Neutron stars within the SU(2) parity doublet model

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Abstract. The equation of state of beta-stable and charge neutral nucleonic matter is computed within the SU(2) parity doublet model in mean field and in the relativistic Hartree approximation. The mass of the chiral partner of the nucleon is assumed to be 1200 MeV. The transition to the chiral restored phase turns out to be a smooth crossover in all the cases considered, taking place at a baryon density of just $2\rho_0$. The mass-radius relations of compact stars are calculated to constrain the model parameters from the maximum mass limit of neutron stars. It is demonstrated that chiral symmetry starts to be restored, which in this model implies the appearance of the chiral partners of the nucleons, in the center of neutron stars. However, the analysis of the decay width of the assumed chiral partner of the nucleon poses limits on the validity of the present version of the model to describe vacuum properties.

1 Introduction

Effective models for the computation of the equation of state of nucleonic matter at finite density must take into account two very important physical aspects: the symmetry properties of QCD, in particular chiral symmetry and its spontaneous breaking in vacuum, and the properties of strongly interacting matter at saturation. Actually this is a difficult task. While from one side the standard sigma model contains a mechanism for the restoration of the chiral symmetry, it does not describe nuclear matter saturation [1]. On the other side, Walecka-type models [2] can successfully describe the properties of nuclear matter but they do not contain the symmetries of QCD. To overcome this problem, many different extensions of the simple sigma model have been proposed including vector mesons [3], the dilaton field [4], non linear realizations of chiral symmetry both in SU(2) and SU(3) [5,6,7,8] and chiral models with a hidden local symmetry for vector mesons [9,10,11,12,13]. The model that we want to investigate here is the SU(2) parity doublet model considered before in Refs. [14,15,16,17,18,19,20,21,22,23]. It has been shown, in ref. [22], that this model can describe correctly both chiral symmetry properties and the properties of nuclear matter at saturation.

The essential new ingredient of this model is an explicit mass term in the Lagrangian, which is chirally invariant due to the special transformation properties of the nucleon field and its chiral partner. The value of this mass parameter, $m_0$, contributes to the mass of the nucleons and therefore, if its value is very large, the breaking of chiral symmetry is responsible only for the mass splitting between the two nucleons. A still open question concerns the identification of the chiral partner of the nucleon: the most likely candidate is the well known $N'(1535)$...
resonance. This possibility has been investigated in the previous works on symmetric matter \cite{14,15,16,17,18,19,20,21} and recently it has been extended to beta-stable matter for the study of the properties of neutron stars \cite{22}. As already pointed out in ref. \cite{22} and as we will discuss in this paper, the assignment for the partner of the nucleon is still uncertain and it is also possible that it is a broad resonance, not yet identified by the experiments, with a mass smaller than the mass of the $^N(1535)$.

The main scope of this paper is to investigate further this hypothesis and we will consider a possible lower mass for the "true" chiral partner of the nucleon in the following, choosing a value of 1200 MeV. Here, in particular, we are interested in studying the properties of the beta-stable equation of state within the parity model and its applications to neutron stars. We will compute the equation of state both at mean field level and using the relativistic Hartree approximation. The latter approach is more complete since it accounts for Dirac sea effects. Interestingly, within the relativistic Hartree approximation, the value of the bare mass $m_0$ is slightly smaller than the value obtained within the mean field approximation. Finally, we will present results showing that the choice of a small value for the mass of the chiral partner of the nucleon allows this particle to be formed at the center of neutron stars. This could have interesting effects on the transport properties of the matter (like viscosities or neutrino opacities) with possible phenomenological applications.

The paper is organized as follows: in Section II we will present the Lagrangian of the parity model for asymmetric matter. In Section III and IV we will compute the equation of state and neutron star structure at mean field level and in relativistic Hartree approximation, respectively. Finally in Section V we draw our conclusions.

2 The parity model for asymmetric matter

In the parity doublet model one uses the so-called “mirror assignment” for the positive and negative parity nucleon states ($N_+$ and $N_-$), in which they belong to the same multiplet. Under the $SU_L(2) \times SU(2)_R$ transformations $L$ and $R$, the two nucleon fields $\psi_1$ and $\psi_2$ transform as:

\begin{equation}
\psi_{1R} \rightarrow R\psi_{1R}, \quad \psi_{1L} \rightarrow L\psi_{1L},
\end{equation}

\begin{equation}
\psi_{2R} \rightarrow L\psi_{2R}, \quad \psi_{2L} \rightarrow R\psi_{2L}.
\end{equation}

This allows for a chirally invariant mass term in the Lagrangian that reads:

\[ m_0(\bar{\psi}_{2\gamma_5}\psi_1 - \bar{\psi}_1\gamma_5\psi_2) = m_0(\bar{\psi}_{2L}\psi_{1R} - \bar{\psi}_{2R}\psi_{1L} - \bar{\psi}_{1L}\psi_{2R} + \bar{\psi}_{1R}\psi_{2L}), \]

where $m_0$ represents a bare mass parameter.

