Learning-Based Symbol Level Precoding: A Memory-Efficient Unsupervised Learning Approach

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Abstract—Symbol level precoding (SLP) has been proven to be an effective means of managing the interference in a multiuser downlink transmission and also enhancing the received signal power. This paper proposes an unsupervised-learning based SLP that applies to quantized deep neural networks (DNNs). Rather than simply training a DNN in a supervised mode, our proposal unfolds a power minimization problem, whose accuracy is bounded by the optimization algorithm. [14].

I. INTRODUCTION

Multiple-input-multiple-output (MIMO) is one of the essential techniques for fifth-generation (5G) wireless communications and has recently attracted a myriad of research. Conventional block-level precoding (BLP) methods that exploit the spatial multiplexing of the multi-user MIMO system, is employed at the base station (BS) to mitigate the multi-user interference (MUI) have proven to be computationally efficient than the optimal dirty paper coding (DPC) but suffer performance deterioration [1]. The method for classifying instantaneous interference into constructive and destructive was first investigated in [2]. The suboptimal precoding methods that exploit constructive interference (CI) were first introduced in [3].

The optimization-based precoding methods are intriguing because of their propensity to deliver various performance targets. The first optimization-based CI precoding was proposed in the context of vector perturbation strategy in [4]. Additional performance is achieved by applying the precoding coefficients on a symbol-by-symbol basis termed symbol level precoding (SLP) that exploits the multi-user interference via CI with the known channel state information (CSI) and converts it into beneficial power at the receiver. Such precoding strategies have been extensively studied over the last five years. [5–8].

More recently, a closed-form optimal precoding design via CI exploitation in the MISO downlink for optimization with both strict and relaxed phase rotations was proposed [9]. Running CI-based precoding methods online on a symbol-by-symbol basis can be computationally taxing despite the outstanding performance they offered.

With relatively low inference complexity, deep learning (DL)-based precoding designs have recently been proposed for MU-MIMO downlink transmission. [10–13]. However, learning-based strategies for wireless physical layer designs use DL model as a function approximator in a supervised learning mode, which requires labeled training data. This labeled training data is obtained from the analytical solution of the optimization problem, whose accuracy is bounded by the optimization algorithm. [14].

The DL model contains millions of trained parameters, which are often stored in a 32-bit floating-point (FP32) numerical format. However, this renders the trained DL model computationally inefficient during inference and challenging to deploy to the edged devices due to resource limitations (memory and power constraints). Scalable deep neural network (DNN) models, whose weights are expressed in lower numerical precision, have been recently attracted a lot of attention [15, 17]. While this idea is not new in computer vision, it has not been adequately explored within the wireless communications domain. A scalable DNN-based MIMO receiver design, where the insignificant neurons were systematically attenuated or removed via monotonically decreasing functions to reduce the network’s size, was first introduced in the work of [18, 19]. However, in this work, we propose an unsupervised, low precision DNN-based SLP framework, where DNN weights are constrained to binary values based on the initial work on scalable learning-based SLP designs [20].

II. SYSTEM MODEL AND SYMBOL LEVEL PRECODING

A. System Model

Consider single cell MU-MISO downlink transmission scenario where K single-antenna users are served by an M BS antennas. Assuming a flat-fading Rayleigh channel $h_i \in \mathbb{C}^{N_t \times 1}$, the received signal at the $i$-th user is expressed as

$$y_i = h_i^T \sum_{k=1}^{K} w_k d_k + n_i,$$  \hspace{1cm} (1)

where $h_i$, $w_k$, $d_k$ and $n_i$ represent the channel vector, precoding vector, data symbol, phase rotation and additive white Gaussian noise for the $i$-th user.

