Estimating Global Household Air Pollution: A Multivariate Hierarchical Model for Cooking Fuel Prevalence

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Abstract

Globally, an estimated 3.8 million deaths per year can be attributed to household air pollution. Information on the proportion of people relying primarily on different polluting fuels for cooking, which acts as a proxy for pollution exposure, is available in the form of nationally-representative household surveys. However, the absence of a modelling framework for comprehensively estimating the use of individual fuels inhibits fuel-specific policy interventions. To address this, we develop a multivariate hierarchical model (GHHEM) for data from the World Health Organization Household Energy Database, spanning the period 1990-2016.

Based on Generalized-Dirichlet-Multinomial distributions, the model jointly estimates trends in the use of eight individual fuels, whilst addressing a number of challenges involved in modelling the data. These include: missing values arising from incomplete surveys; missing values in the number of survey respondents; and sampling bias in the proportion of urban and rural respondents. The model also includes regional structures to improve prediction in countries with limited data. We assess model fit using within-sample predictive analysis and conduct an out-of-sample prediction experiment to evaluate the model’s forecasting performance. Overall, this work substantially contributes to expanding the evidence base for household air pollution, which is crucial for developing policy and planning interventions.

*Keywords:* Multivariate Proportion Data, Missing Values, Sampling Bias, Bayesian Methods, Generalized Dirichlet.
1 Introduction

In 2018, the World Health Organization (WHO) estimated that about 7 million deaths globally could be attributed to air pollution. This total comprises deaths associated with the joint effects of ambient (outdoor) and household exposure to air pollution. For household exposures, the proportion of households in a country relying mainly on various polluting fuels and technologies for cooking is used as an indicator of population exposure to household air pollution. In accordance with WHO guidelines for indoor air quality (household fuel combustion), households mainly cooking with coal, wood, charcoal, dung, crop residues or kerosene are considered exposed [World Health Organization, 2014]. Globally, it is estimated that in 2016, 41% of the world’s population were exposed to household air pollution resulting from cooking with polluting fuels and technologies [World Health Organization, 2018b]. The use of polluting fuels and technologies for cooking is primarily a problem in low and middle income countries where little over half (52%) of people have access to clean fuels, compared with 99% in high income countries.

Although in South East Asia the proportion of the population relying on clean fuels and technologies has doubled over the last two decades, progress in the African region has been much slower, with less than a 4% increase in the population using clean fuels and technologies for cooking [World Health Organization, 2018b]. Despite the apparent increase in the percentage of the population using clean fuels and technologies for cooking, the absolute number of people without access to clean fuels and technologies has stayed fairly constant. Global figures have remained unchanged since 2000, with currently over 3 billion people still relying on polluting fuels and technologies for cooking. To make further progress, through interventions such as encouraging households to adopt cleaner fuels like liquid petroleum gas (LPG) or promoting the use of technologies which make cooking with
polluting fuels safer, it is essential to understand current and past temporal trends in fuel use.

The proportion of populations with primary reliance on clean fuels and technology serves as a key indicator (7.1.2) for monitoring progress towards the Sustainable Development Goal (SDG) 7.1 ‘...to ensure universal access to affordable, reliable and modern energy services’. It also forms an important part of indicator 3.9.1, the mortality rate attributed to the joint effects of ambient and household air pollution, which monitors progress towards SDG 3.9, ‘... to substantially reduce the number of deaths and illnesses from hazardous chemicals and air, water and soil pollution and contamination’. Since the year 2000, all regions have seen progress in access to clean household energy but at varying rates.

Here, we propose a model that estimates trends in the proportion of people relying on individual fuels for cooking in each country, together with associated measures of uncertainty. Relationships between the use of different fuel types are modelled together with the variability associated with survey sampling, which may vary by country. Where data is not available within a country, or is insufficient to produce accurate estimates, the model structure derives information from regional trends. The model allows for different fuel usage in urban and rural areas and is able to produce predictions (with associated uncertainty) of future use of different fuel types, providing policy makers with a baseline against which they can evaluate the effectiveness of future interventions.

