The study of fusion barriers of neutron-rich colliding nuclei using various isospin-dependent potentials

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**Abstract**

A detailed study fusion of neutron-rich colliding nuclei is performed using various isospin dependent potentials. For present study, Three different series namely, Ne-Ne, Ca-Ca, and Zr-Zr are taken into account and N/Z ratio. A monotonous increase (decrease) in the fusion barrier positions (heights) using a unified second order
nonlinear parametrization in the normalized fusion barrier positions and heights with \((N/Z-1)\) is presented. These predictions are in good agreement with the available theoretical as well as experimental results.

1 Introduction

With the availability of radioactive-ion nuclear beams the fusion of colliding nuclei with the excess of neutron/proton ratio or near the drip line has attracted central position in the current days research [1–3]. The neutron-rich radioactive-ion beams have been applied to synthesize new, neutron-rich heavy nuclei. This is because in the synthesis of heavy nuclei with neutron-rich projectile, one expect a higher survival probability of the completely fused system due to its lower fissility and lower excitation energies. Once we go beyond binding energies of colliding nuclei many more phenomena like collective flow, multi fragmentation, stopping and sub-barrier particle production also appeared as dominant modes [4–8].

Several experiential studies have also been undertaken in the literature to study the effect of varying the \(N/Z\)-ratio of the projectile and target nuclei upon the fusion cross sections [6, 7, 10, 11]. In addition, the fusion transfer at the neck region has been suggested [12, 13]. A microscopic description of the formation of neck in the fusion reaction remain a challenge to microscopic theories. The influence on sub-barrier fusion of processes such as, transfer [14] and breakup reaction [9, 15] is not yet clear; moreover, the effect of unusual structure, such as halos and skins [16], is being studied [7, 17]. Recently, Sun et al. [18], suggested that \(N/Z\) may be used as an experimental observable to extract neutron skin for
neutron rich nuclei. All above experimental as well as theoretical information indicates
that the dynamics of neutron/proton-rich nuclei is not fully understood and needs further
attention.

The properties of various neutron-rich nuclei with different N/Z-ratio are studied in the literature e.g.: $^{9-10}\text{He}(N/Z=3.50-4.00)$; where $N$ and $Z$ are the neutron and proton content of the nucleus), $^{6,8,9,11}\text{Li}(N/Z=1.0, 1.67, 2.0, 2.67)$, $^{22}\text{C}(N/Z=2.67)$, $^{26-28}\text{O}(N/Z=2.25-2.50)$, $^{31}\text{F}(N/Z=2.444)$, $^{28,32,34}\text{Ne}(N/Z=1.8, 2.2, 2.40)$, $^{30-32,37}\text{Na}(N/Z=1.727-1.909, 2.364)$, $^{40}\text{Mg}(N/Z=2.333)$, $^{49-51}\text{Ar}(N/Z=1.722-1.833)$, $^{60}\text{Ca}(N/Z=2.0)$, $^{57-60}\text{Mn}(N/Z=1.28-1.4)$, $^{68-78}\text{Ni}(N/Z=1.429-1.786)$, $^{84,86,90,92}\text{Zn}(N/Z=1.8, 1.87)$, $^{90,92}\text{Ge}(N/Z=1.813, 1.875)$, $^{132}\text{Sn}(N/Z=1.64)$, $^{123}\text{Ag}(N/Z=1.617)$, $^{123-128}\text{Cd}(N/Z=1.563-1.667)$[4,32] and proton-rich are $^{6}\text{Be}(N/Z=0.50)$, $^{10}\text{N}(N/Z=0.429)$, $^{12}\text{O}(N/Z=0.50)$, $^{17}\text{F}(N/Z=0.89)$, $^{22}\text{Si}(N/Z=0.571)$, $^{31}\text{Ar}(N/Z=0.722)$, $^{34}\text{Ca}(N/Z=0.70)$, $^{38,39}\text{Ti}(N/Z=0.727, 0.773)$, $^{45,49}\text{Fe}(N/Z=0.731, 0.885)$, $^{48,49,53}\text{Ni}(N/Z=0.714-0.75, 0.893)$, $^{54}\text{Zn}(N/Z=0.80)$, $^{217}\text{U}(N/Z=1.359)$ etc.[19–21].

