**IRAS PSCZ V.S. IRAS 1.2-JY MODEL VELOCITY FIELDS: A SPHERICAL HARMONICS COMPARISON.**

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Abstract

We have used the two IRAS redshift surveys, 1.2-Jy (Fisher et al. 1995) and PSCz (Saunders et al. 2000), to model the linear velocity fields within a redshift of 8000 km s⁻¹ and have compared them in redshift space. The two velocity fields only differ significantly in their monopole components. The monopole discrepancy cannot be solely ascribed to shot-noise errors and incomplete sky coverage. The mismatch seems to arise from incompleteness of the PSCz catalog at fluxes ≤ 1.2 Jy. The 1.2-Jy and PSCz higher order velocity multipoles, particularly the dipole and quadrupole components, appear to be consistent, suggesting that the dipole residuals found by Davis, Nusser and Willick (1996) when comparing 1.2-Jy and MarkIII velocity fields probably originates from the MarkIII velocities calibration procedure rather than from uncertainties in the model velocity field. Our results illustrate the efficiency of the spherical harmonics decomposition techniques in detecting possible differences between real and model velocity fields. Similar analyses shall prove to be very useful in the future to test the reliability of next generation model velocity fields derived from new redshift catalogs like 2dFGRS (Colless et al. 2001), SDSS (York et al. 2000), 6dF and 2MRS.

Subject headings: cosmology — theory — galaxies large-scale structure of the Universe — large-scale dynamics.

1. INTRODUCTION

In the framework of linear gravitational instability theory and linear biasing a comparison between the predicted and observed peculiar velocities allows us to measure the so called β parameter

\[
\beta \equiv \frac{\Omega_m^{0.6}}{b}.
\]

i.e. a combination of the mass density parameter, \(\Omega_m\), and the linear bias parameter, \(b\), which relates the mass overdensity field \(\delta\) to the density fluctuations in galaxy counts \(\delta_g\), through \(\delta_g = b\delta\). Several different techniques have been used to compare measured peculiar velocities (taken from all available catalogs: MarkIII Willick 1997a, SFI Haynes 1999, ENEAR da Costa et al. 2000, SEcat Zaroubi 2002) to predict velocities that, in most cases, have been obtained from two redshift surveys of IRAS galaxies: the 1.2-Jy and the PSCz catalogs.

Most analyses have investigated the consistency between model and observed velocities, while only a few of them have been devoted in comparing different datasets or in studying the consistency of model velocity fields. All recent works show a general consistency between models and data (Willick et al. 1997, hereafter WSDK; Willick and Strauss 1998; Sigad et al. 1998; Dekel et al. 1999; Branchini et al. 1999, hereafter B99; Nusser et al. 2000; Zaroubi et al. 2002) with one noticeable exception represented by the work of Davis, Nusser 
and Willick (1996, hereafter DNW). Their analysis showed that the velocity residuals between the MarkIII peculiar velocities and the 1.2-Jy predictions display a significant spatial correlation. Subsequent analyses by WSDK and Willick and Strauss (1998) were performed using the same dataset, a similar model velocity field but a different comparison technique, called VELMOD, which allows for independent calibrations of MarkIII velocities in each of the MarkIII sub-catalogs. As a result the observed and model velocity fields were brought into better agreement, at least within a redshift distance \(s = 7500\text{ km s}^{-1}\) and returned a calibration inconsistent with the original one performed by Willick et al. (1997a).

While these results suggest that the DNW mismatch originates from calibration problems in the MarkIII catalog, the possibility that systematic errors in the 1.2-Jy model velocities also contribute to the discrepancy has been completely overlooked. In particular, we ignored whether the velocity fields predicted from the deeper IRAS PSCz catalog provides a better match to the MarkIII velocities than the shallower 1.2-Jy catalog, considered by DNW.

The main goal of this work is to check whether the discrepancies between the 1.2-Jy and MarkIII velocities can be ascribed, at least to some extent, to model uncertainties. For this purpose we compare both the overdensity and the radial velocity fields modeled from the redshift space distribution of 1.2-Jy and PSCz galaxies. This work is meant to stress the usefulness of spherical harmonics decomposition techniques in assessing the adequacy of a model velocity (or density) field through velocity-velocity (or density-density) comparisons.

In § 2 we describe how to compare velocity and overdensity fields inferred from different redshift surveys. The galaxy redshift catalogs used in our analyses are described in § 3. In sections 4 and 5 we analyse the two model overdensity fields and radial velocity fields. Our main results are discussed in § 6.
and our main conclusions are presented in § 7.

2. SPHERICAL HARMONIC COEFFICIENTS DECOMPOSITION

The relations between an arbitrary scalar field \( \psi(s) \) and the real–valued spherical harmonic coefficients \( \psi_{lm}(s) \) are

\[
\psi_{lm}(s) = \int \psi(s) Y_{lm}(\hat{s}) d\Omega,
\]

\[
\psi(s) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \psi_{lm}(s) Y_{lm}(\hat{s}),
\]

where \( Y_{lm}(\hat{s}) \{ l=0, \ldots, \infty; |m| \leq l \} \) denote the well known real-valued spherical harmonics, defined as in Baker et al. (1999, hereafter BDSLS) and Bunn (1995) (see also Jackson 1999). For a given \( l \) these functions are normalized to the value of \( Y_{l0} \) at the North Galactic Pole.

