Exclusion of Time in Mermin’s Proof of Bell-Type Inequalities

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Abstract

Mermin states that his nontechnical version of Bell’s theorem stands and is not invalidated by time and setting dependent instrument parameters as claimed in one of our previous papers. We identify deviations from well-established protocol in probability theory as well as mathematical contradictions in Mermin’s argument and show that Mermin’s conclusions are therefore not valid: his proof does not go forward if certain possible time dependencies are taken into account.

1 Introduction

We have presented a critique \cite{1} of a nontechnical version of Bell’s theorem presented by Mermin \cite{2} and forthwith referred to as MI. This critique was based on the introduction of hidden time and setting dependent instrument parameters in addition to the parameters considered by Bell and others \cite{3}. Mermin has responded to this critique \cite{4,5} and has attempted to show that our time and setting dependent instrument parameters fail to undermine his reasoning and that our extended parameter space “collapses” onto his. We demonstrate below that Mermin \cite{5} (referred to as MII) has not properly considered the role of time and the possible stochastic independence of the family of our instrument parameters from the source parameter. When these and other factors are taken into account it becomes clear that our extended parameter space does indeed render his proof invalid.

Owing to the complexity of the problem, we will revert to notation similar to that defined in our previous publications \cite{1,6-8}. However, we denote the random variables with capital letters and use the lower case for the values that these variables may assume. To facilitate the discussion, we provide a one to one correspondence of Mermin’s notation and ours, at least as far as possible. We consider random variables $A = \pm 1$ in station $S_1$ and $B = \pm 1$ in station $S_2$ that describe spin measurements and are indexed by instrument settings that are
characterized by three-dimensional unit vectors \( \mathbf{a}, \mathbf{b}, \mathbf{c} \) in both stations. MI introduces no precise counterpart for the random variables \( A, B \) and only considers green and red detector flashes. These green and red flashes correspond then to values that \( A, B \) assume: +1 which we can identify with green and −1 which we can identify with red. The key assumption of MI in his original proof \cite{2} and of Bell and others \cite{3} is that the random variables \( A, B \) depend only on the setting in the respective station and on another random variable \( \Lambda \) that characterizes the particles emitted from a common source. Instead of \( \Lambda \), MI uses “instruction sets” e.g. GGR meaning flash green for settings \( \mathbf{a}, \mathbf{b} \) (which MI actually labels 1, 2) and flash red for setting \( \mathbf{c} \) (labelled 3 by MI). In our notation this means, for instance, that for a certain value \( \lambda \) (of the parameter \( \Lambda \)) that corresponds to the instruction set GGR we have \( A_\mathbf{a}(\lambda) = A_\mathbf{b}(\lambda) = +1 \) and \( A_\mathbf{c}(\lambda) = −1 \). The possible choices of \( \Lambda \) are restricted by MI \cite{2} and Bell \cite{3} invoking Einstein locality: The source parameter \( \Lambda \) (Mermin’s original \cite{2} instruction sets, as well as their frequency of occurrence) does not depend on the settings.

2 Mermin’s Proof and our Critique

Mermin’s main argument is based on the following facts \cite{5}. “The data accumulated in many runs of this experiment have two important features:

i) In those runs in which the detectors happen to have been given the same settings, the lights always flash the same color.

ii) If all runs are examined without reference to the settings of the detectors, the pattern of flashes is completely random, in particular, the colors flashed are equally likely to be the same or different.”

We first analyze Mermin’s original \cite{2} proof, but using our notation instead of his. The following Table 1 summarizes the eight possible instruction sets and the nine possible different \( AB \) products which are used in MI, MII as a model for EPR-experiments. According to point

