NLL QED CORRECTIONS TO DEEP INELASTIC SCATTERING *

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The $O(\alpha^2 \log(Q^2/m_e^2))$ leptonic QED corrections to unpolarized deeply inelastic electron-nucleon scattering are calculated in the mixed variables.

1 Introduction

Deep inelastic scattering provides us with detailed information on the nucleon structure. In order to extract the parton distribution functions from DIS cross sections and to measure $\alpha_s(M_Z^2)$ with high precision it is crucial to control the QED radiative corrections. The 1– and 2–loop leading–log QED corrections were derived in Ref. [1–3]. Complete 1-loop corrections for DIS were given in Refs. [4]. Furthermore the universal leading logarithmic corrections were derived to $O((\alpha L)^5)$ both for polarized and unpolarized processes in [5], where also the resummation of the $O((\alpha \ln^2(z))^k)$ for polarized scattering was given.

In this paper, we summarize our recent results of NLO leptonic QED corrections in mixed variables [7].

2 Mixed variables

In general radiative corrections do strongly depend on how the kinematic variables are measured. In this paper, we consider the case of mixed variables, i.e. $y = y_h$ is measured from the hadron side and $Q^2 = Q^2_l$ is measured from the lepton side. Then the rescaled variables for initial and final state radiation are given by

$$\hat{y} = \frac{y_h}{z}, \quad \hat{Q}^2 = z Q^2_l, \quad \hat{S} = z S, \quad \hat{x} = zx_m, \quad J^I(z) = 1, \quad z_0^f = \max \{y_h, Q^2_0/Q^2_l\},$$

(1)
FSR : \( \hat{y} = y_h, \hat{Q}^2 = \frac{Q^2}{z}, \hat{S} = S, \hat{x} = \frac{x_m}{z} , \)

\[ J^F(z) = \frac{1}{z}, \quad z_0^F = x_m. \] (2)

Here \( J^{I,(F)}(z) \) are the Jacobians for initial (final) state radiation and \( z_0 \) denotes the lower bound of the rescaling variable. \( Q^2_0 \) is introduced as a cut on \( Q^2_h \) to keep the process duly deep inelastic, i.e. to avoid significant contributions of the Compton peak. In the subsequent section, we frequently use the following shorthand notation for a function with rescaled variables in its argument:

\[ \tilde{F}_{I,F}(y, Q^2) = F \left( y = \hat{y}_{I,F}, Q^2 = \hat{Q}^2_{I,F} \right), \] (3)

where \( I, F \) indicate ISR and FSR rescaling.

3 NLO corrections

We parameterize the \( k \)-th order differential cross section as

\[ \frac{d^2\sigma^{(k)}}{dy_h dQ^2} = \sum_{l=0}^{k} \left( \frac{\alpha}{2\pi} \right)^l \ln^{k-l} \left( \frac{Q^2}{m_e^2} \right) C^{(k,l)}(y, Q^2), \] (4)

with \( C^{(0,0)}(y, Q^2) \) denoting Born cross section. \( C^{(1,0)}(y, Q^2) \) and \( C^{(2,0)}(y, Q^2) \) were calculated in [3]. The \( O(\alpha) \) non-logarithmic term \( C^{(1,1)}(y, Q^2) \) was derived in Ref. [4]. We re-calculated these corrections [7] and agree with the previous results.

NLO corrections \( C^{(1,1)}(y, Q^2) \) are obtained using RG equations for mass factorization and charge renormalization. This method was first implemented in [6] for initial state corrections to \( e^+e^- \) annihilation, a single differential cross section in the \( s \)-channel. We deal with double–differential distributions for a \( t \)-channel process. At first the scattering cross section is decomposed as follows:

\[ \frac{d^2\sigma}{dy_h dQ^2} = \frac{d^2\sigma^0}{dy_h dQ^2} \otimes \left\{ \Gamma^I_{ee} \otimes \hat{\sigma}_{ee} \otimes \Gamma^F_{ee} + \Gamma^I_{\gamma e} \otimes \hat{\sigma}_{e\gamma} \otimes \Gamma^F_{ee} + \Gamma^I_{ee} \otimes \hat{\sigma}_{ee} \otimes \Gamma^F_{e\gamma} \right\} \] (5)
with $\Gamma_{ij}^{I,F}(z, \mu^2/m_e^2)$ the initial and final state operator matrix elements and $\hat{\sigma}_{kl}(z, Q^2/\mu^2)$ the respective Wilson coefficients which obey the representations

$$\Gamma_{ij}^{I,F}(z, \mu^2/m_e^2) = \delta(1-z) + \sum_{m=1}^{\infty} \frac{\alpha}{2\pi} \sum_{n=0}^{m} \Gamma_{ij}^{I,F(m,n)}(z) \ln^{m-n} \left( \frac{\mu^2}{m_e^2} \right)$$

$$\hat{\sigma}_{kl}(z, Q^2/\mu^2) = \delta(1-z) + \sum_{m=1}^{\infty} \frac{\alpha}{2\pi} \sum_{n=0}^{m} \hat{\sigma}_{kl}^{(m,n)}(z) \ln^{m-n} \left( \frac{Q^2}{\mu^2} \right),$$

where $j(l)$ denotes the incoming and $i(k)$ the outgoing particle. In the cross section (5), the $\mu^2$-dependences cancel each other and the final expression expands in $\alpha/2\pi$ and $\ln(Q^2/m^2)$, to $O(\alpha^2 L)$. There are several contributions to NLO corrections:

i. LO initial and final state radiation off $C_{ee}^{(1,1)}(y, Q^2)$

ii. Coupling constant renormalization of $C_{ee}^{(1,1)}(y, Q^2)$

iii. LO initial state splitting of $P_{\gamma e}$ at $C_{e\gamma}^{(1,1)}(y, Q^2)$

iv. LO final state splitting of $P_{e\gamma}$ at $C_{\gamma e}^{(1,1)}(y, Q^2)$

v. NLO initial and final state radiation off $C_{ee}^{(0,0)}(y, Q^2)$.

