Specification and Verification of Distributed Embedded Systems: A Traffic Intersection Product Family

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Distributed embedded systems (DESs) are no longer the exception; they are the rule in many application areas such as avionics, the automotive industry, traffic systems, sensor networks, and medical devices. Formal DES specification and verification is challenging due to state space explosion and the need to support real-time features. This paper reports on an extensive industry-based case study involving a DES product family for a pedestrian and car 4-way traffic intersection in which autonomous devices communicate by asynchronous message passing without a centralized controller. All the safety requirements and a liveness requirement informally specified in the requirements document have been formally verified using Real-Time Maude and its model checking features.

1 Introduction

Distributed embedded systems (DESs) are no longer the exception; they are the rule in many application areas such as avionics, the automotive industry, traffic systems, sensor networks, and medical devices. The specification and verification of such systems poses special challenges: besides modeling the real-time behavior of each distributed component, faithfully modeling the real-time interactions between the different embedded components is just as crucial. This is hard because the asynchronous nature of component interactions makes verification particularly challenging due to state space explosion.

Furthermore, in an industrial setting there are additional reusability requirements for DES specification and verification: (1) the different components of the DES should be specified in as modular and reusable a way as possible; and (2) rather than focusing on a single DES design, the specification and verification effort should be amortized over an entire parametric family of DES designs, which can be used to develop product families in order to cut development cost and shorten time to market without diminishing system quality or compromising safety. This means that component specifications should be both modular and parametric, making it easy to obtain different product designs by composing different components from a library and fixing the various parameters of each component.

This paper reports on an extensive case study about a DES parametric design that can serve as a basis for a product family for a pedestrian and car 4-way traffic intersection that should be totally decentralized (no central controller), should support different traffic light protocols such as the American and European light regimes, should be strongly fault-tolerant, and should have sophisticated features such as detecting the proximity of emergency vehicles and then switching the entire distributed system to an emergency regime in which emergency vehicles have absolute priority over ordinary vehicles and pedestrians.

One important feature of this case study is that the requirements that the specification and verification of this product family should meet were not determined by us: they were provided by an industrial partner (Lockheed-Martin) with no input on our part and with a substantial degree of realism. Furthermore, the traffic intersection product family was viewed as a representative example exhibiting many of the challenges that would be present in other DES product families such as, for example, command and control systems. Specifically, the informal system requirements were specified as a Statement of Work.
(SoW) that should be abided by a system design team. The SoW included a number of important safety and liveness requirements that the system design should meet.

The overall goal of this case study was to evaluate on a realistic and somewhat challenging application the suitability of different modeling languages for the design and analysis of DES product families. Since many system aspects needed to be addressed, including deployment of the DES design on specific hardware platforms, system cost estimation, performance evaluation, and system safety, no single modeling language was envisioned. Instead, the emphasis was on multimodeling [3], with different modeling languages handling those aspects that they do best. Real-Time Maude was chosen to handle all formal modeling and verification aspects, including the verification of all safety, fault-tolerance, and liveness requirements.

1.1 Challenges, Experience, and Contributions

We describe here the formal modeling and verification challenges that had to be addressed in order for Real-Time Maude to successfully handle this case study. For each such challenge we explain how we addressed it. We then finish giving a summary of our overall experience and our contributions.

Modular, Decentralized, and Asynchronous Design. To address this challenge we model the traffic light system as a collection of autonomous concurrent objects that interact with each other by asynchronous message passing. This makes the system highly modular and reusable. In particular, the system has no central controller; instead, the different components interact with each other through messages and act autonomously. The system is also highly parametric: ten different parameters can be specified to obtain different product versions with different special features, such as, for example, support for American or European light regimes, and reaction to the presence of emergency vehicles in the environment. Fault tolerance and recovery issues are explicitly addressed: failures are modeled explicitly under a very general fault model in which multiple devices can fail at any time.

State Space Explosion. It is well-known that concurrency and asynchrony can easily cause an exponential blowup in the search space of a system when model checking its properties. Because of the highly concurrent and asynchronous nature of our traffic light design this poses an important modeling and verification challenge. We have addressed this challenge without sacrificing the good features of modularity, decentralization, autonomy, and asynchrony in the design. It would of course have been much easier to model check a highly centralized synchronous design with much fewer states. But this would have meant sacrificing many good design aspects for the sole purpose of making our verification task easier. Also this would have rendered the design less reusable and would have considerably widened the gap between the model and a realistic deployment. The key approach that we have taken to achieve a design that can be feasibly verified without compromising good design features has been the use of abstraction. This means that the model is as abstract as possible, yet, all relevant features are modeled. For example, since in this example, in the north-south (N-S) and south-north (S-N) directions (similarly for east-west and west-east) the car lights will always have the exact same color, it is not necessary to have two light devices (one for N-S-bound cars, and another for S-N-bound cars). Instead, it is enough to have a single light device for the two (N-S and S-N) directions, in the implicit understanding that such a device will have two simultaneous and identical light displays for both the N-S and S-N directions. In a similar vein, the message-passing communication between devices is assumed to be instantaneous, abstracting out the implicit assumption of a tolerable bound in the network communication delays, given that light changes happen at the level of seconds, whereas network delays can be assumed to be in the order of
milliseconds. However, we believe that all relevant aspects of the system are still modeled, including failures and behavior for emergency vehicles. The only aspects not modeled are those needed for performance estimation purposes, which are not relevant for safety purposes, and that do not preclude certain, more abstract analyses of liveness properties. Specifically, we do not model the exact number of cars or pedestrians, but only their presence or absence near the intersection.

This does not exclude the possibility of developing much more detailed models of this system in Real-Time Maude for simulation and performance estimation purposes. This could easily be done; but their direct formal verification by model checking would be unfeasible. However, they could also be proved correct indirectly, by proving that the model that we present here is a correct abstraction of these more detailed models.

**Formal Verification of Safety and Liveness Properties.** As we explain in more detail in the body of the paper, all the safety properties mentioned in the SoW, as well as a liveness property, have been verified. This means that precise formal specifications in the form of temporal logic formulas have been developed for each of the informal and somewhat vague corresponding requirements in the SoW document; and then they have been verified by model checking in our model. It also means that, as we further explain later, certain inconsistencies between the informal requirements have also been identified and addressed in both the model design and the formal verification.

**Overall Experience and Main Contributions.** Our overall experience can be summarized as follows. Because of its support for distributed objects and asynchronous communication, we have been able to effectively use Real-Time Maude to develop a highly reusable and modular distributed system design for the 4-way traffic intersection system in the form of a parametric product family that could be instantiated in various ways to support a variety of additional features. Perhaps the hardest challenge has been to avoid state space explosion without compromising the faithfulness with which the formal model captures all relevant system aspects. Here, careful use of abstraction to exclude all non-essential aspects from the model has been crucial. This made it possible for us to formally verify all informal safety requirements in the SoW, plus one important liveness requirement. The overall experience has been quite positive, in that all the modeling challenges posed by the case study could be successfully addressed and all the expected verification tasks were accomplished.

