Effective kaon mass in dense baryonic matter: role of correlations

T. Waas 1, M. Rho 2 and W. Weise 1

Physik-Department, Technische Universität München, Institut für Theoretische Physik, D-85747 Garching, Germany

Abstract

We evaluate the effective kaon mass in dense nuclear matter. Pauli blocking and nucleon-nucleon short-range correlations are incorporated. The effects of short-range correlations are shown to be moderate and figure importantly only at densities larger than 2 times normal nuclear density. We discuss the relations between the present results and the results obtained in next-to-next-to-leading order chiral perturbation theory ($\mathcal{O}(Q^3)$, where $Q$ is the characteristic small energy-momentum scale probed). We also discuss mean-field aspects, with some remarks on the relation between the short-range correlations and the four-Fermi contact terms in the chiral effective Lagrangian.

1 Introduction

Much recent attention has been focused on the question how the mass of the light pseudoscalar mesons (pions and kaons) change in nuclear matter with increasing baryon density. In the case of pions, the answer was obvious for a long time. In symmetric nuclear matter to leading order in the density $\rho = \rho_p + \rho_n$ the pion mass shift is $\Delta m_\pi^2 = -4\pi b_0 \rho$ where $b_0$ is the isospin-even $\pi N$ scattering length. A chiral low-energy theorem as well as $\pi N$ data imply that this $b_0$ is very small. Effects of nuclear correlations are therefore important. A useful and successful method to treat such correlations is the effective field approach of Ericson and Ericson 1. The basic idea of this approach is that repulsive two-body correlations between any pair of nucleons in the medium reduce the pion field at the position of each scatterer. Consequently $b_0$ is

1 Work supported in part by GSI and BMBF
2 Permanent address: Service de Physique Théorique, CEA Saclay, F-91191 Gif-sur-Yvette, France; supported in part by the A. v. Humboldt Foundation
replaced by a (negative) effective scattering length $b_{\text{eff}}$. Its repulsive effect leads to a moderate upward shift of the pion mass (by about 10% at $\rho = \rho_0 = 0.17$ fm$^{-3}$, the density of normal nuclear matter). This s-wave repulsion has been verified for many years in pion-nuclear physics (see for example ref. [2]). An impressive confirmation has come very recently through the discovery of deeply bound pion-nuclear states [3].

For kaons, the systematic exploration of in-medium masses has progressed recently on the basis of the chiral SU(3) effective Lagrangian [4–7]. Here the driving s-wave interactions are much stronger than those in the $\pi N$ system. To leading order in the kaon energy $\omega$ (or mass $m_K$) the driving Born terms of the threshold $K^+ N$ and $K^- N$ scattering matrix derived from the chiral meson-baryon Lagrangian are

$$T_{\text{Born}}(K^- p)|_{\text{thr.}} = -T_{\text{Born}}(K^+ p)|_{\text{thr.}} = \frac{m_K}{f^2},$$  \hspace{1cm} (1a) $$T_{\text{Born}}(K^- n)|_{\text{thr.}} = -T_{\text{Born}}(K^+ n)|_{\text{thr.}} = \frac{m_K}{2f^2},$$  \hspace{1cm} (1b) 

where $f \simeq 93$ MeV is the pseudoscalar decay constant. These (Tomozawa-Weinberg, or TW) terms come from vector current interactions. They are therefore repulsive for $K^+ N$ and attractive for $K^- N$ channels. Scalar interactions, attractive in both channels, enter in next-to-leading order. They are proportional to the large kaon-nucleon sigma term $\sigma_{KN}$ of order $m_K^2$, but compete with repulsive $\omega^2$-terms of similar magnitude.

Based on these driving terms, in-medium calculations of kaon self energies have been performed using either chiral perturbation theory [4], or chiral SU(3) dynamics [5,6] in a non-perturbative coupled channels scheme [7]. In the latter calculation the $\Lambda(1405)$ is generated explicitly as a $KN$ $(I = 0)$ quasibound state which quickly dissolves, however, in nuclear matter and is thus of no relevance at higher density. The primary effect in both types of calculations is a splitting of the $K^+$ and $K^-$ masses in medium, a tendency which is indeed already suggested by the TW Born terms (1).

The results of refs. [4–6] include Pauli blocking which turns out to be the dominant medium effect (corrections from nucleon binding and Fermi motion are shown to be small [6]). On the other hand, short-range nucleon-nucleon correlations were also suggested to be important [8] and should therefore be incorporated. The aim of the present paper is to study effects of NN correlations on kaon masses together with Pauli corrections using the effective field method of ref. [1]. The formalism which is needed to adapt this method to the kaon-nuclear case is presented in section 2. Since Pauli corrections have already

* The scattering lengths $a$ are related to the threshold T-matrices by $4\pi(1 + m_K/M)a = T$ where $M$ is the nucleon mass.
been evaluated [5] solving a coupled channel Lippmann-Schwinger equation in medium, we can test the accuracy of the effective field method at that level (section 3) and then continue to include short-range NN correlations (section 4). In section 5 we discuss the relation between this approach and four-Fermi contact terms of the chiral effective Lagrangian used in chiral perturbation theory, and then close with concluding remarks.

