Structural Studies of Many-Body Systems and \((e, e'p)\) Reaction Cross Sections

A.N. Antonov\(^1\), M.K. Gaidarov\(^1\), M.V. Ivanov\(^1\), K.A. Pavlova\(^1\)

\(^1\) Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia 1784, Bulgaria
\(^2\) Departamento de Física Atomica, Molecular y Nuclear, Facultad de Ciencias Físicas, Universidad Complutense de Madrid, Madrid E-28040, Spain

C. Giusti

Dipartimento di Fisica Nucleare e Teorica, Università di Pavia, Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, Pavia, Italy

Abstract

Studies of one-body density matrices (ODM) are performed in various correlation methods, such as the Jastrow method, the correlated basis function method, the Green’s function method and the generator coordinate method aiming to extract the absolute spectroscopic factors and overlap functions (OF) for one-nucleon removal reactions from the ODM of the target nucleus. The advantage of this method is that it avoids the complicated task of calculating the total nuclear spectral function. The procedure for extracting bound-state OF’s has been applied to make calculations of the cross sections of the \((e, e'p)\) reaction on the closed-shell nuclei \(^{16}\text{O}\) and \(^{40}\text{Ca}\) as well as on the open-shell nucleus \(^{32}\text{S}\) consistently (using the same OF’s) with the cross sections of \((p, d)\) and \((\gamma, p)\) reactions on the same nuclei. The analyses of the reaction cross sections and the spectroscopic factors and the comparison with the experimental data show the particular importance of these OF’s, since they contain effects of nucleon correlations (short-range and/or long-range) which are accounted for to different extent in the theoretical methods considered.

1 Introduction

The strong short-range and tensor components of the nucleon-nucleon (NN) interactions induce correlations in the nuclear wave function which are going beyond the independent-particle approximation. Therefore, it has always been a point of experimental and theoretical interest to find observables which reflect these correlations in an unambiguous way. In this sense both, the overlap
functions and single-nucleon spectroscopic factors, have attracted much attention in analyzing the empirical data from one-nucleon removal reactions, such as \((e, e'p)\), \((p, d)\), \((d,^3He)\), and also in other domains of many-body physics, as e.g. atomic and molecular physics\(^1\).

Recently, a general procedure has been adopted\(^2\) to extract the bound-state overlap functions and the associated spectroscopic factors and separation energies on the base of the ground-state (g.s.) one-body density matrix. This makes it possible to investigate the effects of the various types of NN correlations included in the ODM on these structural quantities. Of course, the general success of the above procedure depends strongly on the availability of realistic ODM’s.

Initially, the method for extracting bound-state overlap functions has been applied\(^4\) to a model ODM\(^5\) accounting for the short-range nucleon correlations within the Jastrow correlation method. The resulting OF’s have been used\(^6\) to study one-nucleon removal processes in contrast to the mean-field approaches which account for the nucleon correlations by modifying the mean-field potentials. The results obtained for the differential cross sections of \(^{16}O(p, d)\) and \(^{40}Ca(p, d)\) pick-up reactions at various incident energies demonstrated that the OF’s can be applied as realistic form factors to evaluate absolute cross sections of such reactions. The analysis of single-particle (s.p.) OF’s has been extended to more realistic ODM’s emerging from the correlated basis function (CBF) method\(^7,8\), the Green function method (GFM)\(^10\) and the generator coordinate method (GCM)\(^1,11\). In addition, ODM’s of open-shell nuclei deduced from Jastrow-type calculations have been used\(^12\). We have chosen the CBF theory since it is particularly suitable for the study of the short-range correlations (SRC) in nuclei. The CBF calculations have recently been extended to medium-heavy doubly-closed shell nuclei\(^7,8\) using various levels of the Fermi hypernetted chain approximation\(^7\). The GFM\(^10,13\) provides detailed information on the spectral functions and nucleon momentum distributions predicting the largest effects of the short-range and tensor correlations at high momentum and energy. The results on the one- and two-body density and momentum distributions, occupation probabilities and natural orbitals obtained within the GCM using various construction potentials\(^14\) have shown that the NN correlations accounted for in this method are different from the short-range ones and are rather related to the collective motion of the nucleons.

The main aim of the present work is to study the effects of the NN correlations included in the methods mentioned above on the behavior of the bound-state proton and neutron overlap functions in closed- as well as open-shell nuclei and of the related one-nucleon removal reaction cross sections. Such an investigation allows to examine the relationship between the ODM
and the associated overlap functions within the correlation methods used and also to clarify the importance of the effects of NN correlations on the overlap functions and the reaction cross sections.

