Quantum effect on parametric dispersion in presence of nonuniform size colloids in semiconductors

R Vanshpal¹, S Dubey, S Ghosh
¹School of Studies in Physics, Vikram University, Ujjain-456010 (M.P.) India

E-mail: ravivanshpal@gmail.com

Abstract. Quantum effect on parametric dispersion characteristics in ion implanted semiconductors in presence of nonuniform size colloids is analytically investigated in the present report. Nonuniform size colloids are managed through polynomial distribution function in the analysis. Here the used quantum hydrodynamic model is described by a set of hydrodynamic equations (typically continuity and momentum transfer) that include quantum effects via Bohm potential. Bohm potential modified second order optical susceptibility is obtained through nonlinear induced current density in presence of electrons and negatively charged nonuniform size colloids. It is found that parametric dispersion characteristics are greatly influenced by the quantum modifications. The parametric dispersion of the generated signal mode reduces due to the presence of Bohm potential. The required pump intensity at which one achieves maximum dispersion shifts towards higher value in presence of quantum term. Moreover present study also establishes that quantum effect on colloids is inversely proportional to their size; smaller colloids induce more quantum modifications.

1. Introduction –
Ion implantation techniques are generally employed for the manufacturing of semiconductor components. The implanted ions in a host material can alter their properties through introduction of small quantities of dopant atoms. In this process implanted metal ions directed towards the target and being neutralized during slowing down process (electronic and nuclear stopping) and finally come at rest. The depth profile of dopant atoms may be explained by chemical binding effect associated with ion-ion and ion-target atom interaction at low energy. The energy ranges from several hundred to several million electron volts, resulting in ion distributions with average depth from < 10 nm to 10 µm. The colloid formation of metal ions (such as Ag+, Cu+, Fe+ etc) by ion implantation techniques in SiO₂ glasses has been carried out in a number of laboratory experiments. The presence of metal colloids inside any semiconductor in addition to already present mobile charge carriers (electron and holes) converts the solid into multi-component semiconductor plasma medium. In recent past, the role of charged colloids in searching the novel modes of propagation and/or in modification of wave characteristics of existing modes in ion implanted semiconductor plasma (IISP) has been extensively explored [1-4]. In recent past, a number of workers [5, 6] have theoretically studied the long ranged order formation of colloid particles of small sizes. However, quantum effects were not taken into account in any of these studies.
The quantum effect is included in the present study by employing the quantum hydrodynamic model (QHD) for the carrier dynamics in the IISP medium. The QHD model is described by a set of hydrodynamic equations (typically continuity and momentum transfer) that include quantum effect via Bohm potential and Fermi pressure. The quantum effect on parametric amplification characteristics in piezoelectric semiconductor has been recently reported by the present authors [7]. Quantum effect on parametric dispersion characteristics in IISP in presence of nonuniform size colloids is analytically investigated in the present report. Nonuniform size colloids are managed through polynomial size distribution function in the analysis. In section 2 we first introduce the basic sets of QHD equations which are used to derive the second order susceptibility of IISP medium. Section 3 is devoted to the numerical analysis of the problem under study; the consequence of results are also discussed.

2. Theoretical Formulation

In this section, authors focus on the derivation of second-order optical susceptibility arising due to parametric interaction of a high frequency pump beam with internally generated acoustic mode in presence of nonuniform size colloids in IISP medium. The medium is subjected to a spatially uniform high frequency pump electric field \( \vec{E}_0 \exp(-i\omega_0 t) \) (i.e. pump vector \(|\vec{k}_0| \approx 0\).

When plasma is cooled down to extremely low temperature the de-Broglie wavelength of the plasma particles may become comparable to the Debye length or other scale lengths of the plasma and quantum effects are expected to play a significant role in the behaviour of charged particles. Using the magnetohydrodynamic model of plasma, Haas, Manfredi and others [8] have developed the quantum hydrodynamic (QHD) model of plasma. The quantum statistics is included in the model through the equation of state which takes into account the Fermionic character of the electrons and colloids. The basic equations in such colloid laden semiconductor quantum plasma with different size colloids are as follows:-

\[
\rho \frac{\partial \vec{u}}{\partial t} - 2\rho' \frac{\partial \vec{u}}{\partial t} + \beta \frac{\partial \vec{E}}{\partial \vec{x}} = \gamma \frac{\partial ^2 \vec{u}}{\partial \vec{x}^2}
\]

\[
\frac{\partial \nu_{dil}}{\partial t} + \nu \nu_{dil} = -\frac{Z_i e}{m_i} E_0
\]

