QCD Sum Rules at High Temperature

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We generalize the sum rule approach to investigate the nonperturbative structure of QCD at high temperature. Salient features of the QCD phase above \( T_c \) are discussed, and included in the form of power corrections or condensate insertions, in an operator product expansion of gauge invariant correlators. It is shown that for a plausible choice of condensates, QCD at high temperature exhibits color singlet excitations in the vector channels, as opposed to merely screened quarks and gluons.

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I. INTRODUCTION

The aim of this paper is to merge recent ideas about the nature of the high temperature phase in QCD, with the techniques of QCD sum rules, to obtain information about real time correlation functions at finite temperature. We should emphasize that these response functions can be calculated neither in straightforward perturbation theory (because of infrared divergences) nor by lattice Monte-Carlo techniques (which only provides equilibrium quantities at finite \( T \)). That pure Yang-Mills theories exhibit a phase transition from a low temperature confined phase to a high temperature unconfined one, is by now well established by extensive Monte-Carlo simulations (for a recent review see e.g. [1]). When light, dynamical quarks are included, as in QCD, the situation is more controversial, but it has been rather convincingly demonstrated that chiral symmetry is restored at high \( T \) [1]. Although the high \( T \) phase is characterized by a coupling constant \( g(T) \) which is small because of asymptotic freedom, perturbation theory is still plagued by infrared divergences. The simplest way to understand the origin of these singularities is by noting that at high \( T \) the 4-dimensional pure YM-theory reduces to to a 3-dimensional gauge-Higgs model, where the time component of the gauge potential plays the role of an adjoint Higgs field [2,3,4]. The Higgs theory is super-renormalizable and confining with a non-perturbative mass gap proportional to the (dimensionful) coupling constant \( g^2 T \).

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In the original 4-dimensional theory this mass gap corresponds to a non-perturbatively generated magnetic mass. However, even if one introduces a magnetic mass \( g^2 T \) into the 4-dimensional theory, the range of validity of perturbation theory remains limited [5,3].

In addition to the proper gauge transformations, i.e. those periodic on the imaginary time interval \([0, \beta]\), the action is also invariant under a class of space-independent, non-periodic gauge transformations of the type: \( \Omega(x_4 = i \beta, \vec{x}) = Z \Omega(x_4 = 0, \vec{x}) \), where \( Z \) is an element of the center of the gauge group. The order parameter used to establish the finite \( T \) phase transition in pure YM theories is the Polyakov loop (or Wilson line), \( \text{Tr} W(\vec{x}) = \text{Tr} \text{Pexp} \int_0^{\beta} A_4(\vec{x}, x_4) \), which is invariant under proper gauge transformation, but transforms non-trivially under the center symmetry. In the low temperature confining phase the center symmetry is unbroken, and \( \langle \text{Tr} W \rangle_\beta = 0 \). Above the deconfining phase transition, the center symmetry is broken and the Polyakov loop develops a non-zero expectation value. (For a discussion of the universality classes related to phase transitions in gauge theories we refer to the review article by Svetetsky [6].)

While lattice calculations of the Polyakov loop have clearly established the breaking of the center symmetry above \( T_c \), it is still not clear whether or not the (global) gauge symmetry is also broken. Since any order parameter for this symmetry necessarily vanishes on the lattice, due to Elizur’s theorem, this issue is not so easy to settle. There are, however, several calculations and arguments in the literature which indicate that the global gauge symmetry is indeed spontaneously broken and that \( A_4 \) develops a non-zero expectation value. Since this idea will be very important in our derivation of the finite \( T \) sum rules, we shall give a brief review of the present situation in the next section. Before that, however, we will explain the main ideas of our approach.

Ever since their introduction in the late seventies, the QCD sum rules have been important tools in QCD based phenomenology\(^1\). The basic idea of Shifman, Vainshtein and Zakharov (SVZ) is to extract information about resonance masses, widths, and coupling strengths by connecting them via dispersion relations to various current-current correlation functions in the Euclidian region. These correlation functions are then calculated using a short distance operator product expansion (OPE). The leading term in this expansion corresponds to the usual perturbative contribution, which, because of asymptotic freedom, is well behaved for large enough Euclidian momenta \((Q^2)\). The non-leading, or ”condensate”, terms are present because of the non-perturbative nature of the QCD ground state which implies that e.g. \( \langle \langle 0 | G_{\mu \nu}^a G_{\mu \nu}^a | 0 \rangle \rangle \neq 0 \). Since the condensates are dimensionful, the corresponding contributions to the correlation functions are suppressed by powers of \( Q^2 \). What makes the QCD sum rules phenomenological rather than fundamental, is that a certain amount of arbitrariness goes into the procedure of matching the OPE calculation to the observable singularity structure in the physical region. First, it is necessary to assume a certain number of resonances and thresholds, and weight the sum

\(^1\) For a comprehensive list of references to the early sum rule literature, see [7].
rules so that they give sizable contribution to the dispersion integral. Second, since only few terms in the OPE can be calculated some criteria of "perturbative dominance" (i.e. that the condensate terms are small) must be imposed. In spite of these difficulties, the QCD sum rules have been quite successful in describing a wide range of low $Q^2$ hadronic physics.

It is clearly a challenge to extend the applicability of the QCD sum rules to systems with finite temperature and density, and there have already been several papers on this subject [8,9,10,11]. The main idea has been to calculate the coefficient functions in finite $T$ perturbation theory using the same Feynman graphs as at zero temperature, and then write a sum rule for the retarded commutator of currents. Using this method, and assuming the condensate contributions to be essentially constant, Bochkarev and Shaposhnikov studied the temperature dependence of the $\rho$ mass. In their analysis the $\rho$ resonance disappeared at $T \sim 140 \text{MeV}$, which they interpreted as a signal for the finite $T$ deconfinement phase transition. This conclusion was later challenged by Dosch and Narrison [10] and also by Furnstahl, Hatsuda and Lee [11]. All these papers use the same basic idea, and the discrepancies between them is due to the use of different "sum rule technology". This emphasizes the point made above about the phenomenological nature of the QCD sum rules - the same basic strategy can lead to quite different conclusions. In all these works the sum rules were applied at temperatures around and below the transition temperature. While we will mainly consider the high temperature region, we will also comment on the use of the sum rules at low temperature. Similar comments can also be found in a recent investigation by Adami, Hatsuda and Zahed who critically analyse the breakdown of the sum rule approach at low temperature [12].

