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Nonmesonic Weak Decay Spectra of Light Hypernuclei

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Abstract  The nonmesonic weak decay spectra of light hypernuclei have been evaluated in a systematic way. As theoretical framework we adopt the independent particle shell model with three different one-meson-exchange transition potentials. Good agreement with data is obtained for proton and neutron kinetic energy spectra of $^4\Lambda$He, and $^5\Lambda$He, when the recoil effect is considered. The coincidence spectra of proton-neutron pairs are also accounted for quite reasonably, but it was not possible to reproduce the data for the neutron–neutron pair spectra. It is suggested that the $\pi+K$ meson-exchange model with soft monopole form factors could be a good starting point for describing the dynamics responsible for the decays of these two hypernuclei. The $^4\Lambda$H spectra are also presented.

1 Introduction

Investigations of unusual nuclear properties, such as nontrivial values of flavor quantum numbers (strangeness, charm or beauty), or large isospin (so called neutron rich isotopes) are of continuous interest. A particularly interesting phenomenon in nuclear physics is the existence of nuclei containing strange baryons. The lightest hyperons are stable against strong and electromagnetic decays, and as they do not suffer from Pauli blocking by other nucleons they can live long enough in the nuclear environment to become bound. When a hyperon, specifically a $\Lambda$-hyperon with strangeness $S = -1$, replaces one of the nucleons in the nucleus, the composed system acquires different properties from that of the original one, and is referred as hypernucleus.

One such very important new property is the additional binding. For instance, while the one-neutron separation energy in $^{20}\text{C}$ is 1.01 MeV, it is 1.63 MeV in $^{21}\Lambda\text{C}$, and $^6\Lambda\text{He}$ is bound while $^5\text{He}$ is unbound. As a consequence the neutron drip line is modified, and the extended three-dimensional ($N, Z, S$) domain of radioactivity becomes even richer in elements than the ordinary neutron–proton domain ($N, Z$). Because of this glue attribute of hypernuclei the $\Lambda N$ interaction is closely related to the inquiry on the existence of strange quark matter and its fragments, and strange stars (analogues of neutron stars), which makes the hypernuclear physics also relevant for astrophysics and cosmology.
Another remarkable property of $\Lambda$-hypernuclei is the occurrence of the nonmesonic weak decay (NMWD): $\Lambda N \rightarrow nN$ with $N = p, n$, which is the main decay channel for medium and heavy hypernuclei. This decay takes place only within nuclear environment, and is the unique opportunity that nature offers us to inquire about strangeness–flipping interaction between baryons.

The knowledge of strange hadrons carrying an additional flavour degree-of-freedom is essential for understanding the low-energy regime of the quantum chromodynamics (QCD), which is the field theory of strong interactions among quarks and gluons, and is widely used at very high energies. Yet, at energy scale of the nucleon mass the hadrons represent complex many body systems, making difficult the QCD description due to the non-perturbative nature of the theory.

Ergo the NMWD dynamics is frequently handled by the one-meson-exchange (OME) models [1–9], among which the exchange of full pseudoscalar ($\pi, K, \eta$) and vector ($\rho, \omega, K^*$) meson octets (PSVE) is the most used one. This model is based on the original idea of Yukawa that the $NN$ interaction at long distance is due to the one-pion-exchange (OPE), while the weak coupling constants are obtained from soft meson theorems and $SU(6)$ [1, 2]. The dominant role is being played by the exchange of pion and kaon mesons (PKE). We have recently shown that the $\pi + K$ meson exchange potential with soft dipole form factors (SPKE) reproduces fairly well both $s$-shell [10], and $p$-shell [11] NMWD.

A hybrid mechanism has been also meticulously used by the Tokyo group [12–17] to describe the NMWD in $A = 4$, and 5 hypernuclei. This group represented the short range part of the $\Lambda N$ weak interaction by the direct quark (DQ) weak transition potential, while the longer range interactions are assumed to come from the exchange of $\pi + K$ mesons with soft pion form factor.\(^1\)

