Shock Wave Structure in the Presence of Energetic Particles

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Abstract. Energetic particles that are not equilibrated with the thermal plasma (such as pickup ions (PUIs), anomalous cosmic rays (ACRs) and solar energetic particles (SEPs)) can modify the structure of collisionless shock waves. This is relevant to the inner and outer heliosphere and the Very Local Interstellar Medium (VLISM) where observations of shock waves in the e.g., the inner heliosphere show that the energetic particle component pressure is greater than the both the magnetic field and thermal gas pressure (Lario et al., 2015). Voyager 2 observations revealed that the heliospheric termination shock (HTS) is very broad and mediated by energetic particles. PUIs and SEPs contribute both a collisionless heat flux and a higher-order viscosity. We show that the incorporation of both effects can completely determine the structure of collisionless shocks mediated by energetic ions. Since the reduced form of the PUI mediated plasma model is structurally identical to the classical cosmic ray two-fluid model (Axford et al., (1982)), we note that the presence of viscosity at least formally eliminates the need of a gas sub-shock in the classical two-fluid model, including in that regime where three are possible. By considering parameters upstream of the heliospheric termination shock (HTS), we show that the thermal gas remains relatively cold and the shock is mediated by PUIs. We determine the structure of the weak interstellar shock observed by Voyager 1. We consider the inclusion of the thermal heat flux and viscosity to address the most general form of an energetic particle-thermal plasma two-fluid model.

1. Introduction

Energetic particles such as pickup ions (PUIs), solar energetic particles (SEPs), and anomalous cosmic rays (ACRs, i.e., very energetic PUIs) can modify the structure of shock waves. Recent observations show that some shocks in the heliosphere can be mediated by energetic particles. Lario et al. (2015) [1] observed that the energetic particle pressure is much greater than thermal gas and magnetic field pressure at several interplanetary shocks within 1 AU. The large energetic particle pressure suggests that these interplanetary shocks are mediated by energetic particles and their characteristics are different than expected from a simple MHD description. Voyager 2 measured the properties of the heliospheric termination shock (HTS), finding it was very broad and mediated by energetic particles [2, 3]. A numerical simulation of the HTS was presented by Florinski et al. (2009) [4] which suggested that the HTS is mediated by ACRs since it has a precursor extending about 0.3 AU upstream of the shock. Voyager 1 is now measuring the properties of the very local interstellar medium (VLISM, for a precise definition of the VLISM,
see [5]) region. Burlaga et al. (2013) [6] made the first observations of a shock outside of the heliosphere. The shock was very broad and weak and had a structure completely different than typical shocks in the heliosphere.

Zank et al (2014) [5] estimated the equilibration times for PUIs and thermal gas particles in three different regions (the supersonic solar wind beyond 10 AU, the inner heliosheath, and the VLISM). In these regions, PUIs are not equilibrated with the background plasma and should be treated as a separate component in the system. The approach presented here is applicable to all non-equilibrated plasma systems in which energetic particles such as SEPs, cosmic rays, and ACRs are included [7, 8]. Energetic particles introduce dissipation terms such as a collisionless heat flux and a collisionless stress tensor or viscosity into the plasma system. Dissipation typically determines the structure of shocks. The model developed by Zank et al. (2014) and (2016) [5, 8] is identical structurally to the cosmic ray two-fluid model [9, 10]. Therefore, we compare briefly our PUI modified shock model with the cosmic ray two-fluid model. Drury and Völk (1982) [10] showed that there are no physical solutions in some Mach number regimes connecting an upstream state to a downstream state. In the papers discussed above, the energetic particles are assumed to be nearly isotropic and a heat-conduction like term $\nabla \cdot (K \nabla P_c)$ is present as a dissipation term. The inclusion of the heat conduction term only is sometimes not enough to determine the shock structure and higher order terms should be included. In the case that a non-physical solution exists, a gas sub-shock has to be added [9] which means that the thermal gas is responsible for the dissipation of the system and is heated more than energetic particles at the sub-shock. However, this is not always physical, as appears to be the case at the HTS. Zank et al. (1996) [11] predicted that the thermal gas remains relatively cold with PUIs being heated through the shock. Voyager 2 confirmed the [11] predictions by observing that the thermal gas remains relatively cold [2]. A further limitation of the cosmic ray two-fluid model is that in some Mach number regimes, three different downstream states exist for one specific upstream state, and it is unclear how to choose a unique transition.

