Dark energy, curvature, and cosmic coincidence

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Abstract

The fact that the energy densities of dark energy and matter are similar currently, known as the coincidence problem, is one of the main unsolved problems of cosmology. We present here a model in which a spatial curvature of the universe can lead to a transition in the present epoch from a matter dominated universe to a scaling dark energy dominance in a very natural way. In particular, we show that if the exponential potential of the dark energy field depends linearly on the spatial curvature density of a closed universe, the observed values of some cosmological parameters can be obtained assuming acceptable values for the present spatial curvature of the universe, and without fine tuning in the only parameter of the model. We also comment on possible variations of this model, and realistic scenarios in which it could arise.

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I. INTRODUCTION

The cosmological scenario that has emerged in the last decade indicates that we live in a universe which is almost spatially flat, as indicated by the cosmic microwave background (CMB) experiments [1, 2, 3]. We have also learned from observations [4, 5, 6] that its energy content is comprised by around 4% of baryons, and 26% of cold dark matter [7, 8], both clumped in galaxies and clusters of galaxies. The remaining 70% of its energy is homogeneously distributed in the universe and is causing its observed acceleration [9, 10, 11, 12], a component that was generically dubbed dark energy [13, 14, 15].

This recipe for the universe gives rise to what became known as the cosmic coincidence problem: why are dark energy and dark matter energy densities of the same order of magnitude in the present epoch? This problem arises because dark energy must have a negative pressure to accelerate the universe, and cold dark matter (as well as baryons) has vanishing pressure. Therefore, the ratio of their energy densities $\rho$, for a constant equation of state $\omega_\phi$ of dark energy, must vary as

$$\frac{\rho_M}{\rho_\phi} = \frac{\rho_{M0}}{\rho_{\phi0}} (1 + z)^{-3\omega_\phi} \approx \frac{(1 + z)^3}{2},$$

where $z$ is the redshift, $M$ denotes (barionic plus cold dark) matter, $\phi$ denotes the dark energy field, and the index 0 indicates the present value of a quantity.

Several dark energy candidates have been proposed, from the most obvious, the cosmological constant [16, 17], to modifications of gravity [18, 19, 20, 21]. Following the inflationary idea, "regular" scalar fields, the so-called quintessence models, noncoupled [22, 23, 24, 25, 26, 27, 28] and coupled [29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39] to dark matter content, were also proposed to be the cause of the acceleration, as well as fluids that do not obey the weak energy condition [40, 41, 42, 43, 44, 45], scalar fields with non-canonical kinetic terms [46, 47, 48, 49, 50] and tachyonic fields [51, 52, 53, 54, 55], just to cite a few examples. These two last models have an advantage with respect to others in what concerns the coincidence, in the sense that the equation of state of the field changes to a cosmological constant-like as the background changes from radiation to matter domination, but apparently they have a very small parameter space that can generate relevant cosmological solutions [54, 55].

In this sense, what models do, in general, is to fine tune the overall scale of the potential of dark energy (or the scale in which modifications of general relativity become important)
to be of order of the present critical density, \( \rho_c = 3m_p^2H_0^2 = 8.1h^2 \times 10^{-47} \text{ GeV}^4 \), where \( H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1} \) is the Hubble constant, \( h = 0.72\pm0.08 \) \( ^{[56]} \), and \( m_p = (8\pi G)^{-1/2} = 1.221 \times 10^{18} \text{ GeV} \) is the reduced Planck mass.

This fine tuning emerges even when one is using tracking or scaling \( ^{[57, 58, 59, 60]} \) properties of some potentials, like the exponential potential \( ^{[22, 23, 24, 57, 61, 62]} \) we will focus on in this letter. In this sense, we have recently showed \( ^{[37]} \) that coupling dark energy with dark matter also does not solve the problem, since one has to adjust the value of the potential in very much the same way one does for the uncoupled quintessence.

