MHD simulation of sawtooth events in meridional plane driven by various solar wind velocities

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Abstract. The meridional global MHD modeling of magnetosphere sawtooth events is performed for three values of the solar wind velocity: 200 km/s, 400 km/s and 800 km/s. The tooth cycle period of the open magnetic field specific flux decreases from 6.4 to 2.0 ks, respectively, while the oscillations amplitude changes inessentially. The tail magnetic field topology evolve through all the standard phases and events. In the distant tail the flapping current sheet is disturbed by the Alfvén waves. In the dipole magnetic field domain the night-side attached vortices splitting and their subsequent shedding occur forming the Karman like vortex street. The vortex shedding period falls from 200 s to 110 s with the increase of the solar wind velocity. In the day-side shocked layer the vertically elongated thin magnetic island with the two neutral points is located. The magnetic island accounts for the deflection of the solar plasma inflow around the magnetosphere.

1. Introduction

The MHD approximation is an effective tool for global modeling of the Earth magnetosphere [1]. This approach describes consistently the evolution of magnetic field and plasma stream topology. The fundamental object of the modeling is the multiple magnetic field reconnection. In the dayside magnetosphere the multiple reconnection has been observed by spacecraft missions. The solitary magnetic islands were detected by Geotail spacecraft and Cluster four satellites missions [2, 3]. Single $x$-line and three secondary magnetic islands in the magnetopause were observed by four Magnetospheric Multiscale (MMS) spacecrafts [4]. Hybrid-Vlasov simulation [5] reveals a sequence of magnetic islands including formation of the primary island then its splitting and their merging. MHD ideal simulation [6] with a resistive spot superposed on the $B_z$ reversal at the magnetopause demonstrates field and flow topology evolution with a single $x$-line but similar to a magnetic island mode. The reconnection in the nightside magnetosphere being conformed to the dayside reconnection determines the type of magnetic activity. The temporal behavior of the dayside and nightside reconnection potentials is specific for every type of the magnetospheric events [7].

The statistical analysis of the solar wind driving conditions causing sawtooth events, steady magnetospheric convection periods and isolated substorms is performed in [8]. It was concluded that the solar wind velocity is the leading factor controlling magnetospheric activity. In the global MHD simulation [9, 10] this conclusion was confirmed. In the distant tail the multiple reconnection occurs in the thin magnetic shear [11]. 2D hybrid Vlasov-hybrid code simulation describes multiple
reconnection in the tail current sheet with the islands moving earthward and occasionally merging [12]. Existence of multiple reconnection initiates occurrence of Alfven waves. The Alfven waves generation in the near tail plasma sheet was modeled by the hybrid code [13]. The vortical structures were observed in the equatorial plane by the ISEE satellites and Cluster mission [14 and 15]. Subsequently, the Cluster constellation identified the two vortices in the meridional plane formed under interaction of the ionospheric outflow and the earthward tail convection flow [16].

In the previous work [17] the authors considered the magnetosphere response to the stationary solar wind forcing at various values of the plasma resistivity. In the present paper 2D MHD simulation of sawtooth events in magnetosphere meridional plane is performed for the three values of the solar wind velocities: 200, 400 and 800 km/s. The dayside and the nightside electrodynamic and plasmodynamic topology evolution is described.

2. Problem formulation

The fully ionized hydrogen plasma overflows the infinitely long cylindrical body carrying magnetic dipole. The body has a square cross section with the side face oriented normally to the velocity vector of the incoming flow. The computational area occupies the square with side of 36·10⁶ m extending from −501.5625 R to +98.4375 R in the x direction and from −300 R to 300 R in the z direction. The scale parameter R equals to 6·10⁶ (approximately, the Earth radius). The side of the body cross section with the central point at x=0 and z=0 is 2.8125 R. The magnetic field is created by oppositely directed currents flowing in two infinitely thin parallel wires. Wires are laid along the body symmetrically to the center of coordinates at the distances of 10⁶ m. In each wire the electrical current of 170 MA is supplied. At the sunward boundary of the computational area the solar wind velocity, pressure, density and the interplanetary magnetic field (IMF) are specified. At other boundaries the “soft” conditions, i.e. zero value of the variable normal derivative, are applied. At the body boundaries the zero velocity normal component is preset.

Calculations have been conducted for the three values of the x-component of the solar wind velocity: \( u_0 = \{200, 400 \text{ and } 800\} \text{ km/s} \). We accept the other values of solar wind parameters: z-component of the solar wind velocity \( w_0 = 0 \); IMF components \( B_{0x} = -1 \text{ nT}, B_{0z} = 0 \); pressure \( p_0 = 10^{10} \text{ Pa} \); density \( \rho_0 = 2·10^{20} \text{ kg/m}^3 \); resistivity \( \eta = 10^{14} \text{ Ω·m} \) (on the choice of the plasma resistivity value see our previous work [17]). The resistivity of the rod matter equals to \( 10^{11} \text{ Ω·m} \).

