NEW OPTICAL INSIGHTS INTO THE MASS DISCREPANCY OF GALAXY CLUSTERS: 
THE CASES OF A1689 AND A2218

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ABSTRACT

The discrepancy between the masses estimated from X-ray and from strong gravitational lensing analyses was recently pointed out for several clusters and in particular for A1689 and A2218. We analyze the internal structures of these clusters by applying a recent development of the method of wavelet analysis, which uses the complete information obtained from optical data, i.e. galaxy positions and redshifts. We find that both clusters show the presence of structures superimposed along the line of sight with different mean redshifts and smaller velocity dispersions than that of the system as a whole, suggesting that the clusters could be cases of the on-going merging of clumps.

We compute the masses of the two clusters by adding the optical virial masses of the single clumps, which are supposed to be virialized. By rescaling our masses to the same radii, we compare our results with mass estimates derived from X-ray and gravitational lensing analyses.

In the case of A2218 we find an acceptable agreement with X-ray and gravitational lensing masses.

On the contrary, in the case of A1689 we find that our mass estimates are smaller than X-ray and gravitational lensing ones at both small and large radii. But the observed X-ray temperature in this cluster, and thus the derived X-ray mass, could be enhanced by collision-heated gas during cluster merging. The high central cluster mass obtained from lensing analysis could be explained only by adding the masses of two background galaxy groups possibly aligned with the cluster.

In any case, at variance with earlier claims, there is no evidence that X-ray mass estimates are underestimated.

Subject headings: galaxies: clusters: individual (A1689, A2218) - cosmology: observations

1 INTRODUCTION

The recent literature points out that present estimates of cluster masses derived from X-ray data analysis are smaller than the respective estimates derived from gravitational lensing analysis (e.g. Wu & Fang 1997, hereafter WF97). This discrepancy is especially evident in the case of strong gravitational lensing (see Fig. 2 of Wu & Fang 1996). Indeed, some recent papers outline the presence of this discrepancy in a few galaxy clusters, which show long arcs and arclets produced by gravitational lensing, and for which the radial density distribution and temperature of the intracluster gas are known (e.g. Wu 1994; Miralda-Escudé & Babul 1995, hereafter MB95).

Apart from the intrinsic interest of knowing the masses of galaxy clusters, understanding the mass discrepancy is also important for cosmological reasons. If cluster masses provided by X-ray analysis are really underestimated, the cluster baryon fraction is reduced and one can solve, at least partially, the recently claimed baryon crisis in clusters of galaxies and thus reconcile observations with the value of $\Omega = 1$ (e.g. White et al. 1993; White & Fabian 1995).

The general reliability of cluster mass estimates derived from X-ray analysis, as checked by numerical simulations, is still a matter of discussion (e.g. Schindler 1996; Evrard, Metzler, & Navarro 1996; see Bartelmann & Steinmetz 1996 regarding a systematic mass underestimation). However, the authors agree that, when a strong deviation from hydrostatic equilibrium occurs, the X-ray techniques can strongly overestimate or underestimate the mass depending on, e.g., the presence of shock waves, subclustering and the phase of the merging event, with a tendency to underestimate the total mass throughout the post-merger phase (Roettiger, Burns, & Loken 1996). This last fact, coupled with the suggestion that the probability for a cluster to form large arcs is significantly higher than the norm if the cluster is substructured (Miralda-Escudé 1993; Bartelmann, Steinmetz, & Weiss 1995), could explain the mass discrepancy.

The reliability of gravitational lensing masses also seems to be supported by the agreement between the mass estimates by means of optical and gravitational lensing methods as found in the pioneering work by WF97, who used a sample of 29 lensing clusters. However, they used published velocity dispersions and an isothermal sphere model in order to estimate optical virial masses, without taking into account the internal structures of clusters. Indeed, estimates of velocity dispersions and optical masses for bimodal or complex clusters strongly depend on whether they are treated as single systems or as sums of different clumps (see Girardi et al. 1997, hereafter G97 and references therein).
Recent improvements in three-dimensional structure analysis and its wide application to nearby clusters (Serna & Gerbal 1996; G97) suggest the need to reanalyze the question of optical masses.

