Games in rigged economies

Luis F Seoane

Departamento de Biología de Sistemas, Centro Nacional de Biotecnología (CSIC), C/ Darwin 3, 28049 Madrid, Spain.

Instituto de Física Interdisciplinar y Sistemas Complejos IFISC (CSIC-UIB), Palma de Mallorca, Spain.

Modern economies evolved from simpler human exchanges into very convoluted systems. Today, a multitude of aspects can be regulated, tampered with, or left to chance; these are economic degrees of freedom which together shape the flow of wealth. Economic actors can exploit them, at a cost, and bend that flow in their favor. If intervention becomes widespread, microeconomic strategies of different actors can collide or resonate, building into macroeconomic effects. How viable is a ‘rigged’ economy, and how is this viability affected by growing economic complexity and wealth? Here we capture essential elements of ‘rigged’ economies with a toy model. Nash equilibria of payoff matrices in simple cases show how increased intervention turns economic degrees of freedom from minority into majority games through a dynamical phase. These stages are reproduced by agent-based simulations of our model, which allow us to explore scenarios out of reach for payoff matrices. Increasing economic complexity is then revealed as a mechanism that spontaneously defuses cartels or consensus situations. But excessive complexity enters abruptly into a regime of large fluctuations that threaten the system’s viability. This regime results from non-competitive efforts to intervene the economy coupled across degrees of freedom, becoming unpredictable. Thus non-competitive actions can result in negative spillover due to sheer economic complexity. Simulations suggest that wealth must grow faster than linearly with economic complexity to avoid this regime and keep economies viable in the long run. Our work provides testable conclusions and phenomenological charts to guide policing of ‘rigged’ economic systems.

I. INTRODUCTION

The existence of ‘rigged’ economic scenarios is amply acknowledged. Most notable examples are non-competitive markets [1, 2], legal or illegal, such as cartels, or natural monopolies [3]. In these, all actors usually cooperate to secure similar profits. This entails ‘hand-crafting’ some aspects of the economic games in which they engage. In competitive markets we also find illegal schemes (e.g. inside trading) or innovative, often borderline legal, enterprises to explore unprecedented economic possibilities – e.g. anticipating a broker’s moves with faster internet cables [4]. Such out-of-the-box thinking is part of the economy’s open-ended nature [5, 6]. It redesigns the rules of the game and easily results in a sentiment that “the market is rigged” [4]. Even if all actors stick to the norms and do not innovate, competitive markets are strongly regulated. Some conditions (e.g. demanding a minimum equity to participate) are designed by governments or international institutions. They might change due to democratic consensus or lobbying. If powerful firms bend the rules systematically, regulatory capture happens [7, 8] threatening democracy at large [9, 10]. As transnational markets grow ever more complex and faster, slow public bureaucracies might lag behind and abdicate into nimbler private regulators [11, 12].

Through and through, economies are ‘rigged’. Available games are somewhat manufactured. Once established, they remain open to manipulations that might i) impact costs and rewards of economic games, ii) cap the information available, or iii) limit the number of players allowed to partake. This can be achieved through publicity, bribes, threats, imposing tariffs, etc. More abstractly, we can think of degrees of freedom that can be harnessed in economic systems. Each degree of freedom is a pocket of opportunity that can be exploited, contested or uncontested, at some cost. Envelop theorems assess changes of likely payoffs when a game is altered externally [13, 14]. Wolpert and Grana [17] recently wondered how much an agent should pay if she (and no other actor involved) was given this control before playing a game. The decision boils down to a positive payoff balance versus without intervention. Thus a single agent is offered control, at a cost, over a single economic degree of freedom.

Here we study what happens when multiple actors are allowed, also at a cost, to manipulate several economic degrees of freedom. Different efforts might align or not, yielding uncertain returns. A single agent’s decision to rig one game (as per [17]) might be of limited consequence in isolation. But effects may be amplified, mitigated, or produce emergent phenomena when coupled across games and players. We are interested in how microscopic fates scale up to macroeconomic trends, so we adopt a systemic perspective. More available degrees of freedom result in more complex economies – intervention possibilities grow combinatorially and more external variables become relevant if extra degrees of freedom are left unchecked. How do system-wide dynamics of a rigged economy depend on its complexity? How much can such economies grow? Open-endedly, perhaps? Do they collapse, unable to sustain their participants? How is this affected by the amount of wealth generated and distributed? What is a natural level of intervention depending on these aspects?
Agents can pay to intervene each game’s rule, affecting a rule that randomly determines its winning strategy. n agents who engage in tations into a toy economy. We assume a population of capture essential elements that affect our research ques-

We write complete payoff matrices for some simple scenarios. Their analysis (section III.A) shows that increasing intervention switches isolated degrees of freedom from minority to coordination games. In between, Nash equilibria are mixed strategies, anticipating dynamic struggles. We explore increasing economy size and complexity with simulations based on agents of bounded rationality and Darwinian dynamics to select successful strategies. We argue (section IV) that our results should not depend critically on the agent’s rationality and the Darwinism. We simulate model dynamics for a small, fixed number of degrees of freedom as economy size grows (section III.B). This reveals the same progression: from minority, through dynamic, to coordinating regimes. The later remind us of cartels. Adding degrees of freedom abruptly halts within-game coordination, suggesting an empirical test: increased economic complexity should dissolve cartels spontaneously. We study our toy economy’s viability as its complexity grows large and its size scales appropriately (section III.C). Economies whose size does not grow fast enough with their complexity fall in a large-fluctuations regime that threatens their viability – thus non-competitive actions can have negative spillovers as agents and degrees of freedom become coupled en masse. Our toy model allows us to find limit regimes (e.g. within-game coordination, large-fluctuations, etc.) that emerge from essential elements potentially common to any ‘rigged’ economy. We lay out comprehensive maps of such regimes (section III.D). Their occurrence is tied to a few, yet abstract parameters. In section IV we speculate how we might link them to real-world economies.

II. METHODS

A. Model description

Our toy economy (Figure 1a) consists of a fixed number of games, n; and a population of \( N(t) \in [0, N_{max}] \) agents that changes over time. At each iteration, every agent has to play all games, which admit strategies 0 or 1. The strategies played by agent \( A_i \) are collected in an array: \( a^i \equiv [a^i_k, k = 1, \ldots, n] \). Besides, a second array \( r^i \equiv [r^i_k, k = 1, \ldots, n] \) codifies whether \( A^i \) attempts to rig game \( k \) (\( r^i_k = 1 \)) or not (\( r^i_k = 0 \)). The combination \( (a^i_k, r^i_k) \) constitutes the proper strategy of agent \( A^i \) towards game \( k \). However, to aid the model’s discussion, we use the word ‘strategy’ only to name \( a^i_k \).

At each iteration a rule exists, common to all agents, that determines the winning strategy for each game: \( R(t) \equiv [R_k(t) \in \{0,1\}, k = 1, \ldots, n] \). If any agents at-
tempt to rig game $k$, $R_k(t)$ takes the most common action among those rigging agents (Figure 1b4):

$$R_k(t) = \arg\max_{a \in \{0,1\}} \left( ||\{ A^t, a^t_k = \bar{a}, r^t_k = 1\} || \right).$$

(1)

In case of draw (including no intervention), $R_k(t)$ is set randomly (Figure 1b2-3). Each agent pays an amount $C_R$ for each intervention attempt – successful or not. If $A^t$ has a wealth $w^t(t)$ at the beginning of a round, after setting $R(t)$ this becomes:

$$w^t(t + \Delta t_{rig}) = w^t(t) - C_R \sum_k r^t_k. \quad (2)$$

Each round, an amount $b$ is ruffled at each game – a total wealth $B = nb$ is potentially distributed. The amount allocated to game $k$ is split between all agents who played the winning strategy, $R_k(t)$. After this:

$$w^t(t + \Delta t_{play}) = w^t(t + \Delta t_{rig}) + b \sum_k \delta(a^t_k, R_k(t)) \frac{R_k(t)}{N^w(t)}, \quad (3)$$

where $\delta(\cdot, \cdot)$ is Kronecker’s delta and $N^w(t)$ is the number of winners of game $k$. If $w^t(t + \Delta t_{play}) < 0$, the $i$-th agent is removed, decreasing the population by 1.

