FORWARD MODELING OF REDUCED POWER SPECTRA FROM THREE-DIMENSIONAL K-SPACE

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Received 2014 November 17; accepted 2015 April 17; published 2015 June 10

ABSTRACT

We present results from a numerical forward model to evaluate one-dimensional reduced power spectral densities (PSDs) from arbitrary energy distributions in k-space. In this model, we can separately calculate the diagonal elements of the spectral tensor for incompressible axisymmetric turbulence with vanishing helicity. Given a critically balanced turbulent cascade with \( k_0 \sim k_f^{\nu} \) and \( \alpha < 1 \), we explore the implications on the reduced PSD as a function of frequency. The spectra are obtained under the assumption of Taylor’s hypothesis. We further investigate the functional dependence of the spectral index \( \kappa \) on the field-to-flow angle \( \theta \) between plasma flow and background magnetic field from MHD to electron kinetic scales. We show that critically balanced turbulence asymptotically develops toward \( \theta \)-independent spectra with a slope corresponding to the perpendicular cascade. This occurs at a transition frequency \( f_{2D}(L, \alpha, \theta) \), which is analytically estimated and depends on outer scale \( L \), critical balance exponent \( \alpha \), and field-to-flow angle \( \theta \). We discuss anisotropic damping terms acting on the \( k \)-space distribution of energy and their effects on the PSD. Further, we show that the spectral anisotropies \( k(\theta) \) as found by Horbury et al. and Chen et al. in the solar wind are in accordance with a damped critically balanced cascade of kinetic Alfvén waves. We also model power spectra obtained by Papen et al. in Saturn’s plasma sheet and find that the change of spectral indices inside 9 \( R_p \) can be explained by damping on electron scales.

Key words: methods: numerical – planets and satellites: magnetic fields – plasmas – solar wind – turbulence

Supporting material: tar.gz file

1. INTRODUCTION

Plasma turbulence has been analyzed extensively using in situ measurements of magnetic field and velocity fluctuations in the solar wind (Matthaeus & Goldstein 1982; Tu et al. 1984; Burlaga & Klein 1986; Bruno & Carbone 2005). The statistical properties of these fluctuations are usually interpreted by means of power spectral density (PSD) \( P(f) \) as a function of frequency in spacecraft frame. PSD derived from these in situ measurements of single spacecraft is generally obtained in a one-dimensional (1D) reduced form. This means that fluctuations associated with various wavevectors are observed at the same frequency in spacecraft frame and thus contribute to the spectral density \( P(f) \) at a certain frequency \( f \). However, turbulence models are commonly formulated in three-dimensional (3D) wavevector space, where each wavevector \( k \) is assigned a certain energy. Under the assumption of Taylor’s hypothesis and statistical homogeneity, the observable power spectrum \( P(f) \) can be obtained from the energy distribution in \( k \)-space by a two-dimensional (2D) integration. Although this fact is very well known (e.g., Fredricks & Coroniti 1976; Forman et al. 2011; Wicks et al. 2012; Turner et al. 2012), there is currently no application, where several plasma scales in \( k \)-space have been considered simultaneously. We provide here a numerical tool, which we use to study the effects of various \( k \)-space distributions.

Note that for the calculation of reduced PSDs in frequency space, the applicability of Taylor’s hypothesis and homogeneity needs to be justified for each physical system under investigation, e.g., the solar wind or magnetospheric plasmas. We will return to this issue when we compare observations in the solar wind and Saturn’s magnetosphere with our model results. The transformation from \( k \)-space to the observed frequency space taken in a rest frame moving with respect to the turbulent media is unique. In contrast, the observations taken in the moving frame do not allow us to uniquely determine the spatio-temporal turbulent structure in the rest frame of the media (e.g., Fredricks & Coroniti 1976). Thus, interpretation of observed single spacecraft frequency spectra can only test whether certain \( k \)-space distributions of turbulence are consistent with observations or not, but they cannot uniquely identify one certain \( k \)-space structure.

The statistical properties of turbulent fluctuations have led to the understanding that turbulent fluctuations in plasmas are anisotropic with respect to a background magnetic field \( B_0 \). Matthaeus et al. (1990) observed anisotropy with a composite slab and 2D model consisting of wavevectors parallel and perpendicular to the mean magnetic field, respectively. Bieber et al. (1996) successfully explained the relative ratios of turbulence power in different magnetic field components of \( Helios \) measurements with an 85% 2D and 15% slab model. Saur & Bieber (1999) found that low-frequency turbulent fluctuations in the solar wind could be explained with a combination of 2D and a radial slab model. To explain the anisotropic cascading process, Goldreich & Sridhar (1995) proposed a critical balance between Alfvén wave period and nonlinear time, which leads to spectral anisotropy of the measured PSDs. Although there have been recent results that seem to confirm the critical balance theory (Horbury et al. 2008; Podesta 2009; Chen et al. 2010; Wicks et al. 2010; TenBarge & Howes 2012; He et al. 2013), it is an ongoing debate whether critical balance is an inherent property of plasma turbulence. Tessein et al. (2009), for example, found no dependence of the spectral index on the field-to-flow angle \( \theta \), i.e., the angle between \( B_0 \) and plasma flow \( v \) (relative to the observing spacecraft), using a global background magnetic field in their analysis, and Grappin & Müller (2010) recently
found a θ-independent range, which they claim contradicts the critical balance assumption.

In this paper, we analyze in detail the spectral form of 1D reduced PSD based on critical balance in the MHD and kinetic range of scales. The PSDs are modeled for the first time for oblique angles 0° ≤ θ ≤ 90° from MHD to electron scales based on a 3D energy distribution. In Section 2, we discuss the diagonal elements of the spectral tensor, which depend on two scalar functions for toroidal and poloidal fluctuations, and we show how the tensor reduces to a 1D spectrum if Taylor’s hypothesis holds. In Section 3, we introduce our model for the energy distribution in k-space based on a critically balanced cascade on MHD, ion kinetic, and electron kinetic scales. Further, we discuss potential anisotropic damping terms. In a comparative study in Section 4, we analyze whether the results published by Horbury et al. (2008) in the MHD regime and Chen et al. (2010) in the kinetic regime of the solar wind can be explained by a critically balanced or slab/2D model. In addition to Forman et al. (2011), who made a similar analysis for the results of Horbury et al. (2008), we include kinetic scalings and damping effects. In Section 5, we investigate how spectra observed by von Papen et al. (2014) in Saturn’s magnetosphere are consistent with a critically balanced cascade and damping processes.

We would like to point out that our analysis is also relevant to modeling cosmic-ray transport in turbulent media (e.g., Jokipii 1966; Engelbrecht & Burger 2014, 2015). Cosmic rays are traveling with very large velocities through turbulent plasma fields while being scattered by time-variable fields, which are seen in a Lagrangian frame by the moving particles commonly described in an appropriate guiding center. Thus, cosmic rays are subject to the same sampling effects of anisotropic wavevector distribution as the time-dependent spacecraft measurements commonly also taken at large velocities with respect to the turbulent plasma. The effect of field-to-flow angle on cosmic-ray properties has, for example, been studied by Bieber et al. (1996).

2. DIAGONAL ELEMENTS OF THE SPECTRAL TENSOR

As we are primarily interested in the power spectra of the fluctuations, we will only discuss the diagonal elements of the spectral tensor. Under the assumption of incompressible axisymmetric and mirror symmetric turbulence with regard to the mean magnetic field, a general form of the correlation tensor can be derived (Matthaeus & Smith 1981; Montgomery & Turner 1981). Mirror symmetry leads to vanishing helicity (Oughton et al. 1997) and is therefore equivalent to balanced turbulence, i.e., the same amount of energy in waves parallel and anti-parallel to the background magnetic field (for the imbalanced case, see Lithwick & Goldreich 2003). The diagonal elements of the spectral tensor then depend on only two independent scalar functions Ψ and Φ, which describe the toroidal (shear-Alfvén mode on MHD scales) and poloidal fluctuations (pseudo-Alfvén mode, respectively). The latter are the incompressible limit of the slow mode (Goldreich & Sridhar 1995; Cho et al. 2002).