A suitable set of models are therefore needed to study the dynamics of neutron/proton
-rich colliding nuclei. A large number of theoretical models are available in the literature
based upon the different assumptions [1–3]. Among them, proximity potential due to
Blocki et al. [22], is well known for its simplicity and wider applications in different fields.
Several modifications or refinements over the original proximity potential are also available
in the recent time by including either up-to-date knowledge of the surface energy coefficient
or nuclear radii [3,23]. Various authors, modified or parameterized their approaches
within the proximity concept [3]. All these modifications include new emerging degree of
freedom i.e. isospin, either in radius formula, universal function and/or in surface energy coefficient [3]. Further, The outcome will definitely different if one use such type of models in the isospin plane.

Recently, one of us and collaborator, have carried out a detailed study involving symmetric as well as asymmetric colliding nuclei using 16 proximity-type potentials [3]. Unfortunately, the maximum N/Z content of all the experimentally studied heavy-ion reactions is 1.60 (i.e. $^6\text{He}+^{238}\text{U}$) [7]. On the other hand, the first measurement with the proton drip line nucleus is of $^{17}\text{F} + ^{282}\text{Pb}$ (with N/Z=1.473) [20]. Therefore, a systematic study of nuclei having larger N/Z ratio using new proximity-type potentials is in demand. Further, it gives us a unique possibility to test the validity or accuracy of these models for the nuclei far away from the line of stability. Therefore, a systematic dependence of fusion barrier (heights and positions) and cross sections using various isospin dependent models on neutron excess is needed. Similar study was also presented by Puri et al. [1], where only Ca and Ni series were used. In the present study, we extend the work to include new series like Ne-Ne (with N/Z ratio = 0.6-2.0) and Zr-Zr (with N/Z = 0.75-2.0) along with Ca-Ca (with N/Z = 0.5-2.0) series. The overall domain of N/Z ratio is from 0.5 to 2.0 for all series. The asymmetry parameter $A_s$ (N/Z-1) of the colliding nuclei varies between -0.5 and 1.0. Note that non zero value of $A_s$ will involve complex interplay of the isospin degree of freedom which has strong role at intermediate energies as well [4]. Section 2, deals with fine points of the models in brief, Section 3 contains the results and summary is presented in Section 4.
2 The Model

The total ion-ion interaction potential $V_T(r)$ comprises of nuclear and Coulomb part:

$$V_T(r) = V_N(r) + V_C(r).$$

(1)

Here $V_C(r) = Z_1 Z_2 e^2/r$ is a good approximation, because fusion happens at a distance greater than touching configuration of the colliding pair.

The nuclear part of the ion-ion interaction potential $V_N(r)$ is calculated within the proximity concept. All proximity potentials are based upon the proximity force theorem, according to which [22], "the force between two gently curved surfaces in close proximity is proportional to the interaction potential per unit area between the two flat surfaces". In original proximity potential [22], the nuclear part of the interaction potential $V_N(r)$ can be written as

$$V_N(r) = 4\pi \gamma b \bar{C} \phi\left(\frac{r - C_1 - C_2}{b}\right) \text{MeV}. \quad (2)$$

In this, $\bar{C}$ is the reduced radius with equivalent sharp radius $R_i$ as

$$R_i = 1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3}_{1/3}\text{fm}(i = 1, 2) \quad (3)$$

where $\phi (\xi = \frac{r-C_1-C_2}{b})$ is the universal function that depends on the separation between the surfaces of two colliding nuclei only. Both these factor do not depend on the isospin content. However, the last parameter $\gamma$, the surface energy coefficient, depends upon the neutron/proton excess as

$$\gamma = \gamma_0[1 - K_s(\frac{N - Z}{A})^2] \quad (4)$$
Where \( N, Z \) being the total number of neutrons and protons. In the original version, \( \gamma_0 = 0.9517 \text{ MeV/fm}^2 \) and \( k_s = 1.7826 \) [22]. Noted that for symmetric nuclear matter, \( N = Z, \gamma = \gamma_0 = 0.9517 \text{ MeV/fm}^2 \) indicating maximum strength of the potential. If we move to neutron (proton) -rich colliding nuclei with \( N > Z \) (\( N < Z \)) then \( \gamma \) starts decreasing resulting in comparatively lesser attractive potential. Later on, these coefficients were further improved by Möller and Nix with values \( \gamma_0 = 1.2496 \text{ MeV/fm}^2 \) and \( k_s = 2.3 \) [24]. This is labelled as Prox 88 [2, 25]. In the latest version of proximity potential [23], \( \gamma \) has form based on the precise neutron skin as

\[
\gamma = \frac{1}{4\pi r_0^2} [18.63 \text{(MeV)} - Q\left(t_1^2 + \frac{t_2^2}{2r_0^2}\right)]. \tag{5}
\]