If $w^t(t + \Delta t_{play}) > C_C$, $A^t$ has a child and an amount $C_C$ is subtracted from $w^t$. A new agent is generated which inherits $a^t$ and $r^t$. Each of the bits in these arrays flips once with a probability $p_a$. After this, both arrays remain fixed throughout the new agent’s lifetime. We generate an integer number $j \in [1, N^{max}]$ to allocate the new individual. If $j \leq N(t)$, the new agent becomes $A^t$. The former agent in that position is removed, its wealth is lost, and the population size remains unchanged. If $j > N(t)$, the new individual is appended at the end of the pool and the population grows by 1.

B. Measurements on model dynamics

For each simulation we set model parameters ($C_R = 1$, $C_C = 10$, $p_a = 0.1$, and $N^{max} = 1000$; but variations are explored in Appendix C to show the generality of our results). We explore ranges of $n$ and $B$ to address the main questions – i.e. “how do rigged economies behave as their complexity and size change?”

Model simulations start with a single agent $A^t$ with random strategies and no interventions ($r^t_k = 0, \forall k$). In average, $A^t$ accrues half of the distributed wealth until $w^t(t) > C_C$. As new descents fill the population, reinforcing or competing strategies unfold. After a rapid initial growth, population size and wealth reach an attractor (Figure 1b2). We assess these attractors numerically. Simulations run for 5000 iterations. We take averages (noted $\langle \cdot \rangle$) of diverse quantities over the last 500 iterations. For example, population size $\langle N \rangle$, for which we also report normalized fluctuations $\sigma(N)/\langle N \rangle$, where $\sigma(\cdot)$ indicates standard deviation. Unless $B$ is very small, 5000 iterations suffice to observe convergence (Supporting Figures 13, 14, and 15).

To measure the heterogeneity of strategies in the population, we take the fraction $f_k(t)$ of agents with $a_k = 1$:

$$f_k(t) = \frac{\sum_{i=1}^{N(t)} \delta(a^t_k,1)}{N(t)}, \quad (4)$$

from which we compute the entropy:

$$h^t_k(t) = - \left[ f_k(t) \log_2(f_k(t)) + (1 - f_k(t)) \log_2(1 - f_k(t)) \right] \quad (5)$$

and mean entropy across games (Figure 1b):

$$h^t(t) = \frac{1}{n} \sum_{k=1}^{n} h^t_k(t). \quad (6)$$

If $h^t_k(t) = 0$, all agents play the same strategy in game $k$. This quantity is maximal ($h^t_k(t) = 1$) if the population splits in half around that game. If $h^t(t) = 0$, agents play the same strategy in each game, but not necessarily the same one across games. If $h^t(t) = 1$, agents are split in half at each game, but this split is not necessarily consistent across games.

Finally, we introduce the rigging pressure on a game:

$$r_k(t) = \frac{1}{N(t)} \sum_{i=1}^{N(t)} r^t_k, \quad r_k(t) \in [0,1]; \quad (7)$$

as well as the total rigging pressure (Figure 1d):

$$r(t) = \sum_{k=1}^{n} r_k(t), \quad r(t) \in [0,n]; \quad (8)$$

and average rigging pressure per game $r(t)/n \in [0,1]$.

III. RESULTS

A. Intervention turns minority into majority games

Before looking at model dynamics we can gain some insight from payoff matrices in simple cases. Population size affects these matrices: earnings are split among winners; and more agents imply more distinct, possible correlations between strategies and rigging choices. Hence, utility functions rapidly become very complex. In appendix A we discuss payoff matrices for a single game and one player (Supporting Table I) and for one game and three players (Supporting Tables II, III and IV). All matrices show average earnings over time if strategies, rigging choices, and population size are fixed.

Table II presents the payoff matrix for one game with two players. If $C_R > b/2$, rigging the game is prohibitive.
Then, the system has the Nash equilibria marked in gray – both agents try to take opposite actions \( a^1 \neq a^2 \). With no intervention, we deal with a minority game. For larger populations it pays even more to be in the minority (Supporting Table 1). These equilibria disappear if intervention is cheap enough \( (C_R < b/2) \) but depends on population size. Then, it becomes more profitable for one player to rig the game while playing a minority strategy. But it also becomes better, for the other agent, to parasitize the other’s effort – turning the winning strategy into a majority. If the game were played sequentially, a dynamic scenario ensues alternating minority and majority configurations (gray circuits in table 1). As more players are added, the stakes become higher and the situation more complex. Each player wants to be in the majority among rigging agents, but in the minority among non-rigging ones. For \( n > 3 \), if all agents are intervening (red frame, Supporting Table 1), the sub-game’s Nash equilibrium is a full coordination. This is not a global equilibrium, but large coordinations emerge in our simulations for rising intervention levels (see next section).

Note how all agents rigging a game in an agreed-upon way to share profits resembles a cartel.

Payoff matrices are equal for all games. If \( n \) games were played in isolation (i.e. wealth earned by manipulating a game could not be invested into another), we would observe the same transition to within-game coordination for each degree of freedom as intervention takes hold. What happens when we lift such compartmentalization?

### B. Fixed complexity and growing wealth

We now study model dynamics and stability for a fixed number of games and varying economy size. Discussion of the rich behavior uncovered follows in Appendix B.

Figure 2(a) shows \( \langle N \rangle \) for \( n = 2 \) games. Circles over the plots indicate values of \( B \) for which a stretch of the dynamics is plotted in Supporting Figure 1[5]. Generally, \( \langle N \rangle \) increases with the economy size – i.e. as more money becomes available to sustain more agents or to invest into rigging more games. Indeed, the rigging pressure per game (Figure 2b) grows more or less monotonously. \( \langle N \rangle \) is not so parsimonious. For roughly \( B < 750 \) it grows steadily. At \( B \sim 750 \) it jumps swiftly, then remains similar but slightly declining up until \( B \sim 1600 \), when it undergoes another abrupt increase.

These population boosts seem associated to varying coordination. Figure 2(b) shows that the strategy entropy \( \langle h^a \rangle \) drops sharply before the first boost (shaded area). Before that drop, resources are scarce and rigging the economy is difficult. Either strategy is equally likely to win, so agents playing either option are equally abundant (Supporting Figure 1[4]a). As \( B \) grows, more resources become available to rig the games. Either 1 or 0 becomes the winning strategy over longer time stretches, resulting in temporary selective preferences for one strategy over the other, and oscillatory dynamics ensue (Supporting Figure 1[4]b–c). As \( \langle h^a \rangle \) falls definitely, agents coordinate their strategies (Supporting Figure 1[4]d–e). These dynamics shifts happen simultaneously in all games – as if, so far, payoff matrices were essentially independent for each degree of freedom. By \( B \sim 1000 \) we exhausted all within-game regimes uncovered in payoff matrices: from minority games, through a dynamic struggle, to a mostly majority game. The final population boost at \( B \sim 1600 \) must entail emerging correlations across games – e.g. clustering agents that play the minority vs majority strategies in both games.