Another general assumption when one is modelling dark energy is that the universe is flat, that is, curvature effects can be neglected. The Friedmann equation in this case becomes

\[
H^2 + \frac{k}{a^2} \approx H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{1}{3m_p^2} \left( \rho_M + \rho_R + \rho_\phi \right),
\]

where \( a(t) \) is the cosmic scale factor, dot indicates derivatives with respect to cosmic time \( t \), and \( k \) is related to the curvature via Robertson-Walker metric (+1, 0 or -1 for a closed, flat or open universe, respectively). \( H \) is the Hubble parameter, which present value is the Hubble constant, and \( R \) denotes the radiation component. This equation can then be rewritten as \( \Omega_M + \Omega_R + \Omega_\phi = 1 - \Omega_k \approx 1 \), where \( \Omega_i = \rho_i/\rho_c = \rho_i/3m_p^2H^2 \) are the density parameters, and we have defined \( \rho_k \equiv -3m_p^2k/a^2 \). The energy densities are given by the conservation of the energy-momentum tensor for each component separately, \( \dot{\rho}_i + 3H\rho_i(1 + \omega_i) = 0 \), where \( \omega_M = 0, \omega_R = 1/3 \) and \( \omega_k = -1/3 \). The Lagrangian of the scalar field is the usual one, \( \mathcal{L} = \partial_\mu \phi \partial^\mu \phi/2 - V \), and consequently, for a homogeneous perfect fluid, the equation of state of dark energy is \( \omega_\phi = p_\phi/\rho_\phi = (\dot{\phi}^2/2 - V)/(\dot{\phi}^2/2 + V) \).

The flatness of the universe is one of the main predictions of inflation \( ^{[63]} \), since during an inflationary period \( \Omega_k \) vanishes quasi-exponentially. However, recently some interest has been given to the possibility that the spatial curvature of the universe be non-negligible \( ^{[64, 65, 66]} \), both for theoretical (see, for instance, \( ^{[67, 68, 69, 70, 71]} \) and references therein) and observational reasons \( ^{[4, 5, 64, 65, 72, 73, 74]} \). A positive spatial curvature plus a cosmological constant could, for instance, mimic a phantom regime \( ^{[64]} \). Another interesting point is that CMB experiments \( ^{[1, 2, 3, 6]} \), even when combined with different astronomical data \( ^{[5, 6, 75]} \), present a tendency for some small positive spatial curvature of the universe, result that can be checked soon using other observations \( ^{[76, 77, 78, 79, 80, 81, 82, 83]} \).

In this letter, we explore the possibility that the universe has a global positive spatial...
curvature in the context of a quintessence model. As we will show, if the potential is dependent on the curvature, there is no coincidence in the fact that the transition from a matter dominated universe to a dark energy dominated universe is happening currently, as well as the fact that the dark energy field behaves almost like a cosmological constant. Instead, they are a consequence of the fact the curvature is becoming important in the present epoch. It should be clear, however, we do not intend explaining why the curvature is becoming important right now. If observations really rule out flat models someday (what for sure is not the case today), this fact would be a different coincidence problem, the old flatness problem. We adopt here the approach of checking what kind of solutions one could obtain if observations indicate a non-flat universe.

Once the dark energy attractor regime is reached, after being triggered by the curvature, the universe accelerates forever, and the curvature again become negligible, in a kind of self-flattening process. In the following section we describe the model, stressing its phenomenological nature. We then present an analytical solution for the model, discuss its main properties, and possible variants of it. Finally, in the conclusions some realistic scenarios in which those models could arise are briefly discussed.

II. DARK ENERGY AND CURVATURE

The models presented here are based on the assumptions that the universe presents a small positive curvature, and that the quintessential potential depends linearly on it, that is,

\[ V(\phi, \rho_k) = -\rho_k e^{\lambda \phi / m_p} = -\Omega_k \rho_c e^{\lambda \phi / m_p}, \]

(3)

where the negative sign comes from the fact \( \Omega_k \) is negative for a closed universe, that is what we consider in what follows. Since the "curvature density" obeys the fluid-like equation ~[106], we can write \[ V(\phi, u) = -\rho_{k0} e^{-2u} e^{\lambda \phi / m_p}, \] where \( u = \ln(a/a_0) = -\ln(1 + z), \) and \( \rho_{k0} \equiv -3m_p^2k/a_0^2. \)

Based on the conditions, we will see that it is possible to explain the current cosmological scenario with natural values for the only parameter of the potential, \( \lambda, \) without fine tuning.

