A theoretical development of improved cosine similarity measure for interval valued intuitionistic fuzzy sets and its applications

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Abstract
This study mainly focuses on developing a new flexible technique for interval-valued intuitionistic fuzzy cosine similarity measures, which significantly analyzes the strength of the relationship between two objects. Based on the notion of a cosine similarity measure between IVIFSs, the proposed measure is formulated. Then, the measure is demonstrated to satisfy some essential properties, which prepare the ground for applications in different areas. Finally, the study uses the proposed measure to solve real-world decision problems such as pattern recognition, medical diagnosis, and multi-criteria decision-making problems with interval-valued intuitionistic fuzzy information. The numerical examples of the mentioned applications are delivered to validate the effectiveness of the developed approach in solving real-life problems.

Keywords Interval-valued intuitionistic fuzzy set · Similarity measure · Cosine similarity measure

1 Introduction
In both human nature and real-world situations, uncertainty, vagueness, inconsistency, and imprecision seem to be prevalent. Fuzzy sets (FSs) and their extensions, including intuitionistic fuzzy sets (IFSs) and interval-valued intuitionistic fuzzy sets (IVIFSs), were designed to cope with imprecise or ambiguous scenarios. In Duan and Li (2021), Garg and Singh (2020), using those same tools, numerous methodologies for dealing with uncertainty have already been devised. Zadeh (1965) became the first to invent fuzzy sets in 1965. The belongingness within each component through fuzzy sets has so far been determined by the degree of association or membership, whereas IFSs (formed by Atanassov (1986)) as well as IVIFSs (generated by Atanassov and Gargov (1989)) are assessed by the reference value or membership as well as non-membership degree intervals, respectively. IVIFSs have become formed by combining intervals for membership and non-membership degrees, making them more versatile and extensively relevant to working with actual instances. The increasing complexities of society and insufficient knowledge or information about the problem domains may both be addressed by IVIFSs. They are defined by membership and non-membership functions, the values of which have always been intervals instead of real numbers. Following that, various researchers have actively started working on FSs and their expansions, as well as trying to apply them to a variety of real-world issues (Bakbak et al. 2019; Gao and Zhang 2021; Ulucay et al. 2018, 2019). A few of the relevant papers are as follows:

Joshi (2020) created an innovative multi-criteria decision-making system based on intuitionistic fuzzy data, which she then implemented for machine fault detection. Further to that, adopting IFSs with histogram equalization, Jebadass and Balasubramaniam (2022) developed a low-light enhancement technique for color images. At the age of COVID-19, Ecer published an augmented MAIRCA technique for coronavirus vaccine identification in which it exploits IFSs (Ecer 2022). Based on the Pythagorean fuzzy similarity measure, Premalatha and Dhanalakshmi (2022) developed a significant strategy for image enhancement as well as the categorization of the COVID-19 virus. The importance of a ranking system was then constructed by Bharati, employing IVIFSs to handle a transportation problem (Bharati 2021). Meanwhile, implementing IVIFSs, the author, Percin (2021), presented a circular supplier selection. Then, Wei et al. (2021) examined the use of an
information-based score function for IVIFSs in multiattribute decision making.

On the other hand, a similarity measure is indeed a beneficial tool for predicting the degree of resemblance between two objects (Chai et al. 2021; Ye and Du 2019). A measure of similarity is among the ideal tools for identifying a pattern in many disciplines where the optimum decision or optimality criteria have been specified (Singh and Ganie 2021). Due to its extensive potential applications in many sectors, including machine learning, medical diagnosis, medical image processing, decision-making, pattern recognition, and so on (Nguyen 2021; Xue and Deng 2021), there has been fast progress in the investigation of similarity measures in recent decades. Through the analysis of fuzzy sets and their expansions (Chen and Liu 2022; Gohain et al. 2022; Song et al. 2019), many distinct similarity measures within the IFSs were thoroughly investigated. Boran and Akay (2014) invented a new sort of IFS similarity measure that includes two parameters describing the $L_p$ norm and the amount of uncertainty, respectively. Besides, the similarity measure of IFSs was defined axiomatically by Li and Deng (2012). The connection between entropy and the IFS similarity measure was fully explored in the aforementioned study. It seemed that the similarity measure and the entropy of IFS might be turned into each other by utilizing respective axiomatic definitions. In addition, Garg and Rani (2021) designed a new similarity measure based on modified right-angled triangles among IFSs, including its implementations. The entropy and similarity measures have been suggested by Saeed et al. (2021), and they were applied to the problem of MCDM. Likewise, Ganie and Singh (2021) formed a picture fuzzy similarity measure due to direct operations as well as a new multi-attribute decision-making mechanism. With multi-granular imbalanced linguistic terms, Zhang et al. (2021) built a consensus-reaching scheme for group decision making. The author, Bakbak and Ulucay (2019), Ulucay (2020) recommended the new similarity measures based on trapezoidal fuzzy multi-numbers, which have been applied to solve the MCDM problems. Thao and Chou (2022) announced a new similarity measure, the entropy of IFSs, and its use in software quality assessment. Depending on the IFS, weighted similarity measure, and extended TOPSIS Method, Lee et al. (2021) presented a hybrid MCDM scheme. Additionally, the authors, Thao and Duong (2019), recommended a target market selection based on similar measures from within the IVIFSs. Mishra et al. (2021) built an IVIFS additive ratio evaluation framework based on similarity measures for evaluating and selecting low-carbon tourism.

A cosine similarity measure (CSM) depending upon Bhattacharyya’s distance (Bhattacharyya 1946; Salton and McGill 1983), which may be stated as the inner product of those two vectors divided by the product of their lengths, has one of the plenty of other similarity measures. This is simply the cosine of an angle made up of two fuzzy set vector representations. Later, the author, Ye (2011), made a similar comparison of available IFS similarity measures and developed a cosine similarity measure and even a weighted cosine similarity measure. Harish (2018) and Rajkumar and Merigo (2020) recently developed and improved a CSM for IVIFSs, as well as implemented it to address decision-making challenges. Thereafter, Liu et al. (2017) offered an IVIFS-based WCSM and devised a strategy for solving group decision-making tasks. CSMs were often established by Singh (2012) and Ye (2013) to assess the degree of similarity between two IVIFSs.

