Theory of the oscillatory photoconductivity of a 2D electron gas

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We develop a theory of magnetooscillations in the photoconductivity of a two-dimensional electron gas observed in recent experiments. The effect is governed by a change of the electron distribution function induced by the microwave radiation. We analyze a nonlinearity with respect to both the dc field and the microwave power, as well as the temperature dependence determined by the inelastic relaxation rate.

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Recent experiments have discovered [1] that the resistivity of a high-mobility two-dimensional electron gas (2DEG) in GaAs/AlGaAs heterostructures subjected to microwave radiation of frequency \( \omega \) exhibits magnetooscillations governed by the ratio \( \omega/\omega_c \), where \( \omega_c \) is the cyclotron frequency. Subsequent work [2–6] has shown that for samples with a very high mobility and for high microwave power and that a positive sign is restored for a strong dc bias, as it was assumed in Ref. [7].

We consider a 2DEG (mass \( m \), density \( n_e \), Fermi velocity \( v_F \)) subjected to a transverse magnetic field \( B = (mc/e)\omega_c \). We assume that the field is classically strong, \( \omega_c \tau_{tr} \gg 1 \), where \( \tau_{tr} \) is the transport relaxation time at \( B = 0 \). The photoconductivity \( \sigma_{ph} \) determines the longitudinal current flowing in response to a dc electric field \( E_{dc} \), \( j \cdot E_{dc} = \sigma_{ph}E_{dc}^2 \), in the presence of a microwave electric field \( E_{\omega} \cos \omega t \). The more frequently measured [1–3,5,6] longitudinal resistivity, \( \rho_{ph} \), is given by \( \rho_{ph} \sim \rho_{xy} \sigma_{ph} \), where \( \rho_{xy} \approx eB/n_e c \) is the Hall resistivity, affected only weakly by the radiation.

We start with the formula for the dc conductivity:

\[
\sigma_{dc} = 2 \int d\varepsilon \sigma_{dc}(\varepsilon) [\partial \varepsilon f(\varepsilon)],
\]

where \( f(\varepsilon) = 1/e^{\varepsilon/T} - 1 \) is the electron distribution function, and \( \sigma_{dc}(\varepsilon) \) determines the contribution of electrons with energy \( \varepsilon \) to the dissipative transport. In the leading approximation [10,11], \( \sigma_{dc}(\varepsilon) = \sigma_{dc}^D \nu(\varepsilon) \), where \( \nu(\varepsilon) = \nu(\varepsilon)/\nu_0 \) is the dimensionless DOS, \( \nu_0 = m/2\pi \) is the DOS per spin at zero B (we use \( \hbar = 1 \)), and \( \sigma_{dc}^D = e^2\nu_0 v_F^2/2\omega_c^2 \tau_{tr} \) is the dc Drude conductivity per spin. All interesting effects are due to a non-trivial energy dependence of the non-equilibrium distribution function \( f(\varepsilon) \). The latter is found as a solution of the stationary kinetic equation

\[
\frac{\delta^2 \sigma^D(\omega)}{2\omega^2 \nu_0} \sum_{\pm} \nu(\varepsilon \pm \omega) \left[ f(\varepsilon) - f(\varepsilon + \omega) \right] + \frac{\delta^2 \sigma^D_{dc}(\varepsilon)}{\nu_0 \nu(\varepsilon)} \frac{\partial}{\partial \varepsilon} \nu(\varepsilon) \left[ \frac{\partial \nu(\varepsilon)}{\partial \varepsilon} f(\varepsilon) \right] = \frac{f(\varepsilon) - f_T(\varepsilon)}{\tau_{in}},
\]

where the ac Drude conductivity per spin is given by (we assume \( |\omega| \ll \omega_c \tau_{tr} \gg 1 \))

\[
\sigma^D(\omega) = \sum_{\pm} \frac{e^2\nu_0 v_F^2}{4\tau_{tr}(\omega \pm \omega_c)^2}.
\]
On the right-hand side of Eq. (2), inelastic processes are included in the relaxation time approximation (more detailed discussion of the relaxation time \( \tau_n \) is relegated to the end of the paper), and \( f_F(\varepsilon) \) is the Fermi distribution. The left-hand side is due to the electron collisions with impurities in the presence of the external electric fields. The first term describes the absorption and emission of microwave quanta; the rate of these transitions was calculated in Ref. [11]. This term can be also extracted from the kinetic equation of Ref. [10]. The second term describes the effect of the \( dc \) field and can be obtained from the first one by taking the limit \( \omega \to 0 \).

