Mechanical model of artificial roof overlay deformation elastic thin plate

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Abstract. After presupporting of roof, overlying roof bending deformation law has undergone greatly changed. In order to accurately calculate the top roof of artificial roof movement law after the roof pre-control top. Based on the analysis of the top moving characteristics of the roof and the theory of elastic foundation thin plate, the mechanics model of pre-control top-mining filling body-artificial roof-roof is established. The basic deflection, stress equations and the critical conditions of roof breaking are derived. The impact of roof bending deformation of the key factors affects the artificial roof thickness and elastic modulus. Combination of Wo Hu Shan specific engineering conditions, the calculation equation of tensile stress and sinking displacement of lying tiger hill is deduced and the critical roof thickness is 1.87m. To retain a certain safety factor, the artificial roof thickness of 2.5m is used. The monitoring results show that the maximum roof displacement is 15.80mm, which is consistent with the theoretical calculation.

1. Introduction
Under unstable conditions when mine rock is broken, the stope is prone to collapse, and leading to the loss of mine resources, which increase the difficulty in dealing with the mined area and stope mining around the caving area. Some studies have proposed that the roof can be reinforced after pre-cutting mining [1], that is, “pre-control top” to support roof rock without self-supporting capacity. When the original layered joints of the stope or the rock on the top of the stope are developed with cracks, severe structural influences, strong water absorption, and poor water permeability, if no control measures are taken, the roof of the stope may explode for several days or hours. “Pre-controlling roof” is the use of anchor cables or anchor nets to reinforce surrounding rock and construct an artificial roof, which can effectively support the roof and reduce the displacement of the roof, thereby improving the working environment and accelerating the work cycle [2]. However, the laws of movement and deformation of the stope roof at the top of the pre-control roof are still unknown.

At present, the research methods for the rock movement law mainly include the key layer theory, masonry beam theory, shatter block theory, and continuous beam theory, etc. It is very effective for understanding the movement law of the upper rock strata in the stope, but for the pre-control of the top, breaking the roof has a limited effect [3]. This paper deduces the theory to modify the elastic foundation slab model, calculates the rules of the movement of the pre-control roof covering, and validates with the numerical simulation and the actual measurement results, and provides theoretical guidance for the pre-control roof technology.
2. Pre-controlled top cover movement characteristics

For rock masses of Class III, IV, and V in underground rock masses, the self-stabilization capability of the chamber with a span of more than 10 m is extremely poor, and rock formations will loosen and deform within a few days. As China’s mines gradually enter the deep, the stress and tectonic stress of the stope increase gradually, and the probability of loose deformation of the cracked rock mass increases. In underground mines, after the excavation of the stope, the surrounding rock will be directly exposed to the air. The originally broken rock will soon loosen and deform under the influence of the overburden pressure and air weathering erosion. The top plate fell off and the working face was in danger. The ground pressure of the overlying strata is not controlled in time. With the advancement of the working face, the exposed area gradually increases, the original loose deformation will cause the roof to fall, and the overlying strata will move in large scale, causing geological disasters and waste of resources.

After the stope is excavated, the broken roof shall be timely reinforced with shotcrete and anchor nets, and the concrete in the extremely broken area shall be reinforced by advancement \[4\], that is, “pre-control roof” technology, to construct an artificial roof and close the roof to prevent the roof from being exposed, weathered and corroded. At the same time, the artificial false top and the overlying roof act as active supports, and the artificial roof acts as a permanent support, bearing the pressure of overlying rock formations. The overlying rock formations will only undergo certain bending deformation, and thus slow down. The speed of the overlying strata will reduce the loose deformation of overlying strata and prevent the overburden destruction.

3. Pre-control roof-top plate mechanical model

3.1. Assume the pre-control roof-top plate mechanical model

(1) Basic assumption of elastic sheet.

Pre-control roof can ensure the roof does not break and collapse, and in the absence of large structural stress, the roof meets the basic assumption of the elastic sheet.

(2) Assumption of elastic foundation \[5\].

The artificial roof belongs to the reinforced concrete structure, which is combined with the action of the suspension and composite beam of the upper cover plate through the anchor rod, and is closely combined with the deformation of the overlying rock layer. The artificial roof and the overlying roof can be approximated. Think of a flexible foundation system.

