Simulation-aided walking pattern generation for a humanoid robotic platform

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Abstract. Bipedal locomotion is one of the main research topics in humanoid robotics due to the complexity of generating dynamically stable gait patterns in robots that are generally very unstable. Good walking performance requires diverse and complex control strategies based on a mathematical model of the robot. Here, a methodology to easily generate stable bipedal locomotion patterns in a humanoid robotic platform is presented. Using a simulation environment and a simple programming method, the robot gait, stability and general behaviour are first simulated and then transferred to the actual machine. Three gait patterns are developed according to the biomechanics of human walking and validated using a humanoid robot platform. The robot exhibits good performance in terms of stability, displacement and velocity. The proposed methodology allows the study of locomotion in biped robots and can serve as a basis for developing controls in closed-loop schemes.

1. Introduction

Humanoid robotics has been under research for more than 40 years, since the pioneering study of Kato and Vukobratovic [1, 2]. Owing to the high complexity of such robots and the dynamics involved in performing tasks such as locomotion over uneven terrain, a preliminary simulation of the robot behavior is mandatory [3, 4]; tasks such as path planning, robot control, and teleoperation, are commonly developed and tested via simulation prior to their execution in actual machines [5, 7].

Similarly, machine learning algorithms and methods, such as reinforcement learning, genetic algorithms and neuroevolution, require several tests of the agent to achieve the expected behavior, and this process can be facilitated with an accurate robot simulation [8, 9]. Besides, the online training is susceptible to be extrapolated to the real robot [10, 12]. A promising field for this methodology is represented by legged robots and their applications to biped robots is still open to new developments due to the complexity of generating a control for locomotion on highly unstable machines generally operating in non-ideal environments.
The abovementioned applications have been allowed by the improvement in physical engines implemented in the simulation software. Nowadays, Gazebo, virtual robot experimentation platform (V-REP), and Webots are the most common simulators used in robotics [13, 14], facilitating the assessment of the kinematics and dynamics of a robot even without its mathematical model.

This paper presents a methodology to develop a stable bipedal locomotion with an open-loop control scheme in a humanoid robotic platform (HRP), by using a physical simulation and a simple programming method, which can allow an easy and fast extrapolation of gait patterns to be successively implemented in the hardware without the need for a robot mathematical model.

The primary purpose of the simulation is to assess the machine behavior while performing different tasks, especially bipedal locomotion, over a flat and known surface, implemented without any disturbance or noise; the robot is configured according to the real counterpart characteristics and the gait patterns are developed based on the biomechanics of human walking.

Two gait patterns are developed by simulation and their execution in HRP, which also serves as the basis for a third pattern that significantly improves the robot displacement, is described. The proposed methodology allows studying different locomotion patterns that can form the basis for developing closed-loop controls or used in control schemes with artificial intelligence.

The rest of this paper is structured as follows, it is described the PRH project; then, the methodology proposed to configure and control the robot in a simulation environment and the hardware characteristics, is presented; next, the locomotion tests and both the simulated and experimental results, are reported; the last two sections present the discussion and conclusions of the study.

1.1 HRP Project

The HRP project is currently under development at the Semillero de Investigación en Automática y Diseño A+D, of the Universidad Pontificia Bolivariana (Colombia), and focuses on developing a low-cost humanoid robot for research and education. The platform has 56 degrees of freedom (DOFs), making it modular, easy to assemble and 3D printed. Its anthropometric design is one of its main differentiating features, which includes the same proportions, shape and joint ranges as those of an average human.

This robot is being developed primarily to study the human bipedal locomotion and implement stable gait patterns on a regular flat floor with an open-loop control scheme. For this purpose, it has 12 DOF legs providing the same mobility as that of a person. The robot is 90 cm tall and weighs 4.7 kg, resulting in a lightweight and robust platform [15]. It is sought to develop closed-loop controls and explore the machine learning application in locomotion. The robot is also designed for research on human/robot interfaces, object manipulation, robot interactions with the environment, basic limb control, etc.; its kinematic structure and design are shown in Figure1.
2. Methodology

2.1. Simulation environment
The 3D simulation is performed with the V-REP program, which presents a friendly environment for the development of rigid-body robots and allows the use of four physical tools (Bullet physics library, Open Dynamics Engine, Vortex Dynamics Engine and Newton Dynamics Engine) and the creation of controls in different programming languages, such as C/C++, Java, and Matlab [16, 17]. Moreover, in its distributed architecture, each object of the simulation can be individually controlled by a plug-in, an embedded script, or a remote application programming interface (API) client [18].

