User Coordination for Fast Beam Training in FDD Multi-User Massive MIMO

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Abstract

Massive multiple-input multiple-output (mMIMO) communications are one of the enabling technologies of 5G and beyond networks. While prior work indicates that mMIMO networks employing time division duplexing have a significant capacity growth potential, deploying mMIMO in frequency division duplexing (FDD) networks remains problematic. The two main difficulties in FDD networks are the scalability of the downlink reference signals and the overhead associated with the required uplink feedback for channel state information (CSI) acquisition. To address these difficulties, most existing methods utilize assumptions on the radio environment such as channel sparsity or angular reciprocity. In this work, we propose a novel cooperative method for a scalable and low-overhead approach to FDD mMIMO under the so-called grid-of-beams architecture. The key idea behind our scheme lies in the exploitation of the near-common signal propagation paths that are often found across several mobile users located in nearby regions, through a coordination mechanism. In doing so, we leverage the recently specified device-to-device communications capability in 5G networks. Specifically, we design beam selection algorithms capable of striking a balance between CSI acquisition overhead and multi-user interference mitigation. The selection exploits statistical information, through so-called covariance shaping. Simulation results demonstrate the effectiveness of the proposed algorithms, which prove particularly well-suited to rapidly-varying channels with short coherence time.

Index Terms

Beam selection, massive MIMO, FDD, covariance shaping, training overhead, device-to-device

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I. INTRODUCTION

Massive MIMO (mMIMO) is expected to enable higher performance in 5G and beyond networks through increased data rate, more reliable and power-efficient radio links and reduced multi-user interference [2]. The mMIMO concept originated in a time division duplex (TDD) setting, where exploiting the channel reciprocity through low-overhead orthogonal uplink (UL) sounding led to the design of near-optimal linear precoders [2]. In contrast, downlink (DL) reference signals (RSs) and subsequent UL feedback are required to estimate the DL channels in frequency division duplex (FDD) mode, which makes it considerably more challenging. In general, there exists a one-to-one correspondence between RSs and antenna elements. Therefore, training and feedback overhead are often associated with unfeasibility in the FDD mMIMO regime, where too few resource elements would be in principle left for data transmission [3].

Nevertheless, operating in FDD remains appealing to mobile operators for several reasons, including i) most radio bands below 6 GHz are paired FDD bands, ii) the base stations (BSs) have higher transmit power available for the RSs than the user equipments (UEs), iii) overall deployment, maintenance and operation costs are reduced as fewer BSs are required in FDD networks [3].

A. Related Work

Several papers have proposed methods to cope with the overhead issue in FDD mMIMO. In this section, we provide a short overview of such existing works, which can be divided in four categories: i) second-order statistics-based approaches, ii) compressive sensing (CS)-based approaches, iii) channel extrapolation-based approaches, and iv) grid-of-beams (GoB)-based approaches.

Among the approaches based on second-order statistics, the work such as [4], [5] demonstrated that – under strictly spatially-orthogonal low-rank channel covariances – it is possible to discriminate across interfering UEs with even correlated non-orthogonal pilot sequences, thus reducing the training overhead and the so-called pilot contamination [2]. In [5], the low-rankness allows beamforming with low-dimensional channel state information (CSI) at the BS. However, such condition is seldom experienced in practical scenarios where the channel components are spread over the angular domain and unlikely result in non-overlapping channel eigenspaces [6]. In general, the radio environment has an important role in coloring the channel covariance. The more recent work [7] has introduced a covariance-based precoding method to artificially forge low-dimensional effective channels, independently from the covariance structure. In the MIMO literature, such precoding methods have been known under the term covariance shaping [8–10].
On a different note, CS techniques for estimating high-dimensional sparse channels with only a few measurements have been known for decades [11] and have been applied to FDD mMIMO as well [12]–[15]. The training overhead reduction in all these works relies on the existence of an intrinsic sparse representation of the radio channels, although this is not always found in networks operating at sub-6 GHz bands [6], [16]. Alternative CS-based methods such as [17] capitalize on the angular reciprocity between the DL and the UL channels. In such approaches, the spatial spectrum is estimated from UL sounding and used to design the mMIMO precoder, under the reasonable assumption that the dominant angles-of-departure (AoD) are almost invariant over the spectrum range separating the DL and the UL channels. Similar angle-based methods can be found in [18]–[20]. Nevertheless, the presumed angular correlation can decrease in some practical scenarios, due to e.g. carrier aggregation, and lead to performance degradation.

The intuition behind the channel extrapolation-based approaches is to infer the DL CSI from UL pilot estimates [21]. Therefore, the complete elimination of the DL training overhead is achieved with those approaches, as in a genuine TDD setting. The pioneering work in [22] develops a transform that can infer DL parameters such as path distance and gain from UL channels measured at the BS. In [23], the authors propose to use the super-resolution theory for achieving the DL channel extrapolation. Machine learning (ML)-based techniques have been proposed for channel extrapolation as well. In [24], the DL CSI is predicted from adjacent UL bands through a deep neural network, while in [25] a complex-valued neural network is trained to approximate a deterministic UL-DL mapping function. Further efforts are made in [26], where a DL CSI extrapolation technique using a neural network with simplified input and output is proposed to reduce the learning time.

The GoB approach has recently raised much interest especially within the 3GPP fora, due to its practical implementability [27]. The idea is again to translate high-dimensional channels into low-dimensional representations. Indeed, according to this concept, reduced channel representations are obtained through a spatial transformation based on fixed transmit-receive beams [27]–[29]. Thus, the UEs see low-dimensional effective channels which incorporate the beamforming vectors relative to the beams. In this case, there exists a one-to-one correspondence between RSs and beams in the GoB codebook [27]–[29]. Therefore, estimating such effective channels reduces the training overhead, as it becomes proportional to the codebook size and independent from the number of antenna elements. On the upside, the GoB approach allows a low-dimensional representation even when no sparse representation of the DL channels exists. However, the reduction in training (and feedback) overhead again entails a possibly severe performance degradation [30].
because the mMIMO data precoder is optimized for reduced channel representations which might not capture the prominent characteristics of the actual radio channels.

In order to minimize such losses, an alternative consists in designing the GoB with a larger number of beams, and then training a small subset of them which contains the most relevant channel components [30]–[32]. The number of such components depends on the propagation environment, which is in general beyond the designer’s control. An interesting twist to the story arises when multiple antennas are considered at the UE side as well, as we did in [1]. In fact, in that case, an extra degree of freedom is obtained by letting the UEs steer energy into suitable spatial regions. In particular, statistical beamforming at the UE side (covariance shaping) can be used to excite desirable channel subspaces. In conventional cases, UE-based beamforming focuses on regions where strong paths are located. While this approach makes much sense in a single-user scenario, it fails to exploit all degrees of freedom in a multi-user setting. In fact, [1] set forth the idea that beam selection at the UE side could be designed differently, with the aim to reduce the number of relevant components to estimate (so as to reduce the training overhead). In general, the decision on which beams to activate at both the BS and UE sides is not a trivial one, as several factors participate in the sum-rate optimization problem, including i) the beamforming gain, ii) the multi-user interference and iii) the training overhead. Furthermore, the beam selection is ideally a joint decision problem across the UEs and the BS and a coordination problem ensues. It is precisely the exploration of the novel three-way trade-off arising from the factors i), ii), iii) and underlying coordination mechanisms that form the core ideas of the paper.

B. Contributions

In this work, we propose a coordination mechanism between the UEs to facilitate statistical beam selection for FDD mMIMO performance optimization under the GoB design. We consider multi-beam selection at the UE side with multi-stream mMIMO transmission based on the block diagonalization (BD) precoding. This paper shows that beam selection in FDD mMIMO involves an interesting trade-off between i) selecting the beams that capture the largest channel gains for each UE, i.e. the most relevant channel components, and ii) selecting the beams that might capture somewhat weaker paths but are common to multiple UEs, so as to reduce the training overhead. The essence of such trade-offs is captured in Fig. [1] where UE 2 can capitalize on its weaker paths to reduce the number of activated beams at the BS side. Nevertheless, focusing only on beams that are common to multiple UEs can lead to decreased spatial separability among them and reduce
the gains. To this end, we introduce the so-called generalized correlation matrix distance (GCMD), a metric to evaluate the impact of covariance shaping on the average spatial separation of the UEs.

In order to design the long-term GoB beamformers, we propose a suite of (decentralized) coordinated beam selection algorithms exploring various complexity-performance trade-offs. In practice, the coordination between the UEs is enforced through a message exchange protocol exploiting low-rate device-to-device (D2D) communications. In this respect, we leverage from the forthcoming 3GPP Release 16, which is expected to support point-to-point side-links that facilitate cooperative communications among neighboring UEs with low resource consumption [33], [34].

Our numerical results – under the Winner II channel model – show that beam-domain coordination improves the throughput performance of GoB-based FDD mMIMO as compared with uncoordinated beam selection. In particular, the highest gains over uncoordinated SNR-based beam selection are experienced for rapidly-varying channels, such as the vehicular or the pedestrian ones. For such channels, fast beam training is essential to cope with the short channel coherence time below 20 ms. As a design lesson, we show that shaping the covariance matrices so as to favor the spatial separability (or orthogonality) among the UEs – such as in [8], [10] – is detrimental in such fast channels. This is because such approach neglects the optimization of the pre-log factor related to the training overhead, which has a substantial impact on the effective network throughput.