The amplitude of the \( l \)-multipole \( \psi_l \) is defined by the sum in quadrature over the \((2l+1)\)-\( \psi_{lm}^2 \), where \( \{ m = -l, \ldots, l \} \):

\[
\psi_l(s) = \left[ \sum_{m=-l}^{l} \psi_{lm}^2(s) \right]^{1/2}.
\]

2.1. Peculiar velocity and overdensity fields from the distribution of galaxies in redshift space

Nusser & Davis (1994) show that in the linear regime the peculiar velocity field is irrotational not only in real space but also in redshift space and thus can be expressed as the gradient of a scalar velocity potential: \( \mathbf{V}(s) = -\nabla \Phi(s) \). At a given redshift \( s \), \( \Phi(s) \) and \( \delta(s) \) can be expanded in spherical harmonics and related to each other through a modified Poisson equation:

\[
\frac{1}{s^2} \left( s^2 \Phi_{lm} \right)'' - \frac{1}{1+\beta} \frac{l(l+1)}{s^2} \Phi_{lm} = \frac{\beta}{1+\beta} \left( \delta_{lm} - \frac{1}{s} \frac{d \ln \phi(s)}{d \ln s} \Phi_{lm} \right),
\]

where \( \phi(s) \) and \( \delta(s) \) denote the sample selection function and the spherical components of the overdensity field, respectively. Here prime expresses \( d/ds \). Solving this differential equation requires computing the redshift space density field from the observed galaxy distribution on a spherical grid using Gaussian cells of approximately equal solid angle. These cells are equally distributed in longitude (64 bins) and are centered at the Gaussian-Legendre 32 point quadrature formula in the range \(-1 \leq \cos(b) \leq 1\), where \( b \) is the Galactic latitude. In this work we measure redshifts in the Local Group frame, and use 52 Gaussian radial redshift bins out to a distance of \( s = 18000 \) km s\(^{-1}\). When comparing two or more galaxy catalogs the size of the radial bin is set by requiring equal spatial resolution and the minimal shot noise constant throughout the volume. We do this setting the distance between the centers of the Gaussian cells at distance \( s \) equal to the average galaxy-galaxy separation, \( \sigma(s) = \bar{\sigma}(s) \) in the sparser catalog. The mean number density is estimated as

\[
\bar{n} = \frac{1}{V} \sum_{i} \frac{1}{\phi(s_i)},
\]

where the sum is over the galaxies contained within the volume \( V \), which, in this work, is a sphere of radius 8000 km s\(^{-1}\). Hernquist and Katz (1989) pointed out that there are two possible ways to estimate the smoothed density field: the "scatter" and "gather" approaches. In the former approach each particle is distributed in space, and the density estimate at a given point is the superposition of individual smoothing spheres. In the latter case one defines a smoothing length at a given point and weights the particle in its neighborhood by the resulting kernel. The Gaussian-smoothed galaxy overdensity field at a grid cell centred around \( s_n \) is given by:

\[
1 + \delta(s_n) = \frac{1}{N(2\pi)^{3/2}\sigma_n^2} \sum_{i} \frac{1}{\phi(s_i)} \exp \left[ -\frac{(s_n - s_i)^2}{2\sigma_n^2} \right],
\]

where the sum is over the \( N \) galaxies in the catalog, which clearly is a "gather" approach. The smoothing width at a redshift \( s \), used in this paper is \( \sigma_n = \max[320, \sigma(s)] \) km s\(^{-1}\). Since in this work we compare the \( PSCz \) catalog with the sparser 1.2-Jy catalog, we use the \( \sigma_n \) obtained from the 1.2-Jy catalog shown in Fig. (1). This smoothing scheme is tailored to keep the shot-noise uncertainty roughly constant throughout the sample volume and is similar to the optimal Wiener filtering procedure used by Lahav et al. (1994) and Fisher et al. (1995b). Moreover, the our kernel has a compact support, which simplifies the computational task.

Eqn. (5) is valid in the linear regime, where a one-to-one mapping between distance and redshift is guaranteed. This assumption is not valid in high density regions, such as cluster of galaxies, where shell crossing may occur triple valued-regions may appear. To deal with the latter effect, we adopt the same procedure as Yahil et al. (1991) and collapse the fingers of God associated with the six richest clusters in the sample listed in Table 2 of that work.

In this work we will consider galaxies within \( s_{\text{max}} = 20000 \)
km s$^{-1}$ with the spherical harmonics decomposition limited to $l_{\text{max}} = 15$. Outlying $s_{\text{max}}$ galaxies are assumed to be distributed uniformly according to the mean number density of the parent catalogs.

3. DATASETS

The IRAS PSCz catalog contains $\approx 15\,500$ IRAS PSC galaxies with a 60 $\mu$m flux larger than 0.6 Jy. The average depth of this survey is $\approx 100\,h^{-1}$ Mpc. We restrict our analysis to a subsample of 11,206 galaxies within 20,000 km s$^{-1}$ from the Local Group (LG, hereafter). The areas not covered by the survey amount to 16.0% of the sky and are preferentially located near the galactic plane, in the so-called Zone of Avoidance (Sanders et al. 2000). The empty areas have been filled-in with the cloning procedure described in Branchini et al. (1999).

The 1.2-Jy catalog (Fisher et al. 1995a) contains 5,331 IRAS galaxies with a 60 $\mu$m flux larger than 1.2-Jy within 20,000 km s$^{-1}$. This catalog has a slightly larger sky coverage of ($\approx 87.6\%$) and a smaller median redshift ($\approx 84\,h^{-1}$ Mpc). The same B99 filling technique has been used to restore all sky coverage.

Fisher et al. (1995b) have found that uncertainties in filling-in the empty areas of the 1.2-Jy survey do not appreciably affect the spherical harmonics coefficients with $l \lesssim 15$. Similar conclusions are also valid for the IRAS PSCz survey (Teodoro et al. 1999).

