Short-length storage of intense optical pulses in solid by adiabatic passage

G. G. Grigoryan
Institute for Physical Research, 378410, Ashtarak-2, Armenia
Y. T. Pashayan-Leroy, C. Leroy, and S. Guérin
Institut Carnot de Bourgogne, UMR 5209 CNRS - Université de Bourgogne, BP 47870, 21078 Dijon, France

(Dated: June 23, 2008)

We propose a novel scheme of storage of intense pulses which allows a significant reduction of the storage length with respect to standard schemes. This scheme is particularly adapted to store optical information in media with fast relaxations.

PACS numbers: 42.50.Gy, 42.50.Md

I. INTRODUCTION

The propagation of two pulses in resonant interaction in a Λ-atomic medium has been widely studied during the last decade, in particular with application to quantum information processing (see, e.g., reviews [1]). The possibility of quantum information storage in the gas phase has been demonstrated in cold atoms [2] and in hot ensembles [3]. Achieving information storage in solid-state systems, which are more attractive due to their higher density, compactness, and absence of diffusion, has been pursued [4]. The main drawbacks preventing the efficiency of storage for such solid-state materials are huge inhomogeneous broadenings and high rates of decoherence. For instance, in crystalline films doped with rare-earth metals the rates of transverse relaxations amount to tens of GHz [5]. In practice an efficient storage of information requires rather large optical length such that even a weak loss rate will ruin it [6]. In order to reduce the inhomogeneous broadening, it was proposed in a number of works to use the so-called hole burning technique [7]. It has been shown that media prepared in such a way allow efficient coherent population transfer in Λ systems [8]. However, the hole burning technique faces a so far unsolved problem that leads to the reduction of the optical length of the samples, which will be detrimental in general for the efficiency of the storage.

In the present work we show that, for the storage of optical information (i.e. of classical fields), the recording length can be dramatically reduced with the use of intense pulses. The possibility to store and retrieve optical information in resonant media has mainly been studied in the “linear approximation” with respect to the so-called probe field that has to be stored, i.e. for a weak probe pulse. It was usually assumed that the control pulse propagates in a medium without pulse shape change. Numerical simulation of this problem without restrictions on the probe intensity was performed in [9]. Analytical studies of the problem taking into account the group velocities of both pulses were performed in [10] in the limit of pulses of duration much shorter than all the relaxation times, but sufficiently long to allow adiabatic evolution during the interaction. It has been found that the length of information storage depends remarkably on the ratio between the oscillator strengths of the adjacent transitions that determine the group velocities of the pulses in a medium. The present work aims at a complete analytical study of information storage in case of arbitrary relaxation times and intensities of the probe and control fields. We show analytically and numerically that it is possible, for a proper choice of solid-state medium, to dramatically reduce the optical length needed to store intense short pulses. This would in particular make more efficient the use of the hole burning technique.

The paper is organized as follows: We first describe the model, and analyze the propagation in the adiabatic limit. We next derive the conditions of storage for short lengths of the medium, before concluding.

![Level scheme diagram for a Λ-system interacting with two laser pulses.](image)

FIG. 1: Level scheme diagram for a Λ-system interacting with two laser pulses.

II. THE MODEL

We consider a medium of Λ three-level atoms with two ground states |1⟩, |2⟩ and a metastable excited state |3⟩, of energy ℏω_i, i = 1, 2, 3, and two laser pulses referred to as the probe and control pulses, coupling the transition...
respectively $1 \leftrightarrow 3$ and $2 \leftrightarrow 3$ (see Fig. 1). The interaction Hamiltonian in the rotating wave approximation can be written in the basis $\{ |1\rangle, |2\rangle, |3\rangle \}$ as

$$H = \frac{\hbar}{2} \begin{bmatrix} 0 & 0 & -\Omega_p^* \\ 0 & 2\Delta & -\Omega_c^* \\ -\Omega_p & -\Omega_c & 0 \end{bmatrix}$$

with $\Delta = \omega_3 - \omega_1 - \omega_p = \omega_2 - \omega_1 - \omega_p$ the one-photon detuning, $\omega_p$ ($\omega_c$) the probe (coupling) laser frequency, $\Omega_{p,c} = \mathcal{E}_{p,c} \mu_{p,c} / \hbar$ the Rabi frequencies associated to the corresponding field amplitudes $\mathcal{E}_{p,c}$ and to the dipole moments $\mu_{p,c}$ of the corresponding atomic transitions. We consider an exact two-photon resonance.