2 The s-wave effective field for kaons in nuclear matter

Here we apply the approach of ref. [1] to the s-wave kaon interaction in nuclear matter. In spite of the similar nature of the kaon and the pion we want to give a short review of the arguments which led Ericson and Ericson to their famous optical potential. The reasons are the following:

(i) The kaons have a different isospin structure.
(ii) The charge exchange channels were not included in their first work and only to second order in the later works (to our knowledge), whereas we need to iterate those channels to all orders for our purposes.
(iii) The Pauli principle plays an important role in \( K^- p \) scattering inside nuclear matter [6]. Therefore it has to be included to all orders in the optical potential.

The starting point of ref. [1] is the observation that the meson field \( \phi(r) \) produced by an incident meson wave \( \exp(ik \cdot r) \) on a system of (static) nucleons is the sum of this incident wave and the scattered waves from all the individual nucleons. The scattered wave is simply the effective meson field \( \phi_i^{\text{eff}}(r_i) \) at the (fixed) scatterer \( i \) times the scattering amplitude \( F_i(\omega, k) \) times a spherical outgoing wave. Thus,

\[
\phi(r) = \exp(ik \cdot r) + \sum_i \frac{\exp(i\mu |r - r_i|)}{|r - r_i|} F_i(\omega, k) \phi_i^{\text{eff}}(r_i). \tag{2}
\]

Here \( \omega \) and \( k \) denote the energy and momentum of the meson, and \( \mu = (\omega^2 - m^2)^{1/2} \) with the meson rest mass \( m \). We have suppressed a phase \( \exp(-i\omega t) \), because we are only interested in stationary solutions.

Let us now identify the meson with a kaon. To include the charge-exchange channels of the kaon-nucleon system we recall that the scattering amplitude at scatterer \( i \) has the following isospin structure:

\[
F_i(\omega, k) = f_{i=0}(\omega, k) P_{i=0}^i + f_{i=1}(\omega, k) P_{i=1}^i, \tag{3}
\]
where \( P_i^j \) projects on kaon-nucleon states of definite isospin \( I \). With the (Pauli) isospin-matrices of the \( i \)'th scatterer (nucleon) \( \tau_i \) and kaon \( \tau_K \) they are given as:

\[
P_0^i = \frac{1 - \tau_K \cdot \tau_i}{4}, \quad P_1^i = \frac{3 + \tau_K \cdot \tau_i}{4}.
\] (4)

In a homogeneous medium all effective fields \( \phi_i^{\text{eff}}(r) \) at scatterers \( i \) with the same isospin quantum numbers are the same: \( P_i^1 \phi_i^{\text{eff}}(r) \) can therefore be written as \( P_i^1 \phi_i^{\text{eff}}(r) \), dropping the explicit reference to nucleon \( i \). We introduce the isospin densities

\[
\rho_I(r) \equiv \langle 0 | \sum_i P_i^I \delta^3(r - r_i) | 0 \rangle
\] (5)

of the nuclear ground state \( |0 \rangle \) and rewrite eq. (2) as:

\[
\phi(r) = \exp(i \mathbf{k} \cdot \mathbf{r}) + \sum_{I=0,1} \int \frac{d^3r'}{|r - r'|} \frac{\exp(i \mu |r - r'|)}{|r - r'|} f_I \rho_I(r') \phi_i^{\text{eff}}(r').
\] (6)

The Klein-Gordon equation for \( \phi(r) \) is then:

\[
\left( \omega^2 + \nabla^2 - m_K^2 \right) \phi(r) = -4\pi \sum_{I=0,1} f_I \rho_I(r) \phi_i^{\text{eff}}(r),
\] (7)

where \( m_K \) is now the free kaon mass.

To solve this equation we need the effective field. In complete analogy to eq. (3), the effective field at a scatterer \( i \) is given by the incident wave and the scattered wave from all other particles with the exception of \( i \) itself:

\[
\phi_i^{\text{eff}}(r_i) = \exp(i \mathbf{k} \cdot \mathbf{r}_i) + \sum_{j \neq i} \frac{\exp(i \mu |r_i - r_j|)}{|r_i - r_j|} \mathcal{F}_j(\omega, \mathbf{k}) \phi_j^{\text{eff}}(r_j).
\] (8)

