Geometric approach to finding the best possible solutions based on composition optimization of the mixed aggregate of fine-grained concrete

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Abstract. The work proposes a geometric approach to the search for optimal solutions, based on the hypothesis that the optimal solution is one in which geometric objects, which characterize mutually opposite properties of the investigated process, approach each other most closely. Then the search for the optimal solution is reduced to minimizing the metric characteristics between the simulated geometric entities. In order to bring the initial data into compliance with each other, it is proposed to use their normalization at the stage of preliminary preparation before building a geometric model of the process. There is presented a computational experiment to find an optimal composition of a combined aggregate from industrial wastes to achieve the best physical and mechanical properties of fine-grained concrete, involving construction of corresponding geometric models in the form of response surfaces belonging to 4-dimensional space.

1. Introduction

In engineering and scientific practice, one of the most common tasks is to simulate multi-factor processes and phenomena with their subsequent optimization. The importance of solving optimization problems is emphasized by the demand, in the process of solving problems, for the management of organizational and technical systems [1], in the management at enterprises of the agricultural sector [2], in the optimization of technological and production processes [3-5], in pedagogy and education [6-7], in construction and architecture [8-10], etc. At the same time, optimization is most often understood to be the definition of extreme values of the mathematical model of the process. The general approach to solving optimization problems includes a mathematical model of the process, selection of a function of the target, as well as factors affecting it, and search of extreme values of the target function in a given range of values by mathematical or computational methods. However, extreme values are not always optimal. In practice, conflicting information is often encountered when it is necessary to define a maximum of one objective function with a minimum of another. Such tasks were called minimax [11-13]. For example, when examining the physical and mechanical properties of composite building materials (CBM), it is necessary to determine the composition of components that
provides maximum strength characteristics at minimum density values, and if this is not possible, the closest to them. Then the task is to determine such optimal composition of components, which provides simultaneously not maximum and minimum, but also high values of strength, and low values of density of CBM.

2. Geometric approach to finding optimal solutions
Consider in the complex drawing of V.P. Radishchev two lines characterizing the behavior of some abstract process (Figure 1). The axis $Ox$ corresponds to a factor that affects the behavior of the process. The current point $M_1$ describes the line characterizing some property of the process under study by number 1. This property corresponds to the $Oy$ axis. Then, the current point $M_2$ describes the number 2 property of the process under study, to which the $Oz$ axis corresponds.

![Figure 1. Geometric diagram of search for optimal solutions on the complex drawing of V.P. Radishchev.](image)

3. Hypothesis
The solution is optimal, in which two lines characterizing mutually opposite properties of the investigated process approach each other most closely.

According to this hypothesis, in order to optimize such processes, it is necessary to find a minimum square of length between curves, similar to the least squares method [14]. Since the motion of the current points $M_1$ and $M_2$ is linked to each other by means of an inter-sectional link, it is sufficient to find a minimum square of the difference between the $y$ and $z$ coordinates:

$$\min \left( y_{M_1} - z_{M_2} \right)^2$$  \hspace{1cm} (1)

The difference from the least squares method is that the minimum square of length is between lines, not between points and a line. Another fundamental difference is that the lines characterizing the process carry quite different sense loads, i.e. correspond to different properties of the process under study.

In practice, the natural values of factors and process characteristics under study may have values that differ by several orders of magnitude. In such cases, it is necessary to resort to rationing at the interpolation stage by dividing each of the original values by their sum:

$$\bar{y}_i = \frac{y_i}{\sum_{i=1}^{n} y_i},$$  \hspace{1cm} (2)

where $y_i$ – reference value; $\bar{y}_i$ – rated values; $n$ – quantity of experimental points.

