Simulation of cosmological stochastic background in LISA

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Abstract. A pilot study has been carried out to simulate stochastic gravitational wave background originating from first order phase transitions in the early universe. The space based gravitational wave detector LISA will be operational in the range of \(10^{-4}\) to 0.1 Hz and could be sensitive to the red shifted gravitational waves from cosmological origin. In this study we have modeled the signals from first order phase transitions and we compared the signal both with the expected instrumental noise and realistic simulated foreground signals, originating from the white dwarf population in our galaxy.

1. Introduction
One of the most striking observables from the early universe that has been measured up to now, is most probably the cosmic microwave background radiation (CMB). The CMB provides us with an astonishing picture of the early universe at the period of photon decoupling. The Planck mission, which is scheduled for launch next year will provide an even more accurate measurement of the CMB. With its launch foreseen in the next decade the LISA mission [1] may offer the possibility of measuring the state of the universe in an even earlier phase. It is postulated that first order phase transitions may give rise to violent events which could be a source of gravitational waves [2, 3].

In this paper we describe a pilot study to simulate gravitational waves originating from a primordial plasma. A description of the source is given in the next section along with the expressions for the expected spectra. In section 3 we describe the foreground, while we discuss the Monte Carlo simulations in 4. We conclude in section 5.

2. Gravitational waves from phase transitions
In many cosmological models the state of the universe is described by an effective potential of a scalar field. The potential is symmetric with a single minimum. With the expansion of the universe and the temperature drop, the scalar potential changes shape and reaches a point where it exhibits a barrier between two degenerate minima. At this point, first order phase transitions may occur, when quantum tunneling occurs through the barrier from the high energy state (the so-called “false” vacuum) to a lower level (the “true” vacuum). In contrast to second order phase transitions (when no barrier is tunneled), a first order phase transition leads to bubbles of true vacuum, which expand and merge until the complete plasma is in the broken symmetry state. Collisions between these bubbles are a source of gravitational waves and give rise to subsequent
turbulence in the primordial plasma. Besides bubble collisions, the turbulence of the plasma is an additional—and in most cases a stronger—source of gravitational waves.

The expected spectrum from the stochastic background is well described in the literature [4, 5]. Three important parameters that completely cover the characteristics of the gravitational wave spectrum are the rate of variation of the bubble nucleation rate, called $β$, and the ratio of the latent heat liberated at the phase transition to the energy density in the high energy phase, which is called $α$. The third parameter is the temperature of the plasma ($T$) and sets the scale of the symmetry breaking.

The intensity of stochastic background signals are conventionally expressed in a dimensionless quantity $Ω$ (or $Ωh^2$ to define a parameter which is independent of the actual Hubble expansion rate [6]). The spectra of the gravitational waves is rather featureless and is characterized by the maximum amplitude and the frequency at which the maximum occurs. At lower frequencies the amplitude depends on the frequency $f$ as $f^{2.8}$, and at higher frequencies as $f^{-1.8}$. The dependence of the amplitude frequency is related to the dynamics of plasma and the bubbles, which is detailed in [7, 8, 9].

The maximum intensity of the waves due to bubble collisions is given by:

$$Ω_{\text{coll}}h^2(f_{\text{coll}}) \simeq 1.1 \times 10^{-5} \kappa^2 \left(\frac{H_*}{β}\right)^2 \left[\frac{α}{1 + α}\right]^2 \left[\frac{v_b^3}{0.24 + v_b^2}\right] \left[\frac{100}{g_*}\right]^{1/3}$$

while the peak frequency is given by

$$f_{\text{coll}} \simeq 5.2 \times 10^{-3} \text{mHz} \left[\frac{β}{H_*}\right] \left[\frac{T_*}{100 \text{ GeV}}\right] \left[\frac{g_*}{100}\right]^{1/6}$$

