The effect of fracture density and stress state on the static and dynamic bulk moduli of Westerly granite

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Abstract

Elastic properties are key parameters during the deformation of rocks. They can be measured statically or dynamically, but the two measurements are often different. In this study, the static and dynamic bulk moduli ($K_{\text{static}}$ and $K_{\text{dynamic}}$) were measured at varying effective stress for dry and fluid-saturated Westerly granite with controlled fracture densities under isotropic and differential stress states. Isotropic fracturing of different densities was induced in samples by thermal treatment to 250, 450, 650, and 850°C. Results show that fluid saturation does not greatly affect static moduli but increases dynamic moduli. Under isotropic loading, high fracture density and/or low effective pressure results in a low $K_{\text{static}}/K_{\text{dynamic}}$ ratio. For dry conditions $K_{\text{static}}/K_{\text{dynamic}}$ approaches 1 at low fracture densities when the effective pressure is high, consistent with previous studies. Stress-induced anisotropy exists under differential stress state that greatly affects $K_{\text{static}}$ compared to $K_{\text{dynamic}}$. As a result, the $K_{\text{static}}/K_{\text{dynamic}}$ ratio is higher than that for the isotropic stress state and approaches 1 with increasing axial loading. The effect of stress-induced anisotropy increases with increasing fracture density. A key omission in previous studies comparing static and dynamic properties is that anisotropy has not been considered. The standard methods for measuring static elastic properties, such as Poisson’s ratio, Young’s and shear modulus, involve subjecting the sample to a differential stress state that promotes anisotropy. Our results show that stress-induced anisotropy resulting from differential stress state is a major contributor to the difference between static and dynamic elasticity and is dominant with high fracture density.

1. Introduction

Quantifying the elastic properties of rock is essential in the geomechanical modeling of the subsurface with applications ranging from the understanding of crustal stresses responsible for driving tectonic processes to reducing drilling risk and maximizing reservoir productivity in oil and gas exploration. Elastic properties can be calculated from stress and strain data during slow deformation of a rock, referred to as the static properties, but requires access to the rock mass, which is not always practicable or possible. $P$ and $S$ wave velocity can be inverted also to give the elastic properties, and seismic waves can access all parts of the crust. The inversion of elastic wave velocity gives dynamic elastic properties. However, the static elastic properties are often different from the dynamic elastic properties but are required in mechanical modeling when slow loading predominates. Consequently, interest in quantifying the relationships between the static and dynamic properties has been a goal for more than 100 years [Nagaoka, 1900; Zisman, 1933; Ide, 1936].

The measurement of elastic properties in the laboratory typically involves the assumption that the sample is elastically isotropic. For static elastic property measurements, the standard methodology has been to apply an elastic axial load to a strain-gauged cylindrical specimen and then use the applied load and axial and radial strains to calculate the Young’s modulus and Poisson’s ratio, from which the other elastic parameters can be calculated if isotropy is assumed. It is difficult to characterize even transverse isotropy (with radial symmetry) using static measurements, and even then, several cores in different directions are required [e.g., Amadei, 1996; Liao et al., 1997b; Chou and Chen, 2008; Kim et al., 2012]. For dynamic elastic properties, only the $P$ and $S$ wave velocity in one direction is required to determine the elastic properties of an isotropic sample. In order to characterize transverse isotropy, five independent velocity paths must be known [e.g., Liao et al., 1997a; Mavko et al., 1998].
An understanding of the difference between static and dynamic elastic properties is important, and several studies have tried to identify a relationship between them [King, 1983; Vanheerden, 1987; Eissa and Kazi, 1988; Wang, 2000; Mockovičková and Pandula, 2003]. However, determining a universal relationship has proved elusive, with several reasons being suggested for the differences between the two methods of measurement. These include the magnitude of the strain amplitude applied, the rate of the application of the stress (leading to adiabatic loading for static measurements and isothermal loading for dynamic measurements), the presence of microfractures, the frequency at which the moduli is determined, whether the sample is dry or saturated, and the type of pore fluid [Hammond et al., 1979; Jizba, 1991; Tutuncu et al., 1998a, 1998b; Batzle et al., 2001, 2006; Adam and Batzle, 2008; Adelinet et al., 2010; Müller et al., 2010]. Another factor, which we consider in this work, is the influence of stress-induced anisotropy, produced by the application of an elastic axial load, the effect of which will be sampled differently by static loading and elastic waves and may lead to differences if isotropy is assumed. One way around producing anisotropy is to apply only an isotropic stress state from which the bulk modulus (\(K\), defined as the change in effective stress (confining pressure, \(\sigma_3 - \sigma_2 = \sigma_3\), minus pore pressure) divided by a corresponding change in volumetric strain, \(\Delta V/\Delta V_0\)) can be determined.

