Partially massless gravitons do not destroy general relativity black holes

Richard Brito,1 Vitor Cardoso,1,2,3,4 and Paolo Pani1,5

1CENTRA, Departamento de Física, Instituto Superior Técnico, Universidade Técnica de Lisboa - UTL, Avenida Rovisco Pais 1, 1049 Lisboa, Portugal
2Faculdade de Física, Universidade Federal do Pará, 66075-110, Belém, Pará, Brazil
3Perimeter Institute for Theoretical Physics Waterloo, Ontario N2J 2W9, Canada
4Department of Physics and Astronomy, The University of Mississippi, University, MS 38677, USA.
5Institute for Theory & Computation, Harvard-Smithsonian CfA, 60 Garden Street, Cambridge, MA, USA

Recent nonlinear completions of Fierz-Pauli theory for a massive spin-2 field include nonlinear massive gravity and bimetric theories. The spectrum of vacuum solutions in these theories is rich, and it comprises the same black-hole solutions of Einstein’s gravity enlarged to include a cosmological constant. It was recently shown that Schwarzschild (de Sitter) black holes in these theories are generically unstable against spherical perturbations. Here we show that a notable exception is partially massless gravity, where the mass of the graviton is fixed in terms of the cosmological constant by \( \mu^2 = 2\Lambda/3 \) and a new gauge invariance emerges. We find that general-relativity black holes are stable in this limit. Remarkably, the spectrum of massive gravitational perturbations is isospectral.

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I. INTRODUCTION

Triggered by several circumstances, the last couple of years has witnessed a flurry of activity on theories with a propagating massive graviton. Such theories come under different flavors, such as “nonlinear massive gravity,” a nonlinear generalization of the linear Fierz-Pauli theory [1 2], and “bimetric theories of gravity” which propagate two dynamical spin-2 fields [3 4]. With a very special status, “partially massless (PM) theories” have also been considered for which the graviton mass is constrained to take a specific value dictated by the cosmological constant by \( \mu^2 = 2\Lambda/3 \) and a new gauge invariance emerges [7–13].

The spectrum of vacuum solutions in these theories is rich, and it comprises the same black-hole solutions of Einstein’s gravity [6 14]. However, it was very recently shown that general-relativity BHs are dynamically unstable in these theories [15 16]. The instability is due to a propagating spherically symmetric degree of freedom and affects BHs with or without a cosmological constant [16]. As discussed below, such degree of freedom is absent in PM gravity [5 8 17], so one might wonder whether Schwarzschild–de Sitter (SdS) BHs are stable in such theories. The purpose of this study is to show that this is the case.

II. SETUP

Let us consider the propagation of a massive spin-2 field on a curved spacetime. We assume the background to be a vacuum solution of Einstein’s equations with a cosmological constant \( \Lambda \) [hereafter we use \( G = c = \hbar = 1 \) units]:

\[
\bar{R}_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} + \Lambda \bar{g}_{\mu\nu} = 0, \tag{1}
\]

so that \( \bar{R} = 4\Lambda, \bar{R}_{\mu\nu} = \Lambda \bar{g}_{\mu\nu} \) and bar quantities refer to the background. The most general linearized action describing a massive spin-2 field minimally coupled to gravity in this background reads [see Ref. [18] for a review]

\[
S = \frac{1}{16\pi} \int d^4x \sqrt{|g|} \left[ -\frac{1}{2} \nabla_{\alpha} h_{\mu\nu} \nabla^\alpha h^{\mu\nu} + \nabla_{\alpha} h_{\mu\nu} \nabla^\alpha h_{\mu\nu} - \nabla_{\mu} h \nabla_{\nu} h + \frac{\bar{R}}{4} \left( h_{\mu\nu} h^{\mu\nu} - \frac{1}{2} h^2 \right) - \frac{\mu^2}{2} \left( h_{\mu\nu} h^{\mu\nu} - h^2 \right) \right], \tag{2}
\]

where \( h = \bar{g}_{\mu\nu} h^{\mu\nu} \). From this action the field equations read:

\[
\mathcal{E}_{\mu\nu} h_{\rho\sigma} - \Lambda \left( h_{\mu\nu} - \frac{\bar{g}}{2} \bar{g}_{\mu\nu} \right) + \frac{\mu^2}{2} \left( h_{\mu\nu} - \bar{g}_{\mu\nu} h \right) = 0, \tag{3}
\]