| \( \Lambda \) | \( A_\mathbf{a}B_\mathbf{a} \) | \( A_\mathbf{a}B_\mathbf{b} \) | \( A_\mathbf{a}B_\mathbf{c} \) | \( A_\mathbf{b}B_\mathbf{a} \) | \( A_\mathbf{b}B_\mathbf{b} \) | \( A_\mathbf{b}B_\mathbf{c} \) | \( A_\mathbf{c}B_\mathbf{a} \) | \( A_\mathbf{c}B_\mathbf{b} \) | \( A_\mathbf{c}B_\mathbf{c} \) |
|---|---|---|---|---|---|---|---|---|---|
| RRR | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 |
| RRG | +1 | +1 | −1 | +1 | +1 | −1 | −1 | −1 | +1 |
| RGR | +1 | −1 | +1 | −1 | +1 | −1 | +1 | −1 | +1 |
| GRR | +1 | −1 | −1 | −1 | +1 | +1 | −1 | +1 | +1 |
| GGR | +1 | +1 | −1 | +1 | +1 | −1 | −1 | −1 | +1 |
| GRG | +1 | −1 | +1 | −1 | +1 | −1 | +1 | −1 | +1 |
| RGG | +1 | −1 | −1 | −1 | +1 | +1 | −1 | +1 | +1 |
| GGG | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 | +1 |

Table 1: Possible (but exclusive) \( AB \) products

i) listed above, the columns of \( A_\mathbf{a}B_\mathbf{a}, A_\mathbf{b}B_\mathbf{b} \) and \( A_\mathbf{c}B_\mathbf{c} \) have all entries +1. The main point
of MI, MII is then, that each of the 8 rows of the Table I contains at least five entries +1 and at most 4 entries −1. This conclusion is obtained by considering each entry in the remaining six columns separately. As stated in MII p 144 “Since each of the nine possible pairs of settings is equally likely in any run, any of the six instruction sets in which both colors appear (for example, GGR) will result in the same color flashing 5/9 of the time”. This amounts to averaging each of the eight rows of ±1 of Table I. Using our notation we can complete this step of Mermin’s argument in a slightly more transparent way. Consider the row sums in Table I for each single one of the eight possible instruction sets (i.e. for each given value \( \lambda_s, s = 1, ..., 8 \) that \( \Lambda \) may assume). This sum equals

\[
\sum_{\text{row}} (\lambda_s) := A_aB_a + A_aB_b + A_aB_c + A_bB_a + A_bB_b + A_bB_c + A_cB_a + A_cB_b + A_cB_c \quad (1)
\]

or

\[
\sum_{\text{row}} (\lambda_s) = (A_a + A_b + A_c) \cdot (B_a + B_b + B_c) = (A_a + A_b + A_c)^2 \quad (2)
\]

because from i) given above it follows that \( A_i = B_i \) for \( i = a, b, c \). Each pair of settings occurs with probability 1/9. Thus one obtains for the average of each row

\[
\frac{1}{9} \sum_{\text{row}} (\lambda_s) \geq \frac{1}{9} \quad (3)
\]

as the square equals either 1 or 9. This, according to MII, “is incompatible with feature ii) of the data”.

On the surface, all of this appears to be quite trivially correct. There are, however, subtle contradictions in MII. It is stated on p 143 (point 2) of [5] that we are dealing with “conjectured relations among various hypothetical outcomes of a single experiment to be performed at a single time in one of several different versions (which is what Bell’s theorem is about)” and not with “outcomes of several different versions of an experiment, all of which were actually performed at various different times”. According to this statement, there are no data to be considered because Bell’s theorem is only about “conjectured relations...” and therefore the statement that i) is incompatible with feature ii) of the data makes no sense.

This contradiction in Mermin’s writings has its origins in the fact that Table I can not be directly linked to the experimental data of any EPR experiment because the construction and use of the table does not follow well established protocol of probability theory. Feller [9] states “If we want to speak about experiments or observations in a theoretical way and without ambiguity, we first must agree on the simple events representing the thinkable outcomes; they define the idealized experiment...... By definition every indecomposable result of the (idealized) experiment is represented by one, and only one, sample point. The aggregate of all sample points will be called the sample space.” Following Feller’s prescription for the case of EPR experiments of the type described above, we define the simple (or indecomposable) events that form the sample space \( \Omega \) to be the 72 triplets of the form \((\lambda_s; i, j)\) where \( \lambda_s, s = 1, ..., 8 \) is one of the eight instruction sets and \( i, j \) can assume the values \( a, b, \) or \( c \). Substituting these 72 triplets into the functions \( A \) and \( B \) and subsequently forming the products \( A.B \) we obtain the entries in Table I.
The all important question is: Is it possible to sample Table 1? Reformulated in mathematical terms this question becomes: Is \( A_1(\Lambda)B_3(\Lambda) \) a random variable and what is its expectation value?

