Active Disturbance Rejection Control for Wheeled Mobile Robots with Parametric Uncertainties

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Abstract: In this paper, an efficient controller design method is proposed based on active disturbance rejection control (ADRC) scheme for stabilization problem of wheeled mobile robots with parametric uncertainties, which can make the system converge quickly. By using the extended state observer (ESO), both the system states and the unknown parametric uncertainties could be estimated. In addition, the input-state scaling technique is used to transform the system into two decoupled subsystems. Based on the decoupled subsystems, a switching controller and ADRC are designed. Simulation results show that the proposed scheme can stabilize the wheeled mobile robot system asymptotically despite the presence of parametric uncertainties.

Keywords: Wheeled mobile robots, Parametric uncertainty, Input-state scaling, Active disturbance rejection control, Stabilizing controller

1. INTRODUCTION

Wheeled mobile robots can move autonomously in a complex environment and complete many difficult tasks [Hua and Min (2001); Su et al. (2004)]. Thus, it has been widely used in many fields [Hua (2013)]. However, the motion is under the non-holonomic constraint of pure rolling between the wheels and the ground, which brings challenges to the stabilization problem of wheeled mobile robots [Goldstein et al. (2002)].

The stabilization problem of non-holonomic mobile robots has been widely studied since the seminal work of [Brock et al. (1983)], which establishes the non existence of any continuous state feedback control law that can make system asymptotically stable in the sense of Lyapunov. Therefore, many classical control methods used in holonomic systems fail to deal with non-holonomic systems, which makes the stabilization control of non-holonomic systems very difficult. In order to solve this problem, there are already some existing methods, for example, time-varying continuous controllers [Jiang (1996); Morin and Samson (1997)], and discontinuous controllers [Astolfi (1999); Gao et al. (2015)].

Most of the above studies focus on the deterministic systems. However, a wheeled mobile robot is a complex system with many uncertainties. Due to the existence of these uncertainties, it is usually difficult to obtain the accurate model of the system, which may provoke performance deterioration even instability of the control system, in particular for the presence of parametric uncertainties [Su et al. (2008)]. Thus, to reduce the influence of uncertainty is a key point of controller design. Jiang combines \( \sigma \) transformation and backstepping method to design a robust exponential stabilization controller, in which the higher-order derivations need to be figured out [Jiang (2000)]. A robust stabilization control strategy is proposed by Lucibello to deal with the uncertain wheeled mobile robots with the cost of poor dynamic performance [Lucibello and Oriolo (2001)]. Model reference adaptive control scheme is proposed in [Ashoorirad et al. (2006)] for wheeled mobile robots with bounded parametric uncertainties, where the bound is too small. Guechi designs a stabilizing controller using differential flatness based on the sliding mode technology, in which the design of sliding surface is complicated [Guechi et al. (2012)].

Therefore, to enhance the response speed and uncertainty-rejecting performance of a class of wheeled mobile robots with parametric uncertainties, a new control strategy based on active disturbance rejection control scheme is proposed in this paper. The idea of a controller with the capacity of compensation of internal and external disturbances by means of an extended state observer (ESO) was proposed by Han, and sequentially introduced the concept of active disturbance rejection control (ADRC) [Han (1998)]. ADRC has led to a new paradigmatic view of traditional nonlinear control problems where disturbances, internal and external, are actively estimated and rejected. Furthermore, it has small overshoot, fast response speed,
high accuracy, and strong disturbance-rejection capabilities, having aroused extensive researches [Chen et al. (2013); Huang and Xue (2014); Li et al. (2017); Xue and Huang (2016)]. Up to now, ADRC has been successfully applied to power system, flight control, machine processing, and other fields [Wu et al. (2013); Dong et al. (2012); Zhao and Gao (2013)]. Hence it is natural to resort to ADRC to deal with the parametric uncertainties in wheeled robots [Huang and Su (2019)]. In our method, the system is decoupled to two decoupled subsystems by using input-state scaling technique. After that, the ADRC algorithm is deduced and ESO is used to estimate and compensate for total disturbance. Simulation results show that the proposed controller can stabilize the wheeled mobile robot system quickly and reject uncertainties.

The rest of the paper is organized as follows. Section 2 gives the mathematical model of a kind of wheeled mobile robot with parametric uncertainty. Then the controller design process is given in Section 3. Section 4 presents the qualitative and quantitative simulation results of the controller. Conclusions are drawn in Section 5.

