Constitutive model of shape memory alloy under the effect of martensite plasticity for finite-element applications

Aicheng Zou$^{1,2}$, Yanping Wang$^3$, Qidong Zhang$^4$

$^1$Nanjing University of Aeronautics and Astronautics, Nanjing, People’s Republic of China
$^2$Guilin University of Aerospace Technology, Guilin, People’s Republic of China
$^3$College of Medicine, Xi’an International University, Xi’an, People’s Republic of China
$^4$Zhengzhou Tobacco Research Institute, Zhengzhou, People’s Republic of China

E-mail: jz97011311@aliyun.com

Abstract: This study is focused on the superelastic Ni-based shape memory alloy (SMA) for finite-element applications. SMA has presented excellent characterisations in biomedical application, mechanical application, aerospace application, seismic prevention, disaster reduction and so on. In this study, a SMA constitutive model is proposed that is capable of describing SMA superelastic features and its plasticity effects. Finally, the analysed results show good ability to predict the experimental data.

1 Introduction

Shape memory alloys (SMAs) have been used in many fields for medical applications including stents, endodontics, sutures, medical tweezers, anchors for attaching tendon to bone, implants, aneurism treatments, eyeglass frames and guide wires and other medical devices and equipments in many fields including neurology, orthopedics, cardiology and interventional radiology [1].

Some works are focused on the implementation of constitutive model of SMA into finite-element package [2–4]. However, there are very few constitutive models considering the plastic deformation of the martensite under the conditions of stress-induced martensite transformation. Recently, Saint-Sulpice and Ivshin [5, 6] developed a temperature-dependent three-dimensional phenomenological constitutive model considering the local plastic yield of martensite under high stress, and then successfully implemented into a finite-element package ABAQUS.

In this work, a temperature-independent phenomenological constitutive model is proposed to describe the thermo-mechanical deformation of NiTi alloy combining superelastic and plasticity features. The prediction capability of the proposed model is verified by the experimental results. Finally, the numerical example is given to verify the validity of the implementation.

2 Constitutive model

Generalised plasticity is first employed by Ivshin [2] to depict the thermo-mechanical responses of SMAs. With infinitesimal strain assumption, the additive decomposition of the total strain $\varepsilon$ into an elastic strain $\varepsilon_e$ and an inelastic strain $\varepsilon_i$ yields

$$\varepsilon = \varepsilon_e + \varepsilon_i$$

(1)

Based on the assumption of small deformation, the total inelastic strain $\varepsilon_i$ is the stress-induced martensitic transformation $\varepsilon_t$ and the plasticity transformation $\varepsilon_p$. Therefore, the total inelastic strain $\varepsilon_i$ is expressed as

$$\varepsilon_i = \varepsilon_t + \varepsilon_p$$

(2)

By discretising the equation, the expression is as follows:

$$\Delta \varepsilon_i = \Delta \varepsilon_t + \Delta \varepsilon_p$$

(3)

The stress-induced martensitic transformation and its reverse transformation can be defined by the volume fraction of martensite as the internal variable $\xi$. The elastic stress–strain relation can be then expressed as

$$\sigma = \bar{D}^* \cdot \varepsilon$$

(4)

where $\bar{D}^*$ is the equivalent elastic tensor, and $\Delta \varepsilon_i = 2 \bar{G} \Delta \varepsilon_{\text{in}}$, and

$$\bar{s} = s^* - 2 \bar{G} \Delta \varepsilon_{\text{in}}$$

(5)

As mentioned in [7–9], some phase transformations are pressure dependent. To model such an effect, the Von-Mises-type transformation surfaces are introduced

$$F(\sigma, q) = \sigma_{\text{eq}} - \bar{\sigma}(q) = 0$$

(6)

where $\sigma_{\text{eq}} = \{3/2 \bar{G} \cdot \varepsilon\}^{1/2}$ is the von Mises equivalent stress. The phase transformation to describe the forward transformation and its reverse transformation are introduced as follows:

forward transformation

$$\bar{F}_{\text{AM}}(\sigma, \xi) = \sigma_{\text{eq}} - \sigma_{\text{AM}}^M(\xi) = 0,$$

