Real-time disease prediction with local differential privacy in Internet of Medical Things

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Abstract—The rapid development in Internet of Medical Things (IoMT) boosts the opportunity for real-time health monitoring using various data types such as electroencephalography (EEG) and electrocardiography (ECG). Security issues have significantly impeded the e-healthcare system implementation. Three important challenges for privacy preserving system need to be addressed: accurate matching, privacy enhancement without compromising security, and computation efficiency. It is essential to guarantee prediction accuracy since disease diagnosis is strongly related to health and life. In this paper, we propose efficient disease prediction that guarantees security against malicious clients and honest-but-curious server using matrix encryption technique. A biomedical signal provided by the client is diagnosed such that the server does not get any information about the signal as well as the final result of the diagnosis while the client does not learn any information about the server’s medical data. Thorough security analysis illustrates the disclosure resilience of the proposed scheme and the encryption algorithm satisfies local differential privacy. After result decryption performed by the client’s device, performance is not degraded to perform prediction on encrypted data. The proposed scheme is efficient to implement real-time health monitoring.

Index Terms—Real time, local differential privacy, malicious adversary, Internet of Medical Things, disease prediction.

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I. INTRODUCTION

With the rapid development of data collection via Internet of Medical Things (IoMT), machine learning becomes prevalent in analyzing big data and plays a crucial role in medical diagnosis. Due to the shortage of experts and high cost in manual diagnosis, machine learning improves the quality of healthcare service and avoids expensive diagnosis expenses. With the emergence of personal device applications, various disease prediction systems have been investigated. Existing works adopt wearable IoT in healthcare to diagnose certain diseases including Alzheimer disease [1] and Parkinson’s disease [2].

It is important to develop a high-performance model for real-time and reliable medical diagnosis. Support vector machine (SVM) and neural network (NN), the state-of-the-art machine learning models, have been investigated for disease prediction systems [3], [4]. Personalized healthcare is benefited from the wearable Internet of Things (IoT) devices. Electroencephalography (EEG) and electrocardiography (ECG) sensors are designed to monitor brain-electrical activity and heart activities, respectively [5]. SVM and NN have also been studied to predict diseases such as epileptic seizure and arrhythmias by analyzing EEG and ECG activities [6], [7], [8], [9], [10].

Machine learning-based diagnosis and prediction has also been accompanied by privacy concerns. Health data is considerably sensitive as it contains patient characteristics. For instance, current research shows that EEG can be used to predict viewed images, monitor sleep and reveal user information such as age, gender, and user’s illnesses or additions [11]. It is essential to protect medical information when performing disease prediction. Moreover, the diagnosis result is also sensitive and should be protected. As an example, patients with epilepsy may experience feelings of shame and isolation [12]. Releasing diagnosis result may lead to potential psychological problems.

Privacy preserving machine learning falls into three major categories: homomorphic encryption (HE)-based approaches, randomization-based approaches and differential privacy (DP) techniques. Due to the high cost in terms of encryption computation, HE-based methods achieve high privacy level at the cost of low efficiency [13], [14]. Partially homomorphic encryption is more efficient but usually relies on a trusted third party. Randomization-based approaches such as random projection were studied for data encryption [15], [16]. Matrix encryption technique usually faces disclosure risks such as known input-output attack [17] and thus needs to be studied with through security analysis. DP guarantees the privacy protection by adding additive noise to the original data. The DP-based solutions remain high computational efficiency as all the calculations are conducted on plaintext data [18]. Partial model accuracy is sacrificed as the perturbation inevitably reduces data utility.

In this paper, we focus on real-time health monitoring which should work to detect health emergency timely. Suppose that patient Alice has chronic disease and IoMT device for ECG monitoring is equipped to collect signals frequently, especially during exercise for timely emergency detection. The ECG signal monitoring is especially important to detect chronic heart diseases. We design privacy preserving disease prediction scheme which allows the user to check health status regularly without going to an analysis center or hospital while protecting personal signal record as well as diagnosis result. In the simplest case, the above scenario contains only two parties. One party, referred to as the client, owns a signal generated by personal device. Another party, referred to as the server, owns the medical data and is supposed to provide disease prediction service. We assume model training is performed by the server...

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locally. The server can be a single healthcare provider owning one medical database or a cloud platform with access to multiple medical databases. Various collaborative learning schemes have been proposed to build machine learning models using data provided by different owners \cite{29, 30, 31}. To derive robust results, bootstrap or cross validation is usually applied to build machine learning model and one single record increment to the database does not affect model parameters. It saves time and resources to update prediction model after receiving a batch of records. Moreover, collaborative learning is usually time consuming and restricts real-time applications. It is reasonable to build prediction model prior to the request for real-time disease prediction.

We investigate privacy preserving disease prediction using machine learning model built by the server. Assume that the server is semi-honest and the client is malicious. Matrix encryption is applied for data encryption with efficiency to permit use on relatively large database. The proposed scheme protects confidentiality of the client’s signal record as well as prediction result. The medical database owned by the server is also encrypted and no sensitive information is disclosed. After result decryption, the scheme achieves no accuracy degradation. Our contributions are as follows.

