On the internal consistency of holographic dark energy models

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Abstract. Holographic dark energy (HDE) models, underpinned by an effective quantum field theory (QFT) with a manifest UV/IR connection, have become convincing candidates for providing an explanation of the dark energy in the universe. On the other hand, the maximum number of quantum states that a conventional QFT for a box of size $L$ is capable of describing relates to those boxes which are on the brink of experiencing a sudden collapse to a black hole. Another restriction on the underlying QFT is that the UV cut-off, which cannot be chosen independently of the IR cut-off and therefore becomes a function of time in a cosmological setting, should stay the largest energy scale even in the standard cosmological epochs preceding a dark energy dominated one. We show that, irrespective of whether one deals with the saturated form of HDE or takes a certain degree of non-saturation in the past, the above restrictions cannot be met in a radiation dominated universe, an epoch in the history of the universe which is expected to be perfectly describable within conventional QFT.

Keywords: dark energy theory, black holes, cosmological constant experiments

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The holographic principle [1, 2] is undoubtedly the most amazing ingredient of a modern view of space and time. The most successful realization of the holographic principle, implemented in Maldacena’s discovery of AdS/CFT duality [3], strengthened further this speculative idea about quantum gravity. On the other hand, other closely related concepts forming crucial parts within a new paradigm, like black hole complementarity [4], the UV/IR connection [5], the space–time uncertainty relation [6] and the occurrence of the minimal length scale [7], become completely manifest in the framework of the holographic principle.

In order to encode (via the holographic information) a drastic depletion of quantum states within the effective field-theoretical description, preventing at the same time formation of black holes, the entropy for an effective quantum field theory
\[ S \sim L^3 \Lambda^3, \]
where \( L \) is the size of the region (providing an IR cut-off) and \( \Lambda \) is the UV cut-off, should obey the upper bound [8]
\[ L^3 \Lambda^3 \leq (S_{BH})^{3/4} \sim L^{3/2} M_{Pl}^{3/2}, \tag{1} \]
and \( S_{BH} \sim L^2 M_{Pl}^2 \) is the holographic Bekenstein–Hawking entropy. In an expanding universe \( \Lambda \) should therefore be promoted to a varying quantity (some decreasing function of \( L \)), in order for (1) not to be violated during the course of the expansion, thereby explicitly manifesting the UV/IR correspondence. This gives a constraint on the maximum energy density in the effective theory,
\[ \rho_{\Lambda} \leq \frac{L^3}{M_{Pl}^2}, \]
Obviously, \( \rho_{\Lambda} \) is the energy density corresponding to a zero-point energy and the cut-off \( \Lambda \).

Indeed, this may be seen by calculating the effective cosmological constant (CC) generated by vacuum fluctuations (zero-point energies)
\[ \rho_{\Lambda} \propto \int_{L^{-1}}^{\Lambda} k^2 \sqrt{k^2 + m^2} \sim \Lambda^4 \Lambda \gtrsim m \]
\[ \sim m \Lambda^3 \Lambda \lesssim m, \tag{2} \]
since clearly \( \rho_{\Lambda} \) (as for the entropy) is dominated by UV modes. Although the constraint \( \rho_{\Lambda} \leq L^{-2} M_{Pl}^2 \) stays the same for the two limiting cases, the UV/IR correspondences are different, giving \( L \sim \Lambda^{-2} \) and \( L \sim \Lambda^{-3/2} \), respectively, when the bound is saturated. Such a distinction becomes important in a subsequent discussion.

The origin of (1) stems from the fact that in an effective QFT the entropy scales extensively, \( S \sim L^3 \Lambda^3 \), and therefore (for any \( \Lambda \)) there is a sufficiently large volume for which \( S \) would exceed the absolute bound \( S_{BH} \). Thus, considerations for the maximum possible entropy suggest that ordinary QFT may not be valid for arbitrarily large volumes, unless the UR and IR cut-offs satisfy a bound, \( \Lambda^3 \lesssim M_{Pl}^2 \). However, for saturation, this bound means that an effective QFT should also be capable of describing systems containing black holes, since it necessarily includes many states with Schwarzschild radius much larger than the box size. The arguments for why an effective QFT appears unlikely to provide an adequate description of any system containing a black hole can be found in [8] and in references therein. So, ordinary QFT may not be valid for much smaller volumes, but would apply provided (1) is satisfied.