Simmons and Brace [1965] showed a range of materials subjected to an isotropic stress state (\(\sigma_1 = \sigma_2 = \sigma_3\)) in excess of 200 MPa, gave the same value for static and dynamic bulk compressibility within experimental error. Below 200 MPa, these authors saw differences up to several hundred percent between the statically and dynamically measured bulk compressibility. Simmons and Brace [1965] and Cheng and Johnston [1981] found that the ratio of static to dynamic bulk modulus (\(K_{\text{static}}/K_{\text{dynamic}}\)) of dry Westerly granite is approximately 0.5 at atmospheric pressure and increases to 0.9 at and above 200 MPa effective pressure (confining pressure minus pore pressure). The main reason for the difference between the static and dynamic elastic properties is ascribed to the presence of microfractures which affect the strain of the entire rock specimen during a static measurement more than the propagation characteristics of ultrasonic waves during a dynamic measurement. There is a migration in the \(K_{\text{static}}/K_{\text{dynamic}}\) ratio towards unity due to the closure of the microfractures with increasing effective stress.

In this work, we extend the work of Simmons and Brace [1965] and Cheng and Johnston [1981] to consider the effect of fracture density on the \(K_{\text{static}}/K_{\text{dynamic}}\) ratio under both wet (water saturated) and dry (atmospheric) conditions as a function of effective pressure up to 130 MPa. Isotropic microfracture networks were induced in Westerly granite samples by thermal treatment at 250, 450, 650, and 850°C. Furthermore, we explore the notion that stress-induced anisotropy can affect the static and dynamic elastic properties by introducing elastic differential loads at each effective pressure from which the Young’s modulus and Poisson’s ratio are measured, and the bulk modulus calculated, assuming isotropy.

2. Methodology
2.1. Experimental Procedures

Experiments were carried out on Westerly granite samples from southeast Rhode Island, USA. The samples are generally considered to be isotropic, have low initial fracture density, small grain size (average 1 mm), and total porosity of less than 1%. Westerly granite has an extensive history of laboratory testing and can provide a high level of repeatability under carefully controlled test conditions [Brace, 1965; Nur and Simmons, 1969; Walsh and Brace, 1972; Lockner, 1998; Haimson and Chang, 2000]. The volumetric modal mineralogy of the samples is 27% quartz, 36% microcline, 30% plagioclase, 6% phyllosilicates, and 1% accessory phases [Atkinson, 1984; Meredith and Atkinson, 1985]. For viable data sets and to permit direct comparison, the samples were all cored (20 mm diameter) from the same block and in the same orientation. They were cut so that their length-to-diameter ratios were in excess of 2.5 but less than 3 and precision ground to strict tolerances (±0.02 mm) as to the squareness of their ends in accordance with typical rock mechanics procedures [Paterson and Wong, 2005].

A servo-controlled triaxial deformation apparatus, installed at the Rock Deformation Laboratory, University of Liverpool, was used for the measurements. The apparatus can measure the confining and pore pressure to better than 0.02 MPa, and the internal load cell has a resolution better than 0.1 kN. The confining pressure, pore pressure, and axial load systems are all servo-controlled. The sample arrangement is shown in Figure 1.
A PVC jacket was used to separate the samples from the confining low-viscosity (10 cs) silicon oil. Prior to jacketing the samples, two axial and two radial strain gauges were attached to the central part of the sample with strain gauge adhesive. The strain gauge connections were fed out through a hole made in the jacket, and this hole was sealed with a compliant epoxy resin. The connections were then fed out of the vessel through the high-pressure electrical feedthroughs (Figure 1), and each strain gauge was wired into a Wheatstone bridge configuration. The pairs of axial and radial strain gauge outputs were identical within experimental error and were used to determine the static elastic properties. The strain gauge measurements give similar strain amplitudes as transducer-based measurements, indicating the validity of our results [Heap et al., 2010].

