Continuous-time projected gradient algorithm based optimal dispatch of distributed generators

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Abstract. Traditional dispatch or control method will suffer great difficulty with large scale integration of plug-and-play distributed generators in the power systems. The distributed control method can mitigate the burden of the power systems with distributed generation. In this paper, a continuous-time projection gradient algorithm based optimal dispatch method is proposed to improve the effectiveness of distributed control method for distributed generation. The proposed method does not rely on the known total load of the power system and the artificial initial value of state variables which are used widely in the consensus-based control methods, but only the projection operator is used to constrain the output of the distributed power supply, which can automatically run to the optimal point of the economic dispatch, thus significantly improving the practicability of distributed control method. Case study verifies the effectiveness of the proposed method.

1. Introduction

Nowadays, the power system has become a typical cyber physical system (CPS) [1] where a large number of distributed components having certain sensing, communication and computing functions, such as distributed power generation units, feeder automation terminals and other equipment, have been integrated. With the increasing distributed power generation, the future power grid needs to realize the operation and management mode of distributed power ‘plug and play’ [2]. However, the traditional centralized control mode cannot meet more flexible operation requirements [3]. To be specific, centralized control mode relies on a dispatch center to collect the system information and send the dispatch order to the generators, which will be inefficient when the number of generators is too large. On the other hand, the distributed control mode only needs local communication, which is more flexible than centralized control mode in operation [4].

In recent years, a large number of scholars have devoted themselves to the introduction of distributed control methods into the field of power system operation and control. The researches mainly focus on distributed economic dispatch and frequency/voltage secondary control [5-10]. Among them, the distributed control method based on first-order consensus algorithm received the highest degree of attention due to the simplicity of the algorithm. Literature [11] proposed an incremental cost consensus (ICC) algorithm, and realized a distributed economic dispatch framework in which the dominant generator collects the whole system’s power deviation as feedback signal to drive the variable of incremental rate to reach consensus. Literature [12] used distributed estimation of power deviation on the basis of Literature [11], which improves the lack of a known global power deviation to the leader node (the leader node still exists, but the global power deviation is no longer known). Literature [13]
considered communication noise and network delay, and proposed a distributed control method using a robust consensus algorithm, but the basic framework of the method is still the classic mode proposed in Literature [11]. Literature [14] also designed a consensus estimation algorithm for feedback variables, which could maintain the balance between the system’s total load and power generation in the dynamic process of the algorithm. Although this method can avoid the existence of leader node, it required the initial value of dynamic variables to satisfy the power balance assumption, which was difficult to be guaranteed in actual power system operation. The Literatures [11-14] all used the most common first-order consensus algorithm, and the constructed distributed control system had the advantage of being simple and easy to be implemented in combination with the underlying control. However, the dispatch methods based on the consensus distributed control algorithm relied on assumptions that were difficult to be satisfied in practice. For example, it is necessary for distributed control method based on consensus to assume that a node has the global information of the system, or assume that the initial value of the algorithm can meet the power balance constraint.

The continuous time gradient algorithm [15] and the consensus algorithm have little difference in the order of variables. Because the algorithm [15] uses the original dual variable iteration rules in the optimization problem to realize the distributed control structure, it does not rely on the dominant node to meet the global requirements. Literature [16] proposed a distributed frequency optimal control method for a multi-microgrid interconnected system based on a continuous time gradient algorithm, so that the regulation capacity constraints in the transient process of controllable load power control are always met. Literature [16] uses frequency deviation to reflect the power imbalance deviation of the system, and indirectly solves the power balance constraint problem in the microgrid. However, it has not considered the actual situation of the separation of the current distributed power cooperative control problem and the frequency modulation control on the time scale.

This paper studies the distributed optimal dispatch based on continuous-time projection gradient algorithm, aiming to solve the distributed control problem of the optimal allocation of active power in the scene of large-scale distributed power integration. The proposed method introduces a projection operator on the basis of the continuous time gradient algorithm to solve the upper and lower limit constraints of the distributed power output, avoiding the lack of setting the leader node and knowing the global load demand. In addition, the initial value of the algorithm is not subject to any restrictions, which is more suitable for the power control scenario of the actual power system. The proposed method promotes the feasibility of connecting large-scale distributed power sources to smart grids in terms of active power dispatch.

