Cloud Type Interpretation of Statistical Properties of Human Response Delay in Pendulum Balancing

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Abstract
We present the results of our experiments on studying the probabilistic properties of human response delay in balancing virtual pendulum (stick) with over-damped dynamics. The overdamping eliminates the effects of inertia and, thereby, reduces the dimensionality of the system under control. Two types simulators were employed for studying human response in the stick balancing. One of them hides the stick when it is located in some neighborhood of the upright position, the other just makes the stick unaccessible for subject’s control. It enabled us to measure directly the delay time as the time lag between the moment when the pendulum becomes visible or accessible and the moment when a subject starts to move the mouse. It is demonstrated that the response delay time is characterized by a wide distribution sensitive to the particular details of stick balancing process and its possible correlations in the sequence of actions are ignorable. Besides, in experiments with the second simulator the subject’s anticipation is shown to play a significant role in human control. In particular, the formal delay time can take negative values. It poses a question about the applicability of standard formalism of delayed differential equations to describing human intermittent control.

1 Introduction
During the last decades there have been found much evidence for a novel paradigm of human actions in balancing various systems—human intermittent control (for a review see, e.g., [1, 2, 3, 4, 5]). This paradigm implies that human control is discontinuous, repeatedly switching on and off instead of being always active throughout the process. Thereby such human actions can be conceived of as a sequences of alternate fragments of active and passive behavior, with the active phase admitting the interpretation as open-loop control fragments, at least approximately. According to the current state of the art, human intermittent control being rather efficient on its own is considered to be a natural consequence of human physiology [2].

Among the models proposed for human control intermittency we note the interplay between noise and delays in sensorimotor system [6], the clock-driven (“act-and-wait”) [7, 8, 9] and the event-driven (“drift-and-act”) control. Recently the event-driven models have become widely employed [10, 1, 11, 4]. They are built up on the fact that humans cannot detect small deviations of the controlled system from a desired state and for this reason are not able to control the system dynamics in its close proximity.

Within the modern approach turning to the concept of event-driven control the models of threshold control activation take the leading position. They suppose that the control is switched off as long as the deviation remains below a certain threshold value. Whenever the deviation exceeds the threshold, the control is switched on so that the system is driven back to the desired state. These models allowing for human reaction with time delay accept that a subject respond not to the current system state but the state the system occupied at the previous moment of time \( t = t - \tau \) related to the current time \( t \) by some shift \( \tau \).

To illustrate the key aspects of the this type description we outline, as a characteristic example, a rather simple model for the balancing of inverted pendulum. Within this model the pendulum two-dimensional dynamics is governed by the following equation written for the angle \( \theta(t) \) between the pendulum orientation at the current moment of time and the upright position (for a review see, e.g., [4, 12, 13] and references therein)

\[
\frac{d^2\theta}{dt^2} + k \frac{d\theta}{dt} - \omega^2 \theta = R \left[ \theta, \frac{d\theta}{dt}, \frac{d^2\theta}{dt^2}, \xi(t) \right]_{t-\tau}.
\]  

(1)

The left-hand side of this equation represents the pendulum motion affected by the pendulum inertia, the effect of gravity, and viscous friction, maybe, at the pivot point. Its right-hand side describes subject’s reaction generally depending on the angle \( \theta \), the angular velocity \( d\theta/dt \) and acceleration \( d^2\theta/dt^2 \) of pendulum taken with some time delay \( \tau \). The right-hand side of equation (1) also contains white noise \( \xi(t) \) allowing for random factors in human reaction including also their dependence on the system state in terms of multiplicative noise [6].

The bounded capacity of human perception making the human reaction to small pendulum deviation from the upright position rather problematic is usually taken into account in the concept of reaction threshold \( \theta_c \). In
mathematical terms it is reduced to the right hand side of equation (1) changing step-wise when the angle $\theta$ exceeds the threshold $\theta_c$:

$$ R \left[ \theta, \frac{d\theta}{dt}, \frac{d^2\theta}{dt^2} \right] = \begin{cases} 0, & \text{if } |\theta| < \theta_c, \\ f \left[ \theta, \frac{d\theta}{dt}, \frac{d^2\theta}{dt^2} \right], & \text{if } |\theta| > \theta_c, \end{cases} $$

(2)

where $f(\ldots)$ is some smooth function. It should be noted that such approaches are based on the assumption that human reaction is characterized by one delay time or, at least, its variations are not too significant.

