Higher-order Lorentz-invariance violation, quantum gravity and fine-tuning

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We study radiative corrections in the modified QED with Myers and Pospelov dimension-five operators in the photon sector and standard fermions. We explicitly compute the radiatively induced axial operator from the fermion self energy and we found unsuppressed effects of Lorentz-invariance violation. The calculation is performed using dimensional regularization and considering an arbitrary background where the model can be of higher-order. By applying the minimal subtraction scheme and considering the usual renormalization points of the standard model we found small radiative corrections that allow us to set the limit $\xi < 6 \times 10^{-3}$ on the Lorentz-invariance violating parameter of the model.

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I. INTRODUCTION

New physics at the Planck scale has been hypothesized to show up at low energies as small violations of Lorentz symmetry \cite{1}. This idea applied in local quantum field theory has proven most fertile to compute and contrast the possible Lorentz and CPT symmetry deviations with experiments. Most of the searches have been performed with operators of mass dimension $d \leq 4$, as in the case of the standard model extension \cite{2,3} and other approaches \cite{5} where strong limits on Lorentz symmetry violation have been obtained. On the other hand, operators of higher-order mass dimension have been less studied due to some issues that appear in the quantization \cite{6}. However, they have attracted more attention over the last years \cite{23,24} and several bounds have been putted forward \cite{8,11}. Moreover, a generalization has been constructed to include non-minimal terms in the standard model extension \cite{12}.

Lee-Wick \cite{13} and Cutkosky \cite{14} studied the quantization of higher-order theories using the formalism of indefinite metrics in Hilbert space. By defining a new prescription to compute amplitudes they showed how unitarity can be preserved. The main idea behind the Lee-Wick formulation consist to narrow the physical Hilbert space, that is to consider only states with positive square length in the space of asymptotic states. The success of this approach has stimulated the construction of several higher-order models beyond the standard model \cite{15}. Recently, the Lee-Wick prescription has been generalized to include higher-order theories describing Lorentz-invariance violation \cite{16}.

Myers and Pospelov proposed an effective model based on dimension-five operators to describe possible effects of quantum gravity \cite{17}. The modifications were include in all the sectors of the standard model being characterized with a preferred four-vector $n$ breaking the Lorentz symmetry and defining the higher-order derivatives of the theory \cite{18,19}. The preferred four-vector may be thought to come from a spontaneous symmetry breaking in an underlying theory. One of the original motivations to incorporate such terms was to produce cubic modifications in the dispersion relation, although an exact calculation yields a more complicated structure usually with the gramian of the two vectors $k$ and $n$ involved. The Myers and Pospelov model has become an important arena to study higher-order effects of Lorentz-invariance violation \cite{8,20,21}.

This work aims to contribute to the discussion on the possible Lorentz-invariance fine-tuning in effective quantum field theories \cite{22}. There are different approaches on the subject \cite{23,24}. To this end we explicitly compute the radiatively induced axial operator in the Myers and Pospelov photon modification of QED. A large part of the motivation comes from the observation that in the effective theory all renormalizable operators allowed by symmetry will be generated. Therefore, any lower dimensional operator induced via radiative corrections will need to be appropriately reescaled with the Planck mass $m_{Pl}$ leading to the unobserved Lorentz-invariance violation. As has been mentioned \cite{25}, at a given order, one may be tempted to replace the tensor that is contracted with the Feynman diagram in order to cancel the problematic radiative correction \cite{17}, or just restrict attention to transmuted dimension-five operators \cite{20}. However in both cases the problem comes back at higher-order loop corrections. Here we provide a detailed calculation of these radiative corrections using dimensional regularization and taking into account the higher-order modifications of the theory. An advantage of using dimensional regularization is to preserve unitarity in each step of the calculation extending some early treatments \cite{18}.

The organization of the paper is as follows. In the section II we introduce the higher-order QED Myers-Pospelov model. In section III we compute the radiative corrections together with defining the renormaliza-
tion points of the theory. In the last section we give the conclusions.

II. EXTENDED QED WITH DIMENSION-5 OPERATORS

The Myers-Pospelov Lagrangian extension of QED with modifications in the photon sector can be written as \[ \mathcal{L} = \bar{\psi}(\gamma^\mu \partial_\mu - m)\psi - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} \]
where \( m \) is the Planck mass, \( \xi \) a dimensionless coupling parameter and \( n \) a four-vector defining a preferred reference frame.

