Influence lines for schedule networks

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Abstract. An influence line graphs for project planning show how the changes of a selected activity duration affect the project duration. The name comes from the structural analysis where influence lines show on the ordinates of a graph the variation of the magnitude of, e.g., the bending moment at a specific point as a unit load traverses across the structure. This paper examines the principles of project planning influence lines: it discusses how to depict the six criticality types of activities, and it also shows what kind of transitions are possible among the criticality types. The findings help the greater understanding of the effects of the activity duration changes on the project duration.

Keywords: CPM, Precedence Diagramming Method (PDM), Influence lines

1. Introduction and literature review

One of the most crucial tasks of the planning process is to adjust the project duration to be within the expected project deadline. For speeding up projects, planners usually apply two types of actions: either select some from the critical activities, modify their durations, or change the project's logic, thus creating new and shorter critical paths. The recently developed influence lines can help to speed up this process. The influence line for an activity shows how a selected activity's duration changes affect the project duration. Fig 1 presents an example of the influence line.

Figure 1. An example for influence line.
Figure 1 shows a network with three activities A, B, and C, and four precedence relationships. The project duration is 19 days. The influence line for activity B shows the project duration for different activity durations.

A one-day decrement (from five to four days) in B's duration accelerates the project; two to four days decrements do not result in any further changes; it leaves the project duration unaltered. However, an additional one-day decrement in the duration, from one day to zero, increases the project duration. Increasing the duration of B with any amount results in the same amount of increment in the project duration.

Influence lines for project planning result from Hajdu [1]. The authors have shown that activity criticality types can change, and they introduced the influence lines as a tool to visualize the effects of changing duration on the project duration. Different types of criticality were first discussed by Weist [2]. He has shown that in Precedence Diagramming Method (PDM) [3], [4], [5], changing activity durations can have a diverse effect on project duration. He described only four criticality types in his paper: the normal critical, the reverse critical, the neutral critical, and the bi-critical (called perverse critical in the article). However, contrary to Weist's results, six criticality types exist in PDM, even in the simplest case [1]. The simplest case assumes that activities are non-stretchable and non-splittable, and only the well-known four precedence relationships, namely Start-to-Start (SS), Finish-to-Start (FS), Finish-to-Finish (FF), and Start-to-Finish (SF), are allowed. The influence lines graphically display the effect of changing activity durations on the project duration; therefore, they point out the dynamically changing nature of criticality. However, Hajdu et al. did not discuss the internal rules that define the shape of influence lines. The goal of this research is to fill this research gap.

2. Research goals and limitations
This research aims to define the rules shaping the form of the influence lines in PDM networks. The simplest version of PDM is the subject of the investigation where

- activities are assumed to be linear, non-splittable, and non-stretchable;
- only the four minimal end-point relationships are allowed;
- calendars are not allowed;
- hard constraints (such as Must Start at, Start Earlier than) are not allowed;
- resources and their limitations do not affect the results of the time analysis.

One can see that the different generalizations of the PDM method are not allowed. Extending the scope of the study for these generalizations can serve as possibilities for future research.

3. Depicting criticality types in influence lines
Criticality types of critical activities indicate how the project duration changes if increasing or decreasing the activity's duration by one day. (For the sake of simplicity, duration and relationship lags are given in days.) Table 1. shows the six criticality types.

| Criticality types | Decreasing activity duration | No effect on project duration |
|------------------|-----------------------------|-----------------------------|
| Increasing activity duration | | |
| Increasing project duration | Bi-critical | Normal critical | Increasing normal – decreasing neutral |
| Decreasing project duration | Reverse critical | | |
| No effect on project duration | Increasing neutral – decreasing reverse | Neutral critical | |

Criticality types of critical activities can be defined based on the incoming and outgoing critical paths. In the case of end-point relationships (SS, FF, FS, SF), critical paths can enter a critical activity
at its start, finish, or both points and leave at its start, finish, or both points (Figure 2). The nine possible arrangements define six criticality types. Relationships in red stand for the critical relationships, the black ones represent the non-critical relationships.

![Figure 2](image)

**Figure 2.** Criticality types based on the incoming and outgoing critical paths.

### 3.1. Normal critical activities

Activities are normal critical if the critical path enters at the start of the activity and leaves at its finish (Figure 2a). Increasing normal critical activities' duration by one day increases the project duration by one day. In comparison, a one-day decrement in the activities' duration results in decreasing project duration by one day. Figure 3 illustrates this case.

![Figure 3](image)

**Figure 3.** Example for normal critical activity and its influence line.

### 3.2. Neutral critical activities

An activity is neutral critical if the critical path enters and leaves the activity at the same point, either at the start or at the finish (Figure 2.b-c)). In this case, changing the activity's duration does not affect the project's duration. Figure 4 demonstrates the first case.

![Figure 4](image)
3.3. Reverse critical activities

Reverse critical activities have the critical path of entering at their finish and leaving at their start (figure 2.d). As figure 5 presents, in this case, the project duration moves against the direction of the activity duration.

3.4. Normal for an increase – neutral for a decrease critical activities

This situation emerges as a combination of normal and neutral criticality (figure 2.e-f). The first critical path enters at the start of the activity and leaves at the finish, similar to the normal critical activities. The third critical relationship is either entering and the finish or leaving at the start. The effect on the project duration is also the combination of normal and neural criticalities.

Figure 4. Neutral criticality type based on figure 2.b).

Figure 5. Reverse criticality type.

