Proposed Ranking Function to Solve the Fuzzy Project Management and Network Problem

Huda Fadhil Abbass 1, Idean Hassan AL-kanani 2

University of Baghdad, college of science for women 1.
University of Baghdad ,college of science for women 2.

hoda.fadel1203a@csw.uobaghdad.edu.iq
IdenALKanani@yahoo.com

Abstract. In this paper, we create a relationship between network model and project management by using data of cost to the residential project. We solved these data by using the crisp network method (real data). After that, by using the same data we convert it to fuzzy data then the fuzzy data tested by trapezoidal membership function to be sure it belong to the duration [0,1]. Proposed ranking function had solved the fuzzy network problem to the fuzzy data.

Keywords. Project Network, Ranking Function, Fuzzy Numbers, Critical Path Technique.

1. Introduction

Operations research and management are an essential part of scientific life, because organizations are always looking to study the most effective and productive way to run their business, so the number of studies increased to a large extent in recent years although initial studies were mainly confined to research and management science witnessed a massive increase in all parts of the world, including other developed countries [1].

The project management and network problem are well known in operations research for its wide application in various real life fields. Then the project management and network model developed at the mid-1950s, to produce the critical path technique by two teams. The first team is an American and the second team is British. Zadeh [2] explanation of fuzzy logic and its membership function relationship with it and test the mathematical relations on the fuzzy set. Network model is one of the operation research topics, which is convert the phenomena of life. Which had no mathematical formula into a mathematical problem had a formula and a solution, Gorham [3] used the network model to solve a problem related to military training in order to obtain the lowest cost of training. Project management one of the most important topic that related with network model, Taylor and Moore [4] had been used Q-Gert network model to get less time for the project. Gerry M. Klein [5] illustrate the relationship of properties of network model using the fuzzy set.

Ranking function is method to find the ideal solution of fuzzy number to ordinary set. Goguen [6] this paper explores the foundations of generalizes and continues the work of Zadeh by using ranking function.
The aim of this research is to demonstrate the difference between the traditional network (crisp) solution and the fuzzy network solution by using Proposed algorithm for cost data to the residential project, as we found that the solution using the fuzzy network is more accurate to find the minimum cost to complete the project.

In section two some definitions of fuzzy set theory, in section three trapezoidal fuzzy number, in section four ranking function with the algorithm is explained, in section five includes the partial aspect, in section six the conclusion are made.

2. Fuzzy Set Theory

In this section, dealing with some definition of fuzzy set theory which are as follows:

- **Definition [2]**: Let U be a universe set. A fuzzy set A of U is defined by a membership function \( \mu_A(x) \rightarrow [0, 1] \), where \( \mu_A(x) \), \( \forall x \in U \), indicates the degree of x in A.

- **Definition [7]**: A fuzzy number is a fuzzy subset in support R (real number) which is both "normal" and "convex", where \( \sup(\tilde{A}) = \{ x \in R | \mu_A > 0 \} \).

- **Definition [8]**: The fuzzy subset A of the real line ~, with the membership function \( \text{It:~} \rightarrow [0, 1] \), is a fuzzy number iff:
  
  (a) A is normal, i.e., there exist an element \( x^* \in R \) such that \( \mu(x^*) = 1 \)
  
  (b) A is fuzzy convex, i.e., \( \mu(\lambda x + (1-\lambda)y) \geq \mu_x \wedge \mu_y \), \( \forall x,y \in R \) and \( \forall 0 \leq \lambda \leq 1 \)
  
  (c) \( \mu \) is upper semicontinuous;
  
  (d) \( \text{supp} (\mu) \) is bounded.

3. Trapezoidal Fuzzy number[9]

A trapezoidal fuzzy number A is a fuzzy number with membership function \( \mu_A \) defined by

\[
\mu_A(x) = \begin{cases} 
\frac{x-a_l}{a_l-a_t} & a_l \leq x < a_t \\
1 & a_t \leq x \leq a_m \\
\frac{a_r-x}{a_r-a_m} & a_m < x \leq a_r \\
0 & o.w 
\end{cases} \tag{1}
\]

which can be denoted as a quartet \((a_l, a_t, a_m, a_r)\).

4. Ranking function

It is utilize for ordering the fuzzy numbers. There are many types of ranking function have been used to solving the problems in operation researches.

The ranking function from the set of all fuzzy numbers \( F(R) \), where \( R:F(R) \rightarrow [0,1] \), that means \( F(R) \) is function of collection for fuzzy number define over real line, then the map of every fuzzy number for the real line.
Therefore, it is used for transforms the fuzzy numbers to crisp numbers (traditional numbers) which equivalent to the fuzzy network model, then the optimal solution of the crisp model used as the optimal solution of fuzzy network model.

