Error estimation of discrete formation method for optimal FTN signals

A S Ovsyannikova¹, I I Lavrenyuk¹, S B Makarov¹ and X Wei²

¹Peter the Great St. Petersburg Polytechnic University, 29 Polytechnicheskaya, St. Petersburg, 195251, Russia
²Harbin Engineering University, Harbin, China

E-mail: anyy-ov97@mail.ru

Abstract. One of the branches of Faster than Nyquist (FTN) signaling providing transmission at the rate higher than “Nyquist limit” with insignificant energy losses includes optimal FTN pulses. These pulses have controllable level of intersymbol interference (ISI) and given time and spectral parameters. The efficiency of practical usage of these pulses depends on the errors of discrete formation method. In this work a guidance for estimating the minimum number of samples needed to meet the requirements for spectral and energy characteristics of optimal FTN signals is proposed. Simulation modeling and experiments with the help of vector signal generator have been held. It was found that to form optimal FTN pulse with duration $T_s=2T$ not more than 16 samples are needed. In this case, energy losses relatively to continuous optimal FTN pulse are about 0.2 dB.

1. Introduction
Faster than Nyquist (FTN) signaling provides binary symbols transmission without encoding at the rate $R$ higher than “Nyquist limit” almost without bit-error rate (BER) performance degradation [1-3]. In this case additive white Gaussian noise (AWGN) with average power spectral density $N_0/2$ is used as a transmission channel. Optimal FTN pulses $s_{opt}(t)$ [4, 5] allow to increase symbol rate of transmission with minimum energy losses. The feature of such signals is a controllable level of intersymbol interference (ISI). Due to their application symbol rate $R$ may be increased by several times while energy losses do not exceed 0.5 dB [4] for the case of coherent summarized detection of symbol sequence.

FTN signals formation methods are developing in two directions. Transmitted pulses with duration $T$ are transformed into sequence of symbols with uncontrollable ISI [1]. The requirements for limitation of occupied frequency bandwidth $\Delta F$ and the level of out-of-band emissions are met via linear filtration. However, there is no possibility to set up time parameters of random FTN signals such as given peak-to-average power ratio (PAPR) or minimum energy losses even for complex demodulation algorithms. Limiting factor is increase in complexity of formation algorithms (algorithmic complexity of implementing high-order filter) and detection of such signals when demands for reduction rate of out-of-band emissions become greater.

Optimal FTN pulses [4-6] have given time (pulse duration $T_s \geq T$, value of PAPR) and spectral parameters (occupied frequency bandwidth $\Delta F$, reduction rate of out-of-band emissions $d$). To determine primary characteristics of energy spectrum $|S(f)|^2$ of random sequence of pulses $s_{opt}(t)$ with duration $T_s$ it is necessary to solve optimization problem according to the one or another optimality
criterion [4-10]. Optimal FTN pulses are formed with the help of non-recursive digital (discrete) filters which analogue impulse response is \( h(t) = s_{opt}(T_s-t) \).

In this work it is proposed to consider errors of discrete formation method for optimal FTN signals with duration \( T_s \geq T \), to develop a guidance for estimating the minimum number of samples needed to meet the requirements for spectral and energy characteristics of FTN signals.

2. Simulation model

Let us consider a random sequence of \( M \) optimal binary FTN pulses \( s_{opt}(t) \) with duration \( T_s = LT \) \((L=1,2,3\ldots)\) and energy \( E_{opt} \) (1):

\[
y(t) = \sqrt{E_{opt}/T} \sum_{n=-M/2}^{M/2-1} c_n s_{opt}(t-nT/R).
\]

A random sequence of optimal FTN pulses \( s_{opt}(t) \) with duration \( T_s = 2T \) transmitted at the rates \( R = 5/T \) (a) и \( R = 9.5/T \) (b) is shown in figure 1. Coefficients \( c_n \) (1) are chosen as \(+1, +1, -1, +1\). Bold solid line illustrate the shape of total sequence of optimal FTN pulses. If duration of \( s_{opt}(t) \) becomes longer, \( \Delta F \) reduces. However, it leads to significant ISI. Increasing \( R \) results in a greater number of adjacent optimal FTN pulses which influence on the current one.

