Carrier induced ferromagnetism in diluted magnetic semi-conductors.

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We present a theory for carrier induced ferromagnetism in diluted magnetic semi-conductor (DMS). Our approach treats on equal footing quantum fluctuations within the RPA approximation and disorder within CPA. This method allows for the calculation of $T_c$, magnetization and magnon spectrum as a function of hole, impurity concentration and temperature. It is shown that, sufficiently close to $T_c$, and within our decoupling scheme (Tyablicov type) the CPA for the itinerant electron gas reduces to the Virtual Crystal Approximation. This allows, in the low impurity concentration and low density of carriers to provide analytical expression for $T_c$. For illustration, we consider the case of $Ga_{1-x}Mn_xAs$ and compare our results with available experimental data.

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The discovery of carrier induced ferromagnetism in DMS have attracted considerable attention from both theoreticians and experimentalists. The interest for these material is mainly stimulated by the possible technological applications (e.g. semi-conductor spin devices). For example by doping GaAs with magnetic impurities $Mn^{2+}$, $T_c$ exceeding 100 K has been reached. The doping of a III-V semiconductor compound with Mn impurities introduces simultaneously local magnetic moments ($S = 5/2$) and itinerant valence band carriers ($s = 1/2$). One of the important open issues is to find out whether it is possible to reach critical Curie temperature of order 300 K. Thus it is important to understand theoretically how $T_c$ varies with the impurity concentration, effective mass, hole concentration and exchange integral. Many theoretical approaches have been performed to analyze ferromagnetism in DMS, this includes mean field theory [1], spin wave theory [2], first principle calculations [3] and Monte-Carlo simulations [4]. In contrast to most of the theoretical work, we present a theory is able to treat disorder in a more realistic manner (beyond coarse graining). Our theory includes quantum fluctuations within RPA and disorder is treated within CPA. It should be stressed that in our approach the spin impurities are treated quantum mechanically.

We start with the following minimal Hamiltonian,

$$H = \sum_{i,j,\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i J_i \vec{S}_i \cdot \vec{s}_i \quad (1)$$

The first term stands for the tight binding part of the itinerant free electron gas, $t_{ij} = t$ if i and j are nearest neighbor or 0 otherwise. The second term is the exchange between localized impurities spin and itinerant electron gas, $J_i$ are random variables: $J_i = J$ if site $i$ is occupied by a $Mn^{2+}$ ion or 0. The operator $\vec{s}_i = c_{i\sigma}^\dagger (1/2 \delta_{\sigma,\beta}) c_{i\beta}$ is the spin operator at i of the itinerant electron gas and $\vec{S}_i$ is the spin of the magnetic impurity.

Let us define the Green’s function,

$$G_{ij}^{+-} (t) = -i \delta(t) \langle [S_i^+ (t), S_j^- (0)] \rangle \approx < S_i^+ ; S_j^- > \quad (2)$$

We write the equation of motion and use Tyablicov decoupling [1] (equivalent to RPA) which is suitable for ferromagnetic systems. It consists in closing the system subtracting both eq. (4) and (5) into eq. (3) we immediately find

$$\langle \chi_i \rangle = \frac{2 \langle S_i^z \rangle}{\omega + J_i \langle s^z \rangle} \quad (3)$$

$$\langle s^z \rangle \text{ is the magnetization of the itinerant electron gas and } \langle S_i^z \rangle \text{ the magnetization of a magnetic ion at site i. It is convenient to rewrite the new Green’s function which appears in the right part of the equality in the following form, } \langle s^+_i; S_j^- \rangle = \frac{1}{\pi} \sum_{kq} \delta(\epsilon_k - \epsilon_{k+q}) \Gamma_j^{k+q,k} \text{, where }$$

$$\Gamma_j^{k+q,k} = \langle c_{k+q}^\dagger c_{k,\downarrow}; S_j^- \rangle = \sum_t \frac{1}{2} J_i e^{-i \delta k R_j} G_{ij}^{+-} \quad (4)$$

where,

$$f(k,q,\omega) = \frac{(n_{k+q,\uparrow}) - (n_{k,\downarrow})}{\omega - (\epsilon_k - \epsilon_{k+q}) + c J \langle S_k^z \rangle} \quad (5)$$