Looking more closely at the current IVIFS similarity measures, we can see that creating a robust IVIFS similarity measure is quite difficult. Few among those are unable to fully meet the axiomatic definition of similarity in yielding counterintuitive cases, as evidenced by researchers’ finding of counterintuitive examples for many available measures. Also, other similarity measures have included a lack of defined physical meaning and highly complicated expressions. When most existing measures were introduced, they were initially and foremost considered at the “formula” level. As a consequence, it would be preferable to define an easier-to-understand similarity measure for IVIFS. Moreover, it seems to be noticed that the aforementioned studies (Singh 2012; Ye 2013) did not employ the middle and boundary points of the intervals. Hence, the formulation of a similarity measure remains an open problem that is attracting greater attention. Due to these shortcomings, the measurement procedure cannot accommodate the decision maker’s attitude. It illustrates the incapacity and rigidity of measures in dealing with real-world problems. So, in an IVIFS setting, we require a resilient CSM to accommodate the decision maker’s attitude preferences in the measurement method.

Inspired by the above thoughts, we first propose a new cosine similarity measure for IVIFSs based on the CSM between IVIFSs. Secondly, using the definition of a similarity measure, we verified a proposed similarity measure’s basic and essential properties. Further, the applicability of the proposed approach is studied in various real-life problems with interval-valued intuitionistic fuzzy information. Then, the numerical examples of the proposed measure and other sophisticated measures are compared to show the effectiveness of the proposed one.

The paper is organized as follows. Section 2 briefly reviews the basic concepts related to fuzzy sets, intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy sets, operations on IVIFSs, and similarity measures. In Sect. 3, some existing similarity measures are reviewed. In Sect. 4, we propose the interval-valued intuitionistic fuzzy cosine similarity (IVIFCS) measure between two IVIFSs. And, some properties of the IVIFCS measure are also analyzed. Section 5,
uses the IVIFCS measure to solve real-world problems, include pattern recognition, medical diagnosis, and MCDM with interval-valued intuitionistic fuzzy information, as well as illustrative with a numerical example. Section 6 summarizes the main results and conclusions of the paper.

2 Preliminaries

The basic concepts connected with IVIFSs that have been used in our study are addressed in this section.

Definition 1 (Zadeh 1965) In the universe of discourse, \( X = \{x_1, x_2, \ldots, x_n\} \) a fuzzy set (FS) \( \tilde{P} \) is stated as follows:
\[
\tilde{P} = \{(x, \mu_p(x)) : x \in X\}
\]
where \( \mu_p(x) : X \rightarrow [0, 1] \) is the membership degree.

Definition 2 (Atanassov and Gargov 1989) In the universe of discourse, \( X = \{x_1, x_2, \ldots, x_n\} \) a intuitionistic fuzzy set (IFS) \( \tilde{P} \) is interpreted as follow:
\[
\tilde{P} = \{(x, \mu_p(x), v_p(x)) : x \in X\}
\]
where \( \mu_p(x), v_p(x) : X \rightarrow [0, 1] \) is the membership and non-membership degrees with the condition \( 0 \leq \mu_p(x) + v_p(x) \leq 1 \). The third parameter within the IFS \( \tilde{P} \) is: \( \pi_p(x) = 1 - \mu_p(x) - v_p(x) \), often referred to as the hesitancy degree of whether \( x \) is in \( \tilde{P} \) or not. It can clearly be seen that \( 0 \leq \pi_p(x) \leq 1 \).

Definition 3 (Atanassov 1986) In the universe of discourse, \( X = \{x_1, x_2, \ldots, x_n\} \) a interval-valued intuitionistic fuzzy set (IVIFS) \( \tilde{P} \) is expressed as follow:
\[
\tilde{P} = \{(x, [\mu_p(x), v_p(x)]) : x \in X\}
\]
\[
= \{(x, [\mu_p^-(x), \mu_p^+(x), \nu_p^-(x), \nu_p^+(x)]) : x \in X\}
\]
Here, \( \mu_p(x) \subseteq [0, 1] \) and \( v_p(x) \subseteq [0, 1] \), which satisfies \( 0 \leq \mu_p(x) + v_p(x) \leq 1 \). Further, the intervals \( \mu_p(x) \) and \( v_p(x) \) signify the degrees of both membership and non-membership, respectively. Furthermore, for any \( x \in X \), we will determine the degree of hesitancy is: \( \pi_p(x) = [\pi_p^-(x), \pi_p^+(x)] \) \( = [1 - \mu_p^-(x) - v_p^-(x), 1 - \mu_p^+(x) - v_p^+(x)] \).

Definition 4 (Luo and Liang 2018) For any two IVIFSs, \( \tilde{P} \) and \( \tilde{Q} \), we have the following relations in the finite universe \( X \):
1. \( \tilde{P} \subseteq \tilde{Q} \) iff (for all \( x \in X \)) \( \mu_p^-(x) \leq \mu_Q^-(x), \mu_p^+(x) \leq \mu_Q^+(x) \), \( v_p^-(x) \leq v_Q^-(x) \), and \( v_p^+(x) \leq v_Q^+(x) \).
2. \( \tilde{P} = \tilde{Q} \) iff (for all \( x \in X \)) \( \mu_p^-(x) = \mu_Q^-(x), \mu_p^+(x) = \mu_Q^+(x), \)
\( v_p^-(x) = v_Q^-(x) \), and \( v_p^+(x) = v_Q^+(x) \).
3. \( \tilde{P}^c = \{x, [\nu_p^-(x), \nu_p^+(x)], [\mu_p^-(x), \mu_p^+(x)]\} \)

Definition 5 (Singh 2012) Suppose \( \tilde{P} \) and \( \tilde{Q} \) are IVIFSs in the finite universe \( X \), and then a mapping \( S : IVIFS(X) \times IVIFS(X) \rightarrow [0, 1] \), \( S(\tilde{P}, \tilde{Q}) \), is termed a similarity measure between \( \tilde{P} \) and \( \tilde{Q} \) if \( S(\tilde{P}, \tilde{Q}) \) meets the following criteria:
1. \( 0 \leq S(\tilde{P}, \tilde{Q}) \leq 1 \),
2. \( S(\tilde{P}, \tilde{Q}) = 1 \) iff \( \tilde{P} = \tilde{Q} \),
3. \( S(\tilde{P}, \tilde{Q}) = S(\tilde{Q}, \tilde{P}) \),
4. If \( \tilde{P} \subseteq \tilde{Q} \subseteq R \), then \( S(\tilde{P}, R) \leq S(\tilde{P}, \tilde{Q}) \), and \( S(\tilde{P}, \tilde{R}) \).