Figure 1. Schematic diagram of triaxial apparatus and elastic wave velocity measurement system.
polarization orientation. The through-transmission method was employed to measure the $P$ and $S$ wave velocity of the sample in an axial direction [Birch, 1960]. Sintered stainless steel, 0.125 inch thick with 0.5 μm nominal pore size, was used as a backing material on the piezoelectric ceramic [VanValkenburg, 1983]. The elastic wave impedance of the backing matched the elastic wave impedance of the piezoelectric ceramic resulting in a heavily damped piezoelectric ceramic with wider bandwidth that displays good range resolution [Safari and Akdoğan, 2008]. A negative spike pulser/receiver (JSR DPR300 Pulser/Receiver) was used for excitation of the piezoelectric ceramics and detection of the $P$ and $S$ waveforms. The choice of piezoelectric ceramics to excite or receive is manually controlled by a switch box. A 300 MHz bandwidth digital oscilloscope with 20 ppm time-based accuracy (Tektronix TDS 3032B) was employed for recording and display of the received waveforms which is synchronized with the pulser/receiver. The oscilloscope was also used to average 512 waveforms to increase the signal-to-noise ratio. The pulser/receiver and oscilloscope are coupled to a computer for storage of data and processing of the recorded waveforms for analysis. The American Society for Testing and Materials (ASTM) standard for using ultrasonic testing to determine pulse velocities [ASTM Standard D2845-08, 2008] was adhered to for accurate and reliable measurement of $P$ and $S$ wave velocity. The velocity of the PVC jacket is much less than that of the rock; therefore, there will be no boundary effects that will influence the results. This experimental configuration and methodology resulted in very clear $P$ and $S$ wave arrivals that could easily be identified (see Figure 2). $P$ and $S$ wave velocity are determined as the length of the sample ($L$) divided by the travel time of the waves ($t$), corrected for the change in length due to the applied stress:

$$V_p \text{ or } V_s = \frac{L}{t} \quad (1)$$

where $V_p$ is the $P$ wave velocity and $V_s$ is the $S$ wave velocity. Dynamic elastic properties such as bulk modulus, $K_{\text{dynamic}}$, can be estimated from the $P$ and $S$ wave velocity using the well-known relationships for isotropic materials [e.g., Kuttruff, 1991]:

$$K_{\text{dynamic}} = \rho \left( \frac{V_p^2 - \frac{4}{3} V_s^2}{V_s^2} \right) \quad (2)$$

where $V_p$ is the $P$ wave velocity, $V_s$ is the $S$ wave velocity, and $\rho$ is the density of the sample.
The change in the sample's mass and density, calculated from porosity data [from Nasseri et al., 2007] and volumetric strain \(\epsilon_v = \epsilon_a + 2 \epsilon_r\) which occurs during hydrostatic and axial compression, was accounted for in the calculations.

Experiments were performed on dry and deionized water-saturated samples at 10, 30, 50, and 130 MPa effective stress. Pore pressure pipes are connected into the top and bottom loading platens. The pore pressure connections to the sample were unconnected and left open to ambient laboratory conditions for experiments on dry samples. After completion of dry experiments, the samples were fully saturated with deionized water by injecting the fluid via the upper and lower pore pressure connections at 10 MPa effective pressure for several hours until the pressure across the sample was constant. The pressurization rate was kept at ~10 MPa/min, and the confining pressure was held constant at the required experimental conditions. The pore pressure was servo-controlled at 10 MPa for all saturated experimental conditions. The failure strength of Westerly granite at experimental conditions was first determined (see Table 1). The samples were then loaded axially, \(\sigma_1\), to 25% of their failure strength at a rate of ~8 MPa/min. At regular increments, the load was held constant for a period of ~10 s to record the \(P\) and \(S\) wave data, while the static data were recorded continuously.

A second pressurization cycle was carried out to measure axial and radial strains as a function of effective stress. The method used to calculate static bulk modulus is described in Cheng and Johnston [1981], where the volumetric strain versus effective stress curve is differentiated at 10, 30, 50, and 130 MPa effective stress. For isotropic stress changes the output from the axial and radial strain gauges was checked against each other to ensure both were giving the same value (isotropic strain), and the volumetric strain was computed from the output from one gauge.

For measurements that involved the application of an elastic differential load, the outputs from both strain gauges were used to calculate the bulk modulus. The sample was loaded to 7, 13, 20, and 25% of the sample's failure stress, calculated from the data shown in Table 1. The static Young's modulus and Poisson's ratio of the sample were computed by curve fitting to the loading data, then taking the local slope at the loading level of interest (hence determining the tangent moduli). The standards for determining compressive strength and elastic moduli of intact rock core specimen [ASTM Standard D7012–14, 2014] were followed to calculate the static Young's modulus and Poisson's ratio. Bieniawski [1967a] found that the linear elastic stage finishes at ~35% of the failure strength of the sample, while Brace et al. [1966] found that this stage can end at one third to two thirds of the failure strength of the sample. Hence, in this study, the static Young's modulus and Poisson's ratio were calculated at 7, 13, 20, and 25% of the failure strength. These stress levels will ensure that the static elastic properties are calculated above the crack-closure region and below the crack propagation region of the stress-strain curve [Brace et al., 1966; Bieniawski, 1967a, 1967b, 1967c]. Therefore, the deformation is entirely elastic. For dynamic elastic measurements, the \(P\) and \(S\) wave velocity was measured under the same stress conditions as the static measurements were made; then elastic isotropy was assumed to calculate the bulk modulus, using equation (2).

