Proposed assembly line balancing using mixed integer programming to minimize idle time of union fuselage

Ni Putu Cynthia Sasmita Dewi¹, Dida Diah Damayanti², Murni Dwi Astuti³
Industrial Engineering, School of Industrial and System Engineering
Universitas Telkom, Bandung, Indonesia
¹cynthiasasmita@student.telkomuniversity.ac.id, ²didadiah@telkomuniversity.ac.id, ³murnidwiastuti@telkomuniversity.ac.id

Abstract. This study is aimed to help the company doing an assembly line balancing to minimize station time as not to exceed takt time and minimize idle time between workstation, using Mixed Integer Programming (MIP) method with Multi-manned Assembly Line Balancing (MmALBP) approach using two mathematical model, which 1st model aimed to minimize the cycle time of all workstation, then those cycle time being a parameter in 2nd Mathematical model to specified the optimal number of workers also balancing workload between operators. Result from this research were able to minimize station time from 218.56 hour to 166.3 hour so it didn’t exceed the takt time and also could reduce total idle time until 45% from 905.2 hour in actual condition become 499.49 hour with decreased number of operator from 52 to 22 person, and increase the efficiency of assembly line from 54% in previous to 67% in the proposed line.

1. Introduction
This study is aimed to help the company solving its problem which at this time, the process of assembling fuselage has not meet the production target which is 6 units component per year, currently the department only able to produce 3 units of fuselage per year. After doing an observation there is a workstation at union fuselage assembly line which the station time exceeds the takt time. Beside that, there’s an idle time between all workstation in union fuselage assembly line, total idle time is 905.2 hour. Idle time is a condition when a workstation is being idle due to waiting for input from the previous workstation [1]. One of the causes of an high idle time because the allocation of operations to the operator in a workstation is not optimal [2]. Based on its problem, the purpose of this research is to enhance the index of efficiency and reducing the idle time of assembly line by considering the total of operators used optimally and the allocation of operations to operator on the union fuselage assembly line using mixed integer programming.

2. Literature Review
2.1. Assembly Line Balancing
Assembly line balancing is a process to rebalancing the workload in each workstation with the aim to minimize the idle time and to increase line efficiency [2][3]. Problem classification on assembly line balancing is divided into two, that is SALBP (Simple Assembly Line Balancing Problem) and GALBP (General Assembly Line Balancing Problem) [4][5]. One of classification from GALBP is MALBP (Multi-manned Assembly Line Balancing Problem) which in its application is possible to assign more than one operator to do a different operation at the same time in the workstation based on type and complexity of the product with purpose to determine the optimal number of effectiveness from assembly
line and also reducing the span of line [6][7]. Many researcher address the advantage of MALBP, one of them is multi-manned ALB could decrease the span of line so it also can minimizing the throughput time, minimizing cost and less material handling [8]. There are two technique to solve ALB problem, that is exact and heuristic method. A purpose of exact method is generating feasible solution (optimization), while heuristic generate the solution that close to the best feasible solution [9].

2.2. Mathematical Modelling Formulation
This research is using a mixed-integer programming model that computed using CPLEX Optimization Studio 12.9.0 to solve a MALBP optimally. This study have two phase computation with different mathematical model. 1st model’s was proposed by Roshani and Giglio [10] which main objective is minimizing cycle time and number of operator as a second objective. However using 1st mathematical model only could minimizing cycle time and operator but can’t balancing workload between operator in a same workstation. Therefore, 2nd mathematical model that proposed by Yilmaz and Yilmaz [11] is used to balancing workload between operator in each workstation with a cycle time from 1st model as a parameter. The mathematical model that used in this research is as follows.

