Reionization inference from the CMB optical depth and E-mode polarization power spectra

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ABSTRACT

The Epoch of Reionization (EoR) depends on the complex astrophysics governing the birth and evolution of the first galaxies and structures in the intergalactic medium. EoR models rely on cosmic microwave background (CMB) observations, and in particular the large-scale E-mode polarization power spectra (EE PS), to help constrain their highly uncertain parameters. However, rather than directly forward-modelling the EE PS, most EoR models are constrained using a summary statistic – the Thompson scattering optical depth, $\tau_e$. Compressing CMB observations to $\tau_e$ requires adopting a basis set for the EoR history. The common choice is the unphysical, redshift-symmetric hyperbolic tangent ($\text{Tanh}$) function, which differs in shape from physical EoR models based on hierarchical structure formation. Combining public EoR and CMB codes, 21cmFAST and CLASS, here we quantify how inference using the $\tau_e$ summary statistic impacts the resulting constraints on galaxy properties and EoR histories. Using the last Planck 2018 data release, we show that the marginalized constraints on the EoR history are more sensitive to the choice of the basis set ($\text{Tanh}$ vs physical model) than to the CMB likelihood statistic ($\tau_e$ vs PS). For example, EoR histories implied by the growth of structure show a small tail of partial reionization extending to higher redshifts. However, biases in inference using $\tau_e$ are negligible for the Planck 2018 data. Using EoR constraints from high-redshift observations including the quasar dark fraction, galaxy UV luminosity functions and CMB EE PS, our physical model recovers $\tau_e = 0.0569^{+0.0081}_{-0.0066}$.

Key words: cosmology: theory dark ages, reionization, first stars early Universe cosmic background radiation galaxies: high-redshift intergalactic medium

1 INTRODUCTION

The epoch of reionization (EoR) leaves footprints in the observed cosmic microwave background (CMB) as the photons Thomson scatter off free electrons. These include damping the primary temperature anisotropies, inducing secondary anisotropies from the bulk motion of ionized gas (i.e. the kinetic SunyaevZeldovich effect), and prompting curl-less (i.e. E-mode) polarization from the CMB quadrupole (e.g. Sunyaev & Zeldovich 1980, Vishniac 1987, Hu 2000, Hu & Dodelson 2002, McQuinn et al. 2005, Dvorkin et al. 2009). Of these, the large-scale E-mode polarization anisotropies are a particularly powerful probe of the EoR as they are less plagued by degeneracies and systematics (e.g. Reichardt 2016). Reionization models can therefore constrain their largely uncertain parameters that describe the ionizing emissivity of the early Universe, through forward-modelling the EE autocorrelation power spectra (PS) and comparing against measurements from the Planck satellite (Planck Collaboration et al. 2016, 2019, e.g. Hu & Holder 2003, Mortonson & Hu 2008, Miranda et al. 2017, Hazra et al. 2020).

Nevertheless, most reionization models do not directly use the CMB PS to constrain their parameters. Instead, they use a summary statistic which has become one of the de-facto standard cosmological parameters – the direction-averaged Thompson scattering optical depth, $\tau_e$. In going
from the observed PS to $\tau_c$, one needs to adopt a basis set for the EoR history – a parametrization of the redshift evolution of the comoving number density of free electrons ($n_e$). Early works adopted a simple step function reionization at a given redshift $z_{re}$ (Page et al. 2007; Spergel et al. 2007). Currently, the most common choice has a hyperbolic tangent functional form ($\text{Tanh}$) parametrized by a reionization midpoint ($z_{re}$) and a redshift duration ($\Delta z_{re}$; see equation 1). For example, the latest constraints on $\tau_c$ published by Planck, $\tau_c = 0.0522 \pm 0.0080$ (the $TT+lowE$ reconstruction in Planck Collaboration et al. 2018), were generated by fixing a width of $\Delta z_{re} = 0.5$, sampling a flat prior over $z_{re}$, and comparing against the observed large-scale E-mode PS (and also the temperature autocorrelation).

