The tunneling radiation of a black hole with a $f(R)$ global monopole under generalized uncertainty principle

Lingshen Chen       Hongbo Cheng*
Department of Physics,
East China University of Science and Technology,
Shanghai 200237, China

Abstract

The Parikh-Kraus-Wilczek tunneling radiation of black hole involving a $f(R)$ global monopole is considered based on the generalized uncertainty principle. The influences from global monopole, $f(R)$ gravity and the corrections to the uncertainty appear in the expression of black hole entropy difference. It is found that the global monopole and the revision of general relativity both hinder the black hole from emitting the photons. The two parts as corrections to the uncertainty make the entropy difference of this kind of black hole larger or smaller respectively.

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*E-mail address: libcheng@ecust.edu.cn
I. Introduction

The black holes were thought as perfect absorbers classically without emitting anything [1,2]. More than forty years ago, S. W. Hawking put forward that a black hole can radiate particles quantum-mechanically and the radiation spectrum is purely thermal, while several derivations of this kind of emission emerge [3-5]. Afterwards the Hawking radiation was perceived as a semi-classical tunneling process, leading an alternative method [6-9]. This tunneling formalism due to the imaginary part of action for classically forbidden region of emission across the horizon attracted a lot of attention [10-16]. The discussion on Hawking radiation based on the semi-classical tunneling proposed by Kraus et. al. has been used to explore many kinds of black hole spacetimes such as BTZ black hole [17-20], a series of Taub-NUT black holes [21], Kerr-Newman black holes [22-24], Godel black hole [25], etc..

In the development of quantum gravity some new proposals beyond the previous framework generated. The quantum gravity needs a minimal length of the order of the Planck length [26-32]. Further the generalized uncertainty principle (GUP) as generalization of Heisenberg’s scheme was initiated to modify the quantum mechanics [33]. The following efforts were contributed to the GUP because of the existence of a minimum measurable length scale [27, 34-39].

There has been much interest in the quantum gravitational influence within the frame of GUP. In particular, the relation between the entropy of black hole and a minimal length as quantum gravity scale was derived and estimated [40, 41]. The effects from the GUP on the corrected Beckenstein-Hawking black hole entropy in the higher dimensional spacetime was investigated while the black hole radiation was discussed with the help of the tunneling formalism [40]. The Hawking tunneling radiation from black holes involving GUP corrections was also scrutinized in the world with extra dimensions [41].

During the vacuum phase transition in the early universe, several types of topological defects such as domain walls, cosmic strings and monopoles may generate due to a breakdown of local or global gauge symmetries [42, 43]. As a spherically symmetric topological object, a global monopole appeared in the phase transition of a system involving a self-coupling triplet of a scalar field whose original global $O(3)$ symmetry is spontaneously broken to $U(1)$ [42, 43]. The metric outside a massive source with a global monopole was investigated and the distinct properties were shown that the surroundings has a solid angle leading all light rays deflected at the same angle although the monopole exerts practically no gravitational force on nonrelativistic matter [44]. The theory of $f(R)$ gravity for the acceleration of the universe modifies the description of spacetime significantly [45]. This kind of theory is utilized to explain the accelerated-inflation problem without dark matter or dark energy [46-48]. T. R. P. Carames et al discussed the gravitational field of massive source swallowing a global monopole within the frame of $f(R)$ gravity theory to put forward a parameter $\psi_0$ associated with the corrections from the gravity. It is interesting that this nonvanishing modified parameter also forms an outer horizon as a boundary of the universe subject to the $f(R)$ monopole metric [49, 50]. The thermodynamic quantities of the $f(R)$ monopole black hole were estimated
Recently, F. B. Lustosa et al generalize the metric where $\frac{d f(R)}{dR} = F(R)$ is a power law function of the radial coordinate of the global monopole spacetime. They also analyze the thermodynamics of this kind of black holes [52]. The strong gravitational lensing for a massive source with a $f(R)$ global monopole was discussed analytically [53].

