A Left-Right Mirror Symmetric Model: Common Origin of Neutrino Mass, Baryon Asymmetry and Dark Matter

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Abstract: I suggest a left-right mirror symmetric particle model as the natural and aesthetical extension of the SM. The model can completely account for the common origin of the neutrino mass, the baryon asymmetry and the dark matter, moreover, uncover the profound internal connections among them. The tiny neutrino mass is generated by the radiative mechanism, the leptogenesis arises from the TeV-scale mirror lepton decay, the CDM is a KeV-mass sterile Dirac fermion. In addition, I discuss several feasible approaches to test the model predictions and probe the dark physics by near future experiments.

Keywords: particle model beyond SM; neutrino mass; baryon asymmetry; dark matter
I. Introduction

The standard model (SM) of the fundamental particles has successfully accounted for all kinds of the particle phenomena at or below the electroweak scale, refer to the relevant reviews in Particle Data Group [1]. However, the SM has some shortcomings, aesthetically it is not a left-right symmetric framework, theoretically it can not at all address the three important issues of particle physics and cosmology: the tiny neutrino mass [2], the matter-antimatter asymmetry [3], and the cold dark matter (CDM) [4]. Up to now, particle scientists have established plenty of experimental data for the neutrino physics and the baryon asymmetry [1], but the CDM has not yet been detected by any one terrestrial experiment except for the evidence from cosmic observations [5]. The search for new physics evidences are always the focus of attention of the experimental physicists. Undoubtedly, all of the investigations now reveal that there is a new physics beyond the SM, and also foreshadow the existence of an underlying and more fundamental theory.

A wide variety of theories have been suggested in the last half century since the SM was established. A majority of them only focus on one of the above-mentioned issues and disregard the connections among them, a minority of the theories take into account an integrated solution but they are either unbelievable complexity or unable to be tested. However, some inspiring and outstanding ideas are worth drawing lessons from. The tiny neutrino mass can be generated by the seesaw mechanism [6] or arise from some loop-diagram radiative generation [7]. The baryon asymmetry can be achieved by the thermal leptogenesis [8] or the electroweak baryogenesis [9]. The CDM candidates are possibly the sterile neutrino [10], the lightest supersymmetric particle [11], the axion [12], and so on. In recent years, some interesting models have built some connections among the neutrino mass, the baryon asymmetry and the CDM, for instance, the Scotogenic Model [13], the asymmetric CDM model [14], some sophisticated models [15], and the author’s recent works on this field [16]. Although progresses on the new theory beyond the SM have been made all the time, a realistic and convincing theory is not established as yet. Therefore, finding out the correct new theory beyond the SM becomes the largest challenge for theoretical particle physics.

Based on the universe harmony and the nature unification, it is very reasonable and believable that the tiny neutrino mass, the baryon asymmetry and the dark matter are related to each other and they have a common origin, in other words, a realistic theory beyond the SM should be able to unify the three things into a framework. On the other hand, this new theory should keep the two principles: the simplicity with fewer number of parameters, the feasibility of being tested by future experiments. If one theory is excessive complexity with too many parameters, then it is unbelievable, if it is unable to be tested, then it is also insignificant. After careful considerations, I here suggest the left-right mirror symmetric model as the natural and aesthetical extension of the SM. This model can completely account for the common origin of the above-mentioned things and uncover the profound connections among them by use of fewer number of parameters. In addition, it is very feasible to test the model and probe the dark sector physics by means of the TeV-scale colliders, the neutrino experiments, the $\mu \to e\gamma$ LFV, and the
high-energy cosmic rays.

The remainder of this paper is organized as follows. In Section II, I outline the model and explain the neutrino mass generation. In Section III, I discuss the matter-antimatter asymmetry generation. The dark sector physics is discussed in Section IV. I give the numerical results and the model test approaches in Section V. Section VI is devoted to conclusions.

II. Model and Neutrino Mass

I suggest the natural and aesthetical extension of the SM by introducing the mirror matter corresponding to the SM matter, the model has explicitly the left-right mirror symmetry. Tab. 1 in detail shows the model particle contents and its symmetries, in which I omit the color subgroup $SU(3)_C$ since the strong interaction is not involved in the following discussions of this paper. The SM matter lie in the left-handed sector, while the mirror matter belong to the right-handed sector. The local $U(1)_{Y}$ and the global $U(1)_{B-L}$ symmetries are common for the two sectors. The discrete $Z^M_2$ symmetry is a matter parity, under which the right-handed sector is different from the left-handed sector. Note that $N^0_L$ is filled in the left-handed sector though it is not a SM particle. $\phi^+$ and $\phi^0$ are common for the two sectors and their mirror particles are themselves, in particular, $\phi^0$ is a real scalar field without any charges, but $\phi^+$ has “$+1$" $Z^M_2$ parity and $\phi^0$ has “$-1$” $Z^M_2$ parity. More explanations are in the caption of Tab. 1.

All kinds of the chiral fermions in Tab. 1 are Dirac-type without Majorana-type, moreover, they have three generations as usual. By virtue of the fermion assignments of Tab. 1 and the explicit left-right mirror symmetry of the model, it is easily verified that all of the chiral anomalies are completely cancelled in the model, namely, the model is anomaly-free.

We can now write the invariant Lagrangian of the model which satisfies the above-mentioned symmetries, it is composed of the gauge kinetic energy terms, the Yukawa couplings and the scalar potentials. The gauge kinetic energy terms are

$$\mathcal{L}_G = \mathcal{L}_{\text{pure gauge}} + \sum_{f_L} i \bar{f}_L \gamma_\mu D^\mu f_L + \sum_{f_R} i \bar{f}_R \gamma_\mu D^\mu f_R$$

$$+ (D_\mu H_L)^\dagger D^\mu H_L + (D_\mu H_R)^\dagger D^\mu H_R + (D_\mu \phi^+)^* D^\mu \phi^+ + \frac{1}{2} \partial_\mu \phi^0 \partial^\mu \phi^0,$$

$$D^\mu = \partial^\mu + i g_L W^\mu_L \cdot \frac{\tau^L}{2} + i g_Y B^\mu \frac{Y}{2} + i g_R W^\mu_R \cdot \frac{\tau^R}{2}, \quad (1)$$

where $f_L$ and $f_R$ denote all kinds of fermions in Tab. 1. $g_L, g_Y, g_R$ are three gauge coupling coefficients associated with the model gauge groups. $\tau_i$ are the three Pauli matrices and $Y$ is the $U(1)_Y$ charge operator.
|                      | Left-handed Sector (SM Matter) | Right-handed Sector (Mirror Matter) |
|----------------------|---------------------------------|-------------------------------------|
| Symmetry groups      | $SU(2)_L \otimes U(1)_Y \times SU(2)_R \otimes U(1)^{\text{global}}_{B-L} \otimes Z_2^M$ |                                     |
| Gauge fields          | $W^+_{L}(3,0,1)_0, \quad B^\mu(1,0,1)_0, \quad W^0_{R}(1,0,3)_0$ |                                     |
| Fermion fields        | $q_L = \left( \begin{array}{c} u_L \\ d_L \end{array} \right) (2, \frac{1}{3}, 1)_\frac{1}{3}$ | $q_R = \left( \begin{array}{c} \xi_R \\ \eta_R \end{array} \right) (1, \frac{1}{3}, 2)_\frac{1}{3}$ |
|                      | $d^c_R(1, \frac{2}{3}, 1)_{-\frac{1}{3}}$ | $\eta^c_L(1, \frac{2}{3}, 1)_{-\frac{1}{3}}$ |
|                      | $u^c_R(1, -\frac{1}{3}, 1)_{-\frac{1}{3}}$ | $\xi^c_L(1, -\frac{1}{3}, 1)_{-\frac{1}{3}}$ |
|                      | $l_L = \left( \begin{array}{c} \nu^0_L \\ e^c_L \end{array} \right) (2, -1, 1)_{-1}$ | $l_R = \left( \begin{array}{c} \nu^0_R \\ \chi_R \end{array} \right) (1, -1, 2)_{-1}$ |
|                      | $e^c_R(1, 2, 1)_1$ | $\chi^c_L(1, 2, 1)_1$ |
|                      | $N^0_L(1, 0, 1)_{-1}$ | $N^0_R(1, 0, 1)_{-1}$ |
| Scalar fields         | $H_L = \left( \begin{array}{c} H^0_L \\ H^-_L \end{array} \right) (2, -1, 1)_0$ | $H_R = \left( \begin{array}{c} H^0_R \\ H^-_R \end{array} \right) (1, -1, 2)_0$ |
|                      | $\phi^+(1, 2, 1)_2$, $\phi^0(1, 0, 1)_0$ |                                     |
| $Z_2^M$ parity        | +1 | -1 but +1 for $W^0_R$ and $H_R$ |