The propagation of two laser pulses in the medium is described by the Maxwell equations which, in the slowly-varying-amplitude approximation and in the moving coordinate system

$$\eta = x, \quad \tau = t - x/c$$

(such that the original wave operator $\partial / \partial t + c \partial / \partial x$ becomes $c \partial / \partial \eta$), read [1]

$$\frac{\partial \rho_p}{\partial \eta} = i q_p \rho_{31}, \quad \frac{\partial \rho_c}{\partial \eta} = i q_c \rho_{32}. \quad (2)$$

Here $q_{p,c} = 2 \pi \omega_{p,c} |\mu_{p,c}|^2 \hbar N/c$ are the coupling factors with $N$ the density of atoms. The density matrix elements $\rho_{pq}$ are determined by the system of equations (where the dot denotes the derivative $\partial / \partial \tau$, since $\partial / \partial t = \partial / \partial \eta$)

$$\begin{align} 
\dot{\rho}_{11} &= \gamma_1 \rho_{33} - 2 \text{Im}(\Omega_p^* \rho_{31}) \quad (3a) \\
\dot{\rho}_{22} &= \gamma_2 \rho_{33} - 2 \text{Im}(\Omega_c^* \rho_{32}) \quad (3b) \\
\dot{\rho}_{33} &= -(\gamma_1 + \gamma_2) \rho_{33} + 2 \text{Im}(\Omega_p^* \rho_{31} + \Omega_c^* \rho_{32}) \quad (3c) \\
\dot{\rho}_{31} &= -(\Gamma + i \Delta) \rho_{31} + i \Omega_p (\rho_{11} - \rho_{33}) + i \Omega_c \rho_{21} \quad (3d) \\
\dot{\rho}_{32} &= -(\Gamma + i \Delta) \rho_{32} + i \Omega_c (\rho_{22} - \rho_{33}) + i \Omega_p \rho_{21} \quad (3e) \\
\dot{\rho}_{21} &= -\gamma_c \rho_{21} - i \Omega_p \rho_{32}^* + i \Omega_c^* \rho_{31} \quad (3f)
\end{align}$$

with $\gamma_1$ and $\gamma_2$ the rates of spontaneous emission from state 3 to states 1 and 2 respectively, $\gamma_c$ the dephasing rate between the two ground states, and $\Gamma$ the transverse relaxation rate. We moreover assume pulses of durations much shorter than the characteristic time of decoherence of the ground states (usually microseconds in solids) such that $\gamma_c$ will be neglected. The envelopes of the fields $\mathcal{E}_{p,c}(\eta, \tau)$, the Rabi frequencies $\Omega_{p,c} = \Omega_{p,c}(\eta, \tau)$, and the density matrix components $\rho_{ij} = \rho_{ij}(\eta, \tau)$ depend on the position $\eta = x \in [0, L]$ in the medium of length $L$ and on the time $\tau = t - x/c$. The boundary conditions for the Maxwell equations are the pulses $\mathcal{E}_{p,0}(\tau) = \mathcal{E}_{p,0}(\eta, \tau)$, $\mathcal{E}_{c,0}(\tau) = \mathcal{E}_{c,0}(\eta, \tau)$ given at the entrance $\eta = x = 0$ of the medium. We assume that all atoms in the medium are in state 1 before the interaction with the laser fields.