The selection functions of the two surveys, that quantify the probability of a galaxy at distance $s$, have been determined as maximum-likelihood fits to the smooth function

$$\phi(s) = \begin{cases} \left( \frac{s}{H_0 r_s} \right)^{2\alpha} \left( \frac{r_s^2}{s^2} \right)^{2\beta} & s \geq H_0 r_s, \\ 1 & \text{otherwise}, \end{cases}$$

where $r = s/H_0$ is expressed in $h^{-1}$ Mpc (Yahil et al. 1991). The best-fit parameters, determined using the galaxies within 8,000 km s$^{-1}$ and assuming a flat universe with $\Omega_m = 1$, are shown in Tab. (1). The quantity $r_s$ in Eqn. (8) accounts for the incompleteness of faint galaxies in the innermost regions of the catalogs (Rowan-Robinson et al. 1990, Yahil et al. 1991).

4. THE IRAS PSCz AND 1.2-Jy FIELDS OVERDENSITY

The first step of our analysis consists of applying Eqn. (7) to compute the IRAS 1.2-Jy and PSCz overdensities onto a spherical grid of radius 18,000 km s$^{-1}$ and $l_{\text{max}} = 15$ with the smoothing scheme given by Eqn. (7).

4.1. Overdensity Maps

In Figs. (2–4) we show the Aitoff projections of the PSCz (top panels) and 1.2-Jy (middle panels) galaxy overdensity fields in three spherical shells centred at $s = 1,000$, 3,000 and 5,000 km s$^{-1}$, respectively. The thickness of each shell is defined by the Gaussian width at the distance of the shell. All maps are plotted in Galactic coordinates. The bottom panels show the map of the overdensity residuals $\delta_{\text{res}}(s) = \delta_{0.6}(s) - \delta_{1.2}(s)$, where the subscripts 0.6 and 1.2 refer to PSCz and 1.2-Jy surveys, respectively. In all plots the thick line indicates the zero overdensity (or residual) contour. The overdensity (residual) contour spacing $\Delta \delta$ ($\Delta \delta_{\text{res}}$) depends on the shell’s depth and is indicated in the Figure captions. Detailed cosmographic studies of the structures in the nearby universe have already been performed using both the distribution of 1.2-Jy (Strauss and Willick 1995) and PSCz galaxies$^2$ and will not be repeated here. We only stress that the main cosmic structures like the Virgo Cluster ($l \approx 240^\circ$, $b \approx 75^\circ$, $s = 1000$ km s$^{-1}$), the local void spanning the region $330^\circ \lesssim l \lesssim 120^\circ$, $-90^\circ \lesssim b \lesssim 30^\circ$ at $s = 1000$ km s$^{-1}$, the Hydra-Centaurus complex ($l \approx 270^\circ$, $b \approx 20^\circ$, $s = 3000$ km s$^{-1}$) and ($l \approx 300^\circ$, $b \approx 20^\circ$, $s = 3000$ km s$^{-1}$) and the Perseus-Pisces supercluster ($l \approx 140^\circ$, $b \approx -25^\circ$, $s = 3000$ km s$^{-1}$) are detected in both surveys.

The residuals, calculated at the same gridpoints as the den-

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1Throughout this article we write Hubble’s constant as $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$

2See http://www-astro.physics.ox.ac.uk/~wjs/pscz.html
TABLE 1

SELECTION FUNCTION PARAMETERS FOR IRAS CATALOGS.

| Catalog      | $f_{60\mu m}^{\text{lim}}$ | Ngal$^a$ | $\alpha$ | $\beta$ | $\gamma$ | $r_s^b$ | $r_*$ | $\bar{n}$ |
|--------------|-----------------------------|---------|---------|---------|---------|--------|-------|---------|
|              | [Jy]                        |         |         |         |         | [h$^{-1}$ Mpc] | [h$^{-1}$ Mpc] | [h$^3$ Mpc$^{-3}$] |
| PSCz         | 0.6                         | 13 364  | 0.54    | 1.80    | ...     | 6.0    | 87.00 | 6.12 · 10$^{-2}$ |
| PSCz 0.7     | 0.7                         | 11 170  | 0.57    | 1.80    | ...     | 6.0    | 76.35 | 6.38 · 10$^{-2}$ |
| PSCz 1.2     | 1.2                         | 5 519   | 0.43    | 1.86    | ...     | 5.0    | 50.40 | 4.62 · 10$^{-2}$ |
| 1.2-Jy       | 1.2                         | 6 010   | 0.43    | 1.86    | ...     | 5.0    | 50.60 | 4.67 · 10$^{-2}$ |
| PSCz$^{\text{evo.}}$ | 0.6                     | 13 364  | 0.99    | 3.45    | 1.93    | 6.0    | 76.02 | 5.76 · 10$^{-2}$ |

$^a$ Includes synthetic objects.
$^b$ Parameter kept constant in the likelihood maximization.
$^c$ The selection function parameters have been taken from Canavezes et al. (1999)

FIG. 3.— Same as in Fig. (2). The shell is now centred at $s = 3000$ km s$^{-1}$ and the contour spacings are $\Delta \delta = 0.5$ and $\Delta \delta_{\text{res}} = 0.125$.

FIG. 4.— Same as in Fig. (2). The shell is now centred at $s = 5000$ km s$^{-1}$ and the contour spacings are $\Delta \delta = 0.5$ and $\Delta \delta_{\text{res}} = 0.125$. 
density fields, seem to indicate that, as expected, the largest residuals are located near the galactic plane where uncertainties in the filling-in procedure dominate the errors. In the inner shell, over-densities in the 1.2-Jy galaxy distribution appear to be larger than the PSCz ones while in the most external shell the situation is reversed.