2. PROBLEM DESCRIPTION

A kind of wheeled mobile robots is shown in Fig. 1. It is assumed that there is no sliding between the wheels and the ground. So the constraint can be written as

$$x_c \sin \theta - y_c \cos \theta = 0,$$

where $[x_c, y_c]^T \in \mathbb{R}^2$ represents the position of the robot, and $\theta \in (-\pi, \pi]$ is the angle between the velocity direction of the mobile robot and the $x$-axis.

Therefore, the kinematic model of the wheeled mobile robot can be written as

$$\begin{align*}
\dot{x}_c &= v \cos \theta \\
\dot{y}_c &= v \sin \theta \\
\dot{\theta} &= \omega
\end{align*}$$

where $v \in \mathbb{R}$ and $\omega \in \mathbb{R}$ are the linear velocity and angular velocity of the robot respectively.

Equation (2) belongs to a non-holonomic system model without drift. When parameters are uncertain, the wheeled mobile robot model of (2) becomes

$$\begin{align*}
\dot{x}_c &= d_1 v \cos \theta \\
\dot{y}_c &= d_1 v \sin \theta \\
\dot{\theta} &= d_2 \omega
\end{align*}$$

where $d_1, d_2 \in \mathbb{R}^+$ are uncertain parameters determined by the radius of the wheels and the distance between the wheels. Thus, (3) represents a class of typical wheeled mobile robot model with parametric uncertainties.

Problem description: For the uncertain wheeled mobile robot shown in (3), a controller is designed to stabilize the system to the origin with any non-zero initial value.

3. THE CONTROLLER DESIGN

Assumption 1. The uncertain parameters of the system are bounded, that is, $3c_{11}, c_{12}, c_{21}, c_{22} \geq 0$ such that

$$\begin{align*}
c_{11} &\leq d_1 \leq c_{12} \\
c_{21} &\leq d_2 \leq c_{22}
\end{align*}$$

Remark 1. Since $d_1, d_2 \in \mathbb{R}^+$ are determined by the radius of the wheels and the distance between the wheels, the assumption in (4) is reasonable.

According to our method, the original system (3) is transformed into two decoupled subsystems through two steps.

Step 1: Convert to a chain system

For the uncertain wheeled mobile robot system represented by (3), the following state transformation is introduced

$$
\begin{align*}
x_0 &= \theta \\
x_1 &= x_c \sin \theta - y_c \cos \theta \\
x_2 &= x_c \cos \theta + y_c \sin \theta \\
u_0 &= \omega \\
u_1 &= v
\end{align*}$$

then, (3) can be transformed into a third-order uncertain non-holonomic chain system, that is

$$\begin{align*}
\dot{x}_0 &= d_2 u_0 \\
\dot{x}_1 &= d_2 \dot{u}_2 u_0 \\
\dot{x}_2 &= d_1 u_1 - d_2 x_1 u_0
\end{align*}$$

Step 2: Input-state transformation

The chain system (6) has a potential linear structure. $x_0$-subsystem is only controlled by $u_0$, while the controllability of $x$-subsystem is determined by $u_1$ [Zhu et al. (2006)], so the controllers of $x_0$-subsystem and $x$-subsystem can be designed respectively. In order to realize the decoupling of the two subsystems, the following input-state transformation is carried out for $x$-subsystem:

$$\begin{align*}
z_1 &= x_1 \\
z_2 &= d_2 x_2
\end{align*}$$

Take the derivative of the system states $z_1, z_2$, we can get

$$\begin{align*}
\dot{z}_1 &= z_2 - \frac{u_0}{u_0} z_1 \\
\dot{z}_2 &= d_1 d_2 u_1 - d_2^2 u_0^2 z_1
\end{align*}$$

The $x$-subsystem can be rewritten as a matrix

$$\begin{align*}
z &= A z + b u_1 + B f \\
y &= C^T z
\end{align*}$$

where, $z = \begin{bmatrix} z_1 \\
z_2 \end{bmatrix}$, $A = \begin{bmatrix} -\frac{u_0}{u_0} & 1 \\
0 & 0 \end{bmatrix}$, $B_f = \begin{bmatrix} 1 & 0 \\
0 & 1 \end{bmatrix}$, $f = \begin{bmatrix} 0 \\
d_2^2 u_0^2 z_1 \end{bmatrix}$, $b_0 = d_1 d_2$. It can be found that, through the
discontinuous transformation of (7), \( u_0 \) is no longer coupled with state \( x \), and \( x \)-subsystem is transformed into a second-order nonlinear integral system with disturbance term \( f \). Obviously, in order to ensure the validity of discontinuous transformation (7), \( u_0 \) should satisfy the constraint of \( u_0 \neq 0 \).

The following is the design of controllers for \( x_0 \)-subsystem and \( x \)-subsystem to stabilize the system. Since it is necessary to ensure the validity of discontinuous transformation (7), when \( x_0 = 0 \), switching control is adopted for \( x_0 \)-subsystem.

