Vector meson radiation in relativistic heavy-ion collisions

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Abstract

The \((\sigma, \omega)\) model in mean-field approximation where the meson fields are treated classically, describes much of observed nuclear structure and has been employed to describe the nuclear equation of state up to the quark-gluon phase transition. The acceleration of the meson sources, for example, in relativistic heavy-ion collisions, should result in bremsstrahlung-like radiation of the meson fields. The many mesons emitted serve to justify the use of classical meson fields. The slowing of the nuclei during the collision is modeled here as a smooth transition from initial to final velocity. Under ultra-relativistic conditions, vector radiation dominates. The angular distribution of energy flux shows a characteristic shape. It appears that if the vector meson field couples to the conserved baryon current, independent of the baryonic degrees of freedom, this mechanism will contribute to the radiation seen in relativistic heavy-ion collisions. The possible influence of the quark-gluon plasma is also considered.

Key words: relativistic heavy-ion collisions; vector meson production; bremsstrahlung; quantum hadrodynamics; relativistic mean-field theory

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1 Introduction

The \((\sigma, \omega)\) model of the nucleus \([1,2]\) is a relativistic quantum field theory which describes the nuclear interaction using three fields. They are neutral scalar meson, neutral vector meson and baryon fields. The scalar meson field couples to the scalar density, \(\bar{\psi}\psi\), while the vector meson field couples to the conserved baryon current, \(B_\mu \equiv \bar{\psi}\gamma_\mu\psi\). In the relativistic mean-field approximation (RMFT) where the sources are large, the meson fields can be replaced

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by classical fields and the linearized Dirac equation solved exactly. This model has had several successes, properly describing: bulk properties of nuclear matter, ground state properties of nuclei, the excitation spectrum of nuclei, low energy nucleon-nucleus scattering observables, as well as collective motion. All except the first of the preceding list incorporate spatial dependence for the meson fields and their sources. The last item, collective motion, incorporates a slow time dependence in addition to the spatial dependence.

It is here proposed to use this model to calculate vector meson production during relativistic heavy-ion collisions like those to be seen at RHIC. All of the above successes of the \((\sigma, \omega)\) model have been for low and intermediate energy phenomena. Therefore, some justification is necessary for use of this model in a radically different physical situation. First, using ground state wavefunctions calculated from this model and empirical nucleon-nucleon scattering amplitudes one can obtain excellent agreement with experimental nucleon-nucleus scattering observables in the relativistic impulse approximation up to energies of the order 1 GeV [3]; second, the longitudinal response for \(^{56}\text{Fe}(e,e')\) at momentum transfer of \(|\vec{q}| = 0.55\) GeV has been calculated by Frank [4] in this model and agrees well with data; in addition, the \((\sigma, \omega)\) model taken in RMFT has been employed to describe the nuclear equation of state up to the quark-gluon phase transition [1,5]. Furthermore, the presence of many quanta of the meson fields here serves to validate the classical approximation.

It is not clear what the appropriate degrees of freedom actually are for the RHIC-like collisions. Although the \((\sigma, \omega)\) model deals with hadrons, these results will turn out to be more general. We consider a process whereby two conserved baryon currents pass through each other, are slowed, and thus radiate energy via bremsstrahlung in the form of vector mesons. In the end, the only requirements for this calculation are an empirical knowledge of the initial and final rapidities of the baryon currents, conservation of these currents, and a vector meson that couples to the conserved baryon currents. In the final analysis, it is irrelevant whether the baryon current is carried by nucleons or quarks during the collision, as long as the vector meson couples to the baryon current.

It should be noted that in the work of Mishustin et al [6], where the \((\sigma, \omega)\) model is employed to calculate baryon–anti-baryon production via virtual vector meson bremsstrahlung under RHIC conditions, it is suggested that real \(\omega\)-mesons should be produced via bremsstrahlung and that this process should be relatively soft.

\(^2\) In the work of Mishustin et al [6] it is stated, “One cannot even say what degrees of freedom, hadrons or quarks and gluons, are more suitable for describing these collisions.”

\(^3\) In Ref. [6] it is further observed about vector meson bremsstrahlung that “... it is clear that the same mechanism can produce also mesons. For instance, the real
In this work only central collisions of identical heavy nuclei are considered. Therefore, the lab and equal-velocity frames are equivalent. In Ref. [7] Hanson shows evidence that for central high-energy collisions (rapidity larger than 3), the nuclei are expected to be slowed by the collision, but not completely stopped. This slowing occurs over the short time of the collision. This gives rise to a large deceleration. In this model, the decelerating sources radiate the classical meson fields. This is the same process as classical electromagnetic bremsstrahlung for an accelerated charge.