C. Notation

We use the following notation throughout the paper: bold lowercase letters are reserved for vectors, while bold uppercase letters for matrices. The conjugate operator is denoted with conj(·), the transpose operator is denoted with (·)T, while the Hermitian transpose operator is (·)H. The expectation operator over the random variable X is denoted with ∫X[·]. Tr(·) denotes the trace operator; det(·) denotes the determinant operator, while rank(·) denotes the rank operator. We denote with col(·) (resp. row(·)) the set containing the columns (resp. the rows) of a matrix. The operator card(·) returns the number of elements in a set. The operator vec(·) is the linear transformation which converts a matrix into a column vector, and ⊗ denotes the Kronecker product. All the sets are denoted with calligraphic notation. Furthermore, we use the (·) notation to distinguish the data/digital beamformers from the beam/analog ones. The data beamformers are applied over an effective channel. The same (bar) notation is used to denote the effective channel and its second-order statistics.
II. System Model and Problem Formulation

For the sake of exposition, this paper ignores inter-cell interference effects and focuses on training and interference within a given cell. Consider a single cell mMIMO BS equipped with $N_{BS} \gg 1$ antennas which serves (in downlink transmission) $K \ll N_{BS}$ UEs with $N_{UE}$ antennas each. We assume that the BS uses linear precoding techniques to process the signals before transmitting to all UEs. We consider FDD operation, i.e. the DL and the UL channels are not reciprocal.

Before we detail our mathematical model, let us focus on the toy example shown in Fig. 1, which carries the essence of the intuition behind the proposed trade-off between i) energy, ii) spatial separability, and iii) training overhead. We explore this trade-off through a coordinated beam selection among the UEs, made before the actual training of the beams.

Consider Fig. 1 and the problem of which beams should each UE activate and how it affects which beams are lit up at the BS and the subsequent training overhead. In a conventional strategy, uncoordinated max-SNR based beam selection would collect the highest amount of energy but would result in $M_{BS} = 5$ beams to train at the BS. Instead, UE 2 can opt for the weaker (non-bold light blue beams) $w_{2,1}$ and $w_{2,3}$ while UE 1 continues to activate its three beams. Note that this strategy collects less energy, yet it reduces the training overhead by 40% as the number of activated beams at the BS falls to $M_{BS} = 3$, since beams $v_1$, $v_2$ and $v_5$ at the BS side serve both UE 1 and UE 2 and maintain separability between them. In the rest of the paper, we are interested in designing a coordinated beam selection algorithm that optimizes this trade-off from a throughput perspective. We introduce now our mathematical model.

A. Channel Estimation with Grid-of-Beams

We assume that the GoB approach is exploited at both the BS and UE sides. Let us denote the beam codebooks as $B_{BS}$ and $B_{UE}$, where $\text{card}(B_{BS}) = B_{BS}$ and $\text{card}(B_{UE}) = B_{UE}$, used for GoB precoding and combining, respectively, as follows:

$$B_{BS} \triangleq \{v_1, \ldots, v_{B_{BS}}\}, \quad B_{UE} \triangleq \{w_1, \ldots, w_{B_{UE}}\},$$

where $v_v \in \mathbb{C}^{N_{BS} \times 1}$, $v \in \{1, \ldots, B_{BS}\}$, denotes the $v$-th beamforming vector in $B_{BS}$, and $w_w \in \mathbb{C}^{N_{UE} \times 1}$, $w \in \{1, \ldots, B_{UE}\}$, denotes the $w$-th beamforming vector in $B_{UE}$. To lighten the notation, we assume that $B_{UE}$ is the same across all the UEs.

The algorithms we present in Section V can be easily generalized to different codebooks $B_{UE}^k \forall k \in \{1, \ldots, K\}$ at the UE side.
Fig. 1: Intuitive toy example with $K=2$ UEs highlighting the trade-off between i) energy (i.e. activating strong paths), ii) spatial separability, and iii) training overhead (i.e. lighting up a smaller set of beams at the BS). The blue and orange circles represent the multi-path clusters (or scatterers), which might be shared among some UEs. Stronger paths are marked in bold.

A New Radio (NR)-like OFDM-based modulation scheme is assumed [28]. We consider a resource grid consisting of $T$ resource elements. Among those, $\tau M_{BS}$ are allocated to RSs, and $T-\tau M_{BS}$ to data, where $M_{BS}$ denotes the number of beams that are trained among the ones in $B_{BS}$ and $\tau$ is the duration measured in number of OFDM symbols of their associated RSs (one RS for each beam [28], refer to Fig. 2). The received training signal $Y_k \in \mathbb{C}^{M_{UE} \times \tau}$ at the $k$-th UE, where $M_{UE}$ is the number of activated beams at the UE side, can be expressed as

$$Y_k = \rho W_k^H H_k V S + W_k^H N_k, \quad \forall k \in \{1,\ldots,K\}$$

(2)

where $S \in \mathbb{C}^{M_{BS} \times \tau}$ contains the orthogonal (known) RSs, with $S S^H = I_{M_{BS}}$, $V \triangleq [v_1 \ldots v_{M_{BS,2}}] \in \mathbb{C}^{N_{BS} \times M_{BS}}$ is the normalized training (GoB) precoder common to all the UEs, $H_k \in \mathbb{C}^{N_{UE} \times N_{BS}}$ is the channel between the BS and the $k$-th UE, with $\text{vec}(H_k) \sim \mathcal{CN}(0, \Sigma_k)$ and $\Sigma_k \in \mathbb{C}^{N_{BS} \times N_{BS}}$ the respective channel covariance (assumed to be known), and $W_k \triangleq [w_{k,1} \ldots w_{k,M_{UE}}] \in \mathbb{C}^{N_{UE} \times M_{UE}}$ is the training combiner at the $k$-th UE. Note that both $V$ and $W_k \forall k$ contain beamformers belonging to the predefined GoB codebooks $B_{BS}$ and $B_{UE}$. The matrix $N_k \in \mathbb{C}^{N_{UE} \times \tau}$, whose elements are i.i.d. $\mathcal{CN}(0,\sigma_n^2)$, denotes the receiver noise at the $k$-th UE, while $\rho \triangleq \sqrt{\frac{P}{T}}$, where $P$ is the total transmit power available at the BS in the considered coherent (over both time and sub-carriers) frame.
Following the training stage, the UEs are able to estimate their instantaneous GoB effective channels, defined as

$$\bar{H}_k \triangleq W_k^H H_k V \in \mathbb{C}^{M_{\text{UE}} \times M_{\text{BS}}}, \quad \forall k \in \{1, \ldots, K\}$$

(3)

and whose covariance is denoted with $\bar{\Sigma}_k \in \mathbb{C}^{M_{\text{BS}} \times M_{\text{BS}} \times M_{\text{UE}}}, \forall k \in \{1, \ldots, K\}$.

Remark 1. With respect to the channel estimation, we assume that each UE has (at least) $M_{\text{UE}}$ independent RF chains available. In particular, such assumption implies that each UE can process the incoming training signal at the receive beams $[w_{k,1} \ldots w_{k,M_{\text{UE}}}]$ in parallel.

We introduce now the block diagonal matrix $W \in \mathbb{C}^{K N_{\text{UE}} \times M_{\text{UE}}}$ containing all the GoB combiners $W_k \forall k \in \{1, \ldots, K\}$, as follows:

$$W \triangleq \begin{bmatrix}
W_1 & 0 \\
\vdots & \\
0 & W_K
\end{bmatrix}.$$  (4)

The entire multi-user effective channel matrix $\bar{H} \in \mathbb{C}^{K M_{\text{UE}} \times M_{\text{BS}}}$ can then be expressed as

$$\bar{H} \triangleq W^H H V,$$

(5)

where $H \triangleq [H_1^T \ldots H_K^T]^T \in \mathbb{C}^{K N_{\text{UE}} \times N_{\text{BS}}}$ is the overall multi-user channel.

To close the CSI acquisition loop, each UE feeds back its estimated effective channel to the BS. As a consequence, the BS obtains an estimate $\hat{\bar{H}} \in \mathbb{C}^{K M_{\text{UE}} \times M_{\text{BS}}}$ of the multi-user effective channel $\bar{H}$ which can be used to design the mMIMO data precoder. In this work, we assume that the UEs use the popular linear minimum mean square error (LMMSE) estimator, for which the effective channel estimate $\hat{\bar{H}}_k \in \mathbb{C}^{M_{\text{UE}} \times M_{\text{BS}}}$ at the $k$-th UE reads as follows [35]:

$$\text{vec}(\hat{\bar{H}}_k) = \rho \Sigma_k A^H (\rho^2 A \Sigma_k A^H + \sigma_n^2 \Gamma \Gamma^H)^{-1} \text{vec}(Y_k),$$

(6)

where $A \triangleq (S^T \otimes I_{M_{\text{UE}}}) \in \mathbb{C}^{\tau M_{\text{UE}} \times M_{\text{BS}}}$ and $\Gamma \triangleq (I_\tau \otimes W_k^H) \in \mathbb{C}^{\tau \times M_{\text{UE}}}$.  

The related channel estimation error vector $e_k = \text{vec}(\bar{H}_k) - \text{vec}(\hat{\bar{H}}_k)$ at the $k$-th UE has zero mean elements [35] and associated covariance matrix as given in the next lemma.

