The impact of dark energy perturbations on the growth index

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We show that in clustering dark energy models the growth index of linear matter perturbations, \( \gamma \), can be much lower than in \( \Lambda \)CDM or smooth quintessence models and present a strong variation with redshift. We find that the impact of dark energy perturbations on \( \gamma \) is enhanced if the dark energy equation of state has a large and rapid decay at low redshift. We study four different models with these features and show that we may have \( 0.33 < \gamma(z) < 0.48 \) at \( 0 < z < 3 \). We also show that the constant \( \gamma \) parametrization for the growth rate, \( f = d\ln \delta_m/d\ln a = \Omega_m^0 \), is a few percent inaccurate for such models and that a redshift dependent parametrization for \( \gamma \) can provide about four times more accurate fits for \( f \). We discuss the robustness of the growth index to distinguish between General Relativity with clustering dark energy and modified gravity models, finding that some \( f(R) \) and clustering dark energy models can present similar values for \( \gamma \).

I. INTRODUCTION

The understanding of accelerated expansion of the universe is one of the greatest challenges in physics. It may be caused by a yet unknown form of energy with negative pressure, generically called dark energy, or by a new theory of gravity and space-time. Since different theoretical scenarios and their various models are designed to reproduce the expansion history, from the observational point of view we certainly need to analyze the evolution of cosmological perturbations in order to able to falsify the models and have a better indication of what drives the accelerated expansion.

One particular simple and powerful tool to study the linear growth of cosmic structures and discriminate between the various theoretical possibilities is the growth rate of matter perturbations, \( f = d\ln \delta_m/d\ln a \), which is usually parametrized by \( f = \Omega_m^0 \), where \( \Omega_m \) is the matter density parameter and \( \gamma \) the so-called growth index. It is well-known that \( \Lambda \)CDM and quintessence models (minimally coupled canonical scalar fields) can be well described by a scale independent and constant \( \gamma \approx 0.55 \) with very small corrections \[3\]. Since the underlying theory of gravity in such models is the General Relativity, it is usual to refer to \( \gamma_{GR} \approx 0.55 \) as the value of the growth index in General Relativity. On the other hand, modified gravity models such as \( f(R) \) models may have \( 0.40 \lesssim \gamma \lesssim 0.43 \) at \( z = 0 \), besides being scale dependent \[5\], \[6\], and Dvali-Gabadadze-Porrati (DGP) braneworld gravity model \[7\] has \( \gamma \approx 0.68 \) \[3\]. It has been shown by reference \[8\] that, ongoing observational projects, like Dark Energy Survey \[9\], should be able distinguish a deviation of 0.179 from \( \gamma_{GR} \) and future space based projects, like Euclid \[10\], a deviation of 0.048, which proves the power of the growth index to falsify the various models of cosmic acceleration.

In the context of quintessence models, dark energy perturbations are relevant for dark matter growth only on Hubble scales and then are usually neglected on small scales, which are the observationally relevant for the determination of \( f \). This is due to the fact that quintessence perturbations have unitary effective sound speed (in its rest frame), \( c_{\text{eff}}^2 = (\delta\rho/\delta\rho)_{\text{rest}} = 1 \) \[11\]. Hence its sound horizon, \( \sim c_{\text{eff}} H^{-1} \), is of the order of the Hubble radius and dark energy perturbations are strongly suppressed on smaller scales. However there are models of dark energy based on non-canonical scalar fields in which the effective sound speed is variable and even negligible \[12\] \[10\]. When \( c_{\text{eff}} \ll 1 \) dark energy perturbations grow at the same pace as matter perturbations \[17\] \[18\] and can be as large as them when \( w = p_c/\rho_c \approx 0 \) \[19\], like during the matter dominated period in Early Dark Energy models. Hence it is possible that in such models the growth of matter perturbations is quite different from quintessence or \( \Lambda \)CDM models. In this case we have to ask whether the values of \( \gamma \) could be mistaken for modified gravity models.

