Estimating strong decays of $X(3915)$ and $X(4350)$

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Abstract

Strong decays of $X(3915)$ and $X(4350)$ have been studied by assuming them as P-wave charmonium states. We estimate the $\Gamma(X(3915) \rightarrow J/\psi \omega)$ and $\Gamma(X(4350) \rightarrow J/\psi \phi)$ via $D\bar{D}^{(*)}$ open-charm intermediate states. Our calculation supports that the assignment of $X(4350)$ to the charmonium state while the assignment of $X(3915)$ is disfavored.

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1 Introduction

Two new charmonium-like resonances named $X(3915)$ and $X(4350)$ were observed recently by Belle Collaboration [1, 2] in the processes $\gamma\gamma \to J/\psi\omega$ and $\gamma\gamma \to J/\psi\phi$, respectively. For $X(3915)$, $M = 3915 \pm 3 \text{(stat)} \pm 2 \text{syst} \text{MeV}$ and $\Gamma = 17 \pm 10 \text{(stat)} \pm 3 \text{(syst)} \text{MeV}$; for $X(4350)$, $M = 4350.6^{+1.6}_{-0.9} \text{(stat)} \pm 0.7 \text{syst} \text{MeV}$ and $\Gamma = 13^{+18}_{-9} \text{(stat)} \pm 4 \text{(syst)} \text{MeV}$. The structure of these states has been studied extensively in the literature, the $P$-wave charmonium states [3, 4], the $D_{s0}$ and $D_{s0}^*$ molecule state [5], the $c\bar{c}s\bar{s}$ teraquark state [6], and the scalar $c\bar{c}$ and $D_{s0}^*\bar{D}_{s0}^*$ mixing state [7] and so on.

The purpose of the present paper is to further study strong decays $X(3915) \to J/\psi\omega$ and $X(4350) \to J/\psi\phi$, in order to increase our understanding in the structure of these new resonances. We will treat them as $P$-wave charmonium states, following Ref. [3], and the $J^{PC}$ quantum numbers of theirs are $0^{++}$ for $X(3915)$ and $2^{++}$ for $X(4350)$. Thus their open-charm decay amplitudes, such as $X(3915) \to D\bar{D}$ and $X(4350) \to D_{s0}^*\bar{D}_{s0}^*$ transitions, can be easily obtained using the $3p0$ model [8]. It is obvious that these open-charm intermediate states may rescatter into $J/\psi\omega$ or $J/\psi\phi$ final states, and this rescattering effects can be captured using the method [9] used in [10, 11].

The paper is organized as follows: In Sec. 2 and Sec. 3, we present the formalism used in our study, and give explicit calculations for $X(3915)$ and $X(4350)$. We summarize our results in Sec. 4. The open-charm decay amplitudes of $X(3915)$ and $X(4350)$ are fixed in the Appendix by $3p0$ model. This will help us to determine the coupling constants of $X(3915) \to D\bar{D}$ and $X(4350) \to D_{s0}^*\bar{D}_{s0}^*$, which will be used in the calculations of Sec. 2 and Sec. 3.

2 Calculation for $X(3915)$

The process $X(3915) \to J/\psi\omega$ is OZI rule suppressed, so the final state interaction (FSI) effects may play the central role. We will study if the hidden charm decay $X(3915) \to J/\psi\omega$ mainly arises from the FSI effect of $X(3915) \to D^0\bar{D}^0$ and $X(3915) \to D^+\bar{D}^-$ rather than $X(3915) \to D_{s0}^0\bar{D}_{s0}^0$ or $X(3915) \to D_{s0}^*\bar{D}_{s0}^*$ because $D^*$'s are too heavy.

The strong interactions between $X(3915)$ and $D$'s can be described by the following phenomenological Lagrangian:

$$L_{0^+DD} = g_{0^+DD}(XD^0\bar{D}^0 + XD^+\bar{D}^-)$$

The strong coupling constants $g$ can be calculated by some physical models, for example, the $3p0$ model, which will be shown in the Appendix, the numeral result is $|g_{0^+DD}| = 2760 \text{MeV}$.

The Feynman diagrams for $X(3915) \to J/\psi\omega$ through $D^+$ and $D^-$ is depicted in Figure 1.