\[
\frac{\partial \nu_{eil}}{\partial t} + \nu \nu_{eil} + \left[ \nu_{eil}, \frac{\partial }{\partial \vec{x}} \right] \nu_{eil} = -\frac{Z_i e}{m_i} E_1 - \frac{1}{m_i n_0} \frac{\partial P_i}{\partial \vec{x}} + \frac{\hbar^2}{4 m_i^2 n_{0l}} \frac{\partial ^3 \nu_{eil}}{\partial \vec{x}^3}
\]

\[
\nu_{dil} \frac{\partial \nu_{dil}}{\partial \vec{x}} + \nu_{dil} \frac{\partial \nu_{dil}}{\partial \vec{x}} = -\frac{\partial \nu_{dil}}{\partial \vec{x}}
\]

\[
\frac{\partial E_{1l}}{\partial \vec{x}} + \beta \frac{\partial ^2 \vec{u}}{\partial \vec{x}^2} = -\frac{z_i e}{e} \nu_{eil}
\]

\[
P_i = \frac{m_i \nu_{eil}^3 n_{0l}^3}{3n_0^2}
\]

where \( l = e \) for electrons and \( d \) for colloids. Meaning of all the symbols is given in [7].

Following standard approach [9], with the aid of Eqs. (1) to (6), one gets second order optical susceptibility of the medium as

\[
\chi^{(2)}_l = \frac{eeAk}{2\gamma_e \omega_e \omega_0 \omega_1} \left[ \frac{Z_e \omega_e^2}{m_e} R^{-1} + \frac{Z_d \omega_d \omega_{ped}}{m_d} V^{-1} \right] = \frac{eeAk}{2\gamma_e \omega_e \omega_0 \omega_1} \left[ \chi_e + \chi_d \right]
\]
In which $R = \left[ \frac{\omega_{d}^{2} + k^{2}V_{d}^{2}}{\omega_{p}^{2} + k^{2}V_{p}^{2}} - \omega_{s}^{2} - i\omega_{s}V - \frac{k^{2}E_{d}^{2}}{\omega_{p}^{2} + k^{2}V_{p}^{2}} - \omega_{s}^{2} - i\omega_{s}V \right] \cdot V = \left[ \frac{\omega_{d}^{2} + k^{2}V_{d}^{2}}{\omega_{p}^{2} + k^{2}V_{p}^{2}} - \omega_{s}^{2} - i\omega_{s}V - \frac{k^{2}E_{d}^{2}}{\omega_{p}^{2} + k^{2}V_{p}^{2}} - \omega_{s}^{2} - i\omega_{s}V \right]$ and $\chi_{e,d}$ stand for susceptibility due to electrons and colloids respectively.

Rationalization of Eq. (7) gives the real and imaginary parts of the second-order optical susceptibility. It may be inferred from Eq. (7) that the susceptibility due to implanted charged colloid (\(\chi_d\)) makes an important impact on the total optical susceptibility of the crystal.

2.1 Colloidal size distribution

The quantum effect on electron dynamics are identical due to constant size and mass of the electrons but quantum effect on colloids shall vary with slight change in their size and mass. The grains in colloidal plasma are found in a great variety of sizes, masses and charges. Quantum effect of the particles mainly depends upon their size and mass. In our work we consider a polynomial expressed distribution of colloid particles in quantum colloidal plasma medium consisting of different size colloids. The differential polynomial-expressed distribution function \([8]\) is of the form,

$$N_{\text{tot}} = \left[ n(r)dr \right] , \quad n(r)dr = [a_{0} + a_{1}r + a_{2}r^{2} + a_{3}r^{3} + \ldots \ldots \ldots \ldots ]dr \quad (8)$$

where $N_{\text{tot}}$ is the total number density of colloid grains, $r$ is the radius of colloids in a given range \([r_{\text{min}}, \ r_{\text{max}}]\), $a_{0}, a_{1}, a_{2}, a_{3}, \ldots$ are all constants. Outside the limits $r < r_{\text{min}}$ and $r > r_{\text{max}}$, we considered $n(r) = 0$. The charge and the mass of the $j$-th colloid grain can be expressed as $Z_{j} = k_{2}r_{j}, m_{j} = k_{m}r_{j}^{2}$ where $k_{2} = 4\pi\varepsilon_{0}V_{0}/e$, $k_{m} = 4\pi\rho_{d}/3$, $V_{0}$ is the electric surface potential at equilibrium, $\rho_{d}$ is the mass density of the colloid grains (assumed to be constant and equal for all grains). We assume that the colloid size distribution is given by Eq. (8) and hence we can obtain susceptibility due to colloids as,

$$\chi_{d} = \int_{r_{\text{min}}}^{r_{\text{max}}} \frac{\text{Re}^{3}k_{d}^{3} n(r)}{2\varepsilon_{0}^{2}r_{j}^{2}e_{0}a_{0} b_{0} a_{1}} \left[ \frac{e^{2}k_{d}^{2} n(r)}{2k_{2}K_{d}T_{\text{eff}}} \frac{k^{2}h^{2}}{k_{m}r^{3}} + \frac{k^{2}E_{d}^{2}}{4k_{m}r^{6}} - \alpha_{s}^{2} \right] \frac{dr}{e^{2}k_{d}^{2} n(r) - \frac{k^{2}h^{2}}{k_{m}r^{3}} - \frac{k^{2}E_{d}^{2}}{4k_{m}r^{6}} - \alpha_{s}^{2}}$$

