Rezults on the Optical Transfer Function of the Optical Systems Evaluation by Slanted Edge Method

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Abstract. The study of the Optical Transfer Function of optical systems by the slanted edge method helps us to calculate and to improve the performances of the Digital Camera. Is known from theory, between the subject of photography and his image, there will always be a small difference, caused by the optical transfer function, which may contain various components that limit the resolution. The article describes a new method of studying the optical transfer function to improve their optical resolution. With regard to the optical devices used in security applications, advanced requirements are necessary for image quality control. Optical sensors become more and more powerful and intelligent with advanced image features analysis. If in the past, to check the optical resolution, the microdensitometer was used, now this function can be performed by the digital optical sensor. The dimensions of the photosensitive pixels of optoelectronic chip are sufficiently good to test the resolution of the lens systems.

Keywords: point spread function, line spread function, modulation transfer function, spatial frequency response, slanted edge method, optical system, image resolution

1. Introduction

In the field of digital camera improvement, an important role is to increase the resolution of the entire optical system. The numbers that reflect the camera lens resolution must be treated with prudence, and used for comparative purposes along with other performance metrics, such as distortion, contrast degradation, unwanted glow. One method of evaluating lens performance is measurement of optical transfer function [1,2].

The performance of an optical system is checked with special test targets, consisting of a series of black and white bars equal in width. The slanted edge method for testing optical resolution is also applicable for the thermal camera, in different range of operating wavelengths. The registration principle is the same, usage of the bidimensional pixel array sensor and the use of germanium optics as a particularity for the thermal domain. Thermal imaging systems find applications in science, medicine, industry, defence, surveillance, space, education, leisure and a host of other areas. Monitoring thermal cameras are mainly used in military applications by creating a relative temperature distribution of the observed scene. With the increase in the spatial frequency of the test bars, a limit frequency is reached, the contrast decreases and they become barely visible Figure 1, [3]. The quality of the optical system depends on all its subassemblies, from the precision of the optomechanical mounts to the lens quality.
which are the central elements, better said, the optoelectronic pair between the camera lens and the optical sensor, around which the system is built [4,5]. Technological evolution of the optical sensor [6], increasing resolution due to applied nanotechnologies, implies the need to use high quality lenses [7].

![Figure 1](image1.png)

**Figure 1**, Lens system and transfer function [3]

Even an ideal lens without optical distortion has a PSP transfer function due to Fraunhofer diffraction on the circular aperture of the lens, real lenses, however, are affected by a lot of optical aberrations such as chromatic aberrations, coma, or curvature of the field (barrel distortion, pincushion distortion type) [8,9], which determines the positionally dependent defocusing [10]. There are a variety of methods for characterizing the properties of the lenses and determining their quality. In principle, information about lens aberrations is found in PSF function (point spread function), describes how blurred is the image of a point object [11,20]. Direct measurements of PSF are difficult and time consuming. In practice, the modulation transfer function, MTF, is used as a standard measure of camera lens quality. It can be computed from PSF and encodes information about relative local contrast, frequency and direction dependent. It contains data about how the perfect contrast of the sinusoidal black and white test bars is diminished by the aberration of an optical system, depending on the frequency of the test bars [12]. Most frequently, the MTF are measured from standardized test target photographs, but there are several high-precision techniques, and professionals use specializing optical stands in MTF registration. All these methods share the need for additional equipment, not always suitable, with situations when we do not have this equipment available. This article describes the application of the slanted edge method to various models of slanted edge, for calculating MTF function and their comparative analysis.

2. Optical transfer function

When line pattern is imaged by an optical system, each point in the object will be imaged with some blur. The energy distribution within this blur will depend on the relative aperture of the system and the aberrations present. Since we are dealing with a linear object, the image of each line element can be described by the line spread function, Figure 2, Figure 4, as $A(\delta)$ [12].