1) We investigate a real-time privacy preserving disease prediction system while ensuring security against malicious client and semi-honest server. The secure system is implemented without a third party.

2) The proposed encryption algorithm satisfies local differential privacy, i.e., the probability of encrypted output is roughly the same for any two inputs. The model performance is not degraded after result decryption.

3) Our secure scheme is efficient to implement real-time health monitoring system.

The rest of the paper is organized as follows. Section 2 reviews the related work and Section 3 outlines preliminaries. Section 4 describes the construction of proposed method. Security analysis is given in Section 5. Section 6 provides the performance analysis and evaluations. Finally, Section 7 concludes the paper.

II. RELATED WORK

Existing privacy-preserving machine learning methods fall into two basic types: differential privacy and secure multi-party computation.

**Differential Privacy.** Differential privacy (DP) \cite{32} has been studied to provide privacy guarantees by indistinguishability of the involvement of an individual in the dataset using the released information. DP guarantees that the output distributions any two neighboring databases are statistically close. Numerous approaches have been investigated to achieve DP under various analytical models. DP has been implemented for machine learning \cite{33, 34} and deep learning \cite{35}. Duchi et al \cite{36} extended DP to local DP (LDP). Various problems have been studied under LDP, such as deep learning \cite{37}, high-dimensional data collection and publication \cite{38, 39}. Notably, Johnson-Lindenstrauss (JL) transform, a random projection approach, has been proved to satisfy DP \cite{40}. JL transformation was further investigated to achieve DP for ordinary least squares \cite{41} and high-dimensional data release \cite{42}. Because randomization-based approaches are vulnerable to some well-known disclosure attacks such as known sample attack and known input-output attack \cite{43}, comprehensive security analysis is essential for privacy guarantee.

**Secure Multi-party Computation.** Numerous privacy preserving machine learning models were built with the assistance of secure multi-party computation (SMC). Privacy-Preserving ECG classification using branching programs and neural network was studies in \cite{44}. Using SMC, data owners distribute their private data among two non-colluding servers in SecureML \cite{45}. Plenty of studies proposed privacy preserving SVM for various applications in classification problems \cite{46, 47}. SMC and DP have been combined to guarantee privacy recently \cite{48, 49}. SMC systems tend to be computationally expensive, with iterated encryption, decryption and repeated communication for model updates among participating parties.

**Applications in privacy preserving disease prediction.** Recently, differential privacy and SMC have been applied to design privacy preserving disease prediction system. Using single-layer perceptron, privacy preserving disease prediction scheme was proposed assuming that server was honest-but-curious \cite{50}. Under the setting of honest-but-curious parties, random forest was studied for outsourced disease prediction system \cite{51}. Privacy preserving naive Bayesian classifier was proposed to predict disease risk for honest-but-curious parties \cite{52}. A differentially private K-means clustering scheme was investigated against malicious adversary \cite{53}. The additive noise required to guarantee DP reduced data utility which has significant influence for disease diagnosis and prediction related to individual life and health. Under the setting of honest-but-curious parties, extreme gradient boosting (XGBoost) was studied for privacy preserving medical diagnosis \cite{54}. Secure SVM was proposed for medical diagnosis services assuming honest-but-curious parties \cite{55}. In general, the above secure systems include two phases, the disease model training and remote disease prediction. A remote diagnosis system using ECG signal was proposed in the honest-but-curious model \cite{56}. The ECG classification scheme focused on remote disease diagnosis assuming the classifier parameters were derived locally by the server. The honest-but-curious model with non-collusion restriction in some studies \cite{57, 58} limits practical applications. In this paper, we focus on the remote prediction phase and investigate real-time disease prediction scheme secure against malicious adversary which is stronger than honest-but-curious adversary.

III. PRELIMINARY

EEG and ECG signals have been used for disease monitoring such as real-time heart disease prediction \cite{59, 60}. The signals are pre-processed to eliminate the noise using methods such as discrete wavelet transform (DWT) \cite{61, 62, 63}. In the post-processing phase, dimension reduction is usually applied to project EEG/ECG signals to a low feature space and classification method is then applied for diagnosis \cite{64, 65, 66}. Details of pre-processing methods have been given in
previous literatures and we focus on the post-processing phase. Here, we give a brief description of commonly used dimension reduction algorithms, including Principal Component Analysis (PCA), Linear Discriminant Analysis (LDA) and Independent Component Analysis (ICA), which project data to a lower feature space and perform disease prediction in the new space.

Suppose there are \( n \) signals in the database, \( x_i \in \mathbb{R}^q \) (\( i = 1, 2, \ldots, n \)) and \( X = [x_1, \ldots, x_n] \). Matrix \( X \) contains the training database such that each column corresponds to the signal of one individual.

