An elasto-viscoplastic model for frozen-unfrozen clays for combined problems of temperature and load variations

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ABSTRACT

Frozen soil’s mechanical behavior is characterized by interactions between solid grains, ice and unfrozen water. It is strongly affected by temperature and ice content, indicating pronounced differences between frozen and unfrozen soils. Rate-sensitive behavior of frozen soil is expected, given the highly rate-dependent behavior of ice. The consequence is reflected in the peculiar features seen in frozen soil’s strength and deformation characteristics under transient temperature and load. To capture these features, an elasto-viscoplastic constitutive model for estimating temperature and strain rate-dependent behavior of water-saturated clays is presented that is applicable continuously to both frozen and unfrozen states. This model adopts the \(p\)-\(q\) plane with a Critical State Line (CSL) that moves with temperature and strain rate while converging to a unique unfrozen CSL, thus it is seamlessly continuous to a conventional elasto-viscoplastic critical state model in unfrozen states. This model is based on an isotach over-stress approach with the cryogenic suction as additional state variable, and is potentially capable of describing varying-load and varying-temperature behaviour seen under combined influence of these two factors.

Keywords: frozen soil, temperature and strain rate effects, constitutive modelling

1 INTRODUCTION

Modelling the behaviour of soils at frozen states and transitional states to/from unfrozen states has many engineering applications. In warmer regions, artificial ground freezing techniques have been utilized in geotechnical engineering projects involving shafts, tunnels, and deep foundations with poor ground conditions (Andersland and Ladanyi, 2004). Understanding the behaviour of artificially frozen soils is a prerequisite for application of ground freezing to geotechnical engineering. On the other hand, freeze-thaw process aggravates the ground surface soil condition and leads to problems such as slope failure caused by thawing of frozen soil. By including viscoplastic ice as constituent, the frozen soil exhibits characteristic rheological behaviour. Early studies on the rheological characteristics of frozen soils mainly focused on their creep behaviour under different temperatures and applied stresses, or the stress responses against constant strain rates (Hooke et al., 1972; Ladanyi, 1972). Studies of the frozen soil behaviour under simultaneous and arbitrary variations of the two factors under general conditions and its description have been very few.

The strain rate and temperature have intertwined effects. For example, Akili (1971) and Zhu and Carbee (1984) both found that the peak strength of frozen silts was very sensitive to temperature and applied strain rate at the same time. Wang and Nishimura (2017) conducted frozen-unfrozen parallel triaxial tests on clay at different temperatures and strain rates, and concluded that the shear strength linearly increased with a decrease in the temperature for the range from -10°C to -2°C, and logarithmically increased with an increase in the strain rate for the range from 0.001/min to 0.1/min. Li (2004) also performed uniaxial compressive tests on saturated frozen clay with different dry density at various strain rates and temperatures, and confirmed the dependency of frozen soil’s compressive strength on strain rate and temperature.

The first step in developing a constitutive model for frozen-unfrozen soil in a continuous manner is to identify relevant stress-state variables. Nishimura et al. (2009) proposed an elasto-plastic critical state model for frozen soils by employing the net stress and the cryogenic suction for state description. This model, based on the Barcelona Basic Model (Alonso et al., 1990) for unsaturated soils, reflects an analogy between unfrozen-saturated and frozen-saturated soils, and reduces to an effective stress-based Modified Cam Clay Model under unfrozen conditions. However, strain-rate-dependent stress-strain-strength behaviour was not included in this early model. In order to incorporate the
rate-dependent behaviour of saturated frozen soils, Ghoreishian Amiri et al. (2016) proposed another two stress variables model by introducing the solid phase stress and the cryogenic suction. The elastic-viscoplastic model was developed based on the overstress framework (Perzyna, 1966) and 21 parameters in total were required to be determined. While generalized theories, such as the isotach law (Suklje, 1957) or over-stress concept have been successfully adopted to describe the time-dependent features of unfrozen soils, their applicability remains largely unexplored for frozen soils. Nishimura and Wang (2018) have modified the modelling framework proposed by Nishimura et al. (2009) and adopted the effective stress, p’ and a suction-equivalent stress, s, with a critical state line (CSL) common to both frozen and unfrozen states to describe the soil skeleton behaviour. The present study, while building on the same framework, adopts q-p’ stress space with q being the deviator stress, and isotach/over-stress theory. The model is formulated in a general rate form, and is tested against triaxial tests with constant load or temperature. This paper reports formulation and preliminary validation processes.

