Inverse Seesaw in Supersymmetry

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Abstract

We study a mechanism where tiny neutrino masses arise only from radiative contribution in a supersymmetric model. In each generation, the tree-level light neutrino mass is rotated away by introducing a new singlet neutrino $s_L$ that forms a Dirac mass term with the right-handed neutrino $n_R$. With non-zero Majorana neutrino mass for the right-handed neutrinos $M_Rn_R^\dagger n_R$, the lightest neutrino remains massless at tree level. Supersymmetry ensures that the Majorana neutrino masses $M_Rn_R^\dagger n_R$ and $M_R^*s_L^\dagger s_L$ are not generated simultaneously. There is no exact chiral symmetry to protect the neutrino mass. Consequently, tiny neutrino masses then only arise from radiative contributions and the right-handed neutrino Majorana mass $M_R$ can be at $O$(KeV).
Enormous experimental evidences have shown that neutrinos have tiny masses of $O(10^{-10})$ GeV. Being completely neutral under the unbroken gauge symmetries $SU(3)_c \times U(1)_{EM}$, neutrinos can be Majorana fermions and the origin of neutrino mass may be different from the other SM fermions. In the minimal Higgs boson model, the Dirac neutrino masses can arise by introducing SM singlet fields $n^i_R$ but the fact that dimensionless Yukawa coupling $Y_\nu$ is of order $O(10^{-12})$ is still puzzling. The most elegant mechanism for neutrino mass generation is perhaps the seesaw mechanism [1–3]. Majorana masses of $n_R$ are introduced in addition to the Dirac terms as

$$y_\nu \ell_L n_R H + M_R \bar{n}_R n_R + h.c. ,$$

where $\ell_L$ is the $SU(2)_L$ doublet $(1, 2)_{-1}$, $y_\nu \langle H \rangle = M_D$ after electroweak breaking and the light neutrinos get masses of order of $M_D^2 M_R^{-1} M_D$. The tiny light neutrino masses are realized as a consequence of setting $M_R$ to ultra-high Grand Unification (GUT) scale [1] and the seesaw mechanism can be naturally embedded into various GUT models. However, hierarchy problem arises since the heavy right-handed neutrinos contribute large logarithm corrections to the Higgs mass as $\Delta m^2_h \simeq y^2_\nu M_R^2 \ln(q/M_R)/4\pi^2$ [4]. It is then worth to investigating the possibility of explaining neutrino mass within weak scale. Radiative neutrino mass generation is one of the attempts in this approach. Neutrino masses can only be generated via radiative contribution in various models [5, 6]. Some other models may contain right-handed neutrinos $n_R$ then the tree level mass must be suppressed [7] so that the radiative contribution can dominate the neutrino mass. However, the existence of the radiative contributions in these models implies that the $U(3)_\nu$ chiral symmetry must be broken and the tree level mass cannot be set to zero by setting $y_\nu$. Therefore, in the models [7], the Higgs vacuum expectation value is usually suppressed in the neutrino Yukawa interaction to suppress the tree level contribution. In this paper, we discuss the possibility of suppressing tree level mass without tuning the Yukawa coupling $y_\nu$ or the Higgs vev.

In the Dirac neutrino mass case, if one introduces a new SM singlet $s_L$ to form another Dirac mass term with the right-handed neutrino $n_R$ as [8]

$$y_\nu \ell_L n_R H + M_S \bar{s}_L n_R + h.c. ,$$

The model then contains one massless neutrino and one massive Dirac neutrino per generation. The “inverse seesaw mechanism” [9] extended this model by introducing a Majorana
mass term for the $s_L$ states as
\[ y_\nu \ell_L n_R H + M_S s_L n_R + \epsilon \bar{s}_L s_L + h.c. , \] (3)

The lightest neutrino mass then arises from a small $\epsilon$ as
\[ m_\nu \simeq \epsilon \frac{M_D^2}{M_D^2 + M_S^2} \] (4)
while $\epsilon$ can be identified as soft breaking of $U(1)_{\text{Lept}}$ lepton number symmetry.

