Observables in the Decays of $B$ to Two Vector Mesons

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Abstract

In general there are nine observables in the decay of a $B$ meson to two vector mesons defined in terms of polarization correlations of these mesons. Only six of these can be detected via the subsequent decay angular distributions because of parity conservation in those decays. The remaining three require the measurement of the spin polarization of one of the decay products.

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The decay of the $B$ meson into two vector mesons $B \to V_1 + V_2$, such as $B \to \rho + \Psi$ or $B \to \rho + K^*$, has been calculated in many models \[1–9\]. Here we are concerned with observables from a model-independent viewpoint. We limit this discussion to the case of $B^\pm$ decays or $B^0 (\bar{B}^0)$ in the absence of $B^0 - \bar{B}^0$ mixing effects.

To take the advantage of extracting the $CP$-odd and $CP$-even or $T$-odd and $T$-even components more easily, the angular distribution is often written in the linear polarization (or transversity) basis. Let us define the amplitude of $B \to V_1 V_2$ in the rest frame of $V_1$. According to their polarization combinations, the amplitude can be decomposed into \[1\]

$$A(B \to V_1 V_2) = A_0 \varepsilon_{V_1}^x \varepsilon_{V_2}^x - \frac{A_{\parallel}}{\sqrt{2}} \varepsilon_{V_1}^T \cdot \varepsilon_{V_2}^T - i \frac{A_{\perp}}{\sqrt{2}} \varepsilon_{V_1}^x \times \varepsilon_{V_2}^x \cdot \hat{p}, \tag{1}$$

and similarly for $\bar{B} \to \bar{V}_1 \bar{V}_2$. In Eq. (1), $\varepsilon_{V_1}$ and $\varepsilon_{V_2}$ are the unit polarization vectors of $V_1$ and $V_2$, respectively. $\hat{p}$ is the unit vector along the direction of motion of $V_2$ in the rest frame of $V_1$, $\varepsilon_{V_i}^{xL} \equiv \varepsilon_{V_i}^x \cdot \hat{p}$ and $\varepsilon_{V_i}^{T} = \varepsilon_{V_i}^x - \varepsilon_{V_i}^{xL} \hat{p}$. It is easy to see that $A_{\perp}$ is odd under the parity transformation because of the appearance of $\varepsilon_{V_1}^x \times \varepsilon_{V_2}^x \cdot \hat{p}$, whereas $A_0$ and $A_{\parallel}$ are even.

For $B$ decays, the square of the amplitude $A^* A$ should determine 9 observables proportional to products of the three transversity amplitudes. We can choose these as given by

$$K_1 = |A_0|^2, \quad K_4 = \Re \left( A_0^* A_{\parallel} \right), \quad L_4 = \Im \left( A_0^* A_{\parallel} \right),$$

$$K_2 = |A_{\parallel}|^2, \quad K_5 = \Im \left( A_0^* A_{\perp} \right), \quad L_5 = \Re \left( A_0^* A_{\perp} \right),$$

$$K_3 = |A_{\perp}|^2, \quad K_6 = \Im \left( A_{\parallel}^* A_{\perp} \right), \quad L_6 = \Re \left( A_{\parallel}^* A_{\perp} \right). \tag{2}$$

Then

$$A^* A = K_1 X_1 + K_2 X_2 + K_3 X_3 + K_4 X_4 + L_5 X_5 + L_6 X_6$$

$$+ L_4 Y_4 + K_5 Y_5 + K_6 Y_6$$

The observables $X_i$ and $Y_i$ represent polarizations or polarization correlations of the final vector mesons depending on the vector $\hat{p}$. They are given explicitly in the Appendix.
The polarization state of a spin-1 particle is given in terms of the spin $S_i$ and a second rank traceless tensor $T_{ij}$. Sometimes the tensor polarization is referred to as alignment. The observables can be classified according to their properties with respect to parity $P$ and motion reversal $T$. By “motion reversal” is meant the reversal of all spins and momenta; a nonzero value of a $T$-odd observable signifies time-reversal violation when there are no final state interactions. We find:

|   | P | T |
|---|---|---|
| $X_1$ to $X_4$ | even | even |
| $Y_5$, $Y_6$ | odd | odd |
| $Y_4$ | even | odd |
| $X_5$, $X_6$ | odd | even |

The terms which have opposite behaviour under $P$ and $T$ are necessarily proportional to the spin polarization $S_1$ or $S_2$.

