A cranked self-consistent mean-field description of the triaxially deformed rotational bands in $^{138}$Nd

Yue Shi (石跃)$^1$

$^1$Department of Physics, Harbin Institute of Technology, Harbin 150001, People’s Republic of China

Background Compared to the axially deformed nuclei, triaxially deformed ones are relatively scarce. This is mainly due to the difficulties in the identification of experimental signatures pertaining to the triaxial degree of freedom. In the nucleus $^{138}$Nd, a number of rotational bands have been observed to have medium or high spin values. They have been interpreted in the macroscopic-microscopic method, to be based on triaxial minima. In particular, for a few configurations, the calculations suggested that a re-orientation of the rotational axis may have occurred along the rotational bands.

Purpose The present work aims at a quantitative description of the experimentally observed bands in $^{138}$Nd, using the cranked self-consistent Skyrme-Hartree-Fock (SHF) method or cranked nuclear density functional theory (DFT). Such a study, which is still missing, will provide alternative interpretations of the structure of the bands and hence, shed new lights on the triaxiality issue in connection with experimental data.

Methods The rotational bands are described using cranked self-consistent mean-field method with SLy4 and SkM* Skyrme energy density functionals (EDFs). For SLy4 EDF, the time-odd pieces are included using Landau parameters (denoted with SLy4L). For SkM* EDF, the local gauge invariance argument has been used to determine the time-odd components of the mean-field.

Results The survey of different configurations near Fermi surface of $^{138}$Nd results in 12 lowest configurations, at both positive- and negative-γ deformations. These are calculated to be the energetically lowest configurations. The results show that, for both EDFs, the rotational states based on positive-γ minimum, which is at $\gamma \approx 35^\circ$, are lower than the respective configurations with negative-γ deformation. The general trends of the spin-versus-omega curve, and the energy-versus-spin curve reproduce well those of the experimental data. Further, for the observed bands ‘T1-T8’, the calculated results using SLy4, allows the configurations of the observed bands to be assigned. The calculations predict transitional quadrupole moments, which can be used to compare with future experimental data.

Conclusions The current cranked self-consistent mean-field calculations of the near-yраст high-spin rotational bands in $^{138}$Nd reproduce well the experimental data. The results suggest that the experimentally observed bands can be assigned to the calculated bands with various configurations at the positive-γ deformation. The predictions of the current calculations are complementary to that of the well-know macroscopic-microscopic calculations, both of which await future experiment to verify.

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I. INTRODUCTION

It has been known that the occurrence of rotational bands in a nucleus is due to the break of rotational invariance of the nuclear mean-field Hamiltonian in the intrinsic frame $^1$ $^2$. There are a few fascinating phenomena which can be associated with the triaxial deformation of a nucleus. Among them, the energy inversion of the signature partners, differing from a normal order predicted by rotational models without the triaxial deformation $^3$; the occurrence of identical bands which are presumably due to the break of chiral symmetry in a triaxially deformed mean-field $^4$ $^5$; as well as the bands based on wobbling motions of a rotating nucleus $^6$ are perhaps the most significant examples.

Compared to the axially symmetric deformation, where the characterization has been fairly well established both experimentally and theoretically, physics associated with triaxially deformed nuclei has been rather scarce. This is due to the more subtle experimental sig-
uncertain \[15\].

A much better laboratory for studying the triaxially deformed nucleus may be \(^{138}\)Nd. Indeed, in this nucleus, a multitude of rotational bands have been observed \[16\,17\]. For them, the macroscopic-microscopic model \[12\] has predicted a rather pronounced static triaxial deformation. In particular, the linking transitions from these band members to the energetically known lower-spin states have been observed through precise \(\gamma\)-ray measurements. Consequently, the spin values and the excitation energies of the bands have been determined, which provide much better testing ground for theoretical models compared to the situation of \(^{158}\)Er.

