Avoiding apparent signaling in Bell tests for quantitative applications

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Bell tests have become a powerful tool for quantifying security, randomness, entanglement, and many other properties, as well as for investigating fundamental physical limits. In all these cases, the specific experimental value of the Bell parameter is important as it leads to a quantitative conclusion. However, most experimental implementations aiming for high values of the Bell parameter suffer from the defect of showing signaling. This signaling can be attributed to systematic errors occurring due to weaknesses in the experimental designs. Here we point out the importance, for quantitative applications, to identify and address this problem. We present a set of experiments with polarization-entangled photons in which we point out common sources of systematic errors and demonstrate approaches to avoid them. This allows us to establish a reliable estimate for the Bell parameter.

Motivation.—Bell inequalities were conceived to test whether quantum theory can be replaced by a local realistic theory [1]. However, today the role and purpose of Bell tests has changed and the particular value of the Bell parameter is used to quantify, e.g., the rate of the device-independent secure key generation [2, 3], the degree of entanglement [4], the advantages in communication complexity [5, 6], the robustness of incompatibility [7], the dimension [8], the degree of self-testing [9], and the number of private random bits [10]. The experimental value is also crucial when we investigate the physical limits of correlations [11–13] and test how close we can get to the upper bound according to quantum theory [14] or whether it is even possible to exceed this bound.

Bell tests are based on the assumption of nonsignaling, i.e., that two independent, separated experiments cannot influence each other. This assumption is so natural that there seems to be little value in making the effort of testing whether it is satisfied. However, there are claims that “at least 90% of the Bell experiments have apparent signaling” [15], and, as we demonstrate here, various different systematic errors in experiments can be held responsible for this effect. For certain tasks such as ruling out local hidden variable models, a small amount of signaling does not constitute a problem since there are tools that demonstrate that the conclusion of impossibility of local realism holds even in this case [16–19]. However, for quantitative applications, these tools do not exist and, in general, it is desirable to establishes values for Bell parameters that are universally viable and not tied to specific applications.

The aim of this Letter is to methodically identify common sources of systematic errors which can distort the value of the Bell parameter or cause apparent signaling, and describe how to avoid them. Our list cannot be comprehensive, but it is representative of typical errors. Our aim is to point out that some experimental limitations that were irrelevant for certain purposes in the past are now crucial for the new applications. Although we focus on photonic implementations and the Clauser-Horne-Shimony-Holt (CHSH) scenario [20], which are the most frequent combination for most of the practical applications, our analysis and conclusions apply to other Bell-like scenarios and other experimental implementations.

The CHSH scenario and the nonsignaling conditions.—We consider two independent parties, Alice and Bob, each of them able to perform measurements randomly chosen between $x = 0, 1$ for Alice, and $y = 0, 1$ for Bob. Each measurement has two possible outcomes $a = \pm 1$ for Alice, and $b = \pm 1$ for Bob. We focus on the Bell scenario by CHSH, where the Bell parameter is

$$S = E(0, 0) - E(0, 1) + E(1, 0) + E(1, 1),$$  \hspace{1cm} (1)

with $E(x, y) = \sum_{a,b} P(a, b|x, y)$ and $P(a, b|x, y)$ denoting the joint probability of Alice and Bob obtaining outcomes $a$ and $b$, respectively, when they measure $x$ and $y$, respectively. According to quantum theory $|S| \leq 2\sqrt{2}$ holds as a universal bound [14] and this bound can be saturated already for two-level systems, if both parties share a maximally entangled quantum state, such as $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$.

For the CHSH scenario, the assumption that both parties measure independently implies the nonsignaling conditions $s_{a|x} = P(a, 0|x, 0) - P(a, 0|x, 1) = 0$ for all $x, a$, and $s_{b|y} = P(\perp b|0, y) - P(\perp b|1, y) = 0$ for all $y, b$. Here we wrote $P(a, b|x, y)$ for $\sum_{a,b} (a, b|x, y)$ and similarly for $P(\perp b|x, y)$. Due to the normalization of the correlations, $P(\perp b|x, y) = 1$ for all $x, y$, it is sufficient to consider the nonsignaling conditions only for $a = +1$ and $b = +1$, respectively.

