Failure analysis of variable-stiffness laminate considering manufacturing defects

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Abstract. For the two typical automated fiber placement manufacturing defects, namely the gap and overlap of the variable stiffness laminates, the corresponding finite element model is established. The perfect model, containing gap defect model and overlap defect model was analyzed to predict their load capacity, respectively. The progressive failure was considered by a 2D Camanho degradation model. The numerical result shows that, compared with the prefect laminate, the buckling load and ultimate load of the laminate designed by the completed overlaps strategy were improved by 10.6% and 9.3% respectively; while the buckling load and ultimate load of the laminate designed by the completed gaps strategy were reduced 16.1% and 18.5% respectively.

1. Introduction
In the conventional design of composite laminates, the angle of fiber is constant, such the design cannot make full use of its directivity and limit the designability of composite laminates [1]. Many researchers [2-4] has demonstrated that the buckling load of a laminate can be improved by using variable stiffness structures. Lopes et al. [5] proved that the variable stiffness laminate has higher buckling load and the first layer failure load than the constant stiffness laminate. Du Yu et al. [6-7] confirmed that the variable stiffness laminate has higher failure load than constant one by both Finite Element Analysis and experiments. However, those previous research neglected the influence of defects, overlaps and gaps, caused by manufacturing, while those defects will make the loading and damage problems of variable stiffness laminate more complicated.

Most researchers identify if a point is in defect area by figure out the distance between that point and tow-band center line. Langley et al 1999 [8] used a numerical search method to get the distance of any point to tow-band center line when studying the laying path of its direction angle in a linear change. Yet this method is rarely used because of the calculation cost is very high even when result is not so precisely. Blom et al. [9] and Fayazbakhsh et al. [10] used the circular arc path instead to conduct experiments and finite element analysis. Their method was used a lot because of by using this laying path can make it easier to calculate its distance to any point in a ply.

In this study, a method was proposed to identify whether the points in each layer are in the defect area, where the path direction angle is linear changes, considered the most widely used variable stiffness laminate model and reduced lots of calculate cost. Based on this method, the FEM models was established with and without considering manufacturing defects, and the loading capacity and damage region distribution of these composite laminates models are analyzed.
2. Manufacturing defects of curved fiber laying composites

2.1 Fiber path definition
An automated fiber placement machine is shown in Figure 1. The placement head of the fiber placement machine can soften the prepreg tows and put it on the surface of the mold according to the predetermined path, and then pressed with the compaction roller. Each tow can be cut off and resent independently, the angle of the tow can be changed, and the number of tow in the band can be increased or reduced at any time.

\[
\theta(r) = \theta_0 + \frac{\theta_1 - \theta_0}{d} |r| = \theta_0 + k |r|
\]

(1)

Where \(d\) is half width of the plate, \(\theta_0\) and \(\theta_1\) are the fiber angles at the plate center (\(r=0\)) and the plate edge (\(r=d\)), respectively, and \(k\) is the rate of the angle varies with the width. Based on equation (1) fiber path can be formulated as

\[
s(r) = \begin{cases} 
\frac{1}{k} [-\ln(\cos \theta_0) + \ln(\theta_0 - kr)], & -d \leq r \leq 0 \\
\frac{1}{k} [-\ln(\cos \theta_0) + \ln(\theta_0 + kr)], & 0 < r \leq d
\end{cases}
\]

(2)

2.2 Localization of defect areas
A tow band diagram is shown in Figure 3, a tow band consists of several tows, and a tow contains many fibers. Every curve shown in Figure 3 represents the border of each tow. Among these curve, the central one is the reference path of the tow band as well, while two of the outmost curve are the border of the tow band as well.

The distance from the point in each tow border curve to reference path is constant, so each curve can be formulated based on equation (3)

\[
\begin{pmatrix} r' \\ s' \end{pmatrix} = \begin{pmatrix} 1 & -w \sin \theta \\ 1 & w \cos \theta \end{pmatrix} \begin{pmatrix} r \\ s \end{pmatrix}
\]

(3)

Where \(w\) represents the distance to reference curve.
By using AFP technique tows are cut perpendicular to its direction, resulting in a jagged edge. As shown in Figure 4, when the upper blue tow band is laid, the nether red tow band will be cut along the edge of the blue tow band, there are two feasible different typical cut strategy, complete gaps strategy (Figure 4(a)) and complete overlaps strategy (Figure 4(b)). The grey area represents the gap area and the green area represents the overlap area, those imperfect areas called defect areas.

