Detection and Demultiplexing of Cylindrical Vector Beams Enabled by Rotational Doppler Effect

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Cylindrical vector beams (CVBs) detection is of vital significance in various kinds of studies such as particle observation and mode-division multiplexing (MDM). Herein, a comprehensive detection of CVBs based on the rotational Doppler effect (RDE) including analysis of topological charge (TC), relative amplitude module, and phase for superimposed mode bases is realized. A mode demultiplexing scheme is constructed through the complex amplitude of the harmonic terms in the beating signal. The method resolves both absolute values and signs of the TC of CVB simultaneously, which cannot be simply realized by existing polarization examination techniques. It may be of big potential in related research since an efficient, quantitative, and complete scheme to detect CVBs is verified starting from this work.

1. Introduction

In optics, the polarization state of light relates to spin angular momentum, while the helical wavefront of light relates to orbital angular momentum (OAM).[1,2] They both contribute to a set of higher dimensional bases, cylindrical vector beam (CVB), which is a vectorial solution to the Helmholtz equation[3] and has more complicated properties than the common vortex field with homogeneous polarization. Specifically, a circular polarization state and OAM are classically entangled in CVB,[4] where the cylindrical symmetric inhomogeneous polarization of CVB is an obvious manifestation. Due to the special polarization, there arise many applications in a variety of fields such as super-resolution imaging,[5–7] optical trapping,[8–10] and real-time metrology.[11] Besides, the orthogonality between different CVBs also derives large amounts of applications such as the mode-division multiplexing (MDM).[12–15] high-dimensional encoding,[13,16] and complex Hilbert-space quantum network.[17] MDM usually uses multiple orthogonal beams as carriers of information, where CVB is a potential candidate because of its larger bandwidth capacity than the scalar candidate (e.g., OAM). The state evolution of CVB is correspondent with that of quantum entangled photon pairs, suggesting that CVB as nonseparable states of light can be used to fully characterize quantum entangled channels,[18] then contribute to normal or high-dimensional encoding methods. Apart from these, it is even revealed that real and complex Hilbert-space formulations make different predictions in quantum network scenarios.[17] All these facts confirm a conclusion that a complex spectrum of CVB is necessary to be conducted.

How to effectively conduct a complete CVB spectrum is still to be developed. One simple scheme is to design diffraction operating on left circular polarized (LCP) and right circular polarized (RCP) polarizations. The topological charge (TC) of CVB is recognized by the feature of an intensity image in horizontal (RCP) polarizations. The topological charge (TC) of CVB is recognized by the feature of an intensity image in horizontal polarizations.[19] But this method can only artificially recognize one single-mode CVB at a time. Another useful way is to compose Q-plate and single-mode fiber. A basic Gaussian mode can be coupled into optical fiber only when Q-plate and CVB are successfully coupled.[20,21] This scheme is extended by using cascaded units with Q-plate and interferometer to couple CVBs into optical fibers in different paths.[22] This method can support many CVBs, but the complexity of interferometers restricts its application in practice. In recent years, schemes enabled by deep learning and neural network techniques are developed to detect OAM modes.[23] It is successfully extended to CVB detection,[24] where the response period can reach ≈0.7 ms. Deep learning methods can automatically recognize high-order CVB, but their systems contain an extra training period and only operate on single-mode CVB. A viable scheme to support multimode CVBs is proposed based on a liquid crystal device.[25] It contains spin-dependent geometric transformation and has the function to sort CVBs with different TCs to separate spatial positions. But this scheme needs to fit strict couple conditions of radial modes so is only successful for CVB with special spatial positions. Therefore, it is meaningful to develop a scheme that is valid in a general complex CVB spectrum. Recently, the rotational Doppler effect (RDE) validates plentiful applications in the detection of spinning object[26–29] and is employed to construct OAM spectrum,[30] showing its potential to conduct CVB spectrum. When an OAM light illuminates a spinning object with angular velocity, the reflected light will carry a frequency shift proportional to the TC of OAM.[31–33] Because CVB is composed of two
OAMs with opposite TCs, a similar approach is possible for CVB detection. However, it is predictable that the entangled nature of SAM and OAM in CVB may cause troubles such as parameters of polarization and degeneration of TCs.

