Mathematical modeling of changes in temperature distribution during friction stir welding in massive samples

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Abstract. A mathematical model of temperature distribution in the process of friction stir welding (FSW) of a homogeneous massive body has been developed. The model makes it possible to take into account the geometric dimensions of the tool, the thermophysical properties and the temperature distribution in the sample during the transition of the welded material into the superplastic state. The paper presents comparisons of the temperature distribution of homogeneous thin-sheet and bulky samples. Measurements temperature were carried out at various points of the hemispherical sample during the FSW process. This model is used to estimate the linear velocity of the FSW associated with the thermophysical properties of materials. The calculation results are in satisfactory agreement with the experimental data.

1. Introduction

Heat transfer in friction stir welding (FSW) is of great practical interest. In the process of FSW, the temperature in the tool entry zone and its distribution over the metal significantly affect the technological characteristics, as well as the processes of mass transfer and phase transformations. The method under consideration refers to solid-phase welding methods, where the rate of diffusion processes that provide a reliable connection of the parts to be welded depends very much on the temperature in the welding area.

Some aspects of the FSW process are still poorly understood and require further study. Many experimental researches have already been carried out to adjust the input parameters of the FSW (tool speed, welding speed and tool depth), in contrast to numerical studies that have hardly been used for this purpose. Computational tools can be useful for better understanding and visualizing the influence of input parameters on the FSW process. Visualization and analysis of material flow, temperature field, stresses and deformations associated with the process FSW can be more easily obtained using simulation results than experimental ones. Therefore, to achieve the best weld properties, simulation can help tune and optimize process parameters, and design tool. [1].

One of the main directions of research in FSW is the assessment of the temperature distribution in the welded sample [2,8]. The process temperature is below the melting point of the weld metals, but high enough for phase transformations. Typically, the temperature of the FSW is measured using thermocouples [3,4]. However, the process of measuring the temperature in the zone nugget using the above technique is a very difficult task. Numerical methods can be very effective and convenient for obtaining information on the temperature field, and in fact, over the past several years, they have been used to predict the temperature in the region of the weld in the FSW [5,11]. Riahi and Nazari present...
numerical results showing that a high temperature gradient (for an aluminum alloy) is in the area under the shoulder [5, 6].

Most of the works devoted to the calculation of the temperature distribution in the process of FSW and its dynamics are based on the solution equation of the heat conduction [7,8,9]. To solve this equation, the authors use various numerical methods implemented in the form of computer programs.

When carrying out the FSW of large welded products, it is necessary to take into account mass-dimensional size, since the distribution and outflow of heat is more intense than for thin sheet metal.

2. Mathematical model of temperature dynamics of FSW

It is proposed to carry out computer calculations of the temperature dynamics at FSW taking into account heat losses and a drop in input power based on the following simplified model. Consider a metal sample (blank) in the form of a hemisphere with a disk radius. In the approximation of zero speed of translational motion of the pin relative to the sample, it is also assumed that the heat supplied in the center of the hemisphere propagates axially symmetrically.

During the welding process, heat is released in the area of contact with the workpiece material of a rotating tool, the working part of which consists of a narrow tip (pin) and a wider disc (shoulder). Heat is partially released in the spherical region of the sample near the pin surface (Figure 1a). The paper presents a model of the dynamics of the temperature distribution for the disk (Figure 1b) [10].

Some of the heat is released on the flat surface of the sample in the area of the contact with the shoulder. The mechanical power supplied to the rotating tool is converted primarily into heat power. In the stationary welding mode, the heat power $P_G$ released in the sample depends on time, if we take into account the decrease in the material viscosity with an increase in the sample temperature in the heat release region. In non-stationary modes, the thermal power is also a function of time $P(t)$, determined mainly by the technological mode. It should also be borne in mind that part of the released heat is removed from the sample directly through the instrument. The corresponding proportion depends both on the material of the sample to be welded and on the tool itself. Typically, the share of heat dissipated through the tool is 15% to 25% of the total heat dissipated [2 - 4]. Therefore, in the proposed model of the temperature dynamics of the sample, the value of the thermal power is taken equal to $P = \alpha P_G$, where $\alpha$ is the coefficient, the values of which, depending on the material of the sample, lie in the range 0.75 - 0.85. The heat released during the welding process with power $P$ spreads over time in the radial direction of the disc. This process is accompanied by a temperature-dependent loss of energy from its open surface.