An approach based on governing equations similar to (1) has been developed for describing human reaction in balancing unstable objects with the prediction of their motion, e.g., [1, 14]. The models with prediction assume the information about the system state at time moments $t' < t - \tau$ to be known and uses the internal models to “calculate” system states in the time interval $(t - \tau, t)$. It enables a subject to respond to the pendulum deviation from the upright position in the appropriate way compensating, at least partly, the reaction delay. Recently, a novel concept of noise-driven control activation has been proposed as a more advanced alternative to the conventional threshold-driven activation [15]. It argues that the control activation in humans may be not threshold-driven, but instead intrinsically stochastic, noise-driven, and stems from stochastic interplay between operators need to keep the controlled system near the goal state, on the one hand, and the tendency to postpone interrupting the system dynamics, on the other hand. To justify the noise-driven activation concept a novel experimental paradigm: balancing an overdamped inverted pendulum was employed [15]. The overdamping eliminates the effects of inertia and, therefore, reduces the dimensionality of the system. Within the noise-activation model the human reaction delay is taken into account in an implicit way via renormalization of some system parameters.

As far as the direct measurements of the delay time of human reaction is concerned, the proposed experimental set-up can be also employed. In particular, we have observed that the response delay time characterizing the control activation is a random variable changing in a wide interval about 100–600 ms [16]. It has enabled us to pose a question about, at least, the universality of approaches employing models similar to (1).

The purpose of the present research is to analyze in more detail the experimental data collected previously [16] and to report the preliminary results obtained during our new experiments on balancing inverted pendulum aimed at studying anticipation effects in human reaction. A formalism able to take into account such effects is also discussed.

2 Experimental Setup

As previously [15], the paradigm of balancing an overdamped inverted pendulum was employed. It was implemented via balancing a virtual stick whose dynamics is affected by computer mouse movement (Fig. 1). Namely, the stick dynamics is simulated by numerically solving the ordinary differential equation

$$ \tau_\theta \frac{d\theta}{dt} = \sin \theta - \frac{\tau_\theta}{l} \eta \vartheta \cos \theta, $$

(3)

where $\theta$ is the angular deviation of the stick from the vertical position and $\vartheta$ is the cart velocity. The parameter $\tau_\theta$ determines the timescale of the stick motion: the higher the value of $\tau_\theta$, the faster the stick falls in the absence of human control. The sticks length $l$ determines the characteristic magnitude of the cart displacements required for keeping the stick upright. The cart position was controlled by the operator via a computer mouse. The cofactor $\eta$ determines the feasibility of stick motion correction via the mouse movements; if $\eta = 1$ the stick is accessible and it is not accessible when $\eta = 0$. New features of this balancing simulator are in that (Fig. 1)

a) the stick becomes invisible within the sector $|\theta| < \theta_c$;

b) the stick remains visible but becomes unaccessible within the sector $|\theta| < \theta_c$ (via setting $\eta = 0$) after the current active phase of its correction is finished;

The critical value of the stick angle $\theta_c = 5^\circ$ was used in the experiments within the visible-invisible stick setup (Fig. 1 a) and $\theta_c = 10^\circ$ in the experiments within the accessible-inaccessible setup (Fig. 1 b).

