Vortex-lattice interaction in Pulsar Glitches

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Abstract. In this proceeding we present a quantitative model to study the pinning interaction between a neutron vortex and the nuclear lattice in the inner crust of a neutron star, which is formed by nuclear clusters immersed in a superfluid neutron gas. This is one of the most important quantities in the vortex model for pulsar glitches, where the sudden variations of the rotation velocity observed in some neutron stars are explained in terms of pinning-depinning processes between vortices and the nuclear lattice. The study has been done through an actual counting of the number of pinning sites between the vortex and the nuclear lattice. Moreover, it has been recognized that the vortices can have a generic orientation with respect to the lattice and thus the simple pinned-unpinned scenario (that is characteristic of the aligned case and has been assumed in previous qualitative models) is not acceptable for a realistic calculation. The forces that we have found are more than two orders of magnitude smaller than those calculated so far in the literature, and of the same order as what suggested to explain real Vela-like glitches. They can thus be applied to quantitative models for the vortex dynamics, in order to better simulate and understand these remarkable astrophysical phenomena.

1. Introduction

The rotation of a pulsar is characterized by a regular slow down, which is due to the emission of electromagnetic and gravitational waves. However, in the rotation of many pulsars sudden increases in the spin rate, known as glitches, have been observed. These peculiar phenomena are very important in the framework of the study of nuclear matter, because from their observation it may be possible to extract informations about some matter properties. Indeed they have been observed also in isolated stars and so they can only be related to the behavior of matter in the neutron star interior.

The origin of these phenomena is still debated, but if the smaller glitches observed in younger and hotter stars (e.g. Crab) appear to be linked to starquakes \cite{14}, the giant glitches observed in older and colder pulsars (e.g. Vela) seem to be connected to the presence of bulk superfluidity inside these stars. Indeed, the neutron superfluid cannot follow the rigid rotation of the normal (i.e. non-superfluid) component of the pulsar, since the superfluid flow must be irrotational ($\nabla \times \vec{v}_s = 0$). Instead, it develops an array of vortices, each carrying angular momentum, with macroscopic average velocity field satisfying the rigid rotation relation ($\nabla \times \langle \vec{v}_s \rangle = 2\hat{\Omega}_s$) and macroscopic angular velocity proportional to the vortex density ($\Omega_s \propto n_v$).

According to the ‘vortex model’ of Anderson and Itoh \cite{2}, the glitch phenomenon can be the natural macroscopic outcome of the interaction between these vortices and the Coulomb
Figure 1. Representation of some possible vortex-lattice configurations. a and b are the aligned configurations, respectively vortex bound and free, while c is a possible non-aligned configuration.

lattice of neutron-rich nuclear clusters, which is predicted to coexist with the neutron superfluid in the inner crust [15]. This pinning interaction is expected to bind vortices to the normal component of the star, thus freezing the vortex distribution and then fixing the superfluid angular velocity. As the normal matter slows down, the difference between the angular velocity of the two components of the pulsar grows continuously. This differential motion will be the source of a lift force, called the Magnus force, whose intensity per unit length is proportional to the angular velocity difference \( f_{\text{mag}} \propto \Omega_s - \Omega_n \). Only when the total Magnus force equals the total pinning force, the vortex is detached from the lattice (depinning). Thus free to move, it can transfer its angular momentum to the normal component of the star under the action of drag forces. According to the vortex model, giant glitches are due to the simultaneous depinning of a large number of vortices, which is followed by the rapid transfer of their angular momentum. Furthermore such a storage and trigger mechanism would have a natural periodicity, as indeed observed in Vela [5].

In order to make a quantitative study of the pulsar glitch phenomenon, it is crucial to perform an accurate comparison between the total pinning and Magnus forces acting on the vortex line. In this framework, several models have been proposed in order to calculate the pinning interaction between the vortex and a single nucleus [3, 4, 6, 7, 8, 13]; on the other hand, the Magnus force is a classical hydrodynamical force and so it is well known since the nineteenth century. Anyways, a direct comparison between pinning and Magnus forces is still impossible because the Magnus force is given per unit length, while in the cited works the pinning force is calculated per nuclear site: thus an accurate calculation of the pinning force per unit length is still necessary.