Based on the heavy quark symmetry and chiral symmetry, the effective Lagrangian was constructed as [12, 13, 14]:

$$L_{J/\psi DD} = ig_{J/\psi DD}\psi_\mu(\partial^\mu DD\dagger - D\partial^\mu D\dagger)$$

$$L_{J/\psi DD^*} = -g_{J/\psi DD^*}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu\psi_\nu(\partial_\alpha D^*_{\beta}\dagger + D\partial_\alpha D^*_{\beta}\dagger)$$

1
By the Cutkosky cutting rule, the absorptive part of Fig.1 is written as

$$
A_{1-a} = \frac{1}{2} \int \frac{d^3p_1}{(2\pi)^32E_1} \frac{d^3p_2}{(2\pi)^32E_2} \delta^4(m_X - p_1 - p_2)(2\pi)^4 i g_{p^+DD}(-i)g_{J/\psi DD} p_1 \cdot \epsilon_{\psi}$$

$$+ \frac{i}{\sqrt{2}} g_{DDV} \epsilon_{\omega} \cdot p_2 \frac{q^2 - m_D^2}{m_D^2} \mathcal{F}^2(M_{D^-}, q)$$

(8)
\[ A_{1-c} = \frac{1}{2} \int \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \delta^4(m_X - p_1 - p_2)(2\pi)^4 i g_{0+DD}(-i) g_{J/\psi DD} p_1^\mu \varepsilon_\mu q_{a\mu\nu\alpha\beta} \]

\[ (-\sqrt{2}i) f_{D^*} D^2 \varepsilon_{\rho\kappa\eta}(q)(-i)(- g^{\rho\kappa} + q^\rho q^\kappa) (q^\alpha + p_2^\alpha) \frac{i}{q^2 - m_D^2} F^2(M_{D^*}, q) \]  

where \( F^2(m_i, q) = (\alpha^2 - m_i^2)^2 \), is a form factor which compensate the off-shell effects of mesons at the vertices and are normalized at \( q^2 = m_i^2 \). \( q = p_\psi - p_1 = p_2 - p_\omega \), \( \Lambda(m_i) = m_i + \alpha \Lambda_{QCD} \), \( m_i \) denote the mass of the exchanged particle and we choose \( \Lambda_{QCD} = 220 MeV \) here. \( \alpha \) should not be far from 1. So the total decay amplitude of \( X(3915) \to J/\psi \omega \) is:

\[ \mathcal{M}(X(3915) \to D^+ D^- \to J/\psi \omega) = 2(A_{1-a} + A_{1-c} + A_{1-e} + A_{1-g}) \]  

The factor 2 comes form the fact that the amplitudes of Figure 1-b, 1-d, 1-f, 1-h are the same with Figure 1-a, 1-c, 1-e, 1-g. The relation between decay width of \( X(3915) \) and \( \alpha \) is shown in Figure 2.

![Figure 2: The decay rate for \( X(3915) \to J/\psi + \omega \) via \( D^+ D^- \) and \( D^0 \bar{D}^0 \)](image)

3 Calculation for \( X(4350) \)

Similarly, the hidden charm decay \( X(4350) \to J/\psi \phi \), which is OZI rule suppressed, may occur through \( D_0^+ \), \( D_0^- \) and \( D_0^{*+} \), \( D_0^{*+} \) rescattering.
The strong interaction between X(4350) and $D_s$, $D_s^*$ can be described in a phenomenological way:

$$\mathcal{L}_{2+DD} = g_{2+DD} X^{\mu \nu} \partial_{\mu} D_s^+ \partial_{\nu} D_s^-$$

$$\mathcal{L}_{2+D^*D} = ig_{2+D^*D} \partial^\mu X^{\nu \alpha} \varepsilon_{\mu \nu \rho \beta} (\partial^\beta D_s^{+ \rho} \partial_{\alpha} D_s^- + \partial^\beta D_s^{- \rho} \partial_{\alpha} D_s^+)$$

$$\mathcal{L}_{2+D^*D^*} = g_{2+D^*D^*} X^{\mu \nu} D_s^{+ \mu} D_{s^*}^{- \nu}$$

The absolute value of $g_{2+DD}$, $g_{2+D^*D}$, $g_{2+D^*D^*}$ can be decided by 3p0 model which will be shown in Appendix, the numeral result is $|g_{2+DD}| = 0.002 \text{MeV}^{-1}$, $|g_{2+D^*D}| = 3.30 \times 10^{-7} \text{MeV}^{-2}$, $|g_{2+D^*D^*}| = 700 \text{MeV}$. The Feynman diagrams for $X(4350) \rightarrow \bar{J}/\psi + \phi$ are shown in Figure 3.