Taking the completed gaps method as an example, the gap area would be found between the central curves of both tow bands, as shown in Figure 5 (a). The area between those two curves was divided
into three parts, blue, grey and red respectively. In the r-s coordinate, any point in blue area, like point 
P(r_p, s_p), and any point in red area, like point Q (r_Q, s_Q), have a vertical line P and Q respectively. Both 
points P(r_p, s_p) and Q(r_Q, s_Q) are the intersection of line P and line Q with the bottom border of blue 
tow band respectively, the point Q2(r_Q, s_Q2) is the intersection of line Q with the upper border curve of 
the tow where point Q located. Compared the r-coordinates of these points, point P satisfies the 
condition 1: sp>sp1 ,while point Q satisfies the condition 2: s_Q1>s_Q2>s_Q. 

Enlarging the red framed part, as shown in Figure 5 (b), in the r-s coordinate, any point in grey area, 
like point R(r_R, s_R), have a vertical line R. R1(r_R, s_R1) is the intersection of line R with the bottom 
border of blue tow band, the point R2(r_R, s_R2) is the intersection of line R with the upper border curve 
(drawn as a yellow dotted curve) of the tow where point R located. Compared the r-coordinates of 
these points, point R satisfies the condition 3: s_R2>s_R1>s_R. 

By verifying those three conditions, whether any point in the ply is in the grey area (gap area) can 
be determined. A similar method can determine whether any point in the overlap area too. 
Based on the previous idea, by using the MATLAB function meshgrid () and reshape (), a plate (with a 
size of 80 mm × 100 mm) could be divided into 32,000 grids (with a size of 0.5× 0.5 mm), and all the 
nodes on every grid are verified by this method, and the result of the automatic wire laying defect 
distribution as shown in Figure 6 is obtained. The manufacturing defect distributions are shown in 
Figure 6.

3. Analysis

3.1 Finite element model

The finite element model is a square carbon fiber reinforced composite laminates with a width of 
2a=80 mm and a height of 2b=80 mm. This laminate laid-up with [±<45|90>], and with the thickness 
of each layer is t=0.125 mm. The width of every single tow is w=3.17 mm, and the maximum number of 
tows in a tow band is 2n=8. The maximum width of a tow band tape is w_t=25.36 mm (=3.17 mm × 8). The Elastic properties and strengths of G40-800/5276-1 carbon/epoxy composite is shown in Table 1.

| Table 1. Elastic properties and strengths of G40-800/5276-1 carbon/epoxy composite. |
|---------------------------------------------------------------|
| Modulus       | E1 (GPa) | E2 (GPa) | ν | G12 (GPa) |
|----------------|-----------|-----------|---|-----------|
| 142.7          | 9.1       | 0.3       | 4.82 |
| Strengths      | X1 (MPa)  | X2 (MPa)  | Y1 (MPa) | Y2 (MPa) | Z (MPa) |
| 3013           | 1744       | 90        | 200 | 170 |

As shown in Figure 7, to reduce the computational effort, the full plate model was reduced to 1/4 
and symmetry conditions are applied to two of model edges. The 1/4 plate mesh consists of 1,600 S4R 
elements, 1x 1 mm in size. The direction of fiber in each layer of the element is the fiber direction at 
the center point of the element. In the defective model, if the central point of the element is in the 
defect area, this element would be set as a defect one in this layer. For gap defect elements, the 
thickness of this layer is set to 0.1t =0.0125 mm, and for overlap defect elements, the thickness of this 
layer is set to 2t =0.25 mm. Distribution of gap defect elements and overlap defect elements in 
corresponding models are shown in Figure 8. The yellow and red elements represent the defect 
elements in the <45|90> layer, and the blue and red elements represent the defect elements in the 
-<45|90> layer.

3.2 Degenerate mode and strength criterion

In the Camanho degradation model [11], intermediate state variables are used to indicate the effect of 
material failure on its stiffness, and based on this, the degradation parameters of the main Poisson's
ratio are introduced. The effectiveness of this degradation mode is verified by Liu Yong [12] through computational analysis and experimental comparison. The specific parameters are given in Table 2.

