De Broglie-Bohm Pilot-Wave Theory: Many Worlds in Denial?

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We reply to claims (by Deutsch, Zeh, Brown and Wallace) that the pilot-wave theory of de Broglie and Bohm is really a many-worlds theory with a superfluous configuration appended to one of the worlds. Assuming that pilot-wave theory does contain an ontological pilot wave (a complex-valued field in configuration space), we show that such claims arise from not interpreting pilot-wave theory on its own terms. Specifically, the theory has its own (‘subquantum’) theory of measurement, and in general describes a ‘nonequilibrium’ state that violates the Born rule. Furthermore, in realistic models of the classical limit, one does not obtain localised pieces of an ontological pilot wave following alternative macroscopic trajectories: from a de Broglie-Bohm viewpoint, alternative trajectories are merely mathematical and not ontological. Thus, from the perspective of pilot-wave theory itself, many worlds are an illusion. It is further argued that, even leaving pilot-wave theory aside, the theory of many worlds is rooted in the intrinsically unlikely assumption that quantum measurements should be modelled on classical measurements, and is therefore unlikely to be true.

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1Present address.
1 Introduction

It used to be widely believed that the pilot-wave theory of de Broglie (1928) and Bohm (1952a,b) had been ruled out by experiments demonstrating violations of Bell’s inequality. Such misunderstandings have largely been overcome, and in recent times the theory has come to be widely accepted by physicists as an alternative (and explicitly nonlocal) formulation of quantum theory. Even so, some workers claim that pilot-wave theory is not really a physically distinct formulation of quantum theory, that instead it is actually a theory of Everettian many worlds. The principal aim of this paper is to refute that claim. We shall also end with a counter-claim, to the effect that Everett’s theory of many worlds is unlikely to be true, as it is rooted in an intrinsically unlikely assumption about measurement.

Pilot-wave theory is a first-order, nonclassical theory of dynamics, grounded in configuration space. It was first proposed by de Broglie, at the 1927 Solvay conference (Bacciagaluppi and Valentini 2009). From de Broglie’s dynamics, together with an assumption about initial conditions, it is possible to derive the full phenomenology of quantum theory, as was first shown by Bohm in 1952.

In pilot-wave dynamics, a closed system with configuration $q$ has a wave function $\Psi(q,t)$ — a complex-valued field on configuration space obeying the Schrödinger equation $i\frac{\partial \Psi}{\partial t} = \hat{H}\Psi$. The system has an actual configuration $q(t)$ evolving in time, with a velocity $\dot{q} \equiv dq/dt$ determined by the gradient $\nabla S$ of the phase $S$ of $\Psi$ (for systems with standard Hamiltonians $\hat{H}$). In principle, the configuration $q$ includes all those things that we normally call ‘systems’ (particles, atoms, fields) as well as pieces of equipment, recording devices, experimenters, the environment, and so on.

Let us explicitly write down the dynamical equations for the case of a non-relativistic many-body system, as they were given by de Broglie (1928). For $N$ spinless particles with positions $x_i(t)$ and masses $m_i$ ($i = 1, 2, \ldots, N$), in an external potential $V$, the total configuration $q = (x_1, x_2, \ldots, x_N)$ evolves in accordance with the de Broglie guidance equation

$$m_i \frac{dx_i}{dt} = \nabla_i S \tag{1}$$

(where $\hbar = 1$ and $\Psi = |\Psi| e^{iS}$), while the ‘pilot wave’ $\Psi$ (as it was originally called by de Broglie) satisfies the Schrödinger equation

$$i \frac{\partial \Psi}{\partial t} = \sum_{i=1}^{N} -\frac{1}{2m_i} \nabla_i^2 \Psi + V \Psi. \tag{2}$$

Mathematically, these two equations define de Broglie’s dynamics — just as, for example, Maxwell’s equations and the Lorentz force law may be said to define classical electrodynamics.

\footnote{More generally, $\dot{q} = j/|\Psi|^2$ where $j$ is the current associated with the Schrödinger equation (Struyve and Valentini 2008).}
The theory was revived by Bohm in 1952, though in a pseudo-Newtonian form. Bohm regarded the equation

\[ m_i \frac{d^2 x_i}{dt^2} = -\nabla_i (V + Q) \]  

(3)
as the true law of motion, with a ‘quantum potential’

\[ Q \equiv -\sum_{i=1}^{N} \frac{1}{2m_i} \nabla_i^2 |\Psi| \]

acting on the particles. (Taking the time derivative of (1) and using (2) yields (3).) On Bohm’s view, (1) is not a law of motion but a condition \( p_i = \nabla_i S \) on the initial momenta — a condition that happens to be preserved in time by (3), and which could in principle be relaxed (leading to corrections to quantum theory) (Bohm 1952a, pp. 170–71). One should therefore distinguish between de Broglie’s first-order dynamics of 1927, defined by (1) and (2), and Bohm’s second-order dynamics of 1952, defined by (3) and (2). In particular, Bohm’s rewriting of de Broglie’s theory had the unfortunate effect of making it seem much more like classical physics than it really was. De Broglie’s original intention had been to depart from classical dynamics at a fundamental level, and indeed the resulting theory is highly non-Newtonian. As we shall see, it is crucial to avoid making classical assumptions when interpreting the theory.

Over an ensemble of quantum experiments, beginning at time \( t = 0 \) with the same initial wave function \( \Psi(q,0) \) and with a Born-rule or ‘quantum equilibrium’ distribution

\[ P(q,0) = |\Psi(q,0)|^2 \]

(4)
of initial configurations \( q(0) \), it follows from de Broglie’s dynamics that the distribution of final outcomes is given by the usual Born rule (Bohm 1952a,b). On the other hand, for an ensemble with a ‘quantum nonequilibrium’ distribution

\[ P(q,0) \neq |\Psi(q,0)|^2 \]

(5)
in general one obtains a distribution of final outcomes that \emph{disagrees} with quantum theory (for as long as \( P \) has not yet relaxed to \( |\Psi|^2 \), see below) (Valentini 1991a,b, 1992, 1996, 2001, 2002, 2004a; Pearle and Valentini 2006).

The initial distribution (4) was assumed by both de Broglie and Bohm, and subsequently most workers have regarded it as one of the axioms of the theory. As we shall see, this is a serious mistake that has led to numerous misunderstandings, and is partially responsible for the erroneous claim that pilot-wave theory is really a theory of many worlds.

We shall not attempt to provide an overall assessment of the relative merits of de Broglie-Bohm pilot-wave theory and Everettian many-worlds theory. Instead, here we focus on evaluating the following claim — hereafter referred to as ‘the Claim’ — which has more or less appeared in several places in the literature (Deutsch 1996, Zeh 1999, Brown and Wallace 2005) (author’s paraphrase):
• Claim: If one takes pilot-wave theory seriously as a possible theory of the world, and if one thinks about it properly and carefully, one ought to see that it really contains many worlds — with a superfluous configuration $q$ appended to one of those worlds.

Were the Claim correct, one could reasonably add a corollary to the effect that one should then drop the superfluous configuration $q$, and arrive at (some form of) many-worlds theory.

Deutsch’s way of expressing the Claim has inspired the title of this paper (Deutsch 1996, p. 225):

In short, pilot-wave theories are parallel-universes theories in a state of chronic denial.

We should emphasise that here we shall interpret pilot-wave theory (for a given closed system) as containing an ontological — that is, physically real — complex-valued field $\Psi(q,t)$ on configuration space, where this field drives the motion of an actual configuration $q(t)$. The Claim asserts that, if the theory is regarded in these terms, then proper consideration shows that $\Psi$ contains many worlds, with $q$ amounting to a superfluous appendage to one of the worlds. One might try to side-step the Claim by asserting that $\Psi$ has no ontological status in pilot-wave theory, that it merely provides a mathematical account of the motion $q(t)$. In this case, one could not even begin to make the Claim, for the complete ontology would be defined by the configuration $q$. For all we currently know, this view might turn out to be true in some future derivation of pilot-wave theory from a deeper theory. But in pilot-wave theory as we know it today — the subject of this paper — such a view seems implausible and physically unsatisfactory (see below). In any case, even if only for the sake of argument, let us here assume that the pilot wave $\Psi$ is ontological, and let us show how the Claim may still be refuted.

It will be helpful first to review the distinction between ontological and mathematical structure in current physical theory, and then to give a brief overview of pilot-wave theory interpreted on its own terms.

Generally speaking, theories should be evaluated on their own terms, without assumptions that make sense only in rival theories. We shall see that, in essence, the Claim in fact arises from not interpreting and understanding pilot-wave theory on its own terms.

2 Ontology versus Mathematics

Physics provides many examples of the distinction between ontological and mathematical structure. Let us consider three.

