Deformability analysis of anisotropic rock

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Abstract. Evaluating anisotropic mechanical properties can help predict the behavior of rock materials in analysis, design, and construction, and improves the quality and safety. This research discussed laboratory testing and analytical methods to determine the four values of elastic constants of transversely isotropic slate rock. In this study, a combination of experimental and analytical methods has been used to determine the four elastic constants of transverse isotropic rocks in the laboratory. Analytic calculations were performed to determine the elastic constants of rock material with the assumption of linear, elastic, homogeneous, and transversely isotropic. The strain value was determined at 50% of peak stress on stress-strain curves. Multilinear regression analysis with least squares estimation method was used in determining the linear equation to get the four elastic constants of the rock. The results of uniaxial compression tests revealed that for the slate rock, the deformability in the direction that is normal to the plane of transverse isotropy (θ = 85°) is greater than that is parallel to the plane of transverse isotropy (θ = 5°) or, in other words, E1 > E2.

1. Introduction

Many rocks that are exposed on the surface of the earth have a basic structure in the form of bedding, foliation, fissure, or joint [1]. In general, rocks have different properties (physical, dynamic, thermal, mechanical, and hydraulic) according to their direction and these are called anisotropic properties [2,3]. Knowing the mechanical properties of anisotropic rocks, can help predict rock behavior in design, analysis, and construction also improve quality and safety. The elastic constants of anisotropic rocks also influence deformation and design analysis [4]. Therefore, it is very important to be able to estimate elastic constants from anisotropic rocks quickly and accurately.

In analysis and research, anisotropic material is simplified into orthotropic and transverse isotropes [5]. In transverse isotropic rock analysis there are actually five elastic constants needed in the calculation, namely: Young modulus E1 and Poisson ratio v1 from the transverse isotropic plane, Young modulus E2, Poisson ratio v2 and shear modulus G2 from the normal direction to the transverse isotropic plane, but in this study there are only four elastic constants that can be obtained namely E1, v1, E2, and v2. The shear modulus value of G2 cannot be obtained in the calculation due to limitations on the strain measuring instrument used in laboratory tests. There are several methods that can be used to determine the elastic constants of anisotropic rocks, including in situ tests, laboratory tests and numerical analysis methods [3-5]. Laboratory tests can be divided into tests by dynamic and static methods. Dynamic methods include the resonant bar method and the ultrasonic pulse method. Static methods include the uniaxial compression test, conventional triaxial compression test, true triaxial compression test, hollow cylinder test, bending test, torsion test, and diametral compression test (Brazilian test) [6,7].
The elastic constants of anisotropic rocks can be calculated by substituting the loading force and strain data recorded in the experiment into the stress-strain equation. To determine the five free elastic constants in transverse isotropic rocks, Kim et al made a cylindrical example with one direction of loading required for testing with a uniaxial compression test [2]. In this study, a combination of experimental and analytical methods has been used to determine the four elastic constants of transverse isotropic rocks in the laboratory.

2. Methodology

2.1. Constitutive equations in uniaxial compressive strength tests

In this study the value of elastic constants contained in transverse isotropic material, is determined by constitutive relationships through Hooke's law [3]. One example of transverse isotropic material, among others, is the existence of a bedding plane. The plane of this layer can have different tilt variations, ranging from parallel to perpendicular to the horizontal plane. Under these conditions, rocks will have different responses when receiving stress [7].