The issue of how to differentiate a new energy component from a new gravity theory has been discussed a lot in the literature, e.g., \[20\] \[23\]. Although most smooth dark energy models can be distinguished from modified gravity models, if dark energy can cluster this task can be much more difficult. However reference \[23\] has recently claimed that clustering dark energy models do not impact the growth index severely, and its value remain near \( \Lambda \)CDM one, so these models could be easily distinguished from modified gravity. Moreover it was found that clustering dark energy models with constant equation of state differ only about 5% from \( \gamma_{GR} \) \[24\].

In this paper we will show that if clustering dark energy is allowed to have a large and rapid decay of its equation of state at low redshift, a situation that was not considered in references \[23\] \[24\], then dark energy perturbations will strongly impact the growth index and it would be difficult to distinguish this model from some \( f(R) \) models.

This paper is organized as follows. In section II we present the background evolution of four dark energy

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models that we will use to illustrate the impact of dark energy perturbations on the growth index. In section III we present the equations which we use to evolve the linear perturbations. The results for the growth index and the comparison with some modified gravity models are shown in section IV. We present the conclusions in section V.

II. BACKGROUND EVOLUTION

We study a background evolution given by a flat FRW model with pressureless matter (dark matter plus baryons) and dark energy with equation of state parameter \( w = p_e/\rho_e \), then Friedman equations are given by:

\[
\mathcal{H}^2 = \frac{8\pi G}{3} a^2 (\rho_m + \rho_e) \tag{1}
\]

and

\[
\dot{\mathcal{H}} = -\frac{4\pi G}{3} a^2 [\rho_m + \rho_e (1 + 3w)] , \tag{2}
\]

where \( \mathcal{H} = \dot{a}/a \), the dots represent the derivative with respect to conformal time. As usual, matter and dark energy density parameters are given by \( \Omega_m (a) = \rho_m (a)/\rho_c (a) \) and \( \Omega_e (a) = \rho_e (a)/\rho_c (a) \), where \( \rho_c \) is the critical density, \( \rho_c (a) = 3H^2/8\pi G \), and \( H = \mathcal{H}/a \) is the Hubble parameter as a function of the physical time.

Reference [24] analyzed dark energy models using the Chevallier-Polarski-Linder (CPL) parametrization [25, 26]:

\[
w = w_0 + w_a (1 - a) . \tag{3}
\]

Although it can accurately describe the background evolution of many quintessence models and some modified gravity models, it can not reproduce a rapid transition of \( w \) at low redshift. Incidentally, as we will show, if \( w \) is allowed to have a rapid transition at low redshift, dark energy perturbations may strongly impact the growth of matter perturbations. In order to demonstrate this we will use the following parametrization [27]:

\[
w (a) = w_0 + (w_m - w_0) \frac{1 + \exp \left( \frac{a}{\Delta_m} \right)}{1 + \exp \left( -\frac{a - a_c}{\Delta_m} \right)} , \tag{4}
\]

where \( w_m \) is the limiting value of \( w \) in the matter dominated era, \( w_0 \) is the the value of \( w \) now \( (a = 1 \text{ or } z = 0) \), \( a_c \) is the moment of the transition from \( w_0 \) and \( w_m \) and \( \Delta_m \) is the duration of this transition. The energy density of dark energy is given by:

\[
\rho_e (a) = \rho_c^e \exp \left( -3 \int_1^a \frac{(1 + w (a')) da'}{a'} \right) . \tag{5}
\]

Table I: Table with the parameters for the equation of state of four dark energy models, described by the parametrization in equation (4).

| Model | \( w_0 \) | \( w_m \) | \( a_c \) | \( \Delta_m \) |
|-------|--------|--------|--------|--------|
| A     | -0.8   | -0.2   | 0.5    | 4      |
| B     | -0.8   | -0.2   | 0.5    | 0.05   |
| C     | -0.8   | -0.5   | 0.5    | 0.05   |
| D     | -0.95  | -0.2   | 0.5    | 0.05   |

