Spin–orbital phase synchronization in the magnetic field-driven electron dynamics in a double-well potential

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Abstract
We study the dynamics of an electron confined in a one-dimensional double-well potential in the presence of driving external magnetic fields. The orbital motion of the electron is coupled to the spin dynamics by spin–orbit interaction of the Dresselhaus type. We derive an effective time-dependent model Hamiltonian for the orbital motion of the electron and obtain a condition for synchronization of the orbital and the spin dynamics. We find an analytical expression for the Arnold ‘tongue’ and propose an experimental scheme for realizing the proposed synchronization.

(Some figures may appear in colour only in the online journal)

1. Introduction
Phase synchronization and related phenomena are among the most fascinating effects of nonlinear dynamics. Besides the deep fundamental interest [1–7], phase synchronization has a broad range of applications in chemistry [8], ecology [9], astronomy [10], in the field of information transfer using chaotic signals [11], and for the control of high frequency electronic devices [12]. In nonlinear dissipative systems, phase synchronization occurs if the frequency of the external driving field is close to the eigenfrequency of the system. In this case, for a certain frequency interval of the driving field the oscillations of the nonlinear dissipative system can be synchronized with the perturbing force. Usually, for stronger driving fields the frequency interval for the synchronization becomes broader and the synchronization protocol is more efficient. The broad and growing interest in the phase synchronization calls for the analysis of new possible realizations of this phenomenon. A particularly interesting issue is the application of the synchronization protocols for magnetic nanostructures, which have rich applications [11–16] and exhibit interesting nonlinear dynamical properties that can be exploited as a testing ground for dynamical systems [17, 18].

A principal challenge in nanoscience is to find an efficient procedure for the manipulation of the systems states. A high level of accuracy on the state control is required especially in such applications as quantum computing, where a precise tailoring of the entangled states is highly desirable [19, 20]. With this in mind several physical systems have been considered, e.g. Josephson junction qubits and Rydberg atoms in quantum cavities [19, 20], ion traps [21], single molecular nanomagnets [22], and nanoelectromechanical resonators [23, 24]. Among others, the most promising systems are electron spins confined in two-dimensional quantum dots [25–27] and in one-dimensional nanowires and nanowire-based quantum dots [28–31]. The key element of the corresponding models is the spin–orbit (SO) coupling term, which is linear in the electron momentum. Such momentum-dependent coupling offers a new way of manipulating the spin by changing the electron momentum via a periodic electric field. This is the idea of the electric-dipole spin resonance proposed by Rashba...
and Efros for the electrons confined in nanostructures on the scale of 10 nm [27]. However, the external electric field can strongly affect the orbital dynamics and thus the system is driven out of the linear regime [32]. The nonlinearities usually result in a complex behavior of the affected systems and their dynamics might become complicated and even unpredictable. On the other hand, the nonlinearity may also lead to a number of interesting phenomena.

One of the main issues in the spin manipulation is the dephasing leading to a rapid loss in the spin polarization. For this reason a great deal of attention is paid to the study of the phase synchronization between the spins in nanostructures and external perturbations. In particular it was shown that such a synchronization can be achieved by using trains of optical pulses with the repetition rate synchronized with the spin precession in a magnetic field [33, 34], and the nuclear spin polarization plays an important role in this process [35]. In the above-mentioned works the synchronization in spin ensembles was achieved by optical pumping and the authors concentrated on the coherence issues. In contrast, here we would like to focus on the synchronization effects for single spins with special emphasis on the role of nonlinear dynamical effects.

In spite of the great interest in the systems with SO coupling, some important aspects of the influence of spin dynamics on the orbital motion have yet to be addressed. Here our goal is to investigate the possibilities of controlling the orbital motion of the electron via external magnetic fields acting on its spin and via the spin–orbit coupling. That can be considered as opposite to the electric-dipole spin resonance protocol proposed by Rashba and Efros [27].

Electrons in one-dimensional (1D) nanowire confined via the double-well confinement potential form a model system of ongoing interest, intensively used to describe physical properties of a wide class of nanostructures. Confinement potentials in nanowires can be created using gate voltages, detected and controlled by scanning probe microscopy [36]. An InAs nanowire-based confinement potential was implemented to study fast qubit rotations and single-qubit control [37]. Tuning of the SO coupling in InAs 1D lateral confinement potential was demonstrated in [38], and the electron g-factor in these systems can be controlled by external magnetic field [39]. One-dimensional confinement potentials in Si/Ge heterostructures have been studied in [40]. Recently detecting Majorana fermions with 1D quantum dots in superconducting nanowire was proposed [41].

