A new parametrization of the neutrino mixing matrix
for neutrino oscillations

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Abstract

In this paper we study three active neutrino oscillations, favored by recent data from SuperK and SNO, using a new parametrization of the lepton mixing matrix \( V \) constructed from a linear combination of the unit matrix \( I \), and a hermitian unitary matrix \( U \), that is, \( V = \cos \theta I + i \sin \theta U \). There are only three real parameters in \( V \) including the parameter \( \theta \). It is interesting to find that experimental data on atmospheric neutrino dictates the angle \( \theta \) to be \( \pi/4 \) such that the \( \nu_\mu \) and \( \nu_\tau \) mixing is maximal. The solar neutrino problem is solved via the MSW effect with a small mixing angle, with \( U \) depending on one small parameter \( \epsilon \). The resulting mixing matrix with just two parameters (\( \theta \) and \( \epsilon \)) predicts that the oscillating probabilities for \( \nu_e \to \nu_\mu \) and \( \nu_e \to \nu_\tau \) to be equal and of the order \( 2\epsilon^2 = (0.25 \sim 2.5) \times 10^{-3} \). The measurement of CP asymmetries at the proposed Neutrino Factories would also provide a test of our parametrization.

Flavor mixing in both quark and lepton sectors is one of the mysteries of particle physics. There are abundant experimental data which show that different quark generations mix and so do lepton generations. However, theoretical understanding of the mixing is very poor.
The minimal Standard Model (SM) can accommodate quark mixing through the Cabibbo-Kobayashi-Maskawa mixing matrix [1]. For mixing in the lepton sector, one has to extend the minimal SM, either by introducing a new scalar particle to provide Majorana masses for neutrinos and therefore mixing or by introducing right-handed neutrinos to provide neutrinos with Dirac mass, or a combination of these two. The mixing phenomena in the lepton sector are most prominent in neutrino oscillations and can be described by the Maki-Makagawa-Sakata mixing matrix [2]. These proposals can accommodate mixing phenomena and can be made consistent with experimental data, but there is no understanding why the matrix elements in relevant mixing matrix have the determined values.

In this paper assuming three generations of neutrinos, we consider another way of looking at the mixing matrix problem by dividing the mixing matrix into two pieces. One of them is proportional to the unit matrix $I$ which does not cause any mixing, and the other is proportional to a hermitian unitary matrix $U$ which is responsible for flavor mixing. The mixing matrix $V$ is thus parametrized as $V = \cos \theta I + i \sin \theta U$. This way of looking at the problem has the advantage that the parameter $\theta$ determines the relative weight of the diagonal elements and off diagonal elements in a non-trivial manner. Mathematically, $U$ depends on four real parameters, but due to the rephasing invariance of $V$, there are actually just two real independent parameters in $U$. Similar proposal has been made for the quark mixing matrix which is compatible with experimental data within allowed errors [3]. Here we will concentrate on the lepton sector to analyze mixing phenomena in neutrino oscillations. Neutrino oscillations can provide natural explanations to the atmospheric and solar neutrino data. Recent data from SuperK [4] and SNO [5] favor mixing of active neutrinos. We therefore assume three active neutrino mixing in our analysis. We note that the LSND [6] data cannot be accommodated in this analysis. The required parameter space to explain the atmospheric and solar neutrino data can provide stringent constraints on the parameters in the mixing matrix. Using atmospheric neutrino data, we find that the linear combination angle $\theta$ is determined to be $\pi/4$ indicating that the diagonal and off diagonal parts are equally important. The solar neutrino problem can be solved by the small mixing angle
(SMA) solution with MSW effect. This ansatz also gives interesting predictions for long
base line neutrino experiments and CP violation in neutrino oscillations.

We now provide a detailed analysis of the parametrization proposed here. Unitarity of
$V$ requires

$$VV^\dagger = I = \cos^2 \theta I + \sin^2 \theta UU^\dagger + i \sin \theta \cos \theta (U - U^\dagger),$$

$$V^\dagger V = I = \cos^2 \theta I + \sin^2 \theta U^\dagger U + i \sin \theta \cos \theta (U - U^\dagger).$$

(1)

In general $V$ and $U$ are non-trivially related. However, if $U$ is independent of the linear
combination angle $\theta$, then $U$ has to be unitary and hermitian. The $V$ so obtained is more
restrictive than a general one.