2.2.1. 1st Mathematical Model
In this study we are using mathematical model from Roshani & Giglio (2016) to minimize station time and total operator that used within assembly line. Decision variable that used in this research is:

- \( M_a \): Start time of operation \( a \)
- \( S_a \): Completion time of operation \( a \)
- \( WTB_b \): Completion time of all task that executed at workstation \( b \)
- \( ST \): Cycle time
- \( X_{abc} \): 1 if task \( a \) is executed by operator \( c \) in workstation \( b \) (and 0 otherwise)
- \( Y_{bc} \): 1 if operator \( c \) is assigned to workstation \( b \) (and 0 otherwise)
- \( Z_{a,a1} \): 1 if operation \( a \) is executed earlier than task \( a1 \) (and 0 otherwise)

Objective function of this model is minimizing cycle time and minimizing total number of operator, as follows.

\[
\min \ ST + \frac{1}{n_c \cdot O_{max} + 1} \sum_{b \in B} \sum_{c \in C} Y_{bc}
\]

Constraint that used in this model is as follows.
1. Constraint (2) ensure all task assigned to only one operator at one workstation

\[
\sum_{b \in B} \sum_{c \in C} X_{abc} = 1, \ \forall a \in A
\]

2. Constraint (3) is a precedence constraint to ensure all precedence relations of all task are satisfied.

\[
\sum_{b \in B} \sum_{c \in C} b \cdot X_{a1bc} \leq \sum_{b \in B} \sum_{c \in C} b \cdot X_{abc}, \ \forall a \in A, \ \forall a1 \in P(a)
\]

3. Constraint (4) ensure that \( a1 \) is started after a finished, which \( a \) is immediate predecessor of task \( a1 \).

\[
M_a - M_{a1} + L(1 - \sum_{c \in C} X_{a1bc}) + L(1 - \sum_{c \in C} X_{abc}) \geq t_{a1}, \ \forall a \in A, \ \forall a1 \in P(a), \ \forall b \in B
\]

4. Constraint (5) ensure that \( Y_{bc} \) must be 1 if task \( a \) executed by operator \( c \) in workstation \( b \)

\[
\sum_{a \in A} X_{abc} \cdot n_o \cdot Y_{bc} \leq 0, \ \forall b \in B, \ \forall c \in C
\]

5. Constraint (6) ensure that the sum between start time and operation time of task \( a \) doesn’t exceed the completion time.
\[ M_a + t_a \leq S_a, \; \forall a \in A \]  

(6)

6. Constraint (7) ensure that completion time of all workstation doesn’t exceed the cycle time

\[ W^T_b \leq CT, \; \forall b \in B \]  

(7)

2.2.2. 2nd Mathematical Model

This second phase is using Yilmaz & Yilmaz (2015) model, to specify total of operator used optimally and minimize the gap of operation time of all operator in the same station. This 2nd model have a correlation with 1st model, which is the cycle time from 1st model’ calculation is used as a parameter at this model. Decision variable that used in this model is as follows.

- \( S_a \): Start time of task a
- \( d_{c,c_1} \): Difference of total task time between c and c1 operators
- \( X_{abc} \): 1 if task a is executed by operator c in workstation b (and 0 otherwise)
- \( w_{sb} \): 1 if operator c is used in workstation b (and 0 otherwise)
- \( w_{sb1} \): 1 if c operator are used in workstation b (and 0 otherwise)

This model have a purpose to decrease total of used operator, second term is decreasing total of used workstation, and the last is reducing the difference of total operation time between operator in every station, as follows.

\[
\min \sum_{j \in J} \sum_{k \in K} w_{sj} k + \frac{1}{N_{\text{max}}^n + 1} \sum_{j \in J} \sum_{k \in K} j x_{njk} + \frac{1}{C_{t.n} N_{\text{max}}} \sum_{j \in J} \sum_{k \in K} j x_{njk} \sum_{k \in K} d_{k,k1} 
\]  

(8)

Constraint that used in this model is as follows.

1. Constraint (9) ensure that start and operation time of all task doesn’t exceed cycle time

\[ M_a + t_a \leq S_a, \; \forall a \in A \]  

(9)

2. Constraint (10) and (11) for the different total task time between operator. Constraint (10) active when total operation time of operator c is greater than c1, otherwise constraint (11) become active.

\[
\sum_{a \in A} t(a) X_{abc} - \sum_{a \in A} t(a) X_{abc1} \leq d_{c,c_1} \forall b \in B, \; c \in C
\]  

(10)

\[
- \sum_{a \in A} t(a) X_{abc} + \sum_{a \in A} t(a) X_{abc1} \leq d_{c,c_1} \forall b \in B, \; c \in C
\]  

(11)