The corresponding proximity potential is labelled as Prox 00 [3]. One of us and collaborator [3], modified above potential to include latest radius formula [26] and is denoted as Prox 00N. Note that both Prox 00 and Prox 00N has isospin dependent radius with slightly different constants whereas the factors surface energy coefficient \( \gamma \) and the universal function \( \phi(s) \) are same. In both newer versions of Bass (labelled as Bass 77 and Bass 80 in Ref. [3]) radius is slightly changed to

\[
R_i = 1.16A_i^{1/3} - 1.39A_i^{-1/3} \text{fm}(i = 1, 2) \tag{6}
\]

and then to sharp radius as is used in Prox 77 in the later version (i.e. Bass 80). Both newer versions of Winther (marked as BW 91 and AW 95 in Ref. [3]) has again similar expression for \( \gamma \) as is given in Prox 77 with slight difference that here isospin content is calculated separately for the target/ projectile. Whereas, first version due to Winther (labeled as CW 76 in Ref. [3]) does not have any \( \gamma \) dependence. Even radii are function of mass only. Both versions of Ngô (labeled as Ngô 75 and Ngô 80 in Ref. [3]) do not consider
γ, but latest version of Ngô (Ngô 80) has isospin dependence in radius parameter. On the other hand, a complex isospin dependence in the universal function φ(s) and radius is given in the version of Denisov [27]. Also by using the latest form of radius given in Ref. [26] in Denisov potential resulting in closer agreement with the experimental data for fusion barrier heights and cross-sections. This potential is labeled as Denisov N [3]. All the above mentioned proximity-type potentials are able to reproduce the experimental fusion barriers within ±10 on the average [3].

In total, 8 proximity-type potentials are used in the present study. Among them, three are basic proximity potentials (Prox 77, Prox 88, and Prox 00), three due to Bass (Bass 80), Winther (AW 95) and Ngô (Ngô 80) each, and two newly modified potentials (Prox 00N and Denisov N) are used. The model due to Bass et al., (Bass 80) used in the present analysis is independent of isospin dependence. For the detail of the models reader is refer to Ref. [3]. In most of the potentials and versions, modifications are made either through the surface energy coefficients or in nuclear radii. These two rather being technical parameters can have sizeable effects on the outcome of a reaction [3].

From these brief outlines, it is clear that much stress is made on the surface energy coefficients γ and nuclear radii to incorporate the isospin factor of a potential. Definitely, based on different assumptions and isospin dependence, different versions will respond to the collision of neutron-rich or -deficient nuclei differently compared to N = Z nuclei.
3 Results and Discussion

The present study deals with large variety of above mentioned potentials. Using these potentials, we firstly calculate the total ion-ion interaction potential using Eq. (1). Once total ion-ion interaction potential is calculated, one can extract the barrier height $V_B$ and barrier position $R_B$ using conditions:

$$\frac{dV_T(r)}{dr} |_{r=R_B} = 0; \text{and} \frac{d^2V_T(r)}{dr^2} |_{r=R_B} \leq 0. $$

Here we consider the collisions of three different series namely; $^{A1}Ne + ^{A2}Ne$ (with $N/Z = 0.6$ to $2.0$); $^{A1}Ca + ^{A2}Ca$ (with $N/Z = 0.5$ to $2.0$), and $^{A1}Zr + ^{A2}Zr$ (with $N/Z = 0.75$ to $2.0$) to cover wider mass range. We starts with the collision of $N = Z$ nuclei and then add (or remove) neutrons gradually from either of the colliding pairs till we reach $N = 2Z$ ($N = 0.5Z$) nuclei. In total, 150 such collisions involving different isotopes of different series are taken into account.