2 Overlap functions and their relationship with the one-body density matrix

For a correct calculation of the cross section of nuclear reactions with one-neutron or one-proton removal from the target nucleus, the corresponding OF’s for the neutron and proton bound states must be used in the reaction amplitudes. Here we would like to remind that the single-particle OF’s are defined by the overlap integrals between the eigenstates of the $A$-particle and the $(A-1)$-particle systems:

$$\phi_\alpha(r) = \langle \Psi_{\alpha}^{(A-1)} | a(r) | \Psi^{(A)} \rangle,$$

(1)

where $a(r)$ is the annihilation operator for a nucleon with spatial coordinate $r$ (spin and isospin operators are implied). In the mean-field approximation $\Psi^{(A)}$ and $\Psi_{\alpha}^{(A-1)}$ are single Slater determinants, and the overlap functions are identical with the mean-field s.p. wave functions, while in the presence of correlations both $\Psi^{(A)}$ and $\Psi_{\alpha}^{(A-1)}$ are complicated superpositions of Slater determinants. In general, the overlap functions (1) are not orthogonal. Their norm defines the spectroscopic factor

$$S_\alpha = \langle \phi_\alpha | \phi_\alpha \rangle.$$

(2)

The normalized to unity OF associated with the state $\alpha$ then reads

$$\tilde{\phi}_\alpha(r) = S_\alpha^{-1/2} \phi_\alpha(r).$$

(3)

The ODM can be expressed in terms of the OF’s in the form:

$$\rho(r,r') = \sum_\alpha \phi_\alpha^*(r) \phi_\alpha(r') = \sum_\alpha S_\alpha \tilde{\phi}_\alpha^*(r) \tilde{\phi}_\alpha(r').$$

(4)

The asymptotic behavior of the radial part of the neutron OF for the bound states of the $(A-1)$-system is given by

$$\phi_{nlj}(r) \to C_{nlj} \exp(-k_{nlj} r) / r,$$

(5)

where $k_{nlj}$ is related to the neutron separation energy

$$k_{nlj} = \sqrt{\frac{2m c_{nlj}}{\hbar}}, \quad \epsilon_{nlj} = E_{nlj}^{(A-1)} - E_0^A.$$

(6)
For proton bound states, due to an additional long-range part of the interaction originating from the Coulomb one, the asymptotic behavior of the radial part of the corresponding proton OF’s reads

\[
\phi_{nlj}(r) \rightarrow C_{nlj} \exp[-k_{nlj}r - \eta \ln(2k_{nlj}r)]/r,
\]

where \(\eta\) is the Coulomb (or Sommerfeld) parameter and \(k_{nlj}\) in (6) contains in this case the mass of the proton and the proton separation energy.

Taking into account Eqs. (4) and (5), the lowest \((n = n_0)\) neutron bound-state \(lj\)-overlap function is determined by the asymptotic behavior of the associated partial contribution of the radial ODM \(\rho_{lj}(r, r') (r' = a \rightarrow \infty)\) as

\[
\phi_{n_0lj}(r) = \frac{\rho_{lj}(r, a)}{C_{n_0lj} \exp(-k_{n_0lj}a)/a}.
\]

where the constants \(C_{n_0lj}\) and \(k_{n_0lj}\) are completely determined by \(\rho_{lj}(a, a)\). In this way the separation energy \(\epsilon_{n_0lj}\) and the spectroscopic factor \(S_{n_0lj}\) can be determined as well. Similar expression for the lowest proton bound-state OF can be obtained having in mind its proper asymptotic behavior (7).

3 Results for the cross sections of \((e, e'p)\) reactions on \(^{16}\text{O},^{40}\text{Ca}\) and \(^{32}\text{S}\) nuclei

The inclusion of short-range as well as tensor correlations leads to an enhancement of the values of the overlap functions in the interior region and a depletion in the tail region in the coordinate space\(^{15}\). In the momentum space this leads to a slight redistribution of the strength from the low- to the high-momentum region. The calculated spectroscopic factors (SF), however, differ significantly from the mean-field value. The depletion in the tail region of the OF’s which is due to the NN correlations leads to lower values of the SF’s in comparison with their mean-field values. The SF’s deduced from the calculations with different ODM are listed in Table 1. Only short-range central correlations are included in the ODM in \(^4\text{H},^7\text{He},^1\text{H}\), whereas also tensor correlations are taken into account in \(^1\text{H},^7\text{He}\). It was found that correlation effects on the spectroscopic factor of the hole states are dominated by the tensor channel of the interaction\(^{15}\).

