String effects in SU(2) lattice gauge theory

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We discuss the effective string picture for the confining regime of lattice gauge theories at zero and finite temperature. We present results of extensive Monte Carlo simulations - performed with the Lüscher and Weisz algorithm - for SU(2) Yang-Mills theory in 2+1 dimensions. We also address the issue of "string universality" by comparing our results with those obtained in other lattice gauge theories.

In the confining phase of a pure gauge theory, the correlation function of two Polyakov loops $P(x)$ at a distance $R$ and at a temperature $T = 1/L$:

\[ G(R) \equiv \langle P(x) P^\dagger(x+R) \rangle \equiv e^{-F(R,L)}, \quad \text{(1)} \]

is expected to follow the so called “area law”

\[ F(R,L) \sim \sigma R L + k(L). \quad \text{(2)} \]

Here $\sigma$ denotes the string tension and $k(L)$ is a non-universal constant depending only on $L$. In the rough phase one has to add a correction term (“effective string corrections”) in order to take into account the contribution due to the quantum fluctuations of the flux tube. The simplest way to describe these quantum fluctuations is to model the displacement of the flux tube from its rest position with a 2$d$ free bosonic field. This leads to the following result:

\[ F(R,L) \sim F_q(R,L) = \sigma R L + k(L) + F_q^1(R,L) \quad \text{(3)} \]

with

\[ F_q^1(R,L) = (d-2) \log (\eta(\tau)); \quad -i \tau = \frac{L}{2R}, \quad \text{(4)} \]

where $(d-2)$ is the number of transverse dimensions and $\eta(\tau)$ denotes the Dedekind eta function

\[ \eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n); \quad q = e^{2\pi i \tau}. \quad \text{(5)} \]

The labels $q$ and 1 in $F_q^1$ recall that this is the first order term of the expansion in $(\sigma R L)^{-1}$ of the flux tube quantum fluctuations. Eq.\textsuperscript{(3)} is referred to as the “free bosonic string approximation”. At large enough temperatures, higher order terms of the expansion become important and cannot be neglected. These terms depend on the choice of the effective string action. One of the simplest proposal \textsuperscript{[1,2,3]} is the Nambu–Goto action, which leads to the following subleading contribution

\[ F_q^2(R,L) = -\frac{\pi^2 L}{1152 \sigma R^3} \left[ 2E_4(\tau) - E_2(\tau) \right], \quad \text{(6)} \]

where $E_2$ and $E_4$ are the second and fourth order Eisenstein functions. In addition to these corrections a “boundary term” \textsuperscript{[4]} may also be expected, due to the presence of the Polyakov loops. This term gives a contribution equal to the pure free string action provided one replaces the interquark distance $R$ with \textsuperscript{[3]}

\[ R \rightarrow R^* = \frac{R}{(1 + 2bR)^{\frac{1}{2}}} \quad \text{(7)} \]

where $b$ is a parameter.

The observable which better describes the effective string properties is the the combination

\[ c(R) \equiv -\frac{1}{2} R^3 \frac{1}{L} \log \left( \frac{G(R+1) G(R-1)}{G(R)^2} \right), \quad \text{(8)} \]

which was recently studied in \textsuperscript{[4]} and \textsuperscript{[5]}. 

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1. The model.

We studied the pure SU(2) gauge model, with the standard Wilson action:

$$ S = \beta \sum_p (1 - \frac{1}{2} \text{Tr} U_p) $$

(9)

defined on a 2+1 dimensional cubic lattice of $L$ lattice spacings in the temporal direction and $N_s$ in the spatial ones. We impose periodic boundary conditions in the temporal direction in order to consider the SU(2) Yang-Mills theory at finite temperature. The parameters $\beta$ and $L$ are related to the dimensionful gauge coupling $1/g^2$ and to the temperature $T$ as follows

$$ \frac{4}{g^2} = a \beta, \quad T = \frac{1}{L a}. $$

(10)

Here $a$ is the lattice spacing.

2. Simulation Settings

We have performed numerical simulations at $\beta = 9$, corresponding to a lattice spacing $a \sim 0.072/\text{fm}$ in physical units. Exploiting the recent algorithm proposed by Lüscher and Weisz [6], we have measured with high accuracy the Polyakov loop correlation function in two different temperature regimes.

First, we have studied the low temperature regime, in which we consider very large values of $L$ (we chose to study $L = 42, 48, 54$ and $60$) so as to make the finite temperature corrections to the interquark potential negligible. This allows a high precision test both of the free bosonic string limit (for large values of $R$) and of the higher order corrections (for intermediate values of $R$).

Second, we have considered an intermediate temperature regime in which we chose $L = 8$ which corresponds to $T/T_c \sim 3/4$.

3. Analysis of the low temperature data.

In fig.1 we plot the data obtained for $c(R) - \frac{\pi}{24} (1 - R)$ as a function of $R \sqrt{\sigma}$. The data seem to be well described by a $1/R^3$ behavior; however, a $1/R^2$ correction cannot be completely excluded.

The analysis of the data (see [7] for further details) shows that:

- There is an excellent agreement with those obtained for this same model in [5].
- Higher order corrections are needed to describe our data properly.
- At intermediate distances $R_c < R < 2/\sqrt{\sigma}$ - $R_c$ being the validity threshold of the effective string picture (see [2]) - the data seem to be well described by a $1/R^3$ type corrections.
- If a boundary term exists, it is quite small.

It is important to stress that while the power of the higher order correction that we find is that predicted by the Nambu-Goto model the numerical coefficient is definitely different from the predicted one. This does not necessarily mean that the Nambu-Goto string is the wrong choice (above all in view of the impressive success in the intermediate temperature regime, see below and [2]). The disagreement could be due to the presence for small values of $R$ of contributions due to irrelevant operators which are not related to the effective string. Further investigations are needed to better understand this point.

4. Analysis of the intermediate temperature data.

In this section we report on the results we have obtained in the intermediate temperature regime. In fig.2 we plot our data for $c(R)$ together with the theoretical expectations of the pure bosonic and of the Nambu-Goto string.

The analysis of the data shows that:

- The Lüscher term alone $c(R) = \pi/24$ does not properly describe the behavior of the data. The whole functional form of the correction (see eq.(4) and the dashed line in fig.2) is needed.
- The figure shows an impressive agreement between our data and the truncated Nambu-Goto prediction (see eq.(6)). Interestingly, a similar remarkable agreement has also been recently observed in the 3d Ising model in the same intermediate temperature regime.

5. Comparison with SU(3) and Ising gauge models.

It is very interesting to compare our results with those recently obtained for the SU(3) [4] and the Ising [3] gauge models in 3 dimensions.

In fig.1 we plot the three data sets as a func-
Comparison of SU(2), SU(3) and Ising

Figure 1. Comparison of $c(R) - \pi/24/(1 - R^{-2})$ for several 3d gauge theories: SU(3) Yang-Mills theory (empty squares), the Ising gauge model (filled squares) and our SU(2) sample with $L = 42$ (crosses). The continuous line is a fit obtained assuming a $1/R^3$ correction. The straight vertical line denotes $R_c$.

Figure 2. Comparison of the results of our simulation for $L = 8$ with the theoretical expectations for the free bosonic string (dashed line) and for the Nambu-Goto string truncated at the second order (dotted line). The horizontal continuous line is the coefficient of the Lüscher term $\pi/24$.

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