(1) Classical mechanics. This may be formulated in terms of a Hamiltonian trajectory $(q(t), p(t))$ in phase space. For a given individual system, there is only one real trajectory. The other trajectories, corresponding to alternative initial conditions $(q(0), p(0))$, have a purely mathematical existence. Similarly, in the
Hamilton-Jacobi formulation, the Hamilton-Jacobi function $S(q,t)$ is associated with a whole family of trajectories (with $\dot{q}$ determined by $\nabla S$), only one of which is realised.

(2) A test particle in an external field. This provides a particularly good parallel with pilot-wave theory. A charged test particle, placed in an external electromagnetic field $\mathbf{E}(x,t)$, $\mathbf{B}(x,t)$, will follow a trajectory $x(t)$. One would normally say that the field is real, and that the realised particle trajectory is real; while the alternative particle trajectories (associated with alternative initial positions $x(0)$) are not real, even if they might be said to be contained in the mathematical structure of the electromagnetic field. Similarly, if a test particle moves along a geodesic in a background spacetime geometry, one can think of the geometry as ontological, and the mathematical structure of the geometry contains alternative geodesic motions — but again, only one particle trajectory is realised, and the other geodesics have a purely mathematical existence.

(3) A classical vibrating string. Consider a string held fixed at the endpoints, $x = 0, L$. (This example will also prove relevant to the quantum case.) A small vertical displacement $\psi(x,t)$ obeys the partial differential equation

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial^2 \psi}{\partial x^2}$$

(setting the wave speed $c = 1$). This is conveniently solved using the standard methods of linear functional analysis. One may define a Hilbert space of functions $\psi$, with a Hermitian operator $\hat{\Omega} = -\frac{\partial^2}{\partial x^2}$ acting thereon. Solutions of the wave equation may then be expanded in terms of a complete set of eigenfunctions $\phi_m(x) = \sqrt{2/L} \sin (m \pi x / L)$, where $\hat{\Omega} \phi_m = \omega_m^2 \phi_m$ with $\omega_m^2 = (m \pi / L)^2$ ($m = 1, 2, 3, \ldots$). Assuming for simplicity that $\dot{\psi}(x,0) = 0$, we have the general solution

$$\psi(x,t) = \sum_{m=1}^{\infty} c_m \phi_m(x) \cos \omega_m t \quad \left( c_m \equiv \int_0^L dx \phi_m(x) \psi(x,0) \right)$$

or (in bra-ket vector notation)

$$|\psi(t)\rangle = \sum_{m=1}^{\infty} |m\rangle \langle m| \psi(0) \rangle \cos \omega_m t$$

(where $\hat{\Omega} |m\rangle = \omega_m^2 |m\rangle$). Any solution may be written as a superposition of oscillating ‘modes’. Even so, the true ontology consists essentially of the total displacement $\psi(x,t)$ of the string (perhaps also including its velocity and energy). Individual modes in the sum would not normally be regarded as physically real. One would certainly not assert that $\psi$ is composed of an ontological multiplicity of strings, with each string vibrating in a single mode. Instead one would say that, in general, the eigenfunctions and eigenvalues have a mathematical significance only.

All this is not to say that the question of ontology in physical theories is trivial or always obvious. On the contrary, it is not always self-evident as
to whether mathematical objects in our physical theories should be assigned ontological status or not. For example, classical electrodynamics may be viewed in terms of a field theory (with an ontological electromagnetic field), or in terms of direct action-at-a-distance between charges (where the electromagnetic field is merely an auxiliary field, if it appears at all). Most physicists today prefer the first view, probably because the field seems to contain a lot of independent and contingent structure (see below).

The question to be addressed here is: in the pilot-wave theory of de Broglie and Bohm, if one regards the pilot wave $\Psi$ as ontological (which seems the most natural view at present), does this amount to an ontology of many worlds?

3 Pilot-Wave Theory on its Own Terms

In the author’s view, pilot-wave theory continues to be widely misinterpreted and misrepresented, even by some of its keenest supporters. Here, for illustration, we confine ourselves to de Broglie’s original dynamics for a system of nonrelativistic (and spinless) particles, defined by (1) and (2).

Basic History

Let us begin by setting the historical record straight as historical arguments sometimes play a role in evaluations of pilot-wave theory.

Pilot-wave dynamics was constructed by de Broglie in the period 1923–27. His motivations were grounded in experiment. He wished to explain the quantisation of atomic energy levels and the interference or diffraction of single photons. To this end, he proposed a unification of the physics of particles with the physics of waves. De Broglie argued that Newton’s first law of motion had to be abandoned, because a particle diffracted by a screen does not touch the screen and yet does not move in a straight line. During 1923–24, de Broglie then proposed a new, non-Newtonian form of dynamics in which the velocity of a particle is determined by the phase of a guiding wave. As a theoretical guide, de Broglie sought to unify the classical variational principles of Maupertuis ($\delta \int m v \cdot d\mathbf{x} = 0$, for a particle with velocity $v$) and of Fermat ($\delta \int dS = 0$, for a wave with phase $S$). The result was the guidance equation (1) (at first applied to a single particle and later generalised), which de Broglie regarded as the basis of a new form of dynamics.

At the end of a rather complicated development in the period 1925–27 (including a crucial contribution by Schrödinger, who found the correct wave equation for de Broglie’s waves), de Broglie proposed the many-body dynamics defined by (1) and (2). De Broglie regarded his theory as provisional, much as Newton regarded his own theory of gravity as provisional. And de Broglie regarded the observation of electron diffraction, by Davisson and Germer in 1927, as a vindication of his prediction (first made in 1923), and as clear evidence for his new (first-order) dynamics of particle motion.

\footnote{For a detailed account, see chapter 2 of Bacciagaluppi and Valentini (2009).}
Clearly, de Broglie’s construction of pilot-wave dynamics was motivated by experimental puzzles and had its own internal logic. Note in particular that de Broglie did not construct his theory to ‘solve the measurement problem’, nor did he construct it to provide a (deterministic or realistic) ‘completion of quantum theory’: for in 1923, there was no measurement problem and there was no quantum theory.

Getting the history right is important, for its own sake and also because some criticisms of pilot-wave theory are based on a mistaken appraisal of history. For example, Deutsch (1986, pp. 102–103) has said the following about the theory:

.... to append to the quantum formalism an additional structure .... solely for the purpose of interpretation, is I think a very dangerous thing to do in physics. These structures are being introduced solely to solve the interpretational problems, without any physical motivation. .... the chances of a theory which was formulated for such a reason being right are extremely remote.

But there is no sense in which de Broglie ‘appended’ something to quantum theory, for quantum theory did not exist yet. And de Broglie had ample physical motivation, grounded in experimental puzzles and in a compelling analogy between the principles of Maupertuis and Fermat.

A proper historical account also undermines discussions in which pilot-wave theory is presented as being motivated by the desire to ‘solve the measurement problem’. For example, Brown and Wallace (2005) — who discuss Bohm’s motivations but ignore de Broglie’s — argue that many-worlds theory provides a more natural solution to the measurement problem than does pilot-wave theory. The discussion is framed as if the measurement problem were the prime motivation for considering pilot-wave theory in the first place. As a matter of historical fact, this is false.

The widespread misleading historical perspective has been exacerbated by some workers who present de Broglie’s 1927 dynamics as a way to ‘complete’ quantum theory by adding trajectories to the wave function (D¨ urr et al. 1992, 1996), an approach that furthers the mistaken impression that the theory is a belated reformulation of an already-existing theory. Matters are further confused by some workers who refer to de Broglie’s first-order dynamics by the misnomer ‘Bohmian mechanics’, a term that should properly be applied to Bohm’s second-order dynamics. De Broglie’s dynamics pre-dates quantum theory; and it was given in final form in 1927, not as an after-thought (or reformulation of quantum theory) in 1952.

We may then leave aside certain spurious objections that are grounded in a mistaken version of historical events. In the author’s view, the proper way to pose the question addressed in this paper is: given de Broglie’s dynamics (as it was in 1927), if we examine it carefully on its own terms, does it turn out to contain many worlds?
Basic Ontology

As stated in the introduction, we regard the theory as having a dual ontology: the configuration \( q(t) \) together with the pilot wave \( \Psi[q, t] \). We need to give the relation between this ontology and what we normally think of as physical reality.

De Broglie constructed the theory as a new dynamics of particles: specifically, the basic constituents of matter and radiation (as understood at the time). It is then natural to assume that physical systems, apparatus, people, and so on, are ‘built from’ the configuration \( q \). (In extensions of the theory, \( q \) may of course include configurations of fields, the geometry of 3-space, strings, or whatever may be thought of as the modern fundamental constituents. Further, macroscopic systems — such as experimenters — will usually supervene on \( q \) under some coarse-graining.) This view has been explicitly stated in the literature by several workers — for example Bell (1987, p. 128), Valentini (1992, p. 26), Holland (1993, pp. 337, 350), and others — though perhaps it is not clearly stated in some of the de Broglie-Bohm literature (as Brown and Wallace (2005) suggest). In any case, we shall take this to be the correct and natural viewpoint.