The polarization state of $V_i$ is analyzed via its subsequent decay. If this is a strong or electromagnetic decay into two particles whose angular distribution is measured, then it is impossible to detect the spin polarization $S_i$ and only $T_{ij}$ can be detected. This is a consequence of parity conservation since the final decay cannot depend on $\vec{S} \cdot \vec{k}$, where $\vec{k}$ is the relative momentum of the decay products. As a result, one cannot determine $L_4$, $L_5$, and $L_6$ in this way.

In general, the angular distribution of the decay in the transversity basis can be written as

$$\frac{d^3\Gamma}{d\cos\theta_1 d\cos\theta_2 d\phi} = \sum_{i=1}^{6} K_i f_i(\theta_1, \theta_2, \phi),$$

(3)

where $K_i$’s and $L_i$’s are the amplitude bilinears that contain the dynamics and in the case of $B^0$ would evolve with time$^1$ and $f_i(\theta_1, \theta_2, \phi)$ are the corresponding angular distribution

$^1$The time evolution effects will be addressed in a separate publication [12].
functions which are orthogonal to one another. Here $\theta_1$ (or $\theta_2$) denotes the polar angle of one of the $V_1$ (or $V_2$) decay products measured in the rest frame of $V_1$ (or $V_2$) relative to the motion of $V_1$ (or $V_2$) in the rest frame of the $B$ meson, and $\phi$ is the angle subtended by the two planes formed by decay products of $V_1$ and $V_2$, respectively.

For the case in which the decays of $V_1$ and $V_2$ are both into two pseudoscalar mesons, one can immediately translate the tensor correlations into angular distributions as shown in the Appendix. The resulting normalized angular distribution of the decays $B \to V_1 \to P_1 \to \pi^+ \pi^-$ is:

$$
\frac{1}{\Gamma_0} \frac{d^3 \Gamma}{d \cos \theta_1 d \cos \theta_2 d \phi} = \frac{9}{8\pi \Gamma_0} \left\{ K_1 \cos^2 \theta_1 \cos^2 \theta_2 + \frac{K_2}{2} \sin^2 \theta_1 \sin^2 \theta_2 \cos^2 \phi 
+ \frac{K_3}{2} \sin^2 \theta_1 \sin^2 \theta_2 \sin^2 \phi + \frac{K_4}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \cos \phi 
- \frac{K_5}{2\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \sin \phi - \frac{K_6}{2} \sin^2 \theta_1 \sin^2 \theta_2 \sin 2\phi \right\}.
$$

Here $\theta_1$ ($\theta_2$) is the angle between the $P_1$ ($P_2$) three-momentum vector in the $V_1$($V_2$) rest frame and the $V_1$($V_2$) three-momentum vector defined in the $B$ rest frame, and $\phi$ is the angle between the normals to the planes defined by $P_1 P_1'$ and $\mu^+ \mu^-$ in the $B$ rest frame. An example is the decay $B^- \to K^*-\rho^0 \to (K\pi)^-(\pi^+\pi^-)$.

In order to determine $L_4$, $L_5$, and $L_6$ it is necessary to measure the spin polarization of a decay product. For example, if $V_2$ decays into $\mu^+ \mu^-$ and $V_1$ decays into $P_1 P_1'$ then, as shown in the Appendix, the polarization of either muon is given by

$$
\frac{1}{\sqrt{2}} \sin 2\theta_1 \sin \theta_2 \left( L_4 \sin \phi - L_5 \cos \phi \right) + \frac{L_6}{2} \sin^2 \theta_1 \cos \theta_2,
$$

divided by the sum of the first six terms in Eq. (A11), where $\theta_1$ ($\theta_2$) is, as before, the angle between the $P_1$ ($\mu^-$) three-momentum vector in the $V_1$ ($V_2$) rest frame and the $V_1$ ($V_2$) three-momentum vector defined in the $B$ rest frame, and $\phi$ is the angle between the normals to the planes defined by $P_1 P_1'$ and $\mu^+ \mu^-$ in the $B$ rest frame. For the case of $L_6$ it is only necessary to measure the polarization of one muon independent of observing the decay of $V_1$. The maximum value of the polarizations is 1 for each of the terms proportional to $L_4$, $L_5$, or $L_6$. 


