Analysis of small signal model of DC/DC converter without source and load effects

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Abstract. The small signal model of DC/DC converter without source and load effects is established and analyzed in detail. Characteristics of input and output impedance of converter in frequency domain in the model are applicable to the stability analysis and design in DC multi-converter power system. Investigation shows that the resonant characteristics of input and output impedance under closed-loop control are closely related to the crossover frequency and phase margin of control loop. The input impedance resonant valley and output impedance resonant peak locate at the same frequency as the loop crossover frequency. Furthermore, when the phase margin decreases, the resonance valley of the input impedance becomes lower and the resonance peak of the output impedance becomes higher. Finally, the accuracy of the model is verified by MATLAB Simulink time domain simulation.

1. Introduction
The DC multi-converter power system shown in Figure 1 is widely applied in communication systems, data centers, shipboards, multi-electric aircraft and space stations due to its flexible configuration and high efficiency. In order to ensure the decoupling between converters and realize stable operation in entire power system, scholars have done a lot of research. Analysis methods based on the forbidden region of minor loop gain which is the ratio of source output impedance and load input impedance in small signal model are proposed as shown in Figure 2. These criteria for stability analysis require the small signal model of converter without source and load effects as shown in Figure 3(a), which means the model established must reflect the characteristics of the converter own ports purely. However, a large number of traditional analysis and design methods applied to single converter are based on the assumption that the converter is supplied by an ideal voltage source and loaded by a DC resistance as shown in Figure 3(b). Actually, the dynamic performance of one converter is affected by the output impedance of previous converter and input impedance of subsequent converter as shown in Figure 3(c). Besides, the model based on above assumption has following limitations when analysing or judging the stability of a multi-converter power system:

- The model in complex frequency domain of converter loaded by a resistive lacks disturbance excitation from load side. As a result, no expression of the output impedance for a converter can be obtained and the stability criterion based on impedance analysis method cannot be applied directly;
- The damping ratio $\xi$ in the open-loop model of converter is affected by the load resistance value, which means the model is incapable of reflecting the oscillation characteristics of the converter independently.
Therefore, the paper proposes to establish the small signal model of converter without source and load effects as shown in Figure 3(a), taking a Weinberg converter commonly used in battery power supply system as an example. The method proposed in paper improves the above shortcomings and inapplicability of traditional modeling methods which can reflect the performance of converter independently. Moreover, parasitic parameters of energy storage elements and DC loss parameters of switching device are taken into consideration to reflect the characteristics of the converter more comprehensively.

The outline of paper is as follows. In Section 2, the small signal model of converter without source and load effects is proposed and the impedance characteristics of converter are analysed under voltage mode close-loop control. The modeling method is validated by the results of real-time simulation in Section 3. Conclusions are given in Section 4.

2. Small signal model of converter without source and load effects

This section first investigates the small signal model of open-loop uncoupled converter in DC multi-converter power system. Next, the impedance dynamic performance of converter under voltage mode control is analyzed.

2.1. Small signal model of open-loop uncoupled converter

By ignoring the difference between devices of the same type, the leakage inductance and magnetizing inductance of coupled inductor as well as the leakage inductance and magnetizing inductance of the transformer, the small-signal model of the converter shown in Figure 4 is obtained as (1) by using state-space averaging method and Jacobian linearization. See [4] for the detailed derivation calculation process.

\[
\begin{align*}
\begin{bmatrix} \hat{i}_{in} & \hat{u}_{o} \end{bmatrix}^T &= \mathbf{G}(s)[\hat{u}_{in} \quad \hat{i}_{L} \quad \hat{d}]^T \\
\mathbf{G}(s) &= \begin{bmatrix} Y_{i_{in}o} & A_{L_{o}o} & G_{dl} \\ A_{u_{in}o} & -Z_{o_{in}o} & G_{du} \end{bmatrix}
\end{align*}
\]  

(1)  

(2)
$G(s)$ is transfer function matrix from input variables $\hat{u}_{in}, \hat{i}_L$ and $\hat{d}$ to output variables $\hat{i}_{in}$ and $\hat{u}_o$, its exact definition is as (2).