where we have defined the operator

\[
-2\mathcal{E}_{\mu\nu} = (\delta^\rho_{\mu} \delta^\alpha_{\nu} - \delta^\rho_{\nu} \delta^\alpha_{\mu}) \bar{\Box} + \bar{g}^{\alpha\beta} \bar{\nabla}_\alpha \bar{\nabla}_\beta - 2\delta^\alpha_{(\mu} \bar{\nabla}^\beta_{\nu)} + \bar{\nabla}_{\mu} \bar{\nabla}_{\nu} \bar{\nabla}^\alpha, \tag{4}
\]

and index round brackets denote symmetrization.

Finally, by taking the divergence and the trace of Eq. (3) and using the tensorial relation

\[
(\bar{\nabla}_c \bar{\nabla}_d - \bar{\nabla}_d \bar{\nabla}_c) h_{ab} = \bar{R}_{aceb} h^{ce} + \bar{R}_{beab} h^{ce}, \tag{5}
\]

the linearized equations reduce to the system:

\[
\left\{ \begin{array}{l}
\bar{\Box} h_{\mu\nu} + 2\bar{R}_{\alpha\mu\nu} h^{\alpha\beta} - \mu^2 h_{\mu\nu} = 0, \\
\mu^2 \bar{\nabla}_{\alpha} h_{\mu\nu} = 0, \\
(\mu^2 - 2\Lambda/3) h = 0.
\end{array} \right. \tag{6}
\]
Since we are interested in astrophysical scenarios, here after we assume $\Lambda \geq 0$. The linearized theory generically propagates five degrees of freedom, corresponding to the healthy helicities of a massive spin-2 field, and it does not contain Boulware-Deser ghosts \[19\]. The value
\[ \mu^2 = 2\Lambda/3 \] plays a special role in asymptotically dS spacetimes and it is known as Higuchi limit \[7, 8, 17\]. When $\mu^2 < 2\Lambda/3$, the helicity-0 mode becomes itself a ghost, whereas when $\mu^2 > 2\Lambda/3$ all propagating degrees of freedom are physical. In the Higuchi limit the tracelessness of $h_{\mu\nu}$ is not enforced by the field equations \(6\) and an extra gauge symmetry can be used to eliminate the helicity-0 mode. In this particular case, known as PM gravity \[10, 11\], the graviton propagates only four helicities. The attempt to find a nonlinear completion of PM gravity using the framework of nonlinear massive gravity has been shown to suffer from some obstructions \[11, 13\], nevertheless advances in finding a consistent full nonlinear theory of PM gravity have been recently made in the context of bimetric theories \[3, 20, 22\].

Although the setup above describes a “probe” spin-2 field minimally coupled to standard gravity, it is in fact very generic. Several nonlinear completions of massive gravity – including the recent proposals of Refs. \[1–3\] – admit general-relativity background solutions as in Eq. \(11\). Furthermore, the linearized field equations can be written as Eq. \(4\), where $\mu$ and $\Lambda$ depend on the specific parameters of the theory \[6, 10\]. Finally, Eqs. \(9\) are the only field equations that consistently describe a massive spin-2 field coupled to gravity in generic backgrounds with constant curvature \[23\].

### A. The Schwarzschild-de Sitter geometry

The most general static solution of Eq. \(11\) is the SdS spacetime, described by $ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega^2$, where \[24\]
\[ f = \frac{\Lambda}{3r} (r - r_b)(r_c - r)(r - r_0), \]
with $r_0 = -(r_b + r_c)$, $r_b$ and $r_c > r_b$ being the BH horizon and the cosmological horizon, respectively. The cosmological constant can be expressed as $3/\Lambda = r_b^2 + r_b r_c + r_c^2$ and the spacetime has mass $M = \Lambda r_b r_c (r_b + r_c)/6$. The surface gravity $\kappa_b$ associated with the BH horizon reads \[24\]
\[ \kappa_b \equiv \frac{f'(r_b)}{2} = \frac{\Lambda (r_b - r_c)(r_b - r_0)}{6r_b}. \]

