Electromagnetic diffraction by a circular cylinder with longitudinal slots

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Abstract

A method is presented to investigate diffraction of an electromagnetic plane wave by an infinitely thin infinitely conducting circular cylinder with longitudinal slots. It is based on the use of the combined boundary conditions method that consists on expressing the continuity of the tangential components of both the electric and the magnetic fields in a single equation. This method proves to be very efficient for this kind of problems and leads to fast numerical codes.

I. INTRODUCTION

The problem of the penetration of electromagnetic waves in a conducting circular cavity through a narrow axial aperture has been treated by several authors. Several methods have been used to achieve the determination of the field inside the cavity. Beren \cite{1} used the Aperture Field Integral Equation, the Electric Field Integral Equation and H-field Integral Equation to determine the field around an axially slotted cylinder, while Johnson and Ziolkowski \cite{2} gave a generalized dual series solution for this problem. Mautz and Harrington treated the field penetration inside a conducting circular cylinder through a narrow slot in both TE \cite{3} and TM \cite{4} polarizations. More recently Shumpert and Butler \cite{5,6} proposed three methods to study the penetration in conducting cylinders. In this article, we propose a method to calculate the field inside and around a slotted circular cavity with longitudinal slots. It is based on the combined boundary conditions method introduced first by Montiel and Nevière \cite{7,8}. Section II is dedicated to the description of the theory. In section III we
give some details about the numerical scheme and then compare our results with previous work.

II. THEORY

The structure under study is depicted in Fig 1. The space is divided into two regions, region 1 (exterior region: \( r > R \)) and region 2 (interior: \( r < R \)) that are assumed to be dielectric and homogeneous with relative dielectric permittivities \( \varepsilon_1 \) and \( \varepsilon_2 \), respectively. On the interface between these two media are deposited a finite number of infinitely conducting, infinitely thin circular strips that are invariant along the \( z \) direction. The device is illuminated by a TMz (electric field parallel to the \( z \) axis) or TEz (magnetic field parallel to the \( z \) axis) monochromatic electromagnetic wave under incidence \( \theta_0 \) with vacuum wavelength \( \lambda \).

Throughout this paper we assume an \( \exp(-i\omega t) \) time dependence. The \( z \) component of the electric or the magnetic field will be denoted by \( u(\theta, r) \). We denote by \( \Omega_1 \) the union of the strips and by \( \Omega_2 \) its complementary in \([0, 2\pi]\).

In the exterior region we express the total field as :

\[
 u_1(\theta, r) = \sum_{n \in \mathbb{Z}} a_n J_n(k_1 r) \exp(in\theta) + \sum_{n \in \mathbb{Z}} b_n H_n^{(1)}(k_1 r) \exp(in\theta) \quad (1)
\]

Likewise in the interior region the total field may be expressed as :

\[
 u_2(\theta, r) = \sum_{n \in \mathbb{Z}} c_n J_n(k_2 r) \exp(in\theta) \quad (2)
\]

where \( a_n, b_n \) and \( c_n \) are the amplitudes of the incident, the diffracted and the transmitted waves respectively. We denote \( J_n \) and \( H_n^{(1)} \) the Bessel and the Hankel functions of the first kind. \( k_p = k_0 \sqrt{\varepsilon_p} = \frac{2\pi}{\lambda} \sqrt{\varepsilon_p} \), with \( p = 1, 2 \) and \( \mathbb{Z} \) denotes the usual set of relative integers. Amplitudes \( a_n \) being known, the problem is to determine amplitudes \( b_n \) and \( c_n \) from which the total field can be calculated everywhere. For that purpose one must write the boundary conditions at the interface between both dielectric media. This is done in the next subsections by distinguishing the TMz and the TEz cases of polarization.

