Static potential from spontaneous breaking of scale symmetry

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Abstract

We determine the static potential for a heavy quark-antiquark pair from the spontaneous symmetry breaking of scale invariance in a non-Abelian gauge theory. Our calculation is done within the framework of the gauge-invariant, path-dependent, variables formalism. The result satisfies the 't Hooft basic criterion for achieving confinement.

1 Introduction

The binding energy of an infinitely heavy quark-antiquark pair represents a key tool which is expected to play an important role in the understanding of non-Abelian theories and especially of quark confinement. In fact, the distinction between the apparently related phenomena of screening and confinement have been of considerable importance in order to gain further insight and guidance into this problem.

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As is well known, the problem of confinement in gauge theories has been studied from different viewpoints, like lattice gauge theory techniques [1] and non-perturbative solutions of Schwinger-Dyson’s equations [2,3]. We further note that recently an interesting approach to this problem has been proposed [4], which includes a linear term in the dielectric field that appears in the energy density. Mention should be made, at this point, to the "Cornell potential" [5] which simulates the features of QCD, that is,

\[ V(r) = -\frac{\kappa}{r} + \frac{r}{a^2}, \]  

here \( a \) is a constant with the dimensions of length. According to 't Hooft, confinement is associated to the appearance of a linear term in the dielectric field \( D \) (that dominates for low \( |D| \)) in the energy density:

\[ U(D) = \rho_{\text{str}} |D|, \]

the proportionality constant being the coefficient of the linear potential, that is, \( \rho_{\text{str}} \approx \frac{1}{a^2} \). It is worthwhile remarking at this point that gauge theories with no scale have a symmetry which is associated to this, scale invariance. Thus it follows that the confinement phenomena breaks the scale invariance as the Cornell potential (1) explicitly shows by introducing the scale \( a \).

With these ideas in mind, one of us (E.I.G.) has studied the connection between scale symmetry breaking and confinement in [6]. In effect, we have observed the appearance of the Cornell potential (1) as well as the 't Hooft relation (2) after spontaneous breaking of scale invariance in an Abelian model. As was explained in [7], the scale invariant model under consideration introduces, in addition to the standard gauge fields also maximal rank gauge field strengths of four indices in four dimensions, \( F_{\mu\nu\alpha\beta} = \partial_{[\mu} A_{\nu\alpha\beta]} \) where \( A_{\nu\alpha\beta} \) is a three index potential. The integration of the equations of motion of the \( A_{\nu\alpha\beta} \) field introduces a constant of integration \( M \) which breaks the scale invariance [8]. More specifically, the linear term in the Cornell potential arises from the constant of integration \( M \). Obviously, when \( M = 0 \) the equations of motion reduce to those of the standard gauge field theory.

Motivated by this, it is natural to ask whether a similar thing happens in the case of a non-Abelian model with a spontaneously broken symmetry. In this work we address this question and, as we shall see, the confining potential between quark-antiquark emerges naturally. Here we would mention that confinement arises as an Abelian effect. In general, this picture agrees qualitatively with that of Luscher [9]. More recently it has been related to relativistic membrane dynamics in [10], and implemented through the abelian projection method in [11]. We further observe that some peculiar quantum aspects of the effective long range dynamics of QCD, and certain intriguing analogies with
the Schwinger model, have been considered in [12]. Once again, our procedure fulfills completely the requirements by 't Hooft for perturbative confinement. Our calculation is based on the gauge-invariant but path-dependent variables formalism [13]. According to this formalism, the interaction potential between two static charges is obtained once a judicious identification of the physical degrees of freedom is made.

2 Scale symmetry breaking

In this section, we discuss the connection between the scale symmetry breaking and confinement, introduced in Ref. [7], in the of path integral formulation. For this purpose we restrict our attention to the action

$$S_{YM} = \int d^4x \left(-\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu}\right),$$

(3)

where $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$. We mention in passing that this theory is invariant under the scale symmetry

$$A_\mu^a(x) \mapsto \lambda^{-1} A_\mu^a(\lambda x),$$

(4)

where $\lambda$ is a constant.