2 MODEL DESCRIPTION

2.1 Modelling of unfrozen soil

Watabe et al. (2008) presented an isotach concept using the reference compression curve and the following \( p'c - \dot{\varepsilon}_vp \) relationship (Eq. (1)), where \( p'c \) is the consolidation pressure and \( \dot{\varepsilon}_vp \) is the viscoplastic strain rate.

\[
\ln \frac{p'c - p'cL}{p'cL} = c_1 + c_2 \ln \dot{\varepsilon}_vp
\]

(1)

Here, \( c_1 \) and \( c_2 \) are constants and \( p'cL \) is the lower limit of \( p'c \). When \( \dot{\varepsilon}_vp \) decreases to zero in Eq. (1), \( p'c \) converges to \( p'cL \). Eq. (1) can be changed to:

\[
p'c = p'cL[1 + \exp(c_1) \cdot (\dot{\varepsilon}_vp)^{c_2}]
\]

(2)

The Normal Compression Line (NCL) is expressed in the usual manner:

\[
v = N = N + \lambda \ln p'
\]

(3)

where \( v \) is the specific volume of soil, \( N \) is the intercept of NCL at \( p' = 1 \) kPa, and \( \lambda \) is the slope of NCL.

Watabe et al.’s (2008) equation is used for isotach modelling in this study, with different expression. Instead of \( p'c \), we consider \( N \) as a function of \( \dot{\varepsilon}_vp \) in an equivalent manner. The constant \( N_L \) is defined as \( N \) at \( \dot{\varepsilon}_vp = 0 \). In this paper, the notation for the viscoplastic volumetric strain rate is changed from \( \dot{\varepsilon}_vp \) to \( \dot{\varepsilon}_p^\psi \) in order to distinguish it from viscoplastic shear strain rate, \( \dot{\varepsilon}_q^\psi \). Then the intercept \( N \) can be written as:

\[
N = N_L + \lambda \ln [1 + \exp(c_1) \cdot (\dot{\varepsilon}_p^\psi)^{c_2}]
\]

(4)

With this equation, the shift of NCL for a different viscoplastic volumetric strain rate \( \dot{\varepsilon}_p^\psi \) is easily calculated, as illustrated in Fig. 1. Note that \( \dot{\varepsilon}_p^\psi = 0 \) when \( N < N_L \).

Based on Eq. (4), the viscoplastic volumetric strain rate \( \dot{\varepsilon}_p^\psi \) can be shown as:

\[
\dot{\varepsilon}_p^\psi = \frac{\exp\left((N-N_L)/\exp(c_L)\right)}{\exp(c_L)} \frac{1}{c_2}
\]

(5)

The Modified Cam Clay (MCC) yield surface (e.g. Wood, 1990) is adopted, in the same manner as Nishimura et al., (2009).

\[
f(p', q) = q^2 - M^2[p'(p_0' - p')]
\]

(6)

where \( M \) is the slope of the critical state line (CSL), and \( p_0' \) is the reference size of the yield surface. Also adopting associated flow rule, the viscoplastic volumetric/shear strains are obtained as:

\[
\dot{\varepsilon}_p^\psi = \frac{\partial f(p', q)}{\partial p'} = \phi M^2(2p' - p_0')
\]

(7)

\[
\dot{\varepsilon}_q^\psi = \frac{\partial f(p', q)}{\partial q} = 2\phi q
\]

(8)

where \( \phi \) is the fluidity parameter.

