Understanding concept of students in solving derivative problems based on school origins

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Abstract. This research is based on students' problems in understanding the concept of derivatives in the differential calculus. Learning derivatives always become a difficulty for students, even for they are in the high school. Learning derivatives is the same as studying something abstract, therefore, if students do not understand the concept well, they will continue to experience learning difficulties when solving the problems. However, conceptual understanding is a basic ability that students must have in order to be able to learn and to solve these mathematics problems. This research was conducted to describe the understanding of the students’ concept in solving derivative problems in the differential calculus topic in terms of their school origins. This research is a qualitative descriptive. The subjects of this study were the fourth semester students in Mathematics Education Program of State Islamic Institute of Curup Academic year 2019/2020 selected by purposive sampling based on school origins. The data collection techniques used were tests and interviews. Furthermore, the data is analysed by reducing data, presenting data, and drawing conclusions. The results showed that based on the school origins, the students have concepts understanding in solving derivative problems, but it had various thinking. It can be seen from the concepts of students in a variety representation. In solving derivative problems, there were still found variety of conceptual understanding and concept errors such as factoring of quadratic equations and grouping the terms in solving derivative algebraic functions.

1. Introduction

Carl Friedrich Gauss referred mathematics as the queen of science but unfortunately students fear from this queen, although the subject is very essential to the growth of many other disciplines. The science of mathematics depends on the mental ability. It is the means to develop the thinking power and reasoning intelligence, which sharpens the mind and makes it creative [1]. Mathematics is one of basic knowledge which both applied aspect and reasoning are useful for mastering knowledge and technology [2]. In mathematics learning, conceptual mastery is one of the important things and is the main key. Mathematics has always been considered a difficult subject for students from Elementary School to Higher Education. Calculus is a part of mathematics topics that has a role in the development of technology and other fields of science. Calculus is a basic science that needs to be mastered more broadly and deeply by students, prospective teachers, or prospective scientists [3]. Calculus is known as the introduction to the higher level mathematics needed by mathematicians, doctors, statisticians, engineers, and scientists [4]. In studying calculus must have knowledge of the concept particular mathematics namely trigonometry, high school algebra and geometry [5]. There
were two factors that caused students to experience difficulties in taking calculus courses, namely low learning interest and low basic calculus abilities [6].

Learning calculus in Higher Education has a very important role in developing thinking skills, problem solving and development student independence. Purpose of being taught calculus in a college is (1) organized a scientific mindset that is critical, logical, and systematic; (2) the power of reason is trained and creativity after learning various strategies and (3) trained problem-solving tactics in designing simple mathematical models; (4) skilled in applying techniques standard mathematics supported by concepts, reasoning, formulas, and methods right [7]. A series of calculus courses offer students basic materials mathematics such as functions, limits, derivatives, integrals, and their applications. These basic materials have many applications, calculus is not only studied by students who deals in pure mathematics and mathematics education but also on other fields such as engineering, informatics, and computer science [8].

Calculus has two main parts, differential calculus and the interconnected integral calculus through the basic theorem of calculus. Basic theorem calculus describes the relationship between two operations the centre of calculus, that is, differentiation and integration [9]. The real calculus stuff has been given to students since attending middle school. Therefore, prospective high school mathematics teachers should have mastered calculus before becoming an educator. Even though it has been studied since school, students still think of calculus is a difficult subject, especially with regard to the derivative concept using limits and its rules so that it affects the understanding of the concept. This phenomenon is illustrated by the relatively low acquisition of calculus learning outcomes when compared to the average learning outcomes of other subjects. Likewise with the tasks that are done. Even though the low understanding of the concept of differential calculus courses will result in students becoming obstructed in learning the next course. This can be seen from the final score of the student’s differential calculus course in the 2019/2020 academic year, namely 50% of the total students get a B grade in the range 70-85, 31.25% get an A in the range of 86-100, and the rest of it gets under 70. This data then indicates that there are still learning errors in calculus courses, especially in understanding concepts. Understanding of concepts is a process, method, or action to understand or to know in detail the concepts being studied, this is reflected in students’ learning outcomes [10]. Understanding of concepts that students have is one of the cognitive learning outcomes obtained by students through the learning process [11].

In State Islamic Institute of Curup, majority mathematics education program students come from High Schools (SMA), which has a percentage of 53,855% which has a percentage of 53,855% which is more dominant than the percentage of other students who come from Vocational High Schools (SMK) and Islamic High School (MA). Students' understanding of concepts can also be influenced by the origin of the school. The secondary school background affects students' motivation and academic achievement in college [12]. Erdogan, et.al explained that first year in college is a significant time in students' mathematical understanding and development in which they start to crystallize their understanding of mathematical concepts and to see engineering application of problems in various university mathematics courses starting from Calculus I. At this stage students also get confuse due to unnoticed misconception built upon their high school mathematics [13]. Students often experience confusion when studying in college because they are still carried away with the concepts they got when they were in high school. Therefore, the authors then try to identify these errors based on the initial ability value which is then analyzed based on the errors made when working on derivative problems in terms of school origins.