Let's compose a system of differential equations of the mathematical model of the dynamics of the temperature field of the sample. To do this, mentally select concentric spherical surfaces with non-linearly increasing radius $r_1, r_2, \ldots, r_N$ around the central cylinder (pin) with radius $r_0$ (see Figure 1a). The value of $r_1$ is taken equal to the radius of the shoulder. The radius $r_N$ of the outer cylindrical surface is equal to the outer radius of the hemisphere. Thus, the welded sample is divided into $N$
relatively narrow annular elements. An important feature of the model is that the width of the annular elements sequentially increases from the center of the hemisphere to the periphery as the temperature gradient decreases. This makes it possible to simplify the mathematical model by significantly reducing the number of equations describing the energy balance of each spherical layer bounded by the indicated spherical surfaces.

Spatial discretization of the axially symmetric temperature field is carried out as follows. The temperature of the kth ring \((k = 0, 1, \ldots N – 1)\) is taken as the value of the absolute temperature \(T_k\) on a cylindrical surface with radius \(r_k\).

The enthalpy of the ring is a function of temperature and is expressed as

\[ H_k = m_k c T_k + \text{const}, \]  

where \(c\)- specific isobaric heat capacity of the metal, \(m_k\) - mass of the k-th ring. The rate of change in enthalpy for the selected k-th ring can be represented as an equation energy balance:

\[ m_k c \frac{dT_k}{dt} = w P \delta_{k,0} + (1 - w) P \delta_{k,1} + P^a_k + P^b_k + P^r_k + P^e_k. \]  

Here, \(w\)- part of the power released in the area of contact with the pin sample from the value \(P\) of the power used to heat the sample. The rest of it stands out in the area of contact with the shoulder. \(\delta_{k,k'}\)- the Kronecker delta symbol. The mass of the k-th ring is related to its size and metal density \(\rho\) by the ratio

\[ m_k = \frac{2}{3} \pi (r_{k+1}^3 - r_k^3) \rho. \]  

The terms \(P^a_k\) and \(P^b_k\) on the right side of equation (2) represent the amount of heat received per unit time, respectively, from the \((k - 1)\)-th and \((k + 1)\)-th rings as a result of heat transfer. In accordance with the law of thermal conductivity, we obtain the expressions for them:

\[ P^a_k = -2 \pi r_k \chi \frac{(T_k - T_{k-1})}{(r_k - r_{k-1})} (1 - \delta_{k,0}), \]  

\[ P^b_k = 2 \pi r_{k+1} \chi \frac{(T_k - T_{k+1})}{(r_k - r_{k+1})}, \]  

where \(\chi\)- the thermal conductivity of the metal. Positive values of these quantities correspond to the supply, and negative values correspond to the loss of energy by the k-th ring. The power of the energy loss of the ring for thermal radiation as an absolutely gray body is

\[ P^r_k = -2 \pi \varepsilon (r_{k+1}^2 - r_k^2) \sigma (T_k^4 - T_c^4), \]  

where \(\varepsilon\)- the coefficient of absorption of radiation by the metal, \(\sigma\)- the constant Stefan-Boltzmann, and \(-T_c\)- the temperature of the environment (air). The power lost by the ring surface due to convective heat exchange with the surrounding air is represented in equation (2) by the last term equal to

\[ P^c_k = -2 \pi \omega (r_{k+1}^2 - r_k^2) (T_k - T_c), \]  

where \(\omega\) -coefficient heat transfer the metal-to-air.

Substituting expressions (3) - (7) into equation (2), which describes the energy balance of the ring element sample, we obtain, after transformations, the equation for the temperature dynamics of the k-th element in the form

\[ \frac{dT_k}{dt} = Q_k + \kappa^a_k (T_{k-1} - T_k) + \kappa^b_k (T_{k+1} - T_k) + \beta^r_k (T_k^4 - T_c^4) + \gamma^c_k (T_c - T_k). \]  

For values \(k = 0, 1, \ldots (N - 1)\), we obtain a system of \(N\) equations (8) for the temperature dynamics of spherical elements of the sample, the coefficients of which are determined by the following formulas:
The coefficients of the system of equations (8), expressed by formulas (9) - (12), are determined by the geometric parameters of the sample, the physical properties of its material and the technological conditions of the FSW. Therefore, they can be dependent both on time and on the local temperature of the sample.

The thermal power generated as a result of friction between the tool and the specimen and its part that goes into the specimen, even in the stationary mode of the FSW, can change during the welding process as the area changes temperature in the contact. Due to a decrease in viscosity with increasing temperature, the heat output decreases. This is especially noticeable when the temperature approaches the melting point of the material. Therefore, in order to approximate the mathematical model of the FSW to the real process, in formula (13), the thermal power introduced into the sample will be described by the function of the temperature contacting with the pin of the ring element.