Let us introduce the following parent action functional [15] encoding information about the different phases of the model

$$S_P[\omega, \phi, A_\mu^a, A_{\mu\nu\rho}] = \int d^4x \left[ -\frac{1}{4}\omega^2 + \frac{1}{2}\omega \sqrt{-F_{\mu\nu}^a F^{a\mu\nu}} 
+ \phi(x) \left( \frac{1}{4!} \epsilon^{\mu\nu\rho\sigma} \partial_\lambda (A_{\mu\nu\rho}) - \omega \right) \right],$$

(5)

where $\omega$ and $\phi$ are auxiliary scalar fields, and $A_{\mu\nu\rho}$ is an antisymmetric, rank-three, gauge form. Their scaling transformations are

$$\omega \mapsto \lambda^{-2}\omega(\lambda x)$$

(6)

$$\phi \mapsto \lambda^{-2}\phi(\lambda x)$$

(7)

$$A_{\mu\nu\rho} \mapsto \lambda^{-1}A_{\mu\nu\rho}(\lambda x)$$

(8)

Extended gauge potential have an ubiquitous role in theoretical high energy physics. They act as gauge partner of relativistic extended objects of various
dimensions \[16\], and play a fundamental role in cosmology as well \[17\]. It can be worth to recall that in Yang-Mills theory rank-three gauge potential enters through the topological density term \( F^{* a}_{\mu \nu} F^{a \mu \nu} \) and couples to the membrane boundary of hadronic bubble \[18\]. As an abelian, colorless object, \( A_{\mu \nu \rho} \) is expected to describe the long-range “tail” of QCD \[9\],\[10\].

We notice that the Yang-Mills field enter the action through a Born-Infeld-like term in the strong field limit, suggesting a possible connection with strings/brane formulation of gauge theory \[19\],\[20\].

Dynamics of the whole system is encoded into the partition functional

\[
Z \equiv \mathcal{N} \int \left[ \mathcal{D} A^a_\mu \right] \left[ \mathcal{D} \omega \right] \left[ \mathcal{D} \phi \right] \left[ \mathcal{D} A_{\lambda \mu \nu \rho} \right] \exp \left( -S_P \left[ \omega , \phi , A^a_\mu , A_{\mu \nu \rho} \right] \right) \quad (9)
\]

where \( \mathcal{N} \) is a suitable normalization constant.

Suppose we start by integrating out the three-form gauge potential. We get the following “constrained” partition functional

\[
Z \equiv \mathcal{N} \int \left[ \mathcal{D} \phi \right] \delta \left[ \epsilon^{\lambda \mu \nu \rho} \partial_\lambda \phi \right] \times \exp \left( -\int d^4x \left[ -\frac{1}{4} \omega^2 + \frac{1}{2} \omega \sqrt{-F^{a}_{\mu \nu} F^{a \mu \nu} - \phi(x) \omega} \right] \right) \quad (10)
\]

The Dirac-delta restricts the functional integration over \( \phi \) to constant field configurations only:

\[
\int \left[ \mathcal{D} \phi \right] \delta \left[ \epsilon^{\lambda \mu \nu \rho} \partial_\lambda \phi \right] (\ldots) = \int \left[ \mathcal{D} \phi \right] \delta \left[ \phi - M \right] (\ldots) \quad (11)
\]

Thus, we find

\[
Z_M = \mathcal{N} \int \left[ \mathcal{D} A^a_\mu \right] \left[ \mathcal{D} \omega \right] \times \exp \left( -\int d^4x \left[ -\frac{1}{4} \omega^2 + \frac{1}{2} \omega \left( \sqrt{-F^{a}_{\mu \nu} F^{a \mu \nu}} - M \right) \right] \right) \quad (12)
\]

the integration constant has \((mass)^2\) dimension in natural units. The appearance of a dimensional constant signals the breaking of scale invariance. The \( M \) constant acts as an order parameter for the different phases of the system.