It should be clear that such a dependence on the curvature is phenomenological, although in the last section we point out some contexts in which it may arise. In this section, however, we are interested in verifying what kind of solutions one could obtain when such dependence
is present. Notice also that modifying the exponential potential is not something new in the literature \[84, 85\], although the dependence on the curvature is.

The scalar field equation can then be written as

\[ H^2 \phi'' + \frac{1}{2m_p^2} \left( \rho_M + \frac{2}{3} \rho_R + 2V + \frac{4}{3} \rho_k \right) \phi' = \left( \frac{2}{\phi'} - \frac{\lambda}{m_p} \right) V, \]

where prime denotes derivatives with respect to \( u \), and the Hubble term, equation (2), is given by

\[ H^2 = \frac{(\rho_M + \rho_R + V + \rho_k)/3m_p^2}{1 - \phi'^2/6m_p^2}. \]

### III. RESULTS AND DISCUSSION

The set of equations (4) and (5) can then be solved numerically. It will reach a scaling solution \[58\], and therefore the final solutions are almost independent of the initial conditions. The results for the density parameters of the components of the universe for a typical solution are shown in the top panel of figure 1.

Notice that the transition to the scaling regime is happening around the present epoch, but differently of all models of dark energy, there is no fine tuning in the overall scale of the potential. Instead, the transition is triggered by the fact that curvature is becoming important currently. The system then reaches the accelerating scaling solution, as can be seen in the bottom panel of the same figure, and when this happens the curvature again becomes negligible.

The scaling solution can be obtained from equations (4) and (5) when one realizes the field will dominate completely the energy density of the universe. Since in a scaling solution we have \( \phi'' = 0 \), equation (4) becomes

\[ \frac{\phi'}{m_p} = \frac{2}{\phi'} - \frac{\lambda}{m_p}, \]

which presents a solution,

\[ \frac{\phi'}{m_p} = \frac{\lambda}{2} \left( \sqrt{1 + \frac{8}{\lambda^2}} - 1 \right). \]

A more complete dynamical analysis will be presented elsewhere. Here, we will focus on this solution, that presents scaling properties, that is, the equation of state of the dark energy field is constant and different from the one of background \[58\]. Its value can be obtained using the fact that in the scaling regime both kinetic and potential terms of the scalar field scale in the same way, \( e^{-3(1+\omega_\phi)u} \propto e^{-2u + \lambda \phi/m_p} \), and therefore,

\[ \omega_\phi = -\frac{1}{3} - \frac{\lambda}{3} \frac{\phi'}{m_p} = -\frac{1}{3} - \frac{\lambda^2}{6} \left( \sqrt{1 + \frac{8}{\lambda^2}} - 1 \right). \]
FIG. 1: (Color online) Top panel: Density parameters of the components of the universe as a function of $u = -\ln(1 + z)$ for a typical solution (here, $\lambda = 4$). Notice the transition to a dark energy dominated phase is happening by now, but on the contrary to other models of quintessence, this is due to the fact the universe is non-flat, and not to a fine tuning in the parameters. The upper solid curve indicated by total corresponds to $\Omega_t = \Omega_R + \Omega_M + \Omega_\phi$. Bottom panel: Equation of state of the dark energy. Dashed curve shows the scaling value, equation (7).

The behavior of the equation of state for a typical solution is presented in the bottom panel of figure 1, for $\lambda = 4$. For values of $\lambda \gg 2\sqrt{2}$, the dark energy field practically mimics a cosmological constant. Contrary to what happens in the case of a regular exponential potential [22, 23, 24, 57, 61, 62], where a large fine tuning on $\lambda$ is needed to obtain $\omega_\phi \rightarrow -1$, here almost all values of $\lambda$ generate acceptable values for the equation of state of dark energy. In fact, expanding the square root of (7) for small $8/\lambda^2$ up to second order, one gets that $\omega_\phi \approx -1 + 8/3\lambda^2$, from where one can see it really approaches the cosmological constant value as $\lambda$ increases.

