Investigating Perceived Difficult Concepts in Geometry by Pre-Service Teachers of E. P. College of Education, Bimbilla, Ghana

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Abstract:  
The purpose of this study was to investigate geometry concepts perceived as difficult and likely causes of the difficulties among pre-service teachers of E.P. College of Education, Bimbilla. The population was first year pre-service teachers offering the new Bachelor of Education (B. Ed) programme. A sample size of 103 comprising of 50 males and 53 females were selected through random and convenience sampling techniques. Questionnaire on perceived difficult topics and perceived causes of difficult geometry topics were responded by pre-service teachers. Aside descriptive statistics, an independent sample t-test was employed to find the differences in gender of the perceived difficulty topics in geometry. The results indicate that majority of the pre-service teachers perceived geometry topics to be difficult. The main causes of concept difficulty in geometry according to ranks were non-completion of geometry course outline and study habits after classroom teaching. There was no significant difference between male and female pre-service teachers’ perception of difficult concepts in geometry. This study recommends that weekly professional development sessions by the colleges of education in Ghana should devote significant number of sessions to tackle some of the perceived difficult geometry concepts that impede pre-service teachers’ learning of geometry.

Keywords: Perceived difficult topic, geometry, pre-service teachers

1. Introduction  
The kind of teaching methods adopted by teachers, teacher zeal and commitment and the pace and level at which lessons are presented go a long way to influence perception of difficult topics. Mathematics is a body of knowledge, skills, and procedures that can be used in a rich variety of ways: to describe, illustrate, interpret, predict and to explain patterns and relationships in number, algebra, shape and space, measurements and data. Concepts in mathematics are interrelated and have interconnected elements. The interrelationship of these concepts can be seen in elementary operations such as addition, subtraction, multiplication and division and also translation of word problems and use of symbols across mathematics discourse. According to Robertson and Wright (2014), the interconnected elements are discovery and analysis of pattern, logical reasoning applied to systems and recognition and explanation of the underlying links between these systems. The importance of mathematics in the Ghanaian school curriculum cannot be overemphasized because of its relevance to national development and the fact that it is a compulsory subject that cuts across every other field and also a backbone of all knowledge showing its relevance to all disciplines (Aynila, 2011). Mathematics is also the language of science that allows scientists to communicate ideas using universally accepted terminologies. It is also a requirement for all Senior High School (SHS) graduates to gain admission into higher institutions such as the universities and colleges (Kekere, 2008). Based on this fact, Adegun and Adegun (2013) stressed the need to make mathematics a compulsory subject at the pre-tertiary level of education if scientists, technologists and engineers are to be produced in our universities.

According to Paulina (2007), geometry is a branch of mathematics which deals with the study of different shapes or figures and their properties. The plane shape is a geometrical object with length and width. Plane shapes are also called 2-dimensional shapes such as square, rectangle, circle, and polygon and so on. A solid shape is a geometrical object with
length, width and height. Solid shapes are also called 3-dimensional shapes such as cone, pyramid, sphere, cylinder, prism, cube and cuboid (Salman, 2009). Geometry plays a very important role in pre-tertiary mathematics curricula in Ghana and other countries. According to Battista (1999), geometry provides a rich source of visualization for understanding arithmetical, algebraic and statistical concepts. Volderman (1998) is also of the view that geometry provides a complete appreciation of the world we live in. All these are reasons why pre-service teachers in Ghana and E. P. College of Education in particular should understand concepts in geometry so that they will be able to help improve the achievement of their prospective students in geometry at the basic level of education.

In spite of all these relevance, research exposed the factors that are responsible for pre-service teacher's perceived difficult concepts in geometry to include: lack of background knowledge, poor reasoning skills in geometry, geometric language comprehension, lack of visualization abilities, teachers' method of teaching, and non-availability of instructional materials among others (Mason, 2002; Noraini, 2006; Udousoro, 2011; and Aysen, 2012).

Despite the relevance of mathematics to the nation’s socio-economic and geo-political development, students' performance in the subject in examinations conducted by the West African Examinations Council (WAEC) has remained consistently poor. This poor performance is mostly attributed to candidates' inability to answer questions on geometry. Amazigo (2000) opined that mathematics educators have made selfless efforts to identify the major problems associated with the teaching and learning of mathematics such as poor background in mathematics, lack of incentives for teachers, unqualified teachers in the system and lack of learners' interest. Also students' perception that it is difficult and psychological fear of the subject was identified as factors responsible for the poor performance in the subject. The Institute of Education, University of Cape Coast Chief Examiners' reports July (2015, 2016, 2017, 2018, and 2019) identified geometry as one of the courses in mathematics in which pre-service teachers have not been performing satisfactorily.

