Sizable NSI from the $SU(2)_L$ scalar doublet-singlet mixing and the implications in DUNE

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Abstract

We propose a novel and simple mechanism where sizable effects of non-standard interactions (NSI) in neutrino propagation are induced from the mixing between an electrophilic second Higgs doublet and a charged singlet. The mixing arises from a dimensionful coupling of the scalar doublet and singlet to the standard model Higgs boson. In light of the small mass, the light mass eigenstate from the doublet-singlet mixing can generate much larger NSI than those induced by the heavy eigenstate. We show that a sizable NSI $\varepsilon_{ee}$ ($\sim 0.3$) can be attained without being excluded by a variety of experimental constraints. Furthermore, we demonstrate that NSI can mimic effects of the Dirac CP phase in the neutrino mixing matrix but they can potentially be disentangled by future long-baseline neutrino experiments, such as the Deep Underground Neutrino Experiment (DUNE).
I. INTRODUCTION AND GENERAL MOTIVATIONS

Neutrino oscillations (in the three neutrino framework) are the leading mechanism that explain neutrino flavor transitions observed from neutrinos produced in the sun, the Earth atmosphere, reactors, and accelerators. The parameters accounting for neutrino oscillations, the three mixing angles and the two mass squared differences, have been currently measured within a precision of 8% according to a global fit analysis [1]. One additional parameter, which encodes the violation of the charged parity (CP) symmetry in the lepton sector, is still to be determined. This parameter together with the determination of the neutrino mass ordering (normal or inverted hierarchy) are the two main unknowns in the three neutrino framework. Current and future facilities are aimed to find the two missing pieces and to improve the precision of the oscillation parameters.

Better understanding of uncertainties on both theory and experiment sides is crucial in the completion and improvement of our knowledge of the three active neutrino framework. The reactor mixing angle has been measured within a precision of 5% by ∼ 1km baseline reactor neutrino multidetector experiments [2, 3]. The measurement of the atmospheric parameters (the mixing angle and the mass squared difference) have also been improved in precision thanks to the observations in the disappearance channel by beam-based neutrinos experiments, but still the atmospheric mixing angle is less well-determined among the three mixing angles. However, thanks to the tension in the determination of the reactor mixing angle by current reactor and accelerator experiments, an indication of preferred values for the Dirac CP violating phase have started to emerge [4, 5]. This has opened the possibility of observing CP violation in the lepton sector, which might have an impact in the early Universe. Despite all success so far, upgrades of current and new facilities are needed to probe most of the Dirac CP parameter space, to determine the neutrino mass hierarchy and to improve the precision of the other parameters.

In addition to the standard three neutrino oscillation framework, there are well-motivated scenarios beyond the standard model (SM) that can have phenomenological consequences in neutrino oscillations. This opens the possibility to test new physics along with the standard programs pursued in neutrino oscillation facilities. It would be, for instance, interesting to investigate non-standard neutrino interactions (NSI), non-unitarity neutrino mixing, sterile neutrinos, violation of symmetries, etc. NSI were originally proposed even before neutrino
oscillations were proved \cite{6,9} and still today their phenomenological consequences are being studied. NSI can be a byproduct of neutrino mass models and in general are described by effective four-fermion interaction operators where the strength is characterized by dimensionless couplings (carrying all the flavor information) times the Fermi constant. NSI can be of the charged-current (CC) type or of the neutral-current type (NC) depending on the fields involved, and both of types have distinctive phenomenological consequences. The NSI yield additional contributions to the SM weak interactions and therefore constraints can be derived, for instance, from lepton universality and CKM unitarity \cite{10}. In cases where new physics enter above the electroweak scale, both the charged and the neutral sectors are connected (due to the $SU(2)_L$ symmetry) and thus stringent constraints from charged lepton flavor violating (CLFV) processes can have an impact on the neutral sector \cite{11}. The highlight of this work is to provide a simple mechanism to obtain large NSI effects and simultaneously avoid these constraints such that one can determine the NSI strength via neutrino oscillation experiments. There exist many particular examples in the literature. In general, the CC-like NSI affects neutrino production and detection and can be cleanly probed in experiments where neutrino–matter interactions can be neglected, as in reactor neutrino experiments \cite{12} (see also Ref. \cite{13}). The NC-like NSI affects the neutrino propagation and can be probed in long-baseline neutrino oscillation experiments since the sensitivity is driven by the neutrino–matter interactions. The current NSI constraints, considering neutrino oscillations only, can be found in Ref. \cite{14}. For a general review of the NC-like NSI constraints and phenomenological implications, we refer the reader to Ref. \cite{15} and references therein.

Among future experiments, the Deep Underground Neutrino Experiment (DUNE) is the main project that will determine the neutrino mass ordering and probe most parameter space of the Dirac CP violating phase. DUNE will use a powerful beam to produce a large number of neutrinos in a broad energy range (roughly between 0.5 and 20 GeV) that will be detected in a 40t far detector located at 1300km from the source \cite{16}. As a result, DUNE will be an interesting NSI laboratory. This has been the subject of different studies showing that DUNE will be sensitive to the NC-like NSI with couplings of the order of $0.1 G_F$ (see for instance \cite{17,19}). More importantly, degeneracies between the NSI couplings and the

\footnote{Both CC and NC NSI can be tested at the same time in a long-baseline experiment, however, the large number of parameters will decrease the sensitivity for some NSI couplings.}
standard oscillation parameters might challenge the precise determination of the unknown neutrino parameters. This is the case for the determination of the Dirac CP violating phase; the NSI new phases are a new source of the CP violation and one might observe CP violation effects, which result exclusively from NSI. In a minimal setup, it has been shown that ‘confusion’ can arise with an NSI parameter ($\varepsilon_{e\tau} \sim 0.3$) in T2K and NOvA [20, 21] (see also Ref. [22] in which the ‘confusion’ from $\varepsilon_{e\tau}$ was examined, after the measurement of $\theta_{13}$, at the probability level)\(^2\).