For the case in which $V_1$ and $V_2$ are both $CP$ eigenstates, the amplitude given by $A_\perp$ corresponds to a final state with an opposite $CP$ eigenvalue from the other two. This by itself has nothing to do with $CP$ violation, but it leads to the possibility that the coefficients $K_5, K_6$ involving $A_\perp$ may be good places to look for $CP$ violation.

In particular, in the absence of final state interaction (FSI) it follows from $CPT$ invariance that only “$T$-odd” terms will be odd under $CP$, so that

$$K_i = \bar{K}_i, \text{ for } i = 1, 2, 3, 4;$$

$$K_i = -\bar{K}_i, \text{ for } i = 5, 6; \quad (6)$$

and the only signals of $CP$ violation are $L_4, K_5, \text{ and } K_6$. In fact, in the absence of FSI a nonzero value of $L_4$ or $K_{5,6}$ by itself is a signal of time reversal violation. On the other hand, in the absence of $CP$ violation (as expected in decays like $B \to \Psi K^*$) a nonzero value of $K_{5,6}$ is a signal of significant final state phases.

In order to have $CP$ violation there must be two contributions with different weak phase factors; we label these as $T_\eta$ and $P_\eta$ where $\eta = 0, \parallel, \perp$. Then each of the three amplitudes entering Eq. (1) has the form

$$A_\eta = e^{i\theta_\eta}(T_\eta + P_\eta e^{i\phi_w}e^{i\delta_\eta}). \quad (7)$$

where $\theta_\eta, \delta_\eta$ are strong phases and $\phi_w$ is the relative weak phase between the tree and penguin contributions. We obtain $\bar{A}_\eta$ by changing $\phi_w$ to $-\phi_w$. There are in general 12 parameters: $T_\eta, P_\eta, \delta_\eta, \phi_w$, and two relative phases of the $\theta_\eta$.

In many cases, $\phi_w$ is expected to be large, leading to the possibility of large $CP$ violation. Thus for decays like $B \to K^* \rho$ or $B_s \to \rho \varphi$, $\phi_w = \gamma$ and for $B \to \rho \rho$ or $B_s \to K^* \rho$, $\phi_w = \beta + \gamma$. For the parameters $K_5$ and $K_6$ as well as $L_4$, the $CP$ violation is given by

$$Im[AN_{\eta'}] - Im[\bar{A}_\eta \bar{A}_{\eta'}^*] = 2 \sin \phi_w [P_{\eta}T_{\eta'} \cos(\theta_\eta - \theta_{\eta'} + \delta_\eta) - P_{\eta'}T_{\eta} \cos(\theta_\eta - \theta_{\eta'} - \delta_{\eta'})]. \quad (8)$$

Assuming the strong phases are not very large, the major requirement for a large effect in
the above $CP$ asymmetry quantities is that $P_\eta/T_\eta$ be quite different from $P_{\eta'}/T_{\eta'}$ for $\eta \neq \eta'$.

For the case of $K_1$ to $K_3$, the $CP$ violation is given by

$$\frac{|A_\eta|^2 - |A_{\eta'}|^2}{|A_\eta|^2 + |A_{\eta'}|^2} = \frac{-2T_\eta P_\eta \sin \phi_w \sin \delta_\eta}{T_\eta^2 + P_\eta^2 + 2T_\eta P_\eta \cos \phi_w \cos \delta_\eta}$$

requiring as noted a significant value for $\sin \delta_\eta$. For the parameters $K_4, L_5,$ and $L_6$, the $CP$ violation is measured by

$$Re[A_\eta A_{\eta'}^*] - Re[\bar{A}_\eta \bar{A}_{\eta'}^*] = -2 \sin \phi_w \left[ P_\eta T_{\eta'} \sin(\theta_\eta - \theta_{\eta'} + \delta_\eta) - P_{\eta'} T_\eta \sin(\theta_\eta - \theta_{\eta'} - \delta_{\eta'}) \right].$$

These require a significant value of $\sin(\theta_\eta - \theta_{\eta'})$ or $\sin(\delta_\eta - \delta_{\eta'})$ to have a large effect.

To summarize this paper, we have discussed in a model independent way the observables in the decay of a $B$ meson to two vector mesons and the relations among them. An alternative way of getting the differential angular distribution is provided in the Appendix. We have also explicitly shown how one can determine $L_{4,5,6}$ defined in the text by measuring the polarization of one of the decay products in the final state.

