Dissipative solitons in an atomic medium assisted by an incoherent pumping field

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Abstract
This work models the propagation of an optical pulse in a four-level atomic system in the electromagnetically induced transparency regime. By demonstrating that linear and nonlinear optical properties can be externally controlled and tailored by a continuous-wave control laser beam and an assisting incoherent pump field, it is shown how these media can provide an excellent framework to experimentally explore pulse dynamics in the presence of non-conservative terms, either gain or loss. Furthermore, we explore the existence of stable dissipative soliton solutions, testing the analytical results with computational simulations of both the effective (1+1)-dimensional model and the full Maxwell–Bloch system of equations.

Supplementary material for this article is available online

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1. Introduction

Laser-induced coherent effects between atomic states have been a cornerstone of optics in recent decades. Offering a plethora of effects, from electromagnetically induced transparency [1–3] to light-storage [4–6], the potential of quantum optical systems to constitute an important building block in future technologies has been extensively explored [7–10]. Moreover, quantum enhanced nonlinearities characteristic of the near-resonant regime [4, 11–13] make these systems ideal for exploring nonlinear effects, ranging from the study of the dynamics of temporal [14–16] and spatial solitons [12, 17–19] to the realization of optical analog experiments [20–22]. Still, most of these studies often overlook the non-equilibrium aspects of the physical system. These include the interplay between dissipative effects (that come from relaxation processes), the gain mechanisms (arising from a pump optical beam for example), and even the temporal response and transient regime of atomic populations. Understanding the role played by each of these aspects in different situations is a key challenge for future theoretical and experimental work.

Concerning nonlinear optics, optical solitons are a hallmark, finding technological applications in communications and across all types of optical computing [23–27]. Resulting from a balance between dispersion and nonlinearity, their stable shape is usually lost if the optical medium features dissipation and gain. Remarkably, even in such non-equilibrium conditions it is still possible to observe a class of soliton solutions. These solutions, called dissipative solitons or autosolitons, result from a double dynamical equilibrium between dispersion and nonlinearity and also between dissipation and gain mechanisms [28–32]. While known to exist for a wide diversity of optical systems, in the particular case of coherent media the existence and dynamics of dissipative solitons is still under-explored [31, 32]. In a recent study [32], the existence of temporal dissipative solitons in a hollow core photonic crystal fiber filled with a three-level atomic system was predicted. However, in that case, the dissipative properties of the system can only be varied within a small range due to their dependence on the structure of the fiber, which can be detrimental for an experimental observation of the theoretical predictions. Moreover, the results were only confirmed through numerical simulation of an effective model and not through complete simulations of the Maxwell–Bloch (MB)
equations [33–36], which go well beyond the validity of the effective model and approximate the simulations to real experimental conditions.

In this paper, we propose an experiment-friendlier alternative, which allows the tuning of the linear and nonlinear processes. After the correct choice of parameters, in section 5 we discuss the role of the incoherent pumping in controlling the linear and nonlinear processes, and avoid the occurrence of potentially diverging secular terms [32, 40]. This technique consists of defining the multiscale variables \( t_1 = \delta t \) and \( z_2 = \delta t z \) and introducing the series expansion of the envelope function and density operator

\[
E_p = \sum_{l=1}^{4} \delta^l E_p^{(l)}, \quad \rho = \sum_{l=0}^{2} \delta^l \rho^{(l)}
\]

in equations (1) and (3). Then, a hierarchy of equations is obtained by separating the equations into their dependence on different orders of the parameter \( \delta \). After some cumbersome
calculations outlined in the supplementary materials available online at stacks.iop.org/JPB/53/065401/mmedia⁴, we obtain an equation for the Rabi frequency \( \Omega_p(z, \tau) \) of

\[
i\frac{\partial \Omega_p}{\partial z} + \frac{\beta_p^m}{2} \frac{\partial^2 \Omega_p}{\partial \tau^2} + g r \Omega_p \Omega_p^\dagger = i \beta_p^r \Omega_p - i g' |\Omega_p|^2 \Omega_p = - \frac{\beta_p^m}{2} \frac{\partial^2 \Omega_p}{\partial \tau^2} + \frac{\beta_p^m}{2} \frac{\partial^2 \Omega_p}{\partial \tau^2},
\]