**A. Dimension reduction algorithms**

**PCA.** Principal Component Analysis (PCA) chooses a linear projection that maximizes the variability of all projected samples. Suppose \( S_T \) is the total covariance matrix and \( W \) projection matrix is defined as

\[
W_{\text{pca}} = \arg \max_w |W^T S_T W| = [w_1, \ldots, w_p].
\]

The new feature vectors \( z_i \in \mathbb{R}^p \) are defined by the following linear transformation:

\[
z_i = W_{\text{pca}}^T x_i \quad i = 1, 2, \ldots, n
\]

where \( W_{\text{pca}} \in \mathbb{R}^{q \times p} \) is a matrix with orthonormal rows.

**LDA.** Linear Discriminant Analysis (LDA) provides the highest possible discrimination among different classes in the data. Suppose \( S_W \) is the within class covariance matrix and \( S_B \) is the between class covariance matrix. The projection matrix is defined as

\[
W_{\text{lfa}} = \arg \max_w |W^T S_B W| = [w_1, \ldots, w_p].
\]

The dimension of the signal reduces from \( q \) to \( p \) by

\[
z_i = W_{\text{lfa}}^T x_i \quad i = 1, 2, \ldots, n.
\]

**ICA.** Independent component analysis (ICA) assumes that \( X \) is linearly mixed with source signals such that

\[
X = AS
\]

where \( A \in \mathbb{R}^{q \times q} \) is the weight matrix and \( S \) is the matrix composed of source signals.

ICA reduces the dimension of \( x_i \) from \( q \) to \( p \) as follows.

\[
z_i = W_{\text{ica}}^T x_i
\]

where \( W_{\text{ica}} \in \mathbb{R}^{p \times q} \) is the projection matrix. \( W_{\text{ica}} \) equals the first \( p \) rows of \( A^{-1} \).

The procedure to derive \( W_{\text{ica}} \) was given in [39] and [6].

**B. Local differential privacy**

Local DP (LDP) [25] is deemed to be a state-of-the-art approach for privacy preserving data collection and distribution while parties perform data perturbation before releasing data. LDP is defined as follows.

Let \( f \) be a randomized function with domain \( \Omega_X \) and range \( \Omega_Y \). For a nonnegative \( \epsilon \), the randomized function \( f \) satisfies \( \epsilon \)-LDP if

\[
P(f(t) \in S) \leq e^\epsilon P(f(t') \in S), \quad \forall t, t' \in \Omega_X, \forall S \subseteq \Omega_Y.
\]

LDP is considered to be a strong and rigorous definition of privacy that provides plausible deniability.

**IV. THE PROPOSED METHOD**

The proposed scheme involves two parties: the client who has some symptom information (e.g., EEG, ECG) and the server which has access to the medical database and provides disease risk prediction. The client wants to know the disease risk with EEG/ECG signal obtained by personal devices with privacy protection. If information of the client is disclosed, the insurance companies can restrain the client from coverage and the sensitive information leakage can also lead to some consequences such as public humiliation and losing jobs. On the other hand, the server also considers medical database as private data and is not willing to reveal.

Fig. 1: The overview of privacy preserving disease prediction framework

In this paper, we design privacy preserving scheme for real-time disease prediction. Assume model training is done by the server using medical data already collected from authorized participants. The server does not collect new data in our scheme. Disease prediction is performed in a privacy preserving way such that data transmitted between the client and the server is encrypted before release while result is only accessible to the client. Before disease prediction, medical database and client signal are projected to new feature space by dimension reduction algorithms, i.e., PCA/LDA/ICA. Each image is mapped from the original \( q \)-dimensional image space to a \( p \)-dimensional feature space, where \( p < q \). The new feature vector \( z_i \in \mathbb{R}^p \) is defined by the following linear transformation:

\[
z_i = W x_i \quad i = 1, 2, \ldots, n
\]

where \( W \in \mathbb{R}^{p \times q} \) is the projection. \( W = W_{\text{pca}}^T \) for PCA, \( W = W_{\text{lfa}}^T \) for LDA and \( W = W_{\text{ica}}^T \) for ICA.

Each party conducts two procedures to achieve privacy preserving disease prediction as presented in Figure [1]. The client encrypts signal \( t \) and transmits to the server (E I.1); the server further encrypts received signal to get \( t^* \) and transmits it to the client (E II.1); the server maps the whole database to a new feature space with the projection \( W \) and then encrypts both new database and projection \( W \) and transmits to the client (E II.2); the client projects \( t^* \) to the new feature space using encrypted \( W \) and then performs disease prediction using encrypted database (E II.2).

**A. Data encryption**

**Client signal encryption.** In order to encrypt signal \( t \) held by the client while maintaining data utility, the client and the server generate commutative masking matrices \( (B_{11} \) and \( B_{22} \) using the same encryption key \( B_0 \). Specifically, the encryption key \( B_0 \) is a \( q \times q \) invertible matrix with \( q \) unique
Algorithm 1: Data encryption

**Input:** $q \times q$ invertible matrix $B_0$ with each entry following Gaussian distribution $N(0, \sigma^2_{B_0})$. Let $q$ be the number of unique eigenvalues of $B_0$.

