Baryogenesis in the Zee-Babu model with arbitrary $\xi$ gauge

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We consider the baryogenesis picture in the Zee-Babu model. Our analysis shows that electroweak phase transition (EWPT) in the model is a first-order phase transition at the 100 GeV scale, its strength is about $1 - 4.15$ and the masses of charged Higgs are smaller than 300 GeV. The EWPT is strengthened by only the new bosons and this strength is enhanced by arbitrary $\xi$ gauge. However, the $\xi$ gauge does not make lose the first-order electroweak phase transition. The process of the EWPT kinetics corresponding to B violation is received through the sphaleron probability. By a thin-wall approximation, we assume that sphaleron rate is larger than the cosmological expansion rate at the temperature being higher than the critical temperature; and after the phase transition, the sphaleron process is decoupled. This also suggests that the phase transition is a transition depending at each point in space. This may provide baryon-number violation (B-violation) necessary for baryogenesis in the relationship with non-equilibrium physics in the early universe.

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I. INTRODUCTION

Physics, at present, has entered into a new period, understanding of the early Universe. In that context, almost Cosmology and Particle Physics are on the same way. Being as a central issue of cosmology and particle physics, at present the baryon asymmetry is an interesting problem. If we could explain this problem, we can understand the true nature of the smallest elements and reveal a lot about an imbalances matter-antimatter from the early Universe.

The electroweak baryogenesis (EWBG) is a way to explaining the Baryon Asymmetry of Universe (BAU) in the early universe, has been known by Sakharov conditions, which are B, C, CP violations, and deviation from thermal equilibrium [1]. These conditions can be satisfied when the EWPT must be a strongly first-order phase transition. Because that not only leads to thermal imbalance [2], but also makes a connection between B and CP violation via nonequilibrium physics [3].

The EWPT has been investigated in the Standard Model (SM) [2, 4, 5] as well as its various extended versions [6–12, 14–20]. For the SM, although the EWPT strength is larger than unity at the electroweak scale, but the mass of the Higgs boson must be less than 125 GeV [2, 4, 5]; so the EWBG requires new physics beyond the SM at the weak scale [6].

Many extensions such as the Two-Higgs-Doublet model, the reduced minimal 3-3-1 model, the economical 3-3-1 model or the Minimal Supersymmetric Standard Model, have a strongly first-order EWPT and the new sources of CP violation, which are necessary to account for BAU; triggers for the first-order EWPT in these models are heavy bosons or dark matter candidates [7–11, 17–19]. However, the most researches of the EWPT are the Landau gauge. Recently gauge invariant also made important contributions in the electroweak phase transition as researching in Ref. [20].

The quantity of sphaleron rate which is B violation rate, has been calculated in the SM [2, 4, 5] and the reduced minimal 3-3-1 model [11]. In addition, by using non-perturbative

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lattice simulations, a powerful framework and set of analytic and numerical tools have been developed in Refs. [4, 5].

The Zee-Babu (ZB) model is one of the simplest extensions of the SM which has some interesting features [13]. We have considered the EWPT and sphaleron rate in the ZB model due to its simplicity.

The ZB model has more two charged scalars $h^\pm$ and $k^{\pm\pm}$ in the Higgs potential. The kind of new scalars can play an important role in the early universe. They can be a reason for tiny mass of neutrinos through two loops or three loop corrections [13, 21]. One important property of these particles which will be shown in this paper, is that they can be triggers for the first-order phase transition.

This paper is organized as follows. In Sect. II we give a short review of the ZB model and we drive an effective potential which has a contribution from heavy scalars and the $\xi$ gauge at one-loop level. In Sect. III we find the range mass of charged scalar particles by a first-order phase transition condition. In Sect. IV we offer the solutions of VEV and estimation sphaleron rate by our approximations, and show that this rate can satisfy the decoupling condition. Finally, Sect. V is devoted to constraints on the mass of the charged Higgs boson. In Sect. VI we summarize and describe outlooks.

II. EFFECTIVE POTENTIAL IN THE ZEE-BABU MODEL

In the ZB model, by adding two charged scalar fields $h^\pm$ and $k^{\pm\pm}$ [13], the Lagrangian becomes

\[
\mathcal{L} = \mathcal{L}_{SM} + f_{ab} \bar{\psi}^c_{al} \psi_{bL} h^+ + h'_{ab} \bar{t}^c_{aR} t_{bR} k^{++} + V(\phi, h, k) + (D_\mu h^+)^\dagger (D^\mu h^+) + (D_\mu k^{++})^\dagger (D^\mu k^{++}) + H.c
\]  

(1)

In this model, the Higgs potential has four couplings between $h^\pm$ or $k^{\pm\pm}$ and neutral Higgs [13]:

\[
V(\phi, h, k) = \mu^2 |\phi|^2 + u_1^2 |h|^2 + u_2^2 |k|^2 + \lambda_H |\phi|^2 + \lambda_h |h|^4 + \lambda_k |k|^4 + \lambda_{hk} |h|^2 |k|^2 + 2p^2 |h|^2 |\phi|^2 + 2q^2 |k|^2 |\phi|^2 + (\mu_{hk} h^2 k^{++} + H.c),
\]  

(2)

where

\[
\phi = \begin{pmatrix} \rho^+ \\ \rho^0 \end{pmatrix}
\]  

(3)
and $\rho^0$ has a Vacuum Expectation Value (VEV)

$$\rho^0 = \frac{1}{\sqrt{2}} (v_0 + \sigma + i\zeta). \quad (4)$$

The masses of $h^\pm$ and $k^{\pm\pm}$ are given by

$$m_{h^\pm}^2 = p^2 v_0^2 + u_1^2,$$

$$m_{k^{\pm\pm}}^2 = q^2 v_0^2 + u_2^2.$$

Diagonalizing matrices in the kinetic component of the Higgs potential and retain Goldstone bosons, we obtain

$$m_{H}^2(v_0) = -\mu^2 + \frac{3}{4} \lambda v_0^2,$$

$$m_G^2(v_0) = -\mu^2 + \frac{1}{4} \lambda v_0^2,$$

$$m_Z^2(v_0) = \frac{1}{4} (g^2 + g'^2) v_0^2 = a^2 v_0^2,$$

$$m_W^2(v_0) = \frac{1}{4} g^2 v_0^2 = b^2 v_0^2. \quad (5)$$

A. EFFECTIVE POTENTIAL WITH LANDAU GAUGE

From Eq. (1), ignoring Goldstone bosons, we obtain an effective potential with contributions of $h^\pm$ and $k^{\pm\pm}$ in the Landau gauge:

$$V_{eff}(v) = V_0(v) + \frac{3}{64 \pi^2} \left( m_Z^2(v) \ln \frac{m_Z^2(v)}{Q^2} + 2 m_W^2(v) \ln \frac{m_W^2(v)}{Q^2} - 4 m_t^4(v) \ln \frac{m_t^2(v)}{Q^2} \right)$$

$$+ \frac{1}{64 \pi^2} \left( 2 m_{h^\pm}^4(v) \ln \frac{m_{h^\pm}^2(v)}{Q^2} + 2 m_{k^{\pm\pm}}^4(v) \ln \frac{m_{k^{\pm\pm}}^2(v)}{Q^2} + m_H^4(v) \ln \frac{m_H^2(v)}{Q^2} \right)$$

$$+ \frac{3T^4}{4 \pi^2} \left\{ F_- \left( \frac{m_Z(v)}{T} \right) + F_- \left( \frac{m_W(v)}{T} \right) + 4 F_+ \left( \frac{m_t(v)}{T} \right) \right\}$$

$$+ \frac{T^4}{4 \pi^2} \left\{ 2 F_- \left( \frac{m_{h^\pm}(v)}{T} \right) + 2 F_- \left( \frac{m_{k^{\pm\pm}}(v)}{T} \right) + F_- \left( \frac{m_H(v)}{T} \right) \right\},$$

where $v_\rho$ is a variable which changes with temperature, and at $0^o K$, $v_\rho \equiv v_0 = 246$ GeV.

Here

$$F_\pm \left( \frac{m_\phi}{T} \right) = \int_0^{\frac{m_\phi}{T}} \frac{\alpha J^1_\pm(\alpha, 0) d\alpha}{\alpha J^1_\pm(\alpha, 0)},$$

$$J^1_\pm(\alpha, 0) = 2 \int_0^{\infty} \frac{(x^2 - \alpha^2)^{\frac{1}{2}}}{e^x + 1} dx.$$

B. EFFECTIVE POTENTIAL WITH $\xi$ GAUGE

However, we know that in high levels, the contribution of Goldstone boson cannot be ignored. Therefore, we must consider an effective potential in arbitrary $\xi$ gauge given by
and

\[
\mathcal{V}_1^{T=0}(v) = \frac{1}{4(4\pi)^2}(m_H^2)^2 \left[ \ln \left( \frac{m_H^2}{Q^2} \right) - \frac{3}{2} \right] + \frac{1}{4(4\pi)^2}(m_{\tilde{\chi}_\pm}^2)^2 \left[ \ln \left( \frac{m_{\tilde{\chi}_\pm}^2}{Q^2} \right) - \frac{3}{2} \right] \\
+ \frac{1}{4(4\pi)^2}(m_{\tilde{\chi}_\pm}^2)^2 \left[ \ln \left( \frac{m_{\tilde{\chi}_\pm}^2}{Q^2} \right) - \frac{3}{2} \right] + \frac{2\times 1}{4(4\pi)^2}(m_{\tilde{\chi}_\pm}^2 + \xi m_W^2)^2 \left[ \ln \left( \frac{m_{\tilde{\chi}_\pm}^2 + \xi m_W^2}{Q^2} \right) - \frac{3}{2} \right] \\
+ \frac{1}{4(4\pi)^2}(m_{\tilde{\chi}_\pm}^2 + \xi m_W^2)^2 \left[ \ln \left( \frac{m_{\tilde{\chi}_\pm}^2 + \xi m_W^2}{Q^2} \right) - \frac{3}{2} \right] + \frac{2\times 3}{4(4\pi)^2}(m_{\tilde{\chi}_\pm}^2 + \xi m_W^2)^2 \left[ \ln \left( \frac{m_{\tilde{\chi}_\pm}^2 + \xi m_W^2}{Q^2} \right) - \frac{3}{2} \right] \\
- \frac{3}{4(4\pi)^2}(\xi m_W^2)^2 \left[ \ln \left( \frac{\xi m_W^2}{Q^2} \right) - \frac{3}{2} \right] - \frac{2\times 1}{4(4\pi)^2}(\xi m_W^2)^2 \left[ \ln \left( \frac{\xi m_W^2}{Q^2} \right) - \frac{3}{2} \right] \\
- \frac{1}{4(4\pi)^2}(\xi m_W^2)^2 \left[ \ln \left( \frac{\xi m_W^2}{Q^2} \right) - \frac{3}{2} \right] - \text{“free”},
\]

\( (6) \)

and

\[
\mathcal{V}_1^{T\neq0}(v, T) = \frac{T^4}{2\pi^2} \left[ J_B \left( \frac{m_H^2}{T^2} \right) + J_B \left( \frac{m_{\tilde{\chi}_\pm}^2}{T^2} \right) + 2J_B \left( \frac{m_{\tilde{\chi}_\pm}^2}{T^2} \right) \right] \\
+ \frac{T^4}{2\pi^2} \left[ 2\times J_B \left( \frac{m_{\tilde{\chi}_\pm}^2 + \xi m_W^2}{T^2} \right) + J_B \left( \frac{m_{\tilde{\chi}_\pm}^2 + \xi m_W^2}{T^2} \right) \right] \\
+ \frac{3T^4}{2\pi^2} \left[ 2\times J_B \left( \frac{m_W^2}{T^4} \right) + J_B \left( \frac{m_W^2}{T^4} \right) + J_B \left( \frac{m_W^2}{T^4} \right) \right] \\
- \frac{T^4}{2\pi^2} \left[ 2\times J_B \left( \frac{\xi m_W^2}{T^2} \right) + J_B \left( \frac{\xi m_W^2}{T^2} \right) + J_B \left( \frac{\xi m_W^2}{T^2} \right) \right] - \text{“free”},
\]

\( (7) \)

where “free” represents a free-field subtraction.