In this paper we discuss certain basic questions which are of importance for the understanding of the finite temperature sum rules. Recall that in the usual QCD sum rules the OPE provides a separation of scales in that the coefficient functions which are sensitive to the large external $Q^2$ are assumed to be calculable in perturbation theory, while the condensates, which are sensitive to long wavelengths (i.e. $1/\Lambda_{QCD}$) vacuum fluctuations, must be taken as input parameters. At finite temperature the picture is much more complicated due to the new scale $T$. Another, and as we shall see, related question is the very definition of the condensates at finite temperature. As discussed by SVZ, there is an implicit renormalization point dependence in the condensates, and an important assumption in using the QCD sum rules is that this can be ignored [13]. At temperatures close to and above the transition temperature, the structure of the QCD vacuum changes drastically (deconfinement, chiral symmetry restoration) and it is not clear how the condensates compare with those at zero temperature. It will turn out that the mechanism for deconfinement is quite important. Specifically, if we assume, as mentioned above, that the gauge symmetry is spontaneously broken there will be important new contributions to the sum rules from the "condensate" $\langle A^2 \rangle$.

We shall discuss these questions and offer as a tentative conclusion that the sum rules
are not applicable around $T_c$, but that they might be at much higher temperatures, provided a suitable resummation is performed. We also argue that the sum rules can be applied at low temperature ($T/T_c \ll 1$), but that this will necessarily involve the determination of the $T$ dependence of the condensates (confere [12]).

The possibility of using modified QCD sum rules at temperatures well above the deconfining phase transition is quite interesting since it could provide a probe of the non-perturbative dynamics of the QCD plasma. In particular it gives us a possibility to test DeTar’s conjecture concerning dynamical confinement in the plasma [14]. The later part of this paper, where we investigate a QCD sum rule for the vector channel well above the critical temperature, will be devoted to this problem.

The paper is organized as follows. In the next section we briefly summarize various results that are of relevance to the non-perturbative structure of high temperature QCD with a special emphasis on gauge symmetry breaking. In section 3 we discuss the nature of the OPE at finite temperature. In particular we show how calculating coefficient functions using finite temperature Feynman rules amounts to summing contribution from an infinite set of higher dimensional operators. We also discuss the role played by various mass scales in determining the applicability of the sum rules. In section 4 we formulate the finite temperature sum rules for the p meson channel and in section 5 we discuss the results. Our conclusions are summarized in section 6. Some technical material related to the calculation of coefficient functions and Sorel transforms is collected in two appendices.

II. THE SYMMETRY OF QCD ABOVE $T_c$

In this section, we review the evidence for having a non vanishing expectation value for $A^4_4(\vec{x})$ at finite temperature. First recall that at finite temperature a constant $A^4_4$ has physical significance, since in general it cannot be transformed away by a proper gauge transformation. A more intuitive way to understand the same thing is to realize that a constant $A^4_4$ enters the theory just like an imaginary chemical potential. A two loop calculation of the partition function for pure SU(2) YM theory in a constant background color electric potential, was first performed by Anishetty [15], who noticed that the free energy is minimized for a non zero value $gT$. Such a constant background potential improves the infrared behaviour of the theory, since it generates a mass for those gluons which are orthogonal to it in color charge space. The remaining massless gluons will however generate IR divergences in perturbation theory. Using dimensional reduction to a 3 dimensional gauge-Higgs model as referred to above, Dahlem later argued that the massless modes get a magnetic mass of similar magnitude [16]. His argument was based on Polyakov’s analysis of 3 dimensional compact QED [17], i.e. the magnetic mass of the left over ”photon” (diagonal gluons) was generated by a gas of monopoles. Later Belayev and Eletsky revised Anishetty’s SU(2) calculation [18], and extended it to SU(3) [19] (the one
loop SU(3) result was originally obtained by Weiss [20]). Although these authors differ from Anishetty in the expression for the free energy, they find a non zero minimum at the same value. The SU(3) case is more complicated in that there are two gauge inequivalent directions in color space, corresponding to \( A^3 \) and \( A^8 \). The minimum of the two loop effective potential occurs in the \( A^3 \) direction corresponding to the symmetry breaking pattern \( SU(3) \rightarrow U(1) \otimes U(1) \).

In a very recent paper, Enqvist and Kajantie extended the analysis of [19] to \( \mathcal{O}(g^3) \) by summing the plasmon type ring diagrams for the diagonal (massless) gluons using a general covariant background field gauge. Their result for the effective action is explicitly gauge parameter dependent to \( \mathcal{O}(g^2) \), and exhibits an instability at \( \mathcal{O}(g^3) \) [21].

Needless to say, one must be very careful in interpreting these results. Since there are still potential infrared difficulties due to the remaining massless modes, perturbation theory could very well lead us astray. In this context, Dahlem’s argument, which in its essence was originally proposed by Gross, Yaffe and Pisarski [22], is both interesting and compelling. This scenario has been further discussed in a recent paper by Polonyi and Vazquez [23], who calculated the one loop Higgs potential in the SU(2) case, and showed that the criteria for Polyakov’a semiclassical monopole approximation are fulfilled.

