CURRENT ISSUES IN THE NUCLEAR CHIRAL
PERTURBATION APPROACH

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After outlining the basic ideas of nuclear chiral perturbation theory, I discuss its
application, presenting three examples which I believe are of great current interest:
(1) Exchange currents in those cases where the leading chiral-order terms dominate;
(2) The near-threshold \( p + p \rightarrow p + p + \pi^0 \) reaction, for which the leading order
terms are suppressed and consequently higher-order corrections play a prominent
role; (3) Radiative \( \mu \)-capture on the proton.

1 Introduction

Understanding nuclear dynamics in relation to the fundamental QCD is one of
the most important research frontiers in contemporary nuclear physics. In our
endeavor in this direction chiral perturbation theory (\( \chi \)PT) offers a valuable
guiding principle. In this talk, after giving a minimalist summary of \( \chi \)PT,
I describe three examples of the nuclear physics applications of \( \chi \)PT. I first
discuss the exchange currents in the space components of the electromagnetic
current and in the time component of the axial current. Next I discuss the use
of \( \chi \)PT in describing the \( p + p \rightarrow p + p + \pi^0 \) reaction near threshold. Finally, I
describe an application of \( \chi \)PT to the radiative \( \mu \)-capture on the proton. These
examples, I hope, will illustrate the usefulness and versatility of nuclear \( \chi \)PT.

The basic idea of effective theory, of which \( \chi \)PT is an example, is not
completely foreign to nuclear physicists. For example, in order to truncate
nuclear Hilbert space down to a manageable model space, we often tame the
highly singular \( N-N \) interactions by first summing up the contributions of
high excited states that lie outside the chosen model space. We then use the
resulting \( G \)-matrices as effective interactions acting on the model space. The
basic picture of effective theories is rather similar to this nuclear physics example, and the introduction of \( \chi \)PT follows a general pattern of effective
theories. Let us consider a path integral expression for a vacuum-to-vacuum
amplitude in QCD in the presence of external fields,

\[
e^{iZ[v,a,s,p]} = \int [dg][dq][d\bar{q}] e^{i \int d^4 x \mathcal{L}(q,\bar{q};G; v,a,s,p)}
\]  

\(^a\)invited talk at the APCTP Workshop on Astro-Hadron Physics, Seoul, Korea, October 1997
where $\mathcal{L} = \mathcal{L}^0_{\text{QCD}} + \bar{q} \gamma^\mu [v_\mu(x) - \gamma^5 a_\mu(x)] q - \bar{q} [s(x) - i p(x)] q$. The external fields, $v_\mu$, $a_\mu$, $s$ and $p$, are endowed with appropriate SU($N$) × SU($N$) transformation properties to make $\mathcal{L}$ chiral invariant. (In our case, $N = 2$.) Suppose we are interested in low-energy phenomena of pions with their typical energy scales $E \lesssim \Lambda_\chi \sim 1$ GeV, where $\Lambda_\chi$ is the QCD scale. An effective Lagrangian that describes low-energy behavior of QCD involves the Goldstone bosons and is introduced through

$$ e^{i Z[v,a,s,p]} = \int [dU] e^{i \int d^4 \tau \mathcal{L}_{\text{ch}}(U; v,a,s,p)}, \quad (2) $$

where $U \equiv \exp (i \sum_{\alpha=1}^3 \pi^\alpha \tau^\alpha / f_\pi)$ with $\pi^\alpha$’s representing the pion fields. (Another example of many possible choices of $U$ is $U(x) = \sqrt{1 - |\pi(x)/f_\pi|^2} + i \tau \cdot \pi(x)/f_\pi$.) In $\chi$PT, $\mathcal{L}_{\text{ch}}$ is expanded in powers of $\partial_\mu / \Lambda_\chi$ and the quark mass matrix $\mathcal{M} / \Lambda_\chi$ and, for a given order of expansion, all terms that are consistent with the symmetries are retained. Low energy phenomena may be characterized by a generic pion momentum $Q$, which is small compared to the chiral scale $\Lambda_\chi \sim 1$ GeV. This suggests the possibility of describing low-energy phenomena in terms of $\mathcal{L}_{\text{ch}}$ that contains only rather limited number of terms. This is the basic idea of $\chi$PT. When we try to extend this scheme to the baryon field $N$, we realize that $\partial_0$ acting on $N$ yields $\sim m_N (\approx \text{nucleon mass})$, which unfortunately is not small compared with $\Lambda_\chi$. The heavy-baryon formalism (HBF) was invented to avoid this difficulty. In HBF, instead of the ordinary Dirac field $N$ one works with $B$ defined by $B(x) \equiv e^{i m_N (v \cdot x)} N(x)$ with $v \sim (1,0,0,0)$, shifting the energy reference point from 0 to $m_N$. Insofar as we are only concerned with small energy-momenta $Q$ around this new origin, the antibaryon may be “integrated away”. $\mathcal{L}_{\text{ch}}(B;U; v,a,s,p)$ describing this particle-only world may be defined similarly to Eq. (3). The equation of motion for $B$ resulting from $\mathcal{L}_{\text{ch}}(B;U; v,a,s,p)$ can be rewritten as coupled equations for the large and small components $B_\pm$ defined by $B_\pm \equiv P_\pm B$ with $P_\pm \equiv (1 \pm \gamma^0)/2$. Elimination of $B_-$ in favor of $B_+$ leads to an equation of motion for $B_+$. The HBF Lagrangian $\mathcal{L}_{\text{ch}}^{\text{HB}}$ is defined as an effective Lagrangian that reproduces the equation of motion for $B_+$ and $U$. Since $B_- \approx (Q/m_N) B_+$, $\mathcal{L}_{\text{ch}}^{\text{HB}}$ involves expansion in $\partial_\mu / m_N$ as well as in $\partial_\mu / \Lambda_\chi$ and $\mathcal{M} / \Lambda_\chi$. As $m_N \approx 1$ GeV $\approx \Lambda_\chi$, we usually lump together chiral and heavy-baryon expansions. In this combined expansion scheme, the effective chiral Lagrangian can be organized as

$$ \mathcal{L}_{\text{ch}}^{\text{HB}} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \cdots, \quad \mathcal{L}^{(\nu)} = \mathcal{O}(Q^\nu) \quad (3) $$

The chiral order index $\nu$ in HBF is defined as

$$ \nu = d + (n/2) - 2, \quad (4) $$
where $n$ is the number of fermion lines that participate in a vertex, and $d$ is the number of derivatives (with $M \propto m^2_\pi$ counted as two derivatives). The leading order terms are given as

$$
\mathcal{L}^{(0)} = \frac{f_\pi^2}{4} \text{Tr}[\partial_\mu U^\dagger \partial^\mu U + m^2_\pi (U^\dagger + U - 2)] \\
+ \bar{B}(iv \cdot D + g_A S \cdot u)B - \frac{1}{2} \sum A (\bar{B} \Gamma_A B)^2 
$$