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APPENDIX: CORRELATIONS OF POLARIZATIONS

Analyzing the correlations of polarization vectors appearing in the decay rate provides a way of understanding why only six of the nine amplitude bilinears show up in the differential cross section. The polarization state of a spin 1 particle is described by the density matrix which can be written as a sum of a scalar, vector, and traceless second-rank tensor.

With $a = 1, 2$ for $V_1$ and $V_2$ mesons, these are

\[
1_{ij} = \delta_{ij}, \quad \text{scalar;}
\]
\[
S_i^a = \frac{1}{2i} \varepsilon_{ijk} \epsilon_j^a \epsilon_k^a, \quad \text{vector;}
\]
\[
T_{ij}^a = \frac{1}{2} \left( \epsilon_i^a \epsilon_j^a + \epsilon_i^{a*} \epsilon_j^a - \frac{2}{3} \epsilon \cdot \epsilon^* \delta_{ij} \right), \quad \text{tensor.}
\]

Therefore, we have

\[
\epsilon_i^a \epsilon_j^{a*} = T_{ij}^a + i \varepsilon_{ijk} S_k + \frac{1}{3} 1_{ij},
\]

(A2)

provided that the polarization vector $\vec{\epsilon}$ are normalized to 1.

From Eq. (1), one can get the polarization vector correlations for each of the amplitude bilinears. After simplification, we obtain the following results:

For $|A_0|^2$:

\[
X_1 = \epsilon^{1*L} \epsilon^{1L} \epsilon^{2*L} \epsilon^{2L} \\
= (\hat{p} \cdot \mathbf{T}^1 \cdot \hat{p}) (\hat{p} \cdot \mathbf{T}^2 \cdot \hat{p}) + \frac{1}{3} \hat{p} \cdot (\mathbf{T}^1 + \mathbf{T}^2) \cdot \hat{p} + \frac{1}{9}.
\]

(A3)

For $|A_\parallel|^2$:

\[
2X_2 = \epsilon^{1*\bar{T}} \cdot \epsilon^{2*\bar{T}} \epsilon^{1\bar{T}} \cdot \epsilon^{2\bar{T}} \\
= Tr[\mathbf{T}^1 \cdot \mathbf{T}^2] + (\hat{p} \cdot \mathbf{T}^1 \cdot \hat{p}) (\hat{p} \cdot \mathbf{T}^2 \cdot \hat{p}) - \frac{1}{3} \hat{p} \cdot (\mathbf{T}^1 + \mathbf{T}^2) \cdot \hat{p} \\
- 2 \hat{p} \cdot \mathbf{T}^1 \cdot \mathbf{T}^2 \cdot \hat{p} + \frac{2}{9} - 2 \left( \hat{p} \cdot \mathbf{S}^1 \right) \left( \hat{p} \cdot \mathbf{S}^2 \right) + \mathbf{S}^1 \cdot \mathbf{S}^2.
\]

(A4)

For $|A_\perp|^2$:
\[ 2X_3 = \epsilon^{\bar{i}j} \times \epsilon^{\bar{j}k} \cdot \hat{p} \epsilon^{\bar{i}} \times \epsilon^{\bar{j}} \cdot \hat{p} \]

\[ = \varepsilon_{ijk} \varepsilon_{lmn} T^{1}_{il} T^{2}_{jm} p_k p_{n} - \frac{1}{3} \hat{p} \cdot (T^1 + T^2) \cdot \hat{p} + \frac{2}{9}. \]  

For \( A_0 A_\parallel^* \): (apart from an overall minus sign)

\[ \epsilon^{1*} \epsilon^{2*} \epsilon^{3*} \cdot \epsilon^{2T} = \hat{p} \cdot T^1 \cdot T^2 \cdot \hat{p} - (\hat{p} \cdot T^1 \cdot \hat{p}) (\hat{p} \cdot T^2 \cdot \hat{p}) - i \hat{p} \cdot (T^2 \cdot \hat{p}) \times S^1 \]

\[ -i \hat{p} \cdot (T^1 \cdot \hat{p}) \times S^2 - S^1 \cdot S^2 + (\hat{p} \cdot S^1) (\hat{p} \cdot S^2). \]

