Flow-proof strategy in proof construction on geometry instruction

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Abstract. The purpose of this research is to assess the role of flowproof strategy implementation in improving students’ mathematical proof in similarity and congruence. Proofing similarity and congruence is a prerequisite in developing mathematical skills. The subjects include prospective mathematics teachers, and the study used a qualitative descriptive technique. The implementation of the flow-proof strategy was achieved through four activities, including re-writing existed information, stating what is needed to be proved, using existed rules such as postulate and definition, and writing the results in flowcharts. The findings shows that flow-proof strategy on Geometry construction was implemented appropriately, based on students’ performance in similarity and congruence proofs with percentage of 72% from all research subject.

1. Introduction
Geometry is one of the branches of mathematics with an important role when it comes to teaching other concepts and solving daily problems [1,2,3,4]. It helps to develop critical thinking ability and transforms concrete reasoning to abstract form [5,6,7]. In school, geometry involves similarity and congruence, which are essential in developing relevant skills in learners. In general, congruence helps students understand the connection in geometry, such as the measurement of volume and area and develops advance mathematical thinking [9,10,11]. Additionally, the spatial ability could also be improved by learning congruence [11].

To effectively learn similarity and congruence, students should have the mathematical proof ability [12]. Proofing is essential in constructing mathematical knowledge and develops students’ critical and logical thinking ability and their general understanding [13,14,15,16,17]. According to [18], these benefits include the ability to (1) verify the truth, (2) give insight on why a statement is right, and (3) communicate mathematical ideas.

Kosasih and Rahayu suggest that students make mistakes while dealing with similarity and congruence problems [19, 20]. The errors are attributed to lack of prerequisite tasks given to students, making it difficult for them to understand further concepts and inaccuracies in solving problems [7,21]. Furthermore, a research by [22] stated that students are unable to construct congruence proof using two-column proof. Therefore, there is need to help them in this regard.

Apart from using the two-column proof, there is another strategy of constructing which might be used, and that is flowproof [23]. Generally, this approach involves arranging proof using flowcharts [23]. According to [24], there are two steps of using flowproof, analyzing the known information, and
looking for what is needed to construct a proof. [24] stated that in constructing proof using flowproof, students think back and forward to link conjectures and conclusions. Therefore, the syntax of using flowproof in this research include (1) rewrite known information from given problem, (2) State what’s to be proved from the problem, (3) use the existing rules such as postulate and definition, and (4) write the consequence of applied rules. According to Sari, flowproof helps students construct a proof since it analyzes what is known from a given problem. In line with this assertion, [26] argues that students tend to be more successful using flowproof rather than two-column proof. Employing flowproof helps train students’ critical and logical thinking [27]. The purpose of this research is to assess the role of flowproof strategy implementation in improving students’ mathematical proof in similarity and congruence.

2. Research method

This is a qualitative descriptive study which aims to analyse the implementation of flowproof strategy in geometry among prospective mathematics teacher at grade IX.3 of SMPN 33 Palembang, the academic year 2019/2020. The RPP (lesson plans), LKPD (student worksheets), and a mathematical proof test sheet in the form of problem statements consisting of 3 essays are designed. With the help of the lesson plan, data from field observations through a triangulation approach are analysed. To determine the potential of the flowproof strategy, the results of proof conducted by students through a set of predetermined questions are also assessed. The observations were made by looking at how the technique assist students in constructing proof using the emergence of flowproof syntax, including rewriting known information from a given problem, stating what is to be proved from the problem, using existing rules such as postulate and definition, and writing the results of the applied rules in the form of flowcharts.