For all models that we study we assume that the matter density parameter now is \( \Omega_m^0 = 0.315 \), according to Planck results for a flat \( \Lambda \)CDM model [28]. One of the main advantages in the use of the growth index is that if geometrical probes can tightly constrain the background evolution, and then \( \Omega_m (a) \), different theories of gravitation can be distinguished via the different values they predict for \( \gamma \) [2]. Here we want to show that the impact dark energy perturbations on \( \gamma \) can be as large as in some modified gravity models. However, as we will soon discuss, since one of the key parameters that affects dark energy perturbations is \( w \), which also affects the background evolution, in order to make a realistic comparison, we restrict the evolution of \( w \) in a way that Hubble parameter instantaneous deviation from the \( \Lambda \)CDM one is no larger than 12%. Keeping that in mind, we choose four different sets of parameters, which are shown in Table I. Model A has a smooth transition from \( w_m = -0.2 \) to \( w_0 = -0.8 \), while model B has the same limiting values, but with a fast transition at low redshift. The transition in model C is as fast as in model B, but it has a smaller \( w_m \). Finally, model D has the same parameters as in model B, but with \( w_0 = -0.95 \). The evolution of \( w \) for the four models is shown in figure 1. Assuming the same Hubble constant for all models, in figure 2 we show the percentage difference of the Hubble parameter of a given model, \( H (z)_{mod} \), relative to the one in \( \Lambda \)CDM, \( H (z)_\Lambda \),

\[
\Delta H = 100 \times \left( \frac{H (z)_{mod}}{H (z)_\Lambda} - 1 \right) . \tag{6}
\]

At \( z = 1000 \), model C has \( \Omega_m \sim 10^{-5} \), but models A, B and D have \( \Omega_m \sim 10^{-3} \), therefore these three can be considered as Early Dark Energy models. We will show that the combination of large and rapid decay of \( w \) at low \( z \) can cause important modifications to the growth of matter perturbations. But let us first define the system of equations we will use to evolve the linear perturbations of matter and dark energy and discuss some relevant features of dark energy perturbations.

III. EVOLUTION OF PERTURBATIONS

As already noted in reference [23], dark energy models with anisotropic stress are not sensible to modifications of
the sound speed. Therefore, here we will focus on dark energy without anisotropic stress, hence the system of equations that governs the evolution of matter and dark energy perturbations in the Newtonian gauge, in Fourier space, is given by [29]:

\begin{align}
\dot{\delta}_m + \theta_m &= 3\dot{\phi}, \\
\dot{\theta}_m + \mathcal{H}\theta_m &= k^2\phi, \\
\dot{\delta}_c + 3\mathcal{H}\left(\frac{\delta p_e}{\delta \rho_e} - w\right)\delta_c + (1+w)\theta_c &= 3(1+w)\dot{\phi}, \\
\dot{\theta}_c + \mathcal{H}\left(1-3c_a^2\right)\theta_c &= \frac{(\delta p_e/\delta \rho_e)k^2\delta_c + k^2\phi}{(1+w)}, \\
k^2\phi + 3\mathcal{H}\left(\dot{\phi} + \mathcal{H}\phi\right) &= -\frac{3\mathcal{H}^2}{2}(\Omega_m\delta_m + \Omega_c\delta_c),
\end{align}

where

\[ c_a^2 = \frac{\dot{\rho}_e}{\dot{\rho}_e} = w - \frac{w}{3\mathcal{H}(1+w)} \] (12)

is the squared adiabatic sound speed of dark energy perturbations and the total pressure perturbation is given by [30]:

\[ \delta p_e = c_{\text{eff}}^2\delta \rho_e + 3\mathcal{H}(1+w)\left(c_{\text{eff}}^2 - c_a^2\right)\rho_e \frac{\theta_c}{k^2}. \] (13)

We integrate this system from \( z = 1000 \) to \( z = 0 \).

