Charmonium in finite temperature lattice QCD

T. Umeda*, R. Katayama, H. Matsufuru and O. Miyamura

*Department of Physics, Hiroshima University, 1-3-1 Kagamiyama, Higashihiroshima, 739-8526, Japan
bResearch Center for Nuclear Physics, Osaka University, Mihogaoka 10-1, Ibaraki 567-0047, Japan

We study hadron properties near the deconfining transition in the finite temperature lattice QCD. This paper focuses on the heavy quarkonium states, such as $J/\psi$ mesons. We compare the meson correlators above and below $T_c$ and discuss the possibility of the $c\bar{c}$ bound state by observing the wave function.

1. Introduction

It is widely believed that the Quantum Chromodynamics (QCD) exhibits a phase transition at some temperature $T_c$, and quarks and gluons confined in the low temperature phase are liberated to form “quark gluon plasma”. At the beginning of 2000, CERN reported that the QGP state had been created in the heavy ion collision experiment. In this experiment, the $J/\psi$ suppression is regarded as a key signal of QGP formation. More reliable inspection will be performed in RHIC project at BNL.

On the theoretical side, in spite of various approaches, we are still far from definite understanding of charmonium properties near the transition and the fate of the hadronic states in the plasma phase. We investigate this problem using lattice QCD, which enables us to incorporate the nonperturbative effect of QCD.

2. Our approach

The difficulties in studying the heavy quarkonium state at finite temperature can be categorized into two classes: (i) difficulties to obtain detailed information on the heavy quarkonium correlators at finite temperature, and (ii) analyses of the obtained correlators. Although it is difficult to overcome the latter, the former is possible to be solved by employing the techniques developed in recent years.

At high temperature, sufficient information on the temporal correlator requires high resolution in the temporal direction. To achieve large temporal cutoff with limited computational resources, we employ the anisotropic lattice, on which the temporal lattice spacing is finer than the spatial one. Calculations with a heavy quark need further implementation. In this case, the $J/\psi$ suppression is regarded as a key signal of QGP formation. More reliable inspection will be performed in RHIC project at BNL.

On the theoretical side, in spite of various approaches, we are still far from definite understanding of charmonium properties near the transition and the fate of the hadronic states in the plasma phase. We investigate this problem using lattice QCD, which enables us to incorporate the nonperturbative effect of QCD.

The latter problems, (ii), are much more difficult to be overcome. This is because one is inevitably enforced to extract the spectral properties from the data at the short temporal distance, where high frequency component of the dynamics is significant. At present, there is no well-established way of lattice simulations to attack the spectroscopy at $T > 0$. For this reason, lattice studies of hadrons at finite temperature have been argued on the spatial correlation (screening mass). Recently, QCD-TARO Collaboration (including three of us) analyzed the space-time structure of the mesonic correlators in the temporal direction to examine the temperature effect on them. They caught a sign of the bound state...
even in the deconfined phase, as well as an indication of the chiral symmetry restoration. Their strategy seems one of the best approaches also to the present subject, and is expected to give us novel information on the fate of charminium near the phase transition. Therefore we proceed the analysis with the following steps.

- At \( T = 0 \) we construct good meson operators which have large overlap with the state of interest.
- Temperature effects are observed in the meson correlator between the operators constructed in the previous step.
- We define the “wave function” in the Coulomb gauge at finite temperature, and observe its \( T \)-dependence below and above \( T_c \) to discuss the possibility of the bound state at \( T > T_c \).

3. Fermilab action on anisotropic lattice

The quark action we use is in the following form.

\[
S_F = \sum_{\vec{x},\vec{y}} \bar{q}(\vec{x}) K(\vec{x}, \vec{y}) q(\vec{y}),
\]

\[
K(\vec{x}, \vec{y}) = \delta_{\vec{x}, \vec{y}} - \kappa_\tau \{ (1 - \gamma_4) T_{+4} + (1 + \gamma_4) T_{-4} \} - \kappa_\sigma \sum_i \{ (r - \gamma_4) T_{+i} + (r + \gamma_4) T_{-i} \} - (\kappa_\sigma c_E g_{4i} F_{4i}(x) + r \kappa_\sigma c_B g_{ij} F_{ij}(x)) \delta_{\vec{x}, \vec{y}},
\]

where \( T_{\pm \mu} = U_{\pm \mu}(x) \delta_{\vec{x}, \vec{y}, \gamma \mu \nu} \). The bare anisotropy \( \gamma_F \) of the fermion field appears in the ratio of the spatial and the temporal hopping parameters, \( \kappa_\tau = \gamma_F \kappa_\sigma \). The Wilson parameter in this form is \( r = 1/\xi, \) where \( \xi \) is the renormalized anisotropy: \( \xi = a_\tau/a_\sigma \). The hopping parameters are related to the bare quark mass as \( \kappa_\sigma = 1/2(m_0 + \gamma_F + 3r) \). We also define \( \kappa \) with \( 1/\kappa = 1/\kappa_\sigma - 2(\gamma_F + 3r - 4) \) as the parameter which plays the same role as \( \kappa \) on the isotropic lattice. As the field strength \( F_{\mu \nu} \), we use the standard clover-leaf definition. At the tree level, the coefficients of the clover terms, \( c_E \) and \( c_B \), are unity.

