Quantum-Mechanical Interference over Macroscopic Distances in the $B^0 \bar{B}^0$ System

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Abstract

We argue that the $B^0 \bar{B}^0$ state generated in the decay of $\Upsilon(4S)$ is well suited for performing tests of Einstein–Podolsky–Rosen correlations, i.e., quantum-mechanical interference effects over macroscopic distances. Using measurements of the ratio $R = (#$ like-sign dilepton events)/(# opposite-sign dilepton events) and of the $B_H - B_L$ mass difference we show that already presently existing data strongly favour the contribution of the interference term to $R$, as it is required by the rules of quantum mechanics.

PACS: 13.25.Hw, 14.40.Nd, 03.65.Bz
Keywords: $B^0 \bar{B}^0$ system, dilepton events, entangled state, EPR-correlations, decoherence parameter
1 Introduction

Tests of quantum mechanics are of increasing interest in recent years, in particular, the optical tests of quantum mechanics carried out on systems of two correlated photons. Such systems—showing Einstein–Podolsky–Rosen (EPR) correlations—are suitable to discriminate between quantum mechanics and any local realistic (hidden variable) theory via Bell inequalities [1] (see, e.g., Ref. [2] for a quick introduction into the field). All recent experiments using laser beams confirm quantum mechanics in an impressive way (see, e.g., Refs. [3, 4]) and they teach us that under certain circumstances quantum systems extend over macroscopic scales.

We find it interesting and desireable to perform tests of EPR correlations also with massive particles. Analogously to the entangled photons one can create at factories states of EPR-correlated $B^0\bar{B}^0$ pairs as decay products of the upsilon $\Upsilon(4S)$ resonance (see, e.g., Refs. [5, 6]). More precisely, $B^0_d\bar{B}^0_d$ pairs are produced since $\Upsilon(4S)$ is not heavy enough to decay into $B^0_s\bar{B}^0_s$. We drop the index $d$ for convenience.

$B$ mesons have a lifetime of the order of a picosecond. If a $B^0\bar{B}^0$ pair is produced by the decay of $\Upsilon(4S)$ there is very little kinetic energy left per $B$ meson, namely roughly 10 MeV. Multiplying the corresponding velocity $v$ of such a $B$ meson by its lifetime one obtains $v\tau_{B^0} \approx 3 \times 10^{-2}$ mm. This shows that in average the separation of the decaying $B$ mesons originating in $\Upsilon(4S)$ is macroscopic. The $B^0\bar{B}^0$ system as the decay product of $\Upsilon(4S)$ is a superposition of two states because the $B^0\bar{B}^0$ state inherits the charge conjugation quantum number $C = -1$ of the $\Upsilon(4S)$. This system offers therefore the possibility to test, within particle physics, quantum-mechanical interference over macroscopic distances. Similar tests involving two-kaon systems have been proposed in the past in Refs. [7, 8] and recently for DaΦne in Ref. [9].

To realize the above idea we consider the ratio $R = (#$ like-sign dilepton events)/(# opposite sign-dilepton events) of lepton pairs generated in the decay chain $\Upsilon(4S) \to B^0\bar{B}^0 \to \ell^+\ell^- +$ anything. In order to discriminate between quantum mechanics and local realistic theories we introduce a decoherence parameter $\zeta [9]$ such that the interference term present in the quantum-mechanical calculation of $R$ is multiplied by a factor $1 - \zeta$ where $\zeta$ parameterizes deviations from quantum mechanics. $\zeta$ is called decoherence parameter because at $\zeta = 1$ the interference is totally gone. It turns out that, including this modification, $R$ is a function of $\Delta m/\Gamma$, $\Delta\Gamma/2\Gamma$ and $\zeta$ with $\Delta m$, $\Delta\Gamma$ and $\Gamma$ being mass difference, decay width difference and average decay width, respectively, of the heavy and light neutral $B$ mass eigenstates. There is also a parameter involved characterizing CP violation in $B^0\bar{B}^0$ mixing. We will argue below that this parameter can be set equal to one (no CP violation) and that taking $\Delta\Gamma = 0$ is sufficient for our purpose. The main idea of this paper is to compare the experimental value $R_{\text{exp}}$ of $R$ measured at $\Upsilon(4S)$ with the theoretical expression $R(\Delta m/\Gamma, \zeta)$. Taking $\Delta m$ from independent experiments which study the time dependence of $B^0\bar{B}^0$ mixing and thus interference effects of single $B$ states, the relation $R_{\text{exp}} = R(\Delta m/\Gamma, \zeta)$ allows us to obtain information on $\zeta$ and thus to test the long-range interference effects of quantum mechanics in the $B^0\bar{B}^0$ system.
2 The $B^0 \bar{B}^0$ system