![Figure 4. Weinberg converter which is powered by an ideal voltage source $u_{in}$ and loaded by an ideal current source $i_L$. $r_{L1}, r_{L2}$ are the equivalent series resistance of the coupled inductor $L1$ and $L2$. $r_C$ is the equivalent series resistance of the output capacitor $C$. $r_D1$, $r_D2$ and $r_D3$ are the diode DC on-resistance. $r_{ds1}$ and $r_{ds2}$ are the DC on-resistance of the Mosfets $Q1$ and $Q2$.](image)

Table 1. Transfer functions of converter without source and load effects.

| Transfer function | Expression |
|-------------------|------------|
| $Y_{in-o}$ | $\hat{d}_{i_{in}}/\hat{u}_{in}$ when $i_{in}=0, d=0$ : Open loop input admittance |
| $A_{i-o}$ | $\hat{i}_{in}/\hat{i}_L$ when $i_{in}=0, d=0$ : Open loop current reverse gain |
| $G_{d}$ | $\hat{i}_{in}/\hat{d}$ when $i_{in}=0, d=0$ : Duty cycle to input current transfer function |
| $A_{u-o}$ | $\hat{u}_{in}/\hat{u}_o$ when $i_{in}=0, d=0$ : Audio attenuation rate |
| $Z_{o-o}$ | $-\hat{u}_{in}/\hat{i}_L$ when $i_{in}=0, d=0$ : Open loop output impedance |

$Den^{-1}$: Second-order oscillation element

$Q = \frac{2(L/C)^{1/2}}{r_C + r_D - D r_{ds} + 2(1 + D) r}$, $D$ is duty cycle steady state value

$Y_{in-o} = Den^{-1}K_{Y_{in}}s$ where $K_{Y_{in}} = C(1 + D)^2$

$A_{i-o} = Den^{-1}(1 + D) \omega_C$ where $\omega_C = (Cr_C)^{-1}$

$G_{d} = I + Den^{-1}(1 + D)CsK_{du}$ where $K_{du} = U_{in} + I(r_{ds} - 2r_d)$, $I$ is input current steady state value

$A_{u-o} = Den^{-1}(1 + D) \omega_C$

$Z_{o-o} = Den^{-1}K_{Z_{o-o}}(1 + \frac{s}{\omega_L})(1 + \frac{s}{\omega_C})$ where

$K_{Z_{o-o}} = r_D + 2(1 + D)r_L - D r_{ds}$,

$\omega_L = [r_D + 2(1 + D)r_L - D r_{ds}] / (4L)$
\[ G_{du} = \frac{u_{in}}{d} \quad : \text{Duty cycle to output voltage transfer function} \]

\[ G_{du} = D e n^{-1} K_{d} (1 + s \omega_{C}^{-1}) \]

Table 1 gives the transfer functions of converter without source and load effects. It can be seen that different from the traditional modeling method, the small signal model obtained under proposed method which reflects the characteristics of the converter more accurately and independently is completely determined by the converter own parameters, eliminating the interference of source and load impedance on the damping ratio and quality factor of the converter.

### 2.2. Close loop performance of uncouple converter impedance

When the converter is under voltage mode control as shown in Figure 5, according to Mason gain formula, voltage loop transfer function \( T_v \) is as (3), close loop input impedance is as (4) and close loop output impedance is as (5). Type III compensation network as (6) is selected as \( G_c \).

\[
T_v(s) = H(s)G_c(s)G_m(s)G_{du}(s) \quad (3)
\]

\[
Z_{m-c}^{-1} = \left[ Y_{m-o}(s) - \frac{G_{du}(s)A_{o-o}(s)}{G_{du}(s)} \frac{T(s)}{1+T(s)} \right]^{-1} \quad (4)
\]

\[
Z_{o-c} = \frac{Z_{o-o}}{1+T_v} \quad (5)
\]

\[
G_c(s) = \frac{K(1+s/\omega_c)(1+s/\omega_{z1})}{s(1+s/\omega_{p1})(1+s/\omega_{p2})} \quad (6)
\]

