Heavy flavor molecular states with strangeness

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We proposed a unified framework to describe the interactions of the observed $T_{cc}$, $P_{c}$, and $P_{cs}$ within a quark level interaction in our previous work. In this work, we generalize our framework to the loosely bound hadronic molecules composed of heavy flavor di-hadrons with strangeness. We predict the possible $D^{(*)}D^{(*)}$ molecular states in the SU(3) limit with the masses of the $P_{c}$ states as the inputs. We also investigate the baryon-meson and baryon-baryon systems and consider the SU(3) breaking effect in their flavor wave functions. We generalize our isospin criterion of the formation of heavy flavor di-hadron molecules to the $U/V$ spin case. For a specific heavy flavor meson-meson, baryon-meson, or baryon-baryon system, the interactions for the states with the same flavor and spin matrix elements can be related by a generalized flavor-spin symmetry.

I. INTRODUCTION

The study of exotic hadrons with configurations beyond the conventional quark model has become a hot topic since the discovery of $X(3872)$ [1]. Especially, the $P_{c}$ [2, 3], $P_{cs}$ [4] together with the recently reported $T_{cc}$ [5, 6] are manifestly exotic states. The observation of these states indicates that the interactions between heavy flavor di-hadrons share very similar binding mechanisms.

Since the observed $P_{c}$, $P_{cs}$, and $T_{cc}$ are below the thresholds of the corresponding di-hadron systems from several to several tens MeVs, a natural explanation is that these states are molecules. To describe the possible similarities of the binding mechanisms between the above heavy flavor di-hadron systems, we proposed a hydrogen-like picture and studied the heavy flavor meson-meson ($M-M$), baryon-meson ($B-M$), and baryon-baryon ($B-B$) molecular states in Ref. [7]. We assume that the interactions of heavy flavor di-hadron systems are mainly from the exchange of the light flavor mesons, which is closely related to the flavor and spin structures of light degrees of freedom (d.o.f) in the di-hadron systems.

To address the above molecule picture, we adopt a quark-level Lagrangian [7–10] to describe the interactions of different heavy flavor di-hadron systems. Then the corresponding effective potentials can be parameterized in terms of the light quark-quark coupling constants and the flavor and spin matrix elements which are related to its light d.o.f. In this framework, we obtain a satisfactory description of the $T_{cc}$, $P_{c}$, $P_{cs}$ states, and other possible bound states in the heavy flavor meson-meson, baryon-meson, and baryon-baryon di-hadron systems which arise from the isospin singlet and triplet meson exchange interactions. The interactions of different heavy flavor di-hadron systems ($M-M$, $B-M$, and $B-B$) can be understood in a molecule picture simultaneously.

The simple Lagrangian introduced in our work possesses two important advantages in Ref. [7]. It not only relates the interactions of different heavy flavor di-hadron ($M-M$, $B-M$, and $B-B$) systems via their flavor-spin structures but also relates the interactions of the multiplets in a specific heavy flavor di-hadron system. In Table I, we list the systems which were already studied in Ref. [7]. In this work, we further adopt this framework to study the rest of the doubly heavy flavor di-hadron systems. The inclusion of the rest of the doubly heavy flavor di-hadron systems allows us to present a full analysis of the possible flavor-spin symmetry and the interactions of molecular multiplets in a specific di-hadron systems. This is the main task of this work.

Some of the doubly heavy tetraquark states with strangeness have already been discussed in literature. For example, the strange doubly heavy tetraquark states in molecular configurations have been discussed in Refs. [11–17]. The compact strange tetraquark states were studied via various methods, such as quark potential models [18–21], lattice QCD [22], chromo-magnetic interaction model (CMI) [23–26], and QCD sum rule [27, 28].

The strange doubly heavy baryon-meson and di-baryon systems have also been discussed in literature. The possible pen-

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### TABLE I

| Molecular System | States |
|-----------------|--------|
| $M-M$           | $D\bar{D}$, $D\bar{D}^*$, $D^*D^*$, $D^*D^*$, $D^*_sD^*_s$ |
| $B-M$           | $\Xi_0^*D$, $\Xi_0^*D^*$, $\Xi^*_0\bar{D}$, $\Xi^*_0\bar{D}^*$, $\Xi^*_0\bar{D}^*$, $\Xi^*_0\bar{D}^*$ |
| $B-B$           | $\Xi_0^*\Xi_0^*$, $\Xi_0^*\Xi_0^*$, $\Xi^*_0\Xi^*_0$, $\Xi^*_0\Xi^*_0$, $\Xi^*_0\Xi^*_0$, $\Xi^*_0\Xi^*_0$ |

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This table lists the possible molecular states in a specific di-hadron system. The interactions of the rest of the doubly heavy flavor di-hadron systems allow us to present a full analysis of the possible flavor-spin symmetry and the interactions of molecular multiplets in a specific di-hadron systems. This is the main task of this work.

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I can be calculated by considering the possible contributions from the exchanges of light mesons, which depend on the flavor and spin structures of the heavy di-hadron systems from their light d.o.f. On the other hand, the S-wave interactions of the ground heavy flavor di-hadron systems may arise from the exchange of the $J^P = 0^+$ scalar or $1^+$ axial vector mesons. Thus, we introduce the following quark level Lagrangians [7–10]

$$\mathcal{L} = g_sq\bar{q}Sq + g_a\bar{q}\gamma_\mu\gamma^5A^\mu q$$

(1)

to describe the interactions among the considered ground heavy flavor di-hadron systems. Here, $q = (u, d, s)$, $g_s$ and $g_a$ are two independent coupling constants that encode the non-perturbative low energy dynamics of the di-hadron systems.

From Eq. (1), the effective potential of the light quark-diquark interactions reads

$$V_{qq} = \tilde{g}_s\lambda_1 \cdot \lambda_2 + \tilde{g}_a\lambda_1 \cdot \lambda_2 \sigma_1 \cdot \sigma_2$$

$$= \tilde{g}_s \left( \lambda_1^8 \lambda_2^8 + \lambda_1^7 \lambda_2^7 + \lambda_1^6 \lambda_2^6 \right)$$

$$+ \tilde{g}_a \left( \lambda_1^6 \lambda_2^6 + \lambda_1^5 \lambda_2^5 + \lambda_1^4 \lambda_2^4 \right) \sigma_1 \cdot \sigma_2.$$  

(2)

Here, $i$ and $j$ sum from 1 to 3 and 4 to 7, respectively. The $\lambda_1^i \lambda_2^j$, $\lambda_1^i \lambda_2^j$, and $\lambda_1^i \lambda_2^j$ are the SU(3) flavor matrices of the isospin singlet, triplet, and doublet, respectively. The $\sigma_1$, $\sigma_2$ is the Pauli matrix in spin space. The redefined coupling constants are $\tilde{g}_s = g_s^2/m_5^2$ and $\tilde{g}_a = g_a^2/m_5^2$. The local form of the effective potential $V_{qq}$ is the result of integrating out the exchanged spurions.

