The Relativistic Velocity Transformation

and the Principle of Absolute Simultaneity

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Abstract

The arguments employed by Einstein that led him to the conclusion that “we cannot attach absolute meaning to the concept of simultaneity” are subjected to critical analysis in light of experiments that have subsequently been carried out with atomic clocks. The physical significance of time dilation is a key element in this discussion. It is pointed out that the fact that two clocks at rest in a laboratory disagree on the elapsed time for a given event is normally interpreted to mean that they are not properly synchronized or that they run at different rates, not that the event in question did not occur simultaneously for them. An experiment using GPS technology is outlined to test whether events really do not occur simultaneously for all observers or if instead that their timing results only differ because the clocks employed by them run at systematically different rates. An example of two light pulses moving in opposite directions shows that one can come to a different conclusion about simultaneity depending on whether the Lorentz transformation (LT) of space-time coordinates or the associated relativistic velocity transformation (RVT) is employed as justification. Finally, it is shown that it is possible to satisfy Einstein’s two postulates of relativity while still maintaining the principle of absolute simultaneity of events by introducing an alternative (Global Positioning System) Lorentz space-time transformation (GPS-LT) that is also perfectly consistent with the RVT.
I. Introduction

One of the most significant claims of Einstein’s original paper [1] on the special theory of relativity (STR) was that events that are simultaneous in one inertial frame are not simultaneous in another. He used an example in which two identical rods R₁ and R₂ are initially coincident in a given inertial frame in which two observers O₁ and O₂ have synchronized their respective clocks [2]. Subsequently O₂ moves with R₂ into a different inertial frame (that is, after being accelerated relative to O₁). According to Einstein’s postulates of STR [1], each observer measures the same elapsed time for a light pulse to traverse the rod in his rest frame in both directions (O₁ carries out his measurement for R₁ and O₂ for R₂). If the length of each rod is L, then the elapsed time in both cases is \( T = \frac{2L}{c} \), where c is the speed of light in free space. Observer O₁ also measures the time \( T' \) it takes for the light pulse to traverse R₂. According to STR [1], however, \( T' \neq T \). Specifically, if the relative speed of the two observers is \( u \), then \( T' = \gamma T \), where \( \gamma = (1 - \frac{u^2}{c^2})^{0.5} \). On this basis Einstein concluded [1, 2] that the light pulse did not traverse R₂ simultaneously for O₁ and O₂.

There is another way to explain the results of Einstein’s example, however. The reason that \( T \) does not equal \( T' \) can simply be taken as a consequence of the fact that the respective proper clocks in the rest frames of O₁ and O₂ do not run at the same rate. After all, the same inequality would occur while O₂ and R₂ were still at rest with respect to O₁ and L₁ if some technical problem caused one of the clocks to slow down relative to the other. Since it is well established experimentally that the rates of clocks vary with their state of motion and position in a gravitational field [3, 4], it would seem well advised on this basis to reconsider Einstein’s conclusion about the inevitable non-simultaneity of events for observers in different inertial systems. To this end it is helpful to consider another popular argument that has been given as “proof” of non-simultaneity, as will be done in the following section.
II. Spherical Expansion of a Wave Front

Suppose that a light source flashes at point P in an inertial system S. An observer O at rest at P sees the wave front expand at the speed of light in all directions. Now consider two points A and B equidistant from P and lying diametrically opposite to one another on the x axis of the coordinate system (Fig. 1). If the distance between P and each of these points is L, then O will find that the wave front arrives at both of them at time T=L/c, that is, these two events occur simultaneously for him. According to the Lorentz transformation (LT), however, the two events will not be simultaneous for another observer O’ in S’ who moves at speed u relative to O in the positive x direction. The respective elapsed times t and t’ for a given event are related by the well-known equation of the LT:

\[ t' = \gamma (t - \frac{ux}{c^2}) \]

where x is the location of the object (wave front) relative to O at the time of measurement. Let us refer to the times of arrival (L/c) of the wave front at A and B as measured by O in S as \( t_A \) and \( t_B \), respectively. The corresponding values of \( x_A \) and \( x_B \) are equal to L and –L in this case. We can now use eq. (1) of the LT to determine the corresponding times of arrival (\( t_A' \) and \( t_B' \)) of the wave front at the same two points:

\[ t_A' = \gamma (\frac{L}{c} - \frac{uL}{c^2}) \]

\[ t_B' = \gamma (\frac{L}{c} + \frac{uL}{c^2}). \]

