Multi-dimensional Ambiguity Function for Coherent Pulsed-LFM FDA Radar

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Abstract. Frequency diverse array (FDA) which employs a small frequency increment between adjacent elements can provide a range-angle-dependent beampattern with new application potentials. In this paper, the proposed coherent pulsed-LFM FDA scheme enables full spatial coverage with a single transmit waveform, wherein the transmit beamforming can be performed at the receiver with flexible signal processing means. In the coherent pulsed-LFM FDA radar configuration, the range sidelobe level (SLL) can be reduced significantly. Analyses of the multi-dimensional ambiguity function in terms of the low sidelobe characteristics, spatial coverage capability, and resolution properties in range, angle and Doppler domains are given by ambiguity function profiles. Comparisons with conventional phased array and MIMO (Multiple-Input Multiple-Output) radar are presented in the simulation. Results demonstrate the superiority of the proposed approach in angular coverage and sidelobe level with simplicity in engineering.

1. Introduction

As a new paradigm of phased-array radar, frequency diverse array (FDA) radar has drawn much attention from researchers in recent years [1-2]. The FDA radar adopts a small amount of linear frequency increment across the array elements. As such, the resulting antenna beampattern is a function of time, angle and range, which is of vital importance to provide potential applications such as in moving target detection [3-4] and high-resolution radar imaging [5-6].

The concept of FDA was first proposed in [7] as a method achieving range-dependent beamforming. The continuous beam scanning feature is evaluated by simulation examples in [8] and examined from a design perspective in [9]. In [10], FDA radar is investigated to handle the range ambiguous clutter and improve target detection performance by exploiting the additional degree-of-freedoms (DOFs) in the range domain. In [11], the bistatic FDA radar was introduced in application of low-probability-of-intercept (LPI) radars to limit returns from undesirable range cells. In [12], [13], the phased-MIMO radar with FDA as the transmit array is studied, where the range-angle-dependent beampattern is maintained by dividing the FDA into several subarrays. In [14], a joint range and angle estimation method is devised for the FDA-MIMO radar by using the transmit sub-aperture optimization technique. It is suggested in [15] that the combination of the FDA and space-time adaptive processing (STAP) can suppress clutter and jamming simultaneously and provide significant performance improvement by means of a space-time-range adaptive processing method. The benefit of applying FDA for synthetic aperture (SAR) radar high-resolution imaging was shown in [16]. Furthermore, numerous new forms of FDA have been investigated [17-20].
logarithmically increasing frequency offset to FDA (log-FDA) is presented in [17] which can achieve a beampattern with a single maximum at the target location without defocusing. A uniform linear array FDA using multi-carrier transmission signal (multi-carrier FDA) is discussed in [18] to resolve the range and time coupling problem by convex optimization algorithms. The multi-carrier NLFM FDA system based on pseudo-random frequency offset can be found in [19] which is capable to reduce the 3dB mainlobe width and suppress the sidelobe peaks simultaneously. The convex-multi-log-FDA has been reported to synthesize both single-dot and multi-dot shaped beampatterns at the desired locations in [20].

However, preliminary studies of FDA are most excited with continuous wave in the array radar configuration [12-20], so called CW-FDA. In fact, pulse signals are more often adopted in practical radar systems and engineering applications. In this paper, we proposed the coherent pulsed-LFM FDA framework. Unlike the traditional MIMO radar employing orthogonal waveforms, the coherent pulsed-LFM FDA radar radiates an identical waveform with frequency increment. It enables both a wide illumination and a low SLL with more simplicity in engineering. The multi-dimensional ambiguity function, which is illustrated visually trough the range-angle, angle-angle and range-Doppler profiles, is derived to investigate relevant features of the low sidelobe level, spatial coverage capability and resolution in each domain.