Table 1: The model particle contents and its symmetries. The $SU(3)_C$ subgroup is omitted. The bracket following each field indicates its gauge quantum number and the right subscript of the bracket is its $B - L$ number, and the right superscript of each component is its electric charge. Note that $f^c_R = C \overline{f^c_L}$ is a left-handed anti-fermion and $f^c_L = C \overline{f^c_R}$ is a right-handed anti-fermion, where $C$ is the charge conjugation matrix. $B^\mu, \phi^+, \phi^0$ are common for the left-handed and right-handed sectors, their mirror particles are themselves, moreover, $\phi^0$ is a real scalar field without any charges. After the model symmetry breakings, $\xi, \eta$ and $\chi^-$ will become heavy mirror quark and charged lepton. $\nu^0_L$ and $\nu^0_R$ will combine into a light Dirac neutrino through the loop-diagram radiative generation, which is the hot dark matter (HDM). $N^0_L$ and $N^0_R$ will form into a massive and stable Dirac fermion, which is the CDM in the model. Below the electroweak scale, the dark sector consists of the three light neutral particles $\nu^0_R, N^0, \phi^0$, which are difficult to be detected.
The Yukawa couplings are
\[
\mathcal{L}_Y = q_L^T Y_u u_R^* H_L + q_L^T Y_d d_R^* H_L + l_L^T Y_e e_R^* H_L + \frac{1}{2} l_L^T Y_L \ell_L^* \ell_L + q_R^T Y_e \xi_\ell H^*_R + q_R^T Y_u \eta_\ell H^*_R + l_R^T Y_\ell \chi_\ell H^*_R + \frac{1}{2} l_R^T Y_R \ell_R^* \ell_R + \bar{u}_R^* Y_1 \xi_L \phi^0 + \bar{d}_R^* Y_2 \eta_L \phi^0 + \bar{e}_R^* Y_3 \chi_L \phi^0 + \frac{1}{2} N_R^0 Y_L N_L^0 \phi^0 + \text{H.c.},
\]
where I omit the charge conjugation matrix $C$ for concision which should be sandwiched between two spinors with same chirality. $\epsilon = i \tau_2$ is the two-order antisymmetric tensor. The coupling parameters $Y_u, Y_\ell, Y_1,$ etc., are all $3 \times 3$ complex matrices in the flavor space, however, the leading matrix element of each coupling matrix should naturally be $\sim \mathcal{O}(1)$. Note that because of the spinor anti-commutativity and the $\epsilon$ antisymmetry, $Y_L$ and $Y_R$ must be two antisymmetric matrices for consistency. Obviously, the $Z_2^M$ symmetry forbids the explicit fermion mass terms such as $e_R^T M_{\chi_L} N_R^0 M_{\chi_R}^0$, and also it prohibits the couplings such as $l_L^* N_R^0 H_L$, $l_R^* N_R^0 H_R$, $e_R^T N_R^0 \phi^+$, $\chi_L^T N_R^0 \phi^+$. Similarly, the $B - L$ conservation prohibits the terms such as $l_L^T N_R^0 H^*_L$, $l_R^T N_R^0 H^*_R$, $e_R^* N_R^0 \phi^-$, $\chi_L^* N_R^0 \phi^-$. In short, the model symmetries guarantee that $N_L^0, N_R^0$ can not mix to other fermions except each other coupling, therefore $N^0$ can not mix with $\nu^0$, thus $N^0$ is a stable particle and it will eventually become the CDM.

Eqs. (1) and (2) show the explicit left-right mirror symmetry which is defined as follows,

\[
\begin{align*}
&u_{L,R} \leftrightarrow \xi_{R,L}, \quad d_{L,R} \leftrightarrow \eta_{R,L}, \quad e_{L,R} \leftrightarrow \chi_{R,L}, \quad \nu^0_L \leftrightarrow \nu^0_R, \quad N^0_L \leftrightarrow N^0_R, \\
&W^\mu_L \leftrightarrow W^\mu_R, \quad B^\mu \leftrightarrow B^\mu_R, \quad H_L \leftrightarrow H_R, \quad \phi^+ \leftrightarrow \phi^+, \quad \phi^0 \leftrightarrow \phi^0, \\
&g_L = g_R, \quad Y_u = Y_\xi, \quad Y_d = Y_\eta, \quad Y_e = Y_\chi, \quad Y_L = Y_R, \quad Y_{1,2,3} = Y_{1,2,3}^\dagger, \quad Y_N = Y_N^\dagger.
\end{align*}
\]

This is indeed an aesthetics compared to the SM with many shortcomings. Here we do not pursue the exact left-right mirror symmetry, therefore these Yukawa matrix equalities in Eq. (3) can be invalid. After the model symmetry breakings, the relevant scalar fields will develop their non-vanishing vacuum expectation values, as a result, these Yukawa couplings in Eq. (2) will give rise to all kinds of the fermion masses.

The full scalar potentials are
\[
V_S = \mu^2_L H_L^\dagger H_L + \mu^2_R H_R^\dagger H_R + \mu^2_\phi \phi^+ \phi^- + \frac{1}{2} \mu^2_0 (\phi^0)^2 + \lambda_L(H_L^\dagger H_L)^2 + \lambda_R(H_R^\dagger H_R)^2 + \lambda_+(\phi^+ \phi^-)^2 + \frac{1}{4} \lambda_0 (\phi^0)^4 + 2 \lambda_1 (H_L^\dagger H_L)(H_R^\dagger H_R) + [\lambda_2 H_L^\dagger H_L + \lambda_3 H_R^\dagger H_R](\phi^0)^2 + [2 \lambda_4 H_L^\dagger H_L + 2 \lambda_5 H_R^\dagger H_R + \lambda_6 (\phi^0)^2] \phi^+ \phi^-.
\]

We need constrain these mass-dimensional and coupling parameters in Eq. (4) in order to achieve the model symmetry breaking chain, see the following Eq. (7). We assume that $H_R$ develops a non-zero vacuum expectation value at the scale of $\sim 10^6$ GeV in
earlier period, \( H_L \) and \( \phi^0 \) are successively induced to develop non-zero vacuum expectation values at the low-energy scale in later period, but \( \phi^+ \) can by no means develop a non-zero vacuum expectation value, or it always keeps a vanishing vacuum expectation value. In addition, it is natural and believable that the self-interaction of each scalar field is stronger but the interactions among them are weaker, namely those interactive coupling parameters are much smaller than those self-coupling parameters. On the basis of an overall consideration, all kinds of the parameters in Eq. (4) are therefore constrained as follows,

\[
\begin{align*}
& (\lambda_L, \lambda_R, \lambda_+, \lambda_0) \sim 10^{-1} > 0; \quad 10^{-6} \lesssim (|\lambda_1|, |\lambda_2|, \ldots, |\lambda_6|) \lesssim 10^{-2}, \\
& \mu^2_R \approx -\frac{v^2_R}{\lambda_R} \sim -\left(10^6\right)^2 \text{GeV}^2, \quad \mu^2_L < -\lambda_1 v^2_R, \quad \mu^2_0 < -\lambda_3 v^2_R, \quad \mu^2_+ > -\lambda_5 v^2_R,
\end{align*}
\]

where \( \frac{v_R}{\sqrt{2}} = \langle H_R \rangle \) is the vacuum expectation value of the right-handed doublet scalar, see the following Eq. (6).

