New Implementation Framework for Saturation-Based Reasoning

Alexandre Riazanov
alexandre.riazanov@gmail.com
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Abstract. The saturation-based reasoning methods are among the most theoretically developed ones and are used by most of the state-of-the-art first-order logic reasoners. In the last decade there was a sharp increase in performance of such systems, which I attribute to the use of advanced calculi and the intensified research in implementation techniques. However, nowadays we are witnessing a slowdown in performance progress, which may be considered as a sign that the saturation-based technology is reaching its inherent limits. The position I am trying to put forward in this paper is that such scepticism is premature and a sharp improvement in performance may potentially be reached by adopting new architectural principles for saturation. The top-level algorithms and corresponding designs used in the state-of-the-art saturation-based theorem provers have (at least) two inherent drawbacks: the insufficient flexibility of the used inference selection mechanisms and the lack of means for intelligent prioritising of search directions. In this position paper I analyse these drawbacks and present two ideas on how they could be overcome. In particular, I propose a flexible low-cost high-precision mechanism for inference selection, intended to overcome problems associated with the currently used instances of clause selection-based procedures. I also outline a method for intelligent prioritising of search directions, based on probing the search space by exploring generalised search directions. I discuss some technical issues related to implementation of the proposed architectural principles and outline possible solutions.

1 Introduction

An automatic theorem prover for first-order logic (FOL) is a software system that can be used to show that some conjectures formulated in the language of FOL are implied by some theory. The expressiveness of FOL and its relative mechanisability make automated theorem proving in FOL a useful instrument for such applications as verification [5,4,1,6] and synthesis [19] of hardware and software, knowledge representation [18], Semantic Web [16], assisting human mathematicians [21,3], background reasoning in interactive theorem provers [23], and others.

This paper is concerned with the theorem proving method based on the concept of saturation. Given an input set of formulas, the prover tries to saturate it under all inferences in the inference system of the prover. In order to deal with syntactic objects which allow efficient calculi, the input set of formulas is usually converted into a set of formulas of a special form, called clauses. Demonstrating validity of a first-order formula is thereby reduced to demonstrating unsatisfiability of the corresponding set of clauses. The calculi working with clauses are usually designed in such a way that inferences can only produce clauses (see, e. g., [2,26]).

There are three possible outcomes of the saturation process on clauses: (1) an empty clause is derived, which means that the input set of clauses is unsatisfiable; (2) saturation terminates without producing an empty clause, in which case the input set of clauses is satisfiable (provided that a complete inference system is used); (3) the prover runs out of resources. The saturation method is well-studied theoretically (2,26).

1 Universally quantified disjuncts of literals. A literal is either an atomic formula (possibly depending on some variables), or a negation of such an atomic formula.
2 Sometimes problems coming from applications are already represented in the clausal form or require only minor transformation.
and is implemented in a significant number of modern provers, e.g., E [34], E-SETHEO (the E component), Gandalf [38], Otter [22], SNARK [36], Spass [39], Vampire [32][30], and Waldmeister [14].

In the last decade there has been a sharp increase in performance of such systems, which I attribute to the use of advanced calculi and inference systems (primarily, complete variants of resolution [2] and paramodulation [26] with ordering restrictions, and a number of compatible redundancy detection and simplification techniques), and intensified research on efficient implementation techniques, such as term indexing (see [12] and more recent survey [35]), heuristic methods for guiding proof search (see, e.g., [34]) and top-level saturation algorithms (see, e.g., [13] and [33]). Unfortunately, the initial momentum created by such work seems to have diminished, and nowadays we are witnessing a slowdown in performance progress. Some researchers consider this to be a sign that the saturation-based reasoning technology is reaching its inherent limits. The position I am trying to defend in this paper is that such scepticism is premature. My argumentation is based on a thesis that potential opportunities for a new breakthrough in performance have not been exhausted. Namely, the possibility of adopting new implementation frameworks for saturation, i.e., top-level designs and algorithms, has not been fully explored. To support this claim, I will pinpoint some major weaknesses in the organisation of proof search in the standard approaches to implementing saturation, and propose two concrete ideas on how to overcome these problems.

First, I will analyse some inherent problems with the standard procedures for saturation, based on the implementation of inference selection via clause selection. In particular, I consider the two main procedures, the OTTER algorithm and the DISCOUNT algorithm, based on clause selection. The main problem with the former procedure is the coarseness of inference selection, which translates into insufficient productivity of heuristics and restricts the choice of possible heuristics. The latter procedure implements very fine selection of inferences, but at a high cost in terms of computational resources. I will propose a new procedure based on a flexible high-precision inference selection mechanism with acceptable overhead. A concrete implementation scheme will be outlined.

Second, I will highlight the inadequacy of the popular approaches to prioritising proof search directions, based on syntactic characteristics of separate clauses. As a possible remedy, I propose a method for intelligent prioritising of search directions, based on probing the search space by exploring generalised search directions. I also propose a concrete implementation scheme for the method.

This criticism of the current state of affairs in the saturation architectures originates in my hands-on experience with implementing the saturation-based kernel of Vampire [32][30], and numerous experiments with the system. In fact, I consider the observations related to proof search effectiveness, on which this paper is based, the most valuable lessons learned from the Vampire kernel implementation. However, this paper is only a position paper. As such, it does not present any complete results, either theoretical or experimental. Its aim is to provide a basis and an inspiration for new implementations and experiments.

The rest of this paper is structured as follows. Each of the remaining two sections introduces a new architectural principle. In the beginning of the chapter the relevant aspects of the state-of-the-art designs are criticised. Then, ideas of a possible remedy are formulated, followed by a discussion of related work and a tentative research programme.

