1. Introduction

Dirac semimetals are 3D analogues of 2D graphene first isolated in 2004 [1]. They harbor relativistic fermions with electronic dispersion curves which in the low energy sector are linear in all three momentum directions. The conduction and valence band meet at a single point and at charge neutrality their Fermi surface is point-like. Breaking either time-reversal (TR) or inversion symmetry can lift the two-fold degeneracy of a Dirac cone to produce a pair of Weyl nodes having opposite chirality. Broken TR-symmetry displaces the Dirac cones in momentum space by $\pm Q$ while broken inversion symmetry displaces them in energy by $\pm Q_0$. Weyl nodes where first studied theoretically in pyrochlore iridates [2, 3] and other systems [4, 5] and later suggested to exist in noncentrosymmetric transition-metal monophosphides [6]. This theoretical prediction lead to the discovery of many such materials including TaAs [7–10], TaP [11] and NbAs [12]. An example of broken TR-symmetry is YbMnBi$_2$ [13]. Weyl semimetals (WSM) have many exotic properties [14]. Their surface states have open Fermi arcs which end at the projection on the surface of the bulk Weyl nodes of opposite chirality. In a magnetic field closed cyclotron orbits are possible which involve the arcs on opposite surfaces connected through the bulk Weyl nodes [15–17]. Other anomalous properties include the chiral anomaly [18–23], a negative magnetoresistance [24, 25], Hall effect [26, 27] and other anomalous transport properties [28–30]. The absorptive part of the AC longitudinal optical conductivity [31–35] gives information on the bulk and reflects directly the relativistic, linear in momentum, dispersion curves of the Dirac fermions. In a 3D system this translates to a region of conductivity which is linear in photon
energy [35, 36]. This is the analog of the constant interband background seen in graphene [37, 38]. In more complicated cases [36] there can be more than one quasi-linear region [32]. The chiral anomaly also manifests in optical absorption [39] which contains an image of the transfer of charge, in the presence of an external non-orthogonal electric \( \mathbf{E} \) and magnetic \( \mathbf{B} \) field, from the node of one chirality to the other of the opposite chirality.

Another recent development is the theoretical realization that a new type of Weyl fermions can exist in condensed matter systems that is not part of high-energy physics and an example is WTe\(_2\) [40]. In this case the Dirac cones are tilted with respect to an axis in the Brillouin zone. If the tilt is small enough such that the electronic density of state at the node remains zero, the Weyl node is said to be type I but for large tilt, electron and hole pockets at zero energy can form as a result of a Lifshitz transition and this new phase is referred to as type II [40] with finite density of state at the Weyl point. Many properties of tilted Weyl cones have already been worked out, including its effect on the interband optical background [41, 42], on the AC Hall conductivity [43, 44] and on the absorption of circular polarized light [44]. Tilting leads to the squeezing of the Landau levels [45, 46] on application of an external magnetic field and even the collapse of the spectrum. New transitions appear in the optical spectrum beyond the dipolar ones and the surface Fermi arcs are modified [47].

Central to the present work is the paper of Zyuzin and Tiwari [48] on the intrinsic anomalous Hall effect and very recent discussions of the Nernst effect [49, 50] which apply to WSM when TR-symmetry has been broken. In this work we consider the DC limit of the Hall conductivity valid for a general tilt and chemical potential (\( \mu \)) is taken to be zero. In section 4 we consider the finite \( \mu > 0 \) case. Our final analytic expressions properly reduce to those described in recent literature [49, 50] for the case of only broken TR-symmetry. A discussion and conclusions are given in section 6.

## 2. Formalism

We begin with the minimal continuum Hamilton for a pair of Weyl node of opposite chirality with both TR and inversion symmetry broken. The first displaces the Dirac cone in momentum space by an amount \( \pm \mathbf{Q} \) while the second shifts their energy by \( \pm \mathbf{Q}_0 \). The Hamiltonian is given by the following equation [35, 51, 52],

\[
\hat{H}(k) = C_{1,2}(k - s'Q) + s'\nu\sigma \cdot (k - s'Qe) - s'Q_0
\]

where \( s' = 1 \) for Weyl point indexed by 1 and \( s' = -1 \) for Weyl point indexed by 2. \( C_{1,2} \) describes the amount of tilting of the particular chiral node, \( \nu \) the Fermi velocity and \( e \) the unit vector along the axis \( x_i \) where \( i = x, y, z \). The Pauli matrices are defined as usually by,

\[
\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]

The broken inversion symmetry is introduced through the third term in the Hamiltonian. The energy dispersion corresponding to the above Hamiltonian is described as,

\[
\epsilon_{s,s'} = C_{1,2}k_z - s'C_{1,2}Q - s'Q_0 + s\sqrt{k^2 + 2s'kQ + Q^2}
\]

where \( s = \pm \) stands for conduction(+) and valence(−) bands and \( k^2 = k_x^2 + k_y^2 + k_z^2 \). For a set of values of the parameters \( \nu = 1, Q = 2, Q_0 = 0.5 \) we plot in figure 1 the energy dispersion for the different cases of tilting. When we take the inversion symmetry breaking into account the negative chiral Weyl node gets shifted upward by an amount \( Q_0 = 0.5 \) while the opposite happens for the other chiral node.

In figure 2 we study the evolution of the Weyl nodes when they are tilted parallel to each other for the case \( C_1 = C_2 \) for different amount of tilting. We see that as we increase tilting individual Weyl cones not only bend but also the cone cross-section becomes progressively wider and ultimately the conduction and valence bands merge with the planes \( \epsilon_{s,s'} = C_{1,2}(k_z - s'Q) \) [48] for large tilting (see figure 3) and we can also draw the same inference about their ultimate evolution as shown in figure 4(b).

