at the reference epoch MJD 46,473 (1986 February 12). The localization results are presented in Table 1. According to the uncertainties of each single pulse given by astrometric solving, the results derived by both methods are consistent with each other in a 3σ level. Compared with the two-baseline results, three-baseline results usually yield smaller uncertainties and higher S/N. Among all single pulses presented here, No. 14 corresponds to the one with the highest normalized power in Figure 2 and the smallest uncertainties. Note that its S/N is not the highest; according to our investigation, this is due to its large flux density fluctuation in the image plane. Also note No. 9 in the table, which corresponds to the false detection in Figure 2. Clearly, it yields

8 http://www.atnf.csiro.au/research/pulsar/psrcat
In this section, we analyze the computational cost of both radio imaging and the geodetic VLBI-based single-pulse search pipeline and come to the conclusion that the latter one is more suitable for an FRB search in a real observation.

Figure 4 demonstrates the single-pulse search with the radio imaging pipeline. Visibilities from each baseline are first calibrated and then transformed to the image plane to create fast dumped images with multiple resampling times. Single-pulse candidates are detected in these fast dumped images according to a given threshold. To use fast Fourier transform (FFT) to speed up the transformation process, visibilities are gridded in the \( uv \) plane (Thompson et al. 2001). In the radio imaging search pipeline, the search and localization steps are coupled together: single pulses are detected and localized directly in the fast dumped images. However, such a scheme is only feasible for VLBI systems with not very long baselines.

Take the configuration of the VLA, for example. The longest baseline is 36 km. In the S band (2.2 GHz), the angular resolution is around 0.78\(''\). The diameter of the VLA antenna is 25 m. As a raw estimation, the corresponding FoV is 2248\(''\) (\(\sim 1.22 \lambda/D\)). In radio imaging, the pixel size usually takes a quarter of the angular resolution. To cover 80\% of the FoV (\(\sim \lambda/D\)), the corresponding map size is 5760 \(\times\) 5760, which is reasonable for 2D FFT and the single-pulse detection afterward. However, for a typical VLBI network, e.g., CVN (Zheng 2015), the baseline is as long as 3000 km. By keeping other parameters unchanged, the corresponding map size is 480,000 \(\times\) 480,000, which is two orders of magnitude larger than that of the VLA. Since the computational complexity of the radio imaging pipeline is usually proportional to map size, the computational cost is two orders of magnitude higher. Obviously, this is a huge challenge for the actual operation.

It is possible to compare the computational cost of both pipelines in a qualitative way: by investigating Figures 1 and 4, one may find that both pipelines involve three loops. From outside to inside, they are: DM trials for dedispersion, multiple resampling times, and series of resampled cross spectrum. By selecting the same resampling time, the two pipelines require the same number of iterations. For radio imaging, the innermost operation involves \( uv \) gridding, 2D FFT, and single-pulse selection. The first term is negligible, as it is proportional to the number of samples. In contrast, in the geodetic VLBI pipeline, the most time-consuming part is fringe fitting, which involves 2D FFT to search for single-band delay (SBD) and multiband delay (MBD). We may demonstrate that the size of this 2D array is much smaller than that of radio imaging; e.g., for a typical configuration of the pulsar observation in the above section, Table 3 presents the computational complexity of both pipelines. For the geodetic VLBI pipeline, according to Equation (12) of Liu et al. (2018a), the minimum FFT size for an SBD search is 1746, and the number of frequency channels for an MBD search is six. By rounding them to the power of 2 and taking 4 times extrapolation, the corresponding sizes are 8192 and 32, respectively. One may find that for a baseline length of 3000 km in CVN, the actual computational cost of radio imaging is much higher than that of the geodetic VLBI pipeline. It is not difficult to come to the conclusion that the geodetic VLBI pipeline is more suitable for a real-time FRB search than the radio imaging pipeline.