It is significant to study the radiation of the black holes with $f(R)$ global monopole based on the GUP in addition to their gravitational lensing and thermodynamics. In the case of minimal quantum gravity order, black holes have to be explored while the Heisenberg’s uncertainty is generalized. We relate the revision from quantum gravitation with the deviation of general relativity in the presentation of the entropy difference of this kind of black holes by means of the tunneling formalism. We derive and calculate the tunneling probability consisting of the black hole entropy having something to do with the factors mentioned just now. Our discussions and conclusions are listed in the end.

II. The entropy difference of a radiating black hole with a $f(R)$ global monopole

We are going to investigate the modifications from several directions such as global monopole, $f(R)$ scheme and GUP. Now we start to focus on the entropy of black hole with global monopole in the $f(R)$ theory. The spherical symmetric line element for the gravitational field with global monopole in the $f(R)$ theory is adapted as follow [49, 50, 52],

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$ (1)

where

$$A(r) = B^{-1}(r) = 1 - 8\pi G\eta^2 - \frac{2GM}{r} - \psi_0 r$$ (2)

Here $G$ is the Newton constant. As a monopole parameter, $\eta$ is of the order $10^{16}GeV$ in a typical grand unified theory, leading $8\pi G\eta^2 \approx 10^{-5}$ [43, 44]. $M$ is mass parameter. The factor $\psi_0$ reflects the extension of the standard general relativity. It is obvious that the term $\psi_0 r$ in the metric above is linear, which is certainly different from the structures of de Sitter spacetime and the Reissner-Nordstrom metric, etc. [49, 50]. As roots of $A(r) = 0$ for metric (1), there exists an inner radius,

$$r_- = \frac{1 - 8\pi G\eta^2 - \sqrt{(1 - 8\pi G\eta^2)^2 - 8GM\psi_0}}{\psi_0}$$ (3)

and an outer ones,

$$r_+ = \frac{1 - 8\pi G\eta^2 + \sqrt{(1 - 8\pi G\eta^2)^2 - 8GM\psi_0}}{\psi_0}$$ (4)

If the modified parameter $\psi_0$ vanishes, the outer horizon will disappear. According to Ref. [3-5], the Hawking temperature for the black hole from Eq. (1) and Eq. (2) is a function of variables like $\eta, \psi_0$ etc.,
The Beckenstein-Hawking entropy may be derived from the Hawking temperature in virtue of the following thermodynamics relation [3-5, 9],

\[ T_H = \frac{dE}{dS} \approx \frac{dM}{dS} \]  

The difference between the initial and the final magnitudes of the entropy of the \( f(R) \) black hole containing the global monopole in the emission process can be approximated as,

\[ \Delta S \approx -\frac{4\pi G}{(1 - 8\pi G\eta^2)^2} [M^2 - (M - \hbar \omega)^2] - \frac{16\pi G^2 \psi_0}{(1 - 8\pi G\eta^2)^3} [M^3 - (M - \hbar \omega)^3] \]

where \( \omega \) is a shell of energy moving along the geodesics in the spacetime with metric (1) [9].

The higher order of typical grand unified theory or farther away from standard general relativity will lead larger absolute value of negative entropy difference. According to Ref. [9], the relation probability of the black hole can be demonstrated as,

\[ \Gamma \sim e^{\Delta S} \]

The existence of global monopole in the black hole or the deviation from general relativity damps the emission of the black hole.

III. The entropy difference of a radiating black hole with a \( f(R) \) global monopole under generalized uncertainty principle

Here we turn our discussions in the context of GUP. The Heisenberg’s uncertainty principle is generalized within the microphysics regime as [36, 39, 40, 54-64],

\[ \Delta x \Delta p \geq \frac{\hbar}{2} \left[ 1 - \frac{\alpha \hbar l_p}{\hbar} \Delta p + \left( \frac{\beta \hbar l_p}{\hbar} \right)^2 \Delta p^2 \right] \]

leading

\[ y_- \leq y \leq y_+ \]

where

\[ y_\pm = \left( \frac{l_p}{\hbar} \Delta p \right)_\pm \]

\[ = \frac{1}{2\beta^2} \left( \alpha + \frac{2 \Delta x}{l_p} \right) \pm \frac{1}{2\beta^2} \left( \alpha + \frac{2 \Delta x}{l_p} \right) \sqrt{1 - \left( \frac{2\beta}{\alpha + \frac{2 \Delta x}{l_p}} \right)^2} \]
where \( \alpha \) and \( \beta \) are dimensionless positive parameters, Planck length shown as \( l_p = \sqrt{\frac{2G}{c^3}} \) while the velocity of light \( c \). The terms with the Newtonian constant \( G \) provide the uncertainty with the gravitational effects. According to the procedure of Ref. [40, 41, 65, 66], we introduce,

\[
\Delta p' = \frac{\hbar}{l_p} y_-
\]

