Fermion theories on a 2d torus with Wilson action improved by Pauli – Villars regularization

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Abstract

Vectorial and anomaly free chiral U(1) fermion models on a 2d finite lattice are considered. It is demonstrated both numerically and analytically that introduction of Pauli – Villars type regularization suppresses the symmetry breaking effects caused by the Wilson term.

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1 Introduction

Treating chiral fermions on the lattice still presents serious problem. Due to well known ”no-go” theorem [1] there is no straightforward way to remove fermion spectrum degeneracy without breaking chiral invariance. Several possibilities to overcome this difficulty were proposed (for recent review see [2]), but all of them have certain problems. A really successful model has to provide reasonable results both in perturbative and nonperturbative regions at least for sufficiently small lattice spacing.

Existing computer facilities allow efficient nonperturbative tests only in two-dimensional models, the U(1) gauge model on the torus being a popular example.

In the present paper we apply to the toron model the method, proposed in ref.[3]. The idea of this method is to introduse in the lattice models additional gauge invariant Pauli – Villars (PV) type regularization which supresses the contribution of momenta close to the border of Brillouin zone, $|p| \sim \pi/a$. Pauli – Villars fields mass $M$ introduces a new scale which is choosen to satisfy the condition $M \ll a^{-1}$. With this condition fulfilled any modification of the action at the distances of order $a$ becomes irrele- vant. Therefore, if the additional regularization respects vectorial and chiral gauge invariance, possible symmetry breaking effects due to introduction of Wilson term or some other device removing spectrum degeneracy, are su- pressed and vanish in the continuum limit.

In the paper [3] it was shown that introducing PV type regularization together with the standard Wilson term [4] for anomaly free chiral gauge models on the infinite lattice one gets in the framework of perturbation theory correct continuum results without chiral noninvariant counterterms. Possible chirality breaking effects are of order $a$.

Below we shall check these results nonperturbatively for the two dimen- sional model – U(1) gauge invariant interaction of fermions on the 2d finite lattice. To compare our calculation with known exact results for continuum theory on the torus [5 – 7] the gauge field is chosen to be constant. We
present the results both for vectorial and anomaly free chiral models and compare them with the corresponding calculations for Wilson fermions supplemented by gauge noninvariant counterterms as proposed by the Roma group [8].

2 Vectorial lattice model

We start with the vectorial model on the finite lattice described by the action [4]

\[ I_{VW} = \frac{1}{2} \sum_{x, \mu} \left\{ \bar{\psi}(x) \gamma_\mu U_\mu \psi(x + \hat{\mu}) - \bar{\psi}(x) [\psi(x + \hat{\mu}) - \psi(x)] \right\} + \text{h.c.} \tag{1} \]

Here \(-N/2 + 1 \leq x_\mu \leq N/2, \mu = 0, 1; N\) is the number of lattice sites. The lattice spacing is chosen to be equal to 1. The first term describes gauge invariant interaction of fermions with the constant field

\[ U_\mu = \exp \left( \frac{2\pi i}{N} h_\mu \right). \]

The second term is the Wilson term removing fermion spectrum degeneracy. Being interested in the study of symmetry breaking effects on a lattice we choose the gauge noninvariant Wilson term. Note that chiral invariance is broken both for noninvariant or covariant Wilson terms. The Fermi field \(\psi\) satisfy antiperiodic boundary conditions.

A straightforward calculation gives the following expression for the determinant:

\[ D_{VW} = \prod_{p = -N/2 + 1}^{N/2} \frac{B^2(p, h) + W^2(p)}{B^2(p, 0) + W^2(p)}, \tag{2} \]

where

\[ B_\mu(p, h) = \sin \left( \frac{2\pi}{N} (p_\mu - h_\mu - 1/2) \right), \tag{3} \]

\[ W(p) = \sum_{\mu=0}^{1} \left( 1 - \cos \left( \frac{2\pi}{N} (p_\mu - 1/2) \right) \right). \tag{4} \]
The determinant (2) is normalized to 1 at $h=0$. The corresponding expression in the continuum theory looks as follows [5 – 7]:

$$D_{VC} = e^{-2\pi h_i^2} \prod_{n=1}^{\infty} |F[n, h]F[n, -h]|^2.$$  

(5)

Here

$$F[n, h] = \frac{1 + e^{-2\pi(n-1/2)+2\pi i(h_0+i h_1)}}{1 + e^{-2\pi(n-1/2)}}.$$

