Radiative Leptonic Decays of $D_s^\pm$ and $D^\pm$ Mesons

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Abstract

In this work, we investigate the radiative leptonic decays $D_{(s)}^- \rightarrow \gamma \ell \bar{\nu}$ ($\ell = e, \mu$) at tree level within the non-relativistic constituent quark model and the effective Lagrangian for the heavy flavor decays. We find that the contributions from the three Feynman diagrams are all important. With the full calculation, the decay branching ratios are of the order of $10^{-5}$ for $D_s^- \rightarrow \gamma \ell \bar{\nu}$ ($\ell = e, \mu$) and $10^{-6}$ for $D^- \rightarrow \gamma \ell \bar{\nu}$ ($\ell = e, \mu$), respectively. These decays can be measured at B factories and future CLEO-C experiments to determine the decay constants $f_{D_s}$ and $f_D$. 
1 Introduction

The pure-leptonic decays of heavy mesons are useful to determine the meson decay constants, and they are also sensitive to new physics beyond the Standard Model (SM) [1]. But it is well known that the decays of $D_{(s)}^-$ into light lepton pairs (see, Fig.1) are helicity suppressed by $m_{\ell}^2/m_{D(s)}^2$:

$$\Gamma(D \rightarrow \ell \bar{\nu}) = \frac{G_F^2 |V_{CKM}|^2 f^2_D m_{D(s)}^3 m_{\ell}^2}{8\pi m_{D(s)}^4} \left( 1 - \frac{m_{\ell}^2}{m_{D(s)}^2} \right)^2.$$  \hspace{1cm} (1)

Here $V_{CKM}$ is the corresponding Cabibbo-Kobayashi-Maskawa matrix element. In the case of $D_{s}^- (D^-)$ decay, it is $V_{cs} (V_{cd})$. Fortunately the helicity suppression can be overcome by a photon radiated from the charged particles at the cost of the electromagnetic suppression with coupling constant $\alpha$. It is possible that the radiative decays may be comparable or even larger than the corresponding pure leptonic decays [2].

Several years ago, D. Atwood et al. calculated $B_{\pm} (D_{s}^{\pm}) \rightarrow \gamma \ell \nu$ in a non-relativistic quark model [3], with a large branching ratio. In fact, they only considered one dominant diagram, neglecting other diagrams. They found the order of $10^{-4}$ for the branching ratio of $D_{s}^{\pm} \rightarrow \gamma \ell \nu$ decay.

Later on, Gregory P. Korchemsky et al. calculated these decays in perturbative QCD approach [4]. They gave larger branching ratios. And C. Q. Geng et al. did the calculation in the light front quark model [5]. Their branching ratios are rather smaller.

In this Letter, We will study the radiative leptonic decays $D_{(s)}^- \rightarrow \gamma \ell \bar{\nu} (\ell = e, \mu)$ carefully at tree level, using the non-relativistic constituent quark model, similar to Ref.[3], but including all diagrams. In the following section, we will calculate the processes $D_{(s)}^- \rightarrow \gamma \ell \bar{\nu} (\ell = e, \mu)$ in the framework of the constituent quark model (see, for example [6]). In the third section, we will compare the result with some of the previous calculations[3, 4, 5]. At last we will conclude the calculation briefly.
We begin with the quark diagram calculation of $D_s^- \to \gamma \ell \bar{\nu}$ ($\ell = e, \mu$). There are four charged particle lines in Fig.1, which correspond to four Feynman diagrams contributing to the radiative decays $D_s^- \to \gamma \ell \bar{\nu}$ ($\ell = e, \mu$) at tree level, as shown in Fig.2. However when the photon line is attached to the internal charged line of W boson such as Fig.2d, there is a suppression factor of $m^2_c/m^2_W$. Thus we neglect it for simplicity. To be consistent in the following calculation, we will always neglect the terms suppressed by the factor $m^2_c/m^2_W$.

