Variational Calibration

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Abstract

The approach to the improving the accuracy of the impedance parameter measurements is described. This approach is based on the well-known variations of the influence of the disturbing factors on the results of measurement. Using these variations, measurement circuit provides the additional number of measurements, equal to the number of the disturbing factors. System of equations describes these results of measurements. The solution of this system eliminates the influence of the appropriate uncertainty sources on the results of measurement and gets the true result of the measured value. In addition, the solution of this system also gets the values of the uncertainty components in every measurement and possibility to monitor the properties of the measurement circuit. Examples of the realization of this method for improving the accuracy of the impedance parameter measurements in different bridges are given.

Keywords: impedance, variation calibration, uncertainty, measurement, algorithm, comparison, quadrature, standard, digital synthesis, frequency range, transfer’s function

1. Introduction

History of the electricity science is the history of the development, in sufficient part, of the new methods of measurements. These methods are described perfectly well, for example, in [1]. Widely used replacing and substitution methods entered in all handbooks [2]. Bridge methods are described in many monographs [3, 4]. Monographs [5, 6] describe different methods of bridges’ accurate balance. Many methods of uncertainty correction are described in [7, 8]. All these methods have their widely discussed advantages and disadvantages. There exists no method that could decide all problems, which appears in measuring practice. This chapter describes the method of the variational calibration [9] in the impedance measurements. This method is based on the sequential variation of the influence of the disturbing factors on the
results of measurement. System of equations describes these results. Solution of this system eliminates influences of the disturbing factors and gets the accurate results of measurement. This method significantly simplifies the accurate devices, reducing their weight, dimension and cost, but increases the time of measurement.

2. The variational calibration

2.1. Theoretical basis of the variational calibration

Every measuring circuit (MC) has the input value, which has to be measured and generates measured output value. In an ideal case, the results of measurement depend on the input value and the transfer function $k$ of the MC only.

Formula (1) describes the result of measurement of ideal MC:

$$Z_x = kZ_0$$  \hspace{1cm} (1)

Formula (2) describes the standard uncertainty $\delta_{id}$ of such measurement:

$$\delta_{id} = \sqrt{\delta_0^2 + \delta_s^2}$$  \hspace{1cm} (2)

Here $\delta_0$ and $\delta_s$ are the uncertainties, of the standard $Z_0$ and the uncertainty, caused by the sensitivity of the MC.

In the real MC, the results of measurement $Z_{x0}$ also depend on the complex of the disturbing factors $z_1...z_i...z_j$ as well (for simplicity of the description, these factors on the Figure 1 are shown being out of MC). These factors create proper complex of the uncertainties of measurements $\delta_1...\delta_i...\delta_j$ and shift the appropriate result $Z_{x0}$ of measurement from its ideal value $Z_x$.

![Figure 1. Real measuring circuit.](image-url)
The much more complicated mathematic model (3) of the real MC now describes the results of measurement:

\[ Z_{x0} = \gamma (Z_x; \delta_1 \ldots \delta_i \ldots \delta_j; \delta_0; \delta_s) \]  

(3)

Usually the model (3) is well known from preliminary investigations of the MC.

In the simplest case, every disturbing factor \( z_1 \ldots z_i \ldots z_j \) creates appropriate uncertainty components \( \delta_1 \ldots \delta_i \ldots \delta_j \). In more complicated cases, some disturbing factors \( z_1 \ldots z_i \) can influence some complex \( Z_i \ldots Z_{i+m} \) of the results of measurement. But we know functions \( \delta_i = f_i(z_1 \ldots z_i \ldots z_n) \) and do not know just the constant coefficients, which enters into these dependences.

Formula (4) describes the standard uncertainty \( \delta_r \) of the measurement of the real MC:

\[ \delta_r = \sqrt{\delta_0^2 + \sum_i \delta_i^2 + \delta_s^2} \]  

(4)

To eliminate the influence of the uncertainties \( \delta_1 \ldots \delta_i \ldots \delta_j \) on the results of measurement, the variation method was developed (VM) [9]. **Figure 2** illustrates this method. Here, MC contains \( n \) additional variators \( V_1 \ldots V_j \). Last ones influence the uncertainty sources \( z_1 \ldots z_j \) and change the uncertainty \( \delta_1 \ldots \delta_j \). It creates the output of the proper results of MC measurement \( Z_{x1} \ldots Z_{xj} \).

Variators cannot change the uncertainties \( \delta_0 \) and \( \delta_s \). These uncertainties are supposed to be known or equal to zero during the VM calibration.