From the model building point of view, however, it is very challenging to come up with viable models which can produce such ‘large’ NSI couplings and avoid the constraints from CLFV processes. As a result, the main goal of this work is to introduce a mechanism that generates relative large NSI couplings ($\sim 0.3$) such that the aforementioned confusion can be realized. Future long-baseline neutrino oscillation experiments such as DUNE, can potentially resolve the confusion and investigate the phenomenological implications of the NSI.

We propose a novel and simple mechanism to obtain large NSI $\varepsilon_{e\tau}$. In addition to the SM, there exist an extra $SU(2)$ scalar doublet $\eta$ and a charged $SU(2)$ scalar singlet $\phi$. The pertinent scalar potential, including the SM Higgs doublet $H$, is

$$V \supset \mu_\eta^2 \eta^\dagger \eta + \mu_\phi^2 \phi^+ \phi^- + (\kappa \phi^- H \eta + h.c.). \quad (1)$$

The mixing between $\phi$ and the charged component of $\eta$ arises due to the coupling to the SM Higgs boson, $\kappa \phi^- \langle H \rangle \eta$, where $\kappa$ is a dimensionful coupling and $\langle H \rangle = v$. In the limit of $\mu_\phi^2, \kappa v \ll \mu_\eta^2$, the $\eta - \phi$ mixing is determined by the ratio of $\kappa v$ to $\mu_\eta^2$ while the mass of the light eigenstate $m_1$ is determined by $\mu_\phi$ and the $\kappa v/\mu_\eta$. These two components can cancel each other such that $m_1$ can be treated as an independent parameter from the $\eta - \phi$ mixing angle. The independence is pivotal to achieve large NSI, satisfying various bounds from charged lepton measurements.

Yukawa couplings of $\eta$ to leptons are introduced, obeying an imposed $Z_2$ symmetry, under which $\eta, \phi$ and the right-handed electron $e_R$ are odd. NSI can be generated through charged currents mediated by the charged component $\eta^\pm$, which is a superposition of the two mass eigenstates in light of the $\eta - \phi$ mixing. The light mass eigenstate contribution to NSI can have a large enhancement due to its small mass even if it is suppressed by the mixing angle,

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\(^2\) For an analytic study of the CP determination in the presence of NSI at low energies, see Ref. [23].
in that the mass $m_1$ is independent of the mixing. In other words, the mixing effect induces an additional contribution from the light eigenstate which can be much larger than the heavy state contribution. Furthermore, large NSI realized via cancellation require fine-tuning and as demonstrated below, taking into account various constraints, the level of $10^{-3}$ fine-tuning is needed for have sizable $\varepsilon_{ee}$ ($\sim 0.3$).

NSI mediated by $\eta$ alone are classified as dimension-6 ($d = 6$) operators in Ref [24] which usually comes with hazardous contributions to CLFV processes, while the mixing-induced NSI belong to $d = 8$ operators (in light of extra $\langle H \rangle^2$ compared to the $d = 6$ one) and is usually suppressed with respective to $d = 6$ ones. As pointed out in Refs. [25, 26], some of $d = 8$ operators only induce lepton flavor violation on the neutral sector but not on the charged lepton counterpart such that stringent constraints on charged lepton flavor violation can be escaped. In our model, for instance, $\tau \to 3e$ can be engendered by $\tau^- \to e^-\eta^0 \to e^-e^+e^-$ but not mediated by $\phi^\pm$ (responsible for NSI). Consequently, sizable NSI do not imply large CLFV effects. Furthermore, by virtue of the cancellation within $m_1$, the mixing-induced $d = 8$ operators become dominant over those of $d = 6$.

On the other hand, with the charged singlet $\phi$, the neutrino mass can be produced by adding an interaction $\bar{L}L\phi^+$, as proposed by Zee [27, 28]. The interaction itself can also yield NSI but it has been shown [29] that considerable NSI will demand large couplings of $\bar{L}L\phi^+$, rendering neutrinos too heavy. In contrast, our mechanism is based on cancellation to enhance NSI $\varepsilon_{ee}$ without involving the lepton number violating term $\bar{L}L\phi^+$, which is actually forbidden by the imposed $Z_2$ symmetry. Finally, models with a light gauge boson $Z'$ [30, 32] have been proposed to generate considerable NSI but due to various bounds, $\varepsilon_{ee}$ is constrained to be much less than 0.3.

The paper is organized in the following. In Section II we specify the model setup, followed by discussion of how sizable NSI can be attained via the doublet-singlet mixing in Section III. Various constraints are taken into account in Section IV. Then we perform the numerical analysis in the context of long-baseline neutrino experiments in Section V. Finally, we conclude in Section VI.
II. MODEL

We enlarge the SM particle content by including two scalar fields, one $SU(2)_L$ doublet $\eta$ and one charged singlet $\phi$. Furthermore, we impose a $Z_2$ symmetry under which $\eta$, $\phi$ and the right-handed electron are odd while the rest of SM particles are even:

$$\eta \sim (1, 2, +\frac{1}{2}, -) \ , \ \phi^- \sim (1, 1, -1, -) \ , \ e_R \sim (1, 1, -1, -),$$

where the entries in the parentheses denote the SM $SU(3)_c \times SU(2)_L \times U(1)_Y$ quantum numbers as well as the $Z_2$ parity. The reason of including the $Z_2$ symmetry is to avoid a myriad of experimental constraints from the charged lepton sector. Note that the $Z_2$ symmetry is broken by the SM electron Yukawa coupling and it is arguable that the smallness of the coupling results from the $Z_2$ symmetry protection.