The Green’s function corresponding to the above Hamiltonian is given by,

\[
G(k, z) = \left[I_2z - H(k)\right]^{-1},
\]

where \( I_2 \) is a 2 \( \times \) 2 unit matrix. It is straightforward to show that the Green’s function can be written in the following form,

\[
G_{1,2}(k, \omega_n) = \sum_{s = \pm} \frac{1 - ss'\sigma \cdot N_{s - s'}Q_{e}}{\omega_n - C_{1,2}(k_z - s'Q) + ss'\sqrt{k^2 + 2s'kQ + Q^2}},
\]

where \( N_{s - s'}Q_{e} = \frac{k_{e} + k_{s} + (k_{s - s'}Q_{e})}{\sqrt{k^2 + 2s'kQ + Q^2}} \).

Since in the subsequent sections we will discuss the behavior of anomalous Hall conductivity \( \sigma_{xy} \), we need the corresponding current-current correlation function within the realm of the Kubo formalism which is defined as,
\[ \sigma = \Lambda k + v k + k z. \] Now we let the temperatures and with \(-\) go to zero and \(\infty\) the DC conductance \(\sum f = 2\) \(s\) \(-\) and valence band \((s = -)\) of the Weyl node with negative chirality \((s' = -1)\). Similarly green and brown cones are respectively the conduction band \((s = +)\) and valence band \((s = -)\) of the Weyl node with positive chirality \((s' = 1)\). The two chiral Weyl nodes are separated by a distance \(\pm Q\). The gray shaded plane corresponding to the \(\mu = 0\) plane. We set \(v = 1\). \(Q\) is taken twice the Fermi velocity and \(Q_0\) half of it. (a) Two untilted Weyl nodes separated by \(Q\) in the inversion symmetry preserving case. (b) Two untitled Weyl nodes after we break the inversion symmetry. (c) Two tilted Weyl nodes with \((0)\) of the Weyl node with positive chirality \((s' = 1)\) and \((0)\) of the Weyl node with negative chirality \((s' = -1)\). The actual 3D plot of the energy dispersion showing the orientations of the Weyl cones close to the Fermi energy. Here we plot \((8)\) until the Fermi velocity and \(Q_0\) half of it. (a) Two untilted Weyl nodes separated by \(Q\) in the inversion symmetry preserving case. (b) Two untitled Weyl nodes after we break the inversion symmetry. (c) Two tilted Weyl nodes with \((0)\) but oppositely tilted \((C_1 = -C_2 = -0.5)\).

\[
\Pi_{\delta}\left(\Omega_0, q\right) = T \sum_{\omega_n} \sum_{\omega_n'} \int \frac{d^3k}{(2\pi)^3} J_{\delta,0}\left(k + q, \omega_n + \Omega_0\right) \times J_{\delta,0}\left(k + q, \omega_n\right) = T e \delta^3 \sum_{\omega_n} \sum_{\omega_n'} \int \frac{d^3k}{(2\pi)^3} \sigma_1 G_{1,2}(k + q, \omega_n + \Omega_0) \times \sigma_1 G_{1,2}(k, \omega_n),
\]

where the sum \(\omega_n\) is over the Fermionic Matsubara frequencies and \(\Omega_0\) is an external Bosonic Matsubara frequency. We have used the definition of the current operators,

\[
J_{(\delta,0),\delta'} = s^2 e v_0 (k, \omega_n, \Omega_0).
\]

With these definitions we calculate the expression for the correlation function after setting \(q\) to zero as,

\[
\Pi_{\delta}\left(\Omega_0, 0\right) = e^2 \sum_{\omega_n} \sum_{\omega_n'} \int_{-\omega_n}^{\omega_n} \frac{d^3k}{(2\pi)^3} k_{1,0}^2 \int_{-\omega_n}^{\omega_n} \frac{d^3k}{(2\pi)^3} k_{1,0} \times \left\{ f(C, k _z + v - s' Q_0) - f(C, k _z - v + s' Q_0) \right\} \times k_{1,0} \frac{2i\Omega_0}{\Omega_0^2 + 4v^2 k_{1,0}^2}.
\]

Here \(\Omega_0\) is the Matsubara frequency, \(\Lambda\) the cutoff, \(k_\perp\) is the momentum perpendicular to \(k_z\) and \(f(E) = (e^{(E - \mu)/kT} + 1)^{-1}\) is the Fermi function at finite temperature \(T\) with \(\mu\) the chemical potential. Replacing \(\partial\Omega_0\) with \(\Omega + i\delta\), the DC conductivity is,

\[
\sigma_{\delta,\delta} = -\lim_{\Omega_0 \rightarrow 0} \frac{\Pi_{\delta,\delta}(\Omega_0, 0)}{i\Omega} = e^2 v_0^2 \sum_{\omega_n} \sum_{\omega_n'} \int_{-\omega_n}^{\omega_n} \frac{d^3k}{(2\pi)^3} k_{1,0} \times \left\{ f(C, k _z + v - s' Q_0) - f(C, k _z - v + s' Q_0) \right\} \left[ \frac{1}{4v^2 k_{1,0}^2} + i\pi \delta(4v^2 k_{1,0}^2) \right].
\]

We write the real part of the Hall conductivity as,

\[
R_{\delta,\delta} = \frac{e^2}{8\pi} \sum_{\omega_n} \sum_{\omega_n'} \int_{-\omega_n}^{\omega_n} \frac{d^3k}{(2\pi)^3} k_{1,0} \int_{-\omega_n}^{\omega_n} \frac{d^3k}{(2\pi)^3} k_{1,0} \times \left\{ f(C, k _z + v - s' Q_0) - f(C, k _z - v + s' Q_0) \right\}.
\]

