LOW X INELASTIC ELECTRON SCATTERING FROM GENERALIZED VECTOR DOMINANCE

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Abstract

It is shown that the HERA experimental data on deep inelastic scattering at low values of the scaling variable \( x \leq 0.05 \) are in good agreement with predictions from Generalized Vector Dominance in the full kinematic range from \( Q^2 = 0 \) (photoproduction) to \( Q^2 \approx 350 \) GeV.

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This morning we heard several talks on various aspects of deep inelastic scattering (DIS) of electrons on protons, in particular in the energy range presently explored by the H1 and ZEUS collaborations at HERA in Hamburg. In the talk by Kwiecinski \(^1\) theory and experiment were confronted at all values of the Bjorken scaling variable \( 0 < x_{bj} < 1 \) excluding the \( Q^2 = 0 \) photoproduction limit. Spiesberger \(^2\) and Kalinowski \(^3\) concentrated on the anomalous events observed \(^4\) by the H1 and ZEUS collaborations at HERA at large values of \( x \equiv x_{bj} \). In the present talk, essentially based on a recent paper \(^5\) by Hubert Spiesberger and myself, I will examine the region of small \( x \approx Q^2 \ll 10^{-2} \), including the photoproduction, \( Q^2 = 0 \), limit.

The motivations for this work are twofold:

i) At HERA two interesting experimental results at low \( x \) were established since HERA started operating in 1992: First of all, the proton structure function \( F_2(x,Q^2) \) rises steeply with decreasing \( x \leq 10^{-2} \) and shows a considerable amount of scaling violation \(^6\). Secondly, when analysing the final hadronic state produced, the H1 and ZEUS collaborations found an appreciable fraction of final states (approximately 10% of the total) of typically diffractive nature ("large rapidity

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gap events") with invariant masses of the produced hadronic state up to about 20 GeV [7].

ii) With respect to DIS at small $x$, a long-standing theoretical question concerns the role of the variables $x$ and $Q^2$. This question has been most succinctly posed and discussed by Sakurai and Bjorken, as recorded in the Proc. of the '71 Electron Photon Symposium at Cornell University [8]. It concerns the transition to the hadronlike behaviour of photoproduction, in particular, whether a behaviour similar to photoproduction sets in in the limit of $Q^2 \to 0$ only, or rather in the limit of $x \to 0$ at arbitrarily large fixed values of $Q^2$. Even after the formulation of QCD there is not yet a unique answer to this question. It is likely that the HERA low $x$ data in conjunction with theoretical analysis will resolve this important issue.

In the paper by Spiesberger and myself, we take the point of view that indeed $x$ is the relevant variable, in the sense that $x \leq 10^{-2}$ defines the region in which those features of the virtual photoproduction cross section, $\sigma_{\gamma p}$, become important which show a close similarity to photoproduction and hadron-induced processes (Generalized Vector Dominance [9]).

As a starting point, let me briefly remind you of photoproduction. "Hadronlike" behaviour of photoproduction is quantified by relating [10] the high-energy forward Compton scattering amplitude on protons to the forward amplitude of vector-meson proton scattering, and consequently also to vector-meson forward production extrapolated to $t = 0$ (where $t$ is the four-momentum transfer squared to the initial proton),

$$\sigma_{\gamma p}(W^2) = \sum_{J=0,\omega,\phi,J/\psi} \sqrt{16\pi} \sqrt{\frac{\alpha}{\gamma^2}} \left( \frac{d\sigma}{dt} \bigg|_{t=0} (W^2) \right)^{\frac{1}{2}}.$$  

The photon vector-meson couplings have to be extracted from $e^+e^-$ annihilation, i.e.

$$\frac{\alpha}{\gamma^2} = \frac{1}{4\pi^2\alpha} \sum_{F} \int \sigma_{e^+e^- \to V \to F}(s) ds,$$

where the integral is extended over the peak of the cross section corresponding to production of the vector meson $V$ with subsequent decay into the final state $F$. The sum rule (3) may be rewritten as

$$\sum_{V} r_V = 1$$

with

$$r_V = \frac{1}{\sigma_{\gamma p}} \sigma_{\gamma V} = \frac{1}{\sigma_{\gamma p}} \sqrt{\frac{\alpha}{\gamma^2}} \left( \frac{d\sigma}{dt} \bigg|_{t=0} (\gamma p \to V p) \right)^{\frac{1}{2}}.$$  