For \( A_0^* A_\parallel \): (apart from an overall minus sign)

\[ \epsilon^{1L} \epsilon^{2L} \epsilon^{3T} \cdot \epsilon^{2*} \epsilon^{3T} = \hat{p} \cdot T^1 \cdot T^2 \cdot \hat{p} - (\hat{p} \cdot T^1 \cdot \hat{p}) (\hat{p} \cdot T^2 \cdot \hat{p}) + i \hat{p} \cdot (T^2 \cdot \hat{p}) \times S^1 \]

\[ +i \hat{p} \cdot (T^1 \cdot \hat{p}) \times S^2 - S^1 \cdot S^2 + (\hat{p} \cdot S^1) (\hat{p} \cdot S^2). \]

So the net result for \( A_0 A_\parallel^* \) and \( A_0^* A_\parallel \) is

\[ X_4 = \sqrt{2} \left[ (\hat{p} \cdot T^1 \cdot \hat{p}) (\hat{p} \cdot T^2 \cdot \hat{p}) - \hat{p} \cdot T^1 \cdot T^2 \cdot \hat{p} - S^1 \cdot S^2 + (S^1 \cdot \hat{p}) (S^2 \cdot \hat{p}) \right], \]

\[ Y_4 = \sqrt{2} \left[ \hat{p} \cdot (T^1 \cdot \hat{p}) \times S^2 + \hat{p} \cdot (T^2 \cdot \hat{p}) \times S^1 \right]. \quad (A6) \]

For \( A_0 A_\perp^* \): (apart from an \( i \))

\[ \epsilon^{\bar{i}j} \times \epsilon^{\bar{j}k} \cdot \hat{p} \epsilon^{1*} \epsilon^{2*} \epsilon^{3*} \epsilon^{3T} = \hat{p} \cdot (T^1 \cdot \hat{p}) \times (T^2 \cdot \hat{p}) - \hat{p} \cdot S^1 \times S^2 - i \hat{p} \cdot T^2 \cdot S^1 \]

\[ +i \hat{p} \cdot T^1 \cdot S^2 - i (\hat{p} \cdot T^1 \cdot \hat{p}) (\hat{p} \cdot S^2) + i (\hat{p} \cdot T^2 \cdot \hat{p}) (\hat{p} \cdot S^1). \]

With a similar expression for \( A_0^* A_\perp \), we obtain

\[ X_5 = \sqrt{2} \left[ \hat{p} \cdot T^2 \cdot S^1 - \hat{p} \cdot T^1 \cdot S^2 + (\hat{p} \cdot T^1 \cdot \hat{p}) (S^2 \cdot \hat{p}) - (\hat{p} \cdot T^2 \cdot \hat{p}) (S^1 \cdot \hat{p}) \right], \]

\[ Y_5 = \sqrt{2} \left[ \hat{p} \cdot (T^1 \cdot \hat{p}) \times (T^2 \cdot \hat{p}) - \hat{p} \cdot S^1 \times S^2 \right]. \quad (A7) \]

For \( A_\parallel A_\perp^* \): (apart from an \( i \))

\[ \epsilon^{\bar{i}j} \epsilon^{\bar{j}k} \cdot \hat{p} \epsilon^{1*} \epsilon^{2*} \epsilon^{3T} = -\hat{p} \cdot (T^1 \cdot \hat{p}) \times (T^2 \cdot \hat{p}) + \varepsilon_{ijk} (T^1 \cdot T^2) p_k + \frac{2}{3} \hat{p} \cdot S^1 \]

\[ -\frac{2}{3} i \hat{p} \cdot S^2 + i (\hat{p} \cdot T^1 \cdot \hat{p}) (\hat{p} \cdot S^2) - i (\hat{p} \cdot T^2 \cdot \hat{p}) (\hat{p} \cdot S^1). \]

With a similar expression for \( A_\parallel^* A_\perp \), we obtain
\[
X_6 = \frac{2}{3} \mathbf{p} \cdot (\mathbf{S}^2 - \mathbf{S}^1) + (\mathbf{p} \cdot \mathbf{T}^2 \cdot \mathbf{p}) (\mathbf{S}^1 \cdot \mathbf{p}) - (\mathbf{p} \cdot \mathbf{T}^1 \cdot \mathbf{p}) (\mathbf{S}^2 \cdot \mathbf{p}),
\]
\[
Y_6 = \varepsilon_{ijk} (\mathbf{T}^1 \cdot \mathbf{T}^2)_{ij} \mathbf{p}_k - \mathbf{p} \cdot (\mathbf{T}^1 \cdot \mathbf{p}) \times (\mathbf{T}^2 \cdot \mathbf{p}).
\]
(A8)