Figure 1. The actual 3D plot of the energy dispersion showing the orientations of the Weyl cones close to the Fermi energy. Here we plot the energy dispersion as represented in equation (3). The blue and red cones are respectively the conduction band \((s = +)\) and valence band \((s = -)\) of the Weyl node with positive chirality \((s' = 1)\). Similarly green and brown cones are respectively the conduction band \((s = +)\) and valence band \((s = -)\) of the Weyl node with negative chirality \((s' = -1)\). The two chiral Weyl nodes are separated by a distance \(\pm Q\). The gray shaded plane corresponding to the \(\mu = 0\) plane. We set \(v = 1\). \(Q\) is taken twice the Fermi velocity and \(Q_0\) half of it. (a) Two untilted Weyl nodes separated by \(Q\) in the inversion symmetry preserving case. (b) Two untitled Weyl nodes after we break the inversion symmetry. (c) Two tilted Weyl nodes with \((0)\) but oppositely tilted \((C_1 = -C_2 = -0.5)\).
Figure 2. Evolution of parallel tilted Weyl cones as we change the amount of the tilt. Here we follow the same color code used in figure 1. We also keep the value of the parameters $\nu, Q, Q_0$ unaltered. Additionally we set $k_x = k_y = 0$ and draw the outlines of the positive chiral Weyl nodes by solid lines and that for the negative one with broken lines. Figure (a) shows the two Weyl nodes with the tilt $C_1 = C_2 = 0.5$ for inversion symmetric system. Figure (b) shows the same tilted Weyl nodes with broken inversion symmetry. In figure (c) we show the case for the tilting $C_1 = C_2 = 1$ and in (d) for $C_1 = C_2 = 2$. With increasing tilt the Weyl cones also gradually become wider.

Figure 3. Evolution of two oppositely tilted ($C_1 = -C_2$) Weyl nodes as we change the amount of the tilt. Here we follow the same color code and the parameter set as in figure 2. Figure (a) shows the two Weyl nodes with the tilt $C_1 = -C_2 = -0.5$ for inversion symmetric system. Figure (b) shows the same tilted Weyl cones with broken inversion symmetry. In figure (c) we show the case for the tilting $C_1 = -C_2 = -1$ and in (d) for $C_1 = -C_2 = -2$. With increasing tilt the Weyl cones also gradually become wider.
replace the Fermi functions with the Heaviside step functions \( \Theta \) as shown below,

\[
\Re \sigma_{xy} = -\frac{e^2}{8\pi^2} \sum_{s' = \pm} \int_{-\Lambda - s'Q}^{\Lambda - s'Q} dk_z \\
\times \left[ \text{sgn}(k_z)\Theta(v^2k_z^2 - (C_{s'}k_z - \mu_{s'})^2) + \frac{vk_z}{C_{s'}k_z - \mu_{s'}} \right] \\
\times \left( 1 - \Theta(v^2k_z^2 - (C_{s'}k_z - \mu_{s'})^2) \right),
\]

(11)

where \( \mu_{s'} = \mu + s'Q_0 \) is the chemical potential for the Weyl node with chirality \( s' \) and ‘sgn’ is the sign function. From the above equation we see that the sole effect of the inversion symmetry breaking is coming through the Fermi function where the chemical potential \( \mu \) is getting replaced by \( \mu_{s'} \). We will use this equation for the anomalous Hall conductivity in the subsequent sections of this article.

### 3. Anomalous Hall conductivity at charge neutrality

In this section we discuss the behavior of anomalous Hall conductivity, as described in the last section at charge neutrality. For this we set \( \mu = 0 \) in equation (11) and write it as,

\[
\Re \sigma_{xy} = -\frac{e^2}{8\pi^2} \sum_{s' = \pm} \int_{-\Lambda - s'Q}^{\Lambda - s'Q} dk_z \\
\times \left[ \Theta(v^2k_z^2 - (C_{s'}k_z - s'Q_0)^2) + \frac{vk_z}{C_{s'}k_z - s'Q_0} \right] \\
\times \left( 1 - \Theta(v^2k_z^2 - (C_{s'}k_z - s'Q_0)^2) \right).
\]

(12)

We can divide the above integral into positive and negative range of \( k_z \) and make the variable transformation \( k_z \to -k_z \) in the negative part. Further we see that under the interchange of the chirality \( (s' \to -s') \) the total contribution of the two integrals stays the same. This allows us to drop the \( s' \) index except in the node specific tilt term \( C_{s'} \).

\[
\Re \sigma_{xy} = -\frac{e^2}{8\pi^2} \sum_{s' = \pm} \left[ \int_{0}^{\Lambda - s'Q} dk_z \left\{ \Theta(k_z^2 - (C_{s'}k_z - Q_0)^2) + \frac{k_z}{C_{s'}k_z - Q_0} \right\} \\
\times \left( 1 - \Theta(k_z^2 - (C_{s'}k_z + Q_0)^2) \right) \right].
\]

(13)

Here \( C_{s'} \) denotes the anti-clockwise tilt for the Weyl node with chirality \( s' \), normalized by the Fermi velocity \( v \) and is a dimensionless parameter in our derivation. Similarly \( Q_0' = Q_0/v \) and having the dimension of momentum.

We can differentiate between two cases depending on whether \( C_{s'} \) is less than (WSM type I) or greater than one (WSM type II). First we see what happens when \( C_{s'} < 1 \).

From equation (13) we see that for \( C_{s'} < 1 \) the argument of the \( \Theta \) function is positive only when \( \Lambda - Q > k_z > \frac{Q_0'}{1 + C_{s'}} \) (in the first integral) and \( \Lambda + Q > k_z > \frac{Q_0'}{1 + C_{s'}} \) (in the second integral). In the range \( k_z < \frac{Q_0'}{1 + C_{s'}} \), the quantity \( (C_{s'}k_z - Q_0) \) is \( Q_0' - C_{s'}k_z \). Separating the contributions from the two different Weyl nodes to the conductivity \( \Re \sigma_{xy} \), we write the individual contribution from the \( s' \) node as \( \Re \sigma_{xy}^{s'} \) which is given by,

\[
\Re \sigma_{xy}^{s'} = \frac{e^2}{8\pi^2} \left[ \int_{0}^{\Lambda - s'Q} \frac{k_z dk_z}{C_{s'}k_z + Q_0} + \int_{\frac{Q_0'}{1 + C_{s'}}}^{\Lambda + s'Q} \frac{k_z dk_z}{C_{s'}k_z - Q_0} \right].
\]