That \( \Psi \) is also to be regarded as ontological is often not explicitly stated. A notable exception was Bell (1987, p. 128, original italics):

\[ \ldots \text{the wave is supposed to be just as ‘real’ and ‘objective’ as say the fields of classical Maxwell theory.} \ldots \text{No one can understand this theory until he is willing to think of } \psi \text{ as a real objective field.} \ldots \text{Even though it propagates not in 3-space but in 3N-space.} \]

Could \( \Psi \) instead be regarded as ‘fictitious’, that is, as a merely mathematical field appearing in the law of motion for \( q \)? As already mentioned, this does not seem reasonable, at least not for the theory in its present form, where — like the electromagnetic field — \( \Psi \) contains a lot of independent and contingent structure, and is therefore best regarded as part of the state of the world (Valentini 1992, p. 17; Brown and Wallace 2005, p. 532).

Valentini (1992, p. 13) considered the possibility that \( \Psi \) might merely provide a convenient mathematical summary of the motion \( q(t) \); to this end, he drew an analogy between \( \Psi \) and physical laws such as Maxwell’s equations, which also provide a convenient mathematical summary of the behaviour of physical systems. On this view, ‘the world consists purely of the evolving variables \( X(t) \), whose time evolution may be summarised mathematically by \( \Psi \)’ (ibid., p. 13). But Valentini argued further (p. 17) that such a view did not do justice to the physical information stored in \( \Psi \), and he concluded instead that \( \Psi \) was a new kind of causal agent acting in configuration space (a view that the author still takes today). The former view, that \( \Psi \) is law-like, was adopted by Dürr et al. (1997). They proposed further that the time dependence and contingency of \( \Psi \) — properties that argue for it to be ontological (see Brown and

\[ \ldots \text{the wave function is a component of physical law rather than of the reality described by the law} \] (Dürr et al. 1997, p. 33).
Wallace 2005, p. 532) — may be illusions, as the wave function for the whole universe is (so they claim) expected to be static and unique. However, the present situation in quantum gravity indicates that solutions for $\Psi$ (satisfying the Wheeler-DeWitt equation and other constraints) are far from unique, and display the same kind of contingency (for example in cosmological models) that we are used to for quantum states elsewhere in physics (Rovelli 2004). Should the universal wave function be static — and the notorious ‘problem of time’ in quantum gravity urges caution here — this alone is not enough to establish that it should be law-like: contingency, or under-determination by physical law, is the more important feature. Therefore, current theoretical evidence speaks against the idea. And in any case, our task here is to consider the theory we have now, not ideas for theories that we may have in the future: in the present form of pilot-wave theory, the time-dependence and (especially) the contingency of $\Psi$ makes it best regarded as ontological.

Note that in 1927 de Broglie regarded $\Psi$ as providing — as a temporary measure — a mathematically convenient and phenomenological summary of motions generated from a deeper theory, in which particles were singular regions of 3-space waves (Bacciagaluppi and Valentini 2009, section 2.3.2). De Broglie hoped the theory would later be derived from something deeper (as Newton believed of gravitational attraction at a distance). Should this eventually happen, ontological questions will have to be addressed anew. Alternatively, perhaps de Broglie’s ‘deeper theory’ (the theory of the double solution) should be regarded merely as a conceptual scaffolding which he used to arrive at pilot-wave theory, and the scaffolding should now be forgotten. But in any case, the theory has come to be regarded as a theory in its own right, and the question at hand is whether this theory contains many worlds or not.

Equilibrium and Nonequilibrium

Many workers take the quantum equilibrium distribution as an axiom, alongside the laws of motion and . It has been argued at length that this is incorrect and deeply misleading (Valentini 1991a,b, 1992, 1996, 2001, 2002; Valentini and Westman 2005; Pearle and Valentini 2006). A postulate concerning the distribution of initial conditions has no fundamental status in a theory of dynamics. Instead, quantum equilibrium is to pilot-wave dynamics as thermal equilibrium is to classical dynamics. In both cases, equilibrium may be understood as arising from a process of relaxation. And in both cases, the equilibrium distributions are mere contingencies, not laws: the underlying theories allow for more general distributions, that violate quantum physics in the first case and thermal-equilibrium physics in the second.

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5 One should also guard against the idea — sometimes expressed in this context — that the existence of ‘only one universe’ somehow suggests that the universal wave function cannot be contingent. Equally, in non-Everettian cosmology, there is only one intergalactic magnetic field, and yet it would be generally agreed that the precise form of this field is a contingency (not determined by physical law).

6 Cf. the role played by the ether in electromagnetism, or in Newton’s thinking about gravitation. For a discussion of this parallel, see section 2.3.2 of Bacciagaluppi and Valentini (2009).
Taken on its own terms, then, pilot-wave theory is *not* a mere alternative formulation of quantum theory. Instead, the theory itself tells us that quantum physics is a special case of a much wider ‘nonequilibrium’ physics (with $P \neq |\Psi|^2$), which may exist for example in the early inflationary universe, or for relic particles that decoupled soon after the big bang, or for particles emitted by black holes (Valentini 2004b, 2007, 2008a,b).

**True (Subquantum) Measurements**

The wider physics of nonequilibrium has its own theory of measurement — ‘subquantum measurement’ (Valentini 1992, 2002; Pearle and Valentini 2006). This is to be expected, since measurement is theory-laden: given a (perhaps tentative) theory, one should look to the theory itself to tell us how to perform correct measurements (cf. section 8).

In pilot-wave theory, an ‘ideal subquantum measurement’ (analogous to the ideal, non-disturbing measurement familiar from classical physics) enables an experimenter to measure a de Broglie-Bohm system trajectory without disturbing the wave function. This is possible if the experimenter possesses an apparatus whose ‘pointer’ has an arbitrarily narrow nonequilibrium distribution (Valentini 2002, Pearle and Valentini 2006). Essentially, the system and apparatus are allowed to interact so weakly that the joint wave function hardly changes; yet, the displacement of the pointer contains information about the system configuration, information that is visible if the pointer distribution is sufficiently narrow. A sequence of such operations allows the experimenter to determine the system trajectory without disturbing the wave function, to arbitrary accuracy.

**Generally False Quantum ‘Measurements’ (Formal Analogues of Classical Measurements)**

We are currently unable to perform such true measurements, because we are trapped in a state of quantum equilibrium. Instead, today we generally carry out procedures that are known as ‘quantum measurements’. This terminology is misleading, because such procedures are — at least according to pilot-wave theory — generally *not* correct measurements: they are merely experiments of a certain kind, designed to respect a formal analogy with classical measurements (cf. Valentini 1996, pp. 50–51).

Thus, in classical physics, to measure a system variable $\omega$ using an apparatus pointer $y$, Hamilton’s equations tell us that we should switch on a Hamiltonian $H = a\omega p_y$ (where $a$ is a coupling constant and $p_y$ is the momentum conjugate to $y$). One obtains trajectories $\omega(t) = \omega_0$ and $y(t) = y_0 + a\omega_0 t$. From the displacement $a\omega_0 t$ of the pointer, one may infer the value of $\omega_0$. An experimental operation represented by $H = a\omega p_y$ then indeed realises a correct measurement of $\omega$ (according to classical physics). But there is no reason to expect the same experimental operation to constitute a correct measurement of $\omega$ for a nonclassical system. Even so, remarkably, so-called quantum ‘measurements’ are in general designed using classical measurements as a guide. Specifically, in quantum theory, to measure an observable $\omega$ using an apparatus pointer $y$,
one switches on a Hamiltonian operator $\hat{H} = a\hat{\omega}\hat{p}$. The quantum procedure is obtained, in effect, by ‘quantising’ the classical procedure.

But what does this analogous quantum procedure actually accomplish? According to pilot-wave theory, it merely generates a branching of the total wave function, with branches labelled by eigenvalues $\omega_n$ of the linear operator $\hat{\omega}$, and with the total configuration $q(t)$ ending in the support of one of the (non-overlapping) branches. Thus, for example, if the system is a particle with position $x$, the initial wave function

$$\Psi_0(x, y) = \left( \sum_n c_n \phi_n(x) \right) g_0(y)$$

(where $\hat{\omega}\phi_n = \omega_n \phi_n$ and $g_0$ is the initial (narrow) pointer wave function) evolves into

$$\Psi(x, y, t) = \sum_n c_n \phi_n(x) g_0(y - a\omega_n t) .$$

The effect of the experiment is simply to create this branching.  

From a pilot-wave perspective, the eigenvalues $\omega_n$ have no particular ontological status: we simply have a complex-valued field $\Psi$ on configuration space, obeying a linear wave equation, whose time evolution may in some situations be conveniently analysed using the methods of linear functional analysis (as we saw for the classical vibrating string).

