Universal invariant renormalization of
supersymmetric Yang-Mills theory.

A.A.Slavnov∗

Steklov Mathematical Institute, 117966, Gubkina, 8, Moscow, Russia and
Moscow State University, physical faculty, department of theoretical physics,
117234, Moscow, Russia

and K.V.Stepanyantz†

Moscow State University, physical faculty, department of theoretical physics,
117234, Moscow, Russia

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Abstract

A manifestly invariant renormalization scheme of $N = 1$ nonabelian supersymmetric gauge theories is proposed.

1 Introduction

Construction of a manifestly supersymmetric and gauge invariant renormalization procedure is a nontrivial and in some cases not yet solved problem. Certainly, it is preferrable to have a calculation scheme which preserves supersymmetry at any intermediate stage. However, most popular gauge invariant regularizations like dimensional regularization [1], dimensional reduction [2] or lattice regularization break supersymmetry [3]. Higher covariant derivative regularization [4, 5] can be easily used in Abelian theories [6, 7, 8]. In principle, higher derivative regularization is also applicable to non-Abelian theories [9], but the calculations of quantum corrections are quite involved. In this case it can be convenient to use non gauge invariant regularization. For example, one can regularize the theory by adding higher derivative term with ordinary derivatives, instead of the covariant ones for the Yang-Mills field which simplifies the calculations considerably. Such regularization preserves supersymmetry, but breaks gauge

∗E-mail:slavnov@mi.ras.ru
†E-mail:stepan@theor.phys.msu.su
invariance, which should be restored by a proper choice of a renormalization scheme. For example, in the framework of algebraic renormalization it is done by tuning finite counterterms to provide relevant Slavnov-Taylor Identities (STI) for the renormalized Green functions. This method was applied successfully to SUSY gauge theories in the papers [10, 11], where the invariant renormalizability of $N = 1$ non-Abelian SUSY gauge models was proven in the framework of algebraic renormalization. However a practical implementation of the algebraic renormalization is rather cumbersome as the procedure requires a tuning of a large number of (noninvariant) counterterms.

Recently a new method of invariant renormalization was proposed [12, 13], which provides automatically the renormalized Green functions satisfying STI. This method was generalized to supersymmetric electrodynamics in [14]. However, in order to apply it to non-Abelian supersymmetric theories it is necessary to solve some problems, caused by a more complicated structure of STI. This is done in the present paper.

The paper is organized as follows:

In Section 2 we introduce the notations and remind some information about supersymmetric Yang-Mills theory. The universal invariant renomalization scheme for the model is constructed in Section 3. This scheme is illustrated in Section 4 by calculation of the one-loop $\beta$-function with the simplified (noninvariant) version of higher derivative regularization. The results are discussed in the Conclusion.

2  $N = 1$ supersymmetric Yang-Mills theory.

$N = 1$ supersymmetric Yang-Mills theory may be described by the following action:

\[
S = \frac{1}{2e^2} \text{Re} \text{tr} \int d^4x \ d^2\theta \ W_a C^{ab} W_b + \\
+ \frac{1}{16e^2} \text{tr} \int d^4x \ d^4\theta \left( \bar{c}^+ - \bar{c} \right) \frac{\partial}{\partial \varepsilon} \delta V + \alpha B^+ B - iB^+ \bar{D}^2 V - iBD^2 V, \tag{1}
\]

which is invariant under the BRST transformations:

\[
\delta V = \varepsilon \left[ -\frac{i}{2} V \text{ctg} V \left( c - c^+ \right) + \frac{1}{2} V \left( c + c^+ \right) \right]; \\
\delta B = 0; \quad \delta B^+ = 0; \\
\delta \bar{c} = i\varepsilon B; \quad \delta \bar{c}^+ = -i\varepsilon B^+; \\
\delta c = -\varepsilon c^2; \quad \delta c^+ = -\varepsilon (c^+)^2. \tag{2}
\]

\text{In our notations the metric tensor in the Minkowski space-time has the diagonal elements (1, -1, -1, -1).}
Here we use the following notations:

\[
V = -ieV^a t^a, \quad \text{tr}(t^a t^b) = \frac{1}{2} \delta^{ab},
\]

where \( t^a \) are hermitian generators, so that \( V^a \) are real scalar superfields and for any function \( f(V) = c_0 + c_1 V + c_2 V^2 + \ldots \) we define

\[
f(V)A \equiv c_0 A + c_1 [V, A] + c_2 [V, [V, A]] + \ldots.
\]