Figure 6. Normal for an increase -neutral for a decrease criticality type based on figure 2.f)
Normal criticality governs the changes of the project duration if the duration of the selected activity is increasing. If activity duration is decreasing, then neutral criticality defines the changes of the project duration. Figure 6. explains the joint effect of these two criticalities on the project duration.

3.5. Increasing neutral – decreasing reverse
Increasing neutral – decreasing reverse activities have the critical path entering at their finish and leaving at their start like reverse critical activities have. The third critical relationship can enter at the start (figure 2.g) or leave at the finish (figure 2.h). In this case, increasing the activity duration does not affect the project duration, while decreasing the activity duration results in increasing project duration. This is shown in figure 7.

![Figure 7. Neutral for an increase-reverse for a decrease criticality type based on figure 2.g.](image)

3.6. Bi-critical
If the critical path enters and leaves at both start and finish, then the activity is bi-critical. It is normal, reverse (and neutral) at the same time. Increasing or decreasing a bi-critical activity's duration both result in increasing project duration. This criticality type is presented in figure 8.

![Figure 8. Bi-critical activity type.](image)

4. Rules governing the shape of the influence lines
The transition between criticality types follows specific rules. Increasing and decreasing the duration of the activity can result in changing the criticality type of activities. Table 2. shows the possible transitions from a criticality type to another.
Table 2. Transitions between criticality types.

| Criticality type     | Increasing activity duration | Decreasing activity duration |
|----------------------|------------------------------|-----------------------------|
| Normal critical      | no change                    | normal-neutral or bi-critical|
| Neutral critical     | normal-neutral               | neutral-reverse             |
| Reverse critical     | neutral-reverse or bi-critical| no change                   |
| Normal-neutral       | normal                       | neutral-reverse             |
| Neutral-reverse      | neutral-normal               | reverse                      |
| Bi-critical          | normal                       | reverse                      |

4.1. Possible changes for normal critical activities

Increasing the activity duration of normal critical activities increases the project duration, and the criticality type does not change because the critical path still enters at the start of the activity and leaves at its finish. Decrease in activity duration results in decreasing project duration to a point where the activity becomes normal-neutral or bi-critical: the critical path leaves not just at the start but also at the finish of the activity. Figure 9 illustrates the two possible transitions from normal to normal-neutral (figure 9.a)) and bi-directional criticality types (figure 9.b)).

4.2. Neutral critical

The critical path enters and leaves the activity at the same point, so the project duration does not change until the activity duration is increased or decreased to a point where a third relation becomes critical. An increment in the activity duration results in a normal-neutral (further increment changes it to normal critical) activity, while decreasing the activity duration leads to a neutral-reverse (with further decrement reverse critical) activity. The effects of changing the duration are shown in figure 4., the influence line is shown in figure 10.

4.3. Reverse critical

Reverse critical activities act the opposite way as normal criticals. Increasing a reverse critical activity's duration decreases the project duration until the critical path entering at their finish and leaving at their
start also enters at their start, where the activity becomes neutral-reverse or bi-critical. Decreasing the activity’s duration, on the contrary, increases the project duration. The changing activity durations are shown in figure 5., the influence line changes are presented in figure 11.a) and 11.b).

![Figure 11. Influence line changes for reverse criticality type.](image1)

4.4. Increasing normal – decreasing neutral
Combined critical activities have at least three critical connections. Changing the duration of the activity can result in three or two critical connections. Increasing the activity duration causes the criticality type to become normal critical, with the critical path entering at the start of the activity and leaving at its end. Decreasing the activity duration makes no change in the project duration until the previously non-critical connection becomes critical, and the criticality type changes to neutral-reverse (then reverse). The effect of changing the activity duration can be inspected in figure 6., the influence line is shown in figure 12.

![Figure 12. Influence line changes for normal-neutral criticality type](image2)

4.5. Increasing neutral – decreasing reverse
Increasing neutral – decreasing reverse activities act very similarly to the normal-neutral criticality, only contrariwise. In this case, increasing the activity duration does not affect the project duration until the critical path leaves the activity at the same point and after entering it. At this point, the activity is normal-neutral. Any further increase in the duration changes it to normal critical. Decreasing the activity duration eliminates one of the critical connections: the critical path enters at the finish of the activity and leaves at its start. The criticality type changes to reverse critical. This is presented in figure 7., the influence line is depicted in figure 13.
Figure 13. Influence line changes for neutral-reverse criticality type

Figure 14. Influence line changes for bi-critical criticality type

4.6. Bi-critical
An activity that has the critical path entering and leaving both at the start and the finish points reacts to changing its duration with changing some of its critical relationships to uncritical. Increasing the activity duration results in a normal critical activity, which has the critical path entering at its start and leaving at its finish. In the case of normal critical activities, the project duration increases when the activity duration is increased. A decrement of the activity duration leads to reverse criticality, that again increases project duration. The criticality type with changing activity duration can be inspected in figure 8. The influence line is presented in figure 14.

5. Conclusions and further research
This research established that:
- influence lines are convex;
- they have zero, one, or a maximum of two break-points in the case of end-point relationships
- they are linear or piece-wise linear when activities are linear, non-splittable, non-stretchable, and relationships are the traditional end-point relationships.

Further research direction can be the description of the internal rules of influence lines where the generalizations listed in the ‘Research objectives and limitations’ section are allowed in the schedule network.

Acknowledgment
The research reported in this paper has been supported by the National Research, Development, and Innovation Fund (TUDFO/51757/2019-ITM, Thematic Excellence Program.)

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