Based on the limits of Eq. (5), we can derive the vacuum configurations from the \( V_S \) minimum. The vacua of \( H_L \) and \( H_R \) are necessarily along the directions of their neutral components. The scalar sector will appear three neutral bosons under the unitary gauge. Since the couplings among the different scalars are weaker, the mass mixing among the neutral scalars are actually smaller, namely their mass-squared matrix is approximately diagonal. The detailed results are given as follows,

\[
\begin{align*}
H_L \rightarrow h^0 + v_L \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad H_R \rightarrow \Phi^0 + v_R \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \phi^0 \rightarrow \rho^0 + v_0, \quad \langle \phi^+ \rangle = 0,
\end{align*}
\]

\[
\left( \begin{array}{cc} v^2_L \\ v^2_L \\ v^2_R \end{array} \right) = \left( \begin{array}{ccc} \lambda_0 & \lambda_2 & \lambda_3 \\ \lambda_2 & -\lambda_0 & \lambda_1 \\ \lambda_3 & \lambda_1 & -\lambda_R \end{array} \right)^{-1} \left( \begin{array}{c} -\mu^2_0 \\ -\mu^2_2 \\ -\mu^2_R \end{array} \right),
\]

\[
v_0 \sim 0.1 \text{ MeV} \ll v_L \approx 246 \text{ GeV} \ll v_R \sim 10^6 \text{ GeV},
\]

\[
M_{h^0} \approx \sqrt{2\lambda_L} v_L, \quad M_{\Phi^0} \approx \sqrt{2\lambda_R} v_R, \quad m_{\rho^0} \approx \sqrt{2\lambda_0} v_0, \quad M_{\phi^\pm} \approx \sqrt{\mu^2_+ + \lambda_5 v^2_R},
\]

where \( v_L \) is namely the electroweak breaking scale which has been fixed by the SM physics. \( v_0 \) is the \( Z^M \) violating scale, which can be determined jointly by the neutrino mass and the dark sector physics. \( v_R \) is the left-right mirror symmetry breaking scale, which will be determined by the mirror sector physics. In short, the limits of Eq. (5) are natural and reasonable, they can ensure the vacuum stability and the symmetry breaking chain in the model. In Eq. (6), \( h^0 \) is exactly the SM Higgs boson with \( M_{h^0} \approx 125 \text{ GeV} \). \( M_{\Phi^0} \) is around \( v_R \), so \( \phi^0 \) can not appear in the low-energy phenomena. \( m_{\rho^0} \) is close to \( v_0 \), so \( \rho^0 \) is a light dark scalar, it will play a role in the dark sector physics. \( M_{\phi^\pm} \) is derived from the two contributions which are respectively the original mass \( \mu_+ \) and the induced mass from \( \langle H_R \rangle \), however, its reasonable value should be \( M_{\phi^\pm} \sim 10^4 \text{ GeV} \). \( \phi^\pm \) will play a key role in generating the neutrino mass and the baryon asymmetry.

According to the assignments of Tab. 1 and the relations in Eq. (6), the model
symmetries are spontaneously broken step by step through the following breaking chain,

\[
SU(2)_L \otimes U(1)_Y \times SU(2)_R \otimes U(1)^{global}_B-L \otimes Z_2^M \xrightarrow{(H_R)\sim 10^6 \text{ GeV}} SU(2)_L \otimes U(1)_Y \times SU(2)_R \otimes U(1)^{global}_B-L \otimes Z_2^M \xrightarrow{(H_L)\sim 10^2 \text{ GeV}} U(1)_{Q_e} \otimes U(1)^{global}_{B-L},
\]

\[
Y' = Y + 2I_3^R, \quad Q_e = I_3^L + \frac{Y'}{2} = I_3^L + \frac{Y}{2} + I_3^R,
\]

where \(Y'\) is exactly identified as the SM hypercharge. This breaking chain is also aesthetical, it involves neither super-hierarchy nor superhigh energy scale, all the transitions are very natural. Note that the global \(B-L\) conservation is kept all the while, the residual gauge symmetry is only the local electric charge conservation.

After the step-by-step spontaneous breakings of the model symmetries, all kinds of particle masses and mixings are generated through the Higgs mechanism. In the gauge sector, the masses and mixing of the gauge fields are given by the following relations,

\[
D_\mu \rightarrow \partial_\mu + \frac{ig_L}{\sqrt{2}}(W^+_{L\mu} \tau_L^+ + W^-_{L\mu} \tau_L^-) + \frac{ig_R}{\sqrt{2}}(W^+_{R\mu} \tau_R^+ + W^-_{R\mu} \tau_R^-) + ig_L Z^0_{L\mu} Q_L + i e A^0_{\mu} Q_e + i g_R Z^0_{R\mu} Q_R,
\]

\[
W_{L\mu}^\pm = \frac{W^1_{L\mu} \mp i W^2_{L\mu}}{\sqrt{2}}, \quad W_{R\mu}^\pm = \frac{W^3_{R\mu} \mp i W^2_{R\mu}}{\sqrt{2}}, \quad \begin{pmatrix} Z^0_{L\mu} \\ A^0_{\mu} \\ Z^0_{R\mu} \end{pmatrix} = U_{13} U_{12} U_{23} \begin{pmatrix} W^3_{L\mu} \\ B^3_{\mu} \\ W^3_{R\mu} \end{pmatrix},
\]

\[
U_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}, \quad U_{13} = \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad U_{23} = \begin{pmatrix} c_{13} & 0 & -s_{13} \\ 0 & 1 & 0 \\ s_{13} & 0 & c_{13} \end{pmatrix},
\]

\[
\tan \theta_{23} = \frac{g_Y}{g_R}, \quad \tan \theta_{12} = \frac{g_R}{g_L} \sin \theta_{23}, \quad \tan \theta_{13} \sim \frac{v^2_L}{v_R},
\]

\[
e = g_L \sin \theta_{12}, \quad Q_e = I_3^L + \frac{Y}{2} + I_3^R,
\]

\[
Q_L = \frac{I_3^L - Q_e \sin^2 \theta_{12}}{\cos \theta_{12}}, \quad Q_R = \frac{I_3^L + (I_3^L - Q_e) \sin^2 \theta_{23}}{\cos \theta_{23}}.
\]

\[
M_{W_R} = \frac{v_{L\mu} g_L}{2}, \quad M_{Z_R} = \frac{M_{W_R}}{\cos \theta_{12}}, \quad m_A = 0, \quad M_{Z_L} = \frac{M_{W_R}}{\cos \theta_{23}}, \quad M_{W_R} = \frac{v_R g_R}{2},
\]

where \(c_{ij} = \cos \theta_{ij}, s_{ij} = \sin \theta_{ij}\) are mixing angles. It is easy to see that \(\sin \theta_{12}\) is exactly equal to the Weinberg angle of the SM, so \(\sin \theta_{23} = \sin \theta_W \approx 0.481\) is actually known. If \(g_R = g_L\), then \(\sin \theta_{23} = \tan \theta_{12} = \tan \theta_W\) is also fixed. \(\tan \theta_{13}\) is very small due to \(v^2_L \ll v_R\), so we can ignore it. The mirror gauge bosons \(W^\pm_{R\mu}\) and \(Z^0_{R\mu}\) are \(\sim 10^6 \text{ GeV}\) heavy, they will decay into the mirror quarks or leptons, in addition, \(Z^0_{R\mu}\) can decay into a pair of the SM quark or lepton, for example, \(Z^0_R \rightarrow e^- + e^+\) or \(\nu_L^L + \nu_L^R\), which can be a source of high-energy cosmic rays.
In the Yukawa sector, the Yukawa couplings of Eq. (2) undergo the following three steps of evolutions. After the first step breaking in Eq. (7), \( \langle H_R \rangle \) gives rise to heavy masses of the mirror quarks and charged leptons, the Yukawa couplings become into

\[
\mathcal{L}_Y \rightarrow \begin{align*}
q_L^T Y_u u_R^* H_L^0 + q_L^T Y_d d_R^* c_H L + \bar{H}_L^0 e_R c_L^* e_H L + \frac{1}{2} \bar{H}_L^0 e_L l_L \phi^+ \\
- \xi_L M_\xi \xi_R - \eta_L M_\eta \eta_R - \chi_L M_\chi \chi_R + \nu_R^0 Y_R \chi_R \phi^+ \\
+ \bar{H}_R^0 Y_1 \xi_L \phi^0 + \bar{H}_R^0 Y_2 \eta_L \phi^0 + \bar{H}_L^0 Y_3 \chi_L \phi^0 + \bar{H}_L^0 Y_N \chi_R \phi^0 + H.c.,
\end{align*}
\]

\( M_\xi = \frac{v_R}{\sqrt{2}} \xi_T, \quad M_\eta = \frac{v_R}{\sqrt{2}} \eta_T, \quad M_\chi = \frac{v_R}{\sqrt{2}} \chi_T. \) (9)

\( M_\chi, M_\eta, M_\xi \) should be the scope of several TeVs to hundreds of TeV. However, the mirror quarks and charged leptons can eventually decay into the SM quarks and charged leptons such as \( \xi \rightarrow u + \phi^0, \eta \rightarrow d + \phi^0, \chi^- \rightarrow e^- + \phi^0 \), therefore they are completely decoupling and absence at the low-energy scale. Although they can not be detected at the present colliders, we can search them through the high-energy cosmic rays.