Let us introduce the gauge fixing Lagrangian term,
\[ \mathcal{L}_{G.F} = -B(x)(n \cdot A) , \]
where \( B(x) \) is an auxiliary field. The field equations for \( A_\mu \) and \( B \) derived from the Lagrangian \( \mathcal{L} + \mathcal{L}_{G,F} \) read,
\[ \partial_\mu F^{\mu \nu} + g \epsilon^{\alpha \lambda \sigma} n_\alpha (n \cdot \partial) F_{\lambda \sigma} = B n_\nu , \]
\[ n \cdot A = 0 . \]
Contracting Eq. (3) with \( \partial_\nu \) gives
\[ (\partial \cdot n)B = 0 , \]
which allows us to set
\[ B = 0 . \]
In the same way, the contraction of Eq. (3) with \( n_\nu \) and going to momentum space leads to
\[ k \cdot A = 0 . \]
As seen from the above equations the gauge field is orthogonal to the hyperplane defined by \( k \) and \( n \). We can always select two four-vectors on the orthogonal hyperplane \( e^{(a)}_\mu \) with \( a = 1, 2 \) satisfying \( e^{(a)} \cdot e^{(b)} = -\delta^{ab} \). Combining the two vectors we can form a tensor
\[ -\sum_a (e^{(a)} \otimes e^{(a)})_{\mu \nu} = -(e^{(1)}_\mu e^{(1)}_\nu + e^{(2)}_\mu e^{(2)}_\nu) \equiv \epsilon_{\mu \nu} , \]
and a pseudo tensor
\[ \sum_a (e^{(a)} \wedge e^{(a)})_{\mu \nu} = e^{(1)}_\mu e^{(2)}_\nu - e^{(2)}_\mu e^{(1)}_\nu \equiv \epsilon_{\mu \nu} . \]
In terms of \( k \) and \( n \) we can choose
\[ \epsilon^{\mu \nu} = \eta^{\mu \nu} - \frac{(n \cdot k)}{D} (n^\mu k_\nu - n^\nu k_\mu) + \frac{k^2 n_\nu - n^2 k_\nu}{2D^2} \]
\[ + \frac{k^2 n_\mu - n^2 k_\mu}{2D^2} \]
with \( D = (n \cdot k)^2 - n_\mu n^\mu \). In order to find the transverse photon propagator \( \Delta_{\mu \nu}(k) \) we need to invert the quadratic operator in the Lagrangian
\[ O^{\nu \sigma}(k) = -k^2 \eta^{\nu \sigma} + 2g i \sqrt{D} (n \cdot k)^2 \epsilon^{\nu \sigma} , \]
such that
\[ \Delta_{\mu \nu}(k) O^{\nu \sigma}(k) = \epsilon_{\mu \sigma} . \]
The photon propagator can be shown to be
\[ \Delta_{\mu \nu}(k) = -\sum_{\lambda = \pm} \frac{P_{\mu \nu}^{(\lambda)}(k)}{k^2 + 2g \lambda (k \cdot n) \sqrt{D}} , \]
with
\[ P_{\mu \nu}^{(\lambda)} = \frac{1}{2} (\epsilon_{\mu \nu} + i \lambda \epsilon_{\mu \nu}) . \]
We have that \( P_{\mu \nu}^{(\lambda)} \) is a projector onto the hyperplane orthogonal to \( k \) and \( n \). Above we have used the relations
\[ \epsilon_{\mu \nu} \epsilon^{\nu \alpha} = \epsilon_{\mu \alpha} \]
\[ \epsilon_{\mu \nu} \epsilon^{\nu \nu} = \epsilon_{\mu \nu} \]
\[ \epsilon_{\mu \nu} \epsilon^{\nu \alpha} = -\epsilon_{\mu \alpha} \]

III. FERMION SELF-ENERGY

In this section, we focus on the fermion self energy diagram in order to compute the radiatively induced axial term \( F \bar{\psi} \gamma_5 \psi \).

