Higher Spin Conserved Currents 
in \(Sp(2M)\) Symmetric Spacetime

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Abstract

Infinite set of higher spin conserved charges is found for the \(sp(2M)\) symmetric dynamical systems in \(\frac{1}{2}M(M + 1)\)-dimensional generalized spacetime \(\mathcal{M}_M\). Since the dynamics in \(\mathcal{M}_M\) is equivalent to the conformal dynamics of infinite towers of fields in \(d\)-dimensional Minkowski spacetime with \(d = 3, 4, 6, 10, \ldots\) for \(M = 2, 4, 8, 16, \ldots\), respectively, the constructed currents in \(\mathcal{M}_M\) generate infinite towers of (mostly new) higher spin conformal currents in Minkowski spacetime. The charges have a form of integrals of \(M\)-forms which are bilinear in the field variables and are closed as a consequence of the field equations. Conservation implies independence of a value of charge of a local variation of a \(M\)-dimensional integration surface \(\Sigma \subset \mathcal{M}_M\) analogous to Cauchy surface in the usual spacetime. The scalar conserved charge provides an invariant bilinear form on the space of solutions of the field equations that gives rise to a positive definite norm on the space of quantum states.

1 Introduction

The idea that symplectic superalgebras \(osp(1, 2^n)\) and their subalgebras and contractions are important for understanding dualities and \(M\)-theory is attractive as these algebras contain supergravity algebras in diverse dimensions in a natural way [1, 2, 4, 6, 8, 10, 12, 13, 14]. On the other hand, symplectic superalgebras were argued [5, 6, 11, 12] to play a key role in the theory of massless...
higher spin fields. In particular, Fronsdal emphasized [14] that the set of 4d massless fields of all spins should exhibit $sp(8)$ symmetry and argued that some formulation of their dynamics in the generalized spacetime with matrix coordinates $X^{\alpha\beta} = X^{\beta\alpha}$ ($\alpha, \beta, \ldots = 1, \ldots, 4$) must exist.

Spacetimes with symmetric real matrix coordinates provide a natural realization of the symplectic algebras [19, 15]. Relevant constructions of extended spacetimes were discussed by many authors in different contexts [20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34]. From $M$-theory perspective, relevance of generalized spacetimes with matrix coordinates $X^{\alpha\beta} = X^{\beta\alpha}$ ($\alpha, \beta, \ldots = 1, \ldots, 2p$) is due to observation that once $\alpha, \beta, \ldots$ are interpreted as spacetime spinor indices the matrix coordinates $X^{\alpha\beta}$ provide a set of antisymmetric tensor “central charge coordinates” $x^{n_1\ldots n_k}$ dual to “central charges” $Z^{n_1\ldots n_k}$ and spacetime momenta resulting from the decomposition of the anticommutator of supercharges $\{Q_\alpha, Q_\beta\}$ into irreducible Lorentz tensors

$$\{Q_\alpha, Q_\beta\} = \sum_{k \in S} \gamma_{n_1\ldots n_k}^{\alpha\beta} Z^{n_1\ldots n_k}. \quad (1.1)$$

Here summation is over those values $k \in S$ that totally antisymmetrized products of $\gamma$-matrices $\gamma_{n_1\ldots n_k}^{\alpha\beta}$ are symmetric in the spinor indices $\alpha, \beta$. Since “central charges” $Z^{n_1\ldots n_p}$ characterize branes in superstring theory, a unified treatment of brane dualities requires uniform description of all “central charges”. This is achieved by introducing “central charge coordinates” [20, 24, 26, 27, 28, 29, 30, 31, 32, 34] which together with the usual coordinates are equivalent to the coordinates $X^{\alpha\beta}$. A remarkable observation that supergravity models may result from spontaneous breakdown of symplectic symmetries was made recently in [35, 36] where it was shown that the equations of motion of $11d$ supergravity imply conservation conditions for some $osp(1,64)$ currents while the theory as a whole provides a nonlinear realization of $osp(1,64)$.

The difference between the role of $sp(2p)$ and its superextensions in the $M$-theory setup and in higher spin gauge theory (see [37, 38] for a review and more references on higher spin gauge theories) is that $M$-symmetries $sp(2p)$ are broken (nonlinearly realized) while in the higher spin gauge theory they act linearly and locally on fields as was recently shown in [17]. Higher symplectic symmetries in higher spin gauge theories (e.g., $sp(8)$ is 4d higher spin theory) mix massless fields of all spins. As all massless higher spin fields are gauge fields this implies that a linearly realized higher symplectic symmetry is only possible in an invariant phase exhibiting infinite-dimensional higher spin symmetry that forms an infinite-dimensional extension of the symplectic symmetry. Since higher spin modes in superstring are massive, this explains why higher symplectic symmetries can only show up via nonlinear realization in the low-energy supergravity models.

The idea that higher spin gauge theory is a natural candidate for a most symmetric phase of the theory of fundamental interactions presently identified with superstring theory and $M$-theory provided originally the main motivation for its investigation [39]. A peculiar feature of the higher spin gauge theory is that gauge invariant higher spin interactions require nonzero cosmological constant [40]. This most symmetric phase differs from the phases of superstring with known spectra like in the
flat space \([11]\) or pp wave background \([12]\). The (so far) explicitly solvable phases of superstring therefore require higher spin symmetries to be broken. It is tempting to speculate that \(M\)-theory and superstring theory may result from a higher spin gauge theory via spontaneous breakdown of a higher spin symmetry that contains symplectic symmetry linking together higher spin and lower spin fields. Some ideas on a possible connection between higher spin theory and string theory in the context of \(AdS/CFT\) correspondence were recently discussed in \([13, 14, 15, 17, 16, 17]\). Once a nonlinear symplectic supersymmetry in \(M\)-theory is indeed a manifestation of a higher spin symmetric phase in which it is unbroken, this implies that branes are built of (vev’s of) higher spin gauge fields \([17]\). In other words, higher spin gauge theories are expected to provide a microscopic description of branes.