The result is that \( t_A' \) differs from \( t_B' \), and the conclusion is therefore that the two events do not occur simultaneously for O’ in S’. It is important to note that the LT effect in question is of order \( c^{-2} \), and has never been verified in actual experiments.

We can use standard relativity theory in a different way in order to examine this situation, however, namely to use the relativistic velocity transformation (RVT):

\[ v' = \frac{v - u}{1 - uv/c^2}. \]
In our example, \( v_A = v_B = c \) is the speed of the light wave in both directions relative to P from O’s perspective. Upon substitution in eq. (3), we find that \( v_A' = v_B' = c \) as well, that is, the speed of the light wave relative to P is also equal to c in both directions for O’ (note that although this is probably the strangest result of Einstein’s theory, it has enjoyed wide-spread acceptance among the physics community and has often been verified and never been contradicted by observation).

The indication from experiment [3, 4], however, is that O’s clock rate will generally be different than that employed by O’, but that the **ratio of these two proper clock rates remains constant** so long as the conditions do not change (that is, as long as O and O’ continue moving with constant velocity \( u \) relative to one another and also do not change their respective positions in a gravitational field). This conclusion is also consistent with Newton's First Law (Law of Inertia). This means that in the present example, the **ratio of elapsed (proper) times measured by O and O’, respectively, for the same event will always be the same**. Since O finds that the elapsed times for the two events are equal (\( L/c \)), it therefore follows that O’ will also measure equal times for them (\( L/\gamma c \)). But this means that the wave front arrives simultaneously at A and B for O’ as well as for O. They will differ only in the amount of elapsed time measured in each case (\( L/c \) vs \( L/\gamma c \)). There is thus a contradiction between this result and that following from the LT, even though only standard results of STR [1] and experiment [3,4], namely the RVT and time dilation, have been employed in the second determination.

To attempt to understand the reasons behind the above contradiction, it helps to go back to eq. (1) and ask a simple question. What is the exact definition of \( t' \) in this equation? Einstein made it clear [1] that for him \( t' \) stands for the proper time \( \tau \) (S’) read directly from a local clock in the corresponding rest frame S’. There is another possibility, however, namely to use the corresponding **adjusted time** (\( t'' \)) that takes into account the effects of time dilation on the local clock. The latter value is employed in the methodology of the Global Positioning...
System (GPS) [5], for example (the adjustment actually made also takes into account the
effects of gravity). The basic idea is that the unit of time differs from one inertial system to
another. What O’ believes to be 1 s is actually $\gamma$ s in the system of units employed by O [6].
In order to convert the elapsed proper time $\tau$ (S’) on his clock to the units employed by O, O’
must therefore multiply this quantity by $\gamma$, thereby obtaining the value referred to as $t''$ above.
In the GPS methodology, for example, one has to determine the elapsed time for light to
travel between a satellite and the ground. The only way to do this in practice is to measure
the respective start and end times for the light pulse’s journey on local clocks and then
convert to a common time unit, which is normally the value of 1 s employed by the observer
on the ground.

Since there are (at least) two bona fide values of elapsed time in S’, there is a potential
ambiguity when trying to apply eq. (1) of the LT. As a result, the unit of time in which $t =
\tau$ (S) as expressed in S by O is different than that used by O’ for t’ in S’ if the latter uses his measured proper time $\tau$ (S’) for this quantity. This being the case, it is no longer clear that
eq (1) implies non-simultaneity. The only way to make quantitatively reliable timing
comparisons is to insure that the respective clocks are properly synchronized and run at the
same rate. In other words, if the goal is to prove that an event has not occurred
simultaneously for O and O’, it is necessary to show that the adjusted value $t''$ measured by
O’ differs significantly from $t = \tau$ (S) measured by O, not simply that $t = \tau(S) \neq \tau(S')$.