A. Radiatively induced axial term

Consider the fermion self-energy diagram at one loop order
\[ \Sigma(p) = i \epsilon^2 \int \frac{d^4 k}{(2\pi)^4} \gamma^{\nu} \left( \frac{\varphi - \bar{k} + m}{(p-k)^2 - m^2} \right) \times \gamma^{\mu} \Delta_{\mu \nu}(k) . \]
By symmetry the only term that can generate the axial operator is the one involving the tensor \( \epsilon_{\mu \alpha \nu \rho} \). Hence, let us focus on the part of the photon propagator
\[ \Delta_{\mu \nu}^{\text{pdf}}(k) = -\frac{i}{2} \sum_{\lambda} \lambda \epsilon_{\mu \nu} . \]
and write the odd contributions in two parts
\[ \tilde{\Sigma}(p) = \epsilon_{\mu_0\beta_0\nu_0\gamma_0} \left( \delta^{(\mathbf{\Sigma}_1)_{\beta}(p)} + (\mathbf{\Sigma}_2)_{\beta}(p) \right) \gamma_\nu, \]
with
\[ (\mathbf{\Sigma}_1)_{\beta}(p) = -2g e^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^\beta(p - k)_\sigma}{((k^2)^2 - 4g^2(k \cdot n)^4 D)}, \]
and
\[ (\mathbf{\Sigma}_2)_{\beta}(p) = -2g e^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^\beta n_\sigma}{((k^2)^2 - 4g^2(k \cdot n)^4 D)}. \]
The first integral evaluated at \( p = 0 \) gives
\[ (\mathbf{\Sigma}_1)_{\beta}(0) = F(0, n^2) \delta^{\beta} + R(0, n^2) n^\beta n_\sigma. \]

With the identity \( \epsilon_{\mu_0\beta_0\nu_0\gamma_0} \delta^{\beta} = 3! \sqrt{g} \delta^{\mu},\) the contribution we need comes from the term \( F.\) Multiplying the above equation by \( n^\beta n_\sigma \) and considering its trace produces two independent equations which can be solved to obtain
\[ F(0, n^2) = -\frac{2g e^2}{3n^2} \times \int \frac{d^4k}{(2\pi)^4} \frac{D(n \cdot k)^2}{(k^2 - m^2)((k^2)^2 - 4g^2(k \cdot n)^4 D)}. \]

**B. Dimensional regularization**

In order to compute the integral \[23\] we use dimensional regularization. In \( d \) dimensions we go to Euclidean space via the Wick rotation of \( k_0 \) and perform an analytic continuation to \( n_\mathbf{E} = (in_0, \mathbf{n}) \) and arrive at
\[ F(0, n^2) = \frac{2i g e^2 \mu^{1-d}}{(1-d)n_\mathbf{E}} \int \frac{d^4k_E}{(2\pi)^4} \frac{D_E(n_\mathbf{E} \cdot k_E)^2}{(k_E^2 + m^2)((k_E^2)^2 - 4g^2(k_\mathbf{E} \cdot n_\mathbf{E})^4 D_E)}, \]
where \( D_E = (n_\mathbf{E} \cdot k_E)^2 - k_E^2 n_\mathbf{E}^2.\) Now, using spherical polar coordinates we have
\[ F(0, n^2) = \frac{2i g e^2 n_\mathbf{E}^2 \mu^{1-d}}{(d - 1)n_\mathbf{E}} \times \int \frac{d\Omega}{(2\pi)^d} \sin^2 \theta \cos^2 \theta \]
\[ \times \int_0^\infty d|k_E| |k_E|^{d-1} \frac{M^2}{(|k_E|^2 + m^2)((|k_E|^2 + M^2)} \]
where the notation is \( |k_E| = \sqrt{k_E^2} \) and \( \theta \) for the angle between \( n_\mathbf{E} \) and \( k_E.\) In addition we have defined the mass term
\[ M^2 = \frac{1}{4g^2 n_\mathbf{E}^6 \sin^2 \theta \cos^4 \theta}. \]
The \( |k_E| \) integral of the second line gives
\[ \frac{\pi M^2}{2(M^2 - m^2)^2} \left( \left( \frac{1}{M^2} \right)^{1-d/2} - \left( \frac{1}{m^2} \right)^{1-d/2} \right) \]
\[ \times \csc \left( \frac{d\pi}{2} \right). \]