![Figure 2](image2.png)

**Figure 2**, The energy distribution in the image of a point (a) and a line (b). The line image (b) is generated by summing an infinite number of point images (a) along its length. The line spread function is the cross section of (b).
Figure 3, illustrates the result of the transfer function of light edge, the contrast between the light and dark areas of the image is reduced, the image spread function “rounds off” the “corners” of the image. The modulation transfer function is the ratio of the modulation in the image to that in the object as a function of the frequency (cycles per unit of length) of the sine-wave pattern.

$$MTF(\nu) = \frac{M_l}{M_o}$$

In the past, the MTF has been referred to as frequency response, sine wave response, or contrast transfer function [13,14].

If we assume an object consisting of alternating light and dark bands, the brightness (luminance, radiance) of which varies according to a cosine (or sine) function, as indicated by the upper part of Figure 4, the distribution of brightness can be expressed mathematically as:

$$G(x) = b_0 + b_1 \cos(2\pi\nu x)$$

where \(\nu\) is the frequency of the brightness variation in cycles per unit length, \((b_0+b_1)\) is the maximum brightness, \((b_0-b_1)\) is the minimum brightness, and \(x\) is the spatial coordinate perpendicular to the bands.

The modulation of this pattern is then:

$$M_0 = \frac{(b_0 + b_1) - (b_0 - b_1)}{(b_0 + b_1) + (b_0 - b_1)} = \frac{b_1}{b_0}$$

We now assume that the dimensions \(x\) and \((1/\nu)\) in Eq. (2) are the corresponding dimensions in the image. It is apparent that the image energy distribution at a position \(x\) is the summation of the product of \(G(x)\) and \(A(\delta)\) and can be expressed as

$$F(x) = \int A(\delta)G(x-\delta)d\delta$$

Combining Eqs. (2) and (4), we get

$$F(x) = b_0 \int A(\delta)d\delta + b_1 \int A(\delta)\cos[2\pi\nu(x-\delta)]d\delta$$

After normalizing by dividing by \(\int A(\delta)d\delta\) eq. (5) can be transformed to
Figure 4. Convolution of the object brightness distribution function $G(x)$ with the line spread function $A(\delta)$. (a) The object function, $G(x) = b_0 + b_1 \cos(2\pi v x)$, plotted against $x$. (b) The line spread function $A(\delta)$. Note the asymmetry. (c) Illustrating the manner in which $G(x)$ is modified by $A(\delta)$. A point (or more accurately, a line element) at $x_0$ is imaged by the system as $G(x_0)$ times $A(\delta)$. Similarly at $x_0 + \delta$, the image of the line element is described by $A(\delta)G(x_0 + \delta)$. Thus the image function at a given $x$ has a value equal to the summation of the contributions from all the points whose spread-out images reach $x$. (d) The image function $F(x) = \int A(\delta)G(x - \delta)d\delta$ has been shifted by $\varphi$ and has a modulation $M_f = M_o |A(\nu)|$

$$F(x) = b_0 + b_1 [A(\nu)]\cos(2\pi v x - \varphi) =$$

$$= b_0 + b_1 A_c(\nu)\cos(2\pi v x) + b_1 A_s(\nu)\sin(2\pi v x)$$

where

$$|A(\nu)| = [A_c^2(\nu) + A_s^2(\nu)]^{1/2}$$

and

$$A_c(\nu) = \frac{\int A(\delta)\cos(2\pi \delta)d\delta}{\int A(\delta)d\delta}$$

$$A_s(\nu) = \frac{\int A(\delta)\sin(2\pi \delta)d\delta}{\int A(\delta)d\delta}$$

$$\cos \varphi = \frac{A_c(\nu)}{|A(\nu)|}$$

$$\tan \varphi = \frac{A_s(\nu)}{A_c(\nu)}$$

Note that the resulting image energy distribution $F(x)$ is still modulated by a cosine function of the same frequency $v$, demonstrating that a cosine distribution object is always imaged as a cosine distribution image. If the line spread function $A(\delta)$ is asymmetrical, a phase shift $\varphi$ is introduced. This is a lateral shift of the location of the image (at this frequency). The modulation in the image is given by:
\[ M_i = \frac{b_i}{b_0} |A(v)| = M_0 |A(v)| \] (12)

and \( |A(v)| \) is the modulation transfer function: 
\[ MTF(v) = |A(v)| = \frac{M_i}{M_0}. \]

