Cosmological perturbations in singularity–free, deflationary models

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Abstract

We consider scalar perturbations of energy–density for a class of cosmological models where an early phase of accelerated expansion evolves, without any fine–tuning for graceful exit, towards the standard Friedman eras of observed universe. The quantum geometric procedure which generates such models agrees with results for string cosmology since it works if dynamics is dominated by a primordial fluid of extended massive objects. The main result is that characteristic scales of cosmological interest, connected with the extension of such early objects, are selected.

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1 Introduction

Inflationary “paradigm” can be considered one of the main achievement of recent cosmology since it solves a large amount of shortcomings of standard cosmological model \cite{1,2}. However, it is well known that, among the several formulations of inflation, none is completely satisfactory due to the fine tuning requests of each of them \cite{3}. Sometimes we have to avoid the extremely high rate of magnetic monopole production \cite{4}, sometimes we have to build a suitable scalar field potential in order to allow the slow rolling \cite{5,6}; in any case, we have the ”graceful exit” problem since models continue to inflate without recovering the standard today observed Friedman behaviour \cite{1,3}. Another great problem of many inflationary models is that they are not singularity free (\textit{e.g.} power law inflation \cite{7}) so the main shortcoming of standard model is not solved at all.

Despite of this state of art, inflation seems, up to now, the only mechanism able to produce a perturbation spectrum that, starting from initial quantum fluctuations, could reproduce the observed large scale structures of the universe \cite{8}. However, in all inflationary models, the comparison of generated density perturbations with observational data strongly constrains the model parameters. These limitations follows from the observed isotropy of cosmic microwave background radiation \cite{9}, in particular from the COBE data \cite{10}. Most inflationary models predict that density perturbations are generated by the fluctuations of a scalar field (the inflaton) which are expanded to macroscopic sizes during the inflationary age.

The further issue that any inflationary model has to satisfy is that, during the expansion, perturbations which are inside the Hubble radius $H^{-1}$ at the beginning of inflation expand past the Hubble radius and reenter it at late times as large scale density perturbations. To calculate the amplitude of density perturbations and to study the transition from inflationary to the Friedman era, it is necessary to know how the background geometry change with time.

Therefore, a coherent theory of early universe should:

1. be connected to some unification scheme of all interactions of nature;

2. avoid the initial singularity;

3. evolve smoothly, \textit{i.e.} without fine tuning, from an inflationary stage to a decelerating Friedman era;

4. give rise to a perturbation spectrum in agreement both with the observed microwave background isotropy and with the large scale structures.

In other words, we search for a a cosmological model, connected with some fundamental theory, that, at a certain epoch, acquires a deflationary behaviour \cite{11} reproducing a suitable perturbation spectrum.

In this paper, we face such a problem. By a quantum geometric procedure \cite{12}, we construct a class of cosmological models of deflationary type which smoothly evolves
towards Friedman epochs. Over this background, we analyze the theory of gauge invariant cosmological perturbations for the density contrast $\frac{\delta \rho}{\rho}$ connecting it with the scales of astrophysical interest.

The main hypothesis to build such models is that the early universe is dominated by a fluid of finite–size objects which give rise to a dynamics very similar to that of string–dilaton cosmology [15],[16]. However, the starting point is different from that of string theory since our procedure is just a quantum geometric scheme.

Furthermore, we do not need any scalar field to implement inflation since the proper size of extended objects and the geodesic embedding procedure from an eight–dimensional tangent fiber bundle $M_8$ to the usual $V_4$ manifold of general relativity naturally give rise to an exponential inflationary–like behaviour.

The paper is organized as follows. In Sec.2, we describe the geometric procedure and the background model. Sec.3 is devoted to the discussion of the deflationary behaviour through the matter–energy density acting as the source in the Einstein equations. In Sec.4, we construct the theory of gauge–invariant cosmological perturbations using the above models as background. The analysis is devoted to the large and small scale limits and then to the selection of scales of astrophysical interest. Conclusions are drawn in Sec.5.