We now multiply eq. (8) with \( P_i^j \delta^3(r - r_i) \) and sum over \( i \). Using the isospin structure of \( \mathcal{F}_j \) as in eq. (3), taking the nuclear ground state expectation value and dividing by a common factor \( \rho_I(r) \) one finds:

\[
\phi_1^{\text{eff}}(r) = \exp(i \mathbf{k} \cdot \mathbf{r}) + \sum_{I=0,1} \int \frac{d^3r'}{|r - r'|} \frac{\exp(i \mu |r - r'|)}{|r - r'|} \rho_I(r') [1 + C_{1I}(r, r')] f_I \phi_i^{\text{eff}}(r')
\]

\[
= \phi(r) + \sum_{I=0,1} \int \frac{d^3r'}{|r - r'|} \frac{\exp(i \mu |r - r'|)}{|r - r'|} \rho_I(r') C_{1I}(r, r') f_I \phi_i^{\text{eff}}(r').
\] (9)
Here we have introduced the NN pair correlation function $C_{I,I'}(r,r')$ which is related to the two-particle isospin density $\rho^{(2)}_{I,I'}(r,r')$ as follows:

$$\rho^{(2)}_{I,I'}(r,r') \equiv \langle 0 | \sum_i \sum_{j \neq i} P_i^I P_j^{I'} \delta^3(r-r_i) \delta^3(r'-r_j) | 0 \rangle = \rho_I(r) \rho_{I'}(r') [1 + C_{I,I'}(r,r')] .$$

Let us now consider the long-wavelength limit. In this limit, the pair correlation function is non-vanishing only in a region small compared to the kaon wavelength. The effective field at $r'$ in eq. (9) can then be replaced by its value at $r$. In the case of nuclear matter the densities are constant and the pair correlation function depends only on the distance $|r-r'|$ due to translation invariance. In this limit Eq. (9) simplifies to:

$$\phi^\text{eff}_I(r) = \phi(r) - \sum_{I'=0,1} \xi_{I,I'} \rho_{I'} f_{I'} \phi^\text{eff}_{I'}(r),$$

with constant densities $\rho_{I'}$, where we have defined the correlation parameter $\xi_{I,I'}$ by:

$$\xi_{I,I'}(\mu) = - \int d^3r' \exp(i\mu |r-r'|) C_{I,I'}(|r-r'|).$$

When multiplied with density, this generalizes the isospin-independent inverse correlation length $\langle 1/r \rangle$ of ref. [2]. Note that $\xi_{I,I'}$ is symmetric in $I,I'$ and does not depend on $r$ because of translation invariance, so we can always set $r=0$. Now the two coupled equations (11) can easily be solved. Inserting the effective fields $\phi^\text{eff}_I(r)$ into the wave equation (7) we obtain:

$$\left( \omega^2 + \nabla^2 - m_K^2 \right) \phi(r) = \Pi(\omega, \mathbf{k}) \phi(r),$$

with the self-energy, or optical potential,

$$\Pi(\omega, \mathbf{k}) = 2\omega U_{\text{opt}}(\omega, \mathbf{k}) = -4\pi \frac{f_0 \rho_0 + f_1 \rho_1 - (2\xi_{10} - \xi_{11} - \xi_{00}) f_0 f_1 \rho_0 \rho_1}{1 + f_0 \xi_{00} \rho_0 + f_1 \xi_{11} \rho_1 + (\xi_{00} \xi_{11} - \xi_{10}^2) f_0 f_1 \rho_0 \rho_1}.$$

This is the generalized $s$-wave optical potential of Ericson and Ericson. It includes kaon-nucleon charge exchange and two-body NN correlations to all orders. It has been derived here for infinitely heavy, static nucleons. In actual calculations we multiply the usual kinematic factor $\sqrt{s}/M$ to each of the
amplitudes \( f_1 \), where \( \sqrt{s} \) is the kaon-nucleon center-of-mass energy and \( M \) the nucleon mass:

\[
f_1 \rightarrow \frac{\sqrt{s}}{M} f_1. \tag{15}
\]

The effective kaon mass \( m_K^\star \) is defined by the energy \( \omega \) of a kaon at rest in matter \( (k = 0, \phi(r) = \text{const.}) \). The corresponding (off-shell) s-wave scattering amplitudes are expressed in terms of the energy dependent (off-shell) scattering lengths \( a(\omega) \). Their threshold values \( a(\omega = m_K) \) coincide with the physical scattering lengths. In the sector with strangeness \( S = -1 \):

\[
\begin{align*}
f_0^{S=-1}(\omega, k = 0) &= 2a(K^- p \rightarrow K^- p) - a(K^- n \rightarrow K^- n), \\
f_1^{S=-1}(\omega, k = 0) &= a(K^- n \rightarrow K^- n).
\end{align*} \tag{16}
\]

Analogous relations hold in the \( S = +1 \) sector with the \( K^+ \)N scattering lengths:

\[
\begin{align*}
f_0^{S=+1}(\omega, k = 0) &= 2a(K^+ n \rightarrow K^+ n) - a(K^+ p \rightarrow K^+ p), \\
f_1^{S=+1}(\omega, k = 0) &= a(K^+ p \rightarrow K^+ p).
\end{align*} \tag{17}
\]

In order to calculate the off-shell scattering lengths we use the coupled channels approach of ref. [7] which is based on chiral dynamics and describes all available low-energy kaon-nucleon data together with photoproduction channels.