The relevant terms in the scalar potential read,

$$V \supset \mu_\eta^2 \eta \eta^\dagger + \mu_\phi^2 \phi \phi^\dagger + \phi^- \phi^+ + (\kappa \phi^- H \eta + h.c.) ,$$

(2)

where $H$ is the SM Higgs doublet, $\kappa$ is a dimensionful coupling and $\mu_{\phi, \eta}^2 > 0$. Note that we focus on regions of the parameter space where $\phi$ and $\eta$ do not develop the vacuum expectation value (VEV).

On the other hand, the mixing between $\phi^\pm$ and $\eta^\pm$ arises due to the SM Higgs VEV $v$, and the mass matrix of $\phi^\pm$ and $\eta^\pm$ is given by

$$M_{\eta\phi}^2 = \begin{pmatrix} \mu_{\phi}^2 & \kappa v \\ \kappa v & \mu_{\eta}^2 \end{pmatrix}. $$

(3)

In the limit of $\mu_{\phi} , \sqrt{\kappa v} \ll \mu_{\eta}$, the masses of the two eigenstates $s_1$ and $s_2$ are

$$m_1^2 \simeq \mu_{\phi}^2 - \theta \kappa v + O(\theta^2)$$

$$m_2^2 \simeq \mu_{\eta}^2 + O(\theta)$$

(4)

with $\theta \simeq \kappa v / \mu_{\eta}^2$. Note that because of cancellation between $\mu_{\phi}^2$ and $\theta \kappa v (= \kappa^2 v^2 / \mu_{\eta}^2)$, $m_1$ can be treated as an independent parameter from the mixing angle although, without any fine-tuning, it is expected that $m_1^2 \sim \mu_{\phi}^2 \sim \kappa^2 v^2 / \mu_{\eta}^2$ and $\theta \sim m_1 / m_2$.

Finally, we would like to point out that the neutrino mass can be generated by adding $\bar{L}_c \phi L^c$ (Zee model [27, 28]), which however breaks the $Z_2$ symmetry, or simply by including heavy right-handed neutrinos (Type-I seesaw). The correlation between the neutrino mass mechanism and NSI, however, will not be explored here.
III. NSI

To realize NSI, we couple the $SU(2)_L$ doublet $\eta$ to SM leptons via a renormalizable operator. In light of $Z_2$ under which $\eta$, $\phi$ and the right-handed electron $e_R$ are odd, the only allowed term is

$$\mathcal{L} \supset \lambda_\alpha \overline{L}_\alpha \eta e_R + h.c.,$$

where $\alpha$ is the flavor index, representing $e$ and $\tau$ but not $\mu$ in that we concentrate on effects of $\varepsilon_{e\tau}$, relevant for the confusion mentioned above.

Neglecting the $\eta-\phi$ mixing, the effective operator of the charged current, after integrating out heavy $\eta$, reads

$$\Delta \mathcal{L} \supset \frac{\lambda_\alpha \lambda_\beta^*}{m_\eta^2} (\overline{\nu}_e e_R) (\overline{e_R} \nu_\beta) = \frac{\lambda_\alpha \lambda_\beta^*}{2m_\eta^2} (\overline{\nu}_e \gamma^\mu \nu_\beta) (\overline{e_R} \gamma_\mu e_R),$$

where the second equality comes from Fierz transformation. Comparing with the charged current mediated by the $W$ boson,

$$\Delta \mathcal{L} \supset -2\sqrt{2}G_F (\overline{\nu}_e \gamma^\mu \nu_e) (\overline{e_L} \gamma_\mu e_L),$$

one can obtain

$$\varepsilon_{\alpha\beta}^\eta = -\frac{\lambda_\alpha \lambda_\beta^*}{4\sqrt{2}m_\eta^2 G_F}.$$

After taking into account the $\eta-\phi$ mixing, the NSI from the two mass eigenstates $s_1$ and $s_2$ are

$$\varepsilon_{\alpha\beta}^{s_1} = -\frac{\lambda_\alpha \lambda_\beta^* \theta^2}{4\sqrt{2}m_\eta^2 G_F},$$

$$\varepsilon_{\alpha\beta}^{s_2} = -\frac{\lambda_\alpha \lambda_\beta^*}{4\sqrt{2}m_\eta^2 G_F}.$$

Now, we can estimate the magnitude of the NSI from $s_1$ and $s_2$. The $s_2$-induced contribution, assuming $\lambda_\alpha \sim \lambda_\tau \sim \lambda$, is

$$\varepsilon_{\alpha\beta}^{s_2} = -3.9 \times 10^{-4} \left( \frac{\lambda}{0.16} \right)^2 \left( \frac{\text{TeV}}{m_2} \right)^2,$$

while the $s_1$ contribution can be rewritten as

$$\varepsilon_{\alpha\beta}^{s_1} = -3.9 \times 10^{-4} \left( \frac{\theta}{m_1/m_2} \right)^2 \left( \frac{\lambda}{0.16} \right)^2 \left( \frac{\text{TeV}}{m_2} \right)^2.$$
As we shall see below, due to the constraint on the CLFV process $\tau \rightarrow 3e$, $\lambda$ is restricted to be smaller than 0.16 for TeV $s_2$. It implies the $s_2$-induced NSI contribution can not be considerable. Nevertheless, the $s_1$ contribution can be large since $m_1$ and $\theta$ can be regarded as independent, i.e. $m_1/m_2$ can be quite different from $\theta$. To realize a sizable NSI, one must have

$$\frac{m_1^2}{m_2^2} \lesssim 10^{-3} \theta^2 \Rightarrow \frac{\mu_\phi^2 - \kappa^2 v^2/\mu_\eta^2}{\kappa^2 v^2/\mu_\eta^2} \lesssim 10^{-3}. \quad (12)$$

It implies that in order to obtain a sizable NSI contribution of order $O(0.1)$, the fine-tuning on the cancellation between $\mu_\phi^2$ and $\kappa^2 v^2/\mu_\eta^2$ is required to be around the level of 0.1%.