It is obviously a good idea to try to avoid perturbation theory altogether and look for gauge symmetry breaking by lattice techniques. There are two investigations which both claim numerical evidence for gauge symmetry breaking at high temperature. The first, by Polonyi and Wyld, used unitary gauge to extract expectation values of \( A^3_4 \) and \( A^8_4 \) from measurements of the (untraced) Polyakov loop [24]. They observed a jump in \( \langle A^3_4 \rangle \) at \( T_c \), and after assuming that the perturbative contributions are smooth, they concluded that \( A^3_4 \) develops a non zero expectation value.

In a later paper, Mandula and Olgivie studied lattice propagators in Landau gauge. The \( A_4 \) propagator exhibits a clear jump at the transition temperature, consistent with the interpretation that the gauge propagator picks up a disconnected part, i.e. an expectation value for \( A_4 \). Again, this quantity cannot be measured directly, but assuming the disconnected parts to dominate over the perturbative contributions, the direction of symmetry breaking can be obtained from a study of moments of \( (A_4)^2 \). The numerical results suggest alignment in the direction of \( A^3_4 \) rather than \( A^8_4 \), corresponding to \( SU(3) \) breaking to \( SU(2) \otimes U(1) \) rather than \( U(1) \otimes U(1) \). For our investigation it will not matter which of these alternatives is the correct one.

Clearly much more work has to be done to clear up the situation, but we think there is enough circumstantial evidence in favour of gauge symmetry breaking at high temperature that this possibility warrants serious consideration. In this paper we do so by studying the effects on dynamical quarks with special emphasis on bound state formation.
III. THE OPERATOR PRODUCT EXPANSION AT FINITE T

For a general background to the OPE as applied to QCD sum rules we refer to the articles by Novikov, Shifman, Vainshtein and Zakharov [25,26]. An essential point made by these authors is that in order to define the OPE, it is necessary to fix a renormalisation scale $\mu$. This scale is used to separate hard and soft momenta, the former contributing to the coefficient functions and the latter to the condensates. In particular, this means that there are perturbative contributions to the various condensates. In various toy models these perturbative pieces can be calculated explicitly, and they do not have to be small compared to the non perturbative contributions. For the special case of QCD, Novikov et.al. argue that the renormalisation point can be chosen such that $\alpha_s$, is small enough to allow a low order perturbative calculation of the coefficient functions, and at the same time having only a negligible $\mu$-dependence in the vacuum condensates (they estimate $\leq 20\%$ for $\langle 0|G_{\mu\nu}^aG_{\mu\nu}^a|0\rangle$) [25]. We shall see, that having a heat bath also introduces perturbative contributions to the matrix elements in the OPE, and that these in general cannot be neglected.

To illustrate this basic point we shall derive an OPE in a free massless scalar theory defined by

$$L = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2$$

and the usual canonical commutation relations. We consider the retarded commutator defined by

$$\Pi_R(x) = \left[ : \phi^2(x) : , : \phi^2(0) \right] \theta(x^0)$$

where $: ... :$ denotes normal ordering with respect to the vacuum state. Define the retarded response function at finite temperature by

$$R_\beta(x) = \text{Tr} \left[ e^{-\beta(H-F)} \Pi_R(x) \right],$$

where the thermal average is carried out over all physical states with $F$ being the free energy. By successive commutations one easily derives the following OPE for $\Pi_R(x)$

$$\Pi_R(x) = -2i\Delta_R(x)\tilde{\Delta}(x)$$

$$+ \sum_{\mu_1...\mu_n} x^{\mu_1}...x^{\mu_n} \frac{1}{n!} : \phi(0) \partial_{\mu_1}...\partial_{\mu_n} \phi(0) :$$

where $\tilde{\Delta}(x) = \langle 0|\{\phi(x),\phi(0)\}|0\rangle = -2\text{Im}\Delta_F(x)$. It can be shown (see. e.g. Appendix B in [27]) that the first term correspond to the Feynman diagram in Fig. 1, calculated in Euclidian space, and analytically continued to real time according to the Baym-Kadanoff
theorem. In this simple example we can of course easily evaluate the thermal trace of all the operator matrix elements in (4):

$$\langle \phi(0) \partial_{\mu_1} \ldots \partial_{\mu_n} \phi(0) : \rangle_\beta = \int \frac{d^3k}{(2\pi)^3} \frac{1}{k} n_\beta(i k_{\mu_1} \ldots i k_{\mu_n})$$  \hspace{1cm} (5)

where \( n_\beta(k) \) is the Bose distribution function. Substituting in (4) and summing, we obtain after some algebra,

$$R_\beta(k) = \int \frac{d^4k}{(2\pi)^4} e^{ikx} (1 + n_\beta(k_0)) \text{Im} \Delta_F(k).$$  \hspace{1cm} (6)

Again following [27] one can show that this whole contribution is again nothing but the Feynman graph in Fig. 1, but this time calculated using the Matsubara rules for finite temperature field theory.

The lesson of this simple example is that by calculating coefficient functions with finite temperature Feynman rules, one is in effect resuming contributions from higher order operators. Although we have only considered the most elementary example, we shall take for granted that this holds true in general. Specifically, for QCD we shall assume that by calculating coefficient functions in finite T perturbation theory, we systematically resum order \( T \) (as opposed to order \( gT \)) perturbative effects in the matrix elements of higher dimensional operators.

In the previous section, we presented the arguments for spontaneous breaking of the gauge symmetry in the Euclidian theory describing QCD at high \( T \). We now discuss the effect of this on finite T real time correlation functions of the form

$$\Pi_{\mu\nu}(x) = [J_\mu(x), J_\nu(0)] \theta(x^0)$$  \hspace{1cm} (7)

Here \( 4 \) is a color singlet current, so the operator product expansion of the commutator will only include color singlet operators. As described above, we can now effectively sum contribution from infinitely many higher order operators by calculating the coefficient functions with finite T graph rules in Euclidean space. We are thus led to consider graphs like those in Fig. 2. The general rule for the OPE is that when a low momentum is flowing through any of the internal lines, the graph is considered as a matrix element for the corresponding (gauge invariant) operators. In the figure such a low momentum insertion is denoted by a dot on the corresponding propagator.