For geometric realization of symplectic symmetries, dynamics has to be reformulated in terms of the generalized \(\frac{1}{2}M(M + 1)\)-dimensional spacetime \(\mathcal{M}_M\) with real symmetric matrix coordinates \(X^{\alpha\beta} = X^{\beta\alpha}\) \((\alpha, \beta = 1, \ldots M)\) \([19, 13, 30, 17, 18]\), in which infinitesimal \(Sp(2M)\) transformations are realized by the vector fields \([17]\)

\[
P_{\alpha\beta} = -i \frac{\partial}{\partial X^{\alpha\beta}}, \tag{1.2}
\]

\[
L^\alpha_\beta = 2i X^{\beta\gamma} \frac{\partial}{\partial X^{\alpha\gamma}}, \tag{1.3}
\]

\[
K^{\alpha\beta} = -i X^{\alpha\gamma} X^{\beta\eta} \frac{\partial}{\partial X^{\gamma\eta}}. \tag{1.4}
\]

The (nonzero) \(sp(2M)\) commutation relations are

\[
[L^\alpha_\beta , L^\delta_\gamma] = i \left( \delta^\gamma_\delta L^\beta_\alpha - \delta^\beta_\delta L^\gamma_\alpha \right), \tag{1.5}
\]

\[
[L^\alpha_\beta , P_\gamma^\delta] = -i \left( \delta^\gamma_\delta P^\alpha_\beta + \delta^\beta_\delta P^\alpha_\gamma \right), \quad [L^\alpha_\beta , K^{\gamma\delta}] = i \left( \delta^\gamma_\delta K^{\beta\alpha} + \delta^\beta_\delta K^{\gamma\alpha} \right), \tag{1.6}
\]

\[
[P^\alpha_\beta , K^{\gamma\delta}] = \frac{i}{4} \left( \delta^\gamma_\delta L^\alpha_\beta + \delta^\beta_\delta L^\alpha_\gamma + \delta^\gamma_\alpha L^\beta_\delta + \delta^\delta_\alpha L^\beta_\gamma + \delta^\delta_\beta L^\gamma_\alpha \right). \tag{1.7}
\]

Here \(P^\alpha_\beta\) and \(K^{\alpha\beta}\) are generators of the generalized translations and special conformal transformations. The \(gl_M\) algebra spanned by \(L^\alpha_\beta\) decomposes into the central subalgebra associated with the generalized dilatation generator

\[
D = L^\alpha_\alpha \tag{1.8}
\]

and the \(sl_M\) generalized Lorentz generators

\[
l^\alpha_\beta = L^\alpha_\beta - \frac{1}{M} \delta^\alpha_\beta D. \tag{1.9}
\]

\(A priori,\) it is not obvious how to formulate consistent \(sp(2M)\) invariant dynamical equations compatible with the standard principles of quantum field theory such as unitarity and causality. Some proposals were made, e.g, in \([21, 29, 30]\). An
obvious problem is that it is hard to write down an analog of the Klein-Gordon equation free from the ghost problem because each “central charge momentum” $Z_{n_1 \ldots n_k}$ induces both positive an negative contributions to any Lorentz invariant norm.

On the other hand, the “unfolded formulation” of the free conformal higher spin fields in $d = 3$ [15] and $d = 4$ [17] allowed us to derive a form of $sp(2M)$ invariant equations in $\mathcal{M}_M$ [17] equivalent to the usual higher spin equations compatible with unitarity. As a result of this reformulation, massless fields of all integer spins in four dimensions are described by a single scalar field $c(X)$ in $\mathcal{M}_M$. All half-integer spins are described by a single svector field $c_\beta(X)$. (We use the name “svector” (symplectic vector) to distinguish $c_\beta(X)$ from vectors of the usual Lorentz algebra $o(d-1,1)$. Note that svector fields obey the Fermi statistics [18]). The $sp(2M)$ invariant equations of motion found in [17] which encode 4$d$ massless equations for all spins read

$$\left(\frac{\partial^2}{\partial X^{\alpha\beta}\partial X^{\gamma\delta}} - \frac{\partial^2}{\partial X^{\alpha\gamma}\partial X^{\beta\delta}}\right)c(X) = 0$$

(1.10)

for a scalar field $b(X)$ and

$$\frac{\partial}{\partial X^{\alpha\beta}}c_\gamma(X) - \frac{\partial}{\partial X^{\alpha\gamma}}c_\beta(X) = 0$$

(1.11)

for a svector field $c_\beta(X)$. Note that the same equations were argued in [17, 18] to make sense for any even number $M$ of values taken by svector indices $\alpha, \beta = 1, \ldots M$.

For $M = 2$, because antisymmetrization of any two-component indices $\alpha$ and $\beta$ is equivalent to their contraction with the $2 \times 2$ symplectic form $\epsilon^{\alpha\beta}$, (1.10) and (1.11) coincide with the 3$d$ massless Klein-Gordon and Dirac equations, respectively.