In this article, we design and demonstrate a scheme to detect CVB with full parameters enabled by RDE. We first display the RDE-induced beating signal of single-mode CVB. The beating frequency is proportional to the TC of incident CVB while the initial amplitude and phase are computed with a fast Fourier transform (FFT). Then, the scheme is expanded to include the multimode CVB, where distinguishing two CVBs with opposite TCs is an important problem, which is usually degenerated in the analysis of beating signals. In the third, we specifically design a group of detections of multimode CVBs with unknown power and TCs to reach a theoretically complete demultiplexing of the CVB spectrum. This work enables quantitative, complete, and simultaneous detection of CVB, which will show its advantage in CVB mode demultiplexing, particle observing, and other applications.

2. Single-Mode Detection of Vectorial RDE

Previous research has established a frame, where OAM is scattered by a rough spinning object and the output frequency relies on the TC-shift of incident light. The spinning object can be described with a pure phase modulation \( O(\ell, \phi) \), which can be expressed with the Fourier series

\[
O(\ell, \phi, \xi_n) = \sum_n A_n(\ell, \phi, \xi_n) e^{in\phi} e^{-i\ell \xi_n}
\]

where \( \Omega \) is angular velocity and \( A_n(\ell, \phi, \xi_n) \) is the normalized complex amplitude. If a light beam carrying \( \ell \) OAM per photon illuminates the object, the scattered light will be transformed into many OAM modes with TC = \( \ell' \), where \( \ell' = \ell + n \). Each scattered OAM component has a distinct frequency shift \( \Delta f = (\ell' - \ell)\Omega/2\pi = n\Omega/2\pi \). Our research mainly refers to vectorial RDE, where CVB undertakes the incident light. By composing a higher-order Poincaré sphere (HOPS), the state of arbitrary CVB can be expressed with

\[
\Psi_r(r) = \cos \nu e^{-i\xi} u_r(r) \hat{e}_R + \sin \nu e^{i\xi} u_{-r}(r) \hat{e}_L
\]

where \( u_r(r) = B_{\ell'}(r, z)e^{-i\ell'\phi} \) is the complex amplitude of an OAM mode with TC = \( \ell' \) and \( B_{\ell'}(r, z) \) is the part of the amplitude that is independent of azimuthal angle \( \phi \). \( \hat{e}_R \) and \( \hat{e}_L \) are the unitary vectors corresponding to right and left circular polarization. \((2\nu, 2\gamma)\) are the polar angle and the equator angle of HOPS, respectively. 

As shown in Figure 1, a CVB illuminates the spinning object. The scattered light is an aggregation of OAM modes, and each OAM mode contains two different frequency shifts. Notice that orthogonality of OAM modes guarantees that each scattered OAM mode produces beating signals individually. When the TC of incident CVB is \( \ell' = 2 \) and the spinning object is composed with spiral phase plates with \( n = -4, -2, 0, 2, 4 \), beating signals can be observed in the scattered OAM modes with TC \( \ell' = -2, 0, 2 \). Compared with OAM illumination, CVB produces a more effective beating signal because of its enhanced detection bases.

Frequency shifts induce beating signals in multiple scattered OAM modes, then form an overall intensity signal. Considering the horizontal polarization components of scattered OAM modes, the collected intensity of scattered light is

\[
I_{H} = \frac{1}{2} |B_{\ell'}|^2 (|A_\ell|^2 \cos^2 \nu + |A_{-\ell}|^2 \sin^2 \nu + |A_\ell A_{-\ell}^*| \sin 2\nu \cos(2\ell' \Omega t + 2\gamma - \xi) + I_0)
\]

where \( \xi = \arg(A_\ell A_{-\ell}^*) \), \( A_\ell \) and \( A_{-\ell} \) are customized by complex coefficients \( A_n \) in Equation (1). It is found that the angular frequency of the beating signal is \( 2\ell' \Omega \), which is coincident with twice of TC of CVB. Once we know the angular velocity \( \Omega \) of the spinning object, the TC of the incident CVB can be computed throughout the beating frequency. Notice that the time-variant term in Equation (3) is modified by a factor of \( \cos(2\ell' \Omega t + 2\gamma - \xi) \), which ranges in [0, 1]. We can define visibility \( U \) to describe the magnitude of fluctuation over time of light intensity. Preparing maximum and minimum intensities \( I_{max} = \max(I_H) \), \( I_{min} = \min(I_H) \), we have

\[
U = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}
\]
when  \( \nu = 0, \pi/2 \), the visibility becomes zero, pointing out that the time-varying fluctuation eventually vanishes at the poles of HOPS. Visibility  \( U \) reflects the degree of spatial variation of the polarization. At poles, spatial polarization distribution becomes homogeneous, and the beating effect disappears accordingly. If the complex coefficients of the spinning object satisfy  \( |A_\ell| = |A_{-\ell}| \), Equation (4) reduces to  \( U = \sin 2\nu \). It is convenient to determine the polar angle  \( 2\nu \) by analysing the visibility of the collected signal.

