Vibration Characteristics of PMSM Grinding Motorized Spindle

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Abstract. This paper presents electromechanical vibration characteristics in parallel misalignment of a shaft by analyzing the vibration of PMSM grinding motorized spindle. The unbalanced magnetic pull (UMP) of the PMSM grinding motorized spindle is established as electromechanical model based on the Maxwell stress tensor, which is used to the spindle to cause the electromechanical vibration and calculated under different eccentricities through FEA (Finite Element Analysis). It increases with increasing eccentricity under no load and on load and is maximum value when mixing eccentricity. Applying a Jeffcott rotor model to analyse the dynamic characteristics of the spindle with UMP excitation and mass imbalance. The number of pole-pair is 6 and the number of slots is 36 for the PMSM grinding motorized spindle, the vibrations of 1fr, 2fr and 3fr are generated and the limit cycle is more complicated at 4500rpm. Simulated results agree well with the experimental results, and it indicates the electromechanical vibration with the shaft eccentricity and the UMP is obviously on the shaft.

1. Introduction

A high speed PMSM motorized spindle is the one of the key parts of modern high precision CNC machine tools and an actuator, a transmission and an executor are components of the motorized spindle, which named “nearly zero transmission” [1]. The motorized spindle system has complex structures and the shaft is its core part [2]. For rotating machinery, the dynamic analysis of the rotor can be used as an important feature in designing, operating, and fault diagnosis [3]. In most cases, the machining manufacturing, assembly and material being inhomogeneous, there is a certain deviation between the rotor’s center of mass and the geometric center [4]. The electromechanical system is frequently affected by the electrical fault and its mechanical failures [5]. On the basis of IEEE and EPRI investigates the eccentricity ratio in the mechanical failure of the motor is 22% and 14% respectively [6]. Therefore, the analysis of the electromechanical characteristics of the rotor eccentricity has very important significance for the design and manufacture of the motorized spindle.

Due to the length of the air-gap is not uniform, the unbalanced magnetic pull (UMP) is caused and affects the dynamic characteristics of the system [7]. Yu et al. calculated the UMP of Permanent Magnetic Synchronous Motor (PMSM) with the rotor misalignment by Ansoft [8]. Xu et al. used the Ansoft Maxwell software and the interpolation to simulate and calculate the electromagnetic force in order to study rub impact [9]. James et al. used JMAG FEA tool to study and compare for the parallel misalignment of a shaft with different series and windings of the UMP [10]. Michon et al. proposed the analytical model uses a combined field solution capable of calculating no-load and on-load UMP and studied the effects on bearing forces/life in large permanent magnet machines [11]. Goraj presented a calculation effort efficient analytical formula in order to predict the static UMP in synchronous surface mounted permanent magnet (PM) electrical machines [12]. The calculation of the
UMP of the rotor is very complicated. The air-gap magnetic density contains a large number of harmonic components. Hence, it is difficult to quantitatively calculate the harmonic magnetic field by the traditional analytical analysis method. Although the FEM is complicated in processing and calculation time longer, and not provide analytical solutions, it is a good way to overcome the lack of analytical methods to calculate UMP.

Rotor vibration is an inherent property of rotating machinery due to mass imbalance. Many researchers have also studied the spindle vibrations, which is caused by mass imbalance and eccentric moments [13]. It is a limited ability that used linear theory to describe, explain, and predict nonlinear dynamic phenomena in high speed PMSM motorized spindle. In particularly, nonlinear dynamic phenomena of a high-speed shaft are more complicated under unbalance condition. Hence, nonlinear models are often used to more accurately investigate shaft behavior in rotor-bearing system [14]. Finite element (FE) model is widely applied to simulate and analyze the dynamic of rotor system [15]. Chen et al. proposed the pseudo mode shape method (PMSM) so as to improve the accuracy of analyzed dynamic characteristic in the rotor-bearing-foundation systems [16]. Roy et al. studied modal analysis of higher-order FE model for a viscoelastic rotor supported by the journal bearing and the higher-order model is more important than second order model [17]. Hsu established FE model of rotor-bearing system so as to obtain the natural frequencies, mode shapes, critical speed maps, and bearing stiffness [18]. Jalali et al. carried out full dynamic analysis, used 3D FE model, one-dimensional beam-type model and experimental modal test for a high-speed rotor with certain geometrical and mechanical properties. Experimental results indicated the accuracy of the FE models [19]. Özsahin et al. proposed analytical model was proved by using FEM method for asymmetric multisegment rotor-bearing systems and reduced computational time significantly with maintain calculation accuracy compared with FEM [20].