Due to Lorentz contraction, the scalar meson radiation is greatly suppressed relative to the vector meson radiation. Most of the vector meson radiation is in the form of high energy mesons whose mass is therefore neglected. An attenuation factor is included to account for the strong interaction between the vector mesons and the baryons. The attenuation factor is constructed in a Lorentz invariant manner to maintain the covariance of the theory. This model contains three adjustable parameters: the deceleration time, the rapidity loss and the total cross section for the vector meson-nucleon interaction. The angular distribution of energy flux is characteristic, varying only in magnitude for variations in the cross section and deceleration time. Variations in rapidity loss affect both the magnitude and width of the angular distribution; however, the general shape is robust for parameter variations.

This work indicates that the bremsstrahlung of vector mesons could contribute significantly to the total radiated energy during relativistic heavy-ion collisions. The model used here is very simple; it depends only on the vector mesons coupling to a conserved baryon current. The fact that this model predicts an appreciable amount of radiation with a characteristic angular distribution that is robust against parameter variation suggests that it is deserving of further investigation.

This radiation of mesons during relativistic heavy-ion collisions has been considered before. Weber et. al [8] examined the dynamics of relativistic heavy-ion collisions using the (σ, ω) model in mean field theory. They modeled the collisions within the relativistic Boltzmann-Uehling-Uhlenbeck model using full solutions to the meson-field equations. At the energies they studied (1-20 GeV/A), they found meson radiation to be negligible (< 3 MeV/nucleon). However, the amount of energy radiated increases as the acceleration squared, so their work does not rule out appreciable radiation at higher energies. The production of pions and photons via bremsstrahlung was investigated in Refs. [9,10]. Ivanov [11] calculated ω-meson radiation due to filamentation instabil-

ω-meson can be generated in the bremsstrahlung process when the four-momenta of quanta satisfy the mass shell constraint $p^2 = m^2_{\omega}$. ... These channels are characterized by lower threshold and, therefore, by smaller momentum transfers. Since the corresponding coupling constants are also large, one can expect high multiplicities of mesons coherently produced in ultra-relativistic nuclear collisions.”
ity. The behavior of the meson fields during relativistic heavy-ion collisions was explored in Refs. [12,13]; however, these are at much lower bombarding energies and did not directly look at meson radiation.

It is expected that at the energies available at RHIC these central collisions will produce a quark-gluon plasma. The question then arises, what effect will the creation of the plasma have on this model that uses baryons and mesons? Indeed, would any sign of this hadronic process survive under these conditions? It has been suggested that as individual quarks interact during the collision, color strings or flux tubes will form [14–16]. These flux tubes are stretched out behind the nuclei as they pass through each other (see, for example, figure 4 of Ref. [14]). The breaking of these flux tubes then provides an important contribution to the formation of the quark-gluon plasma. The meson production described in the present work comes exclusively from the forward going baryons that constitute the initial colliding nuclei. Even if the mesons must pass through part of the baryon-rich quark-gluon plasma, there is theoretical evidence that meson-like modes can propagate through a quark-gluon plasma [17]. Such a physical situation may alter the attenuation but could still leave a detectable sign of this mechanism.

This paper attempts to test the limits of the \((\sigma, \omega)\) model in RMFT for a strong time-dependence. Section 2 introduces the framework for the \((\sigma, \omega)\) model and classical bremsstrahlung. Sections 3 to 5 discuss the model for the baryon current, the degree of incoherence in the radiation and the attenuation of the vector meson radiation. The energy spectrum is investigated in section 6. Sections 7 and 8 discuss the results and conclusions.

## 2 Formalism

The basic \((\sigma, \omega)\) model is defined by the lagrangian density:

\[ \mathcal{L} = \frac{1}{4} F^{\mu \nu} F_{\mu \nu} + \frac{1}{2} m_\omega^2 V^\mu V_\mu + \frac{1}{2} \left[ \left( \frac{\partial \phi}{\partial x_\mu} \right)^2 - m_\sigma^2 \phi^2 \right] \]

\[ + \bar{\psi} \left[ i \gamma_\mu \left( \frac{\partial}{\partial x_\mu} - g_\nu V^\nu \right) - (M_N - g_\sigma \phi) \right] \psi, \]

where \(V_\mu\) is the vector field, \(\phi\) is the scalar field and \(\psi\) is the baryon field. The masses of the nucleon, vector meson and scalar meson are \(M_N\), \(m_\omega\) and \(m_\sigma\).