One certainly can sample the column sums of Table 1. For example one could instruct the experimenters in both stations to keep their choices of settings fixed throughout the duration of the experiment. In the special case that each instruction set \( \lambda_s \) occurs with equal probability \( \frac{1}{8} \) we obtain nine expressions of the form \( \frac{1}{8}\sum_{s=1}^{8}A_i(\lambda_s)B_j(\lambda_s) \). This possibility is a special case of the statistical argument discussed in section 3 below. However, it is impossible to sample the row sums of Table 1, which of course is the linchpin in Mermin’s argument given above. Indeed it is impossible to perform experiments or take observations for nine different randomly chosen pairs of settings and to simultaneously have the guarantee that the \( \lambda_s \) remain all the same. Thus

\[
Y := \frac{1}{8} \sum_{i,j=a,b,c} A_i(\Lambda)B_j(\Lambda)
\]  

(4)

is not a random variable. To see this recall that the instruction set \( \Lambda \) is a random variable defined on the sample space \( \Omega \), that is \( \Lambda = \Lambda(\omega) \) where \( \omega \in \Omega \) signifies any of the 72 simple (indecomposable) events [9]. However, at a given time only one of the nine products \( A_iB_j \) listed in Table 1 can be measured. Measurement of each of the nine products will require nine separate experiments. Thus there will be nine different \( \omega \)'s governing these nine experiments. Hence there is no guarantee that the values of \( \Lambda \) at these nine \( \omega \)'s will be the same, nor is there a guarantee that the six terms \( A_a(\Lambda), A_b(\Lambda), \ldots, B_c(\Lambda) \) will be the same at each occurrence. Thus there is no guarantee that \( Y \) as given by Eq. (4) will be a function of a single \( \omega \), only. Consequently \( Y \) is not a random variable. Hence, an application of Fubini’s theorem on double integration shows that \( A_1(\Lambda)B_3(\Lambda) \) is not a random variable and Table 1 cannot be sampled.

These considerations show, that the mathematical operations in MII that link Table 1, Eq.(3) and the experimental data do not properly include the subtle distinctions that are necessary to define random variables. If one wants to apply the inequality of Eq.(3) to the statistics of the experimental data one needs to make sure that all assumptions that constitute the physical and mathematical model are (i) consistent with well established mathematical protocol, (ii) consistent with the procedures of the physical experiment, (iii) are general enough to describe the physical experiment and, most importantly, (iv) lead to a sampling of the complete Table 1 by the experimental procedure. It is the purpose of this paper to show that the assumptions in MII do not fulfill these four requirements and that the inclusion of setting and time dependent instrument parameters prevents the proof that Table 1 is necessarily sampled by EPR experiments. These factors together show that Eq.(3) is not applicable to the statistics of EPR experiments.

We first show that the parameter space used by Mermin is not general enough to describe EPR experiments. MII claims on p 145 [5] that explicit consideration of time can be excluded “because it is obviously irrelevant”. We interpret this statement to mean that Eqs. (1)-(3) can be proven and the sampling of Table 1 can be established with and without consideration of time and setting dependent instrument parameters. Therefore, we first ask the general
question whether time dependencies can somehow be absorbed into the instruction sets of MII. Bell supporters often maintain that in each of their proofs \( \Lambda \) is totally arbitrary. In this context it is frequently claimed that time is certainly “known” also at the source and therefore one must be able to combine any value \( \lambda \) that \( \Lambda \) assumes with the time of measurement \( t_{\text{meas}} \) to form a new source parameter \( \bar{\Lambda} \) with values \( \bar{\lambda} = (\lambda, t_{\text{meas}}) \). However, such a combination runs into the following serious mathematical contradiction. In all Bell-type proofs, at a certain particular step of the proof, one must have the same \( \lambda \) for a sequence of different setting pairs. For example, in Mermin’s proof as given in Eqs. (1) - (3) above, one must have the same \( \lambda_s \) for all nine pairs of settings \( ((i,j) = a, b, c) \). Yet, the measurement times \( t_{ij} \) for different pairs of settings must all be different by a strictly positive amount. Thus, \( \lambda \) and \( t_{\text{meas}} \) follow different and contradicting mathematical requirements and cannot be combined into a new parameter \( \bar{\Lambda} \). Thus, the procedure used in MI, MII to prove Bell-type inequalities prevents the general use of time and time dependencies and therefore restricts the parameter space. The setting and time dependence of our instrument parameters represents an additional generalization of the parameter space and introduces further unsurmountable problems to Bell-type proofs. Note that we have indexed the time of measurement by the settings on both sides. This fact indicates only that the given time of measurement \( t_{ij} \) has been determined by the choice of the given settings \( i, j \) just before the measurement occurred. It certainly does not mean that any nonlocal dependence on both settings is introduced either into \( A \) or into \( B \). The circumstance, however, that determines how \( t_{ij} \) is actually chosen shows that this choice does depend on both settings and illustrates once more the special role of time for EPR experiments.