The remainder of the paper is organized as follows: Section 2 discusses the available data and previous approaches to modelling household fuel use; Section 3 provides details of the proposed modelling approach, including the implementation of the model using Markov chain Monte Carlo (MCMC); and Section 4 presents posterior predictive model checking and a future forecasting experiment. Finally, Section 5 provides an overall summary and a concluding discussion of the model’s impact.
2 Background

Information on the types of technologies and fuels used by households for cooking is regularly collected in nationally-representative household surveys or censuses. These data are compiled in the WHO Household Energy Database (World Health Organization, 2018a) which, as of late 2018, contains over 1100 surveys, with data from 157 countries for the years 1990 to 2016. Survey data are used to calculate the annual percentage of households in each country which use either solid fuels (charcoal, coal, crop waste, dung, rubber and wood) as their primary cooking fuel, liquid fuels (kerosene) or others including gaseous fuels or electricity. While survey coverage is increasing, there are still many countries with an insufficient number of surveys to directly produce reliable estimates.

To address this, statistical models can be used to pool information from other sources, such as covariates, in order to allow for more reliable inference in countries with insufficient survey data. For example, Rehfuess et al. (2006) use regression methods to quantify the association between solid fuel usage and a number of socio-economic factors, in order to predict usage in countries where no data was available. An alternative source of information which can be exploited by statistical models is within-region similarity in cooking fuel use. Figure 1 illustrates regional differences by showing smooth density estimates of the distribution of the proportion of households using wood as the primary cooking fuel by different WHO regions. Regional pooling was adopted by Bonjour et al. (2013), using multi-level models with regional hierarchies to model trends in solid fuel usage, for the purpose of estimating the number of people exposed to household air pollution globally.

Previous approaches using data from the WHO Household Energy Database have focussed on understanding the proportion of population that use solid fuels as a group, rather than individual fuel usage. While a useful indicator for exposure to household air pollution,
Figure 1: Smooth density estimates of the regional variability in the usage of wood as the primary cooking fuel by WHO region, and globally. Regions are AMR = Americas, EMR = Eastern Mediterranean, EUR = Europe, SEAR = South East Asia, WPR = Western Pacific.

the limitation of only estimating the proportion of solid fuel usage in each country is that it inhibits interventions based on specific fuels, such as the deployment of cleaner wood burning stoves, while also failing to take into account the varying levels of harm caused by different fuels. Jointly modelling multivariate proportion data (e.g. from individual fuel types) is a challenging statistical problem in its own right, however there are also three notable features of the data which pose additional challenges:

(i) Many surveys only report values for a subset of the fuels of interest, meaning that the model must allow for surveys of varying levels of completeness. This ensures that inference regarding the use of the other fuels is possible on the basis of partial information and pooling across the data.

(ii) The total number of respondents is only available for approximately 50% of surveys, which prohibits the direct modelling of the number of respondents that use each fuel (e.g. with a Multinomial distribution). Instead, proportions (for which data is available) may be modelled using a multivariate distribution on the simplex, such as the Dirichlet. However, this would assume that individual values are strictly between 0 and 1, whereas there are numerous observations of 0% and 100% in the data considered here, making this impractical.
Although most surveys provide distinct observations for fuel use in urban and rural areas, a large number only provide information for the entire population (both urban and rural). The model must be applicable for both types and, in the case of surveys with only values for overall, allow estimation of the urban and rural values based on information on the proportion of people living in urban and rural areas (in that country). This is made more problematic as, for some countries, surveys include too many or too few urban or rural respondents, which introduces bias in the overall value. To allow for this, the model should also be able to estimate any systematic bias in the proportion of urban and rural people included in surveys, compared to external data on the proportions of the wider population living in urban and rural areas.

3 Model Design

For clarity of exposition, the following explanation relates to $y_i$, the number of respondents in a survey using fuel $i$ as their primary fuel for cooking, ignoring for now any indices related to the country and the year. Here, $i = 1, \ldots, 9$ corresponds to wood, charcoal, coal, crop waste, dung, electricity, l.p.g., kerosene and finally an aggregation of other fuels (e.g. natural gas), which mostly constitute a very small percentage of the total. If we knew the total number of survey respondents $n$ for all data, a first approach to modelling could be to assume that data on $y = \{y_i\}$ arise from a Multinomial$(p, n)$ distribution. Then, $p_i$ would represent the proportion of people in the population using fuel $i$. This assumes that the survey sample is representative of the overall population. In reality, survey samples are imperfect and the Multinomial model may not be sufficiently flexible to capture the extra variability caused by flaws in the survey design. For instance, the survey may not cover
the whole geographical area of interest.

