Model of Optimization solution Based on Generalized Evidence Theory

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Abstract. In order to solve the application of generalized evidence theory in Engineering, a new generalized basic probability assignment GBPA generation method is proposed in this paper. And on this basis, the selection model of civil aircraft optimization scheme is constructed. Then, taking the optimization scheme of a civil aircraft as an example, it is proved that the method is simple in calculation and clear in physical meaning, which is more suitable for practical application.

1. Introduction
With the rapid development of modern data fusion technology[1-6], it plays an increasingly important role in the field of decision-making. Decision making under uncertain factors is very common in practical applications, such as supply chain management in enterprises [7-9], medical services [10,11], unknown risk analysis [12,13], etc. [14-15]. In order to get an optimal goal, many new decision methods are developed based on fuzzy set theory [16,17], probability theory [18-21], rough set theory [22-25], D-S evidence theory [26-28] and other data fusion methods [29-32].

Dempster Shafer evidence theory provides the basic elements of information modeling: recognition framework and basic probability allocation (BPA), and provides combination rules for fusion data. Dempster Shafer evidence theory has been widely used in multiple attribute decision making [33-37], fault diagnosis [38-41], pattern recognition [42-44], controller design [45-47], risk analysis [48, 49] and other [50-52] fields.

Evidence theory is usually based on a complete set, however, in the actual decision-making process, we can't take all the evaluation rules. How to model these incomplete evaluation rules is still an urgent issue to be solved. This issue is addressed in this paper; then a method for generating GBPA is proposed based on the generalized evidence theory (GET)[53], namely, generalized evidential selection model SM FMEA (GESM).

Generalized evidence theory [53] is a more generalized case of Dempst-Shafer evidence theory (D-S evidence theory), which is developed to deal with uncertain information in the open world. D-S evidence theory has been widely studied in the past decades. It is a useful mathematical theory in the practical application of information fusion. Some key problems in D-S evidence theory are still worthy of further study, such as the determination of dependent evidence combination and basic probability assignment [52]. Generalized evidence theory inherits the advantages of D-S evidence theory, and if the recognition framework is incomplete, the generalized basic probability assignment (gbpA) and generalized combination rule (GCR) in get can deal with incomplete knowledge more effectively [53].
In this paper, an empty set is used to represent the incomplete risk factors identified by evaluation rules. The relative importance of these factors is not taken into account. The gesm model based on the traditional SM is a more general model. If the conditions are satisfied, gesm can also degenerate into traditional SM.

The other chapters of this paper are arranged as follows. The second section introduces the basic definition. The third section is a new generalized evidence model (gesm). The fourth section is an experiment based on gesm. Section 5 is the experimental conclusion.

2. Basic definitions

Some basic definitions are briefly introduced in this section, includes Dempster-Shafer evidence theory and General Dempster-Shafer evidence theory. Dempster-Shafer evidence theory D-S theory assumes a fixed, exhaustive set of mutually exclusive events \( \Theta = \{ \theta_1, \theta_2, \ldots, \theta_n \} \). Dempster-Shafer theory is concerned with the set of all subsets of \( \Theta = \{ \theta_1, \theta_2, \ldots, \theta_n \} \), which is the power set \( 2^{\Theta} \), known as the Frames of Discernment (FOD):

\[
\Omega = \{ \emptyset, \{ \theta_1 \}, \{ \theta_2 \}, \ldots, \{ \theta_n \}, \{ \theta_1, \theta_2 \}, \ldots, \{ \theta_1, \theta_2, \ldots, \theta_n \} \}
\]

Mathematically, a basic probability assignment (BPA) or mass function \( m \) is a function from the power set (set of all subsets) of \( \Theta \) to \([0, 1]\), which satisfies:

\[
m(\emptyset) = 0, \quad 0 \leq m(A) \leq 1 \quad \text{and} \quad \sum_{A \in \Theta} m(A) = 1
\]

D-S evidence theory is also called belief function theory [15,16].