One of the laboratory test methods for determining the elastic constants of rocks is the uniaxial compressive strength test [8]. In a uniaxial compressive strength test, the sample only accepts stresses in the direction of its main axis. Therefore, the stresses on the other two axes is zero. The z-axis is considered as the main axis perpendicular to the transverse isotropic plane, while the x and y axes are the axis parallel to the transverse isotropic plane. To get the constitutive equation on the transverse isotropic material equation (1a and 1b) is used:

\[
\varepsilon = [A] [\sigma] \tag{1a}
\]

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{xy} \\
\gamma_{xz} \\
\gamma_{yz}
\end{bmatrix} =
\begin{bmatrix}
1 & -\nu_{xy} & -\nu_{xz} & 0 & 0 & 0 \\
-\nu_{yx} & 1 & -\nu_{yz} & 0 & 0 & 0 \\
-\nu_{zx} & -\nu_{zy} & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{xy}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{xz}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{yz}}
\end{bmatrix}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{xy} \\
\tau_{xz} \\
\tau_{yz}
\end{bmatrix} \tag{1b}
\]

while the z axis is the elastic symmetry axis of rotation.
In equation (1b), it is assumed that the angle formed by the plane of the plane of the horizontal plane is zero, $\theta = 0^\circ$ (Figure 1). Therefore, there will be no function of the angle $\theta$ in the constitutive equation. When the bedding plane forms a certain angle to its horizontal plane (Figure 2), the elastic constants obtained from global coordinates are transformed into their local coordinates.

Because in the uniaxial compressive strength test $\sigma_x = \sigma_y = 0$, the actual matrix equation value sought is [6].
The compliance matrix $[A]$ above is then transformed into its local coordinates with equation (3). So that the elastic constant value is obtained when the layer field has a slope that varies with the horizontal plane.

2.2. Uniaxial compressive strength test

Slate stone was chosen as the rock used in the study. Because this rock has a direction of foliation that appears on its surface, it is categorized as a transversely isotropic material [5]. To get an example with varying angles, two slate stone blocks were cored with a core bit diameter used is 45 mm. Next the rock block is placed under the core drill machine (core drill) in the direction of the slope to the predetermined bedding plane. In the first rock block, we get an example of coring with a slope that is relatively flat and upright to its horizontal plane. Whereas in the second block of rocks, we get examples with slopes varying between $35^\circ$ to $75^\circ$ with respect to the horizontal plane. A splash of water is provided to help simplify the coring process.

Figures 3 and 4 show the variation in the UCS value and the modulus of elasticity with respect to the angle $\theta$ results from the uniaxial compressive strength test in the laboratory. The modulus of elasticity is calculated from the linear portion of the stress-strain curve [4], so that the average modulus of elasticity is obtained.

![Figure 3. Variation of Uniaxial Compressive Strength with respect to angle $\theta$.](image-url)
3. Results and discussion

3.1. Transformation matrix compliance

From Figures 1 and 2 above it can be seen that the x and y axes lie in one plane of elastic symmetry x, y. This means that all mechanical properties contained in the x direction will have properties similar to those in the y direction. Thus, the elastic constant decreases from 9 in equation (1b) to 5 in equation (2) or in its general form [3,6].

The constitutive equation for the transversely isotropic material will be simplified by the following assumptions like equation (2):

\[ E_x = E_y = E_1 \]
\[ E_z = E_2 \]
\[ \nu_{xy} = \nu_{yx} = \nu_1 \]
\[ \nu_{xz} = \nu_{yz} = \nu_2 \]
\[ G_{zx} = G_{zy} = G_2 \]

Following are the steps to transform the compliance matrix \([A]\) from global coordinates (x y z) into their local coordinates (x' y' z'), according to the following equation (3) [6].

\[
[A'] = [P] [A] [P]^T
\] (3)

So from the above equation we can get the matrix equation component \([A']\) which is the result of transformation of matrix compliance \([A]\) from global coordinates into their local coordinates like equation (4) follows:

\[
A_{13} = -\frac{\nu_1}{E_1} \sin^2 \theta - \frac{\nu_2}{E_2} \cos^2 \theta
\]
\[
A_{23} = \frac{\sin^2 2\theta}{4} \left( \frac{1}{E_1} + \frac{1}{E_2} \right) - \frac{\nu_2}{E_2} \left( \sin^4 \theta + \cos^4 \theta \right)
\] (4)
$A_{33} = \frac{\sin^4 \theta}{E_1} + \frac{\cos^4 \theta}{E_2} - \frac{\nu_2 \sin^2 2\theta}{2E_2}$

Eventually a simplified constitutive equation (equation 5) will be obtained, resulting from the transformation of the strain from global coordinates into their local coordinates.