Einstein’s failure to consider this point in his original work [1] invalidates his
conclusion of the non-simultaneity of events for observers in relative motion. It also leaves
open the distinct possibility that the opposite conclusion that emerges above by applying the
RVT in conjunction with the time dilation phenomenon is in fact correct, which in turn would
clearly show that the LT is not a valid space-time transformation.
III. The Global Positioning System Lorentz Transformation (GPS-LT)

Before proceeding further, another aspect of the problem should be discussed. It has recently been pointed out [7, 8] that the LT is not a unique solution to the problem of finding a space-time transformation that satisfies the condition of equal speed of light in free space for all observers (Einstein’s second postulate [1]). Lorentz knew of this situation as early as 1899 [9, 10]. He listed the following set of equations therein:

\[ x' = \varepsilon \gamma (u) (x - u t) \quad (4a) \]
\[ t' = \varepsilon \gamma (u) (t - u x/c^2) \quad (4b) \]
\[ y' = \varepsilon y \quad (4c) \]
\[ z' = \varepsilon z, \quad (4d) \]

where \( \varepsilon \) is an arbitrary factor. The obvious point is that speed (or velocity) is simply a ratio of distance traveled to the corresponding elapsed time, so any value of \( \varepsilon \) will suffice to satisfy the above light speed condition. His main interest was in finding a general space-time transformation that leaves the Maxwell electromagnetic field equations invariant [11], and the above equations are seen to be satisfactory for this purpose as well, regardless of what value is used for \( \varepsilon \). Einstein assumed in his 1905 paper (see p. 900 of Ref. [1]) that \( \varepsilon \) is only a function of the relative speed \( u \) of the two inertial systems, from which a unique result of \( \varepsilon = 1 \) follows. Poincaré made the same argument somewhat earlier [12]. Substitution of this value in eq. (4b) leads to eq. (1) of the LT. Both Einstein and Poincaré interpreted this result as “proof” of the non-simultaneity of events for different observers in relative motion.

The fact is, however, that one can just as well assume that the absolute simultaneity of events must be satisfied in the resulting transformation and use this as the required condition to uniquely specify the value of \( \varepsilon \) in eqs. (4a-d) [7, 8]. To be concrete and also treat the general case, let us assume that the adjusted elapsed time in \( S' \), which has been called \( t'' \) above, is \( Q \) times greater than the proper value of \( t' \) (\( Q = \gamma \) in the above example), that is, \( t'' = t = Q t' \). Substitution in eq. (4b) leads to the following value for \( \varepsilon \):
\[ \epsilon = \frac{t'}{\gamma} (t - u \frac{x}{c^2}) = [\gamma Q(1 - u \frac{x}{c^2}t)]^{-1} = \frac{\eta}{\gamma Q}. \] (5)

The quantity \( \eta = (1 - u \frac{x}{c^2}t)^{-1} = (1 - uvx/c^2)^{-1} \) also appears in the relativistic velocity transformation (RVT) of eq. (3), that is, with \( v=x/t \) and \( v'=x'/t' \). An alternative Lorentz transformation (referred to as the GPS-LT because of its relevance to timing procedures employed in the Global Positioning System methodology [5]) is therefore obtained from eqs. (4a-d) with the value of \( \epsilon \) in eq. (5) as:

\[
\begin{align*}
    x' &= \eta \frac{(x - u t)}{Q} \quad \text{(6a)} \\
    t' &= \frac{t}{Q} \quad \text{(6b)} \\
    y' &= \eta \frac{y}{\gamma Q} \quad \text{(6c)} \\
    z' &= \eta \frac{z}{\gamma Q} \quad \text{(6d)}
\end{align*}
\]