In a magnetic-field-aligned coordinate system with \( \mathbf{B}_0 \) \( \perp \) \( \mathbf{e}_x \) and \( \mathbf{e}_z \), arbitrarily oriented perpendicular unit vectors perpendicular to \( \mathbf{e}_z \), the diagonal elements can be written as (Oughton et al. 1997; Wicks et al. 2012)

\[
S_{xx}(k) = \frac{k_x^2}{k^2} \Psi + \frac{k_y^2k_z^2}{k^2} \Phi
\]

\[
S_{yy}(k) = \frac{k_y^2}{k^2} \Psi + \frac{k_x^2k_z^2}{k^2} \Phi
\]

\[
S_{zz}(k) = \frac{k_z^2}{k^2} \Phi,
\]

where \( k_x^2 = k_x^2 + k_y^2 \) and \( k_z = k_z \). The trace of the tensor is \( \text{tr}(S(k)) = \Psi + \Phi \). Because the general form of Ψ and Φ is not known, we need to introduce reasonable estimates for the scalar functions Ψ and Φ. According to Lithwick & Goldreich (2001), slow-mode waves are passively cascaded by Alfvén waves, which means that the same scaling can be expected for the poloidal fluctuations Φ. Due to the fact that \( k_z \gg k_x, k_y \), which is commonly observed in plasma turbulence (Matthaeus et al. 1990; Bieber et al. 1996), we may estimate the proportionality between the two functions from the measured power anisotropy

\[
\frac{P_\perp}{P_\parallel} \approx \frac{S_\perp}{S_\parallel} = \frac{k_x^2}{k_z^2} \frac{\Psi}{\Phi} + \frac{k_y^2}{k_z^2} \approx \frac{\Psi}{\Phi},
\]

2.1. Power Spectral Density in Frequency Space

In order to compare modeled energy densities in 3D k-space (e.g., Equation (14)) with measured PSD, we need to calculate the reduced 1D spectrum in frequency space \( P(f) \) from the spectral tensor given in Equations (1)–(3). As the modeled spectra are calculated with the assumption of statistically homogeneous fields and Taylor’s hypothesis, the measured data, which the modeled spectra will be compared with, need to be statistically stationary. When our model is compared to observations, both assumptions, i.e., statistical stationarity and Taylor’s hypothesis, need to be verified for the particular properties of the turbulent system and the relative velocity of...
the moving rest frame with respect to the plasma under investigation. For the application of Taylor’s hypothesis, it is required that \( \mathbf{k} \cdot \mathbf{v} \gg \omega \), where \( \mathbf{k} \cdot \mathbf{v} \) is the frequency of structures or fluctuations being convected over the spacecraft with velocity \( \mathbf{v} \) and \( \omega \) is the frequency of the corresponding wave or fluctuation in the plasma frame. Time stationarity of observed turbulent fields is often checked for with stationarity tests (e.g., Matthaeus & Goldstein 1982). Approximately, it can also be estimated with visual inspection of wavelet scalograms.

In the solar wind, where most of our subsequent analysis is applied, Taylor’s hypothesis is well satisfied in the inertial range (Matthaeus & Goldstein 1982; Narita et al. 2013; Howes et al. 2014). However, when the Mach number drops below unity, e.g., close to the Sun or in planetary magnetospheres, Taylor’s hypothesis might not be valid anymore, in general. Violation of Taylor’s hypothesis may also happen at kinetic scales, where phase speeds of kinetic Alfvén waves (KAWs) and whistler waves can become comparable to the relative plasma velocity \( \mathbf{v} \) and frequencies \( \omega \) in the plasma frame strongly increase.

However, strong anisotropy of the fluctuations, e.g., \( k_\perp \gg k_i \), may help to satisfy Taylor’s hypothesis: for KAWs, the phase velocity is strongly reduced for nearly perpendicular propagation as \( v_{ph} \propto \text{KAW} \times \cos(\theta) \). Therefore, Taylor’s hypothesis holds in the solar wind for a critically balanced KAW cascade even in the dissipation range (Howes et al. 2014). The same reasoning applies to strongly anisotropic KAW fluctuations in the magnetospheres of Jupiter and Saturn (Saur et al. 2002; von Papen et al. 2014). Our model is also applicable for non-relativistic cosmic rays. Here typical energies of 10 MeV correspond to \( \sim 4 \times 10^5 \text{ m s}^{-1} \), for which Taylor’s hypothesis is clearly satisfied. Whistler wave turbulence, on the other hand, violates Taylor’s hypothesis and can therefore not be described by our model.

If Taylor’s hypothesis and homogeneity apply, the diagonal elements of the reduced spectral tensor in frequency space, \( P_\parallel(f) \), can be obtained by integrating the 3D energy densities \( S_i(k) \) over a plane perpendicular to the flow direction \( \mathbf{v} \) (Fredricks & Coroniti 1976):

\[
P_\parallel(f) = \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dk^3 S_i(k) e^{-ik_\mathbf{v}t} \\
= \int_{-\infty}^{\infty} d^3k S_i(k) \delta(2\pi f - k_\mathbf{v}\sin(\theta) - k_z\cos(\theta)).
\]

Here we assumed that the \( z \)-axis is parallel to the background magnetic field (\( e_z \parallel B_0 \)) and \( e_x = e_z \times v \), so that \( e_x \) lies in the plane spanned by \( B_0 \) and \( \mathbf{v} \), which is sometimes called the quasi-parallel direction (Bieber et al. 1996).

In order to evaluate Equation (5) numerically, we apply a coordinate transformation that reduces the dimensions of the integration. If we rotate the coordinate system by the angle \( 90^\circ - \theta \) about the \( y \)-axis such that

\[
k'_x = k_x \sin(\theta) + k_z \cos(\theta) \\
\]

\[
k'_y = k_y \]

\[
k'_z = -k_x \cos(\theta) + k_z \sin(\theta),
\]

then \( k'_x \) will be aligned with \( \mathbf{v} \) and the plane of integration lies in the \( y'-z' \) plane. The unprimed parallel and perpendicular wavenumbers now take the form

\[
k_\parallel = \sqrt{(k'_x \sin(\theta) - k'_y \cos(\theta))^2 + k'_z^2} \]

\[
k_\perp = k'_y \cos(\theta) + k'_z \sin(\theta).
\]

In the primed coordinate system Equation (5) becomes

\[
P_\parallel(f) = \frac{1}{v} \int_{-\infty}^{\infty} d^3k' S_i(k') \delta\left(2\pi f - k'_\parallel\right).
\]

Now the Delta function can be evaluated, and the integration over three dimensions in Equation (5) is thus reduced to two dimensions \( k'_y \) and \( k'_z \), which is much easier to calculate numerically. Note that several different wavevectors map to the same frequency in spacecraft frame. In fact, the PSD \( P_\parallel(f) \) is determined by integration over a plane with normal vector \( k_n = (k_x \sin(\theta), 0, k_z \cos(\theta)) \). For the numerical evaluation, the scalar functions \( \psi' \) and \( \Phi' \) are inserted in Equation (11) using Equations (1)–(3) in the rotated form. With this, we are able to calculate the PSDs \( P_\parallel(f) \) in spacecraft frame.

3. RESULTS FOR A CRITICALLY BALANCED TURBULENCE MODEL

In this section, we introduce a \( k \)-space turbulence model ranging from MHD to electron scales based on commonly discussed models in the literature. With this model we calculate 1D reduced PSD for arbitrary measurement geometries. We focus on the theory of a strong critically balanced turbulent cascade as originally proposed by Goldreich & Sridhar (1995) for Alfvénic turbulence. It has been further developed to include the ion kinetic range (Howes et al. 2008; Schekochihin et al. 2009) and the electron kinetic range (TenBarge et al. 2013). Observations indicate that it is suited to describe solar wind turbulence (Horbury et al. 2008; Chen et al. 2010; Wicks et al. 2010), although these observations have not been explicitly compared with reduced spectra rigorously derived from 3D \( k \)-space models. However, with the forward model derived here, we are able to perform such an explicit test and are able to check whether the observations are truly in agreement with critical balance and other suggested properties of 3D \( k \)-space.

3.1. Critically Balanced Cascade on MHD Scales

According to Goldreich & Sridhar (1995), the (kinetic) energy distribution on MHD scales can be described as

\[
E(k) \sim \frac{V_A^2}{k^{10/3}} L^{1/3} f\left(L^{1/3} \frac{k_\parallel}{k_\perp^{2/3}}\right).
\]

where \( V_A = B_0/\sqrt{\rho_0} \) is the Alfvén speed, with \( \rho \) being the mass density of the plasma, \( L \) is the outer scale, and \( f\left(L^{1/3} \frac{k_\parallel}{k_\perp^{2/3}}\right) \) is a positive, symmetric function, that becomes negligibly small, when \( \left|\frac{L^{1/3} k_\parallel}{k_\perp^{2/3}}\right| \gg 1 \) (Goldreich & Sridhar 1995). It is this function which contains the critical balance relation of wave vectors on MHD scales

\[
k_\parallel \sim L^{-1/3} k_\perp^{2/3}.
\]
The energy is supposed to be isotropically injected into the system on the outer scale $L$, where the excitation is assumed to be strong, $\delta v_L \sim V_A$. This relation leads to spectral anisotropy, i.e., different scalings depending on the field-to-flow angle $\theta$. Cho et al. (2002) find that the function that best explains the magnetic fluctuations in their simulation of incompressible MHD turbulence is

$$E_{\text{MHD}} = \left(\frac{B_0^2}{L^{1/3}}\right) k_L^{-10/3} \exp\left(-L^{1/3} \frac{|k_i|}{k_L^{2/3}}\right).$$  \hspace{1cm} (14)

According to Equation (12), this indicates that the magnetic fluctuations on the outer scale $L$ are strong, $\delta B \sim B_0$. However, Forman et al. (2011) find from fits to solar wind data that $B_0^2$ overestimates the energy of the turbulent fluctuations by a factor of two to three. Therefore, the energy level of the modeled power spectra cannot be obtained unambiguously.

The energy distribution given by Equation (14) describes the Alfvénic fluctuations, and we thus set $\Psi = E_{\text{MHD}}$ on MHD scales. According to Equation (4), this constrains the poloidal function to $\Phi = \frac{P_i}{P_\perp} \Psi$.