### III. ELECTROWEAK PHASE TRANSITION IN THE ZEE-BABU MODEL

#### A. EWPT in Landau gauge

Ignoring \( u_1 \) and \( u_2 \) (i.e., \( u_1 \) and \( u_2 \) are assumed to be very small), we can write the high-temperature expansion of the potential \( (6) \) as a quartic equation in \( v \):

\[
V_{\text{eff}}(v) = D(T^2 - T_0^2)v^2 - ET |v|^3 + \frac{\lambda_T}{4} v^4,
\]

\( (8) \)
in which

\[
D = \frac{1}{24v_0^2} \left[ 6m_W^2(v_0) + 3m_{Z_1}^2(v_0) + m_H^2(v_0) + 2m_{h^\pm}^2(v_0) + 2m_{k^{\pm\pm}}^2(v_0) + 6m_t^2(v_0) \right],
\]

\[
T_0^2 = \frac{1}{D} \left\{ \frac{m_H^2(v_0)}{4} - \frac{1}{32\pi^2v_0^2} \left( 6m_W^4(v_0) + 3m_{Z_1}^4(v_0) + m_H^4(v_0) + 2m_{h^\pm}^4(v_0) + 2m_{k^{\pm\pm}}^4(v_0) - 12m_t^4(v_0) \right) \right\},
\]

\[
E = \frac{1}{12\pi v_0^3} \left( 6m_W^4(v_0) + 3m_{Z_1}^4(v_0) + m_H^4(v_0) + 2m_{h^\pm}^4(v_0) + 2m_{k^{\pm\pm}}^4(v_0) \right),
\]

\[
\lambda_T = \frac{m_H^2(v_0)}{2v_0^2} \left\{ 1 - \frac{1}{8\pi^2v_0^2(m_H^2(v_0))} \left[ 6m_W^2(v_0) \ln \frac{m_W^2(v_0)}{a_6T^2} + 3m_{Z_1}^2(v_0) \ln \frac{m_{Z_1}^2(v_0)}{a_6T^2} + m_H^2(v_0) \ln \frac{m_H^2(v_0)}{a_6T^2} + 2m_{h^\pm}^2(v_0) \ln \frac{m_{h^\pm}^2(v_0)}{a_6T^2} + 2m_{k^{\pm\pm}}^2(v_0) \ln \frac{m_{k^{\pm\pm}}^2(v_0)}{a_6T^2} - 12m_t^2(v_0) \ln \frac{m_t^2(v_0)}{a_6T^2} \right] \right\},
\]

where \(v_0\) is the value where the zero-temperature effective potential \(V_{eff}^{\phi K}(v)\) gets the minimum. Here, we acquire \(V_{eff}^{\phi K}\) from \(V_{eff}\) in Eq. (8) by neglecting all terms in the form \(F_+ \left( \frac{m}{T} \right)\).

The minimum conditions for \(V_{eff}^{\phi K}(v)\) are

\[
V_{eff}^{\phi K}(v_0) = 0, \quad \frac{\partial V_{eff}^{\phi K}(v)}{\partial v} \bigg|_{v=v_0} = 0, \quad \frac{\partial^2 V_{eff}^{\phi K}(v)}{\partial v^2} \bigg|_{v=v_0} = \left[ m_H^2(v) \right] \bigg|_{v=v_0} = 125^2 \text{ GeV}^2. \quad (10)
\]

We also have the minima of the effective potential (8)

\[
v = 0, \quad \nu \equiv \nu_c = \frac{2ET_c}{\lambda_{T_c}}, \quad (11)
\]

where \(\nu_c\) is the critical VEV of \(\phi\) at the broken state, and \(T_c\) is the critical temperature of phase transition given by

\[
T_c = \frac{T_0}{\sqrt{1 - E^2/D\lambda_{T_c}}}. \quad (12)
\]

Now let us investigate the phase transition strength

\[
S = \frac{\nu_{co}}{T_c} = \frac{2E}{\lambda_{T_c}} \quad (13)
\]

of this EWPT. In the limit \(E \to 0\), the transition strength \(S \to 0\) and the phase transition is a second-order. To have a first-order phase transition, we requires \(S \geq 1\). We plot \(S\) as a function of \(m_{h^\pm}\) and \(m_{k^{\pm\pm}}\) in Fig. [1].

According to Ref. [28], the accuracy of a high-temperature expansion for the effective potential such as that in Eq. (8) will be better than 5\% if \(\frac{m_{boson}}{T} < 2.2\), where \(m_{boson}\) is the
relevant boson mass. Therefore, as shown in Fig. 1 for $m_{h^\pm}$ and $m_{k^{\pm\pm}}$ which are respectively in the ranges $0 - 350$ GeV, the transition strength is in the range $1 \leq S < 2.4$.

We see that the contribution of $h^\pm$ and $k^{\pm\pm}$ are the same. The larger mass of $h^\pm$ and $k^{\pm\pm}$, the larger cubic term ($E$) in the effective potential but the strength of phase transition cannot be strong. Because the value of $\lambda$ also increases, so there is a tension between $E$ and $\lambda$ to make the first order phase transition. In addition when the masses of charged Higgses are too large, $T_0, \lambda$ will be unknown or $S \to \infty$.

![FIG. 1:](image)

FIG. 1: When the solid contour of $s = 2E/\lambda T_c = 1$, the dashed contour: $2E/\lambda T_c = 1.5$, the dotted contour: $2E/\lambda T_c = 2$, the dotted-dashed contour: $2E/\lambda T_c = 2.4$, even and no-smooth contours: $s \to \infty$

**B. EWPT in $\xi$ gauge**

We can rewrite the high-temperature expansion of the potential (6) and (7) as a like-quartic equation in $v$

$$
\mathcal{V} = (\mathcal{D}_1 + \mathcal{D}_2 + \mathcal{D}_3 + \mathcal{D}_4 + \mathcal{B}_2) v^2 + \mathcal{B}_1 v^3 + \Lambda v^4 + f(T, u_1, u_2, \mu, \xi),
$$

(14)

where

$$
f(T, u_1, u_2, \mu, \xi, v) = \mathcal{C}_1 + \mathcal{C}_2,
$$
\[ \mathcal{D}_1 = \frac{T^2}{24v_0^2} \left( 3m_Z^2(v_0) + 6m_W^2(v_0) + 6m_t^2(v_0) + 2(m_{h^\pm}^2(v_0) - u_1^2) + 2(m_{h^\pm}^2(v_0) - u_2^2) + 6\lambda v_0^2 \right), \]

\[ \mathcal{D}_2 = \frac{1}{32v_0^2\pi^2} \left\{ 3m_Z^4(v_0) + 6m_W^4(v_0) - 12m_t^4(v_0) + 2(m_{h^\pm}^2(v_0) - u_1^2)^2 - 8\pi^2v_0^2m_{H_0}^2 \right. \\
+ 2(m_{h^\pm}^2(v_0) - u_2^2)^2 + 12v_0^4\lambda^2 + 2m_Z^2(v_0)v_0^2\lambda\xi + 4m_W^2(v_0)v_0^2\lambda\xi \right\}, \]