Properties of the equations (1.10) and (1.11) were analyzed in detail in [18] where the dynamics in $\mathcal{M}_M$ described by the equations (1.10) and (1.11) was shown to be consistent with the principles of relativistic quantum field theory including unitarity and microcausality. The most important difference as compared to the usual picture is that, because the system of equations (1.10) and (1.11) is overdetermined, true local phenomena occur in a smaller space $\sigma$ called local Cauchy surface in [15] and identified with the space slice of Minkowski spacetime. The dependence along all “time-like” directions turns out to be fixed at once in terms of appropriate initial data. The full set of “initial data” that fixes a solution of the field equations (1.10) and (1.11) is provided by two functions on a $M$-dimensional “local Cauchy bundle” $E$ having local Cauchy surface $\sigma$ as its base manifold. From the point of view of usual Minkowski geometry the fiber space of $E$ parametrizes spin degrees of freedom of the fields living in the Minkowski spacetime $\sigma \times R$. Note that it is appropriate to describe local phenomena in the Minkowski spacetime $\sigma \times R$ in terms of the local Cauchy bundle $E$ rather than in terms of some $M$-dimensional surface in $\mathcal{M}_M$. The difference is that $E$ is a limit of some surface $\sigma \times \tau$ with the size of $\tau$ tending to zero. The resulting limiting description in terms of $E$ becomes local in terms of $\sigma$ [18].

The formulations of $Sp(2M)$ invariant systems in terms of the generalized spacetime $\mathcal{M}_M$ and usual spacetime are equivalent and complementary. The description
in terms of $\mathcal{M}_M$ provides clear geometric origin for the $Sp(2M)$ generalized conformal symmetry. In particular it provides geometric interpretation of the electromagnetic duality transformations as particular generalized Lorentz transformations $[18]$. The description in terms of the Minkowski spacetime admits standard Cauchy problem but makes some of the symmetries not manifest, namely those which do not leave $E$ invariant. The description in terms of different local Cauchy surfaces $\sigma$ and associated local Cauchy bundles $E$ are equivalent being related by some $Sp(2M)$ transform. One can say that although the dynamics is formulated in $\mathcal{M}_M$ in explicitly $sp(2M)$ invariant manner, the generalized spacetime $\mathcal{M}_M$ is visualized by virtue of local phenomena as some $d-1 \leq M$ dimensional space $\sigma$ times a $M-d+1$ dimensional fiber associated with spin degrees of freedom. This picture has striking similarities with the brane picture. To work out a full-scale correspondence it is necessary to develop full nonlinear theory of higher spin gauge fields in $\mathcal{M}_M$.

In this paper we make a modest step in this direction by constructing conserved charges built of the fields $c(X)$ and $c_\alpha(X)$ in $\mathcal{M}_M$. The constructed currents are in the one-to-one correspondence with the set of generalized higher spin conformal symmetries found in $[17]$ which contain $OSp(L,2M)$ symmetries as a subgroup. Due to specificity of the Cauchy problem in $\mathcal{M}_M$, the corresponding integrals of motion have a form of integrals of certain on-mass-shell closed $M-$forms $\Omega$. Being independent of a choice of a $M-$dimensional integration surface they provide “integrals of motion” that characterize a particular solution of the field equations. In view of the results of $[33,34]$ the conservation conditions of the currents associated with the symplectic superalgebras may be related to the equations of motion in $M$-theory in a higher spin Higgs phase.

The rest of the paper is organized as follows. In section 2 a set of integrals of motions is built provided that a generalized stress tensor satisfying appropriate conservation conditions exists. In section 3 the generalized stress tensor is built in terms of bilinears of the dynamical fields in $\mathcal{M}_M$. In section 4 the expressions for the conserved charges are evaluated in terms of Fourier transformed field variables and it is shown that they reproduce the expressions for the generators of higher spin transformations derived previously in $[17]$. Section 5 contains brief conclusions.

2 General Structure of Currents

The fact $[18]$ that dynamical degrees of freedom associated with the equations (1.10) and (1.11) live on a $M$-dimensional subsurface $S \subset \mathcal{M}_M$ or its limiting $M$-dimensional local Cauchy bundle $E$ suggests that integrals of motion associated with these equations have to be built in terms of some $M$-forms $\Omega(\eta)$ which are closed

$$d\Omega(\eta) = 0, \quad d = dX^{\alpha\beta} \frac{\partial}{\partial X^{\alpha\beta}}$$

(2.1)
as a consequence of the field equations (1.10) and (1.11). Here $\eta$ are some parameters associated with different closed $M$-forms and charges

$$Q(\eta) = \int_S \Omega(\eta).$$

(2.2)
The charges $Q(\eta)$ are independent of local variations of $S$ and, therefore, provide a set of integrals of motion associated with a particular solution of the field equations. In other words, the charges $Q$ are conserved. They generate symmetries by Poisson brackets or commutators upon quantization. (For the quantization rules for the fields $c(X)$ and $c_\alpha(X)$ see [15] and section [4]). The modules $\eta$ are then interpreted as the symmetry parameters associated with the generators $Q(\eta)$. Note that addition to $\Omega(\eta)$ an exact form does not affect charges. This ambiguity characterizes possible “improvements”.

Note that the dimension of a Cauchy surface in the $d$-dimensional Minkowski spacetime is $d-1$. The closed forms $\Omega$ associated with integrals of motion are of degree $d-1$. The conserved currents $J$ are vector fields dual to $\Omega$. In the generalized spacetime $\mathcal{M}_M$ it is natural to formulate the problem in terms of closed $M$-forms rather than conserved dual polyvectors of rank $\frac{1}{2}M(M-1)$. In the sequel on-mass-shell closed forms $\Omega$ are called conserved.