4.2. Density Residuals

A way of quantifying possible systematic differences between the two fields is that of performing a point-to-point comparison. In absence of systematic errors we expect the two IRAS overdensity fields to be linearly related through

$$
\delta_{1.2} = B \delta_{0.6} + A \quad (9)
$$

with $A = 0$ and $B = 1$. $\delta_{0.6}$ and $\delta_{1.2}$ are estimated at Cartesian gridpoint positions. A zero–point offset $A \neq 0$ indicates discrepancies in the mean densities of the two catalogs while a slope $B \neq 1$ reveals the presence of systematic errors. If the values of $\delta_{0.6}$ and $\delta_{1.2}$ are independent and their errors are Gaussian then we can compute the $\chi^2$ statistics

$$
\chi^2 = \sum_{i=1}^{N_{\text{grid}}} \frac{(\delta_{1.2} - A - B \delta_{0.6})^2}{\sigma_{0.6}^2 + \sigma_{1.2}^2 N_{\text{grid}}^2}, \quad (10)
$$

where $\sigma_{0.6}$ and $\sigma_{1.2}$ indicate the typical errors in $\delta$ and $N_{\text{grid}}$ is the total number of points in the comparison. However, since the gridspacing (250 km s$^{-1}$) is smaller than the smoothing length ($\geq 320$ km s$^{-1}$), not all values of $\delta_{0.6}$ (or $\delta_{1.2}$) are independent. Indeed, the effective number of independent points used in the $\chi^2$ statistics, $N_{\text{eff}}$ is given by

$$
N_{\text{eff}} = \frac{N_{\text{grid}} N_{\text{grid}}^{-1}}{\sum_{i=1}^{N_{\text{grid}}} \sum_{j=1}^{N_{\text{grid}}} \exp(-r_{ij}^2/2R_{ij}^2)}, \quad (11)
$$

where $r_{ij}$ is the separation between gridpoints $i$ and $j$ and $R_{ij}$ is the smoothing radius of the Gaussian kernel at the gridpoint $i$. This expression is a generalization of Eqn. (12) in Dekel et al. (1993) to the case of a grid with variable mesh-size since $R_{ij}$ depends on the gridpoint position. The new statistics $\chi^2_{\text{eff}} = (N_{\text{grid}}/N_{\text{eff}}) \chi^2$ is approximately distributed as a $\chi^2$ with $N_{\text{dof}} = N_{\text{eff}} - 1$ degrees of freedom and can be applied to infer the values of $A$, $B$ and their errors.

Fig. (5) shows the $\delta_{1.2}$ vs. $\delta_{0.6}$ scatter diagram obtained using all 112552 gridpoints with $s < 8000$ km s$^{-1}$ and $|b| > 8^\circ$. Within these limits the estimated overdensities are little affected by uncertainties in the filling-in procedure, and we expect that

| Regression | $N_{\text{grid}}$ | $N_{\text{dof}}$ | $A$ | $B$ | $\chi^2_{\text{eff}}/N_{\text{dof}}$ |
|------------|-----------------|----------------|------|-----|----------------------|
| $\delta_{1.2} - \delta_{0.6}$ | 112552 | 156.36 | $-0.011 \pm 0.014$ | $1.025 \pm 0.036$ | 0.78 |
| $u_{1.2} - u_{0.6}$ | $\lesssim$ 156.36 | 60.0 $\pm$ 5.9 | $1.040 \pm 0.019$ | 0.88 |

**Notes:**
- For the $u-u$ comparison $A$ is expressed in km s$^{-1}$.
- Column 2: $N_{\text{grid}}$, the number of gridpoints used for the regression;
- Column 3: $N_{\text{dof}}$, the number of independent volumes;
- Column 4: $A$, the zero–point offset in the linear regression, and its $1-\sigma$ error;
- Column 5: $B$, the slope of the linear regression, and its $1-\sigma$ error;
- Column 6: $\chi^2_{\text{eff}}/N_{\text{dof}}$ from the linear regression.

**Fig. 5.** 1.2-Jy vs. PSCz overdensity. Only 1883 randomly selected gridpoints with $|b| > 8^\circ$ and within $s < 8000$ km s$^{-1}$ are shown in the scatter plot. The continuous line represents the linear best fit characterized by the zeropoint $A$ and slope $B$ indicated in the panel.
shot-noise dominates the total error budget. To estimate the shot-noise errors affecting both PSCz and 1.2-Jy overdensity fields we have generated 100 bootstrap realizations of the observed PSCz and 1.2-Jy surveys. These are obtained by replacing each galaxy (including the ‘synthetic’ galaxies used to fill-in the unobserved regions) with a number of objects drawn from a Poisson deviate with mean unity. Shot noise errors in δ at the generic gridpoint were set equal to the dispersion over the 100 realizations. The solid line in the plot is the best linear fit obtained from the minimization of $\chi^2_{eff}$. As shown in Tab. (2), the slope of the line, $B = 1.025 \pm 0.036$, is consistent with unity, indicating no systematic mismatch. On the other hand the zero-point value $A = -0.011 \pm 0.014$ shows no systematic difference in the mean density of the two IRAS catalogs.

The scatter around the best–fitting line, $\sigma_\delta = 0.16$ is similar to average shot-noise error in the 1.2-Jy density field ($\sigma_\delta = 0.15$). The parameter $S = \chi^2_{eff}/N_{\text{dof}} = 0.78$ indicates that bootstrap errors slightly overestimate the true uncertainties.