2 Geodesic embedding and the background model

The quantum geometric procedure and the background model which we are going to use are treated in detail in [12]. Here, we outline the main features which we need for cosmological perturbations. The starting point is that if we consider dynamics of an extended massive object in general relativity, a limiting maximal acceleration, compatible with the size $\lambda$ of the object and the causal structure of the spacetime manifold, emerges [17]. Such a proper constant acceleration $A$ yields a Rindler horizon at a distance $|A|^{-1}$ from the extremity of the object in the longitudinal direction. In other words, the parts of the object will be in causal contact only if $|A| < \lambda^{-1}$. It is worthwhile to stress that the parameter $A$ (or $\lambda$) is related to the "mass" of the extended object and we are using physical units where $8\pi G = c = 1$.

Let us take into account a Friedman–Robertson–Walker (FRW) spacetime whose scale factor, with respect to the cosmic time $t$, is $a(t)$.

By using the equation of geodesic deviation [12],[13],[14] we get that the size $\lambda$ of the object is compatible with the causal structure if $|\lambda \ddot{a}/a| < 1$.

Consequently, we have a maximal allowed curvature depending on $\lambda$ and the cosmological model becomes singularity free. This fact is in sharp contrast with usual perfect fluid FRW cosmology where curvature and matter–energy density are singular in the limit $t \to 0$. Then, the introduction of finite size objects, instead of pointlike particles, in primordial cosmological background modifies dynamics so that the singular structure of general relativity is easily regularized. It is worthwhile to note that such a feature
does not depend on the particular background geometry which we are considering. More formally, a causal structure in which proper accelerations cannot exceed a given value $\lambda^{-1}$ can be imposed over a generic spacetime $\mathcal{V}_4$ regarding such a manifold as a four-dimensional hypersurface locally embedded in an eight-dimensional tangent fiber bundle $\mathcal{M}_8$, with metric

$$g_{AB} = g_{\mu\nu} \otimes g_{\mu\nu},$$

and coordinates $x^A = (x^\mu, \lambda u^\mu)$, where $u^\mu = \frac{dx^\mu}{ds}$ is the usual four velocity and $\mu, \nu = 1, \ldots, 4$, $A, B = 1, \ldots, 8$ [12], [13].

The embedding of $\mathcal{V}_4$ into $\mathcal{M}_8$, determined by the eight parametric equations $x^\mu = x^\mu(\xi^\alpha)$ and $u^\mu = u^\mu(\xi^\alpha)$, gives rise to a spacetime metric $\tilde{g}_{\mu\nu}(\xi)$, locally induced by the $\mathcal{M}_8$ invariant interval

$$ds^2 = g_{AB} dx^A dx^B = g_{\mu\nu} \left( dx^\mu dx^\nu + \lambda^2 du^\mu du^\nu \right) \equiv \tilde{g}_{\mu\nu} d\xi^\mu d\xi^\nu,$$

where

$$\tilde{g}_{\mu\nu} = g_{\alpha\beta} \left( \frac{\partial x^\alpha}{\partial \xi^\mu} \frac{\partial x^\beta}{\partial \xi^\nu} + \lambda^2 \frac{\partial u^\alpha}{\partial \xi^\mu} \frac{\partial u^\beta}{\partial \xi^\nu} \right).$$

Let us now take into consideration a FRW background modified by such a geodesic embedding.$^1$

