Multi-objective cost optimization for geological risk oriented rail transit route selection

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Abstract: China's rail transit has entered a period of rapid development, but related safety and risk management fail to match with high-speed construction. The complex and uncertain underground space leads to frequent risk accidents of urban rail transit. As the primary task of rail transit development, the reasonable route selection can not only avoid a lot of geological risks, but also greatly save the duration and cost. In order to guide the rail transit route selection work scientifically and reasonably, the cost problems related to geological risk factors by constructing the objective function of risk assessment, section construction cost and duration are analyzed in this study. Combining with heuristic algorithms such as ant algorithm and particle swarm optimization algorithm, an evaluation method of cost optimization of rail transit route selection for the geological risk are put forward. Taking the geological data of a certain section of a proposed rail transit route in Guangzhou as an example, based on this proposed method, the optimal route selection scheme with relatively low risk, short construction duration and low cost is found, which provides theoretical basis and guidance for the route selection and construction of rail transit, and also verifies the rationality of the application of the cost optimization method for the geological risk to multi-objective engineering management.

Key words: Rail transit; Multi-objective optimization; Ant algorithm; Particle swarm optimization algorithm; Cost analysis

1 Introduction

Rail transit route selection is the top priority of rail transit development, which directly affects many issues such as project investment, operating cost, construction difficulty, etc. To save time and labor in the process of rail transit route selection and to save materials and labor in the process of construction, it is necessary to carry out multi-objective optimization evaluation for the geological risk level, construction period and cost of geological risk treatment in the rail transit route selection area, so as to achieve the purpose of reasonable and scientific route selection.

Multi-objective optimization problem is a problem in which multiple objectives interact with each other and jointly determine the engineering results. It can be divided into two kinds: one is the traditional method of solving multi-objective optimization problem based on single objective optimization. Zadeh¹ put forward the multi-objective linear weighting method, which gave certain weight to each objective factor, and synthesized the multi-objective optimization problem into a new single objective optimization problem. However, limited by the factors such as the interrelation among multiple objectives, if one objective is chosen as the best, it often loses the performance of other indicators, resulting in the consequence that the method often has greater randomness and uncertainty. Xiong and Kuang¹ proposed an improved ant colony algorithm with high global search capability, which also gave a certain weight to each objective factor, and transformed the multi-objective cost optimization problem into a single one.

The other one is based on the heuristic algorithm to solve multi-objective optimization problem. Heuristic algorithm mainly includes simulated annealing algorithm, ant algorithm and particle swarm optimization algorithm, etc. In view of the shortcomings of particle swarm optimization, genetic algorithm and immune algorithm were used to provide initial speed and position for particle swarm optimization, and an improved particle swarm optimization algorithm was formed by You¹, Charnes et al.¹ and Tamiz et al.¹ proposed the objective
programming method, which set a threshold value for each objective of the multi-objective optimization problem, and then took the threshold value as the constraint condition of the multi-objective function into the problem of minimizing the absolute difference between the function value and the threshold value. Liu and Rahbar \cite{6} used the theory of "maximum flow and minimum cut" to find "minimum compression set", so as to compress the duration with the minimum cost increase. Naidu \cite{7} used the quality cost theory to solve the problem of quality optimization. According to the Taguchi quality loss function theory, the quality cost was calculated by studying the curve of quality assurance cost and quality loss cost, and to achieve the goal of minimum cost by finding the optimal quality level. Buddhakulsomsiri and Kim \cite{8} and Kolisch and Hartmann \cite{9}, respectively, put forward corresponding heuristic algorithms on the basis of studying the priority sequencing of project activities and the adjustment of project schedule under the condition of resource constraints, providing a new solution for the optimization of duration-cost. Yang et al. \cite{10} established a method to solve the multi-objective optimization problem of duration-cost by combining the comprehensive fuzzy set theory and genetic algorithm, striving to find out the optimal solution of duration-cost under different risk conditions quickly and realize the optimization of multi-objective cost. Tian \cite{11} chose the deviation analysis method to realize the analysis of project cost by establishing an effective link between cost and schedule. Bai et al. \cite{12} proposed an improved ant colony algorithm combined with chaos based on ant colony algorithm. By introducing certainty and uncertainty search rules, the shortcomings of ant colony algorithm, such as premature, stop and local optimum, were solved. The improved ant colony algorithm can effectively improve the global optimization ability of the ant colony algorithm, and the solution of the duration-cost optimization problem can get better results.

