Abstract

We show that the dynamical behavior of the 3D Ising spin glass with Gaussian couplings is not compatible with a droplet dynamics. We show that this is implied from the data of reference [1], that, when analyzed in an accurate way, give multiple evidences of this fact. Our study is based on the analysis of the overlap-overlap correlation function, at different values of the separation r and of the time t.

In a very interesting paper [1] Komori, Yoshino and Takayama discuss the dynamical behavior of the three dimensional (3D) Edwards Anderson (EA) Ising Spin Glass with Gaussian couplings. A number of very accurate numerical evidences are used to try to determine if the 3D EA model behaves in analogy to the Replica Symmetry Broken (RSB) solution of the Mean Field Sherrington Kirkpatrick model [2], or if its behavior is the one typical of coarsening of domain walls [3], in agreement with a picture of separated droplets [4].

The evidences presented in [1] can seem at first view to have mixed signs. For sake of completeness let us try to summarize all the available evidence.

As it is well explained in the paper [1] some of the findings are obviously devastating for a droplet picture. The free energy barriers $B_L$ found on a lattice of linear size $L$ do not grow with $L$ like $L^0$, but with a logarithmic dependence $B_L \propto \log(L)$ (this was already

1
observed by Kisker, Rieger, Santen and Schreckenberg in [5]). Secondly, at variance with the droplet picture, the width of the distribution of the free energy barriers instead of increasing with $L$ turns out to be remarkably constant. These two features do not have any problem with the RSB picture.

Two more findings are not in contradiction with any of the two exemplar behaviors. The relaxation process has an Arrhenius-type form, with the characteristic time of the form $\tau = \tau_0 \exp\left(\frac{B}{T}\right)$, and the decay of the energy density to its infinite time asymptotic value follows a power law. Both the droplet picture and the RSB scheme imply a power law decay of the energy density.

The crucial point, as rightly pointed out in [1], is the dynamical behavior of the overlap-overlap ($q - q$) correlation functions $G(r, t)$ (computed starting from two different random spin configurations, considering overlaps at distance $r$ and equal time $t$). A simple scaling of $G$ as a function of $\frac{r}{\xi(t)}$ would indeed be a strong argument in favor of a droplet behavior, while evidence for a different scaling points to a RSB pattern. The analysis of [1] claims to show a droplet like behavior of $G(r, t)$. Here we show that, on the contrary, the numerical data exclude this simple scaling form. We show that they behave in the non-standard way already discussed in reference [6]. We perform a careful data analysis, and show in detail where the analysis of [1] fails. In this way we show that the observed picture is fully compatible with an RSB behavior, and that none of the available numerical supports a droplet-like behavior: one more crucial piece of evidence, the one concerning the behavior of the overlap-overlap correlation functions, conjures with the observed behavior of the free energy barriers to exclude a dynamical droplet like behavior of $3D$ Ising spin glasses.

We will try to make clear to the reader how delicate this analysis is, how even a careful treatment like the one of reference [1] can misdirected by only apparent straight lines, and why we can get a completely safe evidence of a non-trivial scaling of $G(r, t)$.

As we have already said the crucial difference in the approach to equilibrium among the droplet approach and the RSB theory is in the behavior of the $q - q$ correlation function during the approach to equilibrium.

Let us define

$$G(r) \equiv \lim_{t \to \infty} G(r, t) .$$  \hspace{1cm} (1)

In the droplet model we have

$$\lim_{r \to \infty} G(r) = 7 \neq 0 ,$$ \hspace{1cm} (2)

while in the RSB solution of the Mean Field theory (in the $q \simeq 0$ sector of the theory: we are analyzing this case since we start out from equilibrium on a very large lattice) we find $\ddagger$

$$G(r) \approx r^{-\mu} ,$$ \hspace{1cm} (3)

and therefore

\ddagger A few details about our runs: we have analyzed four samples of a $3D$ Ising spin glass with Gaussian couplings, periodic boundary conditions, on a lattice of linear size $L = 64$. We have computed the overlap-overlap correlation function for runs of time length $t_n = 100 \cdot 2^n$, where $n$ runs from 1 to 13. We average the correlation function over the whole run: this gives a measured $G$ that is numerically slightly different from the one of [1] but has the same scaling behavior. We have used random initial configurations.
The overlap-overlap correlation functions $G(r, t)$ versus $\frac{r}{\xi}$ for different $r$ and $t$ values.

$$\lim_{r \to \infty} G(r) = 0 \, . \quad (4)$$

In reference [6] we have studied this problem and we have concluded that $\mu \approx .5$, and therefore that a droplet-like behavior is inconsistent with the results of numerical simulations.

Reference [1] argues that the numerical data for $r \geq 4$, $T = 0.7$, scale like

$$G(r, t) = M\left(\frac{r}{\xi(t)}\right) \, , \quad (5)$$

where $\xi(t) = t^{\frac{1}{z(t)}}$, $z(T = 0.7) \approx 8.71$ and $M$ is a scaling function. As we have already argued the validity of this analysis would be a good point in favor of a droplet like behavior of $3D$ Ising spin glasses (at variance with the many other evidences of [1], that we have discussed before, that point against the validity of a droplet picture). Since this conclusion is different from the one we had reached in [6] we will analyze here its derivation in better detail. In the rest of the paper (in particular in figures 1, 2 and 3) we use the value $z(T = 0.7) = 8.71$.

