Scalar $a_0(980)$ meson in $\phi \to \pi^0\eta\gamma$ decay

A. Gokalp, A. Küçükarslan, S. Solmaz and O. Yilmaz

Physics Department, Middle East Technical University, 06531 Ankara, Turkey

(October 24, 2018)

Abstract

We study the radiative decay $\phi \to \pi^0\eta\gamma$ within the framework of a phenomenological approach in which the contributions of $\rho$-meson, chiral loop and $a_0$-meson are considered. We analyze the interference effects between different contributions and utilizing the experimental branching ratio and invariant $\pi^0\eta$ mass spectrum for $\phi \to \pi^0\eta\gamma$ decay we estimate the branching ratio of $\phi \to a_0\gamma$ decay.

PACS numbers: 12.20.Ds, 13.40.Hq, 14.40.-n

*agokalp@metu.edu.tr

†oyilmaz@metu.edu.tr
I. INTRODUCTION

The nature and quark substructure of low mass scalar mesons, in particular those of isoscalar \( f_0(980) \) and isovector \( a_0(980) \), have been a subject of controversy in hadron spectroscopy over the years. A variety of interpretations have been proposed for their structure, but, whether they are conventional quark states in quark model [1], \( K\bar{K} \) molecules [2], or exotic multiquark \( \bar{q}q^2 \) states [3] have not been established yet.

It was suggested by Achasov and Ivanchenko [4] that the radiative decays of \( \phi \) meson to pseudoscalar mesons, \( \phi \rightarrow \pi^0\pi^0\gamma \) and \( \phi \rightarrow \pi^0\eta\gamma \), offer the possibility of obtaining information on the nature of \( f_0(980) \) and \( a_0(980) \) mesons, respectively. Close, Isgur and Kumano [5] noted that the radiative decays \( \phi \rightarrow S\gamma \), where \( S = f_0 \) or \( a_0 \), can be utilized to differentiate among various models of the structure of these scalar mesons. They shown that although the transition rates \( \Gamma(\phi \rightarrow f_0\gamma) \) and \( \Gamma(\phi \rightarrow a_0\gamma) \) depends on the unknown dynamics, the ratio of the decay rates \( \Gamma(\phi \rightarrow f_0\gamma)/\Gamma(\phi \rightarrow a_0\gamma) \) will be sensitive to the spatial distribution of quarks and the spatial wavefunctions of scalar mesons, and thus it provides an experimental test which distinguishes between alternative explanations of their structure. In both the \( K\bar{K} \) molecule, and four-quark cluster \( q^2\bar{q}^2 \) pictures of the structure of scalar mesons, it is now generally accepted that the radiative decays \( \phi \rightarrow S\gamma \) proceed by the mechanism in which \( \phi \) and \( S \) both couple to an intermediate \( K^+K^- \) loop in the chain of reactions \( \phi \rightarrow K^+K^- \rightarrow K^+K^-\gamma \rightarrow S\gamma \) [4,5]. The corresponding decay rates were obtained as [5]

\[
BR(\phi \rightarrow f_0(980)\gamma) = BR(\phi \rightarrow a_0(980)\gamma) \simeq (2.0 \pm 0.5) \times 10^{-4} \times F^2(R),
\]

where \( F^2(R) \) is a form factor that depends on the spatial wavefunctions of the scalar mesons, and \( F^2(R) = 1 \) in point-like effective field theory calculations. In obtaining this result it is further assumed that both \( f_0(980) \) and \( a_0(980) \) are isospin eigenstates, and \( g_{SK+K^-}^2/4\pi = 0.58 \, GeV^2 \). If \( f_0(980) \) and \( a_0(980) \) are spatially extended \( K\bar{K} \) molecules with \( R > \Lambda_{QCD}^{-1} \), then \( F^2(R) < 1 \) and the resulting branching ratios are \( BR(\phi \rightarrow S\gamma) \simeq (0.4 - 1) \times 10^{-4} \) [5]. If the meson states are compact four-quark clusters confined within \( R \sim \Lambda_{QCD}^{-1} \) like in \( q^2\bar{q}^2 \)
picture, then the resulting branching ratios are expected as $\text{BR}(\phi \rightarrow S\gamma) \simeq 2 \times 10^{-4}$ [4,5]. In addition to the predictions about the absolute branching ratios of $\phi \rightarrow S\gamma$ decays, there is also the weaker result that follows from Eq. 1, that is theoretically $\text{BR}(\phi \rightarrow f_0\gamma)/\text{BR}(\phi \rightarrow a_0\gamma) = 1$.