- Algorithm of Ranking Function (Proposed Algorithm)

In this section, we use the idea of Yager and Detryniecki in (2001), to study and know the interaction between valuation parameters and ranking function then calculate the valuation of a generalized trapezoidal fuzzy numbers.

If $\tilde{A}=(a, b, c, d; \lambda)$ fuzzy numbers which is generalized from trapezoidal function, then the formula of membership is

$$
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{\lambda(x-a)}{b-a}, & a \leq x < b \\
\frac{\lambda}{b-a}, & b \leq x \leq c \\
\frac{\lambda(d-x)}{d-c}, & c < x \leq d \\
0, & \text{o.w}
\end{cases} \quad \ldots (2)
$$

Where $\lambda$ is weighted function between $0<\lambda<1$. Now, utilizing the anew valuation which symbol by Val ($\tilde{A}$) which achieved the following integral:

$$
\text{Val} (\tilde{A}) = \int_0^\lambda \text{Average} [\tilde{A}(\alpha)] d\alpha \quad \ldots (3)
$$

Where

$$
\tilde{A}(\alpha) = \{ x \mid \tilde{A}(x) \geq \alpha \} \text{ is the level set of } \tilde{A}.
$$

The average $\tilde{A}(\alpha)$ is obtain by $L(\alpha)$ and $U(\alpha)$ by using a generalized trapezoidal membership function which is as follows:

$$
\text{Average} [\tilde{A}(\alpha)] = \frac{L(\alpha) + U(\alpha)}{2} \quad \ldots (4)
$$

Where $L(\alpha)$ is lower $\alpha$-level set of $\tilde{A}$ substituting by $x$ and $U(\alpha)$ is upper $\alpha$-level set of $\tilde{A}$ substituting by $x$. Then $L(\alpha)$ and $U(\alpha)$ is obtain by the following formulas:

$$
\alpha = \frac{\lambda (L(\alpha) - a)}{b-a} \\
\alpha (b - a) = \lambda (L(\alpha) - a) \\
\alpha b - \alpha a = \lambda L(\alpha) - \lambda a \\
\lambda L(\alpha) = \alpha b - \alpha a - \lambda a \quad \ldots (5)
$$
And

\[ \alpha = \frac{\lambda(d - U(\alpha))}{d - c} \]
\[ \alpha(d - c) = \lambda(d - U(\alpha)) \]
\[ \alpha d - \alpha c = \lambda d - \lambda U(\alpha) \]
\[ \lambda U(\alpha) = \lambda d - \alpha d + \alpha c \]
\[ U(\alpha) = d - \frac{\alpha}{\lambda}(d - c) \quad \ldots(6) \]

\[ \text{Average} \left[ \bar{A}(\alpha) \right] = \frac{\frac{\alpha}{\lambda}(b - a) + a + d - \frac{\alpha}{\lambda}(d - c)}{2} \]
\[ \text{Average} \left[ \bar{A}(\alpha) \right] = \frac{(b-a)\alpha + (a+d)\lambda - (d-c)\alpha}{2\lambda} \]
\[ \text{Average} \left[ \bar{A}(\alpha) \right] = \frac{ba - a\lambda + a\lambda + d\lambda - d\alpha + ca}{2\lambda} \]
\[ \text{Average} \left[ \bar{A}(\alpha) \right] = \frac{(b + c)\alpha + a(\lambda - \alpha) + d(\lambda - a)}{2\lambda} \]
\[ \text{Average} \left[ \bar{A}(\alpha) \right] = \frac{(b + c)\alpha + (\lambda - \alpha)(a + d)}{2\lambda} \quad \ldots(7) \]

Then the proposed formula for a class of valuation function which are as follows:

\[ \text{Val} (\bar{A}) = \int_{0}^{\lambda} \text{Average} \left[ \bar{A}(\alpha) \right] \lambda f(\alpha) d\alpha \]
\[ \int_{0}^{\lambda} f(\alpha) d\alpha \quad \ldots(8) \]

Where \( f(\alpha) \) is a mapping from \([0,\lambda]\) to \([0,\lambda]\). Then we proposed two parameters functions from the class of valuation function. The first one is an increasing function with following equation

\[ f : \alpha \rightarrow f(\alpha) = \alpha^{q+1} \quad \text{with} \quad q \geq 0 \quad \ldots(9) \]

The second one is an decreasing function with following equation

\[ f : \alpha \rightarrow f(\alpha) = (1 - \alpha)^{q+1} \quad \text{with} \quad q \geq 0 \quad \ldots(10) \]