VM consists of the following steps:

1. First, MC measures initial value \( Z_{x0} \) of the input value \( Z_x \).
2. Then, MC consequently varies the influence of the disturbing factor \( z_i \) on the well-known value \( \alpha_i \).

Figure 2. Variational measuring circuit.
Variations could be provided in any order. To simplify the system of equations, it is preferable to perform variations sequentially and to switch ON the variation \( \alpha_i \) when all other variations are switched OFF.

Variations could have any law. To simplify the system of equation, it is preferable to provide the multiplicative variation (when we multiply the appropriate uncertainty component \( \delta_i \) on well-known ratio \( \alpha_i (\delta_{iv} = \alpha_i \delta_i) \)) or additive variation (when we add the appropriate well-known uncertainty \( \Delta_v \) to the uncertainty component \( \Delta_{im} (\Delta_{iv} = \Delta_{im} + \Delta_v) \)).

3. After every variation, MC measures the results of the measurement \( Z_{x1} \ldots Z_{xi} \ldots Z_{xj} \).

4. The system of Eqs. (5) describes these measurements:

\[
\begin{align*}
Z_{x0} &= \gamma(Z_x, \delta_1 \ldots \delta_i \ldots \delta_j, \delta_0, \delta_s) \\
Z_{x1} &= \gamma(Z_x, \delta_1, \alpha_1 \ldots \delta_j, \delta_0, \delta_s) \\
Z_{xj} &= \gamma(Z_x, \delta_1 \ldots \delta_i \ldots \delta_j, \alpha_j, \delta_0, \delta_s)
\end{align*}
\] (5)

The system (5) contains \( j + 1 \) unknown quantities: \( Z_x \) and uncertainties of measurement \( \delta_1 \ldots \delta_{i'} \) and \( j + 1 \) results of measurement \( Z_{x0} \ldots Z_{xj} \). Solution (6) of this system gets the true value of the results of measurement \( Z_x \) and the values of the uncertainties \( \delta_1 \ldots \delta_j \) of the measurement:

\[
\begin{align*}
Z_x &= \rho_0[(Z_{x0} - Z_{xj}), (\alpha_1 - \alpha_j), \delta_0, \delta_s] \\
\delta_1 &= \rho_1[(Z_{x0} - Z_{xj}), (\alpha_1 - \alpha_j), \delta_0, \delta_s] \\
\delta_j &= \rho_j[(Z_{x0} - Z_{xj}), (\alpha_1 - \alpha_j), \delta_0, \delta_s]
\end{align*}
\] (6)

Periodical variation calibration lets us to observe the behavior of every disturbing factor, to determine their stability, to monitor measuring circuit and to ensure precision of the period of the variational calibration.

Let the uncertainty caused by the finite sensitivity of the \( i \)-measurement be \( \delta_i \) and the uncertainty of the variation \( \alpha_i \) be \( \delta_{\alpha_i} \). In this case, formula (7) describes the resulting standard uncertainty \( \delta_c \) of the measurement with variation calibration:

\[
\delta_c = \sqrt{\delta_0^2 + \sum_j (\delta_i^2 \delta_{\alpha_i}^2 + \delta_{\alpha_i}^2)}
\] (7)

Eq. (7) shows that the VM sharply decreases influence of the uncertainty components \( \delta_i \) on the common uncertainty of measurement (on the \( 1/\delta_{\alpha_i} \) times).

Let us suppose uncertainty source \( z_i \) creates uncertainty \( \delta_i = 10^{-3} \) and we need to decrease it to the value \( 10^{-6} \). It means that we have to provide appropriate variation with uncertainty better than \( 10^{-3} \) only. It is a very important result of the VM. This effect is restricted only by the stability of the uncertainties \( \delta_1 \ldots \delta_j \) during the time of measurement.

Let us suppose that time of every measurement is \( t_i \). It means that the common time \( t_c \) of measurement increases to the value:
\[ t_c = \sum_{i=0}^{n} t_i \]

Let us suppose that \( \delta \alpha_i = 0 \) and \( \delta_0 = 0 \). In this case, formula (9) describes the standard uncertainty of measurement caused by sensitivity of measurements only:

\[ \delta_c = \sqrt{\sum_{i=0}^{n} \delta_{s_i}^2} \]

Formulas (8) and (9) show that the variation method has two disadvantages:

- Variation method needs \( n + 1 \) measurement instead one only. It sufficiently increases the time of measurement.
- Variation method increases the contribution of measurement sensitivity \( \delta_{s_i} \) in the common uncertainty of measurement.

We can overcome these two disadvantages of the variation method in different ways. Here, we shortly describe time and space clustering of the thesaurus of the uncertainty sources.