In the case of negligible \( c_{\text{eff}} \), dark energy perturbations grow at the same pace of matter perturbations during matter dominated era [17, 18]:

\[ \delta_e = \frac{(1+w)}{(1-3w)}\delta_m. \] (14)

In Early Dark Energy models, which have \( w \approx 0 \) during matter dominated era, dark energy perturbations have the same order of magnitude of matter perturbations. Therefore in order to set the initial conditions we have to take this fact under consideration [19].

Solution [14] is valid on small scales, during matter dominated era and for a constant \( w \), but is also a good order of magnitude estimation for \( \delta_e \), even during the period of transition to dark energy domination and for varying \( w \) [19]. Therefore it is clear that the lower the \( |w| \), the larger the \( \delta_e \). However we must have \( w \lesssim -1 \) at low redshift in order to dark energy to accelerate the cosmic expansion. Therefore dark energy perturbations will necessarily decrease during the transition to the accelerated expansion. If, however, \( w \) has a rapid transition from \( w \approx 0 \) to \( w \approx -1 \) at low redshift, \( \delta_e \) still can strongly influence \( \delta_m \) now. Moreover during the transition from decelerated to accelerated expansion \( \Omega_m \) grows, partially compensating the decrease of \( \delta_e \) in equation (11).

It is important to highlight that the impact of dark energy perturbations strongly depends on the evolution of \( w \). It has been shown that two-parameter descriptions of \( w \) have strong limitations, particularly being not able to follow and constrain rapid evolution [31]. Therefore the use of such parametrizations certainly will not be able describe the potential impact of dark energy perturbations on the matter growth.

As we will show, the impact of dark energy perturbations on matter perturbations is strongest in model B, which has a rapid and large decay of \( w \) at low \( z \). The impact is weaker in model A because of its slower transition of \( w \), which makes the decay of \( \delta_e \) to occur earlier. For model C the impact is even weaker because it has a lower value of \( w \) during matter dominated era. In model D, which has the same parameters of model B, but with a value of \( w \) much closer to \(-1\) now, the impact is only slightly weaker than in model B.

If \( c_{\text{eff}} \) is not negligible, dark energy perturbations are much smaller than matter perturbations on small scales,
therefore their impact on $\delta_m$ will be much weaker. This situation may be considered as smooth or non-clustering dark energy and is a feature of nearly all quintessence models, with exception of models with field potentials that can induce a rapid field oscillation [32]. Indeed, as we will show, when we set $c_{\text{eff}} = 1$ for the models that we consider, the growth index is very similar to the ΛCDM one. For more detailed studies about the phenomenology of dark energy perturbations, in both clustering and non-clustering situations, see for instance [17–19, 33, 34].

In the following section we will study the growth index for the two limiting cases of smooth, $c_{\text{eff}} = 1$, and clustering, $c_{\text{eff}} = 0$, dark energy models. For all the results of the growth index shown in the next section we have fixed the wavenumber to $k = 0.2$ h Mpc$^{-1}$.

IV. GROWTH INDEX

Once we solve the system of equations (7) - (11) we compute $f$ and then determine the growth index as a function of the redshift:

$$\gamma(z) = \frac{\ln f(z)}{\ln \Omega_{m} (z)}. \quad (15)$$

In figure 3 we show, for $c_{\text{eff}} = 1$, the evolution of $\gamma(z)$ for the models A, B, C and D and also the results for ΛCDM model. As is well-known, we see that ΛCDM model has a nearly constant value of the growth index $\gamma \simeq 0.55$ and the values of $\gamma(z)$ in models with smooth dark energy are not too far from it, presenting a mild variation with $z$.

The results for $c_{\text{eff}} = 0$ are shown in figure 4. We can clearly see that in the case of clustering dark energy, large deviations from ΛCDM and smooth dark energy models are present, including a strong evolution with time, which clearly suggests that the usual constant growth index parametrization may not be appropriate in these cases. Since for a given model with smooth or clustering dark energy the background evolution is the same, it is clear that dark energy clustering lowers $\gamma$ and induces an important time variation on it.