The very accurate data that have been obtained from Novosibirsk SND [6,7] and CMD-2 [8] Collaborations give the following branching ratios for $\phi \rightarrow \pi^0\pi^0\gamma$ and $\phi \rightarrow \pi^0\eta\gamma$ decays: $\text{BR}(\phi \rightarrow \pi^0\pi^0\gamma) = (1.221 \pm 0.098 \pm 0.061) \times 10^{-4}$ [4], $\text{BR}(\phi \rightarrow \pi^0\eta\gamma) = (0.88 \pm 0.14 \pm 0.09) \times 10^{-4}$ [7], and $\text{BR}(\phi \rightarrow \pi^0\pi^0\gamma) = (0.92 \pm 0.08 \pm 0.06) \times 10^{-4}$, $\text{BR}(\phi \rightarrow \pi^0\eta\gamma) = (0.90 \pm 0.24 \pm 0.10) \times 10^{-4}$ [8], where the first error is statistical and the second one is systematic. From the analysis of their experimental results, Achasov et al. [6] concluded that the $f_0\gamma$ mechanism dominates the $\phi \rightarrow \pi^0\pi^0\gamma$ decay and the contributions coming from $\sigma\gamma$ and $\rho^0\pi^0$ intermediate states are small. Neglecting these contributions they obtained the branching ratio $\text{BR}(\phi \rightarrow f_0\gamma) = (3.5 \pm 0.3 \pm 1.3) \times 10^{-4}$, where the second errors are systematic and includes uncertainty in the interference terms. This branching ratio was also obtained by Akhmetshin et al. from their data of $\phi \rightarrow \pi^0\pi^0\gamma$ decay as $\text{BR}(\phi \rightarrow f_0(980)\gamma) = (2.90 \pm 0.21 \pm 1.54) \times 10^{-4}$ [8]. The $\phi \rightarrow f_0(980)\gamma$ decay branching ratio is now quoted as an average of these two results which is $\text{BR}(\phi \rightarrow f_0(980)\gamma) = (3.4 \pm 0.4) \times 10^{-4}$ [3]. Achasov et al., on the other hand, by assuming that $a_0\gamma$ intermediate state dominates the $\phi \rightarrow \pi^0\eta\gamma$ decay, and the contributions from other decay mechanisms, for example $\phi \rightarrow \rho^0\pi^0$, $\rho^0 \rightarrow \eta\gamma$, can be neglected, obtained the value for the branching ratio of the decay $\phi \rightarrow a_0(980)\gamma$ as $\text{BR}(\phi \rightarrow a_0(980)\gamma) = (0.88 \pm 0.17) \times 10^{-4}$ from the data of their $\phi \rightarrow \pi^0\eta\gamma$ decay experiment [8]. Therefore, these experimental results suggest for the ratio of the decay rates of the decays $\phi \rightarrow f_0(980)\gamma$ and $\phi \rightarrow a_0(980)\gamma$ the experimental value $\Gamma(\phi \rightarrow f_0\gamma)/\Gamma(\phi \rightarrow a_0\gamma) = 3.8 \pm 1.2$, with which the theoretical result $\Gamma(\phi \rightarrow f_0\gamma)/\Gamma(\phi \rightarrow a_0\gamma) \simeq 1$ is not in agreement.

In this paper, we study the role of the scalar $a_0(980)$ meson in the mechanism of the radiative $\phi \rightarrow \pi^0\eta\gamma$ decay employing a phenomenological framework, in which we try to assess the roles of different processes and the contributions to the decay rate coming from their amplitude in the mechanism of this decay, and this way we attempt to estimate the
branching ratio $BR(\phi \to a_0(980)\gamma)$ of the decay $\phi \to a_0(980)\gamma$ from the experimental data of the radiative $\phi \to \pi^0\eta\gamma$ decay. In our analysis, we use the experimental data of Novosibirsk SND Collaboration [7].

II. FORMALISM

The radiative decay process of the type $V^0 \to P^0P^0\gamma$ where $V$ and $P$ belong to the lowest multiplets of vector ($V$) and pseudoscalar ($P$) mesons have been studied using different approaches. In particular, the branching ratio $BR(\phi \to \pi^0\eta\gamma)$ has been calculated as $BR(\phi \to \pi^0\eta\gamma)_{VDM} = 5.4 \times 10^{-6}$ [10] by considering intermediate vector meson contribution only, and as $BR(\phi \to \pi^0\eta\gamma)_\chi = 3 \times 10^{-5}$ [11] by using chiral loop model within the framework of chiral phenomenological Lagrangians.

In order to include the scalar $a_0$ meson resonance pole in the decay mechanism for the calculation of the decay rate of $\phi \to \pi^0\eta\gamma$ decay in a phenomenological framework we consider two different approaches. In both approaches, we do not make any assumptions about the structure of the scalar $a_0$ meson.

In the first approach, which we name Model I, we include the scalar $a_0$ resonance in an ad hoc manner. In this approach, we assume that the mechanism of the $\phi \to \pi^0\eta\gamma$ decay consists of the reactions shown by the diagrams in Fig. 1. We describe the $\phi a_0\gamma$ and $a_0\pi^0\eta$ vertices in Fig. 1(c) by the phenomenological Lagrangians

$$L_{\phi a_0\gamma} = \frac{e}{M_\phi}g_{a_0\gamma}[\partial^\alpha\phi^\beta\partial_\alpha A_\beta - \partial^\alpha\phi^\beta\partial_\beta A_\alpha]a_0 \tag{2}$$