According to the existing research results, multi-objective comprehensive optimization methods covering quality, duration, cost and other factors have been proposed for the optimization of project cost at home and abroad. However, these optimization studies are mainly aimed at a certain stage of design, construction, operation and maintenance, and only a few of them focus on the full life cycle cost optimization methods, which is unfortunately oriented to the construction field. For the rail transit with long lines and wide areas enclosed, the use of existing cost analysis methods is limited. Therefore, objective of this study is to solve the multi-objective cost optimization problems of geological risk evaluation, section construction duration and cost in rail transit route selection. A method combined the ant algorithm and particle swarm optimization algorithm is established in this paper to solve the multi-objective cost optimization method for rail transit route selection with regard to geological environment risk.

2 Multi-objective cost optimization for geological risk-oriented rail transit route selection

2.1 Multi-objective optimization problem

A multi-objective optimization problem can be constituted by D parameters of decision variables, N objective functions and \( m + n \) constraints, such as equations 1 and 2. In the non inferior solution set, the decision maker can only choose a satisfactory non inferior solution as the final solution according to the specific problem requirements.

\[
\min y = F(x) = (f_1(x), f_2(x), f_3(x), \ldots, f_n(x)) \quad n = 1, 2, \ldots, N \quad (1)
\]

\[
\begin{align*}
g_i(x) & \leq 0 & i = 1, 2, 3, \ldots, m \\
h_j(x) & = 0 & j = 1, 2, 3, \ldots, k \\
x & = (x_1, x_2, \ldots, x_d, \ldots, x_D) \\
x_{d_{min}} & \leq x_d \leq x_{d_{max}} & d = 1, 2, \ldots, D
\end{align*} \quad (2)
\]

Where, \( x \) is the D-dimensional decision vector; \( y \) is the objective vector; \( N \) is the total
number of optimization objectives; \( g_i(x) \leq 0 \) means the \( i \)th inequality constraint; \( h_j(x) = 0 \) represents the \( j \)th equality constraint; \( f_n(x) \) is the \( n \)th objective function. \( g_i(x) \leq 0 \) and \( h_j(x) = 0 \) determine the feasible region of the solution, while \( x_{d_{max}} \) and \( x_{d_{min}} \) are the upper and lower bounds of each dimensional vector search.

2.2 Heuristic multi-objective optimization algorithm

Based on the heuristic algorithm, the multi-objective optimization problem for geological risk-oriented rail transit route selection is solved by the cross fusion of ant algorithm and particle swarm optimization algorithm in this study. Ant algorithm is a kind of self-organized, essentially parallel, positive feedback and strong robustness algorithm. As shown in Figure 1, the corresponding optimal solution is obtained through the ant algorithm. That is, in the process of searching for a path, ants will randomly choose a path when they encounter the cross with the same pheromone; then, the ant will release pheromone which can continue to volatilize in the chosen path. When the path is longer (e.g., route 1 to m in Figure 1), the pheromone will volatilize more, and the remaining pheromone concentration will be lower. The following ants will choose the path with higher pheromone concentration. Thus, the short (optimal) path (e.g., route 1 to 2 in Figure 1) can be obtained. This searching method can provide the starting position of optimal search for the particle swarm optimization algorithm and greatly enhances the operation efficiency of particle swarm optimization algorithm and the effect of finding the optimal solution.