2. Signal model of the coherent pulsed-LFM FDA radar

For simplicity and without loss of generality, consider a uniform linear array (ULA) consisting of \( M \) half-wavelength spaced omni-directional antenna elements. The transmitted signal of the \( m \)th element can be written as

\[
s_m(t) = \sqrt{\frac{E}{M}} \text{rect}\left(\frac{t}{T_p}\right) \varphi(t) \exp \left\{ j2\pi f_m t \right\}
\]

Where \( E \) is the total transmitted energy, \( T_p \) is the pulse duration, \( \varphi(t) \) denotes the unitary-energy baseband transmit waveform, and \( \text{rect}\left(\frac{t}{T_p}\right) = \begin{cases} 1, & |t| \leq T_p/2 \\ 0, & |t| > T_p/2 \end{cases} \) is the rectangular envelope. The carrier frequency at the \( m \)th antenna is \( f_m = f_0 + (m-1)\Delta f \), \( m = 1,2,\ldots, M \), where \( f_0 \) is the carrier frequency of the first element as reference frequency, \( \Delta f \) is the frequency increment across the array elements which is negligible in comparison with \( f_0 \) and the transmitted bandwidth. For this reason, the frequency spectrum of \( s_m(t) \) is largely overlapped in the frequency domain, and the synthetic signal is summed coherently in the far-filed, which is one of the reason why we call the framework coherent FDA. Then, the overall transmitted signal at angle \( \theta \) at time instant \( t \) can be expressed as

\[
s(t,\theta) = \sqrt{\frac{E}{M}} \text{rect}\left(\frac{t}{T_p}\right) \varphi(t) \sum_{m=1}^{M} w_m \exp \left\{ j2\pi d_r (m-1)\sin \theta / \lambda_0 \right\} \exp \left\{ j2\pi f_m t \right\}
\]

Where \( w_m \) denotes the transmit weight of the \( m \)th element which is assumed to be identical, i.e., \( w_m = 1 \), \( d_r = \lambda_0/2 \) is the element spacing, \( \lambda_0 = c/f_0 \) is the reference wavelength, and \( c \) represents the speed of light. Suppose a far-field stationary target at angle \( \theta \) and range \( R \), after backscattering the echo transmitted by the \( m \)th element and received by the \( n \)th element can be ultimately modelled as

\[
s_{m,n}(t-t_{m,n},\theta) = \sqrt{\frac{E}{M}} \text{rect}\left(\frac{t-t_{m,n}}{T_p}\right) \varphi(t-t_{m,n}) w_m \exp \left\{ j2\pi f_m (t-t_{m,n}) \right\}
\]

Where \( t_{m,n} = \tau - d_r (m-1)\sin \theta / c - d_k (n-1)\sin \theta / c \), \( \tau = 2R/c \). For ease of exposition, under the assumption of a narrowband sounding signal, we have \( \varphi(t-t_{m,n}) \approx \varphi(t-\tau) \), \( \text{rect}\left(\frac{t-t_{m,n}}{T_p}\right) \approx \text{rect}\left(\frac{t-\tau}{T_p}\right) \). Therefore, the signal received by the \( n \)th element can be written as
\[ y_n(t, \tau, \theta) = \sum_{m=1}^{M} \sqrt{\frac{E}{M}} \text{rect}\left(\frac{t - \tau_m}{T_p}\right) \exp\left\{j2\pi f_m(t - \tau_m, \theta)\right\} \]

\[ \approx \sqrt{\frac{E}{M}} \text{rect}\left(\frac{t - \tau}{T_p}\right) \exp\left\{j2\pi f_0(t - \tau)\right\} \exp\left\{j2\pi d_k(n-1) \sin \theta / \lambda_0\right\} \quad (4) \]

\[ \times \sum_{m=1}^{M} w_m \exp\left\{j2\pi d_k(m-1) \sin \theta / \lambda_0\right\} \]

Where \( \zeta \) is the target reflection coefficient. Thus, after being down converted, the received signal from \( N \) receiving channels can be concisely expressed as

\[ Y(t, \tau, \theta) = [y_1(\theta, t, \tau), y_2(\theta, t, \tau), \ldots, y_N(\theta, t, \tau)]^T \]

\[ = \sqrt{\frac{E}{M}} \text{rect}\left(\frac{t - \tau}{T_p}\right) \exp\left\{-j2\pi f_0t\right\} P_r(\theta, t, \tau) a_\nu(\theta) \quad (5) \]

Where \( a_\nu(\theta) = \left[1, \exp\left\{j2\pi d_k \sin \theta / \lambda_0\right\}, \ldots, \exp\left\{j2\pi d_k(M-1) \sin \theta / \lambda_0\right\}\right]^T \) is the receive steering vector, \( P_r(t, \tau, \theta) \) is the equivalent transmit beampattern in the form of

\[ P_r(t, \tau, \theta) = \sum_{m=1}^{M} \exp\left\{j2\pi d_k(m-1) \sin \theta / \lambda_0\right\} \]

\[ \sin \left[ M \pi \left(\Delta f(t - \tau) + \frac{d}{\lambda_0} \sin(\theta)\right)\right] \exp\left\{j(M-1)\pi \left(\Delta f(t - \tau) + \frac{d}{\lambda_0} \sin(\theta)\right)\right\} \quad (6) \]