In the literature, the comparison between pinning and Magnus forces has been done several times, but using a simple qualitative calculation of the number of pinning sites per unit length (e.g. [1]). Moreover, these calculations were based on the assumption that the vortices are aligned with the lattice, thus considering just two possible configurations, the vortex totally bound (Fig. 1a) and the vortex totally free (Fig. 1b). In this very simple scenario, the pinning force would be proportional to the number of the pinning sites of the bound configuration \( F \propto N \). This assumption has the merit to make the calculations simpler and faster, but the resulting pinning forces are two or three orders of magnitude larger than those suggested to explain real Vela-like glitches [16].

On the other hand, in the nineties Jones [10, 11, 12] proposed a study of the number of pinning sites per unit length with a generic orientation between the vortex and the lattice (Fig. 1c). In this condition, there are not necessarily configurations in which the vortex is totally free, i.e. no pinning sites with the lattice, and the pinning force can be calculated as proportional to the difference between the maximum and the minimum number of pinning sites.
Figure 2. Representation of the vortex partition.

\( F \propto N_{\text{max}} - N_{\text{min}} \). With this model Jones found that the intensity of the pinning force per unit length would decrease as the considered vortex length increases. He thus concluded that, if the rigid portion of vortex is long enough, the pinning force per unit length becomes negligible.

In this proceeding we present the first quantitative estimate of the pinning force per unit length. In the section 2 we will describe the calculation, while in the section 3 we will present the results obtained with our model.

2. The pinning force per unit length

The first quantity necessary for the pinning force calculation is the vortex length on which perform the study. Indeed, if the estimate done by Jones turns out to be correct, the vortex model would no longer explain the glitch behavior. However, in this and all other calculations found in the literature, the vortex has always been considered as unbendable. But its rigidity is not infinite and so in the star it will bend: thus, it is not correct to perform a calculation on the whole vortex, but it is necessary to develop the study dividing it in smaller parts. The length of each part is calculated to be the maximum length so that it can still be considered as rigid (Fig. 2). Its value is obtained by assuming that the vortex, under tension \( T \), will bend under the influence of the pinning force, and thus equating the energy of two limiting configurations: the rigid vortex and the vortex that has bent in order to have an additional pinning site. With this very simple calculation, we have found that the maximum vortex length is approximately \( L \approx 1000 R_{\text{ws}} \), where \( R_{\text{ws}} \) is the radius of the Wigner-Seitz cell.

In this condition, the pinning force that acts on the entire vortex can be found by summing the contribution of each rigid portion

\[
F = \sum \langle F_L \rangle
\]

where the rigid contributions have been calculated as the mean over all the possible orientations between the vortex and the lattice, orientation defined by standard spherical coordinates \((\theta, \varphi)\) with solid angle \(d\Omega\):

\[
\langle F_L \rangle = \frac{\int F_L(\theta, \varphi) d\Omega}{\int d\Omega}
\]

Finally, in order to have a more useful way to calculate the pinning force, we rewrite the expression (1) as an integral relation

\[
F = \sum \langle F_L \rangle = \int \langle F \rangle dL
\]

where \( \langle F \rangle = \frac{\langle F_L \rangle}{L} \) is the pinning force per unit length.
Figure 3. Representation of the translations for an aligned vortex. We show the initial condition (a) and two possible translations perpendicular to the vortex line (b and c).

Figure 4. Number of pinning sites in unit of the Wigner-Seitz radius. We show two possible cases: the vortex aligned (left) and the vortex non-aligned (right) with the lattice. The number of pinning sites is describe in the sidebar. $dx$ and $dy$ represent the translation of the vortex with respect to the initial condition. It is important to notice that, as Jones supposed, the minimum number of pinning sites in the non-aligned configuration is not zero.