(5)

$$
\mathcal{L}^{(1)} = -\frac{i g_A}{2m_N} \bar{B}(S \cdot D, v \cdot u)B + 2c_1 m^2_\pi \bar{B} B \text{Tr}(U + U^\dagger - 2) \\
+(c_2 - \frac{g_A^2}{8m_N}) \bar{B}(v \cdot u)^2 B + c_3 B u \cdot u B \\
- \frac{c_9}{2m_N} (\bar{B} B)(\bar{B} i S \cdot u B) - \frac{c_{10}}{2m_N} (\bar{B} S^a B)(\bar{B} i u_\mu B). 
$$

(6)

Here $\xi = \sqrt{U}$, $u_\mu \equiv i(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)$, $S_\mu = i\gamma_5 \sigma_{\mu\nu} v^\nu / 2$ and

$$
D_\mu = \partial_\mu + [\xi^\dagger, \partial_\mu u] / 2 - i \xi^\dagger (v_\mu + a_\mu) / 2 - i(\xi(v_\mu - a_\mu)) / 2. 
$$

(7)

The four-velocity parameter $v_\mu$ is in practice chosen to be $v_\mu = (1, 0, 0, 0)$. The low-energy constants $c_1, c_2$ and $c_3$ have been determined from phenomenology\(^b\) and their numerical values will be discussed later. In any practical calculations, one expands $U(x)$ in powers of $\pi(x)/f_\pi$ and and only retains necessary lowest order terms.

The chiral counting above applies to individual vertices. We can also introduce chiral counting for Feynman diagrams; the chiral order $\nu$ of a Feynman diagram is given by

$$
\nu = 2 - N_E + 2L - 2(C - 1) + \sum_i \nu_i, 
$$

(8)

where $N_E$ is the number of external fermion lines, $L$ the number of loops, $C$ the number of disconnected parts, and the sum runs over vertices involved in the Feynman diagram. In fact, in applying this counting rule to nuclear physics, we must exercise caution. As emphasized by Weinberg\(^b\) naive chiral counting fails for a nucleus, because purely nucleonic intermediate states (with no pions in flight) that occur in a nucleus can have very low excitation energies and thereby invalidate the ordinary chiral counting rule. To avoid this difficulty, we need to classify Feynman diagrams into two groups. Diagrams in

\(^b\)The most updated determination of $L_{ch}^{HB}$ can be found in Ref.\(^c\).
which every intermediate state contains at least one meson in flight are called irreducible diagrams, and all others are called reducible diagrams. The $\chi$PT can only be applied to the irreducible diagrams. One can show that Eq. (8) gives a correct chiral order for an irreducible diagram, and that an irreducible diagram of chiral order $\nu$ is typically $O(Q^\nu)$. The contribution of all the irreducible diagrams (up to a specified chiral order) is then used as an effective operator acting on the nucleonic Hilbert space. This second step allows us to incorporate the contributions of the reducible diagrams to infinite order. This two-step procedure may be referred to as nuclear chiral perturbation theory.

Weinberg formulated nuclear $\chi$PT and applied it to a chiral perturbative derivation of the nucleon-nucleon interactions. Further developments by van Kolck et al. succeeded in explaining many basic features of the two-nucleon interactions, but their phenomenological success has not quite reached the level of the traditional phenomenological boson-exchange potentials.

Nuclear $\chi$PT can also be applied to electroweak transition processes. Here a nuclear transition operator is defined as a set of all the irreducible diagrams (up to a given chiral order $\nu$) with an external current inserted. In a completely consistent $\chi$PT calculation, this transition operator is to be sandwiched between the initial and final nuclear states which are governed by the nucleon interactions corresponding to the $\nu$-th order irreducible diagrams. However, the present status of the $\chi$PT derivation of the $N-N$ forces is such that we cannot completely fulfill this formal consistency. In practice, therefore, we use the phenomenological nucleon-nucleon interactions in the nuclear Schrödinger equation to generate the initial and final nuclear wavefunctions. We shall refer to this eclectic method as the hybrid treatment of nuclear $\chi$PT.

2 Exchange currents

First, I wish to talk about $\chi$PT treatments of exchange currents for those pieces of the electroweak currents for which the one-pion exchange contributions are known to be dominant. As discussed in Ref. 9, the space component of the vector current ($V$) and the time component of the axial current ($A_0$) belong to this category. From the $\chi$PT point of view, the pion-exchange current that arises from the low-energy theorem corresponds to the leading-order tree diagram in chiral expansion applied to a two-nucleon system. Furthermore, $\chi$PT provides a systematic framework for organizing exchange-current contributions that have shorter ranges than the one-pion range.

As regards $V$, there is a very clean test in the two-nucleon systems, for which the nuclear wavefunctions are known with sufficient accuracies. Experimentally, the capture rate for the $n(\text{thermal}) + p \to d + \gamma$ process is...
\[ \sigma_{\exp} = 334.2 \pm 0.5 \text{ mb}, \text{ and this value is } \sim 10\% \text{ larger than the impulse } \]

approximation (IA) prediction, \( \sigma_{IA} = 302.5 \pm 4.0 \text{ mb} \). Riska and Brown pointed out that the one-pion exchange current derived from the low-energy theorem can account for \( \sim 70\% \) of the missing capture rate. Another impressive success of the exchange current calculations based on the low-energy theorem is known for the \( e+d \rightarrow e+p+n \) reaction, see e.g. Ref. Park et al. carried out a systematic study of the contributions of the next-to-leading order terms in chiral expansion. According to this study, the inclusion of the next order contributions leads to \( \sigma = 334 \pm 2 \text{ mb} \) in perfect agreement with experiment.

Regarding \( A_0 \) (time component of the axial current), it is not easy to find observables in the two-nucleon system that give clear-cut information about the leading one-pion exchange current. This is mainly because \( A_0 \) must compete with the dominant space component \( A \). We therefore need to go to complex nuclei. This requires careful examinations of the so-called core polarization effects. Despite this non-trivial aspect, there is by now ample evidence that supports the \( \chi PT \) derivation of exchange currents. Warburton et al.'s systematic analyses of the first-forbidden \( \beta \) transitions indicate that, over a very wide range of the periodic table, the ratio of the exchange-current contribution to the 1-body contribution \( \delta_{mec} \equiv \langle A^0(\text{mec}) \rangle / \langle A^0(\text{1-body}) \rangle \) is \( \delta_{\exp}^{\text{mec}} \sim 0.5 \sim 0.8 \).

The leading-order \( \chi PT \) term, i.e. the soft-pion exchange term, can explain the bulk of \( \delta_{\exp}^{\text{mec}} \) and the next-order \( \chi PT \) term gives an additional \( \sim 10\% \) enhancement, bringing the theoretical value closer to the empirical value. The general tendency that \( \delta^{\text{theor}}_{\text{mec}} \) is still somewhat smaller than \( \delta_{\exp}^{\text{mec}} \) has been a subject of recent intensive studies. I wish to mention, however, that in the \( A=16 \) nuclei where the shell model space employed is large enough to get rid of the usual core-polarization corrections and therefore the calculation is deemed most reliable, the lowest-order one-pion current gives good agreement with the data.

Thus, the “extra enhancement” of \( \delta_{\text{mec}} \) for the heavier nuclei should be taken with some caution.

