Data Expansion Method Based on Wavelet Transform

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Abstract. For small sample data, data expansion is an effective means to improve the accuracy of target recognition. To image magnification to enhance image resolution as a starting point, the author puts forward two kinds of image magnification method based on wavelet transform. Qualitative analysis on the experimental data and quantitative analysis using average gradient, cross entropy as performance index, show that the proposed image magnification method has obvious advantages in terms of image definition, maintaining image gray scale information. Finally, the author proposes the method of image expansion based on the image magnification data of the four methods, which greatly enriches the diversification of the training set on resolution, clarity, contrast, integrity and so on, and theoretically analyzes the feasibility of improving the recognition accuracy of the data with few samples and small targets.

Keywords. Wavelet transform; data expansion; mean gradient; cross entropy.

1. Introduction

As is well known, the scale of training set is one of the key factors affecting the target recognition accuracy of deep learning, and data expansion is an effective way to realize the diversification of training set [1-3]. At present, the main methods for data expansion include methods based on image geometric transformation, such as translation and mirroring, and methods based on gray transformation, such as PCA transformation. The author mainly studies data expansion method based on wavelet transform in this paper because wavelet transform can realize image magnification. Compared to other resampling technology or double linear and parabolic interpolation method, method of wavelet transform consciously pay attention to the high frequency information of the image which makes the result of processing as much as possible to reduce the information loss and is better to keep the original structure characteristics, so as to improve the resolution of the image, therefore, data expansion based on wavelet transform can not only improve the accuracy of the target, and is beneficial to the recognition of small target. In this paper, two kinds of image magnification based on wavelet transform are proposed based on the fast construction of multi-base wavelet. Finally, the author proposes the data expansion method based on amplification results is presented, and gives the characteristics of sample diversification brought by the data expansion method.

2. Data Expansion Method Based on Wavelet Transform

2.1. Principle of Multi-band Wavelet Decomposition and Reconstruction for Images

In the process of image zoom in and out, the binary wavelet can only enlarge (or reduce) 2k times (k is positive integer), which can't do to enlarge other times, and 2k times is realized based on 2k-1 times, namely again find the low frequency components on the basis of the low frequency components, so, with the increase of minification times, the details of the loss is bigger. However we can directly...
obtain enlarged image of any integer multiple using multi-band wavelet, and only need to transform original image once.

2.1.1. Multi-scale Analysis of Multi-band Wavelet. The basic theory of multi-band wavelet is still multiscale analysis. Relative to binary wavelet, the difference is that the transformation is carried out in Multi-band (M) discrete rather than binary discrete. The orthogonal decomposition of square integrable function space \( L^2(\mathbb{R}) \) can be obtained by using the theory of multi-band analysis.

By \( V_{j+1} = W_j^* \oplus V_j \) (1 < \( s < M - 1 \)), \( j \in \mathbb{Z} \), for any integer \( N \) and \( M (> 0) \)

\[
V_N = W_{N-1}^* \oplus W_{N-1}^* \oplus \cdots \oplus W_{N-M}^* \oplus V_{N-M}
\]

Thus, for any \( f_N \in V_N \), \( g^s_{N-M} \in W_{N-M}^* \), make \( f_N = g^s_{N-M} + g^s_{N-2M} + \cdots + g^s_{N-MM} \)

Here the closed subspace columns \( V_j \) are generated by the orthogonal basis \( \{ \phi_j^k = M^{j/2} \phi(M^j x - k) | K \in \mathbb{Z} \} \); however \( W_j \) is generated by \( \{ \psi_j^k = M^{j/2} \psi^j(M^j x - k) | 1 \leq s \leq M - 1, s, k \in \mathbb{Z} \} \). Here \( \phi(x) \) is called scale functions, \( \{ \psi^j(s) \} \leq s \leq M - 1 \) is called wavelet functions, they satisfy the following equation:

\[
\phi(x) = \sqrt{M} \sum_{k \in \mathbb{Z}} h_k \phi(Mx - k); \quad \psi_j(x) = \sqrt{M} \sum_{k \in \mathbb{Z}} g^s_k \phi(Mx - k)
\]