If \( M = 0 \) we recover the familiar Yang-Mills theory after integration over the \( \omega \) field:

\[
Z_{M=0} = \mathcal{N} \int \left[ \mathcal{D} A^a_\mu \right] \left[ \mathcal{D} \omega \right] \exp \left( -\int d^4x \left[ -\frac{1}{4} \omega^2 + \frac{1}{2} \omega \left( \sqrt{-F^{a}_{\mu \nu} F^{a \mu \nu}} \right) \right] \right) = \mathcal{N} \int \left[ \mathcal{D} A^a_\mu \right] \exp \left( -\int d^4x \left[ -\frac{1}{4} F^{a}_{\mu \nu} F^{a \mu \nu} \right] \right) \equiv Z_{YM} \quad (13)
\]
On the other hand, if $M \neq 0$ we get an additional Born-Infeld contribution to the Yang-Mills action, coming from the breaking of scale invariance \[6\]

\[
Z_M = \int d^4x \left[ -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{M}{2} \sqrt{-F^a_{\mu\nu} F^{a\mu\nu}} \right],
\] (14)

where field-independent constant has been absorbed into the normalization factor $\mathcal{N}$. In this case the action contains both a Yang-Mills and a Born-Infeld term. It is important to realize that the integration constant $M$ spontaneously breaks the scale invariance, since both $\omega$ and $\sqrt{-F^a_{\mu\nu} F^{a\mu\nu}}$ transform as in Eq.\((6)\) but $M$ does not transform. We also call attention to the fact that $M$ has the same dimensions as the field strength $F^a_{\mu\nu}$, that is, dimensions of $(\text{length})^{-2}$. It now follows that the variation of the $A^a_{\mu}$ field produces the following equation

\[
\nabla_\mu \left[ \left( \sqrt{-F^a_{\alpha\beta} F^{a\alpha\beta}} + M \right) \frac{F^{a\mu\nu}}{\sqrt{-F^b_{\alpha\beta} F^{b\alpha\beta}}} \right] = 0 .
\] (15)

Once again, we observe as an appealing feature of this expression, that the introduction of the unusual $M$ term leads to the generation of confinement. The above equation admits a “trivial vacuum” solution for $M = 0$, which is

\[
M = 0 \quad (16)
\]

\[
F^a_{\mu\nu} = 0 \quad (17)
\]

while, for $M \neq 0$ one finds that the classical field strength is subject to the condition

\[
\sqrt{-F^b_{\alpha\beta} F^{b\alpha\beta}} = -M .
\] (18)

This kind of constrained field configurations has been originally introduced in \[21\] in order to provide a gauge type description of string dynamics. Later in \[19\], \[20\] an in-depth investigation of gauge-type formulation of string theory has been given. It may be worth to recall that the effective string tension is given by \[19\]:

\[
\frac{1}{2\pi \alpha'} = \frac{|M|}{\sqrt{2}}
\] (19)

Confinement is obvious then, since in the presence of two external sources, by symmetry arguments, one can see that such a constant amplitude chromoelectric field must be in the direction of the line joining the two charges. The
potential that gives rise to this kind of field configuration is of course a linear potential. The two solutions (17) and (18) clearly show in which sense scale invariance is “spontaneously broken” and how this effect leads to linear confinement. Notice that this equation gives confinement only if $M < 0$, otherwise the chromoelectric field will be antiparallel to itself, giving that the confinement piece is zero. Accordingly, the term inside the square brackets corresponds to the ”dielectric field $D^{a\mu\nu}$“. Hence we see that the equations of motion would be obtained from an action of the form

$$S = k \int d^4 x \sqrt{-F^{a\mu}_{\nu} F^{a\mu\nu}},$$

(20)

where $k$ is a constant. Such model leads to confinement and to string solutions [12], [20].

In order to compute the interaction energy we need to write the Born-Infeld term in a more tractable form. To achieve this goal we need to introduce an auxiliary field, say $e(x)$. Thus, we get the on-shell equivalent form

$$S \left[ A^a_\mu, e(x) \right] = \int d^4 x \left[ -\frac{M}{4} e(x) F^{a \mu\nu}_{\mu\nu} + \frac{M}{4} \frac{1}{e(x)} \right],$$

(21)

$$S \left[ A^a_\mu, e(x) \right] = S_{BI} \left[ A^a_\mu \right] \longleftrightarrow e(x) = \frac{1}{\sqrt{-F^{a \mu\nu}_{\mu\nu}}}. $$

(22)

By adding the Yang-Mills term we get for the total action

$$S_p \left[ A^a_\mu, e(x) \right] = \int d^4 x \left[ -\frac{1}{4} \left( 1 + M e(x) \right) F^{a \mu\nu}_{\mu\nu} + \frac{M}{4} \frac{1}{e(x)} \right].$$

(23)

The action (23) explicitly shows that we obtained a Yang-Mills theory with an effective, \textit{point-dependent}, dielectric constant

$$\varepsilon_{YM}(x) \equiv 1 + M e(x).$$

(24)

Equation (24) is a consequence of the non-trivial properties of the Yang-Mills vacuum parametrized through the function $e(x)$. To conclude this section we notice that the dimension of $M$ are the same as the string tension:

$$[M] = \left[ 1/\alpha' \right]$$

(25)

Thus, we expect that the appearance of such a constant is a signal of a \textit{stringy} phase of the Yang-Mills field. In the next section we are going to show this
expectation is correct, as \( M \) enters in the linear part of the interaction energy between a pair of static test charges.

