Vibrations of geometrically nonlinear beam subjected to two oppositely moving loads and supported by three equidistant visco-elastic dampers

To cite this article: A Khurshudyan and S Ohanyan 2018 J. Phys.: Conf. Ser. 991 012046

View the article online for updates and enhancements.
Vibrations of geometrically nonlinear beam subjected to two oppositely moving loads and supported by three equidistant visco-elastic dampers

A Khurshudyan* and S Ohanyan**
Institute of Mechanics of the National Academy of Sciences of Armenia, Yerevan, Armenia
E-mail: *khurshudyan@mechins.sci.am, **sergohanyan@mail.ru

Abstract. The bending vibrations of a geometrically nonlinear Euler-Bernoulli beam supported by three equidistant visco-elastic dampers and subjected to two oppositely moving point loads are studied. The Bubnov-Galerkin procedure is used for solving the strong formulation of the problem, and the finite element method is involved for numerical analysis.

List of symbols

- $u_0$ and $w_0$: longitudinal and transverse displacements of the beam
- $\varepsilon_{xx}$, $\varepsilon_{xz}$, $\varepsilon_{zz}$: von Kármán strains
- $N_{xx}$: axial force
- $M_{xx}$: bending moment
- $l$: length of the beam
- $E$ and $\rho$: Young’s modulus and density of the beam
- $S$ and $J$: cross-section area and moment of inertia of the beam
- $P_1$ and $P_2$: intensities of the moving forces
- $v_1$ and $v_2$: velocities of the moving forces
- $t_0$: time instant when the second load starts to move
- $\delta$: Dirac delta function

1. Introduction

Simply supported beams subjected to moving loads are the most common engineering models of bridges [1–3]. The dependence of displacements of the beam on the load parameters is analyzed to estimate the areas of occurrence of critical residual stresses having accumulative nature [4]. To compensate the influence of the load on the beam, different types of dampers and vibration absorbers are usually involved. A tremendous body of literature is devoted to analysis of beams under moving loads supported by various kinds of dampers: elastic [5], viscous [6], linear visco-elastic [7], nonlinear visco-elastic [8, 9]. Linear beam theories of Euler–Bernoulli [3] and Timoshenko [10] are mainly covered. The two-dimensional non-classical theories are also involved [11]. The Fourier method of variable separation, the Bubnov–Galerkin procedure, and Green’s function approach are the main tools for analysis of such problems.

The cases of material or geometrical nonlinearities are considered relatively less (see [12–15] and the references therein). The load-displacement relation is much too complicated to be
Figure 1. Simply supported beam subjected to two oppositely moving point loads and supported by three equidistant visco-elastic dampers.

derived in a closed form, and therefore approximation methods are involved. However, not all approximation methods can be useful in this case, especially because the mathematical model of the problem is formulated in terms of distributions. Due to its variational formulation, the Bubnov–Galerkin procedure [16] is usually applied efficiently.

In this paper, we analyze the load-displacement relation for isotropic, homogeneous, elastic, geometrically nonlinear Euler–Bernoulli beam, which is subjected to point loads of constant intensity uniformly moving along the upper surface of the beam in directions opposite to each other. The beam is supported by three equidistant similar visco-elastic dampers. Numerical analysis reveals the influence of dampers on the longitudinal and transverse vibrations of the beam.

2. Statement of the problem: strong formulation

Let an elastic isotropic homogeneous beam of length $l$ be simply supported at the ends $x = 0$ and $x = l$ and be supported by three visco-elastic dampers placed at $x_1 = l/4$, $x_2 = l/2$, and $x_3 = 3l/4$. The beam is subjected to two constant loads with intensities $P_1$ and $P_2$ perpendicular to the beam axis and moving with constant velocities $v_1$ and $v_2$, respectively, in opposite directions (see figure 1).

The Euler–Bernoulli assumptions are accepted for the beam, and the Kelvin–Voigt model of visco-elastic body is assumed to hold for the dampers. However, the Cauchy relations between the strain tensor and the displacement vector components are assumed to be nonlinear, and the von Kármán strains are introduced. In this case [20],

$$u(x, t) = u_0(x, t) - z \frac{\partial w_0}{\partial x},$$

$$v(x, t) \equiv 0,$$

$$w(x, t) = w_0(x, t),$$

and the only non-zero von Kármán strains are [21]

$$\varepsilon_{xx} = \frac{\partial u_0}{\partial x} - z \frac{\partial^2 w_0}{\partial x^2} + \frac{1}{2} \left[ \left( \frac{\partial u_0}{\partial x} - z \frac{\partial^2 w_0}{\partial x^2} \right)^2 + \left( \frac{\partial w_0}{\partial x} \right)^2 \right],$$

$$\varepsilon_{xz} = -\frac{1}{2} \left( \frac{\partial u_0}{\partial x} - z \frac{\partial^2 w_0}{\partial x^2} \right) \frac{\partial w_0}{\partial x}, \quad \varepsilon_{zz} = -\frac{1}{2} \left( \frac{\partial w_0}{\partial x} \right)^2.$$