and

$$L_{a_0\pi\eta} = g_{a_0\pi\eta}\vec{\pi} \cdot \vec{a_0}, \tag{3}$$

respectively, which also serve to define the coupling constants $g_{\phi a_0\gamma}$ and $g_{a_0\pi\eta}$. Since there are no direct experimental results relating to $\phi a_0\gamma$-vertex, but only an upper limit for the branching ratio of the decay $\phi \to a_0\gamma$ is quoted in Review of Particle Properties as $BR(\phi \to$
we determine this coupling constant in our calculation by employing the experimental branching ratio of the $\phi \to \pi^0 \eta \gamma$ decay. The decay rates for the $\phi \to a_0 \gamma$ and the $a_0 \to \pi^0 \eta$ decays resulting from the above Lagrangians are

$$
\Gamma(\phi \to a_0 \gamma) = \frac{\alpha^2}{24\pi} \frac{(M_\phi^2 - M_{a_0}^2)^3}{M_\phi^5} g_{\phi a_0 \gamma}^2 ,
$$

and

$$
\Gamma(a_0 \to \pi^0 \eta) = \frac{g_{a_0 \pi \eta}^2}{16\pi M_{a_0}} \left[ \frac{1}{2} \left( \frac{M_{\pi^0}^2}{M_{a_0}^2} + \frac{M_\eta^2}{M_{a_0}^2} \right) \right] \left[ \frac{1}{2} \left( \frac{M_{\pi^0}^2 - M_\eta^2}{M_{a_0}^2} \right) \right],
$$

respectively. We use the value $\Gamma_{a_0} = (0.069 \pm 0.011)$ GeV determined by E852 collaboration at BNL [12], and we obtain the coupling constant $g_{a_0 \pi \eta}$ as $g_{a_0 \pi \eta} = (2.32 \pm 0.18)$ GeV.

The $\phi \rho \pi$ vertex in Fig. 1(a) is conventionally described by the phenomenological Lagrangian

$$
\mathcal{L}_{\phi \rho \pi} = g_{\phi \rho \pi} \epsilon^{\mu \nu \alpha \beta} \partial_\mu \phi \partial_\alpha \rho \partial_\beta \pi ,
$$

and the coupling constant $g_{\phi \rho \pi}$ was determined by Achasov and Gubin using the data on the decay $\phi \to \rho \pi \to \pi^+ \pi^- \pi^0$ [8] as $g_{\phi \rho \pi} = (0.811 \pm 0.081)$ GeV$^{-1}$ [13]. The $\rho \pi \gamma$ vertex in Fig. 1(a) is described by the phenomenological Lagrangian

$$
\mathcal{L}_{\rho \pi \gamma} = g_{\rho \pi \gamma} \epsilon^{\mu \nu \alpha \beta} \partial_\mu \rho \partial_\alpha A_\beta \eta ,
$$

and the coupling constant $g_{\rho \pi \gamma}$ is then obtained from the experimental partial width of the radiative decay $\rho \to \eta \gamma$ [8] as $g_{\rho \pi \gamma} = (0.45 \pm 0.07)$ GeV$^{-1}$.

We describe the $\phi KK$ vertex in Fig. 1(b) by the phenomenological Lagrangian

$$
\mathcal{L}_{\phi K^+ K^-} = -i g_{\phi KK} \phi^\mu (K^+ \partial_\mu K^- - K^- \partial_\mu K^+) ,
$$

which results from the standard chiral Lagrangians in the lowest order of chiral perturbation theory [14]. The decay rate for the $\phi \to K^+ K^-$ decay resulting from this Lagrangian is

$$
\Gamma(\phi \to K^+ K^-) = \frac{g_{\phi KK}^2}{48\pi} M_\phi \left[ 1 - \left( \frac{2M_{K^+}}{M_\phi} \right)^2 \right]^{3/2} .
$$
We utilize the experimental value for the branching ratio $BR(\phi \to K^+K^-) = (0.492 \pm 0.007)$ for the decay $\phi \to K^+K^-$ [9], and determine the coupling constant $g_{\phi KK}$ as $g_{\phi KK} = (4.59 \pm 0.05)$. For the four pseudoscalar $KK\pi\eta$ amplitude, we use the result obtained in standard chiral perturbation theory [15] which is

$$\mathcal{M}(K^+K^- \to \pi^0\eta) = \frac{\sqrt{3}}{4f_\pi^2} \left( M_{\pi^0\eta}^2 - \frac{4}{3} M_K^2 \right) .$$

(10)

where $\eta - \eta'$ mixing is neglected, $M_{\pi^0\eta}$ is the invariant mass of the $\pi^0\eta$ system, and we use $f_\pi = 92.4$ MeV. We therefore obtain the amplitude for the diagram in Fig. 1(b) as

$$\mathcal{M} = -\frac{e}{2\pi^2 iM_K^2} \left( [p \cdot k](\epsilon \cdot u) - (p \cdot \epsilon)(k \cdot u) \right) I(a,b) \mathcal{M}(K^+K^- \to \pi^0\eta)$$

