Strong decays of newly observed $D_{sJ}$ states in a constituent quark model with effective Lagrangians

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The strong decay properties of the newly observed states $D_{sJ}(3040)$, $D_{sJ}(2860)$, and $D_{sJ}(2710)$ are studied in a constituent quark model with quark-meson effective Lagrangians. We find that the $D_{sJ}(3040)$ could be identified as the low mass physical state $|2P⟩_L (J^P = 1^+)$ from the $D_s(2P_1)-D_s(2P_1)$ mixing. The $D_{sJ}(2710)$ is likely to be the low-mass mixed state $|(SD)⟩_L$ via the $1^3D_1-2^3S_1$ mixing. In our model, the $D_{sJ}(2860)$ cannot be assigned to any single state with a narrow width and compatible partial widths to $DK$ and $D^*K$. Thus, we investigate a two-state scenario as proposed in the literature. In our model, one resonance is likely to be the $1^3D_1 (J^P = 3^-)$, which mainly decays into $DK$. The other resonance seems to be the $|1D_1⟩_H$, i.e. the high-mass state in the $1^3D_1-1^3D_2$ mixing with $J^P = 2^-$, of which the $D^*K$ channel is its key decay mode. We also discuss implications arising from these assignments and give predictions for their partner states such as $|(SD)⟩_H$, $|2P_1⟩_H$, $2^3P_0$ and $2^3P_2$, which could be helpful for the search for these new states in future experiment.

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I. INTRODUCTION

Experimental progress on the study of $D$ and $D_s$ states in the past few years provides a great opportunity for theory development. Recently a new broad resonance $D_{sJ}(3040)$ with a mass of $(3044 \pm 8_{\text{stat}}^{+30}_{-50})_{\text{syst}}$ MeV and a width of $\Gamma = (239 \pm 35_{\text{stat}}^{+46}_{-42})_{\text{syst}}$ MeV is reported in the $D^*K$ channel [1]. Apart from the $D_{sJ}(3040)$ another two states $D_{sJ}(2710)$ and $D_{sJ}(2860)$, which were observed by BABAR and Belle two years ago [2, 3], are also examined. Their branching ratio fractions between $D^*K$ and $DK$ are measured [1],

$$\frac{D_{sJ}(2710)^+ \rightarrow D^*K}{D_{sJ}(2710)^+ \rightarrow DK} = 0.91 \pm 0.13_{\text{stat}} \pm 0.12_{\text{syst}},$$

$$\frac{D_{sJ}(2860)^+ \rightarrow D^*K}{D_{sJ}(2860)^+ \rightarrow DK} = 1.10 \pm 0.15_{\text{stat}} \pm 0.19_{\text{syst}}.$$  

(1)  

(2)

These new observations stimulate great interest in the understanding of their nature and strong coupling properties in theory. Different theoretical approaches for the study of the strong coupling properties of heavy-light mesons can be found in the literature, such as the heavy quark effective field theory approach (HQEFT) [4, 18], QCD sum rules [16, 20, 30], $3P_0$ model [21, 25], and chiral quark model [26].

In this work, we present an analysis of these $D_s$ states in a constituent quark model with effective Lagrangians for the quark-meson couplings, and try to clarify the following issues: (i) To gain information about the structure of the newly observed state $D_{sJ}(3040)$ according to its strong decay properties. (ii) With the new data for the $D_{sJ}(2710)$ and $D_{sJ}(2860)$, we reanalyze their strong decays and examine their structures again. The quantum numbers of these two states remain controversial. The $D_{sJ}(2710)$ is identified as a state of $J^P = 1^-$ in $B$ decays [3], while it is explained by various models as the $2^3S_1$, $1^3D_1$ admixtures of $2^3S_1-1^3D_1$, molecular structure, or tetraquark state $16, 21, 24, 25, 27$. There are also a lot of solutions proposed for the $D_{sJ}(2860)$. The assignments of $1^3D_3$ or $2^3P_0$ have been discussed in Refs. [21, 24, 25, 28, 31]. A recent comment by Ref. [32] suggests a two-state structure for the $D_{sJ}(2860)$ in order to understand the controversial aspects arising from its strong decays. (iii) The quark-model assignment of these states will result in implications of their partner multiplets. We discuss some of those relevant states, for which the experimental observations would be able to clarify some of those theoretical and experimental issues.
By treating the light mesons (pseudoscalar and vector mesons) as effective fields, we introduce constituent-quark-meson couplings to describe the charmed meson strong decays into a charmed meson plus a light pseudoscalar or vector meson in the final state. The quark-pseudoscalar-meson coupling is given by the chiral quark model at the leading order as proposed by Manohar and Georgi [33]. Its application to pseudoscalar meson photoproduction in the quark model turns out to be promising and many low-energy phenomena can be highlighted in such a framework [34–38]. In particular, the axial current conservation allows one to extract the axial coupling in terms of the meson decay constant and a form factor arising from the microscopic quark model wavefunctions [39]. With an effective quark-vector-meson coupling, one can also extract the vector couplings in a similar way [39–42].

A natural extension of this picture is to apply this effective Lagrangian approach to heavy-light meson strong decays involving light pseudoscalar or vector mesons, which would be a good place to examine the validity of the light axial and vector fields in such a transition. On the one hand, the quark-meson coupling is the same as that defined in meson photoproduction which is proportional to the meson decay constant. On the other hand, the heavy-light meson in the initial and final state would provide information about the coupling form factor and can be calculated in the quark model framework. Thus, one can study the heavy-light meson strong decays by combining dynamical information from meson photoproduction off nucleons.

