Emission of cosmic rays from Pulsar wind nebulae

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Abstract. The goal of the paper is to interpret AMS-02 positron data. Most of the positrons come from secondary production which can fit well the data below 10 GeV. However, above 10 GeV the positron flux detected is much higher than the predictions for the secondary production. This is called ‘positron excess’ whose origin remains unknown, and interpretations including supernova remnants, pulsar wind nebulae (PWNe) or dark matter have been considered. In this paper, we investigate if PWNe can explain this ‘positron excess’ and whether a single pulsar can contribute entirely to it. To do this, we select some powerful pulsars in the ATNF Pulsar Catalogue with specific age and distances from earth and calculate their positron flux. By using a minimization package in Python to adjust the spectral slope of the PWNe and the normalization of the secondaries, we find the best fit to AMS-02 data for each of those pulsars. We can see from the result that if we want to explain the ‘positron excess’ using a single pulsar, Geminga is the best solution in the ATNF Pulsar Catalogue. Moreover, we find that except for the powerful pulsar J0633+1746 (Geminga), B1742-30 and J1741-2054, most of the pulsars we selected need an efficiency much greater than 1 to fit the data. It means that for those pulsars, one pulsar’s energy is not powerful enough to explain the data entirely. Therefore, for pulsars that are not powerful enough, adding more pulsars together provides us with a possible way to interpret this ‘positron excess’ problem.

1. Introduction

There has been a strong interest over the last two decades for the origin of cosmic-ray (CR) positrons for both particle Physics and astrophysics. This interest is associated to the accurate measurements of CRs using space borne detectors like Pamela and Fermi-LAT collaboration. Recently, the Alpha Magnetic Spectrometer (AMS-02) has reported its first data on the positron fraction (the ration between positron flux and the sum of positrons and electrons) for 30 months of data taken on the International Space Station (ISS) [1]. The data covers an energy range from about one GeV up to 500 GeV with improved precise.

As we know, positrons are particles that are very rare and hard to be detected. Most of these positrons come from the secondary productions which originate from the spallation reactions of primary CRs with the nuclei of the interstellar material (ISM). The secondary production can explain the AMS-02 data under 10 GeV, while above 10 GeV there is a significant excess of positrons. We show this positron excess in Figure 1 with the AMS-02 data and a prediction for the secondary production that fits well the data below 10 GeV but that leaves a large flux of unexplained positron above these energies. The origin of the positron flux above 10 GeV is still a mystery and different interpretations have been used. Particularly, supernova remnants, pulsar wind nebulae or dark matter have been considered [2-4]. Solving the ‘positron excess’ problem can help us determine the origin of CRs positrons.
Pulsars, highly magnetized rotating neutron stars creating jets of particles, can also generate positrons through primary production. In this paper we investigate if the positron excess is caused by PWNe. We will first use the burst-like model to calculate and present propagation length of pulsars at a certain age and distance from earth, it can help us determine the age and distance ranges of pulsars whose positron flux may explain the ‘positron excess’. Then we will select pulsars in the ATNF Pulsar Catalogue with specific age and distances from earth to study if positron flux of the secondary production and a single pulsar together can help explain the AMS-02 data. We will use a minimization package in Python to automatically adjust the normalization of secondaries and the spectral slope of PWNe, which can help us to find the best fit to the data. We can then see from the reduced chi-square if the fits are good or not.

The paper is organized as follows: in Section 2, we introduce the sources of galactic CRs positrons, including primary production of positrons accelerated by PWNe and secondary production which produces most of the positrons in galactic. In Section 3, we discuss two propagation mechanisms of primary production in the galaxy, including energy losses and diffusion, and introduce equation describing the propagation process, we also present the propagation length for different sources and give solutions to the propagation equation. In Section 4, we calculate the flux of positrons detected in the device using the burst-like model, we also discuss the continuous injection model that can better describe this process. In Section 5, we present propagation length for pulsars of various ages and distances, which can help us narrow down the search, and then we select some powerful pulsars in the ATNF Pulsar Catalogue, calculate their positron flux and use Python to find the best fit to AMS-02 data for each of those pulsars. Finally, in Section 5, we summarize this work and discuss results presented in Section 4, we also point out the research significance, some existing problems, and suggestions for future work.

2. Sources of Galactic positrons

Cosmic rays (CRs) are very energetic charged particles generated by Galactic and extragalactic sources. Cosmic rays can be divided into primary CRs and secondary CRs. Primary CRs, composed mainly by hydrogen and helium, are accelerated by astrophysics sources while secondary CRs, consisting most of the positrons in the Galaxy, are generated by the spallation of primary CRs with the ISM. Here we want to see if adding the primary CRs can help explain the AMS-02 data.