![Figure 1. Random sequence (1) of optimal FTN pulses.](image)

The shapes of optimal FTN pulses \( s_{opt}(t) \) may be found as a solution to optimization problem [4-10]. The functional determines the problem of maximization of reduction rate of the level of energy spectrum of random sequence (1) with constraints on energy \( E_{opt} \), duration \( T_s \) and correlation coefficient \( K_0 \) or the Euclidean distance for finite sequence of pulses [4, 11]. The last two constraints take into account the level of ISI.

As it was said in [4-6, 8], the optimization problem may be reduced to the problem of searching for the minimum of the function of many variables. To carry out this conversion, Fourier coefficients of the even function \( s_{opt}(t) \) need to be found.

\[
s_{opt}(t) = \frac{s_0}{2} + \sum_{k=1}^{m-1} s_k \cos \left( \frac{2\pi}{T} kt \right).
\]

Limited Fourier series (2) includes \( m \) terms which define the accuracy of optimal function \( s_{opt}(t) \) representation. As shown in [8], the value of \( m \) for FTN pulses should lie within a range of \( 8 \ldots 15 \) to provide standard deviation of approximation (2) from true function \( s_{opt}(t) \) to be about 0.1 %. Then the problem of searching for the global minimum of the function of \( m \) variables may be written as follows:
\[
\min_{\{s_k\}_{k=0}^m} J(\{s_k\}_{k=0}^m), \quad J(\{s_k\}_{k=0}^m) = T_s/2 \sum_{k=0}^m (2\pi k/T_s)^2 s_k^2.
\] (3)

The block diagram of optimal FTN signal formation used in the simulation model is illustrated in figure 2. According to this diagram, the sequence of input symbols of channel alphabet (coefficients \(c_n\) in (1)) goes to the memory block and then goes to the digital delay line at the rate \(R\) determined by clock frequency \(f_T\). Each pulse \(s_{\text{opt}}(t)\) is described by \(N_{\text{samp}}\) samples but the number of taps is \(L\) times greater because of ISI (\(T_s = LT\); \(L=1,2,3\ldots\)). Therefore, the sample rate \(f_{\text{samp}}\) defined at clock interval is equal to \(N_{\text{samp}}/L\). Digital weighing coefficients \(g_i\) \((i=1,2,\ldots LN_{\text{samp}})\) are equal to sample values of \(s_{\text{opt}}(t)\).

![Block diagram of optimal FTN signal formation](image)

**Figure 2.** Block diagram of optimal FTN signal formation.

On the output of the adder step-changing voltage is formed. After wideband low-pass filtration it transforms to analogue shape. These voltage shape should correspond to the shape \(s_{\text{opt}}(t)\) of optimal FTN signal. Using this formation method gives errors which lead to degradation of energy spectrum \(|S(f)|^2\) and correlation properties of transmitted signals.

Thus, we need to develop the guidance for error estimation of formation method for optimal FTN signals. This guidance should take into account and estimate the next properties of block diagram of formation:

- the choice of the minimum number of samples \(N_{\text{samp}}\), which influences on the performance of formation instrument and the level of out-of-band emissions \(|S(f)|^2\);
- the choice of amplitude-frequency characteristic (AFC) of reconstruction low-pass filter (LPF) designed for reconstruction of analogue signal shape;
- the correlation between the necessary number of samples for different symbol rate \(R\).

Let us look at the errors of optimal FTN signal formation. Firstly we have to analyze the errors related to the overlay effect caused by discretization. We will take figure 3(a) as an example of \(s_{\text{opt}}(t)\) spectrum \(S(\omega)\) \((\omega = 2\pi f)\) and figure 3(b) as an example of amplitude-frequency characteristic \(Y_f(\omega)\) of reconstruction filter.