(7a)

reverse transformation

$$\bar{F}_{\text{MA}}(\sigma, \xi) = \sigma_{\text{eq}} - \sigma_{\text{MA}}^M(\xi) = 0,$$

(7b)

where $\sigma_{\text{AM}}^M$ and $\sigma_{\text{MA}}^M$ are the start stresses of the forward transformation and the reverse transformation, respectively.

In this paper, the reversible martensite volume fraction $\xi$ is related to the transformation strain $\varepsilon_t$ by the following expression:
\dot{\varepsilon} = \dot{\varepsilon}_1 \tag{9}

in which, \(\varepsilon_1\) is the maximum phase transformation strain, which is determined by the experiment.

Similar to the normality rule of the plasticity strain, the transformation strain rates obey the normality rule forward phase transformation

\[ \Delta \dot{\varepsilon}^p_1 = \sqrt{\frac{3}{2}} \varepsilon_1 \Delta \dot{\varepsilon}_{\text{AM}}, \Delta \dot{\varepsilon}^p > 0, \tag{10a} \]

reverse phase transformation

\[ \Delta \dot{\varepsilon}^n_1 = \sqrt{\frac{3}{2}} \varepsilon_1 \Delta \dot{\varepsilon}_{\text{MA}}, \Delta \dot{\varepsilon}^n < 0, \tag{10b} \]

with

\[ \dot{n}_{\text{AM}} = \frac{\partial F_{\text{AM}}(\sigma, \dot{\varepsilon})}{\partial \sigma} = \sqrt{\frac{3}{2}} \frac{\dot{\varepsilon}}{\sigma_{eq}} \tag{11a} \]

\[ \dot{n}_{\text{MA}} = \frac{\partial F_{\text{MA}}(\sigma, \dot{\varepsilon})}{\partial \sigma} = \sqrt{\frac{3}{2}} \frac{\dot{\varepsilon}}{\sigma_{eq}} \tag{11b} \]

In this paper, the phase transformation, austenite yield and martensite yield behaviour under different temperature are considered in the process of implicit stress integral solution. Considering the implicit stress integration process of the phase change behaviour, (6) can be further expressed as

\[ s = s' - 2 \bar{G} \Delta \varepsilon_1 \tag{12} \]

By (10a-b), it can be obtained forward phase transformation

\[ \Delta \dot{\varepsilon}^p_1 = \sqrt{\frac{3}{2}} \varepsilon_1 \Delta \dot{\varepsilon}_{\text{AM}}, \Delta \dot{\varepsilon}^p > 0, \tag{13a} \]

reverse phase transformation

\[ \Delta \dot{\varepsilon}^n_1 = \sqrt{\frac{3}{2}} \varepsilon_1 \Delta \dot{\varepsilon}_{\text{MA}}, \Delta \dot{\varepsilon}^n < 0, \tag{13b} \]

in which

\[ \dot{n}_{\text{AM}} = \dot{n}_{\text{MA}} = \dot{n}_1 = \sqrt{\frac{3}{2}} \frac{\dot{\varepsilon}}{\sigma_{eq}} \tag{14} \]

If the loading is continued up to exceed the martensite plastic yield stress after finishing its stress-induced phase transformation, the martensite plastic deformation will occur. In this research, it is assumed that the plastic yield behaviour of martensite subject to von Mises yield criterion, and the plastic yield condition can be expressed as

\[ F_{\text{Mp}}(\sigma, \dot{\varepsilon}) = \sigma_{eq} - \sigma_{\text{M}}^y(\dot{\varepsilon}) = 0 \tag{15} \]

in which, the internal variable \(\dot{\varepsilon}\) is the cumulative plastic strain as

\[ \Delta \dot{\varepsilon} = \left[ \frac{2}{3} \Delta \dot{\varepsilon}_{\text{p}}, \Delta \dot{\varepsilon}_{\text{p}} \right] \frac{1}{(1/2)} \]

and \(\sigma_{\text{M}}^y\) is the plastic yield stress martensite.