Approximately the uncertainty in the momentum is [65],

\[
\Delta p' \approx \frac{\hbar}{\alpha l_p + 2 \Delta x}
\]

We substitute the approximation (13) into the GUP (9) to estimate the distance interval [65],

\[
\Delta x' \approx \Delta x [1 + \frac{(\beta l_p)^2}{2 \Delta x (\alpha l_p + 2 \Delta x)}]
\]

The original size of black hole is chosen as the lower bound on the region like \( \Delta x = 2r_H \) [40, 41], so the Hawking temperature (5) can be changed as follow,

\[
T_H = \frac{1}{2\pi} \left( \frac{1 - 8\pi G \eta^2}{\Delta x} - \psi_0 \right) (15)
\]

If the black hole exists without global monopole and the \( f(R) \) corrections, the temperature will return to that of Schwarzschild case [40, 41]. Like Ref. [65], we choose the distance interval in the temperature (15) as \( \Delta x' \) expressed in Eq. (14) to obtain,

\[
T_H' = \frac{1}{2\pi} \left( \frac{1 - 8\pi G \eta^2}{\Delta x'} - \psi_0 \right)
\approx T_H - \frac{(1 - 8\pi G \eta^2)(\beta l_p)^2}{4\pi \Delta x'^2 (\alpha l_p + 2 \Delta x)}
\]

The corrections from GUP make the Hawking temperature lower. We make use of the thermodynamic relation (6) [3-5, 9] to obtain the corrected entropy difference of the radiating black hole with \( f(R) \) global monopole as,

\[
\Delta S' = \Delta S'(\eta, \psi_0, \alpha, \beta)
= \frac{\pi}{G} (r_H^2 - r_H'^2)
\]

\[
- \frac{\pi}{G} \frac{(1 - 8\pi G \eta^2)(\beta l_p)^2}{16\psi_0 r_H^2 + 4[\psi_0 \alpha l_p - 2(1 - 8\pi G \eta^2)]r_H + [\psi_0 (\beta l_p)^2 - 2(1 - 8\pi G \eta^2)\alpha l_p]} (r_H' - r_H)
\]

where we replace the mass parameter \( M \) in the horizon (3) as \( M - \hbar \omega \) to obtain the horizon at the end of the process that black hole emits a photon [9]. It can be checked that the corrected entropy difference will return to that of Eq. (7) without generalizing the uncertainty. In order to make clear how the generalization of Heisenberg Uncertainty affect the tunneling probability, we compare the
entropy difference $\Delta S'$ with the difference for Schwarzschild black hole like $\Delta S_0 \approx -8\pi GM\hbar \omega$ [9] to obtain,

$$\frac{\Delta S'}{\Delta S_0} = \frac{1}{(1 - 8\pi G\eta^2)^2} + \frac{6GM\psi_0}{(1 - 8\pi G\eta^2)^4}$$

$$- \frac{(\beta \ell_p)^2}{16\psi_0 r_H^2 + 4[\psi_0 \alpha \ell_p - 2(1 - 8\pi G\eta^2)]r_H} + [\psi_0 (\beta \ell_p)^2 - 2(1 - 8\pi G\eta^2)\alpha \ell_p]$$

$$\times \left[ \frac{1}{4GM} + \frac{\psi_0}{(1 - 8\pi G\eta^2)^2} + \frac{6GM\psi_0^2}{(1 - 8\pi G\eta^2)^4} \right]$$

(18)

The influence from GUP on the entropy difference in the case of black hole including $f(R)$ global monopole is shown graphically. The dependence of the ratio $\frac{\Delta S'}{\Delta S_0}$ formulated in Eq. (18) on the variables is plotted in the figures. There are two factors appeared as $\alpha$ - term and $\beta$ - term respectively in the GUP in Eq. (9). It is found that the greater correction denoted as $\alpha$ leads the absolute value of $\Delta S'$ smaller, which retards the radiation of the black holes in Fig. 1. The other figure labelled as Fig. 2 shows that the growth of the other parameter $\beta$ will promote the tunnel process of the black hole because of the larger absolute value of entropy difference.