The optical transfer function, OTF, is the complex function which describes this process. It is a function of the spatial frequency \( v \), of the sine-wave pattern. The real part of the OTF is the modulation transfer function, MTF and the imaginary part is the phase transfer function, PTF. If the PTF is linear with frequency, it is, of course, just a simple lateral displacement of the image (as, for example, distortion), but if it is nonlinear, it can have an effect on the image quality. A phase shift of 180˚ is a reversal of contrast, in that the image pattern is light where it should be dark, and vice versa [13.14].

3. The Slanted Edge Method

Computing of the transfer function MTF through the edge gradient method, also called slanted edge method is a way of calculation of the modulation transfer function, MTF [15], by recording with a digital camera a slanted edge at an angle to the columns or rows of pixels, found experimentally, which ensures a good sampling of the light intensity gradient of the image generated by slanted edge object. To obtain a precise calculation it is necessary to use a perfectly linear slanted edge, to minimize errors. The main feature is the creation of the intelligent 1-D oversampled profile of the edge image, made up of naturally sampled pixels lines in the 2-D space of the optic sensor Figure 5a, the inclined edge intersects each line of pixels at a point called centroid with the coordinates \( x, y \) which are used to calculate the edge slope [16]. From the theoretical standpoint, the described method allows for unequivocal estimates of the MTF beyond the Nyquist frequency of the image capture device, always limited by the number of pixels of the sampling device. A claimed advantage is insensitivity to alignment. This method yields results if the alignment occurs in the accepted angles range. An in-depth description of the method is presented by Stephen E.Reichenbach [17].

The transfer function of the MTF modulation has been set as a performance characterization parameter, it fits well to describe the causes of loss of details in the image, like the blur or focal point quality. The slanted edge MTF calculation was adopted by the International Standard for Digital Camera Performance Measurement, ISO 12233. As a quality standard for digital imaging systems, the MTF concept has been adapted and applied in response to spatial frequency resistance (SFR) [18-21].

4. Results

There have been several light gradients simulated on the computer very similar to the real ones. The difference between them is caused by the absence of image defects, lenses induced distortions, experiment light conditions. Illumination conditions are inherent to each experiment, even the quality of sunlight depends on atmospheric conditions, direct beams, diffuse lighting, etc. On the other hand, optical sensors from different manufacturers have their own interpolation and final imaging algorithms after passing through the Bayer filter that ultimately provides the image.

The study of the light intensity gradient on the optic sensor, formed by the image of the inline edge allows us a quicker estimation of the image quality from which experimental data is extracted for ESF function calculation.

The result of applying the Slanted Edge Algorithm is the calculation of the MTF optical transfer function, representing the quality of the optical system in terms of resolution, and therefore image quality.

For applying this method the following steps are required to be taken:
- Select the slanted edge object. Place the interest region ROI, with a linear edge and a good contrast in the tested optical system sight Figure 5a.
- The region of interest ROI is recorded Figure 5b.
For each pixel line in the ROI region, the centroid is calculated, the point of transition from dark to bright, through which the slanted edge passes Figure 5b.

**Figure 5**, a) Inclined edge object; b) Image Inclined Edge Object and its projection on the pixel matrix of the optical sensor; c) ROI, region of interest, slanted margin on the optical sensor; d) Slanted edge intensity profile, the same contrast, can have more degrees of sharpness, or image sharpness.

Calculate the slope of the slanted edge by linear regression and the smallest square method Figure 5d.

Calculate and construct the ESF, edge spread function, of passing from dark to bright, illustrated in Figure 5d. For this the over-sampling method, meaning the rotation of the coordinate system and the projection of the pixels on the x’ axis of new the coordinate system, of the ROI region, must be applied, Figure 5b, Figure 6b.