Here we analyze clusters A1689 and A2218, which have a large enough number of galaxies with published redshifts (131 and 54, respectively).

MB95 analyzed three clusters at a redshift of 0.17-0.20 and used long arcs to compute the projected mass within the inner core which is then deprojected and extrapolated to be compared to the X-ray mass. They found that the mass derived from strong lensing is larger by a factor of 2-2.5 than the mass estimated from the X-ray emission of the intracluster gas in clusters A1689 and A2218. The results are more ambiguous for the third cluster A2163. The explanation of the discrepancy could reside, e.g., in non-isothermality, inhomogeneity, and the presence of non-thermal pressure support of the gas, as well as in the highly prolate structure of the cluster potential, i.e. of the lens. According to MB95, none of these explanations seems to account for the entire problem. In particular, for clusters which are reasonably elongated along the line of sight, the mass evaluated by means of strong gravitational lensing can be overestimated by only a factor of 1.6-2.0 (Bartelmann 1995). However, in the case of A2218, recent work by Squires et al. (1996) shows a good agreement between X-ray and weak gravitational mass at 0.4 $h^{-1}$ Mpc$^{-1}$. The optical masses of A1689 and A2218, as estimated by Wu & Fang (1997) at several apertures, are generally larger than both respective X-ray and gravitational lensing estimates.

In §2 we present our structure analysis, on which we base our estimates of optical virial masses of the two clusters (§3). In §4 we compare our optical mass estimates with those derived from X-ray and gravitational lensing analyses. In §5 we outline our discussions, and in §6 we summarize our findings and conclusions.

Throughout the paper we give errors at the 68% confidence level (hereafter c.l.) and we use $H_0 = 100 h$ Mpc$^{-1}$ km s$^{-1}$.

2 ANALYSIS OF CLUSTER STRUCTURES

The redshift data of A1689 galaxies come from Teague, Carter, & Gray (1990) and the magnitude data, only partially available, from Gudehus & Hegyi (1991).
The galaxy data for A2218 come from Le Borgne, Pelló, & Sanahuja (1992); in this case only the central cluster region is sampled. The samples we use are suitable for searching for substructures. In fact, the A1689 data consist in a random sample extracted from a complete magnitude sample. Moreover, the A2218 sample shows no correlation between galaxy magnitude and the distance from cluster center. In this way, although a magnitude-complete sample would be preferable, we avoid having some oversampled cluster regions which produce spurious substructures.

Figure 1 shows the velocity distributions and the galaxy density maps of A1689 and A2218 as obtained according to the one- and two-dimensional adaptive kernel methods (e.g. Pisani 1993; see also Girardi et al. 1996 for a wide application to nearby clusters).

However, our structural analysis is based on the recent procedure of structure analysis by G97 which uses combined galaxy and redshift data. This method is an improvement over that described in Escalera & Mazure (1992), and couples the technique of wavelet analysis with local kinematical estimators, which are computed for each galaxy. The technique has the advantage of assigning, within a certain range of probability, the galaxies to the detected structure, thus enabling us to make kinematical and dynamical analyses of the cluster. We refer to G97 for a more detailed description of the method.

First, we roughly extract the main peak (MP) in the velocity distribution. Subsequently, we perform three-dimensional analysis. The structure detected at the largest scale (with respect to the whole sampled region) is the main system MS, or MS1 and MS2 in the case of a bimodal structure. At smaller scales the method uncovers the presence of internal substructures (S). All the structures hereafter mentioned have a confidence level greater than 99.5%; i.e. there are fewer than 5 chances out of 1000 due to random galaxy association. The statistical significances are derived by comparing the wavelet coefficients obtained in the real field to those produced in a series of N replicas which are obtained by drawing independently the positions $X_i$ and $Y_i$ from the $X$ and $Y$ distributions of the sample studied and then by randomly reassigning the velocities (see e.g. Escalera & Mazure 1992).