At zero temperature it is easy to evaluate the diagrams in Fig. 2b and 2c by using the fixed point gauge \( x_\mu A^\mu = 0 \), where the gauge potential can be expanded in the field strength \( F_{\mu\nu} \), and its covariant derivatives[29]. The result is the usual QCD OPE used by Shifman, Vainshtein and Zakharov.

At finite temperature the situation changes. Since we have shown that the OPE expression for the retarded commutator can be calculated from Euclidian finite T field theory, we

\[ ^2 \text{A remark in this respect is also made in [28].} \]
must consider the effects of the spontaneous breaking of the gauge symmetry. As already discussed in connection with the work of Mandula and Olive, this breaking gives rise to a disconnected part in the $A_4$ gluon propagator. We can formally take this effect into account by including the operator $(A_4)^2$ in the OPE, and calculate its coefficient function in finite $T$ perturbation theory. Alternatively we can keep a constant background (Euclidian) $A_4$ potential in the calculation of the coefficient functions which is equivalent to introducing an imaginary chemical potential in the thermodynamical ensemble. The advantage with the first formulation is that the direction of the symmetry breaking never enters (confere the discussion in the previous section). In the second formulation it does, but to leading order we are only sensitive to $((A_4)^2)$. The advantage with the second formulation is that it emphasizes that nothing strange happens to the OPE. We shall elaborate a little bit on this quite important point. Consider QCD in the $A_0 = 0$ gauge, so that the counterpart to (3) will read

$$\langle \ldots \rangle = \mathrm{Tr}[P e^{-\beta(H-F)} \ldots]$$

where $P$ projects on the physical states, i.e. it enforces the Gauss’ law constraint. By a standard procedure we can rewrite this expression as an Euclidian theory with an $A_4$ Lagrange multiplier field that implements the constraint. In the resulting theory, $A_4$ enters as the fourth component of the gauge potential. Thus the whole effect of the dynamics of the $A_4$ field is to implement the constraint on the states in the thermal ensemble. If the infrared behaviour of the resulting effective Euclidian theory is such that the long range fluctuations give rise to a vacuum expectation of $A_4$, this will affect the coefficient functions of the various operators in the OPE. Thus, introducing a non zero $(A_4)^2$ amounts to making an infinite resummation of terms in the OPE, which is necessitated by the vacuum rearrangement above $T_c$ \(^{3}\). It does not imply that a new operator $(A_4)^2$ has appeared in the OPE.

To summarize, by calculating coefficient functions in the OPE, using Euclidian finite $T$ perturbation theory, we sum an infinite number of contributions from higher dimensional operators. Also, in QCD we get new contributions (that superficially look like arising from a new gauge non invariant operator $(A_4)^2$ which again arise from a complicated resummation of terms.

To stress these points we will make use of the formulation in terms of a background colored electric potential, i.e. we shall use the ensemble

$$\langle \ldots \rangle = \mathrm{Tr}[P e^{iA_3 Q^3} P e^{-\beta(H-F)} \ldots]$$

where $Q^a$ is the (global) color generator. We will arbitrarily take the symmetry breaking to be in the 3 direction of SU(3) color space (confere the discussion in section 2).

\(^{3}\) In static gauges these effects are reexpressible in terms of the eigenvalues of the Polyako loop.
We shall now ask under what circumstances the resumptions implied by this formulation can be justified. We have to consider the following scales: the temperature $T$; the zero temperature confining scale $\Lambda_{QCD} \sim T_c$; the electric mass $gT$ (plasmon scale); the magnetic mass $g^2 T$ (nonperturbative scale above $T_c$); the external momentum in the correlation function $Q^2$ and the renormalization point $\mu$.

First consider $T < T_c$. In this case $g$ cannot be assumed to be small and one cannot distinguish between $T$, $gT$ and $g^2 T$. Just as at $T = 0$ one can look for a "window" where the $(T = 0)$ perturbative contribution dominates the OPE. In this case not only the condensate terms, but also terms proportional to powers of $(T^2/Q^2)$ have to be small. It is of course again possible to resum the $((T^2/Q^2)^n)$ terms to obtain coefficient functions calculated in finite $T$ perturbation theory as discussed above. For these low temperatures, however, such a resummation does not seem to make much sense since the thermal fluctuations will dominantly be at low momenta. As such they should not change the coefficient functions, but rather be included in the operator matrix elements. Thus, it is reasonable to believe that at $T < T_c$ the main temperature effects are in the condensates and not in the coefficient functions. Hopefully, improved lattice calculations will enable us to actually calculate the $T$ dependence of the most important low dimensional condensates like $G^2$ and $\bar{q}q$. When the temperature is about $T_c$, some of the condensates are expected to vary rapidly with the temperature. In this range the QCD sum rule approach is likely to break down and lose its predictive power, (compare with reference [12]). These comments are relevant for the calculations presented in references [8,10] and [11].

The discussion so far indicates that the issue of scale separation is problematic at low temperature where the range of applicability of the sum rule approach might be drastically constrained. Fortunately, the situation is much better at high temperature, or more specifically, in the regime of temperatures such that

$$T_c < g^2 T < Q^2 < T$$  \hspace{1cm} (10)

The electric scale $gT$ and the renormalization point $\mu$ will be discussed later. First, since we are at high $T$ we can hope to get some non-perturbative information about the condensates both from lattice calculations [30,31] and estimates in the effective 3 dimensional gauge-Higgs model [32,33]. Secondly, $Q^2$ can hopefully be made small enough to really probe the meson like excitations in the plasma while still keeping the OPE under control (remember that the scale of the finite $T$ condensates is $g^2 T$), i.e. we can hope for a "window". Third, the large $(T^2/Q^2)$ power corrections must be resummed, but this is achieved by the methods described above. Thus we propose that the region specified by (10) is the one that can be studied using the resummed OPE described earlier in this section.