The paper is organized as follows. In Sec. II a brief review of the quark-meson effective Lagrangian approach is given. The numerical results are presented and discussed in Sec. III. Finally, a summary is given in Sec. IV.

II. FRAMEWORK

In Fig. 1, we illustrate the similarity of the quark-meson coupling in meson-baryon and light-meson production in heavy-light meson strong decays. It should be pointed out that since the flavor symmetries beyond the SU(3) are badly broken, the contributions from transitions of treating the final-state heavy-light meson as an effective field are strongly suppressed. We thus can neglect those contributions safely in our approach. An early study of the charmed meson strong decays can be found in Ref. [31].

In the chiral quark model [33], the low energy quark-pseudoscalar-meson interactions in the SU(3) flavor basis are described by the effective Lagrangian [35–38, 43]

$$\mathcal{L}_{P qq} = \sum_j \frac{1}{f_m} \bar{\psi}_j \gamma_\mu \gamma_5 \psi_j \partial^\mu \phi_m,$$  \hspace{1cm} (3)

where $\psi_j$ represents the $j$-th quark field in the hadron, and $\phi_m$ is the pseudoscalar meson field.

The effective Lagrangian for quark-vector-meson interactions in the SU(3) flavor basis is [40–42]

$$\mathcal{L}_{V qq} = \sum_j \bar{\psi}_j (a \gamma_\mu + i \frac{b}{2m_j} \sigma_{\mu\nu} q') V^\mu \psi_j,$$  \hspace{1cm} (4)
where $V^\mu$ represents the vector meson field with four-vector moment $q$. Parameters $a$ and $b$ denote the vector and tensor coupling strength, respectively.

As follows, we provide the quark-pseudoscalar and quark-vector-meson coupling operators in a non-relativistic form. Considering light meson emission in a heavy-light meson strong decays, the effective quark-pseudoscalar-meson coupling operator in the center-of-mass (c.m.) system of the initial meson is

$$H_m = \sum_j \left[ -\left(1 + \frac{\omega_m}{E_j + M_f}\right) \sigma_j \cdot q + \frac{\omega_m}{2\mu_q} \sigma_j \cdot p_j \right] I_j \varphi_m.$$  

In a case that a light vector meson is emitted, the transition operators for producing a transversely or longitudinally polarized vector meson are as follows

$$H_m^T = \sum_j \left\{ \frac{b'}{2m_q} \sigma_j \cdot \left( q \times c \right) + \frac{a}{2\mu_q} p_j \cdot c \right\} I_j \varphi_m,$$  

and

$$H_m^L = \sum_j \frac{aM_q}{|q|} I_j \varphi_m.$$  

In the above three equations, $q$ and $\omega_m$ are the three-vector momentum and energy of the final-state light meson, respectively. $p_j$ is the internal momentum operator of the $j$-th quark in the heavy-light meson rest frame. $\sigma_j$ is the spin operator on the $j$-th quark of the heavy-light system and $\mu_q$ is a reduced mass given by $1/\mu_q = 1/m_j + 1/m'_f$ with $m_j$ and $m'_f$ for the masses of the $j$-th quark in the initial and final mesons, respectively. Here, the $j$-th quark is referred to the active quark involved at the quark-meson coupling vertex. $M_m$ is the mass of the emitted vector meson. The plane wave part of the emitted light meson is $\varphi_m = e^{-i\hat{q} \cdot \hat{r}_f}$, and $I_j$ is the flavor operator defined for the transitions in the SU(3) flavor space. Parameters $a$ and $b$ are the vector and tensor coupling strengths of the quark-vector-meson couplings, respectively. Studies of vector meson photoproduction suggest that $a = g_{\omega qq} = g_{\rho qq} \simeq -3$ and $b' = b - a \simeq 5$. Because of vector current conservation, one has $a = g_{\rho NN} = g_{\omega NN}/3$.

The heavy-light meson wavefunctions have been given in Ref. [31], and some of the decay amplitudes have also been deduced there. In the charmed meson decays, the SU(4) flavor symmetry is broken. Thus, the charm quark is treated as a spectator and the transition amplitude is proportional to the final-state light meson decay constant associated with a form factor arising from the convolution of the initial and final-state charmed meson wavefunctions.

In the calculation, the standard quark model parameters are adopted. Namely, we set $m_u = m_d = 330$ MeV, $m_s = 450$ MeV, and $m_c = 1700$ MeV for the constituent quark masses. The harmonic oscillator parameter $\beta$ is usually adopted in the range of (0.4–0.5) GeV, in this work we take it as $\beta = 0.45$ GeV. The decay constants $f_K = f_\eta = 160$ MeV. As shown in Refs. [31, 44], the flavor symmetry breaking will lead to corrections to the quark-pseudoscalar-meson coupling vertex, for which an additional global parameter $\delta$ is introduced. Here, we fix its value the same as that in Refs. [31, 44], i.e. $\delta = 0.557$. For the quark-vector-meson coupling strength which still suffers relatively large uncertainties, we adopt the values extracted from vector meson photoproduction as mentioned earlier, i.e. $a \simeq -3$ and $b' \simeq 5$. The masses of the mesons used in the calculations are adopted from the PDG.