### 2.1.1. Time clustering

Usually, different uncertainty sources have different typical speeds of drift. We can divide the thesaurus of \( j \) uncertainty sources into clusters, which have congruous time of drift. Figure 3 illustrates this approach. In Figure 3, thesaurus of the \( j \) uncertainty components is divided into three clusters \( T_1, T_2 \) and \( T_3 \) (\( j = m + n + k \)).

The first cluster \( (T_1) \) joins \( m \) of the most stable uncertainty sources. It could be instability of the internal standards or arms ratios in transformer bridges, and so on. MC provides their calibration very seldom, for example, one time per year. For this calibration, MC performs sequential

![Figure 3. Variation calibration with time clustering.](image-url)
variation of all sources of uncertainty and provides $m + n + k + 1$ measurements. The system (5) of equations describes the results of these measurements. Solution (6) of this system gets us values of the $m$ uncertainties of the first cluster.

The second cluster ($T_2$) joins the $n$ less stable sources of the uncertainty. It could be the temperature dependences of the operational amplifiers parameters, and so on. Calibration of these sources is provided more frequently, for example, one time per hour. During this calibration, we suppose that the $m$ uncertainties of the first clusters are stable. Values of these uncertainties enter in the system (5) as constants. To find values of the $n$ uncertainties of the second cluster, MC varies sequentially the uncertainty sources $n + k$, provides proper measurements and solves the system (5). It needs $n + k + 1$ measurements.

The third cluster ($T_3$) joins the $k$ uncertainty sources which change most quickly. This cluster mostly includes the sources, which directly depends on the parameters of the object to be measured. This calibration is aimed to find the true results of measurement and values of the last $k$ uncertainties. During this calibration, we suppose that uncertainties of the first and second clusters are stable. Their appropriate values are entered in system (5) as constants. Calibration now consists of sequential variation of the $k$ uncertainties of third cluster and appropriate measurements. Solution of the system (5) gets us the true results of measurement $Z_x$ and last $k$ uncertainties. This calibration needs $k + 1$ measurements only.

Let us suppose that any measurement needs time $t_i$. Formula (10) describes the weighted average $t_c$ of the measurement with variation calibration:

$$t_c = \Sigma_k t_i \left(1 + \frac{n + k}{m + n + k + 1} \frac{T_k}{T_m} + \frac{k}{m + n + k + 1} \frac{T_k}{T_m} \right)$$

(10)

where $\Sigma_k t_i$ is the time of the $k$ cluster calibration and measurement, $T_n/T_m$ is the ratio of the periods of the second $T_n$ and first $T_m$ clusters calibrations and $T_k/T_m$ is the ratio of the periods of the third $T_k$ and first $T_m$ cluster calibrations.

Formula (10) shows that the time of measurement decreases only slightly during the time of calibration of the third cluster. It means sufficient diminution of the time of measurement.

2.1.2. Space clustering

Sometimes, we do not need to separately study every component of the measurement uncertainty. In this case, we use space clustering. During the space clustering, MC is represented as a complex of the $n$ quadripoles and standards to be compared. Figure 4 shows such decomposition of the measurement circuit.

In Figure 4, $K_1 \ldots K_n$ are the quadripoles of the MC and the $V_1 \ldots V_n$ are the variators used to vary the transfer coefficient of the proper quadripole.

The following formula describes the decomposed MC:

$$Z_{x0} = f(Z_x, K_1, \Delta K_1 \ldots K_i, \Delta K_i \ldots K_n, \Delta K_n)$$

(11)
where \( Z_x \) and \( Z_{x0} \) are the MC input and output values, respectively. \( \Delta K_1 \ldots \Delta K_i \ldots \Delta K_n \) are the uncertainties of the quadripole transfer coefficients \( K_1 \ldots K_i \ldots K_n \).

The following formula expresses the dependence of the measurement uncertainty \( \delta_r \) on the components of the decomposed MC:

\[
\delta_r = \sqrt{\delta_0^2 + \delta_s^2 + \sum_i \Delta K_i^2}
\]  

(12)

where \( \delta_s \) is the uncertainty caused by the finite MC sensitivity.

Let us provide \( n \) well-known variations \( v_1 \ldots v_i \ldots v_n \) of the quadripole transfer coefficients \( K_1 \ldots K_i \ldots K_n \). MC provides the new measurements \( Z_{x0}, Z_{x1} \ldots Z_{xn} \) of the unknown value \( Z_x \) after every variation. The system of Eq. (13) describes these measurements:

\[
Z_{x0} = f(Z_x, K_1, \Delta K_1 \ldots K_i, \Delta K_i \ldots K_n, \Delta K_n)
\]
\[
Z_{x1} = f(Z_x, K_1, \Delta K_1, v_1 \ldots K_i, \Delta K_i \ldots K_n, \Delta K_n)
\]
\[
Z_{xn} = f(Z_x, K_1, \Delta K_1 \ldots K_i, \Delta K_i \ldots K_n, \Delta K_n, v_n)
\]

(13)

Solution of the system (13) of equations gets accurate results of measurement together with all uncertainties of the quadripoles.

