Purification and time-reversal deny entanglement in LOCC-distinguishable orthonormal bases

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We give a simple proof, based on time-reversibility and purity, that a complete orthonormal family of pure states which can be perfectly distinguished by LOCC cannot contain any entangled state. Our results are really about the shape of certain states and processes, and are valid in arbitrary categorical probabilistic theories with time-reversal. From the point of view of the resource theory of entanglement, our results can be interpreted to say that free processes can distinguish between the states in a complete orthonormal family only when the states themselves are all free.

1 Introduction

Quantum theory allows perfect deterministic distinguishability of orthogonal states: for example, there is a POVM (positive operator valued measure) which, given a state from the orthogonal set \( \{ |00\rangle, |11\rangle, |01\rangle + |10\rangle, |01\rangle - |10\rangle \} \) as its input, returns a classical output in the set \( \{ 0, 1, +, - \} \) uniquely identifying the state. When the system under consideration is multipartite (e.g. the bipartite system above), the POVM will in general contain non-separable effects: it is therefore interesting to investigate which limitations are imposed on distinguishability tasks by the requirement that quantum operations be local to the parties involved, but allowing arbitrary amounts of classical communication between the parties, which can inform the classical control of the local quantum operations (i.e. we wish to consider LOCC scenarios).

Investigation on the power of LOCC scenarios was initiated by Peres and Wooters, leading to the development of their much-celebrated teleportation protocol (cf. Peres (2004)). However, it could be argued that the cornerstone result which truly boosted the field was the proof that not all sets of orthogonal product states can be distinguished by LOCC scenarios (cf. Bennett et al. (1999)). This is a very counter-intuitive result, in a sense, as it proves that there is an orthonormal basis of 9 bipartite qutrit states which can be “prepared locally”—as it only consists of product states—but which cannot be distinguished by LOCC.

The issue of LOCC distinguishability has been studied in more general settings throughout the years, with more LOCC-indistinguishable orthonormal families explicitly constructed (cf. Zhang et al. (2016)), and all bipartite qutrit and tripartite qubit families characterised (cf. Feng and Shi (2009); Walgate and Hardy (2002)). It is also known that it is always possible to distinguish between two orthogonal states with certainty by LOCC.
(cf. Walgate et al. (2000)), but that it is generally impossible to distinguish with certainty between three or more states (cf. Ghosh et al. (2001, 2002)). Further research includes connections with the theory of entanglement witnesses (cf. Chefles (2004)), cryptographic applications such as secret sharing and data hiding (cf. DiVincenzo et al. (2003); Hillery and Mii (2003); Lancien and Winter (2012)), as well as a number of alternative approaches to the problem (e.g. cf. Childs et al. (2013); Chitambar et al. (2014a,b); Duan et al. (2009); Roy Moulik and Panigrahi (2016)).

The question of which sets of orthogonal quantum states can be perfectly distinguished by LOCC protocols remains open Chitambar et al. (2014b), as does the quest for the understanding of the physical principles playing a role in this problem. In this work, we focus our efforts on the role played by the physical and informational principles of purity and time-reversal: harnessing their power, we provide a simple alternative proof of a key result by Horodecki et al. (2003), which we furthermore generalise beyond quantum theory. We show that if it is possible to distinguish between the pure states of an orthonormal basis by LOCC alone then the basis can only contain product states. Our results can also be recast from the point of view of resource theory (cf. Coecke et al. (2016)), and specifically of the resource theory of entanglement by Chiribella and Scandolo (2015): within that framework, our result says that free operations cannot distinguish between the states in an orthonormal basis unless the states themselves are all free.

Our proof is carried out in the formalism of string diagrams for categorical quantum mechanics (cf. Abramsky and Coecke (2004, 2009); Backens (2014); Coecke (2009); Coecke and Duncan (2011); Coecke and Kissinger (2015, 2016, 2017); de Beaudrap and Horsman (2017); Horsman (2011)), and more precisely within the recently introduced framework of categorical probabilistic theories (CPTs) by Gogioso and Scandolo (2017). As a consequence, our result extends beyond quantum theory, to any probabilistic theory with time-reversal. The use of diagrammatic methods also highlights an important feature of the result which did not stand out in the original proof of Horodecki et al. (2003): LOCC distinguishability is really a story about the “shape” of certain states and processes, and more specifically a story about how processes of “LOCC shape” fail at distinguishing between states which are not of “LOCC shape”.