However, if one only introduces $M_R n_R$ to the Eq. 2 as in the seesaw mechanism,
\[ y_\nu \ell_L n_R H + M_S s_L n_R + M_R n_R n_R + h.c. , \] (5)
the lightest neutrino will remain massless at tree level and the original seesaw mechanism breaks down. To see this, we write down the neutrino mass matrix in the basis of $(\nu_L, s_L, n_R)$
\[ \mathcal{M} = \begin{pmatrix} 0 & 0 & M_D \\ 0 & 0 & M_S \\ M_D & M_S & M_R \end{pmatrix}. \] (6)

The mass eigenstates consist of one massless state and two massive states which are mixture of Dirac and Majorana spinors
\[ \nu = -\frac{M_S}{\sqrt{M_D^2 + M_S^2}} \nu_L + \frac{M_D}{\sqrt{M_D^2 + M_S^2}} s_L \]
\[ N_\pm = \frac{1}{\sqrt{M_\pm^2 + M_D^2 + M_S^2}} (M_D \nu_L + M_S s_L - M_\pm n_R) \] (7)
with mass eigenvalues as
\[ m_\nu = 0, \quad M_\pm = \frac{1}{2} \left( M_R \pm \sqrt{4M_D^2 + M_R^2 + 4M_S^2} \right). \] (8)

We take one generation to illustrate the features of this model. For the symmetries that only act on neutral fermions, the lagrangian of free fields has accidental symmetries as $U(1)_\nu \otimes U(1)_n \otimes U(1)_s$. The existence of Dirac neutrino mass $M_D$ breaks $U(1)_\nu \otimes U(1)_n$ down to $U(1)_{\nu+n}$ \footnote{If $M_D$ vanishes, the tree level neutrino mass is also massless. However, $y_\nu = 0$ would restore the chiral symmetry and no neutrino mass would be generated radiatively as long as there exists the exact chiral symmetry. On the other hand, the vacuum expectation value (VEV) $\langle H \rangle$ is usually associated with the up-type quark mass like $m_t$ and should not vanish.} and $M_S$ also breaks $U(1)_s \otimes U(1)_n$ down to $U(1)_{s+n}$ \footnote{If $M_S$ vanishes, the model simply becomes the original seesaw mechanism.}. One can redefine...
the two $U(1)$ symmetries and identify one of $U(1)_s$ as $U(1)_{\text{Lep}}$ Lepton Number symmetry under which the fields transform as,

$$\nu_L \rightarrow e^{i\alpha}\nu_L, \quad s_L \rightarrow e^{i\alpha}s_L, \quad n_R \rightarrow e^{-i\alpha}n_R.$$  

(9)

The second $U(1)$ can be identified as $U(1)_{\nu-s}$ under which $\nu_L$ and $s_L$ transform in the same way. However, the $U(1)_{\nu-s}$ is only an approximate symmetry. By writing $U(1)_\nu$ instead of $U(1)_L$, we did not assume the SM gauge symmetry while $\nu_L$ is charged under SM gauge group and $s_L$ is a completely SM singlet, $U(1)_{\nu-s}$ is not respected by the SM gauge interactions or interaction via the Higgs. Therefore, the only exact symmetry is $U(1)_{\text{Lep}}$ which is later broken by the $M_R$ explicitly. One should also notice that at tree level, the Majorana masses of $\nu_L^c s_L$, $s_L^c s_L$ or $\nu_L^c s_L$ all explicitly break the $U(1)_{\nu-s}$.

In principle, once $M_R n_R n_R$ term is generated, the term $M_R^* s_L^c s_L$ will be generated automatically. To forbid the $M_R^* s_L^c s_L$ term, a natural extension is to embed the model into supersymmetric theory. The holomorphic feature of superpotential naturally split the two terms so that they will not be generated simultaneously. Notice supersymmetry does not forbid the $s_L^c s_L$ term. We want to emphasize that this model is only technically natural as a explicit breaking of $U(1)_{\text{Lep}}$ $M_R^* n_R n_R$ won’t automatically generate the Majorana mass of $s_L$ field.

I. MODEL

The superpotential of the model contains

$$W \ni y_{\nu} \ell N^c H_u + y_{e} e^c E^c H_d + \mu H_u H_d + M_S S N^c + M_R N^c N^c,$$

(10)

where $N^c$, $E^c$, $S$ are the chiral superfields. We assume there exists only one explicit breaking of $U(1)_{\text{Lep}}$ as $M_R N^c N^c$ and no $SS$ breaking. In Table I, the $R$-charge and the lepton number $U(1)_{\text{Lep}}$ charges of the fields in the model have been given. Without losing generality, we write the $R$-charge in $SU(5)$ compatible language.