IV. Discussion

The main results of this paper is Eq. (17) representing the entropy difference of the black hole containing $f(R)$ global monopole controlled by the generalized uncertainty principle during its radiation. The discussions show that the global monopole inside the black hole as well as $f(R)$ amendments will decrease the black hole’s tunneling probability. The GUP as a description of the quantum gravitational influence can also be explored in the process of black hole radiation because its deviations have something to do with the tunneling probability [40, 41]. The GUP [36, 39, 40, 54-64] that we employ here consists of two terms modifying the usual uncertainty and the effects of the two terms multiplied by $\alpha$ and $\beta$ respectively are opposite each other. We find that the larger factor $\alpha$ causes the negative term to lower the tunnel radiation of black hole in the case of black hole with a deficit solid angle outside in the $f(R)$ scheme. The emission of this kind of black hole is advanced in favour of the greater parameter $\beta$ as a coefficient of the positive part. Further works that the GUP is discussed in other directions proceed.

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References

[1] M. Carmeli, Classical Fields: General Relativity and Gauge Theory, John Wiley & Sons Inc. 1982

[2] B. Schutz, A First Course in General Relativity, Cambridge University Press, 2009

[3] S. W. Hawking, Nature 248(1974)30

[4] S. W. Hawking, Commun. Math. Phys. 43(1975)199

[5] G. W. Gibbons, S. W. Hawking, Phys. Rev. D15(1977)2752

[6] P. Kraus, F. Wilczek, Nucl. Phys. B433(1995)403

[7] P. Kraus, F. Wilczek, Nucl. Phys. B437(1995)231

[8] P. Kraus, E. Keski-Vakkuri, Nucl. Phys. B491(1997)249

[9] M. K. Parikh, F. Wilczek, Phys. Rev. Lett. 85(2000)5042

[10] K. Srinivasan, T. Padmanabhan, Phys. Rev. D60(1999)24007

[11] S. Shankaranarayanan, T. Padmanabhan, K. Srinivasan, Class. Quantum Grav. 19(2002)2671

