Suppression of dephasing by qubit motion in superconducting circuits

D. V. Averin1,*, K. Xu2, Y. P. Zhong2, C. Song2, H. Wang2,†, and Siyuan Han3†

1Department of Physics and Astronomy, Stony Brook University, SUNY, Stony Brook, NY 11794-3800, USA
2Department of Physics, Zhejiang University, Hangzhou, Zhejiang 310027, China
3Department of Physics and Astronomy, University of Kansas, Lawrence, KS 66045, USA

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We suggest and demonstrate a protocol which suppresses dephasing due to the low-frequency noise by qubit motion, i.e., transfer of the logical qubit of information in a system of $n \geq 2$ physical qubits. The protocol requires only the nearest-neighbor coupling and is applicable to different qubit structures. We further analyze its effectiveness against noises with arbitrary correlations. Our analysis, together with experiments using up to three superconducting qubits, shows that for the realistic uncorrelated noises, qubit motion increases the dephasing time of the logical qubit as $\sqrt{n}$. In general, the protocol provides a diagnostic tool to measure the noise correlations.

Development of superconducting qubits [1–7] have reached the stage when it is interesting to discuss possible architectures of the quantum information processing circuits. The common feature of any quantum computation process of even moderate complexity is the requirement of information transfer between different elements of the qubit circuit. The most straightforward way of achieving this transfer is to physically move the quantum states representing the qubits of information along the circuit. In the case of superconducting qubits, potential for such a direct motion of logical qubits is offered by the so-called nSQUIDs [8, 9], but operation of these circuits in the quantum regime [10] still needs to be demonstrated experimentally. Another method of transferring logical qubits between different physical qubits, already developed in experiments and adopted in this work, is based on creating controlled qubit-qubit interaction through coupling to a common resonator bus [5, 11–13]. The goal of this work is to demonstrate that, in addition to its main function, transfer of information between different circuit elements designed to perform different functions, have an additional notable benefit: suppression of the low-frequency dephasing. We also show that it can be used to measure the noise correlations and, in this way, diagnose the primary sources of the noises.

The basic mechanism of the noise suppression by qubit motion relies on the fact that the low-frequency noise is typically produced by fluctuators - see, e.g., [14, 15], in the form of impurity charges or magnetic moments, localized in each individual physical qubit, and therefore, is not correlated among them. Motion of a logical qubit between different physical qubits limits the correlation time of the effective noise seen by this qubit, and therefore suppresses its decoherence rate. This effect is qualitatively similar to the motional narrowing of the NMR lines [16], with the main difference that it is based on the controlled transfer of the qubit state, not random thermal motion as in NMR. Also, since the effectiveness of this mechanism is sensitive to the noise correlations not only in time, but in space, it can be used to investigate the distribution of the primary sources of noises in quantum circuits, promising a fast and reliable noise diagnostic tool and, ultimately, improving the circuit performance.

Quantitatively, we start with the basic model of dephasing in a system of $n$ physical qubits, where each qubit is coupled to a source of Gaussian fluctuations $\xi_j(t)$, $j = 1, \ldots, n$, of the energy difference between the computational basis states:

$$H_{dec} = -\frac{1}{2} \sum_{j=1}^{n} \sigma_j^z \xi_j(t),$$

$$\langle \xi_j(0) \xi_k(t) \rangle = \int \frac{d\omega}{2\pi} S_{j,k}(\omega) e^{-i\omega t}. \quad (1)$$

Here $\sigma_j^z$ is the $z$ Pauli matrix of the $j$th qubit, $S_{j,j}(\omega) \equiv \text{spectral density of noise } \xi_j(t)$ in the $j$th qubit, the terms $S_{j,k}(\omega)$, with $j \neq k$, account for the noise correlations in different qubits, and we set $\hbar = 1$. The qubits are assumed to be free, i.e., (1) is the only part of the system Hamiltonian that depends on the qubit variables.

