Some Long-Run Correlations of Inflation in Developed Countries

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\begin{abstract}
Using 100+ years of data from 18 developed countries, we use a frequency domain technique to compute “long-run” correlations between inflation on the one hand and money growth and nominal interest rates on the other. The estimated long-run correlations are almost always positive. Their magnitude is relatively substantial for money growth, more modest for interest rates. We conclude that some traditional propositions about monetary neutrality are broadly consistent with the data.
\end{abstract}

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1. Introduction

In this paper, we use frequency domain techniques to estimate the long-run correlation of inflation with money growth and with nominal interest rates. We define “long-run” as relating to periods of 10 years or longer. Our data set consists of 18 developed countries and 100+ years of data, with a special focus on the U.S. An attractive feature of our technique is that we use techniques robust to the order of integration, including whether the series are stationary or have a unit root.

Our estimates of the long-run correlations are quite consistently positive: as expected, in the long-run, higher inflation is associated with higher money growth and higher nominal interest rates. While there is some variation in magnitude from country to country, across different measures of money growth and interest rates, and across sample periods, we find relatively substantial correlations with money growth (generally in the range 0.4 to 0.7) and more modest correlations with nominal interest rates (generally in the range 0.2 to 0.4). One way to interpret this is to note that the square of this correlation is the $R^2$ of a univariate regression of low-frequency inflation on low-frequency money growth. Thus the $R^2$ for money growth is roughly 15% to 50%, and for nominal interest rates is about 5% to 15%.

There is, however, some sensitivity to sample period and country. In our post-World War II sample, the correlations with nominal interest rates are higher than the range just given. As well, in our post-World War II sample, long-run correlations of inflation with growth in U.S. monetary aggregates are below the range just given. This seems to reflect the post-2007 explosion of monetary aggregates with no rise in inflation. In general, however, our estimates are not much affected by data post-Great Financial Crisis.

For both monetary aggregates and interest rates, 68% confidence intervals or credible sets generally exclude 0. Put differently, the estimated correlations generally are significantly different from zero at the 32% level.

We are motivated to compute long-run correlations by some hoary propositions in economics. For money growth, a lengthy literature argues that: (1) Money growth is the dominant factor in price movements, at least over long time periods. And: (2) In the long-run, real interest rates are unaffected by inflation; this implies that in the long-run nominal interest rates move one-to-one with inflation. Lucas (1996) traces related ideas back to a 1752 essay by David Hume. We refer to the second proposition as the long-run Fisher effect.

Recent studies on closely related topics include Benati et al. (2021) and Gao et al. (2020). The papers closest to ours perhaps are Benati (2009) and Haug and Dewald (2012). These papers also apply frequency domain techniques to multi-country data with a long time span to consider the long-run relation between inflation and money growth. In contrast to these two papers, we also consider the long-run Fisher effect. As well, our frequency domain methodology is different. We rely on Müller and Watson (2018, 2020) to estimate long-run correlations. Among other benefits, this allows us (in contrast to Benati, 2009 and Haug and Dewald, 2012) to present confidence intervals robust to the order of integration of the data-stationary, fractionally integrated, or unit root, cointegrated or possibly not cointegrated. A long literature of course has noted the
difficulty of pinning down the order of integration of a given series. That one should be open to
the possibility of fractional integration has been argued in the context of the long-run relationship
between inflation and interest rates in papers such as Jensen (2009) and Caporale and Gil-Alaña
(2019).

All our work is reduced form. Nonetheless, we occasionally stray to make structural sounding
statements, such as our statement above translating estimates of long-run correlations to implied
$R^2$’s of regressions of inflation on money growth or interest rates. The reader may be tempted to
interpret this as the percentage of the long-run variance of inflation explained by money growth
or by interest rates. Of course, here, as always, correlation is not causality. This is illustrated in
the context of low-frequency correlations by Whiteman (1984).

2. Methodology

Let $\pi_t$ be annual inflation in a given country and $x_t$ a correlate of interest—$x_t$ will be either
money growth or a nominal interest rate. The description about to follow of how we compute
long-run correlations is taken in part from Lunsford and West (2019), who also use the Müller
and Watson (2018) methodology.

For $\pi_t$ and for a given correlate $x_t$, we first extract the component of each series corresponding
to frequency domain cycles of 10 years or longer. It may help to point out the goal of this filtering
is, essentially, the opposite of that of the familiar Hodrick and Prescott (1997) filter. The HP
filter removes low and high frequencies, leaving only business cycle frequencies. The Müller and
Watson (2018) “low-pass” filter that we use instead removes business cycle and high frequencies,
leaving only low frequencies.

Figure 1 illustrates the low pass filter, for U.S. CPI inflation. The light dashed line is CPI
inflation in the U.S., annual, 1871–2020. (Data sources are given in Section 3.) The solid

![Figure 1](image)

Figure 1. U.S. CPI inflation and its low-frequency component.
black line is the low-frequency component of inflation extracted by the Müller and Watson filter. One can see that the low-frequency component smooths out year-to-year wiggles in the data. It highlights decadal or longer movements: the rise in inflation from the start of our sample through World War I, and the subsequent fall (1920s)-rise (1930s)-fall (1940s and 1950s)-rise (1960s and 1970s)-fall (1980s-present) pattern.

We also extract the low-frequency component of either money growth or nominal interest rates. We then use the two low-frequency series to compute a “long-run” correlation. Let us use Figure 2 to illustrate. The solid black line is the low-frequency component of U.S. inflation, repeated from Figure 1. The lighter dot-dash line is the low-frequency component of M0 growth in the U.S. If we assume the underlying series on inflation and M0 growth are stationary I(0) series, the long-run correlation we report is simply the correlation between these two series, estimated in the usual way. For the data plotted in Figure 2, the estimate of the correlation happens to be 0.47. This is the value reported in Table 3 below in a column headed “I(0)”. We call this an estimate of the “long-run” correlation between the two series, under the assumption that the order of integration of inflation and M0 growth is zero (i.e., under an I(0) assumption).

We estimate the long-run correlation not only assuming I(0) data but also allowing for various non-zero values for the order of integration. When the order of integration is not 0, the long-run correlation is computed from transformations of the low-frequency components (see Müller and Watson, 2018). Hence the estimates of the correlation vary with the assumed order of integration.

Let $d_\pi$ and $d_x$ be the assumed order of integration of inflation and a correlate. For each correlate, we considered three estimates of the long-run correlation: I(0) ($d_\pi = d_x = 0$: both series are stationary I(0)), I(1) ($d_\pi = d_x = 1$: both series have a unit root), and a Bayesian procedure that, as described in the next paragraph, allows for a range of values for $d_\pi$ and $d_x$. Results are not all that different for different orders of integration. We report results assuming stationary
I(0) and for the Bayesian procedure. Our web appendix presents results assuming unit roots.

The Bayesian procedure allows values for $d_\pi$ and $d_x$ between -0.4 and 1—that is, $-0.4 \leq d_\pi, d_x \leq 1$. This range includes conventional stationary models ($d_\pi = d_x = 0$), unit root models ($d_\pi = d_x = 1$), and stationary and nonstationary fractionally integrated models. The procedure places a uniform prior on possible values $d_\pi$ and $d_x$ within the -0.4 to 1.0 range. Müller and Watson (2018)’s code (used by us) produces among other statistics a posterior mean long-run correlation that we call

$$\rho_{-I(d)}.$$ (1)

Their procedure also produces a credible set that enforces coverage over the entire set of values of $d$ considered, according to what Müller and Watson call an “approximate least favorable distribution”. For additional details, see Müller and Watson (2018).

The reader may wonder about the importance of allowing for fractional integration. Fractional integration has played a prominent role in recent studies of the Fisher effect. See, e.g., Caporale and Gil-Alaña (2019).

The uniform prior on values of $d_\pi$ and $d_x$ between -0.4 and 1 implies that all values of the order of integration in this range are equally appealing a priori. We recognize that to some readers this is an unappealing starting point: the literature cited above has used theoretical arguments or applied careful tests to focus on a particular value or values for the order of integration of inflation and a particular correlate. We recognize the validity of this argument, but view the uniform prior as a convenient starting point.

As noted above, we also report long-run correlations assuming $\pi_t$ and $x_t$ are I(0). We call this estimate

$$\rho_{-I(0)}.$$ (2)

We sometimes relate $\rho_{-I(0)}$ to the coherence between $\pi_t$ and $x_t$ at frequency zero. (“Coherence” in the sense of spectral analysis: a measure of the strength of the relationship between two series at a given frequency (Hamilton, 1994, p. 275). Specifically, in comparison to some related earlier literature, we will compare $|\rho_{-I(0)}|$ to that literature’s estimates of coherence at frequency zero.