The advantage of the proposed approach is the possibility of generalizing it in two directions at once. First, it is easily generalized by increasing the number of optimizable properties of the process under study (for two lines the length of the segment between them is minimized, for three - the area of the triangle, for four - the volume of the tetrahedron, etc.). Second, the number of factors affecting the behavior of the process under study is also generalized to multivariate space through geometric theory of multivariate interpolation. For example, the search for an optimal solution to a two-factor process
can be realized as follows. Let the source data be used to define two interpolation response surfaces that characterize the behavior of the process (Figure 2).

Then, similar to (1), we get:

\[
\left( z_{M_1} - t_{M_2} \right)^2 \rightarrow \text{min.}
\]  

(3)

Thus, the proposed approach can be used to solve optimization problems in different productions, but for each individual task it is necessary to develop a separate solution.

4. Example of construction and optimization of geometric model of dependence of physical and mechanical properties of fine concrete on composition of combined aggregate

The above-mentioned geometric approach is used to optimize the composition of the combined aggregate of fine concrete. First it is necessary to obtain geometric models of dependence of physical and mechanical properties of fine-grained concrete on composition of combined aggregate: Marten slag (MS), granulated slag (GS), burning rock (BR). The initial data for modeling were the results of experimental studies of physical and mechanical properties of fine-grained concrete, described in work [15].

It should be noted that the physical and mechanical properties of fine concrete differ by several orders of magnitude. Thus, the density of the concrete \( \rho \) varies between 1676 and 1997 kg/m\(^3\), and the compressive strength of the \( \sigma \) varies between 2.1 and 9.2 MPa. In such a case, in order to implement a geometric approach to finding optimal solutions, it is necessary to normalize the original data. Using the formula (2), it was obtained (Table 1).

It should be noted that Figure 3 shows a geometric diagram of the simulation of the compressive strength dependence on the composition of the combined aggregate. However, the same scheme is suitable for determining the geometric model of density dependence on the composition of the combined aggregate. Only the strength values are replaced by the density values from Table 1.

According to the geometric diagram (Figure 3), the initial data in the form of 10 experimental points are distributed as follows: first guide line passes through 4 points \( A_i \), the second one – through 3 points \( B_i \), the third – through 2 points \( C_i \), and the fourth – through just one point \( D_i \equiv M_4 \). For analytical determination of the geometric model in the form of a response surface passing through 10 experimental points, we will use a computational algorithm in the form of a point equations sequence of all guiding lines and a response surface generatrix line. According to the geometric theory of multivariate interpolation [17-19], we get:
\[
M_i = A_i \left( \bar{u}^2 - 2,5\bar{u}^2u + \bar{u}^3 \right) + A_i \left( 9\bar{u}^2u - 4,5\bar{u}^2 \right) + \\
+ A_i \left( -4,5\bar{u}^2u + 9\bar{u}^2 \right) + A_i \left( \bar{u}^2u - 2,5\bar{u}^2 + u^3 \right),
\]
\[
M_2 = B_i \bar{v} (1 - 2u) + 4B_i \bar{u} + B_i u (2u - 1),
\]
\[
M_3 = C_i \bar{u} + C_i u,
\]
\[
M = M_1 \left( \bar{v}^3 - 2,5\bar{v}^2v + \bar{v}^3 \right) + M_2 \left( 9\bar{v}^2v - 4,5\bar{v}^2 \right) + \\
+ M_3 \left( -4,5\bar{v}^2v + 9\bar{v}^2 \right) + M_4 \left( \bar{v}^2v - 2,5\bar{v}^2 + v^3 \right),
\]

where \( M \) – current response surface point; \( M_i \) – current points of the response surface guide lines; \( A_i, B_i, C_i \) – initial points, coordinates of which correspond to normalized experimental data; \( u, v \) – response surface point equation parameters that vary from 0 to 1; \( \bar{u} = 1 - u, \bar{v} = 1 - v \) – adding parameters to 1.