2. Literature Review

Bosson-Amenedu (2017) in his study titled “remedial students’ perception of difficult concepts in senior high school core mathematics curriculum in Ghana” found that students identified some mathematics topics such as circle theorem, plane geometry, trigonometry, coordinate geometry, mensuration, bearings and similar triangles as difficult. He also established that Senior High School (SHS) students perceived core mathematics in which geometry is found as the most difficult. The participants were remedial (SHS) students across Ghana who has been unsuccessful in core mathematics with a total sample of 112 comprising of 62 females and 50 males. Another important finding of his study was that SHS students developed their negative perception towards mathematics from JHS level. This is not different from what pre-service teachers of E.P College of Education, Bimbilla went through. In a similar study, analysis of Augustine (2016) showed that proofs of geometrical theorems associated with circles and 3-dimensional problems were difficult to teach. His study used 85 mathematics teachers as the respondents. This is one of the reasons why pre-service teachers have problems with geometry. Furthermore, Fabiyi (2017) study entitled “Geometry concepts in mathematics perceived difficult to learn by senior secondary school students in Ekiti State, Nigeria” used 500 senior secondary school two (SS2) students as the sample and the findings showed that, 8 out of the 23 topics were perceived as difficult concepts. These concepts were congruent triangles, circle theorem, construction and locus, surface areas of solid figures, volume of solid figures, latitude and longitude, coordinates geometry and bearing and distances. Udousoro (2011) study examined secondary school students’ perceived and actual learning difficulties in mathematics. The sample was 120 secondary school students comprising 60 males and 60 females from Akwa Ibom State of Nigeria. His study adopted the survey design approach which utilizes Students’ Perception of Mathematics Difficult Concept Questionnaire (SMDQC) and Mathematics Achievement Test (MAT) as the main instruments. The results indicated that fourteen (14) of the topics learned were identified as difficult based on students’ perceptions. The perceived difficult topics are household arithmetic and commercial arithmetic, approximation, expansion and factorization of algebraic expressions, word problems on algebraic fractions, plane figures and shapes, probability, sets, mensuration, trigonometry, sequence/arithmetic and geometric progression, quadratic equations, statistics, surds and geometrical constructions. Udousoro (2011) concluded that there was a negative significant relationship (r = -0.27) existed between the perceived and actual learning difficulties at 5% level of significance. Finally, Charles-Ogan and George (2015) study purpose was to investigate the perceived difficult concepts in senior secondary school mathematics curriculum. The subjects were 250 SS3 students from river state in Nigeria and the findings revealed the following: longitude and latitude, bearings and mensuration as the perceived difficult topics.

2.1. Statement of the Problem

Geometry plays a very important role in our daily activities, in commerce and industry among others. A close look at the mathematics curriculum reveals the applicability of the Geometrical knowledge in our formal and informal daily activities. The Institute of Education, University of Cape Coast chief examiner’s report has shown that there is about a decade long poor performance of pre-service teachers in geometry despite improved teaching methods and motivational learning strategies.

This trend is frustrating to pre-service teachers’ aspiration to implement effectively the basic school mathematics curriculum. The major gaps identified are that throughout the history of the college and the Nanumba Municipal Assembly, there is no formal study on perceived difficult topics in mathematics across the educational leather. It is an undisputable fact that PSTs are differently gifted and as a result they perceive concepts taught by their tutors differently. There is the need therefore, to know what pre-service teachers perceived about each topic in geometry in terms of difficulty and the degree of difficulty and the likely causes of such difficulties.
2.2. Significance of the Study
It is anticipated that the findings of this study will give curriculum developers new insights into emerging issues on pre-service teachers’ performance and influence the ministry of education on policy formulation. Pre-service teachers are also expected to benefit from the findings because improved performance in geometry will enable them impart positively the knowledge gained onto the basic school child.

2.3. Purpose of the Study
The purpose of this study was to identify geometry concepts perceived as difficult by pre-service teachers of E.P. College of Education, Bimbilla. Also, to determine gender differences on the perceived topics and likely causes of the difficulties.

2.4. Research Questions
- What geometry concepts are perceived to be difficult by the pre-service teachers’?
- What are the causes of the difficulties experienced by the pre-service teachers’ by rank?
- What is the level of difficulty of geometry concepts?

2.5. Research Hypothesis
- Ho: There is no any significant difference between male and female Pre-Service Teachers’ perception of difficult concepts in geometry.

3. Methodology

3.1. Research Design
The design adopted for this study was descriptive research design with purely quantitative approach to data collection on perceived difficult topics in geometry and likely causes of the difficulties.

3.2. Population, Sample and Sampling Procedure
The target population was level 100 (first year) pre-service teachers from E.P College of Education, Bimbilla offering the new B. Ed programme. This study employed two sampling procedures which were convenience and simple random sampling techniques. Convenience sampling is a non-probability sampling method where participants are selected because of their convenience, availability and proximity to the researcher (Castillo, 2009). The researchers are tutors of the college hence the choice of the convenience sampling. Simple random sampling is a process of selecting a sample from the target population where every participant has the same chance of being selected. Random sampling was used to select one hundred and three (103) pre-service teachers comprising fifty (50) males and fifty three (53) females. The level 100 pre-service teachers were chosen because they had just finish the geometry course outline and were about to write their end of first year first semester examinations in Geometry in the 2018/2019 academic year.

3.3. Research Instruments and Pilot
Perceived difficult topics questionnaire and perceived causes of difficult geometry topics were used in the study to collect data. The perceived difficult topics instrument was designed by the researchers after consulting the geometry topics in the Colleges of education mathematics course structure for level 100. The questionnaire composed of twenty (20) items structured in a Likert scale from very difficult, difficult, moderately difficult to not difficult. The second questionnaire on perceived causes of difficult topics was adapted from Charles-Ogan and George (2015) study. The questionnaire contains 10 items using Likert scale of strongly disagree, disagree, neutral, agree to strongly agree. The pilot study was carried out with 30 level 300 PSTs of E.P. College of education-Bimbilla who were not part of this study.