(14)

After evaluating the integrals we get,

\[
\Re \sigma_{xy}^{s'} = \frac{1}{2} + \frac{1}{2} \left\{ \frac{1}{C_{s'}} + \frac{1}{2C_{s'}^2} \ln \left( \frac{1 - C_{s'}}{1 + C_{s'}} \right) \right\} \frac{Q_0'}{Q}.
\]

(15)
Next we see what happens when $C'_{\nu} > 1$. From equation (13) we can find out how the Weyl nodes contribute (we call it $\mathcal{R}_{\sigma_{xy}^I}$). Following the same procedure as already described for the case $C'_{\nu} < 1$, we get

$$\mathcal{R}_{\sigma_{xy}^I} = \frac{e^2}{8\pi^2} \left[ \int_{0}^{\Lambda + Q} k_z dk_z - \int_{0}^{\Lambda - Q} k_z dk_z \right].$$

which can be written as,

$$\mathcal{R}_{\sigma_{xy}^I} = \frac{e^2}{2C'_{\nu}} - \frac{1}{4C'_{\nu}} \left( \ln(C'^2_{\nu} - (C'_{\nu}Q + Q_0)^2) - \ln(Q_0/Q) \right).$$

or

$$\mathcal{R}_{\sigma_{xy}^I} = \frac{e^2}{2C'_{\nu}} - \frac{1}{4C'_{\nu}} \ln\left(\frac{Q_0^2}{C'^4_{\nu}(C'^4_{\nu} - 1)\Lambda^2} \frac{Q_0^2}{Q} \right).$$

where we have assumed that the cut off $\Lambda$ dominates over $Q$ and $Q_0$.

We can also study the effect of tilting the Weyl nodes in the opposite direction (clockwise). For this we have to replace $C'_{\nu}$ with $-|C'_{\nu}|$ in equation (13). Following similar algebra to what we have just done we find

$$\mathcal{R}_{\sigma_{xy}^I} = \frac{e^2}{2C'_{\nu}} - \frac{1}{4C'_{\nu}} \ln\left(\frac{Q_0^2}{C'^4_{\nu}(C'^4_{\nu} - 1)\Lambda^2} \frac{Q_0^2}{Q} \right).$$

and,

$$\mathcal{R}_{\sigma_{xy}^I} = \frac{e^2}{2C'_{\nu}} - \frac{1}{4C'_{\nu}} \ln\left(\frac{Q_0^2}{C'^4_{\nu}(C'^4_{\nu} - 1)\Lambda^2} \frac{Q_0^2}{Q} \right).$$

which shows that changing the tilt from counter clockwise to clockwise changes the sign in the second term of equation (19). For a type II Weyl node we get,

$$\mathcal{R}_{\sigma_{xy}^I} = \frac{1}{2} - \frac{1}{2} \ln\left(\frac{1}{1 + |C'_{\nu}|} \right) \frac{Q_0^2}{Q}.$$

where the second term carries the opposite sign from that in equation (17).

In the next few subsections we will deal with different combinations of tilting of the Weyl nodes. In all these subsections $C'_{\nu}$ stands for the absolute value of the tilts.

3.1. Both the cones are tilted anti-clockwise

3.1.1. WSM type I. Here we have to add $\mathcal{R}_{\sigma_{xy}^I}$ for $s' = \pm$. This gives us from equation (15),

$$\mathcal{R}_{\sigma_{xy}} = \sum_{\nu = \pm} \mathcal{R}_{\sigma_{xy}^I} = e^2Q/2\pi^2 \frac{e^2Q/2\pi^2}{1 + \frac{1}{2} \sum_{\nu = \pm} \left[ \frac{1}{C'_{\nu}} + \frac{1}{2C'^2_{\nu}} \ln\left(\frac{1 - C'_{\nu}}{1 + C'_{\nu}}\right) \right]}.$$

When the magnitude of the tilt are the same $C'_1 = C'_2$ this reduces to,

$$\mathcal{R}_{\sigma_{xy}} = e^2Q/2\pi^2 \frac{e^2Q/2\pi^2}{1 + \frac{1}{2} \sum_{\nu = \pm} \left[ \frac{1}{C'_{\nu}} + \frac{1}{2C'^2_{\nu}} \ln\left(\frac{1 - C'_{\nu}}{1 + C'_{\nu}}\right) \right]}.$$

For both tilts clockwise we would get a minus sign between first and second term of equation (22).

3.1.2. WSM type II. To get the anomalous Hall conductivity we have to add $\mathcal{R}_{\sigma_{xy}^I}$ for $s' = \pm$ according to equation (17). This gives us,

$$\mathcal{R}_{\sigma_{xy}} = \sum_{\nu = \pm} \mathcal{R}_{\sigma_{xy}^I} = e^2Q/2\pi^2 \frac{e^2Q/2\pi^2}{1 + \frac{1}{2} \sum_{\nu = \pm} \left[ \frac{1}{C'_{\nu}} + \frac{1}{2C'^2_{\nu}} \ln\left(\frac{Q_0^2}{C'^4_{\nu}(C'^4_{\nu} - 1)\Lambda^2} \frac{Q_0^2}{Q} \right) \right]}.$$

For $C'_1 = C'_2$ we get

$$\mathcal{R}_{\sigma_{xy}} = e^2Q/2\pi^2 \frac{e^2Q/2\pi^2}{1 + \frac{1}{2} \sum_{\nu = \pm} \left[ \frac{1}{2C'_{\nu}} + \frac{1}{4C'^2_{\nu}} \ln\left(\frac{Q_0^2}{C'^4_{\nu}(C'^4_{\nu} - 1)\Lambda^2} \frac{Q_0^2}{Q} \right) \right]}.$$

and again for both tilts clockwise there would be a minus sign between first and second term of equation (24).