In conformal coordinates $\xi^\mu = (\eta, \vec{x})$, a FRW flat metric is

$$g_{\mu\nu}(\xi) = \text{diag}[a^2(\eta)(1, -1, -1, -1)],$$

where $d\eta = \frac{dt}{a}$ defines the conformal time. The velocity field for an extended object comoving in this background is

$$u^\mu(\xi) = \left( a^{-1}, 0, 0, 0 \right),$$

By Eqs.(2) and (3), the geodesic embedding gives rise to the new metric

$$\tilde{g}_{\mu\nu}(\xi) = \text{diag}a^2 \left( 1 + \lambda^2 \frac{a'^2}{a^4}, -1, -1, -1 \right),$$

corrected by a $\lambda^2$ term with respect to (4). The prime indicates the derivative with respect to $\eta$. The cosmic time results now

$$t = \int d\eta \left( a^2 + \lambda^2 \frac{a'^2}{a^2} \right)^{1/2},$$

$^1$This geometric procedure is called "geodesic embedding" since the velocity field $u^\mu(\xi)$, solution of the geodesic equations, defines the embedding of $\mathcal{V}_4$ into $\mathcal{M}_8$. 
or, in terms of the scale factor only,

\[ t = \lambda \int \frac{da}{a} \left( 1 + \frac{a^4}{\lambda^2 a'^2} \right)^{1/2}. \]  

(8)

The Hubble parameter is now

\[ H = \frac{\dot{a}}{a} = \left( \frac{a'}{a} \right) \left[ a^2 + \lambda^2 \left( \frac{a'}{a} \right)^2 \right]^{-1/2}, \]  

(9)

with the limiting value

\[ H_0 = \lambda^{-1}, \]  

(10)

for \( \left( \lambda^2 \left( \frac{a'}{a} \right)^2 \gg a^2 \right) \).

It is easy to see that the scale factor, with respect to the cosmic time \( t \), has an initial exponential growth which regularly evolves towards a standard Friedman behaviour.

### 3 Deflationary behaviour of energy–density

Modified geometry implies an initial de Sitter behaviour which is not connected with dynamics of some scalar field but it simply comes from the presence of extended (and massive) objects. The e-folding number, i.e. the duration of inflation, and the horizon scale depend on the size \( \lambda \) without any initial value problem or fine tuning.

The natural scale to which to compare perturbations is \( \lambda \): they are inside the Hubble radius if they are smaller than \( \lambda \) while they are outside it if they are greater than \( \lambda \). In other words, \( \lambda \) determines the crossing time (either out of the Hubble radius or into the Hubble radius).

Considering the \((0, 0)\)–Einstein equation for a spatially flat model, we have

\[ H^2 = \frac{\rho}{3}, \]  

(11)

so that

\[ \rho = 3 \left( \frac{a'}{a} \right)^2 \left[ a^2 + \lambda^2 \left( \frac{a'}{a} \right)^2 \right]^{-1}. \]  

(12)

Immediately we see that

\[ \rho \approx \frac{3}{\lambda^2}, \quad \text{for} \quad \lambda^2 \left( \frac{a'}{a} \right)^2 \gg a^2, \]  

(13)

and

\[ \rho \approx 3 \frac{a'^2}{a^4}, \quad \text{for} \quad \lambda^2 \left( \frac{a'}{a} \right)^2 \ll a^2. \]  

(14)
The first case corresponds to an effective cosmological constant \( \Lambda = \frac{3}{\lambda^2} \) selected by the mass (i.e. the size) of the primordial extended objects [12]; the second case is recovered as soon as the universe undergoes the post-inflationary reenter phase. We stress again the fact that such a behaviour does not depend on the specific form of the scale factor \( a \) and the deflationary phase is smooth.

As in [12], we can couple dynamics with ordinary fluid matter in order to obtain a more realistic cosmological scenario. In doing so, we have to consider a perfect fluid state equation

\[
p = (\gamma - 1)\rho,
\]

which, using also the contracted Bianchi identity in FRW spacetime gives the continuity equation

\[
\dot{\rho} + 3H(p + \rho) = 0.
\]

By Eqs(15) and (16), we get

\[
\rho = Da^{-3\gamma}.
\]

For the sake of simplicity, \( \gamma \) is assumed constant. It defines the thermodynamical state of the fluid and it is related to the sound speed being \( \gamma - 1 = c_s^2 \). By inserting this fluid into the Einstein equations, the scale factor of the universe, expressed in conformal time is [11], [12], [18]

\[
a(\eta) = a_0\eta^{2/(3\gamma - 2)},
\]

where \( a_0 \) is a constant depending on \( \lambda \) and \( \gamma \).