However, the transition still is happening in the present epoch, as we clearly can see from figure 1. Because of that, the scaling values have not yet been reached (although they almost
have), and one needs to verify to which values of the curvature reasonable cosmological parameters can be obtained. In what follows, we looked for models with $\Omega_{M0} = 0.3 \pm 0.1$ and $h = 0.72 \pm 0.08$ [4, 5, 56].

In order to do that, we have solved numerically the system of equations varying $\Gamma_{k0} \equiv |\Omega_{k0}| h_{72}^2$ in the range $[10^{-6}; 0.2]$ ($h_{72} = h_n / 0.72$, where $h_n$ is the Hubble constant obtained numerically), and $\lambda$ in the range $[0.01, 120]$, varying both stepsizes to get better resolution and faster calculation. The results are shown in figure 2.

Top panel shows the present value of the equation of state of dark energy versus $\lambda$ for models that satisfy the cited constraints. The values are not exactly the ones given by equation (7), since the transition has not been completed, but they are very close. Bottom panel presents the modulus of the spatial curvature today for the allowed models. A large curvature would imply an equation of state incompatible with observations. However, curious enough, for values of the curvature within the current errors on its measured value, the field presents an equation of state in agreement with observations.
Before concluding, it should be pointed out that similar models might be considered, like for instance a modified Peebles-Ratra potential, \( V = -\rho_k (m_p/\phi) \alpha \), or another variations of the exponential potential like \( V = -\Omega_k m^4 e^{\lambda \phi/m_p} \), or \( V = -\rho_k (m_p/\phi)^\alpha e^{\lambda \phi^2/m_p^2} \), based on a variation of supergravity and supersymmetric models \[86, 87\]. Besides that, it might be clear that similar models can be constructed using \( \Omega_k < 1 \) (open models), although here we have chosen to follow the indications given by the CMB observations.

IV. REALISTIC MODELS

It is important to stress that the dependence of the potential on the curvature is assumed here, and therefore it is crucial to verify whether it could be obtained in the context of a realistic particle physics model or in a modification of general relativity. In this sense, a particular direction which seems promising is obtained in the context of models with a scalar field non-minimally coupled to the Ricci scalar \[88\]. For a Friedmann-Robertson-Walker universe, the Ricci scalar is given by \( \mathcal{R} = -6(\dot{H} + 2H^2 + k/a^2) \), and it is conceivable that it does exist a Lagrangian which is a combination of non-minimal couplings with the scalar invariants (like \( F_1(\phi)\mathcal{R}, F_2(\phi)\mathcal{G}, F_3(\phi)/\mathcal{R} \), etc., where \( F_i(\phi) \) are functions of the field \( \phi \) and \( \mathcal{G} = R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} - 4R_{\mu\nu}R^{\mu\nu} + \mathcal{R}^2 \) is the so-called Gauss-Bonnet term) that can result on a dependence of the potential with the curvature density, but not on \( \dot{H} \) and \( H^2 \). If this is the case, the dependence has the form given in eq. \[107\].

Such kind of terms arise naturally in the extensions of the standard model of particle physics, for instance in effective actions coming from superstring theory \[89, 90, 91, 92, 93, 94, 95\]. In string theory the field \( \phi \) correspond to moduli fields (which generally appears in the form \( \exp(\lambda \phi/m_p) \)), and a combination of the several parameters appearing in such models \[91, 92, 93, 94\] also can potentially lead to the kind of cosmological solutions discussed here.

Another possible way is obtained in the so-called \textit{macroscopic gravity approach} \[96\]. The averaged Einstein equations for a spatially flat, homogenous, and isotropic universe have the form of normal Friedmann equations plus a curvature term induced by the gravitational correlations terms. A comoving observer would therefore measure the universe to be spatially curved. In this way, a coupling of the scalar field with the correlation tensor could in principle lead to a coupling with a curvature-like term as the one studied here.
This is similar to what happens in the case of the *regional averaging* \[97, 98\]. In this case, the curvature-like term is generated by a volume effect and by curvature backreaction \[99, 100\]. Coupling the scalar field with the new terms of the measured cosmological parameters might also result in a potential like the one given by equation \(3\). These and other possibilities are currently under investigation, and seem very promising to generate realistic models with the properties discussed here.

