Research Article

Improved Particle Swarm Optimization Algorithm Based on Last-Eliminated Principle and Enhanced Information Sharing

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In this study, an improved eliminate particle swarm optimization (IEPSO) is proposed on the basis of the last-eliminated principle to solve optimization problems in engineering design. During optimization, the IEPSO enhances information communication among populations and maintains population diversity to overcome the limitations of classical optimization algorithms in solving multiparameter, strong coupling, and nonlinear engineering optimization problems. These limitations include advanced convergence and the tendency to easily fall into local optimization. The parameters involved in the imported “local-global information sharing” term are analyzed, and the principle of parameter selection for performance is determined. The performances of the IEPSO and classical optimization algorithms are then tested by using multiple sets of classical functions to verify the global search performance of the IEPSO. The simulation test results and those of the improved classical optimization algorithms are compared and analyzed to verify the advanced performance of the IEPSO algorithm.

1. Introduction

The development of industrial society has led to the successful application of the optimal design methods to diverse engineering practices, such as path planning, structural design, control theory, and control engineering [1–10]. In 1995, the foraging behavior of bird swarm inspired Kennedy and Eberhart to propose the particle swarm optimization (PSO) algorithm. PSO requires few parameter adjustments and is easy to implement; hence, it is the most commonly used swarm intelligence algorithm [11–20]. However, in practical applications, most problems are complicated design problems with multiple parameters, strong coupling, and nonlinearity. Therefore, improving the global optimization capability of an optimization algorithm is important in solving complex engineering optimization problems. To improve the capability of traditional PSO, many scholars have proposed improvement strategies, including the adjustment of parameters and combinations of various mechanisms.

Shi and Eberhart [21] proposed an inertial weight improvement strategy (SPSO) with strong global search capability at the beginning of an iteration, strong local search capability in the latter iteration, and fine search near the optimal solution. Although the SPSO improves the convergence speed of the algorithm, the “premature” phenomenon remains. Zhang [22] proposed an improved PSO algorithm with adaptive inertial weight that is based on Bayesian technology to balance the development and exploration capability of populations. Ratnawecra [23] proposed a linear adjustment method for learning factors. In the early stages of the iteration, the particle flight was mainly based on the historical information of the particle itself, and the latter particle flight was mainly based on the social information between the particle and the global optimal particle. However, this method still has defects. The best fit for the initial global search is similar to the local optimum. Moreover, convergence is only limited to some optimal regions rather than globally, thereby causing the PSO algorithm to fall into the local extrema. Chen and Ke [24] proposed a chaotic dynamic weight (CDW) PSO (CDW-PSO) algorithm. Chaotic maps and dynamic weights were used to modify the search process. Although CDW-PSO
2. IEPSO

2.1. Standard PSO. The initial population of the PSO algorithm is randomized. The IEPSO updates the position and speed of the particle swarm by adaptive learning, as shown in the following formulas:

\[
\begin{align*}
\vec{v}_{id}^{t+1} &= \omega \vec{v}_{id}^{t} + C_1 R_1 (\vec{p}_{id}^{t} - \vec{x}_{id}^{t}) + C_2 R_2 (\vec{p}_{gd}^{t} - \vec{x}_{id}^{t}), \\
\vec{x}_{id}^{t+1} &= \vec{x}_{id}^{t} + \vec{v}_{id}^{t+1},
\end{align*}
\]

where \( \omega \) is the inertial weight, \( C_1 \) and \( C_2 \) are the acceleration terms, \( R_1 \) and \( R_2 \) are the random variables uniformly distributed in the range of (0, 1), \( \vec{p}_{gd}^{t} \) is the global better position, \( \vec{p}_{id}^{t} \) is the particle that finds the best position in history, \( \vec{x}_{id}^{t} \) is the particle position in the current iteration, and \( \vec{v}_{id}^{t+1} \) is the particle update speed at the next iteration.