### Table 1. Source Data for Normalizing Geometric Models.

| № experience | content in mixture by weight of fillers, % | Physical and mechanical properties of fine concrete |
|--------------|-------------------------------------------|--------------------------------------------------|
|              | MS | GS | BR | \( \rho \) | \( \sigma \) |
| 1            | 0  | 0  | 100 | 0,125     | 0,136     |
| 2            | 33 | 0  | 67  | 0,121     | 0,196     |
| 3            | 67 | 0  | 33  | 0,119     | 0,143     |
| 4            | 100| 0  | 0   | 0,109     | 0,081     |
| 5            | 67 | 33 | 0   | 0,106     | 0,062     |
| 6            | 33 | 67 | 0   | 0,106     | 0,049     |
| 7            | 0  | 100| 0   | 0,106     | 0,045     |
| 8            | 0  | 67 | 33  | 0,12      | 0,055     |
| 9            | 0  | 33 | 67  | 0,109     | 0,145     |
| 10           | 33 | 33 | 33  | 0,105     | 0,087     |

In accordance with [16], we exclude unnecessary combinations of aggregate components from the condition that the sum of the share of all three components (Table 1) is invariably equal to 100% (Figure 3).

![Figure 3. Geometric diagram of simulation of compression strength dependence on combined aggregate composition.](image)

All coefficients of polynomial dependencies of the computational algorithm (4) are constant and, according to the method of their simulation set forth in [20], depend solely on the number of points through which the corresponding curve line passes.
Note that the resulting computational algorithm (4) is universal with respect to the original data and can be effectively used for a similar experiment planning matrix. Thus, it is possible to obtain models both an equation characterizing the dependence of the compression strength and an equation characterizing the dependence of the density of fine concrete on the composition of the combined aggregate from industrial wastes.

\[
\begin{align*}
GS &= 100u(1 - v), \\
MS &= 100v, \\
\rho &= (0.146v^3 - 0.293v^2 + 0.179v - 0.033)u^3 + (-0.03v^3 + 0.104v^2 - 0.11v + 0.036)u^2 + \\
&+ (0.07v^3 - 0.089v^2 + 0.037v - 0.018)u - 0.198v^3 + 0.3v^2 - 0.119v + 0.111, \\
\sigma &= (-2.116v^4 + 4.231v^3 - 2.586v^2 + 0.47)u^3 + (5.268v^3 - 10.247v^2 + 5.958v - 0.979)u^2 + \\
&+ (-3.939v^4 + 7.273v^3 - 3.787v^2 + 0.453)u + 0.959v^3 - 1.401v^2 + 0.386v + 0.136.
\end{align*}
\]

In this case, the polynomial coefficients for the 3rd sign were rounded for a better representation. When calculated in most computer algebra systems, rounding to the 10th sign is used by default to obtain the most accurate calculation results.

As can be seen from the obtained systems of parametric equations, the values of the parameters, and the natural values of the factors GS, MS are in linear dependence, which makes it easy to move from parametric equations to an equation in an explicit form. However, considering that the parameters and vary from 0 to 1, and the factors GS and MS - from 0 to 100, the article gives the systems of parametric equations. Since when replacing parameters with factors, polynomial coefficients will be very small, which does not affect calculations in any computer algebra system, but is completely uncomfortable for presenting the obtained results in a scientific article.

According to the proposed hypothesis, using expression (3), we will search for the optimal solution of the problem. As a result, the minimum distance between the response surfaces was reached at \( u = 0.009 \) and \( v = 0.176 \). Turning to the natural values of factors, we get the following composition of the combined aggregate: GS =0.74%, MS =17.6% and BR=81.66%. At that, compression strength is achieved 7.79 MPa, And concrete density – 1765 kg/m3.

5. Conclusion

The proposed geometric approach for finding optimal solutions can be considered as one of the possible options for optimizing multi-factor processes and phenomena. The obvious advantage of which is the possibility of generalizing both in the direction of increasing the number of optimized properties (characteristics) of the investigated process and in the direction of increasing the number of factors affecting the behavior of the investigated process, which undoubtedly enrich the technical arsenal of the researcher for processing, analyzing and optimizing experimental and statistical information using modern computer technologies.

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