\( W_a \) is a chiral spinor superfield, given by the equation

\[
W_a \equiv \frac{1}{32} \bar{D}(1 - \gamma_5) D \left( e^{-2iV} (1 + \gamma_5) D_a e^{2iV} \right),
\]

where \( D \) is a supersymmetric covariant derivative:

\[
D = \frac{\partial}{\partial \theta} - i \gamma^\mu \theta \frac{\partial}{\partial \epsilon} \delta V.
\]

It is convenient to introduce the generating functional with additional sources \( G \) and \( g \):

\[
Z = \int d\mu \exp \left\{ iS + i \text{tr} \int d^4x d^4\theta \left( V J + G \frac{\partial}{\partial \epsilon} \delta V \right) + i \text{tr} \int d^4x d^2\theta \times \right.

\[
\left. \times \left( j_c c + \bar{j}_c \bar{c} + g \frac{\partial}{\partial \epsilon} \delta c \right) + i \text{tr} \int d^4x d^2\bar{\theta} \left( j^+_c c^+ + \bar{j}^+_c \bar{c}^+ + g^+ \frac{\partial}{\partial \epsilon} \delta c^+ \right) \right\}.
\]

Starting from this generating functional we construct the generating functional for connected Green functions

\[
W = -i \ln Z
\]

and the effective action

\[
\Gamma = W - \int d^4x d^4\theta V J - \int d^4x d^2\theta \left( j_c c + \bar{j}_c \bar{c} \right) - \int d^4x d^2\bar{\theta} \left( j^+_c c^+ + \bar{j}^+_c \bar{c}^+ \right),
\]

where the sources should be expressed via equations

\[
V = \frac{\delta W}{\delta J}; \quad c = \frac{\delta W}{\delta j_c}; \quad \bar{c} = \frac{\delta W}{\delta \bar{j}_c}; \quad c^+ = \frac{\delta W}{\delta j^+_c}; \quad \bar{c}^+ = \frac{\delta W}{\delta \bar{j}^+_c}.
\]

Note, that the effective action will depend on the sources \( G \) and \( g \) as on parameters.
### 3 Invariant renormalization

Performing in the generating functional (7) a substitution (2), it is easy to derive STI, which can be written as

\[
\text{tr} \int d^4x d^4\theta \left[ \frac{\delta \Gamma}{\delta V} \frac{\delta \Gamma}{\delta G} + \frac{2}{\alpha} \left( D^2 V \frac{\delta \Gamma}{\delta \bar{c}} - \bar{D}^2 \frac{\delta \Gamma}{\delta c^+} \right) \right] + \\
+ \text{tr} \int d^4x d^2\theta \frac{\delta \Gamma}{\delta c} \frac{\delta \Gamma}{\delta g} + \text{tr} \int d^4x d^2\bar{\theta} \frac{\delta \Gamma}{\delta \bar{c}} \frac{\delta \Gamma}{\delta g^+} = 0.
\]

Due to invariance of the generating functional under the transformations \( \bar{c} \rightarrow \bar{c} + \epsilon(x) \), where \( \epsilon(x) \) is an arbitrary anticommuting function, the following identities take place:

\[
\langle \frac{1}{32e^2} \bar{D}^2 \frac{\partial}{\partial \epsilon} \delta V + \bar{j}_c \rangle = 0; \quad \langle \frac{1}{32e^2} D^2 \frac{\partial}{\partial \epsilon} \delta V + \bar{j}_c^+ \rangle = 0.
\]

In order to write these STI in terms of the effective action it is convenient to define a functional \( \tilde{\Gamma} \) by subtracting the gauge fixing term from \( \Gamma \):

\[
\tilde{\Gamma} \equiv \Gamma - \frac{1}{32e^2} \text{tr} \int d^4x d^4\theta V \left( D^2 \bar{D}^2 + \bar{D}^2 D^2 \right) V.
\]