2.2. IEPSO. The IEPSO algorithm is mainly based on the last-eliminated principle and enhances the local-global information sharing capability to improve its global optimization performance. The specific implementation of the IEPSO algorithm is shown in Figure 1.

The position and velocity of particles in a population are randomly initialized, and the fitness value of the particles is calculated. Information on the current individual and global optimal particles, including their positions and fitness values, is saved. Then, the particle swarm operation is conducted. In the IEPSO algorithm, Formula (2) is used to update the speed to balance the exploration and exploitation capabilities of the particles in the global optimization process. Formula (3) is the local-global information sharing term:
Formula (2) comprises four parts, namely, the inheritance of the previous speed, particle self-cognition, local information sharing, and “local-global information sharing.”

The IEPSO algorithm is not limited to one-way communication between global and individual particles. The local-global information sharing term \( \varphi_3 \) is added to the
information exchange between the local optimum and global optimal particles obtained by the current iteration, and the population velocity is updated by Formula (2). In the early stage of the algorithm, the entire search space is searched at a relatively high speed to determine the approximate range of the optimal solution; the result is beneficial for global search. In the latter stage, most of the particle search space is gradually reduced and concentrated in the neighborhood of the optimal value for deep search; the result is beneficial for local search.

The particles that have not exceeded the predetermined range after the speed update continue to retain their original speed. The maximum value of the velocity is assigned to the particle that is beyond the predetermined range after the speed is updated. The particles that have not exceeded the predetermined range after the location update continue to retain their original positions. When the particles are beyond the predetermined range, inferior particles are eliminated by adding new particles to the population within the predetermined range, thereby forming a new population. The fitness value of the new population is recalculated, and the information of the individual particle and its global optimal position and fitness value obtained by the current iteration are preserved. In all the algorithms, particles have good global search capability at the beginning of the iteration, and as individual particles move closer to the local optimal particle, the algorithms gradually lose particle diversity. On the basis of the idea of population variation of the traditional genetic algorithm (GA), the last-eliminated principle is applied in the IEPSO algorithm to maintain population diversity. When the PSO satisfies the local convergence condition, the optimal value obtained at this time may be the local optimal value. Particle population diversity is maintained by using the particle fitness function as the evaluation criterion, thereby eliminating particles with poor fitness or high similarity. New particles are added to a new species in a predetermined range, and the particle swarm operations are reexecuted. If the number of the current iteration reaches the required predefined convergence accuracy, the iteration is stopped, and the optimal solution is produced. The complexity and runtime of the algorithm increase due to the increased local-global information sharing and the last-eliminated principle. Nevertheless, experimental results show that the improved method can enhance the accuracy of the algorithm.

3. Experimental Study

Eleven test functions are adopted in this study to test the performance of the proposed IEPSO. In this test, \( f_1 \)–\( f_5 \) are unimodal functions, whereas \( f_6 \)–\( f_{11} \) are multimodal functions. \( f_6 \) (Griewank) is a multimodal function with multiple local extrema, in which achieving the theoretically global optimum is difficult. \( f_7 \) (Rastrigin) possesses several local minima, in which finding the global optimal value is difficult. \( f_9 \) (Ackley) is an almost flat area modulated by a cosine wave to form a hole or a peak; the surface is uneven, and entry to a local optimum during optimization is easy. \( f_{11} \) (Cmfun) possesses multiple local extrema around the global extremum point, and falling into the local optimum is easy. Table 1 presents the 11 test functions, where \( D \) is the space dimension, \( S \) is the search range, and \( CF \) is the theoretically optimal value.

3.1. Parameter Influence Analysis of Local-Global Information Sharing Term. This study proposes the addition of a local-global information sharing term, which involves the parameter \( C_3 \). Therefore, the following exploration is conducted in a manner in which \( C_3 \) is selected by using the 11 test functions.