The determinants (2), (5) satisfy the following symmetry properties:

$$D[h_0, h_1] = D[h_1, h_0] = D[-h_0, h_1] = D[h_0, -h_1].$$

It allows to consider the fields $h_\mu$ in the interval $0 \leq h_0 \leq h_1$. The continuum determinant (5) satisfies also periodicity condition

$$D_{VC}[h_0, h_1] = D_{VC}[h_0 + n_0, h_1 + n_1], \quad n_0, n_1 = 0, \pm 1, \pm 2, \ldots$$

Due to the breaking of gauge invariance by the Wilson term, the lattice determinant satisfies weaker periodicity condition

$$D_{VW}[h_0, h_1] = D_{VW}[h_0 + N/2n_0, h_1 + N/2n_1].$$

Computer simulations were performed for $0 \leq h_0 \leq 0.5, -0.5 \leq h_1 \leq 1.5$ using the lattices with $N=32, 160$. The results are presented at Fig.1 (curves 1, 2). One sees that the lattice results do not agree with the continuum ones in the whole interval of $h_0, h_1$. It shows that although formally the action (1) has the correct continuum limit, symmetry breaking effects due to noninvariant Wilson term change quantum corrections drastically.

Below we show that this discrepancy vanishes if one modifies the action (1) by introducing additional PV regularization. The modified action looks as follows:

$$I_{VR} = I_{VW} + I_{PV},$$

$$I_{PV} = \frac{1}{2} \sum_x \left\{ \sum_{\mu} \left[ \bar{\phi}(x) \gamma_{\mu} U_{\mu} \phi(x + \hat{\mu}) - \bar{\phi}(x) (\phi(x + \hat{\mu}) - \phi(x)) \right] + M \bar{\phi}(x) \phi(x) \right\} + \text{h.c.}$$
Fig.1. Vectorial determinants $D_V$ as functions of $h_1$ at $h_0 = 0.2$:
1 – $D_{VC}$ on the torus; 2 – $D_{VW}$ without PV field, $N = 160$;
3, 4 – $D_{VR}$ with PV field, $M = M_2$; 3 – $N = 32$, 4 – 160;
5, 6 – $D_{VK}$ with counterterm: 5 – $N = 32$, 6 – 160

Here $\phi(x)$ is a bosonic PV field having the same spinorial and internal structure as $\psi$. In the considered model one PV field is sufficient to suppress the contribution of the region near the border of the Brillouin zone.

Regularized determinant may be presented in the form

$$D_{VR} = D_{VW}[h]D_{PV}[h],$$

where $D_{VW}[h]$ is given by eq.(2) and $D_{PV}$ is the corresponding expression for the PV field:

$$D_{PV}[h] = \prod_{p=-N/2+1}^{N/2} \left( \frac{B^2(p,h) + (W(p) + M)^2}{B^2(p,0) + (W(p) + M)^2} \right)^{-1}.$$  (6)
Fig.2. Vectorial determinants $D_V$ as functions of $M$

at $h_0 = 0.2$ and $h_1 = 0.4$:

1 – $D_{VC}$ on the torus; 2, 3 – $D_{VR}$ with PV field:

2 – $N = 160$, 3 – 320

Note that in this equation we defined the determinant of a bosonic PV field as inverse of the corresponding fermion determinant. One can wonder if this determinant may be really presented as a path integral of the corresponding bosonic action $I_{PV}$. A naive integral over bosonic fields of $\exp(-I_{PV})$ does not exist as the action is not positive definite. However as eq.(2) shows the fermionic determinant $D_{VW}$ is positive and can be presented as a path integral of an exponent of a positive definite action. Therefore the corresponding PV determinant also can be written as a path integral of an exponent of a positive action. Its explicit form can be easily read of the eq.(3). In general, if the modulus of a fermion determinant can be presented as a path integral of exponent of a positive local action, analogous representation is valid for the corresponding PV determinant.

The results of calculations for regularized determinant as function of the regularizing mass $M$ for fixed $h_0$, $h_1$ at $N=160$ and 320 are presented at
Fig.2. Exact agreement with the continuum is achieved for two values of $M=M_1$, $M_2$. However in the whole interval $M_1 < M < M_2$ which in our case is of order $M_1=0.01 – 0.03$ and $M_2 \approx 0.3$ the discrepancy is within 10%. So to get the correct result one needs to tune the regularizing mass, but it is not really a fine tuning.