**2.2. Thermal Fracturing of Westerly Granite**

A suite of intact samples were heated at room pressure in a furnace (Carbolite CSF 1200) at a rate of 0.25°C/min to 250, 450, 650, and 850°C where the temperature was kept constant for 2 h. The samples were then cooled at a rate of 0.25°C/min to room temperature. These temperatures will remove moisture within the sample, but some adsorbed water may have reestablished itself on the mineral surfaces upon cooling at room temperature. The low rate of heating and cooling was used to ensure that microfracturing events resulted only from the temperature effect producing fractures from anisotropic thermal expansion of constituent minerals and not due to thermal gradients across the sample. This ensures that the fracture distribution will be as close to homogeneous and isotropic as possible [Cooper and Simmons, 1977].

The volume and mass of the sample increases and decreases, respectively, as the temperature increases during thermal treatment (Table 3). There is a significant increase in the total fracture density and porosity above 573°C as a result of grain boundary opening and cracking and intragranular cracking which is related to the quartz \(\alpha\beta\) phase transition [Glover et al., 1995; Nasseri et al., 2007].

| Effective Stress (MPa) | Effective Differential Failure Stress (MPa) |
|------------------------|--------------------------------------------|
| \(10\)                | \(335\)                                   |
| \(30\)                | \(494\)                                   |
| \(50\)                | \(606\)                                   |
| \(130\)               | \(827\)                                   |

*Table 1. Failure Strength of Westerly Granite at Conditions Commensurate to the Pressure Conditions Used in the Experiments in This Work*

*Failure Strength of Westerly Granite at Experimental Conditions in This Work*
Scanning (backscattered) electron microscope (SEM) images (Figure 3) show that samples thermally treated at 650 and 850°C mainly have grain boundary cracks with mismatched faces, whereas samples thermally treated up to 450°C have hairline cracks. The hairline cracks are along grain boundaries and within grains and are assumed to be easily closed under stress.

Microfracture density was determined for each sample from SEM images by counting the number of microfractures that intersected a line of length 1.5 times the average grain diameter [e.g., Wilson et al., 2003; Mitchell and Faulkner, 2009]. A total of 30 lines were randomly placed on SEM images from which fractures from all minerals that intersected the lines were counted. An average microfracture density was then deduced from the densities of the 30 lines. To determine the average fracture length and aperture, 30 fractures were randomly chosen and traced using polylines (see Table 2). The elastic behavior of the fractures will

![Figure 3. Scanning (backscattered) electron microscope images showing intragranular and grain boundary fractures in (a) untreated and thermally treated Westerly granite samples ((b) TF250°C, (c) TF450°C, (d) TF650°C, and (e) TF850°C thermally treated at 250, 450, 650, and 850°C, respectively).]

| Sample | Fracture Density (number per mm) | Fracture Length (μm) | Apertures (μm) |
|--------|---------------------------------|-----------------------|---------------|
| Untreated | 0.6 ± 0.1 | 74.8 ± 12.4 | 0.6 ± 0.1 |
| TF250 | 1.0 ± 0.1 | 284.8 ± 38.3 | 1.3 ± 0.1 |
| TF450 | 2.2 ± 0.1 | 400.5 ± 29.0 | 1.5 ± 0.1 |
| TF650 | 4.7 ± 0.2 | 486.5 ± 35.9 | 7.9 ± 0.6 |
| TF850 | 5.7 ± 0.3 | 548.5 ± 70.2 | 10.5 ± 0.9 |

Q = Quartz
P = Plagioclase
K = K-feldspar
bt = Biotite.
be sampled by the elastic waves as the wavelength of $P$ (that range from 2.7 to 4 mm) and $S$ waves (that range from 1 to 2.3 mm) are larger than the maximum fracture length and aperture.

The mass loss of the sample treated to 650 and 850°C were investigated using X-ray diffraction to establish if the thermal treatment resulted in the dehydroxylation of biotite or chlorite that typically occurs at temperatures above 400°C [Sanz et al., 1983]. X-ray diffraction analysis (see Figure 4) shows that the biotite peak for the samples treated to 650 and 850°C are diminished, but not completely absent. Figure 4 also shows the shrinkage of chlorite structure at 650°C. At 850°C, the chlorite crystal structure is completely destroyed. These observations are consistent with the mass loss recorded during the thermal treatment of the samples (Table 3).