The plastic strain rate can be expressed as

\[ \Delta \dot{\varepsilon}_{\text{p}} = \Delta \bar{\varepsilon}_{\text{p}} = \Delta \lambda_{\text{M}} \frac{\partial F_{\text{Mp}}(\sigma, \dot{\varepsilon})}{\partial \sigma} \sqrt{\frac{3}{2}} \Delta \dot{\varepsilon}_{\text{M}} \tag{16} \]

martensite plastic yield

\[ \lambda_{\text{M}} \] is the plastic multipliers of the martensite plastic yield. \(\dot{n}_M\) is the direction vector of the martensite plastic yield

\[ \lambda_{\text{M}} = \frac{\| \dot{n}_M \|}{\sigma_{eq}} = \sqrt{\frac{3}{2}} \frac{\dot{\varepsilon}}{\sigma_{eq}} \tag{17} \]

3 Verification of the proposed model

The results of the phase transformation simulations by the proposed model are compared with the experimental data by Kang et al. [10]. To verify the proposed model, the super-elastic stress–strain curves at different temperatures are simulated using the parameters listed as follows to identify the parameters.

\[ E_A = 41.0 \text{ GPa}; \quad E_M = 37.0 \text{ GPa}; \quad \nu_A = \nu_M = 0.33; \quad C_{AM} = 8.0 \text{ MPa/K}; \quad C_{MA} = 8.8 \text{ MPa/K}; \quad k = 0.16; \quad \sigma_{\text{AM}}^y = 353.0 \text{ MPa}; \quad \sigma_{\text{MA}}^y = 381.0 \text{ MPa}; \quad \sigma_{\text{AM}}^p = 141.0 \text{ MPa}; \quad \sigma_{\text{MA}}^p = 122.0 \text{ MPa}; \quad \varepsilon_S = 0.035; \quad T_s = 295 \text{ K}; \quad h_{\text{M}} = 6.7 \text{ GPa}. \]

The uniaxial tension and unloading case of the material is calculated based on the proposed model. The simulation results are shown in Fig. 1. It can be seen that the stress-induced martensitic transformation occurs. With higher applied peak strain, an apparent residual strain can be observed which is caused by the plastic deformation of the martensite. The simulated results show the good agreement with the experimental ones. The results show that the proposed constitutive model can predict the thermodynamic behaviour in the super-elastic NiTi alloy. It can be predicted that the model can also give a reasonable prediction result for other working conditions and other experimental results in the temperature range.

Fig. 2 shows a finite-element model of the tension bar of SMA with multiple elements, with the length of 20 mm, width and height of 1 mm. The fixed boundary condition is applied at one end, and at the other side uniform pressure 370 and 1715 MPa (direction}
along the X-direction) is applied, and then unloaded to zero. It can be seen that the tension bar shows obvious elongation at stage A with the applied uniform pressure 370 MPa. A large deformation about 10% can be observed at the following stage B with the applied uniform pressure 1715 MPa. In this stage, the plastic deformation has been taken place because the equivalent stress exceeds the plastic yielding stress of martensite at 1700 MPa. The unrecoverable strain could also be found in stage C of applied loading as unloaded to zero. Therefore, it can be concluded that the finite-element implementation developed in this work is valid and reasonable.

4 Conclusion
A SMA phenomenological constitutive model considering martensite plasticity with implementation into the finite-element code ANSYS is proposed in the general inelastic framework. The numerical results show the good simulation results compared with the experimental ones. Finally, the numerical analyses for a tension bar of SMA with multiple elements present the valid and reasonable finite-element implementation of the proposed constitutive model.

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6 References
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