The $\Lambda$-hypernuclei are mainly produced by the ($K^-, p^-$) and the ($p^+, K^+$) strong reactions, and disintegrate by the weak decay with the rate

$$\Gamma_W = \Gamma_M + \Gamma_{NM},$$

where $\Gamma_M$ is decay rate for the mesonic (M) decay $\Lambda \rightarrow \pi N$, and $\Gamma_{NM}$ is the rate for the nonmesonic (NM) decay, which can be induced either by one bound nucleon ($1N$), $\Gamma_1(\Lambda N \rightarrow nN)$, or by two bound nucleons ($2N$), $\Gamma_2(\Lambda NN \rightarrow nNN)$, i.e.,

$$\Gamma_{NM} = \Gamma_1 + \Gamma_2,$$

with

$$\Gamma_1 = \Gamma_p + \Gamma_n, \quad \Gamma_2 = \Gamma_{nn} + \Gamma_{np} + \Gamma_{pp}.$$

In the evaluation of $\Gamma_{NM}$, and the corresponding spectra, are used the Shell Model (SM) and the Fermi Gas Model (FGM) for $\Gamma_1$ (within the two-particle phase space) [1–18, 21–24], while for $\Gamma_2$ (within the three-particle phase space) only the later model has been employed so far [21–24].

In our previous work [10] we have analyzed the Brookhaven National Laboratory (BNL) experiment E788 on $^4 \text{He}$ [32], involving: (1) the single-proton spectra $S_p(E)$, and total neutron spectra $S_n(E) = 2S_n(E) + S_p(E)$, as a function of corresponding one-nucleon kinetic energies $E_N$, and (2) two-particle-coinidence spectra, as a function of: (i) the sum of kinetic energies $E_n + E_N \equiv E_{nN}$, $S_{nN}(E)$, and (ii) the opening angle $\theta_{nN}, S_{nN}(\cos \theta)$. This has been done within a simple theoretical framework, based on the independent-particle SM (IPSM) for the $1N$-induced NM weak decay spectra. That is, we have disregarded both the $2N$-NM decay, and the final state interactions (FSIs), which is consistent with the upper limits $\Gamma_2/\Gamma_W \leq 0.097$, and $\Gamma_{FSI}/\Gamma_W \leq 0.11$ established in [32] with a 95% CL\(^2\). Moreover, the calculated spectra were normalized to the experimental ones. For instance, in the case of single proton spectrum $S_p(E)$, the number of protons $\Delta N_p^{exp}(E_i)$ measured at energy $E_i$ within a fixed proton energy bin $\Delta E$, is confronted with the calculated number

$$\Delta N_p(E) = N_p^{exp} S_p(E) \frac{\Delta E}{\Gamma_p}, \quad N_p^{exp} = \sum_{i=1}^{m} \Delta N_p^{exp}(E_i),$$

where $N_p^{exp}$ is the total number of measured protons, while $S_p(E)$ and $\Gamma_p = \int S_p(E) dE$ are evaluated theoretically.\(^3\) Note that: a) while the $\Delta N_p^{exp}(E_i)$ are defined only at $m$ experimental energies $E_i$, the quantity

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\(^1\) McKeever and Gibson employed a very soft $\pi$ of 0.63 GeV [18].

\(^2\) We note that experimentally it is very difficult to distinguish between these two effects and very likely $\Gamma_2 + \Gamma_{FSI}^{E788} / \Gamma_W \sim 10\%$.

\(^3\) Corrections for detection acceptance and threshold are discussed in [10].
$\Delta N_p(E)$ is a continuous function of $E$, and b) the condition $N_p = N_p^{exp}$ is automatically fulfilled when $\Gamma_p = \Gamma_p^{exp}$.

To describe the NMWD dynamics three different OME potentials have been tested in Ref. [10], namely:

(P1) The full PSVE potential that includes the exchanges of nonstrange-mesons $\pi$, $\rho$, $\omega$, and $\eta$, and strange-mesons $K$, and $K^*$ with the weak coupling constants from the Refs. [1,2,4],

(P2) The PKE model, with usual cutoffs $\Lambda_\pi = 1.3$ GeV, and $\Lambda_K = 1.2$ GeV, and

(P3) The SPKE potential, which corresponds to the cutoffs $\Lambda_\pi = 0.7$ GeV and $\Lambda_K = 0.9$ GeV.

