Isocurvature fluctuations in the effective Newton’s constant

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We present a new isocurvature mode present in scalar-tensor theories of gravity that corresponds to a regular growing solution in which the energy of the relativistic degrees of freedom and the scalar field that regulates the gravitational strength compensate during the radiation dominated epoch on scales much larger than the Hubble radius. We study this isocurvature mode and its impact on anisotropies of the cosmic microwave background for the simplest scalar-tensor theory, i.e. the extended Jordan-Brans-Dicke gravity, in which the scalar field also drives the acceleration of the Universe. We use Planck data to constrain the amplitude of this isocurvature mode in the case of fixed correlation with the adiabatic mode and we show how this mode could be generated in a simple two field inflation model.

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Intro. Since its discovery with the analysis of SNIa light curve of the Supernova Cosmology Project [1] and High-Z Supernova Search Team [2], the acceleration of the Universe at $z \sim 1$ has been confirmed by a host of cosmological observations in the last 20 years. A cosmological constant $\Lambda$, which is at the core of the minimal concordance cosmological $\Lambda$CDM model in agreement with observations, is the simplest explanation of the recent acceleration of the Universe, but several alternatives have been proposed either replacing $\Lambda$ by a dynamical component or modifying Einstein gravity (see [3–5] for reviews on dark energy/modified gravity).

If a dynamical component as quintessence, which varies in time and space, drives the Universe into acceleration instead of $\Lambda$ [6–7], not only the homogeneous cosmology is modified, also its fluctuations cannot be neglected and their behaviour can help in distinguishing structure formation in different theoretical models. This dynamical component can, in combination with the other cosmic fluids (radiation, baryons, cold dark matter, neutrinos), lead not only to adiabatic curvature perturbations, but to a mixture which includes an isocurvature component. Isocurvature perturbations appear when the relative energy density and pressure perturbations of the different fluid species compensate to leave the overall curvature perturbations unchanged for scales much larger than the Hubble radius.

In the case of quintessence, it was found that its fluctuations are very close to be adiabatic during a tracking regime in which the parameter of state of quintessence mimics the one of the component dominating the total energy density of the Universe [8]. In the case of thawing quintessence models, in which a tracking regime is absent, isocurvature quintessence fluctuations are instead allowed. From the phenomenological point of view, a mixture of curvature and quintessence isocurvature perturbations is an interesting explanation of the low amplitude of the quadrupole and more in general of the low-ell anisotropies pattern [9–10].

In this Letter we study a new isocurvature mode which is present in scalar-tensor theories of gravity, in which the scalar field responsible for the acceleration of the Universe also regulates the gravitational strength through its non minimal coupling to gravity [11–15]. These models are also known as extended quintessence [11]. We will study the effect of this new isocurvature mode on CMB anisotropies and show that this can be generically excited during inflation with an amplitude allowed by Planck data.

The model. We consider the simplest scalar-tensor gravity theory describing the late time Universe:

\[
S = \int d^4x \sqrt{-g} \left[\frac{\gamma}{2} \sigma^2 R - \frac{g^{\mu\nu}}{2} \partial_\mu \sigma \partial_\nu \sigma - V(\sigma) + \mathcal{L}_m \right]
\]

(1)

where $\mathcal{L}_m$ denotes the matter content (baryon, CDM, photons, neutrinos), $\sigma$ is the Jordan-Brans-Dicke (JBD)
scalar field whose equation of motion is:
\[ \ddot{\sigma} + 3H\dot{\sigma} + \frac{\dot{\sigma}^2}{\sigma} + \frac{\sigma^4}{(1 + 6\gamma)(\sigma^2)} = \frac{\sum_i (\rho_i - 3p_i)}{(1 + 6\gamma)\sigma} \]  

(2)

Note that the above induced gravity Lagrangian can be recast in an extended JBD theory of gravity [15] by a re-definition of the scalar field \( \phi = \gamma\sigma \), and \( \omega_{BD} = (4\gamma)^{-1} \).

