Charged scalar quasi-normal modes for linearly charged dilaton-Lifshitz solutions

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Most available studies of quasi-normal modes for Lifshitz black solutions are limited to the neutral scalar perturbations. In this letter, we investigate the wave dynamics of massive charged scalar perturbation in the background of (3 + 1)-dimensional charged dilaton Lifshitz black branes/holes. We disclose the dependence of the quasi-normal modes on the model parameters, such as the Lifshitz exponent \( z \), the mass and charge of the scalar perturbation field and the charge of the Lifshitz configuration. In contrast with neutral perturbations, we observe the possibility to destroy the original Lifshitz background near the extreme value of charge where the temperature is low. We find out that when the Lifshitz exponent deviates more from unity, it is more difficult to break the stability of the configuration. We also study the behavior of the real part of the quasi-normal frequencies. Unlike the neutral scalar perturbation around uncharged black branes where an overdamping was observed to start at \( z = 2 \) and independent of the value of scalar mass, our observation discloses that the overdamping starting point is no longer at \( z = 2 \) and depends on the mass of scalar field for charged Lifshitz black branes. For charged scalar perturbations, fixing \( m_s \), the asymptotic value of \( \omega_R \) for high \( z \) is more away from zero when the charge of scalar perturbation \( q_s \) increases. There does not appear the overdamping.

I. INTRODUCTION

In black hole physics, quasi normal mode (QNM) is a powerful tool to study the evolution of perturbations in the exterior of black holes [1–4]. The behavior of QNM can be used to identify the black hole existence and disclose dynamical stability of black hole configurations. Besides, QNM can serve as a testing ground of fundamental physics. It is widely believed that QNM can give deeper understandings of the AdS/CFT [4–8], dS/CFT [9] correspondences, loop quantum gravity [10] and also phase transitions of black holes [11] etc.

In this letter we will examine the QNM of the linearly charged dilaton-Lifshitz black brane solutions and try to disclose deep influences of the model parameters on the perturbation wave dynamics and examine the stability of the background configurations. Asymptotic Lifshitz black solutions are interesting duals to many condensed matter systems [12]. They are duals to systems with Schrödinger-like scaling symmetries i.e. \( t \to \lambda^z t \), \( \vec{x} \to \lambda \vec{x} \), where \( z \) is dynamical critical exponent. Lifshitz spacetime is not a vacuum solution of Einstein gravity with or without cosmological constant and some Lifshitz supporting fields are needed. Different Lifshitz supporting fields have been considered in literatures, such as including higher curvature corrections [13–15], inserting massive [16] and massless [17–22] Abelian gauge fields coupled to dilatonic and non-Abelian SU(2) Yang-Mills fields coupled to dilaton [23] etc.

The behavior of neutral scalar perturbations has been extensively studied for Lifshitz solutions in the presence of different Lifshitz supporting fields. In [24, 25], scalar and spinorial perturbations around \((2 + 1)\)-dimensional Lifshitz black holes with \( z = 3 \) in the context of New Massive Gravity (which includes higher curvature terms) have been explored and it has been shown that black holes are stable under both of these perturbations. Moreover, it has been shown that higher-dimensional Lifshitz black branes are stable under massive scalar perturbations in the presence of higher curvature corrections [26, 27]. In [28], \((3 + 1)\)-dimensional Lifshitz black holes with \( z = 0 \) and in the presence of higher curvature corrections have been considered. In the case of massive scalar perturbations minimally coupled to curvature, these black holes are unstable, whereas such black holes with massless perturbations conformally coupled to curvature are stable. In the presence of a massive gauge field including Proca term, scalar massive QNMs for uncharged