The power released as a result of the rotation of the instrument depends on the temperature in the area of its contact with the sample material (see Figure 2). It was calculated using the modified formula Schmidt [2].

\[
P_{Q} = \left(\frac{2\pi}{3}\right)\mu\left[\delta t + (1 - \delta)\mu_{p}R_{p}\right] \left\{R_{S}^{3} - R_{P}^{3} + R_{P}^{2} \left[1 + \frac{R_{0}}{R_{p}} + \left(\frac{R_{0}}{R_{p}}\right)^{2}\right] \sqrt{H^{2} + (R_{P} - R_{S})^{2}} \right\},
\]

where \(\delta\) - the coefficient of slip between the shoulder and the workpiece (\(\delta = 0.31\)), depends on the surface treatment;

\(P_{S}\) - the pressure exerted by the tool shoulder on the workpiece;

\(R_{S}\) - radius tool shoulder;

\(R_{P}\) - the radius of the pin at the base of the tool;

\(R_{H}\) - length tool pin;

\(T\) - the absolute temperature in the area of contact of the tool with the metal.

\[
P(T_{0}) = P_{0} \left[1 - \frac{1}{\pi} \arctg \left(b(T_{0} - T_{m})\right)\right].
\]

Here \(T_{m}\) - the melting point of the sample material, \(b\) - an adjustable constant.

The value of \(b\) is selected so that the calculated characteristics are in the best agreement with the experimental data for a given metal. Acceptable values of this constant are usually in the range \((0.05 - 0.15 \text{ K}^{-1})\). The shape of the curves of the function of the dependence of the input power over temperature in the region adjacent to the pin is shown in Figure 2. The melting point corresponds to the melting point of the aluminum alloy AD1. It is seen that when the melting temperature of the metal is reached, the input power into the welded joint at the FSW tends to zero.
Figure 2. Dependence of the input power on temperature.

Figure 3. Temperature dependence of the specific heat capacity of the aluminum alloy AD1.

The specific heat also depends on the temperature of the material. During phase transitions of a substance, in particular, during melting, an abrupt change occurs in enthalpy, which is equal in magnitude to the heat of the transition. This leads to a temperature dependence of the isobaric heat capacity with a δ-shaped peak at the melting point. Therefore, the specific isobaric heat capacity \(c\), which is included in the coefficients of the system of equations (8) and (15), is modeled by the temperature function of the \(k\)-th elementary ring of the form

\[
c(T) = c' + \frac{\lambda \cdot \delta T}{\pi \cdot \left((T - T_q)^2 + \delta T^2\right)}.
\]  

(15)

where \(c'\) – the specific isobaric heat capacity of the solid phase, \(\lambda\) - the specific heat of fusion, \(\delta T\) - the spread range of the temperature in which melts the metal.

In formula (13), where the coefficient of friction \(\mu\) depends on the temperature of the welded material as a function

\[
\mu(T) = \mu_0 \left[\frac{1}{2} \cdot \frac{1}{\pi} \cdot \arctg(0,01 \cdot (t - T_q))\right].
\]  

(16)

In accordance with formula (13), the pressure exerted by the tool at the FSW is equal to:

\[
P_V = \frac{F}{S} = \frac{F}{\pi \cdot R_s^2},
\]  

(17)

In figure 3 shows the temperature dependence of the specific heat capacity of the aluminum alloy AD1 in the melting region, calculated by formula (15).

2.2. Numerical modeling of the temperature dynamics field of the FSW

The system of differential equations obtained above (8) simulates the dynamics of an axially symmetric discretized temperature field of a hemispherical sample with an FSW under conditions when the heat source is motionlessly fixed at the center of the hemisphere.

As noted above, \(r_0\) – the radius of the pin, and the value of \(r_1\) should be taken equal to the radius of the shoulder. The increasing radius of the remaining cylindrical surfaces can be chosen rather arbitrarily so that the radius \(r_N\) coincides with the overall radius of the end surface. The practice of calculations shows that it is rational to choose the ratio of the current radius to the previous one in the range from 1.4 to 2.0. The number of ring elements \(N = 10\) is quite sufficient for the application of the model under consideration in solving technological problems.

Simulation of the dynamics of the temperature field at FSW was carried out for the following alloys: steel 12H18N10T; aluminum alloy AD1; copper alloy M1; titanium alloy VT-1. Their thermophysical characteristics, taken from reference books [12,13], are given in table. 1.
Table 1. Thermophysical characteristics of materials for modeling the process of heat transfer at FSW

