Note on the charging and spinning of dust particles in complex plasmas in a strong magnetic field

V N Tsytovich¹, N Sato² and G E Morfill³

¹ General Physics Institute, Russian Academy of Science Moscow, Vavilova Street 38, 117942, Moscow, Russia
² Tohoku University, Sendai 980-8579, Japan
³ Max Planck Institute for Extraterrestrial Physics, Giessenbachstrasse 1, 85740 Garching, Germany
E-mail: tsytov@td.lpi.ac.ru, tsyto@mpe.mpg.de, nsato@ecei.tohoku.ac.jp and gem@mpe.mpg.de

New Journal of Physics 5 (2003) 43.1–43.9 (http://www.njp.org/)
Received 10 January 2003
Published 9 May 2003

Abstract. Qualitative analysis of two new effects related to the influence of a strong magnetic field on the state of complex dusty plasmas is given. First, it is shown that the dust shape asymmetry, together with the presence of plasma charging flux, creates an angular momentum flux on a dust grain causing its rotation. The value of the estimated angular velocity is compared with that already observed. The magnetic moment related to the balance of the angular momenta transfer by plasma flux and the friction on neutrals is estimated. It is estimated that the interactions of these magnetic moments are usually much weaker than the Coulomb interactions. Apart from the magnetic moments induced by plasma flux the dust grain can have an intrinsic magnetic moment. Both of them create an average magnetization in complex plasmas. Second, it is shown that an increase in an external magnetic field first decreases the dust charges (when the electrons in the charging process start to be magnetized) and then, with a further increase of the magnetic field, the dust charges increase (when the ions in the charging process start to be magnetized). Only the limiting cases of strong electron and ion magnetization are discussed.
1. Introduction

Recent experiments [1]–[4] on dusty plasmas in different magnetic configurations raise the questions of how the magnetic fields can change the currents on the dust surface and the dust charges, how the magnetic field influences the ion dust drag effect and how the magnetic fields can interact with an intrinsic dust magnetic moment and with the magnetic moments related to induced dust spinning.

2. Estimate of dust spinning

We should point out that, even in the absence of magnetic fields, the process of charging can cause dust rotation. This effect was not noticed before and is described here for the first time. The dust rotation was observed experimentally in [1, 3, 4]. We give here the physical explanation of this effect and propose its possible mathematical treatment.

Dust spinning has been the subject of numerous investigations in astrophysics (see [5]–[9]). All the models considered so far are not related to the dust charging process. The excitation of rotation is thermal (random) and the effect considered is described by the square root of the square of the average rotational velocity. We will show that the charging process can cause a systematic change in rotational velocity and that the average of the increase in dust angular momentum per unit time is not zero for the process considered here. The proposed effect of induced spinning due to the charging process was not mentioned before, but it seems to be one of the most natural effects which could be responsible for the appearance of dust spinning. The proposed physics of the appearance of dust spinning due to dust charging is the following: the process of charging is related to ion and electron currents on the grain surface with the attachment of ions and electrons to the dust surface. This process is continuous and proceeds even after the dust charge reaches its equilibrium value. The main part of the momenta transferred to the grain is related to the ion flux due to its larger mass, as compared to the electron mass. For spherical grains and an isotropic distribution of ions the net momenta transferred to the dust grain is zero, since for any ion with momentum $p$ which is absorbed by the grain there exists another absorbed ion with equal and opposite momentum $-p$. The asymmetry in ion distribution causes the dust drag effect. In any absorption collision of an ion with a grain angular momentum is also transferred. The net effect for this transfer can be zero only in the case of an ideal spherical grain. If the shape of the grain...
is irregular one can estimate the net transfer of the angular momenta as $\eta_{ass} \pi a^2 n_i \infty m_i a v_s v(\infty)$, where $\eta_{ass}$ is the coefficient taking into account the degree of irregularity of the dust shape, $a$ is the average radius of the grain, $v_s$ is the ion velocity at the surface of the grain when they are attached to the grain and $v(\infty)$ is the ion velocity far from the grain. From the conservation law we get $v_s^2 \approx v_{Ti}^2 (1 + 2Z_{de} e^2 / a m_i v_{Ti}^2)$ and by averaging this expression over the thermal ion distribution we get an estimate $v_s^2 \approx v_{Ti}^2 (1 + z / \tau)$, where $v_{Ti} = T_i / m_i$ is the ion thermal velocity, $\tau = T_i / T_e$ is the ratio of the ion to electron temperature (usually about $10^{-2}$ in existing experiments) and $z = Z_d e^2 / a T_e$ is the dimensionless dust charge. The continuous transfer of angular momenta to the dust grain should be compensated by another process of continuous loss of angular momenta which, in a complex low-temperature plasma in the presence of large numbers of neutral atoms, is due to dust friction in the neutral gas. We assume that the friction force for rotation is of the order of the Epstein force $F_{Ep}$ with some coefficient $\eta_{rot}$ of the order of unity. The angular momenta loss force will be of the order of $\eta_{rot} a F_{Ep}$ and therefore, taking into account the known expression for the Epstein force and putting the velocity of the grain equal to the velocity of rotation $\approx a \Omega$, where $\Omega$ is the angular velocity of rotation, we find the rate of the angular velocity losses to be $4 \eta_{rot} n_i \pi a^2 \Omega a^2 / 3 v_{Tn}$, where $n_i$ is the density of neutrals, $T_n$ is the temperature of neutrals and $v_{Tn} = \sqrt{T_n / m_n}$ is the neutral atom thermal velocity. A direct and straightforward calculation of the stopping power for an almost spherical grain gives $\eta_{rot} = 0.728$, but bearing in mind that the asymmetry in the grain shape can be of the order of 1 we will include in the estimate the factor $\eta_{rot}$ as being of the order of 1. Usually in existing experiments $z / \tau \gg 1$ and the balance of angular momenta gives the following estimate of the angular velocity of dust spinning:

$$\Omega = \frac{\eta_{ass} z n_i T_i v_{Tn}}{\eta_{rot} \tau n_i T_n a}.$$  (1)

Since in experiments $z / \tau \approx 10^2$, $n_i T_i / n_i T_n \approx 10^{-6} - 10^{-5}$ we have for the velocity of rotation an estimate of $2 \times 10^2$ cm s$^{-1}$ and for $a = 10$ $\mu$m an estimate for the angular velocity of rotation of $\Omega \approx 200$ rad s$^{-1}$ or the frequency of rotation of about (10–30) Hz which is in reasonable agreement with that observed in experiments [1, 3, 4]. Expression (1) predicts a certain dependence of angular velocity on the roughness of the dust surface, mass of the gas, size of the grain, etc, which can be checked in existing experiments. The friction on the neutral gas can stop the dust rotation if the mechanical angular inertia of the grain is small.

### 3. Induced magnetic moments and magnetic interaction of spinning grains

We denote the dust magnetic moment related to the dust spinning as $\mu_0$. The total magnetic moment of the grain will be $\mu + \mu_0$, where $\mu$ is the intrinsic magnetic moment of the grain. $\mu_0$ can be calculated from the general expression for the magnetic moment of a rotating charge. The result depends on the dust distribution in the grain. In the case where the dust charge is located at its surface we have an exact expression $\mu = -e Z_d \Omega a^2 / 2c$. Usually the dust grain charge distribution is more complex (note that the grains grown in the discharge can even have a fractal structure) and we can introduce some coefficient $\eta_{str}$ which characterizes the charge distribution inside the grain:

$$\mu_0 = -\eta_{str} \frac{e Z_d a^2}{2c} \Omega$$  (2)
where $\Omega = \Omega n$, with $n$ being the unit vector along the rotation axis.

By substituting (1) in expression (2) we get an estimate of the intrinsic magnetic moment of the grain:

$$
\mu_0 \approx -\frac{e Z_d z a \nu T_n}{2c} \frac{\eta_{str} \eta_{ass}}{\eta_{rot}} \frac{T_e n_i}{T_n n_n}.
$$

(3)

It is interesting and necessary to stress that the magnetic moment is independent of grain mass. This is a specific property of complex plasmas with a low degree of ionization when the friction force exceeds the dust inertia force and the dust magnetic moment related to dust spinning depends only on the grain size, grain charge and local plasma parameters, such as the temperature ratio and degree of ionization. Another important point is that, according to relation (3) and $Z_d \propto a$, the magnetic moment is proportional to the square of the dust size $\propto a^2$. Therefore for large dust grains $a > 100 \mu m$, which can be used in future micro-gravity experiments, the magnetic moment effects due to dust spinning can never be neglected.