Formulas (8) and (9) describe the uncertainty and time of measurement when using the space clustering as well. However, the number of measurements in case of space clustering is much less. Error accumulation and common time of measurement are much less as well.

We can decompose the measuring circuit in different ways. Optimal decomposition depends on the structure of the measuring circuit. Here, it is impossible to analyze all these possibilities. In most cases, we are forced to use time and space clustering together.
It should be noted that variation method was used earlier in some measurements (e.g., elimination of the uncertainty caused by self-heating of the resistive thermometer in temperature measurements). Here, we consider generalization and dissemination of this method in different areas, first in impedance measurements.

2.2. Experimental developments of the VM

VM was used in several developments. It is too complicated to analyze all these possible applications. Here, we consider only some applications of this method in very important cases of widely used digibridges and in accurate transformer bridges.

2.2.1. Application of the VM in digibridges

Development of the integral operational amplifiers and microprocessors resulted in the new class of measuring devices—digibridges [10–12]. Nowadays, digibridges cover most part of the specific market of the impedance meters. Now many companies manufacture digibridges (HP, Agilent, TeGam, IetLab, Wine Kerr, etc).

2.2.1.1. Operation and analysis

A usual digibridge consists of two serially coupled impedances \( Z_x \) and \( Z_0 \) (see Figure 5) These impedances are connected between outputs of the generator G and the protecting amplifier A. Negative input of this amplifier is connected to the common point of the impedances \( Z_x \) and \( Z_0 \). Amplifier A creates in this point the potential, close to zero (virtual ground). The same current \( I_x \) flows through both impedances \( Z_x \) and \( Z_0 \) and creates voltages \( U_x \) and \( U_0 \). Differential vector voltmeter DVV, through switcher \( S_0 \), measures these voltages and transfers the

![Figure 5. Structure of the digibridge with variational calibration.](image-url)
results of measurement to microcontroller μC. μC controls the operation of the MC, processes results of the voltages measurements and calculates the ratio of two impedances $Z_x$ and $Z_0$. Display D shows results of measurements.

The amplifier A protects measuring circuit and decreases the influence of the parasitic admittance $Y_c$ between the amplifier inputs on the results of measurement.

In case if gain $K$ is infinite, Eq. (14) describes the process of measurement:

$$\frac{Z_x}{Z_0} = \frac{U_x}{U_0}$$

Let gain $K$ be finite. In this case, admittance $Y_c$ between the amplifier inputs cause one of the biggest sources of the measurement uncertainty. This uncertainty ($\delta Z$) strongly limits the measurements of the high impedances on high frequencies. $\delta Z$ is described by the equation:

$$\delta Z = Y_c Z_0 / (1 + K)$$

If $K \gg 1$, we can write:

$$\delta Z \approx Y_c Z_0 / K$$

Here, the values $Y_c$ and $K$ are the disturbing factors. The quotient of the $Y_c$ and $K$ can be considered as the sole source of the uncertainty. Let us provide the multiplicative variation of the gain $K$ of the amplifier A. To vary $K$ on ratio $\alpha_1$, the divider $D_v$ with transfer coefficient 1 or $\alpha_1$ (Figure 5) is used. After this variation, MC measures the additional voltage $U_{0v}$.

The system of three equations describes the measurements of the voltages $U_x$, $U_o$ and $U_{0v}$.

$$U_x = I_x Z_x; \quad U_0(1 - Y_c Z_0 / K) = I_x Z_0; \quad U_{0v}(1 - Y_c Z_0 / K) = I_x Z_0$$

Solution of this system gets the following formula (18):

$$\frac{Z_x}{Z_0} = U_x [1 - \delta U \cdot \alpha_1 / (1 - \alpha_1)] / U_0$$

where $\delta U = 1 - U_0 / U_{0v}$

Analysis of the formula (18) shows that the uncertainty of the variation calibration has minimal if $\alpha_1=0.5$. Then:

$$\frac{Z_x}{Z_0} = U_x (1 - \delta U) / U_0$$

Formula (19) shows that the ratio $Z_x/Z_0$ does not depend on the quotient of the $Y_c$ and $K$.