Instantaneous interference is categorized into constructive and destructive [21]. As an illustration, Fig. 1(a) shows the QPSK constellations diagram, where the CI area is indicated by the green region with respect to the minimum distance ($\gamma$) from the decision boundaries, allowing the interfering signals to be added constructively with the symbol of interest via precoding vectors. The generic geometrical representation of the CI in Fig. 1(b) shows that if the maximum angle shift ($\phi = 0$) in the CI region is zero, the interfering signals overlap completely on the symbol of interest. Hence, the problem becomes a strict phase angle optimization. However, the strict phase formulation is not appealing because it leads to an additional transmission power compared to the corresponding relaxed counterpart [21]. For simplicity, we define the following variables, $h_i = \sum_{k=1}^{K} \xi_k \langle h_k - s_i \rangle \in \mathbb{C}^{M \times 1}$, $w = \sum_{k=1}^{K} \xi_k w_k \in \mathbb{C}^{M \times 1}$, $h_i = R[h_i]$ , $h_i = \Im{h_i}$, $w_R = R[w]$ and $w_I = \Im{w}$. Similarly, we also let $\Psi = [h_i^T h_i]^T$, $w_1 = [w_R - w_I]^T$, where $\Theta = \begin{bmatrix} O_M & -I_M \\ I_M & O_M \end{bmatrix} \in \mathbb{R}^{2M \times 2M}$. 

Fig. 1(b) shows that if the maximum angle shift ($\phi = 0$) in the CI region is zero, the interfering signals overlap completely on the symbol of interest.
B. Conventional Robust Precoding

In practice, the exact channel state information (CSI) is often unknown; only the estimate is

\[ \hat{h}_i = h_i + e_i, \forall k, \]  

where \( h_i \) is the known CSI estimates at the BS and \( e_i \) denotes the channel error. Given this, the robust traditional recording becomes

\[ \min \{ W_i \geq 0, d_i \geq 0 \} \sum_{i=1}^{K} \text{trace}(\bar{W}_i) \]

\[ \text{s.t.} \]

\[ \hat{h}_i T \hat{h}_i^T - \gamma_{i,n_0} - d_i \delta_i^2 \hat{h}_i T_i - \delta_i^2 I \geq 0, \forall k \]

where \( T_i \triangleq W_i - \Gamma_i \sum_{k=1, k \neq i} W_k \forall k \) and \( W_i = w_i w_i^T \).

C. Robust SLP optimization-Based Power Minimization

The multi-cast CI formulation of the power minimization problem for the worst-case scenario is given by

\[ \min_{\{w\}} \|w\|^2 \]

\[ \text{s.t.} \]

\[ 0 \leq \bar{w}_i \leq \delta_i^2, \forall i. \]

For simplicity, we drop the subscripts in (1) and slit the real and imaginary parts of the constraint into two separate constraints.

\[ \Psi^T w_1 - \Psi^T w_2 \tan \phi + \delta \|w_1 - w_2 \tan \phi\|_2 + \sqrt{\gamma_{1,n_0}} \tan \phi \leq 0, \]  

\[ \Psi^T w_1 - \Psi^T w_2 \tan \phi + \delta \|w_1 + w_2 \tan \phi\|_2 + \sqrt{\gamma_{1,n_0}} \tan \phi \leq 0, \]

where \( \delta \triangleq [e_R \ e_I]^T \) and \( \hat{h} = h_R + jh_I + e_R + je_I \). Then (5) becomes

\[ \min_{\{w_1, w_2\}} \|w_1\|^2 \]

\[ \text{s.t.} \]

Constraints (5) and (6), \( \forall i \)  

where \( w_1 = \Pi w_2 \).