We now subdivide our proof (that the assumptions leading to Table 1 are mathematically and physically inconsistent with the EPR experiments or not general enough to describe them) into two parts (a1) and (b1). The first part (a1) deals with arguments that are made for the rows of Table 1 and the second (b1) for the columns.

(a1) In actual experiments it is impossible to take measurements for nine different pairs of settings and simultaneously to have the guarantee that the \( \lambda_s \) are all the same. Therefore, the relevance of Table 1 to statistical considerations that apply also to the real experiments must first be proven. Only then can a contradiction to the statistical properties of the data follow from Eq. (3).

Indeed, and this is just a variant of Mermin’s characterization of Bell’s theorem as quoted above, it is often argued that Eq. (3) must be valid because one could have made a measurement at the same time with a different setting chosen by a person with free will, all in spite of the fact that only one term of Eq. (1) can be measured at a given time. We have no problem with the counterfactual argument that we can consider what would have happened if for any given term of a row any of the other eight pairs of settings were in force at the analyzer stations. But it is plainly inadmissible, indeed “countersyntaxial” \[10\], to sum, without further justification, all nine possible outcomes for a given instruction set. As an illustration consider the following example. Suppose that a restaurant serves three main dishes \( A_a, A_b, \) and \( A_c \) as well as three side dishes \( B_a, B_b, \) and \( B_c \) for a total of nine items on the menu. Assign the value +1 to
the combination $[A_i, B_j]$ for $i, j = a, b, c$ if it causes no ill effect for a given patron $\lambda_s$ and assign the value $-1$ to the combination $[A_i, B_j]$ if it does. We can not imagine any scenario where it would make sense to add the results of the nine possible outcomes. Yet this is exactly the reasoning behind the statement in MII that Bell’s theorem is dealing with “conjectured relations”. What is the justification? For this, MII seems to point to statistics, as did Bell before. We agree that indeed the argument can be saved by statistical reasoning. However, this salvage operation is only possible for a very limited set of parameters and not for our extended parameter space that properly includes time.

(b1) One can attempt to avoid the row argument and just argue with the columns of Table 1. In order to do this one needs to assume that each column arises from the same setting independent random variable $\Lambda$. In certain special cases such an argument based on the assumptions of MII will work, for instance, if each of the eight instruction sets are assumed with probability $\frac{1}{8}$. However, such an argument clearly does not work if each column would be subject to a different $\Lambda$ (instruction set random variable). If setting dependent instrument parameter random variables are involved in the formation of the instructions then there is no reason why, for example, the joint probability density of these parameter random variables should be the same for each of the nine different pairs of settings. Nor can these joint densities be factorized because of the possible time dependencies on both sides. We will show in section 4 how Einstein local different (for different settings) joint distributions can be constructed by use of two independent computers with equal clock time.

The above discussion indicates various degrees of failure in the reasoning of MII (and actually in all standard Bell-type proofs). Counterfactual arguments, as for example that one could have chosen another setting, may not yet render the proofs invalid. Countersyntaxial arguments, such as the addition of various outcomes that can not co-exist simultaneously, are wrong from the point of view of logic. Conclusions resulting from such arguments may still be correct, but certainly are suspicious. Proofs based on self-contradictory arguments, such as the combination of time and source parameter, always are fatal. The proof in MI, taking into account the source parameter only, is in the best of circumstances just counterfactual and, for a limited class of parameters, can be made rigorous by using the statistical argument and reordering (section 3).