3 Interaction energy

We now examine the interaction energy between external probe sources in the model \( (23) \). This can be done by computing the expectation value of the energy operator \( H \) in the physical state \( |\Phi\rangle \), which we will denote by \( \langle H \rangle_{\Phi} \). The model \( (23) \) cannot be analytically solved in full generality. Thus, we impose spherical symmetry and reduce the problem to an effective \( 1 + 1 \) dimensional model, where the interaction potential can be exactly determined.

Our starting point is the Lagrangian density \( (23) \)

\[
\mathcal{L} = 4\pi r^2 \left\{ -\frac{1}{4} \left( 1 + M\epsilon \right) F^a_{\mu\nu} F^{a\mu\nu} + \frac{M}{4\epsilon} \right\} - A^a_0 J^{a0},
\]

where \( J^{a0} \) is the external current. In passing we note that \( \mu, \nu = 0, 1 \) and \( x^1 \equiv r \equiv x \). As we have noted before, by introducing the auxiliary field \( \varepsilon_{YM}(x) \equiv \frac{1}{V} = 1 + M\epsilon \), expression \( (26) \) then becomes

\[
\mathcal{L} = -4\pi r^2 \left\{ \frac{1}{4V} F^a_{\mu\nu} F^{a\mu\nu} + \frac{M^2}{4} \frac{V}{V - 1} \right\} - A^a_0 J^{a0}.
\]

Once this is done, the canonical quantization of this theory from the Hamiltonian analysis point of view is straightforward. The canonical momenta read \( \Pi^{a \mu} = -4\pi x^1 \frac{1}{V} F^{a0\mu} \), and one immediately identifies the two primary constraints \( \Pi^{a0} = 0 \) and \( P \equiv \frac{\partial \mathcal{L}}{\partial \dot{V}} = 0 \). Standard techniques for constrained systems then lead to the following canonical Hamiltonian:

\[
H_C = \int dx \left( -\frac{V}{8\pi x^2} \frac{\Pi^0}{\Pi^{a0}} \Pi^{0} \Pi^{a_i} + \Pi^0_i \partial^i A^{a0} 
- g f^{abc} \Pi^0_i A^{b0} A^{c_i} + \frac{1}{4} F^a_{ij} F^{a\mu\nu} + A^a_0 J^{a0} + 4\pi x^2 \frac{M^2}{4} \frac{V}{V - 1} \right).
\]

Time conserving the primary constraints \( \Pi^{a0} \approx 0 \) yields the secondary constraints \( \Gamma^{a(1)}(x) \equiv \partial_i \Pi^{a_i} + g f^{abc} A^{b0} \Pi^c - J^{a0} \approx 0 \). The consistency condition for the \( P \) constraint yields no further constraints and just determines the field \( V \),

\[
V = 1 + \frac{|M|}{\sqrt{2}} 4\pi x^2 \frac{1}{\sqrt{\Pi^{a0} \Pi^{a0}}},
\]

7
which will be used to eliminate $V$. The extended Hamiltonian that generates translations in time then reads $H = H_{C} + \int dx \left( c_{0}^{a}(x) \Pi_{0}^{a}(x) + c_{1}^{a}(x) \Gamma_{1}^{a} (x) \right)$, where $c_{0}^{a}(x)$ and $c_{1}^{a}(x)$ are the Lagrange multipliers. Moreover, it follows from this Hamiltonian that $\dot{A}_{a}^{0} (x) = [A_{a}^{0} (x), H] = c_{0}^{a} (x)$, which are arbitrary functions. Since $\Pi_{0}^{a} = 0$, neither $A_{a}^{0}$ nor $\Pi_{0}^{a}$ are of interest in describing the system and may be discarded from the theory. The Hamiltonian then reads

$$H = \int dx \left\{ \frac{\Pi_{ai}^{a} \Pi_{ai}^{a}}{8\pi x^{2}} + \frac{|M|}{\sqrt{2}} + \frac{1}{4} F_{ij}^{a} F_{aij}^{a} + c_{a}^{a} \left( \partial_{i}^{a} A_{ai}^{a} + g f^{abc} A_{bi}^{a} \Pi_{ci}^{a} - J_{i}^{a0} \right) \right\} , \quad (30)$$

where $c_{a}^{a} (x) = c_{1}^{a} (x) - A_{0}^{a} (x)$.