### III. ADIABATIC PROPAGATION

It is well known that the use of a counterintuitive sequence of pulses in two-photon resonance, here the coupling pulse switched on before the probe pulse, leads, in the limit of slow evolution of the pulses, to an adiabatic passage along the dark state (of null eigenvalue) immune to decoherence. The system of linear equations (3) can be recast as $\frac{d}{d\tau} \rho(\epsilon s) = L(\epsilon s) \rho(\epsilon s)$ with $L$ being the associated linear operator and $\epsilon$ the formal (small) adiabatic parameter (we have omitted here the dependence on $\eta$ for simplification). The change of variable $\tau = \epsilon s$ leads to

$$\epsilon \frac{d}{d\tau} \rho(\tau) = L(\tau) \rho(\tau). \quad (4)$$

The components of the eigenvector $\rho^{(0)}(\tau)$ of null eigenvalue satisfying $0 = L(\tau) \rho^{(0)}(\tau)$ read

$$\begin{align} 
\rho^{(0)}_{11} &= \cos^2 \theta, \quad \rho^{(0)}_{22} = \sin^2 \theta, \quad \rho^{(0)}_{33} = 0 \quad (5a) \\
\rho^{(0)}_{21} &= -e^{i(\varphi_p - \varphi_c)} \sin \theta \cos \theta, \quad \rho^{(0)}_{31} = \rho^{(0)}_{32} = 0 \quad (5b)
\end{align}$$

with $\theta \equiv \theta(\eta, \tau)$ as

$$\tan \theta = \frac{\Omega_p}{\Omega_c}, \quad \Omega_{p,c} = |\Omega_{p,c}| e^{i\varphi_{p,c}}. \quad (6)$$

In the adiabatic limit, there is thus no coherence between the ground states and the excited state, which induce no pulse distortion during the propagation [1].

To determine non-adiabatic corrections, we make around the adiabatic solution $\rho(\tau) = \rho^{(0)}(\tau)$ the superadiabatic expansion:

$$\rho = \rho^{(0)} + \epsilon \rho^{(1)} + \epsilon^2 \rho^{(2)} + \cdots. \quad (7)$$

Inserting this expansion in Eq. (4) and identifying the orders of $\epsilon$, we obtain a system of linear equations that can be solved iteratively. For the first order

$$\rho^{(1)} = L \rho^{(0)} \quad (8)$$

denoting $\varphi = \varphi_p - \varphi_c$, we get

$$\begin{align} 
\rho^{(1)}_{11} &= \sin \theta \cos \theta \left( \Gamma \theta - \frac{\Delta \varphi}{2} \sin 2\theta \right), \quad (9a) \\
\rho^{(1)}_{22} &= -\rho^{(1)}_{11}, \quad \rho^{(1)}_{33} = 0 \quad (9b) \\
\rho^{(1)}_{21} &= \frac{e^{i \varphi}}{\Omega^2} \left( \theta (\Gamma \cos 2\theta + i \Delta) \\
&+ \frac{\varphi}{2} \sin 2\theta (-\Delta \cos 2\theta + i \Gamma) \right), \quad (9c) \\
\rho^{(1)}_{31} &= \frac{e^{i \varphi} \cos \theta}{\Omega} \left( i \theta - \frac{\varphi}{2} \sin 2\theta \right), \quad (9d) \\
\rho^{(1)}_{32} &= \frac{e^{i \varphi} \sin \theta}{\Omega} \left( -i \theta + \frac{\varphi}{2} \sin 2\theta \right) \quad (9e)
\end{align}$$

with

$$\Omega^2 \equiv \Omega^2(\eta, \tau) = |\Omega_{p}|^2 + |\Omega_c|^2. \quad (10)$$
Equations (9a) and (9c) show that at the first superadiabatic order the excited state stays unpopulated and that the adiabatic dynamics along the dark state is subjected to a loss of order $\Gamma/\Omega^2 T$ and to a pure dephasing loss of order $\Delta/\Omega^2 T$ with $T$ the characteristic time of interaction. Equations (10a) and (10c) show the transient appearance of coherences between the ground states and the excited state, which depend on the first-order non-adiabatic loss $\theta$ (and also on $\phi$), but not on the relaxation loss. These coherences will be responsible of pulse distortion during the propagation, as the conservation of the photon fluxes during the propagation: $\Omega(x/c)\rightarrow 0$ (see e.g. [10]). However, the system of equations obtained and studied for different regimes of propagation: $n_p(\eta, \tau) = n_0(\tau) \equiv N(|\Omega_p,\tau)|^2/\Omega_p + |\Omega_c,\tau)|^2/q_c$. (17)