If we take the putative extra enhancement in \( \delta_{\text{mec}} \) seriously, what is a possible mechanism for that? Several authors investigated heavy-meson exchange contributions. In particular, Townsell carried out a detailed calculation using the “hard-pion formalism” in conjunction with the Lagrangian that engenders the phenomenological \( N-N \) interactions, and was able to obtain a roughly right amount of extra enhancement to reproduce \( \delta_{\exp}^{\text{mec}} \). Meanwhile, the use of an in-medium chiral Lagrangian that by fiat incorporates the BR scaling was also proposed as a possible solution to the extra enhancement problem. These two explanations in fact may not be as disjoint as they appear. The most prominent heavy-meson pair contribution comes from \( \sigma \)-exchange, whereas this \( \sigma \) contribution can be effectively rewritten as the 1-body term with the nucleon.
mass replaced by an effective mass. Thus the $\sigma$-meson contribution plays a role similar to the BR scaling. So we do have at our disposal some models that can accommodate a certain amount of the “extra enhancement”. What is not clear is their relation to the basic chiral counting. For example, if we are to interpret the $\sigma$-meson as multipion-exchange effects, they would correspond to much elevated chiral orders, and there is at present no consistent way to treat all possible terms of such high chiral orders. Meanwhile, some multi-fermion terms that have higher chiral orders than the next-to-leading-order terms give rise to nucleon mass shift in medium. In this sense the BR scaling has some overlap with higher-order terms in $\chi$PT, but again its systematic treatment in $\chi$PT is at present not available.

Apart from these possible higher chiral order effects, I wish to mention a subtle point that exists even within the next-to-leading-order calculations which scored great success. If we follow faithfully the formal chiral counting rules, transition operators in general can contain terms of the contact multi-fermion type as well as extremely short-ranged terms that reflect the integrated-out high-energy physics. However, in the existing treatments which are based on the hybrid treatment of nuclear $\chi$PT, these singular terms are discarded using the intuitive argument that the short-range $N$-$N$ correlation would strongly suppress these zero-range or extremely short-ranged transition operators.

One of the most urgent problems in nuclear $\chi$PT at present is how to justify, or how to get rid of this intuitive argument. To make progress in this direction, we must learn how to apply $\chi$PT to short-range phenomena. We also need to treat on the same footing the irreducible kernels that generate effective electroweak transition opertaors and the irreducible kernels which are responsible for distortion of the initial and final nuclear wavefunction. As a useful testing ground for this type of fully consistent nuclear $\chi$PT calculations (free from the hybrid treatment), we can consider a system with a characteristic energy low enough for even the pions to be integrated out. In particular, it is interesting to study whether we can describe low-energy two-nucleon phenomena consistently in a version of nuclear $\chi$PT in which the pionic degrees of freedom have been integrated out and their effects are incorporated into the low-energy coefficients of an effective Lagrangian for the nucleons. A $\chi$PT calculation without pions is almost an oxymoron but, in the formal sense of heavy-baryon $\chi$PT, nothing prevents us from integrating out pions from our system. Also, as a first step toward the eventual reintroduction of the pionic degrees of freedom, this nucleon-only $\chi$PT is expected to be a very informative exercise.

As for the low-energy N-N scattering, Kaplan, Savage and Wise recently
attempted to extend $\chi$PT to description of the short-range $N-N$ interactions. The result, however, was not very encouraging in that the chiral expansion turned out to have a poor convergence property. But it was soon realized\textsuperscript{24,25} that a dimensional regularization used in Ref.\textsuperscript{23} is responsible for this difficulty. The use of a cut-off regularization largely eliminates the problem\textsuperscript{25,26}.

A challenge now is to carry out a similar calculation for the two-nucleon observables that involve external probes. Very recently, Park \textit{et al.}\textsuperscript{31} has carried out the first study of this type. Dr. Tae-Sun Park will give a detailed account of that work in his talk right after mine.

3 $p + p \rightarrow p + p + \pi^0$ reaction near threshold

My next topic is the low-energy $p + p \rightarrow p + p + \pi^0$ reaction. The recent high-precision measurements\textsuperscript{29} of the total cross sections for $pp \rightarrow pp\pi^0$ near threshold have invited many theoretical investigations on this process. I attempt here to explain why this reaction is of particular interest from a $\chi$PT point of view. To this end, I first give a quick general survey of the theoretical developments up until 1996\textsuperscript{36} and subsequently I will expound on our latest study\textsuperscript{36}. At the end I present some precautionary remarks concerning the results described in Ref.\textsuperscript{36}.

One expects that threshold pion production occurs via the single-nucleon process (the impulse or Born term), fig.1(a), and the $s$-wave pion rescattering process, fig.1(b).

\textsuperscript{3}Regrettably I must skip here many issues that have been extensively studied by multiple-scattering specialists. For a recent review on the topics omitted here, see \textit{e. g.} Ref.\textsuperscript{39}. I am indebted to Tony Thomas and Boris Blankleider for bringing my attention to the line of work described in this reference.
In the conventional treatment, the $\pi$-$N$ vertex for the impulse term is assumed to be given by the Hamiltonian

$$H_0 = \frac{g_A}{2f_\pi} \bar{\psi} \left( \vec{\sigma} \cdot \vec{\nabla} (\vec{\tau} \cdot \vec{\pi}) - \frac{i}{2m_N} \{\vec{\sigma} \cdot \vec{\nabla}, \vec{\tau} \cdot \vec{\pi}\} \right) \psi,$$  \hspace{1cm} (9)

where $g_A$ is the axial coupling constant, and $f_\pi = 93$ MeV is the pion decay constant. The first term gives $p$-wave pion-nucleon coupling, while the second term accounts for the nucleon recoil effect. The $s$-wave rescattering vertex in Fig.1(b) is customarily described with the phenomenological Hamiltonian

$$H_1 = 4\pi \frac{\lambda_1}{m_\pi} \bar{\psi} \vec{\pi} \cdot \vec{\pi} \psi + 4\pi \frac{\lambda_2}{m_\pi} \bar{\psi} \vec{\tau} \cdot \vec{\pi} \times \vec{\pi} \psi$$  \hspace{1cm} (10)

The coupling constants $\lambda_1$ and $\lambda_2$ determined from the experimental pion-nucleon scattering lengths are $\lambda_1 \sim 0.005$ and $\lambda_2 \sim 0.05$. Thus, $\lambda_1 \ll \lambda_2$, in conformity to an expectation from current algebra. The calculations based on these phenomenological vertices yield cross sections for $s$-wave $\pi^0$ production that are significantly smaller, typically by a factor of $\sim 5$, than the experimental values. To many people this large discrepancy came as a big surprise. Apart from this discrepancy, it is worthwhile to emphasize that the near-threshold $p + p \rightarrow p + p + \pi^0$ reaction is an intrinsically suppressed process for the following reasons. First, in Eq.(9), only the second term contributes to $s$-wave pion production. The suppression factor $\sim m_\pi/m_N$ contained in this term drastically reduces the contribution of the impulse term, Fig.1(a), enhancing thereby the relative importance of the two-body rescattering process, Fig.1(b). On the other hand, the dominant $\lambda_2$ term in Eq.(10) cannot contribute to the $pp\pi^0\pi^0$ vertex in Fig.1(b) due to its isospin structure. Thus the two-body contribution is also hindered here. As we will see below, the fact that the leading terms in the phenomenological Lagrangian, Eqs.(9) and (10), cannot contribute to the near-threshold $p + p \rightarrow p + p + \pi^0$ reaction implies that this reaction is sensitive to higher-order terms in chiral expansion. This is one of the reasons why this reaction is particularly interesting from the $\chi$PT point of view.