Introducing the solid angle element in \( d \) dimensions,
\[ \int d\Omega = \frac{2\pi^{d/2}}{\Gamma(d/2)} \int_0^\pi d\theta (\sin \theta)^{d-2}, \]
and considering \( M^2 \gg m^2 \) leaves us with the angular integrals
\[ \int_0^\pi d\theta \sin^2 \theta (\sin \theta)^{d-2}, \int_0^\pi d\theta (\sin \theta)^{d} \cos^2 \theta. \]
Making \( d = 4 - \varepsilon \) and computing the above angular integrals we finally arrive at
\[ F(0, n^2) = \frac{-ie^2}{\varepsilon \pi^2} \left( \frac{1}{24g(n^2)^2} + \frac{gm^2 n^2}{96} \right) \]
\[ + \frac{ie^2}{1152g \pi^2(n^2)^2} \left[ -24 \ln(g^2 \pi g^2 n^2)^3) - 160 \right] + 24\gamma_E + 8 \ln(64) + \mathcal{O}(g). \]

The radiatively induced axial operator has an finite term which, by applying the minimal subtraction scheme, can be absorbed in the free parameters of QED. However, the term in the second line is a large Lorentz-invariance violating term leading to the additional fine-tuning reported in Ref. \[23\]. In the next subsection we follow the renormalization procedure and we check at the end of the calculation for any large Lorentz-invariance violation in the induced axial operator.

**C. Renormalization points**

In this section we continue the renormalization procedure defining the usual renormalization points of the standard theory. In particular for the mass and residue of the fermion we impose
\[ \Sigma_R(\phi) = \Sigma^{reg}(\phi) - \Sigma^{reg}(m) - (\phi - m) \]
\[ \quad \times \frac{1}{\partial \phi} \Sigma^{reg}(\phi) \bigg|_{\phi = m}. \]
We are interested in the odd contribution which be written as
\[ \tilde{\Sigma}_R(0) = \tilde{\Sigma}^{reg}(0) - \tilde{\Sigma}^{reg}(m) + m \frac{\partial \tilde{\Sigma}^{reg}(\phi)}{\partial \phi} \bigg|_{\phi = m}. \]
That is, in the Euclidean we have to compute the expressions

$$\Sigma_0^\beta(p) = 2ig\epsilon^2 n^2_{E,E} \mu^{4-d} \int \frac{d^d k_E}{(2\pi)^d}$$

\[ \times \frac{k_E^2(p_E - k_E) \cos^2 \theta}{(p_E^2 + k_E^2 + 2(p_E \cdot k_E) + m^2)(k_E^2 + M^2)k_E^2} , \]

and

$$\Sigma_2^\beta(p) = 2ig\epsilon^2 n^2_{E,E} \mu^{4-d} \int \frac{d^d k_E}{(2\pi)^d}$$

\[ \times \frac{k_E^2 m \cos^2 \theta}{(p_E^2 + k_E^2 + 2(p_E \cdot k_E) + m^2)(k_E^2 + M^2)k_E^2} . \]

We start to rewrite the denominator using the Feynman parametrization to arrive at

$$\frac{1}{(p_E^2 + k_E^2 + 2(p_E \cdot k_E) + m^2)(k_E^2 + M^2)k_E^2} = 2! \int_0^1 dx \int_0^{1-x} dy$$

\[ \frac{dA^\beta \cos^2 \theta M^2}{(k_E^2 + (p_E^2 + m^2)x + M^2y + (k_E + p_{E\cdot x})^2 - p_{E\cdot x}^2)^3} , \]

After making the change of variables $k'_E = k_E + p_{E\cdot x}$ and dropping the tilde we have

$$\gamma^\beta(\Sigma_1)^\beta = 4ig\epsilon^2 n^2_{E,E} \mu^{4-d} \int \frac{d^d k_E}{(2\pi)^d}$$

\[ \times \int_0^1 dx \int_0^{1-x} dy A^\beta \cos^2 \theta M^2 (k_E^2 + (p_E^2 + m^2)x + M^2y + (k_E + p_{E\cdot x})^2 - p_{E\cdot x}^2)^3 , \]

and

$$\Sigma_2^\beta = 4ig\epsilon^2 n^2_{E,E} \mu^{4-d} \int \frac{d^d k_E}{(2\pi)^d}$$

\[ \times \int_0^1 dx \int_0^{1-x} dy B^\beta \cos^2 \theta M^2 (k_E^2 + (p_E^2 + m^2)x + M^2y - p_{E\cdot x}^2)^3 , \]

with $Q_1 = m^2 x + M^2 y$ and $Q_2 = m^2 x^2 + M^2 y$. The difference is a finite term

$$\Sigma_1(0) - \Sigma_1(m) = \frac{igm^2 \epsilon^2}{8\pi^3} \int_0^1 dx \int_0^{1-x} dy$$

\[ \times d\Omega M^2 \cos^2 \theta \ln \left( \frac{Q_2}{Q_1} \right) , \]