Several comments on this suggestion, on which this paper is based, are in order. First, it is clear that for the hierarchy (10) to make sense, the temperature must be sufficiently large for $g(T)$ to be small. Since we don’t know the relevant expansion parameter ($g(T)$ or $g(T)/2\pi$ or ...), what is "sufficiently large $T$ cannot be simply answered by e.g. using the
one loop formula for the running coupling constant. Thus, for the time being we can only assume that (10) makes sense at temperatures that might be experimentally accessible. In calculating the coefficient functions, we shall use ordinary finite $T$ Feynman rules. We know, however, that an electric mass is generated at the scale $gT$. We already discussed the resummation of terms of order $(T^2/Q^2)^n$, but if $Q^2$ is not much larger than $(gT)^2$, we must also resume the $(g^2T^2/Q^2)^n$ terms, by introducing an electric mass.\footnote{A first attempt in this direction can be found in [21].} In the following we will for simplicity assume that $Q^2$ can be chosen so that this is not needed. Note that corrections due to magnetic mass insertions will be of the form $(g^2T^2/Q^2)^n$, and are thus included in the matrix elements. At $T = 0$ the operator renormalization scale $\mu$ is chosen so as to minimize perturbative corrections to the condensates. At finite $T$ we would expect this to happen for $g^2T < \mu < Q^2$, but just like at $T = 0$, this cannot be proved but will be assumed.

\section*{IV. SUM RULES}

To investigate the possible occurrence of light bound states above $T_c$ we will construct sum rules for the thermal average of the retarded correlation function for the color singlet vector-isovector $(1^-)$ current $J_\mu = \frac{1}{2} (\bar{u} \gamma_\mu u - \bar{d} \gamma_\mu d)$. The Fourier transform of this correlator reads,

$$\Pi_{\mu\nu}(q_0, \vec{q}) = i \int d^4x e^{i q \cdot x} \text{Tr}(Pe^{i A_3^\mu} Q^3 e^{-\beta(H-F)})[J_\mu(x), J_\nu(0)]$$

Let us recall that $P$ is the projection operator on physical states, and that the term $e^{i A_3^\mu} Q^3$ enforces the spontaneous breaking of the global color symmetry related to the vacuum rearrangement above the critical temperature. Since both $H$ and $P$ commute with the global charge, we can use standard methods to rewrite (2) as an Euclidian path integral in a constant background $A_3^\mu$ potential. We shall use covariant gauge, and as usual the ghosts obey Bose statistics. At finite temperature, the retarded correlator (11) is determined by two invariant form factors,

$$G_{ij} = \left( \delta_{ij} - \frac{Q_i Q_j}{\vec{Q}^2} \right) G_T + \frac{Q_i Q_j}{\vec{Q}^2} Q_0^2 G_L$$

$$G_{00} = \vec{Q}^2 G_L$$

where $q = (iQ_0, \vec{0})$. Both are analytic in the upper $Q^2$ plane. As a result, the longitudinal and transverse form factors satisfy conventional dispersion relations. Specifically, $G_L$ obeys an unsubtracted dispersion relation of the type

$$\text{Re} G_L(Q^2) = \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im} G_L(s)}{s + Q^2}$$
In the frame of the heat bath, \( Q = (Q_0, \vec{0}) \), the transverse form factor \( G_T \) is related to the longitudinal form factor since \( G_T \to Q^2 G_L \) when \( \vec{Q}^2 \to 0 \) [28]. Hence, only the longitudinal form factor will be discussed throughout.

For large space-like \( Q^2 \), the real part (LHS) can be calculated using the OPE. The imaginary part, or spectral density, in the RHS is given by poles and cuts in the time-like region which cannot be approached in perturbation theory. In the sum rule approach this part is conventionally parametrized by a single resonance pole and a continuum threshold. Sum rules are then derived for the position and residue of the resonance, and the position and the strength of the threshold. There are various methods proposed for matching the RHS and LHS in the sum rules. For light resonances the most common procedure is to Borel transform the expressions and look for stability of the difference RHS-LHS as a function of the Borel parameter \( M^2 \). Borel transformation usually improves the convergence of the (asymptotic) perturbative expansion (factorial suppression), and also further suppresses the contributions from the higher dimensional operators. Because of the large uncertainties involved in the present calculation, we have not explored the various other formulations of the sum rules that have been proposed.

Let \( m_\rho \) and \( f_\rho \) be the the mass and residue respectively, of a possible light bound state in the \( \rho \) channel above \( T_c \) and let \( \sqrt{s_0} \) be the threshold energy of the continuum (in this picture we implicitly assume dynamical confinement, in the sense that the resonance is isolated from, rather than imbedded in, the continuum). Then the RHS of (14) can be parametrized as follows

\[
\text{Im} G_L(s) = \pi f_\rho m_\rho^2 \delta(s - m_\rho^2) + \theta(s - s_0) \frac{1}{8\pi} \tanh \frac{\sqrt{s}}{4T} + \frac{2\pi T^2}{3} \delta(s) \tag{15}
\]

As in the conventional sum rules we have chosen to parametrize the \( q\bar{q} \) background, i.e. the strength of the cut, by the (temperature dependent) imaginary part of the perturbative value. The delta function contribution in (15) corresponds to soft scattering of thermal quarks off the resonance (thermal dissociation) [28].