1. When \( C_3 \) takes a constant value, constant 2 is selected.
2. The linear variation formula of \( C_3 \) is as follows:

\[
C_3 = k \left( C_{3,\text{start}} - (C_{3,\text{start}} - C_{3,\text{end}}) \times \frac{t}{t_{\text{max}}} \right),
\]

where \( k \) is the control factor. When \( k = 1 \), \( C_3 \) is a linearly decreasing function; when \( k = -1 \), \( C_3 \) is a linearly increasing function. \( C_{3,\text{start}} \) and \( C_{3,\text{end}} \) are the initial and termination values of \( C_3 \), respectively. \( T \) is the iteration number, and \( t_{\text{max}} \) is the maximum number of iterations.

Tables 2 and 3 and Figure 2 show that \( C_3 \) is a constant that linearly declines and linearly increases in three cases. When the parameter \( C_3 \) of the local-global information sharing term is a linearly decreasing function, the average fitness value of the testing function is optimal, and the convergence speed and capability to jump out of the local extrema are higher than those in the other two cases. When \( C_3 \) takes a constant, the algorithm cannot balance the global and local search, resulting in a “premature” phenomenon. When \( C_3 \) adopts the linearly decreasing form, the entire area can be quickly searched at an early stage, and close attention is paid to local search in the latter part of the iteration to enhance the deep search ability of the algorithm. While \( C_3 \) adopts a linearly increasing form, it focuses on the global-local information exchange in the latter stage of the iteration. Although this condition can increase the deep search ability of the algorithm, it will cause the convergence speed to stagnate. Therefore, compared with the linearly increasing form, the linearly decreasing form shows a simulation curve that converges faster and with higher precision.

Therefore, the selection rules of the parameter \( C_3 \) of local-global information sharing in a decreasing function are investigated in this study. The nonlinear variation formula of \( C_3 \) is as follows:

\[
C_3 = (C_{3,\text{start}} - C_{3,\text{end}}) \times \tan \left( 0.875 \times \left( 1 - \frac{t}{t_{\text{max}}} \right)^k \right) + C_{3,\text{end}},
\]

where \( C_{3,\text{start}} \) and \( C_{3,\text{end}} \) are the initial and termination values of the acceleration term \( C_3 \), respectively, and \( k \) is the control factor. When \( k = 0.2 \), \( C_3 \) is a concave decreasing function; when \( k = 2 \), \( C_3 \) is a convex decreasing function. \( t \) is the iteration number, and \( t_{\text{max}} \) is the maximum number of iterations.
Table 1: 11 test functions.

| No. | Test function | S         | CF    |
|-----|---------------|-----------|-------|
| f1  | Sphere: \( f_1(x) = \sum_{i=1}^{D} x_i^2 \) [10] | \([-100,100]^D\) | 0     |
| f2  | Schaffer: \( f(x,y) = 0.5 + (\sin^2 \sqrt{x^2 + y^2} - 0.5)/\left(1 + 0.001 \left(x^2 + y^2\right)\right)^2 \) [33] | \([-100,100]^D\) | 0     |
| f3  | Step: \( f_3(x) = \sum_{i=1}^{D} \left|x_i + 0.5\right|^2 \) [10] | \([-100,100]^D\) | 0     |
| f4  | SumSquares: \( f_4(x) = \sum_{i=1}^{D} x_i^2 \) [10] | \([-10,10]^D\) | 0     |
| f5  | Zakharov: \( f_5(x) = \sum_{i=1}^{D} x_i^2 + \left(\sum_{i=1}^{D} 0.5 \times i x_i\right)^2 + \left(\sum_{i=1}^{D} 0.5 \times i x_i\right)^4 \) [10] | \([-100,100]^D\) | 0     |
| f6  | Griewank: \( f_6(x) = 1/\sqrt{\sum_{i=1}^{D} x_i^2} - \prod_{i=1}^{D} \cos(x_i/\sqrt{i}) + 1 \) [10] | \([-600,600]^D\) | 0     |
| f7  | Rastrigin: \( f_7(x) = \sum_{i=1}^{D} \left[x_i^2 + 10 \cos(2\pi x_i) + 10\right] \) [10] | \([-5.12,5.12]^D\) | 0     |
| f8  | Alpine: \( f_8(x) = \sum_{i=1}^{D} \left|x_i \sin(x_i) + 0.1x_i\right| \) [6] | \([-10,10]^D\) | 0     |
| f9  | Shubert: \( f_9(x,y) = \sum_{i=1}^{D} \left[\sin \left(i + 1\right)x_i + i\right] \times \left[\sin \left(i + 1\right)y_i + i\right] \) | \([-10,10]^D\) | -186.731 |
| f10 | Ackley: \( f_{10}(x) = -20 \exp\left(-0.2 \sqrt{\sum_{i=1}^{D} \left|x_i\right|^2}\right) - \exp\left(\sum_{i=1}^{D} \cos \left(2\pi x_i\right)\right) + 20 + e \) [10] | \([-32,32]^D\) | 0     |
| f11 | Cmfun: \( f_{11}(x,y) = x \sin \left(\sqrt{|x|}\right) + y \sin \left(\sqrt{|y|}\right) \) | \([-500,500]\) | -837.966 |

