Motion of dust near exterior resonances with planet

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Abstract. Motion of non-spherical cosmic dust particle under the action of electromagnetic radiation and gravitational forces of central star and a planet orbiting the star is investigated. The contribution concentrates on the problem if the particle can be trapped in commensurability resonances with the planet (a particle is in resonance with a planet when the ratio of their orbital periods is the ratio of two small integers). Trapping in the circular restricted three-body problem (planetary orbit is circle) with electromagnetic radiation coming from the star is considered. The effect of radiation plays an important role as for the possible capture in exterior resonances (particle’s orbital period is larger than that for the planet): particle’s optical properties not only define positions of the resonances, but also the capture times and evolution of orbital elements. Results for spherical and non-spherical particles are compared: physical difference between possibilities for resonant captures for spherical and non-spherical dust particles is pointed out.

We concentrate on possible trapping of the grain in commensurability resonances – mean motion exterior resonances – with planet Neptune. The orbital evolution of non-spherical dust grains of radius \(\approx 2\) micrometers is numerically calculated.

Capture of spherical grain in a resonance is characterized by decrease of semimajor axis outside the resonance. Non-spherical grain can exhibit also an increase of semimajor axis outside the resonance. This is the first case when a physically justified force – force given by interaction between electromagnetic radiation and non-spherical dust particle – can generate such a behaviour.

1. Introduction
Commensurability resonances are well-known as Kirkwood gaps since 1857. A body is in resonance with a planet when the ratio of their mean motions (angular velocities in circular orbits) is the ratio of two small integers. Physics of these resonances – mean-motion resonances – with Solar System planets is studied during the last quarter of the century (Wisdom, 1980). The orbital evolution of dust grains near resonances with planets is intensively studied during the last 15 years. Besides gravitational forces the effect of solar electromagnetic radiation in the form of the Poynting-Robertson effect (Robertson, 1937; Klačka 2004) is usually taken into account; see, e. g., Jackson and Zook (1992), Weidenschilling and Jackson (1993), Beaugé and Ferraz-Mello (1994), Marzari and Vanzani (1994), Šidlichovský and Nesvorný (1994), Liou and Zook (1995), Liou et al. (1995), Klačka and Pástor (2004).

Physics of the above mentioned papers is based on the assumption that material within dust grains is distributed in a spherically symmetric way. This assumption enables some analytic calculations of orbital evolution. As a fundamental property one may introduce a decrease
of particle’s semi-major axis before capture into a resonance, which is due to the Poynting-Robertson drag. If the particle is trapped in the exterior mean-motion resonance with a planet, the semi-major axis of the grain is practically constant for a long time – long-term capture. Finally, after elapsing a capture time, the particle escapes from the resonance and semi-major axis of the grain is a decreasing function of time, again.

Poynting-Robertson effect (P-R effect) holds only if a special condition is fulfilled (see Eq. (120) or Eq. (122) in Klačka, 1992; compare Eqs. (40) and (48) in Klačka, 2004). In practice, the particles are assumed to be strictly spherical when dealing with the P-R effect. As a consequence, spherical dust particles spiral toward the Sun and the orbit is planar.

Population of interstellar (Iatì et al., 2004) as well as interplanetary (Lumme, 2000) dust particles exhibits various shapes, sizes and compositions. Therefore, one must take into account the effect of solar electromagnetic radiation in an appropriate form. We are interested in orbital evolution of an arbitrarily shaped dust particle under action of electromagnetic radiation, in the regions close to the mean-motion resonances with a large planet. We are concentrating mainly on fundamental differences between behaviour of spherical and non-spherical dust grains. It turns out that evolution of semi-major axis of the particle near regions of mean-motion resonances and particle’s capture times belong to such fundamental properties. Mean-motion resonances with the planet Neptune are considered in this paper. Neptune is taken as an example of a large planet moving in a circular orbit around a central star (Sun, in our case). Our model corresponds to circular restricted three-body problem with electromagnetic radiation force acting on arbitrarily shaped dust particle.