The matrix $U$ can be parametrized as

$$U = \begin{pmatrix}
-1 + 2|c|^2 & -2b^*c & -2a^*c^* \\
-2bc^* & -1 + 2|b|^2 & -2ab^* \\
-2ac & -2a^*b & -1 + 2|a|^2
\end{pmatrix},$$

(2)

where the magnitudes and the phases of $a$, $b$ and $c$ satisfy the constraints $|a|^2 + |b|^2 + |c|^2 = 1$
and $\phi_a - \phi_b + \phi_c = \pi/2$, respectively. $U$ depends, in general, on two independent moduli and
two independent phases at it stands. However the mixing matrix $V$ is rephasing invariant,
so the two phases in $U$ can be absorbed into lepton phases. $V$ depends on only 3 real
parameters, $\theta$ and the two moduli in $U$.

To determine the parameters in the mixing matrix $V$, we first consider data for atmo-
spheric neutrino from SuperK experiment [1]. To fit the data the muon neutrino $\nu_\mu$ has to
oscillate into another type of neutrino $\nu_x$ with mixing angle $\theta_{2x}$ such that $\sin^2 2\theta_{2x} > 0.88$
which leads to $\theta_{2x} \approx \pi/4$ and also the mass squared difference $|\Delta m_{2x}^2| = |m_2^2 - m_x^2|$ in the
range $(1.5 \sim 5) \times 10^{-3}$ eV$^2$. Here the numbers 1, 2 and 3 correspond to $\nu_e$, $\nu_\mu$ and $\nu_\tau$,
respectively. For three generations of neutrinos, $\nu_\mu$ can only oscillate into a tauon neutrino
$\nu_\tau$ to fit the data. Assuming that electron neutrino $\nu_e$ does not have significant mixing with
muon and tauon neutrinos, then $|V_{11}|$ is fixed to be 1, while $|V_{22}|$, $|V_{23}|$, $|V_{32}|$ and $|V_{33}|$ are
fixed to be $1/\sqrt{2}$. This leads to the following unique solution for $\theta$ and $U$,
\[ \theta = \frac{\pi}{4}, \quad a = \frac{1}{\sqrt{2}}, \quad b = \frac{1}{\sqrt{2}}, \quad c = 0, \]

\[
U = \begin{pmatrix}
-1 & 0 & 0 \\
0 & 0 & -1 \\
0 & -1 & 0
\end{pmatrix}.
\]  

(3)

The mass squared difference \(|\Delta m_{23}^2|\) is constrained to be in the range \((1.5 \sim 5) \times 10^{-3}\)eV\(^2\) at 90\% C.L.. The mixing angle \(\theta = \pi/4\) is a reflection of the fact that the mixing of the \(\nu_\mu\) and \(\nu_\tau\) is maximal. It is interesting that this value of \(\theta\) also implies that the purely diagonal and the non-diagonal pieces have equal weight, that is, \(V = (I + iU)/\sqrt{2}\).

The mixing matrix obtained above can not produce oscillations of electron neutrino into other neutrinos and therefore can not provide a solution to the solar neutrino problem. If the solar neutrino problem is due to neutrino oscillation like the solution to the atmospheric neutrino problem, then the electron neutrino has to mix with other neutrinos such that it can oscillate into them to cause the deficit seen in the measurements on earth. One has to modify the above mixing matrix such that solutions for the solar neutrino problem can be obtained.

To this end we make a small perturbation to the mixing matrix by introducing a non-zero but small value for \(c\) such that \(V_{12,23}\) become non-zero to allow \(\nu_e\) to oscillates into \(\nu_\mu\) and/or \(\nu_\tau\). In fact, in our parametrization, a non-zero value for \(c\) implies that both \(V_{12}\) and \(V_{13}\) are non-zero. Whether the solar neutrino problem is solved by oscillation of \(\nu_e\) into \(\nu_\mu\) or \(\nu_\tau\) depends on the mass squared differences of the neutrino masses which will be discussed later. Denote the perturbations on \(a\), \(b\) and \(c\) as \(a = (1 + \epsilon_a)/\sqrt{2}, b = (1 + \epsilon_b)/\sqrt{2},\) and \(c = i\epsilon\). If \(\epsilon\) is a real number, \(\epsilon_{a,b}\) are also required to be real from the constraint on the phases. A simple choice, satisfying all constraints, is to take \(c = i\epsilon\) with \(a = \sqrt{1-2\epsilon^2}/\sqrt{2}\) and \(b = 1\sqrt{2}\). this gives

\[
V = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 - i(1 - 2\epsilon^2) & \sqrt{2}\epsilon & -\sqrt{2}\epsilon\sqrt{1 - 2\epsilon^2} \\
-\sqrt{2}\epsilon & 1 & -i\sqrt{1 - 2\epsilon^2} \\
\sqrt{2}\epsilon\sqrt{1 - 2\epsilon^2} & -i\sqrt{1 - 2\epsilon^2} & 1 - i2\epsilon^2
\end{pmatrix}.
\]  