Table 2: Unimodal test functions.

| Functions | Criteria | \( C_a = 2 \) | \( C_a = 2 \rightarrow k = -1 \) | \( C_a = 2 \rightarrow k = 1 \) |
|-----------|----------|---------------|-------------------------------|-------------------------------|
| f1        | Mean     | 7.22E + 02    | 1.07E - 06                    | 4.50E - 20                    |
|           | SD       | 3.97E + 04    | 1.11E - 12                    | 3.75E - 16                    |
|           | Best     | 4.05E + 02    | 2.41E - 08                    | 1.55E - 25                    |
| f2        | Mean     | 2.50E - 06    | 2.22E - 17                    | 0                             |
|           | SD       | 2.32E - 12    | 2.59E - 33                    | 0                             |
|           | Best     | 2.85E - 07    | 0                             | 0                             |
| f3        | Mean     | 1.99E + 02    | 8.03E - 07                    | 1.82E - 20                    |
|           | SD       | 2.15E + 04    | 1.04E - 12                    | 1.05E - 39                    |
|           | Best     | 35.81         | 3.95E - 08                    | 3.22E - 24                    |
| f4        | Mean     | 7.110         | 2.45E - 08                    | 8.20E - 20                    |
|           | SD       | 9.57          | 7.95E - 16                    | 5.11E - 38                    |
|           | Best     | 1.47          | 4.08E - 09                    | 8.43E - 26                    |
| f5        | Mean     | 1.74E + 03    | 3.86E - 04                    | 5.56E - 11                    |
|           | SD       | 2.44E + 05    | 1.49E - 07                    | 4.88E - 14                    |
|           | Best     | 8.29E + 02    | 9.78E - 06                    | 3.54E - 11                    |

Table 3: Multimodal test functions.

| Functions | Criteria | \( C_a = 2 \) | \( C_a = 2 \rightarrow k = -1 \) | \( C_a = 2 \rightarrow k = 1 \) |
|-----------|----------|---------------|-------------------------------|-------------------------------|
| f6        | Mean     | 1.10          | 8.18E - 02                    | 4.92E - 02                    |
|           | SD       | 4.66E + 03    | 8.37E - 04                    | 5.96E - 04                    |
|           | Best     | 0.96          | 4.33E - 02                    | 1.23E - 02                    |
| f7        | Mean     | 35.03         | 4.10                         | 1.92E - 04                    |
|           | SD       | 8.44          | 2.5461                       | 5.649                         |
|           | Best     | 29.67         | 2.057                        | 2.25E - 05                    |
| f8        | Mean     | 2.93          | 1.33E - 03                    | 5.28E - 10                    |
|           | SD       | 0.30          | 3.10E - 08                    | 2.23E - 12                    |
|           | Best     | 2.02          | 1.34E - 05                    | 5.83E - 13                    |
| f9        | Mean     | -186.7295     | -186.7309                    | -186.7309                    |
|           | SD       | 1.20E - 06    | 0                            | -186.7309                    |
|           | Best     | -186.7307     | 0                            | -186.7309                    |
| f10       | Mean     | 7.649         | 2.35E - 04                    | 1.84E - 11                    |
|           | SD       | 0.415         | 4.39E - 09                    | 2.27E - 22                    |
|           | Best     | 6.513         | 5.73E - 05                    | 2.50E - 12                    |
| f11       | Mean     | -837.9658     | -837.9658                    | -837.9658                    |
|           | SD       | 4.50E - 09    | 0                            | -837.9658                    |
|           | Best     | -837.9658     | 0                            | -837.9658                    |