The above effective field-theoretical set-up has triggered a novel variable CC approach [9] generically dubbed that of ‘holographic dark energy’ (HDE) [10].
main reason for the above HDE model being so appealing in possible descriptions of dark energy is that when the holographic bound (1) is saturated, \( \rho_\Lambda \) gives the right amount of dark energy in the universe at present, provided \( L \) today is of the order of the Hubble parameter. Moreover, since \( \rho_\Lambda \) is now a running quantity, it also has the potential to shed some light on the ‘cosmic coincidence problem’ [11]. In addition, the original model [8] is capable of satisfying current observations [12], since by construction it has \( \omega_\Lambda = -1 \).

The most pressing problem when dealing with cosmologies based on (1) is certainly that of making the choice for the infrared cut-off \( L^{-1} \). For models based on full saturation in (1), the choice in the form of the inverse Hubble parameter is largely unsatisfactory (for spatially flat universes, as suggested by observations) both for perfect fluids [9] and for interacting fluids [13,14]: in the former case one cannot explain the accelerating expansion of the present universe, while one fails to explain that the acceleration sets in just recently and was preceded by a deceleration era at \( z \gtrsim 1 \) in the latter case. This is easy to see by plugging \( \rho_\Lambda = L^{-2}M^2_{Pl} \) (setting a prefactor to unity for simplicity) into the Friedmann equation for flat space

\[
(HL)^2 = \frac{8\pi}{3}(1 + r),
\]

where \( r = \rho_m/\rho_\Lambda \) and \( \rho_m \) is the matter energy density. Thus, a choice \( L \sim H^{-1} \) would require the ratio \( r \) to be a constant. This is a general statement, holding irrespective of whether a fluid is perfect or not, and even irrespective of whether the Newton constant is varying or not. The interpretations for various cases are, however, different. For perfect fluids, \( r = \text{const} \) means that the equation of state for the dark energy unavoidably matches that of pressureless matter, \( w = 0 \) [9]. Thus, we cannot explain the accelerating expansion of the present universe. For interacting fluids, one is usually able to generate accelerated expansion with \( r = \text{const} \), as now \( a \sim t^{2/3} \), where the parameter \( m \) depends on the interaction term and can be easily made such that \( m < 2/3 \) [13,14] so as to ensure acceleration. The constancy of \( r \) for the flat space case precludes however any transition between the cosmological epochs. Generically, a suggestion of setting \( L \) according to the future event horizon [15] leads to phenomenologically viable models. Even the choice \( L = H^{-1} \) can be retained in models where a certain degree of non-saturation in (1) is allowed in the past [14,16].

In the present paper, we aim to check whether the effective field-theoretical set-up underlying (1) is capable of describing various cosmological epochs consistently—that is, whether (1), besides the late-time acceleration in a dark energy dominated epoch, provides also a consistent description of some earlier epoch, say the radiation dominated universe. The consistency check will be based on the following two requirements: (i) a radiation dominated epoch is considered as a system at a temperature \( T \), having thermal energy \( L^3T^4 \), provided \( L^{-1} < T < \Lambda(L) \), where the UV cut-off \( \Lambda(L) \) is now a running quantity which is to comply with (1); and (ii) the range of validity of the effective QFT is restricted to just those systems not containing black holes. In the following we argue that as soon as we move from the epoch at which the dark energy overwhelmingly dominates over all other forms of energy densities, any consistent description based on (1) is no longer
viable\(^1\). This conclusion remains true irrespective of the choice for \(L\) and the degree of saturation in (1).