3.4. Reliability and Validity
The two questionnaires on perceived difficult topics in geometry and perceived causes of difficult topics in geometry yielded Cronbach alpha of 0.80 and 0.76 respectively after the pilot study. According to Kline (2006) if the alpha value of a reliability test is 0.90 it depicts excellent, 0.80 means very good and 0.70 is said to be acceptable. Hence, the obtained Cronbach alpha coefficients were good for the study. The two questionnaires were given to two colleague tutors from Tamale College of Education to review and validate before piloting it with 30 PSTs. The two mathematics tutors are specialist in mathematics curriculum, measurement and evaluation.

3.5. Data Collection
Data collection was done on 15th June, 2019 where thirty minutes was given as the duration for respondents to fill the two questionnaires. Instructions were read to participants before the questionnaires were filled. For the sake of confidentiality and anonymity, participants did not write their names on the questionnaires. All the one hundred and three (103) respondents returned their questionnaires.

3.6. Data Analysis
The data that was obtained from the perceived difficult topics and perceived causes were analyzed using percentages, frequency, mean, and standard deviation. The criterion that was set for means for identifying perceived difficult topics in geometry and identifying possible causes of the perceived difficulty in geometry were 2.5 and 3.0
respectively. Also, independent t-test was employed to find the differences in gender of the perceived difficult topics in geometry.

4. Results

4.1. Research Question 1: What Geometry Concepts Are Perceived to Be Difficult by The Pre-Service Teachers?

Table 1 shows frequency, percentages, means, and standard deviation of the geometry concepts perceived to be difficult by pre-service teachers. The following rating were used to clarify the items in Table 1: Very difficult (VD) – 4, Difficult (D) – 3, Moderately Difficult (MD) – 2, and Not Difficult (ND) – 1

| Topics                          | Very difficult | Difficult | Moderately difficult | Not difficult | Mean | St. dev | Decision | Rank |
|--------------------------------|----------------|-----------|----------------------|---------------|------|---------|----------|------|
| **Plane Geometry**             |                |           |                      |               |      |         |          |      |
| Angles and lines               | 12(11.7%)      | 15(14.6%) | 23(22.3%)            | 53(51.5%)     | 1.86 | 1.058   | Not Difficult | 19th |
| Triangles                      | 9(8.7%)        | 14(13.6%) | 24(23.3%)            | 56(54.4%)     | 1.77 | .992    | Not Difficult | 20th |
| Theorems on triangles          | 27(26.2%)      | 16(15.5%) | 25(24.3%)            | 35(34%)       | 2.34 | 1.201   | Not Difficult | 12th |
| Similar triangles              | 20(19.4%)      | 25(24.3%) | 33(32.0%)            | 33(32.0%)     | 2.31 | 1.120   | Not Difficult | 14th |
| Congruent triangles            | 26(25.2%)      | 30(29.1%) | 21(20.4%)            | 26(25.2%)     | 2.54 | 1.127   | Difficult     | 8th  |
| Circles                        | 32(31.1%)      | 28(27.2%) | 22(21.4%)            | 21(20.4%)     | 2.69 | 1.120   | Difficult     | 6th  |
| Polygons                       | 15(14.6%)      | 12(11.7%) | 28(27.2%)            | 40(38.8%)     | 2.17 | 1.175   | Not Difficult | 16th |
| Quadrilaterals                 | 23(22.3%)      | 12(11.7%) | 40(38.8%)            | 10(9.7%)      | 3.17 | 1.024   | Difficult     | 1st  |
| Circle theorems                | 54(52.4%)      | 30(29.1%) | 44(42.7%)            | 28(27.2%)     | 2.61 | 1.096   | Difficult     | 7th  |
| Construction and locus         | 20(19.4%)      | 19(18.4%) | 25(24.3%)            | 39(37.9%)     | 2.19 | 1.147   | Not Difficult | 15th |
| **Mensuration**                |                |           |                      |               |      |         |          |      |
| Lengths, Areas and Perimeter of plane figures | 26(25.2%) | 20(19.4%) | 22(21.4%)            | 35(34.0%)     | 2.37 | 1.213   | Not Difficult | 11th |
| Arcs and sectors of circles   | 19(18.4%)      | 28(27.2%) | 29(28.2%)            | 27(26.2%)     | 2.38 | 1.067   | Not Difficult | 10th |
| Chords and segments of circles| 22(21.4%)      | 33(32%)   | 19(18.4%)            | 29(28.2%)     | 2.48 | 1.136   | Not Difficult | 9th  |
| Surface areas of solid figures | 27(26.2%)      | 31(30.1%) | 23(22.3%)            | 22(21.4%)     | 2.61 | 1.096   | Difficult     | 7th  |
| Volume of solid figures        | 30(29.1%)      | 31(30.1%) | 24(23.3%)            | 18(17.5%)     | 2.71 | 1.072   | Difficult     | 5th  |
| Coordinates geometry           | 29(28.2%)      | 15(14.6%) | 19(18.4%)            | 40(38.8%)     | 2.32 | 1.254   | Not Difficult | 13th |
| **Trigonometry**               |                |           |                      |               |      |         |          |      |
| Pythagoras theorem             | 22(21.4%)      | 15(14.6%) | 19(18.4%)            | 47(45.6%)     | 2.12 | 1.207   | Not Difficult | 17th |
| Sine, cosine and tangent of Right angled triangle | 47(45.6%) | 17(16.5%) | 18(17.5%)            | 21(20.4%)     | 2.87 | 1.202   | Difficult     | 3rd  |
| Angles of elevation and Depression | 33(32%)    | 28(27.2%) | 32(31.1%)            | 10(9.7%)      | 2.89 | 1.145   | Difficult     | 2nd  |
| Bearings and Distances         | 45(43.7%)      | 19(18.4%) | 22(21.4%)            | 17(16.5%)     | 2.82 | .998    | Difficult     | 4th  |