Justification of the non-relativistic formulation is not obvious for the light quark sector in the heavy-light meson transitions. This is similar to the case of a non-relativistic quark model for baryons, where the results would rely on the experimental data to tell how far they deviate from reality. Treating the light meson as a chiral field somehow assumes that the light meson is produced at short distance, and the spectators (i.e. the two spectator quarks inside a baryon or the heavy quark in the heavy-light meson transitions) do not respond to the internal structure of the light meson. Instead, the propagation of the light quark pair would feel the hadronic environment from the convolution of initial and final-state heavy-light mesons. Such an implicated assumption means that only the processes with relatively small momentum transfers between the light quarks inside the light meson would dominantly contribute to the transition matrix element. This empirically supports the validity of the non-relativistic formulation as a leading order approximation.
### III. RESULTS AND DISCUSSIONS

#### A. $D_{sJ}(3040)$

The $D_{sJ}(3040)$ is observed in the $D^*K$ mode, while there is no sign of $D_{sJ}(3040) \to DK$ in experiment [1]. This allows its quantum number to be $J^P = 0^-, 1^+,$ etc. The $J^P = 0^-$ state $2^1S_0$ seems not a good candidate since its predicted mass, $\sim 2.7 \text{ GeV}$ [21, 26, 48, 49], is much less than 3.04 GeV. The predicted masses of $J^P = 1^+$ are close to 3.04 GeV. We hence discuss these two possibilities for the $D_{sJ}(3040)$ in this work.

First, we considered it as the $J^P = 1^+$ states $2^1P_1$ and $2^3P_1$. These two states also can decay into $D^*K, DK^*, D_0(2400)K, D_1(2430)K, D_2(2460)K, D_s(2317)\eta, D_s(2460)\eta$. With a mass of 3.04 GeV, we calculate their decay widths, which are listed in Tab. I. From the table, it is found that the decay width of $2^1P_1$ and $2^3P_1$ are $\sim 115 \text{ MeV}$ and $\sim 93 \text{ MeV}$, respectively, which are too small to compare with the data, although the decay mode, dominated by the $D^*K$, is consistent with the observation [1]. Thus, the $D_{sJ}(3040)$ may not be considered as pure $2^1P_1$ or $2^3P_1$ state.

![Figure 2](image_url)

**FIG. 2:** (Color online) The partial decay widths and total width of $|2P_1\rangle_L$ with a mass of 3040 MeV as functions of mixing angle $\phi$. The data are from BABAR [1].

Since the heavy-light mesons are not charge conjugation eigenstates, state mixing between spin $S = 0$ and $S = 1$ states with the same $J^P$ can occur via the spin-orbit interactions [22, 50, 51]. The physical states with $J^P = 1^-$ can then be described as

$$|2P_1\rangle_L = + \cos(\phi)|2^1P_1\rangle + \sin(\phi)|2^3P_1\rangle, \quad (8)$$

$$|2P_1\rangle_H = - \sin(\phi)|2^1P_1\rangle + \cos(\phi)|2^3P_1\rangle, \quad (9)$$

where the subscripts $L$ and $H$ stand for the low mass and high mass of the physical states after the mixing.

Usually, the low mass state has a broad width while the high mass state has a narrow width. We set the mass of $|2P_1\rangle_L$ with 3.04 GeV, and plot its decay width as a function of the mixing angle $\phi$, which is shown in Fig. 2. It shows that when the mixing angle is in the range $\phi \simeq (-40 \pm 12)^\circ$, the total decay width, $\Gamma = (162 \sim 170) \text{ MeV}$, is in the range of the experimental data (close to the lower limit of the data) [1]. The mixing angle predicted here is consistent with the result $\phi \simeq -55^\circ$ in the heavy quark limit [22, 50, 51]. The $D^*K$ governs the decays of $|2P_1\rangle_L$, while the $DK$ channel is forbidden. This is also in agreement with the observations. These results suggest that the $D_{sJ}(3040)$ favors the $|2P_1\rangle_L$ classification.
Apart from the $D^*K$ mode, the $D_1(2430)K$, $D_2(2460)K$, $D_0(2400)K$, $DK^*$, and $D_s^0\eta$ are also important in the decays of $|2P_1\rangle_L$ as shown by Fig. 2. In particular, the partial widths of $D_1(2420)K$, $D_0(2317)\eta$ and $D^*K^*$ turn out to be sizable. A search for those channels would be useful for clarifying the property of the $D_s(3040)$. With the mixing angle $\phi \simeq -55^\circ$, the relative decay ratios among those decay channels are $D^*K : D_1(2430)K : D_2(2460)K : D_0(2400)K : D_1(2420)K : DK^* : D_s(2317)\eta : D_s^0\eta : D^*K^* = 78 : 17 : 19 : 4 : 8 : 4 : 11 : 2$.

Since the mass of $D_s(3040)$ still has a large uncertainty, it may bring uncertainties to the theoretical predictions on the decay widths. To investigate this effect, we plot the decay widths as a function of the mass in Fig. 3 with the mixing angle fixed at $\phi = -50^\circ$. It shows that the mass uncertainty gives rise to an uncertainty of about $\sim 70$ MeV in the total decay width. The predicted widths are much closer to the central value of the data with the increasing mass. The sensitivity of different decay modes to the mass can also be seen clearly in the plot.

**FIG. 3:** (Color online) The partial decay widths and total width of $|2P_1\rangle_L$ as functions of mass. The data are from BABAR.