\[4\] Throughout this paper natural units are used, \(c = \hbar = 1\). In these units \(0.197\ \text{GeV} = 1 \text{ fm}^{-1}\). The metric is that of Bjorken and Drell and Ref. [2].
respectively. The field tensor is defined as

$$F^{\mu\nu} = \frac{\partial V^\nu}{\partial x^\mu} - \frac{\partial V^\mu}{\partial x^\nu}. \quad (2)$$

This lagrangian neglects non-linear self-couplings of the scalar field. In the nuclear ground state, the vector and scalar fields are of comparable strength [1]. Under a Lorentz boost the scalar density is invariant; however, the vector density is enhanced by a factor of $\gamma$, defined as

$$\gamma = (1 - \beta^2)^{-1/2}, \quad (3)$$

with $\beta$ the velocity. For the dynamics of the collisions investigated in this paper, this factor varies between 10 and 100. The radiated energy is proportional to the field squared; therefore, the vector meson radiation is at least a factor of 100 greater than the scalar meson radiation. Consequently, the scalar field can be neglected.

The energy-momentum tensor is,

$$T^{\mu\nu} = -\mathcal{L} g^{\mu\nu} + \frac{\partial \mathcal{L}}{\partial (\partial q/\partial x^\mu)} \frac{\partial q}{\partial x^\nu} = \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} - F^\mu_\sigma \frac{\partial V^\sigma}{\partial x^\nu}. \quad (4)$$

This form of the energy-momentum tensor works well for calculating the total energy and momentum of the system. However, it contains total divergences that will give incorrect results for the energy flux. Therefore, the symmetric energy-momentum tensor must be used [18],

$$\Theta^{\mu\nu} = \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} + F^{\mu\sigma} F_{\sigma\nu}. \quad (5)$$

The power radiated is

$$\frac{dP(t')}{d\Omega} = R^2 [S(t) \cdot \vec{n}]_{\text{ret}} \quad (6)$$

where $P(t')$ is the power radiated at time $t'$ and $S_i (\equiv \Theta^{0i})$ is the Poynting vector. The observer is located a distance $R$ from the source of the radiation along the unit vector $\vec{n}$. The notation, $[]_{\text{ret}}$, means evaluate at $t' = t - R(t')$.

The equation of motion for the vector meson field is

$$\partial_\nu F^{\mu\nu} + m_\omega^2 V^\mu = g_\rho B^\mu. \quad (7)$$

5 The development of the radiated energy follows closely that in Jackson [19].
Since the baryon current is conserved, this reduces to \((\partial^2 + m^2) V^\mu = g_\nu B^\nu\). This is the inhomogeneous Klein-Gordon equation for a massive particle which has the solution,

\[
V^\mu(x) = \frac{1}{(2\pi)^4} \int d^4k \frac{e^{-ik\cdot x}}{m^2 - k^2 - i\eta\varepsilon} g_\nu B^\nu(k),
\]

In the above equation, \(x\) is the four-position \((t, \vec{x})\), \(k\) is the four-momentum \((\varepsilon, \vec{k})\) and \(B^\mu(k)\) is the Fourier transform of the current. A convergence factor, \(i\eta\varepsilon\), is introduced to eliminate the singularity at \(k^2 = m^2\). The choice of \(-i\eta\varepsilon\) gives the retarded solution. The solution above can be converted to an integral over \(d^4x'\) and gives

\[
V^\mu(x) = \int d^4x' G_r(x - x') B^\mu(x')
\]

where

\[
G_r(x - x') = \frac{1}{(2\pi)^4} \int d^4k \frac{e^{-ik\cdot (x - x')}}{m^2 - k^2 - i\eta\varepsilon}
\]

\[
= \frac{\delta(\tau - \mathcal{R})}{4\pi \mathcal{R}} - \frac{m_\omega}{4\pi \sqrt{\tau^2 - \mathcal{R}^2}} J_1(m_\omega \sqrt{\tau^2 - \mathcal{R}^2}) \Theta(\tau - \mathcal{R}),
\]

where \(\tau = t - t'\) and \(\mathcal{R} = |\vec{x} - \vec{x}'|\). \(J_1\) is the first order Bessel function and \(\Theta(y)\) is the Heavyside step function defined as \(\Theta(y) \equiv (1 + |y|/y)/2\).

If the baryon current is replaced by the electromagnetic current, the coupling constant changed to the fine structure constant, \(g_\nu^2/4\pi \rightarrow \alpha\), and the mass, \(m_\omega\), goes to zero, this reduces to electromagnetic bremsstrahlung. Note that as \(m_\omega \rightarrow 0\) the second term in equation (9) vanishes. It is argued in section 6 that neglecting the mass of the vector meson does not alter the results appreciably, and we will take \(m_\omega = 0\) for the rest of this work.