To explain major issues involved in the more recent theoretical developments, it is convenient to introduce what we call the typical threshold (TT) kinematics. Consider Fig.1(b) in the center of mass (CM) system with the initial and final interactions turned off. At threshold, $(q_0, \vec{q}) = (m_\pi, \vec{0})$, $p'_{10} = p'_{20} = m_N$, $p'_1 = \vec{p}'_2 = \vec{0}$, so that $p_{10} = p_{20} = m_N + m_\pi/2$, $k_0 = m_\pi/2$, $\vec{p}_1 = k = -\vec{p}_2$ with $|k| = \sqrt{m_\pi m_N + m_\pi^2/4}$. (Of course, even for $\vec{q} = 0$, the actual kinematics for the transition process may differ from the TT kinematics.)
because of the initial- and final-state interactions.) Now, for the $TT$ kinematics, $k^2 = -m_N m_π \neq m_π^2$, indicating that the rescattering diagram is sensitive to the off-shell $πN$ amplitudes. However, there is no guarantee that $H_1$ of Eq. (14) describes the off-shell amplitudes adequately. Hernández and Oset suggested that the s-wave amplitude enhanced for off-shell kinematics could increase the rescattering contribution sufficiently to reproduce the experimental cross sections. However, Ref. 32 used phenomenological off-shell extrapolations, the reliability of which requires further examination. We will come back to this issue later. Another point is that $k^2 = -m_N m_π$ for $TT$ kinematics implies that the rescattering process typically probes inter-nucleon distances $\sim 0.5$ fm. The process then can be sensitive to exchange of the heavy mesons which play an important role in the phenomenological meson-exchange $N-N$ potentials. Lee and Riska studied the possible enhancement of the $pp \rightarrow ppπ^0$ cross section due to shorter-range meson exchanges. This enhancement, however, turns out to be very sensitive to how one evaluates the “basic” diagrams, fig. 1(a) and (b). We will see below that a $\chi$PT calculation of these diagrams does not necessarily support the idea of heavy-meson exchange enhancement.

As emphasized in Introduction, $\chi$PT serves as a consistent framework to describe the low-energy $πN$ scattering amplitudes for off-shell as well as on-shell kinematics. Park et al. (to be referred to as PM3K) and Cohen et al. (to be referred to as CFMK) carried out the first $\chi$PT calculations for the $pp \rightarrow ppπ^0$ reaction. The results of these two groups essentially agree with each other on the following major points: (1) the pion rescattering term in a $\chi$PT treatment is significantly larger than in the conventional treatment; (2) the sign of the rescattering term in a $\chi$PT treatment is opposite to that obtained in the conventional approach; (3) the enhanced rescattering term in a $\chi$PT treatment almost cancels the impulse term, leading to theoretical cross sections much smaller than the observed values; (4) the smallness of the amplitude corresponding to Fig. 1(a)+(b) implies that the addition of the heavy-meson exchange diagrams does not result in large enough interference to reproduce the experimental cross sections. Thus $\chi$PT treatments of the off-shell $πN$ scattering amplitudes indeed drastically change the near-threshold $pp \rightarrow ppπ^0$ cross section.

We need to mention, however, that the calculations in Refs. 34, 35, which rely on coordinate space representation, involve potentially problematic approximations on the kinematical variables appearing in the $πN$ scattering amplitudes. Namely, there the $r$-space representation of the two-body transition operator [fig. 1(b)] was derived from the Feynman amplitude corresponding to the $TT$ kinematics by Fourier-transforming this particular amplitude with respect to $\vec{p}_1, \vec{p}_2, \vec{p}_1', \vec{p}_2'$, while keeping all the other kinematical variables fixed at
their $TT$ kinematics values. Although this type of kinematical simplification is commonly used in nuclear physics for deriving effective r-space operators, it is expected to be much less reliable for the threshold $pp \rightarrow pp\pi^0$ reaction. The reason is two-fold: First, the energy-momentum exchange due to the initial and final-state interactions is essentially important for this process. Secondly, the destructive interference between the one-body and two-body terms found in Refs. 34,35 implies that even a rather moderate change in the two-body term can influence the cross sections significantly. In view of these problems, Sato et al. 36 (to be referred to as SLMK) have recently carried out a $\chi$PT calculation for $pp \rightarrow pp\pi^0$ in the momentum representation, which allowed them to avoid the above-mentioned kinematical simplifications. This improvement is found to affect strongly the calculated cross section. We give below a brief summary of SLMK’s work. The necessity to discuss the basic ingredients of SLMK also offers an opportunity to explain the earlier $\chi$PT treatments 34,35 in more detail than the above itemized summary allows.

In heavy-baryon $\chi$PT, in order to generate the one-body and two-body diagrams depicted in figs. 1(a), 1(b), we minimally need terms with $\bar{\nu} = 1$ and 2 in $L^{\text{HB}}_{\text{ch}}$, Eq.(3). The relevant pion-nucleon interaction Hamiltonian is $H_{\text{int}} = H^{(0)} + H^{(1)}$ with

$$H^{(0)} = \frac{g_A}{2f_\pi}\bar{B}[\sigma \cdot \nabla(\tau \cdot \pi)]B + \frac{1}{4f_\pi}\bar{B}\tau \cdot \pi \times \hat{\pi}B$$

$$H^{(1)} = -\frac{ig_A}{4m_Nf_\pi}\bar{B}[\sigma \cdot \nabla, \tau \cdot \hat{\pi}]B$$

$$+ \frac{1}{f_\pi}[2c_1m_\pi^2\hat{\pi}^2 - (c_2 - \frac{g_A^2}{8m_N})\hat{\pi}^2 - c_3(\partial \pi)^2]\bar{B}B. \quad (12)$$

Here $H^{(\bar{\nu})}$ represents the term of chiral order $\bar{\nu}$. The standard values of the low-energy coefficients (LEC), $c_1$, $c_2$ and $c_3$, may be taken from Ref. 2, where these parameters were determined from the experimental values of the pion-nucleon $\sigma$ term, the nucleon axial polarizability $\alpha_A$ and the isospin-even s-wave $\pi N$ scattering length $a^+$. Their values are

$$c_1 = -0.87 \pm 0.11 \text{ GeV}^{-1}, \quad c_2 = 3.34 \pm 0.27 \text{ GeV}^{-1}, \quad c_3 = -5.25 \pm 0.22 \text{ GeV}^{-1}. \quad (13)$$