Next to the exponential function used in Equation (14), there are several different possibilities to describe $f(u)$, e.g., Dirac Delta, Heaviside, or Gauss functions. The particular choice of $f(u)$, however, results in similar spectral anisotropies as found in the case of MHD scales by Forman et al. (2011) and Turner et al. (2012). For $\theta = 90^\circ$ the 1D PSD scales as $P(f) \propto f^{-5/3}$, and for $\theta = 0^\circ$ as $P(f) \propto f^{-2}$ in the inertial range. Exact power laws in frequency space only occur for these two extreme cases. In the literature, however, it is often implicitly assumed that the spectra in the intermediate range $0^\circ < \theta < 90^\circ$ also follow power laws with spectral slopes $5/3 < \kappa < 2$ and that the spectral index $\kappa$ of the reduced spectra is—similar to the spectral index of 3D $k$-space—indepedent of frequency. In this paper, we will see that these assumptions are incorrect.

3.2. Transition from MHD to Kinetic Scales

When the observed Doppler-shifted frequencies reach scales close to the characteristic ion scales, such as the ion gyroradius $\rho_i$ or the ion inertial length $\lambda_i$, the MHD approximation breaks down and one needs to take into account kinetic effects to describe the plasma dynamics. For the remainder of this paper, we use the gyroradius as the controlling kinetic scale with an associated critical balance on ion kinetic scales

$$k_i \sim L^{-1/3} \rho_i^{-1/3} k_L^{1/3},$$  \hspace{1cm} (15)

according to Howes et al. (2008). Here we drop the factor $(\beta_i + 2/(1 + T_e/T_i))^{1/6}$, where $\beta_i$ is the ion plasma beta and $T_i$ and $T_e$ are the temperatures of ions and electrons, respectively, because the factor is of order unity in the solar wind and Saturn’s magnetosphere. Note that our forward model can similarly be formulated for other theories or with the inertial length as the controlling scale.

We model the transition from MHD to kinetic scales as an abrupt change in the function $E_{\text{MHD}}(k)$ at $k_{\perp}\rho_i = 1$ without energy loss, e.g., due to ion resonances. Let us denote the function $E_{\text{MHD}}$ given in Equation (14), as the one applicable for $k_{\perp}\rho_i \ll 1$ and $E_{\text{KAW}}$ as the one applicable for $k_{\perp}\rho_i \gg 1$. At $k_{\perp}\rho_i = 1$, both energy distributions are equal, $E_{\text{MHD}}(k_{\perp}\rho_i = 1) = E_{\text{KAW}}(k_{\perp}\rho_i = 1)$. Also, the kinetic energy distribution, $E_{\text{KAW}}$, is assumed to scale as $P(f) \propto f^{7/3}$ for $\theta = 90^\circ$ (Howes et al. 2008). These requirements are fulfilled in the expression

$$E_{\text{KAW}} = \left(\frac{B_0^2}{L^{1/3} \rho_i^{1/3}}\right) k_L^{-11/3} \exp\left(-L^{1/3} \rho_i^{1/3} \frac{|k_i|}{k_L^{1/3}}\right).$$  \hspace{1cm} (16)

For the parallel cascade ($\theta = 0^\circ$), this leads to a scaling of $P(f) \propto f^{-5}$.

At even smaller scales, we approach the electron gyroradius $\rho_e$ and the energy distribution changes once again from $E_{\text{KAW}}$ to an electron or dissipation (ED) range distribution $E_{\text{ED}}$. Here Landau damping is assumed to weaken the cascade so that there is no more parallel transfer of energy (Sridhar & Goldreich 1994; Howes et al. 2008). The electron or dissipation range fluctuations are modeled with an associated critical balance (TenBarge et al. 2013) of

$$k_i \sim L^{-1/3} \rho_i^{-1/3} \rho_e^{-1/3}.$$

(17)

A functional form that satisfies the equality of $E_{\text{KAW}}$ and $E_{\text{ED}}$ at $k_{\perp}\rho_i = 1$ can be given by

$$E_{\text{ED}} = \left(\frac{B_0^2}{L^{1/3} \rho_i^{1/3}}\right) k_L^{-11/3} \exp\left(-L^{1/3} \rho_i^{1/3} \frac{|k_i|}{k_L^{1/3}}\right).$$  \hspace{1cm} (18)

It can be shown that this leads to a scaling of $P(f) \propto f^{-8/3}$ for the perpendicular cascade ($\theta = 90^\circ$) and an exponential decay $P(f) \propto \exp(-f)$ for the parallel cascade.

The energy distribution in $k$-space according to our model is shown in Figure 1 for the parameters given in Table 1. It shows logarithmically equidistant iso-contours of energy densities according to Equations (14), (16), and (18) in a double-logarithmic plot as a function of $k_i$ and $k_z$ normalized by the ion
gyroradius \( \rho_i \). The characteristic slopes of the critical balance relations, 2/3, 1/3, and 0 in the MHD, KAW, and ED regime, respectively, can be seen as the boundary, where the energy becomes negligible (dark blue). Note that the energy density decreases with \( k_i \) and \( k_{\perp} \), which leads to a seemingly negative slope in the electron dissipation range.

Recall from Equation (5) and subsequent discussion that the reduced 1D spectrum is calculated by integrating over a plane given by

\[
k_x \sin(\theta) = \frac{2\pi f}{v} - k_z \cos(\theta).
\]

Projections of these planes into the \( k_x - k_z \) plane are shown as dashed lines in Figure 1 for an angle \( \theta = 1^\circ \) at logarithmically equidistant frequencies \( f = 10^{-4} - 10 \text{ Hz} \). For \( \theta = 0^\circ \), these dashed lines would be horizontal and parallel to the \( k_x \)-axis, and for \( \theta = 90^\circ \), they would be vertical and parallel to the \( k_z \)-axis. However, due to the double-logarithmic nature of the plot, the dashed lines for intermediate angles \( \theta \) are curved. Note that even for a very small angle \( \theta = 1^\circ \), the plane of integration turns quasi-perpendicular to the \( k_z \)-axis, i.e., lies parallel to planes expected from integration for \( \theta = 90^\circ \), for sufficiently large \( k_z \).

### 3.3. Cascade toward Quasi-perpendicular Spectra

While Equation (11) can be integrated analytically for angles \( \theta = 0^\circ \) and \( \theta = 90^\circ \), it can only be numerically evaluated for intermediate angles \( 0^\circ < \theta < 90^\circ \). Here we carry out this numerical integration for the first time considering all scales from the MHD to the electron kinetic regime. Results of this calculation using the parameters given in Table 1 are shown in Figure 2. We plot the resulting PSD for several field-to-flow angles as a function of frequency \( f \) in spacecraft frame. Note that frequency can be transformed into normalized wavenumber according to \( \frac{2\pi f}{v} = k \rho_i \), particularly for parameters in Table 1: \( f \approx 0.95 \cdot k \rho_i \).

For \( \theta = 90^\circ \) the spectral breaks of the PSD, which mark the transition from MHD to KAW and KAW to ED range, are located at the Doppler-shifted frequencies corresponding to \( \rho_i \) and \( \rho_e \) (denoted by vertical dashed lines). Here the spectral slope of the reduced PSD steepens from 5/3 to 7/3 and from 7/3 to 8/3, respectively. For smaller angles \( \theta < 90^\circ \) these spectral breaks are less pronounced and occur at lower frequencies. The expected spectral slopes for \( \theta = 0^\circ \) and \( \theta = 90^\circ \) are shown as black lines in Figure 2 to guide the eye. Here and throughout this paper, we use plasma parameters characteristic of the solar wind at 1 AU as given in Table 1, which we adopt from the work of Alexandrova et al. (2009), Schekochihin et al. (2009), and Chen et al. (2010).

We note a particularly interesting feature in Figure 2: the spectra for oblique angles are steepened in a short frequency range around the first spectral break at \( 2\pi f \left( \sin(\theta) \right) \rho_i = k_{\perp} \rho_i \sim 1 \) and then flatten out toward a slope of 7/3 at higher frequencies. The smaller the angle \( \theta \), the longer the range, where PSDs are steeper than 7/3. Such an asymptotic behavior has been predicted for the MHD range by Forman et al. (2011), however, without specifying the frequency range where this transition could be expected. Here we show for the first time that this transition happens on a short frequency range around the spectral break in the solar wind. We emphasize the importance of this result as it contradicts the commonly assumed constancy of the spectral index with frequency. Also, it implies that PSDs in the kinetic range are almost exclusively observed with their perpendicular slope (here 7/3 and 8/3) if not subject to damping. In the remainder of this section, we explain this feature as a geometrical or sampling effect.

The flattening of the PSD, although puzzling at first, can be explained with the anisotropic distribution of energy in \( k \)-space. Because of the linear relation between \( k_i \) and \( k \), in Equation (19), the point of maximum curvature of the integration plane in Figure 1 grows faster than the critical balance relations \( k_i \sim k^{1/3} \) in the MHD \((\alpha = 2/3)\), KAW \((\alpha = 1/3)\), and ED \((\alpha = 0)\) range. Hence, for increasing frequencies \( f \), the plane of integration given by Equation (19) effectively has only contributions from the part quasi-perpendicular to the \( k_z \)-axis in the double-logarithmic plot of Figure 1. Therefore, at frequencies above a transition frequency \( f_{3D} \), the power spectrum at a given field-to-flow angle \( \theta \) will have a slope within a small error \( \Delta \kappa \) to that at \( \theta = 90^\circ \) appropriate to MHD \((5/3)\), KAW \((7/3)\), or ED \((8/3)\). The transition frequency \( f_{3D} \) thus marks the boundary where the anisotropy of the \( k \)-space energy distribution becomes so large that it is indistinguishable from a 2D distribution in the reduced PSD within measurement error \( \Delta \kappa \). It is this feature that causes
the spectral slope of the PSD to asymptotically approach its perpendicular value at high frequencies.