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} = \begin{bmatrix} A_{13} \\ A_{23} \\ A_{33} \end{bmatrix} \cdot \sigma_z \tag{5}$$

3.2. Matrix compliance calculation

All strain measurements were analysed simultaneously. Let $n$ be for all strain measurements (where $n \geq 4$) for the sixteen samples tested. In accordance with equations (4) and (5) that each strain measurement is linearly related to the four unknown compliance components ($1/E_1$ $1/E_2$ $\nu_1/E_1$ $\nu_2/E_2$). In the matrix form, strain measurements can be shown in relation to the four complications as follows:

$$[\varepsilon] = [T] \cdot [C] \tag{6a}$$

where:

$[\varepsilon] =$ matrix (n x 1) of strain measurements, $n$ is the total number of measurements ($n > 4$)

$[T] =$ matrix (n x 4) associated with the angle $\theta$

$[C] = (1/E_1$ $1/E_2$ $\nu_1/E_1$ $\nu_2/E_2)$

$$[\varepsilon] = [T] \cdot \begin{bmatrix} 1 \\ \frac{1}{E_1} \\ \frac{1}{E_2} \\ \frac{\nu_1}{E_1} \\ \frac{\nu_2}{E_2} \end{bmatrix} \tag{6b}$$

Equation (6b) is then solved by estimating the least squares for the four compliance values by multilinier regression analysis [9]. The advantage of this approach is that all strain measurements are included in the calculation when determining the compliance values. In addition, this method can be developed for examples in sufficient quantities.

Because the test does not measure deformation in the $y$ direction, the matrix compliance equation for $\varepsilon_y$ is not included in the analysis. As an example illustration in example no. 1 where:

$\theta = 85^\circ$

$\sigma_c = 40.58$ MPa $\Rightarrow$ $50 \%. \sigma_c = 20.29$ MPa

In the matrix calculation the data entered is as follows:
Substitute the data in equation (6b), it will be obtained:

$$
\begin{bmatrix}
1/E_1 \\
1/E_2 \\
v_1/E_1 \\
v_2/E_2 \\
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & -0.001170 & -20.135415 & -0.154122 \\
19.982464 & 0.003490 & -0.305902 \\
1/E_1 & 1/E_2 & v_1/E_1 & v_2/E_2 \\
\end{bmatrix}
$$

The method described above is used in the sixteen slate rock samples, in the uniaxial compressive strength test. The strain used in the entire analysis is determined at 50% of the peak stress.

### 3.3. Calculation of elastic constants with multi linear regression

From the above data it is known that n = 32 observations and k = 4 variables. By eliminating the assumption of b_0, the regression equation is linear [9].

$$
y_i = b_1 x_{1i} + b_2 x_{2i} + b_3 x_{3i} + b_4 x_{4i} + e_i
$$

(7)

The concept of least squares is used to obtain estimates of b_1, b_2, b_3 and b_4. By lowering the sum of the squares of errors with respect to each coefficient b_1, b_2, b_3, b_4 and the result is equal to zero, we will get a normal equation in the form of a matrix.