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Lifshitz black branes with $z = 2$ have been studied in [29]. It was shown that the QNMs are purely damping (the real part of QNM vanishes) and Lifshitz black branes are always stable. It was further shown that quasi-normal frequencies of scalar massive perturbation around topological Lifshitz black holes with $z = 2$ [30] are always purely imaginary and negative, supporting that these black holes are always stable [31]. In [32], scalar QNMs of 2- and 3-dimensional uncharged Lifshitz black branes with $z = 3$ in the context of New Massive Gravity and 4-dimensional uncharged Lifshitz black branes with $z = 2$ in the presence of massive and massless gauge fields coupled to dilaton have been studied. It has been shown that quasi-normal frequencies are purely negative imaginary, reflecting that these solutions are globally stable. Considering dilaton field and massless gauge fields coupled to dilaton as Lifshitz have been studied. It has been shown that quasi-normal frequencies are purely negative imaginary, reflecting that these solutions are globally stable. Considering dilaton field and massless gauge fields coupled to dilaton as Lifshitz have been studied. It has been shown that quasi-normal frequencies are purely negative imaginary, reflecting that these solutions are globally stable. Considering dilaton field and massless gauge fields coupled to dilaton as Lifshitz have been studied. It has been shown that quasi-normal frequencies are purely negative imaginary, reflecting that these solutions are globally stable. Considering dilaton field and massless gauge fields coupled to dilaton as Lifshitz have been studied. It has been shown that quasi-normal frequencies are purely negative imaginary, reflecting that these solutions are globally stable.

It was further found that QNMs for black branes can become overdamping when the dynamical critical exponent $z$ and angular momentum take some special values. In [37], retarded Green functions of the current and momentum operator of a Lifshitz field theory have been investigated and it was shown that there exists a massive QNMs with an effective mass linearly proportional to temperature in non-vanishing momentum case.

Most available studies of QNMs for Lifshitz black solutions are limited to the neutral scalar perturbation. Some extensions to the Fermionic and electromagnetic perturbations for Lifshitz solutions have been reported recently [38, 39]. It is of great interest to generalize the discussion to the charged scalar perturbation in the background of Lifshitz solutions. The charged scalar perturbation has been studied extensively in other contexts [40–45]. There has been examples showing that charged scalar perturbation can result in the spacetime instabilities in the Reissner-Nordström-de Sitter configuration and the anti-de Sitter charged black holes in [41, 42] and [43–45, respectively. In the AdS space, the charged scalar field can condensate onto the AdS black hole to form a new hairy black hole. In the present letter, we will examine the dynamics of massive charged scalar perturbation in the charged Lifshitz black solution backgrounds. We will discuss the dependence of the QNMs on the model parameters, such as the mass and charge of the scalar perturbation field, the charge of the background spacetime etc. Especially we will examine in the Lifshitz configuration, whether the instability of the background configuration can appear and how the instability will depend on model parameters including the Lifshitz exponent $z$. For simplicity in our discussion, we will focus on the $(3+1)$-dimensional Lifshitz spacetime and leave the study on the higher dimensional influence on QNMs for our future work.

The letter is organized as follows. In the next section, we will review solutions of the four-dimensional linearly charged Lifshitz configurations. Then we will write out the wave equations for the charged scalar perturbation and explain the method of the numerical computation we are going to employ. In Section IV, we will report results of QNMs. In the last section we will conclude our results.

II. REVIEW ON 4-DIMENSIONAL LINEARLY CHARGED LIFSHITZ SOLUTIONS

In this section we will review the 4-dimensional charged dilaton-Lifshitz solutions. The line element of 4-dimensional Lifshitz black solutions can be written as [17]

$$\begin{align*}
ds^2 &= -\frac{r^{2z}}{f^{2z}} f(r) dt^2 + \frac{l^2}{r^2} \frac{dr^2}{f(r)} + r^2 d\Omega_k^2,
\end{align*}$$

(1)

where $z$ is dynamical critical exponent and

$$d\Omega_k^2 = \begin{cases} 
d\theta^2 + \sin^2(\theta) d\phi^2 & k = 1 \\
\frac{d\theta^2 + d\phi^2}{k = 0} \\
\frac{d\theta^2 + \sinh^2(\theta) d\phi^2}{k = -1}
\end{cases}$$

(2)

represents a 2-dimensional hypersurface with constant curvature $2k$. $f(r)$ in the line element (1) has a solution in the context of Einstein-dilaton gravity in the presence of linear Maxwell electrodynamics and two Lifshitz supporting gauge fields

$$S = -\frac{1}{16\pi} \int_M d^4x \sqrt{-g} \left[ R - 2(\nabla \Phi)^2 - 2\Lambda + e^{-2\Lambda_1 \Phi} F - \sum_{i=2}^{3} e^{-2\Lambda_i \Phi} H_i \right],$$