Eq. (13) can be rewritten as $\partial n_0/\partial \eta = 0$ and interpreted as the conservation of the photon fluxes during the propagation:

$$\theta(0, \tau) \equiv \theta_0(\tau).$$

Denoting the photon fluxes $(j = p, c)$

$$n_j \equiv n_j(\eta, \tau) = c|\mathcal{E}_j(\eta, \tau)|^2/2\pi\hbar\omega_j = N|\Omega_j(\eta, \tau)|^2/q_j,$$

respectively the two-photon transition strength and the group velocity, and the boundary condition

$$\theta(0, \tau) \equiv \theta_0(\tau).$$

Note that this holds only in the limit of no population in the excited state (i.e. at the first order of the superadiabatic expansion) [10].

The obtained system of equations coincides with that studied in detail in literature, where pulses of durations much shorter than all relaxation times were considered (see e.g. [10]). However, the system of equations obtained in previous papers (e.g. in [12]) is valid under condition of interaction adiabaticity $\Omega T \gg 1$, whereas at high relaxation rates the inequality [12] should be fulfilled. Solutions of the system of equations [13] have been obtained and studied for different regimes of propagation in a number of works (see, e.g., the review [1]). The method of characteristics, recalled in Appendix A allows one to determine them.

In particular, equal transition strengths $q_p = q_c$ lead to a constant value $Q = q_p$, and from Eq. (13) to $\Omega$ independent of $\eta$, thus of shape invariance during the propagation: $\Omega(\eta, \tau) \equiv \Omega_0(t - x/c)$, as well as $u$: $u(\eta, \tau) \equiv u_0(t - x/c) = \Omega_0^2(t - x/c)/q_p$ with $\Omega_0(t)$ given at the entrance of the medium. The pulses [through their Rabi frequencies $\Omega_p = e^{i\varphi_p}\Omega_0(t - x/c)\sin \theta$ and $\Omega_c = e^{i\varphi_c}\Omega_0(t - x/c)\cos \theta$] propagate in a complicated manner through $\Omega_0$ and the mixing angle of solution [see Eq. (A4)]

$$\theta(\eta, \tau) = \theta(0, \xi) \equiv \theta_0(\xi)$$

where $\xi \equiv \xi(x, t)$ is defined by (A7) (where $t$ should be replaced by $\tau = t - x/c$) [12]

$$\int_{\xi}^{t-x/c} d\tau \frac{\Omega^2_0(\tau)}{q_p} = x.$$
When $\Omega_0$ is additionally constant, the excitation propagates at constant velocity satisfying $1/u = 1/c + q_p/\Omega_0^2$. This propagation regime is known as adiabatic [12].

In the more general case of unequal transition strengths, Eq. (13a) entails that the mixing angle still reads [10]

$$\theta(\eta, \tau) = \theta(0, \xi) = \theta_0(\xi)$$

and from (14a) that $Q(\eta, \tau) = Q(0, \xi) \equiv Q_0(\xi)$ with $u(\eta) = Q_0^2(\xi) \eta_0(\tau)/Nq_pq_c$, and $\xi = \xi(x, t)$ determined from Eq. (17) (where $t$ should be replaced by $\tau = t - x/c$):

$$\int_{\xi}^{t-x/c} d\tau \eta_0(\tau) = \frac{Nq_pq_c}{Q_0^2(\xi)} x.$$  

(21)

$\Omega^2$ propagates in a more complicated manner than in case of equal transition strength as $\Omega^2(\eta, \tau) = Q_0(\xi) \eta_0(t - x/c)/N$. Since $Q_0$ and $n_0$ do not propagate at the same velocity, $\Omega(\eta, \tau)$ is not any more shape-invariant during the propagation.