A flexible extension of this approach is to model \( y \) using a Generalized-Dirichlet-Multinomial(\( \alpha, \beta, n \)) (GDM) distribution, a mixture of the Generalized-Dirichlet with pdf:

\[
p(p_1, p_2, \ldots, p_k \mid \alpha, \beta) = p_k^{\frac{\alpha_i - 1}{\beta_i}} \prod_{i=1}^{k-1} \left[ p_i \left( \frac{\sum_{j=i}^{k} p_j}{B(\alpha_i, \beta_i)} \right)^{\beta_i - (\alpha_i + \beta_i)} \right] \tag{1}
\]

and the Multinomial distribution, so that

\[
p \sim \text{Generalized-Dirichlet}(\alpha, \beta); \quad y \mid p \sim \text{Multinomial}(p, n) \tag{2}
\]

with pdf:

\[
p(y_1, y_2, \ldots, y_k \mid \alpha, \beta, n) = \frac{\Gamma(n + 1)}{\Gamma(y_k + 1)} \prod_{i=1}^{k-1} \left[ \frac{\Gamma(y_i + \alpha_i) \Gamma(\sum_{j=i+1}^{k} y_j + \beta_i)}{B(\alpha_i, \beta_i) \Gamma(y_i + 1) \Gamma(\alpha_i + \beta_i + \sum_{j=i}^{k} y_j)} \right] \tag{3}
\]

This means that any additional variability caused by flawed sampling can be potentially captured by the Generalized-Dirichlet component. The Generalized-Dirichlet also has a very flexible covariance structure compared to the Dirichlet, which it reduces to in the special case that \( \beta_i = \alpha_{i+1} + \beta_{i+1} \) for \( i \in 1, \ldots, k - 2 \) and \( \beta_{k-1} = \alpha_k \).

Recall that for most of the available data, only the proportion \( x = \{y_i/n\} \) of respondents using each fuel is available, with the total number of respondents \( n \) being unknown. This means that the GDM cannot be used to directly model the number of respondents primarily using each fuel, if one wishes to use all of the available data. However, here the principal interest lies in estimating the fuel usage proportions \( x \), so an alternative approach would be to model the proportions themselves, for example using a Generalized-Dirichlet distribution. In that case, though, the presence of many 0% and 100% fuel usage observations (which fall outside the range space of the Generalised-Dirichlet) make this impractical. Instead, we opt for an approximate procedure for modelling \( x \), namely by
transforming observations of \( x_i \) into conceptual counts \( v_i \), out of a user-chosen total \( N \). To ensure that the sum of the transformed counts does not exceed \( N \), one can compute \( v_i = \lfloor nx_i \rfloor \) (as opposed to rounding). The counts \( v \) can then be modelled as \( \text{GDM}(\alpha, \beta, N) \) so that predictions are based on \( v_i/N \).

Using this method, we can obtain approximately the same inference for the underlying usage in the wider population as if we had modelled \( y \) directly. In addition, the flexibility of the GDM means that we can still capture the distribution of \( x \) well. This is because any variability lost or gained from the Multinomial component, by respectively using a larger or a smaller \( N \) compared to the original \( n \), can be accounted for by appropriate adjustment in the parameters of the underlying Generalized-Dirichlet component. Here we opted for \( N = 1000 \) so that the contribution to the variability of \( v/N \) from the Multinomial component is relatively small, bearing in mind that the GD component can absorb any additional variation associated with smaller sample sizes.

### 3.1 Conditional models

Recall that one of the main issues with the given data is that quite often, a value \( x_i \) (and thus \( v_i \)) for at least one individual fuel is missing (for a given country-year combination). To model this data in a way that inference is made possible on the missing fuel proportions, it makes sense to implement the GDM using the implicit conditional densities rather than the joint one. Specifically, for counts \( v \) and total \( N \), the conditional distribution of (fuel)
Given the others is:

\[ v_1 \mid \alpha, \beta \sim \text{Beta-Binomial}(\alpha_1, \beta_1, n_1 = N - v_1) \]  
(4)

\[ v_i \mid v_1, \ldots, v_{i-1}, \alpha, \beta \sim \text{Beta-Binomial}(\alpha_i, \beta_i, n_i = N - \sum_{j=1}^{i-1} v_j) \text{ for } i = 2, \ldots, k \]  
(5)

\[ p(v_i \mid v_1, \ldots, v_{i-1}, \alpha, \beta) = \frac{\binom{n_i}{v_i} B(v_i + \alpha_i, n_i - v_i + \beta_i)}{B(\alpha_i, \beta_i)} \]  
(6)

Fitting this model in a Bayesian setting implies that any missing values \( v_i \) can be sampled using Markov Chain Monte Carlo (MCMC).

