Nearly Bi-Maximal Neutrino Mixing, Muon g-2 Anomaly and Lepton-Flavor-Violating Processes

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Abstract

We interpret the newly observed muon g-2 anomaly in the framework of a leptonic Higgs doublet model with nearly degenerate neutrino masses and nearly bi-maximal neutrino mixing. Useful constraints are obtained on the rates of lepton-flavor-violating rare decays $\tau \to \mu \gamma$, $\mu \to e \gamma$ and $\tau \to e \gamma$ as well as the $\mu$-$e$ conversion ratio $R_{\mu e}$. We find that $\Gamma(\mu \to e \gamma)$, $\Gamma(\tau \to e \gamma)$ and $R_{\mu e}$ depend crucially on possible non-zero but small values of the neutrino mixing matrix element $V_{e3}$, and they are also sensitive to the Dirac-type CP-violating phase. In particular, we show that $\Gamma(\mu \to e \gamma)/m_{\mu}^5$, $\Gamma(\mu \to e \gamma)/m_{\tau}^5$ and $\Gamma(\tau \to e \gamma)/m_{\tau}^5$ are approximately in the ratio $1:2|V_{e3}|^2:2|V_{e3}|^2$ if $|V_{e3}|$ is much larger than $\mathcal{O}(10^{-2})$, and in the ratio $2(\Delta m_{\text{atm}}^2)^2:(\Delta m_{\text{sun}}^2)^2:(\Delta m_{\text{sun}}^2)^2$ if $|V_{e3}|$ is much lower than $\mathcal{O}(10^{-3})$, where $\Delta m_{\text{atm}}^2$ and $\Delta m_{\text{sun}}^2$ are the corresponding mass-squared differences of atmospheric and solar neutrino oscillations.
Recently the Muon g-2 Collaboration has reported a precise measurement of the muon anomalous magnetic moment \( \mu \),
\[
a_\mu^{(\text{exp})} = \frac{g_\mu - 2}{2} = 11659202(16) \times 10^{-10},
\]
which deviates from the standard-model (SM) prediction by 2.6\( \sigma \):
\[
\Delta a_\mu \equiv a_\mu^{(\text{exp})} - a_\mu^{(\text{SM})} = 43(16) \times 10^{-10}.
\]

To interpret the nonvanishing and positive value of \( \Delta a_\mu \), a lot of scenarios of new physics have been proposed [2]–[4]. In Ref. [4], Ma and Raidal emphasized that the new physics responsible for \( \Delta a_\mu \) and other lepton-flavor-violating processes (e.g., \( \mu \to e\gamma \)) might be related to that responsible for non-zero neutrino masses indicated by the evidence of neutrino oscillations. They illustrated this important point in the framework of a leptonic Higgs doublet model [5] with nearly degenerate neutrino masses and bi-maximal lepton flavor oscillations. They illustrated this important point in the framework of a leptonic Higgs doublet model proposed in Ref. [5]. The neutrino mass matrix \( M_{\nu} \) and \( \tilde{M}_{\nu} \) of solar and atmospheric neutrino oscillations.

Let us concentrate on the leptonic Higgs doublet model proposed in Ref. [5]. The neutrino mass matrix \( M_{\nu} \), defined in the flavor basis where the charged lepton mass matrix is diagonal, can be derived from the following Lagrangian through a simple seesaw mechanism:
\[
\frac{1}{2} M_{\nu} N_{\tilde{R}}^2 + h_{ij} \bar{N}_{\tilde{R}} \left( \nu_j \eta^0 - l_j \eta^+ \right) + \text{h.c.}.
\]