We will consider the case of a non-tracking potential as \( V(\sigma) \propto \sigma^4 \) [17] in the following (see [18] for other monomial potentials). The background cosmological evolution is displayed in Fig. 1: deep in the radiation era, \( \sigma \) is almost frozen, since it is effectively massless; during the subsequent matter dominated era, \( \sigma \) is driven by non-relativistic matter to higher values. These two stages of evolution are quite model independent for a nearly massless \( \sigma \) within the coupling \( \gamma\sigma^2R \): the potential \( V(\sigma) \) kicks in only in a third stage at recent times determining the rate of the accelerated expansion (see Fig. 1).

The evolution of linear perturbations in the adiabatic initial conditions has been considered for the most recent constraints on this class of scalar-tensor theories [18, 19]. The so-called adiabatic initial condition [20] are the regular solution to the Boltzmann, Klein-Gordon and metric equations in scalar-tensor gravity characterized by a constant curvature perturbation for scales much larger the Hubble radius during the radiation dominated epoch.

In this Letter we wish to present the original result for a more general initial condition which include a mixture of the adiabatic and a new isocurvature solution between the relativistic degrees of freedom and the scalar field. The latter is a new solution which is obviously absent in \( \Lambda \)CDM and is an example of the generic new independent growing solution within scalar-tensor theories of gravity.

The initial conditions In the following we use the synchronous gauge for metric fluctuations [20] and we denote by \( \delta_i \) the energy density contrast, \( \theta _i = ik_j v_j^i \) the velocity potential and \( \sigma _i \) the neutrino anisotropic pressure. The indices \( i = b, c, \gamma, \nu \) denote baryons, CDM, photons, and neutrinos respectively. The adiabatic plus the new isocurvature solution, in the background considered, is given by:

\[
\begin{align*}
\delta _\gamma &= \delta _\nu = C \left[ -\frac{2}{3} k^2 \tau^2 + \frac{2\nu}{15} k^2 \tau^3 \right] + D \left[ -1 - \frac{2\nu}{3} \tau + \frac{3(15\gamma + 2)\omega^2 + 4k^2}{24}\tau^2 \right], \\
\theta _\gamma &= C \left[ -\frac{k^4 \tau^3}{36} + \frac{(5(1 + 6\gamma)R_b + R_\nu)(k^4 \tau^4)}{240R_\gamma} \right] + D \left[ -\frac{k^2}{4} \tau + \frac{\omega}{48} \left( \frac{1}{R_\gamma} - 4 \right) k^2 \tau^2 \right], \\
\delta _b &= C \left[ -\frac{k^2}{2} \tau^2 + \frac{\omega}{10} k^2 \tau^3 \right] + D \left[ -\frac{\omega}{2} \tau + \frac{1}{8} \left( \frac{3(15\gamma + 2)\omega^2}{4k} + k \right) k^2 \tau^2 \right], \\
\delta _c &= C \left[ -\frac{k^2}{2} \tau^2 + \frac{\omega}{10} k^2 \tau^3 \right] + D \left[ -\frac{1}{2} \omega \tau + \frac{3}{32} (15\gamma + 2) \omega^2 \tau^2 \right], \\
\delta _\nu &= C \left[ -\frac{2}{3} k^2 \tau^2 + \frac{2}{15} k^2 \tau^3 \omega \right] + D \left[ -1 - \frac{2\omega}{3} \tau + \frac{3(15\gamma + 2)\omega^2 + 4k^2}{24}\tau^2 \right], \\
\theta _\nu &= C \left[ -\frac{(4R_\nu + 23)}{18(4R_\nu + 15)} k^4 \tau^4 + \frac{\omega}{120(2R_\nu + 15)} \left( 8R^2_\nu + 60\nu(5 - 4R_\nu) + 50R_\nu + 275 \right) \right] + D \left[ -\frac{k^2}{4} \tau^2 - \frac{1}{12} \omega k^2 \tau^2 \right], \\
\sigma _\nu &= C \left[ -\frac{4k^2 \tau^2}{3(4R_\nu + 15)} + \frac{(1 + 6\gamma)(4R_\nu - 5)k^2 \tau^3}{3(4R_\nu + 15)(2R_\nu + 15)} \right] + D \left[ \frac{k^2 \tau^2}{6(4R_\nu + 15)} - \frac{2\omega(1 + 6\gamma)(R_\nu + 5)k^2 \tau^3}{3(2R_\nu + 15)(4R_\nu + 15)} \right], \\
\tau &= \left[ -\frac{2}{3} \frac{(4R_\nu + 5)}{6(4R_\nu + 15)} k^2 \tau^2 \right] + D \left[ -\frac{\omega}{6} \tau + \frac{16k^2(R_\nu + 5) + 3(15\gamma + 2)(4R_\nu + 15)\omega^2}{96(4R_\nu + 15)} \right], \\
\delta \sigma &= C \left[ -\frac{1}{4} \frac{\omega k^2 \tau^3 + \gamma \omega^2}{40} (4 + 15\gamma)k^2 \tau^4 \right] + D \left[ -\frac{1}{2} + \frac{3}{4} \gamma \omega \right],
\end{align*}
\]