For ease of interpretation, it makes sense to re-parameterize the conditional distributions in terms of their expectations \( \nu_i \) and variance parameters \( \phi_i \):

\[ \alpha_i = \nu_i \phi_i; \quad \beta_i = (1 - \nu_i) \phi_i \]  
(7)

The relative mean \( \nu_i \) is interpreted as the mean proportion of households using fuel \( i \) out of those not using any of the fuels higher up the hierarchy \( 1, \ldots, i-1 \). For example, \( \nu_1 \) is the underlying proportion who use wood from the whole population, \( \nu_2 \) is the proportion who use charcoal from the population who do not use wood and \( \nu_3 \) is the proportion who use coal from the population who use neither wood nor charcoal. Through parameter \( \phi_i \), the model is able to compensate for any gain or loss of variance in the conditional Multinomial component caused by the introduction of the “artificial” total \( N \).

It is noted that for the SDG indicators, the primary quantity of interest is the marginal mean vector of proportions \( \mu = \{\mu_i\} \) of households relying on each fuel \( i \). This can be
recovered from the relative means $\nu_i$:

$$\mu_1 = \nu_1$$

$$\mu_2 = \nu_2(1 - \nu_1)$$

$$\vdots$$

$$\mu_k = \nu_k \prod_{i=1}^{k-1} (1 - \nu_i)$$

Now introducing indices for a survey conducted in WHO region $r$, country $c$ and year $t$, the characterisation of the relative mean $\nu_{i,r,c,t}$ is defined by:

$$\log \left( \frac{\nu_{i,r,c,t}}{1 - \nu_{i,r,c,t}} \right) = \gamma_{i,r,1} + \delta_{i,c,1} + (\gamma_{i,r,2} + \delta_{i,c,2})t$$

where the logistic transformation ensures that $\nu_{i,r,c,t} \in (0, 1)$. Fixed effects $\gamma_{i,r} = (\gamma_{i,r,1}, \gamma_{i,r,2})$ capture regional (linear) changes over time in mean fuel use. Constrained (random) effects $\delta_{i,c,1}$ and $\delta_{i,c,2}$ allow countries to deviate from the regional trend, if there is sufficient evidence in the data that they should. We return to the model for the variance parameters $\phi_i$ later on.

### 3.2 Rural and Urban variability

A further source of variability in fuel usage arises from differences between rural and urban areas. The model captures this by allowing the regional trends as well as the country differences in (11) to be different for rural/urban areas. It is likely that these differences within a country are correlated, so to capture this we model each pair $\delta_{i,c,j} = (\delta_{i,c,j}^{urban}, \delta_{i,c,j}^{rural})$ with a Multivariate-Normal distribution:

$$\begin{pmatrix} \delta_{i,c,j}^{urban} \\ \delta_{i,c,j}^{rural} \end{pmatrix} \sim \text{Normal}(\mathbf{0}, \Sigma_{i,r,j}) \text{ for } j = 1, 2$$

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where the covariance matrix $\Sigma_{i,r,j}^\delta$ is allowed to differ between regions. Unfortunately, while most surveys in the data report both urban and rural values, this is not always the case. Some only report an overall value for the whole sample. So that we can still use this information, we incorporate a layer in the model to constrain the marginal mean proportions as follows:

$$
\mu_{r,c,t}^{\text{overall}} = \pi_{c,t} \mu_{r,c,t}^{\text{urban}} + (1 - \pi_{c,t}) \mu_{r,c,t}^{\text{rural}} \tag{13}
$$

$$
\log \left( \frac{\pi_{c,t}}{1 - \pi_{c,t}} \right) = \log \left( \frac{P_{c,t}}{1 - P_{c,t}} \right) + f_c(t) \tag{14}
$$