The effect of cosmic ray viscosity on the structure of a cold thermal gas shock was first studied by Jokipii & Williams (1992) [12]. However, they considered only a cold plasma in the absence of a magnetic field and all the shock transitions for a cold plasma are smooth, meaning there is no need for a gas sub-shock.

In this paper, we use the Zank et al. (2014, 2016) model [5, 7] and consider the dissipative role of both the PUI collisionless heat flux and collisionless viscosity in determining the structure of shock waves. We show that the incorporation of both PUI heat flux and viscosity is sufficient to smooth and so determine the shock structure. The comparison of the results from a PUI mediated plasma model with the Voyager 2 observations shows a good agreement. During the HTS transition the PUIs are heated much more than the thermal gas. Both the HTS and a shock in the VLISM are numerically simulated and compared with the Voyager 2 and 1 observations. For the HTS, we show that almost all the internal energy is converted to PUIs and so the thermal gas remains relatively cold during the transition. The flow remains supersonic with respect to the thermal gas downstream of the HTS. However, the flow changes from supersonic to subsonic with respect to the combined thermal gas and PUI Mach number. The HTS results are consistent with observations of the HTS ([2, 13]). The shock in the VLISM is found to be very weak and broad. We also consider a general case where both thermal gas and PUIs are weakly non-isotropic and have included thermal gas dissipation terms (i.e., heat flux and viscosity). The numerical simulation of the general case is discussed in section 5. Finally, we summarize our results about shock structure when mediated by energetic particles such as PUIs, ACRs, and SEPs.
2. Pickup Ion Mediated Plasma Model

PUIs in the supersonic solar wind beyond 10 AU, subsonic solar wind in the inner heliosheath (IHS), and the VLISM are not equilibrated with the background thermal plasma and should be treated as a separate component. Therefore, a multi-component plasma model is required to consider thermal solar wind electrons and ions, and a superthermal ion component (e.g., PUIs, SEPs, or ACRs). Complete details of the model were presented in [5, 7, 8]. The continuity, momentum and energy equations governing the electrons and protons are assumed to have Maxwellian distribution functions and are given by

\[
\frac{\partial n_{e,s}}{\partial t} + \nabla \cdot (n_{e,s} u_{e,s}) = 0; \tag{1}
\]

\[
m_{e,s} n_{e,s} \left( \frac{\partial u_{e,s}}{\partial t} + u_{e,s} \cdot \nabla u_{e,s} \right) = -\nabla (P_{e,s}) - en_{e,s} (E + u_{e,s} \times B); \tag{2}
\]

\[
\frac{\partial P_{e,s}}{\partial t} + u_{e,s} \cdot \nabla P_{e,s} + \gamma_{e,s} P_{e,s} (\nabla \cdot u_{e,s}) = 0, \tag{3}
\]

where \( n_{e,s}, P_{e,s}, u_{e,s}, \) and \( \gamma_{e,s} \) are the macroscopic fluid variables for the electron (e) and proton (s) number density, pressure, velocity, and the adiabatic index, respectively. \( B \) and \( E \) are the magnetic field and electric field.

On the other hand, PUIs or energetic particles drive streaming instabilities and experience pitch angle scattering, which introduces a collisionless heat flux and a collisionless stress tensor or viscosity. The energetic particle distribution function can be written up to a second order correction as

\[
f \approx f_0 + \mu f_1 + \frac{1}{2} (3\mu^2 - 1)f_2, \tag{4}\]

where \( \mu \) is the cosine of the particle pitch angle and the \( f_1 \) and \( f_2 \) are functions of the isotropic distribution, \( f_0 \). After some calculations, the pressure tensor and heat flux terms can be expressed as

\[
(P_{ij}) = P_p (\delta_{ij}) + \left( \begin{array}{ccc} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right) \frac{\eta \epsilon}{2} \left( \frac{\partial U_k}{\partial x_\ell} + \frac{\partial U_\ell}{\partial x_k} - \frac{2}{3} \delta_{k\ell} \frac{\partial U_m}{\partial x_m} \right) \equiv P_p I + \Pi_p; \tag{5}
\]

\[
q_{ij}(x,t) = -\frac{1}{2} K_{p,ij} \frac{\partial P_p}{\partial x_j}, \tag{6}\]

where the pressure tensor is the sum of an isotropic PUI scalar pressure \( P_p \) and a stress tensor \( \Pi_p \) (i.e., collisionless viscosity) and \( K_p \) represents the diffusion coefficient (i.e. the collisionless heat flux). The inclusion of \( \Pi_p \) and \( K_p \) is due to the non-isotropic PUIs or energetic particles present in the system.