Comparing Eqs. (11), (12) with the phenomenological effective Hamiltonian $H_0 + H_1$, Eqs. (1), (10), we note that the first term in $H^{(0)}$ combined with the first term in $H^{(1)}$ exactly reproduces $H_0$. As for the $\pi\pi NN$ vertices, we can associate the second term in $H^{(0)}$ to the $\lambda_2$ term in $H_1$, and second term in $H^{(1)}$ to the $\lambda_1$ term in $H_1$. Thus, as mentioned earlier, the threshold $pp \rightarrow pp\pi^0$ reaction is indeed sensitive to the higher chiral-order terms. Concentrating on
the $\lambda_1$ term, which is of direct relevance here, we recognize the correspondence:

$$4\pi \lambda_1/m_\pi \leftrightarrow \kappa(k, q),$$

where

$$\kappa(k, q) \equiv \frac{m_\pi^2}{f_\pi^2} \left[ 2c_1 - (c_2 - \frac{g_A^2}{8m_N} \omega q \omega k - c_3 \frac{q \cdot k}{m_\pi^2}) \right], \quad (14)$$

with $q = (\omega_q, q)$ and $k = (\omega_k, k)$, see fig.1(b). Now, for on-shell low energy pion-nucleon scattering, i.e., $k \sim q \sim (m_\pi, 0)$, we equate

$$\left(\frac{4\pi \lambda_1}{m_\pi}\right)_{\text{on-shell}}^{\chi PT} = \kappa_0 \equiv \kappa(k=(m_\pi, 0), q=(m_\pi, 0)), \quad (15)$$

which gives $\kappa_0 = (0.87 \pm 0.20) \text{ GeV}^{-1}$. On the other hand, in the conventional approach, $\lambda_1$ is determined from the $a^+$ term in Eq.(16) alone. Namely

$$\left(\frac{4\pi \lambda_1}{m_\pi}\right)_{\text{conventional}} = -2\pi \left(1 + \frac{m_\pi}{m_N}\right) a^+ = (0.43 \pm 0.20) \text{ GeV}^{-1}, \quad (17)$$

which corresponds to the “literature value” $\lambda_1 = 0.005$. Thus the $\chi PT$ value $\kappa_0 = 0.87 \text{ GeV}^{-1}$ is about twice as large as the conventional value. To pursue further comparison between the traditional and $\chi PT$ approaches, let us go back to Eq.(14). Obviously, $\kappa(k, q)$ cannot be fully simulated by a constant $\lambda_1$. To illustrate how the $q$ and $k$ dependence in $\kappa(k, q)$ affects the rescattering vertex [fig.1(b)], let us consider the value of $\kappa(k, q)$ corresponding to the $TT$ kinematics and denote it by $\kappa_{TT}$.

$$\kappa_{TT} = \frac{m_\pi^2}{f_\pi^2} \left[ 2c_1 - \frac{1}{2} \left(c_2 - \frac{g_A^2}{8m_N}\right) \frac{c_3}{2} \right] = -1.5 \text{ GeV}^{-1} \quad (18)$$

We can see the $s$-wave $\pi$-$N$ interaction is much stronger here than in the on-shell cases [Eqs.(16), (17)], and the sign of $\kappa_{TT}$ is opposite to that of $\kappa_0$. This flip in sign changes drastically the interference pattern of the Born and rescattering terms.

Adopting nuclear $\chi PT$, we write the transition amplitude for the $pp \rightarrow pp\pi^0$ reaction as

$$T = \langle \Phi_f | T | \Phi_i \rangle, \quad (19)$$

where $|\Phi_i \rangle$ ($|\Phi_f \rangle$) is the initial (final) two-nucleon state distorted by the initial-state (final-state) interaction. For formal consistency, if $T$ is calculated up to order $\nu$, the nucleon-nucleon interactions that generate $|\Phi_i \rangle$ and $|\Phi_f \rangle$ should
also be calculated by summing up all irreducible two-nucleon scattering diagrams up to order $\nu$. In practice, however, it is common to use the phenomenological $N-N$ interactions that reproduce measured two-nucleon observables, and here again we use this hybrid version of nuclear $\chi$PT. The lowest-order contributions to the impulse and rescattering terms come from the $\nu = -1$ and $\nu = 1$ terms, respectively. (Here we are using the counting rule Eq. (8)). Therefore we have

$$\mathcal{T} = \mathcal{T}^{(-1)} + \mathcal{T}^{(+1)} \equiv \mathcal{T}^{\text{Imp}} + \mathcal{T}^{\text{Resc}}$$

The use of the Hamiltonian in Eqs. (11) and (12) leads to momentum-space matrix elements

$$\mathcal{T}^{(-1)} = \frac{i}{(2\pi)^{3/2}} \frac{g_A}{\sqrt{2\omega_q}} \frac{1}{2f_\pi} \sum_{j=1,2} \left[ -\vec{\sigma}_j \cdot \vec{q} + \frac{\omega_q}{2m_N} \vec{\sigma}_j \cdot (\vec{p}_j + \vec{p}_j') \right] \tau_j^0,$$

$$\mathcal{T}^{(+1)} = -\frac{\kappa(k_j,q)}{(2\pi)^{9/2}} \frac{g_A}{f_\pi} \sum_{j=1,2} \kappa(k_j,q) \vec{\sigma}_j \cdot \vec{k}_j \tau_j^0 \frac{k_j^2 - m_N^2}{k_j^2},$$

where $\vec{p}_j$ and $\vec{p}_j'$ ($j = 1, 2$) denote the initial and final momenta of the $j$-th proton (see fig. 1). The four-momentum of the exchanged pion is defined by the nucleon four-momenta at the $\pi NN$ vertex: $k_j = p_j - p_j'$, where $p_j = (E_{p_j}, \vec{p}_j), p_j' = (E_{p_j'}, \vec{p}_j')$ with the definition $E_p = (p^2 + m_N^2)^{1/2}$.

The transition amplitude of the $pp \rightarrow pp\pi^0$ reaction is evaluated by taking the matrix element of the production operator $\mathcal{T}$ defined above between the initial ($\chi^{(+)}$) and the final ($\chi^{(-)}$) $pp$ scattering wavefunctions. In terms of this transition matrix element, the total cross section is given by

$$\sigma_{pp \rightarrow pp\pi^0}(W) = \frac{(2\pi)^4}{2v_i} \int d\vec{p}_f d\vec{q} \delta(\sqrt{4E_{p_f}^2 + \vec{q}^2} + \omega_q - W) \times \frac{1}{4} \sum_{m_{s_1}, m_{s_2}, m_{s_1}', m_{s_2}'} |<\chi^{(-)}_{p_1, m_{s_1}, m_{s_1}'}; \vec{q}_f | \mathcal{T} | \chi^{(+)}_{p_1, m_{s_1}, m_{s_2}}; \vec{q}_i>|^2,$$

where $\vec{p}_i$ and $\vec{p}_f$ are the asymptotic relative momenta of the initial and final $pp$ states, respectively, $W = 2E_{p_i}$ is the total energy, $v_i = 2|\vec{p}_i|/E_{p_i}$ is the asymptotic relative velocity of the two initial protons, and $m_{s_j}$ is the $z$-component of the spin of the $j$th nucleon. Near threshold the transition matrix appearing in Eq. (23) can be expressed using only one partial wave amplitude. The corresponding reduced matrix element for the impulse term is

$$\frac{1}{\sqrt{4\pi}} <p_f|^{[1S_0]} \mathcal{T}_{\text{Imp}}^{\text{Imp}}(q) |p_i|[3P_0]> = -\frac{i}{\sqrt{(2\pi)^3 2\omega_q}} \frac{g_A}{f_\pi} \int \int \frac{d\vec{p}_f' d\vec{p}_i'}{4\pi} R_{1S_0 p_f}(p_f')$$
while the reduced matrix element of the rescattering term is given by
\[
\frac{1}{\sqrt{4\pi}} < p_f | {^{1}S_0} | \mathcal{T}_{s=0}^{\text{Resc}}(q) | p_i | ^{3}P_0 > = \frac{i}{\sqrt{(2\pi)^32\omega_q}} \frac{2g_A}{f_\pi} \int \int \frac{d\tilde{p}' d\tilde{p}}{4\pi} R_S(p_0,p_f(p'))
\times \frac{\kappa(k,q)}{(2\pi)^3 k^2 - m_N^2} R_S(p_0,p_f(p)).
\]
(25)