The exact value of $α$ and $β$ (as well as the fluid velocities $v_b$ and $u_s$ and the function $κ$) depend on the details of the symmetry breaking potential, but typical values for $β/H_*$ (hereafter we used the dimensionless parameter $β/H_*$ instead of $β$) range up to a few $10^3$ [10]. Likewise, $α$ is found in the range 0–1. The parameter $g_*$ is the number of relativistic degrees of freedom at $T_*$. The number of degrees of freedom is model dependent and is in principle a free parameter in our study. However since the dependence of $Ω_{\text{coll}}h^2$ on $g_*$ is only $g_*^{1/3}$ we fix the parameter to the Standard Model value of 100 at or above the electroweak temperature. Note that some of the parameters are labeled ($*$) to denote their values at the transition temperature $T_*$. Gravitational waves from bubble collisions are accompanied by another and possibly a more dominant source of gravitational waves, i.e. the turbulence of the primordial plasma. The maximum amplitude and the peak frequency of this source is described in a similar way as the gravitational waves originating from bubble collisions:

$$Ω_{\text{turb}}h^2(f_{\text{turb}}) \simeq 1.4 \times 10^{-4} u_s^2 v_b^2 \left[\frac{H_*}{β}\right]^2 \left[\frac{100}{g_*}\right]^{1/3}$$

while the peak frequency is given by;

$$f_{\text{turb}} \simeq 3.4 \times 10^{-3} \text{mHz} \left[\frac{u_s}{v_b}\right] \left[\frac{β}{H_*}\right] \left[\frac{T_*}{100 \text{ GeV}}\right] \left[\frac{g_*}{100}\right]^{1/6}$$

Here, the amplitude at lower frequencies to the frequency with $f^2$ and at higher frequencies $f^{-7/2}$.

Note that the red-shift of the signal to the present epoch has been included in the description of the signal intensity and frequencies.
3. Foreground and noise

The stochastic background signal as discussed in the previous section is expected to be weak in comparison with the foreground signals and noise experienced in the spacecraft. A study of the stochastic background signal requires therefore detailed knowledge on the instrumental noise and foreground. The foreground signals are dominated by the white dwarfs binaries and are described in many papers [11, 12]. White dwarf binaries are expected to be numerous. They will give rise to individual waveforms, but the addition of many binaries will add up to a stochastic foreground. This foreground signal has a broad bandwidth, ranging from $10^{-4}$ down to $10^{-5}$ Hz. Individual binaries may be resolved or may stand out in form of peaks on top of a otherwise broad foreground signal. Because the white dwarf binaries are located in the galactic plane, the foreground is relatively high (compared to the instrumental noise), but due to LISA’s orbit around the Sun, the intensity will fluctuates with time with an annual periodicity. This periodicity may prove to be a handle to discriminate the foreground, since the stochastic background from cosmological origin is expected to be an isotropic signal.

4. Simulations and results

Since the existing science simulators for the LISA missions, such as the ‘LISA simulator’ [13] or ‘Synthetic LISA’ [14], only allow for point sources to be generated, simulation of an isotropic stochastic background cannot be done yet. A way out is to mimic an isotropic signal as the superposition of many isotropically distributed point sources. This solution is however relatively CPU time consuming and we therefore use the analytic description of the response of LISA [15] and convolve this with the expected stochastic background signal. An advantage of this method is that the generation of a signal spectrum is relatively fast, which allows us to generate the signals for the various input parameters. Given the number of input parameters a fast scan of the parameter space is advantageous. A drawback of the method is that no time trace is generated, which means that filtering algorithms cannot be tested on the signal of interest. A second

The annual averaged LISA response is shown in figure 1. Shown here is the frequency dependent response for the time delay interferometry parameter $X$. Details of the time delay interferometry (TDI) as used for the LISA mission can be found in [16].

![Figure 1](image)

**Figure 1.** Annual averaged response of the LISA mission for the TDI variable $X$. The analytical expression for the response is taken from [15].
In figure 2 we show the stochastic background spectrum for several possible values of the three free parameters as discussed in section 2.

![Figure 2.](image)

Figure 2. The fractional frequency fluctuation spectrum of the TDI variable $X$ for various signals in the LISA mission. The colored lines indicate several possible realizations of the cosmological stochastic background. The foreground signals from the white dwarf binaries are in black, while the grey line represents the instrumental noise.