$$
\begin{bmatrix}
\sum_{i=1}^{n} x_{1i}^2 & \sum_{i=1}^{n} x_{1i} x_{2i} & \sum_{i=1}^{n} x_{1i} x_{3i} & \sum_{i=1}^{n} x_{1i} x_{4i} \\
\sum_{i=1}^{n} x_{2i} x_{1i} & \sum_{i=1}^{n} x_{2i}^2 & \sum_{i=1}^{n} x_{2i} x_{3i} & \sum_{i=1}^{n} x_{2i} x_{4i} \\
\sum_{i=1}^{n} x_{3i} x_{1i} & \sum_{i=1}^{n} x_{3i} x_{2i} & \sum_{i=1}^{n} x_{3i}^2 & \sum_{i=1}^{n} x_{3i} x_{4i} \\
\sum_{i=1}^{n} x_{4i} x_{1i} & \sum_{i=1}^{n} x_{4i} x_{2i} & \sum_{i=1}^{n} x_{4i} x_{3i} & \sum_{i=1}^{n} x_{4i}^2 \\
\end{bmatrix}
\begin{bmatrix}
b_1 \\
b_2 \\
b_3 \\
b_4 \\
\end{bmatrix}
= \begin{bmatrix}
g_1 = \sum_{i=1}^{n} x_{1i} y_i \\
g_2 = \sum_{i=1}^{n} x_{2i} y_i \\
g_3 = \sum_{i=1}^{n} x_{3i} y_i \\
g_4 = \sum_{i=1}^{n} x_{4i} y_i \\
\end{bmatrix}
$$

(8)

The results of the analysis of the 32 strain measurements (n = 32) produce elastic constants below:

$$
b_1 = \frac{1}{E_1} = 0.000199
\Rightarrow E_1 = 5028,995 \text{ MPa}
$$

$$
b_2 = \frac{1}{E_2} = 0.000234
\Rightarrow E_2 = 4274,661 \text{ MPa}
$$

$$
b_3 = \frac{v_1}{E_1} = 0.000060
\Rightarrow v_1 = 0.302
$$
Where the value of $E_1/E_2 = 1,176$ means that on the rock, the ability of deformation in the normal direction to the plane of transverse isotropy is greater than the ability of the deformation in the direction parallel to the transverse isotropy plane.

The following chart is a comparison between stress-strain laboratory test results and the results of the equation model in example no. 1.

**Figure 5.** Comparison of stress and strain curve obtained from laboratory test results – model.

4. Conclusion

In this research, we discuss laboratory and analytical testing methods to determine the four elastic constant values of transverse isotropic rocks. Where rock material is assumed to be linear, elastic, homogeneous, continuous and transverse isotropic. The rock used as an example in a laboratory test is a slate rock which is a foliating rock in the direction of the foliation that appears on its surface, so it is treated as a transverse isotropic material. In the uniaxial compressive strength test, the slope of the foliation plane to the horizontal plane is varied from $5^\circ$ - $85^\circ$ from the horizontal plane.

In this study, the measuring instrument used in measuring the amount of deformation that occurs in rocks is the dial gauge. Dial gauge works based on indications of pressure that occurs on the spindle as a result of changes in shape on the surface of the rock, which will play certain readings on the scale of the rotation counter as the value of deformation that occurs on the rock. Therefore, in the process of data acquisition, the deformation value needed to calculate the magnitude of the shear modulus $G_2$ cannot be obtained. This is because the measuring instrument used is unable to measure at the same time the strain that occurs in two different coordinate directions. For this purpose the measuring instrument used is in the form of a strain gauge, where the strain gauge is able to measure the magnitude of the strain that occurs based on the principle of the ratio of the change in length to the original length. So that the strain value in two different coordinate directions can be measured at the same time to get the $G_2$ modulus value from the rock.
From the results of the uniaxial compressive strength test for the sixteen slate rock samples in Figure 4 shows the variation of the value of the UCS with respect to the angle $\theta$. Where in the picture it appears that the maximum compressive strength appears when the force is applied in a direction parallel to the isotropic plane ($\theta = 85^\circ$). From the strain value measured at 50% of the peak stress on the stress-strain curve, the elastic constant value is determined. Equation (6a) is used to calculate secant elastic constants with the least squares estimation, as many as the number of tests (n = 32) and obtained values of $E_1 = 5028.995$ MPa, $E_2 = 4274.661$ MPa, $\nu_1 = 0.302$ and $\nu_2 = 0.106$. The value of $E_1/E_2 = 1.176$ which indicates that for the rock, the ability of deformation in the normal direction of the plane of transverse isotropy is greater than the ability of the deformation in the parallel direction of transverse isotropy.

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