The dependence of the regularized determinant on the $h_1$ at $h_0=0.2$ for value $M=M_2$ are shown at Fig.1 (curves 3, 4). One sees that for $-0.5 \leq h_\mu \leq 0.5$ there is a very good agreement with the continuum results both for $N=32$ and 160. To get a good agreement for larger $h$ one needs a bigger lattice: for $0.5 \leq h_1 \leq 1.5$ $N=160$ provides a good agreement whereas $N=32$ gives a sizable mistake.

The reason for that is easily understood. PV regularization supresses the contribution of momenta for which $B^2 + W^2 \gg M^2$. One sees from the eq.(3) that $B_\mu$ depends only on the difference $p_\mu - h_\mu$. Hence for $h$ big enough $B^2$ can be much bigger than $M^2$ even for relatively small $p$. In this way part of a physical region will be also supressed. This effect obviously disappears with increasing of $N$. It seems to be a common problem for any non gauge invariant lattice scheme. In particular our calculations show that the same phenomenon occurs in Roma approach [8]. In the particular model one can get better agreement by using other approaches e.g. overlap formalism [6] or staggered fermions as was done in somewhat different context in ref.[9]. In our case we could easily avoid the problem by using a gauge invariant Wilson term. In this case our regularized action is manifestly gauge invariant and the determinant is periodic with the period 1. Hence if there is an agreement with the continuum in the interval $-1/2 \leq h_\mu \leq 1/2$ it is continued automatically to any $h$.

However for general chiral models considering large nonsmooth external fields really makes a problem. One way to avoid it is to supress the contribution of such fields by some additional regularization. In the paper [3] it was shown that in perturbation theory higher covariant derivative for gauge field may play this role. It would be important to check if it also in nonperturbative calculations.
Fig. 3. Values $-\delta_V$ and $0.2\delta_C$ as functions of $h_0^2 + h_1^2$: 

1 – $N = 160$, $h_0 = 0$ and $0.2$; 2 – $32$, $0$.

The alternative way to improve the agreement with the continuum results is, following the Roma approach [8], to introduce to the action (1) a gauge noninvariant counterterm. Introducing the counterterm $k_V (h_0^2 + h_1^2)$, one gets for the determinant

$$D_{VK} = D_{VW} \exp \left[ k_V (h_0^2 + h_1^2) \right].$$

To determine the coefficient $k_V$ we calculated the value $\delta_V = -\ln(D_{VW}/D_{VC})$ as a function of $h_0^2 + h_1^2$, the result being presented at Fig.3. It gives $k_V = -1.6877$. Substituting this value to eq.(7) one gets for $D_{VK}$ the curves presented at Fig.1 (curves 5, 6). Qualitatively we have the same situation as in the case of PV regularization: for $N=32$ there is a good agreement in the interval $-0.5 \leq h_\mu \leq 0.5$, for $N = 160$ the interval of agreement extends to $-0.5 \leq h_\mu \leq 1.5$.

Therefore in the model under consideration both PV regularization and
Roma approach give analogous results. The advantages of PV approach are twofold. First of all in this case the only source of gauge symmetry breaking of the action is the Wilson term and no gauge noninvariant counterterms are needed. One can avoid gauge symmetry breaking completely by choosing other methods of spectrum degeneracy removing. SLAC-model supplemented by PV regularization is the example of manifestly gauge invariant (although nonlocal) model without spectrum doubling [10]. Moreover, gauge symmetry preserving PV type regularization may be constructed also for anomaly free chiral models [3, 10, 11].

Secondly, in more complicated four dimensional models in the framework of Roma approach one would need to introduce more gauge noninvariant counterterms, whereas PV type regularization cuts all unwanted effects of momenta $p \sim \pi/a$, provided the Yang–Mills fields are also regularized by means of higher covariant derivatives (see the discussion in [3]). It is worthwhile to mention that even in a two-dimensional models a simple mass renormalization may be not sufficient to get a good agreement with the continuum [9].

The numerical results obtained above are in a good agreement with the analytical estimates. One can show that all the lattice diagrams with more than two external lines differ from the corresponding diagrams in the continuum toron model by the terms of order $O(1/MN)$. At the same time

$$\Pi_{VW}(0) = \Pi_{VC}(0) - \frac{16\pi^2}{27\sqrt{3}},$$

where $\Pi_{VC}(0) = 2\pi$ is the mass gap in the continuum toron model. Therefore to get the correct continuum result without PV regularization one has to add to determinant $D_{VW}$ the counterterm with the coefficient $k_V = -8\pi^2/27\sqrt{3} \approx -1.6884$, to be compared with the numerical value $-1.6877$.