(4)
Experimentally, from $e^+e^-$-annihilation and photoproduction, one finds that $\rho^0, \omega$ and $\phi$ fail to saturate the sum rule at the level of 22%, as \[ \sum_{\rho^0,\omega,\phi} r_V = 0.78. \] (5)

Generalized Vector Dominance \[ \text{starts from the hypothesis that the 22\% deficiency in the sum rule (3) is made up by the contribution of the more massive states directly produced in } e^+e^- \text{-annihilation. Indeed, the propagation of these more massive states increases their weight at spacelike four-momenta of the (virtual) photon considerably, compared with the } \rho^0, \omega, \phi \text{ contributions which are of minor importance, once } Q^2 \text{ becomes large compared with the mass of the } \rho^0 \text{ meson, } Q^2 \gg m_{\rho}^2. \text{ Accordingly, in low } x \text{ DIS in Generalized Vector Dominance, one expects an appreciable signal for diffractive production of high-mass states. Indeed, the HERA experiments found such a signal \[.\] Moreover, shadowing in inelastic scattering from complex nuclei, as a result of diffractive production of high-mass states, was expected to persist at low } x \text{ and large } Q^2, \text{ which is indeed the case \[ in semiquantitative agreement with the predictions \[.\] These features of DIS, persistance of shadowing for spacelike } Q^2 \text{ and diffractive production of high-mass states, are suggestive of treating low-} x \text{ DIS quantitatively from the point of view of Generalized Vector Dominance. \[}

In the diagonal approximation, the transverse part of the photon absorption cross section, $\sigma_T(W^2, Q^2)$, reads \[ \sigma_T(W^2, Q^2) = \int \rho_T(W^2, m^2) dm^2 \frac{m^4}{(m^2 + Q^2)^2}, \] (6)

where the spectral weight-function, $\rho_T$, is proportional to the product of the cross section of $e^+e^-$-annihilation into hadrons at the energy $m^2$ and the hadronic cross section for the scattering of the state of mass $m^2$ on the nucleon,

$$\rho_T(W^2, m^2) = \frac{1}{4\pi^2\alpha} \sigma_{e^+e^-}(m^2) \sigma_{\text{hadr}}(W^2, m^2).$$ \] (7)

The cross section $\sigma_{\text{hadr}}$ is clearly to be identified with the total cross section for scattering on the nucleon of the state of mass $m$, which apart from being producible in $e^+e^-$-annihilation, should also be visible in diffractive production by (virtual) photons on protons ("large rapidity gap events").

In the recent paper \[ we concentrated on evaluating \[ in the high-energy limit, $W \geq 60$ GeV, where hadronic and photoproduction cross sections rise with increasing energy. We adopted an ansatz with a logarithmic rise,

$$\rho_T(W^2, m^2) = N \frac{\ln(W^2/am^2)}{m^4},$$ \] (8)
which obviously fails to describe experimental data a lower energy. The $m^{-4}$ dependence contains an $m^{-2}$ factor from $e^+e^-$ annihilation and an $m^{-2}$ factor from the subsequent interaction of this state with the nucleon. This latter $m^{-2}$ factor has to be considered as a theoretical input which is enforced, if powerlike (in $Q^2$) scaling violations of the proton structure function $F_2$ are to be excluded. Bjorken argues \[8, 14\] that jet alignment may be the origin of the $m^{-2}$ decrease in the scattering of the state of mass $m \gg m_\rho$ from the nucleon, even though in $e^+e^-$ annihilation the 2-jet $(q\bar{q})$ configurations are not very pronounced in the mass range ($m < 20$ GeV) relevant at present HERA energies. The threshold mass $m_0 \approx m_\rho$ in (4) in principle is determined by $e^+e^-$ annihilation, while the normalization $N$ and the scale $a$ of the energy dependence are determined by the $Q^2 = 0$ photoproduction limit of $\sigma_T$ in (6) upon substitution of (8). In the recent paper [5], these parameters were actually determined by a fitting procedure which included $Q^2 \neq 0$ electron-scattering data as well as $Q^2 = 0$ photoproduction.