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**TABLE I:** The decay widths (MeV) for the $D_{sJ}(3040)$ as $1^D_1D_2$, $1^D_3D_2$, $2^D_1P_3$ and $2^D_3P_1$ candidates.

|                | $D^*K$ | $DK^*$ | $D_s^0\eta$ | $D_1(2430)K$ | $D_1(2420)K$ | $D_s(2317)\eta$ | $D_s^0\eta$ | $D_s(2317)\eta$ | total |
|----------------|--------|--------|-------------|--------------|--------------|----------------|-------------|----------------|-------|
| $1^D_1D_2$    | 197    | 27     | 2           | 25           | 2            | 0.4            | 4           | 3              | 608   |
| $1^D_3D_2$    | 256    | 21     | 33          | 34           | 1            | 18             | 0.01        | 0.05           | 3.4   |
| $2^D_1P_3$    | 44     | 9      | 0.3         | 5.5          | 0.02         | 0.01           | $7.5 \times 10^{-3}$ | 0.1   |
| $2^D_3P_1$    | 41     | 2      | 2.5         | 7            | 24           | 7              | 0.5         | 0.002          | 0.09  |

Finally, we discuss the possibilities of $D_{sJ}(3040)$ as a $J^P = 2^-$ candidate. There are two states, $1^D_1D_2$ and $1^D_3D_2$, with $J^P = 2^-$. If $1^D_1D_2$ and $1^D_3D_2$ have a mass of 3.04 GeV, they can decay into the following channels, $D^*K$, $DK^*$, $D_s^0\eta$, $D_s^0\phi$, $D_0(2400)K$, $D_1(2430)K$, $D_1(2420)K$, $D_2(2460)K$, $D_s(2317)\eta$, and $D_s(2400)\eta$. We calculate these partial decay widths and list the results in Tab. 1. It shows that $D^*K$ and $D_2(2460)K$ are the two main decay channels. The total widths for both $1^D_1D_2$ and $1^D_3D_2$ are very broad, i.e., $\Gamma \sim 608$ MeV and $\sim 879$ MeV, respectively. They are too large to compare with the data $\Gamma = (239 \pm 35)$ MeV. Nevertheless, it shows that the admixtures between $1^D_1D_2$ and $1^D_3D_2$ are unable to give a reasonable explanation of the decay properties of $D_{sJ}(3040)$ as well. Thus, the $D_{sJ}(3040)$ as a $J^P = 2^-$ candidate is not favored.
In brief, the $D_{sJ}(3040)$ seems to favor a $|2P_1⟩_L$ state with $J^P = 1^+$, which is an admixture of $2^1P_1$ and $2^3P_1$ with a mixing angle $\phi \simeq -(40 \pm 12)^\circ$. Our conclusion is in agreement with that of a $3P_0$ model analysis [23]. The semiclassical flux tube model [52] and relativistic quark model [49] mass calculations also support this picture.

**B. $D_{sJ}(2710)$**

The $D_{sJ}(2710)$ was first reported by BABAR [2], and its quantum number $J^P = 1^-$ was determined by Belle [3]. Recently, the decay ratios of the $D_{sJ}(2710)$ have also been reported [1], which is very useful for understanding its nature. According to the classification of the quark model, only two states $2^1S_1$ and $1^3D_1$ with the quantum number $J^P = 1^-$ are located around the mass range (2.7 $\sim$ 2.8) GeV. This state is studied by various models, e.g. as a $2^3S_1$ state [17, 49], $1^3D_1$ state [27], or admixture of $2^3S_1$-$1^3D_1$ [21]. It should mention that in our previous work [31] an error occurred in the partial decay amplitude of $1^3D_1 \rightarrow DK$, which led to a rather small width for the assignment of the admixture of $2^3S_1$-$1^3D_1$. Here we correct the formulation and reanalyze the mixing scenario for the $D_{sJ}(2710)$.

| $D^*K$ | $D^*K^*$ | $D^*K^0$ | $D^*\eta$ | $D_s^0\eta$ | $D_s^0\eta$ total | $Γ(D^*K)/Γ(DK)$ |
|-------|---------|--------|---------|---------|----------------|-----------------|
| $1^3D_1$ | 15 | 17.9 | 17.8 | 18.5 | 14 | 0.9 | 200 | 0.24 |
| $2^3S_1$ | 5.4 | 5.6 | 9.0 | 9.1 | 1.7 | 0.7 | 31 | 1.65 |

TABLE II: The decay widths (MeV) for the $D_{sJ}(2710)$ as $1^3D_1$ and $2^3S_1$ candidates.

We first assign the $D_{sJ}(2710)$ as the $2^3S_1$ and $1^3D_1$ states and calculate its decay widths. The results are listed in Tab. II, respectively. For the assignment of the $2^3S_1$ state, the total decay width and the decay branching ratio fraction between $D^*K$ and $DK$ channels are

$$\Gamma \simeq 31 \text{ MeV}, \quad \frac{Γ(D^*K)}{Γ(DK)} \simeq 1.65. \quad (10)$$

It shows that the predicted width $Γ \simeq 31$ MeV is too narrow to compare with the data, and the predicted decay ratio $D^*K/DK \simeq 1.67$ is much larger than the measurement $D^*K/DK \simeq 0.91 \pm 0.13 \pm 0.12$ [1]. The calculations of Ref. [24] also tend to give a small width $Γ \simeq 32$ MeV for the $2^3S_1$ configuration. The predicted branching ratio fraction is also inconsistent with the observations [1]. In Ref. [21], it is also found that the large branching ratio fraction $D^*K/DK \simeq 3.55$ does not support the $D_{sJ}(2710)$ as a pure $2^3S_1$ state.

On the other hand, if the $D_{sJ}(2710)$ is considered as a $1^3D_1$ state, the decay width and branching ratio fraction will be

$$\Gamma \simeq 200 \text{ MeV}, \quad \frac{Γ(D^*K)}{Γ(DK)} \simeq 0.24. \quad (11)$$

In this case, the branching ratio fraction $Γ(D^*K)/Γ(DK) \simeq 0.24$ is too small though the decay width $Γ \simeq 200$ MeV is roughly consistent with the upper limit of the data [1, 13]. These results suggest that either $1^3D_1$ or $2^3S_1$ is not a good assignment for the $D_{sJ}(2710)$.