The matter–energy density results, from Eq.(12),

\[
\rho = 3\left(\frac{2}{3\gamma - 2}\right)^2 \frac{1}{\eta^2} \left[ a_0^2 \eta^{3\gamma - 2} + \lambda^2 \left(\frac{2}{3\gamma - 2}\right)^2 \frac{1}{\eta^2}\right]^{-1},
\]

from which \( \rho \sim \text{constant for } \frac{\lambda}{\eta^{6\gamma/(3\gamma - 2)}} \gg 1 \) and \( \rho \sim \eta^{\frac{6\gamma}{3\gamma - 2}} \) in the opposite case. The standard situations for \( \gamma = 4/3 \) (radiation dominated regime) and \( \gamma = 1 \) (matter dominated regime) are easily recovered. It is interesting to see that it is not only the specific value of \( \gamma = 0 \), as usual, that allows to recover inflation but, mainly, the scale \( \lambda \). In the regime \( \frac{\lambda}{\eta} \gg 1 \), the constant matter density value is independent of \( \gamma \).

In the next section, we shall study the density contrast \( \frac{\dot{\rho}}{\rho} \) which gives the perturbation spectrum. Due to the smooth transition from the inflationary to the FRW regime, the perturbation scale lengths do not need any cut–off and can be parametrized in all their evolution by the parameter \( \lambda \) which has to be compared with Hubble causal horizon \( H^{-1} \).
4  Gauge–invariant cosmological perturbations

In the gauge-invariant formalism, the conformal invariance and the frame-independence are requested for variables connected to perturbations in order to eliminate the pure gauge modes \[8\]. Furthermore, any generalized theory of gravity can be recast into the Einstein theory plus one or more than one additional scalar fields \[19\]. In some sense, our quantum geometric procedure can be seen as a modified theory of gravity.

We can turn now to consider the scalar perturbations. For a spatially flat FRW metric, the line element is \[8\]

\[
\begin{align*}
    ds^2 &= a^2(\eta) \left[ (1 + 2\phi) d\eta^2 - 2B_{ij} dx^i d\eta - dx^i dx^j \left( 2E_{ij} + (1 - 2\psi) \delta_{ij} \right) \right].
\end{align*}
\]  

(20)

It is always possible to construct combinations of the scalar quantities \(\phi, \psi, E, B\) which are invariant under general coordinate transformations as \(x^\alpha \rightarrow \tilde{x}^\alpha + \xi^\alpha\). A useful combination, which gives rise to the invariant perturbation potentials, is

\[
\begin{align*}
    \Phi &= \phi + \frac{1}{a} [(B - E') a''] \quad \text{(21)}
    \\
    \Psi &= \psi - \frac{a'}{a} (B - E'). \quad \text{(22)}
\end{align*}
\]

Such a choice simplifies the evolution equations for density perturbations and, as we shall see below, furnishes quantities with a clear physical meaning. In the same way, we can construct perturbed Einstein equations which are invariant under general coordinate transformations and, consequently, we get gauge invariant quantities. These equations are generally written in terms of \(\Phi\) and \(\Psi\). Furthermore, the symmetries of the stress–energy tensor can give additional simplifications. In fact, as it is clearly shown in \[8\], if the source stress–energy tensor is symmetric, we have \(\Phi = \Psi\), so that we need just one evolution equation (plus, however, the gauge choice).

Usually, \(\Phi\) is called the "gauge-invariant potential" and characterizes the amplitude of scalar density perturbations. It is a function of the conformal time \(\eta\) and the spatial coordinates \(x\). It is important to note that below the Hubble radius \(H^{-1}\), \(\Phi\) has the role of a Newtonian potential for the density contrast yielded by perturbations.

