Constraining $qqtt$ operators from four-top production: a case for enhanced EFT sensitivity

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Abstract: Recently, experimental collaborations have reported $\mathcal{O}(10)$ upper limits on the signal strength of four-top production at the LHC. Surprisingly, we find that the constraining power of four-top production on the $qqtt$ type of operators is already competitive with the measurements of top-pair production, even though the precision level of the latter is more than two orders of magnitude better. This is explained by the enhanced sensitivity of the four-top cross section to $qqtt$ operators, due to multiple insertion of operators in the squared amplitude, and to the large threshold energy of four-top production. We point out that even though the dominant contribution beyond the standard model comes from the $\mathcal{O}(C^4/\Lambda^8)$ terms, the effective field theory expansion remains valid for a wide range of underlying theories. Considering the possible improvements of this measurement with higher integrated luminosity, we believe that this process will become even more crucial for probing and testing the standard model deviations in the top-quark sector, and will eventually provide valuable information about the top-quark properties, leading to significant improvements in precision top physics.

Key words: Top quark, LHC, effective field theory

PACS: 14.65.Ha, 13.85.Hd

1 Introduction

As a top-quark factory with more than six million top-quark pairs produced at Run-I and much more to expect in the future, the LHC is an ideal place to probe the top-quark properties. In proton-proton collision, most top quarks are produced in $t\bar{t}$ pairs. Single top production has the second largest cross section, which is about one third of the $t\bar{t}$. More recently, associated production modes such as $t\bar{t}X$ and single $tX$, where $X$ is a gauge boson or the Higgs boson, have also been extensively studied. These are the main channels that are now pushing the top-quark physics into a precision era [1].

Attention has also been paid to the four-top production mode, $pp \rightarrow ttt\bar{t}$, which, despite its tiny rate ($\approx 9$ fb [2, 3]) in the standard model (SM), i.e. five orders of magnitude lower than $t\bar{t}$ production (832 pb, [4, 5]), is particularly sensitive to new physics. It has been noticed that the total rate of this process can be enhanced significantly in many scenarios beyond the standard model (BSM) [6–17]. This can be due to the direct production of new resonant states which subsequently decay into tops, or to the contribution from contact four-top operators, which rises as the energy grows. These operators are not directly constrained by other processes at the tree level, and therefore the four-top channel may be the first place to see their effects.

Nevertheless, a comprehensive model-independent study of four-top production in the context of the standard model effective field theory (SMEFT) approach [18–20] has not yet appeared in the literature. This is not surprising. The SMEFT framework aims to probe the indirect effects from BSM models that are beyond the direct reach of the LHC. These effects are expected to show up as relatively small deviations from the SM prediction, and therefore the most powerful approach is to combine all available precision measurements and perform global analyses. In the top-quark sector, such analyses are often based on the most precise ones, such as top-pair and single-top cross sections and distributions, branching ratio measurements, and recently also on associated production modes such as $t\bar{t}Z$ and $t\bar{t}\gamma$, see e.g. Refs. [21, 22] for a recent global fit. The four-top production, on the other hand, is still far from being precise. The process has been searched for in a series of experimental reports [23–32], and the best upper limit from Ref. [30] is about 4.6 times the SM signal. Naïvely, one would not expect an $\mathcal{O}(10)$ upper bound to provide competitive information with respect to all the other precise measurements, except for the four-top operators that are not directly probed elsewhere.

The goal of this work is to demonstrate that this is not the case. For a very important class of operators, namely the contact four-fermion interactions with two top quarks and two light quarks, $qqtt$, we will show that the four-top process, with only a $\mathcal{O}(10)$ upper bound, is
as powerful as $t\bar{t}$ measurements with a percentage error. This constraining power is due to an enhanced sensitivity of four-top production, which comes from the fact that its cross section can depend on up to the fourth power of the operator coefficients, which scales like $(CE^2/\Lambda^2)^4$, where $E$ is the energy of the process, and $C/\Lambda^2$ is the coefficient of an $q\bar{q}tt$ operator. Given the large energy scale related to this process, and the current limits on the coefficient $C/\Lambda^2$, the factor $(CE^2/\Lambda^2)^4$ significantly enhances the sensitivity of four-top process to the $q\bar{q}tt$ operators. We will also show that the validity of SMEFT and its perturbativity can be guaranteed by imposing an analysis cut on the center of mass energy of the process at a few TeV, without reducing the enhancement factor too much, and thus the resulting constraints apply to BSM theories that live above this energy scale, if certain assumptions are made to justify the omission of operators at dim-8 and higher.

For comparison, we also consider the $t\bar{t}$ observables at the LHC, and study the corresponding exclusion limit on the same class of operators. These observables have been incorporated in a global fit by the authors of Refs. [21, 22]. In this work, however, the approach we follow is quite different, mainly because we are interested in the enhancement effect of higher powers of $CE^2/\Lambda^2$. Even in $t\bar{t}$ measurements, the squared term from dim-6 operators cannot be neglected with the current precision, and therefore instead of the four linear combinations of $q\bar{q}tt$ operators used in Refs. [21, 22] (defined in Ref. [33]), we will have to include the complete set of 14 $q\bar{q}tt$ operators. A global fit, including the main cross section and asymmetry measurements, as well as a differential cross section measurement, will be performed to derive the global constraints in these 14 directions. These constraints will then be compared with those from the four-top production in the same directions. For the latter process, we will also consider the impact of including the full set of $tttt$ type four-fermion operators, which might be generated together with the $q\bar{q}tt$ operators, when heavy mediator particles in the full theory are integrated out. Note that RG-induced constraints are also available on the $q\bar{q}tt$ operators [34], but they are typically considered as indirect constraints.

In this work, our numerical approach will be fully based on the MadGraph5_@NLO framework [3]. We use NNPDF3.0 parton distribution functions (PDF) [35]. A UFO model [36] that contains all 14 $q\bar{q}tt$ operators and 4 $tttt$ operators is generated using the FeynRules package [37]. All calculations are done at the leading order (LO). For the four-top production, we assume that a SM $K$-factor of about 1.4 [2, 3] at the next-to-leading order (NLO) can be applied also to the operator contributions. This might not be a good approximation (see Ref. [38] for an example), but is the best we can do given that the NLO prediction for all $q\bar{q}tt$ operators are not yet available. The corresponding theoretical error at the NLO is about $\sim 30\%$, much smaller than the experimental ones, and so they will be neglected throughout. Similarly, for the top-pair production mode, we always rescale the cross sections to the state-of-the-art theory predictions, except for the asymmetries, where the SM contribution is an NLO effect, while those from the four-fermion operators are from LO. We therefore only use the LO asymmetries from the dim-6 contributions.