![Figure 1. Schematic diagram of ant algorithm search.](image)

Particle swarm optimization can search for non inferior solutions in parallel with efficient clustering method, and generate multiple non inferior solutions in each iteration process of particles in various groups; in the process of algorithm implementation, particles in each group have memory function, so that particles can track the optimal solution in the group and their own optimal solution during space search. The algorithm does not depend on the function features of the optimization problem and the formal features of the solution. The combination of ant algorithm can provide a better search starting position for particle swarm optimization, and then improve the convergence speed of each subordinate particle swarm. Particle swarm optimization can make up for the shortcoming that ant algorithm falls into local optimum in the process of solving multi-objective optimization solution after solving single objective non inferior solution.

3 Realization of multi-objective cost optimization

The establishment of multi-objective cost optimization model for geological risk-oriented rail transit route selection is mainly divided into two parts. Firstly, the ant algorithm is used to optimize the geological risk assessment, section construction period and section construction cost involved in rail transit route selection, separately. Then, these optimization objectives obtained by ant algorithm are inserted into particle swarm optimization as three subordinate.
particle swarms; one principal particle swarm is also set to realize the final multi-objective cost optimization. The specific implementation for the cost optimization model of rail transit route selection oriented to geological risk is shown in Figure 2.

**Figure 2.** Modeling process of cost optimization model of rail transit route selection for geological risk.

### 3.1 Objective functions for geological risk assessment, section construction cost and duration

The combined influences of geological risk assessment, section construction cost and duration are explored in this paper. The related objective functions are as follows.

The objective function of the geological risk assessment is,

\[
\min f_R(x) = \frac{\sum_{j=1}^{n} H_j}{n}
\]

where, \( f_R(x) \) represents the eigenvalue of the average risk level of \( n \) sections of rail transit; \( n \) is the number of sections of the route; \( H_j \) is the eigenvalue of the risk level in the \( j^{th} \) interval, which is calculated according to Wu et al.\(^{[13]}\)

The objective function of section construction duration is,

\[
\min f_D(x) = \max_{k=1}^{K} \left\{ L_{k\times n} \cdot t_{ij} \left(x_{ij}\right) \right\}
\]

where, \( f_D(x) \) represents the time spent in handling the risk of unfavorable geological environment and construction in section \( j \), \( i \) route selection scheme; \( L_{k\times n} \) represents a path from the start point to the end point. \( x_{ij} \) represents the \( j \) interval of the \( i \) route selection scheme.

The objective function of section construction cost is,

\[
\min f_C(x) = \sum_{i=1}^{n} d_{C_{ij}} \left(x_{ij}\right) \max_{k=1}^{K} \left\{ L_{k\times n} \cdot t_{ij} \left(x_{ij}\right) \right\} \times I_c
\]

where, \( d_{C_{ij}} \) represents the direct costs of personnel, materials, machinery, etc. for handling the adverse geological environment risks and construction in the \( j \) interval of the \( i \) route selection scheme; \( I_c \) represents indirect cost.

### 3.2 ant algorithm

The ant algorithm is utilized to realize the optimal selection of single objective, i.e., geological risk assessment, construction cost and duration for the rail transit route selection. Five steps are included,

1) Initialization: initializing pheromone, searching taboo list, setting reasonable target threshold and establishing solution library.

2) Path construction by ant. At time \( t \), the transfer probability of ant \( k \) from one route...
scheme in section \( i \) to one line scheme in section \( j \) is \( p^k_{ij}(t) \), which can be calculated according to equation 6.

\[
p^k_{ij}(t) = \begin{cases} 
\sum_{\text{settabuk}} \left[ \alpha \eta_{ij}(t) \right]^\beta & \text{if } j \notin \text{tabuk} \\
0 & \text{else}
\end{cases}
\]

(6)

Where, \( \text{tabuk} \) is the taboo list of ant \( k \), \( \alpha \) is the information heuristic factor to reflect the relative importance of ant's trajectory; \( \beta \) is the expected heuristic factor; \( \eta_{ij}(t) \) is heuristic function. \( \eta_{ij}(t) = \frac{1}{d_{ij}} \) represents the expected degree of ants from one section of route scheme \( i \) to one section of line scheme \( j \) at time \( t \), and \( d_{ij} \) is the “distance” from one section of route scheme \( i \) to one section of line scheme \( j \) at time \( t \) by ants.