We report $R^2$ for $\rho_{-I(0)}$ and the posterior mean $R^2$ for $\rho_{-I(d)}$. $R^2$ for $\rho_{-I(0)}$ is merely the square of the estimate of the long-run correlation $\rho_{-I(0)}$. The posterior mean $R^2$ for $\rho_{-I(d)}$ is a weighted average of the posterior squared correlations where the weights are the posterior probabilities. For concreteness and simplicity, we sometimes interpret $R^2$ as a measure of how much of $\pi_t$ is explained by a given correlate. But, formally, upon recalling that in a bivariate regression such as ours, $R^2$ is a monotonic function of the t-statistic on the correlate, $R^2$ supplements the confidence interval or credible set as an indicator of the statistical strength of the relationship.

Following Müller and Watson (2018) and Lunsford and West (2019), we report 68% confidence intervals (for our I(0) estimates) or credible sets (for our Bayesian I(d) estimates). We report 68% rather than 90% or 95% intervals or sets because, even with our 100+ years of data, we do not have many non-overlapping 10 year observations. Our web appendix reports 90% confidence intervals and credible sets. As well, for a few of our series, we experimented with computing long-run correlations after extracting cycles of 15 years or longer (rather than 10 years or longer). Results were virtually unchanged, and are reported in our web appendix.
A note on terminology: from this point forward, we will use “confidence interval” but not “credible set” to refer to our measure of uncertainty, even when discussing our Bayesian I(d) estimates.

3. Data

3.1 Data Sources

Most of our data comes from the Jordà-Schularick-Taylor (2017) Macrohistory Database. The data are annual, for the 18 developed countries listed in Table 1. The maximum data span is 1870–2017. Taking log differences to convert price levels to inflation and money supply levels to growth rates reduced the maximum span to 1871–2017.

Detailed descriptions of data sources can be found in Jordà et al. (2017). Briefly: the price level is the CPI; narrow money is M0; broad money is M3; short term interest rates are for Treasury bills in recent years but for private debt in earlier years; long term interest rates are government debt, for example 10 year bonds in the U.S. For GDP weighting to construct multicountry aggregates (see below), PPP adjusted GDP is used.

For the U.S., we also use an updated version of the data in Lunsford and West (2019). The interest rate series are similar to that for the U.S. in the 18-country dataset. But narrow money is M1 and broad money is M3. We look at GDP inflation as well as CPI inflation. The data for M1 end in 2019. All other data extend through 2020.

3.2 Sample Periods

We report results for four sample periods. Per the blank entries in Table 1, a number of series have missing values. In each sample period, we only use countries for which data are continuously available over the indicated period (apart, perhaps, from a few observations at the beginning or end of the period). The four periods are:

- 1871–2017 (1871–2020, for the U.S.). The longest sample.
- 1871–1913. The classical gold standard era.
- 1919–2017 (1919–2020, for the U.S.). The post-WWI era.
- 1948–2017 (1948–2020, for the U.S.). The post-WWII era.

We consider various samples to allow for regime shifts. These can reflect changes in structure (e.g., gold standard vs. fiat money regime), one-time events that potentially dominate certain time periods (e.g, the Great Depression or World War I or World War II), or gradual drifts that cumulate to a discernable change in behavior. Our choice of three but not more subperiods of our 1871–2017 sample partly reflects our desire to have at least several decades worth of data in a given subsample. (A technical note: due to a limitation in our code, for 1871–2017 samples, the frequency cut-off in our low-pass filter is 11 years rather than 10.)
Table 1
Data availability.

| Year Interval | M0 | M3 | \(i^S\) | \(i^L\) | M0 | M3 | \(i^S\) | \(i^L\) | M0 | M3 | \(i^S\) | \(i^L\) | M0 | M3 | \(i^S\) | \(i^L\) |
|--------------|----|----|---------|--------|----|----|---------|--------|----|----|---------|--------|----|----|---------|--------|
| Australia    | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      |
| Belgium      |    |    |         |        |    |    |         |        |    |    |         |        |    |    |         |        |
| Canada       | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      |
| Switzerland  | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      |
| Germany      |    |    |         |        |    |    |         |        |    |    |         |        |    |    |         |        |
| Denmark      | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      |
| Spain        |    |    |         |        |    |    |         |        |    |    |         |        |    |    |         |        |
| Finland      | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      |
| France       |    |    |         |        |    |    |         |        |    |    |         |        |    |    |         |        |
| UK           | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      |
| Ireland      | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      |
| Italy        |    |    |         |        |    |    |         |        |    |    |         |        |    |    |         |        |
| Japan        | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      |
| Netherlands  | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      |
| Norway       | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      |
| Portugal     | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      |
| Sweden       | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      |
| USA          | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      | x  | x  | x       | x      |
| n            | 11 | 12 | 9      | 13     | 17 | 16 | 14     | 16     | 13 | 14 | 15     | 18     | 17 | 16 | 18      | 18     |

Notes: 1. The table indicates series that are available to compute long-run correlations with inflation over the given sample period. 2. Notation: M0 and M3 are growth in narrow and broad money; \(i^S\) and \(i^L\) are short and long-term nominal interest rates. 3. For a small number of series, the sample start date is slightly later given in the table; two series end in 2016; U.S. data continue to 2020.

3.3 Multicountry Aggregates

In addition to country-by-country estimates, we estimate long-run correlations for weighted averages of individual country variables. These are constructed by taking (smoothed) GDP weighted averages of the underlying data. The weights are computed from averaging GDP in the previous five years. For simplicity, let us temporarily refer to these as “world” variables.

We compute a different world measure for each sample period and correlate, using the countries as indicated in Table 1. For example, to compute the correlation between world inflation and world M0 growth for 1871–2017, we construct world variables from the 11 countries with an “x” in the “1871–2017 / M0” column in Table 1. (For samples beginning in 1871, five years of GDP data are available only by 1875. GDP weights for 1871–1874 are constructed averaging data back to 1871.) To construct those variables for the 1919–2017 sample we add in the two additional countries with an “x” in the “1919–2017 / M0” column in Table 1. Note that our measure of world inflation used to compute a long-run correlation with (say) M0, 1919–2017, is different from the measure used to compute a long-run correlation with (say) M3, 1919–2017. This is because M3 data is available for an additional country, namely, Denmark. This means that Denmark’s GDP weighted inflation is included in world inflation when computing the long-run correlation with world M3. Indeed, world inflation generally differs for different correlates in a given sample. The lone exception is the one case (1948–2017, M0 and \(i^L\) ) in which data coverage is identical for two correlates.
4. Empirical Results

4.1 Overview

We report detailed results for our world variables and for the U.S. We present summary results of country-by-country estimation for our 18 countries, with country-by-country results reported in Appendix A. We also summarize results of country-by-country estimation for the half of our countries that are not part of the Euro area. These 9 non-Euro area results are of course a subset of the results for all 18 countries. We report a separate summary for them because country-specific monetary aggregates, and possibly interest rates, are less meaningful after the introduction of the Euro. Hence we provide results not reliant on those aggregates (though in the end, median estimates of the long-run correlation, and statistical significance, are pretty similar for Euro and non-Euro countries, even in samples that include post-2001 data). As just noted, Appendix A has country-by-country results for all countries, Euro and non-Euro, that are the basis of the summary results presented in our tables.

We consider in turn narrow money growth, broad money growth, short term interest rates, and long term interest rates. For each of the four variables we first present results for CPI inflation in our 18-country database, with longest sample 1871–2017. We then present U.S. results for CPI and for GDP inflation, with longest sample 1871–2020.

4.2 Money Growth

Tables 2 and 3 present results for narrow money growth. All our results are presented in a format similar to Tables 2 and 3. Table 2 presents results from our 18-country dataset, and Table 3 presents results for the U.S.

Table 2 includes three panels. Panel A gives results for “world” variables constructed as described above. Column (1) in panel A gives the number of countries that goes into the weighted average. The list of countries can be seen in Table 1. For example, in the first column of panel A, \( n = 11 \) for 1871–2017; the 11 countries are those listed for M0 in the 1871–2017 panel of Table 1. For each sample period, panel A presents estimates and 68% confidence intervals of the long-run correlation (columns (2a) and (3a)), along with \( R^2 \) (columns (2b) and (3b)). Note that the country coverage changes from sample period to sample period.

Panel B presents median estimates of individual country results for the same countries that go into the GDP weighted averages in panel A. For example, the 0.76 figure for the 1871–2017 sample in column (2a) in panel B means that 0.76 is the median value across the 11 estimates for the 11 different countries for which continuous M0 data was available 1871–2017. Panel B, the “# sig” entry gives the number of countries for which the estimate is positive and the 68% confidence interval does not include zero. For example, for the 1871–2017 sample in panel B, the figure of “11” in the “# sig” row means that for all 11 countries, the 68% confidence interval excludes 0.