The decay amplitudes corresponding to the other three diagrams are

$$\mathcal{H}_{a+b} = -i\sqrt{2}G_F e V_{cs} \bar{c} \left[ Q_c \not\!{\gamma} \frac{\not{p}_c - \not{p}_e + m_c}{(p_c \cdot p_\gamma)} \gamma_\mu P_L + Q_s P_R \gamma_\mu \frac{\not{p}_s - \not{p}_\gamma + m_s}{(p_s \cdot p_\gamma)} \not{\gamma} \right] s \ (\ell \gamma^\mu P_L \nu),$$

$$\mathcal{H}_c = -i\sqrt{2}G_F e V_{cs} (\bar{c}\gamma^\mu P_L s) \left[ \bar{\ell} \not\!{\gamma} \frac{\not{p}_\ell + m_\ell}{(p_\ell \cdot p_\gamma)} \gamma_\mu P_L \nu \right].$$

(2)

As mentioned in the Introduction, we will use the constituent quark model to reduce the amplitudes into the ‘hadronic level’. In this simple model, both of the quark and anti-quark inside the meson move with the same velocity. Thus we have

$$p_\mu^c = (m_c/m_{D_s}) p_\mu^{D_s}, \quad p_\mu^s = (m_s/m_{D_s}) p_\mu^{D_s}.$$  

(3)

We use further the interpolating field technique [7] which relate the hadronic matrix elements to the decay constants of the mesons. The decay constant $f_P$ for a charged pseudoscalar meson is defined by [8]:

$$<0|A_\mu(0)|P(q)> = i f_P q_\mu.$$  

(4)

In the case of $D_s^-$, we have

$$<0|\bar{c}\gamma^\mu \gamma_5 s|D_s> = i f_{D_s} p_\mu^{D_s}.$$  

(5)

The whole decay amplitude for $D_s^- \to \gamma \ell \bar{\nu}$ ($\ell = e, \mu$) is derived from eqs.(2,3,5) by
neglecting the terms suppressed by $m_l/m_c^1$

$$\mathcal{A} = \frac{\sqrt{2}eG_FV_{cs}}{6(p_{D_s} \cdot p_\gamma)} f_{D_s} \left[ \left( \frac{m_{D_s}}{m_s} - 2 \frac{m_{D_s}}{m_c} \right) i\epsilon_{\mu\nu\alpha\beta} p_\mu^\nu p_\alpha^\beta \right. $$

$$+ \left( 6 - \frac{m_{D_s}}{m_s} - 2 \frac{m_{D_s}}{m_c} \right) (p_{\gamma\nu} \epsilon_{\gamma\mu} - p_{\gamma\mu} \epsilon_{\gamma\nu}) p_\mu^\nu \right] \left( \bar{e}\gamma^\mu P_L \nu \right). \tag{6}$$

In the $D_s^-$ rest frame, the differential decay width [8] is

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32(M_{D_s})^3} |\mathcal{A}|^2 d\hat{s} d\hat{t}, \tag{7}$$

Neglecting the mass of light leptons, we get the differential decay width:

$$\frac{d\Gamma}{d\hat{s} d\hat{t}} = \frac{\alpha G_F^2 |V_{cs}|^2 f_{D_s}^2 f_{D_s} m_{D_s}^3 (m_{D_s}^2 - \hat{s} - \hat{t})^2 + x_c \hat{t}^2]}{144\pi^2 m_{D_s}^2 (m_{D_s}^2 - \hat{s})^2}, \tag{8}$$

with

$$x_s = \left( 3 - \frac{m_{D_s}}{m_s} \right)^2, \quad x_c = \left( 3 - 2 \frac{m_{D_s}}{m_c} \right)^2. \tag{9}$$

The $\hat{s}, \hat{t}$ are defined as $\hat{s} = (p_\ell + p_\nu)^2$, $\hat{t} = (p_\ell + p_\gamma)^2$. Integrating eqn.(8) in phase space, we obtain the decay width

$$\Gamma = \frac{\alpha G_F^2 |V_{cs}|^2 f_{D_s}^2 m_{D_s}^3}{2592\pi^2} \left[ x_s + x_c \right]. \tag{10}$$

Using $\alpha = 1/137$, $m_c = 1.5$ GeV, $m_{D_s} = 1.97$ GeV, $|V_{cs}| = 0.974$ [8], we get

$$\Gamma(D_s \to \gamma\ell\bar{\nu}) = 2.3 \times 10^{-17} \times \left( \frac{f_{D_s}}{230\text{MeV}} \right)^2 \text{GeV}. \tag{11}$$

For the lifetime $\tau(D_s) = 0.5 \times 10^{-12}$ s [8], and the decay constant used as $f_{D_s} = 230\text{MeV}$ [3], the branching ratio is found to be $1.8 \times 10^{-5}$. From eqs.(9,10), we can easily see that the decay width is sensitive to the decay constant $f_{D_s}^2$, and the constituent quark mass $m_c$ (or $m_s$). Any changes of the two input parameters, will result in a big change in the decay amplitude. Therefore the prediction of branching ratios remain the accuracy at the order of magnitude, unless we can precisely determinate the input parameters.