In figure 5 we plot the anomalous Hall conductivity $\mathcal{R}_{\sigma_{xy}}$ in the units of $e^2Q/2\pi^2$ against the magnitude of tilt $C'_1$ for different values of $Q_0/Q$. Here we have assumed that both the cones are tilted by the same amount. We show results for both WSM type I ($C'_1 < 1$) as well as WSM type II ($C'_1 > 1$). This figure also shows the behavior of anomalous Hall conductivity when both nodes are tilted anti-clockwise (for positive $C'_1$) and clockwise (for negative $C'_1$). This covers all four cases discussed in this subsection. In frame (a) we set the cut off $\Lambda/Q = 20$ while in frame (b) it is increased to 200. There is no qualitative changes introduced from the change in cut off and the quantitative differences are largest as the region $C'_1 = \pm 1$ is approached. This is precisely the region where the change from type I to type II occurs and there is a Lifshitz transition from a point like Fermi surface to the existence of electron and hole pockets at the Weyl point and our linear model is no longer realistic. These issues are further elaborated upon in [48] and [49]. As a result here we have blacked out this region in figure 5 with thick solid vertical black lines.

3.2. $s' = +$ node is tilted clockwise and $s' = -$ node is tilted anti-clockwise

3.2.1. WSM type I. To get the anomalous Hall conductivity in this case we have to add $\mathcal{R}_{\sigma_{xy}^-}$ and $\mathcal{R}_{\sigma_{xy}^+}$ as they appear in equations (15) for $s' = -$ and (19) for $s' = +$ respectively.
Figure 5. We plot with dotted lines the anomalous Hall conductivity $\mathcal{R}_{xy}$ in the units of $e^2 Q/2\pi^2$ against the amount of tilt $C'_s$ for different values of $Q_0/Q$ at charge neutrality $\mu = 0$. The absolute value of the amount of tilt $C'_s$ is same for both of the Weyl cones. This diagram covers both WSM type I as well as type II when both are tilted either clockwise or anti-clockwise. Blue circular dotted line is for $Q_0/Q = 0$, green square dotted line for 0.05, violet diamond dotted line for 0.1, red star dotted line for 0.2, black plus dotted line for 0.3 and magenta cross dotted line for $Q_0/Q = 0.5$. We also show the effect of the cutoff on the anomalous conductivity $\mathcal{R}_{xy}$ by choosing two values of the cutoff, in (a) $\Lambda/Q = 20$ and in (b) $\Lambda/Q = 200$.

Here $C'_s$ stands for the absolute value of the tilts. This gives us,

$$\frac{\mathcal{R}_{xy}}{e^2 Q/2\pi^2} = \frac{\mathcal{R}_{xy}^I}{e^2 Q/2\pi^2} + \frac{\mathcal{R}_{xy}^II}{e^2 Q/2\pi^2}$$

$$= 1 - \frac{1}{2} \left\{ \left( 1 - \frac{1}{C'_s} \right) + \frac{1}{2C'_s^2} \ln \left( \frac{1 - C'_s}{1 + C'_s} \right) \right\} \frac{Q'_0}{Q}.$$

(25)

Which shows that the Hall conductivity is truly universal when the absolute value of the tilts are the same, giving us,

$$\frac{\mathcal{R}_{xy}}{e^2 Q/2\pi^2} = 1$$

(26)

for whatever the value of $Q_0$ may be.

3.2.2. WSM type II. To get the anomalous Hall conductivity in this case we have to add $\mathcal{R}_{xy}^{II}$, and $\mathcal{R}_{xy}^{II, -}$ as they appear in equations (17) (for $s' = -$) and (20) (for $s' = +$) respectively. Here $C'_s$ stands for the absolute value of the tilts. This gives us,

$$\frac{\mathcal{R}_{xy}}{e^2 Q/2\pi^2} = \frac{\mathcal{R}_{xy}^{II}}{e^2 Q/2\pi^2} + \frac{\mathcal{R}_{xy}^{II, -}}{e^2 Q/2\pi^2}$$

$$= \frac{1}{2} \left\{ \left( 1 + \frac{1}{C'_s} \right) - \frac{1}{4C'_s^2} \ln \left( \frac{Q'_0}{C'_s(C'_s - 1)\Lambda^2} \right) \right\} \frac{Q'_0}{Q}.$$

(27)

For same magnitude of tilt we can further simplify it to get,

$$\frac{\mathcal{R}_{xy}}{e^2 Q/2\pi^2} = \frac{1}{C'_s},$$

(28)

again independent of $Q_0$.

4. Anomalous Hall conductivity for finite chemical potential

The generalization to the finite chemical potential case is straightforward and requires the mapping of $Q_0$ of the previous section to $s'\mu_\nu$. The anomalous Hall conductivity for a pair of Weyl nodes for a general value of the distance in momentum space between the nodes ($Q$), shift in energy of the nodes ($Q_0$) due to the violation of inversion symmetry, for a general tilt ($\mu_\nu$) and magnitude of this tilt $C'_s$ taken to be always positive is,

$$\frac{\mathcal{R}_{xy}}{e^2 /2\pi^2} = \sum_{s' = \pm} \left[ \frac{Q}{2C'_s} + \frac{t_{\nu} s'}{4C'_s^2} \ln \left( \frac{\mu^2_{\nu}}{C'_s^2(C'_s - 1)\Lambda^2} \right) \right] \mu_{\nu}.$$

(29)

for $C'_s < 1$ (type I) and

$$\frac{\mathcal{R}_{xy}}{e^2 /2\pi^2} = \sum_{s' = \pm} \left[ \frac{Q}{2C'_s} + \frac{t_{\nu} s'}{4C'_s^2} \ln \left( \frac{\mu^2_{\nu}}{C'_s^2(C'_s - 1)\Lambda^2} \right) \right] \mu_{\nu}.$$

(30)

for $C'_s > 1$ (type II). Here we have assumed that the tilt index $\mu_\nu = \pm 1$ for counterclockwise and clockwise tilt respectively, irrespective of $s'$ where $s'$ is the index on the chirality of the Weyl nodes.