When the current stress state is \( (p', q) \), and the specific volume is \( v \), the viscoplastic strains are calculated in the following manner. Firstly, consider a yield surface that pass through the current stress state point and obtain \( p_0' \) based on \( f(p', q) = 0 \):

\[
p_0' = \frac{q^2 + M^2p'^2}{M^2p'}
\]

(9)

Based on Eq. (3), \( N \) for this state is:

\[
N = v + \lambda \ln p_0'
\]

(10)

Then the viscoplastic strain rate at the stress state \( (p_0', q) \) is obtained by Eq. (5) and Eq. (7) as:

\[
\dot{\varepsilon}_p^\psi = \frac{\partial f(p', q)}{\partial p'} \bigg|_{(p_0', q)} = \frac{\exp((N-N_L)/\exp(c_L))}{\exp(c_L)} \frac{1}{c_2}
\]

(10)

\[
\phi = \frac{1}{M^2p_0'} \left(\frac{\exp((N-N_L)/\exp(c_L))}{\exp(c_L)}\right)^{1/c_2}
\]

(11)

By following Bodas Freitas et al. (2011), it is assumed that the fluidity parameter \( \phi \) is constant on a given yield surface. Therefore at \( (p', q) \):

\[
\dot{\varepsilon}_p^\psi = \left(\frac{\partial f(p', q)}{\partial p'} \bigg|_{(p_0', q)}\right) \frac{M^2(p'^2 - p_0')}{M^2p_0'} \left(\frac{\exp((N-N_L)/\exp(c_L))}{\exp(c_L)}\right)^{1/c_2}
\]

(12)

\[
\dot{\varepsilon}_q^\psi = \left(\frac{\partial f(p', q)}{\partial q} \bigg|_{(p_0', q)}\right) = \frac{2q}{M^2p_0'} \left(\frac{\exp((N-N_L)/\exp(c_L))}{\exp(c_L)}\right)^{1/c_2}
\]

(13)
2.2 Modelling of frozen soil

In this paper, the effective stress $p'$ is adopted to describe soil skeleton behaviour of frozen soil and defined by following Bishop-type expression:

$$p' = p - \chi p_l - (1 - \chi) p_i$$  \hspace{1cm} (14)

where $p_l$, $p_i$ and $\chi$ are the liquid pressure, ice pressure and a state parameter, often identified with the degree of liquid saturation (=volume of liquid water/volume of void), $S_l$. While adopting the mean effective stress $p'$, for the horizontal axis in the stress space, total deviator stress, $q$, is used for the vertical axis, which involves all the contributions from different phases in the frozen soil system and can be directly measured in experiments.

Based on the experiment results by Wang et al. (2017), a family of Critical State Lines (CSL) for different $s$ values is expressed by the following equations:

$$q_{cs} = q_f + M_f p_{cs}$$  \hspace{1cm} (15)

$$q_f = a_q s$$  \hspace{1cm} (16)

$$M_f = M + a_M s$$  \hspace{1cm} (17)

where $a_q$ and $a_M$ are constants. In this approach, the subscript $cs$ refers to Critical State. The $q_{cs}$ represents the ultimate $q$ controlled by soil skeleton, pore ice and the interactions between them, and has an intercept value, $q_f$, as illustrated below ($p' = 0$ means soil particles are floating in the ice). The suction, $s$, is strictly a function of the temperature, $T$, the liquid pressure, $p_l$, and the ice pressure, $p_i$, via Clausius-Clapeyron equation, but it is predominantly governed by temperature $T$, and approximated by the following equation:

$$s [\text{MPa}] = -1.2 T [\degree C]$$  \hspace{1cm} (18)