3 Some existing similarity measures

In this section, we review some existing similarity measures. Let \( \tilde{P} = \{(x_i, [\mu_p^-(x_i), \mu_p^+(x_i)), [v_p^-(x_i), v_p^+(x_i)]) : x_i \in X\} \) and \( \tilde{Q} = \{(x_i, [\mu_Q^-(x_i), \mu_Q^+(x_i)), [v_Q^-(x_i), v_Q^+(x_i)]) : x_i \in X\} \) are two IVIFSs defined in the universe \( X = \{x_1, x_2, \ldots, x_n\} \). The following Formulas (5)-(10) are similarity measures based on IVIFSs:

Singh’s cosine similarity measure (Singh 2012)
\[
S_C(\tilde{P}, \tilde{Q}) = \frac{1}{n} \sum_{i=1}^{n} \frac{\alpha_i \beta_i + \delta_i \gamma_i}{\sqrt{\alpha_i^2 + \beta_i^2} \sqrt{\delta_i^2 + \gamma_i^2}}
\]
\[
\alpha_i = (\mu_p^-(x_i) + \mu_p^+(x_i)) , \quad \beta_i = (\mu_p^-(x_i) + \mu_p^+(x_i)) ,
\]
\[
\delta_i = (v_p^-(x_i) + v_p^+(x_i)), \quad \gamma_i = (v_Q^-(x_i) + v_Q^+(x_i)).
\]
Xu’s similarity measure (Xu and Chen 2008)
\[
S_1(\tilde{P}, \tilde{Q}) = 1 - \frac{1}{4n} \sum_{i=1}^{n} \left[ |\mu_p^-(x_i) - \mu_Q^-(x_i)|^p + |\mu_p^+(x_i) - \mu_Q^+(x_i)|^p + |v_p^-(x_i) - v_Q^-(x_i)|^p + |v_p^+(x_i) - v_Q^+(x_i)|^p \right]
\]
Wei’s similarity measure (Wei et al. 2011)
\[
S_2(\tilde{P}, \tilde{Q}) = 1 - \frac{1}{4n} \sum_{i=1}^{n} \max \left\{ |\mu_p^-(x_i) - \mu_Q^-(x_i)|^p, \right\}
\]
\[
|\mu_p^+(x_i) - \mu_Q^+(x_i)|^p, |v_p^-(x_i) - v_Q^-(x_i)|^p, |v_p^+(x_i) - v_Q^+(x_i)|^p \right\}
\]
Mathematical formulation of a new similarity measure based on IVIFS

In this section, we propose an innovative mechanism for formulating IVIFS similarity measure that enables one to create an IVIFS similarity measure by modifying an existing IVIFS measure. The similarity measure of IVIFSs given by the article (Singh 2012), in particular, promotes a novel similarity measure of IVIFSs. Then, the flowchart of the proposed study is presented in Fig. 1.
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\[ \beta_3 = ((1 - \mu_{\bar{Q}}(x_i)) + (1 - \mu_{\bar{Q}}(x_i))) \]

\[ \delta_3 = ((1 - \nu_{\bar{Q}}(x_i)) + (1 - \nu_{\bar{Q}}(x_i))) \]

\[ \gamma_3 = ((1 - \nu_{\bar{Q}}(x_i)) + (1 - \nu_{\bar{Q}}(x_i))) \]

Moreover, the proposed similarity measure satisfies the following axiomatic requirements:

1. \( 0 \leq S(\bar{P}, \bar{Q}) \leq 1 \)
2. \( S(\bar{P}, \bar{Q}) = 1 \) if and only if \( \bar{Q} \leq \bar{P} \)
3. \( S(\bar{P}, \bar{P}) = S(\bar{Q}, \bar{Q}) \)
4. If \( \bar{P} \leq \bar{Q} \leq \bar{R} \), then \( S(\bar{P}, \bar{R}) \leq S(\bar{P}, \bar{Q}) \)

Theorem 1 \( 0 \leq S_{IVIFS}(\bar{P}, \bar{Q}) \leq 1 \)

Proof By induction hypothesis, for any \( c_1 \geq 0, c_2 \geq 0, \theta_0 \geq 0 \), we have \( (c_1 \theta_0 + c_2 \theta_0^2) \leq (c_1 \theta_0^2 + c_2 \theta_0^2) \) if \( \theta_0 \geq 0 \)

\[ \begin{align*}
0 & \leq \sqrt{(c_1 \theta_0^2 + c_2 \theta_0^2)} \leq \sqrt{(c_1 \theta_0^2 + c_2 \theta_0^2)} \leq \sqrt{(c_1 \theta_0^2 + c_2 \theta_0^2)} \leq 1. 
\end{align*} \]

Theorem 2 \( S_{IVIFS}(\bar{P}, \bar{Q}) = 1 \) if and only if \( \bar{P} = \bar{Q} \)

Proof Let us now consider the following induction,

\[ S_{IVIFS}(\bar{P}, \bar{Q}) = \begin{cases} 1 & \text{if and only if } (S_1^{IVIFS}(\bar{P}, \bar{Q}) + S_2^{IVIFS}(\bar{P}, \bar{Q}) + S_3^{IVIFS}(\bar{P}, \bar{Q})) = 3 \\
S_1^{IVIFS}(\bar{P}, \bar{Q}) & \text{if and only if } (S_1^{IVIFS}(\bar{P}, \bar{Q}) + S_2^{IVIFS}(\bar{P}, \bar{Q}) + S_3^{IVIFS}(\bar{P}, \bar{Q})) = 1 \\
S_1^{IVIFS}(\bar{P}, \bar{Q}) & \text{if and only if } (S_1^{IVIFS}(\bar{P}, \bar{Q}) + S_2^{IVIFS}(\bar{P}, \bar{Q}) + S_3^{IVIFS}(\bar{P}, \bar{Q})) = 2 \\
S_1^{IVIFS}(\bar{P}, \bar{Q}) & \text{if and only if } (S_1^{IVIFS}(\bar{P}, \bar{Q}) + S_2^{IVIFS}(\bar{P}, \bar{Q}) + S_3^{IVIFS}(\bar{P}, \bar{Q})) = 4 \\
S_1^{IVIFS}(\bar{P}, \bar{Q}) & \text{if and only if } (S_1^{IVIFS}(\bar{P}, \bar{Q}) + S_2^{IVIFS}(\bar{P}, \bar{Q}) + S_3^{IVIFS}(\bar{P}, \bar{Q})) = 0 \\
\end{cases} \]

Theorem 3 \( S_{IVIFS}(\bar{P}, \bar{Q}) = S_{IVIFS}(\bar{Q}, \bar{P}) \)

Proof By symmetry, \( S_{IVIFS}(\bar{P}, \bar{Q}) = S_{IVIFS}(\bar{Q}, \bar{P}) \) and \( S_{IVIFS}(\bar{Q}, \bar{P}) = S_{IVIFS}(\bar{Q}, \bar{P}) \) Since

\[ S_{IVIFS}(\bar{P}, \bar{Q}) = \frac{1}{3}(S_1^{IVIFS}(\bar{P}, \bar{Q}) + S_2^{IVIFS}(\bar{P}, \bar{Q}) + S_3^{IVIFS}(\bar{P}, \bar{Q})) \]