### 4.3. Radial Density Contrast

An alternative way to characterize possible discrepancies between the two IRAS density fields is that of computing the differential radial density contrast in shells with identical thickness $\Delta s$ for both galaxy distributions:

$$\frac{\rho(s)}{\bar{\rho}} = \frac{1}{\sum_i N_i \phi(s_i)} \sum_i N_i \frac{W(s, s_i, \Delta s)}{\phi(s_i)},$$

(12)

where the window associated to the spherical shell $W_{\delta\rho}/\phi(s, s_i, \Delta s)$ is the Heaviside function $H(s + \Delta s/2 - s_i)H(s + \Delta s/2 - s)$, $s_i$ is the redshift of the object $i$, $V$ is the volume used to compute the mean number density $\bar{n}$ and the sum runs over the $N_p$ galaxies of the catalog. Fig. (6) shows the quantity $\rho(s)/\bar{\rho}$ for 1.2-Jy (light, dotted histogram) and PSCz (thick, continuous histogram) galaxies within $s = 15000\text{ km s}^{-1}$, computed in in spherical shells of thickness $\Delta s = 200\text{ km s}^{-1}$. Although the general shapes of two radial density contrasts are similar (with a bump at $s \sim 1100\text{ km s}^{-1}$ due to Virgo and Fornax clusters, a dip at $s \sim 3500\text{ km s}^{-1}$ induced by the Sculptor Void and a second prominent peak at $s \sim 5000\text{ km s}^{-1}$, at the same redshift as the Perseus-Pisces supercluster and the Great Attractor), a discrepancy between the two curves exists in the range $2000 \lesssim s \lesssim 6000\text{ km s}^{-1}$ where the PSCz density is larger than the 1.2-Jy one.

The two smoothed curves in Fig. (6) show the monopole terms of the two overdensity fields $\delta_{\text{rad,mon}}$ as a function of radius, obtained from the spherical harmonics decomposition. Both curves interpolate the histograms rather well (hence showing the adequacy of our spherical harmonics analysis) and confirm the existence of a PSCz vs. 1.2-Jy density mismatch between the two catalogs.

5. THE IRAS PSCz AND 1.2-Jy PECULIAR VELOCITY FIELDS

To perform a similar analysis on the 1.2-Jy and PSCz model velocity fields we have integrated Eqn. (5) using $\beta = 0.6$, a value consistent with recent determinations based on several velocity-velocity comparisons (e.g. Nusser et al. 2000), a recent density-density analysis (Zaroubi et al. 2002) and the study of mean relative velocities of galaxy pairs (Juszkiewicz et al. 2000).

### 5.1. Radial Peculiar Velocity Maps

To obtain maps of the radial velocity field we have computed the gradient of the velocity potential projected along the line of sight: $u(s) = s \cdot \nabla \Phi$ at the same gridpoint positions used for the analysis of the density field.

The top end mid panels of Figs. (7-9) are similar to those of figs. 2–4 and show the radial velocity maps in the same shells. The bottom panels show the maps of the radial velocity residuals $u_{\text{res}}(s) \equiv u_{0,6}(s) - u_{1.2}(s)$. The thick line represents the zero radial velocity (residual) contour. The velocity (residual) contour spacing $\Delta u (\Delta u_{\text{res}})$ varies with the redshift and is indicated in the Figure captions.

The main feature seen at all redshifts in both 1.2-Jy and PSCz radial velocity maps is the dipole pattern resulting from the LG reflex motion toward the CMB apex ($l \approx 276^\circ$, $b \approx 30^\circ$, Kogut et al. 1993).

Local deviations from this pattern arise from peculiar motions, like the infall onto Perseus-Pisces superclusters at ($s \approx 3000\text{ km s}^{-1}$, $l \approx 120^\circ$, $b \approx -40^\circ$). The largest velocity residuals are located near the galactic plane, but their contours are broader due to the intrinsic non-locality of the peculiar velocity field. The most striking feature is perhaps the fact that velocity residuals, that are positive across a large fraction of the sky in the first shell, becomes mostly negative in the remaining two shells, suggesting possible systematic errors in either PSCz or 1.2-Jy velocity models.

![Fig. 6.](image-url)
5.2. Velocity Residuals

To perform a more quantitative comparison between the two radial velocity fields we have repeated the analysis of § 4.2 and performed a linear regression between $u_{1.2}$ and $u_{0.6}$ by minimizing

$$\chi^2 = \sum_{i=1}^{N_{\text{grid}}} \left( \frac{u_{1.2} - A - B\sigma_{u,0.6}}{\sigma_{u,1.2}^2 + B^2\sigma_{u,0.6}^2} \right)^2,$$

where $N_{\text{grid}}$ runs over the same points used for the density-density comparison. Linear velocities are obtained by integrating the mass density fields over large scales causing them to be correlated on scales larger than the radius of the Gaussian kernel. This means that the number of independent gridpoint $N_{\text{eff}}$ is smaller than the estimate given by Eqn. (11). Intrinsic large scale smoothing also affects the velocity error estimate, $\sigma_u$, since uncertainties in modeling the galaxy distribution in the unobserved areas now add to shot noise errors.

The shot noise errors have been computed using the same bootstrap resampling technique described in § 4.2. Uncertainties in the filling-in procedure have been evaluated from 20 independent $PSCz$ and 1.2-Jy mock catalogs extracted from the $N$-body simulations performed by Cole et al. (1997) simulating a volume of $\approx 170 h^{-1}$ Mpc in an Einstein-de Sitter universe with a non-vanishing ($\Omega_\Lambda = 0.7$) cosmological constant and density fluctuations normalized to the observed cluster abundance (Eke, Cole & Frenk 1996). Model velocity fields from each IRAS mock catalog were compared with velocities obtained from ideal, all-sky IRAS mock catalogs. The variance over 20 mocks at each gridpoint quantifies the uncertainties in the filling procedure. Total uncertainties were obtained by adding in quadrature the two errors.