A. TMz polarization

The boundary conditions applied to the tangential components of the electromagnetic field at the interface defined by \( r = R \) lead to :

\[
 u_1(\theta, R) = u_2(\theta, R) , \ \forall \theta \in [0, 2\pi] \quad (3a)
\]

\[
 \frac{du_1}{dr} \bigg|_{(\theta, R)} = \frac{du_2}{dr} \bigg|_{(\theta, R)} , \ \forall \theta \in \Omega_2 \quad (3b)
\]

With the supplementary condition that the electric field must vanish on the strips:

\[
 u_1(\theta, R) = u_2(\theta, R) = 0, \ \forall \theta \in \Omega_1 \quad (4)
\]
Following Montiel and Nevière [7,8], equations (3b) and (4) can be combined in a single equation that holds for every $\theta$ in $[0, 2\pi]$:

$$(1 - \chi(\theta)) u_2(\theta, R) + g\chi(\theta) \left[ \left( \frac{du_2}{dr} \right)_{(\theta, R)} - \left( \frac{du_1}{dr} \right)_{(\theta, R)} \right] = 0, \quad \forall \theta \in [0, 2\pi] \quad (5)$$

where $\chi(\theta)$ is the characteristic function of set $\Omega_2$:

$$\chi(\theta) = \begin{cases} 1 & \text{if } x \in \Omega_2 \\ 0 & \text{elsewhere} \end{cases}$$

and $g$ is some numerical parameter introduced for dimensional and numerical purposes.

Remark that the set of Eqs. (3a), (3b) and (4) is equivalent to the set of Eqs. (3a) and (5). Since $\chi(\theta)$ is $2\pi$-periodic it can be expanded in Fourier series:

$$\chi(\theta) = \sum_{p \in \mathbb{Z}} \chi_p \exp(ip\theta) \quad (6)$$

Reporting equations (1) and (2) into equation (3a) and projecting on the $(\exp(in\theta))_{n \in \mathbb{Z}}$ basis gives:

$$a_n J_n(k_1 R) + b_n H_n^{(1)}(k_1 R) = c_n J_n(k_2 R), \quad \forall n \in \mathbb{Z} \quad (7)$$

then reporting equations (1), (2) and (6) into equation (5) and projecting on the $(\exp(in\theta))_{n \in \mathbb{Z}}$ basis leads to:

$$c_n J_n(k_2 R) - \sum_{p \in \mathbb{Z}} \chi_{n-p} c_p J_p(k_2 R) + g \sum_{p \in \mathbb{Z}} \chi_{n-p} \left[ k_2 \left( c_p J'_p(k_2 R) \right) - k_1 \left( a_p J'_p(k_1 R) + b_p H_p^{(1)}(k_1 R) \right) \right] = 0, \quad \forall n \in \mathbb{Z} \quad (8)$$

where the primes denote derivation with respect to $r$. From Eq(7) one can extract $c_n$:

$$c_n = \frac{J_n(k_1 R)}{J_n(k_2 R)} a_n + \frac{H_n^{(1)}(k_1 R)}{J_n(k_2 R)} b_n, \quad \forall n \in \mathbb{Z} \quad (9)$$

and report its expression into Eq(8) to obtain the following linear system linking the amplitudes $a_n$ and $b_n$:

$$a_n J_n(k_1 R) + \sum_{p \in \mathbb{Z}} \chi_{n-p} a_p \left[ -J_p(k_1 R) + g k_2 \frac{J_p(k_1 R)}{J_p(k_2 R)} J'_p(k_2 R) - g k_1 J'_p(k_1 R) \right] =$$

$$-b_n H_n^{(1)}(k_1 R) + \sum_{p \in \mathbb{Z}} \chi_{n-p} b_p \left[ H_p^{(1)}(k_1 R) - g k_2 \frac{H_p^{(1)}(k_1 R)}{J_p(k_2 R)} J'_p(k_2 R) + g k_1 H_p^{(1)}(k_1 R) \right] \quad (10)$$

The solution of the linear system (10) gives the unknown amplitudes $b_n$ and then Eq.(9) gives the amplitudes $c_n$. Thus the field can be computed everywhere in space using Eqs.(1) and (2).
B. TEz polarization