The MII proof, attempted with setting and time dependent instrument parameters, contains the whole gamut of problems ranging from counterfactual to countersyntaxial to containing mathematical contradictions. To show the contradictions in the MII proof in the clearest possible way, we proceed below as follows. We do not start with Eqs. (1)-(3) or Table 1 but instead with the statistical arguments that permit the use of Eqs. (1)-(3) or Table 1. We show then that our extended parameter space does not collapse onto Mermin’s and that the statistical arguments can not be made for our extended parameter space.
3 The Statistical Argument and Reordering

We reformulate Bell’s main statistical idea for Mermin’s original example [2]. Assume that the number \( M \) of values that the hidden parameter \( \Lambda \) can assume is finite (extension to a countably infinite number is easy). In MI, MII we have \( M = 8 \). Denote this set of values by \( \{ \lambda_s \} \) and the probability \( P(\Lambda = \lambda_s) \) by \( p_s \), all with \( s = 1, \ldots, M \). Let \( N \) be the number of experiments performed. If \( N \) is large, then we have by the strong law of large numbers that with probability 1 the number of occurrences of \( \lambda_s \) is approximately equal to \( N \cdot p_s \), \( s = 1, \ldots, M \). Assume now with MII that each pair of settings \( (a, a) \), \( (a, b) \), etc. occurs with probability 1/9. The fact that the measurements can only be made in sequence plays no role, because now one can reorder the measurements into rows for a given \( \lambda_s \) and one could recognize such rows in the experiments if \( \lambda_s \) could somehow be made visible. To be more specific, for each \( s = 1, \ldots, M \) we would have observed about \( \frac{1}{9} N p_s \) times the value \( \lambda_s \) with each of the nine pairs of settings because of the stochastic independence of \( \Lambda \) and the settings. Thus, after reordering the data we would have accumulated about \( \frac{1}{9} N p_s \) rows of the form as given in Eq.(1), each of them corresponding to the value \( \lambda_s \). For each of these rows Eq.(3) holds. Of course, there may be some terms left over. However, by the strong law of large numbers, the number of such incomplete rows is negligible for large \( N \). This possibility of reordering is equivalent to the point of view that the column sums in Table I and in a metaphorical sense Table I itself are actually sampled or accumulated by the measurement process. We recall in passing that tables similar to Table I and their rows are used in various proofs of the Bell inequalities, often without regard to the main points of Bell which show under which circumstance and how these rows can be used to prove these inequalities. To just claim that because Bell’s inequalities follow from Table I that this will prove Bell’s inequalities is circular logic. The proof of Bell’s theorem is only then complete if a proof is given that in some appropriate way Table I is actually statistically sampled by the EPR-type experiments.

4 Time and Setting Dependent Instrument Variables

Key to all Bell-type proofs is that the source parameter \( \Lambda \) is independent of the instrument settings in the stations which is guaranteed by the design of the experiments (delayed choice of settings after particles have left the source). We have shown that the argument involving Eqs.(1) and (3) is countersyntaxial, except when salvaged by a statistical argument that involves reordering of the elements of these equations. As a consequence, this type of proof of Bell’s theorem eliminates only a small class of hidden variables. Bell and his followers maintain that in their proofs they eliminate all Einstein local parameters. We show below that the proof of the statistical argument given in section 3 comes to a halt when Einstein local time and setting dependent instrument parameters are included even in the special case where the distribution of the source parameter \( \Lambda \) does not depend on time.

We introduce these setting and time dependent instrument variables by using the example of two independent computers \( C_1 \) in \( S_1 \) and \( C_2 \) in \( S_2 \) with identical clock time. The computers contain evaluation routines \( A, B \) that map the source parameter \( \Lambda \) and local instrument
parameters $\Lambda_a^*(t)$ (that are generated by computer $C_1$) and $\Lambda_b^{**}(t)$ (that are generated by computer $C_2$) into $\pm 1$. These instrument parameters can be thought of as arbitrarily complicated numerical routines that supply output $\pm 1$ from the input of setting and time, the only condition being that for equal time and setting we must have, using Mermin’s convention,

$$A_i = B_i \text{ for } i = a, b, c \quad (5)$$

We maintain that any proof of Bell-type theorems needs to be able to accommodate such parameters if it should be taken seriously. Notice that we have now at least seven random variables $\Lambda, \Lambda^*_a(t)$ and $\Lambda^{**}_b(t)$ with $(i, j) = a, b, c$; in fact we have $\Lambda$ plus six families of random variables.