The rotor eccentricity generates the UMP leading to the vibration of the motorized spindle. In particular, it directly affects the performance of the machine tool and processing quality. To study the electromechanical coupling vibration of the PMSM grinding motorized spindle for the purpose of research the rotor eccentricity. There are the constructions of paper. Section II will propose an analytical method to analyze the radial UMP with the different eccentricities. The relationship between permeance and air-gap will be given by an equation, and the magnetic flux density distribution of the air-gap is represented by the magneto motive force (MMF). The radial electromagnetic force density will be calculated based on the Maxwell stress tensor and the analytical expression of the UMP will be obtained in the x-direction and the y-direction. Section III will use the Motor-CAD software to calculate and compare the UMP of the rotor under different eccentricities. The relative UMP of the eccentricity is studied. Section IV will discuss the vibration of the rotor based on Jeffcott rotor model and study the electromagnetic vibration characteristics of the rotor. Section V will apply to the PMSM grinding motorized spindle of type YKZ7230 with eccentricity for verification. Lastly, a conclusion can be given.

2. Electromagnetic Model

The air-gap length is greatly important to calculate the UMP and analyze the vibration of the motorized spindle. As Fig.1 shows the geometrical centers of the stator and rotor are the \( O_s \), \( O_r \), and the initial geometrical center of rotor is \( O \). The air-gap length of mixed eccentricity (ME) for an eccentric rotor can be approximately expressed by the following equation:

\[
\delta(a,t) = \delta_0 - r \cos(\alpha - \varphi) - r_0 \cos(\alpha - \theta)
\]  

(1)

Where \( r_0 \) is the static eccentricity (SE); \( r \) represents the dynamic eccentricity (DE); \( \delta_0 \) is the average the air-gap length when the rotor is center; \( \alpha \) is the air-gap angle with reference to the x-axis; \( \varphi \) and \( \theta \) are the angles of SE and DE with reference to the x-axis respectively.

The Fourier series of air-gap permeance is

\[
\Lambda(a, t) = \sum_{n=0}^{\infty} \Lambda_n \cos[n(\alpha - \varphi)]
\]  

(2)
\[
\Lambda_n = \begin{cases} 
\frac{\mu_0}{k}\sqrt{1-e^2} & (n=0) \\
\frac{2\mu_0}{k}\sqrt{1-e^2} & (n>0)
\end{cases}
\]

(3)

Where \(\mu_0\) is the vacuum magnetic permeability, \(k\) is the magnetic saturation coefficient, \(e = r_d/\delta_0\) is the relative eccentricity.

The motorized spindle is powered by the PWM voltage. The stator armature current is approximately a three-phase symmetric sinusoidal current ignored harmonic effects. In accordance with the winding theory of the motor, the fundamental MMF can be expressed as:

\[
F_s(\alpha, t) = F_{\mu s} \cos(\omega_s t - \alpha) = 1.35 \frac{Nk}{2p} I_m (\omega_s t - \alpha)
\]

(4)

Where \(F_{\mu s}\) is the amplitude of the fundamental MMF of the armature reaction current of the stator and related to winding current, number of turns and winding type, \(N\) represents the number of series turns per phase winding, \(k_w\) represents the fundamental winding distribution coefficient, \(\omega_s\) is the electric angular frequency, and \(p\) is the number of pole-pair.

MMF generated by a permanent magnet is non-sinusoidally distributed and its expression is:

\[
F_r(\alpha, t) = F_{\mu rm} \cos(\omega_r t - \alpha) = \frac{4}{\pi} \frac{B h_m}{\mu_n} \sin \left(\frac{\pi \alpha_r}{2}\right) (\omega_r t - \alpha)
\]

(5)

Where \(F_{\mu rm}\) is the amplitude of the resultant MMF for the rotor, \(h_m\) is the magnetizing direction thickness of the permanent magnet, \(\alpha_r\) is the pole arc coefficients.

Fig.2 shows the spatial positional relationship between the MMF of the stator and the rotor, and the air-gap synthetic MMF is obtained:

\[
F(\alpha, t) = F_{\mu s} \cos(\omega_s t - \alpha - \theta) + F_{\mu r} \cos(\omega_r t - \alpha) = F_{\mu} \cos(\omega t - \alpha)
\]

(6)

Figure 1. Schematic diagram of static and dynamic mixed eccentricity

Figure 2. PMSM motorized spindle running vector diagram

3. Analysis of UMP Due to Eccentricity (FEA)

To be able to calculate the UMP, we modeled the motorized spindle in two dimensions utilizing proprietary Motor-CAD software. The simulation parameters are displayed in Table1. For this purpose, we simulated the machine under different static eccentricities, different dynamic eccentricities, and mixed eccentricities for no load and on load operating conditions. Fig3. shows the relationship between the UMP and the eccentricity. UMP increases with increasing eccentricity.

Fig 4. and Fig 5. show the UMP on the rotor of the motorized spindle under different eccentricities. As the eccentricity increases, the UMP also increases. In fact, the air-gap length is minimum so that the radial component of magnetic local traction is maximum. Under on load, the UMP is greater than
under no load.