1. **Client**
   1. Generate a random coefficient vector $(b_{11}, b_{12}, \cdots, b_{1q})$;
   2. $B_{11} = \sum_{j=1}^{q} b_{1j} B_0^j$ and compute $B_{11} t$;
   3. Send $B_{11} t$ to the server;

2. **Server**
   1. Apply PCA/LDA/ICA to compute the optimal projection $W$;
   2. Permute the columns of $X$;
   3. Generate a random coefficient vector $(b_{21}, b_{22}, \cdots, b_{2q})$ and $B_{22} = \sum_{j=1}^{q} b_{2j} B_0^j$;
   4. Generate a $p \times p$ random invertible matrix $A$ with each entry following $N(0, \sigma^2_A)$ and let $A_1 = (AA^T)^{-1}$;
   5. Compute $W^* = AWB_{22}$ and $X^*_W = AWX$;
   6. Compute $t^* = B_{22}^{-1} B_{11} t$ and sends to the client;
   7. Send $W^*$, $X^*_W$ and $A_1$ to the client;

13. **Client**
   1. Compute $t^*_W = W^* B_{11}^{-1} t^*$;

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The columns of $X$ (each column corresponds to EEG/ECG signal from one authorized participant) are further permuted to enhance the privacy protection. The server generates random invertible matrix $A$ to encrypt projection $W$ and $WX$. The encrypted data $W^* = AWB_{22}$ and $X^*_W = AWX$ are transmitted to the client. To derive accurate result, the server further computes $A_1 = (AA^T)^{-1}$ and sends to the client.

**Medical database encryption.** The server applies PCA/LDA/ICA to compute the corresponding optimal projection $W$. The columns of $X$ (each column corresponds to EEG/ECG signal from one authorized participant) are further permuted to enhance the privacy protection. The server generates random invertible matrix $A$ to encrypt projection $W$ and $WX$. The encrypted data $W^* = AWB_{22}$ and $X^*_W = AWX$ are transmitted to the client. To derive accurate result, the server further computes $A_1 = (AA^T)^{-1}$ and sends to the client.

Algorithm 1 provides details of the proposed encryption method. Figure 2 depicts data communication details between two parties.

### B. Identification

We design decryption method such that the client derives accurate disease prediction with encrypted data. Support vector machine (SVM) and neural network (NN) are investigated for the classification and the accuracy is not degraded following the proposed decryption method.

We first define notations for signal $x_i$ in database $X$ and test signal $t$: $x_iW = Wx_i$, $x_i^*W = Ax_iW$, $tW = Wt$, $t^*_W = AtW$.

**Algorithm 1:**

| Client | Server |
|--------|--------|
| Step 0 | $B_0$, $t$ | $B_0$, $W$, $X$ |
| E I.1 | $B_{11} t$ | $B_{22}$ |
| E II.1 | $t^*$ | $t^*$ |
| E II.2 | $t^*_W$ | $W^*$, $X^*_W$ |

**Fig. 2:** Privacy preserving face recognition procedures. In each step, each party generates or has access to matrices in the boxes. Matrices transferred between two parties are above or below the arrows.

$x_i^*W$ and $t_i^*W$ are released to the client while $A_I = (AA^T)^{-1}$ is also sent to the client for decryption.

1. **Support vector machine (SVM)**

   In the model training stage, the server builds SVM on new feature database $WX$. To predict the client’s disease status, the server sends model parameters $\beta_0$ and $\alpha_i y_i (i = 1, \cdots, n)$ to the client. A SVM classifier \[40\] calculates the classification result of projected test signal $t_W$ using

   $$
   f(t_W) = \sum_{i=1}^{n} \alpha_i y_i K(t_W, x_iW) + \beta_0
   $$

   where $y_i$ is the class label of the training signal $x_iW$. The most common kernel functions are (1) linear kernel function: $K(t_W, x_iW) = t_W^T x_iW$, (2) polynomial function: $K(t_W, x_iW) = (t_W^T x_iW + 1)^d$ where $d$ is the degree, (3) Gaussian radial basis function: $K(t_W, x_iW) = \exp(-\gamma ||t_W - x_iW||^2)$ for $\gamma > 0$, (4) hyperbolic tangent function: $K(t_W, x_iW) = \tanh(\kappa t_W^T x_iW + c)$ for some $\kappa$ and $c$.

   For the client, the two formulas below ensure that $f(t)$ calculated using the encrypted data is identical as using original data $WX$ and $Wt$.

1. $t_W^T A_I x_i^*W = t_W^T A_I A x_iW = t_W^T x_iW$.

2. $\sqrt{(x_i^*W - t_W^*)^T A_I (x_i^*W - t_W^*)} = \sqrt{(Ax_iW - At_W)^T A_I (Ax_iW - At_W)} = \sqrt{(x_iW - t_W)^T (x_iW - t_W)} = ||x_iW - t_W||_2$. 