[12] K. Srinivasan, Phys. Rev. D67(2003)084026

[13] S. Iso, H. Umetsu, F. Wilczek, Phys. Rev. D74(2006)044017

[14] R. Banerjee, B. R. Majhi, Phys. Lett. B662(2008)62

[15] R. Banerjee, B. R. Majhi, S. Samanta, Phys. Rev. D77(2008)124035

[16] R. Kerner, R. B. Mann, Phys. Lett. B665(2008)277

[17] S. Hemming, E. Keski-Vakkuri, Phys. Rev. D64(2001)044006

[18] M. Agheben, M. Nadalini, I. Vanzo, S. Zerbini, JHEP 0505(2005)014

[19] S. Wu, Q. Jiang, JHEP 0603(2006)079

[20] R. Li, J. Ren, Phys. Lett. B661(2008)370

[21] R. Kerner, R. B. Mann, Phys. Rev. D73(2006)104010

[22] J. Zhang, Z. Zhao, Phys. Lett. B638(2006)110

[23] Q. Jiang, S. Wu, X. Cai, Phys. Rev. D73(2006)064003

[24] R. Li, J. Ren, Class. Quantum Grav. 25(2008)1265016
[25] R. Kerner, R. B. Mann, Phys. Rev. D75(2007)084022
[26] D. J. Gross, P. F. Mende, Nucl. Phys. B303(1988)407
[27] D. Amati, M. Ciafaloni, G. Veneziano, Phys. Lett. B216(1989)41
[28] M. Kato, Phys. Lett. B245(1990)43
[29] M. Maggiore, Phys. Lett. B304(1993)65
[30] L. G. Garay, Int. J. Mod. Phys. A10(1995)145
[31] S. de Haro, JHEP 9810(1998)023
[32] M. I. Park, Phys. Lett. B659(2008)698
[33] R. J. Adler, D. I. Santiago, Mod. Phys. Lett. A14(1999)1371
[34] A. Kempf, G. Mangano, R. B. Mann, Phys. Rev. D52(1995)1108
[35] A. Kempf, J. Phys. A30(1997)2093
[36] A. Kempf, G. Mangano, Phys. Rev. D55(1997)7909
[37] L. N. Chang, D. Minic, N. Okamura, T. Takeuchi, Phys. Rev. D65(2002)125027
[38] L. N. Chang, D. Minic, N. Okamura, T. Takeuchi, Phys. Rev. D65(2002)125028
[39] S. Das, E. C. Vagenas, A. F. Ali, Phys. Lett. B690(2010)407
[40] M. Dehghani, A. Farmany, Phys. Lett. B675(2009)460
[41] M. Dehghani, Phys. Lett. B749(2015)125
[42] T. W. B. Kibble, J. Phys. A9(1976)1387
[43] A. Vilenkin, Phys. Rep. 121(1985)263
[44] M. Barriola, A. Vilenkin, Phys. Rev. Lett. 63(1989)341
[45] H. A. Buchdahl, Mon. Not. R. Astron. Soc. 150(1970)1
[46] S. Nojiri, S. D. Odintsov, Phys. Rev. D68(2003)123512
[47] S. M. Carrol, V. Duvvuri, M. Trodden, M. S. Turner, Phys. Rev. D70(2004)043528
[48] S. Fay, R. Tavakol, S. Tsujikawa, Phys. Rev. D75(2007)063509
[49] T. R. P. Carames, E. R. B. de Mello, M. E. X. Guimaraes, Int. J. Mod. Phys. Conf. Ser. 03(2011)446
[50] T. R. P. Carames, E. R. B. de Mello, M. E. X. Guimaraes, Mod. Phys. Lett. A27(2012)1250177

[51] J. Man, H. Cheng, Phys. Rev. D87(2013)044002

[52] F. B. Lustosa, M. E. X. Guimaraes, C. N. Ferreira, J. L. Neto, 2015, arXiv: 1510.08176

[53] J. Man, H. Cheng, Phys. Rev. D92(2015)024004

[54] R. Adler, P. Chen, D. Santiago, Gen. Relativ. Grav. 33(2001)2101

[55] L. Xiang, Phys. Lett. B638(2006)519

[56] Z. Ren et.al., Phys. Lett. B641(2006)208

[57] K. Nouicer, Phys. Lett. B646(2007)63

[58] Y. Kim, Y. Park, Phys. Lett. B655(2007)63

[59] X. Han, H. Li, Y. Ling, Phys. Lett. B666(2008)121

[60] D. Amati, M. Ciafaloni, G. Veneziano, Phys. Lett. B197(1987)81

[61] S. Hossenfelder er.al., Phys. Lett. B584(2004)109

[62] S. Hossenfelder er.al., Phys. Rev. D73(2006)105013

[63] A. Farmany, S. Abbasi, A. Naghipour, Phys. Lett. B650(2007)33

[64] A. Farmany, S. Abbasi, A. Naghipour, Phys. Lett. B659(2008)913

[65] A. Farmany, S. Abbasi, A. Naghipour, Phys. Lett. B682(2009)114

[66] M. A. Anacleto, F. A. Brito, E. Passos, Phys. Lett. B749(2015)181
Figure 1: The solid, dashed, dot curves of the dependence of the ratio $\frac{\Delta S'}{\Delta S_0}$ on $\alpha$ for $\psi_0 = 0.01, 0.05, 0.08$ respectively with $\beta = 3.5$ and for simplicity $8\pi G \eta^2 = 0.1, G = 1 = M = l_p = 1$. 
Figure 2: The solid, dashed, dot curves of the dependence of the ratio $\frac{\Delta S'}{\Delta S_0}$ on $\beta$ for $\psi_0 = 0.01, 0.05, 0.08$ respectively with $\alpha = 3.5$ and for simplicity $8\pi G\eta^2 = 0.1, G = 1 = M = l_p = 1$. 