where \( \tau = t - \beta_p^p z, \beta_p^r = \beta_p^r + i \beta_p^r \) is the propagation constant, and \( g = g' + ig' \) is related to the nonlinear susceptibility of the atomic medium. The two remaining parameters \( \beta_p^r = \beta_p^r + i \beta_p^r \) and \( \beta_p^r = \beta_p^r + i \beta_p^m \) are second-order and third-order expansion terms, respectively, and are related to the group velocity and to the group velocity dispersion as \( v_g \equiv 1/\beta_p^m \) and \( \text{GVD} \equiv 1/\beta_p^m \). Furthermore, we have separated each of the complex-valued parameters into their real and imaginary parts, which detach the conservative terms (associated with the real part) from the non-conservative terms (that can be either loss or gain and are associated with the pure imaginary parts).

All these quantities have an intricate dependency on the physical properties of the atomic system, which is discussed in more detail in the supplementary material (see footnote 4). Still, we emphasize that they depend on the characteristics of the physical system (specifically on the decay and dephasing rates), as well as on a set of experimentally tunable parameters, namely the incoherent pumping rate \( P \) and the detunings of the coherent fields \( \Delta_p \) and \( \Delta_c \). Thus, in principle, the optical response of the atomic system can be tailored externally in real experiments, which allows exploration of the existence of dissipative solitons in regions of the space parameters where the double dynamical equilibrium between dispersion and nonlinearity and between dissipation and gain processes is achieved.

3. Dissipative soliton solutions

The multiscale approach introduced in the last section and outlined in the supplementary material (see footnote 4) allowed the transformation of the MB equation system into a simplified (1+1)-dimensional model, which corresponds to the well-known complex cubic Ginzburg–Landau equation (CGLE). By rescaling \( \tau \rightarrow \tau / \sqrt{|\beta_p^m|} \) and \( \Omega_p \rightarrow \sqrt{|g'|} \Omega_p \), we can get the adimensional form of the former equation as

\[
i\frac{\partial \Omega_p}{\partial z} + \frac{s_d}{2} \frac{\partial^2 \Omega_p}{\partial \tau^2} + s_g \Omega_p \Omega_p^\dagger = i s_\alpha \Omega_p + i \epsilon \frac{\partial \Omega_p}{\partial \tau} + i e |\Omega_p|^2 \Omega_p + \xi \frac{\partial \Omega_p}{\partial \tau^2},
\]

with \( s_d \equiv \beta_p^m / |\beta_p^m| \) and \( s_g \equiv g'/|g'| \) where \( \sigma(q) \equiv q/|q| \) is the signal function, \( \alpha = -\beta_p^r, \xi = -\beta_p^r / (2|\beta_p^m|), e = -g'/|g'|, \) and \( \xi \equiv \beta_p^r / \sqrt{|\beta_p^m|^2} \). The CGLE can be found in a myriad of physical systems and, qualitatively, the terms on the left-hand side of the equation describe the dynamics of an envelope function in a nonlinear medium, while the extra terms in the right-hand side account for the dissipative or gain phenomena.

Neglecting the terms on the right-hand side of equation (7), it reduces to the well-studied nonlinear Schrödinger equation, which admits stable localized wave solutions called solitons when an equilibrium between dispersion and nonlinear effects is achieved. Depending on the signals of the dispersion and of the nonlinearity, these can be either of the bright type (if \( \sigma(\beta_p^m) \sigma(g') = 1 \)) or the dark type (if \( \sigma(\beta_p^m) \sigma(g') = -1 \)). Usually in optics, the bright solitons are more relevant, and for the standard nonlinear Schrödinger equations the family of solutions can be simply expressed as

\[
\Omega_p(z, \tau) = \text{asinh}[a(z - v\tau)] e^{i\alpha z - i|a|^2 - v^2 \tau^2 / 2}
\]

where \( a \) and \( v \) are both free parameters associated with the amplitude and the velocity, respectively. However, in the presence of the additional terms on the right-hand side, these solutions become unstable. Remarkably, it has been proven that equation (7) can still admit a special type of soliton-like solution called dissipative solitons or autosolitons, if conditions of equilibrium between gain and loss are met, in addition to the balance between dispersion and nonlinearity [39, 41–44].