Thus the modified real part of total susceptibility becomes

$$\text{Re}(\chi_{d}) = \frac{\text{e}c_{0}A k}{2\gamma_{s} \varepsilon_{0} a_{0} \omega_{0} \omega_{1}} \left[ \text{Re}(\chi_{e}) + \int_{r_{\text{min}}}^{r_{\text{max}}} \chi_{d} \left( j \right) dr \right] \quad (9)$$

It may be inferred from Eq. (9) that the total optical susceptibility is influenced and modified by the presence of implanted metal colloids and their size distribution; hence colloids and their size distribution is responsible for the modifications in dispersion characteristics of the scattered wave in a parametric process.

3. Results and Discussion

To have some numerical appreciation of the above formulations and to study the quantum effect on parametric dispersion in an IISP medium with non uniform size colloids, we have assumed this typical medium duly irradiated by a nano second pulsed 10.6 $\mu$m CO2 laser. All the relevant parameters are given in [7].
Figure 1. Variation of \( \text{Re}(\chi_I) \) with \( E_0 \)

Figure 2. Variation of \( \text{Re}(\chi_I) \) with \( E_0 \) in two radius ranges.

Figure 1 show the variation of real part of total crystal susceptibility \( \text{Re}(\chi_I) \) with pump field amplitude \( E_0 \) in the presence and absence of quantum correction term. Here we have assumed range of colloid radius from 0.1 to 1 nm. The quantum correction shows the effective modification in total optical susceptibility in the crystal. The magnitude of \( \text{Re}(\chi_I) \) first increases and reaches to a maximum value. The value of \( E_0 \) at which one gets maximum magnitude of \( \text{Re}(\chi_I) \) shifts towards higher side whereas maximum possible magnitude of \( \text{Re}(\chi_I) \) reduces due to the presence of quantum correction. A little departure from this peak point sharply diminishes the value of \( \text{Re}(\chi_I) \) to zero. Then it crosses over to negative quadrature and approaches to a minimum. Again the magnitude of \( E_0 \) at which one gets minimum \( \text{Re}(\chi_I) \) shifts towards higher point due to the presence of quantum correction term. Further a forward tuning of \( E_0 \) after the dip increases the \( \text{Re}(\chi_I) \) which saturates afterwards. This saturation regime is achieved at \( E_0 \) where quantum effects become negligible.

Figure 2 shows the variation of real part of total optical susceptibility with \( E_0 \) in two different colloid radius ranges in IISP medium. The polynomial size distribution function gives the flexibility to choose the desired range of colloid radius. Larger colloid radius range increases the magnitude of total optical susceptibility \( \text{Re}(\chi_I) \) as illustrated in the figure.

4. Conclusions

Therefore we can conclude that quantum correction term including Bohm potential effectively modifies the parametric dispersion characteristics of IISP medium having nonuniform size colloids. Modified electron plasma frequency and dust plasma frequency is responsible for these improved dispersion characteristics. Present study reports that polynomial size distribution function appears very useful in determining effect of nonuniform size distribution. The favourable range of radius of colloids can be obtained to have a better IISP medium for the fabrication of semiconductor components.
5. Acknowledgements
The financial assistance from the Madhya Pradesh Council of Science and Technology, Bhopal, India, under a research project is gratefully acknowledged.

6. References
[1] Ghosh S and Khare P 2005 European Phys. J. D 35 521
[2] Ghosh S and Thakur P 2006 European Phys. J. D 37 417
[3] Ghosh S and Thakur P 2006 Ind. J. Pure & Appl. Phys. 44 188
[4] Ghosh S and Khare P 2006 Acta Physica Polonica A 109 187
[5] Salimullah M, Shukla P K, Ghosh S K, Nitta H and Hayashi Y 2003 J. Phys. D: Appl. Phys. 36 958
[6] Salimullah M, Ehsan Z, Zubia K, Shah H A and Murtaza G 2007 J. Appl. Phys. 102 053301
[7] Ghosh S, Dubey S and Vanshpal R 2010 Phys. Lett. A 375 43
[8] Haas F, Garcia L G, Geodert J and Manfredi G 2003 Phys. Plasmas 10 3858
[9] Guha S, Sen P K, Ghosh S1979 Phys. Stat. Sol. (a) 52 407.
[10] He G J, Duan W S and Tian D X 2008 Phys. Plasmas 15 043702.