Concluding this introduction, I would like to ask the reader to be tolerant to some presentational problems with this text. I am trying to keep this paper informative for experts in the implementation of saturation-

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3 A good benchmark is Otter which has not changed much since 1996. Compare its relative performance in CASC-13 (http://www.cs.miami.edu/~tptp/CASC/13/) and CASC-20 (http://www.cs.miami.edu/~tptp/CASC/20/).

4 Compare the performance of the best provers in CASC-20 (http://www.cs.miami.edu/~tptp/CASC/20/) with the previous year winners.
based provers and, at the same time, acceptable for a superficial reading by a broader audience. Some negative consequences of such conflict of intentions seem to be inevitable.

2 General preliminaries

For the sake of self-containedness, I will reproduce a number of standard definitions here.

I am assuming that the reader is familiar with the syntax and semantics of first-order predicate logic with equality. In what follows, ordinary predicate symbols will be denoted by \( p, q \) and \( r \), the equality predicate will be denoted by \( \sim \), function symbols will be denoted by \( f, g \) and \( h \), individual constants will be denoted by \( a, b \) and \( c \), variables will be denoted by \( x, y \) and \( z \), possibly with subscripts, and the letters \( s \) and \( t \), possibly with subscripts, will denote terms.

We are mostly interested in a special kind of first-order formulas called clauses. A clause is a disjunction \( L_1 \lor \ldots \lor L_n \), where all \( L_i \) are literals, i.e. atoms (positive literals) or negated atoms (negative literals). The order of the literals in a clause is usually irrelevant, so I will often refer to clauses as finite multisets of literals. The empty multiset of literals will also be considered a clause which is false in any interpretation.

Substitutions are total functions that map variables to terms. They will be denoted by \( \theta \) and \( \sigma \), possibly with subscripts. Substitution application is extended to complex expressions, such as terms, atoms, literals and clauses, in an obvious way: if \( E \) is an expression, \( E\theta \) is obtained by replacing each variable \( x \) in \( E \) by \( x\theta \). A substitution \( \theta \) is a unifier for two expressions \( E_1 \) and \( E_2 \) if \( E_1\theta = E_2\theta \). It is the most general unifier, if for any other unifier \( \theta_1 \), there exists a substitution \( \theta_2 \), such that \( E_1\theta_1 = (E_1\theta)\theta_2 \).

We will say that a clause \( C \) subsumes clause \( D \) if there is a substitution \( \theta \) such that (the multiset of literals) \( C\theta \) is a submultiset of \( D \).

We are interested in implementation of calculi based on resolution and paramodulation (see, e.g., [2,26]). (Unrestricted) binary resolution is the following deduction rule:

\[
\frac{C \lor A \quad D \lor \neg B}{(C \lor D)\theta}
\]

where \( \theta \) is the most general unifier of the atoms \( A \) and \( B \).

Paramodulation is the following rule:

\[
\frac{C \lor s \sim t \quad D[u]}{(C \lor D[u])\theta}
\]

where \( \theta \) is the most general unifier of the terms \( s \) and \( u \), and \( u \) is not a variable.

A resolution-based reasoner usually applies restricted variants of these rules together with some auxiliary rules to demonstrate unsatisfiability of an input clause set by deriving an empty clause from it. Such derivations are called refutations of the corresponding clause sets.

Saturation-based reasoners are called so because of the way they search for refutations. In an attempt to derive an empty clause, a reasoner tries to saturate the initial set with all clauses derivable from it. Roughly speaking, at some steps of the saturation process the reasoner selects a possible inference between some clauses in the current clause set, applies the inference and adds the resulting clause to the current clause set. Other steps of the process usually prune the search space by removing redundant clauses, i.e. clauses that are not strictly necessary to find a refutation. For details on the concept of saturation modulo redundancy, the reader is referred to [2].
3 Fine inference selection at affordable cost

3.1 Background: inference selection via clause selection

When one has to search in an indefinitely large space, the ability to explore more promising search directions before the less promising ones is a key to success. In saturation-based reasoning the mechanism responsible for deciding on which direction to promote first is known as inference selection. Ideally, the inference selection should be able to name one single inference to be deployed at every step of saturation, and the decision should be based on the (heuristically evaluated) quality of the resulting clause. In practice, most of the working saturation-based systems adopt a simpler but coarser mechanism known as clause selection. Instead of selecting a single inference at a time, we select a clause and oblige to deploy immediately all possible inferences between the clause and all active (previously selected) clauses. Clauses of better heuristically evaluated quality are given higher priority for selection, in the hope that they will produce heuristically good inferences. The algorithm realising inference selection via clause selection is known as given-clause algorithm. Its variants have been used in provers since as early as 1974 [27] (see also [20]), although its current monopoly seems to be mostly due to the success of Otter [22]. Other provers based on variants of given-clause algorithm include E, Gandalf, SNARK, Spass, Vampire and Waldmeister, i.e., practically all modern saturation-based systems.

In order to illustrate the main idea behind the given-clause algorithm, namely the implementation of inference selection via clause selection, it is sufficient to consider only deduction inferences. So, the algorithm presented in Figure 1 performs no simplification steps.

\[
\text{procedure GivenClause(input : set of clauses)} \\
\text{var new, passive, active : sets of clauses} \\
\text{var current : clause} \\
\text{active := \emptyset} \\
\text{passive := input} \\
\text{while passive \neq \emptyset do} \\
\quad \text{current := select(passive)} \\
\quad \text{passive := passive \setminus \{current\}} \\
\quad \text{active := active \cup \{current\}} \\
\quad \text{new := infer(current, active)} \\
\quad \text{if new contains empty clause} \\
\qquad \text{then return refutable} \\
\qquad \text{passive := passive \cup new} \\
\text{od} \\
\text{return failure to refute}
\]