One should note that, independently on the way the model is generated, the general behavior of the models described here will be maintained. This can be seen from the fact that its main property (the fact that the spatial curvature triggers the dark energy dominance) comes only from the fact that the energy densities of matter, “curvature”, and of the potential energy of the dark energy field scales respectively as \(a^{-3}, a^{-2}, \text{ and } a^0\). These scalings are independent of the gravity model, since two of them come only from the conservation of the energy-momentum tensors, and the other one only from the fact the curvature scales as \(a^{-2}\), which is a dimensional argument. Therefore, the conclusions of the present work are expect to hold even for more general theories and modifications of gravity, and should be considered as a possible alternative to understand the coincidences which plague the present cosmological model.

V. CONCLUSIONS

We have presented a phenomenological model of dark energy in which its potential depends on the positive spatial curvature of the universe, assumed to be closed. The model presents a scaling behavior that is triggered by the fact the curvature is becoming non-negligible around the present epoch, and, in this sense, there is no coincidence in the fact \(\Omega_{\phi0} \approx \Omega_{M0} \[108\]. Notice that it is a testable model, since the scenario would lose its appeal if observations indicate \(\Omega_{k0}\) is zero with enough accuracy.

In this sense, an important feature of the model is that values of the spatial curvature that give rise to reasonable cosmological parameters (like \(\Omega_{M0}, \omega_{\phi0}\) and \(h\)) are within current uncertainties in the observations of \(\Omega_{k0}\). Besides that, for all values of \(\lambda\) sufficiently high (\(\lambda \gtrsim 2\sqrt{2}\)) the model behaves almost like a cosmological constant, showing there is no fine tuning on this parameter. Further investigations are needed to check if such a model would still survive to tests from different observations, and more realistic models will allow one to
infer different features of such a scenario.

Of course it is important to be cautious and to keep in mind that, until now, all the cosmological data are totally compatible with a flat universe which energy density is dominated by the cosmological constant. That is probably the simplest scenario one can think to current cosmology, ignoring the coincidence. Whether alternatives like the one described here are cosmologically reasonable is something that depends on the forthcoming observations, especially the ones able to probe a possible deviation from flatness on the spatial curvature of universe.