The Yukawa couplings in Eq. (9) can certainly generate a effective Dirac neutrino coupling through the loop diagram shown as Fig. 1. Note that \( \chi^- \) changes its chirality...
due to $M_\chi$ insert in Fig. 1. The careful calculation gives the following results,

$$
\mathcal{L}^{\text{eff}}_{\text{Neutrino}} = \frac{\sqrt{2}}{v_L} l_L^T Y_\nu \nu_R^0 H^*_L \phi^0 + \text{H.c.},
$$

$$(Y_\nu)_{\alpha\beta} = \frac{v_L}{16\pi^2 \sqrt{2}} \sum_i \left( Y_{L_i} Y_e^* Y_{3i} \right)_{\alpha\beta} M_{\chi_i}(Y_H^i)_{\beta\beta}(C_0 - p^2_{H_L} D_{11} + \phi^* \bar{\psi} H_L D_{12} - \bar{\psi} \psi H_L D_{13})$$

$$\approx \frac{1}{16\pi^2} \sum_i \left( Y_{L_i} M(Y_3)_{\alpha\beta} M_\chi_i (Y_H^i)_{i\beta} \right) \frac{f(M^2_{\chi_i})}{M^2_{\phi^-}} \sim 10^{-6},$$

$$C_0[ (p_L - p_{H_L})^2, p^2_\nu, p^2_{\phi^0}, M^2_{\phi^-}, M^2_{\phi^-}, M^2_{\chi_i}] = \frac{1}{M^2_{\phi^-}} f(M^2_{\chi_i}),$$

$$f(M^2_{\chi_i}) = \frac{\ln M^2_{\chi_i}}{M^2_{\phi^-}} - 1 \frac{\text{i} 2\pi \Theta(M^2_{\chi_i}/M^2_{\phi^-} - 1)}{M^2_{\phi^-}} \sim 1,$$  \quad (10)

where $M_{\chi_i} = \frac{v_L}{\sqrt{2}} Y_{\chi_i} (i = 1, 2, 3)$ are three eigen-masses of the diagonalized mass matrix $M_\chi$, and $M_\nu = \frac{v_L}{\sqrt{2}} Y_e^T$ is the charged lepton mass matrix (see the following (11)). $C_0$ and $D_{1i}$ are respectively the three-point and four-point functions of Passarino-Veltman \cite{17}. Because the $D_{1i}$ terms are much smaller than the $C_0$ term, for example, $p^2_{H_L} D_{11} \ll C_0$, we can ignore all the $D_{1i}$ terms for the neutrino mass generation. $\Theta(x)$ is the step function, so $\text{Im}[C_0] = \frac{\text{i} 2\pi}{M^2_{\phi^-}} \neq 0$ only when $M^2_{\chi_i} > M^2_{\phi^-}$. For $M_{\chi_1} < M_{\chi_2} < M_{\chi_3} \sim M_{\phi^-} \sim 10^4$ GeV and $M_\nu \sim 1$ GeV, we can estimate $Y_\nu \sim 10^{-6}$, obviously, this is a weak coupling compared to those couplings in Eq. (9).

The second step breaking in Eq. (7) is namely the electroweak breaking, $\langle H_L \rangle$ generates the SM particle masses, at the same time, the effective neutrino coupling of Eq. (10) is developed into the normal neutrino coupling. Thus the couplings of Eqs. (9) and (10) further become into

$$\mathcal{L}_Y \frac{\langle H_L \rangle}{\sqrt{2} v_L} M_u u_L - \bar{d}_R M_d d_L - \bar{e}_R M_e e_L + \nu^0 L Y_e L \phi^0 + \bar{\nu}_L Y_L \phi^0 - \bar{\nu}_L Y_L \phi^0 + \bar{\nu}_L Y_L \phi^0 + H.c.,$$

$$M_u = -\frac{v_L}{\sqrt{2}} Y_u^T, \quad M_d = \frac{v_L}{\sqrt{2}} Y_d^T, \quad M_e = \frac{v_L}{\sqrt{2}} Y_e^T,$$  \quad (11)

where the neutrino coupling is naturally brought into $\mathcal{L}_Y$. By this stage, the three light neutral particles $\phi^0, N^0, \nu^0$ are gradually separated from the rest of the model particles, they completely disappear into the dark sector after $\nu^0_L$ lastly decouples from the SM at the temperature of $\sim 1$ MeV.

The last step breaking in Eq. (7) is the $Z^M$ parity violating in the dark sector, $\langle \phi^0 \rangle$ generates light masses of $N^0$ and $\nu^0$, and tiny mixings between the SM quark(charged lepton) and the mirror quark(charged lepton). Eventually, all of the fermion masses are
Since quark(charged lepton) are actually very small, we can neglect them completely obtained as follows,

$$\mathcal{L}_Y \overset{(\phi^0)}{\rightarrow} - (\overline{u}_{R}, \xi_{L}) \left( \begin{array}{c} M_u \\ 0 \end{array} \right) \left( \begin{array}{c} -v_0 Y_1 \\ \xi_{L} \end{array} \right) - (\overline{d}_{R}, \eta_{L}) \left( \begin{array}{c} M_d \\ 0 \end{array} \right) \left( \begin{array}{c} -v_0 Y_2 \\ \eta_{L} \end{array} \right) - (\overline{e}_{R}, \chi_{L}) \left( \begin{array}{c} M_e \\ 0 \end{array} \right) \left( \begin{array}{c} -v_0 Y_3 \\ \chi_{L} \end{array} \right).$$

$$M_N = -v_0 Y_N, \quad M_{\nu} = -v_0 Y_{\nu}.$$  \tag{12}

Since \(v_0 \ll v_L \ll v_R\), the mixings between the SM quark(charged lepton) and the mirror quark(charged lepton) are actually very small, we can neglect them. \(M_N\) is close to \(v_0 \sim 0.1\) MeV, its three mass eigenvalues will be denoted by \(M_{N_i}(i = 1, 2, 3)\). \(N^0\) has no mixing with \(\nu^0\), but there is a weak interaction between them via the \(\phi^0\) mediation. Therefore, \(N^0\) is genuinely both a sterile neutral fermion and a stable WIMP, it will eventually become the CDM in the model. We will specially discuss the dark sector physics in Sec. IV. \(M_{\nu}\) embraces the full information of the neutrino mass and mixing which have been measured by the experiments. Here we only work out the neutrino mass, regardless of its mixing. Under the exact left-right mirror symmetry, there are \(Y_L = Y_R\), diagonal \(Y_{e_i} = Y_{\chi_i}\), and Hermitian \(Y_{\nu}\), so \(Y_{\nu}\) excluding the factor \(f(M_{\chi_i}^2/M_{\phi^-}^2)\) is a Hermitian matrix, thus we can derive the following results,

$$\text{Tr} M_{\nu} = \sum_i m_{\nu_i} = -\frac{v_0}{16\pi^2} \sum_i \frac{(Y_R^\dagger Y_L M^2_{\chi_i})_{ii} M_{\chi_i}}{M_{\phi^-}^2} f\left(\frac{M_{\chi_i}^2}{M_{\phi^-}^2}\right),$$

$$\implies m_{\nu_i} \sim \frac{v_0 m_{\tau} M_{\chi_i}}{16\pi^2 M_{\phi^-}^2} \left| f\left(\frac{M_{\chi_i}^2}{M_{\phi^-}^2}\right) \right| \lesssim 10^{-10} \text{ GeV},$$ \tag{13}

where \(m_{\nu_i}\) is three mass eigenvalues of \(M_{\nu}\) and \(M_{\tau} = 1.777\) GeV is the largest eigenvalue of the charged lepton mass matrix \(M_{\nu}\). Because of the \(v_0\) smallness and the \(M_{\phi^-}\) suppression, \(m_{\nu_i}\) is only Sub-eV. In short, the model can naturally generate the tiny neutrino mass and elegantly explain its origin, obviously, this mechanism is very different from a wide variety of seesaw ones \[18\].