CVBs with the same  \( |\ell| \) in Equation (2) share the same beating frequency and thus each component of the CVB spectrum is twofold degenerate. A complex CVB spectrum should include nondegenerated TCs, amplitudes, and phases. To describe a complete spectrum with all possible states of CVBs, two CVBs with opposite signs are required. [36] Figure 2 exhibits two HOPSSs sharing the same beating frequency and two samples of polarization distribution on two spheres.

Different from common RDE detection, vectorial RDE is affected by the variation of polarization state, which can be utilized to distinguish the degeneration of TC of CVB. [39] For two CVBs with opposite TCs, the intensity variance over time makes the beating signal, a one-to-one mapping is expected between TC and beating frequencies. Cross-terms are meaningless and need to be eliminated. In Equations (1) and (2), the coefficients in Equation (7) satisfies  \( \int_0^\infty B_\ell B'_\ell A_\ell A'_\ell \sin^2 \nu e^{-i(\ell - \ell')\Omega t} + A_{-\ell} A'_{-\ell} \sin^2 \nu e^{i(\ell - \ell')\Omega t} + A_\ell A'_{-\ell} \cos \nu \sin e^{-i\nu(\ell - \ell')\Omega t} + A_{-\ell} A'_{\ell} \cos \nu \sin e^{i\nu(\ell - \ell')\Omega t} \)

Here  \( \ell' \neq \ell \), the overall intensity now contains frequencies proportional to cross-terms between different TCs.

To obtain the mode spectrum of the incident beam through the beating signal, a one-to-one mapping is expected between TC and beating frequencies. Cross-terms are meaningless and need to be eliminated. In Equations (1) and (2), the coefficients contain radial Gaussian functions so that they operate on a finite area. Thus, if the radial integration of coefficients in Equation (7) satisfies  \( \int_0^\infty B_\ell B'_\ell A_\ell A'_\ell \sin^2 \nu e^{-i(\ell - \ell')\Omega t} + A_{-\ell} A'_{-\ell} \sin^2 \nu e^{i(\ell - \ell')\Omega t} + A_\ell A'_{-\ell} \cos \nu \sin e^{-i\nu(\ell - \ell')\Omega t} + A_{-\ell} A'_{\ell} \cos \nu \sin e^{i\nu(\ell - \ell')\Omega t} \), the overall intensity can concentrate oscillate at the frequencies proportional to the TC of the incident CVB. Incidentally, this method can also apply to the vertical polarization component of the incident CVB. By replacing  \( \gamma \) with  \( \gamma + \pi/2 \) in Equation (7), the intensity of the vertical component can be detected, whose main term differs from that of the horizontal component by only a sign. Detailed calculation of the difference value between the two components shows that

\[
I_1 = \frac{I_H - I_V}{2} = \sum_{\ell = 1}^\infty \sin 2\nu D_\ell \cos(2\nu\Omega t + \Xi_\ell) \tag{8}
\]

where  \( D_\ell^2 = |C_\ell|^4 + |C_{-\ell}|^4 + 2|C_\ell|^2|C_{-\ell}|^2 \cos 4\gamma, \tan \Xi_\ell = 4|C_\ell|^2 - |C_{-\ell}|^2 + |C_{-\ell}|^2 \) stand for the amplitude and phase of each trigonometric component of the beating signal. Each absolute value of TC eventually corresponds to a unique harmonic term of  \( 2\nu\Omega \). The Fourier transform of Equation (8) can be written as

\[
\mathcal{F}(I_1) = \sum_{\ell = 1}^\infty \sin 2\nu D_\ell \left[ \delta(\omega - 2\nu\Omega)e^{-i\Xi_\ell} + \delta(\omega + 2\nu\Omega)e^{i\Xi_\ell} \right] \tag{9}
\]

If the rotational speed of the spinning object is known, the TC spectrum can be obtained through the real amplitude of the Fourier transform spectrum of Equation (8). In this step, the frequency degeneracy of CVB modes with opposite TCs is still
to be considered. To demonstrate such a degenerate problem more precisely, one can consider a special case where input mode is a superposition of CVBs of TCs \( \ell = \pm 1 \). In this case, the Fourier spectrum has two equal needle-like values at \( \Omega \), whose power ratio is still undetermined. So phase analysis in the Fourier spectrum is required. When \( \tan 2\gamma \neq 0 \), the ratio between amplitudes of CVBs of opposite TCs is given by

\[
\frac{C_l}{C_{-l}} = \sqrt{\frac{1 + \tan \varphi_r / \tan 2\gamma}{1 - \tan \varphi_r / \tan 2\gamma}} \quad (10)
\]

After obtaining the relative superimposed amplitude between CVBs with the same absolute value of TC by Equation (9), the CVB spectrum can be easily calculated.

