EFFECTS OF THE ($\rho$, $\omega$, $\phi$) MIXING ON THE DIPION MASS SPECTRUM IN 
$e^+e^-$ ANNIHILATION AND $\tau$ DECAY

M. BENAYOUN
LPNHE Paris VI/VII, IN2P3/CNRS, F-75252 Paris, France

in collaboration with P. DAVID, L. DELBUONO, O. LEITNER and H. B. O’CONNELL

A way to explain the puzzling difference between the pion form factor as measured in $e^+e^-$ annihilations and in $\tau$ decays is discussed. We show that isospin symmetry breaking, beside the already identified effects, produces also a full mixing between the $\rho^0$, $\omega$ and $\phi$ mesons which generates an isospin 0 component inside the $\rho^0$ meson. This effect, not accounted for in current treatments of the problem, seems able to account for the apparent mismatch between $e^+e^-$ and $\tau$ data below the $\phi$ mass.

1 Introduction

In order to get a theoretical estimate of the muon anomalous magnetic moment $g - 2$, one needs to estimate precisely the photon vacuum polarization (see Jegerlehner\textsuperscript{1} for a comprehensive review). Its leptonic part can be computed theoretically to a high precision from QED, but the dominance of non–pertubative effects in the low energy region prevents to perform likewise starting from QCD in order to estimate the hadronic part. This is instead done by means of a dispersion integral involving the measured cross section $\sigma(e^+e^- \rightarrow \text{hadrons})$; however, the integration kernel is such that the low energy region contribution is enhanced by a $\sim 1/s^2$ factor. Because of this, the non–pertubative region provides, by far, the largest contribution to the hadronic vacuum polarization (VP). Additionally, the annihilation process $e^+e^- \rightarrow \pi^+\pi^-$ alone happens to provide more than 60 % of the total hadronic VP.

As one has to rely on data in order to estimate this contribution, the precision of the measured $\sigma(e^+e^- \rightarrow \pi^+\pi^-)$ is clearly an important issue. For this purpose, several sets of data collected in $e^+e^-$ annihilations at low energies during a long period of time are available. This covers the former data sets collected by the OLYA, CMD and DM1 Collaborations – which are
gathers in the review by Barkov et al.\textsuperscript{2} – and the data sets more recently collected by the CMD2\textsuperscript{3,4,5} and SND\textsuperscript{6} Collaborations. Additional data sets taking advantage of the initial state radiation mechanism have also been collected by the KLOE, BaBar and Belle Collaborations and are expected to become available soon.

Moreover, high statistics data on the decay $\tau^\pm \rightarrow \nu_\tau \pi^\mp \pi^0$ are also available from the ALEPH\textsuperscript{7} and CLEO\textsuperscript{8} Collaborations. As the pion form factor (i.e. the dipion mass spectrum) in $\tau$ decays and in $e^+e^-$ annihilations are related by the Conserved Vector Current (CVC) assumption, these data are expected to be useful in order to improve the estimate of the photon hadronic VP. Indeed, these two kinds of data can only differ by isospin breaking effects which are subject to accurate estimates.

Isospin symmetry breaking effects have been especially studied in order to include the $\tau$ data in the estimation of the photon hadronic VP. This covers non trivial effects specific of the $\tau$ decay like the short range\textsuperscript{9} and long range\textsuperscript{10,11} isospin breaking factors, but also more standard effects easier to account for : mass differences between charged and neutral pions and charged and neutral $\rho$ mesons, the $\rho^\pm - \rho^0$ width difference, or the $\omega$ and $\phi$ contributions to the $e^+e^-$ annihilation amplitude (see, for instance, the work by Davier et al.\textsuperscript{12,13}).

As a preliminary step in the process of including $\tau$ data in estimating the photon hadronic VP, the comparision has been done of the pion form factor as measured in $e^+e^-$ annihilations and as derived from $\tau$ decays while accounting for all known isospin breaking effects appropriately\textsuperscript{12,13}. This comparison, however, clearly exhibits an unexpected $s$-dependence of the difference between the $e^+e^-$ data and the pion form factor function reconstructed from $\tau$ data, as reported still recently by Davier\textsuperscript{14}.