Notice that the observables \(Y_4, X_5,\) and \(X_6\) are linear in \(\mathbf{S}^1\) or \(\mathbf{S}^2\). As a result, as discussed in the text, they cannot be detected via the angular distribution of the decays of \(V_1\) and \(V_2\). However, in principle they may be observed in more complicated decays, or in decays like \(B \to K^* (\to \ell^+ \ell^-) J/\Psi (\to l^+ l^-)\) by measuring the spin of one of the leptons. In particular \(X_6\) contains \(\mathbf{p} \cdot \mathbf{S}^2\) and \(\mathbf{p} \cdot \mathbf{S}^1\) and so could be observed by measuring the polarization of one of the mesons from the decay of \(V_1\) or \(V_2\) without observing the other decay.

The above results can be directly applied to the decays \(B \to V_1 (\to PP) V_2 (\to PP)\) to obtain the angular distribution, Eq. (4), which is uniquely determined by the tensor polarizations of the vector mesons. The angular distributions of \(B \to V_1 (\to PP) V_2 (\to l^+ l^-)\) and \(B \to V_1 (\to P \gamma) V_2 (\to P \gamma)\) can be obtained by taking into account that the lepton or photon motions must be perpendicular to the parent particle polarization vector and all possible spins are summed over.

For the case of decay into pseudoscalars \(B \to V_1 (\to P_1 P_1') V_2 (\to P_2 P_2')\), one can go directly from \(X_1-X_4, Y_5,\) and \(Y_6\) to the angular distribution Eq. (4). The polarization vectors of \(V_2\) directly convert to the outgoing relative momentum vectors of the pseudoscalars. Choosing \(\mathbf{p} = (0, 0, 1),\) the momentum of \(P_1, k_1 = (\sin \theta_1, 0, \cos \theta_1)\), the momentum of \(P_2, k_2 = (\sin \theta_2 \cos \phi, \sin \theta_2 \sin \phi, \cos \theta_2)\), we have

\[
\begin{align*}
\vec{\varepsilon}_1 &\to (\sin \theta_1, 0, \cos \theta_1), \\
\vec{\varepsilon}_2 &\to (\sin \theta_2 \cos \phi, \sin \theta_2 \sin \phi, \cos \theta_2), \\
\mathbf{T}^1 &\to \begin{pmatrix}
\sin^2 \theta_1 - \frac{1}{3} & 0 & \sin \theta_1 \cos \theta_1 \\
0 & -\frac{1}{3} & 0 \\
\sin \theta_1 \cos \theta_1 & 0 & \cos^2 \theta_1 - \frac{1}{3}
\end{pmatrix},
\end{align*}
\]
Putting Eq. (A9) into Eqs. (A3-A8) one can immediately get Eq. (4). Notice that terms involving $S^0$ make no contribution to the result.

For the case of the decay $B \rightarrow V_1 (\rightarrow P_1 P'_1) V_2 (\rightarrow l^+ l^-)$, suppose we observe that $l^-$ is a right-handed particle and comes out in the direction $k_2 = (\sin \theta_2 \cos \phi, \sin \theta_2 \sin \phi, \cos \theta_2)$ with $\hat{p} = (0, 0, 1)$ and the momentum of $P_1$, $k_1 = (\sin \theta_1, 0, \cos \theta_1)$, we have instead

\[
\begin{align*}
\epsilon^1 &\rightarrow (\sin \theta_1, 0, \cos \theta_1), \\
\epsilon^2 &\rightarrow \frac{1}{\sqrt{2}}(\cos \theta_2 \cos \phi - i \sin \phi, \cos \theta_2 \sin \phi + i \cos \phi, - \sin \theta_2),
\end{align*}
\]