It can be concluded in (6) that the transmit beampattern of the coherent pulsed-LFM FDA is time-invariant as a function of time, angle, range and frequency, so it is unreasonable to neglect the time factor in some existing work, namely, \( t = 0 \). From the view point of electromagnetics, the equiphase surface of the forward-propagating electromagnetic wave changes continuously over time. Every range bin actually undergoes the same phase history in sequence. After non-adaptive receive beamforming, the echoes in (6) can be expressed as

\[ z(t, \tau, \theta) = w_\nu(\theta_0) Y(t, \tau, \theta) = \sqrt{\frac{E}{M}} \text{rect}\left(\frac{t - \tau}{T_p}\right) \exp\left\{-j2\pi f_0t\right\} P_r(\theta, t, \tau) P_\nu(\theta) \quad (7) \]

Where \( w_\nu(\theta) = \left[1, \exp\left\{j2\pi d_k \sin \theta_0 / \lambda_0\right\}, \ldots, \exp\left\{j2\pi d_k(N-1) \sin \theta_0 / \lambda_0\right\}\right]^T \) is the non-adaptive weight vector at beamformed direction \( \theta_0 \), \( P_\nu(\theta) \) is the receive beampattern formulated as

\[ P_\nu(\theta) = \frac{\sin\left[N \pi \frac{d}{\lambda_0} \left(\sin(\theta) - \sin(\theta_0)\right)\right]}{\sin\left[\pi \frac{d}{\lambda_0} \left(\sin(\theta) - \sin(\theta_0)\right)\right]} \exp\left\{j(N-1)\pi \frac{d}{\lambda_0} \left(\sin(\theta) - \sin(\theta_0)\right)\right\} \quad (8) \]

It is can be observed that the receive beampattern is time-invariant. Therefore, the transmit beampattern and receive beampattern are not reciprocal, the latter is independent of time or range factor. Besides, the overall beampattern \( P(t, \theta) \) can be expressed as \( P(t, \theta) = P_r(t, \theta) P_\nu(\theta) \). As stated above, the coherent pulsed-LFM FDA radar enjoys the advantages of full spatial illumination. It is capable of achieving mainlobe auto-scanning during a single pulse repetitive interval (PRI) which enhances the ability of the radar system considerably.

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3. Multi-dimensional ambiguity functions

The analysis of the multi-dimensional AF is of great importance, from which, the characteristics of the applied waveforms can be evaluated. Because of the angle-time dependency in the complex radiated signal, the standard range-Doppler ambiguity function in conventional phased array is no longer sufficient. As a result, we employ the multi-dimensional ambiguity function including the presumed angle \( \theta \), the beamforming angle \( \theta_0 \), the time delay \( t \) and the Doppler frequency \( f_d \) in the following analysis to provide insight in resolution capability, spatial coverage and sidelobe level. In general, the multi-dimensional AF can be given in form of the modular square

\[
\Psi(\theta, \theta_0, t, f_d) = \sum_{m=0}^{M} \sum_{n=1}^{N} \exp\left(j2\pi d/\lambda(m-1)\sin\theta \right) \exp\left(-j2\pi d/\lambda(n-1)\sin\theta_0 \right) \int \phi_m(t)\phi_n^*(t+t) e^{j2\pi f_d \tau} dt
\]

(9)

Where \(( \ )^* \) stands for the conjugate operator. Then, substitute (2) into (9) and neglect the constant terms, (9) can be further expanded as

\[
\Psi(\theta, \theta_0, t, f_d) = \left| \sum_{m=0}^{M} \sum_{n=1}^{N} \exp\left(j2\pi d/\lambda(m-1)\sin\theta \right) \exp\left(-j2\pi d/\lambda(n-1)\sin\theta_0 \right) \int \varphi_m(t)\varphi_n^*(t+t) e^{j2\pi f_d \tau} dt \right|^2
\]

(10)

As is shown in (10), term1 is analogous to the traditional non-adaptive beamforming in phased array, it means that the frequency shift has no influence on the angular resolution in coherent-pulsed FDA which is the same as that in phased array. Term2 is similar to the standard 2-D AF, which consists of time delay and Doppler shift.