Using the simulator of the type a (Fig. 1) the experiments were implemented as a sequence of stick balancing trials. Within one trial the stick is initially placed by the computer inside the sector of invisibility and
its further motion is controlled by a subject during the next 5 s or is terminated earlier if the stick has fallen. After the following 3 s designated for subject’s rest the system position is restored, the cart is put in the middle of the screen and the stick is automatically returned into the sector of invisibility. Then, the subject again continues the balancing process for the next trial. The total number of trials was about 300 for each subject. Two versions of these experiments were conducted. Within the first one, to be referred to as the “Random” set-up, at the beginning of each trial the stick is placed at an arbitrary chosen position inside the sector of invisibility, whereas within the second “One-side” set-up its initial position can be chosen only from one side of the upward position (Fig. 1). Within the “Random” set-up subjects cannot predict the side on which the stick will appear, whereas for the “One-side” set-up it is determined.

Within one trial the delay time in the human response is measured as the time lag between the moment when the stick becomes visible for the first time and the moment when a subject starts to move the mouse, i.e., when the mouse speed exceeds some threshold introduced to cut-off noise effects. The characteristic time velocity pattern of the mouse motion found in the conducted experiments is shown in Fig. 2.

Using the simulator of the type b (Fig. 1) the experiments were conducted following the previous scenario [15] of continuous balancing of the stick with its possible falls. The change of the feasibility of stick motion control via the mouse movements is implemented as follows. Initially the stick is placed inside the sector of stick inaccessibility and the value $\eta$ is set equal to $\eta = 0$. Its further changes are governed by the following rules of system update $n \rightarrow n + 1$ implemented at each step of recording the stick dynamics:

$$\text{if } |\theta_n| \geq \theta_c \text{ then } \eta_{n+1} = 1,$$
$$\text{else if } \eta_n = 0 \text{ then } \eta_{n+1} = 0,$$
$$\text{else if } |\theta_n| < \theta_c \text{ then } \eta_{n+1} = 0,$$
$$\text{else } \eta_{n+1} = 1,$$
end if

where the cart velocity threshold $\theta_c$ is specified based on the velocity time patterns as a value enabling us to separate noise effects from the subject’s intentional actions. These rules lead to the initiation of the stick motion control when the stick goes outside the sector of inaccessibility. Then this control is halted when, first, the stick is returned by a subject to some proximity to the upward position and, second, the corresponding fragment of active phase is also ended and the subject’s current behavior matches the passive phase of human intermittent control.

Figure 3 illustrates this dynamics. When during its motion $\theta(t)$ the stick crosses the boundary of the inaccessibility sector $\theta = \pm \theta_c$ the variable $\eta$ changes its value from 0 to 1, which makes the stick motion control possible (Fig. 3a, b). Then a subject becomes able to return the stick inside the sector $|\theta| < \theta_c$ by moving the mouse. When this task is implemented and the stick has gotten some relatively small neighborhood of its upright position the subject finishes, the subject finishes the mouse movement. As a result, when mouse speed drops below the threshold $\vartheta_c$ the variable changes
Fig. 4: The characteristic histograms and action sequence patterns of response delay time obtained based on the conducted experiments. Blue lines represent the results of the “Random” set-up, red lines match the “One-side” set-up.
back from 1 to 0 and the subject loses the possibility of affecting the stick motion via the mouse until the stick angle $\theta$ attains the critical value $\theta_c$ before the subject finishes the mouse movement the stick control remain active and subject is able to correct the stick position again. Figure 3c also illustrates how the subject’s delay time was measured in the conducted experiments.

Totally eight right-handed healthy male students participated in the experiments. The set of 5-minute exercise sessions enabled them to get familiar with the simulator manipulations before the main experiments. As far as the experiments with the accessible-inaccessible stick simulator are concerned, below we will present pilot results obtained by one subject in experiments of total duration about 30 minutes and the number of records about 700. The velocity cutoff was set equal to $\vartheta = 0.01$; for comparison, the characteristic amplitude of velocity variation was about 0.5. The time parameter $\tau_\theta = 0.7$ s was used, which corresponds to the intermediate difficulty of stick balancing under the given conditions.

3 Results and Discussion

The results obtained using the simulator shown in Fig. 1a are represent by the following plots.

Figure 4 demonstrates the characteristic histograms (plots I, II, VII, VIII) and patterns of the reaction delay time in a sequence of balancing actions (plots III–VI, IX–XII); Figure 5 exhibits the corresponding correlation functions of the reaction delay time. In Fig. 4 the data collected within the “One-side”/“Random” set-up are shown in red/blue.