Based on the discussions in this Section, finally, we summarize that the full particle mass spectrum in the model should be such relations as

\[
\begin{align*}
m_A &= 0 < m_{\nu_i} \lesssim 0.05 \text{ eV} \ll m_{N_i} \sim 0.01 \text{ MeV} < (m_{\phi}, m_{\phi}) \sim 0.1 \text{ MeV} \\
\begin{array}{c}< (M_e, M_\rho) \sim (10^{-3} - 1) \text{ GeV} < (M_{W_L}, M_{Z_L}, M_{h^0}, M_\eta) \sim 100 \text{ GeV} \\
< M_{\chi_i} \sim (1 - 10) \text{ TeV} < (M_{\phi^-}, M_{\chi_3}, M_\eta, M_\xi) \sim (10 - 10^2) \text{ TeV}
\end{array} \\
< (M_{W_R}, M_{Z_R}, M_{h^0}) \sim 10^3 \text{ TeV}. \tag{14}
\end{align*}
\]

In the following Sections, we will see that the mass relations of Eq. (14) can lead to successful explanations for the matter-antimatter asymmetry and the CDM.

III. Baryon Asymmetry
\[ \chi_1^- \rightarrow \nu_R^0 + l_L + l_L. \] The big black vertex indicates the effective neutrino coupling. This is a CP asymmetric and out-of-equilibrium decay but the \( B - L \) number is conserved, after this decay \( \nu_R^0 \) is decoupling and disappears into the dark sector. As a consequence, the decay simultaneously and equivalently generates the \( \nu_R^0 \) asymmetry in the dark sector and the \( B - L \) asymmetry in the SM sector, the former is namely equal to \(-(B - L)\) asymmetry in the dark sector, the latter will be partly converted into the baryon asymmetry through the electroweak sphaleron effect.

In the model, the matter-antimatter asymmetry occurs at the time of \( \nu_R^0 \) decoupling and disappearing into the dark sector, it originates from the characteristic decays of the lightest mirror charged lepton \( \chi_1^- \), namely the mirror particle of \( e^- \), whose mass is about several TeVs. From those couplings in Eq. (9), we can see that \( \chi_1^- \) has only two decay modes at the tree level, i) the two-body decay \( \chi_1^- \rightarrow e^- + \phi^0 \), which is dominant, ii) the three-body decay \( \chi_1^- \rightarrow \nu_R^0 + l_L + l_L \) via the \( \phi^- \) mediation, which is suppressed since \( M_{\phi^-} \) is an order of magnitude heavier than \( M_{\chi_1} \). After the effective neutrino coupling of Eq. (10) is taken into account, the three-body decay should also add a loop-diagram contribute. Fig. 2 draws the tree and loop diagrams of \( \chi_1^- \rightarrow \nu_R^0 + l_L + l_L \) on the basis of the couplings in Eqs. (9) and (10). It is however emphasized that the decay processes always conserve the \( Y' \) charge, the \( B - L \) number and the \( Z^M \) parity, so the model can only provide the other two of the Sakharov’s three conditions [19].

The decay in Fig. 2 has the following three characteristics. Firstly, the decay rate of \( \chi_1^- \rightarrow \nu_R^0 + l_L + l_L \) is different from one of its CP conjugate process \( \chi_1^+ \rightarrow \nu_R^0 + l_L^c + l_L^c \) due to the interference between the tree diagram and the loop one, so this is a CP asymmetric decay. The decay Feynman amplitude is given by

\[
|M|^2 = \frac{m_{12}^2}{2M_{\phi^-}^4} \text{Tr}[Y_L^\dagger Y_L](Y_R^\dagger Y_R)_{11} - \frac{2m_{12}^2m_{23}^2}{M_{\phi^-}^2 v_L^2} \text{Re}[\text{Tr}[Y_\nu Y_\nu^\dagger(M_{\chi_1})C_{12}^\alpha]] ,
\]

\[
Y_{\alpha\beta}(M_{\chi_1}) = \frac{1}{16\pi^2}(Y_L M_L^\dagger Y_3)_\alpha i \text{Re} (Y_{\nu R}^\dagger)_{i\beta} , \quad Y_\nu = \sum_i Y(M_{\chi_1}) C_0(M_{\chi_1}) ,
\]

\[
\text{Im}[C_{12}(m_{17}^2, m_{23}^2, M_{\chi_1}^2, m_{e}^2, M_{H_L}^2, m_{\phi^0}^2)] = -\frac{i\pi}{M_{\chi_1}^2} \left[ 1 + \frac{M_{H_L}^2}{M_{\chi_1}^2} \ln(1 + \frac{M_{\chi_1}^2}{M_{H_L}^2}) \right] \approx -\frac{i\pi}{M_{\chi_1}^2} ,
\]
where $m_{12}^2 = (p_l + p_n)^2$, $m_{23}^2 = (p_n + p_{
u e n})^2$, and we have defined the functional matrix $Y(M_{\chi_1})$ for concision. In the Feynman amplitude, the first term is pure tree-diagram result, the second term is the interference term. $Y(M_{\chi_1})$ is a Hermitian matrix in the case of the left-right mirror symmetry, then $\text{Tr}[Y(M_{\chi_1})Y^\dagger(M_{\chi_1})]$ is certainly real. Provided by $M_{\chi_1} > M_{\phi^-}$, then $\text{Im}[C_0(M_{\chi_1})] = \frac{12\pi}{M_{\chi_1}^3} \neq 0$, thus $\text{Im}[C_0(M_{\chi_1})]$ and $\text{Im}[C_{12}]$ can jointly generate a CP asymmetry of the decay rate, in other words, the decay CP asymmetry completely and purely arises from the two loop-diagram radiative effects in Fig. 1 and Fig. 2, it has nothing to do with the CP-violating sources in the Yukawa sector (namely Eq. (2)). The decay rate and its CP asymmetry are therefore calculated as follows,

\[
\Gamma[\chi_1^\rightarrow e^- + \phi^0] = \frac{M_{\chi_1}}{32\pi}(Y^\dagger Y)_{11},
\]

\[
\Gamma[\chi_1^\rightarrow \nu^0_e + l_L + l_L] = \frac{M_{\chi_1}}{768(2\pi)^3}(M_{\phi^-})^4\text{Tr}[Y^\dagger Y_L](Y^\dagger Y_R)_{11},
\]

\[
\Gamma_{total}[\chi_1^\rightarrow e^- + \phi^0] = \Gamma[\chi_1^\rightarrow e^- + \phi^0] + \Gamma[\chi_1^\rightarrow \nu^0_e + l_L + l_L] \approx \Gamma[\chi_1^\rightarrow e^- + \phi^0],
\]

\[
\varepsilon = \frac{\Gamma[\chi_1^\rightarrow \nu^0_e + l_L + l_L] - \Gamma[\chi_1^\rightarrow \nu^0_R + l_L^c + l_L^c]}{\Gamma_{total}[\chi_1^\rightarrow e^- + \phi^0]},
\]

\[
\sim - \frac{M_{\chi_1}^4}{24v_L^2M_{\phi^-}^2(Y^\dagger Y)_{11}} \text{Tr}[Y^\dagger Y_L](Y^\dagger Y_R)_{11},
\]

\[
\sim - \frac{m_{\nu}^2M_{\chi_1}^2}{(16\pi)^2v_L^2M_{\phi^-}^2(Y^\dagger Y)_{11}},
\]

where the Yukawa matrices $Y_L$, $Y_3$, $Y_R$ are $\sim O(1)$ but the matrix elements $(Y^\dagger Y)_{11}$, $(Y^\dagger Y_{11})_{11}$ are only $\sim 10^{-6}$, and the trace of multiple matrix multiplication is also $\sim O(1)$ in the last approximation. In fact, the three-body decay rate is $\sim 10^{-7}$ times smaller than the two-body one because of the twofold suppressions of the phase space factor and the $\left(\frac{M_{\chi_1}}{M_{\phi^-}}\right)^4$ factor. For $(Y^\dagger Y)_{11} \sim 10^{-6}$, $\frac{M_{\chi_1}}{M_{\phi^-}} \sim 0.1$ and $\frac{M_{\chi_1}}{M_{\phi^-}} \gtrsim 1$, then we can obtain $\varepsilon \sim 10^{-8}$, which is a suitable value for the successful leptogenesis.