If we assume constant phases $\varphi_{p,c}$ at the entrance of the medium, one can conclude from Eq. (13a) that they will thus not change during the propagation. Note that this statement is only valid at the exact two-photon resonance. Evolution of the phase self-modulation for a two-photon detuning different from zero has been analyzed in [10].

In the linear approximation (with respect to the probe field), i.e., in the first order with respect to $\theta$, one has $Q = q_c$, $\Omega^2 = \Omega_c^2$, $u = \Omega_c^2/q_p$. We remark that in case of equal strengths of adjacent transitions $q_p = q_c$, similar expressions are obtained with $\Omega^2$ in place of $\Omega_c^2$. Thus, in case of equal transition strengths the nonlinearity in the probe field results in replacement of the Rabi frequency of the control field by the generalized Rabi frequency $\Omega^2$. The nonlinearity is more important when the adjacent transition strengths are unequal and can lead to formation of shock-wave fronts and violation of the interaction adiabaticity, as shown in [10].

### IV. EFFICIENT STORAGE

The storage of the probe pulse is achieved when its time profile is mapped into the spatial distribution of the atoms in the medium. More precisely, the probe pulse can be encoded in the spatial distribution of the angle $\theta_0(\xi(x, t \to +\infty))$ after the passage of the pulses through the medium, inducing for instance a storage in the coherence between the lower states (in the adiabatic limit):

$$\rho_{21}(x, t \to +\infty) \approx -\frac{1}{2} e^{-i\varphi} \sin[2\theta_0(\xi(x, t \to +\infty))]$$

(22)

An efficient storage requires that the excitation, through $\theta(x, t)$, propagates adiabatically until it eventually stops, which necessitates a long enough medium. It is usually achieved for a strong control pulse switched on before and switched off after a weak probe. We present below a novel pulse configuration with a strong probe and a weak control that allows one to shorten significantly the length of storage. The storage requires the stopping of the ex-
probe pulse, as is well known. On the other hand, adiabaticity of the interaction is required. Criterion for this adiabaticity has been analyzed in [10], where it has been shown that, under the certain condition adiabaticity is broken with formation of shock-wave fronts for distances exceeding the critical length \( x_0 \) estimated as

\[
x_0 \sim \frac{T_p \left( \Omega_{p, \text{max}}^2 + \Omega_{c, \text{max}}^2 \right)}{q_p(1-q)}.
\]  

(24)

The latter approximation holds for a probe pulse of duration \( T_p \) shorter than the control. This shows that reducing \( q \) to lower \( x_{\text{max}} \) will however also shorten \( x_0 \). Preserving adiabaticity usually requires thus a strong control pulse and an efficient storage will take place when

\[
x_{\text{max}} \leq x_0.
\]

(25)

Through their asymmetry in \( \Omega_{p, \text{max}}^2 \) and \( \Omega_{c, \text{max}}^2 \) with respect to \( q \), Eqs. (23) and (24) show that we can shorten \( x_{\text{max}} \) with the use of a weak control pulse in addition to using a small \( q \) parameter, while preserving the condition of adiabaticity using a strong probe (see Fig. 2 for the pulse scheme).

We have performed numerical calculations of the full set of the density-matrix and Maxwell equations for both pulses taking into account a fast relaxation \( GT_p = 200 \). It indeed shows that the storage of the probe field in inhomogeneously broadened media is more efficient when we use atoms of different adjacent transition strengths. Fig. 3b presents the temporal evolution of the probe pulse at different propagation lengths (normalized as \( z \equiv xq_p/\Gamma \)) for the novel scheme of pulses at the entrance of the medium shown in Fig. 2b. The chosen parameters lead to \( \Omega_{c, \text{max}}T_c = \Omega_{p, \text{max}}T_p \approx 44 \), \( \Omega_{c, \text{max}}T_c/\Gamma \approx 1 \), and \( \Omega_{p, \text{max}}^2T_p/\Gamma \approx 10 \), which satisfy the adiabatic conditions [11] and (12) at the entrance of the medium (here the time of interaction is \( T \sim T_p \)). For the comparison we present in Fig. 3a the dynamics for the usual scheme of pulses used for the optical information storage. One can see from Figs. 3a and 3b that in the case of the novel scheme the length of information storage is dramatically reduced as compared with that of the usual case. Namely, for the novel scheme at the propagation length \( z \) of about 1 the probe pulse is completely absorbed by the medium while at the same length in the usual scheme the probe pulse remains still unabsorbed.