Let the discernibility framework \( \Theta \) be a set of N mutually exclusive global exhaustion events. The elements in \( \Theta \) is defined as \( \Theta = \{ \theta_1, \theta_2, \ldots, \theta_n \} \). The power set of \( \Theta \) is denoted by \( 2^{\Theta} \), and its elements are called hypothesises or a propositions. Based on the above two concepts, the definition of the mass function can be described. A mass function, also called basic probability assignment (BPA), is a mapping \( m \) from \( 2^{\emptyset} \) to \([0, 1]\), which is given below:

\[
m : 2^{\emptyset} \rightarrow [0,1]
\]

Satisfying:

\[
m(\emptyset) = 0
\]

\[
\sum_{A \in \Theta} m(A) = 1
\]

The value \( m(A) \) represents the belief degree distributed to hypothesis \( A \). Note that \( m(\emptyset) = 0 \) means that there is no belief degree assigned to the empty set, which is required in the closed world.

While in the open world [30], the criterion is not required, and \( m(\emptyset) \) can be bigger than zero. All subsets \( A \) of \( \Theta \) that satisfying \( m(A) > 0 \) are called focal elements. Dempster’s combination rule, also called the orthogonal sum, is defined as follows:
\[
m(C) = m_X + m_Y
\]
\[
= m_X \oplus m_Y
\]
\[
= \begin{cases} 0, X \cap Y \neq \emptyset \\ \sum_{X \cap Y = C, X, Y \subseteq \Theta} m_X \times m_Y \times (1 - K), X \cap Y \neq \emptyset \end{cases}
\]

(3)

K is called the conflict factor between \(m(X)\) and \(m(Y)\), which is defined below:

\[
K = \frac{\sum_{X \cap Y = \emptyset, \forall X, Y \subseteq \Theta} m_X \times m_Y}{X \cap Y \neq \emptyset}
\]

(4)

When there are more than two pieces of evidence, these can be combined in the following form:

\[
m = m_1 \oplus m_2 \oplus \cdots \oplus m_n
\]

(5)

**General D-S evidence theory**

This section mainly introduces the basic concepts of the generalized evidence theory (GET) [53]. The generalized basic probability assignment (gbpA) in generalized evidence theory corresponds to the basic probability assignment (BPA) in D-S evidence theory. It is used to represent data and modeling. The conflicting, inconsistent, or uncertain evidence bodies are combined by generalized combination rules (GCR), which is derived from Dempster’s combination rules.

**Definition 1.** Suppose that \(\Theta\) is a frame of discernment in the open world. Its power set, \(2^U\), is composed of \(2^U\) propositions, \(\forall A \subseteq U\); a mass function is a mapping \(m_g: m_g : [0,1] \rightarrow [0,1]\), satisfying [53]

\[
\sum_{A \subseteq 2^\Theta} m_g(A) = 1
\]

(6)

where \(m_g\) is the GBPA of the frame of discernment, \(\Theta\). The difference between GBPA and BPA is the restriction of 0. In GET, \(m_g(\emptyset) = 0\) is not necessary in GBPA. In other words, the empty set can also be a focal element. If \(m_g(\emptyset) = 0\), the GBPA degenerates to a conventional BPA in D-S evidence theory.

**Definition 2.** Given a GBPA, the generalized belief function is GBel: \(2^U \rightarrow [0,1]\), which satisfies [53]

\[
\text{GBel}(A) = \sum_{B \subseteq A} m(B)
\]

\[
\text{GBel}(\emptyset) = m(\emptyset)
\]

(7)