But here increases component of the uncertainty, caused by the increased number of measurements. VV measures quadrature components $a$ and $b$ of three voltages: $U_x$, $U_o$ and $U_{0v}$. Let us suppose that effective input noise of the VV in all these measurement has the same value $\Delta$ and the results of measurement are not correlated. In this case, the following formulas are justified:
\[ U_x = (a_x + \Delta) + j(b_x + \Delta); \quad U_0 = (a_0 + \Delta) + j(b_0 + \Delta); \quad U_{0v} = (a_{0v} + \Delta) + j(b_x + \Delta) \] (20)

Let us substitute formula (20) in (14) and (19). It gets the following formulas for two cases:

Without variational calibration:
\[ \delta_m \approx \sqrt{2} \delta_n \quad \text{and} \quad \Delta_a \approx \sqrt{2} \delta_n \] (21)

With variational calibration:
\[ \delta_m \approx \sqrt{5} \delta_n \quad \text{and} \quad \Delta_a \approx \sqrt{2} \delta_n \] (22)

where \( \delta_m \) and \( \Delta_a \) are the multiplicative and additive uncertainties caused by the relative noise \( \delta_n \) of the VV.

Formulas (21) and (22) show that the additive uncertainty \( \Delta_a \) caused by the relative noise \( (\delta_n = \Delta/U_0) \) in both cases is the same. But these formulas also show that due to the variational calibration, the multiplicative random uncertainty \( \delta_m \) increases 1.6 times.

Calculation of the uncertainty by the formula (16) has the truncation error \( \delta_t \) caused by inequality \( K \gg 1 \). This error sharply increases when \( K \) on high frequencies is low, so that calibration practically does not work when \( K \to 1 \). If amplifier gain \( K \) is so low, we cannot consider value \( Y_c/K \) as the sole source of the uncertainty. As a result, we have to provide two separate variations: multiplicative variation of the gain \( K \) and additive variation of the admittance \( Y_c \) (using variational admittance \( Y_v \) and switcher \( S_v \)). DVV measures sequentially voltages \( U_x, U_0 \) and \( U_{0v}, U_{00} \) after multiplicative variation of the gain \( K \) and additive variation of the admittance \( Y_v \).

System of three equations describes these four measurements:
\[
\begin{align*}
U_x/U_0 &= Z_x/Z_0[1 + Y_cZ_0/(1 + K)] \\
U_x/U_0 &= Z_x/Z_0[1 + Y_cZ_0/(1 + \alpha_1 K)] \\
U_x/U_0 &= Z_x/Z_0[1 + (Y_c + Y_v)Z_0/(1 + K)]
\end{align*}
\] (23)

Solution of the system (23) gets following two equations:
\[
\begin{align*}
Y_cZ_0 &= \left( A' - 1 \right) \left( \alpha_1 K + 1 \right) (K + 1)/(K(1 - \alpha_1)) \\
aK^2 + bK + c &= 0
\end{align*}
\] (24)

here: \( a = [(1 + \alpha_1) - \alpha_1(A' - 1)](A'' - 1), \quad b = (Y_vZ_0 + A')A'' - A', \quad c = (A' - 1)(A'' - 1) \), \( A' = U_{0v}/U_{0v} \), \( A'' = U_0/U_0 \)

Solution of the Eqs. (24) and substitution of these results in (15) gets the accurate results of measurement which absolutely does not depend on the values \( Y_c \) and \( K \).
The described approach could be used for the accurate calibration of any amplifier with positive or negative gain, followers, gyrators, and so on. It could be used for calibration of any control system as well.

3. Experimental results

The earlier described approach was used in digibridge MNS1200. This digibridge was developed for Siberian Institute of Metrology (Novosibirsk), to be used in working inductance standard. Its short specification is as follows.

MNS1200 operates in frequency range of DC to 1 MHz.

Frequency set discreteness $2 \times 10^{-5}$.

Capacitance range of measurement (F) $10^{-17}$–$10^{5}$.

Resistance range of measurement (R) $10^{-6}$–$10^{14}$.

Inductance range of measurement (H) $10^{-12}$–$10^{10}$.

Dissipation factor $\mathrm{tg} \delta$ ($\mathrm{tg} \varphi$) $10^{-6}$–1.0.

Main uncertainty (ppm) 10.

Sensitivity (ppm) 0.5

Inner standard instability (24 hours, ppm) ± 2.

Weight (kg) 4

MNS1200 appearance is shown in Figure 6.

Instability of the MNS1200 inner standard can achieve $10^{-4}$ in a long period of time. To get maximal accuracy, MNS1200 can be calibrated by arbitrary R,L,C outer standard. In this case,

Figure 6. Digibridge MNS1200.
uncertainty of measurement depends on short-time stability of inner standards. Results of the 24-hour 1 Ohm standard measurements are shown in Figure 7.