\[ \mathcal{D}_3 = \frac{1}{32\pi^2} \left\{ 2p^2u_1^2 \ln \left[ \frac{a_bT^2}{p^2u_1^2 + u_1^2} \right] + 2q^2u_2^2 \ln \left[ \frac{a_bT^2}{q^2v_0^2 + u_2^2} \right] \\
- 3\lambda\mu^2 \ln \left[ \frac{a_bT^2}{3v_0^2\lambda - \mu^2} \right] - \lambda\mu^2 \ln \left[ \frac{a_bT^2}{v_0^2(\lambda + a^2\xi) - \mu^2} \right] \\
- 2\lambda\mu^2 \ln \left[ \frac{a_bT^2}{v_0^2(\lambda + b^2\xi) - \mu^2} \right] - a^2\xi\mu^2 \ln \left[ \frac{a_bT^2}{v_0^2(\lambda + a^2\xi) - \mu^2} \right] \\
- 2b^2\xi\mu^2 \ln \left[ \frac{a_bT^2}{v_0^2(\lambda + b^2\xi) - \mu^2} \right] \right\} , \]

\[ \mathcal{D}_4 = \frac{1}{32\pi^2} \left( 2p^2u_1^2 + 2q^2u_2^2 - 6\lambda\mu^2 - a^2\xi\mu^2 - 2b^2\xi\mu^2 \right), \]

\[ \Lambda = \frac{1}{64\pi^2} \left\{ 2p^4 \ln \left[ \frac{a_bT^2}{u_1^2 + p^2v_0^2} \right] + 2q^4 \ln \left[ \frac{a_bT^2}{u_2^2 + q^2v_0^2} \right] + 3a^4 \ln \left[ \frac{a_bT^2}{a^2v_0^2} \right] + 6b^4 \ln \left[ \frac{a_bT^2}{b^2v_0^2} \right] \\
- 12k^4 \ln \left[ \frac{a_fT^2}{k^2v_0^2} \right] + 9\lambda^2 \ln \left[ \frac{a_bT^2}{3v_0^2\lambda - \mu^2} \right] + 8\pi^2v_0^2m_{H_0}^2 \\
- a^2\xi^2 \ln \left[ \frac{a_bT^2}{a^2\xi v_0^2} \right] - 2b^4\xi^2 \ln \left[ \frac{a_bT^2}{b^2\xi v_0^2} \right] \\
+ a^4\xi^2 \ln \left[ \frac{a_bT^2}{v_0^2(\lambda + a^2\xi) - \mu^2} \right] + 2b^4\xi^2 \ln \left[ \frac{a_bT^2}{v_0^2(\lambda + b^2\xi) - \mu^2} \right] \\
+ 2a^2\lambda\xi \ln \left[ \frac{a_bT^2}{v_0^2(\lambda + a^2\xi) - \mu^2} \right] + 4b^2\lambda\xi \ln \left[ \frac{a_bT^2}{v_0^2(\lambda + b^2\xi) - \mu^2} \right] \\
+ \lambda^2 \ln \left[ \frac{a_bT^2}{v_0^2(\lambda + a^2\xi) - \mu^2} \right] + 2\lambda^2 \ln \left[ \frac{a_bT^2}{v_0^2(\lambda + b^2\xi) - \mu^2} \right] \right\} , \]

\[ \mathcal{B}_1 = \frac{T^3}{12\pi v_0^3} \left( -3m_Z^3(v_0) - 6m_W^3(v_0) + m_Z^3(v_0)\xi^{3/2} + 2m_W^3(v_0)\xi^{3/2} \right), \]

\[ \mathcal{B}_2 = T \left( \frac{-p^2u_1 + p^2v_2}{6\pi} - \frac{q^2u_2 + q^2v_2}{6\pi} - \frac{\lambda\sqrt{3\lambda v^2 - \mu^2}}{4\pi} - \frac{\lambda\sqrt{\lambda v^2 + a^2\xi v^2 - \mu^2}}{12\pi} - \frac{a^2\xi\sqrt{\lambda v^2 + a^2\xi v^2 - \mu^2}}{12\pi} \right. \\
- \frac{\lambda\sqrt{\lambda v^2 + b^2\xi v^2 - \mu^2}}{6\pi} - \frac{b^2\xi\sqrt{\lambda v^2 + b^2\xi v^2 - \mu^2}}{6\pi} \right), \]
\[ C_1 = -\frac{T u_1^2 \sqrt{u_1^2 + p^2 v^2}}{6\pi} - \frac{T u_2^2 \sqrt{u_2^2 + q^2 v^2}}{6\pi} - \frac{T^2 \mu^2}{6} + \frac{3\mu^4}{32\pi^2} + T \mu^2 \sqrt{3\lambda v^2 - \mu^2} + T \mu^2 \sqrt{\lambda v^2 + a^2 \xi v^2 - \mu^2} + T \mu^2 \sqrt{\lambda v^2 + b^2 \xi v^2 - \mu^2} + \frac{u_1^4}{32\pi^2} + \frac{u_2^4}{32\pi^2} \mu^4 \ln \left[ \frac{a_\mu T^2}{v_0^2} \right] \mu^4 \ln \left[ \frac{a_\mu T^2}{v_0^2 (\lambda + a^2 \xi)} \right] \mu^4 \ln \left[ \frac{a_\mu T^2}{v_0^2 (\lambda + b^2 \xi)} \right] \mu^4 \ln \left[ \frac{a_\mu T^2}{v_0^2 (\lambda + b^2 \xi)} \right] \]

\[ C_2 = \frac{T^2 u_1^2}{12} + \frac{3u_1^4}{64\pi^2} + \frac{T^2 u_2^2}{12} + \frac{3u_2^4}{64\pi^2} + \delta \Omega, \]

\[ \delta \Omega = -\frac{1}{128\pi^2} \left( -4p^2 u_1^2 v_0^2 - 4q^2 u_2^2 v_0^2 + 3a^4 v_0^4 + 6b^4 v_0^4 - 12k^4 v_0^4 + 2p^4 v_0^4 + 2q^4 v_0^4 \right. \]

\[ + 12v_0^4 \lambda^2 + 2a^2 v_0^4 \lambda \xi + 4b^2 v_0^4 \lambda \xi + 12v_0^2 \lambda \mu^2 + 2a^2 v_0^2 \xi \mu^2 + 4b^2 v_0^2 \xi \mu^2 \]

\[ + 4u_1^4 \ln \left[ \frac{u_1^2 + p^2 v_0^2}{p^2 v_0^2} \right] + 4u_2^4 \ln \left[ \frac{u_2^2 + q^2 v_0^2}{q^2 v_0^2} \right] + 2\mu^4 \ln \left[ \frac{3v_0^2 \lambda - \mu^2}{3v_0^2 \lambda} \right] \]

\[ + 2\mu^4 \ln \left[ \frac{v_0^2 \lambda + a^2 v_0^2 \xi - \mu^2}{v_0^2 (\lambda + a^2 \xi)} \right] + 4\mu^4 \ln \left[ \frac{v_0^2 \lambda + b^2 v_0^2 \xi - \mu^2}{v_0^2 (\lambda + b^2 \xi)} \right] - 16\pi^2 v_0^2 m_{H_0}^2 \]

The potential (14) is not a quartic equation because \( B_2, D_3, D_4 \) and \( f(T, u_1, u_2, \mu, \xi, v) \) depend on \( v, \xi \text{ and } T \). The above effective potential has seven variables \( (u_1, u_2, p, q, \mu, \lambda, \xi) \). If Goldstone bosons are neglected and \( \xi = 0 \), the effective potential will be reduced to those in the Landau gauge. So, the efficiency is distorted by \( u_1, u_2, p, q, \xi \) but not so much and the electroweak phase transition can be slightly altered in the case of \( \xi = 0 \). Because (14) is, in form, similar to (8), the minimum conditions for (14) are still like (10) with just a condition: \( m_{H_0} = -\mu^2 + 3\lambda v_0^2 = 125 \text{ GeV} \).