A1689 — By using the one-dimensional adaptive kernel method, Girardi et al. (1996) found that cluster A1689 consists of two strongly overlapping peaks in the velocity distribution (see Figure 1). In particular, the method assigns 16 and 41 galaxies to the two peaks, but 15 galaxies assigned to one peak have a high probability of belonging to the other peak. In Fig. 1, which shows only the range of radial velocities between $\sim 50000 - 70000 \ km \ s^{-1}$ of the whole field, also two background (not significant) peaks are shown. Instead, A1689 appears to be regular in the galaxy density map.

By applying the multi-scale analysis of G97, A1689 appears to be a condensed structure including three distinct groups, which fully overlap with one another, but are extremely well separated in terms of velocity: $\Delta V_{1-2} \sim 1200 \ km \ s^{-1}$ and $\Delta V_{2-3} \sim 1500 \ km \ s^{-1}$. The first group (S1) is a coherent structure, compact and regular. On the con-
trary, the second one (S2) is a collection of intermediate galaxies, namely three pairs and a loose object. Actually S2 presents only two of its seven galaxies within the supposed virialization radius and so we do not consider S2 in the mass estimate of A1689 (see § 3). The third one (S3) is the most extended and populated; thus it appears to be the dominant system. Since substructures represent most of the galaxy population, we define this cluster as complex according to the G97 morphological classification.

### Table 1

| Name | Center (1950) | V | R | σ |
|------|--------------|---|---|---|
|      |              | (km/s) | (Mpc) | (km/s) |
| A1689 |              |      |      |      |
| MS=MP | 57 130855.9−010504 | 54477 | 2.26 | 1429\(^{+115}_{−96}\) |
| S1   | 12 130855.0−010442 | 52638 | 0.38 | 321\(^{+57}_{−29}\) |
| S2   | 7 130839.6−010320 | 53827 | 0.62 | 243\(^{+84}_{−28}\) |
| S3   | 20 130856.8−010526 | 55300 | 0.87 | 390\(^{+47}_{−47}\) |
|      |              |      |      |      |
| A2218 |              |      |      |      |
| MP   | 48 163541.0+661854 | 52549 | 0.23 | 1405\(^{+163}_{−129}\) |
| MS1  | 11 163547.4+661845 | 54397 | 0.17 | 660\(^{+129}_{−124}\) |
| MS2  | 25 163543.0+661905 | 52229 | 0.16 | 788\(^{+128}_{−68}\) |
| S    | 5 163541.8+661900 | 51672 | 0.06 | 449\(^{+288}_{−16}\) |

In the lensing model of A1689, MB95 assumed the existence of two clumps, which correspond to a large central concentration around galaxy no. 82 in Gudelhs & Hegyi (1991) and to a smaller concentration of galaxies lying 1′ to the north-east. Also the centers of our structures S1 and S3 are aligned along the north-east direction, but they do not correspond to those of MB95. The center of S1 roughly coincides with galaxy no. 82, though the galaxy itself belongs to S3. The center of S3 lies about 50'' to the south-west of S1.

**A2218** — The one-dimensional adaptive method applied to the cluster A2218 gives a velocity distribution with a single peak, slightly asymmetrical (skewness = −0.44 ± 0.35), while the cluster shows two concentrations in the galaxy density map (see Figure 1).

The redshift information on A2218 allows us to study only the central cluster region, which consists of two main structures (MS1 and MS2) separated by about 2000 km s\(^−1\) and superimposed along the line of sight. A further small substructure (S) is detected in the more populated MS2 structure and contains the cD galaxy. We define this cluster as bimodal according to the G97 morphological classification.

In the lensing model of A2218, MB95 (see also Kneib et al. 1995) assumed as principal clumps those centered on the cD galaxy and on galaxy no. 244 in Le Borgnë et al. (1992), which roughly correspond to the two most significant clumps in our two-dimensional galaxy distribution (Figure 1). Also our structures are aligned along the east-west direction, but the more heavily populated one, MS2, lies eastward and contains both the cD galaxy and galaxy no. 244.