Table 1: Geometry Concepts Perceive to Be Difficult by the Pre-Service Teachers’

The mean value for difficult concept in geometry was set at X ≥ 2.5 otherwise not difficult. From Table 1, under plane geometry the following were identified as the difficult geometry concepts congruent triangles (χ²: 2.54 ≥ 2.50), Circles (χ²: 2.69 ≥ 2.50) and Circle theorems (χ²: 3.17 ≥ 2.50). However, the easiest under the plane geometry topics are angles and lines, triangles and polygons with mean values less than 2.50. Again from Table 1 under mensuration topics the difficult concepts are surface areas of solid figures (χ²: 2.61 ≥ 2.50) and volume of solid figures (χ²: 2.71 ≥ 2.50). The two easiest among the mensuration topics are coordinate geometry and lengths, areas and perimeter of plane shapes with mean values of 2.32 and 2.37 respectively. Finally, under trigonometry the identified difficult geometry topics are sine, cosine and tangent of right angled triangle (χ²: 2.87 ≥ 2.50), angles of elevation and depression (χ²: 2.89 ≥ 2.50) and bearings and distances (χ²: 2.82 ≥ 2.50). The not difficult concept under trigonometry is Pythagoras theorem with a mean value of 2.12.
4.2. Research Question 2: What Are the Causes of The Difficulties Experienced by the Pre-Service Teachers’ in Geometry by Rank?

Table 2 is perceived causes of difficult geometry concepts which are presented by means of frequency, percentages, means, and standard deviation. The scale for Table 2 is as follows: Strongly Disagree=1, Disagree=2, Neutral=3, Agree=4, Strongly Agree=5.

| Causes of concept difficulty in mathematics | SD | D | N | A | SA | Mean | SD | Decision | Rank |
|--------------------------------------------|----|---|---|---|----|------|----|----------|------|
| Lack of mathematics teachers in the school | 33(32%) | 27(26.2%) | 17(16.5%) | 11(10.7%) | 15(14.6%) | 2.50 | 1.413 | Disagree | 10th |
| Non-completion of geometry scheme of work/course outline | 15(14.7%) | 5(4.9%) | 14(13.7%) | 29(28.4%) | 39(38.2%) | 3.71 | 1.404 | Agree | 1st |
| Lack of relating geometry concepts to real life activities | 14(13.6%) | 13(12.6%) | 21(20.4%) | 24(23.3%) | 31(30.1%) | 3.44 | 1.391 | Agree | 3rd |
| Deliberate skipping of some geometry concepts by teachers | 18(17.5%) | 27(26.2%) | 12(11.7%) | 20(19.4%) | 26(25.2%) | 3.09 | 1.476 | Agree | 8th |
| Dominant use of discussion teaching method by teachers in geometry | 17(16.5%) | 22(21.4%) | 19(18.4%) | 16(15.5%) | 29(28.2%) | 3.17 | 1.465 | Agree | 7th |
| There are some geometry concepts that do not interest me | 15(14.6%) | 22(21.4%) | 22(11.7%) | 27(26.2%) | 27(26.2%) | 3.28 | 1.431 | Agree | 4th |
| Insufficient problem solving in geometry concepts | 12(11.7%) | 25(24.3%) | 14(13.6%) | 28(27.2%) | 24(23.3%) | 3.26 | 1.365 | Agree | 5th |
| Non-marking and correction of assignment to find out students' strengths and weaknesses in geometry concepts | 26(25.2%) | 29(28.2%) | 11(10.7%) | 20(19.4%) | 17(16.5%) | 2.74 | 1.448 | Disagree | 9th |
| I do study geometry after classroom teaching | 12(11.7%) | 17(16.5%) | 17(16.5%) | 12(11.7%) | 37(35.9%) | 3.45 | 1.334 | Agree | 2nd |
| I have the belief that geometry is difficult | 16(15.5%) | 22(21.4%) | 15(14.6%) | 19(18.4%) | 31(30.1%) | 3.26 | 1.475 | Agree | 6th |