This functional satisfies the following equations:

\[
\text{tr} \int d^4x d^4\theta \frac{\delta \tilde{\Gamma}}{\delta V} \frac{\delta \tilde{\Gamma}}{\delta G} + \text{tr} \int d^4x d^2\theta \frac{\delta \tilde{\Gamma}}{\delta c} \frac{\delta \tilde{\Gamma}}{\delta g} + \text{tr} \int d^4x d^2\bar{\theta} \frac{\delta \tilde{\Gamma}}{\delta \bar{c}} \frac{\delta \tilde{\Gamma}}{\delta g^+} = 0; \quad (14)
\]

\[
- \frac{1}{32e^2} \bar{D}^2 \frac{\delta \tilde{\Gamma}}{\delta G} + \frac{\delta \tilde{\Gamma}}{\delta \bar{c}} = 0; \quad - \frac{1}{32e^2} D^2 \frac{\delta \tilde{\Gamma}}{\delta G} + \frac{\delta \tilde{\Gamma}}{\delta c^+} = 0.
\]

Note, that up to now we did not set any field equal to 0. Moreover, identity (14) is not linear. So, to treat these identities the following algorithm should be used:

1. In order to find relations between different Green functions it is necessary to differentiate identities (14) and (15) with respect to the arguments and then set all fields equal to 0.

2. Identity (14) should be expanded over the Plank constant \( \hbar \), that corresponds to the loop expansion. Let us present the effective action as follows:

\[
\tilde{\Gamma} = S + \frac{1}{\hbar} \Delta \Gamma^{(1)} + \frac{\hbar^2}{2} \Delta \Gamma^{(2)} + \ldots
\]

Then in the lowest order this identity is satisfied due to BRST-invariance of the classical action. At the arbitrary order one has

\[
\Delta \Gamma^{(n)} : S + S \cdot \Delta \Gamma^{(n)} = - \sum_{m=1}^{n-1} \Delta \Gamma^{(m-n)} \cdot \Delta \Gamma^{(m)}
\]
where \( \cdot \) denotes an operator, which appears after differentiation of (14):

\[
A \cdot B \equiv \text{tr} \int d^4x \, d^4\theta \left( \frac{\delta A \delta B}{\delta V \delta G} + \frac{\delta B \delta A}{\delta V \delta G} \right) + \\
\text{tr} \int d^4x \, d^2\theta \left( \frac{\delta A \delta B}{\delta c \delta g} + \frac{\delta B \delta A}{\delta c \delta g} \right) + \text{tr} \int d^4x \, d^2\theta \left( \frac{\delta A \delta B}{\delta c^+ \delta g^+} + \frac{\delta B \delta A}{\delta c^+ \delta g^+} \right) = 0. \tag{18}
\]

The identity (17) holds if a regularization preserves gauge invariance. If a noninvariant regularization is used the invariance may be restored by adding local counterterms to the proper vertices. This is the procedure adopted in the algebraic renormalization scheme, where the main problem is to find the renormalized action \( S_{\text{ren}} \), providing the effective action which satisfies ST identities. As the explicit determination of all noninvariant counterterms is a difficult problem, we adopt another approach, which allows to write directly the gauge invariant renormalized effective action by solving ST identities.

Our goal is to get the renormalized effective action \( \tilde{\Gamma}_R \), which satisfies STI by construction:

\[
\tilde{\Gamma}_R = S_{\text{ren}} + \hbar \Delta \gamma^{(1)} + \hbar^2 \Delta \gamma^{(2)} + \ldots. \tag{19}
\]

Here \( \Delta \gamma^{(i)} \) are some functions, which are constructed from \( \Delta \Gamma^{(i)} \) so that the renormalized effective action satisfies renormalized STI. The renormalized action is given by

\[
S_{\text{ren}} = S - \hbar \Delta \gamma^{(1)}_{\text{div}} - \hbar^2 \Delta \gamma^{(2)}_{\text{div}} + \ldots, \tag{20}
\]

so that

\[
\tilde{\Gamma}_R = S + \hbar \Delta \Gamma^{(1)}_R + \hbar^2 \Delta \Gamma^{(2)}_R + \ldots, \tag{21}
\]

where

\[
\Delta \Gamma^{(i)}_R \equiv \Delta \gamma^{(i)} - \Delta \gamma^{(i)}_{\text{div}}. \tag{22}
\]