We are not aware of other industrially-based case studies in which full formal specification and verification of DES systems with comparable degrees of concurrency, modularity, and parametricity have been carried out. In this regard we view our main contribution to be a novel and convincing demonstration that formal specification and verification of nontrivial DES product families is indeed possible in spite of the challenges involved; and that rewriting-based methods such as those supported by Real-Time Maude are indeed effective in addressing these challenges. A subsidiary contribution is the use of a distributed object-based formal modeling to achieve a decentralized and highly reusable DES product family design.

### 2 Real-Time Maude

A Real-Time Maude timed module specifies a real-time rewrite theory of the form \((\Sigma, E, IR, TR)\), where:

- \((\Sigma, E)\) is a membership equational logic \([2]\) theory with \(\Sigma\) a signature \([3]\) and \(E\) a set of confluent and terminating conditional equations. \((\Sigma, E)\) specifies the system’s state space as an algebraic data type, and must contain a specification of a sort \(\text{Time}\) modeling the (discrete or dense) time domain.

\(^{1}\text{That is, } \Sigma \text{ is a set of declarations of sorts, subsorts, and function symbols.}\)
- IR is a set of (possibly conditional) labeled instantaneous rewrite rules specifying the system’s instantaneous (i.e., zero-time) local transitions, written \( rl \ [l] : t \Rightarrow t' \), where \( l \) is a label. The rules are applied modulo the equations \( E \).

- TR is a set of tick (rewrite) rules, written \( rl \ [l] : \{i\} \Rightarrow \{i'\} \) in time \( \tau \), that model time elapse. \( \{ \_ \} \) is a built-in constructor of sort \( \text{GlobalSystem} \), and \( \tau \) is a term of sort \( \text{Time} \) that denotes the duration of the rewrite.

The initial state must be a ground term of sort \( \text{GlobalSystem} \) and must be reducible to a term of the form \( \{i\} \) using the equations in the specifications.

The Real-Time Maude syntax is fairly intuitive. For example, function symbols, or operators, are declared with the syntax \( \text{op} \ f : s_1 \ldots s_n \rightarrow s \). \( f \) is the name of the operator; \( s_1 \ldots s_n \) are the sorts of the arguments of \( f \); and \( s \) is its (value) sort. Equations are written with syntax \( \text{eq} t = t' \), and \( \text{ceq} t = t' \) if \( \text{cond} \) for conditional equations. The mathematical variables in such statements are declared with the keywords \( \text{var} \) and \( \text{vars} \). We refer to [2] for more details on the syntax of Real-Time Maude.

In object-oriented Real-Time Maude modules, a class declaration

\[
\text{class } C \mid \text{att}_1 : s_1, \ldots, \text{att}_n : s_n .
\]

declares a class \( C \) with attributes \( \text{att}_1 \) to \( \text{att}_n \) of sorts \( s_1 \) to \( s_n \), respectively. An object of class \( C \) in a given state is represented as a term \(< C : C \mid \text{att}_1 : \text{val}_1, \ldots, \text{att}_n : \text{val}_n > \) of sort \( \text{Object} \), where \( O \), of sort \( \text{Obj} \), is the object’s identifier, and where \( \text{val}_1 \) to \( \text{val}_n \) are the current values of the attributes \( \text{att}_1 \) to \( \text{att}_n \), respectively. In a concurrent object-oriented system, the state is a term of the sort \( \text{Configuration} \). It has the structure of a multiset made up of objects and messages. Multiset union for configurations is denoted by a juxtaposition operator (empty syntax) that is declared associative and commutative, so that rewriting is \( \text{multiset rewriting} \) supported directly in Real-Time Maude. The dynamic behavior of concurrent object systems is axiomatized by specifying its transition patterns by rewrite rules. For example, the rule

\[
rl \ [l] : m(0, w) < \text{O : C} \mid \text{a}_1 : x, \text{a}_2 : O', \text{a}_3 : z > \Rightarrow
\]

\[
< \text{O : C} \mid \text{a}_1 : x + w, \text{a}_2 : O', \text{a}_3 : z > m'(0', x) .
\]

defines a family of transitions in which a message \( m \), with parameters \( 0 \) and \( w \), is read and consumed by an object \( 0 \) of class \( C \). The transitions have the effect of altering the attribute \( \text{a}1 \) of the object \( 0 \) and of sending a new message \( m' (0', x) \). “Irrelevant” attributes (such as \( \text{a}3 \)) need not be mentioned in a rule.

**Formal Analysis.** A Real-Time Maude specification is executable, and the tool offers a variety of formal analysis methods. In this paper we focus on temporal logic model checking. Real-Time Maude extends Maude’s linear temporal logic model checker to check whether each behavior, possibly up to a certain time bound, satisfies a temporal logic formula. State propositions are terms of sort \( \text{Prop} \), and their semantics is defined by (possibly conditional) equations of the form \( \{\text{statePattern}\} \models \text{prop} = b \), with \( b \) a term of sort \( \text{Bool} \). Such equations define the state proposition \( \text{prop} \) to hold in exactly those states \( \{i\} \) where \( \{i\} \models \text{prop} \) evaluates to \( \text{true} \). A temporal logic \text{formula} is constructed by state propositions and temporal logic operators such as \( \text{True} \), \( \text{False} \), \( \lnot \) (negation), \( \land \) (conjunction), \( \rightarrow \) (implication), \( [] \) ("always"), \( <> \) ("eventually"), and \( \U \) ("until"). The model checking command is written \( \text{(mc } t \models_{u \text{ formula} .} \) for \( t \) the initial state and \( \text{formula} \) the temporal logic formula.

Finally, Real-Time Maude provides some metric temporal logic model checking commands for non-hierarchical object-oriented models, such as the traffic intersection. For example, the bounded response model checking command \( \text{(br } t \models p \Rightarrow <>\text{le}(\tau) q .) \) investigates whether each \( p \)-state will be followed by a \( q \)-state in \( \tau \) time units or less [4].

\( E = E' \cup A \), where \( A \) is a set of equational axioms such as associativity, commutativity, and identity, so that deduction is performed modulo \( A \). Operationally, a term is reduced to its \( E' \)-normal form modulo \( A \) before any rewrite rule is applied.
3 Overview of the Requirements Specification

This section gives a brief overview of selected parts, namely, those concerning functionality rather than performance, of the statement of work (SoW) for the Easily Deployable Traffic Congestion Management System for Four-Way Intersections (EDeTCMS-4) that was provided to us by Lockheed Martin.

The overall goal is to design a four-way traffic intersection solution that is easily customizable to enable rapid deployment in the US and Europe. The aim of the system is to reduce commute times and traffic jams, but not at the expense of safety and reliability aspects which keep motorists and pedestrians safe on the road. In particular, a main goal is to reduce the emergency vehicle commute times which have become more and more unacceptable with the rising number of vehicles on the road.

Some assumptions about the intersections where EDeTCMS-4 will be deployed are: (i) pedestrians walk in all possible directions except diagonally through the intersection; (ii) traffic enters the intersection from any possible direction, and exits in either a left, straight, or right direction; (iii) emergency vehicles may require the intersection to be cleared at any time, allowing them to enter from any direction and exit in either a left, straight, or right direction; and (iv) there are no cross-traffic dedicated turn lanes.

