The truncation of stellar discs. A theoretical model

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Abstract. The truncation of stellar discs is not abrupt but characterized by a continuous distancing from the exponential profile. There exists a truncation curve, \( t(r) \), ending at a truncation radius, \( r_t \). We present here a theoretical model in which it is assumed that the magnetic hypothesis explaining the flat rotation curve also explains the truncation. Once stars are born, the centripetal magnetic force previously acting on the progenitor gas cloud is suddenly interrupted, and stars must move to larger orbits or escape. The agreement between theoretical and observed truncation curves is very satisfactory. Parameters defining the disc gas rotation curve should therefore be related to those defining the truncation. It is predicted that rotation curves that quickly reach the asymptotic value \( \theta_0 = \theta(r = \infty) \) would have small truncation radii. On the contrary, \( r_t \) and \( \theta_0 \) itself, would be uncorrelated quantities.

Key words: Galaxies: magnetic fields – spiral – structure

1. Introduction

The interest in studying the truncation of stellar discs lies in the fact that this is a phenomenon present in all spirals and that no theoretical model has yet been advanced.

The truncation of stellar discs was discovered by van der Kruit (1979) and was the object of a preliminary but noticeably precise description by van der Kruit & Searle (1981a,b; 1982a,b) by means of photographic photometry. Some basic facts were established in these pioneer works and are noted here: a) as the involved intensities are very low, truncations are better observed in edge-on galaxies; b) the truncation radius, \( r_t \), is about 4.2 times the radial scale length, \( R \); c) the truncation is not a sharp cut-off, but the radial e-folding drops to about 1 kpc; therefore, there is a truncation curve, \( t(r) \), with \( t(r_t) = \infty \), which will be precisely defined later.
It is evident that this phenomenon reveals important dynamic effects, particularly if it is as common as it seems. More than a decade after the work by van der Kruit & Searle, the subject was reconsidered by Barteldrees & Dettmar (1994) who have renewed the interest in this topic. In this CCD photometry study, it was proposed that $r_t/R < 3$, noticeably lower than the corresponding value obtained by van der Kruit & Searle.

Studies by Hamabe (1982), Sasaki (1987) and Hamabe & Wakamatsu (1989) have also considered truncation. In our Galaxy, Habing (1988) found $r_t$ to be 9.5 kpc; Robin et al. (1992), 14 kpc; Ruphy et al. (1996), 15 kpc; Porcel et al. (1997) found $r_t \leq 15$ kpc, assuming the truncation interpretation for the near infrared COBE data, following a discussion by Freudenreich et al. (1994).

The truncation of the stellar disc is a common feature of all spiral galaxies. For instance, van der Kruit & Searle (1982a) detected it in the four galaxies studied, as did Barteldrees & Dettmar in a sample of 27 edge-on galaxies. Although it must be observed in noisy conditions, it is clear that it is a universal phenomenon. The large sample of galaxies in nearby clusters by Gavazzi et al. (1990, 1994, 1995) and Gavazzi & Randone (1994) gives an approximate truncation frequency of around 0.6, considering just edge-on non interacting galaxies.

Therefore, even if the relatively sharp truncation takes place at very low surface brightness (greater than about 24 mag arcsec$^{-2}$), it is clear that it is a universal phenomenon. Hence, it is highly interesting. This interest is in contrast with the scarce number of statistical studies of truncation reported in the literature. Considering that it is a feature related to dominant dynamic effects at the periphery of spiral galaxies, it is also remarkable that such limited attention has been paid by theoretical studies to explain it.

Previous theoretical hypotheses concerning truncation have been summarized by de Grijs (1997). Fall & Efstathiou (1980) suggested that truncation takes place at those radii at which shear by differential rotation overcomes selfgravity, so that gravitational collapse and star formation are inhibited, but this idea did not provide agreement with real truncation radii (van der Kruit & Searle, 1982a). Larson (1976) considered slow disc formation, so that the truncation radius would just reflect the present age of the galaxy. These early hypotheses were not developed with theoretical models. Recently, Bottema (1996) proposed tides in interacting galaxies as a cause of truncation, which could explain some but not all the observed truncated discs. It can therefore be stated that, at present, no compelling theory exists to explain this important dynamic phenomenon.