If the theory is anomaly free, it is always possible to find such counterterms \( \Delta \gamma^{(i)}_{\text{div}} \) that

\[
\Delta \gamma^{(i)} - \Delta \gamma^{(i)}_{\text{div}} = \Delta \Gamma^{(i)} - \Delta \Gamma^{(i)}_{\text{div}}, \tag{23}
\]

in the limit, when a regularization is removed. Here \( \Delta \Gamma^{(i)} \) and \( \Delta \Gamma^{(i)}_{\text{div}} \) correspond to \( \Delta \gamma^{(i)} \) and \( \Delta \gamma^{(i)}_{\text{div}} \) in case of invariant regularization. Below we shall show that by solving STI one may get an expression for \( \Delta \Gamma^{(i)}_R \) without finding explicitly noninvariant counterterms \( \Delta \gamma^{(i)}_{\text{div}} \).
We shall use the eq.(17) to determine the renormalized Green function of order \( n \) assuming that according to \( R \)-operation the subtractions in all divergent subgraphs, corresponding to \( \Gamma^{(m)}, \ m < n \) are done in agreement with the eq.(17).

We shall work with the equation (17) assuming that it is differentiated with respect to the fields, and then all the fields are put equal to zero. In this way we get a relation which expresses an arbitrary Green function with at least one chiral external gauge line in terms of other Green functions which are of the same order in \( h \) but have one external gauge line less and lower order Green functions. Symbolically

\[
D_x^2 \Gamma^n(x...) = R(\Gamma^m)
\]  

(24)

where \( R(\Gamma_m) \) denotes a product of lower order Green functions and the Green functions of the same order but having one external gauge line less. We assume that all Green functions of order \( m < n \) are renormalized in agreement with the eq.(17) and accordingly the divergent subgraphs in the function \( \Gamma_n \) are subtracted following the \( R \)-operation. Then the eq.(24) fixes the overall subtraction in the function \( \Gamma_n \) so that the renormalized function satisfies STI. Indeed let us present the function \( \Gamma_n \) in the form

\[
\Gamma_n(\theta, p) = \sum_i B_i(\theta, p)F_i(p)
\]  

(25)

where the functions \( B_i \) are polynomials in \( \theta, p \) and \( F_i \) depend only on \( p \). The renormalization is achieved by subtracting a polynomial of \( F_i(p) \). We will choose the polynomials \( B_i \) so that their chiral parts

\[
Q_i = D_x^2 B_i
\]  

(26)

are either linear independent, or equal to zero.

The r.h.s. of eq.(24) may be decomposed over the polynomials \( Q_i \)

\[
R(\Gamma_m) = \sum_i Q_i(\theta, p)R_i(p)
\]  

(27)

In practice to find the functions \( R_i \) explicitely one has to perform a multiplication of the coefficients functions of the Green functions \( \Gamma_m \), entering the r.h.s. of eq.(24) using the supersymmetry algebra.

According to our assumption all functions at the r.h.s. of eq.(24) are finite. Using linear independence of coefficient functions \( Q_i \), we find that the renormalized function \( \Gamma_n \) may be written as follows

\[
\Gamma_n^R = \sum_i B_i(\theta, p)R_i(p) + \sum_j B_j(\theta, p)F_j(p)
\]  

(28)

where the second sum runs over \( j \) for which \( Q_j = 0 \). It completes the renormalization.

Such renormalization can be made in the following sequence:

At the first step we should calculate one-loop correction to the two-point Green functions of the ghost fields:
\[
\frac{\delta^2}{\delta c^+ \delta c} \Delta \Gamma_R^{(1)} ; \quad \frac{\delta^2}{\delta c \delta \bar{c}} \Delta \Gamma_R^{(1)} .
\] (29)

STI do not impose any restrictions on the renormalization of these functions. Therefore, they can be renormalized in arbitrary way, say, by subtraction at some fixed normalization point \( \mu_c \). At the next step we calculate functions

\[
\frac{\delta^2}{\delta c^+ \delta G} \Delta \Gamma_R^{(1)} ; \quad \frac{\delta^2}{\delta c \delta \bar{G}} \Delta \Gamma_R^{(1)} .
\] (30)