Supporting Figures 1 and 2 compare \( \langle N \rangle \), \( \langle r \rangle / n \), and \( \langle h^a \rangle \) as economy size grows for different, fixed \( n \). With more games, more discrete jumps in \( \langle N \rangle \) appear. These arise, potentially, from the combinatorially growing coordination possibilities across games. They happen after the oscillatory phases (Supporting Figures 1[3] and 1[5]c–e for \( n = 3 \)). This again suggests that within-game coordination happens first, simultaneously for all games; then degrees of freedom start coupling with each other. Some regimes have similar \( \langle N \rangle \) for different \( n \) (horizontal dashed lines, Supporting Figure 1[4]), suggesting that they are effectively similar. Population boosts succeed each other more rapidly for larger \( n \), approaching a continuous buildup instead of discrete jumps (Supporting Figure 1[3]d). This is not reflected by \( \langle h^a \rangle \), which only drops once due to within-game coordination. The \( \langle h^a \rangle \) plateau is higher for larger \( n \) (Supporting Figure 1[5]d), indicating

| \( a^1 = 0, r^1 = 0 \) | \( a^1 = 1, r^1 = 0 \) | \( a^1 = 0, r^1 = 1 \) | \( a^1 = 1, r^1 = 1 \) |
|---------------------|---------------------|---------------------|---------------------|
| \( a^2 = 0, r^2 = 0 \) | \( b/4 \) | \( b/2 \) | \( b/2 - C_R \) |
| \( a^2 = 1, r^2 = 0 \) | \( b/2 \) | \( b/4 \) | \( 0 \) |
| \( a^2 = 0, r^2 = 1 \) | \( b/2 - C_R \) | \( b/2 \) | \( 0 \) |
| \( a^2 = 1, r^2 = 1 \) | \( b - C_R \) | \( b/2 - C_R \) | \( b/2 - C_R \) |

**TABLE 1 Payoff matrix of one game with two players.** Table entries are labeled by each agent’s strategy \( a^1 \) and \( a^2 \) and rigging choice \( r^1 = 0 \) or \( 1 \). Each cell displays average payoff with no death or reproduction for fixed options. Gray cells are Nash equilibria if \( C_R > b/2 \). Gray circuits indicate possible dynamic situations that emerge for \( C_R < b/2 \).
that across-game correlations weaken or interrupt within-game coordination. Above, we compared such coordination to cartels: games are consensually rigged to favor most actors. Our results suggest that increasing economic complexity prevents the formation of such consensus, defusing cartels, even with rising intervention levels. This is a testable conclusion of our model.

**C. Growing wealth and economic complexity**

We now change the number of games as wealth scales as $B = B(B_0, n)$. The constant $B_0$ is a normalizing factor to facilitate comparisons. We explore four cases:

**I**: A fixed wealth $B_I = B_0$ is split evenly between all games: $b_I = B_0/n$. Returns per game drop as the economy becomes more complex.

**II**: Each game distributes a fixed amount $b_{II} = B_0$, total wealth grows linearly $B_{II} = B_0 \cdot n$. Returns per game remain constant against growing complexity.

**III**: Each degree of freedom revalues previously existing games logarithmically: $b_{III} = B_0 (\log(n) + 1)$. Total wealth grows as $B_{III} = B_0 \cdot n (\log(n) + 1)$.

**IV**: Each degree of freedom revalues previously existing games linearly: $b_{IV} = B_0 \cdot n$. Total wealth grows quadratically: $B_{IV} = B_0 \cdot n^2$.

Figure 3a-b shows $\langle N \rangle$ for each scenario. Extreme cases I (black curves) and IV (green) are relatively uninteresting: Stable population size declines quickly for I. As the economic complexity grows and returns per game drop, more intervention is needed to secure the same earnings. Such rigged economies collapse if they become too complex. For IV, wealth grows so quickly with $n$ that, promptly, population saturates.

Intermediate cases II (blue curves) and III (red) are more interesting. With $B_0 = 50$ (Figure 3a) and $B_0 = 100$ (Figure 3b), $\langle N \rangle$ in declines slowly for II. Thus, in general, a rigged economy’s wealth must grow faster than linearly with its complexity to remain viable. In case III, $\langle N \rangle$ saturates for $B_0 = 100$; but not for $B_0 = 50$, for which population seems stagnant.

For case II, and III with $B_0 = 50$, fluctuations in population size reveal the existence of thresholds, $n^*_N/NN(B_0)$, at which system dynamics change abruptly (Figure 3c-d). This affects $\langle N \rangle$ marginally (arrows in Figure 3a-b), but the increase in $\sigma(N)/\langle N \rangle$ is always salient. For $n < n^*$, fluctuations are small ($< 5\%$). For $n > n^*(B_0)$ large fluctuations ($\sim 25\%$ for case II and $\sim 15\%$ for case III) set in. There is an absorbing state at $N(t) = 0$, thus fluctuations of $15 - 25\%$ system size can compromise its viability.

We explore the transition to large fluctuations by simulating case II below ($n = 40, 60$) and above ($n = 80, 100$) their onset at $n^*$. We ran the model for 5000 iterations and discarded the first 1000. Figure 3a shows the probability of finding the system with population $N$. Below $n^*$ we see a neat Gaussian; above, the distribution presents two balanced modes. Transitions between them contribute to the large fluctuations. We also plot total wealth (Figure 3b) and wealth per agent (Figure 3g). Their averages fall, first, and grow, eventually, as $n$ increases. Below $n^*$ clear Gaussians appear again. Above, we observe broad tails, indicating inequality. Despite risking collapse, the average agent can be wealthier in the large-fluctuations regime.

**D. Charting rigged economies**

We run simulations of case I ($B_I \equiv B_0$, $b_I \equiv B_0/n$) for ranges of economic complexity, $n$, and distributed wealth, $B_0$. This renders maps (Figure 4) where the four scalings above can be read as curved sections. Trivially, case I traces a horizontal line (solid, red; bottom of each map). Case II traces a line with slope $B_0$ (dashed black lines). Cases III (dotted black) and IV (dash-dotted black) trace curves growing faster than linearly. Reading $\langle N \rangle$ (Figure 4c) or $\sigma(N)/\langle N \rangle$ (Figure 4b-e) along such curves renders the plots from figures 3a-b and 3c-d. Results for fixed $n$ and growing $B$ from Figure 2 result from vertical cuts of the map. Other possible progressions $B = B(n)$ can be charted similarly.
The large-fluctuations regime is a salient anomaly (Figure 4b-c) expanding up- and right-wards (perhaps unboundedly) over a broad range of \((n, B)\) values. Its contour constraint dependencies, \(B = B(n)\), that could avoid this regime. Its upper bound seems to grow faster than \(n \cdot \log(n)\) suggesting that case III with \(B_0 = 50\) will not escape large-fluctuations despite sustained growth.

Supporting Figure 6 shows maps for \(\langle h^s \rangle\) and \(\langle r \rangle / n\). The dent of low \((h^s)\) due to within-game coordination in simple yet wealthy setups is notable (Supporting Figure 6b). We argued that such cartel-like cases could be defused by increasing complexity. But this map shows that, if \(n\) grows too much without raising \(B\), \((h^s)\) drops gradually – consensus strategies build up again. It is intuitive that \((r) / n\) grows alongside \(B\) (Supporting Figure 6b), since more available resources can be dedicated to rigging the games. Less intuitively, our map shows \((r) / n\) growing with \(n\) as well, even if returns per game diminish.

We speculate that, for low \(n\), different agents meddling are likely to collide, resulting in uncertain returns. With larger \(n\), different agents can intervene different degrees of freedom, lowering the chance of mutual frustration.

**IV. DISCUSSION**

It is difficult to pinpoint what an ‘unrigged’ economy is. We model economies as containing degrees of freedom that can be controlled at a cost by its actors. Unchecked degrees favor economic agents at random. An economy with more ‘riggable’ facets is more complex. We studied dynamics, stability, and viability of a rigged economy toy model as its complexity and total wealth change.

Simple scenarios allow a study of equilibria in payoff matrices. We find that individual degrees of freedom turn from minority into majority games, through a dynamical
phase, as intervention raises. Agent-based simulations confirm these regimes. They also show new behaviors as synergies develop between degrees of freedom. These new behaviors (difficult to capture with payoff matrices) halt within-game coordination. Within-game coordination in simple yet wealthy markets resembles cartels: most economic actors with decision power bend the rules homogeneously in their favor. Our results suggest that this consensus is spontaneously defused if the system becomes complex enough, which can be empirically tested.

We study our toy economies as their complexity increases and the wealth they distribute remains constant, grows linearly, or faster than linearly with the number of economic degrees of freedom. In general, wealth should grow faster than linearly. Against raising complexity, stagnant or slowly growing wealth only sustains a decreasing ensemble of actors sharing ever more meager resources. An unlucky fluctuation can kill them off. This becomes more pressing as our model predicts that large fluctuations build up abruptly above a complexity threshold. These large fluctuations remind us of chaotic regimes in the El Farol and similar problems [18–21]. In them, agents with sufficient rationality anticipate a market, but their own success turns the market unpredictable. In our model, above a complexity threshold, non-competitive intervention choices become intertwined across games. Birth and death of agents ripple system-wide, making successful strategies hard to track. Even though agents are exploring non-competitive strategies, large fluctuations (~15 – 20% population size) ensue, compromising the system’s viability – thus non-competitive actions can result in negative spillover by sheer market complexity. This is another testable conclusion.