(3) The elastic sheet shape assumption. According to the theory of the elastic sheet \[6\], the elastic sheet will satisfy the following conditions:

\[
\frac{1}{80} \leq \frac{h}{a} \leq \frac{1}{5}
\]  

Where \(h\) is the thickness of the sheet, m; \(a\) is the width of the sheet, m.

The stope of underground mines is arranged along the strike or vertical direction. The width is generally 15–30m, the length is 30–60m, the pre-control top thickness is 1–3m, and the middle section is divided into more than ten stopes along the strike. The thickness of the pre-control roof and the single stope or the entire middle section of the mining area all conform to the assumption of the elastic sheet shape and can be regarded as an elastic sheet \[7\].

3.2. Pre-control roof-top plate mechanical model

The upper part of the roof is subjected to uniformly distributed overburden loads \(q\), the lower part is supported by an elastic foundation, and the artificial roof supporting force is \(k_w\), and the anchoring force of the anchor rod can be directly transmitted to the ceiling through an artificial false ceiling \[8\]. Therefore, the top plate can be regarded as an upper elastic plate with four sides fixedly supported by the elastic roof artificial roof. The mechanical model of the elastic plate is shown in Figure 1. Figure 2 is a sectional view of the mechanical model of the elastic plate.
Take the inner corner point of the plate $O$ as the far point, the direction along the direction of the ore body as the axis, the direction of the ore body as the axis, and establish the coordinate system, as shown in Figure 1. Among them, $b$ is the length of the middle pre-control roof, $a$ is the width of the pre-control roof, $q$ is the uniform load of the overlying rock on the roof, $k$ is the elastic foundation coefficient, and $w$ is the deflection of the roof.

4. Pre-controlled bending deformation of the roof covering

4.1. Top plate bending deformation analysis
According to the principle of equivalent replacement [9], the system with four sides fixedly clamped is simplified as a simplified system that uses four sides to simply support and four sides receive the bending moments $M_x(x)$ and $M_y(y)$ effects with the coordinates. The bending moment $M(x)$ is distributed on both sides of $y=0$ and $y=a$ vary with the position of each point $x$; The moments are distributed on both sides of $x=0$ and $x=b$ vary with the positions of the points $y$.

The plane stress component of the plate is:

$$
\begin{align*}
\sigma_x &= \frac{12M_{x, z}}{h^3} \\
\sigma_y &= \frac{12M_{y, z}}{h^3} \\
\tau_{xy} &= \frac{12M_{xy, z}}{h^3}
\end{align*}
$$

(2)

According to the common method used to solve the plate problem in mechanics, the double sine series adopted by Navier is used as the bending surface equation of the plate:

$$
w = \sum_{m=1,3,5,\ldots}^{\infty} \sum_{n=1,3,5,\ldots}^{\infty} a_{mn} \sin \frac{m\pi x}{b} \sin \frac{n\pi y}{a}
$$

(3)

Again, we see the top plate and the pre-control top elastic foundation as a whole system [10].

The uniform load on the overburden $q$, the fixed edge $M(x)$, $M(y)$ and the virtual work performed $\delta T$ are:
\[ \delta T = \int_0^b q \delta w \, dx \, dy + \int_0^b M(x) \, dx \delta \left( \frac{\partial w}{\partial y} \right)_{y=0} + 2 \int_0^a M(y) \, dy \delta \left( \frac{\partial w}{\partial x} \right)_{x=0} \]
\[ = \int_0^b q \delta a_{mn} \sin \frac{m \pi x}{b} \sin \frac{n \pi y}{a} \, dx \, dy + 2 \int_0^a M(x) \, dx \sin \frac{m \pi x}{b} \, \delta a_{mn} \]
\[ + 2 \left( \frac{m \pi}{b} \right)^2 \int_0^b M(y) \, dy \frac{n \pi y}{a} \, \delta a_{mn} \]
\[ = \int_0^b q \delta a_{mn} \sin \frac{m \pi x}{b} \sin \frac{n \pi y}{a} \, dx \, dy + 2 \int_0^a M(x) \, dx \sin \frac{m \pi x}{b} \, \delta a_{mn} \]
\[ + 2 \left( \frac{m \pi}{b} \right)^2 \int_0^b M(y) \, dy \frac{n \pi y}{a} \, \delta a_{mn} \]  
(4)

The sum in the above equation of \( E_m \) and \( F_m \) is the coefficient when expanding the bending moment \( M(x) \) and \( M(y) \) to the Fourier series [11].