When we fit the numerical solution of $f$ with a constant growth index, $\gamma_0$, we find that smooth dark energy models have $\gamma_0 \simeq 0.56$ with an accuracy better than 0.5%. For the clustering models, however, since $\gamma(z)$ has a large variation with $z$, the constant $\gamma_0$ parametrization is much more inaccurate. In figure 4 we show, only for the clustering models, the percentage difference relative to the numerical solution of $f$,

$$\Delta f = 100 \times \left( \frac{f_p}{f} - 1 \right), \quad (16)$$

where $f_p$ is the corresponding parametrized expression. We observe that the constant $\gamma_0$ parametrizations for models A and C present errors about 2%, but reaching almost 6% for models B and D at very low $z$.

Given this inaccuracy of the constant $\gamma_0$ parametrization in the clustering models, we also study the following redshift dependent parametrization for $\gamma$ [35]:

$$\gamma(z) = \gamma_\infty + \gamma_\delta e^{-z/0.61}. \quad (17)$$

In figure 5 we show that the error in this fit is much smaller, always bellow the percent level, expect for very low $z$ in models B and D. Again we note that clustering dark energy models may not be very accurately described by parametrizations for the growth index, especially when presenting a large and fast decay of its equation of state, like in model B and D. The results with the best fit parameters for the constant and redshift dependent growth index parametrizations are show in table [11].
the indication of modified gravity is clear regardless of the presence of clustering dark energy.

On the other hand, if future observations point to low values of $\gamma$, the indication of modified gravity is not clear anymore. In some $f(R)$ models $\gamma(z) \lesssim 0.42 \ [35, 36]$, which is not very far from the values we have found for clustering dark energy models. Furthermore the slope of the growth index in these $f(R)$ models is positive, just as we found for clustering dark energy. However, since $\gamma$ is basically monotonically decreasing with $z$ in $f(R)$, whereas in clustering dark energy it eventually reaches a minimum (no lower than 0.33 for models B and D), precise measurements for intermediate redshifts can in principle discriminate between these two scenarios.

In Table II we present the fitted values for the constant $\gamma_0$ and variable $\gamma$, equation (17), parametrizations. It is clear that in the presence of clustering dark energy perturbations the values of $\gamma_0$ can be well bellow the $\Lambda$CDM. When considering the variable $\gamma$ parametrizations we can see that the values of $\gamma_b$, which is associated with the time dependence of $\gamma$, are one order of magnitude larger than in smooth dark energy models, or clustering dark energy with slowly varying equation of state [23]. This significant time variation of $\gamma$ is actually an important feature of these clustering dark energy models, which however may also be present in $f(R)$ models [6].

Therefore we have showed that when dark energy is allowed to cluster the growth index alone may not be a valid way to distinguish between General Relativity and modified gravity. However, this strongly depends on the time variation of $w$. Then we have to stress that in order to dark energy perturbations to cause an strong impact on the growth rate, dark energy equation of state must have a rapid and large decay at low $z$. For instance, in model B, the one with largest difference relative to $\Lambda$CDM, we have a rapid decay from $-0.2$ to $-0.8$ happening around $z = 1$.

Another interesting point that can be analyzed with the growth index is its potential ability to distinguish between smooth and clustering dark energy models. In the context of Early Dark Energy models it has been shown that galaxy cluster abundances in clustering dark energy models are more similar to the $\Lambda$CDM one than the non-clustering models with the same background evo-

One interesting aspect that can be analyzed in this redshift dependent parametrizations is the sign and magnitude of $\gamma_b$, which provides information about the slope of the growth index. It has been noted that in General Relativity the slope is positive [29, 35, 36]. For all clustering dark energy models that we consider we obtain positive slope for the growth index, $\gamma_b > 0$ (see table [1]). Therefore this feature can still be used to discriminate between General Relativity and DGP models, which have negative slope. Moreover DGP model has much higher values of the growth index ($\simeq 0.68$), whereas clustering dark energy much lower, e.g., $\gamma(z) < 0.48$ for models A and B. Hence if future observations point to high values of $\gamma$ the indication of modified gravity is clear regardless of