Panel C is formatted identically to panel B. It is based on the subset of panel B results for countries whose currency is not the Euro after 2002.
Table 2
Long-run correlations of CPI inflation with narrow money growth.

| A. Weighted average of n countries | B. Median ρ and R² across n countries | C. Median ρ and R² across n non-Euro countries |
|-----------------------------------|--------------------------------------|-----------------------------------------------|
| (1) | (2a) | (2b) | (3a) | (3b) | n | ρ | I(0) | I(d) |
| (1) | (2a) | (2b) | (3a) | (3b) | n | ρ | I(0) | I(d) |
| (1) | (2a) | (2b) | (3a) | (3b) | n | ρ | I(0) | I(d) |
| 1871–2017 | 11 | 0.75 | 0.56 | 0.69 | 0.50 | 11 | 0.76 | 0.58 | 0.72 | 0.53 | 8 | 0.68 | 0.47 | 0.64 | 0.43 |
| | (0.64, 0.82) | (0.56, 0.82) | # sig | 11 | 11 | # sig | 8 | 8 |
| 1871–1913 | 17 | 0.83 | 0.68 | 0.72 | 0.55 | 17 | 0.57 | 0.32 | 0.35 | 0.22 | 9 | 0.57 | 0.32 | 0.43 | 0.26 |
| | (0.63, 0.90) | (0.53, 0.89) | # sig | 11 | 10 | # sig | 5 | 5 |
| 1919–2017 | 13 | 0.61 | 0.37 | 0.50 | 0.30 | 13 | 0.67 | 0.46 | 0.60 | 0.39 | 8 | 0.60 | 0.36 | 0.48 | 0.28 |
| | (0.43, 0.73) | (0.30, 0.71) | # sig | 11 | 11 | # sig | 6 | 6 |
| 1948–2017 | 18 | 0.31 | 0.10 | 0.23 | 0.12 | 18 | 0.48 | 0.23 | 0.27 | 0.14 | 9 | 0.54 | 0.29 | 0.34 | 0.17 |
| | (0.04, 0.52) | (-0.03, 0.55) | # sig | 12 | 9 | # sig | 6 | 6 |

Notes: 1. Here and throughout, data are annual and long-run correlations are estimated from the components of inflation and short-term rates corresponding to frequencies longer than 10 years. See text for details.
2. In each of the three panels, columns (2a) and (3a) present estimates and information on statistical significance for long-run correlations between CPI inflation and growth in M0 over the sample period given in column (1). Column (2a) is constructed under the assumption that both variables are I(0). Column (3a) is the posterior mean estimate of a Bayesian procedure that allow inflation and money growth to have any order of integration between -0.4 and 1.0, where 0 is the usual stationarity assumption and 1 corresponds to unit roots. See Müller and Watson (2018) for details.
3. In each of the three panels, columns (2b) and (3b) give the R² of a regression of inflation on money growth, where both variables are standardized to have unit variance. In (3b), this is again the posterior mean from a Bayesian procedure that allows orders of integration between -0.4 and 1.0.
4. Panel A presents results for a GDP-weighted aggregate of inflation and growth in M0. See text for details. The number of countries used to construct the aggregates is given in the column labeled n. For example, for 1871–2017, the 11 countries used to construct the aggregates are the 11 countries given in the M0 column of the 1871–2017 panel of Table 1. In columns (2a) and (3a) in panel A, the figures given in parentheses are 68% confidence intervals or credible sets.
5. Panels B and C present medians across the set of individual country estimates. For example, n = 11 and ρ̂ = 0.46 in the I(0) column of the 1871–2017 row of panel B indicates that 0.76 is the median value of 11 estimates of ρ made under an I(0) assumption. Panel C presents results in which Euro area countries have been removed from panel B.
6. In panels B and C, the “# sig” entries give the number of countries in which ρ̂ is positive and the lower bound of the 68% confidence interval is positive.
Table 3
Long-run correlations of inflation with narrow money growth in the U.S.

|     | A. CPI inflation | B. GDP inflation |
|-----|------------------|------------------|
|     | (1) (2a) (2b) (3a) (3b) | (2a) (2b) (3a) (3b) |
|     | I(0) I(d) | I(0) I(d) |
|      | \( \hat{\rho} \) | \( \hat{\rho} \) |
| \( R^2 \) | \( \hat{R}^2 \) | \( \hat{R}^2 \) |
| 1871–2020 | 0.47 0.22 | 0.41 0.21 |
|          | (0.29, 0.60) | (0.21, 0.59) |
| 1871–1913 | 0.67 0.45 | 0.63 0.44 |
|          | (0.38, 0.81) | (0.42, 0.83) |
| 1919–2020 | 0.16 0.02 | 0.11 0.07 |
|          | (-0.07, 0.36) | (-0.12, 0.35) |
| 1948–2020 | 0.08 0.01 | 0.05 0.07 |
|          | (-0.19, 0.33) | (-0.21, 0.32) |

Notes: 1. In panel A, narrow money is measured as in the previous table, as M0. In panel B, M1 is used instead, and the last available observation is 2019 rather than 2020. Some entries are missing in panel B because of lack of data on M1.

2. In this and other tables with results for the U.S.: a. CPI inflation is used in panel A, GDP inflation in panel B; b. Results in panel A are not quite identical to the results for the U.S. that are included in the previous table’s 18 country results because the present table includes extra years after 2017.

3. See notes to Table 2.

Now that we have described the structure of the table, let us summarize the numbers presented. For the 1948–2017 period, the estimates of the long-run correlation \( \rho \) range from around 0.2 to 0.5; for the other three sample periods, the range is about 0.4 to 0.8. Correspondingly, \( R^2 \) varies from 10% to 30% for 1948–2017, from about 30% to around 70% in other periods. The majority of estimates are “significant”—that is, the 68% confidence intervals exclude 0. The estimates that assume the data are I(0) (column (2a) in each panel) yield slightly higher estimates of \( \rho \) than allow a range of orders (column (3a) in each panel).

Table 3 presents results for the U.S., for M0. Panel A relies on the data also used in Table 2, but extended through 2020. Panel B relies on the Lunsford and West (2019) dataset updated through 2020. The M1 data are not available back to 1871, which accounts for the blank lines in panel B for the 1871–2020 and 1871–1913 sample periods.

It continues to be the case that estimates of the long-run correlation \( \rho \) are a little smaller when one allows for I(d) models (column (3a) in each panel) rather than assuming I(0) (columns (2a) in each panel). The point estimates and \( R^2 \) tend to be a little smaller for the U.S. (Table 3) than for the broader set of countries in Table 2. As well, the estimates 1948–2020 generally are insignificant. The U.S. thus is a little anomalous, especially in recent years, in that it has a somewhat weaker low-frequency link between inflation and M0 growth than most of the other countries in our sample.

To see whether this is in part a reflection of the post-2007 combination of stable inflation and rapidly growing monetary aggregates, we re-estimated for the U.S., 1948–2007. Here are the results. The second line (1948–2020) repeats what is in Table 3, for convenience.
One can see that the results are affected by the data 2008–2020. Under an I(0) assumption, omitting 2008–2020 causes the point estimate of the long-run correlation to rise dramatically, to 0.48, a value typical for the 18-country data set in Table 2. The rise is less marked under an I(d) assumption, with a estimated $\rho$ rising from 0.05 to 0.16. We conclude that for the 1948–2020 sample, the observations at the far right end of Figure 2 pull the U.S. estimate downwards, perhaps dramatically (I(0)), perhaps modestly (I(d)). Hence that period partly explains why U.S. estimates are lower than those of most of the other countries in our sample.

We also did a quick check to see whether estimates for our other 17 countries are substantially affected by the 2008–2017 period. We use the low-frequency component extracted from the 1948–2017 sample, and assume I(0) data. To our surprise, the typical answer is that there is little effect of the 2008–2017 period. Across the 17 countries, the median change in the estimate of $\rho$-I(0) from 1948–2017 to 1948–2007 is 0.00; the changes are equally split between 1948–2007 having a lower (8 countries) and a higher (9 countries) estimate than 1948–2017; only 5 of the 17 changes are greater than 0.05 in absolute value. Hence the U.S. correlation is unusually strongly affected by the post-2007 period.

This ends our discussion of results for M0. We turn to results for growth in M3, in Tables 4 and 5. Table 4 presents results from our 18-country dataset. The point estimates range from a little below 0.4 to a little below 0.9. They are almost always significant. In Table 5 we see comparable, though slightly lower point estimates, with all confidence intervals excluding negative values.