### B. Field equations in PM gravity

In a spherically symmetric background, Eqs. \(6\) can be conveniently decomposed in a complete basis of tensor spherical harmonics. Perturbations separate into two sets, which are called “axial” and “polar” according to their parity. In Fourier space this can be written as \[16, 26\]
\[ h^{\mu\nu}(t, r, \theta, \phi) = \sum_{l, m} \int_{-\infty}^{+\infty} e^{-i\omega t} \left[ h^{\text{axial},lm}_{\mu\nu}(\omega, r, \theta, \phi) + h^{\text{polar},lm}_{\mu\nu}(\omega, r, \theta, \phi) \right] d\omega, \]
where
\[ h^{\text{axial},lm}_{\mu\nu}(\omega, r, \theta, \phi) = \begin{pmatrix} h_{00}^{lm}(\omega, r) \csc \theta \partial_\phi Y_{lm}(\theta, \phi) & -h_{10}^{lm}(\omega, r) \sin \theta \partial_\theta Y_{lm}(\theta, \phi) \\ 0 & h_{11}^{lm}(\omega, r) \csc \theta \partial_\phi Y_{lm}(\theta, \phi) & -h_{10}^{lm}(\omega, r) \sin \theta \partial_\theta Y_{lm}(\theta, \phi) \\ 0 & 0 & h_{20}^{lm}(\omega, r) \sin \theta \partial_\phi Y_{lm}(\theta, \phi) \end{pmatrix}, \]
\[ h^{\text{polar},lm}_{\mu\nu}(\omega, r, \theta, \phi) = \begin{pmatrix} f(r) H_0^{lm}(\omega, r) Y_{lm} & f(r)^{-1} H_2^{lm}(\omega, r) Y_{lm} & \eta_0^{lm}(\omega, r) \partial_\phi Y_{lm} \\ f(r)^{-1} H_2^{lm}(\omega, r) Y_{lm} & f(r) H_0^{lm}(\omega, r) Y_{lm} & \eta_1^{lm}(\omega, r) \partial_\phi Y_{lm} \\ 0 & 0 & \eta_2^{lm}(\omega, r) \partial_\phi Y_{lm} \end{pmatrix}, \]
\[ W_{lm}(\theta, \phi) = \partial_\theta^2 Y_{lm} - \cot \theta \partial_\phi Y_{lm} - \csc^2 \theta \partial_\phi^2 Y_{lm}. \]

As we showed in a recent work, SdS geometries are generically unstable against a monopole fluctuation \[16\]. In the asymptotically flat case, the instability is equiva-
lent to the Gregory-Laflamme instability \cite{laflamme_GR} of a black string \cite{benn}. Thus, general-relativity BHs cannot describe static solutions in theories whose linearized equations reduce to Eqs. (10).

However, a notable exception to this outcome is given by PM theories, which are defined by the tuning \cite{pm}. In this case, the helicity-0 mode which is responsible for the instability can be gauged away and the theory propagates four degrees of freedom \cite{pm,pm1,pm2}.

The approach we developed in Ref. \cite{pm} can be easily extended to obtain a set of coupled master equations that fully characterize the linear stability properties of the background. The axial sector is described by the following system:

\begin{equation}
\frac{d^2}{dr^2} Q + \left[ \omega^2 - f \left( \frac{\lambda + 4}{r^2} - \frac{16M}{r^3} \right) \right] Q = S_Q, \tag{15}
\end{equation}

\begin{equation}
\frac{d^2}{dr^2} Z + \left[ \omega^2 - f \left( \frac{\lambda - 2}{r^2} + \frac{2M}{r^3} \right) \right] Z = S_Z, \tag{16}
\end{equation}

where \( \lambda = l(l+1) \) and we have defined the tortoise coordinate \( r_* \) via \( dr/dr_* = f \). The functions \( Q(r) \equiv f(r)h_1 \) and \( Z(r) \equiv h_2/r \) are combinations of the axial perturbations, whereas the source terms are given by

\[ S_Q = (\lambda - 2) \frac{2f(r - 3M)}{r^3} Z, \quad S_Z = \frac{2}{r^2} f Q. \tag{17} \]