It is possible to estimate the role of the interaction of the dust magnetic moments related to dust spinning. Since the magnetic field of the magnetic moment decreases with inter-dust distance much faster than the Coulomb field it is reasonable to estimate the maximum possible magnetic interactions at the lowest separation of grains of the order of the dust grain size:

$$
U_M \approx \frac{\mu_0^2}{a^3} \approx \frac{e^2 Z_d^2 \nu^2 T_n}{a^3} \frac{\tau^2}{c^2}.
$$

(4)

This expression illustrates that the magnetic interaction in existing experimental conditions is always much lower than the Coulomb interactions and it is easy to estimate that they will also be much lower than the collective attraction interaction of grains [10]. The intrinsic magnetic moment $\mu$ should be made at least three orders of magnitude larger than $\mu_0$ for the magnetic interaction to be important in crystal formation.

The magnetic moments related to dust spinning will be distributed randomly for random distribution of the irregularities of the grain shapes in the case where the external magnetic field is absent. In the presence of strong magnetic fields they can be aligned in the direction of the magnetic field, causing a collective diamagnetic field. An estimate of this field is, for the present experimental conditions, about 1 G for the size of complex plasma of the order of 10 cm (in all three directions).

In this case the dust grains are magnetic (they have their intrinsic magnetic moment in the absence of spinning) and the field of those aligned grains is larger if their magnetic moment is substantially larger than the magnetic moment related to grain spinning.

4. Change of dust charge by strong magnetic field

For not very strong fields where the dust size is much smaller than the electron gyro-radius the change of dust charges is relatively small and is determined by the known charging equation which, for $\tau = T_i / T_e \ll 1$ (met in the present experiments), has a simple form

$$
\exp(-z) = \sqrt{\frac{m_e}{m_i \tau}}^{z}.
$$

(5)
where, as before, \( z = Z_d e^2 / a T_e \). Equation (5) describes the balance of ion and electron currents. The electrons are deflected by the dust particles (the exponent in the left-hand side shows that only fast electrons are taking part in charging and their average cross section is smaller than the geometrical cross section \( \pi a^2 \)). The ions are attracted (their effective cross section is \( z/\tau \gg 1 \) larger than the geometrical cross section \( \pi a^2 \)). This makes the balance of thermal currents possible although the electron thermal velocity is much larger than the ion thermal velocity.

As soon as the magnetic field becomes larger than the critical field where the electron gyro-radius is equal to the collection radius of electrons on dust grains the electrons in the charging process can be considered as moving straight towards the dust particles but still the low energy electrons are reflected backwards (not deflected as usual in the absence of a magnetic field). The large negative charge of the dust particles creates the reflected electron flow in the opposite direction but along the magnetic lines, and only the fast electrons will be able to reach the dust and charge it. The cross section for magnetized electrons is changed, namely \( \omega_{Be} = eB_0 / m_e c \gg v_e / a \) becomes equal to \( \pi a^2 \). But by using the energy conservation for electrons moving along a magnetic field we find that the absorbed electrons should have a velocity larger than the critical velocity, namely \( v_e^2 / 2v_{Te} > z \), and the electron current on the grain will be determined by the electrons with velocities larger than the critical velocity. A simple calculation of this current for a thermal distribution \( \propto \exp(-v_e^2 / 2v_{Te}^2 - v_{\perp e}^2 / 2v_{Te}^2) \) shows that the electron current is reduced by a factor of four compared to that in the absence of a magnetic field. On the other hand, if the magnetic field is not so strong that the ion gyro-radius is still much larger than the ion dust attraction size \( \approx a \sqrt{z/\tau} \), the ions will be attracted to the dust grain with approximately the same rate and their effective cross section will be much larger than the geometrical cross section \( \pi a^2 \) and will be approximately equal to that in the absence of a magnetic field. The ion current on the grain will then not be substantially changed. The reduction of electron current means a decrease of the dust charges. The charging equation then takes the form

\[
\exp(-z) = 4z \sqrt{m_e / m_i \tau}.
\]

(6)

In deriving relation (6) it was assumed that for electrons the cross section coincides with the geometrical cross section and the electrons (neglecting the rotation on magnetic field lines) move straight along the magnetic field lines. The charges on dust particles become somewhat smaller (see figure 1 and the explanations given below). A simple estimate for the condition necessary to neglect the \( E \times B \) drift of electrons in the vicinity of the grain (caused by the electric field of the grain) gives that the ratio of the dust size to the electron gyro-radius should be larger than \( z \). The values of \( z \) are of the order of one, as found from equation (6) for the dust charge. Thus the criterion of electrons to be magnetized is not much changed by the effect of \( E \times B \) drift in the vicinity of the grain.