\[
\begin{align*}
T_1 &\rightarrow \begin{pmatrix}
\sin^2 \theta_1 - \frac{1}{3} & 0 & \sin \theta_1 \cos \theta_1 \\
0 & -\frac{1}{3} & 0 \\
\sin \theta_1 \cos \theta_1 & 0 & \cos^2 \theta_1 - \frac{1}{3}
\end{pmatrix}, \\
S^2 &\rightarrow \frac{1}{2}
\begin{pmatrix}
\sin \theta_2 \cos \phi \\
\sin \theta_2 \sin \phi \\
\cos \theta_2
\end{pmatrix}, \\
T^2 &\rightarrow \frac{1}{2}
\begin{pmatrix}
\cos^2 \theta_2 \cos^2 \phi + \sin^2 \phi - \frac{2}{3} & - \sin^2 \theta_2 \sin \phi \cos \phi & - \sin \theta_2 \cos \theta_2 \cos \phi \\
- \sin^2 \theta_2 \sin \phi \cos \phi & \cos^2 \theta_2 \sin^2 \phi + \cos^2 \phi - \frac{2}{3} & - \sin \theta_2 \cos \theta_2 \sin \phi \\
- \sin \theta_2 \cos \theta_2 \sin \phi & - \sin \theta_2 \cos \theta_2 \cos \phi & \sin^2 \theta_2 - \frac{2}{3}
\end{pmatrix},
\end{align*}
\]

Putting Eq. (A10) into Eqs. (A3-A8) one can immediately get the differential angular distribution for the decay $B \rightarrow V_1 (\rightarrow P_1 P_2) V_2 (\rightarrow l^+ l^-)$ with a right-handed $l^-$ coming out in the final state:

\[
\frac{1}{\Gamma_0} \frac{d^3 \Gamma}{d \cos \theta_1 d \cos \theta_2 d \phi} = \frac{9}{16 \pi \Gamma_0} \left\{ K_1 \cos^2 \theta_1 \sin^2 \theta_2 + \frac{K_2}{2} \left( \sin^2 \theta_1 \cos^2 \theta_2 \cos^2 \phi + \sin^2 \theta_1 \sin^2 \phi \right) \right. \\
+ \frac{K_3}{2} \left( \sin^2 \theta_1 \cos^2 \theta_2 \sin^2 \phi + \sin^2 \theta_1 \cos^2 \phi \right) + \frac{K_4}{2 \sqrt{2}} \sin 2 \theta_1 \sin 2 \theta_2 \cos \phi \\
- \frac{K_5}{2 \sqrt{2}} \sin 2 \theta_1 \sin 2 \theta_2 \sin \phi - \frac{K_6}{2} \sin^2 \theta_1 \sin^2 \theta_2 \sin 2 \phi
\]
\[ + \frac{L_4}{\sqrt{2}} \sin 2\theta_1 \sin \theta_2 \sin \phi - \frac{L_5}{\sqrt{2}} \sin 2\theta_1 \sin \theta_2 \cos \phi + \frac{L_6}{2} \sin^2 \theta_1 \cos \theta_2 \} \quad (A11) \]

Notice that terms involving $S^1$ do not contribute to the result. To obtain the result for the other possible final state with an left-handed outgoing $l^-$, one only needs to flip the sign of $S^2$ and thus the signs of the coefficients of $L_4$, $L_5$, and $L_6$ (namely, the signs of $Y_4$, $X_5$, and $X_6$). The muon polarization is equal to the sum of the terms $L_4$, $L_5$, $L_6$ divided by the sum of the other 6 terms. For the case of $L_6$ it is seen that the polarization does not vanish after integrating over $\theta_1$ and $\phi$ and so the observation can be made without observing the $V_1$ decay.
REFERENCES

[1] A.S. Dighe, I. Dunietz, H.J. Lipkin, and J.L. Rosner, Phys. Lett. B369, 144 (1996).
[2] A.S. Dighe, I. Dunietz, and R. Fleischer, Eur. Phys. J. C6, 647 (1999).
[3] G. Kramer and W.F. Palmer, Phys. Rev. D45, 193 (1992).
[4] G. Kramer and W.F. Palmer, Phys. Lett. B279, 181 (1992).
[5] G. Kramer, W.F. Palmer, and H. Simma, Nucl. Phys. B428, 77 (1994).
[6] G. Kramer, T. Mannel, and W.F. Palmer, Z. Phys. C55, 497 (1992).
[7] R. Fleischer, Phys. Rev. D 60, 07008 (1999).
[8] D. London, N. Sinha, and R. Sinha, Phys. Rev. D 60, 074020 (1999).
[9] C.W. Chiang and B. Tseng, hep-ph9905338.
[10] W. Lakin, Phys. Rev. 98, 139 (1955); B.A. Robson, The Theory of Polarization Phenomena, Oxford University Press, 1974.
[11] R.G. Sachs, The Physics of Time Reversal, Chicago University Press, 1987.
[12] C.W. Chiang and L. Wolfenstein, in preparation.