Table 1 presents the results from the multi-scale structure analysis and related kinematical analysis (the structure type is indicated by the symbols described above). For each structure we list the number \(N_{gal}\) of galaxies involved; the coordinates of the galaxy density center computed by using the two-dimensional adaptive kernel method; the mean biweight velocity \(V\); the projected radius \(R\); and the robust velocity dispersion \(σ\) with the respective bootstrap errors computed by using the ROS-TAT routines by Beers, Flynn, & Gebhardt (1990). The structures we found in clusters are highlighted in Figures 1 and 2.

The centers of the two clumps are only 0.1 and 0.06 h\(^−1\) Mpc away, in projection, for A1689 and A2218, respectively, so that, considering the possible errors on the center estimate, the clumps can be aligned along the line of sight. Therefore A1689 and A2218 clusters could be cases of head-on clumps merging.

It is not clear whether the redshift separation between the two clumps comes from a difference in distance or rather it is due to a relative motion by analyzing the magnitudes of galaxies. The magnitudes of galaxies belonging to the two clumps in A1689, S1 and S3, and A2218, MS1 and MS2, are not different both according to the Kolmogorov-Smirnov test (e.g. Ledermann 1982, differing at 14% and 20% c.l., respectively) and the Median-test (Siegel 1956, differing at 60% c.l.). But, if the mean velocity difference between clumps were simply due to Hubble flow, the expected difference in magnitude would be so small (∼ 0.1 mag) to be comparable with errors associated to the mean values of magnitudes of substructure members.

### 3 Optical Mass Estimates

When the cluster appears far from dynamical equilibrium, showing two or more clumps, it is more appropriate to compute its mass by adding the masses of single clumps supposed to be virialized (e.g. Mohr, Geller, & Wegner 1996). In particular, in the case of the head-on clumps merging, the velocity dispersion and the cluster mass are strongly overestimated (e.g. G97; see also Pinkney et al. 1996 and Roettgering et al. 1996 for simulated clusters).

The **standard** virial theorem, applied to galaxies within the virialization radius \(R_{vir}\), estimates the cluster mass as \(M = 3π/2 · σ^2 R_{vir}/G\), where \(R_{vir}\) is the projected virial radius which depends on the projected distance \(r_{ij}\) between any pair of galaxies, i.e. \(R_{vir} = N^2 / \sum_{ij} r_{ij}^{-1}\) where \(N\) is the number of galaxies (Limber & Mathews 1960; here we use the luminosity-unweighted expression). The virial mass estimate does not require any assumptions about the isotropy of galaxy orbits (at least in the context of spherical systems) but is strictly valid only if the mass distribution follows galaxy distribution (e.g. Merritt 1988). Virialization radii were computed similarly as in Bird (1995), by assuming \(R_{vir} ∝ σ\) and by scaling to Coma values (see § 5.4 of G97).
In the case of A2218, the sampled region is much smaller than $R_{vir}$. Thus, we have to assume the $\sigma$ computed in the central region as the value of global velocity dispersion and to apply a more complex procedure in the estimate of $RV$. We assume a hydrostatic-isothermal surface density profile for the galaxy distribution $\Sigma(r) = \Sigma_0[1 + (r/R_c)^2]^{-\alpha}$, where $\Sigma_0$ is the central projected galaxy density and $R_c$ is the core radius; this distribution corresponds to a volume-density $\rho(r) \propto r^{-2\alpha - 1}$ for $r \gg R_c$. We adopted a value of $\alpha = 0.8$, which gives a better fit to observational data than the traditional King model (Girardi et al. 1995; see also Bahcall & Lubin 1994). Then, adopting $R_c = 0.17 \, h^{-1} \, Mpc^1$, i.e. the mean value obtained for a sample of 90 nearby clusters, we computed $RV$ at $R_{vir}$ by means of the $RV - R_c$ relation proposed by Girardi et al. (1995, eq. A8).

Note that this procedure, applied to A1689, gives a value of $1.27 \, 10^{14} \, h^{-1} \, M_\odot$, which agrees, within the errors, with the value of $1.45 \, 10^{14} \, h^{-1} \, M_\odot$ obtained by directly applying the virial theorem.