(11)

where $(u,p)$ and $(\epsilon,k)$ are the polarizations and four-momenta of the $\phi$ meson and the photon, respectively, and $a = M_\phi^2/M_K^2$, $b = M_{\pi^0\eta}^2/M_K^2$ with the invariant mass of the final $\pi^0\eta$ system given by $M_{\pi^0\eta}^2 = (q_1 + q_2)^2 = (p - k)^2$. The loop function is defined as [16,17]

$$I(a,b) = \frac{1}{2(a-b)} - \frac{2}{(a-b)^2} \left[ f\left(\frac{1}{b}\right) - f\left(\frac{1}{a}\right) \right] + \frac{a}{(a-b)^2} \left[ g\left(\frac{1}{b}\right) - g\left(\frac{1}{a}\right) \right]$$

(12)

where

$$f(x) = \begin{cases} -\left[ \arcsin\left(\frac{1}{\sqrt{2x}}\right) \right]^2, & x > \frac{1}{4} \\ \frac{1}{4} \left[ \ln\left(\frac{x}{\sqrt{1-x}}\right) - i\pi \right]^2, & x < \frac{1}{4} \end{cases}$$

$$g(x) = \begin{cases} (4x - 1)^{1/2} \arcsin\left(\frac{1}{\sqrt{2x}}\right), & x > \frac{1}{4} \\ \frac{1}{2} (1 - 4x)^{1/2} \left[ \ln\left(\frac{x}{\sqrt{1-x}}\right) - i\pi \right], & x < \frac{1}{4} \end{cases}$$

$$\eta_\pm = \frac{1}{2x} \left[ 1 \pm (1 - 4x)^{1/2} \right] .$$

(13)

On the other hand, the introduction of the $a_0$ amplitude as in Fig. 1(c) may be considered not to be very realistic since this diagram implies direct quark transition and it thus makes a very small contribution because of OZI supression. Indeed, it has been shown that the scalar resonances $f_0(980)$ and $a_0(980)$ can be excited from the chiral loops, with the loop iteration provided by the Bethe-Salpeter equation using a kernel from the lowest order chiral
Furthermore, it has been argued that the experimental data obtained in Novosibirsk give reasonable arguments in favour of the one-loop mechanism for $\phi \to K^+ K^- \to a_0\gamma$ and $\phi \to K^+ K^- \to f_0\gamma$ decays \cite{19}. Therefore, in our second approach to study the radiative $\phi \to \pi^0 \eta \gamma$ decay, which we name Model II, we assume that this decay proceeds through the reactions $\phi \to \rho^0 \pi^0 \to \pi^0 \eta \gamma$, $\phi \to K^+ K^- \gamma \to \pi^0 \eta \gamma$, and $\phi \to a_0\gamma \to \pi^0 \eta \gamma$ where the last reaction proceeds by a two-step mechanisms with $a_0$ coupling to $\phi$ with intermediate $K\bar{K}$ states. We show the processes contributing to the $\phi \to \pi^0 \eta \gamma$ decay amplitude diagramatically in Fig. 2. We note that we do not make any assumptions about the structure of the $a_0$ meson, and only assume that the $\phi$ and $a_0$ mesons both couple strongly to the $K^+ K^-$ system, as a result of which there is an amplitude for the decay $\phi \to a_0\gamma$ to proceed through the charged kaon loop independent of the nature and the dynamical structure of $a_0$ meson. The only new ingredient required in our second approach is the $K^+ K^- a_0$ vertex which we assume is described by the phenomenological Lagrangian

$$\mathcal{L}_{a_0 K^+ K^-} = g_{a_0 K^+ K^-} K^+ K^- a_0 .$$

The decay width of $a_0$ that follows from this Lagrangian is

$$\Gamma(a_0 \to K^+ K^-) = \frac{g_{a_0 K^+ K^-}^2}{16 \pi M_{a_0}} \left[ 1 - \left( \frac{2 M_K}{M_{a_0}} \right)^2 \right]^{1/2}$$

which is usually considered to define the coupling constant $g_{a_0 K^+ K^-}$. In our second approach, we calculate the decay rate of $\phi \to \pi^0 \eta \gamma$ decay using the diagrams shown Fig. 2, and by utilizing the experimental value of this decay rate determine the coupling constant $g_{a_0 K^+ K^-}$.

In our calculation of the invariant amplitudes we make the replacement $p^2 - M^2 \to p^2 - M^2 + i M \Gamma$ in $a_0$ and $\rho^0$ propagators and use the experimental value $\Gamma_{\rho} = (150.2 \pm 0.8)$ MeV \cite{9} for $\rho^0$ meson, because using a $q^2$-dependent width did not affect our results appreciably. However, since the mass of the $M_{a_0}$ meson is very close to the $K^+ K^-$ threshold, this induces a strong energy dependence in the width of the $a_0$-meson. We follow the widely accepted option to deal with this problem that was proposed by Flatté \cite{20} based on a coupled channel ($\pi \eta, K\bar{K}$) description of the $a_0$ resonance, and parametrize the $a_0$ width as
\[
\Gamma^{a_0}(q^2) = \Gamma^{a_0\eta\pi_0}(q^2) \theta(\sqrt{q^2} - (M_{\pi^0} + M_\eta)) \\
+ ig_{K\pi} \sqrt{M_K^2 - q^2/4} \theta(2M_K - \sqrt{q^2}) + g_{K\pi} \sqrt{q^2/4 - M_K^2} \theta(\sqrt{q^2} - 2M_K),
\]
(16)

where

\[
\Gamma^{a_0\eta\pi_0}(q^2) = \frac{g_{a_0\pi^0\eta}}{16\pi(q^2)^{3/2}} \sqrt{[q^2 - (M_{\pi^0} + M_\eta)^2][q^2 - (M_{\pi^0} - M_\eta)^2]}. \tag{17}
\]

