Constraining TeVeS Gravity as Effective Dark Matter and Dark Energy

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The phenomena customly described with the standard ΛCDM model are broadly reproduced by an extremely simple model in TeVeS, Bekenstein’s (2004) modification of General Relativity motivated by galaxy phenomenology. Our model can account for the acceleration of the universe seen in SNeIa distances without a cosmological constant, and the accelerations seen rotation curves of nearby spiral galaxies and gravitational lensing of high-redshift elliptical galaxies without cold dark matter. The model is consistent with BBN and the neutrino mass between 0.05eV to 2eV. The TeVeS scalar field is shown to play the effective dual roles of Dark Matter and Dark Energy with the amplitudes of the effects controled by a $\mu$-function of the scalar field, called the $\mu$–essence here. We also discuss outliers to the theory’s predictions on multi-imaged galaxy lenses and outliers on sub-galaxy scale.

Keywords: Dark Matter; Cosmology; Gravitation

1. Introduction

As with the start of the last century, a rethinking of fundamental physics is forced upon us by a set of experimental surprises, only difference with this time is that the whole universe is the laboratory. Einstein’s General Relativity together with the ordinary matter described by the standard model of particle physics is well-tested in the solar system, but fails miserably in accounting for astronomical observations from just the edge of solar system to cosmological distance, e.g., fast-rotating galaxies like ours would have been escaped the shallow gravitational potentials of their luminous constituents (stars and gas). Standard physics also cannot fully explain the cosmological observations of the cosmic acceleration seen in Supernovae type Ia data and the angular scales seen in the anisotropy spectrum of the Cosmic Microwave Background Radiation(CMBR) (Spergel et al. 2006). The remedy is usually introducing two exotic components to dominate the matter-energy budget of the
Universe with a split of about 25% : 74% to the universe energy budget: Dark Matter (DM) as a collisionless and pressureless fluid described by perhaps the SUSY physics, and Dark Energy (DE) as a negative pressure and nearly homogeneous field described by unknown physics.

1.1. Challenges to dark matter and dark energy

In spite of the success of this concordance model the nature of dark matter and dark energy is one of the greatest mysteries of modern cosmology. For example, it has long been noted that on galaxy scales dark matter and baryonic matter (stars plus gas) have a remarkable correlation, and respects a mysterious acceleration scale $a_0$ (Milgrom 1983, McGaugh 2005). The Newtonian gravity of the baryons $g_b$ and the dark matter gravity $g_{DM}$ are correlated through an empirical relation (Zhao & Famaey 2006, Angus et al. 2006, Famaey et al. 2006) such that the light-to-dark ratio

$$\frac{g_b}{g_{DM}} = \frac{g_{DM} + \alpha g_b}{a_0}, \quad a_0 = 1\,\text{Asec}^{-2}$$

where $a_0$ is a dividing gravity scale, and $0 \leq \alpha \leq 1$ is a parameter, experimentally determined to fit rotation curves.¹ Such a tight correlation is difficult to understand in a galaxy formation theory where dark matter and baryons interactions enjoy a huge degrees of freedom. Equally peculiar is the amplitude of dark energy density $\Lambda$, which is of order $10^{120}$ times smaller than its natural scale. It is hard to explain from fundamental physics why DE starts to dominate the Universe density only at the present epoch, hence marking the present as the turning point for the universe from de-acceleration to acceleration. This is related to the fact that $\Lambda \sim a_0^2$, where $a_0$ is a characteristic scale of DM. Somehow DE and DM are tuned to shift dominance when the DM energy density falls below $\frac{a_0^2}{8\pi G}$. These empirical facts should not be completely treated as random coincidences of the fundamental parameters of the universe.

Given that the dark sector and its properties are only inferred indirectly from the gravitational acceleration of ordinary matter, one wonders if the dark sector are not just a sign of our lack of understanding of gravitational physics. Here we propose to investigate whether the role of DM and DE could be replaced by the scalar field in a metric theory called TeVeS.

2. TeVeS Framework

TeVeS is a co-variant theory proposed by Bekenstein which in the weak field limit reduces to the phenomenologically successful but non-covariant MOND theory of Bekenstein & Milgrom (1983). The co-variant nature of TeVeS makes it ready to be analyzed in a general setting.