Based on the obtained results we can draw the following conclusions.

(i) The human response delay time recorded in these experiments is practically a random variable distributed inside a wide interval. The lower boundary $\tau_l$ of this interval can be less than 50 ms (within the obtained accuracy); its upper boundary $\tau_u$ is about 500–600 ms. This estimate of $\tau_l$ is rather close to the limit response time determined by human physiology (for a resent review see, e.g., [17] and the following discussion), whereas the found value of $\tau_u$ is typical for human response delay during complex balancing tasks (see, e.g., [18]).

(ii) For different subjects the histograms can exhibit a strong as well as weak dependence on the predictability of the stick motion. Namely, the side on which the stick appears for the first time after the initial system position having been restored. Two subjects, whose actions are illustrated by the diagram shown in the lower right corner in Fig. 4, demonstrated a strong dependence on this factor. Their histograms in the case of the “One-side” set-up are remarkably wider than in the case of the “Random” set-up due to the considerable contribution of the region of small values less than 200–300 ms. This region of rather short response delay may be related to the automatic mechanism of human reaction [18].

(iii) The patterns (Fig. 4, III–VI, IX–XII) and the corresponding correlation functions (Fig. 5, I–VI) demonstrate the weak correlations (up to their absence) in the response delay time in a stream of subject actions in pendulum balancing. The found correlations for some subjects admits the interpretation, e.g., as certain variations in subject’s attention during the balancing task.

During the last decades there have being ongoing de-
bates about the existence of two types of cognitive processes that are fast, automatic, and unconscious and those that are slow, deliberative, and conscious. Moreover they may be assumed to occur from two architecturally (and evolutionarily) distinct cognitive systems. Correspondingly one of these systems must be reflexive, automatic, fast, affective, associative, and primitive, and the second one should be deliberative, controlled, slow, cognitive, propositional, and more uniquely human. Besides, there are accounts assuming the dual-processes to arise parallel and compete with each other, however, there are also arguments against the dual system of decision-making; for a review and discussion of the evidence supporting both sides of the debate a reader may be referred to Refs. [19, 20]. The found dependence of the constructed histograms on subject’s individuality argue for the fact that the two cognitive systems are comparable in influence on response delay in human intermittent control. Therefore the response delay must be actually a certain rather complex function of the human state affected by the current situation. It is in a qualitative agreement with the statement that in complex balancing tasks human response may indicate flexible, variable delay and intentional mechanisms associated with central processing [18].

As far as the experiments using the stick balancing simulator shown in Fig. 1b are concerned, Figure 6 shows the preliminary results for the histogram of the delay response time calculated with respect to the time moment when this stick crosses the boundary of the stick accessibility and mouse movements by a subject affects the stick dynamics. Because of subject’s anticipation the response delay can take negative values which is clear visible in Fig. 6.

It should be emphasized that the subject’s skill and individuality can be responsible for that some of the reported properties will not be found in the actions of a given individual. However, this feature does not mitigate the obtained results because the actual purpose of the present research was to demonstrate that human response delay does exhibit a complex behavior not matching the fixed delay time paradigm. No assumption about the universality in subjects’ action was made and used.

4 Summary to the experimental results

The demonstrated complex properties of human response delay argue that the delay time, at least in human intermittent control, has to be treated as a random variable which is simultaneously characterized by the following features:

- The response delay time characterizing the control activation is a random variable changing in a wide interval from lower values less than 100 ms up to upper values about 600 ms. It should be noted that the lower and upper values are typical delay times of human response to predictable and unpredictable events, see respectively [2].

- The values of delay time practically are not correlated in the sequence of control activation events so may be treated as a Markov process whose mean values admit time variations reflected changes in the local “strategy” of subject’s actions.

- The found distributions of response delay time for several subjects contain at least two different fragments, which argues for the hypothesis about the existence of two mental systems affecting together human decision-making.

