Modeling and analysis of silicon-embedded MEMS toroidal inductors

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Abstract. This paper presents the modeling and analysis of three-dimensional silicon-embedded toroidal inductors designed for power converter applications. Special attention is given to modeling phenomena associated with the presence of silicon, namely an increase in loss and parasitic capacitance. Silicon-embedded inductors can be fabricated with silicon inside the donut-shaped toroidal core and inside the donut hole, as well as with silicon above, below and outside the inductor. It is argued here that, with the exception of the losses in the core at high doping densities, the losses in the silicon can be tolerated in many power applications, making fully-integrated silicon-embedded air-core inductors viable for power applications. An equivalent circuit model is presented for such inductors which captures the stored magnetic energy, the parasitic electric energy stored between the windings and the silicon, the loss in the toroidal windings, and the electrically- and magnetically-driven losses inside the silicon. The model developed here is verified against experimental data, and the comparison shows a good match over the frequency range of interest to power electronics applications.

1. Introduction
Efforts to embed microfabricated solenoidal and toroidal inductors were previously reported in [1, 2] where fabrication limitations led to low inductances near several nH. Additionally, the inductors were widely separated from the surrounding silicon or suspended on a membrane over an open cavity to minimize losses [2]. Higher inductance and less restrictive fabrication is required for next-generation integrated power converters operating at frequencies near 10-100 MHz [3]. Correspondingly, this paper focuses on embedded inductors such as those fabricated more tightly within a deep recess in the silicon substrate as reported recently in [4]. There, a 50-nH inductor was demonstrated; its inductance was extendable to hundreds of nH using more turns. Analytic models for such inductors are essential for understanding the impact of their surrounding silicon, and for their optimized design. Models of the stored magnetic energy, the winding conduction loss, and the losses induced by adjacent windings have already been developed in [5, 6] for non-embedded inductors. These models can also be used for silicon-embedded inductors. Therefore, this paper focuses on modeling the additional phenomena that arise due to the presence of the silicon substrate. This includes losses in the silicon and increased parasitic capacitance. The original and extended models are combined to form an equivalent circuit model for silicon-embedded inductors.
2. Losses

As seen in Figure 1(a), the winding in a toroidal inductor spirals around the toroidal core in the poloidal direction as the spiral progresses in the toroidal direction. Thus, there exists a single-turn toroidal current which creates poloidal flux outside the toroidal core and a multi-turn poloidal current which creates toroidal flux through the toroidal core. Both fluxes drive currents and hence losses in the silicon. Additionally, the magnitude of the potential of the winding turns progresses linearly from one turn to the next in the toroidal direction. Capacitive coupling of this temporally- and spatially-varying potential also drives losses in the silicon. The geometry shown in Figure 1(b) is used to model these losses.

The magnetically-driven losses in the silicon are modeled under the assumption that the skin depth in silicon is comparable to or larger than $R_o$ or smaller dimensions at the operating frequencies of interest; $R_o$ is typically 5 mm. Assuming a doping density of $10^{16}$ cm$^{-3}$ and an operating frequency of 30 MHz, the skin depth of silicon is approximately 7 mm, and so the large skin depth assumption is justified. At a doping density of $10^{18}$ cm$^{-3}$, for example, the skin depth drops to 1 mm, and magnetic diffusion must be considered.

The magnetically-driven losses in the toroidal core, Region 2 ranging over $R_i < r < R_o$, is modeled under the assumption that the skin depth of silicon is larger than $R_i$ and $R_o$, respectively. In these regions, a nominally

$$P_{M2} = \frac{\sigma \omega^2 \mu_0^2 N^2 I^2 D^3 \ln(R_o/R_i)}{(48 \pi)}$$

where $NI$ is the peak number of Ampere-Turns around the core. As a typical example, for a toroidal inductor with $N = 25$, $R_o = 5$ mm, $R_i = 0.5$ mm, $D = 1$ mm, a silicon doping density of $10^{18}$ cm$^{-3}$ which yields $\sigma = 5 \cdot 10^3$ S/m, $\omega = 2\pi \times 30$ MHz, and $I = 1$ A, (1) yields $P_{M2} = 2.7$ W, whereas a doping density of $10^{16}$ cm$^{-3}$ which yields $\sigma = 200$ S/m, yields $P_{M2} = 0.1$ W. Therefore, due to the significant loss for doping densities above approximately $10^{16}$ cm$^{-3}$, the option of having a silicon core is eliminated and not considered further here.