We have chosen to use categorical probabilistic theories, rather than the more established operational probabilistic theories (OPTs, cf. Chiribella (2014); Chiribella et al. (2010, 2011, 2016); D’Ariano et al. (2017); Hardy (2001, 2011a,b, 2016)), for two main reasons. Firstly, the categorical framework is process-oriented, rather than probability-oriented: in CPTs, classical control and classical outcomes can be talked about in an explicitly compositional way, and are naturally related by time-reversal; in OPTs, on the other hand, classical systems are introduced externally using indices, in a non-compositional way. Secondly, the categorical framework provides us with additional flexibility when decomposing “normalised” processes (e.g. CPTP maps and quantum instruments) in terms of “unnormalised” building blocks (e.g. generic CP maps and families thereof), especially when it comes to dealing with the time-reversal of the building blocks themselves (which might not be individually sub-normalised); conversely, OPTs impose very specific requirements on the building blocks allowed in processes, introducing the need for unnecessary additional checks and restrictions.

2 Categorical probabilistic theories

Categorical Probabilistic Theories. When talking about a categorical theory, we mean a symmetric monoidal category which captures physical systems and the composi-
tional structure of processes between them. In the general spirit of categorical quantum mechanics, we avoid restricting our attention to physical processes alone, but instead we allow the presence of a whole spectrum of idealised, abstract processes that provide building blocks for the physical processes themselves, or otherwise help in reasoning about them. When talking about a categorical probabilistic theory (CPT), we mean a symmetric monoidal category which includes at least the classical systems amongst its ranks, and which is furthermore compatible with a couple of basic operational features—namely probabilistic structure and marginalisation. We briefly motivate our requirements below.

1. Every probabilistic theory has classical probabilistic systems under the hood: because these are themselves physical systems, we model them explicitly. In particular, their interface with other systems (e.g. measurements and preparations) can be talked about in compositional terms.

2. It makes no sense to talk about a probabilistic theory if the probabilistic structure does not extend from classical systems to arbitrary systems. If this were not the case, one would not necessarily be able to work with scenarios in which multiplexed processes are controlled by a classical random variable, or to condition a process based on a classical output.

3. Every probabilistic theory with operational aspirations should include a notion of localisation of states and processes. Indeed, the absence of a notion of local state compatible with classical marginalisation renders most protocol specifications meaningless, a fate they share with the notion of no-signalling and with the probabilistic foundations of thermodynamics. Sensible theories without a notion of local state do exist (e.g. Everettian quantum theory), but they present a number of operational challenges, and they are not probabilistic in nature.

We now proceed to formalise these requirements in categorical terms. When talking about classical theory, we mean the symmetric monoidal category $\mathbb{R}^+\text{-Mat}$ which has finite sets $X$ as systems, and where processes $X \to Y$ are the $Y$-by-$X$ matrices with entries in the non-negative reals $\mathbb{R}^+$. Processes $X \to Y$ in classical theory form a convex cone, i.e. they are closed under $\mathbb{R}^+$-linear combinations. Because we have explicit linear structure, we are allowed to write the following resolution of the identity:

$$X \quad = \quad \sum_{x \in X} X \otimes [x] \otimes [x] \otimes X$$

Classical marginalisation gives rise to a family of effects $(\mathbb{1}_{\cdot X} : X \to \mathbb{1})_{X \in \text{obj} \mathbb{R}^+\text{-Mat}}$ which canonically localises states and processes, and are therefore known as the discarding maps:

$$X \quad = \quad \sum_{x \in X} X \otimes [x]$$

In the string diagrams literature this is often known as an environment structure Coecke and Kissinger (2016); Coecke and Lal (2013), because it satisfies the following equations:

$$X \otimes Y \quad = \quad X \otimes [1] \otimes [1] \quad = \quad [1] \otimes [1] \otimes [1]$$

The empty diagram on the right hand side of the right equation is simply the scalar 1. Any choice of environment structure singles out a sub-category of normalised processes, namely those processes $f$ satisfying the following condition:

$$\quad = \quad \quad \quad$$
In classical theory, the normalised processes are exactly the stochastic matrices, and in particular the normalised states are the probability distributions.