The typical Feynman diagram giving rise to \( \Delta a_\mu \) and \( l_i \to l_j \gamma \) is shown in Fig. 1. Following Ma and Raidal [4], we assume that the masses of three \( N_{\tilde{R}} \)'s are equal, and the coupling matrix \( (h_{ij}) \) takes the form
\[
(h_{ij}) = 2 \begin{pmatrix}
    h_1 & 0 & 0 \\
    0 & h_2 & 0 \\
    0 & 0 & h_3
\end{pmatrix} V^T,
\]

where \( h_i \) (for \( i = 1, 2, 3 \)) are the eigenvalues of \( (h_{ij}) \), and \( V \) is the neutrino mixing matrix in the chosen flavor basis. The neutrino mass eigenvalues \( m_i \) can then be calculated by using
Figure 1: The Feynman diagram giving rise to $\Delta a_\mu$ and $l_i \rightarrow l_j \gamma$ in the leptonic Higgs doublet model, where the photon can be attached to any charged line.

the seesaw relation $M_\nu = (h_{ij})^T (h_{ij}) / M$, where $M$ is the heavy mass scale common for three $N_R$'s. Explicitly one obtains $m_i = 4 h_i^2 \langle \eta^0 \rangle^2 / M$. Different from Ref. [4], the neutrino mixing matrix $V$ is taken to be nearly bi-maximal in this paper:

$$V = \begin{pmatrix}
\frac{c}{\sqrt{2}} & \frac{c}{\sqrt{2}} & -is \\
-\frac{1}{2} & \frac{A^*}{2} & \frac{c}{\sqrt{2}} \\
\frac{A}{2} & -\frac{1}{2} & \frac{s}{\sqrt{2}}
\end{pmatrix}, \quad (5)$$

where $s \equiv \sin \theta$, $c \equiv \cos \theta$, and $A = 1 + i s$ [4]. The mixing angle $\theta$ measures a slight coupling between solar and atmospheric neutrino oscillations. The current experimental constraint on the magnitude of $s$ is $s \leq 0.2$, obtained from CHOOZ and Palo Verde reactor experiments of neutrino oscillations [4]. Note that only the Dirac-type CP-violating phase (of a special but instructive value $90^\circ$) has been taken into account in $V$, since the Majorana-type phases have no effect on the rare processes to be discussed subsequently.

For simplicity, we follow Ref. [4] to make two more assumptions: (a) the masses of three light neutrinos are nearly degenerate, i.e., $m_3 \approx m_2 \approx m_1 \equiv m$ or equivalently $h_3 \approx h_2 \approx h_1 \equiv h$; and (b) the universal mass of $N_R$'s is identical to the mass of $\eta$. While assumption (b) helps to make the value of $\Delta a_\mu$ as large as possible within the model under consideration, assumption (a) is necessary for the suppression of $\Gamma(\tau \rightarrow \mu \gamma)$ relative to $\Delta a_\mu$.

Calculating Fig. 1, one obtains

$$\Delta a_\mu = \sum_i \frac{|h_{ij}|^2}{192 \pi^2} \cdot \frac{m_\mu^2}{m_\eta^2}, \quad (6)$$

With the help of Eqs. (4) and (5) as well as the assumptions made above, we can simplify Eq. (6) and arrive at

$$m_\eta \approx \frac{m_\mu}{2 \sqrt{3 \pi \Delta a_\mu}} \sqrt{\alpha_h}, \quad (7)$$

where $\alpha_h \equiv h^2 / (4 \pi)$. Using the 90% confidence-level limit $\Delta a_\mu > 215 \times 10^{-11}$ [4], we then obtain $m_\eta < 371 \sqrt{\alpha_h}$ GeV, a mass scale which is not far away from being discovered or ruled out in the future high-energy collider experiments [4]. Furthermore, the branching ratio of $l_i \rightarrow l_j \gamma$ is found to be

$$B(l_i \rightarrow l_j \gamma) \approx \frac{\alpha}{3072 \pi G_F^2 m_\eta^4} \left| \sum_k (h_{kl_i} h^*_{kl_j}) \right|^2 B(l_i \rightarrow l_j \nu_i \nu_j), \quad (8)$$
in the leptonic Higgs doublet model, where \( \alpha = 1/137 \), and the relevant assumptions have been taken into account. For rare decays \( \tau \rightarrow \mu \gamma \), \( \mu \rightarrow e\gamma \) and \( \tau \rightarrow e\gamma \), we obtain