Table 4 shows that when \( C_a \) is a convex function, the precision and robustness of the algorithm can obtain satisfactory results on \( f_1\)–\( f_5 \). Table 5 shows that when \( C_a \) is a convex function, the algorithm obtains a satisfactory solution and shows a fast convergence rate on \( f_6, f_9, f_{10}, f_{11} \) and \( f_{11} \). In the unimodal test function, the IEPSO algorithm does...
Figure 2: Continued.
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Figure 2: 11 test functions: (a) $f_1$ sphere function; (b) $f_2$ Schaffer function; (c) $f_3$ step function; (d) $f_4$ SumSquares function; (e) $f_5$ Zakharov function; (f) $f_6$ Griewank function; (g) $f_7$ Rastrigin function; (h) $f_8$ alpine function; (i) $f_9$ Shubert function; (j) $f_{10}$ Ackley function; (k) $f_{11}$ Cmfun function.

Table 4: Unimodal test functions.

| Functions | Criteria | $C_3 = 2$ $k = 0.2$ | $C_3 = 2$ $k = 2$ | $C_3 = 2$ $k = 1$ |
|-----------|----------|----------------------|----------------------|----------------------|
| $f_1$     | Mean     | $2.66E-20$           | $5.51E-10$           | $4.50E-20$           |
|           | SD       | $2.65E-39$           | $2.87E-19$           | $3.75E-16$           |
|           | Best     | $9.12E-24$           | $1.38E-11$           | $1.55E-25$           |
| $f_2$     | Mean     | 0                    | 0                    | 0                    |
|           | SD       | 0                    | 0                    | 0                    |
|           | Best     | 0                    | 0                    | 0                    |
| $f_3$     | Mean     | $6.21E-19$           | $6.04E-10$           | $1.82E-20$           |
|           | SD       | $2.63E-36$           | $7.79E-19$           | $1.05E-39$           |
|           | Best     | $1.81E-27$           | $3.08E-11$           | $3.22E-24$           |
| $f_4$     | Mean     | $1.70E-21$           | $2.42E-11$           | $8.20E-20$           |
|           | SD       | $1.31E-41$           | $4.40E-22$           | $5.11E-38$           |
|           | Best     | $2.82E-29$           | $4.36E-12$           | $8.43E-26$           |
| $f_5$     | Mean     | $1.65E-10$           | $2.83E-11$           | $5.56E-11$           |
|           | SD       | $3.30E-20$           | $3.59E-11$           | $4.88E-14$           |
|           | Best     | $2.17E-11$           | $1.00E-11$           | $3.54E-11$           |
not show its advantages because of its strong deep search capability. In the complex multimodal test function, when the convex function is used in $C_0$, the downward trend is slow in the early stage, thus benefiting the global search, and the downward speed increases in the later stage, thus benefiting the local search. When the concave function is used for $C_0$, the descent speed is fast in the early stage. Although the search speed is improved, the coverage area of the search is reduced, thereby leading to the convergence of the algorithm to the nonoptimal value. From the simulation diagrams (f)–(k), the convergence speed is observed to be slightly slow when $C_3$ is a convex function, but its ability to jump out of the local extremum and the accuracy of the global search are higher than those in the other two cases. When $C_3$ is a concave function, the convergence speed is faster than those in the other two cases, and the search accuracy is lower than that when $C_3$ is a convex function.