To ensure non-zero eigenvalues of Eq. (10) at least one of the Majorana mass terms

$$\nu_L^c s_L + s_L^c s_L + \nu_L^c \nu_L$$

(11)

\footnote{If the $U(1)_{\text{Lep}}$ is exact symmetry, the lightest mass eigenstate will be exactly massless to all orders.}
| Field | $\ell$ | $E^c$ | $N^c$ | $S$ | $H_u$ | $H_d$ | $\theta$ |
|-------|-------|-------|-------|-----|-------|-------|-------|
| $R$-charge | 1/5 | 3/5 | 1 | 1 | 4/5 | 6/5 | 1 |
| $U(1)_{lep}$ | 1 | -1 | -1 | 1 | 0 | 0 | 0 |

TABLE I: Charge assignment of leptons and Higgses in the Model

| $W_{\text{eff}}$ | $\mathcal{L}$ | $R$-charge of $\mathcal{L}$ | $U(1)_l$ charge |
|-----------------|---------------|---------------------------|------------------|
| $N^cN^c$ | $\bar{n}^c_Rn_R$ | $1 + 1 - 2\theta = 0$ | 1 |
| $\ell SH_u$ | $\bar{\nu}_L^c s_L$ | $\frac{4}{5} + 1 + \frac{4}{5} - 2\theta = 0$ | 2 |
| $\ell\ell H_uH_u$ | $\bar{\nu}_L^c \nu_L$ | $\frac{4}{5} + 1 + \frac{4}{5} + \frac{4}{5} - 2\theta = 0$ | 2 |
| $SS$ | $\bar{s}^c_L s_L$ | $1 + 1 - 2\theta = 0$ | 2 |

TABLE II: Properties of Lepton number violation terms in the model. $\mathcal{L}$ stands for the fermion mass terms in the lagrangian.

should be at present. We summarize the properties of these terms in Table II. If the right-handed Majorana mass $\bar{n}^c_R n_R$ carries $U(1)_{lep}$ Lepton number charge as $-2$, the terms $\bar{\nu}_L^c \nu_L$, $\bar{s}_L s_L$ or $\bar{\nu}_L^c s_L$ all carry lepton number $+2$. None of the Majorana neutrino mass terms breaks $R$-symmetry. Therefore, the corresponding soft SUSY breaking terms

$$\mathcal{L}_{\text{soft}} \ni B_R \bar{n}_R n_R + B_u \bar{\nu}_L \nu_L + B_S \bar{s}_L s_L + A_s \bar{\ell}_L \tilde{s}_L H_u$$ (12)

all break $U(1)_{lep}$ as well as the $R$-symmetry and the fermion masses due to Eq. 12 then require loops involving gaugino mass insertion.

A very similar philosophy has been employed in the uplifted MSSM [10]. The holomorphic feature of superpotential and the anomaly cancellation conditions require the supersymmetric SM to be a Two-Higgs-doublet-model (2HDM) with $\langle H_u \rangle$ and $\langle H_d \rangle$ responsible for generation of $m_u$ and $m_d$ (or $m_e$) respectively. In MSSM, contributions to $m_e$ or $m_d$ from the $\langle H_u \rangle$ are then similar to our model. In that case, all the chiral symmetries associated with the SM fermions are broken by the Yukawa couplings. When Peccei-Quinn (PQ) symmetry and $R$-symmetry have been broken, effective operators

$$y_d' Q d \dagger H_u^\dagger + y_e' \ell L e \dagger H_u^\dagger$$ (13)

can then be radiatively generated.
In the presence of the new singlet $s_L$ which is charged under $U(1)_{\text{lep}}$, the gravitational anomaly ($\text{Tr}[U(1)_{B-L}]$) and cubic anomaly $[U(1)_{B-L}]^3$ become non-vanishing and $U(1)_{B-L}$ symmetry is then anomalous. In minimal model, there is no additional gauge interaction besides the SM gauge symmetries.

