An Analysis of Correlation of the Distances between Turbines in a Turbine Farm with their Power and Cost

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Abstract

Abstract. In Indonesia with the increasing demand of electrical energy, fuel production from coal and crude oil will reach its limitation in just a few decades, along with the occurrence a number of extreme weather caused by the global warming. Therefore, the search for renewable energy sources, particularly wind energy that involving wind turbine becomes a hot topic to be discussed. Irrenewable natural resources such as coal and natural gas become the main energy sources all over the world. It is ironic, that those resources are also dangerous to our environment. This also become a considerable reason to make wind as our potential energy resource since it is renewable and less causing pollution. The wind turbine is currently developing in Indonesia; it is call Pembangkit Listrik Tenaga Bayu (PLTB). One of them, namely PLTB Sidrap, already operated.

1. Introduction

In Indonesia, with the increasing demand of electrical energy, the fuel production which using coal and oil will reach its limitation in just a few decades, along with the occurrence a number of extreme weather caused by the global warming. Therefore, the search for renewable energy sources, particularly wind energy that involving wind turbine becomes a hot topic to be discussed. Irrenewable natural resources such as coal and natural gas become the main energy sources all over the world. It is ironic, that those resources are also dangerous to our environment. This also become a considerable reason to make wind as our potential energy resource since it is renewable and less causing pollution. The wind turbine is currently developing in Indonesia; it is call Pembangkit Listrik Tenaga Bayu (PLTB). One of them, namely PLTB Sidrap, already operated.

2. Wind Energy Potency in Indonesia

The Indonesian government keeps improving the usage of renewable energies. PLTB Sidrap for example, it had been constructed since 2012. With 75 MW power capacity, the save electricity generator has 30 wind turbines; each contributes power 2.5 MW. It build in Pabbaresseng, which involves area in two villages, with wind speed estimation 7 m/s. Each turbine has a pole with height 80m, 3 blades with a length 57 m and weight 20 ton so that the maximum height of the windmill is 137 m. There are 166 locations in Indonesia had been researched and measured its wind velocities. The result shows that 35 sites have a good wind potency, with annual average wind velocity above 6 m/s. Besides that, 34 other sites also have potential wind energy; with annual average wind velocity 3-4 m/s. After collecting data for 15 years, it concluded that some areas in Indonesia contain wind with high velocity,
which in other words suitable for the wind farm. Table 1 contains the resume of potential sites for the wind farm in Indonesia.

Table 1. The Resume of Potential Sites for Wind Farm in Indonesia. Source: wind data from LAPAN [5].

| Site               | V (m/s) | Weibul parameter | Wind Energy Density W/m² | Height Calculation | Resolution Calculation |
|--------------------|---------|-------------------|--------------------------|--------------------|------------------------|
| Baron, DIY         | 6.13    | 6.29              | 2.24                     | 245                | 50                     | 150                   |
| Lebak, Banten      | 5.58    | 6.3               | 2.06                     | 198                | 50                     | 150                   |
| Nusa Penida, Bali | 2.73    | 3.1               | 1.66                     | 30                 | 20                     | 130                   |
| Oelbubuk, NTT      | 6.1     | 6.9               | 1.6                      | 301                | 30                     | 160                   |
| Bantul, DIY        | 4       | 4.7               | 1.87                     | 91                 | 50                     | 130                   |
| Sukabumi, Jawa Barat | 6.27   | 7.1               | 2.08                     | 272                | 50                     | 180                   |
| Purworejo, Jawa Tengah | 5.16  | 5.7               | 1.5                      | 231                | 60                     | 150                   |
| Garut, Jawa Barat  | 6.57    | 7.4               | 2.89                     | 268                | 50                     | 100                   |
| Sidrap, Sulawesi Selatan* | 6.43 | 7.3               | 2.05                     | 320                | 50                     | 100                   |
| Jeneponto, Sulawesi Selatan* | 7.69 | 9.0               | 2.51                     | 491                | 50                     | 100                   |
| Selayar, Sulawesi Selatan | 4.6   | 5.2               | 1.83                     | 143                | 24                     | 100                   |

Considering these tables, it is clear that some areas in Indonesia have good potency to be developed to become wind farms as the source for electricity.

3. The Distance between Turbines Problem

To design a wind farm in a particular area one faces a direct problem, which is how to produce maximum power with minimum cost possible. There are several factors determining this, for example, the size of the turbine. If one uses small turbines instead of big ones; he pays a lower cost but will get less power. The other factor is how to arrange wind turbines. This matters because of wake effect phenomena, the wind that passed a turbine will spread and lose some energies due to the spreading and the converted kinetic energy from the wind to the blade. Consequently, placing another turbine behind the first one will reduce the power gained with the same cost. Let alone placing an array of turbines.