### 4. Implementation

An experimental configuration is implemented to verify the CVB detection scheme. Figure 3 illustrates the scheme of the experiment. The generation of CVBs is the first step for all three experiments. Recently, we report a new passive generation technique using a polarization-selective Gouy phase shifter, filling the gap of efficient generation of multiple arbitrary CVBs.\(^{[38]}\) But Q-plate\(^{[41-44]}\) is still the simplest way to generate CVBs of low order. Figure 3b is the practical setup to generate single-mode CVB, in which the Q-plate (QP) is set on \( q = 1/2 \). The polar angle of CVB is adjusted by a quarter wave plate (QWP) before the Q-plate. The equator angle of CVB is adjusted by two HWPs (HWP2, HWP3) after the Q-plate. The equator angle of CVB is adjusted by a quarter wave plate (QWP) before the Q-plate. The equator angle of CVB is adjusted by a quarter wave plate (QWP) before the Q-plate. The equator angle of CVB is adjusted by a quarter wave plate (QWP) before the Q-plate.

First, the detection of single-mode is demonstrated by six featured points on the HOPS of \( \ell = 1 \). Figure 4a displays the polar and equator angles of each point and their polarization distributions. A, B1, B2, B3 are four points set on the equator of HOPS with \( 2\alpha = 90^\circ \) and \( 2\gamma = 0^\circ \), \( 40^\circ \), \( 80^\circ \) and \( 120^\circ \) in sequence; C is a point set on \( (50^\circ, 0^\circ) \), which has the same equator angle with radial polarization; D is a point set on \( (50^\circ, 40^\circ) \) indicating the ability to detect arbitrary point on HOPS. The scattered light of screen passes through a pinhole (P) to get a low-noise light, then a charge-coupled device (CCD) collects final beating intensity.

![Figure 3](image-url)

**Figure 3.** a) General setup of the experiment. The combination of HWP and polarizing beam splitter is used to regulate the input light intensity. b) Generation setup for single-mode CVB with \( \ell = 1 \). Quarter wave plate (QWP) and two HWPs are used to adjust the polarization parameter of the CVB. c1) Generation set up for superposition modes of \( \ell = 1, 2 \) with their relative amplitude adjusted by IA. QP1 is a Q-plate whose \( q = 1/2 \) and QP2 is a Q-plate whose \( q = 1 \). c2) Generation setup for superposition modes of \( \ell = -1, 1 \). HWP2 and HWP3 are used to adjust the equator angle of the generated CVBs. Then, CVBs are expanded by two lenses and illuminate an spatial light modulator (SLM). The scattered light of SLM is directed to a pinhole (P) and collected by charge-coupled device (CCD).

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\( \alpha \) is the aligned angle between the fast axis of HWP and the horizontal direction. Thus two HWPs transform the CVB to \((2\alpha, 2\gamma + 4\Delta\alpha)\), where \( \Delta\alpha = d - \alpha \) is the difference of the aligned angles of two HWPs. Figure 3c1,c2 schematically displays the setup to generate multimode CVB. (c1) is the generation setup for superposition CVBs of \( \ell = 1, 2 \), where QP1 is a Q-plate whose \( q = 1/2 \) and QP2 is a Q-plate whose \( q = 1 \). When a Gaussian beam illuminates a Q-plate, the outputting mode with OAM is hypergeometric Gaussian mode.\(^{[45, 46]}\) In addition to a Gaussian function, the radial amplitude distribution of a hypergeometric Gaussian beam is modified by an extra factor of confluent hypergeometric function and radius power term,\(^{[47, 48]}\) and their effect is counteracted by the harmonic coefficients of the spinning object. (c2) is the generation setup for superposition CVBs of \( \ell = -1, 1 \). HWP1 is employed to flip the TC of CVB, while HWP2 and HWP3 are used to adjust the equator angle of the CVB. In multimode CVB detections, the ratio of amplitudes is initialized with \( C_1 : C_2 = 1 : 1 \), which can be controlled by an intensity attenuator (IA). After one of the generations, a CVB illuminates the screen of the spatial light modulator (SLM), which loads a computational hologram to simulate the customized spinning object.\(^{[38]}\) The scattered light of the screen passes through a pinhole (P) to get a low-noise light, then a charge-coupled device (CCD) collects final beating intensity.
red lines are their fitting results. It is found that most of the collected intensity values fit well with the cosine curve, showing that the beating signal in experiments is valid. Each curve undergoes two periods when the spinning object rotates one cycle and their initial phase changes with the equator angle $2\gamma$ of CVB, which agrees well with the prediction in Equation (3). Several intensity values have a large deviation because the employed Q-plate is defective in some angular positions making the collected intensity abruptly decrease at some specific positions. The results also indicate that the elliptically polarized field has a smaller amplitude of fluctuation than linear polarization in the performance of the beating signal. Figure 4c1,c3 quantitatively shows the visibility and phase differences of the selected CVBs. The phase of the intensity signal is acquired by an FFT of the beating signal. FFT is an almost instant information processing method from the beating single, so our scheme possesses an almost instant processing speed.