3) Adjustment of pheromone. When all ants have completed a cycle, the pheromone content of each path will be updated and adjusted according to the following equation 7.

\[
\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \Delta \tau_{ij}
\]

(7)

Where, \( \rho \) is the pheromone volatilization factor; \( 1-\rho \) is the pheromone residue factor; \( \Delta \tau_{ij} \) is the sum of the pheromone increments of the path \( (i, j) \) after the \( k \)th ant passes through the cycle of path \( (i, j) \), and \( \Delta \tau_{ij} = \sum_{k=1}^{m} \Delta \tau_{ij}^k \).

4) Verification of terminal conditions. After all kinds of group search, if the calculated target value is greater than the predetermined target threshold, the solution is added to the non-dominated solution set. Otherwise, the pheromone is updated and the search continues.

5) Output of optimal value. The optimal objectives of three groups, namely, geological risk assessment, section construction duration and section construction cost of rail transit route selection, are output respectively.

3.3 particle swarm optimization algorithm

The particle swarm optimization algorithm is then used to make the geological risk assessment target, section construction cost and duration approach the overall target optimally, which can be divided into four steps, as shown in Figure 3.

![Figure 3. Implementation flow of particle swarm optimization algorithm.](image-url)
1) Three subordinate particle swarms are set up for the geological risk evaluation, cost optimization and duration optimization based on the results from ant algorithm, and one main particle swarm is built at the same time, and the external reserve set is also added.

2) The solutions of multi ant algorithms are used to initialize each subordinate particle and determine the initial velocity \( v \) and initial position \( x \) of each particle. The non inferior solution obtained by ant algorithm search process is stored in the external reserve set.

3) The main particle swarm optimization search is started to realize the parallel and efficient search subordinate between particle swarm and main particle swarm to improve the search efficiency and calculation accuracy. The specific model of speed and position search is shown in equations 8 and 9.

\[
v_{ij}(t + 1) = \omega v_{ij}(t) + r_1 C_1 \left( p_{ij}(t) - x_{ij}(t) \right) + r_2 C_2 \left( g_j(t) - x_{ij}(t) \right) \tag{8}
\]

Where, \( v_{ij}(t) \) and \( v_{ij}(t + 1) \) represent the update speed of the subordinate particle swarm with \( t \) and \( t + 1 \) iterations, respectively; \( \omega \) is the inertia factor, that is, the global optimization ability is strong and the local optimization ability is weak when \( \omega \) is large; \( C_1 \) and \( C_2 \) are acceleration constants, generally, \( C_1 = C_2 \in [0, 4] \). \( r_1 \) and \( r_2 \) represent random number on interval \([0, 1]\); \( p_{ij}(t) \) represents the optimal position coordinate of the \( i^{th} \) particle in the \( j^{th} \) dimension; \( g_j(t) \) represents the global optimal position coordinate of the whole particle swarm in the \( j^{th} \) dimension.

\[
x_{ij}(t + 1) = x_{ij}(t) + v_{ij}(t + 1) \tag{9}
\]

Where, \( x_{ij}(t) \) and \( x_{ij}(t + 1) \) represent the update location of the subordinate particle swarm with \( t \) and \( t + 1 \) iterations, respectively.

4 Case study

Taking a certain route of a proposed rail transit in Guangzhou as an example, the geological data\(^{(14)}\) in the preliminary exploration stage are analyzed for the route selection for cost optimization oriented to geological risk. The primary factor to select the route is the cost as well as considering the reasonable construction duration and the geological risk through the method discussed in sections 2 and 3. The route is between two stations, with a total length of 2.38km, and divided into four sections, i.e., section 1, 2, 3 and 4 in Table 1. Each section has three alternative route schemes (1, 2, 3). From top to bottom, the strata in this area are miscellaneous fill, silt, fine sand, silty clay, fine sandstone, siltstone and coarse sandstone. Among them, the risks faced by section 3 and section 4 are mainly karst caves and faults. The section cost in this study mainly includes the construction cost and the indirect management cost. Table 1 shows the risk level, section duration, cost and data survey summary of route selection.