In the above, \(\tilde{p}\) and \(\tilde{p}'\) stand for the relative momenta of the two protons before and after the pion emission, respectively, and \(k = (k_0,\vec{k}) = (E_{\vec{p}} - E_{\vec{p}'},-q/2,\vec{p} - \vec{p}' + q/2)\). The radial functions for the \(pp\) scattering states that appear in Eqs. (24), (25) are to be generated with the use of realistic nucleon-nucleon interactions. SLMK used the Argonne V18 potential. If we take the limit \(q \to 0\) limit and freeze \(k_0\), the energy variable of the exchanged pion, at the fixed value \(k_0 = m_N/2\) corresponding to the threshold pion production, then we are back with the TT kinematics results. This simplified treatment is equivalent to the fixed kinematics approximation used in PM3K.

The upshot of Sato et al. ’s results is as follows. The rescattering contribution \(< p_f | {^{1}S_0} | \mathcal{T}_{s=0}^{\text{Resc}}(q) | p_i | ^{3}P_0 >\) estimated in the full \(p\)-space calculation has the same sign as the result of the fixed kinematics approximation but, as far as their magnitudes are concerned,

\[
| < p_f | {^{1}S_0} | \mathcal{T}_{s=0}^{\text{Resc}}(q) | p_i | ^{3}P_0 > | \approx | < p_f | {^{1}S_0} | \mathcal{T}_{s=0}^{\text{Imp}}(q) | p_i | ^{3}P_0 > | _{\text{full}} \approx | < p_f | {^{1}S_0} | \mathcal{T}_{s=0}^{\text{Imp}}(q) | p_i | ^{3}P_0 > | _{\text{kin}}.
\]
(26)

In PM3K, \(< p_f | {^{1}S_0} | \mathcal{T}_{s=0}^{\text{Resc}}(q) | p_i | ^{3}P_0 >\approx - < p_f | {^{1}S_0} | \mathcal{T}_{s=0}^{\text{Imp}}(q) | p_i | ^{3}P_0 >\), leading to an almost perfect cancellation between the impulse and rescattering terms. SLMK find instead

\[
< p_f | {^{1}S_0} | \mathcal{T}_{s=0}^{\text{Resc}}(q) | p_i | ^{3}P_0 > \approx - 3 < p_f | {^{1}S_0} | \mathcal{T}_{s=0}^{\text{Imp}}(q) | p_i | ^{3}P_0 >
\]
(27)

Therefore, although the rescattering and impulse terms interfere destructively in this case also, the dominance of the rescattering term leads to a much larger cross section than in the fixed kinematics approximation. Yet, \(\sigma_{\text{full}}\) obtained in SLMK is still significantly smaller than the observed cross sections.

SLMK also examined to what extent the uncertainties in the low-energy coefficients, \(c_1\), \(c_2\) and \(c_3\) affect their calculational results. They found that the near-threshold \(pp \to pp\pi^0\) cross sections are sensitive to the value of \(c_1\) \(^d\)SLMK checked that the use of, e. g. , the Reid soft-core potential gives essentially the same results.
and change significantly as $c_1$ varies within the currently accepted error bars [see Eq. (13)]. It was found, however, that even with the most favorable choice of $c_1$ the calculated cross sections are significantly smaller than the observed values.

To place the calculation of SLMK in an appropriate context, the following three remarks are in order.

(i) $T^{(+1)}$ in Eq. (22) represents only the tree-diagram contribution, Fig.1(b); loop corrections to the $\nu = -1$ impulse term generate transition operators of order $\nu = 1$. These additional contributions generate an effective $\pi NN$ vertex form factor for the impulse term, Fig.1(a). SLMK ignored these effects quoting a rough estimate in PM$^3$K which indicates that the net effect of the loop corrections after renormalization is less than 20% of the leading-order impulse term. It is obviously desirable to reexamine this issue, and we are currently carrying out a calculation that includes all the $\nu = 1$ contributions. Qualitatively speaking, the inclusion of this effect is expected to reduce the contribution of the impulse term, further enhancing the dominance of the rescattering term.

(ii) The nuclear chiral counting scheme employed above is in fact best applicable when energy-momentum transfers to a nucleus are small, whereas the near-threshold $pp \to pp\pi^0$ reaction involves significant energy transfers $q_0 \sim m_\pi$. We therefore must exercise caution in applying Weinberg's counting rule, Eq.(3), to this case. In SLMK as well as in PM$^3$K, in the stage of constructing the transition operators, the energy-momentum transfer due to the final pion is ignored (i.e., the external pion is taken to be soft, $q_\mu \approx 0$) in order to utilize Weinberg's original counting rule. The physical value of $q_\mu$ is used only at the stage of calculating the phase space integral. The approximate nature of this approach is particularly evident for the impulse term, where the pion-emitting nucleon is off-shell by $\sim m_\pi$. This off-shell nucleon must interact at least once with a second nucleon before losing its off-shell character. It is then sensible to treat an impulse diagram accompanied by subsequent one-pion exchange as an irreducible diagram, even though the original Weinberg classification would treat it as a reducible diagram. CFMV proposed a modified chiral counting rule that takes account of this feature. In addition, these authors argued that the $\Delta$ degree of freedom should be taken into account explicitly in $\chi$PT since the $N-\Delta$ mass difference $\sim 2m_\pi$ is small on the chiral scale $\Lambda$ (see also Ref.38). All these are important issues that deserve further investigations. In particular, if one uses the CFMV's counting rule the expansion parameter is not any longer $m_\pi/M_N$ but $\sqrt{m_\pi/M_N}$. This indicates a possible slow convergence of chiral expansion. (The insufficiency of the lowest-order rescattering diagram for describing $pp \to ppp^0$ was discussed in a different context in Ref.3.)
This last remark is rather technical but it may contain an important message. SLMK find that the rescattering transition matrix element is sensitive to uncomfortably high momentum components of the nuclear $(pp)$ wavefunction; as a matter of fact, the relevant range of momentum is not much smaller than the chiral scale $\Lambda$. This disturbing feature is in fact shared also by other known applications of nuclear $\chi$PT. A satisfactory solution to this problem probably requires the study of terms with higher chiral orders than considered so far.

Despite all the caveats stated above, it seems almost certain that, in any reasonably realistic $\chi$PT calculations, the rescattering term dominates over the impulse term and their signs are opposite to each other. This implies that the heavy-meson exchange contributions considered in Ref.\textsuperscript{31} cannot be invoked as a possible mechanism to enhance $\sigma_{\text{full}}$ to bring it closer to the observed cross sections. Since it is established that these heavy-meson contributions have the same sign as the impulse term, the addition of these extra contributions to the transition amplitude obtained in the full $p$-space calculation would result in a destructive interference. Thus the heavy-meson contributions such as considered in Ref.\textsuperscript{31} suppresses the cross section instead of enhancing it. Most recently, van Kolck, Miller and Riska\textsuperscript{37} considered yet another diagram involving $\rho - \omega$ exchanges. But, again, since this extra contribution has the same sign as the impulse term, one encounters the same difficulty as above. Thus, the near-threshold $pp \rightarrow pp\pi^0$ reaction awaits further detailed investigations.