**Definition 3.** Given a GBPA, the generalized belief function is GPI: \(2^U \rightarrow [0,1]\), which satisfies [53]

\[
\text{GPl}(A) = \sum_{B \subseteq A} m(B)
\]

\[
\text{GPl}(\emptyset) = m(\emptyset)
\]
Definition 4. In generalized evidence theory, \( \emptyset \cap \emptyset = \emptyset \), which means that the intersection between two empty sets is still an empty set. Given two GBPAs \((m_1, m_2)\) the generalized combination rule (GCR) is defined as follows\([53]\):

\[
m(A) = \frac{(1 - m(\emptyset)) \sum_{B \cap C = A} m_1(B) \cdot m_2(C)}{1 - K}
K = \sum_{B \cap C = \emptyset} m_1(B) \cdot m_2(C)
\]

\(m(\emptyset) = m_1(\emptyset) \cdot m_2(\emptyset)\),

\(m(\emptyset) = 1\), if \(K = 1\)

3. Selection Model of Optimization solution

In this section, a new GBPA generation method and a new selection model are introduced.

A new GBPA generation method

A new GBPA generation method assigns the part that each solution could not satisfy to the empty set \(\emptyset\). Suppose there are \(w\) alternative solutions, and \(n\) selection criterias, Set the valuation of each solution based on each evaluation criterion as \(V_1, V_2, \ldots, V_w\), and \(V \in [0, 1]\). Then set the valuation of the empty set \(\emptyset\) as :

\[
V(\emptyset) = 1 - \max|V_1, V_2, \ldots, V_w|
\]

Then the GBPA is generated as follows :

\[
m_w = \frac{V}{V_1 + V_2 + \ldots + V_w}
\]

A numerical example is given to illustrate the new GBPA generation method. The valuation of each solution based on each evaluation criterion is \(V(A), V(B)\) and \(V(C)\). According to Equation (10), the empty set can be calculated:

\[
V(\emptyset) = 1 - \max[V(A), V(B), V(C)]
\]

The GBPAs according to Equation (11) are given below :

\[
m(A) = \frac{V(A)}{V(A) + V(B) + V(C) + V(\emptyset)}
\]

\[
m(B) = \frac{V(B)}{V(A) + V(B) + V(C) + V(\emptyset)}
\]
\[ m(C) = \frac{V(C)}{V(A) + V(B) + V(C) + V(\varnothing)} \]

This is to say, one evidence get from that criterion is \( m : m(A), m(B), m(C), m(\varnothing) \).

Step 1: Set the selection criteria according to the characteristics of the product

Step 2: Give the valuation of each solution according to the selection criteria for each solution

Step 3: Get GBPA based on the proposed method

Step 4: Data fusion based on GCR

Step 5: Decision making

End

Fig. 1. The flowchart on new GESM

If the value of \( m(\varnothing) \) is large, the risk of all alternative solutions is great; If the value of \( m(\varnothing) \) is small, it shows that the alternative solution is very good.

The proposed selection model

The new selection model is applied to the optimization of civil aircraft product, as shown in Figure 1:

Step 1: Set the selection criteria according to the characteristics of the product:
Step 2: Give the valuation of each solution according to the selection criteria for each solution

Step 3: Get GBPA based on the proposed method;

Step 4: Data fusion based on GCR

Step 5: Decision making based on the result of data fusion

4. A Case Study

In this section, a real case study on a civil aircraft system is need to optimize, and there are three alternative solutions. There are five selection criteria for these alternative solutions, and based on these selection criteria, the scoring results given by the expert system are as follows (Table 1):

Table 1 The scoring results based on five selection criteria

| Criteria   | Solution A | Solution B | Solution C |
|------------|------------|------------|------------|
| Criteria 1 | 0.9        | 0.95       | 0.9        |
| Criteria 2 | 0.85       | 0.9        | 0.95       |
| Criteria 3 | 0.9        | 0.95       | 0.85       |
| Criteria 4 | 0.8        | 0.7        | 0.9        |
| Criteria 5 | 0.8        | 0.7        | 0.7        |

Introduce the generation of GBPA with criteria as an example. We can calculate the value of $V(\emptyset)$ according to equation (10).

$$V_1(\emptyset) = 1 - \text{Max}[0.9, 0.95, 0.9] = 0.05$$

With the result of equation (10), the GBPA can be get as follows:

$m_1(A) = \frac{V_1(A)}{V_1(A) + V_1(B) + V_1(C) + V_1(\emptyset)} = \frac{0.9}{0.9 + 0.95 + 0.9 + 0.05} = 0.3214$

$m_1(B) = \frac{V_1(B)}{V_1(A) + V_1(B) + V_1(C) + V_1(\emptyset)} = \frac{0.95}{0.9 + 0.95 + 0.9 + 0.05} = 0.3393$

$m_1(C) = \frac{V_1(C)}{V_1(A) + V_1(B) + V_1(C) + V_1(\emptyset)} = \frac{0.9}{0.9 + 0.95 + 0.9 + 0.05} = 0.3214$

$m_1(\emptyset) = \frac{V_1(\emptyset)}{V_1(A) + V_1(B) + V_1(C) + V_1(\emptyset)} = \frac{0.05}{0.9 + 0.95 + 0.9 + 0.05} = 0.0179$

This is to say, one evidence get from that criterion is:

$m_1(A) = 0.32, m_1(B) = 0.33, m_1(C) = 0.32, m_1(\emptyset) = 0.02$

With the above method, five groups of evidence can be obtained by Table 1, as shown in table 2:

Table 2 The evidences get from the table 1

| Evidence | Solution A | Solution B | Solution C | $V(\emptyset)$ |
|----------|------------|------------|------------|----------------|
| $m_1$    | 0.32       | 0.34       | 0.32       | 0.02           |
| $m_2$    | 0.31       | 0.33       | 0.34       | 0.02           |
Combine $m_1, m_2, m_3, m_4$ and $m_5$ based on GCR, the combination result is shown in Table 3:

| Evidence | Solution A | Solution B | Solution C | (Ø) |
|----------|------------|------------|------------|-----|
| $m_1, m_2$ | 0.4098 | 0.4447 | 0.1451 | 0.0004 |
| $m_1, m_2, m_3$ | 0.4472 | 0.4879 | 0.0649 | 0.0000 |
| $m_1, m_2, m_3, m_4$ | 0.4787 | 0.4869 | 0.0344 | 0.0000 |
| $m_1, m_2, m_3, m_4, m_5$ | 0.5047 | 0.4789 | 0.0163 | 0.0000 |

From the fusion results of Table 3, it can be drawn that solution A is the best. Although, the GBPA of solution A is not much larger than the other two programs, the GBPA of solution (Ø) is also very small, which shows that the risk of the three alternative solutions is small. The three solutions are good ones.

5. Conclusion
The development of generalized evidence theory has been faster, but the application in engineering practice needs to be strengthened. This paper presents a method to generate GBPA based on generalized evidence theory, and constructs the selection model of civil aircraft optimization solutions based on the new GBPA. The data fusion results of a real study show that the new model works well. However, there is still much room for the development of generalized evidence theory in engineering.

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References
[1] Tang Yongchuan, Zhou Deyun, Jiang Wen. A New Fuzzy-Evidential Controller for Stabilization of the Planar Inverted Pendulum System[J]. Plos One, 2016, 11(8): e0160416. doi:10.1371/journal.pone.0160416.
[2] Zhou Deyun, Tang Yongchuan, Jiang Wen. A Modified Model of Failure Mode and Effects Analysis Based on Generalized Evidence Theory[J]. Mathematical Problems in Engineering, 2016, 2016(2016):1-11.doi: 10.1155/2016/4512383.
[3] Zhou Deyun, Tang Yongchuan, Jiang Wen. An improved belief entropy and its application in decision-making [J]. Complexity, 2017, 2017(2017);Article ID 4359195, in press.
[4] Xiaoge Zhang, SankaranMahadevan, Xinyang Deng. Reliability analysis with linguistic data: An evidential network approach [J]. Reliability Engineering& System Safety, 2017, 162:111-121.doi: dx.doi.org/10.1016/j.ress.2017.01.009.
[5] Xiaoge Zhang, SankaranMahadevan. Aircraft Re-routing Optimization and Performance Assessment under Uncertainty [J].Decision Support Systems, 2017, in press.
[6] Han, D.Q.; Han, C.Z.; Yang, Y. Multi-class SVM classifiers fusion based on evidence combination. InProceedings of the International Conference on Wavelet Analysis and Pattern Recognition, Beijing, China,2–4 November 2007; Volume 2, pp. 579–584.
[7] E. K. Zavadskas, J. Antucheviciene, S. H. R. Hajiagha, S. S. Hashemi, The interval-valued intuitionistic fuzzy multimoora method for group decision making in engineering, Mathematical Problems in Engineering 2015 (2015) 560690.