The polar sector can be simplified by using an extra gauge symmetry arising in the Higuchi limit \cite{higuchi}. In this case the field equations \cite{higuchi} are invariant under \cite{higuchi}

\[ h_{\mu\nu} \rightarrow h_{\mu\nu} + \left( \nabla_\mu \nabla_\nu + \frac{\Lambda}{3} g_{\mu\nu} \right) \xi \tag{18} \]

where \( \xi \) is a generic scalar gauge function of the spacetime coordinates. The symmetry above can be used to enforce \( \eta_0 \equiv 0 \) in the decomposition \cite{pm}. In this gauge, the polar sector is fully described by a system of two coupled ordinary differential equations:

\begin{equation}
f^2 \frac{d^2 \eta_1}{dr^2} + \hat{\alpha}_1 \frac{d \eta_1}{dr} + \hat{\beta}_1 \eta_1 = S_{\eta_1}, \tag{19}
\end{equation}

\begin{equation}
f^2 \frac{d^2 G}{dr^2} + \hat{\alpha}_2 \frac{d G}{dr} + \hat{\beta}_2 G = S_G, \tag{20}
\end{equation}

where the source terms are given by

\[ S_{\eta_1} = (\lambda - 2) \hat{\sigma}_1 \frac{dG}{dr} + (\lambda - 2) \hat{\rho}_1 G, \tag{21} \]

\[ S_G = \hat{\sigma}_2 \frac{d\eta_1}{dr} + \hat{\rho}_2 \eta_1. \tag{22} \]

The coefficients \( \hat{\alpha}_i, \hat{\beta}_i, \hat{\sigma}_i, \hat{\rho}_i \) are radial functions which also depend on \( \omega, l, r_b \) and \( r_c \).

The regular asymptotic solutions of the axial and polar systems are ingoing and outgoing waves, \( \Psi \rightarrow e^{\pm i\omega r_*} \), at the BH horizon and at the cosmological horizon, respectively [\( \Psi \) collectively denotes the master functions \( Q, Z, \eta_1 \) and \( G \)]. The complex eigenfrequencies \( \omega = \omega_R + i\omega_I \), that simultaneously satisfy these boundary conditions are called quasinormal modes \cite{qnm}.
III. RESULTS

A. The near-extremal SdS geometry

The above equations are in general not analytically solvable. Fortunately, the geometry is sufficiently rich so that it admits a special limit where one can indeed considerably simplify the equations and solve them analytically. This regime is the near extremal SdS BH, defined as the spacetime for which the cosmological horizon \( r_c \) is very close (in the \( r \) coordinate) to the BH horizon \( r_h \), i.e. \( \frac{r_c - r_h}{r_h} \ll 1 \). As shown in Ref. [24], it is possible to solve analytically a large class of Schrödinger-type equations in this background by adopting a perturbative approach in powers of \( r_c - r_h \). In the near-extremal limit, one can make the following approximations: 

\[
0 \sim -2r_b^2; \quad \Lambda \sim r_b^{-2}; \quad M \sim \frac{r_h}{3}; \quad \kappa_b \sim \frac{r_c - r_h}{2r_b^2}.
\]

The key point of the approximation is to realize that in the near-extremal regime \( r \sim r_h \sim r_c \), as we are interested only in the region between the two horizons. Then, \( r - r_0 \sim r_h - r_0 \sim 3r_0 \) and thus

\[
r \sim \frac{r_c e^{2\kappa_b r} + r_h}{1 + e^{2\kappa_b r}}, \quad f \sim \frac{(r_c - r_h)^2}{4r_b^2 \cosh(\kappa_b r)}\quad (24)
\]

Finally, the equations for massive gravitational perturbations of near-extremal SdS geometries reduce to

\[
\frac{d^2 \Psi}{d r^2} + \left[ \omega^2 - \frac{\kappa_b^2 U_0}{\cosh(\kappa_b r)} \right] \Psi = 0, \quad (25)
\]

where \( \Psi \) can be any of the metric variables. The potential \( U_0 \) is the same for the axial metric functions \( Q, Z \) but it is different for the polar functions \( \eta_1, G \). We find

\[
U_0 = \begin{cases} 
-4/3 + \lambda, & \text{axial} \\
27\lambda^3 + 36\lambda(9r_c^2\omega^2 - 1) - 16, & 3(9\lambda^2 + 12\lambda + 36r_c^2\omega^2 + 4) \\
& 27\lambda^3 - 36\lambda + 72r_c^2\omega^2 - 16, & \text{polar I} \\
& 3(9\lambda^2 + 12\lambda - 36r_c^2\omega^2 + 4), & \text{polar II}
\end{cases}
\]