Note that the energy of a \( K^- \) in matter is generally complex, its imaginary part being related to processes in which \( KN \) decays into pion-hyperon channels. But as we will see, this imaginary part is always small compared with the real part. Therefore, the effective kaon mass \( (m_K^\star \equiv \text{Re} \omega) \) is determined to a good approximation through the simpler relation:

\[
(m_K^\star)^2 - m_K^2 = 2m_K^\star \text{ Re } U_{\text{opt}}(m_K^\star, k = 0). \tag{18}
\]

The effective kaon decay width \( \Gamma = -2 \text{ Im } \omega \), can then be approximated by:

\[
\Gamma = -2 \text{ Im } U_{\text{opt}}(m_K^\star, k = 0). \tag{19}
\]

### 3 Pauli principle effects

In this section we calculate the average inverse correlation length for a Fermi-gas. In this model the nucleons do not interact and the correlation length is
entirely due to the Pauli exclusion principle. The nuclear ground state $|0\rangle$ is a Slater determinant of plane-waves. All proton (neutron) levels are filled up to the Fermi momentum $p_F^p$ ($p_F^n$). The nucleon states are characterized as usual by their momentum $p$, spin $s = \pm \frac{1}{2}$ and isospin $t = \pm \frac{1}{2}$ for (p,n). In the following $\alpha$ and $\beta$ denote occupied states.

The isospin densities $\rho_I$ defined in eq. (5) are given by:

$$
\rho_0 = \frac{1}{4} (\rho_p + \rho_n) - \frac{\tau_K^3}{4} (\rho_p - \rho_n) \\
\rho_1 = \frac{3}{4} (\rho_p + \rho_n) + \frac{\tau_K^3}{4} (\rho_p - \rho_n),
$$

where $\rho_p = (p_F^p)^3/3\pi^2$ and $\rho_n = (p_F^n)^3/3\pi^2$ are the proton and neutron densities, and $\tau_K^3 = \pm 1$ for $K^+$ or $K^-$, respectively.

In the Fermi gas model the two-particle isospin density (10) becomes

$$
\rho_{I,I'}(r, r') = \sum_{\alpha\beta} \langle \alpha \beta | P_I^1 P_{I'}^2 \delta^3(r - r_1) \delta^3(r' - r_2) | \alpha \beta \rangle \rho_I \rho_{I'} [1 + C_{I,I'}^{Pauli}(|r - r'|)]
$$

where the single particle matrix elements are understood to involve integrals over $r_1, r_2$. The result for the Pauli correlation function is:

$$
C_{I,I'}^{Pauli}(r) = -\frac{1}{2} \sum_{t,t'} c_t(r) c_{t'}(r) \frac{\langle t | P_I | t' \rangle \langle t' | P_{I'} | t \rangle}{\rho_I \rho_{I'}}
$$

where

$$
c_t(r) = \frac{3j_1(p_F^t r)}{p_F^t r} \rho_t
$$

with the spherical Bessel function $j_1$, and the indices $t, t'$ distinguish between protons and neutrons.

Inserting this into eq. (12) we determine the correlation parameters $\xi_{I,I'}$ for the following cases:

7
i) $\mathbf{K}^+$ or $\mathbf{K}^-$ in symmetric nuclear matter

Here we have $p_F^p = p_F^n = p_F$, $\rho_p + \rho_n = \rho$ and $\rho_0 = \rho_1/3 = \rho/4$. One finds

$$\xi \equiv \xi_{00} = 3 \xi_{11} = \frac{9}{p_F^2} F(0, \mu, p_F), \quad \xi_{10} = \xi_{01} = 0,$$

where we have introduced

$$F(R, \mu, p_F) = 4\pi \int_R^\infty \frac{dr \exp(i\mu r) j_1^2(p_F r)}{r}$$

and recall that $\mu(\omega) = \sqrt{\omega^2 - m_K^2}$. The optical potential (14) for this case becomes

$$2\omega U_{\text{opt}} = -4\pi \left[ \frac{1}{4} \frac{f_0\rho}{1 + \frac{2}{3}f_0\rho} + \frac{3}{4} \frac{f_1\rho}{1 + \frac{4}{3}f_1\rho} \right].$$

To leading order in $\xi$ one finds

$$2\omega U_{\text{opt}} = -4\pi \mathcal{F}_{\text{eff}} \rho$$

with the effective amplitude (including the $\sqrt{s}/M$ factors):

$$\mathcal{F}_{\text{eff}} = \frac{\sqrt{s}}{4M}(f_0 + 3f_1) - \frac{s}{16M^2}(f_0^2 + 3f_1^2)\xi\rho + O(\xi^2).$$

