Pseudoscalar Meson Temporal Correlation Function for Finite Momenta in HTL approach*

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Abstract

The temporal pseudoscalar meson correlation function in a QCD plasma is investigated in a range of temperatures exceeding $T_c$ and first time for a finite momenta which is of the experimental interest. The imaginary time formalism is employed for the finite temperature calculations. The behavior of the meson spectral function and of the temporal correlator is studied in the HTL approximation, where one replaces the free thermal quark propagators with the HTL resumed ones.

Key words: Finite temperature QCD, Quark Gluon Plasma, Meson correlation function, Meson spectral function, Finite momentum, HTL approximation.

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1 Introduction

Present contribution is based on the article [1] done in collaboration with W.M. Alberico, A. Beraudo and A. Molinari. We compute the Meson Spectral Function (MSF) in the HTL approximation. The degrees of freedom we deal with are light quarks (massless) and gluons and our results refer to the case of zero chemical potential, a condition which is expected to be realized in the heavy ion

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experiments at RHIC and even better in the future experiments at LHC. The calculations are performed in the imaginary time formalism, taking at the end a proper analytical continuation to real frequencies when required. The zero momentum MSF at small values of the energy displays Van Hove singularities, which exist because of the presence of the plasmino mode in the HTL quark propagator. Divergences appear in the density of states giving rise to peaks in the MSF. The possible experimental relevance of such singularities was analyzed in Ref. [4], where the back-to-back dilepton production rate was evaluated in terms of the zero momentum MSF in the vector channel. The full information on the momentum and temperature behavior of the mesonic excitations in the QGP phase is encoded in their spectral function for any energy and momenta.

2 Finite temperature meson spectral function

The analytical expression for MSF is:

\[
\sigma_{HTL}^{ps}(\omega, p) = 2N_c \int \frac{d^3k}{(2\pi)^3} (e^{\beta \omega} - 1) \int_{-\infty}^{+\infty} d\omega_1 \int_{-\infty}^{+\infty} d\omega_2 \tilde{n}(\omega_1) \tilde{n}(\omega_2) \delta(\omega - \omega_1 - \omega_2) \times
\]

\[
\times \left\{ (1 + \hat{k} \cdot \hat{q}) \left[ \rho_+ (\omega_1, k) \rho_+ (\omega_2, q) + \rho_- (\omega_1, k) \rho_- (\omega_2, q) \right] + \\
+ (1 - \hat{k} \cdot \hat{q}) \left[ \rho_+ (\omega_1, k) \rho_- (\omega_2, q) + \rho_- (\omega_1, k) \rho_+ (\omega_2, q) \right] \right\},
\]

where \( \beta = 1/T \) and \( \tilde{n}(\omega) \) is a Fermi distribution. The HTL quark spectral function \( \rho_{\pm} (\omega, k) \), whose expression in given in Ref. [1], reflects the singularities of the quark propagator in the complex \( \omega \)-plane. These lie on the real \( \omega \)-axis. At a given value of the spatial momentum \( k \), in the time-like domain (\( \omega^2 > k^2 \)) discrete poles are associated to quasiparticle excitations; a cut for space-like momenta (\( \omega^2 < k^2 \)) is related to the Landau damping.

The thermal meson propagator can be expressed through the spectral function \( \sigma_{HTL} \):

\[
G_{HTL}(\tau, p) = -\frac{1}{\beta} \sum_{n=-\infty}^{+\infty} e^{-i\omega_n \tau} \int_{-\infty}^{+\infty} d\omega \frac{\sigma_{HTL}(\omega, p)}{i\omega_n - \omega} = \int_{0}^{+\infty} d\omega \sigma_{HTL}(\omega, p) K(\omega, \tau)
\]

with \( \tau \in [0, \beta] \), the kernel \( K(\omega, \tau) = \frac{\cosh[\omega(\tau - \beta/2)]}{\sinh(\omega\beta/2)} \) and the sum over the Matsubara frequencies with a standard contour integration in the complex \( \omega \) plane \[5, 6\] is performed \[3\].

We shall denote with \( q_+ (\bar{q}_+) \) the normal quark (antiquark) mode, with \( q_- (\bar{q}_-) \) the plasmino (antiplasmino) mode and with \( M \) the excitation carrying the quantum numbers of a pseudoscalar meson.
Figure 1: The zero momentum HTL and free MSF (divided by $\omega^2$) as a function of $\omega$ for two different temperatures. The asymptotic high energy plateau is also shown.