Incidentally, the macroscopic-microscopic calculations in Ref. \[18\] showed that the structural changes in a pair of the rotational bands in \(^{138}\)Nd were due to the deformation change from a positive-\(\gamma\) value to a negative-\(\gamma\) one. These findings may constitute the first observation of the re-orientation effect of the rotational axis with increasing rotational frequency. It is then desirable to see the predictions from alternative theoretical models, such as the self-consistent mean-field methods \[13\,20\], which is still missing.

The present work aims at a description of the observed bands in Refs. \[17\,18\] using a cranked nuclear density functional theory (DFT). The alternative description of the observables is supposed to shed new lights on the un-derstanding of triaxial deformation in nuclear structure. In section \[\text{III}\] I describe the model, and the parameter used in the current work. Section \[\text{III}\] presents detailed results and discussions, before a summary which is presented in section \[\text{IV}\].

II. THE MODEL

In the present work, the cranked SHF calculations are performed with symmetry-unrestricted solver HFODD (version 2.49t \[21\]). In this model, the total energies (or Routhians in the rotational frame \[22\]) are represented as functional of various densities \[24\]. The single-particle Hamiltonian is expanded in terms of 969 deformed harmonics oscillator basis, which are \(\sqrt{\hbar}\omega_x = \hbar\omega_y = 0.4945\), and \(\hbar\omega_z = 0.4499\) MeV.

The SkM* \[23\] and SLy4 \[24\] energy density functionals (EDFs) are used in the particle-hole channel of the cranked SHF problem. For cranking calculations, one deals with non-zero time-odd densities and fields. The coupling constants are determined using local gauge invariance arguments \[25\], and Landau parameters \[26\], for SkM* and SLy4 EDFs, respectively. The latter is then denoted with SLy4L. The current choice of the parameterizations including the time-odd part is identical with those appeared in Ref. \[13\,14\] for consistency.

To obtain the solutions corresponding to the local minima that are interesting for the current work, calculations with constraints on the quadrupole moments, \(Q_{20}\) and \(Q_{22}\), are first performed. The constraints on \(Q_{20}\) and \(Q_{22}\) deformations are then removed by zeroing the respective Lagrangian multipliers. The earlier constraint solutions are used to warm start the deformation unconstrained calculations.

In the current principal-axis-cranking calculations, parity and signature symmetries are always enforced. Each single-particle level could be labeled by parity and signature quantum numbers. For details, see the caption of Table \[\text{II}\].

Before I end this section, some explanations about (1) the sign of the \(\gamma\) value which is used to measure the degree of triaxial deformation, and (2) the relation between the commonly accepted convention (Lund convention) and the one used in the current work is necessary.

In the Lund convention, the rotation is chosen to be around the \(x\)-axis and the deformation with \(\gamma\) value in the interval of \((0,60^{\circ})\) corresponds to a triaxially deformed nucleus that rotates around its short axis \[2\].

In the HFODD code \[22\], the one-dimensional cranking is around the \(y\)-axis, and the \(Q_{22}\) value is defined as \(\sqrt{3}(x^2-\bar{y}^2)\). In the current calculation, the minima with positive \(Q_{22}\) values are identified as minima with positive \(\gamma\) values. This definition is consistent with the Lund convention in the fact that, for both conventions, a positive \(\gamma\) value indicates rotation around its short axis.

To fully conform with the Lund convention which uses \(x\)-axis as the cranking axis, a final step one needs to do is the following relabelling of the axes in the HFODD code: \((x,y,z)\rightarrow(y,x,-z)\). This results in the change of the sign of \(Q_{22}\) value. Such a convention transformation in our calculations has been carefully considered. Note the negative sign in front of \(Q_{22}\) in Figure 3 of Ref. \[12\], and in the current work.

III. RESULTS AND DISCUSSIONS

Table \[\text{II}\] lists the configurations in \(^{138}\)Nd studied in this work. Note that for each configuration, there are two states corresponding to the minima with positive and negative \(\gamma\) values. In the following discussions, the ‘01’ configuration with a positive-\(\gamma\) minimum is denoted with ‘pg01’, the negative-\(\gamma\) one is denoted with ‘ng01’. The same rule of naming is applied to other configurations.