To study the equation of state of beta-stable matter, in the Lagrangian of ref. \cite{22} we add the vector-isovector meson $\vec{\rho}$ which couples to the isospin current:

\[ \mathcal{L} = \psi_1 i\sigma^\mu \partial_\mu \psi_1 + \psi_2 i\sigma^\mu \partial_\mu \psi_2 + m_0 (\bar{\psi}_{2\gamma_5}\psi_1 - \bar{\psi}_1\gamma_5\psi_2) + a\bar{\psi}_1 (\sigma + i\gamma_5 \tau \cdot \pi) \psi_1 \\
+ b\bar{\psi}_2 (\sigma - i\gamma_5 \tau \cdot \pi) \psi_2 - g_\omega \bar{\psi}_1 \gamma_\mu \omega^\mu \psi_1 - g_\rho \bar{\psi}_2 \gamma_\rho \omega^\rho \psi_2 \\
- \lambda_\pi g_\rho \bar{\psi}_1 \gamma_\mu \tau \cdot \rho^\mu \psi_1 - \frac{\lambda_\pi}{2} g_\rho \bar{\psi}_2 \gamma_\rho \tau \cdot \rho^\rho \psi_2 + \mathcal{L}_M, \]

where $a$, $b$, $g_\omega$, and $g_\rho$ are the coupling constants of the mesons fields ($\sigma$, $\pi$, $\omega$ and $\rho$) to the baryons $\psi_1$ and $\psi_2$ and the mesonic Lagrangian $\mathcal{L}_M$ contains the kinetic terms of the different meson species, and potentials for the scalar and vector fields:

\[ \mathcal{L}_M = \frac{1}{2} \partial_\mu \sigma^\mu \partial_\nu \sigma_\nu + \frac{1}{2} \partial_\mu \pi^\mu \partial_\nu \pi_\nu - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} \\
- \frac{1}{4} \rho_{\mu\nu} \rho^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_{\mu} \omega^{\mu} + \frac{1}{2} m_\rho^2 \rho_{\mu} \rho^{\mu} \\
+ g_\rho^4 (|\omega_{\mu} \omega^{\mu}|^2) + \frac{1}{2} \rho^2 (\sigma^2 + \pi^2) \\
- \frac{\lambda}{4} (\sigma^2 + \pi^2)^2 + \epsilon \sigma, \]
Table 1. Parametrization and results for the two possible configurations within the MFT approximation considering $M_{N_{-}} = 1200$ MeV and $m_0 = 790$ MeV.

|               | P1 | P2 |
|---------------|----|----|
| $g_4 (MeV)$   | 0  | 3.76 |
| $m_{\sigma} (MeV)$ | 318.56 | 302.01 |
| $g_{\omega}$  | 6.08 | 6.77 |
| $g_{\rho}$    | 4.22 | 4.18 |
| $K (MeV)$     | 436.09 | 374.62 |
| $M_{\text{max}} (M_{\odot})$ | 1.85 | 1.39 |
| $\rho_{\text{crit}} (fm^{-3})$ | 0.32 | 0.31 |

where $\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ and $\rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$ represent the field strength tensors of the vector fields. The parameters $\lambda$, $\bar{\mu}$ and $\epsilon$ are as in ref. [22]:

$$
\lambda = \frac{m_\sigma^2 - m_\rho^2}{2 \sigma_0^2},
$$
$$
\bar{\mu}^2 = \frac{m_\sigma^2 - 3m_\rho^2}{2},
$$
$$
\epsilon = m_\pi^2 f_\pi,
$$

(6)

with $m_\pi = 138$ MeV, $f_\pi = 93$ MeV. The vacuum expectation value of the sigma field is $\sigma_0 = f_\pi$ and its vacuum mass $m_\sigma$ is taken as a parameter. This can be done because this meson has its origin in reproducing multiple pion-exchange in the nucleon interactions and doesn’t represent a genuine particle. In this sense, its mass plays the role of an adjustable parameter. If one computes the sigma effective mass at saturation density, as done in ref. [22], a rather small value is obtained, which could cause problems with density profiles in nuclei. The vacuum mass of the $\omega$ field is $m_\omega = 783$ MeV, while the $g_4$ term for the $\omega$ field represents a free parameter with finite values causing a softening of the equation of state. Concerning the $\bar{\rho}$ field, we choose, among the possible SU(2) chiral invariant terms for the vector meson self-interaction, the one that has no self-interaction for the $\rho$ meson and no $\omega - \rho$ mixing term [25]. This choice leads to a stiffer Equation of State (EoS) besides being in agreement with the observed small mixing of the two mesons. The vacuum mass of the $\bar{\rho}$ meson is $m_\rho = 761$ MeV.