The results of the linear regression are summarized in Tab. (2) which shows the slope of the best fitting line $B = 1.040 \pm 0.019$ and the value of the zero-point, $A = 60.0 \pm 5.9$ km s$^{-1}$. While the slope is still consistent with unity, at the 2.1-$\sigma$ confidence level, the zero point is significantly different from zero, hence corroborating the evidence for a mismatch in the average densities of $PSCz$ and 1.2-Jy catalogs. The dispersion around the fit...
is, $\sigma_u = 88 \text{ km s}^{-1}$ is similar to the typical velocity error computed from the IRAS mocks and the bootstrap resampling catalogs ($\sigma_{m,0} = 71 \text{ km s}^{-1}$ and $\sigma_{m,2} = 140 \text{ km s}^{-1}$). The parameter $\chi^2_{\text{eff}}/N_{\text{dof}}$ is less than unity which either indicates that errors are overestimated or that the value $N_{\text{eff}}$, taken from Eqn. (11), does significantly overestimate the true number of degrees of freedom.

5.3. Velocity Multipoles

The spherical harmonics decomposition allows us to better investigate and characterize possible discrepancies between 1.2-Jy and PSCz velocity fields. Here we are only interested in the first three ($l = 0, 1, 2$) components, corresponding to the monopole, dipole and quadrupole terms, which have already been investigated in previous analyses (e.g. BDSLS). It is worth stressing that the monopole and dipole components at a given redshift are only sensitive to the mass distribution within the corresponding distance and thus can be directly related to the structures within the volume considered, while the quadrupole components are also sensitive to the mass distribution beyond such a distance [see Eqn. (9) in Nusser & Davis 1994].
Both velocity monopoles display the same general features, as expected given the similarities between the density monopoles \( \delta_{00}(s) \) shown in Fig. (6). There is a significant radial inflow around \( s = 1800 \text{ km s}^{-1} \) corresponding to the infall motion toward the Virgo-Fornax complex, an outflow in the range \( 1800 \lesssim s \lesssim 4000 \text{ km s}^{-1} \) due to the presence of the local void and another inflow beyond this radius, due to the Hercules, Hydra, Centaurus and Perseus-Pisces supershells. Differences between the two velocity monopoles are detected with a significance level larger than \( 1-\sigma \) at very small radii \( (s < 1200 \text{ km s}^{-1}) \) and in the range \( 3500 \text{ km s}^{-1} < s < 7500 \text{ km s}^{-1} \).

Fig. (12) shows the total amplitude (lower left panel) and the three Cartesian components (remaining panels) of the two IRAS velocity dipoles. The total amplitude of the \( PSC_z \) dipole (continuous line) is systematically smaller that the 1.2-Jy one (dashed line) but the discrepancy is well within the expected errors, in agreement with previous studies (e.g. Schmoldt et al. 1999).

Finally, Fig. (13) shows the five components of the \( PSC_z \) (continuous line) and 1.2-Jy (dashed line) velocity quadrupoles, \( u_{2m}(s) \) (where \( m = -2, ..., 2 \)) along with total amplitude (bottom-left panel). As for the dipole case, the two sets of quadrupole components agree to within \( 1-\sigma \) from a small disagreement in the the \( m = +2 \) and \( m = -2 \) components beyond \( s = 7000 \text{ km s}^{-1} \). Indeed, beyond \( s \approx 6000 \text{ km s}^{-1} \) the amplitude of the \( PSC_z \) quadrupole decreases, while the 1.2-Jy one increases monotonically.

It is worth noticing that our 1.2-Jy velocity multipoles agree with those computed by BDSLS except that their multipole components exhibit sharper features at small radii \( (s < 500 \text{ km s}^{-1}) \). This mismatch derives from our use of a Gaussian smoothing kernel of radius \( 330 \text{ km s}^{-1} \) which is somewhat larger than that applied by BDSLS.

6. DISCUSSION.

Perhaps the most surprising result of our analysis is the disagreement between the \( PSC_z \) and 1.2-Jy velocity monopoles in the \( 1800 \lesssim s \lesssim 4000 \text{ km s}^{-1} \) range. Before investigating the possible causes of this mismatch, it is worth noticing that none of our IRAS monopoles is consistent with an outward flow of magnitude \( \approx 400 \text{ km s}^{-1} \) at a distance of 7000 km s\(^{-1}\) like the one detected by Zehavi et al. (1998) using a sample of 44 Type Ia supernovae. That result is also at variance with the velocity model obtained from the distribution of ORS galaxies obtained by BDSLS which means that, if confirmed, it would be difficult to reconcile with the Gravitational Instability scenario.

A possible clue to understand the origin of the monopole mismatch is provided by the analysis of BDSLS that returns a velocity monopole somewhere between the \( PSC_z \) and 1.2-
Jy ones in the range of interest. This seems to suggest that our monopole discrepancy originates from systematic errors in one of the IRAS catalogs, possibly some incompleteness of the largest catalog at faint fluxes.

The results of Tadros et al. (1999) seem to confirm that PSCz catalog might be statistically incomplete at fluxes $f_{\text{60} \mu \text{m}} \leq 0.7$ Jy. To test whether this is indeed the case we have extracted a brighter PSCz subsample (called PSCz$_{0.7}$) by discarding all object with $f_{\text{60} \mu \text{m}} \leq 0.7$ Jy and have repeated our spherical harmonics decomposition analysis. The results, shown in Fig. (14) indicate that the PSCz$_{0.7}$ velocity monopole (dotted line) basically coincide with that of the complete sample. It is only when discarding galaxies with fluxes smaller than 1.2 Jy that the velocity monopole (dashed line) agrees with the 1.2-Jy one.

