Magnon-mediated NMR quantum gates in a 1-D antiferromagnet

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We propose a method of controlling a quantum logic gate in a solid-state NMR quantum computer. A switchable inter-qubit coupling can be generated by using the longitudinal component of the Suhl-Nakamura interaction induced by a local singlet-triplet excitation in a 1-D antiferromagnet with a spin gap.

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An implementation of a quantum computer (QC) has been a chief research target in many fields of the materials science. The concept of the QC is quite general, which allows us to suppose many quantum systems to be potential candidates for the QC. Among them, a nuclear spin system is regarded as one of the most promising candidates because of a good isolation from the environment and an excellent controllability by the well-established technique of nuclear magnetic resonance (NMR). Actually, the first 2-qubit QC’s were implemented by solution NMR [1, 2], which proved a great promise of the NMR-QC. It is unfortunate, however, that the solution QC has a difficulty in its scalability, because the number of available qubits is limited by the number of nuclei in one molecule. In order to increase the number of qubits systematically, a solid-state (crystal) NMR-QC using a magnetic field gradient was proposed [3]. Although there are some practical issues in the original proposal [3], its scalability is attractive to realize the multi-qubit QC. It is our purpose to pursue the possibility of the solid-state NMR-QC by introducing new concepts into it.

One of the crucial processes in the QC implementation is to provide a controlled-NOT (CN) gate. The CN gate is a two-qubit operation in which a target qubit changes its logic according to the state of the control qubit. It holds the essential part of the quantum computation, where a quantum entanglement plays a crucial role. In the NMR-QC, the entanglement is provided by an inter-nuclear coupling so that it is the key issue to provide the appropriate inter-nuclear coupling. In fact, the characteristics of the coupling considerably affect the fundamental properties of the QC, such as a design and a performance. In particular, we find that a direct nuclear dipole coupling, which is regarded as a prospective inter-qubit coupling [3], poses serious problems on the QC properties, as shown below.

From the practical point of view, the following two properties are the crucial requisites for the inter-qubit coupling. One is to be capable of switching. The coupling needs to be strong enough to finish the logic gate in an appropriate time scale, but to be removable whenever unnecessary. The nuclear dipole coupling is fortunate in that it is available whenever nuclear spins are put close to each other, while unfortunate in that the decouplings for a huge number of inter-nuclear couplings are formidable. It is desirable that the coupling be switched-on only when necessary. The other property is to reach rather long range so that qubits in the field gradient can be distinguished from each other in the frequency domain. The larger inter-qubit distance in the real space makes the frequency difference between adjacent NMR lines wider in the frequency domain, so that the constraint on the field gradient can be relieved by using a long-range coupling. Unfortunately, the nuclear dipole coupling reaches at most a few lattice points [4], which motivates us to invoke some long-range indirect interactions via electrons.

In this letter, we present a switchable inter-qubit coupling realized by the Suhl-Nakamura (SN) interaction mediated by magnons [5, 6] in a 1-D antiferromagnet with a spin gap. The coupling is switched on selectively by local magnon excitations across the spin gap, which works as a switch for the CN gate. The proposed system is intuitive in that the computation starts in a silent environment rather than the turbulence of interactions, which simplifies the designs of the logic gates. Moreover, since the SN interaction reaches rather long distance, one can take the inter-qubit distance longer and relieve the constraint on the field gradient significantly.

The main idea of the gate switching is illustrated in Fig. 1. Suppose a device with an antiferromagnetic electron spin chain placed in a magnetic field gradient produced by a micromagnet fabricated outside the spin chain. The spin chain is supposed to have a singlet ground state (| ssz = 1/2⟩) with a finite gap to the lowest triplet branch of the magnon modes because of a quantum many body effect. Examples can be found in spin ladder and Haldane systems. One could also use dimer and spin-Peierls systems. Suppose that nuclear spins (I = 1/2) serving as qubits can be placed periodically, e.g., every 10a (a: lattice spacing), with the rest of the sites be occupied with I = 0 nuclei. The field gradient enables us to access each qubit individually by tuning the NMR frequency. The rather long distance between qubits is effective both to diminish the nuclear dipole couplings between qubits and to distinguish each qubit in the frequency domain.

Since there are no unpaired electron spins in the ground state, the interactions between qubits are absent.
and 1/2 (= qubits) nuclei are shown by circles and arrows, respectively. (a) Before the microwave irradiation, all the inter-nuclear couplings are switched off. (b), (c) A magnon packet (hatched part) is excited between the control and the target qubits, which creates the entanglement between them. The additional magnetic field produced by the packet at the target qubit depends on the state of the control qubit \( (h = h_{tr} \pm h_{SN}) \) corresponding to the NMR frequencies of \( \omega_k = \gamma_n(H_0 + h_{tr} \pm h_{SN}) \). The rf field with the frequency \( \omega_\pm \) can rotate the target spin in the case of (b), but not in (c). The states of (b) and (c) are superposed in the actual computation.

at low enough \( T \), which means that each qubit is isolated from other qubits as well as the environment (= electron spin system). Then, the inter-qubit coupling needed for the CN-gate is switched on through the SN interaction by the \( k = 0 \) triplet magnons, which are created locally by the singlet-triplet excitation across the spin gap. Although the excitation is primarily forbidden for the usual electric dipolar transition, it often becomes possible in the actual systems because of some higher order effects.