**Definition 1 (Categorical Probabilistic Theories (Gogioso and Scandolo (2017))).**

A categorical probabilistic theory is a symmetric monoidal category $\mathcal{C}$ which satisfies the following three requirements:

1. There is a chosen full subcategory of $\mathcal{C}$, denoted by $\mathcal{C}_K$, together with a chosen $\mathbb{R}^+$-linear monoidal equivalence between $\mathcal{C}_K$ and the category $\mathbb{R}^+$-$\text{Mat}$.

2. The category $\mathcal{C}$ has $\mathbb{R}^+$-linear structure compatible with that which the chosen subcategory $\mathcal{C}_K$ inherits from $\mathbb{R}^+$-$\text{Mat}$.

3. The category $\mathcal{C}$ has a chosen environment structure $(\mathcal{H} : \mathcal{H} \to \mathbb{1})_{\mathcal{H} \in \text{obj } \mathcal{C}}$ compatible with that which the chosen subcategory $\mathcal{C}_K$ inherits from $\mathbb{R}^+$-$\text{Mat}$.

In the context of a specific CPT, we refer to $\mathcal{C}_K$ as the classical theory, to its object as the classical systems and to its morphisms as the classical processes. In particular, $\mathbb{1}$ is the tensor unit for $\mathcal{C}$, and the scalars of $\mathcal{C}$ form the probabilistic semiring $\mathbb{R}^+$. A generic process in a CPT can involve both classical systems and more general systems:

\[
\begin{array}{c}
\text{generic input} \\
\downarrow \quad \downarrow \quad \downarrow \\
M \\
\uparrow \quad \uparrow \quad \uparrow \\
\text{classical input} \\
\end{array}
\]

\[
\begin{array}{c}
\text{generic output} \\
\downarrow \quad \downarrow \quad \downarrow \\
\text{classical output} \\
\end{array}
\]

In line with the nomenclature adopted for classical systems, we refer to the $\mathcal{H}$ maps involved in the environment structure as the discarding maps, and to those processes $M$ satisfying the following equation as normalised:

\[
\begin{align*}
\ldots \overrightarrow{M} \ldots & \quad \overrightarrow{\mathbb{1}} \\
\overleftarrow{\mathbb{1}} & \quad \overleftarrow{M} \\
\ldots \overleftarrow{M} \ldots & \quad \overleftarrow{\mathbb{1}} \\
\end{align*}
\]

**LOCC instruments.** This work is concerned with a very special class of processes which can be captured by the CPT framework, namely that of LOCC instruments. In its most generic form, an LOCC instrument is a process taking the following shape:

\[
\begin{array}{c}
\text{Global classical operations} \\
\hline \\
\uparrow \quad \uparrow \quad \uparrow \\
\cdots \quad \cdots \quad \cdots \\
\downarrow \quad \downarrow \quad \downarrow \\
M \\
\cdots \\
\uparrow \quad \uparrow \quad \uparrow \\
\text{Local instruments} \\
\hline \\
\end{array}
\]

\[
\begin{align*}
\sum_{x} \sum_{x'} p_{x'x} & \quad q_{y'yt} \\
\ldots & \quad \ldots \\
\sum_{y} \sum_{y'} q_{y'y} & \quad \sum_{y} \sum_{y'} q_{y'y} \\
\end{align*}
\]

A product family $M_1 \otimes \cdots \otimes M_N$ of processes—the local instruments—is sandwiched between two global classical processes $\sum_{x} \sum_{x'} p_{x'x} |x_1...x_N\rangle \langle x'...x'|$ and $\sum_{y} \sum_{y'} q_{y'y} |y_1...y_N\rangle \langle y_1...y_N|$—the global classical operations—which we don’t label explicitly for reasons of notational convenience. The global classical processes are allowed to act on the classical inputs and classical outputs of the local instruments, but leave all other inputs and outputs invariant—a fact which we denote by drawing the non-classical wires “passing overhead”. Note that
using global classical operations is equivalent to allowing classical communication between the parties: we can always implement one such global classical operation by allowing all parties to communicate their respective classical inputs to a distinguished party, who then performs the operation and sends the classical outputs back to the respective parties.

**Purity.** Purity is a feature arising at the interface between quantum theory and thermodynamics Chiribella and Scandolo (2015); Chiribella et al. (2010): pure processes can broadly be interpreted as not involving any probabilistic mixing due to non-trivial interactions with a discarded environment. To be precise, a process is pure if it cannot be decomposed in any way as a non-trivial $\mathbb{R}^+$-linear combination of other processes:

$$M' = \sum p_i M_i \Rightarrow M = \sum p_i M_i$$ (8)

It should be noted that purity is a notion that applies to both normalised and unnormalised processes: it is simply a statement of extremality in the convex cone of processes from a fixed input system to a fixed output system. When the process is normalised, the $\mathbb{R}^+$-valued coefficients will sum to 1, and $\mathbb{R}^+$-linear combinations reduce to the usual notion of probabilistic mixtures.

