RESIDUAL ENERGY IN MAGNETOHYDRODYNAMIC TURBULENCE

YUXUAN WANG1, STANISLAV BOLDYREV1,2, AND JEAN CARLOS PEREZ1,3

1 Department of Physics, University of Wisconsin, Madison, WI 53706, USA
2 Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA
3 Space Science Center and Department of Physics, University of New Hampshire, Durham, NH 03824, USA

Received 2011 July 23; accepted 2011 August 19; published 2011 September 27

ABSTRACT

There is mounting evidence in solar wind observations and in numerical simulations that kinetic and magnetic energies are not in equipartition in magnetohydrodynamic (MHD) turbulence. The origin of their mismatch, the residual energy \( E_r = E_k - E_B \), is not understood well. In the present work this effect is studied analytically in the regime of weak MHD turbulence. We find that residual energy is spontaneously generated by turbulent dynamics, and it has a negative sign, in good agreement with the observations. We find that the residual energy condenses around \( k_1 = 0 \) with its \( k_1 \)-spectrum broadening linearly with \( k_\perp \), where \( k_1 \) and \( k_\perp \) are the wavenumbers parallel and perpendicular to the background magnetic field, and the field-perpendicular spectrum of the residual energy has the scaling \( E_r(k_\perp) \propto k_\perp^{-1.5} \) in the inertial interval. These results are found to be in agreement with numerical simulations. We propose that residual energy plays a fundamental role in Alfvénic turbulence and it should be taken into account for the correct interpretation of observational and numerical data.

Key words: magnetohydrodynamics (MHD) – methods: statistical – plasmas – turbulence – waves

Online-only material: magnetohydrodynamics (MHD) – methods: statistical – plasmas – turbulence – waves

1. INTRODUCTION

Turbulence plays an important role in a variety of astrophysical phenomena, including amplification of magnetic fields in planetary and stellar interiors, magnetization and angular momentum transport in accretion disks, scattering of cosmic rays, formation of small-scale density structures in the interstellar medium, and heating of the solar corona and the solar wind. Theoretical description of magnetized plasma turbulence is a complicated task. Valuable guidance for phenomenological modeling has been provided by high-resolution numerical simulations (e.g., Brandenburg & Nordlund 2011; Kritsuk et al. 2011). On the analytical side, significant progress has been made in the limit of weak turbulence, that is, turbulence consisting of weakly interacting linear waves (e.g., Newell et al. 2001; Galtier et al. 2000, 2002), which provides a test bed for fundamental ideas in the theory of turbulence, such as scale invariance, locality of interactions, energy cascades, and anisotropy. Weak magnetohydrodynamic (MHD) turbulence is also interesting on its own as it may play a role in the interstellar medium, in the solar corona and the solar wind, and in planetary and stellar magnetospheres (e.g., Bhattacharjee & Ng 2001; Saur et al. 2002; Luo & Melrose 2006; Rappazzo et al. 2008; Chandran 2010; Chandran et al. 2010).

In the present Letter we address the problem of mismatch between kinetic and magnetic energies recently reported in observations of the solar wind turbulence. Our interest is also motivated by a similar mismatch found in recent numerical simulations of MHD turbulence, suggesting that the phenomenon may, in fact, have a fundamental nature rather than reflect a possible non-universality of solar wind turbulence. By employing the framework of weak MHD turbulence we present the first direct analytical derivation of residual energy generation in MHD turbulence. We demonstrate that kinetic–magnetic equipartition gets spontaneously broken by interacting random Alfvén waves, even if it is present initially. Our results indicate that this effect plays a fundamental role in the energy cascade and it should be taken into account in phenomenological modeling of MHD turbulence.

For our analysis we write the incompressible MHD equations in terms of the Elsasser variables:

\[
\left( \frac{\partial}{\partial t} + \mathbf{v}_A \cdot \nabla \right) \mathbf{z}^\pm + (\mathbf{z}^\mp \cdot \nabla) \mathbf{z}^\pm = -\nabla P, \tag{1}
\]