This mismatch is an important issue as the photon hadronic VP as reconstructed from $e^+e^-$ data leads to a theoretical prediction for the muon $g - 2$ at $\sim 3.3$ $\sigma$ from its measured value\textsuperscript{15}; in contrast, the prediction derived from $\tau$ data fed with all currently identified isospin symmetry breaking effects provides an expectation in close agreement\textsuperscript{14} with the $g - 2$ value directly measured at BNL\textsuperscript{15}. Therefore, the question is whether the $(e^+e^- - \tau)$ mismatch can be explained by physics only connected with isospin symmetry breaking and then something is missing – or if it calls for another kind of physics effect (actually hard to identify). Responding this question by leaving $e^+e^-$ data beyond any doubt may point towards a new physics effect exhibited by the muon anomalous magnetic moment.

2 A missing piece of isospin symmetry breaking?

As it is clear that all identified (and listed above) effects produced by isospin symmetry breaking should be considered, the question is about a possible missing piece in the isospin symmetry breaking procedure (or a piece not appropriately accounted for).

Actually, a clue towards the solution we propose has been given by Maltman\textsuperscript{16}; using sum rules derived from an OPE input, he concluded that the $\rho$ part of the $e^+e^-$ form factor data was inconsistent with being isospin 1, in contrast with the corresponding information provided by $\tau$ data. This statement implies that either the quality of the available $e^+e^-$ data\textsuperscript{b} can be questioned or that the $\rho^0$ meson is not a (pure) isospin 1 object.

Up to now, the model amplitudes used to describe the neutral and charged $\rho$ mesons may differ by feeding their propagators with different masses and widths to be fit with data; of course, the pion mass difference is also fed in, together with the $\omega$ (and $\phi$) meson(s) propagators, generally Breit–Wigner formulae. However, as the $e^+e^-$ data clearly exhibit\textsuperscript{2,3,4,5} the narrow (isospin 1 part of the) $\omega$ interfering with the broad $\rho^0$, one may ask oneself about the existence of a (broad) isospin 0 part of the $\rho^0$ meson which might make it differing from its charged

\textsuperscript{a}See also A. Hoecker, http://moriond.in2p3.fr/QCD/2008/MorQCD08Prog.html, Moriond QCD, March 2008.

\textsuperscript{b}One may note that these data has now been confirmed by several new and precise measurements.
partner beyond genuine mass effects. Stated otherwise, the question is whether mass and width differences for the ρ mesons exhaust isospin symmetry breaking in the pion form factor.

3 The Pion Form Factor at One Loop

In order to make our statements explicit, we have found it appropriate to work in the framework of the Hidden Local Symmetry (HLS) model \(^{17,18}\). However, our arguments are certainly valid in most other VMD–like models. In the HLS model, the pion form factor for both \(e^+e^-\) annihilation and \(τ\) decay writes:

\[
F_\pi(s) = (1 - \frac{a}{2}) - \frac{f_\rho g_{\rho\pi\pi}}{D_V(s)}, \quad (f_\rho = ag_f^2, \quad g_{\rho\pi\pi} = \frac{ag}{2}),
\]

(1)

where \(a\) is a parameter specific of this model – close to 2 – and \(g\) is the universal vector coupling – close to 5.5 (from QCD sum rules). \(D_V(s) = s - m_\rho^2\) is the inverse ρ bare propagator \((m_\rho^2 = ag_f^2 f_\rho^2)\). While including one loop effects, \(D_V(s)\) acquires a pion (and kaon) loop term \(Π_\rho(s)\) which shifts the ρ pole off the real \(s\)-axis. The transition amplitude from \(γ/W\) to (neutral/charged) \(ρ\) is also dressed by loop effects; this turns out to perform the change:

\[
f_\rho \rightarrow F_\rho = f_\rho - Π_{W/γ}(s)
\]

(2)

in the expression \(^d\) for \(F_\pi(s)\). The 3 loop functions \(Π_\rho(s)\) and \(Π_{W/γ}(s)\) just defined fulfill each a dispersion relation \(^{19}\) and their imaginary parts are influenced by SU(3) flavor symmetry breaking. Each of these carries a subtraction polynomial, which has been chosen of degree 2 and vanishing at the origin. Additionally, it has been possible to relate the subtraction polynomials for \(Π_W(s)\) and \(Π_γ(s)\). All this lessens significantly the model parameter freedom.