According to the usual procedure we introduce a supplementary condition on the vector potential such that the full set of constraints becomes second class. A useful choice is found to be \[13\]:

$$\Gamma_{a}^{(2)} (x) = \int_{0}^{1} d\lambda \left( x - \xi \right)^{i} A_{ai}^{(a)} (x) \delta \approx 0, \quad (31)$$

where $\lambda$ ($0 \leq \lambda \leq 1$) is the parameter describing the spacelike straight path $x^{i} = \xi^{i} + \lambda \left( x - \xi \right)^{i}$, and $\xi$ is a fixed point (reference point).

This supplementary condition is the non-Abelian generalization of the gauge condition discussed in \[14\], which leads to the Poincaré gauge. There is no essential loss of generality if we restrict our considerations to $\xi^{i} = 0$. In this case, the only nontrivial Dirac bracket is

$$\{ A_{i}^{a} (x) , \Pi^{bj} (y) \}^{\ast} = \delta^{ab} \delta_{i}^{j} \delta^{(3)} (x - y) - \int_{0}^{1} d\lambda \left( \delta^{ab} \frac{\partial}{\partial x^{i}} - g f^{abc} A_{i}^{c} (x) \right) x^{j} \delta^{(3)} (\lambda x - y) . \quad (32)$$

In passing we note the presence of the last term on the right-hand side which depends on $g$.

Now we are in a position to be able to compute the interaction energy between pointlike sources in the theory under consideration, where a fermion is localized at the origin $0$ and an antifermion at $y$. As we have already indicated, we will calculate the expectation value of the energy operator $H$ in the physical state $|\Phi\rangle$. From our above discussion we then get for the expectation value

$$\langle H \rangle_{\Phi} = \text{Tr} \langle \Phi | \int dx \left( \frac{\Pi_{ai} \Pi_{ai}}{8\pi x^{2}} + \frac{|M|}{\sqrt{2}} \sqrt{\Pi_{ai} \Pi_{ai}} + \frac{1}{4} F_{ij}^{a} F_{aij}^{a} \right) |\Phi\rangle . \quad (33)$$
It is easy to see that the first term inside the bracket comes from the usual Yang-Mills theory while the second one is a correction which comes from the square root modification.

Now we recall that the physical state can be written as [13],

$$|\Phi\rangle = \bar{\psi}(y) U(y,0) \psi(0) |0\rangle,$$

where

$$U(y,0) \equiv P \exp \left( ig \int_0^y dz A^a_i(z) T^a \right).$$

As before, the line integral is along a spacelike path on a fixed time slice, $P$ is the path-ordering prescription and $|0\rangle$ is the physical vacuum state. As in [13], we again restrict our attention to the weak coupling limit.

From this and the foregoing Hamiltonian discussion, we then get

$$\langle H \rangle_\Phi = \langle H \rangle_0 + V_1 + V_2,$$

where $\langle H \rangle_0 = \langle 0 | H | 0 \rangle$, and the $V_1$ and $V_2$ terms are given by:

$$V_1 = \text{Tr} \langle \Phi | \int dx \left( \frac{\Pi^a_i \Pi^a_i}{8\pi x^2} \right) | \Phi \rangle,$$

and

$$V_2 = \frac{|M|}{\sqrt{2}} \text{Tr} \langle \Phi | \int dx \sqrt{\Pi^a_i \Pi^a_i} | \Phi \rangle.$$

For more technical details about the derivation of (36) we refer to the following papers [22].