We proceed in two steps by analogy with the case of usual conformal higher spin currents considered in [49]. Firstly, we observe that, once there is a totally symmetric multisvector $T_{\alpha_1...\alpha_n}$, $n \geq M$ satisfying certain generalized conservation conditions, the $M$-form

$$\Omega(\eta) = \epsilon_{\gamma_1...\gamma_M} dX^{\gamma_1 \alpha_1} \wedge ... dX^{\gamma_M \alpha_M} \eta_{\beta_1...\beta_t}^{\alpha_{M+1}...\alpha_{M+s}} X^{\alpha_{M+s+1}} ... X^{\alpha_{M+1}} T_{\alpha_1...\alpha_{M+s+t}}$$

(2.3)

is closed. Here $\epsilon_{\gamma_1...\gamma_M}$ is the totally antisymmetric symbol. It is introduced to get rid of the set of totally antisymmetric indices from the parameter $\eta_{\beta_1...\beta_t}^{\alpha_{M+1}...\alpha_{M+s}}$ being an arbitrary totally symmetric $X$-independent multisvector in lower and upper indices. At the second stage we will explicitly construct the generalized stress tensors in terms of bilinears in $c(X)$ and $c_\alpha(X)$.

The generalized conservation condition can be written in the following three equivalent forms

$$\frac{\partial}{\partial X^{\gamma_\eta}} T_{\alpha_\beta_3...\alpha_n} = \frac{\partial}{\partial X^{\gamma_\alpha}} T_{\eta_\beta_3...\alpha_n} - \frac{\partial}{\partial X^{\gamma_\beta_\eta}} T_{\alpha_\gamma_3...\alpha_n} + \frac{\partial}{\partial X^{\gamma_\alpha}} T_{\gamma_\eta_3...\alpha_n} = 0 \rightleftharpoons (2.4)$$

or

$$\lambda^{\alpha_1...\alpha_n,\beta_1\beta_2} \frac{\partial}{\partial X^{\beta_1\beta_2}} T_{\alpha_1...\alpha_n} = 0 \rightleftharpoons (2.5)$$

for any $\lambda^{\alpha_1...\alpha_n,\beta_1\beta_2}$ having the symmetry properties of two-row Young scheme with two cells in the second row (i.e., $\lambda^{\alpha_1...\alpha_n,\beta_1\beta_2}$ is separately symmetric in the indices $\alpha_i$ and $\beta_j$ and symmetrization of any $n+1$ indices gives zero), or

$$\frac{\partial}{\partial X^{\gamma_\eta}} T_{\alpha_1...\alpha_n} = U_{\alpha_1...\alpha_n,\gamma,\eta} + U_{\alpha_1...\alpha_n,\eta,\gamma} \rightleftharpoons (2.6)$$

where $U_{\alpha_1...\alpha_{n+1},\eta}$ is some multisvector totally symmetric in the $n+1$ indices $\alpha$. The conditions (2.4)-(2.6) are equivalent to each other because the derivative $\frac{\partial}{\partial X^{\gamma_\eta}} T_{\alpha_1...\alpha_n}$ decomposes into three irreducible parts associated with two-row Young schemes with

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Each of the conditions (2.4), (2.7) and (2.6) implies that the part described by the Young scheme with two cells in the second row vanishes.

To prove that the $M -$form (2.3) is closed one observes that, by virtue of (2.6), the part of $d\Omega$ due to differentiation of $T$ vanishes because it is proportional to $dX^{\gamma_1 \alpha_1} \wedge \ldots \wedge dX^{\gamma_{M+1} \alpha_{M+1}} U_{\alpha_1 \ldots \alpha_{M+1} \ldots \alpha_{n+1}} \eta$ which expression contains total antisymmetrization over $M + 1$ vector indices $\gamma_i$ taking only $M$ values. The part due to differentiation of the factors of $X^{\alpha \beta}$ gives rise to the product of $M + 1$ differentials $dX^{\alpha \gamma}$, each having one index contracted with the totally symmetric multisvector $T_{\alpha_1 \ldots \alpha_{M+1}}$. It therefore also vanishes because of total antisymmetrization of the rest $M + 1$ vector indices in the anticommuting differentials.

The multisvector $T_{\alpha_1 \ldots \alpha_n}$ is a generalization of the spin 1 current, spin 3/2 supercurrent, spin 2 stress tensor and their higher spin extensions \[50, 49\] in the conformal field theory in Minkowski spacetime. It is notable that the set of the symmetry parameters $\eta_{\beta_1 \ldots \beta_t \alpha_1 \ldots \alpha_s}$ is in the one-to-one correspondence with the set of parameters of the generalized higher spin conformal symmetries found in \[17\] in the form

$$\eta(a, b) = \sum_{n,m=0}^{\infty} \eta_{\beta_1 \ldots \beta_t \alpha_1 \ldots \alpha_s} a_{\alpha_1} \ldots a_{\alpha_s} b^{\beta_1} \ldots b^{\beta_t},$$

where $a_{\alpha}$ and $b^{\beta}$ are auxiliary oscillators

$$[a_{\alpha}, a_{\beta}] = 0, \quad [a_{\alpha}, b^{\beta}] = \delta^{\beta}_{\alpha}, \quad [b^{\alpha}, b^{\beta}] = 0$$

being generating elements of the star product algebra realization of the generalized conformal higher spin symmetry. Note that the expressions for the full set of generalized higher spin conformal currents (2.3) are much simpler than those for the conformal higher spin currents in Minkowski spacetime \[49\].

