The Mathematical Model and an Genetic Algorithm for the Two-Echelon Electric Vehicle Routing Problem

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Abstract. In order to cope with the challenges of high cargo load and high timeliness distribution in logistics industry, as well as to alleviate the current situation of oil resource depletion and air pollution, this study established a mathematical model of two-echelon electric vehicle routing problem (2E-EVRP) and design a heuristic algorithm. The 2E-EVRP can be divided into the multiple depot vehicle routing problem (MDEVRP) and the split delivery vehicle routing problem (SDVRP). The proposed genetic algorithm is used to solve the MDEVRP, and the actual case of a logistics company in Beijing is taken as the calculation experiment, so as to verify the feasibility of the proposed algorithm and provide decision-making reference for the development of logistics enterprises. The results show that the total path length obtained by the proposed algorithm is optimized by 20.82 kilometers compared with the traditional simulated annealing algorithm.

Keywords: Two-echelon electric vehicle routing; Genetic algorithm; Transportation.

1. Introduction
The distribution mode of the logistics industry can be divided into direct distribution and multi-echelon distribution according to different levels of distribution system. Direct distribution refers to the direct distribution of goods from the distribution center to the customer through means of transportation, and multi-echelon distribution refers to the introduction of transit nodes between the distribution center and the customer. With the expansion of city scale and increasing logistics demand, the distance between distribution center and customer point is getting farther, the establishment of multi-echelon distribution system is the inevitable trend of the development of logistics industry.

Dantzig and Ramser [1] introduced vehicle route problem (VRP) for the first time in 1959. The VRP is to optimize the route for a fleet of vehicles from the depot to customers of different demands, with a certain aim such as the shortest distance, the smallest cost, or the least time consuming, etc. Some well-known heuristic algorithms include simulated annealing, tabu search, genetic algorithm and ant colony algorithm [2].

Vehicle routing problems have different extensions and changes in practical applications, which are mainly divided into the following categories: capacitated vehicle routing problem (CVRP) [3], which restricts the maximum vehicle load; vehicle routing problem with time windows (VRPTW) [4][5], which limit service within the time required by the customer; vehicle routing problem with multiple depots (MDVRP) [6], which can transport goods from multiple depots; the vehicle routing problem with pick-up and delivery (VRPPD) [7], that is, considering both the distribution of goods from the distribution center to the customer point and the goods delivered back to the distribution center from the customer point; the vehicle routing problem with split deliveries (SDVRP) [8], that is, multiple...
vehicles can serve one customer so as to make full use of the vehicle's load capacity; the stochastic vehicle routing problem (SVRP) appears when some elements of the problem are random [9], the stochastic factors include road conditions and periodic customer time windows etc.

Scholars initially studied the electric vehicle routing problem (EVRP) in 2012, then Afroditi et al. (2014) [10] and Pelletier et al. (2016) [12] considered the technical limitations of electric vehicles, including load capacity, battery capacity, charging station location, charging policy, charging station available time and capacity restrictions, and electricity prices in different periods. Some researches reviewed the green vehicle routing problem [11].

There are less researches on the two-echelon vehicle routing problems compared with traditional vehicle routing problems. Feliu et al. [13] and Perboli et al. [14] initially proposed the two-echelon vehicle routing problem (2E-VRP) and conducted computational experiments on small-scale data. Hemmelmayr et al. [15] proposed an adaptive large neighborhood search for the 2E-VRP and conducted the experiment of 200 customers and 10 satellites.

In this study, we introduced the two-echelon electric vehicle routing problem (2E-EVRP) and designed an genetic algorithm for application, which is a practical-based extension of the two-echelon vehicle routing problem (2E-VRP) and electric vehicle routing problem (EVRP).

2. Problem Description

The proposed 2E-EVRP is composed of one depot, multiple satellites, charge stations and customers. In the first echelon, the traditional vehicles are used to transport goods from the depot to the satellites. In the second echelon, the electric vehicle are used to transport goods from the satellites to the customers, as shown in Figure 1.