![Feynman Diagrams](image)

Figure 3: The diagrams for $X(4350) \rightarrow \bar{J}/\psi + \phi$ via $D_s$ and $D_s^*$ assuming $X(4350)$ is $\chi_{c2}''$

The absorptive part of Fig.2 is written as

$$A_{2-a} = \frac{1}{2} \int \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \delta^4(m_X - p_1 - p_2)(2\pi)^4(-i)g_{2+DD} p_1 \cdot p_2 \varepsilon_X^{\mu \nu}

(-i g_{\bar{J}/\psi DD}) \varepsilon_a^*(\psi)(p_1^a - q^a)(-i)g_{DDV} (q_3 + p_2) \varepsilon_\phi^* \frac{i}{q^2 - m_{D_s}^2} F^2(M_{D_s}, q)$$

(16)
\[
A_{2-e} = \frac{1}{2} \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \delta^4(m_X - p_1 - p_2)(2\pi)^4 i g_{2+DD} \kappa p_1 \kappa p_2 \phi \\
\quad \quad \quad \times \left[2(p_2^r + q^r) \right]
\]
\[
\frac{i}{q^2 - m_D^2} F^2(M_{D^*}^-, q)
\]

(17)

\[
A_{2-g} = \frac{1}{2} \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \delta^4(m_X - p_1 - p_2)(2\pi)^4 (-i) g_{2+DD} \kappa p_1 \kappa p_2 \phi \\
\quad \quad \quad \times \left[2(p_2^r + q^r) \right]
\]
\[
\frac{i}{q^2 - m_D^2} F^2(M_{D^*}^+, q)
\]

(18)

\[
A_{2-j} = \frac{1}{2} \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \delta^4(m_X - p_1 - p_2)(2\pi)^4 (-i) g_{2+DD} \kappa p_1 \kappa p_2 \phi \\
\quad \quad \quad \times \left[2(p_2^r + q^r) \right]
\]
\[
\frac{i}{q^2 - m_D^2} F^2(M_{D^*}^+, q)
\]

(19)

\[
A_{2-l} = \frac{1}{2} \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \delta^4(m_X - p_1 - p_2)(2\pi)^4 (-i) g_{2+DD} \kappa p_1 \kappa p_2 \phi \\
\quad \quad \quad \times \left[2(p_2^r + q^r) \right]
\]
\[
\frac{i}{q^2 - m_D^2} F^2(M_{D^*}^+, q)
\]

(20)

\[
A_{2} = \frac{1}{2} \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \delta^4(m_X - p_1 - p_2)(2\pi)^4 (-i) g_{2+DD} \kappa p_1 \kappa p_2 \phi \\
\quad \quad \quad \times \left[2(p_2^r + q^r) \right]
\]
\[
\frac{i}{q^2 - m_D^2} F^2(M_{D^*}^+, q)
\]

(21)
\[
A_{2-m} = \frac{1}{2} \int \frac{d^3p_1}{(2\pi)^32E_1} \frac{d^3p_2}{(2\pi)^32E_2} \delta^4(m_X - p_1 - p_2)(2\pi)^4ig_{2+D^*D^*\varepsilon_X^\lambda(-i)g_{J/\psi DD^*\varepsilon^{\mu\alpha\beta}}p_{\mu\varepsilon^*_\psi\lambda}p_{1\alpha}
\]
\[
\left(-i\right)2f_{D^*D^*V}\varepsilon_{\rho\sigma\eta}(q_\tau + p_2\varepsilon^*_\sigma\rho\eta(-g_{\kappa\lambda} + \frac{p_{\mu\varepsilon^*_\psi\lambda}}{m_{D^*}^2})(-g_{\lambda\eta} + \frac{p_{\lambda\eta}}{m_{D^*}^2})
\]
\[
\frac{i}{q^2 - m_{D^*}^2}F^2(M_{D^*}, q)
\]

\[
A_{2-o} = \frac{1}{2} \int \frac{d^3p_1}{(2\pi)^32E_1} \frac{d^3p_2}{(2\pi)^32E_2} \delta^4(m_X - p_1 - p_2)(2\pi)^4ig_{2+D^*D^*\varepsilon_X^\lambda(-i)g_{J/\psi DD^*\varepsilon^{\mu\alpha\beta}}p_{\mu\varepsilon^*_\psi\lambda}p_{1\alpha}
\]
\[
\left(-i\right)4f_{D^*D^*V}\varepsilon_{\rho\sigma\eta}^\beta(-i\beta - i\beta^2(-g_{\sigma\eta} + \frac{p_{\sigma\eta}}{m_{D^*}^2}))
\]
\[
\frac{i}{q^2 - m_{D^*}^2}F^2(M_{D^*}, q)
\]