¹Note $\alpha$ mimics the role of the mass-to-light, hence inherits some of its uncertainty.
Just like Einstein’s theory, Bekenstein’s theory is a metric theory, in fact, it has two metrics. The first metric $g_{\mu\nu}$ is minimally coupled to all the matter fields in the Universe. We shall call the frame of this metric the “Matter Frame” (MF). All geodesics are calculated in terms of this MF metric. For example, in a quasi-static system like a galaxy with a weak gravitational field, we can define a physical coordinate system $(t,x,y,z)$ such that

$$d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu = (1 + 2\Phi) dt^2 - (1 - 2\Phi) (dx^2 + dy^2 + dz^2).$$

Here the potential $\Phi = \Phi_b + \phi$ where the field $\phi$ replaces the usual role of the potential of the Dark Matter.

Another metric of TeVeS $\tilde{g}_{\mu\nu}$ has its dynamics governed by the Einstein-Hilbert action

$$S_{\tilde{g}} = \frac{1}{16\pi G} \int d^4x \sqrt{-\tilde{g}} \tilde{R},$$

where $\tilde{R}$ is the scalar curvature of $\tilde{g}_{\mu\nu}$. We shall call the frame of this metric the “Einstein Frame” (EF). It is related to the MF metric through

$$g_{\mu\nu} = e^{-2\phi}(\tilde{g}_{\mu\nu} + A_\mu A_\nu) - e^{2\phi} A_\mu A_\nu$$

(notations of tildes here are opposite of Bekenstein), which involves the unit timelike vector field $A_\mu$ (can often be expressed as $(\sqrt{-g^{00}}, 0, 0, 0)$ for galaxies or FRW cosmology), and a scalar field $\phi$, which is governed by the action $S = \int d^4x \sqrt{-\tilde{g}} L$, where according to Bekenstein (2004) and Skordis et al. (2006), the Lagrangian density

$$L = -\Lambda + \frac{1}{16\pi G} \left[ \mu_{Sk} \frac{dV}{d\mu_{Sk}} - V(\mu_{Sk}) \right]$$

where $\Lambda$ is a constant of integration, equivalent to cosmological constant, and $V$ is a free function of $\mu_{Sk}$, which is an implicit function of the scalar field $\phi$ through

$$(\tilde{g}^{\mu\nu} - A^\mu A^\nu) \phi_{,\mu} \phi_{,\nu} = - \frac{dV}{d\mu_{Sk}}.$$  \number{4}

By picking an expression and parameters for the scalar field Lagrangian density $L$ or potential $V$, one picks out a given TeVeS theory. Note in all above $G \equiv (1 - K/2)G_\odot$ is a to-be-determined bare gravitational constant related to the usual experimentally determined value $G_\odot \approx 6.67 \times 10^{-11}$ through the coupling constant $K$ of TeVeS (Skordis, private communication).

### 3. Connecting galaxies with cosmology

Bekenstein’s original proposal was to construct the Lagrangian density with $L$ as one-to-one function of the $\mu_{Sk}$ (cf. Skordis et al 2006). Such one-to-one construction has the drawback that the Lagrangian necessarily has unphysical “gaps” such that a sector is reserved for space-like systems (e.g. from dwarf galaxies to the solar system in $0 < \mu_{Sk} < \mu_0$) and a disconnected sector is reserved for time-like systems (e.g., expanding universe in $\mu_{Sk} > 2\mu_0$). While viable mathematically, such disconnected universe would not permit galaxies to collapse out of the Hubble expansion. The particular function that Bekenstein used also result in an interpolation function, to
be computed from a non-trivial implicit function of the scalar field strength, which is found to overpredict observed rotation curve amplitudes when the gravity is of order $a_0$ (Famaey & Binney 2005).

In an effort to re-connect galaxies with the expanding universe Zhao & Famaey (2006) proposed to construct the Lagrangian as a one-to-one function of the scalar field $\phi$ through $\Upsilon$, where $\Upsilon > 0$ in galaxies, and $\Upsilon < 0$ for cosmic expansion. This way allows for a smooth transition from the edge of galaxies where $\Upsilon \sim 0$ to the Hubble expansion. Zhao & Famaey also suggested to extrapolate the Lagrangian for galaxies to predict cosmologies to minimize any fine-tuning in TeVeS. The counterpart of the ZF model in dark matter language would be Eq. 1, which they used to fit to rotation curves, and found that both $\alpha = 1$ and $\alpha = 0$ give reasonable fits, with some preference on the former.