For this purpose it is necessary to use STI, which is obtained by differentiating equation (15) over \( \rho \). Then we proceed to the renormalization of the function

\[
\frac{\delta^3 \Delta \Gamma^{(1)}}{\delta c^+ \delta \bar{c} \delta V},
\] (31)

using the corresponding STI. Having constructed functions (30) and (31) it is possible to renormalize the function

\[
\frac{\delta^3 \Delta \Gamma^{(1)}}{\delta c^+ \delta \bar{G} \delta V}.
\] (32)

In each case the STI, which are needed for the renormalization, are obtained by differentiating of (14) or (15). Then we differentiate all STI over \( V \) and proceed to the renormalization of functions with larger powers of \( V \), starting from

\[
\frac{\delta^3 \Delta \Gamma^{(1)}}{\delta c \delta \bar{G} \delta V}.
\] (33)

Renormalization of the two-point Green function for gauge field is made similar to the QED case [14]: If we introduce the function \( \Pi \)

\[
(2\pi)^4 \delta^4 \left( (p_1)_\mu + (p_2)_\mu \right) \Pi \left( \theta_{x_1}, p_1), (\theta_{x_2}, -p_1) \right) \equiv
\]
\[
\equiv \int d^4 x_1 d^4 x_2 \frac{\delta^2 \tilde{\Gamma}}{\delta V_{x_1} \delta V_{x_2}} \bigg|_{\phi, \bar{\phi}, V = 0} \exp \left( i (p_1)_\mu x_1^\mu + i (p_2)_\mu x_2^\mu \right),
\] (34)

then

\[
\Pi \left( \theta_x, p \right) \left( \theta_y, -p \right) = F_1(p) \frac{p^2}{2} \delta^4(\theta_x - \theta_y) + F_2(p) \delta^4(\theta_x - \theta_y),
\] (35)

where

\[
\Pi_{1/2} = -\frac{1}{16 \partial^2} D^a D^2 C_{ab} D^b.
\] (36)

In this case

\[7\]
\[ B_1(\theta, p) = p^2 \Pi_{1/2} \delta^4(\theta_x - \theta_y); \quad B_2(\theta, p) = \delta^4(\theta_x - \theta_y) \]  

The renormalized Green functions are obtained by subtracting from \( F_i(p) \) some polynomials chosen to provide STI for the function \( \Pi^r \)

\[ \Pi^r[\theta, p] = \sum_i B_i(\theta, p) \left( F_i(p) - P_i(p) \right), \]  

For the two-point Green function substitution of (35) into the STI

\[ \bar{D}_y^2 \frac{\delta^2 \Delta \Gamma^{(1)}}{\delta V_x \delta V_y} = 0, \]  

which is obtained by differentiating the linearized STI (14) over \( c^+ \) and \( V \), gives the following equation:

\[ \left( F_2(p) - P_2(p) \right) D_x^2 \delta^4(\theta_x - \theta_y) = 0. \]  

Therefore,

\[ P_2(p) = F_2(p), \]  

while the function \( P_1 \) can not be defined from STI and corresponds to a gauge invariant counterterm. It is convenient to choose

\[ P_1(p) = F_1(\mu_\pi), \]  

where \( \mu_\pi \) is a normalization point. Then the renormalized two-point Green function can be written as

\[ \Pi^r[\theta_x, p), (\theta_y, -p)] = \left( F_1(p) - F_1(\mu_\pi) \right) p^2 \Pi_{1/2} \delta^4(\theta_x - \theta_y). \]  

This Green function satisfies the equation

\[ D_x^2 \Pi^r[\theta_x, p), (\theta_y, -p)] = 0, \]  

which is a supersymmetric generalization of transversality condition in the usual quantum electrodynamics.

Renormalization of the three-point function for the gauge field can be made according to the above algorithm using the corresponding STI.