The overall mean proportion of fuel usage $\mu_{r,c,t}^{\text{overall}}$ is a vector of individual fuel usage, as defined in (8)–(10), for each region $r$, country $c$ and year $t$. This is then defined as a weighted sum in (13), of the rural and urban mean proportions. The weights $\pi_{c,t} \in (0, 1)$ represent the mean proportion of survey respondents living in an urban area, in country $c$ and year $t$. Furthermore, $P_{c,t}$ are estimates of the proportion of people living in an urban area for each country and year [United Nations 2018] and they are used as offsets in a model for $\pi_{c,t}$. For each country, systematic deviations from these estimates are modelled using a smooth function $f_c(t)$, to allow for potential under- or over-sampling of urban populations in the survey data. Since $f_c(t)$ is a correcting factor, it should ideally be flexible enough to capture any systematic sampling biases with respect to the UN estimates $P_{c,t}$. However, from a modelling perspective, this introduces extra degrees of freedom for the model to capture the overall survey observations well. Therefore, to avoid over-fitting, we model $f_c(t)$ using penalised low-rank thin-plate splines [Crainiceanu et al. 2005], using a different smoothing penalty parameter $\sigma_c$ for each country. This allows $f_c(t)$ to capture non-linear deviations from $P_{c,t}$ over time, but only when there is ample evidence of non-linearity in the data for a given country.

We complete the model specification by defining the following structure in the variance
parameters $\phi_{i,r,c}$:

\[
\log(\phi_{i,r,c}) = \psi_{i,r} + \epsilon_{i,c}
\]  

(15)

where $\psi_{i,r}$ are fixed regional effects and $\epsilon_{i,c}$ are country-level deviations from these. This allows for the fact that average survey size and representativeness can vary between countries, affecting the variance of the observed fuel proportions. As with the means, we believe the variance parameters for urban, rural and overall survey values to be correlated within a country, and so to capture this we use a Multivariate-Normal model for the triples $\epsilon_{i,c} = (\epsilon_{i,c}^{urban}, \epsilon_{i,c}^{rural}, \epsilon_{i,c}^{overall})$:

\[
\begin{pmatrix}
\epsilon_{i,c}^{urban} \\
\epsilon_{i,c}^{rural} \\
\epsilon_{i,c}^{overall}
\end{pmatrix}
\sim \text{Normal}(0, \Sigma_{i,r})
\]  

(16)

Such that, as with the model for the $\delta_{i,c,j}$, the covariance structure is allowed to differ between regions.

### 3.3 Prior distributions and implementation

For the regional (fixed) effects $\gamma_{i,r}$ and $\psi_{i,r}$, we chose to specify weakly informative Normal$(0, 10^2)$ and Normal$(0, 2^2)$ prior distributions, respectively. These reflect our limited prior knowledge of regional trends. For more efficient MCMC sampling, the country effects $\delta_{i,c,j}$ and $\epsilon_{i,c}$ were centred on the regional effects, so that their Multivariate-Normal means were (instead of 0 as in (12) and (16)) $\gamma_{i,r,j}$ and $\psi_{i,r}$, respectively, with:

\[
\log \left( \frac{\nu_{i,r,c,t}}{1-\nu_{i,r,c,t}} \right) = \delta_{i,c,1} + \delta_{i,c,2} t
\]  

(17)

\[
\log(\phi_{i,r,c}) = \epsilon_{i,c}
\]  

(18)
For the Multivariate-Normal covariance matrices, we chose conjugate Inverse-Wishart prior distributions with informative marginal distributions for the variances and correlations, with more prior density over positive correlations than negative correlations. For the smoothing penalty parameters \( \sigma_c \) we assigned Half-Normal(0, 1) prior distributions. This reflects our belief that smaller values for \( \sigma_c \) (corresponding to a stronger penalty) are more likely than larger ones.

All code was written and executed using R (R Core Team (2018)) and the model was implemented using NIMBLE (de Valpine et al., 2017), a facility for highly flexible implementation of Markov Chain Monte Carlo (MCMC) models. For this application, we needed to add the Beta-Binomial distribution to NIMBLE, which was straightforward using only a few lines of R code. Four MCMC chains were run for 80,000 iterations from different randomly generated initial values and with different random number generator seeds. The first 40,000 samples were then discarded as burn-in and, to limit system memory usage, the remaining samples were thinned by 10. Convergence of the MCMC chains is discussed in Appendix A. The model was applied to a subset of the data consisting of 993 surveys. Survey selection criteria are discussed in Appendix C. All the associated code and data required to implement the model and to reproduce all results are included as supplementary material.

4 Model Checking

The task of assessing the validity of the statistical model is divided into two parts. The first comprises basic procedures to check that the model has no systematic issues with reproducing the observed data, while the second assesses the ability of the model to predict future fuel usage values.
4.1 Posterior predictive checking

Given the Bayesian implementation of the model, assessing the fit to both in-sample and out-of-sample data is based on posterior predictive model checking (Gelman et al., 2014). For in-sample data, this involves simulating from the posterior distribution of parameters and random effects (samples of which are already available from MCMC) and then simulating $v_i$ from the conditional Multinomial distribution to obtain samples of the posterior predictive distribution for replicates $\tilde{x}_{i|\tilde{x}}$ of the observed fuel proportions $x$. The statistical properties of these replicates can then be compared to properties of the corresponding observations. For brevity, we present predictive checking for wood usage in this subsection and for all of the other fuels in Appendix B.