III. ROBUST LOW-BIT DNN-BASED SLP FOR POWER MINIMIZATION PROBLEM

This section presents robust binary and ternary DNN-based SLP models (RSLP-BDNet and RSLP-TDNet). We begin first by formulating the full-precision DNN-based SLP counterpart (RSLP-DNet). From (7), we define the following:

\[ Q_1 = (\Theta - \tan \phi I) \]  

and \[ Q_2 = (\Theta + \tan \phi I) \]. Therefore, constraints (5) and (6) can be written as

\[ \Lambda^T Q_1 w_2 + \delta \|Q_1 w_2\|_2 + \sqrt{\gamma_{1,n_0}} \tan \phi \leq 0 \]  

\[ \Lambda^T Q_2 w_2 + \delta \|Q_2 w_2\|_2 + \sqrt{\gamma_{1,n_0}} \tan \phi \leq 0 \]

Following this, (7) is thus

\[ \min_{\{w_2\}} \|w_2\|^2 \]

\[ \text{s.t.} \]

Constraints (8) and (9), \( \forall i \).

A. SLP using Interior Point Method

We begin by unfolding (10) using an IPM ‘log’ barrier function and transform it to its equivalent unconstrained sequence of sub-problems per user

\[ \min_{w \in \mathbb{R}^{|2Mx1|}} f(w) + vB(w), \]

where \( B(\cdot) = -\sum \ln(\cdot) \) is the logarithmic barrier function, \( v \) is the Lagrangian multiplier for inequality constraints. The learning framework is derived by defining a proximity operator of (11)

\[ \text{prox}_{\gamma vB}(w_2) = \arg\min_{w_2 \in \mathbb{R}^{|2Mx1|}} \frac{1}{2} \|w_0 - w_2\|^2_2 + \gamma vB(w_2), \]

where \( \gamma \in (0, +\infty) \) is the training step-size, \( w_0 \) is the initial predicing vector and \( v \) is the Lagrange multiplier of the inequality constraint.

1) Euclidean Constraint: It can be observed that the constraints (8) and (9) are bounded by the \( \ell_2 \)-norm of the form

\[ C = \{z \in \mathbb{R}^n \mid \|z - x\|_2 \leq \alpha \}, \]

where \( \alpha > 0 \) and \( x \in \mathbb{R}^n \). The ‘log barrier function is given by

\[ B(z) = -\ln(\alpha - \|z - x\|^2_2), \text{ if } \|z - x\|_2 < \alpha \]

\[ +\infty, \text{ otherwise.} \]

Based on (13), the barrier function for (12) is expressed at the bottom of this page. Similar expression can also be written for (9). Therefore, the effective barrier function for the two constraints is the sum of the individual barrier functions

\[ B(w_2) = B_1(w_2) + B_2(w_2). \]

It can be seen that the upper bounds of the two constraints \( (8) \) and \( (9) \) are zeros. Therefore, combining (8) and (9), we obtain

\[ (\delta^2 - \Psi^T \Psi) G \|w_2\|^2 + 4\Psi^T \Psi \tan \phi \sqrt{\gamma_{1,n_0}} \leq 2\Gamma_{1,n_0} \tan^2 \phi \]
where $G = Q_1^T Q_1 + Q_2^T Q_2$. Consequently, for each $w_2$, the proximity operator of the barrier $\gamma \nu B$ is

$$
\Psi(w_2, \gamma, \nu) = \frac{2T \nu \tan^2 \phi - \chi(w_2, \gamma, \nu)^2}{2T \nu \tan^2 \phi - \|\Psi(w_2, \gamma, \nu)\|_2^2 + 2\gamma \nu} w_2 
$$

where $\chi(w_2, \gamma, \nu)$ is the analytical solution of the cubic equation \[14\]. The robust deep-unfolded model is derived according to the derivatives of \[17\] with respect to $w_2$, $\gamma$, and $\nu$ as follows

$$
\Psi(w_2, \gamma, \nu) = \frac{2T \nu \tan^2 \phi - \Psi(w_2, \gamma, \nu) \|_2^2 + 2\gamma \nu}{2T \nu \tan^2 \phi - \|\Psi(w_2, \gamma, \nu)\|_2^2 + 2\gamma \nu} M(w_2, \gamma, \nu),
$$