2.1. Primary production of positrons accelerated by PWNe

A supernova remnants (SNR) is generated by a supernova explosion that releases in the Galactic environment about $10^{51}$ erg. This material travel initially at a fraction of the speed of light towards the

![Figure 1](https://example.com/figure1.png)

**Figure 1.** This figure shows the positron flux of AMS-02 data detected and a prediction for the flux from secondary production.

2.1. Primary production of positrons accelerated by PWNe

A supernova remnants (SNR) is generated by a supernova explosion that releases in the Galactic environment about $10^{51}$ erg. This material travel initially at a fraction of the speed of light towards the
interstellar medium creating a forward and reverse shock. What remains of the collapsed core is a dense, highly magnetized and periodically fast rotating neutron star whose surface’s intense magnetic fields are believed to be able to drive an energetic wind of particles and magnetic field confined by the surrounding ejecta [5].

In the region between the wind termination shock and the ejecta, pulsar wind nebulae (PWNe), a relativistic hot magnetized hot plasma, are generated by the neutron star and its huge magnetic fields inside the shells of supernova remnants.

Pulsar is the real engine of PWNe, particles ejected by the pulsar are then accelerated in the termination shock and enter the PWNe where they get trapped by the PWNe magnetic field until they are accelerated to relativistic properties and escape the PWNe. Then about 50 kyr after the nebula formation, we assume that those accelerated particles are released into the interstellar medium (ISM) completely.

\( Q(E) \), the spectrum of positrons emitted by PWNe, can be modeled as a power-law with an exponential cut-off energy \( E_c \)

\[
Q(E) = Q_0 \left( \frac{E}{E_0} \right)^{-\gamma} \exp \left( \frac{-E}{E_c} \right)
\]  

The total energy emitted in positrons and electrons is calculated by integrating \( Q(E) \) from the minimum of energy \( E_i \) to infinity

\[
E_{\text{tot}} = \int_{E_i}^{\infty} dEEQ(E)
\]

On the other hand, the total energy of the pulsar is

\[
W_0 = \tau_0 \dot{E} \left( 1 + \frac{t^*}{\tau_0} \right)^2
\]

and is determined by the pulsar’s age \( t_{\text{obs}} \) (\( t^* = t_{\text{obs}} + d/c \) where \( t^* \) is the actual age, \( t_{\text{obs}} \) the observed age, \( d \) the distance and \( c \) the velocity of light), the typical pulsar decay time \( \tau_0 \) (\( \tau_0 = 12 \text{ kyr} \)) and the spin down luminosity \( \dot{E} \) denotes how much the energy is converted from rotational kinetic energy into cosmic rays and light emitted by the pulsar [6]. The age and \( \dot{E} \) of a specific pulsar is taken from the ATNF Pulsar Catalogue, so each pulsar has a specific certain \( W_0 \).

\[
E_{\text{tot}} = \eta W_0
\]

\( E_{\text{tot}} \) is connected to \( W_0 \) by \( \eta \). As we know, positrons are generated by PWNe and energy emitted into positrons is only a fraction of that rotational kinetic energy. Here \( \eta \) is the efficiency which denotes how much the energy is converted to electron and positrons. In the python code, \( \eta \) denotes the normalization for the flux of positrons from PWNe. So after multiplying \( W_0 \) by \( \eta \), we get \( E_{\text{tot}} \).

Then from equation (2) and (4) we have

\[
\int Q(E) EdE = \eta W_0
\]

After substituting equation (1) into equation (5) we get \( Q_0 \int E^{-\gamma} \exp(-E/E_c) dE = \eta W_0 \) which can be rewritten as \( Q_0 = \frac{\eta W_0}{\int E^{-\gamma} \exp(-E/E_c) dE} \). It is clear that if data is determined, \( Q_0 \) is a definite
number, therefore, we only need to adjust $\eta$ to find the positron flux that can best fit the AMS-02 data.

