Data-driven Reduced Order Model for prediction of wind turbine wakes

G.V. Iungo\textsuperscript{1}, C. Santoni-Ortiz\textsuperscript{1}, M. Abkar\textsuperscript{2}, F. Porté-Agel\textsuperscript{2}, M.A. Rotea\textsuperscript{1} and S. Leonardi\textsuperscript{1}

\textsuperscript{1} The University of Texas at Dallas, Mechanical Engineering Department, 75080 Richardson, TX
\textsuperscript{2} Ecole Polytechnique Fédérale de Lausanne (EPFL), Wind Engineering and Renewable Energy (WIRE) Lab, Lausanne, Switzerland

E-mail: valerio.iungo@utdallas.edu

Abstract. In this paper a new paradigm for prediction of wind turbine wakes is proposed, which is based on a reduced order model (ROM) embedded in a Kalman filter. The ROM is evaluated by means of dynamic mode decomposition performed on high fidelity LES numerical simulations of wind turbines operating under different operational regimes. The ROM enables to capture the main physical processes underpinning the downstream evolution and dynamics of wind turbine wakes. The ROM is then embedded within a Kalman filter in order to produce a time-marching algorithm for prediction of wind turbine wake flows. This data-driven algorithm enables data assimilation of new measurements simultaneously to the wake prediction, which leads to an improved accuracy and a dynamic update of the ROM in presence of emerging coherent wake dynamics observed from new available data. Thanks to its low computational cost, this numerical tool is particularly suitable for real-time applications, control and optimization of large wind farms.

1. Introduction

Computational fluid dynamics (CFD) techniques with different level of complexity and accuracy have been developed over the last decade in order to simulate wind turbine wake flows \cite{1, 2}. Although a large improvement has been achieved to predict wake fluid dynamic features and power production, the computational cost connected to high fidelity LES simulations of wind turbine arrays still remains far too high, which makes them inapplicable for operational purposes and real-time applications. For this reason, simple analytical wake models are still considered as a standard procedure for predictions of power harvesting from a wind power plant, and for siting of new wind farms. Recently improved wake models, such as the dynamic wake meandering model, have been proposed in order to capture the large-scale downstream motion of wind turbine wakes \cite{3, 4}.

An in-depth comprehension and characterization of the physical processes occurring in intra-wind farm wake flows is the substrate to develop new concepts for wind turbine wake prediction. The fluid motion is an infinite-dimensional dynamic system governed by the well known Navier-Stokes equations, which are nonlinear partial differential equations. Therefore, the analysis and prediction of fluid dynamics and instabilities result to be quite challenging tasks. In the domain of wind turbine wakes, in \cite{5, 6} wake instabilities were predicted through linear stability analysis.
performed on the time-averaged velocity field obtained from wind tunnel experiments. For this technique, the Navier-Stokes equations are linearized on the base flow, and small perturbations are applied in order to analyze their spatial and temporal evolution.

The dynamic modes of a fluid motion can also be detected by analyzing time-resolved observations, obtained through experiments or numerical simulations [7, 8], and by applying different modal decomposition techniques, such as proper orthogonal decomposition (POD). The POD modes are characterized by spatial orthogonality, but they have multi-frequential temporal content. This means that different wake dynamics acting at completely separated time scales, thus connected to different phenomena, cannot be distinguished if they have comparable energy. In wind energy, POD has been used to investigate wind turbine wake dynamics in [9, 10].

In the present work, LES simulation data of wind turbine wakes are analyzed through the dynamic mode decomposition (DMD) [11]. For DMD time-resolved data are processed in order to evaluate the dominant physical processes, which are denoted as DMD modes. The latter may be non-orthogonal, but each DMD mode captures a single spectral contribution, thus an isolated physical phenomenon. DMD modes are characterized as for their spectral content (i.e. frequency and spatial wavelength), growth rate (i.e. a stable mode is going to be damped by proceeding in space and time, conversely an unstable mode will be amplified and add kinetic energy to the system), and energy. All those information are essential to highlight the main contributions to wake morphology and dynamics, which in turn affect power production and fatigue loads of wind turbines. DMD modes can be sorted accordingly to different objective functions, rather than their merely energy contribution to wake dynamics, as for POD. DMD modes can be sorted as a function of their spectral contribution and characteristic length scale. This feature can turn out to be very effective for investigations on wake interactions and added fatigue loads of waked wind turbines. Indeed, the overall life cycle of a wind turbine is mainly affected by dynamic loads acting within a well defined spectral range, which neglects very small and very large scales of the turbulent atmospheric wind with respect to the rotor diameter. In the same spirit, DMD modes can be sorted as a function of their growth rate. This criterion is useful for design of wind farm layout, for which only the DMD modes with a relevant energy at a certain downstream distance are considered. Therefore, accordingly to the specific criterion adopted, only a subset of DMD modes is selected in order to formulate a reduced order model (ROM).