$\langle n_{k,\sigma} \rangle$ is the occupation number of $(k, \sigma)$ state. $c$ is the impurity concentration, $\langle S_k^z \rangle$ is the averaged magnetization of $Mn^{2+}$, and $\epsilon_k$ denotes the hole’s dispersion. Inserting both eq. (4) and (5) into eq. (3) we immediately find,

$$G_{ij}^{+-} = c g_i \delta_{ij} + \sum_\ell \phi_\ell G_{ij}^{+-} \quad (6)$$

where the T dependent locator $g_i$ is defined as,

$$g_i (\omega) = \frac{2 \langle S_i^z \rangle}{\omega + J_i \langle s^z \rangle} \quad (7)$$

$\phi_\ell = \frac{1}{2} J_i J_{\ell i} \chi_{\ell i}^0 (\omega)$ and $\chi_{\ell i}^0 (\omega)$ is the Fourier transform of the polarized susceptibility $\chi_{\ell i}^0 (q', \omega)$.
\[ \chi^0(q', \omega) = \frac{1}{L} \sum_k \frac{\langle n_{k+q', \uparrow} \rangle - \langle n_{k, \uparrow} \rangle}{\omega - \epsilon_k - \epsilon_{k+q'} + cJ(S_A^z)} \] (8)

Note that eq. (8) still contains the disorder through \( \phi_{il} \) and \( g_i \). It is also interesting to mention that the previous equation can be interpreted as the propagator of a free particle moving on a disordered medium. \( g_i \) is the random on-site potential and \( \phi_{il} \) the long range-hopping terms. Note also that \( \phi_{il} \) is energy dependent through \( \chi^0_{il} (\omega) \). To solve the problem we have to write the equation of motion for the Green’s function \( K_{ij, \sigma} = \langle \epsilon_{i, \sigma} ; \epsilon_{j, \sigma} \rangle \). After decoupling we get,

\[ (\omega - \frac{1}{2} z_{ij} J_c(S_A^z)) K_{ij, \sigma} = \delta_{ij} + \sum_l t_{il} K_{lj, \sigma} \] (9)

One can recognize the propagator of the Anderson model, with on-site random potential depending on the spin \( \sigma \): \( \epsilon_{i, \sigma} = \frac{1}{2} z_{ij} J_c(S_A^z) \). Since in our model the potential is temperature dependent through \( \langle S_A^z \rangle \) then sufficiently close to \( T_c \), we will always be in the metallic regime \( k_f l_c \gg 1 \) [2]: \( l_c \approx \frac{1}{J_c \langle S_A^z \rangle^2} \). This is in contrast with the standard Anderson model where the impurities are static. Equations (8) and (9) (\( \sigma = \pm 1 \)) provide a closed system of equations which have to be solved self-consistently within CPA.

The simplest is to start with eq. (8). Indeed, it is straightforward to get the solution with the standard CPA since it contains only diagonal disorder. The averaged Green’s function is

\[ K_{k, \sigma} = \frac{1}{\omega - \epsilon(k) - \Sigma_{\sigma}(\omega)} \] (10)

where the self-energy is,

\[ \Sigma_{\sigma}(\omega) = V_{\sigma} - (\epsilon_{A, \sigma} - \Sigma_{\sigma}(\omega)) K_{\sigma}^{00}(\omega) (\epsilon_{B, \sigma} - \Sigma_{\sigma}(\omega)) \] (11)

where \( \epsilon_{A, \sigma} = \frac{1}{2} z_{ij} J_c(S_A^z) \), \( \epsilon_{B, \sigma} = 0 \) and \( V_{\sigma} \) is the average value \( \langle \epsilon_{i, \sigma} \rangle = \frac{1}{2} z_{ij} J_c(S_A^z) \) and \( K_{\sigma}^{00}(\omega) = \frac{1}{2} \sum_{\omega} \omega - (\sigma - \epsilon_{\omega, \sigma}) \).