3. Constraint (12) ensure total operator that assigned to workstation j

\[
\sum_{c \in C} c w_{sb} - \sum_{c \in C} w_{sb} = 0, \; \forall b \in B
\]  

(12)

4. Constraint (13) ensure operator assigned to the workstation is as needed

\[
Y_{b(c+1)} - Y_{bc} \leq 0 \; \forall b \in B, \; \forall c \in C
\]  

(13)

3. Result and Discussion

3.1. Actual Assembly Line

The actual total of used workstation in union fuselage is 9 workstation with 20 available operator in there. Total number of operation in union fuselage assembly line is 195. These operations are assigned in 9 different jigs (workstation), where each operation that has been allocated to each jig can’t be moved to another jig. Total station time of actual line is 1061.84 hour and total takt time is 200 hours. One of the workstation that has a station time exceeding takt time is union fuselage with actual station time is
18.56 hour, total operation in this workstation is 33 task that assigned to 8 operator, with precedence diagram as Figure 1 below.

![Figure 1. Precedence Diagram Union Fuselage](image)

3.2. Mixed Integer Programming Calculation

The problem of this research is solved using CPLEX Optimizer Studio 12.9.0 with the computation using 1st mathematical model aimed to minimizing station time and the number of operator, result of computation using 1st mathematical is as in Table 1 below

| Task | Operator | Start Time (Hour) | Completion Time (Hour) | Task | Operator | Start Time (Hour) | Completion Time (Hour) |
|------|----------|------------------|------------------------|------|----------|------------------|------------------------|
| 1    | 1        | 0                | 4                      | 18   | 2        | 142.76           | 144.29                 |
| 2    | 1        | 4                | 8                      | 19   | 2        | 144.29           | 160.29                 |
| 3    | 1        | 8                | 22                     | 20   | 1        | 155.55           | 163.55                 |
| 4    | 1        | 22               | 36                     | 21   | 2        | 117.13           | 129.13                 |
| 5    | 2        | 36               | 52                     | 22   | 2        | 129.13           | 141.13                 |
| 6    | 1        | 36               | 36.66                  | 23   | 2        | 1.93             | 2.89                   |
| 7    | 1        | 52               | 64                     | 24   | 1        | 132.33           | 138.23                 |
| 8    | 1        | 64               | 76                     | 25   | 2        | 141.13           | 142.76                 |
| 9    | 2        | 0                | 0.77                   | 26   | 2        | 0.77             | 1.93                   |
| 10   | 1        | 76               | 88                     | 27   | 1        | 142.92           | 146.92                 |
| 11   | 1        | 88               | 93.13                  | 28   | 1        | 146.92           | 149.55                 |
| 12   | 1        | 93.13            | 102.13                 | 29   | 1        | 149.55           | 155.55                 |
| 13   | 1        | 102.13           | 111.13                 | 30   | 1        | 138.23           | 142.86                 |
| 14   | 2        | 160.29           | 166.29                 | 31   | 1        | 111.13           | 116.33                 |
| 15   | 2        | 111.13           | 117.13                 | 32   | 1        | 163.55           | 165.71                 |
| 16   | 2        | 2.89             | 3.77                   | 33   | 1        | 165.71           | 166.29                 |
| 17   | 1        | 116.33           | 132.33                 |      |          |                  |                        |

Based on the result of computation in Table 1 above, station time of union fuselage has decreased from 218.56 hours into 166.3 hours, and the number of operator’s required has decreased from 8 to 2 operator. However the allocation of operation to operator still unbalanced as seen in Table 2 below.

| Operator | Operation | Duration |
|----------|-----------|----------|
| 1        | 1,2,3,4,6,7,8,10,11,12,13,17,20,24,27,29,30,31,32,33 | 150.89 |
| 2        | 5,9,14,15,16,18,19,21,22,23,25,26 | 74.93 |

Although the number of operator assigned is decreased from 8 to 2 operator but the allocation of operations to the operators in each workstation is still unbalanced, operator 2 works for 74.93 hours
while operator 1 works for 150.89 hours there is a gap between operator for about 75.96 hour. There is an imbalance of workload between operators in a workstation, so it must be rebalanced by using 2nd mathematical model.