In a recent study, Facão and co-workers investigated the existence of temporal optical dissipative solitons in a gas-filled fiber with a three-level atomic medium [32]. The family of solutions proposed is an extension of those previously predicted in the literature for cases where \( \beta_p^r = 0 \) [39], and are given by

\[
\Omega_p(z, \tau) = \text{asinh}[\sqrt{B} (\tau - \frac{\tau}{v})] e^{i\alpha z - iD\tau}
\]

where

\[
A = \frac{3B^2 (1 + 4\epsilon^2)}{2(2\epsilon^2 - \epsilon s_d)},
\]

\[
B = \sqrt{\frac{\alpha + \epsilon^2}{4\epsilon}},
\]

\[
C = B^2 \left[ 2v_d - \left( s_d (d^2 - 1) \right) \right] = s_d D^2
\]

\[
D = - \frac{s_d}{2\epsilon},
\]

\[
v = s_d v_d.
\]

and with the chirp parameter \( d \) equal to

\[
d = -3(1 + 2\epsilon \xi) + \sqrt{8(2\epsilon \xi - s_d \epsilon)^2 - 9(1 + 2\epsilon \xi)^2}.
\]

It is important to notice that contrary to what happens with the non-dissipative case, the parameters are now fixed by the optical properties of the system, and this family of solutions is
only stable [32] provided that the following conditions are met simultaneously

\[ \zeta > 0 \]  
\[ \alpha + \frac{\epsilon^2}{4\zeta} > 0 \]  
\[ \epsilon > \frac{\zeta(3\sqrt{1 + 4\epsilon^2} - 1)}{4 + 18\zeta^2}. \]  

However, while the soliton solution is stable per se if these conditions are satisfied, it turns out that they coincide exactly with the instability conditions for the background, which occurs in the complementary condition of (12), i.e. for \( \alpha + \frac{\epsilon^2}{4\zeta} < 0 \), as was previously noted in other studies on analytical solutions of the CGLE [42, 44]. This means that the stability of the solution is limited by the growth rate of the background, which can be proven to be significant at distances \( \zeta_{\text{inst}} \sim \left( \alpha + \frac{\epsilon^2}{4\zeta} \right)^{-1} \), thus constituting a boundary for the propagation of these dissipative soliton solutions.

In the previous work [32], Facio et al. propose that the linear loss usually present in a three-level atomic medium can be transformed into a gain by exploiting the transverse confinement of light by the structure of the hollow crystal fiber, which means that it is dependent on the structural design of the system and therefore not controllable. In the next section, we demonstrate how the use of the additional incoherent pumping in the four-level atomic system introduced in this paper can result in an experiment-friendly setup for exploring the phenomenology of temporal dissipative solitons in optical systems, as it allows easier control of the optical properties of the system over a wider parameter range, which turns out to be also helpful in controlling the stability of the solution.

### 4. Tuning the optical properties

As discussed in section 2, the optical properties of the system can be controlled externally by an appropriate choice of experimentally tunable parameters, namely the incoherent pumping rate \( P \) and the detunings of the coherent fields \( \Delta_p \). Furthermore, the four-level system proposed here can be realized with a multitude of atomic species and, just to give a practical example of real experimental values, we consider for now the hyperfine structure of the D line of \(^{85}\text{Rb}\) atoms [45] filling a rectangular waveguide of dimensions \( L_x = L_y = 10 \mu \text{m} \) at a fixed atomic concentration of \( \eta = 10^{12} \text{ cm}^{-3} \). Assigning the hyperfine structure levels \( 5S_{1/2}(F = 1), 5S_{1/2}(F = 2), 5P_{1/2}(F = 2) \) and \( 5P_{3/2}(F = 1) \) to \([1],[2],[3] \) and \([4]\), respectively, the dipole matrix elements are given by \( \mu_{13} \approx 6.74 \times 10^{-30} \text{ C m} \) and \( \mu_{23} \approx 2.24 \times 10^{-30} \text{ C m} \). Moreover, in typical room temperature experiments, the decoherence rates are approximately the same for every excited level, meaning that \( \gamma_3 \approx \gamma_4 \approx \gamma_5 \approx \gamma = 3.6 \times 10^7 \text{ s}^{-1} \). Finally, we will also consider the case for which the control beam has a constant spatial intensity with \( |\Omega_2| = \gamma \).