4 Radiative muon capture on proton

My third topic is concerned with the latest experiment on the radiative $\mu$-capture (RMC) on the proton: $\mu^- + p \rightarrow n + \nu_\mu + \gamma$. It has long been a tremendous experimental challenge to observe RMC on the proton, $\mu^- + p \rightarrow n + \nu_\mu + \gamma$, because of its extremely small branching ratio. Recently, an experiment at TRIUMF\textsuperscript{41} finally succeeded in measuring $\Gamma_{\text{RMC}}$, the proton RMC rate. To be more precise, the TRIUMF experiment determined the partial capture rate $R(> 60\text{MeV})$, corresponding to emission of a photon with $E_\gamma > 60$ MeV. The matrix element of the hadronic charged weak current $h^\lambda = V^\lambda - A^\lambda$ between a free proton and a free neutron is given by

$$\langle n(p_f)|V^\lambda - A^\lambda|p(p_i)\rangle = \bar{u}(p_f)\left[ f_V(q^2)\gamma^\lambda + \frac{f_M(q^2)}{2m_N}\sigma^{\lambda\mu}q_\mu + f_A(q^2)\gamma^\lambda\gamma_5 + \frac{f_P(q^2)}{m_\pi}q^{\lambda\gamma_5}\right] u(p_i), \quad (28)$$

where $q \equiv p_i - p_f$, and the absence of the second-class current is assumed. Of the four form factors appearing in Eq.\textsuperscript{(28)}, $f_P$ is experimentally the least well
known. Although ordinary muon capture (OMC) on a proton, $\mu^- + p \rightarrow n + \nu_\mu$, can in principle give information on $f_P$, its sensitivity to $f_P$ is intrinsically suppressed. This is due to the fact that the momentum transfer involved in OMC, $q^2 = -0.88m_\mu^2$, is far from the pion-pole position $q^2 = m_\pi^2$, where the contribution of $f_P (q^2)$ becomes most important. The radiative $\mu$-capture provides a more sensitive probe of $f_\nu$ than OMC, because the three-body final state in RMC allows kinematical regions that are close to the pion pole.

To relate the measured RMC rate, $\Gamma_{RMC}$, to $f_\nu$, the TRIUMF group used a theoretical formula of Fearing. Fearing’s formula was derived essentially by invoking minimal substitution to generate the transition matrix for RMC from that for OMC. Thus, after extracting the pion pole factor from $f_\nu$ to write $f_\nu (q^2) = \tilde{f}_\nu (q^2 - m_\pi^2)$, one replaces every $q$ in Eq.(28) with $q - eA$ ($A$ is the electromagnetic field) except the $q$ appearing in the $q^2$ dependence in $f_\nu$, $f_\lambda$, and $f_M$. By treating $\Gamma_{RMC}^{\text{theor}}$ obtained this way as a function of $\tilde{f}_\nu$, $\tilde{f}_\nu$ is optimized to reproduce $\Gamma_{RMC}^{\text{exp}}$. (Speaking more precisely, the photon spectrum with a cut-off $E_\gamma \geq 60$ MeV was fitted.) The result expressed in terms of $g_\nu = f_\nu (q^2 = -0.88m_\mu^2)$, the value of $q^2$ relevant to OMC, is $g_\nu = (10.9 \pm 0.9 \pm 0.3) f_\lambda (0)$. This value of $g_\nu$ is $\sim 1.5$ times the value expected from PCAC. This surprising result should be contrasted with the fact that $g_\nu$ measured in OMC is consistent with the PCAC prediction although the experimental uncertainties are large.

The result reported in Ref. motivates us to ask: How reliable is $\Gamma_{RMC}^{\text{theor}}$ used in deducing $g_\nu$ from $\Gamma_{RMC}^{\text{exp}}$? The theoretical framework of Fearing is rather operational in character and it is desirable to reassess its reliability. For example, we know that the nucleon matrix element of the charged weak current can contain covariants other than those that appear in Eq.(28), if the initial and/or final nucleons are off-shell. This means that Eq.(28) may be too restrictive for the description of RMC. That is, for off-shell nucleons, there are many possible form factor expressions to which one could apply minimal substitution $q \rightarrow q - eA$. Then, even within the framework of minimal substitution, there is ambiguity that cannot be lifted with phenomenological approaches. In this context, even the statement that RMC is sensitive to the pseudoscalar formfactor $f_P$ should be taken with some caution.

Here again, a systematic calculation based on $\chi$PT is very useful because (up to certain chiral orders) $\chi$PT uniquely gives all the necessary vertices for electromagnetic coupling as well as for strong interactions. Thus we can avoid applying a phenomenological minimal-coupling substitution at the level of the transition amplitude. Furthermore, $\chi$PT enables us to satisfy the gauge-invariance and chiral-symmetry requirements in a transparent way. Muon capture is a favorable case for applying $\chi$PT since momentum transfers involved

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<sup>a</sup>Dr. Cheon’s talk at this meeting addresses this type of problem.
here do not exceed \( m_\mu \), and \( m_\mu \) is small compared to the chiral scale \( \Lambda \sim 1 \) GeV, indicating the possibility of a reasonably rapid convergence of the chiral expansion. In the case of OMC, Bernard \textit{et al.} \cite{51} and Fearing \textit{et al.} \cite{52} used heavy-baryon \( \chi PT \) to evaluate \( f_\mu \) with better accuracy than achieved in the PCAC approach. In the case of RMC, a \( \chi PT \) calculation provides a natural extension of the classic work of Adler and Dothan \cite{43} based on the low-energy theorems. For instance, the \( \mathcal{O}(kq) \) terms that remained undetermined within the framework of the low energy theorems can be evaluated in \( \chi PT \). We also note that, in the case of threshold pion photo- and electroproduction, chiral loop corrections lead to a significant deviation of \( E_{0+} \) from the low-energy theorem value \( \mathcal{O}(kq) \).

Recently, Thomas Meissner, Fred Myhrer and myself (MMK) carried out a systematic \( \chi PT \) calculation of \( \Gamma_{\text{heAV}} \) to next-to-leading order. An extension of this treatment to sub-sub-leading order seemed to be a real challenge to us but this has already been achieved by Ando and Min \cite{53}. Here I wish to describe briefly the work of the USC group. It is to be noted that \( \chi PT \) applied to this single-nucleon process is free from the aforementioned extra complications that afflict \textit{nuclear} \( \chi PT \).