As we have noted before, the $V_1$ term is similar to the energy for the Yang-Mills theory. Nevertheless, in order to put our discussion into context it is useful to summarize the relevant aspects of the analysis described previously [13]. From (37) we then get an Abelian part (proportional to $C_F$) and a non-Abelian part (proportional to the combination $C_F C_A$). As we have noted before, the Abelian part takes the form

$$V_1^{(g^2)} = \frac{g^2}{2} \text{Tr} (T^a T^a) \int_0^y dz^1 \int_0^y dz^1 \frac{\delta (z^1 - z'^1)}{4\pi (z^1)^2},$$
Writing the group factor $\text{Tr}(T^a T^a) = C_F$, the expression (39) is given by

$$V_1^{(g^2)} (L) = - \frac{1}{8\pi} g^2 C_F \frac{1}{L}, \quad (40)$$

where $|y| \equiv L$. Next, the non-Abelian part may be written as

$$V_1^{(g^4)} = \text{Tr} \int \frac{dx}{8\pi x^2} (0| (I^1)^2 |0) , \quad (41)$$

where

$$I^1 = g^2 f^{abc} T^b \int_0^y dz^1 \int_0^1 d\lambda A_1 \left( z^1 \right) z^1 \delta \left( x^1 - \lambda z^1 \right). \quad (42)$$

Expression (41) reduces to

$$V_1^{(g^4)} (L) = \frac{1}{4} g^4 C_A C_F \left( - \frac{1}{L} \right) \int_0^y dz^1 \int_0^y d\lambda D_{11} \left( z^1, z'^1 \right). \quad (43)$$

Here $D_{11}(z^1, z'^1)$ stands for the propagator, which is diagonal in color and taken in an arbitrary gauge. Following our earlier discussion, we choose the Feynman gauge. As a consequence, expression (43) then becomes

$$V_1^{(g^4)} (L) = - g^4 \frac{1}{8\pi^2} C_A C_F \frac{1}{L} \log (\mu L), \quad (44)$$

where $\mu$ is a cutoff. Then, the $V_1$ term takes the form

$$V_1 = - g^2 C_F \frac{1}{8\pi L} \left( 1 + \frac{g^2}{\pi} C_A \log (\mu L) \right). \quad (45)$$

The task is now to evaluate the $V_2$ term, which is given by Eq.(38). Since the field $\Pi^{ai}$ is basically the electric field $F^{a0i}$, we restrict ourselves to constant electric fields (in color space) in order to handle the square root in expression (38). In such a case, we can write $\Pi^{ai} = v^a \Pi^{ai}$, where $v^a$ is a constant vector in color space. In this way, color invariance is explicitly broken. The presence of a constant chromomagnetic field breaks Lorentz invariance as well. However, both symmetries are recovered at large distance by averaging over a large number of randomly oriented vacuum domains. In any case, the confining features of the model are not affected by the averaging procedure. In fact, this same aspect has been discussed recently in the context of (2 + 1) - dimensional reformulated $SU(2)$ Yang-Mills theory [23].
In this case, the only nontrivial contribution is the Abelian one, that is,

$$V_2 = \frac{|M|}{\sqrt{2}} g \text{Tr} \left( v^{1a} e^{1T^a} \right) L .$$  \hspace{1cm} (46)$$

where $e^1$ is a unit vector along the $z^1$.

By putting together equations (45) and (46), we obtain for the total interquark potential

$$V = -g^2 C_F \frac{1}{8\pi L} \left( 1 + \frac{g^2}{\pi} C_A \log (\mu L) \right) + \frac{|M| g}{\sqrt{2}} \text{Tr} \left( v^{1a} e^{1T^a} \right) L ,$$  \hspace{1cm} (47)$$

which has the Cornell form. Notice that confinement is obtained at a finite value of the strong coupling just as claimed by 't Hooft [4].

4 Conclusions

Let us summarize our work. We have shown that the non-Abelian generalization of the model considered in Ref. [7] leads to the Cornell confining potential. More interestingly, it was found that confinement is basically an Abelian effect. In general, this picture agrees qualitatively with that of Luscher [9]. Once again, the gauge-invariant formalism has been very economical in order to obtain the interaction energy, this time showing a confining effect in the context of a non-Abelian effective model. We also draw attention to the fact that the model satisfies indeed the 't Hooft basic criterion for achieving confinement, even with finite coupling constant. In such a case, the necessary term for the dependence of the energy density for low dielectric field (linear in $|D|$) discussed by 't Hooft is here obtained as a result of spontaneous breaking of scale invariance which introduces the constant $M$.

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