However, the redshift-symmetric evolution given by $\text{Tanh}$ differs in shape from both physical and empirical models of EoR history, based on the growth of cosmic structure and/or fit to observed galaxy luminosity functions (LFs; e.g. Choudhury & Ferrara 2006; Raskutti et al. 2012; Koh & Wise 2018; Greig & Mesinger 2017; Qiu et al. 2017; Kulka rni et al. 2019; Gorce et al. 2018; Roy et al. 2018; Qin et al. 2020). Therefore, computing a likelihood using the $\tau_c$ summary statistic instead of directly forward-modelling the CMB PS can bias EoR model constraints (e.g. Douspis et al. 2015; Planck Collaboration et al. 2016a; Miranda et al. 2017; Hazra et al. 2020). For example, [Miranda et al. 2017] and Heinrich & Hu (2018) claimed (though see Millea & Bouchet 2018 and Planck Collaboration et al. 2018) that the Planck 2015 E-mode PS prefers reionization histories with an extended tail of partial reionization towards very high redshifts ($z>15$). This would have significant implications for our understanding of the very first galaxies. However, quantifying such claims is difficult without directly performing inference on galaxy model parameters.

In this work, we use a physically-motivated EoR model to quantify how inference using the $\tau_c$ summary statistic (instead of directly forward-modelling the E-mode PS) impacts the resulting constraints on galaxy properties and EoR histories. Using the last Planck data release, we show that the marginalized constraints on the EoR history are far more sensitive to the choice of the basis set ($\text{Tanh}$ vs physical model) than to the CMB likelihood statistic ($\tau_c$ vs PS). Specifically, we use the latest v3.0.0 release (Murray et al. in prep.) of 21cmFAST (Mesinger & Furlanetto 2007; Mesinger et al. 2011), whose parametrization for galaxy properties is informed by high-redshift UV LFs (Park et al. 2019). To calculate the CMB PS for a given reionization history and compute the corresponding likelihood, we add the Cosmic Linear Anisotropy Solving System (CLASS; Lesgourgues 2011) Boltzmann solver and the Planck likelihood codes (PLC; Planck Collaboration et al. 2016a) to the upcoming v1.0.0 release of the public 21CMMC Markov chain Monte Carlo (MCMC) framework (Greig & Mesinger 2015, 2017). All codes developed here are publicly available.

This paper is organized as follows. We present our analysis of the Planck data and discuss the difference between the 2015 and 2018 results in Section 2. In Section 3 we briefly introduce our EoR model (Sec. 3.1) and show the resulting constraints inferred from CMB and other observations. We quantify the bias from the choice of basis set for EoR histories in Sec. 3.2 and the choice of likelihood statistic in Sec. 3.3. We summarize our results and conclusions in Section 6.
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Planck 2015 (Planck Collaboration et al. 2016) PS observations (see also Hazra et al. 2020). The top panel of Fig. 1 shows their best-fit models, where

(i) Pop-II only considers UV ionizing photons from Pop-II star-dominated galaxies;
(ii) Pop-III considers UV ionizing photons from both Pop-II and Pop-III star-dominated galaxies; and
(iii) Pop-III, self-regulated is similar to the previous model but assumes a significant contribution from Pop-III stars in the early universe (z ~ 20) before their formation becomes completely quenched when ne/nH reaches 0.2.

Also shown is a Tanh model that assumes a fairly sharp transition at z_e ~ 10 (see equation 1). Comparing the corresponding large-scale E-mode polarization PS of these models against the Planck 2015 measurement (Planck Collaboration et al. 2016a), Miranda et al. (2017) concluded Planck 2015 might favour a significant UV ionizing photon contribution from Pop-III star-dominated galaxies in the early Universe and the resulting optical depth is much higher than what the default Tanh parametrization suggests. This example illustrates how a reionization model informed only by the CMB optical depth could result in biased constraints, compared to using the PS directly in the inference.

2.1.1 From Planck 2015 to 2018

From their 2015 to 2018 data release, the Planck collaboration has made tremendous efforts in improving the characterization and removal of systematic uncertainties affecting the polarization data of the Planck High Frequency Instrument (HFI) on large angular scales (Planck Collaboration et al. 2016b). With such improvements, the Planck collaboration has shown that the mean value and uncertainties of the optical depth deduced from the low-ℓ data assuming the Tanh EoR model have significantly decreased from τ = 0.078 ± 0.010 (the TT+lowP reconstruction in Planck Collaboration et al. 2016a) to τ = 0.052±0.0080 (TT+lowE in Planck Collaboration et al. 2018).