The magnetic hypothesis of the rotation curves (Nelson, 1988; Battaner et al., 1992; Florido & Battaner, 1995) provides a very straightforward explanation of the truncation discs: the outer disc has no star because they escape once they are formed. Under
the magnetic hypothesis, the rotating gas is subject to the centrifugal force in equilibrium with two centripetal forces: gravitational and magnetic. When gas forms stars, the centrifugal and the gravitational forces remain the same but the magnetic suddenly disappears. Then the star migrates to another orbit with a larger radius or even escapes.

The purpose of this paper is therefore to quantify this idea and to show how real truncation curves are reproduced by the magnetic model.

2. An analytical model

We first need a rotation curve for the gaseous disc becoming flat at large radii, and another curve for the stellar disc, coincident with the gas curve for small radii and Keplerian for large radii. We then propose

\[ \varphi_{\text{gas}} = \frac{\theta_{\text{gas}}}{\theta_0} \]  
(1)

\[ \varphi_{\text{stars}} = \frac{\theta_{\text{stars}}}{\theta_0} \]  
(2)

\[ x = \frac{r}{R_F} \]  
(3)

\[ \varphi_{\text{gas}} = 1 - e^{-x} \]  
(4)

\[ \varphi_{\text{stars}} = \left(1 - e^{-x}\right) e^{-\left(\frac{R_F}{R_K}\right)^2 x^2} + Cx^{-\frac{3}{2}} \left(1 - e^{-\left(\frac{R_F}{R_K}\right)^2 x^2}\right) \]  
(5)

where \( R_F \) is a constant, a typical radial length which indicates how slowly the gas rotation curve becomes flat; \( R_K \) is another constant, a typical radial length indicating where the stellar rotation curve becomes Keplerian; \( C \) is another constant, providing information on the point central mass once \( \varphi_{\text{stars}} \) become Keplerian.

We therefore assume a corotation region for \( r \ll R_F \). For \( r > R_K \) the star rotation curve is Keplerian and the gas rotation curve is flat. As we are working under the magnetic hypothesis we do not need the presence of a dark halo. The Keplerian region would specify the velocity of true stars in a steady orbit at radius \( r \), if stars really existed in this region. However, we will see that this region is devoid of stars with stationary orbits, at least for a large range of the parameters involved.

Neither the disc gas rotation curve nor the disc stellar rotation curve are directly observable. In the innermost region the bulge dominates both curves, and we only consider the disc component. In the Keplerian region, star velocities are unobserved, because either the luminosity is too low to be appreciated with present techniques, or because of the complete absence of stars. However, observations very much restrict our choices of both curves. The parameter \( \theta_0 \) is observational and is known for most flat rotation galaxies; \( R_F \) is more or less related to the radius at which \( \theta \) no longer depends on \( r \); \( R_K \) must
Fig. 1. Rotation curves for the Milky Way. The dominant curve at small radii is observational (Burton et al., 1992). The continuous line represents the assumed disc gas rotation curve. The dotted line represents the assumed disc stellar rotation curve. Values of the parameters: $\theta_0 = 250\,\text{km\,s}^{-1}$, $R_F = 2.5\,\text{kpc}$, $R_K = 60\,\text{kpc}$, $C = 2.5$

be larger than the optical disc and $C$ is related to the mass $M$ (bulge and disc) of the galaxy through

$$C \approx \frac{(GM)^{\frac{1}{2}}}{R_F \theta_0} \tag{6}$$

Therefore, even if we are using a set of 4 parameters, observations very much reduce the choice, which must be different for each galaxy.

Figure 1 illustrates the different rotation curves considered here for the particular case of the Milky Way. The observational curve has been taken from Burton et al. (1992). Figure 2 is another example for NGC 5023. In this case, there is no practical difference between the observational (Bottema et al., 1996) and the assumed disc gas curves.