### Table 3. Percentage Change in Volume and Mass of Samples as They Are Thermally Treated to a Prescribed Temperature

| Temperature (°C) | % Increase in Volume | % Decrease in Mass |
|------------------|----------------------|--------------------|
| 250              | 0.01                 | 0.12               |
| 450              | 0.36                 | 0.14               |
| 650              | 1.69                 | 0.31               |
| 850              | 2.98                 | 0.47               |

3. Results

#### 3.1. Application of Isotropic Effective Stress

Figure 5 shows volumetric strain and $P$ and $S$ wave velocity as a function of effective stress for both dry and saturated conditions from which static and dynamic bulk moduli are
calculated. The data collected consistently show that the dynamic bulk moduli are greater than the static bulk moduli (Figure 6). There is an increase in both the static and dynamic bulk moduli with increasing isotropic effective stress (Figures 6a and 6b). At 130 MPa effective stress, the bulk modulus of samples heat treated up to 450°C are approximately equal to that of the untreated sample. The static and dynamic bulk moduli decrease systematically with samples that have higher fracture density, with the decrease significant for samples thermally treated to 650 and 850°C. Treatment to higher temperatures affects the static moduli more compared to the dynamic bulk moduli. The S wave velocity could not be measured at 10 MPa isotropic effective stress for dry samples heat treated to 650 and 850°C because the waveform was attenuated and within the noise; hence, the dynamic bulk modulus could not be determined. The static bulk modulus shows no significant difference between the dry and fluid-saturated conditions, which is corroborated by results documented by Heap et al. [2009]. The dynamic bulk moduli under fluid-saturated conditions are greater than those in dry conditions and are largely not stress sensitive, apart from the samples treated to the highest temperatures, which show a small decrease at the very lowest effective pressures (Figure 6b).

Figure 7 shows the effect of these changes on the ratio of $K_{\text{static}}/K_{\text{dynamic}}$. The ratio of $K_{\text{static}}$ to $K_{\text{dynamic}}$ for untreated Westerly granite under dry conditions agrees with the results of Simmons and Brace [1965] and Cheng and Johnston [1981], where the $K_{\text{static}}$ is much lower than $K_{\text{dynamic}}$ at low pressure, but converge at higher pressure. The same is true for the samples heat treated to 250 and 450°C, although the $K_{\text{static}}/K_{\text{dynamic}}$ ratios at low pressures become progressively smaller (Figure 7a). Under saturated conditions, the same pattern is evident, although the $K_{\text{static}}/K_{\text{dynamic}}$ ratio is always smaller than under dry conditions (Figure 7b).
3.2. Application of Differential Stress

An example of the stress-strain and velocity data for the untreated sample is shown in Figure 8 from which static and dynamic bulk moduli are calculated. We also calculated the static and dynamic shear moduli (see Appendix A). Figures 9a and 9b show that the untreated Westerly Granite sample, for both dry and saturated conditions, has a small pressure dependence of the static and dynamic bulk modulus. Again, the dynamic measurements give higher values. The application of an axial load to 7% and 25% of the failure stress increases the moduli slightly but does not greatly affect the overall pattern, apart making the sample slightly less pressure sensitive.

For a sample with higher fracture density (treated to 650°C), the static bulk modulus is not greatly affected by saturation, but the application of axial stress clearly increases this value (Figure 9c). The dynamic bulk modulus shows a similar pattern for the dry case, but, when saturated, the pressure sensitivity of the bulk moduli disappears and perhaps shows a slight negative dependence at low pressures (Figure 9d).

Figure 10 shows the $K_{\text{static}}/K_{\text{dynamic}}$ ratio for untreated and treated samples as a function of effective mean stress $[(\sigma_1 + \sigma_2 + \sigma_3)/3 - \text{pore pressure}; \text{where } \sigma_1 > \sigma_3 \text{ and } \sigma_3 = \sigma_2]$ under dry conditions. There is a striking lack of sensitivity of $K_{\text{static}}/K_{\text{dynamic}}$ to the effective mean stress for the samples with a low fracture density, with the ratio always close to 1. With samples with higher initial fracture density (TF450, TF650, and TF850), the $K_{\text{static}}/K_{\text{dynamic}}$ ratio starts to show pressure sensitivity at lower pressures. For TF850, the highest axial load shows a very high $K_{\text{static}}/K_{\text{dynamic}}$ ratio at the lowest effective stress. Under saturated conditions, the $K_{\text{static}}/K_{\text{dynamic}}$ ratio shows similar trends, although slightly more pressure sensitivity is evident (Figure 11).