Before discussing the difference between EoR inference from the CMB optical depth and PS, we revisit the 4 best-fit models from Miranda et al. (2017) mentioned above, using the updated measurement from Planck 2018 (Planck Collaboration et al. 2019) to see if the latest data still supports an earlier reionization and larger optical depths.

We show the PS of the E-mode polarization anisotropies using the Miranda et al. (2017) EoR models in the bottom panel of Fig. 1 assuming Planck 2018 TT+lowE cosmology (Ω_m,Ω_b,Ω_Λ,h,σ_8 = 0.321, 0.04952, 0.679, 0.6688, 0.8118; Planck Collaboration et al. 2018). We present the relative \( \chi^2 = 2 \ln \mathcal{Z} \) and the corresponding optical depth in Table 1 together with the results using Planck 2015 TT+lowP cosmology (Ω_m,Ω_b,Ω_Λ,h,σ_8 = 0.315, 0.04904, 0.685, 0.6731,

Note that the updated optical depths from the 4 models presented in Fig. 1 using Planck 2018 cosmology are all higher than the reported value (0.0522 ± 0.0080; Planck Collaboration et al. 2018). This is due to the fact that the model parameters were chosen to fit the 2015 data, not the 2018. Here we do not vary the parameters of the Miranda et al. (2017) models but instead use a different parametrization introduced below.

Table 1. Revisiting the models from Miranda et al. (2017), using both Planck 2015 and 2018 data. The models correspond to those shown in Fig. 1; the parameters are fixed and are not allowed to vary so as to better fit the 2018 data. For each model, we list the optical depth and the difference in the reduced \( \chi^2 \) with respect to the Pop-I model (computed using the Planck likelihood code).

| Model                  | Planck 2015      | Planck 2018      |
|------------------------|------------------|------------------|
|                        | \( \tau_e \)     | \( \chi^2 - \chi^2_{\text{popI}} \) | \( \tau_e \)     | \( \chi^2 - \chi^2_{\text{popI}} \) |
| Tanh                   | 0.0792           | 1.13             | 0.0788           | -1.14             |
| Pop-I\(^c\)           | 0.0832           | 0                | 0.0827           | 0                 |
| Pop-III\(^d\)         | 0.0926           | -0.96            | 0.0921           | 5.47              |
| Pop-III_self-regulated | 0.1049           | -2.27            | 0.1043           | 16.10             |

\(^a\) This column assumes cosmological parameters (i.e. the density and Hubble constant) from the TT+lowP reconstruction in Planck Collaboration et al. (2016c), and calculates \( \chi^2 \) using the low\( Z \) likelihood in PLC 2.0 (Planck Collaboration et al. 2016c). This likelihood considers both EE, TT, TE and BB components.
\(^b\) This column uses Planck 2015 cosmology, and the low\( Z \) likelihood in PLC 2.0 (Planck Collaboration et al. 2016c). This likelihood allows users to consider only the EE PS, which is used in this work.
\(^c\) For each model, there are slight changes in the resulting optical depth when comparing the Planck 2015 and 2018 columns. This is due to the variation in the Hubble constant (see equation 2) when different cosmological parameters are adopted.
\(^d\) \( \chi^2_{\text{popI}} = 10492.45 \) (407.50) for Planck 2015 (2018).

$\chi^2_{\text{popI}}$
forms a 3D EoR simulation \cite{Worseck2011, Worseck2016}. This is mainly driven by the reduced amplitude at multipole \(\ell \sim 10 \sim 20\) in the updated Planck E-mode PS (see the bottom panel of Fig. [1]). Consequently, models with a better fit to the updated PS measurement also return an optical depth closer to the reported value from Planck Collaboration et al. \cite{2018PlanckXXIV} 2018). We introduce some basic characteristics of our model evolution following an excursion-set approach based on the power-law scaling relations with respect to their host halo mass \cite{Mesinger2007, Mesinger2011, Murray2016}. We conclude that, unlike Planck 2015, the 2018 measurement no longer prefers an earlier reionization or a significant contribution from Pop-III stars to early reionization – the likelihood decreases when ionization starts at earlier times \cite{Mills2018}. In this work, we do not consider ionization by X-ray photons or earlier objects such as minihalo-hosted galaxies, and their corresponding parameters. Although efficient at heating the IGM before reionization, X-rays and minihalos are expected to have a very minor contribution to the EoR for reasonable galaxy models \cite{2005MNRAS.356..770R, 2013MNRAS.435.1725R}. The volume-averaged EoR history of that simulation is passed to CLASS, which then computes the corresponding CMB PS.