(18)

$$
\Delta \phi \left|_{(\nu)} \right. = -2\nu \frac{2T \nu \tan^2 \phi - \|\Psi(w_2, \gamma, \nu)\|_2^2 + 2\gamma \nu}{2T \nu \tan^2 \phi - \|\Psi(w_2, \gamma, \nu)\|_2^2 + 2\gamma \nu} M(w_2, \gamma, \nu),
$$

(19)

$$
\Delta \phi \left|_{(\gamma)} \right. = -2\gamma \frac{2T \nu \tan^2 \phi - \|\Psi(w_2, \gamma, \nu)\|_2^2 + 2\gamma \nu}{2T \nu \tan^2 \phi - \|\Psi(w_2, \gamma, \nu)\|_2^2 + 2\gamma \nu} M(w_2, \gamma, \nu),
$$

(20)

where $M(w_2, \gamma, \nu)$ is as defined in \[14\].

We use the proximity operator of the barrier to obtain the variable update function as follows

$$
w_2^{(r+1)} = \text{prox}_{\nu \delta f(w_2^{(r)}, \lambda^{(r)})} \left( w_2^{(r)} - \nu \Delta f(w_2^{(r)}, \lambda^{(r)}) \right),
$$

(21)

where $f(w_2^{(r)}, \lambda^{(r)}) = \|w_2\|_2^2 + \lambda w_2$. We define the update function $D$ as

$$
D(w_2^{(r)}, \gamma^{(r)}, \nu^{(r)}, \lambda^{(r)}) = \text{prox}_{\nu \delta f(w_2^{(r)}, \lambda^{(r)})} \left( w_2^{(r)} - \nu \Delta f(w_2^{(r)}, \lambda^{(r)}) \right),
$$

(22)

and $\Delta = \frac{\partial f(w_2^{(r)}, \lambda^{(r)})}{\partial w_2^{(r)}}$.

2) Loss Function: The training loss function is the Lagrangian function of \[17\] obtained as

$$
\mathcal{L}(w_2, v_1, v_2) = \frac{1}{N} \sum_{i=1}^{N} \|w_2\|_2^2 + \nu_1 \sum_{i=1}^{N} \delta_1^2 \|Q_1 w_2\|_2^2 - \left( \sqrt{T \nu \tan \phi - \Psi^T Q_1 w_2 \right) \|_2^2 + \nu_2 \sum_{i=1}^{N} \delta_2^2 \|Q_2 w_2\|_2^2 - \left( \sqrt{T \nu \tan \phi - \Psi^T Q_2 w_2 \right) \|_2^2
$$

$$
+ \frac{\mu N}{K} \sum_{i=1}^{N} \sum_{s=1}^{L} \|\Omega_i\|_2^2,
$$

(23)

where $v_1$ and $v_2$ are the Lagrangian multipliers of the two inequality constraints. The $\Omega_i(s)$ are the trainable parameters of the $i$-th layers and $\mu > 0$ is the penalty parameter that controls the bias and variance of the learnable parameters. Note that are associated with the barrier term and are randomly initialized from a uniform distribution. The model is trained in an unsupervised mode to update $v$, $\lambda$, $\gamma$ and $w_2$ such that the loss function is minimized. By minimizing \[23\] with respect to $w_2$, we obtain the optimal precoder

$$
(1 + (v_1 \|Q_1\|_2^2 + v_2 \|Q_2\|_2^2) \left( \delta_1^2 - \Psi^T \Psi \right)) w_2 = - \left( v_1 Q_1 + v_2 Q_2 \right) \Psi \sqrt{T \nu \tan \phi}.
$$

(24)

For clarity, we let $\|Q_1\|_2^2 = \bar{Q}_1$, $\|Q_2\|_2^2 = \bar{Q}_2$ and $[v_1 \ v_2] = \bar{v}$. Hence, \[24\] is reduced to

$$
(\bar{I}_M + \bar{Q}_1 \bar{v}^T \left( \delta^2 - \Psi^T \Psi \right)) w_2 = -\Psi \bar{Q} \bar{v}^T \sqrt{T \nu \tan \phi}
$$

(25)

The optimal transmit precoder is finally obtained as

$$
w_2 = -\Psi \bar{Q} \bar{v}^T \bar{P} \left( \delta^2 \bar{I}_M - \Psi^T \Psi \right),
$$

(26)

where $\bar{P} = (\bar{I}_M + \bar{Q}_1 \bar{v}^T \left( \delta^2 \bar{I}_M - \Psi^T \Psi \right))$.