Since the beam material is supposed to be linear elastic, with neglecting rotary and longitudinal inertia, as well as shearing deformations, the balance of forces and moments gives

$$\frac{\partial N_{xx}}{\partial x} = 0,$$

$$\frac{\partial^2 M_{xx}}{\partial x^2} + \frac{\partial}{\partial x} \left( N_{xx} \frac{\partial w_0}{\partial x} \right) - \rho S \frac{\partial^2 w_0}{\partial t^2} - D(x, t) + F(x, t) = 0,$$
where $F$ characterizes the influence of the loads, and $D$ characterizes that of the dampers. Here

$$N_{xx} \approx ES \left[ \frac{\partial u_0}{\partial x} + \frac{1}{2} \left( \frac{\partial w_0}{\partial x} \right)^2 \right],$$

$$M_{xx} \approx -EJ \frac{\partial^2 w_0}{\partial x^2}.$$

Substituting the obtained relations into (3), we obtain

$$\frac{\partial}{\partial x} \left[ \frac{\partial u_0}{\partial x} + \frac{1}{2} \left( \frac{\partial w_0}{\partial x} \right)^2 \right] = 0,$$

$$EJ \frac{\partial^4 w_0}{\partial x^4} - ES \left\{ \left[ \frac{\partial u_0}{\partial x} + \frac{1}{2} \left( \frac{\partial w_0}{\partial x} \right)^2 \right] \frac{\partial^2 w_0}{\partial x^2} \right\} + \rho S \frac{\partial^2 w_0}{\partial t^2} = F(x, t) - D(x, t).$$

(4)

Since there are no any longitudinal loadings acting on the beam, the boundary conditions for $u_0$ are

$$u_0(0, t) = \frac{\partial u_0}{\partial x} \bigg|_{x=l} = 0.$$  

(5)

Integration of the first equation gives

$$\frac{\partial u_0}{\partial x} + \frac{1}{2} \left( \frac{\partial w_0}{\partial x} \right)^2 = f(t),$$

(6)

which together with the second boundary condition in (5) leads to

$$f(t) = \frac{1}{2} \left( \frac{\partial w_0}{\partial x} \right)^2 \bigg|_{x=l}.$$  

Integration of (6) once more gives

$$u_0(x, t) = -\frac{1}{2} \int_0^x \left( \frac{\partial w_0}{\partial \xi} \right)^2 d\xi + \frac{1}{2} \left( \frac{\partial w_0}{\partial x} \right)^2 \bigg|_{x=l} \cdot x,$$

(7)

where the first boundary condition in (5) is taken into account.

Thus,

$$\frac{\partial u_0}{\partial x} = -\frac{1}{2} \left( \frac{\partial w_0}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial w_0}{\partial x} \right)^2 \bigg|_{x=l},$$

(8)

whose substitution into the second equation in (4) reduces to a single fourth order nonlinear ordinary differential equation with respect to $w_0$

$$EJ \frac{\partial^4 w_0}{\partial x^4} - \frac{ES}{2} \left( \frac{\partial w_0}{\partial x} \right)^2 \bigg|_{x=l} \frac{\partial^2 w_0}{\partial x^2} + \rho S \frac{\partial^2 w_0}{\partial t^2} = F(x, t) - D(x, t).$$

(9)

2.1. Influence of the loads and dampers

Involving the theory of generalized functions, the influence of moving loads can be described by the potential [18, 19, 22]

$$F(x, t) = P_1 \delta(x - vt_1) + P_2 \delta[x - l + v_2(t - t_0)],$$

(10)
at this (for generality), we assume that the load of intensity \( P_2 \) starts moving from the end \( x = l \) of the beam at time moment \( t_0 \geq 0 \). (The case \( t_0 = 0 \) corresponds to the case of simultaneous start of motion of the loads.)