where the Elsasser variables are defined as \( \mathbf{z}^\pm = \mathbf{v} \pm \mathbf{b} \). \( \mathbf{v} \) is the fluctuating plasma velocity, \( \mathbf{b} \) is the fluctuating magnetic field normalized by \( \sqrt{4\pi\rho_0} \), \( \mathbf{v}_A = \mathbf{B}_0/\sqrt{4\pi\rho_0} \) is the Alfvén velocity corresponding to the uniform magnetic field \( \mathbf{B}_0 \), \( P = (p/\rho_0 + b^2/2) \) includes the plasma pressure \( p \) and the magnetic pressure, \( \rho_0 \) is the constant mass density, and we neglected the terms describing viscous and resistive dissipation. One observes that when \( \mathbf{z}^\pm(\mathbf{x}, t) \equiv 0 \), an arbitrary function \( \mathbf{z}^\pm(\mathbf{x}, t) = F^\pm(\mathbf{x} \pm \mathbf{v}_A t) \) is an exact solution of (1), which represents a non-dispersive Alfvén wave propagating parallel or antiparallel to \( \mathbf{B}_0 \) with the Alfvén speed. Nonlinear interactions are the result of collisions between counterpropagating Alfvén wave packets. This qualitative picture plays a central role in phenomenological models of MHD turbulence; comprehensive reviews of recent analytical and numerical results can be found in, e.g., Galtier (2009), Sridhar (2010), Mininni (2011), Newell & Rumpf (2011), and Tobias et al. (2011).

The ideal MHD system (1) conserves the two independent Elsasser energies, \( E^\pm = \langle |\mathbf{z}^\pm|^2 \rangle/4 \), related to the total energy and cross-helicity, \( E = E^+ + E^- \) and \( H_c = E^+ - E^- \), respectively. In a turbulent state, when energy is supplied to the system at large scales, both energies \( E^\pm \) cascade toward small scales where they are converted into heat by viscosity and resistivity. Theories and numerical simulations of MHD turbulence address the Fourier spectra of the energies \( E^\pm(k) \) in the inertial range of scales, that is, scales much smaller than the forcing scales and much larger than the dissipation scales.

In the picture of counterpropagating Alfvén modes it is often assumed that energy spectra are the same for both kinetic and
magnetic fluctuations, since such energy equipartition holds for individual Alfvén wave packets. However, recent solar wind observations demonstrate contradictions to this assumption: the Fourier energy spectra of magnetic and velocity fluctuations appear to have different slopes in the inertial interval (Podesta et al. 2007; Salem et al. 2009; Tessier et al. 2009; Chen et al. 2011a, 2011b; Wicks et al. 2011; Li et al. 2011). This is somewhat similar to the mismatch in the spectra found in numerical simulations of strong MHD turbulence (Müller & Grappin 2005; Boldyrev et al. 2011). Most intriguing, however, is the fact that a significant difference between magnetic and kinetic energies was detected in numerical studies of weak MHD turbulence (Boldyrev & Perez 2009) consisting of mostly independent Alfvén waves for which equipartition between magnetic and kinetic energies holds exactly.

The mismatch between magnetic and kinetic energies, the so-called residual energy \( E_r(k) = E_m(k) - E_v(k) \), was noted in previous works (e.g., Grappin et al. 1983; Zank et al. 1996; Müller & Grappin 2005), however, it has not been fully explored in models of MHD turbulence, since such models are often based on the conserved quantities \( E^\parallel, E, H_\perp, \) which do not contain information about the residual energy. In this work, we present the analytic derivation of the residual energy in the case of weak MHD turbulence. We demonstrate that residual energy is spontaneously generated by interacting Alfvén waves even if it is absent initially, plays a fundamental role in the turbulent dynamics, and provides a qualitative explanation for the breakdown of magnetic and kinetic equipartition observed in the solar wind.

2. THE ORIGIN OF THE RESIDUAL ENERGY

We concentrate on turbulence of shear-Alfvén waves whose polarizations are normal to the guide field, \( \mathbf{z}^\pm \perp B_0 \). We assume that the guide field is strong; its strength with respect to the magnetic and velocity fluctuations is characterized by a small parameter \( \epsilon \sim z^\pm/v_A \ll 1 \). We also assume that the turbulence is weak, that is, the wave spectrum in the field-parallel direction is broad enough to ensure that the nonlinear terms are small, \( k_\parallel v_A \gg k_\perp z^\pm \), where \( k_1 \parallel \) and \( k_1 \perp \) are typical wavenumbers characterizing the spectral width in the field-parallel and field-perpendicular directions and \( z^\pm \) are typical (rms) values of the fluctuations. The Elsasser Fourier energies are given by the correlation functions

\[
\langle z^\pm(k, t) \cdot z^\pm(k', t) \rangle = \epsilon^2 v_A^2 q^\pm(k_1, k_\perp, t) \delta(k + k'),
\]

while the residual energy is related to

\[
\langle z^+(k, t) \cdot z^-(k', t) \rangle = \epsilon^2 v_A^2 Q^0(k_1, k_\perp; t) \delta(k + k').
\]