The key element of the corresponding models is the spin–orbit (SO) coupling term, which is linear in the electron momentum. Such momentum-dependent coupling offers a new way of manipulating the spin by changing the electron momentum via a periodic electric field. This is the idea of the electric-dipole spin resonance proposed by Rashba and Efros for the electrons confined in nanostructures on the scale of 10 nm [27]. However, the external electric field can strongly affect the orbital dynamics and thus the system is driven out of the linear regime [32]. The nonlinearities usually result in a complex behavior of the affected systems and their dynamics might become complicated and even unpredictable. On the other hand, the nonlinearity may also lead to a number of interesting phenomena. In spite of the huge interest in the systems with SO coupling, the influence of the spin dynamics on the orbital motion has not yet been addressed in full detail. With the present work we would like to bridge this gap. Our goal is to investigate the possibilities of controlling the orbital motion of the electron via external magnetic fields acting on its spin and via the spin–orbit coupling.

This proposal is opposite to the electric-dipole spin resonance proposed in [27]. Although [27] concentrates on 2D electron gas, the electric-dipole spin resonance is a very general spin–orbit coupling phenomenon applicable to electrons confined in nanowires and quantum dots [45].

We will demonstrate that: (i) by using an external driving field one can achieve a sufficient degree of control over the orbital motion and (ii) as a result, one can design a very efficient synchronization protocol of the orbital motion and the spin dynamics based on the application of a pulsed external magnetic field.

2. Theoretical model

We consider a model system of a single electron confined in a double-well confinement potential of the form $U(x) = U_0 \left[ -2 \left( \frac{x}{d} \right)^2 + \left( \frac{x}{d} \right)^4 \right]$. Here $U_0$ is the energy barrier separating two minima with $2d$ being the distance between them. A confinement potential can be easily created in the experiment using gate voltages [37]. We assume the system is dissipative, and the dissipation is a thermal effect appearing due to a coupling to the environment. The dissipation, which impacts mainly on the orbital motion, is essential for the synchronization processes we are going to discuss later in the text. For strong driving magnetic fields, the influence of the dissipation on the spin dynamics is negligibly small and can be ignored. In addition, we assume that the temperature is low enough to prevent the thermal noise effects on the over-the-barrier motion. For the particular value of $U_0 \sim 20$ meV the low-temperature regime means $T < 100$ K. For the GaAs-based structure with the electron effective mass $m$ being 0.067 of the free electron mass and $d \sim 100$ nm, the tunneling probability is small and we can neglect under-the-barrier motion. To justify the applicability of the classical picture, we estimate parameter $P^{\text{max}}_x = \sqrt{2mU_0d}/\hbar$, where $P^{\text{max}}_x$ is the maximum classical momentum for a particle with energy $E = 0$ in the given potential. For the above-chosen system parameters this ratio is larger than 20, justifying the applicability of the classical approach [46].

To quantify the SO interaction we use a coupling term of the Dresselhaus type $H_\alpha = \alpha P_x \sigma^\alpha$, where $P_x$ is the momentum of the electron and $\sigma^\alpha$ is the Pauli matrix. Therefore, the Hamiltonian of the one-dimensional system reads:

$$H = \frac{P_x^2}{2m} + U(x) + \alpha P_x \sigma^x + \mu_B g_B \frac{\sigma^z}{2} + \mu_B g_B \frac{\sigma^z}{2}.$$  

(1)
where $\mu_B$ is the Bohr magneton and $g$ is the electron Landé factor. Here $B_t(t) = B_0 \sum_{n=0}^{\infty} \delta_t(t - nT)$ is an infinite series of external magnetic field pulses with the pulse strength $B_0$, which is applied along the $z$ axis. The temporal width of the pulses applied along the $z$-axis is smaller than the interval between the pulses $\tau \ll T$ (in what follows we set $T = 1$). On the other hand, for the pulses along the $x$-axis $B_x(t) = B_0 \sum_{n=0}^{\infty} \delta_x(t - \tau n)$ the pulse duration is larger than the interval between pulses $\tau \ll T$ (see figure 1). A different route to the control of the spin dynamics in double quantum dots via electric field pulses is outlined in [42].