Thus, we consider the possibilities of the $D_{sJ}(2710)$ as a mixed state of $2^3S_1$-$1^3D_1$, for which the physical states can be expressed as [21]

$$|⟨S⟩⟩_L = \cos(\phi)|2^3S_1⟩ + \sin(\phi)|1^3D_1⟩, \quad (12)$$

$$|⟨S⟩⟩_H = -\sin(\phi)|2^3S_1⟩ + \cos(\phi)|1^3D_1⟩, \quad (13)$$

where the physical partner in the mixing is included. Assuming that the low mass state $|⟨S⟩⟩_L$ corresponds to the $D_{sJ}(2710)$ [21], we plot the decay properties of $|⟨S⟩⟩_L$ as functions of the mixing angle $ϕ$ in Fig. 4. It shows that with the mixing angle $ϕ \simeq (-54 \pm 7)^\circ$, the decay width and branching ratio fraction are

$$\Gamma \simeq (133 \pm 22) \text{ MeV}, \quad \frac{Γ(D^*K)}{Γ(DK)} \simeq 0.91 \pm 0.25, \quad (14)$$

which are in a good agreement with the data [1, 3].

Following this scheme, one can examine the high-mass partner $|⟨S⟩⟩_H$, of which the expected mass is $\sim 2.81$ GeV [21]. Taking into account the mass uncertainties of a region $M \simeq (2.71 \sim 2.88)$ GeV, we plot the mass-dependence of the partial and total widths in Fig. 4. It shows that the $|⟨S⟩⟩_H$ also has a broad width $\sim (120 \pm 10)$ MeV, and
the $DK$ channel is dominant over others. In contrast, the partial width of $D_s\eta$ is also sizable, while the $D^*_s\eta$ width is negligible. Around $M = 2.81$ GeV, the predicted branching ratio fractions are

$$\frac{\Gamma(D_s\eta)}{\Gamma(DK)} \simeq 0.15, \quad \frac{\Gamma(D^*K)}{\Gamma(D_s\eta)} \simeq 0.06. \quad (15)$$

The above mixing scheme is consistent with Ref. [21] for the low-mass state while the predicted suppression of the $D^*K$ decay mode is different from that of Ref. [21]. In Ref. [21] a very broad high-mass state is predicted and would dominantly decay into both $DK$ and $D^*K$. In our scheme, the predicted decay width for $|\langle SD\rangle_l\rangle_H$ is $\sim (120 \pm 10)$ MeV. As a consequence, one would expect that it should appear in the $DK$ spectrum similar to the $D_s(2710)$ signal. Taking into account the still undetermined mass for $|\langle SD\rangle_l\rangle_H$, one possible explanation would be that the $|\langle SD\rangle_l\rangle_H$ mass may be larger than $M \simeq 2.88$ GeV. If so, its total width would be larger than we estimated above and become much broader, thus, cannot be easily identified in the present $DK$ spectrum. Interestingly, a recent study of the $D_s$ spectrum suggests a larger mass for the $1^3D_1$ state [49].

It should be noted that different methods seem to lead to different conclusions on the $D_{sJ}(2710)$ state. In Refs. [16, 17], the $D_{sJ}(2710)$ is assigned as a $2^3S_1$ state. However, the recent observation of $D_{sJ}(2710) \rightarrow D^*K$ does not support this picture. It is also proposed to be a $J^P = 2^3S_1$ state [53]. Therefore, additional information for $D_{sJ}(2710) \rightarrow D_s\eta$ and $D^*_s\eta$, as well as a search for the $|\langle SD\rangle_l\rangle_H$ partner in experiment would be useful for understanding the property of the $D_{sJ}(2710)$.

FIG. 4: (Color online) The partial decay widths, total width, and the decay branching ratio fraction $\Gamma(D^*K)/\Gamma(D^*K)$ of $|\langle SD\rangle_l\rangle_L$ as functions of mass, respectively. The data are from BABAR [1].

C. $D_{sJ}(2860)$

The situation about the $D_{sJ}(2860)$ is still controversial and different solutions have been proposed in the literature. In Ref. [28], the $D_{sJ}(2860)$ is assigned as a $J^P = 0^+$ state. However, the recent observation of $D_{sJ}(2860) \rightarrow D^*K$ does not support this picture. It is also proposed to be a $J^P = 3^-$ state [24, 29, 31]. However, although the decay width
and decay mode are consistent with the observation, the predicted ratio $D^*K/DK \simeq 0.4$ is too small to compare with the data $D^*K/DK \simeq 1.1 \pm 0.1$. 

Since the $D_{sJ}(2860)$ is observed in both $D^*K$ and $DK$ channels, the allowed quantum numbers would be $1^3D_3$, $2^3P_2$, and $1^3F_2$. We calculate the total and partial widths for these configurations and list the results in Tab. III.

More specifically, as the $1^3D_3$ state, the predicted width and branching ratio fraction between the $D^*K$ and $DK$ channel are

$$\Gamma \simeq 36 \text{ MeV}, \quad \frac{\Gamma(D^*K)}{\Gamma(DK)} \simeq 0.4.$$  \hfill (16)

The predicted ratio $\Gamma(D^*K)/\Gamma(DK)$ differs from the measurement $D^*K/DK \simeq 1.1 \pm 0.1$ at the level of three standard deviations, although the decay width is in agreement with the data. Our predictions are consistent with those of Refs. [17, 29]. It should be mentioned that the QCD-motivated relativistic quark model can not well explain the mass of $D_{sJ}(2860)$ if it is considered as the $1^3D_3$ state [49]. This could be a signal indicating the chiral symmetry in association with the heavy quark symmetry in the heavy-light meson transitions.