3 Generalized Stress Tensors

To present expression for $T_{\alpha_1 \ldots \alpha_n}$ in terms of the dynamical fields it is useful to introduce a set of chains of totally symmetric multisvector fields $c^k_{\alpha_1 \ldots \alpha_n}$ of all ranks $n$ which satisfy the conditions

$$\frac{\partial}{\partial X^{\beta_1 \beta_2}} c^k_{\alpha_1 \ldots \alpha_n}(X) = c^k_{\beta_1 \beta_2 \alpha_1 \ldots \alpha_n}(X).$$

The Chan-Paton index $k$ enumerates different chains and can take an arbitrary number of values. Let us set

$$T^{kl}_{\alpha_1 \ldots \alpha_n}(X) = \sum_{m=0}^{n} a((-1)^m, n) \frac{i^m n!}{m!(n - m)!} c^k_{\alpha_1 \ldots \alpha_m}(X)c^l_{\alpha_{m+1} \ldots \alpha_n}(X)$$

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with totally symmetrized indices $\alpha$ on the right hand side and arbitrary normalization coefficients $a((-1)^m, n)$. The key fact is that the multivector $T_{\alpha_1 \cdots \alpha_n}^{kl}$ defined this way satisfies the generalized conservation condition. To see this it is most convenient to use its form (2.5). It follows

$$
\lambda_{\alpha_1 \cdots \alpha_n, \beta_1 \beta_2} \frac{\partial}{\partial X^{\beta_1 \beta_2}} (c_{\alpha_1 \cdots \alpha_m} X^{\alpha_1 \cdots \alpha_n - m} = \lambda_{\alpha_1 \cdots \alpha_m, \gamma_1 \cdots \gamma_n - m} \partial_{\alpha_1 \cdots \alpha_m + 2} c_{\alpha_1 \cdots \alpha_m + 2} \gamma_1 \cdots \gamma_n - m
$$

$$
= \frac{\partial_{\alpha_1 \cdots \alpha_m + 2} \gamma_{\cdots \gamma_n - m}}{(m + 2)(m + 1)} \lambda_{\alpha_1 \cdots \alpha_m + 2 \gamma_1 \cdots \gamma_n - m} \partial_{\alpha_1 \cdots \alpha_m + 2} \gamma_{\cdots \gamma_n - m}
$$

(3.3)

where the property that symmetrization over any $n + 1$ indices in $\lambda_{\alpha_1 \cdots \alpha_n, \beta_1 \beta_2}$ gives zero was used. As a result, because of a sign factor produced by the factor of $i^m$ in (3.2) all terms in (2.5) cancel pairwise.

The set of quantities $c_{\alpha_1 \cdots \alpha_n}^k$ satisfying (3.1) is provided by the higher derivatives of the dynamical scalar and vector fields

$$
c_{\alpha_1 \cdots \alpha_2p} = \frac{\partial}{\partial X^{\alpha_1 \alpha_2}} \frac{\partial}{\partial X^{\alpha_3 \alpha_4}} \cdots \frac{\partial}{\partial X^{\alpha_{2p-1} \alpha_{2p}}} c^k(X),
$$

(3.4)

$$
c_{\alpha_1 \cdots \alpha_{2p+1}} = \frac{\partial}{\partial X^{\alpha_1 \alpha_2}} \frac{\partial}{\partial X^{\alpha_3 \alpha_4}} \cdots \frac{\partial}{\partial X^{\alpha_{2p-1} \alpha_{2p}}} c_{\alpha_{2p+1}}(X).
$$

(3.5)

Such defined quantities $c_{\alpha_1 \cdots \alpha_n}^k$ are totally symmetric in $\alpha_1 \cdots \alpha_n$ as a result of the equations of motion (1.10) and (1.11) which imply that any antisymmetrization of vector indices in higher derivatives of the dynamical fields gives zero (note that every solution of (1.11) satisfies (1.10) (15)).

In fact, the equation (3.1) is just the unfolded form of the equations (1.10) and (1.11) from which they were derived in (17). In terms of the generating function

$$
C^k(b|X) = \sum_{n=0}^{\infty} \frac{1}{n!} c_{\alpha_1 \cdots \alpha_n}^k(X) b^{\alpha_1} \cdots b^{\alpha_n}
$$

(3.6)

the equation (3.1) reads

$$
\frac{\partial}{\partial X^{\beta_1 \beta_2}} C^k(b|X) = \frac{\partial^2}{\partial b^{\beta_1} \partial b^{\beta_2}} C^k(b|X).
$$

(3.7)

The generating function $C^k(b|X)$ was interpreted in (17) as a set of Fock modules

$$
|C^k(b|X)\rangle = C^k(b|X)|0\rangle |0\rangle
$$

(3.8)

$(k = 1, 2, 3 \ldots)$ generated by the creation operators $b^{\alpha}$ from the vacuum state $|0\rangle |0\rangle$ satisfying

$$
a_{\alpha}|0\rangle |0\rangle = 0, \quad |0\rangle |0\rangle b^{\alpha} = 0.
$$

(3.9)

In terms of the analogous generating function for $T$

$$
T^{kl}(b|X) = \sum_{n=0}^{\infty} \frac{1}{n!} c_{\alpha_1 \cdots \alpha_n}^{kl}(X) b^{\alpha_1} \cdots b^{\alpha_n}
$$

(3.10)

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the formula (3.2) with \( a((-1)^m, n) = 1 \) gets remarkably simple form

\[
T^{kl}(b|X) = C^k(i b|X)C^l(b|X) .
\] (3.11)

A proof of the conservation condition (2.5) is now obvious because, in terms of the generating function

\[
\Lambda^{\alpha_1\beta_2}(a) = \sum \frac{1}{n!} \lambda^{\alpha_1...a_n, \beta_1\beta_2} a_{a_1}...a_{a_n} ,
\] (3.12)

the Young property of \( \lambda^{\alpha_1...a_n, \beta_1\beta_2} \) is equivalent to

\[
\lambda^{\alpha_1...a_n, \beta_1\beta_2} a_{a_1}...a_{a_n} a_{\beta_1} = 0 .
\] (3.13)