2) Neural network (NN)

After dimension reduction, neural network is trained on the new feature space in the model training stage. Consider a two-layer NN in which the inputs are connected to the hidden layer and this hidden layer is connected to the output layer. Suppose there are \( M \) neurons in the hidden layer. Neuron \( i \) (\( i = 1, \cdots, M \)) consists of a vector of model parameters \( \alpha_i = (\alpha_{i1}, \alpha_{i2}, \cdots, \alpha_{i\ell})^T \) and a bias \( \alpha_{i0} \). After the input \( x_W = (x_{W(1)}, x_{W(2)}, \cdots, x_{W(p)})^T \) is provided, neuron \( i \) calculates \( z_i = \sigma(\alpha_{i0} + \alpha_i^T x_W) \) where \( \sigma(v) = 1/(1+e^{-v}) \). The output layer consists of a parameter vector \( \beta = (\beta_1, \beta_2, \cdots, \beta_M)^T \) and a bias \( \beta_0 \). After receiving the hidden layer result \( z = (z_1, z_2, \cdots, z_M)^T \), the output is computed as \( f(x) = g(\beta_0 + \beta^T z) \) where \( g \) is the activation function in the output layer.

After modeling training stage, the server sends \( \beta, \beta_0, (A^{-1})^T \alpha_i \) and \( \alpha_{i0} \) (\( i = 1, 2, \cdots, M \)) to the client. To predict disease status, the client enters the projected signal \( t_W \) to neural network model using received model parameters. Different from SVM, the client does not require \( WX \) for disease prediction using NN.

V. SECURITY ANALYSIS

| Dataset   | Client | Server |
|-----------|--------|--------|
| Original  | \( t \) | \( X \) |
| Intermediate | \( B_0, B_{11} \) | \( W, A, B_0, B_{22}, WX \) |
| Transmitted | \( B_{11}t \) | \( AWX, AWB_{22}, B_{22}^{-1}B_{11}t \) |

Next, we show that there is no disclosure risk of the transmitted data, i.e., the original and intermediate datasets are secure given transmitted data. Table I summarizes data transmitted between two parties. We consider the security analysis from two views, the security analysis of the encryption technique as described in Algorithm I and information loss of the encrypted images. Theory analysis shows that our encryption technique is secure and no information of original/intermediate datasets is disclosed.

A. Threat model

**Honest-but-curious server.** We assume the server aims to obtain the client signal, but performs the scheme honestly and correctly. More specifically, the server faithfully executes the delegated computations but may be curious about the intermediate data and try to learn or infer any sensitive information.

**Malicious client.** Assume the client is malicious, i.e., the client may arbitrarily deviate from the protocol specification and use any efficient attack strategy. Consider a situation that a malicious client gains control of the legitimate client’s device, e.g., by hacking it or compromising it. In other words, the stored encryption key of the client is available to the adversary, but the legitimate client’s signal is not available since it is not stored locally in the device. We consider two main types of such malicious clients. The first type aims to obtain database features stored in the server by maliciously manipulating the system. Another type of malicious client tries to get authenticated by maliciously manipulating the system without information of the signal or feature vector of the legitimate client.

B. Privacy proof of the encryption technique

We first address the privacy guarantee of the encryption technique in Algorithm I. Theorem I, Theorem 2, and Theorem 3 explain privacy protection for original and intermediate databases in detail. More specifically, the first two theorems guarantee that the client does not learn sensitive information related to medical database \( X \) (i.e., \( X, W, WX \)) owned by the server and Theorem 3 guarantees the server cannot recover the client’s signal \( t \). The server/client acts as the intruder when trying to recover data owned by the client/owner. The outside intruder is assumed to have no access or access to a subset of transmitted data and so has less probability to recover data than the server and the client.

We first illustrate that using the same encryption key \( B_0 \) for commutative matrices \((B_{11}, B_{22})\) generation does not increase the disclosure risk.

**Lemma 1.** \( B_{ii} \) is determined by \( b_{ij} \) \( (j = 1, \cdots, q) \) where \( B_{ii} \) is generated by procedures given in Algorithm I.

**Proof.** Details are given in Appendix A.

Assume each element of \( b^{(1)} \) and \( b^{(2)} \) is randomly generated from uniform distribution \( \mathcal{U}(-\alpha, \alpha) \) by the client and the server respectively where \( \alpha > 0 \) is set to be big. So \( P(B_{ii}|B_0) \) follows multivariate uniform distribution \( (i = 1, 2) \). Because \( b^{(1)} \) and \( b^{(2)} \) are independently generated by the client and the server, \( P(B_{i1}|B_{11}, B_0) = P(B_{i1}|B_0) \) and \( P(B_{22}|B_{11}, B_0) = P(B_{22}|B_0) \). In other words, knowing the encryption key \( B_0 \) for commutative matrix generation is not useful to recover invertible masking matrix.