The effective potential in Eq. (2) consists of three sets of flavor related operators and three sets of flavor-spin related operators. For convenience, we adopt the following notations

$$O_1 = \lambda_1^i \lambda_2^j,$$

$$O_4 = \lambda_1^i \lambda_2^j \sigma_1 \cdot \sigma_2,$$

$$O_2 = \lambda_1^i \lambda_2^j,$$

$$O_5 = \lambda_1^i \lambda_2^j \sigma_1 \cdot \sigma_2,$$

$$O_3 = \lambda_1^i \lambda_2^j.$$  

(3)

In our previous work [7], we only consider the di-hadron systems without the strange meson exchange contribution. The coupling constants $\tilde{g}_s$ and $\tilde{g}_a$ are obtained from a solvable Lippmann-Schwinger equation (LSE). For the heavy flavor di-hadron systems containing strange quark(s), the contributions from the SU(3) flavor octet, the physical masses of the exchanged mesons that belong to the flavor isospin doublets are close to the masses of their isospin singlet or triplet mesons. Thus, we take the SU(3) limit and set the coupling parameters $\tilde{g}_s$ and $\tilde{g}_a$ to be equal to those of the isospin singlet or triplet mesons.

The effective potential for a specific heavy flavor di-hadron system $H_{1}H_{2}$ system with total isospin $I$ and total angular momentum $J$ can be written as

$$V_{[H_1H_2]_J} = [V_{qq}[H_1H_2]_J],$$

(4)

where $[H_1H_2]_J$ denotes the quark-level spin-flavor wave function of the $H_1H_2$ system. Note that as an approximation, the correction from heavy degree of freedom is omitted. The operators $\lambda_1 \cdot \lambda_2$ and $\sigma_1 \cdot \sigma_2$ only act on the flavor and spin wave functions of the light quark components. As can be seen from Eq. (2), the interactions of the considered di-hadron systems only depend on the flavor $(\lambda_1, \lambda_2)$ and spin $(\sigma_1, \sigma_2)$ matrix elements, implying a flavor-spin symmetry for the interactions of the heavy flavor di-hadron systems.

In the following, we will further discuss the properties of this symmetry.

### A. The spin matrix elements

The operator $\sigma_1 \cdot \sigma_2$ can be easily calculated at hadron level with the technique of angular momentum theory. At hadron level, the matrix elements of the $l_1 \cdot l_2$ spin-spin operator in the light d.o.f. for the discussed $M-M$, $B-M$, and $B-B$ systems can be obtained via a spin rearrangement procedure. The di-hadron systems can be related to the states with a specific total light spin and a heavy spin by the relations [35]

$$l_1h_1 S_1 l_2 h_2 S_2 J M$$

$$= \sum_{L,H} \sqrt{(2S_1 + 1)(2S_2 + 1)(2L + 1)(2H + 1)}$$

$$L \times H \rightarrow S_1 S_2 J$$

$$l_1 l_2 H J M,$$  

(5)

where $l_i$ and $h_i$ are the light and heavy spin for $H_i$ hadron, respectively. $L$ and $H$ are the total light spin and total heavy spin of the di-hadron system, respectively. $S_i$ is the total spin of the $H_i$ hadron. Then the matrix elements of the operator $l_1 \cdot l_2$ can be obtained as

$$\langle l_1 \cdot l_2 \rangle = \sum_{L,H} [L (L + 1) - l_1 (l_1 + 1) - l_2 (l_2 + 1)]$$

(6)
\[ \frac{1}{2} (2S_1 + 1) (2S_2 + 1) (2L + 1) (2H + 1) \]
\[ \times \left\{ \begin{array}{c} l_1 \ l_2 \ \ L \\ h_1 \ h_2 \ \ H \\ S_1 \ S_2 \ \ J \end{array} \right\}^2 \]

Since the \( h_i \) is identical for all the single heavy flavor hadrons, we use a more brief notation \( |l_1, S_1; l_2, S_2; J \rangle \) to specify the spin wave function of a \( |H_1 H_2 \rangle \) di-hadron system. In Table II, we collect the results of \( \langle l_1 \cdot l_2 \rangle \) matrix elements for the discussed \( M-M, B-M, \) and \( B-B \) di-hadron systems.

**B. The flavor matrix elements in the SU(3) limit**

Now we discuss the quark-level flavor wave functions of the considered di-hadron systems. In the SU(3) limit, the two anti-charmed mesons belong to the flavor 3 representation and have

\[ 3 \otimes 3 = 6 \oplus \bar{3}. \]  

Similarly, a charmed baryon and an anti-charmed meson can form the following representations

\[ (3 \otimes 3) \otimes 3 = (6 \otimes 3) \oplus (3 \otimes 3) \]
\[ = (8 \oplus 10) \oplus (1 \oplus 8'). \]  

Two singly charmed baryons have four light quarks. The product of four 3 representations can be divided into

\[ (3 \otimes 3) \otimes (3 \otimes 3) = (6 \otimes 6) \oplus (6 \otimes 3) \oplus (3 \otimes 6) \oplus (3 \otimes 3) \]
\[ = (15_1 \oplus 15_2 \oplus 6_1) \oplus (15_1 \oplus 3_1) \oplus (15_2 \oplus 3_1) \oplus (3_2 \oplus (6_2). \]  

We construct the explicit forms of flavor wave functions for the multiplets of different heavy flavor di-hadron systems in the Eqs. (6)-(8) using the SU(3) Clebsch-Gordan (CG) coefficients [36]. We collect the results of the \( \lambda_1 \lambda_2 \) matrix elements for the different di-hadron systems in their corresponding SU(3) representations in Table III. For the di-hadron systems in the same SU(3) multiplet, their \( \lambda_1 \lambda_2 \) matrix elements have the same value.

Combining the results in Table II and Table III, we emphasize that the flavor-spin symmetry in Eq. (2) demonstrates that if two heavy flavor di-hadron systems have the same spin structure \( |l_1, S_1; l_2, S_2; J \rangle \) and belong to the same SU(3) multiplet, they have the same effective potentials. Correspondingly, if the potential can provide enough attractions to form a bound state, their binding energies should have the same size.

In addition, we can extract many other useful information numerically from Table II and Table III. For example, as presented in Table III, in the \( B-B \) systems, the matrix elements \( \lambda_1 \lambda_2 \) for the 15 and 6\( _2 \) multiplets are the same. Thus, if their spin matrix elements turn out to be the same (it is easy to find two different spin states that have the same \( |l_1 \cdot l_2 \rangle \) in Table II), they should have the same effective potentials. Thus, the systems which share the same values of the \( \lambda \) and \( \sigma \) matrix elements should have the same effective potential and are related by this generalized flavor-spin symmetry.