**Lemma 1.** The covariance $\Sigma_{e_k} \in \mathbb{C}^{M_{\text{BS}} M_{\text{UE}} \times M_{\text{BS}} M_{\text{UE}}}$ of the LMMSE channel estimation error at the $k$-th UE can be expressed as follows:

$$\Sigma_{e_k} = \left( \Sigma_k^{-1} + \kappa A^H (\Gamma \Gamma^H)^{-1} A \right)^{-1},$$

(7)

having defined the scalar $\kappa \triangleq \rho^2 / \sigma_n^2$. 

Proof. From the definition of the error covariance $\Sigma_{e_k} \triangleq \mathbb{E}_{H_k}[e_k e_k^H]$, we have:

$$
\Sigma_{e_k} = \bar{\Sigma}_k - \rho \bar{\Sigma}_k A^H \left( \rho^2 A \bar{\Sigma}_k A^H + \sigma_n^2 \Gamma \Gamma^H \right)^{-1} \rho A \bar{\Sigma}_k \\
= \bar{\Sigma}_k - \left( \bar{\Sigma}_k^{-1} + \kappa A^H (\Gamma \Gamma^H)^{-1} A \right)^{-1} \kappa A^H (\Gamma \Gamma^H)^{-1} A \bar{\Sigma}_k,
$$

where (a) is due to the Woodbury identity. We can then rewrite the error covariance as

$$
\Sigma_{e_k} = D^{-1} \left( \bar{\Sigma}_k - \kappa A^H (\Gamma \Gamma^H)^{-1} A \bar{\Sigma}_k \right) \\
= D^{-1},
$$

where $D = \left( \bar{\Sigma}_k^{-1} + \kappa A^H (\Gamma \Gamma^H)^{-1} A \right)$, as given in (7). \hfill $\square$

B. Data Signal Model

The data transmission phase (over the effective channels) follows the training and UE feedback stages. Let us consider a single resource element, and denote with $x_k \triangleq [x_{1,1}, \ldots, x_{1,L_k}] \in \mathbb{C}^{L_k \times 1}$ the data vector transmitted to the $k$-th UE. Thus, $x \triangleq [x_1 \ldots x_K] \in \mathbb{C}^{L \times 1}$ is the overall data vector, where $L \triangleq \sum_k L_k$ is the total number of transmitted data symbols and $\mathbb{E}[xx^H] = I_L$. The received data signal $\hat{x}_k$ at the $k$-th UE can be expressed as

$$
\hat{x}_k = \rho \bar{W}_k^H \bar{H}_k \bar{V} x + \bar{W}_k^H \bar{n}_k, \quad \forall k \in \{1, \ldots, K\} \\
= \rho \bar{W}_k^H \bar{H}_k \bar{V} x_k + \sum_{j \neq k} \rho \bar{W}_k^H \bar{H}_k \bar{V} x_k \bar{V} x_j + \bar{W}_k^H \bar{n}_k,
$$

where $\bar{V} \triangleq [\bar{V}_1 \ldots \bar{V}_K] \in \mathbb{C}^{M_{\text{in}} \times L}$ is the normalized mMIMO (digital) data precoder, with $\bar{V}_k \triangleq [\bar{V}_{k,1} \ldots \bar{V}_{k,L_k}]$, $\bar{H}_k$ is the effective channel between the BS and the $k$-th UE after GoB precoding and combining, $\bar{W}_k \in \mathbb{C}^{M_{\text{out}} \times L_k}$ is the mMIMO (digital) data combiner at the $k$-th UE, and $\bar{n}_k \triangleq \bar{W}_k^H \bar{n}_k \in \mathbb{C}^{M_{\text{in}} \times 1}$ denotes the filtered receiver noise at the $k$-th UE.

The instantaneous spectral efficiency (SE) $R_k(\bar{V}, \bar{V}, \bar{W}, \bar{W})$ relative to the $k$-th UE can then be expressed as follows:

$$
R_k(\bar{V}, \bar{V}, \bar{W}, \bar{W}) \triangleq \log_2 \det \left( I_{L_k} + \rho^2 K_k^{-1} \bar{W}_k^H \bar{H}_k \bar{V}_k \bar{V}_k^H \bar{H}_k \bar{W}_k \right),
$$

where $K_k \triangleq \rho^2 \sum_{j \neq k} \bar{W}_k^H \bar{H}_k \bar{V}_j \bar{V}_j^H \bar{H}_k \bar{W}_k + \sigma_n^2 \bar{W}_k^H \bar{W}_k^H \bar{W}_k^H \bar{W}_k$ is the interference plus noise covariance relative to the $k$-th UE, and where we recall that the dependence on $V$ and $W$ is because $\bar{H} \triangleq W^H H V$. 
C. Optimal Precoders and Combiners

In order to design a processing scheme which achieves the optimal effective network throughput, the mutual optimization of the (constrained) GoB and (unconstrained) mMIMO data beamformers should be considered. Let us first define the overall training overhead as follows.

**Definition 1.** Let \( \mathbf{V} \in \mathbb{C}^{N_{\text{BS}} \times M_{\text{BS}}} \) be the GoB precoder at the BS. The training overhead \( \omega(\mathbf{V}) \in [0,1] \) in terms of pilot resource elements is defined as follows:

\[
\omega(\mathbf{V}) \triangleq \frac{\tau}{T} \text{card}(\text{col}(\mathbf{V})).
\]  

(12)

Note that the training overhead depends on how the GoB precoder \( \mathbf{V} \) is designed. \( \text{Col}(\mathbf{V}) \) consist indeed of the beams to train in the channel estimation phase (refer to Eq. (2) and Fig. 2).

Therefore, the achievable effective network throughput \( \mathcal{R} \) can be expressed as

\[
\mathcal{R}(\mathbf{V},\bar{\mathbf{V}},\mathbf{W},\bar{\mathbf{W}}) \triangleq (1-\omega(\mathbf{V})) \sum_{k=1}^{K} \mathcal{R}_k(\mathbf{V},\bar{\mathbf{V}},\mathbf{W},\bar{\mathbf{W}}).
\]  

(13)

The optimal beamformers \( (\mathbf{V}^*,\bar{\mathbf{V}}^*,\mathbf{W}^*,\bar{\mathbf{W}}^*) \) are then found as follows:

\[
(\mathbf{V}^*,\bar{\mathbf{V}}^*,\mathbf{W}^*,\bar{\mathbf{W}}^*) = \arg\max_{\mathbf{V},\bar{\mathbf{V}},\mathbf{W},\bar{\mathbf{W}}} \mathbb{E}_{\mathbf{H}} \left[ \mathcal{R}(\mathbf{V},\bar{\mathbf{V}},\mathbf{W},\bar{\mathbf{W}}) \right],
\]

(P*)

subject to \( \text{col}(\mathbf{V}) \in \mathcal{B}_{\text{BS}}, \forall \mathbf{W}_k \in \mathcal{B}_{\text{UE}}, \forall k = \{1,\ldots,K\} \).

Finding the global optimum for the optimization problem (P*) is not trivial and often found to be intractable, even without considering the pre-log factor relative to the training overhead \([36],[37]\).

A common and viable approach consists in decoupling the design, as the GoB beamformers can be optimized through long-term statistical information, whereas the mMIMO data beamformers can depend on the instantaneous CSI \([37]\). The same approach is followed in this work. In particular, we consider two different timescales:

- **Small timescale** (*channel coherence time*): within which the instantaneous channel realization \( \mathbf{H}_k, \forall k \) is assumed to be constant and a single training phase is carried out;

- **Large timescale** (*beam coherence time*): within which the covariance matrices \( \Sigma_k, \forall k \) are assumed to be constant and the GoB beamformers are designed (*beam selection*).

In the following section, we will focus on the design of the mMIMO data precoder and combiners with given multi-user effective channel \( \bar{\mathbf{H}} \). Later, the design of the long-term GoB beamformers will be considered assuming fixed mMIMO data beamformers.
Fig. 2: CSI-RS locations in a DL NR resource block for $M_{\text{BS}}=8$. When GoB precoding is used, the radiated beams are mapped to one precoded CSI-RS each, sent over $\tau M_{\text{BS}}$ non-overlapping resource elements on distinct antenna ports [27]. Therefore, less resource elements are available for transmitting data to the UEs, leading to throughput degradation.

III. DATA BEAMFORMERS DESIGN

Since we consider multi-beam processing at the UE side, i.e. $M_{\text{UE}} > 1$, the complete diagonalization of the effective channel $\mathbf{H}$ at the BS side is suboptimal [38]. The BD approach is a popular method to design near-optimal beamformers that eliminate the multi-user interference in such scenarios. In particular, the mMIMO data precoder $\mathbf{V}$ at the BS side aims to produce a block-diagonal $\mathbf{H} \mathbf{V}$ where no multi-user interference is experienced. The eventual remaining inter-stream interference can then be suppressed at the UE side through a proper combining operation. In this section, we review the complete procedure to perform the BD [38], which will allow for a simplified SE expression depending on the long-term GoB beamformers only.