Table 2 gives the parameters of power stage and control loop. By selecting different parameters in \( G_c \), frequency domain characteristics of closed-loop input impedance \( Z_{m-c} \) is shown as Figure 6 and Figure 7. In Figure 6, the loop phase margin PM is 30° and \( f_{cr} \), the crossover frequency of the loop gain \( T_v \) ranges from 1.21kHz to 7.95kHz. It can be seen that the resonance valley value of \( Z_{m-c} \) locates at \( f_{cr} \), which means the larger the bandwidth of the converter is, the higher frequency closed-loop input impedance \( Z_{m-c} \) resonant valley value locates. This conclusion is meaningful for the impedance decoupling between the previous and subsequent converters in a multi-converter power system. In Figure 7, \( f_{cr} \) is 3kHz and the loop phase margin PM ranges approximately from 20° to 65°. It can be seen that the resonant valley value of \( Z_{m-c} \) becomes higher when PM becomes larger, which means for
a single converter with no source and load effects, the more stable the control loop is, the higher the $Z_{\text{in}_c}$ resonant valley value is. Positive correlation exists between the stability of a single converter and a multi-converter power system.

By selecting different parameters in $G_c$, frequency domain characteristics of closed-loop output impedance $Z_{o_c}$ is shown as Figure 8 and Figure 9. In Figure 8, the loop phase margin PM is $30^\circ$ and $f_{cr}$.

Table 2. Parameters of power stage and control loop.

| Description                 | Symbol | Value       |
|-----------------------------|--------|-------------|
| Switch frequency            | $f_s$  | 25kHz       |
| Input voltage               | $U_{in}$ | 68V         |
| Output voltage              | $U_o$  | 100V        |
| Load current                | $I_L$  | 5A          |
| Inductor $L_1, L_2$         |        | 30μH        |
| Output capacitor $C$        | $C$    | 450.66μF    |
| ESR of inductor $r_{L1}, r_{L2}$ | 8mΩ   |
| ESR of output cap $r_C$     |        | 87.2mΩ      |
| Diode DC on-resistance      | $r_{D1}, r_{D2}, r_{D3}$ | 20 mΩ |
| DC on-resistance of the Mosfet | $r_{ds1}, r_{ds2}$ | 9 mΩ |
| PWM modulator gain          | $G_m$  | 1           |
| Sample gain of output voltage | $H$   | 1           |

Figure 6. Behavior of $Z_{\text{in}_c}$ when PM is $30^\circ$ and $f_{cr}$ increases from 1.21kHz to 7.95kHz.

Figure 7. Behavior of $Z_{o_c}$ when $f_{cr}$ is 3kHz and PM increases from $19.1^\circ$ to $64.1^\circ$. 
the crossover frequency of the loop gain $T_v$ ranges from 1.21kHz to 7.95kHz. It can be seen that the resonance peak value of $Z_{o_c}$ locates at $f_{cr}$, which means the larger bandwidth of the converter is, the higher frequency closed-loop output impedance $Z_{o_c}$ resonant peak value locates. In Figure 9, the crossover frequency $f_{cr}$ is 3kHz and the loop phase margin PM ranges approximately from 20° to 65°. It can be seen that the resonant peak value of $Z_{o_c}$ decreases with the increase of phase margin PM, which means the more stable the control loop is, the lower the $Z_{o_c}$ resonant peak value is.

3. Simulation
To demonstrate the validation of proposed modeling method, a Weinberg converter under voltage mode control is simulated using MATLAB Simulink. Besides parameters given in Table 2, parameters in $G_c$ are as follows: $K=250$, $\omega_{z1}=\omega_{z2}=4044$ rad/s, $\omega_{p1}=22513$ rad/s, $\omega_{p2}=37699$ rad/s. By small signal sweeping, the analytical predictions and small signal simulaton results of close loop output impedance $Z_{o_c}$ and loop gain $T_v$ are compared as in Figure 10 and Figure 11. It can be seen that the theoretical prediction results are in well agreement with the simulated frequency sweeping results. It should be pointed out that the difference in the phase between the analytical prediction and simulated result is caused by the 180° phase shift, which has no effect on the results comparsion.
4. Conclusion
The small signal model of DC/DC converter without source and load effects is established and analyzed. Studies indicate that the resonant characteristics of the converter input and output impedance under closed-loop control are closely related to the crossover frequency and phase margin of control loop. The input impedance resonant valley and output impedance resonant peak locate at the same frequency as the loop crossover frequency. Furthermore, when the phase margin decreases, the resonance valley of the input impedance becomes lower and the resonance peak of the output impedance becomes higher. The simulation results verify the correctness of the model.

References
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