Concept understanding is a thing which is very important, because with mastery of concepts will make it easier for students in studying a subject matter [14]. Mathematical concepts arranged hierarchically, structured, logical and systematic starting from the simplest concept arrived at the most complex concept. Understanding the concept is an ability owned by individuals so it can provide an understanding of a study [15]. Understanding of mathematical concepts indicators that can be used as a basis for by teachers in developing material learning. The ability indicator mathematical understanding, namely: 1) restate existing concepts learned; 2) classify objects based on mathematical
concepts; 3) apply the concept algorithmically; 4) provide examples or counter examples at learned concepts; 5) presenting concepts in a variety representation; and 6) connect various mathematical concepts internally or externally [16].

2. Methods
This research is a qualitative research with a descriptive approach to describe students’ understanding of the derivative problem solving concept in differential calculus based on school origins. In this study, researchers selected subjects using purposive sampling. Research subjects were selected based on representatives from each school origins who had a score of more than 70 in differential calculus course. They were three students in the fourth semester in Mathematics Education Program of State Islamic Institute of Curup in the academic year 2019/2020. Data collection techniques used were midterm learning outcomes test and interviews, then the data were processed by reducing data, presenting data, and drawing conclusions.

3. Results and Discussion
The instrument used for the selection of research subjects was a test of conceptual understanding in the form of a description. The research subjects consisted of three people who were divided into three categories based on the origin of the school, namely the first category came from Senior High School (SMA), the second category came from Vocational High School (SMK), and the third category came from Islamic High School (MA). Furthermore, the results of the student's work were analysed in order to find out how students understood concepts and what mistakes were made by students in understanding these concepts when working on differential calculus problems. The three subjects are presented in Table 1.

| No | Initial | Initial of School Origin |
|----|---------|--------------------------|
| 1  | DS      | SMK N X                  |
| 2  | AB      | MAN Y                    |
| 3  | SA      | SMA N Z                  |

Indicators of concept understanding in this study, namely: 1) restate existing concepts learned; 2) classify objects based on mathematical concepts; 3) apply the concept algorithmically; and 4) presenting concepts in a variety representation.

1. **Noted that:** \( f'(x) = \lim_{t \to x} \frac{f(t) - f(x)}{t - x} \), then find the derivative of \( f(x) = x^2 - 3x \)!

Indicator: Restate existing concepts learned and Presenting concepts in a variety representation analysis of student’s concept understanding for the first question, shown in Figure 1.
In the answers sheet of the DS, it was found that there were errors in answers due to misconceptions. DS actually uses the concept of changing \( f(x) \) to \( f(t) \) well. However, DS cannot factorize the quadratic equation, namely \( t^2 - x^2 \) to \( (t - x)(t + x) \) and \( -(3t - 3x) \) to \( -3(t - x) \) so that it has the equal divider, that is \( t - x \). DS divides against \( (t - x) \) but does not comply with the rules. As a result, the results obtained were not correct.

On the other hand, based on the answer of AB, there are no misconceptions, so the AB is considered to understand the concept. AB has correctly used the concept of defining limit to find the derivative of a known algebraic function. The concepts to pay attention to in this problem are the form \( f(t) \) and \( f(x) \). If you don't understand the concept well, you can't change \( f(x) \) to \( f(t) \) and can't find the correct answer to the problem. This problem also has a quadratic equation factoring concept, namely the form \( t^2 - x^2 \) and \( -(3t - 3x) = -3(t - x) \), as shown in Figure 2.

![Figure 2. The answers sheet of the AB.](image-url)

Based on the results of research by Parrot on understanding the concept of calculus, it was found that many studies have demonstrated that students’ difficulty in understanding calculus are caused by their weak understanding of functions and the inability to use functions to reason and represent relationships [17]. The results of these studies also occur in this research that is, changing the form of the function \( f(x) \) to \( f(t) \).

The SA answered the question perfectly (Figure 3 below). There are no misconceptions in it. In contrast to the two previous subjects, SA factored the form \( t^2 - 3t - x^2 + 3x \) to \( (t - x)(t + x - 3) \) with just one time and did very well without having to factor it into two different parts. Based on the work on question number 1 about finding the derivative using the concept of limit definition and the indicators above, it can be seen that the three subjects can do it well, it's just that the way are different. AB and SA did well, while the DS still made misconceptions in factoring a quadratic equation. It can be seen that the derivative problem does not only require the concept of a limit definition alone but also uses other algebraic concepts that can help solve the problem. So, here the concept of quadratic equations needs to be properly understood again, as shown in Figure 3.