**II. NEUTRINO MASSES**

To realize neutrino masses, one should look at the corrections to the zero entries in the Eq. 6. Even though there is no symmetry to protect these terms from being generated, the magnitude of these terms depends on mediation. As being argued, with the second SM singlet $s_L$ which is also charged under $U(1)_{\text{lep}}$, the $U(1)_{B-L}$ can no longer be gauge symmetry in the minimal model and the model cannot be embedded into minimal $E_6$, there is no other $U(1)'$ gauge interactions. The $n_R$ and $s_L$ fields then become completely gauge singlet. The gauge singlet $s_L$ only couple through gravity interaction. Consequently, the terms involving $s_L$ field as $\bar{\nu}_L s_L$ or $\bar{s}_L s_L$ will then only arise from gravity mediation with $1/M_{\text{Pl}}$ suppression. The leading correction in Eq. 6 is then $\bar{\nu}_L \nu_L$ type which originates from $\ell_L \ell_L H_u H_u$.

For exactly the same reason, the lepton number violation terms in the soft SUSY breaking lagrangian Eq. 12 can not be generated via gauge mediation but only from gravity interaction. The bilinear lepton number violation $B$-terms are then of gravitino mass $O(m_{3/2})$. The $R$-breaking contributions to neutrino masses are realized at the order of $m_{3/2}^2/M_\lambda$ where the $M_\lambda$ is the wino or bino masses.

To generate neutrino masses radiatively in this model, the chiral symmetries $U(1)_\nu \times U(1)_n \times U(1)_s$ as well as their remanent $U(1)_{\text{lep}}$ must be broken. As being argued, the $U(1)_{\nu-s}$ is only an approximate symmetry since $\nu_L$ and $s_L$ can be easily distinguished through SM gauge interaction or interacting with Higgs, for instance the one-loop contribution as in Fig. 1. The processes involving weak gauge boson can be realized at even higher orders.

Figure 1 shows that the contribution is proportional to the Yukawa coupling $y_\nu$ squared.

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4 In gauge mediated SUSY breaking (GMSB) models, it would be interesting if the $R$-breaking contribution were the leading since the tiny neutrino mass would then be identified as a consequence of gravitational interaction with $1/M_{\text{Pl}}$ suppression. However, $R$-breaking contribution is not the leading contribution here.
as $y_\nu$ appears in two vertices. The loop induced mass vanishes in the limit of vanishing $y_\nu$ which restores $U(1)_\nu \times U(1)_n$ symmetries. $M_R$ dependence has two pieces. There is one mass insertion as $M_R$ and the fermion propagators of $n$ fields also contain $M_R$. For simplicity, we want to discuss only the qualitative feature here. The neutrino mass due to Fig. 1 is proportional to

$$M_\nu \propto \frac{1}{16\pi^2} \lambda y_\nu^2 \langle H_u \rangle^2 M_R \sum_{\phi_i} \frac{R_i}{M_{\phi_i}^2 - M_R^2} \ln \left( \frac{M_{\phi_i}^2}{M_R^2} \right).$$

(14)

In evaluating the loop contribution in Fig. 1 one should compute the diagrams in the mass eigenstates. In MSSM, the $H_u$ is actually a mixture state of several neutral scalars $h, H, A$ and they can all run into the loops. We use $R_i$ to denote the mixing factor for each scalar $\phi_i$. $\lambda$ stands for the Higgs quartic coupling which is $(g_1^2 + g_2^2)/8$ in MSSM. The interesting feature of this radiatively generated mass is that $M_R$ appears in both numerators and denominators as

$$\frac{M_R}{M_{\phi_i}^2 - M_R^2}.$$  

(15)

With the scalar masses $M_{\phi_i} \sim M_{\text{EW}}$ of $\mathcal{O}(10^2)$ GeV, to obtain the tiny neutrino mass, one can take two different limit of $M_R$:

- $M_R \gg M_{\phi_i}$ where correction then reduces to

$$M_\nu \propto y_\nu^2 \langle H_u \rangle^2 / M_R$$

(16)

as in the conventional seesaw mechanism with additional loop suppression.

- $M_R \ll M_{\phi_i}$ where the

$$M_\nu \propto M_R.$$  

(17)

Since there is additional loop suppression, it requires $M_R \sim \mathcal{O}(\text{KeV})$. 