Secondly, the three-body decay rate in Eq. (16) is surely smaller than the universe Hubble expansion rate, namely

\[
\Gamma[\chi_1^\rightarrow \nu^0_R + l_L + l_L] < H(T = M_{\chi_1}) = \frac{1.66\sqrt{g_*}M_{\chi_1}^2}{M_{Pl}},
\]

where $M_{Pl} = 1.22 \times 10^{19}$ GeV, $g_*(T)$ is the effective number of relativistic degrees of freedom. At the temperature of $T = M_{\chi_1}$, the relativistic states include all the SM particles and the light dark particles $\phi^0, N^0, \nu^0_1$, so we can easily figure out $g_* = 123.5$. Eq. (17) explicitly indicates that the decay in Fig. 2 is out-of-equilibrium.

Thirdly, the out-of-equilibrium three-body decay directly leads that $\nu^0_R$ decouples from the rest of the model and disappears into the dark sector. As a consequence, the decay in Fig. 2 simultaneously and equivalently generates a $B - L$ asymmetry in the SM sector and a $-(B - L)$ asymmetry in the dark sector which is namely equal to the $\nu^0_R$ asymmetry,
note that $N^0$ can not be generated an asymmetry in the dark sector due to its distinctive coupling, so it is always a symmetric CDM. After this, the generated $B - L$ asymmetry in the SM sector will be partly converted into the baryon asymmetry through the sphaleron effect above the electroweak scale [20]. Put them together, these asymmetries normalized to the entropy are therefore given by the following relations [21],

$$Y_{B-L}^{SM} = -Y_{B-L}^{DS} = Y_{\nu_R} = \frac{n_{\nu_R} - \overline{n_{\nu_R}}}{s} = \kappa \frac{-\varepsilon}{g_*},$$

$$Y_B = c_s Y_{B-L}^{SM},$$

where $c_s = \frac{28}{79}$ is the sphaleron conversion coefficient in the SM sector. $s$ is the total entropy density in the SM and dark sectors. $\kappa$ is a dilution factor, it can be taken as $\kappa \approx 1$ because the dilution effect is in fact very weak. It should be pointed out that the effective neutrino coupling of Eq. (10) in which $\nu_R^0$ is involved is more severely out-of-equilibrium due to the $Y$, smallness and the $v_L$ suppression, so it can not dilute the above-mentioned asymmetries. In addition, the weak neutrino coupling in Eq. (11) is presence only below the electroweak scale, through which the $\nu_R$ asymmetry and the $\nu_L$ one can be erased in later time, but the sphaleron process was already closed and the baryon asymmetry has been locked down.

After the $\chi^{\pm}$ decays are over, all of the mirror particles including $\nu_R^0$ are completely decoupling, and the stable baryon asymmetry has been generated. As the universe temperature drops to the electroweak scale, then the universe comes into the SM epoch and the known evolutions. In the present-day universe, the baryon asymmetry and its density are therefore given by

$$\eta_B = \frac{n_B - \overline{n_B}}{n_\gamma} = \frac{s(T_0)}{n_\gamma(T_0)} Y_B \approx 6.1 \times 10^{-10},$$

$$\Omega_B h^2 = \frac{m_p \eta_B n_\gamma(T_0)}{\rho_c} h^2 \approx 0.0223,$$

where $T_0 \approx 2.73$ K is the present-day temperature of the CMB, $n_\gamma(T_0) \approx 411$ cm$^{-3}$ is the photon number density, and $\frac{s(T_0)}{n_\gamma(T_0)} = 3.6$ since only the photon is still relativistic and the massive neutrino has become non-relativistic. $m_p = 0.938$ GeV is the proton mass, $\rho_c = 1.054 \times 10^{-5} h^2$ GeV cm$^{-3}$ is the critical energy density [1]. $\eta_B \approx 6.1 \times 10^{-10}$ is the current baryon asymmetry from multiple experiments [1, 22], and $\Omega_B h^2 \approx 0.0223$ is the current baryon density [1]. In short, the model can clearly explain the origin of the baryon asymmetry, in particular, the matter-antimatter asymmetry only takes place at the TeV scale, which is very possibly tested in the future experiments.

IV. Dark Sector Physics

Below the electroweak scale, the dark sector consists of $\nu_R^0$, $N^0$, $\phi^0$ in the model. There are two portals connecting the dark sector and the SM one, by which the SM sector can communicate with the dark one. One is the $\lambda_2$ scalar coupling in the Eq. (4) potentials.
For $10^{-3} \lesssim \lambda_2 \lesssim 10^{-2}$, the reaction rate of $h^0 + h^0 \leftrightarrow \phi^0 + \phi^0$ will become smaller than the universe expansion rate at $T \approx 10$ GeV or so, refer to [22], $\phi^0$ thus decouples from the SM Matter below this temperature. The other one is the neutrino coupling in Eq. (11), which is developed from the Eq. (10) effective coupling after the electroweak breaking. At $T_D \approx 1$ MeV or so, however, $\nu^0_L$ also decouples from the SM sector and disappears into the dark sector, which becomes the last member of the dark sector. Eventually, the dark sector and the SM one are isolated from each other as the universe cooling.

Below $T_D \approx 1$ MeV, the SM sector and the dark sector are separately evolving, therefore the entropy in each sector is respectively conserved, thus we can derive the effective temperature of $\nu^0$ as follows,

\[
\frac{s^{DS}(T_D) a^3(T_D)}{s^{SM}(T_D) a^3(T_D)} = \frac{s^{DS}(T_0) a^3(T_0)}{s^{SM}(T_0) a^3(T_0)} = \frac{g^\nu_s(T_D)}{g^\nu_s(T_0)} = \frac{g^\nu_s(T_D)}{g^s(T_0)} \frac{g_s(T_0)}{g_s(T_0)} = \frac{T_\nu}{T_0} = \left( \frac{16}{21} \right)^{\frac{1}{4}}, \quad T_\nu \approx 2.49 \text{ K},
\]

where $a(T)$ is the scale factor of the universe expansion. At $T_D \approx 1$ MeV the dark particles are all massless states since the $Z^M_2$ symmetry is yet unbroken. At $T = v_0 \approx 0.1$ MeV, the $Z^M_2$ parity violating generates the light masses of the dark particles, only $\nu^0$ is still relativistic state. Here $T_\nu \approx 2.49$ K is higher than $Y_\nu \approx 1.95$ K in the SM, this a prediction of the model.

In the light of the couplings of the third line in Eq. (11), the main evolutions inside the dark sector are the following processes,

\[
\phi^0 \rightarrow N_1 + \overline{N}_1, \quad N_{2,3} + \overline{N}_{2,3} \rightarrow N_1 + \overline{N}_1, \quad N_1 + \overline{N}_1 \rightarrow \nu + \overline{\nu},
\]

\[
N_1 + N_1 \rightarrow N_1 + N_1, \quad N_1 + \overline{N}_1 \rightarrow N_1 + \overline{N}_1, \quad N_1 + \nu \rightarrow N_1 + \nu.
\]

$\phi^0$ decaying into $N_1 + \overline{N}_1$ is always permitted by kinematics, so it is absence in the present-day dark sector. $N_i$ can not decay and is stable, so the pair annihilation of $N_i + \overline{N}_i$ via the s-channel $\phi^0$ mediation is the only way out, Fig. 3 shows the relevant Feynman diagrams. Because $N_{2,3} + \overline{N}_{2,3} \rightarrow N_1 + \overline{N}_1$ has a very strong cross-section, which is $\sim 10^{10}$ times larger than the cross-section of $N_i + \overline{N}_i \rightarrow \nu + \overline{\nu}$, the heavier pairs of $N_{2,3} + \overline{N}_{2,3}$ are all annihilating exhaustion, so they are also absence in the present-day dark sector. However, $N_1 + \overline{N}_1 \rightarrow \nu + \overline{\nu}$ has exactly a weak cross-section on account of the weak neutrino coupling $Y_\nu$, therefore the lightest pairs of $N_1 + \overline{N}_1$ can not be annihilating exhaustion, their sizeable relics are thus left in the dark sector, which are namely the CDM in the present-day universe.