The general gauge–invariant evolution equation for scalar adiabatic perturbations is \[8, 20\]

\[
\Phi'' + 3\mathcal{H}(1 + c_s^2)\Phi' - c_s^2 \nabla^2 \Phi + \left[ 2\mathcal{H}' + (1 + 3c_s^2)\mathcal{H}^2 \right] \Phi = 0,
\]

(23)

\(\mathcal{H}\) is the Hubble parameter in the conformal time defined as

\[
\mathcal{H} = \frac{a'}{a}.
\]

(24)

c_s, as above, is the sound speed. It is interesting to note that Eq.\((23)\) can be recast in terms of the scale factor \(a\) by the variable change \(dt = a d\eta\) and \(da/dt = aH\) \[21\].
In this way, the information contained in the evolution equation is directly related to the background. However, for our purposes, it is better to use the “conformal time picture” since it immediately shows when the sizes of perturbations are comparable to the characteristic scale length $\lambda$.

Another important step is the decomposition of the perturbation potential into spatial Fourier harmonics

$$\Phi(\eta, x) = \int d^3k \tilde{\Phi}(\eta, k)e^{i k \cdot x}, \quad (25)$$

where $k$ is the wavenumber. Essentially, this decomposition consists in replacing $\nabla^2 \rightarrow -k^2$ in the dynamical equation (23). It allows to follow the evolution of a single mode. In our case, considering a specific mode, we can follow it from the inflationary deSitter stage to the deflationary Friedman era. For example, if $k \ll H$, we have long wavelength modes which furnish the spectrum of perturbations during inflation. In our case, it is interesting to compare such modes with the "natural" scale of the model, that is $H_0 = \lambda^{-1}$.

Before performing the Fourier analysis, it is useful to simplify the dynamical problem by a suitable change of variables. Eq. (24) can be reduced to the simpler form

$$u'' - c_s^2 \nabla^2 u - \frac{\theta''}{\theta} u = 0, \quad (26)$$

where $\theta$ is

$$\theta = \frac{1}{a} \left( \frac{\rho_0}{\rho_0 + p_0} \right)^{1/2} = \frac{1}{a} \left( \frac{1}{1 + p_0/\rho_0} \right)^{1/2} = \frac{1}{a \sqrt{\gamma}}, \quad (27)$$

and the gauge–invariant gravitational potential $\Phi$ is given by

$$\Phi = \frac{1}{2} \left( \rho_0 + p_0 \right)^{1/2} u. \quad (28)$$

From now on, the subscript "0" will indicate the unperturbed quantities.

The density perturbations are given by

$$\frac{\delta \rho}{\rho_0} = \frac{2 \left[ \nabla^2 \Phi - 3H\Phi' - 3H^2\Phi \right]}{3H^2}. \quad (29)$$

In the specific case we are considering, using the solution (18), we get

$$\theta(\eta) = \left[ \frac{2}{H_0 \sqrt{\gamma} (3\gamma - 2)} \right] \eta^{2/(3\gamma-2)}, \quad (30)$$

and

$$\frac{\theta''}{\theta} = \left[ \frac{6\gamma}{(3\gamma - 2)^2} \right] \frac{1}{\eta^2}. \quad (31)$$

After the Fourier transform, Eq. (26) becomes

$$u''_k + \left[ c_s^2 k^2 - \frac{6\gamma}{(3\gamma - 2)^2 \eta^2} \right] u_k = 0, \quad (32)$$
which is nothing else but a Bessel equation. The density perturbations can be rewritten as
\[
\frac{\delta \rho}{\rho_0} = -(\rho_0 + p_0)^{1/2} \left[ \left( 1 + \frac{k^2}{H^2} \right) u_k(\eta) + \frac{u_k'(\eta)}{H} \right],
\] (33)
where
\[
H = \left( \frac{2}{3\gamma - 2} \right)^{1/2}. \tag{34}
\]

The general solution of (32) is
\[
u = \pm \frac{3\gamma + 2}{2(3\gamma - 2)}, \quad z = c_s k \eta \tag{36}
\]
where \( \nu = \pm \frac{3\gamma + 2}{2(3\gamma - 2)} \), \( z = c_s k \eta \).