The procedure described above may be interpreted as finding local counterterms, which can be written as

\[ \Delta S = \frac{1}{16e^2} \text{tr} \int d^4x d^4\theta \left( \bar{c}^+ - \bar{c} \right) \left( i \sum_{n=1}^\infty a_n V^n (c - c^+) + \sum_{n=0, \neq 1}^\infty b_n V^n (c + c^+) \right) + \text{tr} \int d^4x d^4\theta G \left( i \sum_{n=0}^\infty f_n V^n (c - c^+) + \sum_{n=0}^\infty h_n V^n (c + c^+) \right) + \text{tr} \int d^4x d^4\theta F[V]. \]
After adding these counterterms the renormalized effective action \( \Delta \Gamma_R^{(1)} \) will satisfy renormalized STI. However, the above algorithm allows to avoid writing and tuning of all terms in (45), that may cause considerable technical difficulties.

4 One-loop renormalization with the simplest PV-regularization

In order to illustrate the scheme, presented above, we consider \( N = 1 \) supersymmetric Yang-Mills theory and calculate its \( \beta \)-function using regularization by (usual) higher derivatives, complemented by Pauli-Villars fields for regularization of one-loop divergences.

The regularization is introduced by adding the higher derivative term, so that the regularized action is equal to

\[
S_{\text{reg}} = \frac{1}{2e^2} \text{Re} \int d^4x \, d^2\theta \, W_a C^{ab} \left( 1 + \frac{\partial^{2n}}{\Lambda^{2n}} \right) W_b + \]
\[
+ \frac{1}{16e^2} \text{tr} \int d^4x \, d^4\theta \left( \left( \bar{c}^+ - \bar{c} \right) \frac{\partial}{\partial \varepsilon} \delta V + \alpha B^+ B - iB^+ \bar{D}^2 V - iB D^2 V \right), \tag{46}
\]

However, the higher derivative term does not regularize one-loop divergences. Therefore, it is necessary to add in the generating functional PV-fields, that can be made by the following way:

\[
Z = \int d\mu \prod_i \left( \text{det} \, PV(V, M_i) \right)^{\alpha_i} \prod_i \left( \text{det} \, pv(V, m_i) \right)^{\beta_i} \times \]
\[
\times \exp \left\{ iS_{\text{reg}} + i \text{tr} \int d^4 x \, d^4 \theta \left( VJ + G \frac{\partial}{\partial \varepsilon} \delta V + i \text{tr} \int d^4 x \, d^2 \theta \times \right. \right.
\]
\[
\times \left( j_C^+ c + \bar{j}_C^+ \bar{c} + g \frac{\partial}{\partial \varepsilon} \delta c \right) + i \text{tr} \int d^4 x \, d^2 \theta \left( j_C^+ c^+ + \bar{j}_C^+ \bar{c}^+ + g^+ \frac{\partial}{\partial \varepsilon} \delta c^+ \right) \} \]. \tag{47}

where

\[
\left( \text{det} \, PV(V, M) \right)^{-1} = \int DW \exp \left\{ \frac{i}{2} \int d^4 x \, d^4 \theta_x \, d^4 y \, d^4 \theta_y \, W^a_x \frac{\delta^2 S_{\text{reg}}}{\delta V^a_x \delta V^b_y} W^b_y - \frac{i}{4} M^2 \int d^4 x \, d^4 \theta \, W^2 \right\},
\]

\[
\left( \text{det} \, pv(V, m) \right)^{-1} = \int DC \, D\bar{C} \exp \left\{ \frac{i}{16e^2} \text{tr} \int d^4 x \, d^4 \theta \left( \left( \bar{c}^+ - \bar{c} \right) \frac{\partial}{\partial \varepsilon} \delta V(V, C, C^+) \right. \right.
\]
\[
\left. + \right. \int \ldots \right\}.
\]
\[ + \frac{im}{2e^2} \text{tr} \int d^4x \, d^2\bar{\theta} \, \bar{C} \, C + \frac{im}{2e^2} \text{tr} \int d^4x \, d^2\bar{\theta} \, \bar{C}^+ C^+ \], \quad (48) \]

and the coefficients \( \alpha_i \) and \( \beta_i \) satisfy the following equations:

\[
\sum_i \alpha_i = 1; \quad \sum_i \alpha_i M_i^2 = 0; \\
\sum_i \beta_i = 1; \quad \sum_i \beta_i m_i^2 = 0. \quad (49)\]

Here \( W^a, C \) and \( \bar{C} \) are PV fields, \( M_i \) are masses of PV-fields, corresponding to \( V \) superfield and \( m_i \) are masses of PV-fields, corresponding to ghosts. We will assume, that all \( M_i \) and \( m_i \) are proportional to the parameter \( \Lambda \) in the higher derivative term.