Behavioral economics offers a prominent chance to test our findings. We see stable states with raising rigging pressure as expected returns grow. This is consistent with empirical data on cheating: while different profiles exist (including people hardly corrupted), cheating eventually ensues for large enough rewards [24], especially after removing the concern of being caught [25]. Further experiments reveal that cheating is more likely as a partnership [26]. This resounds with our model’s “cartels” in simple yet wealthy economies. Such simple experiments are perfect to test our predictions for growing complexity: Does coordination fall apart swiftly? Does rigging pressure grow with complexity in the long run? More ambitiously, we could emulate recent implementations of Prisoner dilemmas and other simple games [27–30].

This work did not aim at specific realism, but at capturing elements that we find essential about ‘rigged’ economies, and thus derive qualitative regimes and wealth-complexity scalings that keep our toy economies viable. Exploring lesser model parameters (Appendix C), the same phenomenology features consistently. This encourages us to think that we are unveiling general results of ‘rigged’ economies. But we made important simplifications to keep our model tractable. All agents participate of all games, while real economic actors might walk out or be banned from a specific market. We model all degrees of freedom with a similar game. Real manipulations might treat agents with a same strategy differently. Some pay off only the first intervention, others reward non-linearly a varying investment. Exploring these and other alternatives is easy and might uncover new systemic regimes. Our results constitute solid limit behaviors that should be recovered under appropriate circumstances.

In our model, wealth is generated externally – the economic games merely distribute it. An important variation should create wealth organically, depending on population size, strategies explored, and degrees of freedom available. These, like technological niches, are developed and sustained at a cost. Rigged economies might then correct themselves by losing complexity if necessary. Similar feedbacks can poise complex systems near critical regimes [37–44], which proved relevant to rationalize some phenomenology in economics [21, 23, 45] – at criticality we observe fat tails in wealth distributions or dynamic turnover of complex markets.

An important design choice are the Darwinian dynamics that propagate successful strategies. We could have modeled boundedly rational agents that learn, similarly spreading successful behaviors. A key parameter then would be a learning rate, instead of our replication cost, $C_C$. Similar models show that certain regimes depend tangentially on the cognitive mechanism [18–21]. Different implementations might move around the onset of unpredictable regimes (as $C_C$ does, Supporting Figure 7). When unpredictability is intrinsic to the phenomenology studied, rational agents cannot perform better either. Our results suggest that rigged economies might be intrinsically uncomputable in certain limits.

Our work is designed in economic terms, but it has an obvious political reading – e.g. construction and control of power structures. More pragmatically, in our model wealth redistribution is achieved through low rigging pressures. Empirical measurements of redistribution might help us map real economies into our framework, as has been done for similarly abstract models [22, 23]. At large, the evolutionary stability of fair governance [13] is under scrutiny. In ecosystems that bring together wealth, people, and economic games, all subjected to Darwinism: What lasting structures emerge? Do fair rules survive? Under which circumstances does unfairness prevail?

Acknowledgments

The author wishes to thank Roberto Enríquez (Bob Pop) for his deep insights in socio-economic systems, which prompted this work. The author also wishes to thank Juan Fernández Gracia, Victor Eguiluz, Carlos Melián, and other IFISC (Institute for Interdisciplinary Physics and Complex Systems) members, as well as Víctor Notivol, Paolo Barucca, David Wolpert, Justin Grana, and, very especially, Federico Curci for indispensable feedback about economic systems. This work has
been funded by IFISC as part of the María de Maeztu Program for Units of Excellence in R&D (MDM-2017-0711), and by the Spanish National Research Council (CSIC) and the Spanish Department for Science and Innovation (MICINN) through a Juan de la Cierva Fellowship (IJC2018-036694-I).

References

[1] Tirole J, The theory of industrial organization. MIT press (1988).
[2] Willig RD, Schmalensee R, Handbook of industrial organization. Elsevier (1989).
[3] Laffont JJ, Tirole J, A theory of incentives in procurement and regulation. MIT press (1993).
[4] Lewis M, Flash Boys: A Wall Street Revolt. W. W. Norton Company (2015).
[5] Schumpeter JA, Capitalism, socialism and democracy. Routledge, (1942).
[6] Ferguson N, The ascent of money: A financial history of the world. Penguin (2008).
[7] Stigler G, The economic theory of regulation. RAND J. Econ. 2(1), 3-21 (1971).
[8] Tirole J, Hierarchies and bureaucracies: On the role of collusion in organizations. J. L. Econ. & Org. 2, 181 (1986). ACHTUNG!!
[9] Laffont JJ, Tirole J, The politics of government decision-making: A theory of regulatory capture. Q. J. Econ. 106(4), 1089-1127 (1991).
[10] Acemoglu D, Johnson S, Robinson JA, Institutions as a fundamental cause of long-run growth. In Handbook of economic growth 1. Aghion P, Durlauf S, eds., 385-472 (2005).
[11] Acemoglu D, Naiki S, Restrepo P, Robinson JA, Democracy does cause growth. J. Political Econ. 127(1), 47-100 (2019).
[12] Draghi M, Sovereignty in a globalised world. Speech on the role of European Union in promoting the rule of law, University of Bologna, 22, (2019).
[13] Barruca P, A Fair Governance: On Inequality, Power and Democracy. Topoi, 1-6 (2020).
[14] Milgrom P, Shannon C, Monotone comparative statics. Econometrica, pp.157-180 (1994).
[15] Caputo MR, The envelope theorem and comparative statics of Nash equilibria. Games Econ. Behav. 13(2), pp.201-224.
[16] Acemoglu D, Jensen MK, Aggregate comparative statics. Games Econ. Behav. 81, 27-49 (2013).
[17] Wolpert D, Grana J, How much would you pay to change a game before playing it? Entropy 21(7), p.686 (2019).
[18] Gintis H, Game theory evolving: A problem-centered introduction to modeling strategic behavior. Princeton university press (2000).
[19] Whitehead D, The El Farol bar problem revisited: Reinforcement learning in a potential game. ESE discussion papers, 186 (2008).
[20] Buchanan M, The social atom: Why the rich get richer, cheaters get caught, and your neighbor usually looks like you. Bloomsbury Publishing USA (2008).
[21] Sornette D, Why stock markets crash: critical events in complex financial systems. Princeton University Press (2017).
[22] Devitt-Lee A, Wang H, Li J, Boghosian B. A nonstandard description of wealth concentration in large-scale economies. SIAM J. Appl. Math. 78(2), pp.996-1008 (2018).
[23] Li, J, Boghosian BM, Li C. The Affine Wealth Model: An agent-based model of asset exchange that allows for negative-wealth agents and its empirical validation. Physica A 516, pp.423-442 (2019).
[24] Hillig BE, Thielmann I. Does everyone have a price? On the role of payoff magnitude for ethical decision making. Cognition 163, 15-25 (2017).
[25] Kajackaitė A, Gneezy U. Incentives and cheating. Games Econ. Behav. 102, 433-444 (2017).
[26] Weisberg O, Shalvi S. The collaborative roots of corruption. Proc. Nat. Acad. Sci. 112(34), 10651-10656 (2015).
[27] Cassar A, Coordination and cooperation in local, random and small world networks: Experimental evidence. Games Econ. Behav. 58(2), 209-230 (2007).
[28] Kirchkamp O, Nagel R. Naïve learning and cooperation in network games. Games Econ. Behav. 58(2), 269-292 (2007).
[29] Traulsen A, Semmann D, Sommerfeld RD, Krambeck HJ, Milinski M. Human strategy updating in evolutionary games. Proc. Nat. Acad. Sci. 107(7), 2962-2966 (2010).
[30] Grujić J, Fosco C, Araujo L, Cuesta JA, Sánchez A. Social experiments in the mesoscale: Humans playing a spatial prisoner’s dilemma. PLoS one 5(11), (2010).
[31] Suri S, Watts DJ. Cooperation and contagion in web-based, networked public goods experiments. ACM SIGecom Exchanges 10(2), 3–8 (2011).
[32] Gracia-Lázaro C, Ferrer A, Ruiz G, Tarancón A, Cuesta JA, Sánchez A, Moreno Y. Heterogeneous networks do not promote cooperation when humans play a Prisoner’s Dilemma. Proc. Nat. Acad. Sci. 109(32), 12922-12926 (2012).
[33] Grujić J, Gracia-Lázaro C, Milinski M, Semmann D, Traulsen A, Cuesta JA, Moreno Y, Sánchez A. A comparative analysis of spatial Prisoner’s Dilemma experiments: Conditional cooperation and payoff irrelevance. Sci. Rep. 4, p.4615.
[34] Rand DG, Nowak MA, Fowler JH, Christakis NA. Static network structure can stabilize human cooperation. Proc. Nat. Acad. Sci. 111(48), 17093-17098 (2014).
[35] Mao A, Dworkin L, Suri S, Watts DJ. Resilient cooperators stabilize long-run cooperation in the finitely repeated Prisoner’s Dilemma. Nat. Commun. 8(1), 1-10 (2017).
[36] Sánchez A. Physics of human cooperation: experimental evidence and theoretical models. J. Stat. Mech. 2018(2), 024001 (2018).
[37] Bak P, Tang C, Wiesenfeld K. Self-organized criticality: An explanation of the 1/f noise. Phys. Rev. Let. 59(4), p.381 (1987).
[38] Bak P, Tang C, Wiesenfeld K. Self-organized criticality. Phys. Rev. A 38(1), p.364 (1988).
[39] Bak P, Sneppen K. Punctuated equilibrium and criticality in a simple model of evolution. Phys. Rev. Let. 71(24), p.4083 (1993).
[40] Kauffman SA. The origins of order: Self-organization and selection in evolution. Oxford University Press, USA (1993).
[41] Kauffman S. At home in the universe: The search for the laws of self-organization and complexity. Oxford univer-
Appendix A: Payoff matrices for simple cases

