A New Execution Model for the logic of hereditary Harrop formulas

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Abstract: The class of first-order Hereditary Harrop formulas (fohh) is a well-established extension of first-order Horn clauses. Its operational semantics is based on intuitionistic provability.

We propose another operational semantics for fohn which is based on game semantics. This new semantics has several interesting aspects: in particular, it gives a logical status to the read predicate.

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1 Introduction

The logic of first-order hereditary Harrop formulas is a well-established extension to the logic of Horn clauses. Its operational semantics is based on intuitionistic provability. In the operational semantics based on provability such as uniform provability [6, 7], solving the universally quantified goal \( \forall x G \) from a program \( P \) simply terminates with a success if \([c/x]G\) is solvable from \( P \) where \( c \) is a new constant.

Our approach in this paper involves a modification of the operational semantics to allow for more active participation from the user. Executing \( \forall x G \) from a program \( P \) now has the following two-step operational semantics:

- Step (1): the machine tries to prove \( \forall x G \) from a program \( P \). If it fails, the machine returns the failure. If it succeeds, goto Step (2).

- Step (2): the machine requests the user to choose a constant \( c \) for \( x \) and then proceeds with solving the goal, \([c/x]G\).

It can be easily seen that our new semantics is more “constructive” than the old semantics. In particular, it gives a logical status to the read predicate in Prolog.
As an illustration of this approach, let us consider the following program which computes the cube of a natural number.

\[ \forall x \forall y (\text{cube}(x, y) : \neg \text{nat}(x) \land y \text{ is } x \times x \times x) \]

Here, : \neg represents reverse implication. As a particular example, consider a goal task \( \forall x \exists y (\text{nat}(x) \supset \text{cube}(x, y)) \). This goal simply terminates with a success in the context of \([7]\) as it is solvable. However, in our context, execution does more. To be specific, execution proceeds as follows: the system requests the user to select a particular number for \( x \). After the number -- say, 5 -- is selected, the system returns \( y = 125 \).

As seen from the example above, universally quantified goals in intuitionistic logic can be used to model interactive tasks.

In this paper we present the syntax and semantics of this language. The remainder of this paper is structured as follows. We describe \( fohh \) logic in the next section. Section 3 describes the new semantics. Section 4 concludes the paper.

2 First-Order Hereditary Harrop Formulas

The extended language is a version of Horn clauses with some extensions. It is described by \( G \)- and \( D \)-formulas given by the syntax rules below:

\[
G ::= A \mid G \land G \mid \exists x \; G \mid \forall x \; G \mid D \supset G
\]

\[
D ::= A \mid G \supset A \mid \forall x \; D
\]

In the rules above, \( A \) represents an atomic formula. A \( D \)-formula is called a \( fohh \).

In the transition system to be considered, \( G \)-formulas will function as queries and a set of \( D \)-formulas will constitute a program. We will present the standard operational semantics for this language as inference rules \([1]\). The rules for executing queries in our language are based on uniform provability \([7]\). Below the notation \( D; \mathcal{P} \) denotes \( \{D\} \cup \mathcal{P} \) but with the \( D \) formula being distinguished (marked for backchaining). Note that execution alternates
between two phases: the goal reduction phase (one without a distinguished clause) and the backchaining phase (one with a distinguished clause).

**Definition 1.** Let $G$ be a goal and let $\mathcal{P}$ be a program. Then the task of proving $G$ from $\mathcal{P} - pv(\mathcal{P}, G)$ – is defined as follows:

1. $pv(A; \mathcal{P}, A)$. % This is a success.
2. $pv((G_0 \supset A); \mathcal{P}, A)$ if $pv(\mathcal{P}, G_0)$. % backchaining
3. $pv(\forall x D; \mathcal{P}, A)$ if $pv([t/x]D; \mathcal{P}, A)$.
4. $pv(\mathcal{P}, A)$ if $D \in \mathcal{P}$ and $pv(D; \mathcal{P}, A)$. % solving an atomic goal
5. $pv(\mathcal{P}, G_0 \land G_1)$ if $pv(\mathcal{P}, G_0)$ and $pv(\mathcal{P}, G_1)$.
6. $pv(\mathcal{P}, D \supset G)$ if $pv(\{D\} \cup \mathcal{P}, G)$.
7. $pv(\mathcal{P}, \forall x G)$ if $pv(\mathcal{P}, [y/x]G_1)$ where $y$ is a new free variable.
8. $pv(\mathcal{P}, \exists x G)$ if $pv(\mathcal{P}, [t/x]G)$ where $t$ is a constant or a variable.

### 3 An Alternative Operational Semantics

Adding game semantics requires fundamental changes to the execution model. To be precise, our new execution model – adapted from [3] – now requires two phases:

1. the proof phase: This phase builds a *proof tree*. This proof tree encodes all the possible execution sequences.
2. the execution phase: This phase actually solves the goal relative to the program using the proof tree.