**C. The wave functions of heavy flavor di-hadron systems including the SU(3) breaking effect**

In the previous section, we assume a perfect SU(3) symmetry when we construct the flavor wave functions of the heavy flavor di-hadron systems. To discuss the physical di-hadron systems, we need to distinguish the \( s \) quark from the \( u, d \) quarks when we construct the corresponding flavor wave functions. In Table IV, we collect the quark-level flavor wave functions of the charged hadrons considered in this work.

Then we construct the total wave functions of the considered heavy-flavor baryon-meson systems as

\[ |[H_1 H_2]^I J \rangle = \sum_{m_{I1}, m_{I2}} C^{I, J}_{I_1, I_2, m_{I1}, m_{I2}} \phi_{I_1, m_{I1}} \phi_{I_2, m_{I2}} \]
\[ \times \sum_{m_{S1}, m_{S2}} C^{I, J}_{S_1, S_2, m_{S1}, m_{S2}} \phi_{S_1, m_{S1}} \phi_{S_2, m_{S2}}. \]

Here, the spin wave functions \( \phi_{I_1, m_{I1}} \) and \( \phi_{I_2, m_{I2}} \) can be constructed directly using the SU(2) CG coefficients. For the flavor wave functions \( \phi_{H_1, m_{H1}} \) and \( \phi_{H_2, m_{H2}} \), we adopt the flavor wave functions listed in Table IV and use the SU(2) CG coefficients to construct the total flavor wave functions of considered \( B-M \) systems. Unlike using the SU(3) CG coefficients, the construction in Eq. (9) ensures that the \( s \) quark can not interchange with the \( u \) or \( d \) quark and acts as a SU(3) flavor singlet.

For the flavor wave functions of the \( M-M \) and \( B-B \) systems, we first construct the quark-level \( |H_1 H_2 \rangle \) and \( |H_2 H_1 \rangle \) flavor wave functions with SU(2) CG coefficients. Then we combine them with a (an) symmetric (antisymmetric) factor to partly include the symmetric properties introduced from the \( s \) quark. Collectively, the flavor-spin wave functions for the heavy flavor meson-meson and baryon-baryon systems are

\[ |[H_1 H_2]^{I S/A} \rangle = \sum_{m_{I1}, m_{I2}} \frac{1}{\sqrt{2}} \left( C^{I, J}_{I_1, I_2, m_{I1}, m_{I2}} \phi_{I_1, m_{I1}} \phi_{I_2, m_{I2}} \phi_{H_1, m_{H1}} \phi_{H_2, m_{H2}} \right) \]
\[ \pm C^{I, J}_{I_2, I_1, m_{I2}, m_{I1}} \phi_{I_2, m_{I2}} \phi_{I_1, m_{I1}} \phi_{H_1, m_{H1}} \phi_{H_2, m_{H2}} \]
\[ \times \sum_{m_{S1}, m_{S2}} C^{I, J}_{S_1, S_2, m_{S1}, m_{S2}} \phi_{S_1, m_{S1}} \phi_{S_2, m_{S2}}. \]  

(10)
TABLE III. The matrix elements $\langle l_1 \cdot l_2 \rangle$ for the discussed $M-M$, $B-M$, and $B-B$ di-hadron systems with spin wave functions $|l_1, S_1; l_2, S_2; J \rangle$.

| Spin $l_1 \cdot l_2$ | $\langle l_1 \cdot l_2 \rangle$ |
|------------------|------------------|
| $M-M$            | $\langle 1, 0; 0, 1 \rangle$ | 0 |
|                  | $\langle 1, 1; 1, 2 \rangle$ | -2 |
| $B-M$            | $\langle 0, 1; 0, 1 \rangle$ | 0 |
|                  | $\langle 1, 1; 1, 2 \rangle$ | -4 |
| $B-B$            | $\langle 0, 1; 0, 1 \rangle$ | 0 |
|                  | $\langle 1, 1; 1, 2 \rangle$ | 2 |

TABLE IV. The flavor wave functions for the charmed hadrons considered in this work.

| Hadron $|I \rangle$ | $\phi_{I_\Lambda}^{(s)}$ | Hadron $|I \rangle$ | $\phi_{I_\Lambda}^{(c)}$ |
|------------------|------------------|------------------|------------------|
| $D^{(s)+}$       | $|\frac{1}{2}, \frac{1}{2}\rangle$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ |
| $D^{(s)-}$       | $|0, 0\rangle$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ |
| $\Sigma^{(s)+}$  | $|\frac{1}{2}, \frac{1}{2}\rangle$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ |
| $\Sigma^{(s)-}$  | $|1, -1\rangle$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ |
| $\Xi^{(s)+}$     | $|\frac{1}{2}, -\frac{1}{2}\rangle$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ |
| $\Xi^{(s)-}$     | $|1, 1\rangle$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ |

SU(3) $|RYT\rangle = |8'01\rangle$ triplet in the $B-M$ sector, where $R$, $Y$, and $T$ denote the dimension of the SU(3) representation, hypercharge, and isospin, respectively. They have the same $\langle \Lambda_1 \cdot \Lambda_2 \rangle$ matrix elements. On the contrary, in most cases, the flavor functions constructed from Eqs. (9)-(10) do not have corresponding SU(3) symmetry states. We need to further calculate their $\langle \Lambda_1 \cdot \Lambda_2 \rangle$ matrix elements. We will present our results in the next section.

After we deduce the effective potentials for all the studied heavy flavor di-hadron systems, we iterate the corresponding effective potentials into the Lippmann-Schwinger equation (LSE),

$$T(p',p) = V(p',p) + \int \frac{d^3q}{(2\pi)^3} \frac{V(p',q) T(q,p)}{E - \frac{q^2}{2m} + i\epsilon},$$

where $m$ is the reduced mass of the di-hadron system, $p$ and $p'$ are the momentum of the initial and final states in the center of mass frame, respectively.

We introduce a hard regulator to exclude the contribution from higher momenta [37, 38],

$$V(p',p) = \left. V_{[H_1H_2]}^{[\Lambda]} \right| \Theta(\Lambda - p) \Theta(\Lambda - p').$$

After some deductions, we can solve the following equation to find the possible bound state of the $[H_1H_2]^{[\Lambda]}$ system,

$$1 - \frac{m}{\pi^2} V_{[H_1H_2]}^{[\Lambda]} \left[ -\Lambda + k\tan^{-1} \left( \frac{\Lambda}{k} \right) \right] = 0,$$

with $k = \sqrt{-2mE}$.

III. NUMERICAL RESULTS AND DISCUSSION

In our model, we have three free parameters: the quark coupling constants $\bar{g}_s$, $\bar{g}_a$, and the momentum cutoff parameter $\Lambda$. In our previous work, we use the masses of the $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ to construct an equation set and precisely solve the above parameters with $\bar{g}_s = 11.739$ GeV$^{-2}$, $\bar{g}_a = -2.860$ GeV$^{-2}$, and $\Lambda = 0.409$ GeV. With these parameters, we reproduce the masses of the recently observed $P_{c0}$ and $T_{cc}$ states very well. One can refer to Ref. [7] for more details. In our convention, a positive (negative) $V_{[H_1H_2]}^{[\Lambda]}$ corresponds to a(n) repulsive (attractive) force. In this section,
we present our numerical results of the meson-meson, baryon-meson, and baryon-baryon systems case by case.