The requirements on the light operations include: (v) the system shall turn green a pedestrian light only when pedestrians are waiting to cross in that direction; (vi) the system should turn green a vehicular light only when there are vehicles waiting to go in the direction controlled by the light, or when turning the light green does not prohibit any other cars from proceeding through the intersection; (vii) the system must be fault-tolerant, and, except for transient faults, must ensure the proper functioning of the lights; (viii) the system must ensure failure recovery and safe car and pedestrian conditions also under failures; (ix) under no circumstances (including in an emergency clearance) will there be unsafe situations for cars or pedestrians; and (x) the maximum pedestrian wait time shall be less than 5 minutes.

4 The Real-Time Maude Model of the EDeTCMS-4

This section presents our Real-Time Maude model of the EDeTCMS-4 intersection. The entire executable model, including the model checking commands is Section 5 is available at [http://www.ifi.uio.no/RealTimeMaude/TrafficLight](http://www.ifi.uio.no/RealTimeMaude/TrafficLight).

4.1 Overview and Assumptions

We have defined a model of the behavior of the traffic lights that should function correctly under the following very general conditions:

- Emergencies may happen at any time, and may end at any time thereafter.
- Any device may fail at any time. We assume that there is a minimum time interval between the repair of a device and the next time that device can fail. This minimum interval is a parameter of the system, and can be set to 1 to get a completely nondeterministic failure model.

Our model focuses on modularity and autonomy, in the sense that each traffic light should operate as independently as possible. In our model, devices only communicate through message passing.

We have defined an object-based model of EDeTCMS-4. Each intersection has four objects modeling the traffic lights:

1. One object models the (controller for the) car lights in the east-west direction.
2. One object models the (controller for the) car lights in the north-south direction.
3. One object models the (controller for the) pedestrian lights in the east-west direction.
4. One object models the (controller for the) pedestrian lights in the north-south direction.

Since there are no turn signals, we did not see any reason why the car lights in opposite directions (say, north and south) should not always show the same color. From this, it follows that both pedestrian crossings in the same direction should have the same color. This is also how we have observed traffic lights; e.g., you push the button on one pedestrian light pole, and all four buttons for that direction are lit. Therefore, we assume one controller for the car lights in the north/south direction, and so on.

In addition to the controllers, we model the environment as follows:

- One environment object nondeterministically generates new cars and new pedestrians at each time instant. That is, at each moment in time, this environment object may or may not generate a new pedestrian/car in a certain direction.
- One environment object generates emergency and emergency-over signals. Such signals can be generated at any time, with the emergency-over signal following at any time after one or more emergency signals.
- For each device/controller, there is a corresponding environment object which generates failures for that device at any time. After a failure, it may generate a repair message for the device at any time after a minimum separation between a repair and a new failure.

By including/excluding certain of the above environment objects in the initial state of the system, the system can be analyzed in the presence/absence of the corresponding emergencies/failures. Section 5 illustrates this feature by analyzing the system without emergencies and failures, with emergencies and without failures, with the failures of a subset of the devices, and so on.

4.1.1 Parametricity and Reusability

The requirements place great emphasis on being able to use the controller in different intersections in both Europe and the U.S. Our model supports substantial reuse in the following ways:

- It is defined for both American and European traffic light configurations. A parameter can be set to denote European or American deployment.
- It is parametric in important parameters such as:
  - the amount of time during which the light in a given direction should be green/red in a round,
  - the safety margin during which the car lights in both directions should be red,
  - the time during which the light should be yellow in each round,
  - the minimum time it is assumed to take for a pedestrian to cross the street, and so on.

This allows the system to be deployed under varying circumstances. For instance, we have noticed as lone walkers in Cherry Hill, NJ, that the system assumes that pedestrians cross the street faster than an Olympic sprinter, whereas in Europe people typically have more time to cross. Likewise, as the speed limit increases, so should the safety margin and the time the light stays yellow.

- Since our model is designed to work under very general failure and emergency assumptions, it can be deployed in all kinds of places, including in poorer communities where devices often fail and where repair is rarely available, as well as in places where the members of the plutocracy activate the emergency clearance with a high frequency and long durations.

- Our object-oriented model is also parametric in the number of intersections, so that we can deploy and analyze a set of intersections by changing the initial state.
4.1.2 Inconsistent Requirements

During the formalization effort we discovered that the requirements (vi) (a car light should turn green only when there are cars waiting) and (x) (no pedestrian should wait for more than five minutes to cross) seem to be in contradiction with each other: if no car is driving in a certain direction, then the cars coming from the other direction should not be prohibited from proceeding through the intersection, so the poor pedestrian has to wait forever. We therefore model a system where a car light can turn green if either cars or pedestrians are waiting to cross in that direction.

4.1.3 Overview of Car Lights in Normal Operation

In a setting without failures, the car lights operate almost independently. The only communication between the two car light (controllers) happens when one car light is about to turn green, but detects that no car or pedestrian is waiting to cross in that direction. This car light must then send a message to the other car light, informing that other car light that it can stay green for another round.

The parameters for the car lights are: yellowTime (the time that the light is yellow after being green and before turning red), safetyMargin (the time during which both car lights should be red), and redTime_{NS} and greenTime_{NS} (the duration of the red, resp. green, light in the NS direction in each round). The operation of a car light controller in direction D can be summarized as follows:

1. Assume that the car light for direction D just got red.
2. It stays red for time redTime_{D} - (\Delta + yellowTime + safetyMargin), for some small value \Delta > 0.
3. It then checks the sensors to see if there is a car present at the approach, and also checks whether it has recorded a signal from the associated pedestrian light about the pedestrian button having been pushed lately. If a car is waiting or a pedestrian button push has been recorded, the car light shows red for additional time (\Delta + yellowTime + safetyMargin), unless it is an European light, in which case it waits for only time \Delta + yellowTime before turning both yellow and red. If neither car nor pedestrian is waiting, then the car light sends a signal to the opposing car light that it will not turn green this time, and resets its timer to an entire round, and remains red.
4. After waiting for the duration given in item (3), the red light turns green. If the car light has recorded a pedestrian push lately, it sends a message to “its” pedestrian light to turn the light green.
5. After showing green for time greenTime_{D}, the light turns yellow unless it has received a signal that the other car light does not need to turn green, in which case it stays green for another round.
6. After showing yellow for time yellowTime, it turns red, and starts from item (1) above.
7. In addition, the car light treats ’pedestrian pushed button’ messages from the pedestrian light.

Given redTime_{NS} and greenTime_{NS}, the corresponding values for the EW direction can be computed.

4.1.4 Pedestrian Lights During Normal Operation

The operation of a pedestrian light is fairly simple:

- A pedestrian light turns green when it receives a ’pedestrian go’ message from its car light.
- When time walkTime remains of the time period for which the car light has promised to stay green, the pedestrian light starts blinking.
• After the light has been blinking for time `walkTime`, it turns red.
• When a new pedestrian arrives, and the light is red or blinking, the pedestrian is assumed to push the button on the pedestrian light pole unless this is lit. If the button is not lit, it becomes lit, and a message is sent to the car light.