Before turning to the analysis of the experimental data, the extension of (6) to production of hadrons by longitudinal photons, $\sigma_L(W^2, Q^2)$, has to be given. Introducing the ratio $\xi$ of longitudinal-to-transverse (on-mass-shell) scattering of the state of mass $m$, we have

$$\sigma_L(W^2, Q^2) = \int_{m_0^2}^m dm^2 \frac{\xi \rho_T(W^2, m^2)m^4}{Q^2(m^2 + Q^2)} \frac{Q^2}{m^2},$$

where the factor $Q^2/m^2$ originates from the coupling of the hadronic vector state of mass $m$ to a conserved source \[15\]. The integration in (6) and (9) may be carried out in closed form. For the resulting expressions we refer to the original paper [5].

From the fit to the H1 and ZEUS data [11] we obtained

$$N = 0.187 \times 4\pi^2\alpha = 0.054,$$
$$m_0^2 = 0.89\text{GeV}^2,$$
$$\xi = 0.171,$$
$$a = 15.1.$$ (10)

The fact that the threshold mass $m_0$ is somewhat larger than the $\rho^0$ mass, $m_\rho$, is presumably due to our very simplified ansatz which does not discriminate between the different thresholds associated with the light-quark and the charm-quark masses. Figure 1 shows remarkably good agreement for

$$\sigma_{\gamma\rho^0} \equiv \sigma_T + \sigma_L$$ (11)

with the experimental data over the full $Q^2$ range from photoproduction, $Q^2 = 0$, to $Q^2 \approx 350\text{GeV}^2$ at energies from $W \approx 60$ GeV to $W \approx 245$ GeV, corresponding to values of the scaling variable $x \leq 0.05$. 
Fig. 1. Generalized Vector Dominance prediction for $\sigma_{\gamma^* p}$ compared with the experimental data from the H1 and ZEUS collaborations at HERA.

Figure 2 shows the proton structure function

$$F_2(W^2, Q^2) \simeq \frac{Q^2}{4\pi^2\alpha}(\sigma_T + \sigma_L)$$

as a function of $x \simeq Q^2/W^2$ for various values of $Q^2$. This Figure explicitly shows that the theoretical prediction for the transverse part of $F_2$, due to
σ_T, in our simple ansatz has reached its scaling limit for \( Q^2 > 12 \text{GeV}^2 \).
The rise of \( F_2(W^2, Q^2) \) with increasing \( Q^2 \) for \( Q^2 > 12 \text{GeV}^2 \) is due to the influence of \( \sigma_L \). For details we refer to the original publication [5].

Fig. 2. Generalized Vector Dominance prediction for \( F_2 \) as a function of \( x \) compared with the HERA data for different values of \( Q^2 \). The dotted lines show the contribution to \( F_2 \) due to transverse virtual photons (\( \xi = 0 \)). The dashed curves indicate the region \( W \lesssim 60 \text{GeV} \) in which the present model becomes inadequate.
Various refinements of the present work are to be carried out, such as the extension of this high-energy model to lower values of $W$, the incorporation of the charm threshold, an analysis of how additional scaling violations in $\sigma_T$ may appear within the present non-perturbative ansatz, etc. Further experimental tests consist of separating $\sigma_L$ and $\sigma_T$ experimentally, a project for the more distant future, while more detailed experimental studies of diffractive production in the nearby future will provide tests of the underlying theoretical assumptions.

Various parametrizations of the experimental data on low $x$ deep inelastic scattering, including photoproduction, exist in the literature, either based on [16, 17, 18] modifications of Regge theory or on a combination [19] of $\rho^0, \omega, \phi$ dominance with the parton-model approach. The fit of the data presented in [20] is of interest in the context of the present paper, as logarithmic $Q^2$ and $x$ dependences only are employed in the fit. An analysis of the data which in its spirit is similar to the one of the present paper, even though different in detail, is given in [21].

In summary, it has been shown that the framework of Generalized Vector Dominance is able to provide a unified representation of photoproduction and the low-$x$ proton structure function in the kinematic range accessible to HERA which is in good agreement with experiment. Details are subject to improvement and change in close collaboration between theory and experiment. The principal dynamical ansatz, relating $\sigma_{\gamma^*p}$, or, equivalently, $F_2$ at low values of $x$ to diffractive scattering (via unitarity) of the states produced in $e^+e^-$ annihilation, is likely to stand the test of time.

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