3 Mean field approximation

In a first approximation, to study dense cold matter, we neglect the fluctuations around the constant vacuum expectation values of the mesonic field operators. Only the time-like component of the isoscalar vector meson $\bar{\omega} \equiv \omega_0$ and the time-like third component of the isovector vector meson $\bar{\rho}_3^0$ (where the upper index refers to isospin component and the lower index refers to the Lorentz component) of the $\bar{\rho}$ field remains (see for instance ref. [26]). Additionally, parity conservation demands $\bar{\pi} = 0$. The mass eigenstates for the parity doubled nucleons, the $N_+$ and $N_-$, are determined by diagonalizing the mass matrix, eq. (3), for $\psi_1$ and $\psi_2$. Writing the coupling constants $a$ and $b$ as functions of the mass of the positive parity nucleons $M_{N_+} = 939$ MeV, the mass of the negative parity nucleons $M_{N_-}$, the vacuum value of the scalar condensate $\sigma_0$ and the the bare mass term $m_0$, the effective masses of the baryons are given by:

$$
M_{N_{\pm}}^* = \sqrt{\left[\frac{(M_{N_+} + M_{N_-})^2}{4} - m_0^2\right] \frac{\sigma^2}{\sigma_0^2} + m_0^2 \pm \frac{M_{N_+} - M_{N_-} \sigma}{\sigma_0^2}},
$$

(7)
It is easy to realize that in the limit $\sigma \to 0$, for which chiral symmetry is restored, the two nucleons have the same mass $m_0$ and for $\sigma \to \sigma_0$, for which chiral symmetry is broken, the two nucleons have different masses, equal to their vacuum values.

The grand canonical partition function in mean field approximation is

$$
\frac{\Omega}{V} = -\mathcal{L}_M + \sum_i \frac{\gamma_i}{(2\pi)^3} \int_{k_F} d^3 k (E_i^*(k) - \mu_i^*) ,
$$

and the mesons term reads

$$
\mathcal{L}_M = \frac{1}{2} m_i^2 \omega_i^2 + g_4^4 \omega_0^4 + \frac{1}{2} \mu_i^2 - \frac{\lambda}{4} \sigma^4 + \epsilon \sigma + \frac{1}{2} m_\rho^2 (\rho_0^3)^2 ,
$$

where $i \in \{n_+, n_-, p_+, p_-\}$ denotes the nucleon type (positive and negative parity neutrons and positive and negative parity protons), $\gamma_i$ is the fermionic degeneracy, $k_F$ are the Fermi momenta, $E_i^*(k) = \sqrt{k^2 + M_i^2}$ the energy, and $\mu_i^* = \mu_i - g_\omega \omega_0 - g_\rho \rho_0^3 I_3 = \sqrt{k_F^2 + M_i^2}$ the corresponding effective chemical potential where $I_3$ is the third component of the isospin (1/2 for the positive and negative parity proton and -1/2 for the positive and negative parity neutron). The single particle energy of each parity partner $i$ is given by $E_i(k) = E_i^*(k) + g_\omega \omega_0 + g_\rho \rho_0^3 I_3$. Altogether there are six unknown parameters: $g_\omega$, $g_\rho$, $m_\sigma$, $g_4$, $m_0$, $M_{N_-}$. The first three are determined by the basic nuclear matter saturation properties, i.e., the stable minimum of the grand canonical potential for $\mu_B = 923$ MeV has to meet three conditions:

$$
E/A(\mu_B = 923\text{MeV}) - M_N = -16\text{MeV},
\rho_0(\mu_B = 923\text{MeV}) = 0.16 \text{ fm}^{-3} ,
\alpha_{\text{sym}} = 32.5\text{MeV} ,
$$

\[10\]
Fig. 2. Equations of state of symmetric matter computed within the parity doublet model for the P1 and P2 parameter sets and the GM3 model as in fig. 1. The shaded box corresponds to limits obtained from the analysis of heavy ion collisions at intermediate energies presented in ref. [30].