To begin with we discuss the quantum mechanics of the $B^0 \bar{B}^0$ system. Phenomenologically, there are the two independent amplitudes

$$\mathcal{A}(B^0 \to f) \equiv A \quad \text{and} \quad \mathcal{A}(\bar{B}^0 \to f) \equiv B$$

which enter into the description of the decays of the neutral $B$ mesons into an arbitrary final state $f$. The mass eigenstates of the neutral $B$ mesons are given by

$$|B_H\rangle = p|B^0\rangle + q|\bar{B}^0\rangle,$$
$$|B_L\rangle = p|B^0\rangle - q|\bar{B}^0\rangle,$$

with $|p|^2 + |q|^2 = 1$ and

$$\frac{q}{p} = \frac{\Delta m - \frac{i}{2} \Delta \Gamma}{2(M_{12} - \frac{i}{2} \Gamma_{12})} = \frac{2(M_{12}^* - \frac{i}{2} \Gamma_{12}^*)}{\Delta m - \frac{i}{2} \Delta \Gamma} = \sqrt{\frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}}},$$

where $\Delta m = m_H - m_L > 0$ ($H$=heavy, $L$=light), $\Delta \Gamma = \Gamma_H - \Gamma_L$ and $M_{12} - \frac{i}{2} \Gamma_{12}$ is the off-diagonal matrix element in the effective time evolution in the $B^0 \bar{B}^0$ space. The positivity of $\Delta m$ fixes the sign of the square root in Eq. (3). The $B^0 \bar{B}^0$ pair produced in the decay of $\Upsilon(4S)$ is in the state

$$\Psi(t = 0) = \frac{1}{\sqrt{2}} \left( |B^0\rangle \otimes |\bar{B}^0\rangle - |\bar{B}^0\rangle \otimes |B^0\rangle \right)$$

with charge conjugation quantum number $C = -1$ because the $\Upsilon(4S)$ has quantum numbers $J^{CP} = 1^{--}$ and its decay into $B^0 \bar{B}^0$ proceeds via strong interactions. The subsequent time evolution of (4) is given by

$$|B^0(t)\rangle = g_+(t)|B^0\rangle + \frac{q}{p} g_-(t)|\bar{B}^0\rangle,$$
$$|\bar{B}^0(t)\rangle = \frac{p}{q} g_-(t)|B^0\rangle + g_+(t)|\bar{B}^0\rangle$$

with

$$g_\pm(t) = \frac{1}{2} e^{-i(m \mp \frac{1}{2} \Gamma)t} \left[ e^{-\frac{i}{2}(\Delta m - \frac{i}{2} \Delta \Gamma)t} \pm e^{\frac{i}{2}(\Delta m - \frac{i}{2} \Delta \Gamma)t} \right]$$

and

$$m = \frac{1}{2}(m_H + m_L), \quad \Gamma = \frac{1}{2} (\Gamma_H + \Gamma_L).$$