![Figure 5: Evolution of $\Delta f$, equation (16), for the constant growth index parametrization: $\Lambda$CDM model (solid black line), model A (dotted red line), model B (short-dashed blue line), model C (dot-dashed green line) and model D (long-dashed magenta line).](image1)

![Figure 6: Evolution of $\Delta f$, equation (16), for the redshift dependent $\gamma$ parametrization, equation (17), for model A (dotted red line), model B (short-dashed blue line), model C (dot-dashed green line) and model D (long-dashed magenta line). With this parametrization $\Delta f$ for $\Lambda$CDM model is below 0.01% level and is indistinguishable from the horizontal axis.](image2)

| Model | $\gamma_0$ | $\gamma_\alpha$ | $\gamma_b$ |
|-------|------------|-----------------|------------|
| A, $c_{\text{eff}} = 0$ | 0.4544 | 0.4096 | 0.0756 |
| B, $c_{\text{eff}} = 0$ | 0.4295 | 0.3336 | 0.1568 |
| C, $c_{\text{eff}} = 0$ | 0.5074 | 0.4844 | 0.0367 |
| D, $c_{\text{eff}} = 0$ | 0.4564 | 0.3453 | 0.1714 |
| A, $c_{\text{eff}} = 1$ | 0.5697 | 0.5737 | -0.0069 |
| B, $c_{\text{eff}} = 1$ | 0.5684 | 0.5807 | -0.0206 |
| C, $c_{\text{eff}} = 1$ | 0.5618 | 0.5628 | -0.0017 |
| D, $c_{\text{eff}} = 1$ | 0.5619 | 0.5764 | -0.0227 |

Table II: Table with the fitted parameters for $f$ with the constant growth index parametrization, $\gamma_0$, and the redshift dependent parametrization, equation (15).
olution [19]. As we can see in figure 1, dark energy perturbations strongly impacts the growth index, in its values and redshift dependence, making the distinction from the smooth dark energy models, figure 3, easier.

V. CONCLUSIONS

We have showed that clustering dark energy models with a rapid and large decay of equation of state at low redshift have a growth index much lower than in ΛCDM or quintessence models. When fitting a constant growth index we found values in the range $0.43 < \gamma_0 < 0.51$ for the four representative clustering models that we have considered. However we also have observed that the constant $\gamma_0$ fit is not very accurate to represent the numerical solution for the growth rate, $f$, being almost 6% inaccurate for our models B and D.

When fitting $f$ with the variable $\gamma$ redshift dependent parametrization proposed in reference [35] the accuracy is about four times better, but can still be as large as 1%. In this parametrization, the $\gamma_0$ parameter, associated with the time dependence of $\gamma$, is always positive, and one order of magnitude larger for clustering dark energy models when compared to their non-clustering versions. This behavior can be useful in order to distinguish between smooth and clustering dark energy.

The general trend we have found is that clustering dark energy lowers $\gamma$ and induces an important time variation to it. This is the same behavior found in some $f(R)$ models [5, 6]. Therefore we conclude that if future data on the growth index points to values much lower than in ΛCDM model, the interpretation of the result as a clear evidence of modified gravity is not straightforward.