### A. Meson-Meson system

We take the \(|\bar{D}D^\dagger_1|^{S/A}\) (or equivalently \(|\bar{D}D^\dagger_2|^{S/A}\) states as examples to show more details of our calculations. From Eq. (10), the quark-level flavor-spin wave functions for the \(|\bar{D}D^\dagger_2|^{S/A}\) systems can be directly written as

\[
\langle \bar{u}c\bar{s}\bar{c}\rangle \lambda_1 \cdot \lambda_2 \rho | \bar{u}c\bar{s}\bar{c}\rangle \lambda_1 \cdot \lambda_2 \rho = \frac{1}{\sqrt{2}} (\bar{u}c\bar{s}\bar{c} + \bar{s}c\bar{u}\bar{c}) \otimes \frac{1}{\sqrt{2}} (\uparrow \downarrow \uparrow \downarrow - \downarrow \uparrow \downarrow \uparrow),
\]

(14)

and

\[
\langle \bar{u}c\bar{s}\bar{c}\rangle \lambda_1 \cdot \lambda_2 \rho | \bar{u}c\bar{s}\bar{c}\rangle \lambda_1 \cdot \lambda_2 \rho = \frac{1}{\sqrt{2}} (\bar{u}c\bar{s}\bar{c} - \bar{s}c\bar{u}\bar{c}) \otimes \frac{1}{\sqrt{2}} (\uparrow \downarrow \uparrow \downarrow - \downarrow \uparrow \downarrow \uparrow).
\]

(15)

Here, we only present their \(I_3 = 1/2\) and \(J_3 = 1\) components. The other components with the same \(I\) and \(J\) will give the identical matrix elements. Then we substitute Eqs. (14)-(15) into Eq. (3), we will encounter the direct terms

\[
\langle \bar{u}c\bar{s}\bar{c}\rangle \lambda_1 \cdot \lambda_2 | \bar{u}c\bar{s}\bar{c}\rangle \lambda_1 \cdot \lambda_2 = \frac{1}{\sqrt{2}} (\bar{u}c\bar{s}\bar{c} + \bar{s}c\bar{u}\bar{c}) \otimes \frac{1}{\sqrt{2}} (\uparrow \downarrow \uparrow \downarrow - \downarrow \uparrow \downarrow \uparrow),
\]

(16)

that only have non-vanishing matrix elements from the \(\lambda_1^D \lambda_2^s\) term and the cross terms

\[
\langle \bar{u}c\bar{s}\bar{c}\rangle \lambda_1 \cdot \lambda_2 | \bar{s}c\bar{u}\bar{c}\rangle \lambda_1 \cdot \lambda_2 = \frac{1}{\sqrt{2}} (\bar{u}c\bar{s}\bar{c} - \bar{s}c\bar{u}\bar{c}) \otimes \frac{1}{\sqrt{2}} (\uparrow \downarrow \uparrow \downarrow - \downarrow \uparrow \downarrow \uparrow),
\]

(17)

that only receive contributions from the \(\lambda_1^D \lambda_2^s\) components. Thus, for the \(|\bar{D}D^\dagger_2|^{S/A}\) systems, they do not have interactions from the exchange of the isospin triplet \((\lambda_1^D \lambda_2^s)\) fields.

Similarly, the isospin triplet related operators \(O_2(\lambda_1^D \lambda_2^s)\) and \(O_5(\lambda_1^D \lambda_2^s \sigma_1 \cdot \sigma_2)\) do not contribute to the interactions of the \(\bar{D}D\) and \(D^* D^*\) systems. Besides, the interactions of the \(\bar{D}^* D^*\) systems only arise from the \(O_1\) and \(O_4\) operators. In Table V, we present the matrix elements of the \(O_1, O_3, O_4, O_5\) for the \(\bar{D}D, D^* \bar{D}^*, D D^*\) and \(\bar{D}^* D^*\) di-meson systems.

Then the effective potentials \(V_{[H_1H_2]}^{(s)}\) in Eq. (3) for the \(\bar{D}D, D^* \bar{D}^*, D D^*\) system can be collectively written as

\[
V_{[H_1H_2]}^{(s)} = \hat{g}_s (O_1 + O_3) + \hat{g}_s (O_4 + O_5).
\]

(18)

With the parameters solved in Ref. [7], we substitute \(V_{[H_1H_2]}^{(s)}\) into Eq. (13) to search for possible bound states and present them in Table VI. We do not find molecular candidates in the \(\bar{D}^* D^*\) systems since their corresponding effective potentials are all repulsive (positive), which can be easily checked from Table V and Eq. (18). As presented in Table VI, we obtain five possible molecular states in the \(\bar{D}^* D^*\) systems. Note that for the \(|\bar{D}^* D^*|^{S/A}\) system, the corresponding attractive force is just enough to form a bound state with a very tiny binding energy. Besides, a common feature of these five molecular candidates is that their flavor wave functions are all anti-symmetric.

### Table V. The matrix elements of the operators \(O_1(\lambda_1^D \lambda_2^s), O_3(\lambda_1^D \lambda_2^s \sigma_1 \cdot \sigma_2), O_4(\lambda_1^D \lambda_2^s \sigma_1 \cdot \sigma_2),\) and \(O_5(\lambda_1^D \lambda_2^s \sigma_1 \cdot \sigma_2)\) for the considered heavy flavor di-meson systems listed in Table I.

| System               | \(O_1\) | \(O_3\) | \(O_4\) | \(O_5\) |
|----------------------|--------|--------|--------|--------|
| \(|\bar{D}D|^{S/A}\) | \(-\frac{2}{3}\) | 2 | 0 | 0 |
| \(|\bar{D}D|^{A}\)  | \(-\frac{2}{3}\) | 2 | 0 | 0 |
| \(|\bar{D}D^* (\bar{D}^* D^*)|^{S/A}\) | \(-\frac{2}{3}\) | 2 | 0 | 0 |
| \(|\bar{D}D^* (\bar{D}^* D^*)|^{A}\)  | \(-\frac{2}{3}\) | 2 | 0 | 0 |
| \(|\bar{D}^* D^*|^{S/A}\)  | \(-\frac{2}{3}\) | 2 | 0 | 0 |
| \(|\bar{D}^* D^*|^{A}\)   | \(-\frac{2}{3}\) | 2 | 0 | 0 |

### Table VI. The predicted masses and binding energies (BE) for the considered di-meson \((H_1H_2)^{(s)}\) systems. These bottomed partners are also presented. We adopt the isospin averaged masses for the single-charm (bottom) hadrons [39]. The values are all in units of MeV.