2.2. Secondary positrons production
Primary CR species are mostly protons and particles accelerated by PWNe and SNR interacting with the nucleus of interstellar material which are made mostly of hydrogen and helium through the spallation reactions and create other particles called secondary production. This process can be abstracted to a model calculating the secondary production [2]

$$q_{e^\pm}(x, E_e) = 4\pi n_{\text{ISM}}(x) \int dE_{\text{CR}} \Phi_{\text{CR}}(x, E_{\text{CR}}) \frac{d\sigma}{dE_e}(E_{\text{CR}}, E_e)$$

(6)

Here $q_{e^\pm}$ is the flux of secondary production, $n_{\text{ISM}}$ denotes the density of atoms in the interstellar medium, $\Phi_{\text{CR}}$ is the flux of cosmic rays that are accelerated by PWNe and SNR, $\frac{d\sigma}{dE_e}$ shows how strong this process goes and by fixing the proton and helium primary flux to the new AMS-02 data we can get (6).

3. Propagation of primary CRs in the Galaxy
Primary CR positrons after being accelerated by the PWNe and SNRs encounter a process that is called propagation. This is mainly due to two mechanisms: energy losses and diffusion [7].

3.1. Energy losses
Electrons and positrons (above 1 GeV) lose energy mainly by synchrotron radiation interacting with the Galactic magnetic field. This energy loss is due to the fact that charged particles (such as proton or electron) emit radio waves when they travel through a magnetic field. These cosmic particles lose energy also for inverse Compton (IC) scattering on the interstellar radiation field (ISRF), which consists of three photon fields—Cosmic Microwave Background (CMB), Infrared radiation (IR) and Starlight (SL). In this paper we neglect the energy losses for Bremsstrahlung, ionization losses and Coulomb scattering since they only affect electrons and positrons below 1 GeV.

The energy-loss rate can be parametrized as

$$b(E) = b_0 e^{\alpha} = \frac{E_0}{\tau_j} e^{\alpha} \quad \text{with} \quad \epsilon = \frac{E}{E_0} = 1\text{GeV}$$

(7)

where $b_0$ is the normalization of energy-loss process, $\epsilon$ is the energy of electron, $\alpha$ is the slope of energy losses and we take 2 as its value, $\tau_j$ is the characteristic energy-loss time that tells after how much time the energy of pulsars are completely dissipated [8].

In the following equation, $E_i$ is the initial energy and $E_i$ is the energy detected, so after knowing the energy of electron and the time $T$, we can calculate the energy of the electron at any given time as follows

$$\frac{dE}{dt} = b(E) = b_0 E^\alpha$$

$$\int_{E_i}^{E_f} \frac{dE}{E^\alpha} = b_0 \int_0^T dt$$

$$\frac{1}{-\alpha + 1} \left( E_i^{2/\alpha + 1} - E_0^{\alpha + 1} \right) = b_0 T$$

$$E_f = \exp\left[(-\alpha + 1)\log\left( b_0 T (\alpha + 1) + E_0^{\alpha + 1}\right)\right]$$
3.2. Diffusion
The diffusion takes into consideration how positrons travel in the irregular Galactic magnetic field.

\[ K(E) = \beta K_0 \left( \frac{R}{1 \text{GV}} \right)^{\delta} = K_0 \epsilon^{\delta} \]  

(8)

Here \( \beta = \frac{v}{c} \) is the Lorentz factor (\( \beta = 1 \) as our analysis consider only the relativistic electrons), \( R \) is the particle rigidity, \( \epsilon \) is the energy of electron. \( K_0 \) is the normalization of diffusion coefficient \( K(E) \) and \( \delta \) the slope, both of them are found by fitting AMS-02 data for protons helium and boron carbon ratio \( \frac{B}{C} \) [8].

3.3. Propagation length
This propagation can be described as a current conservation as \( \hat{D} \mathcal{N} = Q(E, x, t) \) where the transport operator \( \hat{D} \) can be expanded as

\[ \partial_t \mathcal{N} - \nabla \cdot \{ K(E) \nabla \mathcal{N} \} + \partial_E \left\{ \frac{dE}{dt} \mathcal{N} \right\} = Q(E, x, t) \]  

(9)

Here \( Q(E, x, t) \) is the source term that is the number of electrons and positrons emitted from the Galactic source (SNRs and PWNe) and \( \mathcal{N} = \mathcal{N}(E, x, t) = \frac{dn}{dE} \) is the electron number density per unit of energy. The operator \( \hat{D} \) takes into account the diffusion part of particles interact with Galactic magnetic field and energy losses part where particles interacting with Galactic magnetic field and ISRF. \( K(E) \) is the diffusion coefficient which depends on how CRs travels in the irregular magnetic field in the Galaxy and can be written as (8), \( \frac{dE}{dt} = b(E) \) denotes the loss of energy completely dissipated as is shown in (7). Here we neglect convection and reacceleration [9].