At threshold ($\omega = m_K$) we have $\sqrt{s}/M = 1 + m_K/M$, $\mu = 0$, and $\xi\rho$ reduces to the familiar inverse correlation length [1,2]:

$$[\xi_{00}\rho_0]_{\omega = m_K}^{\mu = 0} = \left( \frac{1}{\rho} \right) = -\int d^3r \frac{C_{\text{Pauli}}(r)}{r} \rho_0 = \frac{3p_F}{2\pi}.$$

ii) $\mathbf{K}^+$ in neutron matter

Here we have $p_F^p = 0$, $p_F^n = p_F$ and $\rho_0 = \rho_1 = \rho_n/2$. One finds

$$\xi \equiv \xi_{00} = \xi_{11} = \xi_{10} = \xi_{01} = \frac{9}{2p_F^2} F(0, \mu, p_F).$$
The K$^+$ self-energy in neutron matter becomes

$$\Pi_n = -4\pi \frac{\mathcal{F}\rho_n}{1 + \mathcal{F}\xi\rho_n}$$  \hspace{1cm} (31)

with

$$\mathcal{F} = \frac{\sqrt{s}}{2M}(f_0 + f_1)$$  \hspace{1cm} (32)

iii) K$^-$ in neutron matter

In this case only the $I = 1$ amplitude contributes and we have (with $p_F^0 = p_F$, $\rho_0 = 0$, $\rho_1 = \rho_n$):

$$\xi \equiv \xi_{11} = \frac{9}{2p_F^2} F(0, \mu, p_F).$$  \hspace{1cm} (33)

The K$^-$ self-energy in neutron matter has the same form as $\Pi_n$ of eq. (31), but now with

$$\mathcal{F} = \frac{\sqrt{s}}{M} f_1.$$  \hspace{1cm} (34)

This completes our exposition of Pauli exchange effects on the kaon self-energies. Using those self-energies we have solved the dispersion relations for the effective kaon masses (and decay widths) in matter. The results for K$^+$ and K$^-$ in symmetric nuclear matter are shown in Fig. 1, those for neutron matter in Fig. 2. Turning on the Pauli correlations moves the K$^+$ and K$^-$ masses upward from the values they would have with just the leading order self-energies $\Pi = -4\pi(\sqrt{s}/M)[f_0\rho_0 + f_1\rho_1]$. Note that with the TW Born terms (1) alone we would have

$$\Pi_{\text{Born}}^\pm = \pm \frac{\omega}{f^2} \left[ \frac{3}{4}(\rho_p + \rho_n) + \frac{1}{4}(\rho_p - \rho_n) \right]$$  \hspace{1cm} (35)

for K$^+$ or K$^-$, respectively. This already accounts for the qualitative feature of K$^+$/K$^-$ mass splitting in matter. Iterations of the Born amplitudes as in [3] reduce the repulsive K$^+$N and enhance the attractive K$^-$N interactions, leading to the patterns seen in Figs. 1 and 2. The Pauli correlations act repulsive on both the K$^+$ and K$^-$ branches.

In Fig. 1 and 2 we also compare the present treatment of Pauli correlations to the results of ref. [3] where these effects have been treated by solving an in-medium coupled channels Lippmann-Schwinger equation. The agreement
between the present effective field approach and the previous full coupled-channels result is quite remarkable, especially in view of the fact that the Lippmann-Schwinger coupled channels calculation does not make use of the fixed scatterer approximation.

4 Short-range correlations

The Pauli exchange correlations discussed in the previous section give an impression of the long-range part of the nucleon-nucleon pair correlation function. As a next step we study the influence of short-range repulsive NN correlations. To start with, consider a schematic hard core. Assume that the two-particle density \( n(r) \) vanishes at distances smaller than \( R \) and that the pair correlation function is therefore simply

\[
C_{I, I'}(r) = -1 \quad \text{for} \quad r \leq R
\]

(36)

independent of spin and isospin.

It is instructive to compare this with the Pauli pair correlation functions at zero distance \( (r = r') \). For example, in the symmetric nuclear matter case they are:

\[
C^{\text{Pauli}}_{00}(0) = -1, \quad C^{\text{Pauli}}_{10}(0) = C^{\text{Pauli}}_{01}(0) = 0, \quad C^{\text{Pauli}}_{11}(0) = -\frac{1}{3}.
\]

(37)

Therefore the main contributions of the short-range correlations to the pair correlation functions are in the sectors \((I, I') = (0, 1), (1, 0), (1, 1)\). In the sector \((I, I') = (0, 0)\) the Pauli principle already prevents the two nucleons to come close to each other, so the repulsive core is nearly inactive.

For distances larger than \( R \) we assume that the pair correlation function is completely described by Pauli exchange effects. In this simple model the correlation parameter \( \xi_{I, I'} \) is given by:

\[
\xi_{I, I'}(R, \omega, p_F) = G(R, \mu, p_F) + 9p_F^{-2}(1 - I - I' + \frac{4}{3} I I') F(R, \mu, p_F)
\]

(38a)

for symmetric nuclear matter and by

\[
\xi_{I, I'}(R, \omega, p_F) = G(R, \mu, p_F) + \frac{9}{2}p_F^{-2} F(R, \mu, p_F)
\]

(38b)
for K$^+$ in neutron matter (for the K$^-$ in neutron matter, multiply $F$ by $\delta_{I\delta I'}$).