The proposed tool for wind turbine wake prediction is meant to be suitable for real-time applications, thus it should be characterized by a relatively low computational cost, but at the same time by an adequate accuracy. For this reason a data-driven approach is considered, where new available data are injected into the simulation tool in order to provide a more accurate wake prediction. This operation is performed by embedding the ROM obtained via DMD in a Kalman filter [12]. The Kalman filter provides an efficient computational recursive algorithm to estimate the state of a process by minimizing the mean square error.

The paper is organized as follows. The database obtained from high fidelity LES simulations of wind turbines is described in section 2. The DMD formulation is recalled in section 3, together with a description of the Kalman filter. Two different applications are then presented in this paper: for section 4 the proposed numerical tool is applied to highly detailed LES data, which are obtained through the actuator line model and simulating also the presence of the wind turbine tower and nacelle. In section 5, wind turbine wake is simulated through the actuator disc model, but a more realistic atmospheric wind field under neutral thermal stability regime is generated through precursor simulations. Final discussion and conclusions are reported in section 6.

2. Description of the LES database
Two different databases have been used to developed the reduced order model, one produced at UT Dallas and another one at EPFL. For the former, the flow around the wind turbine blades are
modeled using the actuator line model. The wind turbine tower and nacelle are simulated via an immersed boundary technique described in [14]. This method allows the flexibility of imposing complex geometries inside the domain without the need of body fitted grids, thus reducing the computational time. The computational box is $12.5d \times 2.1d \times 3d$ ($d$ is the rotor diameter of 0.89 m), i.e. $1024 \times 512 \times 512$ grid points, in the streamwise, wall normal and spanwise direction, respectively. A uniform velocity profile is used as inflow condition. For these simulations, the rotational frequency of the rotor is equal to 10.7 Hz, while the time-averaged incoming wind at hub height, $U_\infty$, is 10 m/s, which produces a tip speed ratio equal to 3. The axial velocity field is evaluated over the vertical symmetry plane of the wake for 214 snapshots acquired with a sampling frequency of 1124 Hz, which represent about 2 full rotations of the rotor disc.

A second set of LES simulations performed by the WIRE Lab at EPFL, which will be analyzed in section 5, employed the scale-dependent Lagrangian dynamic model and to parameterize the turbine-induced forces, the actuator-disk model with rotation is used. It is worth mentioning that the grid resolution is chosen such that 10 points in the spanwise direction and 16 points in the vertical direction cover the turbine rotor diameter. In these simulations, the Vestas V80-2MW wind turbine, with a rotor diameter of 80 m and a hub-height of 70 m, is immersed in a neutral boundary layer flow. The wind turbine operates with a mean angular velocity of about 0.27 Hz. The code is run for a long-enough time to guarantee quasi-steady conditions [15].

### 3. Dynamic mode decomposition and reduced order model

The DMD algorithm is briefly outlined in the following. At each timestamp, the measurements, $\psi$, denoted as snapshot, collected with a fixed sampling frequency, $F_{\text{samp}}$, are arranged as columns of a data matrix. The number of measurement points for each snapshot is equal to $M$, and the total number of snapshots is equal to $N$. From the snapshot sequence, two data matrices are generated:

$$
\Psi_0 := [\psi_0, \psi_1, ..., \psi_{N-1}] ; \quad \Psi_1 := [\psi_1, \psi_2, ..., \psi_N].
$$

(1)

The DMD is based on the assumption that temporal consecutive snapshots are generated through a linear, time-invariant operator applied to the the previous snapshot. In other therm:

$$
\psi_{t+1} = A \psi_t ; \quad t = [0, ..., N-1].
$$

(2)