The absolute value of optimal FTN pulse spectrum in the necessary bandwidth \(\Delta F_n\) is constant and declines in occupied frequency bandwidth \(\Delta F_o\) in accordance to this expression:

\[
|S(\omega)| \leq |S(0)| \cdot (\pi \Delta F_o)^s / |\omega|^d,
\] (4)
AFC of reconstruction filter should provide constant transfer ratio and linear phase-frequency characteristic in the bandwidth $\Delta F_o$:

$$Y_r(\omega) = \exp(-j\Delta \omega t_r), \quad |\omega| \leq \pi \Delta F_o,$$  \hspace{1cm} (5)

where $t_r$ - filter delay, $\Delta F_o \geq \Delta F_w$. Let us set $|Y_r(\omega)| \leq B/|\omega|^\nu$, where $B$ is a constant value for $|\omega| \leq \pi \Delta F_o$, as a result we have:

$$|Y_r(\omega)| \leq (\pi \Delta F_o)^\nu / |\omega|^\nu, \quad |\omega| \geq \pi \Delta F_o.$$  \hspace{1cm} (6)

It is rational to estimate the difference between the formed spectrum $S(\omega)$ and required $S(\omega)$ one separately in occupied frequency bandwidth $|\omega| \leq \pi \Delta F_o$ and in the area $|\omega| > \pi \Delta F_o$. In this case the estimation of standard deviation (SD) $\delta^2$ of the spectrums which characterizes the difference between the shapes of formed and required signal essentially is reasonable. At the same time in the area $|\omega| > \pi \Delta F_o$ the spectrums should be compared by relative difference in their absolute values, because exactly this value defines the raise of the spectrum level caused by discretization. Such method of spectrum comparison in the area $|\omega| > \pi \Delta F_o$ is rational in terms of necessity of meeting the requirements for the emission level, e.g. in adjacent transmission channels of multichannel communication systems with frequency division multiplexing.

Taking into account (4), (5) and (6), we have the next SD in the area $|\omega| \leq \pi \Delta F_o$

$$\delta^2 = \int_0^{\pi \Delta F_o} |S(\omega)\exp(-j\omega t_r) - S(\omega)|^2 d\omega \int_0^{\pi \Delta F_o} |S(\omega)|^2 d\omega.$$  \hspace{1cm} (7)

The factor $\exp(-j\omega t_r)$ is needed to compensate the delay in reconstruction LPF while calculating $\delta^2$. After some conversions, (7) transforms into (8):

$$\delta^2 \leq \left( \Delta F_o \over \Delta F_w \right) \left[ \frac{1}{3} \left( 1 - \frac{\sin(\pi \Delta F_o \Delta t / 2)}{\pi \Delta F_o \Delta t / 2} \right)^2 + \frac{2}{d} \left( \Delta F_o \over \Delta F_w \right)^{2d} \right].$$  \hspace{1cm} (8)

In the area $|\omega| > \pi \Delta F_o$ distortions of the spectrum leading to significant raise of the level of $S(\omega)$ relatively to $|S(\omega)|$ become unwanted. For the case of applying reconstruction LPF which characteristic is determined by (6), spectrum suppression in the considered area becomes greater with higher frequencies. That is why it is enough to estimate the emission level just in the area $\pi \Delta F_o < \omega < (2\pi/\Delta t + \pi \Delta F_o)$. We will assume that the level of spectrum distortion is appropriate if $S(\omega)$ in the area $\pi \Delta F_o < \omega < (2\pi/\Delta t + \pi \Delta F_o)$ does not exceed the level of original spectrum $|S(\omega)|$ at the borders of the bandwidth $|\omega| = \pi \Delta F_o$. This condition may be represented as follows:

![Figure 3](image-url)
\[
\left(\frac{\pi \Delta F_o}{\omega} \right)^2 \left| S_o(\omega) \right| \sum_{l=-\infty}^{\infty} S \left( \omega - \frac{2\pi l}{\Delta t} \right) \leq \left| S(0) \right| \left( \Delta F_o \right)^2 \left( \Delta F_o \right)^2,
\]

where \( \psi_{\Delta t}(t) = 1 \) for \( 0 \leq t \leq \Delta t \) and \( \psi_{\Delta t}(t) = 0 \) for \( t < 0, t > \Delta t \). \( \Delta t = LT/N_{\text{samp}} \) – sample interval of optimal FTN pulse. \( S_o(\omega) \) - spectrum of \( \psi_{\Delta t}(t) \).

3. Results of simulation modelling and experiment

Let us consider the results of simulation modelling of optimal FTN pulses and estimate the errors of formation method based on the non-recursive discrete filter (figure 2) with the help of proposed guidance.

First of all, we estimate the errors of formation of optimal FTN pulses obtained as a result of solving optimization problem with constraint on coefficient of mutual correlation \( (K_0=0.01) \) for \( R = 2T \) [6]. In figure 4 the shapes of step-changing optimal FTN pulse \( s_{\text{opt}}(t) \) with duration \( T_s = 2T \) consisting of 16 samples (a) and normalized values of \( |S(\omega)| \) (b).

![Figure 4. Pulse shape (figure 2) (a) and spectrum \( |S(\omega)| \) (b) for \( N_{\text{samp}}=16 \).](image)

The first replications of the spectrum (on the right and on the left from the main spectrum) with maximum level -27 dB may be seen in figure 4(b). The presence of replicas is proved by experimental research (figure 5).

![Figure 5. Optimal FTN signal (a) and corresponding spectrum (b) of the signal.](image)

| Table 1. Comparison of results obtained during simulation and experiment |
|---------------------------------------------------------------|
| Simulation | Experiment |
| The maximum level of spectrum replicas | -27 dB | -25 dB |
| Frequency separation between spectrum replicas | 370 kHz | 368 kHz |
Experimental research was held with the use of vector signal generator Agilent E8267D PSG. The sequence of optimal FTN pulses was transmitted at the central frequency 415 MHz with binary phase-shift keying (BPSK). The results of simulation match those of experiment (table 1) and their difference in the maximum level of spectrum replicas is not more than 5%. Based on the calculations, simulation modelling and experimental researches it is enough to use relatively small number of samples when FTN signal is formed to meet the requirements for SD of absolute values of formed signal spectrums. Thus, to provide symbol rate \( R = 2/T \) we can use just 16 samples for doubled pulse duration \( (T_s = 2T) \).

Now we should estimate the errors of optimal FTN pulse formation when symbol rate \( R = 9.5/T \) and constraint on the Euclidean distance \( D_{\text{min}} = 0.99 \) are used to solve optimization problem for \( T_s = 2T \) (figure 1(b)). The values of SD of \( |\hat{S}(\omega)| \) from original analogue shape \( |S(\omega)| \) depending on \( N_{\text{samp}} \) may be found in table 2. SD does not exceed 58 % (theoretical estimation for \( N_{\text{samp}} = 10 \)) and 16 % (simulation for \( N_{\text{samp}} = 10 \)). As expected, increasing the number of samples up to \( N_{\text{samp}} = 100 \) gives smaller values of SD (2.9e−2 % in theory and 1.2e−3 % for the case of simulation).

| \( N_{\text{samp}} \) | SD (\( \delta^2 \) in (8)) | SD (simulation result) |
|-----------------|--------------------------|-------------------------|
| 100             | 2.98e-4                  | 1.26e-5                 |
| 80              | 7.14e-4                  | 3.22e-5                 |
| 40              | 10.52e-3                 | 5.30e-4                 |
| 20              | 12.41e-2                 | 0.77e-2                 |
| 16              | 24.08e-2                 | 2.03e-2                 |
| 10              | 5.82e-1                  | 1.64e-1                 |

The shape of \( s_{\text{opt}}(t) \) on the output of adder (figure 2) for symbol rate \( R=9.5/T \) (a) and dependency of the Euclidean distance on symbol rate for \( N_{\text{samp}} \approx 40 \) are shown in figure 6.