The propagation scale (diffusion scale) can be formulated as

\[ \lambda^2 = 4 \int_{E_i}^{E_f} dE \frac{K(E)}{b(E)} \]  

(10)

Here \( E_i \) is the initial energy emitted by the source and \( E \) is the energy detected, so \( \lambda \) is decided by the ratio of \( K(E) \) to \( b(E) \). Moreover, \( K(E) \propto E^{\delta} \) and \( b(E) \propto E^{\epsilon} \), then \( \lambda \propto E^{\delta-\epsilon+1} \) [8].

In Figure 2, we show the propagation length \( \sqrt{\lambda} \) for sources of different ages (30, 60, 100 and 300 kyrs) as a function of the electron energy. Here the cut-off energy is the maximum energy that can be detected for a give source with a given age. We see that for younger sources we have electrons up to tens of TeV energy while from older sources we can detect electrons only up to 1 TeV.
Figure 2. Propagation length $\sqrt{\lambda}$ for sources of different ages (30, 60, 100 and 300 kyrs) as a function of the electron energy.

3.4. Solution for the propagation equation

The steady-state solution of propagation equation (9) is the Green function assuming that spatial diffusion and energy losses are isotropic and homogeneous disregarding the energy-loss features for the moment [8]

$$\bar{D}_T G = \delta^3 (x - x_s) \delta(E - E_s)$$

(11)

$$G(x, E \leftarrow x_s, E_s) = \frac{1}{b(E)(\pi \lambda^2)^{3/2}} \exp \left( -\frac{(x - x_s)^2}{\lambda^2} \right)$$

(12)

Moreover, the solution is found within a zone that can be modeled as a cylindrical slab where radius $R = R_{\text{disk}} = 20 \text{kpc}$, a vertical half-height of $L = 1-15 \text{kpc}$, and a vertical extend of $Z_{\text{disk}} = 0.1 \text{kpc}$ [9]. The subscript $s$ flags quantities at source ($E_s \geq E$), the energy-loss rate is $b(E) = -\frac{dE}{dt}$ and the diffusion scale (10) we have mentioned above.

4. Flux of positrons detected in the device

Flux is the number of particles detected per unit of surface, time and energy, the flux of positrons on earth can be calculated as

$$\phi(E) = \frac{E \beta c}{4\pi} \times \int dx_1 \int dx_0 \int dt E dE_1 dE_0 G(E, x_\circ \leftarrow t, E_\circ, x_s) Q(t, E_s, x_s)$$

(13)

Where $G$ is Green function, $Q(t, E_s, x_s)$ is source term and in order to get the flux in 1/GeV/cm2/s/sr, we add the conversion factor by multiplying $\frac{1}{4\pi}$. $\beta$ is the ratio of the velocity of particles to light which we take for 1 and $c$ is the velocity of light. After integrating in time, space and energy, we get the flux of positrons [8].

To explain it further, the source term $Q(E) = Q_0 \left( \frac{E}{E_0} \right)^{-\gamma} \exp \left( -\frac{E}{E_\circ} \right)$ containing positrons generated by the pulsar is the only function of energy, in a supernova explosion, as all particles are generated at that time $t$ and that certain location $x_s$, it is better to add the $\delta$ function to get
\( Q(t_s, E_s, \mathbf{x}_s) = Q(E_s) \delta(t_s) \delta(\mathbf{x}_s) \) which shows the energy \( E_s \), explosion time \( t_s \) and location \( \mathbf{x}_s \) of a specific source. In other words, the specific PWNe is located in one point of the Galaxy and produce positrons and electrons at a given time while in other time or space the source term is equal to zero.

After put the source term \( Q(t_s, E_s, \mathbf{x}_s) = Q(E_s) \delta(t_s) \delta(\mathbf{x}_s) \) into (13), we get the positron flux of a specific source in the galaxy that is formulized as

\[
\psi(\mathbf{x}_s, E, t) = \frac{b(E)}{b(E)} \frac{1}{(\pi \lambda^2)^{\frac{3}{2}}} \exp \left( -\frac{\|\mathbf{x}_s - \mathbf{x}_s\|^2}{\lambda^2} \right) Q(E_s)
\]  

(14)

Here \( \mathbf{x}_s \) indicates the source position, \( \mathbf{x}_s - \mathbf{x}_s \) is the distance of the source which can be taken from the ATNF catalog, \( E_s \) is the energy at source (initial energy of electrons and positrons emitted from the source) and \( E \) is the energy of electrons and positrons detected.

