Aspects of brane-antibrane inflation

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Abstract: I describe a dynamical mechanism for solving the fine-tuning problem of brane-antibrane inflation. By inflating with stacks of branes and antibranes, the branes can naturally be trapped at a metastable minimum of the potential. As branes tunnel out of this minimum, the shape of the potential changes to make the minimum shallower. Eventually the minimum disappears and the remaining branes roll slowly because the potential is nearly flat. I show that even with a small number of branes, there is a good chance of getting enough inflation. Running of the spectral index is correlated with the tilt in such a way as to provide a test of the model by future CMB experiments.

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1. Introduction

Significant progress has been made recently in obtaining inflation from specific string theory constructions, notably in type IIB string theory in which moduli are stabilized by fluxes. Aspects of racetrack inflation, brane-antibrane inflation, and the problem of reheating and cosmic string production in the latter, have been reviewed elsewhere [1], and I will not repeat that material here. Instead I focus on a newer idea, multibrane inflation [2], which provides a simple and novel solution to the problem of fine-tuning of the inflaton potential in the KKLMMT [3] proposal for brane-antibrane inflation. It is a straightforward generalization of the KKLMMT construction shown in figure 1, where branes are attracted to antibranes which live in the bottom of a Klebanov-Strassler [4] warped throat.

The Lagrangian for the inflaton field $\psi$, which is the brane-antibrane separation, is

$$\mathcal{L} = \frac{1}{(2\sigma - \psi^2)^2} \left( 6\sigma \dot{\psi}^2 - \frac{\epsilon \tau}{1 + \left(\frac{\psi - \psi_0}{\tau}\right)^4} \right)$$

where $\epsilon$ is the fourth power of the warp factor at the bottom of the Klebanov-Strassler throat, $\tau$ is the unwarped 3-brane tension, $\sigma$ is the Kähler modulus of the Calabi-Yau manifold, and $\psi_0$ is the position of the antibranes at the bottom of the throat. It was noted in [5] that, depending on the values of the parameters $\tau$, $\epsilon$, $\psi_0$, the inflaton may be trapped in a metastable minimum near $\psi = 0$. For inflation, one needs to tune the parameters to avoid this minimum, because otherwise the inflaton does not roll, but rather undergoes old inflation with its infamous graceful exit problem. A tuning at the level of 1 part in 1000 was needed to obtain sufficient inflation.

In this discussion, it was assumed that there is just a single brane and antibrane driving inflation. However in string theory it is natural to consider stacks of coincident branes or antibranes. If we make

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this simple generalization, then a qualitatively new effect can occur. The potential (1) generalizes to
\[
\mathcal{L} \rightarrow \frac{1}{(2\sigma - \sum_i \psi_i^2)^2} \left( \sum_i 6\sigma \dot{\psi}_i^2 - \frac{N\epsilon}{\sum_i (\psi_i - \psi_0)} \right)
\]  

(2)

For some limited range of the parameter \( b \equiv b \equiv \sqrt{2\sigma \epsilon / \psi_0^6} \), this potential has the interesting property that for large a number \( N \) of branes, there is a metastable minimum near \( \psi = 0 \), but the minimum disappears as \( N \) decreases. This behavior is illustrated in figure 2. Therefore if one starts with a large enough number of branes, then the successive tunneling of single branes from the metastable minimum will cause the potential to automatically tune itself to become increasingly flat, until the point at which the branes can start to slowly roll and drive inflation. During the tunneling phase, “old inflation” is occurring, which by itself is not a successful inflationary scenario, but this is unimportant for us since the last phase of old inflation is followed by inflation driven by the slowly rolling branes.

One way to understand the origin of this behavior of the potential is to consider its curvature at the origin. There are competing effects which depend differently on \( N \): one comes from the brane-antibrane Coulomb-like potential, and the other comes from a supergravity effect, with the entire flat-space potential being divided by the Calabi-Yau volume, \( 2\sigma - N\psi^2 \), as modified by the presence of the branes. The curvature of the potential at \( \psi = 0 \) is proportional to
\[
V'' \sim \frac{1}{4\sigma} - 5/2 \frac{\epsilon}{\psi_0^2 (\psi_0^4 + N\epsilon)} + 4 \frac{N\epsilon^2}{\psi_0^2 (\psi_0^4 + N\epsilon)^2}
\]  

(3)

Clearly this is always positive for large enough \( N \). But when \( N = 0 \), it is proportional to
\[
V'' \sim \frac{1}{4\sigma} - 5/2 \frac{\epsilon}{\psi_0^6}
\]  

(4)

which will be negative if \( b^2 \equiv 2\sigma \epsilon / \psi_0^6 > 0.2 \) Thus the curvature of the potential changes sign as a function of \( N \). This turns out to be just one condition on \( b \) which must be satisfied in order to get the right behavior. We will show how to get around this restriction later.