![Figure 1. The schematic diagram of 2E-EVRP.](image)

Given a mixed graph $G=(N, E, A)$, where the vertex set $N = \{0\} \cup N_S \cup N_C \cup N_R$. Vertex 0 denotes the depot, $N_S = \{1, 2, \ldots, n_s\}$ denotes $n_s$ satellites, $N_C = \{n_s + 1, \ldots, n_s + n_c\}$ denotes $n_c$ customers, and $N_R = \{n_s + n_c + 1, \ldots, n_s + n_c + n_r\}$ denotes $n_r$ charging stations. The edge set $E = \{(0, j) : j \in N_S\} \cup \{(i, j) : i, j \in N_S, i < j\}$ and the arc set $A = \{(i, j) : i, j \in N_S \cup N_C \cup N_R, i \neq j\} \setminus \{(i, j) : i, j \in N_S\} \setminus \{(i, j) : i, j \in N_R\}$. Each customer $i$ requires $q_i$ goods. $m^1$ traditional vehicles of $Q_1$ capacity in the first echelon and $m^k$ electric vehicles of $Q_2$ capacity in the second echelon. In summary, symbol definitions are shown in Table 1.
Table 1. Symbol definitions in 2E-EVRP model.

| Symbol | definitions                              | Symbol | definitions                              |
|--------|------------------------------------------|--------|------------------------------------------|
| \( N_c \) | Set of customers                         | \( C_j \) | Battery loss coefficient                  |
| \( N_s \) | Set of satellites                        | \( R_{1k} \) | Set of first-echelon routes through satellite \( k \) |
| \( N_R \) | Set of charge station                    | \( C_r \) | The cost of first-echelon route \( r \)     |
| \( R_1 \) | Set of first-echelon routes               | \( q_{kr} \) | The load of vehicles transport from first-echelon route \( r \) to satellite \( k \) |
| \( R_2 \) | Set of second-echelon routes              | \( L_k \) | Battery capacity at satellite \( k \)       |
| \( m^j \) | The number of fuel vehicles               | \( C_d \) | Charge cost per unit at satellite          |
| \( Q_1 \) | The load of fuel vehicles                 | \( q_i \) | The demand of customer \( i \)              |
| \( m^k \) | The number of electric vehicles in satellite \( k \) | \( d_{ij} \) | The distance between \( i \) and \( j \) |
| \( Q_2 \) | The load of electric vehicles             | \( Q_j^L \) | The load of electric vehicle when leaving from satellite \( j \) |
| \( L \) | The battery capacity of electric vehicles | \( L_j^L \) | The remaining battery capacity when arriving at satellite \( j \) |
| \( C_1 \) | Charge cost per unit at charge station    | \( x_{ij} \) | binary variable to indicate whether the route from \( i \) to \( j \) exists or not |
| \( C_2 \) | charging loss of charging station per unit| \( y_r \) | binary variable to indicate whether the route \( r \) exists or not |
| \( L_j \) | Charge capacity at charge station \( j \)  |                    |                                              |

3. Mathematical Model

Since the establishment of the model in this study was based on the practical situation of the logistics distribution industry, reasonable and ingenious assumptions are helpful to simplify the complexity of the mathematical model and greatly improve the feasibility of the algorithm. Therefore, the following assumptions are made in this study:

- Traditional fuel vehicles (first-echelon) can visit satellites more than once.
- Traditional fuel vehicles (first-echelon) must depart from depot and return back to depot.
- The unloading time of fuel vehicles and electric vehicles is fixed.
- The charge time is fixed and each time the battery is charged full capacity.
- Take no account of changes in road traffic conditions.
- The energy consumption of electric vehicle is linearly related to travel distance.
- Given the coordinates of each node, the Euclidean distance is used to measure the distance.

Based on the above problem description and model assumption, the objective function can be established as follows:

\[
\min W = \sum_{r \in R} C_r \times y_r + \sum_{j \in N_g} C_1 \times L_j + C_2 \sum_{j \in N_g, j \in N_y, j \in N_y} x_{ij} + \sum_{k \in N_s} C_4 \times q_k \tag{1}
\]

The objective function is composed of four parts, namely, distribution cost of fuel vehicle, charge cost in charge station, energy consumption cost of electric vehicles and charge cost at satellites.

Constraints:

\[
\sum_{r \in R} y_r \leq m^r \tag{2}
\]

\[
\sum_{j \in N_g} x_{ij} \leq m^i, \forall k \in N_s \tag{3}
\]

\[
\sum_{r \in R} q_{rk} = \sum_{j \in N_y, \cap N_g} x_{ij} (Q_j + q_i), \forall k \in N_s \tag{4}
\]
In the above model, Constraints (2) and (3) restrain the number of vehicles. Constraint (4) denotes the continuity of the number of goods at satellites. Constraints (5) and (6) limit the load of vehicles. Constraint (7) and (8) limit the number of times that each customer and charge station can be visited. Constraint (9) denotes the continuity of the number of vehicles at each node. Constraint (10-15) denote the continuity of battery capacity at each node. Constraint (16) and (17) denotes the continuity of the number of goods at customers and charge stations. Constraint (18) denotes the non-negative decision variables. Constraint (19) denotes the binary decision variables.