Figure 3. Localization of every individual single pulse (SP) and the average result. For comparison, two reference positions are also presented. They are calculated by evolving the positions at the reference epoch to the date of the pulsar observation according to given proper motions. The error bars of the reference positions are calculated by combining the uncertainties of the positions and proper motions: \( \sigma = \sqrt{\sigma_{\text{pos}}^2 + (\sigma_{\text{pm}} \Delta t)^2} \). Here \( \sigma_{\text{pos}} \) and \( \sigma_{\text{pm}} \) are the uncertainties of the positions and proper motions in the R.A. and decl. directions given by references, and \( \Delta t \) is the time between the reference epoch and date of pulsar observation. Ellipses are drawn according to the shape and directions given by references, and the positions given by the two solving methods are inconsistent with each other.

In Figure 3, we plot the localization result of every individual single pulse and the average positions given by two methods. To evaluate the absolute localization precision of the whole data processing pipeline, we also present two reference positions (Sekido et al. 1999; Brisken et al. 2002; see Table 2 for details). The two positions are derived by evolving the reference positions at the reference epochs to the pulsar observation date according to the reference proper motions. As demonstrated in the figure, the reference positions are roughly consistent with the single-pulse localization result. All single pulses, except one derived by the astrometric solving method, distribute in a 200 mas \(\times\) 200 mas area. The scatters of the single-pulse locations derived by the radio imaging and astrometric solving methods are 53.2 and 65.1 mas, respectively, which can be regarded as the absolute localization precision of this work.

The scatter is estimated by combining the standard deviations in the R.A. and decl. directions: \( \sigma = \sqrt{\sigma_{\text{pos}}^2 + \sigma_{\text{pm}}^2} \).

\[ \Delta \alpha^* = \Delta \alpha \cos \delta. \]
Table 2
Reference Positions (J2000.0) and Proper Motions of PSR J0332+5434 at Their Respect Reference Epochs

| Reference | \( \alpha \) | \( \delta \) | \( \mu_\alpha^* \) (mas yr\(^{-1}\)) | \( \mu_\delta \) (mas yr\(^{-1}\)) | Reference Epoch |
|-----------|-------------|-------------|-------------------------------|-------------------------------|-----------------|
| Sekido et al. (1999) | 03\(^{\mathrm{h}}\)32\(^{\mathrm{m}}\)59\(^{\mathrm{s}}\)3760 ± 0.0010 | 54\(^{\circ}\)34\('\)50\(''\)5040 ± 0.0070 | 17.30 ± 0.80 | −11.50 ± 0.60 | 1995.0 |
| Brisken et al. (2002) | 03\(^{\mathrm{h}}\)32\(^{\mathrm{m}}\)59\(^{\mathrm{s}}\)3862 ± 0.0017 | 54\(^{\circ}\)34\('\)50\(''\)5051 ± 0.0150 | 17.00 ± 0.27 | −9.48 ± 0.37 | 2000.0 |

Note. The proper motion in the R.A. direction is the tangent plane projection: \( \mu_\alpha^* = \mu_\alpha \cos \delta \).

5. Discussion

5.1. Subtraction of Constant Sources

A single-pulse search is usually carried out as a commensal task in regular VLBI observations. In this case, how to remove the influence of the target source and other constant sources is a problem that must be solved. The “realfast” pipeline deals with this problem by subtracting the mean visibility in time on timescales less than the VLA fringe rate. Similar treatment is suitable for our geodetic VLBI-based pipeline, too. Besides that, we propose another scheme that is specially designed for the fringe fitting pipeline: to carry out a single-pulse search, the clock is well adjusted (fringe rate less than \( 10^{-3} \) Hz), and the MBD and SBD for the target source do not change too much in the whole scan. We may skip the target source and the surrounding area in the MBD and SBD search matrix. However, this scheme can only be verified with data in which fast transients present together with a constant source, which is not available at present. In this case, a mean visibilities subtraction scheme might be more reasonable.