As a candidate of the $2^3P_2$ state, the decay width and branching ratio fraction of $D_{sJ}(2860)$ are

$$\Gamma \simeq 8 \text{ MeV}, \quad \frac{\Gamma(D^*K)}{\Gamma(DK)} \simeq 1.53,$$  \hfill (17)

where both the predicted width and ratio are inconsistent with the data. It is interesting to mention that our predicted ratio agrees with the estimation of Ref. [32].

If the $D_{sJ}(2860)$ is a $1^3F_2$ state, the predicted width and branching ratio fraction are

$$\Gamma \simeq 49 \text{ MeV}, \quad \frac{\Gamma(D^*K)}{\Gamma(DK)} \simeq 0.005,$$  \hfill (18)

where the decay mode of $D^*K$ turns out to be negligible in comparison with the $DK$ mode, and disagrees with the experimental observation.

FIG. 5: (Color online) The partial decay widths and total width of $|(SD)^j_H|$ as functions of mass.

| $D^*K$   | $D^*K^*$ | $D*\eta$ | $D_s\eta$ | $DK$   | total | $\Gamma(D^*K)/\Gamma(DK)$ |
|----------|----------|----------|-----------|--------|-------|--------------------------|
| $1^3D_3$ | 12.3     | 11.8     | 5         | 4.7    | 1.7   | 0.3                      | 0.2 | 36 | 0.40                        |
| $2^3P_2$ | 1.3      | 1.3      | 2.1       | 1.9    | 0.01  | 1.7                      | 0.02 | 8  | 1.53                        |
| $1^3F_2$ | 21.9     | 21.4     | 0.1       | 0.1    | 5.5   | 0.02                     | 0.005 | 49 | 0.005                       |
It can be seen from the above analysis that a simple assignment of the \( D_{sJ}(2860) \) to be a pure \( 2^3P_0 \), \( 1^3D_3 \), \( 2^3P_2 \) or \( 1^3F_2 \) cannot well explain the data. We also point out that the \( 2^3P_2 \) and \( 1^3F_2 \) mixing is unable to overcome the problem either because of the narrow width of the \( 2^3P_2 \) state or small branching ratio fraction \( \Gamma(D^*K)/\Gamma(DK) \approx 0.005 \) of \( 1^3F_2 \).

In Ref. [32], van Beveren and Rupp recently proposed an alternative solution that there might exist two largely overlapping resonances at about 2.86 GeV, i.e. a radially excited tensor \( (2^+) \) and a scalar \( (0^+) \) \( c\bar{s} \) state. Following this two-state assumption, one would expect that one state \( D_{sJ_1}(2860) \) dominantly decays into \( DK \), while the other one \( D_{sJ_2}(2860) \) dominantly decays into \( D^*K \). Both states have a mass around 2.86 GeV, and comparable width \( \Gamma \sim 50 \) MeV.
MeV. This idea may shed some light on the controversial issues. As follows, we shall investigate such a possibility in our approach.

It shows that the decays of $2^3P_0$, $1^3D_3$ and $1^3F_2$ is dominated by the $DK$ channel, while the decay of $1^3D_2$, $1^1D_2$, $2^3P_2$ is dominated by the $D^*K$ channel. We shall identify which states are more appropriate candidates in the two-state scenario.

First, we analyze the states dominated by $DK$ decays, i.e. $2^3P_0$, $1^3D_3$ and $1^3F_2$. In Fig. 8 the total and partial decay widths for the $2^3P_0$ state are revealed. It shows that the $2^3P_0$ possesses a broad decay width $\Gamma \simeq 115$ MeV at about 2.86 GeV, which is inconsistent with the data. The $1^3F_2$ is not considered as a good candidate of $D_{sJ}(2860)$. Furthermore, our earlier analysis suggests that the $D_{sJ}(3040)$ may favor a configuration of $|2P_1⟩_L$ such that the mass of the $1^3F_2$ should be larger than the $P$ wave state $|2P_1⟩_L$ as a consequence. In contrast, we find that the $1^3D_3$ could be a good candidate for $D_{sJ}(2860)$ since it is dominated by the $DK$ decay mode and has a narrow width $\Gamma \simeq 36$ MeV. The calculation results for the total and partial decay widths have been listed in Tab. III.

Candidates for the $D_{sJ}(2860)$ could be $1^3D_2$, $1^1D_2$, or $2^3P_2$ which dominantly decay into $D^*K$. As discussed earlier in this section and shown in Tab. III the $2^3P_2$ is not a good candidate since its total width is too small to compare with the data. Nevertheless, its expected mass should be larger than 2.86 GeV [26, 49].

If the $D_{sJ}(2860)$ is considered as pure $1^3D_2$ or $1^1D_2$ state, their decay widths would be $\Gamma \simeq 170$ MeV and $\Gamma \simeq 130$ MeV, respectively, which are inconsistent with the data as well. In fact, the physical states should be the admixtures between $1^3D_2$ and $1^1D_2$ due to the presence of the spin-orbit interactions [22, 50, 51]. Thus, the mixed states can be expressed as

$$|1D_2⟩_L = \cos(\phi)|1D_2⟩ + \sin(\phi)|1^3D_2⟩,$$

$$|1D_2⟩_H = -\sin(\phi)|1D_2⟩ + \cos(\phi)|1^3D_2⟩,$$

where the subscripts $L$ and $H$ denote the low-mass and high-mass state due to the mixing. Usually, the $|1D_2⟩_H$ have a narrow width [22, 50, 51]. We thus consider the $|1D_2⟩_H$ as the $D_{sJ}(2860)$ in the calculation. In Fig. 7 the decay properties as a function of the mixing angle are plotted. We see that around $\phi = -65^\circ$ or $\phi = -35^\circ$ the decay width is $\Gamma \simeq 40$ MeV, which is compatible with the observation, and the decay mode is dominated by the $D^*K$. With $\phi = -35^\circ$, the corresponding decay branching ratio fractions are