It is worth to point out that the assumption of a linear system for the DMD time-marching algorithm is not equivalent to perform a linearization of the dynamic system, as performed for instance within linear stability analysis. For DMD all the non-linearities in the evolution of the system dynamics are still kept, only the advancement in time is linearized. By combining Eq. 1 with Eq. 2, it is obtained:

$$
\Psi_1 = [\psi_1, \psi_2, ..., \psi_N] = [A \psi_0, A \psi_1, ..., A \psi_{N-1}] = A \Psi_0.
$$

(3)

If $r$ is the dimension of the aimed reduced order model (ROM), with $r < N$, a matrix $F$ can be defined through DMD in order to be the optimal representation of the matrix $A$ in the basis spanned by the POD modes of $\Psi_0$,

$$
A \approx U F U^*.
$$

(4)

$U^*$ denotes the complex-conjugate-transpose of the POD modes of the snapshot matrix $\Psi_0$, obtained with an economic-size singular value decomposition:

$$
\Psi_0 = U \Sigma V^*;
$$

(5)

where $\Sigma$ is the $r \times r$ diagonal matrix of the singular values. The matrix $F$ can be determined by minimizing the Frobenius norm of the difference between $\Psi_1$ and $A \Psi_0$, thus by injecting Eqs. 4 and 5 in Eq. 3,

$$
\minimize_{F} \| \Psi_1 - A \psi_0 \Sigma V^* \|_F^2.
$$

(6)
The matrix \( F \) can be expressed in a diagonal form, by means of its eigenvalues, \( \mu_i \), and eigenvectors, \( y_i \),

\[
F = \begin{bmatrix} y_1 & \cdots & y_r \end{bmatrix} \begin{bmatrix} \mu_1 & \cdots & \mu_r \\ \vdots & \ddots & \vdots \\ \mu_1 & \cdots & \mu_r \end{bmatrix} = YD\mu Z^*,
\]

(7)

where \( z_i \) are the eigenvectors of \( F^* \), corresponding to the eigenvalues \( \mu_i \) (the eigenvectors are bi-orthogonal \( z_i^* y_j = \delta_{ij} \)). It is possible to evaluate each snapshot in the form:

\[
\psi_t = A \psi_{t-1} = U F^* U \psi_{t-1} = U Y D\mu Z^* U \psi_{t-1} = U Y D\mu^t Z^* U \psi_0.
\]

(8)

The DMD modes, \( \phi_i \), are then obtained:

\[
\Phi = U Y.
\]

(9)

Thus, Eq. 8, can be reformulated as a linear combination of the DMD modes:

\[
\psi_t = \sum_{i=1}^r \phi_i \mu_i^t \alpha_i,
\]

(10)

and \( \alpha_i \) are the amplitude of each DMD mode. In matrix form:

\[
\begin{bmatrix} \psi_0 & \psi_1 & \cdots & \psi_{N-1} \end{bmatrix} \approx \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_r \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_r \end{bmatrix} = \begin{bmatrix} 1 & \mu_1 & \cdots & \mu_r^{N-1} \\ 1 & \mu_2 & \cdots & \mu_r^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \mu_r & \cdots & \mu_r^{N-1} \end{bmatrix} = \Phi D\alpha V_{and},
\]

(11)

where \( V_{and} \) is the Vandermonde matrix of the eigenvalues of the matrix \( F \). The determination of the DMD amplitudes, \( \alpha_i \), is obtained through the following optimization problem:

\[
\text{minimize} \| \Psi_0 - \Phi D\alpha V_{and} \|_F^2.
\]

(12)

The solution of this problem is detailed in [13]. The different dynamics of a wind turbine wake flow are then properly characterized through the DMD modes, \( \phi_i \), defined in Eq. 9, whose amplitudes are \( \alpha_i \), frequency equal to \( F_{samp}/(2\pi)\Im[\log(\mu_i)] \), and growth rate equal to \( F_{samp} \times \Re[\log(\mu_i)] \) (\( \mu_i \) are the eigenvalues of the matrix \( F \)). A reduce order model (ROM) for prediction of the wind turbine wake flow can be formulated by using the matrix \( F \) as linear operator of a time-marching algorithm. However, a ROM can be formulated by only selecting a subset of DMD modes that fulfill a particular criterion, such as for their energy content, frequency, growth rate, or a combination of multiple criteria. Thus, the columns of the matrix \( Y \) corresponding to the selected DMD modes produce the matrix \( Y^{ROM} \). The same procedure is applied to the matrix \( Z \) of the eigenvectors of \( F^* \), producing the matrix \( Z^{ROM} \). Finally, the respective eigenvalues constitute the diagonal matrix \( D^{ROM}_\mu \). Therefore, the linear operator used for the ROM is:

\[
F^{ROM} = Y^{ROM} D^{ROM}_\mu Z^{ROM}.
\]

