PARITY ODD BUBBLES IN HOT QCD

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We consider the topological susceptibility for an SU(N) gauge theory in the limit of a large number of colors, $N \to \infty$. At nonzero temperature, the behavior of the topological susceptibility depends upon the order of the deconfining phase transition. The most interesting possibility is if the deconfining transition, at $T = T_d$, is of second order. Then we argue that Witten's relation implies that the topological susceptibility vanishes in a calculable fashion at $T_d$. As noted by Witten, this implies that for sufficiently light quark masses, metastable states which act like regions of nonzero $\theta$ — parity odd bubbles — can arise at temperatures just below $T_d$. Experimentally, parity odd bubbles have dramatic signatures: the $\eta'$ meson, and especially the $\eta$ meson, become light, and are copiously produced.

Further, in parity odd bubbles, processes which are normally forbidden, such as $\eta \to \pi^0\pi^0$, are allowed. The most direct way to detect parity violation is by measuring a parity odd global asymmetry for charged pions, which we define.

1 Introduction

In this paper we give a pedagogical introduction to recent work of ours. We consider an SU(N) gauge theory in the limit of a large number of colors, $N \to \infty$. This is, of course, a familiar limit. We use the large $N$ expansion to investigate the behavior of the theory at nonzero temperature, especially for the topological susceptibility. The results depend crucially upon the order of the deconfining phase transition; if it is first order, nothing very interesting happens. If the deconfining transition is of second order, however, the topological susceptibility vanishes in a calculable fashion.

This implies that metastable states, which act like regions with nonzero $\theta$, can appear. Parity is spontaneously broken in such parity odd bubbles, and produces novel physics. The $\eta$ meson becomes very light, at most a few
hundred $MeV$, and so is easily produced. As parity is broken, the $\eta$ can decay into two pions, instead of the usual three. We also propose a global variable which can be used to measure an asymmetry in parity.

2 Large $N$

Holding the number of fermion flavors fixed as $N \to \infty$, the large $N$ limit is very much a gluonic limit, as the $\sim N^2$ gluons totally dominate the $\sim N$ quarks. This is the basis for most of the conclusions which we can draw at large $N$: given what happens to the gluons, what happens to the quarks follows almost immediately.

The standard assumptions at large $N$ is that the physics is like that for $N = 3$: confinement occurs, so at low temperature we can speak entirely of mesons and glueballs. Their masses are assumed to be of order one as $N \to \infty$; as usually occurs in any large $N$ limit, interactions between either mesons and/or glueballs are suppressed by powers of $1/N$.

It is also natural to assume that the degeneracy of mesons and glueballs is of order one. This is, after all, what we mean by confinement: all trace of the color indices disappear, leaving bound states which are characterized only by spin, parity, etc. Thus in the low temperature phase at a temperature $T$, since each meson or glueball has a free energy which is $\sim N^0 T^4$, the total free energy is also of order one. In contrast, in the deconfined phase at high temperature, the free energy is of order $\sim N^2$. As pointed out by Thorn, this allows us to use the free energy itself as an order parameter for the phase transition: the transition at a temperature $T_d$, occurs when the term in the free energy $\sim N^2$ turns on.

Rigorously, the true order parameter for the deconfining phase transition is associated with the spontaneous breaking of the global $Z(N)$ symmetry above $T_d$; this symmetry becomes $O(2)$ as $N \to \infty$.

We also make a further assumption, namely that any other phase transitions occur at the same time as deconfinement, at $T_d$. We have no proof of this statement, although we suspect that a proof can be constructed in the limit of large $N$. Nevertheless, it strains credulity to image that at the point where this huge increase in the free energy occurs, that that alone doesn’t force any other phase transitions in the theory.

In particular, assume that we couple massless quarks to the gluons. At zero temperature, it is known that the quarks’ chiral symmetry must be broken in the familiar pattern, to a diagonal subgroup of flavor. Then we assume that the chiral symmetry is restored at a temperature $T_\chi$, with $T_\chi = T_d$.

We take the scale of the deconfining transition to be the same as for the
glueball masses; thus $T_d$ is of order one as $N \to \infty$. This turns out to be a remarkably powerful assumption. Consider the large $N$ limit of a theory without confinement, such as a $N$-component vector with coupling $g^2$, holding $g^2 N$ fixed as $N \to \infty$. We assume that the masses of the fields are of order one. Then the only way for a transition to occur in what is, after all, free field theory, is to go to temperatures which grow with $N$; a simple one loop estimate gives $T_\chi \sim 1/\sqrt{g^2} \sim \sqrt{N}$ (this is also the scale of $f_\pi$, which is natural). What happens in a confining theory is far more dramatic: the transition occurs at temperatures of order one, not $\sim \sqrt{N}$. This implies that the hadronic phase is “cold” at large $N$: interactions are small, so that effects from the thermal bath, such as the loss of manifest Lorentz invariance, can be neglected.