(3)
in which $\mathcal{R}$ is the Ricci scalar on manifold $\mathcal{M}$, $\Phi$ is the dilaton field and $F$ and $H_i$’s are the Maxwell invariants of electromagnetic fields $F_{\mu\nu} = \partial_\mu A_\nu$ and $(H_i)_{\mu\nu} = \partial_\mu (B_i)_{\nu}$, where $A_\mu$ and $(B_i)_{\nu}$’s are the electromagnetic potentials. $\Lambda$, $\lambda_1$ and $\lambda_i$’s are constants. The solution for $f(r)$ in Einstein-dilaton gravity governed by the action (3), is \cite{17, 18}

$$f(r) = 1 + \frac{kl^2}{z^{2z+2}} - \frac{m}{r^{\frac{2z+2}{2z+1}}} + \frac{q^2l^{2z+2}(z-1)}{z^2l^{2z+1}},$$

(4)

where $m$ and $q$ are two constants which are, respectively, related to total mass and total charge of the black hole, the dilaton field is

$$\Phi(r) = \sqrt{z-1} \ln \left( \frac{r}{b} \right),$$

(5)

in which $b$ is a constant and the gauge potentials are

$$A_t = \frac{qbl^{2(z-1)}}{z} \left( \frac{1}{r^2} - \frac{1}{r^2} \right),$$

$$(B_2)_t = \frac{qr^{z+2}}{(z+2)b^2}, \quad (B_3)_t = \frac{rql^2}{z^2b^2}.$$

(6)

where $r_+$ is the largest root of metric function $f(r)$, called event horizon. The constants of the model have been fixed as

$$\lambda_1 = -\sqrt{z-1}, \quad \lambda_2 = \frac{2}{\sqrt{z-1}}, \quad \lambda_3 = \frac{1}{\sqrt{z-1}},$$

$$q_2^2 = \frac{(z-1)(z+2)}{2b^{2z+2}}, \quad q_3^2 = \frac{kb^2(z-1)}{2(z-1)^2},$$

$$\Lambda = -\frac{(z+1)(z+2)}{2l^2}.$$  

(7)

It is clear from (4) that $f(r)$ tends to 1 at spatial infinity and therefore the metric (1) is asymptotically Lifshitz. Looking at $q_2^2$ in (7), we find that the $k = -1$ case where the constant curvature at horizon hypersurface is negative, causes an imaginary charge except for the AdS case with $z = 1$ \cite{17, 18}. Since we will discuss the Lifshitz solutions with $z > 1$, we will consider the cases $k = 0$ (black brane) and $k = 1$ (black hole) in following studies.

The Hawking temperature can be obtained as \cite{17, 18}

$$T = \frac{r_+^{z+1} f'(r_+)}{4\pi l^{z+1}} = \frac{1}{4\pi} \left( \frac{(z+2)m}{l^{z+1}r_+^{z+2}} - \frac{2r_+^{-2k}}{z^2l^{z-1}} - \frac{2q^2(z+1)}{zl^{2z+1}b^2(z-1)} \right),$$

$$= \frac{1}{4\pi} \left( \frac{kr_+^{z-2}}{zl^{z-1}} + \frac{(z+2)r_+^2}{l^{z+1}} - \frac{q_2^2l^{2z-1}b^2(z-1)}{r_+^{z+2}} \right),$$

(8)

where $m$ has been inserted by using the fact that $f(r_+) = 0$. As the charge of black hole approaches the extreme value

$$q_2^{ext} = \frac{kr_+^{2z}}{zl^{2z-2}b^2(z-1)} + \frac{(z+2)r_+^{z+2}}{l^{2z}b^2(z-1)},$$

(9)

the Hawking temperature tends to zero.

In the next section, we will consider a massive charged scalar perturbation around our Lifshitz solutions and discuss the numerical method to obtain the corresponding QNM frequencies.

### III. WAVE EQUATIONS FOR CHARGED SCALAR PERTURBATION AROUND LIFSHITZ SOLUTIONS

Here, we intend to consider a massive charged scalar perturbation around our Lifshitz solutions. The dynamical wave equation for this scalar field perturbation is