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[1] D. N. Spergel et al., Astrophys. J. Supp. 148, 175 (2003), astro-ph/0302209.
[2] C. J. MacTavish et al., astro-ph/0507503.
[3] D. N. Spergel et al., astro-ph/0603449.
[4] W. L. Freedman and M. S. Turner, Rev. Mod. Phys. 75, 1433 (2003), astro-ph/0308418.
[5] M. Tegmark et al., Phys. Rev. D 69, 103501 (2004), astro-ph/0310723.
[6] U. Seljak, A. Slosar and P. McDonald, astro-ph/0604335.
[7] J. M. Overduin and P. S. Wesson, Phys. Rept. 402, 267 (2004), astro-ph/0407207.
[8] G. Bertone, D. Hooper and J. Silk, Phys. Rept. 405, 279 (2005), astro-ph/0404175.
[9] A. G. Riess et al., Astron. J. 116, 1009 (1998), astro-ph/9805201.
[10] S. Perlmutter et al., Astrophys. J. 517, 565 (1999), astro-ph/9812133.
[11] A. G. Riess et al., Astrophys. J. 607, 665 (2004), astro-ph/0402512.
[12] P. Astier et al., astro-ph/0510447.
[13] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. 75, 559 (2003), astro-ph/0207347.
[14] T. Padmanabhan, Phys. Rept. 380, 235 (2003), astro-ph/0212290.
[15] E. J. Copeland, M. Sami and S. Tsujikawa, astro-ph/0603057.
[16] S. Weinberg, Rev. Mod. Phys. 61, 1 (1989).
[17] S. M. Carroll, Living Rev. Rel. 4, 1 (2001), astro-ph/0004075.
[18] C. Deffayet, G. Dvali and G. Gabadadze, Phys. Rev. D 65, 044023 (2002), astro-ph/0105068.
[19] F. Perrotta, C. Baccigalupi and S. Matarrese, Phys. Rev. D 61, 023507 (2000), astro-ph/9906066.
[20] C. Baccigalupi, S. Matarrese and F. Perrotta, Phys. Rev. D 62, 123510 (2000), astro-ph/0005543.
[21] S. Matarrese, C. Baccigalupi and F. Perrotta, Phys. Rev. D 70, 061301(R) (2004), astro-ph/0403480.
[22] C. Wetterich, Nucl. Phys. B 302, 668 (1988).
[23] P. G. Ferreira and M. Joyce, Phys. Rev. Lett. 79, 4740 (1997), astro-ph/9707286.
[24] P. G. Ferreira and M. Joyce, Phys. Rev. D 58, 023503 (1998), astro-ph/9711102.
[25] B. Ratra and P. J. E. Peebles, Phys. Rev. D 37, 3406 (1988).
[26] P. J. E. Peebles and B. Ratra, Astrophys. J. Lett. 325, L17 (1988).
[27] J. A. Frieman, C. T. Hill, A. Stebbins and I. Waga, Phys. Rev. Lett. 75, 2077 (1995), astro-ph/9505060.
[28] R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. 80, 1582 (1998), astro-ph/9708069.
[29] G. W. Anderson and S. M. Carroll, astro-ph/9711288.
[30] J. A. Casas, J. García-Bellido and M. Quirós, Class. Quantum Grav. 9, 1371 (1992), astro-ph/9204213.
[31] G. R. Farrar and P. J. E. Peebles, Astrophys. J. 604, 1 (2004), astro-ph/0307316.
[32] M. B. Hoffman, astro-ph/0307350.
[33] L. Amendola, Phys. Rev. D 62, 043511 (2000), astro-ph/9908023.
[34] L. Amendola and D. Tocchini-Valentini, Phys. Rev. D 64, 043509 (2001), astro-ph/0011243.
[35] L. Amendola, Mon. Not. R. Astron. Soc. 342, 221 (2003), astro-ph/0209494.
[36] D. Comelli, M. Pietroni and A. Riotto, Phys. Lett. B 571, 115 (2003), hep-ph/0302080.
[37] U. França and R. Rosenfeld, Phys. Rev. D 69, 063517 (2004), astro-ph/0308149.
[38] T. Biswas, R. Brandenberger, A. Mazumdar and T. Multamaki, hep-th/0507199.
[39] S. Nojiri and S. D. Odintsov, hep-th/0506212.
[40] R. R. Caldwell, Phys. Lett. B 545, 23 (2002), astro-ph/9908168.
[41] V. K. Onemli and R. P. Woodard, Class. Quant. Grav. 19, 4607 (2002), astro-ph/0204065.
[42] R. R. Caldwell, M. Kamionkowski, and N. N. Weinberg, Phys. Rev. Lett. 91, 071301 (2003), astro-ph/0302506.
[43] S. M. Carroll, M. Hoffman and M. Trodden, Phys. Rev. D 68, 023509 (2003), astro-ph/0301273.
[44] V. K. Onemli and R. P. Woodard, Phys. Rev. D 70, 107301 (2004), gr-qc/0406098.
[45] B. Feng, X. Wang and X. Zhang, Phys. Lett. B 607, 35 (2005), astro-ph/0404224.
[46] C. Armendariz-Picon, V. Mukhanov and P. J. Steinhardt, Phys. Rev. Lett. 85, 4438 (2000), astro-ph/0004134.
[47] C. Armendariz-Picon, V. Mukhanov and P. J. Steinhardt, Phys. Rev. D 63, 103510 (2001), astro-ph/0006373.
[48] T. Chiba, Phys. Rev. D 66, 063514 (2002), astro-ph/0206298.
[49] M. Malquarti, E. J. Copeland and A. R. Liddle, Phys. Rev. D 68, 023512 (2003), astro-ph/0304277.
[50] R. J. Scherrer, Phys. Rev. Lett. 93, 011301 (2004), astro-ph/0402316.
[51] T. Padmanabhan, Phys. Rev. D 66, 021301(R) (2002), astro-ph/0204150.
[52] M. Sami, P. Chingangbam and T. Qureshi, Phys. Rev. D 66, 043530 (2002), hep-th/0205179.
[53] J. S. Bagla, H. K. Jassal and T. Padmanabhan, Phys. Rev. D 67, 063504 (2003), astro-ph/0212198.
[54] L. R. W. Abramo and F. Finelli, Phys. Lett. B 575, 165 (2003), astro-ph/0307208.
[55] L. R. W. Abramo, Int. J. Theor. Phys. 43, 563 (2004).
[56] W. Freedman et al., Astrophys. J. 553, 47 (2001), astro-ph/0012376.
[57] E. J. Copeland, A. R. Liddle and D. Wands, Phys. Rev. D 57, 4686 (1998), gr-qc/9711068.
[58] A. R. Liddle and R. J. Scherrer, Phys. Rev. D 59, 023509 (1999), astro-ph/9809272.
[59] I. Zlatev, L. Wang and P. J. Steinhardt, Phys. Rev. Lett. 82, 896 (1999), astro-ph/9807002.
[60] P. J. Steinhardt, L. Wang, I. Zlatev, Phys. Rev. D 59, 123504 (1999), astro-ph/9812313.
[61] U. França and R. Rosenfeld, JHEP 0210, 015 (2002), astro-ph/0206194.
[62] C. Rubano et al., Phys. Rev. D 69, 103510 (2004), astro-ph/0311537.
[63] For reviews, see, e.g., A. R. Liddle and D. Lyth, *Cosmological Inflation and Large Scale Structure*, Cambridge University Press, UK (2000);
D. Lyth and A. Riotto, Phys. Rept. 314, 1 (1999), [hep-ph/9807278];