In these formulas we assume spatial homogeneity and average over a statistical ensemble. The Elsasser energies are real, while the residual correlation function satisfies \( Q^0(k_1, k_\perp; t) = Q^0(-k_1, k_\perp; t) \). Its real part is then the residual energy \( e_r(k) = \Re Q^0 \). In practical applications one also uses the phase-volume-compensated residual energy defined as \( E_r(k) = \epsilon^2 v_A^2 e_r(k) 2\pi k_\perp \). In previous treatments (Galtier et al. 2000, 2002), the residual energy was assumed to be zero as it would be the case for independent \( z^+ \) and \( z^- \) waves. In the present study we do not make such an assumption; rather, we derive the equation for the residual energy.

The derivation is performed perturbatively in \( \epsilon \), following the standard procedure (e.g., Galtier et al. 2002). When the nonlinear terms in Equation (1) are neglected, both \( z^+(k) \) oscillate in time and so does the residual energy, \( Q^0(k, t) = q^0(k) \exp(2ik_1v_A t) \). When the nonlinear interaction is taken into account, \( q^0 \) acquires small corrections that one finds, to the first non-vanishing order, by integrating the equation

\[
\partial_t q^0(k, t) = -\epsilon^{-2i k_1 v_A t} \epsilon^2 \int R_{k, pq}[q^+(q)\{q^-(p) - q^-(k)\} + q^-(q)[q^+(p) - q^+(k)]] \delta(k - p - q) d^3p d^3q, \tag{4}
\]

where \( R_{k, pq} = (\pi v_A/2)(k_1 \times q_\perp)^2(k_\perp \cdot p_\perp)(k_\perp \cdot q_\perp)/(k_1^2 p_\perp^2 q_\perp^2) \), and we introduce a “smeared” delta-function \( \delta(k_1) = (1 - \exp(-2ik_1 v_A t))/(i\pi k_1) \), which approaches \( \delta(k_1) + (i/\pi)^3\delta(1/k_1) \) at large \( t \). Since we are interested in spontaneous generation of the residual energy, we assume that it is absent initially; therefore, in the right-hand side of Equation (4) we neglected the terms proportional to \( q^0 \). The derivation of Equation (4) is standard although lengthy, and it will be presented elsewhere. In what follows we discuss the main consequences of the obtained result.

3. THE SPECTRUM OF THE RESIDUAL ENERGY

First, we notice that the integral in the right-hand side of Equation (4) is essentially nonzero. It therefore describes the generation of the residual energy by Alfvén waves. The important conclusion is that the residual energy is generated by nonlinear interactions even if it is zero initially. Moreover, one can directly verify that for all the spectra of interest, the integral in the right-hand side of Equation (4) is positive. As we shall see below, this means that the generated residual energy is negative, that is, the magnetic energy exceeds the kinetic one.

A simple evaluation of the integral in Equation (4) can be performed for the case of balanced MHD turbulence, that is, for \( q^+ = q^- \). In this case, the Elsasser energy spectra have the well-known scaling (e.g., Ng & Bhattacharjee 1996; Galtier et al. 2000) \( q^+ = q^- = f(k_1)k_\perp^{-3} \), where \( f(k_1) \) is arbitrary, and we assume that it is smooth at least initially. For the reasons that will be clear momentarily, we estimate the integral in the right-hand side of Equation (4) at \( k_\perp = 0 \):

\[
\int R_{k, pq} \ldots \delta(q_\perp) \delta(k - p - q) d^3p d^3q = \alpha k_\perp^{-2} v_A, \tag{5}
\]

where the numerical factor is \( \alpha \sim f^2(0) \). Non-vanishing of this term at \( k_\perp = 0 \) implies that Equation (4) has a solution diverging in time, which is unphysical. The reason is that when \( q^\perp \) becomes large one cannot neglect the nonlinear terms containing \( q^\perp \) in Equation (4). As we shall see, in order to obtain the convergent result, it is enough to retain the terms linear in \( q^\perp \), which are proportional to \( q^\perp q^\perp \). Those terms describe relaxation of the residual energy due to its interaction with the Elsasser energies. Their structure can be found from a simple estimate, if one notices that these terms should have the same dimensional structure as the right-hand side term in Equation (4) if one replaces one of \( q^\perp \) by \( q^\parallel \). The corresponding relaxation term can thus be added to the right-hand side of Equation (4) in the

4 This result is a statistical effect of many independent collisions of Alfvén waves. It does not imply that a particular collision of two Alfvén wave packets necessarily produces negative residual energy.