which are the measured values for the binding energy per nucleon, the baryon density and a phenomenologically reasonable value for the symmetry energy at saturation. The nuclear matter compressibility at saturation is defined as

\[ K = 9\rho_B^2 \frac{\partial^2 E/A}{\partial \rho_B^2} \bigg|_{\rho_B = \rho_0} = 9\frac{\partial p}{\partial \rho_B} \bigg|_{\rho_B = \rho_0} = 9\rho_B \frac{\partial \mu_B}{\partial \rho_B} \bigg|_{\rho_B = \rho_0}, \quad (11) \]

where \( p \) is the pressure. Considering that there is not much information from the physics inside neutron stars, it is expected that it should agree with finite nuclei and heavy ion collisions data. The problem is that in the first case the surface effects are not negligible even for large nuclei, while in the second case the system does not come into equilibrium. Because of these differences we should keep in mind that the values of compressibility coming from those experiments should only be used as a guideline to constrain neutron star EoSs, as suggested in ref. [27]. We will fix at first \( m_0 = 790 \text{ MeV} \) (as in ref. [22,24]) and \( M_{N_\pi} = 1200 \text{ MeV} \), and consider the equations of state P1 and P2 for \( g_4 = 0, 3.8 \) respectively as done in ref. [22] for symmetric matter. The values of the parameters for these two cases are reported in Table 1.

The large value of the parameter \( m_0 \) is fixed in order to obtain reasonable values for the compressibility. It is important to stress that a large \( m_0 \) implies that most of the nucleons mass is given by the mixing with its chiral partner and not by the chiral condensate. On the other hand, in ref. [21], within the assumption that the chiral partner of the nucleon is the \( N'(1535) \) resonance, a value of \( m_0 = 270 \text{ MeV} \) is found from the measured decay amplitude of the process \( N' \rightarrow N_\pi \pi \). However, in their model they use a simple “ansatz” for the potential of the sigma meson which produces too low values for the compressibility. In our model we use the standard linear sigma model, including also the explicit chiral symmetry breaking term, and for the same value of \( m_0 \) we would obtain compressibilities much larger than what is indicated by the phenomenology [28]. For this reason the identification of the \( N'(1535) \) as the chiral partner of the nucleon is problematic. To solve this problem, as in ref. [22], it is possible that the chiral partner actually is another particle with a lower mass and which escaped experimental
Fig. 3. The ratio $\sigma/\sigma_0$ is shown as a function of the baryon density for the P1 and P2 parameter sets for both symmetric matter and neutron star matter. In the case of neutron star matter, due to the beta stability and charge neutrality conditions, the beginning of the restoration of chiral symmetry takes place slightly before with respect to the case of symmetric matter. The transition is in all cases a smooth crossover.

detection. Here we adopt this suggestion and we consider the mass of the chiral partner to be 1200 MeV.

The mean meson fields $\hat{\sigma}, \hat{\omega} = \omega_0$ and $\hat{\rho} = \rho_0^3$ are determined by extremizing the grand canonical potential $\Omega/V$:

\[
0 = -\mu^2 \hat{\sigma} + \lambda \hat{\sigma}^3 - \epsilon + \sum_i \rho_i \frac{\partial M_i^*}{\partial \hat{\sigma}} \bigg|_{\hat{\sigma}},
\]

\[
0 = -m^2 \hat{\omega} - 4g^4 \hat{\omega}^3 + g_\omega \sum_i \rho_i (\hat{\sigma}, \hat{\omega}, \hat{\rho}) = 0,
\]

\[
0 = -m_\rho^2 \hat{\rho} - g_\rho (\rho_{n+} (\hat{\sigma}, \hat{\omega}, \hat{\rho}) + \rho_{n-} (\hat{\sigma}, \hat{\omega}, \hat{\rho}) - \rho_{p+} (\hat{\sigma}, \hat{\omega}, \hat{\rho}) - \rho_{p-} (\hat{\sigma}, \hat{\omega}, \hat{\rho})) .
\] (12)

The energy density is obtained from the grand canonical potential:

\[
\epsilon = -L_M + \sum_i \frac{\gamma_i}{(2\pi)^3} \int_0^{k_F_i} \frac{d^3 k}{(2\pi)^3} (E_i^* (k) - \mu_i^*) + \rho_i \mu ,
\] (13)

and the baryon and scalar densities for each particle are given by the usual expressions:

\[
\rho_i = \gamma_i \int_0^{k_F_i} \frac{d^3 k}{(2\pi)^3} = \frac{\gamma_i k_F_i^3}{6\pi^2} ,
\]

\[
\rho_{i,S} = \gamma_i \int_0^{k_F_i} \frac{d^3 k}{(2\pi)^3} \frac{M_i^*}{E_i^*} = \frac{\gamma_i M_i^*}{4\pi^2} \left[ k_F_i E_i^* - M_i^* 2 \ln \left( \frac{k_F_i + E_i^*}{M_i^*} \right) \right] .
\] (14)

After fixing the model parameters for symmetric matter, as explained before, we can compute the equation of state for beta-stable and charge-neutral hadronic matter suitable for application in neutron stars. Defining $\mu_n, \mu_p$ and $\mu_e$ as the chemical potential of the doublets of neutrons and protons and the electrons, beta stability and charge neutrality are met if the following conditions are satisfied:

\[
\mu_n = \mu_p + \mu_e ,
\]

\[
\rho_e = \rho_{p+} + \rho_{p-} .
\] (15)