Our aim here is to check whether the suggestions of Zhao & Famaey (2006) lead to reasonable galaxy rotation curves and cosmologies. To minimize fine-tuning, we consider an extremely simple Lagrangian density governing the scalar field

$$L(\Upsilon) = \int_0^\Upsilon \frac{d\Upsilon}{8\pi G_0 a_0 \exp(-\phi_0)} \sqrt{|\Upsilon|}, \quad \Lambda = 0. \tag{5}$$

In quasi-static systems $\sqrt{|\Upsilon|} = |\nabla \phi| \exp(-\phi)$, where the constant $\phi_0$ is the present day cosmological value of the scalar field $\phi$. With this, the Poisson’s equation reduces to $-\nabla \left( \frac{|\nabla \exp(\phi_0 - \phi)|}{a_0} \nabla \phi \right) = 4\pi G_0 \rho = -\nabla \cdot g_\phi$. So in spherical approximations we have

$$g_\phi = \frac{|\nabla \phi|}{a(\phi)}, \quad a(\phi) \equiv a_0 \exp(-\phi_0). \tag{6}$$

Clearly the above Lagrangian or TeVeS Poisson’s equation for the scalar field essentially recovers eq. (1) in the $\alpha = 0$ case in the Dark Matter language if we identify that the scalar field $\nabla \phi \rightarrow g_{DM}$, hence playing the role of gravity of the dark matter $g_{DM}$ at present day when $\phi = \phi_0$. Interestingly the characteristic acceleration scale $a(\phi)$ varies with redshift together with the scalar field $\phi(t)$.

| Data                        | Pros (+) | Cons (-) | References                        |
|-----------------------------|----------|----------|-----------------------------------|
| Pioneer Anamoly             | -        | +        | Sanders (2006)                    |
| Rotation Curves HSB/LSB     | -        | ++       | Zhao & Famaey (2006)              |
| Lensing by Ellipticals      | ++       | +/-      | Zhao, Bacon, Taylor, Horne (2006) |
| Dynamics of X-ray Clusters  | ++       | +/-      | Sanders (2003), Pointecouteau & Silk (2006) |
| Hubble expansion and CMB    | ++       | +/-      | Zhao et al. (2006), Skordis et al. (2005) |

A summary of how well TeVeS/MOND or CDM fits data on all scales is given in Table 1. To illustrate, two sample fits to rotation curves of a dwarf galaxy and a high surface brightness spiral galaxy are shown in Fig.1a,b, including the possible
effects of imbedding the galaxies in a large neutrino core. We also repeat the exercise of Zhao, Bacon, Taylor & Horne (2006), and fitting the lens Einstein radii with Hernquist models in a $\alpha = 0$ modified gravity. We show in Fig1c,d that the CASTLES gravitational lenses (mostly high redshift ellipticals) are mostly consistent with TeVeS predicted Einstein ring size within plausible uncertainties of the mass-to-light ratios. Note that the critical gravity ($c^2/D_l)(D_s/D_ls)$ is always much stronger than $10^{-10} \text{m/s}^2$ at Einstein radii of elliptical galaxy lenses, so the Einstein rings are insensitive to MONDian effects, hence insensitive to $a_0$. Some of the outliers are known to in galaxy clusters (RXJ0921 and SDS1004), where a neutrino density core of a few times $10^{-6} \text{M}_\odot \text{kpc}^{-3}$ might help to reduce the discrepancy. Given that the $\alpha = 0$ model is reasonable consistent with spiral galaxy rotation curve data and Einstein rings of high redshift ellipticals, next we wish to study cosmology in this TeVeS model. The important thing to note here is that the cosmological constant $\Lambda$ is set to zero, so the zero point of the Lagrangian coincide with where the scalar field is zero.

4. Hubble expansion and late time acceleration

TeVeS is a metric theory, the uniform expanding background can be described by the FRW metric. Assume a flat cosmology with a physical time $t$ and scale factor $a(t)$, we have

$$ds^2 = -dt^2 + a^2(t) \left[d\chi^2 + \chi^2 (d\theta^2 + \sin^2 \theta d\phi^2)\right].$$

The Hubble expansion can be modelled with

$$\rho_\phi + \rho_b + \rho_r = \frac{3H^2}{8\pi G_\odot \Gamma},$$

where the first term is the scalar field effective energy density $\rho_\phi = \frac{Y}{\frac{d\phi}{dt}} - \mathcal{L} = \frac{8\sqrt{2}}{3} \exp(5\phi) \left(\frac{d\phi}{dt}\right)^3 (8\pi G_\odot a_0 \exp(-\phi_0))^{-1}$ in the matter frame. The correction factor

$$\Gamma = \frac{\exp(-4\phi)}{\left(1 + \frac{d\phi}{da}\ln a\right)^2} \approx \exp(4\phi_{BBN} - 4\phi),$$

such that the expansion rate is very close to that LCDM at the epoch of BBN where the radiation density $\rho_r$ dominates, i.e., no corrections at BBN. Note that TeVeS would mimic Dark Matter and Dark Energy if we identify

$$\rho_\phi \Gamma \rightarrow \rho_\Lambda, \quad (\rho_b + \rho_r)(\Gamma - 1) \rightarrow \rho_{DM},$$

where in the LCDM framework the Hubble expansion is normally modeled with

$$\rho_\Lambda + \rho_{DM} + \rho_b + \rho_r = \frac{3H^2}{8\pi G_\odot}, \quad H = \frac{da}{dt}.$$
Using the mirror-imaged $\alpha = 0$ Lagrangian of the scalar field, we derive the following 2nd order ODE for the scalar field $\phi$.