2. Power cooperative control based on consensus algorithm

2.1. Graph theory foundation
The consensus algorithm relies on graph theory. Let G be the topological graph of distributed power communication network, graph G is composed of vertex set V and edge set E, denoted as G=(V,E), where V is a finite set composed of all vertices in graph G, denoted as $V = \{1,2,\cdots,n\}$, and E is an ordered two-tuple composed of elements in V. If there is a communication line between vertex i and vertex j, it is recorded as $(i,j) \in E$. In this article, it is assumed that the graph G is a connected undirected graph, that is, there is a path between any two vertices in the graph. The adjacency matrix of graph G is defined as $A = \{a_{ij}\}$, where, $a_{ij} > 0, \forall i \neq j \cap (i,j) \in E$ and $a_{ii} = 0$.

The neighboring node of node i refers to the node that has a connection relationship with node i, which is represented by a set $N_i = \{j \in V : (i,j) \in E\}$. Let $d_i = \sum_{j \in N_i} a_{ij}$ denote the in (out) degree of node i, and define the diagonal matrix $D = \text{diag}\{d_i\}$ as the degree matrix of graph G. The Laplacian matrix of graph G is defined as $L = D - A$, according to the definition in this article, the Laplacian matrix L is a positive semidefinite matrix. In addition, the geometric and algebraic multiplicity of the 0 characteristic root are both 1, and its corresponding right characteristic $1^{'}$ is a column vector with all elements of 1.
2.2. Power cooperative control of distributed generation

Assuming that a certain partial subsystem of the smart grid is connected to multiple distributed power sources, without loss of generality, the number of independently controllable distributed power sources is recorded as \( n \). On the time scale of the three-time control of active power, the distributed power cooperative control problem is essentially an economic dispatch problem of active power, which can be expressed as the following optimization problem:

\[
\min \sum_{i=1}^{n} C_i(P_i) \\
\text{s.t.} \quad \sum_{i=1}^{n} P_i = \sum_{j=1}^{m} d_j \\
P_{i,\min} \leq P_i \leq P_{i,\max}
\]

where, \( P_i \) is the active power emitted by distributed power source \( i \), \( C_i(P_i) \) represents the cost function of distributed power generation. According to the approximate method of most documents [11-14], this paper uses a quadratic function \( \alpha P_i^2 + \beta P_i + \gamma_i \) to approximate the cost function \( C_i(P_i) \); Constraint (2) is the power balance constraint and \( d_j \) is the load; Constraint (3) is the upper and lower limit of the power adjustment of the distributed power supply.

When the Constraint (3) does not reach the boundary value, the optimal solution of the optimization problem (1) (2) satisfies the equal incremental rate criterion [11], namely:

\[
\frac{dC_i(P_i)}{dP_i} = \frac{dC_2(P_2)}{dP_2} = \ldots = \frac{dC_n(P_n)}{dP_n}
\]

(4)

2.3. Power cooperative control method based on consensus algorithm

It is assumed that there is a communication network between all the distributed generators in this paper, and its adjacency matrix and Laplacian matrix are both in the form defined in Section 2.1.

Based on the above assumption, the most common power cooperative control method using the leader node in the literatures is introduced here [11]. The nodes of distributed generators are divided into leader nodes and non-leader nodes. For non-leader nodes, the state variables \( \lambda_i \) denote the incremental rate (see (4) in the Section 2.2) of each generator respectively, and the dynamic equation about \( \lambda_i \) is shown in (5):

\[
\dot{\lambda}_i = -\sum_{j=1}^{m} l_{ij}\lambda_j
\]

(5)

where, \( l_{ij} \) is an element of the Laplace matrix \( L \), and \( l_{ij} \neq 0 \) means that there is mutual communication between distributed power sources \( i \) and \( j \). For the leader node \( i \), the dynamic Equation (6) denotes the dynamic of state variable \( \lambda_i \), where the feedback signal of the system's whole power imbalance deviation is added:

\[
\dot{\lambda}_i = -\sum_{j=1}^{m} l_{ij}\lambda_j + \varepsilon \left[ \sum_{j=1}^{m} d_j - \sum_{j=1}^{m} P_j \right]
\]

(6)

where, \( \varepsilon \) is the proportional coefficient of the power imbalance deviation feedback signal. The larger the value of \( \varepsilon \), the faster the algorithm’s convergence speed, but the consensus of the state variables will also be more affected because the steady-state error of the algorithm is determined by the value of \( \varepsilon \). Generally, \( \varepsilon \) is set to a smaller value.