IV. CONSTRAINTS

Due to the existence of the couplings of $\phi$ and $\eta$ to the SM leptons, we here consider constraints involving charged leptons $e$ and $\tau$ from various measurements.

- **LEP constraints on the mass of $s_1$.**

  From the LEP measurements on the $Z$ decay width, the non-SM contribution are bounded below 2.9 MeV [33], which requires that $m_1$ should be larger than half of the $Z$ mass to kinetically forbid $Z$ decay into $s_1^+ s_1^-$. Besides, the LEP charged Higgs ($H^\pm$) searches [34] based on $e^+e^- \rightarrow Z \rightarrow H^+H^-$, followed by $H^\pm \rightarrow \tau^\pm \nu$, set a limit of $m_{H^\pm} > 80$ GeV in the context of two Higgs doublet models. It also applies to $s_1$ in our model. Therefore, we have $m_1 \gtrsim 80$ GeV.

- **LEP $e^+e^- \rightarrow \ell^+\ell^-$ constraints.**

  The LEP measurements on the cross-section of $e^+e^- \rightarrow \ell^+\ell^-$ can be translated into constraints on the new physics scale in the context of effective four-fermion interactions [35]

  $$\mathcal{L}_{\text{eff}} = \frac{4\pi}{(1 + \delta) \Lambda^2} \sum_{i,j=L,R} \eta_{i,j} \bar{e}_i \gamma_\mu e_i \bar{f}_j \gamma^\mu f_j,$$

  where $\delta = 0 \ (1)$ for $f \neq e \ (f = e)$ and $\eta_{i,j} = 1 \ (-1)$ corresponds to constructive (destructive) interference between the SM and new physics processes.
In our model, $e^+e^- \rightarrow \ell^+\ell^-$ processes will be mediated by solely $\eta^0$ of mass $m_{\eta} \simeq m_2$, which can be described by effective operators

$$L_{\text{eff}} = \frac{|\lambda_e|^2}{2m_2^2} (\bar{e}_L\gamma^\mu e_L) (\bar{e}_R\gamma_\mu e_R) + \frac{|\lambda_\tau|^2}{2m_2^2} (\bar{e}_R\gamma_\mu e_R) (\bar{\tau}_L\gamma^\mu \tau_L).$$

(14)

Since $\Lambda = 9.1$ TeV for $e^+e^- \rightarrow e^+e^-$ and $\Lambda = 10.2$ TeV for $e^+e^- \rightarrow \tau^+\tau^-$, we infer $\lambda_e/m_2 \lesssim 0.39$ TeV and $\lambda_\tau/m_2 \lesssim 0.49$ TeV.

- **LEP mono-photon constraints.**

Finally, the last LEP constraint comes from DM searches based on the mono-photon signal [36]: $e^+e^- \rightarrow$ DM DM $\gamma$ where $\gamma$ comes from the initial state radiation or the internal bremsstrahlung. In our model, we have similar mono-photon events from $e^+e^- \rightarrow \nu_{e,\tau}\nu_{e,\tau}\gamma$ via the $s_1$-exchange. The constraint on DM searches can be translated as

$$\frac{1}{\Lambda_{\text{DM}}^4} \gtrsim \frac{\theta^4}{16 m_1^4} \left([|\lambda_e|^4 + 2|\lambda_e\lambda_\tau|^2 + |\lambda_\tau|^4]\right),$$

(15)

where the coefficient $1/16$ on the right-hand side is to account for the fact only the right-handed $e^-$ (left-handed $e^+$) and left-handed $\nu$ (right-handed $\bar{\nu}$) are involved, i.e., $(1/2 \times 1/2)^2$, and $\Lambda_{\text{DM}} \simeq 320$ GeV for very small DM masses [36].

In the limit of $\lambda_e \sim \lambda_\tau$, the constraint is reduced to

$$\frac{1}{\Lambda_{\text{DM}}^2} \gtrsim \frac{\theta^2 \lambda^2}{2m_1^2},$$

(16)

and from Eq. (9), it implies the maximum NSI is

$$|\varepsilon^{s_1}| = \frac{1}{2\sqrt{2}G_F} \left(\frac{\lambda^2\theta^2}{2m_1^2}\right) \lesssim 0.3,$$

(17)

which is consistent with results in Refs [25, 37] based on $e^+e^- \rightarrow \nu_{e,\tau}\nu_{e,\tau}\gamma$ analysis in the context of NSI. Note that the mono-photon bound on NSI is unavoidable in the model since it is the same interactions that contribute to both NSI and the mono-photon signals.

The bound derived above is actually more stringent than needed since all the relevant processes in question are $t$-channel ones, and so one has, for the propagator, $|1/((p_e - p_\nu)^2 - m_1^2)| \simeq |1/(2p_e \cdot p_\nu - m_1^2)| \lesssim 1/m_1^2$, where $p_e$ ($p_\nu$) is the four momentum of the initial electron (final neutrino). The contribution to mono-photons from $s_2$ also
exists but is highly suppressed due to $1/m_2^2 \lesssim 10^{-3} \theta^2/m_1^2$ necessitated for large NSI as explained in Eq. (12).

- $\tau \to 3e$ limit.