Equation (2) suggests convenient dimensionless units for the strength of the \( ac \) and \( dc \) fields:

\[
\mathcal{P}_\omega = \frac{\tau_{in}}{\tau_{tr}} \left( \frac{\varepsilon E_\omega v_F}{\omega} \right)^2 \frac{\omega_c^2 + \omega^2}{(\omega^2 - \omega_c^2)^2}, \tag{4a}
\]

\[
\sigma_{dc} = -\frac{2\tau_{in}}{\tau_{tr}} \left( \frac{\varepsilon E_\omega v_F}{\omega_c} \right)^2 \left( \frac{\pi}{\omega_c} \right)^2. \tag{4b}
\]

Note that \( \mathcal{P}_\omega \) and \( \sigma_{dc} \) are proportional to \( \tau_{in} \) and are infinite in the absence of inelastic relaxation processes.

We consider first the case of overlapping Landau levels (LLs), with the DOS given by \( \nu = 1 - 2\delta \cos \frac{2\pi\varepsilon}{\omega_c}, \) where \( \delta = \exp(-\pi/\omega_c, \tau_0) \ll 1 \). Here \( \tau_0 \) is the zero-\( B \) single-particle relaxation time, which is much shorter than the transport time in high-mobility structures, \( \tau_0 \ll \tau_c \) (because of the smooth character of a random potential of remote donors). The existence of a small parameter \( \delta \) simplifies solution of the kinetic equation (2). To first order in \( \delta \), we look for a solution in the form

\[
f = f_0 + f_{osc} + O(\delta^2), \quad f_{osc} = \delta \Re \left[ f_1(\varepsilon) e^{iQ\omega} \right]. \tag{5}
\]

We assume that the electric fields are not too strong \( |\mathcal{P}_\omega(\omega/T)^2 \ll 1 \) and \( \sigma_{dc}(\omega/T)^2 \ll 1 \), so that the smooth part \( f_0(\varepsilon) \) is close to the Fermi distribution \( f_F(\varepsilon) \) at a bath temperature \( T \gg \omega_c \); otherwise, the temperature of the electron gas is further increased due to heating. Smooth functions \( f_{0,1}(\varepsilon) \) change on a scale of the order of temperature. We obtain

\[
f_{osc}(\varepsilon) = \delta \frac{\omega_c}{2\pi} \frac{\partial f_F}{\partial \varepsilon} \sin \frac{2\pi\varepsilon}{\omega_c} \frac{\mathcal{P}_\omega \sigma_{ph}}{\omega_c} \sin 2\frac{\pi\varepsilon}{\omega_c} + 4\mathcal{Q}_{dc} \tag{6}
\]

and substitute Eq. (6) into Eq. (1). Performing the energy integration in Eq. (1), we assume (in conformity with the experiment) that \( T \) is much larger than the Dingle temperature, \( T \gg 1/2\pi\tau_{in} \). The terms of order \( \delta \) in Eq. (1) are exponentially suppressed \( \delta \int d\varepsilon \partial_\varepsilon f_F \cos \frac{2\pi\varepsilon}{\omega_c} \propto \delta \exp(-2\pi^2 T/\omega_c) \ll \delta^2 \) and can be neglected. The leading \( \omega \) dependent contribution to \( \sigma_{ph} \) comes from the \( \delta^2 \) term generated by the product of \( \partial_\varepsilon f_{osc}(\varepsilon) \propto \delta \cos \frac{2\pi\varepsilon}{\omega_c} \) and the oscillatory part \( -2\delta \cos \frac{2\pi\varepsilon}{\omega_c} \) of \( \nu(\varepsilon) \). This term does survive the energy averaging, \( -\int d\varepsilon \partial_\varepsilon f_T \cos^2 \frac{2\pi\varepsilon}{\omega_c} \propto 1/2 \). We thus find

\[
\frac{\sigma_{ph}}{\sigma_{dc}} = 1 + 2\delta^2 \left[ 1 - \mathcal{P}_\omega \frac{2\pi\varepsilon}{\omega_c} \sin \frac{2\pi\varepsilon}{\omega_c} + 4\mathcal{Q}_{dc} \right]. \tag{7}
\]