The OPE is now used to evaluate the LHS of (14) in the deep Euclidean regime \( (Q^2 \to 0) \). At high temperature, however, nontrivial resummations are needed. The rationale for this has been described in section 3. In this spirit, we will evaluate the Wilson coefficients in Euclidean space using the Matsubara techniques and analytically continue them to Minkowski space using the Baym-Kadanoff prescription. Figs. 2b and 2c show the leading nonperturbative contributions to the causal correlator as implied by the OPE. In these figures, the blob can mean either \( \langle B^2 \rangle \), \( \langle E^2 \rangle \) or \( \langle A_4^2 \rangle \). The \( A_4^2 \) insertions are due to the complex colored chemical potential inserted in (11) to account for the spontaneous breakdown of global gauge invariance. The \( E^2 \) and \( B^2 \) insertions correspond to the usual

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5 For a rationale of the Borel method we refer to the original papers by SVZ and for a recent discussion of these matters in the context of finite T sum rules to [10].
electric and magnetic condensates which are still present above $T_c$. These need not be the same as at $T = 0$ since the heat bath defines a preferred Lorentz frame which means that we have the two $O(3)$ invariants $E^2$ and $B^2$ rather than the single $O(4)$ invariant $G^2$. Higher dimensional operators are suppressed by powers of $Q^2$. Remember that chiral symmetry is restored above $T_c$ so, at least for massive quarks, there are no quark condensate insertions.

The results for the Wilson coefficients are

$$C_1 = \frac{1}{8\pi^2} \int_0^\infty ds \left(1 + 8n_\beta \frac{s}{Q^2}\right) \frac{1}{s + Q^2/4}$$

$$C_{A_4} = \frac{\alpha_s}{12\pi} \int_0^\infty ds 2n_\beta \frac{2 - 3Q^2/4}{(s + Q^2/4)^3}$$

$$C_{E^2} = -\frac{\alpha_s}{6\pi Q^4} - \frac{\alpha_s}{72\pi Q^2} \int_0^\infty ds 2n_\beta \frac{2 - 3Q^2/4}{(s + Q^2/4)^3}$$

$$C_{B^2} = +\frac{\alpha_s}{36\pi Q^2} \int_0^\infty ds 2n_\beta \frac{2 - 3Q^2/4}{(s + Q^2/4)^3}$$

where $n_\beta = n(\sqrt{s}/T)$ is the Fermi distribution. A derivation of these coefficients is given in Appendix A using a variant of the fixed point gauge.

The Wilson coefficients for $E^2$ and $B^2$ corresponding to Fig. 2b are infrared divergent both at zero and finite temperature, since the momentum in the dressed quark line can become soft. At zero temperature Lorentz invariance and gauge invariance conspire to the vanishing of Fig. 2b in the chiral limit, but this is no longer true for massive fermions. Fortunately, this soft contribution can always be reabsorbed into $\bar{m}q$ at the operator level using the equations of motion. This prescription, which is due to Smilga [29], is shortly discussed in Appendix A, and will be understood throughout. As a result, the contribution of Fig. 2b will be set to zero even at finite temperature since we are all the time working in the chiral limit.

Following the conventional recipe used for for light $Q^2$ systems at zero temperature, we equate the Borel transform of the RHS with that of the the LHS, to get the following sum rule,

$$f_\rho^2 m_\rho^2 e^{-m_\rho^2/M^2} = \frac{1}{8\pi^2} R(M^2, T)$$

where $R$ is given by
\[ R(M^2, T) = \int_0^{\infty} ds \, e^{-s/M^2} \tanh\left(\frac{\sqrt{s}}{4T}\right) \]
\[ - \frac{16\pi\alpha_s}{3} \frac{\langle (A_4)^2 \rangle}{M^2} \int_0^{\infty} ds \, n_\beta \left(\frac{\sqrt{s}}{2T}\right) \left( -3 + \frac{2s}{M^2} \right) e^{-s/M^2} \]
\[ - \frac{8\pi\alpha_s}{9} \frac{\langle E^2 \rangle}{M^2} \int_0^{\infty} ds \, n_\beta \left(\frac{\sqrt{s}}{2T}\right) \]
\[ - \frac{4\pi\alpha_s}{3} \frac{\langle (E^2 - 2B^2) \rangle}{M^4} \int_0^{\infty} ds \, n_\beta \left(\frac{\sqrt{s}}{2T}\right) B(M, s) \]  
(21)

and

\[ B(M, s) = \frac{M^4}{s^2} \left( 1 - e^{-s/M^2} \left( 1 + \frac{s}{M^2} + \frac{2s^2}{M^4} \right) \right) \]  
(22)

which satisfies \( \int_0^{\infty} ds B = -1 \). The pertinent Borel transforms for the finite temperature expressions can be found in Appendix B. Note that the threshold parametrization used in (15), implies a large cancellation between the continuum contributions on the two sides of the sum rule.

V. NUMERICAL RESULTS

By now it should be clear that we are not in a position to make any firm predictions based on the sum rule (20). In addition to all the assumptions made in deriving it, we also have the additional difficulties related to the finite \( T \) condensates. At \( T = 0 \) the condensates can be determined phenomenologically while here we must rely on theoretical estimates. With all this in mind, we will nevertheless try to estimate our parameters using available information, plug everything into (20) and look for signs of above \( T_c \) resonances. We believe, that at this point, nothing better can be done, and that the results below represent a possible scenario. Needless to say, we have not spent any time trying to optimize our fits, explore different forms of the sum rules, do systematic searches in parameter space etc.. Such efforts would be wasted on a numerical calculation which is primarily illustrative. For the same reasons we shall also not present any details of our fits, but only the basic results.

From the lattice measurements of the free energy and pressure, it is possible to make an estimate of the electric and magnetic condensates above \( T_c \). The analysis of Adami, Hatsuda and Zahed [12] gives \( \langle (\alpha_s/\pi)B^2 \rangle \sim (196 \pm 10 \text{ MeV})^4 \) from lattice calculations in the range \( T_c < T < 2T_c \). Above \( T_c \) the electric and magnetic condensates are about half their values at \( T = 0 \). These estimates are consistent with the recent SU(2) lattice calculation by Lee [31].
Numerical estimates for the Higgs condensate are unfortunately not available. The lattice calculations of Mandula and Olgovie were done in Landau gauge and thus cannot be directly used here. For lack of better, we shall nevertheless take \( \langle (\alpha_s/\pi A_0)^2 \rangle \sim +0.02 \text{GeV}^2 \), which is consistent with their results [33].