![Figure 6](image-url)  
Figure 6. Shape of \( s_{\text{opt}}(t) \) and the Euclidean distance vs. symbol rate.

It can be noticed from figure 6(b) that the Euclidean distance reduces to the value 0.98 that leads to energy losses about 0.2 dB. It is the minimum value which may be obtained for considered \( N_{\text{samp}} \) (table 2). Therefore energy losses relatively to the original pulse with \( R=9.5/T \) do not exceed 0.2 dB.

4. Conclusion
As a result of carried out researches it is shown that the errors of discrete formation method for optimal FTN signals cause degradation of energy spectrum of transmitted signal. Nevertheless, these errors has insignificant impact on the spectrum shape and time characteristics of signal. In particular, it was found that the minimum number of samples of FTN pulse with duration \( T_s = 2T \) is not more than 16. Then in the case of \( R=9.5/T \) energy losses do not exceed 0.2 dB.
Acknowledgments
The results of the work were obtained under the State contract № 8.2880.2017/ПЧ with Ministry of Education and Science of the Russian Federation and used computational resources of Peter the Great Saint-Petersburg Polytechnic University Supercomputing Center (http://www.scc.spbstu.ru).

References
[1] Anderson J B, Rusek F and Öwall V 2013 Faster-Than-Nyquist signaling Proceedings of the IEEE 101 1817
[2] Darwazeh I, Ghannam H and Xu T 2018 The first 15 years of SEFDM: a brief survey 11th International Symposium on Communication Systems, Networks & Digital Signal Processing (CSNDSP) I 1
[3] Ozan W, Haigh P A, Tan B and Darwazeh I 2018 Experimental SEFDM pipelined iterative detection architecture with improved throughput IEEE 87th Vehicular Technology Conference (VTC Spring) I 1
[4] Ovsyannikova A S, Zavjalov S V and Volvenko S V 2018 Influence of correlation coefficient on spectral and energy efficiency of optimal signals 10th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT) I 1
[5] Ishkaev I R, Shevelev A E, Ovsyannikova A S, Zavjalov S V, Volvenko S V and Makarov S B 2018 Possibility of peak-to-average power ratio reduction by application of optimal signal for transmitter based on SDR HackRF One IEEE International Conference on Electrical Engineering and Photonics (EExPolytech) I 141
[6] Zavjalov S V, Ovsyannikova A.S and Volvenko S V 2018 On the necessary accuracy of representation of optimal signals Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), I 153
[7] Gelgor A, Gorlov A and Popov E 2015 On the synthesis of optimal finite pulses for bandwidth and energy efficient single-carrier modulation, Internet of Things Smart Spaces and Next Generation Networks and Systems Springer International Publishing I 655
[8] Sadovaya Y and Gelgor A 2018 Synthesis of signals with a low-level of out-of-band emission and peak-to-average power ratio IEEE International Conference on Electrical Engineering and Photonics (EExPolytech) I 103
[9] Rashich A and Urvantsev A 2018 Pulse-shaped multicarrier signals with nonorthogonal frequency spacing IEEE International Black Sea Conference on Communications and Networking (BlackSeaCom) I 1
[10] Rashich A, Kisliatsyn A and Gorbunov S 2018 Trellis Demodulator for Pulse Shaped OFDM IEEE International Black Sea Conference on Communications and Networking (BlackSeaCom) I 1
[11] Vasilyev D and Rashich A 2018 SEFDM-signals Euclidean distance analysis IEEE International Conference on Electrical Engineering and Photonics (EExPolytech) I 75