The above solution for \(V^\mu\) and equation (5) gives a distribution of energy as a function of outgoing energy, \(\varepsilon\), and angle of, (compare equation (14.70) of Ref. [19])

\[
\frac{d^2E}{d\varepsilon d\Omega} = \frac{\varepsilon^2 g_\nu^2}{4\pi^2} \left| \int_0^\infty dt \int d^3x \vec{n} \times [\vec{n} \times \vec{B}(\vec{x}, t)] e^{i(\varepsilon(t - \vec{n} \cdot \vec{x}))} \right|^2.
\]

Again, \(\vec{n}\) is the unit vector from the point of radiation to the observer.

The radiation is strongly peaked in the direction of the baryon cluster's motion. Thus, we concentrate now on just one of the incident nuclei. Assuming the current is that of a collection of point baryons traveling with identical velocity,

\[
\vec{B}(\vec{x}, t) = \sum_j \delta(\vec{x} - \vec{x}_j(t)) \vec{\beta}(t)
\]
we may do the spatial integration. The baryon’s position is given as
\[ \vec{x}_j(t) = \vec{X}(t) + \vec{r}_j(t) \] where \( \vec{X}(t) \) is the position of the center-of-mass and \( \vec{r}_j(t) \) is the baryon’s position relative to the center-of-mass. The relative position of the baryon will vary with time, for example, due to changing Lorentz contraction and the empirically seen spreading of final rapidity; however, in this simple model this motion will be neglected relative to the much more important center-of-mass motion. The baryons will this be considered as “frozen” relative to the baryon clusters as they pass through each other and decelerate. The baryon’s position thus becomes,
\[ \vec{x}_j(t) = \int_{-\infty}^{t} \vec{\beta}(t') \, dt' + \vec{r}_j(-\infty) \equiv \vec{X}(t) + \vec{r}_j. \] (12)

Separating out the dependence on \( j \) we may write,
\[ \frac{d^2 E}{d\varepsilon \, d\Omega} = \frac{\varepsilon^2 g^2}{4\pi^2} \left| \int_{-\infty}^{\infty} dt \, \vec{n} \times \left[ \vec{n} \times \vec{\beta}(t) \right] e^{i\varepsilon(t-\vec{n} \cdot \vec{X}(t))} \sum_j e^{-i\varepsilon \vec{n} \cdot \vec{r}_j} \right|^2. \] (13)

Taking the sum outside the square gives a coherence factor
\[ \mathcal{P} \equiv \sum_{j=1}^{A} \sum_{k=1}^{A} e^{-i\varepsilon (\vec{r}_j - \vec{r}_k)}. \] (14)

One must now take a statistical average over the positions of the baryons in the incident nucleus; this average is weighted by the square of the ground state wave-function. If the exponent is large, as argued in section 4, the off-diagonal elements average to zero and the coherence factor is approximately \( A \), the number of baryons.

We now wish to find the angular distribution of energy flux and the total energy radiated. Equation (13) must therefore be integrated over all frequencies. Equation (13) may be rewritten as
\[ \frac{d^2 E}{d\varepsilon \, d\Omega} = \frac{g^2}{4\pi^2} A \left| \int_{-\infty}^{\infty} dt \, \vec{n} \times \left[ \vec{n} \times \vec{\beta}(t) \right] e^{i\varepsilon(t-\vec{n} \cdot \vec{X}(t))} \right|^2. \] (15)

where we have assumed the velocity and acceleration are collinear to write
\[ \frac{d}{dt} \left[ \vec{n} \times \left[ \vec{n} \times \vec{\beta}(t) \right] \right] = \frac{\vec{n} \times \left[ \vec{n} \times \vec{\beta}(t) \right]}{(1 - \vec{n} \cdot \vec{\beta}(t))^2}. \] (16)
Changing variables to $\tau = t - \vec{n} \cdot \vec{X}(t)$ and expanding the square we may now do the $\varepsilon$ integration. The integration should be over positive energies; however, the integrand is even in $\varepsilon$ so we take $1/2$ the integral over all energies. The angular distribution of energy flux is

$$\frac{dE}{d\Omega} = \frac{g_v^2}{4\pi} A \int_{-\infty}^{\infty} dt \frac{||\vec{n} \times [\vec{n} \times \vec{\beta}(t)]||^2}{(1 - \vec{n} \cdot \vec{\beta}(t))^5}. \quad (17)$$

The angular integration can also be done and this gives

$$E = \frac{2g_v^2}{3} A \int_{-\infty}^{\infty} dt \gamma^6 |\vec{\beta}(t)|^2. \quad (18)$$

Note that the last two results are just the single particle result (see equations (14.38) and (14.43) of Ref. [19]) times the coherence factor, $P \approx A$.