It needs to be emphasized that eq. (6b) does not mean that observers in the primed and unprimed inertial systems (S and S’) actually obtain the same values for the time of an event on their respective local uncompensated proper clock, i.e. \( \tau (S) \) and \( \tau (S') \). Rather, it means that only after the timing results have been converted to a common system of units do the two values become equal, i.e., \( t'' \) in S’ equals \( t \) in S. The latter is the practice in the GPS technology [5, 7, 8]. The clock on the satellite when a light signal is transmitted must first be adjusted (“pre-corrected”) to take account of the effects of relativistic time dilation (and also gravitation in actual practice) before its reading can be meaningfully compared with that of the corresponding clock that records the arrival of the light signal on the ground.

In this way it is entirely possible that both time dilation and simultaneity can be observed in the same experiment. Einstein by contrast simply assumed [1] that \( t' \) in eq. (1) of the LT is the proper time \( \tau (S') \) read from a stationary clock in S’, and that when this value is not equal to \( \tau (S) \), the corresponding proper time read on a stationary clock in S, non-simultaneity is proven. In so doing, he tried to derive both time dilation and non-simultaneity from the same equation. The relativistic velocity transformation (RVT),

\[ v_{x'} = \eta (v_x - u) \] (7a)
\[ v_y' = \eta \frac{v_y}{\gamma} \]  
\[ v_z' = \eta \frac{v_z}{\gamma} \]

is derived from Lorentz’s general form of the transformation in eqs. (4a-d), and the same obviously holds true for both the special cases of the LT (\( \varepsilon = 1 \)) and alternative GPS-LT transformation (\( \varepsilon = \frac{\eta}{\gamma Q} \)) of eqs. (6a-d). One simply has to divide through by the expression for \( t' \) in each case. Conversely, the GPS-LT can be derived from the RVT of eqs. (7a-c) by applying the condition of simultaneity and therefore multiplying all three of its equations by \( t' = \frac{t}{Q} \), i.e. with \( v_x = \frac{x}{t} \) and \( v_x' = \frac{x'}{t'} \) and analogous definitions for \( v_y, v_z, v_y' \) and \( v_z' \). Only the RVT has actually been confirmed in direct experiments [13], specifically in studies of the light aberration at the zenith and the Fizeau light drag phenomenon (Fresnel formula). An additional experiment is required to distinguish between the LT and the GPS-LT, however, namely one that explicitly tests the simultaneity condition of eq. (6b) or its antithesis in eq. (1) of the LT. Einstein unfortunately overlooked this point in his original work [1], and his conclusion of the inevitable non-simultaneity of events for observers in relative motion as a direct consequence of his postulates has gone largely unchallenged by the physics community ever since. However, as shown in a companion article [7], it is straightforward to design an experiment on a GPS satellite that would prove that the LT prediction of non-simultaneity is incorrect. This conclusion follows from the fact that the ratio of the readings on the proper clock and its pre-corrected counterpart always has the same constant value on the satellite.

This failure to correctly predict the results of experiment therefore demonstrates that the LT is not a valid space-time transformation, as strange as this conclusion might seem coming over a century after Einstein’s original paper [1]. An alternative has been found that not only satisfies Einstein’s two postulates, but also insists upon the absolute simultaneity of events. Eqs. (6a-d) satisfy all three of these conditions and also have other advantages over the LT as well. As discussed elsewhere [7,8,14], it puts relativity theory back on a strictly objective basis. This characteristic is easily demonstrated by inversion of eq. (6b). The result
is simply: \( t = Q t' \). In other words, if the clock rate in \( S \) is \( Q \) times that in \( S' \), it follows automatically (i.e., by straightforward algebraic manipulation) that the \( S' \) rate is only \( 1/Q \) times that in \( S \). The relationship between the rates of clocks in relative motion is *reciprocal* rather than symmetric, in contrast to the predictions of the LT. Hence, the GPS-LT in eqs. (6a-d) may aptly be called the *reciprocal* LT.