4.1.5 Emergency Clearance

In the literature about the treatment of emergencies in traffic systems, there are different ways of signaling and sensing emergencies, such as acoustic sensing, and there are different ways of responding to emergencies, one of which is to turn all lights red, another which is to turn some light green.

Our model has been influenced by thinking about acoustic sensing, in that we assume that one signal is sent when the emergency situation first appears (the car lights detect the sirens), and one is sent when the emergency is over (it suddenly becomes quiet again). Furthermore, in an acoustic setting we did not find it natural to assume that only one car light detects the emergency signals.

Given that there is a real problem of people trying to fool the sensors when the light turns green during emergency, we have opted for what we think is the most natural solution: all lights turn red during emergencies. Since drivers hear the emergency signals, they will hopefully make way for emergency vehicles even if that implies violating red lights if needed. However, in order not to add collisions when emergency vehicles arrive, we think it is natural to turn a green light yellow before turning it red during emergencies. When the emergency is over, the system restarts from a standard starting state, which basically means that the prioritized direction gets the green light first, if there are cars or pedestrians waiting; otherwise the direction with lowest priority restarts with green lights.

4.1.6 Failures

Each device (for which an error generator is included in the initial state of the system) may fail at any time, and will be repaired after an arbitrary amount of time. Upon detecting a failure in a device, the failed device notifies the other devices about the failure, and all devices go into error mode as follows:

• The car lights in the prioritized direction start blinking yellow.
• The car lights in the other direction start blinking red.
• All pedestrian lights are turned off.

Each device must keep track of how many devices are currently in failed state, so that they only go to normal mode after all the failed devices have been repaired. When all devices have been repaired, the system restarts from a “neutral” position.

4.1.7 Communication

Given the short distances between the devices and the relatively large time scale of the changes in the traffic lights, we abstract from communication delays and assume instantaneous and reliable asynchronous message passing communication between the devices, and from the “environment” to the devices.

4.2 The Real-Time Maude Model

This section presents fragments of our Real-Time Maude model of the controllers.
Tunable Parameters. The following defines some system parameters. Additional parameters, such as redTime and greenTime, are given as parameters to the initial state as shown in Section 5.1.

--- American or European crossing?
ops americanXing europeanXing : -> Bool .
eq americanXing = true .
eq europeanXing = false .

--- A small amount of time:
op Delta : -> NzTime .
eq Delta = 1 .

--- Safety margin is the time that both lights should be red:
op safetyMargin : -> NzTime .
eq safetyMargin = 1 .

--- Duration of yellow light before turning red:
op yellowTime : -> NzTime .
eq yellowTime = 1 .

--- The shortest time it takes to cross the street for a pedestrian:
op walkTime : -> NzTime .
eq walkTime = 2 .

--- Minimum duration of green and red car lights that ensures that pedestrian will see some green before blinking:
ops minRedTime minGreenTime : -> NzTime .
eq minGreenTime = walkTime + 1 .
eq minRedTime = safetyMargin + minGreenTime + yellowTime + safetyMargin .

Object Identifiers. We envision that cities will have multiple intersections, and our model supports multiple (independent) intersections. Therefore, each object’s name includes the name of the intersection. For example, the name of a car light controller could be carLight("SpitsB-2", NS), denoting the car lights for the north-south direction in the intersection called "SpitsB-2". The corresponding pedestrian light controller object should be named pedLight("SpitsB-2", NS). Furthermore, approach(xing, dir) is the name of the sensor which senses whether some car is traveling in the direction dir in the intersection xing; and so on:

vars CN CN' : CrossingName . var DIR : Direction .
ops NS EW : -> Direction [ctor] .
op opposite : Direction -> Direction .
eq opposite(NS) = EW .
eq opposite(EW) = NS .

eq opposite(carLight(CN, DIR)) = carLight(CN, opposite(DIR)) .
eq opposite(pedLight(CN, DIR)) = pedLight(CN, opposite(DIR)) .

Messages. The following messages define the interface between the objects: a continueGreen(carL) message signals to the receiving car light carL that it should remain green for another round; the pedGo message signals to the receiving pedestrian light that it should turn green and remain green/blinking for the given amount of time; and the pedsWaiting message is sent from a pedestrian light to the corresponding car light to signal that a pedestrian has pushed the unlit button.
newPed(pedLight) and newCars(approach(xing,dir)) messages are generated by the environment to denote that new (non-empty sets of) pedestrians and cars have arrived at the given place.

emergency(xing) and emergencyOver(xing) messages are generated by the environment and signal, respectively, the start of a period of emergency and the end of such a period. Given that we assume acoustic sensing of sirens, we assume that both car lights hear any emergency signal. Therefore, emergency messages are only sent to the crossing; the two equations below then “distribute” such a message to each of the two car lights in the crossing:

\[
\text{eq emergency} (\text{CN}) = \text{emergency} (\text{carLight(CN, EW)}) \quad \text{emergency} (\text{carLight(CN, NS)}) .
\]

\[
\text{eq emergencyOver} (\text{CN}) = \text{emergencyOver} (\text{carLight(CN, EW)}) \quad \text{emergencyOver} (\text{carLight(CN, NS)}) .
\]

The following messages are sent from a car light to the corresponding pedestrian light to signal, respectively, that an emergency is detected; and when the emergency is over, whether to turn red or green.

\[
\text{msg emergency resumeRed} : \text{PedLightId} \rightarrow \text{Msg} .
\]

\[
\text{msg resumeGreen} : \text{PedLightId} \text{ NzTime} \rightarrow \text{Msg} .
\]

When an emergency is over, the car lights need to restart. In our setting, one of the directions is defined to be the prioritized direction which should turn green first after an emergency. However, given requirement (vi) in the SoW, this prioritized light cannot turn green if no car or pedestrian is waiting in that direction. Therefore, the main car light must signal to the other car light how to restart after the emergency:

\[
\text{msgs reStartRed reStartGreen} : \text{CarLightId} \rightarrow \text{Msg} .
\]

Finally, the following messages are used for failure generation and repairs. The environment generates error messages of the form \((\text{to d error}(d))\). Once device \(d\) reads such an error message, it must also inform the three other controllers about its failure by sending a \((\text{to d_k error}(d))\) message to each other device \(d_k\) in the intersection. The messages for repairs work in the same way.