We conclude from Tables 2, 3, 4 and 5 that, overall, the long-run correlation between money growth and inflation is substantial. U.S. correlations tend to lower than the typical country in our dataset. In contrast to Benati (2009), we do not find that correlations are lower during the gold standard era than in the post-WWII era.

### 4.3 Interest Rates

Let us turn now to interest rates. Table 6 has results from our 18-country dataset for short term interest rates. Compared to correlations for either narrow or broad money growth, the estimates of the long-run correlation $\rho$ are lower for three of our samples (1871–2017, 1871–1913, 1919–2017) but comparable for our final sample (1948–2017). The same pattern applies for the U.S. (Table 7). In general, significance is less marked than for money growth. The preceding description applies as well for long term interest rates (Tables 8 and 9). Apart from the post-World War II sample, the range for estimates of $\rho$ is about 0 to about 0.5; for the post-World War II the range is about 0.4 to about 0.8. The generally smaller magnitude comes with greatly diminished significance in the gold standard era (1871–1913); otherwise, the confidence intervals often but not always exclude negative values.
### Table 4
Long-run correlations of CPI inflation with broad money growth.

| (1) | (2a) | (2b) | (3a) | (3b) | (1) | (2a) | (2b) | (3a) | (3b) | (1) | (2a) | (2b) | (3a) | (3b) |
|-----|------|------|------|------|-----|------|------|------|------|-----|------|------|------|------|
| n   | ρ    | R²   | ρ    | R²   | n   | ρ    | R²   | ρ    | R²   | n   | ρ    | R²   | ρ    | R²   |
|-----|------|------|------|------|-----|------|------|------|------|-----|------|------|------|------|
| 1871–2017 | 12  | 0.87 | 0.77 | 0.85 | 0.73| 12  | 0.79 | 0.63 | 0.75 | 0.58| 9   | 0.73 | 0.54 | 0.74 | 0.56|
|     | (0.81, 0.91) | (0.80, 0.90) |       |       |     | (0.81, 0.91) | (0.80, 0.90) |       |       |     | (0.81, 0.91) | (0.80, 0.90) |       |       |
|     | # sig | 12  |       |       |     | # sig | 12  |       |       |     | # sig | 8   |       |       |
| 1871–1913 | 16  | 0.48 | 0.23 | 0.36 | 0.21| 16  | 0.53 | 0.28 | 0.39 | 0.23| 9   | 0.55 | 0.31 | 0.42 | 0.25|
|     | (0.11, 0.69) | (0.08, 0.63) |       |       |     | (0.11, 0.69) | (0.08, 0.63) |       |       |     | (0.11, 0.69) | (0.08, 0.63) |       |       |
|     | # sig | 12  |       |       |     | # sig | 12  |       |       |     | # sig | 8   |       |       |
| 1919–2017 | 14  | 0.87 | 0.75 | 0.84 | 0.71| 14  | 0.75 | 0.56 | 0.67 | 0.47| 9   | 0.72 | 0.52 | 0.64 | 0.43|
|     | (0.79, 0.91) | (0.75, 0.92) |       |       |     | (0.79, 0.91) | (0.75, 0.92) |       |       |     | (0.79, 0.91) | (0.75, 0.92) |       |       |
|     | # sig | 14  |       |       |     | # sig | 14  |       |       |     | # sig | 9   |       |       |
| 1948–2017 | 17  | 0.78 | 0.61 | 0.50 | 0.30| 17  | 0.67 | 0.45 | 0.41 | 0.23| 9   | 0.58 | 0.34 | 0.38 | 0.20|
|     | (0.64, 0.86) | (0.28, 0.85) |       |       |     | (0.64, 0.86) | (0.28, 0.85) |       |       |     | (0.64, 0.86) | (0.28, 0.85) |       |       |
|     | # sig | 17  |       |       |     | # sig | 17  |       |       |     | # sig | 9   |       |       |

Notes: 1. Broad money is measured by M3. 2. See notes to Table 2.

### Table 5
Long-run correlations of inflation with broad money growth in the U.S.

| (1) | (2a) | (2b) | (3a) | (3b) | (1) | (2a) | (2b) | (3a) | (3b) | (1) | (2a) | (2b) | (3a) | (3b) |
|-----|------|------|------|------|-----|------|------|------|------|-----|------|------|------|------|
| ρ   | R²   | ρ    | R²   | ρ    | R²   | ρ    | R²   | ρ    | R²   | ρ    | R²   | ρ    | R²   |
|-----|------|------|------|------|-----|------|------|------|------|-----|------|------|------|------|
| 1871–2020 | 0.67 | 0.44 | 0.62 | 0.40| 0.72 | 0.52 | 0.68 | 0.48| 0.60 | 0.80 | (0.54, 0.80) | (0.54, 0.80) | (0.54, 0.80) | (0.54, 0.80) |
|     | (0.53, 0.76) | (0.50, 0.75) |       |      | (0.53, 0.76) | (0.50, 0.75) |       |      | (0.53, 0.76) | (0.50, 0.75) |       |      |
| 1871–1913 | 0.40 | 0.16 | 0.31 | 0.17| 0.67 | 0.45 | 0.71 | 0.54| 0.38 | 0.81 | 0.59 | 0.87 |       |      |
|     | (0.03, 0.64) | (0.01, 0.54) |       |      | (0.03, 0.64) | (0.01, 0.54) |       |      | (0.03, 0.64) | (0.01, 0.54) |       |      |
| 1919–2020 | 0.69 | 0.47 | 0.64 | 0.43| 0.73 | 0.53 | 0.65 | 0.45| 0.59 | 0.81 | 0.51 | 0.80 |       |      |
|     | (0.54, 0.78) | (0.48, 0.78) |       |      | (0.54, 0.78) | (0.48, 0.78) |       |      | (0.54, 0.78) | (0.48, 0.78) |       |      |
| 1948–2020 | 0.47 | 0.22 | 0.28 | 0.14| 0.53 | 0.28 | 0.34 | 0.17| 0.29 | 0.69 | 0.10 | 0.70 |       |      |
|     | (0.21, 0.64) | (0.00, 0.70) |       |      | (0.21, 0.64) | (0.00, 0.70) |       |      | (0.21, 0.64) | (0.00, 0.70) |       |      |

Notes: 1. In panel A, broad money is measured as in the previous table, as M3. In panel B, M2 is used instead. 2. See notes to Tables 2 and 3.
Table 6
Long-run correlations of CPI inflation with short-term nominal interest rates.

| A. Weighted average of n countries | B. Median $\rho$ and $R^2$ across n countries | C. Median $\rho$ and $R^2$ across n non-Euro countries |
|-----------------------------------|-----------------------------------------------|---------------------------------------------------|
| (1) I(0) (2a) I(0) (2b) I(d) (3a) I(d) (3b) | (1) I(0) (2a) I(d) (2b) I(0) (3a) I(d) (3b) | (1) I(0) (2a) I(d) (2b) I(0) (3a) I(d) (3b) |
| $n$ | $\hat{\rho}$ | $R^2$ | $\hat{\rho}$ | $R^2$ | $n$ | $\hat{\rho}$ | $R^2$ | $\hat{\rho}$ | $R^2$ | $n$ | $\hat{\rho}$ | $R^2$ | $\hat{\rho}$ | $R^2$ |
| 1871–2017 | 9 | 0.39 | 0.15 | 0.26 | 0.12 | 9 | 0.39 | 0.15 | 0.25 | 0.10 | 5 | 0.39 | 0.15 | 0.25 | 0.10 |
| (0.21, 0.54) | (0.0, 0.70) | # sig 6 | 5 | # sig 4 | 3 |
| 1871–1913 | 14 | 0.37 | 0.14 | 0.54 | 0.35 | 14 | 0.22 | 0.06 | 0.13 | 0.11 | 7 | 0.18 | 0.04 | 0.07 | 0.12 |
| (-0.01, 0.62) | (0.0, 0.85) | # sig 5 | 5 | # sig 2 | 2 |
| 1919–2017 | 13 | 0.38 | 0.14 | 0.25 | 0.11 | 13 | 0.43 | 0.18 | 0.25 | 0.11 | 5 | 0.46 | 0.21 | 0.26 | 0.12 |
| (0.15, 0.55) | (-0.20, 0.70) | # sig 7 | 4 | # sig 4 | 3 |
| 1948–2017 | 16 | 0.73 | 0.53 | 0.42 | 0.23 | 16 | 0.67 | 0.45 | 0.44 | 0.24 | 7 | 0.66 | 0.43 | 0.42 | 0.22 |
| (0.56, 0.83) | (0.23, 0.80) | # sig 16 | 13 | # sig 7 | 6 |

Notes: 1. Short-term interest rates are measured by Treasury debt in recent years but commercial debt in early years. 2. See notes to Table 2.