Table 2: Difficulties Experienced by the Pre-Service Teachers’ in Geometry by Rank
From Table 2, a mean value for agree is $X \geq 3.0$ otherwise disagree was set as the criterion for causes of concept difficulty in geometry. The PSTs agreed to 8 out of the 10 causes of concept difficulty in geometry by rejecting lack of mathematics teachers in the school with (58.2%) disagreement and non-marking and correction of assignment to find out students strengths and weaknesses in geometry concepts (53.4%) disagreement as the causes of concept difficulty in geometry. The following were the agreed causes of the perceived difficult geometry topic by rank are (1) Non completion of geometry scheme of work / course outline ($x^2: 3.71 \geq 3$) representing 68 (66.6%), (2) I do not study geometry after classroom teaching ($x^2: 3.45 \geq 3$) representing 49 (47.6%), (3) Lack of relating geometry concepts to real life activities ($x^2: 3.44 \geq 3$) representing 55 (53.4%), (4) There are some geometry concepts that do not interest me ($x^2: 3.28 \geq 3$) representing 54 (52.4%). The rest are (5) Insufficient problem solving in geometry concepts ($x^2: 3.26 \geq 3$) representing 52 (50.5%), (6) I have the believe that geometry is difficult ($x^2: 3.26 \geq 3$) representing 50 (48.5%), (7) Dominant use of discussion teaching method by teachers in geometry ($x^2: 3.17 \geq 3$) representing 45 (43.7%), and (8) Deliberate skipping of some geometry concepts by teachers ($x^2: 3.09 \geq 3$) representing 46 (43.6%).

4.3. Research Question 3: What Is The Level of Difficulty of Geometry Concepts?

For Table 3, very difficult, difficult, and moderately difficult were collapsed to form difficult which was matched against not difficult to measure the degree level. This is presented by means of frequency and percentage.

| S. No. | Topics                                      | Difficult | Not difficult |
|--------|---------------------------------------------|-----------|---------------|
| Plane Geometry |                                             |           |               |
| 1      | Angles and lines                            | 50(48.5%) | 53(51.5%)     |
| 2      | Triangles                                   | 47(45.6%) | 56(54.4%)     |
| 3      | Theorems on triangles                       | 68(66%)   | 35(34%)       |
| 4      | Similar triangles                           | 70(60%)   | 33(32.0%)     |
| 5      | Congruent triangles                         | 77(74.8%) | 26(25.2%)     |
| 6      | Circles                                    | 82(79.6%) | 21(20.4%)     |
| 7      | Polygons                                   | 59(57.3%) | 44(42.7%)     |
| 8      | Quadrilaterals                              | 63(61.2%) | 40(38.8%)     |
| 9      | Circle theorems                             | 93(90.3%) | 10(9.7%)      |
| 10     | Construction and locus                      | 64(62.1%) | 39(37.9%)     |
| Sub Total | Plane Geometry                        | 673(65.1%) | 357(34.9%)    |
| Mensuration |                                             |           |               |
| 11     | Lengths, Areas and Perimeter of plane figures | 68(66%)   | 35(34.0%)     |
| 12     | Arcs and sectors of circles                 | 76(73.8%) | 27(26.2%)     |
| 13     | Chords and segments of circles              | 74(71.8%) | 29(28.2%)     |
| 14     | Surface areas of solid figures              | 81(78.6%) | 22(21.4%)     |
| 15     | Volume of solid figures                     | 85(82.5%) | 18(17.5%)     |
| 16     | Coordinates geometry                        | 63(61.2%) | 40(38.8%)     |
| Sub Total | Mensuration                             | 447(72.3%) | 171(27.7%)    |
| Trigonometry |                                             |           |               |
| 17     | Pythagoras theorem                          | 56(54.4%) | 47(45.6%)     |
| 18     | Sine, cosine and tangent of Right angled triangle | 82 (79.6%) | 21(20.4%)     |
| 19     | Angles of elevation and Depression          | 93 (90.3%)| 10(9.7%)      |
| 20     | Bearings and Distances                      | 86 (83.5%)| 17(16.5%)     |
| Sub Total | Trigonometry                               | 317(77%)   | 95(23%)       |
| Grand Total |                                        | 1437(69.8%) | 623(30.2%)    |

Table 3: Level of Difficulty of Geometry Topics

From Table 3, 75% and above was set as the criterion for difficult topics. The difficult geometry topics under plane geometry as perceived by the PSTs are congruent triangles, circles and circle theorems with 75%, 80% and 90%
respectively all converted to the nearest whole percentage. Also, the PSTs perceived surface areas of solid figures with 79% and volume of solid figures with 83% under mensuration as the perceived difficult topics. Furthermore, topics under trigonometry perceived difficult were sine, cosine and tangent of right-angled triangle with 80% agreement. Another perceived difficult topic is angles of elevation and depression with 90% endorsement. Finally, bearings and distance received 84% endorsement as difficult topic.

In conclusion, 673 (65.1%) said plane geometry concepts are difficult while 357 (34.9%) said they are not difficult. Also, for mensuration 447 (72.3%) said their concepts are difficult while 171 (27.7%) said they are not difficult. For trigonometry, 317 (77%) said their concepts are difficult and 95 (23%) said they are not difficult. Finally, 1437 responses representing 69.8% perceived all geometry topics to be difficult while 623 responses representing 30.2% said geometry is not difficult.

4.5. Research Hypotheses

- Ho: There is no any significant difference between male and female Pre-Service Teachers’ perception of difficult concepts in geometry in terms of: plane geometry; mensuration; and trigonometry.