$$\frac{d}{dt} \left[ \mu_s \left( \frac{d\phi}{dt} \right) \right] = (\rho_b + \rho_r)a^3, \quad dt = \frac{d\ln a}{H}, \quad \mu_s \equiv \frac{\exp(5\phi + \phi_0)a^3}{\sqrt{2\pi G_{\odot}a_0}} \left( \frac{d\phi}{dt} \right)$$

(12)

Note the similarity of this 1D equation with the 3D Poisson’s equation.

We can integrate the above equation to solve for $\phi$ as a function of $\ln a$ or the physical synchronous time $t$. We note at current epoch $\phi \sim \phi_0$, $\mu_s \sim s$, $\rho_b, \rho_r \sim (s^2/8\pi G_{\odot} \rho_b)^{-1/2}$, where $s \sim \sqrt{g_{tt}t\phi t}$.

We then aim to test if the cosmology specified by the above non-fine-tuned Lagrangian in TeVeS could match the behavior of a LCDM universe. We solve the equations numerically by iteration of the bare constant $G$. Assuming a value for $G$, the initial $\phi$, and $\frac{d\phi}{d\ln a}$ are then set by the fact that $G_{\text{eff}} \approx G_{\odot} = 6.67 \times 10^{-11}$ at BBN in order to be consistent with the number of relativistic degrees of freedom at temperature of 1 MeV. The parameters $A$ and $K$ are determined by the boundary condition at present day such that we recover the normalisation in the MONDian Poisson’s equation in galaxies and in solar system. Typically the scalar field tracks the matter density, and $L$ and $\phi$ are slow varying functions of redshift. We then iterate the parameter $G$ such that the sound horizon angular size at LSS ($z = 1000 - 1100$) matches that of LCDM. The parameters typically converge in 20-30 iterations. The Hubble constant and cosmic acceleration come out without any tuning.

In Fig. 2a we show a model with a present day matter density $\omega_b = 0.024$. This is consistent with the baryon density at BBN, hence there is no non-baryonic matter in the present model. This model invokes neither cosmological constant nor dark matter. The resulting model has $H_0 = 77\text{km/s/Mpc}$. The expansion history is almost the same as LCDM; slight difference exists in the energy density (hence the expansion rate) in the future $a > 1$.

To understand whether the above explanation for late acceleration and dark matter is unique we have also run models with a more general Lagrangian.

$$L(\Upsilon) = \int_0^\Upsilon \frac{d\Upsilon}{8\pi G_{\odot}} \frac{s}{1 - \alpha s}, \quad s \equiv \frac{\sqrt{|\Upsilon|}}{a_0 \exp(-\phi_0)}, \quad \Lambda = 0.$$  

(13)

This is so constructed that we recover eq. (1) for any value of $\alpha$ by identifying dark matter gravity with the scalar field $\nabla \phi$. This whole sequence of models are largely consistent with Dark Matter phenomenology on galaxy scales, with a slight preference for $\alpha = 1$ models in galaxies. Models with non-zero $\alpha$ also have interesting effects on the solar system. For example, a model with $\alpha = 0.2$ would predict (cf. eq. 1) a constant, non-Keplerian acceleration of $a_P = a_0 \alpha^{-1} \sim 6 \times 10^{-10}\text{msec}^{-2}$ in the solar system, consistent with the Pioneer Anomaly (although an non-gravitational origin is hard to excluded). Such a constant gravity would cause a gravitational redshift of $10^{-13}$ (D/100AU) between the solar system bodies of separation $D$, which could be testable with experiments with accurate clocks in the future (see
these proceedings). Calculating the Hubble expansion for models with increasing $\alpha$, we are able to match LCDM in all cases in terms of BBN, LSS, SNeIa distances, and late acceleration. For all these models we have also varied initial conditions and found the solutions are stable. The acceleration continues into far future with $b = a \exp(\phi) \rightarrow \text{cst}$. The amplitude of the modification function $\mu$ also decreases with expansion. Compared to model with $\alpha = 0$, however, larger $\alpha$ drives up the present-day Hubble constant, unless the present day matter density $w_b$ is also increased. This is effectively achieved by allowing for relatively massive neutrinos. E.g., for $\alpha = 0.2$ would require the matter density parameter $w_b$ twice the nominal value 0.024, implying the need to include massive neutrinos of 0.8 eV. For $\alpha = 1$ would require 2 eV neutrinos as needed for explaining galaxy cluster data (Sanders 2003, Pointecouteau & Silk) and the CMB (Skordis et al. 2006). The latter model is shown in Fig.2b.