The calculated spectra shown in Figs. 2, 3, and 4 in Ref. [10] correspond to the parametrization P3, but almost identical spectra are obtained with the remaining two parametrizations, which is due to the fact that the shapes of all spectra are basically tailored by the kinematics, depending very weakly on the dynamics, which cancels out almost wholly in (1) when $S_p(\bar{E})$ is divided by $\Gamma_p$. Moreover, the fairly good agreement between the data and calculations indicates that the framework employed (IPSM plus the two-particle phase space and the recoil effect) could be appropriate for describing the kinematics in the NMWD of $^4_A$He. It indicates as well that, neither the FSI, nor the two-nucleon induced decay processes play a very significant role. Needless to stress that we study the NMWD in order to understand its mechanism, and to disentangle in this way the strangeness-flipping interaction among baryons, while the FSIs, together the $2N$-NM decay, are to some extent just undesirale and inevitable complications.

To test not only the kinematics but also the dynamics the comparison between the experimental and calculated spectra has to be done differently, as already pointed out in Refs. [11,29,30]. This is done by confronting straightforwardly the corresponding spectra. For instance, for the case considered above one compares the calculated spectrum $S_p(E)$ with the experimental one defined as:

$$S_p^{exp}(E_i) = \frac{\Gamma_W \Delta N_p(E_i)}{\Delta E} \bigg|_{BNL,FINUDA} = \frac{\Gamma_{NM} \Delta N_p(E_i)}{N_{NM} \Delta E} \bigg|_{KEK}. \quad (2)$$

The difference between the first and second terms in the right side is due to different normalizations of $\Delta N(E_i)$. The BNL and FINUDA groups do it in relation with the total number of weak decays $N_W$, while the KEK data are normalized to the number of NM decays $N_{NM}$. In both cases the experimental proton transition rate, derived from the proton kinetic energy spectrum $S_p(E)$, is defined as

$$\Gamma_p^{exp} = \Delta E \sum_i S_p^{exp}(E_i), \quad (3)$$

and similarly those obtained from $S_{nt}(E)$, $S_{nN}(\cos \theta)$, $S_{nN}(E)$, and $S_{nN}(P)$. The neutron decay rate derived from the total neutron kinetic energy spectrum is defined as: $\Gamma_n^{exp} = (\Gamma_{nt}^{exp} - \Gamma_p^{exp})/2$. For $\Gamma_W$ we use the relationship

$$\Gamma_W(A) = (0.990 \pm 0.094) + (0.018 \pm 0.010) A,$$

which was determined in Ref. [39] for all measured hypernuclei in the mass range $A=4$–12.

We will analyze here as well the spectra $S_{nN}(P)$ as a function of the center of mass (c.m.) momentum $P \equiv P_{nN} = |P_{nN}| = |p_n + p_N|$. More, we will seize the opportunity to discuss the spectra of other two light hypernuclei, namely of $^4_A$H and $^5_A$He, taking care of measurements done on the latter at KEK [33–35], and FINUDA [37]. Theoretical expressions for different NM weak decay spectra within the IPSM that are used here have been presented previously [10,26–30] and will not be repeated here.

2 Results

To describe the NMWD dynamics, we will employ the OME potentials that are listed above. The short range correlations (SRC) acting on final $nN$ states are incorporated a posteriori phenomenologically through Jastrow-like SRC functions, as used within both finite nuclei [2,4,5,7,8], and FGM calculations [19–25]. The size parameter $b$, which is the most important nuclear structure parameter [28], was estimate to be $b = \frac{1}{\sqrt{3}}(R_N + R_\Lambda)$ where $R_N$ and $R_\Lambda$ are, respectively, the root-mean-square distances of the the nucleons and the $\Lambda$ from the center of mass of the hypernucleus. This yields $b(^4_A$H$) = 1.57$ fm, $b(^4_A$He$) = 1.53$ fm, and $b(^5_A$He$) = 1.33$ fm [38].
In Fig. 1 are shown the theoretical results for: a) the proton kinetic energy $S_p(E)$ developed by the decay $\Lambda p \rightarrow np$ (left panel), and b) the total neutron spectrum $S_n(E)$, induced both by protons, $\Lambda p \rightarrow np$, and by neutron, $\Lambda n \rightarrow nn$, yielding, respectively, the spectra $S_p(E)$, and $S_n(E)$. The BNL data for $^4\Lambda$He [32], and the KEK [33], and FINUDA [37] data for $^5\Lambda$He are shown as well. Within the parametrizations P1 and P2 the theory strongly overestimates all the data. In turn the SPKE potential reproduces both $^4\Lambda$He spectra quite well, but not so well in the case of $^5\Lambda$He. Here the spectra are undervalued for energies $E \gtrsim 0.04$ GeV, while they are somewhat overvalued for higher energies. This relatively minor discrepancy could be attributed to lack of FSIs in the theory whose main effect is to shift a portion of the transition strength $\Gamma_1$ from high energies towards low energies. The inclusion in the calculations of $\Gamma_2$, which contributes dominantly at low energies, could also improve the agreement.