\( A_0 \) and \( B_0 \) are arbitrary constants.

Actually, we are interested in the asymptotic behaviour of \( \Phi \), that is \( u_k \), since it, by (29), determines the large scale structures of the universe.

The large scale limit is recovered as soon as \( k^2 \ll \theta''/\theta \), or \( k \ll H \). This means that the solution (35) becomes
\[
\begin{align*}
u \nu & = u_k(\eta) = \eta^{1/2} \left[ A_0 J_\nu(z) + B_0 Y_\nu(z) \right], \tag{35}
\end{align*}
\]
where \( J_\nu(z) \) and \( Y_\nu(z) \) are Bessel functions and
\[
u \nu = \frac{3\gamma + 2}{2(3\gamma - 2)} \quad \text{and} \quad z = c_s k \eta \tag{36}
\]

For different values of \( \gamma \), the index \( \nu \) can be positive or negative determining growing or decaying modes.

In the vacuum–dominated era \( (\gamma = 0) \), we have, for \( k \to 0 \),
\[
u \nu = u_k(\eta) \sim \eta^{1/2} \left[ \frac{A_0}{\Gamma(\nu + 1)} \left( \frac{c_s k \eta}{2} \right)^\nu - \frac{B_0 \Gamma(\nu)}{\pi} \left( \frac{c_s k \eta}{2} \right)^{-\nu} \right]. \tag{37}
\]

This is a nice feature since the spectrum of perturbations is a constant with respect to \( \eta \) as it must be during inflation, when dynamics is frozen \(^2\). As we pointed out, we recover the case \( \gamma = 0 \) any time that \( H_0 = \lambda^{-1} = k_\lambda \), that is the feature of the spectrum is fixed by the natural scale of the model \(^2\).

\(^2\) To be more precise, by using (33), we get
\[
u \nu = \frac{\delta \rho}{\rho_0} \sim \sqrt{\rho_0 + p_0} \left( 1 + \frac{k^2}{3H^2} \right) u_k(\eta).
\]

As soon as \( k^2 \ll H^2 \), in particular \( k^2 \ll k_\lambda^2 \), the long wavelength perturbations go beyond the horizon and their dynamics results frozen. This feature is always present during inflation. In our case, it is recovered without any fine–tuning.
If \( \gamma \) is any, in particular \( \gamma = 1, 4/3, 2 \), corresponding to the cases "dust", "radiation" and "stiff matter" respectively, we get

\[
  u_k(\eta) \sim \frac{A_0}{\Gamma(\nu + 1)} \left( \frac{c_s}{2} \right)^{\nu} \eta^{(\nu + 1/2)} k^{\nu} - \frac{B_0 \Gamma(\nu)}{\pi} \left( \frac{c_s}{2} \right)^{-\nu} \eta^{(1/2 - \nu)} k^{-\nu}.
\]

In particular, for \( k \to 0 \), only the second term survives. The density contrast, in the same limit, is

\[
  \frac{\delta \rho}{\rho_0} \sim \frac{B_0 \Gamma(\nu)}{\pi} \sqrt{\rho_0 + p_0} \left( \frac{2}{c_s} \right)^{\nu} \frac{(3\gamma - 2)^2}{12} \eta^{(2 - \nu)} k^{(2 - \nu)}.
\]

It is interesting to note that, for \( \gamma = 1, \nu = 5/2 \) and

\[
  \frac{\delta \rho}{\rho_0} \propto k^{-1/2},
\]

that is we lose the time dependence also if the scales are reentered the horizon (for \( \gamma = 1 \) we are in the Friedman regime).