5. Conclusion

In summary, we have focused on one very specific model in the Bekenstein theory. We have shown that it may be possible to satisfy some of the most stringent cosmological observations without the need to introduce/fine-tune dark matter nor dark energy. The TeVeS scalar field $\mu$-function (called $\mu$-essence here) can be fixed by galaxy rotation curves, and it predicts the right amount of cosmic acceleration, the size of the horizon at $z = 1000$, and the present Hubble constant without fine-tuning. The ultimate test of the model should come from simulating the evolution of linear perturbations on this background and CMB. By fitting galaxy cluster data and the 3rd peak of CMB we could break the degeneracy of models of different $\alpha$, and constrain the neutrino mass.

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References

1. Bekenstein J., 2004, Phys. Rev. D., 70, 3509
2. Famaey B., Binney J., 2005, MNRAS, 363, 603
3. Milgrom M., 1983, ApJ, 270, 365
4. Pointecouteau E., Silk J., 2005, MNRAS, 364, 654
5. Sanders R.H., 2003, MNRAS, 342, 901
6. Skordis C., Mota D.F., Ferreira P.G., Boehm C., 2006, Phys. Rev. Lett., 96, 1301
7. Zhao H.S., Bacon D., Taylor A.N., Horne K.D., 2006, MNRAS, 368, 171
8. Zhao H.S., Famaey B., 2006, ApJ, 638, L9
9. Zhao H.S., Qin B., 2006, ChJAA, 6, 141
Fig. 1. Left panel shows TeVeS fits to rotation curves of a gas-rich dwarf galaxy NGC1560 and a gas-poor larger spiral galaxy NGC4157 (solid curves), adopting $a_0 = 1.2 \times 10^{-8}$, $\alpha = 0$ $\mu$-function model without neutrinos; the Newtonian rotation curve by baryons for the assumed stellar $(M/L)_{*}$ are shown as well (dashed lines). Right panel similar to the left, except for assuming the $\alpha = 1$ $\mu$-function and assuming that galaxies are imbeded in a neutrino over-density of $200 \times \frac{3H_0^2}{8\pi G} \sim 2.7 \times 10^4 M_\odot$ kpc$^{-3}$ or $5000 \times \frac{3H_0^2}{8\pi G} \sim 6.7 \times 10^5 M_\odot$ kpc$^{-3}$ (the two values brackets the typical gas density of x-ray clusters on average and in the centres). The Newtonian rotation curves of the constant neutrino cores are also shown (dotted lines).

Fig. 2. Shows the values for the TeVeS $a_0$ parameter derived to fit individual strong lensing Einstein radius of 50 CASTELS multi-imaged systems, assuming $(M/L)_{*} = 4$ (circles); also shown are the effects of raising/lowering $(M/L)_{*}$ by a factor of 2 (solid vertical lines) or a factor of 4 (dotted vertical lines). A few outliers are labeled. The right panel shows the Newtonian acceleration $GM_{*}/R_{E}^2$ vs. the critical gravity (related to the critical surface density $(c^2/4\pi GD_{l})(D_{ls}/D_{l})$ in GR) is the minimal local gravity for a lens to form Einstein rings. The dashed line is a prediction for point lenses in TeVeS $\alpha = 0$ model.
Fig. 3. compares $\Lambda$CDM (dashed) with Zero-$\Lambda$-TeVeS flat cosmologies (solid) (left panel assumes zero mass for neutrinos and a $\mu$-essence with $\alpha = 0$; right panel assumes 2eV neutrinos and $\alpha = 1$ model). Shown are the co-moving distance $D_{\text{com}}$ vs. the physical scale factor $a$ in log-log diagram overplotted with SNIa data (small symbols) up to redshift 2. Likewise shows the horizon, the Hubble parameter $H$ in units of $(\text{Mpc}^{-1} c)$ in two theories. The evolution of the scalar field $\phi$ and $\mu$ can be inferred from (thin solid lines) $a \exp(\phi)$ and $\mu^{-1}$ with the cutoff of $\mu^{-1} = 0.005$ be adopted for numerical reasons.