The LSF function, the line spread function, of the Gaussian type, is calculated by the numerical derivation of the ESF function defined at p. 5. Hamming Window noise filtering methods are applied.

Finally, the MTF function, modulation transfer function, is calculated by applying the DFT method, discrete Fourier transform, on the LSF function, defined at p. 6, the decomposition of Fourier series of spatial frequencies. The function expresses the performance of the analyzed system quantitatively.

The slanted edge method is useful in that it offers the ability to build the slanted edge transfer function between the object and its image. The numerical derivation of the ESF function allows us to construct the LSF (Gaussian line spread function), the decomposition in spatial frequency components by the DFT (discrete Fourier transform) method allows us to calculate the spatial frequencies, so we can calculate the limits of resolution of the optical system, and other optical parameters.

The photopixel dimensions of 2-4 μm of the optical sensor allow us to study the effects of Fraunhofer diffraction on the circular aperture or rectangular slit. Description of the fundamental optical phenomena that occur in the optical instrument.

Computerized Slanted Edge Simulation is useful for improving and verifying measurement techniques, noise reduction.

Researchers have noticed that by applying the slanted edge method, over-sampling can be achieved, a discretization a few times, smoother at subpixel level, in our case 1/4 pixel, Figure 6b. Being aware of optical imperfections, experimental data contain various errors, for example, internal hardware, image processing algorithms can amplify contrast, resulting in artificially changing the image (to impress the viewer) and loss of valuable information about the energy of the electromagnetic field of incident radiation. The image of the slanted edge, in reality, is not perfectly linear, sometimes there may be image defects that create non-uniformities. In the Study, based on the experimental data obtained, concrete shapes are formed from the inclined edge image to study the relation between the light intensity gradient and the form of the optical transfer function MTF. Slanted edge images are represented by matrices of pixels in digital memory. For their processing, operators and MatLab code sequences were used, a program containing libraries of necessary functions adapted to matrix operations, image converters. For example:
Figure 6. Pixel oversampling, x’axis, perpendicular to the light intensity gradient line. Rotating the coordinate system and pixel design on the x’coordinate axis allows a sampling interval smaller than the pixel size.

$$[A, \text{map}] = \text{imread} (\text{filename, fmt})$$; where: $\text{imageread}$ - function, read image file content; $\text{filename}$ - file name experimental data; $\text{fmt}$ - image format, (jpg, tif, png, bmp ...); $\text{map}$ - colormap variable associated with matrix A; $A$ - image data, returns as a m-by-n array.

Image matrices representing image pixels can be transformed in the most different ways according to the proposed experimental tasks, in our case examining the optical transfer functions, ESF, LSF, MTF.

Simulation of the light intensity gradient is useful to estimate the quality of the real optical system. Which is better, the one extended over a greater distance or the one restrained over a shorter distance. This knowledge helps us to quickly estimate the optical performance of an optical instrument lens.

A series of samples of slanted edge patterns, different widths of the margins recorded, were analyzed.

Figure 7, Samples of slanted edge patterns, from high contrast - a) to low contrast – f)
Figure 8. Calculation of ESF – a1), LSF – a2), MTF – a3), functions describing the slanted edge from fig. 7a)

Figure 9. Calculation of ESF – b1), LSF – b2), MTF – b3), functions describing the slanted edge from fig. 7b)

Figure 10. Calculation of ESF – c1), LSF – c2), MTF – c3), functions describing the slanted edge from fig. 7c)

Figure 11. Calculation of ESF – d1), LSF – d2), MTF – d3), functions describing the slanted edge from fig. 7d)
The edge transfer function, ESF, was calculated using the operators and functions of the MatLab calculation program. The function \( p = \text{polyfit} (x, y, n) \) was used; returns the polynomial coefficients for \( p(x) \) of degree \( n \), in our case we have an inclined line, \( n = 1 \), which approximates the inclined edge line (the smallest square method) for the data in the matrix \( y \). The coefficients in \( p \) are in decreasing order and the length of \( p \) is \( n + 1 \).