The magnetically-driven losses in Regions 1 and 3 are modeled under the assumption that the skin depth of silicon is larger than $R_i$ and $R_o$, respectively. In these regions, a nominally
\( \hat{z} \)-directed time-varying poloidal \( \vec{B} \) drives a \( \hat{\phi} \)-directed \( \vec{E} \). The resulting losses are analyzed here based on the inductance \( L_p \) [9], associated with the energy stored in the exterior poloidal magnetic fields given by

\[
L_p = 0.5 \mu_o (R_o + R_i) \left( \ln(8(R_o + R_i)/(R_o - R_i)) - 2 \right) .
\]

(2)

Assuming that the skin depth of silicon is larger than \( R_i \), the peak poloidal flux \( \Lambda_p \), and the magnitude of the approximately uniform poloidal magnetic field in Region 1, \( r < R_i \), can be determined from

\[
\Lambda_p = \pi R_i^2 |\vec{B}| = L_p I .
\]

(3)

Faraday’s Equation in combination with (2) and (3) then yields \( |\vec{E}| \) from which the magnetically-driven Ohmic loss in Region 1, \( P_{M1} \), may be computed. This computation yields

\[
P_{M1} = \sigma W \omega^2 L_p^2 I^2 / (16 \pi) .
\]

(4)

In Region 3, the magnitude of the magnetic flux density is assumed to fall off as \( B_o/(R_o/r)^3 \). Thus it decays in \( r \) as it would away from a magnetic dipole assuming that the skin depth of silicon is larger than \( R_o \). The coefficient \( B_o \) can then be determined from

\[
\Lambda_p = \int_{R_o}^\infty B_o (R_o/r)^3 2\pi rdr = 2\pi B_o R_o^2 L_p I .
\]

(5)

Faraday’s Equation in combination with (2) and (5) then yields \( |\vec{E}| \) from which the magnetically-driven Ohmic loss in Region 3, \( P_{M3} \), may be computed. This computation yields

\[
P_{M3} = D \sigma \omega^2 L_p^2 I^2 / (8 \pi) .
\]

(6)

The summation of (4) and (6) is used to model the magnetically-driven losses in silicon.

The electrically-driven losses are driven by the time- and spatially-varying winding turn potentials coupled capacitively to the silicon through the insulator that separates the turns from the silicon. Modeling the associated electric fields and Ohmic loss begins by expressing the electric potential around the inductor in the \( \hat{\phi} \) direction as a Fourier sum of single harmonic functions. Following [7], this potential is then used to determine two-dimensional \( \hat{r}, \hat{\phi} \) potential solutions to Laplace’s Equation in Regions 1, 3 and 4. The electric fields and the electrically-driven Ohmic losses can then be found directly from the potentials in the three regions. The details of this approach are given in [8]. For brevity, only the results are detailed here.

In Regions 1 and 3, the electric potential is modeled as a Fourier sum of terms taking the form \( (\phi_+ r^m + \phi_- r^{-m}) \sin(m\phi) \), where \( m \) is the Fourier expansion index, and \( \phi_+ \) and \( \phi_- \) are coefficients. A separate expression having this form is used for each silicon region and each insulating silicon-dioxide region between the silicon and the winding turns. The solutions are stitched together through boundary conditions at material interfaces, and the coefficients are determined by matching boundary conditions. The \( \hat{r} \) - and \( \hat{\phi} \)-directed electric fields in the silicon, and the associated Ohmic losses, may then be calculated directly from the potential solutions. For Region 3, \( r > R_o \), this approach yields

\[
P_{E3} = \frac{2\pi \sigma V^2 \omega^2 \epsilon_i^2}{m (\Gamma_{\omega e}^2 + \Gamma_\sigma^2)} , \quad \Gamma_\sigma = \sigma_{Si} \left( \left( \frac{R_o + \Delta}{R_o} \right)^m - \left( \frac{R_o}{R_o + \Delta} \right)^m \right) , \quad \Gamma_{\omega e} = \omega \left( \frac{R_o}{R_o + \Delta} \right)^m (\epsilon_1 - \epsilon_{Si}) + \omega \left( \frac{R_o + \Delta}{R_o} \right)^m (\epsilon_1 + \epsilon_{Si})
\]

(7)

for the electrically-driven loss \( P_{E3} \) where \( V \) is the terminal voltage of the inductor, \( \epsilon_{Si} \) is the permittivity of silicon, and \( \epsilon_1 \) is the permittivity of the silicon-dioxide insulator. The same
Figure 2. (a) Semi-distributed reactive circuit model for a silicon-embedded inductor. (b) Simplified equivalent circuit for a silicon-embedded inductor.

approach can be used for computing the loss $P_{E1}$ in Region 1, $r < R_i$. Indeed, $P_{E1}$ can be found from (7) by replacing $(R_o + \Delta)$ with $(R_i - \Delta)$ and by replacing $R_o$ alone with $R_i$ alone.