The self-energy \( \Sigma_{\sigma}(\omega) \) can be re-expressed,

\[ \Sigma_{\sigma}(\omega) = V_{\sigma} [1 + (1 - c) \frac{1}{2} z_{ij} J_c(S_A^z) K_{\sigma}^{00}(\omega)] \] (12)

We see that when \( T \to T_c \), \( \Sigma_{\sigma}(\omega) \to \Sigma_{\sigma}^{\text{VCA}}(\omega) = V_{\sigma} \). Thus, in the framework of our decoupling scheme, close enough to \( T_c \), the CPA for eq. (10) reduces to the VCA.

The final step of the calculation consists in solving eq. (9). In order to provide analytical form for \( T_c \) we use the similar approximation (VCA) for \( G_{ij}^{\text{VCA}} \) as done above for \( K_{ij, \sigma} \). We expect this approximation to be reasonable in the limit of both low impurity concentration and low density of itinerant carriers. To get the averaged Green’s function, we use the well-known Blackman-Esterling-Beck formalism [13,14]. By contrast with standard CPA, this approach is suitable for non diagonal disorder problems. It is based on a 2 \( \times \) 2 matrix Green’s function formalism for binary alloys using locator expansion. Within VCA approximation, one gets for the averaged Green’s function of an atom of type A,

\[ G_A^{\text{VCA}}(k, \omega) = \frac{c}{g_A^* - c \alpha(k, \omega)} \] (13)

where \( \alpha(k, \omega) = \frac{1}{2} J^2 \chi^0 (k, \omega) \).

Note also that since the Ga atoms have no magnetic moment, it implies \( G_B^{\text{VCA}}(k, \omega) = g_B = 0 \). The Mn\(^{2+}\) ion propagator can be rewritten, \( G_A^{+}(k, \omega) = \frac{1}{2} J^2 \omega^2 q^2 E(q) \), where \( E = \frac{\omega}{\omega(q)} \). The dispersion \( E(q) \) is solution of,

\[ E(q) = -J \langle S_z \rangle + \frac{1}{2} J^2 \chi^0(q, E(q)) \langle S_z \rangle \] (14)

According to ref. [15], the magnetization can be expressed in the following form,

\[ \langle S_A^z \rangle = \frac{\langle \epsilon - \phi \rangle (1 - \phi)^{S+1} + (\epsilon + \phi)^{S+1}}{1 + \phi^{2S+1} - \phi^{2S+1}} \] (15)

where \( \phi = \frac{1}{2} \sum q \frac{1}{\omega - q^2} \).

When \( T \to T_c \), \( \phi = \frac{1}{2} \sum q \frac{1}{\omega - q^2} \). This implies for \( T_c \) the standard RPA form,

\[ T_c = \frac{1}{3} \frac{S(S+1)}{N} \langle S_z \rangle \] (16)

This expression is similar to the one obtained in the clean limit for the Kondo Lattice Model [16,17]. In the vicinity of \( T_c \) the dispersion \( E(q) \) is,

\[ E(q) = \frac{1}{8 \pi^2} \frac{J^2}{t} \langle k_f - 1 q^2 + 2 q^2 \rangle \ln \frac{q + 2 k_f}{q - 2 k_f} \] (17)

Note that, below \( T_c \), the eq. (14) should be solved numerically in order to get \( E(q) \) as a function of the temperature. This is required to calculate \( \langle S_z \rangle \) as function of \( T \). According to eq. (10) \( T_c \) is given by,

\[ T_c = \frac{S(S+1) J_c^2}{24 \pi^2} \frac{1}{t} \langle N \sum_q C(q, k_f) \rangle^{-1} \] (18)