Aiming at assessing the performance of the multi-dimensional ambiguity function, a class of reduced-dimension profiles involving those in angle, range and Doppler domains are portrayed in detail. Specifically, it includes the following three categories: \( \Psi(\tau, \theta)_{\tau=0, \theta=0} \) is the range-angle map at a fixed beamforming direction \( \theta_0=0^\circ \); \( \Psi(\theta, \theta_0)_{\tau=0, \theta=0} \) is the angle-angle map which represents the transmit diagram in condition of a matched range; \( \Psi(\tau, f_d)_{\theta=0, \theta=0} \) is the range-Doppler map to explore the resolution in range and Doppler matched domains. We would analyse the multi-dimensional AF in the following three aspects: (a) Spatial coverage. There doesn’t exist obvious fluctuation along the main diagonal in the angle-angle AF cut, from which it could be inferred that the physically radiated signal remains uniform for all observed angular sectors. In other words, the transmitted energy is evenly distributed across the whole airspace. Moreover, it is remarkable that the wide spatial coverage can be obtained by the way of a single waveform. (b) Resolution. The range resolution in coherent FDA radar is directly concerned with the number of transmit array elements. As such, it is degraded by a factor equal to \( M \). Because the width of the mainlobe focused for a given probing angle is \( \pi/M \) in the typical conventional beampattern. Consequently, only the fraction \( 1/M \) of the bandwidth is sent in each direction. (c) Low sidelobes. In reality, the components of the summation in (10) influence each other via sidelobes, beneficially, the sidelobes can compensate each other for most of the ranges. Due to the positive impact of the sidelobes’ interaction, the range sidelobe level can be very low in coherent pulsed-LFM FDA. Whereas, the angular sidelobe remains at the same level as that in phased array, which can be mitigated by means of windowing weighting, such as Hamming, Hanning, Kaiser, and Dolph-Chebyshev windows. However, the sidelobes are reduced at the cost of a wider mainlobe and a lower processing gain. What’s more, artificial intelligence optimization algorithms, say, genetic algorithms, particle swarm optimization and artificial bee colony can be applied while the system complexity would increase to a certain extent.

4. Simulation examples

In this section, numerical examples are carried out to demonstrate the effectiveness of the proposed coherent pulsed-LFM FDA radar. For the sake of fair comparison, we consider two other radar models, that is, the phased array radar and the MIMO radar. We consider a uniform array with 10 antennas.
The carrier frequency is 1GHz and the bandwidth B is 20 MHz. Frequency increment is set to be 10 kHz.

4.1. Range-angle AF profiles

Figure 1. Range-angle ambiguity function $\Psi(\tau, \theta)_{\tau=0, \theta=\theta_0}$, (a) Phased array radar, (b) MIMO radar transmitting random phase codes, (c) Coherent pulsed-LFM FDA radar transmitting LFMcs, (d) Coherent pulsed-LFM FDA radar transmitting NLFMs.

Figure 1. represents the range-angle ambiguity function $\Psi(\tau, \theta)_{\tau=0, \theta=\theta_0}$ for three tested models. From the point of physics, every vertical cut indicates the one dimensional range profile which is the same as that via matched filtering. To be clear, all the directions of the digital beamforming point to $0^\circ$. In Figure. 1(a), the range sidelobe is relatively low, and the peak sidelobe is almost below -30 dB. The angular mainlobe is located at $0^\circ$. Note that the range and angle are independent which is similar to that in MIMO radar as seen in Figure. 1(b). However, the range sidelobe in MIMO radar is relatively high caused by the high cross-correlation of the signal transmitted in every channel. Due to the fact, the ideally complete orthogonality is hardly achievable in practical applications. Better orthogonality can improve the channel isolation and reduce the sidelobe level as well. As is shown in Figures. 1(c) and (d), the peak of the range-angle ambiguity function along the probing angle is located at the aiming direction $\theta_0 = 0^\circ$ which could verify the range-angle dependence. As can be seen in Figure. 1(c), the clean straight section demonstrates the range-angle dependency in the coherent pulsed-LFM FDA radar, whose vertical slices describe a rather homogeneous range profile at a certain chosen observation direction. Actually, it is the traditional correlation function when performing beamforming at $\theta = 0^\circ$ in essence. The peak sidelobe level across the range is reduced to -40 dB which is far below the SLL of the LFMcs, namely, -13.2 dB. Because the aggregate signal is the summation of the
frequency shift components, whose range sidelobe could compensate each other. Nevertheless, as stated above, the width of the main lobe in range domain is broadened compared with the other two methods. The wide spatial coverage is at the cost of the range resolution degradation.