Let us note that the model is actually grounded on game theory by building payoff matrices for simple scenarios.

Take one game \((n = 1)\) and a fixed population of one player \(N(t) = 1\) (i.e. even if the agent accumulates wealth, she does not have descendants, so she never pays \(C_R\); she is not removed either if she accumulates negative wealth). Supporting Table I shows the average payoff that a player earns if she plays the same game repeatedly with fixed behavior (i.e. fixed strategy \(a_1\) and rigging choice \(r_1\)). If there is no intervention \((r_1 = 0)\), the winning rule \(R(t)\) is set randomly at each iteration and the expected payoff per round is \(b/2\). If the agent attempts to rig the game \((r_1 = 1)\), she always succeeds (because there is no opposition) and sets \(R(t) = a_1\). Thus she ensures earning an average \(b\) per round, from which \(C_R\) must be subtracted. The optimal strategy is to intervene if \(C_R < b/2\).

Things become more interesting if we add another agent \((N(t) = 2)\) while, again, playing only one game. This case was discussed in the main text. Let us take a closer look. There are three scenarios worth considering separately, and each corresponds to a \(2 \times 2\) block matrix from Table I:

- **No player attempts any rigging** (upper-left block matrix in Table I). In this case the winning rule is set randomly, so that both players win half of the time. If they play the same strategy, whenever they win (i.e. half of the rounds), they must split the earnings. If they play different strategies, each agent still wins half of the time but they always get to keep all the earnings. In other words, in this case the model reduces to a minority game. If the winning rule behaves randomly because there is no intervention, the preferable strategy is to stay in the minority. This is true also when there are more players (see below), since the only varying factor that reduces a player’s earnings is the number of others with a same strategy, among whom the benefit is split.

- **Only one of the players attempts to rig the game** (either off-diagonal block matrices in Table I). The intervening agent pays \(C_R\) to ensure that the winning strategy \(R(t)\) always matches her own. Assuming that only one agent (e.g. agent 1, thus look at the top-right block matrix in Table I) is given the option to rig the game, doing it becomes always favorable if \(C_R < b/4\), disregarding of what action agent 2 takes. If \(b/4 < C_R < b/2\), then rigging the game is favorable only if agent 2 plays a different strategy. If \(C_R > b/2\), it never becomes favorable to rig the game.

- **Both agents attempt to rig the game** (bottom-right block matrix in Table I). Both agents pay \(C_R\)
| Player's behavior | a = 0, r = 0 | a = 1, r = 0 | a = 0, r = 1 | a = 1, r = 1 |
|------------------|-------------|-------------|-------------|-------------|
| Payoff           | b/2         | b/2         | b - C_R     | b - C_R     |

TABLE I Payoff matrix for one game with one player. The agent’s behavior is coded by two bits. A first one \( a \) indicates the agent’s strategy (0 or 1). A second bit \( r \) indicates whether the agent attempts to rig the game or not.

in this case. But they only intervene the game successfully if both play the same strategy. Note that this has the effect of turning the minority game into a neutral one regarding the agent’s strategies \( a_1^i \): If both players are attempting to intervene the game, they will always receive the same payoff disregarding of whether \( a_1^i = a_1^j \) or not. The received payoff is always less than the best scenario with no intervention. But, if \( C_R < b/4 \), it is better than the scenarios with no intervention and matching strategies.

It becomes cumbersome to write payoff matrices when more players are involved, but it is still feasible for \( N(t) = 3 \). We do so in Supporting Tables I, III, and IV. In them, we group up the behaviors of players 2 and 3, assuming that, whenever one of the three agents plays a different strategy (i.e. not all \( a_1^i \) are the same), it is always player 1 (either \( a_1^1 = 0 \) and \( a_1^{2,3} = 1 \) or \( a_1^1 = 1 \) and \( a_1^{2,3} = 0 \)). We call player 1 the minority player and players 2 and 3 the majority players.

Supporting Table II shows the average payoff matrix when only the minority player is allowed to rig the game. If she is not meddling with the rules (left half of Supporting Table II), we deal again with a minority game. The Nash equilibria of this subgame \( (r_1^i = 0 \forall i \text{ and } a_1^i \neq a_1^{2,3}) \) are Nash equilibria of the whole game if \( C_R > b/4 \). For cheaper cost of rigging, the global Nash equilibria disappear as it becomes favorable to one of the majority agents to intervene (for which we have to look at Supporting Table III). This sets on a dynamic situation similar to the one discussed in the main text. Finally, Supporting Table IV has both majority players attempting (and succeeding, since they are in the majority) to rig the game. Interestingly, if all three agents are trying to manipulate the winning rule (red frame), the model turns into a majority game in which all three players earn \( b/3 - C_R \). If an agent decided to change its strategy \( a_1^i \), this would put her in the minority, in which (according to Supporting Table IV), it would earn 0 per round – thus full coordination is a Nash equilibrium of the subgame in which everybody intervenes. This, however, is not a Nash equilibrium of the complete game: in full coordination, it would pay off to a single agent to stop rigging the game. This suggests that the way that our model reaches large levels of coordination (as discussed in the main text) is a tragedy-of-the-commons scenario.

In a static situation (i.e. population is fixed and agents always choose the same actions and whether to rig each game or not), games are independent of each other. We could take these payoff matrices and compute averages over many games. The situation becomes more difficult when dynamics are included. Because new agents can be born and older ones may die, averages over time should keep into account that agent’s actions feed back on each other. For example, a possible good strategy for agent 1 may be to rig games that favor a third agent (say, agent 3) who, in turn, is rigging games that favor agent 1. In such a way, games can become coupled to each other and result in much more complicated payoff functions.

Appendix B: Supporting plots and discussion for increasing economy size and fixed complexity

Despite its simplicity, the model turned out to have very rich dynamics. Its behavior changes, sometimes drastically, with the economy complexity (as measured by the number of games, \( n \)) or with its size (as measured by wealth distributed at each round, \( B \)). In this appendix we take a closer look to what happens when the number of games is fixed, but the wealth distributed in each game grows. We saw an example of this (with \( n = 2 \)) in the main text, and we saw that increasing the number of resources drives agents to coordinate with each other in different manners around the available strategies and whether to rig them or not.