Here the function

$$G(R, \mu, p_F) = 4\pi \int_0^R dr r \exp(i \mu r) = \frac{4\pi}{\mu^2} [(1 - i \mu R) \exp(i \mu R) - 1]$$

(39)

describes the short-range correlations (36). Later we will discuss a different parameterization of the short-range part, one which approximates the realistic G-matrix derived from Brueckner Hartree Fock calculations.

Inserting the correlation parameters (38a,b) into the optical potential (14), the resulting solutions of the kaon dispersion relations (18,19) are shown in Fig. 3 (symmetric nuclear matter) and in Fig. 4 (neutron matter). For comparison we show in these Figures also the effective kaon masses calculated in the previous section with Pauli blocking only ($R = 0$). The effect of the short-range correlations starts out small at low densities, becoming sizeable only at densities larger than normal nuclear matter density, $\rho_0 = 0.17$ fm$^3$. The short-range correlations increase both kaon and antikaon masses as compared with the results including Pauli blocking only.

A better parameterization of the pair correlation function in symmetric nuclear matter is given in [11]. There the short-range part is chosen to approximate the Brueckner G-Matrix and found to be quite well described by a spherical Bessel function:

$$C_{SR}^{I, I'}(\omega, r) = -j_0(q_c r); \quad q_c = 3.93 \text{ fm}^{-1} \approx m_\omega,$$

(40)

where $m_\omega$ is the mass of the $\omega$-meson. To incorporate the Pauli part of the pair correlation function (22) we assume that these two correlations contribute multiplicatively in the two-particle density:

$$\rho_1^{(2)}(r, r') = \rho_1(r) \rho_1(r') \left[ 1 + C_{I, I'}^{\text{Pauli}}(|r - r'|) \right] \left[ 1 + C_{I, I'}^{\text{SR}}(|r - r'|) \right].$$

(41)

Therefore the full pair correlation function is

$$C_{I, I'}(r) = C_{I, I'}^{\text{Pauli}}(r) + C_{I, I'}^{\text{SR}}(r) + C_{I, I'}^{\text{Pauli}}(r) C_{I, I'}^{\text{SR}}(r)$$

(42)

in this model. The effective kaon masses, calculated with these pair correlation functions, are also shown in Fig. 3 and differ only marginally from the ones calculated with the simpler ansatz (36).

While the repulsive effect of short range correlations is moderate, we expect that it will be important for the issue of kaon condensation in neutron stars.
unless certain nonperturbative effects discussed below (which are not included in the present treatment) bring large corrections. Kaon condensation can occur once the effective kaon mass gets equal to or smaller than the electron chemical potential \[\mu\]. Without the short-range correlations this matching will occur around \(\rho \approx 3\rho_0\) if one uses the electron chemical potential employed in [1]. With the inclusion of the short-range correlations this matching will be shifted to higher density \(\rho > 3\rho_0\) (see Fig. 5) or if hyperons are present in the dense matter as suggested in some recent papers [4,7], may not even take place. In the latter case, the electron chemical potential can drop at densities \(\rho > 3\rho_0\) due to the appearance of \(\Sigma^-\)– and \(\Xi^-\)–hyperons in the neutron star matter. This matter is currently under debate and deserves to be clarified.

5 Comparison with higher-order chiral perturbation and mean-field calculations

We shall now compare the present calculation which, as we should stress, is very tightly constrained [1] by the full ensemble of kaon-nucleon data, to the chiral perturbation calculation of Lee et al. [12] and the mean-field treatment of [14], both of which are less constrained by on-shell data. Some features between them overlap and some do not. Understanding the similarities and differences between these approaches will be crucial in confronting data on the properties of kaons in dense matter coming from extrapolations of kaonic atom data as well as the GSI experiments of the KaoS and FOPI collaborations currently in progress. It will also be important for establishing whether or not kaons can condense at low enough density as to be relevant to “nuclear star” matter [13].

To streamline the discussion, we first recall the key ingredients of the present treatment and of ref. [12]. In the present paper (call it A), the potential for kaon-nuclear interactions is constructed with “irreducible graphs” calculated to \(\mathcal{O}(Q^2)\) in chiral perturbation theory and then inserted as the kernel into the Lippmann-Schwinger equation for both free-space and in-medium kaon-nucleon interactions. The solution of the integral equation correctly accounts for the infrared singular “reducible” graphs to all orders of the chiral expansion in a way consistent with the general strategy of chiral perturbation theory in the presence of bound states or resonances. Pauli correlations are readily taken into account at the level of solving the Lippmann-Schwinger equation or as, in this paper, in the effective field method. While these two approaches are found to give the same result, the latter has the advantage that the short-range correlations can also be simply implemented.

