Kernel spectral angle mapper

G. Camps-Valls

This communication introduces a very simple generalisation of the familiar spectral angle mapper (SAM) distance. SAM is perhaps the most widely used distance in chemometrics, hyperspectral imaging, and remote sensing applications. It is shown that a nonlinear version of SAM can be readily obtained by measuring the angle between pairs of vectors in a reproducing kernel Hilbert space. The kernel SAM generalises the angle measure to higher-order statistics, it is universal, and it has consistent geometrical properties that permit deriving a metric easily. We illustrate its performance in target detection problem using very high resolution imagery. Excellent results and insensitivity to parameter tuning over competing methods make it a valuable choice for many applications.

Introduction: The spectral angle mapper (SAM) is a method for directly comparing image spectra [1]. Since its seminal introduction, SAM is widely used in chemometrics, astrophysics, imaging, hyperspectral image analysis, industrial engineering, and computer vision applications [2-5], just to name a few. The measure achieved widespread popularity thanks to its implementation in software packages, such as ENVI or ArcGIS. The reason is simple: SAM is invariant to (unknown) multi-plicative scalings of spectra due to differences in illumination and angular orientation [6]. SAM is widely used for fast spectral classification, as well as to evaluate the quality of extracted endmembers, and the spectral quality of an image compression algorithm.

The popularity of SAM is mainly due to its simplicity and geometrical interpretability. However, the main limitation of the measure is that it only considers second-order angle dependences between spectra. This Letter generalises the SAM measure to the nonlinear case by means of kernels [7]. This short note aims to characterise the kernel SAM (KSAM) theoretically, and to show its practical convenience over the widely used linear kernel, it is universal, and it has consistent geometrical properties that permit deriving a metric easily. We illustrate its performance in target detection problem using very high resolution imagery. Excellent results and insensitivity to parameter tuning over competing methods make it a valuable choice for many applications.

Property 1 (Reproducing property): Let \( \mathcal{H} \) be a vector space with basis \( \{ e_i \} \). Then a mapping \( \mathbf{f} : \mathcal{H} \to \mathbb{R} \) is called reproducing if \( \mathbf{f}(x) = \sum_{i=1}^{\infty} \alpha_i f_i(x) \) for some \( \{ \alpha_i \} \). In particular, if \( \mathcal{H} = \mathbb{R}^d \) and \( \{ e_i \} = \{ e_i \}_{i=1}^d \) is the standard basis of \( \mathbb{R}^d \), then \( f \) is called reproducing if \( f(x) = \sum_{i=1}^d \alpha_i x_i \).

Property 2 (KSAM is a valid Mercer’s Kernel): The KSAM is a valid reproducing kernel because the arccos-operation simply expands the argument in an infinite sum of SAM distances weighted by positive scalars \( \alpha_i \), i.e., \( \theta_{K} = \sum_{k=0}^{\infty} \alpha_k \theta^k \).

Property 3 (KSAM is universal): The KSAM is a universal kernel on every compact subset of the input domain \( X \) because of the previous series expansion [8].

Geometrical interpretation of KSAM: Kernel methods may appear elusive because the mapping \( \phi \) is not explicitly defined, and the vector coordinates in the new feature spaces are not accessible. However, the framework allows to compute distances, angles, displacements, averages, and covariances implicitly in \( \mathcal{H} \) from the available data [7]. In addition, and very importantly, we show here that one can compute the metric associated to the used kernel.

For any positive definite kernel, we assume that the mapped data in \( \mathcal{H} \) are distributed in a surface \( S \) smooth enough to be considered a Riemannian manifold [9]. The line element of \( S \) can be expressed as

\[
\mathbf{ds}^2 = g_{\alpha \beta} \mathbf{d} \theta^\alpha (x) \mathbf{d} \theta^\beta (x) = g_{\mu \nu} \mathbf{d} \theta^\mu \mathbf{d} \theta^\nu,
\]

where superscripts \( \alpha, \beta \) and \( \mu, \nu \) correspond to the vector space \( \mathcal{H} \). \( g_{\alpha \beta} \) is the induced metric, and the surface \( S \) is parameterised by \( x \). Computing the components of the (symmetric) metric tensor only need the kernel function

\[
g_{\mu \nu} = \frac{1}{2} \partial_{\mu} \partial_{\nu} K(x, x) - \langle \partial_{\mu} K(x, x), \partial_{\nu} \rangle_{\mathcal{H}}.
\]

For the RBF kernel with \( \sigma \) parameter, this metric tensor becomes flat, \( g_{\mu \nu} = \delta_{\mu \nu} \sigma^2 \), and the squared geodesic distance between \( f(x) \) and \( f(x') \) simply becomes

\[
\| f(x) - f(x') \|^2 = 2 \left( 1 - \exp \left( -\frac{\| x - x' \|^2}{2 \sigma^2} \right) \right) = 2 \left( 1 - K(x, x') \right).
\]

Note that the metric solely depends on the original spectra yet computed implicitly in a higher dimensional feature space \( \mathcal{H} \), whose notion of distance is controlled by the parameter \( \sigma \): the higher the \( \sigma \) the smoother (linear) is the space. Actually, \( \sigma \to \infty \) reduces the RBF kernel to approximately compute the Euclidean distance between vectors, the metric tensor reduces to \( g_{\mu \nu} = 0 \).