Let us begin with models saturating (1). As mentioned earlier, apart from a small set of models presented in [14,16], the bulk of the models have employed (1) in its saturated form. The HDE density in the latter case is conveniently parametrized as \(\rho_s = (3c^2/8\pi)L^{-2}M_{\text{Pl}}^2\) [10], with a parameter \(c^2\) of the order of unity. The corresponding Schwarzschild radius for a box of volume \(L^3\) dominated by \(\rho_\Lambda\),

\[
R_s \sim M_{\text{Pl}}^{-2}(L^3 \rho_\Lambda) \sim L, \tag{4}
\]

always sets the system at the brink of collapse to a black hole. It is important to note that this contribution of \(\rho_\Lambda\) in \(R_s\) is always such throughout the history of the universe, regardless of which form of energy dominates a particular epoch. This means that every epoch preceding a dark energy dominated phase, where \(\rho_\Lambda\) necessarily represents a subdominant component in the total energy density of the universe, would set \(R_s\) at \(R_s \gg L\). That is, in epochs when \(\rho_\Lambda\) is subdominant, \(R_s\) would rise, \(R_s \gg L\), since \(R_s\) is determined by the total energy density. In that way, if the same effective QFT is to describe cosmological epochs besides the current one, it necessarily includes many states with \(R_s\) much larger than the box size. In order not to contradict standard cosmology, one should assume that in a radiation dominated universe \(\rho_{\text{rad}} \sim T^4\), and thus with \(\rho_{\text{rad}} \gg \rho_\Lambda\) one necessarily includes many states with \(R_s \gg L\), which are not expected to be describable within conventional QFT. One can hope to remedy the situation by applying the same recipe as led to (1), which brings HDE to a domain describable within QFT. This amounts to saying that the total energy of the system of size \(L\) should not exceed the mass of a black hole of the same sized, i.e., \(L^3T^4 \lesssim \Lambda M_{\text{Pl}}^2\). Apparently, radiation states would be safe, as now \(R_s \lesssim L\). The same constraint would however preclude radiation from being the dominant component, as \(\rho_{\text{rad}} \sim T^4 \lesssim L^{-2}M_{\text{Pl}}^2 \sim \rho_\Lambda\). So, we see that any saturated HDE model precludes either description of radiation within ordinary QFT or a transition between the cosmological epochs\(^2\).

Arguably much better prospects can be expected for models consistent with (1) but, at the same time, allowing a certain degree of non-saturation in past epochs. In these so-called non-saturated HDE models [14,16], the parameter \(c^2\) is promoted to being a function of cosmic time, \(c^2(t)\). The function \(c^2(t)\) should satisfy \(c^2(t_0) \rightarrow 1\) (dark energy dominance), while \(c^2(t) \ll 1\) during the radiation dominated epoch. A promising set-up,

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\(^1\) It was already noted in the background paper [8] that within an effective QFT, requiring cut-offs which do obey (1), there is no possibility of simultaneously addressing the CC problem and complying with computations relevant for current laboratory experiments. To address the CC problem with a naive estimate, \(\rho_\Lambda \approx \Lambda^4\), one requires a UV cut-off of the order of \(10^{-2.5}\) eV. Such a cut-off would induce a discrepancy in the calculation of \((g-2)\) for the electron, between a framework relying on (1) and a conventional one performed in an infinite box, which is unacceptably large; cf equation (5) in [8]. Here we focus on a consistent description within cosmology only.

\(^2\) It is remarkable that even the absolute Bekenstein–Hawking bound can be saturated in the radiation dominated epoch. Take for instance the popular Li model [10] and solve for \(\rho_{\text{rad}} \gg \rho_\Lambda\). One obtains \(\rho_\Lambda \approx \rho_{\text{rad}} a^{-3}\), where the subscript ‘0’ denotes the present-day value. This determines, in turn, the IR cut-off as \(L \approx M_{\text{Pl}}(\rho_{\text{rad}} a^{-3})^{-1/2}\). Equipped with these relationships, and \(T \sim a^{-1}\), we find that the absolute bound \(L^3T^4 \lesssim \Lambda M_{\text{Pl}}^2\) is saturated at \(T \sim 10^9\) GeV. This gives an interesting limit on the post-inflation reheating temperature, i.e. the temperature at the beginning of the hot big bang universe. Since Li’s model employs a saturated version of (1), when the absolute bound is saturated we have \(R_s \gg L\), and therefore it is clear that this interesting bound cannot be fully trusted.
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in which each epoch represents a system not containing black holes, would be