A. Linde, Particle Physics and Inflationary Cosmology, Harwood, Chur, Switzerland (1990), [hep-th/0503203].

[64] E. V. Linder, 2005 Astropart. Phys. 24, 391 (2005), [astro-ph/0508333].

[65] D. Polarski and A. Ranquet, Phys. Lett. B 627, 1 (2005), [astro-ph/0507290].

[66] J. C. N. de Araujo, Astropart. Phys. 23, 279 (2005).

[67] M. Barnard and A. Albrecht, [hep-th/0409082].

[68] G. Ellis and R. Maartens, Class. Quant. Grav. 21, 223 (2004), [gr-qc/0211082].

[69] B. Freivogel, M. Kleban, M. R. Martinez and L. Susskind, [hep-th/0505232].

[70] R. J. Adler and J. M. Overduin, Gen. Rel. Grav. 37, 1491 (2005), [gr-qc/0501061].

[71] K. Lake, Phys. Rev. Lett. 94, 201102 (2005), [astro-ph/0403119].

[72] M. White and D. Scott, Astrophys. J. 459, 415 (1996), [astro-ph/9508157].

[73] G. Efstathiou, Mon. Not. Roy. Astron. Soc. 343, L95 (2003), [astro-ph/0303127].

[74] J. P. Uzan, U. Kirchner and G. F. R. Ellis, Mon. Not. Roy. Astron. Soc. 344, L65 (2003), [astro-ph/0302597].

[75] D. Rapetti, S. W. Allen and J. Weller, Mon. Not. Roy. Astron. Soc. 360, 555 (2005), [astro-ph/0507290].

[76] D. J. Eisenstein, W. Hu and M. Tegmark, Astrophys. J. 518, 2 (1999), [astro-ph/9807130].

[77] C. Blake and K. Glazebrook, Astrophys. J. 594, 665 (2003), [astro-ph/0301632].

[78] T. Matsubara, Astrophys. J. 615, 573 (2004), [astro-ph/0408349].