\[
\left| \sum_i (h_{i\nu} h_{i\mu}^*) \right|^2 \approx c^2 h^4 \left( \Delta m_{\text{atm}}^2 \right)^2 / m^4,
\]

\[
\left| \sum_i (h_{i\mu} h_{i\nu}^*) \right|^2 \approx c^2 / 2 h^4 \left[ \frac{(\Delta m_{\text{sun}}^2)^2}{m^4} + 4 s^2 \frac{(\Delta m_{\text{atm}}^2)^2}{m^4} \right],
\]

\[
\left| \sum_i (h_{i\nu} h_{i\mu}^*) \right|^2 \approx c^2 / 2 h^4 \left[ \frac{(\Delta m_{\text{sun}}^2)^2}{m^4} + 4 s^2 \frac{(\Delta m_{\text{atm}}^2)^2}{m^4} \right],
\]

(9)

in which \( \Delta m_{\text{sun}}^2 \equiv |m_2^2 - m_1^2| \sim 10^{-5} \text{eV}^2 \) and \( \Delta m_{\text{atm}}^2 \equiv |m_3^2 - m_1^2| \approx |m_3^2 - m_2^2| \sim 10^{-3} \text{eV}^2 \) are the mass-squared differences of solar and atmospheric neutrino oscillations [4], respectively. It is clear that \( \Gamma(\mu \rightarrow e\gamma) \) or \( \Gamma(\tau \rightarrow e\gamma) \) will get comparable contributions from the terms associated with \( \Delta m_{\text{sun}}^2 \) and \( \Delta m_{\text{atm}}^2 \), if \( s \sim \mathcal{O}(10^{-2}) \) holds. For larger values of \( s \), the second term may dominate over the first term, leading to the following interesting relationship:

\[
\frac{\Gamma(\tau \rightarrow \mu\gamma)}{m^5_{\tau}} : \frac{\Gamma(\mu \rightarrow e\gamma)}{m^5_{\mu}} : \frac{\Gamma(\tau \rightarrow e\gamma)}{m^5_{\tau}} \approx c^2 : 2s^2 : 2s^2 .
\]

(10)

If the value of \( s \) is much lower than \( \mathcal{O}(10^{-3}) \), we will arrive at

\[
\frac{\Gamma(\tau \rightarrow \mu\gamma)}{m^5_{\tau}} : \frac{\Gamma(\mu \rightarrow e\gamma)}{m^5_{\mu}} : \frac{\Gamma(\tau \rightarrow e\gamma)}{m^5_{\tau}} \approx 2c^2(\Delta m_{\text{atm}}^2)^2 : (\Delta m_{\text{sun}}^2)^2 : (\Delta m_{\text{sun}}^2)^2 .
\]

(11)

This result is consistent with that obtained by Ma and Raidal [3] in the bi-maximal neutrino mixing case with \( s = 0 \). It becomes transparent that \( \Gamma(\mu \rightarrow e\gamma) \) and \( \Gamma(\tau \rightarrow e\gamma) \) are dependent crucially upon the magnitude of \( s \). In contrast, \( \Gamma(\tau \rightarrow \mu\gamma) \) is insensitive to small values of \( s \), no matter whether neutrino mixing is bi-maximal or nearly bi-maximal.