(13)

The main feature of the ROM is to enable operating on a lower dimensional space \( R^r \), rather than for the DMD operating with the matrix \( A \), which has size \( M \times M \), and most probably
impossible to calculate on standard computers because of memory requirements. Therefore, the ROM allows achieving very fast computational algorithms, and the DMD is now applied as

\[ \tilde{\psi}_{t+1} = F^{\text{ROM}} \tilde{\psi}_t \]  

(14)

with \( \tilde{\psi}_t \in \mathbb{R}^r \), which is obtained

\[ \tilde{\psi}_t = U^* \psi_t. \]  

(15)

The ROM is then embedded in a Kalman filter in order to produce a time-marching algorithm for wake prediction, and it also enables data-assimilation of new available observations. The Kalman filter addresses the general problem to estimate the state \( \tilde{\psi} \in \mathbb{R}^r \) of a process governed by the linear stochastic difference equation

\[ \tilde{\psi}_{t+1} = F^{\text{ROM}} \tilde{\psi}_t + B \tilde{w}_t. \]  

(16)

The variables \( \tilde{w} \) represents the process noise, and the respective linear operator \( B \) is evaluated by applying the DMD to the residual of the analyzed data not extracted through the ROM. With a Kalman filter, a measurement of the system \( \tilde{z} \in \mathbb{R}^r \) can be considered

\[ \tilde{z}_t = H \tilde{x}_t + \tilde{v}_t; \]  

(17)

thus, when a new observation of the system is available, \( \tilde{z}_t \), it is embedded in the system in order to improve the accuracy of the ROM. The variable \( \tilde{v} \) is the measurement noise, which is neglected by assuming an ideal measuring system. The observation matrix, \( H \), is an \( r \times r \) identity matrix, because all the LES data obtained from each snapshot are used for data-assimilation. The Kalman filter allows to evaluate the so-called Kalman gain, \( M \), which is an \( r \times r \) matrix. Each time a new observation, \( \tilde{z}_t \) is available, the forecast of the process, \( \tilde{\psi}_{t+1} \) is obtained as follows:

\[ \tilde{\psi}_{t+1} = F^{\text{ROM}} (\tilde{\psi}_t + M (\tilde{z}_t - \tilde{\psi}_t)). \]  

(18)

4. ROM for wind turbine wakes simulated with the actuator line model

In this section the ROM is evaluated for the wind turbine wake obtained from LES simulations carried out by the CFD Lab at UT Dallas by means of the actuator line model. The time-averaged axial velocity field evaluated over the wake vertical symmetry plane is shown in Fig. 1. The velocity deficit produced by the presence of the turbine tower and nacelle is evident in the very near-wake. Then, the whole wake, which is generated by the blade rotation, the turbine tower and nacelle, gradually recovers by proceeding downstream.

DMD has been applied to the full data set, and 210 DMD modes were extracted in order to achieve a detailed analysis of the wake dynamics. Their respective frequency, growth rate and energy are summarized in Fig. 2. Specifically, frequency is reported on the horizontal axis, the opposite value of the growth rate on the vertical axis, and energy is represented by both size of the dots and the colorbar reported therein. All the DMD modes have a negative growth rate, which characterizes stable modes with a reducing energy by proceeding downstream.

\[ \]
general damping of the DMD modes is a consequence of the uniform incoming wind, and thus wake dynamics are dissipated as they evolve downstream. The scenario is different for the case of a realistic atmospheric boundary layer flow presented in the next section, for which unstable modes are triggered by the incoming wind.

DMD highlights the presence of dominant modes, which are sorted as function of their physical origin, i.e. connected to the instability of the helicoidal tip vortices, the shedding of the trailing vortices, or to the vortex shedding from the turbine tower. In Fig. 3, the four most energetic DMD modes for each of those clusters are shown, and their characteristics are reported in Table 1.