As expected, the theoretical spectra $S_p(E)$, are peaked around the half of the corresponding liberated energy ($Q$-value): $\Delta_p = \Delta + \epsilon_{\Lambda} + \epsilon_N$, where $\Delta = M_\Lambda - M = 0.1776$ GeV is the $\Lambda - N$ mass difference, and $\epsilon$’s are the single-particle energies. However, in light hypernuclei the spectrum shape is not exactly that of a symmetric inverted bell, since the single-proton kinetic energy reaches rather abruptly its maximum value

$$E_{\text{max}}^p = \frac{A - 2}{A - 1} \Delta_p,$$

with $A$ being the nuclear mass number, which is significantly smaller than the corresponding $Q$-value, as a consequence of energy conservation, and the recoil effect. One gets, in units of GeV:

| $\Lambda$-nucleus | $\Delta_p$ | $E_{\text{max}}^p$ |
|-------------------|------------|-----------------|
| $^4$H             | 0.175      | 0.117           |
| $^4\Lambda$He     | 0.165      | 0.110           |
| $^5\Lambda$He     | 0.154      | 0.115           |

In the absence of the recoil effect $E_{\text{max}}^p \equiv \Delta_p$, and therefore it can be concluded that this effect is very important in defining the shape of the light hypernuclei spectra. Similar results are obtained for the neutron spectrum $S_n(E)$, as well as for the total neutron spectrum $S_{nt}(E)$.  

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4 There is an mistake in Eq. (13) of Ref. [10].
It is worthy of note that, when the SPKE potential is used, the predicted $S_{nn}(E)$ spectrum in $^4\Lambda$H turns out to be of the same order of magnitude as those in $^4\Lambda$He, and $^5\Lambda$He. Therefore it would be very interesting to measure the $^4\Lambda$H neutron spectrum to verify whether this prediction is fulfilled.

In Fig. 2 are exhibited the calculations of opening angle distribution for the $np$ (left panel) and $nn$ (right panel) pairs, which, as expected, emerge roughly back-to-back with a separation angle near $\theta_{np} = 180^\circ$. First, it should be noticed that the approaches P1 and P2 yield virtually identical results for $S_{nn}(\cos \theta)$ in $^4\Lambda$H and $^5\Lambda$He, to the point that they can not be distinguished visually. Something very similar happens with $S_{np}(\cos \theta)$ in $^4\Lambda$H, and $S_{nn}(\cos \theta)$ in $^4\Lambda$He within the parametrizations P2 and P3. The interpretation of the similarity between the P1 and P2 results is that the contributions of heavy mesons have little effect in this case. Similarly, one can say that when P2 and P3 models produce similar results the size of the form factors is of little importance. All these overlays will be repeated in other spectra in coincidence.

Again the best agreement between the theory, and: i) the BNL spectrum $S_{np}(\cos \theta)$ in $^4\Lambda$He [32], and ii) both KEK spectra $S_{nn}(\cos \theta)$ in $^5\Lambda$He [34,35] is achieved with the SPKE potential, which yet overestimates slightly the latter data. On the other hand, the theory is not able to account for the $nn$ coincidence spectra measured in the BNL experiment on $^4\Lambda$He [32], nor for the KEK data [34,35] in $^5\Lambda$He at angles $\theta_{nn} < 90^\circ$. We do not believe that the FSIs and/or the 2N-NM decay are capable to solve these problems, since these effects were not able to correctly describe the $S_{nn}(\cos \theta)$ spectra in $^4\Lambda$C (see Fig. 4 in Ref. [25], and Fig. 7 in Ref. [30]). At this point it might be interesting to compare the concordance obtained in Fig. 3 of Ref. [10] for the $nn$ angular coincidences in $^4\Lambda$He with the strong disagreement for the same observable exhibited in Fig. 2. The answer to this apparent contradiction can be found in the fact that in [10] has been used the relationship (1) where all information on the dynamics is washed out almost entirely.