Table 4, sort to find out whether gender differences exist under plane geometry section of the perceived difficult topics. The table is making use of mean, standard deviation, degree of freedom, t-value and significant value to make the decision.

| Gender | N  | Mean | SD   | df   | T     | sig  |
|--------|----|------|------|------|-------|------|
| Female | 50 | 2.35 | 0.444| 88.34| 0.497 | 0.62 |
| Male   | 53 | 2.29 | 0.705|       |       |      |

Table 4: Independent Samples T-Test on Perceived Difficult Concepts on Plane Geometry

From Table 4, the results of the Independent samples t-test showed no significant difference between female (M = 2.35, SD = .444) compared to male (M = 2.29, SD = .705) at (t(88.34) = .497, p = .620 > .05) on their perceived difficult concepts under plane geometry. Even though there is no statistically significant differences the female PSTs perceived topics on plane geometry difficult than their male counterparts.

Table 5, sort to find out whether gender differences exist under mensuration section of the perceived difficult topics. The table is making use of mean, standard deviation, degree of freedom, t-value and significant value to make the decision.

| Gender | N  | Mean | SD   | df   | T     | sig  |
|--------|----|------|------|------|-------|------|
| Female | 50 | 2.42 | 0.743| 101  | -.683 | .496 |
| Male   | 53 | 2.53 | 0.812|       |       |      |

Table 5: Independent Samples T-Test on Perceived Difficult Topics in Mensuration

The results of Independent samples t-test samples in Table 5 showed a no statistically significant difference in perceived difficult topics in geometry with (t(101) = -.683, p = .496 > .05) between female and male PSTs on perceived difficult topics in geometry topics under mensuration. The female recorded a lower mean of (M = 2.42, SD = .743) compared with their male counterparts with (M = 2.53, SD = .812) hence the male PSTs perceived mensuration concepts in geometry difficult than their female counterparts.

Table 6, sort to find out whether gender differences exist under trigonometry section of the perceived difficult topics. The table is making use of mean, standard deviation, degree of freedom, t-value and significant value to make the decision.

| Gender | N  | Mean | SD   | df | T     | sig  |
|--------|----|------|------|----|-------|------|
| Female | 50 | 2.66 | .797 | 101| -.229 | .820 |
| Male   | 53 | 2.69 | .900 |    |       |      |

Table 6: Independent Samples T-Test on Perceived Difficult Topics on Trigonometry.

From Table 6, the results of the Independent t-test showed female (M = 2.66, SD = .797) compared to male (M = 2.69, SD = .900) at (t(101) = -.229, p = .820 > .05) signifies no significant difference between their perceived difficult geometry topics under trigonometry. Even though there is no statistically significant differences the male PSTs perceived topics on trigonometry difficult than their female counterparts.

Table 7, sort to find out whether gender differences exist in perceived difficult topics in geometry as a whole. The table is making use of mean, standard deviation, degree of freedom, t-value and significant value to make the decision.

| Gender | N  | Mean | SD   | df  | T     | Sig  |
|--------|----|------|------|-----|-------|------|
| Female | 50 | 2.43 | .440 | 91.238| .011 | .991 |
| Male   | 53 | 2.43 | .657 |     |       |      |

Table 7: Independent Samples T-Test on Perceived Difficult Topics on All Geometry Topics in the Study
From Table 7, the results of the Independent t-test showed no significant difference between female and male PSTs on their perceived difficult geometry topics with female ($M = 2.43, SD = .440$) compared to male ($M = 2.43, SD = .657$) at ($t(91.238) = .011, p = .991 > .05$). This result is unique in the sense that both gender perceived difficult geometry concepts equally although their mean values were equal.

Table 8 makes use of mean, standard deviation, degree of freedom, t-value and significant value to determine whether significant difference existed in each of the items perceived difficult topics in geometry even though the overall decision indicated no difference among male and female PSTs.

| SNo | Topics                              | Gender | N  | Mean | SD   | Df  | t   | Sig   |
|-----|-------------------------------------|--------|----|------|------|-----|-----|-------|
| 1   | Angles and lines                    | Female | 50 | 1.94 | 1.096| 101 | .706| .402  |
|     |                                     | Male   | 53 | 1.79 | 1.026|     |     |       |
| 2   | Triangles                           | Female | 50 | 1.58 | .758 | 90.546 | -1.903| .060  |
|     |                                     | Male   | 53 | 1.94 | 1.151|     |     |       |
| 3   | Theorems on triangles               | Female | 50 | 2.40 | 1.125| 101 | .492| .624  |
|     |                                     | Male   | 53 | 2.28 | 1.277|     |     |       |
| 4   | Similar triangles                   | Female | 50 | 2.42 | 1.108| 101 | .961| .339  |
|     |                                     | Male   | 53 | 2.21 | 1.133|     |     |       |
| 5   | Congruent triangles                 | Female | 50 | 2.56 | 1.110| 101 | .142| .887  |
|     |                                     | Male   | 53 | 2.53 | 1.154|     |     |       |
| 6   | Circles                             | Female | 50 | 2.72 | 1.144| 101 | .269| .789  |
|     |                                     | Male   | 53 | 2.66 | 1.108|     |     |       |
| 7   | Polygons                            | Female | 50 | 2.04 | 1.124| 101 | .365| .716  |
|     |                                     | Male   | 53 | 1.96 | 1.037|     |     |       |
| 8   | Quadrilaterals                      | Female | 50 | 2.38 | 1.176| 101 | 1.739| .085  |
|     |                                     | Male   | 53 | 1.98 | 1.152|     |     |       |
| 9   | Circle theorems                     | Female | 50 | 3.20 | .969 | 101 | .242| .809  |
|     |                                     | Male   | 53 | 3.15 | 1.081|     |     |       |
| 10  | Construction and locus              | Female | 50 | 2.24 | 1.080| 101 | .392| .696  |
|     |                                     | Male   | 53 | 2.15 | 1.215|     |     |       |