With the introduced characteristic maximum intensity of the electron $P_x^{\text{max}}$ we can estimate the maximum precession rate of the spin due to the SO coupling $\Omega_{so}^{\text{max}} = (2\alpha/\hbar) \sqrt{2mU_0}$, while the magnetic field pulse of the amplitude $B = B_0$ induces a spin precession with the rate $\Omega_B = |g|B_0/\hbar$. Therefore, if $\Omega_B > \Omega_{so}^{\text{max}}$ we can neglect the pulse duration of the SO coupling and the spin is completely controlled by the external driving fields. We need a protocol with two driving fields in order to fulfill the synchronization requirements as discussed later in the text. Namely, for the control of the spin dynamics via the external driving fields, the amplitudes of the fields should be large, $B_0 > (2\alpha/|g|)\sqrt{2mU_0}$. On the other hand, a strong constant magnetic field produces a high frequency precession of the spin $\Omega_B = |g|B_0/\hbar$, while for the synchronization we need to tune the precession frequency up or down keeping fixed the strong driving field amplitude. Below we will show that the optimal conditions for the synchronization are realized using two types of driving pulses. Applying short pulses along the $z$-axis $\tau \ll T$, $B_z(t) = B_0 \sum_{n=0}^{\infty} \delta_z(t - nT)$ and long pulses along the $x$-axis $B_x(t) = B_0 \sum_{n=0}^{\infty} \Delta_x(t - \tau n)$, we can realize a spin precession with a frequency that is inversely proportional to the time interval between the short pulses $\Omega \sim 1/T$ independently from the driving field strength $B_0$. In what follows, for convenience we use dimensionless units via the transformations $E \rightarrow E/4U_0$, $x \rightarrow x/d$, $P_x \rightarrow P_x/2\sqrt{mU_0}$, $\varepsilon \rightarrow \alpha \sqrt{m/2U_0}$.

3. Dissipative system and the problem of phase synchronization between orbital and spin motion

3.1. Spin dynamics in pulsed magnetic fields

As was stated above, the synchronization can occur if the frequency of the driving field is close to the eigenfrequency of the nonlinear dissipative system. If this is the case, in the particular frequency interval, the oscillations of the nonlinear dissipative system and the field can be synchronized. With the increase in the driving field amplitude, the synchronization can occur in a broader frequency interval, and the synchronization protocol becomes more efficient. Our aim is to develop a method for the synchronization of the dynamics of the electron spin and the orbital motion, using an external driving magnetic field and SO coupling. Although the magnetic field is not coupled to the orbital motion directly, a sufficiently strong field influences the orbital motion through the spin dynamics if the SO coupling is present. If the SO term is relatively small $\Omega_{so}^{\text{max}} < \Omega_B$, that is

$$\Omega_B = \frac{|g|B_0}{\hbar} > \Omega_{so}^{\text{max}} = 2\alpha \sqrt{2mU_0},$$

the spin and, correspondingly, the orbital motion can be controlled externally. From equation (1) it is easy to see that in between the short pulses the electron spin rotates around the $x$-axis and the equations of motion for the electron spin in this case read

$$\dot{\sigma}^x = 0, \quad \dot{\sigma}^z = -\Omega_B \sigma^z, \quad \dot{\sigma}^y = \Omega_B \sigma^y.$$  (3)

On the other hand, during the short pulses we have

$$\dot{\sigma}^z = -\Omega_B \sigma^y, \quad \dot{\sigma}^y = \Omega_B \sigma^z, \quad \dot{\sigma}^x = 0.$$  (4)

Considering the dynamics due to the pulse acting on the spin at the moment of time $t = t_0$, we can split the evolution operator $\hat{T}_\text{ev}$ defined as

$$\hat{\sigma}(t_0 + T) = \hat{T}_\text{ev} \hat{\sigma}(t_0^-)$$

into two parts: $\hat{T}_\text{ev} = \hat{T}_R \times \hat{T}_\sigma$, where

$$\hat{\sigma}(t_0^+^-) = \hat{T}_\sigma \hat{\sigma}(t_0^-), \quad \hat{\sigma}(t_0^+ + T) = \hat{T}_R \hat{\sigma}(t_0^+).$$