### 4.2.1 Car Lights During Normal Operation

We first present the behavior of the car lights during normal operation. Car light controllers are modeled as object instances of a class CarLight with the following attributes:

- **lights** denotes the lights shown (in both directions) by the car light. It is a set of colors, because in Europe it may show both red and yellow at the same time.
- **The timer attribute is set to expire when the next time-triggered action must be taken.**
- **state** denotes the “internal” state of the objects, as seen in the rules below.
- **redTime** and **greenTime** denote the length of time the light stays red, respectively green, in a round. Since these are class attributes and hence represented in the state, they can be changed during system execution, for example to increase the **greenTime** of the busier direction.
- **pedWaiting** denotes whether or not the car light has received a signal from the pedestrian light that a pedestrian has pushed the button. This is needed to avoid turning a pedestrian light green if no pedestrians are waiting in that direction.
- **defaultStarter** is true if this car light has priority to turn green after failures and emergencies.

\[
\text{class CarLight | lights : ColorSet, timer : TimeInf, state : CLState, redTime : NzTime, greenTime : NzTime, pedWaiting : Bool, defaultStarter : Bool} .
\]

\[
\text{sorts CLState NormalCLState} . \quad \text{subsort NormalCLState < CLState} .
\]

\[
\text{ops red toRedYellow toGreen green yellow : -> NormalCLState [ctor]} .
\]

\[
\text{op emergency : -> CLState [ctor]} .
\]
The following rules apply when the timer expires and the object is in state red. The car light must then check whether some cars are waiting in the corresponding approach, or whether it has any record of a pedestrian having pushed the button lately. If neither car nor pedestrian is waiting, rule dontGoGreen can be applied, and the car light stays in state and color red but resets its timer to expire an entire round later (redTime + greenTime + yellowTime). It also sends a continueGreen message to the other car light. If, on the other hand, some pedestrian or car is waiting (B1 or B2), rule redToSafetyMargin is applied. Depending on the type of crossing, the timer is set to expire either when the light should turn green, or when it should turn red and yellow. The light itself is not updated, and stays red. Only the “internal” state and the timer are updated. We first declare the variables used in the rules:

```
var C : Color . var CL : CarLightId . var PL : PedLightId .
vars RT GT : NzTime . vars B B1 B2 : Bool . var TI : TimeInf .
var T : Time . var DIR : Direction . var CN : CrossingName .
var S : CLState . var NORMAL : NormalCLState .
```

```
rl [dontGoGreen] :
  < approach(CN, DIR) : XingApproach | carsPresent : false >
  < carLight(CN, DIR) : CarLight | state : red, timer : 0, pedWaiting : false, redTime : RT, greenTime : GT >
  =>
  < carLight(CN, DIR) : CarLight | timer : GT + RT + yellowTime >
  < approach(CN, DIR) : XingApproach | >
  continueGreen(carLight(CN, opposite(DIR))) .
```

```
crl [redToSafetyMargin] :
  < approach(CN, DIR) : XingApproach | carsPresent : B1 >
  < carLight(CN, DIR) : CarLight | state : red, timer : 0, pedWaiting : B2 >
  =>
  if americanXing then
    < carLight(CN, DIR) : CarLight | state : toGreen, timer : Delta + yellowTime + safetyMargin >
  < approach(CN, DIR) : XingApproach | >
  else --- european, timer only to Delta and another state:
  < carLight(CN, DIR) : CarLight | state : toRedYellow, timer : Delta + yellowTime >
  < approach(CN, DIR) : XingApproach | > fi
  if (B1 or B2) .
```

In the following rule, the car light turns green and the timer is set so that the light stays green for time GT (which is the variable denoting the value of the attribute greenTime). If a pedestrian is recorded to be waiting, a pedGo message is also sent to the corresponding pedestrian light:

```
rl [redToGreen] :
  < CL : CarLight | state : toGreen, greenTime : GT, timer : 0, pedWaiting : B >
  =>
  < CL : CarLight | state : green, timer : GT, lights : green, pedWaiting : false >
  (if B then pedGo(pl(CL), GT) else none fi) .
```

When the car light is in state green and the timer has expired, the light should turn yellow and stay yellow for time yellowTime. Then, it should turn red:

```
rl [greenToYellow] :
  < CL : CarLight | state : green, timer : 0 >
```
The following two rules deal with receiving messages from the pedestrian light when someone wants to cross the intersection. If the car light in the same direction as the pedestrian crossing is green, and there is enough time for the pedestrian to cross during the car light’s remaining time in green \((TI \geq walkTime)\), then the car light sends a pedGo signal to the pedestrian light with the time remaining of the green light (rule \text{buttonPressedTurnedOn}). If the car light in the same direction as the pedestrian crossing does not show green or if there is not enough time remaining for the green light to allow the pedestrian to cross safely, the car light remembers that a pedestrian is waiting by setting \text{pedWaiting} to true:

\begin{align*}
\text{crl } \text{[buttonPressedTurnOn]} : \\
(to \text{ CL } \text{pedsWaiting}) \\
< \text{ CL } : \text{CarLight} \mid \text{state} : \text{green}, \text{timer} : \text{TI} > \\
=> \\
< \text{ CL } : \text{CarLight} \mid \text{pedGo(pl(CL), TI)} \\
\text{if } \text{TI} \geq \text{walkTime} .
\end{align*}

\begin{align*}
\text{crl } \text{[rememberButtonPressed]} : \\
(to \text{ CL } \text{pedsWaiting}) \\
< \text{ CL } : \text{CarLight} \mid \text{state} : S, \text{timer} : \text{TI} > \\
=> \\
< \text{ CL } : \text{CarLight} \mid \text{pedWaiting} : \text{true} > \\
\text{if } (S \neq \text{green}) \text{ or } (\text{TI} < \text{walkTime}) .
\end{align*}

Finally, if the car light receives a \text{continueGreen} message, it knows that it can stay green for another round, and increases its timer by a whole round:

\begin{align*}
\text{rl } \text{[continueGreen]} : \\
\text{continueGreen(CL)} \\
< \text{ CL } : \text{CarLight} \mid \text{state} : \text{NORMAL}, \text{timer} : \text{T}, \text{greenTime} : \text{GT}, \text{redTime} : \text{RT}, \text{pedWaiting} : \text{B} > \\
=> \\
< \text{ CL } : \text{CarLight} \mid \text{timer} : \text{T} + \text{GT} + \text{RT} + \text{yellowTime}, \text{pedWaiting} : \text{false} > \\
\text{if } \text{B} \text{ then } \text{pedGo(pl(CL), T} + \text{GT} + \text{RT} + \text{yellowTime} \text{ else } \text{none fi} .
\end{align*}

### 4.2.2 Pedestrian Lights During Normal Operations

Our pedestrian lights work by sending \text{pedWaiting} messages to the car light when a button is pushed and the pedestrian cannot cross, by receiving \text{pedGo} messages from the car light, which turns the pedestrian light green, and by starting blinking and then turning red at appropriate times thereafter.

In the following declaration of the class \text{PedLight}, the attribute \text{color} shows the color of the light, where we have now added \text{blinking} as a “color”; \text{buttonLit} is true if some pedestrian has pushed the button and has not been able to cross since; and \text{mode} denotes the mode the system is assumed to be in, which is \text{normal} during normal operations:
class PedLight | timer : TimeInf, color : Color, buttonLit : Bool, mode : PLMode.

sort PLMode.  ops normal emergency : -> PLMode [ctor].