The LHS and RHS of the sum rule (20) can be made to match above \( T_c \). For \( T = 1.5 T_c \), we find a Borel "window", \( 0.6 \text{GeV}^2 < M^2 < 1.2 \text{GeV}^2 \), where the sum rule is saturated by a resonance with mass \( m_\rho \sim 1 \text{GeV} \) and strength \( f_\rho \sim 0.03 \) plus a threshold at \( s_0 \sim 1.5 \text{GeV} \). Recall that the resonance strength is a measurement of the rho coupling above \( T_c \), i.e. \( g_\rho^2/4\pi = 1/\pi f_\rho \sim 2.5 \).

For the range of parameters quoted above, the vacuum contribution is about three times the contribution of the condensates. More specifically, the electric and magnetic condensates contribute about 1% of the RHS while the \( (A_4)^2 \) condensate contributes about 30%. This shows that the perturbative approach is consistent. Moreover, the continuum contribution to the sum rule is about 20% of the resonance contribution, which justifies the single resonance dominance approximation.

VI. CONCLUSIONS

We have shown how to use present theoretical ideas about the high temperature phase of QCD in order to calculate finite temperature real time correlation functions, and extract information about light relativistic bound states. A first crude attempt to a numerical analysis of the sum rules gives a clear indication for a resonance structure in the rho channel. Since chiral symmetry is restored above \( T_c \), we expect the same signal to occur in the \( A_1 \) channel. Other light multiplets are also expected. We again emphasize that the correlators we consider can be obtained neither in perturbation theory, because of the infrared problems above \( T_c \) nor in present lattice calculations because of their limitation to static quantities. (Attempts to analytically continue numerical results from high temperature lattice results are questionable.) Our results indicate that the high temperature phase of QCD has excitations which are reminiscent of those at zero temperature. The constituents of the plasma are not just screened quarks and gluons but also propagating color singlet excitations as first conjectured by DeTar[14]. Such light resonances, if confirmed, might have important phenomenological implications. Our work can be extended in several ways. First, direct lattice calculations of the lowest dimensional condensates are needed to put any numerical analysis of the sum rules on a firm basis. Second, more systematic investigations of the resonance parameters can be carried out for a wide range of temperatures above \( T_c \), to study the behavior of the expected light chiral multiplets. Third, the analysis can be extended to baryons.
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Appendix A

In this Appendix we outline the calculation of the Wilson coefficients (16)-(19) given in section 4. At finite temperature, and in the spontaneously broken phase, it is convenient to use the following gauge,

$$\partial_0 A_0 = 0 \quad \text{and} \quad \vec{x} \cdot \vec{A} = 0 \quad (A1)$$

which is a combination of the static gauge and the 0(3) version of the Fock-Schwinger, or fixed point gauge. The calculation is performed in Minkowski space and the final result converted to Euclidian (note $\langle A_2^2 \rangle = -\langle A_0^2 \rangle > 0$). It can be easily checked that the gauge conditions (A.1) are both reachable and complete, and imply the following relations,

$$A_0(t, \vec{x}) = A_0(\vec{x})$$
$$A_j(t, \vec{x}) = \int_0^1 d\alpha x^i A_{ij}(t, \alpha \vec{x}) \quad (A2)$$

The second of these equations shows that the vector potential can be expressed solely in terms of the magnetic field and its covariant derivatives, i.e.

$$A_j(t, \vec{x}) = \sum_0^\infty \frac{1}{k!} (k + 2) x^i x^{j_1} \ldots x^{j_k} \nabla_{j_1} \ldots \nabla_{j_k} A_{ij}(t, \vec{0}) \quad (A3)$$

Where $A_{ij} = A_0^a T^a$ is the (matrix-valued) colormagnetic field tensor. In this gauge the momentum space fermion propagator reads,

$$S(q) = \frac{\phi}{q^0} - \frac{\phi^0}{q^4} g A_0 - \frac{\phi}{q^0} q^0 g D_j A_0 - \frac{1}{4 q^2} ( \sigma^{ij} \phi + \phi \sigma^{ij}) g A_{ij} + \frac{\phi^0}{q^0} - \frac{(g A_0)^2 + \ldots}{q^4} \quad (A4)$$

The ... stands for higher dimensional contributions. By relating the background potential to the (Euclidian space) “condensate” according to

$$A_a^0 A_0^b = -\frac{1}{8} \delta^{ab} \langle A_4^2 \rangle \quad (A5)$$

a straightforward calculation yields the following contribution to the trace of the polarization tensor from the diagrams 2b and 2c:

$$\Pi^{(2b)}_{\mu\nu} = 4 g^2 \langle A_2^2 \rangle (-3q_0 I_{21}^0 + 4 q_0 I_{31}^{00} + 2 I_{21}^{00} - I_{11}) \quad (A6)$$
$$\Pi^{(2c)}_{\mu\nu} = 2 g^2 \langle A_2^2 \rangle (2 q_0^2 I_{22}^{00} - q_0^2 I_{12} + 2 q_0 I_{12}^0 + I_{02}) \quad (A7)$$
In both cases we used the following notation

\[ I_{mn}^{\mu_1 \ldots \mu_j} = \sum_\beta \frac{k^{\mu_1} \cdots k^{\mu_j}}{k^{2m}(k + q)^{2n}} \] (A8)

The sum in (A.8) is over discrete energies \( k_0 = 2\pi/\beta \) and continuous momenta \( \vec{k} \). Some useful properties of these integrals can be found in reference [34]. Relating \( G_L \) to \( \Pi_\mu^\mu \) we obtain the coefficient function (16 -19) in the text. An alternative way to calculate the coefficient function for the \( A_4^2 \) insertion, is to use Feynman rules corresponding to an (imaginary) chemical potential.