Then \( S_{IVIFS}(\bar{P}, \bar{Q}) = S_{IVIFS}(\bar{Q}, \bar{P}) \) and \( S_{IVIFS}(\bar{Q}, \bar{P}) = S_{IVIFS}(\bar{Q}, \bar{P}) \)

Theorem 4 \( S_{IVIFS}(\bar{P}, \bar{R}) \leq S_{IVIFS}(\bar{P}, \bar{Q}) \) a n d \( S_{IVIFS}(\bar{Q}, \bar{R}) \leq S_{IVIFS}(\bar{Q}, \bar{P}) \) if \( \bar{P} \subseteq \bar{Q} \subseteq \bar{R} \)

Proof Let us consider the function \( g \) with \( g(\theta_1) = \frac{e^3\theta_1^2 + e^2\theta_2^2}{(\theta_1^2 + \theta_2^2)^2} \) and \( g(\theta_2) = \frac{e^3\theta_1^2 + e^2\theta_2^2}{(\theta_1^2 + \theta_2^2)^2} \) if \( \theta_1 \leq \theta_2 \)

\[ \begin{align*}
S_{IVIFS}(\bar{P}, \bar{R}) & = \frac{\Delta E + ZT}{\sqrt{\Delta^2 + Z^2\sqrt{E^2 + T^2}}} \\
& \leq \frac{\sqrt{\Delta^2 + Z^2\sqrt{E^2 + T^2}}}{\sqrt{\Delta^2 + Z^2\sqrt{E^2 + T^2}}} \\
& = S_{IVIFS}(\bar{P}, \bar{Q}) \\
\end{align*} \]

Remark 1 Consider \( \bar{P} = \{x, [0.0.1], [0.0.1]\} \) and \( \bar{Q} = \{x, [0.0.3], [0.0.4]\} \) are the two IVIFSs, the CSM for
IVIFSs $\tilde{P}$ and $\tilde{Q}$ that is $S_{IVIFS}(\tilde{P}, \tilde{Q}) = 1$. Since $P \neq Q$, but the degree of similarity is equal to 1. Therefore, the condition (ii) in Definition 5 is not satisfied. On the other hand, $S_{IVIFS}$ between IVIFSs $\tilde{P}$ and $\tilde{Q}$ as $S_{IVIFS}(\tilde{P}, \tilde{Q}) = 0.9869$. Hence, it satisfies the condition (ii) in Definition 5.

**Remark 2** If $\tilde{P} = \{x, [0.0, 0.0], [0.1, 0.2]\}$ and $\tilde{Q} = \{x, [0.1, 0.2], [0.0, 0.0]\}$ are two IVIFSs, then the CSM between IVIFSs $\tilde{P}$ and $\tilde{Q}$ is equal to zero. This is the unreasonable result. Meanwhile, applying the proposed similarity measure can obtained the result $S_{IVIFS}(\tilde{P}, \tilde{Q}) = 0.6433$, it is reasonable.

**Note 1** From Remarks 1 and 2, it shows that $S_{IVIFS}$ between IVIFSs is more effective and reasonable than CSM between IVIFSs.

**Example 1** Supposing that $\tilde{P}_i$ and $\tilde{Q}_i$ are two IVIFSs, we can compute the similarity measures between $\tilde{P}_i$ and $\tilde{Q}_i$ by different similarity measures listed in Table 1.

In Table 1, by comparing the first column and the second column, we can find that $S_{1}(\tilde{P}_1, \tilde{Q}_1) = S_{1}(\tilde{P}_2, \tilde{Q}_2)$ (i = 1, 2) when $\tilde{P}_1 = \tilde{P}_2$, $\tilde{Q}_1 \neq \tilde{Q}_2$. Similarly, by comparing the third column and the fourth column, we can find $S_{2}(\tilde{P}_4, \tilde{Q}_4) = S_{2}(\tilde{P}_3, \tilde{Q}_3)$ when $\tilde{P}_3 = \tilde{P}_4$, $\tilde{Q}_3 \neq \tilde{Q}_4$. Therefore, we can determine that the similarity measure $S_{1}$ and $S_{2}$ is not reasonable. Meanwhile, we find that $S_{D}(\tilde{P}_1, \tilde{Q}_1) = 1$ when $\tilde{P}_1 \neq \tilde{Q}_1$, which is not satisfy the second axiom of the definition of similarity measure. Most importantly, we can observe that the proposed similarity measure $S_{IVIFS}$ and the other measure $S_{D}$ can overcome these drawbacks. But, from the Table 1, the proposed measure value is highly compared to other sophisticated techniques. Therefore, our novel similarity measure for IVIFSs is more reasonable than other measures.

**5 Applications**

In this section, the suggested similarity measure has been widely applied to various problems in the IVIFS environment, and the findings were also compared to those of other similarity measures.

| Table 1 Comparison of similarity measures in the environment of IVIFSs |
|---------------------------------------------------------------|
|                  | 1                | 2                | 3                | 4                |
| $\tilde{P}_i$    | ([0.2,0.3], [0.4,0.6]) | ([0.2,0.3], [0.4,0.6]) | ([0.2,0.3], [0.3,0.5]) | ([0.2,0.3], [0.3,0.5]) |
| $\tilde{Q}_i$    | ([0.3,0.4], [0.4,0.6]) | ([0.3,0.4], [0.4,0.6]) | ([0.3,0.4], [0.4,0.6]) | ([0.3,0.4], [0.4,0.6]) |
| $S_{1}$          | 0.90             | 0.90             | 0.90             | 0.95             |
| $S_{2}$          | 0.90             | 0.90             | 0.90             | 0.90             |
| $S_{D}$          | 1.00             | 0.98             | 0.95             | 0.94             |
| $S_{P}$          | 0.95             | 0.90             | 0.80             | 0.94             |
| $S_{C}$          | 0.98             | 0.96             | 0.99             | 0.98             |
| $S_{IVIFS}$      | 0.99             | 0.97             | 0.95             | 0.98             |