In rule turnGreen, the pedestrian light receives a pedGo message with a time T during which it can be green or blinking. It sets the timer to T minus walkTime (where \(x \text{ minus } y = \max(x - y, 0)\)) and turns the pedestrian light green. When it is time to start blinking, the rule startBlinking turns the pedestrian light to blinking and sets the timer to walkTime. When the blinking time is over, the pedestrian light turns red and turns off the timer. The pedestrian light therefore stays red until it receives a pedGo message:

rl [turnGreen]:
  pedGo(PL, T)
  =>
    < PL : PedLight | mode : normal >

rl [startBlinking]:
  < PL : PedLight | timer : 0, color : green >
  =>
    < PL : PedLight | timer : walkTime, color : blinking >.

rl [stop]:
  < PL : PedLight | timer : 0, color : blinking >
  =>
    < PL : PedLight | timer : INF, color : red >.

The following rules treat the arrival of new pedestrians. If the light is blinking or red (\(C =/= \text{green}\)), the pedestrian does not cross; if in addition the button is not lit, the pedestrian pushes the button, with the result that the button is lit and a pedsWaiting message is sent to the corresponding car light (rule newPedestrian1). If the light is green when the pedestrian arrives, (s)he just crosses the street, and if the button is lit, (s)he just joins the other waiting pedestrians. In neither of these cases does the new pedestrian cause any change in the resulting state (rule newPedestrian2):

crl [newPedestrian1]:
  newPed(pedStop(CN, DIR))
  < pedLight(CN, DIR) : PedLight | buttonLit : false, color : C >
  =>
    < pedLight(CN, DIR) : PedLight | buttonLit : true >
    (to carLight(CN, DIR) pedsWaiting)
    if (C = /= \text{green}).

crl [newPedestrian2]:
  newPed(pedStop(CN, DIR))
  < pedLight(CN, DIR) : PedLight | buttonLit : B, color : C >
  =>
    < pedLight(CN, DIR) : PedLight | >
    if (C == \text{green}) or B.

4.2.3 Cars

Most intersections have sensors that sense if a car is close to the intersection. For each direction, we model this sensor as an object of the class
class XingApproach | carsPresent : Bool.

whose carsPresent attribute is true if one or more cars are present close to the intersection. The treatment of new cars from the environment is thus modeled by the following rule:

\[\text{rl [newCars]} : \]
\[\text{newCars(CA)} \]
\[< \text{CA : XingApproach} | > \]
\[=\]
\[< \text{CA : XingApproach} | \text{carsPresent : true} > .\]

In addition, when the car light is green, all cars at the intersection may be able to cross the intersection. The following rule is nondeterministic in the sense that it is not triggered by the arrival of a message or by the expiration of a timer. It therefore may or may not be applied when enabled:

\[\text{rl [allCarsPass]} : \]
\[< \text{approach(CN, DIR) : XingApproach} | \text{carsPresent : true} > \]
\[< \text{carLight(CN, DIR) : CarLight} | \text{lights : green} > \]
\[=\]
\[< \text{approach(CN, DIR) : XingApproach} | \text{carsPresent : false} > \]
\[< \text{carLight(CN, DIR) : CarLight} | > .\]

4.2.4 Emergency Handling

The system handles an emergency by turning all lights red, after first turning a green light yellow to avoid further accidents. After the end of the emergency, the lights in the prioritized direction turn green, provided that some car or pedestrian wants to go in that direction. We present below two of the eight rules that model the emergency-related behavior of the car lights.

The following rule handles the arrival of an emergency signal when the car light is in a normal (i.e., non-emergency and non-failure) internal state. If the light is green, then it must first turn yellow before turning red after time yellowTime. However, if the light is already yellow, the remaining time that it stays yellow doesn’t change. In addition, an emergency message is sent to the corresponding pedestrian light, and the car light controller goes to internal state emergency:

\[\text{rl [newEmergency]} : \]
\[\text{emergency(CL)} \]
\[< \text{CL : CarLight} | \text{state : NORMAL}, \text{timer : T} > \]
\[=\]
\[< \text{CL : CarLight} | \text{state : emergency}, \text{timer : (if (NORMAL == green) then yellowTime else (if NORMAL == yellow then T else INF fi) fi)}, \text{lights : (if NORMAL == green or NORMAL == yellow then yellow else red fi)} > \]
\[\text{emergency(pl(CL))} . \quad \text{--- send emergency message to ped light}\]

The following rule models the arrival of an emergencyOver message. The car light in the prioritized direction (defaultStarter : true) is supposed to turn green, provided that cars are present in its direction in the intersection or that it has recorded that pedestrians are waiting (B1 or B2). The car light turns green, and tells the other car light to restart by showing red (reStartRed). In addition, the car light has to signal to its pedestrian light that the emergency is over by sending either a resumeGreen message or a resumeRed message, depending on whether or not pedestrians are waiting in this direction:
4.2.5 Device Failures

Any device may fail at any time. Upon a failure, a device must immediately signal to all the other devices in the crossing, so that the system begins with the failure treatment. We choose to deal with failures by letting the car lights in the prioritized direction blink yellow, by letting the car lights in the other direction blink red, and by turning off the pedestrian lights. The following new “colors” are therefore introduced:

ops blinkingYellow blinkingRed off : -> Color [ctor] .

Since any subset of the four devices in an intersection may fail at any time, each object must keep track of the number of temporarily failed devices. We therefore add a new constructor for keeping track of errors in the internal state of the controllers:

sort ErrorState . subsort ErrorState < CLState .
op error : NzNat -> ErrorState [ctor] .
op errorRecovery : -> CLState [ctor] .

We present below two of the thirteen rules that define the treatment of failures and failure recovery.

The following rule defines the treatment of an error message when the system is not already in an error state (not (S :: ErrorState); however, the controller may be in the emergency state). The light controller goes into state error(1), and if it is the light in the prioritized direction, it starts blinking yellow, otherwise it starts blinking red. If it is the device itself that has failed (the argument of the error message denotes the failed device), it must notify the three other devices about its failure:

crl [somethingBroken1] :
  (to CL error(DID))
  < CL : CarLight | defaultStarter : B, state : S >
=>
  < CL : CarLight | lights : (if B then blinkingYellow else blinkingRed fi),
  state : error(1),timer : INF >
--- If broken device was my device: send messages to other devices:
  (if CL == DID then (to opposite(CL) error(CL)) (to pl(CL) error(CL))
  (to pl(opposite(CL)) error(CL))
  else none fi)
  if not (S :: ErrorState) .

When the last failed device has been repaired, the lights go back to the normal state. However, to avoid inconsistent situations, in which a message from the environment may be read just before or after
the last repair, we have to enforce a regime where messages (other than error and repair messages) are ignored for a time Delta after the last repair.