We use the Flatté parameter \(g_{K\bar{K}}\) as \(g_{K\bar{K}} = (0.22 \pm 0.04)\) which was determined by E852 Collaboration at BNL by a fit to data in their experiment in which they determined the parameters of the \(a_0\) meson \([12]\). In all our calculations and in the analysis of the experimental invariant \(\pi^0\eta\) mass spectrum of the \(\phi \to \pi^0\eta\gamma\) decay, therefore, for the mass of \(a_0\) meson we use the value \(M_{a_0} = (0.991 \pm 0.0025)\) GeV as determined in this experiment by E852 Collaboration \([12]\).

We express the invariant amplitude \(\mathcal{M}(E_\gamma, E_1)\) for the decay \(\phi \to \pi^0\eta\gamma\) as \(\mathcal{M} = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c\) in Model I and as \(\mathcal{M} = \mathcal{M}_a' + \mathcal{M}_b' + \mathcal{M}_c'\) in Model II, where \(\mathcal{M}_a, \mathcal{M}_b, \text{ and } \mathcal{M}_c\) are the invariant amplitudes resulting from the diagrams (a), (b), and (c) in Fig. 1 respectively, describing the Model I, and \(\mathcal{M}_a', \mathcal{M}_b'\) and \(\mathcal{M}_c'\) are the invariant amplitudes corresponding to the diagrams (a), (b) and (c) in Fig. 2, respectively, defining the Model II. This way we take the interference between different reactions contributing to the decay \(\phi \to \pi^0\eta\gamma\) into account. Then the differential decay probability for \(\phi \to \pi^0\eta\gamma\) for an unpolarized \(\phi\) meson at rest is given as

\[
\frac{d\Gamma}{dE_\gamma dE_1} = \frac{1}{(2\pi)^3} \frac{1}{8M_\phi} |\mathcal{M}|^2,
\]
(18)

where \(E_\gamma\) and \(E_1\) are the photon and pion energies respectively. We perform an average over the spin states of \(\phi\) meson and a sum over the polarization states of the photon. The decay width \(\Gamma(\phi \to \pi^0\eta\gamma)\) is then obtained by integration

\[
\Gamma = \int_{E_\gamma,min.}^{E_\gamma,max.} dE_\gamma \int_{E_1,min.}^{E_1,max.} dE_1 \frac{d\Gamma}{dE_\gamma dE_1},
\]
(19)

where the minimum photon energy is \(E_{\gamma,min.} = 0\) and the maximum photon energy is given as \(E_{\gamma,max.} = [M_\phi^2 - (M_{\pi^0} + M_\eta)^2]/2M_\phi\). The maximum and minimum values for pion energy \(E_1\) are given by
\[
\begin{align*}
\frac{1}{2(2E_\gamma M_\phi - M_\phi^2)} & \left[ -2E_\gamma^2 M_\phi - M_\phi (M_\phi^2 + M_{\pi^0}^2 - M_\eta^2) + E_\gamma (3M_\phi^2 + M_{\pi^0}^2 - M_\eta^2) \right. \\
& \pm E_\gamma \sqrt{4E_\gamma^2 M_\phi^2 + M_\phi^4 + (M_{\pi^0}^2 - M_\eta^2)^2 - 2M_\phi^2 (M_{\pi^0}^2 + M_\eta^2) + 4E_\gamma M_\phi (-M_\phi^2 + M_{\pi^0}^2 + M_\eta^2)}] \\
\end{align*}
\]

### III. RESULTS AND DISCUSSION

In order to determine the coupling constants \( g_{\phi a_0 \gamma} \) in Model I and \( g_{a_0 K^+ K^-} \) in Model II, we use the experimental value of the branching ratio for the radiative decay \( \phi \rightarrow \pi^0 \eta \gamma \) [9] in our calculation of this decay rate, and this way we obtain a quadric equation for the coupling constant \( g_{\phi a_0 \gamma} \) in Model I and another quadric equation for the coupling constant \( g_{a_0 K^+ K^-} \) in Model II. In these quadric equations the coefficient of quadric term results from the \( a_0 \) meson amplitude contribution, and the coefficient of the linear term from the interference of the \( a_0 \) meson amplitude with the vector meson dominance and the loop amplitudes. We then predict and study the invariant mass distribution \( dBR/dM_{\pi^0 \eta} \) for the radiative decay \( \phi \rightarrow \pi^0 \eta \gamma \) in our phenomenological approach using the values of coupling constants \( g_{\phi a_0 \gamma} \) in Model I and \( g_{a_0 K^+ K^-} \) in Model II that we obtain and compare our results with the experimental invariant \( \pi^0 \eta \) mass spectrum for the decay \( \phi \rightarrow \pi^0 \eta \gamma \) [9].