### 3 Baryon Current Models

The only quantity required to evaluate equation (10) is the baryon current, $\vec{B}(\vec{x}, t)$. In the present covariant approach, this is related to the particles’ velocity, $\vec{\beta}$, in the lab frame by

$$B_\mu(\vec{x}, t) = \sum_{\lambda=1,2} \rho_B^{(\lambda)}(\vec{x}) U_\mu^{(\lambda)}(t) \quad (19)$$

where 1 and 2 are the two nuclei, $\rho_B$ is the rest-frame baryon density and $U_\mu = (\gamma, \gamma \vec{\beta})$ is the four-velocity of the nucleus. The baryon current is conserved;

$$\frac{\partial}{\partial x_\mu} B_\mu = 0, \quad (20)$$

and transforms as a four-vector, $\Lambda_\mu^{\nu} B_\nu(\Lambda x) = \sum_{\lambda=1,2} \rho_B^{(\lambda)}(\Lambda x) \Lambda_\mu^{\nu} U_\nu^{(\lambda)}(\Lambda x)$.

The baryon density should be determined from the ground-state nuclear wavefunctions, computed in the Dirac-Hartree approximation, summed over the occupied orbitals. This procedure is explained in detail in Ref. [20] and implemented in Ref. [21]; however, it is argued in sections 2 and 4 of this paper that the nucleons radiate incoherently and the energy flux is insensitive to the exact baryon distribution. Therefore, one can approximate the true baryon density by a constant density inside the nuclear radius, $\rho_B(r) \equiv \rho_o \Theta(R_N - r)$. 


The motion of the baryons will be modeled as having a smooth transition from their initial speed to their final speed. A Fermi-type parameterization of the speed is used [6]:

$$
\beta(t) = \beta_f + \frac{\beta_i - \beta_f}{1 + e^{t/\tau}},
$$

where $\tau$ is the stopping parameter. Call the unit vector along the beam axis $\vec{z}$. Therefore, the first nucleus is traveling with velocity $\beta \vec{z}$ and the second with $-\beta \vec{z}$. The initial and final speeds are fixed from experimental results and the stopping parameter is allowed to vary in this model.

The beam energy per nucleon is $E_{\text{beam}}/A$. The initial rapidity is related to the beam energy by $M_N \cosh y_i = E_{\text{beam}}/A$, and the initial speed by $\beta_i = \tanh y_i$.

The energy lost during the collision is characterized by the rapidity loss, $\delta y = y_i - y_f$. Mishustin et. al [6] claim a rapidity loss of $\delta y = 2.4 \pm 0.2$ for central Au+Au collisions at RHIC energies of $E_{\text{beam}}/A = 100$ GeV. The same value is used for this study of Pb+Pb at the same energy. The stopping parameter is allowed to vary between 5 and 20 fm.

4 Incoherence

The initial speed of the nucleons for $E_{\text{beam}}/A = 100$ GeV is $\beta_i \approx 0.99996$. The velocity of the nucleons relative to the center-of-mass is small compared to the velocity of the center-of-mass. This means that the collision time between nuclei will be much shorter than the interaction time between nucleons in the same nucleus. The nuclei then can be treated as static collections of nucleons with uniform longitudinal motion. This is similar to the frozen approximation used in the parton model.

The baryon current can be represented as a collection of point particles:

$$
\vec{B}(\vec{x}, t) = \vec{\beta}^{(1)}(t) \sum_{j=1}^{A} \delta[\vec{x} - \vec{X}_1(t) - \vec{r}_j] + \vec{\beta}^{(2)}(t) \sum_{j=1}^{A} \delta[\vec{x} - \vec{X}_2(t) - \vec{r}_j] (22)
$$

where the first (second) term represents the first (second) nucleus, $\vec{X}_i(t)$ is the position of the center of mass of the $i^{\text{th}}$ nucleus and $\vec{r}_j$ is the position of the $j^{\text{th}}$ baryon relative to the center of mass at $t = -\infty$. Since the lab frame coincides with the equal-velocity frame, $\vec{X}_1 = -\vec{X}_2 \equiv \vec{X}$. The position is the integral of

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6 Investigations up to 50 fm still maintain the characteristic shape of the angular energy flux distribution.
the velocity with the boundary condition that the velocity asymptote to the free particle solution at \( t = \pm \infty \). The exact form used is

\[
\vec{X}(t) = \int t \, dt' \, \vec{z} = \beta_t \vec{z} + (\beta_f - \beta_i) \tau \ln(1 + e^{t/\tau}) \vec{z}.
\]