Table 1. Motorized spindle parameters

| Parameter                             | YKZ7230 |
|---------------------------------------|---------|
| Phase                                 | 3       |
| Pole-pair                             | 3       |
| Rated power /Kw                       | 25      |
| Mass of the stator /Kg                | 22.9    |
| Mass of the rotor /Kg                 | 4.3     |
| Outer diameter of the stator /mm      | 116     |
| Inner diameter of the stator /mm      | 84.2    |
| Outer diameter of the rotor /mm       | 83.2    |
| Inner diameter of the rotor /mm       | 58      |
| Length of the stator and rotor /mm    | 200     |
| Length of the spindle /mm             | 517     |
| Inertia of the rotor / kg·m²          | 0.021   |

Figure 3. The relationship between the UMP and the eccentricity.
(a) Static eccentricity
(b) Dynamic eccentricity

Figure 4. The UMP on the rotor under different static eccentricities
(a) No load
(b) on load
Figure 5. The UMP on the rotor under different dynamic eccentricities. (a) No load. (b) on load.

4. Analysis of the Electromagnetic Vibration by Eccentricity
A Jeffcott rotor model can be used to study the electromagnetic vibration characteristics of the PMSM grinding motorized spindle. The transverse vibration of the rotor system can be written as differential equations.

\[
\begin{align*}
mx'' + cx' + kx - F_x^{ue} &= m\omega^2 \cos(\omega t) \\
m\dot{y}' + cy' + ky - F_y^{ue} &= m\omega^2 \sin(\omega t)
\end{align*}
\]  

(7)

Where \( m, k \) and \( c \) are mass, spring coefficient and damping coefficient of the rotor system, respectively. \( e \) is the mass eccentricity, \( \omega \) represents the rotating angular velocity. The vibration displacements in both directions are represented by \( x \) and \( y \), respectively.

It is assumed that air-gap is uniform, then the geometric center of the rotor coincides with the geometric center of the stator and the UMP is zero. The periodic unbalanced mass excitation produces periodic responses. The rotor is only subjected to centrifugal force. Fig 6. displays the spectrum by calculating the centrifugal force without eccentricity at \( n=4500 \) rpm, \( n=8000 \) rpm and \( n=10000 \) rpm. According to the vibration spectrum of the rotor, the amplitudes of the rotating frequencies close to different rotational speeds are sequentially decreased. The results show that the motorized spindle is running at a lower speed, the \( f_r \) is close to the natural frequency, thus a resonance phenomenon occurs. In this case, the vibration of the rotor weakens as the rotational speed increases. The motions of spindle at steady state are indicated in Fig 7.

Figure 6. Vibration frequency without eccentricity.  Figure 7. Limit cycle without eccentricity at \( n=4500\)8000\10000rpm.
Fig. 8. shows the vibration response and spectrum of the rotor in the x-direction under the UMP action at \( n = 4500 \text{rpm} \). It can be seen from the figure that the vibration of the rotor in the x-direction is mainly close to \( 3f_r \), and the amplitude of the vibration close to \( 1f_r \) and \( 2f_r \) is also large.

![Figure 8. The UMP with \( \delta = 0.2 \text{mm} \) on the rotor at \( n = 4500 \text{rpm} \).](image)

(a) Time-domain response  
(b) Spectrum of rotor vibration

Fig. 9. shows the vibration response and spectrum of the rotor in the x-direction under the UMP and mass centrifugal force action at \( n = 4500 \text{rpm} \). It can be seen from the figure that the vibration frequency of the rotor in the x-direction are 2.93Hz, 69.21Hz, 74.91Hz, 141.36Hz and 225Hz.

![Figure 9. The UMP with \( \delta = 0.2 \text{mm} \) on the rotor at \( n = 4500 \text{rpm} \).](image)

(a) Time-domain response  
(b) Spectrum of rotor vibration

5. Conclusion

This research study electromechanical vibration characteristics of the rotor of PMSM grinding motorized spindle with 6 poles and 36 slots when the eccentricity is generated. First of all, the UMP was analyzed with equations corresponding to the electromagnetic model and calculated by the Motor-CAD software. The results showed that the eccentricity becomes small, the UMP decreases and the UMP generated is greatest under ME. As well, for the rotor system, a Jeffcott rotor model was adopted to study the electromechanical vibration. The amplitudes of the rotating frequencies close to different rotational speeds are sequentially decreased without eccentricity, the vibration of the rotor in the x-direction is mainly close to \( 3f_r \), and the amplitude of the vibration close to \( 1f_r \) and \( 2f_r \) is also large only the UMP act on rotor with eccentricity at 4500rpm, however, the vibration frequency are 2.93Hz, 69.21Hz, 74.91Hz, 141.36Hz and 225Hz, respectively and close to \( 1f_r \), \( 2f_r \), and \( 3f_r \) under the UMP and
mass centrifugal force action. Their limit cycles were more complicated. In conclusion, this paper shows the vibrations of 1-time, 2-time and 3-time components of the mechanical rotation frequency are generated. There is electromechanical vibration with the rotor eccentricity and the UMP is obviously on the rotor through the test with the model YKZ7230 motorized spindle. It is helpful for analyzing and detecting the rotor eccentricity of PMSM motorized spindle. Particularly, it will be helpful for detecting faults of motorized spindles that have frequency eccentricity according to development, assembly or use.

6. References

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