In figure \[\text{I}\] I show the calculated single-particle routhians for SLy4L calculated with configuration ‘pg01’ listed in table \[\text{II}\]. It can be seen that the configuration ‘pg01’ contains no particle-hole excitations for the frequency interval of \(\sim 0.0-0.5\) MeV. It has to be noted that, even though the configuration ‘pg01’ appears to be the lowest energetically, since the single-particle levels below the Fermi surfaces of protons and neutrons are occupied, it does not necessarily indicate that the obtained total energy of this configuration in the rotating frame is the lowest. This is so because a state with another configuration may acquire additional binding through self-consistent process. In addition, a state with another configuration may be
TABLE I: The SHF configurations in $^{138}$Nd studied in this work. Each configuration is described by the number of states occupied in the four parity-signature ($\pi,\rho$) blocks, in the convention defined in Ref. [27]. For configurations ‘07’ and ‘08’, the ‘18→19’ means that the neutron is moved from the 18th to the 19th level. The transition quadrupole moment, $Q_{t}$ values, are calculated using the SLy4L EDF at $\hbar\omega = 0.6$ MeV for the positive-\greek{$gamma$} deformed minimum. The $Q_{t}$ values are calculated through the relation $Q_{t} = Q_{20}^{\pi} + \sqrt{2}Q_{22}^{\pi}$.

| Label | Configuration | $\pi$ | $\rho$ | $Q_{t}$ (eb) |
|-------|---------------|-------|-------|-------------|
| 01    | $\nu[20,20,19,19] \otimes [16,16,14,14]$ | +   | +1   | 2.7         |
| 02    | $\nu[19,21,19,19] \otimes [16,16,14,14]$ | +   | -1   | 3.2         |
| 03    | $\nu[20,20,19,19] \otimes [16,15,15,15]$ | -   | -1   | 3.0         |
| 04    | $\nu[20,21,19,18] \otimes [16,15,15,14]$ | +   | -1   | 3.5         |
| 05    | $\nu[21,20,18,19] \otimes [16,14,14,14]$ | -   | +1   | 2.6         |
| 06    | $\nu[21,20,19,18] \otimes [16,14,14,14]$ | -   | -1   | 2.6         |
| 07    | $\nu[21,20,18→19,19] \otimes [16,16,14,14]$ | -   | +1   | 2.2         |
| 08    | $\nu[21,20,19,18→19] \otimes [16,16,14,14]$ | -   | -1   | 2.3         |
| 09    | $\nu[20,20,19,19] \otimes [15,16,14,15]$ | -   | -1   | 2.7         |
| 10    | $\nu[20,21,19,18] \otimes [16,14,14,14]$ | +   | +1   | 3.0         |
| 11    | $\nu[20,21,18,19] \otimes [16,14,14,14]$ | -   | -1   | 3.0         |
| 12    | $\nu[20,20,19,19] \otimes [15,16,14,15]$ | -   | +1   | 3.0         |

lower in energy by having a slightly different deformation compared to the configuration ‘pg01’.

Nevertheless, it is reasonable to state that configuration ‘pg01’ is among the lowest in total Routhian for frequency interval of $\sim 0.0-0.5$ MeV, which will be shown in the energy-versus-spin plot in later discussions. In figure 1 it can be seen that, for $\hbar\omega > 0.5$ MeV, a state with ($\pi, \rho$) = ($+i$, $-i$) penetrates below the Fermi surface, and crosses with the 20th level in the $(+i, -i)$ block. This crossing corresponds to the characteristic backbending observed in the experimental data which will be discussed later. The behaviors of spin and energy values as a function of rotational frequency of configuration ‘pg01’ are rather typical. Hence, the quality of the description of these behaviors are particularly important for assessing the usefulness of the current calculations.