**Time-reversal.** From a compositional perspective, the action of a generic notion of time-reversal can be described as some way of sending processes to other processes which have inputs and outputs swapped, while at the same time respecting their sequential/parallel compositional structure and their probabilistic structure. Categorically, this is captured by asking that time-reversal is a contravariant $\mathbb{R}^+$-linear monoidal functor on the CPT, i.e. a $\mathbb{R}^+$-linear dagger functor (in string diagram language) which coincides with the transpose on classical processes. Extremely relevant to this work is the duality established by any such notion of time-reversal between normalised processes and unital processes. In a CPT with time-reversal (as described above), a unital process is one satisfying the following equation (where $M^\dagger$ is the time-reverse of $M$):

$$M = \sum p_i M_i$$ (9)

The reversed discarding maps are (up to proportionality factor) the uniform probability distribution (on classical systems) and the maximally mixed state (on generic systems). The relationship between normalised and unital processes established by time-reversal can then be summarised as follows:

$$\cdots M \cdots = \cdots M^\dagger \cdots \quad \Leftrightarrow \quad \cdots M \cdots = \cdots M^\dagger \cdots$$ (10)

In particular, the normalised processes which are invariant under time-reversal are exactly those which are both normalised and unital, such as the RaRe introduced by Chiribella and Scandolo (2015).

3 Main result

**The distinguishing task.** Consider the following game between $N$ players, each player $j$ being in possession of a finite-dimensional quantum system $\mathcal{H}_j$. The players share a pure quantum state $\psi_b$, which they are guaranteed to be drawn at random from an orthonormal
basis \((\psi_b)_{b \in B}\) of the joint quantum system \(\bigotimes_{j=1}^N H_j\). Here \(B\) is a finite set of labels, with cardinality \(|B| = \prod_{j=1}^N \dim H_j\). They are tasked to identify the state with certainty using only local operations and classical communication. Our main result will state that the presence of even a single entangled state in the basis makes the task certainly impossible. Note that the task might be impossible even when the basis states are product states, cf. Bennett et al. (1999).

**Theorem 2** *(Time-reversal and purity deny entanglement).* If the players can succeed in the state distinguishing task by using any LOCC protocol (possibly involving unnormalised instruments/operations), then the complete orthonormal family \((\psi_b)_{b \in B}\) cannot contain any entangled states. Conversely, if the complete orthonormal family \((\psi_b)_{b \in B}\) contains only product states, then the players can succeed in the state distinguishing task using a unital LOCC protocol, although the latter might involve unnormalised instruments/operations.

**Proof.** We begin by considering a generic LOCC protocol (possibly involving unnormalised instruments/operations) that the player might be using to accomplish their distinguishing task. One such protocol would include some number \(R > 0\) of rounds, each round \(r\) consisting of a global classical operation shared amongst the parties, followed by local instruments \(M_i^{(r)}\) performed by the individual parties \(i = 1, \ldots, N\). For the first round, we just assume this to be a shared global classical state, with no inputs. After the last round, a global post-processing operation is applied to the classical outputs of the local instruments, producing a classical output in the set \(B\) which identifies the state that the players believe they were given. Overall, the protocol takes the following shape:

\[
\begin{array}{c}
\text{Shared global classical state} \\
\mathcal{H}_1 \\
\vdots \\
\mathcal{H}_{N-1} \\
\mathcal{H}_N
\end{array}
\begin{array}{c}
\text{Global classical operations} \\
M_1^{(1)} \\
\vdots \\
M_{N-1}^{(1)} \\
M_N^{(1)}
\end{array}
\begin{array}{c}
\text{Local instruments} \\
M_1^{(2)} \\
\vdots \\
M_{N-1}^{(2)} \\
M_N^{(2)}
\end{array}
\begin{array}{c}
\text{Global classical operations} \\
M_1^{(R)} \\
\vdots \\
M_{N-1}^{(R)} \\
M_N^{(R)}
\end{array}
B
\]