No longer does one have to believe as the LT demands that two clocks can both be running slower than one another at the same time, something that is in fact contradicted by the transverse Doppler experiments carried out with ultracentrifuges [15]. Instead, measurement is totally objective and all observers agree on the ratio of any two clock rates or any other pair of measured quantities. They simply may express their respective measured values in different sets of physical units dependent on their state of relative motion and thus obtain different *numerical* values for the same quantity.

The above considerations suggest that Galileo's Relativity Principle (RP) needs to be amended as follows [16]: *The laws of physics are the same in all inertial systems but the units in which they are expressed can and do vary in a systematic manner from one rest frame to another.* Each observer finds that his proper clocks run at the standard rate, for example, but his timing results always differ from those obtained with proper clocks in a different rest frame by a fixed ratio. A simple way to express this relationship between proper clocks in different rest frames is to assume that their respective units of time are not the same.

**IV. Relative Velocities in General**

The discussion in the previous two sections based on light speed measurements gives strong support to the principle of absolute simultaneity of events for all observers, but it needs to be emphasized that the same conclusion follows quite generally for the motion of any object. It can be shown from the RVT of eqs. (7a-c) that the *relative* velocity of the light wave as it proceeds from \( P \) to \( A \) or \( P \) to \( B \) in Fig. 1 is exactly the same for both observers \( O_1 \) and \( O_2 \).
and O’ even though they are in relative motion themselves. The simultaneity conclusion then follows directly from the fact that the rates of the respective local clocks employed by O and O’ are known to have a constant ratio. The latter conclusion has been verified directly in both the Hafele-Keating [3] and Vessot-Levine [4] experiments, and is a key assumption in the GPS procedure [5]. In other words, if O finds that the times for two events are equal, then O’ must find that the corresponding two times measured on his clock are also equal because of the aforementioned fixed ratio [7]. The fact that this result holds for the motion of other objects than light pulses will be discussed in detail below.

As in Fig. 1, let us assume as above that O’ moves along the x axis with speed u relative to O. In the general case, it is always possible to define a coordinate system which fulfills this condition. It is simply necessary to have this relationship in order to apply the RVT of eqs. (7a-c) directly. O’ measures the velocity of an object moving relative to him to have the following components: \( v_x' = wc\theta \) in the parallel (x) direction and \( v_y' = ws\theta \) in the perpendicular direction (y). The RVT can be used to obtain the corresponding velocity \( v \) for the object's motion relative to the S' observer from the vantage point of the stationary observer in S. There are two steps to be followed for each component of \( v \): first, the velocity \( w_S \) of the object relative to S is computed; secondly, the desired relative velocity \( v \) of the object to S' is obtained based on the value of \( w_S \).

The RVT eq. (7a) gives the following value for the x-component of \( w_S \), namely
\[
w_x^S = (w\cos\theta + u)(1 + wc^2\cos\theta)^{-1}
\]
Next, the difference \( v_x \) between \( w_x^S \) and \( u \) needs to be computed, again using eq. (7a), and not simply subtracting \( u \) from \( w_x^S \):
\[
v_x = (w_x^S - u)(1 - uw_x^Sc^2)^{-1}
\]
\[
= [(w\cos\theta + u)(1 + wc^2\cos\theta)^{-1} - u][1 - uc^2(u + wc\theta)(1 + wc^2\cos\theta)^{-1}]^{-1}
\]
\[
= (w\cos\theta + u - u - wc^2\cos\theta)(1 + wc^2\cos\theta - u^2c^2 - uwc^2\cos\theta)^{-1}
\]
\[
= w\cos\theta(1 - u^2c^2)(1 + u^2c^2)^{-1}
\]
The corresponding calculation of \( v_y \) using the RVT eq. 7b in two steps is given next, whereby

\[
w_y^S = w_y \sin \theta (1 - u^2 c^{-2})^{0.5} (1 + wc^2 \cos \theta)^{-1}:
\]

\[
v_y = w_y \sin \theta (1 - u^2 c^{-2})^{0.5} \left[ 1 - uc^{-2}(u + wc \cos \theta)(1 + wc^2 \cos \theta)^{-1} \right]^{-1}
\]

\[
= \sin \theta (1 - u^2 c^{-2})(1 + wc^2 \cos \theta - u^2 c^{-2} - uwc \cos \theta)^{-1} \tag{9}
\]

\[
= \sin \theta (1 - u^2 c^{-2})(1 - u^2 c^{-2})^{-1} = \sin \theta
\]

\[
= v'_y.
\]