5.2. Large Search Area

One might doubt that it is possible to detect single pulses efficiently in the whole FoV with our method. In Liu et al. (2018a), we pointed out that the fringe fitting process is somewhat similar to that of coherent beam forming, but without the computational expense to form a great number of beams to cover the whole FoV of telescopes. For localization, traditionally, we only carry out narrow-field imaging.\(^{10}\) The corresponding searching area is much smaller than the FoV. The main reason is that the delay model is (usually) calculated for the center of the FoV but applied to the whole FoV. The residual delay rate is large at the edge of the FoV. The signal degrades quickly as the integration becomes long. In regular VLBI correlation, the integration time is as long as 1 s. In contrast, in our single-pulse detection method, the maximum integration time is no more than 32 ms and usually as short as 4 ms. This short integration time makes it possible to investigate the whole FoV with only one image. Actually, this is somewhat similar to the implementation of the multiple-phase center in the modern VLBI correlator (Deller et al. 2011; Keimpema et al. 2015). For instance, in SFXC, for each subintegration period (25 ms), a phase shift is performed for each phase center, so as to compensate for the phase change due to the large residual delay rate in that position. We know that although the signal will not degrade very much within such a short time, the corresponding S/N is low. In this work, we have demonstrated that it is still possible to detect signals for such a short integration time.

For the localization of a single pulse in the whole FoV, one of the drawbacks of radio imaging is that the computational cost increases significantly when the imaging area becomes large in long baseline observation. In contrast, in the geodetic VLBI search scheme, this is not a problem. We have to admit that when the single pulse is far from phase center, e.g., close to \( \frac{1}{2} \theta_{\text{FWHM}} \), the performance of the search pipeline is still not clear. Possible problems include the decrease of detection sensitivity, the increase of localization uncertainty, etc. Although these problems also exist in the radio imaging–based search scheme, we have not seen their related descriptions and solutions. Therefore, we propose to carry out further VLBI observation to test the pipeline, for instance, by placing the pulsar in the FoV with different offsets to the phase center, such that we may plot the power and localization precision of the detected single pulses as a function of offset to the FoV center.

5.3. Localization in Geodetic VLBI Observation

The VLBI observation that provides the pulsar data set used in this work is carried out in phase reference mode, which

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\(^{10}\) By assuming the burst is a point source, mathematically, the radio imaging and astrometric solving methods are equivalent.
makes it possible to calibrate the extracted single pulses with a phase reference source. However, geodetic VLBI observation takes a totally different approach: to cover as large a sky area as possible, sources distribute evenly in the sky. In this case, it is still possible to localize the burst: the target source itself is a perfect phase reference source. Since the burst and the target source always appear in the same FoV, it is not even necessary to extrapolate the fringe fitting solution to the burst time. We may just derive MBD, SBD, delay rate, and residual phase for the target source in the scan and then calibrate the visibilities with these quantities. A single-pulse search is carried out with calibrated visibilities. Once a single pulse is detected, the derived delay can be used directly for localization with the astrometric method. No further calibration with the phase reference source is needed.

5.4. Dispersion Measure Search

One thing we want to point out is that we do not carry out a dispersion measure search in this work. The DM value provided by the ATNF pulsar database is used for dedispersion. The main reason is that the DM value of PSR J0332 provided by the ATNF pulsar database is used for dedispersion. Dispersion measure search in this work. The DM value derived delay can be used directly for localization with the calibrated visibilities. Once a single pulse is detected, the derived delay can be used directly for localization with the astrometric method. No further calibration with the phase reference source is needed.