B. RSLP-DNet and the Generic NN Architecture

Intuitively, we can form NN cascade layers from \[21\] as follows

$$
w_2^{(r+1)} = \text{prox}_{\nu \delta f(w_2^{(r)}, \lambda^{(r)})} \left( w_2^{(r)} - \nu \Delta f(w_2^{(r)}, \lambda^{(r)}) \right),
$$

(27)

where $1 \in \mathbb{R}^{1 \times 2M}$ is a vector of ones. By letting $W_i = I_{2M} - 2\gamma \bar{v} \bar{b}_i^T \bar{I}^T$, and $\Xi = \text{prox}_{\nu \delta f(w_2^{(r)}, \lambda^{(r)})}$, the $l$-layer network \[l \cdot l \cdot l \cdot l \cdot l \] will correspond to the following

$$
\Xi_0 \left( W_0 + b_0 \right), \ldots, \Xi (W_l + b_l),
$$

(28)

where $W_i$ and $b_i$ represent weight and bias parameters, respectively, and $\Xi_i$ describes the nonlinear activation functions. Finally, based on this formulation, RSLP-DNet is built as shown in Fig.\[15\] and its internal NN designs are summarized in Tables \[16\] and \[17\].

| TABLE I: Proximity Barrier Term DNN Design |
|------------------------------------------|
| Layer | Parameter, kernel size = $3 \times 3$ |
| Input Layer | Input size (B, 1, 2M, K) |
| Layer 1: Convolutional | Size (B, 20, 2M, K); zero padding |
| Layer 2: Average Pooling | Size (1, 1), stride = (1, 1) |
| Layer 3: Activation | Soft-Plus |
| Layer 4: Flat | Size (B x 40 x K^2) |
| Layer 5: Fully-connected | Size (B x 40 x K^2, 1) |
| Layer 5: Activation | Soft-Plus function |

| TABLE II: A PPU DNN Design |
|-----------------------------|
| Layer | Parameter, kernel size = $3 \times 3$ |
| Input Layer | Input size (B, 1, 2M, K) |
| Layer 1: Convolutional | Size (B, 16, 2M, K), dilatation = 1 and unit padding |
| Layer 2: Batch Normalization | eps = $10^{-6}$, momentum = 0.1 |
| Layer 3: Activation | PReLU/kbit function |
| Layer 4: Convolutional | Size (B, 8, K, 2KM), dilatation = 1 and unit padding |
| Layer 5: Batch Normalization | eps = $10^{-5}$, momentum = 0.1 |
| Layer 6: Activation | PReLU/kbit function |
| Layer 7: Convolutional | Size (B, 1, 2M, 1), dilatation = 1 and unit padding |

$$
B_1(w_2) = \left\{ -\ln \left( -\sqrt{T \nu \tan \phi - \left( \Psi^T Q_1 w_2 + \delta \|Q_1 w_2\|_2 \right) \|_2 \right), \right.
$$

(15)

if $\Psi^T Q_1 w_2 + \delta \|Q_1 w_2\|_2 < -\sqrt{T \nu \tan \phi}$

otherwise
C. Low-bit DNN Weights

Traditionally, DNN is designed with full-precision weights and activations. The quantization schemes have been proposed to design low-bit DNN models to address the problems of limited storage capacity and reduce hardware requirements during model deployment.