In fig. 1, the equations of state are shown for the different parameters sets. The parametrization P2, that has a considerable high fourth-order self-interaction coupling constant for the
vector mesons, gives a softer equation of state in comparison with P1. This shows that the increase of the self-interaction coupling constant or, equivalently, the decrease of the vector-isoscalar field itself, that represents the repulsive part of the strong force, causes the pressure of the system to decrease. From the comparison with a relativistic mean field equation of state GM3 [29], it is also possible to notice the softening of the equation of state predicted within the parity doublet model as the chiral symmetry starts to be restored. In fig. 2 we compare the equations of state obtained for symmetric matter with the constraints obtained from the analysis of heavy ion collisions at intermediate energies presented in ref. [30]. Both the equation of states P1 and P2 are in a good agreement with the experimental constraints for densities larger than 2.5\(\rho_0\). At lower densities the P1 parametrization exceeds the experimental limit due to the large value of the corresponding compressibility at saturation (see Table I). Interestingly our equation of state shares similarities with the equation of state obtained in ref. [13] by using the chiral dilaton model: also in that model the equation of state is rather stiff at saturation but softens at large densities due to the restoration of chiral symmetry.

In fig. 4 we show the scaled expectation value of the chiral condensate as a function of the baryon density. It is interesting to notice that considering beta equilibrium the chiral symmetry restoration occurs at lower values of densities with respect to the case of symmetric matter. This effect is due to the conditions of beta stability together with charge neutrality, eq. [15] which move at larger densities the appearance of negative parity protons and at lower densities the appearance of negative parity neutrons with respect to the case of symmetric matter. Since in the parity doublet model there is a strict link between the appearance of the chiral partners and the chiral symmetry restoration, the isospin asymmetry has an effect also on the chiral restoration density. Moreover it affects also the order of the phase transition: for asymmetric matter the phase transition is smoother than for symmetric matter. The density for the beginning of the chiral symmetry restoration turns out to be very low, \(\sim 2\rho_0\), for the P1 and P2 equations of state while it would have been higher if we have used a more massive chiral partner. It can also be seen in fig. 3 that the effect of the vector-isoscalar meson self-coupling in the chiral restoration is practically negligible. In fig. 4 the number density fractions for the various particles are shown as functions of the baryon density. Notice the different thresholds for the appearance of negative parity protons and negative parity neutrons.

Now we use the above described equations of state to compute the mass-radius relations and the structure of neutron stars by solving the Tolman-Oppenheimer-Volkov equations. We use the parity doublet model equations of state down to baryon densities of 0.05fm\(^{-3}\) while for lower densities we use the recent equation of state presented in ref. [31]. The results are shown in the left panel of fig. 5 for the different cases. In the same plot, we also indicate an horizontal line corresponding to the mass \(M_{\text{max}} = 1.44M_\odot\) of the Hulse-Taylor binary pulsar, which is still the largest precisely known neutron star mass. Interestingly, considering the \(M_{\text{max}}\) limit we can rule out the P2 equation of state, indicating that in our model a self-interaction term for the \(\omega\) meson renders the equation of state too soft. Taking into account the self-interaction
of the $\rho$ meson or the mixing between the $\omega$ and the $\rho$ meson would make the equation of state even softer.

Let us now study whether chiral symmetry is restored in neutron stars. The results are shown in the right panel of fig. 5 where the masses as functions of the central baryon densities are plotted. The stars on the curves indicate the densities corresponding to the onsets of chiral symmetry restoration. We find that stars having a mass larger than $1.2 M_\odot$ have core of a partially chiral restored phase for both cases. The same effect would not happen considering a more massive chiral partner, like for example with $M_{N_-} = 1535$ MeV. In that case the stars are unstable before the central density reaches the chiral symmetry restoration threshold (see ref. [24]).

4 Relativistic Hartree approximation

The Relativistic Hartree Approximation (RHA) goes beyond mean field by accounting for the effect of the baryonic Dirac sea as one sums over the baryonic tadpole diagrams [32]. In any consistent relativistic field theory, one should account for the negative-energy baryon states. Those contributions are an important part of a fully relativistic description of nuclear structure. It is, actually, impossible to construct a meaningful nuclear response or consistent nuclear currents without them (see [2]). It is, therefore, natural to ask about the role of those corrections from the filled Dirac sea on top of the mean-field ground state, which by itself is causal and consistent with Lorentz invariance and thermodynamics. In more modern views of effective hadronic field theory, one must include loop contributions that contain negative-energy baryon wave functions, since it is important to maintain the completeness of the Dirac basis, which is fundamental in the field theory [33].