A derivation of an analytically approximated expression \( f_{\text{3D}}^{\theta} \) (see Equation (39)) can be found in the Appendix. The frequencies, where the PSDs turn quasi-perpendicular, are shown in Figure 2 as colored dots assuming a pure KAW cascade. Note that the transition frequency \( f_{\text{3D}}^{\theta} \), in contrast to the spectral break, shifts to higher frequencies for smaller angles \( \theta \). Assuming a plain MHD cascade, the approximate transition frequency from Equation (39) gives \( f_{\text{3D}}^{\text{MHD}}(\theta = 20^\circ) = 0.2 \text{ Hz} \). The corresponding transition frequencies for \( \theta = [1^\circ, 5^\circ] \) reach into the kinetic range of scales. The transition to quasi-perpendicular scaling is always found at frequencies below the electron spectral break, \( k_{\perp}\rho_e \sim 1 \), which is why frequencies \( f_{\text{3D}}^{\theta} \) are not shown in the ED range.

We find that the transition for KAW toward a quasi-perpendicular cascade for almost all field-to-flow angles (namely, \( \theta > 4^\circ \); see Figure 14 in the Appendix) occurs already below 1 Hz, where the spectral break is observed at typical solar wind conditions (Sahraoui et al. 2010; Alexandrova et al. 2012). We conclude that spectra of a critically balanced KAW cascade without damping on kinetic scales will almost exclusively be observed with spectral indices close to 7/3 and 8/3 in the ion and electron kinetic range, respectively. Only for small angles and in a short frequency range between the first spectral break and 1 Hz might a measurably steeper slope be observed in the solar wind.

We stress the importance of this result: we have shown that the spectral index in a critically balanced cascade for oblique field-to-flow angles \( 0^\circ < \theta < 90^\circ \) is not constant with growing frequency. Instead, it evolves toward a quasi-perpendicular slope as a function of frequency. Therefore, we cannot expect to observe significantly steeper slopes than 7/3 and 8/3 over a broad frequency range in the KAW and ED range, respectively, for a critically balanced cascade in the solar wind. Indeed, we see in Section 3.4 that such slopes only appear if the fluctuations are additionally subject to damping. This transition of reduced PSD toward quasi-perpendicular spectra contrasts a common understanding in the literature, where it is often implicitly assumed that the PSDs have a power-law shape with a constant spectral index that only depends on the field-to-flow angle. It is interesting to note that a 3D direct numerical simulation with strong guide field by Grappin & Müller (2010) produced a qualitatively similar transition toward a perpendicular cascade with a \( \theta \)-independent slope, although the flattening of their spectra does not follow Equation (39).

### 3.4. Anisotropic Damping

Several mechanisms have been proposed to explain the dissipation of fluctuations on small scales, e.g., ion-cyclotron damping, Landau damping, and current sheet formation (Matthaeus et al. 1990; Leamon et al. 1999; Dmitruk et al. 2004; Howes 2009; Schekochihin et al. 2009; TenBarge et al. 2013). Most of the proposed mechanisms are anisotropic with regard to the background magnetic field. In this section, we analyze how possible anisotropic damping terms that act on the energy distribution in 3D \( k \)-space change the characteristics of reduced 1D spectra. Here we assume that damping leads to an exponential decay of the energy in \( k \)-space associated with the damped wavevectors.

From our assumption of critical balance, it follows that \( k_{\perp} \gg k_{||} \). Therefore, damping at a fixed scale affects the turbulent cascade first through perpendicular wavevectors and only later through wavevectors parallel to the mean magnetic field. It has been argued that the ion-cyclotron resonance at \( k_{||} \sim \Omega_{ic}/V_A \), where \( \Omega_{ic} = \frac{eB}{m_i} \) is the ion-cyclotron frequency, has only minor influence on a critically balanced cascade as it is reached only at very high perpendicular wavenumbers \( k_{\perp} \), where Landau damping already dominates (Howes et al. 2008; Schekochihin et al. 2009; Cranmer & van Ballegooijen 2012). With our synthetic spectra, we are now able to test this argument quantitatively.

#### 3.4.1. Damping of Parallel Wavevectors

For the ion-cyclotron resonance, the damping rate is low for \( k_{||} V_A \ll \Omega_{ic} \), and increases rapidly as \( k_{||} V_A \sim \Omega_{ic} \) approaches unity (Howes et al. 2008; Cranmer & van Ballegooijen 2012). We may therefore model the ion-cyclotron resonance as an exponential decay according to

\[
E_{\text{ic}}(k) = E(k) \cdot \exp\left(-\frac{k_{||} V_A}{\Omega_{ic}}\right).
\]

To quantify the effective damping, we model damped spectra \( P_{\text{ic}} \) using the parameters in Table 1 and find only minor changes compared to their undamped counterparts \( P_0 \).

In Figure 3 we show the ratio \( R_0/P_{\text{ic}} \) as a function of normalized wavenumber for several angles \( \theta \). Both \( P_0 \) and \( P_{\text{ic}} \) cover all three ranges from MHD to KAW to electron kinetic scales. For quasi-parallel spectra \( (\theta = 1^\circ) \), a peak factor of 1.34 is observed around the spectral break. Power spectra \( P_{\text{ic}} \) for larger field-to-flow angles \( \theta \geq 5^\circ \) only differ a factor of 1.2 from PSDs without damping. On electron scales, the difference between damped and undamped spectra remains constant as there is no more parallel transfer along the cascade. The asymptotic factor \( R_0/P_{\text{ic}} \) for \( 2\pi f/V \gg \rho_e^{-1} \) and \( \theta = 90^\circ \) can be
calculated by integration of Equation (18) over \( k_z \) according to

\[
\frac{R_i}{P_{ic}} \left( 2\pi f / V \right) \rho_i^{-1}, \theta = 90^\circ
\]

\[
= \int_{-\infty}^{\infty} dk_z \exp \left( -L_{\perp}^3 \rho_i^{1/3} \rho_i^{1/3} \rho_i^{1/3} |k_z| \right)
\]

\[
\int_{-\infty}^{\infty} dk_z \exp \left( -L_{\perp}^3 \rho_i^{1/3} \rho_i^{1/3} \rho_i^{1/3} |k_z| - |k_z| \frac{V_A}{\beta_{ic}} \right)
\]

\[
\approx 1.21.
\]

This shows that the influence of cyclotron damping on the spectral shape can be neglected in the solar wind for critically balanced turbulence. However, one might think of systems in which the ion-cyclotron resonance does change the form of the PSD significantly. Such systems could be characterized by (1) low plasma density, which leads to large Alfvén speeds; (2) a critical balance exponent \( \alpha \) close to unity; and/or (3) a much smaller outer scale than the \( L \sim 10^6-10^{10} \text{m} \) found in the solar wind (Howes et al. 2008; Schekochihin et al. 2009), so that high \( k_i \) values are reached earlier in the cascade. An enhanced ion-cyclotron damping then leads to a visible decrease of spectral power around the spectral break.

3.4.2. Damping of Perpendicular Wavevectors

Let us now turn to damping of perpendicular wavevectors. Alexandrova et al. (2012) found that reduced power spectra measured in the solar wind can empirically be described by

\[
P(k_\perp) \propto k_\perp^{-\kappa} \exp \left( -k_\perp \rho_i \right)
\]

(22)

with a spectral index of \( \kappa = 8/3 \). This result is consistent with numerical gyrokinetic simulations (Howes et al. 2011; Ten-Barge et al. 2013), and we may treat the exponential damping term of Equation (22) as a proxy for electron Landau damping. Note that we apply the damping term of Equation (22) to 3D

\[
P_{\text{damp}}(k) = P(k) \cdot \exp \left( -k \rho_i \right)
\]

(23)

The corresponding distribution in \( k \)-space according to

\[
E_{\text{damp}}(k) = E(k) \cdot \exp \left( -k \rho_i \right).
\]

The corresponding distribution in \( k \)-space is shown in Figure 4, where damping of both parallel and perpendicular wavevectors (Equations (20) and (23)) is included. While the power at a certain frequency is obtained by integration over a 2D plane, the damping term depends on \( k_\perp \) and is therefore axisymmetric with respect to \( k_z \). This means that the integration along \( k_z \) includes perpendicular wavenumbers larger than \( k_\perp = 2\pi f / (V \sin(\theta)) \). Consequently, the effect of the exponential damping term is weaker when applied to 3D \( k \)-space compared to a reduced 1D spectrum. Therefore, we are able to produce similar results to those of Equation (22) with \( \kappa = 8/3 \), although we apply a less steep spectral index of only 7/3 in the ion kinetic range (see Figure 5).

3.4.3. Reduced PSD Subject to Damping

Figure 5 shows damped PSD according to Equations (20) and (23) with the same plasma parameters (Table 1) as in Figure 2. As expected, the high-frequency part is strongly affected and the damping leads to a characteristic exponential decay. The spectra are dominated by damping of perpendicular wavevectors, and the damping terms are found to be generally more effective at small angles \( \theta \). The latter can be understood considering the plane of integration given by Equation (19). The power at a certain frequency primarily stems from perpendicular wavenumbers \( k_\perp \). For small angles \( \theta \), this involves larger \( k_\perp \), which are more strongly damped than a corresponding spectrum with angles close to 90°.

