Simulation of the electrovortex flow in a linear approximation under the action of the external magnetic field

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Abstract. Electrovortex flows appear in a liquid metal when a non-uniform current interacts with its own magnetic field. They are very important in different electrometallurgical processes connected with melting of metals. We describe the electrovortex flow in a hemispherical container which is connected with experiment which is carried in Joint Institute of High Temperature. Our main aim is to study the influence of the external magnetic field, because the experiment shows that even weak Earth magnetic field can change the characteristics of the flow.

To solve this problem we formulate a system of equations using Stokes' approximation. They are solved with Thoms' boundary conditions using the time marching method. Our numerical scheme gives us the opportunity to parallelize the computing process. For the calculations we use the CUDA technology. It allows us accelerate the calculation process, even if we use low-power user graphics cards. We hope that our experience can be used to solved another similar problems.

Introduction

The electrovortex flow (EVF) is an important process in different fundamental and applied problems in the fluid mechanics. It is the result of the interaction of an inhomogeneous electric current propagating in an electrically conductive medium with its own magnetic field [1]. The importance of research of such processes is connected with the role of EVF in different branches of the metallurgy [2]. The generation of such flows is a significant effect which often is connected with electric melting. Such mechanisms change the flow of liquid metal, which can influence the characteristics of the industrial process.

From the practical point of view, it is interesting to study the flow of the liquid metal in the hemispherical container. It is a good simplified model for different technological processes which take place in metallurgy [3]. In this case, a conductive medium is situated between two concentric hemispherical electrodes. The electric current propagates from a small electrode to the large one, and its density becomes smaller as the distance from the centre increases. The changing current density creates the magnetic field. The electromagnetic force (which is the vector product of the current density and the magnetic field) induces the vortex motions in the conductive medium.
The theoretical description of the electrovortex flow, just like another problems of the fluid mechanics, is based on the solutions of the Navier-Stokes equation. Of course, we can solve them directly, but it is quite difficult both from the analytical and numerical point of view. So, usually different approximations are used to simplify the mathematical formulation of the problem. In this case usually the Stokes approximation is used, which takes into account that the motions are quite slow and we can neglect the nonlinear terms.

It is necessary to emphasize, that for some simplified cases the solution of the corresponding equations can be found analytically using the series expansion [4]. However, the summation of them takes considerable time, which for applied problems is comparable with the time of the numerical solution [5]. Another problem is that the simplified conditions are used. So, to solve this problem it is better to use different numerical methods.

Previous theoretical works [4] describe the electrovortex flow neglecting the external magnetic field. (It can be connected both with the Earth magnetism and the field which is created specially by the laboratory or industrial equipment.) However, the experiments show that the magnetic field can significantly change the character of the flow [6,7]. So it would be quite important to take into account its influence.

To solve this problem we write the equations for the electrovortex flow with an external magnetic field. If it has vertical direction, its contribution is connected with the equation for the azimuthal velocity component. Another two components of the field can be found using the vector potential of the velocity and the vorticity vector. The equations are solved numerically using the CUDA technology, which is connected with using the resources of graphics cards. It allows us significantly speed up the process of the modelling in comparison with linear programs which a carried on singular processors.

1. Problem formulation
Here we describe the electrovortex flow in the hemispherical bowl (fig.1), which is a model of the experimental setup which is situated in the Joint Institute for High Temperatures of RAS [8]. It is filled with the In-Ga-Sn alloy, which has a lot of advantages from the experimental point of view (for example, the melting temperature for this fluid is about 11°C, so main experiments can be carried out at a room temperature).