In the framework of the heavy-baryon \( \chi PT \) MMK use the effective Lagrangian \( \mathcal{L}_{\text{ch}} \) in Eqs.\((5),(6)\). As explained earlier, \( \mathcal{L}_{\text{ch}} \) is written in the most general form involving pions and heavy nucleons in external electromagnetic and weak fields consistent with chiral symmetry. Limiting themselves to a next-to-leading chiral order (NLO) calculation, MMK only keep terms with \( \bar{\nu} = 0 \) and \( \bar{\nu} = 1 \). To this chiral order, only tree diagrams need to be considered, and then \( \mathcal{L}^{(1)}_{\pi N} \) simply represents \( 1/M \) “nucleon recoil” corrections to the leading “static” part \( \mathcal{L}^{(0)}_{\pi N} \).

We consider all possible Feynman diagrams up to chiral order \( \nu = 1 \) which contribute to the process \( \mu^- + p \rightarrow n + \nu + \gamma \). The leptonic vertices in these Feynman diagrams are of course well known. The hadronic vertices are obtained by expanding the \( \chi PT \) Lagrangian \( \mathcal{L}_{\text{ch}} \) in terms of the elementary fields \( \mathcal{B}, \pi, \mathcal{V} \) and \( A \) and their derivatives. The evaluation of the transition amplitudes corresponding to these Feynman diagrams is straightforward, and their explicit expressions can be found in Ref.\cite{44}. MMK use the Coulomb gauge, which assures \( v \cdot \epsilon(\lambda) = 0 \). The use of this relation combined with a number of kinematical approximations consistent with the accuracy of the \( \nu = 1 \) \( \mathcal{CPT} \) calculation drastically simplifies the calculation since many of the hadronic radiation diagrams become \( \mathcal{O}(1/M^2) \) and hence negligible.

Although this is certainly not a place for going into details, several salient features of MMK’s results are worth emphasizing. The pion-pole diagrams originate from \( \mathcal{L}_{\pi}^{(0)} \) due to the coupling of the axial vector to the pion. In
\( \chi PT \) the pion-pole contributions, which arise automatically from a well-defined chiral Lagrangian, contain no ambiguity. The fact that they need not be introduced by hand constitutes a major advantage of the \( \chi PT \) approach over the phenomenological approaches which have been used in the earlier calculations. MMK also report that some of the pion pole terms that appear in \( \chi PT \) expansion have no counterpart in the phenomenological minimum substitution approach. In this connection it is also worthwhile to mention that in \( \chi PT \) calculations the transition matrix for RMC need not be directly related to the pseudoscalar coupling \( gP \) itself. The Lagrangian \( L_{\chi} \) uniquely determines \( gP \) as well as the RMC amplitude. In this sense \( gP \) and \( \Gamma_{RMC} \) are of related to each other but their relation is not as trivial as indicated by the phenomenological treatment. Furthermore, MMK observe that, with the use of the same Coulomb gauge, the behavior of the contributions of certain diagrams can differ between \( \chi PT \) and the phenomenological treatment (e. g., that in Ref. 45).

As far as numerical computations are concerned, MMK considered only the RMC from the \( \mu p \) atomic state with the hyperfine states averaged over. The spin-averaged total capture rate is given by

\[
\Gamma_{RMC} = \left( \frac{eG}{\sqrt{2}} \right)^2 |\Phi(0)|^2 \frac{1}{4} (2\pi)^4 \int \frac{d^3n}{(2\pi)^3} \int \frac{d^3\nu}{(2\pi)^3} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} \delta^4(n + \nu + k - p - \mu) \sum_{\sigma\sigma' ss' \lambda} |M|^2 ,
\]

where the sum is over all spin and polarization orientations, and \( M = \sum_{i=1}^{6} M_i \) with \( M_i \) representing six distinct hadronic radiation amplitudes; \( \Phi(0) \) is the value of the \( \mu p \) atomic wavefunction at the proton position.

The \( \nu = 1 \) \( \chi PT \) calculation of MMK gives the total capture rate \( \Gamma_{RMC} = 0.075 s^{-1} \), of which 0.061 \( s^{-1} \) comes from the leading-order \( O((1/M)^0) \) terms, and 0.014 \( s^{-1} \) from the \( O(1/M) \) terms due to \( L_{\pi N}^{(1)} \). If the contributions of the pion-pole diagrams are dropped, the resulting total capture rate would be \( \Gamma_{RMC|\text{no } \pi} = 0.053 s^{-1} \), of which 0.043 \( s^{-1} \) comes from the \( O((1/M)^0) \) terms and 0.010 \( s^{-1} \) from the \( O(1/M) \) terms. Thus about 30% of the \( \Gamma_{RMC} \) comes from the pion-pole diagrams. MMK's result for the total capture rate \( \Gamma_{RMC} = 0.075 s^{-1} \) is close to the value given in Ref. 44, \( \Gamma_{RMC} = 0.069 s^{-1} \), and practically identical to \( \Gamma_{RMC} = 0.076 s^{-1} \) reported in Ref. 45. The \( O(1/M) \) recoil corrections are found to account for about 20% of the leading order \( O((1/M)^0) \) contribution, which indicates a reasonable convergence of the chiral expansion. By contrast, the size of the \( 1/M \) corrections is noticeably larger in the approach of Ref. 45.
MMK did not consider capture from the singlet and triplet hyperfine states separately, or capture from the $p\mu p$ molecular state. Their results therefore cannot be directly compared with the experimental data. I also repeat that MMK included only up to the next-to-leading chiral order (NLO) contributions. To perform next-to-next-to-leading order (NNLO) calculations, one must include the $\bar{\nu} = 2$ chiral Lagrangian, $\mathcal{L}^{(2)}_\pi$ and $\mathcal{L}^{(2)}_{\pi N}$, and also loop corrections arising from $\mathcal{L}^{(0)}_\pi$ and $\mathcal{L}^{(0)}_{\pi N}$. Since chiral expansion for muon capture is characterized by the expansion parameter $m_\mu/M$, we expect a reasonably rapid convergence. Indeed, in the case of OMC, where the $\nu = 2$ calculation is much less involved, explicit evaluations show that the NNLO contributions amount only to a few percents. This feature is likely to persist for RMC as well, but it is reassuring to check it explicitly. It should also be mentioned that the formalism of Bernard et al. used by MMK used does not contain the explicit $\Delta$ degree of freedom in contrast to the approaches of Ref. The inclusion of the $\Delta$ particle may turn out to be more important than the NNNO extension.

Ando and Min’s calculation for RMC not only includes the NNLO contributions but also project out the hyperfine states for both atomic and molecular capture. As far as the spin-averaged $\Gamma_{\text{RMC}}$ is concerned, the result of Ando and Min is very close to that of MMK, indicating that chiral expansion for RMC indeed converges rapidly. Ando and Min conclude that $\Gamma_{\text{exp}}^{\text{RMC}}$ reported in Ref. is extremely difficult to understand from the $\chi^\text{PT}$ point of view. In this situation it seems important to me to examine the contribution of the $\Delta$ particle, which is still missing in the existing $\chi^\text{PT}$ calculations for RMC. It is also to be mentioned that Fearing and his collaborators are now undertaking a most systematic $\chi^\text{PT}$ evaluation of RMC.

**Acknowledgments**

It is my great pleasure to participate in this Workshop organized to celebrate Dr. Mannque Rho’s 60th birthday. I wish to take this opportunity to express my deep gratitude to Mannque for his warm friendship over more than a quarter of a century. This work is supported in part by the National Science Foundation, Grant No. PHYS-9602000.
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