Regarding the experimental limits on the four-top process, we will only consider the ones on the SM four-top production signal strength. This implies that the SM signal shape is always assumed. Ideally, an experimental analysis tailored to SMEFT operators, with various cuts on the center-of-mass energy to ensure the validity of the effective theory expansion, would be the best for our purpose. This however has not been done for the $q\bar{q}tt$ operators. If one naively applies the bound on the SM cross section, the limits on BSM will be more conservative, as in general the effective operators lead to harder energy distributions. This is indeed the case for the four-top operators, as have been considered in the experimental analyses in Refs. [25–28]. Furthermore, applying the upper bound of the total cross section on the fiducial cross section below some center-of-mass energy cut to ensure the SMEFT validity will also make the results conservative. Still, even these conservative constraints on $q\bar{q}tt$ operator coefficients already compete with those from $t\bar{t}$ measurements, so they are sufficient for the goal of this work. One should keep in mind that further improvements from the experimental side are possible.

The paper is organized as follows. In Section 2 we present the relevant dim-6 operators in this work. In Section 3 we explain the enhanced sensitivity of the four-top process, and discuss the validity range of the EFT. We compare the constraining powers of the four-top and $t\bar{t}$ cross sections in Section 4. Section 5 is devoted to a global fit using $t\bar{t}$ measurements, which will be compared with the fully marginalized constraints from four-top cross section. In Section 6 we conclude.

2 The four-fermion operators

In this work we are interested in the four-fermion operators that involve two top quarks and two light quarks. This is an important class of operators, as they are common in BSM models where new heavy states couple to both $t\bar{t}$ and $q\bar{q}$, or $q\bar{t}$ and $t\bar{q}$ currents. Assuming an $U(2)^{(u,d,s)}$ flavor symmetry for the first two generations, the full set of $q\bar{q}tt$ operators at dim-6...
can be written as follows

\[
\begin{align*}
\mathcal{O}_{Qq}^{(8,3)} &= (Q_L \gamma_\mu T^a T^\tau \bar{q}_L) (\bar{q}_L \gamma^\mu T^a T^\tau q_L), \\
\mathcal{O}_{Qq}^{(8,1)} &= (Q_L \gamma_\mu T^a Q_L) (\bar{q}_L \gamma^\mu T^a q_L), \\
\mathcal{O}_{tt}^{(8)} &= (\bar{t}_R \gamma_\mu T^a t_R) (\bar{d}_R \gamma^\mu T^a d_R), \\
\mathcal{O}_{tu}^{(8)} &= (\bar{t}_R \gamma_\mu T^a t_R) (\bar{u}_R \gamma^\mu T^a u_R), \\
\mathcal{O}_{tq}^{(8)} &= (\bar{t}_L \gamma_\mu T^a t_L) (\bar{q}_L \gamma^\mu T^a q_L), \\
\mathcal{O}_{Qd}^{(8)} &= (Q_L \gamma_\mu T^a Q_L) (\bar{d}_R \gamma^\mu T^a d_R), \\
\mathcal{O}_{Qu}^{(8)} &= (\bar{Q}_L \gamma_\mu T^a Q_L) (\bar{u}_R \gamma^\mu T^a u_R), \\
\mathcal{O}_{Qq}^{(3,3)} &= (Q_L \gamma_\mu \tau T^a Q_L) (\bar{q}_L \gamma^\mu \tau T^a q_L), \\
\mathcal{O}_{Qq}^{(3,1)} &= (Q_L \gamma_\mu T^a Q_L) (\bar{q}_L \gamma^\mu \tau T^a q_L), \\
\mathcal{O}_{tt}^{(3)} &= (\bar{t}_R \gamma_\mu T^a t_R) (\bar{d}_R \gamma^\mu \tau T^a d_R), \\
\mathcal{O}_{tu}^{(3)} &= (\bar{t}_R \gamma_\mu T^a t_R) (\bar{u}_R \gamma^\mu \tau T^a u_R), \\
\mathcal{O}_{tq}^{(3)} &= (\bar{t}_L \gamma_\mu T^a t_L) (\bar{q}_L \gamma^\mu \tau T^a q_L), \\
\mathcal{O}_{Qd}^{(3)} &= (Q_L \gamma_\mu T^a Q_L) (\bar{d}_R \gamma^\mu \tau T^a d_R), \\
\mathcal{O}_{Qu}^{(3)} &= (\bar{Q}_L \gamma_\mu T^a Q_L) (\bar{u}_R \gamma^\mu \tau T^a u_R), \\
\end{align*}
\]

where \(Q_L\) represents the left-handed doublet for the 3rd generation, and \(q_L, u_R\) and \(d_R\) represent the 1st and the 2nd generation quarks. The operators are summed over the first two generations, but we omit the flavor indices. Other four-fermion operators are excluded by the flavor symmetry.

For later convenience we have written the 14 operators in the form of a top-quark vector current (color singlet or octet) contracted with a light-quark vector current. Their contributions to both \(q\bar{q} \to t\bar{t}\) and \(q\bar{q} \to t\bar{t}t\bar{t}\) are independent of each other. One could also count the 14 degrees of freedom in a more physical way:

- Both the light and the heavy quark currents can be either left- or right-handed. This counts 4 degrees of freedom.
- The light quark can be up/down or charmed/down. This leads to 8 in total.
- \(SU(2)_L\) symmetry requires that \(u_L u_L t_R t_R\) and \(d_L d_L t_R t_R\) interactions have the same coefficient. This reduces the number to 7.
- With two possible color structures, i.e. singlet and octet, the total number of degrees of freedom is 14.

In the \(t\bar{t}\) process, the cross section can be written as a quadratic function of 14 operator coefficients:

\[\sigma = \sigma_{SM} + \sum_i \frac{C_i}{\Lambda^2} \sigma_i + \sum_{i,j} \frac{C_i C_j}{\Lambda^4} \sigma_{ij}\]

If one truncates the function and keeps only the interference term, then the 8 color-singlet operators, Eqs. (8)-(14), do not give any contribution at the LO. Furthermore, without information from the decay of the tops, the LLLL (LLRR) interactions cannot be distinguished from the RRRR (RRLL) operators. Therefore only 4 degrees of freedom can be observed [33], which significantly simplifies the analysis. However, the current limits on the operator coefficients \(C/\Lambda^2\) indicate that the dim-6 squared terms are not negligible, and so the full set of 14 operator need to be included in \(t\bar{t}\) production. The four-top production mode is similar, and in particular, there the dominant terms may come from the fourth power of dim-6 coefficients.