Experimental evidence: A QuickBird image of a residential neighborhood of Zürich, Switzerland is used for illustration purposes. The image size is \( 329 \times 347 \) pixels. A total of 40,762 pixels were labelled by photointerpretation and assigned to nine landuses (Fig. 2). Four target detectors are compared in the task of detecting the class ‘Soil’: orthogonal subspace projection (OSP) [10], its kernel counterpart (KOSP) [11], standard SAM [1], and the extension KSAM are presented here.

Fig. 1 shows the receiver operating characteristic (ROC) curves, and the area under the ROC curves as a function of the kernel lengthscale parameter \( \sigma \). KSAM shows excellent detection rates, especially remarkable in the inset plot (note the logarithmical scale). Perhaps more importantly, the KSAM method is relatively insensitive to the selection of the kernel parameter compared with the KOSP detector, provided that a large enough value is specified.
The latter experiments allow us to use a traditional prescription to fix the RBF kernel parameter $\sigma$ for both KOSP and KSAM as the mean distance among all spectra, $d_M$. Note that after data standardisation and proper scaling, this is a reasonable heuristic $\sigma \approx d_M = 1$. The thresholds were optimised for all methods. OSP returns a decision function strongly contaminated by noise, while the KOSP detector results in a correct detection. Fig. 2 shows the detection maps and the metric learned. The (linear) SAM gives rise to very good detection but with strong false alarms in the bottom left side of the image, where the roof of the commercial centre saturates the sensor and thus returns a false positive rate.

Fig. 2 Detection results for different algorithms (accuracy, kappa statistic)

Conclusions: A nonlinear generalisation of the SAM metric was introduced. The KSAM replaces the dot product between spectra by a positive-definite kernel function. It is shown the metric induced by the kernel function and provided a geometrical intuition for tuning the kernel parameter. Experiments on a real multispectral VHR multispectral image illustrated the validity and effectiveness of the proposed method.

Acknowledgment: This paper has been partially supported by the European Research Council (ERC) under the ERC-CoG-2014 project 647423, http://www.erc.europa.eu/

© The Institution of Engineering and Technology 2016
Submitted: 3 March 2016 E-first: 13 June 2016
doi: 10.1049/el.2016.0661
One or more of the Figures in this Letter are available in colour online.

G. Camps-Valls (Image Processing Laboratory (IPL), University of València, València, Spain)
✉ E-mail: gustau.camps@uv.es

References
1 Kruse, F.A., Leikoff, A.B., Boardman, J.W., Heidebrecht, A.T., Shapiro, K.B., Barloon, P.J., and Goetz, A.F.H.: ‘The spectral image processing system (SIPS) – interactive visualization and analysis of imaging spectrometer data’, Remote Sens. Environ., 1993, 44, (2/3), pp. 145–163
2 Ball, J.E., and Bruce, L.M.: ‘Level set hyperspectral image classification using best band analysis’, Trans. Geosci. Remote Sens., 2007, 45, (10), pp. 3022–3027
3 Hecker, C., van der Meijde, M., van der Werff, H., and van der Meer, F. D.: ‘Assessing the influence of reference spectra on synthetic SAM classification results’, Trans. Geosci. Remote Sens., 2008, 46, (12), pp. 4162–4172
4 Garcia-Allende, P.B., Conde, O.M., Mirapeix, J., Cubillas, A.M., and Lopez-Higuera, J.M.: ‘Data processing method applying principal component analysis and spectral angle mapper for imaging spectroscopic sensors’, Sens. J., 2006, 8, (7), pp. 1310–1316
5 Cho, M.A., Debba, P., Mathieu, R., Naidoo, L., van Aardt, J., and Asner, G.P.: ‘Improving discrimination of savanna tree species through a multiple-endmember spectral angle mapper approach: canopy-level analysis’, Trans. Geosci. Remote Sens., 2010, 48, (11), pp. 4133–4142
6 Keshava, N.: ‘Distance metrics and band selection in hyperspectral processing with applications to material identification and spectral libraries’, Trans. Geosci. Remote Sens., 2004, 42, (7), pp. 1552–1565
7 Camps-Valls, G., and Bruzzone, L. (Eds.): ‘Kernel methods for remote sensing data analysis’ (Wiley & Sons, UK, 2009)
8 Micchelli, C.A., Xu, Y., and Zhang, H.: ‘Universal kernels’, J. Mach. Learn. Res., 2006, 6, pp. 2651–2667
9 Burges, C.J.C.: ‘Geometry and invariance in kernel based methods’, in Schölkopf, B.S.B., and Burges, C.J.C. (Eds.): ‘Advances in Kernel methods – support vector learning’ (MIT Press, Cambridge, USA, 1999)
10 Harsanyi, J.C., and Chang, C.-I.: ‘Hyperspectral image classification and dimensionality reduction: an orthogonal subspace projection’, Trans. Geosci. Remote Sens., 1994, 32, (4), pp. 779–785
11 Kwon, H., and Nasrabadi, N.: ‘A comparative analysis of kernel subspace target detectors for hyperspectral imagery’, EURASIP J. Adv. Signal Process., 2007, 2007, (29250), pp. 1–13