In Model I, we obtain for the coupling constant \( g_{\phi a_0 \gamma} \) the values \( g_{\phi a_0 \gamma} = (0.24 \pm 0.06) \) and \( g_{\phi a_0 \gamma} = (-1.3 \pm 0.3) \). In Fig. 3 we plot the distribution \( dBR/dM_{\pi^0 \eta} \) choosing the coupling constant \( g_{\phi a_0 \gamma} = (0.24 \pm 0.06) \) in which we also indicate the contributions coming from the different reactions shown diagramatically in Fig. 1, as well as the contribution of the total amplitude which includes the interference term as well. Our Model gives a reasonable prediction for the spectrum over most of the range of the invariant mass \( M_{\pi^0 \eta} \) except in its higher part where the expected enhancement due to the contribution of \( a_0 \) resonance is not produced. The distribution \( dBR/dM_{\pi^0 \eta} \) we obtain for the other root, that is for \( g_{\phi a_0 \gamma} = (-1.3 \pm 0.3) \), is even poorer which we do not show. We obtain the branching ratio for the decay \( \phi \rightarrow a_0 \gamma \) using for the coupling constant \( g_{\phi a_0 \gamma} = (0.24 \pm 0.06) \) as \( BR(\phi \rightarrow a_0 \gamma) = (0.2 \pm 0.1) \times 10^{-5} \). However, because of the fact that Model I does
not produce a satisfactory description of the experimental invariant $M_{π^0\eta}$ mass spectrum for the decay $φ \rightarrow π^0ηγ$, we cannot consider the value of the coupling constant $g_{φa_0γ} = (0.24 \pm 0.06)$ and the resulting branching ratio $BR(φ \rightarrow a_0γ) = (0.2 \pm 0.1) \times 10^{-5}$ too seriously. In this connection, we like to note that the coupling constant $g_{φa_0γ}$ has been calculated [21] employing QCD sum rules and utilizing $ωφ$-mixing by studying the three point $φa_0γ$-correlation function as $g_{φa_0γ} = (0.11 \pm 0.03)$. Furthermore, the photoproduction cross section of $ρ^0$ mesons on photon targets near threshold has been shown [22] to be mainly given by $σ$-exchange, and assuming vector meson dominance of the electromagnetic current the value of the coupling constant $g_{ρσγ} = 2.71$ was deduced. In the study of the structure of the $φ$ meson photoproduction amplitude on nucleons near threshold based on the one-meson exchange and Pomeron-exchange mechanism [23], this value of the coupling constant $g_{ρσγ}$ was used to calculate the coupling constant $g_{φa_0γ}$ by invoking unitary symmetry arguments as $g_{φa_0γ} = 0.16$ by assuming that $σ$, $f_0$, and $a_0$ are numbers of a unitary nonet, which is not without problems. In the light of our discussion of Model I and the resulting poor invariant mass spectrum of the decay $φ \rightarrow π^0ηγ$, it may be argued that all these values for the coupling constant $g_{φa_0γ}$ should be considered with some caution.

We follow the same procedure in Model II, and obtain the coupling constant $g_{a_0K^+K^-}$ as $g_{a_0K^+K^-} = (−1.5 \pm 0.3) \text{ GeV}$ and $g_{a_0K^+K^-} = (3.0 \pm 0.4) \text{ GeV}$ utilizing the experimental value of the $φ \rightarrow π^0ηγ$ decay rate. We then plot the resulting invariant mass distribution for the decay $φ \rightarrow π^0ηγ$ and compare it with the experimental result [5]. In Fig. 4 and in Fig. 5 we plot the distribution $dBR/dM_{π^0η}$ for the radiative decay $φ \rightarrow π^0ηγ$ in our phenomenological approach choosing coupling constant $g_{a_0K^+K^-} = −1.5 \text{ GeV}$ and $g_{a_0K^+K^-} = 3.0 \text{ GeV}$, respectively, as a function of the invariant mass $M_{π^0η}$ of the $π^0η$ system. In these figures, as before, we also indicate the contributions coming from the different reactions $φ \rightarrow ρ^0π^0 \rightarrow π^0ηγ$, $φ \rightarrow K^+K^-γ \rightarrow π^0ηγ$, and $φ \rightarrow a_0γ \rightarrow π^0ηγ$ in Model II shown diagramatically in Fig. 2, as well as the contribution of the total amplitude which includes the interference terms as well. We note that for $g_{a_0K^+K^-} = −1.5 \text{ GeV}$ our prediction for the invariant mass spectrum is in good agreement with the experimental result and
not only the overall shape and feature but also the enhancement due to the contribution of the $a_0$ resonance is well produced. Therefore, from the analysis of the spectrum obtained with the coupling constants $g_{a_0K^+K^-} = -1.5$ GeV and $g_{a_0K^+K^-} = 3.0$ GeV in Fig. 4 and 5, respectively, we may decide in favour of the value $g_{a_0K^+K^-} = -1.5$ GeV, and we may state that the experimental data within the framework of our phenomenological approach in Model II, suggest for the coupling constant $g_{a_0K^+K^-}$ the value $g_{a_0K^+K^-} = (-1.5 \pm 0.3)$ GeV. Furthermore, we note that Model II provides a better way as compared to Model I in order to incorporate the $a_0$ meson into the mechanism of the $\phi \to \pi^0\eta\gamma$ decay, and thus our result gives further support for the approach in which $a_0$ meson state arises as a dynamical state and for the two-step mechanism for its coupling to $\phi$ meson with intermediate $K\bar{K}$ states. In a previous work [24], we studied the radiative $\phi \to \pi^0\pi^0\gamma$ decay also within the framework of a phenomenological approach in which we considered the contributions of $\sigma$ meson, $\rho$ meson and $f_0(980)$ meson. We analyzed the interference effects between different contributions using the experimental results of SND Collaboration. In that analysis, the coupling constant $g_{f_0K^+K^-}$ appeared with positive sign, that is $g_{f_0K^+K^-} > 0$. In the present analysis, we obtained the coupling constant $g_{a_0K^+K^-}$ as $g_{a_0K^+K^-} < 0$. We like to note that this result about the relative phase between $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$ is consistent with the result obtained in the $q^2\bar{q}^2$ model where $g_{f_0K^+K^-} = -g_{a_0K^+K^-}$ [3,4,25].