We further assume an exponential gas distribution (Freeman, 1970), which is reasonable for large radii, so that the number of stars born at a given radius throughout the whole history can in turn be assumed to be exponential

$$\rho_B = Ae^{-\frac{\theta}{R}} = Ae^{-\beta x} \tag{7}$$

where $A$ is a constant related to the gas-star formation efficiency, its precise value being unimportant for our present purposes. $R$ is the radial scale length of the exponential disc and $\beta = R_F / R$ can be adopted as a parameter, not free because $R$ can be deduced from observations. The precise definition of $\rho_B$ is: $\rho_B(r)\,dr$ gives the number of stars
Fig. 2. Rotation curves for NGC 5023. The assumed disc gas rotation curve is slightly above the observational rotation curve. The dotted line again represents the assumed disc stellar rotation curve. Values of the parameters: $\theta_0 = 86 \text{km s}^{-1}$, $R_F = 1.5 \text{kpc}$, $R_K = 50 \text{kpc}$, $C = 2$

born in a ring between $r$ and $r + dr$, throughout the whole history of the galaxy. It would coincide with the present distribution of stars if stars were not able to move to other rings.

The velocity of these $\rho_B(r)dr$ hypothetical stars would be given by eq. (4) and, as a relation between $r$ and $\theta$ exists, we are able to calculate $\Gamma(\theta)d\theta$, the number of stars born with a velocity in the range $[\theta, \theta + d\theta]$

$$\rho_B(r)dr = \Gamma(\theta)d\theta$$  \hspace{1cm} (8)

$$\Gamma(\theta) = \frac{\rho_B}{d\theta/br} = Ae^{-\beta x} \frac{R_F}{\theta_0} e^{x} = \frac{AR_F}{\theta_0} e^{x(1-\beta)} = \frac{AR_F}{\theta_0} \left(1 - \frac{\theta}{\theta_0}\right)^{\beta-1}$$  \hspace{1cm} (9)

Real stars would conserve the speed they had at birth. Therefore, the distribution of stars in the velocity space would be the same and is given by eq. (9). For real stars however, the relation between $\theta$ and $r$ is different. This relation is now accounted for by eq. (5). The real distribution of stars in the position space $\rho$ would be obtained by

$$\rho(r)dr = \Gamma(\theta)d\theta$$  \hspace{1cm} (10)

therefore

$$\rho(r) = \Gamma(\theta) \frac{d\theta}{dr} = \frac{AR' \theta_0 d\varphi}{\theta_0 R' dx} \left(1 - \frac{\theta}{\theta_0}\right)^{\beta-1} \frac{d\varphi}{dx}$$  \hspace{1cm} (11)
where \( \frac{d\varphi}{dx} \) would be calculated using (5) although it is not necessary to write it explicitly.

Truncation would take place for \( \rho(r) = 0 \), i.e. for \( \frac{d\varphi}{dx} = 0 \). i.e. the maximum of the function \( \varphi(x) \). The equation \( \frac{d\varphi}{dx} = 0 \) is transcendental and the value of the truncation radius \( r_t \) cannot be found analytically. We will therefore adopt numerical techniques for given real spiral galaxies.

Let us consider

\[
\mu_B = -2.5 \log \frac{\rho_B}{\rho_C} = -2.5 \log \rho \quad \mu = -2.5 \log \frac{\rho}{\rho_C} \tag{12}
\]

where \( \mu_B \) and \( \mu \) are in mag arcsec\(^{-2} \) and \( \rho_C \) is a constant. Here, \( \mu \) is the real stellar luminosity profile and \( \mu_B \) would correspond to the luminosity profile that would be observed if all born stars had maintained their orbit radius until the present. In the most internal regions, where magnetic forces are negligible with respect to gravitational forces, the distribution \( \mu_B \) would be the actual luminosity profile. It is easily checked with eq. (7), that \( \mu_B \) depends linearly on \( x \) (or on the galactocentric radius, \( r \)) defining the assumed exponential profile. However, for very large \( x \) values \( \mu_B \) no longer represents the real profile, as magnetic forces produce a difference between the real luminosity and the extrapolated exponential luminosity, i.e. truncation develops. Therefore, let us define the truncation curve as

\[
t(r) = \mu - \mu_B = -2.5 \log \frac{\rho_B}{\rho} \tag{14}
\]

Therefore

\[
t(r) = -2.5 \log \frac{\rho}{\rho_C} \left( \frac{1 - \frac{\varphi}{\beta}}{\varphi} \right)^{\beta-1} \frac{d\varphi}{dx} = 2.5 \log \frac{e^{-\beta x}}{(1 - \varphi)^{\beta-1} \frac{d\varphi}{dx}} \tag{15}
\]

where again \( \frac{d\varphi}{dx} \) is obtainable from (5).