Table 7
Long-run correlations of inflation with short-term interest rates in the U.S.

| A. CPI inflation | B. GDP inflation |
|------------------|------------------|
| (1) I(0) (2a) I(0) (2b) I(d) (3a) I(d) (3b) | (2a) I(0) (2b) I(d) (3a) I(d) (3b) |
| $\hat{\rho}$ | $R^2$ | $\hat{\rho}$ | $R^2$ | $\hat{\rho}$ | $R^2$ | $\hat{\rho}$ | $R^2$ |
| 1871–2020 | 0.28 | 0.08 | 0.17 | 0.10 | 0.23 | 0.05 | 0.15 | 0.09 |
| (0.08, 0.44) | (-0.04, 0.47) | (0.03, 0.40) | (-0.07, 0.46) |
| 1871–1913 | -0.50 | 0.25 | -0.23 | 0.14 | -0.28 | 0.08 | -0.10 | 0.11 |
| (-0.71, -0.15) | (-0.75, 0.05) | (-0.56, 0.10) | (-0.60, 0.21) |
| 1919–2020 | 0.48 | 0.23 | 0.31 | 0.15 | 0.44 | 0.20 | 0.25 | 0.12 |
| (0.28, 0.62) | (0.05, 0.75) | (0.24, 0.60) | (0.0, 0.55) |
| 1948–2020 | 0.78 | 0.60 | 0.52 | 0.32 | 0.77 | 0.60 | 0.55 | 0.35 |
| (0.63, 0.86) | (0.30, 0.85) | (0.62, 0.86) | (0.32, 0.85) |

Notes: 1. In panel A, short-term rates are measured as in the previous table, as Treasury bills in recent years but commercial debt in early years. In panel B, Treasury bills are also used in recent years, but with a different commercial debt series in early years. 2. See notes to Tables 2 and 3.
Table 8
Long-run correlations of CPI inflation with long-term interest rates.

| Year Range | n | \( \hat{\rho} \) | \( R^2 \) | \( \hat{\rho} \) | \( R^2 \) |
|------------|---|----------------|------------|----------------|------------|
|1871–2017   | 13 | 0.31           | 0.09       | 0.19           | 0.09       |
|            |    | (0.11, 0.47)   | (0.0, 0.60) |                |            |
|1871–1913   | 16 | -0.30          | 0.09       | 0.05           | 0.10       |
|            |    | (-0.57, 0.08)  | (-0.40, 0.38) |                |            |
|1919–2017   | 15 | 0.25           | 0.06       | 0.16           | 0.09       |
|            |    | (0.02, 0.45)   | (-0.25, 0.65) |                |            |
|1948–2017   | 18 | 0.67           | 0.45       | 0.36           | 0.18       |
|            |    | (0.47, 0.79)   | (0.0, 0.75)  |                |            |

Notes: 1. Long-term interest rates are generally measured by 10 year government debt. 2. See notes to Table 2.

Table 9
Long-run correlations of inflation with long-term interest rates in the U.S.

| Year Range | CPI inflation | GDP inflation |
|------------|---------------|---------------|
|            | A. CPI inflation | B. GDP inflation |
|            | \( \rho \) | \( R^2 \) | \( \hat{\rho} \) | \( R^2 \) | \( \rho \) | \( R^2 \) |
| 1871–2020  | 0.42           | 0.18           | 0.02           | 0.13           | 0.37           | 0.14           | 0.24           | 0.12           |
|            | (0.24, 0.57)   | (0.0, 0.65)    |                |          | (0.18, 0.53)   | (0.0, 0.65)    |                |          |
| 1871–1913  | -0.57          | 0.33           | -0.35          | 0.02          | -0.50          | 0.25           | -0.26          | 0.15          |
|            | (-0.75, -0.24) | (-0.75, -0.03) |                |            | (-0.71, -0.15) | (-0.65, 0.01) |                |            |
| 1919–2020  | 0.51           | 0.26           | 0.30           | 0.14          | 0.45           | 0.20           | 0.24           | 0.12          |
|            | (0.31, 0.65)   | (0.07, 0.75)   |                |            | (0.24, 0.60)   | (0.0, 0.65)    |                |            |
| 1948–2020  | 0.72           | 0.51           | 0.42           | 0.22          | 0.69           | 0.48           | 0.41           | 0.22          |
|            | (0.53, 0.82)   | (0.20, 0.80)   |                |            | (0.50, 0.80)   | (0.18, 0.80)   |                |            |

Notes: 1. In both panels, long-term interest rates are measured by 10 year U.S. Treasury bonds. 2. See notes to Tables 2 and 3.
In Tables 7 and 9 entries for the gold standard era (1871–1913), the U.S. estimates of the long-run correlation between inflation and interest rates are negative, with tight confidence intervals. The negative connection may seem surprising. But during the gold standard era, a negative long-run correlation between U.S. inflation and both short- and long-term nominal rates is also found in the band-spectral regressions in Summers (1982). (Summers uses different data than do we, and his sample period is 1870–1900.) In work not reported in the tables, we found that if we begin the sample in the early 1880s, the correlation turns positive. See Barsky (1987) for a discussion.

We investigated sensitivity of interest rate estimates to the post-Great Financial Crisis era with a quick check similar to the check we did for M0 growth. We used the low-frequency component extracted from the 1948–2017 sample, computing \( \rho_{\text{I}(0)} \) using data only from 1948–2007. Some specifics:

**Estimates of \( \rho_{\text{I}(0)} \), 1948–2007 relative to 1948–2017.**

|                          | \( i^S \) | \( i^L \) |
|--------------------------|-----------|-----------|
| Number of countries in which 1948–2007 estimate is smaller | 15 | 16 |
| Median fall in the estimate | 0.06 | 0.05 |
| Reference: median estimate in Tables 6 and 8 (1948–2017) | 0.66 | 0.71 |
| Number of countries used in the calculations | 16 | 18 |

Relative to the 1948–2017 sample reported in the tables, in the 1948–2007 sample, the I(0) estimate of the long-run correlation is virtually always smaller, though the typical fall is small relative to the typical value of the estimate in the 1948–2017 sample. We conclude that stable inflation and nominal interest rates post-Great Financial Crisis strengthened the long-run Fisher effect, though to an economically small degree.

### 4.4 Comparison to Simple Correlations

Given that we are measuring the strength of the long-run correlation between inflation and other variables by the correlation across low-frequency components, it is natural to compare our estimates to simple correlations that do not focus on any particular frequency. For simplicity, we do so for our I(0) estimates, recognizing that such estimates potentially are fraught since some the series arguably are not I(0).

Using our 18-country dataset, we compute simple correlations between inflation and one of our correlates for each of the countries and samples also used to compute long-run correlations. For example, we compute the simple correlation between CPI inflation and M0 growth, 1871–2017, for the 11 countries listed in the “1871–2017 / M0” column in Table 1. By “simple correlation” we mean, well, the usual correlation. With \( x \) a correlate (either money growth or nominal interest rates), the estimate of the simple correlation is, in what we hope is obvious notation:

\[
\hat{\rho}_{\text{I}(0)} = \sum_t \frac{(\pi_t - \bar{\pi})(x_t - \bar{x})}{\sigma_\pi \sigma_x}.
\]  

We include “I(0)” to emphasize that the correlation is computed from the levels rather than a transformation of \( \pi \) and \( x \).
If, indeed, money growth affects inflation especially over long horizons, and the Fisher effect is particularly strong at long horizons, then one would expect the simple correlations to be smaller in magnitude than the long-run correlations. And that indeed is almost always the case. We compute 232 simple correlations, where 232 is the sum of the numbers in the $n$ row of Table 1. Of these, the overwhelming share are smaller than their $\rho-I(0)$ counterparts. Specifically, 211, or 91% of the 232 estimates, are smaller for simple than long-run correlations. We do not have a story for the 21 ($= 232 - 211$) series for which the estimate of $r-I(0)$ is larger. 14 of these 21 occur in the 1871–1913 sample. Otherwise there appears to be no particular pattern, and may simply reflect chance.

The long-run correlations are not only larger, but are substantially larger, at least for money growth. For each correlate and sample, the median values across countries are reported in Table 10. For convenience, columns (1a), (2a), (3a) and (4a) repeat the median estimates of $\rho-I(0)$ reported in earlier tables. Columns (1b), (2b), (3b) and (4b) report the medians of the simple correlations.

To our eyes, for M0 growth, the differences between the long-run correlation $\rho-I(0)$ and the simple correlation (columns (1a) and (1b)) are substantial. For example, for the 1948–2017 sample, if one were to square the indicated figures to get the value of the $R^2$ of a regression of inflation on money growth, the resulting implied $R^2$ is 10 times larger for the low-frequency component than for the series as a whole (i.e., $0.48^2 \approx 10 \times 0.15^2$).