Note that a proof tree can be represented as a list and this idea is used here. Now, given a program $\mathcal{P}$ and a goal $G$, a proof tree of $\mathcal{P} \supset G$ is a list of tuples of the form $\langle E, i \rangle$ or $\langle E, (i, j) \rangle$ where $E$ is a (proof) formula and $i, j$ are the distances to $F$’s children in the proof tree. Below, $a_1 :: \ldots :: a_n :: nil$ represents a list of $n$ elements.

**Definition 2.** Let $G$ be a goal and let $\mathcal{P}$ be a program. Then the task of proving $\mathcal{P} \supset G$ and returning its proof tree $L$ – written as $pv(\mathcal{P} \supset G, L)$ – is defined as follows:
\(pv(E, \langle E, \rightarrow \rangle :: \text{nil})\) if \(E = A; \mathcal{P} \supset A\). % This is a leaf node.

\(pv(E, \langle E, 1 \rangle :: L)\) if \(E = \neg\left(G_0 \supset A\right); \mathcal{P} \supset A\) and \(pv(\mathcal{P} \supset G_0, L)\).

\(pv(E, \langle E, 1 \rangle :: L)\) if \(E = \forall x D; \mathcal{P} \supset A\) and \(pv([t/x]D; \mathcal{P} \supset A, L)\).

\(pv(E, \langle E, 1 \rangle :: L)\) if \(E = \mathcal{P} \supset A\) and \(D \in \mathcal{P}\) and \(pv(D; \mathcal{P} \supset A, L)\).

\(pv(E, \langle E, (m + 1, 1) \rangle :: L_2)\) if \(E = \mathcal{P} \supset (G_0 \land G_1)\) and \(pv(\mathcal{P} \supset G_0, L_0)\) and \(pv(\mathcal{P} \supset G_1, L_1)\) and \(\text{append}(L_0, L_1, L_2)\) and \(\text{length}(L_1, m)\).

\(pv(E, \langle E, 1 \rangle :: L)\) if \(E = \mathcal{P} \supset (D \supset G)\) and \(pv((\{D\} \cup \mathcal{P}) \supset G, L)\).

\(pv(E, \langle E, 1 \rangle :: L)\) if \(E = \mathcal{P} \supset \exists x G\) and \(pv(\mathcal{P} \supset [t/x]G, L)\) where \(t\) is a constant or a variable.

Once a proof tree is built, the execution phase actually solves the goal relative to the program using the proof tree. In addition, to deal with the universally quantified goals properly, the execution needs to maintain an input substitution of the form \(\{y_0/c_0, \ldots, y_n/c_n\}\) where each \(y_i\) is a variable introduced by a universally quantified goal in the proof phase and each \(c_i\) is a constant typed by the user during the execution phase.

**Definition 3.** Let \(i\) be an index and let \(L\) be a proof tree and \(F\) is an input substitution. Then executing \(L_i\) (the \(i\) element in \(L\)) with \(F\) – written as \(ex(i, L, F)\) – is defined as follows:

1. \(ex(i, L, F)\) if \(L_i = (E, \rightarrow)\). % no child
2. \(ex(i, L, F)\) if \(L_i = (\mathcal{P} \supset G_0 \land G_1, (n, m))\) and \(ex(i - n, L, F)\) and \(ex(i - m, L, F)\). % two children
3. \(ex(i, L, F)\) if \(L_i = (\mathcal{P} \supset \forall x G, 1)\) and \(L_{i-1} = (\mathcal{P} \supset [y/x]G, n)\) and \(\text{read}(k)\) and \(ex(i - 1, L, F \cup \{y/c\})\) where \(c\) is the user input (the value stored in \(k\)).
4. \(ex(i, L, F)\) if \(L_i = (\mathcal{P} \supset \exists x G, 1)\) and \(L_{i-1} = (\mathcal{P} \supset [t/x]G, n)\) and \(ex(i - 1, L', F)\) where \(c = F(t)\) and \(L'\) is identical to \(L\) except that \(L'_{i-1} = (\mathcal{P} \supset [c/x]G, n)\). Hence the term \(t\) must be replaced by \(c\) to ensure correct operation.
Now given a program $P$ and a goal $G$, $L$ is initialized to the proof tree of $P \supset G$, and $F$ is initialized to an empty substitution and $n$ is initialized to the length of $L$.

4 Conclusion

In this paper, we have considered a new execution model for $fohh$. This new model is interesting in that it gives a logical status to the $read$ predicate in Prolog. We plan to connect our execution model to Japaridze’s Computability Logic \cite{3,4} in the near future.

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