In a straightforward test of experimental data, one can consider these four independent nonsignaling conditions. For example, $s_{+1,0}^A$ is obtained by replacing the correlations in $s_{+1,0}^A$ by the empirical frequencies. Even in an ideal experiment, the condition $s_{+1,0} = 0$ will not be satisfied exactly, but rather fluctuate with a standard...
Experimental sources of apparent signaling.—Here we identify experimental sources of apparent signaling and address them by adding components to the initial setup and optimizing the data acquisition procedures. We start by considering a common configuration for Bell experiments using polarization-entangled photon pairs generated through a down-conversion process. It consists of a free-space down-conversion source and two measurement stations, each including a polarizer and one single-mode fiber (SMF) connected to a single-photon detector. This is the specific configuration in which the highest values for \( S \) to date have been achieved \([11–13]\). Nevertheless, it suffers two key limitations that lead to apparent signaling in the results:

(S1) Drifts in the pump laser power. The measurements are performed with only one detector on each side and the different outcomes for each measurement are measured by choosing orthogonal settings for the polarizer. Hence, all measurement outcomes are recorded sequentially. If during this process the pump laser intensity drifts, then the frequencies of outcomes do not yield the correlations as the fair sampling assumption is not satisfied.

(S2) Polarization-dependent collection efficiency. Any optical element can slightly displace the beam if moved or rotated during the measurement. This is particularly true for wave plates or polarizers used to change the measurement settings. Careful alignment and particularly good components may reduce the problem, but if SMFs are used to collect photons, this issue will still affect the coincidence rates depending on the measurement setting. The difficulty here is that SMFs have a core diameter of only a few micrometers, while multi-mode fibers (MMF) at the measurement stations would collect photons with different spatial modes and thus decrease the visibility.

A clean and straightforward solution for issue (S1) is to use a polarization beam splitter and two detectors on each measurement station. Then the normalization is naturally identical for all outcomes, independent of the laser intensity. Regarding issue (S2), a practical solution consists in having SMFs between the source and the measurement stations, since here no component is moved or rotated during the measurement. MMFs are then used for collecting photons for the detection. The SMF acts as spatial mode filter, while the MMFs are effectively insensitive to small beam displacements due to their core being at least ten times wider. Such a setup, depicted in Fig. 1, is our starting point, since it naturally does neither suffer (S1) nor (S2).

In the first experiment we proceed as follows: First we maximize the visibility and the collection efficiency. After that, every setting is continuously measured for 1000 seconds. The order of the settings is \((x, y) = (0, 0), (0, 1), (1, 1), \) and then \((1, 0)\). The results of this first experiment are presented in Fig. 2 a). While the obtained value for \( S \) is very high—within one sigma from the quantum
FIG. 2. Effect of different measures taken to address the systematic errors, as function of the data collection time. Left panels: CHSH parameter $S$ (solid blue line) with the corresponding statistical uncertainty (orange shaded area, one standard deviation) and the quantum maximum $S = 2\sqrt{2}$ (dashed red line). Right panels: Statistical significance of the apparent signaling in standard deviations $\sigma$. a) Simple setup with low setting reproducibility and asymmetric collection efficiency. b) Improved setup addressing setting reproducibility by using precise motors and repeating the settings 10 times. c) Improved setup addressing asymmetric collection efficiency and also using precise motors, but not repeating the settings. d) Setup used to obtain our experimental value, using precise motors, symmetric collection efficiency, and repeating the settings 200 times.

maximum—, the statistical significance of the apparent signaling grows to be about 40 standard deviations. The main reasons for this are the following:

(A1) Measurement setting precision and measurement setting reproducibility. While a low accuracy will result in lower visibility and hence in a lower value of $S$, low precision in repeating a measurement setting also yields apparent signaling, as this is equivalent to Bob setting his polarizer differently according to Alice’s measurement setting. In our first experiment, motors with a precision of $0.2^\circ$ were used in order to simulate manual rotation of the wave plates.

(A2) Asymmetric collection efficiency. This may violate the fair sampling assumption. If the photons are collected with different efficiency at the two outputs of the same measurement station, signaling may appear in the data. To get some intuition, suppose that one of the detectors has unit efficiency, while the other has zero efficiency. Clearly, then the marginal probabilities will show signaling. In our first experiment, the collection efficiency was maximized for each setting, resulting in a different efficiency for each detector.

The second experiment aims to address (A1). An obvious countermeasure to the reproducibility issue is to use more precise motors. Therefore, in the second experiment we used $0.02^\circ$ precise stepper motors. Nevertheless, no matter how precise those motors are, they will always stop slightly short or long of the desired position, therefore yielding, for large number of samples, a violation of the nonsignaling conditions. A further approach to circumvent (A1) is to measure each setting with a short collection time and to repeat the experiment many times. If we assume that the motor precision is sufficiently symmetric around a central position, then the deviations from the desired angle will average out. Therefore, in the second experiment, each setting has been repeated 10 times. The results of the second experiment are presented in Fig. 2 b), and show that the improvements have been effective in reducing the apparent signaling. Still, unequal collection efficiency causes signaling to grow as the collection time increases.