After having introduced the basic formalism we now come to the point where we modify the result of ordinary quantum mechanics and subject this modification to a comparison with experimental results. The class of observables we are interested in is the
probability that $\Psi$ decays into final states $f_1$ and $f_2$ with momenta $\vec{p}$ and $-\vec{p}$, respectively, in its restframe. This probability is calculated by the integral \[11\]

Furthermore, the relation

The last term in Eq. (8) is the usual quantum-mechanical interference term as it results from the two summands of the wave function \[12\] modified by a factor $1 - \zeta$. In the following we will rather arbitrarily assume that $0 \leq \zeta \leq 1$ to incorporate quantum mechanics with $\zeta = 0$ at one end of the interval and no interference corresponding to $\zeta = 1$ at the other end. Our aim is to test which range of $\zeta$ is experimentally allowed if we use information on semileptonic decays of the $B^0\bar{B}^0$ system. To apply Eq. (8) we have to perform the integrals and we arrive at the general formula

with

and $x$ and $y$ are defined as

Furthermore, the relation $I_{++} = (I_{+-})^*$ is valid.
In principle, measurements of $N(f_1, f_2)$ for any $f_1$, $f_2$ could be used to obtain information on $x$ (see Ref. [12]), $y$ and $\zeta$. In this case one would have to know the quantities $|A_1 B_2 - B_1 A_2|$, $|\frac{p}{q} A_1 A_2 - \frac{q}{p} B_1 B_2|$, etc. which, in general, require additional experimental information. However, for semileptonic decays the situation is very simple because in lowest order in weak interactions only the tree-level $W$ exchange graphs are responsible for such decays. In addition, since the quark content of $B^0 \bar{B}^0$ is given by $B^0 = (b \bar{d})$ and $\bar{B}^0 = (\bar{b} d)$ the lepton $\ell^+$ in the final state tags $B^0$ whereas $\ell^-$ tags $\bar{B}^0$. Therefore, with $f_+ \equiv X \ell^+ \nu_\ell$ and $f_- \equiv \bar{X} \ell^- \bar{\nu}_\ell$ and the labels $+, -$ pertaining to $f_+, f_-$, respectively, we have

$$|A_+| = |B_-| \quad \text{and} \quad B_+ = A_-= 0.$$  \hspace{1cm} (13)

In these final states $X$ denotes an arbitrary kinematically allowed hadronic state and $\bar{X}$ its charge-conjugate counterpart. Defining $N_{++} \equiv N(f_+, f_+)$, etc., and using Eq. (13), we obtain the following very simple expression for $N(f_1, f_2)$, Eq. (9), in the case of semileptonic decays:

$$N_{++} = \frac{1}{2} |A_+|^2 \frac{|p|^2}{|q|} (I_2 + 2\zeta |I_{+-}|^2), \quad (14)$$

$$N_{--} = \frac{1}{2} |B_-|^2 \frac{|q|^2}{|p|} (I_2 + 2\zeta |I_{+-}|^2), \quad (15)$$

$$N_{+-} = N_{-+} = \frac{1}{2} |A_+|^2 |B_-|^2 (I_1 + 2\zeta \Re (I_{+-})^2). \quad (16)$$

Defining the ratio of like-sign dilepton events to opposite-sign dilepton events [13, 14]

$$R \equiv \frac{N_{++} + N_{--}}{N_{+-} + N_{-+}}$$  \hspace{1cm} (17)

the amplitudes cancel and we find $R$ as a function of $|p/q|$, $x$, $y$ and $\zeta$:

$$R = \frac{1}{2} \left( \frac{|p|^2}{|q|^2} + \frac{|q|^2}{|p|^2} \right) \frac{x^2 + y^2 + \zeta \left[ y^2 \frac{1+x^2}{1-y^2} + x^2 \frac{1-y^2}{1+x^2} \right]}{2 + x^2 - y^2 + \zeta \left[ y^2 \frac{1+x^2}{1-y^2} - x^2 \frac{1-y^2}{1+x^2} \right]}.$$  \hspace{1cm} (18)