(4)
There are other choices for $\epsilon_{a,b}$ which can also satisfy $|a|^2 + |b|^2 + |c|^2 = 1$, such as the general case $a = \sqrt{1 - 2(1 + \alpha)\epsilon^2 / \sqrt{2}}$ and $b = \sqrt{1 + 2\alpha\epsilon^2 / \sqrt{2}}$, with $\alpha$ a number of order one so that $\epsilon_{a,b}$ are small. However, to the leading order in $\epsilon^2$ the general case gives the same results for neutrino oscillations. Without loss of generality, we will carry out the rest of the analysis using $V$ given above.

There are four types of solution to the solar neutrino problem [7]. Three of them are based on the MSW mechanism [8]. They are the large mixing angle (LMA), the long wavelength (LOW) large mixing angle, and the small mixing angle (SMA) solutions. There is one solution which does not depend on the MSW effect, the “just so” vacuum (VAC) oscillation solution. Recent data from SuperK favors the LMA solution [7]. However, at present it is premature to rule out the other three solutions. Among the four solutions, our parametrization can only accommodate the SMA solution. Fitting data we determine the parameter $\epsilon^2$ and the mass squared difference of $\nu_e$ and the neutrino oscillated into $\nu_x$ to be

$$\epsilon^2 = (0.125 \sim 1.25) \times 10^{-3},$$

$$|\Delta m_{1x}^2| = (0.35 \sim 1) \times 10^{-5} \text{eV}^2.$$  

(5)

Because the mixing matrix elements $V_{12}$ and $V_{13}$ to the leading order in $\epsilon$ are equal, present constraints on various neutrino oscillation data can allow the neutrino $\nu_x$ to be $\nu_\tau$ or $\nu_\mu$ depending on the pattern of the neutrino masses. There are two scenarios: a) if $\nu_e$ oscillates into $\nu_\tau$, the neutrino mass hierarchy will be $m_{\nu_\mu} > m_{\nu_e} > m_{\nu_\tau}$; and b) if $\nu_e$ oscillates into $\nu_\mu$, then $m_{\nu_\tau} > m_{\nu_\mu} > m_{\nu_e}$.

The mass hierarchy patterns for the cases a) and b) have different implications in general for long base line experiments. The probability for a $l$ type of neutrino to oscillate into a $k$ type of neutrino in vacuum is given by

$$P(l \rightarrow k) = \delta_{lk} - 4 \sum_{i<j} Re(V_{ki}^* V_{li} V_{lj}^* V_{kj}) \sin^2 \left( \frac{1.27 \Delta m_{ij}^2}{E} L \right)$$

$$- 2 \sum_{i<j} Im(V_{ki}^* V_{li} V_{lj}^* V_{kj}) \sin \left( \frac{2.54 \Delta m_{ij}^2}{E} L \right),$$  

(6)
where $\Delta m^2_{ij}$ is in eV$^2$, $E$ in GeV and $L$ in km. The first two terms are CP conserving and symmetric in $l$ and $k$ while the last term is CP violating and anti-symmetric in $l$ and $k$. For anti-neutrino oscillation the last term will change sign.

In our new parametrization, to order $\epsilon^2$ we find the following oscillation probabilities

$$P(\nu_e \rightarrow \nu_e) = 1 - 4\epsilon^2[\sin^2(\frac{1.27\Delta m^2_{12}}{E}L) + \sin^2(\frac{1.27\Delta m^2_{13}}{E}L)],$$

$$P(\nu_\mu \rightarrow \nu_\mu) = P(\nu_\tau \rightarrow \nu_\tau)$$

$$= 1 - 2\epsilon^2[\sin^2(\frac{1.27\Delta m^2_{12}}{E}L) + \sin^2(\frac{1.27\Delta m^2_{13}}{E}L)] - (1 - 2\epsilon^2)\sin^2(\frac{1.27\Delta m^2_{23}}{E}L),$$

$$P(\nu_\mu \rightarrow \nu_\mu) = P(\nu_\tau \rightarrow \nu_e) = 2\epsilon^2[\sin^2(\frac{1.27\Delta m^2_{12}}{E}L) + \sin^2(\frac{1.27\Delta m^2_{13}}{E}L)] + P^-, $$

$$P(\nu_\mu \rightarrow \nu_\tau) = (1 - 2\epsilon^2)\sin^2(\frac{1.27\Delta m^2_{23}}{E}L) + P^-, $$

$$P^- = -4\epsilon^2\sin(\frac{1.27\Delta m^2_{12}}{E}L)\sin(\frac{1.27\Delta m^2_{13}}{E}L)\sin(\frac{1.27\Delta m^2_{23}}{E}L).$$  