Here we introduced the notations $t_0^+ = t_0 + 0$ and $t_0^- = t_0 - 0$. The operator $\hat{T}_R$ describes the rotation of the electron spin around the $x$-axis produced by the long pulse of the external magnetic field $B_x(t) = B_0 \sum_{n=0}^{\infty} \Delta_x(t - \tau n)$, which is applied along the $x$-axis and $\hat{T}_\sigma$ corresponds to the evolution produced by the short pulses $B_z(t) = B_0 \sum_{n=0}^{\infty} \delta_z(t - nT)$ applied along the $z$-axis. Integrating equations (4) for a short time interval $t \in (t_0^-, t_0^+)$ we obtain

$$\hat{T}_\sigma(\sigma^z) = \sigma^z(t_0^-) \cos(\Omega_B \tau) - \sigma^\prime z(t_0^-) \sin(\Omega_B \tau),$$

$$\hat{T}_\sigma(\sigma^y) = \sigma^y(t_0^-) \sin(\Omega_B \tau) + \sigma^\prime z(t_0^-) \cos(\Omega_B \tau).$$

Integrating equation (3) during the time interval $t \in (t_0^-, t_0^+ + T)$ of long applied pulse we find

$$\hat{T}_R(\sigma^z) = \sqrt{1 - (\sigma^x(t_0^+))^2} \cos(\Omega_B \tau).$$

Figure 1. Schematic illustration of the infinite series of external magnetic field pulses applied to the system. A series of short pulses $B_t(t) = B_0 \sum_{n=0}^{\infty} \delta_t(t - nT)$ with the pulse width $\tau$ and time interval between pulses $T$, is applied along the $z$ axis. A series of pulses with larger width $T$ and a shorter interval between the pulses $\tau$, $B_t(t) = B_0 \sum_{n=0}^{\infty} \Delta_t(t - \tau n)$ is applied along the $x$ axis. The amplitude of the pulses $B_0$ is the same in both cases.
is accomplished by changing \( \tau \). After \( N = 1000 \) iterations, the Lyapunov exponent is negative \( \lambda(\sigma_0^x) < 0 \), meaning that the spin dynamics is regular. \( T = 1, \Omega_B \tau = 1, \Omega_B T = 20 \). At these conditions, the dependence is weak.

\[
\hat{F}_R(\sigma^z) = \sqrt{1 - (\sigma^z(0))^2} \sin(\Omega_B T). \tag{10}
\]

Combining equations (7) with (9) we finally can reconstruct the complete picture of the full time evolution of the electron spin:

\[
\begin{align*}
\sigma_{n+1}^y &= \sqrt{1 - (\sigma_{n+1}^x)^2} \cos((n + 1)\Omega_B T), \\
\sigma_{n+1}^z &= \sqrt{1 - (\sigma_{n+1}^x)^2} \sin((n + 1)\Omega_B T), \tag{11} \\
\sigma_{n+1}^x &= \sigma_n^x \cos(\Omega_B \tau) - \sqrt{1 - (\sigma_n^x)^2} \sin(\Omega_B \tau) \cos(\Omega_B \Omega_B T).
\end{align*}
\]

The recurrent relations equations (11) describe the spin dynamics. The relation (11) contains the width of the short \((\tau)\) and long \((T)\) pulses. Therefore one can adjust the spin precession frequency either by tuning \( T \), while a fine-tuning is accomplished by changing \( \tau \). This fine-tuning is important because the nonlinear synchronization requires the possibility to tune precession frequency independently of the driving amplitude. Accuracy of the employed approximations may be checked by the validity of the normalization condition \( \sigma^2 = 1 \). In order to identify, whether the nonlinear map (11) is chaotic or regular, we evaluate the Lyapunov exponent for the spin system [43]. Taking into account the peculiarity of the system (11), which is the fact that the equation for the \( x \) component \( \sigma_{n+1}^x \) is self-consistent \( \sigma_{n+1}^x = f(\sigma_n^x) \), we deduce for the Lyapunov exponent

\[
\lambda(\sigma_0^x) = \lim_{N \to \infty} \frac{1}{N} \ln \left| \frac{f^N(\sigma_0^x + \delta \sigma) - f^N(\sigma_0^x)}{\delta \sigma} \right|.
\]