In the rule lastDeviceFixed, the car light has recorded that one device is still broken (error(1)), and a repaired message arrives. The car light goes to the errorRecovery state and stays there for a short time Delta. If the car light is in the prioritized direction (defaultStarter is true), it turns green, otherwise red. (In this case, there is no check whether or not pedestrians are waiting, since in error mode, messages from the pedestrian lights are ignored.) As always, if the repaired device was this device (CL == DID), it has to notify the other devices about the repair:

\[
\text{rl [lastDeviceFixed] :} \\
\text{(to CL repaired(DID))} \\
\text{< CL : CarLight | state : error(1), defaultStarter : B >} \\
\text{=>} \\
\text{< CL : CarLight | state : errorRecovery, timer : Delta,} \\
\text{lights : (if B then green else red fi) >} \\
\text{(if CL == DID then (to opposite(CL) repaired(CL))} \\
\text{(to pl(CL) repaired(CL))} \\
\text{(to pl(opposite(CL)) repaired(CL))} \\
\text{else none fi)} .
\]

After time delta in errorRecovery mode, the system goes back to normal mode (not shown).

4.2.6 Modeling Environments

The following class PeriodicEnv is used to periodically generate nondeterministically a subset (which may be empty) of a set of messages:

\[
\text{class PeriodicEnv | frequency : NzTime, timeToNextEvents : TimeInf,} \\
\text{possibleEvents : NEMsgConfiguration .}
\]

The frequency attribute states at what interval the environment object should generate a subset of the set of messages in the possibleEvents attribute. (If frequency equals 1 and the time domain is discrete, then a subset is generated at any moment in time.)

The environment generating new cars and pedestrians is defined by the following rule, where the multiset MsgSET1 MsgSET2 is the total set of messages that can be generated. The rule generates the message set MsgSET1 when the timer expires. This set MsgSET1 could be any subset of MsgSET1 MsgSET2, since message multisets are defined with a multiset union operator that is associative and commutative and has null as its identity.

\[
\text{var E : Oid . vars MsgSET1 MsgSET2 : Configuration . var NZT : NzTime .}
\]

\[
\text{rl [generateSubsetAndReset] :} \\
\text{< E : PeriodicEnv | timeToNextEvents : 0 , frequency : NZT,} \\
\text{possibleEvents : MsgSET1 MsgSET2 >} \\
\text{=>} \\
\text{< E : PeriodicEnv | timeToNextEvents : NZT >} \\
\text{MsgSET1 .}
\]

The generator of emergency and emergencyOver messages is similar, with the difference being that emergency and emergencyOver messages should be generated in an alternating way, so that an emergencyOver message is generated when emergencyOn is true. The environment that generates failures and repairs of the devices is defined in a similar way, with the difference being that there should be some minimum time interval between the repair of a device and the next failure of the same device.
4.2.7 Time Advance Behavior

The time advance behavior of the system is defined as for most object-oriented Real-Time Maude specifications (see [8] and the executable Real-Time Maude model), and is not further explained here. The tick rule definition ensures that time advance stops when a timer expires, and that time does not advance if there are messages in the state. This forces all messages to be read without delay.

5 Analyzing the Model

This section shows how our model can be analyzed by using linear temporal logic (LTL), including metric LTL, model checking to analyze all possible behaviors from a given initial state.

5.1 Defining Initial States

In the spirit of the SoW, we have defined initial states that are parametric in the duration of the lights, the number of flawed devices, the possibility of emergencies, the duration between possible failures, etc.

First, we define the light objects in an intersection. The term \texttt{lights}(name, dir, greenTime, redTime) defines the car light and pedestrian light objects for an intersection called name, where direction dir is the prioritized direction, and where the green time and red time of this light is, respectively, greenTime and redTime:

\begin{verbatim}
eq lights(CN, DIR, GREENTIME, REDTIME) =
  < carLight(CN, DIR) : CarLight | lights : green, timer : GREENTIME, redTime : REDTIME,
  greenTime : GREENTIME, state : green,
  pedWaiting : false, defaultStarter : true >
  < carLight(CN, opposite(DIR)) : CarLight | lights : red,
  timer : (GREENTIME monus Delta),
  redTime : (GREENTIME + yellowTime +
  safetyMargin + safetyMargin),
  greenTime : (REDTIME monus (yellowTime +
  safetyMargin + safetyMargin)),
  state : red, pedWaiting : false,
  defaultStarter : false >
  < pedLight(CN, DIR) : PedLight | timer : INF, color : red, buttonLit : false, mode : normal >
  < pedLight(CN, opposite(DIR)) : PedLight | timer : INF, color : red,
  buttonLit : false, mode : normal >
  < approach(CN, NS) : XingApproach | carsPresent : false >
  < approach(CN, EW) : XingApproach | carsPresent : false > .
\end{verbatim}

The environment object that generates subsets of cars and pedestrians every NZTth time unit in a crossing CN is defined as follows:

\begin{verbatim}
op carsAndPeds : CrossingName NzTime -> Configuration .
eq carsAndPeds(CN, NZT) =
  < carsAndPeds(CN) : PeriodicEnv | frequency : NZT, timeToNextEvents : 0,
  possibleEvents : (newCars(approach(CN, NS))
  newCars(approach(CN, EW))
  newPed(pedStop(CN, NS))
  newPed(pedStop(CN, EW)) ) > .
\end{verbatim
Emergency generators and error generators can be defined in the same way.

A term \( \text{init}(\text{XING}, \text{GREENTIME}, \text{REDTIME}, T, \text{CARF}, \text{PEDF}, N_1, N_2) \) defines an initial state, with an intersection called \( \text{XING} \), where NS is the prioritized direction, and where the NS light has green time \( \text{GREENTIME} \) and red time \( \text{REDTIME} \). The time between each nondeterministic “choice” of whether or not to generate an emergency or emergencyOver message is denoted by \( T \) (if this number is 0, then no emergencies are generated). The number of potentially faulty car lights and pedestrian lights is denoted by, respectively, \( \text{CARF} \) and \( \text{PEDF} \). Finally, \( N_1 \) denotes how often errors may occur, and \( N_2 \) denotes the minimum time between the repair of a device and the next failure of that device:

\[
\text{eq init}(CN, \text{GREENTIME}, \text{REDTIME}, T, \text{CARF}, \text{PEDF}, N_1, N_2) = \\
\{ \text{lights}(CN, NS, \text{GREENTIME}, \text{REDTIME}) \\
\quad \text{carsAndPeds}(CN, 1) \\
\quad \text{if } T /= 0 \text{ then } \text{emergencyEnv}(CN, T) \text{ else } \text{none fi} \\
\quad \text{if } \text{CARF} == 2 \text{ then } \text{bothCarErrors}(CN, N_1, N_2) \\
\quad \text{else if } \text{CARF} == 1 \text{ then } \text{errors(carLight}(CN, NS), N_1, N_2) \text{ else } \text{none fi} \text{ fi} \\
\quad \text{if } \text{PEDF} == 2 \text{ then } \text{errors(pedLight}(CN, NS), N_1, N_2) \\
\quad \text{errors(pedLight}(CN, EW), N_1, N_2) \\
\quad \text{else if } \text{PEDF} == 1 \text{ then } \text{errors(pedLight}(CN, EW), N_1, N_2) \text{ else } \text{none fi} \text{ fi} \}.
\]

5.2 LTL and Metric LTL Model Checking

We can now use Real-Time Maude’s LTL model checker to analyze our model with respect to the requirements given in the SoW.