Let us start with the simplest case now, with \( n = 1 \). We plot average population size in the steady state (Supporting Figure 1a), rigging pressure over the only game (Supporting figure 1b), and the strategy entropy (Supporting Figure 1c) as more resources become available. Circles over these plots show values of \( B \) for which we show 1000 iterations of the dynamics in Supporting Figure 13.

As we saw for \( n = 2 \) in the main text, if there are very few resources, spending them in intervening the economy is not a favored behavior in the steady state. This implies that the winning strategy is randomly 0 or 1, likely changing from one iteration to the next. As a consequence, the steady population does not settle for either strategy. The second row of Supporting Figure 13a shows \( f_k \) the fraction of agents playing 1 over time in game \( k = 1 \). We see that this number moves around 0.5, indicating that roughly half the population is choosing 1 and the other half is choosing 0. This results in a high strategy entropy \( \langle h_k^p \rangle \) in game \( k = 1 \), as shown in the third row of Supporting Figure 13b. The fourth row shows that, indeed, the rigging pressure is negligible for low values of \( B \).

Above some amount of available resources, an effective level of rigging pressure starts to build up periodically (lower row of Supporting Figure 13b-c). Conceive a situ-
TABLE II Payoff matrix of one game with three players – only the minority player can rig. We assume that player 1 is in the minority when there is no consensus. In this table, only player 1 is allowed to rig the game, so she always succeeds. Entries marked in gray are global Nash equilibria when rigging is very expensive $C_R \gg b$.

| Agent 2, 3 | Agent 1 |
|-----------|---------|
| $a^2, a^3 = 0, r^2, r^3 = 0$ | $a^1 = 0, r^1 = 0$ | $a^1 = 1, r^1 = 0$ | $a^1 = 0, r^1 = 1$ | $a^1 = 1, r^1 = 1$ |
| $b/6$ | $b/4$ | $b/2$ | $b/3 - C_R$ | $b - C_R$ |
| $b/4$ | $b/2$ | $b/6$ | 0 | $b/3 - C_R$ |

TABLE III Payoff matrix of one game with three players – only one of the majority players (player 2) rigs. This is the only situation in which the symmetry between the majority players is broken.

| Agent 1 | Agent 1 |
|---------|---------|
| $a^2, a^3 = 0, r^2, r^3 = 0$ | $a^1 = 0, r^1 = 0$ | $a^1 = 1, r^1 = 0$ | $a^1 = 0, r^1 = 1$ | $a^1 = 1, r^1 = 1$ |
| $b/3 - C_R$ | $b/3$ | $b/2 - C_R$ | 0 | $b/3 - C_R$ |
| $b/2 - C_R$ | $b/3 - C_R$ | $b/4$ | $b/2 - C_R$ | $b/3 - C_R$ |

| Agent 3 | Agent 1 |
|---------|---------|
| $a^2, a^3 = 0, r^2, r^3 = 0$ | $a^1 = 0, r^1 = 0$ | $a^1 = 1, r^1 = 0$ | $a^1 = 0, r^1 = 1$ | $a^1 = 1, r^1 = 1$ |
| $b/3$ | $b/2$ | 0 | $b/3 - C_R$ | $b/4$ |
| $b/2$ | 0 | $b/3$ | $b/2 - C_R$ | $b/3 - C_R$ |

...


**TABLE IV** Payoff matrix of one game with three players – both majority players rig the game. Since they are in the majority, they always succeed in their attempt to set the winning rule. If all three players rig the game simultaneously (red frame), the model turns into a majority game – i.e. the best strategy is to play what everybody else is playing. Gray squares indicate Nash equilibria of this sub-game.

| Agent 1 | Agent 2 |
|---------|---------|
| $a^2, a^3 = 0, r^2, r^3 = 1$ | $a^1 = 0, r^1 = 0$ |
| $a^2, a^3 = 1, r^2, r^3 = 1$ | $a^1 = 1, r^1 = 0$ |
| $b/3 - C_R$ | $b/3 - C_R$ |
| $b/2 - C_R$ | $b/2 - C_R$ |
| $b/3 - C_R$ | $b/3 - C_R$ |
| $b/3 - C_R$ | $0$ |

- Show that for both cases the amount of rigging in both games is very similar (bottom row). But $\langle h_a \rangle$ reveals important asymmetries which, furthermore, change as we increase $B$. For the lowest $B$ value shown with cyclic behavior ($B = 500$, Supporting Figures 14a), in average, the population does not converge on persistent homogeneous strategies for neither of the games. But it does not stay divided randomly either (as it happens for $B = 300$, Supporting Figure 14b). In the second example with oscillating behavior ($B = 700$, Supporting Figure 14c), in average, the population has converged regarding the strategy of one of the games. The dynamics move towards converge for the other game as well, but they fail periodically or, if they succeed, then the strategy for the other game (formerly homogeneous) breaks apart. Summing up: while the level of intervention is similar in both games, this symmetry is broken regarding how homogeneous the population is about each strategy.

After the cyclic behavior, Figure 2 and Supporting Figure 3 still show two more regimes separated each by a large boost in stable population size. As noted in the main text, this last regime shift is not accompanied by large changes in action entropy, $\langle h_a \rangle$, of neither game. Hence, we conclude that within-game coordination has been exhausted and that new kinds of correlations, now across games, are taking place. Supporting Figure 14d-g are samples of the dynamics in those two regimes after the cyclic behavior. Panels 14d-e sample the regime between $B \sim 800$ and 1600, and panels 14f-g sample the regime for $B > 1700$. The second row shows that in all four cases the strategies have become homogeneous across the population for both games – still with reservoirs of agents playing a minority strategy. The level of homogeneity remains roughly the same for both regimes – indicated by $\langle h_a \rangle$ as well, which remains low throughout. We appreciate the population boost already discussed in the main text (average population in the top panels of Supporting Figure 14d-e is lower than in Supporting Figure 14f-g). We also appreciate that the fluctuations in population is larger when the population is smaller. The other significant difference between these two last regimes is that the rigging pressure becomes notably higher in the regime with larger $B$. This suggests that the difference between both regimes lies in a more efficient coordination between the rigging efforts across games, allowing the population to extract more wealth in average. For example, we see that both games sustain a minority of agents playing the minority option even if the population has broadly converged about each game’s strategy. But these minority-playing agents might not be the same in both games if the conditions allow it. A transition to a higher across-game coordination might happen if the agents playing the minority in both games become the same.

Finally, for $n = 3$ too, we show average population size in the steady state (Supporting Figure 3a), rigging pressure over each game (Supporting Figure 3b), and ac-
SUP. FIG. 2 Fixed economy complexity, $n = 2$, and growing distributed wealth, $B$. Average population size in the steady state. b Average rigging pressure per game. c Average strategy entropy over the two games. Circles over the plots curves indicate values of $B$ for which we show samples of the dynamics in Supporting Figure 14. Error bars indicate the standard deviation over the last 500 iterations of the corresponding simulation.

SUP. FIG. 3 Fixed economy complexity, $n = 3$, and growing distributed wealth, $B$. Average population size in the steady state. b Average rigging pressure per game. c Average strategy entropy over the three games. Circles over the plots curves indicate values of $B$ for which we show samples of the dynamics in Supporting Figure 15.

As briefly discussed in the main text, adding more games has two different effects: One the one hand more regimes seem to become available (i.e. as $B$ grows). We observe more regime shifts (as identified by boosts in population size) than for $n = 2$, which is compatible with more available games and more possibilities for across-game coordination.

Supporting Figure 15 shows samples of the dynamics for various of these regimes, including the oscillatory regime. We see again that the rigging pressure over all three games is simultaneous, even if the symmetry regarding strategy coordination is broken. In the example shown we see that, in average, the population has converged regarding the strategies of two out of three games. As for $n = 2$, when the population converges also about the third game, one of the former agreements breaks apart. There are also values of $B$ for which there is convergence of, at most, one game in average (not shown). Again, the extra regime shifts (which we identify by abrupt boosts of $\langle N \rangle$) happen after within-game coordination has been exhausted. This is consistent with the idea that more games bring in more possible across-game coordination, which are explored only for large values of $B$.