Here, \( \sigma_0^x \) is the initial value of the spin projection and the small increment of the initial values \( \sigma_0^x + \delta \sigma \) quantifies the sensitivity of the recurrence relations (11) with respect to the slight change in the initial conditions. After some algebra from equations (11) and (12) we finally obtain:

\[
\lambda(\sigma_0^x) = \lim_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N-1} \ln \left| \frac{df(\sigma_n^x)}{d\sigma_n^x} \right|
= \frac{1}{N} \sum_{n=0}^{N-1} \ln \left| \cos(\Omega_B) + \frac{\sigma_n^x}{\sqrt{1 - (\sigma_n^x)^2}} \right|
\times \sin(\Omega_B) \cos(n\Omega_B). \tag{13}
\]

The results of the numerical calculations are presented in figures 2 and 3. From figure 2 we see that the Lyapunov exponent is negative, \( \lambda(\sigma_0^x) < 0 \) and therefore the spin dynamics is regular, since the initial distance between two neighboring trajectories starting from the initial points \( \sigma_0^x \) and \( \sigma_0^x + \delta \sigma \) is not increasing asymptotically after an infinite number of iterations \( \delta \sigma e^{N\lambda(\sigma_0^x)} \). Therefore, from figures 2 and 3 we conclude that the dynamics of the electron spin is controlled by the magnetic field pulses, thus following equation \( \sigma^x(t) = \sigma_0^x \cos(\Omega t) \) we achieve the spin manipulation by magnetic fields. The spin rotation frequency is determined by the time interval between the short pulses \( \Omega \approx 2\pi/T \).

Although the above discussion is based on a designed periodic sequence of pulses, it is not restricted to this time-dependent external perturbation. One can obtain controlled and periodical in time spin dynamics by using a constant magnetic field. If the magnetic field \( B \) is strong enough compared to the effective spin–orbit field to govern the spin precession, that is \( B > 2\alpha \sqrt{\Delta n_c U_0/\mu_B g} \), the spin dynamics is described by \( \sigma_z(t) = \sigma_z^{(0)} \cos(\Omega_B t) \). Consequently instead of the spin precession frequency defined by the time interval between short pulses \( \Omega \approx 2\pi/T \), in the case of applied constant magnetic field, precession frequency is proportional to the amplitude of the magnetic field. For the efficient synchronization protocol, in order to evaluate synchronization...
frequency interval, it is highly desirable to have rotation frequency independent of the field amplitude. Since \( \Omega_B = \frac{\mu_B |g| B}{\hbar} \), it may happen that amplitude of the magnetic field is too weak to control spin dynamics and suppress the influence of the SO coupling. The advantage of the pulse scheme is the absence of these theoretically imposed restrictions on the precession frequency. However, in the experiment one can get the same effect with the constant magnetic field.

3.2. Synchronization of the spin and the orbital motion

With the spin dynamics discussed in section 3.1, for the orbital motion of electron we can write the following effective Hamiltonian assuming that \( \sigma^z(t) \approx \cos(\Omega t) \):

\[
H = \frac{p_x^2}{2m} + U(x) + \alpha P_x \cos(\Omega t). \tag{14}
\]

The equation of motion for the system (14) has the form:

\[
\ddot{x} + \gamma \dot{x} - x + x^3 = -\beta \sin(\Omega t), \tag{15}
\]

where two new dimensionless quantities are introduced: \( \beta = \alpha \Omega \sqrt{m/4U_0} \) and \( \gamma = \gamma/\sqrt{4U_0 \hbar} \). We seek a solution of equation (15) using the following ansatz

\[
x(t) = \frac{1}{2} A(t) e^{i\Omega t} + \frac{1}{2} A^*(t) e^{-i\Omega t}. \tag{16}
\]

Assuming that the amplitude \( A(t) \) in equation (16) is a slow variable the following condition applies

\[
\dot{A}(t) e^{i\Omega t} + A^*(t) e^{-i\Omega t} = 0. \tag{17}
\]

Taking into account equations (16) and (17) from equation (15) we deduce:

\[
i\Omega \dot{A}(t) e^{i\Omega t} - \left( \frac{\Omega^2}{2} A(t) e^{i\Omega t} + c.c. \right) + \gamma \left( \frac{\Omega}{2} A(t) e^{i\Omega t} + c.c. \right) = - \left( \frac{1}{2} A(t) e^{i\Omega t} + c.c. \right) + \frac{1}{8} \left( A^3(t) e^{3i\Omega t} + 3|A(t)|^2 A(t) e^{i\Omega t} + c.c. \right) = \frac{\beta}{2i} (e^{i\Omega t} - c.c.). \tag{18}
\]