The dressed propagator of a baryon $i$ is obtained by solving the Dyson-Schwinger equation:

$$ G^H_i = G^0_i(k) + G^0_i(k)\Sigma^H_i(k)G^H_i(k) , $$

where $G^0_i(k)$ is the free propagator and $\Sigma^H_i(k)$ the Hartree self-energy, which contains the scalar ($\Sigma^S$) and vector ($\Sigma^V$) parts

$$ \Sigma^H_i = \Sigma^S_i - \gamma_\mu(\Sigma^V_i)\mu . $$

The solution of the Dyson-Schwinger equation is

$$ [G^H_i(k)]^{-1} = \gamma^\mu k_\mu - M^*_i , $$
Fig. 6. Compressibility at saturation as a function of bare mass $m_0$ for the mean field and Hartree approximations for the P1 parameter set. If the compressibility has a value of 270 MeV, as indicated in ref. [28], $m_0 \sim 850$ MeV.

or, in an equivalent way,

$$G_i^H(k) = \langle \gamma^\mu \bar{\kappa}_\mu + M_i^* \rangle \left[ \frac{1}{k^2 - M_i^{*2}} + i\epsilon + \frac{i\pi}{E_i^*(k)} \delta(k_0 - E_i^*(k)) \theta(k_F,i - |k|) \right]$$

$$= (G_i^H)^F(k) + (G_i^H)^D(k), \quad (19)$$

with $E_i^*(k) = \sqrt{k^2 + M_i^{*2}}$, and the shifted four-momentum $\bar{k}_i = k_i + \Sigma_i^V$ and mass $M_i^* = M_i + \Sigma_i^S$. In the Hartree approximation, the Feynman part $(G_i^H)^F$ describes the propagation of virtual positive- and negative-energy quasinucleons, while the density-dependent part $(G_i^H)^D$ allows for quasinucleon holes inside the Fermi sea correcting $(G_i^H)^F$ for the Pauli exclusion principle.

Then, the scalar and vector contributions are

$$\Sigma_i^S = \frac{g_{S_i}^2}{m_{S_i}^2} \int \frac{d^4k}{(2\pi)^4} \text{Tr} (G_i^H(k)) = i \frac{g_{S_i}^2}{m_{S_i}^2} \int \frac{d^4k}{(2\pi)^4} [(G_i^H)^F(k) + (G_i^H)^D(k)]$$

$$= \frac{g_{S_i}^2}{m_{S_i}^2} \left[ i \gamma_i \int \frac{d^4k}{(2\pi)^4} \frac{M_i^*}{k^2 - M_i^{*2} + i\epsilon} - \rho_{i,S} \right], \quad (20)$$

$$(\Sigma_i^V)^\mu = \frac{g_{V_i}^2}{m_{V_i}^2} \int \frac{d^4k}{(2\pi)^4} \text{Tr} (\gamma^\mu G_i^H(k)) = i \frac{g_{V_i}^2}{m_{V_i}^2} \int \frac{d^4k}{(2\pi)^4} \text{Tr} [\gamma^\mu ((G_i^H)^F(k) + (G_i^H)^D(k))]$$

$$= \frac{g_{V_i}^2}{m_{V_i}^2} \left[ i 2 \gamma_i \int \frac{d^4k}{(2\pi)^4} \frac{\bar{k}_\mu}{k^2 - M_i^{*2} + i\epsilon} - \delta^{\mu\nu} \rho_{i \nu} \right], \quad (21)$$

where $\rho_{i,S}$ and $\rho_i$ are the scalar and baryon density, $g_{S_i}$ and $g_{V_i}$ the scalar and vector coupling constants and $m_{S_i}$ and $m_{V_i}$ the scalar and vector meson masses for each baryon $i$. The mean field contribution corresponds to neglect the contribution from the Feynman part $(G_i^H)^F$ (antibaryons) and take only into account the contributions from the density-dependent part $(G_i^H)^D$ (filled Fermi sea), i.e., only considering the scalar and baryon densities.
Table 2. Parametrization and results for six possible configurations within the Hartree approximation considering $M_{N} = 1200$ MeV and $g_{4} = 0$.

| $m_{0}$(MeV) | 300  | 600  | 750  | 790  | 850  | 900  |
|--------------|------|------|------|------|------|------|
| $m_{\sigma}$(MeV) | 657  | 484  | 362  | 322  | 251  | 164  |
| $g_{\omega}$ | 9.35 | 8.37 | 6.59 | 5.74 | 3.94 | 0.59 |
| $g_{\rho}$ | 4.05 | 4.10 | 4.25 | 4.25 | 4.30 | 4.35 |
| $K$(MeV) | 651.27 | 554.72 | 421.68 | 360.27 | 257.40 | 129.32 |
| $M_{\text{max}}$(M⊙) | 2.34 | 2.18 | 1.91 | 1.78 | 1.44 | 1.02 |
| $\rho_{\text{crit}}$(fm$^{-3}$) | 0.47 | 0.38 | 0.35 | 0.38 | 0.43 | 0.58 |