![Figure 1. The Wake Effect Scheme.](image)

A turbine caused the wind to reduce its velocity according to this equation.

\[
u = u_0 \left[ 1 - \left( \frac{2}{3} \left( \frac{r_r}{\alpha x + r_r} \right)^2 \right) \right]
\]

(3.1)

Where:
- \(u\) = the reduced wind velocity (m/s)
- \(u_0\) = the initial velocity (m/s)
\( r_r = \) blade length (m)
\( x = \) the distance between turbines (m)
\( a = \) axial induction factor (dimensionless constant)

From eq. (3.1), we can see that for two collinear turbine, the one behind gets only up to 1/3 of the initial velocity. However, based on [6], it can be said that the turbine lay out in a wind farm has practical distance, namely 4 times blade diameter on its side and 7 times diameter backward.

4. Wind Power, Cost and Objective Function

4.1. Wind Power

Based on classical physics, the power that can be extracted from a wind flow that is to be converted into wind turbine kinetic energy and converted furthermore into electricity is

\[
P = \frac{1}{2} A v^3 C_p = \frac{1}{2} A v^3 \times 0.6 = 0.3 A v^3
\]

(4.1)

Where \( A \) is the cross-sectional area that swept by the blade. So, the total power of the entire turbine in the wind farm is given by

\[
P_{tot} = \sum_{i=1}^{N} 0.3 v_i^3
\]

(4.2)

4.2. Cost

According to [7], the cost is modelled in a way such that only the number of turbine is considerable. Cost is assumed as a dimensionless quantity, annual cost for a single turbine is 1, and the reduction up to 1/3 of the total cost can be gotten if the a large number of turbine is involved. The formula that match these conditions is:

\[
Cost = N_t \left[ \frac{2}{3} + \frac{1}{3} e^{-0.00174 N_t^2} \right]
\]

(4.3)

Where:

\( N_t \) = The number of wind turbines involved.

4.3. Objective Function

Objective function is a function in which the power resulted by the wind and the cost are related. We are looking for the maximum power with minimum cost. Here we are using the objective function according to [8, 9],

\[
Objective Function = \frac{1}{Total\ Power} k_1 + \frac{Cost}{Total\ Power} k_2
\]

(4.4)

Where the total power is the amount of energy produced, \( k_1 \) and \( k_2 \) are constants so that both term in eq. (4.4) has the same dimension.

5. Methods of the Research

5.1. General Setting of the System

The first step is that we define initial parameters necessary, such as initial distance between turbines, initial lay out, power function, cost function, and objective function. Meanwhile the focus parameter in this research is the ability of the system to optimize the power and minimize the cost needed. The
variables that determines the change of the system condition are: (a) the change of distance between turbines parallel to the wind direction and (b) the change of distance between turbines sideward.

The distance between turbines is assumed to have minimum value 200 m, and maximum value 2,000 m, considering the area and turbines used (we assumed that the blade diameter is 40 m and the hub height is 60 m) so that there will be no crash between neighbouring turbine blades. The distance is determined initially before the system run. The distance is varying for a number of iteration to find the local bests. Then these the local bests are then compared to find the global best, namely the result in which the power produced is maximum and the cost is minimum.

![Figure 2](image.png)

**Figure 2.** The scheme of wind farm area in this research.

It is assumed, as shown in Figure 2, that the wind farm area is assumed to be $2 \times 2 \text{ km}^2$ wide. And it will be divided into $10 \times 10$ smaller areas, each of $200 \times 200 \text{ m}^2$ wide.

### 5.2. Conceptual Setting of the System

By calculating the equation of power function, cost function and the reduction of wind energy due to wake effect, the result will be produced.

a. Determining the distance between turbines as an input of the system. D1 as backward distance and D2 as sideward distance, with minimum distance and maximum distances are 200 m and 2 km respectively.

b. Determining the number of turbines that can be built in the corresponding area by dividing the area into smaller areas, each of $200 \times 200 \text{ m}$ wide.

c. Calculating the cost function for each configuration using the equation:

$$Cost = N_t \left[ \frac{2}{3} + \frac{1}{3} e^{-0.00174 N_t^2} \right]$$  \hspace{1cm} (5.1)

d. Calculating the reduction of wind energy using the equation:

$$P_{total} = \sum_{i=1}^{N_t} 0.3 v_i^3$$ \hspace{1cm} (5.2)

e. Calculating the objective function

$$Objective \ Function = \frac{1}{Total \ Power} k_1 + \frac{Cost}{Total \ Power} k_2$$ \hspace{1cm} (5.3)

and its minimum value.

f. Comparing which turbine configuration gives the minimum value of the objective function.