2. Using the rules in derivatives, solve it!

   a. \( y = (x^2 + 2)(x^2 + 1) \)
   b. \( y = \frac{-4}{x^6} \)
   c. \( y = \frac{x^3 - 2x + 5}{x^2 + 2x - 3} \)

   Indicator: Classify objects based on mathematical concepts; Apply the concept algorithmically; and Presenting concepts in a variety representation

   The DS has answered question a correctly, but did not use the product derivative rule. DS uses the rules of power and addition only. In fact, if you want to be more effective, you should just use the product rule because the problem is in the form of multiplying two functions. In problem b also does not use the derivative rule for division of two functions. For problem c, using the derivative rule, it is
just that there is a misconception in grouping the terms and performing arithmetic operations. This means that the DS actually still does not understand the concept of algebra well, as shown in Figure 4.

![Figure 3. The answers sheet of the SA.](image)

![Figure 4. The answers sheet of the DS.](image)

In question number 2 there are three forms of question being asked, namely the form of multiplication of two functions, division of two functions where the denominator is the power function, and the division function. To find the derivative of these three forms, students can use the product rule, quotient, and power rule. When solving problem a, AB has answered according to the concept, namely using the product derivative rule, likewise for problems b and c, AB has used the quotient rule and the power rule well. In solving the problem, there was no operating or principle error. For problem c, AB forgot to write the form v2 on changing the form of the question into the form of the derivative rule. This is not a misconception, only the forgetting factor, because in the next section, AB has rewritten it. After being confirmed through interviews, AB also admitted that he forgot to write down, as shown in Figure 5.
The SA has answered question a correctly and has used the rule of product derivative. However, in problem b, SA uses the power rule. In fact, you should use the quotient rule. This means showing that the SA actually understands the derivative rules just wants to try a shorter alternative. These are the results of the interview with the SA. As for question c, SA has been able to answer it and there are no errors found, as shown in Figure 6.

Based on the analysis of question number 2, it is found that the DS often does not use derivative rules in solving it, but he can answer with in another way that is by factoring and then doing for derivative. Whereas the AB and SA have answered according to the derivative rules, but there are still answers in their own way. In addition, there are also factors of forgetfulness and inadequacy in the process.

In State Islamic Institute of Curup, mathematics education program has students backgrounds from High School (SMA), Vocational High School (SMK), and Islamic High School (MA) which are quite influencing the students' calculus learning patterns. Based on the results of this research, it is found that students who have a High School (SMA) background apparently have a good previous material memory, but often careless. For the students of Islamic High School (MA) often forget to write symbol in the equation. While, for the vocational students (SMK), had difficulty understanding algebraic concepts such as factoring of quadratic equations and grouping the terms. Lerner confirmed that common mistakes done by children in doing the math, that is the lack of knowledge about the symbol, the lack of understanding of the value of the place, the use of the process wrong, miscalculations, and writing that cannot be read so that the learner made a mistake because no longer able to read his own writing. The condition is caused by several factors. The factors that cause errors in the math homework covers the causes of fault location, cause this type of error factors, factors causing this type of error concept, factors causing this type of error owned operations, factors causing this type of error principle [18]. In addition, when studying in high school, students who study in high school get more mathematics lessons compared to other students who are not from high school,
namely such as derivative basic material and examples in everyday life have been studied in secondary school.

Various learning innovations developed to help students master calculus. Especially for prospective students Mathematics teachers, the basic concepts of calculus must be well understood. The concept or material is an expansion or deepening of the material that has been studied. It becomes very disastrous when students, especially teachers, have a wrong or inaccurate understanding of a mathematical concept [19]. Mairead Greene and Paula Shorter also explained that it is important that a student truly understands each mathematical concept and is able to reason from that understanding to answer questions — as opposed to simply memorizing (or mimicking) a few methods utilizing that concept [20]. Understanding the concept must understand the concepts being learned, not memorizing formulas. One way to improve students' understanding of concepts in calculus learning is to use technology such as Maple Software. The results showed that using maple software in calculus courses could help students understand concepts and make it easier to solve differential calculus problems [21]. In addition to the use of software, to improve students' understanding of calculus concepts, it can also be done using the Mind Mapping method. The results showed that students had a positive attitude towards the use of the mind mapping method and there was an increase in the ability to understand mathematical concepts of students who used mind mapping better than students who used conventional learning seen from their initial mathematical abilities (low, medium, and high) [22].

4. Conclusion
Based on the results of this research, it is found that students who have a High School (SMA) background apparently have a good previous material memory, but often careless. For the students of Islamic High School (MA) often forget to write symbol in the equation. While, for the vocational students (SMK), had difficulty understanding algebraic concepts such as factoring of quadratic equations and grouping the terms. Basically, based on the school origins, the students have concepts understanding in solving derivative problems, but it had various thinking. It can be seen from the concepts of students in a variety representation.

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