**Fig. 1.** Given-clause algorithm (without simplifications)

It is also convenient to represent the algorithm with a more abstract dataflow diagram as in Figure 2. In this picture, the boxes denote operations performed on clauses. The rounded boxes denote sets of clauses. The shallow ones correspond to the sets that typically contain very few clauses, while the deep ones correspond to the sets that can grow large. The arrows reflect the information flow for different operations. Arrows labeled with the same number belong to the same operation/processing phase. In Figure 2 label 1 corresponds to the line \text{passive := input} in the pseudocode from Figure 1 phase 2 is clause selection \text{(current := select(passive) and passive := passive \setminus \{current\})}, 3 corresponds
to \( \text{active} := \text{active} \cup \{\text{current}\} \), 4 is the generation of deduction inferences between \( \text{current} \) and \( \text{active} \) (\( \text{new} := \text{infer}(\text{current}, \text{active}) \)), and 5 is the integration of newly derived clauses into \( \text{passive} \) (\( \text{passive} := \text{passive} \cup \text{new} \)). The thin solid arrows show the movements of clauses between clause sets and from operations to the sets. A dashed arrow from a set to an operation indicates that the operation depends on the clauses from the set.

My experience with Vampire and, to some extent, with other provers allows me to see a number of soft spots of the given-clause algorithm:

- The selection is based on the properties of eligible clauses, which are only vaguely related to the properties of the enabled inferences.

   A “good” clause may interact with many previously selected “not-so-good” clauses and produce many “not-so-good” inferences. The set of selected clauses often contains such heuristically bad clauses for a number of reasons. In particular, we cannot completely avoid selecting bad clauses because in general it leads to incompleteness. Moreover, in practice we often cannot even significantly restrict the selection of heuristically bad clauses since such strategy easily leads to loss of solutions (in the practical sense, i.e., solutions that can be obtained with given resources). Another reason why bad clauses get into the set of selected clauses is the relativity of the heuristic estimation of clause quality: a clause selected as relatively good in the beginning of the proof search, can become relatively bad later if many better clauses have been derived.

   Another problem with clause property-based selection is that even two “good” clauses can easily have “not-so-good” inferences between them. This happens when the clause quality criteria do not sufficiently penalise clauses containing “bad” parts available for inferences. If our quality criteria are too strict with respect to clauses with “bad” parts, the prover also postpones the inferences involving “good” parts of such clauses.

- The newly selected clause may, and often does, interact with very many parts of very many active clauses.

   This often leads to pathological situations of the following kind: a prolific clause is selected and the processing of inferences between this clause and many active ones takes all available time, whereas a few inferences with other clauses would lead to a solution.

In sum, the coarseness of the clause selection principle deprives us of control over the proof search process to a great extent, which translates into poor productivity of heuristics, restricts the choice of heuristics that can be implemented, and leads to littering the search state with too many “undesirable” clauses.
There are two main variants of the given-clause algorithm: the Otter algorithm\(^5\) and the DISCOUNT algorithm\(^6\), which differ in the way the passive (waiting to be selected) clauses are treated.

![Fig. 3. Otter algorithm](image)

In the Otter algorithm, presented as a dataflow diagram in Figure\(^3\), the passive clauses are subject to simplification by the newly derived clauses, can be discarded as redundant with the help of the newly derived clauses, and themselves can be used to simplify/discard the newly derived clauses. Newly derived clauses are subject to forward simplification which may transform them or even discard them completely.\(^7\) Note that in the Otter algorithm forward simplification uses both passive and active clauses as simplifiers (see the dashed arrows labeled with 5 in the diagram). Backward simplification also affects passive clauses as well as the active ones (see the broad arrows labeled with 7).

In the DISCOUNT algorithm (see Figure\(^4\)), only active clauses can be simplified/discarded, or used to simplify/discard new clauses. So, there are no dashed lines between the box passive and the forward and backward simplification boxes. Note also that the clause in current is subject to forward simplification (arrows labeled with 3), and it is used to simplify the active clauses (arrows labeled with 4). This is done to keep the set of active clauses as simple as possible.

In the DISCOUNT algorithm passive clauses are constructed practically exclusively for evaluation of their properties which have to be known for controlling the inference selection. One may argue that the set of passive clauses is just a representation of all (potentially non-redundant) one-step inferences from the active clauses, and from this point of view the DISCOUNT algorithm implements the idealistic notion of inference selection described in the beginning of Section 3.1. In other words, the DISCOUNT algorithm allows the prover to observe the space of all possible one-step inferences between active clauses, which is a good thing by itself. However, the algorithm also obliges the system to do so by explicitly making all such inferences and storing the resulting clauses as passive. The cost of good inference selection becomes very high. Typically, a thousand active clauses may generate hundreds of thousands inferences, and a great

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\(^5\) Implemented, in particular, in Gandalf, Otter, SNARK, Spass and Vampire.

\(^6\) Implemented, in particular, in E, Vampire and Waldmeister.

\(^7\) In diagrams on Figures\(^2\) and\(^4\) a broad arrow from an operation to a set indicates that the operation modifies the set by removing or replacing some clauses.

\(^8\) Simplicity here is, of course, relative to the features of the used inference system, in particular, the redundancy criteria.
deal of the resulting clauses may be non-redundant with respect to the active ones, and, as such, have to be stored as passive. Since the passive clauses are not used for anything but selection, the work spent on constructing a clause may be frozen for a long time, while the clause remains passive, and this work is lost if the prover exhausts a given time or memory limit and terminates. Storing huge numbers of passive clauses may additionally require a lot of memory.

The Otter algorithm is not completely immune from any of these problems too. In addition, the cost of simplification operations grows with the growth of the set of passive clauses.