\(\varepsilon_X\) denote the polarization tensors of \(X(4350)\) which can be constructed from the polarization vector of massive vector bosons as follows:

\[
\varepsilon_{\mu\nu}^\lambda = \{\sqrt{2}\varepsilon_\mu^+\varepsilon_\nu^+, (\varepsilon_\mu^0\varepsilon_\nu^0 + \varepsilon_\mu^0\varepsilon_\nu^0), \frac{1}{\sqrt{3}}(\varepsilon_\mu^+\varepsilon_\nu^- + \varepsilon_\mu^-\varepsilon_\nu^+ - 2\varepsilon_\mu^0\varepsilon_\nu^0), (\varepsilon_\mu^0\varepsilon_\nu^0 + \varepsilon_\mu^0\varepsilon_\nu^0), \sqrt{2}\varepsilon_\mu^-\varepsilon_\nu^-\}
\]

The polarization tensor is traceless, transverse and orthogonal: \((\varepsilon_\lambda^\mu)^\mu = 0, k_\mu\varepsilon_\mu^\lambda = 0, \varepsilon_\mu_\nu^\lambda\varepsilon_\mu_\nu^\lambda = 2\delta_\nu^\sigma, \lambda\) could be 2 or 1 or 0 or -1 or -2. And

\[
\sum_{\lambda=1}^5 \varepsilon_{\mu\nu}^\lambda\varepsilon_{\alpha\beta}^\lambda = B_{\mu\nu,\alpha\beta}(k).
\]

\[
B_{\mu\nu,\alpha\beta}(k) = (g_{\mu\alpha} - \frac{k_\mu k_\alpha}{m^2})(g_{\nu\beta} - \frac{k_\nu k_\beta}{m^2}) + (g_{\mu\beta} - \frac{k_\mu k_\beta}{m^2})(g_{\nu\alpha} - \frac{k_\nu k_\alpha}{m^2})
\]
\[
-\frac{2}{3}(g_{\mu\nu} - \frac{k_\mu k_\nu}{m^2})(g_{\alpha\beta} - \frac{k_\alpha k_\beta}{m^2})
\]

It is obvious that \(k_\mu B_{\mu\nu,\alpha\beta} = 0, B_{\mu,\alpha\beta} = 0\), for details, see [16] (The Appendix part)

Similar with \(X(3915), A_{2-b} = A_{2-a}, A_{2-c} = A_{2-d}, A_{2-f} = A_{2-e}, A_{2-h} = A_{2-g}, A_{2-j} = A_{2-i}, A_{2-l} = A_{2-k}, A_{2-n} = A_{2-m}, A_{2-p} = A_{2-o}\), so:

\[
M(4350) = 2(A_{2-a} + A_{2-c} + A_{2-e} + A_{2-g} + A_{2-j} + A_{2-l} + A_{2-m} + A_{2-o})
\]
We are unable to calculate the width of $X(4350) \rightarrow J/\psi \phi$ since we can’t decide the relative phase between $g_{2+DD}$, $g_{2+D^*D}$ and $g_{2+D^*D^*}$. But we can check which channel is dominance by assuming $X(4350)$ decay to $J/\psi$ and $\phi$ in one of the three way. We show the result in Figure 4. It is clear that $D_s^{*+}D_s^{*-}$ is the dominance channel.

![Graph](image)

Figure 4: (Color on line) The decay rate for $X(4350) \rightarrow J/\psi + \phi$ via $D_s^{*+}D_s^{*-}$ (green line), $D_s^+D_s$ (blue line), and $D_s^{*+}D_s^{*-}$ (red line).