The magnetic contributions to the trace of the response function is shown in Fig. 2b and 2c, and follows from the magnetic insertion appearing in the fermion propagator (A.4). Specifically, the contribution of Figure 2b and 2c are

\[
\Pi^{(2b)}_{\mu\mu} = \frac{32g^2}{3} \langle B^2 \rangle (2q_0I_{01}^0 + I_{31}^0 + q_0^2I_{41}^0 - I_{40}^0) \\
\Pi^{(2c)}_{\mu\mu} = -\frac{q^2}{3} \langle B^2 \rangle (-2I_{12} - q_0^2I_{22} + 4q_0I_{22}^0 + 4I_{22}^{00}) \] (A9) (A10)

The contribution to (A.9) diverges in the infrared at zero temperature. The reason for this is rather simple. Each magnetic field insertion brings two powers of \( q^2 \) in the denominator turning the process divergent in the infrared. As discussed by Smilga, this process can be regarded as a condensate effect and can be reabsorbed into \( \bar{m}q \) at zero temperature using the equations of motion. We will assume the same to hold true at finite temperature. Since the fermion condensate vanishes above \( T_c \), the contribution of Fig. 2b will be set equal to zero.

At this stage, we should point out that if we were to use the operator formalism rather than the background field method, and include the short distance effects only in the coefficient functions, the infrared problem we encountered would never have occurred. That it did, shows that spurious effects can occur when coefficient functions are calculated using the background field method. Similar arguments apply to the electric contribution to the trace of the polarization function. Combining these results with (A.6 - A.7) and ignoring (A.9) we have\(^6\)

\[
\Pi_{\mu\nu} = -4g^2\langle E^2 \rangle (2I_{12} - q_0^2I_{22}) \\
+ \frac{4g^2}{3} \langle E^2 + B^2 \rangle (2I_{12} - q_0^2I_{22} + 4q_0I_{22}^0 + 4I_{22}^{00}) \] (A11)

The asymmetry between the electric and magnetic contributions to \( \Pi_{\mu\nu} \) is due to the breaking of Lorentz invariance introduced by the heat bath. Indeed, if we denote by

---

\(^6\) The terms involving \( E^2 \) were calculated using the ordinary fixed point gauge. The terms involving \( B^2 \) were calculated using the ordinary fixed point gauge and the gauge (A.1). For the latter both gauges lead the same answer as expected.
\( n^\mu = (1, \vec{0}) \) the rest frame of the heat bath, we can decompose the product of two field strengths as

\[
G^a_{\mu\nu} G^{b\alpha\beta} = -\frac{\delta^{ab}}{24} (g^\alpha_\mu g^\beta_\nu - g^\alpha_\nu g^\beta_\mu) E^2 + \frac{\delta^{ab}}{24} \epsilon_{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\rho\sigma} n^\sigma n^\tau (E^2 + B^2) \tag{A12}
\]

It is rather clear that at zero temperature only the electric part contributes since the vacuum is Lorentz invariant (with \( \langle E^2 \rangle = -\langle B^2 \rangle \)). At finite temperature this no longer true. Notice that the magnetic effects are frame dependent.

The summation in the \( I \) integrals can be carried out by separating the zero and finite temperature contributions. The zero temperature part can be calculated using conventional dimensional regularization and the results are

\[
2I_{12} - q_0^2 I_{22} = \frac{i}{8\pi^2} \frac{1}{q_0^2}
\]

\[
2I_{12} - q_0^2 I_{22} + 4q_0 I_{22} + 4I_{22}^{00} = 0 \tag{A13}
\]

The Lorentz breaking term in (A.11) follows the decomposition (A.12) and vanishes at zero temperature as it should. At finite temperature both terms in (A.11) contribute. Using

\[
2I_{12} - q_0^2 I_{22} = 4q_0 I_{22}^{00} + 4I_{22}^{00} = \frac{2}{\pi^2} \int dx x n_\beta \frac{4x^2 + 3q_0^2}{(4x^2 - q_0^2)^3} \tag{A14}
\]

in (A.11) and substituting \( q_0 \to -iQ_0 \) yield the quoted results for the coefficient functions.

**Appendix B**

In this appendix we derive two useful expressions for the Borel transforms of inverse powers of polynomials needed in deriving (21). If we denote by \( L_M \) the Borel transformation then

\[
L_M \left( \frac{1}{(s + Q^2)^d} \right) = \lim_{n \to \infty} \frac{(Q^2)^n}{(n-1)!} \left( -\frac{d}{dQ^2} \right)^n \left( \frac{1}{s + Q^2} \right)^d
\]

\[
= \lim_{n \to \infty} \frac{d(d+1)...(d+n-1)}{(n-1)!} \left( \frac{1}{Q^2} \right)^d \left( \frac{1}{1 + s/(nM^2)} \right)^{d+n}
\]

\[
= \frac{1}{(d-1)!} \left( \frac{1}{M^2} \right)^d e^{-s/M^2} \tag{B1}
\]

where \( Q^2 = nM^2 \) and the last equality follows from Stirling’s approximation in the limit of large \( n \). We also have

\[
L_M \left( \frac{1}{Q^2(s + Q^2)^d} \right) = \frac{1}{s} L_M \left( \frac{1}{Q^2(s + Q^2)^{d-1}} \right) - \frac{1}{s} L_M \left( \frac{1}{(s + Q^2)^d} \right)
\]

\[
= \frac{1}{s^d M^2} - \frac{1}{M^{2d+2}} \sum_{k=1}^{d} \frac{1}{(d-k)!} \left( \frac{M^2}{s} \right)^k e^{-s/M^2} \tag{B2}
\]
The last equality follows from (37) by successive iterations. Using (B.1) and (B.2) it is straightforward to derive the Borel transforms of the LHS of (14) from the expressions (16)-(19) for the Wilson coefficients. The Borel transform of the RHS is standard.
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Figure Caption

Fig. 1. Vacuum contribution to the retarded current-current correlator for free scalars as defined in Eq. (2).

Fig. 2. (a) Vacuum contribution to the vector current; (b) and (c) Soft gluon contributions to the vector current.
FIG. 1.

FIG. 2.