Breaking isospin symmetry turns out to multiply \(|F_\pi^\rho(s)|^2\) by some specific factors \(^9,10,11\) not discussed here (see \(^{19}\)) and add the \(ω\) and \(φ\) contributions to \(F_\pi^ρ(s)\). This, however, has been shown insufficient in order to restore consistency between \(e^+e^-\) and \(τ\) data \(^{12,13,14}\).

4 A model for \(ρ^0\), \(ω\), \(φ\) mixing

As it is clear that the isospin symmetry breaking effects listed above have to be taken into account, the question is rather about a missing piece in the scheme outlined in Section 3. While working at one loop order, the HLS model provides self–masses already referred to for the ρ meson propagators. However, it also contains the piece:

\[
\frac{imag}{4z_A} \left[(\rho^0_I + ω_I - \sqrt{2}z_Vφ_I) K^- \overleftrightarrow{∂} K^+ + (\rho^0_I - ω_I + \sqrt{2}z_Vφ_I) K^0 \overleftrightarrow{∂} \overline{K}^0\right]
\]

(3)

which – through kaon loops – generates transitions \(^e\) among the so–called ideal (bare) fields \(ρ^0_I\), \(ω_I\) and \(φ_I\) with no counter part affecting the \(ρ^\pm\) field. We have:

\[
\begin{aligned}
Π_{ωφ}(s) &= -g_{ωKKφKK} [Π_{±}(s) + Π_0(s)] \\
Π_{ρω}(s) &= g_{ρKKωKK} [Π_{±}(s) - Π_0(s)] \\
Π_{ρφ}(s) &= -g_{ρKKφKK} [Π_{±}(s) - Π_0(s)]
\end{aligned}
\]

(4)

\(^1\)In this paper, we only outline the method and refer the reader to \(^{19}\) for detailed information.

\(^d\)\(Π_\rho(s)\) refers to the pion form factor in \(e^+e^-\) annihilation, which will be denoted \(F_\rho^e(s)\), while \(Π_W(s)\) refers to the pion form factor in \(τ\) decay correspondingly denoted \(F_τ^ω(s)\).

\(^e\)The anomalous and the Yang–Mills pieces of the full HLS Lagrangian contribute also to the mechanism we outline here in a quite analogous manner \(^{19}\); these contributions will be skipped from now on. The constants \(z_A\) and \(z_V\) in Eq. 3 are SU(3) breaking parameters which have to be determined by fit.
as transition amplitudes between the (ideal) \( \rho_0^0, \omega_I \) and \( \phi_I \). \( \Pi_\pm(s) \) and \( \Pi_0(s) \) denote, resp. the charged and neutral kaon loops amputated of the coupling constants factored out for sake of clarity. These loops are defined by dispersion integrals over their imaginary parts and contain subtraction polynomials \( (P_\pm(s) \) and \( P_0(s) \)) real for real \( s \), the invariant mass squared flowing through the vector meson lines. These polynomials are chosen of degree 2 and vanishing at \( s = 0 \); their coefficients have to be fixed by external conditions. If isospin symmetry is conserved one may assume that \( P_\pm(s) = P_0(s) \) and, then, \( \Pi_{\rho_0}(s) \) and \( \Pi_{\rho\phi}(s) \) identically vanish; when isospin symmetry is broken this condition is certainly no longer fulfilled. Therefore, the HLS model which always predicts \( \omega_I - \phi_I \) transitions (as \( \Pi_{\omega\phi}(s) \) never vanishes identically), predicts additionally \( \rho_0^0 - \omega_I \) and \( \rho_0^0 - \phi_I \) transitions when isospin symmetry is broken.

