A TRANSIENCE CONDITION FOR A CLASS OF ONE-DIMENSIONAL SYMMETRIC LÉVY PROCESSES

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An \( \mathbb{R}^d \)-valued, \( d \geq 1 \), Lévy process \( \{L_t\}_{t \geq 0} \) defined on a probability space \((\Omega, \mathcal{F}, \mathbb{P})\) is said to be transient if \( \lim_{t \to \infty} |L_t| = \infty \) \( \mathbb{P} \)-a.s. and recurrent if \( \liminf_{t \to \infty} |L_t| = 0 \) \( \mathbb{P} \)-a.s. It is well known that every Lévy process is either transient or recurrent.

Further, every Lévy process \( \{L_t\}_{t \geq 0} \) can be completely and uniquely characterized through the characteristic function of a single random variable \( L_t, t > 0 \), that is, by the famous Lévy-Khintchine formula we have

\[
\mathbb{E}\left[\exp\left\{i\langle \xi, L_t \rangle\right]\right] = \exp\{-t\psi(\xi)\} \quad \text{for all} \quad t \geq 0,
\]

where

\[
\psi(\xi) = i\langle \xi, b \rangle + \frac{1}{2} \langle \xi, c\xi \rangle + \int_{\mathbb{R}^d} \left(1 - \exp\left\{i\langle \xi, y \rangle\right\} + i\langle \xi, y \rangle 1_{\{|y| \leq 1\}}(y)\right) \nu(dy).
\]

The characterization of the transience and recurrence property in terms of the characteristic exponent \( \psi(\xi) \) is given by the well-known Chung-Fuchs criterion: A Lévy process \( \{L_t\}_{t \geq 0} \) is transient if and only if

\[
\int_{\{|\xi| < a\}} \text{Re}\left(\frac{1}{\psi(\xi)}\right) d\xi < \infty \quad \text{for some} \quad a > 0.
\]

In many cases this criterion is not applicable, that is, it is not always easy to compute the above integral. In this talk, we present a transience condition for a class of one-dimensional symmetric Lévy processes in terms of the Lévy measure \( \nu(dy) \).

**Theorem 1.** Let \( \{L_t\}_{t \geq 0} \) be a one-dimensional symmetric Lévy process with the Lévy measure \( \nu(dy) = f(y)dy \) or \( \nu(n) = p_n \), where \( f(y) \) is such that \( f(y) > 0 \) a.e. and \( \{p_n\}_{n \geq 1} \) is such that \( p_n > 0 \) for all \( n \geq 1 \). Then, \( \{L_t\}_{t \geq 0} \) is transient if

\[
\int_1^{\infty} \frac{dy}{y^3f(y)} < \infty \quad \text{or} \quad \sum_{n=1}^{\infty} \frac{1}{n^3p_n} < \infty.
\]

As a simple consequence of Theorem 1 we get a new proof for the transience property of one-dimensional symmetric stable Lévy processes.

**Corollary 1.** A one-dimensional symmetric \( \alpha \)-stable Lévy process is transient if \( \alpha < 1 \).

Also, let us remark that the analogous transience condition holds for one-dimensional symmetric random walks.

**References**

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