4. Algorithm Design

Coding is a key step in designing genetic algorithm, namely, the feasible solution of a problem is transformed from its solution space to the search space that can be handled by genetic algorithm.

For the MDVRP, using binary coding mode to represent chromosome coding is not easy to reflect the structural features of the problem. Therefore, our study applied floating-point coding method to improve the genetic algorithm, which means that the length of chromosome coding depends on the decision variables. The proposed coding mode is suitable for high accuracy requirement model, and has higher search performance for large-scale datasets. The set of customers is coded as a one-dimensional array, and each customer node can only appear once. The order in the array illustrates the solution of the MDVRP, so each one-dimensional array denotes a chromosome.
In the MDVRP model established in our study, each chromosome in the population corresponds to a solution route, and each solution route can calculate its corresponding distance. Therefore, the objective function $f(x)$ is the total distance of the solution route, where the Euclidean distance is used to measure the distance between nodes. In the process of population evolution, we hope that the fitness value of the population can be larger, so the fitness function can be specified as $Fit(f(x)) = -f(x)$.

In order to avoid increasing the number of similar individuals in the population, leading to a halt in evolution, we exclude the roulette wheel method. In order to retain as many individuals with good fitness as possible to the next generation, the elitist preservation method is adopted to carry out the survival of the fittest operation, that is, the individuals with the highest fitness in the current population do not participate in crossover and mutation operations, and it is used to replace the least adaptive individuals in the current generation group after crossover, mutation and other operations.

The crossover operator is the main method to generate new individuals, including two contents of determining the crossover position and performing partial gene exchange. In order to improve the search ability of the algorithm and make full use of the population characteristics and distribution, this paper uses the XO algorithm to perform crossover operations. Taking the generation process of p1 as an example, the schematic diagram of the XO algorithm is shown in Figure 2.

**Figure 2.** The schematic diagram of the XO algorithm.

From the perspective of the genetic algorithm framework, mutation operation has randomness. However, when combined with selection and crossover operators, mutation operation can maintain the diversity of the population. In this study, the simple mutation was adopted, that is, perform mutation operation on the individual code string with a certain mutation probability, and randomly designate the value of a certain locus within a certain range for mutation operation.

For the second echelon composed of the satellites and the customers, $M$ sub-paths starting from the satellites and passing through customers can be obtained by the proposed genetic algorithm. The specific steps to insert charge stations for each sub-path are described as follows:

**Step 1** Insert the charge station from the first sub-path.

**Step 2** Based on the energy consumption assumptions, traverse sequentially in the order of customers in the current sub-path.

**Step 3** Determine whether the remaining energy can reach the next customer, if the battery capacity requirement is met, continue to the next step, if not, skip to Step 5.

**Step 4** Update the current node, determine whether the current node is a satellite, if it is, skip to Step 6, if not, return to Step 3.

**Step 5** Calculate the sum of the distance from the current node to all charging stations and the distance from each charge station to the next node. Similarly, calculate the sum of the distance from the previous node to all charging stations and the distance from each charging station to the next node. Update the node with the minimum total distance as the current node, and update the remaining energy, then return to Step 3.

**Step 6** Determine whether the current sub-path $m$ is equal to $M$, if it is, the algorithm ends, if not, update $m=m+1$, return to Step 2.

5. **Computational Experiments**

5.1. **Instance Description**

The depot of a certain logistics company is located in the southeast of Beijing. In order to alleviate the
pressure of warehousing costs in the urban area of Beijing, two satellites was set up to form a two-
echelon electric vehicle routing delivery system. The experiment instance consists of 53 customers and
10 charge stations.
Based on the actual location of the customers, satellites and the depot, we convert it to the coordinate
system in a certain proportion, and part of the coordinate data after conversion is shown in Table 2:

| Nodes | X-coordinate | Y-coordinate | Nodes | X-coordinate | Y-coordinate |
|-------|--------------|--------------|-------|--------------|--------------|
| 39    | 212.81       | 405.33       | 48    | 250.47       | 256.86       |
| 40    | 148.53       | 178.7        | 49    | 368.84       | 143.58       |
| 41    | 100.35       | 380.2        | 50    | 264.23       | 287.24       |
| 42    | 250.94       | 339.03       | 51    | 144.23       | 254.03       |
| 43    | 388.88       | 374.74       | 52    | 177.44       | 220.49       |
| 44    | 222.55       | 165.5        | 53    | 501.66       | 200.62       |
| 45    | 276.62       | 179.48       | 54    | 69.97        | 220.57       |
| 46    | 192.59       | 122.9        | 55    | 164.03       | 263.89       |
| 47    | 326.21       | 140.61       | 56    | 221.06       | 320.71       |

According to the field investigation, in the first echelon, the load of fuel vehicles Q₁ takes the value of
5000, the average unloading time is 0.5 hour, the average speed is 60 km/h, and the transportation cost
per kilometer is 5 yuan. In the second echelon, the load of electric vehicle Q₂ takes the value of 1200,
the average unloading time is 0.25 hour, the average speed is 44.7 km/h, the average charging time is
0.5 hour , the maximum travel distance of electric vehicles is 80 kilometers, the energy consumption
rate is 0.4 kwh/km, and the average charge cost is 2 yuan/kwh.