Therefore, in the general case of isospin symmetry breaking, there are transitions among the ideal vector fields. If one defines the physical vector fields as eigenstates of the vector mass matrix, as the amplitudes in Eqs. 4 provide non–vanishing entries in the vector meson squared mass matrix, these cannot coincide with their ideal partners. Let us define the vector \( V \) and \( V_I \) as the vectors constructed with (resp.) the (physical) \( \rho^0, \omega \) and \( \phi \) fields on the one hand, and the (ideal) \( \rho_0^0, \omega_I \) and \( \phi_I \) fields the other hand. Then the mass eigenstates of the vector meson squared mass matrix and the ideal fields are related by \( V = R(s) V_I \) and \( V_I = \overline{R}(s) V \) with:

\[
R = \begin{pmatrix}
1 & \frac{\epsilon_1}{\Pi_{\pi\pi}(s) - \epsilon_2} & -\frac{\mu \epsilon_1}{(1 - z \gamma)m^2 + \Pi_{\pi\pi}(s) - \mu^2 \epsilon_2} \\
\frac{\epsilon_1}{\Pi_{\pi\pi}(s) - \epsilon_2} & 1 & \frac{\mu \epsilon_2}{(1 - z \gamma)m^2 + (1 - \mu^2) \epsilon_2} \\
\frac{1}{(1 - z \gamma)m^2 + \Pi_{\pi\pi}(s) - \mu^2 \epsilon_2} & \frac{\mu \epsilon_2}{(1 - z \gamma)m^2 + (1 - \mu^2) \epsilon_2} & 1
\end{pmatrix}
\]

where \( \epsilon_1 = \Pi_{\rho_0}(s) \) and \( \epsilon_2 = \Pi_{\rho\phi}(s) \) are functions of \( s \), real below \( s \sim 1 \text{ GeV}^2 \). Indeed, the loop imaginary parts start at the corresponding two–kaon thresholds. One neglects terms of second order in \( \epsilon_1 \) and/or \( \epsilon_2 \). \( \Pi_{\pi\pi}(s) \) is the pion loop representing the bulk of the \( \rho \) self–energy and \( m^2 = a g^2 f_\pi^2 \) is the unperturbed \( \rho \) meson mass squared.

Performing the change to physical fields into the HLS Lagrangian generates isospin symmetry violating couplings of the \( \omega \) and \( \phi \) fields to \( \pi^+\pi^- \), while leaving the \( \rho_0^0 \) coupling to \( \pi^+\pi^- \) identical to that of its ideal partner at leading (first) order in the \( \epsilon_i \). In contrast, the \( \gamma - \rho_0^0 \) transition amplitude (named \( f_\rho \) in Eq. 1) is modified to \( f_\rho + \delta f_\rho(s) \) at leading order in \( \epsilon_i \), while the \( W - \rho \pm \) transition is obviously unaffected. We thus get, using obvious notations:

\[
\left\{
\begin{array}{ccc}
\gamma \rightarrow \rho_0^0 & = & ag f_\pi^2 \\
\gamma \rightarrow \rho_\mp & = & \frac{\epsilon_1}{\Pi_{\pi\pi}(s) - \epsilon_2} + \frac{\mu^2 \epsilon_3}{3(1 - z \gamma)m^2 + \Pi_{\pi\pi}(s) - \mu^2 \epsilon_2}
\end{array}
\right.
\]

Therefore, because of one–loop effects, isospin symmetry breaking introduces a \( s \)–dependent difference between the \( \gamma - \rho_0^0 \) and \( W - \rho_\mp \) transitions; this is entirely due to the fact that ideal neutral vector fields cease to coincide with physical neutral vector fields, when defined as mass matrix eigenstates. Loop effects always affect the (\( \omega, \phi \)) sector, but the whole (\( \rho_0^0, \omega, \phi \)) is affected only when, additionally, isospin symmetry is broken. Clearly, this effect has not been accounted for in previous analyses of the pion form factor in \( e^+e^- \) and \( \tau \) data.

5 How to work out the model?

The issue now is whether the (\( \rho_0^0, \omega, \phi \)) mixing we just sketched is able to account numerically for the long standing mismatch between \( e^+e^- \) and \( \tau \) data. From the point of view of data analysis, the number of parameters (coupling constants, U(3)/SU(3) breaking parameters,
subtraction parameters from dispersion integrals...) in our HLS based model is too large to hope fixing them reasonably well using only the $e^+e^-$ and $\tau$ data. Fortunately, there is a way out.