|                        | Steel 12H18N10T | Aluminum alloy AD1 | Copper alloy M1 | Titanium alloy VT-1 |
|------------------------|-----------------|--------------------|-----------------|---------------------|
| Density, $\rho$ (kg/m³) | 7.8·10³          | 2.71·10³           | 8.9·10³         | 4.5·10³             |
| Specific heat, $c$ (J/kg·K) | 447          | 880                | 390             | 540                 |
| Specific heat of fusion, $\lambda$ (J/kg) | 82·10³       | 390·10³            | 205·10³        | 358·10³             |
| Thermal conductivity, $\chi$ (W/m·K) | 45.4          | 209.3              | 389.6           | 21.9                |
| Melting point, $T_q$ (K) | 1823          | 933.32             | 1357.6          | 1668                |
| Coefficient absorption material, $\varepsilon$ | 0.37           | 0.15               | 0.64            | 1.28                |
| Coefficient heat transfer metal-air $\omega$ (W/m²·K) | 13            | 13                 | 13              | 13                  |
| Friction coefficient, $\mu$ | 0.7             | 0.47               | 0.35            | 0.79                |
| Shear stress, yield strength $\tau$ (MPa) | 40-55          | 6 (13-15)          | 36-38           | 45-70               |

The dynamics of the discretized temperature field of the sample is described by solving a system $N+1$ equations (8) and (15) for given initial values of the temperature of the ring elements $T_k(0)$, where $k=0, 1, ..., N$.

The results of numerical simulation for a hemisphere with a radius of 100 mm of aluminum alloy AD1 with tool rotation $\omega=700$ rpm, $F=3500\_N$ are shown in graphical form in Figure 4a. The plot of temperature versus time for a disk-shaped sample in Figure 4b with the same parameters [10].

![Graph](image)

**Figure 4.** Dependence of the temperature of ring elements on time for an aluminum alloy: a massive sample (a) and a sample in the form of a disk (b)

Figure 4 it can be seen that for the disk the temperature is 930 K, and for the hemisphere 690 K. When heat is introduced from the tool into a massive body, a more intense redistribution of heat occurs, since the surface area of the rings is correspondingly equal to the surface area of the hemisphere. For a disc, the area is limited by the thickness of the disc.

It has been found that the intensity of the temperature rise of the second ring located under the shoulder of the materials depends on the thermophysical properties of the materials being welded and changes monotonically in accordance with the exponential law.
Figure 5. Dependence of temperature for the second radius ($r_1$) of the materials to be welded: a massive specimen (a) and a specimen in the form of a disk (b)

The temperature difference for the massive material and the disk was 350 K for the VT-1 alloy. The smallest difference was for the aluminum alloy AD1 45 K, since aluminum has low thermophysical characteristics in comparison with other investigated materials.

2.3. Comparison with experiment

To confirm the calculations, the temperature of the aluminum alloy AD1 was measured while moving on the surface of a cylindrical tool with a flat end with a radius of 7 mm for 30 seconds. The temperature was measured by the thermocouple method using a device that displays data. The thermocouple was a chromel-alumel wire in diameter 0.25 mm. The experiment was repeated three times. Based on this, average temperatures were obtained. The thermocouple was attached to the surface of the metal to be welded by capping at a distance of 3 mm and 5 mm from axis the input heat.

When comparing the calculated and experimental data (Figure 6), it was found that the model is accurate (starting from the moment the maximum temperature is reached, the discrepancy is less than 10%). A somewhat lower accuracy is achieved when describing the process on the ascending heating branch, which is associated moves with the difficulty of determining the changes in the heat transfer of the transition layer when the heat source.

Figure 6. Graphs of the heating and cooling process of aluminum alloy AD1: $r_{1mod}$ and $r_{2mod}$ - terms simulated for 1 and 2 rings, $r_{1exp}$ and $r_{2exp}$ - terms experimentally obtained for 1 and 2 rings

3. Conclusions

1. Based on primary principles in the approximation of a hemispherical massive body. By solving the equation heat balance in differential form, an analytical model for describing the temperature field, maximum temperatures, and cooling rate for a stationary point source operating for a limited time is obtained.
2. The mathematical model was created under the initial and boundary conditions of a decrease in the power of the heat flux when the melting temperature is reached and an increase in the specific heat capacity during the phase transition (solid-liquid). To estimate the heating rate of the weld section in contact with the pin, the linear welding speed was calculated by using the system of heat balance equation minus the heat sinks into the pin and shoulder of the tool, as well as due to convection and radiation.

3. The analysis of two extreme cases of the mathematical model of a thin disk and a hemispherical massive body is carried out. The temperature difference was 240 K. When heat is introduced from the tool into a massive body, a more intense redistribution of heat occurs, since the surface area of the rings is correspondingly equal to the surface area of the hemisphere. For a disc, the area is limited by the thickness of the sheet metal.

4. Analysis of the temperature change curves calculated using the model for different materials differing in plasticity, thermal conductivity and heat capacity, melting points, differing at times (aluminum alloy - 692 K, copper alloy - 592 K, steel - 1115 K, titanium alloy - 1064 K) showed that in spite of significant differences in the thermophysical characteristics of materials, the achievement of temperatures close to the maximum temperatures on average lies within 5 seconds.

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