With this interface, we use the MCMC driver 21cmFAST \cite{Mesinger2007, Mesinger2011, Murray2016} (see below); and (ii) the commonly used, two-parameter \(Tanh\) model \cite{Mesinger2013, Behroozi2018}. A cell is ionized if \(n_{\text{ion}} > (1 + n_{\text{rec}})\).

### 3 EoR INFERENCE FROM THE CMB

To compute the impact of realistic reionization histories on the CMB PS, we connect 21cmFAST \cite{Mesinger2007, Mesinger2011, Murray2016} to CLASS \cite{Lesgourgues2011}. Specifically, for a given set of cosmological and astrophysical parameters, 21cmFAST performs a 3D EoR simulation\footnote{We do not explicitly model helium reionization. Instead we assume helium to be singly ionized following the same rate as hydrogen before becoming fully ionized at \(z=3\) \cite{Hogan1997, Worseck2011, Worseck2016}.}. The volume-averaged EoR history of that simulation is passed to CLASS, which then computes the corresponding CMB PS.

#### 3.1 Modelling the EoR

Our EoR model describes galaxy properties mostly using power-law scaling relations with respect to their host halo masses \cite{Park2016} and calculates the 3D reionization evolution following an excursion-set approach based on the cumulative number density of ionizing photons and recombinations \cite{Furlanetto2004, Sobacchi2014}. We introduce some basic characteristics of our model below and refer interested readers to the aforementioned references for more details.

We start from an initial Gaussian realization of the density and velocity fields \cite{Mesinger2007} in a large-volume (250Mpc), grid-based, high-resolution (\(\sim\)0.65Mpc; i.e. 250Mpc/384) simulation box assuming periodic boundary conditions. These fields are then evolved according to second-order Lagrangian perturbation theory \cite{Scoccimarro1998}, and re-gridded to a lower resolution (\(\sim\)1.95Mpc; i.e. 250Mpc/128) for the sake of computing efficiency. Then, for each cell centred at a spatial position and redshift of \((r, z)\), we compare the cumulative number per baryon of ionizing photons \((n_{\text{ion}})\) to that of recombinations \((n_{\text{rec}})\) in spheres with decreasing radii \cite{Furlanetto2004, Sobacchi2014}. A cell is ionized if at any radius \(R\),

\[
\hat{n}_{\text{ion}}(r, z | R, \delta_{\text{vir}}) = \int dM_{\text{vir}} \phi f_{\text{duty}} f_{\gamma} \frac{\Omega_b}{\Omega_m} \frac{M_{\text{vir}}}{\rho_b} n_\gamma \frac{\tau_{\text{esc}}}{\bar{n}_\gamma} (4)
\]

where \(\delta_{\text{vir}} = \rho_h / \bar{\rho}_h - 1\) is the average overdensity within the spherical region, \(\rho_h\) and \(\bar{\rho}_h\) represent the baryonic density and its cosmic mean. In equation (4),

(i) \(M_{\text{vir}}\) and \(\phi (M_{\text{vir}}, z | R, \delta_{\text{vir}})\) are the halo mass and halo mass function;

(ii) \(f_{\text{duty}} (M_{\text{vir}}) = \exp \left(-\frac{M_{\text{turn}}}{M_{\text{vir}}}\right)\), with a characteristic mass \(M_{\text{turn}}\) as a free parameter, accounts for a decreasing occupation fraction of star forming galaxies inside smaller halos due to inefficient cooling, photo-heating feedback \cite{Efstathiou1992, Shapiro1994, Thoul1996, Hui1997, Sobacchi2014, Sobacchi2013, Behroozi2014};

(iii) \(f_{\gamma} (M_{\text{vir}}) = \min [1, f_{\gamma,10} \left(\frac{M_{\text{vir}}}{10^{10} M_\odot}\right)^{\alpha_{\gamma}}]\) is the fraction of galactic gas in stars and is assumed to scale with the host halo mass \cite{Moster2013, Sun2016, Mutch2016, Tacchella2018, Behroozi2019};

(iv) \(n_\gamma\) is the number of ionizing photons intrinsically emitted per stellar baryon; and

(v) \(f_{\text{esc}} (M_{\text{vir}}) = \min [1, f_{\text{esc,10}} \left(\frac{M_{\text{vir}}}{10^{10} M_\odot}\right)^{\alpha_{\text{esc}}}]\) is the ionizing escape fraction defined as the number ratio of photons reaching the IGM to those emitted in the galaxy, and is also assumed to scale with the halo mass \cite{Ferrara2013, Kimm2014, Paardekooper2015, Xu2016}.