It indeed happens that the radiative decays ($PV\gamma$ and $P\gamma\gamma$), which are accounted for by the anomalous sector of the HLS Lagrangian, depend on a large part of the parameters involved in our model and can serve to fix them quite reliably, even by fitting them in isolation. If one adds to this data set the leptonic decay information for the $\omega$ and $\phi$ mesons on the one hand, and two–pion decay information of the $\phi$ meson on the other hand, the minimization program becomes numerically well defined. This additional data set will be referred to as "decay data".

Therefore, the resolution method we propose is to consider the $e^+e^-$ and $\tau$ data together with the decay data. One should stress that the form factor $F_{\tau\pi}(s)$ is entirely determined, from a numerical point of view, by the $e^+e^-$ and decay data in isolation, since actually all parameters it depends on are already involved in the decay widths considered or in $F^e_{\pi}(s)$. Stated otherwise, $F_{\tau\pi}(s)$ can be predicted from our model using only the $e^+e^-$ and decay data. We actually consider this last property as the main test of validity of our approach.

Before closing this Section, one should mention that the HLS model, as currently known, involves the pseudoscalar meson ($P$) and only the lowest lying vector meson nonet ($V$). This prevents, for the time being, to include in fit procedures invariant mass region influenced by higher mass vector mesons, such as the $\rho(1450)$ or $\rho(1700)$. Therefore, the fits to form factors will be restricted to the $s < 1$ GeV$^2$ region, where the corresponding effects should be limited.

6 Brief analysis of fit results

Detailed fit information can be found in Ref. where they are lengthily presented and discussed. Here, we limit ourselves to the most relevant. One should also mention that data on the pion form factor in the close spacelike region are included in our fits.

| Table 1: Global fit information with special emphasis on pion form factor data subset contributions. |
|---------------------------------|-----------------|-----------------|-----------------|
|                                | Full Fit         | Excluding $\tau$ data | No $\rho^0 - \rho^\pm$ mass shift |
| $\chi^2/dof$                  | 313.83/331       | 257.73/274        | 321.75/332      |
| Probability                   | 74.4%            | 75.2%             | 64.7%           |
| All Timelike Data             |                 |                 |                 |
| ($\chi^2$/points)             | 187.15/(209)     | 176.70/(209)      | 192.38/(209)    |
| $\tau$ ALEPH                  |                 |                 |                 |
| ($\chi^2$/points)             | 23.86/(33)       | 42.27/(33)        | 24.28/(33)      |
| $\tau$ CLEO                   |                 |                 |                 |
| ($\chi^2$/points)             | 26.06/(25)       | 26.16/(25)        | 28.55/(25)      |

Table 1 clearly shows that the description of the global data set is quite satisfactory. The second data column gives mostly the $\chi^2$ distance of the model to the $\tau$ data points left out from the fit procedure; this clearly illustrates that $F_{\tau\pi}(s)$ is indeed numerically derived from the HLS model together with data independent of the $\tau$ form factor. One can even remark that CLEO data are as well accounted for as when including them in the fit procedure! Comparing the first and third data columns, one may also remark that, allowing a different mass for the

\footnote{The $e^+e^-$ spectrum in the region of the $\phi$ meson is not available as such in the data published by the various Collaborations at Novosibirsk.}

\footnote{Except for a parameter $\delta m^2$ which may account for a (possible) mass difference between $\rho^0$ and $\rho^\pm$.}
charged and neutral $\rho$ mesons provides a marginal improvement. Actually, the corresponding mass difference (at the edge of statistical significance) is visible only in ALEPH data and needs confirmation by forthcoming data sets.

Therefore, one may conclude that introducing the effects of isospin symmetry breaking (a non–zero $\epsilon_1(s)$, substantially) on vector meson mixing, together with the already reported effects, is enough to reconcile the $e^+e^-$ and $\tau$ data. A missing piece in the current isospin symmetry breaking procedure is then identified as the effects of the isospin 0 component of the $\rho^0$ meson which has no counterpart inside the $\rho^\pm$ meson.

![Figure 1: Distributions of fit residuals for the ALEPH and CLEO $\tau$ data from the full fit with an allowed mass difference between charged and neutral $\rho$ mesons. The arrows indicate the upper limit of the fit region. The insets magnify the $\rho$ peak invariant mass region.](image)

Other information is provided by Figs. 1 and 2 which exhibit the fit residuals. One can clearly consider them as structureless in the region below 0.9 GeV as no obvious $s$–dependent effect can be observed. One also clearly sees the effects of higher mass vector mesons starting as early as around the GeV region.