The generalized conservation property (2.3) is equivalent to

\[
\langle \Lambda^{\beta_1\beta_2}(a) \rangle \frac{\partial}{\partial X^{\beta_1\beta_2}} [T^{kl}(b|X)] = 0 ,
\] (3.14)

where \( \langle \Lambda^{\beta_1\beta_2}(a) \rangle = |0\rangle\langle 0| \Lambda^{\beta_1\beta_2}(a) \) and \( [T^{kl}(b|X)] = T^{kl}(b|X)|0\rangle\langle 0| \). One gets

\[
\langle \Lambda^{\beta_1\beta_2}(a) \rangle \frac{\partial}{\partial X^{\beta_1\beta_2}} [T^{kl}(b|X)] = \langle \Lambda^{\beta_1\beta_2}(a) \rangle \left[ C^k(i b|X) \frac{\partial^2}{\partial b^{\beta_1} \partial b^{\beta_2}} (C^l(b|X)) \right] - \left[ C^k(i b|X) C^l(b|X) \right] [0\rangle\langle 0| .
\] (3.15)

The property (3.13) implies that a total derivative with respect to \( \frac{\partial}{\partial X^{\beta_1\beta_2}} \) does not contribute, thus allowing to “integrate by parts”, that immediately proves (3.14).

The formula (3.2) describes several different cases. If \( n \) is even (odd), the charges (2.2) have commuting (anticommuting) parameters \( \eta \). Let us define generalized spin \( s = \frac{n}{2} \). Then \( T_{\alpha_1...\alpha_{2s}}^{kl} \) generate symmetries and supersymmetries for \( s \) integer and half-integer, respectively.

Using the ambiguity in the coefficients \( a((-1)^m, n) \) in (3.2) one can fix statistics of the fields in \( T_{\alpha_1...\alpha_{2s}}^{kl} \), arriving at the following different cases

\[
T_{BB\alpha_1...\alpha_{2s}}^{kl} = \sum_{q=0}^{s} \frac{(-1)^q (2s)!}{(q)!(2s-2q)!} c_{\alpha_1...\alpha_{2q+1}}^k c_{\alpha_{2q+2}...\alpha_{2s}}^l \, \text{s integer} ,
\] (3.16)

\[
T_{FF\alpha_1...\alpha_{2s}}^{kl} = \sum_{q=0}^{s-1} \frac{i(-1)^q (2s)!}{(q+1)!(2s-2q-1)!} k_{\alpha_1...\alpha_{2q+1}}^k c_{\alpha_{2q+2}...\alpha_{2s}}^l \, \text{s integer} ,
\] (3.17)

\[
T_{BF\alpha_1...\alpha_{2s}}^{kl} = \sum_{q=0}^{s-\frac{1}{2}} \frac{(-1)^q (2s)!}{(2q+1)!(2s-2q-1)!} k_{\alpha_1...\alpha_{2q+1}}^k c_{\alpha_{2q+2}...\alpha_{2s}}^l \, \text{s half-integer} ,
\] (3.18)

\[
T_{FB\alpha_1...\alpha_{2s}}^{kl} = \sum_{q=0}^{s-\frac{1}{2}} \frac{i(-1)^q (2s)!}{(2q+1)!(2s-2q-1)!} k_{\alpha_1...\alpha_{2q+1}}^k c_{\alpha_{2q+2}...\alpha_{2s}}^l \, \text{s half-integer} .
\] (3.19)
One observes that, for s integer, \( T \) is symmetric in the inner indices for even spins and antisymmetric for odd spins

\[
T_{a_1 \ldots a_{2s}}^{kl} = (-1)^s T_{a_1 \ldots a_{2s}}^{lk}.
\]  

This formula holds both for the bosonic case of \( T_{BB} \) and for the fermionic case of \( T_{FF} \) provided that bosonic and fermionic fields are, respectively, commuting and anticommuting as required by microcausality \[18\]. These properties are in agreement with the standard symmetry properties of usual spin 1 current, spin 2 stress tensor and their higher spin generalizations. For the fermionic case one gets

\[
T_{BF; a_1 \ldots a_{2s}}^{kl} = i(-1)^{s+\frac{1}{2}} T_{FB; a_1 \ldots a_{2s}}^{lk}.
\]  

The following comment is now in order. According to (3.1) the components \( c_{\alpha_1 \ldots \alpha_n} \) themselves satisfy the conservation property (2.5) and, therefore, can be used as \( T_{\alpha_1 \ldots \alpha_n} \) in the construction of currents. By virtue of antisymmetrization over \( M + 1 \) indices, taking into account the field equations (3.1), one can see that all forms containing parameters \( \eta_{\beta_1 \ldots \beta_t \alpha_1 \ldots \alpha_s} \) with \( s > 0 \) are equivalent modulo exact forms to the analogous forms containing traces of \( \eta \). This allows one to consider only the case with \( s = 0 \), i.e., only the forms \( \Omega(\eta) \) with the parameters \( \eta_{\beta_1 \ldots \beta_t} \) with \( t = 0, 1, 2, \ldots \) parametrize the cohomology class associated with nontrivial integrals of motion of this type. This result is expected because such parameters are associated with the shifts

\[
\delta c(X) = \eta + \eta_{\alpha \beta} X^{\alpha \beta} + \eta_{\alpha \beta \gamma} X^{\alpha \beta} X^{\gamma \eta} \ldots , \quad \delta c_{\alpha}(X) = \eta_{\alpha} + \eta_{\alpha \beta \gamma} X^{\gamma \beta} \ldots ,
\]  

which are obvious symmetries of the equations (1.10) and (1.11).