Before showing the calculated total angular momenta and energies as a function of rotational frequency, in figure 1 show the total-Routhian-surfaces of $^{138}$Nd for configuration ‘01’ in table I calculated with SLy4L [21] (a), and SkM* [22] (b) EDFs, at a rotational frequency of 0.6 MeV. For both EDFs, the minima at the positive and negative $\gamma$ deformations can be seen. The minima with positive $\gamma$ values are deeper, and are larger in terms of $|\gamma|$ value. For SLy4L, the positive-$\gamma$ minimum is deeper than that of the negative-$\gamma$ one by $\sim 3.0$ MeV in energy. Whereas the energy difference for the two minima of SkM* is only $\sim 1.5$ MeV. The barrier separating the two minima in the $\gamma$ direction is lower for SkM* compared to that of SLy4L. Note that for SkM* EDF, the crossing with the lowest level from $N = 6$ states has already happened (see blow for the crossing rotational frequency). For SLy4L EDF, this crossing happens at a larger rotational frequency ($\hbar\omega \approx 0.7$ MeV) for this configuration.

Figure 3 shows the total spin values for the configurations listed in table I that are at the positive-$\gamma$ deformation (see figure 2), as a function of rotational frequency. It can be seen that the angular momenta are in the range of $0.4-0.8$ MeV are ($Q_{20}, Q_{22}$) $\approx (10,0.5,3)$ b. The levels with positive and negative parities are indicated by black and light gray lines, respectively. The levels with +i and $i$ signatures are indicated by solid and dashed lines, respectively. One could find the order of the level within its ($\pi, \rho$) block at the beginning and the end of each line.

FIG. 1: Single-particle routhians from cranked SHF calculations with SLy4L EDF for $^{138}$Nd. The configuration is ‘pg01’ in table I. The quadrupole moments for rotational frequencies between 0.4-0.8 MeV are ($Q_{20}, Q_{22}$) $\approx (10,0.5,3)$ b. The levels with positive and negative parities are indicated by black and light gray lines, respectively. The levels with +i and $i$ signatures are indicated by solid and dashed lines, respectively. One could find the order of the level within its ($\pi, \rho$) block at the beginning and the end of each line.
FIG. 2: Total Routhian surfaces for $^{138}$Nd for configuration ‘pg01’ in table I. Calculations are performed with Skyrme EDF SLy4L (a), and SkM* (b) at a rotational frequency of 0.6 MeV. Contour lines are 0.4 MeV apart in energy.

It has to be noted that, in this mean-field study, the total spin values are determined by summing the individual contributions from occupied single-particle levels. Hence, the total spin values are rather sensitive to the calculated deformation and the specific configurations. This can be seen by comparing figure 3 and figure 4 which correspond angular momenta variations as functions of rotational frequency at two different deformations. It can be seen that the total spin values for the same EDF, at the same frequency values, and for the same configuration can differ up to $\sim 10\hbar$. The variation as functions of rotational frequency is also different for the same EDF, and the same configuration.

Figure 5 displays the total energies of $^{138}$Nd for configurations listed in table I at the positive-$\gamma$ deformation, calculated with SLy4L, and SkM* EDFs. For each configuration, the energy of an arbitrary rotor has been subtracted from the total energy. This is to bring the curves less steep for viewing the detail of the curves more clearly. It can be seen that both EDFs predict similar trends for same configurations. In addition, the alignments that have been seen in the spin-versus-frequency figure (figure 3) are reflected in these energy-versus-spin curves, as a subtle change of the slopes. This can be understood as the sudden increase of the moments of inertia which results in the decrease of energies that are needed to provide the same amount of spin increase.

Figure 6 shows the same quantities as those in figure 5 except for negative-$\gamma$ values. It can be seen that the changes due to the abrupt spin changes are absent in these bands. Comparing the energies of the same configuration, it can be seen that the energies on the negative-$\gamma$ side are in general 1-2 MeV higher than that of their positive-$\gamma$ states. For SkM* EDF, the ‘ng02’ shows a large change. This is because the self-consistent calculation follows the positive-$\gamma$ minimum.

Figure 7 compares the experimental spin values (a) of the rotational bands, extracted from Ref. 19, with the present cranked DFT calculations using SLy4L EDF (b). The calculated spin values are extracted from figure 3(a) and figure 4(a). It can be seen that the calculations display general consistencies with data in terms of both the trend and the absolute values. In particular, both data and the calculations show characteristic upbends at certain rotational frequency. Specifically, the spin values of ‘T7’ band upbend at $\hbar\omega \approx 0.75$ MeV. The calculations underestimate this rotational frequency by about 0.15 and 0.25 MeV (see figure 3) for SLy4L(a) and SkM*(b) EDFs, respectively.