Table 1: Spectroscopic factors for the \(p_{1/2}\) and \(p_{3/2}\) quasihole states in \(^{16}\text{O}\): column I--deduced from the calculations with different ODM of \(^{16}\text{O}\); column II--additional reduction factors determined through a comparison between the \((e, e'p)\) data of \(^{17}\) and the reduced cross sections calculated in DWIA with the different overlap functions; column III--total spectroscopic factors obtained from the product of the factors in I and II.
The reduced cross sections of \((e, e'p)\) knockout reactions have been calculated with the code DWEEPY\(^{18}\), which is based on the nonrelativistic distorted wave impulse approximation (DWIA) description of the nucleon knockout process and includes final-state interactions and Coulomb distortion of the electron waves\(^{19}\). The latter has been treated with a high-energy expansion in inverse powers of the electron energy\(^{18}\). In the standard DWIA approach, however, phenomenological s.p. wave functions were used, with some parameters fitted to the data. In this paper the results have been obtained with theoretically calculated overlap functions which do not include free parameters.

The reduced cross sections \(\rho(p_m)\) for the \(^{16}\)O\((e, e'p)\) reaction as a function of the missing momentum \(p_m\) and for the transitions to the \(1/2^-\) ground state is displayed in Figure 1. As can be seen the cross sections are sensitive to the shape of the various overlap functions used. The differences are considerable at large values of \(p_m\), where the cross section is several orders of magnitude lower than in the maximum region. The deviations of the various results at large values of \(p_m\) are related to different accounting for the short-range NN correlations within the correlation methods used. SRC are particularly important in one-nucleon emission at large missing momenta and energy\(^{20,21}\). At high missing energies, however, other competing processes are also present and a clear identification of SRC can better be made by means of two-nucleon knock-out reactions. At low missing-energy values measurements over an extended range of missing momenta, in particular at large values, where the SRC effects seem to be more sizable, can test the various s.p. overlap functions and NN correlations.

In order to reproduce the size of the experimental cross section a reduction factor has been applied to the theoretical results in Fig. 1. These factors, which have been obtained by a fit of the calculated reduced cross sections to the data over the whole missing-momentum range considered in the experiment, are also listed in Table I (column II). In general, a fair agreement with the
shape of the experimental distribution is achieved. The results, however, are also sensitive to details of the various overlap functions. The best agreement with the data, for both transitions, is obtained with the overlap functions emerging from the ODM calculated within the GFM. This is due to the substantial realistic inclusion of short-range as well as tensor correlations in the ODM. The calculations based on the Green function theory have shown that about 10% of the 1p strength is removed by these correlations. An excellent agreement with the data is obtained also by using the overlap function from the GCM, which does not include tensor correlations. The reduced cross section calculated on the base of the OF from is in accordance with the data for the 1/2− state. The shape of the experimental reduced cross sections can adequately be described also by the HF wave functions, in particular for \( p_m \leq 150 \text{ MeV/c} \).

The fact that in general our results overestimate the data (and a reduction factor has to be applied to the calculated cross sections to reproduce them correctly) may be explained having in mind that our overlap functions are deduced from calculations including only SRC but not long-range correlations (LRC). The reduction factor can thus be considered as a further spectroscopic factor reflecting the depletion of the quasihole state produced by LRC. Of course, the discrepancy with the data can be due also to other effects not included or not adequately described by the theoretical treatment. We note, however, that the reduction factors applied here to the calculated cross sections are not the result of a precise theoretical calculation. They have been obtained by a fit to the data and have only an indicative meaning. Small variations within 10-15% around their values would not significantly change the comparison with data. In any case the reduction factors should mostly be ascribed to LRC, but for the HF wave function, which does not contain any kind of correlations. For this wave function the reduction factor accounts for both LRC and SRC. It is interesting to note that in the calculations with the correlated overlap functions the reduction factors for the 1p1/2 state turn out to be close to the spectroscopic factor (0.83) obtained in the theoretical approach of where only LRC are included.

In Table I we give, in addition, in column III the factor obtained by the product of the two factors in columns I and II. This factor can be considered as a total spectroscopic factor and can be attributed to the combined effect of SRC and LRC. Indeed for 1p1/2 these factors are in reasonable agreement with the spectroscopic factor value (0.76) calculated in where both SRC and LRC are included.