5 A more precise numerical integration gives \( \alpha \approx 0.065 \pi^2 f^2(0) \); however, our discussion of scaling properties of the residual energy does not require the knowledge of numerical coefficients with such precision.
form $\gamma q^0$, with the relaxation rate $\gamma \sim -e^2 v_A k_\perp^2 q^0(k)$. The estimate of the relaxation rate at $k_\parallel = 0$ gives

$$\gamma(0, k_\perp) = -\beta e^2 k_\perp v_A,$$  
where $\beta \sim f(0)$.

Collecting results (5) and (6), we rewrite Equation (4) in the form

$$\partial_t q^0(k, t) = -\frac{a e^2 k_\perp^2}{\beta e^2 k_\perp - 2i k_\parallel} e^{-2i k_\parallel v_A t},$$

which has to be solved together with the initial condition $q^0 = 0$. The solution is given by

$$q^0(k, t) = -\frac{a e^2 k_\perp^2}{\beta e^2 k_\perp - 2i k_\parallel} e^{-2i k_\parallel v_A t},$$

and the residual energy is found as

$$e_r(k_\parallel, k_\perp) = \Re Q^0 = -\frac{a e^2 k_\perp^2}{\beta e^2 k_\perp + 4k_\parallel^2}. $$

The residual energy can be rewritten as

$$e_r(k_\parallel, k_\perp) = -a \pi e^2 k_\perp^2 \Delta(2k_\parallel),$$

where the $\Delta$ function has support inside a narrow wedge-shaped domain:

$$k_\parallel \leq \beta e^2 k_\perp / 2.$$  

According to the result $\lim_{\epsilon \to 0} \epsilon / (\epsilon^2 + \epsilon^3) = \pi \delta(x)$, the function $\Delta$ would turn into the $\delta$-function in the limit $\epsilon \to 0$. This explains why the region around $k_\parallel = 0$ was relevant in our estimates (5) and (6). We will refer to such a spectrum as “condensate.” It is also convenient to define the field-perpendicular spectrum of the residual energy,

$$E_r(k_\perp) = \int e^2 v_A^2 e_r(k_\parallel, k_\perp) 2\pi k_\parallel dk_\parallel = -a \pi e^2 k_\perp^2 k_\perp^2.$$  

Expressions (9)–(12) are the main results of the present work.

4. NUMERICAL RESULTS

In this section, we test our analytic predictions in numerical simulations. In the presence of a strong guide field, the universal properties of MHD turbulence can be studied in the framework of reduced MHD, which can be effectively used in numerical simulations of both strong and weak turbulence (e.g., Dmitruk et al. 2003; Oughton et al. 2004; Galtier & Chandran 2006; Perez & Boldyrev 2008; Rappazzo et al. 2010). In our numerical setup, the $z^+$ and $z^-$ fields are independently driven by Gaussian-distributed random, short-time correlated forces with zero mean and prescribed variances, so that no residual energy is supplied by the driving routine. The forces are applied at $k_\parallel = 1, \ldots, 16$ and $k_\perp = 1, 2$ in the Fourier space. The box is elongated in the field-parallel direction according to $L_\parallel / L_\perp = B_0 / b_{\text{ms}}$. We employ a fully dealiased pseudo-spectral method; the run has a resolution of 512$^3$, the Reynolds number $Re = 2400$, and it is averaged over 41 large-scale eddy turnover times. We choose viscosity equal to magnetic diffusivity. The details on the code and the numerical setup can be found in Perez & Boldyrev (2010).

We find that the residual-energy condensate is indeed generated in weak balanced MHD turbulence, as shown in Figure (1). The condensate is negative and its broadening with $k_\parallel$ shown in Figure (2) is consistent with the linear law predicted in Equation (11). The scaling behavior of the field-perpendicular spectrum of the residual energy is also consistent with the analytic prediction in Equation (12); see Figure (2). When we repeated the computation for the imbalanced case, that is, the case when $q^+ \neq q^-$ (not presented here), we did not detect essential changes in the structure and scaling of the condensate.