Figure 6. (a) Static and (b) dynamic bulk modulus versus isotropic effective stress. Measurements were conducted on untreated and thermally treated Westerly granite samples (TF250, TF450, TF650, and TF850 thermally treated at 250, 450, 650, and 850°C, respectively). Solid and dashed lines represent dry and saturated condition, respectively. The static and dynamic error is less than 7 and 1%, respectively.

Figure 7. $K_{\text{static}}/K_{\text{dynamic}}$ ratio versus isotropic effective stress for (a) dry and (b) fluid-saturated conditions. Measurements were conducted on untreated and thermally treated Westerly granite samples (TF250, TF450, TF650, and TF850 thermally treated at 250, 450, 650, and 850°C, respectively).
4. Discussion

Clear changes in the ratio between static and dynamic measurements are seen in the results presented, despite such factors as strain amplitude and frequency difference between the static and dynamic measurements effectively being held constant in our measurements. This indicates that while these factors may play a part in the difference between static and dynamic measurements, the fracture density and the stress state have a dominant role.

4.1. The Effect of Fractures and Isotropic Stress on the $K_{\text{static}}/K_{\text{dynamic}}$ Ratio

Fractures are opened and closed depending on the stress state to which they are subjected. The application of an isotropic stress state to isotropic rocks should close fractures uniformly in all directions. In this work, the isotropic closure of fractures is shown to increase the $K_{\text{static}}/K_{\text{dynamic}}$ ratio towards unity. The increase of $K_{\text{static}}/K_{\text{dynamic}}$ ratio with increasing effective pressure of the untreated sample corroborates the results previously documented by Simmons and Brace [1965] and Cheng and Johnston [1981]. However, the $K_{\text{static}}/K_{\text{dynamic}}$ ratio presented in this paper is slightly higher at low effective stress because these bulk moduli were measured during the second pressurization cycle whereas the above mentioned authors calculated the bulk moduli during the first pressurization cycle [Bernabe, 1986; Faulkner and Rutter, 1998; Armitage et al., 2011].

The fracture density within the Westerly granite samples clearly plays a major role in determining $K_{\text{static}}/K_{\text{dynamic}}$ ratio. The work presented here has extended the studies of Simmons and Brace [1965] and Cheng and Johnston [1981] to include fracture density as a variable. The fracture density varies with the heat treatment applied and...

Figure 8. (a) Differential stress-radial strain, (b) differential stress-axial strain, and (c) $P$ wave and (d) $S$ wave velocity at varying effective stress ($\sigma_3^{\text{eff}}$) of untreated Westerly granite sample. Solid line and dashed lines represent dry and fluid-saturated conditions, respectively. The clusters in the stress-strain curves are where we held the differential stress constant to measure $P$ and $S$ wave data.
also with the effective pressure. As pressure is applied, fractures are closed so that the rock mass behaves as though they were not there, and hence, the fracture density may be viewed as decreasing, even though the fractures are still present.

4.2. The Effect of Stress-Induced Elastic Anisotropy

Subjecting the rock to a differential stress state causes fractures that are oriented perpendicular and obliquely to the axial loading direction ($\sigma_1$) to close and those oriented parallel to $\sigma_1$ to open. The aperture of the axially oriented fractures will increase elastically with increasing axial load which predominantly adds to the radial strain and affects the $S$ wave velocity, as the $S$ wave is polarized perpendicular to these fractures. However, once the aperture of perpendicular and obliquely oriented fractures is closed, increasing the axial loading will not add to the axial strain or affect the $P$ and $S$ wave velocity. Axially loading the rock causes it to become anisotropic, although it is loaded within the elastic region of the stress-strain curve. This stress-induced anisotropy appears to have a greater effect on the static bulk modulus compared to the dynamic bulk modulus, causing the static bulk modulus to be greater and resulting in an increase in the $K_{\text{static}}/K_{\text{dynamic}}$ ratio (Figures 10 and 11). A possible explanation is because dynamic elastic properties are calculated from wave propagation that interacts with fractures only within the propagating path, while the static elastic properties are influenced by all of the fractures within the sample.

