Application of the time-space distributed model for the description of Q-switched regime in solid-state lasers with a saturable absorber

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Abstract. One-dimension time-space distributed model for Nd:YAG laser with saturable absorber Cr<sup>4+</sup>:YAG is considered. The dependencies for peak power, pulse width and pulse repetition rate on pump power and resonator length are introduced.

1. Introduction

Q-switched regime in solid-state lasers has been researched sufficiently wide [1]. Nevertheless, we still face an urgent problem of laser parameters optimization and tuning for a predefined values range of output parameters. One of the ways of this tuning is varying the resonator geometry, particularly, varying the resonator length in an even wide range, from a few millimeters up to several meters [2, 3, 4].

A major difficulty raises when one tries to exploit concentrated models that are based on resonator volume averages [5, 6]. These models either completely neglect spatial evolution of amplified radiation inside the resonator or reduce it to some effective values (e.g. per one round trip time [7]). Recently, there is a raise of spacially distributed dynamic models that allow a direct simulation of an amplified radiation pulse propagation inside the resonator [5, 6, 8, 9]. These models allow us to reveal a variety of considerable details and peculiarities of amplified pulse formation [5, 6], while concentrated models may fail.

2. Theoretical model

We present a dynamic model of formaion of amplified radiation with one spatial dimension, free of limits for the active media length, saturable absorber length, total length, and their mutual arrangement.

Let us consider an example configuration of a laser: a Nd:YAG active media of length $L_g$, a saturable absorber Cr<sup>4+</sup>:YAG of length $L_s$ positioned inside a resonator of total length $L_{rez}$ closely to each other and to the input mirror, and an output mirror at a distance of $L_3$ from the absorber, that is $L_{rez} = L_g + L_s + L_3$. We have chosen the materials for active media and saturable absorber as their full characteristics were available in paper [10], still the model is not limit to this case. We consider transverse pumping, and base on the model [10] that is stated in terms of dimensionless values of the photon flux density ($u^+$ and $u^-$) and the population inversion of the active media ($D$) and the absorber ($D_s$).
The proposed model has the following major differences: (i) radiation amplification and absorption happens only in the active media and the absorber, while inside the range $L_3$ radiation propagates freely, (ii) a dimensionless spatial coordinate is introduced (the input mirror and the output mirror ($r_2 = 0.99$) correspond to coordinates $x = 0$ and $x = 1$ respectively, and (iii) a term describing the contribution of the amplified radiation to the lasing is accounted. Thus, we express the final equation system as follows:

$$\frac{dD}{dt} = \frac{\tau_c}{\alpha_L} \gamma (1 - (1 + (u^+ - u^-))D)$$

$$\frac{dD_s}{dt} = \frac{\tau_c}{\alpha_L} \gamma_1 (-1 - (1 + \alpha(u^+ + u^-))D_s)$$

$$\frac{\alpha_L}{\tau_c} \frac{\partial u^+}{\partial t} - \frac{\partial u^+}{\partial x} = (-1 + AD + A_1 D_s)u^+ + R_{lum}|D|$$

$$\frac{\alpha_L}{\tau_c} \frac{\partial u^-}{\partial t} + \frac{\partial u^-}{\partial x} = (-1 + AD + A_1 D_s)u^- + R_{lum}|D|$$

The boundary conditions are:

$$u^+(0, t) = r_1 u^-(0, t); u^-(1, t) = r_2 u^+(1, t); u^\pm(x, 0) = 0$$

The constants in the system (1)-(2) are following:

$$D = \frac{(1 + \gamma_a \tau_g I) \cdot G}{2L_g N_2 \gamma I \tau_g}, D_s = \frac{G_s}{2L_s N_0 \sigma_a} u^+ = \frac{\gamma_c \tau_g R^\pm}{1 + \gamma_a \tau_g I}, \tau_c = \frac{2(L_g + L_s)n}{c}$$

$$G = 2L_g \gamma_a N_2, G_s = 2L_s(\sigma_e N_{2a} - \sigma_a (N_0 - N_{2a})), \alpha = \frac{\sigma_c \tau_g}{\gamma_c \tau_g} (1 + \gamma_a \tau_g I)$$

$$\gamma = \frac{\tau_c}{\tau_g \alpha_L}, \gamma_1 = \frac{\tau_c}{\tau_s \alpha_L}, R^\pm = \frac{u^\pm + 1 + \gamma_a \tau_g I}{\gamma_c \tau_g}$$

where $N_2$ and $N_{2a}$ are populations of exited levels of the active media and the saturable absorber respectively, $R^\pm$ are values of the generation photon flux density ($u^+$ and $u^-$ respectively). The value of pump power is determined as $P_p = I \nu_p S_p$, where $\nu_p$ is the frequency of the pump radiation, $S_p$ is the cross-section of the pump.

Then, the lasing power is defined as $P = R^\pm h \nu_{gen} S_{gen}$, where $\nu_{gen}$ is the frequency and $S_{gen}$ is the cross-section of the lasing. The numeric values are given in the table below.

The system (1)-(3) has been solved via explicit Euler’s method for the balance equations and an explicit two-step method [11] for wave equations.
3. Numerical simulation results

In figure 1 we plot a representative output power dependance \( P_{\text{max}} \) on pumping power \( P_p \) for several absorber widths.

We point out two regions. While pumping power is rather small \( P_{\text{max}} \) is almost linear, as it was observed experimentally [4]. The pumping power further being raised, the dependence becomes nonlinear, reaches some critical value where the output power is maximal. With the increase of saturable absorber length this maximum moves towards higher pumping and output power (fig.1).

We also observe an opposite dependence of pulse duration \( \tau \) on pumping power: with the increase of \( P_p \) the duration starts to decrease down its to minimal value that corresponds to output power maximum. A similar correspondence of an output power maximum and a pulse duration minimum is found in the paper [12]. We found the length dependencies to be monotonous, moreover, if pulse duration increases almost linearly (fig.2), both the peak power and the pulse frequency decrease (fig.3). E.g. \( L_{\text{rez}} \) increase from 0.1 m. to 1.04 m. leads to a more than 20% \( P_{\text{max}} \) decrease. This result may be validated via comparison with an analytic expression for the pulse power [7], where it is stated that \( P_{\text{max}} \) has to be inversely proportional to the round trip time, and, respectively, to the resonator length. Validity of the obtained dependence is also supported by theoretical papers [7, 13] and experimental data [13, 14].

4. Conclusion

Our results show that the proposed model is applicable for passive Q-switched laser description and its parameters optimization. Despite it requires far more computational resources, the proposed model reveals several new results, associated with the correspondence of the maximal and minimal values of photon flux density, pulse duration and with the influence of the saturable absorber length.

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**Figure 1.** Generation peak power and pulse width vs. the pump power ($L_3 = 0.1$ m, $L_s = 2.4$ (1); 2.5 (2); 2.7 (3) mm).

**Figure 2.** Generation peak power and pulse width vs. the resonator length $L_3$ ($P_p = 190$ mWt, $L_s = 0.25$ (1); 0.26 (2); 0.27 (3) mm).

**Figure 3.** Pulse repetition rate vs. the resonator length $L_3$ ($P_p = 190$ mWt, $L_s = 0.25$ (1); 0.26 (2); 0.27 (3) mm).