The calculation using 2nd mathematical model has an objective to specify total of used operator optimally and balancing the workload of operator in a workstation by using cycle time from 1st mathematical model. Based on computational result, 2nd mathematical model able to balancing the workload between operator in each workstation as in figure 2 below on union fuselage station (workstation 8) the total operator that assigned to workstation same as solution from 1st mathematical model but the allocation of operation to operator has balanced there is no gap of allocation between operator that assignet to this workstation.

![Allocation of Operator & Station Time](image)

**Figure 2. Allocation of Operator and Station Time**

Calculation using mixed integer programming applied to all workstation in union fuselage assembly line, based on data at Table 3, these model able to minimizing the total of operator that assigned to each workstation and also balancing workload between operator in each workstation, the complete result and comparision with actual line is as in Table 3 below.

| No | Workstation               | Actual Station Time | Proposed Station Time | Actual Operator | Proposed Operator |
|----|---------------------------|---------------------|-----------------------|-----------------|-------------------|
| 1  | Lower Panel Center Fuselage | 153.84              | 111.08                | 6               | 2                 |
| 2  | Upper Panel Center Fuselage (FO) | 138.48              | 138.44                | 6               | 2                 |
| 3  | Upper Panel Center Fuselage (GO) | 44.88               | 49.96                 | 4               | 2                 |
| 4  | Rear Cone                 | 149.93              | 148.9                 | 4               | 2                 |
| 5  | Skin Panel Rear Fuselage LH | 68.96               | 67.96                 | 6               | 2                 |
| 6  | Skin Panel Rear Fuselage RH | 78.8                | 78.8                  | 6               | 2                 |
| 7  | Upper Panel Rear Fuselage | 86.08               | 85.11                 | 4               | 2                 |
| 8  | Fuselage Union            | 218.56              | 166.3                 | 8               | 2                 |
| 9  | Center Fuselage           | 122.32              | 150.66                | 8               | 6                 |
|    | TOTAL                     | 1061.84             | 997.21                | 52              | 22                |

There’s another alternative of research which is adjust between the number of operator, total operation time and takt time. For workstation wich operation time is lower than takt time we set the number of operator in the station is as much as 1 operator so that the operation time is same as station time, this alternative is aimed to minimizing the number of operator. From the data, with the value of takt time is 200 hours, there is 7 workstation with operation time lower than takt time, the result is total station time has decreased into 1058.17 hours, and the number of operator’s required has decreased to 15 person.

With the proposed of assembly line, then we calculate the performance index which is compared with the actual state of assembly line. From the performance result we know that the proposed assembly line give a better performance than actual assembly line as we seen at Table 4.
Table 4. Comparison of Actual and Proposed Assembly Line Performance

|                          | Actual  | Proposed 1 | Proposed 2 |
|--------------------------|---------|------------|------------|
| Largest Station Time (Hour) | 218.56  | 166.3      | 166.3      |
| Total Idle Time (Hour)    | 905.2   | 499.49     | 438.53     |
| Operator Required         | 52      | 22         | 15         |
| Production Capacity (unit)| 5       | 7          | 7          |
| Balance Delay             | 46%     | 33%        | 29%        |
| Smoothness Index          | 337.941 | 204.516    | 189.762    |
| Line Efficiency           | 54%     | 67%        | 71%        |

After rebalancing the assembly line, the result shows that the station time is decreasing in the proposed assembly line, so there is no station with time that exceeds the takt time. Also from the result there is 2 solution that give a better performance than actual condition, then we compared the utility of operator between these 2 solution. The result is solution 1 with total operator used 22 person the average of utility among operator is 56% while solution 2 give the higher value average utility that is 59% with total number of used operator is 15 person.

4. Conclusion
This study is aimed to minimize the station time so it don’t exceed the takt time also minimize the idle time of assembly line using *mixed integer programming*. After doing rebalancing of assembly line, there is no workstation with station time that longer than takt time. Performance index such as smoothness index and balance delay is decreased, and the line efficiency and production capacity is increased so it means the proposed assembly line give a better solution than actual condition. This solution needs to be reconfigured if there is an increase of target or demand so that the department can fulfill the target/demand.

5. References
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