FIG. 1: One loop contribution of neutrino mass in non-SUSY.
Without breaking the $R$-symmetry, this supersymmetric model has another Lepton number violation vertex which is proportional to $M_R$. Given the superpotential of the model as

$$ W = \ell N^c H_u + M_R N^c N^c + M_S S N^c . $$

(18)

In the scalar potential,

$$ V = \left| \frac{\partial W}{\partial N^c} \right|^2 = |(\tilde{\ell} H_u + M_S \tilde{s} + M_R \tilde{n})|^2 $$

$$ \ni M_R^* \tilde{n}^* \tilde{\ell} H_u + M_R^* M_S \tilde{n}^* \tilde{s} . $$

(19)

Both $M_R^* \tilde{n}^* \tilde{\ell} H_u$ and $M_R^* M_S \tilde{n}^* \tilde{s}$ are proportional to $M_R$ and violate $U(1)_{Lep}$. However, since the sfermion $\tilde{s}$ do not participate in the gauge interactions as we have argued, the contribution is then suppressed. The contribution to neutrino mass due to the vertex $M_R^* \tilde{n}^* \tilde{\ell} H_u$ is shown in Fig. 2.

\[
\begin{tikzpicture}
  \node (Hu) at (0,0) {$\langle H_u \rangle$};
  \node (n) at (1,1) {$\tilde{n}$};
  \node (nu) at (1,0) {$\tilde{\nu}$};
  \node (nubar) at (1,-1) {$\bar{\nu}$};
  \node (nu_L) at (-1,-1) {$\nu_L$};
  \node (nubar_L) at (-1,1) {$\bar{\nu}_L$};
  \node (nu_Lbar) at (-1,0) {$\nu_L^c$};
  \node (nubar_Lbar) at (-1,-1) {$\bar{\nu}_L^c$};
  \node (chi0) at (0,-2) {$\chi_0^0$};
  \node (chi1) at (0,-3) {$\chi_1^0$};
  \draw[->] (Hu) -- (n);
  \draw[->] (n) -- (nu);
  \draw[->] (nu) -- (nubar);
  \draw[->] (nubar) -- (Hu);
  \draw[->] (n) -- (chi0);
  \draw[->] (n) -- (chi1);
  \draw[->] (nubar) -- (chi0);
  \draw[->] (nubar) -- (chi1);
\end{tikzpicture}
\]

FIG. 2: One loop contribution of neutrino mass in SUSY.

Again, the Yukawa coupling $y_{\nu}$ appears in the vertex of Higgsino/right-handed sneutrino and the vanishing $y_{\nu}$ leads to the vanishing mass. Figure 2 contributions also involve the right-handed sneutrinos in the loops and the right-handed sneutrino mass is

$$ m_{\tilde{n}}^2 \simeq M_R^2 . $$

(20)

$M_R$ will again appear in both vertices and propagators and the behavior is very similar to the previous case in Eq. 15.

\footnote{Since the right-handed neutrino is completely gauge singlet, the right-handed sneutrino mass will not receive a $M_{\text{SUSY}}$ level contribution if the SUSY breaking is not gravity mediated.}
In this paper, we want to take the second limit as $M_R \ll M_{\text{SUSY}} \sim M_{\text{EW}}$ for various reasons. First of all, it is to ensure there is no large correction in Kähler potential to Majorana mass terms involving $s_L$ fields without making any additional assumption. Secondly, with weak scale $M_S$ and $M_D$, a tiny $M_R$ not only explains the light neutrino mass but also predicts two nearly degenerated weak scale pseudo-Dirac neutrinos which can in principle be produced at the CERN Large Hadron Collider (LHC). In the end, as being argued in “inverse seesaw mechanism” [9], a tiny $M_R$ can be identified as a soft breaking of $U(1)_{\text{Lep}}$.

III. CONCLUSIONS

In this paper, we study the scenario where tiny neutrino mass arises as radiatively corrections. To suppress the tree level mass without restoring the chiral symmetry or tuning the VEV, we introduce one singlet field in addition to the right-handed neutrino and rotate away the tree level neutrino mass. We employ the same philosophy as in the inverse seesaw model that there exists a tiny scale around KeV due to the soft breaking of $U(1)_{\text{Lep}}$ Lepton number symmetry. Unfortunately, the spectrum predicted in this model is almost identical to the inverse seesaw mechanism and it will be difficult to distinguish this model from the inverse seesaw model.

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