It is therefore seen that both components of \( v \) and \( v' \) are equal, as was to be proved.

In other words, the RVT leads to the conclusion that two observers always agree on the relative speed of an object to one of them. This result can easily be extended to apply to the relative velocity of any two objects. This finding is thus consistent with the above conclusion based on the light-speed postulate that the unit of velocity is the same for the respective stationary observers in \( S \) and \( S' \), even though their units of time generally differ.

It is the nature of relativity theory that it is not possible to use ordinary vector analysis to relate the components of the above two relative velocity predictions. A similar problem prevents one from proving that all Lorentz transformation matrices form a group, not just those in which the velocity of the objects is in the same direction as the relative velocity of the two inertial systems [17].

There is also another argument that can be made based on Einstein’s second postulate that supports the thesis that the relative velocity of any two objects is the same for all observers located at the same gravitational potential. The latter assumes that the speed of light between two points is the same for each such observer. When an object moves along the same vector with speed \( v_{21} \) for observer \( O \), the only way that \( v'_{21} \) could have a different value is if \( v_{21}/c \) does not equal \( v'_{21}/c \). Such ratios are dimensionless, however, and therefore must be relativistic invariants. The same argument has been used in previous work [18] to explain why highly accelerated muons in the upper atmosphere are able to reach the Earth in such large concentrations [19]. One simply assumes that the speed \( v \) of the muons relative to the
Earth is the same for observers there as for someone travelling with the muons. Since it is known that clocks run slower in the muon rest frame \((S')\) by a definite ratio \(Q=\gamma(v)\), it follows that the distance \(L'=v\tau(S')\) travelled by the muons on the way to the Earth must be found to be smaller in \(S'\) by the same ratio. Thus, the muons have less time to decay based on the \(S'\) proper clocks than on their counterparts in \(S\). It has not been appreciated that the same argument implies that the muons arrive simultaneously on the Earth for both observers. Once the proper time \(\tau(S')\) read on the muon clocks is adjusted to account for time dilation in \(S'\), it will be exactly the same \(\tau''\) as that read on the proper clocks, i.e. \(\tau(S)\), in the Earth’s rest frame.

V. Conclusion

Although the Lorentz transformation (LT) leads unequivocally to the position that events generally do not occur simultaneously for different observers in relative motion, the fact is that the opposite conclusion results when one takes into account experimental findings that have been obtained with atomic clocks since Einstein’s original paper [1] on the special theory of relativity (STR) first appeared. The example of a wave front of light proceeding in opposite directions (Figs. 1a-b) has been used to illustrate this situation. There is ample experimental evidence to support the view that all observers at the same gravitational potential will agree that the speed with which the wave front propagates in all directions is equal to \(c\), the speed of light in free space, as predicted explicitly by the relativistic velocity transformation (RVT). However, the rates of atomic clocks employed by different observers are known to depend on both their state of motion and position in a gravitational field. Most importantly in the present context, it is possible to predict to high accuracy what the ratio of the respective clock rates will be, as for example is done in the GPS procedure for measuring distances on the Earth’s surface. This means that if observer O’s proper clocks run \(Q\) times faster than those employed by his counterpart O’, his measured times will always be \(Q\) times
greater than the latter’s. The consequences of this relationship are unavoidable when it comes to questions of simultaneity. Whatever is simultaneous for O, must therefore also be simultaneous for O’ [7]. The two observers will differ only in the amount of time they measure for each of two such events to occur (L/c vs. L/Qc in the present example). They will not disagree that the events occur at the same time.