If a logical qubit, $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$, is prepared at time $t = 0$ as an initial state of the $j$th physical qubit and is kept there for a period $\tau$, it will decohere due to the noise $\xi_j(t)$. This decoherence process can be characterized quantitatively by the function $F(\tau)$, defined as

$$F(\tau) = \frac{\sigma_j(\tau)}{\sigma_j(0)}, \quad \sigma_j(\tau) = \text{Tr}\{\sigma_j^z(\tau)\rho\}, \quad (2)$$

where $\rho$ is the initial density matrix of the system, which consists of the qubit part and the part $\rho_{env}$ describing the noise source:

$$\rho = |\Psi\rangle\langle \Psi | \otimes \prod_{k \neq j} |0\rangle\langle 0 | \otimes \rho_{env}. \quad \text{Time dependence of the raising Pauli matrix, } \sigma_j^{+} = (\sigma_j^{x} + i\sigma_j^{y})/2, \text{ of the } j\text{th qubit is governed by the Heisenberg equation of motion that follows from the Hamiltonian (1): } \sigma_j^{+}(t) = -i\xi_j(t)\sigma_j^{+}(t), \text{ and gives, as usual, }$$

$$F(\tau) = \langle T \exp\{-i \int_{0}^{\tau} \xi_j(t) dt\} \rangle = \exp\{-\int_{0}^{\tau} dt \int_{0}^{t} d\tau' \langle \xi_j(t)\xi_j(t') \rangle \}. \quad (3)$$
Here $T$ denotes the time-ordering operator, and $\langle \ldots \rangle$ – averaging over the noise source $\rho_{\text{env}}$. Experimentally, the function $F(\tau)$ is obtained by measuring the Ramsey fringes.

On the other hand, we can arrange the situation, when the logical qubit $|\Psi\rangle$, instead of staying just in one physical qubit for the entire time interval $\tau$, is transferred successively from qubit 1 to qubit $n$ spending the time $\tau/n$ in each of them, while the transfer processes themselves are done much faster than $\tau/n$. Such transfers can be achieved, e.g., by applying SWAP gates to the successive pairs of physical qubits. Then, if the transfers are done accurately, so that the dephasing during them is negligible, the decoherence of the logical qubit $|\Psi\rangle$ in the total time $\tau$ is

$$F(\tau) = \exp \left\{- \frac{1}{\pi} \int_0^{\tau} d\tilde{\omega} \frac{\sin^2(\omega \tau/2n)}{\omega^2} \sum_{j=1}^n S_j(\omega) \right\}.$$  \tag{4}$$

If the noises are low-frequency and uncorrelated at different qubits, decoherence is suppressed with increasing number $n$ of the physical qubits. Indeed, in this regime, it is appropriate to neglect the quantum part of the noise and the second sum in Eq. (4) which reduces to

$$F(\tau) = \exp \left\{- \frac{1}{\pi} \int_0^{\tau} d\omega \frac{\sin^2(\omega \tau/2n)}{\omega^2} \sum_{j=1}^n S_j(\omega) \right\}.$$  \tag{5}$$

The low-frequency dephasing is obtained then by expanding sine in $\omega$ and keeping the first term:

$$F(\tau) = \exp \left\{- \frac{\tau^2}{2n^2} \sum_{j=1}^n W_j^2 \right\}, \quad W_j^2 = \int_0^{\omega_h} \frac{d\omega}{2\pi} S_j(\omega).$$  \tag{6}$$

For the experimentally relevant $1/f$ noise, $S_j(\omega) = A_j/|\omega|$, the last approximation applies directly if the high-frequency cutoff of the noise $\omega_h$ satisfies the condition $\tau/n \ll 1/\omega_h$. As shown in the Supplementary Material [17], even in the opposite regime, there are only weak logarithmic correction to scaling of the dephasing time with $n$, and the main conclusion remains the same. The low-frequency cutoff $\omega_h$ can be estimated as inverse of the time of the experiment, and $W_j^2 = (A_j/\pi) \ln(\omega_h/\omega_i)$. If all physical qubits have the same decoherence properties, $W_j = W$, we can rewrite Eq. (5) as

$$F(\tau) = e^{-(\tau/\tau_d)^2}, \quad \tau_d = \sqrt{2n/W},$$  \tag{7}$$

and see that the dephasing time $\tau_d$ of the moving qubit increases in comparison to the stationary qubit as $\sqrt{n}$.

