Reconstruction of Algebraic-Exponential Data from Moments

Joint work with M. Putinar

THURSDAY, May 15, 2014, at 4:30 PM
Eckhart 133, 5734 S. University Avenue

ABSTRACT

Let \( G \) be a bounded open subset of Euclidean space with real algebraic boundary \( \Gamma \). In a first part of the talk we consider the case where \( G = \{ x : g(x) \leq 1 \} \) for some quasi-homogeneous polynomial \( g \) and derive several properties of \( G \) as well as the non-Gaussian integral \( \int \exp(-g)dx \). In particular we show that the volume of \( G \) is a convex function of the coefficients of \( g \).

Next, we consider a more general case and under the assumption that the degree \( d \) of \( \Gamma \) is given, and the power moments of the Lebesgue measure on \( G \) are known up to order \( 3d \), we describe an algorithmic procedure for obtaining a polynomial vanishing on \( \Gamma \). The particular case of semi-algebraic sets defined by a single polynomial inequality raises an intriguing question related to the finite determinateness of the full moment sequence. The more general case of a measure with density equal to the exponential of a polynomial is treated in parallel. Our approach relies on Stokes theorem and simple Hankel-type matrix identities.