It cannot be sufficiently stressed that, generally speaking, by means of this procedure one has not measured anything (so pilot-wave theory tells us). In quantum theory, if the pointer is found to occupy the $n$th branch, it is common to assert that therefore ‘the observable $\omega$ has the value $\omega_n$’. But in pilot-wave theory, all that has happened is that, at the end of the experiment, the system trajectory $x(t)$ is guided by the (effectively) reduced wave function $\phi_n(x)$. This does not usually imply that the system has or had some property with value $\omega_n$ (at the end of the experiment or at the beginning), because in pilot-wave theory there is no general relation between eigenvalues and ontology.

Thus, a so-called ‘ideal quantum measurement of $\omega$’ is not a true measurement (a notable exception being the case $\omega = x$). And in general, it is usually incorrect to identify eigenvalues with values of real physical quantities: one must beware of ‘eigenvalue realism’.  

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7Over an ensemble, if $x$ and $y$ have an initial distribution $P_0(x, y) = |\Psi_0(x, y)|^2$, one of course finds that a fraction $|c_n|^2$ of trajectories $q(t) = (x(t), y(t))$ end in the support of the $n$th branch $\phi_n(x)g_0(y - a\omega_n t)$.

8Because the branches have separated in configuration space, it follows from de Broglie’s equation of motion that the ‘empty’ branches no longer affect the trajectory.

9For example, the eigenfunction $\phi_E(x) \propto (e^{ipx} + e^{-ipx})$ of the kinetic-energy operator $\hat{p}^2/2m$ has eigenvalue $E = p^2/2m \neq 0$; and yet, the actual de Broglie-Bohm kinetic energy vanishes, $\frac{1}{2}m\dot{x}^2 = 0$ (since $\partial S/\partial x = 0$). If the system had this initial wave function, and we performed a so-called ‘quantum measurement of kinetic energy’ using a pointer $y$, then the initial joint wave function $\phi_E(x)g_0(y)$ would evolve into $\phi_E(x)g_0(y - aEt)$ and the pointer would indicate the value $E$ — even though the particle kinetic energy was and would remain equal to zero. The experiment has not really measured anything.
4 Some Examples of the Claim

Before evaluating the Claim, let us quote some examples of it from the literature. First, Deutsch (1996, p. 225) argues that parallel universes are

.... a logical consequence of Bohm’s ‘pilot-wave’ theory (Bohm [1952]) and its variants (Bell [1986]). .... The idea is that the ‘pilot-wave’ .... guides Bohm’s single universe along its trajectory. This trajectory occupies one of the ‘grooves’ in that immensely complicated multi-dimensional wave function. The question that pilot-wave theorists must therefore address, and over which they invariably equivocate, is what are the unoccupied grooves? It is no good saying that they are merely a theoretical construct and do not exist physically, for they continually jostle both each other and the ‘occupied’ groove, affecting its trajectory (Tipler [1987], p. 189). .... So the unoccupied grooves’ must be physically real. Moreover they obey the same laws of physics as the ‘occupied groove’ that is supposed to be ‘the’ universe. But that is just another way of saying that they are universes too. .... In short, pilot-wave theories are parallel-universes theories in a state of chronic denial.

Zeh (1999, p. 200) puts the matter thus:

It is usually overlooked that Bohm’s theory contains the same ‘many worlds’ of dynamically separate branches as the Everett interpretation (now regarded as ‘empty’ wave components), since it is based on precisely the same (‘absolutely real’) global wave function .... . Only the ‘occupied’ wave packet itself is thus meaningful, while the assumed classical trajectory would merely point at it: ‘This is where we are in the quantum world.’

Similarly, Brown and Wallace (2005, p. 527) write the following:

.... the corpuscle’s role is minimal indeed: it is in danger of being relegated to the role of a mere epiphenomenal ‘pointer’, irrelevantly picking out one of the many branches defined by decoherence, while the real story — dynamically and ontologically — is being told by the unfolding evolution of those branches. The ‘empty wavepackets’ in the configuration space which the corpuscles do not point at are none the worse for its absence: they still contain cells, dust motes, cats, people, wars and the like.

In the case of Zeh, and of Brown and Wallace, the key assertion is that pilot-wave theory and many-worlds theory contain the same multitude of wave-function branches, and that in pilot-wave theory the ‘empty’ branches nevertheless constitute parallel worlds (which ‘still contain cells, dust motes, cats, people, wars and the like’).
Deutsch’s argument leads to the same assertion — if one interprets his word ‘grooves’ to mean what are normally called ‘branches’. However, Deutsch may in fact have used ‘grooves’ to mean the set of de Broglie-Bohm trajectories, in which case his version of the Claim states that pilot-wave theory is really a theory of ‘many de Broglie-Bohm worlds’\(^\text{10}\) (This version of the Claim is addressed in section 7.) In any case, in essence Deutsch argues that the unoccupied grooves are real, and that they ‘obey the same laws of physics’ as the occupied groove, thereby constituting a ‘multiverse’.

Today, it is often said that in Everettian quantum theory the notion of parallel ‘worlds’ or ‘universes’ applies only to the macroscopic worlds defined (approximately) by decoherence. Formerly, it was common to assert the existence of many worlds at the microscopic level as well. Without entering into any controversy that might still remain about this, here for completeness we shall address the Claim for both ‘microscopic’ and ‘macroscopic’ cases.

5 ‘Microscopic’ Many Worlds?

In pilot-wave theory, is there a multiplicity of parallel worlds at the microscopic level? To see that there is not, let us consider some examples.

(1) **Superposition of eigenvalues.** Let a single particle moving in one dimension have the wave function \(\psi(x,t) \propto e^{-iEt} (e^{ipx} + e^{-ipx})\), which is a mathematical superposition of two distinct eigenfunctions of the momentum operator \(\hat{p} = -i\partial/\partial x\). Are there in any sense two particles, with two different momenta \(+p\) and \(-p\)? Clearly not. While the field \(\psi \propto \cos px\) has two Fourier components \(e^{ipx}\) and \(e^{-ipx}\), there is only one single-valued field \(\psi\) (as in our example of the classical vibrating string). And a true (subquantum) measurement of the particle trajectory \(x(t)\) would reveal that the particle is at rest (since \(S = -Et\) and \(\partial S/\partial x = 0\)). In a so-called ‘quantum measurement of momentum’, at the end of the experiment \(x(t)\) is guided by \(e^{ipx}\) or \(e^{-ipx}\): during the experiment the particle is accelerated and acquires a momentum \(+p\) or \(-p\), as could be confirmed by a true subquantum measurement. Any impression that there may be two particles present arises from a mistaken belief in eigenvalue realism.

(2) **Double-slit experiment.** Let a single particle be fired at a screen with two slits, where the incident wave function \(\psi\) passes through both slits, leading to an interference pattern on the far side of the screen. Are there in any sense two particles, one passing through each slit? Again, clearly not. There is a single-valued field \(\psi\) passing through both slits, and there is one particle trajectory \(x(t)\) in 3-space, passing through one slit only (as again could be tracked by a true subquantum measurement).

\(^{10}\)Deutsch cites the rather confused paper by Tipler (1987), which argues among other things that de Broglie-Bohm trajectories must affect each other in unphysical ways. Tipler’s critique is mostly aimed at a certain stochastic version of pilot-wave theory. While it is not really relevant to Deutsch’s argument, for completeness we note that, as regards conventional (deterministic) pilot-wave theory, Tipler’s critique stems from an elementary misunderstanding of the role of probability in the theory.
Superposition of ‘Ehrenfest’ packets for a hydrogen atom. Finally, consider a single hydrogen atom, with a centre-of-mass trajectory $x(t)$ and with a wave function that is a superposition

$$\psi = \frac{1}{\sqrt{2}} (\psi_1 + \psi_2)$$

of two localised and spatially-separated ‘Ehrenfest’ packets $\psi_1$ and $\psi_2$. Each packet, with centroid $\langle x \rangle_1$ or $\langle x \rangle_2$, follows an approximately classical trajectory, and let us suppose that the actual trajectory $x(t)$ lies in $\psi_2$ only. Is there any sense in which we have two hydrogen atoms? The answer is no, because, once again, a true subquantum measurement could track the unique atomic trajectory $x(t)$ (without affecting $\psi$).

This last example has a parallel in the macroscopic domain, to be discussed in the next section. Before proceeding, it will prove useful to consider the present example further. In particular, one might argue that each packet $\psi_1$ and $\psi_2$ behaves like a hydrogen atom, under operations defined by changes in the external potential $V$. Specifically, the motion of the empty packet $\psi_1$ will respond to changes in $V$, in exactly the same way as will the motion of the occupied packet $\psi_2$. One might then claim that, if one regards each packet as physically real, one may as well conclude that there really are two hydrogen atoms present. But this argument fails, because the similarity of behaviour of the two packets holds only under the said restricted class of operations (that is, modifying the classical potential $V$). In pilot-wave theory, in principle, other experimental operations are possible, under which the behaviours of $\psi_1$ and $\psi_2$ will be quite different.