To ensure a block-diagonal $\mathbf{H} \mathbf{V}$, the precoding matrix $\mathbf{V}_k$ has to be designed such that
\[ \mathbf{H}_j \mathbf{V}_k = 0, \quad \forall j \neq k. \] (14)

Introducing the matrix $\bar{\mathbf{H}}_{/k} \in \mathbb{C}^{(K-1)M_{\text{UE}} \times M_{\text{BS}}}$ as
\[ \bar{\mathbf{H}}_{/k} \triangleq \begin{bmatrix} \mathbf{H}_1^\top & \mathbf{H}_{k-1}^\top & \mathbf{H}_{k+1}^\top & \cdots & \mathbf{H}_K^\top \end{bmatrix}^\top, \] (15)
the condition in (14) is enforced by letting $\bar{\mathbf{V}}_k$ lie in null($\bar{\mathbf{H}}_{/k}$). Whenever card(null($\bar{\mathbf{H}}_{/k}$)) $\neq 0$, which holds when rank($\bar{\mathbf{H}}_{/k}$) $< M_{\text{BS}}$, the BS can send (multi-user) interference-free data to the $k$-th UE.
As a first step, the singular value decomposition (SVD) is performed on $\bar{H}/k$. We can write

$$\bar{H}/k = \bar{U}/k \bar{S}/k \left[ \bar{M}^{(1)}_k \bar{M}^{(0)}_k \right]^H,$$

where $\bar{M}^{(1)}_k$ contains the first $\bar{M}_k = \text{rank}(\bar{H}/k)$ right singular vectors of $\bar{H}/k$, while $\bar{M}^{(0)}_k$ contains the last $(\bar{M}_{BS} - \bar{M}_k)$ ones. Thus, we know that

$$\bar{H}_j \bar{M}^{(0)}_k = 0, \, \forall j \neq k.$$  

The BD of the overall multi-user effective channel $\bar{H}$ can then be expressed as

$$\bar{H}_{BD} = \begin{bmatrix} \bar{H}_1 \bar{M}^{(0)}_1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \bar{H}_K \bar{M}^{(0)}_K \end{bmatrix}.$$  

To achieve optimal SE, further SVD-based processing is carried out \cite{38}. Since $\bar{H}_{BD}$ is block diagonal, we can perform an individual SVD for each UE rather than decomposing the overall large matrix $\bar{H}_{BD}$. In particular, we can write

$$\bar{H}_k \bar{M}^{(0)}_k = \left[ \bar{U}^{(1)}_k \bar{U}^{(0)}_k \right] \bar{S}_k \left[ \begin{array}{c} 0 \\ \bar{M}^{(1)}_k \bar{M}^{(0)}_k \end{array} \right]^H.$$  

The product $\bar{M}^{(0)}_k \bar{M}^{(1)}_k$ produces an orthogonal basis with dimension $L_k = \text{rank}(\bar{H}_k \bar{M}^{(0)}_k)$ and can be used as the multi-user interference-nulling data precoder for the $k$-th UE, i.e. $\bar{V}_k = \bar{M}^{(0)}_k \bar{M}^{(1)}_k$. In order to send interference-free data to the $k$-th UE, $\text{rank}(\bar{H}_k \bar{M}^{(0)}_k) \geq 1$ is needed. The receive data combiner $\bar{W}_k$ relative to the $k$-th UE is then designed as $\bar{W}_k = \bar{U}^{(1)}_k$.

**Lemma 2.** The condition $\text{rank}(\bar{H}_k \bar{M}^{(0)}_k) \geq 1$ is respected when there exists at least one vector in $\text{row}(\bar{H}_k)$ that is linearly independent (LI) of $\text{row}(\bar{H}/k)$.

**Proof.** Let us assume that $\exists \, k \in \text{row}(\bar{H}_k)$ that is LI of $\text{row}(\bar{H}/k)$. Then, since $\bar{M}^{(0)}_k$ is a basis for $\text{null}(\bar{H}/k)$, we have $k \bar{M}^{(0)}_k \neq 0$. Therefore, $\text{rank}(\bar{H}_k \bar{M}^{(0)}_k) \geq 1$. \qed

Note that inverting the entire $\bar{H}$ at the BS side through e.g. Zero-Forcing (ZF) precoding requires that each vector in $\text{row}(\bar{H}_k)$ is LI of $\text{row}(\bar{H}/k)$. The BD approach offers thus more freedom for designing the GoB beamformers $V$ and $W$, although a higher value for $\text{rank}(\bar{H}_k \bar{M}^{(0)}_k)$ would still be beneficial for increasing the SE (more available streams for the $k$-th UE).
Proposition 1 ([38]). When all the interference cancellation conditions are met, the instantaneous SE after BD precoding $R_{k}^{BD}(V, W)$ relative to the $k$-th UE can be written as follows:

$$R_{k}^{BD}(V, W) \triangleq \log_2 \det(I_{L_k} + \kappa \bar{S}_k^H \bar{S}_k),$$

(20)

where the dependence on $V$ and $W$ is hidden in the linear transformation ([19]).

Therefore, fixing BD as the mMIMO data precoder allows to reformulate $(P^{\star})$ as a long-term joint transmit-receive beam selection problem, where the optimum GoB beamformers $(V^{(P0)}, W^{(P0)})$ are found as follows:

$$(V^{(P0)}, W^{(P0)}) = \arg\max_{V, W} E[H] \left[ (1 - \omega(V)) \sum_{k=1}^{K} R_{k}^{BD}(V, W) \right],$$

(P0)

subject to col($V$) $\in B_{BS}$

$$\text{col}(W_k) \in B_{UE}, \forall k = \{1, \ldots, K\}.$$  

The problem $(P0)$ is a discrete optimization problem with a non-convex objective function. The solution for this class of problems is often hard to find and requires combinatorial search, alternating minimization algorithms or relaxation techniques, which are however demanding to put into practice. In this work, we aim instead to design heuristic beam selection algorithms. In the next section, we will thus deal with the design of the long-term GoB beamformers $V$ and $W$.

IV. GRID-OF-BEAMS BEAMFORMERS DESIGN

In general, it can be seen through inspecting the objective function in $(P0)$ that designing proper GoB beamformers $V$ and $W$ implies i) harvesting large effective channel gain, ii) avoiding catastrophic multi-user interference, and iii) minimizing the training overhead. In this section, we investigate such conditions in detail so as to set the requirements for an effective GoB beamformers design. In particular, for each condition, we introduce a related beam selection optimization problem which approximates $(P0)$ and whose practical implementation will be discussed in Section V. To this end, we define the notion of relevant channel components and take a closer look at the beam reporting procedure defined in the current 5G NR specifications. Furthermore, we highlight the role of coordinating UEs in reducing the multi-user interference and the training overhead in the considered FDD mMIMO scenario.
A. Harvesting Large Effective Channel Gain

In the classical GoB implementation all the beams in the grid are trained regardless of their actual relevance, i.e. $M_{BS} = B_{BS}$. As pointed out in Section I, such an operating mode is feasible for small GoBs only (refer to Fig. 2), although employing a small GoB, in turn, leads to a high performance loss [31]. In order to avoid exchanging performance for overhead, the intuition is to use a large GoB and leverage the knowledge of the long-term statistical information to train a few (accurately) selected beams to train, so as to keep $\omega = (\tau / T) M_{BS}$ small. In particular, in order to gather as much beamforming gain as possible, the idea is to capitalize on the so-called relevant channel components, whose number depends on the propagation environment.

Remark 2. This intuition has been exploited, to a large extent, to optimize single-user mmWave communications. Owing to the sparse mmWave radio environment, few beams are enough to obtain an accurate and profitable low-dimensional representation of the actual channel [39].

Definition 2. We define the set $\mathcal{M}_k$ containing the relevant channel components (or relevant beam pairs) of the $k$-th UE as follows:

$$\mathcal{M}_k \triangleq \left\{ (v,w) : \mathbb{E}_{H_k} \left[ |w^H H_k v|^2 \right] \geq \xi \right\},$$

(21)

where $\xi$ is a predefined power threshold.

Remark 3. The set $\mathcal{M}_k$ is solely dependent on the second order statistics of the channel $H_k$, for fixed $B_{BS}$ and $B_{UE}$. In particular, we refer to the notion of beam coherence time to denote the coherence time of such statistics. The beam coherence time $T_{beam}$ – which depends on the beam width, UE speed and other factors – is much longer than the channel coherence time $T_{coh}$ [40].

The following lemma establishes the mathematical relation between the relevant channel components and the second order statistics of the channel (channel covariance matrix).

Lemma 3. Let $\Sigma_k \triangleq \mathbb{E}_{H_k} \left[ \text{vec}(H_k) \text{vec}(H_k)^H \right] \in \mathbb{C}^{N_{BS}N_{UE} \times N_{BS}N_{UE}}$ be the channel covariance matrix relative to the $k$-th UE. The set $\mathcal{M}_k$ containing the relevant channel components can be equivalently expressed as

$$\mathcal{M}_k = \left\{ (v,w) : b_{v,w}^H \Sigma_k b_{v,w} \geq \xi \right\},$$

(22)

where $b_{v,w} \triangleq (\text{conj}(v_v) \otimes w_w) \in \mathbb{C}^{N_{BS}N_{UE} \times 1}$. 
Proof. From the properties of the vec(·) operator, we can write: \( \text{vec}(w^H H v) = (v^T \otimes w^H) \text{vec}(H) \). Now, from the definition in (21), we have:

\[
\begin{align*}
\mathbb{E}_{H_k} \left[ |w^H H_k v|_v^2 \right] &= \mathbb{E}_{H_k} \left[ \text{vec}(H)^H b_{v,w} b_{v,w}^H \text{vec}(H) \right] \\
&= \mathbb{E}_{H_k} \left[ \text{Tr} \left( b_{v,w} b_{v,w}^H \text{vec}(H) \text{vec}(H)^H \right) \right] \\
&= \text{Tr} \left( b_{v,w}^H \mathbb{E}_{H_k} \left[ \text{vec}(H) \text{vec}(H)^H \right] b_{v,w} \right) \\
&= b_{v,w}^H \Sigma_k b_{v,w},
\end{align*}
\]

(23)
as given in (22).