In RHA we consider both contributions, but the integral of $\text{Tr}[^{\gamma^{\mu}}(G_{H}^{H})^{F}]$ in eq. (21) vanishes, i.e., the Dirac sea contributes only to the scalar part of the self-energy. This is a divergent integral which is rendered finite by dimensional regularization, introducing the appropriate counter-terms, as done in ref. [32,34]. Then, after regularization, the finite scalar self-energy reads

$$(\Sigma_{i}^{S})_{\text{finite}} = -\frac{g_{S}^{2}}{m_{S}^{2}} \Delta \rho_{i,S}, \quad (22)$$

where the additional contribution to the scalar density for each baryon species $\Delta \rho_{i,S}$ is given by

$$\Delta \rho_{i,S} = -\frac{\gamma_{i}}{4\pi^{2}} \left[ M_{i}^{*} \ln \left( \frac{M_{i}^{*}}{M_{i}} \right) + M_{i}^{2} (M_{i} - M_{i}^{*}) - \frac{5}{2} M_{i} (M_{i} - M_{i}^{*})^{2} + \frac{11}{6} (M_{i} - M_{i}^{*})^{3} \right]. \quad (23)$$

As a consequence, the grand canonical potential is modified inducing changes in the pressure, energy density and the meson field equations. In the parity doublet model, the energy density can be evaluated as

$$\epsilon_{RHA} = \epsilon_{MFT} + \Delta \epsilon, \quad (24)$$

with $\epsilon_{MFT}$ being the mean field result of eq. (13). The contribution to the energy density from the Dirac sea, $\Delta \epsilon$, reads

$$\Delta \epsilon = -\sum_{i} \frac{\gamma_{i}}{16\pi^{2}} \left[ M_{i}^{*2} \ln \left( \frac{M_{i}^{*}}{M_{i}} \right) + M_{i}^{2} (M_{i} - M_{i}^{*}) - \frac{7}{2} M_{i}^{2} (M_{i} - M_{i}^{*})^{2} + \frac{13}{3} M_{i} (M_{i} - M_{i}^{*})^{3} - \frac{25}{12} (M_{i} - M_{i}^{*})^{4} \right]. \quad (25)$$

The pressure is

$$p_{RHA} = p_{MFT} - \Delta \epsilon, \quad (26)$$

and the field equation for the scalar meson field $\sigma$ is then modified to

$$\frac{\partial (\Omega/V)}{\partial \sigma} \bigg|_{RHA} = \frac{\partial (\Omega/V)}{\partial \sigma} \bigg|_{MFT} + \sum_{i} \frac{\partial M_{i}^{*}}{\partial \sigma} \Delta \rho_{i,S} = 0, \quad (27)$$

where the mean field contribution is found in eq. (12). The coupling constants $g_{\omega}$, $g_{\rho}$ and $m_{\sigma}$ have to be re-fitted in order to obtain the nuclear saturation properties.

Let us now apply this formalism to compute the equation of state. We start from the P1 set of parameters. In fig. [6] we show a comparison between the compressibility obtained in mean field approximation and in relativistic Hartree approximation as a function of the bare mass $m_{0}$. Note that when $m_{0}$ decreases, the scalar potential increases according to eq. (7). To keep the
Fig. 7. Equations of state for the doublet model in the four cases discussed in the text for the P1 parametrization.

saturation properties, the increase of the scalar potential must be balanced by an increase of the vector potential which in turn induces a higher value of the compressibility (eq. [11]). This effect is quite pronounced in the mean field approximation as we can notice in the figure. On the other hand, in the relativistic Hartree approximation the scalar mesons are enhanced and the vector mesons are suppressed (ref. [35,36]), causing the equation of state to be softer and, consequently, the compressibility to be smaller (fig. 6).

For the applications to neutron star matter, we choose four different bare mass values keeping the mass of the chiral partner \( M_{N_c} \) = 1200 MeV and ignoring a self-coupling for the vector mesons \( g_4 = 0 \), for the reasons described before. The parametrizations are: \( m_0 = 300, 600, 790, 900 \) MeV (see Table 2 for numerical values of the parameters for these \( m_0 \)s together with \( m_0 = 750 \) MeV and \( m_0 = 850 \) MeV). The four parametrizations are shown in fig. 7. It can again be seen that smaller bare masses generate stiffer equations of states. Although for small bare masses of \( m_0 = 300 \) MeV and \( m_0 = 600 \) MeV the compressibility is too high according to phenomenology, these parametrizations are still presented in the plots just for illustrative purposes. On the other hand, high bare masses generate EoSs with high nucleon effective masses and, hence, smaller values for the nucleon scalar potential. While saturation properties are still described due to a simultaneous reduction of the nucleon vector potential, this causes problems in reproducing spin-orbit splittings in finite nuclei. This problem could still be corrected by the adjustment of the corresponding tensor coupling, as shown by Furnstahl in ref. [36], but for the parity model work in this direction is still missing.