### 6. Results

Given the distances between turbines, we calculate the power, cost and the related minimum function. We then varied the distances so we have several values of minimum function, which we chose the smallest value as the best turbine layout. Here are data tables and plots for calculations of the power, cost and minimum function, given the distance between turbines, D1 and D2, along with their variation:
Table 2. Cost, power and minimum function values for D1 = 400 m and varied D2.

| D1  | D2  | Cost(US) | Power(W)  | Objective function | Total number of Turbines |
|-----|-----|----------|-----------|-------------------|--------------------------|
| 400 | 200 | 77,333   | 14,849,901| 5.275             | 116                      |
| 400 | 400 | 40,698   | 8,081,057 | 5.160             | 61                       |
| 400 | 600 | 26,922   | 5,373,520 | 5.196             | 39                       |
| 400 | 800 | 23,654   | 4,019,751 | 6.133             | 33                       |
| 400 | 1000| 21,052   | 4,019,751 | 5.486             | 28                       |
| 400 | 1200| 17,826   | 2,665,982 | 7.061             | 22                       |
| 400 | 1400| 14,761   | 2,665,982 | 5.912             | 17                       |
| 400 | 1600| 14,761   | 2,665,982 | 5.912             | 17                       |
| 400 | 1800| 14,761   | 2,665,982 | 5.912             | 17                       |
| 400 | 2000| 4,929    | 3,834,582 | 1.546             | 5                        |

7. Analysis
We only show a few tables here. We think that these tables are enough to represent the results and we can conclude without losing the points emphasized. As we can see, table 2 shows that the minimum value of the objective function is generated from the turbine configuration where the side-to-side distance is 400m and back 2000. Table 3 shows that the minimum objective function value is generated from a turbine configuration where D1 is 1200m and D2 is 200m. The result in tables 2, 3 and 4 show that the minimum objective function is in table 4 which is the distance between D1 2000 and D2 200.

8. Conclusion
Comparing all of the tables (with those which are not showed here as well), we see that the optimum distances for turbines are 200 m sideward and 2000 m backward. This should be logical since the placement of turbines right behind another turbine causes it produces power less than the front turbine, and placing another behind these two causes the reduction to go further while the cost stays the same. This effect does not happen to sideward turbine. The power is not reduced due to the wake effect. Therefore, the best layout is the one in which the distance sideward is as small as possible, while the distance backward is as far as possible.

Table 3. Cost, power and minimum function values for D1 = 1200 m and varied D2.

| D1   | D2  | Cost(US) | Power(W)  | Objective function | Total number of Turbines |
|------|-----|----------|-----------|-------------------|--------------------------|
| 1200 | 200 | 28,650   | 25,417,197| 1.167             | 42                       |
| 1200 | 400 | 17,826   | 13,364,705| 1.409             | 22                       |
| 1200 | 600 | 12,651   | 8,543,709 | 1.598             | 14                       |
| 1200 | 800 | 11,113   | 6,133,210 | 1.975             | 12                       |
| 1200 | 1000| 9,468    | 6,133,210 | 1.707             | 10                       |
| 1200 | 1200| 7,719    | 3,722,712 | 2.342             | 8                        |
| 1200 | 1400| 5,879    | 3,722,712 | 1.848             | 6                        |
| 1200 | 1600| 5,879    | 3,722,712 | 1.848             | 6                        |
| 1200 | 1800| 5,879    | 3,722,712 | 1.848             | 6                        |
| 1200 | 2000| 5,879    | 3,722,712 | 1.848             | 6                        |
### Table 4. Cost, power and minimum function values for \( D_1 = 2000 \text{ m} \) and varied \( D_2 \).

| \( D_1 \) | \( D_2 \) | Cost (US) | Power (W) | Objective function | Total number of Turbines |
|---|---|---|---|---|---|
| 2000 | 200 | 23,129 | 26,535,901 | 0.909 | 32 |
| 2000 | 400 | 14,761 | 13,924,057 | 1.132 | 17 |
| 2000 | 600 | 10,304 | 8,879,320 | 1.273 | 11 |
| 2000 | 800 | 8,606 | 6,356,951 | 1.511 | 9 |
| 2000 | 1000 | 7,719 | 6,356,951 | 1.372 | 8 |
| 2000 | 1200 | 5,879 | 3,834,582 | 1.794 | 6 |
| 2000 | 1400 | 4,929 | 3,834,582 | 1.546 | 5 |
| 2000 | 1600 | 4,929 | 3,834,582 | 1.546 | 5 |
| 2000 | 1800 | 4,929 | 3,834,582 | 1.546 | 5 |
| 2000 | 2000 | 4,929 | 3,834,582 | 1.546 | 5 |

### Table 5. Cost, power and minimum function values for \( D_1 = 1000 \text{ m}, 1200 \text{ m}, 1400 \text{ m}, 1600 \text{ m}, 1800 \text{ m}, 2000 \text{ m}, \) and \( D_2 = 1000 \text{ m}, 1200 \text{ m}, 1400 \text{ m}, 1600 \text{ m}, 1800 \text{ m}, 2000 \text{ m} \) along with their plots.
D1 = 1800, D2 = 1800

D1 = 2000, D2 = 2000

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