![Figure 7. Load and Boundary conditions.](image1)

![Figure 8. Distribution of (a) gap defect elements (b) overlap defect elements.](image2)

### Table 2. 2D degradation mode based on Camanho model.

| Failure type                      | Degradation parameter based on Camanho mode |
|----------------------------------|---------------------------------------------|
| Matrix tension failure           | $E_{yy} = 0.2E_{yy}$, $v_{xy} = 0.15v_{xy}$, $G_{xy} = 0.22G_{xy}$, |
| Matrix compression failure       | $E_{yy} = 0.4E_{yy}$, $v_{xy} = 0.15v_{xy}$, $G_{xy} = 0.4G_{xy}$, |
| Fiber tension failure            | $E_{xx} = 0.07E_{xx}$, $E_{yy} = 0.07E_{yy}$, $v_{xy} = 0.07v_{xy}$, $G_{xy} = 0.07G_{xy}$, |
| Fiber compression failure        | $E_{xx} = 0.14E_{xx}$, $E_{yy} = 0.14E_{yy}$, $v_{xy} = 0.14v_{xy}$, $G_{xy} = 0.14G_{xy}$, |

4. Result and discussion

#### 4.1 Load-displacement curves

The load-displacement curves of the plate obtain gap defects, no defect and overlap defects, are shown in Figure 9. From this figure, it can be seen that the compressive load increases linearly in the initial stage. When reached the initial buckling load, the slope of the curve decreases until the ultimate load is reached. After that, the failure of the plate began to occur, and the load capacity began to decrease nonlinearly. That resulted by the occurrence of multiple failure modes and the nonlinear variation of the geometric configuration.

Compared with the perfect plate, the plate with gap defects has the minimum stiffness, buckling load and ultimate load, while the plate with overlap defects owns the maximum buckling load and ultimate load, and those load were listed in Table 3. In order to facilitate comparison, the value of those load has normalized on the basis of the perfect plate. It can be seen that, compared with the defect free plate, the buckling load and the ultimate load of the plate with gaps are reduced by 16.1% and 18.5%, while the buckling load and the ultimate load of the plate with overlaps are increased by 10.6% and 9.3% respectively.
Figure 9. Load-displacement curve of compression loading.

Table 3. Buckling load and ultimate load of laminate.

| Laminate                          | Buckling load (N) | Ultimate load (N) |
|----------------------------------|-------------------|-------------------|
| Perfect laminate                 | 1257.37           | 4988.00           |
| (1)                              | (1)               |
| Laminate with gap defects        | 1054.75           | 4063.10           |
| (0.839)                          | (0.815)           |
| Laminate with overlap defects    | 1391.04           | 5451.21           |
| (1.106)                          | (1.093)           |

Table 4. Lamina failure distribution at the end of loading.

| Lamina                        | Perfect lamina | Gap defect lamina | Overlap defect lamina |
|-------------------------------|----------------|-------------------|-----------------------|
| <45|90>                          | -<45|90>           | <45|90>           | -<45|90>           |
| Matrix tension failure        | ![Image](image1) | ![Image](image2) | ![Image](image3) | ![Image](image4) |
| Matrix compression failure    | ![Image](image5) | ![Image](image6) | ![Image](image7) | ![Image](image8) |
| Fiber tension failure         | ![Image](image9) | ![Image](image10) | ![Image](image11) | ![Image](image12) |
| Fiber compression failure     | ![Image](image13) | ![Image](image14) | ![Image](image15) | ![Image](image16) |
The failure distribution at the end of loading of each plate is listed in table 4, where the blue elements represent no corresponding failure occurred and the red elements represent corresponding failure occurred. It can be seen that the main failure mode is matrix compression failure, next to the fiber compression failure. Furthermore, the failure distributions of each layer of those three plates are compared, the distribution of the damage regions is similar generally, since the scale of the defective areas is small. Compared with Figure 8, it can be seen that material failure is more likely to occur in the gap regions or in the vicinity of the overlap regions.

5. Conclusions
1) According to the present AFP technology, the defect area of the laminates is determined, and the manufacturing defects are considered in the finite element model, which makes the result more reliable when analyzing and studying the mechanical properties of the laminate. It is helpful to apply this technology to practical research and popularization.
2) The models contained completed gap strategy, completed overlap strategy, and idealized perfect model are analyzed under compressive load. The thickness of laminates made by completed overlap strategy at the area of overlap increased, those extra part act as ribs on the surface of the plate, hence those laminates possess the maximum stiffness, buckling strength and ultimate strength. And the laminates made by completed overlap strategy no longer contain fibers in the gap area, leaving grooves on the surface, hence those laminates possess the minimum stiffness, buckling strength and ultimate strength.
3) Four kinds of failure modes and the distribution of failure region of the three kinds of laminates under the compression load are compared and analyzed. The results show that the failure of the laminate is mainly related to the macrostructure of the laminate and is more likely to occur in the failure-sensitive area (the gap areas or in the vicinity of the overlap areas).

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