To determine the electrically-driven loss $P_{E4}$ in the silicon layer underneath the inductor in Region 4, a structure involving three layers is considered. Beneath the winding turns is a thin insulating layer of silicon dioxide having a thickness of $\Delta$ above a silicon layer having a thickness $Y$, which is in turn above an infinitely-thick air layer. An approach similar to that used above for Regions 1 and 3 is used here as well. This approach is detailed in [8]. The analytical results for the underlying layer are not included here for brevity as they are quite long.

3. Equivalent Circuit & Capacitance

Equivalent circuits for sinusoidal-steady-state inductor operation are useful for power circuit analysis, co-optimization and design. They also capture what is naturally measured as the inductor impedance. Figure 2(a) shows a semi-distributed equivalent reactive circuit modeling a silicon-embedded toroidal inductor. This circuit treats the inductor as a leakage-free $N$-winding transformer with one transformer winding per inductor turn. A lumped parasitic capacitor $C$ exists between each turn and the silicon adjacent to that turn; the silicon substrate is treated as a common node. Analysis of the inductor in Figure 1(b) and the circuit in Figure 2(a) leads to an equivalent capacitance of

$$C_{eq} = C \left( N^2 - 1 \right) / \left( 12N \right) , \quad C = 2 \pi F \varepsilon_1 \left( R_o D + R_i D + 0.5 \left( R_o^2 - R_i^2 \right) \right) / \left( N \Delta \right) . \tag{8}$$

as used in the equivalent circuit in Figure 2(b) where $F$ is the lateral filling factor fraction of the area of the winding turns above the silicon. The inductance $L_{eq}$ accounts for the toroidal and poloidal magnetic flux [5, 8]. Ohmic winding losses [5, 8], and $P_{M1}$ and $P_{M3}$ are driven by the inductor current and properly appear as parts of $R_{eq1}$ in series with $L_{eq}$ as shown in Figure 2(b). Their total time-average loss is divided by $I^2/2$ to determine their contributions to $R_{eq1}$. Losses $P_{E1}$, $P_{E3}$, and $P_{M4}$ are driven by $V$, and properly appear as parts of $R_{eq2}$ in parallel with $L_{eq}$ and $R_{eq1}$, along with $C_{eq}$. Their total time-average loss is divided by $V^2/2$ to determine their contributions to $R_{eq2}$. In all cases, the losses are frequency dependent, and so are evaluated at the frequency of power circuit operation or impedance measurement prior to determining $R_{eq}$.

4. Experiments & Model Comparisons

The measured impedance of toroidal inductors embedded in 1-10 $\Omega$-cm standard-resistivity (SR) wafers and 2-3 k$\Omega$-cm high-resistivity (HR) wafers, having skin depths of 1.6 cm and 46 cm, respectively, was reported in [4]. This data is shown in Figure 3(a), together with theoretical predictions based on the models presented above. A 0.65-m$\Omega$ contact resistance per vertical via joint is added to $R_{eq1}$ to improve the experiment-to-theory match. The embedded inductors have $N = 25$, $W = 500 \mu$m, $D = 300 \mu$m, $R_o = 6$ mm, $R_i = 2$ mm, $\Delta = 12 \mu$m and an average copper winding thickness of $T = 30 \mu$m. The inductors exhibit a measured $\Im(Z)/\omega$ of 45 nH at
Figure 3. (a) Measured and modeled impedance of embedded inductors in SR and HR wafers. (b) Decomposition of modeled losses for the cases of SR and HR wafers.

low frequency and a DC resistance of 300 mΩ. Figure 3(b) decomposes the modeled losses into winding and silicon losses. The inductors in the SR and HR wafers exhibit peak quality factors of 16 near 40 MHz and 22 near 80 MHz, respectively. Note that the silicon losses in the two inductors rise quickly near 40-50 MHz. Further discussion of these results appears in Section 5.

5. Conclusions
Several important conclusions can be drawn from the work described here. First, the equivalent circuit model matches well the measured behavior of a silicon-embedded toroidal inductor. Second, as long as it is removed from the core of the inductor at high doping densities, the close proximity of silicon need not significantly degrade inductor quality factor. Winding loss dominates the quality factors modeled and measured here for frequencies up to 40 MHz, making winding optimization more important than silicon selection; see Figure 3. For higher frequencies, quality factor is a strong function of silicon resistivity as expected, making its selection more important than winding design. At these frequencies, electrically-driven silicon losses appear to be dominant, making large Δ an important fabrication objective.

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