6. Conclusions

In this paper, we present the astrometric solving–based single-pulse localization method. By applying this method to a VLBI pulsar observation data set, we demonstrate that the localization results for each single pulse derived by both radio imaging and astrometric solving are consistent with each other in a 3σ level. Most of the single pulses, together with the reference positions, distribute in a 200 mas × 200 mas area. The scatters of localization results using both methods are less than 70 mas, which can be regarded as the absolute localization precision. Our work proves that it is possible to derive single-pulse positions with reasonable precision based on just three or even two baselines and 4 ms integration in a VLBI observation. The localization method, together with the single-pulse search method in Liu et al. (2018a), builds up the complete geodetic VLBI-based single-pulse search and localization pipeline. We further demonstrate that the computational cost of the radio imaging pipeline is much higher than that of the geodetic VLBI-based pipeline. Therefore, for a cross spectrum–based FRB search in a VLBI observation, the geodetic VLBI pipeline might be a better choice. We name our pipeline VOLKS and have made it publicly available. We hope this will be helpful for radio transient studies.

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Appendix
Least-squares Solutions

The derivation of the offset (Δα, Δδ) to an a priori position is divided into two steps.

(a) Fitting residual delay. For every single pulse, the delay τ for each baseline is derived by fitting the fringe phase φ after phase reference calibration as a function of frequency fk:

$$\phi_k = 2\pi f_k \tau + \phi_0.$$  \hspace{1cm} (2)

The fit of the above linear equation by using the amplitude of each frequency point f_k as weight is available in most mathematical libraries. After fitting, we obtain the delay τ_i and the corresponding uncertainties σ_i for baseline i. The relation between σ_i and the scatter of the fringe phase is explained in Takahashi et al. (2000). Note that when the source is far from an a priori position, fringe-phase ambiguity exists even after calibration. In this case, we have to compensate for an initial delay value to remove ambiguity before fitting.\footnote{When the scatter of the fringe phase is large, simply unwarping the fringe phase and then doing linear fitting might lead to an incorrect result.}

(b) Derive the position offset. This is to solve the linear equation

$$y = Ax.$$  \hspace{1cm} (3)

Table 3
Computational Complexity of Two Single-pulse Search Pipelines

| Radio Imaging | Geodetic VLBI |
|---------------|---------------|
| N: 480,000    | N_1: 8192, N_2: 32 |
| 2D FFT       | 2D FFT (per baseline) |
| Finding peak | N_1 log_2(N_1) × N_2 + N_1 N_2 log_2(N_2) × N_1 |
| Total        | Total (three baselines) |
|              | N_1N_2 log_2(N_1N_2) + 3N_1N_2 |
|              | (5.43 × 10^7) |
The square roots of $\Sigma_{p,11}$ and $\Sigma_{p,22}$ correspond to the uncertainties of $\Delta \alpha$ and $\Delta \delta$, respectively.

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Here $y = (\tau_1, \tau_2, ..., \tau_n)^T$ is the delay vector for $n$ baselines, $x = (\Delta \alpha, \Delta \delta)^T$ is the position offset vector, and $A = (A_1, A_2, ..., A_n)^T$ is the partial derivative matrix $A_i = \left( \frac{\partial \tau_1}{\partial \alpha}, \frac{\partial \tau_1}{\partial \delta}, \frac{\partial \tau_2}{\partial \alpha}, \frac{\partial \tau_2}{\partial \delta}, ..., \frac{\partial \tau_n}{\partial \alpha}, \frac{\partial \tau_n}{\partial \delta} \right)$. The least-squares solution of the above equations is

$$\hat{x} = (A^T WA)^{-1} A^T Wy.$$  

Here $W$ is the weight matrix, $W = \Sigma^{-1}$, and $\Sigma = \text{diag}(\sigma_1^2, \sigma_2^2, ..., \sigma_n^2)$ is the delay error matrix. The estimation parameter error matrix is

$$\Sigma_p = (A^T \Sigma^{-1} A)^{-1}.$$  

The square roots of $\Sigma_{p,11}$ and $\Sigma_{p,22}$ correspond to the uncertainties of $\Delta \alpha$ and $\Delta \delta$, respectively.