This action is obtained by generalizing the form in [4] to the anisotropic lattice, and by setting \( r = 1/\xi \). This choice of the Wilson term is adopted so that the action restores the axial intercharge symmetry in the physical unit as \( m_0 \to 0 \).

| \( \kappa \) | \( \gamma_F \) | \( m_P \) \([\text{GeV}]\) | \( m_V \) \([\text{GeV}]\) |
|---|---|---|---|
| 0.0971 | 3.629 | 3.51(16) | 3.56(16) |
| 0.1013 | 3.703 | 3.07(14) | 3.13(14) |
| 0.1055 | 3.765 | 2.67(12) | 2.74(12) |

Table 1

The quark parameters and the meson masses in the physical unit. The error of the meson masses include the statistical error of the scale \( a_\tau^{-1} \).

Therefore we can use the same form in the light quark region as the \( O(a) \) improved Wilson quark action [4]. This is the reason that our form of the action is different from the form used in [3].

On the anisotropic lattice, we need additional tuning of the parameters called as “calibration”, to ensure the renormalized anisotropy is same for the gauge and the fermion fields [4]. We define the fermionic anisotropy \( \gamma_F \) using the dispersion relation of the pseudoscalar and the vector mesons and tune the value of \( \gamma_F \) so that \( \gamma_F \) coincides with the gauge field anisotropy \( \xi \).

We incorporate the mean-field improvement [4]: \( U_i \to U_i/u_{0\sigma} (i = 1,2,3) \) and \( U_4 \to U_4/u_{0\tau} \) with the mean-field values of the spatial and the temporal link variables, \( u_{0\sigma} \) and \( u_{0\tau} \) defined in the Landau gauge.

4. Result at zero temperature

Numerical calculation is done on the lattice of sizes \( 12^3 \times N_t \) where \( N_t = 72, 20, 16 \) and 12, with the standard Wilson gauge action at the coupling \( \beta = 5.68 \) and the bare anisotropy \( \gamma_G = 4.0 \) in the quenched approximation [3]. On \( N_t = 72 \) lattice, the renormalized anisotropy and the spatial lattice spacing from the string tension are determined as \( \xi = 5.3(1) \) and \( a_\sigma^{-1} = 0.85(3) \) GeV. \( N_t = 20, 16 \) and 12 lattices correspond to the temperatures \( T \simeq 0.93 T_c, 1.15 T_c \) and 1.5 \( T_c \), respectively. We calculate the meson correlators at \( T = 0 \) with the three sets of the parameters \((\kappa, \gamma_F)\) listed in Table [4].

Each value of \( \gamma_F \) is determined by the calibration procedure mentioned above. Observing the resultant meson masses summarized in Table [4], \((\kappa, \gamma_F) = (0.1013, 3.703)\) roughly corresponds to the charm quark. In the successive analysis, we
Figure 1. Effective mass plot for the diagonalized correlators in the vector channel.

use this set of parameters.

To optimize the meson correlator at $T = 0$, we employ the variational analysis. This is implemented by diagonalizing the correlator matrix

$$C_{ij}(t) = \sum_{\vec{x}} \langle O_i(\vec{x}, t) O_j^\dagger(0) \rangle.$$  

(3)

The operator $O_i(\vec{x}, t)$ is defined with the smearing function $\varphi$ in the Coulomb gauge as

$$O_i(\vec{x}, t) = \sum_{\vec{y}} \bar{\psi}(\vec{x} + \vec{y}, t) \varphi_i(\vec{y}) \Gamma q(\vec{x}, t),$$  

(4)

where the $4 \times 4$ matrix $\Gamma$ specifies the quantum number of the meson. As the smearing function $\varphi_i$, we use the eigenfunction obtained by solving the Schrödinger equation for the $S$ state,

$$\left[ -\frac{1}{2m_R} \frac{d^2}{dr^2} + V(r) \right] y(r) = Ey(r),$$  

$$y(r) = r\varphi(r),$$  

(5)

where $V(r)$ is the static quark potential obtained on the same lattice, and $m_R = 1.5/2$ [GeV] is used.

Figure 2 shows the effective mass plot at finite temperatures in the vector channel. Below $T_c$ ($N_t = 20$), the plateaus of the effective mass seem to appear. Although the effective mass of the ground state is slightly larger than that at $T = 0$, it is difficult to identify the plateau precisely and determine the mass quantitatively with present statistics. More detailed analysis with higher statistics may open a stage to discuss the potential mass shift of charmonium near to the $T_c$.