If the PV fields are introduced the following expressions for polarization operator can be derived:

$$\Pi_{VR}(0) = 2\pi + O(1/MN) + O(M \ln^2 N), \quad \text{if } M \to 0 \text{ when } N \to \infty;$$

$$\Pi_{VR}(0) \to 2\pi, \quad \text{if } M = 0.307 \text{ when } N \to \infty.$$
These values are in a good agreement with the numerical results presented at Fig.2. Analogous estimates are done for anomaly free chiral models.

3 11112 lattice model

In this section we apply the approach described above to anomaly free chiral models on the finite lattice. The possibility to suppress the contribution of momenta close to the border of Brillouin zone in anomaly free 4d models by introducing a chiral gauge invariant PV type regularization is related to the fact that in this case divergent diagrams (with less than 5 external vector lines) contribute only to the modulus of the determinant. From the point of view of these diagrams the theory is essentially vector like. In two dimension the anomalous diagram is the polarization operator, and the only way to get rid off anomaly is to use a combination of left-handed and right-handed interactions in which the terms proportional to $\gamma_5$ cancel. In particular one can consider 11112 model including four left-handed fermions with charge 1 and one right-handed fermion with charge 2.

The action of this model can be written in the form \[[4]\]

$$I_{CW} = \frac{1}{2} \sum_{k=1}^{4} \left\{ \left[ \sum_{x, \mu} \overline{\psi}_{k+}(x) \gamma_\mu (P_+ U_\mu + P_-) \psi_{k+}(x + \mu) - \overline{\psi}_{k+}(x) \times \right. \right.$$

$$\times \left. \left( \psi_{k+}(x + \mu) - \psi_{k+}(x) \right) \right\} + \frac{1}{2} \sum_{x, \mu} \overline{\psi}_-(x) \gamma_\mu (P_+ + U^2_\mu P_-) \times$$

$$\times \psi_-(x + \mu) + \overline{\psi}_-(x) (\psi_-(x + \mu) - \psi_-(x)) \right\} + \text{h.c.} \quad (8)$$

Here $P_\pm = \frac{1}{2} (1 \pm \gamma_3)$. To suppress the contribution of the region near the border of the Brillouin zone we introduce the following interaction of PV fields:

$$I_{PV} = \frac{1}{2} \sum_{x, \mu} \left\{ \overline{\Phi}(x) \gamma_\mu (P_R + U^2_\mu P_L) \Phi(x + \mu) - \overline{\Phi}(x) \times \right.$$

$$\times \left. [\Phi(x + \mu) - \Phi(x)] \right\} + \frac{1}{2} \sum_x M \overline{\Phi}(x) \tau_1 \Phi(x) + \text{h.c.} \quad (9)$$
Here $\Phi(x)$ is doublet of bosonic PV fields $\varphi_+, \varphi_-$ having the same spinorial structure as $\psi_+, \psi_-$ and satisfying antiperiodic boundary condition; $P_L = \frac{1}{2}(1 + \gamma_3 \tau_3)$, $P_R = \frac{1}{2}(1 - \gamma_3 \tau_3)$; $\tau_1, \tau_3$ are Pauli matrices. The regularized action

$$I_{CR} = I_{CW} + I_{PV}$$

in the formal continuum limit, when the Wilson terms are neglected, is invariant under chiral gauge transformations of the fields $\psi_\pm, \Phi$.

The action $I_{CR}$ generates the following propagators:

$$S_{\psi_+\psi_+} = S_{\psi_-\psi_-} = \frac{i\hat{B}(p) + W(p)}{B^2(p) + W^2(p)}, \quad (10)$$

$$S_{\Phi\Phi} = \frac{i\hat{B}(p) + W(p) + M}{B^2(p) + (W(p) + M)^2} \cdot \frac{1 + \tau_1}{2} + \frac{i\hat{B}(p) + W(p) - M}{B^2(p) + (W(p) - M)^2} \cdot \frac{1 - \tau_1}{2}, \quad (11)$$

where $\hat{B}(p) = \sum_{\mu=0}^{1} B_\mu(p)\gamma_\mu$, $B_\mu(p) = B_\mu(p, 0)$ is given by eq.(3) and $W(p)$ is given by (4).