4.2. Angle-angle AF profiles

Figure 2. Angle-angle ambiguity function $\Psi(\theta, \theta)$, (a) Phased array radar, (b) MIMO radar transmitting random phase codes, (c) Coherent pulsed-LFM FDA radar transmitting LFMs, (d) Coherent pulsed-LFM FDA radar transmitting NLFMs.

Figure 2. portrays the angle-angle cuts of the ambiguity function taken at $r = 0, f_d = 0$. In fact, every single vertical slice in the angle-angle AF represents an equivalent transmit beampattern at a definite angle $\theta_0$, which can be formed by signal processing on the receiver. The fluctuation along the main diagonal physically stands for the difference of gain on transmission. As illustrated in Figure 2(a), the high gain area mainly concentrates in the mainlobe centered at $0^\circ$. Meanwhile, the sidelobe level along the main diagonal is below -30 dB. Note that the phased array would focus the energy in a narrow beam so that it can’t cover a wide angular sector. In Figure 2(b), the energy is mostly distributed along the main diagonal which is in accordance with the wide transmit beam. Whereas the amplitude variation is more than 5 dB among all directions, and the gain of the mainlobe would vary with angle. It would give rise to an inconsistent performance. It can be noted from Figures 2(c) and (d) that there is no severe amplitude variation along the main diagonal, which means that the level of the physically radiated signal remains uniform for all the observation angles. Besides, the target can be detected at any angular direction owing to its advantage of wide spatial coverage and the capability of forming multiple beams simultaneously in the coherent pulsed-LFM FDA. Basically, the angular resolution in coherent pulsed-LFM FDA is determined by the array aperture which is the same as that in PA.
4.3. **Range-Doppler AF profiles**

![Figure 3](image)

Figure 3 Range-Doppler ambiguity function $\Psi(r, f_d)_{\theta=\theta_0, \phi_0}$, (a) Phased array radar, (b) MIMO radar transmitting random phase codes, (c) Coherent pulsed-LFM FDA radar transmitting LFM, (d) Coherent pulsed-LFM FDA radar transmitting NLFM.

Figure 3. shows an example of the Doppler-range profile $\Psi(r, f_d)_{\theta=\theta_0, \phi_0}$ . Every vertical slice stands for the Doppler response in a range bin. As can be seen in Figure. 3(a), the helical blade Doppler-range AF in conventional phased array which is determined by the transmitted chirps can provide satisfactory Doppler tolerance. This is in contrast to the orthogonal waveform scenario in Figure. 3(b) where the Doppler tolerance is poor. The gain would drop dramatically in a large Doppler shift with a performance loss more than 20 dB. Moreover, the Doppler sidelobe is obviously higher. Hence, this radar system does not apply to the high moving radar platform and high moving target detection. In comparison with the first two models, the coherent pulsed-LFM FDA radar can basically preserve the characteristics of the emitted waveform as evidenced from Figures. 3(c) and (d), whereas the widened mainlobe in the range domain is also retained. It can be observed that the SLL can be suppressed to nearly -45 dB without window weighting for most of the ranges in case of transmitting LFM in Figure. 3(c). Since the sidelobe could compensate each other, the range sidelobe level can be very low. In Figure. 3(d), the Doppler-range AF approximates the “thumbtack-like” when transmitting the NLFM.

5. **Conclusion**

In this paper, a novel framework of coherent pulsed-LFM FDA radar is proposed and the multi-dimensional ambiguity function has been derived to evaluate the relevant performance. Thorough analyses of the multi-dimensional ambiguity function in terms of the low sidelobe characteristics, spatial coverage capability, and resolution properties in range, angle and Doppler domains, the range-
angle, angle-angle and range-Doppler profiles are given to evaluate the specific features. It can be concluded that the proposed coherent pulse-FDA radar can generate a range-angle-dependent transmit beampattern with a wide illumination as well as a low sidelobe level in range domain in the utilisation of a single waveform. Thus, it has simple practicality and high reliability in practice.

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