2.2. Robot configuration on V-REP
Each segment of the robot, designed on a CAD program, is exported into the simulation environment as a mesh piece and transformed into a convex hull configuration, which preserves its external shape and contact points without the detailed mechatronic design, simplifying the interactions between the parts and speeding-up the simulation. The pieces are configured with parameters defining the dynamics in the actual robot, such as mass, center of mass (CoM) position, and moments of inertia; collision detection is also included.

The kinematic chain and general structure of the robot are configured on a hierarchical tree. Revolute joints are used, whose parameters, including torque, speed, and proportional–integral–derivative control properties, are set according to the actuator characteristics. The robot in the simulation environment is shown in Figure 2.

2.3. Control Method
The robot is controlled via a Matlab script, which works as a remote API, communicates with the V-REP environment through a child script related to the HRP hierarchical tree, and controls the execution time of each simulation step.

The main task of the Matlab program is to deliver (write) the angles for each actuator; similarly, the actual joint angles must be read in the simulation to determine the possible deviations due to the robot dynamics and interaction with the environment.

The commands sent to the simulation are manually generated and indicate the set of angles to be executed by the robot for each time period. Then, these data are interpolated in the program with the shape-preserving piecewise cubic Hermite interpolating polynomial method to execute the commands every 50 ms.
The four physics tools provided by V-REP have been successively tested with the robot (Figure 3), and the Bullet Physics Library, which presents a stable and reliable contact between the robot feet and floor, is selected. The other three show similar behaviors, but the contact with the ground exhibits slight vibrations and movements, destabilizing the machine.

2.4. Joints torques

Servomotors are chosen as the robot actuators and connected to the joints directly or through a reduction system. To determine the static torque required in the joints for a stable locomotion, a kinematic analysis is carried based on the robot position; for this analysis, HRP is supported on one foot, whose ankle is considered the origin of the coordinate system and anchored to the ground.
As regards the frontal plane, the following five joints are analyzed: the roll movement of the supporting foot ankle, two joints of the legs with the hip, and two joints in the shoulders. For the sagittal plane, there are seven joints of the pitch movement corresponding to the one supporting the foot ankle, two knees, two legs, and two arms.

The total torque is calculated as the sum of individual torques, derived from the weight and distance between the origin and the robot nodes,

\[ \tau_T = \sum_{i=0}^{n} \tau_i \]  

\[ \tau_i = F_i l_i (\theta_0, l_0, \ldots, \theta_k, l_k) \cos(\theta_i(\theta_0, l_0, \ldots, \theta_k, l_k)) \]  

where \( \tau_T, \tau_i, F_i, l_i, \theta_i \) are, respectively, the total torque on the joint, the individual torques, the force due to the node weight, and the node distance and angle, which are both functions of the \( \theta_0 \) to \( \theta_k \) and distances \( l_0 \) to \( l_k \). The analysis is performed considering the main segments of the limbs not exceeding a weight of 375 g, for a total robot weight of 4.7 kg.

The projection of the robot COM on the ground is generally found, considering it as a particle system [19], where its extremities are represented by links whose mass is concentrated in a predefined node. The torque generated in the frontal and sagittal planes of the supporting foot ankle joint is determined based on (2). Then, the system CoM [20] is calculated as,

\[ r_{cmi} = \frac{1}{M} \sum_{i=0}^{n} \tau_i \]  

\[ r_{cmi} = \frac{1}{M} \sum_{i=0}^{n} m_i |r_i| \sin(\theta_i) \]  

where \( \tau_i, m_i, \theta_i, r_i \) are the torque, mass, angle and distance, respectively, of each particle and \( M \) the total mass of the system; thus, the COM position in \( i \) and \( j \) is obtained. Based on the previous equations, a program is developed in Matlab to find the maximum torque in each joint and calculate the robot COM; in particular, two functions are elaborated for the sagittal and frontal planes, where robot characteristics such as its limbs weight and length, node position and general geometry are configured. The inputs are the desired joint angles.