\[
T^4 \sim c_{\text{rad}}^2(t) L^{-2} M_{\text{Pl}}^4, \tag{5}
\]

\[
\rho_\Lambda \sim c_\Lambda^2(t) L^{-2} M_{\text{Pl}}^2, \tag{6}
\]

where the holographic bound is saturated asymptotically only by \( c_\Lambda^2 \) (in order to ensure the current dark energy dominance). On the other hand, \( c_{\text{rad}}^2 \gg c_\Lambda^2 \) (also \( c_{\text{rad}}^2 \lesssim 1 \)) in the radiation dominated era. The set-up as given by equations (5) and (6), with \( c^2 s \lesssim 1 \), is only a formal account of a system free of black holes. Note that with the choice (6) the UV/IR correspondence becomes more complicated, now depending on the particular choices for \( c_\Lambda^2 \). The bottom line, however, is that, by UV/IR mixing,\( \rho_\Lambda \) should always (irrespective of the form of \( c_\Lambda^2 \)) acquire the form given by equation (2). The era of radiation dominance therefore imposes a constraint, \( T^4 > \rho_\Lambda \). On the other hand, in a conventional QFT with some infrared limitation, a system at a temperature \( T \) has an energy \( L^3 T^4 \) (and therefore energy density \( T^4 \)), provided \( L^{-1} < T < \Lambda \). If the mass scale is negligible with respect to the UV cut-off, \( \rho_\Lambda \sim \Lambda^4 \) (see equation (2)), the above constraints are impossible to satisfy simultaneously throughout the radiation dominated era, showing thus internal inconsistency. With a more realistic estimate, \( \rho_\Lambda \sim m_\Lambda^3 \) (see equation (2)), one obtains

\[
\Lambda > T > m^{1/4} \Lambda^{3/4}. \tag{7}
\]

Thus, when \( m > \Lambda \), equation (7) would entail (via the case (b) of equation (2)) \( m < \Lambda \), showing internal inconsistency again. We have thus seen that though with some degree of non-saturation of the holographic bound, both systems (dark energy and radiation) can be made free of states lying within their Schwarzschild radius, a consistent description of takeover of the dominance by radiation within the same QFT is not possible.

Perhaps the situation is even worse than stated above. If an effective QFT is to encompass the standard model particles (\( m \gtrsim 100 \text{ GeV} \)), the present-day UV cut-off is much smaller than \( 10^{-2.5} \text{ eV} \). Indeed, from \( m \Lambda_0^3 \sim 10^{-11} \text{ eV}^4 \), one obtains \( \Lambda_0 \sim 10^{-7} \text{ eV} \). This means that even in the present epoch (dominated by dark energy fluid) a consistent description of (CMBR) radiation is not viable since the present temperature of the universe \( T_0 \sim 10^{-4} \text{ eV} \).

In conclusion, we have shown that an effective QFT, with a proposed relationship between UV and IR cut-offs for eliminating the need for fine-tuning in the ‘old’ cosmological constant problem and furthermore explaining dark energy at present, cannot consistently describe a radiation dominated universe. Such a framework is particularly compelling in description of an expanding universe since without a corresponding UV/IR mixing, conventional QFT may not be valid for arbitrarily large volumes. Although in a radiation dominated epoch the UV/IR correspondence can be made virtually arbitrary, takeover by radiation cannot yet be obtained. Our results are quite generic in that they do not depend on the pressing problem of the choice of the IR cut-off. Because of the absence of a prominent energy scale (connected to microphysics), disparate mass scales as well as the possibility that the underlying framework may not be QFT (i.e. a black hole fluid), we cannot a priori draw the same conclusion for a matter dominated epoch. Still, on similar grounds to the above it is seen that saturated HDE models would compromise a consistent description of that epoch as well. Our overall conclusion is therefore that the basic framework underlying all HDE models seems too ad hoc to have any real explanatory value, which leaves us still in need of a firmer theoretical background.
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