The potential in \( (25) \) is the well known Pöschl-Teller potential \( [27] \). The solutions of the corresponding Schrödinger-like equations were studied and they are of the hypergeometric type, (for details see Section 4.1 in Ref. [25]). The eigenfrequencies are given by \( [24, 27] \)

\[
\frac{\omega}{\kappa_b} = -\left( n + \frac{1}{2} \right) i + \sqrt{U_0 - \frac{1}{4}}, \quad n = 0, 1, \ldots
\]

Using Eq. \( (26) \), we obtain

\[
\frac{\omega}{\kappa_b} = -\left( n + \frac{1}{2} \right) i + \sqrt{l(l + 1) - \frac{19}{12}}, \quad n = 0, 1, \ldots
\]

for both axial and polar perturbations. Thus, we get the surprising result that in this regime all perturbations are isospectral \([24, 28]\). Because the polar potential \( U_0 \) is frequency-dependent, we also find a second, spurious, root at \( \omega = \pm i \frac{3 + \sqrt{3}}{6r_0} \). It is easy to check analytically that at these frequencies the wavefunction is not regular at one of the horizons, thus it does not belong to the spectrum.

B. Numerical results

Using two independent techniques (a matrix-valued direct integration and a matrix-valued continued-fraction method \([27]\)), we have numerically obtained the quasinormal spectrum for generic SdS geometries, looking explicitly for unstable modes, i.e., modes for which \( \text{Im}(\omega) > 0 \) and which therefore grow exponentially in time while being spatially bounded. Our results are summarized in Fig. \( 1 \), where we overplot the near-extremal analytical result (denoted by a black dashed line) and the \( \Lambda \rightarrow 0 \) limit (denoted by horizontal lines). Since \( \Lambda \propto \mu^2 \), this limit corresponds to the massless limit of PM gravity, i.e. to general relativity.

Except for \( l = 1 \) modes, axial and polar perturbations are grouped in two different families which, in the \( \Lambda \propto \mu^2 \rightarrow 0 \) limit, reduce to gravitational and electromagnetic modes of a Schwarzschild BH, respectively. As predicted by our near-extremal analysis, the two families merge in the \( r_c \rightarrow r_h \) limit. For \( l = 1 \) there is only one single family which reduces to the dipole electromagnetic modes of a Schwarzschild BH in the \( \Lambda \propto \mu^2 \rightarrow 0 \) limit.

An intriguing result, which would merit further study is the fact that axial and polar modes have exactly the same quasinormal-mode spectrum for any value of the cosmological constant, up to numerical accuracy. We were not able to produce an analytical proof of this. Isospectrality guarantees that the entire quasinormal spectrum can be obtained from the axial equations \([15] \) and \([16] \) only.

To summarize, our numerical results are in excellent agreement with independent analytical/numerical analysis on two opposite regimes, the general-relativity limit when the cosmological constant vanishes and the near-extremal limit when the two horizons coalesce. We found no hints of instabilities in the full parameter space.

IV. DISCUSSION

In addition to the spectrum of stable modes presented above, we have searched for unstable, exponentially-growing modes and found none. Thus, our analysis provides solid evidence for the linear stability of SdS BHs in PM gravity. This is in contrast with generic theories of massive spin-2 fields (including massive gravitons) in which SdS BHs are unstable \([13, 16] \).

From our results and from those of Refs. \([13, 16] \) the following interesting picture emerges. If a theory of massive gravity allows for the same BH solutions of general
relativity, the latter are unstable against spherical perturbations. This is the case for Schwarzschild BHs in any consistent theory of a massive spin-2 field, including the recent nonlinear massive gravity [1, 2] and bimetric theories [3], with or without a cosmological constant. The end-state of the instability is an interesting open problem but it is likely that the instability triggers static BHs to develop “graviton hair”.

We have shown here that a notable exception to this picture is represented by PM gravity, which is obtained by enforcing the constraint (1). The unstable monopole is absent in this theory and a complete analysis of non-spherical modes has revealed no instability. Remarkably, the spectrum of massive gravitational perturbations is isospectral in this theory, which is another piece of evidence for its special role within the family of massive gravities.

Note added in proof: After this work has been accepted, a paper appeared [30] which argues against the existence of a consistent nonlinear PM completion in bimetric theories (see also Ref. [31]). Modifying the kinetic term of the full nonlinear action could be a possible way out as recently proposed in [32]. Note however that our linear analysis applies to any nonlinear generalization of the Fierz-Pauli theory.

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