\[
p(x) = p_1x^n + p_2x^{n-1} + \ldots + p_nx + p_{n+1}
\]

The equation of the searched line is of type \( f(x) = a + bx \). For an experimental data set \( x_1, x_2, \ldots x_n; y_1, y_2, \ldots y_n \), the value of \( a \) is the slope, and the value of \( b \) is the \( y \) intercept. The LSF function is calculated based on the data described by the ESF function by numerical derivation of the ESF function, with the formula 

\[
f'(x) = \frac{f(x+1) - f(x-1)}{2}
\]

The actual calculation was performed by convolution of the ESF function with a 1D array: \([-0.5, 0, 0.5]\). The MatLab function \( w = \text{conv} (u, v) \) was used; returns the result of the convolution of the vectors \( u \) and \( v \), the result, the LSF function of the Gaussian type.

\[
w(k) = \sum_j u(i)v(k - j + 1)
\]

\[
w(1)=u(1)*v(1)
\]
\[
w(2)=u(1)*v(2)+u(2)*v(1)
\]
\[
w(3)=u(1)*v(3)+u(2)*v(2)+u(3)*v(1)
\]
\[
\ldots
\]
\[
W(n)=u(1)*v(n)+u(2)*v(n-1)+\ldots+u(n)*v(l)
\]
\[
w(2*n - 1)=u(n)*v(n)
\]

The MTF function is built based on the LSF function calculation data. For the calculation, the MatLab program was used, the function \( y = \text{fft} (x, n) \), where \( x \) represents the numerical input vector, or the data describing the LSF function, in Gaussian form, which can be decomposed into Fourier series. The DFT process, the basic method in spectral analysis, transforms data describing spatial distribution into
data describing distribution in the frequency space. DFT for a vector \( x \) of length \( n \) is another vector \( y \) of length \( n \) such that:

\[
y_{p+1} = \sum_{j=0}^{n-1} \omega^j x_{j+1}, \quad \omega = e^{-\frac{2\pi i}{n}}, \quad p \text{ and } j \text{ are used for indexing from } 0 \text{ to } n-1, \quad i \text{ for the imaginary part.}
\]

The data of the vector \( x \) is separated by a constant interval in space \( ds = 1 / f_s \), where \( f_s \) is the sampling frequency. The Fourier transform contains the complex component. The absolute value of \( y_{p+1} \) measures the amount of frequency \( f = p (f_s / n) \) present in the numerical vector analyzed.

When light passes through a lens system, loss of intensity, light diffusion on optical inhomogeneities, loss due to diffraction, occurs, all this reduces the resolution and contrast. The MTF curve is an important feature to quantify these losses.

The calculation was performed, using the Matlab multi-paradigm numerical computing environment from MathWorks Inc; sfrmat3 Author: Peter Burns, International Imaging Industry Association; and Microsoft Excel spreadsheet program, a grid interface to organize scientific data and formulas, to perform calculations. Many thanks to the creators of these wonderful work tools.

Conclusion

As a result of the experiment proposed in this paper, some conclusions will be drawn.

The study looked at the dependence of the gradient model on the aspect of its width and the amount of spatial frequencies contained in the model. The computerized simulation of the light intensity gradient model on the optical sensor is more than just constructing a virtual reality, it is a verification technique of the calculation, and of the possible errors in the calculation methods, of checking the sampling procedure, model.

The constructed MTF charts show us how that the narrower the edge gradient is, the better the transmission of spatial frequency or the MTF is going to be, and as the edge gradient is wider the MTF curve is steeper, the high frequency transmission decreases to zero amplitude, the optical system resolution decreases.

The pixel of the optical sensor is like the pixel of the monitor we look at each day, except that the first one absorbs the light, and the latter emits light, it can be said that they are pixel matrices with perfectly inverse functionality, but which are subject to the same pixel matrix processing rules, so monitor pixels help us to imagine pixel processing patterns on the sensor.