The relevant channel components relative to the \( k \)-th UE can thus be found through linear search over \( B_{BS} B_{UE} \) elements, provided that the second order statistics of \( H_k \) are known.

Note that when the UEs exploit multi-beam covariance shaping, the set of relevant channel components can be altered\(^2\). Indeed, applying some receive beams means focusing on specific relevant beam pairs and neglecting some others. To this end, we define the subset \( M_{k}^{BS} \subseteq M_k \) as follows.

**Definition 3.** We define the set \( M_{k}^{BS} \subseteq M_k \) containing the relevant channel components (or, equivalently, beam pairs) of the \( k \)-th UE, when the \( k \)-th UE adopts \( W_k \) as its receive GoB combiner, as follows:

\[
M_{k}^{BS} (W_k) \triangleq \{ (v,w) \in M_k : w \in W_k \},
\]

(24)
where we have introduced the notation \( M_{k}^{BS}(\cdot) \) to highlight that the set \( M_{k}^{BS} \) depends on the selected GoB combiner \( W_k \).

In more detail, for given GoB beamformers \( V_k \in \mathbb{C}^{N_{BS} \times M_{BS}} \) and \( W_k \in \mathbb{C}^{N_{UE} \times M_{UE}} \), an effective channel covariance \( \Sigma_k \) can be defined. Furthermore, \( \Sigma_k \) can be expressed in closed form as a function of the channel covariance \( \Sigma_k \), as highlighted in the following lemma.

**Lemma 4.** Let \( \Sigma_k \triangleq \mathbb{E}_{H_k} \left[ \text{vec}(W_k^H H_k V_k) \text{vec}(W_k^H H_k V_k)^H \right] \in \mathbb{C}^{M_{BS} M_{UE} \times M_{BS} M_{UE}} \) be the effective channel covariance relative to the \( k \)-th UE. \( \Sigma_k \) can be equivalently expressed as

\[
\Sigma_k = B_k^H \Sigma_k B_k,
\]

(25)
where \( B_k \triangleq (\text{conj}(V_k) \otimes W_k) \in \mathbb{C}^{N_{BS} N_{UE} \times M_{BS} M_{UE}} \).

\(^2\)With the exception of spatially-uncorrelated channels, where the same gain is expected from all spatial directions.
Proof. Based on \( \text{vec}(W^H H V) = (V^T \otimes W^H) \text{vec}(H) \), we have:
\[
\Sigma_k \triangleq \mathbb{E}_{H_k} \left[ \text{vec}(W_k^H H_k V_k) \text{vec}(W_k^H H_k V_k)^H \right] = B_k^H \mathbb{E}_{H_k} \left[ \text{vec}(H_k)^H \text{vec}(H_k) \right] B_k
\]
\[= B_k^H \Sigma_k B_k, \tag{26} \]
as given in (25).

Let us now consider the single-user optimal SVD precoding \([41]\) over the effective channels. We can express the achievable SE at the \( k \)-th UE as follows:
\[
R_{\text{SVD}}^k (V_k, W_k) \triangleq \log_2 \det \left( I_{M_{\text{UE}}} + \kappa \Lambda_k^H \Lambda_k \right), \tag{27} \]
where we recall that \( \kappa \triangleq \rho^2 / \sigma_n^2 \) and where \( \Lambda \triangleq \text{diag} (\lambda_1, \ldots, \lambda_{M_{\text{UE}}}) \), with \( \lambda_1, \ldots, \lambda_{M_{\text{UE}}} \) being the singular values of the effective channel \( \bar{H}_k \triangleq W_k^H H_k V_k \).

**Proposition 2.** The average SE achievable at the \( k \)-th UE in a single-user scenario with SVD precoding can be upper bounded as follows:
\[
\mathbb{E}_{H_k} \left[ R_{\text{SVD}}^k (V_k, W_k) \right] \leq M_{\text{UE}} \log_2 \left( 1 + \kappa M_{\text{UE}}^{-1} \text{Tr}(\bar{\Sigma}_k) \right), \tag{28} \]
where \( \bar{\Sigma}_k \) is the effective channel covariance relative to the \( k \)-th UE.

*Proof.* We first rewrite \( R_{\text{SVD}}^k (V_k, W_k) \) as follows, using the properties of the \( \det \) operator:
\[
R_{\text{SVD}}^k (V_k, W_k) = \sum_{m=1}^{M_{\text{UE}}} \log_2 (1 + \kappa \lambda_m^2). \tag{29} \]
According to *Jensen’s inequality*, we have:
\[
\sum_{m=1}^{M_{\text{UE}}} \log_2 (1 + \kappa \lambda_m^2) \leq M_{\text{UE}} \log_2 \left( 1 + \kappa \sum_{m=1}^{M_{\text{UE}}} \lambda_m^2 \right) = M_{\text{UE}} \log_2 \left( 1 + \kappa M_{\text{UE}}^{-1} \text{Tr}(\bar{H}_k \bar{H}_k^H) \right). \tag{30} \]
Now, considering the expectation and again exploiting *Jensen’s inequality*, we can write:
\[
\mathbb{E}_{H_k} \left[ M_{\text{UE}} \log_2 \left( 1 + \kappa M_{\text{UE}}^{-1} \text{Tr}(\bar{H}_k \bar{H}_k^H) \right) \right] \leq M_{\text{UE}} \log_2 \left( 1 + \kappa M_{\text{UE}}^{-1} \mathbb{E}_{H_k} \left[ \text{Tr}(\bar{H}_k \bar{H}_k^H) \right] \right)
\]
\[= M_{\text{UE}} \log_2 \left( 1 + \kappa M_{\text{UE}}^{-1} \mathbb{E}_{H_k} \left[ \text{Tr} \left( \text{vec}(\bar{H}_k) \text{vec}(\bar{H}_k)^H \right) \right] \right)
\]
\[= M_{\text{UE}} \log_2 \left( 1 + \kappa M_{\text{UE}}^{-1} \text{Tr}(\bar{\Sigma}_k) \right), \tag{31} \]
as given in (28).
Corollary 1. The upper bound of the average SE in (28) is maximized when the effective channel covariance $\bar{\Sigma}_k$ is shaped through a beam selection based on the relevant beams.

Proof. From Lemma 4 we have:

$$
\text{Tr}(\bar{\Sigma}_k) = \text{Tr}(B_k^H \Sigma_k B_k) = \sum_{m=1}^{M_{BS}M_{UE}} b_m^H \Sigma_k b_m.
$$

(32)

The sum in (32) is maximized when the first $M_{BS}M_{UE}$ strongest channel components are selected for transmission, among the ones in $\mathcal{M}_k$.

Fig. 3 shows that (28) is a good upper-bound of the actual average SE, that can be used to perform beam selection. Thus, as a first approximation towards the maximization of the overall effective network throughput as in (P0), we formulate the uncoordinated beam selection problem (P1) which aims to maximize instead the sum SE defined as $\sum_{k=1}^{K} R_{k}^{\text{SVD}}(V_k, W_k)$:

$$
(V^{(P1)}, W^{(P1)}) = \arg \max_{V, W} \sum_{k=1}^{K} M_{UE} \log_2 \left( 1 + \kappa M_{UE}^{-1} \text{Tr}(\bar{\Sigma}_k) \right),
$$

subject to $\text{col}(V) \in \mathcal{B}_{BS}$

$$
\text{col}(W_k) \in \mathcal{B}_{UE}, \forall k = \{1, ..., K\}.
$$

Since the objective function in (P1) is disjoint with the UEs, (P1) can be solved through letting each UE maximizing its own related term in the sum. In particular, the $k$-th UE shapes its channel covariance using the beams in $\mathcal{M}_k$. Such a task requires a linear search over the $B_{BS}B_{UE}$ elements in the GoB codebooks. The precoder matrix at the BS side (common to all the UEs) is then constructed as $\text{col}(V) = \bigcup_{k=1}^{K} \text{col}(V_k)$. The relevant channel components (or beams) offers thus a straightforward method to design the GoB beamformers. Note that (P1) represents the baseline performance for the algorithms that we will introduce in the following. Uncoordinated SNR-based approaches like (P1) can be found in previous works related to hybrid beamforming, which apply to FDD mMIMO as well. For example, the authors in [29] focus on single-user beam selection and propose strategies based on the received signal strength. Likewise, in [37], a multi-user beam selection method is proposed, where the analog beams are chosen according to the strongest paths at each UE. Although our derivation of (P1) falls within covariance shaping and differs from previous works, the resulting beam selection can be considered de facto identical.
Fig. 3: Average SE vs SNR for a single-user case where beam selection is based on the relevant beams. The simulation parameters are as follows: $N_{\text{BS}} = 64$, $N_{\text{UE}} = 4$, $M_{\text{BS}} = 5$, $M_{\text{UE}} = 3$. The upper bound obtained in (28) can be used as a tight approximation for the actual SE.