As a first step, we check the effect of addition or removal of neutrons on the nuclear part of the interaction potential $V_N(r)$ in different models. In Fig. 1, we display $V_N(r)$ as a function of internuclear distance $r$ for the reactions of $^{16}Ne + ^{16}Ne$, $^{16}Ne + ^{20}Ne$, $^{20}Ne + ^{20}Ne$, $^{20}Ne + ^{28}Ne$ and $^{28}Ne + ^{28}Ne$ using eight proximity type potentials. Based upon the different assumptions used in different models, shape as well as strength of the potential differ accordingly. From this plot we can compare the different models to check the isotopic dependence of the interaction potential. From this plot it is evident that the $V_N$ becomes deeper with the addition of neutrons whereas the reverse is true for the removal of neutrons. At the same time, the general shape at the surface region is same. In particular, Bass 80, Prox 00, and Prox 00N have no repulsive core at shorter
distances, whereas AW 95 follow Woods-Saxon type form. On the other hand, Ngô 80, Denisov N, Prox 77, and Prox 88 have repulsive core at shorter distances. In addition, four effects of addition (removal) neutrons to N = Z nuclei are clearly visible: (i) barrier height is decreased (ii) barrier position is increased (iii) depth of the potential is increased in all potentials except Denisov N, and (iv) diffuseness of the potential is also changes. These effects will definitely influence the fusion probability at below barrier energies. Therefore, before discussing the enhancement of the fusion cross section for the neutron-rich fusion reaction, we investigate the systematic dependence of fusion barriers on the neutron asymmetry parameter $A_s(= N/Z-1)$. Using the above sets of models, fusion barrier heights and positions are calculated for 150 colliding pairs using 8 sets of models.

Here the proton-rich systems show the deeper pocket compared to neutron-rich systems. This is due to the reason that the form of radius used in Denisov N has very complex dependence on the mass number $A$. In Fig. 2, we have plotted $V_N$ as a function of internuclear distance $r$, but for Zr-Zr reactions. In this case again same results are obtained as in the case of Ne-Ne. Again Denisov N shows exceptional behavior. These two graphs basically shows the same trend with the addition of neutrons/protons. As we move from the lighter systems to heavier ones i.e., from Ne to Zr series, the scattering around the mean values decreases. This implies that as we move from the lighter to heavier systems, the role of neutron content diminishes.

The total potential containing the nuclear and coulomb parts will also be affected by the neutron contents. In Fig. 3, we have plotted total potential $V_T(r)$ as a function of internuclear distance $r$ for Zr-Zr colliding pairs. With the addition of neutron barrier height decreases and the barrier position increases. Whereas the reverse is true for the
removal of neutrons. As a result, the fusion probability increases with the addition of neutrons. Again some discrepancies have been noticed for Denisov N in the repulsive part of potential. Similarly, we have plotted the total potential $V_T$ as a function of $r$ for the Ne-Ne and Ca-Ca (not shown here). from these plots we observed that as we move from the lighter to heavier ones i.e., from Ne to Zr series, explicit mass dependence is more visible. This indicates that the conclusion based on island of the periodic table can be misleading.

For a model independent analysis, we see the same effect in all these models. Some differences in various potentials have been noticed at the surface regions. We have noticed that the modal ingredients such as nuclear radii, reduced radius, surface energy coefficient and universal function, have sizable effect on the interaction potential as well as on the fusion probability. We can simply say that different potential use different radii, surface energy coefficient and universal function leading to different mass dependence. Some of these reactions along with barrier heights and barrier positions are summarized in tables. In these Figs., we have observed that Bass 80, Prox 00, and Prox 00N do not have any pocket. This is due the reason that Prox 00 and Prox 00N are derived only for the distances greater that touching configuration and Bass 80 is based on the classical assumptions. All other potentials have pockets. We have also notices that the neutron/proton content also affect the diffuseness of the pocket. The nuclear interaction potential is more attractive for Prox 88 compared to other potentials. The shape of the different potentials is different because according to Proximity theorem, The nuclear part of the potential i.e., $V_N(r)$ depends on $\phi(s)$ and $\overline{C}$. And the shape of the potential depends on the universal function $\phi(s)$. All the potentials, except Bass 80, have isospin dependence in $\overline{C}$. As a result, the
contribution of $\mathcal{C}$ is different in all potential. Also we have noticed that the contribution of $\mathcal{C}$ due to isotopic dependence is stronger for the proton-rich colliding nuclei compared to neutron-rich nuclei. Also the change in the neutron content also affect the depth of the pocket of the potential. We have observed that the pocket is less deep in the case of proton-rich nuclei. This implies that these reactions are less favorable for the fusion of colliding nuclei.