The main restriction of detection speed is described by the rotation speed of the spinning object. In the experiment, the rotation speed is controlled by switching a sequence of pictures on the screen of SLM, so it is restricted by the response time of SLM and CCD. For the sake of robustness, we applied a 0.25 s delay time between two pictures, which have a rotation of $\pi/100$ rad, thus the average rotation speed in our experiment is $\Omega = \pi/(100 \times 0.25) \approx 0.126$ rad s$^{-1}$. Theoretically, the response time of SLM and CCD approximates to 25 ms, so the detection speed of this scheme can reach about $2\pi/\Omega = 5.00$ s. If SLM is substituted by a faster digital micromirror device (DMD), whose response time is 13.4 $\mu$s, the detection speed can even reach 2.68 ms.

Notice that, in Figure 4c1, the visibility of radial polarization (A point) is about 0.25, which is the result of an extra constant $I_0$ of Equation (3) and some imperfect modulations of SLM.
Figure 4c3, the phase of radial polarization is not exact zero, because all the employed optical devices introduce tiny noise phases. Therefore, an exact detection should include a device-dependent calibration before it performs on arbitrary CVB mode. The calibration is achieved by one detection of a known CVB mode such as radial polarization. The measurement data of the calibration light serve as a reference. The equator and polar angle of an arbitrary CVB are then accurately retrieved by subtracting and dividing the phase and visibility of the calibration data respectively. Such calibration is performed only one time for a detection system to be successfully compensated. Figure 4c2,c4 presents the results of the modified polarization parameters, where A point is selected as the point of calibration. The result after calibration fits well with the theoretical expectations of Equation (3), proving our detection scheme is valid for the single-mode CVB.

The second experiment is implemented to verify the previous discussion about the degeneracy between two CVBs with opposite TCs. In theory, the two CVBs are set on two different HOPSs of \( \ell = 1 \) and \( \ell = -1 \), respectively, as shown in Figure 2. In the first experiment, the beating signals of four points (A, B1, B2, and B3) on the equator of HOPS of \( \ell = 1 \) are collected. To construct a comparison, another four points (A', B1', B2', and B3'), which are set in the same positions as the former four points, are examined to analyze the phase deviations on HOPS of \( \ell = -1 \). In Figure 5a1–a6, all the eight beating signals are divided into six groups, where points A and A' are set for references marked with blue dots, and points B1 – B3 and B1' – B3' are marked with red dots. By comparing B3 with A, it is found that the phase of FFT is shifted with 0.6497 – (−0.00296) = 0.6793(rad). Similarly, the FFT phase of B2 and B1, A', B1', B2', and B3' are 1.3250, 2.0526, 0.0745, −0.5697, −1.2025, and −1.8842, respectively. In the experiment, two HWPs (HW2P1, HW2P2) are set to control the evolution of points on a HOPS. Figure 5b gathers all the measured phases versus relative angles between HW2P1 and HW2P2 and fits them with two lines corresponding to the two groups of points set on HOPSs of \( \ell = 1 \) and \( \ell = -1 \). The two lines demonstrate that a CVB on the HOPS of \( \ell = 1 \) experiences phase delay while a CVB on the corresponding sphere with \( \ell = -1 \) undergoes phase advance. If the sign of TC of CVB is unknown, one can recognize it by the sign of the slope of the fitting line. In the third, a series of detections of multimode CVBs are composed to demonstrate the ability to analyze CVB spectrum.