In equation (14), \( E_s \) in \( Q(E_s) \) can derives from

\[
\Delta \tau(E_s, E_s) = \int_{E}^{E_s} \frac{dE}{b(E)} = t - t_s
\]

where \( E_s \) is the initial energy of \( e^-\) that cool down to \( E \) in a loss time \( \Delta \tau \), then after substituting the propagation scale \( \lambda^2 = \lambda^2(E_s, E_s) \equiv 4 \int_{E}^{E_s} dE \frac{K(E)}{b(E)} \) which we get from (10) into (14), we calculated the flux of positrons [8].

Above-mentioned is a model of burst-like, however, it is not the best physical motivated model since the pulsars create energy also after the explosion, they continue to emit particles for ages. So a better model is the continuous injection model, which means after the explosion of supernova, particles emitted and accelerated from the PWNe continuously instead of producing them at one time. Those two models are quite similar in the low energy part while the continuous injection model has larger flux in the high energy part above 1TeV.

To modify the equation of burst-like model, we integrate equation (14) with respect to time and get the model of continuous injection [6]

\[
\mathcal{N}_s(E, r, t) = \int_0^t dt_s b(E, t_s) \frac{1}{(\pi \lambda^2(t_s, t_s, E))^\frac{3}{2}} \exp \left( -\frac{\|\mathbf{r}_s - \mathbf{r}\|^2}{\lambda^2(t_s, t_s, E)^2} \right) Q(E_s(t_s))
\]  

(16)

Where the injection spectrum is

\[
Q(E, t) = L(t) \left( \frac{E}{E_0} \right)^\gamma \exp \left( -\frac{E}{E_s} \right)
\]

(17)

and the total energy emitted by the source is

\[
E_{tot} = \int_0^t dt \int_{E_0}^E dE Q(E, t) = \int_0^t dt L(t)
\]

(18)

the magnetic dipole barking describing what is the time-dependence of the emission is given by

\[
L(t) = \frac{L_0}{\left(1 + \frac{t}{\tau_0}\right)^2}
\]

(19)
5. Results

5.1. Results for different ages and distances
In order to find what kind of pulsars can explain the ‘positron excess’, we use the burst like model to calculate and make pictures showing the propagation length of pulsars at a fixed age and a certain distance from earth. It shows the age and distance range of pulsars whose positron flux can explain the ‘positron excess’, which helps us reduce the scope of the search.

We can see from the left picture of Figure 3, the distance is fixed to 0.5 kpc and the age changed from 10 kyrs to 10^4 kyrs, we can see from the picture that the graphs move to low energy part, the flux of positrons and the energy of the pulsar decreases during its aging since the longer time the particles travel in the Galaxy, the more energy they lose [9].

In the right picture of Figure 3 that a pulsar is created 20 kyrs ago and the distance varies from 0.01 to 10 kpc, it is obvious that the positrons’ flux decreases quickly as the distance increases. The graphs move to high energy part.

![Figure 3](image)

Figure 3. Flux of positrons as a function of energy of electrons for different distance and age.

Plot (a) represents the flux for a fixed distance of 0.5 kpc and source age between 10 kyrs and 10^4 kyrs. The different lines from left to right represent older to younger sources. Plot (b) is the flux at a fixed age of 20 kyrs and for source distances between 0.01 to 10 kpc. The different lines are represented from the left to the right which is closer to farther sources.

5.2. Positron flux for the brightest pulsars
Now we estimate if a single pulsar can explain entirely the positron excess. To do this we take the brightest pulsars in the ATNF catalog. Particularly, we select the pulsars reported in Table 2 in the paper [2]. Those are close and powerful pulsars with high value for the spin-down luminosity.

We calculate the flux of the sources using (14) and we fit the AMS-02 data with the flux of positrons from secondary and pulsar wind nebulae. In the process of fitting the data, we leave free the normalization of the secondaries and spectral slope of the PWN. By using a minimization package that is the scipy.optimize.fmin tool in Python, we find the minimum \( \chi^2 \). The results are listed in Table 1.

The fitting figure that we used Python code to find the best fit to the AMS data for each pulsar are shown below as Figure 4-12.