In Fig. 4., We have used only three versions of Proximity potential these versions differ due to different forms of $\gamma$. Here the contribution due to the surface energy coefficient, $\gamma$ at barrier heights and positions is taken. Alternatively, we can say that the isotopic dependence of any quantity can examined more clearly by plotting different quantity against asymmetric term $A_s$. We have observed that if we start from $N=Z$ i.e., symmetric system and add or remove neutrons from symmetric systems the contribution of $\gamma$ decreases on the both sides of symmetric point. The contribution of $\gamma$ is much stronger in the case of proton rich nuclei as compared to the neutron rich nuclii. the contribution of $\gamma$ towards the Prox 88 potential is much stronger compared to remaining two versions. Similarly in Fig. 5., we display the universal function $\phi(s)$, calculated at barrier position, versus $A_s$ for all the potential. The variation of $\phi(s)$ is smooth throughout the variation of neutron content in the case of AW 95 and Denisov N. Whereas Bass 80 and Ngô 80 show slight isotopic dependence as we move away from the $N=Z$ symmetric line. The variation in $\phi(s)$ is different for neutron neutron-rich nuclei compared to proton-rich nuclei. This variation of $\phi(s)$ is more for neutron-deficient nuclei whereas the variation of $\phi(s)$ is almost saturates in the case of neutron-rich colliding nuclei. As we move from lighter to heavier system e.g., from Ne-Ne to Zr-Zr, all the potential converge to same results as
shown in Fig. 5, indicating the mass independent observation.

The variation in the fusion barrier positions with neutron/proton content is analyzed in Figs. 6 and 7. Here, we display the variation of $\Delta R_B(\%)$ and $\Delta V_B(\%)$ defined as

\[
\Delta R_B(\%) = \frac{R_B - R_B^0}{R_B^0} \times 100, \\
\Delta V_B(\%) = \frac{V_B - V_B^0}{V_B^0} \times 100,
\]

as a function of asymmetry parameter $A_s (= N/Z - 1)$ using eight sets of potentials discussed above. Here, $R_B^0$ and $V_B^0$ are, respectively, the fusion barrier position and height corresponding to $(N = Z)$ colliding nuclei and $R_B$ and $V_B$ refer for neutron/proton-rich colliding nuclei. The main advantage of these normalized variation is that it gives mass independent picture. For the present picture, we start with the collision of $N=Z$ nuclei, then gradually add/remove neutrons from either of the colliding pair. For example, we started with the collision of $^{40}Ca+^{40}Ca$, then add neutrons gradually, by keeping charges $Z_1$ and $Z_2$ always fixed. In this series, at the end of the chain we have the collision of $^{60}Ca+^{60}Ca$. Similarly, if we remove the neutrons then we have at the end the collision of $^{30}Ca+^{30}Ca$. It is clear from the figures, that all the models follow a unified non-linear second order parametrization given as:

\[
\Delta R_B(\%) = a\left(\frac{N}{Z} - 1\right) + b\left(\frac{N}{Z} - 1\right)^2, \\
\Delta V_B(\%) = c\left(\frac{N}{Z} - 1\right) + d\left(\frac{N}{Z} - 1\right)^2,
\]

Here, $a$, $b$, $c$, and $d$ are the constants varies from model to model and its values are displayed in Figs 2 and 3. The above results are in agreement with the recent work due to Puri et al., for Ca and Ni series [1], The available experimental as well as theoretical
data is also displayed. All the available theoretical as well as experimental data follow our parametrization pattern very well except few points due to experimental uncertainty in different experimental setups. The theoretical data is taken from Refs. [28], whereas, the experimental data is taken from Refs. [29]. The above pattern indicates that, with the addition of neutron to N = Z nuclei, fusion barrier position is increased and barrier height decreased, whereas, reverse happen for neutron-deficient nuclei. All models do not show much scattering from the middle curve as one move from N = Z to very neutron-rich and -deficient colliding nuclei. Bass 80 and Ngô 80 show slight scattering for neutron-rich and -deficient nuclei. It may be due to the reason that the isospin dependence included in Ngô 80 model have not much effect on N/Z ratio, whereas, Bass 80 is independent of such kind of dependence. We further note that the slopes of the central line also varies from model to model, whereas, the overall pattern is nicely explained by the above parametrization. It may be due to the different assumptions used in different models.

Along with \( \Delta R_B(\%) \) and \( \Delta V_B(\%) \), we also studied \( \Delta V_N(\%) \) and \( \Delta V_C(\%) \) using the same set of models and series (not shown here). We see that these variations show larger scattering due to the different assumptions resulting from different forms of radii, universal function, surface energy coefficients etc. The diffuseness parameter ‘a’ is also different in some potentials.