Note that this alignment in the present calculation is mainly due to the crossing of the $N = 6$ state with the lower state in the same ($\pi, \rho$)-block. It is rather sensitive to the parameter used, and in particular, the predicted deformations of the minima. Hence, the critical frequency at which the alignment occurs could be used as a criterion for assessing the usefulness of a given parameter set used. One may use this quantity to adjust parameterization if one needs to constrain on the single-particle properties.

The transitions linking the high-spin states to states with lower spins have been observed in Refs. 19. This allows for the excitation energies and the spin values to be determined. To compare these data with the calculations, one needs to calculate the ground state of $^{138}$Nd where pairing interaction is present. In table II I list the calculated ground state total energies, and the pairing properties for $^{138}$Nd with SLy4L and SkM* EDFs. The calculation setup is identical to those used to calculate the rotational bands, except that for the calculations of the ground states, the pairing correlations are included through the Skyrme-Hartree-Fock-Bogoliubov method 28. The pairing cutoff energies are $E_{\text{cutoff}} = 60$ MeV, for both protons and neutrons. The
FIG. 3: The total angular momenta as a function of rotational frequency for different configurations at positive $\gamma$ values as a function of rotational frequency calculated with SLy4$_L$ and SkM* EDFs. The solid, dashed, dotted, and dotted-dashed lines denote states with $(\pi, \rho) =$ $(+ ,0)$, $(+ ,+1)$, $(- ,0)$, and $(- ,+1)$, respectively.

FIG. 4: The total angular momenta for different configurations at negative $\gamma$ values as a function of rotational frequency calculated with SLy4$_L$ and SkM* EDFs.

Pairing strengths are determined to reproduce the experimental odd-even staggering energies, which are 1.07, and 1.15 MeV for neutrons, and protons, respectively.

In figure 8 I compare the observed excited energies [19] (a) with the calculated ones with SLy4$_L$ EDF, the latter of which are extracted from figure 5(a) and figure 6(b). It can be seen from figure 8 that the present calculations [figure 8(b)] systematically underestimate the observed excited energies of the bands [figure 8(a)] by $\sim$ 1.5 MeV for SLy4$_L$ EDF. It should be noted that the calculations with SLy4$_L$ EDF predict prolate ground state, whereas the results using SkM* EDF suggest a well deformed triaxial deformation for the ground state. One needs to note that the current description may be insufficient for this nucleus near ground state which has soft potential-energy surface. Additional effects such as the vibrational motions may invite further correlations on the current mean-field results.

Compared to the absolute energies, better indicators of a successful description of rotational bands are (1) the relative energies between the bands, and (2) the trend of the total energies as a function of spin values. Taking into account of these two criteria [see figure 8], together with the spin-versus-\(\omega\) plots [see figure 3] allow one to assign the experimentally observed bands [19] ‘T1’, ‘T2’, ‘T3’, ‘T4’, ‘T5’, ‘T6’, ‘T7’, and ‘T8’ to the calculated ones with configurations ‘pg05’, ‘pg06’, ‘pg10/ng10’, ‘pg11/ng11’, ‘pg07’, ‘pg08’, ‘pg01’, and ‘pg03’, respectively. For pos-
FIG. 5: Total energies for different configurations at positive $\gamma$ values. Results are calculated with SLy4 and SkM* EDFs.

FIG. 6: Same as figure 5, except that the curves correspond to the configurations with negative $\gamma$ values.