where \( \omega \equiv \sqrt{\frac{\rho_{mat0}}{\rho_{rad0}(1 + 6\gamma)\sigma_i}} \), and \( \sigma_i \) is the value of \( \sigma \) deep in the radiation era.

In the above equations \( C (D) \) encodes the primordial power spectrum for curvature (isocurvature) perturbations. This new isocurvature mode is entirely due to the presence of the scalar field and corresponds to a compensation between the relativistic degrees of freedom and scalar field energy densities (note that this mode is essentially independent on the potential \( V(\sigma) \) since \( \sigma \) drives the Universe into acceleration at late times). To complete the characterization of this new isocurvature mode as done for Einstein gravity [21], we note that the (gauge-invariant) curvature perturbation and the Newtonian potentials, as defined in [20], are given at leading order by \( R = -\frac{5(R_\nu + 4)}{4(4R_\nu + 15)} \omega \tau, \Psi = -\frac{(R_\nu + 5)}{4(4R_\nu + 15)} \) and \( \Phi = \frac{2(R_\nu + 5)}{4(4R_\nu + 15)} \) respectively.
FIG. 1. Example of an evolution for $\sigma/\sigma_0$ (left panel) and $\Omega_i$ (right panel) as function of $a$ for different choices of $\gamma$ for a quartic potential. The value $\sigma_0$ of the scalar field at present is fixed consistently with the Cavendish-type measurement of the gravitational constant $G = 6.67 \times 10^{-8} \text{ cm}^3 \text{g}^{-1} \text{s}^{-2}$: $\gamma_0^2 = \frac{1}{8\pi G \chi^4}$.

Impact on CMB anisotropies In order to derive the predictions for the CMB anisotropy angular power spectra we have used an extension to the publicly available Einstein-Boltzmann code CLASS\footnote{www.class-code.net}, called CLASSig\cite{19}. CLASSig has been developed to derive the predictions for cosmological observables in induced gravity, and more in general scalar-tensor theories, solving for the perturbations but also for the background in order to derive the initial scalar field parameters which provide the cosmology in agreement with the measurements of the gravitational constant in laboratory Cavendish-type experiments. CLASSig has been modified to include the initial conditions for the new isocurvature mode presented. In Fig. 2 we show the comparison of the new mode with the adiabatic and standard isocurvature modes in the $\Lambda$CDM model within Einstein gravity. Fig. 2 also shows the weak dependence of the new isocurvature mode on $\gamma$ at least for the small values consistent with the current cosmological 95\%CL upper bound $\gamma \lesssim \eta$, and for Solar system constraints $\gamma \lesssim 6 \times 10^{-5}$\cite{24}.

The impact of a mixture of curvature and the new isocurvature initial conditions admitting a non-vanishing correlation $\theta$ on CMB anisotropies defined as \cite{25} $C_\ell = C^\text{ADI}_\ell + f^2_\text{ISO} C^{\text{ISO}}_\ell + 2 f_\text{ISO} \cos \theta \, C^{\text{CORR}}_\ell$ is displayed in Fig. 3 ($f_\text{ISO}$ being the relative fraction of isocurvature). Overall, the effect of this new isocurvature mode seems larger and on a wider range of multipoles than the quintessence isocurvature mode in Einstein gravity studied in\cite{9}.

