Dynamical screening in a quark gluon plasma

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Abstract. We calculate the screening potential of a fast parton moving through a quark-gluon plasma in the framework of semi-classical transport theory. We found an anisotropic potential showing a minimum in the direction of the parton velocity. Possible consequences of this potential on binary states in a quark-gluon plasma are discussed.

Screening of charges in a plasma is an important collective effect in plasma physics. In a classical isotropic and homogeneous plasma the screening potential of a point-like test charge \( Q \) at rest is modified from Coulomb potential into a Yukawa potential [1] as

\[
\phi(r) = \frac{Q}{r} \exp(-m_D r)
\]

with the Debye mass (inverse screening length) \( m_D = \sqrt{\frac{\hbar}{c k_B T}} = 1 \). In the quark-gluon plasma the Debye mass of a chromoelectric charge follows from the static limit of the longitudinal polarization tensor which in the high temperature limit and one loop order is given by \([2, 3, 4, 5]\),

\[
\Pi_{00}(\omega = 0, k) = -m_D^2 = -g^2 T^2 \left(1 + \frac{n_f}{6}\right),
\]

where \( g \) is the strong coupling constant and \( n_f \) the number of light quark flavors in the QGP with \( m_q \ll T \).

The modification of the confinement potential below the critical temperature into a Yukawa potential above the critical temperature might have important consequences for the discovery of the QGP in relativistic heavy-ion collisions. Bound states of heavy quarks, in particular the \( J/\psi \) meson, which are produced in the initial hard scattering processes of the collision, will be dissociated in the QGP due to screening of the quark potential and break-up by energetic gluons [6]. Hence the suppression of \( J/\psi \) mesons have been proposed as one of the most promising signatures for the QGP formation [7]. On the other hand, the formation of colored bound states, e.g., \( qq, \bar{q}q, gg \), of partons at rest has also been claimed [8] above the critical temperature \((2T_c - 3T_c)\) by analyzing lattice data, indicating that the plasma behaves as a strongly coupled quark-gluon plasma.

Earlier in most calculations of the screening potential in the QGP, the test charge was assumed to be at rest. However, quarks and gluons coming from initial hard processes receive a transverse momentum which causes them to propagate through

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the QGP \cite{9}. In addition, hydrodynamical models predict a radial outward flow in the fireball \cite{10}. Hence, it is of great interest to estimate the screening potential of a parton moving \cite{11} relatively to the QGP. Chu and Matsui \cite{12} have used the Vlasov equation to investigate dynamic Debye screening for a heavy quark-antiquark pair traversing a quark-gluon plasma. They found that the screening potential becomes strongly anisotropic.

The screening potential of a moving charge $Q$ with velocity $v$ follows from the linearized Vlasov and Poisson equations as \cite{13, 14}

$$
\phi(\vec{r}, t; \vec{v}) = \frac{Q}{2\pi^2} \int d^3k \frac{\exp[-i\vec{k} \cdot (\vec{r} - \vec{v}t)]}{k^2 \text{Re}[\epsilon_l(\omega = \vec{k} \cdot \vec{v}, k)]}.
$$

(3)

The dielectric function following from the semi-classical Vlasov equation describing a collisionless plasma is related to the high-temperature limit of the polarization tensor. For example, the longitudinal dielectric function following from the Vlasov equation is given by \cite{3, 4, 5}

$$
\epsilon_l(\omega, k) = 1 - \frac{\Pi_{00}(\omega, k)}{k^2} = 1 + \frac{m_D^2}{k^2} \left(1 - \frac{\omega}{2k} \ln\frac{\omega + k}{\omega - k}\right),
$$

(4)

where the only non-classical inputs are Fermi and Bose distributions instead of the Boltzmann distribution. The gluon self-energies derived within the hard thermal loop approximation \cite{15} have been shown to be gauge invariant and the dielectric functions obtained from these are therefore also gauge invariant.

For small velocities, $v \to 0$, i.e. $\omega \ll k$, we obtain

$$
\epsilon_l(\omega \ll k) = 1 + \frac{m_D^2}{k^2},
$$

(5)

from which again the (shifted) Yukawa potential results \cite{14}

$$
\phi(\vec{r}, t; \vec{v}) = \frac{Q}{|\vec{r} - \vec{v}t|} \exp(-m_D|\vec{r} - \vec{v}t|).
$$

(6)

In the general case, for parton velocities $v$ between 0 and 1, we have to solve \cite{3} together with \cite{4} numerically. Since the potential is not isotropic anymore due to the velocity vector $\vec{v}$, we will restrict ourselves only to two cases, $\vec{r}$ parallel to $\vec{v}$ and $\vec{r}$ perpendicular to $\vec{v}$, i.e., for illustration we consider \cite{11} the screening potential only in the direction of the moving parton or perpendicular to it.

In Fig.1 the screening potential $\phi/Q$ in $\vec{v}$-direction is shown as a function of $r' = r - vt$, where $r = |\vec{r}|$, between 0 and 6 fm for various velocities. For illustration we have chosen a strong fine structure constant $\alpha_s = g^2/(4\pi) = 0.3$, a temperature $T = 0.25$ GeV, and the number of quark flavors $n_f = 2$. The shifted potentials depend only on $v$ and not on $t$ as it should be the case in a homogeneous and isotropic plasma. For $r' < 1$ fm one observes that the fall-off of the potential is stronger than for a parton at rest. The reason for this behavior is the fact that there is a stronger screening in the direction of the moving parton due to an enhancement of the particle density in the rest frame of the moving parton.