In order to find two-point Green function for the gauge superfield it is necessary to calculate Feynman diagrams, presented at Figure 1. The result appeared to be

\[
\Delta \Gamma_{2\text{-point}}^{(1)} = -ie^2 c_1 \int \frac{d^4p}{(2\pi)^4} V^a(-p) \partial^2 \Pi_{1/2} V^a(p) \times \]
\[
\times \left\{ \frac{1}{2} \left( 1 + (-1)^n p^{2n}/\Lambda^{2n} \right) \int \frac{d^4k}{(2\pi)^4} \left( \frac{1}{k^2(k+p)^2(1 + (-1)^n k^{2n}/\Lambda^{2n})} \right) - \right. \\
- \sum_k \alpha_k \left( k^2 - M_j^2 \right) \left( (k+p)^2 - M_j^2 \right) \left( 1 + (-1)^n k^{2n}/\Lambda^{2n} \right) \right\} + \\
+ \frac{1}{4} \int \frac{d^4k}{(2\pi)^4} \left( \frac{1}{k^2(k+p)^2} - \sum_k \alpha_k \int \frac{d^4k}{(2\pi)^4} \left( \frac{1}{(k^2 - M_j^2)((k+p)^2 - M_j^2)} \right) \right) \}
\]
\[
+ e^2 c_1 \int \frac{d^4p}{(2\pi)^4} V_a(-p) V_a(p) \times \]
\[
\times \left\{ \sum_j \frac{i\alpha_j}{3} \int \frac{d^4k}{(2\pi)^4} \left( \frac{1}{k^2 - m_j^2} \right) - \sum_j \frac{i\alpha_j}{8} \int \frac{d^4k}{(2\pi)^4} \left( \frac{2(k+p)^2 + p^2}{(k^2 - M_j^2)((k+p)^2 - M_j^2)} \right) + \\
+ \sum_j \frac{i\alpha_j}{4} \int \frac{d^4k}{(2\pi)^4} \left( \frac{M_j^2(1 + (-1)^n (k+p)^{2n}/\Lambda^{2n})}{(k^2 - M_j^2)((k+p)^2 - M_j^2)} \right) - \\
- \sum_j \frac{i\beta_j}{12} \int \frac{d^4k}{(2\pi)^4} \left( \frac{1}{(k^2 - m_j^2)^2} \right) + \sum_j \frac{i\beta_j}{8} \int \frac{d^4k}{(2\pi)^4} \left( \frac{p^2}{(k^2 - m_j^2)((k+p)^2 - m_j^2)} \right) \right\} \quad (50)\]

Substituting this expression to equation (34) we obtain the function \( \Pi \), which is equal to (35) with

\[
F_1(p) = ie^2 c_1 \left\{ \left( 1 + (-1)^n p^{2n}/\Lambda^{2n} \right) \int \frac{d^4k}{(2\pi)^4} \left( \frac{1}{k^2(k+p)^2(1 + (-1)^n k^{2n}/\Lambda^{2n})} \right) - \\
\right. \]

\[ + \frac{im}{2e^2} \text{tr} \int d^4x \, d^2\bar{\theta} \, \bar{C} \, C + \frac{im}{2e^2} \text{tr} \int d^4x \, d^2\bar{\theta} \, \bar{C}^+ C^+ \right\}, \quad (48)\]
\[-\sum_k \alpha_k \left( k^2 - M_j^2 \right) \left( (k+p)^2 - M_j^2 \right) \left( 1 + (-1)^n k^{2n}/\Lambda^{2n} \right) \]  
\[+ \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \left( \frac{1}{k^2(k+p)^2} - \sum_k \alpha_k \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - M_j^2)((k+p)^2 - M_j^2)} \right) \]