**Theorem 1.** Our scheme does not release any information of the projection \( W \).

**Proof.** Knowing \( B_0 \) does not threaten the security of \( W \) by released \( AWB_{22} \). Since \( B_{22} \) is determined by \( b_{2j}(j = 1, \cdots, q) \) (Lemma 1 Appendix A), we have \( AWB_{22} = AW(\sum_{j=1}^q b_{2j}B_0^j) \). For each element of \( AWB_{22} \), there are \( 2p+q \) unknown elements \( p \) elements from \( A \), \( p \) elements from \( W \) and \( q \) from \( B_{22} \). There are always more unknown elements than the number of equations in any combinations of the equations which guarantees that the solutions to any partial unknown elements are infinite.

\( WX \) is the medical data projected to the new feature space. Because \( W \) is derived using the entire medical data \( X \), it is reasonable to assume that other parties do not know any element in \( WX \). In other words, the adversary cannot conduct known plaintext attack to recover sensitive information from \( AWX \). Moreover, brute-force attack is not effective according to Theorem 2.
**Theorem 2.** The encryption scheme does not disclose $WX$.

**Proof.** Let $O$ denote the set consisting of all the invertible matrices. The restricted support of $WX$ given $AWX$ is $S_{WX}(AWX) = \{U : \exists A \in O \text{ such that } AU = AWX\}$. $A^*WX \in S_{WX}(AWX)$ for any invertible matrix $A^*$ which indicates that there are infinite solutions given $AWX$. □

Known plaintext attack is not effective to recover information given $AWX$. Consider an extreme scenario that a subset of $X$ is disclosed to the attacker. $X_i^t$ is encrypted to a lower dimensional space since the number of rows ($p$) in $X_i^t$ is smaller to the number of rows ($q$) in $X$. For general scenario, $2p - 1 \leq q$. Based on [16], there is no linear method that can separate out information of $X$.

**Theorem 3.** Transferring $B_{11t}$ among two parties does not disclose $t$.

**Proof.** Let $t^* = B_{11t}$. The intruder has equation $t^* = (\sum_{j=1}^q b_1^j B_0^j) t$ by knowing $t^*$ and $B_0$. There are $q$ equations and $2q$ unknown elements ($q$ elements from $t$ and $q$ from $B_{11}$). Each equation contains these $2q$ unknown elements which leads that solutions to any partial unknown elements are infinite. So it is impossible to recover $t$. □

The proposed scheme is resilient to collusion attack since signal $t$ is encrypted by different $B_{11}$ in each request.

The server sends $A_I = (AA^T)^{-1}$ to the client for result decryption. Releasing $A_I$ does not disclose $A$. For any orthogonal matrix $P$, the variance matrix of $AP$ is $AP(A^P)^T = AA^T$. Given $A_I$, the restricted support of $A$ includes any orthogonal transformation $AP$.

**Theorem 4.** Encryption function $f(x) = Bx$ achieves LDP using encryption matrix $B$ with entries following Gaussian distribution $N(0, \sigma^2)$.

**Proof.** Consider any two records, $x^{(1)}$ and $x^{(2)}$, that randomly selected from input domain (all the possible image/feature vectors). Since $B$ follows Gaussian distribution, each entry $x^{(1)}_{i,j}$ in $Bx^{(1)}$ follows $N(0, ||x^{(1)}||^2 \sigma^2)$ and each entry $x^{(2)}_{i,j}$ in $Bx^{(2)}$ follows $N(0, ||x^{(2)}||^2 \sigma^2)$. So

$$P(x^{(1)}_{i,j} \in (-t,t)) = \frac{erf(t/(\sqrt{2}||x^{(1)}||\sigma))}{erf(t/(\sqrt{2}||x^{(2)}||\sigma))}$$

where $erf$ is Gaussian error function. For any given $||x^{(1)}||_2, ||x^{(2)}||_2$ and $t$, there exists a $\sigma \rightarrow 0$ such that $P(x^{(1)}_{i,j} \in (-t,t)) \rightarrow 1$. So the encryption function $f(x)$ achieves LDP. □

To protect image vector $t$, the encryption key $B_0$ is generated randomly with each entry following Gaussian distribution $N(0, \sigma_{B_0})$. Released data $B_{11}t = b_1^1 B_0^1 + b_1^2 B_0^2 + \cdots + b_{1q} B_0^{q-1} B_0 t_0$ is encrypted by two encryption mechanisms $f_1(X) = B_0 t_0$ and $f_2(f_1(X)) = (b_1^1 + b_1^2 B_0 + \cdots + b_{1q} B_0^{q-1}) f_1(t)$. Based on Theorem □

$f_1(t)$ satisfies LDP with $\sigma_{B_0} \rightarrow 0$. According to closure under postprocessing [41], the proposed encryption method achieves LDP.

The server generates random invertible matrix $A$ to encrypt medical databases. We use $Z$ as the notation of the pre-processed database (i.e., $W, WX$). Similar to the above analysis, $AZ$ achieves LDP for $\sigma_A \rightarrow 0$.