In order to use equation (2) for the calculation of the pinning force per unit length, it is necessary to evaluate the quantity $F_L(\theta, \varphi)$. Its calculation is done through a classical approach

$$F_L(\theta, \varphi) \propto \frac{\Delta E(\theta, \varphi)}{D(\theta, \varphi)} = \frac{E_p(N_{\text{free}}(\theta, \varphi) - N_{\text{bound}}(\theta, \varphi))}{D(\theta, \varphi)}$$  \hspace{1cm} (4)

where $\Delta E(\theta, \varphi)$ is the energy difference between the configurations on which the vortex is free and bound, which are respectively characterized by a number of pinning sites equal to $N_{\text{free}}$ and $N_{\text{bound}}$. $E_p$ is the pinning energy per site and $D(\theta, \varphi)$ is the distance between the vortex position of these two configurations.

The study of the number of pinning sites in the two configurations, bound and free, is done with an actual counting of the number of pinning sites between the vortex and the lattice, taken to have a bcc structure. Indeed, considering an initial position (Fig. 3a) and all the possible parallel translations of the vortex line with a given orientation $(\theta, \varphi)$ (e.g. Fig. 3b and 3c), it is possible to obtain graphs of the number of pinning sites with respect to vortex translations (Fig. 4). Furthermore, studying those graphs for some non-aligned cases it has been possible to understand that the configurations characterized by the minimum number of pinning sites are very few, thus the vortex can migrate just avoiding them. In conclusion, we have supposed that if the vortex can reach the configurations with an average energy (and so an average number of pinning sites), it can have a sufficient number of configurations available to be considered free. On the other hand, as expected, it is bound when the number of pinning sites is maximum. Thus it is possible to rewrite the equation (4) as

$$F_L(\theta, \varphi) \propto \frac{E_p(N_{\text{av}}(\theta, \varphi) - N_{\text{max}}(\theta, \varphi))}{\frac{1}{2}D_{\text{max}}(\theta, \varphi)} \propto E_p(N_{\text{av}}(\theta, \varphi) - N_{\text{max}}(\theta, \varphi)) \sqrt{B_{\text{max}}(\theta, \varphi)}$$  \hspace{1cm} (5)

where we have considered that the mean distance between the bound and the free configuration can be taken as equal to the half of the average distance between the bound configurations

$$D(\theta, \varphi) \approx \frac{1}{2}D_{\text{max}}(\theta, \varphi) = \frac{1}{2}n_{\text{max}}^{-1/2}(\theta, \varphi) \propto B_{\text{max}}^{-1/2}(\theta, \varphi)$$  \hspace{1cm} (6)

here $n_{\text{max}}$ and $B$ are respectively the mean density and the number of bound configurations.
The previous equations refer to an attractive interaction between the vortex and the nuclei of the inner crust, but depending on the matter density this interaction can be also repulsive [6, 7]. In the literature the pinning interaction is called Nuclear or Interstitial respectively, when it is attractive or repulsive.

In the interstitial pinning scenario the calculation of the pinning force is very similar to that described for the nuclear pinning regions (5). The only important difference is in the pinning energy per site: since the interaction is repulsive, the pinning energy per site is positive. Therefore, the vortex will be bound to the configurations characterized by a minimum number of pinning sites (those colored in white in figure 4). On the other hand, as in the nuclear pinning case, the vortex can be considered free if it can reach the configurations with an average number of pinning sites. In conclusion, in the interstitial pinning regions

$$F_L(\theta, \varphi) \propto E_p (N_{av}(\theta, \varphi) - N_{min}(\theta, \varphi)) \sqrt{B_{min}(\theta, \varphi)}$$

(7)

3. Results

The results that will be presented are obtained using different quantities from the literature (Tab. 1). Some of these concern the lattice composition (the elements that compose the cells, the Wigner-Seitz radius and the nuclear radius) and they are obtained from [15]. The others, concerning the superfluid properties, are obtained from [7]. These last values have an important dependence from a factor $\beta$ that describe the intensity of the pairing gap

$$\Delta = \frac{\Delta_0}{\beta}$$

(8)

where $\Delta_0$ is the bare BCS gap. This factor is related to the polarization effects of matter on the nuclear interaction. The case $\beta = 1$ describe the non-polarized situation, while the case $\beta = 3$ describe the situation in which the effect of the polarization is maximum. When $\beta = 1$ the mean pairing gap has a maximum of about 3 $MeV$, which corresponds to the strong pairing scenario, while when $\beta = 3$ the mean pairing gap has a maximum of about 1 $MeV$, as usually assumed in the weak pairing scenario.