![Figure 2: Residual distribution for all the $e^+e^-$ new timelike data over the whole invariant mass interval. The inset magnifies the $\rho$ peak invariant mass region.](image)

As final conclusion, one may indeed consider that $e^+e^-$ and $\tau$ data do not exhibit any mis-
match once all consequences of isospin symmetry breaking are indeed considered, including the isospin 0 component generated inside the $\rho^0$ meson. Then, it follows from this work that the predicted value of the muon anomalous moment derived using $e^+e^-$ data is indeed reliable and that the actual mismatch is between the prediction of the muon $g - 2$ and its direct (BNL) measurement\textsuperscript{15}, rather than between $e^+e^-$ and $\tau$ data. Therefore, getting an improved measurement\textsuperscript{25} of the muon anomalous magnetic moment becomes a key issue, possibly a window on some New Physics.

References

1. F. Jegerlehner, *Nucl. Phys. Proc. Suppl.* **126**, 325 (2004), *Nucl. Phys. Proc. Suppl.* **131**, 213 (2004).
2. L. M. Barkov et al, *Nucl. Phys.* B **76**, 512 (1978).
3. R.R. Akhmetshin et al, *Phys. Lett.* B **578**, 285 (2004).
4. R.R. Akhmetshin et al, *Phys. Lett.* B **648**, 28 (2007).
5. R.R. Akhmetshin et al, *JETP Lett.* **84**, 413 (2006).
6. M.N. Achasov et al, *J. Exp. Theor. Phys.* **103**, 380 (2006).
7. S. Schael et al, *Phys. Rept.* **421**, 191 (2005).
8. S. Anderson et al, *Phys. Rev.* D **61**, 11202 (2000).
9. W.J. Marciano and A. Sirlin, *Phys. Rev.* D **71**, 3629 (1993).
10. V. Cirigliano, G.Ecker and H. Neufeld, *Phys. Lett.* B **513**, 361 (2001), *JHEP* **08**, 002 (2002).
11. A. Flores-Tlalpa and G. Lopez Castro, *Phys. Rev.* D **72**, 113003 (2005), *Phys. Rev.* D **74**, 071301 (2006).
12. M. Davier, S. Eidelman, A. Hocker and Z. Zhang, *Eur. Phys. J.* C **27**, 497 (2003), *Eur. Phys. J.* C **31**, 503 (2003).
13. M. Davier, A. Hocker and Z. Zhang, *Rev. Mod. Phys.* **78**, 1043 (2006).
14. M. Davier, *Nucl. Phys. Proc. Suppl.* **169**, 288 (2007).
15. G. W. Bennett et al, *Phys. Rev.* D **73**, 072003 (2006).
16. K. Maltman, *Phys. Lett.* B **633**, 512 (2006) *Int. J. Mod. Phys.* A **21**, 813 (2006).
17. M. Bando, T. Kugo and K. Yamawaki, *Phys. Rept.* **164**, 217 (1988).
18. M. Harada and K. Yamawaki, *Phys. Rept.* **381**, 1 (2003).
19. M. Benayoun, P. David, L. DelBuono, O. Leitner and H. B. O’Connell, ArXiv 0711.4482 [hep/ph]; *Eur. Phys. J.* C ,to be published.
20. T. Fujiwara, T.Kugo, H. Terao, S. Uehara and K. Yamawaki, *Prog. Theor. Phys.* **73**, 926 (1985).
21. M. Benayoun, L. DelBuono, S. Eidelman, V.N. Ivanchenko and H. B. O’Connell, *Phys. Rev.* D **59**, 114027 (1999).
22. M. Benayoun, L. DelBuono, Ph. Leruste and H. B. O’Connell, *Eur. Phys. J.* C **17**, 303 (2000).
23. S.R. Amendolia et al, *Nucl. Phys.* B **277**, 168 (1986).
24. E.B. Dally et al, *Phys. Rev. Lett.* **48**, 375 (1982).
25. B. Lee Roberts et al, ArXiv 0510056[hep/ex]; *Nucl. Phys. Proc. Suppl.* **155**, 372 (2006).