| System               | Mass | BE   | System               | Mass | BE   |
|----------------------|------|------|----------------------|------|------|
| \(|\bar{D}D|^{S/A}\) | 3834.6 | -1.6 | \(|\bar{D}D^* (\bar{D}^* D^*)|^{S/A}\) | 3976.4 | -2.1 |
| \(|\bar{D}D^* (\bar{D}^* D^*)|^{A}\)  | 4106.1 | -14.7 | \(|\bar{D}^* D^*|^{S/A}\)  | 4012.8 | -8.0 |
| \(|\bar{D}^* D^*|^{A}\)   | 4102.8 | -0.0 | \(|\bar{D}^* D^*|^{S/A}\)  | 4012.8 | -0.0 |

Since the expressions of the flavor wave functions constructed from Eq. (10) are identical to that in the SU(3) 3 and 6 flavor multiplets, the \(|\bar{D}^* D^*|^{S/A}\) systems provide us a perfect platform to discuss the possible flavor-symmetry of their interactions. In Table VII, we collect the total effective potentials for all the \(\bar{D}^* D^*\) systems obtained from Ref. [7] and this work.

From Table VII, we find that the flavor-spin symmetry manifests itself clearly in the \(|\bar{D}^* D^*|^{S/A}\) systems. In flavor space, the anti-symmetric \(\bar{D}^* D^*\) systems belong to the SU(3) 3 representation, while the symmetric \(\bar{D}^* D^*\) \(|\bar{D}^* D^*|^{S/A}\) systems belong to the 6 representation. As presented in Table VII, the doubly charmed di-meson system with the same spin structure \([l_1, s_1;l_2, s_2;j]\) and belong to the same SU(3) multiplet have the same effective potential.

In our previous analysis, there exist only two molecular states in the \(|\bar{D}^* D^*|^{S/A}\) systems, i.e., the \(|\bar{D}D^*|^{S/A}\) and \(|\bar{D}^* D^*|^{S/A}\) states. Correspondingly, in this work, we obtain their strange partners the \(|\bar{D}D^*|^{S/A}\) and \(|\bar{D}^* D^*|^{S/A}\) states, respectively. In addition, the \(\bar{D} D^*\) and \(\bar{D}^* D^*\) are no longer identical particles. Thus, in the \(|\bar{D}^* D^*|^{S/A}\) systems, there exist three
The total effective potentials $V_{[H_1H_2]^j}$ for all the $D_s^{(*)}\bar{D}_s^{(*)}$ di-meson ($[H_1H_2]^j$) systems obtained from Ref. [7] and this work. The corresponding flavor wave functions belong to 3 or 6 representation and have spin assignment $|l_1,S_1;l_2,S_2;J>$. The states in boldface are forbidden due to the selection rule.

| $\lambda$ | $V_{[H_1H_2]^j}$ |
|---------|-----------------|
| 3       | $[\bar{D}D]_{0,3}^{[1]}$ | $-\frac{2}{3}g_s$ |
| 3       | $[\bar{D}D]_{1,3}^{[1]}$ | $-\frac{2}{3}g_s$ |
| 3       | $[\bar{D}D']_{1,3}^{[1]}$ | $\frac{8}{3}g_s + \frac{16}{3}g_a$ |
| 1       | $[\bar{D}D]_{0,1}^{[1]}$ | $-\frac{2}{3}g_s + \frac{4}{3}g_a$ |
| 1       | $[\bar{D}D']_{1,1}^{[1]}$ | $\frac{8}{3}g_s - \frac{2}{3}g_a$ |
| 1       | $[\bar{D}D']_{0,1}^{[1]}$ | $\frac{8}{3}g_s - \frac{2}{3}g_a$ |

The exchange of the isospin triplet or doublet light mesons due to their special flavor components. The matrix elements $\mathcal{O}_2$, $\mathcal{O}_3$, $\mathcal{O}_5$, and $\mathcal{O}_6$ vanish. Thus, the total effective potentials for the considered systems are

$$V_{[BM]^j} = g_s\mathcal{O}_1 + g_a\mathcal{O}_4.$$  

We substitute the total effective potentials of the considered $B$–$M$ systems into Eq. (13) and find that some of the charmed $B$–$M$ systems have marginal attractive interactions that are just enough to form bound states. Namely, the results for the considered $B$–$M$ systems are quite sensitive to the adopted parameters $\bar{g}_s$ and $\bar{g}_a$.

In Ref. [7], we use the central values of the experimental masses of the $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$ states to solve the parameters $\bar{g}_s$ and $\bar{a}$. Note that the uncertainties from the experimental masses should be small due to the relatively small experimental errors [3], and the $\bar{g}_s$ dominates the total effective potentials since we obtained $|\bar{g}_s|/|\bar{g}_a| \approx 4.0$. Thus, we assume that the determined $\bar{g}_s$ has at most 20% deviation and check the possible bound states of the $B$–$M$ systems by adjusting the value of $\bar{g}_s$.

We present the results of charmed and bottomed $B$–$M$ systems that have negative (attractive) total effective potentials in Table IX. Here, the three sets of results are obtained with the parameters $(0.8\bar{g}_s, \bar{g}_a)$, $(\bar{g}_s, \bar{g}_a)$, and $(1.2\bar{g}_s, \bar{g}_a)$. As shown in Table IX, in the charmed sector, when we adopt the parameters $(\bar{g}_s, \bar{g}_a)$, we only find one molecular state $[\Sigma_c^0\bar{D}_s^+]_1^{[1]}$ with a very tiny binding energy $-0.1$ MeV. As presented in Table VIII, the spin-iso-spin related interaction ($\mathcal{O}_4$) provides an extra attractive force in this system. This tiny interaction together with the attraction from the exchange of isospin singlet field is just enough to form a bound state. When we enlarge the coupling parameter to $1.2\bar{g}_s$, the marginal attractive forces are also magnified. Thus we obtain a few more molecular candidates. If we slightly decrease the coupling parameter into $0.8\bar{g}_s$, we find no bound state solutions.

The significant role of the $b$ quark in stabilizing the double-bottom $B$–$M$ bound states are demonstrated in Table IX. When we adopt the lower limit of our parameter set

TABLE VII. The total effective potentials $V_{[H_1H_2]^j}$ for all the $D_s^{(*)}\bar{D}_s^{(*)}$ di-meson ($[H_1H_2]^j$) systems obtained from Ref. [7]

more molecular candidates, i.e., the $[\bar{D}D]_{1}^{[1]}$, $[\bar{D}D]_{0}^{[1]}$, and $[\bar{D}D]_{0}^{[3]}$. The last state may not exist due to its tiny binding energy and theoretical uncertainties.