Recently there have been (at least) two attempts to address some of these issues. Vampire implements the Limited Resource Strategy [33], which is intended to minimise the amount of work on generating, processing and keeping passive clauses in the Otter algorithm, which is wasted when the time limit is reached. This is done by discarding some non-redundant but heuristically bad clauses and inferences. Waldmeister implements a sophisticated scheme to reduce the memory requirements by the DISCOUNT algorithm [13,8]. In both cases, the adjustments of the top level algorithms led to a great improvement in the effectiveness of the systems. This gives me hope that a radically different approach to inference selection may result in a real performance breakthrough.

3.2 Finer selection units with graded activeness

**Finer selection units.** The inherent problems with the given-clause algorithm motivated me to look for a scheme that can facilitate better control of search at an affordable cost. Instead of selecting clauses, we are going to select some particular parts (literals or subterms) of clauses and make them available for some particular kinds of inferences. Such triples (clause + clause part + inference rule) will be the new selection units. This will help us to avoid premature invocation of less promising clause parts.

Stronger heuristics become available for evaluating the quality of selection units since such evaluation can take into account more than just integral characteristics of a whole clause. For example, a selection unit with a generally good clause, but with a bad literal or subterm intended for a prolific inference rule may now be given a low priority. On the one hand, this allows us to delay inferences with a bad part of the clause. On the other hand, we don’t have to delay all inferences with the clause simply because one of its parts is bad.

As an illustration, consider the unit clause $p(f(x, y), f(a, b))$. If some form of paramodulation is allowed, the subterm $f(x, y)$ is available for paramodulation into. This selection unit is extremely prolific.
since \( f(x, y) \) unifies with all terms starting with \( f \), and it makes a good sense to delay paramodulations into this term without postponing other inferences with the clause, e.g., paramodulations into \( f(a, b) \).

Another example of a highly promising heuristic which is enabled by the proposed approach, is to give higher priority to binary resolution than to paramodulation, since the latter is often much more prolific than the former. This heuristic already works very well (at least) in Vampire. The prover never enables inferences with positive equalities in a clause if there are literals of other kinds. Although generally successful, this strategy often fails if all the other literals are relatively bad, e.g., if they can generate many inferences. Consider the clause \( p(x, y) \lor q(a, b) \lor f(a, b) \simeq a \). The literal \( p(x, y) \) is likely to be more prolific than \( f(a, b) \simeq a \), since \( p(x, y) \) unifies with any atom starting with \( p \). The proposed scheme allows us to give very high priority to \( q(a, b) \), lower priority to \( f(a, b) \simeq a \) since this is a positive equality, and a very low priority to the overly prolific literal \( p(x, y) \).

Also, simplification inferences and redundancy tests can be treated in the same way as deduction inferences. In the example above, we could make the term \( f(a, b) \) available for rewriting immediately, and postpone the integration of \( f(x, y) \) into the corresponding indexes until much later. By delaying simplification inferences on stored clauses in a controlled manner we can achieve behaviours combining the properties of the Otter and DISCOUNT algorithms. If simplification inferences are given higher priority, the behaviour of our procedure will be closer to that of the Otter algorithm. If simplification inferences have priority comparable to the priority of deduction inferences such as resolution and paramodulation, we can expect the new procedure to behave similar to the DISCOUNT algorithm.

**Graded activeness.** Apart from changing the subject of selection, I propose to change the notion of selection itself. The given-clause algorithm divides the search state in two parts. One part contains active clauses, and the other one contains passive clauses that are not yet available for deduction inferences. If a clause gets into the active set, it becomes available for all future inferences regardless of its quality. To overcome this problem, I propose to use finer gradation of selection unit activeness.

Intuitively, all selection units would become potentially available for inferences almost immediately, but some would be “more available” than the others. **Less active selection units would be available for inferences with more active ones.** High degree of activeness of a selection unit would indicate higher priority of this unit for proof search. In the new procedure, the units containing parts of newly generated clauses initially receive the minimal degree of activeness, but later are gradually promoted to higher degrees of activeness. When a promotion step takes place, the selection unit becomes available for new inferences with some units which have not been eligible so far due to insufficient activeness. To give higher priority to inferences with heuristically better selection units, the promotion frequency for different units should vary according to their quality. Thus, we will be able to delay inferences between heuristically bad inference units.

To illustrate this rather general scheme, I will outline a simple implementation scheme. For this implementation the nature of the used selection units is irrelevant, i.e., they can be clauses as well as the finer selection units proposed above. However, the implementation relies on the assumption that the quality of selected units is reflected by a special real-valued coefficient which takes positive values.

If \( v \) is a selection unit, the corresponding coefficient will be denoted as \( \text{quality}(v) \). The intuitive meaning of the quality coefficient is the relative frequency of promotion. If \( v_1 \) and \( v_2 \) are two selection units, at each promotion step the probability of selecting \( v_1 \) for promotion relates to the probability of selection \( v_2 \) as \( \text{quality}(v_1) \) relates to \( \text{quality}(v_1) \). Practically, we can select units for promotion randomly according to the distribution explicitly specified with the quality coefficients. This selection discipline is known in the area of Genetic Algorithms as roulette-wheel selection \([11]\).

To realise the idea of graded activeness, I propose to partition the set of all available selection units into \( n + 1 \) sets \( Y_0, Y_1, \ldots, Y_n \). The indexes of the sets reflect the activeness of selection units contained in...
them: units in $\mathcal{Y}_{i+1}$ are more active than units in $\mathcal{Y}_i$. More specifically, for $i > 0$, $v \in \mathcal{Y}_i$ implies that all possible inferences between $v$ and units from $\mathcal{Y}_{n-i+1}, \ldots, \mathcal{Y}_n$ have been made and no inferences between $v$ and units in $\mathcal{Y}_0, \ldots, \mathcal{Y}_{n-i}$ have been considered yet. $\mathcal{Y}_0$ contains absolutely passive selection units, i.e. units that have not participated in any inferences yet. This invariant, illustrated in Figure 5, is maintained by the following procedure. As soon as some selection unit is constructed, it is placed in $\mathcal{Y}_0$. At each macrostep of the procedure some selection unit $v$ is selected for promotion as outlined above. If $v$ happens to be in $\mathcal{Y}_i$, where $i < n$, its promotion means that $v$ is removed from $\mathcal{Y}_i$, all possible inferences between $v$ and selection units from $\mathcal{Y}_{n-i}$ are made, and $v$ is placed in $\mathcal{Y}_{i+1}$. Selection units from $\mathcal{Y}_n$ are not promoted, they have the maximal activeness.