### 4 Fourier Transform and Invariant Norm

The equations (1.10) and (1.11) were analyzed in [18] by means of Fourier transform. For a scalar field

\[
c(X) = c_0 \exp i k_{\alpha \beta} X^{\alpha \beta}
\]  

(1.10) requires

\[
k_{\alpha \beta} = \pm \xi_{\alpha} \xi_{\beta}
\]  

with an arbitrary commuting real vector \( \xi_{\alpha} \). The equation for a svector field \( c_{\alpha}(X) \) fixes in addition a polarization factor so that the generic solution of the equations (1.10) and (1.11) has the form

\[
c(X) = c^+(X) + c^-(X) , \quad c^{\pm}(X) = \frac{1}{\pi^{\frac{M}{2}}} \int d^M \xi b^{\pm}(\xi) \exp \pm i \xi_{\alpha} \xi_{\beta} X^{\alpha \beta} ,
\]  

\[
c_{\gamma}(X) = c_{\gamma}^+(X) + c_{\gamma}^-(X) , \quad c_{\gamma}^{\pm}(X) = \frac{1}{\pi^{\frac{M}{2}}} \int d^M \xi \xi_{\gamma} f^{\pm}(\xi) \exp \pm i \xi_{\alpha} \xi_{\beta} X^{\alpha \beta} .
\]  

The space of solutions is parametrized by two functions of \( M \) variables \( \xi_{\alpha} \) both for the scalar \( c(X) \) and for the svector \( c_{\alpha}(X) \). Scalar and svector therefore have equal
numbers of on-mass-shell degrees of freedom. Because odd functions \(b^\pm(\xi)\) and even functions \(f^\pm(\xi)\) do not contribute to (4.3) and (4.4), respectively, we demand
\[
b^\pm(\xi) = b^\pm(-\xi), \quad f^\pm(\xi) = -f^\pm(-\xi).
\] (4.5)

In [18] it was shown that charges generating \(osp(1,2M)\) symmetry and its higher spin extension admit simple realization in terms of Fourier components \(b^\pm(\xi)\) and \(f^\pm(\xi)\)
\[
Q^{B,F}(\eta) = \int d^M \xi \left( b^+(\xi)\eta^{B,F}(\xi, \frac{\partial}{\partial \xi}) b^-(\xi) + f^+(\xi)\eta^{B,F}(\xi, \frac{\partial}{\partial \xi}) f^-(\xi) \right).
\] (4.6)

Depending on the oddness properties, the parameters
\[
\eta(\xi, \frac{\partial}{\partial \xi}) = \sum_{kl} \eta^{\alpha_1 \ldots \alpha_k \beta_1 \ldots \beta_l} \xi_{\alpha_1} \ldots \xi_{\alpha_k} \frac{\partial}{\partial \xi_{\beta_1}} \ldots \frac{\partial}{\partial \xi_{\beta_l}}
\] (4.7)
are bosonic or fermionic
\[
\eta^B(-\xi, -\frac{\partial}{\partial \xi}) = \eta^B(\xi, \frac{\partial}{\partial \xi}), \quad \eta^F(-\xi, -\frac{\partial}{\partial \xi}) = -\eta^F(\xi, \frac{\partial}{\partial \xi}).
\] (4.8)

Taking into account the commutation relations introduced in [18]
\[
[b^\pm(\xi_1), b^\pm(\xi_2)] = 0, \quad [b^-(\xi_1), b^+(\xi_2)] = \frac{1}{2} (\delta(\xi_1 - \xi_2) + \delta(\xi_1 + \xi_2)),
\] (4.9)
\[
[f^\pm(\xi_1), f^\pm(\xi_2)]_+ = 0, \quad [f^-(\xi_1), f^+(\xi_2)]_+ = \frac{1}{2} (\delta(\xi_1 - \xi_2) - \delta(\xi_1 + \xi_2)),
\] (4.10)
where \([\cdot, \cdot]_+\) denotes anticommutator, it is easy to see that the charges (4.6) generate all generalized higher spin transformations.

The analogy between the symmetry parameters (4.7) and the parameters (2.8) in the closed form (2.3) suggests that the expression for the generators (4.6) must result from the charges (2.2). Let us show that this is indeed true. Inserting (4.3) and (4.4) into (3.4) and (3.5) we get
\[
c_{\alpha_1 \ldots \alpha_q}^{\pm k} = (\pm i)^{[q/2]} \frac{1}{\pi^{1/2}} \int d^M \xi \xi_{\alpha_1} \ldots \xi_{\alpha_q} \exp \pm i \xi_\gamma \xi_\beta X^{\gamma \beta},
\] (4.11)
where \(c^\pm(\xi) = b^\pm(\xi)\) for \(q\) even and \(c^\pm(\xi) = f^\pm(\xi)\) for \(q\) odd (\([r]\) denotes the integer part of \(r\)). Inserting this into (2.3), (2.2) with
\[
a((-1)^m, n) = i^{[\frac{m+1}{2}] \cdot \frac{1}{4} (-1)^{[m+1]}},
\] (4.12)
we find for \(\eta = 1\)
\[
Q(c^+, c^-) = \epsilon_{\gamma_1 \ldots \gamma_M} \int_S dX^{\gamma_{\alpha_1}} \wedge \ldots \wedge dX^{\gamma_{M\alpha_M}} \int d^M \xi d^M \xi' c^+(\xi) c^-(\xi') \exp -i X^{\gamma \beta} (\xi, \xi')_{\alpha} \exp -i X^{\gamma \beta} (\xi, \xi')_{\beta}.
\] (4.13)
Let an integration surface be a hyperplane parametrized by some coordinates $y^i$ with $i = 1 \ldots M$, i.e. $X^{\alpha\beta} = U_i^{\alpha\beta} y^i$ where the matrix $U_i^{\alpha\beta}$ has rank $M$. It follows

$$Q(c^+, c^-) = \int d^M y \int d^M \xi d^M \xi' \det V_n^\alpha(\xi + \xi') \delta^M \left( (\xi_\alpha - \xi'_\alpha) V_n^\alpha(\xi + \xi') \right) c^+(\xi) c^-(\xi'),$$