Finally we use the above described equations of state to compute the mass-radius relations and the structure of neutron stars by solving the Tolman-Oppenheimer-Volkov equation. The results are shown in the left panel of fig. 8 for the different cases. In the case of the highest bare mass value, the maximum mass of neutron stars is too low and, therefore, the corresponding equation of state is ruled out. We suggest that the equation of state compatible with the observed neutron star masses and nuclear matter data correspond to the case of bare mass of 850 MeV. In this case a star with mass higher than 1.4 \( M_\odot \) is obtained with a compressibility smaller than 300 MeV. The results for P2 in the relativistic Hartree approximation would be similar to the ones for P1, because the self-interaction of the \( \omega \) meson does not change
Fig. 8. Left panel: mass radius relations for the different choices of the bare mass parameter. Right panel: masses as functions of the central baryon density. The stars on the curves denote the onset of chiral symmetry restoration.

We can also use the bare mass parameters of the six parametrizations to study vacuum properties, as the pion-nucleon scattering or the decay width of the chiral partner, as was already done for $N'(1535)$. We use the formula of ref. [14] to compute the decay width $\Gamma$ of $N_- \rightarrow N_+ \pi$ as a function of $m_0$. The result is shown in fig. 9, where it can be seen that the decay width increases quadratically as a function of $m_0$, but, even for the higher bare mass value $m_0 = 900$ MeV, the width is still around 100 MeV. This value is too small to justify the assumption that the "true" chiral partner of the nucleon is an undetected resonance with a mass of 1200 MeV. For a more massive chiral partner as the $M_{N_-} = 1379$ MeV, which is the limiting case for the formation of chiral partners inside neutron stars, we get values of the order of 300 MeV for the width. Probably such a resonance is still not broad enough to have escaped experimental detection. This indicates the importance of improving our model to reconcile the finite density matter properties with the microphysics of the interaction of the chiral partner with the nucleon and the pion. Working along this line is in progress.

5 Conclusions

We have studied the equation of state of nucleonic matter at zero temperature within the SU(2) parity doublet model by using first the mean field approximation. We assume that the chiral partner of the nucleon is a resonance, not yet detected, which has a mass of 1200 MeV. The parameters of the model are fixed by fitting the properties of nuclear matter at saturation. To obtain a reasonable value of the nuclear matter compressibility, the mixing parameter between the nucleon and its chiral partner $m_0$ turns out to be large, higher than 790 MeV, thus indicating that the chiral condensate gives a minor contribution to the mass of the nucleon. We then studied the equation of state of beta-stable and charge neutral matter suitable for the applications to neutron stars. We have shown that a significant softening of the equation of state is realized due to the transition from a chiral broken phase to a partially chiral restored phase at a density of roughly $2\rho_0$, which corresponds also to the threshold for the appearance of the chiral partners of the nucleons.

We then used the equations of state for the computation of the mass-radius relations and structure of neutron stars. Taking into account the neutron star mass measurements, we have ruled out the possibility of having self-interaction terms for vector mesons in the Lagrangian since they render the equation of state too soft. Finally we have shown that for neutron stars with masses larger than roughly $1.2M_\odot$, chiral symmetry starts to be restored in their core and, therefore, the chiral partners of the nucleons appear.
As a second step we repeated the calculations by using the relativistic Hartree approximation. In this case, smaller values of $m_0$, down to 750 MeV, can still reproduce reasonable values of the compressibility due to the suppression on the vector meson sector, which is a characteristic of this kind of approximation. Also within this approximation, we predict that the chiral partners of the nucleon could be formed at the center of neutron stars. The astrophysical implications of these results could be interesting. For instance the late cooling of neutron stars could be modified if these new particles appear as they are opening new cooling processes.

The hypothesis that the chiral partner of the nucleon is a very broad and therefore still undetected resonance with a relatively low mass leads to the population of chiral partners in neutron stars. However, as we have shown, within the present model, we obtain rather small values of the width of this particle. We need to improve the doublet parity model adopting, for instance, a gauged linear sigma model, as done in ref. [23], in which the physics of the vacuum is described more accurately. If also in the improved version of the model the hypothetical resonance at 1.2 GeV turns out to be narrow, one should abandon the assumption of a low mass chiral partner of the nucleon. This would imply that the chiral partners cannot be formed in neutron stars and, therefore, their existence cannot be tested by using astrophysical observations.

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