Fortunately, as we will see in Section 4, the SMEFT analysis with all 14 operators can be simplified by observing that these operators can be divided into three categories according to the flavor of the light quarks, without any interference effect across:

1. \(u_R\):
   \[\mathcal{O}_{uu}^{(8)}, \mathcal{O}_{uu}^{(1)}, \mathcal{O}_{Qq}^{(8)}, \mathcal{O}_{Qq}^{(1)};\]
2. \(d_R\):
   \[\mathcal{O}_{td}^{(8)}, \mathcal{O}_{td}^{(1)}, \mathcal{O}_{Qd}^{(8)}, \mathcal{O}_{Qd}^{(1)};\]
3. \(q_L\):
   \[\mathcal{O}_{Qq}^{(8,3)}, \mathcal{O}_{Qq}^{(8,1)}, \mathcal{O}_{Qq}^{(3,3)}, \mathcal{O}_{Qq}^{(1,3)}, \mathcal{O}_{Qq}^{(1,1)}, \mathcal{O}_{Qq}^{(8)}, \mathcal{O}_{Qq}^{(1)};\]

Furthermore, the operators in the first two categories can be easily related to those in the last category by parity. This implies that one analysis with 14 operators can be simplified into two independent ones, each with only 4 operators, from the 1st or the 2nd category, and parity can be used to derive results for the 3rd category. This is one of the reasons for choosing the operator basis given by Eqs. (1)-(14). As we will see, this simplification is very important for analyzing the four-top production process, as there the cross section is a quartic function of 14 operators, with a large number of interference terms.

We also consider the operators that consist of four top quarks. These four-top operators are important because unlike the \(q\bar{q}t\) ones, they are bound to be generated as long as there are BSM particles coupled to the top quark. The four-top production is the first process to directly probe them (see, for example, discussions in Refs. [7, 8, 16, 40]). In this work we will also provide constraints on these operators. Note that our main goal is to derive constraints on the \(q\bar{q}t\) operators, however, reliable constraints need to be obtained by marginalizing over other operators that enter the same process. This is the main reason to study the contribution from four-top operators, as we want our conclusion to be independent of their sizes.

Five such operators exist in the so-called Warsaw basis [33]:

\[\mathcal{O}_{qq}^{(3),3333}, \mathcal{O}_{qq}^{(3),3333}, \mathcal{O}_{uu}^{3333}, \mathcal{O}_{uu}^{(3),3333}, \mathcal{O}_{uu}^{(3),3333}.\]
Among them only four are independent in four-top production, which we define as
\[ O_{\text{QQ}}^{(+)} = \frac{1}{2} O_{qq}^{(1)(3333)} + \frac{1}{2} O_{qq}^{(3)(3333)}, \]  
\[ O_{tt} = O_{uu}^{(3333)}, \]  
\[ O_{tq}^{(1)} = C_{qu}^{(1)(3333)}, \]  
\[ O_{tq}^{(8)} = C_{qu}^{(8)(3333)}, \]
while the remaining degree of freedom is chosen as
\[ O_{\text{QQ}}^{(-)} = \frac{1}{2} O_{qq}^{(1)(3333)} - \frac{1}{2} O_{qq}^{(3)(3333)}, \] with no contribution to the process.

These four-top operators in general interfere with the other \( qqt \) operators in the four-top production. For a complete analysis, one will have to consider each category in Eqs. (16)-(18) together with these four operators. Parity relation still holds, under which the new physics resides. This is not a model-independent statement.

The relation between our four-fermion operator basis and the more standard basis, i.e., the Warsaw basis in Ref. 14, is given in Appendix A.

Finally, we briefly explain the notation used in this work. The coefficients of dim-6 operators are denoted as \( C/\Lambda^2 \). One should keep in mind that the \( C \) and \( \Lambda \) individually do not have any physical meaning. Only their combination is a physical quantity. We define
\[ \tilde{C}_i \equiv \frac{C_i (1 \text{ TeV})^2}{\Lambda^2}, \] so that constraints on SM deviations can be conveniently quoted in terms of \( \tilde{C} \). The values of \( \tilde{C} \) are constrained by experiments and are model-independent. On the other hand, we use \( \Lambda_{NP} \) to denote the characteristic scale at which the new physics resides. This is not a model-independent quantity, but it is useful for defining the range of validity of the EFT expansion, which requires \( E < \Lambda_{NP} \), where \( E \) is the typical energy transfer in the process of interest.

### 3 Sensitivity and EFT validity

To briefly explain the sensitivity of the four-top process to four-fermion \( qqt \) operators, let us take \( O_{tu}^{(8)} \) as an example. This operator represents a contact interaction between a color octet right-handed up-quark current and a color octet right-handed top-quark current. We first consider the \( t \bar{t} \) process. \( t \bar{t} \) measurements so far impose the tightest bounds on \( qqt \) operators. The LO cross section at 8 TeV, rescaled to the next-to-next-to-leading order (NNLO) prediction including the resummation of next-to-next-to-leading logarithmic (NNLL) soft gluon terms \([4,5]\), is numerically given by (in pb):
\[ 252.9 + 2.94 \tilde{C}_{tu}^{(8)} + 0.411 \tilde{C}_{tu}^{(8)2}. \]

Using the combined ATLAS and CMS measurement on \( t \bar{t} \) inclusive cross section \([72]\), we find the following bounds
\[ -11.8 < \tilde{C}_{tu}^{(8)} < 4.6 \]
at the 95\% confidence level (CL). Interestingly, both the upper and the lower limits on \( \tilde{C}_{tu}^{(8)} \) come only from the upper bound of the cross section. In particular for the lower limit \( \tilde{C}_{tu}^{(8)} = -11.8 \), the squared term in Eq. (26) already dominates over the interference.

As we have mentioned in the introduction, it is this same effect, i.e., the dominance of terms with higher powers in \( \tilde{C} \), that enhances the EFT sensitivity of four-top production. In particular, at the LO, the 14 \( qqt \) type operators can be inserted at most twice in the amplitude. The squared amplitude at LO is thus a quartic function with 14 arguments:
\[ O = O_{SM} + \sum_i \frac{C_i}{\Lambda^2} O_i + \sum_{i \leq j} \frac{C_i C_j}{\Lambda^4} O_{ij} + \sum_{i \leq j \leq k} \frac{C_i C_j C_k}{\Lambda^6} O_{ijk} + \sum_{i \leq j \leq k \leq l} \frac{C_i C_j C_k C_l}{\Lambda^8} O_{ijkl}, \]
where \( O \) represents any observable. Focusing again on \( O_{tu}^{(8)} \), without worrying about EFT validity for the moment, the LO total cross section is (in fb)
\[ 6.1 + 0.10 \tilde{C}_{tu}^{(8)} + 0.081 \tilde{C}_{tu}^{(8)2} + 0.016 \tilde{C}_{tu}^{(8)3} + 0.0048 \tilde{C}_{tu}^{(8)4}. \]