If the noises at different physical qubits are correlated, one needs to take into account both sums in Eq. (4). In this case, under the same assumptions as above, the dephasing time in Eq. (7) can be written as

$$\frac{1}{\tau_d} = \frac{W^2}{2n^2} \left[ n + 2 \sum_{j<k} r_{j,k} \right],$$  \tag{8}$$

where the coefficient $r_{j,k}$ describe the degree of noise correlations between the $j$th and the $k$th qubit. They are defined by the relation $S_{j,k}(\omega) = r_{j,k} S(\omega)$, and have the property $|r_{j,k}| \leq 1$, with $r_{j,j} = 1$ corresponding to full correlations, and $r_{j,k} = -1$ describing full anticorrelations [18]. Equation (8) shows that if all noises are fully correlated, then $\tau_d = \sqrt{2/W}$, and qubit motion does not produce any suppression of dephasing. If the noises are completely anticorrelated between the nearest neighbors along the qubit array, the dephasing is suppressed even more strongly than in the absence of correlations. In this case, Eq. (8) gives fully suppressed dephasing for even $n$, and $\tau_d = \sqrt{2n/W}$ for odd $n$.

To test experimentally the mechanism of dephasing suppression by qubit motion as discussed above, we perform the Ramsey fringe experiments (Fig. 1(b)) [19] using up to three superconducting qubits, among which the initial logical qubit state $|\Psi\rangle = |0\rangle - i |1\rangle$ (here and below we ignore the normalization constant) is relayed and its phase information is probed after the total relay time $\tau$. We use two types of superconducting circuits in which dephasing noises differ very much in magnitude: one features three phase qubits, each capacitively coupled to a common resonator [20, 21] (Fig. 1(a)), and the other one
For the phase qubit circuit, $\omega^*/2\pi = 6.22$ GHz and $\lambda/2\pi = 15.5$ MHz. The operation frequencies of qubits $q_1, q_2,$ and $q_3$ are chosen at 5.99, 6.04, and 6.06 GHz, respectively, for their dephasing times $T_2^*$ to be about the same. Corresponding energy relaxation times $T_1$ are 512 $\pm$ 6, 538 $\pm$ 6, and 488 $\pm$ 4 ns. The dephasing times $T_2^*$ are $173 \pm 2, 177 \pm 1,$ and $176 \pm 2$ ns by fitting to $\ln[P_1(\tau)] \propto -\tau/2T_1 - (\tau/T_2^*)^2$, where $P_1$ is the $\{1\}$-state probability in the Ramsey fringe experiment [22]. Since three qubits have similar $T_2^*$ values, we expect that the noise power spectral densities $S_j(\omega)$ ($j = 1, 2,$ and 3), which characterize the flux-noise environments of these qubits, are approximately at the same level [22–24].

At its operation frequency each qubit is effectively decoupled from the resonator. If qubit $q_1$ is in $|0\rangle - i|1\rangle$ and resonator $r$ is in $|0\rangle$, we can turn on the qubit-resonator interaction by rapidly matching the qubit frequency to that of the resonator for a controlled amount of time, fulfilling an iSWAP gate [25] to transfer the state from $q_1$ to resonator $r$, i.e., $(|0\rangle - i|1\rangle)_q_1 |0\rangle_r \rightarrow |0\rangle_q_1 (|0\rangle + i|1\rangle)_r$. Immediately after the first iSWAP gate, we bring qubit $q_2$, originally in $|0\rangle$, on resonance with resonator $r$ for another iSWAP gate. As such, other than a phase factor, we effectively relay the logical qubit state between the two qubits, i.e., $(|0\rangle - i|1\rangle)_q_1 |0\rangle_q_2 \rightarrow |0\rangle_q_1 (|0\rangle + i|1\rangle)_q_2$. [21]

For the phase qubit circuit, an iSWAP gate takes about $16$ ns and the total time for transferring the state from one qubit to the other qubit is about $32$ ns. We measure the Ramsey fringe of the logical qubit which spends equal amount of time in each of the $n \geq 2$ physical qubits. The sequences are illustrated in Fig. 1(c). The obtained Ramsey fringe is fitted according to [22],

$$P_1(\tau) = A \exp \left[ -\frac{\tau}{2T_1^{ave}} - \left( \frac{\tau}{\tau_d} \right)^2 \right] \cos(\omega R \tau + B) + C,$$

(10)

where $T_1^{ave}$ is fixed as the average of all qubits involved and $\tau_d$ is the effective dephasing time for the logical qubit as obtained from the fit (so are the constants $A, B, C,$ and $\omega_R$). Representative experimental data and fitting curves are shown in Fig. 2.