Although strong damping sets in not before electron scales in wavevector space, the KAW range of the PSD, \( \rho_e^{-1} < k < \rho_e^{-1} \), is already affected: Figure 5 shows the undamped spectrum for \( \theta = 90^\circ \) as a black dashed line, and it is fairly visible that the spectrum subject to damping is steeper. In fact, we measure a spectral slope of 2.63 (shown by the black dashed line) in the ion kinetic range \( 1 < k \rho_i < 42 \) for \( P_{\text{damp}}(\theta = 90^\circ) \), although the corresponding energy in \( k \)-space scales with 7/3.
Figure 6. Spectral anisotropy in the inertial range for different values of the outer scale L for MHD scaling, i.e., integration of (11) over expression (14). The spectral index is obtained in frequency range \( f = [10^{-4}, 0.1] \) Hz. A smaller outer scale results in a transition of the spectral index from 2 to 5/3 at larger angles \( \theta \).

If \( T_i/T_e \) decreases or the electron gyroradius increases, the exponential decay moves to lower frequencies and further steepens the spectra. The two opposed mechanisms, flattening toward perpendicular slope and damping, can cancel each other and may lead to a range of seemingly constant slope. This shows that an approximate power law of the measured PSD is not equivalent to the absence of damping. Further, it shows that the observed spectral slope of the reduced spectrum on ion kinetic scales is not necessarily the spectral index predicted for this range by the underlying theory.

4. APPLICATION TO SOLAR WIND OBSERVATIONS

By numerically evaluating Equation (11), it is possible to calculate PSDs \( P_i(f) \) for any component \( i=x, y, z \) and given plasma parameters (plasma speed, outer scale, field-to-flow angle, background magnetic field, Alfvén speed, gyroradii). Here we compare our results to in situ measurements in the solar wind made by Horbury et al. (2008) in the MHD range and Chen et al. (2010) in the kinetic range. Podesta (2009) and Wicks et al. (2010) have presented similar results in the MHD range, which are consistent with those from Horbury et al. (2008). We show that the measured spectral anisotropies are in accordance with our model and can thus be described by a critically balanced cascade. However, the influence of damping turns out to be more important than previously thought.

4.1. MHD Turbulence

From the assumption of critical balance it follows that the reduced PSD scales as \( P \propto f^{-5/3} \) for \( \theta = 90^\circ \) and \( P \propto f^{-2} \) for \( \theta = 0^\circ \). Figure 6 shows exemplary for the case of MHD, i.e., Equation (14) only, that the transition between these two scalings is controlled by the outer scale \( L \). Here we have calculated the spectral slope from a least-squares fit in the frequency range \( f = [10^{-4}, 0.1] \) Hz. Note that the spectral index slightly varies in this frequency range and, therefore, no exact power law is observed. However, the result shown in Figure 6 impressively characterizes the influence of the outer scale on the cascade. It can be seen that the perpendicular scaling is reached only at very large angles \( \theta \) for small outer scales \( L \). This reflects the evolution of the turbulent cascade along the critical balance path. While the energy is isotropic at the outer scale, it grows increasingly anisotropic as it cascades to smaller scales.

We can use this functional dependence of the spectral anisotropy to check whether our critically balanced \( k \)-space distribution generates results in agreement with Horbury et al. (2008) and, if so, which value of \( L \) fits the observations best. For several outer scales \( L \), we calculate the spectral anisotropy and fit the spectral indices \( \kappa(L, \theta) \) to their results. To evaluate the critically balanced power spectra, we use the complete set of energy distributions, \( E_{\text{MHD}}, E_{\text{KAW}}, \) and \( E_{\text{ED}} \), together with anisotropic damping, i.e., exponential damping at the ion-cyclotron resonance and at electron scales according to Equations (20) and (23), respectively.

Horbury et al. (2008) used magnetic field data from the Ulysses spacecraft at 1.4 AU in the fast solar wind (\( V = 750 \) km s\(^{-1} \)). From McComas et al. (2000), we estimate the proton gyroradius as \( r_p = 190 \) km, and due to lack of a better estimate, we assume \( T_i = T_e \) to calculate the electron gyroradius. For comparison, we also calculate the best fit for a slab+2D turbulence model, where we use a spectral index of 2 for the slab component and 5/3 for the 2D component (Horbury et al. 2011). To quantify the goodness of fit, we use a reduced error

\[
\chi = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \frac{(\bar{\kappa}_i - \kappa_i)^2}{\bar{\sigma}_i^2}}, \quad (24)
\]

where \( \bar{\kappa}_i \) and \( \bar{\sigma}_i \) are the spectral indices and their corresponding errors, respectively, taken from Horbury et al. (2008), \( N \) is the number of angle bins, and \( \kappa_i \) are the modeled spectral indices, which are obtained by a least-squares fit in the same frequency range, 15–98 mHz, as those used by Horbury et al. (2008). The spectral anisotropies in the frequency range 15–98 mHz for the best-fit parameters are shown in Figure 7.

In general, the fit is better for the critically balanced turbulence model (\( \chi = 2.8–3.0 \)) than for the slab+2D turbulence model (\( \chi = 4.6 \)). This can be seen in Figure 7, where we show the observed and modeled spectral anisotropies. We find that an outer scale of \( L = 10^9 \) m in the undamped case and \( L = 10^{10} \) m for the damped cascade give the best results. These values are in accordance with observations of the transition from \( f^{-1} \)-spectra to \( f^{-5/3} \)-spectra in the solar wind, which is believed to mark the end of the energy injection scale (Schekochihin et al. 2009). The spectra at small angles \( \theta \) are found to be steeper than \( \kappa = 2 \). This is not caused by damping. The difference between damped and undamped cascade is, according to Figures 1 and 4, very small.
Instead, the steep slopes result from the fact that the fitting range \( f = 15-98 \text{ mHz} \) includes the steeper kinetic range cascade, where slopes are significantly steeper than \( \kappa = 2 \). For \( \theta = 5^\circ \), e.g., \( f = 98 \text{ mHz} \) corresponds to \( k_f \rho_i = 1.8 \).

For slab+2D turbulence, we find that the best fit has 30% slab and 70% 2D turbulence, which is also in accordance with results obtained in the solar wind (Bieber et al. 1996). Note, however, that we use two different spectral indices for slab and 2D turbulence, while Bieber et al. (1996) used the same slope for both slab and 2D turbulence.

The anisotropy of the power, \( P(\theta)/P(\theta = 5^\circ) \), measured at a fixed frequency of \( f = 61 \text{ mHz} \) is shown in Figure 8. The results of the damped critically balanced cascade well reproduce the anisotropy found by Horbury et al. (2008) for angles \( \theta < 40^\circ \) but overestimate the power at larger angles. The undamped cascade generally shows a weaker anisotropy but is in agreement with the measurements at \( \theta \sim 90^\circ \). Similar to what has been found for the spectral index, a critically balanced cascade fits the data much better than slab+2D turbulence.

Given the simplicity of our parameters, the presented fits to the results of Horbury et al. (2008) can be regarded as a qualitatively successful reproduction of the observed data. Although Forman et al. (2011) showed that the observed spectral anisotropy is in agreement with a critical balance on MHD scales, this is the first time that these results have been analyzed with a model including kinetic range scalings and damping terms. The quality of the fits indicates that the spectral anisotropy observed by Horbury et al. (2008) is consistent with a critically balanced cascade.

### 4.2. Kinetic Range Turbulence

For an analysis of kinetic range magnetic field fluctuations, Chen et al. (2010) used data from CLUSTER during fast solar wind conditions with moderate ion plasma \( \beta_i \sim 1 \). They calculated the spectral index using a wavelet-based method described by Horbury et al. (2008). Ion and electron gyroradii, as well as the bulk plasma velocity, are given in Table 1 of Chen et al. (2010), and the power anisotropy is given as \( P_\perp/P_\parallel \sim 20 \). The spectral anisotropy is calculated separately for parallel and perpendicular fluctuations. Here we limit our fit to the perpendicular fluctuations \( P_\perp \). The spectral anisotropy of parallel fluctuations cannot be reproduced sufficiently well, which is consistent with the findings of Forman et al. (2013). For the modeled spectra, we choose for the outer scales \( L = 10^6 \text{ m} \) for the undamped and \( L = 10^{10} \text{ m} \) for the damped cascade based on our fit to the Horbury et al. (2008) data.

In Figure 9 we show the results obtained from our forward calculation and those presented in Chen et al. (2010) for the ion kinetic range \( 1.5 \leq k_f \rho_i \leq 6 \). Additional to the damped KAW cascade with \( \kappa = 7/3 \), we show results for a damped critically balanced cascade with an assumed spectral index of 8/3 and critical balance exponent \( \alpha = 1/3 \). This scenario is included as a hypothetical case for visual orientation only. Although the modeled results for damped and undamped spectra do not fit the data within error tolerances, the damped cascades show qualitatively similar spectral anisotropies. Here the variation from \( \theta = 0^\circ \) to \( \theta = 90^\circ \) is smooth and not as abrupt as in the undamped case. For cross-comparison, we point the reader to Figure 5, where it can be seen that PSDs are steeper for smaller field-to-flow angles \( \theta \). The undamped cascade, in contrast, leads to spectral slopes of \( \sim 7/3 \) at almost all angles (cf. Figure 2). Even at \( \theta = 5^\circ \), where the observed spectral index is \( \sim 3.3 \), the spectral index of the undamped cascade is already 2.4. This is the consequence of the transition toward a quasi-perpendicular slope as was elaborated in Section 3.3. The spectral index of the damped KAW cascade, on the other hand, is 3.2 at \( \theta = 5^\circ \) and thus much closer to the observation.