[8] M. A. Sodenkamp, M. Tavana, D. D. Caprio, Modeling synergies in multi-criteria supplier selection and order allocation: An application to commodity trading, European Journal of Operational Research 254 (2016) 859–874.

[9] E. Siozinyte, J. Antucheviciene, V. Kutut, Upgrading the old vernacular building to contemporary norms: multiple criteria approach, Journal of Civil Engineering & Management 20 (2014) 291–298.

[10] J. Antucheviciene, Z. Kala, M. Marzouk, E. R. Vaidogas, Solving civil engineering problems by means of fuzzy and stochastic mcdm methods: Current state and future research, Mathematical Problems in Engineering 2015 (2015) 362579.

[11] M. K. Ghorabaee, E. K. Zavadskas, M. Amiri, J. Antucheviciene, A new method of assessment based on fuzzy ranking and aggregated weights (AFRAW) for MCDM problems under type-2 fuzzy environment, Economic Comp.

[12] T. Aven, Supplementing quantitative risk assessments with a stage addressing the risk understanding of the decision maker, Reliability Engineering & System Safety 152 (2016) 51–57.

[13] T. Aven, E. Zio, Some considerations on the treatment of uncertainties in risk assessment for practical decision making, Reliability Engineering & System Safety 96 (2011) 64–74.

[14] Z. S. Chen, K. S. Chin, Y. L. Li, Y. Yang, Proportional hesitant fuzzy linguistic term set for multiple criteria group decision making, Information Sciences 357 (2016) 61–87.

[15] W. Feller, An introduction to probability theory and its applications (2nd ed.), New York: Wiley, 1957.

[16] X. Wang, J. Zhu, Y. Song, L. Lei, Combination of unreliable evidence sources in intuitionistic fuzzy mcdm framework, Knowledge-Based Systems 97 (2016) 24–39

[17] A. P. Dempster, Upper and lower probabilities induced by a multivalued mapping, Annals of Mathematical Statistics 38 (1967) 325–339.