3. Results

Figure 3 represents the truncation curve obtained from the observations and the theoretical one, obtained with a set of reasonable values of the different parameters involved, for the galaxies NGC 5023 (fig. 3a), NGC 891 (fig. 3b) and NGC 4013 (fig. 3c). The observational truncation curve has been adopted from van der Kruit & Searle (1982a) for NGC 5023 and NGC 4013, and from van der Kruit (1981b) for NGC 891. For the Milky Way, no truncation curve is available and we represent in fig. 4 the theoretical curves for \( C = 2.5 \) and \( C = 3 \). To adopt the rotation curve we have taken observations from Sofue (1996) and Bottema (1996) for NGC 891 and NGC 4013 respectively. The
Fig. 3. Truncation curves of different galaxies. Dotted line, observational curve; continuous line, this model. Fig. 3a: NGC 5023, with parameters $R_F = 1.5$ kpc, $R_K = 50$ kpc, $R = 2$ kpc, $C = 2$, $\theta_0 = 86 \text{ km s}^{-1}$. Fig. 3b: NGC 891, with parameters $R_F = 6$ kpc, $R_K = 75$ kpc, $R = 4.9$ kpc, $C = 1$, $\theta_0 = 230 \text{ km s}^{-1}$. Fig. 3c: NGC 4013, with parameters $R_F = 3$ kpc, $R_K = 70$ kpc, $R = 2.3$ kpc, $C = 2$, $\theta_0 = 175 \text{ km s}^{-1}$.
Fig. 4. Predicted truncation curve for the Milky Way for \( C = 2.5 \) and \( C = 3 \). The remaining parameters were assumed to be: \( R_F = 2.5 \text{ kpc} \), \( R_K = 60 \text{ kpc} \), \( R = 2.5 \text{ kpc} \), \( \theta_0 = 250 \text{km} \text{s}^{-1} \). \( R_F \) and \( \theta_0 \) were adopted taking into account Burton et al (1992). \( R \) was adopted from Porcel et al. (1997)

Once the validity of the model has been confirmed, we are able to predict or compare the dependence of the truncation curve, and in particular the truncation radius, on the value of the different observable parameters characterizing the rotation curve.

We predict that the truncation radius is very sensitive to the value of \( R_F \), as shown in Fig. 5. Those galaxies having a rotation curve slowly reaching the constant rotation velocity (larger \( R_F \)) would have a more extended stellar disc (larger \( r_t \)), with an approximate relation \( r_t \approx 4R_F \).

As \( \varphi(x) \) is independent of \( \theta_0 \), we see from eq. (14) that \( t(r) \) and \( r_t \) do not depend on \( \theta_0 \). Therefore, our analytical model predicts no statistical relation of \( r_t \) and \( \theta_0 \), the rotation velocity at infinity.

4. Conclusions

The rotation curve of the gaseous disc determines the truncation of the stellar disc. We have shown how the rotation scale length of the gas disc, \( R_F \), which represents a typical length for the disc to reach the outer flat rotation velocity is very closely related to the truncation radius. However, we find that the truncation radius is insensitive to the asymptotic rotation velocity at large radii.
To confirm these predictions a larger statistical basis is needed. Though the sample used by Barteldrees and Dettmar (1994) contained as many as 27 edge-on galaxies, their rotation properties are mostly unknown. When these properties are studied by observations, the low luminosities at which truncation is observed may pose difficult problems in many galaxies, and hence in the statistical analysis. At large radii, stars are not observed either because of a physical truncation or due to sensitivity limitations. In most cases, truncation takes place at those radii where good photometry is able to detect it. Nevertheless, large truncation radii could be unobservable because of sensitivity limits, thus introducing a bias. For instance, when the radial scale length, $R$, is very small, a high $r_t$ could be undetectable.

Observations to detect a Keplerian regime of the stellar disc at large radii may be unsuccessful, as such a region could in general be devoid of stars or they could possess transient orbits.

The truncation of stellar discs is a fundamental concept in understanding the evolution and structure of spiral galaxies. Some of the assumptions adopted here are reasonable, though modifiable, but it can be firmly concluded that the magnetic scenario explaining flat rotation curves also provides a clear, simple and natural explanation for this phenomenon. Other truncation models based on alternative hypotheses could clarify in the future our understanding of this neglected but important dynamic feature.

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