It is well known that a deviation of $|p/q|$ from 1 is a signal for CP violation in $B^0 \bar{B}^0$ mixing. A suitable measure for $|p/q|$ and CP violation in mixing is thus given by [14]

$$A_{CP} \equiv \frac{N_{++} - N_{--}}{N_{++} + N_{--}} = \frac{|\frac{p}{q}|^2 - |\frac{q}{p}|^2}{|\frac{p}{q}|^2 + |\frac{q}{p}|^2}.$$  \hspace{1cm} (19)

To derive this formula, Eqs. (14), (15) and (16) have been used which correspond to odd relative angular momentum of the $B^0 \bar{B}^0$ pair. It is easy to show with the methods expounded here that the same formula (19) is valid for even relative angular momentum. Moreover, Eq. (13) is also valid for any statistical mixture of odd and even [15] and does not depend on the parameter $\zeta$ which could even be different for odd and even. This
shows that it is consistent to take any measurement of $A_{CP}$ and use it as information on $|p/q|$ in $R$ \([18]\). A recent measurement of the CDF Collaboration \([10]\) gives $A_{CP} = (2.4 \pm 6.3 \text{ (stat)} \pm 3.3 \text{ (sys)}) \times 10^{-2}$. The factor in front of $R$ which depends on $|p/q|$ is expressed by $A_{CP}$ as

$$\frac{1}{2} \left( \left| \frac{p}{q} \right|^2 + \left| \frac{q}{p} \right|^2 \right) = (1 - A_{CP}^2)^{-1/2} \approx 1 + \frac{1}{2} A_{CP}^2.$$ \hspace{1cm} (20)

With the above value of $A_{CP}$ the quantity \((2)\) differs less than a percent from 1. In view of the experimental errors associated with $R$ and $x$ we will simply set \((20)\) equal to 1 in the rest of this paper.

3 Discussion of the experimental data

Having disposed of $|p/q|$, there remain three variables in $R$, namely $x = \Delta m/\Gamma$, $y$ and $\zeta$. To test the quantum-mechanical interference term, i.e., to get information on $\zeta$, we want to take $x$ from measurements of the time dependence of $B^0\bar{B}^0$ mixing \([17, 18, 19, 20]\) and compare $R$ with $R_{\text{exp}}$ measured at the $\Upsilon(4S)$ \([21, 22]\). In the concrete, we apply the following procedure. We take the values of $\Delta m$ from the results of the LEP experiments ALEPH \([17]\), DELPHI \([18]\), L3 \([19]\) and OPAL \([20]\) which are $\Delta m = 0.436 \pm 0.033 \text{ h/ps}$, $\Delta m = 0.531 \pm 0.046 \pm 0.078 \text{ h/ps}$, $\Delta m = 0.496 \pm 0.050 \pm 0.051 \pm 0.043 \text{ h/ps}$ and $\Delta m = 0.548 \pm 0.050 \pm 0.023 \pm 0.019 \text{ h/ps}$, respectively. The first error is the statistical and the second one the systematic. For each experiment, we simply add the squares of the statistical and systematic error (we select the larger value where positive and negative errors are different) and use the law of combination of errors to get the combined value of $\Delta m$. After division by $\tau_{B^0} = (1.56\pm0.06) \text{ ps} \ [23]$ we arrive at the final value $\bar{x} = 0.74\pm0.05$ which will be used in the figures. As for $R$ we take the experimental input $R_{\text{exp}} = 0.194 \pm 0.062 \pm 0.054$ obtained by ARGUS \([21]\) and $R_{\text{exp}} = 0.187 \pm 0.022 \pm 0.025 \pm 0.040 \pm 0.030$, the result of the CLEO Collaboration \([22]\), where the third error reflects a $\pm 15 \%$ uncertainty in the assumption that charged and neutral $B$ pairs contribute equally to dilepton events. Performing the same steps as for $\Delta m$ we obtain $R_{\text{exp}} = 0.189 \pm 0.044$.