Figure 6a–f presents beating signals of multimode CVBs of \( \ell = 1 \), 2. The setup for generation is shown in Figure 3c1. With the help of IA, relative amplitudes are set in 1:1 \((C_1 : C_2))\) 1:2 and 2:1 in sequence. The blue dots represent normalized intensities recorded by CCD and the red line represents the theoretical beating signals. In a–c), the equator angles of CVB are set in \( 2 \gamma = 0 \) for both \( \ell = 1 \) and 2. In d–f), the equator angles of CVB are set in \( 2 \gamma = \pi \). These intensities fit well with the beating signal, indicating the collected dots can be successfully employed to acquire beating frequencies 2, 4Ω and related TCs 1, 2. Incidentally, the amplitude of each CVB can be directly computed by FFT meaning that our scheme is applicable for unknown power ratios. Figure 6g,h shows the performance of our scheme in demultiplexing two commonly degenerated CVBs of \( \ell = -1 \), 1 with their equator angle \( \gamma = 40^\circ \). The employed generating setup is shown in Figure 3c2. In this group, IA is used to accurately control the relative amplitude of CVBs of \( \ell = 1 \) and \( \ell = -1 \). In the experiment, the relative amplitudes \(|C_\ell|/|C_{-\ell}|\) in input light is set in 0.498, 0.666, 0.742, 0.830, 0.903, 1.101, 1.201, 1.354, 1.509, and 2.016 in sequence, which is directly collected and measured by an optical power meter. These values are used to compute \( \Xi \) by relationship tan \( \Xi = (|C_\ell|^2 - |C_{-\ell}|^2)/(|C_\ell|^2 + |C_{-\ell}|^2) \tan2\gamma \). Exactly, \( \Xi = -1.282, -1.136, -1.017, -0.801, -0.516, 0.573, 0.884, 1.104, 1.207, \) and 1.331 in sequence. Using Equation (10), an analytical prediction is computed, which is marked with red line. Beating signals of ten selected points of \( \Xi \) are also collected. By a series of calculation with Equations (9) and (10), one can acquire experimental values of relative amplitude, specifically \(|C_\ell|/|C_{-\ell}| = 0.480, 0.626, 0.648, \)
0.838, 0.880, 1.034, 1.125, 1.352, 1.489, and 2.098 in sequence, which are marked with blue triangles. Experimental values agree well with the analytical line, so our scheme is effective in demultiplexing degenerated CVBs. Composing multimode CVBs of $l = 1, 2$ and degenerated CVBs of $l = 1, -1$, the absolute value and sign of TC are both exactly detected, so our scheme is effective to examine a complete CVB spectrum.

Sincerely, many detection schemes are integrated into on-chip systems, which is a trend in the evolution of modern optical applications such as optical communications. In our scheme, the rotating object and polarization detection are possible to be harnessed into an integrated platform. Though the present implementation is composed of bulk volumes, an integrated scheme is prospective in the future.

5. Conclusion

In summary, our research demonstrates a method to detect and demultiplex CVB enabled by RDE. The RDE of CVB introduces a beating frequency in the scattered light beam which is employed to compute parameters of CVB including TC, amplitude, and polarization. Where the polarization is determined by polar and equator angles of HOPS. Three groups of experiments are implemented to examine the validity in the detection of arbitrary single-mode CVB, multimode CVB with opposite TCs, and multimode CVB with different absolute values of TC. CVB with opposite TCs which share the same beating frequency can be distinguished because of opposite phase shifts when their equator angle changes. For multimode CVB, the correspondence between beating signals and mode spectrum is no longer one-to-one due to the presence of cross-beating terms, which can be removed by exploiting a rotating object whose harmonic components form an orthogonal relationship with the radial amplitude distribution of the incident beam. The FFT phase of the intensity difference between the horizontal and vertical polarization of the scattered light is used to demultiplex two degenerate modes. Single-mode CVB and nondegenerated multimode CVB compose the complete detection and demultiplexing of the CVB spectrum. Based on this work, an arbitrary CVB can be simply detected by a group of beating signals. Moreover, the construction of a complete CVB spectrum makes contributions to further
researches aspect of CVB-based long-distance detection, microscopic particle operation, optical communication, and so on.

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Conflict of Interest
The authors declare no conflict of interest.

Data Availability Statement
The data that support the findings of this study are available from the corresponding author upon reasonable request.

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