(4.14)

where

$$V_n^\alpha(\xi + \xi') = (\xi_{\beta} + \xi'_{\beta}) U_n^\alpha\beta.$$  

(4.15)

The factor of $\det V_n^\alpha(\xi + \xi')$ guarantees that, generically, the zeros of the delta function in (4.14) at $\xi_{\beta} + \xi'_{\beta} \to 0$ do not contribute. As a result, only the zeros $\xi_{\beta} - \xi'_{\beta} \to 0$ have to be taken into account. The integration measure in (4.14) therefore amounts to $\delta^M(\xi - \xi')$. As a result one is left with the expected expression

$$Q(c^+, c^-) = \int d^M \xi c^+(\xi) c^-(\xi).$$  

(4.16)

It remains to note that, $Q(c^+, c^\pm) = 0$ because the bilinear form in $\xi$ and $\xi'$ in the exponential $\exp \pm i X^{\alpha\beta}(\xi_{\gamma} \xi'_{\gamma} + \xi_{\gamma} \xi_{\beta})$ is semi-definite and, therefore, the argument of the delta-function resulting from the integration over $y_n$ has no nontrivial zeros.

The integral of motion (4.16) produces an invariant norm on the space of solutions. It gives rise to a positive-definite invariant norm

$$A(b^+, f^+; b^-, f^-) = \int d\xi^M \left( b^+ (\xi) b^- (\xi) + f^+ (\xi) f^- (\xi) \right)$$  

(4.17)

on the one-particle quantum states

$$\int d^M \xi \left( b^- (\xi) b^+ (\xi) + f^- (\xi) f^+ (\xi) \right) |0\rangle \langle 0|,$$

(4.18)

parametrized by the functions $b^-$ and $f^-$. The charges with multispinor parameters $\eta_{\beta_1 \ldots \beta_t}{}^{\alpha_1 \ldots \alpha_s}$ give rise to the higher spin charges (4.6) because every factor of $X^{\alpha\beta}$ in (2.3) is equivalent to the second derivative over $\xi$.

5 Conclusion

It is shown that $sp(2M)$ invariant equations of motion in the generalized spacetime $\mathcal{M}_M$ with matrix coordinates suggested in [17] admit conserved (super)charges associated with the infinite-dimensional higher spin superextension of $osp(1, 2M)$. The (super)charges are integrals of on-mass-shell closed $M$-forms bilinear in the dynamical fields. This provides one more manifestation of the fact that nontrivial independent degrees of freedom of the $sp(2M)$ invariant dynamics live on $M$-dimensional surfaces [18]. The scalar charge gives rise to an invariant norm on the space of solutions of the field equations which turns out to be equivalent to the positive-definite norm on the Fock space of one-particle states. The proposed construction has a good chance to admit a generalization to less trivial (not necessarily flat) geometries.
It is straightforward to write down conformally invariant Noether current interactions as

\[ S^{\text{Noether}} = \int_{\mathcal{S}T} \Omega(A), \]

(5.1)

where \( \Omega(A) \) is the \( M+1 \) form obtained from the \( M \)-form (2.3) via replacement of the parameter \( \eta_{\alpha_1 \ldots \alpha_n}^{\beta_1 \ldots \beta_m} \) by the higher spin conformal gauge 1-form \( A_{\alpha_1 \ldots \alpha_n}^{\beta_1 \ldots \beta_m} \). The integration in (5.1) is performed over a \( M+1 \)-dimensional surface to be interpreted as spacetime [13].

The dynamics described by (1.10) and (1.11) is equivalent [17, 18] to the conformal dynamics in Minkowski spacetime for specific (infinite for \( M > 2 \)) sets of usual relativistic fields contained in scalar and vector fields in \( \mathcal{M}_M \). Conserved charges built in this paper amount to certain sets of conserved charges in the associated Minkowski spacetime. In fact they give rise to infinite towers of different higher spin charges built of various conformal fields in the usual Minkowski spacetime. (Recall that infinite towers of fields in Minkowski space result from “Kaluza-Klein” modes on the fiber of the local Cauchy fiber bundle.) An interesting problem is to analyze the content of the conserved higher spin currents associated with the set of symmetry parameters \( \eta_{\beta_1 \ldots \beta_t}^{\alpha_1 \ldots \alpha_s} \) from the perspective of Minkowski spacetime. It should be taken into account that the conformal Minkowski fields hidden in \( c(X) \) and \( c_\alpha(X) \) contain higher derivatives when expressed in terms of gauge potentials. For example, \( c(X) \) describes scalar field in the spin 0 sector, Maxwell field strength in the spin 1 sector, Weyl tensor in the spin 2 sector etc. Generically, a spin \( s \) field contains order \( [s] \) derivatives of the respective conformal gauge potential. This is in agreement with the fact that, for example, the generalized conformal stress tensor in the spin two sector is built from the Weyl tensor, thus corresponding to the stress tensor of the \( \mathcal{C}^2 \) conformal (Weyl) gravity.

Technically, to derive explicit form of all conformal higher spin currents built from various pairs of Minkowskian conformal fields is going to be a hard problem which requires a knowledge of a content of the decomposition of a totally symmetric tensor product of an arbitrary number of spinor representations into irreducible representations of the Lorentz algebra. To the best of our knowledge a solution of this problem is yet unknown for generic \( M \). This problem can be avoided however in case one manages to operate in totally \( sp(2p) \) invariant terms of \( sp(2p) \) multiplets or rather \( osp(N, 2p) \) supermultiplets.

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