$\eta^0$ will induce $\tau^- \to e^+e^-e^-$ decay\(^3\) and the decay width normalized to the $W$-mediated $\tau^- \to \mu^-\nu_\tau\bar{\nu}_\mu$ is constrained by null $\tau \to 3e$ results from Belle Collaboration \(^{38}\),

$$\frac{\Gamma_{\tau \to 3e}}{\Gamma_{\tau \to \mu \nu}} = \left| \frac{\lambda_e \lambda_\tau}{4\sqrt{2} m_2^2 G_F} \right|^2 \lesssim \frac{2.7 \times 10^{-8}}{0.17},$$

which can be rewritten as

$$\frac{\sqrt{\lambda_e \lambda_\tau}}{m_2} \lesssim 0.16 \text{ TeV},$$

and is stronger than the LEP constraints on $e^+e^- \to \ell^+\ell^-$. Note that one can similarly impose the bound from $\mu \to 3e$ measurements if $\lambda_\mu$ is switched on. Due to the fact $\text{Br}(\mu \to 3e) < 1.0 \times 10^{-12}$, one will have $\sqrt{\lambda_e \lambda_\mu}/m_2 \lesssim 8.12 \times 10^{-3}/\text{TeV}$. It implies that in order to achieve a sizable $\epsilon_{e\mu}$, the fine-tuning between $\mu_\phi^2$ and $\kappa^2 v^2/\mu_\eta^2$ mentioned in Eq. (12) has to be at the level of $10^{-5}$.

As mentioned above, sizable NSI induced via the $\eta - \phi$ mixing fall into the category of $d - 8$ operators for which there is not always direct correlation between NSI and CLFV interactions: $\tau^- \to e^+e^-e^-$ does not receive the same enhancement from the $s_1$-exchange as NSI.

- $\tau \to e\gamma$ bound.

$\eta^0$ will also radiatively induce $\tau \to e\gamma$ and the process actually stems from the process $\tau \to 3e$ by closing the $e^+$ and $e^-$ lines with a photon insertion. Therefore, $\tau \to e\gamma$ is suppressed by two powers of the electric coupling constant as well as a loop factor, which amount to $10^{-3}$ or so compared to $\tau \to 3e$. Given that the experimental constrains on these two processes are similar, $\text{Br}(\tau \to e\gamma) \lesssim 3.3 \times 10^{-8}$ \(^{39}\) versus $\text{Br}(\tau \to 3e) \lesssim 2.7 \times 10^{-8}$ \(^{38}\), we will not include the $\tau \to e\gamma$ limit here.

The constraints are summarized in Fig. 1 where we choose $\theta = 0.3$ and $m_2 = 10$ TeV. The purple area is excluded by the LEP searches on the charged Higgs, the light red area

\(^3\) Note that $\phi^\pm$ will not induce $\tau^- \to e^-\nu_\tau\bar{\nu}_e$ at tree level since $\eta$ only couples to $e_R$ but not $\tau_R$. Therefore, we will not consider bounds from the $\tau^- \to e^-\nu_\tau\bar{\nu}_e$ branching ratio measurements.
FIG. 1. The summary plot of the constraints where we assume $\lambda_e = \lambda_\tau$, $\theta = 0.3$ and $m_2 = 10$ TeV. The purple, red and crosshatched areas are excluded by the LEP charged Higgs searches, Belle $\tau \to 3e$ limit and LEP mono-photon bound, respectively. To realize $\varepsilon_{e\tau} = 0.3$, $m_1$ has to range from 80 to 105 GeV.

is eliminated by the Belle $\tau \to 3e$ bound which are more stringent than the LEP bounds on $e^+e^- \to \ell^+\ell^-$, while the crosshatched region will be disfavored by the LEP mono-photon searches. To achieve sizable NSI of 0.3, $m_1$ is constrained to be between 80 and 105 GeV.

Finally, we comment on the constraint from the electron magnetic dipole moment and implications on the IceCube experiment. At one-loop level, the electron anomalous magnetic moment ($g-2$) receives an additional radiative contribution from loops of $\phi^-$ and $\nu_e$. The contribution can be estimated as:

$$\Delta a_e \equiv \frac{g-2}{2} \approx \frac{e \lambda^2 \theta^2 m_e^2}{16\pi^2 m_\phi^2} \sim 10^{-14},$$

for the region of interest in Fig. 1. It is much smaller than the difference between the experiment result and the SM prediction: $a^{\text{exp}}_e - a^{\text{SM}}_e = -(1.06 \pm 0.82) \times 10^{-12}$ [40–43]. Therefore, the new contribution to $\Delta a_e$ is negligible.

As pointed out in Ref. [44], the resonance enhancement with a single scalar leptoquark can be used to increase the very high energy shower event rates at the IceCube. In our model, high energy neutrinos can interact with electrons and produce $\phi$ or $\eta$, which later decays into neutrinos and charged leptons. To have $\phi$ or $\eta$ on-shell, one must have $\sqrt{2m_eE_\nu} \gtrsim m_{\phi,\eta}$, requiring $E_\nu \sim 10$ and $10^5$ PeV for $m_\phi \sim 100$ GeV and $m_\eta \sim 10$ TeV. The flux of such high
energy cosmic neutrinos is then highly suppressed. Moreover, the relevant coupling for the \( \phi \)-exchange has to be of \( \mathcal{O}(1) \) \[44\] so that the new contribution is comparable with that of the SM. Nonetheless, the coupling in this model is simply \( \lambda^4 \theta^4 \), that is much smaller than the unity for regions of interest. As a result, one can not account for PeV events at the IceCube with the resonance enhancement of \( \phi \) or \( \eta \).

V. NSI OSCILLATIONS AT DUNE

A NC-like NSI interaction can be parametrized as four-fermion effective operators of the form:

\[
L_{NC}^{NSI} = -2\sqrt{2} G_F \sum_f \varepsilon_{\alpha\beta}^{f,P} [\bar{\nu}_\alpha \gamma^\rho L \nu_\beta] [\bar{f}_\gamma P f]
\]  

(21)

where \( G_F \) is the Fermi constant, \( \varepsilon_{\alpha\beta}^{f,P} \) are the NSI dimensionless couplings whose absolute value represent the relative NSI strength, \( P = (L, R) \) is the chiral projector, and \( f \) is the SM fermion of the first family: \( e, u, \) and \( d \).