## 4 Summary

Using 3p0 model to fix the coupling constants, we studied the hidden charm decays of $X(3915) \rightarrow J/\psi \omega$ and $X(4350) \rightarrow J/\psi \phi$ by assuming $X(3915)$ and $X(4350)$ are P-wave charmonium states. It seems that this assumption works well for $X(4350) \rightarrow J/\psi \phi$. While things are not very good for $X(3915)$ since the experimental results implied that $\Gamma(X(3915) \rightarrow J/\psi \omega)$ should be around 1 MeV [1]. Thus $X(3915)$ may not be regarded as a pure charmonium state. This has been also pointed out [17] and [4].
Appendix: The coupling constant

The 3p0 model use to do a good job in calculating the OZI allowed decay width in meson’s strong decay. So we can use it to study the amplitude of \( X(3915) \rightarrow D^+D^- \) and \( X(4350) \rightarrow D_s^+D_s^- \), then decide the coupling constants \( g_{0+DD}, g_{2+DD}, g_{2+D^*D} \) and \( g_{2+D^*D^*} \). When a heavy quark meson like \( X(4350) \) or \( X(3915) \) decays, a pair of light quark-antiquark created from the vacuum with the vacuum’s quantum number 0\(^{++} \). The pair should be singlet in color, flavor, of zero momentum and zero total angular momentum. The parity should be positive. So \( L = 1, S = 1 \). Then the quark-antiquark separate and enter into different mesons. See Figure 5. The transition matrix \( T \) is defined as:

\[
S = 1 - 2\pi i\delta(E_f - E_i)T
\]

and express \( T \) in the form of nonrelativistic limit:

\[
T = -\int d^3p_3 d^3p_4 [3\gamma\delta(p_3 + p_4) \sum_m <1,m,1,-m|0,0> |k|Y_{1,-m}(k\varphi_0\omega_0)|b\dagger(p_3)d\dagger(p_4)|A.2>
\]

\( \varphi_0 \) is for \( SU(3)_F \) singlet \( \varphi_0 = -(u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3} \), \( \omega_0 \) for color-singlet and \( \chi_{1,-m} \) for triplet state of spin \( \frac{8}{3} \). \( Y_{1,m} \) is the spheric harmonic function, reflecting the \( L = 1 \) orbital angular momentum of the pair. The quark-antiquark pair is created by \( b\dagger(p_3) \) and \( d\dagger(p_4) \). \( k = (m_i k_j - m_j k_i)/(m_i + m_j) \) is the relative momentum between the quark and the antiquark within a meson. \( \gamma \) is a dimensionless constant that correspond to the strength of the transition. The state of the meson \( A \) can be written as:

\[
|A(J,M_J, L, S) > = \sqrt{2E_A} \sum_{M_L,M_S} <LM_L.SM_S|J M_J > \int d^3p_1d^3p_2\delta(k_A - p_1 - p_2) \Psi_{nLM_L}(k)\chi_{SM_S}\varphi_A\omega_A\Psi_{p_1\dagger}p_2\dagger|0 >
\]

The states of B and C are similar. \( \Psi_{nLM_L}(k) \) is the spatial part of a meson’s function. In this work, we use the simple harmonic oscillator (SHO) potential as the interaction potential function inside a meson. So \( \Psi_{nLM_L}(k) \) could be expressed as [18]:

\[
\Psi_{nLM_L}(k) = (-1)^n(-i)^L R^{3/2} \sqrt{2n!\Gamma(n + L + 3/2)}(kR)^L exp(-2k^2R^2)L_{n+1/2}(k^2R^2)Y_{LM_L} \quad (A.4)
\]
\(L_n^{L+1/2}(k^2R^2)\) is the Laguerre polynomial. R is a parameter comes from the potential function. If we take the center of mass frame of meson A, the element of the T matrix can be expressed as:

\[
<BC|T|A> = \sqrt{8E_AE_BE_CE_C}\sum_{M_{LA},M_{SA},M_{LC},M_{SC},m} L_AM_{LA}S_AM_{SA}|J_AM_JA>
\]

\[
<LM_BLM_SM_B|J_BM_JB><LM_CLC_SM_C|J_CM_JC><1m;1-m|000>
\]

\[
<\varphi_B^\dagger\varphi_C|\varphi_A\varphi_0><\chi_SM_SM_SM_S|\chi_SM_SM_S\chi_1-\delta_m>
\]

\[
<\omega_B(1,3)|\omega_C(2,4)|\omega_A(1,2)P(3,4)> I_{ML_A,m}^{LM_B,M_{LC}}(k)
\]