It is also likely that in the case of $^5\Lambda$He the kinematics of the final state does not reflect the the real situation. In fact, one should also keep in mind that in the decay channel $^4\text{He} \rightarrow (2p) + n + n$ the remaining two protons are not in a bound state, and that one is faced with the so-called four-body problem [31]. It is much harder to find a plausible explanation for the discrepancy in $^5\Lambda$He, which may be indicative of $nn$ coincidences originated from sources other than $\Lambda n$ decays, as already suggested in Ref. [32].

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[3 The decay of $^4\Lambda$He and $^5\Lambda$He via two-body channels $^4\Lambda$He $\rightarrow d + d$, $p + t$, and $^5\Lambda$He $\rightarrow d + t$, have been measured recently by FINUDA [40] to be of a few percent of the one-proton induced decay. Nothing is said in this work about an unbound four-nucleon final state.]
Next, we discuss the spectra for the kinetic energy sums \( E \equiv E_{nN} = E_n + E_N \) in NMWD \( \Lambda N \rightarrow nN \). As demonstrated by Barbero et al. [26] within the IPSM these spectra contain one or more peaks, the number of which is equal to the number of shell-model orbitals \( 1s_{1/2}, 1p_{3/2}, 1p_{1/2}, \ldots \) that are either fully or partly occupied. Before including the recoil, all these peaks would be just spikes at the corresponding \( \Delta_N \).

With the recoil effect they develop rather narrow widths \( \sim \sqrt{b^2 M(A-2)} \). For the \( s \)-shell hypernuclei there is only one such peak, and the spectra behave in this case as

\[
S_{nN}(E \cong \Delta_N) \sim \sqrt{(\Delta_N - E)(E - \Delta'_N)} e^{-M(A-2)(\Delta_N - E)b^2},
\]

where

\[
\Delta'_N = \Delta_N - \frac{A-2}{A},
\]

is the minimum kinetic energy of the emitted pair nN. That is, the spectra \( S_{nN}(E) \) are restricted within the energy intervals \( \Delta'_N < E < \Delta_N \).

In the left and right panels of Fig. 3 are displayed, respectively, the calculated correlated spectra as a function of sums of kinetic energies of \( np \) pairs, \( S_{np}(E) \), and \( nn \) pairs, \( S_{nn}(E) \). The measurements done so far at BNL for \( ^4\Lambda He \) [32] are also exhibited. The experimental \( S_{np}(E) \) spectrum of \( ^4\Lambda He \) agrees fairly well with the theory when the parametrization P3 is used. Moreover, as seen from the right panel of Fig. 3, the data of the \( ^4\Lambda He \) spectrum \( S_{nn}(E) \) differ strongly from the theoretical calculation, which is consistent with the similar deviation of the spectrum \( S_{np}(\cos \theta) \) in Fig. 2. Again, the reason for this discrepancy may be \( nn \) coincidences that do not come from NMWD. In this regard, one should remember that is valid the relationship

\[
\Gamma_n = \int S_{nn}(\cos \theta)d\cos \theta = \int S_{nn}(E)dE,
\]

as well as, that the experimental \( \Gamma_n \) for \( ^4\Lambda He \) is very small (\( \Gamma_n/\Gamma_W \leq 0.018 [32] \)).

One has to mention as well that the KEK group has measured the \( nN \) distributions of the sums of kinetic energies in \( ^5\Lambda He [34,36] \). But we can not compare quantitatively their results with our calculations, since in their work the number of measured pairs are not normalized to the number of NM decays \( N_{NM} \). Nevertheless, we can say that, while theoretically both \( S_{nN}(E) \) spectra in \( ^5\Lambda He \) are peaked up at \( \sim 0.15 \) GeV with a width of \( \cong 20 \) MeV, experimentally is observed a sharp peak at \( \sim 0.14 \) GeV in the \( np \) spectrum, while a wide bump within the energy range 0.08–0.15 GeV has been detected in the \( nn \) spectrum.