For example, suppose one first carries out an ideal subquantum measurement, which shows that the particle is in the packet $\psi_2$. One may then carry out an additional experiment — say an ordinary quantum experiment, using a piece of macroscopic apparatus — designed to find out whether or not a given packet is occupied. One may predict that, in the second experiment, if the operation is performed on packet $\psi_1$ the apparatus pointer will point to ‘unoccupied’, while if the operation is performed on $\psi_2$ the pointer will point to ‘occupied’. It will then become operationally apparent that $\psi_1$ consists solely of a bundle of the complex-valued $\psi$-field, whose centroid happens to be simulating the approximately classical motion of a hydrogen atom in an external field (under the said restricted class of operations).

It is of course hardly mysterious that in some circumstances one may have an ontological but empty $\psi$-packet whose motion approximately traces out the trajectory of a classical body — just as, in some circumstances, a localised classical electromagnetic pulse travelling through an appropriate medium (with variable refractive index) might trace out a trajectory similar to that of a moving body. In both cases, it would be clear from other experiments that the moving pulse is not really a moving body.

\[11\] In quantum theory too, of course, the second experiment will always give different results for the two packets. But the outcome will be random, making the operational difference between the packets less clear.
Let us now ask if there is any sense in which pilot-wave theory contains many worlds at the ‘macroscopic’ level.

We shall begin with an utterly unrealistic example, involving a superposition of two ‘Ehrenfest’ packets each (supposedly) representing a classical macroscopic world. This example has the virtue of illustrating the Claim in what we believe to be its strongest possible form. We shall see that, even for this example, the Claim may be straightforwardly refuted, along the lines given in the last section for the case of the hydrogen atom.

We then turn to a further unrealistic example, involving a superposition of two delocalised ‘WKB’ packets which, again, are each supposed to represent a classical macroscopic world. This example has the virtue of showing that, if one cannot point to some piece of localised ‘Ψ-stuff’ following an alternative classical trajectory, then the Claim simply cannot be formulated. The lesson learned from this example is then readily applied to realistic cases with decoherence, for which the wave functions involved are also generally delocalised, and for which, therefore, the Claim again cannot be formulated.

The Claim in a ‘Strong Form’

Let us again consider an ‘Ehrenfest’ superposition

\[ \Psi(q, t) = \frac{1}{\sqrt{2}} \left( \Psi_1(q, t) + \Psi_2(q, t) \right), \]

where now the configuration \( q \) represents not just a single hydrogen atom but all the contents of a macroscopic region — for example, a region including the Earth, with human experimenters, apparatus, and so on. We shall imagine that the centroids \( \langle q \rangle_1, \langle q \rangle_2 \) of the respective packets \( \Psi_1, \Psi_2 \) follow approximately classical trajectories, corresponding to alternative histories of events on Earth. This is of course not at all a realistic formulation of the classical limit for a complex macroscopic system: wave packets spread, and they do so particularly rapidly for chaotic systems. But we shall ignore this for a moment, because the example is nevertheless instructive.

Let us assume that \( \Psi \) consists initially of a single narrow packet, and that the subsequent splitting of \( \Psi \) into the (non-overlapping) branches \( \Psi_1, \Psi_2 \) occurs as a result of a ‘quantum measurement’ with two possible outcomes +1 and −1. (See Fig. 1.) One might imagine that, at first, the branches \( \Psi_1, \Psi_2 \) develop a non-overlap with respect to the apparatus pointer coordinate \( y \), which then generates a non-overlap with respect to other (macroscopic) degrees of freedom — beginning, perhaps, with variables in the eye and brain of the experimenter who looks at the pointer. We may imagine that it had been decided in advance that if the outcome were +1, the experimenter would stay at home; while if the outcome were −1, the experimenter would go on holiday. These alternative histories for the experimenter are supposed to be described by the trajectories of the narrow packets \( \Psi_1 \) and \( \Psi_2 \) (whose arguments include all the relevant...
variables, constituting the centre-of-mass of the experimenter, his immediate environment, the plane he may or may not catch, and so on). Let us assume that the actual de Broglie-Bohm trajectory \( q(t) \) ends in the support of \( \Psi_2 \), as shown in Fig. 1.

One could of course extend the example to superpositions of the form \( \Psi = \Psi_1 + \Psi_2 + \Psi_3 + \ldots \), where \( \Psi_1, \Psi_2, \Psi_3 \ldots \) are non-overlapping narrow packets that trace out — in configuration space — approximately classical motions corresponding to alternative macroscopic histories of the world, with each history containing, in the words of Brown and Wallace, ‘cells, dust motes, cats, people, wars and the like’.

Now, with these completely unrealistic assumptions, the Claim seems to be at its strongest. For if \( \Psi \) is ontological, then in the example of Fig. 1 the narrow packets \( \Psi_1 \) and \( \Psi_2 \) are both real objects moving along approximately classical paths in configuration space. There is certainly something real moving along each path. One of the paths has an extra component too — the actual configuration \( q(t) \) — but even so the fact remains that something real is moving along the other path as well.

This situation seems to be the strongest possible realisation of the Claim. One might say, for example with Brown and Wallace (section 4 above), that ‘[t]he ’empty wavepackets’ in the configuration space which the corpuscles do not point at are none the worse for its absence \(^{12}\) One might assert that here there really are two macroscopic worlds, one built from \( \Psi_1 \) alone, and one built from \( \Psi_2 \) together with \( q \). And again, as in the case of the hydrogen atom discussed in section 5, one might argue that there is no difference in the behaviour of these two worlds, and that the motion of \( \Psi_1 \) represents a world every bit as \textit{bona fide} as the world represented by \( \Psi_2 \) (together with \( q \), which one might assert is

\(^{12}\)This is not to suggest that Brown and Wallace, or other proponents of the Claim, actually make the Claim in the ‘strong’ form given here. We consider this form first, because it seems to us to be the strongest possible version of the argument.
superfluous).

But again, as in the case of the hydrogen atom, pilot-wave theory tells us that a remote experimenter with access to nonequilibrium particles could in principle track the true history \( q(t) \), without affecting \( \Psi \). Further, once it is known which packet is empty and which not, the experimenter could perform additional experiments showing that \( \Psi_1 \) and \( \Psi_2 \) (predictably) behave differently under certain operations. Again, the empty packet is merely simulating a classical world, and the simulation holds only under a class of operations more restrictive than those allowed in pilot-wave theory. The situation is conceptually the same as in the case of the single hydrogen atom.

We conclude that the Claim fails, even in a ‘strong form’.

**The Claim in a ‘Weak Form’**

Before considering more realistic approaches (with decoherence), it is instructive to reconsider the above scenario in terms of a different — and equally unrealistic — approach to the classical limit, namely the WKB approach, in which the amplitude of \( \Psi \) is taken to vary slowly over relevant lengthscales. It is often said that the resulting wave function may be ‘associated with’ a family of classical trajectories, defined by the equation \( p = \nabla S \) giving the classical momentum \( p \) in terms of the phase gradient. (This approach is frequently used, for example, in quantum cosmology.) Where such trajectories come from is not clear in standard quantum theory, but in pilot-wave theory it is clear enough: in the WKB regime, the de Broglie-Bohm trajectory \( q(t) \) (within the extended wave) will indeed follow a classical trajectory defined by \( p = \nabla S \).

Now let the superposition

\[
\Psi(q, t) = \frac{1}{\sqrt{2}} (\Psi_1(q, t) + \Psi_2(q, t))
\]

be composed of two non-overlapping ‘WKB packets’ \( \Psi_1, \Psi_2 \), that formed from the division of a single WKB packet \( \Psi \), where again \( q \) represents the contents of a macroscopic region including the Earth. As in the earlier example, we imagine that the division occurred because a quantum experiment was performed, with two possible outcomes indicated by a pointer coordinate \( y \): and again, \( \Psi_1 \) corresponds to the outcome \(+1\), while \( \Psi_2 \) corresponds to the outcome \(-1\), and the actual \( q(t) \) ends in the support of \( \Psi_2 \). Unlike the earlier example, though, in this case the packets \( \Psi_1, \Psi_2 \) are narrow with respect to \( y \) but broad with respect to the other (relevant) degrees of freedom — so broad, in fact, that with respect to these other degrees of freedom the packets are effectively plane waves. The only really significant difference between \( \Psi_1 \) and \( \Psi_2 \) is in their support with respect to \( y \). (See Fig. 2.)

To be sure, this is not a realistic model of the macroscopic world, no more than the Ehrenfest model was. But it is instructive to see the effect this alternative approach has on the Claim.