The equation for the curved surface of the plate is:

\[ w = \sum_{m=1,3,5,7} \sum_{n=1,3} \left( \frac{4 \pi b}{m \pi^2} + \frac{n \pi y}{a} b E_m + \frac{m \pi y}{b} b F_n \right) \sin \frac{m \pi x}{b} \sin \frac{n \pi y}{a} \]
\[ \frac{4 \pi^2 b}{a} \sin \frac{m \pi^2}{b^2} + \frac{n \pi y}{a} b E_m + \frac{m \pi y}{b} b F_n \]  
(5)

Substituting the boundary conditions into the equation, find the coefficients \( M(x) \) and \( M(y) \) when Fourier series \( E_m \) and \( F_n \) in the formula (4), and the slope of the edge \( y = 0, x = 0 \).

The four sides of the board are fixed:

\[ \sum_{m=1,3} \sum_{n=1,3} \left( \frac{4 \pi b}{m \pi^2} + \frac{n \pi y}{a} b E_m + \frac{m \pi y}{b} b F_n \right) \sin \frac{m \pi x}{b} \sin \frac{n \pi y}{a} \]
\[ \frac{4 \pi^2 b}{a} \sin \frac{m \pi^2}{b^2} + \frac{n \pi y}{a} b E_m + \frac{m \pi y}{b} b F_n = 0 \]  
(6)

In the equations of equation (6), in each special case, the Fourier coefficients \( E_1, E_3, E_5, E_7 \ldots \)
and \( F_1, F_3, F_5, F_7 \ldots \) can be obtained from these two equations by the successive approximation method.

According to the relationship between elastic modulus, stress, strain and elastic foundation coefficient [12]:

\[ \begin{cases} \sigma = \frac{E z h}{E} \\ \sigma = kw \\ w = \varepsilon H \end{cases} \]  
(7)

In the formula, \( w \) - roof deflection
\( E_{zh} \) - Artificial False Top - Elastic Modulus of Backfill Body Composite Material
\( H \) - mining height
\( \sigma, \varepsilon \) - Stress and strain in composite materials of artificial false tops and backfills

Equation (7) can be used to derive the relationship between the elastic foundation coefficient and the elastic modulus of the composite material:

\[ E_{zh} = H k \]  
(8)
From the deformation coordination conditions of composite materials in material mechanics, the elastic modulus of the artificial roof-filled body combination can be obtained as follows:

$$E_{zh} = \frac{E_z A_z + E_h A_h}{A_z + A_h} = \varphi E_z + (1 - \varphi)E_h$$ \hspace{1cm} (9)

In the formula: $E_z$ is the artificial roof elastic modulus; $E_h$ is the filling body elastic modulus; $A_z$ is the artificial roof cross-sectional area; $A_h$ is the filling body cross-sectional area; $\varphi$ is the proportion of cross-sectional area for artificial roof.

Among them:

$$\varphi = \frac{A_z}{A_h} = \frac{H_z \times a}{H \times a} = \frac{H_z}{H}$$ \hspace{1cm} (10)

Substituting Equations (9) and (10) into Equation (8) yields the relationship between the elastic foundation coefficient and the artificial roof and the elastic modulus of the filling body:

$$k = \frac{H_z E_z + H_a E_h}{H^2}$$ \hspace{1cm} (11)

Bring equation (11) to (5) to obtain the top plate deflection equation:

$$w = \sum_{m=1,3}^{\infty} \sum_{n=1,3}^{\infty} \frac{4q}{mb^4} \frac{bE_m + m\pi a F_n}{b^2 + \frac{a^2}{m^2}} \sin \frac{m\pi x}{b} \sin \frac{n\pi y}{a}$$