For much stronger magnetic fields where the ion gyro-radius becomes smaller than the dust size both the ion and electron currents are modified and we find the equation for the dust charges:

\[
\exp(-z) = \tau \sqrt{m_e / m_i \tau}.
\]

(7)

The intermediate case where the ion gyro-radius is less than the ion collection radius \( a \sqrt{z/\tau} \) but larger that the dust geometric radius can only be treated numerically.
Figure 1. Dependence of the dimensionless dust charge \( z \) (vertical axis) on the parameter \( \mu = \sqrt{m_i \tau / m_e} \) (horizontal axis); the full curve \( z(\mu) \) describes the case of the absence of a magnetic field; the dotted curve \( zBe(\mu) \) describes the case of an intermediate magnetic field, where the size of the dust particle is larger than the electron gyro-radius and is smaller than the ion gyro-radius; the broken curve \( zBi1(\mu) \), the chain curve \( zBi01(\mu) \) and the upper full curve \( zBi0005(\mu) \) describe the case of a strong magnetic field where the size of the dust particle is less than the ion gyro-radius for \( \tau = 1, 0.01 \) and \( 0.0005 \), respectively.

Only the parameter \( m \) enters into equations (5) and (6) where

\[
m = \sqrt{\frac{m_i \tau}{m_e}}
\]

while equation (7) also contains another parameter \( \tau \). Therefore we show on figure 1 the dependence of the dust charge on the parameter \( m \) for solutions of (5) and (7) and we show three values of \( \tau = 1, 0.01, 0.0005 \) for the solution of equation (7). In experiments for heavy ions the parameter \( m \) can reach a value 100 or even larger but for not very heavy ions and low ion temperatures the parameter \( m \) is rather small—about 3–4. As can be seen from figure 1 the dust value of the dust charge in a strong magnetic field can be substantially larger (up to 12 times) than in the absence of a magnetic field or in a weak magnetic field. In the latter case the criterion
for neglecting the ion $E \times B$ drift in the vicinity of the grain will be more restrictive (the ratio of the dust size to the ion gyro-radius should be larger than the value of $z$ which is enlarged by the strong magnetic field).

5. Critical magnetic fields

The inequality requiring that the size of the dust particle is larger than the electron gyro-radius and can be written in practical units as follows:

$$B_{cr}^e (\text{kG} a(\mu m)) > 41.37 \sqrt{\frac{T_e (\text{eV})}{3 (\text{eV})}}. \tag{9}$$

For $a \approx 10$ $\mu$m it is about 4 kG. The corresponding critical field, where the size of the dust grain is larger than the ion gyro-radius, is $\mu$ times larger. As can be seen from figure 1 for fields larger than that given by expression (9) the dust charge $z$ is reduced not very substantially (not more than a factor of 1.5). A more substantial change in the dust charges occurs for $B > B_{i cr}^i$. An increase of the charge of the dust particle cannot lead to dust braking in parts of the magnetic field used in the present experiments (this can only occur in astrophysical conditions where very strong magnetic fields are present). The conditions $B > B_{i cr}^i$ can be satisfied in the present experiments for rather large dust particles and not very heavy ions.

6. Drag force and the Lorentz force

The drag force is determined by ion scattering on dust grains and is less affected by magnetic field strength (the strength of the magnetic field enters only under the sign of the Coulomb logarithm). The magnetic field can change the ion motion and therefore change the drag force acting on the dust grain due to ion–dust collisions. The corresponding critical magnetic field $B_{cr}^{dr}$ can be found by comparing the Lorentz force acting on ions with the ion friction related to dust drag. We can write this expression in practical units:

$$B_{cr}^{dr} (\text{kG}) = 14 \left( \frac{a}{\lambda_{Di}} \right) \left( \frac{0.02}{\tau} \right) \sqrt{n_i/10^9 (\text{cm}^{-3})} P \ln \Lambda \tag{10}$$

where $\lambda_{Di}$ is the ion Debye radius, $P$ is the Havnes parameter $P = n_d Z_d / n_i$ and $\ln \Lambda$ is the Coulomb logarithm which takes into account the collective effects and large angular scattering of ions. For $a / \lambda_{Di} \approx 7$, $P \approx 1$ and $\ln \Lambda \approx 3$ this critical field is of the order of that given by expression (9). Nevertheless it seems that the critical field (10) is one of the most important since the curvature of ions for $B > B_{cr}^{dr}$ can cause the dust rotation while for $B > B_{cr}^e$ there occurs no drastic change of dust charges. We notice that both critical fields depend strongly on dust size and in the opposite way—the field $B_{cr}^e$ increases with decreasing the dust size while the critical field $B_{cr}^{dr}$ decreases with a decrease in dust size. Therefore we write expression (9) in a form similar to (10):