(7)

For the large $\nu_\mu \rightarrow \nu_\tau$ oscillation, further tests can be carried out to check the oscillation mechanism at the long base line neutrino experiments [9] at K2K, Minos and Opera in a controlled fashion. In these experiments, it may also be possible to carry out some tests for the predictions of the ansatz proposed here. In both cases a) and b), it is feasible to have the right combination of $L/E$ such that $1.27\Delta m^2_{12}/E$ or $1.27\Delta m^2_{13}/E$ are of order unity so that oscillations between $\nu_e$ and $\nu_\mu$, and $\nu_e$ and $\nu_\tau$ can be observed. The long base line neutrino oscillation experiments K2K, Minos and Opera have $L$ around 250km, 730km, and 730km with $E$ around $\sim 1.4$ GeV, $\sim 3$, 7, 15 GeV, and $\sim 17$ GeV, respectively. It is possible using $E_{\nu_\mu} = 3$ GeV from Minos to have $1.27\Delta m^2_{12}/E$ or $1.27\Delta m^2_{13}/E$ as large as $\pi/2$. The maximal probability at the far end of the long base line detector for the appearing of $\nu_e$ can be as large as $2\epsilon^2 = (0.25 \sim 2.5) \times 10^{-3}$ from $\nu_\mu \rightarrow \nu_e$ oscillation. The appearing probability of $\nu_\mu$ can be in the range $(0.25 \sim 2.5) \times 10^{-3}$ from $\nu_e \rightarrow \nu_\mu$ oscillation. Future experimental data will provide important information about the mixing matrix. Unfortunately, the cases a) and b), to the leading order, give identical predictions for the above mentioned experiments and can not be distinguished.

Another interesting prediction of the new mixing matrix parametrization is CP violation.
in neutrino oscillations. Tests of CP violation in neutrino oscillations may be feasible at Neutrino Factories [10][11]. Proposed Neutrino Factories have a high intensity muon storage ring with long sections along which the muons decay, $\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e$, to deliver high intensity $\nu_\mu$ and $\bar{\nu}_e$ neutrino beams. With these neutrino beams one can carry out disappearing and appearing neutrino experiments to test different neutrino mixing scenarios. Defining CP violating asymmetry,

$$A(\nu_l \rightarrow \nu_k) = \frac{P(\nu_l \rightarrow \nu_k) - P(\bar{\nu}_l \rightarrow \bar{\nu}_k)}{P(\nu_l \rightarrow \nu_k) + P(\bar{\nu}_l \rightarrow \bar{\nu}_k)},$$

we have

$$A(\nu_e \rightarrow \nu_\mu) = A(\nu_\tau \rightarrow \nu_e) = -A(\nu_\mu \rightarrow \nu_e) = -A(\nu_e \rightarrow \nu_\tau)$$

$$= -2 \sin(\frac{1.27 \Delta m^2_{31}}{E} L) \sin(\frac{1.27 \Delta m^2_{12}}{E} L) \sin(\frac{1.27 \Delta m^2_{23}}{E} L) \left( \sin^2(\frac{1.27 \Delta m^2_{31}}{E} L) + \sin^2(\frac{1.27 \Delta m^2_{23}}{E} L) \right).$$

In the above we have given the modes which may have large CP violation. In principle $A(\nu_i \rightarrow \nu_i)$, $A(\nu_\mu \rightarrow \nu_\tau)$ and $A(\nu_\tau \rightarrow \nu_\mu)$ can have non-zero values. However there is an additional suppression factor of order $\epsilon^2$ for CP violation in these modes and they would be difficult to measure.

It is interesting to note that in the above expression the mixing parameter $\epsilon$ has canceled out, and also that all the above modes have CP violation of the same magnitude. Therefore uncertainties associated with the mixing parameters do not enter and allow clear interpretations. These unique features can provide good tests for the new parametrization studied here.