| Topics                              | Gender | N  | Mean | SD   | Df  | t   | Sig   |
|-------------------------------------|--------|----|------|------|-----|-----|-------|
| 11 Lengths, Areas and Perimeter of plane figures | Female | 50 | 2.50 | 1.233| 101 | 1.066| .289  |
|                                     | Male   | 53 | 2.25 | 1.191|     |     |       |
| 12 Arcs and sectors of circles      | Female | 50 | 2.46 | 1.073| 101 | .750 | .455  |
|                                     | Male   | 53 | 2.30 | 1.067|     |     |       |
| 13 Chords and segments of circles   | Female | 50 | 2.54 | 1.199| 101 | .556 | .580  |
|                                     | Male   | 53 | 2.42 | 1.082|     |     |       |
| 14 Surface areas of solid figures   | Female | 50 | 2.42 | 1.071| 101 | -1.741| .085  |
|                                     | Male   | 53 | 2.79 | 1.098|     |     |       |
| 15 Volume of solid figures          | Female | 50 | 2.40 | 1.125| 95.716 | -2.928| .004  |
|                                     | Male   | 53 | 3.00 | .941 |     |     |       |
| 16 Coordinates geometry             | Female | 50 | 2.22 | 1.217| 101 | -.788| .433  |
|                                     | Male   | 53 | 2.42 | 1.292|     |     |       |

| Topics                              | Gender | N  | Mean | SD   | Df  | t   | Sig   |
|-------------------------------------|--------|----|------|------|-----|-----|-------|
| 17 Pythagoras theorem               | Female | 50 | 2.06 | 1.168| 101 | -.460| .647  |
|                                     | Male   | 53 | 2.17 | 1.252|     |     |       |
| 18 Sine, cosine and tangent of Right angled triangle | Female | 50 | 2.78 | 1.183| 101 | -.768| .445  |
|                                     | Male   | 53 | 2.96 | 1.224|     |     |       |
| 19 Angles of elevation and Depression | Female | 50 | 2.84 | .976 | 101 | .241 | .810  |
|                                     | Male   | 53 | 2.79 | 1.026|     |     |       |
| 20 Bearings and Distances           | Female | 50 | 2.94 | 1.077| 101 | .401 | .689  |
|                                     | Male   | 53 | 2.85 | 1.215|     |     |       |

Table 8: Independent Sample T-Test for Gender Differences in Perceived Difficulty of PstS Geometry Topics

Statistically significant difference was found at the 0.05 alpha level for 1 out of the 20 topics. Item 15 topic which is Volume of solid figures indicates ($M=2.40, SD=1.125$) for female and ($M=3.00, SD=.941$) for male at $t(95.716) = -2.928$. 
p = .004 < 0.05). This implies that the male PSTs perceived Volume of solid figures difficult than their female counterparts. No statistically significant differences were found for the rest of the topics at the 0.05 level. This latter result implies that both female and male students equally found these topics at the same level of difficulty. The female perceived these items 1,3,4,5,6,7,8,9,10,11,12,13, 19 and 20 as difficult geometry topics slightly higher their male counterparts. While their male counterparts also perceived items 2, 14,15,16,17 and 18 as difficult geometry concepts slightly higher their female counterparts.

5. Discussion of Results

The purpose of this study was to identify geometry concepts perceived as difficult and also examine the level of the perceived difficulties by Pre-Service Teachers of E.P. College of Education, Bimbilla. Finally, to determine gender differences on the perceived topics and likely causes of the difficulties.

Research question one sort to determine geometry concepts perceived to be difficult by the Pre-Service Teachers. The identified difficult geometry concepts are congruent triangles, circles and circle theorems which is similar to Fabiyi (2017), Bosson-Amedenu (2017) studies where those same topics were found to be difficult under plane geometry. For congruent triangles problems it means PSTs lacked combination of ideas on (1) Side-Angle-Side (2) Side - Side -Side (3) Angle -Side -Angle (4) Angle -Angle -Side (5) Leg- Leg theorem (6) Leg –Acute Angle theorem (7) hypotenuse- Acute Angle theorem (8) Acute Angle-Leg postulate. Also, PSTs could not identify and apply the required theorems of circle to solving problems. The theorems include the following: (1) The angles between two tangents to a circle is supplementary with the angle formed between the two radii. (2) Equal chords subtend equal angles at the centre of a circle. (3) The angle between a chord and a tangent is congruent to the interior angle directly opposite to the chord. (5) The angles subtended at the circumference of a circle by the ends of at the same chord are congruent. (6) The angle subtended at the circumference of a circle by the ends of a diameter is 90°. (7) The angle formed between a radii and a tangent is 90° (they are orthogonal). (8) The interior opposite angles of a cyclic quadrilateral are supplementary. (9) The angle subtended at the circumference of a circle by radii is one-half the angle formed between the radii. (10) Exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