[79] H. J. Seo and D. J. Eisenstein, Astrophys. J. 633, 575 (2005), [astro-ph/0507338].

[80] L. Knox, Phys. Rev. D 73, 023503 (2006), [astro-ph/0503405].

[81] G. Bernstein, Astrophys. J. 637, 598 (2006), [astro-ph/0503276].

[82] L. Knox, Y. S. Song and H. Zhan, [astro-ph/0605563].

[83] J. Guzik and G. Bernstein, [astro-ph/0605694].

[84] A. Albrecht and C. Skordis, Phys. Rev. Lett. 84, 2076 (2000), [astro-ph/9908085].

[85] M. C. Bento, O. Bertolami and N. C. Santos, Phys. Rev. D 65, 067301 (2002), [astro-ph/0106405].

[86] P. Brax and J. Martin, Phys. Lett. B 468, 40, (1999), [astro-ph/9905040].

[87] A. de la Macorra, Phys. Rev. D 72, 043508 (2005), [astro-ph/0409523].

[88] See, e. g., S. Capozziello et al., Int. J. Mod. Phys. D 4, 767 (1995);
N. Bartolo and M. Pietroni, Phys. Rev. D 61, 023518 (2000), hep-ph/9908521;
E. Gunzig et al., Class. Quantum Grav. 17, 1783 (2000);
D. F. Torres, Phys. Rev. D 66, 043522 (2002), astro-ph/0204504;
V. Faraoni, Annals Phys. 317, 366 (2005) gr-qc/0502015; and references therein.

[89] T. Damour and A. M. Polyakov, Gen. Rel. Grav. 26, 1171 (1994), gr-qc/9411069.
[90] I. Antoniadis, J. Rizos and K. Tamvakis, Nucl. Phys. B 415, 497 (1994), hep-th/9305025.
[91] J. Hwang and H. Noh H, Phys. Rev. D 61, 043511 (2000), astro-ph/9909480.
[92] C. Cartier, J. Hwang and E. J. Copeland, Phys. Rev. D 64, 103504 (2001), astro-ph/0106197.
[93] J. Hwang and H. Noh, Phys. Rev. D 71, 063536 (2005), gr-qc/0412126.
[94] G. Calcagni, B. de Carlos and A. De Felice, hep-th/0604201.
[95] S. Nojiri, S. D. Odintsov and M. Sami, hep-th/0605039.
[96] A. A. Coley, N. Pelavas and R. M. Zalaletdinov, Phys. Rev. Lett. 95, 151102 (2005) gr-qc/0504115.
[97] T. Buchert and M. Carfora, Class. Quantum Grav. 19, 6109 (2002), gr-qc/0210037.
[98] T. Buchert and M. Carfora, Phys. Rev. Lett. 90, 031101 (2003), gr-qc/0210045.
[99] T. Buchert and M. Carfora, gr-qc/0312621.

[100] S. Rasanen, Class. Quantum Grav. 23, 1823 (2006), astro-ph/0504005.
[101] G. F. R. Ellis, D. H. Lyth and M. B. Mijic, Phys. Lett. B 271, 52 (1991).
[102] A. Linde and A. Mezhlumian, Phys. Rev. D 52, 6789 (1995), astro-ph/9506017.
[103] G. F. R. Ellis et al., Gen. Rel. Grav. 34, 1445 (2002), gr-qc/0109023.
[104] A. Linde, JCAP 0305, 002 (2003), astro-ph/0303245.
[105] A. Lasenby and C. Doran, Phys. Rev. D 71, 063502 (2005), astro-ph/0307311.

One should keep in mind that curvature is not a real fluid, and the fact it obeys a fluid-like
equation comes only from the fact it scales as $a^{-2}$, not from the conservation of a stress-energy
tensor, like the other fluids.

[107] I acknowledge Raul Abramo for discussions on this point.

[108] Although, as stated before, one would need to explain why $\Omega_k$ is different from zero only
recently. Notice that this could be achieved, for instance, using closed inflationary models:
see, e.g., [68, 101, 102, 103, 104, 105] and references therein.