The continuity, momentum, energy, and pressure equations governing the energetic particles or PUIs are

\[
\frac{\partial \rho_p}{\partial t} + \nabla \cdot (\rho_p U_p) = 0; \tag{7}
\]

\[
\frac{\partial}{\partial t} (\rho_p U_p) + \nabla \cdot (\rho_p U_p U_p + I P_p + \Pi_p) = \epsilon_n p_p (E + U_p \times B); \tag{8}
\]

\[
\frac{\partial}{\partial t} \left( \frac{1}{2} \rho_p U_p^2 + \frac{P_p}{\gamma_p - 1} \right) + \nabla \cdot \left[ \frac{1}{2} \rho_p U_p U_p \right] + \frac{\gamma_p}{\gamma_p - 1} P_p U_p + \Pi_p U_p - \frac{K_p}{3(\gamma_p - 1)} \cdot \nabla P_p \right] = \epsilon_n p_p U_p \cdot E; \tag{9}
\]

\[
\frac{\partial P_p}{\partial t} + U_p \cdot \nabla P_p + \gamma_p P_p (\nabla \cdot U_p) = \frac{1}{3} \nabla \cdot (K_p \cdot \nabla P_p) - (\gamma_p - 1) \Pi_p : (\nabla U_p), \tag{10}\]
where the $\gamma_p$ is the adiabatic index for PUIs.

Equations (1)-(3) and (7)-(9) or (10), together with Maxwell’s equations describe the multi-fluid system of equations. However, in most cases solving the multi-fluid model is difficult and by making some assumptions it can be reduced to a single-fluid-like model (see [5, 7] for more details). The reduced single-fluid-like model equations together with Maxwell’s equations may be expressed as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0; \quad (11)$$

$$\rho \left( \frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} \right) = - \nabla (P_g + P_p) - \nabla \cdot \Pi_g + \mathbf{J} \times \mathbf{B}; \quad (12)$$

$$\frac{\partial P_g}{\partial t} + \mathbf{U} \cdot \nabla P_g + \gamma_g P_g \nabla \cdot \mathbf{U} = 0; \quad (13)$$

$$\frac{\partial P_p}{\partial t} + \mathbf{U} \cdot \nabla P_p + \frac{P_p}{\gamma_p - 1} \nabla \cdot \mathbf{U} = \frac{1}{3} \nabla \cdot \left( K_p \cdot \nabla P_p \right) - (\gamma_p - 1) \Pi_p : (\nabla \mathbf{U}); \quad (14)$$

$$\frac{1}{\mu_0} \frac{\partial \mathbf{B}}{\partial t} = - \nabla \times \mathbf{E}; \quad \mu_0 \mathbf{J} = \nabla \times \mathbf{B}; \quad \nabla \cdot \mathbf{B} = 0, \quad (16)$$

where $\rho$, $\mathbf{U}$, and $P_g$ are the fluid density, velocity, and thermal gas pressure, respectively. We denote all quantities pertaining to the background thermal gas with the subscript “g” and those related to the energetic particles, such as PUIs, with the subscript “p”.

3. Pickup Ion Mediated Shock Structure
The PUI mediated plasma model derived in the last section has many applications, one of which is determining the structure of shocks mediated by PUIs. The hydrodynamic form of the one-dimensional single-fluid-like PUI mediated plasma model is studied here. The viscosity term in one-dimensional form may be expressed as $\Pi_p = -4 \eta_p \frac{\partial \mathbf{U}}{\partial x}$. The $\eta_p$ is the coefficient of the PUI collisionless viscosity and is typically assumed to be constant. The thermal gas is isotropic and PUIs are non-thermal particles. The continuity, momentum and pressure equations governing the PUIs and the thermal background gas are given by

$$\rho \left( \frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} \right) = - \frac{\partial P_g}{\partial x} \frac{\partial \mathbf{U}}{\partial x} + \frac{4}{3} \eta_p \frac{\partial^2 \mathbf{U}}{\partial x^2}; \quad (18)$$

$$\frac{\partial P_g}{\partial t} + \frac{\partial P_g}{\partial x} + \gamma_g P_g \frac{\partial \mathbf{U}}{\partial x} = 0; \quad (19)$$

$$\frac{\partial P_p}{\partial t} + \frac{\partial P_p}{\partial x} + \gamma_p P_p \frac{\partial \mathbf{U}}{\partial x} = \frac{1}{3} \frac{\partial}{\partial x} \left( K_p \frac{dP_p}{dx} \right) + \frac{4}{3} \eta_p (\gamma_p - 1) \left( \frac{\partial \mathbf{U}}{\partial x} \right)^2. \quad (20)$$