Mermin [5] claims that our model collapses onto his because of Eq. (5). We shall show presently that this claim is false. Let $t_{ij}^{(l)}$ be the times when $A$ and $B$ are measured with settings $i = a, b, c$ in $S_1$ and $j = a, b, c$ in $S_2$. Here $l = 1, 2, ..., L$. By the strong law of large numbers we can assume that $L$ is about the same for all nine pairs of settings $(i, j)$. Eq. (6) thus becomes:

$$A_i(\Lambda, \Lambda^*_a(t_{ii}^{(l)})) = B_i(\Lambda, \Lambda^{**}_a(t_{ii}^{(l)})) \quad (6)$$

On page 145 of MII it is stated about microsettings (our instrument parameters) that “no matter how strongly correlated the microsettings may otherwise be, the expanded instruction sets for that run must assign the same color to every microsetting underlying a given setting...i.e. for each of the three settings”. Expressed mathematically this statement results in:

$$A_a(\Lambda) = A_a(\Lambda, \Lambda^*_a(t_{aa}^{(l)})) = A_a(\Lambda, \Lambda^*_a(t_{ab}^{(l)})) = A_a(\Lambda, \Lambda^*_a(t_{ac}^{(l)})) \quad (7)$$

as well as

$$B_a(\Lambda) = B_a(\Lambda, \Lambda^{**}_a(t_{aa}^{(l)})) = B_a(\Lambda, \Lambda^{**}_a(t_{ba}^{(l)})) = B_a(\Lambda, \Lambda^{**}_a(t_{ca}^{(l)})) \quad (8)$$

and similar relations with cyclical exchange of $a, b, c$. Of course Eqs. (6), (7) and (8) imply

$$A_a(\Lambda) = B_a(\Lambda) \quad (9)$$

Thus MII postulates that if two functions $A$ and $B$ assume equal values on a finite set of points $t_{ii}^{(l)}$, $l = 1, ..., L$ then $A$ and $B$ must be identical constants (remember that the measurement times must all be different for different setting pairs). We are not aware of any theorem of calculus that would allow for such a sweeping statement. Nor does the physics involved permit any such conclusions. The instrument parameters are locally determined and will depend on the settings. Einstein locality and the delayed choice of the settings does not impose any further conditions in contrast to the situation with a source parameter only (that must not depend on the settings).

Mermin argues for the extended parameter space exactly in the same way he argues when only a source parameter is used. He states about the parameters of the extended space on page 145 [5]: “these must exist in every run, whether or not the detectors do end up with the same setting...and the particles have to be prepared for every one of these possibilities when they leave the source”. The whole point of our work is, of course, that we include setting and time dependent instrument parameters in the extended instruction set. Therefore the
statement “these must exist...whether or not the detectors do end up with the same setting” is lacking logic. Mermin’s statement would only make sense for the source parameter Λ, because the instrument settings are rapidly changed and are chosen only after the particles have left the source. Therefore, it would be correct to state for source parameters that “these must exist...whether or not the detectors do end up with the same settings”.

For instrument parameters (detector parameters), the flapping about of the instrument settings is, of course, locally known and they are determined by the settings at which the detectors do end up. It is important to realize that the random variables $A_i(Λ, Λ^{*}_i)$ and $B_j(Λ, Λ^{**}_j)$ can not be rewritten as $\bar{A}_i(\bar{Λ})$ and $\bar{B}_j(\bar{Λ})$ with the time $t$ being absorbed into $\bar{Λ}$. We have demonstrated in section 2 that this argumentation is not valid even when only source parameters and time are considered. A fortiori, this argument fails when time and setting dependent instrument parameters are added. Therefore the statement of MII that “The expanded instruction sets of Hess and Philipp must thus collapse back to the instruction sets of my example” is refuted.

We now turn to the question whether our expanded instruction sets, assuredly different from those of MII and Bell, will permit the all important statistical argument.