\( h_k \) is called filter coefficient, \( g^s_k \) is the wavelet coefficients, which satisfy the following orthogonal relations among them:

\[
\sum_{k \in \mathbb{Z}} h_k \overline{h}_{k+m} = \delta_{1,0}, \quad \sum_{k \in \mathbb{Z}} g^s_k \overline{g}^s_{k+m} = 0, \quad \sum_{k \in \mathbb{Z}} g^s_k \overline{g}^{s+j}_{k+m} = \delta_{m2}, \delta_{1,0}
\]

(1)

2.1.2. Decomposition and Reconstruction of M-band Wavelet Based on Fundamental Matrix.

Assuming that the image matrix is represented by \( F \), the multi-band wavelet decomposition of the image based on the basic matrix is represented by

\[
F_1 = F^* A \quad F_2 = A^* F_1
\]

(2)

\( F_1 \) is the image decomposed row by row, \( F_2 \) is the image decomposed column by column, which are also the image obtained after single multi-band wavelet decomposition.

Reconstruction of multi-band wavelet for image:

\[
E_1 = A^* F_2 \quad E_2 = E_1^* A^*
\]

(3)

\( E_2 \) is the final reconstructed image.

2.1.3. Construction of the Fundamental Matrix A. Defining a fundamental matrix \( M \times M \),

\[
A_M = \begin{pmatrix}
    h_0 & h_1 & \cdots & h_{M-1} \\
    g_0^{(1)} & g_1^{(1)} & \cdots & g_{M-1}^{(1)} \\
    \vdots & \vdots & \cdots & \vdots \\
    g_{M-1}^{(M-1)} & g_1^{(M-1)} & \cdots & g_{M-1}^{(M-1)}
\end{pmatrix}
\]

(4)

In this matrix, the first row is called the filter coefficient and the second to last row is called the wavelet coefficient. For the sake of simplicity, we denote the filter coefficient as \( h_0 = \cdots = h_{M-1} = 1/\sqrt{M} \), and the matrix rows satisfy the orthogonal relations (1). It can be seen from the construction process of the fundamental matrix that, when the filtering coefficient is set, the wavelet coefficient, namely the orthogonal normalized vector \( g^{(s)}_k, k = 0, 1, \ldots, (M-1), s = 1, 2, \ldots, (M-1) \),
can be calculated by the orthogonal relation equations (1), and finally the fundamental matrix \( A_M \) is obtained. When \( M \) increases, the rapid construction is of great significance.

It is found that when \( M \) can be factorized, a recursive method is adopted to construct a high-order fundamental matrix from a low-order fundamental matrix, which can realize the rapid construction of \( M \)-band fundamental matrix. The realization of recursive relation is mainly based on matrix direct product operation. The recursive relation used here is: if \( A_p \) and \( A_Q \) is the low-order fundamental matrix, then there is

\[
A_R = A_p \otimes A_Q
\]

Satisfy \( A^T_R A_R = I \) or \( A^{-1}_R = A^T_R \).

When \( P = Q \), the high-order fundamental matrix can be constructed quickly by using the power relation of the low-order fundamental matrix. When \( P \neq Q \), we can get the high-order fundamental matrix whose order can be factorized quickly. Generally speaking, when factorization is decomposed into 2 or 3 order of the fundamental matrix as far as possible, for example, in order to construct a 24 order of the fundamental matrix \( A_{24} \), because 24 order can be decomposed into \( 2^3 \times 3 \), so we don't have to construct \( A_{24} \) by cumbersome matrix orthogonal standardized process directly, \( A_{24} \) can be realized very conveniently by \( A_{24} = A_2 \otimes A_2 \otimes A_2 \otimes A_3 \). Therefore, the construction of high order fundamental matrix is realized by using low order fundamental matrix, which obviously reduces the complexity of calculation greatly. Here, the fundamental matrix of second and third order used is:

\[
A_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \end{bmatrix}
\]