In Fig. 8. and Fig. 9., we have plotted \( \Delta V_C(\%) \) and \( \Delta V_N(\%) \) as a function of asymmetry parameter \( A_s(= N/Z-1) \). These normalized quantities are given by:

\[
\Delta V_C(\%) = \frac{V_C - V_C^0}{V_C^0} \times 100, \tag{12}
\]

\[
\Delta V_N(\%) = \frac{V_N - V_N^0}{V_N^0} \times 100, \tag{13}
\]
In the case of neutron-deficient nuclei, not only the nuclear potential becomes more attractive, but at the same time, the Coulomb forces become stronger, therefore, their mutual dominance decides about the fate of the barrier. It is clear from the Fig. 8. that the increase in Coulomb potential is much more compared to the corresponding nuclear potential, therefore, enhancing the fusion barriers when neutron are removed. Nuclear potential is different for different colliding series. Nuclear potential that also includes geometrical factor has a monotonic isotopic removal of neutrons. This result is in contradiction to the couple of results calculated earlier where it was discussed that the nuclear part of the potential is more attractive with addition of neutrons and leading to the reduced barrier. In general, we can say that all the different models converge to nearly same results. More experiments are needed to verify our prediction. A considerable mass dependence is seen in the case of Bass 80 and Ngô 80. Aw95 and Denisov N also show slight mass dependence but Prox 77, Prox 88, Prox 00 and Prox 00N potentials indicate a mass independent isotopic effect in fusion dynamics. Similarly in Fig. 9. large scattering is observed in all the cases. Again here Bass 80, Ngô 80, AW95 and Denisov N show large scattering compared to Prox 77, Prox 88, Prox00 and Prox 00N potentials.

4 Summary

We analyze the fusion of three different series namely, Ne, Ca, and Zr by covering the wider mass spectrum with N/Z ratio between 0.5 and 2.0. We analyzed the systematic dependence of fusion barriers on neutron excess and presented a unified second order non linear quadratic parametrization in fusion barrier heights and positions with \((N/Z-1)\)
Figure 1: The nuclear part $V_N$(MeV) as a function of internuclear distance $r$ for the reactions of $^{16}\text{Ne}+^{16}\text{Ne}, ^{16}\text{Ne}+^{20}\text{Ne}, ^{20}\text{Ne}+^{20}\text{Ne}, ^{20}\text{Ne}+^{28}\text{Ne}$ and $^{28}\text{Ne}+^{28}\text{Ne}$ using eight sets of proximity-type potentials.
using eight isospin dependent proximity-type models for three different series. Our results are in good agreement with the available theoretical as well as experimental results. A linear dependence in the fusion probabilities is also presented. Further, the enhancement
Figure 3: The total interaction potential \( V_T(r)(\text{MeV}) \) is plotted as a function of internuclear distance \( r \) for Ne-Ne colliding pairs using eight different potentials. Here neutron as well as proton-rich colliding pairs are considered. For one pair we also display barrier height \( V_B \) and position \( R_B \).
Figure 4: Variation of surface energy coefficient $\gamma$(MeV/fm$^2$) as a function of asymmetry parameter for three different series using three versions of Proximity potentials.

In fusion cross sections for neutron-rich nuclei due to lowering of fusion barrier heights is clearly seen, whereas, reverse happen for proton-rich nuclei. Along with this, our
Figure 5: Variation of universal function $\phi$ as a function of asymmetry parameter $A_S$ for three different series using four versions of Proximity potentials.

Parametrization pattern is independent of the colliding nuclei as well as model and isospin content. At near barrier energies, $N/Z$ content plays dominant role, whereas, the effect
is insignificant at above barrier energies. More experiments are needed to verify our predications.

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Figure 6: The normalized barrier positions $\Delta R_B(\%)$ [Define in Eq. (8)] as a function of $A_s=\frac{N}{Z}-1$. We display the results of our calculations for the collisions of $^4\text{He}+^4\text{He}$, $^4\text{He}+^6\text{He}$, and $^4\text{He}+^8\text{He}$ series using eight models along with other available theoretical and experimental values. The theoretical as well as experimental data reported here is taken from Refs. [28] and [29], respectively. The shaded areas denotes the deviation from the central solid lines.
Figure 7: Same as Figure 6, but for $\Delta V_B(\%)$.[Define in Eq. (9)]
Figure 8: The normalized Coulomb barriers potential is plotted as a function asymmetry $A_s = N/Z-1$ using eight different potential for three series. Here non-linear second order fit is applied.
Figure 9: The normalized nuclear potential $\Delta V_N(\%)$ is plotted as a function of asymmetry $A_s = (N/Z - 1)$ by using eight different potential for three series. Here non-linear second order fit is applied.