2. Simulation of dust dynamics in the resonance region

All our numerical simulations are based on equation of motion written in the form (see Klačka and Kocifaj (2001) for more details)

\[
\frac{d\vec{v}}{dt} = -\frac{GM_\odot}{r^2} \vec{e}_R - \frac{G m_P}{|\vec{r} - \vec{r}_P|^3} \left( \frac{\vec{r} - \vec{r}_P}{|\vec{r} - \vec{r}_P|^3} + \frac{\vec{r}_P}{|\vec{r}_P|^3} \right) + \beta \frac{GM_\odot}{r^2} \sum_{j=1}^{3} \frac{Q_{prj}}{Q_{pr1}} \left( \left( 1 - \frac{2 \vec{v} \cdot \vec{n}_l}{c} + \frac{\vec{v} \cdot \vec{n}_j}{c} \right) \vec{n}_j - \frac{\vec{v}}{c} \right),
\]

where \( \vec{r} \) is position vector of the particle and \( \vec{r}_P \) is position vector of the planet (Neptune) which moves in circular orbit (\( m_P \) is mass of the planet, \( M_\odot \) is mass of the Sun); \( \vec{n}_j = (1 - \vec{v} \cdot n_{j/l}/c) n_{j/l} + \vec{v}/c, j = 1, 2, 3, n_{k/l} \cdot n_{l'} = \delta_{kl}, \) where \( k, l \in \{1, 2, 3\} \). The case \( Q_{pr2} = Q_{pr3} = 0, Q_{pr1} = constant \ of \ motion \) is the well-known Poynting-Robertson effect.

The numerical calculations were performed for the real cosmic dust particle shape which is identical to the particle from NASA collection archived under identification code U2015 B10 (Clanton et al., 1984). The particle shape appears to be representative of cosmic dust, since its aspect ratio (equal to 1.4) coincides well with the results of mid-infrared spectropolarimetry (Hildebrand and Dragovan 1995).

We have made simulations for various initial conditions of the particle: position vector, velocity vector, orientation of particle’s rotational axis, orientation of the particle with respect to the Sun (central star), optical properties of the particle. An orientation of the particle’s rotational axis relatively to the incident radiation was recalculated in each new orbital point, and the process of particle rotation was taken into account. It means, the radiation scattering diagram was computed for each individual particle orientation (i.e. during rotation in fixed position, as well as in each new point of particle trajectory). The averaged components of the
vector of mean propagation of scattered radiation for all directions, radial and both transversal, were calculated according to Draine and Flatau (2003). Numerical solution of the interaction of radiation with the particle was based on so-called Discrete Dipole Approximation (DDA, Draine, 1988; Draine and Flatau 1994). The calculation accuracy for our cosmic dust particle can be characterized by the quantity \( |n_r - i\kappa|kd \approx 0.6 \), where \( n_r - i\kappa \) is complex refractive index of the particle, \( k = 2\pi/\lambda \) is the magnitude of the wave vector, and \( d \) is a lattice spacing within particle model expressed as follows:

\[
d = a_{eff} \left( \frac{3N}{4\pi} \right)^{-1/3}.
\]

The quantity of \( N \) is total number of cubes in the digitized particle’s model and \( a_{eff} \) is effective radius of a volume identical sphere. Most conservative criterion of DDA method, \( |n_r - i\kappa|kd < 0.5 \), was satisfied almost in all cases. This will ensure that differential scattering cross sections \( dC_{sca}/d\Omega \) are accurate to within a few percent of the average differential scattering cross section \( C_{sca}/4\pi \).

\[
\text{Figure 1. The digitized model of cosmic dust particle U2015 B10.}
\]

The basic model of our particle consists of 14154 cubes, where position of zero point inside the coordinate system (geometric center of the particle) is \([27, 16, 16]\) (Fig 1). Center of gravity referred to this geometric center is \([0, 1, -5]\). The model was rendered using ray-tracing method (with the help of 3D-Studio MAX by Kinetix). To adapt the particle size we have developed new numerical codes: the modules implement the fine- (1) and coarse-routines (2). This functionality is helpful if the above mentioned DDA condition isn’t fulfilled and we need to increase the number of cubes (1). Shape of an examined particle is not changed, but we can replace one cube by 4, 9, etc. subcubes. On the other hand, the calculation may be rapidly accelerated by decreasing of number of cubes (2) (using backtracking procedure, i.e. 4, 9,... cubes are replaced by one cube of the same volume), but we must ensure that accuracy constraints are not violated (e.g. \( |n_r - i\kappa|kd \) should be smaller than 1).

Extensive simulations were performed for particles of effective radius 2 \( \mu m \); they are most representative of the lower fraction of zodiacal dust population (the size of these particles varies
from 1 µm to 5 µm). Several materials with different optical properties were considered: ice (mass densities 1 g/cm³ and 2 g/cm³), magnesium-rich silicate (mass densities 2 g/cm³ and 4 g/cm³), and iron-rich silicate (mass densities 6 g/cm³ and 8 g/cm³). Central acceleration $- GM_\odot(1 - \beta) \vec{e}_R/r^2$ is used in calculations of orbital elements.