Inside the HII regions, we estimate the local, average...
photoionization rate with (Sobacchi & Mesinger 2014):
\[
\frac{\dot{n}_{\text{ion}}(r, z)}{\dot{n}_{\text{ion}}}(1 + z)^2 R_{\text{H}} \frac{\sigma_{\text{UVB}}}{\beta_H + \sigma_{\text{UVB}}} \frac{\dot{\rho}_b}{m_p} \hat{n}_{\text{ion}},
\]
where \(\sigma_{\text{UVB}} = 6.3 \times 10^{-18} \text{cm}^2\) and \(\beta_H \sim 2.75\) are the photoionization cross-section at Lyman limit and its spectral dependence; \(\alpha_{\text{UVB}}\) and \(\hat{n}_{\text{ion}}\) are the spectral indices of a stellar-driven UV ionizing background (Thoul & Weinberg 1996), the mass of a proton, and the local production rate of ionizing photons. Assuming the typical star formation timescale is \(t_\star H^{-1}(z)\), with \(t_\star\) being a free parameter, we calculate the average star formation rate (SFR) of galaxies in halos of a given mass at a given redshift by
\[
\text{SFR}(M_{\text{vir}}, z) = \frac{M_\star}{t_\star H(z)^{-1}}.
\]
We convert it to the non-ionizing UV luminosity via
\[
\frac{L_{1500}}{\text{SFR}} = 8.7 \times 10^{27} \text{erg s}^{-1} \text{Hz}^{-1} M_\odot^{-1} \text{yr}^{-1} \text{Madau & Dickinson 2014},
\]
and estimate \(\hat{n}_{\text{ion}}\) using equation (4) with \(M_\star\) being replaced by the SFR.

In summary, our model consists of the following six as-
trophysical parameters that we sample within the MCMC:

- \(\alpha_{\text{s}}\), the power-law index of the stellar-to-galactic gas mass relation at \(M_{\text{vir}} = 10^{10} M_\odot\), sampled with a flat prior in log space between 10\(^{-3}\) and 1;
- \(\alpha_{\text{c}}\), the power-law index of the ionizing escape fraction to halo mass relation at \(M_{\text{vir}} = 10^{10} M_\odot\), sampled with a flat prior in log space between 10\(^{-3}\) and 1;
- \(\alpha_{\text{c}}\), the power-law index of the ionising escape fraction to halo mass relation, sampled with a flat prior between -1 and 0.5;
- \(M_{\text{turn}}\), the turnover halo mass below which the number density of halos hosting star-forming galaxies drops exponentially, sampled with a flat prior in log space between \(10^8\) and \(10^{10} M_\odot\);
- \(t_\star\), the star-formation timescale as a fraction of the Hubble time, sampled with a flat prior between 0 and 1.

We then compute the cumulative number of recombinations in each cell with a spatial position and redshift of \((r, z')\) as well as an overdensity of \(\Delta_{\text{cell}}\) by
\[
\hat{n}_{\text{rec}}(r, z') = \int d\Delta_{\text{sub}} \phi_{\text{sub}}(z', \Delta_{\text{sub}}|\Delta_{\text{cell}}) \sigma_{\text{sub}} f_H \frac{\dot{\rho}_b}{m_p} \Delta_{\text{cell}} (1 - x_{\text{HI,sub}})^2.
\]
Here \(\Delta_{\text{sub}}\), \(\phi_{\text{sub}}(z', \Delta_{\text{sub}}|\Delta_{\text{cell}})\), \(\alpha_{\text{sub}}\), \(f_H\) and \(x_{\text{HI,sub}}(z', \Delta_{\text{sub}}, T_F, \Gamma_{\text{ion}})\) are the sub-grid (unresolved) overdensity, its probability distribution function (PDF; e.g. Miralda-Escudé et al. 2000) within our large-scale simulation cell (∼2Mpc), case-B recombination coefficient evaluated at \(10^9\) K, number fraction of hydrogen in the Universe, and the neutral hydrogen fraction of the sub-grid gas element, respectively. We assume photoionization equi-
librium in the ionized IGM, accounting for the attenuation of the local photoionization rate, \(\Gamma_{\text{ion}}\), according to the radiative transfer simulations from Rahmati et al. (2013).