Also, this study found that surface areas of solid figures and volume of solid figures which is under mensuration are difficult which also tally with studies by Fabiyi (2017), Charles-Ogan and George (2015) and Bosson-Amedenu (2017). The implication is PSTs could not solve problems on surface areas of cubes, cuboids, cylinder, pyramids, prisms, cones and spheres as well as problems on volumes of cubes, cuboids, cylinders, cones, prisms and right pyramids and spheres. For the trigonometry section, (i) Sine, cosine and tangent of Right angled triangle, (ii) Angles of elevation and Depression (iii) Bearings and Distances were found to be difficult which is also consistent with Udousoro (2011), Fabiyi (2017), Charles-Ogan and George (2015) and Bosson-Amedenu (2017) studies. This means that PSTs had problems with the following concepts: (1) Application of trigonometric ratios in finding angles in the four quadrants. (2) Drawing trigonometric graphs and solving associated problems. (3) Simplifying and evaluating trigonometric expressions. (4) Solving simple trigonometric equations. This finding also signifies that PSTs faced difficulties in solving problems on back-bearings and applying the concept of bearings in real life problems as well as solving questions on angles of elevation and depression.

Research question two was to determine causes of concept difficulty in geometry by the PSTs. The PSTs agreed to all causes of concept difficulty in geometry except (i) lack of mathematics teachers in the school and (ii) non marking and correction of assignment to find out students strengths and weaknesses in geometry concepts. They agreed that non completion of geometry scheme of work / course outline by their tutors is one of the main causes of the perceived difficult concepts. They also indicated that they do not study geometry after classroom teaching. They also affirmed that another causes is their tutors not relating geometry concepts to real life activities. Again they asserted that some geometry concepts that do not interest them. Furthermore, they agreed that insufficient problem solving in geometry concepts is one of the causes hence they believe that geometry is difficult to study. They finally concluded that dominant use of discussion teaching method by teachers in geometry and deliberate skipping of some geometry concepts by teachers does not help them. This finding also corroborates with Charles-Ogan and George (2015) where their study findings tallies with this study.

Research question three which sort to examine the level of the perceived difficult concepts of geometry. The findings has indicated that the PSTs 673 responses representing 65.1% said plane geometry concepts notably congruent triangles, circles and circle theorems are difficult while 357 responses representing 34.9% said they are not difficult. Also, PSTs 447 responses representing 72.3% said mensuration concepts notably surface areas of solid figures and volume of solid figures are difficult while 171 responses representing 27.7% said they are not difficult.

Furthermore, PSTs 317 responses representing 77% said trigonometry concepts such as sine, cosine and tangent of right angled triangle, angles of elevation and depression, and bearings and distances are difficult and 95 responses representing 23% said they are not difficult. Finally, PSTs 1437 responses representing 69.8% perceived geometry topics to be difficult while 623 responses representing 30.2% said geometry topics are not difficult. This finding is also consistent with Fabiyi (2017) who also found more students having difficulty in his study.

The research hypothesis sorts to find significant difference between male and female Pre-Service Teachers’ perception of difficult concepts in plane geometry, mensuration, trigonometry and overall geometry. The findings revealed that there was no significant difference between male and female Pre-Service Teachers’ perception of difficult concepts in plane geometry, mensuration, trigonometry and overall geometry. This study finding does not tally with Fabiyi (2017) and Bosson-Amedenu (2017) studies which revealed significant difference in favor of female students when students were exposed to mensuration and 3-dimensional mathematics instructional material respectively.
6. Conclusions

From the results, the following conclusions were made:
The PSTs perceived eight out of the twenty geometry concepts as difficult and these are congruent triangles, circles, circle theorems, surface areas of solid figures, volume of solid figures, sine, cosine and tangent of right angled triangle, angles of elevation and depression and finally bearings and distances.

The PSTs agreed that the causes of concept difficulty by rank in geometry are as follows: (1) Non completion of geometry scheme of work / course outline. (2) I do not study geometry after classroom teaching. (3) Lack of relating geometry concepts to real life activities. (4) There are some geometry concepts that do not interest me. (5) Insufficient problem solving in geometry concepts. (6) I have believed that geometry is difficult. (7) Dominant use of discussion teaching method by teachers in geometry (8) Deliberate skipping of some geometry concepts by teachers.

The findings revealed that there was no significant difference between male and female Pre-Service Teachers' perception of difficult concepts in plane geometry, mensuration, trigonometry and overall geometry.

7. Recommendations

Based on the findings of this study, the researchers recommended the following:

- The weekly professional development sessions by the colleges of education in Ghana should devote some sessions to tackle the perceived difficult geometry concepts that impede PSTs learning of geometry.
- College Mathematics tutors endeavor to always relate geometry concepts to real life situations, complete the scheme of work and refrain from skipping difficult geometry topics.
- College Mathematics Tutors should appreciate the perceptions of their PSTs and implement teaching methods to make the perceived difficult topics easy for the PSTs.

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