The NSI effective interactions modify the effective matter potential that accounts for the neutrino–matter interactions. Therefore, there is a dependency on the fermion density in the medium. For long baselines below 2000 km, one can assume the matter density is constant simplifying the expression for the Hamiltonian in presence of the NSI, which can be written as:

\[
H_{\text{int}} = V \begin{pmatrix}
1 + \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\
\varepsilon_{e\mu}^* & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\
\varepsilon_{e\tau}^* & \varepsilon_{\mu\tau}^* & \varepsilon_{\tau\tau}
\end{pmatrix}
\]  

(22)

with \( V = \sqrt{2} G_F N_e \), where \( N_e \) is the electron density on Earth. Notice that the ‘1’ in the interaction Hamiltonian corresponds to the SM neutrino–matter interactions. By adding the NSI coupling in the formalism, we have increased the number of real parameters by eight since, in the diagonalization, one of the diagonal parameters can be rephased out. It is worth to mention that long baseline neutrino oscillations are sensitive to a combination of NSI couplings defined in Eq. (21):

\[
\varepsilon_{\alpha\beta} = \sum_{f=e,u,d} \left( \frac{Y_f}{Y_e} \right) \varepsilon_{\alpha\beta}^f = \varepsilon_{\alpha\beta}^e + Y_u \varepsilon_{\alpha\beta}^u + Y_d \varepsilon_{\alpha\beta}^d
\]  

(23)
where $Y$ is the abundance of each fermion in the medium. In Eq. (23) the effective NSI couplings are a weighted combination of the Lagrangian parameters.

The vacuum neutrino oscillations are governed by the usual Hamiltonian

$$H_0 = \frac{1}{2E} \left[ U \text{ diag}\{\Delta m_{21}^2, \Delta m_{31}^2, 0\} U^\dagger \right]$$

(24)

where $U$ is the lepton mixing matrix, $\Delta m_{ki}^2$ are the two measured mass squared differences, and $E$ is the energy of the incoming neutrino. The total Hamiltonian describing neutrino oscillations in matter is the sum of Eq. (22) and Eq. (24).

In our model, we have NSI in the lepton sector only, i.e., $\varepsilon^u = \varepsilon^d = 0$ and thus $\varepsilon = \varepsilon^e$, since the imposed $Z_2$ symmetry forbids Yukawa couplings of $\eta$ to quarks. To simplify the analysis, we set $\lambda_e = \lambda_\tau^4$ where $\lambda_\alpha$ is the Yukawa coupling of $\eta$ defined in Eq. (5). From Eq. (8), we have the following relations:

$$\varepsilon_{ee} = \varepsilon_{\tau\tau} \equiv -|\varepsilon|$$

$$\varepsilon_{e\tau} = |\varepsilon| \exp (i \phi),$$

(25)

which are similar to those of a recent work [45], that features a light gauge boson $Z'$ corresponding to the $U(1)_B$ or $U(1)_{B-L}$ gauge symmetry and can also generate large NSI, including $\varepsilon_{e\tau}$.

For the numerical analysis we have used the GLoBES library [46, 47] and the NSI tool (prepared for the study in Ref. [48]) with the official implementation of the DUNE experiment from Ref. [49]. In the analysis we have included the full DUNE implementation, i.e. the four oscillation channels for (anti-)neutrino appearance and disappearance running 3.5 years in each mode with the optimized neutrino beam. Also, we included the effect of the systematical errors in our analysis. Finally, as `true` parameters, we used the best-fit values for the standard oscillation parameters from Ref. [11] except for the reactor mixing angle, whose value was fixed to the Daya Bay result from Ref. [2]. The atmospheric mixing angle is assumed maximal but large errors on the atmospheric parameters (with the current precision) were implemented as penalties in the $\chi^2$ statistical analysis. Our analysis is based

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4 Since one of the diagonal NSI parameters is irrelevant in the diagonalization of the Hamiltonian in Eq. (22), one can set $\varepsilon_{\mu\mu}$ equal to zero which implies $\lambda_\mu = 0$. However, $\lambda_\mu \neq 0$ also induces two off-diagonal terms in addition to diagonal one. The resulting off-diagonal terms in principle affect the oscillation NSI analysis although the effect is small. Therefore, in our analysis we have assumed $\lambda_\mu = 0$ without significantly affecting the CP degeneracy mentioned above.
FIG. 2. The left panel corresponds to the DUNE sensitivity to the NSI parameter space defined in Eq. (25). Most of the parameters not shown in the plot were marginalized over. See text for details. In the right panel, we show the bi-rate plots that identifies the parameter degeneracies. The solid line corresponds to the case with SM interactions and for all Dirac CP phase values. The dashed and dotted curves correspond to the NSI case, for all possible NSI phase $\phi$ values, and for CP conserving values of the Standard CP phase. The SM case with $\delta_{CP} = -\pi/2$ denoted by the cross is also shown including the statistical uncertainty as a reference. In this case, the standard oscillation parameters not shown were fixed to their best-fit values.

on the normal neutrino mass hierarchy (NH) and we commented on the relevant differences in the case of the inverted mass hierarchy (IH) at the end.

Initially, we have extracted a constraint on the NSI couplings in our simplified setup by assuming only standard oscillation parameters as the ‘true’ parameters and by testing the NSI couplings. The results are shown in the left panel of Fig. 2 where the dependency on the ‘true’ Dirac CP phase value is also shown. All the parameters not shown in the plot have been marginalized over except for the solar oscillation parameters and the reactor mixing angle that were fixed to their best-fit values\(^5\). We have obtained the allowed interval $\varepsilon \in [-0.16, 0]$ at the 90% confidence level for 1 d.o.f.. This limit can not be directly compared with with existing works in Refs. [18, 19] due to the correlations in Eq. (25) from our model. Notice that, in our model, only the $\varepsilon^e$ couplings are predicted and therefore NSI constraints

\(^5\) The central values and uncertainties for the oscillation parameters $(\theta_{ij}, \Delta m^2_{k1})$, that are marginalized over, are obtained from the global fit analysis [1] assuming standard interactions only, i.e., in the absence of NSI.
from neutrino–electron scattering also apply. However, the bound extracted from DUNE simulated data is compatible with the scattering NSI bounds in Refs. [50–52] by identifying $\varepsilon^e = \varepsilon^{eR} + \varepsilon^{eL}$.