Figure 1. Flowproof strategies designed by researchers
3. Result and discussion
From the observations of the activities of prospective teachers, it shows that all flowproof strategy steps are implemented, only at the first meeting the implementation is not in order. Prospective teachers start by rewrite known information from a given problem and then jump to the third step, use the existing rules such as postulate and definition. Whereas at the second meeting all teacher activities were carried out in accordance with the flowproof strategy. Teachers’ activities are shown in Table 1.

| Table 1. Prospective Teachers’ Activities |
|------------------------------------------|
| Flowproof Steps                          | Meeting          |
|                                          | First Meeting    | Second Meeting |
| Step 1 rewrite known information from given problem | ✓                | ✓              |
| Step 2 state what’s to be proved from the problem | ✗                | ✓              |
| Step 3 use the existing rules such as postulate and definition | ✗                | ✓              |
| Step 4 write the consequence of applied rules | ✗                | ✓              |

✓: well implemented
✗: not well implemented

For the first meeting, prospective teachers didn’t follow the order of designed learning by researchers. But prospective teachers carried out all the steps of flowproof strategy. The irregularities made by prospective teachers in using flowproof strategies occurred because they want to make it easier for students to prove what was asked. This is in line with research conducted by [25] that in constructing a proof using flowproof, students will think back and forth to connect between conjecture and conclusions. In general, the application of flowproof strategies in learning geometry of congruence material in SMP Negeri 33 Palembang by prospective mathematics teachers has been carried out according to what was developed by researchers. Prospective teachers begin by giving the theorem "If the two sides of a triangle are congruent, then the angle opposite that side will be congruent too", then the prospective teacher use the help of ∆ABC figure by following the theorem before constructing a proof [28].

![Figure 2. Theorem 1 in Triangles](image2)

After that, Teacher drew a parallelogram on the board and asked "what information is known in the picture?". Students answered that 2 sides are congruent. Next students are asked to write two congruent sides in LKPD namely $\overline{AC} \cong \overline{BC}$ shown in figure 3.

![Figure 3. Known information of the triangle](image3)
The second step of flowproof was skipped but implemented by the prospective teacher. The prospective teacher immediately jumped to step 3 using the previous rule by reminding the definition of congruence and told students that there are 3 postulates about two congruent triangles. In the third step, the students struggled in making two triangles, it’s because the students didn’t understand prerequisite material. Prerequisite material is also an obstacle in research conducted by [29]. Therefore, the teacher must provide a stimulus by reminding about the lines for the angles to obtain the line CD, and the coincide lines. With teacher’s guidance, students could write the used rules, namely the bisector and write in the parallelogram box, $\angle C \cong \angle C$, $\overline{CD} \cong \overline{CD}$. The use of rules showed in figure 4.

Furthermore, the Flowproof step is continued by inviting students to determine what will be proven from the theorem. However, students did not know what will be proven from the theorem. This is in line with research conducted by [29], stating that one of the difficulties students have in constructing a proof is determining what will be proven. This is because students did not understand the sentence "If - then" in the theorem, since students did not comprehend mathematical logic [30]. The teacher had to explain the mathematical logic related to "if - then" until students understand the theorem that will be proven, which was two mutually congruent angles, namely $\angle A \cong \angle B$, shown by Figure 5.

In the last flowproof step, which is to write down the effects of what is used, the prospective teacher asked students to use the number of postulates from the existing answers. Students answer "Postulate 13. Two triangles will be congruent if two corresponding sides are the same length and one angle is flanked both sides are equal (side-angle-side)" and the teacher reminds about the definition of two congruent triangles in which there are congruent angles and sides that are also congruent. The following answers to students in Figure 6.
After filling in the existing diagram. The final stage prospective teacher also asks students to rewrite their proofs in paragraphs assisted by flowproof strategy. The following is the result of student’s work:

![Flowproof Diagram](image)

**Figure 7.** Student constructs a proof in paragraph

Based on Figure 7, the students were able to construct a proof, starting from writing what is known, which is the first step of the flowproof strategy then writing what is proven obtained from the second step of the flowproof strategy. Then using the rules in accordance with the third step, and finally writing down the results of the rules used.

4. Conclusion

Prospective mathematics teachers have implemented a flowproof strategy in learning congruence appropriately in class IX of SMP Negeri 33 Palembang. This is shown from the worksheets of students using this approach to prove the existing theorem of congruence. However, at the first meeting, the teacher did not carry out the learning as designed by the researcher. The researchers make two main suggestions to be considered. First, there is a need to prove starts from junior high school to improve mathematical proof ability. Second, further research should be conducted to perfect the flowproof strategy steps. Also, other researchers are expected to innovate ways of using flowproof strategies and use other material in its application.

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