In our previous work, we propose the isospin criterion to explain why the experimentally observed $T_{cc}^+$, $P_c$, and $P_a$ molecular candidates prefer the lowest isospin numbers. As presented in the above discussion of the $\bar{D}_s^{(*)}\bar{D}_s^{(*)}$ systems, this isospin criterion can be extended to the $I/U/V$ criterion. When the considered $\bar{D}_s^{(*)}\bar{D}_s^{(*)}$ systems have the lowest $I/U/V$ spins, they belong to the SU(3) 3 multiplet and have the strongest attractive force from the exchange of the light mesons.

Since the $D^+$ and $D_s^+$ is stable against the strong or electromagnetic decay, the predicted $[DD]_{10}^{[1]}$ is expected to be a stable state. Besides, the $D_s^*$ and $D_s^*$ have a very narrow width, similar to the observed $T_{cc}^+$ state, the predicted $[DD]_{10}^{[1]}$ and $[DD]_{10}^{[3]}$ should also be narrow in a loosely bound molecular picture.

In our framework, we neglect the corrections from the heavy quarks. The total effective potentials are all from the interactions of their light d.o.f. Thus, the corresponding $T_{bba}$ states have the same effective potentials. In Table VI, we also present the binding energies of the molecular candidates in the $B_s^{(*)}B_s^{(*)}$ systems. As can be seen from Table VI, the more non-relativistic bottomed di-meson systems are seen tightly bound.

B. Baryon-meson systems

For the baryon-meson systems considered in this work, we present their corresponding matrix elements in Table VIII. Note that the $B$–$M$ systems listed in Table VIII do not allow

TABLE VIII. The matrix elements of the operators $\mathcal{O}_1$ and $\mathcal{O}_4$ for the considered heavy flavor baryon-meson systems ($[H_1H_2]^j$) listed in Table I.
(0.8\bar{g}_s, \bar{g}_b), the considered double-bottom \(B-M\) systems all have bound state solutions. Thus, a tiny attractive force together with a large reduced mass may lead to a bottomed \(B-M\) bound state. Therefore, it is not uncommon to find bound state solutions in the doubly bottomed \(B-M\) systems.

After studying the systems listed in Table VIII, we go back to the previously discussed \(\Lambda_c \bar{D}^{(*)}\), \(\Sigma_c^{(*)} \bar{D}^{(*)}\), \(\Xi_c^{(*)} \bar{D}^{(*)}\), and \(\Xi_c^{(')(*)} \bar{D}^{(*)}\) systems in Ref. [7]. We find that there exist several molecular candidates in the \(\Sigma_c^{(*)} \bar{D}^{(*)}, \Xi_c^{(*)} \bar{D}^{(*)}\), and \(\Xi_c^{(')(*)} \bar{D}^{(*)}\) systems. In these systems, the exchange of the isospin triplet mesons is allowed and its contribution dominates the whole effective potentials.

Compared with the \(B-M\) systems studied in this work, the \(\Sigma_c^{(*)} \bar{D}^{(*)}\), \(\Xi_c^{(*)} \bar{D}^{(*)}\), and \(\Xi_c^{(')(*)} \bar{D}^{(*)}\) systems are three particularly interesting systems in the whole double-charm \(B-M\) dihadron community. They not only allow the exchange of the isospin singlet mesons but also the exchange of the isospin triplet mesons, which may contribute enough attractions to form bound states. On the other hand, when we construct the wave functions of the considered \(B-M\) systems, we consider the SU(3) breaking effects. Although the constructed flavor wave functions of the \(\Sigma_c^{(*)} \bar{D}^{(*)}\), \(\Xi_c^{(*)} \bar{D}^{(*)}\), and \(\Xi_c^{(')(*)} \bar{D}^{(*)}\) systems may not have their corresponding SU(3) symmetry states, we can still relate the \(B-M\) states with the same flavor and spin matrix elements by a generalized flavor-spin symmetry.

To further emphasize this point, we collect the molecular candidates in the \(\Sigma_c^{(*)} \bar{D}^{(*)}\), \(\Xi_c^{(*)} \bar{D}^{(*)}\), and \(\Xi_c^{(')(*)} \bar{D}^{(*)}\) systems into several sets and present them in Table X. Each set of molecular candidates share the same total effective potentials extracted from their light d.o.f. For example, in Ref. [7] we assign the \(P_c(4312)\) and \(P_c(4380)\) as the \(\Sigma_c^{(*)} \bar{D}^{(*)}\) and \(\Sigma_c^{(*)} \bar{D}^{(*)}\) states, respectively. Their binding energies calculated from the obtained total effective potentials listed in Table X are \(-8.9\) MeV and \(-9.1\) MeV, respectively, which are in good agreement with the LHCb results. Besides, we predict that the binding energies of the \(\Xi_c^{(*)} \bar{D}^{(*)}\) and \(\Xi_c^{(*)} \bar{D}^{(*)}\) states should be very close to the observed \(P_c(4440)\) and \(P_c(4457)\) states if their assignments are \(\Sigma_c^{(*)} \bar{D}^{(*)}\) and \(\Sigma_c^{(*)} \bar{D}^{(*)}\) states, respectively. The other sets of possible molecular states that have very close binding energies among the \(\Sigma_c^{(*)} \bar{D}^{(*)}\), \(\Xi_c^{(*)} \bar{D}^{(*)}\), and \(\Xi_c^{(')(*)} \bar{D}^{(*)}\) systems are also presented in Table X.
The interactions in the systems consisting of two single heavy flavor baryons may arise from the $\mathcal{O}_1$ to $\mathcal{O}_6$ operators. Their total effective potentials can be written as

$$V_{[H_1 H_2]} = \tilde{g}_s (\mathcal{O}_1 + \mathcal{O}_2 + \mathcal{O}_3) + \tilde{g}_a (\mathcal{O}_4 + \mathcal{O}_5 + \mathcal{O}_6).$$  \hspace{1cm} (20)

We present the results of the $\mathcal{O}_1$-$\mathcal{O}_6$ operators for the considered di-baryon systems in Table XI.

Note that we adopt Eq. (10) to construct the wave functions of the considered $B-B$ systems. The symmetric property of the $u$ and $d$ quarks are described by the SU(2) CG coefficients, and we further introduce a symmetric factor to partly include the symmetric property of the $s$ quark. As presented in Table XI, the signs of the matrix elements for the $\mathcal{O}_3$ or $\mathcal{O}_6$ operator are related to the exchange of the strange isospin doublet fields are opposite for each $S/A$ di-baryon systems. For the systems that do not have the contributions from the exchange of the isospin doublet fields, we list the results of their symmetric/antisymmetric ($S/A$) systems together since their corresponding $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_4$, and $\mathcal{O}_5$ matrix elements have the identical signs.