Multiplying equation (18) by the exponent \( e^{-i\Omega t} \) and averaging it over the fast phases we find:

\[
\ddot{\dot{A}}(t) + \left( \frac{\Omega^2 + 1}{2\Omega} \right) \dot{A}(t) + \frac{\gamma}{2} A(t) - \frac{3i}{8\Omega} |A(t)|^2 A(t) = - \frac{\beta}{2\Omega}. \tag{19}
\]

Introducing the notations

\[
\Delta = \frac{\Omega^2 + 1}{\gamma \Omega}, \quad \varepsilon = \frac{\beta}{2\Omega \sqrt{3}}, \tag{20}
\]

we can rewrite equation (19) in a more compact form

\[
\ddot{z}(\tau) + \Delta z(\tau) = -z(\tau) + \frac{3i}{\Omega} |z(\tau)|^2 z(\tau) + \varepsilon. \tag{21}
\]

Figure 4. Arnold tongue diagram in terms of the decay constant \( \gamma \) and field frequency \( \Omega \), plotted using the conditions (25). Shadowed regions define parameter values for which the synchronization is possible.

Inserting \( z(\tau) = R(\tau) e^{i\varphi(\tau)} \) into equation (21), for the real and imaginary parts we obtain

\[
\dot{R}(\tau) = -R + \varepsilon \cos \varphi(\tau), \quad \dot{\varphi}(\tau) + \Delta = \frac{3}{\Omega} R^2(\tau) - \frac{\varepsilon}{R(\tau)} \sin \varphi(\tau). \tag{22}
\]

Using equations (22) and setting \( \dot{R} = 0, \dot{\varphi} = 0 \) for the stationary solutions we obtain:

\[
f(\xi) = \xi + \xi \left( \frac{3}{\Omega^2} \xi - \Delta \right)^2 = \varepsilon^2, \tag{23}
\]

\[
\xi_{1,2} = \frac{\Omega}{3} (2\Delta \pm \sqrt{\Delta^2 - 3}), \tag{24}
\]

where \( R^2 = \xi \) and \( \xi_{1,2} \) are roots of the equation \( df(\xi)/d\xi = 0 \). In order to identify the Arnold tongue [44], which shows the regions in which a synchronization is possible, we utilize the standard condition \( df(\xi)/d\xi = 0 \). From equation (24) one can see that the roots of equation \( df(\xi)/d\xi = 0 \) are real if \( \Delta > \sqrt{3} \). Taking into account that \( \Delta = (\Omega^2 + 1)/\gamma \Omega \) we can rewrite the inequality in the form \( \Omega^2 + 1 > \gamma \Omega \sqrt{3} \). Consequently we obtain the following criteria for the synchronization

\[
0 < \Omega < \frac{1}{2} (\frac{1}{\gamma} \sqrt{3} - \sqrt{3\gamma^2 - 4}), \quad \Omega > \frac{1}{2} (\frac{1}{\gamma} \sqrt{3} + \sqrt{3\gamma^2 - 4}), \quad \gamma > \frac{2}{\sqrt{3}}. \tag{25}
\]

Graphical representation of the conditions (25) is shown in figure 4.

Criteria (25) define the synchronization area in terms of the external field frequency \( \Omega \) and the dissipation constant \( \gamma \). The minima points of the function \( f(\xi) \), (23) do not depend on the SO coupling constant \( \alpha \). Therefore criteria (25) are independent of the values of the SO coupling...
Figure 5. Arnold tongue diagram in terms of the parameters \( \Delta = (\Omega^2 + 1)/\gamma \Omega, (\Delta, \varepsilon) \) plotted using equation (26). Point b belongs to the domain where a synchronization is not possible, while point a belongs to the synchronization domain.

strength as well. Nevertheless, inserting the roots \( \xi_1, \xi_2 \) of the equation \( df/d\xi = 0 \) into equation (23) one can derive more illustrative and precise criteria in the form of the parametrical curve:

\[
\frac{2}{\sqrt{\Omega}} \left( \pm 3\sqrt{\Delta^2 - 3} + 9\Delta + 3 \mp \Delta^2\sqrt{\Delta^2 - 3} \right)
- \left( \frac{\varepsilon}{\sqrt{\Omega}} \right)^2 = 0.
\]