In ref. [12], the in-medium kaon-nuclear interaction is treated to one-loop order (or \(\mathcal{O}(Q^3)\)) with the quasi-bound \(\Lambda(1405)\) introduced as a local interpolating
field. While the Λ(1405) as an elementary local field may not be accurate enough for K⁻-proton scattering near threshold, it can however be an adequate way of introducing the Λ(1405) degree of freedom for in-medium physics (a discussion on this point is found in [15] where a chiral expansion to \( \mathcal{O}(Q^2) \) is made. To be more predictive, one would have to do an \( \mathcal{O}(Q^3) \) calculation which is not feasible at the moment). To the chiral order considered, the Λ(1405) can contribute in two ways: one, in the off-shell elementary kaon-nucleon scattering amplitude and two, in an effective four-Fermi interaction involving both nucleon and Λ(1405). In this approach (that we shall refer to as B), a certain class of two-body correlations including Pauli-principle effects are included but the short-range correlations calculated in A are not.

Let us now compare quantitatively the two (A and B) approaches. For in-medium K⁺ properties, the results are quite similar: the effective kaon mass \( m_K^\ast \) scales in density at about the same rate up to the normal matter density and increases somewhat more rapidly in A than in B. The reason for this similarity is simple: since the Λ(1405) does not figure importantly in this channel, its ramifications in A (i.e., the proper treatment of the binding mechanism of the Λ(1405)) and in B (i.e., the four-Fermi interaction involving Λ(1405)) are unimportant.

The situation with K⁻-nuclear interaction is, however, considerably different. This is more pronounced in symmetric nuclear matter than neutron matter. The approach A in symmetric nuclear matter predicts a decrease in the K⁻ mass (or equivalently the attraction in the kaon-nuclear potential) of about 120 MeV whereas in the approach B, the attraction is sizeable greater, say, about 200 MeV. This large attraction in B is obtained by fixing by fiat a parameter associated with four-Fermi interactions involving the Λ(1405) to agree with the attraction suggested by Friedman, Gal and Batty [16] from their analysis of kaonic atoms. It is not a prediction of the theory. The resulting attraction at densities greater than that of nuclear matter found in B is thus simply explained by this additional term. The four-Fermi interaction in question corresponds to an \( \mathcal{O}(Q^3) \) irreducible graph in the chiral expansion and is not included in the approach A. One can think of the short-range correlations calculated in A as a four-Fermi interaction involving nucleons only, but in a naive chiral counting it would appear at \( \mathcal{O}(Q^5) \). Since in B not all the four-Fermi interaction terms that can contribute are included, the additional attraction in B over and above that accounted in A can be thought of as the net effect of the irreducible \( \mathcal{O}(Q^3) \) terms not taken into account in A. That it is an additional attraction is a consequence of the fitting. As such, it will be subject to further empirical constraints.

The mean-field approach of [14] exploits the fact that in medium, the Λ(1405) plays an insignificant role (due to the dissociation of the Λ(1405) in the approach A or due to a weak coupling of the Λ(1405) with no kinematic en-
hancement near threshold in the approach B). As such, this approach is not
constrained by the on-shell kaon-nucleon data as in the cases of A and B.
Limiting to the $O(Q^2)$ chiral order and ignoring the role of the $\Lambda(1405)$, all
dominant in-medium higher-order correlation effects (particularly, the many-
Fermi interactions in the scalar channel) are then subsumed into the scaling
of the effective pseudoscalar decay constant $f_\pi^\star$. This leads to an effective
constituent quark – or mean-field – counting that predicts that the effective
scalar and vector potentials in kaon-nuclear interactions – the kaon containing
only one valence chiral quark – are 1/3 of the corresponding nucleon-nuclear
interactions that involve three light quarks. This description which correctly
accounts for the nucleon mean field in nuclear matter provides a simple expla-
nation for the large ($\sim 200$ MeV) attraction for kaonic atoms of ref.[16], and
seems to be consistent with the (preliminary) KaoS and FOPI data of kaon
properties in dense nuclear medium [18].

Whether or not such an additional higher order chiral contribution with a net
attraction is unambiguously needed for understanding in-medium kaon proper-
ties should be settled by careful analyses of the ongoing experiments on kaons
in dense medium. These experiments will also provide a crucial constraint on
the role of kaons in neutron-rich medium and determine in particular whether
kaon condensation does actually figure in the structure of compact stars. If
such an attraction is established, it would help in going beyond the present
treatment in a systematic chiral perturbation calculation that can account
simultaneously for kaon-nucleon and kaon-nuclear interactions.