Above $T_c$ ($N_t = 16$ and 12), the correlators do not show any clear plateau in the effective mass plot. They decrease more rapidly than expected from the behavior at $N_t = 20$ in both of the correlators correspond to the ground and the first excited states at $T = 0$. The Ps channel shows similar feature, although the decrease of the effective mass is slightly milder. These behavior at least signal significant change in the nature of

5. Result at finite temperature

We start the argument at $T > 0$ with the temperature dependence of the correlator between the optimized operators at $T = 0$ defined in the previous section. Figure 2 shows the effective mass plot at finite temperatures in the vector channel. Below $T_c$ ($N_t = 20$), the correlators show plateaus beyond $t \sim 10$, though the statistical fluctuation is large for the correlator corresponding to the first excited state. We fit these data at $t = 16$--36 and find the meson masses in $2S$ states as $m_{Ps(2S)} = 0.7677(77)$ and $m_{V(2S)} = 0.7794(88)$ in the lattice unit. Although these give $1S$-$2S$ splitting smaller than the experimental value, considering the large systematic uncertainty on this coarse lattice, they are not inconsistent results. We regard the operator corresponds to the ground state in this analysis as the optimized meson operator at $T = 0$.  


the correlators when the system crosses $T_c$. In these figures, the (effective) masses increase as $T$ in the pseudoscalar and the vector channels. The observed behavior in present work, however, shows qualitatively different nature of the correlators.

To discuss the spatial correlation between quark and antiquark at $T > 0$, we observe the “wave function” normalized at the spatial origin, 

$$\phi(\vec{r}, t) = w_T(\vec{r}, t)/w_T(\vec{0}, t),$$  \hspace{1cm} (6)  

$$w_T(\vec{r}, t) = \sum_{\vec{x}} \langle \bar{q}(\vec{x} + \vec{r}, t) \Gamma q(\vec{x}, t) O(0) \rangle.$$  \hspace{1cm} (7)  

If there is no bound state, like as with the free quark propagators, this wave function $\phi(\vec{r}, t)$ broadens with $t$ for any source smearing function. In contrast, the existence of the stable shape of $\phi(\vec{r}, t)$ give us a hint on the existence of the bound state.

Figure 3 shows the $t$-dependence of the wave function normalized at the spatial origin in the vector channel at $N_t = 20$ and 12. The source function of these correlators are taken to be wider than the optimized one, to show clearly that $\phi(\vec{r}, t)$ keeps narrow shape. At the both $N_t$, the wave function gradually decrease as $t$, in contrast to the case with the free quark propagators shown together. This indicates that the quark and antiquark tend to stay together even in the deconfined phase, up to $1.5 T_c$.

These two observations are interesting, and at the same time puzzling. Obviously, the phase transition causes significant change in the charmonium correlators, especially in the vector channel. On the other hand, the result of the wave function suggests the persistence of the bound states. One natural picture is that the mesonic spectral function still have a peak with rather large width above $T_c$. This explains both of our observations. Direct extraction of the spectral function from the lattice data may give us further information. For the definite understanding of the fate of charmonium at the phase transition, we need more systematic studies and development of procedures.

The simulation has been done on Intel Paragon XP/S and NEC HSP at INSAM, Hiroshima University. This work is supported by the Grant-in-Aide for Scientific Research by Monbusho, Japan (No.10640272, No.11440080)

REFERENCES

1. NA50 Collaboration, Phys. Lett. B 477 (2000) 28.
2. T. Matsui and H. Satz, Phys. Lett. B 178 (1986) 416.
3. F. Karsch, Nucl. Phys. B 205 (1986) 285.
4. A. El-Khadra, A.S. Kronfeld and P.B. Mackenzie, Phys. Rev. D 55 (1997) 3933.
5. C. DeTar and J.B. Kogut, Phys. Rev. D 36 (1987) 2828.
6. QCD-TARO Collaboration (Ph. de Forcrand et al.), Nucl. Phys. B (Proc. Suppl.) 73 (1999) 420; B (Proc. Suppl.) 83-84 (2000) 411; hep-lat/9901017.
7. B. Sheikholeslami and R. Wohlert, Nucl. Phys. B 259 (1985) 572.
8. T.R. Klassen, Nucl. Phys. B (Proc. Suppl.) 73 (1999) 918.
9. P. Chen, hep-lat/0006019.
10. G.P. Lepage and P.B. Mackenzie, Phys. Rev. D 48 (1993) 2250.
11. T. Hashimoto et al, Phys. Rev. Lett. 57 (1986) 2123.
12. M. Oevers, in these proceedings.