We then compute the cumulative number of recombinations for each cell (see equation 3) by integrating \(\hat{n}_{\text{rec}}\) from the time the cell was ionized to the redshift of interest.

In summary, our model consists of the following six astrophysical parameters that we sample within the MCMC:
\[ \tau_e = 0.0556 \pm 0.0080 \] (for the astrophysical model). However, we note that the astrophysical model results in an asymmetric PDF of \( \tau_e \), with a tail extending towards high values.

The reason for this is apparent looking at the recovered EoR histories in the bottom left panel. The two distributions are comparable around the midpoint of reionization, where a large fraction of the EE power is imprinted. However, the astrophysical model results in asymmetric EoR histories with tails towards high redshifts. In Fig. 3, we show the marginalized posteriors for the astrophysical parameters (panels in lower left corner), together with the corresponding: (1) EE PS; (2) EoR history; and (3) \( \tau_e \) (upper right). We run three different MCMC simulations corresponding to different combinations of observational data sets used for the likelihood:

(i) \textit{DarkFraction}$_{LF}$, including high-redshift LFs and the QSO dark fraction upper limit on \( x_{HI}(z = 5.9) \). This run does not consider any CMB observations;

(ii) \textit{DarkFraction}_\text{tau LF}, based on \textit{DarkFraction}$_{LF}$, but including an additional constraint on \( \tau_e = 0.0522 \pm 0.0080 \), which is taken from the \textit{TT+lowE} reconstruction in Planck 2018 [Planck Collaboration et al. 2018] and was generated using a \textit{Tanh} basis set; and

(iii) \textit{DarkFraction}$_{EE, LF}$, based on \textit{DarkFraction}$_{LF}$, but also forward-modelling the low-\( \ell \) E-mode PS using class \cite{Lesgourgues2011} and includes the low-\( \ell \) EE likelihood from Planck 2018 \cite{PlanckCollaboration2019}. These are our “flagship” constraints.

Looking at the posterior of \textit{DarkFraction}$_{LF}$, we see...
that some of our astrophysical parameters are already constrained by galaxy and QSO observations. As pointed out in Park et al. (2019) (see also e.g. Tacchella et al. 2018 Behroozi et al. 2019 Yung et al. 2019), the observed high-redshift UV LFs already constrain the SFR-to-halo mass relation to within factors of ~2 (i.e. \( f_{*, 10}/t_* \) and \( \alpha_* \)) and provide an upper limit on the characteristic halo mass below which the galaxy occupancy fraction starts decreasing (\( \tau_{\text{turn}} \lesssim 10^9 \Msun; 2\sigma \)). Additionally, the dark fraction measurement of QSO spectra sets a lower limit for the ionizing
escape fraction normalization ($f_{\text{esc,10}} > 0.02; 2\sigma$), requiring the bulk of reionization to occur before $z \sim 6$ (panel 2) and setting a lower limit on $\tau_e$ (panel 3). However, without the CMB, the early stages of reionization are unconstrained (c.f. Greig & Mesinger 2017). This is evident from the broad range of EoR histories allowed beyond $z \gtrsim 8$ (panel 2), as well as the broad distributions of $\tau_e$ (panel 3) and the PS (panel 1). These early EoR models generally correspond to the high $f_{\text{esc}} + \text{low } \alpha_{\text{esc}}$ corner of astrophysical parameter space.

Including CMB observations rules out early reionizing models. Both DarkFraction$_{\tau \Lambda}$ LF and DarkFraction$_{EE \text{ LF}}$ posteriors disfavour the high $f_{\text{esc}}$ and low $\alpha_{\text{esc}}$ corner of parameter space. The EE PS, EoR history and $\tau_e$ distributions all shrink.