An elliptic yield surface that is consistent with the idea described so far is drawn in the figure above. By connecting the stress space origin and the yield surface crown, $M_{f,sec}$ (the secant $M$) is defined:

$$M_{f,sec} = \left( q_f + \frac{M_f p_{cs}}{2} \right) / \frac{p_0}{2}$$  \hspace{1cm} (19)

By using this $M_{f,sec}$, the usual equation for the Modified Cam Clay (MCC) yield surface can be directly used:

$$f = q^2 - M_{f,sec}^2 [p'(p_0 - p')]$$  \hspace{1cm} (20)

In this way, the rest of the formulation (associated flow law, the NCL, etc.) is assumed to be the same as MCC, as illustrated in Fig. 2.

The existing experimental data suggests greater viscosity for frozen soils than for unfrozen soils. It then follows that additional time-dependent submodel is necessary to reflect the enhanced viscosity in frozen soils. The high viscosity of frozen soils against shear deformation is most likely to derive from the shear viscosity of the pore ice. It is then logical to consider increased shear resistance due to increased plastic shear strain rate. One possible form is, by adopting the same form as Eq. (2) and combining with Eq. (16):

$$q_f = a_q s [1 + \exp(c_3) (p/c_4)^{c_4}]$$  \hspace{1cm} (21)

where $c_3$ and $c_4$ are additional viscosity parameters.

As for shear stiffness of frozen soil, Wang et al. (2019) modelled the shear stiffness of frozen clay as a coupled multi-phase system. In this study at the moment, their experimental results are very much simplified into the following expression:

$$G_f = G - \beta T$$  \hspace{1cm} (22)

where $G$ is the shear modulus of soil skeleton (i.e. unfrozen stiffness), and $\beta$ is a coefficient with a unit [MPa/°C]. The elastic strains are calculated as:

$$\dot{e}_q^e = dq/3G_f$$  \hspace{1cm} (23)

$$\dot{e}_p^e = dp'/K = \kappa dp'/vp'$$  \hspace{1cm} (24)

where $\kappa$ is the slope of unloading-reloading line. By combining the equations so far, $p'$ at the Critical State and the Critical State strength, $q_{cs}$, is expressed as:
\[ p_{cs}' = \frac{1}{2} p_0 = \frac{1}{2} \exp \left[ N_L + \lambda \ln \left( 1 + \exp(c_1) \right) \right] \cdot \left( M_{f,sec} \frac{\epsilon_0}{c_2} - v - \kappa \ln 2 \right) / \lambda \]  
\[ q_{cs} = q_f + \frac{1}{2} M_f \cdot p_{cs}' \]

Note that Eq. (19) and Eq. (25) need to be iteratively solved to obtain \( M_{f,sec} \) when \( s \neq 0 \).

3 SIMULATION RESULTS AND DISCUSSION

In order to examine the ability of the model to simulate the behavior of both unfrozen and frozen soils in an acceptable way, Wang et al.’s (2017) tests are firstly simulated. Wang et al. (2017) conducted some triaxial tests on high-plasticity unfrozen Kasaoka Clay. Its specific gravity was 2.65, and the liquid limit and plastic limit were 62% and 28%, respectively. The samples were \( K_0 \) reconstituted from slurry with water content of 100%. The preconsolidated cake was trimmed to form a triaxial specimen with a diameter of 30mm for frozen tests and 50mm for unfrozen tests. For detail, see Wang et al. (2017).

3.1 Determination of the model parameters

The proposed model requires 12 parameters in total, among which 7 are used for describing the behaviour of unfrozen soil, and additional 5 parameters are required to account for the frozen state.

The results of an isotropic compression test on an unfrozen specimen in \( v \cdot d \ln p \) plane can determine \( \kappa \) and \( \lambda \). The lower limit, \( N_L \), can be estimated from either a strain-rate-controlled test or creep tests, as described by Watebe et al. (2008). By also referring to the values by Watabe et al. (2008), \( c_1 \) and \( c_2 \) can be estimated for typical clays. An undrained triaxial shear test at an unfrozen state of the soil can be used for determining the slope of the critical state line, \( M_c \), and the shear modulus, \( G \) (and then Poisson’s ratio, \( \mu \)).