For this case of polarization, the continuity of the tangential components of the electromagnetic field at the interface defined by \( r = R \) leads to:

\[
\frac{1}{\varepsilon_1} \left( \frac{du_1}{dr} \right)_{(\theta, R)} = \frac{1}{\varepsilon_2} \left( \frac{du_2}{dr} \right)_{(\theta, R)}, \quad \forall \theta \in [0, 2\pi] \tag{11a}
\]

\[
u_1(\theta, R) = u_2(\theta, R), \quad \forall \theta \in \Omega_2 \tag{11b}
\]

With the supplementary condition that the electric field must vanish on the strips:

\[
\frac{1}{\varepsilon_1} \left( \frac{du_1}{dr} \right)_{(\theta, R)} = \frac{1}{\varepsilon_2} \left( \frac{du_2}{dr} \right)_{(\theta, R)} = 0, \forall \theta \in \Omega_1 \tag{12}
\]

Here again we can replace equations (11b) and (12) by:

\[
(1 - \chi(\theta)) \frac{1}{\varepsilon_2} \left( \frac{du_2}{dr} \right)_{(\theta, R)} + g\chi(\theta)[u_2(\theta, R) - u_1(\theta, R)] = 0, \forall \theta \in [0, 2\pi] \tag{13}
\]

Reporting equations (1) and (2) into Eq.(11a) and projecting on the \((\exp(in\theta))_{n \in \mathbb{Z}}\) basis gives:

\[
a_n J_n'(k_1 R) + b_n H_n^{(1)\prime}(k_1 R) = \frac{k_2}{k_1} \frac{\varepsilon_1}{\varepsilon_2} c_n J_n'(k_2 R), \quad \forall n \in \mathbb{Z} \tag{14}
\]

Remark that the set of Eqs (11a), (11b) and (12) are equivalent to the set of Eqs (11b) and (13). Reporting equations (1),(2) and (6) into Eq.(13) and projecting on the \((\exp(in\theta))_{n \in \mathbb{Z}}\) basis leads to:

\[
\frac{k_2}{\varepsilon_2} c_n J_n'(k_2 R) - \frac{k_2}{\varepsilon_2} \sum_{p \in \mathbb{Z}} \chi_{n-p} c_p J_p'(k_2 R) +
\sum_{p \in \mathbb{Z}} \chi_{n-p} \left[ c_p J_p(k_2 R) - \left( a_p J_p(k_1 R) + b_p H_p^{(1)}(k_1 R) \right) \right] = 0, \quad \forall n \in \mathbb{Z} \tag{15}
\]

From Eq.(14) one can extract \(c_n\):

\[
c_n = \frac{k_1}{k_2} \frac{\varepsilon_2}{\varepsilon_1} \left( \frac{J_n'(k_1 R)}{J_n'(k_2 R)} a_n + \frac{H_n^{(1)\prime}(k_1 R)}{J_n'(k_2 R)} b_n \right), \quad \forall n \in \mathbb{Z} \tag{16}
\]

and report its expression into Eq.(13) to obtain the following linear system linking the amplitudes \(a_n\) and \(b_n\):

\[
a_n \frac{k_1}{\varepsilon_1} J_n'(k_1 R) + \sum_{p \in \mathbb{Z}} \chi_{n-p} a_p \left[ -\frac{k_1}{\varepsilon_1} J_p'(k_1 R) + \frac{k_1}{\varepsilon_2} J_p(k_2 R) J_p'(k_1 R) - g J_p(k_1 R) \right] =
\]

\[
-b_n \frac{k_1}{\varepsilon_1} H_n^{(1)\prime}(k_1 R) + \sum_{p \in \mathbb{Z}} \chi_{n-p} b_p \left[ \frac{k_1}{\varepsilon_1} H_p^{(1)\prime}(k_1 R) - \frac{k_1}{\varepsilon_2} J_p(k_2 R) H_p^{(1)\prime}(k_1 R) + g H_p^{(1)}(k_1 R) \right] \tag{17}
\]

The Solution of the linear system (17) gives the unknown amplitudes \(b_n\) and then Eq.(16) gives the amplitudes \(c_n\). Thus the field can be computed everywhere in space using Eqs.(1) and (2).
III. NUMERICAL RESULTS