To calculate the valuation procedure of generalized trapezoidal fuzzy numbers which define as follows:

\[ \bar{A} = (\text{left support}, \text{left core}, \text{right core}, \text{right support}) \]
Therefore, applying the valuation formula in equation(8) by subsisting equation (7) and function (9) to reach to the following formula:

\[
\text{Val}(\bar{\lambda}) = \frac{\int_0^\lambda \left[ \frac{(b+c)\alpha + (\lambda - \alpha)(a+d)}{2\lambda} \right] \lambda \alpha^{q+1} d\alpha}{\int_0^\lambda \alpha^{q+1} d\alpha}
\]

\[
\text{Val}(\bar{\lambda}) = \frac{\int_0^\lambda \frac{\alpha^{q+2}}{\alpha^{q+1}} d\alpha}{\int_0^\lambda \alpha^{q+1} d\alpha} + \frac{\int_0^\lambda \frac{(\lambda - \alpha)(a+d)}{2\lambda} \alpha^{q+1} d\alpha}{\int_0^\lambda \alpha^{q+1} d\alpha}
\]

\[
\text{Val}(\bar{\lambda}) = \frac{1}{2} (b+c) \frac{\alpha^{q+3}/(q+3)}{\alpha^{q+2}/(q+2)} + \frac{1}{2} \alpha^{q+1} d\alpha - \frac{1}{2} (a + d) \frac{\alpha^{q+2} d\alpha}{\alpha^{q+1} d\alpha}
\]

\[
\text{Val}(\bar{\lambda}) = \frac{1}{2} (b+c) \frac{\lambda^{q+3}/(q+3)}{\lambda^{q+2}/(q+2)} + \frac{1}{2} \lambda (a+d) - \frac{1}{2} (a + d) \frac{\lambda^{q+2}/(q+2)}{\lambda^{q+1}/(q+2)}
\]

\[
\text{Val}(\bar{\lambda}) = \frac{1}{2} (b+c) \frac{q+2}{q+3} + \frac{1}{2} \lambda (a+d) - \frac{1}{2} \lambda \frac{q+2}{q+3}
\]

\[
\text{Val}(\bar{\lambda}) = \frac{\lambda^{q+3}/(q+3)}{\lambda^{q+2}/(q+2)}
\]

Where \( w = \lambda^{q+2}/(q+3) \) and \( 1-w = (\lambda - \lambda^{q+2}/(q+3)) \)

Finally, computing the \( w \) and \( (1-w) \), where \( w \) is weighted function between 0 < \( w \) < \( \lambda \).

\[
w = \frac{\int_0^\lambda \alpha f(\alpha) d\alpha}{\int_0^\lambda f(\alpha) d\alpha}
\]

\[
w = \frac{\int_0^\lambda \alpha^{q+2} d\alpha}{\int_0^\lambda \alpha^{q+1} d\alpha} = \frac{\alpha^{q+3}/(q+3)l_0^\lambda}{\alpha^{q+2}/(q+2)l_0^\lambda}
\]

\[
w = \frac{\lambda^{q+3}}{q+3} \frac{q+2}{\lambda^{q+2}} = \lambda \frac{q+2}{q+3}
\]

\[
(1-w) = \left( \lambda - \lambda \frac{q+2}{q+3} \right)
\]
Now, substituting valuation of $\tilde{A}$ in the ranking function getting the following ranking function:

$$R(\tilde{A}) = \frac{1}{2} (b + c)w + \frac{1}{2} (a + d)(1 - w) \ldots (11)$$

5. **Partial Aspect [10]**

The modern world is testify a great development in the field of civil projects, commercial, housing. In the developing countries made a rebound in the field of planning which led to treatment of the problems which represent obstacle in the work. Project construction had been development depend on planning technique. Previously, companies had used Gantt technique which determine the time between each of the two activities and the start and end times of the project. Gantt technique had been development to Pert technique, it determines the start time for each activity. After that Pert technique development to critical path technique to specify the end time for each activity.

The project is about a residential project consist of 96 Residence units on a land area of 5630 meters square. Every unit consist of four floors, 38 months is the duration of finish the project which estimated the origin time regards as the Gantt technique. Now, demonstrate the project description and the cost of each stage.