In order to investigate the effect of asymmetric collection efficiency (A2), we perform a third experiment in which we compensate the different efficiencies for each party (D1 vs. D2 and D3 vs. D4) by using variable attenuators in front of the MMF couplers. For this, we define as $C_{ij}(\theta)$ the rate of coincident clicks of detectors $D_i$ and $D_j$ when Alice’s wave plate is at angle $\theta$. Then, we compare the rates $\gamma = C_{13}(0^\circ) + C_{23}(0^\circ)$ and $\gamma' = C_{13}(45^\circ) + C_{23}(45^\circ)$; see Fig. 3. If $\gamma \neq \gamma'$, then D1 and D2 have different detection rates. In this case,
we use a variable attenuator in the path with higher detection rate in order to approach $\gamma = \gamma'$. In our third experiment we implement this procedure for Alice and Bob, we use the more precise motors $(0.02^\circ)$, but we do not repeat the settings. The results are shown in Fig. 2 c) and show that compensating asymmetric collection efficiencies reduced the apparent signaling to 2.1 standard deviations.

In the fourth experiment we improved the third experiment by also repeating each of the four settings 200 times. The results of this fourth experiment are shown in Fig. 2 d). There, the statistical significance of the apparent signaling is 1.3 standard deviations, which indicates that no major source of signaling is present for the amount of samples taken. The results of all four experiments are summarized in Table I.

**Estimate for the systematic errors.**—In the analysis so far, we considered the significance of the apparent signaling as an indicator for systematic errors dominating the experimental evaluation. However, conversely, when we do not observe any significant apparent signaling this does not purge us of a quantitative analysis of sources of systematic errors in the experiment, which may affect our estimate of the CHSH parameter $S$. In our experiments the number of samples was such that the statistical uncertainty originating from Poissonian shot noise was of the order of 0.003. We compare this to the following sources of systematic errors:

1. **Motor precision.** As we demonstrated, the motor precision makes a major contribution to the total systematic error. Whenever a setting has to be repeated (that is, a wave plate has to go back to a previous position after having been moved), an uncertainty in the wave plate angle affects the result of the experiment. When low precision motors were used, in our first experiment, then the propagated uncertainty on the CHSH parameter due to their $0.2^\circ$ precision is 0.02, which is larger than the statistical error. For the other experiments, motors with precision of $0.02^\circ$ were used. This gives a propagated systematic error on $S$ of 0.002 and is comparable to the statistical uncertainty. However, when repeating the measurement settings $N$ times, this error is suppressed by a factor of $\sqrt{N}$.

2. **Detector dark counts.** Each avalanche photodiode used in the experiment has a dark count rate of approximately 500 Hz. The rate of coincidences coming from a true detection and a dark count can then be estimated to be of the order of $10^{-11}$ Hz and is therefore negligible.

3. **Higher order down-conversion events.** At the low pump power used in the experiment, the rate of so-called accidental coincidences was fairly minimal, of the order of 0.1 Hz or less. This can be estimated by counting coincident events between detectors in the same measurement station. Such a low rate does nevertheless affect the final result by lowering visibilities and therefore the violation. This can be seen as if the source were more noisy. Therefore, we do not consider this as a systematic error and we do not to subtract this noise in our evaluation.

**Conclusions.**—Experimental setups aiming for high values of the Bell parameter are susceptible to systematic errors causing apparent signaling and therefore making previous quantitative estimation of the Bell parameter questionable. Specifically, these systematic errors cannot be explained by shot noise, but rather appear due to common limitations in the experimental setups and typical methods for acquiring the experimental data. For modern quantitative applications of Bell tests, these problems have to be addressed and accounted for to obtain universally viable estimates for the Bell parameter. Here we have experimentally identified the most common sources of systematic errors leading to apparent signaling and described how to address (that is, minimize the impact of) them. We have obtained a value for the Bell CHSH parameter of $S = 2.812 \pm 0.003$ and negligible systematic errors. This is the highest value we know that is free of the signaling loophole. For future Bell experiments aimed at quantitative applications, we should avoid the sources of systematic errors described in this paper as much as possible and understand and estimate residual systematic errors.

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