B. Minimizing Multi-User Interference

As well-captured in Fig. 1, the uncoordinated selection of the GoB beamformers as in (P1) can lead to overall inefficient strategies in terms of training overhead and multi-user interference reduction. As opposed to uncoordinated approaches, clever coordinated beam selection strategies can help shaping the effective channel subspaces so as to optimize the multi-user transmission. In this section, we will show that a proper beam selection can be made so as to take multi-user interference into account within the covariance shaping process.

As seen in Section III, the BD approach imposes two crucial conditions on the overall effective channel $\bar{H} \triangleq W^H H V$ for transmitting data without multi-user interference:

- No inter-user interference $\iff |\text{null}(\bar{H}_{jk})| \neq 0$;
- No inter-stream interference $\iff \text{rank}(\bar{H}_k M^{(0)}_{jk}) \geq 1$.

From Lemma 2, we know that the second condition requires at least one vector in row($\bar{H}_k$) that is LI of row($\bar{H}_{jk}$). Opposite to TDD mMIMO, where the LMMSE can return LI channel estimates depending on the propagation environment [42], the estimates in (6) are LI almost surely, due to independent channel realizations and estimation processes at the UE side. Therefore, the second condition for multi-user interference cancellation is always respected in the case of downlink training with LMMSE estimation at the UE side.
On the other hand, whenever \((K - 1)M_{\text{UE}} < M_{\text{BS}}\), then \(\left| \text{null}(\bar{H}_k) \right| \geq M_{\text{BS}} - (K - 1)M_{\text{UE}} > 0\). Therefore, in such a case, it is always possible (for whatever \(V\) and \(W\)) to find a matrix \(\bar{M}_{j/k}^{(0)}\) in (16) different from the null matrix 0 and as such, to remove multi-user interference. In this case, the minimum training overhead becomes proportional to \(K\) and comparable to the one needed in TDD operation (although in TDD such overhead is generated in the uplink channel). The bottom line is that \((K - 1)M_{\text{UE}} < M_{\text{BS}}\) is the only condition that the BD precoding imposes on the GoB beamformers design in order to suppress multi-user interference.

Nevertheless, the BD precoding affects the received gain at the generic \(k\)-th UE. In particular, depending on how much the effective channels in \(\bar{H}\) are spatially-separated, the application of the precoding matrix \(\bar{M}_{j/k}^{(0)}\) on \(\bar{H}_k\) can lead to a drastic gain loss compared to the single-user case (refer to Proposition 2). In order to infer such loss, the so-called correlation matrix distance (CMD) can be used. The CMD has been exploited in [10] to increase the spatial separability among the UEs through covariance shaping at the UE side. The authors in [10] consider a 2-UE case. For multiple UEs, we introduce the generalized correlation matrix distance (GCMD) as follows.

**Definition 4.** We define the GCMD \(\delta_k(\Sigma_1, \ldots, \Sigma_K) \in [0,1]\) between the channel covariance \(\Sigma_k\) of the \(k\)-th UE and the channel covariance \(\Sigma_j\) of the \(j\)-th UE, where \(j \in \{1, \ldots, K\} \setminus \{k\}\) as

\[
\delta_k(\Sigma_1, \ldots, \Sigma_K) \triangleq 1 - \frac{1}{K - 1} \sum_{j=1, j \neq k}^{K} \frac{\text{Tr}(\Sigma_k \Sigma_j)}{\|\Sigma_k\|_F \|\Sigma_j\|_F}.
\]

(33)

Note that the spatial orthogonality condition, i.e. \(\text{Tr}(\Sigma_k \Sigma_j) = 0\), \(\forall j \neq k\), which was exploited in several other studies related to FDD mMIMO optimization [4], [5] is equivalent to \(\delta_k(\Sigma_1, \ldots, \Sigma_K) = 1\), \(\forall k\). This is a desirable spatial condition for which the BD incurs no reduction of the channel gain. On the other hand, the GCMD becomes zero when the covariance matrices of the UEs are equal up to a scaling factor. Both these extreme conditions are seldom experienced in practical scenarios [6], [42]. Nevertheless, when the channel covariances are shaped through statistical beamforming, resulting in some effective channel covariances, the GCMD can be used as a metric to evaluate how the covariance shaping affects the spatial separability of the UEs.

In particular, we use the GCMD evaluated on the effective covariances \(\Sigma_k \triangleq B_k^H \Sigma_k B_k\), \(\forall k\), where \(B_k \triangleq (\text{conj}(V) \otimes W_k)\), to introduce a penalty factor in \(\mathcal{R}_k^{\text{SVD}}(V_k, W_k)\), so as to approximate the SE in (20) achieved after BD precoding. In this case, the GoB beamformers \((V^{(P2)}, W^{(P2)})\)
are obtained through solving the coordinated beam selection problem (P2), as follows:

\[
(V^{(P2)}, W^{(P2)}) = \arg\max_{V, W} \sum_{k=1}^{K} M_{UE} \log_2 \left( 1 + \kappa M_{UE}^{-1} \text{Tr}(\Sigma_k) \delta_k(\Sigma_1, \ldots, \Sigma_K) \right),
\]

subject to \((K-1)M_{UE} < M_{BS} \)

\[
\text{col}(V) \in B_{BS}
\]

\[
\text{col}(W_k) \in B_{UE}, \ \forall k = \{1, \ldots, K\}.
\]

In (P2), the beam decision at the \(k\)-th UE influences the beam decisions at all the other UEs. Therefore, a central coordinator knowing all the large-dimensional channel covariances \(\Sigma_k, \forall k\) and which dictates the beam strategies to each UE is needed to solve this problem. In Section V, we will propose a hierarchical approach to circumvent this issue. Note that in both (P1) and (P2) we have proposed approximations of the SE which neglect the pre-log factor relative to the training overhead. We will now look into the third condition required for an effective GoB beamformers design in the FDD mMIMO regime, which is the minimization of the training overhead.

**C. Minimizing Training Overhead**

As seen in Section II-C there is a direct relation between the training overhead and the design of the GoB precoder \(V\) (refer to Definition 1). In particular, under the GoB assumption, the training overhead \(\omega(V)\) ranges in \([1, (\tau/T)B_{BS}]\), where the right extreme is experienced when all the beams in the codebook \(B_{BS}\) are trained. The more beams are trained, the more spatial degrees of freedom are obtained (higher spatial multiplexing and beamforming gain). However, when considering the training overhead, adding more and more beams is likely to result in diminishing returns [31].

The alternative is to train the relevant channel components [31], [32], as those relate to the spatial subspaces which give the strongest gain. In this case, the design of the precoder \(V\) is not separated from the design of the combiners \(W\), as well captured in (24). In the current 3GPP specifications, a beam reporting procedure is designed to assist the BS in the precoder selection [27]. Such procedure has a direct impact on the performance of the DL SE. In particular, the \(k\)-th UE reports to the BS the set \(\mathcal{M}_{k}^{BS}(W_k)\) – also known as precoding matrix indicator (PMI) [27] – following an appropriate GoB combiner (beam) selection. To this end, we reformulate the definition of the training overhead, depending on the beam decisions carried out at the UE side.
Definition 5. Let $W \in \mathbb{C}^{K_{\text{UE}} \times K_{\text{UE}}}$ be the overall GoB combiner as in (4). The training overhead $\omega(W)$ is defined as follows:

$$\omega(W) \triangleq \frac{\tau}{T} \text{card} \left( \bigcup_{k=1}^{K} M_{k}^{\text{BS}}(W_k) \right).$$  (34)

In the 3GPP implementation, the beam decisions carried out at each UE have thus a central role in affecting the training overhead under the GoB approach. Note that $\omega(W)$ can increase and approach the extreme value $(\tau/T)B_{\text{BS}}$ in heterogeneous propagation environments with rich scattering, due to the growing number of relevant beams to activate at the BS side [31]. In this respect, adopting approaches such as (P1) or (P2) for selecting the beams can undermine the application of the GoB approach in multi-user scenarios.

On the other hand, the largest training overhead reduction is achieved when the UEs coordinate in the beam domain so that to activate the smallest possible number of beams at the BS. In general, a balance between achievable beamforming gain and required training overhead, as well as multi-user interference, has to be considered in the beam decision process and combiner selection at the UEs. In the following, we formulate two optimization problems which take the pre-log factor relative to the training overhead into account. In the first one, the pre-log term is added in the objective function of the optimization problem (P1). Thus, we introduce the coordinated beam selection problem (P3), where both the achieved channel gain and the training overhead are taken into account, as follows:

$$\left( V^{(P3)}, W^{(P3)} \right) = \arg \max_{V,W} (1-\omega(W)) \sum_{k=1}^{K} M_{\text{UE}} \log_2 \left( 1 + \frac{\kappa M_{\text{UE}}^{-1} M_{\text{BS}}^{\text{col}}(V)}{\text{Tr}(\bar{\Sigma}_k)} \right),$$  \hspace{1cm} (P3)

subject to $(K-1)M_{\text{UE}} < M_{\text{BS}}$

$$\text{col}(V) \in B_{\text{BS}}$$

$$\text{col}(W_k) \in B_{\text{UE}}, \forall k = \{1,\ldots,K\},$$

where we recall that $(K-1)M_{\text{UE}} < M_{\text{BS}}$ relates to the crucial condition that the BD precoding imposes on the GoB beamformers design in order to suppress multi-user interference.