Our problem is that there are many variables and some of them, for example, \( u_1, u_2, p, q, \mu \) play the same role. However, \( \xi \text{ and } \lambda \) are two important variables and have different roles. Therefore, in order to reduce a number of variables, we simplify the latter, meanwhile, do not lose the generality of the problem.

We can see that \( \mu \) reduces the strength of phase transitions. Because components containing \( \mu \) in the effective potential (14) (i.e., the term \( -\mu^2 \)) in \( \Lambda, B_2 \) are always associated with the negative sign, so \( \mu \) gives negative contributions. As a consequence, we will ignore \( \mu \) or fix \( \mu \ll \lambda v^2 \).

In \( D_2, B_2 \) factors, \( u_1, u_2 \) are the components of the Goldstone and gauge bosons. Therefore they make the contribution of these particles to be depended on \( v, u_1 \text{ and } u_2 \), so \( u_1 \text{ and }
\( u_2 \) always follow \( v \). However the effective potential is a function of \( v \), thus these two values \((u_1, u_2)\) also distort the shape of phase transitions. Hence, we ignore \( u_1, u_2 \).

More importantly we want to find the main contribution of the \( \xi \) gauge to the phase transition. However \( u_1, u_2, p, q, \mu \) are in the mass composition of the particles. As we pointed out in the previous section, when \( \xi = 0 \), heavy particles which were sufficiently stocked, make the strength of phase transition greater than one in this model. So we look for the simplest approximation to investigate the role of \( \xi \), i.e., we retain \( \xi \) and simplify the role of \( u_1, u_2, \mu \).

Furthermore \( \xi \) and \( u_1, u_2, \mu \) are not mutually exclusive, except for the \( D_3 \) factor which is a factor of \( v^2 \), containing \( T \). Thus the strength of phase transition depends directly on \( B \) and \( \Lambda \). As a result we want to estimate a contribution of the \( \xi \) gauge and we simplify \( D_3 \) being a compound function of \( T, \mu, u_1, u_2 \) by ignoring the action of \( u_1, u_2, \mu \).

Let us summarize our arguments. Firstly, ignoring \( u_1, u_2, \mu \), we obtain
\[
f(T, u_1, u_2, \mu, \xi, v) = 0, \quad D_3 = D_4 = 0 \quad \text{and} \quad \delta \Omega \approx \text{const}.
\]
Secondly, we can approximate \( \mu \ll \lambda v^2, u_1^2 \ll p^2 v^2 \) and \( u_2^2 \ll q^2 v^2 \) in \( B_2 \) actually, this case is quite similar to ignoring \( u_1, u_2, \mu \) and rewrite \( B_2 \) as follows:
\[
B_2 \approx B_3 = Tv \left( \frac{p^3}{6\pi} + \frac{q^3}{6\pi} - \frac{\lambda \sqrt{3\lambda}}{4\pi} + \frac{\lambda \sqrt{\lambda + a^2 \xi}}{12\pi} \right.
\]
\[
- \frac{a^2 \xi \sqrt{\lambda + a^2 \xi}}{12\pi} - \frac{\lambda \sqrt{\lambda + b^2 \xi}}{6\pi} - \frac{b^2 \xi \sqrt{\lambda + b^2 \xi}}{6\pi}
\]
\[
= \frac{T}{2v_0^3} \left( -2(m_{h^\pm}(v_0) - u_1^2)^{3/2} - 2(m_{k^\pm}(v_0) - u_2^2)^{3/2} \right.
\]
\[
-3v_0^3 \lambda \sqrt{3\lambda} - v_0^2 \lambda \sqrt{v_0^2 \lambda + m_Z^2(v_0) \xi} - 2v_0^2 \lambda \sqrt{v_0^2 \lambda + m_W^2(v_0) \xi} - m_Z^2(v_0) \xi \sqrt{v_0^2 \lambda + m_W^2(v_0) \xi} \right)
\]

Finally, we can rewrite (14) as follows:
\[
\mathcal{V} = (D_1 + D_2) v^2 + B v^3 + \Lambda v^4, \quad (15)
\]
where \( B = B_1 + B_3 \). If we ignore Goldstone boson and fix \( \xi = 0 \), Eq. (15) will return to Eq. (6). We do not write \( \delta \Omega \) in (15) because it is a constant.

The potential (15) is a quartic equation that likes (8). So the form of the critical temperature and strength EWPT also are like (12) and (13). As well as the comments in the previous section, two charged Higgs bosons have played the same role in the effective po-
tential. In order to ease calculating, we accept that \( m_{h^\pm} = m_{k^{\pm\pm}} \). Therefore, the effective potential only depends on \( \lambda, \xi \) and \( m_{h^\pm} \).

In figures from Fig. 2 to Fig. 5, we have plotted \( S \) as a function of \( m_{h^\pm}, \xi \) with \( \lambda = 0.1, 0.2, 0.35, 0.5 \).

FIG. 2: \( \lambda = 0.1 \) when the solid contour of \( S = 1 \), the dashed contour: \( S = 1.5 \), the dotted contour: \( S = 2 \), the dotted-dashed contour: \( S = 4.15 \), even and no-smooth contours: \( S \rightarrow \infty \)

According to Figures 2-5, if \( m_{h^\pm} = m_{k^{\pm\pm}} \) is smaller than 300 GeV, we have the first order phase transition. We also found that the larger \( \Lambda \) be, the smaller maximum strength of EWPT will be. In this case, the strength of the EWPT is in range, \( 1 < S < 4.15 \).