The results in Figure 9 show that an undamped cascade leads to a sharp increase of the spectral index at small field-to-flow angles, which appears rather step-like. This result has interesting implications for the interpretation of critically balanced plasma turbulence. It is often assumed that a critically balanced cascade in the ion kinetic range can lead to any spectral slope, \( 7/3 \leq \kappa \leq 5 \), for intermediate field-to-flow angles \( \theta \). However, we have shown in Section 3.3 and in Equation (39) that the spectral index is \( \sim 7/3 \) for most angles. A smooth or slow variation of the spectral index, as seen in the results of Chen et al. (2010), can therefore not be caused by critical balance alone. Instead, the effect of damping is essential to obtain steeper spectra at non-zero angles, which means that
the observed spectral anisotropy is to a large degree determined by the damping mechanisms.

5. MODELING SPECTRAL DENSITIES AT SATURN

Recently, it has been shown that magnetic fluctuations in Saturn’s plasma sheet form a turbulent cascade (von Papen et al. 2014). These fluctuations have different properties compared to the solar wind. They are embedded within Saturn’s strong background magnetic field and propagate as Alfvén waves along magnetic field lines until they are reflected by density gradients at the plasma sheet or the ionosphere. Measurements by the Cassini spacecraft during its first seven orbits around Saturn indicate that observed power-law spectra can be interpreted as a critically balanced cascade of KAW. On MHD scales, however, power spectra indicating an Alfvén wave cascade with Kolmogorov-like power law are observed only sporadically because large-scale magnetospheric processes dominate this range of scales. As Taylor’s hypothesis holds in Saturn’s magnetosphere for KAWs with $k_r \gg k_i$, we are able to test the observations with our forward model and to investigate the spatial distribution of observed spectral indices as a function of radial distance to Saturn.

As the turbulent cascade is predominantly observed on kinetic scales, we restrict our model to the reproduction of kinetic range spectra. We analyze the radial distribution of spectral indices reported in Figure 12 of von Papen et al. (2014) and focus on an explanation for the change of spectral slopes inside $9 R_s$, where $1 R_s = 60,268$ km is the planetary radius of Saturn. Inside $9 R_s$, increasingly flatter spectra are observed, which so far could not be explained. As there are two distinct electron populations in Saturn’s magnetosphere, we also analyze which dissipation scales control the onset of damping. Comparison of modeled and observed spectra may thus shed more light on the physics of the turbulence and may help us understand which electron population is energized by the turbulent cascade in Saturn’s magnetosphere.

To model synthetic spectra in Saturn’s magnetosphere, we use the parameters listed in Table 2 as reported in von Papen et al. (2014), where we use $2H_w \sim L$ as the outer scale. These parameters fluctuate in time and vary strongly with radial distance to Saturn. The basic plasma parameters at Saturn—namely, velocity, scale height, ion temperature, and density—are based on observations by Thomsen et al. (2010) for water group ions, which are the main constituent of Saturn’s magnetosphere. To calculate the electron gyroradii, which control the onset of the empirical damping term in Equation (23), we use the electron temperature models given by Schippers et al. (2008, Table 1) derived from CAPS/MIMI measurements (Krimigis et al. 2004; Young et al. 2004). The field-to-flow angle $\theta$ is calculated assuming the plasma flow to be in azimuthal direction only. We further use the measured power anisotropy $P_{\perp}/P_{\parallel}$, averaged in the kinetic range, to estimate the ratio $\psi/\theta$ between the energy of toroidal and poloidal fluctuations according to Equation (4). The slopes of the modeled spectra are calculated from the trace of the synthetic spectral tensor.

Equatorial temperatures have been derived by Schippers et al. (2008) for the cold or thermal (<100 eV) and hot (>100 eV) electron populations. These temperatures control the electron gyroradii according to

$$\rho_{e,c/h} = \frac{\sqrt{2m_e k_B T_{e,c/h}}}{eB},$$

where $T_{ec}$ and $T_{eh}$ denote the cold and hot electron temperatures, respectively. The radial profiles of the temperatures and densities are shown in the top and bottom panels of Figure 10, respectively, together with the water group ion temperatures and densities for comparison. The model for the hot electrons is valid for distances up to $18 R_s$. The temperature of the cold population is more variable outside $15 R_s$ and thus only provided inside of that distance. However, due to the lack of a better estimate, we approximate the cold electron
populations also outside of 15 $R_s$ with this model. Both temperatures similarly peak at around 9 $R_s$.

In the following, we model reduced spectra using electron gyroradii from the cold and hot electron populations and compare both to the observed spectra. The larger electron gyroradius of the hot electron population leads to an earlier onset of damping, i.e., the damping term is larger and its effects are already important at lower frequencies compared to damping on cold electron scales. This causes the slope of the hot electron spectrum to be slightly steeper than the cold electron spectrum.

In order to compare our modeled results with the observations, we need to add noise to the synthetic data and assume that the excitation of turbulence is not strong, i.e., we assume $\delta V_s < V_A$. The latter is as expected from turbulence excited within a strong planetary magnetic field (von Papen et al. 2014). The noise levels are computed as the sum of magnetometer instrument noise $n_i$, quantization noise $n_q$, and aliasing. The instrument noise is measured and must therefore be added to the spectrum before calculating the aliasing. This leads to a synthetic spectrum $P'$ given by

$$P'(f) = P(f)/c_E + n_i/f + n_q + A(f) \cdot \left[ P(f)/c_E + n_i/f \right],$$

where $P(f)$ is the synthetic spectrum without noise, $c_E$ is a factor that considers the weaker excitation discussed above, and the aliasing function

$$A(f) = \sum_{n=1}^{\infty} \left( \frac{f/f_{ny}}{2n - f/f_{ny}} \right)^\kappa + \sum_{n=1}^{\infty} \left[ \frac{f/f_{ny}}{2n + f/f_{ny}} \right]^\kappa$$

is determined according to Podesta et al. (2006) for a power law of $\kappa = 7/3$ and Nyquist frequency $f_{ny}$. In our case, this function takes on a value of 1.3 at $f = f_{ny} = 3.6$ Hz and drops rapidly for lower frequencies.

The second term on the right-hand side of Equation (26) is the instrument noise, which is $n_i = 75$ pT$^2$ Hz$^{-1}$ according to Dougherty et al. (2004) and goes with 1/f (Russell 1972). The third term is the quantization noise, which can be estimated as $n_q = \frac{1}{2} \Delta B^2 \Delta t$ (Russell 1972) with sampling period $\Delta t = 0.14$ s and the quantization of the magnetic field data, e.g., $\Delta B = 4.9$ pT in fluxgate magnetometer (FGM) range 0 (Dougherty et al. 2004). The factor $c_E$ effectively corrects the energies of the synthetic spectra to the observed ones; however, it does not change the spectral index. The factor $c_E = (P_{obs}/P)$ is on average 50 and is calculated as the geometrical mean of the ratio of observed $P_{obs}$ to synthetic spectra $P$ in the normalized wavenumber range $k_{\perp} \rho_W = [2, 50]$, where

$$k_{\perp} = \frac{2\pi f}{V \sin(\theta)}$$

and $\rho_W$ is the water group ion gyroradius, for signal-to-noise ratios (S/N) $> 5$.

We carry out a forward modeling for each of the 1136 observed 10-minute time series and compute the spectral slopes in the same frequency ranges as determined for the observed data, i.e., in the range $2 < k_{\perp} \rho_W < 50$, and for S/N $> 5$ of the respective measurement (von Papen et al. 2014). As an example, we show an observed spectrum from 9.5 $R_s$ and its corresponding synthetic spectra in Figure 11. The black line shows the observed spectrum, the blue line the spectrum with dissipation controlled by the gyroradius of the cold electrons, and the red line the spectrum with dissipation controlled by the gyroradius of the hot electrons. All of these spectra include noise with the exception of the red dashed line, which shows the spectrum $P_{obs}^{\text{hot}}$ without noise for comparison.

Inside the fitting range, depicted by two vertical dashed lines, the three spectra are very similar. Their spectral indices are given in the legend of the figure, and the modeled results ($\kappa_{\text{cold}} = 2.32$ and $\kappa_{\text{hot}} = 2.61$) are close to the observed slope of $\kappa = 2.46$. However, at high frequencies the spectrum controlled by the gyroradius of the hot electrons clearly fits the observation better. At frequencies higher than the fitting range, the noise level shown by the dotted line leads to a flattening of the synthetic spectra. On average, this leads to a slight decrease of the spectral index within the fitting range by 0.06 and 0.07 for cold and hot electron spectra, respectively, compared to spectra without noise (see red dashed line in Figure 11).

Figure 12 shows the radial distribution of observed spectral indices in Saturn’s magnetosphere as black crosses. Spectral indices of synthetic spectra are shown as dots for damping on cold (blue) and hot (red) electron scales. Damping on hot electron scales explains qualitatively the change of slopes inside of 9 $R_s$. 

![Figure 11](image1.png)

**Figure 11.** PSD of a 10-minute time series measured in Saturn’s magnetosphere at a radial distance of 9.5 $R_s$ (black line). The blue and red lines show synthetic spectra for damping by the gyroradius of cold and hot electrons, respectively. Associated PSDs are labeled $P_{cold}$ and $P_{hot}$ respectively. Vertical dashed lines show the fitting range in which the spectral index is calculated. The determined values are given in the legend. The dotted line shows the noise level corresponding to $P_{obs}$, and the dashed red line shows the spectrum $P_{obs}^{\text{hot}}$ without noise.