The method is useful for studying MTF curve trends for various light gradient models.

References

[1] Hecht E., *Optics* 5ed., Adelphi University, Boston, Columbus, Indianapolis, New York, San Francisco, Pearson Education Limited, Edinburg Gate, Harlow Essex CM20 2JE England, 2017.

[2] Lazar B., et al. *Simulating delayed pulses in organic materials. In: International Conference on Computational Science and Its Applications*. Springer, Berlin, Heidelberg, 2006. p. 779-784.

[3] https://photographylife.com/what-is-a-decentered-lens.

[4] Bauer M., et al., *Automatic Estimation of Modulation Transfer Functions*, Tubingen, Germany, University of Cambridge United Kingdom, 2018.

[5] Sterian A. R., *Computer modeling of the coherent optical amplifier and laser systems*. In: International Conference on Computational Science and Its Applications. Springer, Berlin, Heidelberg, 2007. p. 436-449.

[6] Maitre H., *From Photon to Pixel* 2nd Ed, London, Hoboken, NJ 07030: ISTE Ltd and John Wiley & Sons, Inc., 2017.
[7] P. H. KG, *Bright light conditions: the limiting resolution*, 2018. Available: http://stanfordcomputeroptics.com/technology/optical-resolution/limiting-resolution.html.

[8] GUenther B.D., et al. *Encyclopedia of Modern Optics*, Volume One, Duck University Durham, NC, USA.

[9] Stefanescu E., et al. *Study on the fermion systems coupled by electric dipol interaction with the free electromagnetic field*. In: Advanced Laser Technologies 2004. International Society for Optics and Photonics, 2005. p. 160-166C.M.

[10] Tsai C.M., *Evaluation of Geometrical Modulation Transfer Function in Optical Lens System*, Mathematical Problems in Engineering, Volume 2015, Hindawi Publishing Corporation, 2014.

[11] D Panduru, N Craciunoiu, E N Patru, M Bica, *Study on cutting temperature and surface roughness during the turning process of pure titanium*, Scientific Bulletin of Naval Academy, Vol. XXI 2018, pg. 195-202.

[12] Wilcox, M. *How to measure MTF and other properties of lenses*. Optikos Corporation, Cambridge, MA, USA, Tech. Rep(1999): 4-04.

[13] Smith W. J., *Modern Optical Engineering*, New York: McGraw-Hill, 2000.

[14] Sterian A., Sterian P., *Mathematical models of dissipative systems in quantum engineering*. Mathematical Problems in Engineering, 2012, 2012.

[15] Burns P., *Slanted-edge Analysis for digital camera and scanner*, http://losburns.com/imaging/software/SFRedge/sfrmat3_post/index.html, International Imaging Industry Association, 12 May 2015.

[16] D. Williams, Benchmarking of the ISO 12233 Slanted-edge Spatial Frequency Response Plugin, in IS&T’s 1998 PICS Conference.

[17] Reichenbach S.E., et al. *Characterizing digital image acquisition devices*, Optica Engineering 30(2), 170-177, 1991.

[18] A. N. Atodiresei, E. Bautu, A. Bautu, *Automatically deploy a local positioning system based on open-source software and commodity hardware*, Scientific Bulletin of Naval Academy, Vol. XXI 2018, pg. 194-201.

[19] Paul Vasiliu, *Automatically determines the length of the dual system to a bivalent system*, Scientific Bulletin of Naval Academy, Vol. XXI 2018, pg. 50-57.

[20] Jacobson Ralph E., *The Manual of Photography, ninth edition*, Focal Press, Oxford, Auckland Boston., 2000.

[21] Williams Don, Burns P. D., *Evolution of Slanted Edge Gradient SFR Measurement*, in Image Quality and System Performance XI (2014), Proc. SPIE Vol. 9016, NY USA, 2014.