When the annihilate rate of $N_1 + \overline{N}_1 \rightarrow \nu + \overline{\nu}$ is smaller than the universe expansion rate, the annihilation process is frozen, thus $N_1$ and $\overline{N}_1$ are non-relativistic decoupling and become the CDM, at the same time, $\nu$ and $\overline{\nu}$ are relativistic decoupling and become the HDM. The annihilate cross-section and the freeze-out temperature are calculated by
Figure 3: (a) \( N_{2,3} + \overline{N}_{2,3} \rightarrow N_1 + \overline{N}_1 \) has a very strong cross-section, so the heavier pairs of \( N_{2,3} + \overline{N}_{2,3} \) are all annihilating exhaustion. (b) \( N_1 + \overline{N}_1 \rightarrow \nu + \overline{\nu} \) has a weak cross-section, which exactly fits the “WIMP Miracle”, so the lightest pairs of \( N_1 + \overline{N}_1 \) can remain sizeable relics. After the annihilation is frozen, \( N_1 \) is non-relativistic decoupling and becomes the CDM, at the same time, \( \nu \) is relativistic decoupling and becomes the HDM.

The following relations,

\[
\Gamma[N_1 + \overline{N}_1 \rightarrow \nu + \overline{\nu}] = \langle \sigma v_r \rangle n_{N_1} = H(T_f),
\]

\[
\langle \sigma v_r \rangle_{T_f} = a + b (v^2_{r})_{T_f} = a + b \frac{6T_f}{m_{N_1}}, \quad n_{N_1}(T_f) = 2T_f^3 \left( \frac{m_{N_1}}{2\pi T_f} \right)^2 e^{-\frac{m_{N_1}}{T_f}},
\]

\[
a = 0, \quad b = \frac{\sum_i m_{\nu_i}^2}{128\pi v_0^4 (1-y)^2}, \quad y = \left( \frac{m_{\phi}}{2m_{N_1}} \right)^2,
\]

\[
\frac{m_{N_1}}{T_f} = \frac{1}{x} \approx 11.4 + \ln \frac{m_{N_1} \text{(MeV)}}{\sqrt{g^*_{\nu} (T_f)}} + \ln \frac{\langle \sigma v_r \rangle_{T_f} \text{(GeV}^{-2})}{10^{-10}},
\]

\[
\Rightarrow \langle \sigma v_r \rangle_{T_f} \sim 5 \times 10^{-9} \text{ GeV}^{-2}, \quad \frac{m_{N_1}}{T_f} \sim 10,
\]

for \( v_0 \sim 0.1 \text{ MeV}, \ y \sim 10, \ m_{N_1} \sim 0.01 \text{ MeV}, \) \hspace{1cm} (22)

where \( v_r = 2\sqrt{1 - \frac{4m_{N_1}^2}{\phi}} \) is a relative velocity of \( N_1 \) and \( \overline{N}_1 \), and \( g_*(T_f) = g^*_{\gamma} + g^*_{\nu} (T_f) = 10 \). \langle \sigma v_r \rangle_{T_f} \) is the thermal average annihilate cross-section, note that the s-wave contribution to it is actually vanishing. For the above parameter values, \( \langle \sigma v_r \rangle_{T_f} \) is exactly a weak interaction cross-section, which is namely the so-called “WIMP Miracle” \[24\], in addition, the freeze-out temperature is \( T_f \sim 1 \text{ KeV} \).

In the present-day universe, the density of \( N_1 \) as the CDM is given by the following equations \[25\],

\[
\Omega_{\text{CDM}} h^2 = \Omega_{N_1} h^2 = \frac{2m_{N_1} n_{N_1}(T_0)}{\rho_c} h^2 = \frac{0.87 \times 10^{-10} \text{ GeV}^{-2}}{\sqrt{g_*(T_f) x(a + 3bx)}} \approx 0.119,
\]

\[
n_{N_1}(T_0) = \frac{g_*(T_0) T_0^3}{g_*(T_f) T_f^3} n_{N_1}(T_f).
\]

\hspace{1cm} (23)
By use of $a, b, x$ determined in Eq. (22), we can correctly reproduce $\Omega_{\text{CDM}} h^2 \approx 0.119$ which is the current density of the CDM from multiple observations [1, 20]. The neutrino $\nu$ as the HDM has only a tiny mass, its density is calculated by the following relations,

$$\Omega_{\text{HDM}} h^2 = \Omega_{\nu} h^2 = \frac{n_{\nu}(T_{\nu}) \sum m_{\nu_i}}{\rho_c} h^2 \approx 3.5 \times 10^{-3},$$

$$n_{\nu}(T_{\nu}) = \left( \frac{T_{\nu}}{T_f} \right)^3 n_{\nu}(T_f) = \frac{1.2}{\pi^2} g'_\nu T^3_{\nu} \approx 469 \text{ cm}^{-3},$$

where $g'_\nu = \frac{3}{4} \times 4 = 3$ for one generation of Dirac neutrino. Here the neutrino number density $n_{\nu}$ is about four times as large as $n_{\nu} \approx 112 \text{ cm}^{-3}$ in the SM, and it exceeds $n_{\gamma}(T_0) \approx 411 \text{ cm}^{-3}$. The above density value of the HDM neutrino is another one prediction of the model, see the following Eq. (26). However, both the CDM $N_1$ and the HDM $\nu$ are in the dark sector, which have only feeble connections with the SM sector, so it is very difficult to detect them.

Finally, it is worthy to be pointed out that $N_1$ is in fact the CDM with a self-interaction. The elastic scatterings of $N_1 + N_1 \rightarrow N_1 + N_1$ and $N_1 + \overline{N}_1 \rightarrow N_1 + \overline{N}_1$ in Eq. (21) are implemented via the t-channel $\phi^0$ mediation. These elastic scatterings are still equilibrium after the weak annihilation of $N_1 + \overline{N}_1 \rightarrow \nu + \overline{\nu}$ is frozen, therefore this self-interaction among the CDM can drive its density distribution and has important impact on small scale structure of the universe [27], we will specially discuss this problem in another paper. In addition, the elastic scattering of $N_1 + \nu \rightarrow N_1 + \nu$ in Eq. (21) is also a significant process, it will be discussed in the following section. In short, we can see from the above discussions that the model not only explains the origin of the dark matter but also paves the way for the fruitful dark physics world.

V. Numerical Results and Model Test

We now demonstrate and summarize the model by some concrete numerical results. All kinds of the parameters in the SM sector have essentially been fixed by the current experimental data [1]. Some key parameters in the mirror sector, also including the dark sector, can be determined jointly by the current data of the tiny neutrino mass, the baryon asymmetry, and the CDM abundance. Therefore the key parameters of the model are chosen as follows,

$$v_L = 246 \text{ MeV}, \quad v_R = 10^6 \text{ GeV}, \quad \text{Tr} (Y^\dagger_L Y_L) = \text{Tr} (Y^\dagger_R Y_R) = \text{Tr} (Y^\dagger_3 Y_3) = 1, \quad (Y^\dagger_3 Y_3)_{11} = (Y^\dagger_3 Y_3)_{11} = 10^{-6},$$

$$M_{\phi^0} = 2 \times 10^4 (5 \times 10^4) \text{ GeV}, \quad v_0 = 0.05 (0.1) \text{ MeV}, \quad m_{N_1} = 0.1 v_0,$$

$$\frac{M_{N_1}}{M_{\phi^0}} = 0.135 (0.125), \quad \frac{M_{N_2}}{M_{\phi^0}} = 0.176 (0.19), \quad \frac{M_{N_3}}{M_{\phi^0}} = 3.62 (2.9), \quad \frac{m_{\phi^0}}{2m_{N_1}} = 3.78 (1.98),$$

(25)

where those values in the first two lines are fixed as benchmark, the last two lines are two sets of typical values in the parameter space. For the two sets of values of $M_{\phi^0}$ and
\( \nu_0 \), the neutrino \( m_{\nu} \) is dominated by the ratio of \( \frac{M_{\chi_i}}{M_{\phi^-}} \), so \( \frac{M_{\chi_1}}{M_{\phi^-}} \) and \( \frac{M_{\chi_2}}{M_{\phi^-}} \) are determined by the two mass-squared differences of the neutrino, while \( \frac{M_{\chi_3}}{M_{\phi^-}} \) is determined by fitting the baryon asymmetry \( \eta_B \), lastly, the CDM density \( \Omega_{CDM} h^2 \) determines the ratio of \( \frac{m_{\phi_0}}{2m_{N_1}} \).

Obviously, all of the parameter values in Eq. (25) are consistent and reasonable, and without any fine-tuning, they are completely in accordance with the model requirements and the previous discussions, see Eq. (14).