As we are interested in the observation of dissipative solitons of the family proposed in section 3, we shall now look to the regions of the space parameters \( (P, \Delta_p) \) that fulfill conditions (11)–(13) (herein referred to as conditions A) as well as \( \sigma(\beta^p_p) > 0 \) \& \( \sigma(\gamma^p_p) > 0 \) (herein conditions B), while maximizing \( \zeta_{\text{inst}} \sim \left( \alpha + \frac{\epsilon^2}{4\zeta} \right)^{-1} \) in order to limit the effects of the background instability. Figure 2(a) represents the regions of the parameter space where the conditions are met, while figure 2(b) shows the dependence of the magnitude of \( \zeta_{\text{inst}} \) in the region where both conditions are satisfied. A graphical analysis of figure 2(b), together with a numerical search on the parameter space suggests that \( \zeta_{\text{inst}} \) is maximized near the boundary of the intersection of the two conditions, which happens to coincide also with the line defined by the condition \( \beta^p_0 = 0 \). Remarkably, this case corresponds to the absence of linear absorption/gain and therefore to a case of perfect electromagnetically induced transparency. This characteristic is uncommon and distinct from what is observed with the most common three-level \( \Lambda \) atomic system, which usually features a linear absorption due to the dephasing rate between levels \([2] \) and \([1]\). As previously suggested in the literature [38], the suppression of the normal linear optical absorption in the four-level system can be attributed to a gain mechanism introduced by the incoherent pumping to the additional level. While not the main result of this work, this interesting and unusual response is important for the observation of dissipative solitons, but can also be relevant for the study of other problems, including the case of spatial dissipative solitons [38, 46].

Considering a control beam intensity with \( |\Omega_2| = \gamma \), the choice of detuning and incoherent pumping rate as \( \Delta_p = -4\gamma \) and \( P = 0.174\gamma \), respectively, together with the expressions given in the supplementary material (see footnote 4), allow us to obtain \( \zeta = 1.49, \xi = 1.55 \) and \( \epsilon = -5.42 \) with \( \zeta_{\text{inst}} \approx 2.4 \). Therefore, this choice of parameters corresponds to a system which features no linear gain/absorption and only includes effects of nonlinear absorption and spectral filtering [32]. Moreover, for this set of values we obtain a group velocity for the pulse of 0.3% of the vacuum speed of light. As discussed in the previous section, if the conditions A and B are met, it is possible to observe a class of
dissipative soliton solutions that we will test in the next section through numerical simulations.

5. Numerical results and discussion

To test the existence and stability of dissipative solitons in the atomic system proposed section 4, we have performed numerical simulations of both the full MB and the effective CGLE models. In both cases, we used high performance numerical tools based on general purpose Graphical Processing Unit (GPU) programming frameworks, an approach already employed in previous works [20, 21, 47, 48]. The MB system was solved using a leap-frog method and a finite-differences scheme [48], while the CGLE solver uses a standard beam propagation technique based on the split-step Fourier method (SSFM) [47].