Fig. 1. The KKLMMT brane-antibrane inflation setup in a Calabi-Yau space stabilized by fluxes and containing a warped throat.
We have explored the parameter space of $\sigma$ and $\epsilon$ (the size of the internal manifold and the warping in the inflationary throat) to determine how many branes are needed, and how much inflation results. The correlation between the number of $e$-foldings and the number of branes is shown in figure 3, which was obtained by uniformly sampling the parameter space $-5 < \log_2 0\epsilon < -2$ and $1 < \log_3 0(2\sigma) < 4$. This corresponds to a moderately warped inflationary throat, and large Calabi-Yau volumes ($2\sigma$ is the size of the manifold in string units), which is required for consistency of the low-energy supergravity description. The figure shows the actual points as well as an empirical formula which explains the complicated structure rather well. The origin of the steep lines of points is due to the fact that, if $N$ could be noninteger, it would be possible to potentials with perfectly flat spots (where $V'$ and $V''$ vanish simultaneously) leading to very long periods of inflations. The lines are sequences of points in which the actual value of $N$ becomes increasingly close to the ideal value.

It is interesting to note that, even though we consider the possibility of large numbers of branes, there is a high probability of getting enough inflation $N_e \gtrsim 60$ even with a relatively small number of branes, of order 10. This is shown in figure 4, which is a close-up of the small-$N$ region of figure 3. Nevertheless, brane stacks of up to 100 or even 1000 could be used to satisfy the tadpole conditions of the flux compactification scenario, given a Calabi-Yau manifold with large enough Euler characteristic [6]. Examples with $\chi = 10^5$ are known.

I come back to the fine-tuning issue. If we had no restriction on the combination $b \equiv \sqrt{2\sigma \epsilon}/\psi_0^2$, this mechanism would be a complete solution to the problem of fine-tuning. The fact that $b$ must lie in a narrow range makes it less compelling. However, we have found that by adding corrections to the potential which are generically expected to be present, the restriction on $b$ can be removed altogether. These take the form of corrections to the superpotential and Kähler potential. The potential generalizes to

$$V \to \frac{N\delta(\psi^2 - \psi_0^2)^2 - \frac{N\epsilon \tau}{1 + N\epsilon(\psi - \psi_0)}\tau}{(2\sigma - N(\psi^2 + d\psi^4))^2}$$

Then using, for example, $d = -0.5\psi_0^{-2}$ and $\delta = 0.1\epsilon\tau\psi_0^{-4}$, we get the required self-flattening behavior of the potential for any value of $b$. The question of whether this theory of inflation can be distinguished from others by the data is difficult. In our scan of parameter space, we find a wide range of predictions for properties of the scalar and tensor perturbation spectra relevant for the Cosmic Microwave Background. For example,
Fig. 3. Number of e-foldings of inflation versus critical number of branes needed for a flat potential, from scanning over the $\sigma$-t parameter space.

Fig. 4. Close-up of small-$N$ region of figure 3.
applying the COBE normalization to the spectrum implies that a range of energy scales for the potential is allowed, $V^{1/4} < 10^{-3} M_p$, with the inequality being saturated only for the rare cases with $N \sim 1$ branes. For this reason, a measurable tensor component in the CMB is not a generic prediction of the model. Moreover, we find a wide range of spectral tilts, $0.93 < n_s < 1.15$, although the majority of realizations has $n_s < 1$. Thus there is no smoking-gun prediction for the spectral index in this model.

However, there is an interesting test which could allow future CMB experiments to rule out the models under consideration. There is presently a hint in the WMAP data of strong running of the spectral index, $dn_s/d \ln k \sim -0.1$. Our models were not able to produce such a large amount of running, and the exact value awaits experimental confirmation because the errors are still large. But if upcoming measurements were able to confirm even a much smaller level of running, $dn_s/d \ln k \sim -0.01$, it would effectively rule out the model because of the very definite correlation between $dn_s/d \ln k$ and the spectral index itself, $n_s$, as shown in figure 5. The predicted points fall along the solid line. Although many of these points lie within the experimentally allowed region [7], those with significant running do not.

In summary, we have presented a novel mechanism for solving the fine-tuning problem of obtaining a flat inflaton potential, within a popular string theoretic model of inflation. The self-flattening feature of the brane-antibrane potential does not naturally occur in field theory models of inflation, but it appears quite generically in the present stringy context. It is also encouraging that at least one experimental test can potentially falsify the model.

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