The CMS search presented in Ref. [30] gives an upper bound on the signal strength of four-top process, \( \mu < 4.6 \). Naively applying this result to Eq. (29), we find the following constraints
\[ -8.8 < \tilde{C}_{tu}^{(8)} < 7.1, \]
which are already complementary to the previous constraints from \( t \bar{t} \). Note, however, that when these constraints are saturated, it is the \( \tilde{C}_{tu}^{(8)4} \) term that gives the dominant contribution. Had we truncated Eq. (29) to, say, the linear term in \( \tilde{C}_{tu}^{(8)} \), the resulting constraints would have been more than one order of magnitude worse. This implies that the four-top process has an enhanced sensitivity to \( qqt \) operators, due to the contribution from higher power terms in \( \tilde{C} \), and this is why such a process with only a \( O(10) \) upper bound on its signal strength can beat the \( t \bar{t} \) measurement with a precision at the percentage level. Note that which term dominates depends on the size of the \( \tilde{C}_{tu}^{(8)} \), and is therefore related to the current experimental bounds. The quartic term dominates if \( |\tilde{C}_{tu}^{(8)}| > 4.1 \), while the quadratic one
The above observation however leads to two questions: why the high power terms dominate, and whether the SMEFT expansion is still valid. The first question is mostly explained by the large energy scale related to the four-top process. The threshold of four-top production is 4m_t = 690 GeV. Most signal events have a typical center of mass energy of ≳ O(1) TeV, depending on the value of the operator coefficients, as illustrated in Figure 1. The series in Eq. (29) comes from multiple insertion of the four-fermion effective interaction in the squared amplitude, and by power counting each insertion corresponds to a factor of CE^2/Λ^2, where E is the characteristic energy of the process. The current constraints on C/Λ^2 then implies that

\[ \frac{C E^2}{\Lambda^2} > 1 \]  

(31)

and so terms with the highest power in C/Λ^2 are supposed to dominate[1] Note that this is not true for all operators. For example, another important operator that enters both t$t and four-top production channels is the top-quark chromo-magnetic dipole operator,

\[ O_{\text{tg}} = y_t g_s (\bar{Q} o^{\mu \nu} T^A t) \tilde{\phi} C_{\mu \nu} \Delta \]  

(32)

The contribution of this operator does not scale as CE^2/Λ^2 because of the Higgs vev. It is also better constrained by t$t in the gg initiated channel. As a result, the four-top limit on C_{tg} cannot compete with the one from the t$ measurement, and so we will not consider it in this work.

The second question is more crucial. The fact that higher power terms in Eq. (29) dominate seems to imply the breakdown of the EFT expansion, as one could ask whether the contributions from dim-8 and higher operators can be safely ignored in an EFT expansion, given that they scale the same way in 1/Λ as the higher-power terms in Eq. (29). Therefore the validity of the EFT expansion itself needs to be justified. Here to make things clear, it is important to distinguish between two kinds of “expansions”. The EFT expansion comes from integrating out heavy degrees of freedom at the energy scale Λ_{NP}, (to be distinguished from the non-physical Λ), a procedure whose legitimacy is related to E/Λ_{NP} < 1. This means that for a given process one could always truncate the SMEFT Lagrangian at a certain dimension [11]. This is however different than the “expansion” in Eq. (29), which instead comes from multiple insertion of dimension-six effective interaction and squaring the amplitude. In this case the “expansion parameter” is CE^2/Λ^2 > 1. However, this second “expansion” is not related to EFT validity, and is strictly speaking not even an expansion: there are no more terms after the fourth power of CE^2/Λ^2 (at LO, with on-shell tops and no further radiations), so there is no need to truncate. Simply put, when CE^2/Λ^2 > 1 is allowed by experimental constraints, one should only truncate the expansion in the dimension of operators, but keep all terms in a series of CE^2/Λ^2. The relative theory error due to neglecting higher order terms is then controlled by E^2/Λ_{NP}^2 < 1, instead of CE^2/Λ^2 > 1.

As a simple example of the above argument, it has been discussed in Refs. [45] [46] that in a wide class of BSM models with strongy couplings, the contribution from dim-8 operators is subleading with respect to dim-6 squared terms, without invalidating the EFT expansion. An explicit example has been given in Ref. [46], where a 2 → 2 scattering process is considered. The SM contribution is of order g^2_{SM}^2, while a dimension-six operator coming from integrating out the heavy mediator can be as large as g^2_{SM} E^2/Λ_{NP}^2, where g_{SM} is the BSM coupling of the mediator to the SM particles. We have

\[ \frac{C}{\Lambda^2} \sim \frac{g_{SM}^2}{\Lambda_{NP}^2} . \]  

(33)

If the coupling g_{SM} is much larger than the SM coupling g_{SM} (which is often the case when experimental constraints are saturated, if Λ_{NP} is kept larger than E), the BSM contribution dominates the SM contribution when

\[ E \approx \sqrt{s} \]  

(31)

with E ≈ \sqrt{s} tends to overestimate the effective contribution. The reason is that the energy transfer at the effective vertices is often less than \sqrt{s}. The only configuration where the energy transfer is equal to \sqrt{s} is the case where the two initial quarks enter the same effective vertex, which then produces t$ → t$t, but in this case the squared amplitude can depend on at most two powers of Cs/Λ^2. Still, in this process either (CE^2/Λ^2) with E < \sqrt{s} or (CE^2/Λ^2)^2 represents a large factor.
(g_*/g_{SM})^2 \frac{E^2}{\Lambda_{N_P}^2} > 1$, and similarly the BSM squared term dominates over the interference term between SM and BSM. The EFT expansion is still valid if $E/\Lambda_{N_P} < 1$, because a $2 \rightarrow 2$ scattering at the tree level can be enhanced at most by $g_0^2$, and therefore no $g_*/g_{SM}$ factor exists between dimension-six and dimension-eight operators. In general the validity of the EFT expansion is not spoiled by a large $g_*$, or a large $C/\Lambda^2$, because the maximum power of $g_*$ in a given process is fixed. As a physics case, in reality the LHC sensitivities to the triple-gauge-boson couplings are completely dominated by dim-6 squared contributions, while the global EFT analyses can be performed without including dim-8 operators [47, 48].