By considering

$$\ln \left( \frac{Q_2}{Q_1} \right) = \ln \left( 1 - \frac{m^2 x(1-x)}{M^2 y + m^2 x} \right)$$

\[ \approx - \frac{m^2 x(1-x)}{M^2 y + m^2 x} , \]

to the lowest order integrating in $x$ and $y$ we find

$$\Sigma_{reg}(0) - \Sigma_{reg}(m) = \frac{gm^2 \epsilon^2 \gamma^2 \epsilon \gamma^5}{8\pi}$$

\[ \times (3 - 12\ln(2) + 2\ln(4\epsilon^2 m^2 n^2_{E,E})) , \]

where $\alpha = \epsilon^2 / 4\pi$. From both integrals [36] and [37] we can find the derivative contribution of [31]

$$\frac{\partial \Sigma_{reg}(p)}{\partial \phi} \bigg|_{\phi=m} = \left( \epsilon_{\mu\alpha\beta\nu} \gamma^\mu \gamma^\beta \gamma^\nu n^\alpha \right)$$

\[ \times \left( \frac{\partial \Sigma_1(\phi)}{\partial \phi} + \frac{\partial \Sigma_2(\phi)}{\partial \phi} \right) \bigg|_{\phi=m} . \]

We find

$$m \frac{\partial \Sigma_1(p)}{\partial \phi} \bigg|_{\phi=m} = - \frac{4ig\epsilon^2 m n^2_{E,E}}{(2\pi)^d} \int_0^1 dx \int_0^{1-x} dy$$

\[ \times d\Omega M^2 \cos^2 \theta \int d|k_E||k_E|^3(-2m)x(1-x) \]

\[ \frac{(k_E^2 + M^2y + m^2 x^2)^3}{(k_E^2 + 2m^2 y + m^2 x^2)^3} , \]

$\Sigma_1(0) - \Sigma_1(m) = - \frac{4ig\epsilon^2 m n^2_{E,E}}{(2\pi)^d} \int_0^1 dx \int_0^{1-x} dy$

\[ \times \int d\Omega M^2 \cos^2 \theta \int d|k_E||k_E|^3(-2m)x \]

\[ \frac{1}{(k_E^2 + Q_1)^3} - \frac{1}{(k_E^2 + Q_2)^3} , \]

Solving both integrals we have

$$m \frac{\partial \Sigma_1(p)}{\partial \phi} \bigg|_{\phi=m} = - \frac{5ig\epsilon^2 m^2 n^2_{E,E}}{216\pi^2} , \]

and

$$m \frac{\partial \Sigma_2(p)}{\partial \phi} \bigg|_{\phi=m} = \frac{3ig\epsilon^2 m^2 n^2_{E,E}}{32\pi^2} . \]
Reorganizing all the contributions the dominant contribution comes from

\[ \Sigma_R(0) = \frac{g m^2 n^2 \alpha \ln(2g m)}{2 \pi} \eta R. \]  

(50)

From the bound \( \Sigma_R(0) < 10^{-31} \text{ GeV} \), due to a torsion pendulum experiment \( \xi < 2 \times 10^{-3} \).

IV. CONCLUSIONS

Effective field theory provides a very powerful framework in order to search for new physics arising at the Planck scales. One effective model is the Myers and Pospelov model introducing possible quantum gravity effects through dimension five operators. It introduces a preferred four-vector responsible to break the Lorentz symmetry and to define the higher-order character of the model. For a general background the model can lead to ghost states associated to a negative metric that may affect in a nontrivial way the analytic structure of Feynman diagrams. We have investigated the role of these higher-order degrees of freedom in the radiative corrections of the Myers and Pospelov QED. For this we have computed the radiatively induced axial term using dimensional regularization and we have found large Lorentz-invariance violations. However, we have shown that following the standard renormalization procedure, that is applying minimal subtraction and defining the usual renormalization points, leads to acceptable small Lorentz-invariance violation corrections.

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