Literature [14] improved the feedback signal in the dynamic system (6). By assuming that each distributed power source can sense the loads near its location, an additional feedback variable of dynamic system can be used to realize the distributed sensing of the system's total power deviation.

With the distributed sensing of the total load, the dynamic equation is simply summarized as
\[\dot{\lambda}_i = -\sum_{j=1}^{n} l_{ij} \lambda_j + \varepsilon f_i, \]
\[\dot{f}_i = -\sum_{j=1}^{n} l_{ij} f_j + (d_i - \dot{P}_i) \]

where, \(f_i\) is an additional feedback variable for distributed sensing power imbalance. However, this method requires strict requirements on the initial value of the dynamic system, that is, at the initial moment \(\sum f_i + P_i - d_i = 0\).

When the upper and lower limits of power (3) is reached, the state variable \(\dot{\lambda}_i\) in the above algorithm becomes a virtual incremental rate. For distributed power generators that reach the upper and lower limits of output, they can be kept at the boundary power according to their own output constraints, and the system uses feedback to eliminate the power imbalance deviation, so the virtual increment rate can still be synchronized (the actual increment rate is no longer synchronized). The relationship between the actual power of the distributed power generation and the virtual increase rate variable is as follows:

\[P_i = \left[ (\dot{\lambda}_i - \beta_l)/2\alpha_l \right] \]  

where, \([x]_{\alpha_{\text{max}}, \alpha_{\text{min}}}^p\) is a projection operator in the form of a saturated link, when \(x \geq P_{\text{max}}\) its output is \(P_{\text{max}}\) or when \(x \leq P_{\text{min}}\) its output is \(P_{\text{min}}\), otherwise the output is \(x\).

The current consensus algorithm-based power cooperative control method collects the global power deviation by setting the leader node, which obviously destroys the basic characteristics of distributed control, and the method of using distributed feedback to sense the power deviation requires setting the initial value to satisfy harsh constraints.

### 3. Power cooperative control based on continuous-time projection gradient algorithm

#### 3.1. Continuous-time projection gradient algorithm for distributed dispatch

In addition to the consensus algorithm, the continuous-time gradient algorithm is also a common method in the field of distributed control. This method adopts the iterative rule of gradient descent for decision variables [15]. In this paper, the optimization problems (1)-(3) are regarded as optimization problems of decision variables \(P_i\) and auxiliary variables \(\dot{\lambda}_i\) and \(\dot{z}_i\). On this basis, the upper and lower limit constraints in the iterative process of \(P_i\) are added by the projection operator to ensure the optimization result to satisfy the Constraint (3). Using the continuous-time projection gradient algorithm defined in the Literature [17] to solve the distributed power cooperative power control problem (1)-(3), the dynamic system can be obtained as:

\[\dot{\lambda}_i = -\sum_{j=1}^{n} l_{ij} \lambda_j - \sum_{j=1}^{n} l_{ij} \dot{z}_j + (d_i - P_i)\]
\[\dot{z}_i = \sum_{j=1}^{n} l_{ij} \dot{\lambda}_j\]
\[\dot{P}_i = P_{\text{th}} \left( P_i - 2\alpha_i P_i - \beta + \dot{\lambda}_i \right) - P_i\]

where, \(P_{\text{th}}(x) = \arg \min_{y \in \Omega} \|x - y\|\) represents the projection operator, \(\Omega\) is the convex set projected onto, which corresponds to the Constraint (3) in this paper.

#### 3.2. The revised control method based on continuous-time gradient algorithm

The damping of the nonlinear dynamic system (9) may be insufficient when the power imbalance is large. To overcome the shortcoming, we increase the damping of the nonlinear dynamic system (9) by adding a parameter \(\eta\) in (10), and illustrate the effectiveness of the revised method in the following section.
Moreover, under the condition of linear constraint (3), the above-mentioned projection operator can be further characterized by the saturation link where the output is 0 at the threshold, so the algorithm (9) can be further expressed as the following formula:

\[
\hat{\lambda}_i = -\sum_{j=1}^{n_i} l_{ij} \lambda_j - \sum_{j=1}^{n_i} l_{ij} z_j + (d_i - P_i)
\]

\[
\hat{z}_i = \eta \sum_{j=1}^{n_i} l_{ij} \lambda_j
\]

\[
\hat{P} = \eta \left[ -2\alpha P - \beta + \lambda \right] P_{\text{sat}} - P
\]

where, \(x_{\text{sat}} = 0\) if \(x \leq 0\) and \(P \leq P_{\text{sat}}\), otherwise \(x_{\text{sat}} = x\). This is slightly different from the saturation link in the previous section. The difference is that the saturation link here outputs 0 at the threshold. \(P\) denotes the output power of the distributed generator \(i\), which is also obtained by the distributed control as a state variable in the control system (10). \(\lambda_i\) can still be regarded as a kind of virtual incremental rate variable, but it forms an iterative form similar to the second-order consensus algorithm together with \(z_j\), and adds the feedback term of the power imbalance deviation to the dynamic equation.