Conducting a single multi-step constant-temperature and a single multi-step constant-rate frozen triaxial compression tests is sufficient to determine the frozen state-related material constants, \( a_q \), \( a_m \), \( \beta \), \( c_3 \) and \( c_4 \); see Wang et al. (2017).

3.2 Shear behaviour at reference unfrozen states

The specimens consolidated to 100, 200 and 400 kPa were subjected to undrained triaxial compression at 24°C. For over-consolidated (OC) specimens, they were unloaded to swell by a volumetric strain corresponding to 9% void ratio increase prior to undrained shear, resulting in an over-consolidation ratio (OCR) of 4.0-6.7. Comparison of the undrained effective stress paths between the test and simulation from the normal and over-consolidated states are shown in Fig. 3 and Fig. 4. The material parameters used for simulation are listed in Table 1. Fig. 5 shows the calculated stress-strain curves and effective stress paths for the NC specimens. Some degrees of rate-dependency can be seen on the stress-strain relationship for this particular clay. Since no visible effect of the strain rate for the OC specimen is
Table 1. Input model parameters for unfrozen Kasaoka Clays.

| Definition                                         | Parameter value |
|----------------------------------------------------|------------------|
| \(q/p'\) value at CS                               | \(M=1.08\)       |
| Slope of normal compression line                   | \(\lambda=0.20\) |
| Slope of unloading-reloading line                   | \(\kappa=0.05\)  |
| The lower limit of N                                | \(N_c=2.98\)     |
| Poisson’s ratio                                     | \(\mu=0.20\)     |
| Constant in Watabe et al.’s (2006) equation        | \(c_1=0.95\)     |
| Constant in Watabe et al.’s (2006) equation        | \(c_2=0.09\)     |

Table 2. Input model parameters for frozen Kasaoka Clays (additional to Table 1).

| Definition                                         | Parameter value |
|----------------------------------------------------|------------------|
| Constant related to viscosity                       | \(c_3=1.50\)     |
| Constant related to viscosity                       | \(c_4=0.15\)     |
| Material constant in Eq. (16)                       | \(a_s=0.09\)     |
| Material constant in Eq. (17), \(\Delta P\) MPa/\(^\circ\)C | \(a_m=0.0001\)   |
| Material constant in Eq. (22), MPa/\(^\circ\)C       | \(\beta=8000\)   |

![Fig. 5](image1.png)

Fig. 5. (a) Effective stress paths and (b) Stress-strain relationships for NC unfrozen specimens (p_c'=400kPa) at different axial strain rates.

![Fig. 6](image2.png)

Fig. 6. Comparison between the measured and simulated stress-strain curves for frozen Kasaoka clay (0.1%/min) under different temperatures (-2°C, -5°C and -10°C).

![Fig. 7](image3.png)

Fig. 7. Comparison between the measured and simulated stress-strain curves for frozen Kasaoka clay (-10°C) under different strain rates.

3.3 Shear behaviour and strength at frozen states

The specimens consolidated to 400kPa were tested at three freezing temperatures (-2, -5 and -10°C) at a strain rate of 0.1%/min. The stress-strain curves of both laboratory data and simulation results are shown in Fig. 6. For p_c'=400kPa, three orders of strain rates (0.1%/min, 0.01%/min and 0.001%/min) were applied at T=10°C. The stress-strain curves are shown in Fig. 7. The values for additional material parameters required for describing frozen states are listed in Table 2. Although the progressive reduction of pre-failure stiffness was not well reproduced, the temperature and the strain-rate effects on the strength are well captured.