### Table 5: Multimodal test functions.

| Functions | Criteria | $C_3 = 2 \sim 0$ $k = 0.2$ | $C_3 = 2 \sim 0$ $k = 2$ | $C_3 = 2 \sim 0$ $k = 1$ |
|-----------|----------|-----------------------------|-----------------------------|-----------------------------|
|           | Mean     | $4.19E - 02$                | $4.79E - 02$                | $4.92E - 02$                |
|           | SD       | $3.43E - 04$                | $7.07E - 04$                | $5.96E - 04$                |
|           | Best     | $1.25E - 02$                | $5.7E - 03$                 | $1.23E - 02$                |
| $f_6$     | Mean     | $4.46E - 03$                | $5.00E - 05$                | $1.9Ee - 04$                |
|           | SD       | $1.73E - 04$                | $3.03E - 06$                | $5.649$                     |
|           | Best     | $2.31E - 12$                | $3.89E - 11$                | $2.25E - 05$                |
| $f_7$     | Mean     | $2.42E - 10$                | $3.74E - 10$                | $5.28E - 10$                |
|           | SD       | $6.74E - 20$                | $2.47E - 12$                | $2.23E - 12$                |
|           | Best     | $3.71E - 16$                | $4.36E - 11$                | $5.83E - 13$                |
| $f_8$     | Mean     | $1.13E - 11$                | $2.05E - 10$                | $1.84E - 11$                |
|           | SD       | $2.21E - 22$                | $4.37E - 12$                | $2.27E - 22$                |
|           | Best     | $5.06E - 14$                | $1.75E - 10$                | $2.50E - 12$                |
| $f_9$     | Mean     | $-837.9658$                 | $-837.9658$                 | $-837.9658$                 |
|           | SD       | 0                           | 0                           | 0                           |
|           | Best     | $-837.9658$                 | $-837.9658$                 | $-837.9658$                 |
| $f_{10}$  | Mean     | $-186.7309$                 | $-186.7309$                 | $-186.7309$                 |
|           | SD       | 0                           | 0                           | 0                           |
|           | Best     | $-186.7309$                 | $-186.7309$                 | $-186.7309$                 |
| $f_{11}$  | Mean     | $-837.9658$                 | $-837.9658$                 | $-837.9658$                 |
|           | SD       | 0                           | 0                           | 0                           |
|           | Best     | $-837.9658$                 | $-837.9658$                 | $-837.9658$                 |

3.2. Comparison of Test Results. The 11 test functions in Figure 1 are used to compare the IEPSO algorithm with classical PSO, SPSO, differential algorithm (DE), and GA. The DE, GA, and PSO algorithms are all stochastic intelligent optimization algorithms with population iterations. The evaluation criteria of algorithm performance include speed of convergence and size of individual population search coverage. The differential optimization algorithm has a low space complexity and obvious advantages in dealing with large-scale and complex optimization problems. The GA has good convergence when solving discrete, multipeak, and noise-containing optimization problems. Based on the traditional PSO algorithm, the SPSO algorithm achieves the balance between global search and local search by adjusting the inertial weight (Figures 3 and 4).

The experimental parameters of the five algorithms are set, as shown in Table 6. Each test function is run independently 10 times, and the average is recorded to reduce the data error. The iteration is stopped when the convergence condition meets the convergence accuracy. The best average fitness value of the five algorithms is blackened. The standard deviation, average fitness, and optimal value of each algorithm are shown in Tables 7 and 8; Figures 5 and 6 plot the convergence curves of the 11 test functions.