In addition, a minimum in the screening potential at $r' > 1$ fm shows up. For example, for $v = 0.8$ this minimum is at about 1.5 fm with a depth of about 8 MeV. A minimum in the screening potential is also known from non-relativistic, complex plasmas, where an attractive potential even between equal charges can be found if the finite extension of the charges is considered \cite{16}. A similar screening potential was found for a color charge at rest in Ref.\cite{17}, where a polarization tensor

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beyond the high-temperature limit was used. However, this approach has its limitations as a gauge dependent and incomplete (within the order of the coupling constant) approximation for the polarization tensor was used \[15\]. Obviously, a minimum in the interparticle potential in a relativistic or non-relativistic plasma is a general feature if one goes beyond the Debye-Hückel approximation by either taking quantum effects, finite velocities, or finite sizes of the particles into account.

We also note that Chu and Matsui \[12\] did not report the existence of a minimum in the potential of a quark traversing the QGP. However, in their Fig.1(d) a negative value of the potential of a fast quark \((v = 0.9)\) in the direction of the quark velocity is shown. Since the potential has to tend to zero for large distances, this implies a minimum in the screening potential. This minimum was not found because the potential was plotted only for the limited range \(0 < r < 1/m_d \approx 0.35\) fm for our choice of the parameters.

The minimum could give rise to bound states, e.g., of diquarks, if thermal fluctuations do not destroy them. The two-body potential, associated with the dipole fields created by two test charges \(Q_1\) and \(Q_2\) at \(\vec{r}_1\) and \(\vec{r}_2\) with velocities \(\vec{v}_1\) and \(\vec{v}_2\), can be written as

\[
\Phi(\vec{r}_1 - \vec{r}_2, \vec{v}_1 - \vec{v}_2, t) = \frac{Q_1 Q_2}{4\pi^2} \int d^3k \left\{ \frac{e^{i\vec{k} \cdot (\vec{r}_1 - \vec{r}_2) - (\vec{v}_1 - \vec{v}_2)t}}{k^2 \text{Re} [\epsilon_l(\omega = \vec{k} \cdot \vec{v}_1, k)]} + \frac{e^{i\vec{k} \cdot (\vec{r}_1 - \vec{r}_2) - (\vec{v}_1 - \vec{v}_2)t}}{k^2 \text{Re} [\epsilon_l(\omega = \vec{k} \cdot \vec{v}_2, k)]} \right\}.
\]

For comoving quarks, \(\vec{v}_1 = \vec{v}_2\), this two-body potential reduces to the one-body potential \[3\], showing the attraction between the quarks which could give rise to a bound state. Colored bound states, e.g., diquarks, of partons at rest have also been claimed by analyzing lattice data \[8\]. For a comoving diquark system, where the interaction between two color charges \(Q_1 Q_2 > 0\), the two-body potential shows an attractive minimum and may therefore indicate a possible appearance of a diquark bound state as compared to partons at rest where the two-body potential is only repulsive. On the other hand, for a quark-antiquark system, where \(Q_1 Q_2 < 0\), the two-body potential is inverted, showing a maximum. This may lead to short living mesonic resonances and an enhancement of the attraction between quarks and antiquarks of mesonic states moving through the QGP.

The results for \(\vec{r}\) perpendicular to \(\vec{v}\) are shown in Fig.2, where the potential is shown as a function of \(|\vec{r} - \vec{v}t| = \sqrt{r^2 + v^2t^2}\) between 0.1 and 1 fm. Here we consider only the case \(t = 0\) since at \(t > 0\) and \(v > 0\) there is no singularity in the potential due to \(\sqrt{r^2 + v^2t^2} > 0\) for all \(r\). Hence the potential is cut-off artificially at small distances if plotted as a function of \(\sqrt{r^2 + v^2t^2}\). In contrast to the parallel case (Fig.1) the fall-off of the potential at larger values of \(v\) is less steep, i.e. the screening is reduced as it is expected since the formation of the screening cloud is suppressed at large velocities. Also no minimum in the potential is found.

Summarizing, we have calculated the screening potential of a fast color charge moving through the QGP from semi-classical transport theory corresponding to the high-temperature limit. As in Ref. \[12\] the potential is found to be strongly anisotropic. The screening is reduced in the perpendicular direction of the moving parton but increased in the direction of the moving parton, which may lead to a modification of the \(J/\psi\) suppression. In addition, we found a new feature of the screening potential of a fast parton in a QGP: a minimum in the potential shows up which could give rise to
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Figure 1. Screening potential parallel to the velocity of the moving parton in a QGP as a function of $r - vt$ (0 to 6 fm) for $v = 0$, 0.5, and 0.8.

Figure 2. Screening potential perpendicular to the velocity of the moving parton in a QGP as a function of $|\vec{r} - \vec{vt}|$ (0.1 to 1 fm) for $v = 0$, 0.5, and 0.8.

bound states of, for example, diquarks if not destroyed by thermal fluctuations. For a quark-antiquark pair this minimum turns into a maximum which could cause short living mesonic resonances. Combining the effect of reduced screening in perpendicular direction and the presence of a maximum in parallel direction we expect a stronger binding of moving $J/\psi$ mesons with respect to the QGP than of $J/\psi$ mesons at rest. The consequences, e.g., for the $J/\psi$ yield should be investigated in more detail using hydrodynamical models or event generators for the space-time evolution of the fireball. Finally, let us note that our results also apply to other ultrarelativistic plasmas such as an electron-positron plasma in Supernova explosions. In this case one simply has to replace the Debye mass $m_D$ by $eT$, where $e$ is the electron charge.

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