For a four-level atom with \( \Delta_p = -4\gamma \) and \( P = 0.174\gamma \) (which correspond to the values \( \delta = 0 \), \( \zeta = 1.49 \), \( \xi = 1.55 \) and \( \epsilon = -5.42 \) in the CGLE effective model), equations (10) give the parameters for a dissipative solition solution of the form of equation (9), which results in \( A = 0.28 \), \( B = 0.087 \), \( C = -0.128 \) and chirp \( d = 5.75 \). The evolution of this dissipative soliton was calculated using the MB (see figure 3(a)) and SSFM (see figure 3(c)) solvers. The results obtained are qualitatively similar, despite a slight difference that can be identified by comparing \( \Omega_p \) at \( z = 0.3 \) m for both methods (see figure 3(b)). While the results from the SSFM preserve the initial intensity profile as it is expected for a dissipative soliton, the more complete MB method predicts a small reduction of the intensity amplitude \( |\Omega_p| \), suggesting that this might not be a true dissipative soliton. Therefore, to clarify if we do indeed have a dissipative soliton, we have performed numerical simulations of non-dissipative solitons given by equation (8) with \( a = 0.3 \) and \( \lambda = 0.6 \) (figure 4). Contrary to what happens in the dissipative soliton case, the absorption is now easily identified for small propagation distances, and the pulse amplitude features an appreciable decay. Moreover, it should be noticed that the case of figure 4(a) corresponds to a soliton solution of the non-dissipative nonlinear Schrödinger equation with the approximate amplitude of the dissipative soliton. However, while the amplitude is approximately the same, the dissipative soliton is usually broader as compared to the non-dissipative case [39, 42], as can be seen in figure 4(c), which signifies a different balance between gain and loss due to spectral filtering processes. Furthermore, figure 4(d) shows the total power that reaches the distance \( z \), normalized to the initial conditions and defined by

\[
N(z) \equiv \frac{\int_{-\infty}^{+\infty} |\Omega_p(z, \tau)|^2 d\tau}{\int_{-\infty}^{+\infty} |\Omega_p(0, \tau)|^2 d\tau}.
\]

It is straightforward to observe that \( N \) is conserved in the case of dissipative soliton conditions, contrary to what happens for the other initial conditions, where a strong decay is present. Together with the observations made earlier, this constitutes strong evidence that the type of dissipative soliton solution proposed can be observed in the atomic system under study. Additional simulations have shown that these solutions are stable up to distances of \( z \approx 4 \) m, which is consistent with previous results [32] and relevant for possible experimental observations.

Compared to the previous work on temporal dissipative solitons in atomic systems [32], our work introduces a system to allow full control of the optical properties and, therefore, to extend the parameter space of the CGLE to explore real experiments. In fact, the assisting incoherent pump allows an easy tuning of the gain and loss parameters, to an extent previously unavailable. Moreover, we also stress that our work extends the previous results, verifying the predictions with numerical simulations of not only the effective model but also of the more complete MB system, and are far more complete in terms of the dynamics of the optical system in real experiments. These simulations reveal small but still significant...
differences, possibly related to the breakdown of the perturbative approach. Indeed, the maximum amplitude of the dissipative soliton is $\Omega_0 \approx 0.16\gamma$, almost of the order of the other parameters of the system, which means that the dissipative soliton solution is near the boundary of the limits of the approximation. We have verified that this also happened in the case explored in previous works [14], and therefore constitutes an argument that full MB simulations are important to address the full dynamics of the system for future studies on the existence of dissipative solitons in real atomic systems.

6. Conclusions

In summary, in this work we have developed a theoretical model for the propagation of a temporal optical pulse in a four-level N-type atomic gas, driven by a continuous-wave electromagnetic field and an assisting incoherent pump. Employing a multiscale approach, it is shown that the propagation of a weak optical probe beam inside the two-dimensional waveguide can be described by an effective (1+1)-dimensional model in the form of a complex cubic Ginzburg–Landau equation, whose parameters are related to the optical properties of the system. We have shown that the N-type configuration together with the additional incoherent pumping rate allow a control of the optical response of the system that extends far beyond that previously studied, not only in terms of the linear and nonlinear optical properties but also of the gain and loss processes. Testing a family of dissipative soliton solutions, we have confirmed the possibility of observing them with computational simulations of both the effective (1+1)-dimensional model and the full Maxwell–Bloch system of equations, which is far more complete as it includes most of the dynamical aspects of a real system. Therefore, extending the parameter space available and providing more complete simulations, these results can motivate further studies on temporal dissipative solitons in atomic systems and can constitute a useful reference for experimental demonstrations of the existence of temporal dissipative solitons in these atomic systems.