(26)

The parametrical curve defined by equation (26) represents the border of the synchronization domain, see figure 5. Taking into account that \( \beta = \alpha\Omega\sqrt{m/4U_0}, \varepsilon = \beta/2\Omega\gamma^{3/2} \) we obtain

\[
\frac{\varepsilon}{\sqrt{\Omega}} = \frac{\alpha}{2\sqrt{\Omega}\gamma^{3/2}} \sqrt{\frac{m}{4U_0}}.
\]

(27)

From equation (27) we see that the parameters of figure 5 depend on the oscillation frequency that can be easily controlled by tuning the time interval between pulses \( \Omega \approx 2\pi/T \). All other parameters in equation (27), such as the SO coupling constant \( \alpha \), barrier height \( U_0 \), and the electron effective mass \( m \) are internal characteristics of the system whereas the decay constant \( \gamma \) is related to the thermal effects.

Using equation (26) and figure 5 one can synchronize the electron orbital motion with its spin dynamics (see figure 6).

Figure 5 defines values of parameters for which the synchronization occurs. The suggestion for the experiment is therefore straightforward. Orbital and spin dynamics can be synchronized using either constant magnetic field or pulses. If constant magnetic field \( B_0 \) producing spin precession with the desired frequency from the Arnold tongue area \( \Omega_t = \mu_B|g|B/\hbar \) is strong enough, \( B_0 > 2\alpha\sqrt{2mU_0}/\mu_B|g| \) for the synchronization in the experiment, one can use this field. In the opposite regime synchronization can be achieved by tuning the time interval between pulses \( T = 2\pi/\Omega_1 \). Also a constant field can be more easily realized in experiment, it imposes restrictions on the system parameters. A synchronization protocol based on the sequence of pulses being more complicated is, however, more universal and works for a broad range of the parameters.

4. Conclusions

We have investigated the classical electron dynamics in a double-well confinement potential, with the spin of the electron being controlled by external magnetic fields. We have shown that the orbital electron dynamics can be controlled...
very effectively by the field in the presence of a spin–orbit coupling. Using the proposed protocol of magnetic field pulses of different duration we have shown that it is possible to synchronize the spin and the orbital motion of the electron. In particular, if the driving field amplitude is large enough, $B_0 > (2\pi/\mu_B|g|)\sqrt{2mE_0}$, the spin dynamics is periodical in time. Then $\sigma^2(t) = \sigma_0^2 \cos(2t)$, where the oscillation frequency is inversely proportional to the time interval between pulses $\Omega \approx 1/T$ and can be tuned independently from the amplitude of the pulses $B_0$. As a consequence the orbital dynamics can be studied with a reduced, effective, time-dependent, one-dimensional model. By using this model we found the synchronization condition between the orbital and the spin dynamics. Furthermore, we derived an analytical expression for the Arnold tongue that defines the values of the parameters for which a synchronization is possible. Since in the designed protocol the spin precession rate is determined by the interval between the applied pulses we believe that it can be realized in future experiments on semiconductor nanowire quantum devices.

The prototype system studied in the paper [37] can be relevant for the synchronization experiment with a GaAs- or InAs-based semiconductor quantum wire, where one can confine an electron with an appropriately chosen gate as well as bias voltages. Taking into account typical values of the parameters $a \sim 10^5 \text{ cm}^{-1}$, $U_0 \approx 25 \text{ meV}$ one can estimate the amplitude of the driving magnetic pulses (see figure 1) necessary to control the spin precession as $B_0 > 1.5 \text{T}$. Then by proper choice of the pulse durations $T = 20\text{ps}, \ T = 100 \text{ps}$ and $\tau = 5 \text{ps}$ one can synchronize orbital motion with spin dynamics. This means that the electron will oscillate with a frequency equal to the spin precession frequency, while the spin precession frequency is defined via the time interval between short pulses $\Omega \approx \frac{2\pi}{T}$. As a result, in the synchronization domain, changing $T$ one can control and modify electron oscillation frequency. The system in the experiment can be easily shifted in and out of the synchronization domain by tuning the width of the short pulses $\tau$.

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