The influence of the dampers, again within the theory of generalized functions, can be expressed by [18, 19, 22]:

\[
D(x,t) = \left( \alpha w_0 + \beta \frac{\partial w_0}{\partial t} \right) \sum_{k=1}^{3} \delta(x - x_k), \quad x_k = \frac{kl}{4} \quad (11)
\]

2.2. Boundary and initial conditions

As was supposed above, the beam is simply supported, therefore the corresponding boundary conditions read as

\[
w_0(0,t) = w_0(l,t) \equiv 0,
\]

\[
\frac{\partial^2 w_0}{\partial x^2} \bigg|_{x=0} = \frac{\partial^2 w_0}{\partial x^2} \bigg|_{x=l} \equiv 0, \quad t \geq 0. \quad (12)
\]

To derive a unique solution, the following initial conditions are given:

\[
w_0(x,0) = w_i(x), \quad \frac{\partial w_0}{\partial t} \bigg|_{t=0} = w_1^i(x), \quad 0 \leq x \leq l. \quad (13)
\]

We assume that \( w_i \) and \( w_1^i \) are continuous on \([0,l]\). For simplicity, we can take \( w_i = w_1^i \equiv 0 \), i.e., before \( P_1 \) starts to act on the beam, it was in equilibrium.

2.3. Bubnov–Galerkin procedure

To derive \( w_0 \) from (4)$_2$ (since \( w_0 \) is well defined in terms of \( w_0 \) from (4)$_1$), we involve the Bubnov–Galerkin procedure [16]. Because of its weak or variational nature, it is not required to pass to the weak formulation of the problem, as it is usually done [17]. An approximate solution of (4)$_2$, (12) is sought in the form

\[
w_0N(x,t) = \sum_{n=1}^{N} \lambda_n(t) \mu_n(x), \quad \mu_n(x) := \sin(n \pi x). \quad (14)
\]

Following the Bubnov–Galerkin procedure, \( \lambda_n, n = 1, \ldots, N, \) is determined from the system of nonlinear ordinary differential equations of second order

\[
\rho S \ddot{\lambda}_\nu(t) + \sum_{n=1}^{N} \left\{ k A_\nu \beta \dot{\lambda}_n(t) + (k A_\nu \alpha + B_\nu E J) \lambda_n(t) + \frac{ES}{2} \pi^4 n^2 C_\nu \lambda_n(t) \sum_{k=1}^{N} k \cos(\pi kl) \lambda_k(t) \right\}^2 \quad (15)
\]

where

\[
k A_\nu = \mu_n(x_k) \mu_\nu(x_k), \quad B_\nu = \int_0^l \mu_\nu^V(x) \mu_\nu(x) \, dx, \quad C_\nu = \int_0^l \mu_\nu^V(x) \mu_\nu(x) \, dx.
\]

Note that (13) provides the initial conditions on \( \lambda_n \):

\[
\lambda_n(0) = \int_0^l w_i(x) \sin(n \pi x) \, dx, \quad \dot{\lambda}_n(0) = \int_0^l w_1^i(x) \sin(n \pi x) \, dx, \quad n = 1, \ldots, N. \quad (16)
\]

Thus, determining the beam displacements is reduced to solving the Cauchy problem (15), (16). In this study, we involve a finite difference scheme to determine the expansion coefficients \( \lambda_n, n = 1, \ldots, N, \) without linearizing (15), as is usually the case [12–14].
3. Numerical results and discussions

Numerical analysis is carried out when the dampers are equally spaced, however, in [18, 19], it is shown that, in that way, one can attain the maximal damping rate and the minimal value of absolute value of the displacement.

We assume, that at initial moment, the beam is in equilibrium, i.e., \( w_i = w_i^0 \equiv 0 \). Let the cross-section of the beam be a solid rectangle of unit area, and let the beam material be steel with \( E = 2 \times 10^{11} \text{ kg/m}^2 \) and \( \nu = 0.3 \). The transverse and longitudinal displacements of the beam are examined for different combinations of the load parameters. We conventionally consider the following two cases. In the first case, we study the transverse and longitudinal displacements of the point \( x_0 = 12 \text{ m} \) (the system of reference is at the left end of the beam, and \( l = 100 \text{ m} \)), while in the second case, we study the same displacements of the point at which the loads meet.

The parameters are chosen in the following ranges: \( P_1, P_2 \in \{10^5, 2 \times 10^5, 4 \times 10^5\} \text{ kg} \), \( v_1, v_2 \in \{4, 8, 32, 64\} \text{ m/s} \), and \( t_0 \in \{0, 2, 4\} \text{ s} \). The cases considered below are numbered as \( m.n.k.l \), where \( m \) corresponds to \( P_1 \), \( n \) — to \( P_2 \), \( k \) — to \( v_1 \), and \( l \) — to \( v_2 \). So, when we write Case 1.2.3.4, we mean \( P_1 = 10^5 \text{ kg}, P_2 = 2 \times 10^5 \text{ kg}, v_1 = 32 \text{ m/s}, v_2 = 64 \text{ m/s}, \) and \( t_0 \) is usually fixed.