Now substitute Eq. (25) into the relevant equations of the model, then we can correctly reproduce the following desired results,

\[
\begin{align*}
m_{\nu_1} & = 0.0155 (0.0119) \text{ eV}, \\
m_{\nu_2} & = 0.0178 (0.0147) \text{ eV}, \\
m_{\nu_3} & = 0.0534 (0.0522) \text{ eV}, \\
\Delta m_{21} & \approx 7.52 (7.58) \times 10^{-5} \text{ eV}^2, \\
\Delta m_{32} & \approx 2.54 (2.51) \times 10^{-3} \text{ eV}^2, \\
\frac{\Gamma}{H} & = 0.427 (0.136), \\
\eta_B & \approx 6.12 (6.07) \times 10^{-10}, \\
\Omega_B h^2 & \approx 0.0224 (0.0222), \\
\Omega_{CDM} h^2 & \approx 0.119 (0.119), \\
\Omega_{HDM} h^2 & \approx 0.00386 (0.00351),
\end{align*}
\] (26)

where \( \Delta m_{ij} = m_{\nu_i}^2 - m_{\nu_j}^2 \), and \( \frac{\Gamma}{H} \) is the ratio of \( \Gamma[\chi_1^- \rightarrow \nu_{R\alpha}^0 + l_L + l_L] \) to \( H(M_{\chi_1}) \). The values of \( \frac{\Gamma}{H} \) are smaller than one, this thus confirms that the decay is indeed out-of-equilibrium which is a necessary prerequisite. Explicitly, all the results of Eq. (26) are very well in agreement with the current experimental data \[1\]. In conclusion, only by use of these simple and natural parameters in Eq. (25), the model can completely account for the three outstanding puzzles of the neutrino mass, the baryon asymmetry, and the dark matter, therefore, this fully demonstrates that the model is very successful and believable.

In the end, we simply discuss about the model test. The heavy mirror particles can not be produced at the present colliders, but we can search them through high-energy cosmic rays, for example, the latest news from DAMPE collaboration about the cosmic ray spectrum from 40 GeV to 100 TeV \[28\]. On the basis of those couplings of the model, Fig. 4 shows three feasible approaches by which we can test the model predictions and probe the dark sector in the future. The (a) diagram process can directly produce the lightest mirror charged lepton pair at the future line collider with \( \sqrt{s} = 10 \) TeV such as ILC \[29\], then the \( CP \) asymmetric decay of \( \chi^\pm_1 \) will further generate an asymmetric number of the SM lepton and anti-lepton. Therefore this method can directly test the leptogenesis mechanism in the model.

The (b) diagram shows the elastic scattering of \( \nu_L + N_1 \rightarrow \nu_R + N_1 \) via the t-channel \( \phi^0 \) mediation, note that the \( \nu \) chirality is changed in the process. If a beam of \( \alpha \)-flavor left-handed \( \nu_{L\alpha} \) is produced at the laboratory, in the travel to the distant detector, its tiny part is scattering off the surrounding CDM and converted into the dark right-handed \( \nu_R \), which thus escapes detection, therefore we can detect the model dark sector by use
Figure 4: (a) The pair production of the lightest mirror charged lepton at the future $e^- + e^+$ collider with $\sqrt{s} = 10$ TeV, then the CP-asymmetric decay of $\chi_1^\pm$ can generate the SM lepton asymmetry. (b) The SM $\nu_L$ scattering off the CDM $N_1$, by which $\nu_L$ is converted into the dark $\nu_R$ and thus escapes detection. (c) The LFV process of $\mu \to e\gamma$ through the $\chi_1^-$ and $\phi^0$ mediation.

of this method. The scattering cross-section is given by

$$\sum_\beta \sigma[\nu_{L\alpha} + N_1 \to \nu_{R\beta} + N_1] = \left(\frac{M_{\nu_L}^\dagger M_\nu}{64\pi v_0^2 E_{\nu_L}^2}\right) f(E_{\nu_L}),$$

$$M_{\nu_L}^\dagger M_\nu = U_{\nu_L} \text{Diag}(m_{\nu_1}^2, m_{\nu_2}^2, m_{\nu_3}^2) U_{\nu_L}^\dagger,$$

$$f(E_{\nu_L}) = \int_{t_1}^{t_0} dt \frac{t(t - 4m_{N_1}^2)}{(t - m_{\phi^0}^2)^2}, \quad t_0 = 0, \quad t_1 = -\frac{4E_{\nu_L}^2 m_{N_1}}{2E_{\nu_L} + m_{N_1}},$$

(27)

where $U_{\nu_L}$ is the $\nu_{L\alpha}$ mixing matrix and $E_{\nu_L}$ is the $\nu_{L\alpha}$ energy in the laboratory frame. If we can use the electronic neutrino beam with $E_{\nu_L} = 1$ MeV, then $f(E_{\nu_L}) \approx -t_1$, thus we can estimate $\sigma \sim 10^{-10}$ GeV$^{-2}$ provided by $v_0 = 0.1$ MeV and $m_{N_1} = 0.1v_0$, which is also a weak interaction cross-section. Of course, the scattering rate also depends on the local density of the CDM $N_1$. By measuring the $\nu_{L\alpha}$ disappearance rate, however, we can learn the $v_0$ and $m_{N_1}$ information of the dark physics. In fact, the cosmic neutrino source is a better laboratory, for instance, we can detect the $\nu_L$ stream emitted by a distant supernova, it will travel through the CDM in the galactic halo before it can arrive to the earth, its tiny part will be scattering off and converted into the dark $\nu_R$, thus the $\nu_L$ stream which is eventually received is certainly less than the expected value. This detection is very similar to one of the flavor oscillation of the solar neutrino. In a word, this method can not only detect the dark sector physics, but also indirectly shed light on the neutrino mass origin in the model.

The (c) diagram is a LFV process of $\mu \to e\gamma$ through the $\chi_1^-$ and $\phi^0$ mediation. Its branch ratio is estimated as $\sim 10^{-14}$ provided by $(Y_3^\dagger)_{12}(Y_3)_{11} \sim 10^{-4}$, which is one order of magnitude lower than the current limit [1], so this process is very promising to be detected in the near future. In short, the above suggestions can become new subjects and goals of the experimental physicists which are endeavoring to search new physics evidences beyond the SM [3, 30]. Although it will be very large challenges to actualize
them, it is not impossible, moreover, its scientific significance is beyond all doubt.

VI. Conclusions

In summary, I suggest the left-right mirror symmetric particle model as the most natural and aesthetical extension of the SM. This model has the $SU(2)_L \times U(1)_Y \times SU(2)_R$ gauge symmetry, the global $B - L$ conservation and the $Z_2^M$ matter parity. The $SU(2)_R$ breaking gives rise to the heavy mirror particle masses at the $\sim 10^6$ GeV scale, while the $Z_2^M$ violating generates the light dark particle masses at the $\sim 0.1$ MeV scale. The SM electroweak breaking is just between the two scale. The tiny neutrino mass arises from the weak effective neutrino coupling which is generated by the loop diagram radiation. The lightest mirror charged lepton three-body decay leads to the $B - L$ asymmetry in SM sector and the $\nu_R$ asymmetry, the former is partly converted into the baryon asymmetry through the sphaleron effect, the latter disappears into the dark sector due to the $\nu_R$ decoupling. The dark sector consists of all of the light particles except the photon, note that $\nu_L$ eventually disappears into the dark sector after it decoupling from the SM sector. The dark Dirac fermion $N_1$ with $\sim 10$ KeV mass is a desirable CDM candidate. $N_1 + \overline{N}_1$ can annihilate into $\nu + \overline{\nu}$ via the dark scalar $\phi^0$ mediation, its cross-section exactly fits the “WIMP Miracle”. The fruitful dark physics is waiting for us to explore further.

In short, the simple model can completely account for the common origin of the tiny neutrino mass, the baryon asymmetry and the dark matter, moreover, uncover the profound internal connections among them. In addition, the model gives some interesting predictions, for instance, the lightest mirror charged lepton mass is about several TeVs, the dark physics scale is $\sim 0.1$ MeV, the $\nu$ effective temperature is 2.49 K, the HDM neutrino density is $\sim 3.5 \times 10^{-3}$, and so on. Finally, I give several feasible approaches to test the model by means of the TeV line collider, the neutrino experiments, the detection for $\mu \rightarrow e\gamma$, and the search for high-energy cosmic rays. In the near future, it is very possible that we shall ushered in a new physics era beyond the SM and open the door of the dark universe.

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