![Fig. 5. Graded activeness implementation](image)

Special arrangements may have to be made if we admit selection units that need not interact with other selection units to produce inferences. For example, we may decide that selection units intended for binary factoring ([2]) with a particular literal in a particular clause do not need a counterpart unit, i.e. if we decide to deploy such a unit, we will have to make all possible factoring inferences with the specified literal within the specified clause. One possibility of dealing with such selection units is to designate some activeness $i > 0$ as a threshold, so that when a selection unit reaches $\mathcal{Y}_i$, all inferences requiring this unit alone are immediately made.

As a whole, the proposed inference selection scheme allows for much better control over inference selection which may translate into higher productivity of heuristics and enables the use of new heuristics which could not be used with the given-clause algorithm. Apart from other things, the extra flexibility of inference selection will enhance the diversity of available strategies. These advantages come at an affordable cost. The only involved overhead, caused by the need to store large numbers of selection units, is compensated by lower numbers of heuristically bad clauses which have to be created and stored only to maintain completeness.

I would like to add one final consideration here. The calculi used in the state-of-the-art saturation-based provers are designed with the aim of reducing search space. Partially, they do this by restricting the applicability of resolution and paramodulation rules. Often this is done by prohibiting inferences with

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9 The proposed design is strictly more flexible than the standard ones since it is possible to implement it in such a way that both the Otter and DISCOUNT algorithms can be simulated by appropriate parameter settings.

10 My experience suggests that this is a very important factor as in 2002–2005 the multitude of strategies supported by the Vampire kernel has been a major, if not the main, contributor to the growth of performance of the whole system.
certain parts of clauses. For example, ordered resolution with literal selection (see [2]) prohibits resolving non-maximal positive literals. However, restricting the shape of eligible derivations also means restricting the number of eligible solutions, and *simple solutions are often thrown away* if they do not satisfy the restrictions.

It is possible, in principle, to relax the restrictions by allowing some redundant inferences with some heuristically good parts of clauses. For example, we may want to resolve large (i.e., containing many symbols) positive non-maximal literals with the aim of obtaining smaller resolvents. However, adjusting prover architectures based on the standard variants of the given-clause algorithm makes very desirable the introduction of new ad hoc mechanisms for regulating the *proportion of redundant and non-redundant inferences*. The scheme proposed in this paper seems to have sufficient flexibility to accommodate such control mechanisms for free. For example, we can allow selection units with large positive non-maximal literals, and assign to them higher quality measure than to non-redundant selection units, if we are eager to derive small clauses earlier. If we choose to be more conservative and want to avoid most redundant inferences except a small number of very heuristically promising ones, we can always assign higher quality measure to non-redundant selection units.

### 3.3 Methodological considerations

The proposed scheme for finer inference selection is completely compatible with the modern theory of resolution and paramodulation, and requires no theoretical analysis. The difficult part of the job is to find an adequate design and to do the actual implementation.

One of the implementation options is to adjust an existing system. To investigate this possibility, I looked through the code of the kernel of Vampire, v7.0, with the purpose of estimating the amount of work required to adjust it to the new scheme. This investigation has convinced me that at least one third of the code would have to be rewritten completely, and at least another third of it would have to be heavily adjusted to accommodate the new code. This is hardly surprising, taking into account that the proposed changes target the top level design as well as some key data representations and some mid-level functionality such as indexing.

The main conclusion of my inspection of the Vampire kernel code is that the amount of work required for a transition to the new scheme is likely to exceed the cost of creating a rather advanced brand new prototype. An implementation from scratch can also be better tailored to the new design. Considering this additional advantage, my preference is clear. However, I do not dismiss the possibility of implementing the new scheme on the base of other advanced saturation-based provers.

The nature of the proposed architectural principles is such that their advantages can only be fully demonstrated if a significant effort is invested in design and assessment of search heuristics. Indeed, the main advantages of the new inference selection approach are the higher productivity of existing heuristics and the possibility of using new heuristics.

This extra flexibility in directing proof search can only be fully exploited by means of *tuning*. Therefore, very extensive experimentation will be necessary to find generally good combinations of parameters of heuristics, as well as strategies specialised for important classes of problems. Strong tuning infrastructure to support such experimentation seems highly desirable. Developing such an infrastructure may be itself an interesting research problem.

Finally, I would like to add a note about term indexing. The finer gradation of activeness divides clauses and their parts into many logically separate sets. An initial implementation may adapt existing techniques

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11 For experiments one can use the TPTP library [37], which is at the moment the largest and most diverse collection of first-order proof problems. It would also be very useful to look at more specialised large problem sets coming from applications in order to demonstrate the tunability of the proposed architecture.
to index these sets separately. However, better specialised indexing solutions may exist, and, if the proposed
design proves viable in the initial experiments, it may give rise to a new line of research in term indexing.

4 Generalisation-based prioritising of search directions

4.1 Background: local syntactic relevancy estimation

Blind search in indefinitely large spaces is usually not effective enough for most applications, so, all modern
saturation-based provers try to predict the relevancy of particular search directions by using various
heuristics. In a saturation process state, the available search directions are identified by the accumulated
clauses (e. g., the contents of the sets passive and active in the pick-given clause algorithm presented in
Figure[1]). The most common heuristics prioritise search directions by giving some clauses higher priority
for participating in inferences, than the others. The estimation of relevancy of a clause is based on such character-istics of the clause as its structural complexity (e.g., simpler clauses get higher priority) or its potential
for participating in inferences (e.g., very prolific clauses get very low priority).