3.1. Application of the VM in transformer bridge

Accurate comparison and unit dissemination of the impedance parameters are provided using many different, very complicated manual bridges with numerous different standards. The main world-renowned laboratories (BIPM, NIST, NML, NPL, PTB, VNIIM, etc.) in developed countries have their own primary standards, based on the calculable capacitor [13, 14] and the appropriate transformer bridges [15, 16], on the quantum hall resistance [17] and the appropriate bridges [18, 19] and very accurate quadrature transformer bridges for comparison of different impedance parameters [20, 21], that have original constructions. All these bridges contain complicated set of devices and have long and intricate handle balancing processes. In addition, these bridges and standards are of different kinds and are located in various laboratories. The process of calibration and traceability is, therefore, complicated and very expensive. Uncertainty of the measurement of these bridges achieves $10^{-8}$–$10^{-9}$. It makes them an excellent instrument for fundamental investigations.

For practical needs of the metrologic calibration, it is enough to provide measurements with uncertainty about $10^{-6}$. In this case, the equipment have to be universal, to compare arbitrary standards, to have low cost and weight and to be transportable. The complex of bridges described later satisfies these demands. Complex consists of autotransformer and quadrature bridges. Both of them are based on the variational calibration. Autotransformer bridge provides unit transfers in the whole range of the impedance of the C,L,R standards. Quadrature bridge provides cross transfers of the units. Last bridge is described in [22, 23].

This chapter describes the part of the results of this project, covering the development of the transformer bridge-comparators which transfer units of the resistance, inductance, capacitance...
and dissipation factor in a whole range of measurements and reciprocal transfer of any units. Balance and calibration of these bridges are based on the variational method.

### 3.1.1. Autotransformer bridge: description and analysis

Early autotransformer bridges were described in [24, 25]. These bridges have been widely used up to now [15, 16]. To eliminate the influence of the cable impedance (yoke) on the results of measurement, double autotransformer bridges are used [3, 5]. The wide-range double autotransformer bridge contains two inductive dividers, simultaneously controlled for bridge balance. For accurate measurements, these inductive dividers usually are of a two-stage design at least. Every stage of these inductive dividers [26] consists of a lot of turns and appropriate complicate switchers. They have to have multidigit capacity (up to seven or eight digits). This quite complicates the bridge.

Development of the variational bridge has to solve two problems:

- to eliminate the Yoke ($Z_n$) influence on the results of measurement without using the double autotransformer bridge;
- to decrease sharply the number of the autotransformer divider decades without loss in the accuracy of measurement.

The simplified measuring circuit of the automatic variational bridge (PICS) [27], which solves these problems, is shown in **Figure 8**.

The bridge consists of the supply unit (the generator $G$ connected to the voltage transformer $TV$), the main autotransformer $AT$ and the variationally balanced $90^\circ$ phase shifter [28], which is calibrated through calibration circuit $CC$. The vector voltmeter $VV$ (through the preamplifier $PA$ and switchers $S_1$ and $S_2$) measures the bridge ($U_1$, $U_2$) and the calibration circuit $CC$ ($U_c$)

![Figure 8. Circuit diagram of the autotransformer bridge.](image-url)
unbalances the signals. The differential voltage follower 1:1 compensates the voltage drop $U_n$ on the cable impedance $Z_n$. The microcontroller $\mu$C transfers the results of the VV measurements to the personal computer PC and controls the bridge balance and calibration of the phase shifter $90^\circ$. The autotransformer AT Carries on its core windings $m_2$, $m_{1c}$ and $m_{1k}$. These windings are used to balance the bridge by the main ($m_{1c}$) and secondary ($m_{1k}$) parameters. The standards to be compared $Z_1$ and $Z_2$ are connected serially by the cable (yoke) and by their high potential ports, to voltage transformer TV and to the windings $m_{1c}$ and $m_2$ of the autotransformer AT.

The output of the $90^\circ$ phase shifter is connected in series with the winding $m_{1c}$ to create the balance winding $m_1 = m_{1c} + jm_{1k}$.

The drop of the voltage $U_n$ acts on the impedance $Z_n$ of the cable which connects $Z_1$ and $Z_2$. This voltage is applied to the input of the differential voltage follower 1:1.

The two-channel VV has two digital synchronous demodulators, proper LF digital filters and $\Sigma$-$\Delta$ ADC. It simultaneously measures two orthogonal components of the bridge unbalance signals. This voltmeter has high selectivity (equivalent Q-factor is higher than $10^5$). Its integral nonlinearity is better than $10^{-4}$ and relative sensitivity is better than $10^{-5}$. The VV is calibrated automatically and periodically by variational algorithm, described in [29].