(23)

Using this form of the current in equation (10) gives

\[
\frac{d^2 E}{d\varepsilon \, d\Omega} = \frac{\varepsilon^2 g^2_v}{\pi^2} \mathcal{P} \int dt \int dt' \, [\beta(t) \sin \psi(t)][\beta(t') \sin \psi(t')] \, e^{i\varepsilon(t-t')} \times \sin[\varepsilon \phi(t)] \sin[\varepsilon \phi(t')]
\]

(24)

where \( \sin \psi(t) = \vec{n} \times (\vec{n} \times \vec{z}) \) at time \( t \), \( \phi(t) = \vec{n} \cdot \vec{X}(t) \) and \( \mathcal{P} \) is the coherence factor defined in equation (14).

If in the coherence factor, \( |\varepsilon \, \vec{n} \cdot (\vec{r}_j - \vec{r}_k)| \ll 1 \) then the phases are all near zero. If, on the other hand, \( |\varepsilon \, \vec{n} \cdot (\vec{r}_j - \vec{r}_k)| > 1 \) then the phases are large and nearly random. In the end, when a statistical average over all possible configurations is taken using the ground-state wave-functions, the first case leads to \( \mathcal{P} \approx A^2 \) corresponding to the nucleons radiating as one large baryon of “baryonic charge” \( A \). For the latter case the coherence factor becomes \( \mathcal{P} \approx A \) corresponding to each nucleon radiating independent of its neighbors. Since neither of these conditions hold absolutely, the coherence factor will be some power of the atomic weight, \( A^\alpha \) where \( 1 \leq \alpha \leq 2 \). To neglect the vector meson mass, its outgoing energy must be on the order of several GeV. The inter-particle distance is on the order of 1 fm. The latter condition holds and the nucleons should radiate incoherently, \( \alpha = 1 \).

5 Attenuation

Throughout this work the vector meson field has been treated analogously to the electromagnetic field with just a change in coupling strength. However, there is one aspect where there is a significant difference. The vector mesons interact strongly with the baryons; therefore, there will be an attenuation of vector mesons as they pass through either the nucleus from which they are radiated or the other nucleus. The attenuated energy flux can appear as radiation through another channel or could be used to heat the nuclear material. Since most of the radiation will occur early during the collision [equation (17) is dominated by the denominator which increases during the collision] and is directed in the forward direction, this attenuation is non-negligible. This reduction is accounted for by including a multiplicative factor,
\[ \mathcal{A}^2(t), \text{ in the radiated power}, \]
\[ \frac{dP(t')}{d\Omega} = R^2[\mathcal{A}^2(t)\mathcal{S}(t) \cdot \vec{n}]_{ret}. \] (25)

Forms of equations (17) and (18) involving the attenuation factor follow the development in section 2.

This attenuation factor\(^7\) is taken as,
\[ \mathcal{A}(t) = \frac{\int d^3x \rho_B(\vec{x}) e^{-l\sigma \rho_B} e^{-\chi(t)\gamma'\sigma \rho_B}}{\int d^3x \rho_B(\vec{x})}, \] (26)

where \(l\) and \(\chi\) are the distances traveled through the radiating nucleus and the second nucleus respectively, \(\sigma\) is the vector meson-nucleon total cross-section, and \(\rho_B\) is the baryon density. Equation (26) can be evaluated in the rest frame of the radiating nucleus making \(l\) time-independent. To maintain the covariance of the model the attenuation factor is evaluated along the axis of motion and taken to be independent of outgoing angle. Since most of the radiation is in the far forward direction, this results in only a small overestimation of the attenuation.

To evaluate equation (26), consider the radiation coming from the nucleus traveling in the positive \(\vec{z}\) direction (the argument is identical for the other nucleus). Move the origin for the integration to the center of mass in nucleus’ rest frame. The distance traveled inside this nucleus depends only on where the radiation originates (remember, the attenuation is assumed independent of outgoing angle). Using cylindrical coordinates \((\rho, \phi, z)\) to take advantage of the azimuthal symmetry, we find
\[ l = -z + \sqrt{R_N^2 - \rho^2}, \] (27)

where \(R_N\) is the nuclear radius. Since the radiation must come from inside the nucleus the following condition always holds: \(z^2 + \rho^2 \leq R_N^2\). The form of the equation for the distance through the target nucleus is more involved since the nucleus is moving. Defining the distance between the two centers of mass