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1By neglecting terms proportional to $m_l/m_c$, we drop the infrared divergence terms, which should be canceled by the radiative corrections of the pure leptonic decay $D_s^- \to \ell\bar{\nu}$ [9].
It is worthy of considering the differential spectrum for experimental purposes. Deriving from eqn.(8), we obtain

\[ \frac{m_{D_s} \frac{d\Gamma}{\Gamma}}{dE_\gamma} = \frac{1}{\Gamma} \frac{d\Gamma}{d\lambda} = 24\lambda_\gamma(1 - 2\lambda_\gamma), \]  

(12)

with \( \lambda_\gamma = E_\gamma/m_{D_s} \). This result is the same as Ref.[3]. We show the photon energy spectrum in Fig.3 as the solid line. This is clearly distinct from the bremsstrahlung photon spectrum.

The lepton energy distributions are

\[ \frac{1}{\Gamma} \frac{d\Gamma}{d\lambda_\nu} = \frac{36}{x_\nu + x_c} \left\{ x_c (1 - 2\lambda_\nu) \left[ 2\lambda_\nu + (1 - 2\lambda_\nu) \ln(1 - 2\lambda_\nu) \right] \right. \]
\[ + x_\nu \left[ 2\lambda_\nu(3 - 5\lambda_\nu) + (1 - 2\lambda_\nu)(3 - 2\lambda_\nu) \ln(1 - 2\lambda_\nu) \right] \}, \]

(13)

\[ \frac{1}{\Gamma} \frac{d\Gamma}{d\lambda_\ell} = \frac{36}{x_\ell + x_e} \left\{ x_\ell(1 - 2\lambda_\ell) \left[ 2\lambda_\ell + (1 - 2\lambda_\ell) \ln(1 - 2\lambda_\ell) \right] \right. \]
\[ + x_e \left[ 2\lambda_\ell(3 - 5\lambda_\ell) + (1 - 2\lambda_\ell)(3 - 2\lambda_\ell) \ln(1 - 2\lambda_\ell) \right] \}, \]

(14)

where \( \lambda_\nu = E_\nu/m_{D_s}, \lambda_\ell = E_\ell/m_{D_s} \). We show the neutrino energy spectrum \( \frac{1}{\Gamma} \frac{d\Gamma}{d\lambda_\nu} \), and the charged lepton (e, \( \mu \)) energy spectrum \( \frac{1}{\Gamma} \frac{d\Gamma}{d\lambda_\ell} \) in Fig.3 as dashed and dash-dotted lines, respectively. Eqs.(13,14) are consistent with Ref.[3], if we consider only the diagram in Fig.2a with the photon connecting the strange quark line like the case in that paper.

The formulas above can be applied to the case of \( D^- \) decay i.e. \( D^- \to \gamma \ell \bar{\nu} \ (\ell = e, \mu) \), directly. We get the decay width easily:

\[ \Gamma(D^- \to \gamma \ell \bar{\nu}) = \frac{\alpha G_F^2 |V_{cd}|^2}{2592\pi^2} f_{D^-}^2 m_{D^-}^3 [x_d + x_c]. \]

(15)

with

\[ x_d = \left( 3 - \frac{m_{D^-}}{m_d} \right)^2, \quad x_c = \left( 3 - \frac{2m_{D^-}}{m_c} \right)^2. \]

Using \( m_d = 0.37 \text{ GeV}, \ m_{D^-} = 1.87 \text{ GeV}, \ |V_{cd}| = 0.22 \ [8] \), we get

\[ \Gamma(D^- \to \gamma \ell \bar{\nu}) = 2.9 \times 10^{-18} \times \left( \frac{f_{D^-}}{230\text{MeV}} \right)^2 \text{ GeV}. \]

(16)

For \( \tau(D^-) = 1.05 \times 10^{-12} \text{ s} \ [8] \), the decay branching ratio is \( 4.6 \times 10^{-6} \) with the decay constant \( f_{D^-} = 230\text{MeV} \ [5] \). Again, without the precise determination of the constituent quark mass \( m_d \), the decay branching ratio is only meaningful at the order of magnitude.
In the case of $D^-$ decay, the formulas for the differential spectra of photon and lepton energy distribution are the same as the $D_s$ decay, except replacing $x_s$ with $x_d$ in eqs.(12,13,14). Their numerical results are shown in Fig.4 as solid, dashed and dash-dotted lines for $\gamma, \nu, \ell$ respectively. From Fig.3 and Fig.4, we can see that the differential spectra of $D^+_s$ and $D^\pm$ radiative leptonic decays are very similar. Only the endpoints of leptonic energy spectra are different.