Higher dimensional operators can be constructed in different ways. One can use the first expansion parameter in Eq. (34), $D_\mu/\Lambda_{N_P}$, to increase the dimension without changing the field content. This is like expanding a heavy mediator propagator $(p^2 - M^2)^{-1} = -M^{-2}(1 + p^2/M^2 + p^4/M^4 + \ldots)$, where $M \approx \Lambda_{N_P}$, so the expansion parameter is simply $E^2/\Lambda_{N_P}^2$. In this case neglecting higher-dimensional operators is justified. Alternatively, one can also use $g_* f_{L,R}/\Lambda_{N_P}^{3/2}$ or $g_* H/\Lambda_{N_P}$ to increase the dimension, and the expansion parameter is enhanced by $g_*$, so higher-dimensional operators have a chance to contribute more. This however cannot be done repetitively, because at some point the operator will contain more than six fields and become irrelevant (assuming LO amplitude dominates, and neglecting the vev as we are interested in the high-energy regime). The question is where to stop this $g_*$ enhanced expansion. Note that the dim-6 contribution is dominated by amplitudes like Figure 2 (b) which already scale like $g_0^2 E^4/\Lambda_{N_P}^2$. The first relevant operator that is enhanced by $g_0^4$ is a dim-10 operator, $g_0^4 f^a D$, whose contribution scales like $g_0^4 E^6/\Lambda_{N_P}^2$. This is still subdominant. For illustration we give an example in Figure 2 (c) and (d), in a model with a heavy mediator with coupling strength $g_*$. Note that the two-to-four process can be enhanced at most by $g_0^4$ at the tree level, and a SMEFT operator that contains six fermions is at least at dim-10, because odd-dimensional operators do not exist in the SM if B and L number violating operators are ignored [50, 51]. On the other hand, dim-8 operators are enhanced at most by 3 powers of $g_*$. Since both the dim-8 and dim-10 contributions are less than $g_0^4 E^4/\Lambda_{N_P}^2$, and further enhancement with $g_*$ beyond dim-10 is not possible without adding more particles, we conclude that, under the above assumption, truncating the SMEFT at dim-6 is justified.

![Fig. 2. The $q\bar{q} \rightarrow t\bar{t}H$ amplitudes that are enhanced by four powers of BSM coupling $g_*$. Blue lines are heavy mediators. Double lines represent the top quarks. The square represents a $g_*$ coupling, and the blue blob represents effective operators, coming from integrating out the mediators. Diagrams (a), (c) describe the amplitudes in the underlying theory, while in the EFT they respectively correspond to (b) and (d). Diagrams (a) and (b) correspond to two insertions of dim-6 operators. They scale like $g_0^4 E^4/\Lambda_{N_P}^2$. Diagrams (c) and (d) correspond to one insertion of a dim-10 operator. They scale like $g_0^4 E^6/\Lambda_{N_P}^2$.](image-url)
It is important to keep in mind that this conclusion is a model-dependent one. In practice one could come up with theories with more than one scales or couplings, where the dim-8/10 contributions might be important. As a general rule, when interpreting results obtained with a dim-6 SMEFT in specific models, one always needs to check the validity of the EFT by estimating the impact of higher dimensional contributions.

It remains to show that the validity condition, $E/\Lambda_{NP} < 1$, can be taken under control. As proposed in Ref. [49], the standard way to deal with this in a hadron collider is to apply a mass cut $M_{cut}$ on the center of mass energy of the event, or some other observable that characterizes the energy scale of the process. Results of the analysis should be provided as functions of $M_{cut}$. The SMEFT approach is then valid if results are interpreted to $M_{cut}$, and theory errors due to missing higher dimensional terms can be estimated by $M_{cut}^2/\Lambda_{NP}^2$. As we have mentioned in the introduction, since the experimental search is not carried out with this kind of strategy, we will simply apply various $M_{cut}$ of order a few TeV on the center-of-mass energy in our cross section calculation. The resulting fiducial cross sections are required to be less than the upper bound set on the SM total cross section, which then gives conservative constraints.

![Fig. 3. Center-of-mass energy distribution of four-top production, to illustrate the validity of EFT. Results are shown for the EFT, the SM, and for $M_V = 5, 6, 8$ TeV respectively.](image)

(35) to make sure the loop corrections due to additional insertion of effective operators are not important. This will be checked for typical values of $M_{cut}$. A related issue is that jets in the final state may allow additional powers of $CE^2/\Lambda^2$, if loop corrections are important. In this case dim-8 operators are also needed for a consistent theory prediction, as in general two dim-6 operators can mix with a dim-8 one. Given the precision level of the process, we simply impose the following perturbativity condition

$$\frac{CE^2}{(4\pi)^2\Lambda^2} < \frac{CM_{cut}^2}{(4\pi)^2\Lambda^2} < 1.$$ 

The discussion in this section is based on LO accuracy. The $qq\to ttt\bar{t}$ amplitude can go beyond the fourth power of $CE^2/\Lambda^2$, if loop corrections are important. In this case dim-8 operators are also needed for a consistent theory prediction, as in general two dim-6 operators can mix with a dim-8 one. Given the precision level of the process, we simply impose the following perturbativity condition

$$\frac{CE^2}{(4\pi)^2\Lambda^2} < \frac{CM_{cut}^2}{(4\pi)^2\Lambda^2} < 1.$$ 

4 The signal process

The cross section of four-top production is a quartic function of the 14 $qgqt$ operator coefficients. Such a function in general has $C_{4+4}^{4+4} = 3060$ terms. Numerically determining this function will then require at least 3060 independent simulations at different parameter space points, which is huge amount of work. Fortunately, as we have explained in Section [2], the procedure can be simplified into two steps. The first step is to determine the cross section as functions of operators in the first two categories, separately, which requires a minimum of only $C_{4+4}^{4+4} + C_{4+4}^{4+4} - 1 = 139$ independent simulations. The second is to derive the cross section as a
function of operators in the third category, with the help of parity. Namely, if one imposes
\[ C_{Q_{tu}}^{(8)} = C_{Q_{4\ell}}^{(8)}, \quad C_{Q_{tu}}^{(1)} = C_{Q_{4\ell}}^{(1)}, \]
then the cross section is invariant under the following transformation
\[ C_{Q_{tu}}^{(a)} = C_{Q_{4\ell}}^{(a)} \rightleftharpoons C_{Q_{tu}}^{(a)}, \]
\[ C_{Q_{tu}}^{(a)} = C_{Q_{4\ell}}^{(a)} \rightleftharpoons C_{Q_{tu}}^{(a)} \pm C_{Q_{tu}}^{(a)}, \]
\[ C_{Q_{tu}}^{(a)} \rightleftharpoons \frac{1}{2} \left( C_{Q_{tu}}^{(a)} + C_{Q_{4\ell}}^{(a)} \right), \]
where \( a = 1, 8 \). Using these relations, the dependence on the third category operators can be derived from that of the first two.