Another possibility of understanding the monopole mismatch is that of invoking a strong evolution of IRAS galaxies as a function of the redshift, which we have neglected in our calculations so far. Roughly speaking evolution can be distinguished into two different types. Pure density evolution changes only the normalization of the luminosity function while its shape is invariant with redshift. Pure luminosity evolution on the other hand describes a possible change of the intrinsic luminosities of the sources but not their number density. Both forms of evolution occur simultaneously in the Universe. For instance, galaxy merging reduces the number density of galaxies causing thus evolution. Besides, such events also cause different luminosities of the final systems and hence leading to luminosity evolution. Other physical mechanism for galaxy evolution can be devised. Some evidences for a sizable evolution of IRAS galaxies have been reported by a number of authors, although there seem to be much controversy about the amplitude of the effect (see Springel 1996 and references therein).

Canavezes et al. (1998) expressed evolution in terms of a generalization of the selection function introduced by Yahil et al. (1991) given by

$$
\phi(s)_{\text{evol}} = \begin{cases} 
\left( \frac{s}{s_0} \right)^{\alpha} \left( \frac{r_r^2 + r_s^2}{r_r^2 + r_s^2} \right)^{\beta/\gamma}, & s \geq H_0 r_s, \\
1, & \text{otherwise.}
\end{cases}
$$

(14)

Springel (1996) showed that it indeed provides a rather accurate modeling of both IRAS data-sets. The estimated parameters using a maximum-likelihood procedure are listed in Tab. (1) and are denoted by PSCz$_{\text{evol}}$. The effect of evolution are shown in Fig. (14) in which the velocity monopole term derived from the PSCz galaxy distribution in which evolution effects have been accounted for through the new selection function $\phi(s)_{\text{evol}}$ (dot-dashed line) is plotted against the unevolved model. The evolutionary effects are very minor, as was somewhat expected given the locality of our galaxy sample, and do not help in reducing the PSCz vs. 1.2-Jy velocity monopole mismatch.

Overall, our analysis seems to indicate that the discrepancy between 1.2-Jy and PSCz velocity monopoles cannot be ascribed to errors in the modeling of the selection function which, as we have seen, would not change the monopole terms appreciably. Catalog incompleteness at very low fluxes has to be ruled out too. We can only conclude that either the PSCz in-completeness extend at objects with fluxes comparable to 1.2 Jy or that the monopole mismatch does reflect a genuine difference in the average density of faint vs. bright IRAS galaxies at $s \leq 4000$ km s$^{-1}$.

A second important result found in our analysis is that the 1.2-Jy and PSCz dipoles agree within the 1-$\sigma$ errors. This means that the use of the PSCz rather than the 1.2-Jy catalog does not help in reducing the Mark III-IRAS dipole residuals found by Davis, Nusser & Willick (1996). Indeed, both GX and GY components of the PSCz velocity dipole are much too large with respect to Mark III ones (see Fig 15 of Davis, Nusser & Willick 1996). Similar results were also obtained from the ORS vs. Mark III dipole velocity comparison (BDSLS). We conclude that the discrepancy between the Mark III and 1.2-

![FIG. 14.— Velocity monopoles for the 1.2-Jy (dark gray line) PSCz (light-gray), PSCz$_{0.7}$ (dot-dashed), PSCz$_{1.2}$ (long-dashed) PSCz$_{\text{evol}}$ (dotted) samples.](image)

**TABLE 3**  
**SELECTION FUNCTION PARAMETERS IN THE IRAS MOCK CATALOGS.**

| Mock catalog | $\alpha$ | $\beta$ | $r_s$ | $r_e$ |
|--------------|---------|---------|------|------|
| PSCz         | 0.53    | 1.90    | 10.90| 86.40|
| 1.2-Jy       | 0.48    | 1.79    | 5.00 | 50.40|


Jy velocity fields cannot be alleviated by reducing the shot noise errors of the model velocity field.

The agreement between the *PSCz*, 1.2-Jy and ORS dipoles is reassuring since it guarantees that uncertainties in model velocity fields arising from within the galaxy samples that derive from incompleteness in one (or more) catalogs, incorrect treatment of non-linear velocities or non-linear non-uniform biasing do not generate systematic errors apart from a the monopole mismatch in the range $1800 \text{ km s}^{-1} < s \leq 4000 \text{ km s}^{-1}$.

Unlike the case of monopole and dipole moments, possible differences between the 1.2-Jy and *PSCz* quadrupole components can also be attributed to incorrect determination and dilute sampling of the density field beyond the sample. The fact that 1.2-Jy and *PSCz* velocity quadrupole agree within the expected errors suggests that differences in the quadrupole components of the two fields can be understood in terms of shot noise errors and uncertainties in filling the empty regions.

With this respect it is worth investigating whether the denser sampling of the *PSCz* catalog at large radii allows a better modeling of the large scale contribution to the model velocity field. Indeed, when comparing the model 1.2-Jy velocity field to *Mark III* velocities in the framework of the VELMOD analysis, WSDK have found that velocity residuals exhibit a quadrupole pattern of the form $u_0 \propto \nabla_P r \cdot \mathbf{r}$, where $\nabla_P$ is a traceless, symmetric $3 \times 3$ tensor. They have attributed this “VELMOD quadrupole” residuals to uncertainties in modeling the IRAS density field beyond $3000 \text{ km s}^{-1}$. In particular they found that the most important sources of uncertainties were the smoothing procedure (based on a Wiener Filtering technique) and the shot noise errors, with the errors deriving from having ignored mass inhomogeneities beyond $12,000 \text{ km s}^{-1}$ playing a minor role. Using *PSCz* instead of 1.2-Jy catalog reduces the shot noise errors in the density field beyond $3000 \text{ km s}^{-1}$ and thus should improve the agreement with the *Mark III* velocities. In other words one would expect the quadrupole of the residual velocity field $u_0 - u_{12}$, where one only takes in account the mass beyond $3000 \text{ km s}^{-1}$, to be similar to the “VELMOD quadrupole” of WSDK.