**5.1 Pattern recognition**

**5.1.1 Algorithms for pattern recognition**

If we take $X = \{x_1, x_2, \ldots, x_n\}$ to become a finite universe of discourse, we can see that there are $m$ patterns labeled by IVIFSs $\tilde{P}_j = \{(x_1, [\mu^-_{\tilde{P}_j}(x_1), \mu^+_{\tilde{P}_j}(x_1)], [v^-_{\tilde{P}_j}(x_1), v^+_{\tilde{P}_j}(x_1)])\}, \ldots, (x_n, [\mu^-_{\tilde{P}_j}(x_n), \mu^+_{\tilde{P}_j}(x_n)], [v^-_{\tilde{P}_j}(x_n), v^+_{\tilde{P}_j}(x_n)]) : x_1, \ldots, x_n \in X)\}$ (j=1,2,...,m) and a test sample that was classified with an IVIFS $\tilde{Q} = \{(x_1, [\mu^-_{\tilde{Q}}(x_1), \mu^+_{\tilde{Q}}(x_1)], [v^-_{\tilde{Q}}(x_1), v^+_{\tilde{Q}}(x_1)])\}, \ldots, (x_n, [\mu^-_{\tilde{Q}}(x_n), \mu^+_{\tilde{Q}}(x_n)], [v^-_{\tilde{Q}}(x_n), v^+_{\tilde{Q}}(x_n)]) : x_1, \ldots, x_n \in X)\}$. The procedure for recognition is as described in the following:

**Step 1.** Compute the $S(\tilde{P}_j, \tilde{Q})$ measure of similarity with $\tilde{P}_j (j = 1, \ldots, m)$ and $\tilde{Q}$.

**Step 2.** Select the highest $S(\tilde{P}_j, \tilde{Q})$ value from $S(\tilde{P}_j, \tilde{Q})$ (j = 1, 2, ...,m), i.e., $S(\tilde{P}_j, \tilde{Q}) = \max \{S(\tilde{P}_1, \tilde{Q}), \ldots, S(\tilde{P}_m, \tilde{Q})\}$. Following that, the test sample $\tilde{Q}$ becomes classified to it in the pattern $\tilde{P}_j$.

**5.1.2 Applications for pattern recognition**

**Example 2** Suppose that in the area formed by the coal mine company, there were four kinds of ores $P_i$ (i=1,2,3,4), in which the corresponding feature information would be indicated by IVIFSs and $\tilde{P}_i = \{(x_1, [\mu^-_{\tilde{P}_i}(x_1), \mu^+_{\tilde{P}_i}(x_1)], [v^-_{\tilde{P}_i}(x_1), v^+_{\tilde{P}_i}(x_1)])\}, \ldots, (x_4, [\mu^-_{\tilde{P}_i}(x_4), \mu^+_{\tilde{P}_i}(x_4)], [v^-_{\tilde{P}_i}(x_4), v^+_{\tilde{P}_i}(x_4)]) : x_1, \ldots, x_4 \in X)\}$, as can be seen in Table 2. Now there is an unknown ore, $\tilde{Q}$, so our goal is just to categorize $\tilde{Q}$ under one of the four types of ores described above.

Estimate the $S(\tilde{P}_i, \tilde{Q})$ measure of similarity among both $\tilde{P}_i$ and $\tilde{Q}$. By examining the obtained results in Table 3, we will clearly determine that if $S_1$ is employed in pattern recognition, authors can get $S_1(\tilde{P}_1, \tilde{Q}) = S_1(\tilde{P}_2, \tilde{Q}) = S_1(\tilde{P}_4, \tilde{Q}) > S_1(\tilde{P}_3, \tilde{Q})$. As a consequence, we are unable to properly classify the sample $\tilde{Q}$ into a specific pattern. $S_1(\tilde{P}_2, \tilde{Q}) = S_1(\tilde{P}_1, \tilde{Q}) > S_1(\tilde{P}_3, \tilde{Q})$ can be acquired if $S_1$ has become applied in pattern recognition. In this way, we cannot be certain whether the sample $\tilde{Q}$ belongs to $\tilde{P}_2$ and $\tilde{P}_4$. We can also achieve $S_1(\tilde{P}_3, \tilde{Q}) = S_1(\tilde{P}_4, \tilde{Q}) > S_1(\tilde{P}_2, \tilde{Q}) >$
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Table 2 Feature matrix of $\hat{P}_1$, $\hat{P}_2$, $\hat{P}_3$, $\hat{P}_4$ and $\hat{Q}$

| Feature 1 | Feature 2 | Feature 3 | Feature 4 |
|-----------|-----------|-----------|-----------|
| $\hat{P}_1$ | ([0.10,0.50], [0.20,0.30]) | ([0.10,0.30], [0.00,0.20]) | ([0.30,0.50], [0.20,0.30]) | ([0.10,0.50], [0.20,0.30]) |
| $\hat{P}_2$ | ([0.20,0.40], [0.15,0.35]) | ([0.20,0.60], [0.05,0.15]) | ([0.20,0.40], [0.50,0.60]) | ([0.20,0.40], [0.15,0.25]) |
| $\hat{P}_3$ | ([0.15,0.30], [0.30,0.40]) | ([0.20,0.60], [0.15,0.35]) | ([0.35,0.60], [0.05,0.30]) | ([0.25,0.45], [0.30,0.40]) |
| $\hat{P}_4$ | ([0.20,0.35], [0.10,0.65]) | ([0.15,0.30], [0.40,0.55]) | ([0.15,0.25], [0.45,0.55]) | ([0.15,0.40], [0.20,0.50]) |
| $\hat{Q}$   | ([0.30,0.40], [0.10,0.50]) | ([0.10,0.40], [0.25,0.40]) | ([0.30,0.50], [0.00,0.20]) | ([0.20,0.30], [0.10,0.35]) |

Table 3 Pattern recognition results under different similarity measures

| $S(P_1, Q)$ | $S(P_2, Q)$ | $S(P_3, Q)$ | $S(P_4, Q)$ | Classification results |
|-------------|-------------|-------------|-------------|------------------------|
| $S_1$       | 0.87        | 0.87        | 0.86        | 0.87                   | N.A.                  |
| $S_2$       | 0.75        | 0.76        | 0.79        | 0.76                   | N.A.                  |
| $S_W$       | 0.78        | 0.79        | 0.78        | 0.79                   | N.A.                  |
| $S_D$       | 0.82        | 0.86        | 0.88        | 0.88                   | N.A.                  |
| $S_P$       | 0.82        | 0.81        | 0.88        | 0.75                   | $\hat{P}_3$          |
| $S_{IVIFS}$ | 0.95        | 0.95        | 0.97        | 0.92                   | N.A.                  |
| $S_{IVIFS}$ | 0.97        | 0.95        | 0.98        | 0.94                   | $\hat{P}_3$          |

N.A. not applicable

and Fig. 2, we conclude that, the measures $S_P$ and $S_{IVIFS}$ are mostly give the same results, but $S_{IVIFS}$ value is high than $S_P$ measure. Hence, the pattern $P_3$ can sometimes be assigned to sample $\hat{Q}$.