### Table 3

| Name | $R_{vir}$ | $RV$ | $M_{VT}$ | $M$ | $M$ range |
|------|----------|------|----------|-----|----------|
|      | (Mpc)    | (Mpc)| (10^{14} M_\odot) | (10^{15} M_\odot) |
| S1   | 0.40     | 0.37 | 0.59±0.17 | 0.46±0.16 | 0.23-0.51 |
| S3   | 0.49     | 0.45 | 0.86±0.21 | 0.81±0.21 | 0.40-0.91 |
| A1689| 1.45     | 0.38 | 1.27±0.20 | 0.63-1.42 |

In order to consider the possibility that the galaxy distribution of A2218 is characterized by a different core radius, we compute masses by considering two boundary values for $R_c$: $0.01 \, h^{-1} \, Mpc^{-1}$ and $0.5 \, h^{-1} \, Mpc^{-1}$. Moreover, we compute the same values also for A1689 to take into account the possibility that the dark matter distribution differs from the galaxy distribution. In particular, small radii are suggested to be characteristic of dark matter distributions (Durret al. 1994).

In Tables 2 and 3, for each clump of A1689 and A2218, we list $R_{vir}$, $RV$, the virial mass $M$, as obtained using galaxy distribution, with the errors induced by $\sigma$, and the mass range corresponding to the above-mentioned range in $R_c$ ($0.01$-$0.50 \, h^{-1} \, Mpc^{-1}$). The mass values are given for each system as a whole, too. For A1689 we also listed the virial mass, $M_{VT}$, as obtained by directly applying the virial theorem, with the respective jackknife error.

### Table 4

| Name | $R_{vp}$ | $M_{opt}$ | $M_X$ | $M_{gl}$ | $M$ range |
|------|----------|----------|-------|----------|----------|
|      | (Mpc)    | (10^{14} M_\odot) | (10^{15} M_\odot) | (10^{15} M_\odot) |
| A1689| 0.095    | 0.14(0.06-0.26) | 1.14±0.75 | 1.8 | 1.2 |
| A1689| 0.74     | 2.22(0.82-3.84) | 7.75 | - | 3 |
| A1689| 1.5      | 3.43(1.12-6.54) | 24.6±2.22 | 27 | 1.2 |
| A2218| 0.0425   | 0.12(0.04-0.72) | 0.16±0.08 | 0.305 | 1.4 |
| A2218| 0.4      | 4.40(3.04-4.44) | 2.6±1.16 | >3.9+0.7 | 5 |

REFERENCES. 1 Wu & Fang 1997; 2 Tyson & Fischer 1995; 3 White & Fabian 1995; 4 Kneib et al. 1995; 5 Squires et al. 1996.

NOTES: All the masses, except for those in the second line, refer to projected values.

In Table 4 we compare our optical mass estimates, $M_{opt}$, with the X-ray and gravitational lensing masses, $M_X$ and $M_{gl}$, as presented in the literature within the respective projected radius $R_{vp}$. The values of $M_{gl}$ refer to strong gravitational lensing when they are given at small $R_{vp}$; otherwise they refer to weak gravitational lensing. As for our masses, we report also the mass range obtained varying $R_c$ within the two boundary limits (0.01-0.50 $h^{-1} \, Mpc^{-1}$); moreover, all these mass values are affected by the errors due to velocity dispersion, $\sim 25\%$ and $\sim 40\%$ for A1689 and A2218, respectively (see Tables 2 and 3).

In the case of A2218, our optical mass estimates agree, within the errors, with those derived from X-ray and gravitational lensing analyses, and are significantly smaller in the case of A1689.

### 5 DISCUSSION

As discussed in this section, the most reliable interpretation of the results of our structure analysis is that A1689 and A2218 are cases of on-going merging along the line of sight.