Figures 2 and 3 indicate the influence of the right load on the beam horizontal and vertical deflection at point \( A \) for fixed values of the parameters of the left load: more precisely, the cases 1.1.1.1, 1.1.1.4, 1.1.2.4, 1.1.4.1, respectively.

Figure 2 indicates a certain shift of \( u_0 \) displacement in the horizontal direction depending on \( t_0 \): the greater \( t_0 \), the greater the shift. The upper two graphs show that an increase in the velocity of the right load leads to an increase in (absolute) \( u_0 \) for \( t_0 = 2\text{ s} \) and to a decrease in (absolute) \( u_0 \) for \( t_0 = 4\text{ s} \) (relative to \( t_0 = 0\text{ s} \)). An increase in the velocity of the left load leads to an increase in the (absolute) \( u_0 \) for \( t_0 = 0\text{ s} \) and to a decrease in (absolute) \( u_0 \) for both \( t_0 = 2\text{ s} \) and \( t_0 = 4\text{ s} \).

The same dependence of (absolute) \( w_0 \) on \( v_1 \) and \( v_2 \) is observed in figure 3.

Figures 4 and 5 show the influence of the right load on the beam horizontal and vertical deflection at point \( B \) for fixed values of the parameters of the left load: more precisely, the cases 1.1.1.1, 1.1.1.4, 1.1.2.4, 1.1.4.1, respectively. Figure 4 indicates the same behavior of (absolute) \( u_0 \) on \( v_1 \) and \( v_2 \), while figure 5 reveals a different picture. An increase in \( v_2 \) leads to an increase in (absolute) \( w_0 \) for \( t_0 = 4\text{ s} \) only (relative to \( t_0 = 0\text{ s} \) and \( t_0 = 2\text{ s} \)). On the other hand, an increase in \( v_1 \) increases the maximal (absolute) value of \( w_0 \).

Figures 6, 7 and 8, 9 graphically express the dependence of \( u_0 \) and \( w_0 \) of points \( A \) and \( B \), respectively, on \( P_1 \) and \( P_1 \) (\( v_1 \) and \( v_2 \) are fixed: the same behavior is seen for all \( v_1 \) and \( v_2 \)). A shift depending on \( t_0 \) is detected in all cases. Unlike the previous case, neither \( P_1 \) nor \( P_2 \) lead to an increase or a decrease in \( u_0 \) and \( w_0 \) for different \( t_0 \).

It is planned to study the possibility of optimal location of the dampers under the beam in order to reduce its vibrations in the minimal possible time. The results will be reported in subsequent papers.

Conclusions

The influence of two different point loads moving on a beam in opposite directions is considered. The beam is simply supported at its ends and is also supported by three equidistant visco-elastic dampers. The von Kármán theory is accepted with respect to the beam, and it is assumed that the dampers are filled with Kelvin–Voight visco-elastic material. Assuming that the loads start to move at different time moments, the dependence of the beam vertical displacements on external parameters (loads intensities, velocities and starting time moment) is studied. Fully nonlinear problem is studied and strongly nonlinear behavior is observed. Future goals are set up.
Figure 2. Horizontal deflection of point A in the cases 1.1.1.1, 1.1.1.4, 1.1.2.4, 1.1.4.1, respectively.

Figure 3. Vertical deflection of point A in the cases 1.1.1.1, 1.1.1.4, 1.1.2.4, 1.1.4.1, respectively.
Figure 4. Horizontal deflection of point $B$ in the cases 1.1.1.1, 1.1.1.4, 1.1.2.4, 1.1.4.1, respectively.

Figure 5. Vertical deflection of point $B$ in the cases 1.1.1.1, 1.1.1.4, 1.1.2.4, 1.1.4.1, respectively.
Figure 6. Horizontal deflection of point A in the cases 1.4.1.1, 4.1.1.1, 4.2.1.1, respectively.

Figure 7. Vertical deflection of point A in the cases 1.4.1.1, 4.1.1.1, 4.2.1.1, respectively.
Figure 8. Horizontal deflection of point $B$ in the cases 1.4.1.1, 4.1.1.1, 4.2.1.1, respectively.