TABLE II: The calculated total energies, pairing gaps, Fermi energies, and quadrupole moments for the ground state of $^{138}$Nd using SHFB method.

|            | SLy4 | SkM* |
|------------|------|------|
| $E_{tot}$ (MeV) | -1145.207 | -1141.870 |
| $\Delta_n$ (MeV)  | 1.03   | 1.06   |
| $\Delta_p$ (MeV)  | 1.06   | 1.10   |
| $\lambda_n$ (MeV) | -9.806 | -9.696 |
| $\lambda_p$ (MeV) | -4.520 | -4.018 |
| $Q_{20}$ (e$\cdot$b) | 2.62   | 2.90   |
| $Q_{22}$ (e$\cdot$b) | 0.00   | 1.15   |

above and below the Fermi surfaces, as shown in figure 1. To create a positive one-particle-one-hole state, a neutron or proton needs to be excited across the opposite states above the Fermi surface, which results in high excitation energies.

There are a few interesting points worth mentioning along with the assignments. First, the present calculations predict a yrast band with ‘pg01’ configuration for the spin values in the interval of $\sim$24-30\hbar. For spin values lower than 24\hbar, configuration ‘pg06’ is lower in energy. For higher spin values ($I \geq 30$\hbar), the yrast status of ‘pg01’ is taken over by signature partners ‘pg10,11’. This is consistent with the data, as shown in figure 8. For the near yrast states, the good agreement between the calculations and the data is important for evaluating the predictions of the current calculations. For spin values between 20\hbar and 25\hbar, the
FIG. 7: Experimental angular momenta as a function of rotational frequency (a) for ‘t1-t8’ bands from Ref. [19]. The $\hbar \omega$ values for $E(I)$ values are evaluated as $\frac{1}{2}E_{\gamma}(I + 1 \rightarrow I - 1)$. The calculated values (b) are extracted from figures 3(a) and 4(a). The proposed configuration assignment of the current work for the experimental data are indicated by labelling the respective calculated curve with the same type of markers. See text for the case of ‘t3’ and ‘t4’ bands.

FIG. 8: Total energies as a function of angular momenta extracted from experimental data [16, 19, 29], compared with the SLy4L shown in figure 5. The energy values are plotted with respect to the energy of the 22$^+$ state of ‘T7’. The calculated values (b) are extracted from figures 5(a) and 6(a).

Experimental information on band ‘t2’ is still missing, which prevents a direct comparison between experimental data (‘t2’) and the calculations (‘pg06’), see figure 8.

Second, a close examination of the experimental curve ‘T7’ and the curve with ‘pg01’ configuration, reveals a change of the slope in the spin range of 30-35 $\hbar$. This is, in the current calculation, interpreted to be due to the alignment of neutrons at $\hbar \omega \approx 0.7$ MeV (see figure 8). The interpretation is similar for ‘T1’, and ‘T2’ bands, where the calculations predict that the variations in the energy curves are due to alignment. This differs from the interpretation in Ref. [18], where the macroscopic-microscopic calculations suggested a transition of the minimum from a positive- to a negative-$\gamma$ deformation.

Third, if one assigns ‘T3’, and ‘T4’ to ‘pg10’, and ‘pg11’, respectively, then one finds that the energy curves
show slower increase with increasing spin values compared to the experiment, as shown in figure 8. The calculated angular momenta seem to have a smaller slope compared to data, see figure 7. However, the energy curves and the spin curves corresponding to the negative-γ deformation seem to overestimate the slopes of the curves of experimental data.

TABLE III: Calculated proton quadrupole moments in 138Nd for different configurations, with SLy4E, EDF. The ‘pg’, and ‘ng’ denote positive- and negative-γ minima, respectively.