Matlab Simulink is used to improve the user interface; specifically, a robot representation is generated by a SimMechanics module and sliders allow its position configuration based on the joint angles. The maximum torque generated in each joint is obtained from these simulations according to a set of fundamental positions in the locomotion process, mainly in the single-support stages.

3. Results

The results are divided into two main sections that present, respectively, the performance of locomotion patterns generated in the simulation environment and their validation in the real HRP.

3.1. Locomotion simulation tests

In a first method, the maximum and minimum tilts in the frontal and sagittal planes executed by the robot without losing balance, achieved through the gradual increase in the ankle and hip joint angles while keeping the trunk vertical, are determined. A second method with a graphical estimation is also elaborated based on the COM position and its projection in the floor.

The results obtained with the two methods are presented in Table 1; although the values are similar in most cases, the simulated ones tend to be smaller due to the dynamics involved when the robot is moving to the commanded angles. These results are useful to determine the basic limits of stability in some locomotion phases, but the dynamics involved in the walking cycle may further limit these
values. The similarly determined angle, giving the COM projection at the center of the supporting foot polygon, is $7.74^\circ$ for the frontal plane and $3.19^\circ$ for the sagittal one.

### Table I

Minimum and maximum angles obtained via the 2d method and the virtual robot experimentation platform (v-rep)

| Actuator                              | 2D Method ($^\circ$) | V-REP ($^\circ$) |
|---------------------------------------|----------------------|------------------|
| Sagittal plane minimum                | 4.19                 | 4.50             |
| Sagittal plane maximum                | 10.96                | 7.20             |
| Frontal plane minimum                 | -2.47                | -1.50            |
| Frontal plane maximum                 | 8.89                 | 8.88             |

For the locomotion tests, two patterns are generated based on the biomechanics of human gait, including the typical joint angles. Although the robot is anthropometrically designed, these angles cannot be extrapolated directly, but must be first adapted in a 2D environment and then edited in V-REP until achieving a dynamically stable gait. In these tests, only 10 of the 56 DOFs, corresponding to the pitch/roll rotation of the ankles, knees, and hip, are used.

The first gait pattern searches a very human-like locomotion, implying mainly knee-stretched, heel-contact and toe-off motion, i.e., the supporting leg is maintained with low knee flexion and the toes are used as additional support in the pre-swing stage. Figure 4 shows from the support phase, until terminal stance and pre-swing, also showing the COM projection remaining within the feet support.

Multiple tests are performed for adjusting the different angles according to the results obtained in the 3D simulation until achieving satisfactory and stable locomotion at all times. In Figure 5, the locomotion on V-REP, where 4 steps corresponding to 2 gait cycles are executed for 200 s, is presented. The motion of the toes is illustrated in Figure 6, showing their additional support to the robot at some gait phases, as the pre-swing stage.

The thus-generated pattern achieves a stable locomotion and a very human-like walking. However, slight changes in the angle commands can easily destabilize the machine due to the nature of this gait type; therefore, the sequence is executed slowly to counteract the effects of the overall dynamics. On the other hand, the execution of the initial oscillation (pre-swing), when the supporting foot is
switched and a new phase of single support initiates, implies a very complex series of movements in the joints.

![Sequence of the first gait pattern implemented.](image)

**Figure 5.** Sequence of the first gait pattern implemented.

The second gait pattern aims to reduce the machine instability via a bipedal locomotion performed with the knees flexed and without the additional support of the toes. Such gait maintains a low COM and allows better control of parameters such as the zero-moment point. The commands are also generated from the 2D environment and then edited in the 3D simulation.

![Additional support of the toes in the pre-swing stage.](image)

**Figure 6.** Additional support of the toes in the pre-swing stage.

The two gait cycles are executed successfully on V-REP during 195 s, as shown in Figure 7; the stability increases with a knee flexion of 25°, although the COM height is decreased by only 2.03 %. Moreover, with the second gait pattern, the process of COM transfer from one foot to another in the pre-swing phase is performed easily as compared to the first pattern, because the knee flexion allows a smoother and more stable transition.