It remains to discuss $y$ in the context of the determination of the decoherence parameter. The Standard Model predicts a very small difference between the lifetimes of the heavy and the light neutral $B$ meson such that $|y|/x \lesssim 10^{-2}$ (see, e.g., Ref. \([24]\)). This alone would already constitute a strong motivation for putting $y = 0$ in $R$ \([18]\). Furthermore, plotting $R$ as a function of $y$ and $\zeta$ numerically reveals that with increasing $y^2$ the restriction on $\zeta$ gets stronger. Therefore, for our purpose of getting...
information on the quantum-mechanical interference term it is sufficient to study $R$ as a function of $\zeta$ with $y = 0$.

This is done in Figs. 1 and 2. The three curves in Fig. 1 correspond to $R$ with $y = 0$ and the three $x$ values $\bar{x} - \Delta \bar{x}$ (lower curve), $\bar{x}$ (middle curve) and $\bar{x} + \Delta \bar{x}$ (upper curve). The horizontal lines indicate the mean value $\bar{R}_{\text{exp}}$ and $\bar{R}_{\text{exp}} \pm \Delta \bar{R}_{\text{exp}}$. In Fig. 2 we have again plotted $R$ and $R_{\text{exp}}$ but the error bands correspond to 1.64 standard deviations or 90 % CL if the distributions are Gaussian.

As a side-remark we want to stress that the method discussed here can also be used to get a bound on $|y|$. For simplicity we assume quantum mechanics to be valid ($\zeta = 0$) and compare $R$ as a function of $y$ with $R_{\text{exp}}$. Then we obtain $|y| \leq 0.40$ at 90 % CL. This shows that $y = \Delta \Gamma/2\Gamma$ and thus the difference in the decay widths of the heavy and light neutral $B$ mesons is only mildly restricted by present data. There is still a large gap between experimental information and the Standard Model prediction for $y$.

4 Conclusions

We observe that the overlap of the allowed areas of $R$ and $R_{\text{exp}}$ restricts the decoherence parameter to $\zeta \leq 0.26$ in Fig. 1 and $\zeta \leq 0.53$ in Fig. 2. This result conforms nicely with quantum mechanics and leaves little room for local realistic theories ($\zeta = 1$). Of course, our statistical analysis is rather crude and the experimental errors of $x$ and $R_{\text{exp}}$ are large. Nevertheless, there is a clear sign of long-range interference effects in $\bar{B}^0\bar{B}^0$ in agreement with quantum mechanics. This is not so surprising in view of the overwhelming success of quantum mechanics. We expect that with the improvement of the experimental errors the bound on $\zeta$ will become much tighter in the future. However, we also notice that the mean value of $R$ at $\zeta = 0$ is slightly higher than the mean value of $R_{\text{exp}}$. This acts in favour of smaller bounds on $\zeta$ (see figures) and has to be kept in mind when considering their above numerical values. Adding a note of scepticism concerning tests like the one discussed here, we want to remark that changing quantum mechanics in one point, in the present case in the two-particle interference term, but assuming its validity in all other domains, e.g., one-particle interference terms from which $\Delta m$ is extracted, is an arbitrary procedure. However, since no consistent local theory encompassing quantum mechanics is known, all parameterizations of deviations from quantum mechanics involve a certain amount of arbitrariness.

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Figure 1: $R$ as a function of the decoherence parameter $\zeta$ for $x = \bar{x}, \bar{x} \pm \Delta \bar{x}$. The horizontal lines indicate the mean value $\bar{R}_{\text{exp}}$ and $\bar{R}_{\text{exp}} \pm \Delta \bar{R}_{\text{exp}}$ of the experimental measurement of $R$. The shaded region shows the overlap of the two error bands.

Figure 2: The same as in Fig. 1 but the error bands correspond to $1.64 \sigma$ or 90 % CL.