In Fig. (15) we compare the VELMOD quadrupole at $s = 2000 \text{ km s}^{-1}$ (rescaled from $\beta = 0.492$ to $\beta = 0.6$, upper panel) to the $u_{06} - u_{12}$ quadrupole residual (lower panel). The VELMOD quadrupole reaches its maximum value at $(l \approx 165^\circ, b \approx 55^\circ)$ and in the opposite direction of the sky. The $u_{06} - u_{12}$ residual quadrupole exhibits a rather different pattern, reaching its maximum amplitude at $(l \approx 156^\circ, b \approx -74^\circ)$. The value of the quadrupole components of 1.2-Jy and *PSCz* as well as those of the VELMOD and $u_{06} - u_{12}$ quadrupole residuals computed at a distance of $2000 \text{ km s}^{-1}$ are shown in Tab. (4). This comparison is at best qualitative since (i) VELMOD quadrupole is computed in real space while both IRAS quadrupoles refer to redshift space instead, and (ii) the smoothing scheme used in WDKS is different from with ours.

7. CONCLUSIONS.

We have compared the model velocity and density fields of the *PSCz* survey to those derived from the sparser 1.2-Jy sam-

![FIG. 15.— VELMOD (upper panel) and $u_{06} - u_{12}$ (lower panel) quadrupole residuals at $s = 2000 \text{ km s}^{-1}$. Stars denote outflowing from us (LG frame); circles denote infall to us. The size of the symbol is proportional to the amplitude of the flow (notice the different velocity scales in the two panels). The maximum amplitude of the “VELMOD quadrupole” (top panel) is $\approx 181 \text{ km s}^{-1}$, while for $u_{06} - u_{12}$ is $\approx 23 \text{ km s}^{-1}$.]

Our model predictions are based on the Nusser & Davis (1994) spherical harmonics decomposition technique and assumes Zeldovich approximation and linear biasing.

The IRAS overdensity fields are reconstructed with a smoothing proportional to the 1.2-Jy inter-galaxy separation and exhibit the same general features corresponding to the known cosmic structures in the local universe. The model velocity field, computed in the LG frame, has a dipole–like appearance, with galaxies moving away from us toward Perseus-Pisces region and approaching the LG from the opposite direction.

The analysis of the velocity multipoles has revealed a significant discrepancy between the *PSCz* and 1.2-Jy velocity monopoles in the range $1800 \lesssim s \lesssim 4000 \text{ km s}^{-1}$ in which *PSCz* radial velocities are systematically larger than 1.2-Jy ones, that cannot be explained by uncertainties in the IRAS selection functions or incompleteness of the *PSCz* catalog at faint fluxes. Also, the mismatch cannot be ascribed to errors derived from shot–noise and treatment of unobserved areas since both have been accounted for with the help of realistic mock IRAS catalogs and the extensive application of bootstrap resampling techniques to both IRAS datasets.

Both IRAS dipole and quadrupole moments are in good agreement within the sampled volume apart from a small mismatch in the $u_{10}$, $u_{22}$ and $u_{12}$ quadrupole components beyond $s = 7000 \text{ km s}^{-1}$. The agreement between the dipole moments and their consistency with the velocity dipole computed by BD-
SLS from the ORS catalog suggests that deviations from homogeneous linear biasing prescriptions have to be small and cannot be advocated to explain the aforementioned mismatch between the IRAS monopoles. Another remarkable consequence of the agreement among the various dipoles is that the 1.2-Jy-Mark III residuals cannot be alleviated by using the PSCz catalog instead of the 1.2-Jy or the ORS ones. In fact, this suggests that the origin of the 1.2-Jy-Mark III dipole resides in the Mark III calibration procedure; a suggestion corroborated by the fact that the dipole mismatch disappears when adopting the alternative VELMOD procedure to calibrate -Mark III velocities (WDSK). The VELMOD analysis requires a sizable external quadrupole component within 3000 km s⁻¹. This quadrupole accounts for uncertainties errors in the 1.2-Jy model density field beyond that distance. We found that using the PSCz sample provides part of the required external tidal field.

As a final remark, we would like to stress that the present analysis illustrates once more the importance of spherical-harmonics decomposition techniques to compare observed and model density and velocity fields obtained from all-sky samples, to reveal possible data-model or model-model inconsistencies and to assess model adequacy. These techniques will prove very useful when the next generation of all-sky redshift surveys [e.g. the 6dF (http://www.mso.anu.edu.au/6dFGS) and 2MRS (http:cfa-www.harvard.edu/huchra/2mass) datasets] will become available.

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### Table 4

| VELMOD | PSCz | 1.2-Jy | PSCz - 1.2-Jy |
|--------|------|--------|---------------|
| [km s⁻¹] | [km s⁻¹] | [km s⁻¹] | [km s⁻¹] |
| ψ₂ (x,x) | 45 | 57 | 69 | -13 |
| ψ₂ (y,y) | 44 | -22 | -15 | -7 |
| ψ₂ (z,z) | -89 | -35 | -54 | 20 |
| ψ₂ (x,y) | 18 | -150 | -156 | 6 |
| ψ₂ (x,z) | 138 | 57 | 48 | 10 |
| ψ₂ (y,z) | -29 | -106 | -102 | -5 |

NOTE.-(1) We have rescaled the “VELMOD quadrupole” components presented in WSDK’s Tab. (2) to the numerical value of β = 0.6.

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