The first contribution $C_i^{(2,1)}(y, Q^2)$ is

$$C_i^{(2,1)}(y, Q^2) = \int_0^1 dz P_{ee}^0 \left[ \theta(z - z_0^i) J_I \tilde{C}_I^{(1,1)}(y, Q^2) - C^{(1,1)}(y, Q^2) \right] + \int_0^1 dz P_{ee}^0 \left[ \theta(z - z_0^F) J_F \tilde{C}_F^{(1,1)}(y, Q^2) - C^{(1,1)}(y, Q^2) \right],$$

where $P_{ee}^0(z)$ is the LO splitting function:

$$P_{ee}^0(z) = \frac{1 + z^2}{1 - z}.$$ (9)

The QED coupling is renormalized as

$$\alpha(\mu^2) = \alpha(m_e^2) \left[ 1 - \frac{\beta_0}{4\pi} \ln \left( \frac{\mu^2}{m_e^2} \right) \right],$$

with $\beta_0 = -4/3$ and the second contribution $C_0(y, Q^2)$ is given by

$$C_0^{(2,1)}(y, Q^2) = -\frac{\beta_0}{2} C^{(1,1)}(y, Q^2).$$ (11)
In $C^{(2,1)}_{ii,iv}(y, Q^2)$ there appear new subprocesses:

\[ C^{(2,1)}_{ii}(y, Q^2) = \int_{z_0}^1 dz P^0_{\gamma e}(z) J^I(z) \tilde{C}^{(1,1)}_{\gamma e}(y, Q^2) \]

\[ C^{(2,1)}_{iv}(y, Q^2) = \int_{z_0}^1 dz P^0_{e\gamma}(z) J^F(z) \tilde{C}^{(1,1)}_{\gamma e}(y, Q^2), \]

where $P^0_{\gamma e}$ and $P^0_{e\gamma}$ are LO off-diagonal splitting functions

\[ P^0_{\gamma e} = 1 + (1 - z)^2, \quad P^0_{e\gamma} = z^2 + (1 - z)^2. \]

$C^{(1,1)}_{\gamma e}(y, Q^2)$ and $C^{(1,1)}_{\gamma e}(y, Q^2)$ are defined in the same way as $C^{(1,1)}(y, Q^2)$ and their explicit expressions are given in [7].

The last contribution $C^{(2,1)}_v(y, Q^2)$ is given by

\[ C^{(2,1)}_v(y, Q^2) = \int_0^1 P^{1,NS,OM}_{ee,S}(z) \left[ \theta(z - z'_0) J^I(z) \tilde{C}^{(0,0)}_I(y, Q^2) - C^{(0,0)}(y, Q^2) \right] + \int_0^1 P^{1,PS,OM}_{ee,S}(z) J^I(z) \tilde{C}^{(0,0)}_I(y, Q^2) \]

\[ + \int_0^1 P^{1,NS,OM}_{ee,T}(z) \left[ \theta(z - z'_0) J^F(z) \tilde{C}^{(0,0)}_F(y, Q^2) - C^{(0,0)}(y, Q^2) \right] + \int_0^1 P^{1,PS,OM}_{ee,T}(z) J^F(z) \tilde{C}^{(0,0)}_F(y, Q^2). \]

Here $P^{1,NS,OM}_{ee,S,T}(z)$ and $P^{1,PS,OM}_{ee,S,T}(z)$ denote the space– and time–like NLO QED splitting functions of the non–singlet (NS) and pure–singlet (PS) channels in the on–mass–shell scheme which are obtained from the $\overline{\text{MS}}$-scheme [8] by

\[ P^{1,NS,OM}_{ee,S,T}(z) = P^{1,NS,\overline{\text{MS}}}_{ee,S,T}(z) + \frac{\beta_0}{2} \Gamma^{S,T,(1,1)}_{ee}(z), \]

\[ \Gamma^{S,T,(1,1)}_{ee}(z) = -2 \left[ \frac{1 + z^2}{1 - z} \left( \ln(1 - z) + \frac{1}{2} \right) \right], \]

and $P^{1,PS,OM}_{ee,S,T}(z) = P^{1,PS,\overline{\text{MS}}}_{ee,S,T}(z)$. We would like to remark that both the lepton–hadron interference term and the pure hadronic QED corrections, although apparently not widely known, are small. Already in $O(\alpha)$ their inclusion will only lead to a marginal change of the present result. In the case of the purely hadronic corrections details are explained e.g. in [1a].
4 Conclusions

We calculated the $O(\alpha^2 L)$ leptonic QED corrections to deep inelastic electron-nucleon scattering in the mixed variables. With the help of the RGE decomposition, the corrections are expressed as the convolutions of the splitting functions with the Born or 1-loop cross sections. This method generalizes earlier investigations for ISR in $e^+e^-$ annihilation [6] and includes both space- and time-like splitting functions and Wilson coefficients.

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