The motion of the liquid is described by the Navier Stokes equation:

\[
\rho \left( \frac{\partial \vec{V}}{\partial t} + (\vec{V}, \nabla) \vec{V} \right) = -\nabla p + \rho \nu \Delta \vec{V} + \vec{F},
\]

where \( \vec{V} \) is the flow velocity, \( p \) is the pressure, \( \rho \) is the density, \( \vec{F} \) is the electromagnetic force, \( \nu \) is the kinematic viscosity coefficient.
To solve the Navier – Stokes equation for slow stationary motions, the Stokes approximation is often used. For this case, we can neglect the left part of the equation (because the time derivative is assumed zero and the convective term is proportional to the squared small velocity). Neglecting the convective terms in the equation for azimuthal velocity leads to the disappearance of the vortex effect, i.e. an increase in the spin velocity in the axial region, nevertheless, this formulation of the problem is also of interest:

$$-\bar{V}p + \rho\bar{V} = \bar{F} = 0.$$  \hspace{1cm} (2)

Here \(\bar{F} = \bar{j} \times (\bar{B} + \bar{B}_{\text{ext}})\), where:

$$\bar{j} = \frac{1}{2\pi r z} \bar{e}_r.$$  

\(\bar{B}\) is the self-magnetic field of the current:

$$\bar{B} = -\frac{\mu_0 l(1 - \cos \theta)}{2\pi r \sin \theta} \bar{e}_\phi,$$

\(\bar{B}_{\text{ext}}\) is the axial magnetic field (\(\bar{B}_{\text{ext}}\) is directed along axis \(z\), if we consider cylindrical coordinates), so in spherical coordinates:

$$\bar{B}_{\text{ext}} = B_0 (\cos \theta \bar{e}_r - \sin \theta \bar{e}_\theta)$$

This equation contains the pressure which is quite difficult to be found. So, for the incompressible fluid, we can present the velocity as the following combination:

$$\bar{V} = \text{curl} (\psi \bar{e}_\phi) + \psi \bar{e}_\phi,$$ \hspace{1cm} (3)

where \(\psi\) is the azimuthal component of the vector velocity potential, \(u\) is the azimuthal component of the velocity. Also it is useful to introduce the azimuthal component of the vorticity:

$$\omega = -\Delta \psi + \frac{\psi}{r^2 \sin^2 \theta}.$$ \hspace{1cm} (4)

The equation for the azimuthal component of the velocity can be written as a \(\phi\) – projection of the equation (2). The equation for the vorticity can be obtained taking rotor of both parts of the equation (2), and the equation for the vorticity is (4). In the generalized form the equations for the flow are the following:

$$\Delta \omega - \frac{\omega}{r^2 \sin^2 \theta} = -\frac{A(1 - \cos \theta)}{r^4 \sin \theta};$$ \hspace{1cm} (5)
The equations are dimensionless. The distances are measured in units of radius of the outer electrode, and two dimensionless parameters are introduced. Parameter \( A = \frac{I\mu_0^{1/2}}{2\pi^2\nu r^{1/2}} \) characterizes the current \( I \) is the current) and parameter \( \alpha = \frac{b\beta_0}{\mu_0} \) describes the action of the external axial magnetic field (here \( b \) is the radius of the outer electrode). The full description of this model is given in the work [4].

For boundary conditions for vorticity we use Thoms’ conditions, which describe mathematically the solid wall condition in terms of vorticity and vector potential (here \( a \) is the radius of the inner electrode, we consider \( b=1 \) and \( \Delta r \) and \( \Delta \theta \) are small parameters which can be associated with steps of the numerical scheme) [9]:

\[
\omega|_{r=a} + \frac{2}{\Delta r^2} \psi|r=a+\Delta r = \omega|_{r=1} + \frac{2}{\Delta r^2} \psi|r=1-\Delta r = \omega|_{\theta=\pi/2} + \frac{2}{\Delta \theta^2} \psi|_{\theta=\pi/2-\Delta \theta} = 0. \tag{8}
\]

For another conditions we can take so-called Dirichlet boundary conditions: \[
\psi|_{r=a} = \psi|_{r=1} = \psi|_{\theta=\pi/2} = 0; \tag{9}
\]
\[
u|_{r=a} = \nu|_{r=1} = \nu|_{\theta=\pi/2} = 0. \tag{10}
\]

2. Numerical solution

The system (5) – (7) is a system of elliptic equation. In mathematical modelling, the pseudo-transient method is often used for solution of such equations. It is connected with changing the stationary functions by time-dependent ones. So, we can introduce the time derivatives and suppose that for large time the solution will pass to the stationary ones. It is based on the maximum principle.