3. The most important results
To cover all possible situations, the particles were homogeneously distributed along circular orbit around the planet Neptune (initial position of each grain was always in the plane of the planet’s orbit). In our computations we sampled the initial distances from the Sun in the range (≈ 1.0, 1.5 )-times the distance between the Sun and the planet. It was considered that particle rapidly rotates around an axis which is assumed to have a fixed orientation in space (Kocifaj and Klačka, 2004). As for wide range of possible particle orientations, we performed complex computations of various particle’s spins. In addition, the simulations for various optical properties were also realized (as described in previous section).

Application of Eq. (1) for the case $Q'_{pr2} = Q'_{pr3} = 0$ yielded standard positions for resonances (see Eq. (3) below: the fraction of orbital periods $T/T_P$ defines type of the resonance). However, real particles exhibit non-zero values for $Q'_{pr2}$ and $Q'_{pr3}$ and their values may be much more important than the ratio $v/c$ present in the P-R effect (Klačka and Kocifaj, 2001; Klačka, 2004). Our simulations for orbital evolution of micron-sized dust particles which allow for non-zero values of $Q'_{pr2}$ and $Q'_{pr3}$ show several temporary captures of grains in exterior resonances with the planet Neptune (capture with time larger than $4 \times 10^4$ years was not found for ice particles).

We have succeeded in finding various types of resonant captures in exterior mean-motion resonances for non-spherical particles: first-order resonances, defined by relation $T : T_P = (j + 1) : j$, and, second-order resonances defined by relation $T : T_P = (j + 2) : j$, where $T$ is orbital period of the particle, $T_P$ is orbital period of the planet and $j$ is an integer number [it is assumed that $(j + 2)$ and $j$ have no common divisors]. As for the first-order resonances for non-spherical particles, we have not observed a decrease of capture time with an increasing value of $j$, as it is well-known for spherical particles. As for another interesting result, our simulations for non-spherical particles show that capture time for the second-order resonances may be several times larger than capture time for the first-order resonances – capture times for spherical particles in resonances 2:1 and 5:3 are almost equal.

![Figure 2. Time evolution of semimajor axis and eccentricity for non-spherical iron-rich silicate dust grain of effective radius 2 microns. The plots show exterior resonance 5/3 with Neptune.](image-url)
Fig. 2 illustrates time evolution of semimajor axis and eccentricity for non-spherical iron-rich silicate dust grain. While spherical particle is characterized by a decrease of semimajor axis outside resonance (P-R effect), non-spherical dust grain may exhibit also an increase of semimajor axis outside the resonance; analogous situation holds for eccentricity. The presented real commensurability resonance 5/3 corresponds to semimajor axes $41.81 \text{ AU} \leq a \leq 42.02 \text{ AU}$, while its theoretical value for $\beta = 0.0243$ equals to $a = 41.908 \text{ AU}$, according to Eq. (3). As for the same type of mean-motion resonance 5/3 with Neptune, capture of spherical particle with $\beta = 0.0243$ corresponds to $41.726 \text{ AU} \leq a \leq 42.093 \text{ AU}$, minimum value of eccentricity is 0.009 and capture time is greater than $2 \times 10^8$ years. The comparison shows that the resonant width in semi-major axis is larger for spherical particle than for non-spherical grain. This may be an unexpected result. In reality, it is a general property of spherical particles that very long-term captures exhibit large widths in semimajor axis. On the other hand, the length of the capture presents no surprise: capture time for the non-spherical particle is much shorter than for the corresponding spherical grain.

4. Discussion
There are important differences in orbital evolution for spherical and non-spherical particles.

Spherical particles always move in the plane of the planetary orbit, if the initial orbital angular momentum vector of the particle was normal to the plane of the planetary orbit. On the other hand, orbital motion of non-spherical particles is not planar. This effect is generated by accelerations containing factors $Q'_{pr2}$ and $Q'_{pr3}$.

Another important physical difference between the orbital evolution of spherical and non-spherical particles concerns the values of parameter $\beta$. This parameter is a constant of motion for spherical particles. Thus, the third Kepler’s law yields

$$\frac{a}{a_P} = (1 - \beta)^{1/3} \left( \frac{T}{T_P} \right)^{2/3} \left( 1 + \frac{m_P}{M_\odot} \right)^{-1/3} ,$$

for the semimajor axes and periods of revolution around the Sun of the particle and the planet (subscript $P$). Defining any resonance by the ratio $T/T_P$, Eq. (3) yields immediately the ratio $a/a_P$. However, parameter $\beta$ is not conserved during the motion of non-spherical particle. If its relative change is $\varrho_\beta$, then (approximately)

$$\varrho_a = \frac{\beta}{3 \left( 1 - \beta \right)} \varrho_\beta .$$

The consequence of this equation is, that several first and higher order resonances may overlap for larger values of $\beta$. Moreover, Eq. (4) shows that the change of semimajor axis $a$ in orbital resonance caused by the change of parameter $\beta$ may be relatively large. Thus, one should await that resonant capture times for non-spherical particles are smaller than the capture times for spherical grains.