We then calculate the decay rate $\Gamma(\phi \to a_0\gamma)$ by assuming the two-step one-loop mechanism $\phi \to K^+K^- \to a_0\gamma$ for the decay $\phi \to a_0\gamma$. We show this mechanism diagramatically in Fig. 6. The decay rate that follows from these diagrams is given by

$$\Gamma(\phi \to a_0\gamma) = \frac{\alpha g_{a_0K^+K^-}^2}{3(2\pi)^4} \frac{E_\gamma}{M_\phi^2} |(a-b)I(a,b)|^2 \quad (20)$$

where $a = M_\phi^2/M_\pi^2$, $b = M_{a_0}^2/M_K^2$, and the loop function $I(a,b)$ is defined in Eq. 11. We use the coupling constant $g_{a_0KK} = (-1.5 \pm 0.3)$ GeV and then obtain the decay rate for $\Gamma(\phi \to a_0\gamma)$ decay as $\Gamma(\phi \to a_0\gamma) = (0.51 \pm 0.09)$ KeV and the branching ratio as $BR(\phi \to a_0\gamma) = (1.1 \pm 0.2) \times 10^{-4}$. This result when combined with the experimental value $BR(\phi \to f_0\gamma) = (3.4 \pm 0.4) \times 10^{-4}$ gives the ratio $\Gamma(\phi \to f_0\gamma)/\Gamma(\phi \to a_0\gamma) = (3.0 \pm 0.9)$, which is still
larger than the theoretical value of 1. The value $BR(\phi \to a_0\gamma) = (1.1 \pm 0.2) \times 10^{-4}$ we obtain for the branching ratio of the decay $\phi \to a_0\gamma$, although not in disagreement, is somewhat larger than the result of Achasov et al., which is $BR(\phi \to a_0\gamma) = (0.88 \pm 0.17) \times 10^{-4}$. Consequently, we obtain a smaller value for the experimental ratio $\Gamma(\phi \to f_0\gamma)/\Gamma(\phi \to a_0\gamma)$ than the previous result which is $\Gamma(\phi \to f_0\gamma)/\Gamma(\phi \to a_0\gamma) = (3.8 \pm 1.2)$. We like to note, however, that Achasov et al. assumed that $a_0\gamma$ intermediate state dominates the $\phi \to \pi^0\eta\gamma$ decay and they neglected the contributions coming from other intermediate states in their analysis. On the other hand, in our analysis we include the contributions coming not only from $a_0\gamma$ intermediate state but also from $\rho^0\pi^0$ and $K^+K^-$ intermediate states as well as from their interference.

In order to obtain the branching ratio for $\phi \to a_0\gamma$ decay utilizing the experimental value of the decay rate and the experimental invariant $\pi^0\eta$ mass spectrum for the $\phi \to \pi^0\eta\gamma$ decay, we use a phenomenological approach. In our analysis, we employ point-like effective field theory and we deduce from the experimental data the coupling constant of point-like $a_0$ meson coupled to point-like $K$ meson. The coupling constant that we obtain $g_{a_0K^+K^-} = (-1.5 \pm 0.3)$ GeV results in the branching ratio $BR(\phi \to a_0\gamma) = (1.1 \pm 0.2) \times 10^{-4}$ for $\phi \to a_0\gamma$ decay using one loop $\phi \to K^+K^- \to a_0\gamma$ mechanism. We can, therefore, suggest that the branching ratio for the decay $\phi \to a_0\gamma$ that is used in the literature should be revised as $BR(\phi \to a_0\gamma) = (1.1 \pm 0.2) \times 10^{-4} \times F^2(R)$. We can also assert that our analysis suggests a lower value $(3.0 \pm 0.9)$ for the ratio $\Gamma(\phi \to f_0\gamma)/\Gamma(\phi \to a_0\gamma)$ than used previously.