### 3.1 Controller design of \( x_0 \)-subsystem

**Theorem 1.** For \( x_0 \)-subsystem, the following controller is designed,

\[
 u_0 = \begin{cases} 
 -\lambda_0 x_0, & x_0(0) \neq 0 \\
 \alpha, & t \leq t_s, \\
 -\lambda_0 x_0, & t > t_s, x_0(0) = 0,
\end{cases}
\tag{10}
\]

where, \( t_s > 0 \) is the switching time of the controller, \( \lambda_0 > 0, \alpha > 0 \). Then, \( x_0 \)-subsystem can be globally exponentially regulated at the origin.

**Proof.** The theorem is proven in two cases.

1. \( x_0(0) \neq 0 \)

   Since \( \dot{x}_0 = d_2 u_0 + u_0 = -\lambda_0 x_0 + c_{21} \leq d_2 \leq c_{22} \), it can be concluded as follows according to Gronwall-Belorman. [Khalil (2002)]

   When \( x_0(0) > 0 \),
   \[
   x_0(0)e^{-\lambda_0 c_{22}t} \leq x_0(t) \leq x_0(0)e^{-\lambda_0 c_{21}t},
   \tag{11}
   \]

   When \( x_0(0) < 0 \),
   \[
   x_0(0)e^{-\lambda_0 c_{21}t} \leq x_0(t) \leq x_0(0)e^{-\lambda_0 c_{22}t}.
   \tag{12}
   \]

   In this case, \( x_0 \)-subsystem is globally exponentially regulated at the origin.

2. \( x_0(0) = 0 \)

   When \( 0 \leq t \leq t_s \), there is \( u_0 = \alpha \), we have
   \[
   0 \leq x_0(t) \leq \alpha c_{22} t,
   \tag{13}
   \]

   thus, \( x_0(t_s) \leq \alpha c_{22} t_s \) is a bounded positive number.

   When \( t > t_s \), similar to case (1), we have
   \[
   x_0(t_s)e^{-\lambda_0 c_{22}(t-t_s)} \leq x_0(t) \leq x_0(t_s)e^{-\lambda_0 c_{21}(t-t_s)}.
   \tag{14}
   \]

   In this case, \( x_0 \)-subsystem is globally exponentially regulated at the origin.

In combination of the above two cases, theorem 1 is proven. \( \square \)

**Remark 2.** From the proof of theorem 1, it can be seen that \( x_0(t) \neq 0 \), thus \( u_0(t) \neq 0 \), and \( x_0(t) \) will keep its sign invariance, thus ensuring the validity of discontinuous transformation (7).

### 3.2 Controller design of \( x \)-subsystem

For \( x \)-subsystem, we propose a controller design method based on ADRC. It is composed of ESO and the state error feedback control law. The structure of the controller is shown in Fig. 2.

**Extended state observer** The state of the system and the total disturbance can be observed by the ESO, then a third-order full-dimensional extended state observer is designed as (17),

\[
\begin{align*}
\dot{\hat{x}}_1 &= \hat{x}_2 - \beta_1 (\hat{x}_1 - \xi_1) \\
\dot{\hat{x}}_2 &= \hat{x}_3 - \beta_2 (\hat{x}_2 - \xi_1) + \hat{b}_0 u_1 \\
\dot{\hat{x}}_3 &= -\beta_3 (\hat{x}_3 - \xi_1)
\end{align*}
\tag{18}
\]

where, \( \hat{x}_1 \) and \( \hat{x}_2 \) are the estimated values of state \( x_1 \) and \( x_2 \) respectively, and \( \hat{x}_3 \) is the estimated value of the total disturbance \( f_{\text{total}} \). \( \beta_1, \beta_2, \beta_3 \) are the parameters of ESO to be adjusted. The bandwidth method proposed by [Gao (2006)] is adopted to place the observer pole at \(-\omega_0\), i.e.

\[
\beta_1 = 3\omega_0, \quad \beta_2 = 3\omega_0^2, \quad \beta_3 = \omega_0^3,
\tag{19}
\]

where, \( \omega_0 \) represents the bandwidth of the observer. The ESO is converged under this parameter configuration [Guo and Zhao (2011)], i.e.

\[
\xi_1 \rightarrow \hat{x}_1, \hat{x}_2 \rightarrow \hat{x}_2, \hat{x}_3 \rightarrow f_{\text{total}}.
\tag{20}
\]

**Remark 3.** Since the uncertainty or disturbance that affect the output can be observed from the output, they are observable, thus such uncertainty or disturbance can be extracted from the output.