Fig. 3 (Color online) Calculations of the kinetic energy sum correlations \( S_{nN}(E) \) of \( np \) (left panel) and \( nn \) (right panel) pairs for three different parametrizations are confronted with the measurements done so far at BNL for \( ^4\Lambda He [32] \).
The results for the coincidence spectra as a function of the c.m. momentum $P$ are shown in the Fig. 4. Theoretically, they behave as

$$S_{NN}(P) \sim P^2 \sqrt{P^2 N - P^2 e^{-(Pb)^2/2}},$$

from $P = 0$ up to the maximum values of c.m. momenta

$$P_N = 2 \sqrt{\frac{A - 2}{A} \Delta N}.\tag{9}$$

The maximum of the correlation spectra $S_{NN}(P)$ occurs at the value of $P$ equal to

$$P_N^\uparrow = \left[\frac{b^2 P_N^2 + 3 - \sqrt{b^4 P_N^4 - 2b^2 P_N^2 + 9}}{2b^2}\right]^{1/2}.\tag{10}$$

For example, the $^4\Lambda^4\text{He}$ spectrum $S_{np}(P)$ goes up to the maximum value of the c.m. momentum $P_p = 0.546$ GeV, while its maximum occurs at $P_p^\uparrow = 0.165$ GeV. It is unfortunate that in the literature there are no data on the spectra $S_{nn}(P)$ for any of the three hypernuclei in order to make comparison with theoretical predictions.

The identity (7) and similar relationships for protons allow us to relate the experimental transition probabilities derived from different spectra through the Eq. (3) and those corresponding to other observables. They are compared in Table 1, and although labelled as $\Gamma_p$ and $\Gamma_n$, they include not only contributions arising from $\Gamma_1$ but also from $\Gamma_2$, as well as the effects of the FSIs. As can be see the results coming from different spectra differ considerably, and the answer to these discrepancies must come from the experimental side. For the sake of completeness in the same table we also show the values of the "bared transitions rates", which were extracted from the experimental data [32,35,36] excluding the effects of both the $2N$-$\text{NM}$ decay, and the FSIs.

3 Final Remarks

Single and coincidence spectra of the NM weak decay of light hypernuclei have been evaluated in a systematic way for the first time. We have considered the $1N$ induced processes only, omitting entirely the $2N$ induced events, as well as the effects of the FSIs. However, we have discussed in detail the recoil effect, showing that
it is very important in the description of the spectra of light hypernuclei. For the theoretical framework we have used the IPSM with three different parametrizations for the transition potential. The comparison with data suggests that the soft $\pi + K$ exchange model, introduced in Ref. [10], could be a good starting point to describe the dynamics of the NMWD in $^3\Lambda$He, and $^5\Lambda$He hypernuclei. Such a statement is supported by the results shown in Fig. 1 for the kinetic energy spectra, and in Figs. 2, and 3 for $pn$ coincidence spectra. In spite of this agreement we feel that a more realistic description of the SRC, such as done in Refs. [12–17] by means of the DQ weak transition potential, may help us to better understand the baryon–baryon strangeness–flipping interaction.

Definitively, the coincidence $nn$ data can not be explained theoretically neither in $^4\Lambda$He nor in $^5\Lambda$He. On the other hand, due to the lack of data nothing can be said regarding $^4\Lambda$H. We note, however, that both the single and coincidence neutron spectra in this case are of the same order of magnitude as in the other two hypernuclei, and that therefore it would be extremely useful to measure them experimentally.

Despite having achieved a relatively fair agreement with the data for several exclusive observable, such as proton and neutron kinetic energy spectra and the $pn$ coincidence spectra, the question arises whether it is legitimate to use the SM, and in particular the harmonic oscillator wave functions for systems as small as $^3\Lambda$H, $^4\Lambda$He, and $^5\Lambda$He. Moreover, as pointed out by Jun [31], the residual nucleons are in an unbound state for some channels, and therefore they are very different from the initial nucleons which do not take part in the decay process. Then, it could be appropriate to treat the four- and five-body nature of these NMWD explicitly, as was done for instance in the evaluation of separation energies in single and double strange hypernuclei [41–47], and in ab initio calculations of four-nucleon scattering [48–50]. (The shell model yields quite nice results for light $S = −1$, −2 hypernuclei [51,52].) It would be very interesting to analyze whether it is possible to account for the experimental nn coincidence spectra through such microscopic calculations. An undertaking of this nature is, however, beyond our present means.

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