Fig. 8 shows that Eqs. (25) and (26) describe increase of the shear strength of both unfrozen and frozen clays with an increase in the strain rate in a log-log scale for the range higher than 10^-4/s at a given temperature. With the adopted parameters, slightly greater viscosity (i.e. steeper log q_max – log \(\varepsilon_{\alpha}\) relationships) is achieved in frozen states. Li et al. (2004) found that the unconfined compressive strength of a frozen clay with medium dry density increased with a different gradient against \(\varepsilon_{\alpha}\) at 0.01%/min or greater (i.e. viscosity increases at warmer temperature in frozen states), as shown in Fig. 9, while

![Fig. 8](image4.png)
the proposed model outputs almost parallel trends. Future work will involve reproducing this feature in this model.

4 CONCLUSIONS

In this paper, an elasto-viscoplastic model with two sets of stress-state variables was proposed for simulating the behavior of unfrozen-frozen clays. The model was developed based on the isotach/over-stress theory and Critical State framework. In unfrozen states, the model becomes a conventional elasto-viscoplastic critical state model. Thus, this model can simulate the behavior from a frozen state to an unfrozen state with a single set of parameters. The model was validated against available frozen-unfrozen parallel triaxial tests at different strain rates and temperatures. The model, constructed with a very minimum set of parameters, captures well the stress-strain behavior of unfrozen and frozen clay under different constant conditions. Future work involves validating the model with experiments under transient temperature and strain rates.

REFERENCES

1) Akili, W. (1971): Stress-strain behavior of frozen fine-grained soils, *Highway Research Record*, (360).
2) Alonso, E. E., Gens, A. and Josa, A. (1990): A constitutive model for partially saturated soils, *Géotechnique*, 40(3), 405-430.
3) Andersland, O. B. and Ladanyi, B. (2004): Frozen ground engineering, John Wiley & Sons.
4) Freitas, T. M. B., Potts, D. M. and Zdravkovic, L. (2011): A time dependent constitutive model for soils with isotach viscosity, *Computers and Geotechnics*, 38(6), 809-820.
5) Ghoreishian Amiri, S. A., Grimstad, G. and Kadivar, M. (2016): An elastic-viscoplastic model for saturated frozen soils, *European Journal of Environmental and Civil Engineering*, 1-17.
6) Hooke, R. L., Dahlin, B. B. and Kauper, M. T. (1972): Creep of ice containing dispersed fine sand, *Journal of Glaciology*, 11(63), 327-336.
7) Li, H., Zhu, Y., Zhang, J. and Lin, C. (2004): Effects of temperature, strain rate and dry density on compressive strength of saturated frozen clay, *Cold regions science and technology*, 39(1), 39-45.
8) Nishimura, S., Gens, A., Olivella, S. and Jardine, R. J. (2009): THM-coupled finite element analysis of frozen soil, formulation and application, *Géotechnique*, 59(3), 159-171.
9) Nishimura, S. and Wang, J. (2018): A simple framework for describing strength of saturated frozen soils as multi-phase coupled system, *Géotechnique*, 69(8), 659-671.
10) Perzyna, P. (1966): Fundamental problems in viscoplasticity, *Advances in applied mechanics*, 244-377.
11) Suklje, L. (1957): The analysis of the consolidation process by the isochrones method, *Proceedings of the 4th International Conference on Soil Mechanics and Foundation Engineering*, London, 200-206.
12) Wang, J., Nishimura, S. and Tokoro, T. (2017): Laboratory study and interpretation of mechanical behavior of frozen clay through state concept, *Soils and Foundations*, 57(2), 194-210.
13) Watabe, Y., Udaka, K. and Morikawa, Y. (2008): Strain rate effect on long-term consolidation of Osaka bay clay, *Soils and Foundations*, 48(4), 495-509.
14) Wood, D. M. (1990): Soil behaviour and critical state soil mechanics, Cambridge University Press.
15) Zhu, Y. and Carbee, D. L. (1984): Uniaxial compressive strength of frozen silt under constant deformation rates, *Cold regions science and technology*, 9 (1), 3-15.