Controlled motion of the logical qubit are attempted under various experimental conditions. Table I lists $\tau_d$ values of the logical qubit obtained using different experimental sequences and different qubit combinations. The Ramsey fringe measurements using two qubits (three qubits) show that dephasing times of the logic qubits are extended to about $244.0 \pm 3.1$ ns ($297.2 \pm 5.5$ ns), averaged a gain by a factor of $1.392 = 0.984\sqrt{2}$ ($1.695 = 0.979\sqrt{3}$) compared with those from the single-qubit measurements ($175.3 \pm 2.3$ ns). As expected from Eq. (5), the logic qubit dephasing time $\tau_q$ scales very well with the square root of the number of physical qubits, $\sqrt{n}$. The similar scaling is also observed using two Xmon qubits, where the single-qubit $T_2^*$ values are about $1$ $\mu$s, achieving a gain of $1.405 = 0.993\sqrt{2}$ (Ramsey fringe data not shown).

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![FIG. 2: (color online) The Ramsey fringe experimental data for sequences shown in Fig. 1. Red lines are fits according to Eq. (10). $T_2^*$ for single qubit can be directly compared with $\tau_d$ for multiple qubits. Statistical errors, from the measured probability spread of $\sim 2\%$, are omitted for display clarity, but are used to estimate the standard deviations of $T_2^*$ and $\tau_d$.](image)

**TABLE I: $T_2^*$ values for single qubit and $\tau_d$ values for different experimental sequences and different qubit combinations as outlined in Fig. 1. Numbers in brackets represent standard deviations.**

| 1-qubit $T_2^*$ (ns) | 2-qubit $\tau_d$ (ns) | 3-qubit $\tau_d$ (ns) |
|----------------------|----------------------|----------------------|
| q1: 173 (2)         | q2→q1: 249 (3)       | q3→q2→q1: 298 (4)    |
| q2: 177 (1)         | q3→q1: 243 (3)       | q2→q3→q1: 295 (4)    |
| q3: 176 (2)         | q1→q2: 245 (3)       | q3→q1→q2: 306 (4)    |
|                     | q3→q2: 242 (3)       | q1→q3→q2: 290 (5)    |
|                     | q1→q3: 244 (2)       | q2→q1→q3: 298 (4)    |
|                     | q3→q2→q3: 241 (4)    | q1→q2→q3: 296 (4)    |
| average: 175.3 (2.3)| 244.0 (3.1)          | 297.2 (5.5)          |

**features two Xmon qubits with much reduced dephasing noise, each as well capacitively coupled to a common resonator. The Hamiltonian of these quantum circuits is**

$$H = -\frac{1}{2} \sum_{j=1}^{n} \omega_j^0 \sigma_j^z + \omega^* a^\dagger a + \sum_{j=1}^{n} \lambda_j (a\sigma_j^+ + a^\dagger \sigma_j^-),$$

(9)

where the resonator frequency $\omega^*$ is fixed by circuit design, the qubit frequency $\omega_j^0$ is individually tunable, $\lambda_j$ ($\equiv \lambda$ under the homogeneous condition and $\ll \omega^*, \omega_j^0$) describes the qubit-resonator coupling strength whose magnitude is also fixed by circuit design, and $a^\dagger (a)$ is the creation (annihilation) operator of the resonator field. $n (= 1, 2,$ or 3) refers to the total number of physical qubits involved in each experimental sequence.
Our result clearly demonstrates that dephasing caused by uncorrelated low-frequency noises can be reduced by a factor of $\sqrt{n}$ by moving the logic qubit state along an array of $n \geq 2$ physical qubits.