The evolutionary equations for the pseudo-transient method will become the following:

\[
\frac{\partial \omega}{\partial t} = \Delta \omega - \frac{\omega}{r^2 \sin^2 \theta} + \frac{A(1 - \cos \theta)}{r^4 \sin \theta}; \tag{11}
\]
\[
\frac{\partial \psi}{\partial t} = \Delta \psi + \frac{\psi}{r^2 \sin^2 \theta} + \omega; \tag{12}
\]
\[
\frac{\partial \nu}{\partial t} = \Delta \nu + \frac{\nu}{r^2 \sin^2 \theta} + \frac{\alpha \sin \theta}{2 \sigma^2}. \tag{13}
\]

The solution for large time will be asymptotically close to the solution of the problem (5) – (7).

To solve these equations we construct the grid, which described the discrete values of the coordinate \( r \), angle \( \theta \) and time \( t \):

\[ r_i = i \Delta r; \quad \theta_j = j \cdot \Delta \theta; \quad t_k = k \cdot \Delta t; \]

where \( \Delta r \), \( \Delta \theta \) and \( \Delta t \) are the steps of the difference scheme.

The equations can be rewritten in finite differences:
\[
\frac{\omega_{i,j}^{k+1} - \omega_{i,j}^{k}}{\Delta t} = \frac{\omega_{i+1,j}^{k} - 2\omega_{i,j}^{k} + \omega_{i-1,j}^{k}}{\Delta r^2} + \frac{1}{r_i} \frac{\omega_{j+1,j}^{k} - \omega_{i,j-1}^{k}}{\Delta r} + \frac{1}{(r_i)^2 \tan \theta_j} \frac{\omega_{i,j+1}^{k} - \omega_{i,j}^{k}}{2\Delta \theta} + \frac{1}{(r_i)^2} \frac{\omega_{i,j+1}^{k} - \omega_{i,j}^{k}}{\Delta \theta^2} - \frac{\omega_{i,j-1}^{k}}{(r_i)^2 \sin^2 \theta_j} + \frac{\alpha}{(r_i)^2} \frac{\sin \theta_j}{2\pi(r_i)^2}.
\]

The solution of the equations takes quite large computational time. However, this method can be easily parallelized. For the parallel computing we use the CUDA technology, which uses GPUs. It gives quite important acceleration in comparison with linear program which uses only one processor [11].

3. Results
In our calculation below we consider \(a=0.1, A=1, \alpha=1\). The results of computing are given on figures 2, 3 and 4. They show the distributions of the vorticity, the vector potential of the velocity and the azimuthal component of the velocity. Figure 5 contains the dependence of the azimuthal velocity for different depths \(z\) on the radius \(R\) in the cylindrical coordinates.

The poloidal velocity lines can be associated with the contour lines on the figure for the vector potential. So, the velocity of the fluid will have spiral structure.
Figure 2. Vorticity distribution distribution (here and below data calculated in spherical coordinates are shown in cylindrical one).

Figure 3. Vector potential distribution
The radial velocity dependence for different angles is shown on fig.5.

Figure 5. Azimuthal velocity for different depths $z$ along radius $R$ in the cylindrical coordinates. $1$ - 0.1, $2$ - 0.3, $3$ - 0.5, $4$ - 0.9, $5$ - 0.7

So we can say that the magnetic field changes the characteristics of the flow which can be found experimentally.

4. Conclusions
In this paper, we have studied the basic features of the influence of the external magnetic field on the electrovortex flow. We have shown that it is connected with the generation of the azimuthal component of the velocity which can be associated with the motion of the fluid around the axis. It gives us the opportunity to explain many effects which are connected with the external magnetic field and its influence on the flow.

It is necessary to emphasize, that our code uses CUDA technology, which is connected with programming on graphics cards, and it allows us make the calculation much faster.

Of course, it would be important to take in future nonlinear terms of the equations, which can help us to describe another effects of the electrovortex flows.

5. References
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