From Fig. 6 and 7, if \( \xi \) increases, \( \mathcal{E} \) increases and \( \lambda \) decreases, this means that the larger \( \xi \) be, the stronger strength of EWPT will be. The contributions from new particles \( (h^\pm, k^{\pm\pm}) \) make of the first order phase transition that the Standard Model cannot. In addition, we found that \( \xi \) will not lose the first order phase transition as mentioned in Ref. 20. From Fig. 6 we do not find the value of \( \xi \) that makes \( \mathcal{B} = 0 \). Moreover, ignoring \( \xi \) we still search a first order EWPT in the previous section.
FIG. 3: $\lambda = 0.2$ when the solid contour of $S = 1$, the dashed contour: $S = 1.5$, the dotted contour: $S = 2$, the dotted-dashed contour: $S = 4.15$, even and no-smooth contours: $S \rightarrow \infty$

FIG. 4: $\lambda = 0.35$ when the solid contour of $S = 1$, the dashed contour: $S = 1.5$, the dotted contour: $S = 2$, the dotted-dashed contour: $S = 4.15$, even and no-smooth contours: $S \rightarrow \infty$
FIG. 5: $\lambda = 0.5$ when the solid contour of $S = 1$, the dashed contour: $S = 1.5$, the dotted contour: $S = 2$, the dotted-dashed contour: $S = 4.1$, even and no-smooth contours: $S \rightarrow \infty$

FIG. 6: The dependence of $\mathcal{B}$ on $\xi$
IV. SPHALERON RATE IN THE ZEE-BABU MODEL

When temperature drops below $T_1$, the effective potential appears a non-zero VEV. The transition from a zero vacuum to a non-zero vacuum through two ways. The first way which cross-over a barrier, was called sphaleron. The second way is quantum tunneling, was called instanton.

The $B$ violation can be seen through lepton number violation. Lepton violation can be read through Chern-Simon number or Winding number [4]. But we obtain the kinetic processes of Higgs field in EWPT which can be described by the transition rate between two VEVs. This rate is sphaleron rate. So sphaleron rate or Chern-Simon number is different from zero, driving $\Delta B$ will not be zero [4].

The sphaleron rate is also important for determining whether the baryon asymmetry produced at the bubble exterior, then diffusing into the bubble interior or not.

The criteria $S > 1$ is a very approximate condition, subject to considerable theoretical uncertainties. We see that $S > 1$ is required for a first order phase transition but one can certainly have a first order phase transition for $S < 1$. However the sphaleron rate may be too large to preserve the baryon asymmetry in this case. Therefore we state that $S > 1$
is required for a first order phase transition and the sphaleron rate must be satisfied the sphaleron decoupling.

In addition, the sphaleron decoupling condition must be imposed at the temperature $T_E$ lower than $T_C$ where the EWPT terminates. In practice, since it is difficult to determine this temperature, we can substitute $T_N$, a nucleation temperature of the critical bubble, in place of $T_E$. The critical bubble is defined as the bubble whose surface energy and volume energy becomes balanced. Only such bubbles can nucleate and expand in the symmetric phase.

From above analysis we see that the heavy particles (with masses larger or equal mass of the $W^\pm$ boson) give main contribution to the sphaleron rate. Therefore, to study the sphaleron processes in the ZB model, we begin from the Lagrangian of the gauge-Higgs system

$$\mathcal{L}_{\text{gauge-Higgs}} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi). \quad (16)$$

From Eq.(16), the energy functional being the sum of the kinetic and potential constituents in the temporal gauge, will be the following form

$$\mathcal{E} = \int d^3 x \left[ (D_\mu \phi)^\dagger (D^\mu \phi) + V(\phi) \right], \quad (17)$$

here we assume that the least energy has the pure-gauge configurations. Thus $F_{0i}^a = 0$. Assuming that the EWPT processes occur in the form of nucleation bubbles, and using the temperature expansion of the effective potential at one loop given in the section II, we can rewrite the energy functional in the spherical coordinate system as follows

$$\mathcal{E} = 4\pi \int_0^\infty d^3 x \left[ \frac{1}{2} \left( \nabla^2 v \right)^2 + V_{\text{eff}}(v; T) \right]. \quad (18)$$

Using the static field approximation, i.e., VEV variable do not change in time, as follows

$$\frac{\partial v}{\partial t} = 0, \quad (19)$$

we obtain

$$\mathcal{E} = \int d^3 x \left[ \frac{1}{2} (\partial_i v)^2 + V_{\text{eff}}(v; T) \right]. \quad (20)$$

From the Lagrangian (16), it follows the equation of motion for the VEV $v_\rho$:

$$\ddot{v} + \nabla^2 v - \frac{\partial V_{\text{eff}}(v, T)}{\partial v} = 0. \quad (21)$$
When VEV does not change in time according to the condition (19), we can rewrite Eq. (21) in spherical coordinates

$$\frac{d^2 v}{dr^2} + \frac{2}{r} \frac{dv}{dr} - \frac{\partial V_{eff}(v, T)}{\partial v} = 0. \quad (22)$$

Lastly, from Eq. (18) and (22), the sphaleron energy in the EWPT process, $\mathcal{E}_{sph, su(2)}$, has the following form

$$\mathcal{E}_{sph} = 4\pi \int \left[ \frac{1}{2} \left( \frac{dv}{dr} \right)^2 + V_{eff}(v, T) \right] r^2 dr. \quad (23)$$

Let us consider the electroweak phase transition in the form of the formation of bubble nucleation. In order to calculate the energy, we must solve the equation of motion (22) for the VEV of the Higgs field and obtain $v(r)$. Eq. (22) does not have an exact solution because the factor $\frac{2}{r} \frac{dv}{dr}$ oscillates very strongly at $r \to 0$. Therefore, we should use approximations presented in the next subsections. The sphaleron rate per unit time, per unit volume, $\Gamma / V$, is characterized by a Boltzmann factor, $\exp \left(-\mathcal{E} / T\right)$, as follows [4, 24, 25]:

$$\Gamma / V = \kappa_{brok} \alpha^4 T^4 \exp \left(-\mathcal{E} / T\right), \quad (24)$$

where $V$ is the volume of universe, $T$ is the temperature, $\mathcal{E}$ is the sphaleron energy, and $\alpha = 1/30$. $\kappa_{brok}$ specifies the strength of EWPT. In this model, the strength is about 1, so $\kappa_{brok} \approx 1$. We will compare the sphaleron rate with the Hubble constant describing the cosmological expansion rate at the temperature $T$ [29, 30]

$$H^2 = \frac{\pi^2 g T^4}{90 M_{pl}^2}, \quad (25)$$

where $g = 106.75$, $M_{pl} = 2.43 \times 10^{18}$ GeV.

In order to have B violation, the sphaleron rate must be larger than the Hubble rate at the temperatures above the critical temperature (otherwise, B violation will become negligible during the Universe’s expansion); however, the sphaleron process must be decoupled after the EWPT to ensure the generated BAU is not washed out [26].