5. DISCUSSION

In this Letter, we demonstrated that residual energy is spontaneously generated by nonlinearly interacting Alfvén waves and it accumulates in a narrow region of phase space (11). This explains the phenomenon of “condensate” of the residual energy numerically observed in Boldyrev & Perez (2009). Although the total (phase-space integrated) residual energy is small, its presence is crucial for turbulent dynamics for the following reason. Inside region (11), that is, for $k_\parallel \approx 0$, the residual energy is comparable to the Elsasser energies and it follows the same scaling, $e_r(0, k_\perp) \propto q^\pm(0, k_\perp) \propto k_\perp^2$. Since the turbulent cascade of Alfvén waves crucially depends on the modes with $k_\parallel \approx 0$
turbulence (Galtier et al. 2000, 2002), it was assumed that
see, e.g., Equation (4). In previous treatments of weak MHD
\[ q \]
shallow spectrum holds in the inertial interval and breaks down
(e.g., Montgomery & Turner 1981; Sridhar & Goldreich 1994;
Montgomery & Turner1981; Sridhar & Goldreich1994; Bhattacharjee &
Galtier, S. 2009, Nonlinear Process. Geophys., 16, 83
Galtier, S., & Chandran, B. D. G. 2006, Phys. Plasmas, 13, 114505
Galtier, S., Nazarenko, S. V., Newell, A. C., & Pouquet, A. 2000, J. Plasma
Physics, 63, 447
Galtier, S., Nazarenko, S. V., Newell, A. C., & Pouquet, A. 2002, ApJ, 564, L49
Grappin, R., Pouquet, A., & Leorat, J. 1983, A&A, 126, 51
Kritsuk, A. G., Nordlund, Å., Collins, D., et al. 2011, ApJ, 737, 13
Li, G., Miao, B., Hu, Q., & Qin, G. 2011, Phys. Rev. Lett., 106, 125001
Luo, Q., & Melrose, D. 2006, MNRAS, 368, 1151
Mininni, P. D. 2011, Annu. Rev. Fluid Mech., 43, 377
Montgomery, D., & Turner, L. 1991, Phys. Fluids, 24, 825
Müller, W., & Grappin, R. 2005, J. Geophys. Res., 101, 17093
Podesta, J. J., Roberts, D. A., & Goldstein, M. L. 2007, ApJ, 664, 543
Rappazzo, A. F., Velli, M., & Einaudi, G. 2010, ApJ, 722, 65
Rappazzo, A. F., Velli, M., Einaudi, G., & Dahlburg, R. B. 2008, ApJ, 677, 1348
Salem, C., Mangeney, A., Bale, S. D., & Veltri, P. 2009, ApJ, 702, 537
Saur, J., Politano, H., Pouquet, A., & Matthaeus, W. H. 2002, A&A, 386, 699
Sridhar, S. 2010, Astron. Nachr., 331, 93
Sridhar, S., & Goldreich, P. 1994, ApJ, 432, 612
Tessein, J. A., Smith, C. W., MacBride, B. T., et al. 2009, ApJ, 692, 684
Tobias, S. M., Cattaneo, F., & Boldyrev, S. 2011, arXiv:1103.3138
Wicks, R. T., Horbury, T. S., Chen, C. H. K., & Schekochihin, A. A. 2011, Phys.
Rev. Lett., 106, 045001
Zank, G. P., Matthaeus, W. H., & Smith, C. W. 1996, J. Geophys. Res., 101, 17093