On the low impedance ranges, the drop $U_n$ of the voltage on the cable impedance increases. This increases the uncertainty of the bridge unbalance measurement. To decrease this effect, the voltage follower 1:1 is used. This follower places the named drop of the voltage between low potential pins of the windings $m_1$ and $m_2$. It decreases the effective cable impedance from $Z_n$ to the equivalent value $Z_{ne} = Z_n \delta$, where $\delta$ is the uncertainty of the transfer coefficient of the voltage follower.

To decrease the number of the decades of the autotransformer divider and eliminate the influence of the $Z_n$ on the results of measurement, the bridge operates in a non-fully balance mode and use twice variational balance [27].

In compliance with developed variational algorithm, VV measures sequentially the bridge unbalance signals $U_1$ and $U_2$. After that, $\mu$C varies the turns of the winding $m_1$ on $\Delta m_v$ and VV measures the variational signal $U_{2v}$.

The system of Eqs. (30) describes these three measurements:

$$
U_0(Z_1/Z_c) - U_0[1 - Z_n(1 + \delta)/Z_c]m_1/(m_1 + m_2) - U_1 = 0
$$

$$
-U_0[1 - Z_n(1 + \delta)/Z_c]m_2/(m_1 + m_2) + U_0Z_2/Z_c + U_2 = 0
$$

$$
-U_0[1 - Z_n(1 + \delta)/Z_c]m_2/(m_1 + m_2 + \Delta m_v) + U_0Z_2/Z_c + U_{2v} = 0
$$

(25)

where $Z_c = Z_1 + Z_2 + Z_{nr}$ and $\delta$ is the uncertainty of the voltage follower 1:1, $U_0$ is the supply voltage.

The formula (26) gives the solution of the system (25):

$$
\delta Z = -\frac{\Delta m_1 + m_2}{2} \left( C + \frac{m_1 - m_2}{m_1 + m_2} D \right) /[1 + (C + D)\delta_v]
$$

(26)

where
\[ C = \frac{(U_2 + U_1)}{(U_{2v} - U_2)}; \quad D = \frac{(U_2 - U_1)}{(U_{2v} - U_2)}; \quad \delta_v = \frac{\delta m}{(1 + \delta m)}; \quad \delta m = \frac{\Delta m_v}{(m_1 + m_2)} \]

\( \mu \)C uses the results of the calculation of the bridge unbalance \( \delta Z_c \) by described algorithm in two stages:

- in the first stage, \( \mu \)C makes quick, automatic balance of the bridge on the four high-order decades (balance stage);
- in the second stage, \( \mu \)C increases the sensitivity of the voltmeter VV on \( 10^4 \) and decreases the value of the variation \( \Delta m_v \) of the \( m_1 \) turns in the same ratio. Then, \( \mu \)C repeats the measurements by described algorithm. Results of these measurements and calculations by formula (26) determine the balance point coordinates and find the impedance ratio:

\[ \frac{Z_1}{Z_2} = \frac{m_1}{m_2} - \delta Z \]  \hspace{1cm} (27)

The final result is given in 8.5 digits.

The bridge balance and data processing by described variational algorithm reduce the number of the autotransformer dividers to only one and sharply (twice) reduce the number of the digits of this divider.

The 90° phase shifter and the calibration circuit CC do not contain accurate internal standards of capacitance or resistance. To get good accuracy, we use the special phase shifter calibration procedure based on the variational method. Simplified structure of this phase shifter is shown in Figure 9.

Phase shifter consists of serially connected inverter I and proper phase shifter PS. Firstly, calibrating circuit (resistors \( R_1 \) and \( R_2 \) and switchers \( S_1 \) and \( S_2 \)) are used to calibrate inverter I. Secondly, calibrating circuit (resistor \( R_1 \) and capacitor \( C_1 \)) and switchers \( S_3 \) and \( S_4 \) are used to calibrate the phase shifter PS. Vector voltmeter VV, through switcher \( S_5 \) measures unbalance signals of the first or second calibration circuits and translates the results of measurements to microcontroller \( \mu \)C. Finally, one controls all calibration procedure and calculates PS real transfer coefficient.