The DMD modes 59 and 60 are connected to the vortex shedding from the wind turbine tower, and they are characterized by a frequency of 5.2 Hz. The DMD modes 63 and 64 are also connected to the same phenomenon, but they are characterized by a frequency of 10.4 Hz, which is double than the previous one. This harmonic contribution can be connected to the alternate vortex shedding, which produces a double frequency at the wake vertical symmetry plan. It is interesting to observe that this harmonic frequency is roughly equal to the rotational frequency of the rotor, \( f_{hub} \), which leads to suppose that the blade rotation triggers, or at least, affects the vortex shedding from the turbine tower. This remark is also endorsed by the slanted structure of the vortices, which indicates a perturbation produced by the rotor.
Table 1. Characteristics of the DMD modes selected for the reduced order model.

| DMD modes | Energy | Freq. [Hz] | Growth rate |
|-----------|--------|------------|-------------|
| 59-60     | 6.6    | 5.2        | -2.2        |
| 63-64     | 6.4    | 10.4       | -3.8        |
| 71-72     | 4.6    | 31.5       | -2.8        |
| 77-78     | 4.6    | 36.1       | -5.1        |
| 91-92     | 3.3    | 62.6       | -4.6        |
| 111-112   | 3.3    | 99         | -6.8        |

Figure 4. Flow reconstruction through the selected DMD modes: a) instantaneous axial velocity field; b) instantaneous velocity minus the time-averaged axial velocity; c) extracted velocity field; d) extracted velocity plus the time-averaged velocity field.

The DMD modes 71, 72, 77, and 78, are characterized by frequencies roughly equal to $3f_{hub}$, which are connected with the blade rotation, thus to the shedding of trailing and tip vortices. Finally, the DMD modes 111 and 112, are characterized by a frequency about $9f_{hub}$, and the DMD modes 91 and 92 by a frequency of about $6f_{hub}$. Their main energy contribution is localized at the rotor top tip height, which corresponds to the area where the helicoidal tip vortices are shed. All those features suggest that these DMD modes are associated with instabilities of the helicoidal tip vortices.

In order to evaluate the effectiveness of the selected DMD modes to constitute a ROM, the projection of the velocity field on those DMD modes is considered. Indeed, in Fig. 4(a) a snapshot of the axial velocity at a specific instant is reported, while in Fig. 4(b) the time-averaged velocity field is subtracted from the instantaneous one in order to highlight the wake dynamics. The instantaneous velocity field is projected on each of the selected DMD modes, then all the projections are added in order to produce the reconstructed velocity field. The latter is shown in Fig. 4(c), which clearly enables to capture the main flow dynamics shown in Fig. 4(b). Moreover, the accuracy in the flow reconstruction is even more evident when the time-averaged velocity field is added, as for Fig. 4(d). Therefore, it has been shown that the selected DMD modes are suitable to produce a ROM in order to represent the main dynamics present within the turbine wake.

The ROM is then embedded in a Kalman filter, as described above in section 3. The performance of the ROM is evaluated by calculating the global percentage error between the
predicted velocity field, \( \hat{\psi} \), and the original data, \( \psi \):

\[
E(t) = \frac{1}{M} \sum \sum \left| \psi(x_i, z_j, t) - \hat{\psi}(x_i, z_j, t) \right| < \psi(x_i, z_j) > ,
\]

where \( < \psi(x_i, z_j) > \) is the time-averaged velocity at a specific location. The global error is first calculated for the projection of the original data on the selected DMD modes, which is reported in Fig. 5 with black squares. This parameter is a reference for the approximation performed with the ROM. Then, the ROM embedded in the Kalman filter is used to predict the wind turbine wake flow by providing as only input the projection of the first snapshot on the selected DMD modes. Therefore, the velocity field is predicted for all the timestamps from 2 till \( N=214 \). The percentage global error for the wake prediction is reported in Fig. 5 with red dots, and it overlaps to close approximation with the error performed with the projection of the original data on the selected DMD modes. This result highlights the high accuracy obtained with the time-marching algorithm obtained by embedding the ROM in the Kalman filter.

In case the original velocity field is provided as initial condition of the ROM, then the ROM performance are further enhanced. Indeed, the global error, reported in Fig. 5 with a blue circles, shows that an even better prediction of the wake velocity field is obtained by using as initial condition the original snapshot of the axial velocity field.