The crucial thing which we do not know about the large $N$ limit is the order of the deconfining phase transition. (The effect of quarks can be neglected, since the gluons dominate the free energy above $T_d$; this will be elaborated later.) For this we must look to the lattice, which as always provides the true intellectual basis for our understanding.

In the early days of Monte Carlo simulations on the lattice, it was generally agreed that the deconfining phase transition is of first order when $N = 4$. It is not clear, however, if these simulations are definitive. In particular, they were done at $n_t = 4$, where $n_t$ is the number of steps in the imaginary time direction. For the standard Wilson action, at this value of $n_t$ there is a bulk transition close to the finite temperature transition. The bulk transition can be avoided by going to larger values of $n_t$, but at the time this was computationally difficult to do. Recently, however, Ohta and Wingate have computed for $N = 4$ and $n_t = 6$; they find that the strong first order transition at $n_t = 4$ is gone for $n_t = 6$. Of course, to really establish that there is a true second order phase transition is a difficult matter, requiring lengthy study. But these results do suggest that it may be hasty to conclude from $n_t = 4$ that the deconfining phase transition is of first order.

There are also results on the large $N$ limit of gauge theories on the lattice. Such reduced models appear to reliably predict the ratio of the critical temperature to the square root of the string tension. They predict a first order transition, but only under the technical assumption that the coupling between spacelike plaquettes can be neglected. It is not apparent to us how strong this assumption is.

Previously, Pisarski and Tytgat suggested that the large $N$ deconfining phase transition is of second order. Their argument was rudimentary: the easiest way to understand why the deconfining transition is weakly first order for $N = 3$ is if the large $N$ expansion is a good approximation, and if the transition is of second order for $N = \infty$. Then the cubic invariant, which
drives the transition first order at $N = 3$, is suppressed by $\sim 1/N$. Of course the first assumption is rather strong: perhaps the large $N$ expansion is not a good guide to thermodynamic properties.

In the following we assume that the deconfining transition is of second order for all $N \geq 4$, but comment upon how our results change if the transition is of first order.

The principal object we are interested in is the topological susceptibility. From the topological charge density,

$$Q(x) = \frac{g^2}{32\pi^2}tr(G_{\alpha\beta}\tilde{G}^{\alpha\beta}) = \partial_\alpha K^\alpha. \quad (1)$$

The current $K^\alpha$ is gauge dependent. The topological susceptibility is the two point function of $Q$,

$$\lambda_{YM}(T) \equiv \partial^2 F(\theta, T)/\partial \theta^2 = \int d^4x Q(x)Q(0) ; \quad (2)$$

$F(\theta, T)$ is the free energy, and the $\theta$ parameter is conjugate to $Q$. At zero temperature, the free energy reduces to the energy, $F(\theta, 0) = E(\theta)$.

Since $Q$ is a total derivative, $\lambda_{YM}(T)$ vanishes order by order in perturbation theory. It receives contributions entirely from nonperturbative effects, such as instantons. The action of a single instanton with fixed scale size is $8\pi^2/g^2$. In the large $N$ limit, $g^2N$ is held fixed as $N \to \infty$, so the contribution of an instanton to the topological susceptibility is $\lambda_{YM}(T) \sim e^{\exp(-aN)}$, with $a = 8\pi^2/(g^2N)$. Thus the contribution of instantons vanishes exponentially in the large $N$ limit. This naive argument assumes that the integral over instanton scale size is well behaved. This is certainly true in the limit of high temperature; then the theory is weakly coupled, and instantons are suppressed by the Debye screening of electric fluctuations. This naive picture was verified, at all temperatures, by Affleck in a soluble asymptotically free theory, the $CP^N$ model in 1 + 1 dimensions.

Thus at large $N$, in the deconfined phase the topological susceptibility is exponentially small in $1/N$, and so essentially vanishes. At zero temperature, Witten suggested that instead of semiclassical fluctuations, that quantum fluctuations generate a nonzero value for $\lambda_{YM}(0) \sim N^0$. It is natural to assume that the topological susceptibility is $\sim N^0$ throughout the deconfined phase, and changes to $\sim e^{\exp(-aN)}$ at the deconfining phase transition. This was previously argued by Affleck and by Davis and Matheson.

How it changes depends upon the order of the phase transition. If the deconfining transition is of first order, then as the hadronic phase is “cold”, the most natural possibility is that the topological susceptibility is essentially
constant in the hadronic phase, and changes discontinuously to zero at $T_d$. Recent lattice results in full QCD with four flavors indicate a sharp drop for the topological susceptibility across the phase transition, and thus seem to support this conjecture.