As briefly discussed in the main text, adding more games has two different effects: One the one hand more regimes seem to become available (as duly noted); and on the other hand, more consecutive regimes seem to be visited within a smaller range of $B$. This means that, as we increase $B$, regimes progress more rapidly into each other (Supporting Figure 4). This effect is exaggerated if even more games are available (Supporting Figure 5). So much so that, instead of regime shifts, we approximate a continuous progression. The increase in rigging pressure per game becomes parsimonious as well (while for a small number of games it presented some discrete boosts associated to regime shifts).

Supporting Figure 4c shows that exhausting the within-game coordination results in a drop of strategy entropy, $\langle h^a \rangle$. We see that this drop becomes less accentuated for larger $n$ (Supporting Figure 5c). This suggests that the onset of interactions across games somehow thwarts within game coordination. In other words, a more complex economy seems to enable populations with more diverse strategies within single games.
Furthermore, we hoped that the most interesting phe-
nomenology would depend on $B$ and $n$. These param-
ters capture respectively the economy’s size (as measured
by distributed wealth) and complexity, which are at
the center of our research questions. Luckily, as we show in
this appendix, the observed phenomenology is not much altered
when toying with the remaining parameters. This
suggests that the regimes and phenomenology discovered
for varying economy size and complexity should be found
over again for a range of model options – which speaks
strongly in favor of the minimalism of our approach.

First we note that the cost of rigging a game $C_R$ sets
a scale with respect to the wealth allotted to each game
$b = B/n$. In our simulations we set $C_R = 1$. If we
would try a different value of $C_R$, we could normalize
$b \equiv b/C_R$, $C_C \equiv C_C/C_R$, and $\tilde{C}_R \equiv C_R/C_R = 1$ and
map the parameter choice back to a case that we have
already studied. Thus actually our model has only 5 free
parameters.

Supporting Figure 7 shows what happens to $\langle N \rangle$ and
$\langle \sigma(N)/\langle N \rangle \rangle$ as $C_C$ changes. Interestingly, the effect in
$\langle N \rangle$ seems negligible for the values explored (Figure Sup-
porting Figure 2a–c). More notably, this parameter has
the effect of displacing the onset of the large-fluctuations
regime (Supporting Figure 2d–f). If $C_C$ is smaller, this
regime ensues for a lower economy complexity, $n$. The pa-
parameter $C_C$ tells us how cheap it is to have descendants.
SUP. FIG. 6 Comprehensive maps of strategy entropy and rigging pressure. a Average strategy entropy, ⟨h^s(t)⟩, shows a dent for small n and large B—a situation that we compared to cartels in the main text. This coordination regime falls apart swiftly as complexity grows a little bit. If we move to very large n without increasing B, coordination starts to build up again—yet very smoothly. b Rigging pressure per game, ⟨r(t)/n⟩. Unsurprisingly, it grows with the amount of wealth distributed. More interestingly, it also grows with the number of degrees of freedom in the system.

When it is cheaper, it is easier to trigger large fluctuations; probably because a large descent explores more behaviors (both in rigging decisions and game strategies) simultaneously, as well as it displaces a bigger proportion of former agents. Both these actions result in major disruptions of the winning rules. Thus, cheaper descent more easily brings up a scenario in which agents are continuously deceiving each other into bankruptcy. If this is correct, other parameters that promote behavior diversity or population renewal among agents should have a similar effect.

Supporting Figure 5 shows, indeed, that the mutation rate p_M prompts this expected outcome too. As for C_C, ⟨N⟩ is mostly unaffected by variations of p_M (not shown), but increasing p_M has the predicted result of advancing the onset of the large-fluctuations regime. In Supporting Figure 5b we show a range of p_M with C_C = 20. This choice of C_C is different from the value taken in simulations in the main text (C_C = 10) to show that, with a large enough p_M, we can advance the onset of large fluctuations to the point where it was with C_C = 10 and p_M = 0.1.

Supporting Figure 5a shows the effect of C_C on rigging pressure and rigging pressure per game. The effect is mostly uninteresting, as it just smoothly or slightly displaces a map similar to the settings commented in the main text (Supporting Figure 5b). The key outcome is that rigging pressure grows both if more resources, B, and more games, n, are available—as discussed in the main text. A similar structure is found for different values of p_M (not shown). The strategy entropy (not shown) is largely unchanged by C_C and p_M—while large enough p_F (i.e. very large noise) has the expected effect of weakening the convergence to a majority strategy for low n and large B (arrows in Supporting Figure 5a).

The model is a bit more sensitive to the parameter N^max that sets an external maximum size to the population. This parameter makes sense as population size might be constrained by actual physical limits—e.g., the amount of people that can occupy a territory. It could also be seen as a manufactured (‘rigged’) limit to the size of the market. This is an important kind of economic manipulation not studied in the model. Even if we were to look at N^max from this perspective, model agents cannot modify it, so we would not be studying such manipulation organically, as we do with other degrees of freedom.

A more technical reason to set a parameter N^max is that it solves parsimoniously the problem of a maximum average lifespan. In the limit N^max → ∞, the first agent (who does not rig any game, hence does not pay C_R) never dies because it never ends up with negative wealth. The same would happen to any descent that does not rig any games. This is unrealistic and undesired. In the current model, agents that last too long are naturally and randomly replaced by newborns thanks to the finite N^max. If we would set an infinite N^max, we would need to introduce other mechanisms to remove unrealistically long-lasting agents—e.g. an average life-time or removing agents with a wealth below a threshold w^- > 0. It would be interesting to try these and other variations in the future—noting that they introduce new parameters nevertheless.

Ideally, we would like to find intrinsic limits to population size that emerge out of the model dynamics alone. To achieve this, we would need to simulate the model in the large N^max limit. The value chosen to report our results (N^max = 1000) is a compromise between a fairly large maximum population size and an ability to run simulations within a reasonable time. We ran one additional simulation for N^max = 5000 and another one for N^max = 10000 (reported next). Each of these took more than ten days in a fairly powerful computer cluster.

Supporting Figure 10 shows ⟨N⟩ for N^max = 100, 1000, 5000, 10000. Some additional structure seems to emerge for intermediate values of n (around n ≈ 20) and B (around B ∈ [1000, 2000]) for large N^max. It might be interesting to look at this, but it is not so relevant for our discussion. We see that the region associated to the large-fluctuations regime for large n, one of the most salient features of the model, is not very much affected by N^max. Average population sizes in the steady state in this regime (Figure 4) as well as its fluctuations (Figure 6b–c) fall well below the chosen N^max = 1000.
SUP. FIG. 7 Comprehensive maps of $\langle N \rangle$ and $\sigma(N)/\langle N \rangle$ for varying replication cost, $C_C$. Maps result from simulating case I (i.e. $B_I = B_0$, $b_I = B_0/n$) for ranges of economic complexity, $n$, and distributed wealth, $B_0$. Black curves represent trajectories $B = B(B_0 = 50, n)$ for cases II, III, and IV (dashed, dotted, and dot-dashed respectively). Horizontal red lines at the bottom of each map represent case I with $B_0 = 50$. a-c shows $\langle N \rangle$ and d-f show $\sigma(N)/\langle N \rangle$. a, d, $C_C = 5$; b, e, $C_C = 10$; c, f, $C_C = 20$.

SUP. FIG. 8 Comprehensive maps of $\sigma(N)/\langle N \rangle$ for varying mutation, $p_\mu$. Maps result from simulating case I (i.e. $B_I = B_0$, $b_I = B_0/n$) for ranges of economic complexity, $n$, and distributed wealth, $B_0$. Black curves represent trajectories $B = B(B_0 = 50, n)$ for cases II, III, and IV (dashed, dotted, and dot-dashed respectively). Horizontal red lines at the bottom of each map represent case I with $B_0 = 50$. a, $p_\mu = 0.05$; b, $p_\mu = 0.10$; c, $p_\mu = 0.15$; d, $p_\mu = 0.20$; e, $p_\mu = 0.25$; f, $p_\mu = 0.30$.