Such approaches have natural limitations. The syntactic characteristics of a clause, used in the estimation,
often fail to reflect the usefulness of the clause adequately. For example, a structurally complex clause
may be absolutely indispensable for any solution of the problem at hand, but it will be suspended for a long
time. Another problem is that the estimation is done locally, i.e. only one clause is analysed and global properties of the current search state are not taken into account. For example, an absolutely irrelevant clause, i.e.,
participating in no minimal unsatisfiable subset of the current clause set, may be given high priority because of its simplicity.

4.2 Generalisation-based prioritising of proof-search directions

To address the issues raised above, I propose a method for intelligent prioritising of search directions. The idea is as follows. We will estimate the potential of a clause to participate in solutions of the whole problem at hand by interacting with other currently available clauses. Precise estimation is impossible since it would require finding all, or at least some, solutions of the problem, so we are looking for a good approximation.

General method. I suggest to probe the search space by exploring a substantially simpler search space. The latter is obtained from the former by generalising some search directions. This is done by replacing (preferably large) clusters of similar clauses with their common generalisations. If we find a solution of the simplified problem, which involves the generalisation of a particular cluster, this is a good indication that at least some of the clauses in the cluster can be relevant. More importantly, the clauses whose generalisations have not yet proved useful, can be suspended as potentially irrelevant. Additionally, the closer a resolved generalisation is to a particular clause in its cluster, the better chances the clause has to participate in a solution and the bigger priority it should be given.

Generalisations can be defined semantically: a clause \( C \) can be called a generalisation of clause \( D \) if \( C \) logically implies \( D \). For our purposes, however, it is convenient to use a simpler, syntactically defined notion of generalisation, based on subsumption. In what follows, we will call \( C \) a generalisation of \( D \) if \( C \) subsumes \( D \), i.e. \( C \theta \subseteq D \)[12], where \( C \theta \) and \( D \) are viewed as sets of literals.

Implementation with naming and folding. Technically, the general approach described above can be realised by means of a combination of dynamic naming and folding. This combination is called decomposition rule in [17], but for the purposes of this paper it is convenient to consider the rules separately.

[12] More restrictive multiset-based variant of subsumption, where \( C \theta \) is required to be a submultiset of \( D \), can also be used.
The idea should be clear from the following example. Suppose we have a clause $C_1 = p(f(a, b)) \lor p(g(b, a)) \lor q(a)$. We decide that this clause is too specific and its generalisation $\Gamma_1(x_1, x_2) = p(f(x_1, x_2)) \lor p(g(x_2, x_1))$ should be explored first. To this end, we introduce a new binary (according to the number of variables in $\Gamma_1$) predicate $\gamma_1$ and make it the name for $\Gamma_1$. Logically, this can be viewed as introduction of the definition $\forall x_1, x_2. \gamma_1(x_1, x_2) \Leftarrow \Gamma_1(x_1, x_2)$. We immediately transform $C_1$ by folding this definition into the following clause $C'_1 = \gamma(a, b) \lor q(a)$. Moreover, if there are other clauses, currently stored or derived in the future, which are instances of the generalisation $\Gamma_1$, we can apply folding to them as well, thus recognising that the clauses are covered by the generalisation $\Gamma_1$. For example, if the clause $C_2 = p(f(h(a), b)) \lor p(g(b, h(a))) \lor r(b)$ is derived, it will be replaced by the clause $C'_2 = \gamma_1(h(a), b) \lor r(b)$. The generalisation $\Gamma_1$ is injected into the search space in the form of the clause $\Gamma_1(x_1, x_2) \lor \neg \gamma_1(x_1, x_2)$, which is a logical consequence of the definition for $\gamma_1$.

In order to obtain the behaviour prescribed by the general scheme, clauses containing $\gamma$-predicates (i.e., predicates which are generalisation names) are given special treatment. Namely, if a clause contains negatively the name $\gamma_1$ for the generalisation $\Gamma_1$, it means that the clause was derived from the clause $\Gamma_1(x_1, x_2) \lor \neg \gamma_1(x_1, x_2)$ representing the generalisation $\Gamma_1$ in the search space. In such clauses, we prohibit all inferences involving negative $\gamma$-literals (i.e., literals with $\gamma$-predicates) if there is at least one literal of a different kind. Roughly, in the clause $\Gamma_1(x_1, x_2) \lor \neg \gamma_1(x_1, x_2)$ we want to resolve the generalisation part $\Gamma_1(x_1, x_2)$ before we touch the literal $\neg \gamma_1(x_1, x_2)$. Until this happens, the literal $\neg \gamma_1(x_1, x_2)$ only accumulates the substitution which solves $\Gamma_1(x_1, x_2)$.

When a clause containing only negative literals with generalisation names is derived, this indicates that some generalisations “fired”, i.e. they contradict each other and some ordinary input clauses. We will call such clauses $\gamma$-contradictions and their inferences $\gamma$-refutations.

Clauses containing positive $\gamma$-literals are suspended (temporarily removed from the search state) until all of the corresponding generalisations have proved useful, i.e. every participating $\gamma$-predicate belongs to at least one $\gamma$-contradiction. When we can no longer suspend such a clause, we still block any inferences involving its non-$\gamma$ literals. A resolution inference between such a clause and a $\gamma$-contradiction indicates that the clause is compatible with the corresponding $\gamma$-refutation, and it represents an attempt to (gradually) refine the $\gamma$-refutation into a solution for the original problem. If some form of paramodulation is used, we have to allow paramodulation into the positive $\gamma$-literals in an attempt to make them compatible with available $\gamma$-contradictions.