For illustration, we calculate \( B(\mu \rightarrow e\gamma) \) and \( B(\tau \rightarrow \mu\gamma) \) as functions of the neutrino mass \( m \) and the flavor mixing parameter \( s \), and plot the numerical results in Fig. 2, where \( \Delta m_{\text{sun}}^2 = 3 \times 10^{-5} \text{eV}^2 \), \( \Delta m_{\text{atm}}^2 = 3 \times 10^{-3} \text{eV}^2 \), and \( \alpha_h / m_\eta^2 = (371 \text{GeV})^{-2} \) have been typically input. The chosen value of \( \alpha_h / m_\eta^2 \) implies that the relevant results shown in Fig. 2 should be understood as lower bounds in the case of nearly bi-maximal neutrino mixing. Note that the branching ratio of \( \tau \rightarrow e\gamma \) can straightforwardly be obtained from that of \( \mu \rightarrow e\gamma \), as the relationship \( B(\tau \rightarrow e\gamma) = B(\mu \rightarrow e\gamma) \cdot B(\tau \rightarrow e\nu_\tau \bar{\nu}_e) \) with \( B(\tau \rightarrow e\nu_\tau \bar{\nu}_e) \approx 17.83\% \) hold [10].

From Fig. 2 one can see that \( B(\mu \rightarrow e\gamma) \) will be close to its current experimental upper limit of \( 1.2 \times 10^{-11} \) [11], if \( m \approx 0.2 \text{eV} \) and \( s \approx 0 \) or if \( m \approx 1 \text{eV} \) and \( s \approx 0.1 \). Because of \( s \leq 0.2 \) [3], no fine tuning is required to achieve \( B(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11} \) for \( m \leq 2 \text{eV} \). Neutrinos of \( m \sim (1 - 2) \text{eV} \) may have important cosmological implications [12]. To accommodate the nonobservation of the neutrinoless double beta decay in the scenario of neutrino masses under discussion, additional Majorana phases should be included into the neutrino mixing matrix \( V \) [13].

Finally it is worth mentioning that the leptonic Higgs doublet model can also lead to the \( \mu - e \) conversion in nuclei. The relevant assumptions made above allow us to simplify the
Figure 2: The lower bounds on $B(\tau \to \mu \gamma)$ and $B(\mu \to e \gamma)$ in the leptonic Higgs doublet model with nearly degenerate neutrino masses and nearly bi-maximal neutrino mixing: (a) $s = 0$, (b) $s = 0.01$, (c) $s = 0.03$, and (d) $s = 0.1$.

The formula of the $\mu$-e conversion ratio $R_{\mu e}$ given in Ref. [4]. We then obtain

$$R_{\mu e} \approx \frac{\alpha^5 m_\mu^9 Z_{\text{eff}}^4 Z |\bar{F}_p(p_e)|^2}{18432 \pi^4 \Gamma_{\text{capt}} q^4 m_\eta^4} \left| \sum_i (h_{ij} h_{ie}^*) \right|^2,$$  \hspace{1cm} (12)$$

where $q^2 \approx -m_\mu^2$, $Z_{\text{eff}} = 11.62$, $\bar{F}_p = 0.66$, and $\Gamma_{\text{capt}} = 7.1 \times 10^5 \text{ s}^{-1}$ for Al [9]. Comparing between $R_{\mu e}$ and $B(\mu \to e \gamma)$, one may easily find that they depend upon $m$ and $s$ in the same way. The magnitude of $R_{\mu e}$ is smaller than that of $B(\mu \to e \gamma)$ by two orders.

In summary, we have discussed the muon g-2 anomaly and the lepton-flavor-violating rare decays in the framework of a leptonic Higgs doublet model with nearly degenerate neutrino masses and nearly bi-maximal neutrino mixing. We demonstrate the sensitivity of $\Gamma(\mu \to e \gamma)$, $\Gamma(\tau \to e \gamma)$ and $R_{\mu e}$ to possible non-zero but small values of $|V_{e3}|$. It is obvious that they are also sensitive to the Dirac-type CP-violating phase in $V$, which has been taken to be $90^\circ$ in our specific nearly bi-maximal neutrino mixing pattern. One may of course apply other nearly bi-maximal neutrino mixing patterns [14] to the models giving rise to $\Delta a_\mu$ and $\mu \to e \gamma$. We expect that new and more precise data to be accumulated from a variety of experiments on neutrino oscillations and lepton-flavor-violating rare decays will help to pin down the self-consistent models (even the true theory) of neutrino masses, lepton flavor mixing and CP violation.

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