The result demonstrated above is based on the fact that noises at different qubits were completely uncorrelated. In general, degree of the noise suppression by qubit motion method depends on the noise correlation magnitude. Assuming that the noise environments $S_j(\omega)$, $j = 1$ and 2, for two qubits are at the same level, Eq. (8) is reduced to

$$\frac{1}{\tau_d^2} = \frac{W^2}{4} (1 + r_c), \quad \tau_d = \sqrt{\frac{2}{1 + r_c}} T^*_2, \quad (11)$$

where $T^*_2 = \sqrt{2}/W$ and $r_c \equiv r_{1,2}$. The monotonous dependence of $\tau_d$ on the correlation coefficient $r_c$ of the two-qubit noises provides a much needed guide for measuring noise correlations. Since the Ramsey fringe measurement is much faster than the conventional two-point correlation measurement [23, 27], the latter of which usually takes at least a few hours in order to cover a wide range in spectrum, the mechanism of qubit motion may provide a fast and reliable diagnostic tool for identifying the primary sources of noises in complex quantum circuits.

We experimentally emulate the monotonous dependence of $\tau_d$ on $r_c$ in Eq. (11) using two Xmon qubits, where the much reduced intrinsic dephasing noises make it easier to inject controllable noises [26]. Here the intrinsic noises refer to any noises associated with the device or measurement setup, in contrast to the extrinsic ones that specifically refer to our controlled noises. We first set the operation frequencies of the two Xmon qubits, exposing them to the same level of intrinsic noise environments, characterized by $S_j^I(\omega)$, $j = 1$ and 2. We then apply strong low-frequency noises, digitally synthesized with an adjustable correlation coefficient $r_c$, to the two qubits so that both qubits’ dephasing rates are dominated by these extrinsic noises. It is verified that for each qubit $T^*_2$ is reduced to about 220 ns due to the combination of the noise powers of $S_j^I(\omega)$ and $S_j^E(\omega)$, $j = 1$ or 2, where $S_j^E(\omega)$ characterizes the synthesized noise power spectral densities (see Supplemental Material [17]). Synthesized noises are simultaneously applied during the 2-qubit Ramsey fringe experiments. Resulted Ramsey fringes shown in Fig. 3(a) can be used to estimate $\tau_d^E$, the logic-qubit dephasing time determined by both $S_j^E(\omega)$ and $S_j^I(\omega)$, $j = 1$ and 2. It is observed that $\tau_d^E$ increases monotonically when the correlation coefficient $r_c$ changes from 1 (perfectly correlated) to −1 (anti-correlated), in agreement with Eq. (11). In Fig. 3(b) we plot $(1/\tau_d^2)^2 - (1/\tau_d^E)^2$ versus $r_c$ (black squares with error bars), where $\tau_d^E$ is the logic-qubit dephasing time dominated only by $S_j^I(\omega)$, $j = 1$ and 2, as measured with the 2-qubit Ramsey sequence under no extrinsic noises. Also shown in Fig. 3(b) are the numerical simulation results.

The experimental and simulation data are slightly different from the prediction by Eq. (11), likely due to the fact that experimentally synthesized extrinsic noises only cover down to 10 kHz in spectrum as limited by hardware resource (see Supplemental Material [17]), while Eq. (12) works better for lower-frequency noises.

To summarize, we propose a new scheme to suppress low-frequency induced dephasing of logic qubit states by moving the quantum information along an array of $n \geq 2$ physical qubits. We have shown that in general qubit motion can make dephasing time $\tau_d$ of the logic qubits longer than that of the physical qubits $T^*_2$, as long as noises on physical qubits are not completely correlated. For uncorrelated noises, our model predicts a simple scaling $\tau_d = \sqrt{n} T^*_2$. We have experimentally implemented the qubit motion scheme to suppress dephasing using superconducting circuits with up to $n = 3$ Josephson qubits. The results agree very well with that expected from the model. Furthermore, using synthesized noises we have demonstrated that measuring the ratio $\tau_d/T^*_2$ allows one to determine quantitatively...
the correlation coefficient $r_c$ between noises on two physical qubits, which is difficult to obtain with conventional methods. The qubit motion method can be readily applied to more qubits to further suppress dephasing and it is straightforward to incorporate qubit motion with quantum gate operations on logic qubits. Our results thus open a new venue for improving performance of logic qubits and gaining insight on low-frequency noises in complex quantum information processing circuits.

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* Electronic address: dmitri.averin@stonybrook.edu
† Electronic address: hhwang@zju.edu.cn
‡ Electronic address: han@kansas.edu

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