These equations are the central results of this work. The first term in these equations is directly proportional to the distance in momentum space of the two nodes $Q$ and does not depend on doping $\mu \neq 0$ and on the energy shift due to inversion symmetry breaking $Q_0$. The second term is a correction due to $\mu$ and $Q_0$ but does not depend on $Q$ in the approximation used here namely that the cut off $\Lambda$ is much larger than both $Q$ and $Q_0$ in momentum unit. While for simplicity we have assumed that the tilt of the negative chirality node is counterclockwise, changing to clockwise simply changes the sign of this contribution in the second term of both equations (29) and (30) and will not be explicitly treated beyond this comment. It is instructive to write down separately the case of the positive
chirality node tilted to the left (counterclockwise) and to the right (clockwise). In the first case we have
\[\frac{\mathcal{R}_{\sigma_y}}{e^2/2\pi^2} = Q + \frac{1}{2} \sum_{s' = \pm} s' \left\{ \frac{1}{C_{s'}} + \frac{1}{2C_{s'}^2} \ln \left( \frac{1 - C_{s'}}{1 + C_{s'}} \right) \right\} \mu'_s,\]
for \(C_{s'} < 1\) (type I) and
\[\frac{\mathcal{R}_{\sigma_y}}{e^2/2\pi^2} = Q + \frac{1}{2} \sum_{s' = \pm} s' \left\{ \frac{1}{C_{s'}} + \frac{1}{2C_{s'}^2} \ln \left( \frac{1 - C_{s'}}{1 + C_{s'}} \right) \right\} \mu'_s,\]
(31)
for \(C_{s'} > 1\) (type II) and
\[\frac{\mathcal{R}_{\sigma_y}}{e^2/2\pi^2} = Q - \frac{1}{2} \sum_{s' = \pm} s' \left\{ \frac{1}{C_{s'}} + \frac{1}{2C_{s'}^2} \ln \left( \frac{1 - C_{s'}}{1 + C_{s'}} \right) \right\} \mu'_s,\]
for \(C_{s'} < 1\) (type I) and
\[\frac{\mathcal{R}_{\sigma_y}}{e^2/2\pi^2} = Q - \frac{1}{2} \sum_{s' = \pm} s' \left\{ \frac{1}{C_{s'}} + \frac{1}{2C_{s'}^2} \ln \left( \frac{1 - C_{s'}}{1 + C_{s'}} \right) \right\} \mu'_s,\]
(33)
for \(C_{s'} > 1\) (type II). These equations simplify if we assume the magnitude of the tilt is the same for each Weyl node but we have written the formula for the case when, either both are type I or both type II. For the second case when the negative chirality cone is tilted counterclockwise while the positive chirality cone is clockwise we have instead
\[\frac{\mathcal{R}_{\sigma_y}}{e^2/2\pi^2} = Q - \frac{1}{2} \sum_{s' = \pm} s' \left\{ \frac{1}{C_{s'}} + \frac{1}{2C_{s'}^2} \ln \left( \frac{1 - C_{s'}}{1 + C_{s'}} \right) \right\} \mu'_s,\]
\[\frac{\mathcal{R}_{\sigma_y}}{e^2/2\pi^2} = Q - \frac{1}{2} \sum_{s' = \pm} s' \left\{ \frac{1}{C_{s'}} + \frac{1}{2C_{s'}^2} \ln \left( \frac{1 - C_{s'}}{1 + C_{s'}} \right) \right\} \mu'_s,\]
(32)
for \(C_{s'} > 1\) (type II). We have not assumed that the magnitude of the tilts is the same for each Weyl node but we have written the formula for the case when, either both are type I or both type II. For the second case when the negative chirality cone is tilted counterclockwise while the positive chirality cone is clockwise we have instead
\[\frac{\mathcal{R}_{\sigma_y}}{e^2/2\pi^2} = Q - \frac{1}{2} \sum_{s' = \pm} s' \left\{ \frac{1}{C_{s'}} + \frac{1}{2C_{s'}^2} \ln \left( \frac{1 - C_{s'}}{1 + C_{s'}} \right) \right\} \mu'_s,\]
\[\frac{\mathcal{R}_{\sigma_y}}{e^2/2\pi^2} = Q - \frac{1}{2} \sum_{s' = \pm} s' \left\{ \frac{1}{C_{s'}} + \frac{1}{2C_{s'}^2} \ln \left( \frac{1 - C_{s'}}{1 + C_{s'}} \right) \right\} \mu'_s,\]
(34)
for \(C_{s'} > 1\) (type II). These equations simplify if we assume the magnitude of the tilt to be the same for each of the two nodes. Denoting the tilt simply by \(C\) we get for \(C < 1\),
\[\frac{\mathcal{R}_{\sigma_y}}{e^2/2\pi^2} = Q + \left[ \frac{1}{C} + \frac{1}{2C^2} \ln \left( \frac{1 - C}{1 + C} \right) \right] Q_0,\]
which is independent of the chemical potential. For \(C > 1\)
\[\frac{\mathcal{R}_{\sigma_y}}{e^2/2\pi^2} = Q + \frac{1}{C} + \frac{1}{2C^2} \ln \left( \frac{|\mu'^2 - Q_0^2|}{C^2(C^2 - 1)\Lambda^2} \right) Q_0 + \ln \frac{\mu' + Q_0}{\mu' - Q_0} \mu',\]
(35)
which applies to parallel tilts. For opposite tilting of the two nodes we get instead for \(C < 1\)
\[\frac{\mathcal{R}_{\sigma_y}}{e^2/2\pi^2} = Q - \left[ \frac{1}{C} + \frac{1}{2C^2} \ln \left( \frac{1 - C}{1 + C} \right) \right] \mu'_s,\]
which is now independent of the inversion symmetry breaking energy shift \(Q_0\). For \(C > 1\)
\[\frac{\mathcal{R}_{\sigma_y}}{e^2/2\pi^2} = Q - \frac{1}{C} + \frac{1}{2C^2} \ln \left( \frac{|\mu'^2 - Q_0^2|}{C^2(C^2 - 1)\Lambda^2} \right) \mu'_s + \ln \frac{\mu' + Q_0}{\mu' - Q_0} Q_0,\]
(36)
where \(\mu'\) is the chirality and \(\tau_{\text{inter}}\) is an intervalley relaxation time. This contribution is equivalent to a finite \(Q_0\) value of magnitude (given in equation (39)) \(\mu_p\). For small tilt \(C \ll 1\) and the parallel tilt case we need not correct equation (39) for tilt to lowest order and the anomalous Hall conductivity, in our units, will be equal to \(\left( 1 - \frac{C}{6} \frac{\mu_p}{\mu_0} \right)\). For \(\mu_p \sim 10\) meV [39],
Figure 6. We plot with dotted lines the anomalous Hall conductivity $\mathcal{R}\sigma_{xy}$ in the units of $e^2/2\pi^2$ against the amount of tilt $C_1$ for the finite chemical potential ($\mu \neq 0$). It is assumed that the Weyl nodes are tilted parallel to each other and the absolute value of the amount of tilt is same for both of the Weyl nodes and assumed to be $C_1$. This diagram covers both WSM type I as well as type II when both are tilted either clockwise or anti-clockwise. In panel (a) we fix $Q_0^\prime/Q = 0.5$ and generate plots with dotted lines for different values of the chemical potential normalized by $Q$. Blue circular dots are for $\mu^\prime/Q = 0$, red square dots for 0.2, green diamond dots for 0.4, violet star dots for 0.6, magenta plus dots for 0.8 and black cross dots for $\mu^\prime/Q = 1$. In panel (b) we fix $\mu^\prime/Q$ at 0.5 and plot for different values of $Q_0^\prime/Q$ namely blue circular dotted line for $Q_0^\prime/Q = 0$, red square dotted line for 0.2, green diamond dotted line for 0.4, violet star dotted line for 0.6, magenta plus dotted line for 0.8 and black cross dotted line for $Q_0^\prime/Q = 1$.