In a very recent paper [26] the KLOE Collaboration presented the result of their experimental study of $\phi \to \pi^0\eta\gamma$ decay. In their analysis, they conclude that $\phi \to \pi^0\eta\gamma$ decay is dominated by the process $\phi \to a_0\gamma$ and from a fit to $\pi^0\eta$ invariant mass spectrum they find the branching ratio $BR(\phi \to a_0\gamma) = (7.4 \pm 0.7) \times 10^{-5}$ for the $\phi \to a_0\gamma$ decay. In view of this finding, our result $BR(\phi \to a_0\gamma) = (1.1 \pm 0.2) \times 10^{-4} \times F^2(R)$ implies that $F^2(R) = (0.7 \pm 0.2)$. This value supports the view that the structure of $a_0$ meson is a combination of a $K\bar{K}$ molecule with a compact $q^2\bar{q}^2$ core [27].

11
REFERENCES

[1] E. van Beveren, T. A. Rijken, K. Metzger, C. Dullemond, G. Rupp, J. E. Ribeiro, Z. Phys. C30, 615 (1986); N. A. Törnqvist and M. Roos, Phys. Rev. Lett. 76, 1575 (1996).

[2] J. Weinstein, N. Isgur, Phys. Rev. D 41 (1990) 2236.

[3] R. L. Jaffe, Phys. Rev. D 15 (1977) 267; 281; D 17 (1978) 1444.

[4] N. N. Achasov, V. N. Ivanchenko, Nucl. Phys. B 315 (1989) 465.

[5] F. E. Close, N. Isgur, S. Kumana, Nucl. Phys. B389 (1993) 513.

[6] M. N. Achasov et al., Phys. Lett. B 485 (2000) 349.

[7] M. N. Achasov et al., Phys. Lett. B 479 (2000) 53.

[8] R. R. Akhmetshin et al., Phys. Lett. B 462 (1999) 380.

[9] Review of Particle Properties, D. E. Groom et al., Eur. Phys. J. C15 (2000) 1.

[10] A. Bramon, A. Grau and G. Pancheri, Phys. Lett. B 283 (1992) 416.

[11] A. Bramon, A. Grau and G. Pancheri, Phys. Lett. B 289 (1992) 97.

[12] S. Teige et al., Phys. Rev. D 59 (1998) 012001.

[13] N. N. Achasov, V. V. Gubin, Phys. Rev. D 63 (2001) 094007.

[14] F. Klingl, N. Kaiser, W. Weise, Z. Phys. A 356 (1996) 193.

[15] A. Bramon, R. Escribana, J. L. Lucio M., M. Napsuciale, and G. Pancheri, Phys. Lett. B 494 (2000) 221.

[16] J. A. Oller, Phys. Lett. B 426 (1998) 7.

[17] J. Lucio, J. Pestiau, Phys. Rev. D 42 (1990) 3253; D 43 (1991) 2447(E).

[18] E. Marco, S. Hirenzaki, E. Oset and H. Toki, Phys. Lett. B 470 (1999) 20.
[19] N. N. Achasov, hep-ph/0201299 and references therein.

[20] S. M. Flatté, Phys. Lett. B 63 (1976) 224.

[21] A. Gokalp and O. Yilmaz, Phys. Lett. B 515 (2002) 273.

[22] B. Friman and M. Soyeur, Nucl. Phys. A 600 (1996) 477.

[23] A. I. Titov, T. -S. H. Lee, H. Toki and O. Streltrova, Phys. Rev. C 60 (1999) 035205.

[24] A. Gokalp and O. Yilmaz, Phys. Rev. D 64 (2001) 053017.

[25] N. N. Achasov, S. A. Devyanin and G. N. Shestakov, Phys. Lett. B 96 (1980) 168.

[26] A. Aloisio et al., The KLOE Collaboration, hep-ex/0204012.

[27] F. E. Close and N. A. Törnqvist, hep-ph/0204205.
FIG. 1. Diagrams for the decay $\phi \rightarrow \pi^0 \eta \gamma$ in Model I.
FIG. 2. Diagrams for the decay $\phi \to \pi^0\eta\gamma$ in Model II.
FIG. 3. The $\pi^0\eta$ invariant mass spectrum for the decay $\phi \to \pi^0\eta\gamma$ in Model I. The contributions of different terms are indicated.
FIG. 4. The $\pi^0\eta$ invariant mass spectrum for the decay $\phi \rightarrow \pi^0\eta\gamma$ for $g_{a_0K^+K^-} = -1.5$ GeV in Model II. The contributions of different terms are indicated.
FIG. 5. The $\pi^0\eta$ invariant mass spectrum for the decay $\phi \rightarrow \pi^0\eta\gamma$ for $g_{a_0K^+K^-} = 3.0$ GeV in Model II. The contributions of different terms are indicated.
FIG. 6. Diagrams for the decay $\phi \rightarrow a_0\gamma$. 