Figure 2: Scatter plots comparing posterior means of wood usage replicates $\tilde{x}_{1,r,c,t}$ to their corresponding observed values.

In the first instance, scatter plots comparing the posterior means of the replicates with the observed values can give an indication of any systematic issues. These are shown for wood in Figure 2 and, for the most part, there are no obvious systematic issues. Also shown are coverage values, the proportion of observed values which lie within the 95% posterior predictive intervals of the corresponding replicates. A coverage substantially lower than
95% would mean a high proportion of observed values are extreme values with respect to the posterior predictive model, implying a poor fit. In this case, the coverage values for the 95% credible intervals were higher than 95% for all fuels and areas. Taken together, these two checks indicate that the model captures the observed data well.

Figure 3: Mean predicted fuel usage trends with associated 95% posterior predictive intervals for Ethiopia. Coloured points represent survey observations and black points represent removed surveys. For each fuel, the left, central and right plots show urban, rural and overall usage, respectively.

Another way of checking the model is to compare predicted trends to survey observations on an individual country basis. Figure3 shows the mean predicted trend for the proportion
using each fuel in each segment (urban, rural and overall) of Ethiopia, with associated 95% posterior predictive intervals. Here it can be seen that the mean trend lines follow the observed trends well, with prediction intervals that envelop a reasonable number of surveys. Moreover, by examining the tightness of the prediction intervals with respect to the variance of the observations, we can verify that the high coverage values obtained for the replicate prediction intervals are not simply caused by excessively high model uncertainty.

Figure 4: Mean predicted fuel usage trends with associated 95% posterior predictive intervals for South Africa.

The same plots can serve as a useful tool for identifying surveys which don’t align with
the general pattern in a given country. Figure 4 shows the predicted trends for South Africa and it can be seen that all surveys report substantial use of electricity for cooking, except for one survey which reports zero usage. The model has sought to capture the conflicting information with a mean trend line between the two, accompanied by wide prediction intervals. While not completely implausible, the plot indicates this may warrant further investigation.

Note that to check the model reproduces the observed data well, the overall predictions in Figures 3 and 4 incorporate the model’s prediction of any systematic deviation from the UN estimates of urban and rural proportions, in the sampling of urban and rural respondents. Instead, predictions of overall fuel usage can be based solely on the UN estimates of urban and rural proportions (rather than based on the proportions in the surveys), which may constitute a more robust summary of fuel usage in a given country. This is achieved by removing $f_c(t)$ from (14) during simulation. Fuel usage plots for both the prediction of new surveys (which include sampling variation and potential sampling biases, as in Figures 3 and 4) and the prediction of the underlying fuel usage in the population $(\mu_{i,r,c,t})$ are available for all countries in the supplementary material.

The model’s ability to capture systematic biases in the proportions of urban and rural respondents in the survey samples, relative to UN estimates, can also be inspected; Figure 5 shows the proportion of respondents recorded as urban in the fuel surveys for India (left) and Malawi (right) compared to UN estimates and predicted values from the model. The plot for India shows evidence of systematic over-sampling of urban respondents between 1997 and 2007, compared to the UN estimates. The plot for Malawi, meanwhile, shows limited evidence of any systematic deviation. It is likely that ignoring potential systematic biases would result in a less reliable inference for the relationship between urban, rural and overall surveys. In both of these cases, the spline incorporated in (14) appears to capture
Figure 5: Plot of the urban proportions of fuel survey respondents in India (left) and Malawi (right), shown as points, compared to the U.N. estimates of the proportion of the respective populations living in an urban environment. Also plotted are the model’s predictions of the change in each country’s mean urban proportion of the surveys, with 95% credible intervals.

any differences well and therefore the associated bias can be mitigated when predicting and forecasting fuel usage.