Calibration procedure consists of two stages.

\[ C = \frac{(U_2 + U_1)}{(U_{2v} - U_2)}; \quad D = \frac{(U_2 - U_1)}{(U_{2v} - U_2)}; \quad \delta_v = \frac{\delta m}{(1 + \delta m)}; \quad \delta m = \frac{\Delta m_v}{(m_1 + m_2)} \]
4. Calibration of the inverter I

To calibrate the inverter, the VV measures three signals of the calibration circuit $R_1-R_2$:

- The initial output signal of the calibration circuit $U_{i1}$;
- The signal $U_{i2}$ after the variation of the inverter transfer coefficient on the value $\delta_{iv}$;
- The signal $U_{i3}$ after the inversion of the connection of the calibration circuit between the input and output of the inverter I by the switchers $S_1$ and $S_2$.

Complex of these signals is described by proper system of equations. Solution of this system (formula 38) gets the accurate deviation $\delta_i$ of the inverter transfer coefficient from its nominal value “1.”

$$\delta_i = \delta_{iu} (1 + \delta_{kia})$$  \hspace{1cm} (28)

where,

$$\delta_{iu} \approx \frac{\delta_{iv} U_{i1} + U_{i1}}{2 U_{i2} - U_{i1}}, \delta_{kia} \approx \frac{\delta_{iv} U_{i2} - U_{i1}}{2 U_{i3} - U_{i1}}, \delta_{iu}$$ and $\delta_{kia}$ are the approximate values of the transfer coefficients of the inverter I and calibration circuit $R_1-R_2$.

5. Calibration of the phase shifter PS

To calibrate the phase shifter PS, the VV measures three signals of the calibration circuit $R_3-C_1$:

- The initial output signal $U_{p1}$ of the calibration circuit, when calibration circuit is connected between input and output of the phase shifter;
- The signal $U_{p2}$ after the variation of the phase shifter PS transfer coefficient in the value $\delta_{pv}$;
- The signal $U_{p3}$ after the inversion of the calibration circuit and connection of this circuit between the input of the inverter I and output of the phase shifter PS by the switchers $S_1$ and $S_2$.

Complex of these signals is described by proper system of equations. Solution of this system (formula (29)) gets the accurate deviation $\delta_p$ of the phase shifter PS transfer coefficient from its nominal value “1”:

$$\delta_p = \delta_{pu} (1 + \delta_{kpa})$$  \hspace{1cm} (29)

where:

$$\delta_{pu} \approx \frac{\delta_{pv} jU_{p3} + U_{p1}}{2 U_{p2} - U_{p1} - \delta_k 1 + \delta_i} \text{ and }$$

$$\delta_{kpa} \approx \frac{\delta_{pv} jU_{p3} - U_{p1}}{2 U_{p2} - U_{p1} - \delta_k 1 + \delta_i}$$

$\delta_{pu}$ and $\delta_{kpa}$ are the approximate values of the transfer coefficients of the inverter PS and calibration circuit, respectively.
After the calibration procedure, we know the real value of the phase shifter transfer coefficient with an uncertainty better than 1–3 ppm. \( \mu \text{C} \) makes this calibration procedure automatically at least every hour.

Figure 10. Some results of experimental investigations.
5.1. Experimental results

All results of the theoretical investigations shown earlier were used to develop the comparator PICS.

PICS very short specification is given as follows.

Short PICS Specification.

PICS operates on frequencies 1.00 and 1.59 kHz.

Frequency set discreteness $5 \times 10^{-5}$.

Capacitance range of measurement ($F$) $10^{-19}$–$10^{-3}$.

Resistance range of measurement ($R$) $10^{-7}$–$10^{8}$.

Inductance range of measurement ($H$) $10^{-12}$–$10^{3}$.

Dissipation factor $\tan \delta$ ($\tan \varphi$) $10^{-6}$–1.0.

Main uncertainty (ppm) 1.0.

Sensitivity (ppm) 0.02–0.05

Weight (kg) 5

PICS was tested in USA (NIST) and Russia (VNIIM), in Germany (PTB) and Poland (GUM), in Ukraine (Ukrmetrteststandard) and Byalorussia (Center of metrology).

Some results of these tests are shown in Figure 10.

Appearance of the PICS, together with intermediary thermostated standards, is shown in Figure 11.
6. Conclusion

Variational calibration sharply increases the accuracy of measurement. In case of variation correction, for precision measurements, we can use simple and cheap measuring circuits with rather high uncertainty. Variational calibration diminishes the uncertainty of such circuits on thousands or even more times. It does not need too accurate variational standards. Time and space clustering in significant measure overcomes disadvantages of this calibration—increasing the time of measurement. Experimental investigations of the comparator PICS have shown that uncertainty of measurement on main ranges is lower than $10^{-6}$ and sensitivity is better than $10^{-7}$–$10^{-8}$. Variational calibration also decreases the weight and cost of the accurate equipment.

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