The study presented here is done considering all the densities reported in table 1. Moreover our estimate of the maximum length for vortex rigidity is quite crude, so that it is important to consider some different vortex lengths in order to cover a greater range of possibilities ($100 R_{ws} < L < 3000 R_{ws}$).

The pinning forces per unit length are reported in figure 5. Here it is possible to see that there is a strong dependence on the maximum vortex length assumed in the calculation of the number of pinning sites. This behavior give a verification of Jones’ hypothesis [10, 11, 12] that

| Region | $\rho$ | Element | $R_{ws}$ | $R_N$ | $E_p$ | $\xi$ |
|-------|-------|---------|---------|-------|-------|-------|
|       |       | $^{98}\text{Zr}$ | 44.0    | 6.0   | 2.6 | 0.2 | 6.7 | 20.0 |
| 2     | 9.6 $\times 10^{12}$ | $^{110}\text{Sn}$ | 35.5    | 6.7   | 1.6 | 0.3 | 4.3 | 13.0 |
| 3     | 3.4 $\times 10^{13}$ | $^{180}\text{Sn}$ | 27.0    | 7.3   | −5.2 | −2.7 | 5.2 | 15.4 |
| 4     | 7.8 $\times 10^{13}$ | $^{150}\text{Zr}$ | 19.4    | 6.7   | −5.1 | −0.7 | 11.2 | 33.5 |
| 5     | 1.3 $\times 10^{14}$ | $^{68}\text{Ge}$  | 13.8    | 5.2   | −0.4 | −0.0 | 38.8 | 116.4 |

Table 1. Quantities used in the calculations. The density ($\rho$) is given in $g cm^{-3}$, the pinning energy per site ($E_p$) is given in $MeV$, while the Wigner-Seitz radius ($R_{ws}$), the nuclear radius ($R_N$) and the coherence length ($\xi$) are given in $fm$. 

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Figure 5. Pinning force per unit length as a function of the density of matter in the inner crust of a neutron star. We show both the strong pairing (left) and the weak pairing (right) scenario. The straight lines represent the pinning force per unit length calculated using three different vortex length, $L = 100 \ R_{ws}$, $L = 1000 \ R_{ws}$ and $L = 3000 \ R_{ws}$, while the dashed lines represent the aligned case, as it is described in the literature. The quantities are given in $cgs$ unit.

pinning goes to zero for infinite vortex rigidity. However, using a realistic maximum vortex length the pinning force is still not negligible.

Moreover a comparison between our results and those found in the literature (Fig. 5) shows that the maximum pinning forces per unit length obtained in this work are two or three orders of magnitude lower than those found for an aligned vortex. Thus, they are of the same order of magnitude as what has been suggested to explain the real behavior of a pulsar glitch ($\langle F \rangle \approx 10^{15} \text{erg cm}^{-2}$) [16]. Furthermore the pinning forces obtained with our model survive in a larger range of densities. Indeed in the presented calculations we have found that there is a considerable contribution to the pinning force also in the regions characterized by interstitial pinning, while in the same regions the existing calculations considered the vortex as free.

4. Conclusions
In this proceeding we have presented a quantitative and realistic calculation of the pinning force per unit length.

With this study it has been possible to verify the strong dependence of the pinning force on the length of vortex assumed as rigid vortex [10, 11, 12]. Indeed we find a decrease of the pinning force as the vortex length increases. On the other hand it has been possible also to disprove the conclusions given by Jones. As a matter of fact, considering a realistic portion of vortex it has been found that the pinning forces per unit length are still not negligible.

Moreover the maximum pinning forces per unit length found with this model are two or three order of magnitude smaller than those reported in literature, where the vortex has been considered as aligned with the lattice. Actually, the forces found in this work are of the same order of magnitude as those suggested to explain the behavior of a real pulsar glitch. Therefore, they can be used in realistic models of vortex dynamics in order to understand what really happens in a pulsar glitch [9].

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