1) Binary Weights:
The real-valued weights are converted to \( W_b \in \{+1, -1\}^n \). A full-precision 32-bit weight matrix is binarized such that the weights \( W \) are converted to their equivalent binary by the following function

\[
W_b = \text{sign}(W) = \begin{cases} +1 & \text{if } W \geq 0 \\ -1 & \text{otherwise} \end{cases}
\]

A more robust binarized network “BWN” is proposed in [15] as an extension of a straightforward binary network (Binary Connect) by introducing a real scaling factor \( \beta \in \mathbb{R}^+ \) such that \( W \approx \beta W_b \) by solving an optimization problem

\[
J(W_b, \beta) = \min_{W_b, \beta} \| W - \beta W_b \|_2^2,
\]

and this yields

\[
W_b^* = \text{sign}(W) \quad \beta^* = \frac{1}{n} \| W \|_1
\]

2) Weighted Ternary Weights:
A ternary weighted network (TWN) is the one in which an extra 0 state is introduced during model deployment.

\[
W_t = \begin{cases} +1 & \text{if } W > \rho \\ 0 & \text{if } |W| \leq \rho \\ -1 & \text{if } W < -\rho,
\end{cases}
\]

where \( \rho = \frac{0.2}{n} \sum_{i=1}^{n} |W| \) and \( \beta^* = \frac{1}{n} \sum_{i=1}^{n} |W| \), \( I_0 = \{|W| > \rho\} \) is the cardinality of set \( I_0 \). As an illustration, Fig. 3 depicts how the weight matrices are quantised based on (31) and (33).

D. RSLP-BDNet Training and Inference

The RSLP-BDNet has two central units; the parameter update unit (PUU) and the post-processing unit (PPU). The parameter unit has three core components; \( \rho \) (associated with the barrier term), \( \lambda \) and \( \gamma \) that are wired across the network (see Fig. 2). The barrier term is formed with one convolutional layer, an average pooling layer, a fully connected layer, and a softPlus layer to satisfy the positive inequality constraint. The PUU has \( r \)-th blocks, each representing a layer, and is trained block-wise for \( l \)-th iterations. Similarly, the PPU is made up three convolutional layers with batch normalisation layers in between them except for the last layer, and is trained for \( k \)-th iterations. The number of training iterations for the PPU may not necessarily be the same as that of the PUU. We train the PUU unit for 15 iterations and the PPU iterations with Adam optimizer [25]. We assume a single cell in a downlink scenario where the BS having four antennas (\( M = 4 \)) serves \( K \), single users. We generate 50000 training and 2000 test samples of channel coefficients, respectively. The transmit symbols are modulated using a QPSK modulation. The training SINR is randomly generated from uniform distribution \( \Gamma_{\text{train}} \sim U(\Gamma_{\text{low}}, \Gamma_{\text{high}}) \) to allow training over wide range of SINR values. Parametric rectified linear unit (PRelu) activation function is used in RSLP-DNet instead of the traditional ReLu function to mitigate the effect of dying gradient due to the saturation of neurons. After every iteration, the learning rate is reduced by a factor \( \alpha = 0.65 \) to facilitate learning convergence. The simulation parameters are summarized in Table 1. We implement the models in Pytorch 1.7.1 and Python 3.7.8 on a computer with the following specifications: Intel(R) Core (TM) i7-6700 CPU Core, 32.0GB of RAM.
average transmit power increases with the SNR against R-SLP-DNet [14] robust SLP optimization-based average transmit power of RSLP-BDNet and RSLP-TDNet. We use 4 and its quantized counterparts (RSLP-BDNet and RSLP-TDNet). We use 4 and 2 users MISO system with CSI error bounds \( \delta \), and QPSK modulation scheme. We compare the performance evaluation of conventional RBLP, RSLP optimization-based, binary and ternary DNN-based models under \( M = 4, K = 4 \) and \( \delta = 0.0002 \). 

In this subsection, we consider a full-precision RSLP-DNet and its quantized counterparts (RSLP-BDNet and RSLP-TDNet). We use 4 \times 4 MISO system with CSI error bounds \( \delta = 10^{-4} \), and QPSK modulation scheme. We compare the average transmit power of RSLP-BDNet and RSLP-TDNet against R-SLP-DNet [14] robust SLP optimization-based [9] and conventional [22] BLP methods. Fig. 4 depicts how the average transmit power increases with the SNR thresholds.