For both long-run and simple correlations, magnitudes are larger for M3 growth than for M0 growth (columns (2a) vs. (1a) and (2b) vs. (1b) in Table 10). But the long-run correlation again is notably larger.

For the interest rate series (columns (3a), (3b), (4a) and (4b)), the median value for the long-run correlation again is always larger than that for the simple correlation. However, the arithmetic difference between the two tends to be smaller than for M0 or M3 growth. This is

### Table 10

|       | (1a) | (1b) | (2a) | (2b) | (3a) | (3b) | (4a) | (4b) |
|-------|------|------|------|------|------|------|------|------|
|       | $\hat{\rho}$-I(0) | $\hat{r}$-I(0) | $\hat{\rho}$-I(0) | $\hat{r}$-I(0) | $\hat{\rho}$-I(0) | $\hat{r}$-I(0) | $\hat{\rho}$-I(0) | $\hat{r}$-I(0) |
| 1871–2017 | 0.76 | 0.47 | 0.79 | 0.58 | 0.39 | 0.30 | 0.43 | 0.33 |
| 1871–1913 | 0.57 | 0.27 | 0.53 | 0.33 | 0.22 | 0.11 | 0.07 | 0.02 |
| 1919–2017 | 0.67 | 0.35 | 0.75 | 0.57 | 0.43 | 0.34 | 0.44 | 0.36 |
| 1948–2017 | 0.48 | 0.15 | 0.67 | 0.43 | 0.67 | 0.58 | 0.63 | 0.53 |

**Notes:**
1. Columns (1a), (2a), (3a) and (4a) present the median estimates of long-run correlations under an I(0) assumption. These are repeated from earlier tables. For example, the values in column (1a) repeat the values in the column (2a) in panel B in Table 2. The number of countries over which the median is calculated is given in the $n$ row of Table 1.
2. Columns (1b), (2b), (3b) and (4b) report the median values of simple correlations ($\hat{r}$-I(0)) between the indicated variables in the indicated sample period, computed as in equation (4.3). For example, the value of 0.47 in column (1b) for 1871–2017 means: The simple correlation between CPI inflation and M0 growth is calculated for the 11 countries listed in the 1871–2017 / M0 column of Table 1. The median value across these 11 estimates is 0.47.
consistent with the fact that the magnitudes are smaller than for M0 or M3 growth, with the exception of the 1948–2017 period.

We conclude that, indeed, there is a relatively strong low-frequency, or long-run, connection between money growth and inflation and between nominal interest rates and inflation. That the connection is relatively strong at low frequencies is also the finding for U.S. data in Lucas (1980). In our data, the relative strength of the long-run connection is particularly marked for money growth.

4.5 Selective Comparison to Some Earlier Literature

Our results for money growth are similar to those in Benati (2009) and Haug and Dewald (2012). Those papers also use long-run annual data from developed countries, with samples ending modestly earlier than ours. Their econometric techniques to extract long-run components are different from ours, and they assume that money growth and inflation are stationary. They, too, find substantial low-frequency correlations between the two series. Benati’s estimates of coherence tend to be above 0.9 (Benati, 2009, tables 2 and 4) and thus tend to be higher than the values given in our tables. (Indeed, according to the country-by-country point estimates and confidence intervals in Appendix A, a value of 0.9 is generally above the upper limit of our 68% confidence intervals.) Hence while we agree with Benati qualitatively (substantial correlation), quantitatively we tend to get lower figures. Haug and Dewald’s estimates, given in panels B and C of their table 3, perhaps fall in roughly the same ranges as ours, though they find much larger variation across countries than do we.

While there is a fair amount of recent work on the Fisher effect, it appears that little of it focuses on the long-run as we have defined it. Exceptions include Lucas (1980) and Müller and Watson (2018), who also find positive correlations between the low-frequency components of U.S. inflation and nominal interest rates (though our U.S. findings are hardly independent of Müller and Watson, 2018, since our U.S. data is similar and we use their technique). Our finding of a modest but positive long-run correlation perhaps is consistent with recent literature on the Fisher relation that is also modestly supportive (e.g., Caporale and Gil-Alaña, 2019 or Kruse et al., 2017).

5. Conclusion

Using a frequency domain technique that is robust to various orders of integration, we find positive correlations between the long-run, or low-frequency, components of inflation on the one hand and of money growth or long- or short-run nominal interest rates on the other. The correlations are more substantial for money growth than for interest rates. The implied $R^2$ of a regression of low-frequency inflation on the low-frequency components of either of these variables (or of the reverse regression) is, however, well below 1. Thus other factors have played important roles in low-frequency variations in these series. Understanding what these other factors are, and tying the various factors together in a structural economic models, are important tasks for future research.
Appendix A

This appendix presents the country-by-country results that are summarized in panels B and C in Tables 2, 4, 6 and 8. The results for the U.S. given here are slightly different than those given in earlier tables because the Appendix results rely on data that end in 2017 rather than 2020. The figures given in parentheses are 68% confidence intervals or credible sets.
### Table A.1
Long-run correlations of CPI inflation with narrow money growth.

| Country     | 1871-1917 | 1871-1913 | 1919-2017 | 1948-2017 |
|-------------|-----------|-----------|-----------|-----------|
|              | \(\hat{\rho}\) | \(R^2\) | \(\hat{\rho}\) | \(R^2\) | \(\hat{\rho}\) | \(R^2\) | \(\hat{\rho}\) | \(R^2\) | \(\hat{\rho}\) | \(R^2\) |
| Australia   | 0.56      | 0.32      | 0.25      | 0.12      | 0.02      | 0.12      | 0.11      | 0.55      | 0.30      | 0.21      | 0.56      | 0.31      | 0.47      | 0.27      |
| Belgium     | n.a.      | n.a.      | n.a.      | 0.56      | 0.31      | 0.34      | 0.21      | n.a.      | n.a.      | n.a.      | 0.09      | 0.01      | 0.08      | 0.07      |
| Canada      | 0.52      | 0.27      | 0.44      | 0.23      | 0.88      | 0.78      | 0.71      | 0.55      | 0.38      | 0.15      | 0.26      | 0.12      | 0.09      | 0.01      | 0.05      | 0.06      |
| Switzerland | 0.61      | 0.37      | 0.56      | 0.33      | 0.57      | 0.32      | 0.43      | 0.26      | 0.21      | 0.05      | 0.15      | 0.07      | -0.03     | 0.00      | -0.01     | 0.06      |
| Germany     | n.a.      | n.a.      | n.a.      | 0.70      | 0.49      | 0.56      | 0.37      | n.a.      | n.a.      | n.a.      | -0.03     | 0.00      | 0.01      | 0.07      |           |
| Denmark     | n.a.      | n.a.      | n.a.      | -0.20     | 0.04      | -0.18     | 0.12      | n.a.      | n.a.      | n.a.      | 0.54      | 0.29      | 0.34      | 0.17      |           |
| Spain       | n.a.      | n.a.      | n.a.      | 0.39      | 0.15      | 0.26      | 0.16      | n.a.      | n.a.      | n.a.      | -0.03     | 0.00      | 0.01      | 0.07      |           |
| Finland     | 0.86      | 0.74      | 0.85      | 0.72      | 0.44      | 0.20      | 0.31      | 0.19      | 0.83      | 0.69      | 0.79      | 0.64      | 0.82      | 0.68      | 0.73      | 0.55      |
| France      | n.a.      | n.a.      | n.a.      | 0.66      | 0.43      | 0.35      | 0.21      | 0.78      | 0.61      | 0.75      | 0.57      | 0.82      | 0.67      | 0.76      | 0.60      |           |
| UK          | 0.84      | 0.70      | 0.81      | 0.66      | 0.79      | 0.62      | 0.65      | 0.48      | 0.70      | 0.49      | 0.60      | 0.39      | 0.61      | 0.38      | 0.50      | 0.29      |
| Ireland     | n.a.      | n.a.      | n.a.      | 0.76      | 0.56      | 0.88      | 0.44      | 0.85      | 0.55      | 0.79      | 0.45      | 0.76      | 0.40      | 0.75      | 0.30      | 0.71      |           |
| Italy       | 0.90      | 0.62      | 0.88      | 0.78      | 0.93      | 0.87      | 0.87      | 0.78      | 0.88      | 0.78      | 0.85      | 0.74      | 0.47      | 0.22      | 0.24      | 0.13      |
| Japan       | 0.88      | 0.67      | 0.87      | 0.76      | 0.36      | 0.13      | 0.24      | 0.15      | 0.81      | 0.87      | 0.81      | 0.78      | 0.75      | 0.56      | 0.66      | 0.47      |
| Netherlands | n.a.      | n.a.      | n.a.      | 0.60      | 0.36      | 0.46      | 0.29      | n.a.      | n.a.      | n.a.      | 0.22      | 0.05      | 0.12      | 0.08      |           |
| Norway      | 0.76      | 0.58      | 0.72      | 0.53      | 0.74      | 0.54      | 0.59      | 0.41      | 0.64      | 0.41      | 0.60      | 0.39      | 0.49      | 0.24      | 0.30      | 0.15      |
| Portugal    | 0.76      | 0.58      | 0.72      | 0.53      | -0.50     | 0.25      | -0.37     | 0.22      | 0.74      | 0.54      | 0.69      | 0.50      | 0.64      | 0.41      | 0.38      | 0.20      |
| Sweden      | 0.76      | 0.57      | 0.72      | 0.53      | 0.28      | 0.08      | 0.19      | 0.13      | 0.67      | 0.46      | 0.56      | 0.36      | 0.72      | 0.52      | 0.43      | 0.24      |
| USA         | 0.45      | 0.20      | 0.40      | 0.20      | 0.67      | 0.45      | 0.63      | 0.44      | 0.16      | 0.03      | 0.12      | 0.08      | 0.09      | 0.01      | 0.06      | 0.08      |
### Table A.2
Long-run correlations of CPI inflation with broad money growth.