2.2. Data Expansion Scheme Based on Multi-band Wavelet Transform

At present, there are three methods of image magnification based on wavelet transform. Firstly, the original image is decomposed after binary wavelet transform, high frequency components are amplified by interpolation [4], at the same time the original image take the place of low frequency component, and then reconstructed image by using inverse wavelet transformation, this method has caused many unnecessary details edges because its high frequency components are different from high frequency components of the original image; Secondly, the image is enlarged by interpolation method and then decomposed by wavelet transform [5]. Following, the image is reconstructed by inverse wavelet transform after the low-frequency part is replaced by original image. The above two methods will cause darkening the average gray level of the enlarged image. Thirdly, to solve the problem of darkening the average gray level of the enlarged image, a method of copying adjacent pixels [6] is proposed. However, enlarged image by using this method is of serious fuzziness with the increase of the magnification times. So this author uses the first two method, and makes two improvements. One, multi-band wavelet transform method is used instead of the binary wavelet transform method, so under different magnification, original image is decomposed by only once wavelet decomposition, which avoids low frequency decomposition again based on low frequency components, two, aiming at darkening problem in view of the average gray level of image, the image after inverse transformation is processed by Histogram Specification using histogram of original image specification as reference. And the last data expansion is realized in multi enlarged images through translation and mirroring transform, and copying. The overall idea is shown in figure 1.
2.3. The Evaluation Index of Expand Data

2.3.1. Average Gradient. Average gradient is the measurement of image clarity. Generally speaking, the higher the average gradient is, the richer the image detail information will be.

\[ T = \frac{1}{(M - 1)(N - 1)} \sum_{i=1}^{M} \sum_{j=1}^{N} \sqrt{(\Delta_x^2 + \Delta_y^2)} \]

where \( \Delta_x = F(i + 1, j) - F(i, j) \) and \( \Delta_y = F(i, j + 1) - F(i, j) \).

2.3.2. Cross Entropy. The definition of cross entropy:

\[ \text{CEN}(P, Q) = \sum_{i=1}^{m} p_i \ln \left( \frac{p_i}{q_i} \right) \]

where \( P = \{p_1, p_2, \cdots, p_m\} \) and \( Q = \{q_1, q_2, \cdots, q_i, \cdots q_m\} \), represents the gray distribution of the image before and after amplification respectively. The smaller the cross entropy is, the smaller the difference between the enlarged image and the original image.

2.4. Experimental Results and Analysis

Three images were selected for algorithm analysis in the experiment, namely, images in figure 2, the starry sky image and the urban area image. The first image was the commonly used test image, while the latter two images were characterized by large imaging area, relatively small target in the whole image, and great difficulty in target identification. For these small target images, adopting the scheme of data expansion in this paper can not only improve the resolution of the image target, but also be very effective for the diversified expansion of sample data, which can improve the target recognition accuracy of small target or small sample data.

In the experiment of scheme 1 and scheme 2, the wavelet transform constructed in this paper was used to compare the effects of two different schemes on image amplification. From figures 3-5, it can be seen that compared with Refs. [4, 5], no matter scheme 1 or Scheme 2, the enlarged image is superior to the original image in maintaining the gray value of the image compared with Refs. [4, 5], and the gray value of the original image is well maintained. Seen from the figures 3-5e and 5f, the histogram equalization distribution effect of scheme 1 is better than that of Scheme 2. Seen from the low gray value end and high gray value end, the contrast of enlarged image of scheme 1 is better than that of scheme 2, which is further proved in figures 3-5b and 5d. Secondly, as can be seen from the comparison between figures 4 and 5, when the target becomes small, many false edges appear in the enlarged image of scheme 2, and the fuzziness becomes more obvious, which we do not want.