The first property to check is requirement (v) (“the system shall turn green a pedestrian light only when there are pedestrians waiting to cross in that direction”). In our model, the fact that a pedestrian is waiting is represented by the fact that the button on the pedestrian light pole is “lit.” The LTL formula in the following model checking command states that, for all states reachable from the initial state, and for both directions NS and EW, if the pedestrian light is red and the button is not lit (\( \text{pedLightRed}(EW) \land \neg \text{buttonPushed}(EW) \)), then the light remains red forever, or until a pedestrian arrives (\( \text{pedLightRed}(EW) \text{W pedArriving}(EW) \)). This property is checked by the following Real-Time Maude command in the presence of arbitrary emergency signals and with no failed devices:

\[
\langle \text{mc init}("Spitsbergen", \text{minGreenTime} + 2, \text{minRedTime}, 2, 0, 1, 1) \\
\quad \text{|=} \text{u} \\
\quad ((\text{pedLightRed}(EW) \land \text{buttonPushed}(EW)) => (\text{pedLightRed}(EW) \text{W pedArriving}(EW))) \\
\quad \And \\
\quad ((\text{pedLightRed}(NS) \land \text{buttonPushed}(NS)) => (\text{pedLightRed}(NS) \text{W pedArriving}(NS))) ..)
\]

The proposition \( \text{pedLightRed}(d) \) holds if the pedestrian light in direction \( d \) shows red:

\[
\text{op pedLightRed : Direction -> Prop [ctor]} .
\]
\[
\text{eq \{REST < pedLight(0, DIR) : PedLight | color : C >\} |=} \text{pedLightRed(DIR) = (C == \text{red})} .
\]

The proposition \( \text{pedArriving} \) is true if the state contains a message \( \text{newPed} \) for the given direction:

\[
\text{op pedArriving : Direction -> Prop [ctor]} .
\]
\[
\text{eq \{REST \text{newPed(pedStop("Spitsbergen", DIR))}\} |=} \text{pedArriving(DIR) = true} .
\]

\(^3\text{Remember that } \phi => \theta \text{ is an abbreviation for } [\langle \cdot \rangle]_\phi(\theta).\)
The proposition \texttt{buttonPushed} can be defined in a similar way. The model checking command above returned the expected result \texttt{true} after executing for 212 seconds on a server.

The next property we check is a modification of requirement (vi) in the SoW, namely, that the system should turn a vehicular light green only if there are vehicles or pedestrians waiting to go in that direction. The following formula, again checked in the presence of emergencies, says that if the NS car light is red, and no pedestrians or cars are waiting, then this light will be red until a pedestrian or a car arrives:

\[
\text{(mc init("Spitsbergen", minGreenTime + 2, minRedTime, 2, 0, 0, 1, 1)}
\]

\[
\quad |u \quad
\quad \text{(carLightRed(NS) /\ ~ buttonPushed(NS) /\ ~ carWaiting(NS))}
\]

\[
\quad => \quad \text{(carLightRed(NS) W (pedArriving(NS) /\ carArriving(NS))) .)}
\]

We refer to the Real-Time Maude specification for the definition of the propositions in this formula. The result of executing the command above is \texttt{true}, and the execution took 153 seconds.

Another requirement of the system is that “The system shall be fault-tolerant.” We have defined a set of properties, including some of the properties described below, which together demonstrate that the system is fault-tolerant. The specific liveness property we map to this item says that if each failure is eventually repaired (\(\[ \] \text{(failure -> <> repair)}\)), then cars in the NS direction will be able to pass infinitely often (\(\[ <> \] \text{carLightGreen(NS)}\)) if cars arrive in this direction infinitely often. This property is checked with one faulty device and where the smallest time interval between a repair and another failure is 9:

\[
\text{(mc init("Spitsbergen", minGreenTime + 2, minRedTime, 0, 1, 0, 2, 9)}
\]

\[
\quad |u \quad
\quad \text{(([] <> carArriving(NS)) -> ([] <> carLightGreen(NS))) .)}
\]

The executions of this command returned \texttt{true} in 148 seconds.

The SoW requires that “The system shall be fail-safe, ensuring both failure recovery, and safe car and pedestrian conditions also under failure.” We check whether cars can collide with pedestrians in the presence of one faulty device (and no emergencies):

\[
\text{(mc init("Spitsbergen", minGreenTime + 2, minRedTime, 0, 1, 0, 2, 9)}
\]

\[
\quad |u \quad
\quad \text{([] (~ (walking(NS) \/ driving(EW))) /\ (~ (walking(EW) \/ driving(NS))) .)}
\]

The propositions \texttt{walking} and \texttt{driving} are again defined in a straightforward way:

\[
\text{ops walking driving : Direction -> Prop [ctor] .}
\]

\[
\text{eq \{REST < pedLight(O, DIR) : PedLight | color : C >\} |= walking(DIR) = (C == green or C == blinking) .}
\]

\[
\text{eq \{REST < carLight(O, DIR) : CarLight | lights : green >\} |= driving(DIR) = true .}
\]
Model checking this crucial safety property resulted in the answer true. We have also checked the crucial property that it is not possible for cars to legally drive in both directions at the same time.

One QoS requirement in the SoW states that no pedestrian should wait for more than five minutes. This requirement is analyzed with the following bounded response command, that states that each time a pedestrian arrives in the NS direction, (s)he will be walking within 15 time units, in a setting without failures or emergencies (since there is no upper limit on the duration of an emergency or a failure of a device):

\[
\text{br init("Spitsbergen", minGreenTime + 2, minRedTime, 0, 0, 1, 1, false, 0) = pedArriving(NS) => <>le(15) walking(NS).}
\]

This command returned true in 168 seconds. Executing the command for 14 time units returned a counterexample, so for the given parameter values, 15 time units is the best time guarantee we can give.

6 Conclusions

We have presented a highly decentralized and modular parametrized Real-Time Maude model of a four-way traffic intersection, in which autonomous devices communicate by asynchronous message passing without a centralized controller. All the safety requirements and a liveness requirement informally specified in the SoW requirements document have been formally verified. We believe that this work shows how formal specification and verification can be inserted within a design environment for DES product families. And that this can be done at the stage where this matters most, namely, at the design stage, since design errors are much more costly than coding errors. In particular, our modeling and formal analysis have allowed us to both obtain a fully verified model with respect to the required safety properties, and to identify nontrivial inconsistencies in the requirements document.

This positive experience should not be exaggerated and should be treated with caution. Scalability is still a very serious challenge for DES verification. We feel that in this case study we were at the limit of the kind of distributed system complexity that can be directly verified with Real-Time Maude, which uses a state-of-the-art explicit-state model checker. However, some recent methods look promising to verify properties of more complex DES system designs either indirectly or under statistical guarantees, including: (i) more aggressive uses of abstraction methods such as the PALS methodology [7,10,11,5,6], which can greatly decrease system complexity and verification cost by reducing the verification of asynchronous real-time systems to that of a much simpler synchronous version under some conditions; and (ii) the use of more scalable statistical model checking [11,9] techniques that trade off full verification of system properties by statistical guarantees on the satisfaction of such properties when full model checking verification becomes unfeasible.

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