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13 Except, one might argue, if one is talking about the ‘whole universe’. One could restrict the argument to approximately-independent regions; this does not seem an essential point.
Under the above assumptions, the actual trajectory \( q(t) \) will be approximately classical (except in the small branching region), and might be taken to correctly model the macroscopic history with outcome \(-1\) and the experimenter going on holiday. But is there now any other discernible realisation of an alternative classical macroscopic motion, such as the experimenter staying at home? Clearly not. While the empty branch \( \Psi_1 \) is ontological, it is spread out over all degrees of freedom except \( y \), so that its time evolution does not trace out a trajectory corresponding to an approximately classical alternative motion. The experimenter ‘staying at home’ is nowhere to be seen. Unlike in the Ehrenfest case, one cannot point to some piece of localised ‘\( \Psi \)-stuff’ following an alternative classical trajectory.

Of course, different initial configurations \( q(0) \) (with the same initial \( \Psi \)) would yield different trajectories \( q(t) \). And the ‘information’ about these alternative paths certainly exists in a mathematical sense, in the structure of the complex field \( \Psi \). But there is no reason to ascribe anything other than mathematical status to these alternative trajectories — just as we saw in section 2, for the analogous classical case of a test particle moving in an external electromagnetic field or in a background spacetime geometry. The alternative trajectories are mathematical, not ontological.

Realistic Models (with Environmental Decoherence)

A more realistic account of the macroscopic, approximately classical realm may be obtained from models with environmental decoherence. (For a review, see Zurek (2003).)

Consider a system with configuration \( q \), coupled to environmental degrees of freedom \( y = (x_1, x_2, ..., x_N) \). For a pure state the wave function is \( \Psi(q, y, t) \), and one often considers mixtures with a density operator

\[
\hat{\rho}(t) = \sum_{\alpha} p_{\alpha} |\Psi_{\alpha}(t)\rangle\langle \Psi_{\alpha}(t)|.
\]
(For example, in ‘quantum Brownian motion’, the system is a single particle in a potential and the environment consists of a large number of harmonic oscillators in a thermal state.) By tracing over \( y \) one obtains a reduced density operator for the system, with matrix elements

\[
\rho_{\text{red}}(q, q', t) = \sum_{\alpha} p_{\alpha} \int dy \, \Psi_{\alpha}(q, y, t) \Psi_{\alpha}^*(q', y, t),
\]

from which one may define a quasi-probability distribution in phase space for the system:

\[
W_{\text{red}}(q, p, t) = \frac{1}{2\pi} \int dz \, e^{ipz} \rho_{\text{red}}(q - z/2, q + z/2, t)
\]

(the reduced Wigner function). In certain conditions, one obtains an approximately non-negative function \( W_{\text{red}}(q, p, t) \) whose time evolution approximates that of a classical phase-space distribution.

For some elementary systems, such as a harmonic oscillator, the motion of a narrowly-localised packet \( W_{\text{red}}(q, p, t) \) can trace out a thin ‘tube’ approximating a classical trajectory in phase space (Zurek et al. 1993). However, such simple quantum-classical correspondence breaks down for chaotic systems, because of the rapid spreading of the packet: even an initial minimum-uncertainty packet spreads over macroscopic regions of phase space within experimentally-accessible timescales (Zurek 1998). On the other hand, at least for some examples it can be shown that, even in the chaotic case, the evolution of \( W_{\text{red}}(q, p, t) \) approximates the evolution of a classical phase-space distribution \( W_{\text{class}}(q, p, t) \) (a Liouville flow with a diffusive contribution from the environment), where both distributions rapidly delocalise (Habib et al. 1998; Zurek 2003, pp. 745–47).

In pilot-wave theory, a mixed quantum state is described by a preferred decomposition of \( \hat{\rho} \) into a statistical mixture (with weights \( p_{\alpha} \)) of ontological pilot waves \( \Psi_{\alpha} \) (Bohm and Hiley 1996). For a given element of the ensemble, the de Broglian velocity of the configuration is determined by the actual pilot wave \( \Psi_{\alpha} \). (A different decomposition generally yields different velocities, and so is physically distinct at the fundamental level.) Now, the pilot-wave theory of quantum Brownian motion has been studied by Appleby (1999). Under certain conditions it was found that, as a result of decoherence, the de Broglie-Bohm trajectories of the system become approximately classical (as one might have expected). While Appleby made some simplifying assumptions in his analysis, pending further studies of this kind it is reasonable to assume that Appleby’s conclusions hold more generally.

We may now evaluate the Claim in the context of realistic models. First of all, as in the unrealistic examples considered above, the Claim fails because an ideal subquantum measurement will always show that there is just one trajectory \( q(t) \); and, further experiments will show that empty wave packets (predictably)

\[14\] The examples are based on the weak-coupling, high-temperature limit of quantum Brownian motion. The system consists of a single particle moving in one dimension in a classically-chaotic potential.
behave differently from packets containing the actual configuration. This alone suffices to refute the Claim. Even so, it is interesting to ask if it is possible to have localised ontological packets (‘built out of Ψ’) whose motions execute alternative classical histories: that is, it is interesting to ask if the ‘strong form’ of the Claim discussed above — which in any case fails, but is still rather intriguing — could ever occur in practice in realistic models. The answer, again, is no.

For an elementary non-chaotic system, one can obtain a narrow ‘Wigner packet’ $W_{\text{red}}(q, p, t)$ approximating a classical trajectory, and one could also have a superposition of two or more such packets (with macroscopic separations). One might then argue that, since $W_{\text{red}}$ is built out of Ψ, we have (in a realistic setting, with decoherence) something like the ‘strong form’ of the Claim discussed above. However, the models usually involve a mixture of Ψ’s, of which $W_{\text{red}}$ is not a local functional. So the ontological status of a narrow packet $W_{\text{red}}$ is far from clear. But even glossing over this, having a narrow packet $W_{\text{red}}$ following an approximately classical path is in any case unrealistic in a world containing chaos, where, as we have already stated, one can show only that $W_{\text{red}}$ — an approximately non-negative function, with a large spread over phase space — has a time evolution that approximately agrees with the time evolution of a classical (delocalised) phase-space distribution; that is, $W_{\text{red}}$ follows an approximately Hamiltonian or Liouville flow (with a diffusive contribution). Again, one cannot obtain anything like ‘localised ontological Ψ-stuff’ (or something locally derived therefrom) executing an approximately classical trajectory — not even for one particle in a chaotic potential, and certainly not for a realistic world containing turbulent fluid flow, double pendulums, people, wars, and so on.

One can obtain localised trajectories from a quantum description of a chaotic system, if the system is continuously measured — which in practice involves an experimenter continuously monitoring an apparatus or environment that is interacting with the system (Bhattacharya et al. 2000). Such trajectories for the Earth and its contents might in principle be obtained by monitoring the environment (the interstellar medium, the cosmic microwave background, etc.), but in the absence of an experimenter performing the required measurements it is difficult to see how this could be relevant to our discussion. And in any case, in a pilot-wave treatment, there is no reason why such a procedure would yield ‘localised ontological Ψ-stuff’ executing the said trajectories.

In a realistic quantum-theoretical model, then, the outcome is a highly delocalised distribution $W_{\text{red}}(q, p, t)$ on phase space, obeying an approximately Hamiltonian or Liouville evolution (with a diffusive contribution). As in the unrealistic WKB example above, in pilot-wave theory there will be one trajectory for each system. And, while different initial conditions will yield different trajectories, there is no reason to ascribe anything other than mathematical status to these alternatives — just as in the analogous classical case of a test particle moving in an external field or background geometry. Once again, the alternative trajectories are mathematical, not ontological.

Of course, given such a distribution $W_{\text{red}}(q, p, t)$, if one wishes one may
identify the flow with a set of trajectories representing parallel (approximately classical) worlds, as in the decoherence-based approach to many worlds of Saunders and Wallace. This is fair enough from a many-worlds point of view. But if we start from pilot-wave theory understood on its own terms, there is no motivation for doing so: such a step would amount to a reification of mathematical structure (assigning reality to all the trajectories associated with the velocity field at all points in phase space). If one does so reify, one has constructed a different physical theory, with a different ontology; one may do so if one wishes, but from a pilot-wave perspective there is no special reason to take this step.

**Other Approaches to Decoherence**

Finally, decoherence and the emergence of the classical limit has also been studied using the decoherent histories formulation of quantum theory. In these treatments, there will still be no discernible ‘localised ontological Ψ-stuff’ following alternative classical trajectories, for realistic models containing chaos. Therefore, again, the ‘strong form’ of the Claim (which in any case fails by virtue of subquantum measurement) could never occur in practice.