The above situation is not restricted to the observation of light waves. The relative velocity that is measured for any two objects is independent of the state of motion of the observers (again, as long as the latter are at the same gravitational potential). This theoretical argument has been used to explain why the *same fraction of muons* produced in cosmic ray collisions in the upper atmosphere arrives at the Earth’s surface for observers at rest there as for those travelling with the muons. Once their *respective clocks have been adjusted to account for their different rates*, it therefore follows that the elapsed time for the muons to travel the distance from their original position down to the Earth’s surface will be exactly the same for both.

The same principle is used in the GPS procedure. There it is assumed that the absolute time of emission of a light signal from a given satellite is exactly the same there as it is on the Earth’s surface. Therefore, one has to adjust the local proper time on the satellite’s clock for the effects of time dilation and the gravitational red shift in order to obtain the corresponding result that would be obtained with the proper clock that is used on the Earth to measure the arrival time of the same light signal there. *The fact that the GPS procedure has been applied with high accuracy to measure distances on the Earth’s surface therefore constitutes strong evidence that events do occur simultaneously for observers in relative motion independent of their respective positions in a gravitational field.* One simply has to accept the principle that timing comparisons for different proper clocks can only be carried out on a meaningful basis *after adjustments have been made* to take account of the known effects of time dilation and
gravity on their respective rates. This simply amounts to insisting that both observers base their measurements on the same unit of time.

The above discussion clearly raises questions about the validity of the LT itself. The RVT can be derived from the LT, so it is often claimed that an experimental test of one is a verification of the other. Lorentz pointed out that the condition of the constancy of the speed of light only fixes the space-time transformation to within a constant factor, however. There is another version, the GPS-LT of eqs. (6a-d) [7, 8, 14], that is consistent with both the principle of the absolute simultaneity of events as well as Einstein’s two postulates of STR. In this case the relationship between the two time variables is strictly proportional, namely $t = Qt'$. It reduces to the simple times' relationship of the Galilean transformation when adjustment is made for the different clock rates, not the mixing of space and time demanded by the LT. As in the GPS procedure, the local proper clock readings in the two rest frames will generally differ, but they always become equal after adjustment is made for the effects of time dilation on the respective clocks.

Einstein used the LT to derive two different effects, time dilation and non-simultaneity. The former has been observed in subsequent experiments, but this in itself should not be construed as proof of the LT’s validity, especially since the observed ratio of elapsed times is not always equal to the value of $\gamma$ predicted explicitly by the LT for the different rest frames [3,4]. The LT also claims that there is a relativistic symmetry principle, whereby two clocks can each run slower than the other at the same time, although this phenomenon has never actually been observed in practice and is actually contradicted by transverse Doppler measurements [15]. The GPS-LT, by contrast, treats time dilation as a completely separate detail of timing measurements, but insists that all events occur simultaneously for all observers, regardless of their state of motion or position in a gravitational field. It gives relationships between space and time variables in different rest frames, but always expressed in their respective proper set of physical units in each case. In
the last analysis, one only needs the RVT (to obtain the speed of the object) to make a reliable prediction for the elapsed time required for an object to pass between two fixed points separated by a known distance in a given rest frame, and then to make a suitable conversion of units in order to make the corresponding predictions for other observers. This position is perfectly consistent with experimental observations, and thus provides strong confirmation of the principle of absolute simultaneity of events so long denied by STR [1].
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**Figure Captions**

Fig. 1. Diagram illustrating the motion of a wave front of light as it propagates outward from P to equally spaced points B and A. Observer O is at rest with respect to P, while O’ is moving with constant speed u toward the right. At the time of initiation of the wave front both O and O’ are located at P (Fig. 1a). The situation at a later time when the wave front has reached A and B is shown in Fig. 1b. As discussed in the text, although O’ is closer to B than to A at this point, each observer finds that the wave front has reached both these positions simultaneously according to their respective clocks.
Figures

Fig. 1a

Fig. 1b