Appendix

From the master equation (3) and the Hamiltonian (4) introduced in section 2, we can obtain a system of equations describing the dynamics of the four-level atomic system interacting with the coherent electromagnetic fields and assisted with incoherent pumping, as depicted in figure 1. Under the rotating wave approximation, it is possible to obtain

$$\dot{\rho}_{11} = \gamma_3 \rho_{33} + \rho_{44}(\gamma_4 + P) - P\rho_{11} + i(\Omega_{0r}\rho_{31} - \Omega_{p}\rho_{13})$$

$$\dot{\rho}_{21} = -\frac{1}{2}(\gamma_2 + 2i(\Delta_r - \Delta_p) + P)$$

$$\times \rho_{21} + \frac{i}{2}[\Omega^*_{0r}\rho_{31} - \Omega_{p}\rho_{23}]$$

$$\dot{\rho}_{31} = -\frac{1}{2}(2i\gamma_3 - 2i\Delta_p + P)\rho_{31}$$

$$+ i[\Omega_{0r}(\rho_{11} - \rho_{33})] + i\Omega_{p}\rho_{21}$$

$$\dot{\rho}_{41} = -(\gamma_4 + P + i(\Delta_r - \Delta_p))\rho_{41} - i\Omega_{p}\rho_{43}$$

$$\dot{\rho}_{22} = \gamma_3 \rho_{33} + \gamma_4 \rho_{44} + i[\Omega^*_{0r}\rho_{32} - \Omega_r\rho_{23}]$$

$$\dot{\rho}_{32} = (i\Delta_r - \gamma_3)\rho_{32} + i[\Omega_{p}\rho_{12} + \Omega_r\rho_{22} - \rho_{33}]$$

$$\dot{\rho}_{42} = -(\gamma_4 + P)\rho_{42} - i\Omega_{r}\rho_{43}$$

$$\dot{\rho}_{33} = -2\gamma_3 \rho_{33} + i[\Omega_{p}\rho_{13} + \Omega_r\rho_{23} + \Omega^*_{0r}\rho_{31} + \Omega^*_{p}\rho_{32}]$$

$$\dot{\rho}_{43} = -(i\Delta_r + \gamma_3 + \gamma_4 + P)\rho_{43} - i[\Omega^*_{0r}\rho_{41} + \Omega^*_{p}\rho_{42}]$$

$$\dot{\rho}_{44} = P\rho_{11} - (2\gamma_4 + P)\rho_{44}$$

$$\rho_{ij} = \rho_{ij}^*,$$  \hspace{1cm} (A.1)

which shall also respect the closure relation $\sum_i \rho_{ij} = 1$. The parameters introduced are related to the system as follows: $\Omega_{t}$ are the Rabi frequencies for the transitions defined as $\Omega_{p} = \mu_3 E_p/\hbar$ and $\Omega_{r} = \mu_3 E_r/\hbar$; $\Delta_p \equiv \omega_p - (\omega_1 - \omega_2)$ corresponds to the detunings of the probe beam with the transition $|1\rangle \rightarrow |3\rangle$; $\Delta_r \equiv \omega_r - (\omega_3 - \omega_2)$ is the control beam detuning, which in our case and for simplicity we have chosen to satisfy $\Delta_r = \Delta_p$. Also, $\gamma_3$ and $\gamma_4$ are the decay rates from state $|3\rangle$ and $|4\rangle$ to the ground states, respectively, and $\gamma_{21}$ is the dephasing rate related to the loss of coherence $\rho_{21}$, in this work considered all equal to $\gamma$, which is the most common situation in room temperature experiments. To obtain this set of equations we also made use of a rotating frame,

$$\tilde{\rho}_{12} = e^{i(\omega_1 - \omega_2)t}\rho_{12}$$

$$\tilde{\rho}_{13} = e^{-i\omega_1 t}\rho_{13}$$

$$\tilde{\rho}_{14} = e^{i(\omega_1 - \omega_2)t}\rho_{14}$$

$$\tilde{\rho}_{23} = e^{-i\omega_1 t}\rho_{23}$$

$$\tilde{\rho}_{24} = \rho_{24}$$

$$\tilde{\rho}_{34} = e^{i\omega_1 t}\rho_{34},$$  \hspace{1cm} (A.2)

which eliminates the explicit time dependency of the equation system.

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