The infinite linear systems (10) and (17) are truncated to a finite size by retaining only 
\((2N + 1)\) coefficients and solved to obtain a representation of the field at truncation order 
\(N\). The convergence of the results has been checked by increasing integer \(N\) and using 
the usual criteria of energy balance (optical theorem) and reciprocity. We have also verified 
that the boundary conditions are fulfilled, for instance the nullity of the tangential electric 
field on the strips. In all the calculations carried in this paper we set \(g = -10^{-3}\). However, 
as mentioned in [9], numerical experiments show that only the sign of \(g\) is of importance: the 
umerical scheme is more stable with a negative value of \(g\). All the computations reported 
have been obtained on a Personal Computer (200 MHz processor with 32 Mo of RAM), only 
a few seconds are necessary to perform each result shown here.
In the following we provide some numerical examples and compare our results with those 
obtained in previous works [3], [4], [5] and [6].
In our first example, we consider a circular cavity with a single longitudinal narrow slot 
(see Fig. 2) with \(\phi = 5^\circ\) and we compute the interior field on \(x\) axis. Figures 3 (a) and (b) 
show the magnitude of the normalized electric field in both the TM\(_z\) and the TE\(_z\) cases of 
polarization. It can be seen that our results are in excellent agreement with those published 
recently by Shumpert and Butler [3], [5], see for instance figure 6 in this last reference. In the 
second example, we consider a circular cavity with an aperture such that \(\phi = 5^\circ\). In figures 
4 (a) and (b) are plotted the normalized electric field amplitude at the center of the cylinder 
for various values of the parameter \(k_1 R\) for both the TM\(_z\) and the TE\(_z\) cases of polarization. 
These curves agree with those obtained by Mautz and Harrington (see references [10] and 
[11]). It is worth noticing that the resonances in these plots correspond to the modes of the 
cavity.
Finally we give the map of the electric field around and inside the slotted cylinder when 
excited by a plane wave such that \(k_1 R\) corresponds to a mode of the closed cylinder. We 
can see in figures 5 (a) and (b) that the modes TM\(_{01}\)\((k_1 R = 2.404)\) and TM\(_{11}\)(\(k_1 R = 3.832)\) 
are excited inside the structure.

IV. CONCLUSION

We have developed a very efficient and fast method adapted to study diffraction of an elec-
tromagnetic wave by a finite number of infinitely thin, infinitely conducting strips deposited 
on a dielectric cylinder. It is based on the combined boundary conditions method. The 
method is very low CPU-time consuming. The numerical examples that have been given to 
illustrate the method are not restrictive. One can use as an incident radiation a beam of any 
shape. It suffices to calculate its corresponding incident amplitudes \(a_n\). It is also possible 
to study the radiation pattern of a source located at the center of the cylinder by making 
slight changes in the equations.
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Figure captions

**Figure 1**: Geometry of the problem: a TEz or a TMz polarized plane wave illuminates the slotted cylinder.

**Figure 2**: Electromagnetic penetration into a circular cavity through a narrow slot.

**Figure 3**: (a) Magnitude of normalized electric field on the x axis of a slotted circular cylinder excited by TMz plane wave \(k_1 R = 0.7, \theta_0 = 180^\circ, \phi = 5^\circ\)  
(b) Magnitude of normalized electric field on the x axis of slotted circular cylinder excited by TEz plane wave \(k_1 R = 0.7, \theta_0 = 0^\circ, \phi = 5^\circ\).

**Figure 4**: (a) Normalized electric field amplitude at the center of the cylinder for various \(k_1 R \) with \(\theta_0 = 0^\circ, \phi = 5^\circ\)  
(b) Normalized electric field amplitude at the center of the cylinder for various \(k_1 R \) with \(\theta_0 = 0^\circ, \phi = 5^\circ\).

**Figure 5**: Map of the electric field for values of \(k_1 R \) corresponding to the modes: \(\text{TM}_{01}\) and \(\text{TM}_{11}\) of the cylinder.
Figure 1
Figure 2
Figures 3 (a) and (b):
Figure 4 (a), and (b)
Figures 5