MB95 identified two clumps in the two-dimensional galaxy distribution of clusters A1689 and A2218. However, they concluded that these clumps are too dissimilar in size and not well enough aligned to produce a strong
mass overestimate by means of gravitational lensing analysis. The structures we detect do not exactly correspond to the structures seen in the two-dimensional maps, since they are comparable in size and superimposed along the line of sight, but they are rather different in redshift. The presence of on-going merging in cluster A2218 is also supported by the X-ray map and CCD images in the study of Kneib et al. (1995, see also Squires et al. 1996) and by its X-ray elongated shape (Wang & Ulmer 1997).

The immediate consequence of our structure analysis is that our optical mass estimates are lower than the WF97 ones, which are based on very high velocity dispersion, i.e. 1989 km s$^{-1}$ for A1689 (Teague, Carter, & Gray 1990) and 1370 km s$^{-1}$ for A2218 (Le Borgne, Pelló, & Sanahuja 1992).

In the case of A1689, our optical mass estimates are significantly smaller than those derived from X-ray and gravitational analyses.

The high value of X-ray temperature, 10.9 KeV, measured in A1689 (White & Fabian 1995), which corresponds to the very high velocity dispersion of 1350 km s$^{-1}$, could be due to the collision-heated gas, as expected in numerical simulations (e.g. Schindler & Mueller 1993; Burns et al. 1995). There are some analogies with the case of A754, a cluster with a X-ray temperature of 9 keV, whose two clumps have a velocity dispersion of about 400-500 km s$^{-1}$ and are colliding in the plane of the sky (Zabludoff & Zaritsky 1995, G97). But, in our cases, the merging is seen along the line of sight rather than in the plane of the sky. This fact leads to an apparent enhancement of the observed velocity dispersion and can mask the typical features of merging, which usually appear in two-dimensional galaxy distribution and X-ray images (e.g. Schindler & Mueller 1993). The high X-ray mass estimate could be a direct consequence of the overestimate of X-ray temperature. Note that Cirimele, Nesci, & Trevese (1997), who used the lower X-ray temperature of 10.1 KeV (David et al. 1993) found a value for the X-ray mass of 5.9 h$^{-1} M_{\odot}$ within 0.75 h$^{-1}$ Mpc$^{-1}$, which is smaller than the estimate of White & Fabian (1995) although still larger than our optical estimate.

Any structure along the line of sight is particularly relevant for masses derived from arcs, since the superposition of a modest mass group can increase the surface mass density up to the critical value. Since possible contribution could come from all the structures between the source and A1689, we also considered the whole cluster field. A1689 appears well aligned along the line of sight with other structures. Three foreground groups (with $\sigma \sim 500$ km s$^{-1}$) are obvious and were already pointed out (Teague et al. 1990, fig. 4). This fact suggests the presence of a large structure filament well aligned along the line of sight. Indeed, our analysis of velocity distribution (see § 2 and Figures 1 and 2) detects, at a low significance level, two other background structures roughly at the limit of the redshift sample. When assuming the reliability of these two structures (with $\sigma \sim 750$ km s$^{-1}$), we can obtain a rough estimate of their mass. Their contribution to the projected mass within the lens-arc radius, i.e. $0.6 \times 10^{14} h^{-1}$ M$_{\odot}$ or $1.5 \times 10^{14} h^{-1}$ M$_{\odot}$ in the extreme case, can allow us to explain the value of the lens-arc estimate. In this framework, also the sparse structure S2 can contribute a little, even if its contribution to the cluster mass is negligible. Only a larger and deeper redshift sample could allow us to draw firm conclusions, especially with regard to the presence of a large-scale structure filament.

In the case of A2218, Squires et al. (1996), whose X-ray mass agrees with ours within errors, estimated an X-ray temperature of 3-5 keV, which corresponds to $\sigma \sim 700-900$ km s$^{-1}$. These authors pointed out that their estimate of temperature is smaller than that of ASCA and GINGA, and suggested the presence of gas components with different temperatures. If this is the case, the higher temperature could describe the gas heated by the collision between the merging clumps, and the colder temperature would refer to the gas already roughly in dynamical equilibrium. In this scenario, the observed galaxy clumps, which are more stable than the gas and can also survive the first encounter of the clumps (McGlynn & Fabian 1984), still preserve the traces of the parent clumps.