Let us now study what value $A(\nu_\mu \rightarrow \nu_e)$ can have. One can tune $L/E$ to maximize the detection probability. Tuning $1.27 \Delta m^2_{21} L/E = \pi/2$ for case a) and substituting $1.27 \Delta m^2_{31} L/E = (\Delta m^2_{31}/\Delta m^2_{21}) \pi/2$, we have

$$A(\nu_e \rightarrow \nu_\mu) = -2 \sin(\frac{\Delta m^2_{31}}{\Delta m^2_{21}} \frac{\pi}{2}) \approx -2 \frac{\Delta m^2_{31}}{\Delta m^2_{21}} \frac{\pi}{2}$$

$$= -(0.6 \sim 0.6) \times 10^{-2}. \quad (10)$$
In case b), one would get the same magnitude, but opposite sign. This fact can be used to
distinguish the cases a) and b). To have a 3σ detection of CP asymmetry, one would need
\((10^{10} - 10^8)\) neutrino oscillation events.

When neutrino beams travel through the earth to reach the other side to be detected,
due to the fact that the earth is composed of matter and not anti-matter, the MSW effect for
the neutrino and anti-neutrino beams is different. This results in additional asymmetry. To
have some idea about the additional asymmetry due to matter effects, we consider case b)
using \(m_1 = 0, m_2 = 2.5 \times 10^{-3} \text{ eV}, m_3 = 6 \times 10^{-2} \text{ eV and } \epsilon^2 = 5 \times 10^{-4}\) as the input vacuum
values for the neutrino masses and mixing for illustration. The matter effect is proportional
to the electron density \(N_e\) in the earth. The parameter that enters in the calculation is
\(A = 2\sqrt{2}G_F N_e E_{\nu}\). For neutrino oscillation, \(A\) takes the positive sign and for anti-neutrino
oscillation it takes the minus sign. For the typical neutrino energy of the Neutrino Factories
and the electron density in the earth, \(A\) is in the range of \(10^{-7} \sim 10^{-2} \text{ eV}^2\) [12]. We find
that in vacuum for the above values of parameters, the CP asymmetry is predicted to be
0.0055. But with a non zero \(A\), the result can be very different. For example for \(A = 10^{-3}\)
\(\text{eV}^2\), the asymmetry would be 0.035. One should note that the asymmetry caused by the
matter effect does not imply CP violation in the mixing matrix. To pin down the effect
due to CP violation, one needs to have a good understanding of the profile of the earth
to eliminate the asymmetry due to the MSW effect on neutrino oscillations. This is very
difficult. However, we would like to point out that \(A(\nu_l \rightarrow \nu_k) = [P(\nu_l \rightarrow \nu_k) - P(\bar{\nu}_l \rightarrow \bar{\nu}_k)]/[P(\nu_l \rightarrow \nu_k) + P(\bar{\nu}_l \rightarrow \bar{\nu}_k)]\) can still provide important information about the neutrino
mass hierarchy. Because in case a) and case b) the signs for \(A(\nu_e \rightarrow \nu_\mu)\) and \(A(\nu_e \rightarrow \nu_\tau)\) are
different, and matter effects do not change the signs of the asymmetries for \(A\) in the range
\(10^{-7} \sim 10^{-2} \text{ eV}^2\).

We have considered a new parametrization for neutrino mixing matrix \(V\) using the ansatz
\(V = \cos \theta I + i \sin \theta U\) with the value \(\theta = \pi/4\) which gives equal weight to the diagonal \((I)\) and
non-diagonal \((U)\) pieces in \(V\). This value of \(\theta\) is a natural consequence of the atmospheric
neutrino data which requires maximal \(\nu_\mu \rightarrow \nu_\tau\) oscillation. The matrix \(U\) depends on only
one small parameter $\epsilon \sim 10^{-2}$ which gives the SMA solution to the solar neutrino problem. It also predicts that $P(\nu_e \rightarrow \nu_\mu) = P(\nu_\tau \rightarrow \nu_e)$ to the leading order.

We find that to order $\epsilon^2$, the CP asymmetry $A(\nu_e \rightarrow \nu_\ell)$ is independent of $\epsilon$. Moreover, we find that the sign of $A(\nu_e \rightarrow \nu_\mu)$ and $A(\nu_e \rightarrow \nu_\tau)$ could be used to distinguish between the two possible mass hierarchies, $m_{\nu_\mu} > m_{\nu_\tau} > m_{\nu_e}$ and $m_{\nu_\tau} > m_{\nu_\mu} > m_{\nu_e}$ permitted by the SMA solution to the solar neutrino problem and other oscillation data, even though matter effects may affect the magnitude of the asymmetry significantly. These asymmetries could be detected at Neutrino Factories.

It is remarkable that our mixing matrix $V$ which contains only two parameters ($\theta$ and $\epsilon$) can explain the atmospheric and solar neutrino data. CP asymmetry measurements in neutrino oscillations could provide future tests of our parametrization.

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