The small scale limit is recovered as soon as in (26) or (32) \( k^2 \gg \theta''/\theta \). The solution can be written as

\[
  u_k(\eta) \sim \sqrt{\frac{2}{\pi c_s k}} \left[ A_0 \cos(c_s k \eta) + B_0 \sin(c_s k \eta) \right],
\]

and looking at (33), also the density contrast is an oscillating function in \( \eta \). From a cosmological point of view, this limit is not very interesting since it is not directly connected to dynamics of inflation.

### 5 Discussion and conclusions

In this paper, we have constructed the theory of gauge–invariant cosmological perturbations for a model in which, by a geometric procedure of local embedding, the metric is modified.

Such a modification can be read as the effect of a fluid of extended primordial objects whose dynamics alters the cosmological background. A very important point is that the size of the objects gives rise to an inflationary period that smoothly evolves toward a Friedman era.

Also the cosmological perturbations are affected by such a dynamics since the scales (i.e. the wavenumbers \( k \)) are regulated by the size \( \lambda \) which is a natural scale giving the Hubble horizon \( H_0 = \lambda^{-1} \) during inflation. Then the limits to compare very large scale structures and small large scale structures are \( k \ll H_0 \) and \( k \gg H_0 \). In other words, \( \lambda \) fixes the time at which perturbations cross the horizon and reenter, enlarged, into it without any fine–tuning. This point has to be discussed in detail comparing it with the standard method used to calculate the amplification of perturbations after reenter.
In the limit $k \ll H$ and for adiabatic perturbations, the quantity
\[
\zeta = \frac{2}{3\gamma} \left( \Phi + H^{-1} \dot{\Phi} \right) + \Phi,
\]
(44)
or its Fourier transform
\[
\tilde{\zeta} = \frac{2}{3\gamma} \left( \tilde{\Phi} + \mathcal{H}^{-1} \tilde{\Phi}' \right) + \tilde{\Phi},
\]
(45)
is conserved \citep{8,21}.

In such a limit, Eq. (23) corresponds to $\dot{\zeta} = 0$, so that, for long wavelengths, the use of $\zeta$ to obtain the evolution of $\Phi$ is justified. However, this position holds only on scales larger than Hubble radius (when $c_s^2 \nabla^2 \Phi$ is negligible) and not for all dynamics. At very early and very late times, it is realistic to neglect also the derivative $\dot{\Phi}$ \citep{23}, so that we have
\[
\Phi(t_f) = \left[ \frac{1 + \frac{2}{3\gamma} \gamma^{-1}(t_f)}{1 + \frac{2}{3\gamma} \gamma^{-1}(t_i)} \right] \Phi(t_i),
\]
(46)
which means that the amplitudes of perturbations crossing out from the Hubble radius and reentering it later are related. The net change is due to the state equation $p = (\gamma - 1) \rho$ describing the model before crossing and after reenter. As $\gamma \to 0$, the amplification becomes huge solving the problem that microscopic perturbations enlarge to macroscopic (astronomical) sizes.

In any case, $t_i$ must be taken well before the beginning of inflation and well after its end. Then, if Eq. (10) is a useful tool to calculate how inflation enlarges the amplitude of primordial perturbations, it gives rise to a further fine-tuning problem since $t_i$ and $t_f$, and the relative $\gamma(t_i)$ and $\gamma(t_f)$, must be chosen with a lot of care.

Our model bypasses such a shortcoming since inflation smoothly comes to an end and also the amplitude of perturbations smoothly evolves towards the Friedman era. However, due to the presence of extended objects at very beginning, the model starts as inflationary and singularity-free so that, from a cosmological point of view, we do not have an epoch before inflation. Besides, the size $\lambda$ triggers the scales at which galaxies and cluster of galaxies should form \citep{22}.

The quantum–geometric procedure which we used acquires physical meaning only if we suppose that, in an early phase, the contribution of finite–size objects becomes dominant.

In a forthcoming paper, we shall discuss some physically motivated examples of such dynamics.
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