A. An upper bound of the sphaleron rates

From the equation of motion (22), it can be seen that the VEV of the Higgs field cannot be equal at every point in space. So we suppose that the VEV of the Higgs fields dose not
change from point to point. Due to this supposition, we have \( \frac{dv}{dr} = 0 \). Hence, from Eq. (22) we obtain

\[
\frac{\partial V_{\text{eff}}(v)}{\partial v} = 0. \tag{26}
\]

Eq. (26) shows that \( v_\rho \) is the extremes of the effective potential. Therefore, the sphaleron energy (23) can be rewritten as

\[
E_{\text{sph}} = 4\pi \int V_{\text{eff}}(v, T) r^2 dr = \left. \frac{4\pi r^3}{3} V_{\text{eff}}(v_\rho, T) \right|_{v_m}, \tag{27}
\]

where \( v_m \) is the VEV at the maximum of the effective potentials. From Eq. (27), the sphaleron energy is equal to the maximum heights of the potential barriers.

The Universe’s volume at a temperature \( T \), after the inflation and re-heating epoch, is given by \( V = \frac{4\pi r^3}{3} \sim \frac{1}{T^3} \). Because the whole Universe is an identically thermal bath, the sphaleron energies are approximately

\[
E_{\text{sph}} \sim \frac{E^4 T}{4\lambda_f^4}. \tag{28}
\]

From the definition (24), the sphaleron rates take the form

\[
\Gamma \sim \alpha_\mu^4 T \exp \left(-\frac{E^4 T}{4\lambda_f^4 T} \right). \tag{29}
\]

For the heavy particles, \( E, \lambda \) are constants. Hence, the sphaleron rate in this approximation is the linear function of temperature. From Eq. (29), we estimate the value of the sphaleron rates as follows

\[
\Gamma \sim 10^{-4} \gg H \sim 10^{-13}. \tag{30}
\]

This value is very large, so we assume it as an upper bound of the sphaleron rate. Therefore, sphaleron decoupling condition cannot be satisfied. For instance, as the temperature drops below the phase-transition temperature \( T_c \) and the Universe switches to the symmetry-breaking phase, the sphaleron rate is still much larger than the Hubble constant, and this makes the B violation washed out. By this consequence, the sphaleron process cannot occur identically in large regions of space; it can only take place in the microscopic regions or at each point in space.

B. Sphaleron rate in a thin-wall approximation

At every point in the early Universe, the effective potential varies as a function of VEV of the Higgs field and temperature, as illustrated in Fig. 8. If the temperature at a spatial
location is higher than $T_1$, then $V_{eff}(v)$ at this location has only one the zero minimum, and this location is in a symmetric phase region.

As the temperature goes below $T_1$, the second minimum of $V_{eff}(v)$ gradually forms, and a potential barrier which separates two minima gradually appears. The VEV can be transformed by thermal fluctuations. The phase transition occurs microscopically, resulting in a tiny bubble of broken phase where the Higgs field $v$ gets a nonzero vacuum expectation value.

As the temperature goes to $T_c$, $V_{eff}(v)$ at two minimums are equal each other. But when the temperature goes below $T_c$, the second minimum becomes the lower one corresponding to a true vacuum, while the first minimum becomes the false vacuum. Such tiny true-vacuum bubbles at various locations in the Universe can occur randomly and expand in the midst of false vacuum. If the sphaleron rate is larger than the Universe's expansion rate, the bubbles can collide and merge until the true vacuum fills all space. However, if the sphaleron decoupling condition is satisfied after the transition, the sphaleron rate must be smaller than the cosmological expansion rate when the temperature goes from $T_N$ ($T_N$ lower than $T_C$, at which the EWPT terminates, $T_C < T_N < T_0$).

FIG. 8: The dependence of the effective potential on temperature, $m_{h^\pm} = m_{k^\pm} = 220\text{GeV}$, $\lambda = 0.35$, $\xi = 2$

The broken phase of the EWPT can be started at $T_1 \approx 120.5 \text{ GeV}$. At $T_1$, the sphaleron
rate can be larger than the Hubble rate \((H = 2.1231 \times 10^{-14} \text{ GeV})\). As the temperature drops below \(T_1\), the EWPT really starts at \(T_C\), the sphaleron rate is still larger than the Hubble rate and this lasts until the temperature reaches the nucleation temperature \(T_N\) when the transition ends. As the temperature goes down \(T_N\), the sphaleron rate is smaller than the Hubble rate.

In a bubble of the EWPT, we have the following approximation

\[
\frac{\partial V_{\text{eff}}(v)}{\partial v} \approx \frac{\Delta V_{\text{eff}}(v)}{\Delta v} = \text{const} \equiv M,
\]

(31)

here \(\Delta v = v_c\), \(\Delta V_{\text{eff}}(v) = V_{\text{eff}}(v_c) - V_{\text{eff}}(0)\), and \(v_c\) is a second minimum of the effective potential for the phase transition.

Now, we solve the equations of motion \((22)\) for the VEV \(v\) by the approximation \((31)\). Rewriting Eq. \((22)\) in this approximation, we have

\[
d^2 v \over dr^2 + \frac{2}{r} dv \over dr = M.
\]

(32)

In the cases that \(r \to \infty\) (the spatial locations are in the symmetric phase) or \(r \to 0\) (the spatial locations are in the broken phase), the VEVs must satisfy the boundary conditions

\[
\lim_{r \to \infty} v(r) = 0; \quad \frac{dv(r)}{dr} \bigg|_{r=0} = 0.
\]

(33)

In the bubble walls, the solutions of Eq. \((32)\) take the form

\[
v(r) = \frac{M}{6} r^2 - A/r + B,
\]

(34)

where \(A, B\) are the parameters to be specified.

The continuity of the scalar fields in the bubble results in the following system of equations:

\[
\begin{cases}
\frac{M}{6} R_b^2 - A/R_b + B = v_c, \\
\frac{M}{6} (R_b + \Delta l)^2 - A/(R_b + \Delta l) + B = 0,
\end{cases}
\]

(35)

where \(R_b\) and \(\Delta l\) are, respectively, the radius and the wall thickness of a bubble nucleated.

Solving the systems of Eq. \((35)\), we obtain the solutions \(v\), which are of the forms

\[
v(r) = \begin{cases} 
v_c; & \text{when } r \leq R_b, \\
\frac{M}{6} r^2 - A/r + B; & \text{when } R_b < r \leq R_{b,su(2)} + \Delta l, \\
0; & \text{when } R_b + \Delta l < r.
\end{cases}
\]

(36)
The system (35) has four equations, so we have to specify four unknown parameters \([A, B, \Delta l, \text{ and } R_b]\). Therefore the decoupling condition in which the sphaleron rate is equal to the Hubble rate at \(T_N\), has been used. This supposition relies on the requirement for avoiding the washout of the generated BAU after a phase transition, by which the sphaleron rate must be larger than the Hubble rate at temperatures above \(T_N\), but the sphaleron rate must be smaller than the Hubble rate at temperatures below \(T_N\).