![Figure 12](image2.png)

**Figure 12.** Radial distribution of observed spectral indices in Saturn’s magnetosphere as black crosses. Spectral indices of synthetic spectra are shown as dots for damping on cold (blue) and hot (red) electron scales. Damping on hot electron scales explains qualitatively the change of slopes inside of 9 $R_s$. 

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The spectral slopes for damping on cold electron scales (blue dots) are generally less steep than the observations and close to the undamped spectral index of \(7/3\). This shows that the corresponding synthetic spectra have not yet reached dissipation scales, where the spectral energy decreases significantly. Damping on hot electron scales, on the other hand, leads to steeper spectra that agree much better with the observations. Clearly, the decrease of the electron temperature inside \(9 R_e\) leads to shallower spectra because the damping is reduced for smaller electron gyroradii. This indicates that the change of slopes inside \(9 R_e\) can be explained by damping effects. However, the radial profiles of both electron temperatures are nearly identical so that the difference between the two gyroradii can in principle be compensated by a simple factor \(c > 1\) in the exponent of the damping term, i.e., \(\exp(-c \cdot k_B f_k)\) in Equation (23). Note that for larger distances \((>12 R_e)\) the spectral indices \(\kappa_{\text{hot}}\) become shallower than the observations, which indicates either the presence of additional effects on the turbulent cascade or magnetospheric variations.

In summary, the forward modeling of kinetic range spectra has provided a possible explanation for the change of spectral slopes inside \(9 R_e\), which is solely based on first principles of a KAW cascade and an empirical term to describe damping on electron scales. Our results indicate that the energy transferred along the kinetic range cascade is preferably deposited into the hot electron population. The heating examined here could thus be the primary process, which maintains the hot electron population in Saturn’s magnetosphere.

6. CONCLUSION

We present results from a numerical forward model to evaluate reduced PSD from an arbitrarily distributed energy density in 3D \(k\)-space.\(^2\) Given a critically balanced \(k\) -space distribution of energy, we investigate the functional dependence of the reduced spectra on several parameters, such as the field-to-flow angle or the outer scale. Such an analysis has been carried out for the first time covering the complete range from MHD to electron kinetic scales. We show that for intermediate field-to-flow angles \(\theta\), the spectral slope of undamped critically balanced turbulence is not constant, i.e., the reduced PSD is not an exact power law (see Figure 2). Instead, the reduced spectra evolve with frequency in the undamped case toward a quasi-perpendicular spectrum. This is a pure sampling effect of highly Doppler-shifted measurements at any angle \(\theta > 0^\circ\) caused by the anisotropy \(k_\perp \gg k_\parallel\) increasing with frequency. Power spectra in this quasi-perpendicular cascade range have a spectral slope corresponding to the spectral index of the perpendicular cascade \(\kappa (\theta = 90^\circ)\). For PSDs that are additionally subject to damping (Figure 5), this transition to quasi-perpendicular spectra is masked by Landau damping on electron kinetic scales. Here the spectra become steeper and steeper at higher frequencies.

The transition frequency, where the change of slope to \(\kappa \approx \kappa (\theta = 90^\circ)\) is reached for the undamped cascade, can be approximated by a simple expression \(f_{\text{3D}}^2\) given in Equation (39). Under typical solar wind conditions, \(f_{\text{3D}}^a\) for the ion kinetic regime is smaller than or on the same order of the observed spectral break at \(f_s \sim 0.3\) Hz. Therefore, significantly steeper slopes than the perpendicular spectra on kinetic scales can only be explained by additional damping effects. We find that an empirical damping term at electron scales of the form \(\exp(-k_\perp f_k)\) (Alexandrova et al. 2012) already affects the spectral index in the ion kinetic range \(\rho^{-1}_i < k < \rho^{-1}_e\) of the reduced spectrum and steepens it measurably.

We apply our model to in situ measurements in the solar wind and show that turbulent fluctuations measured by Horbury et al. (2008) are in good agreement with a critically balanced cascade and less so with a combined slab+2D turbulence. In the kinetic range we find good agreement with the results of Chen et al. (2010), which can be explained by a damped critically balanced KAW cascade. While the spectral break and damping terms have only minor influence on the fit to the observations of Horbury et al. (2008), the inclusion of a damping term is essential to explain the results in the kinetic range. Indeed, spectral slopes for an undamped cascade differ strongly from the Chen et al. (2010) results and are approximately \(7/3\) for nearly all field-to-flow angles in the ion kinetic regime. This is caused by the transition toward a quasi-perpendicular slope. This means that damping is an integral part of the kinetic range cascade and the dominating factor in the observed spectral anisotropy.

The forward modeling technique is further applied to observations of turbulent magnetic field fluctuations in Saturn’s magnetosphere. Here we show that the measured reduced spectra are qualitatively in agreement with a critically balanced KAW cascade, which further corroborates the interpretation of von Papen et al. (2014). The observation of shallower spectra inside \(9 R_e\) can be reproduced by damping on electron scales controlled by the hot electron population. This indicates that the dissipation of turbulent magnetic fluctuations predominantly heats the hot electron population at Saturn. This additional insight on magnetic turbulence in Saturn’s magnetosphere shows the advantage of applying a first principles forward modeling technique to determine the shape of reduced spectra.

We would like to thank R. A. Burger for pointing out the relevance of our study to cosmic-ray transport and the referee for constructive comments. The authors also acknowledge fruitful discussions with R. Wicks and M. Forman.

APPENDIX

ESTIMATION OF TRANSITION FREQUENCY TO QUASI-PERPENDICULAR SPECTRA

Here we derive an approximate expression for the frequency \(f_{\text{3D}}^2\), at which the reduced power spectrum of an undamped critically balanced cascade turns quasi-perpendicular, i.e., the PSD is characterized by a spectral slope \(\kappa = \kappa (\theta = 90^\circ) + \Delta \kappa\) with a small error \(\Delta \kappa\). For mathematical tractability we derive this approximate expression under additional assumptions. Later, we show by comparison with our full 3D model that the resultant expression still provides a good estimator when these assumptions are relaxed. The assumptions we use for our derivation are a 2D \(k\)-space \((k_\parallel, k_\perp)\) and a critically balanced cascade that is controlled by a Dirac Delta function instead of the exponential function in Equation (14). The latter seems to be a strong simplification, but Forman et al. (2011) and Turner et al. (2012) have shown that there are only minor differences between the spectral anisotropies of PSDs controlled by an exponential and a Dirac Delta function.

In order to estimate the implications of our assumption of a 2D \(k\)-space, it is important to clarify the 3D geometry of the

\(^2\) The forward code is available as a tar.gz file.
problem. In Figure 1, we see a slice along the $k_x$-$k_z$ plane. While the planes of integration (white dashed lines) are 2D and expand straight in $k_x$ direction, the energy distribution is 3D and axisymmetric with respect to $k_z$. This means that the plane of integration intersects with both Dirac Delta surfaces (for $k_z > 0$ and $k_z < 0$) in a complicated curve, which depends on outer scale $L$, field-to-flow angle $\theta$, and critical balance exponent $\alpha$. However, most of the energy along these curves resides at $k_z = 0$ and decreases with growing wavenumber in $y$-direction. Therefore, we assume that the assumption of a 2D wavevector space is suitable for our following derivation. This is subsequently corroborated by the agreement of the such-derived expression with our fully 3D model results.

In two dimensions, the energy distribution of critical balance controlled by a Dirac function may be written as

$$ E_{2D}(k_{\perp}, k_i) \propto k_{\perp}^{-\kappa} \delta\left(\left|k_{\|}\right| - L^{-1/\alpha} \rho^{\beta} \left|k_i\right|^{2/\alpha}\right), \quad (28) $$

where $\kappa = [5/3, 7/3, 8/3]$ are the 1D spectral indices and $\alpha = [2/3, 1/3, 0]$ the critical balance exponents on MHD, KAW, and ED scales, respectively (see also Grappin & Müller 2010). For the sake of simplicity, we use only one kinetic scale $\rho$ in Equation (28): in the KAW range $\rho = \rho_1$, while the result for the ED range can be obtained by setting $\rho = \sqrt{\rho_1 \rho_2}$. The distribution of energy density in $k$-space that contributes to the integral curve toward lower $k_{\perp}$-values is shown in Figure 13. The intersection of the two Dirac Delta branches with the line of integration given by Equation (19) yields two points:

$$ \pm k_{||} = L^{-1/\alpha} \rho^{\beta} \left|k_{\|}\right|^{2/\alpha}, \quad (29) $$

positive $k_{||}(k_{\perp,1})$ and negative $k_{||}(k_{\perp,2})$. The locations of these points are depicted as red circles in Figure 13. Field-to-flow angles smaller than $90^\circ$ yield $k_{\perp,1} < k_{\perp,0} < k_{\perp,2}$, where

$$ k_{\perp,0} = \frac{2\pi f}{v \sin(\theta)} \quad (30) $$

is the wavenumber corresponding to $k_0 = 0$. Note that we do not need to make any assumptions regarding the energy at zero parallel wavenumber. The two locations $k_{\perp,1/2}$ are uniquely defined by Equations (19) and (29). It follows that

$$ k_{\perp,1/2} = \frac{2\pi f}{v \sin(\theta)} \mp k_{\perp,1/2} \cot(\theta) = \frac{2\pi f}{v \sin(\theta)} \mp L^{-1/\alpha} \rho^{\beta} \left|k_{\|}\right|^{2/\alpha} \cot(\theta). \quad (31) $$

With the particular choices of $\alpha = [2/3, 1/3]$, this equation can be written as a cubic polynomial with real solutions $k_{\perp,1/2}$. Even though a cubic algebraic equation can be solved analytically, we search for an approximate solution to achieve a mathematically simpler expression, which is easier to work with.