### Computational Complexity Evaluation

The RSLP optimization-based is observed to show a significant power savings of more than 60% compared to the conventional RBPL solution. Similarly, the proposed RSLP-BDNet and RSLP-TDNet show considerable power savings of 40% – 58% against the conventional RBPL but lower than the RSLP optimization-based solution.

Furthermore, we study the effect of the CSI error bounds on the transmit power at 30dB. Fig. 5 depicts the variation of the transmit power with increasing CSI errors. A significant increase in transmit power can be observed where the channel uncertainty lies within the region of CSI error bounds of \( \delta^2 = 10^{-3} \). Interestingly, like the RSLP optimization-based, by exploiting the CI, the proposed methods show a descent or moderate increase in transmit power.

| Parameters                                      | Values       |
|-------------------------------------------------|--------------|
| Training Samples                                | 50000        |
| Batch Size (B)                                  | 200          |
| Test Samples                                    | 2000         |
| Training SINR range                             | 0.0dB - 45.0dB |
| Test SINR range (i-th user SINR)                | 0.0dB - 35.0dB |
| Initial Learning Rate \( \eta \)               | 0.001        |
| Learning Rate decay factor \( \vartheta \)     | 0.65         |
| Number of blocks in the PUU                    | \( B_r = 2 \) |
| Training Iterations for each block of the PUU  | 15           |
| Training iterations for PPU                    | 10           |

### Table III: Simulation settings

| Parameters | Values |
|------------|--------|
| SINR       | 30 dB  |
| Training Iterations for PPU                   | 10     |
| Training Iterations for each block            | 15     |
| Number of blocks in the PUU                   | \( B_r = 2 \) |
| Learning Rate decay factor \( \vartheta \)    | 0.65   |
| Initial Learning Rate \( \eta \)              | 0.001  |
| Test SINR range (i-th user SINR)              | 0.0dB - 35.0dB |
| Training SINR range                           | 0.0dB - 45.0dB |
| K users                                         | 2, 3, 4, 5, 6, 7 |
| SINR (dB)                                       | 0, 5, 10, 15, 20, 25, 30, 35 |
| Error bound \( \delta^2 \)                     | 0.0002  |
parameters, expressed as $\frac{1}{2}W_b + W_f$, where $W_b$ and $W_f$ are the binary and floating-point weights, respectively. Table IV presents the summary of the inference memory requirements, where we observe that RSLP-BDNet and RSLP-TDNet provide considerable memory savings up to $\sim 21\times$ and $\sim 13\times$ compared to the RSLP-DNet, respectively.

V. Conclusion

This paper proposes robust binary and ternary unsupervised learning-based SLP designs for downlink power minimization optimization. The real-valued NN weights are converted to binary values, allowing the operations between the inputs and weights tensors to be performed in binary operations. We use domain knowledge to design unsupervised learning architectures by unfolding the proximal interior point method barrier function for a relaxed phase rotation. The performance is within the range of 89% – 95% of the RSLP optimization-based solution with a substantial computational complexity reduction. Therefore, our proposals demonstrate an indispensable balance between the performance and the computational complexity involved.

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**TABLE IV: Inference memory utilization comparison**

| Models        | Weights | Activations | Memory usage (MB) | Memory saving |
|---------------|---------|-------------|-------------------|---------------|
| RSLP-DNet     | $(32 \times bit) \in \mathbb{R}$ | $(32 \times bit) \in \mathbb{R}$ | 0.1898 | — |
| RSLP-BDNet    | $(-1,+1)$ | $(-1,+1)$ | 0.0089 | 21.33x |
| RSLP-TDNet    | $(-1,0,+1)$ | $(-1,+1)$ | 0.0146 | 13x |

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