Example 3 To exemplify the suggested similarity measure, a pattern recognition application involving the categorization of building materials is utilized. Assuming that here in the feature space $X = \{x_1, x_2, \ldots, x_{12}\}$, there have been four major classes of building materials designated by the IVIFSs $\hat{P}_j = \{(x_1, [\mu_{\hat{P}_j}(x_1), \mu_{\hat{P}_j}(x_1)]), [\nu_{\hat{P}_j}(x_1), \nu_{\hat{P}_j}(x_1))]\), \ldots, (x_{12}, [\mu_{\hat{P}_j}(x_{12}), \mu_{\hat{P}_j}(x_{12})], [\nu_{\hat{P}_j}(x_{12}), \nu_{\hat{P}_j}(x_{12})])) : x_1, \ldots, x_{12} \in X\}$ ($j = 1, \ldots, 12$) and that there is an unknown pattern $\hat{Q}$. 

Fig. 2 Comparison graph of pattern recognition problem
Subsequently, implementing the proposed measure formula, we evaluate the similarity measure $S(\hat{P}_j, \tilde{Q})$ among IVIFSs $\hat{P}_j$ ($j = 1, 2, 3, 4$) and $\tilde{Q}$. The similarity measure is seen in the literature (Luo and Liang 2018) as a special feature of $S_1$ and $S_2$, as well as the quantified outcome being identical to (Luo and Liang 2018). However, according to the recognition principle, Table 4 and Fig. 3, the unknown pattern in $\hat{P}_4$ may be properly classified by finding the similarity measure. This finding is consistent with that of Luo and Liang (2018).

5.2 Medical diagnosis

5.2.1 Application for medical diagnosis

Researchers proposed a variety of approaches from various perspectives to address medical diagnosis issues. In this part, pattern recognition algorithms have been employed to tackle medical diagnosis challenges, i.e., patients represent unknown test samples, diseases are different patterns, and the symptom set seems to be the set universe of discourse.

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**Table 4** Pattern recognition results under different similarity measures

|                  | $S(\hat{P}_1, \tilde{Q})$ | $S(\hat{P}_2, \tilde{Q})$ | $S(\hat{P}_3, \tilde{Q})$ | $S(\hat{P}_4, \tilde{Q})$ | Recognition results |
|------------------|--------------------------|--------------------------|--------------------------|--------------------------|---------------------|
| $S_1$            | 0.59                     | 0.58                     | 0.81                     | 0.97                     | $\hat{P}_4$         |
| $S_2$            | 0.53                     | 0.53                     | 0.79                     | 0.94                     | $\hat{P}_4$         |
| $S_W$            | 0.48                     | 0.47                     | 0.74                     | 0.94                     | $\hat{P}_4$         |
| $S_D$            | 0.64                     | 0.56                     | 0.83                     | 0.98                     | $\hat{P}_4$         |
| $S_P$            | 0.60                     | 0.58                     | 0.85                     | 0.97                     | $\hat{P}_4$         |
| $S_C$            | 0.69                     | 0.65                     | 0.83                     | 0.97                     | $\hat{P}_4$         |
| $S_{IVIFS}$      | 0.72                     | 0.69                     | 0.88                     | 0.99                     | $\hat{P}_4$         |

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Fig. 3 Comparison graph of pattern recognition problem
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Table 5 Disease-symptom matrix

|        | x1 (temperature) | x2 (cough) | x3 (headache) | x4 (stomach pain) |
|--------|------------------|------------|---------------|-------------------|
| A1 (viral fever) | ([0.8,0.9], [0.0,0.1]) | ([0.7,0.8], [0.1,0.2]) | ([0.5,0.6], [0.2,0.3]) | ([0.6,0.8], [0.1,0.2]) |
| A2 (typhoid)     | ([0.5,0.6], [0.1,0.3]) | ([0.8,0.9], [0.0,0.1]) | ([0.6,0.8], [0.1,0.2]) | ([0.4,0.6], [0.1,0.2]) |
| A3 (pneumonia)   | ([0.7,0.8], [0.1,0.2]) | ([0.7,0.9], [0.0,0.1]) | ([0.4,0.6], [0.2,0.4]) | ([0.3,0.5], [0.2,0.4]) |
| A4 (stomach problem) | ([0.8,0.9], [0.0,0.1]) | ([0.7,0.8], [0.1,0.2]) | ([0.7,0.9], [0.0,0.1]) | ([0.8,0.9], [0.0,0.1]) |

Table 6 Computed results under different similarity measures

|        | $S(P_1, Q)$ | $S(P_2, Q)$ | $S(P_3, Q)$ | $S(P_4, Q)$ | Recognition results |
|--------|-------------|-------------|-------------|-------------|---------------------|
| $S_1$  | 0.81        | 0.89        | 0.86        | 0.84        | $\overline{P}_2$   |
| $S_2$  | 0.73        | 0.80        | 0.78        | 0.73        | $\overline{P}_2$   |
| $S_w$  | 0.82        | 0.80        | 0.79        | 0.77        | $\overline{P}_2$   |
| $S_D$  | 0.82        | 0.91        | 0.86        | 0.84        | $\overline{P}_2$   |
| $S_P$  | 0.83        | 0.89        | 0.87        | 0.85        | $\overline{P}_2$   |
| $S_C$  | 0.93        | 0.97        | 0.96        | 0.94        | $\overline{P}_2$   |
| $S_{IVIFS}$ | 0.94      | 0.98        | 0.97        | 0.96        | $\overline{P}_2$   |

Fig. 4 Comparison graph of medical diagnosis problem

Our intention is to classify patients as having one of the disorders.

Example 4 Letting $P = \{P_1, P_2, P_3, P_4\}$ where $P_1 =$Viral fever, $P_2 =$Typhoid, $P_3 =$Pneumonia, and $P_4 =$Stomach problem represent a set of diagnoses and $X = \{x_1($ Temperature $), x_2 ($ Cough $), x_3 ($ Headache $), x_4 ($ Stomach pain $)\}$ symbolize a collection of symptoms. After that, Table 5 lists the disease-symptom matrix with IVIFSs representation. Suppose that the patient $Q$ may be described as the following: $\overline{Q} = ((x_1, [0.4, 0.5], [0.1, 0.2]), (x_2, [0.7, 0.8], [0.1, 0.2]), (x_3, [0.9, 0.9], [0.0, 0.1]), (x_4, [0.3, 0.5], [0.2, 0.4]))$.

Our goal is to determine whether or not patient $Q$ has one of the diseases $\overline{P}_1, \overline{P}_2, \overline{P}_3$ and $\overline{P}_4$. Thereafter, in the context of IVIFSs, we may expect the following outcomes, as shown by Table 6. Further, the tabulated values are plotted as graphical form, which is shown in Fig. 4.