These results have significant implications for the standard methods that are used to determine static elastic properties. As a differential stress state is employed to measure the static elastic properties, such as Young’s modulus, Poisson’s ratio and shear modulus, and isotropy is assumed, the resultant elastic properties could be subject to a significant error as differential stress state promotes anisotropy. This, coupled with poor characterization of the fracture density of rocks, is likely one of the primary reasons why correlating static and dynamic elastic properties has proved so problematic in previous studies.

Figure 9. Static and dynamic bulk modulus sample data set at 7 and 25% of the failure strength with varying effective stress for (a and b) low and (c and d) high fracture density. Solid and dashed lines represent dry and saturated condition, respectively. The static and dynamic error is less than 7 and 1%, respectively.
Figure 12 schematically summarizes the above discussion. The application of a differential stress causes transverse isotropy around the $\sigma_1$ axis which results in a slight linear increase in the $K_{\text{static}}/K_{\text{dynamic}}$ ratio with increasing effective mean stress when the fracture density is low (see Figure 12 (ii)). At high fracture density, the $K_{\text{static}}/K_{\text{dynamic}}$ ratio increases significantly with increasing effective mean stress and becomes higher than those at low fracture density. The $K_{\text{static}}/K_{\text{dynamic}}$ ratio behaves differently when the samples are isotropically loaded (see Figure 12 (i)) where it increases exponentially with increasing effective stress. An increase in the fracture damage causes a downward shift in the $K_{\text{static}}/K_{\text{dynamic}}$ ratio.

4.3. The Effect of Strain Amplitude During Differential Loading

Static elastic properties are determined by increasing stress in steps which is accompanied by large strain amplitudes, generally of the order $>10^{-3}$, whereas dynamic elastic properties are determined by wave propagation with extremely small strain amplitude, generally of the order $<10^{-6}$. A number of studies have...
suggested the differences in strain amplitude between static and dynamic measurement as a primary cause for the difference between static and dynamic elastic properties [Hilbert et al., 1994; Martin and Haupt, 1994; Tutuncu et al., 1994; Tutuncu et al., 1998a]. They found that as the strain amplitudes decrease to levels similar to ultrasonic wave propagation, the Young’s modulus increases to a value close to the dynamic value.

Even though the samples were loaded within the elastic region, it may be expected that some inelastic reversible strain, linked to sliding between the faces of closed cracks that are oriented obliquely to the axial loading direction, may take place [Walsh, 1965; Bieniawski, 1967b; Kachanov, 1982; David et al., 2012]. The strain amplitude is of key importance, as large strain amplitudes will promote sliding along cracks, whereas cracks will be unaffected by small strain amplitudes. In our results, the differential load applied is kept small in order to minimize the effect of sliding along oblique cracks. The very minor amount of hysteresis in the stress-strain curves suggests that the effect is minimal. Nevertheless, any sliding of cracks will contribute to the axial and

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**Figure 11.** $K_{\text{static}}/K_{\text{dynamic}}$ ratios as a function of effective mean stress for saturated condition. $K_{\text{static}}/K_{\text{dynamic}}$ ratios were measured at 0, 7, 13, 20, and 25% of the failure strength. The sample is subjected to an isotropic stress state when there is no axial load. During axial loading, the sample is subjected to a differential stress state at varying effective stress. The static and dynamic error is less than 7 and 1%, respectively.
radial strains, and hence, the static Young modulus and Poisson’s ratio decreases and increases, respectively, which leads to an increase in the static bulk modulus. The amount of strain associated with sliding increases with increasing applied axial loading and increasing fracture density, as crack surface area increases with temperature during thermal treatment of samples.

4.4. The Effect of Thermal Treatment

Thermally treating Westerly granite samples generally causes isotropic fracturing due to inhomogeneous strain caused by the anisotropic thermal expansion of randomly oriented minerals that generate internal stresses that drive fracture growth [Cooper and Simmons, 1977]. Glover et al. [1995] carried out acoustic-emission experiments that monitor the process of thermal fracturing as it occurs during heating and found a strong peak of fractures at the \( \alpha - \beta \) phase transition temperature for quartz (\( \approx 573^\circ \text{C} \)) and another clear peak of fractures at \( \approx 800^\circ \text{C} \) that may be attributed to oxidation-dehydroxylation reactions of hornblende and chlorite. This corroborates well with our results where we (1) observed significant reduction in the \( K_{\text{static}}/K_{\text{dynamic}} \) ratio for samples thermally treated to 650 and 850°C in comparison to the samples.