If the deconfining transition is of second order, a more extended analysis is necessary. Generalizing the results of Witten and Veneziano to nonzero temperature, and using results on the anomalous couplings of mesons, we find that the free energy depends upon $\theta$ as

$$F(\theta, T) \sim (1 + c\theta^2)(T_d - T)^{2-\alpha}.$$  \hspace{1cm} (3)

Here $\alpha$ is the critical exponent for the deconfining phase transition, $\alpha \approx -0.013$. We then use Witten’s formula for the $\eta'$ mass to conclude that the $\eta'$ mass vanishes at $T_d$,

$$m_{\eta'}^2(T) = 4N_f \frac{f_{\pi}^2(T)}{f_{\pi}(T)} \lambda_{YM}(T) \sim (T_d - T)^{1-\alpha}.$$  \hspace{1cm} (4)

Implicitly, we have used the fact that the hadronic phase is cold, so that zero temperature formulas, such as (4), generalize trivially.

3 Parity odd bubbles

We now use this result on the $\eta'$ mass to investigate the nature of the theory in the hadronic phase, just below $T_d$. At zero temperature, a successful phenomenology of the $\eta'$ was developed with a chiral lagrangian formalism. For $N_f$ flavors, a $U(N_f)$ matrix $U$ is introduced, satisfying $U^\dagger U = 1$. $U$ describes the $N_f^2 - 1$ pions and the $\eta'$. The effects of the anomaly are represented solely by a mass term for the $\eta'$, $(tr \ln U)^2$.

This is stark contrast to how effects of the anomaly due to instantons are included. Consider a linear sigma model with a field $\Phi$. Then the effects of the anomaly enter exclusively through a term $\sim det(\Phi)$. As in the nonlinear sigma model, when the chiral symmetry is spontaneously broken, this term generates a mass for the $\eta'$. However, in the nonlinear sigma model, at large $N$, there is only a mass term for the $\eta'$; four point interactions between $\eta'$'s are induced by the anomaly, but are suppressed by higher powers of $1/N^2$. As emphasized by Witten, a term $\sim det(\Phi)$ violates this large $N$ counting. This is subject to the trivial qualification that $N_f \geq 4$, so that there are quartic interactions between the $\eta'$'s.

For $U$ fields which are constant in spacetime, the potential for $U$ is

$$V(U) = \frac{f_{\pi}^2}{2} (tr (M(U + U^\dagger)) - a(tr \ln U - \theta)^2) ;$$  \hspace{1cm} (5)
The pion decay constant $f_\pi = 93$ MeV, while $M$ is the quark mass matrix. When $M = 0$, $m^2_{\eta'} \sim a$, so $a \sim \lambda^2_{\eta'}/N$.

Taking $M_{ij} = \mu^2_i \delta_{ij}$, any vacuum expectation value (v.e.v) of $U$ can be assumed to be diagonal, $U_{ij} = e^{i \phi_i} \delta_{ij}$. The potential reduces to

$$V(\phi_i) = f_\pi^2 \left( -\sum_i \mu_i^2 \cos(\phi_i) + \frac{a}{2} (\sum_i \phi_i - \theta)^2 \right). \quad (6)$$

This is minimized for

$$\mu_i^2 \sin(\phi_i) + a (\sum_i \phi_i - \theta) = 0. \quad (7)$$

Note that as $\sum \phi_i$ arises from $tr \ln U$, it is defined modulo $2\pi$.

Previously, several authors studied how the v.e.v.’s of the $\phi$’s change as a function of $\theta$. In the present work, we consider $\theta = 0$, but consider how the $\mu_i$ and $a$ change with temperature. Witten pointed out that when the anomaly term $a$ becomes small, metastable states in the $\phi$’s can arise. From our arguments in the previous section, this happens naturally if the phase transition at large $N$ is of second order.

The presence of these metastable states can be easily understood for a single flavor, as discussed by Witten (for a recent discussion see). From (6), the v.e.v. arises from a balance between a term $\sim \sin(\phi)$ and a term $\sim \phi$. For large $a$, the term linear in $\phi$ wins, and there is no possibility for a metastable point. Now consider the opposite limit, of vanishing $a$: then there automatically other solutions besides $\phi = 0, 2\pi, 4\pi, \text{ etc.}$ These solutions are equivalent to the trivial vacuum, and so there is nothing new. But for small values of $a$, the term linear in $a$ will only move the stationary point a little bit from $2\pi, 4\pi, \text{ etc.}$ Because $a$ is nonzero, they will become metastable, distinct from the usual vacuum.