$v_c \sim 3 \times 10^5$ m s$^{-1}$ [24] and $Q \sim 0.08$ Å [8], $\mu_p/hv_cQ$ is of the order 0.06 which is small.

5. Nernst and thermal Hall

In a recent paper Ferreiros et al [49] have discussed the effect of tilt on the Nernst and thermal Hall coefficient in Weyl semimetals. Here we generalize their work to noncentrosymmetric materials. Both Nernst ($\alpha_{xy}$) and thermal Hall conductivity ($\kappa_{xy}$) depend on the Hall conductivity $\mathcal{R}\sigma_{xy}$ of the previous sections which from here on we will denote simply by $\sigma_{xy}$ suppressing the real part index. They are [49]

$$\alpha_{xy} = \frac{\pi^2 k_B^2 T}{6e} \frac{d\sigma_{xy}}{d\mu}$$

and

$$\kappa_{xy} = \frac{\pi^2 k_B^2 T}{3e^2} \sigma_{xy}$$

with $k_B$ the Boltzmann constant, $e$ the electronic charge and $T$ the temperature. The thermal Hall coefficient in equation (41) is simply proportional to the Hall conductivity and is therefore given by equations (35) to (38) multiplied by $\pi^2 k_B^2 T$. As we have discussed in the previous section for type I Weyl, the Hall conductivity has a logarithmic singularity (equations (35) and (36)) as the Lifshitz transition at $C = 1$ is approached. Our results are no longer expected to be quantitatively accurate in this region. For type II there is a related logarithmic singularity at $\mu = Q_0$. This is expected since the effective chemical potential of the negative chirality node is zero at $\mu = Q_0$ and we are again probing the nodal region for which our continuum Hamiltonian requires modification. Here we have introduced a cut off in momentum $\Lambda$ to regularize our results. In view of this limitation we will consider only the case $Q_0/\mu < 1$ and $\mu/Q_0 < 1$. Both logarithms can be expanded in these two limiting cases. Equation (36) for parallel tilt then takes the form,

$$\mathcal{R}\sigma_{xy} \propto \frac{Q}{C} + \frac{1}{2C^2} \left\{ \ln \left( \frac{\mu^2}{C(C^2-1)\Lambda^2} \right) + 2 \right\} Q_0^\prime$$

and

$$\alpha_{xy} = \frac{e k_B^2 T}{6} \left( \frac{Q_0^\prime}{\mu^\prime} \right)^3 \left( \frac{1}{C^2} \right) \left( \frac{\mu^\prime}{Q_0^\prime} \right)^2$$

with $\kappa_{xy} = \pi^2 k_B^2 T \sigma_{xy}$.

This means that for $C < 1$ (type I) (see equation (35)) the Nernst coefficient is zero for parallel tilts and for $C > 1$ it is,

$$\alpha_{xy} = \frac{e k_B^2 T}{6} \left( \frac{Q_0^\prime}{\mu^\prime} \right)^3 \left( \frac{1}{C^2} \right) \left( \frac{\mu^\prime}{Q_0^\prime} \right)^2$$

with $\kappa_{xy} = \pi^2 k_B^2 T \sigma_{xy}$.

For $Q_0 = 0$ we recover the result found in [49] that $\alpha_{xy} = 0$. At finite $Q_0$ with $Q_0/\mu < 1$ the Nernst coefficient is finite with leading order proportional to $Q_0/\mu$ and first correction of order $Q_0^\prime/\mu^3$. For $\mu/Q_0 < 1$, $\alpha_{xy}$ is also finite and to leading order in $\mu/Q_0$, is proportional to $\mu$ and inversely proportional to $Q_0$.