### 7 Further Remarks

**Many de Broglie-Bohm Worlds?**

In the Saunders-Wallace approach to many worlds, one ascribes reality to the full set of trajectories associated with the reduced Wigner function $W_{\text{red}}(q,p,t)$ in the classical limit (for some appropriately-defined macrosystem with configuration $q$). This raises a question. Why not also ascribe reality to the full set of de Broglie-Bohm trajectories outside the classical limit, for arbitrary (pure) quantum states, resulting in a theory of ‘many de Broglie-Bohm worlds’?

After all, just as $W_{\text{red}}$ has a natural velocity field associated with it (on phase space), so an arbitrary wave function $\Psi$ has a natural velocity field associated with it (on configuration space) — namely, de Broglie’s velocity field derived from the phase gradient $\nabla S$ (or more generally, from the quantum current). In both cases, the velocity fields generate a set of trajectories, and one may ascribe reality to them all if one wishes. Why do so in the first case, but not in the second?

Furthermore, if the results due to Appleby (1999) (mentioned in section 6) for quantum Brownian motion hold more generally, the parallel de Broglie-Bohm trajectories will reduce to the parallel classical trajectories in an appropriate limit; in which case, the theory of ‘many de Broglie-Bohm worlds’ will reproduce the Saunders-Wallace multiverse in the classical limit, and will provide a simple and natural extension of it outside that limit — that is, one will have a notion

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15 See, for example, Gell-Mann and Hartle (1993) and Halliwell (1998), as well as the reviews in this volume.

16 Such a theory has, in effect, been considered by Tipler (2006).
of parallel worlds that is defined generally, even at the microscopic level, and not just in the classical-decoherent limit.\[17\]

However, since the de Broglie velocity field is single-valued, trajectories $q(t)$ cannot cross. There can be no splitting or fusion of worlds. The above ‘de Broglie-Bohm multiverse’ then has the same kind of ‘trivial’ structure that would be obtained if one reified all the possible trajectories for a classical test particle in an external field: the parallel worlds evolve independently, side by side. Given such a theory, on the grounds of Ockham’s razor alone, there would be a conclusive case for taking only one of the worlds as real.

On this point we remark that, in Deutsch’s version of the Claim, if his word ‘grooves’ is interpreted as referring to the set of de Broglie-Bohm trajectories, then the Claim amounts to asserting that pilot-wave theory implies the de Broglie-Bohm multiverse. But again, because the parallel worlds never branch or fuse, it would be natural to reduce the theory to a single-world theory with only one trajectory.

A theory of many de Broglie-Bohm worlds, then, can only be a mere curiosity — a foil, perhaps, against which to test conventional Everettian ideas, but not a serious candidate for a physical theory. On the other hand, it appears to provide the basis for an argument against the Saunders-Wallace multiverse. For as we have seen, it is natural to extend the Saunders-Wallace multiverse to a deeper and more general de Broglie-Bohm multiverse. And this, in turn, reduces naturally to a single-universe theory — that is, to standard de Broglie-Bohm theory. Thus, we have an argument that begins by extending the Saunders-Wallace worlds to the microscopic level, and ends by declaring only one of the resulting worlds to be real.

Quantum Nonequilibrium and Many Worlds

Since pilot-wave theory generally violates the Born rule, while conventional many-worlds theory (apparently) does not, on this ground alone any attempt to argue that the two theories are really the same must fail. Further, if such violations were discovered then Everett’s theory would be disproved and that of de Broglie and Bohm vindicated.

On the other hand, it might be suggested that violations of the Born rule could be incorporated into an Everett-type framework, by adopting the theory of ‘many de Broglie-Bohm worlds’ sketched above. Restricting ourselves for simplicity to the pure case, if one assumes a nonequilibrium probability measure $P_0 \neq |\Psi_0|^2$ on the set of (parallel) initial configurations $q(0)$, then for as long as relaxation to quantum equilibrium has yet to occur completely, one will obtain

\[17\]One need not think of this as ‘adding’ trajectories to the wave function; one could think of it as an alternative reading of physical structure already existing in the ‘bare’ wave function.

\[18\]It might be claimed that, outside the nonrelativistic domain, such an extension is neither simple nor natural. However, the (deterministic) pilot-wave theory of high-energy physics has achieved a rather complete (if not necessarily final) state of development — for recent progress see Colin (2003), Colin and Struyve (2007), Struyve (2008), Struyve and Westman (2007), and Valentini (2008c).

\[19\]See Valentini (2007, 2008a,b) for recent discussions of possible experimental evidence.
a nonequilibrium set of parallel trajectories $q(t)$, and one expects (in general) to find violations of the Born rule within individual parallel worlds. If one accepts this, then observation of quantum nonequilibrium would not suffice to disprove many worlds (though of course conventional Everettian quantum theory would be disproved). On the other hand, however, as stated above it is natural to reduce the theory of many de Broglie-Bohm worlds to a single-world theory, and this is equally true in the nonequilibrium case. Therefore, the de Broglie-Bohm multiverse would not provide a plausible refuge for the Everettian faced with nonequilibrium phenomena.

Even so, it might be worth exploring the theory of many de Broglie-Bohm worlds with a nonequilibrium measure, in particular to highlight the assumptions made in the Deutsch-Wallace derivation of the Born rule (Deutsch 1999, Wallace 2003a).

On Arguments Concerning ‘Structure’

One might argue that the mathematical structure in the quantum state that is reified by many-worlds theorists plays such an explanatory and predictive role that it should indeed be regarded as real. To quote Wallace (2003b, p. 93):

A tiger is any pattern which behaves as a tiger. .... the existence of a pattern as a real thing depends on the usefulness — in particular, the explanatory power and predictive reliability — of theories which admit that pattern in their ontology.

However, the behaviour of a system depends on the allowed set of experimental operations. If one considers subquantum measurements, the patterns reified by many-worlds theorists will cease to be explanatory and predictive. From a pilot-wave perspective, then, such mathematical patterns are explanatory and predictive only in the confines of quantum equilibrium: outside that limited domain, subquantum measurement theory would provide a more explanatory and predictive framework.

At best, it can only be argued that, if approximately classical experimenters are confined to the quantum equilibrium state, so that they are unable to perform subquantum measurements, then they will encounter a phenomenological appearance of many worlds — just as they will encounter a phenomenological appearance of locality, uncertainty, and of quantum physics generally.

On Arguments Concerning Computation

It might be argued that quantum computation provides evidence for the existence of many worlds (Deutsch 1985, 1997). Deutsch asks ‘how’ and ‘where’ the supposedly huge number of parallel computations are performed, and has challenged those who doubt the existence of parallel universes to provide an explanation.
explanation for quantum-computational processes such as Shor’s factorisation algorithm (Deutsch 1997, p. 217).

However, while it often used to be asserted that the advantages of quantum computation originated from quantum superposition, the matter has become controversial. Some workers, such as Jozsa (1998) and Steane (2003), claim that entanglement is the truly crucial feature. Further, the ability to find periods seems to be the mechanism underlying Shor’s algorithm, and this is arguably more related to the ‘wave-like’ aspect of quantum physics than it is to superposition (Mermin 2007).

Leaving such controversies aside, we know in any case that, in quantum equilibrium, pilot-wave theory yields the same predictions as ordinary quantum theory, including for quantum algorithms. In an assessment of precisely how pilot-wave theory provides an explanation for a specific quantum algorithm, it should be borne in mind that: (a) the theory contains an ontological pilot wave propagating in many-dimensional configuration space; (b) the theory is nonlocal; and (c) with respect to quantum ‘measurements’, the theory is contextual. There is then ample scope for exploring the pilot-wave-theoretical account of quantum-computational processes, if one wishes to do so, just as there is for any other type of quantum process.

8 Counter-Claim: A General Argument Against Many Worlds

We have refuted the Claim, that pilot-wave theory is ‘many worlds in denial’. Here, we put forward a Counter-Claim:

- **Counter-Claim:** The theory of many worlds is unlikely to be true, because it is ultimately motivated by the puzzle of quantum superposition, which arises from a belief in eigenvalue realism, which is in turn based (ultimately) on the intrinsically unlikely assumption that quantum measurements should be modelled on classical measurements.

We saw in section 3 that quantum theorists call an experiment ‘a measurement of $\omega$’ only because it formally resembles what would have been a correct measurement of $\omega$ had the system been classical. Thus, the system-apparatus interaction Hamiltonian is chosen by means of (for example) the mapping

$$H = a\omega p_y \longrightarrow \hat{H} = a\hat{\omega}\hat{p}_y,$$

so that quantum ‘measurements’ are in effect modelled on classical measurements. That this is a mistake is clear from a pilot-wave perspective. But the key point is more general, and does not depend on pilot-wave theory. In fact, it was made by Einstein in 1926 (see below).