4.2. Top plate bending stress analysis

By equation (13) analysis, the position of the maximum bending moment on the fixed side midpoint, i.e., $x = \frac{b}{2}, y = \frac{a}{2}$. Substituting the moment type (7) represented by the Fourier series into (2) gives the maximum tensile stress equation at the top of the equation:
The critical conditions that must be met when the top panel is controlled to break are:

\[
\begin{align*}
\sigma_{x_{\text{max}}} &= \frac{6M_x}{h^2} = \frac{6}{h^2} \sum_{m=1,3-}^{\infty} E_m \sin \frac{m \pi x}{b} \\
\sigma_{y_{\text{max}}} &= \frac{6M_y}{h^2} = \frac{6}{h^2} \sum_{n=1,3-}^{\infty} F_n \sin \frac{n \pi y}{a}
\end{align*}
\]  

(14)

The relationship between the maximum tensile stress of the top plate and the thickness and elastic modulus of the artificial roof is obtained by bringing \(E_1, E_3, F_1, F_3\) and the elastic foundation coefficient \(k\) into the maximum tensile stress equation (14) of the top plate (the simplified form is omitted and the smaller term is omitted):

\[
\begin{align*}
\sigma_{x_{\text{max}}} &= \frac{6.48 \times 10^3 h^2 + 9.51 \times 10^4 h + 2.67 \times 10^5}{4.78 h^2 + 3.63 h + 0.695} \\
\sigma_{y_{\text{max}}} &= \frac{1.15 \times 10^5 h^2 + 5.99 \times 10^6 h + 3.11 \times 10^7}{0.53 h^3 + 2.63 h^2 + 2.72 h + 1.06}
\end{align*}
\]  

(15)

5. Case studies
The above deduced the deflection equation, bending moment equation and maximum stress of the artificial roof, and the main factors affecting the deformation and stress of the prestressed roof are the length, elastic modulus and bending stiffness of the roof along the inclination and strike. Overlying uniform load, elastic modulus, cross-sectional area ratio of filling material and artificial roof, and stope height.

It is known that the length, breadth and height of the stope in the middle section of the 40-foot section of Wohushan Mine are respectively 40m×12.5m×16m. Stope vertical layout. Twenty stope filling and filling operations have been completed. The length is 250m, the width is 40m, mining height (recovery height and pre-control roof thickness) is 16m, pre-control roof height is less than 3.5m, filling body height is more than 12.5m, the elastic modulus \(E_z\) of the artificial roof is 3 GPa, the elastic modulus \(E_h\) of the filling body is 8 GPa. The roof of the artificial roof is made of limestone and has a thickness \(h\) of 3 m. Its elastic modulus \(E\) is 11.67 GPa, its Poisson’s ratio is \(\mu\) 0.26, its flexural rigidity \(D\) is 8.3×10^{12} N⋅m, its tensile strength \(\sigma_t\) is 4.1 MPa, and its compressive strength \(\sigma_c\) is 14.78 MPa. The uniform overburden load on the overlying strata \(q\) in the roof is 8.7 MPa (the height of the rock from the roof to the surface is 330m, and the average bulk density of the rock \(\gamma\) is 26.8KN/m3).

5.1. Tensile stress analysis
The first two terms of the Fourier coefficient representing the fixed bending moment are approximated by \(E_m\) and \(F_n\). Substituting the above data into equation (6), calculate \(E_1, E_3, F_1, F_3\). The relationship between the maximum tensile stress of the top plate and the thickness and elastic modulus of the artificial roof is obtained by bringing \(E_1, E_3, F_1, F_3\) and the elastic foundation coefficient \(k\) into the maximum tensile stress equation (14) of the top plate (the simplified form is omitted and the smaller term is omitted):
The top plate \( \sigma_y \) is 4.1 MPa. From equation (16), the minimum thickness of the top plate in both cases can be calculated as 1.58m and 1.87m. Under the condition that the thickness of the top plate is 1.87m, the top plate can theoretically meet the requirement of no cracking.

The variation of maximum tensile stress \( \sigma_x \) and \( \sigma_y \) in the top plate with artificial roof thickness is shown in Figure 2(a) and (b) below. As can be seen from the figure, with the increase of the artificial roof thickness, the maximum tensile stress in the roof gradually decreases, and the decreasing speed of the maximum tensile stress gradually decreases as the thickness of the artificial roof increases, and when the value increases to a certain value, the maximum tensile stress in the roof gradually tends to be stable, indicating that the top plate plays the most significant supporting role when the thickness is small, and it can well control the stress in the top plate. The critical point where the roof does not break in both cases has been marked in the figure.