All this suggests that the reported $\langle N \rangle$ are intrinsic limits emerging from the model.

The onset of the large-fluctuations regime, however, is more affected (Supporting Figure 11). Increasing $N_{\text{max}}$ results in a displaced onset of this regime. It happens for notably larger values of $n$ and $B$ for increasing $N_{\text{max}}$. Within the plotted range, however, we still appreciate fluctuations as large as 25% of population size for some $(n, B)$ combinations. Note that for $N_{\text{max}} = 10000$ the maximum population size is around two orders of magnitude bigger than the stable population size ($\sim 100$ for the affected area). This strongly suggests that large-fluctuations are an intrinsic regime of the model, even if its precise location in the map is affected by $N_{\text{max}}$.

Nevertheless, it is affected by $N_{\text{max}}$, which indicates that this phenomenon is enhanced by having a finite, maximum population size. Let us compare the shift of this onset with $N_{\text{max}}$ to the shifts observed when we var-
SUP. FIG. 9 Comprehensive maps of rigging pressure per game for varying replication cost, $C_C$. Maps result from simulating case I (i.e. $B_I = B_0, b_I = B_0/n$) for ranges of economic complexity, $n$, and distributed wealth, $B_0$. Black curves represent trajectories $B = B(B_0 = 50, n)$ for cases II, III, and IV (dashed, dotted, and dot-dashed respectively). Horizontal red lines at the bottom of each map represent case I with $B_0 = 50$. a, $C_C = 5$; b, $C_C = 10$; c, $C_C = 20$.

SUP. FIG. 10 Comprehensive maps of $\langle N \rangle$ for varying maximum population size, $N_{max}$. Maps result from simulating case I (i.e. $B_I = B_0, b_I = B_0/n$) for ranges of economic complexity, $n$, and distributed wealth, $B_0$. Black curves represent trajectories $B = B(B_0 = 50, n)$ for cases II, III, and IV (dashed, dotted, and dot-dashed respectively). Horizontal red lines at the bottom of each map represent case I with $B_0 = 50$. a, $N_{max} = 100$; b, $N_{max} = 1000$; c, $N_{max} = 5000$; d, $N_{max} = 10000$.

ied $C_C$ (Supporting Figure 7) and $p_\mu$ (Supporting Figure 8). About these, we argued that the onset of the regime was advanced by mechanisms that result in more diverse strategies competing closer together or a higher population turnover. Thus, lower $C_C$ (cheaper reproduction) and higher $p_\mu$ (increased mutation) both advanced the onset of the regime because an array of diverse strategies is promptly forced to compete, potentially altering the winning rules in an unpredicted fashion. A larger $N_{max}$ has the effect of diluting this competence because there is less replacement of older agents. Accordingly, higher $N_{max}$ displaces the large-fluctuations regime to larger values of $n$ and $B$. Oppositely, note that the population renewal introduced by smaller $N_{max}$ results in a higher uncertainty about winning strategies. This is consistent, as discussed in the main text, with an onset of large fluctuations associated to a cognitive transition of the population as a whole: at some point, it becomes cognitively impossible to keep track of winning strategies as agents attempt to deceive each other.

Another sensible area of the map is that with large $B$ and small $n$, which showed the transition to within-game coordination that results in most of the population converging to a same strategy for each game. Supporting Figure 10 shows that for such combination of parameters (large $B$ and small $n$), there are enough resources to sustain a very large average population size, often saturating even for large $N_{max}$ values. Supporting
SUP. FIG. 11 Comprehensive maps of $\sigma(N)/\langle N \rangle$ for varying maximum population size, $N^{\text{max}}$. Maps result from simulating case I (i.e. $B_I = B_0$, $b_I = B_0/n$) for ranges of economic complexity, $n$, and distributed wealth, $B_0$. Black curves represent trajectories $B = B(B_0 = 50, n)$ for cases II, III, and IV (dashed, dotted, and dot-dashed respectively). Horizontal red lines at the bottom of each map represent case I with $B_0 = 50$. a, $N^{\text{max}} = 100$; b, $N^{\text{max}} = 1000$; c, $N^{\text{max}} = 5000$; d, $N^{\text{max}} = 10000$.

SUP. FIG. 12 Comprehensive maps of strategy entropy, $\langle h^a \rangle$ for varying maximum population size, $N^{\text{max}}$. Maps result from simulating case I (i.e. $B_I = B_0$, $b_I = B_0/n$) for ranges of economic complexity, $n$, and distributed wealth, $B_0$. Black curves represent trajectories $B = B(B_0 = 50, n)$ for cases II, III, and IV (dashed, dotted, and dot-dashed respectively). Horizontal red lines at the bottom of each map represent case I with $B_0 = 50$. a, $N^{\text{max}} = 100$; b, $N^{\text{max}} = 1000$; c, $N^{\text{max}} = 5000$; d, $N^{\text{max}} = 10000$.

Figure 12 shows that this homogeneous regime remains present as we increase $N^{\text{max}}$. Some isolated cases in $N^{\text{max}} = 10000$ (Supporting Figure 12d) and, more notably, $N^{\text{max}} = 5000$ (Supporting Figure 12c) show up. These cases appear in the plots as outstanding pixels of huge $\langle h^a \rangle$ in their otherwise smoother neighborhood with lower entropy (further supporting that these are oddballs).
All in all, the results summarized in this appendix strongly suggest that the relevant phenomenology of the model is the one reported in the main text, and that this phenomenology is robust against reasonable variations of all parameters. Furthermore, changes on the onset of this phenomenology as we vary these extra parameters are parsimonious and follow logical explanations. All this, once again, is a strong reassurance of the minimalism of the model. This phenomenology likely underlies more complicated models that could study additional effects such as those discussed in the final section of the paper. We would expect to find at least similar regimes and regime shifts to the ones described here, even if the exact numerical values at which they happen are altered.
SUP. FIG. 13 Samples of the model dynamics for fixed economy complexity \((n = 1)\) and different economy sizes. The model dynamics over 1000 iterations are shown for one game and economy sizes \(B = 300\) (a), \(B = 500\) (b), \(B = 700\) (c), and \(B = 1500\) (d). Panels in the top row show the population size over time, second row shows the fraction \(f_1(t)\) of agents playing strategy 1, third row shows the action entropy \(h^a = -f_1 \cdot \log_2(f_1) - (1 - f_1) \cdot \log_2(1 - f_1)\) which is maximal when the agents disagree maximally over their strategies regarding the one game, bottom row shows the rigging pressure over time.
SUP. FIG. 14 Samples of the model dynamics for fixed economy complexity \( (n = 2) \) and different economy sizes. The model dynamics over 1000 iterations are shown for one game and economy sizes \( B = 300 \) (a), \( B = 500 \) (b), \( B = 700 \) (c), \( B = 1000 \) (d), \( B = 1500 \) (e), \( B = 2000 \) (f), and \( B = 2500 \) (g). Panels in the top row show the population size over time, second row shows the fraction \( f_k(t) \) of agents playing strategy 1 in the \( k \)-th game, third row shows the corresponding action entropy \( h^a = -f_k \cdot \log_2(f_k) - (1 - f_k) \cdot \log_2(1 - f_k) \) which is maximal when the agents disagree maximally over their strategies regarding the one game, bottom row shows the rigging pressure over time.
SUP. FIG. 15 Samples of the model dynamics for fixed economy complexity \( (n = 2) \) and different economy sizes. The model dynamics over 1000 iterations are shown for one game and economy sizes \( B = 300 \) (a), \( B = 600 \) (b), \( B = 1200 \) (c), \( B = 2000 \) (d), and \( B = 3000 \) (e). Panels in the top row show the population size over time, second row shows the fraction \( f_k(t) \) of agents playing strategy 1 in the \( k \)-th game, third row shows the corresponding action entropy \( h^a = -f_k \cdot \log_2(f_k) - (1 - f_k) \cdot \log_2(1 - f_k) \) which is maximal when the agents disagree maximally over their strategies regarding the one game, bottom row shows the rigging pressure over time.