As for the mass computed within the lens-arc, the large errors do not allow us to draw firm conclusions about the discrepancy between lens and X-ray masses, although we suspect that, for the aforesaid reasons, the X-ray mass may not be reliable in the very central region. Note that the mass based on weak gravitational lensing at a larger radius (Squires et al. 1996) is in good agreement with our mass estimate.

Also for the third cluster reported by MB95, A2163, the data analysis shows evidence of recent merging (Elbaz, Arnaud, & Böhringer 1995). Another case of mass discrepancy, the A370 cluster (Wu 1994), shows a bimodal structure (Kneib et al. 1993) and CL 0500-24 shows the presence of two peaks in the velocity distribution (Infante et al. 1994). The presence of so many likely merging clusters at moderate redshift is remarkable when compared with the percentage of nearby clusters which show two or more peaks in the velocity distributions (about 10%, Fadda et al. 1996) or the presence of strong substructures (about 14%, G97). Indeed, there is evidence that the probability for a cluster to form long arcs is significantly larger if the cluster is substructured (Bartelmann, Steinmetz, & Weiss 1995; Bartelmann & Steinmetz 1996), in particular if the cluster is elongated along the line of sight (Miralda-Escudé 1993). Note, however, that a good agreement of the mass determinations from the strong gravitational lensing methods is found for the lowest redshift cluster (z≈ 0.10) PKS 0745-191, which appears to be a regular and relaxed cluster (Allen, Fabian, & Kneib 1996). Indeed, only better statistics would make it possible to know whether clusters which show long arcs are very atypical or whether distant clusters are more substructured than nearby ones. This question is of considerable importance for cosmology, since strong evidence of a substructure-distance correlation could significantly constrain theories of large-scale structure formation.
6 SUMMARY AND CONCLUSIONS

We summarize here our principal results:

i) By applying a recent development of the method of wavelet analysis, which uses the complete information obtained from optical data, i.e. galaxy positions and redshifts, we find that both clusters A1689 and A2218 show the presence of structures superimposed along the line of sight, suggesting that these clusters are cases of head-on merging.

ii) Our optical virial masses, estimated by adding the masses of single clumps, are lower than the estimates of WF97, who did not take into account the internal structure of clusters.

iii) When comparing our optical masses with those derived from X-ray and gravitational lensing analyses, we find a reasonable agreement in the case of A2218, but we estimate a lower optical mass in the case of A1689.

iv) We suggest that the high X-ray mass of A1689 is due to the overestimated X-ray temperature, which could be due to gas collision phenomena, and we can explain the high projected gravitational lensing mass only by adding the masses of two background galaxy systems.

We stress that optical studies of clusters should be a default complement of X-ray and gravitational lensing analyses. However, only optical analyses which consider the internal cluster structure can allow one to draw reliable conclusions. In particular, high values of velocity dispersion for some clusters are probably due to the presence of substantial subclustering along the line of sight. In these cases, the use of galaxy redshifts makes optical analysis a very suitable method, while the standard X-ray and gravitational lensing analyses suffer from strong deviation of gas hydrostatic equilibrium and from the very atypical lens geometry, respectively.

For the clusters we analyze there is no evidence that present cluster X-ray masses are underestimated; on the contrary, the mass of A1689 appears overestimated. If we generalize our results, we are still far from solving the so-called baryon crisis in clusters of galaxies by increasing the estimates of cluster masses.

Moreover, there is increasing evidence that well-analyzed clusters which have long gravitational arcs show the presence of a strong substructure. This fact suggests that they are not the best targets for making estimates of mass and baryon fraction, but rather it opens new questions. One needs to compare distant clusters selected on the basis of lens-arc presence to a sample selected only on X-ray and/or optical basis (e.g., the CNOC redshift survey of distant clusters by Carlberg et al. 1994) in order to understand whether they are unbiased examples of distant clusters and to draw conclusions about cluster evolution.

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