When the four-top operators are included, in each category one has to consider together the 4 \( qqt\ell \) operators and the 4 \( tttt \) operators. The \( tttt \) operators can be inserted only once in the amplitude, if the \( qqt\ell \) operators are not inserted twice in the same amplitude. This increases the total number of independent terms to 705, which is still manageable. The parity relations can still be used to derive the dependence on the third category operators, provided
\[ C_{Q_{tu}}^{(+) \rightleftharpoons C_{tt}} \]
is added to Eqs. (37)- (39).

Following the procedures described above, to determine the dependence of the four-top cross section on the 14 \( qqt\ell \) and 4 \( tttt \) operator coefficients, we have randomly generated \( \sim \mathcal{O}(1000) \) points in the parameter space, and computed the cross section at these points, applying \( M_{\text{cut}} = 2, 3, 4 \) TeV respectively. These points are uniformly distributed roughly within the experimentally allowed region of the coefficients. Results are then fitted to the polynomial described above. We have checked that the prediction of the fitted function at all these sampled points agree with the simulation within 3% error.

With this function we are ready to evaluate the constraining power of the signal process, and compare the constraints from four-top production with those obtained from \( tt \) measurements. For this purpose we first consider single measurements on the \( tt \) total cross sections at the LHC, including:

- 8 TeV ATLAS, \( \sigma = 242.9 \pm 8.8 \) pb \( [52] \).
- 13 TeV CMS, \( \sigma = 888_{-32}^{+34} \) pb \( [53] \).

Corresponding theoretical predictions at NNLO+NNLL are taken from \( [1, 5] \). For the four-top production, we consider the current upper bound with signal strength \( \mu < 4.6 \) \( [30] \), applying the \( M_{\text{cut}} = 3 \) TeV cut on the center-of-mass energy. We further consider the projection for an integrated luminosity at 300 fb\(^{-1} \), \( \mu < 1.87 \), estimated by Ref. \( [32] \), and apply \( M_{\text{cut}} = 2 \) TeV and 3 TeV respectively.

In Figure 4 we show the resulting constraints at 95% confidence level, for the operators in the first category (i.e. those that couple to \( u_R \)), with two operators turned on at a time. Results for the other two categories are similar and are given in Appendix B. From these plots, our observations are the following:

- Current constraints from four-top production already provide competitive constraints (black dashed), which are close to, and in some cases better than, the constraints from the 13 TeV \( tt \) measurement with only a 4% error (green dashed).
- The 8 TeV \( tt \) measurement so far gives better constraints (green shaded), but even these will be superseded in the future by an improved search/measurement of four-top production at 300 fb\(^{-1} \) luminosity with a projected \( \mu < 1.87 \) upper bound (black solid), assuming \( M_{\text{cut}} = 3 \) TeV.
- Lowering \( M_{\text{cut}} \) to 2 TeV will give somewhat looser constraints (blue solid), but results can be applied to more underlying models where the BSM scales are not so heavy. On the other hand, increasing this cut can further improve the constraints.

![Fig. 4. Constraints from four-top cross section and individual \( tt \) cross section measurements, on the operator coefficients in the first category (\( C_{tu}^{(8)}, C_{tu}^{(1)}, C_{tu}^{(8)}, C_{tu}^{(1)} \)), assuming two coefficients to be nonzero at a time.](image)

It is important to point out that the \( tt \) measurement is limited by systematic errors, and further improvements with higher luminosity is difficult. On the
other hand, there is still a lot of room for the four-top search/measurement to be improved in the future. Our results indicate that, in the near future, the four-top process could even take place and provide more crucial information on $qqtt$ operators. One should keep in mind that this requires a relatively large cut, $M_{\text{cut}} \gtrsim 2 \sim 3$ TeV, to be applied in the analysis, so only the BSM models that live above $\sim 3$ TeV are subject to these new constraints. This is however not a drawback from the SMEFT point of view, because new states below this energy scale are likely to be excluded by explicit resonance searches \cite{1}. 

## 5 Global fit

In the previous section we have shown that the future four-top measurement can have a better sensitivity to four-fermion operators compared with individual $t\bar{t}$ cross section measurements. However, the $t\bar{t}$ production is such an important process that has been measured extensively at many different energies (Tevatron, LHC 7, 8, 13 TeV) and in many different ways (cross section, asymmetries, distributions, etc.) A global fit of all available measurements thus provides so far the best available limits. In this section we will investigate whether the four-top process can add useful information on top of the global top measurement program.

The recent global fit performed by the authors of Refs. \cite{21,22} has included the four-fermion operators. The fit is based on four linear combinations of their coefficients, called $C_{u,d}^{1,2}$, which are the only independent degrees of freedom at the dim-6 interference level \cite{33}. However, the theoretical set up in this work is different, in that we expect that the dim-6 squared terms can dominate the current bounds. This has been confirmed by, for example, a fit for four-fermion operators in Ref. \cite{54}, where this domination has been interpreted as the SMEFT being invalid. However, as explained in Section 3 in this work we distinguish the $CE^2/\Lambda^2$ expansion from the true EFT expansion $E^2/\Lambda_{NP}^2$, and in any case we expect higher powers of $CE^2/\Lambda^2$ to dominate in four-top production and to enhance its sensitivity to BSM. The actual EFT validity is then guaranteed by the assumption $\Lambda_{NP} > M_{\text{cut}} \sim 2 \sim 3$ TeV, which is more than enough for most $t\bar{t}$ measurements.

In light of the above considerations, the fit we will perform is different than the previous ones, in that the dim-6 squared terms as well as the interference between two dim-6 operators will be fully incorporated. We will have to abandon the $C_{u,d}^{1,2}$ language, as this simplification breaks down at the dim-6 squared level. Furthermore, the color-singlet operators cannot be neglected due to vanishing interference term. The fit will then include 14 independent operators. A complete analysis including every existing measurements is certainly beyond the scope of this work. Given that the goal is to evaluate the relative constraining powers of $t\bar{t}$ and four-top production, we will follow the approach in Ref. \cite{54}, where the most relevant measurements on cross sections and asymmetries have been included. The most recent LHC 13 TeV cross section measurements will be added as well. Furthermore, unlike Ref. \cite{54}, we shall also consider the differential $m_t$ distribution measurement to constrain the possible shape change from four-fermion operators \cite{16}. In Table 1 we list the measurements that will be used in our fit, together with the corresponding SM predictions. We believe these observables represent the most sensitive ones to $qqtt$ operators that have been measured so far.