Diagram 12 above shows the situation in which the players apply their \(R\)-round LOCC protocol to the initial state \(\psi_b\): the correctness requirement for the protocol is captured

\[
\psi_b \\
\vdots \\
M_1^{(1)} \\
\vdots \\
M_{N-1}^{(1)} \\
M_N^{(1)}
\begin{array}{c}
\text{Local instruments} \\
M_1^{(2)} \\
\vdots \\
M_{N-1}^{(2)} \\
M_N^{(2)}
\end{array}
\begin{array}{c}
\text{Global classical operations} \\
M_1^{(R)} \\
\vdots \\
M_{N-1}^{(R)} \\
M_N^{(R)}
\end{array}
B = \{ b \} \rightarrow B \]
by the fact that this process deterministically results in the correct label \( b \in B \). Now we consider the time-reversal of the entire protocol used by the players, i.e. we take the dagger of all global classical operations (which are simply transposed) and local instruments (which are transformed in some other way, depending on the exact choice of dagger functor implementing the time-reversal):

Note that the time-reversed LOCC protocol of Diagram 13 need not be normalised, even when the original LOCC protocol of Diagram 11 is normalised (in the latter case, however, the time-reversed protocol is certainly unital). Under time-reversal, the protocol success condition of Equation 12 turns into a state preparation condition:

By repeatedly invoking Equation 7, i.e. by inserting classical resolutions of the identity on all classical wires, the LHS of Equation 14 turns into a mixture of product states. But RHS of Equation 14 is a pure state, and by the very definition of purity we have that the mixture on the LHS is necessarily trivial. We conclude that \( \psi_b \) must be a pure product state, for each choice of \( b \in B \): under the assumption that the players can deterministically distinguish all states in the basis, we have shown that the basis cannot contain any entangled states.

Conversely, assume that the family \( (\psi_b)_{b \in B} \) contains only product states, and write \( \psi_b = \otimes_{i=1}^N \psi_{b,i} \). Without loss of generality, assume that all the local states \( \psi_{b,i} \) are pure (because \( \psi_b \) is) and normalised (because \( \psi_b \) is normalised, and hence all \( \psi_{b,i} \) must be normalisable). Then the family can be prepared by using a normalised LOCC protocol as follows:

The time-reversal of the LOCC protocol described by Diagram 15 is a unital LOCC protocol which implements the state distinguishing task for the family \( (\psi_b)_{b \in B} \). The time-reversed
LOCC protocol is normalised (resp. unital) if and only if the original protocol described by Diagram 15 is unital (resp. normalised).

Finally, we can put these results together. If the players can perfectly distinguish between the states of a complete orthonormal family by a normalised unital LOCC protocol, then the family can be prepared by the time-reversed LOCC protocol, which is unital and normalised, and hence it cannot contain any entangled states. Vice versa, if the states in a complete orthonormal family can be perfectly prepared by a normalised unital LOCC protocol, then the family does not contain any product states and it can furthermore be distinguished by the time-reversed LOCC protocol, which is unital and normalised.

4 Discussion

We have provided a simple argument showing that any complete orthonormal family of multipartite pure states which can be perfectly distinguished by LOCC protocols cannot contain any entangled states. Our proof is diagrammatic and theory-independent, and straightforwardly applies to both quantum theory and any post-quantum theory which can be modelled by our categorical framework. A number of well-established results in LOCC-distinguishability arise as a corollary of our work: for example, the fact that the four two-qubit Bell states are not LOCC-distinguishable Ghosh et al. (2001), or the fact that the four two-qubit states \{\ket{00}, \ket{11}, \ket{01} + \ket{10}, \ket{01} − \ket{10}\} are not LOCC-distinguishable Ghosh et al. (2002). Contrary to the majority of previous results, our treatment is independent of the number of parties and of the dimension of quantum systems involved.

Our proof shows that LOCC-distinguishability of complete orthonormal families is really about purity and time-reversal, bearing no relation to normalisation and causality. In this sense, it is a story about the shape of processes, rather than their inner workings. The result also finds a particularly fitting interpretation in the resource theory of entanglement: free processes can distinguish between states of a complete orthonormal family only when the states themselves are all free.

Future work will focus on extending our theory-independent diagrammatic approach to more general problems of interest in LOCC-distinguishability and the resource theory of entanglement. For example, we propose to investigate the complete orthonormal family of Bennett et al. (1999) from a process shape perspective, hopefully shedding further light on an otherwise counter-intuitive result.

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