5. ROM for wind turbine wake simulated with the actuator disc model and a neutral atmospheric boundary layer

The analysis of the LES data provided by the WIRE Lab at EPFL has been performed for the axial velocity field evaluated over the horizontal plane at hub height. The time-averaged velocity field is shown in Fig. 6. A total number of 2000 snapshots acquired with a sampling frequency of 9.7 Hz has been processed through DMD, producing the DMD spectrum presented in Fig. 7.

For this investigation only unstable mode with a positive growth rate have been considered for the ROM in order to investigate the dominant wake dynamics that might affect fatigue loads on possible downstream wind turbines. The selected unstable DMD modes are shown in Fig. 8, and their characteristics are listed in Table 2. It interesting to observe that all the unstable DMD modes have a wavelength equal or larger than the rotor diameter. This result suggests that the large turbulent scales present in the incoming atmospheric wind excite the wind turbine wake flow producing far-wake dynamics, such as wake meandering [16].

The unstable DMD modes are now used to evaluate the ROM as presented in the previous sections. The first snapshot of the LES data is used as initial condition of the time-marching algorithm obtained through the Kalman filter, and a closer-look to a specific timestamp is presented in the following. The instantaneous axial velocity field obtained from the LES data...
is reported in Fig. 9(a), while the same one minus the time-averaged velocity field is in Fig. 9(b). For the same timestamp, the velocity field predicted through the ROM shows in Fig. 9(c) the presence of coherent structures that evolve moving downstream with an increasing cross-width. Finally, in Fig. 9(d) the predicted velocity field clearly shows that the ROM formulated through the unstable DMD modes enables to predict the far-wake meandering excited by the large coherent structures present in the incoming atmospheric boundary layer.
6. Conclusions
A novel approach for fast and accurate prediction of wind turbine wakes has been presented, which consists in a reduced order model (ROM) embedded in a Kalman filter. The ROM is evaluated by means of dynamic mode decomposition performed on high fidelity LES experiments of wind turbines. This data-driven algorithm enables data assimilation of new measurements simultaneously to the wake prediction, which leads to an improved accuracy and a dynamic update of the ROM in presence of emerging coherent wake dynamics observed from new available data. It has been shown that the entire wake flow can be predicted with a global accuracy of about 4% by only providing time-resolved data of the wake flow, and then using as initial condition only for the prediction only one snapshot. It has been shown that only specific phenomena can be predicted by formulating opportunely the ROM, as for instance the prediction of far-wake meandering. The low computational cost of the proposed algorithm makes it suitable for real-time applications, control and optimization of large wind farms.

Acknowledgments
C. Santoni-Ortiz and S. Leonardi were supported by NSF PIRE Award IIA 1243482. TACC is acknowledged for providing computational time.

References
[1] Martinez-Tossas L A, Churchfield M J and Leonardi S 2014 Wind Energy, published online
[2] Porté-Agel F, Wu Y-T and Chen C-H 2013 Energies, 6 (10) 5297-5313
[3] Larsen G C, Madsen H A, Thomsen K and Larsen T J 2008 Wind Energy, 11 377-395
[4] Madsen H A, Larsen G C, Larsen T J, Trolldborg N and Mikkelsen R 2010 J. Solar energy Eng., 132 041014
[5] Iungo G V, Viola F, Camarri S, Porté-Agel F and Gallaire F 2013 J. Fluid Mech. 737 499-526
[6] Viola F, Iungo G V, Camarri S, Porté-Agel F and Gallaire F 2014 J. Fluid Mech. 750 R1
[7] Iungo G V, Wu Y-T and Porté-Agel F 2013 J. Atmos. Ocean. Technol. 30 274-287
[8] Iungo G V and Porté-Agel F 2014 J. Atmos. Ocean. Technol. 31 (10) 2035-2048
[9] Hamilton N, Tutkun M and Cal R B 2015 Wind Energy. 18 297-315
[10] Bastine D, Witha B, Wachter M and Peinke J 2015 Energies. 8 895-920
[11] Schmid P 2010 J. Fluid Mech. 656 5-28
[12] Kalman R E 1960 Trans. ASME J. Basic Eng. 82 Series D 35-45
[13] Jovanovic M R, Schmid P J and Nichols J W 2014 Physics of Fluids 26 024103
[14] Orlandi P and Leonardi S 2006 J. of Turbulence 7 53
[15] Abkar M and Porté-Agel F 2015 Physics of Fluids in press
[16] Espana G, Aubrun S, Loyer S and Devinant P 2011 Wind Energy 14 923-937