The two as-generated gait patterns can be modified to enhance the locomotion, and hence, improve the stability and execution speed. These tests also confirm that the HRP is mechanically functional, since the torque in the joints allows the correct development of the walking movements.
3.2. Locomotion tests with the real hardware

To validate the robot locomotion patterns, the actuation system, electronics, and control protocol are preliminarily tested. The two walking patterns, evaluated in the 3D simulation, are executed to assess the performance of complex movements in the platform. Figure 8 shows the acceleration exhibited by the femur in the hip flexion/extension, which does not exceed 4 m/s², and results in a smooth and continuous movement of the joint.

In the bipedal locomotion process, the inclination movement in the frontal plane, where the robot COM is transferred to the supporting foot, is fundamental to maintain. The simulation gives a maximum inclination of 8.88°, with an optimum value of 7.74°; therefore, in a process similar to that in V-REP, the inclination is gradually incremented while keeping the robot supported by both feet; an optimum value ranging between 7° and 8° is found (Figure 9).
Figure 10 shows the response of FSRs, revealing how the inclination movement transfers the robot COM to one foot, since the total force on it increases and that on the other one decreases. Moreover, during this process, the COP is kept in an appropriate position in the foot receiving the COM, maintaining its balance (Figure 11).

Based on these preliminary results, the bipedal locomotion in HRP is explored without the full support by the walking frame, although this support has to be manually moved to prevent the hindering of the robot free movements by the cables, used as security elements.

First, the locomotion is tested with the first simulated gait pattern by editing only some angles referred to the inclination in the frontal plane. Figure 12 shows the frontal view of a four locomotion-step sequence, which allows an approximately 65 cm displacement in 200 s. The gait is correctly executed with sufficient spacing between each step, fulfilling the planned phases including the additional support provided by the toes. However, between the initial and final oscillation stages of the leg, the feet rise few millimeters from the ground, often causing a foot dragging, and hence, destabilizing HRP.

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The data from the inertial sensors show no significant difference in the extremity acceleration between the supported and unsupported execution, as shown in Figure 13, for the case of the femur segment, suggesting that the actuators work similarly in both conditions, i.e., the locomotion on the ground does not require a greater effort from the joint; nevertheless, a slight increase in the lateral vibration is perceived due to the corresponding gait movement.

Then, the second simulated gait pattern is validated in the robot, and unlike the first one, it achieves a displacement of only 25 cm in 200 s for a total of two gait cycles and deficient stabilization is also observed.

Owing to the limitations exhibited by the two gait patterns, which perform well in the simulation, a new pattern is developed. The process to generate the gait pattern is carried out similarly to the previous ones, based on the simulation and angle commands.

![Figure 11. Center of pressure position in the inclination test between -7° and 7° over the foot boundaries.](image)

![Figure 12. First locomotion pattern implemented on the HRP.](image)
The new locomotion pattern uses a completely upright posture without pre-flexing the knees, which should allow a greater displacement despite the less stability, as well as additional support of the toes. The generated sequence comprises two gait cycles, i.e., a total of four steps in 140 s; it is successfully executed in HRP without the total support of the walking frame, as shown in Figure 14. A more stable gait is observed, with a step length of 20 cm. The transition in the pre-swing and initial swing phases is also significantly improved; a permanent contact with the toes is maintained (Figure 15), allowing the other foot to rise more than 1.5 cm above the ground. The robot covers a distance of 85 cm in the 140 s programmed, which is an improvement of approximately 30% in both displacement and speed compared to the previous patterns.

The new locomotion pattern is tested with different locomotion speeds, showing good performance for a speed up to 2.15 times the original, completing two gait cycles in 65 s. Despite a greater hip movement during these tests at increased speed, the acceleration at the extremities does not significantly change, as shown in Figure 16, for the femur and lower leg segments.

Figure 13. Acceleration in the femur segment during the supported and unsupported execution of the first locomotion pattern.

Figure 14. Locomotion implemented with the new (third) gait pattern.
4. Discussion

Although the locomotion patterns are based on the human gait biomechanics, which gives the robot a human-like walking, they must be adjusted online in the simulation to improve the stability and increase the displacement. Nevertheless, the simulation environment and programming method provide an easy and quick way to edit and test the walking without the need for an explicit mathematical model of the robot. In this way the simulation is easily set up and the locomotion execution takes a few iterations to be accomplished.