By implementing the maximum similarity principle, we may deduce that the similarity measure among both $P_2$ and $Q$ seems to be the biggest. Meanwhile, the similarity measures $S_2$ were unable to determine which would have been high among both $P_1$ and $P_4$. As a consequence of the recognition principle, we may identify that patient $Q$ has sickness $\overline{P}_2$. Therefore, we may conclude that the patient has typhoid. Finally, the mentioned Table 6 and Fig. 4 shows the performance of the proposed measure.

5.3 Multi-criteria decision making problems

5.3.1 Algorithm for MCDM

In this subsection, we proposed similarity measure is to solve a multiple criteria decision making problem. Let $\tilde{H} = \{\tilde{H}_1, \tilde{H}_2, \ldots, \tilde{H}_m\}$ be the set of alternative, $\tilde{C} = \{\tilde{C}_1, \tilde{C}_2, \ldots, \tilde{C}_n\}$ be the set of criteria. The alternative $\tilde{H}_i$ of the criteria $\tilde{C}$, represented by IVIFS as follows: $\tilde{H}_i = ([\tilde{C}_j, [\mu^+_{i,j}, \mu^-_{i,j}], [v^+_{i,j}, v^-_{i,j}]] : \tilde{C}_j \in \tilde{C}), i = 1, 2, \ldots, m$.

Using the proposed measure (11), we present the following algorithm to solve multiple criteria decision making problem:

Step 1: Determine the both positive ideal solution ($H^+$) and negative ideal solution ($H^-$):

$H^+ = ([\mu^+_{1+} - \mu^-_{1-}], [v^+_{1+} - v^-_{1-}], [\mu^+_{2+} - \mu^-_{2-}], [v^+_{2+} - v^-_{2-}], \ldots, ([\mu^+_{m+} - \mu^-_{m-}], [v^+_{m+} - v^-_{m-}]]$)

$H^- = ([\mu^+_{1-} - \mu^-_{1+}], [v^+_{1-} - v^-_{1+}], [\mu^+_{2-} - \mu^-_{2+}], [v^+_{2-} - v^-_{2+}], \ldots, ([\mu^+_{m-} - \mu^-_{m+}], [v^+_{m-} - v^-_{m+}])]$
where, \( j = 1, 2, \ldots, n \), \( (\mu_{j+}^{m}, \mu_{j+}^{n}), (v_{j+}^{m}, v_{j+}^{n}) = \\
(max_{j+}, max_{j+}, min_{j+}, min_{j+}), (min_{j+}, min_{j+}, max_{j+}, max_{j+}) \).

**Step 2:** Estimate the degree of similarity for both positive ideal solution \( S(\hat{H}, H^+) \) and negative ideal solution \( S(\hat{H}, H^-) \) by using the formula as follows:

\[
S(\hat{H}, H^+) = \frac{1}{3} (S^1_{IVIFS}(\hat{H}, H^+) + S^2_{IVIFS}(\hat{H}, H^+) + S^3_{IVIFS}(\hat{H}, H^+))
\]

(12)

\[
S(\hat{H}, H^-) = \frac{1}{3} (S^1_{IVIFS}(\hat{H}, H^-) + S^2_{IVIFS}(\hat{H}, H^-) + S^3_{IVIFS}(\hat{H}, H^-))
\]

(13)

**Step 3:** Employ Eqs. (12)–(13), to calculate the relative similarity measure \( S(\hat{H}) \) of \( H \) with respect to \( H^+ \) and \( H^- \) as follows:

\[
S(\hat{H}) = \frac{S(\hat{H}, H^+)}{S(\hat{H}, H^-) + S(\hat{H}, H^+)}
\]

(14)

where \( i = 1, 2, \ldots, n \).

**Step 4:** Select the maximum degree of similarity \( S(\hat{H}) \) from \( S(\hat{H}) \) where \( i = 1, 2, \ldots, n \). Conclusively, \( S(\hat{H}) \) is the best choice.

\[
\hat{H}_1 = (\bar{T}_1, [0.4, 0.5], [0.2, 0.3]), (\bar{T}_2, [0.6, 0.8], [0.1, 0.2]), (\bar{T}_3, [0.4, 0.5], [0.2, 0.4]), \\
(\bar{T}_4, [0.8, 0.9], [0.1, 0.1]), (\bar{T}_5, [0.2, 0.6], [0.2, 0.3]), (\bar{T}_6, [0.5, 0.7], [0.1, 0.2])
\]

\[
\hat{H}_2 = (\bar{T}_1, [0.5, 0.7], [0.1, 0.2]), (\bar{T}_2, [0.6, 0.8], [0.1, 0.2]), (\bar{T}_3, [0.3, 0.4], [0.4, 0.6]), \\
(\bar{T}_4, [0.8, 0.9], [0.0, 0.1]), (\bar{T}_5, [0.2, 0.5], [0.3, 0.4]), (\bar{T}_6, [0.1, 0.2], [0.4, 0.5])
\]

\[
\hat{H}_3 = (\bar{T}_1, [0.2, 0.3], [0.6, 0.7]), (\bar{T}_2, [0.4, 0.5], [0.3, 0.4]), (\bar{T}_3, [0.7, 0.8], [0.1, 0.2]), \\
(\bar{T}_4, [0.2, 0.5], [0.1, 0.2]), (\bar{T}_5, [0.7, 0.8], [0.0, 0.1]), (\bar{T}_6, [0.5, 0.6], [0.2, 0.4])
\]

\[
\hat{H}_4 = (\bar{T}_1, [0.5, 0.6], [0.1, 0.2]), (\bar{T}_2, [0.3, 0.4], [0.2, 0.3]), (\bar{T}_3, [0.5, 0.8], [0.1, 0.2]), \\
(\bar{T}_4, [0.6, 0.7], [0.1, 0.2]), (\bar{T}_5, [0.3, 0.4], [0.3, 0.4]), (\bar{T}_6, [0.1, 0.2], [0.7, 0.8])
\]

\[
\hat{H}_5 = (\bar{T}_1, [0.4, 0.5], [0.3, 0.4]), (\bar{T}_2, [0.8, 0.9], [0.0, 0.1]), (\bar{T}_3, [0.6, 0.8], [0.1, 0.2]), \\
(\bar{T}_4, [0.8, 0.9], [0.0, 0.1]), (\bar{T}_5, [0.7, 0.8], [0.1, 0.2]), (\bar{T}_6, [0.5, 0.6], [0.1, 0.2])
\]