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[1] L.-M. Wang and P. J. Steinhardt, Astrophys. J. 508, 483 (1998), astro-ph/9804015.
[2] E. V. Linder, Phys. Rev. D72, 043529 (2005), astro-ph/0507263.
[3] E. V. Linder and R. N. Cahn, Astropart.Phys. 28, 481 (2007), astro-ph/0701317.
[4] S. Tsujikawa, A. De Felice, and J. Alcaniz, JCAP 1301, 030 (2013), 1210.4239.
[5] S. Tsujikawa, R. Gannouji, B. Moraes, and D. Polarski, Phys.Rev. D80, 084044 (2009), 0908.2669.
[6] R. Gannouji, B. Moraes, and D. Polarski, JCAP 0902, 034 (2009), 0809.3374.
[7] G. Dvali, G. Gabadadze, and M. Porrati, Phys.Lett. B485, 208 (2000), hep-th/0005016.
[8] A. F. Heavens, T. Kitching, and L. Verde, Mon.Not.Roy.Astron.Soc. 380, 1029 (2007), astro-ph/0703191.
[9] T. Abbott et al. (Dark Energy Survey) (2005), astro-ph/0510346.
[10] R. Laureijs et al. (EUCLID Collaboration) (2011), 1110.3193.
[11] W. Hu, Astrophys. J. 506, 485 (1998), astro-ph/9801234.
[12] T. Chiba, T. Okabe, and M. Yamaguchi, Phys.Rev. D62, 023511 (2000), astro-ph/9912463.
[13] C. Armendariz-Picon, V. F. Mukhanov, and P. J. Steinhardt, Phys.Rev. D63, 103510 (2001), astro-ph/0006373.
[14] L. P. Chimento and R. Lazkoz, Phys.Rev. D71, 023505 (2005), astro-ph/0404494.
[15] P. Creminelli, G. D’Amico, J. Norena, and F. Vernizzi, JCAP 0902, 018 (2009), 0811.0827.
[16] E. A. Lim, I. Sawicki, and A. Vikman, JCAP 1005, 012 (2010), 1003.5751.
[17] L. R. Abramo, R. C. Batista, L. Liberato, and R. Rosenfeld, Phys. Rev. D79, 023516 (2009), 0806.3461.
[18] D. Sapone, M. Kunz, and M. Kunz, Phys. Rev. D80, 083519 (2009), 0909.0007.
[19] R. Batista and F. Pace, JCAP 1306, 044 (2013), 1303.0414.
[20] M. Kunz and D. Sapone, Phys.Rev.Lett. 98, 121301 (2007), astro-ph/0612452.
[21] B. Jain and P. Zhang, Phys.Rev. D78, 063503 (2008), 0709.2375.
[22] E. Bertschinger and P. Zukin, Phys.Rev. D78, 024015 (2008), 0801.2341.
[23] J. Dossett and M. Ishak (2013), 1311.0726.
[24] E. Bertschinger and A. Riotto, Phys.Lett. B668, 171 (2008), 0807.3343.
[25] M. Chevallier and D. Polarski, Int. J. Mod. Phys. D10, 213 (2001), gr-qc/0009008.
[26] E. V. Linder, Phys. Rev. Lett. 90, 091301 (2003), astro-ph/0208512.
[27] P. S. Corasaniti and E. J. Copeland, Phys. Rev. D67, 063521 (2003), astro-ph/0205544.
[28] P. Ade et al. (Planck Collaboration) (2013), 1303.5076.
[29] C.-P. Ma and E. Bertschinger, Astrophys. J. 455, 7 (1995), astro-ph/9506072.
[30] R. Bean and O. Dore, Phys. Rev. D69, 083503 (2004), astro-ph/0307100.
[31] B. A. Bassett, P. S. Corasaniti, and M. Kunz, Astrophys.J. 617, L1 (2004), astro-ph/0407364.
[32] M. A. Amin, P. Zukin, and E. Bertschinger, Phys.Rev. D85, 103510 (2012), 1108.1793.
[33] G. Ballesteros and J. Lesgourgues, JCAP 1010, 014 (2010), 1004.5509.
[34] E. Sefusatti and F. Vernizzi, JCAP 1103, 047 (2011), 1101.1026.
[35] J. Dossett, M. Ishak, J. Moldenhauer, Y. Gong, A. Wang, et al., JCAP 1004, 022 (2010), 1004.3086.
[36] P. Wu, H. W. Yu, and X. Fu, JCAP 0906, 019 (2009), 0905.3444.