![Graph showing maximum tensile stress in X and Y](image)

**Figure 2. Maximum tensile stress of the top plate**

5.2. Roof settlement displacement analysis

From the symmetry of the board, it can be seen that the displacement in the center of the board sinks the most, ie, \( x = 125m, y = 20m \). Substituting the equation of the curved surface of the top plate (12) yields the relationship between the maximum subsidence of the top plate \( w_{\text{max}} \) and the thickness and elastic modulus of the artificial top plate. Since the relational form is too complex, it is not listed here. The theoretical calculation value of the maximum roof subsidence under different artificial roof thicknesses is shown in Figure 3. Here, the polynomial fitting theory calculation result of \( y = B_3x^3 + B_2x^2 + B_1x + A \) is used, the fitting factor \( R^2 \) is 0.99, and the fitting degree is very high.
As can be seen from the figure, the settlement displacement of the roof also gradually decreases with the increase of the thickness of the artificial roof, and the speed is getting slower and slower. When the thickness reaches a certain critical value, the settlement displacement of the roof is stable near a certain value, indicating that the artificial roof is opposite to the roof. The control of the settlement displacement has a good effect, and the displacement of the roof can be controlled by adjusting the thickness of the artificial roof and the elastic modulus (the ratio of the concrete, etc.). The relationship between the artificial roof thickness and the settlement displacement of the roof can be calculated. When the artificial roof thickness is the critical value of 1.87m, the settlement displacement of the roof is 22.85mm.

6. Engineering practice

6.1. Project overview
The ore section basically adopts the method of pre-controlling the top-to-the-minute sub-section miner’s backfill and mining method. The vertical ore body of the mine is oriented towards the height of 60m and is divided into 4 subsections. The span of the mine house is 12.5m, the height is 15m, and the length is the ore body. Thickness, roof support in each mine roof, namely pre-control roof, pre-control ceiling height 3m.

The spatial relationship of the stope-artificial roof-artificial roof is shown in the figure 4 below. Each roof of the stope is provided with a monitoring point to monitor the sinking displacement of the roof.

6.2. Pre-control top height
After the above theoretical calculation, the thickness of the artificial roof is 1.87m, the roof will theoretically not break, and the sinking displacement is 22.85mm. After numerical simulation and engineering practice, the mine reserves a certain safety factor and does not change the condition of the concrete used for filling and artificial false ceilings. The artificial roof used at present has a thickness of 2.5m. At this time, the maximum tensile stress of the roof is 2.5 MPa. The sinking displacement is 16.17mm.
6.3. Measurement results
A dynamic displacement monitor is set in the center of each stope in the middle section, and the settlement of the roof during the middle stopage recovery is monitored. The relationship between the sinking displacement of the roof and the middle section mining distance is shown in Figure 5. The maximum value is 15.80mm, which is very close to the theoretical calculation value. At the same time, after the roof of the roof is pre-controlled, there is no major damage to the joints and cracks in the roof, and the roof still maintains good integrity.

![Figure 5. The regular relationship of the settlement displacement of the roof plate with the middle-distance mining distance](image)

7. Conclusions
(1) Contrasting and analyzing the traditional mining methods and pre-controlling top mining, the characteristics of the roof moving on the stope. The method of pre-controlling the roof protects the roof from weathering and erosion. At the same time, the artificial roof and the overlying roof act as active supports to reduce the loose deformation of the overlying rock and prevent overburden damage.

(2) Using the “thin plate model” theory of elastic foundation, the mechanical model of the backfill-pre-control roof-top plate was established, and the relationship between the top plate deflection equation and the maximum tensile stress equation and the artificial roof height was deduced, and the top plate was further introduced. Breaking critical conditions occurred.

(3) Combining with the specific engineering example of Wohushan Mountain, the maximum tensile stress equation of the roof is theoretically deduced, and the critical condition for ensuring that the roof does not break is determined to be 1.87m. Based on theoretical calculations and data fitting, the equation of settlement displacement of the roof is deduced. Under the critical condition, the displacement of the roof is 22.58mm. Analyze the influence factors and change laws of stress and subsidence displacement of the roof.

(4) Combination with engineering practice, the theoretically calculated roof subsidence displacement value is very close to the field engineering monitoring value. And there is no major damage to the joint cracks in the roof of the pre-control roof, and the stability of the roof is good.

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