In order to evidence the parameter degeneracies after the inclusion of the NSI couplings, we have made use of the total signal rates shown in the a bi-rate plot in the right panel of Fig. 2. In the same spirit of Ref. [20], the ‘true’ Dirac CP phase values were assumed to be CP conserving to explore the possibility that the new phase, coming from the NSI, could mimic the effect of the standard Dirac CP phase – what we call the ‘confusion’. For comparison, the standard oscillation case is also shown with a benchmark point $\delta_{CP} = -\pi/2$ including the statistical errors. This point is one of the probable values within the allowed range of the Dirac CP phase determined by the T2K and NOvA [4, 5] measurements after including the reactor mixing angle determined at reactors. In the case the value $\delta_{CP} = -\pi/2$ were measured at DUNE a potential of ‘confusion’ might arise after the inclusion of the NSI. In particular, this is more evident for the case of $|\varepsilon| = 0.1$ with $\delta_{CP} = \pi$ since the maximum neutrino rates are comparable with the SM prediction with $\delta_{CP} = -\pi/2$. Notice however that if one includes also the statistical errors for the NSI and SM ellipses, there is an ample room for the confusion to happen. In other words, considering that the current preferred values span the complete negative region of the parameter $\delta_{CP}/\pi$, there is a potential of confusion for values of the CP violating phase within the interval $[-1, 0]$.

We now are in a position to quantify the degree of ‘confusion’ in the establishment of CP violation in the lepton sector in DUNE. The magnitude of the NSI couplings regarded as a ‘true’ parameter is fixed to $|\varepsilon| = 0.1$ and the SM CP phase is chosen to be $\delta_{CP}^{\text{True}} = \pi$. Statistical fluctuations in the ‘true’ rates are included and we test the standard oscillation rates. In the left panel of Fig. 3 we show the distribution of the best-fit value of $\delta_{CP}$ for different values of the new phase $\phi$. For comparison, we also display the case without NSI, which appears distributed around $\delta_{CP}^{\text{True}} = \pm \pi$ as expected. Given the chosen values of the NSI phase $\phi (\phi = -\pi, -11\pi/12, -5\pi/6, -\pi/2)$, the histograms cover the interval $[-\pi, \pi/2]$. Except for the case of $\phi = -\pi/2$, the NSI histograms are centered around the particular value of $-3\pi/4 (-135^\circ)$. Even though none of the NSI histograms is centered around the reference value ‘$-\pi/2$’ (the current best-fit value), there is however some potential of ‘confusion’. After all, with the current data, the allowed range of $\delta_{CP}/\pi$ is $[-0.996, -0.124]$ at the 90% confidence level for the normal hierarchy [53]. Thus, if DUNE measures a
FIG. 3. Results assuming $\delta_{\text{CP}}^{\text{True}} = \pi$. In the left panel, the best-fit distributions for the Dirac CP phase are shown for both the SM and NSI cases. We have fixed the NSI magnitude to the value $|\varepsilon| = 0.1$ (see also Eq. (25)) and for the different NSI phases $\phi$ showed in the plot. In the right panel the minimum $\chi^2$ distributions are shown for the SM and NSI cases showed in the left panel. All not shown parameters were marginalized over, see text for details of the analysis.

value of the CP violating phase close to $-3\pi/4$ (away from the current best-fit value), the degeneracy will persist. Otherwise, DUNE might break the degeneracy, strongly depending on its precision on the CP violating phase measurement.

The minimum $\chi^2$ distributions for each of standard and NSI cases in the left panel of Fig. 3 are shown in the right panel of the same figure. Except for the case of $\phi = -\pi/2$, the histograms of the NSI and the standard cases are centered around $\chi^2_{\text{min}} \sim 260$, which is compatible with the number of bins minus that of the fitted parameters, and is within a deviation of less than ten $\chi^2_{\text{min}}$ units. This evidences the possibility that DUNE might not have the ability to distinguish the origin of the CP violation if the measured CP phase happens to be around $-3\pi/4$.

Finally, in the case of the IH, the constraint on $|\varepsilon|$ is similar to that of the NH case, shown in the left panel of Fig. 2 and it is even stronger for certain values of $\delta_{\text{CP}}^{\text{true}}$. The parameter degeneracy shown in the right panel of Fig. 2 for NH is also present in the IH case for different NSI parameters, in particular for $|\varepsilon| = 0.12$ with $\delta_{CP} = 0$. This is due to the fact that for IH in DUNE, with $\delta_{CP} = -\pi/2$, lower neutrino rates and higher antineutrino rates
are expected in comparison to the NH case. With $|\varepsilon| = 0.12$ and $\phi = \pi/3$, the histogram of relative frequency (the left panel of Fig. 3 for NH) will be centered around $\delta_{CP} = \pi/4$ with $\delta_{CP}^{\text{true}} = 0$ in the case of IH. Clearly, the value is in tension with the current preferred one $\delta_{CP} \sim -\pi/2$ and therefore it is likely that DUNE will break the degeneracy. Notice that we here have fixed $\delta_{CP}^{\text{true}} = 0$ but one can have mixed sources of CP violation from both the SM and NSI. In contrast, in the NH case even for a CP conserving value $\delta_{CP}^{\text{true}} = \pi$, there exists the degeneracy which may not be resolved by DUNE, as demonstrated in Fig. 3.