The robot anthropometry results in a high that acts like an inverted pendulum, where small differences in the set point can totally destabilize the machine; hence, to reduce the overall robot dynamics, the gait is designed to be slow.

Simulation is a tool for generally testing the machine behavior and the simulated patterns confirm the possibility of a stable walking on HRP; however, it is not guaranteed that the simulated patterns always work correctly on the real robot and effectively achieve a stable locomotion due to the differences between simulation and real hardware and the unstable nature of the robot.

The execution of the locomotion patterns on the real HRP demonstrates an acceptable transfer from the simulation to the real hardware, validating the methodology proposed as easy and quick to implement in the robot. Although, the first two patterns do not achieve perfect walking, they provide, in conjunction with the simulation, a basis to develop the final locomotion pattern on HRP, which exhibits good displacement, stability, and speed performance.
As mentioned before, the walking patterns are developed in an open-loop control scheme and the good stability achieved can provide a fundamental basis for developing closed-loop control algorithms able to correct errors under perturbations. In addition, these generated patterns can further serve as a base to optimize the locomotion through machine learning, and thus, to produce a dynamically stable movement.

The interactions of the upper body movements should be analyzed in future works since it can significantly improve the stability and error correction as well as decrease unwanted accelerations in the hip sway and surge. In the long term, locomotion is expected to be generated by machine learning through neuroevolution or reinforcement learning.

5. Conclusions

A methodology to easily edit and develop bipedal locomotion patterns for a humanoid robotic platform using a physical simulation environment, where they are tested before being transferred to the real hardware, was presented.

Two gait patterns were generated through a simple programming method that facilitated their adjusting and validation in the simulation. Then, these patterns were tested on the real hardware and achieved good performance in the robot gait. Based on the results, a third pattern was developed, providing better performance, in terms of speed and displacement, of around 30% compared to the previous patterns generated, a better stability is also observed.

Although the walking patterns were developed in an open-loop control scheme with preprogrammed sequences executed directly in the simulation, which required a previous configuration of the robot characteristics, no mathematical model of the machine was required.

References

[1] M. Vukobratovic and B. Borovac, “ZERO-MOMENT POINT — THIRTY FIVE YEARS OF ITS LIFE,” International Journal of Humanoid Robotics, vol. 01, no.01, pp. 157–173, mar 2004. [Online]. Available: http://www.worldscientific.com/doi/abs/10.1142/S0219843604000083
[2] M. Sherman and C. Dembia, “DARPA Robotics Challenge”, DARPA, pp. 1–42, 2012. [Online]. Available: https://www.darpa.mil/program/darpa-robotics-challenge
[3] Y. Kakiuchi, K. Kojima, E. Kuroiwa, S. Noda, M. Murooka, I. Kumagai, R. Ueda, F. Sugai, S. Nozawa, K. Okada, M. Inaba, “Development of humanoid robot system for disaster response through team nedo-jsk’s approach to darpa robotics challenge finals”, 2015 IEEE-RAS 15th International Conference on Humanoid Robots (Humanoids). IEEE, 2015. p. 805-810. [Online]. Available: https://ieeexplore.ieee.org/abstract/document/7363446/
[4] P. Oh, K. Sohn, G. Jang, Y. Jun, and B. K. Cho, “Technical Overview of Team DRC-Hubo@UNLV’s Approach to the 2015 DARPA Robotics Challenge Finals”, Journal of Field Robotics, vol. 34, no. 5, pp. 874–896, aug 2017. [Online]. Available: http://doi.wiley.com/10.1002/rob.21686
[5] J. Ennglsberger, A. Werner, C. Ott, B. Henze, M. Roa, G. Garofalo, R. Burger, A. Beyer, O. Eiberger, K. Schmid, and A. Albu-Schaffer, “Overview of the torque-controlled humanoid robot TORO”, 2014 IEEE-RAS International Conference on Humanoid Robots. IEEE, 2014. p. 916-923. [Online]. Available: https://ieeexplore.ieee.org/abstract/document/7041473/
[6] Q. Huang, K. Yoko, S. Kajita, K. Kaneko, H. Aral, N. Koyachi and K. Tanie, “Planning walking patterns for a biped robot”, IEEE Transactions on Robotics and Automation, vol. 17, no. 3, pp. 280–289, 2001. [Online]. Available: https://ieeexplore.ieee.org/abstract/document/938385/
[7] M. Wahlde and J. Pettersson, “A Brief Review of Bipedal Robotics Research,” Proceedings of the 8th UK Mechatronics Forum International Conference (Mechatronics 2002), pp. 480–
488, 2002.