Let us now look at the energy $E_{2D}$ in $k$-space that contributes to the integration:

$$ E_{2D} \propto k_{\perp,1}^{-\kappa} + k_{\perp,2}^{-\kappa}, \quad (32) $$

which is essentially the summation over both critical balance branches. In the following, we show that this energy asymptotically approaches $E_0 \propto 2k_{\perp,0}^{-\kappa}$ with increasing frequency, thus leading to a scaling according to $\theta = 90^\circ$. At a certain frequency $f_{2D}^0$, the measurement uncertainty will be larger than the difference between the energies $E_{2D}$ and $E_0$ and, therefore, the observed PSD will scale like the perpendicular cascade. We proceed to estimate the frequency for which the reduced PSD turns quasi-perpendicular by demanding that the ratio $E_{2D}/E_0$ be almost unity. Because the spectral index is obtained from the logarithms of the power spectral energy, we demand

$$ \ln \left| \frac{k_{\perp,1}^{-\kappa} + k_{\perp,2}^{-\kappa}}{2k_{\perp,0}^{-\kappa}} \right| < \epsilon, \quad (33) $$

where $\epsilon > 0$ is chosen according to typical measurement uncertainties $\Delta k$ for spectral slopes in solar wind observations.

To proceed further, we approximate

$$ k_{\perp,1/2} \approx k_{\perp,0} = L^{-1/\alpha} \rho^{\beta} \left|k_{\|}\right|^{2/\alpha} \cot(\theta). \quad (34) $$

The geometrical interpretation of this approximation is shown in Figure 13. Instead of the exact solutions shown by the red circles, we now evaluate the integral at the approximated locations shown by the blue circles. These are slightly shifted along the integration curve toward lower $k_{\perp}$-values. Now, we can write Equation (31) as

$$ k_{\perp,1/2} \approx k_{\perp,0} \mp L^{-1/\alpha} \rho^{\beta} \left|k_{\|}\right|^{2/\alpha} \cot(\theta) = k_{\perp,0} \mp \Delta. \quad (35) $$

Inserting this into Equation (33), we get

$$ \left(1 - \frac{\Delta}{k_{\perp,0}}\right)^{-\kappa} + \left(1 + \frac{\Delta}{k_{\perp,0}}\right)^{-\kappa} < 2\epsilon. \quad (36) $$
If we expand the two terms on the left-hand side of Equation (36) to second order around \( \Delta/k_{\perp,0} = 0 \), we find that
\[
\frac{\Delta}{k_{\perp,0}} < \sqrt{\frac{2\epsilon - 2}{\kappa(\kappa + 1)}}.
\]  
(37)

Inserting \( \Delta = L^{-1/3}\rho_{\perp}^{-2/3}k_{\perp,0}^2\sin(\theta) \) and \( k_{\perp,0} = 2\pi f/(v\sin(\theta)) \) into this equation finally yields
\[
f > \frac{v\sin(\theta)}{2\pi} L^{1/3}\rho_{\perp}^{-2/3}2\epsilon - 2 \left( \frac{\kappa(\kappa + 1)}{\kappa} \right)^{1/3} \sin(\theta)^{1/3} \cot(\theta)^{1/3}.
\]  
(38)

For an appropriately chosen \( \epsilon \), the difference between energies \( E_{2D} \) and \( E_0 \) for all frequencies \( f > f_{2D}^{a} \) is so small that the scaling will be quasi-perpendicular within measurement errors \( \Delta \kappa \). Note that the change of slope in frequency space is a pure sampling effect and does not mean that the nature of the turbulent cascade in wavevector space is changing at this frequency. For \( \epsilon \ll 1 \), we can further simplify Equation (38) to determine the approximate transition frequency that marks the boundary to quasi-perpendicular scaling as
\[
f_{2D}^{a} = \frac{v\sin(\theta)}{2\pi} \left( L^{1/3}\rho_{\perp}^{-2/3}2\epsilon - 2 \left( \frac{\kappa(\kappa + 1)}{\kappa} \right)^{1/3} \sin(\theta)^{1/3} \cot(\theta)^{1/3} \right).
\]  
(39)

Note that Equation (39) can additionally be used to test whether an observed steep slope in a reduced spectrum of critically balanced turbulence can be explained by spectral anisotropy only, i.e., without damping, or if additional damping is needed to explain the steep slope.

In order to check in which turbulent range \( f_{2D}^{a} \) lies, i.e., in the MHD, the KAW, or the ED range, the following procedure might be used: One chooses the appropriate critical balance exponent for the observed range of scales, namely, \( \alpha = 2/3 \) for MHD, \( \alpha = 1/3 \) for KAW, or \( \alpha = 0 \) for electron kinetic scales. Inserting \( \alpha \) into Equation (39), together with the relative plasma velocity \( v \), observation angle \( \theta \), outer scale \( L \), and controlling kinetic scale \( \rho \) (\( \rho = \rho_{i} \) for KAW and \( \rho = \sqrt{\rho_{i}\rho_{e}} \) for ED), one obtains the transition frequency \( f_{2D}^{a} \) for the observation geometry under consideration. All frequencies \( f > f_{2D}^{a} \) will be characterized by a quasi-perpendicular scaling.

To estimate in which range (MHD, KAW, ED) the transition toward quasi-perpendicular spectra occurs, one can calculate the corresponding perpendicular wavenumber according to \( k_{\perp} = 2\pi f_{2D}^{a}/(v\sin(\theta)) \). If the resultant \( k_{\perp} \) falls outside the range used to calculate \( f_{2D}^{a} \), then either the transition toward quasi-perpendicular spectra will happen on smaller scales (if \( k_{\perp} \) is too large for the corresponding range) or the transition has already happened on larger scales (if \( k_{\perp} \) is too small). The latter case, e.g., is found for the ED range in the solar wind as the transition toward quasi-perpendicular spectra already occurs on KAW scales. To find the correct transition frequency, the procedure then needs to be repeated with the \( \alpha \) and \( \rho \) values from the alternative range.

Note that the error parameter \( \epsilon \) must be determined according to the measurement accuracy, which usually requires modeling the corresponding spectra. We find that \( \epsilon = 0.05 \) in Equation (39) represents an uncertainty of the spectral index of \( \Delta \kappa = \pm 0.1 \) in the KAW and \( \Delta \kappa = \pm 0.02 \) in the MHD range for typical solar wind conditions. For the KAW range, this uncertainty is on the order of the measurement error in solar wind observations.

In the following, we test the approximate expression \( f_{2D}^{a} \) given in Equation (39) against our 3D model results. For that matter, we determine the transition frequencies \( f_{2D}^{mod} \) from modeled PSDs without a break at \( k_{\perp,0} \rho_{i} \sim 1 \) or \( k_{\perp,0} \rho_{e} \sim 1 \), i.e., for Equations (14) and (16) separately, each extending over the whole range of wavevectors. Because of divergence at low wavenumber \( k_{\perp} \), this method is numerically not stable if we use an energy distribution in \( k \)-space, which is solely given by Equation (18). From the modeled PSD in the MHD and KAW range, we calculate the spectral slopes between two consecutive data points \( (f_{n+1}/f_{n} \approx 1.3) \) as \( \kappa = \kappa/\theta = 90^\circ + \Delta \kappa \), where \( \kappa(\theta = 90^\circ) = [5/3, 7/3] \) in the MHD and KAW regime, respectively. We then define \( f_{2D}^{mod} \) as the frequency where \( \Delta \kappa \) decreases below thresholds of \( \Delta \kappa = \pm 0.1 \) in the KAW and \( \Delta \kappa = \pm 0.02 \) in the MHD range.

In Figure 14, we show the such-derived \( f_{2D}^{mod} \) (crosses) for parameters given in Table 1 compared to the analytical approximation \( f_{2D}^{a} \) (solid lines) given in Equation (39) for critical balance values \( \alpha = [2/3, 1/3, 0] \). Solid lines show \( f_{2D}^{mod} \) calculated from the approximate expression in Equation (39) using \( \epsilon = 0.05 \), and crosses show \( f_{2D}^{mod} \) obtained numerically from modeled PSD for an uncertainty in the spectral index of \( \Delta \kappa = 0.02 \) on MHD and \( \Delta \kappa = 0.1 \) on ion kinetic scales.

![Figure 14. Transition frequencies as a function of \( \theta \) for critical balance values \( \alpha = [2/3, 1/3, 0] \). Solid lines show \( f_{2D}^{mod} \) calculated from the approximate expression in Equation (39) using \( \epsilon = 0.05 \), and crosses show \( f_{2D}^{mod} \) obtained numerically from modeled PSD for an uncertainty in the spectral index of \( \Delta \kappa = 0.02 \) on MHD and \( \Delta \kappa = 0.1 \) on ion kinetic scales.](image-url)
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