---

This can be understood as follows. N-Fermi interactions increase the chiral or-
der as $Q^{N/2}$. Thus higher-N Fermi interactions are generally suppressed. There is,
however, one possible exception: when the interaction is mediated by a scalar me-
son, then the corresponding N-Fermi interactions can give a coherent effect when
the effective mass of the scalar meson becomes light as it is expected to happen in
dense matter (or hot matter near the chiral phase transition). Such a scalar-induced
higher-Fermi interaction then acts like a scalar tadpole on hadrons and can give rise
to a decreasing hadron mass or equivalently to the $f_\pi^\star$. [17]
Acknowledgements

One of the authors (MR) acknowledges the support of the Humboldt Foundation through its “Forschungspreis” and the generous hospitality of the Institut für Theoretische Physik, Technische Universität München where this work was done. He is grateful for illuminating comments from Gerry Brown and C.-H. Lee.

One of us (WW) gratefully acknowledges support as a recipient of a Humboldt-Mutis award. He thanks Enlogio Oset for his kind hospitality and discussions at the University of Valencia where this paper was completed.

Two of us (TW and WW) thank Avraham Gal for helpful discussions and comments.
Figure Captions

**Fig. 1:** Effective $K^-$ and $K^+$ masses in symmetric nuclear matter as a function of the density $\rho = \rho_p + \rho_n$, in units of nuclear matter density $\rho_0 = 0.17$ fm$^{-3}$. The full curves are the results of this work, including only Pauli correlations. The dashed lines are the results of the in-medium coupled channels calculation of ref. [5]. The dotted lines show results without Pauli correlations. The dashed–dotted lines show for reference the effective masses obtained with the Tomozawa-Weinberg Born terms (1) only.

**Fig. 2:** Effective $K^-$ and $K^+$ masses in neutron matter as a function of the neutron density $\rho_n$. Notations as in Fig. 1.

**Fig. 3:** Effective $K^-$ and $K^+$ masses in symmetric nuclear matter as a function of density $\rho$. Dashed lines are the results including only Pauli blocking, full lines include also the short-range correlation functions (36). The dashed–dotted lines are calculated with the short-range correlation function (40) for comparison. The lower section shows the $K^-$ width in matter (scaled by a factor 100).

**Fig. 4:** Effective $K^-$ and $K^+$ masses in neutron matter as a function of the neutron density. Dashed lines are the results including only Pauli blocking and the full curves include also short-range correlations as in eq. (36). The lower section shows the $K^-$ width in matter (scaled by a factor 100).

**Fig. 5:** The effective $K^-$ mass in neutron star matter (full line: with short range correlations; dashed line: without short range correlations) versus the electron chemical potential (dotted line). For demonstration we have used the chemical potential from ref. [15] calculated with one of their parametrizations of the symmetry energy (case “$F(u) = u$”).

References

[1] M. Ericson and T. E. O. Ericson, *Ann. Phys.* **36** (1966) 323; M. Krell and T. E. O. Ericson, *Nucl. Phys.* **B11** (1969) 521

[2] T. E. O. Ericson and W. Weise, *Pions and Nuclei* (Clarendon Press, Oxford, 1988)

[3] T. Yamazaki et al., *Z. Physik* **A355** (1996) 219
[4] G.E. Brown, K. Kubodera, M. Rho, V. Thorsson, *Phys. Lett.* B291 (1992) 355; G.E. Brown, C.H. Lee, M. Rho, V. Thorsson, *Nucl. Phys.* A567 (1994) 937

[5] T. Waas, N. Kaiser and W. Weise, *Phys. Lett.* B379 (1996) 34.

[6] T. Waas, N. Kaiser and W. Weise, *Phys. Lett.* B365 (1996) 12.

[7] N. Kaiser, P.B. Siegel and W. Weise, *Nucl. Phys.* A594 (1995) 325; N. Kaiser, T. Waas and W. Weise, *Nucl. Phys.* A (1996), in print.

[8] V.R. Pandharipande, C.J. Pethick and V. Thorsson, *Phys. Rev. Letters* 75 (1995) 4567

[9] J. Schaffner and I.N. Mishustin, *Phys. Rev.* C53 (1996) 1416.

[10] M. Prakash, I. Bombaci, Manju Prakash, P.J. Ellis, J.M. Lattimer and R. Knorren, *nucl-th/9603042*.

[11] G.E. Brown, S.O. Bäckman, E. Oset and W. Weise, *Nucl. Phys.* A286 (1977) 191.

[12] C.-H. Lee, G.E. Brown, D.-P. Min and M. Rho, *Nucl. Phys.* A385 (1995) 401.

[13] H.A. Bethe and G.E. Brown, *Astrophys. J* 445 (1995) L125.

[14] G.E. Brown and M. Rho, *Nucl. Phys.* A596 (1996) 503.

[15] C.-H. Lee, D.-P. Min and M. Rho, *Nucl. Phys.* A602 (1996) 334.

[16] E. Friedman, A. Gal and C.J. Batty, *Nucl. Phys.* A579 (1994) 518.

[17] G.E. Brown and M. Rho, *Phys. Rev. Lett.* 66 (1991) 2720.

[18] G.E. Brown, private communication (July 1996).
Figure 3

Figure 4
Figure 5