Figure 9. Vertical deflection of point $B$ in the cases 1.4.1.1, 4.1.1.1, 4.2.1.1, respectively.
Acknowledgments

Acknowledgements to our teachers Prof. Dr. Eduard Kh. Grigoryan (1937–2016) and Dr. Hamlet V. Hovhannisyan (1956–2016), who unexpectedly passed away before realizing their great scientific ideas.

References

[1] Frýba L 1999 Vibration of Solids and Structures under Moving Loads (London: Thomas Telford)
[2] Bajer C and Dyniewicz B 2012 Numerical analysis of vibrations of structures under moving inertial load (Berlin - Heidelberg: Springer)
[3] Ouyang H 2011 Moving-load dynamic problems: A tutorial (with a brief overview) Mech. Syst. Signal Process. 25 2039–60
[4] Savin E 2001 Dynamic amplification factor and response spectrum for the evaluation of vibrations of beams under successive moving loads J. Sound Vibr. 248 (2) 267–88
[5] Thambiratnam D and Zhuge Y 1996 Dynamic analysis of beams on an elastic foundation subjected to moving loads J. Sound Vibr. 198 (2) 149–69
[6] Museros P and Martinez-Rodrigo M D 2007 Vibration control of simply supported beams under moving loads using fluid viscous dampers J. Sound Vibr. 300 (1-2) 292–315
[7] Di Lorenzo S, Di Paola M, Failla G, and Pirrotta A 2016 On the moving load problem in Euler–Bernoulli uniform beams with viscoelastic supports and joints Acta Mech. 228 (3) 805–21
[8] Samani F S and Pellicano F 2009 Vibration reduction on beams subjected to moving loads using linear and nonlinear dynamic absorbers J. Sound Vibr. 325 (4-5) 742–54
[9] Samani F S and Pellicano F 2012 Vibration reduction of beams under successive traveling loads by means of linear and nonlinear dynamic absorbers J. Sound Vibr. 331 (10) 2272–90
[10] Sapountzakis E J and Douarakopoulos J A 2009 Nonlinear dynamic analysis of Timoshenko beams by BEM. Part I: Theory and numerical implementation Nonlin. Dyn. 58 295–306
[11] Dyniewicz B, Pisasri D, and Bajer C I 2017 Vibrations of a Mindlin plate subjected to a pair of inertial loads moving in opposite directions J. Sound Vibr. 386 265–82
[12] Yoshimura T, Hino J, and Anantharayana N 1986 Vibrational analysis of a non-linear beam subjected to moving loads by using the Galerkin method J. Sound Vibr. 104 (2) 179–86
[13] Chang T-P and Liu Y-N 1996 Dynamic finite element analysis of a nonlinear beam subjected to a moving load Int. J. Solids Struct. 33 (12) 1673–88
[14] Yanmeni Wayou A N, Tchoukuegno R, and Woafo P 2004 Non-linear dynamics of an elastic beam under moving loads J. Sound Vibr. 273 (4-5) 1101–8
[15] Mamandi A, Kargarnovin M H, and Younessian D 2010 Nonlinear dynamics of an inclined beam subjected to a moving load Nonlin. Dyn. 60 (3) 277–93
[16] Mikhlin S M 1991 Error Analysis in Numerical Processes (Chichester - New York: Wiley & Sons)
[17] Liu C-S, Liu D, and Jhao W-S 2017 Solving a singular beam equation by using a weak-form integral equation method Appl. Math. Lett. 64 51–8
[18] Khursudyan Am Zh and Khursudyan As Zh 2014 Optimal distribution of viscoelastic dampers under elastic finite beam under moving load Izv. Nats. Akad. Nauk Armenii. Mekh. 67 (3) 56–67
[19] Khursudyan As Zh 2015 The Bubnov–Galerkin method in control problems for bilinear systems Automat. Remote Contr. 76 (8) 1361–8
[20] Reddy J N 2015 An Introduction to Nonlinear Finite Element Analysis: with Applications to Heat Transfer, Fluid Mechanics, and Solid Mechanics, 2nd Edition (Oxford: Oxford University Press) p 768
[21] Ciarlet F G 1997 Mathematical Elasticity, Vol. II: Theory of Plates (Amsterdam: North-Holland)
[22] Avetisyan A S and Khursudyan As Zh 2016 Green’s function approach in approximate controllability problems Izv. Nats. Akad. Nauk Armenii. Mekh. 69 (2) 3–20
[23] Avetisyan A S and Khursudyan As Zh 2017 Green’s function approach in approximate controllability for nonlinear physical processes Modern Phys. Lett. A 32 (21) 1730015