3 Comparison with other calculations

In Ref.[3], D. Atwood et. al. made the calculation within the non-relativistic constituent quark model like us. As stated in the introduction part that they just considered the contribution of the emission of photon from the strange quark, i.e. Fig.2a, for they made an analogy with $B^-$ decay directly. After our careful calculation, we conclude that the contribution of the Feynman diagram where the photon is emitted from the initial light quark is dominant enough to neglect the other diagrams in the case of $B^-$ decay, but not for the case of $D^-_s$ or $D^-$. It can be seen at Table.1 that the contribution of the other two diagrams corresponding to Fig.2b and 2c must be considered because the interference among the three Feynman diagrams is large and destructive. That is the reason why their branching ratio of $\text{Br}(D^-_s \to \gamma\ell\bar{\nu})(\ell = e, \mu)$ decay [3] is about four times of ours.

Gregory P. Korchemsky et al. used the perturbative QCD method to calculate B and D meson radiative decays. Their result for $D^-$ decay is very large. In fact, the perturbative QCD approach [10] is good for the B meson decays since the energy release is very large there, but may not be good for the lighter D meson decays. In addition, our result is consistent with that of [5] within the light front quark model in the case of $D^-_s \to \gamma e\bar{\nu}$. It is instructive to calculate with various models, and the accuracy of various models will be tested in future experiments.
Table 1: Decay width with different diagrams and their relative size. Considering only the diagram where the photon is emitted from the initial light quark (Fig. 2a), heavy quark (Fig. 2b) or lepton (Fig. 2c), we get $\Gamma_a$, $\Gamma_b$, $\Gamma_c$, respectively. And considering the three diagrams together, we get $\Gamma_{a+b+c}$.

|     | $\Gamma_a$  | $\Gamma_b$  | $\Gamma_c$  | $\Gamma_{a+b+c}$ | $\Gamma_a : \Gamma_b : \Gamma_c : \Gamma_{a+b+c}$ |
|-----|--------------|--------------|--------------|-------------------|--------------------------------------------------|
| $B^-$| $1.7 \times 10^{-18}$ | $5.7 \times 10^{-22}$ | $5 \times 10^{-20}$ | $1.2 \times 10^{-18}$ | $1.40 : 0.0005 : 0.04 : 1$ |
| $D_s$| $3.4 \times 10^{-16}$ | $8 \times 10^{-17}$  | $4 \times 10^{-16}$ | $2.3 \times 10^{-17}$ | $14.72 : 3.47 : 17.32 : 1$ |
| $D^-$| $2.1 \times 10^{-17}$ | $2.7 \times 10^{-18}$ | $1.8 \times 10^{-17}$ | $2.9 \times 10^{-18}$ | $7.30 : 0.94 : 6.03 : 1$ |

4 Summary

We have calculated $D_{(s)}^- \rightarrow \gamma \ell \bar{\nu}$ ($\ell = e, \mu$) decay in non-relativistic constituent quark model. We included all the three Feynman diagrams, and found that none of them is small in $D_{(s)}$ decays. We obtained the decay branching ratios of $D_{s}^- \rightarrow \gamma \ell \bar{\nu}$, $D^- \rightarrow \gamma \ell \bar{\nu}$ ($\ell = e, \mu$) are of order $10^{-5}$ and $10^{-6}$ respectively. Such a branching ratio for the radiative leptonic decays can be measured in the two B factories and the future CLEO-C Experiments.

Eqs.(10,15) indicate that the decay rate of $D_{(s)}^- \rightarrow \gamma \ell \bar{\nu}$ ($\ell = e, \mu$) is proportional to $f_{D_{(s)}}^2$, so one can use it to determine the decay constant $f_{D_{(s)}}$. On the other hand, it is seen that these processes can also be used to test the $|V_{cs}|$ and $|V_{cd}|$ if $f_{D_s}$ and $f_D$ are known.

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Figure 1: Feynman diagram in standard model for $D_s^- \to l \bar{\nu}$ decay.

Figure 2: Feynman diagrams in standard model for $D_s^- \to \gamma l \bar{\nu}$ decay.
Figure 3: Normalized energy spectra of the decay $D_s^- \rightarrow \gamma \ell \bar{\nu}$. The solid line is for the photon energy spectrum, the dashed line is for the neutrino energy and the dash-dotted line is for the lepton energy spectrum, respectively.

Figure 4: Normalized energy spectra of the decay $D^- \rightarrow \gamma \ell \bar{\nu}$. The solid line is for the photon energy spectrum, the dashed line is for the neutrino energy and the dash-dotted line is for the lepton energy spectrum, respectively.