To illustrate this, I continue the example. Suppose we have derived the $\gamma$-contradiction $\neg \gamma_1(a, b)$. The clauses $C'_1 = \gamma(a, b) \lor q(a)$ and $C'_2 = \gamma_1(h(a), b) \lor r(b)$ can no longer be suspended. The clause $C'_1$ is directly compatible with the $\gamma$-refutation, which results in a derivation of the clause $q(a)$. The clause $C'_2$ is not compatible with the $\gamma$-refutation since $\gamma_1(h(a), b)$ is not unifiable with $\gamma_1(a, b)$. However, in presence of the unit equality clause $h(a) \simeq a$, we can rewrite $C'_2$ into $\gamma_1(a, b) \lor r(b)$, which is compatible with the $\gamma$-refutation, and then derive $r(b)$. Note, that the work spent on refuting the generalisation $\Gamma_1$ (modulo some ordinary input clauses) is utilised: we do not repeat the same inferences with the generalised literals from the original clause $C_1$. Moreover, the results of this work are shared with another clause – $C_2$, and, potentially, with many other clauses covered by the generalisation $\Gamma_1$. Such sharing of work on similar parts of potentially very many different clauses can be an additional advantage.

Note that the proposed naming- and folding-based scheme is rather flexible. It allows many variants which may differ, e.g., in the way suspended clauses are treated, how selection of inferences is done with the $\gamma$-literals, how generalisations are chosen, how many generalisations can be applied to a single clause.
and whether they can be overlapping\(^\text{[13]}\) etc. The description above is only intended to provide a general framework for formulating such variants. Moreover, it is obviously not the only possible framework for implementing the general scheme presented in the beginning of this section.

The proposed implementation scheme offers another advantage for free. The user gets an additional means of controlling proof search by specifying in the input which clauses he would like to make named generalisations from the start. This can be viewed as a way of hinting at useful lemmas (of a restricted kind since only clauses are named rather than arbitrary formulas) or suppressing search directions which do not seem promising to the user.

For example, by analysing some previous proof attempts the user may conclude that many clauses of the form \(\neg p(g(a, b)) \lor C\) are generated. If the user has reasons to believe that the literal \(\neg p(g(a, b))\) can be solved, i.e. \(p(g(a, b))\) is logically implied by the input clauses, he may want to try proving \(p(g(a, b))\) as a lemma, and later use it to resolve with the clauses \(\neg p(g(a, b)) \lor C\). Practically, this can be done by making \(\neg p(g(a, b))\) a generalisation and giving it some name, e.g. \(\gamma_3\). Refuting \(\neg p(g(a, b)) \lor \neg \gamma_3\) corresponds to proving the lemma \(p(g(a, b))\), and resolutions between the \(\gamma\)-contradiction \(\neg \gamma_3\) and clauses of the form \(\gamma_3 \lor C\) correspond to applications of the lemma. Such lemma hinting may be beneficial because it allows to share the work on solving literals \(\neg p(g(a, b))\) in many different clauses instead of solving them separately.

If the user has reasons to believe that the literal \(\neg p(g(a, b))\) cannot be solved, and thus all the clauses \(\neg p(g(a, b)) \lor C\) are redundant, it still makes sense to make \(\neg p(g(a, b))\) a generalisation. This will keep the generalised clauses \(\neg p(g(a, b)) \lor C\) away from inferences without completely discarding them. Only if the user’s intuition was incorrect, i.e. \(\neg p(g(a, b))\) can actually be solved, the generalised clauses are reintroduced in the search space.

4.3 Related work

Static relevancy prediction. My original idea was to use some sort of clause abstractions for dynamic suppressing of potentially irrelevant search directions in the framework of saturation-based reasoning. This idea was inspired by \([7]\) where the authors propose to use various clause abstractions for statically identifying input clauses which are practically irrelevant, i.e. can not be useful in a proof attempt of acceptable complexity. Roughly, this is done by applying abstractions to an input clause set, exploring the space of all proofs of restricted complexity with the abstracted clause set, and throwing away the input clauses whose abstractions do not participate in any of the obtained proofs with the abstracted set.

Iterative generalisation-refinement. Some time ago \([29]\) drew my attention to the simplest kind of clause abstractions – generalisations, which seems convenient for our purposes. The method works roughly as follows. A resolution prover is parameterised by a generalisation function on clauses, i.e. a function which computes several, possibly overlapping, generalisations for a given clause. When the prover is run on a problem, the generalisation mechanism replaces suitable clauses by their generalisations. The whole scheme works as iteration through levels of generalisation strength. First, the prover is run with a strong generalisation function to enumerate all refutations with depth below a certain limit. Then the generalisation function is weakened\(^\text{[14]}\) and the prover uses the previously found refutations to guide the enumeration of refutations with the new generalisation function. The key idea is that the refutations with the weaker generalisation function

\(^{13}\) Intuitively, two generalisations of a clause \(C\) overlap if they cover some common literals in \(C\). For example, \(C_1 = p(f(a, b)) \lor p(g(b, a)) \lor q(a)\) has overlapping generalisations \(p(f(x_1, x_2)) \lor p(g(x_2, x_1))\) and \(p(g(x_1, x_2)) \lor q(x_2)\) because the literals \(p(g(x_2, x_1))\) and \(p(g(x_2, x_1))\) both generalise the literal \(p(g(b, a))\).

\(^{14}\) Roughly, a weaker generalisation function produces more specific generalisations of a given clause.
are in a certain (strict) sense refinements of the refutations obtained with the stronger generalisation function. Such refinement is performed repeatedly and at some point the prover tries to refine a refutation from the previous step into a refutation which uses no generalisation.