$$\frac{\Gamma(D^*K)}{\Gamma(D_{sJ}^*\eta)} \simeq 1.2,$n

$$\frac{\Gamma(D^*K)}{\Gamma(D_{sJ}^*\pi)} \simeq 13,$n

which fit in the experimental data quite well. This result turns out to support the $|1D_2⟩_H$ to be a candidates of $D_{sJ}(2860)$ in the two-state scenario. In the range of $\phi = -65^\circ \sim -35^\circ$ the partial widths do not change drastically with the mixing angle. In contrast, the suggested value is consistent with that ($\phi = -50.7^\circ$) obtained in the heavy quark effective theory [22, 49, 51].

In brief, it seems likely that the abnormal property with the $D_{sJ}(2860)$ arises from two overlapping resonances with the same mass but different decay modes. One is $1^3D_3$ and the other is $|1D_2⟩_H$ from the $1^3D_2$ and $1^1D_2$ mixing. The $1^3D_3$ state mainly decays into $DK$ and the $|1D_2⟩_H$ into $D^*K$. With these two largely overlapping resonances at about 2.86 GeV, we can understand both the observed decay widths and branching ratio fractions of the $D_{sJ}(2860)$. It shows that the $1^3D_3$ has a sizable partial width in the $D_{sJ}\eta$ channel, while the $|1D_2⟩_H$ also turns out to be measurable. Further measurements of $\Gamma(D_{sJ}^*\eta)/\Gamma(D_{sJ}^*K)$ and $\Gamma(DK)/\Gamma(D_{sJ}\eta)$ may be able to distinguish the $1^3D_3$ and $|1D_2⟩_H$ and test the two-state scenario in experiment.

D. $D_{sJ}(2|P_{1}^{0}\rangle_H)$, $D_{sJ}(2^3P_0)$ and $D_{sJ}(2^3P_2)$

In this subsection we discuss the implications of other states following the consequence of the assignments for the $D_{sJ}(3040)$, $D_{sJ}(2710)$ and $D_{sJ}(2860)$.

Since the $D_{sJ}(3040)$ seems to favor a $P$ wave with $J^P = 1^+$ $(|2P_1⟩_L)$, experimental evidences for the other $P$ waves, $D_{sJ}(2|P_{1}^{0}\rangle_H)$, $D_{sJ}(2^3P_0)$ and $D_{sJ}(2^3P_2)$, would be important to establish the spectrum. In particular, its high-mass partner $|2P_1⟩_H$ should be searched in experiments. Supposing that the $|2P_1⟩_H$ has a mass in the range of (3.04 $\sim$ 3.2) GeV, we plot in Fig. 8 the decay widths as functions of the mass with the mixing angle $\phi = -50^\circ$ fixed by $D_{sJ}(3040)$. It shows that the $|2P_1⟩_H$ width is indeed relative narrower around $M = 3.04$ GeV, although we should note that the decay width increases fast with the increasing mass. The decay channels, $D_0(2400)K$, $D_2(2460)K$ and $D_2(2317)\eta$,
are predicted to be the dominant ones, which can be investigated in experiments. In contrast, the $D^*K$ channel plays a less important role in the decays.

We further study the $D_{sJ} (2^{3} P_0)$ in detail here. The decay widths as a function of the possible mass range $M = (2.8 \sim 2.9)$ GeV are plotted in Fig. [9] in this range the total decay width is $\Gamma \approx (90 \sim 140)$ MeV, and increases with the increasing mass. It shows that the $DK$ channel dominates its decays. Taking the mass of the $D_{sJ} (2^{3} P_0)$ as $M \approx (2.82 \sim 2.84)$ GeV [21, 54], the total width and branching ratio fractions between $D_s\eta$ and $DK$ are

$$\Gamma \approx (101 \pm 5) \text{ MeV, } \frac{\Gamma(D_s\eta)}{\Gamma(DK)} \approx 0.08.$$  

It should be pointed out that the decay properties of $2^{3} P_0$ are similar to those of $|SD'_{1,1}H|$ in the mass range $M < 2.9$ GeV (see Fig. [5] and Fig. [9]). Both of them have comparable decay widths $\Gamma \sim 100$ MeV, and mainly decay into $DK$. To distinguish them from each other, the measurements of their decay ratio $\Gamma(D_s\eta)/\Gamma(DK)$ are important. We also note that a recent calculation suggests a larger mass of $M \approx 3.054$ GeV for $2^{3} P_0$ [49]. As a consequence of this scenario, its total decay width would become much broader than we estimated above. Thus, it may not be easily isolated in experiment.