A consistent analysis of the cross sections of \((e,e'p)\) and \((\gamma,p)\) on \(^{40}\text{Ca}\) by means of the same overlap functions is given in Figures 2 and 3, respectively.
Figure 1: Reduced cross section of the $^{16}\text{O}(e, e'p)$ reaction as a function of the missing momentum $p_m$ for the transition to the $1/2^-$ ground state of $^{15}\text{N}$ in parallel kinematics, with $E_0 = 520.6$ MeV and an outgoing proton energy of 90 MeV. The optical potential is from $^{24}$ (see Table III of $^{17}$). Overlap functions are derived from the ODM of GFM $^{10}$ (solid line), CBF $^{9}$ (long-dashed line), CBF $^{7}$ (dot-dashed line), JCM $^{4}$ (double dot-dashed line) and GCM $^{16}$ (short-dashed line). The dotted line is calculated with the HF wave function. The experimental data are taken from $^{17}$. The theoretical results have been multiplied by the reduction factor given in column II of Table I.

The calculated cross sections (presented by solid lines in Fig. 2 and Fig. 3) are generally in good agreement with the experimental data $^{25,26}$. The important role of the additional reduction spectroscopic factors applied to the present calculations, namely 0.55 for the $3/2^+$ ground state and 0.50 for the $1/2^+$ excited state, is pointed out. Apart from the short-range and tensor correlations studied in previous papers $^{15,27}$, in this work we looked into the role of correlations caused by the collective nucleon motion. The results indicate that the effects of NN correlations taken into account within our approach and which are of long-range type are of significant importance for the correct analysis of the processes considered.

An example of electron induced proton knockout from $^{32}\text{S}$ for the transition to the ground $2s_{1/2}$ state of $^{31}\text{P}$ is illustrated in Figure 4. In the figure the result obtained with the proton OF for the $2s$ state of $^{32}\text{S}$ and the optical potential from $^{24}$ is compared with the NIKHEF data from $^{29}$. A reasonable agreement with the experimental data for the reduced cross section is obtained. We emphasize that in the present work the OF theoretically calculated on the basis of the Jastrow-type ODM of $^{32}\text{S}$ does not contain free parameters. It
can be seen from Fig. 4 that our spectroscopic factor of 0.5648 gives a good agreement with the size of the experimental cross section and, in addition, it is in accordance with the integrated strength for the valence $2s_{1/2}$ shell in $^{32}$S which amounts to $65(7)$% of the SP strength obtained using the shell-model bound-state function. The result for the $^{32}$S($e,e'p$) cross section obtained with the harmonic-oscillator bound-state wave function is also illustrated in Fig. 4. It has been computed with the same oscillator parameter value $b=2$ fm for the $2s_{1/2}$ ground-state wave function as in the original calculations of the ODM without SRC. In order to perform a consistent comparison with the result when considering theoretically calculated overlap function, we have applied the same spectroscopic factor of 0.5648. In this case, the size of the reduced cross section is also reproduced, but the HO wave function gives much worse description of the experimental data. Thus, the comparison made in Fig. 4 shows the important role of the SRC accounted for in our approach for the correct description of knockout reactions.

4 Conclusions

The s.p. overlap functions calculated on the basis of ODM for the ground state of closed- and open-shell nuclei emerging from different correlation methods have been used to calculate the cross sections of the ($e,e'p$), ($\gamma,p$) and ($p,d$) reactions. The theoretical results for the cross sections show that they are sensitive to the shape of the different OF’s and are generally able to reproduce the shape of the experimental cross sections. In order to describe correctly the experimental data on the reduced cross sections a reduction factor must
be applied to the calculated cross sections. The fact that it is consistent in
different nucleon removal reactions gives a more profound theoretical meaning
to this parameter. The results indicate that the effects of SRC correlations
taken into account within CBF and GFM and of correlations accounted for in
GCM which are of long-range type are of significant importance for the correct
analysis of the processes considered.

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Figure 4: Reduced cross section of the $^{32}\text{S}(e,e'p)^{31}\text{P}$ reaction as a function of the missing momentum $p_m$ for the transition to the $1/2^+$ ground state of $^{31}\text{P}$. The proton overlap function is derived from the ODM (solid line). The result with the uncorrelated (HO) wave function is given by dotted line. The experimental data (full circles) are taken from $^{29}$. 

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