| Country  | 1871-1913 | 1919-2017 | 1948-2017 |
|----------|------------|------------|------------|
|          | I(0) R² | I(0) R² | I(0) R² | I(0) R² | I(0) R² | I(0) R² | I(0) R² | I(0) R² |
|          | 1871-2017 | 1871-1913 | 1919-2017 | 1948-2017 |
| Australia | 0.73 0.54 | 0.63 0.43 | 0.47 0.22 | 0.36 0.21 | 0.72 0.51 | 0.59 0.39 | 0.72 0.53 | 0.67 0.48 |
| Belgium  | n.a. 0.50 | 0.78 0.11 | 0.69 0.03 | 0.65 0.57 | 0.81 0.44 | 0.75 0.44 | 0.77 0.51 | 0.83 0.48 |
| Canada   | 0.73 0.53 | 0.65 0.44 | 0.68 0.46 | 0.51 0.33 | 0.72 0.52 | 0.64 0.43 | 0.69 0.47 | 0.48 0.29 |
| Switzerland | 0.62 0.38 | 0.56 0.34 | 0.51 0.26 | 0.27 0.18 | 0.58 0.34 | 0.49 0.20 | 0.58 0.30 | 0.38 0.20 |
| Germany  | n.a. 0.42 | 0.72 0.08 | 0.74 0.08 | 0.50 0.10 | 0.40 0.70 | 0.71 0.03 | 0.36 0.20 | 0.16 0.75 |
| Denmark  | 0.77 0.60 | 0.74 0.56 | 0.59 0.34 | 0.42 0.25 | 0.76 0.57 | 0.74 0.56 | 0.58 0.34 | 0.36 0.18 |
| Spain    | n.a. 0.65 | 0.84 0.76 | 0.26 0.08 | 0.67 0.63 | 0.83 0.65 | 0.82 0.35 | 0.72 0.13 | 0.75 0.04 |
| Finland  | 0.83 0.69 | 0.80 0.64 | 0.45 0.20 | 0.19 0.19 | 0.66 0.58 | 0.63 0.69 | 0.73 0.56 | 0.56 0.45 |
| France   | n.a. 0.71 | 0.88 0.68 | 0.50 0.04 | 0.70 0.70 | 0.87 0.60 | 0.89 0.60 | 0.73 0.54 | 0.64 0.45 |
| UK       | 0.81 0.65 | 0.76 0.59 | 0.67 0.45 | 0.51 0.33 | 0.70 0.49 | 0.53 0.32 | 0.57 0.30 | 0.30 0.16 |
| Ireland  | n.a. 0.68 | 0.85 0.78 | 0.37 0.81 | 0.27 0.76 | 0.55 0.79 | 0.34 0.80 | 0.34 0.72 | 0.00 0.75 |
| Italy    | 0.91 0.83 | 0.89 0.80 | 0.79 0.62 | 0.65 0.47 | 0.89 0.78 | 0.86 0.74 | 0.53 0.29 | 0.36 0.19 |
| Japan    | 0.72 0.51 | 0.74 0.56 | 0.50 0.25 | 0.18 0.10 | 0.82 0.67 | 0.77 0.60 | 0.77 0.60 | 0.72 0.54 |
| Netherlands | 0.60 0.60 | 0.81 0.71 | 0.39 0.82 | 0.55 0.52 | 0.36 0.36 | 0.43 0.18 | 0.42 0.13 | 0.17 0.61 |
| Norway   | 0.89 0.79 | 0.87 0.76 | 0.55 0.31 | 0.73 0.50 | 0.85 0.72 | 0.79 0.64 | 0.77 0.59 | 0.56 0.36 |
| Portugal | 0.83 0.69 | 0.80 0.64 | 0.55 0.21 | 0.17 0.21 | 0.76 0.50 | 0.80 0.70 | 0.81 0.41 | 0.50 0.20 |
| Sweden   | 0.81 0.66 | 0.79 0.62 | 0.67 0.44 | 0.76 0.50 | 0.76 0.67 | 0.76 0.64 | 0.25 0.06 | 0.05 0.65 |
| USA      | 0.67 0.45 | 0.62 0.40 | 0.40 0.16 | 0.31 0.17 | 0.72 0.52 | 0.66 0.46 | 0.52 0.27 | 0.30 0.15 |

(Standard errors in parentheses)
Table A.3
Long-run correlations of CPI inflation with short-term nominal interest rates.