For oppositely tilted Weyl nodes (tilt inversion symmetry applies) we get for type I

$$\alpha_{xy} = \frac{-e k_B^2 T}{6C} \left[ 1 + \frac{1}{2C} \ln \left\{ \frac{1 - C}{1 + C} \right\} \right].$$
which in the limit of the tilt going to zero, gives $e k_F^2 T C/18$. This has the opposite sign to that in [49] because we are dealing here with a counterclockwise tilt on the negative chirality node with the tilt of the positive chirality node clockwise. For type II a similar set of equations to equations (42) and (43) are obtained from equation (38) by changing the overall sign of the second term in these equations and switching the variable $Q_0$ and $\mu$. For $Q_0/\mu < 1$

$$\alpha_{xy} \cong -\frac{e k_F^2 T}{12 C^2} \left[ \ln \left( \frac{\mu'^2}{C^2 (C^2 - 1)^2} \right) + 2 - \left( \frac{Q_0}{\mu'} \right)^2 \right], \quad (47)$$

which in the $Q_0 = 0$ limit gives

$$\alpha_{xy} \cong -\frac{e k_F^2 T}{6 C^2} \left[ \ln \frac{\mu'}{\sqrt{C^2 - 1}} + 1 \right]. \quad (48)$$

This agrees with [49]. Here we have an additive correction for finite $Q_0$ equal to $\frac{e k_F^2 T}{12 C^2} \left( \frac{\mu}{\mu'} \right)^2$. For the opposite limit $\mu/\mu_Q < 1$ to leading order

$$\alpha_{xy} \cong -\frac{e k_F^2 T}{6 C^2} \left[ \ln \frac{Q_0}{\sqrt{C^2 - 1}} + 1 - \frac{1}{2} \left( \frac{\mu'}{Q_0} \right)^2 \right]. \quad (49)$$

which is, in the leading order, independent of chemical potential $\mu'$.

6. Discussion and conclusions

We have studied the anomalous DC Hall conductivity ($\sigma_{xy}$) in a continuum Dirac model Hamiltonian which additionally includes a tilt ($C$) as well as time reversal (TR) and inversion (I) symmetry breaking terms. These last two terms lift the two fold degeneracy of the Dirac cone creating a pair of Weyl nodes of opposite chirality ($\gamma' = \pm$). For broken TR symmetry the Weyl nodes are displaced in momentum space by $-\gamma' Q$ while for inversion symmetry breaking the displacement is in energy $-\gamma' Q_0$. We employ a Kubo formula to calculate the Hall conductivity and derive simple analytic expressions for $\sigma_{xy}$ which depend on tilt, $Q$ and $Q_0$ as well as on chemical potential $\mu$. For the case of charge neutrality and $Q_0 = 0$ (centrosymmetric case) we recover the known result when the magnitude of the tilt is taken to be the same on each nodes. For type I Weyl ($C < 1$) $\sigma_{xy} = e^2 Q/2\pi^2$ and for type II ($C > 1$) $\sigma_{xy} = e^2 Q/2\pi^2 C$. Both are proportional to the momentum space displacement $Q$. For type I the result is universal independent of tilt while for type II it is inversely proportional to $C$. Adding finite $\mu$ and/or finite $Q_0$ (noncentrosymmetric case) does not change in any way the contribution proportional to $Q$ but adds a correction which depends on $C$, $Q_0$ and $\mu$ but not on $Q$. The relative orientation of the tilts affects this contribution. It matters whether the tilts are parallel to each other which violates tilt inversion symmetry or they oppose each other which respect tilt inversion symmetry. For centrosymmetric Weyl with parallel tilts we recover the known result that there is no correction for finite $\mu$ while for oppositely oriented tilts it is proportional to $\mu$ for type I with a further logarithmic correction dependent on a momentum cut off $\Lambda$ multiplying $\mu$ for type II which makes this contribution non linear. For the general case the analytic expressions describing the $\mu$, $Q_0$ correction has the same general form for both tilt configuration except that the two variables $Q_0$ and $\mu$ are interchanged and the overall sign of this contribution is reversed. For oppositely tilted Weyl cones (tilt inversion symmetry is respected) and type I the correction to $e^2 Q/2\pi^2$ contribution is linear in $\mu$ and independent of $Q_0$ with coefficient dependent only on tilt magnitude. For parallel tilted Weyl this contribution is now linear in $Q_0$ and completely independent of $\mu$. For type II both variables $Q_0$ and $\mu$ feature in the correction term which is non linear and contains the momentum cut off $\Lambda$ on the $k_z$ integration as recorded in equations (36) and (38). Expansions for $\frac{\pi \sigma_{xy}}{\mu^2/2}$ valid for $\mu/\mu_Q < 1$ and for $Q_0/\mu < 1$ are derived and given in equations (42) and (43) for parallel tilts. Equivalent equations for oppositely directed tilts can be obtained from these through an overall change in sign and switching the role of $Q_0$ and $\mu$.

We have also considered the effect of broken inversion symmetry on the Nernst effect and on the thermal Hall conductivity. Recently Ferreiros et al [49] have considered these transport coefficients when only time reversal symmetry is violated. Our results properly reduce to those of [49] when we set $Q_0 = 0$. For finite $Q_0$ we find corrections. The thermal Hall follows directly from the results that we have just summarized for the Hall conductivity after multiplying by $\pi^2 k_F^2 T/3 e^2$. The Nernst requires taking a derivative of $\sigma_{xy}$ with respect to the chemical potential. For parallel tilts we find that the Nernst effect remains zero in type I Weyl but it is no longer zero for type II with leading correction instead of order $Q_0/\mu < 1$ and $Q_0/\mu < 1$. For oppositely oriented tilts there is no correction to the Nernst coefficient due to $Q_0$ for type I, but for type II there is a correction of order $Q_0/\mu^2$ for $Q_0/\mu < 1$. For $\mu/\mu_Q < 1$, to leading order $\sigma_{xy}$ depends logarithmically on $Q_0$ and on the momentum cut off $\Lambda$ but not on $\mu$. A first correction is of order $(\mu/\mu_Q)^2$.

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