4.2 Forecasting experiment

Samples from posterior predictive distributions for out-of-sample data are obtained in the same way as in-sample data, albeit using future time points as covariate values. The model’s ability to predict (forecast) can be assessed using out-of-sample predictive testing. This is particularly important for this model to evaluate its suitability for forecasting future fuel use. To emulate a hypothetical forecasting scenario, the model was fitted only to surveys up to and including year 2012, therefore excluding 4 years (approximately one third of the data). We then used the model to predict 4 years into the future and produce predictive distributions for the out-of-sample data. Note that forecasting future overall
survey observations would involve forecasting how any systematic trends in the sampling of urban and rural respondents will progress in the future. While this may be possible it is not our primary interest, and here we focus on checking the out-of-sample prediction of urban and rural surveys.

Figure 6: Scatter plots of mean predicted fuel usage values from 2013 onwards, versus their observed values, from the model which was only supplied data from 2012 or earlier.

Figure 6 shows scatter plots comparing the out-of-sample survey values to the mean predicted values from the model. While there are some values which are not captured well
(some potentially due to erroneous data), generally the model does not seem to systematically over or under-predict. The only exception to this is crop waste (urban), where there does appear to be some systematic deviation from the diagonal. Notably, the coverage values tend to be quite high, indicating that the model produces reliable uncertainty estimates when predicting into the future.

Figure 7: Mean predicted fuel usage trends with associated 95% posterior predictive intervals for Nepal, from the model where surveys from 2013 onwards were excluded. The black points from 2013 onwards represent values from excluded surveys.

To guard against high coverage values through unreasonably uncertain prediction intervals, we can assess the model’s performance when forecasting by examining predictive
plots for individual countries. Figure shows predictive fuel usage plots for Nepal, from the model where surveys from 2013 onwards are excluded. Though for some fuels the excluded surveys deviate from the mean predicted trend, they are generally well within the 95% predictive intervals, which are not so wide that they are impractical. Looking at urban areas, we can see that the surveys up to 2012 suggest the use of LPG might continue to increase from 2013 onwards, although the model appears to correctly predict the plateauing that is present in the results from the excluded surveys.

5 Discussion

In this paper, we have developed a multivariate hierarchical model, based on Generalized-Dirichlet-Multinomial distributions, to model trends in the use of polluting and clean fuels for cooking across the world. The work was motivated by the need to expand the evidence base related to household use of individual fuels that is crucial when developing policy and planning interventions. The principal aim was to estimate changes in the use of individual fuels within the period 1990-2016. This was achieved for each country, with distinction between urban and rural areas, together with predictions of future fuel usage. The proposed approach addresses the inherent difficulty in jointly modelling multivariate proportion data, and several other challenges associated with modelling the data from the WHO Household Energy Database. These challenges included missing values for the use of some fuels within surveys, the total number of respondents only being available for approximately half of all surveys, some surveys not distinguishing between urban and rural areas, and biases in the sampling of urban and rural respondents.

The resulting global hierarchical household energy model (GHHEM) is implemented within a Bayesian hierarchical framework. Trends in the proportions of populations using
each fuel are estimated for each country, based primarily on information from surveys within that country. Where data are not available within a country, or are insufficient to produce accurate estimates, the model structure ‘borrows’ information from regional trends and, in such cases, the associated uncertainty is increased. The model also takes into account, and estimates, any systematic biases in the sampling of urban and rural respondents. The primary output of the model is the underlying fuel usage in the sampled population, represented by the \( \mu_{r,c,t} \) in Equation (13). This constitutes a more robust and stable measure on which to base policy decisions than using individual surveys.

Predicting future patterns of fuel usage from the model using the estimated trends provides a baseline representation of what might be expected in the absence of intervention and provides a comparison against which future surveys conducted post-interventions could be compared. The advantage of modelling the relative fuel means (\( \nu_{r,c,t} \), in Equation (11)) as linear in time (on the logistic scale) is that it is possible to extrapolate observed trends arbitrarily far into the future. However, this should be done in the context of the forecasting experiment in Section 4.2, which suggests forecasts might be restricted to a few years into the future, beyond which it is possible that the logistic-linear approximation may not be reasonable.

The model has been used by the WHO to produce estimates of the number of people in each country who use polluting fuels for cooking, to provide proxy for exposure to household air pollution and to assess the take-up of clean fuel technologies (World Health Organization, 2018b). During its development, the model has played, and will continue to play, an important role in highlighting data points which appear to be out-of-line with general country-level patterns and may warrant further investigation. These data may correctly reflect the effect of policy interventions or changes in societal conditions, but in many cases they have proved to be the result of issues with recording or classification and
were subsequently corrected in the database.

