1 Introduction ............................................. 487
2 Main Results ............................................. 487
3 Conclusion ................................................ 490
References ............................................... 490

1. Introduction

Here we mean a finite and undirected graph with \( p \) vertices and \( q \) edges. The labeling in graph was introduced by Rosa in 1960 [5]. Magic labeling was defined by Sedlacek [9]. In 1996, Ringel and Llado [3] initiated this labeling as edge magic. Edge bimagic labeling of graphs was defined by Babujee [1] in 2004. Graham and Sloane introduced harmonious labeling [4]. Dushyant Tanna [2] defined some graph labeling techniques. For more detailed, we use dynamic survey of graph labeling by Gallian [5]. If \( G \) is of order \( n \), the corona of \( G \) with \( H, G \odot H \) is the graph obtained by taking one copy of \( G \) and \( n \) copies of \( H \) and joining the \( i^{th} \) vertex of \( G \) with an edge to every vertex in the \( i^{th} \) copy of \( H \) [8]. A graph \( G = (V, E) \) with \( p \) vertices and \( q \) edges is said to be edge bimagic harmonious if there exists a bijection \( f : V \cup E \rightarrow \{1, 2, 3, \ldots, p + q\} \) such that for each edge \( xy \) in \( E(G) \), the value of \( [(f(x) + f(y))(\mod q) + f(xy)] \) is equal to the constants \( k_1 \) or \( k_2 \), called two distinct magic constants [8]. An edge bimagic harmonious labeling is said to be super edge bimagic harmonious labeling if the graph \( G \) has the further property that the vertex labels are 1 to \( |V| \). In this paper we prove that the corona graphs \( C_n \odot \bar{K}_2, P_n \odot \bar{K}_2 \) and \( P_n \odot \bar{K}_2 \) are super edge bimagic harmonious graphs.
Using these labelings, there exist two magic constants for each edge $xy \in E$, $|(f(x) + f(y)) \mod q| + f(xy)$ yields any one of the magic constant $k_1 = 4n + 2$ and $k_2 = 7n + 2$.

Therefore, the graph $C_n \odot \tilde{K}_2$ admits an edge bimagic harmonious labeling for even $n$.

**Case 2: $n$ is even**

Define a bijection $f : V \cup E \to \{1, 2, 3, ..., 6n\}$ such that

- $f(u_i) = i, 1 \leq i \leq n$
- $f(v_i) = n + i, 1 \leq i \leq n$
- $f(u_{i+1}) = 2n + i, 1 \leq i \leq n$
- $f(u_1u_i) = 6n - 2i - 1, 1 \leq i \leq n - 2$
- $f(u_1u_{i+1}) = 6n - 2i - 2, \frac{n+2}{2} \leq i \leq n - 1$
- $f(u_1v_1) = 5n - 2$
- $f(u_1v_i) = 5n - 2i - 1, 1 \leq i \leq n - 1$
- $f(u_nv_i) = 6n - 1$
- $f(u_1w_i) = 4n - 2i, 1 \leq i \leq n - 2$
- $f(u_1w_{i+1}) = 7n - 2i, \frac{n}{2} \leq i \leq n$

Using these labelings, there exist two magic constants for each edge $xy \in E$, $|(f(x) + f(y)) \mod q| + f(xy)$ yields any one of the magic constant $k_1 = 6n$ and $k_2 = 6n - 1$.

Therefore, the graph $C_n \odot \tilde{K}_2$ admits an edge bimagic harmonious labeling for even $n$.

**Corollary 2.2.** The graph $C_n \odot \tilde{K}_2$ admits a super edge bimagic harmonious labeling for all $n$.

**Proof.** We proved that the graph $C_n \odot \tilde{K}_2$ admits an edge bimagic harmonious labeling for all $n$. The labeling given in the proof of Theorem 2.1, the vertices get labels 1, 2, 3, ..., $3n$.

Since the graph $C_n \odot \tilde{K}_2$ has 3$n$ vertices and the $3n$ vertices have labels 1, 2, 3, ..., $3n$ for all $n$, the graph $C_n \odot \tilde{K}_2$ is a super edge bimagic harmonious for all $n$.

**Example 2.3.** Super edge bimagic harmonious labeling of the graph $C_9 \odot \tilde{K}_2$ and $C_8 \odot \tilde{K}_2$ are given in figure 1 and figure 2.

**Theorem 2.4.** The graph $P_n \odot \tilde{K}_2$ admits an edge bimagic harmonious labeling for all $n$.

**Proof.** Let $V = \{u_i, v_i, w_i/1 \leq i \leq n\}$ be the vertex set and $E = \{u_iu_{i+1}/1 \leq i \leq n - 1\} \cup \{u_iw_i/1 \leq i \leq n\} \cup \{v_iw_i/1 \leq i \leq n\}$ be the edge set of the graph $P_n \odot \tilde{K}_2$. The graph $P_n \odot \tilde{K}_2$ has 3$n$ vertices and $4n - 1$ edges.

**Case 1: $n$ is odd**

Define a bijection $f : V \cup E \to \{1, 2, 3, ..., 7n - 1\}$ such that

- $f(u_i) = i, 1 \leq i \leq n$
- $f(v_i) = n + i, 1 \leq i \leq n$
- $f(w_i) = 2n + i, 1 \leq i \leq n$
- $f(u_{i+1}) = 7n - 2i - 1, 1 \leq i \leq n - 1$
- $f(u_iw_i) = 6n - 2i - 2, 1 \leq i \leq \frac{n-1}{2}$
Using these labelings, there exist two magic constants for each edge $xy \in E$. $[(f(x) + f(y)) \mod q + f(xy)]$ yields any one of the magic constant $k_1 = 7n - 1$ and $k_2 = 7n - 2$.