Equation (7) is our central result. It describes the photoconductivity in the regime of overlapping LLs, including all non-linear (in \( E_\omega \) and \( \mathcal{E}_{dc} \)) effects. Let us analyze it in more detail. In the linear-response regime (\( \mathcal{E}_{dc} \to 0 \)) and for a not too strong microwave field, Eq. (7) yields a correction to the dark \( dc \) conductivity \( \sigma_{dc} = \sigma_{dc}^0 (1 + 2\delta^2) \) which is linear in the microwave power:

\[
\frac{\sigma_{ph} - \sigma_{dc}}{\sigma_{dc}} = -4\delta^2 \mathcal{P}_\omega \frac{\pi\omega}{\omega_c} \sin \frac{2\pi\omega}{\omega_c}, \tag{8}
\]

in agreement with Ref. [11]. It is enlightening to compare Eq. (8) with the contribution of the effect of the \( ac \) field on the impurity scattering [8–10]. The analytic result, Eq. (6.11) of Ref. [10], in the notation of Eq. (4) is

\[
\frac{\sigma_{ph}}{\sigma_{dc}} = -12\frac{\tau_{in}}{\pi q} \delta^2 \mathcal{P}_\omega \frac{\pi\omega}{\omega_c} \sin \frac{2\pi\omega}{\omega_c} \sin 2\frac{\pi\omega}{\omega_c}. \tag{9}
\]

This result has a similar frequency dependence as Eq. (8); however, its amplitude is much smaller at \( \tau_{in} \gg \tau_q \), i.e., the mechanism of Refs. [8–10] appears to be irrelevant. Physically, the effect of the \( ac \) field on the distribution function is dominant because it is accumulated during a diffusive process of duration \( \tau_{in} \), whereas Refs. [8–10] consider only one scattering event.

![FIG. 1. Photoresistivity (normalized to the dark Drude value) for overlapping Landau levels vs \( \omega_c/\omega \) at fixed \( \omega_{in} = 2\pi \). The curves correspond to different levels of microwave power \( \mathcal{P}_{\omega}^{[10]} = \{0.24, 0.8, 2.4\} \). Nonlinear \( I - V \) characteristics at the marked minima are shown in Fig. 2.](image-url)
vicinity of the cyclotron resonance harmonics $\omega = k\omega_c$ ($k = 1, 2, \ldots$), and allows the photo-induced correction to exceed in magnitude the dark conductivity $\sigma_{dc}$. In particular, $\sigma_{ph}$ around minima becomes negative at $P_\omega > P^*_\omega > 0$, with the threshold value given according to Eq. (7) by $P^*_\omega = \left(4\delta^2 \pi \omega_c^2 \sin \frac{n\omega_c}{\omega_c} - \sin^2 \frac{n\omega_c}{\omega_c}\right)^{-1}$. The evolution of a $B$ dependence of the photoresistivity $\rho_{ph}$ with increasing microwave power $P_\omega^{(0)} = P_\omega (\omega_c = 0)$ is illustrated in Fig. 1.

Let us now fix $\omega/\omega_c$ such that $P^*_\omega > 0$, and consider the dependence of $\sigma_{ph}$ on the $dc$ field $\mathcal{E}_{dc}$ at $P_\omega > P^*_\omega$. As follows from Eq. (7), in the limit of large $\mathcal{E}_{dc}$ the conductivity is close to the Drude value and thus positive, $\sigma_{ph} = (1 - 6\delta^2)\sigma_{dc}^D > 0$. Therefore, $\sigma_{ph}$ changes sign at a certain value $\mathcal{E}_{dc}^*\omega_{dc}$ of the $dc$ field, which is determined by the condition $Q_{dc} = (P_\omega - P^*_\omega)/P^*_\omega$, see Fig. 2. The negative-conductivity state at $\mathcal{E}_{dc} < \mathcal{E}_{dc}^*$ is unstable with respect to the formation of domains with a spontaneous electric field of the magnitude $\mathcal{E}_{dc}^*$ [7].