**State error feedback control law** Since ESO estimates the total disturbance in real time, then we choose the
control law $u_1$ to compensate it, so as to achieve good performance. The expression for $u_1$ is as follows

$$u_1 = \frac{u - \dot{x}_3}{b_0},$$

(21)

where $u$ is some form of control component. Since $\dot{x}_3 \rightarrow f_{total}$, substituting (21) into (17) we can get,

$$\ddot{x}_1 = \dot{b}_0 u_1 + f_{total} = u - \dot{x}_3 + f_{total} \approx u.$$  

(22)

In this way, the system with uncertainty is reduced to a second-order integral system. Then, the control component $u$ is designed as

$$u = k_1 (r - \dot{x}_1) - k_2 \ddot{x}_2.$$  

(23)

According to (20), (22), and (23), the closed loop differential equation of the system can be written as

$$\ddot{x}_1 + k_2 \dot{x}_1 + k_1 x_1 = k_1 r.$$  

(24)

For the stabilization problem in this paper, we have $r = 0$, then the differential equation of the closed loop system is $x_1 + k_2 \dot{x}_1 + k_1 x_1 = 0$. Obviously, the system states $x_1, x_2, (\dot{x}_2 = \dot{x}_1)$ are asymptotically stabilized. Thus, it can be concluded that the original system is asymptotically stabilized by combining (7), (9), and (15).

Since Peaking will occur in the initial stage of ESO [Esfandiar and Khalil (1992)], this paper adopts the strategy of not controlling in the initial stage, that is

$$u_1 = \begin{cases} 0, & t \leq t_u \\ \frac{-k_1 \dot{x}_1 - k_2 \ddot{x}_2 - \dot{x}_3}{b_0}, & t > t_u \end{cases},$$

(25)

where, $t_u > 0$ is the switching time of the controller. Similarly, according to the bandwidth method, the controller parameters are written as,

$$k_1 = \omega_c^2, k_2 = 2\zeta \omega_c,$$

(26)

where, $\omega_c$ is the bandwidth of the controller and $\zeta$ is the damping ratio.

Based on the above design process, we can get the controller based on ESO (refer to (18))

$$w_0 = \left\{ \begin{array}{ll}
-\lambda_0 x_0, & x_0(0) \neq 0 \\
\alpha, & t \leq t_s \\
-\lambda_0 x_0, & t > t_s, x_0(0) = 0
\end{array} \right. ,$$

$$w_1 = \left\{ \begin{array}{ll}
0, & t \leq t_u \\
-\frac{k_1 \dot{x}_1 - k_2 \ddot{x}_2 - \dot{x}_3}{b_0}, & t > t_u, x_0(0) \neq 0 \\
0, & t \leq t_s + t_u \\
-\frac{k_1 \dot{x}_1 - k_2 \ddot{x}_2 - \dot{x}_3}{b_0}, & t > t_s + t_u, x_0(0) = 0
\end{array} \right. ,$$

(27)

which enables the system to have a good ability of rejecting uncertainty and quick stabilization performance.

### 4. SIMULATION COMPARISON AND ANALYSIS

#### 4.1 Parameters tuning of ADRC

In general, we have the following experience of parameter tuning.

1. The larger $\omega_o$ is, the more accurately the ESO can observe the extended state, but this will increase the sensitivity to noise at the same time. Therefore, $\omega_c$ should be gradually increased from a smaller value until the good performance is achieved.

2. (a) The curve of state by our method
   (b) The curve of mobile robot's trajectory by our method
   (c) The curve of control input $\omega$ by our method
   (d) The curve of control input $v$ by our method

Fig. 4. Stabilization control of uncertain non-holonomic mobile robot based on our method with initial conditions $\theta(0) = 1, x_c(0) = 1, y_c(0) = 1$. 

3. The larger $\omega_c$ is and the smaller $\dot{b}_0$ and $\zeta$ are, the faster the response of the system will be. However, the control value will become larger and the overshoot will become more serious simultaneously. Therefore, $\omega_c$ should not be too small, $\dot{b}_0$ and $\zeta$ should not be too large. In general, $\zeta$ is taken as 1, then $\omega_c$ and $\dot{b}_0$ are adjusted gradually.

4. The curve of state by Jiang's method
   (b) The curve of mobile robot's trajectory by Jiang's method
   (c) The curve of control input $\omega$ by Jiang's method
   (d) The curve of control input $v$ by Jiang's method

Fig. 3. Stabilization control of uncertain wheeled mobile robot based on Jiang's method with initial conditions $\theta(0) = 1, x_c(0) = 1, y_c(0) = 1$. 