The polarization operator is a sum of the terms which have a form

$$\Pi_{\mu\nu} \sim \sum_{p=-N/2+1}^{N/2} q^2 \text{Tr} \left( \gamma_\mu P_\pm V_\mu(p)S(p)\gamma_\nu P_\pm V_\nu(p)S(p) \right),$$

where $V_\mu(p)$ is a lattice vertex function and the propagators $S$ are given by eqs.(10), (11).

Consider firstly the contribution of physical fields. As the model is anomaly free, the terms proportional to $\gamma_3$ cancel and one gets

$$\Pi_{\mu\nu}^{CW} \sim \sum_{p=-N/2+1}^{N/2} 4 \text{Tr} \left( \gamma_\mu V_\mu(p)\frac{i\hat{B}(p) + W(p)}{B^2(p) + W^2(p)}S(p)\gamma_\nu V_\nu(p)\frac{i\hat{B}(p) + W(p)}{B^2(p) + W^2(p)}S(p) \right).$$

Following the reasonings of ref.[3] one can show that analogous expression for PV fields cancel the contribution of momenta close to the border of the Brillouin zone and the remaining part in the limit $N \to \infty$ coincides with the gauge invariant continuum result. Separating the summation domain into two parts $V_{in}$: $|p| < N\gamma$, $\gamma < \frac{1}{2}$, and $V_{out}$: $|p| \geq N\gamma$, and choosing $M \sim N^\delta$, $\delta < \gamma$, one sees that in the domain $V_{out}$ one can expand the PV
fields propagators in terms of $M^2$. The zero order term coincides with the propagator of $\psi$-fields and due to our choice of charges the leading terms in the expansion of $\Pi^{PV}_{\mu\nu}$ cancel the value $\Pi^{CW}_{\mu\nu}$. The next term is majorated by $N^{-2\gamma}M^2$ and vanishes in the limit $N \to \infty$.

In the domain $V_m$ one can expand the Wilson term in power of $p$. The leading term in the limit $N \to \infty$ gives the gauge invariant continuum expression and the higher order terms are majorated by $N^{4\gamma-2} \to 0$ when $N \to \infty$.

By the same reasoning all higher order diagrams, which correspond to power counting convergent integrals in the limit $N \to \infty$ coincide with the corresponding continuum expressions. So we expect that simulations of the 11112 model (8) regularized by the PV action (9) has to provide a reasonable approximation to the continuum toron model.

A straightforward calculation gives for the regularized lattice 11112 determinant normalized on 1 at $h=0$ the following expression:

$$D_{CR} = D_{CW}[h]D_{PV}[h],$$

$$D_{CW} = D^4_{+W}[h]D^*_W[2h],$$

$$D_{+W}[h] = \prod_{p=-N/2+1}^{N/2} \frac{G[p,h]}{H[p,0]},$$

$$D_{PV}[h] = \prod_{p=-N/2+1}^{N/2} \left( \frac{|G[p,2h]|^2 + M^2[B^2(p,2h) + B^2(p,0) - 2W^2(p) + M^2]}{H[p,M]H[p,-M]} \right)^{-1}.$$

Here

$$G[p,h] = [B_0(p,h) + iB_1(p,h)][B_0(p,0) - iB_1(p,0)] + W^2(p),$$

$$H[p,M] = B^2(p,0) + (W(p) + M)^2,$$

$B_\mu(p,h), B^2(p,h)$ and $W(p)$ are defined above. One sees that the regularization changes only modulus of the lattice determinant whereas its argument remains intact, i.e. $\text{Arg } D_{CR} = \text{Arg } D_{CW}$. 

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Fig. 4. 11112 determinant arguments \( \text{Arg} \ D_C \) as functions of \( h_1 \) at different \( h_0 \):
1 – \( \text{Arg} \ D_{CC} \) on the torus;
2, 3 – \( \text{Arg} \ D_{CR}=\text{Arg} \ D_{CK}=\text{Arg} \ D_{CW} \): 2 – \( N = 32 \), 3 – 160

The corresponding expression in the continuum toron 11112 theory looks as follows \([5 – 7]\):

\[
D_{CC}[h] = D_{+C}^4[h]D_{+C}^*[2h].
\] (13)

Here

\[
D_{+C}[h] = e^{i\pi h_1(h_0 + ih_1)} \prod_{n=1}^{\infty} F[n, h]F[n, -h],
\]

and expression for the \( F[n, h] \) is given above.