In order to compare the performance of the algorithms in more detail, average gradient and cross entropy are selected as indicators to compare the algorithms, as shown in table 1. As can be seen from table 1, after image enlargement, the average gradient of Refs. [4, 5] was significantly smaller than that of the original image with unclear edges, and the cross entropy was significantly increased, indicating that the correlation with the original image was weakened. In this paper, the average
gradient values of the two proposed schemes are obviously increased, but there are a lot of pseudo-edges in scheme 2, so the edge clarity is not better. In scheme 1, not only the edge clarity is improved, but also the cross entropy is significantly lower than that in Refs. [4, 5], indicating that the original image information is better maintained. It can be seen that the image magnification effect of scheme 1 is the best, and scheme 2 are suboptimal, followed by Ref. [5] and Ref. [4].

Figure 2. Original image.

Figure 3. Magnification comparison of original image 1.
Figure 4. Magnification comparison of original image 2.

Figure 5. Magnification comparison of original image 3.
Table 1. Performance index comparison.

| Original Image | Performance index | Original image | Ref. [5] | Scheme one | Ref. [4] | Scheme two |
|----------------|-------------------|----------------|---------|------------|---------|------------|
| Image 1        | Average gradient  | 23.0530        | 7.4041  | 14.1948    | 12.8713 | 23.4367    |
|                | Cross entropy     | -0.7461        | 0.3316  | -0.9807    | 0.3122  | 8.9271     |
| Image 2        | Average gradient  | 9.2020         | 3.0493  | 5.7034     | 5.1996  | 8.9271     |
|                | Cross entropy     | -2.0055        | 0.3206  | -1.9948    | 0.2828  | 20.8683    |
| Image 3        | Average gradient  | 22.2541        | 7.2382  | 13.7263    | 11.6272 | 20.8683    |
|                | Cross entropy     | -1.5156        | 0.0861  | -1.6068    | 0.0718  |            |

In the process of image data expansion, we should give full consideration to all aspects which sample image becomes poor, so we take full advantage of the enlarged image of scheme 1, 2, Ref. [4], and Ref. [5], because the enlarged image of the above four methods are different in clarity and contrast. Then we can effectively implement expansion of less sample data through translation transform, mirroring translation, and cropping image according to a certain size, which copying may also lead to the un-integrity of the target. At the same time, for the small target image, the method in this paper can be used to expand the data with better resolution, so as to improve the detection accuracy of small target. It can be seen that the data expansion method in this paper greatly enriches the diversity of the training set in resolution, clarity, contrast, level of completeness and so on.

3. Conclusion

Aiming to the present problems existing on the image magnification method based on wavelet transform, the author presents a structure method of multi-band wavelet quickly, then proposes two scheme of image magnification based on multi-band wavelet transformation, and deeply analyzes merit and demerit of the improved algorithm by qualitative and quantitative analysis, finally, based on the results of image magnification of two typical literature algorithm and two kinds of improved algorithm, implementation method of image expansion are given. The expansion image samples obtained in this paper mainly reflect diversified changes on resolution, clarity, contrast and target integrity, and target rotation changes of the expansion images are deserved to be further studied.

References

[1] Chen L, Zhang F and Jiang S 2019 Sheng deep forest learning for military object recognition under small training set condition Journal of CAEIT 14 232-37.
[2] Li P 2018 Training Convolutional Neural Network with Only Less Positive Samples to Build Model of Multiobjective Recognition (Fujian Normal University) Master Thesis.
[3] Zhang J, Shao K and Luo X 2018 Small sample image recognition using improved convolutional neural network Journal of Visual Communication and Image Representation 55 640-647.
[4] Shi J, Guo B 1998 A new image interpolation scheme-subband interpolation Journal of Xidian University 25 684-88.
[5] Wang W and He Y-B 2000 The application of the wavelet decomposition in image zooming Computer Engineering and Applications 43-45.
[6] Wang Y 2008 Research on image magnification based on wavelet transform Software Guide 7 143-145.