$$B_{cr}^e (\text{kG}) = 0.7 \left( \frac{\lambda_{Di}}{a} \right) \left( \frac{0.02}{\tau} \right) \sqrt{n_i/10^9 \text{ cm}^{-3}} \tag{11}$$

and find the ratio of the two critical fields in the form

$$\frac{B_{cr}^{dr}}{B_{cr}^e} = 20 \left( \frac{a}{\lambda_{Di}} \right)^2 P \ln \Lambda \left( \frac{0.02}{\tau} \right) \tag{12}$$
which illustrates how small \( a/\lambda_{Di} \) should be so that, with an increase of the magnetic field strength the change of the ion motion (taken together with the ion drag) will precede the dust charge changes. In the case that the dust charges are reduced by the magnetic field \( (B > B_{cr}^e) \) the drag effect should be changed by substituting in the drag force the dust charges according to the values given in figure 1.

7. Conclusions

This investigation shows the following.

- Dust spinning [3, 4] can create appreciative magnetic moments on dust grains which can be independent of the dust mass and have a random distribution in their direction.

- The external magnetic field can align the spinning moment but this effect is unable to create a strong magnetic field.

- The magnetic moment interaction is rather weak and can exceed the Coulomb interaction and collective dust interactions only if the dust grains have intrinsic magnetic moments about three orders of magnitude larger than the magnetic moment related to dust spinning.

- The influence of the magnetic field will be mainly related to a change of the ion drag and dust–dust collective interaction for fields larger than \( B_{cr}^{dr} \). The latter effect was estimated in [6], where it is shown that for \( B > B_{cr}^{dr} \) the magnetic field increases the potential wells for dust collective attraction and thus increases the temperature for crystal melting.

- Therefore, it is predicted that, for plasma crystals created in a strong magnetic field, first the melting occurs perpendicular to the magnetic field, forming a system of chaotic rods and then it occurs along the magnetic field where these rods are melted (see the results of [11]).

- A change of the dust charge when it is transferred from the region of weak magnetic field to the region of strong magnetic field can be used in experiments.

- The change of dust charges by magnetic fields will be important in dust shocks in strong magnetic fields where the value of the magnetic field suddenly changes at the surface of the shock.

- In a strong magnetic field the reflected electrons can create a two-stream instability which can also affect the charging process.

We mention in conclusion that the spinning of dust grains can be created by a dust charging process even for an almost perfect grain surface if the ion sticking coefficient is inhomogeneous along the grain surface (see [12] where an exact solution for such a model was given and the equation for dust spinning was solved).

The problems of complex plasmas in strong magnetic fields qualitatively discussed here will be an important issue for future experiments and the future theory of complex plasmas.
Acknowledgments

VNT wishes to acknowledge the support of an MPG senior visiting fellowship.

References

[1] Sato N, Uchida G, Kaneko Y, Dhimizu S and Iizuka S 2001 Phys. Plasmas 8 1786
[2] Konopka U, Samsonov D, Ivlev A, Goree J, Stenberg V and Morfill G 2000 Phys. Rev. E 61 1890
[3] Sato N 2002 Magnetic effects in dusty plasmas Proc. ICPDP-2002 Invited talks (Durban, South Africa, 2002)
[4] Ishihara O and Sato N 2001 On the rotation of dust particles in an ion flow in a magnetic field IEEE Trans. Plasma Sci. 29 179
[5] Spitzer L 1978 Physical Processes in the Interstellar Medium (New York: Wiley)
[6] Drane B and Salpeter E 1984 Astrophys. J. 285 89
[7] Rouan D, Lager A, Omont A and Giard M 1992 Astron. Astrophys. 253 498
[8] Anderson N and Watson W 1993 Astron. Astrophys. 270 477
[9] Drain B and Lazarian A 1998 Astrophys. J. 508 157
[10] Tsytovich V and Morfill G 2002 Plasma Phys. Rep. 28 171
[11] Tsytovich V and Morfill G 2002 Physics of collective dust–dust attraction and dust structure formation Proc. ICPDP-2002 Invited talks (Durban, South Africa, 2002)
[12] Vladimirov S V and Tsytovich V N 2003 Spinning of dust spherical grains in dusty plasmas Phys. Plasmas at press