The conclusion of [1] is based on the plot of the raw data for $G(r, t)$ as a function of $\frac{r}{\xi(t)}$ for $r \geq 4$ (figure 5 of [1]). So at first, mainly to exclude the possibility of programming errors, we compare our raw data ([3, 4]) to the ones of figure 5 of [1]. We plot in figure 1 our raw data for $G(r, t)$ versus $\frac{r}{\xi}$: by comparing with [1] one can see they are not substantially
different, so we can feel safe about the quality of the numerical data of both [1] and [6, 7]. In figure 1 we have selected the same $x$ and $y$ scales than in figure 5 of [1], to make the comparison of the two sets of data easier. In order to make a visual comparison easier we have also selected time values as close as possible to the ones of [1]. In figure 1 of this note like in figure 5 of [1] the data can seem at first view well aligned on a straight line, i.e. decreasing with a simple exponential behavior. We will show now that this is just not true, and that these data clearly follow a non-exponential behavior.

The fact that the decay is not purely exponential can be clearly seen, for example, by plotting only the points with $r = 4$ on the same semilogarithmic scale: we do that in figure 2. Here it is clear that the points are not on a straight line. It is just the dispersion of the many points in figure 1 that make them looking like being on a straight line: they are not, and figure 2 shows it clearly.

We continue our analysis by trying to double check the conclusion of [6] leading to the functional dependence

$$G(r, t) = \frac{g}{r^{1/2}} \exp \left[ - \left( \frac{r}{\xi(t)} \right)^{3/2} \right]. \quad (6)$$

We start by plotting in figure 3 the same data of figure 2 versus $\left( \frac{r}{\xi(t)} \right)^{3/2}$, together with the (very good) best fit to the form
Figure 3: $G(r = 4, t)$ versus $\left(\frac{4}{\xi}\right)^{3/2}$. The dashed curve is a straight line.

\[ G(r) \exp\left(-\frac{A(r)}{\xi(t)^{3/2}}\right). \]  
(at fixed $r$ as a function of $t$ for the two parameters $G$ and $A$). It is clear from figure 3 and from the quality of the best fit that this is the scale where the data follow a straight line, not the one of figure 2.

We repeat the procedure of figure 3 for different distances (finding always a very good best fit), and we fit the results to a power law (see figure 4). We fit

\[ G(r) = r^{\epsilon_1}, \quad A(r) = r^{\epsilon_2}, \]  
and for the best fit $\epsilon_1 = -0.54$ and $\epsilon_2 = 1.47$, in very good agreement with our initial Ansatz.

Finally, in figure 5, we show $G$ as a function of $r$ for our larger available time ($t = 819200$), and the best fit to the form $a \, r^{-\frac{3}{2}} \exp\left(-b \, r^\frac{3}{2}\right)$. The best fit is again very good. It is interesting to note that, as a final confirmation of our point, the behavior of the function plotted in figure 5 is clearly not the one of a simple exponential, but that in a limited $r$ region it can be mistakenly fitted with a simple exponential.

We believe that these data and the data of [1], when correctly analyzed, give strong evidence in favor of a Replica Symmetry Breaking like dynamical behavior of 3D spin glasses, and that they show beyond any doubt that the droplet model cannot not describe the numerical data one obtains for the $q - q$ correlation function $G(r, t)$. 

5
Figure 4: $G(r)$ and $A(r)$ versus $r$ and the best fit (dashed lines) to a power law (see the text for more details).

Figure 5: $G(r, t = 819200)$ versus $r$. The dashed line is our best fit to $\frac{a}{r^{1.2}} \exp(-b \cdot r^{0.8})$. 
References

[1] T. Komori, H. Yoshino and H. Takayama, *Numerical Study on Aging Dynamics in the 3D Ising Spin Glass Model. I. Energy Relaxation and Domain Coarsening*, cond-mat/9904143.

[2] G. Parisi, Phys. Lett. A 73, 154 (1979); Phys. Rev. Lett. 43, 1754 (1979); J. Phys. A: Math. Gen. 13, L115 (1980); 1101; 1887; Phys. Rev. Lett. 50, 1946 (1983).

[3] A. J. Bray, Adv. Phys. 43, 357 (1994).

[4] W. L. McMillan, J. Phys. C 17, 3179 (1984); A. J. Bray and M. A. Moore, in *Heidelberg Colloquium on Glassy Dynamics*, edited by J. L. Van Hemmen and I. Morgenstern (Springer Verlag, Heidelberg, 1986), p. 121; D. S. Fisher and D. A. Huse, Phys. Rev. Lett. 56, 1601 (1986); Phys. Rev. B 38, 386 (1988).

[5] J. Kisker, L. Santen, M. Schreckenberg and H. Rieger, Phys. Rev. B 53, 6418 (1996); H. Rieger, J. Phys. A 26, L615 (1993); in *Annual Reviews of Computational Physics II* (World Scientific 1995, Singapore) p. 295.

[6] E. Marinari, G. Parisi, J. J. Ruiz-Lorenzo and F. Ritort, Phys. Rev. Lett 76, 843 (1996).

[7] E. Marinari, G. Parisi and J. J. Ruiz-Lorenzo, in preparation.