| Country     | 1871-2017 |          | 1871-1913 |          | 1919-2017 |          | 1948-2017 |          |
|-------------|-----------|----------|-----------|----------|-----------|----------|-----------|----------|
|             | I(0)      | I(d)     | R²        | I(0)      | I(d)     | R²       | I(0)      | I(d)     |
|             | ˆρ        | ˆρ       | R²        | ˆρ        | ˆρ       | R²       | ˆρ        | ˆρ       |
| Australia   | n.a.      | n.a.     | n.a.      | n.a.      | n.a.     | n.a.     | n.a.      | n.a.     |
| Belgium     | n.a.      | n.a.     | 0.57      | 0.32      | 0.37     | 0.22     | 0.10      | 0.01     |
|             | (0.23, 0.75) | (0.04, 0.80) | (-0.14, 0.31) | (0.13, 0.22) | (0.47, 0.78) | (0.45, 0.39) | (0.07, 0.60) | (0.21, 0.32) |
| Canada      | n.a.      | n.a.     | n.a.      | 0.63      | 0.39     | 0.37     | n.a.      | 0.19     |
| Switzerland | 0.14      | 0.02     | 0.11      | 0.06      | 0.39     | 0.23     | 0.12      | 0.02     |
|             | (-0.05, 0.33) | (-0.08, 0.34) | (0.30, 0.78) | (0.11, 0.80) | (-0.11, 0.34) | (-0.18, 0.35) | (0.45, 0.78) | (0.27, 0.80) |
| Germany     | n.a.      | n.a.     | 0.25      | 0.06      | 0.39     | 0.23     | 0.12      | 0.02     |
|             | (0.12, 0.54) | (-0.23, 0.60) | (0.46, 0.79) | (0.10, 0.75) | (0.50, 0.33) | (0.50, 0.34) | (0.07, 0.75) | (0.30, 0.75) |
| Denmark     | 0.46      | 0.21     | 0.32      | 0.14      | 0.13     | 0.02     | -0.13     | 0.12     |
|             | (0.28, 0.60) | (0.11, 0.70) | (-0.27, 0.47) | (-0.50, 0.50) | (0.37, 0.69) | (0.16, 0.75) | (0.67, 0.87) | (0.32, 0.85) |
| Spain       | 0.42      | 0.18     | 0.32      | 0.15      | 0.14     | 0.12     | 0.11      | 0.43     |
|             | (0.24, 0.57) | (0.07, 0.75) | (-0.24, 0.46) | (-0.20, 0.47) | (0.21, 0.59) | (-0.05, 0.70) | (0.53, 0.82) | (0.32, 0.85) |
| Finland     | 0.17      | 0.03     | 0.11      | 0.05      | 0.04     | 0.00     | -0.01     | 0.15     |
|             | (-0.03, 0.35) | (-0.03, 0.50) | (-0.32, 0.38) | (-0.35, 0.32) | (-0.09, 0.36) | (-0.23, 0.50) | (0.33, 0.71) | (0.01, 0.70) |
| France      | n.a.      | n.a.     | 0.58      | 0.53      | 0.35     | -0.02     | 0.00      | 0.02     |
|             | (0.24, 0.76) | (0.30, 0.79) | (-0.25, 0.20) | (-0.23, 0.27) | (0.01, 0.50) | (-0.05, 0.55) | (0.16, 0.70) | (0.00, 0.65) |
| UK          | 0.47      | 0.22     | 0.33      | 0.14      | 0.59     | 0.35     | 0.48      | 0.46     |
|             | (0.29, 0.60) | (0.15, 0.70) | (0.26, 0.76) | (0.19, 0.73) | (0.25, 0.62) | (0.01, 0.70) | (0.31, 0.70) | (0.00, 0.65) |
| Ireland     | n.a.      | n.a.     | n.a.      | n.a.      | n.a.     | n.a.     | n.a.      | n.a.     |
|             | (0.44, 0.73) | (0.10, 0.75) | (0.58, 0.84) | (0.58, 0.74) | (0.72, 0.71) | (0.53, 0.53) | (0.00, 0.53) | (0.00, 0.53) |
| Italy       | n.a.      | n.a.     | n.a.      | n.a.      | n.a.     | n.a.     | n.a.      | n.a.     |
|             | (0.20, 0.51) | (-0.15, 0.48) | (0.54, 0.62) | (0.11, 0.39) | (0.36, 0.56) | (0.03, 0.60) | (0.11, 0.20) | (0.00, 0.11) |
| Japan       | n.a.      | n.a.     | n.a.      | 0.18      | 0.03     | 0.15     | 0.12      | n.a.     |
|             | (-0.22, 0.51) | (-0.15, 0.48) | (0.00, 0.37) | (0.00, 0.40) | (0.27, 0.47) | (0.00, 0.40) | (0.00, 0.37) | (0.00, 0.40) |
| Netherlands | 0.20      | 0.04     | 0.12      | 0.07      | 0.63     | 0.40     | 0.51      | 0.33     |
|             | (0.00, 0.37) | (-0.10, 0.40) | (0.32, 0.79) | (0.27, 0.76) | (-0.03, 0.41) | (-0.17, 0.55) | (0.09, 0.56) | (-0.03, 0.60) |
| Norway      | n.a.      | n.a.     | n.a.      | 0.10      | 0.01     | 0.07     | 0.10      | n.a.     |
|             | (-0.27, 0.43) | (-0.25, 0.40) | (-0.27, 0.43) | (-0.25, 0.40) | (0.26, 0.62) | (0.03, 0.75) | (0.61, 0.86) | (0.27, 0.85) |
| Portugal    | 0.45      | 0.20     | 0.32      | 0.15      | 0.25     | 0.06     | 0.17      | 0.14     |
|             | (0.27, 0.59) | (0.10, 0.75) | (-0.20, 0.58) | (-0.18, 0.52) | (0.32, 0.66) | (-0.20, 0.75) | (0.63, 0.86) | (0.27, 0.85) |
| Sweden      | 0.39      | 0.15     | 0.25      | 0.10      | 0.19     | 0.04     | 0.04      | 0.10     |
|             | (0.20, 0.53) | (0.03, 0.60) | (-0.19, 0.50) | (-0.30, 0.50) | (0.24, 0.61) | (-0.20, 0.70) | (0.50, 0.80) | (0.23, 0.80) |
| USA         | 0.28      | 0.08     | 0.16      | 0.10      | -0.50    | 0.25     | -0.23     | 0.14     |
|             | (0.09, 0.45) | (-0.03, 0.46) | (-0.71, -0.15) | (-0.75, 0.05) | (0.26, 0.62) | (0.03, 0.75) | (0.61, 0.85) | (0.27, 0.85) |
Table A.4
Long-run correlations of CPI inflation with long-term interest rates.

|           | 1871-1917 | 1871-1913 | 1919-2017 | 1948-2017 |
|-----------|-----------|-----------|-----------|-----------|
|           | I(0)  R^2 | I(d)  R^2 | I(0)  R^2 | I(d)  R^2 |
| Australia| 0.50  0.25 | 0.31  0.14 | 0.15  0.02 | 0.05  0.10 | 0.44  0.19 | 0.25  0.12 | 0.43  0.19 | 0.23  0.11 |
|           | (0.33, 0.63) | (0.10, 0.75) | (-0.22, 0.47) | (-0.27, 0.55) | (0.22, 0.60) | (0.00, 0.65) | (0.17, 0.62) | (-0.20, 0.60) |
| Belgium   | n.a.     | n.a.     | n.a.     | n.a.     | 0.11  0.01 | 0.05  0.04 | 0.65  0.42 | 0.35  0.18 |
| Canada    | 0.52  0.27 | 0.32  0.15 | 0.01  0.00 | 0.16  0.11 | 0.56  0.31 | 0.32  0.15 | 0.62  0.38 | 0.32  0.16 |
| Switzerland| 0.25  0.06 | 0.15  0.06 | -0.16  0.03 | -0.05  0.12 | 0.16  0.02 | 0.14  0.08 | 0.86  0.74 | 0.73  0.56 |
|           | (-0.02, 0.50) | (-0.52, 0.27) | (-0.55, 0.33) | (-0.08, 0.37) | (-0.30, 0.39) | (0.75, 0.91) | (0.54, 0.93) |
| Germany   | n.a.     | n.a.     | n.a.     | n.a.     | 0.05  0.00 | 0.09  0.10 | n.a.     | n.a.     |
|           | (0.31, 0.39) | (0.21, 0.43) | (0.17, 0.39) | (0.08, 0.17) |
| Denmark   | 0.53  0.28 | 0.36  0.16 | -0.19  0.04 | -0.03  0.08 | 0.62  0.39 | 0.40  0.19 | 0.85  0.72 | 0.65  0.45 |
| Spain     | n.a.     | n.a.     | n.a.     | n.a.     | 0.09  0.01 | 0.15  0.12 | n.a.     | n.a.     |
|           | (0.37, 0.65) | (0.22, 0.75) | (0.32, 0.25) | (0.45, 0.74) |
| Finland   | n.a.     | n.a.     | n.a.     | n.a.     | -0.28  0.42 | -0.18  0.49 | n.a.     | n.a.     |
|           | (-0.13, 0.54) | (-0.18, 0.55) | (-0.18, 0.55) | (0.13, 0.59) |
| France    | 0.20  0.04 | 0.13  0.09 | 0.26  0.07 | 0.40  0.24 | 0.06  0.08 | 0.08  0.08 | 0.49  0.24 | 0.34  0.16 |
| UK        | 0.60  0.36 | 0.42  0.19 | 0.01  0.00 | 0.07  0.10 | 0.65  0.42 | 0.45  0.25 | 0.77  0.60 | 0.69  0.50 |
| Ireland   | n.a.     | n.a.     | n.a.     | n.a.     | 0.25  0.06 | 0.13  0.11 | n.a.     | n.a.     |
|           | (0.45, 0.70) | (0.32, 0.70) | (0.34, 0.36) | (0.48, 0.76) |
| Italy     | 0.22  0.05 | 0.16  0.08 | 0.39  0.15 | 0.28  0.16 | 0.15  0.10 | 0.22  0.07 | 0.83  0.69 | 0.65  0.46 |
| Japan     | -0.09  -0.17 | 0.07  -0.28 | -0.17  -0.08 | -0.22  0.10 | 0.16  0.07 | 0.83  0.69 | 0.46  0.91 |
|           | (-0.28, 0.11) | (-0.42, 0.20) | (-0.56, 0.10) | (-0.12, 0.41) |
| Netherlands| 0.33  0.11 | 0.19  0.08 | 0.21  0.05 | 0.34  0.20 | -0.31  0.10 | 0.22  0.10 | 0.53  0.28 | 0.38  0.19 |
| Norway    | 0.33  0.11 | 0.20  0.09 | 0.04  0.00 | 0.14  0.10 | 0.35  0.12 | 0.21  0.10 | 0.60  0.36 | 0.33  0.17 |
| Portugal  | 0.14  0.49 | -0.01  0.55 | -0.16  0.52 | -0.08  0.50 | 0.12  0.53 | 0.25  0.65 | 0.38  0.74 | 0.06  0.75 |
| Sweden    | 0.46  0.22 | 0.39  0.20 | 0.11  0.01 | 0.04  0.10 | 0.61  0.38 | 0.43  0.23 | 0.80  0.64 | 0.52  0.32 |
| USA       | 0.43  0.18 | 0.26  0.13 | -0.57  0.33 | -0.35  0.20 | 0.51  0.26 | 0.30  0.14 | 0.71  0.50 | 0.41  0.22 |
|           | (0.24, 0.57) | (0.00, 0.60) | (-0.75, -0.24) | (-0.75, -0.03) |

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