The following table is represented the basic activate for project and the cost of this project:

| Activity                              | Cost in $  | Symbol of activity |
|---------------------------------------|------------|--------------------|
| Equipping and installing work teams and equipment | 1250       | A                  |
| Primary sanitation works              | 1750       | B                  |
| Excavation and foundation drilling    | 8064       | C                  |
| Asphalt concrete infrastructure       | 77414.3775 | D                  |
| Waterway service                      | 30000      | E                  |
| Equipped with water and gas pipelines| 12500      | F                  |
| Equipped electricity                  | 25110.52   | G                  |
| Asphalt concreter superstructure      | 50000      | H                  |
| Black paint ceilings                  | 30805.45   | I                  |
Carpentry
Blacksmithing work
Building
Coating the walls with cement
Preparing the external perimeter of the units
Paint
Glass processing
Cover the floor with tiles

Now, TABLE1. can be graph by using the network model and project management with cost by designer in following chart.

**CHART1.** The Network for Activity Time in Weeks
5.1 The Crisp Algorithm

The traditional (crisp) algorithm for the network model and project management of a residential project from the CHART1, then the minimum-cost capacitated as the following:

Then, the minimum-cost capacitated network algorithm is $(234858.2875)\$$. 

5.2 The Fuzzy Algorithm

If the fuzzy algorithm applying for the network model and project management of a residential project from CHART1 too. After that using fuzzy numbers for applying the trapezoidal membership function to be sure that data is belong to the duration $[0,1]$. The formula of Fuzzy number is $(a - \Delta_1, b + \Delta_1, c - \Delta_2, d + \Delta_2)$ where $\Delta_1 = 100$and $\Delta_2 = 200$, for example:

Fuzzy numbers $(1250-100, 1250+100, 1250-200, 1250+200)$=$(1150,1350,1050,1450)$, and so on for the other activity cost in\$. therefore applying proposed ranking function by using equation (11) as follows where the values of fuzzy numbers $\hat{A}=(a,b,c,d,\lambda)$ let $\lambda=[0.1,0.9]$ is:

$$a=1150, b=1350, c=1050, d=1450$$

and substitute the weight function $w \in [0,1]$ then choosing $w=0.1$ now, the results of ranking function for activity cost in\$, put it in the following table as follows:

| Activity | Real number | Fuzzy numbers | $W=0.1$ | $W=0.9$ |
|----------|-------------|---------------|---------|---------|
| 1250     | $(1150,1350,1050,1450)$ | 1290          | 1210    |
which is find that the critical path techniques for the fuzzy numbers when \((w=0.1)\) is \((235177.8825)\).
The minimum-cost capacitated network algorithm for fuzzy cost is $(234498.2875)_w$ for $w=0.9$.

6. Conclusions

In this paper shows the relationship between project management and network model by using fuzzy numbers with ranking function ,if we employ the critical path technique for algorithms (crisp algorithm , fuzzy algorithm )the minimum-cost capacitated network algorithm for fuzzy cost had been found for fuzzy algorithm when $w=0.9$ ,so the  fuzzy algorithm is better if $w \in [0.6,0.9]$ .

References

[1] C. Bilir, C. Cu‘ngo‘r, and (O.)‘K_okalan(2020) , Operation Research \ Management Science Research in Europe:A bibliometric overview Hindawi , Advance in operation research, pp.1-14.
[2] Yu-Jie Wang and Hsuan-Shih Lee(2008) , The revised method of ranking fuzzy numbers with an area between the centroid and original points , Computers Mathematics with Applications,Vol. 55;Iss.9.
[3] W.Gorham(1963) , An Application of a Network Flow Model to Personnel Planning,IEEE transaction on Engineering Management.
[4] B. W. Taylor and L.J. Moore (1980 ) , R&D Project Planning with Q-GERT Network Modeling and Simulation,Institute for Operations Research and the Management Sciences (INFORMS) INFORMS is located in Maryland USA,Vol. 26NO.1.
[5] Gerry M. Klein, Circuits(1990), Networks, and Power Systems, Math/ Compu. Modelling, Vol. 14,p p. 336-339.
[6] J. A. Goguen(1967) , L-Fuzzy Sets , Journal of Mathematical Analysis and Applications, Vol. 18,pp145-174.
[7] C.H. Cheng and D.L. Mon(1993) , Fuzzy system reliability analysis by confidence interval, Fuzzy Sets and Systems 56,29-35.
[8] Przemyslaw Grzegorzewski(1998), Metrics and orders in space of fuzzy numbers, Fuzzy Sets and Systems,Vol.97;Iss.1.
[9] Yu-Jie Wang(2015), Ranking triangle and trapezoidal fuzzy numbers based on the relative preference relation, Applied Mathematical Modelling, Vol. 39; Iss.2.
[10] Shamcham Hafiza(2014) , TheDeference between Traditional and Modern Networking Models Project Planning and planning Case study: Social housing project Biskra Faculty of Economics, Commerce Management Sciences,pp107-143.
[11] Hamdy A.Taha(2017) , Operations Research an Introduction, tenth edition,pp247-288.