5. Similarly, according to the bandwidth method, the controller parameters are written as,

$$k_1 = \omega_c^2, k_2 = 2\zeta \omega_c,$$

(26)

where, $\omega_c$ is the bandwidth of the controller and $\zeta$ is the damping ratio.
4.2 Simulation

In order to verify the effectiveness of the proposed scheme, system (3) is simulated numerically. The sampling time is 0.02s, and the parameters of the system are as follows: \( d_1 = d_2 = 1.5 \), refer to [Jiang (2000)]. After parameters tuning, the parameters of the extended state observer and controller are selected as \( \lambda_0 = 0.6, \alpha = 0.7, \tau_s = 1, \omega_0 = 60, \omega_c = 5, \zeta = 1, b_0 = 2, t_u = 0.12 \). Then we compare our method with the method proposed by Jiang in two different initial conditions.

![Fig. 5. Stabilization control of uncertain non-holonomic mobile robot based on Jiang’s method with initial conditions \( \theta(0) = 0, x_c(0) = 1, y_c(0) = 1 \).](image)

The initial conditions are \( \theta(0) = 1, x_c(0) = 1, y_c(0) = 1 \).

The curve of state, mobile robot’s trajectory, and control input by Jiang’s method are shown in Fig. 3. The curve of state, mobile robot’s trajectory, and control input by our method are shown in Fig. 4.

It can be seen from Fig. 3 and Fig. 4 that the method proposed in this paper has good dynamic performance and steady-state performance, and the convergence speed is obviously better than Jiang’s method.

The initial conditions are \( \theta(0) = 0, x_c(0) = 1, y_c(0) = 1 \).

The curve of state, mobile robot’s trajectory, and control input by Jiang’s method are shown in Fig. 5. The curve of state, mobile robot’s trajectory, and control input by our method are shown in Fig. 6.

It can be seen from the Fig. 5 and Fig. 6 that although the method proposed by Jiang could finally stabilize the system to the origin, however, in the beginning, \( x \) and \( y \) have large overshoot and there is a large value of control input, which makes it difficult to apply in practice. The method proposed in this paper has no overshoot and a faster convergence speed, and the value of control input is also within the allowable range.

![Fig. 6. Stabilization control of uncertain non-holonomic mobile robot based on our method with initial conditions \( \theta(0) = 0, x_c(0) = 1, y_c(0) = 1 \).](image)

4.3 Quantitative analysis

Table 1 and Table 2 show the performance comparison of the two control methods under different initial conditions.

As can be seen from Table 1 and Table 2, the settling time of the method proposed in this paper is obviously better than that of Jiang’s method. More importantly, the method proposed in this paper is simple in design, does not have overshoot, and has a reasonable control value, which is of more practical application value.

| Index                  | Jiang’s method | Our method |
|------------------------|----------------|------------|
| Setting time of \( x_c \) (s) | 1.73           | 0.75       |
| Setting time of \( y_c \) (s) | 1.07           | 0.61       |
| Setting time of \( \omega \) (s) | 4.00           | 3.33       |
| Steady state error(m)   | 0              | 0          |
| Is there an overshoot    | no             | no         |
| Maximum \( \omega_{max} \) (rad/s) | -0.50       | -0.60      |
| Maximum \( v_{max} \) (m/s)   | -7.17           | -11.92     |

| Index                  | Jiang’s method | Our method |
|------------------------|----------------|------------|
| Setting time of \( x_c \) (s) | 3.66           | 1.95       |
| Setting time of \( y_c \) (s) | 2.22           | 1.76       |
| Setting time of \( \omega \) (s) | 3.40           | 4.38       |
| Steady state error(m)   | 0              | 0          |
| Is there an overshoot    | yes            | no         |
| Maximum \( \omega_{max} \) (rad/s) | 0.20         | 0.70       |
| Maximum \( v_{max} \) (m/s)   | 99.80          | -11.11     |
5. CONCLUSIONS

This paper proposes a controller design method for the stabilization problem of wheeled mobile robot with parametric uncertainties based on ADRC scheme. The whole design is divided into two parts, the switching controller for $x_0$-subsystem and the ADRC scheme for $x$-subsystem. The switching controller is designed to stabilize the $x_0$-subsystem exponentially and ensure the validity of input-state transformation. Besides, the ADRC scheme provides accurate estimation of the total disturbance and rejects the uncertainties well. Therefore, the whole controller is capable of making the system converge asymptotically while effectively reject the uncertainty. According to the simulation results, it is shown that the system designed with the proposed controller achieves the excellent steady-state and dynamic performance even in the face of the severe challenge of parametric uncertainties.

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