| Band | hω (MeV) | $Q_{2\gamma}$ (eb) | $-Q_{2\gamma}$ (eb) | $Q_{1\gamma}$ (eb) | J (h) |
|------|----------|-------------------|-------------------|------------------|-------|
| 01   | 0.4      | 4.3 4.8 2.2       | -1.6 3.0 5.7      | 24.2 19.0        |
|      | 0.5      | 4.2 4.7 2.3       | -1.5 2.9 5.6      | 26.1 23.2        |
|      | 0.6      | 4.0 4.6 2.3       | -1.4 2.7 5.4      | 27.8 27.1        |
|      | 0.7      | 3.9 4.3 2.4       | -1.2 2.5 5.0      | 29.4 32.3        |
|      | 0.8      | 4.0 4.5 2.9       | -1.2 2.3 5.2      | 37.3 44.3        |
| 05   | 0.4      | 3.8 4.6 2.0       | -1.3 2.6 5.4      | 22.5 20.8        |
|      | 0.5      | 3.6 4.5 2.1       | -1.1 2.4 5.1      | 24.2 25.6        |
|      | 0.6      | 3.5 4.2 2.2       | -0.9 2.2 4.7      | 25.7 30.5        |
|      | 0.7      | 3.4 4.0 2.3       | -0.8 2.1 4.6      | 32.2 34.9        |
| 06   | 0.4      | 3.7 4.6 2.0       | -1.3 2.5 5.4      | 21.6 21.0        |
|      | 0.5      | 3.6 4.4 2.1       | -1.1 2.4 5.0      | 23.1 26.0        |
|      | 0.6      | 3.5 4.1 2.1       | -1.0 2.3 4.7      | 25.1 31.5        |
|      | 0.7      | 3.3 3.7 2.1       | -1.0 2.1 4.3      | 25.9 37.1        |
|      | 0.8      | 3.3 3.5 2.5       | -1.4 2.3 3.3      | 37.6 43.7        |
| 10   | 0.4      | 4.7 4.1 2.2       | -1.4 3.4 4.9      | 30.9 20.1        |
|      | 0.5      | 4.5 4.3 2.3       | -1.1 3.2 4.9      | 33.3 27.7        |
|      | 0.6      | 4.4 4.0 2.4       | -1.0 3.0 4.6      | 35.5 33.1        |
|      | 0.7      | 4.2 3.6 2.4       | -1.1 2.8 4.2      | 37.5 38.3        |
|      | 0.8      | 4.1 3.2 2.5       | -1.1 2.7 3.8      | 39.5 42.2        |
| 11   | 0.4      | 4.7 4.6 2.2       | -1.3 3.4 5.4      | 31.1 22.2        |
|      | 0.5      | 4.5 4.4 2.3       | -1.1 3.2 5.0      | 33.5 27.4        |
|      | 0.6      | 4.4 4.1 2.4       | -1.0 3.0 4.7      | 35.7 31.8        |
|      | 0.7      | 4.2 3.9 2.4       | -0.9 2.8 4.4      | 37.8 35.7        |
|      | 0.8      | 4.0 3.6 2.5       | -1.0 2.6 4.2      | 39.9 44.4        |

To verify the second and third interpretations raised by the current calculations, one needs to conduct experimental measurements of the transitional quadrupole moments or life time on the bands ‘T12347’, as well as a few other states for references. Table III lists the transitional quadrupole moments calculated with SLy4E at both positive- and negative-γ values for a few configurations. Incidentally, a recent experiment 50 performed interesting life-time measurement on the rotational bands near ground state of 138Nd. The current work suggests that the life-time data in the spin intervals of 30–45ℏ are highly desirable. A comparative study among the bands with spin increase, together with theoretical calculations will reveal better modelings, better parameterizations, as well as suitable testing grounds for phenomena such as shape transitions, rotational axis re-orientations, alignments with triaxial shape, all in the same nucleus.

IV. SUMMARY

In the current work, cranked self-consistent Skyrme-Hartree-Fock (SHF) calculations were applied to the description of the rotational bands recently observed in 138Nd in the spin range of 20–45ℏ. For the configurations near yrast line, the calculations with both SkM* and SLy4E parameters predicted pronounced triaxial minima at positive-γ deformation and comparatively soft triaxial minima at negative-γ deformation. The configurations with positive-γ deformation were calculated to be lower in energy compared to those with negative-γ deformation for both parameters. By comparing the calculated total angular momenta as a function of rotational frequency and the relative total energies as a function of spin values for different configurations with experimental data, the configurations of the observed bands 19 ‘T1–8’ were assigned in the current cranked SHF calculations. It needs to be noted that for some configurations, the current interpretations of structural changes differ from those of previous macroscopic-microscopic calculations 19. The predicted transitional quadrupole moments were provided for future life-time measurement to verify the current interpretations.

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