The last optimization problem that we introduce aims at balancing the three conditions for an effective GoB beamformers design that we have considered in this section. As such, the long-term beam selection problem (P4) includes the pre-log factor relative to the training overhead in the
objective function of the problem (P2). The optimum GoB beamformers \((V^{(P4)}, W^{(P4)})\) are thus obtained through solving the following optimization problem:

\[
(V^{(P4)}, W^{(P4)}) = \arg \max_{V, W} (1 - \omega(W)) \sum_{k=1}^{K} M_{UE} \log_2 \left( 1 + \kappa M_{UE}^{-1} \text{Tr} (\Sigma_k) \delta_k (\Sigma_1, \ldots, \Sigma_K) \right),
\]

subject to \((K-1)M_{UE} < M_{BS}\)

\[
\begin{align*}
\text{col}(V) \in B_{BS} \\
\text{col}(W_k) \in B_{UE}, \forall k = \{1,\ldots,K\}.
\end{align*}
\]

The same conclusions drawn for the optimization problem (P2) are valid for (P3) and (P4). In particular, to solve (P4), the central coordinator needs to know the PMIs \(M_{BS}^k(W_k), \forall k\) in addition to the channel covariances \(\Sigma_k, \forall k\).

Fig. 4 compares the effective network throughput \(R\) as in (P0) with its approximations in (P1)-(P4). The approximated objective function of the optimization problem (P4) gives the tightest upper bound to the actual effective network throughput as expected. We summarize the proposed optimization problems and their considered sub-problems as introduced above in Table I.

In the next section, we will propose a framework exploiting D2D communications which will allow for a decentralized implementation of a series of beam selection algorithms based on the problems (P2)-(P4) described above. The nature of such problems is such that (P1)-(P4) offer and explore various complexity-performance trade-offs interesting from the implementation perspective.

| Problem          | max channel gain | min multi-user interference | min training overhead |
|------------------|------------------|-----------------------------|-----------------------|
| Uncoordinated (P1) | ✓                | □                           | □                     |
| Coordinated (P2)  | ✓                | ✓                           | □                     |
| Coordinated (P3)  | ✓                | □                           | ✓                     |
| Coordinated (P4)  | ✓                | ✓                           | ✓                     |

V. DECENTRALIZED COORDINATED BEAM SELECTION ALGORITHMS

Although no instantaneous information is needed to solve (P2)-(P4), such problems still require a central coordinator that knows the channel covariances \(\Sigma_k, \forall k\) and dictates the beam strategies to each UE. This is because the beam decisions at the generic \(k\)-th UE affect the beam decisions at all the other UEs. In this respect, collecting such large-dimensional statistical information at a central coordinator as e.g. the BS involves additional resource overhead.
Fig. 4: Comparison of the actual effective network throughput $\mathcal{R}$ as in (13) and its approximations defined in (P0)-(P4), $K = 7$ UEs. The approximation (P4) is the closest to the actual throughput.

In order to achieve decentralized coordination, we propose to use a hierarchical information structure requiring small overhead. In particular, an (arbitrary) order among the UEs is established\(^3\) for which the $k$-th UE has access to some long-term statistical information of the (lower-ranked) UEs $1,...,k-1$. This configuration is obtainable through e.g. D2D communications. In this respect, the 3GPP Release 16 is expected to support point-to-point side-links which facilitate cooperative communications among the UEs with low resource consumption \([33], [34]\). The recently-specified NR side-link is thus a cornerstone for the proposed scheme. We further assume that such exchanged information is perfectly decoded at the intended UEs.

The full signaling sequence of the proposed hierarchical beam selection is given in Fig. 5. The core part of the procedure resides in the long-term beam decision made at each UE on a beam coherence time basis so that the respective objective function is maximized. Based on the objective functions in (P2)-(P4), we consider 3 different beam decision policies, as in (36). Such policies have different requirements concerning the statistical information to exchange through D2D side-links. In Table II we summarize the differences between the proposed beam selection policies based on (P1)-(P4) with respect to the required information at the $k$-th UE.

\(^3\)The hierarchical order of the UEs has a clear impact on the performance of the proposed scheme. In this paper, we will consider a random order and leave aside further analysis on how such hierarchy is defined and maintained.
TABLE II: The proposed algorithms and their required information at the $k$-th UE. The information relative to the (lower-ranked) UEs $1,...,k-1$ is exchanged through D2D side-links.

| Algorithm          | Required local information | Required information to be exchanged through D2D |
|--------------------|-----------------------------|-------------------------------------------------|
| Uncoordinated (P1) | $\Sigma_k$                 | Nothing                                         |
| Coordinated (P2)   | $\Sigma_k$                 | $\Sigma_j, j \in \{1,...,k-1\}$               |
| Coordinated (P3)   | $\Sigma_k$                 | $\mathcal{M}^\text{BS}_j(W_j), j \in \{1,...,k-1\}$ |
| Coordinated (P4)   | $\Sigma_k$                 | $\Sigma_j, \mathcal{M}^\text{BS}_j(W_j), j \in \{1,...,k-1\}$ |

Fig. 5: Signaling sequence of the proposed coordinated beam selection [P3] for $K = 3$. The beam decision made at each UE leverages the D2D-enabled long-term statistical information (the PMI $\mathcal{M}^\text{BS}_k(W^*_k)$, for [P3]) coming from the higher-ranked UEs in a hierarchical fashion.

Let us consider w.l.o.g. the beam selection at the $k$-th UE, i.e. at the $k$-th step of the algorithm, for the algorithm [P4]. The algorithms [P2]-[P4] can be regarded as a sub-case of [P4]. We define the set $\mathcal{W}_{k-1} \triangleq \{W^*_1,...,W^*_k\}$ containing the beam decisions which have been fixed prior to the $k$-th step. According to the hierarchical structure, the $k$-th UE knows the set $\mathcal{B}^\text{fix}(\mathcal{W}_{k-1}) \triangleq \bigcup_{j=1}^{k-1} \mathcal{M}^\text{BS}_j(W^*_j)$ and the effective channel covariances $\Sigma_j, \forall j \in \{1,...,k-1\}$. Therefore, the $k$-th UE can i) evaluate a partial GCMD $\delta_k(\Sigma_1,...,\Sigma_k)$ and ii) construct a partial GoB precoder $\mathbf{V}_{k-1}$ containing the precoding vectors relative to the indexes in $\mathcal{B}^\text{fix}(\mathcal{W}_{k-1})$, i.e. $\text{col}(\mathbf{V}_{k-1}) = \{v_v \in \mathcal{B}_\text{BS} : (v,w) \in \mathcal{B}^\text{fix}(\mathcal{W}_{k-1})\}$. Likewise, the $k$-th UE can compute a partial $\omega(\mathcal{W}_{k-1})$. 
The proposed decentralized beam selection $W^*_k$ at the $k$-th UE can be then expressed in a recursive manner as follows:

$$W^*_k = \arg\max_{W_k} f_k\left([V_k V_{k-1}] ; \{W_k, W_{k-1}\}\right), \tag{35}$$

where $\text{col}(V_k) = \{v \in B_{\text{BS}} : (v, w) \in M_{BS}^k(W_k)\}$, and

$$f_k(V, W) \triangleq \begin{cases} M_{\text{UE}} \log_2 (1 + \kappa_M^{-1} \text{Tr}(\Sigma_k) \delta_k(\Sigma_1, \ldots, \Sigma_k)) & \text{(P2)} \\ (1 - \omega(W)) M_{\text{UE}} \log_2 (1 + \kappa_M^{-1} \text{Tr}(\Sigma_k)) & \text{(P3)} \\ (1 - \omega(W)) M_{\text{UE}} \log_2 (1 + \kappa_M^{-1} \text{Tr}(\Sigma_k) \delta_k(\Sigma_1, \ldots, \Sigma_k)) & \text{(P4)} \end{cases} \tag{36}$$

The intuition behind the proposed scheme is to let the $k$-th UE select the $W_k \in B_{\text{UE}}$ maximizing the $k$-th term of the sum in the respective objective function, in a greedy manner. The remaining constraint $(K - 1)M_{\text{UE}} < M_{\text{BS}}$ can be enforced at the BS through e.g. activating predefined beams.

A. On Algorithm and Information Exchange Complexity

The proposed decentralized problem can be addressed using linear (exhaustive) search in the codebook $B_{\text{UE}}$ at each UE. In particular, the linear search does not involve a large computational burden as the $k$-th UE has to evaluate the respective objective function in (36) in $(B_{\text{UE}} M_{\text{UE}})$ points, which is generally small in practical implementations [28]. On the other hand, the direct solving of the optimization problems (P2)-(P4) requires combinatorial (exhaustive) search. Note that the complexity of the proposed solution is independent from the number of antennas at the BS and at the UE side. However, the complexity increases with the number of beams $M_{\text{UE}}$ at the UE side, which is somewhat dependent on $N_{\text{UE}}$ (to avoid bad spatial resolution) [31].