Here we use the residuals and the reduced chi-square to determine the goodness-of-fit. If chi-square is comparable with the number of degrees of freedom, that is, the reduced chi-square is close to 1, then the fit is good.
Table 1. Powerful pulsars selected from the ATNF Pulsar Catalogue.

| Name       | Dist(kpc) | Age(kyr) | Edot(erg/s) | γ  | η   | Norm | Reduced chi-square |
|------------|-----------|----------|-------------|----|-----|------|--------------------|
| B1742-30   | 0.2       | 546      | 8.5e+33     | 1.90 | 11.61 | 0.97 | 1.29               |
| B1749-28   | 0.2       | 1100     | 1.8e+33     | 1.84 | 61.30 | 0.93 | 4.25               |
| J0633+1746 | 0.19      | 342      | 3.2e+34     | 1.94 | 2.89 | 1.04 | 1.20               |
| J1741-2054 | 0.3       | 386      | 9.5e+33     | 1.94 | 9.86 | 1.05 | 1.10               |
| B0959-54   | 0.3       | 443      | 6.8e+32     | 1.93 | 134.77 | 0.88 | 1.06               |
| B0940-55   | 0.3       | 461      | 3.1e+33     | 1.93 | 29.38 | 1.03 | 1.05               |
| B0834+06   | 0.19      | 2970     | 1.3e+32     | 1.74 | 1159.30 | 1.07 | 39.15              |
| J1918+1541 | 0.80      | 2310     | 2.0e+33     | 1.77 | 71.87 | 1.19 | 26.92              |
| B1822-09   | 0.30      | 233      | 4.5e+33     | 1.98 | 22.20 | 0.96 | 1.26               |

Figure 4. B1742-30.

In the left, we show the fitting results of AMS-02 data, here the + shape icon is the AMS-02 positron data we observed, the red dots denote a prediction for the flux from secondary production, the blue dots are positron flux produced by PWN and black line shows the total positron flux of secondary production and PWN contributor. Then we show the residuals of the fit on the right. The deviation from 0 tells us the difference between the prediction of the model and actual observation. The less the deviation from 0, the better the fit. We can see from the picture that the fit is good from 10 to 100 GeV, while above 100 GeV, it has larger deviation, indicating the fit is not good in this range.
Figure 5. B1749-28.

Figure 6. J0633+1746.

Figure 7. J1741-2054.
**Figure 8.** B0959-54.

**Figure 9.** B0940-55.

**Figure 10.** B0834+06.
6. Discussion and conclusions
The current research was designed to examine if PWNe can help explain the 'positron excess', we used Python to find the best fit to AMS-02 data of secondary production and the flux of positrons from a single energetic pulsar which we selected from the ATNF catalog.

In the process of fitting the AMS-02 data with positrons from PWNe and secondary production, we can see from the reduced chi-square in Table 1 that except for pulsars B0834+06 and J1918+1541, all the pulsars listed in the table provide a decent fit to the data. However, we found that except for the powerful pulsar Geminga, B1742-30 and J1741-2054, most of the pulsars we listed above have the efficiency much greater than 1 which is unrealistic, because efficiency $\eta$ denotes how much the energy is converted from a specific pulsar to positrons. Therefore, we can use a single pulsar to explain the 'positron excess', but only for some limited powerful pulsars like Geminga, B1742-30 and J1741-2054. Moreover, Geminga is the best option in the ATNF Pulsar Catalogue to explain the data using only one PWNe source. Recently there was the detection of gamma-ray halo around Geminga and that could motivate even more that this pulsar is a possible bright contributor of the positron data [10]. However, most of the pulsars listed in Table 1 have efficiency much greater than 1 to fit the data, which means interpretation with only one of these pulsars is implausible since one pulsar's energy is not powerful enough to explain the data entirely. Later, for those pulsars that are not powerful enough,
we want to find if we can interpret this ‘positron excess’ by adding more pulsars together. What's more, we can use the continuous injection as a more accurate model. Solving the ‘positron excess’ problem will help us to gain a better understanding of the origin of CR positrons in the universe.

Also, we can see from Figure 4-12 that most of the pulsars do not fit the positron data in the entire energy range. To be more specific, we didn't explain the 'positron excess' above the cut off energy, because the cut off energy is decided by the maximum energy detected at earth from a specific pulsar with given age and distance from earth. Dark matter, one major component in the Universe, can also generate positrons by the pair annihilation or decay of DM particles [11]. Therefore, we can try to add the dark matter annihilation contribution to explain the data above the cut off energy, we believe dark matter and primary astrophysical sources together will better interpret the AMS-02 data [12]. Moreover, if we can interpret the AMS-02 data very well by adding the dark matter part, it will help us in revealing the outcome or mechanism of a DM pair annihilation or decay event which remains a mystery.

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