- The individual difference in the response delay time distribution found for several subjects in experiments with predictable and unpredictable stick appearance demonstrates the dependence of the human reaction delay on the system dynamics and different mechanics of its mental analysis.

- The subject anticipation of the stick motion can make the cumulative delay time equal to zero or even negative (cf., e.g., [17]).

The found results fit well the observations of human reaction delay in other systems reported in a number of publications. However in the present work we have demonstrated that human behavior can exhibit all of them dealing within one system. It makes the generalization of the standard model of human reaction based on delayed differential equations rather doubtful and raises the question about the necessity of changing the paradigm of theoretical description of such human actions.
Towards a Cloud Type Description of Human Response

The gist of our approach to describing human response, we put forward for discussion, is based on the concept of specious present developed in psychology and philosophy of mind. It assumes that humans cannot discriminate time moments separated by rather short gaps and perceive them as simultaneous. It concerns not only visual recognition but also mental evaluation of events and their contribution to observed phenomena including spatial and temporal dimensions.

To take into account the specious present in human analysis of the dynamics of such systems we have proposed the concept of cloud states \[ \psi \] \[ (\theta,t) \] \[ \{ \psi_x(\theta,t), \psi_v(\theta,t) \} \]—the cloud state—distributed in space and comprising two components representing the stick spatial position and its motion. In this case in order to compare two possible-cloud like objects with respect to their equivalence we have to turn to the formalism of quantum physics and measure it as

\[ \langle \psi_1 | \psi_2 \rangle \]

naturally the identity

\[ \langle \psi | \psi \rangle \equiv 1 \]

has to hold always. In there terms the subject actions in the control activation admit the description in terms of the value \[ \langle \psi^s | \psi^b \rangle \] quantifying the proximity of the moving stick \[ \psi^s(\theta,t) \] to the desired position \[ \psi^b(\theta) \]. To describe the cloud state dynamics of the pendulum under control we propose a certain governing equation that can be written in the general form as

\[ i \partial_t \psi^s = \mathcal{H}\{\psi^s\} \psi^s + V_s(\theta) \psi^s. \]

Here the nonlinear Hamiltonian depending on \[ \psi^s \] describes the specious present in human cognition and evaluation of stick motion, whereas the effective potential \[ V_s(\theta) \] represents the visual perception of the stick on its own.

Here the nonlinear Hamiltonian \[ \mathcal{H}\{\psi^s\} \] depending on \[ \psi^s \] describes the effects of specious (complex) present and fuzzy points in evaluating the passed events and anticipated ones, whereas the effective potential \[ V_s(\theta) \] represents the spatial and temporal limitations of physiological mechanisms in human perception via the five senses. In other words the Hamiltonian \[ \mathcal{H}\{\psi^s\} \] is caused by the mental activity, whereas the potential \[ V_s(\theta) \] is due to physiological processes in the human body.

The construction of this approach could allow us to make the next step in modeling intentional human behavior.

References

[1] P. Gawthrop, I. Loram, M. Lakie, and H. Gollee, “Intermittent control: a computational theory of human control,” Biological Cybernetics, vol. 104, no. 1-2, pp. 31–51, 2011.
[2] I. Loram, H. Gollee, M. Lakie, and P. Gawthrop, “Human control of an inverted pendulum: is continuous control necessary? Is intermittent control effective? Is intermittent control physiological?,” The Journal of Physiology, vol. 589, no. 2, pp. 307–324, 2011.
[3] R. Balasubramaniam, “On the Control of Unstable Objects: The Dynamics of Human Stick Balancing,” in Progress in Motor Control: Neural, Computational and Dynamic Approaches (M. Richardson, M. Riley, and K. Shockley, eds.), pp. 149–168, New York: Springer Science + Business Media, 2013.
[4] J. G. Milton, “Intermittent Motor Control: The “drift-and-act” Hypothesis,” in Progress in Motor Control: Neural, Computational and Dynamic Approaches (M. J. Richardson, M. A. Riley, and K. Shockley, eds.), pp. 169–193, New York: Springer Science+Business Media, 2013.
[5] Y. Asai, S. Tateyama, and T. Nomura, “Learning an Intermittent Control Strategy for Postural Balancing Using an EMG-Based Human-Computer Interface,” PLoS ONE, vol. 8, p. e62956 (19 pages), mar 2013.
[6] J. L. Cabrera and J. G. Milton, “On-Off Intermittency in a Human Balancing Task,” Physical Review Letters, vol. 89, no. 15, pp. 158702(1–4), 2002.
[7] K. Craik, “Theory of the human operator in control systems. I. The operator as an engineering sys-
tem,” *British Journal of Psychology. General Section*, vol. 38, no. 2, pp. 56–61, 1947.