The gasdynamic part of the problem is described by the Euler equations with the MHD source terms:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial z} (\rho w) = 0
\]

\[
\frac{\partial}{\partial t} (\rho u) + \frac{\partial}{\partial x} (\rho u^2 + \rho w^2) + \frac{\partial}{\partial z} (\rho uw) = -\frac{\partial p}{\partial x} + j_z B_z
\]

\[
\frac{\partial}{\partial t} (\rho w) + \frac{\partial}{\partial x} (\rho uw) + \frac{\partial}{\partial z} (\rho w^2) = -\frac{\partial p}{\partial z} - j_z B_z
\]

\[
\frac{\partial}{\partial t} (\rho (\varepsilon + \frac{u^2 + w^2}{2})) + \frac{\partial}{\partial x} (\rho (\varepsilon + \frac{u^2 + w^2}{2})) + \frac{\partial}{\partial z} (\rho w (\varepsilon + \frac{u^2 + w^2}{2})) = -\frac{\partial}{\partial x} (\rho u) - \frac{\partial}{\partial z} (\rho w) + j_z E_x
\]

\[
\varepsilon = \frac{1}{\gamma - 1} \frac{p}{\rho}
\]

The medium is considered as an inviscid non-heat-conducting perfect gas consisting of protons and electrons with the specific heat ratio \( \gamma = 5/3 \).

The electrodynamics is described by the Maxwell equations and the Ohm’s law (the Hall effect is neglected):

\[
j_z = \frac{1}{\mu} \left( -\frac{\partial B_x}{\partial x} + \frac{\partial B_z}{\partial z} \right)
\]
3. The results of simulation

The global modeling of magnetosphere describes variation the open magnetic field flux per unit cylinder length. In Figure 1 the sawtooth events are presented for the solar wind velocity magnitudes of 200 km/s, 400 km/s and 800 km/s. The increase of the solar wind velocity essentially shortens the periods of the tooth cycles. When the solar wind velocity changes from 200 km/s to 800 km/s the cycle period decreases from 6.4 ks (the second cycle) to 2.0 ks (the third cycle). Meanwhile the tooth pikes are not substantially changed. In the statistical survey on sawtooth events observed by satellites missions [8] the periodicity of sawtooth cycles is determined in the interval of 10.8 – 7.2 ks.

Figure 1. Open magnetic flux per unit body length versus time for 200 km/s (left panel), 400 km/s (middle panel) and 800 km/s (right panel). The growth, expansion and recovery phases are colored, respectively, in yellow, purple and light blue.

The magnetic field and plasma stream topology evolution for the solar wind velocity 200 km/s is presented in Figure 2 (the second cycle extended over 7.6 ks – 14 ks period). The onsets of the growth, expansion and recovery phases are illustrated. In the first panel row the magnetic force lines and colored current density are drawn.

At the beginning of the growth phase the tail magnetic field contains the expanding dipole domain, the outgoing secondary magnetic island and the back of the primary magnetic island (the last two formed in the previous cycle). The flapping current sheets appear between these formations. The reconnection occurs at the 10.4 ks moment and around -16 R x-coordinate. The recovery phase starts with the magnetic island separation from the dipole. The separation is localized approximately at -18 R x-coordinate.

The plasma flow evolution in the near tail is shown in the lower row. It presents splitting the attached flow vortices and their subsequent shedding outward. According to the general mechanism
[20] the splitting originates at the point where the plasma velocity reduces to zero and the length-wise net force (-grad\(p+\mathbf{j} \times \mathbf{B}\)) alternates its sign what stretches the vortex out and tears it in two parts. In the expansion phase the like Karman vortex street shedding period equals to 200 s. The vortex street is confined within the dipole domain and quenches when the magnetic island separates from the dipole.

\[
j_y, \text{nA/m}^2
\]

\[
\begin{array}{cccc}
-1.50 & -1.35 & -1.20 & -1.00 \\
-1.00 & -0.83 & -0.67 & -0.50 \\
-0.33 & -0.17 & 0.00 & 0.17 \\
0.33 & 0.50 & 0.67 & 0.83 \\
1.00 & 1.17 & 1.33 & 1.50 \\
1.67 & 1.83 & 2.00 & \\
\end{array}
\]

rot\(\mathbf{\omega}\), 1/ms

\[
\begin{array}{cccc}
-30 & -27 & -24 & -21 \\
-19 & -16 & -13 & -10 \\
-7 & -4 & 1 & 4 \\
7 & 10 & 13 & 16 \\
19 & 21 & 24 & 27 \\
30 & & & \\
\end{array}
\]

Figure 2. The evolution of magnetic field and plasma stream topology for the solar wind velocity 200 km/s at onset moments of the growth, expansion and recovery phases (left, middle and right columns, respectively); magnetic force lines and color current density (upper row), plasma streamlines and colored vorticity (lower row).