VI. CONCLUSIONS

We come up with a novel way to achieve sizable NSI of order $O(0.3)$ at the cost of fine-tuning, which is required to be at the level of $10^{-3}$. In addition to the SM particles, the extra $SU(2)$ doublet and charged singlet scalars, denoted by $\eta$ and $\phi$ respectively, are introduced as well as the $Z_2$ symmetry, under which $\eta$, $\phi$ and the right-handed electron $e_R$ are odd. The charged component of $\eta$ mixes with $\phi$ via the dimensionful coupling $\kappa$ to the Higgs boson, $\kappa \phi \eta H$ with $\langle H \rangle = v$. Note that the $Z_2$ symmetry is explicitly broken by the electron Yukawa coupling and it is plausible that the smallness of the coupling is protected by the $Z_2$ symmetry.

If there exists the mass hierarchy, $m_\eta^2 \gg \kappa v \gg m_\phi^2$, the mass of the light mass eigenstate $s_1$ can be treated as an uncorrelated parameter from the $\phi - \eta$ mixing angle at the price of fine-tuning. As a result, one can have a very small mass of $s_1$, $m_1$, but a relatively large mixing angle $\theta$. Note that without fine-tuning one has $\theta \sim m_1/m_2$, where $m_2$ is the mass of the heavy eigenstate $s_2$.

An analogy can be drawn between this model and hybrid models of the Type-I plus Type-II seesaw mechanism, where light neutrino masses similarly receive two contributions from the mixing with heavy right-handed neutrinos and from the VEV of the $SU(2)_L$ triplet scalar. The light-heavy neutrino mixing angle is merely determined by the heavy neutrino mass and the Yukawa coupling but is not related to the triplet VEV.

NSI can be generated by coupling $\eta$ to $e_R$ and the $SU(2)_L$ lepton doublets $L_\alpha$ ($\alpha = e, \tau$), i.e., $\lambda_\alpha \overline{L_\alpha} \eta e_R$ which obeys the $Z_2$ symmetry. The new Yukawa coupling will give rise to NSI via the $\eta^\pm$-exchange. $\lambda_\mu$ is not considered here since it has a little impact on the ‘confusion’ we look for. In light of the $\phi - \eta$ mixing, NSI from $s_1$ is $\lambda_\alpha \lambda_\beta \theta^2/m_1^2$, which can be sizable.
if $\theta^2/m_1^2 \gg 1/m_2^2$. Taking into account various experimental bounds such as the LEP measurements on the $e^+e^- \rightarrow \ell^+\ell^-$ cross-section, upper limits on $\tau \rightarrow 3e$, $\tau \rightarrow e\gamma$ branching ratios, $\lambda_{e,\tau}$ are constrained to less than 0.16 for TeV $s_2$, while the LEP searches on the charged Higgs demand $m_1$ to be greater than 80 GeV. All in all, one needs $m_1^2/m_2^2 \sim 10^{-3}$ such that $\varepsilon_{e\tau}$ can be as large as 0.3 with $m_1 \sim 100$ GeV, given $\theta \sim 0.3$ and $m_2 \gtrsim 10$ TeV. The inevitable upper bound on $\varepsilon_{e\tau}$ comes from the LEP mono-photon searches since in our model both NSI and mono-photon signals result from exactly the same interactions.

The extra particles $\eta$ and $\phi$ in the model are within the reach of future experiments. First, the LEP mono-photon search has limited $\varepsilon_{e\tau}$ to be smaller than 0.3. Future electron colliders such as ILC [54, 55] or FCC-ee (formerly known as TLEP,) [56, 57] can significantly improve the mono-photon bound or spot the signal, which is an indirect evident of the charged $\phi$. Second, the accessible branching fractions for $\tau \rightarrow 3e$ at the superKEKB/Belle II will reach the level of $\mathcal{O}(10^{-10})$ [58, 59], discovery of $\tau \rightarrow 3e$ or $\tau \rightarrow e\gamma$ will implicitly indicate the presence of the neutral component $\eta^0$. Third, the direct detection of $e^+e^- \rightarrow e^+\tau^- \text{ or } e^-\tau^+$ at high-luminosity ILC and FCC-ee will be a smoking gun for the $\eta^0$ existence.

We have also discussed the phenomenological implications from the induced NSI. One of the main objectives of the future neutrino program is to establish if there is CP violation in the lepton sector with the help of current and future facilities. DUNE is one of the future facilities that will shed light on the current unknowns in the three neutrino framework and in particular on the determination of the CP violating phase. In this work we have determined DUNE sensitivity to the generated NSI by taking a simple limit of $\lambda_e = \lambda_\tau$, which results in the correlations in Eq. (25). We have extracted the bound $\varepsilon \in [-0.16, 0]$ at the 90% confidence level Since DUNE is sensitive to an NSI at the $\sim 10\%$ level, we also studied the NSI impact on the determination of the CP violating phase. To this purpose, we have exploited the parameter degeneracies that arise due to the new parameters coming form the NSI. One of the main consequences is the possible ‘confusion’ in terms of the source of the CP violation. We have study the degree of ‘confusion’ at DUNE experiment by setting the Dirac CP phase to a CP conserving value and allowing the $\varepsilon_{e\tau}$ NSI phase to generate the observed CP violation. We have found that if DUNE measures a phase close to $-3\pi/4$ ($-135^\circ$) instead of $-\pi/2$, DUNE will not be able to determine the origin of the measured CP violation. Otherwise, if DUNE measures a CP phase different from $-3\pi/4$ with a precision better than $\sim 10^\circ$ then it will be able to break the standard and NSI CP
Finally, we would like to point out that by having $\eta$ couple to quarks, one also get large NSI from quark-neutrino interactions. It is possible to realize the “dark-side” solution for solar neutrinos proposed in Ref \cite{60, 61} as an alternative to the standard LMA solution based on the Mikheev-Smirnov-Wolfenstein mechanism \cite{6, 62}. Bounds on NSI from LHC mono-jet searches \cite{63}, however, will come into play in this case. Some models \cite{30, 31, 45} have recently been proposed to realize such large NSI.

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