The determinants \((12), (13)\) satisfy the following symmetry properties:

\[
D[h_0, h_1] = D^*[h_1, h_0] = D^*[-h_0, h_1] = D^*[h_0, -h_1],
\]

\[
D_{CC}[h_0, h_1] = D_{CC}[h_0 + n_0, h_1 + n_1], \quad n_0, n_1 = 0, \pm 1, \pm 2, \ldots
\]

\[
D_{CR}[h_0, h_1] = D_{CR}[h_0 + Nn_0, h_1 + Nn_1].
\] (14)
Due to these properties it is sufficient to take the external fields in the interval $0 \leq h_0 \leq h_1 \leq N/2$.

It follows from (14), that only diagrams with more than two external lines contribute to the arguments of the lattice and continuum determinants. Such diagrams on the lattice differ from the corresponding continuum ones by the terms of order $O(1/N)$. So in the framework of perturbation theory

$$\text{Arg } D_{CR} = \text{Arg } D_{CW} \rightarrow \text{Arg } D_{CC}, \quad \text{when } N \rightarrow \infty. \quad (15)$$

Computer simulations of the determinant argument were performed for $N=32, 160$ and are presented at Fig.4. One sees that eq.(15) is valid in the interval $-0.5 \leq h_1 \leq 1.5$.

Since the higher order diagrams are convergent the best agreement for the determinant modulus is achieved at the values of $M$ for which the
Fig.6. 11112 determinant modulus $|D_C|$ as functions of $h_1$ at $h_0 = 0.2$:
1 – $|D_{CC}|$ on the torus; 2, 3, 4 – $|D_{CR}|$ with PV field, $M = M_0$:
2 – $N = 32$, 3 – 160, 4 – 320;
5, 6 – $|D_{CK}|$ with counterterm: 5 – $N = 32$, 6 – 160

regularized polarization operator $\Pi_{CR}(0)$ is closest to the continuum value $\Pi_{CC}(0)=8\pi$. Fig.5 shows that the ratio $\Pi_{CR}(0)/\Pi_{CC}(0)$ for $N \geq 160$ differs from 1 less than by 2% if $M = M_0(N) \sim N^{-3/4}$. This gives $M_0 = 0.03 - 0.05$ at $N=160 - 320$. It agrees with the analytical estimates which gives

$$\Pi_{CR}(0) = 8\pi + O(1/MN) + O(M^2\ln^2 N), \quad \text{if } M \to 0 \text{ when } N \to \infty.$$  

The dependence of the 11112 regularized determinant modulus on $h_1$ at $h_0=0.2$ for these values of $M$ is given at Fig.6 (curves 1 – 4). One sees that results obtained for the lattice Wilson action with PV regularization agree with the continuum theory in a certain interval. When the value $N$ grows from 32 to 320 this interval expands from $|h_1| \leq 0.25$ to $-0.5 \leq h_1 \leq 0.9$.

As in the case of vectorial theory another way to achieve the agreement...
of lattice and continuum 11112 theories is to introduce to the action (8) a real gauge noninvariant counterterm. Then we get for the 11112 lattice determinant

\[ D_{CK} = D_{CW} \exp \left[ k_C (h_0^2 + h_1^2) \right], \]

where \( D_{CW} \) defined above. Choosing \( k_C = 6.4005 \) we get the curves 5, 6 shown at Fig.6. At \( N = 160 \) one has an agreement of the theory with counterterm and continuum one in the interval \( -0.5 \leq h_1 \leq 1.5 \).

4 Discussion

In this paper we showed that introducing to the fermion lattice action additional Pauli – Villars type regularization one can suppress the gauge symmetry breaking effects caused by the Wilson term both in perturbative and nonperturbative regime. No gauge noninvariant counterterms are needed to get the correct continuum result. For a finite lattice spacing there are symmetry breaking effects of order \( a \). In principle one can avoid the symmetry breaking completely by introducing a manifestly chiral gauge invariant action like SLAC model supplemented by PV regularization. Our calculations show that in the vectorial model it produces a manifestly gauge invariant result in a good agreement with the continuum expression. However in the anomaly free chiral models there are some problems related to the nonlocality of the SLAC model. These problems will be discussed in a separate publication.

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