Using Eqs. (4), we obtain

$$
\mathcal{E}_{dc}^* = \sqrt{\mathcal{E}_{dc}^2 - (\mathcal{E}_{dc}^*)^2} \left[ \frac{\omega_c^4 (\omega^2 + \omega_c^2)}{2n^2 (\omega^2 - \omega_c^2)^2} \right]^{1/2} \times \frac{1}{\pi} \mathrm{Re} \left( \frac{4 \pi \omega_c^2}{\omega} \sin \frac{2\pi \omega_c}{\omega_c} - \sin^2 \frac{\pi \omega}{\omega} \right)^{1/2},
$$

with $\mathcal{E}_{dc}^*$ being the threshold value of the $ac$ field at which the zero-resistance state develops. Equation (10) relates the electric field formed in the domain (measurable by local probe [6]) with the excess power of microwave radiation. It is worth noticing that this relation does not include the rate of the inelastic processes.

![FIG. 2. Current–voltage characteristics (dimensionless current $j_x = (\sigma_{ph}/\sigma_{dc}) \mathcal{E}_{dc}$ vs dimensionless field $\mathcal{E}_{dc} = Q_{dc}^{1/2}$) at the points of minima marked by the circles in Fig. 1. The arrows show the $dc$ field $\mathcal{E}_{dc}^*$ in spontaneously formed domains.](image)

We use Eqs. (1) and (2) to evaluate the $OPC$ at $Q_{dc} \to 0$ to first order in $P_\omega$ and estimate the correction of the second order. We obtain

$$
\frac{\sigma_{ph}}{\sigma_{dc}^D} = \frac{16\omega_c}{3\pi^2} \left(1 - P_\omega \frac{\omega_c^4}{\Gamma^2} \right) \times \left\{ \sum_n \Phi \left( \frac{\omega - n\omega_c}{\Gamma} \right) + O \left( \frac{\omega_c P_\omega}{\Gamma} \right) \right\},
$$

(12)

The photoresitivity for the case of separated LLs, Eq. (12), is shown in Fig. 3 for several values $P_\omega$ of the microwave power. Notice that a correction to Eq. (12) of second order in $P_\omega$ is still small even at $P_\omega > P^*_\omega = \Gamma^2/\omega_c^2$, since $\omega_c P_\omega/\Gamma = \Gamma/\omega \ll 1$. This means that it suffices to keep the linear-in-$P_\omega$ term only even for the microwave power at which the linear-response resistance becomes negative.

![FIG. 3. Photoresistivity (normalized to the dark Drude value) for separated Landau levels vs $\omega_c/\omega$ at fixed $\omega_c \tau_\ell = 16\pi$. The curves correspond to different levels of microwave power $P_\omega^{(0)} = \{0.004, 0.02, 0.04\}$.](image)

As in the case of overlapping LLs, a negative value of the linear-response conductivity signals an instability leading to the formation of domains with the field $\mathcal{E}_{dc}^*$ at which $\sigma_{ph}(\mathcal{E}_{dc}) = 0$. It turns out, however, that for separated LLs the kinetic equation in the form of Eq. (2) yields zeros (rather than expected positive) conductivity in the limit of strong $\mathcal{E}_{dc}$. This happens because elastic impurity scattering between LLs, inclined in a strong $dc$ field, is not included in Eq. (2). The inter-LL transitions become efficient in $dc$ fields as strong as $\mathcal{E}_{dc}^* \simeq (\tau_\ell/\tau_n)^{1/2} \omega_c^2/\epsilon_T$ [10], which actually gives the strength of the field in domains.

Finally, we calculate the inelastic relaxation time $\tau_n$. Of particular importance is its $T$ dependence which in turn determines that of $\sigma_{ph}$. At not too high $T$, the dominant mechanism of inelastic scattering is due to electron-electron ($e$-$e$) collisions. It is worth emphasizing that the
e-e scattering does not yield relaxation of the total energy of the 2DEG and as such cannot establish a steady-state dc photoconductivity. That is to say the smearing of \( f_0(\varepsilon) \) in Eq. (5), which is a measure of the degree of heating, is governed by electron-phonon scattering. However, the e-e scattering at \( T \gg \omega_c \) does lead to relaxation of the oscillatory term \( f_{sc} \) [Eq. (6)] and thus determines the \( T \) behavior of the oscillatory contribution to \( \sigma_{ph} \).