The proposed privacy preserving schemes are resilient to brute-force attack. $A$ is generated randomly for each face recognition request. The encrypted $AWX$ belongs to the infinite support $S_{WX}(AWX)$ (Theorem 2). The infinite support makes brute-force attack invalid. $AWB$ is also secure to brute-force attack since the commutative matrix $B$ has degree of freedom $q$ and $A$ is a random invertible matrix.

The proposed schemes are resilient to differential attack. Differential attack uses a relationship of the differences in input and output to guess the original input using the knowledge of the output. Let $B^{(1)}t$ and $B^{(2)}t$ be two encrypted signals by two disease prediction requests for the same signal $t$. The difference is $(B^{(1)} - B^{(2)})t$. Because $B^{(1)}$ and $B^{(2)}$ are required to be commutative, the support of $(B^{(1)} - B^{(2)})t$ is $\{B_At : B_A = \sum_{j=1}^q b_j B_0^j\}$ where $(b_1, b_2, \ldots, b_q)$ is a random coefficient vector. The degree of freedom $q$ makes the possible values of encrypted signal infinite.

To summarize, the malicious client and honest-but-curious server can not derive any sensitive information. Lemma 1 guarantees the scheme resilience for the malicious client. Even has access to the encryption key $B_0$, the client cannot recover invertible masking matrix generated by the server. The invertible matrix encryption further achieves LDP. Moreover, our scheme is resilient to brute-force attack and satisfies differential attack.

VI. PERFORMANCE ANALYSIS

In our privacy preserving face recognition, the LDP is achieved by matrix encryption which can be corrected and does not degrade scheme performance. Specifically, the invertible matrix is generated for the encryption with each entry following Gaussian distribution $N(0, \sigma^2)$. For any $t \in \mathbb{R}$, there exists $\sigma \rightarrow 0$ such that each entry in the encrypted data is within $(-t, t)$ with probability 1. With $t \rightarrow 0$, each entry in the encrypted data is close to each other which ensures that any input is indistinguishable.

Next we focus on the computation and communication efficiency.

A. Computation complexity

The computation complexity of masking matrix generation and multiplication for data encryption depends on the dimension of database. One way to reduce the cost is to partition $B_{11}, B_{22}$ and $A$ into block diagonal matrices. Partitioning $B_{11}$ and $B_{22}$ are equivalent to partition $B_0$ since they are both generated by $B_0$. Suppose we partition $B_0$ with block size $q^*$ so that $B_0$ is generated as block matrix with the diagonal elements as square matrices of size $q^*$ and the off-diagonal elements being 0. In other words, $B_0 = \text{diag}(B_{11}, \ldots, B_{q^*q^*})$ where $B_i$ is $q^* \times q^*$ invertible matrix $(i = 1, \ldots, q^*/q^*)$. Then the bottleneck of the computation cost $O(q^3)$ and $O(q^2)$ reduce to $O(q(q^*q^*)$ and $O(q^2q^*)$, respectively. As an example, suppose $q = 1000$ and $q^* = 100$. Each party generates $10$ invertible matrices with dimension $100 \times 100$ instead of one $1000 \times 1000$ matrix. A simple way to generate masking matrices is
to use the same matrix $B$ for these 100 diagonal elements in $B_0$. In other words, $B_{11} = diag(q \sum_{j=1}^{q^*} b_{1j}^{(1)} B^j, \ldots, q \sum_{j=1}^{q^*} b_{1j}^{(10)} B^j)$ and $B_{22} = diag(q \sum_{j=1}^{q^*} b_{2j}^{(1)} B^j, \ldots, q \sum_{j=1}^{q^*} b_{2j}^{(10)} B^j)$ where $q^*$ is the number of $B$'s unique eigenvalues, $(b_{11}^{(i)}, b_{12}^{(i)}, \ldots, b_{1q^*}^{(i)})$ and $(b_{21}^{(i)}, b_{22}^{(i)}, \ldots, b_{2q^*}^{(i)})$ are random coefficient vectors ($i = 1, \ldots, 10$).

The proposed scheme is efficient with real-time computation. Assuming $n = 10000$, $q = 1000$ and $p = 50$, it takes approximately 1 second to predict disease status of the client. A running time of less than 1 second is considered to be enough for real life applications as discussed in [13]. Considering that our scheme is secure against malicious adversary, it is feasible to implement the proposed method in real life application. For database with large number of records, parallel computing can be applied to further reduce computation time.

B. Communication complexity

The communication complexity depends on the dimension of the database. One advantage of our proposed schemes is that the dimension of databases transmitted between parties are reduced by the dimension reduction projection.

