On modelling transitional turbulent flows using under-resolved direct numerical simulations: the case of plane Couette flow

1 Introduction

The ‘laminar–turbulent’ transition in globally subcritical flows is far from being fully understood. This is due to its abrupt and hysteretic nature and to the fact that phase space coexistence, typical of a subcritical bifurcation, has a nontrivial counterpart in physical space, with laminar flow and turbulence coexisting in separate regions of the flow domain. Here, we focus on plane Couette flow (PCF), the flow of a viscous fluid with kinematic viscosity $v$ sheared between two parallel plates at a distance $2h$, translating in opposite directions at speeds $\pm U$. This flow configuration is free from global advection. The laminar flow is known to be linearly stable for all values of the Reynolds number $R = Uh/v$, whereas under usual conditions turbulent flow takes place for $R$ large enough, typically $R \sim 400$.

In fact, the transition can be examined in both directions, ‘laminar$\rightarrow$turbulent’ (direct) and ‘turbulent$\rightarrow$laminar’ (reverse). Many early studies have dealt with the direct transition, and especially with the dynamics of turbulent spots, by means of laboratory experiments or numerical simulations. More recently, experiments performed at Saclay [1] have shown that the reverse transition is marked by the occurrence of oblique turbulent...
bands, only observable in very large aspect ratio\textsuperscript{1} systems, in some range \( R_g < R < R_t \). In the lowest part of this range,\textsuperscript{2} near \( R_g \simeq 325 \), the turbulent bands become fragmented and turn into spots of irregular shape before decaying after long transients when \( R \) is further decreased below \( R_g \). Hysteresis is observed and sustained turbulent spots can be obtained by triggering the laminar flow with sufficiently large local perturbations when \( R > R_g \), whereas the laminar profile can be maintained up to much higher values of \( R \) provided that the experiment is sufficiently clean. At the upper end of the transitional range, the pattern disappears progressively and the transition from the turbulent bands to \textit{featureless turbulence} at \( R_t \simeq 410 \) is continuous. The term ‘featureless’ used here is borrowed from [2] where it served to describe the high-\( R \) turbulent regime beyond spiral turbulence in Taylor–Couette flow which corresponds to the oblique turbulent band pattern in PCF.

Besides laboratory experiments, numerical simulations of the Navier–Stokes equations (NSE) have provided invaluable information. An important output of early computations was the concept of minimal flow unit (MFU) of size just necessary to maintain turbulence in a wall-bounded flow [3], a fundamental ingredient in the elucidation of the mechanisms sustaining turbulence [4]. Later, the MFU context was extensively used to study the decay of turbulence within the framework of dynamical systems theory [5]. Simultaneously, numerical simulations were also performed in wider domains, which lead to the discovery of a large-scale streamwise structures in turbulent PCF [6] and other wall-bounded flows at Reynolds number somewhat beyond the transitional range defined above [7].

Numerical studies specially dedicated to the problem of oblique turbulent bands are recent. Soon after the experiments that put them at the forefront, Barkley and Tuckerman [8–10] succeeded in reproducing the fact by simulating the NSE in domains elongated in the expected direction of the pattern’s wavevector but narrow in the complementary in-plane direction. These simulations gave useful information on the pattern, properly accounting for the essential features of the laminar-turbulent alternation. The mechanism producing the bands has however remained elusive up to now, and it is not clear whether periodic boundary conditions a few MFUs apart along the short dimension of these domains do not handicap our understanding of it. Although the occurrence of bands appears to be an extremely robust phenomenon, as our study will confirm, it thus seemed interesting to consider cases where the long-range streamwise coherence of the large-scale streaky structures commonly observed in wall-bounded flows [7] was sufficiently well embraced. The coherence length of these structures being indeed at least one order of magnitude larger than the streamwise length of the MFU, this revives simulations in large aspect ratio, conventionally oriented domains. Such simulations again showed the occurrence of oblique turbulent bands [11,12].

The computationally demanding character of these fully resolved numerical experiments calls for the exploration of alternate approaches involving some more or less well-controlled level of approximation. This perspective was taken in [13] where the flow was modelled using a Galerkin expansion of the NSE in the cross-stream direction \( y \) in terms of well-chosen ad hoc polynomials. The main characteristics of the transition were recovered at much lower numerical cost from a truncation of the expansion at first significant order, which permitted simulations in very large aspect-ratio domains [14]. However, the transitional range was lowered by a factor of two with respect to the experiments as a result of insufficient energy transfer and dissipation in the cross-stream direction. Furthermore, the oblique bands in the upper part of the transitional range were not obtained, presumably another effect of the lowered cross-stream resolution.

The purpose of the work presented here is not to improve the model mentioned above by truncating it at higher orders, which is possible but very cumbersome and opaque, but to test this resolution effect in the context of direct numerical simulations, thus considering the deliberate decrease in the spatiotemporal resolution as a systematic modelling strategy. Our motivation is basically that, since qualitative and quantitative comparisons of solutions obtained at different resolutions are easy, the degree of approximation can be evaluated with some confidence. Having tested the reliability of this procedure, we may expect to obtain clues on the mechanisms of band formation and decay directly from the NSE at reduced numerical cost, in much the same way as lowering the size of the computational domain down to the dimensions of the MFU has helped towards the understanding of the self-sustaining process [15]. Finally, if quantitatively reliable low-resolution simulations can be performed, studying the statistics of the upper transition at \( R_t \) as well as the lower transition at \( R_g \) will be possible in larger domains, during longer periods of time (the so-called thermodynamic limit involved in analogies with thermodynamic phase transitions [17]), which will go in the same direction as in [14] but without the limitations of the model used in that work. Encouragement to follow the program sketched above

\textsuperscript{1} The aspect ratio is the dimensionless size of the set-up, i.e. its lateral size in units of the cross-stream half-gap \( h \).

\textsuperscript{2} Subscript ‘\( g \)’ used hereafter stands for ‘global’ in the sense of ‘global stability threshold’, the value of \( R \) below which the laminar base flow profile is unconditionally recovered in the long term, i.e. whatever the strength of the perturbation brought to the flow.