Therefore, the graph $P_n \circ K_2$ admits an edge bimagic harmonious labeling for odd $n$.

**Case 2:** $n$ is even

Define a bijection $f : V \cup E \rightarrow \{1, 2, 3, \ldots, 7n-1\}$ such that

\[
\begin{align*}
    f(u_i) &= i, 1 \leq i \leq n \\
    f(v_i) &= 2n + i, 1 \leq i \leq n \\
    f(w_i) &= n + i, 1 \leq i \leq n \\
    f(u_iu_{i+1}) &= 7n - 2i - 1, 1 \leq i \leq \frac{n}{2} \\
    f(u_iu_{i+1}) &= 7n - 2i - 2, \frac{n+2}{2} \leq i \leq n - 1 \\
    f(u_iv_i) &= 5n - 2i - 1, 1 \leq i \leq n - 1 \\
    f(u_nv_n) &= 7n - 1 \\
    f(u_iw_i) &= 6n - 2i - 1, 1 \leq i \leq n \\
    f(v_iw_i) &= 4n - 2i - 1, 1 \leq i \leq \frac{n-2}{2} \\
    f(v_iw_i) &= 8n - 2i - 2, \frac{n-1}{2} \leq i \leq n
\end{align*}
\]

Using these labelings, there exist two magic constants for each edge $xy \in E$. $[(f(x) + f(y)) \mod q + f(xy)]$ yields any one of the magic constant $k_1 = 7n - 1$ and $k_2 = 7n - 2$.

Therefore, the graph $P_n \circ K_2$ admits an edge bimagic harmonious labeling for even $n$. From cases (1) and (2), the graph $P_n \circ K_2$ admits an edge bimagic harmonious labeling for all $n$.

**Corollary 2.5.** The graph $P_n \circ K_2$ admits a super edge bimagic harmonious labeling for all $n$.

**Proof.** We proved that the graph $P_n \circ K_2$ admits an edge bimagic harmonious labeling for all $n$. The labeling given in the proof of Theorem 2.4, the vertices get labels 1, 2, 3, ..., $3n$.

Since the graph $P_n \circ K_2$ has $3n$ vertices and the $3n$ vertices have labels 1, 2, 3, ..., $3n$ for all $n$, the graph $P_n \circ K_2$ is a super edge bimagic harmonious for all $n$. □

**Example 2.6.** Super edge bimagic harmonious labeling of the graph $P_7 \circ K_2$ and $P_6 \circ K_2$ are given in figure 3 and figure 4.
Using these labelings, there exist two magic constants for each edge $xy \in E, \{(f(x) + f(y)) (mod \ q) + f(xy)\}$ yields any one of the magic constant $k_1 = 6n - 1$ and $k_2 = 6n$.

Therefore, the graph $P_n \odot \bar{K}_2$ admits an edge bimagic harmonious labeling for even $n$. From cases (1) and (2), the graph $P_n \odot \bar{K}_2$ admits an edge bimagic harmonious labeling for all $n$.

**Corollary 2.8.** The graph $P_n \odot \bar{K}_2$ admits a super edge bimagic harmonious labeling for all $n$.

**Proof.** We proved that the graph $P_n \odot \bar{K}_2$ admits an edge bimagic harmonious labeling for all $n$. The labeling given in the proof of Theorem 2.7, the vertices get labels $1, 2, 3, \ldots, 3n$.

Since the graph $P_n \odot \bar{K}_2$ has $3n$ vertices and the $3n$ vertices have labels $1, 2, 3, \ldots, 3n$ for all $n$, the graph $P_n \odot \bar{K}_2$ is a super edge bimagic harmonious for all $n$.

**Example 2.9.** Super edge bimagic harmonious labeling of the graph $P_7 \odot \bar{K}_2$ and $P_6 \odot \bar{K}_2$ are given in figure 5 and figure 6.

![Figure 5: $P_7 \odot \bar{K}_2$ with $k_1 = 41$ and $k_2 = 40$.](image1)

![Figure 6: $P_6 \odot \bar{K}_2$ with $k_1 = 35$ and $k_2 = 36$.](image2)

### 3. Conclusion

In this paper we proved that the corona graphs $C_n \odot \bar{K}_2$, $P_n \odot \bar{K}_2$ and $P_n \odot \bar{K}_2$ are super edge bimagic harmonious graphs.

### References

[1] J. Baskar Babujee, On edge bimagic Labeling, *Journal of Combinatorics Information & System Sciences*, 28(1-4)(2004), 239-244.

[2] Dushyant Tanna, Harmonious Labeling of Certain Graphs, *International Journal of Advanced Engineering Research and Studies*, (2013), 46-68.

[3] H. Enomoto, Anna S. Llado, Tomoki Nakamigawa and Gerhard Ringel, On Super Edge Magic Graphs, *SUT Journal of Mathematics*, 34(1998), 105-109.

[4] R. L. Graham and N. J. A. Sloane, On additive bases and harmonious graphs, *SIAM, Journal on Algebraic and Discrete Methods*, 1(1980), 382-404.

[5] Joseph A. Gallian, A dynamic survey of graph labeling of some graphs, *The Electronic Journal of Combinatorics*, (2018).