\[F_2(p) = 2e^2c_1 \left\{ \sum_j \frac{i\alpha_j}{3} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - M_j^2)} - \sum_j \frac{i\alpha_j}{8} \int \frac{d^4k}{(2\pi)^4} \frac{2(k+p)^2 + p^2}{(k^2 - M_j^2)((k+p)^2 - M_j^2)} \right. \]
\[+ \sum_j \frac{i\alpha_j}{4} \int \frac{d^4k}{(2\pi)^4} \frac{M_j^2 \left( 1 + (-1)^n (k+p)^2n/\Lambda^{2n} \right)}{(k^2 - M_j^2)((k+p)^2 - M_j^2)} \]
\[- \sum_j \frac{i\beta_j}{12} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_j^2)} + \sum_j \frac{i\beta_j}{8} \int \frac{d^4k}{(2\pi)^4} \frac{p^2}{(k^2 - m_j^2)((k+p)^2 - m_j^2)} \right\} \]  \hspace{1cm} (51)

In order to calculate the integrals in this expression, first it is necessary to perform the Wick rotation. Then it is easy to see, that

\[F_1(p) = -\frac{3}{16} e^2c_1 \left( \ln \frac{\Lambda}{p} + \text{finite terms} \right) \]  \hspace{1cm} (52)

It is easy to verify, that the function \(F_2\) is finite and proportional to \(p^2\). After taking a limit \(\Lambda \to \infty\) (and therefore, \(M_j \to \infty\) and \(m_j \to \infty\)) we obtain, that

\[
\Pi[(\theta_x, p), (\theta_y, -p)] = -\frac{3}{16\pi^2} e^2c_1p^2\Pi_{1/2}\delta^4(\theta_x - \theta_y) \left( \ln \frac{\Lambda}{p} + \text{finite terms} \right) + 
+ e^2c_1\delta^4(\theta_x - \theta_y)p^2C \left( \alpha_j, \beta_j, M_j, \Lambda, m_j \right) \]  \hspace{1cm} (53)

where the constant \(C\) is given by

\[p^2C = \lim_{\Lambda \to \infty} F_2(p) \]  \hspace{1cm} (54)

and in the general case is not 0, that can be easily verified. Then the renormalized function \(\Pi'\), obtained according to prescription (43), is

\[
\Pi[(\theta_x, p), (\theta_y, -p)] = -\frac{3}{16\pi^2} e^2c_1p^2\Pi_{1/2}\delta^4(\theta_x - \theta_y) \left( \ln \frac{\mu}{p} + \text{finite terms} \right) \]  \hspace{1cm} (55)

and corresponds to the following renormalized contribution to the effective action:

\[
\Delta \Gamma_R^{(1)} = \frac{3e^2}{32\pi^2} c_1 \int \frac{d^4p}{(2\pi)^4} V^a(-p) \partial^2 \Pi_{1/2} V^a(p) \left( \ln \frac{\mu}{p} + \text{finite terms} \right) \]  \hspace{1cm} (56)

11
One-loop correction to the ghost propagator corresponding to the first diagram, presented in Fig. 2 is found to be zero. We also verified, that the one-loop ghost-gluon vertex, corresponding to the other diagrams in this figure, is finite. Therefore, the one-loop $\beta$-function

$$
\beta(\alpha) \equiv \frac{d}{d \ln \mu} \left( \frac{e^2}{4\pi} \right),
$$

(57)
calculated in the noninvariant regularization, described above is

$$
\beta(\alpha) = -\frac{3\alpha^2 c_1}{2\pi} + O(\alpha^3)
$$

(58)
and agrees with the calculations made by DRED. However, it is interesting to find the two-loop contribution to the $\beta$-function (57) in the considered regularization, because in SUSY QED \cite{15, 16} such contribution is different from the DRED result and is equal to zero.

\section{Conclusion.}

In this paper we presented a renormalization procedure for SUSY Yang-Mills theories, which guarantees BRST-invariance of the renormalized theory for any intermediate regularization. SUSY Slavnov-Taylor identities are incorporated into subtractions, which allows to avoid the appearance of noninvariant counterterms. The proposed procedure is illustrated by the calculation of the one-loop $\beta$-function for the $N = 1$ SUSY Yang-Mills theory with the noninvariant version of higher derivative regularization. However it would be interesting to calculate two-loop quantum corrections and compare them with the correspondent result in SUSY QED \cite{15, 16}.

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Figure 1: One-loop contribution to the two-point Green function of the gauge superfield.

Figure 2: One-loop contributions to the ghost propagator and ghost-gluon vertex.