V. CONCLUSION

We have shown that the optical information storage of a probe in inhomogeneously broadened media of \( \Lambda \) atoms occurs for short medium length when strong probe pulses, weak control pulses and very different strengths of adjacent transitions are used. For such a scheme the optical length may be of the order of unity in contrast to the conventional scheme where the optical length must well exceed unity. The proposed scheme would allow one to reduce the unavoidable losses caused by decoherence. The required \( \Lambda \) systems with very different strengths of adjacent transitions are expected to be found in, e.g., rare-earth doped solid-state materials. Study to elaborate efficient schemes to retrieve the stored pulse in this strong regime is in progress.

APPENDIX A: CHARACTERISTICS METHOD

We briefly recall here the characteristics method (see for instance [13]) to solve the Maxwell equation

\[
\frac{\partial \theta}{\partial x} + \frac{\partial \theta}{\partial t} = 0, \quad \theta(0, t) = \theta_0(t)
\]

(A1)

in the case of a non-linear group velocity of the form: \( u \equiv u(x,t) \). We want to transform this linear first order partial differential equation into an ordinary differential equation along the characteristic curves \( (x(s), t(s)) \). Using the chain rule \( \frac{d}{ds}\theta(x(s), t(s)) = \frac{\partial \theta}{\partial x} \frac{dx}{ds} + \frac{\partial \theta}{\partial t} \frac{dt}{ds}, \) we ob-
tain
\[ \frac{d}{ds} \theta(x(s), t(s)) = 0, \]  \tag{A2} 
if we set (choosing \( x(0) = 0 \) and denoting \( \xi := t(0) \))
\[ \frac{dx}{ds} = 1, \quad \text{i.e.} \quad x(s) = s, \]  \tag{A3a}
\[ \frac{dt}{ds} = \frac{1}{u}, \quad \text{i.e.} \quad t(s) = \int_0^s \frac{ds'}{u(x(s'), t(s'))} + \xi. \]  \tag{A3b}

Equation \((A2)\) leads to the solution
\[ \theta(x(s), t(s)) = \theta(x(0), t(0)) = \theta(0, \xi) = \theta_0(\xi) \]  \tag{A4}
which is thus a constant along the characteristics. \( \xi \) allows one to label a characteristic \( t(x) \) rewritten from Eq. \((A3b)\) as (since \( s = x \))
\[ t(x) = \int_0^x \frac{dx'}{u(x', t(x'))} + \xi. \]  \tag{A5}

The non-linear velocity \( u \) derived from \((A5)\):
\[ \frac{1}{u} = \frac{dt}{dx}. \]  \tag{A6}

\[ \text{corresponds to the instantaneous velocity along a characteristic.} \]

When \( u \) is independent of \( t \), i.e. \( u \equiv u(x) \), one has
\[ \xi = t - \int_0^x \frac{dx'}{u(x')}, \]  which becomes \( \xi = t - x/u \) if \( u \) is also independent of \( x \).

When \( u \) depends only on \( t \), i.e. \( u \equiv u(t) \), Eq. \((A3b)\) can be rewritten as
\[ \int_\xi^t dt'u(t') = s = x, \]  \tag{A7}
which gives in principle \( \xi \equiv \xi(x, t) \).

Acknowledgments

We acknowledge support from INTAS 06-100001-9234, the Research Project ANSEF PS-opt-1347 of the Republic of Armenia, the Agence Nationale de la Recherche (ANR CoMoC) and the Conseil Régional de Bourgogne.