(a2) The reason why the statistical argument as the one given in section 3 (justifying the use of the same $\lambda$ in the rows) is invalid if time and setting dependent instrument parameters are included is the following. Assume just for simplicity the special case that the distribution of the source parameter $Λ$ does not depend on time. Then there will be still about $\frac{1}{9}N_p$s rows of data $A_iB_j (i, j = a, b, c)$ each corresponding to the values $\lambda_s$ that the source parameter $Λ$ can assume. However, the terms

$$A_i(\lambda_s, \lambda^{*}_i(t^{(u)}_{ij}))B_j(\lambda_s, \lambda^{**}_j(t^{(u)}_{ij}))$$

with $u = 1, 2, ..., \frac{1}{9}N_p$s may be all different because they contain different measurement times for different settings. Here, the difference of the role of time and the role of the source parameter becomes very clear. While the measurement times can under no circumstance be equal for different settings, the information $\lambda_s$ that is sent out from the source and relates to the spin must be independent of the settings and appears $\frac{1}{9}N_p$s times. As mentioned before MII maintains (“time is irrelevant”) that the pair $\lambda, t$ can be concatenated to new source parameter values $\bar{Λ}$. However, the mathematical conditions for the values $\lambda_s$ and the measurement times $t_{ij}$ contradict each other and prevent such combination. This is further illustrated by the fact that the pair $Λ, Λ^{*}_i(t_{ij})$ may depend on the setting $i$ in $S_1$ and the pair $Λ, Λ^{**}_j(t_{ij})$ on the setting $j$ in $S_2$ without violating Einstein locality because the instrument parameters are allowed to depend on the local setting.

(b2) It is very important to realize that one certainly can obtain different averages over $u = 1, 2, ..., \frac{1}{9}N_p$s of the terms in Eq.(10) for the nine different pairs of settings $i, j$ because (in the best of circumstances) these averages converge respectively to

$$\int \int \int A_i(\lambda, \lambda^{*}_i)B_j(\lambda, \lambda^{**}_j)\rho(i, j, \lambda, \lambda^{*}_i, \lambda^{**}_j)d\lambda d\lambda^{*}_i d\lambda^{**}_j$$

(11)
However, here the density $\rho$ will not only be setting dependent but also time dependent. Thus there is no guarantee that $\rho$ is a common factor of the nine integrands and that the argument leading to Eq. (3) will work.

It is often erroneously believed that a setting dependent density $\rho$ automatically means Einstein non-locality. This is clearly incorrect when time and setting dependent instrument parameters are involved. Indeed it would be ironic if, even in the trivial case where $\Lambda, \Lambda^*_i$ and $\Lambda^{**}_j$ are stochastically independent, their joint density, being now the product of the three marginal densities, would not be allowed to depend on the settings. Of course it is well known that the proof of the Bell inequalities does work for this special case [3]. However, it is easy to give an example of time and setting dependent instrument parameters $\Lambda^*_i(t)$ and $\Lambda^{**}_j(t)$ in the two stations that obey Einstein locality but are not stochastically independent because of their common time dependence. The corresponding density $\rho$ will depend on the settings and Bell’s proof will not work. The following construction of setting and time dependent instrument random variables should be instructive.

The particles emitted from the source may, in general, carry not only information related to the spin but also other information. As a consequence of all physical properties of the particle pair and as a consequence of the choice $i, j$ of settings by the experimenters, a pair of closely linked measurement times $t_i$ in station $S_1$ and $t_j$ in station $S_2$ emerges for each measurement. Assume as before $t_i = t_j = t_{ij}$. Different measurement times for a given correlated pair are addressed below. The fact that we have equal measurement times for any given correlated pair and given pair of settings gives us the possibility to construct parameter random variables $\Lambda_i^*(t)$ and $\Lambda^{**}_j(t)$ that are not independent in spite of the fact that they may be stochastically independent of the source parameter $\Lambda$ that contains some information on the spin. $\Lambda_i^*(t)$ and $\Lambda^{**}_j(t)$ may, for example, exhibit a “stroboscopic” randomness constructed on two computers with equal clock time in the following way. Both stations contain three stacks of files denoted by $\lambda_a^*(t), \lambda_b^*(t)$ and $\lambda_c^*(t)$ in station $S_1$ and entirely identically arranged stacks of files denoted by $\lambda_a^{**}(t), \lambda_b^{**}(t)$ and $\lambda_c^{**}(t)$ in station $S_2$. Given a particular setting pair and measurement time $t_{ij}$, two actual files are picked, one in each station. Because the stacks are identically arranged we have for all times