The masses of heavy particles \((h^\pm, k^{\pm\pm})\) are unknown so far. However, we can estimate their mass regions which satisfy the first-order phase transition conditions, and we choose any values in these regions for calculating the sphaleron energy. Although the strengths of the first-order EWPT are sufficiently strong \((>1)\), they are not so strong \((<4.15)\), hence the coefficients \((\Lambda, B, D)\) in the effective potential are not meaningfully different for the different values in these regions. Here, as an example, we choose \(m_{h^\pm} = m_{k^{\pm\pm}} = 220\text{ GeV}\) (our choices are random), to illustrate the determination of the radius of nucleation.

In Fig. 9 our respective solution \(v(r)\) is not as smooth as those in Refs. [25–27]. Because in this work, the bubble walls were been considered very thin, i.e., \(\Delta l \ll 1/T\) (while in Ref. [27], for instance, \(\Delta l \gg 1/T\)). Inside the thin walls of bubbles, \(\frac{d\phi}{dr}\) is very large; this allows the Higgs field \(\phi\) to change their values over potential barriers. Therefore, the thinner the bubble walls, the larger the sphaleron rates.

![FIG. 9: The solutions \(v(r)\) with \(m_{h^\pm} = m_{k^{\pm\pm}} = 220\text{ GeV}, \lambda = 0.35, \xi = 2\). The regions in grey portray the thin walls of vacuum bubbles nucleated in each phase transition.](image)

In order to estimate \(R_b\) at \(T_N\) (the radius of nucleation), we accept that \(\Delta l = \frac{R_b}{10}\). In addition, it is difficult to determine \(T_N\), we only know that \(T_0 < T_N < T_C\). Therefore, we can
TABLE I: Sphaleron rate with $m_{h\pm} = m_{k\pm\pm} = 220\text{ GeV}$, $\lambda = 0.35$, $\xi = 2$

| $T$ [GeV] | $R[10^{-4} \times \text{ GeV}^{-1}]$ | $R/\Delta l$ | $\varepsilon$ [GeV] | $\Gamma[10^{-14} \times \text{ GeV}]$ | $H[10^{-14} \times \text{ GeV}]$ | $T_N$ | Bound  |
|-----------|--------------------------------------|-------------|-------------------|------------------|------------------|-------|--------|
| 116.964 ($T_C$) | 8.82948 | 10 | 2659.49 | 2 | 2 | $T_N = T_C$ | Upper |
| 96.25 ($T_0$) | 4.77064 | 10 | 2207.27 | 1.3 | 1.3 | $T_N = T_0$ | Lower |

only estimate the upper and lower bounds of the radius of nucleation. They are estimated in Table II.

From Table II, in this case, $m_{h\pm} = m_{k\pm\pm} = 220\text{ GeV}$, $\lambda = 0.35$, $\xi = 2$, the largest and smallest radius are $8.82948 \times 10^{-4} \times \text{ GeV}^{-1}$ and $4.77064 \times 10^{-4} \times \text{ GeV}^{-1}$ respectively.

V. CONSTRAINTS ON COUPLING CONSTANTS IN THE HIGGS POTENTIAL

In order to have the first order phase transition, $m_{h\pm}$ and $m_{k\pm\pm}$ must be smaller than 350 GeV. Therefore, we obtain

$$p^2v_0^2 < (300 \text{ GeV})^2,$$

and

$$q^2v_0^2 < (300 \text{ GeV})^2.$$

From the above equations, we obtain $0 < p < 1.22$ and $0 < q < 1.22$. However, we need to have other considerations in order to find these accurate values of $m_{h\pm}$ and $m_{k\pm\pm}$.

In the ZB model, the tiny masses of neutrino are generated at two loops, so $m_{h\pm}$ and $m_{k\pm\pm}$ cannot be very heavy [22]. From the experimental point of view it is interesting to consider new scalars light enough to be produced at the LHC, theoretical arguments introduce that the scalar masses should be a few TeVs, to avoid unnaturally large one-loop corrections to the Higgs mass which would introduce a hierarchy problem. Therefore, these upper bounds of new scalar masses can be 2 TeVs [23]. Contacting to neutrino oscillation data, in the decay $k^{\pm\pm} \rightarrow ll$, the branching ratio to $\tau\tau$ is very small in the ZB model, less than about 1%. Then, a conservative limit is $m_{k^{\pm\pm}} > 200$ GeV. In the ZB model, we can have the decay $k^{\pm\pm} \rightarrow h^{\pm}h^{\pm}$, so $2m_{h\pm} < m_{k^{\pm\pm}}$. Therefore, our results in Eqs. (37), (38) are consistent with the above estimates.

Recently, the experimental groups at LHC (ATLAS and CMS Collaborations) [32] have
reported an experimental anomaly in diboson production with apparent excess in boosted jets of the $W^+W^-, W^\pm Z$ and $ZZ$ channels at around 2 TeV invariant mass of the boson pair.

In addition the calculation the Higgs coupling to photons (due to charged particles in the loop diagram) can be related to neutrino mass and CP violation which are the key of matter and antimatter asymmetry. This study will be investigated in a future publication.

VI. CONCLUSION AND OUTLOOKS

In this paper we have investigated the EWPT and sphaleron rate in the ZB model using the high-temperature effective potential. The EWPT is strengthened by the new scalars to be the strongly first-order, the phase transition strength is in the range $1 - 4.15$. By using the sphaleron decoupling condition, we also propose a way to estimate the radius of the nucleation. $h^\pm, k^{\pm\pm}$ are triggers for the first-order EWPT. Our results may be further than the results in Ref. [31].

In addition, the EWPT can be calculated in a different way as in [20]. In order to determine $T_N$ or $T_E$, we will examine this problem in conjunction with the CP-violation.

In the ZB model, the tiny mass of neutrino which can be explain in two loops interactions of charged Higgs with neutrino, can be a reason of the matter-antimatter asymmetry and CP-violation. The behavior of charged Higgs is also very interested. Therefore, in the next works, we can investigate the ratio $m_{h^\pm/k^{\pm\pm}}$ by using neutrino data. We will investigate the CP-violation and beyond issues of the baryon asymmetry problem through neutrino physics.

With this region of self couplings in the Higgs potential, we can serve as basis for the calculation of cross section of the decay Higgs to photons and evaluate and extend the Zee-Babu model when connected to the data of LHC.

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