Comparison with data. We now present the constraints on the new isocurvature amplitude with Planck data. Since the effect of isocurvature perturbations on the CMB anisotropy power spectra does not depend significantly on $\gamma$ for $\gamma \lesssim 10^{-3}$ (see Fig. 2), we fix $\gamma = 5 \times 10^{-4}$ to contain the computational cost of our investigation. Such a value is either compatible with current cosmological observations\cite{18} and conservatively close to the values tested in the comparison of Einstein-Boltzmann codes dedicated to JBD theories reported in\cite{20}. We consider separately the three cases of correlations between adiabatic and isocurvature perturbation $\cos \theta = -1, 0, 1$ as in\cite{27}. As data we consider Planck
modes in Einstein gravity\(^2\), although scale similarly with the degree of correlation \(^2\).

**Isocurvature perturbations in the effective Newton’s constant from inflation.** We now show that the amplitude of the new isocurvature mode in \(\delta\sigma\) compatible with current data could be easily obtained in minimal inflationary models within scalar-tensor gravity. We consider a two-field dynamics in which the scalar field \(\sigma \equiv \Phi_2\) responsible for the late-time acceleration was present during the inflationary stage driven by the inflaton \(\phi \equiv \Phi_1\):

\[
S = \int d^4x \sqrt{-g} \left[ \frac{\gamma \sigma^2 R}{2} - \frac{g^{\mu\nu}}{2} \partial_\mu \Phi_1 \partial_\nu \Phi_1 - V(\phi, \sigma) \right].
\]

where the indices \(i = 1, 2\) are meant to be summed and \(V(\phi, \sigma) = V_{\text{inf}}(\phi) + V(\sigma)\).

Since \(V(\sigma_0)/(3\gamma \sigma^2_0 H_0^2) \sim 0.7\), \(\sigma\) is effectively massless during inflation. We assume that after inflation \(\phi\) decays in ordinary matter and dark matter, which are coupled to \(\sigma\) only gravitationally through the term \(\gamma R \sigma^2\). Once the Universe is thermalized, the evolution during the radiation and matter dominated era matches with what previously described for the background and perturbations: indeed, isocurvature perturbations in \(\sigma\) are nearly decoupled from curvature perturbations during the radiation dominated period in which \(\sigma\) is frozen.

The two field dynamics in Eq. (1) and the corresponding generation of curvature and isocurvature fluctuations have been previously studied \(^3\) either in the original Jordan frame or in the mathematically equivalent Einstein frame. Since in general curvature and isocurvature perturbations are not invariant under conformal transformations \(^4\), we work within the original frame in Eq. (1) consistently with the late time cosmology previously described. Under the assumption that \(\sigma\) is subdominant during inflation, we find to lowest order in \(\gamma\) and in the slow-roll parameters the isocurvature fraction on scale much larger than the Hubble radius \(^5\):

\[
\frac{P_S(k_*)}{P_R(k_*)} \lesssim e^{(n_S-1-n_T-320\gamma)/(N_\pi-\frac{1}{2})},
\]

where \(n_S\) (\(n_T\)) is the scalar (tensor) tilt, \(N_*\) is the number of \(e\)-folds from the Hubble crossing of the pivot scale \(k_*\), and the inequality stands for the adiabatic limit used for the curvature power spectrum. Note that we assume that the value of the scalar field at the end of inflation \(\sigma_*\) is equal to the one at the beginning of radiation era \(\sigma_i\). By considering the scalar tilt consistent with Planck results \((n_S = 0.968\pm0.06\text{ at } 68}\% \text{ CL}\)\(^7\) and \((\sigma_\pi)\) as a range for \(N_*\), we find that the isocurvature fraction at the end of inflation in Eq. (5) is of the same order of magnitude of the upper bound obtained from data.

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\(^2\) https://github.com/brinckmann/montepython_public

\(^3\) Note that the relation \(\beta = \frac{f_{\text{ISO}}}{1+f_{\text{ISO}}}\) between the isocurvature fraction \(\beta\) in \(^7\) and \(f_{\text{ISO}}\) holds.
Conclusions

On concluding, we have presented a new growing independent solution in scalar-tensor gravity corresponding to an isocurvature mode between the scalar field which determines the evolution of the effective Newton’s constant and the relativistic degrees of freedom. We have constrained with Planck data this new isocurvature mode when the scalar field is also responsible for the current acceleration of the Universe and we have shown how this mode can be generated during inflation. It will be interesting to see how the most recent models can constrain this phenomenological aspect of scalar tensor theories. Work in this direction is in progress.

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