$$\lambda_i^*(t) = \lambda_i^{**}(t) \text{ for } i = a, b, c \quad (12)$$

Then pairs of settings $(i, j)$ are chosen sequentially and at random according to an arbitrary distribution and the measurements are performed during certain small time periods labelled as measurement time. The possible setting and time dependencies i.e. the order of the files in the stacks can be determined by arbitrary algorithms (including e.g. appropriate elements of the history of past experiments) and determine then the setting and time dependence of the joint probability density of $\lambda_i^*(t)$ and $\lambda_j^{**}(t)$ for different settings $i, j$. Thus, for the special case of a time independent source parameter, the instrument parameters may be stochastically independent of the source parameter. In addition virtually arbitrary stochastic dependencies of instrument and source parameters can be constructed. Next we chose the functions $A, B = \pm 1$ identical to each other and we see that Eqs. (3) and (4) hold. Note that this construction is not the most general one since Eq. (12) is only sufficient for Eqs. (3) and
to hold, but not necessary. In summary, our extended parameter set does not collapse onto Mermin’s nor can Mermin’s proof proceed in our extended space. All his essential objections against our work that we discussed so far are therefore without basis.

We add just two final remarks. MII tries to refute the possibility of time and setting dependent instrument parameters also on additional grounds by the following statement on p 146 [4]: “According to quantum mechanics...the statistical character of the data...is unaffected if the two detections of a given entangled pair are separated by arbitrary long time intervals” and “To maintain this in a Hess-Philipp expansion...requires, in the absence of spookily conspiratorial...exchange of information between the two detectors...that the choice of...microsettings for each setting must be the same for all times”. This statement contains several problems. First, to separate the detections of a given entangled pair by arbitrary long times $\Delta_t$ is experimentally very difficult. For experiments that involve optics, one would need to try separations over a time period $\Delta_t$ covering about the whole duration of the experiments that is necessary to accumulate the statistical data, i.e. at least minutes. One minute multiplied by the velocity of light corresponds to almost inter-planetary distances for the measurements. As a consequence, such measurements are difficult to perform and have not been performed. Second, it is logically inconsistent to come up with conditions that are not contained in the mathematics of Bell-type proofs and to drag in quantum mechanics in absence of any experimental confirmation. Third, and most importantly, our parameter space can be adapted to derive even this additional quantum mechanical result and without the necessity of “spookily conspiratorial” elements. Our argument is as follows. Any change of the measurement in one station (wing) by a time interval $\Delta_t$ requires experimental changes that may have some causal effect on both the source parameter $\Lambda(t)$ and the instrument parameter $\Lambda_i(t)$ of that wing. If, for instance, the change occurs in the wing of station $S_1$, then all that needs to be done is to replace $t$ by $t \pm \Delta_t$ in the time related arguments $\{\}$ of the function $A_\Lambda(\{\},\{\},\{\},\{\})$. A similar reasoning can be applied for the delay of measurements in the wing of station $S_2$ or even for both wings. The changes of the experimental configuration that need to be made to accomplish the time shift $\Delta_t$ for a given wing in the experiments can be taken as the causal reasons for the transformation. This shows that Mermin’s suspicion of “spookily conspiratorial” elements is without basis. There certainly is the possibility that the “conspiracy” may just be a standard cause-effect situation.

Note finally that under Mermin’s assumptions our extensive mathematical model [7] can indeed be simplified. However, these simplifications deal only with the calculations. For example, we can replace the B-splines by more elementary functions. The basic construction remains the same and is sketched on the first four pages of a recent manuscript [11]. Because we did not carry out a detailed specialization of our model as it relates to MII, Mermin infers that our model “must contain errors, as has been argued directly elsewhere”. In [12] and [13] we have refuted the papers [14] and [15] to which Mermin points.

Acknowledgement: the work was supported by the Office of Naval Research N00014-98-1-0604. The authors would also like to thank Salvador Barraza-Lopez for valuable suggestions to improve the presentation.
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