As discussed earlier the $D_{sJ} (2860)$ does not favor the assignment of $2^{3} P_0$. Thus, we investigate its decay properties and implications of experimental measurement. We also plot its total and partial decay widths as functions of the mass in the possible range $M = (3.04 \sim 3.2)$ GeV in Fig. [9]. If $2^{3} P_0$ has a mass larger than 3.04 GeV, decay channels, $DK$, $D^*K$, $DK^*$, $D_s^*\eta$, $D_s\phi$, $D_s\eta$, $D_1(2430)K$, $D_1(2420)K$, $D_2(2460)K$, $D_0(2460)\eta$, will open in which $D_1(2430)K$ and $DK$ channels are dominant. In Fig. [9] we do not show the results for the $D_s^*\eta$, $D_s\phi$ and $D_s(2460)\eta$ channels since they are negligibly small (< 1 MeV). If we adopt the mass $\sim 3.15$ GeV as predicted by Refs. [26, 49, 54], the predicted width is $\Gamma \approx 140$ MeV, and the relative decay strengths are $DK : D^*K : D_1(2430)K : D_1(2420)K : D_2(2460)K : DK^* : D_s^*K^* : D_s\eta \approx 41 : 9 : 50 : 13 : 11 : 6 : 7 : 4$. It suggests that the $DK$, $D_1(2430)K$, $D_1(2420)K$ channels may be the optimal ones for searching for the $D_{sJ} (2^{3} P_0)$ state in experiment.
FIG. 9: (Color online) The partial decay widths and total width of $2^3P_2$ as functions of mass.

E. Sensitivity to the harmonic oscillator parameter

It should be mentioned that model-dependent feature of our model arises from the simple treatment of harmonic oscillator potential for the heavy-light quark system. Therefore, uncertainties with the theoretical results are present in the choice of the quark model parameter values. The most important parameter in our model should be the harmonic oscillator strength $\beta$, which controls the size effect or coupling form factor from the convolution of the heavy-light meson wavefunctions. The commonly adopted range of this quantity is $\beta = (0.4 \sim 0.5)$ GeV, and we apply $\beta = 0.45$ GeV in the above calculations.

In order to examine the sensitivity of the calculation results to $\beta$, we plot the decay widths and ratios of $2^3S_1$, $1^3D_3$, mixed state $|SD\rangle_L$ of $2^3S_1$-$1^3D_1$, and mixed state $|2P_1\rangle_L$ of $2^1P_1$-$2^3P_1$ as a function of $\beta$ in Fig. 10. It shows that the decay widths of these excited $D_s$ states exhibit some sensitivities to the parameter $\beta$. Within the range of $\beta = (0.45 \pm 0.05)$ GeV, about 30% uncertainties of the decay widths would be expected. This is a typical order of accuracy for the constituent quark model, and can be regarded as reasonable.

The ratio $\Gamma(D^*K)/\Gamma(DK)$ appears to behave differently. For the $2^3S_1$, the sensitivity of the ratio to $\beta$ is apparent. In contrast, the ratios of $|SD\rangle_L$ and $1^3D_3$ are quite insensitive to $\beta$. The ratio $\Gamma(D^*K)/\Gamma(DK)$ is not shown for $|2P_1\rangle_L$ since its decay into $DK$ is forbidden.

In brief, although the harmonic oscillator parameter $\beta$ can bring some uncertainties to the final results, within the range of $\beta = (0.4 \sim 0.5)$ GeV, our major conclusions will still hold.

IV. SUMMARY

In this work we investigate the strong decays of several newly observed charmed mesons in a constituent quark model with effective Lagrangians for the quark-meson interactions. The decay amplitudes are extracted for light pseudoscalar meson or vector meson productions via axial or vector current conservation between the quark-level and hadronic level couplings. The quark-meson couplings can then be determined by independent measurements such as meson photoproduction and meson-baryon scatterings.

We find that the new state $D_{sJ}(3040)$ can be identified as the low mass physical state $|2P_1\rangle_L$ from the $D_s(2^1P_1)$-$D_s(2^3P_1)$ mixing with a mixing angle $\phi \simeq -(40 \pm 12)^\circ$. Further experimental search for decay modes of $D_1(2430)K$, $D_2(2460)K$, $D_0(2400)K$, $DK^*$, and $D_s^*\eta$ should be able to disentangle its property and test our model predictions.

The $D_{sJ}(2710)$ seems to favor a low mass physical state $|(SD)\rangle_L$ from the $2^1S_1$-$1^3D_1$ mixing with a mixing angle $\phi \simeq (-54 \pm 7)^\circ$. Both the ratio and width are in a good agreement with the data. The decay properties of its heavy partner $|(SD)\rangle_H$ are also discussed. It has a broad width $\Gamma \simeq (110 \sim 140)$ MeV at the 2.8 GeV mass region, and
dominated by the $DK$ mode. We also point out that the $|SD⟩^H$ state may be searched in the $DK$ spectrum as the $D_{sJ}(2710)$ if its mass is $\sim 2.8$ GeV. Whether the present data have contained its signal could be a crucial criteria for various model predictions.

The $D_{sJ}(2860)$ cannot be easily explained by a single configuration of $2^3P_0$, $2^3P_2$, $1^3F_2$ or $1^3D_3$. To overcome this problem we follow the proposal of a two-state picture by Ref. [32] and assume that two narrow resonances may have been observed around 2.86 GeV with a width $\Gamma \simeq (40 \sim 50)$ MeV. It shows that one resonance seems to be the $1^3D_3$, which mainly decays into $DK$. The other resonance could be the $|1D_2⟩^H$, which is the high-mass state from the $1^1D_2$-$1^3D_2$ mixing, and dominantly decays into $D^*K$. Further theoretical and experimental efforts are needed to disentangle the mysterious properties about this state.

We also study the implications arising from the assignments for those observed resonances, e.g. their partner states in the mixing. In particular, if the $D_{sJ}(3040)$ is indeed a $P$-wave state $|2P⟩^L$, the other three $P$-wave states $D_{sJ}(2P^⟩^H)$, $D_{sJ}(2^3P_0)$ and $D_{sJ}(2^3P_2)$ may also have measurable effects in experiment. Their strong decay properties are predicted, which could be useful for future experimental studies.

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