[1] M.D. Lukin, Rev. Mod. Phys. 75, 457 (2003); M. Fleischhauer, A. Imamoglu, J.P. Marangos, Rev. Mod. Phys. 77, 633 (2005).
[2] C. Liu, Z. Dutton, C.H. Behroozi, L.V. Hau, Nature 409, 490 (2001); T. Chaneliere, D.N. Matsukevich, S.D. Jenkins, S.Y. Lan, T.A.B. Kennedy, A. Kuzmich. Nature 438, 833, (2005).
[3] M.D. Lukin, D.F. Phillips, A. Fleischhauer, A. Mair, R.L. Walsworth, Phys. Rev. Lett. 86, 783 (2001); A.S. Zibrov, A.B. Matsko, O. Kocharovskaya, Y.V. Rostovtsev, G.R. Welch, M.O. Scully, Phys.Rev.Lett. 88, 103601 (2002); M.D. Eisaman, A. Andre, F. Massou, M. Fleischhauer, A.S. Zibrov, M.D. Lukin, Nature 438, 837, (2005); R. Pugatch, M. Shuker, O. Firstenberg, A. Ron, N. Davidson, Phys. Rev. Lett. 98, 203601 (2007); P. K. Vudyasetu, R. M. Camacho, J. C. Howell Phys. Rev. Lett. 100, 123903 (2008).
[4] E. Kuznetsova, O. Kocharovskaya, Ph. Hemmer, M.O. Scully, Phys. Rev. A 66, 063802 (2002); A.V. Turukhin, V.S. Sudarshanam, M.S. Shahrir, J.A. Musser, B.S. Ham, P.R. Hemmer. Phys. Rev. Lett., 88, 023602 (2002); S.E. Yellin, P.R. Hemmer, Phys. Rev. A, 66, 013803 (2002); L. Alexander, J. J. Longdell, M. J. Sellars, and N. B.Manson, Phys. Rev. Lett. 96, 043602 (2006); M. U. Staudt, S. R. Hastings-Simon, M. Nilsson, M. Afzelius, V. Scarani, R. Ricken, H. Suche, W. Sohler, W. Tittel, and N. Gisin, Phys. Rev. Lett. 98, 113601 (2007); Sørensen, M. D. Lukin, and R. L. Walsworth, Phys. Rev. Lett. 98, 243602 (2007).
[5] M. Johnsson, K. Mølmer, Phys. Rev. A 70, 032320 (2004).
[6] I. Novikova, A. V. Gorshkov, D. F. Phillips, A. S. Sørensen, M. D. Lukin, R. L. Walsworth, Phys. Rev. Lett. 98, 243602 (2007); A. V. Gorshkov, A. André, M. Fleischhauer, A. S. Sørensen, M. D. Lukin, Phys. Rev. Lett. 98, 123601 (2007).
[7] 7. M.S. Shahriar, P.R. Hemmer, S. Lloyd, P.S. Bhatia, A. Craig, Phys. Rev. A, 66, 032301 (2002); M. Nilsson, L. Rippe, S. Kroll, R. Klieber, D. Sutter, Phys. Rev. B, 70, 214116 (2004).
[8] 8. J. Klein, F. Beil, T. Halfman, Phys. Rev. Lett. 99, 113003 (2007); H. Goto and K. Ichimura, Phys. Rev. A 75, 033404 (2007).
[9] T.N. Dey, G.S. Agarwal, Phys. Rev. A 67, 033813 (2003).
[10] G.G. Grigoryan, Y.T. Pashayan, Phys. Rev. A 64, 013816 (2001); V.O. Chaltykyan, G.G. Grigoryan, G.V. Nikoghosyan. Phys. Rev. A, 68, 013819, (2003).
[11] 14. A.B. Matsko, Y.V. Rostovtsev, O. Kocharovskaya, A.S. Zibrov, M.O. Scully, Phys. Rev. A 64, 043809 (2001).
[12] R. Grobe, F.T. Hioe, and J.H. Eberly, Phys. Rev. Lett. 73, 3183 (1994).
[13] R. Courant, Partial Differential Equations (New York, 1962).