Additionally, we adjust the bit-length of the elements in the databases to reduce the communication cost. However, bit-length needs to be chosen carefully to avoid losing computation accuracy. We use three different measures (absolute difference, Euclidean distance and cosine similarity) to measure distances between the original data and data after reducing bit-length of the databases. A 200-dimensional test vector ($p = 200$, $n = 1000$) are randomly simulated and each column is normalized for recognition. The bit-adjusted databases are generated by reducing the bit-length of each element in each database. Let $D$ be the original database with $d_{ij}$ be its element and $t$ be the original test vector. $\hat{D}$ denotes the new database with $\hat{d}_{ij}$ be its element and $\hat{t}$ denotes the new test vector. The distance between original database and new database is measured by $T_A = \max_{i,j}|d_{ij} - \hat{d}_{ij}|$, $T_B = \max_{i,j}|T_1(\hat{t}, \hat{d}_i) - T_1(t, d_i)|$ and $T_C = \max_{i,j}|T_2(\hat{t}, \hat{d}_i) - T_2(t, d_i)|$ where $T_1(a,b)$ denote Euclidean distance of $a$ and $b$ and $T_2(a,b)$ denote cosine similarity of $a$ and $b$.

| Bit-length | $T_A$ | $T_B$ | $T_C$ |
|------------|-------|-------|-------|
| 8          | 2.47  | 0.30  | 0.04  |
| 12         | $4 \times 10^{-3}$ | $2 \times 10^{-4}$ | $9 \times 10^{-4}$ |
| 13         | $10^{-3}$ | $5 \times 10^{-4}$ | $2 \times 10^{-4}$ |
| 16         | $10^{-4}$ | $6 \times 10^{-5}$ | $3 \times 10^{-5}$ |
| 24         | $5 \times 10^{-7}$ | $3 \times 10^{-7}$ | $10^{-7}$ |
| 32         | $2 \times 10^{-9}$ | $9 \times 10^{-10}$ | $4 \times 10^{-10}$ |

Table II presents the computation accuracy change with the bit-length change of database elements. Bit-length 8 lowers the communication cost but also deviates from original results and losses partial information. We use bit-length 12 due to the high precision (Table II). When the dimension of new feature space $p$ is fixed, the number of images in the biometric database does not have a significant impact on the communication cost.

![Fig. 3: Communication cost using 12-bit database at $q = 1000$.](image)

With the bit-length adjustment and dimension reduction procedures (PCA/LDA/ICA), the proposed scheme is efficient with less than 1 MB communication cost to perform disease prediction for a medical database with 10000 records (Figure 3). The rapid development in 5G technology enables real-time communication using wearable devices [42].

VII. CONCLUSIONS

In this paper, we propose real-time privacy preserving disease prediction scheme that guarantees security against malicious clients and an honest-but-curious server. No third party is required to perform secure diagnosis. The proposed scheme satisfies local differential privacy and the privacy of medical data is guaranteed. Support vector machine and neural network are investigated to analyze the encrypted data. Result decryption procedure ensures no accuracy degradation. The proposed scheme is efficient for large database. Assuming there are 10000 signal records in medical database, the proposed scheme takes approximately 1 second to perform the prediction. For efficient communication as well as model accuracy preservation, we adjust bit-length to 12 for elements in transmitted data between two parties. It requires less than 1 MB to perform the prediction with 10000 training records. Future work includes investigating more efficient scheme to further decrease the communication cost.

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APPENDIX A

PROOF OF LEMMA 1

As pointed out in Section 3.3 of [43], a polynomial $f(v)$ is said to annihilate matrix $B$ if $f(B) = 0$. The minimal polynomial of $B$ is the monic polynomial of minimum degree that annihilates $B$. The minimal polynomial $f_B(v)$ is unique and $f_B(\lambda) = 0$ if and only if $\lambda$ is an eigenvalue of $B$. For given matrix $B$ and any monic polynomial $f(v)$, $f(B) = 0$ if and only if there exists monic polynomial $h(v)$ such that $f(v) = h(v)f_B(v)$ where $f_B(v)$ is the minimal polynomial of $B$. For polynomial $f(v)$ such that $f(B) = 0$, all the eigenvalues of $B$ are the roots of $f(v)$. 
Lemma 1. $B_{ij}$ is determined by $b_{ij}$ ($j = 1, \ldots, q$) where $B_{ii}$ is generated by procedures given in Algorithm 1.

Proof. Let $f_1(v) = \sum_{i=1}^{q} b_{1i} v^i$, $f_2(v) = \sum_{i=1}^{q} b_{2i} v^i$ and $f(v) = f_1(v) - f_2(v)$. We have $B_{11} = f_1(B_0)$ and $B_{22} = f_2(B_0)$. $B_{11} = B_{22}$ is equivalent to $f(B_0) = 0$. So $B_{11} = B_{22}$ if and only if all the eigenvalues of $B_0$ are the roots of $f(v)$. Suppose $\lambda_i$ ($i = 1, \ldots, q$) are the unique eigenvalues of $B_0$. In order to get $B_{11} = B_{22}$, $\Lambda$ is Vandermonde matrix with $\text{rank} (\Lambda) = q$. Given $\Lambda$ and $b_{1i}$, there is only one solution for $b_{2i}$ since $\text{rank} (\Lambda, \Lambda b_{1i}^{(1)}) = q$. So $P(B_{11} = B_{22}) = P(b_{2i} \mid b_{1i})$ which indicates that $B_{ii}$ is determined by $b_{ii}$.

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