[18] G. Shafer, A Mathematical Theory of Evidence, Princeton University Press, Princeton, 1976.
[31] J. Wang, Y. Hu, F. Xiao, X. Deng, Y. Deng, A novel method to use fuzzy soft sets in decision making based on ambiguity measure and Dempster–Shafer theory of evidence: An application in medical diagnosis, Artificial Intelligence in Medicine 69 (2016) 1–11.
[32] C. Fu, D. L. Xu, Determining attribute weights to improve solution reliability and its application to selecting leading industries, Annals of Operations Research 245 (2014) 401–426.
[33] B. Kang, Y. Hu, Y. Deng, D. Zhou, A new methodology of multicriteria decision-making in supplier selection based on Z-numbers, Mathematical Problems in Engineering 2016 (2016) 8475987.
[34] X. Deng, X. Zheng, X. Su, F. T. S. Chan, Y. Hu, R. Sadiq, Y. Deng, An evidential game theory framework in multi-criteria decision making process, Applied Mathematics & Computation 244 (2014) 783–793.
[35] Z.-G. Liu, Q. Pan, J. Dezert, A. Martin, Adaptive imputation of missing values for incomplete pattern classification, Pattern Recognition 52(2016) 85–95.
[36] D. Han, W. Liu, J. Dezert, Y. Yang, A novel approach to pre-extracting support vectors based on the theory of belief functions, Knowledge-Based Systems 110 (2016) 210–223.
[37] Z. G. Liu, Q. Pan, J. Dezert, G. Mercier, Credal c-means clustering method based on belief functions, Knowledge-Based Systems 74 (2015)119–132.
[38] J. Ma, W. Liu, P. Miller, H. Zhou, An evidential fusion approach for gender profiling, Information Sciences 333 (2015) 10–20.
[39] Z. G. Liu, Q. Pan, J. Dezert, A new belief-based k-nearest neighbor classification method, Pattern Recognition 46 (2013) 834–844.
[40] W. Jiang, C. Xie, M. Zhuang, Y. Shou, Y. Tang, Sensor data fusion with z-numbers and its application in fault diagnosis, Sensors 16 (2016)1509.
[41] K. Yuan, F. Xiao, L. Fei, B. Kang, Y. Deng, Modeling sensor reliability in fault diagnosis based on evidence theory, Sensors 16 (2015) 1–13.
[42] X. Su, Y. Deng, S. Mahadevan, Q. Bao, An improved method for risk evaluation in failure modes and effects analysis of aircraft engine rotor blades, Engineering Failure Analysis 26 (2012) 164–174.
[43] W. Jiang, B. Wei, X. Qin, J. Zhan, Y. Tang, Sensor data fusion based on a new conflict measure, Mathematical Problems in Engineering 2016 (2016) 5769061.
[44] W. S. Du, B. Q. Hu, Attribute reduction in ordered decision tables via evidence theory, Information Sciences 364 (2016) 91–110
[45] C. Fu, Y. Wang, An interval difference based evidential reasoning approach with unknown attribute weights and utilities of assessment grades, Computers & Industrial Engineering 81 (2015) 109–117.
[46] K. S. Chin, C. Fu, Y. Wang, A method of determining attribute weights in evidential reasoning approach based on incompatibility among attributes, Computers & Industrial Engineering 87 (2015) 150–162.
[47] C. Fu, J. B. Yang, S. L. Yang, A group evidential reasoning approach based on expert reliability, European Journal of Operational Research 246 (2015) 886–893.
[48] Y. M. Wang, T. M. S. Elhag, A comparison of neural network, evidential reasoning and multiple regression analysis in modelling bridge risks, Expert Systems with Applications 32 (2007) 336–348.
[49] W. Jiang, C. Xie, B. Wei, D. Zhou, A modified method for risk evaluation in failure modes and effects analysis of aircraft turbine rotor blades, Advances in Mechanical Engineering 8 (2016) 1–16.
[50] Y. Tang, D. Zhou, W. Jiang, A new fuzzy-evidential controller for stabilization of the planar inverted pendulum system, PloS ONE 11 (2016) e0160416
[51] R. R. Yager, D. P. Filev, Including probabilistic uncertainty in fuzzy logic controller modeling using dempster–shafer theory, IEEE transactions on systems, man, and cybernetics 25 (1995) 1221–1230.
[52] J. Ma, W. Liu, S. Benferhat, A belief revision framework for revising epistemic states with partial epistemic states, International Journal of Approximate Reasoning 59 (2015) 20–40.
[53] K. Zhou, A. Martin, Q. Pan, Z.-G. Liu, Median evidential c-means algorithm and its application to community detection, Knowledge-Based Systems 74 (2015) 69–88.
[54] Y. M. Wang, J. B. Yang, D. L. Xu, K. S. Chin, Consumer preference prediction by using a hybrid evidential reasoning and belief rule-based methodology, Expert Systems with Applications 36 (2009) 8421–8430.
[55] Y. Deng, ”Generalized evidence theory,” Applied Intelligence, vol. 43, no. 3, pp. 530–543, 2015.