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Appendix A Convergence of MCMC Chains

Assessing the convergence of the MCMC chains is made challenging by the extremely high number of parameters (tens of thousands) in the model. Recall that the intercept and slope effects $\delta_{i,c,j}$ and variance effects $\epsilon_{i,c}$ were centred around the regional fixed effects. This means that the fuel usage means $\mu_{i,r,c,t}$ are completely defined by $\delta_{i,c,j}$ (and in the case of overall fuel usage, the urban proportions $\pi_{c,t}$) and that the variance parameters $\phi_{i,r,c}$ are completely defined by $\epsilon_{i,c}$. Therefore we can assess the convergence of the model by assessing the convergence of $\delta_{i,c,j}$, $\pi_{c,t}$ and $\epsilon_{i,c}$.

One way to assess the convergence of a parameter is to compute the Potential Scale Reduction Factor (PSRF) ([Brooks and Gelman 1998](#)). This compares the variance between the MCMC chains to the variance within the chains. A PSRF of 1 is obtained when the two variances are the same, so starting the chains from different initial values and obtaining a PSRF close to 1 (typically taken to be less than 1.05) gives a good indication that the chains have converged to the parameter’s posterior distribution.

We computed the PSRF for the (6208) intercept and slope effects $\delta_{i,c,j}$, the (993) urban proportions $\pi_{c,t}$ and the (4656) variance effects $\epsilon_{i,c}$ and Figure 8 presents them respectively in frequency histograms. For all three sets of parameters, the overwhelming majority of
Figure 8: Histograms of the Potential Scale Reduction Factor (PSRF) computed for the intercept and slope effects $\delta_{i,c,j}$ (left), the urban proportions $\pi_{c,t}$ (centre) and the variance effects $\epsilon_{i,c}$ (right).

the values lie in the closest bin to 1, suggesting that the model has converged.

**Appendix B  Further Model Checking**

As discussed in Section 4.1, it is important to verify that the model is able to reproduce the observed data well. We do this by comparing replicates (predictions) of the observed data to the actual observations. In Section 4.1 we checked the replicates of wood usage and here we check the remaining fuels.

Figure 9 shows scatter plots comparing the mean predicted replicates for wood, charcoal, crop waste and coal to their corresponding observed values and Figure 10 shows the same plots for dung, electricity, l.p.g. and kerosene. In general the points are scattered about the diagonal line fairly evenly, indicating a good model fit for the different fuels. However, there are some patterns among the different fuels worth discussing. First, the variation around the diagonal differs considerably between the fuels. For example, the differences between the predictions and observed values are more variable for l.p.g. than for wood. This suggests that the model is more precise when predicting wood usage than l.p.g., though
in both cases the coverage of the 95% intervals is very high.

Figure 9: Scatter plots comparing the posterior means of wood, charcoal, cropwaste and coal usage replicates to their corresponding observed values.

Additionally, there is some notable systematic deviation from the diagonal in the plot of overall electricity values, where a string of observed values exceeds the mean predicted replicates. Upon closer inspection, this was found to be the survey values for electricity in South Africa, where the model is distorted by a single outlier (as discussed in Section 4.1).
Figure 10: Scatter plots comparing the posterior means of dung, electricity, l.p.g. and kerosene usage replicates to their corresponding observed values.

Appendix C  Survey Selection

The model was applied to a selection of the WHO Household Energy Database. Surveys were excluded from the analyses if:

- They only reported the usage of 'solid fuels' as a group, rather than the usage of at least one individual fuel.
They included a high proportion (>15%) of respondents who either reported that they cook with an unlisted fuel, do not cook at all or who failed to respond. These surveys were deemed to be excessively 'incomplete' and were not included for modelling. This threshold was chosen subject to sensitivity analysis.

Surveys removed for exceeding the incompleteness threshold are shown as black points in the Supplementary Material.

SUPPLEMENTARY MATERIAL

**Country Check:** PDF document containing survey prediction interval plots for all countries with at least one survey observation.

**Country Underlying:** PDF document containing credible interval plots of the underlying fuel usage proportions, for all countries.

**HAP Master:** R script from which all analysis is done and the other scripts are executed.

**HAP Data Setup:** R script which loads in and prepares the data for modelling.

**HAP Model:** R script which implements the model for a given dataset.

**HAP Plots:** R script containing functions for interactive plotting.

**HAP Function:** R script containing miscellaneous functions.

**HAP Data:** R data file containing the survey data, UN estimates of urban and rural proportions and country information.

**Data Dictionary:** PDF document describing some key variables in the data objects.
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