where we define \( C(q, k_f) = \frac{1}{2} \langle k_f - \frac{1}{2} k_f^2 \rangle \ln \frac{q^2 + 2 k_f^2}{q^2 - 2 k_f^2} \). This implies that \( T_c \) is proportional to \( J^2 \) and to the effective mass \( 1/t \). The dependence on the hole concentration is only contained in \( C(q, k_f) \). We define the hole concentration as \( n_h = c \gamma \) where \( \gamma \leq 1 \). This is the simplest way to take into account the presence of Asantisites [13]. In Fig. 1 we show the variation of \( T_c \) as a function of \( \gamma \).
FIG. 1. $T_c$ as a function of $\gamma$ for $c = 0.05$ and $J^2 = 10.5$ eV. The continuous line represent the VCA calculation, the dashed line corresponds to $T_c$ within the mean-field approximation, and the dotted line is obtained by approximating the dispersion by $E(q) = D q^2$, D is the spin stiffness. The inset shows $\log(T_c)$ versus $\log(\gamma)$.

We observe that in the low hole concentration regime, $T_c$ agrees very well with the mean field result (this is more clear in the inset log-log plot). In the mean field regime the magnon excitation spectrum is dispersionless: $E_{MF}^{MF}(q) = \lim_{q \to \infty} E(q) = \frac{1}{8\pi^2} J^2 k_f$ where $k_f = (3\pi^2 c)\gamma^{1/3}$. In this limit,

$$T_c = \frac{1}{24} \left[ \frac{3}{\pi^2} \right]^{1/3} S(S + 1) J^2 \gamma^{1/3} C^{4/3} \quad (19)$$

When increasing $\gamma$, $T_c$ strongly deviates from the mean field results and shows a broad maximum. Such a maximum was also observed in ref. [3]. By further increase of $\gamma$ the Curie temperature starts to decrease [19]. As we observe it from Fig. 1, for very large $\gamma$, $T_c$ agrees very well with the case where the magnon spectrum is approximated by $E(q) = E_{MF}(q) = D q^2$ where the stiffness D is given by $D = \frac{1}{18\pi^2} J^2 \frac{1}{k_f}$, this regime is denoted “stiffness” regime. In this regime we find,

$$T_c = \frac{1}{144} \left[ \frac{18\pi^4}{1} \right]^{1/3} S(S + 1) J^2 \gamma^{-1/3} C^{2/3} \quad (20)$$

The existence of a maximum can be understood in the following way: Like in the RKKY situation [20], the exchange oscillate with typically length scale $l_{osc} \propto 1/k_f$. Thus it is expected that when the length scale gets sufficiently large (larger than the average distance between impurities) some Mn-Mn bonds are coupled antiferromagnetically. The induced frustration has for immediate consequence a decrease of $T_c$. In Fig. 2 we illustrate the previous discussion by showing the dispersion as a function of $k/k_c$ where $k_c$ is chosen in order to conserve the volume of the Brillouin zone ($v = (2\pi)^3$). The results are shown for the 3 different regions: “mean field”, “intermediate” and “stiffness” regime. We observe that in all cases the dispersion goes to 0 (when $q \to 0$), as expected when the Goldstone theorem is fulfilled.

FIG. 2. Magnon dispersion for $c = 0.05$ in the 3 different regime, “mean field” (continuous line), “intermediate” (dotted line) and “stiffness” regime (dashed line). $E(q)$ is rescaled by a factor $\lambda = \frac{J^2}{4\pi} (3\pi^2 c)^{1/3}$.

In Fig. 3, we show the region for which $T_c$ reaches its maximum as a function of $c$ ($T_c^{max}(c)$) and the region where MF formula provides a good approximation for $T_c$, it corresponds to $|T_c - T_c^{MF}| \leq 0.1$. First we see that the region of validity of the MF result (dashed area) corresponds to a very narrow region typically $\gamma \leq 0.05$. A good approximated value of the $\gamma$ for which $T_c$ is maximum can be obtained by taking the intersection point between the MF and “stiffness” values. This leads to $\gamma_{max} c = n_{max} = 0.016$.

FIG. 3. The dashed area represents region where mean-field result for $T_c$ is valid, the symbols are calculated points and the dashed line a fit. The continuous curve represents values of $(\gamma, c)$ for which $T_c$ is maximum ($T_{c^{max}}$). In the inset we have plotted $T_{c^{max}}$ as a function of $c$ assuming $J^2 = 10.5$ eV.