$$D'^\mu D_\mu \Psi = m_\Psi^2 \Psi,$$

(10)
where \( D' = \nabla' - i q_s A' \). The wave function can be separated into
\[
\Psi = e^{-i\omega t} R(r) Y(\theta, \phi),
\]
and the differential equation (10) can be written into angular part and radial part by using the metric (1)
\[
\nabla^2 Y(\theta, \phi) = -Q Y(\theta, \phi),
\]
\[
f R'' + \left( f' + \left( \frac{3+z}{r} f \right) \right) R' + \left( \frac{\omega + q_s A_t}{r^{z+1}} \right)^2 \frac{R}{f} - \left( m_s^2 + \frac{Q}{r^2} \right) \frac{R}{f} = 0,
\]
where \( Q = \ell (\ell + 1) \) and \( \ell = 0, 1, 2, \cdots \). Hereafter we will fix the spacetime radius \( l \) to 1. Defining
\[
R(r) = \frac{K(r)}{r},
\]
and the tortoise coordinate
\[
\frac{dr_*}{dr} = \left[ r^{z+1} f(r) \right]^{-1},
\]
we can rewrite (13) into the Schrödinger form
\[
\frac{d^2 K(r_*)}{dr_*^2} + \left[ (\omega + q_s A_t) - V(r) \right] K(r_*) = 0,
\]
where the effective potential
\[
V(r) = r^{2z} f(r) \left[ r f'(r) + \frac{Q}{r^2} + m_s^2 + (z+1)^2 \right] .
\]

At spatial infinity \( f(r) \to 1 \) and \( f'(r) \to 0 \), it is easy to see that \( V(r) \to \infty \) (note that \( z \geq 1 \)) and therefore we need to impose the Dirichlet boundary condition i.e. \( R(r) \to 0 \) at the boundary of spatial infinity.

We are going to employ the improved asymptotic iteration method (AIM) [46] to solve (13) numerically. In order to do so, we rewrite (13) in terms of \( u = 1 - r_+/r \)
\[
\frac{r_+^{z+1} f(u)}{(1-u)^{z+1}} R''(u) + \frac{r_+^{z+1} R'(u)}{(1-u)^z} \left[ (1-u)f'(u) + (z+1)f(u) \right] + \frac{R(u)(1-u)^{z-1}}{r_+^{z-1} f(u)} (q_s A_t(u) + \omega)^2 - \frac{R(u)r_+^{z+1}}{(1-u)^{z+1}} \left( m_s^2 + \frac{Q(1-u)^2}{r_+^2} \right) = 0.
\]

Considering the asymptotic behaviors of \( R(u) \) satisfying (18), at horizon \( u = 0 \) we have \( f(0) \approx uf'(0) \) and \( A_t(0) = 0 \), so that (18) reduces to
\[
R''(u) + \frac{R'(u)}{u} + \frac{\omega^2 R(u)}{u^2 r_+^{2z} f'(0)^2} = 0,
\]
which has the solution
\[
R(u \to 0) \sim C_1 u^{-\xi} + C_2 u^\xi, \quad \xi = \frac{i\omega}{r_+^{z} f'(0)},
\]
in which we have to set \( C_2 = 0 \) to respect the ingoing condition at horizon.

Near infinity \( (u = 1) \), we have
\[
R''(u) + \frac{(z+1) R'(u)}{1-u} - \frac{m_s^2 R(u)}{(1-u)^2} = 0,
\]
as asymptotic form of (18) (note that \( z \geq 1 \)). The above differential equation has the solution
In this case, we should set $D_2 = 0$ in order to satisfy Dirichlet boundary condition $R(r \to \infty) \to 0$.

Now, the desired ansatz for (18) can be defined as

$$R(u) = u^{-\xi}(1-u)^{z+2\Pi}\chi(u).$$

Putting (24) into (18), we have

$$\chi'' = \lambda_0(u)\chi' + s_0(u)\chi,$$

where

$$\lambda_0(u) = \frac{2i\omega}{ur^+ f'(0)} - \frac{f'(u)}{f(u)} + \frac{1 + \Pi}{1 - u},$$

and

$$s_0(u) = \frac{r^{-2(z+1)} f'(0)^{-2}}{2(u-1)^2 u^2 f(u)^2} \left[ -2f'(0)^2 r^2 u^2(1-u)^{2z} (q_s A_1(u) + \omega)^2 + f'(0) uf(u) r^2 \left( m_s^2 r^2 + Q(u-1)^2 \right) - r^2 (u-1) f'(u) \left( f'(0) u (\Pi + z + 2) r^2 - 2i(u - 1) \right) + 2r^2 f(u)^2 \left( - f'(0)^2 m_s^2 u^2 r^2 + i f'(0) (u-1) \omega r^2 (u\Pi + 1) + (u - 1)^2 \omega^2 \right) \right].$$

(25) can be solved numerically by employing the improved AIM. In the following, we will set $l = b = 1$ and $Q = 0$. In [33–36], QNMs corresponding to neutral massive scalar perturbations around dilaton-Lifshitz solutions have been obtained by the improved AIM. Here, we will calculate the charged scalar perturbations and examine the influences of different model parameters on the real and imaginary parts of quasinormal frequencies around Lifshitz black solutions.