[8] B. F. Allen and P. Faloutsos, “Evolved controllers for simulated locomotion”, in Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), vol. 5884 LNCS, 2009, pp. 219–230. [Online]. Available: http://link.springer.com/10.1007/978-3-642-10347-6-20

[9] J. Clune, B. E. Beckmann, C. Ofria, and R. T. Pennock, “Evolving coordinated quadruped gaits with the hyperNEAT generative encoding,” in 2009 IEEE Congress on Evolutionary Computation, CEC 2009, 2009, pp. 2764–2771.

[10] P. Manoonpong, T. Geng, T. Kulvicius, B. Porr, and F. Wörgöter, “Adaptive, fast walking in a biped robot under neuronal control and learning”, PLoS Computational Biology, vol. 3, no. 7, pp. 1305–1320, 2007. [Online]. Available: http://dx.plos.org/10.1371/journal.pcbi.0030134

[11] R. Tedrake, T. Zhang, and H. Seung, “Learning to walk in 20 minutes”, Proceedings of the Fourteenth Yale Workshop on Adaptive and Learning Systems. 2005. p. 1939-1412. [Online]. Available: http://groups.csail.mit.edu/robotics-center/public papers/Tedrake05.pdf

[12] J. Yosinski, J. Clune, D. Hidalgo, S. Nguyen, J. Zagal, and H. Lipson, “Evolving Robot Gaits in Hardware: the HyperNEAT Generative Encoding Vs. Parameter Optimization,” Proceedings of the European Conference on Artificial Life (ECAL), pp. 1–8, 2011.

[13] M. Freese, S. Singh, F. Ozaki, and N. Matsuhira, “Virtual robot experimentation platform V-REP: A versatile 3D robot simulator,” in Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), vol. 6472 LNAI, 2010, pp. 51–62. [Online]. Available: http://link.springer.com/10.1007/978-3-642-17319-68

[14] N. Koenig and A. Howard, “Design and use paradigms for gazebo, an open-source multi-robot simulator,” in 2004 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), vol. 3, pp. 2149–2154. [Online]. Available: http://ieeexplore.ieee.org/document/1389727/

[15] S. Zapata, I. D. Mora, and G. Suarez, “Mechatronic design of PRH: A NEW 3D printed and anthropometric humanoid robot platform,” in 2017 IEEE 3rd Colombian Conference on Automatic Control, CCAC 2017 - Conference Proceedings, vol. 2018-Janua. IEEE, oct 2018, pp. 1–6. [Online]. Available: http://ieeexplore.ieee.org/document/8276477/

[16] E. Rohmer and S.P. Singh, “V-REP: a Versatile and Scalable Robot Simulation Framework”, in Proc. of The International Conference on Intelligent Robots and Systems (IROS), 2013. [Online]. Available: https://ieeexplore.ieee.org/abstract/document/6696520

[17] Coppelia Robotics Switzerland, “Coppelia Robotics v-rep: Create. Compose. Simulate. Any Robot,” 2018. [Online]. Available: http://www.coppeliarobotics.com/

[18] M. Mendez and S. Kannan, “V-REP & ROS Testbed for Design, Test, and Tuning of a Quadrotor Vision Based Fuzzy Control System for Autonomous Landing,” Proceedings of The International Micro Air Vehicle Conference and Competition 2014, 2014. [Online]. Available: http://publications.uni.lu/handle/10993/19225

[19] J. B. Marion, Dinámica clásica de las partículas y sistemas, 1996.

[20] J. Millard F. Beatty, Principles of Engineering Mechanics Volume 2 Dynamics - The Analysis of Motion, 2006.