[8] I. Loram and M. Lakie, “Human balancing of an inverted pendulum: position control by small, ballistic-like, throw and catch movements,” *The Journal of Physiology*, vol. 540, no. 3, pp. 1111–1124, 2002.

[9] T. Insperger, “Stick balancing with reflex delay in case of parametric forcing,” *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 4, pp. 2160–2168, 2011.

[10] J. Milton, J. Townsend, M. King, and T. Ohira, “Balancing with positive feedback: the case for discontinuous control,” *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 367, no. 1891, pp. 1181–1193, 2009.

[11] P. Kowalczyk, P. Glendinning, M. Brown, G. Medrano-Cerda, H. Dallali, and J. Shapiro, “Modelling human balance using switched systems with linear feedback control,” *Journal of The Royal Society Interface*, vol. 9, no. 67, pp. 234–245, 2012.

[12] J. Milton, T. Insperger, and G. Stepan, “Human Balance Control: Dead Zones, Intermittency, and Micro-chaos,” in *Mathematical Approaches to Biological Systems: Networks, Oscillations, and Collective Motions* (T. Ohira and T. Uzawa, eds.), pp. 1–28, Tokyo: Springer Japan, 2015.

[13] T. Insperger, J. Milton, and G. Stepan, “Semidiscretization for Time-Delayed Neural Balance Control,” *SIAM Journal on Applied Dynamical Systems*, vol. 14, no. 3, pp. 1258–1277, 2015.

[14] T. Insperger and J. Milton, “Sensory uncertainty and stick balancing at the fingertip,” *Biological Cybernetics*, vol. 108, no. 1, pp. 85–101, 2014.

[15] A. Zgonnikov, I. Lubashevsky, S. Kanemoto, T. Miyazawa, and T. Suzuki, “To react or not to react? Intrinsic stochasticity of human control in virtual stick balancing,” *Journal of The Royal Society Interface*, vol. 11, p. 20140636, 2014.

[16] T. Suzuki, T. Miyazawa, S. Kanemoto, and I. Lubashevsky, “Statistical Properties of Human Response Delay: Analysis of Virtual Stick Balancing Experiments,” in *Proceedings of the 46th ISCIE International Symposium on Stochastic Systems Theory and Its Applications, Kyoto, Nov. 1-2, 2014*, (Kyoto), pp. 236–241, Institute of Systems, Control and Information Engineers (ISCIE), 2015.

[17] R. Nijhawan, “Visual prediction: Psychophysics and neurophysiology of compensation for time delays,” *Behavioral and Brain Sciences*, vol. 31, no. 02, pp. 179–198, 2008.

[18] I. Loram, M. Lakie, and P. Gawthrop, “Visual control of stable and unstable loads: what is the feedback delay and extent of linear time-invariant control?,” *The Journal of Physiology*, vol. 587, no. 6, pp. 1343–1365, 2009.

[19] J. Evans, “Dual-processing accounts of reasoning, judgment, and social cognition,” *Annual Review of Psychology*, vol. 59, pp. 255–278, 2008.

[20] A. Rustichini, “Dual or unitary system? Two alternative models of decision making,” *Cognitive, Affective, & Behavioral Neuroscience*, vol. 8, no. 4, pp. 355–362, 2008.

[21] I. Lubashevsky, *Physics of the Human Mind* (Springer, Cham, 2017).