The position of the excited magnons along the chain can be specified in the field gradient. The energy diagrams of the \( k = 0 \) magnon excitations in the magnetic field are shown in Fig. 1. The magnon excitation energy to \( |1-1\rangle \) are different position by position along the chain because of the field gradient, which provides us with a spatial resolution of the magnon excitation region. The microwave frequency is adjusted so as to specify the pair of qubits to be coupled. The process creates a packet of the superposition of \(|00\rangle \) and \(|1-1\rangle \), corresponding to a soliton-like magnon excitation with \( k \sim 0 \).

The packet is localized on the chain due, once again, to the magnetic field gradient. A mismatch in the magnon excitation energies between adjacent regions along the chain prohibits the packet from moving to the lower field region. On the other hand, the continuum excitations near the one-magnon excitations at \( k=0 \) is absent, so that it is difficult for the packet to move to the higher field region unless the process of the energy release by phonon emissions is considerable. Consequently, the magnons are confined in the region where they are excited, and the SN interaction is produced only between the qubit pair of interest.

The triplet states make an additional field \( (h_{tr}) \) at the target qubit, which should be distinguished from that from the control qubit via the SN interaction \( (h_{SN}) \). \( h_{tr} \) can be measured by observing the NMR frequency shift of the target qubit during the microwave irradiation, while saturating the control qubit by applying the rf field continuously. The saturation of the control qubit results in \( h_{SN} = 0 \) at the target qubit site, so that the observed shift directly corresponds to \( h_{tr} \).

The SN coupling between the two qubits \( H_{SN} \) is given by \( H_{SN} = W_{ij} I_i \gamma \gamma_j \) with,

\[
W_{ij} = \frac{\gamma_n A}{N} \sum_{k \neq k'} (\epsilon_{k'} - \epsilon_k) \cos((k - k')r_{ij}).
\]

Here, \( n_k \) and \( \epsilon_k \) are, respectively, the population and the energy of the magnon with the wave number \( k \), and \( r_{ij} \) is the distance between the two qubits of interest. \( A \) and \( N \) are the hyperfine coupling constant and the number of sites (including the \( I = 0 \) sites) inside the packet, respectively.

Suppose the magnon dispersion of the spin ladder, as an example, in the form, \( \epsilon(k_n) = C + J_i (j_1 - j_1^2/4) \cos(k_n) + ... \), where \( k_n = 2\pi n / N, j_1 = J_1 / J \) with \( J_1 \) and \( J \) being the intra- and inter-chain exchange interactions, and \( C \) is the part independent of \( k_n \). Recalling that the microwave irradiation excites only the \( k = 0 \) magnons, i.e., \( n(k) = 0 \) for \( k \neq 0 \), the range function \( W_{ij} \)
can be calculated as,

\[ W_{ij} = \left( \frac{\gamma_n A_{ij}}{N} \right)^2 \sum_{n=1}^{N} \frac{2n(0)}{\epsilon(k_n) - \epsilon(0)} \cos(k_n r_{ij}) \]

\[ = \frac{2\gamma_n^2 A_{ij}^2}{J(j_1 - \frac{1}{2}j_1^2)N} \sum_{n=1}^{N} \frac{\cos(k_n r_{ij})}{\cos(k_n) - 1} \]  

(2)

Assuming \( N = 20, r_{ij} = 10\alpha, A_{ij} = 100 \text{kOe/}\mu_B, \gamma_n/(2\pi) = 4.3 \text{MHz/kOe} \) (\( ^1H \) as an example), \( J = 50 \text{ K}, j_2 = 0.2 \) and \( n(0)/N = 0.01 \), one obtains \( W_{ij} = 15 \text{ kHz} \), which is the same order of magnitude as the nuclear dipole coupling acting between nuclei 3 A apart from each other [1, 2], and one to three orders of magnitude greater than the J-couplings used in the solution NMR-QC’s [3, 4]. Note that \( W_{ij} \) can be controlled by the microwave intensity \( n(0) \), i.e., \( n(0) \) is determined by the balance between excitation and relaxation; \( dn(0)/dt = W_{ex} - n(0)/T_s \), with \( W_{ex} \) being the transition probability per unit time and \( T_s \) the magnon lifetime. In the steady state, \( dn(0)/dt = 0 \), so that \( n(0) = W_{ex}T_s \).

The CN gate is achieved by an NMR technique shown in Fig. 3. After the gate operation, the coupling is switched-off by shutting-off the microwave irradiation. The triplet state relaxes to the singlet ground state by the time scale of \( T_s \). Through these processes, the microwave irradiation works as a switch for the CN gate.

In summary, we have described the magnon-mediated quantum gate in a 1-D antiferromagnet. The switching capability of the gate is useful in designing the logic gate of the solid-state NMR-QC, whereas the long-range nature of the coupling significantly relieves the constraint on the field gradient required to distinguish each qubits. We are indebted to M. Kitagawa for fruitful discussion. This work has been supported by CREST of JST (Japan Science and Technology Corporation).

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