21In the classical limit of pilot-wave theory, emergent effective degrees of freedom have a purely mathematical correspondence with linear operators acting on the wave function. Physicists trapped in quantum equilibrium have made the mistake of taking this correspondence literally (Valentini 1992, pp. 14–16, 19–29; 1996, pp. 50–51).
The Argument

Everett’s initial motivation for introducing many worlds was the puzzle of quantum superposition, in particular the apparent transfer of superposition from microscopic to macroscopic scales during a quantum measurement (Everett 1973, pp. 4–6). While our understanding of the theory today differs in many respects from Everett’s, it is highly doubtful that the theory would ever have been proposed, were it not for the puzzle of quantum superposition.

Now, the puzzle of superposition stems from what we have called ‘eigenvalue realism’: the assignment of an ontological status to the eigenvalues of linear operators acting on the wave function. For if an initial wave function

$$\psi_0(x) = \sum_n c_n \phi_n(x)$$

is a superposition of different eigenfunctions $\phi_n(x)$ of $\hat{\omega}$ with different eigenvalues $\omega_n$, then if one takes eigenvalue realism literally it appears as if all the values $\omega_n$ should somehow be regarded as simultaneous ontological attributes of a single system.

Why do so many physicists believe in eigenvalue realism? The answer lies, ultimately, in their belief in the quantum theory of measurement. For example, it is widely thought that an experimental operation described by the Hamiltonian operator $\hat{H} = a\hat{\omega}\hat{p}_y$ constitutes a correct measurement of an observable $\omega$, as indicated by the value of the pointer coordinate $y$. To see that this leads to a belief in eigenvalue realism, consider a system with wave function $\phi_n(x)$. Under such an operation, the pointer $y$ will indicate the value $\omega_n$. Because the experimenter believes that this pointer reading provides a correct measurement, the experimenter will then believe that the system must have a property $\omega$ with value $\omega_n$ — that is, the experimenter will believe in eigenvalue realism.

Now, why do so many physicists believe that an operation described by (for example) $\hat{H} = a\hat{\omega}\hat{p}_y$ constitutes a correct measurement of $\omega$, for any observable $\omega$? The answer, as we have seen, is that the said operation formally resembles a classical measurement of $\omega$, via the mapping (6).

We claim that this is the heart of the matter: it is widely assumed, in effect, that classical physics provides a reliable guide to measurement for nonclassical systems. We claim further that this assumption is intrinsically unlikely, so that the conclusions stemming from it — eigenvalue realism, superposition of properties, multiplicity of worlds — are in turn intrinsically unlikely (Valentini 1992, pp. 14–16, 19–29; 1996, pp. 50–51).

The assumption is unlikely because, generally speaking, one cannot use a theory as an accurate guide to measurement outside the domain of validity of the theory. For experiment is theory-laden, and correct measurement procedures must be laden with the correct theory. As an example, consider what might happen if one used Newton’s theory of gravity to interpret observations close to a black hole: one would encounter numerous puzzles and paradoxes, that would be resolved only when the observations were interpreted using general relativity. It is intrinsically improbable that measurement operations taken from an older,
superseded physics will remain valid in a fundamentally new domain for all possible observables. It is much more likely that a new domain will be better understood in terms of a new theory based on new concepts, with its own new theory of measurement — as shown by the example of general relativity, and indeed by the example of de Broglie’s nonclassical dynamics.22

‘Einstein’s Hot Water’

This very point was made by Einstein in 1926, in a well-known conversation with Heisenberg (Heisenberg 1971, pp. 62–69). This conversation is often cited as evidence of Einstein’s view that observation is theory-laden. But a crucial element is usually missed: Einstein also warned Heisenberg that his treatment of observation was unduly laden with the superseded theory of classical physics, and that this would eventually cause trouble (Valentini 1992, p. 15; 1996, p. 51).

During the conversation, Heisenberg made the (at the time fashionable) claim that ‘a good theory must be based on directly observable magnitudes’ (p. 63). Einstein replied that, on the contrary (p. 63):

.... it is quite wrong to try founding a theory on observable magnitudes alone. In reality the very opposite happens. *It is the theory which decides what we can observe.* [Italics added.]

Einstein added that there is a long, complicated path underlying any observation, which runs from the phenomenon, to the production of events in our apparatus, and from there to the production of sense impressions. And theory is required to make sense of this process:

Along this whole path .... we must be able to tell how nature functions .... before we can claim to have observed anything at all. Only theory, that is, knowledge of natural laws, enables us to deduce the underlying phenomena from our sense impressions.

Einstein’s key point so far is that, as we have said, there is no *a priori* notion of how to perform a correct measurement: one requires some knowledge of physics to do so. If we wish to design a piece of apparatus that will correctly measure some property $\omega$ of a system, then we need to know the correct laws governing the interaction between the system and the apparatus, to ensure that the apparatus pointer will finish up pointing to the correct reading. (One cannot, for example, design an ammeter to measure electric current without some knowledge of electromagnetic forces.)

Now, Einstein went on to note that, when new experimental phenomena are discovered — phenomena that require the formulation of a new theory — in practice the old theory is at first assumed to provide a reliable guide to interpreting the observations (pp. 63–64):

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22In contrast with Bohr’s unwarranted claim: ‘The unambiguous interpretation of any measurement must be essentially framed in terms of the classical physical theories, and we may say that in this sense the language of Newton and Maxwell will remain the language of physicists for all time’ (Bohr 1931).
When we claim that we can observe something new, we ought really to be saying that, although we are about to formulate new natural laws that do not agree with the old ones, we nevertheless assume that the existing laws — covering the whole path from the phenomenon to our consciousness — function in such a way that we can rely upon them and hence speak of ‘observations’.

Note that this is a practical necessity, for the new theory has yet to be formulated. However — and here is the crucial point — once the new theory has been formulated, one ought to be careful to use the new theory to design and interpret measurements, and not continue to rely on the old theory to do so. For one may well find that consistency is obtained only when the new laws are found and applied to the process of observation. If one fails to do this, one is likely to cause difficulties. That Einstein saw this very point is clear from a subsequent passage (p. 66):

I have a strong suspicion that, precisely because of the problems we have just been discussing, your theory will one day get you into hot water. .... When it comes to observation, you behave as if everything can be left as it was, that is, as if you could use the old descriptive language.

Here, then, is Einstein’s warning to Heisenberg: not to interpret observations of quantum systems using the ‘old descriptive language’ of classical physics. The point, again, is that while observation is in general theory-laden, in quantum theory observations are incorrectly laden with a superseded theory (classical physics), and this will surely lead to trouble.

We claim that the theory of many worlds is precisely an example of what one might call ‘Einstein’s hot water’. Specifically, the apparent multiplicity of the quantum domain is an illusion, caused by an over-reliance on a superseded (classical) physics as a guide to observation and measurement — a mistake that is the ultimate basis of the belief in eigenvalue realism, which in turn led to the puzzle of superposition and to Everett’s valiant attempt to resolve that puzzle.

9 Conclusion

Pilot-wave theory is intrinsically nonclassical, with its own (‘subquantum’) theory of measurement, and it is in general a ‘nonequilibrium’ theory that violates the quantum Born rule. From the point of view of pilot-wave theory itself, an apparent multiplicity of worlds at the microscopic level (envisaged by some theorists) stems from the generally mistaken assumption that eigenvalues have an ontological status (‘eigenvalue realism’), which in turn ultimately derives from the generally mistaken assumption that ‘quantum measurements’ are true and proper measurements.

At the macroscopic level, it might be thought that the universal (and ontological) pilot wave can develop non-overlapping and localised branches that
evolve just like parallel classical worlds. But in fact, such localised branches are unrealistic (especially over long periods of time, and even for short periods of time in a world containing chaos). And in any case, subquantum measurements could track the actual de Broglie-Bohm trajectory, so that in principle one could distinguish the branch containing the configuration from the empty ones, where the latter would be regarded merely as concentrations of a complex-valued configuration-space field.

In realistic models of decoherence, the pilot wave is delocalised, and the identification of a set of parallel (approximately) classical worlds does not arise in terms of localised pieces of actual ‘Ψ-stuff’ executing approximately classical motions. Instead, such identification amounts to a reification of purely mathematical trajectories — a move that is fair enough from a many-worlds perspective, but which is unnecessary and unjustified from a pilot-wave perspective because according to pilot-wave theory there is nothing actually moving along any of the trajectories except one (just as in the classical theory of a test particle in an external field or background spacetime geometry). In addition to being unmotivated, such reification begs the question of why the mathematical trajectories should not also be reified outside the classical limit for general wave functions, resulting in a theory of ‘many de Broglie-Bohm worlds’ (which in turn naturally reduces to a single-world theory).

Properly understood, pilot-wave theory is not ‘many worlds in denial’: it is a different physical theory. Furthermore, from the perspective of pilot-wave theory itself, many worlds are an illusion. And indeed, even leaving pilot-wave theory aside, we have seen that the theory of many worlds is rooted in the intrinsically unlikely assumption that quantum measurements should be modelled on classical measurements, and is therefore in any case unlikely to be true.

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