5.2. Analysis of Results
According to the algorithm solution results, for the first-echelon, 3 fuel vehicles are required and the
total travel distance is 392.09 km. It takes 3.48 hours to complete the transportation task from the
depot to the satellites. For the second echelon, 11 electric vehicles are required and the total travel
distance is 885.21 km. It takes 4.49 hours to complete the transportation task from satellites to 53
customers. To sum up, the optimal routes obtained by the proposed algorithm of 2E-EVRP model is
1,277.30 km, and the total time to realize customer coverage is 6.59 hours. The specific solution is
shown in Table 3:

| Routes    | Total distance (km) | Load (Pcs) | Loading rate(%) | Cost( ¥) | Delivery time(h) |
|-----------|---------------------|------------|-----------------|----------|-----------------|
| 0-54-0    | 96.24               | 5000       | 100.00          | 481.20   | 2.10            |
| 0-55-0    | 146.85              | 5000       | 100.00          | 734.25   | 2.95            |
| 0-54-55-0 | 149.00              | 1743       | 33.87           | 745.00   | 3.48            |
| 54-3-51-57-14-38-35-36-61-54 | 158.71       | 1176       | 98.03           | 126.97   | 4.49            |
| 54-40-12-18-9-54 | 49.67       | 1058       | 88.13           | 39.74    | 2.11            |
| 55-26-5-6-7-50-55 | 60.52       | 1075       | 83.93           | 48.42    | 2.60            |
| 55-17-2-42-30-1-55 | 53.57       | 931        | 94.71           | 42.86    | 2.44            |
| 55-13-19-32-8-15-55 | 77.31       | 1125       | 88.91           | 61.85    | 2.97            |
| 55-33-10-43-11-58-44-55 | 88.49       | 1171       | 89.55           | 70.79    | 3.61            |
| 55-41-27-21-37-49-58-55 | 94.08       | 1173       | 77.46           | 75.26    | 3.73            |
| 54-39-53-34-29-45-54 | 70.96       | 1007       | 93.75           | 56.77    | 2.66            |
| 54-28-23-52-20-60-22-54 | 86.96       | 1136       | 97.61           | 69.57    | 3.49            |
| 54-4-25-24-48-65-47-54 | 102.22      | 1067       | 97.72           | 81.78    | 3.91            |
| 55-31-16-46-55 | 42.72       | 666        | 55.51           | 34.18    | 1.71            |
The comparison between the traditional simulated annealing algorithm (SA) and our proposed genetic algorithm (GA) is shown in Table 4:

| Algorithm | Total distance (km) | Total cost (¥) | Total delivery time (h) |
|-----------|---------------------|----------------|------------------------|
| SA        | 1298.12             | 2685.28        | 44.45                  |
| GA        | 1277.30             | 2657.87        | 42.25                  |

Clustering algorithm is commonly used in traditional heuristic algorithms to solve the MDVRP, that is, transform the MDVRP to the traditional VRP by assigning customers to corresponding satellite first. The proposed genetic algorithm assigns customers to corresponding satellites optimally according to the current load of electric vehicles, which greatly improves the integrity of the algorithm and optimizes the solution results. The total travel distance is optimized by 20.82 km.

6. Conclusion

In view of the increasing freight demand and shopping experience in the logistics industry, our study points out the necessity of establishing a two-echelon vehicle routing distribution system. Considering the current situation of oil resource depletion and air pollution, our study applied electric vehicles in the second echelon to solve the "last kilometer" problem in the urban center. In order to verify the feasibility and effectiveness of the proposed algorithm, an actual logistics company was taken as an experiment instance. Based on the 2E-EVRP model established in this paper, an genetic algorithm was proposed to solve the instance, and the calculation results were analyzed and compared in terms of the total travel distance and timeliness of the delivery. The main contributions are as follows:
- The two-echelon electric vehicle routing problem (2E-EVRP) were introduced and the mathematical model for the 2E-EVRP was established, in order to better meet the needs of today's logistics industry.
- The genetic algorithm was proposed and tested in the practical instance. The results showed that the integrity and efficiency of the algorithm are improved compared with traditional algorithms.

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