For simplicity, we only consider the SM prediction uncertainties as theoretical uncertainties. We add the theoretical and experimental errors in quadrature, and when the errors are not symmetric, we take the larger one for both sides. We further assume all uncertainties are not correlated, except for the theory ones that come from the same prediction. The fit for the $m_t$ distribution should be considered at most as a “toy fit” given that correlations between different bins are not available from the experimental report. We have dropped the last bin due to the normalization constraint. A K-factor rescaling will not improve the normalized distribution. For this reason, we use LO prediction and consider various theory errors. The scale uncertainty is from variation of $\mu_R$ and $\mu_F$ by a factor of 2. The PDF uncertainty is taken from the envelope of three PDFs, including the NNPDF \cite{25}, MMHT \cite{55}, and CT14 \cite{56} PDF sets with their own uncertainty bands. We have checked that the differences between LO and NLO predictions are within these errors.
### Table 1. Measurements used in the global fit, with corresponding theory predictions and uncertainties.

|                        | SM prediction | Measurement       |
|------------------------|---------------|-------------------|
| Cross section, Tevatron 1.96 TeV, CDF+D0 | 7.35$^{+0.26}_{-0.33}$ pb [4] | 7.60±0.41 pb [57] |
| Cross section, LHC 8 TeV, ATLAS+CMS          | 252.9$^{+13.3}_{-14.5}$ pb [4] | 241.5±8.5 pb [42] |
| Cross section, LHC 13 TeV, CMS                 | 832$^{+40}_{-45}$ pb [5] | 888$^{+33}_{-34}$ pb [57] |
| Cross section, LHC 13 TeV, ATLAS               | 832$^{+40}_{-45}$ pb [5] | 818$^{+36}_{-35}$ pb [58] |
| $A_{FB}$, Tevatron 1.96 TeV, CDF+D0           | 0.095 ± 0.007 [59] | 0.128 ± 0.025 [60] |
| $A_{C}$, LHC 8 TeV, ATLAS                      | 0.0111 ± 0.0004 [61] | 0.009 ± 0.005 [62] |
| $A_{C}$, LHC 8 TeV, CMS                        | 0.0111 ± 0.0004 [61] | 0.0033 ± 0.0042 [63] |

$m_{tt}$ distribution, LHC 8 TeV, ATLAS        | MadGraph5@NLO+PYTHIA6 [64] | Ref. [65] |

A $\chi^2$ is constructed based on the information in Table 1 to derive the 95% CL limits on operator coefficients. These limits are compared with the projection of four-top measurement at 300 fb$^{-1}$. Some results are shown in Figure 5 with two operators turned on at a time. Our observations are the following:

- In general, with a 3 ~ 4 TeV cut, constraints from four-top cross section are very similar to those from $m_{tt}$ measurement (black dashed vs blue and red). In rare cases they are complementary.

- In most cases, combining the $t\bar{t}$ inclusive measurements, i.e. the cross sections and the asymmetries, provides the most constraining limits, as expected. This is illustrated by the three plots in the first row in Figure 5. Results from $m_{tt}$ differential measurements and four-top cross sections provide slightly weaker bounds. The $t\bar{t}$ global fit, including cross sections, asymmetries, and $m_{tt}$, is indicated by the green shaded area.

- Exceptions are the directions that are not effectively constrained by asymmetry measurements. In this case both $m_{tt}$ differential measurements and the four-top cross section provide better limits. These cases are illustrated in the second row in Figure 5. The diagonal directions roughly correspond to flat directions between the LLLL/RRRR operators and the LLRR/RRLL operators, whose contributions to $A_{FB}$ and $A_{C}$ have the opposite sign.

The asymmetry measurements thus do not provide useful information in these directions. When two operators are turned on, there can be four such directions, as the dominant contributions are coming from dim-6 squared terms. This can be seen in Figure 5 second row. Clearly, in these cases both the $m_{tt}$ and the four-top measurements help to further improve our reach in SM deviations.

Given that the four-top measurement provides almost the same information as the $m_{tt}$ differential cross section, we do not expect the four-top to give better constraints than a global fit on the $qqtt$ operators. It is however a valuable add to the precision top physics at the LHC, given that the $m_{tt}$ distribution is already one of the most sensitive observables to four-fermion operators. In particular, in directions that are not sensitive to asymmetry measurements, information from four-top process is useful. For this reason we expect that marginalized constraints from a $t\bar{t}$ global fit and those from the four-top process are comparable. However, to really confirm this point in a model-independent way, we need to take into account the four-top operators given in Eqs. (20)-(23), to derive the fully marginalized constraints from the four-top process. Naively, one would expect that further marginalizing over the additional $tttt$ operators would make the constraints on $qqtt$ operators weaker. We will show that, while this is indeed the case, the effect is not large enough to qualitatively change our conclusion.
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Fig. 5. Selected results for $t\bar{t}$ global fit, compared with projected constraints from the four-top production at high luminosity, at 95% CL. Black solid and dashed contours represent constraints from $t\bar{t}$ inclusive measurements (i.e. cross sections and asymmetries) and $m_{t\bar{t}}$ differential measurement respectively. The green shaded area is the combined result. Constraints from $t\bar{t}t\bar{t}$ with $M_{\text{cut}} = 3$, 4 TeV are given by the blue and red curves separately.

Fig. 6. Marginalized constraints on four-top operators, using the current bound as well as the projection for 300 fb$^{-1}$, for several $M_{\text{cut}}$ values.

For illustration, in Figure 6 we present the constraints on four-top operators. These are marginalized over other four-top operators, but not the $qqt\bar{t}$ operators. The current constraints are derived using $M_{\text{cut}} = 3$ TeV, while the projected ones for 300 fb$^{-1}$ are given with $M_{\text{cut}} = 2$, 3, 4 TeV, respectively. The constraints are more conservative than those directly extracted from a tailored experimental analysis, e.g. Ref. [28]. This is expected because we assumed SM signal shape and only use the cross section below $M_{\text{cut}}$. Also note that even for the most constraining limits, the dim-6 squared contribution already dominates over the interference. For this reason, including these operators in our analysis should not significantly affect the constraints on the other 14 $qqtt$ operators.

In Figure 7 we present the most important results of this work: a comparison of fixed (i.e. one operator at a time) and fully marginalized (i.e. all other operators floated) constraints for all $qqtt$ operators, from the four-top measurement and from the $t\bar{t}$ measurements. The $t\bar{t}$ constraints are from our global fit, including cross sections, asymmetries, and $m_{t\bar{t}}$ distribution, while the four-top constraints are from the 300 fb$^{-1}$ projection, $\mu < 1.87$, with different $M_{\text{cut}}$ values applied. Perturbativity in the EFT requires Eq. (35) to hold. This leads to an upper bound of $|\tilde{C}| < 39, 18$, and 9.9 respectively for $M_{\text{cut}} = 2, 3, 4$ TeV. The latter two are shown in Figure 7 by the vertical dotted lines.
We want to emphasize that the four-top constraints obtained in this work are in general conservative, and in practice better results can be expected from a tailored experimental analysis. For example, in the case of four-$t_R$ operator $O_{tt}$, by assuming the spectrum via the EFT model, the constraints can be enhanced by a factor of $\sim 2$ compared with assuming the SM signal shape [28]. One could imagine that a similar factor applies also to $qqtt$ operators, and in this case the four-top cross section could even be more constraining than a $t\bar{t}$ global fit. In addition, in the future combining searches/measurements in different channels could further improve the reach.