Quantitatively, the effect of electron-electron interaction is taken into account by replacing the right-hand side of Eq. (2) by \(-\sigma_{ee} \{f\}\), where the collision integral \( \sigma_{ee} \{f\} \) is given by

\[
\sigma_{ee} \{f\} = \int d\varepsilon' \int dE A(E) \tilde{v}(\varepsilon_+) \tilde{v}(\varepsilon_-) \quad (13)
\]

\[
\times [ -f(\varepsilon) h_\varepsilon(\varepsilon_-) f(\varepsilon_-) h_\varepsilon(\varepsilon_-) + h_\varepsilon(\varepsilon-) f(\varepsilon-) f(\varepsilon_-) f(\varepsilon_-) ] ,
\]

and \( h_\varepsilon(\varepsilon) \equiv 1 - f(\varepsilon) \). \( \varepsilon_+ = \varepsilon + E, \varepsilon_- = \varepsilon - E \). The function \( A(E) \) describes the dependence of the matrix element of the screened Coulomb interaction on the transferred energy \( E \),

\[
A(E) = \frac{1}{2\pi \epsilon_F} \ln \frac{\epsilon_F}{\epsilon_F} \max [E, \omega_c(\omega, \tau_{tr})]^{1/2} ,
\]

where \( \epsilon_F \) is the Fermi energy. Thus \( A(E) \) differs from the corresponding dependence for a clean 2DEG at zero \( B \) only by a change in the argument of the logarithm (a more detailed discussion will be given elsewhere).

We linearize the collision integral and solve Eq. (2). For overlapping LLs, we put \( \tilde{v} = 1 \) in accord with the accuracy of Eq. (7). Then only out-scattering processes contribute to the relaxation of the oscillatory part of the distribution function (6); the result is obtained by replacing \( \tau_{in} \rightarrow \tau_{ee}(\varepsilon, T) \) in Eq. (2) with [12]

\[
\frac{1}{\tau_{ee}} = \frac{\pi^2 \tau_{2} + \varepsilon^2}{4\pi \epsilon_F} \ln \max \left[ T, \omega_c(\omega, \tau_{tr}) \right]^{1/2} , \quad (14)
\]

We turn now to the case of separated LLs. In this case, due to oscillation of \( \tilde{v} \), even the linearized collision integral gives rise to a non-trivial integral operator. Analytical solution of the kinetic equation with this collision operator does not seem feasible. However, up to a factor of order unity, we can replace the exact collision integral with the relaxation-time approximation, thus returning to Eq. (2) with

\[
\frac{1}{\tau_{in}} \sim \frac{\omega_c}{\Gamma} \frac{T^2}{\epsilon_F} \ln \max \left[ T, \Gamma(\omega_c \tau_{tr}) \right]^{1/2} , \quad (15)
\]

One sees that in both cases of overlapping and separated LLs the inelastic relaxation rate is proportional to \( T^2 \), so that the OPC \( \sigma_{ph} - \sigma_{dc} \) in the linear-in-\( \mathcal{P}_c \) regime [Eqs. (8), (12)] scales as \( T^{-2} \).

Our results are in overall agreement with the experimental findings [2,3]. The observed \( T \) dependence of the photoresistivity at maxima compares well with the predicted \( T^{-2} \) behavior. Typical parameters \( \omega/2\pi \sim 50 - 100 \text{ GHz}, \tau_{q} \sim 10 \text{ ps} \) yield \( \tau_{q} \tau_{2} / 2\pi \sim 0.5 - 1 \) (overlapping LLs), and the experimental data indeed closely resemble Fig. 1. For \( T \sim 1 \text{ K} \) and \( \epsilon_F \sim 100 \text{ K} \) we find \( \tau_{in}^{-1} \sim 10 \text{ mK}, \) much less than \( \tau_{q}^{-1} \sim 1 \text{ K}, \) as assumed in our theory. Finally, for the microwave power \( \sim 1 \text{ mW} \) and the sample area \( \sim 1 \text{ cm}^2 \), we estimate the dimensionless power \( \mathcal{P}_c^{(0)} \sim 0.005 - 0.1 \), which agrees with characteristic values for separated LLs (Fig. 3) but is noticeably less than the prediction for overlapping LLs (Fig. 1). The reason for this discrepancy remains to be clarified.

To summarize, we have presented a theory of magnetooscillations in the photoconductivity of a 2DEG. The parametrically largest contribution to the effect is governed by the microwave-induced change in the distribution function. We have analyzed the nonlinearity with respect to both the microwave and \( dc \) fields. The result takes an especially simple form in the regime of overlapping LLs, Eq. (7). We have shown that the magnitude of the effect governed by the inelastic relaxation time increases as \( T^{-2} \) with lowering temperature.

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