The noise and white dwarf binaries foreground are taken from the Mock LISA Data Challenge (MLDC) [17]. This is a project that provides training and blind data sets containing combinations of realistic simulated LISA noise and the signal from typical gravitational wave sources. Participants of this project have to perform data analysis to extract the signals from the noise, and correctly identify the gravitational wave properties. We have used two data sets as representatives for noise and foreground. The first data set contains consists the LISA instrumental noise (MLDC round 1.3), while the second data set (MLDC round 2.1) consists of both instrumental noise and monochromatic signals from about 30 million white dwarf binaries. Both data sets are obtained in the form of a time trace of $2^{22}$ data points. Since the cadence of the time trace of the data is 15 seconds, the highest frequency in the spectrum is 0.033 Hz. In figure 2 the frequency spectra of the MLDC are shown together with the expected signal from the stochastic background. The MLDC data sets are transformed using the tools provided by the Synthetic LISA software package to obtain a frequency spectrum.

From figure 2 it is clear that the stochastic background spectra differ widely for the various input parameters and that the sensitivity for these signals varies with free parameters in the phase transition models. A complete simulation of one point in the parameter space involves a number of steps. The simulation is started by choosing a point the parameters space. In this study we varied the three parameters: $\alpha$, $\beta/H_*$ and the temperature $T_*$. The resulting frequency spectrum for each point in parameter space is then convoluted with the response of LISA and added to the data from the LISA Mock Data Challenge. The spectra with and without signal added are compared using a Kolmogorov-Smirnov test [18] for a large range of input parameters. Using this test has the advantage that the spectra can be compared without having to assume an analytical expression for the expected distribution. For each point in the parameter space the Kolmogorov-Smirnov test returned the probability that the null hypothesis (i.e., that the two spectra are the same) is false. Scanning the parameter space in this way provides the knowledge on the details of the phase transition for which the LISA would be sensitive. In figure 3 we
show the excluded values of $\beta/H_*$ as a function of the temperature $T_*$ for two values of $\alpha$. Two exclusion plots are shown: in figure 3 (a) stochastic background is compared with instrumental noise only, while in (b) the signal of interest is compared with the white dwarf binaries. In figure

![Figure 3. Excluded values for $\beta_*/H$ as function of the temperature $T_*$ at the phase transition for two values of $\alpha$. In (a) the spectrum from stochastic background is compared to LISA noise only, while in (b) the white dwarf foreground is added. The excluded region is on the shaded side of the solid lines.](image1)

![Figure 4. Excluded values for $\alpha$ as function of the temperature $T_*$ at the phase transition for two values of $\beta_*/H$. In (a) the spectrum from stochastic background is compared to LISA noise only, while in (b) the white dwarf foreground is added. The excluded region is on the shaded side of the solid lines.](image2)
5. Conclusion

We have studied and simulated the stochastic background from first order phase transitions in the early universe. Using the analytical expression for the response of LISA we have been able to compare the signal of interest to a realistic simulated dataset of noise sources and signals from astrophysical sources. Data sets that included the noise and foreground, were taken from the Mock LISA Data Challenge. To get a first estimation of LISA’s sensitivity to the gravitational waves from cosmological origin, the statistical significance has been determined between the spectra containing the noise sources and foreground signals with and without the signal of interest.

Based on the study we arrive at the following conclusions. The stochastic background signal from cosmological origin may be observed with LISA if the released latent heat is strong enough (i.e. large value for $\alpha$) in combination with a large bubble nucleation rate (i.e. low value for $\beta_*/H$). Moreover we see that LISA’s sensitivity is the largest for the temperatures at which the phase transition occurs, i.e. below a few TeV. At higher temperatures the spectrum from the stochastic background waves shift to higher frequencies and out of the sensitive region of LISA. Due to a cadence of the time trace in the data under study, the maximum frequency in our study was 0.033 Hz. A similar study with a smaller value for cadence could provide a better determination of the sensitivity at higher frequencies.

Finally we have shown that the suppression of the white dwarf foreground substantially increases the discovery potential of the mission for gravity-waves from symmetry breaking in the early universe, in particular if the symmetry breaking occurs at an energy scale between 100 GeV and 1 TeV.

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