The perturbativity in general is not a problem for $M_{cut} = 2, 3$ TeV, while for 4 TeV some of the marginalized constraints start to approach or even go beyond the perturbative limit. Unitarity gives further constraints. Following Ref. [60], we find that the following constraints apply to color singlet and octet operators respectively. These values imply that the limits obtained with $M_{cut} = 2$ to 3 TeV are more or less safe, while the improvements due to including events from 3 to 4 TeV are not reliable, even though these improvements are quite small already. It should be noted that the unitarity problem with $M_{cut} = 4$ TeV is at most temporary, as the experimental precision will continue to improve. In fact, considering the possible improvements discussed in the previous paragraph, it is likely that going to 4 TeV is safe already with the assumed luminosity.

In any case, we conclude that in the near future, compared with a $t\bar{t}$ global fit, the four-top production can provide competitive constraints, due to its enhanced sensitivity to four-fermion $qqtt$ operators. Including this process in the global top-fitting program will definitely improve our reach in SM deviations in the top sector. Compared with $t\bar{t}$ inclusive measurements which are dominated by systematic errors, the high-mass $t\bar{t}$ production and the four-top production have more room to improve. In the long term, they should become the crucial ones that determine our final reach at the LHC.

Before concluding, we remind the reader that the cost of such an enhanced sensitivity is a relatively large value of $M_{cut}$, which implies that the results are applicable only to BSM scenarios above this scale. In the long term, however, we believe that in any case new states below this energy scale are likely to be excluded by explicit resonance searches. One should also keep in mind that when these results are interpreted with explicit BSM models, it typically implies that a large BSM coupling is allowed, and so one should always check the sizes of higher di-
mensional operators, to make sure that the truncation of the SMEFT at dim-6 is valid.

6 Conclusion

Precision measurements are not just for precision itself. The ultimate goal is the higher reach in testing new physics and the ability to exclude deviations from the SM, and therefore sensitivity to SM deviations is crucial. An observable with an enhanced sensitivity to SM deviations, even poorly measured, may have a chance to play an important role. We have demonstrated this last point, using the four-top production process in the top EFT context. As a benchmark to assess its sensitivity, we use the top-pair production measurements for comparison. We have found that, as far as the dim-6 four-fermion operators are concerned, the current upper bound on the four-top signal strength at the level is already as powerful as the $t\bar{t}$ cross section measurements which have percentage level precision. Furthermore, using the projected bounds for 300 fb$^{-1}$ at 13 TeV, the four-top measurement can even compete with a global fit using $t\bar{t}$ measurements, including the $m_{t\bar{t}}$ differential distributions. This comparison is remarkable as the four-top cross section was never considered as a precision measurement like the $t\bar{t}$ process.

The origin of the enhanced sensitivity of four-top cross section comes from the fact the four-fermion operators can be inserted up to four times in the squared amplitude, each time with a factor $CE^2/\Lambda^2$ enhancing their contribution to the cross section. This factor can be larger than one given the current limits on $C/\Lambda^2$, and the typical center-of-mass energy of the four top quarks produced. We have shown that the validity of the EFT, or in other words the validity of expansion in higher dimensional operators, can be controlled by $E^2 < M_{cut}^2 < \Lambda^2_{NP}$, without spoiling the enhancement effect for $M_{cut} \sim$ a few TeV, and that the EFT perturbativity $CM_{cut}^2/\Lambda^2 < (4\pi)^2$ is also satisfied in general. The four-top measurement can thus provide useful bounds for underlying BSM models that live at a scale $\gtrsim$ a few TeV, and is therefore a valuable add to the precision top physics at the LHC, in particular, given that there is still a lot of room for this process to improve in the future. On the other hand, for BSM scenarios below a few TeV, these results may not apply, but in any case one expects that there the explicit resonant searches provide better exclusion.

For comparison purpose we performed a global fit for the most relevant $t\bar{t}$ measurements, including a differential measurement on $m_{t\bar{t}}$, to which the four-fermion operators are sensitive. Unlike previous studies, our fit is done including all dim-6 squared terms as well as interference effects between all 14 dim-6 operators. We have also included the four-top operators in our analysis, and have demonstrated that marginalizing over these operators do not qualitatively change our conclusion. Compared with our $t\bar{t}$ global fit, the four-top process gives comparable limits on all operator coefficients. One should however keep in mind that these limits are still relatively conservative, given that the upper bound on the total cross section assumes SM signal shape, and is used regardless of the value of the $M_{cut}$. We expect that future experimental analyses following the SMEFT strategy will further improve the sensitivity of this process to SM deviations.

Finally, we would like to point out that potentially other processes can have a similar enhanced sensitivity, provided that the following conditions are satisfied: 1) there are multiple heavy particles in the final state, so that the process is naturally related with a large energy scale; 2) multiple insertion of dim-6 operators are allowed, and thus potentially leading to more powers of $CE^2/\Lambda^2$ enhancing the EFT contribution; and 3) the contribution of dim-6 operators goes like $E^2/\Lambda^2$, i.e. not suppressed by any mass or Higgs vev factors. Of course, the validity of EFT has to be checked carefully as one starts to approach the boundary of its applicability. Still, we hope that this study could inspire new ideas about using observables that are not so precisely measured, to further push the frontier of precision measurements in the EFT context.

Acknowledgements

CZ thanks Gauthier Durieux and Fabio Maltoni for their invaluable advice.

Appendix A: Operator basis

Here we present the relations between the coefficients of our four-fermion operators and those of the basis operators.
Appendix B: More results

We present constraints from four-top production and $t\bar{t}$ cross section measurements, similar to Figure 4 but for the operators in the 2nd and the 3rd categories, i.e. in Eqs. (17) and (18). They are displayed in Figure 8 and Figure 9, respectively.
Fig. 9. Constraints from four-top cross section and individual $t\bar{t}$ cross section measurements, on the operator coefficients in the third category ($\tilde{C}_{Qq}^{(1)}, \tilde{C}_{Qq}^{(1,1)}, \tilde{C}_{Qq}^{(1,3)}, \tilde{C}_{tq}^{(3)}, \tilde{C}_{tq}^{(1)}$), assuming two coefficients to be nonzero at a time.

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