From (7), these states will act like regions of nonzero $\theta$. Parity and CP are both violated spontaneously in such a region.

The condition for metastable states to arise with more than one flavor is not apparent, and a new result of our analysis. It is easiest understood by analogy. At zero temperature, and nonzero $\theta$, if any quark mass vanishes, the $\theta$ parameter can be eliminated by a chiral rotation through that quark flavor. Thus it is the lightest quark mass which controls $\theta$ dependence. We found a similar phenomenon for metastable states: they only occur when the anomaly term is small relative to the lightest quark masses.

This means that metastable states only arise when the anomaly term becomes very small. At zero temperature, the anomaly term is on the order of
the strange quark mass. The previous argument indicates that it must become on the order of the up and down quark masses. Putting in the numbers, we find that metastable states only arise when the anomaly term becomes on the order of 1% of its value at zero temperature. Clearly this is a strong variation of the topological susceptibility with temperature; nevertheless, it is interesting to investigate the possible implications for phenomenology.

Most notably, when the anomaly term $a$ becomes small, there is maximal violation of isospin. At zero temperature, the nonet of pseudo-Goldstone bosons — the π’s, K’s, η, and η’ — are, to a good approximation, eigenstates of SU(3) flavor. It is not often appreciated, but this is really due to the fact that the anomaly term is large, splitting off the η’ to be entirely an SU(3) singlet. When the anomaly term becomes small, however, while the charged pseudo-Goldstone bosons remain approximate eigenstates of flavor, the neutral ones do not. Without the anomaly, the π⁰ becomes pure $\bar{u}u$, the η pure $\bar{d}d$, and the η’ pure $\bar{s}s$. Consequently, these three mesons become light. This is especially pronounced for the η, as it sheds all of its strangeness, to become purely $\bar{d}d$. Thus the η and η’ would be produced copiously, and would manifest itself in at least two ways. First, light η’s and η’’s decay into two photons, and so produce an excess at low momentum. Secondly, these mesons decay into pions, which would be seen in Bose-Einstein correlations. Further, through Dalitz decays, the enhanced production of η’s and η’’s will enhance the yield of low mass dileptons.

This maximal violation of isospin is true whenever the anomaly term becomes small. There are other signals which only appear when parity odd bubbles are produced. Since parity is spontaneously violated in such a bubble, various decays, not allowed in the parity symmetric vacuum, are possible. Most notably, the η can decay not just to three pions, as at zero temperature, but to two pions. Because of the kinematics, in a parity odd bubble, $\eta \rightarrow \pi^0\pi^0$ is allowed, but $\eta \rightarrow \pi^+\pi^-$ is not.

There is another measure of how parity may be violated. We first argue by analogy. Consider propagation in a background magnetic field. As charged particles propagate in the magnetic field, those with positive charge are bent one way, and those with negative charge, the other. This could be observed by measuring the following variable globally, on an event–by–event basis:

$$\mathcal{P} = \sum_{\pi^+\pi^-} \frac{(\vec{p}_{\pi^+} \times \vec{p}_{\pi^-}) \cdot \vec{z}}{|\vec{p}_{\pi^+}||\vec{p}_{\pi^-}|};$$

(8)

Here $\vec{z}$ is the beam axis, and $\vec{p}$ are the three momenta of the pions. If the quarks were propagating through a background chromo-magnetic field, then $\mathcal{P}$, which is like handedness in jet physics, is precisely the right
quantity. However, a parity odd bubble is not directly analogous to a background chromo–magnetic field: $\pi^+$’s and $\pi^-$’s propagate in a region with constant but nonzero $\phi$ in the same fashion. Consider, however, the edge of the parity odd bubble: in such a region, $U^\dagger \partial_\mu U$ is nonzero, and does rotate $\pi^+$ and $\pi^-$ in opposite directions. Thus it is the edges of parity odd bubbles which contribute to the parity odd asymmetry of (8). Purely on geometric grounds, this suggests that a reasonable estimate for the maximal value of $\mathcal{P}$ is on the order of a few percent.

We conclude by noting that what appears to be a rather technical subject — the $\theta$ dependence of the free energy — is related to interesting and novel experimental signatures in heavy ion collisions. Within our assumptions, we find that parity odd bubbles only arise very near the point of the phase transition. This is very much tied to the fact that we limit ourselves to an analysis at large $N$. For finite $N$, it is a long standing question of how to reconcile the known limit at large $N$ with periodicity in $\theta$, with period $2\pi$. A probable solution involves “glued” potentials, which are a sum of cosines; see and recently.\[ The precise form of the potential at finite $N$ could dramatically alter our results, and, as discussed by Halperin and Zhitnitsky, make the emergence of parity odd bubbles far more likely.

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