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\(^7\) It is assumed here that the vector meson will interact with the baryons immediately after it is created. If the nucleus demonstrates color transparency, then this attenuation factor will be reduced.
as \( d = -2\gamma'(t)X(t) \), the distance traveled through the moving nucleus is

\[
\chi(t) = \begin{cases} 
\frac{2}{\gamma'} \sqrt{R^2 - \rho^2} & ; \quad d - R_N/\gamma' > z \\
-(z - d) + \frac{1}{\gamma'} \sqrt{R_N^2 - \rho^2} & ; \quad d - R_N/\gamma' < z < d + R_N/\gamma' \\
0 & ; \quad \text{otherwise}
\end{cases}
\]  

(28)

where \( \gamma' \) is the Lorentz factor for the target moving in the rest frame of the radiating nucleus. It is related to the Lorentz factor in the lab frame [equation (3)] by

\[
\gamma'(t) = \gamma^2(t)(1 + \beta^2) = \frac{1 + \beta^2}{1 - \beta^2}.
\]  

(29)

6 Energy Spectrum

Throughout this paper the vector meson mass has been assumed small compared to the particles’ outgoing energy, and hence neglected. It is the purpose of this section to justify this assumption. Equation (24) gives the energy distribution of the radiated mesons. If the observer is assumed far away, the angle to the observer, \( \psi(t) \), will change little over the time of interest. The \( \sin \psi \) factor may be taken outside the time integral as a constant. We define the meson energy spectrum by keeping only the energy and time dependent factors in equation (24),

\[
\mathcal{I}^2(\epsilon) = \left\vert \epsilon \int_{-\infty}^{\infty} dt \, \beta(t) e^{i\epsilon t} \sin[\epsilon \vec{n} \cdot \vec{X}(t)] \right\vert^2. \tag{30}
\]

Note that \( \mathcal{I}^2(\epsilon) \) is dimensionless. Again assuming the angle to the observer changes little and the velocity remains near one, the integral can be evaluated analytically giving:

\[
\mathcal{I}^2(\epsilon) = (\epsilon \tau)^2 \left[ B[i\epsilon \tau(1 - \beta_i \cos \psi), i\epsilon \tau(\beta_f \cos \psi - 1)] + B[i\epsilon \tau(1 + \beta_i \cos \psi), -i\epsilon \tau(\beta_f \cos \psi + 1)] \right]^2, \tag{31}
\]

where \( B[x, y] \) is the beta function which is related to the gamma or factorial function by

\[
B[x, y] \equiv \frac{\Gamma(x)\Gamma(y)}{\Gamma(x + y)}. \tag{32}
\]
Fig. 1. The dimensionless energy spectrum, $\mathcal{I}^2(\epsilon)$, as defined in equation 30 as a function of outgoing vector meson energy.

As can be seen in figure 1 the peak occurs at $\epsilon = 0$, and decreases slowly as $\epsilon$ increases. This slow decrease means that most of the radiation is above the vector meson mass and that the massless assumption is acceptable. Cutting the spectrum off at the meson mass $\epsilon = m_\omega = .783$ GeV, reduces the integrated spectrum by less than 1%.

7 Results

Throughout this work, only central collisions of identical nuclei at a bombarding energy achievable at RHIC of $E_{\text{beam}}/A = 100$ GeV [22] have been considered. A short discussion is still needed concerning the value of the required coupling constant, $g_v$. There is no solid evidence to direct a choice of the coupling constant for the type of reaction that this work describes. A value that reproduces static nuclear properties has been chosen [1]; $g_v^2/4\pi = 10.8$. Using a value of $g_v^2/4\pi$ between eight and nine, Gross, Van Orden and Holinde [23] were able to reproduce free nucleon-nucleon phase shifts to several hundred MeV. Since the $(\sigma, \omega)$ model is not an asymptotically-free theory, the coupling constant may even increase at higher energy and momentum transfer; this would lead to even more radiation than is shown below. The paper by Mishustin et. al [6], uses a coupling constant of $g_v^2/4\pi = 15.1$, stronger than what is used here. As long as the coupling constant is of at least order 1, this model predicts appreciable amounts of energy radiated through this channel.
Fig. 2. Angular distribution of energy flux as a function of lab angle for outgoing vector mesons. The rapidity loss is fixed at \( \delta y = 2.4 \) and the vector meson-nucleon total cross-section is fixed at \( \sigma = 30 \text{ mb} \). It is shown for \( \tau = 5.0 \text{ fm} \) (solid line), \( \tau = 10 \text{ fm} \) (dashed line) and \( \tau = 20 \text{ fm} \) (dotted line). The \( A \) and \( g^2/4\pi \) factors have been divided out.

This is a crude model that has incorporated several simplifying assumptions. It would be inconsistent to think of the predicted numbers here as more than a rough guide to the order of magnitude of what may actually be seen. To remove any ambiguity in the coupling constant, the angular distributions are shown with the coupling constant divided out.