As presented in Table XI, the $\Lambda_c$ and $\Xi_c$ baryons can form four different states, i.e., the $[\Lambda_c \Xi_c]^{\frac{1}{2}}_s$, $[\Lambda_c \Xi_c]^{\frac{1}{2}}_t$, $[\Lambda_c \Xi_c]^{\frac{1}{2}}_a$, and $[\Lambda_c \Xi_c]^{\frac{1}{2}}_b$. The matrix elements of the $\mathcal{O}_2$ and $\mathcal{O}_5$ operators are 0 since the exchange of the isospin triplet fields is forbidden. Moreover, although the matrix elements of the $\mathcal{O}_1$ and $\mathcal{O}_3$ operators are nonzero, the contributions of their spin-related operators $\mathcal{O}_1$ and $\mathcal{O}_6$ vanish since the $\Lambda_c$ and $\Xi_c$ contain the spin-0 light di-quarks. In our framework, the total effective potentials are extracted from their light d.o.f. Thus, the $[\Lambda_c \Xi_c]^{\frac{1}{2}}_{S/A}$ systems with $J = 0$ and 1 are degenerate and we present them together in Table XI. The inclusion of the interactions from their heavy d.o.f may distinguish their total spins. The matrix elements of the $\mathcal{O}_1$-$\mathcal{O}_6$ operators for the other considered heavy flavor di-baryon systems can be understood in a similar way.

With the obtained effective potentials, we further check whether they have bound state solutions. We present the possible molecular candidates of the considered di-baryon systems in Table XII. In our previous work, we only consider the di-baryon systems that can only exchange the isospin singlet and isospin triplet fields. We find that the repulsive or attractive forces from the exchanges of isospin singlet mesons are very small, while the contributions from the exchange of isospin triplet fields dominate the total effective potentials. And the heavy flavor di-baryon systems with lower isospin will have larger attractive forces from the exchanges of the isospin triplet fields. Thus, the heavy flavor di-baryon systems with lower isospin quantum numbers are more likely to form bound states.

In this work, we further include the di-baryon systems that may also exchange isospin doublet fields. As can be seen from Table XI, for the di-baryon systems that have negative total effective potentials and can exchange the isospin singlet, triplet and doublet fields simultaneously, the contributions from the exchanges of isospin singlet fields are still trivial. In contrast, the contributions from the exchanges of the isospin triplet and doublet fields are dominant. The exchanges of the isospin doublet and triplet fields play a similar role. The contributions of these two type of interactions compete with each other in the overall effective potentials. Thus, the di-baryon system with a higher isospin number is still possible to form a bound state if it receives enough attractions from the exchanges of the isospin doublet fields. For example, the $[\Sigma_c \Xi_c]^{\frac{1}{2}}_{1,2}$ and $[\Sigma_c \Xi_c]^{\frac{1}{2}}_{1,2}$ states listed in Table XII are all good molecular candidates.

For the studied di-baryon systems, based on the flavor and spin structures of their light d.o.f, we can also collect the states that have identical $(\lambda_1 \cdot \lambda_2)$ and $(\sigma_1 \cdot \sigma_2)$ matrix elements, and relate them with the generalized flavor-spin symmetry. In Table XIII, we collect several sets of molecular candidates that have the same total effective potentials obtained from Ref. [7] and this work. Each set of molecular states are expected to have very close binding energies.

IV. SUMMARY

In our previous work, we proposed a unified formalism to describe the observed $T_{cc}$, $P_{cc}$, and $P_{cs}$ states. Their interactions are related via a quark-level Lagrangian and we obtain a satisfactory description of their masses. We also study the possible molecular candidates in the other systems composed of two single heavy flavor hadrons (meson-meson, baryon-meson, and baryon-baryon systems) that can only exchange the isospin singlet and triplet scalar and (or) axial vector fields. In this work, we further include the systems that can also exchange the isospin doublet fields. The inclusion of these systems allows us to further explore the possible symmetry properties of the interactions in the heavy flavor meson-meson, baryon-meson, or baryon-baryon systems. This is the main task of this work. These two works provide a complete and general discussion of the possible molecule community composed of two single heavy flavor di-hadron systems.

For the ground light flavor scalar and axial-vector mesons, the masses of the isospin doublet mesons are close to those of the isospin singlet or triplet mesons. Thus, we neglect the difference between the isospin doublet mesons and isospin singlet or triplet mesons and only introduce two coupling parameters $\tilde{g}_s$ and $\tilde{g}_a$ to describe the interactions from the exchanges of the scalar and axial vector octet mesons, respectively. The Lagrangian has a strict SU(3) symmetry. However, in order to distinguish the physical states from the SU(3) flavor states, we retain the SU(2) symmetry and take the $s$ quark as a flavor singlet to construct the flavor wave functions of the considered di-hadron systems. For the $M-M$, and $B-B$ systems, we partly include the symmetry properties of the $s$ quark by constructing the $[H_1 H_2]^{s'}$ and $[H_3 H_1]^{s'}$ flavor wave functions and combine them with a symmetry factor.

For the $\bar{D}^e(s)\bar{D}^n(s)$ systems, the constructed flavor wave functions are identical to the expressions of the SU(3) flavor 3 and 6 multiplets. The antisymmetric 3 states correspond to
the \( \bar{D}^{(s)} \bar{D}^{(s)} \) states with the lowest \( I/U/V \) spins and have enough attractive forces to form bound states. The interactions of the states that belong to the flavor 3 or 6 multiplet with a specific \( |l_1, S_1; l_2, S_2; J \rangle \) spin structure can be related by the flavor-spin symmetry induced from our Lagrangian. The molecular candidates in the \( \bar{D}^{(s)} \bar{D}^{(s)} \) systems are also discussed.

Our analysis of the \( B-M \) systems suggests that the previously studied \( \Xi_c \bar{D}^{(s)} \), \( \Xi_c^{(s)} \bar{D}^{(s)} \), and \( \Sigma_c^{(s)} \bar{D}^{(s)} \) systems are three particularly important systems. Apart from the exchanges of the isospin singlet fields, the exchanges of the isospin triplet fields are also allowed, which dominate the total effective potentials of the molecular states in these systems. Among them, we use a generalized flavor-spin symmetry to relate the states with the same \( \langle \lambda_1 \cdot \lambda_2 \rangle \) and \( \langle \sigma_1 \cdot \sigma_2 \rangle \) matrix elements and present them in Table X. In contrast, the \( B-M \) systems studied in this work can only exchange the isospin singlet fields. Even if the corresponding contribu-

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**TABLE XI.** The matrix elements of the operators \( \mathcal{O}_1 - \mathcal{O}_6 \) for the considered heavy flavor baryon-baryon systems \( ([H_1 H_2]_{\bar{U}}) \) listed in Table I.