The cost optimization model of rail transit route selection based on ant algorithm and particle swarm optimization algorithm for geological environment risk is adopted. The optimization analysis mainly considers the geological risk level (eigenvalue H), the total construction duration and the total cost. The target threshold is set as total eigenvalue \( f_H(x) \geq 2.36 \), total construction period \( f_T(x) \leq 1370d \), total cost \( f_C(x) \leq 156 \text{ million RMB} \). The valid data within the threshold range is picked out, that is, from 81 combination schemes, 7 combination schemes that satisfy the target threshold, as shown in Table 2. Compared with these 7 schemes with the cost as the first consideration, the 4\(^{th} \) scheme with the lowest total cost and satisfactory geological risk level and total construction duration is recommended. However, the total construction duration is 15 days longer than the shortest one.
Table 1. Summary of the geological risk level, construction duration and cost for different route sections.

| Route section | Route scheme | Risk level (eigenvalue H) | Construction duration/(day) | Construction cost/(million RMB) |
|---------------|-------------|--------------------------|-----------------------------|--------------------------------|
| 1             | 1           | Medium risk (0.547)      | 321                         | 39.022                         |
|               | 2           | Medium risk (0.537)      | 318                         | 36.832                         |
|               | 3           | Medium risk (0.545)      | 328                         | 37.564                         |
| 2             | 1           | Low risk (0.601)         | 340                         | 39.960                         |
|               | 2           | Medium risk (0.575)      | 345                         | 38.892                         |
|               | 3           | Medium risk (0.592)      | 363                         | 40.363                         |
| 3             | 1           | Medium risk (0.585)      | 349                         | 39.452                         |
|               | 2           | Low risk (0.611)         | 343                         | 37.250                         |
|               | 3           | Medium risk (0.573)      | 370                         | 40.520                         |
| 4             | 2           | Medium risk (0.542)      | 351                         | 39.582                         |
|               | 3           | Low risk (0.662)         | 356                         | 40.416                         |

Noted that eigenvalue H is 0~1. The values of 0~0.35, 0.35~0.6, 0.6~0.85 and 0.85~1.0 represent high, medium, low and small risk.

Table 2. Optimization results of route selection scheme in geological risk treatment of rail transit.

| Combination scheme | Route section | Total eigenvalue | Total duration/(day) | Total cost/(million RMB) | Recommendations |
|--------------------|---------------|------------------|----------------------|--------------------------|----------------|
| 1                  | ①②②③        | 2.395            | 1365                 | 155.58                   | --             |
| 2                  | ②①②①        | 2.374            | 1347                 | 154.132                  | --             |
| 3                  | ②①②③        | 2.411            | 1357                 | 154.458                  | --             |
| 4                  | ②②②③        | 2.385            | 1362                 | 153.39                   | recommend      |
| 5                  | ②③②①        | 2.365            | 1370                 | 154.535                  | --             |
| 6                  | ③①②①        | 2.382            | 1357                 | 154.864                  | --             |
| 7                  | ③①②③        | 2.419            | 1367                 | 155.19                   | --             |

5 Conclusions

In this paper, a multi-objective cost optimization method for rail transit route selection is established. Aiming at the geological risk, cost and duration problems in rail transit route selection, the objective functions of geological risk assessment, section construction cost and duration are constructed. This multi-objective cost optimization problem is solved through the
cross fusion modeling of ant algorithm and particle swarm optimization algorithm. According to the geological data of a certain section of a proposed rail transit route in Guangzhou, in order to avoid the possible geological risks for rail transit, based on the multi-objective cost optimization model of rail transit route selection, a relatively low-risk, low-cost, short-term, scientific and reasonable route selection scheme is recommended, which has guiding significance for the selection and construction of rail transit.

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