**Octopus approach.** The Octopus system \([25]\) runs a large number of sessions of the prover Theo \([24]\) distributed over a cluster of computers. Each Theo session first runs on a *weakening* of the original problem, obtained by *replacing one of the clauses with one of its generalisations*. If one of the sessions succeeds in solving the weakened problem, the solution is used to direct the search for a solution of the original problem in two ways:

- The unmodified clauses from the original problem formulation, which participate in the solution of the weakened problem, are considered to be heuristically more relevant. In the future searches for solutions of the original problem, these clauses are given higher priority.
- Some clauses in the obtained refutation of the generalised clause set, which were derived from unmodified clauses, are added as lemmas to the problem formulation.

The main difference between my approach and the static relevancy prediction approach of \([7]\), and also the Octopus approach \([25]\), is that our clause generalisations are introduced *dynamically*, and can be used on derived clauses. This allows a good degree of adaptivity.

My approach is closer to, and can be viewed as an attempt to revive the line of work presented in \([29]\). I hope to improve on this approach mainly by enumerating generalised refutations lazily, thus *avoiding any artificial limits on the complexity of refutations* and the need to enumerate a whole, potentially large, set of generalised refutations before we try to use these refutations. Also, my approach is more *semantic* in its nature since we do not try to refine generalised refutations by following their structure. We are interested in *existence* of \(\gamma\)-refutations rather than their shape. This allows much easier integration with various variants of resolution- and superposition-based inference systems. Additionally, my approach imposes no restrictions on how the generalisation functions are specified and implemented. In particular, the generalisation mechanism can be adaptive. For example, the strength of generalisation may depend on various properties of clauses being generalised, or even on some global properties of the current search state.

The general method is also partially inspired by, and shares some philosophical ideas with \([28]\) and \([10]\). The use of naming and folding is a natural continuation of our joint work with Andrei Voronkov on implementing *splitting without backtracking* \([31]\) and also partially stems from an unfinished attempt by the author to mimic *tableaux without backtracking* \([9]\) in the context of saturation. Recently I have discovered that \([17]\) proposes to use exactly the same combination of naming and folding, under the name of *decomposition rule*, for deciding two description logics and query answering in one of them.

**Semantic guidance in the style of SCOTT.** To conclude the overview of relevant work, I would like to mention another approach which is technically unrelated to the one proposed here, but which also provides an alternative to local syntactic relevancy estimation.

The *semantic guidance* approach, developed within the SCOTT project \([15]\), is roughly as follows. The prover tries to establish satisfiability of several sets of stored clauses (in SCOTT this is done with the help of an external model builder). Ideally, these sets must approximate their maximal satisfiable supersets as closely as possible. The sets are used for guiding clause selection roughly as follows: clauses participating in fewer such satisfiable sets are given higher priority for selection. The intuition behind this approach is that a clause is more likely to be redundant if it participates in many satisfiable sets. This heuristic is supported by the fact that if a clause is in every maximal consistent subset, then it is definitely redundant.
The applicability of the semantic guidance approach seems limited because it relies on the costly operation of establishing satisfiability of large clause sets. This overhead may be acceptable in solving very hard problems when the user can afford to run a prover for hours or even days. Many applications, however, require solving large numbers of simpler problems and much quicker response. I hope that generalisation-based guidance can be more useful for this kind of applications because the associated overhead seems more manageable due to the flexibility of generalisation function choice. Anyway, a meaningful comparison of the two approaches can only be done experimentally, when at least one variant of the generalisation-based method is implemented.

4.4 Methodological considerations

Certain theoretical effort is required to formulate the method in full detail. It makes sense to consider a number of variants of the method and try to predict their strengths and weaknesses. It is also essential to have a clear picture of how the proposed use of generalisations will interact with the popular inference systems based on resolution, paramodulation and standard simplification techniques. In particular, it is necessary to consider the search completeness issues.

The effectiveness of the method is likely to depend strongly on the choice of generalisation functions and, therefore, a significant effort to find adequate heuristics would be well justified. In particular, anybody implementing the method is very likely to encounter the problem of overgeneralisation. Working with too strong generalisations of clauses may potentially lead to numerous \( \gamma \)-refutations that are not compatible with any of the covered clauses. For example, if we fold the definition \( \forall x. \gamma(x) \leftrightarrow p(x) \) into the clause \( p(f(a)) \), transforming it into \( \gamma(f(a)) \), we may later derive some \( \gamma \)-refutation \( \neg \gamma(b) \) which is incompatible with \( \gamma(f(a)) \). The work on deriving \( \neg \gamma(b) \) is potentially wasted, unless, of course, there are other clauses compatible with \( \neg \gamma(b) \). Another problem with overgeneralisation is that \( \gamma \)-refutations compatible with many clauses may be found quickly, and will activate the corresponding clauses. In such cases the work spent on creating generalisations themselves and their application to clauses, is wasted because the generalisations do not fulfill their mission of suspending clauses. On the other hand, too weak generalisations may also be bad, e. g., because they cover too small sets of clauses, in which case their construction is not properly amortised. I hope these considerations illustrate the thesis about importance of searching for heuristics for choosing effective generalisation functions.

In contrast with the fine inference selection scheme which essentially requires creating a new implementation, the generalisation-based search guidance can be relatively easily integrated into some existing provers, especially if it is implemented with naming and folding as outlined earlier. My experience with implementing splitting-without-backtracking [31] (see also Chapter 5 in [30]) in the Vampire kernel suggests that only a moderate effort is required to implement naming and folding on the base of a reasonably manageable implementation of forward subsumption, which is a standard feature in advanced saturation-based provers.

The most difficult task is likely to be the design and implementation of a flexible, yet manageable, mechanism for specifying generalisation functions, and to provide a higher-level interface for this mechanism which would enable productive use of heuristics. The reliance on heuristics also implies that very extensive experimentation will be required to assess the general effectiveness of the method and to compare its variants.
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