**5.3.2 Application for MCDM**

**Example 5** Assume that the problem of distributor selection for a product. Six evaluating indexes are consider by the company for the problem. They are price \((\bar{T}_1)\), deadline \((\bar{T}_2)\), quality \((\bar{T}_3)\), the level of technology \((\bar{T}_4)\), service \((\bar{T}_5)\) and the future cooperation \((\bar{T}_6)\). Given there are five distributors for this problem \(\hat{H}_i (i = 1, 2, 3, 4, 5)\). By using (Wei et al. 2011) above indexes, suppliers are analysed by experts and the analysed results are given as follows:

| Measure | Measure value | Ranking | The best one |
|---------|--------------|---------|--------------|
| \(S_1\) | 0.57, 0.49, 0.50, 0.45, 0.66 | \(S(\hat{H}_1) > S(\hat{H}_2) > S(\hat{H}_3) > S(\hat{H}_4) > S(\hat{H}_5)\) | \(\hat{H}_5\) |
| \(S_2\) | 0.52, 0.50, 0.49, 0.48, 0.56 | \(S(\hat{H}_1) > S(\hat{H}_2) > S(\hat{H}_3) > S(\hat{H}_4) > S(\hat{H}_5)\) | \(\hat{H}_5\) |
| \(S_w\) | 0.55, 0.46, 0.53, 0.48, 0.64 | \(S(\hat{H}_1) > S(\hat{H}_2) > S(\hat{H}_3) > S(\hat{H}_4) > S(\hat{H}_5)\) | \(\hat{H}_5\) |
| \(S_D\) | 0.54, 0.50, 0.49, 0.48, 0.58 | \(S(\hat{H}_1) > S(\hat{H}_2) > S(\hat{H}_3) > S(\hat{H}_4) > S(\hat{H}_5)\) | \(\hat{H}_5\) |
| \(S_P\) | 0.50, 0.49, 0.50, 0.49, 0.50 | \(S(\hat{H}_1) > S(\hat{H}_2) > S(\hat{H}_3) > S(\hat{H}_4) > S(\hat{H}_5)\) | \(\hat{H}_5\) |
| \(S_C\) | 0.54, 0.48, 0.51, 0.48, 0.56 | \(S(\hat{H}_1) > S(\hat{H}_2) > S(\hat{H}_3) > S(\hat{H}_4) > S(\hat{H}_5)\) | \(\hat{H}_5\) |
| \(S_{IVIFS}\) | 0.54, 0.50, 0.51, 0.49, 0.55 | \(S(\hat{H}_1) > S(\hat{H}_2) > S(\hat{H}_3) > S(\hat{H}_4) > S(\hat{H}_5)\) | \(\hat{H}_5\) |
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We acquire the values of the suggested similarity measures and decision outcomes by using derived Eqs. (12)–(13), that are reported in Table 7 for comparison. The tabulated findings are then presented in a graphical format, as illustrated in Fig. 5. Besides, Table 7 reveals that a few of the ranking results have become consistent, with the best being \( \tilde{H}_5 \), demonstrating the efficiency of the devised MCDM process based on a recommended similarity measure. The majority of the ranking orders in Table 7 are similar to the specific case of the illustrated example in the IVIFS environment. The rank ordering predicated on the suggested MCDM approach implementing IVIFS similarity measures in Table 7 varies from some of those depending on the MCDM method utilizing IVIFS similarity measures in Table 7. As a result, the influence on alternative ranking reflects its significance in proposed MCDM applications. However, although many state-of-the-art decision-making processes in IVIFS settings (Dhivya and Sridevi 2018; Luo and Liang 2018; Singh 2012; Wei et al. 2011; Xu and Chen 2008) have not yet been able to handle MCDM issues with IVIFS data, the latest sophisticated approaches do not include middle and complementary terms for membership and non-membership degrees. Also, they have not yet been included in the measure. Then, Table 7 and Fig. 5 evidenced that the proposed MCDM framework based on the similarity measure of IVIFSs may be applied to the MCDM issues with IVIFSs and which makes the decision findings more valuable and realistic in decision making implementations.

### 5.3.2.1 Merits of the proposed method

The following are typical benefits of working in the context of an IVIFS-based similarity measure:

1. IVIFS improves the reliability measure of intervals of membership and non-membership degrees, which makes IFS more trustworthy. As a consequence, the recommended IVIFS incorporates far more valuable information than IFS.
2. The pattern recognition, medical diagnosis, and MCDM problems solved using the suggested similarity measure of IVIFSs in this work strengthen the validity and subjectivity of pattern, medical, and decision-making outcomes as well as illustrate its effectiveness over existing IVIFS-based similarity measures.

3. The benefit of this technique is that it seems to be uncomplicated to compute the similarity degree. Likewise, the strengths of this study include a higher degree of resemblance to real-world situations.

4. Moreover, the fundamental benefit of this strategy is that the degree of similarity is derived by taking into account the terms’ intervals of degrees of membership, non-membership, middle of both, and complement of both terms, respectively. Therefore, our suggested technique may be more accurate than other advanced procedures.

### 6 Conclusion

In this study, we developed a novel and resilient technique for analyzing the similarity of IVIFSs. Using the idea of an existing CSM, the study found a new interval-valued intuitionistic fuzzy cosine similarity measure as well as demonstrated some of its fundamental and crucial characteristics. Further, the newly established similarity measure has been employed to address a wide variety of real-world challenges such as pattern recognition, medical diagnosis, as well as MCDM. Furthermore, numerical examples have been furnished to demonstrate the stated application process. In each application, the existing and addressed similarity measures were analyzed. Finally, we can notice that the proposed technique delivers more information for all of the applications listed since it returns higher values than other similarity measures owing to the maximum similarity principle.

Also, the suggested method has a few drawbacks as well. The suggested similarity measure may be used in cases where its degrees of membership and non-membership have numerical values on an interval scale. However, linguistic variables are employed to express qualitative information in many real-life situations. This similarity measure cannot be used in a linguistic context. Further research into this similarity measure involving linguistic interval-valued intuitionistic fuzzy information is required.

More advanced formulations, including probabilities, power averages, moving averages, and so on, are expected to be applied for future research. Social network
decision-making, consensus, medical imaging, and large-scale decision-making are all relevant topics to examine. Since we all know, consensus measures are extremely important in group decision-making problems. Before a solution can be reached, there must be broad consensus among specialists. We will also concentrate on the creation of several consensus measures based on the suggested similarity measure, as well as their applications in social network decision-making and large-scale decision-making challenges in various uncertain circumstances. Based on the prior studies (Gao and Zhang 2021; Zhang et al. 2021), we are keen to implement the recommended measure to cope with consensus-reaching challenges in group decision-making via IVIFSs.

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Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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