| System     | \( \mathcal{O}_1 \) | \( \mathcal{O}_2 \) | \( \mathcal{O}_3 \) | \( \mathcal{O}_4 \) | \( \mathcal{O}_5 \) | \( \mathcal{O}_6 \) |
|------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \[\Lambda, \Xi \] \( \rho \to \rho \) | 0/0             | 0/0             | 0/0             | 0/0             | 0/0             | 0/0             |
| \[\Lambda, \Xi \] \( \rho \to \rho \) | 0/0             | 2/2             | 0/0             | 0/0             | 0/0             | 0/0             |
| \[\Lambda, \Xi \] \( \rho \to \rho \) | 0/0             | 0/0             | 0/0             | 0/0             | 0/0             | 0/0             |
| \[\Lambda, \Xi \] \( \rho \to \rho \) | 0/0             | 0/0             | 0/0             | 0/0             | 0/0             | 0/0             |
| \[\Lambda, \Xi \] \( \rho \to \rho \) | 0/0             | -2/2            | 0/0             | 0/0             | 0/0             | 0/0             |

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The table continues with similar entries for different systems...
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System \[\Lambda, \Xi\] \[\Sigma^0, \Omega^0\] \[\Sigma^0, \Xi\] \[\Xi^\ast, \Omega^\ast\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Omega^0\] \[\Xi^0, \Omega^0\] \[\Xi^0, \Omega^0\] \[\Xi^0, \Omega^0\] \[\Xi^0, \Omega^0\] \[\Xi^0, \Omega^0\]

Mass 4750.8 4975.9 4989.4 4994.8 4987.6 5017.2 5028.7 5017.2 5028.7 5021.3 5059.0

BE -5.1 -5.8 -28.0 -28.5 -44.8 -15.2 -3.6 -3.6 -78.2 -40.5

System \[\Sigma, \Xi\] \[\Sigma^0, \Xi^0\] \[\Sigma^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\]

Mass 5085.7 5096.8 5096.8 5133.1 5144.8 4959.2 5018.7 5056.4 5083.1 5094.2

BE -13.8 -2.7 -13.8 -13.8 -2.7 -15.7 -3.9 -28.4 -78.2 -40.6 -13.9 -2.7

System \[\Xi, \Xi\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\]

Mass 5083.1 5094.2 5073.5 5083.1 5102.1 5130.0 5145.8 5149.1 5155.4 5162.7 5154.8

BE -13.9 -2.7 -90.6 -81.0 -62.0 -34.1 -18.3 -15.0 -8.7 -1.4 -18.3

System \[\Xi, \Xi\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\] \[\Xi^0, \Xi^0\]

Mass 5149.1 5155.4 5162.7 5199.0 5210.3 5257.7 5269.7 5326.2 5326.2 5337.8

BE -15.0 -8.7 -1.4 -14.4 -2.9 -16.3 -4.3 -15.0 -3.4

System \[\Lambda, \Xi\] \[\Lambda, \Omega^0\] \[\Xi, \Xi^0\] \[\Xi, \Xi^0\] \[\Xi, \Xi^0\] \[\Xi, \Xi^0\] \[\Xi, \Xi^0\] \[\Xi, \Xi^0\] \[\Xi, \Xi^0\] \[\Xi, \Xi^0\] \[\Xi, \Xi^0\]

Mass 11396.3 11645.1 11563.2 11644.9 11683.9 11716.1 11731.1 11716.1 11731.1 11670.1 11709.0

BE -20.3 -20.6 -46.9 -103.2 -64.3 -32.1 -17.0 -32.1 -17.0 -98.3 -59.4

System \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\]

Mass 11738.3 11753.2 11738.3 11753.2 11827.0 11842.1 11582.6 11669.2 11708.1 11737.4 11752.3

BE -30.2 -15.2 -30.2 -15.2 -32.2 -17.1 -47.0 -98.3 -59.4 -30.2 -12.5

System \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\]

Mass 11737.4 11752.3 11677.3 11687.1 11706.5 11735.7 11752.9 11756.7 11764.3 11775.3 11752.9

BE -30.2 -15.2 -110.6 -100.8 -81.3 -52.2 -35.0 -31.2 -23.6 -12.5 -35.0

System \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\] \[\Xi, \Xi\]

Mass 11756.7 11764.3 11775.3 11848.3 11863.3 11948.8 11963.8 11971.0 11986.0

BE -31.2 -23.6 -12.5 -30.3 -15.3 -32.3 -17.3 -30.5 -15.5

TABLE XII. The predicted masses and binding energies (BE) for the charmed di-baryon \([H_1 H_2]^\ast_j\) systems listed in Table I. We adopt the isospin averaged masses for the single-charm hadrons [39]. The values are all in units of MeV.

LHCb collaboration may have the potential to look for these systems in the future.

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TABLE XIII. The possible sets of di-baryon molecular states from Ref. [7] and this work that have identical total effective potentials extracted from their light d.o.f.

| Systems | $V_{\{H_1,H_2\}}$ |
|---------|------------------|
| $[\Lambda_{c},\Xi_{c}]_{S=0}^{A},[\Xi_{c},\Xi_{c}']_{S=0}^{A},[\Xi_{c},\Xi_{c}']_{S=1}^{A}$ | $\frac{8}{3}g_A$ |
| $[\Sigma_{c},\Sigma_{c}']_{S=0}^{A},[\Sigma_{c},\Sigma_{c}']_{S=0}^{A}$ | $-\frac{20}{3}g_A + \frac{100}{9}g_8$ |
| $[\Sigma_{c},\Xi_{c}]_{S=0}^{A},[\Xi_{c},\Xi_{c}']_{S=0}^{A}$ | $-\frac{8}{3}g_A - \frac{32}{9}g_8$ |
| $[\Sigma_{c},\Xi_{c}]_{S=1}^{A},[\Sigma_{c},\Xi_{c}']_{S=1}^{A}$ | $-\frac{8}{3}g_A - \frac{32}{9}g_8$ |
| $[\Sigma_{c},\Xi_{c}']_{S=0}^{A},[\Sigma_{c},\Xi_{c}']_{S=0}^{A}$ | $-\frac{20}{3}g_A + \frac{100}{9}g_8$ |
| $[\Sigma_{c},\Xi_{c}']_{S=1}^{A},[\Sigma_{c},\Xi_{c}']_{S=1}^{A}$ | $-\frac{8}{3}g_A - \frac{16}{9}g_8$ |
| $[\Xi_{c},\Sigma_{c}]_{S=0}^{A},[\Xi_{c},\Sigma_{c}']_{S=0}^{A}$ | $\frac{20}{3}g_A + \frac{100}{9}g_8$ |
| $[\Xi_{c},\Sigma_{c}']_{S=1}^{A},[\Xi_{c},\Sigma_{c}']_{S=1}^{A}$ | $-\frac{8}{3}g_A + \frac{88}{27}g_8$ |
| $[\Xi_{c},\Sigma_{c}]_{S=0}^{A},[\Xi_{c},\Sigma_{c}']_{S=0}^{A}$ | $-\frac{20}{3}g_A + \frac{100}{9}g_8$ |
| $[\Xi_{c},\Sigma_{c}']_{S=1}^{A},[\Xi_{c},\Sigma_{c}']_{S=1}^{A}$ | $-\frac{8}{3}g_A - \frac{16}{9}g_8$ |

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