3 Results

In Fig. 1 we plot the spectral function divided by $\omega^2$. Two different temperatures are considered. One can appreciate the dramatic difference in the behavior at low energy between the free curves (which vanish at $\omega = 0$) and the HTL ones (which diverge for $\omega \to 0$). One can also recognize the Van Hove singularities in the HTL curves arising from a divergence in the density of states due to the minimum in the plasmino dispersion relation. All the curves approach for large values of $\omega$ the same high energy, temperature independent, plateau. We now investigate how things change at finite spatial momentum. In Fig. 2 we plot the pseudoscalar MSF for $p = 1$fm$^{-1}$ at different temperatures. The non-interacting result vanishes on the light-cone, at variance with the HTL curves, which stays finite there. On the other hand when $p$ is finite both the free and the interacting curves diverge as $\omega \to 0$. Furthermore in the HTL case the Van Hove singularities are washed-out by the angular integration. Finally both the free and the HTL finite momentum MSF approach the asymptotic plateau for large values of $\omega$.

We can also see Fig. 3 how the different processes contribute to the pole-pole term [1], at a given value of the temperature and of the spatial momentum, to the HTL MSF. In analogy to the zero-momentum case studied in Ref. [2, 3], it turns out that the dominant process at low energy is the decay $q_+ \to q_- + M$; then there is a wide gap for an intermediate range of energy till when the plasmino-antiplasmino annihilation starts contributing. Such a process initially gives a quite large contribution due to the large density of states; then it decreases rapidly with the energy because of the very small value of the plasmino residue.
Figure 2: The $p = 1$ fm$^{-1}$ HTL and free MSF (divided by $\omega^2$) as a function of $\omega$ for two different temperatures. The asymptotic high energy plateau is also shown.

Figure 3: The major processes contributing to the pole-pole term of the HTL MSF. The plot is given for the case $p = 1$ fm$^{-1}$ and $T = 2T_c$. 
It appears that, for large enough frequencies, the dominant role is played by the quark-antiquark annihilation. Such a process starts contributing for $\omega$ larger than a threshold depending on the thermal gap mass $m_\text{q}$ acquired by the quarks in the thermal bath. On the other hand the processes $\bar{q}_- \to \bar{q}_- + M$ and $q_- \to q_+ + M$ turn out to be totally negligible, due to the low value of the plasmino residue and to the very small available phase space.

The finite momentum temporal correlator is defined in Eq. (2). In order to assess the impact of the interaction it is convenient to consider the ratio $G_{\text{HTL}}(\tau)/G_{\text{free}}(\tau)$ between HTL and a free cases. Such a ratio turns out to be finite for every value of $\tau$. The above ratio approaches 1 for $\tau \to 0$ (or $\beta$). In fact, due to the structure of the thermal kernel given in Eq. 2, in this limit the correlator $G(\tau)$ is dominated by the high energy behavior of the MSF. We start by considering the zero momentum case, left part of Fig. 5, showing the ratio for a range of temperatures from $T = T_c$ to $T = 10T_c$. Then we move to the finite momentum case, $p = 4\text{fm}^{-1}$, right part of Fig. 4. As the temperature increases the ratio moves closer to 1, reflecting the running of the coupling, and the bump at $\tau = \beta/2$ is smeared out.

Figure 4: The ratio $G_{\text{HTL}}(\tau)/G_{\text{free}}(\tau)$ for different temperatures at $p = 0\text{fm}^{-1}$. 

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Figure 5: The ratio $G_{HTL}(\tau)/G_{free}(\tau)$ for different temperatures at $p = 4\text{fm}^{-1}$.

4 Conclusions

We have examined the impact of a finite value of the spatial momentum on the spectral density and on the temporal correlation function of a pseudoscalar meson for temperatures above the deconfinement phase transition and zero chemical potential. This amounts to study the properties of an excitation carrying the quantum number of a meson and propagating in the heat-bath frame. In our treatment we employed HTL resummed quark propagators. This has allowed us to perform many calculations analytically [1] and to identify the different physical processes contributing to the MSFs and to the temporal correlators, a task impossible to achieve, of course, when the same quantities are evaluated on the lattice.

At zero momentum, the main features of the MSF are the presence of the Van Hove singularities and the radically different low energy behavior of the HTL predictions with respect to the free case. Such a contrast results particularly visible in the plot of $\sigma(\omega, 0)/\omega^2$, where, for $\omega \to 0$ the HTL curve diverges while the free result vanishes. At finite spatial momentum it turns out that the Van Hove singularities so prominent in the zero momentum case are smoothed out by the angular integration. Another finding worth to be pointed out is that while the free MSFs vanish on the light-cone, the HTL curves stay finite. The impact of a finite value of the spatial momentum has been investigated also
for the HTL temporal correlator $G(\tau, p)$, for which we plotted the ratios with respect to the free results. We found that the ratio of the HTL result with respect to the non-interacting correlator differs from one just for a few percent.

We hope our work can provide a complementary and independent approach to the studies of MSFs and temporal correlators performed on the lattice.

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