Figures 2, 3 and 4 show the angular distribution of radiated energy in the form of vector mesons. Following the analysis of section 4, the energy flux is calculated as the incoherent sum of 2A nucleons radiating as point particles,

\[
\frac{dE}{d\Omega} = \frac{g_v^2}{4\pi} A \int_0^\infty dt \frac{A^2(t)\beta(t)^2(t)\sin^2 \theta}{(1 - \beta(t)\cos \theta)^5} + \frac{A^2(t)\beta(t)^2(t)\sin^2 \theta}{(1 + \beta(t)\cos \theta)^5}.
\]  

(33)

The factor \( (g_v^2/4\pi)A \) is divided out in the figures to remove any ambiguity\(^8\)

\(^8\) The dimensions of the remaining expression are converted by \( 1 \text{ fm}^{-1} = 0.197 \text{ GeV} \).
Fig. 4. Same as figure 2 except stopping parameter is fixed at $\tau = 10$ fm and the rapidity loss takes the values $\delta y = 2.2$ (solid line), $\delta y = 2.4$ (dashed line) and $\delta y = 2.6$ (dotted line).

In each of the figures, two of the parameters are held constant while the third is varied over reasonable values. The peak of energy flux occurs around $2.5^\circ$ off the beam-axis. It is seen that variations in the stopping time, $\tau$, and the cross-section, $\sigma$, only affect the magnitude and not the characteristic angular distribution shape. Changes in the rapidity loss, $\delta y$, affect both the magnitude and shape of the distribution with larger rapidity losses in this range causing less radiation and causing the peak to be moved further off axis and broadened. It is interesting to note that the smallest rapidity loss used in this study, $\delta y = 2.2$, results in the most energy being radiated. This can be explained by considering the circumstances under which most of the energy is radiated. At relativistic speeds, the energy radiated is dominated by the factor of $(1-\vec{n} \cdot \vec{\beta})^{-5}$ in equation (17). A smaller rapidity loss results in the nucleus spending more time traveling faster, resulting in more radiation. Of course this only holds over a small region. At some point the smallness of the acceleration overcomes the smallness of the denominator and at zero rapidity lost, there is no radiation.

For the intermediate values of the parameters ($\tau = 10$ fm, $\sigma = 30$ mb and $\delta y = 2.4$) and $A = 208$ (Pb beams) the total energy radiated, $E_{\text{rad}}$, is 353 GeV. The total energy available to be radiated in this collision is $3.78 \times 10^4$ GeV. Reducing the stopping parameter to $\tau = 5$ fm gives $E_{\text{rad}} = 730$ GeV. A rapidity loss of $\delta y = 2.2$ and a stopping parameter of $\tau = 10$ fm gives $E_{\text{rad}} = 516$ GeV out of a total available energy of $3.70 \times 10^4$ GeV. On the order of 1% of the total energy loss is through this channel.
8 Conclusions

In this paper a model for treating the bremsstrahlung radiation of neutral vector mesons coupled to a baryon current during central relativistic heavy-ion collisions has been developed. This model treats the nuclei as clusters of baryons frozen in relative position over the time of the collision. The clusters’ velocities are modeled as changing smoothly with time. Lorentz contraction greatly increases the baryon density relative to the scalar density; hence, the scalar meson radiation becomes negligible at high energies. Most of the energy is radiated as highly energetic vector mesons, allowing for their mass to be neglected. The vector mesons interact strongly with the baryons and therefore, an attenuation factor is included. To maintain the covariance of the model, this attenuation factor is assumed independent of angle.

The modeling of the flow of the baryon current is the only freedom in the model. By using a smooth connection between the initial rapidity of the nuclei with the experimentally measurable final rapidity, the baryon current depends on just two parameters, the rapidity loss (obtained from the initial and final rapidities) and a stopping parameter which is allowed to vary freely. Although this model is used well outside its tested domain, in the end only the vector field coupling to the conserved baryon current is seen. The model predicts a characteristic shape for the angular distribution of radiation that is robust against parameter variations. It appears this mechanism may contribute to meson production in the next generation of relativistic heavy-ion collisions like those at RHIC. This possibility would seem to be worth exploring experimentally.¹⁹

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¹⁹ If this process were to be observed, it could provide a diagnostic for the baryon flow during such collisions. For example, only straight line motion of the baryons has been considered. The radiation is focused sharply around the instantaneous motion of the baryon current so deviations from the angular distribution shown here could be used to track the intermediate flow of baryons during the collision. See also Ref. [11].
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