Spherical particles are described by the P-R effect. P-R effect does not hold, in general, for non-spherical particles. While P-R effect exhibits decrease of semi-major axis before trapping in a resonance, interaction of electromagnetic radiation with non-spherical grain may initiate also an increase of semi-major axis close to the resonant region. The increase of semi-major axis has to be generated by non-zero values of $Q'_{pr2}$ and $Q'_{pr3}$, since no other significant components of electromagnetic radiation force exist.

5. Conclusion
Our numerical simulations have shown that resonant captures of dust grains exist for exterior resonances with the planet Neptune for micron-sized non-spherical particles in circular restricted
three-body problem with solar electromagnetic radiation: see Fig. 2. The existence of resonant capture can be influenced by four facts:

i) interaction of electromagnetic radiation with non-spherical particle changes inclination of particle’s orbit,

ii) resonances are not defined in a unique way – several resonances may overlap due to the changing value of parameter $\beta$ for its greater values (and, even for spherical grains, capture times are very small for similar values of $T$ and $T_P$),

iii) the change of $\beta$ during particle’s motion can cause significant change of semimajor axis (see Eq. 4), and, thus, also decrease of capture time,

iv) non-zero values of $Q'_{pr2}$ and $Q'_{pr3}$ may be even more important for capture time than variable value of $\beta$.

Resonant captures of dust grains exist for exterior resonances with large planets for micron-sized non-spherical particles in circular restricted three-body problem with star’s electromagnetic radiation. Spherical grains can be captured for longer time than non-spherical particles. Ejection of arbitrarily shaped dust grain from the mean-motion resonance is due to the grain’s close encounter with the planet. While capture of spherical grain in a resonance is characterized by decrease of semimajor axis outside the resonance, non-spherical grain can exhibit also an increase of semimajor axis outside the resonance. Resonant trapping is not normally expected for diverging orbits such as the trapping presented in Fig. 2: this is the first case when a physically justified force can generate such a behaviour. The increase of semi-major axis near resonant region is caused by non-zero values of efficiency factors $Q'_{pr2}$ and $Q'_{pr3}$.

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References
Beaugé C, and Ferraz-Mello S 1994 Icarus 110 239
Clanton V S, Gooding J L, Mckay D S, Robinson G A, Warren J L, and Watts L A 1984 Cosmic dust catalog (particles from collection flag U2015), NASA, Johnson Space Center, (Houston, Texas) 70/1 10
Draine B T 1988 Astrophys. J. 333 848
Draine B T, Flatau P J 2003 User Guide for the Discrete Dipole Approximation Code DDSCAT 6.0 Freeware, http://arxiv.org/abs/astro-ph/0309069
Draine B T, Flatau P J 1994 J. Opt. Soc. of America A 11 1491
Hildebrand R H, Dragovan M 1995 Astrophys. J. 450 663
Iatía M A, Giusto A, Saija R, Borghese F, Denti P, Pestellini C C, Aiello S 2004 Astrophys. J. 615 286
Jackson A A, and Zook H A 1992 Icarus 97 70
Klačka J 1992 Earth, Moon, and Planets 59 41
Klačka J 2004 Cel. Mech. and Dynam. Astron. 89 1
Klačka J, and Kocifaj M 2001 J. Quant. Spectrosc. Radiat. Transfer 70 595
Klačka J, and Pátos P 2004 http://xxx.lanl.gov/abs/astro-ph/0411691
Kocifaj M, Klačka J 2004 J. Quant. Spectrosc. Radiat. Transfer 89 165
Liou J-Ch, and Zook H A 1995 Icarus 113 403
Liou J-Ch, Zook H A, and Jackson A A 1995 Icarus 116 186
Lumme K 2000 In: Light scattering by nonspherical particles. Theory, measurements, and applications (Eds. Mishchenko M I, Hovenier J W, and Travis L D) (Academic Press, San Diego, San Francisco, New York, Boston, London, Sydney, Tokyo) pp. 555-584.
Marzari F, and Vanzani V 1994 Astron. Astrophys. 283 275
Robertson H P 1937 Mon. Not. Roy. Astron. Soc. 97 423
Šídlíčkovský M, and Nesvorný D 1994 Astron. Astrophys. 289 972
Weidenschilling S J, and Jackson A A 1993 Icarus 104 244
Wisdom J 1980 Astron. J. 85 1122