Comparing the DarkFraction$_{\tau \Lambda}$ LF and DarkFraction$_{EE \text{ LF}}$ posteriors in Fig. 3 quantifies the bias of using the $\tau_e$ summary statistic, generated with a different EoR model, instead of directly forward-modelling the CMB PS. There is a small difference in the EE PS with DarkFraction$_{EE \text{ LF}}$ allowing for a slightly earlier EoR. In general however the two posteriors are nearly identical. This indicates that, with the Planck 2018 data, the bias in using $\tau_e$ for the likelihood instead of the EE PS directly is negligible.

Finally, Fig. 4 presents the constrained photionization rate ($\Gamma_{\text{ion}}$; see equation 5). This serves to further illustrate how a physical model allows us to predict additional IGM properties. We only show DarkFraction$_{\tau \Lambda}$ LF, as DarkFraction$_{EE \text{ LF}}$ is nearly identical, while not including the CMB observations allows $\Gamma_{\text{ion}}$ to become unrealistically large. The rise in $\Gamma_{\text{ion}}$ with redshift is determined by the formation of structure in our galaxy model, with the “flattening” seen at $z \lesssim 6$–7 being due to photo-heating suppression of gas accretion onto galaxies following reionization (e.g. Sobacchi & Mesinger 2013). We see that while our prediction is consistent with the measured UV ionizing background at $z \approx 5$ to 6 (Bolton & Haehnelt 2007; Calverley et al. 2011; Wyithe & Bolton 2011; Calverley et al. 2011) observational data lies on the lower boundary. Additionally including $z \sim 5$–6 Lyα transmission statistics constrains the upper envelope of $\Gamma_{\text{ion}}$ significantly, though the results are more model dependent than those presented here (Qin et al. in prep).

We should caution however that our findings here are valid in the context of a flat CDM Universe. An important follow-up to this study will be to generalize these trends to alternative cosmologies (e.g. Paolletti et al. 2020). It is possible that in some exotic cosmologies, the correlation between parameters describing exotic physics and reionization exist at the level of $\tau_e$ but are broken once the full power of the CMB PS is included, rendering the necessity of joint analysis of reionization data and CMB observations.

8 We note that if our prior ranges were extended even further, reionization would be allowed at even higher redshifts. However, an important benefit of using an astrophysical EoR model is that it allows us to place physically-motivated priors on the parameters. For example, an ionizing escape fraction cannot be higher than unity, nor can star formation occur efficiently inside halos whose virial temperature is smaller than available gas cooling channels. This is not the case for non-physical or so-called model-independent constraints, for which it can be difficult to choose reasonable priors on the model parameters.
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DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.

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It is well known that there exists a strong degeneracy between the optical depth to reionization $\tau_e$ and the amplitude of the primordial power spectrum $A_s$ in the high-$\ell$ ($\ell \gtrsim 30$) $TT, TE, EE$ power spectrum, such that the parameter combination well-constrained by these data is $A_s \exp(-2\tau_e)$ (Planck Collaboration et al. 2016e). Once the physical densities $\Omega_m h^2$, $\Omega_b h^2$ and the tilt of the primordial power spectrum $n_s$ are fixed, there is a direct correspondence between $A_s$ and $\sigma_8$, and hence $\sigma_8$ inherits this degeneracy with $\tau_e$. As a result, one might worry that fixing $\sigma_8$, as we have done for computational convenience, would over-constrain the optical depth and the EoR reconstructed from the low-$\ell$ $PS$. To test this, in Fig. A1 we plot the analogous quantities from Fig. 2 using the $Tanh$ parametrization, which is much less time-consuming than 21cmFAST. In orange, we show the same model as in Fig. 2 generated by fixing $\sigma_8$. In blue we show the posterior of the $Tanh$ model, but also allowing $\sigma_8$ to co-vary with a prior on the $\sigma - \tau_e$ relation inherited from the high-multipole data (see Fig. 42 and discussion in Planck Collaboration et al. 2016e). The fact that the orange and blue posteriors are virtually indistinguishable suggests that our conclusions are unaffected by our choice of fixing cosmological parameters while performing inference.
Figure A1. Same as Fig. 2 but for the Tanh EoR model with (blue) and without co-varying $\sigma_8$ (orange). For the posterior shown in blue, we put a prior on the $\sigma_8 - \tau_e$ relation inferred from the high-$\ell$ data (see Fig. 42 in Planck Collaboration et al. 2016e). The difference between the two results is negligible.