Figure 12. Summary figure illustrating how stress and initial fracture density affects the \( K_{\text{static}}/K_{\text{dynamic}} \) ratio. The block diagrams illustrate schematically how the initially isotropic fracture networks develop with stress under isotropic loading (i) and elastic differential loading (ii). The graphs show the evolution of the \( K_{\text{static}}/K_{\text{dynamic}} \) ratio at the loading and fracture density conditions.
treated at lower peak temperatures (450 and 250°C) and (2) observed that the chlorite crystal structure is completely destroyed for sample heated to 850 in the XRD analysis (Figure 4).

The $K_{\text{static}}/K_{\text{dynamic}}$ ratio at 130 MPa effective stress of samples thermally treated at 650 and 850°C is much lower than those of the other samples, suggesting that either the cracks are being propped open and withstanding the high stress or that more equant pores are being produced that are difficult to close under pressure, perhaps by the breakdown of hydrous phases within the rock. This interpretation is supported by the mass loss of TF650 and TF850 (Table 3) and also the loss of chlorite and biotite peaks in the XRD analysis (Figure 4).

5. Summary

Elastic properties are strongly influenced by fracture damage and state of stress within the rock. The static and dynamic moduli were measured under two different stress states—iso- tropic stress and differential stress. We found that the fracture density increases as a result of thermal treatment which is significant for samples thermally treated to 650 and 850°C. Fractures affect the static bulk modulus more than the dynamic bulk modulus. Under isotropic stress state, fractures are closed uniformly in all directions which results in an increase in the $K_{\text{static}}/K_{\text{dynamic}}$ ratio toward unity due to the closure of the microfractures with increasing effective stress. As the fracture density increases, the $K_{\text{static}}/K_{\text{dynamic}}$ ratio decreases systematically.

An application of a differential stress state causes fractures that are oriented perpendicular and parallel to the axial loading direction to close and open, respectively. This leads to stress-induced anisotropy even though the isotropic samples are loaded within the elastic region of the stress-strain curve. Under differential stress state, the $K_{\text{static}}/K_{\text{dynamic}}$ for samples that have a low fracture density (below 2.2) is less than those under isotropic stress state. For samples with fracture density above 2.2, the $K_{\text{static}}/K_{\text{dynamic}}$ ratio is greater. Increasing the axial load causes $K_{\text{static}}/K_{\text{dynamic}}$ to approach 1. Although differential loading brings the static and dynamic bulk modulus closer together, it shows a significant difference when compared to $K_{\text{static}}/K_{\text{dynamic}}$ measured under isotropic stress conditions, indicating that stress-induced anisotropy is responsible for the change.
Fluid saturation of the samples resulted in a significant increase in the $P$ wave velocity which led to an increase in the dynamic bulk modulus. In the saturated condition the $K_{\text{static}}/K_{\text{dynamic}}$ ratio is less than and follows the same trend as the dry case.

**Appendix A**

One of the principal aims of this work was to compare elastic properties determined under two stress states—hydrostatic loading and differential loading. The bulk modulus is the only elastic parameter that can be measured during both hydrostatic load and differential load. However, the shear modulus is an important parameter, and hence, included below is a summary of the changes in shear modulus in the experiments.

The static shear modulus ($\mu_{\text{static}}$) can only be measured under differential loading and is calculated from the measured Young's modulus and Poisson's ratio, assuming isotropy. Figures 13 shows the results of the shear modulus under a differential stress state. The dynamic measurements ($\mu_{\text{dynamic}}$) give higher values compared to the static ones. The application of an axial load to 7% and 25% of the failure stress does not cause a significant change for sample with low fracture density (untreated sample) but increases the moduli slightly for high fracture density (sample treated to 650°C) at low effective mean stresses. Water saturation of the samples causes the dynamic moduli to increase and be less sensitive to axial loading. Figure 14 shows the ratio of $\mu_{\text{static}}/\mu_{\text{dynamic}}$ as a function of effective mean stress for dry and saturated condition. There is lack of sensitivity of $\mu_{\text{static}}/\mu_{\text{dynamic}}$ to the effective mean stress. Pressure sensitivity is only seen for samples with high fracture density at low effective mean stress.
The application of an axial load causes the sample to become anisotropic which predominantly increases the radial strain. In general, the shear moduli behave similar to bulk moduli under differential stress state, but the changes are not as pronounced because the component of radial strain ($\varepsilon_r$) in the calculation of the static bulk modulus (equation (A1)) is tripled compared to the static shear modulus (equation (A2)).

$$G_s = \frac{\sigma_1 - \sigma_3}{3(\varepsilon_1 - 2\varepsilon_3)} \quad (A1)$$

$$\mu_s = \frac{\sigma_1 - \sigma_3}{2(\varepsilon_1 + \varepsilon_3)} \quad (A2)$$

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