REFERENCES

Bhattacharjee, A., & Ng, C. S. 2001, ApJ, 548, 318
Boldyrev, S., & Perez, J. C. 2009, Phys. Rev. Lett., 103, 225001
Boldyrev, S., Perez, J. C., Borovsky, J. E., & Podesta, J. J. 2011, arXiv:1106.0700
Brandenburg, A., & Nordlund, Å. 2011, Rep. Prog. Phys., 74, 046901
Chandran, B. D. G. 2010, ApJ, 720, 548
Chandran, B. D. G., Li, B., Rogers, B. N., Quataert, E., & Germschewski, K. 2010, ApJ, 720, 503
Chen, C. H. K., Bale, S. D., Salem, C., & Mozer, F. S. 2011a, ApJ, 737, L41
Chen, C. H. K., Mallet, A., Youssef, T. A., Schekochihin, A. A., & Horbury, T. S. 2011b, MNRAS, 415, 3219
Dmitruk, P., Gómez, D. O., & Matthaeus, W. H. 2003, Phys. Plasmas, 10, 3584
Galtier, S. 2009, Nonlinear Process. Geophys., 16, 83
Galtier, S., & Chandran, B. D. G. 2006, Phys. Plasmas, 13, 114505
Galtier, S., Nazarenko, S. V., Newell, A. C., & Pouquet, A. 2000, J. Plasma
Physics, 63, 447
Galtier, S., Nazarenko, S. V., Newell, A. C., & Pouquet, A. 2002, ApJ, 564, L49
Grappin, R., Pouquet, A., & Leorat, J. 1983, A&A, 126, 51
Kritsuk, A. G., Nordlund, Å., Collins, D., et al. 2011, ApJ, 737, 13
Li, G., Miao, B., Hu, Q., & Qin, G. 2011, Phys. Rev. Lett., 106, 125001
Luo, Q., & Melrose, D. 2006, MNRAS, 368, 1151
Mininni, P. D. 2011, Annu. Rev. Fluid Mech., 43, 377
Montgomery, D., & Turner, L. 1991, Phys. Fluids, 24, 825
Müller, W., & Grappin, R. 2005, Phys. Rev. Lett., 95, 114502
Newell, A. C., Nazarenko, S., & Biven, L. 2001, Physica D: Nonlinear Phenom., 152, 520
Newell, A. C., & Rumpf, B. 2011, Annu. Rev. Fluid Mech., 43, 59
Ng, C. S., & Bhattacharjee, A. 1996, ApJ, 465, 845
Oughton, S., Dmitruk, P., & Matthaeus, W. H. 2004, Phys. Plasmas, 11, 2214
Perez, J. C., & Boldyrev, S. 2008, ApJ, 672, L61
Perez, J. C., & Boldyrev, S. 2010, ApJ, 710, L60
Podesta, J. C., & Boldyrev, S. 2008, ApJ, 672, L61
Rappazzo, A. F., Velli, M., & Einaudi, G. 2010, ApJ, 722, 65
Rappazzo, A. F., Velli, M., Einaudi, G., & Dahlburg, R. B. 2008, ApJ, 677, 1348
Salem, C., Mangeney, A., Bale, S. D., & Veltri, P. 2009, ApJ, 702, 537
Saur, J., Politano, H., Pouquet, A., & Matthaeus, W. H. 2002, A&A, 386, 699
Sridhar, S. 2010, Astron. Nachr., 331, 93
Sridhar, S., & Goldreich, P. 1994, ApJ, 432, 612
Tessein, J. A., Smith, C. W., MacBride, B. T., et al. 2009, ApJ, 692, 684
Tobias, S. M., Cattaneo, F., & Boldyrev, S. 2011, arXiv:1103.3138
Wicks, R. T., Horbury, T. S., Chen, C. H. K., & Schekochihin, A. A. 2011, Phys.
Rev. Lett., 106, 045001
51164, and the NSF Center for Magnetic Self-organization in Laboratory and Astrophysical Plasmas at the University of Wisconsin–Madison. High Performance Computing resources were provided by the Texas Advanced Computing Center (TACC) at the University of Texas at Austin under the NSF-Teragrid Project TG-PHY080013N.

We established that the spontaneously generated residual energy is always negative. Although this result is obtained in the framework of weak MHD turbulence, it provides the first analytic explanation for the observational and numerical findings that magnetic energy exceeds kinetic energy in the inertial interval of MHD turbulence (e.g., Müller & Grappin 2005; Podesta et al. 2007; Boldyrev & Perez 2009).

We derived that the residual energy has the field-perpendicular spectrum \[ E_r(k_z) \propto k_z^{-1} \] (12). This relatively shallow spectrum holds in the inertial interval and breaks down at sufficiently large \( k_z \) when the nonlinear broadening of the residual energy spectrum in the field-parallel direction (11) becomes comparable to the width of the field-parallel energy spectra of the Alfvén waves, \( q^\parallel(k_z) \). It is easy to see, however, that this is precisely the scale beyond which the weak interaction approximation breaks down, and the turbulence becomes strong. In our future work we will extend our analysis of residual energy to the case of strong MHD turbulence.

This work was supported by U.S. Department of Energy awards DE-FG02-07ER54932, DE-SC0003888, DE-SC0001794, NSF grant PHY-0903872, NSF/DOE grant AGS-1003451, and in part by NSF grant No. NSF PHY05-