IV. NUMERICAL RESULTS

In this section, we report our numerical results of the QNMs of the charged scalar perturbation around Lifshitz black brane solutions.

Let us start with the imaginary part of QNMs shown in Fig. 1. Fixing the mass of the scalar field $m_s$ and increasing the charge $q$ of the background configuration, we observe that the absolute value of the imaginary part of quasi-normal frequency $|\omega_I|$ decreases. This behavior holds for all chosen values of the charge of the scalar field $q_s$. This property is shown in Fig. 1. Furthermore we find that fixing the charge of the black hole $q$ and the mass of the scalar field $m_s$, the bigger value of $q_s$ leads to the smaller $|\omega_I|$. But when $q = 0$, the $q_s$ influence on the imaginary part of the
the scalar field \( q \) here for zero black hole charge, we observe a purely damping mode for charged scalar perturbation. The oscillation when \( |\omega_I| \) shows that the perturbation outside black hole will persist longer period of time to decay completely. From holographic point of view, it means that it takes longer time for the dual system to go back to equilibrium [6]. When the charge of the black holes is increased, the \( |\omega_I| \) is even smaller which shows that the perturbation can last even longer to finally vanish outside the black hole. This result is consistent with the Reissner-Nordström anti-de Sitter (RN-AdS) black hole case where it was found in [7] that the absolute value of the imaginary part of quasinormal frequency \( |\omega_I| \) decreases when the black hole charge approaches to the extreme value. In RN-AdS black hole background when the black hole becomes extreme, \( |\omega_I| \) tends to zero, which makes the extreme RN-AdS black hole marginally unstable. But in the dilaton-Lifshitz black hole background, the imaginary part of the quasinormal frequency will not tend to zero except for some big (small) enough value of the scalar field charge (mass) of the perturbation field. This exhibits that the compared with the usual RN-AdS black hole, the stability of the dilaton-Lifshitz black hole is more easily to be protected. It is important to stress that the black hole stability found here is only stability against scalar perturbations in a certain region of parameters.

Now, we turn to discuss the behavior of the real part of the quasi-normal frequency \( \omega_R \) exhibited in Fig. 1. When the Lifshitz exponent \( z \) is close to the unity, we find that for fixed \( m_s \) and \( q_s \), \( \omega_R \) monotonically decreases with the increase of the charge \( q \) in the background configuration. Besides, fixing \( m_s \) and \( q \), we observe that \( \omega_R \) is smaller when \( q_s \) becomes bigger, as shown in Fig. 1(a). It is worth mentioning that the smaller value of the real part of the quasinormal frequency \( \omega_R \) shows that the scalar perturbation have less energy. When \( q = 0 \), the \( q_s \) influence disappears as well for the real part of the frequency. When \( z \) deviates a bit away from the unity, for small \( q_s \), \( \omega_R \) still decreases with the increase of black hole charge \( q \). But when the scalar field is more charged with bigger \( q_s \), \( \omega_R \) behaves differently and does not monotonically decrease with the increase of \( q \). Instead, there is a barrier in the value of \( \omega_R \) so that it increases when \( q \) increases from zero, but then decreases when \( q \) is over a critical value (Fig. 1(b)). When \( z \) is much bigger, from Fig. 1(c), we see that \( \omega_R \) increases from zero and flattens later with the increase of \( q \) from zero when \( q_s \) is small. For bigger \( q_s \), the real part of the frequency will increase from zero to a maximum value and then slowly decreases when black hole charge approaches to the extreme value. Comparing with the small \( z \) case, here for zero black hole charge, we observe a purely damping mode for charged scalar perturbation. The oscillation adds when \( q \) is nonzero. When the mass of the scalar field is bigger, the difference caused by the small and large \( q_s \) will be enlarged in \( \omega_R \) when the black hole charge is small. When the black hole charge is big enough, the influence of the mass of scalar field on \( \omega_R \) fades away.