Table 7 shows that the IEPSO has the best performance on $f_1$, $f_2$, $f_3$, and $f_4$. The IEPSO algorithm obtains the theoretical optimal value on $f_2$. DE can search the global solution on $f_5$. The deep search capability of the IEPSO algorithm is considerably higher than that of the PSO and SPSO algorithms due to the increased global-local information sharing term and the last-eliminated principle. The crossover, mutation, and selection mechanisms make the DE algorithm perform well in the early stage of the global search. However, the diversity of the population declines in the latter stage because of population differences. The
Figure 4: Continued.
Figure 4: 11 test functions: (a) $f_1$ sphere function; (b) $f_2$ Schaffer function; (c) $f_3$ step function; (d) $f_4$ SumSquare function; (e) $f_5$ Zakharov function; (f) $f_6$ Griewank function; (g) $f_7$ Rastrigin function; (h) $f_8$ alpine function; (i) $f_9$ Shubert function; (j) $f_{10}$ Ackley function; (k) $f_{11}$ Cmfun function.
simulation diagrams (a)–(e) show that although the DE algorithm converges rapidly in the early stage, its global search performance in the later stage becomes lower than that of the IEPSO algorithm. When the GA is used to solve optimization problems, the individuals in the population fall into the local optimum and do not continue searching for the optimum solution. Therefore, in Figure 5, the simulation curve of the GA converges to the local optimum.

The test results in Table 8 indicate that the IEPSO has the best performance on $f_{6}$, $f_{7}$, $f_{8}$, $f_{10}$, and $f_{11}$, and that the DE and GA can obtain the theoretical optimal value on $f_{6}$ and $f_{11}$. Although the GA and IEPSO algorithm can obtain the global optimal value on $f_{6}$, the IEPSO algorithm is more robust than the GA is. As shown in the simulation curve of Figure 6, the diversity of the population is maintained because the supplementary particles in the population are stochastic when the local optimal solution converges gradually. The IEPSO algorithm can jump out of the local extrema points in the face of complex multimodal test functions, and the number of iterations required is correspondingly reduced.
Figure 5: Unimodal functions: (a) $f_1$ sphere function; (b) $f_2$ Schaffer function; (c) $f_3$ step function; (d) $f_4$ SumSquares function; (e) $f_5$ Zakharov function.
Figure 6: Multimodal functions: (a) $f_6$ Griewank function; (b) $f_7$ Rastrigin function; (c) $f_8$ alpine function; (d) $f_9$ Shubert function; (e) $f_{10}$ Ackley function; (f) $f_{11}$ Cmfun function.
Table 9: Three improved particle swarm algorithm test results.

| Functions | Criteria | IEPSO | DMSDL-PSO [25] | BHPSOWM [26] |
|-----------|----------|-------|----------------|--------------|
|           | Mean     | 8.92E-22 | 4.73E-10 | 42.40 |
|           | SD       | 2.65E-39 | 1.81E-09 | 52.11 |
| f_3       | Mean     | 6.21E-19 | 2.37E+03 | 7.61 |
|           | SD       | 2.63E-36 | 5.71E+02 | 0.07 |
| f_6       | Mean     | 4.19E-02 | 8.66E-05 | —   |
|           | SD       | 3.43E-04 | 2.96E-04 | —   |
| f_7       | Mean     | 4.46E-03 | 9.15E+01 | 76.18 |
|           | SD       | 1.73E-04 | 1.80E+01 | 26.75 |
| f_8       | Mean     | 2.42E-10 | 1.31E+02 | —   |
|           | SD       | 6.74E-20 | 5.82E+01 | —   |
| f_10      | Mean     | 1.13E-11 | 1.01E+00 | 1.72 |
|           | SD       | 2.21E-22 | 2.71E-01 | 0   |

In summary, the comparative results of the simulation analysis reveal that, with the application of the last-eliminated principle and the local-global information sharing term to the IEPSO, the proposed algorithm effectively overcomes the disadvantages of the classical algorithms, including their precarious convergence and tendency to fall into the local optimum. The IEPSO shows an ideal global optimization performance and indicates a high application value for solving practical engineering optimization problems.

### Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

### Conflicts of Interest

The authors declare that there are no conflicts of interest.

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