Table II summarizes the required information at each UE for the proposed beam selection policies based on (P1)-(P4). The proposed scheme relies on acquiring and tracking channel second order statistics at each UE, on a beam coherence time basis, as described in Fig. 5. In this respect, the beam coherence time must be long enough to avoid large side-link overhead, and to ensure the feasibility of the proposed scheme. The authors in [40] perform a thorough investigation on the behavior of the beam coherence time for practical use cases. For example, the beam coherence time is found to be higher than 1 second under a vehicular scenario with UEs speed around 100 km/h communicating in the 60 GHz band. Thus, it seems reasonable to assume that the beam coherence time is long enough for the purposes of the proposed scheme.
VI. SIMULATION RESULTS

We evaluate here the performance of the proposed decentralized beam selection algorithms. We assume $N_{\text{BS}} = 64$ and $N_{\text{UE}} = 4$. The beamforming vectors in $B_{\text{BS}}$ and $B_{\text{UE}}$ are discrete Fourier transform (DFT)-based orthogonal beams, according to the codebook-based transmission in 3GPP NR [28]. Furthermore, we assume that the UEs are allowed to indicate at most 4 relevant beam pairs each to the BS, i.e. the PMI $M_{k}^{BS}(W_{k}^*)$ is truncated to its 4 strongest elements $\forall k$. This is equivalent to the Type II CSI reporting in NR [28]. We assume that the UEs use the popular minimum mean square error (MMSE) method to estimate their instantaneous effective channels (refer to (6) and (7)), which are then fed back to the BS for BD-based precoder design (refer to Fig. 5). The Zadoff-Chu sequences are used for training the effective channel [28]. In our simulation, we consider transmission over 25 adjacent resource blocks, each one consisting in 12 sub-carriers and 14 OFDM symbols [28]. The total considered bandwidth is 5 MHz, and the total number of available resource elements in the channel coherence time is $T \triangleq (14 \cdot 12 \cdot 25)T_{\text{coh}}$, where $T_{\text{coh}}$ is expressed in ms. All the metrics in the next plots are averaged over 10000 Monte-Carlo iterations with varying network scenarios.

A. Winner II Channel Model

The channel model used for the simulations is the cluster-based Winner II model, which extends the 3GPP spatial channel model. In particular, we consider the urban micro-cell scenario operating at 2.1 GHz. In urban micro-cell scenarios, both the BS and the UEs are assumed to be located outdoors in an area where the streets are laid out in a Manhattan-like grid. The UE speed is set according to a uniform distribution in the interval $[5,50]$ km/h. This scenario considers both line-of-sight and non-line-of-sight links. Like in all cluster-based models, the channel realizations are generated through summing the contributions of the multiple paths within each cluster. Those paths come with their own small- and large-scale parameters such as amplitude, AoD and AoA.

B. Results and Discussion

In what follows, we show and discuss the performance achieved via the proposed algorithms. For benchmarking, we consider – where appropriate – the relative TDD configuration with both

\[\text{In actual networks, control information occupies a portion of the available resource elements. We assume that such portion is negligible and that all resource elements are used for either CSI-RS or data transmission.}\]
perfect and imperfect CSI[^1]. Moreover, we will consider two different network scenarios: the i) with randomly-located UEs, and the ii) with closely-located UEs (highly spatially-correlated channels).

We start with configuration i). In Fig. 6a, we show the average effective network throughput as a function of the transmit SNR for $K=7$ UEs and a channel coherence time $T_{coh}=15$ ms. As expected, higher throughput is achieved under TDD settings compared to the proposed FDD solutions [30]. In particular, the gain over FDD increases with the SNR. With respect to the proposed algorithms, both (P3) and (P4) outperform the uncoordinated benchmark (P1), with equal average effective throughput values obtained with up to 10 dBs less. Since $T_{coh}$ is small, the pre-log factor dominates the log factor in (13). Therefore, just a small performance gap divides (P3) from (P4) and, as such, according to Table II (P3) is preferable in this case (since much less information needs to be shared among the UEs). On the other hand, the coordinated algorithm (P2) performs even worse than the uncoordinated benchmark (P1). In particular, when the channel coherence time $T_{coh}$ is small, shaping the covariances so as to maximize the spatial separability of the UEs is counter-effective. Some more insights on this are given in the next paragraph. Since the training overhead increases with $K$, the performance gain achieved by the coordinated algorithms (P3) and (P4) surges in Fig. 6b for $K=11$. For the same reason, the gap between (P2) and all other solutions increases.

Fig. 7a shows the average throughput gain over the uncoordinated benchmark (P1) as a function of $T_{coh}$ for $K=7$ UEs. In particular, two areas can be identified:

1) $T_{coh} < 20$ ms, i.e. vehicular or fast pedestrian channels: where (P3) and (P4) have high gains compared to the other solutions (up to 45%) and where the coordinated algorithm (P2) performs even worse than the uncoordinated (P1). Indeed, as we can see in Fig. 8, in order to achieve greater spatial separation across the UEs, the algorithm (P2) activates a much greater number of beams at the BS side, which is detrimental under fast-varying channels.

2) $T_{coh} \geq 20$ ms, i.e. pedestrian channels: where the gap between (P3)-(P4) and (P1) reduces (up to 15%). In particular, (P3) converges to the uncoordinated benchmark (P1). This is because the training overhead becomes negligible for long channel coherence times, and it is more important to focus on the log factor in (13). For the same reason, (P2) experiences gains over the uncoordinated solution (P1) for $T_{coh} \geq 20$ ms. The coordinated algorithm (P4) converges to (P2).

[^1]: The curves relative to the TDD setting are obtained after BD precoding at the BS, under the assumption of perfect channel reciprocity. The precoder is applied over the full-dimensional $KN_{UE} \times N_{BS}$ channel. In this respect, a clear advantage is experienced over the proposed FDD solutions, which consider the transmission over an effective channel, as a result of the GoB precoding.
Fig. 6: Average effective network throughput vs SNR for (a) $K = 7$ and (b) $K = 11$ randomly-located UEs. $M_{\text{UE}} = 3$ beams activated at each UE. $T_{\text{coh}} = 15$ ms. The coordinated algorithms (P3) and (P4) outperform the uncoordinated (P1), as opposed to (P2).

Therefore, for long channel coherence times, (P2) allows to avoid some additional coordination overhead, according to Table II and, as such, is preferable.

The same reasoning holds for Fig. 7b with $K = 11$ UEs, where the positive and negative behaviors described above are intensified. In particular, for $T_{\text{coh}} < 20$ ms, (P3) and (P4) achieve up to 120% gain over (P1), while 20% loss is achieved with (P2).

Fig. 7: Average effective throughput gain over uncoordinated beam selection (P1) vs $T_{\text{coh}}$ for $K = 7$ UEs. The transmit SNR is 11 dB. Taking the pre-log factor into account is essential for an effective coordinated beam selection under fast-varying channels where $T_{\text{coh}} < 20$ ms.
Let us now focus on the configuration $ii)$, where higher spatial correlation is found among the UEs. Fig. 9 shows the average throughput gain over the uncoordinated benchmark as a function of the channel coherence time $T_{coh}$. We can see that now (P2) outperforms (P1) for all the considered values of $T_{coh}$. Indeed, due to the increasing spatial correlation among the UEs, the multi-user interference becomes non-negligible even for small channel coherence times below 20 ms. Moreover, in this case, the performance gain obtained through (P4) justifies more the need to exchange some additional long-term information compared to the other solutions (refer to Table II).

Fig. 9: Average effective throughput gain over uncoordinated beam selection (P1) vs $T_{coh}$ for $K = 7$ closely-located UEs. The transmit SNR is 11 dB. Owing to high spatial correlation among the UEs, (P2) achieves high gains compared to the solutions which neglect the multi-user interference.
C. Effect of Feedback Quantization

In this section, we take into account the impact of limited feedback from the UEs to the BS on the overall DL performance. In particular, we assume that the estimated channels are uniformly-quantized with $q_B$ bits (element-wise quantization). For a fairer comparison between FDD and TDD operation, we assume that, under TDD, the quantization is applied as well on the (effective) channels which are fed back to the UE for coherent detection. Fig. 10 shows the effective network throughput as a function of the quantization bits $q_B$ for an SNR equal to 11 dB, and a $T_{coh} = 18$ ms. As expected, reducing $q_B$ entails a sharp throughput loss. However, it should be noted that such a loss starts when $q_B \leq 7$, except under TDD settings. With respect to the proposed algorithms, adopting a small $q_B$ reduces the performance gaps as well. At the same time, quantizing with a smaller $q_B$ reduces the feedback overhead in the UL band. In this respect, feedback overhead and coordination performance must be balanced.

![Graph showing effective network throughput vs quantization bits for different channel conditions](image)

(a) $K=7$ UEs  
(b) $K=11$ UEs

Fig. 10: Average effective network throughput vs $q_B$ for $K=7$ UEs. The transmit SNR is 11 dB. $T_{coh} = 18$ ms. Reducing $q_B$ reduces the feedback overhead, but decreases performance sharply.

VII. CONCLUSIONS

In this paper, we have shown that beam-domain coordination between the UEs offers a convenient means to improve the performance in FDD mMIMO networks under the GoB design assumption. We have proposed a decentralized beam selection algorithm exploiting long-term statistical information and its exchange through D2D side-links. The proposed scheme explores the interesting trade-off between i) harvesting large channel gain, ii) avoiding multi-user interference (low spatial separation among the UEs), and iii) minimizing the training overhead, which arises in
this context. Simulation results demonstrate the effectiveness of the proposed algorithm. In particular, under fast pedestrian or vehicular channels where the channel coherence time is below 20 ms, the proposed scheme gives consistently better performance in terms of overall network throughput than uncoordinated beam selection or schemes that do not properly address the above trade-off.

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