(A.5)

The spatial integral is:

\[
I_{ML_A,m}^{LM_B,M_{LC}}(k) = \int d^3p_1d^3p_2d^3p_3d^3p_4\delta(p_1+p_2)\delta(p_3+p_4)\delta(k_B-p_1-p_3)\delta(k_C-p_2-p_4)
\]

\[
\times\Psi^*_{n_BLM_B}(p_1,p_3)\Psi^*_{n_CLC,M_{LC}}(p_2,p_4)\Psi_{n_ALA,LM_A}(p_1,p_2)|\frac{p_3-p_1}{2}|Y_{lm}
\]

(A.6)

In this way, the amplitudes of \(X(3915) \rightarrow D^+D^-\) or \(D^0\bar{D}^0\) and \(X(4350) \rightarrow D_s^0D_s^+\) can be expressed like:

\[
\mathcal{M}(X(3915) \rightarrow D^+D^-) = \frac{\sqrt{2}}{3\sqrt{3}}\sqrt{E_AE_BE_CE_C}\gamma[2I_{0,0}^{1,-1} - I_{0,0}^{0,0}]
\]

(A.7)

\[
\mathcal{M}(X(3915) \rightarrow D^0\bar{D}^0) = \frac{\sqrt{2}}{3\sqrt{3}}\sqrt{E_AE_BE_CE_C}\gamma[2I_{0,0}^{1,-1} - I_{0,0}^{0,0}]
\]

(A.8)

\[
\mathcal{M}(X(4350) \rightarrow D_sD_s) = \frac{2}{3\sqrt{15}}\sqrt{E_AE_BE_CE_C}\gamma[I_{0,0}^{1,-1} + I_{0,0}^{0,0}]
\]

(A.9)

\[
\mathcal{M}(X(4350) \rightarrow D_s^0D_s^+) = \frac{2}{3\sqrt{10}}\sqrt{E_AE_BE_CE_C}\gamma[I_{0,0}^{1,-1} + I_{0,0}^{0,0}]
\]

(A.10)

\[
\mathcal{M}(X(4350) \rightarrow D_s^+D_s^-) = \frac{2\sqrt{2}}{9}\sqrt{E_AE_BE_CE_C}\gamma[2I_{0,0}^{1,-1} - I_{0,0}^{0,0}]
\]

(A.11)

\(\gamma = 6.3^{\pm 1.9}\) for the quark-antiquark pair creation of \(u\bar{u}\) and \(d\bar{d}\). \(\gamma = 6.3/\sqrt{3}\) for \(s\bar{s}\) \[20\]. We take \(m_c = 1.6 \text{GeV}, m_u = m_d = 0.22 \text{GeV}, m_s = 0.419 \text{GeV}\). According to \[19\], \(R_D = 1.52 \text{GeV}^{-1}, R_{D_s} = 1.41 \text{GeV}^{-1}, R_{D_{s}^*} = 1.69 \text{GeV}^{-1}\). It was decided in \[3\] that \(R_X(3915) = 1.80 \sim 1.99 \text{GeV}^{-1}, 1.92 \text{GeV}^{-1}\) for the central value of decay width and \(R_X(4350) = 1.8 \sim 3.0 \text{GeV}^{-1}\), central value is not available. We will choose \(R_X(3915) = 1.92 \text{GeV}^{-1}, R_X(4350) = 1.90 \text{GeV}^{-1}\) in this work. The decay rates could be calculated in this way: \(\Gamma(X(3915) \rightarrow D^+D^-) = 8.446\text{MeV}, \Gamma(X(4350) \rightarrow D_{s}^{+}D_{s}^{-}) = 0.518\text{MeV}, \Gamma(X(4350) \rightarrow D_{s}^{+}D_{s}^{-}) = 2.915\text{MeV}, \Gamma(X(4350) \rightarrow D_{s}^{+}D_{s}^{-}) = 1.282\text{MeV}\). Then we find the numerical result for the coupling constant is

\[
\begin{align*}
|g_{0+DD}| &= 2370\text{MeV} \\
|g_{2+DD}| &= 0.002\text{MeV}^{-1} \\
|g_{2+D^+D^-}| &= 3.30 \times 10^{-7}\text{MeV}^{-2} \\
|g_{2+D^+D^-}| &= 700\text{MeV}
\end{align*}
\]

(A.12)
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