The electrodynamic and flow topology evolution for the solar wind velocity 400 km/s is shown in Figure 3 for the third tooth cycle. The traveling downstream large plasmoid, magnetic island, born in the previous tooth is seen in the left panel of the upper row. The plasmoid is separated from the growing dipole night sector by the IMF. The IMF lines are strongly stretched sunward resulting in the intense current sheet. In the sheet the secondary magnetic island arises (see the middle panel of the same row). At the time moment of 9.9 ks the magnetic field reconnection sets in. Inside of the dipole night sector the magnetic island is growing what increases the sector dimensions. In the distant tail the flapping IMF current sheet is well-disturbed by the Alfvén waves.

\[
j_y, \text{nA/m}^2
\]

\[
\begin{array}{cccc}
-1.50 & -1.35 & -1.20 & -1.00 \\
-1.00 & -0.83 & -0.67 & -0.50 \\
-0.33 & -0.17 & 0.00 & 0.17 \\
0.33 & 0.50 & 0.67 & 0.83 \\
1.00 & 1.17 & 1.33 & 1.50 \\
1.67 & 1.83 & 2.00 & \\
\end{array}
\]

rot\(\mathbf{\omega}\), 1/ms

\[
\begin{array}{cccc}
-30 & -27 & -24 & -21 \\
-19 & -16 & -13 & -10 \\
-7 & -4 & 1 & 4 \\
7 & 10 & 13 & 16 \\
19 & 21 & 24 & 27 \\
30 & & & \\
\end{array}
\]

Figure 3. The evolution of magnetic field and plasma stream topology for the solar wind velocity 400 km/s at onset moments of the growth, expansion and recovery phases (left, middle and right columns, respectively); magnetic force lines and color current density (upper row), plasma streamlines and colored vorticity (lower row).
In the middle panel of the upper row at \( x = -121 \) R the observed wave phase velocity and plasma velocity equal to 3.9 km/s and 1.8 km/s, respectively. This results in 2.1 km/s for the Alfvén velocity, that is in a good agreement with the computed magnitude. The magnetic island gets detached from the dipole night sector at the time 11.6 ks starting the recovery phase.

The flow evolution is kinematicaly similar to the previous flow evolution for the solar wind velocity of 200 km/s but differs in the shortened periods of pulsations. Thus, the vortex shedding period is reduced to 170 s in the expansion phase.

The magnetic field and plasma stream topology evolution for the solar wind velocity 800 km/s is presented in Figure 4. The periodic events are basically repeated with the respective frequency growth. The distinctive feature is seen in the middle panel of the upper row where the regular Alfvén waves propagate in the distant tail. The vortex shedding period decreases to 110 s.

![Figure 4](image)

**Figure 4.** The evolution of magnetic field and plasma stream topology for the solar wind velocity 800 km/s at onset moments of the growth, expansion and recovery phases (left, middle and right columns, respectively); magnetic force lines and color current density (upper row), plasma streamlines and colored vorticity (lower row).

The dayside electrodynamic and hydrodynamic topology for various solar wind velocities is presented in Figure 5. The bow shock advances towards the body when solar wind velocity increases. The field and flow parameters distributions are practically steady and symmetric in the magnetosheath area. Between the magnetosheath and the magnetosphere the magnetic islands are formed (upper panel row). The elongate vertically magnetic islands are located roughly in the middle of the shock layer. Each magnetic island has two x-points and separates the initial north and south open magnetic force lines.

Downstream the magnetic island the two vortices, northern and southern, are generated (lower panel row of Figure 5). The essential factor contributing to the vortex formation and evolution is the total body force. The vortices are unstable and split and recover again periodically in the growth and expansion phases.

**4. Conclusions**

The solar wind velocity is one of the basic drivers which determine magnetospheric evolution. Provided that the solar wind magnetic field is directed southward the sawtooth event is feasible for a certain magnitude of plasma resistivity. The 2D MHD simulation of magnetosphere response to the solar wind impact is performed for the solar wind velocities of 200 km/s, 400 km/s and 800 km/s in the meridional plane. At every velocity the standard tooth cycles with growth, expansion and recovery phases are realized.
The sequence of the tooth cycles is presented by the temporal profile of the open magnetic field flux per rod unit length. The cycle period decreases with the increase of the solar wind velocity. In the dayside sector of magnetosphere the solar wind velocity rise reduces the bow shock standoff distance. The standoff distance is equal to 22 R, 10.5 R and 6.5 R, respectively. In the shock layer, approximately in its middle, the magnetic island is formed. The thin elongated island is oriented normally to the solar wind flow. The island spikes are superposed with the two neutral points indicating the dayside dual reconnection. The magnetic island assists in the magnetosheath flow deceleration and deflection around the magnetosphere. The MHD overflow of the square bar is convectively unstable. In the near tail the vortex street is generated.

The north and south attached vortices are splitted by the total body force comprising of the pressure gradient and the Lorentz force and then are shed staggered within the dipole night domain. The shedding period is decreasing with the rise of the solar wind velocity. In the distant IMF current sheet Alfven waves propagate.
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