In Fig. 1(a), we can see that for some choices of parameters, the imaginary frequency can approach zero and can even jump to be above zero which shows that the stability of the background configuration can be destroyed. This phenomenon happens when the charge of the background black brane solution is high enough, which corresponds to the low temperature. The instability is consistent with the description of the charged scalar field condensation to make the original Lifshitz black brane to be a new hairy configuration. In usual RN-AdS black hole, it was found in [8] that when the black hole becomes extreme, the imaginary quasinormal frequency tends to zero, indicating that the extreme RN-AdS black hole is marginally unstable, since the perturbation will not die out and always persist outside the black hole background. Then if there comes another stronger wave of perturbation, the original black hole background is more likely to be destroyed. This was confirmed in [7]. Here in the dilaton-Lifshitz black hole background, we observed that when the scalar field is highly charged, the imaginary quasinormal frequency can even change to be positive. This shows that instead of the decay of the perturbation outside the dilaton-Lifshitz black hole background, the highly charged scalar perturbation can even blow up outside the black hole and destroy the original black hole spacetime structure. This effect to make the spacetime unstable brought in by the highly charged
perturbation field was also observed in the stability analysis in Reissner-Nordström black hole in de Sitter background [41].

As one can see from Fig. 1(a), when the Lifshitz exponent \( z \) is close to unity, and decreasing \( q_s \) or increasing \( m_s \) can keep the imaginary frequency to be negative to ensure the stability. When \( z \) is more away from unity (Figs. 1(b) and 1(a)) for given values of \( m_s \) and \( q_s \), there is no possibility to destroy the original background configuration, since even when \( q \) takes the extreme value, the imaginary frequency is negative. For chosen value of the Lifshitz exponent \( z \), we can find threshold values for the mass and charge of scalar perturbation field to ensure the stability of the original Lifshitz black brane when its charge \( q \) is nearly extreme. These threshold values are listed in table I. For fixing \( m_s \), when \( q_s \) is below the corresponding threshold values for the chosen \( z \), the black brane can keep to be stable. When we fix \( q_s \), the mass of the scalar field is above the corresponding threshold value for the chosen \( z \) can ensure the stability. \( m_s^2 \) has a lower bound \(- (z + 2)^2/4\) according to Eq. (23). Thus \( m_s \) cannot be reduced infinitely so that in table I at some \( z \) the decrease of \( m_s^2 \) stops.

**TABLE I:** The threshold values of \( m_s \) and \( q_s \) for chosen \( z \) to keep the stability of the black brane with extreme charge value \( q \) and \( m = 3 \).

| \( m_s \) = 0.6 | \( q_s \) = 8.8 |
| --- | --- |
| \( z \) | 1 | 1.2 | 1.6 | 2 | 3 | \( m_s^2 \) | 6.25 | 2.25 | 0.36 | 3.42 |
| \( q \) | 5.3 | 7.3 | 8.8 | 12.3 | 22 | \( m_s^2 \) | 6.25 | 2.25 | 0.36 | 3.42 |

It is clear from above discussions that increasing \( z \) makes the stability more easily to be protected. This result is physically reasonable from holographic point of view since it is in agreement with the behavior of Lifshitz superconductors where it was found that increasing dynamical critical exponent \( z \) makes superconductor more difficult to be formed [47].

In Fig. 2, the behaviors of real and imaginary parts of quasi-normal frequencies with the change of Lifshitz exponent \( z \) have been illustrated. In [33], it was shown that for \((3 + 1)\)-dimensional uncharged dilaton-Lifshitz black branes, independent of the value of \( m_s \), QNMs corresponding to neutral scalar perturbations start overdamping (with \( \omega_R = 0 \)) at \( z = 2 \). Here, we find that for charged black branes, the point at which the real part of quasi-normal frequency of neutral scalar perturbation starts to vanish is no longer at \( z = 2 \), but depends on the value of \( m_s \) (see red and blue lines in Fig. 2(a)). For fixed \( m_s \), when \( q_s \) is small, \( \omega_R \) reduces and finally flattens when the Lifshitz exponent \( z \) becomes big. The value of \( \omega_R \) for high \( z \) can approach to zero (see red and green lines in Fig. 2(a)). For bigger \( q_s \), \( \omega_R \) finally will be above zero (see purple line in Fig. 2(a)).