Compared with the distributed control algorithm (7) (8) based on the consensus algorithm, the power cooperative control based on the continuous-time projection gradient algorithm (10) proposed in this paper does not increase the number of equations of the dynamic system, but only changes (8) from the algebraic equation describing the conversion relationship between power and increase rate to a differential equation based on the relationship between the power change rate and the increment rate deviation, so that the order of the system is not significantly increased. In addition, the algorithm does not need to set up a leader node, and it is better than the algorithm of the previous version of (6) in maintaining the characteristics of the distributed control structure; at the same time, because the power of the distributed dispatch could not violate the upper and lower limits during the iterative process, so the algorithm proposed in this paper does not need to artificially set the initial value, but it can realize the optimal allocation of active power by automatically running according to the practical operating state.

4. Case study

The main purpose of the case study is to verify the effectiveness of power cooperative control based on continuous-time projection gradient algorithm (10). The modified IEEE33-node system is used for simulation analysis. The power transmission network and communication topology are shown in Figure 1. The total load in the distribution system is supplied by the distributed power supply DG1-3 together, and the load information is collected distributedly by the DGs based on the partition in Figure 1. The total load of the two areas within the red line are collected by DG2 and DG3 respectively, and DG1 is responsible for the rest load of the system.

In this paper, the weight of the communication topology is 1, that is, \(a_{ij} = 1\), \(i \neq j\), \((i, j) \in E\), and the other parameters are shown in Table 1 and defined in Section 2.1. The implementation platform of
the control algorithm is Matlab2017b, and the effectiveness of the method proposed in this paper is verified by constructing a dynamic system in Simulink.

Table 1. Data of distributed generators.

|  |  |  |  |
|---|---|---|---|
|  | $\alpha$ | $\beta$ | limits/MW | $d_i$ |
| 1 | 0.0046 | 13.0650 | [100, 400] | 300 |
| 2 | 0.0111 | 5.2950 | [150, 450] | 400 |
| 3 | 0.0099 | 11.3700 | [80, 320] | 200 |

The three curves in Figure 2 represent the response trajectory of the DG1-3 power variable in the continuous-time projection gradient algorithm, where the control parameter value $\eta$ is set as 50. When the system reaches the steady state, the output powers of the three distributed power sources are 400, 180, and 320MW respectively, which meets the total demand of the system's global load. At the same time, the initial values of the three power variables at the initial moment of the algorithm are set to 150, 200, and 100MW, respectively. The continuous-time algorithm dynamic iteration proves that the balance point of the constructed cooperative control algorithm system can converge to the optimal solution of the economic dispatch optimization problem, and has a faster convergence speed.

![Figure 2. Response curve of output power with $\eta=50$.](image1)

The Literature [17] studied the convergence problem of the continuous-time projection gradient algorithm, but did not discuss the convergence speed of the algorithm and the nonlinear oscillation problem of the algorithm itself, that is, [17] only studied the algorithm when the parameter value $\eta$ is 1. The power response curve shown in Figure 3 is the dynamics of the continuous-time projection gradient algorithm proposed in [17]. Compared with Figure 2, the power curve in Figure 3 has a larger oscillation amplitude, and slower system response speed. The algorithm proposed in this paper improves the system response speed by increasing the parameter $\eta$, and suppresses the oscillation of the response curve, which improves the dynamic performance of the algorithm proposed in [17].

![Figure 3. Response curve of output power with $\eta=1$.](image2)

5. Conclusions

This paper proposes a distributed optimal dispatch method based on the continuous-time projection gradient algorithm. Without additional leadership nodes and unnecessary restrictions on the initial value of the control algorithm, the distributed optimal dispatch of distributed generation is realized. The
proposed method provides a valuable exploration for large-scale distributed sources to connect to the smart grid by operating in ‘plug and play’ mode.

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