So far, for our discussion we did not have to specify the values of the parameters $t$ and $J$. In order to check
the validity of our theory we compare our results with available experimental data. GaAs is known to have a fcc structure with a lattice constant $a_0 \approx 5.6A$. For simplification in our calculation we have assumed a simple cubic structure thus the lattice constant which has to be taken in our calculation is $a_1 = \frac{a_0}{\sqrt{3}}$ in order to conserve the volume for the unit cell. By also assuming an effective mass for the holes $m = 0.5 \ m_e$ one gets $t = 0.63 \ eV$. The remaining free parameter $J$ will be chosen in order to fit the experimental data of ref. [2]. For that purpose we calculate $\gamma$ for each sample according to the measured experimental values of the hole concentration given in Fig. 2 of ref. [2]. The results are depicted in Fig. 4. As it can be seen we find a very good agreement with the experimental data if $\frac{J}{t} = 10.5 \ eV$, this implies $|J| = 2.58 \ eV$ [22]. Note that the deviations observed at low $c$ are due to the uncertainty on the hole concentration value (see the huge error bars in Fig. 2 of ref. [2]).

From the experimental measurements, there is no clear consensus concerning the correct value of this parameter. Indeed, recent core level photoemission has provided $J = -1.2 \pm 0.2 \ eV$ [23]. Whilst, from Magneto-transport measurements a value of $|J| = 2.4 \pm 0.9 \ eV$ was suggested. [22] And within first principle calculations Sanvito et al. [4] have found $J \approx -4.65 \pm 0.25 \ eV$. In order to proceed to a better estimation of the parameters $J$ one should compare theoretical calculations with other data, for instance transport measurements data [23]. However, it is interesting to note that the band splitting at $T = 0K$ ($\Delta = JcS$) obtained within our calculations agrees with the experimental value reasonably well [22]. In the inset of Fig. 3, assuming $\frac{J}{t} = 10.5 \ eV$ we show $T_c$ as function of $c$ taking $\gamma$ on the line of “maximum of $T_c$”. For instance if $c \approx 0.1$ and $\gamma \approx 0.2$ a $T_c$ of order 230 K can be reached.

![FIG. 4. $T_c$ in Kelvin as function of $c$. The full square corresponds to experimental values taken from ref. [2]. The full circles represent the calculated values, the $\gamma$’s where also taken from the same reference.](image)

Let us proceed further on and compare the calculated magnetization with the measured one. In the experimental data the concentration of $Mn^{2+}$ is $c = 0.053\%$ and the parameter $\gamma$ is estimated to be 0.3 (see ref. [2]). In Fig. 5, we show the magnetization as function of the $T$ where $m^* = 0.5 \ m_e$ and $J = 2.58 \ eV$. We observe that for sufficiently high temperature $T \leq 0.5 \ T_c$ there is a very good agreement with the measured $Mn^{2+}$ magnetization. When decreasing $T$ some deviation appears, this suggests that the VCA treatment is not good enough in this region, which was expected.

To conclude, we have presented a general theory for carrier induced ferromagnetism in DMS. Our approach allows to treat the disorder beyond simple coarse graining within full CPA treatment. It goes beyond mean-field and includes quantum fluctuations in the RPA approximation. We have shown that, within our decoupling scheme and sufficiently close to $T_c$, the CPA for the itinerant gas reduces to VCA, which allowed us to provide analytical results for $T_c$ in the low impurity concentration and hole density regime. We have also discussed its dependence on the hole concentration. We have also shown that the mean field approximation is only valid for very low carrier concentration. Additionally, for illustration of our theory a comparison with available experimental data on $Ga_{1-c}Mn_cAs$ was done. We find a very good agreement with the experimental results assuming a single band for itinerant carriers and a large exchange constant $J = -2.58 \ eV$. Finally, this work provide a good starting point for higher decoupling scheme.

**Note added:** After this work was completed we became aware of Yang et al. comment [17]. By analogy with the Kondo Lattice Model [16] (no disorder) they proposed a similar expression for $T_c$ to the one derived in eq. [17].
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