Now, we discuss the imaginary part of quasi-normal frequency as shown in Fig. 2. For small \( q_s \), the absolute value of the imaginary part is always bigger than the corresponding value with bigger \( q_s \). With the increase of the Lifshitz exponent \( z \), the absolute value of \( \omega_I \) increases. For large \( z \), \( |\omega_I| \) has little dependence on \( q_s \) and it is mainly influenced by the value of \( m_s \).

We have mainly reported QNM behaviors of the charged scalar perturbation in the Lifshitz black brane background with \( k = 0 \). For the Lifshitz black hole case with \( k = 1 \) we have found similar results. We do not repeat explaining these similar results here for Lifshitz black holes.

**V. SUMMARY AND CONCLUSION**

One of the important subject in black hole physics is to investigate the resonances for the scattering of incoming waves by black holes. Quasi-normal modes (QNMs) of a black hole spacetime are indeed the proper solutions of the perturbation equations. It is a general belief that QNMs carry unique footprints to directly identify the black hole existence.

Most available studies on QNMs corresponding to Lifshitz solutions are restricted to neutral scalar perturbations. In this letter, we have extended the study to the charged scalar perturbation in the background of \((3 + 1)\)-dimensional charged dilaton Lifshitz branes/holes. Asymptotic Lifshitz solutions are interesting duals to many condensed matter systems.

We have used the improved asymptotic iteration method (AIM) to calculate the charged scalar perturbations numerically and examine the influences of different model parameters on the imaginary and real parts of quasi-normal frequencies of the charged scalar field perturbations around Lifshitz black solutions. In contrast to the case of neutral scalar perturbations, we found that it is possible to destroy the Lifshitz configuration. To be more clear, we observed that for suitable choices of model parameters, imaginary part of QNM frequencies can be positive near extreme charge value of Lifshitz configuration where temperature is low. For a chosen \( z \), there are threshold values for mass and
charge of scalar perturbation to guarantee the stability of the Lifshitz black configuration. Fixing $q_s$ and taking the mass of the scalar field above the corresponding threshold value, or fixing $m_s$ and keeping the charge of the scalar field below the threshold value, the black brane/hole stability can be protected. It is remarkable to note that here by stability of black hole, we mean only the stability against scalar perturbations in a certain region of parameters. In terms of $z$, it was shown that when it is more away from unity, it is more difficult for the Lifshitz background to become unstable. This result is consistent with the observation that Lifshitz superconductors are more difficult to be formed for greater values of Lifshitz exponent $z$ [47]. We have also observed rich dependence of the quasinormal frequencies on the mass of the scalar field $m_s$, the charge $q$ of the background configuration and the charge of the scalar field $q_s$.

We have also investigated the behaviors of the real and imaginary parts of quasi-normal frequencies in terms of the Lifshitz exponent $z$. We found out that for charged black branes, the point at which neutral QNMs start overdamping is no longer at $z = 2$ as what was claimed in [33] for uncharged Lifshitz black branes, but it has dependence on the value of $m_s$. For fixed $m_s$, when $q_s$ is small, $\omega_R$ reduces and finally flattens when the Lifshitz exponent $z$ becomes big. The value of $\omega_I$ for high $z$ can finally approach to zero, whereas for bigger $q_s$, this possibility does not exist and $\omega_I$ will be finally above zero. The rich dependence of the imaginary part $|\omega_I|$ on the Lifshitz exponent $z$, the scalar field charge $q_s$, mass $m_s$ has also been investigated carefully. We have found that the QNMs properties of the Lifshitz black brane background with $k = 0$ hold as well for the Lifshitz black hole case with $k = 1$. To be concise, we do not repeat the discussion for the Lifshitz black hole here.

Finally, it is worth mentioning that in this letter we have only considered the gauge field as the linear Maxwell field. It is interesting to generalize this study to other nonlinear gauge fields such as power-law Maxwell, Born-Infeld, exponential and logarithmic nonlinear gauge fields in the background of Lifshitz spacetime. Besides, we studied the $(3+1)$-dimensional charged dilaton Lifshitz black solutions. It is also interesting if one could extend the study to higher dimensional Lifshitz spacetime. In addition, we investigated the scalar perturbations. It is worthwhile to extend this investigation to the vector and tensor perturbations in this context. These issues are now under investigation and the results will appear in our future works.

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