The shock structure equation can be derived by combining the steady-state form of equations (17)-(20), which yields

$$\frac{d^2 y}{dx^2} + \left[ \frac{3}{4 \eta_p} \rho_1 U_1 \left( \frac{y}{y_{s+1}} - 1 \right) - \frac{3}{K_p} U_1 y \right] \frac{dy}{dx} = \frac{9 \eta_p \rho_1 U_1^2}{4K_p \eta_p \gamma_g M_{s+1}^2} (1 - y) P(y), \quad (21)$$
where $y$ is the inverse compression ratio and

$$P(y) \equiv \gamma_y M_{c1}^2 \frac{\gamma_p + 1}{2\gamma_p} (y - \mu_p^2) - \left( 1 + \frac{P_{pl}}{P_{g1}} \frac{(\gamma_y - \gamma_p) (1 - y^{\gamma_y - 1})}{\gamma_p(\gamma_g - 1) (1 - y)} \right),$$

$$\mu_p^2 \equiv \frac{(\gamma_p - 1)^2}{(\gamma_p + 1)^2} \text{ and } y_{\gamma_y}^{\gamma_y + 1} = \frac{1}{M_{c1}^4}. \text{ The normalized form of the shock structure equation may be written as}$$

$$\frac{d^2 y}{dx^2} + \frac{1}{Sc_p} \left[ \frac{3}{4} \left( \frac{y}{y_{\gamma_y}} \right)^{y_{\gamma_y} + 1} - 1 \right] - 3Sc_p y = \frac{1}{S_{c_p} 4 \gamma_g M_{c1}^2} \frac{\gamma_p}{1 - y} P(y),$$

where $\pi$ is normalized to the diffusion length scale, i.e. $\pi \equiv x/L$, $L \equiv (K_p/U_1)$, and $S_{c_p} \equiv (\eta_p/\rho_1 K_p)$ is the Schmidt number. Equation (23) determines the shock structure in the presence of the PUI heat flux and PUI viscosity. Further discussion related to the shock structure was presented in Mostafavi et al. (2016, 2017) [14, 15].

Axford et al. (1982) [9] investigated cosmic ray mediated shock structure in the presence of only the dissipation term associated with energetic particles (i.e., the heat flux). In this case, the shock structure equation exhibits two kinds of solutions. One solution yields a smooth transition connecting an upstream state to a downstream state. This solution describes an energetic particle fully mediated shock. An example of a smooth transition is presented in Fig. (1a). The second kind of solution exhibits double-valued solutions and is not physical. In this case, there is no smooth transition from an upstream state to a downstream state (Fig. (1b)). To solve this issue, Drury & Völk (1981) and Axford et al. (1982) [9, 10] introduced a gas sub-shock to reach a downstream state. Since, in this regime, the energetic particle heat flux was insufficient to smooth the shock transition, they assumed that there should be a dissipation mechanism associated with the thermal gas (a gas sub-shock) to determine the shock transition. This assumption is unlikely to be correct in all cases. For example, the HTS converts most of the kinetic energy to PUI heating and the thermal gas remains relatively cold and is only heated adiabatically [11]. Voyager 2 observed the thermal background gas to only 180,000 K and not $10^6 K$ [2]. We will discuss the HTS in greater detail below.

The structure of shock with the inclusion of PUI viscosity is given by equation (23), which is a second-order differential equation. This equation does not possess singular points that introduce double-valued solutions. Therefore, all transitions between upstream and downstream states are smooth and there is no need to add a gas sub-shock (see Fig. (2a)-(2d)). Figures (2b) and (2d) use the same parameters as figures (1b) and (1d), for which both needed gas sub-shocks. However, figures (2b) and (2d) show a smooth transition from the upstream to the downstream state because of the inclusion of PUI viscosity. Fig. (1b) shows an example for relativistic particles such as ACRs. In this case, the downstream state determined by the inclusion of a gas sub-shock is completely different that the downstream state determined from the inclusion of energetic particle viscosity. By introducing gas sub-shock, it is assumed than an unspecified thermal dissipation mechanism acts only on the thermal gas. However, Fig. (2d) shows that when non-relativistic particles such as PUI are considering then the downstream state with either PUI viscosity or a gas sub-shock included is the same. The reason for that is for both PUIs and the thermal gas have the same adiabatic index $\gamma = 5/3$. Figures (3a)-(3d) plot particle pressure, showing that as the Mach number increases, the PUI pressure become dominate. Shocks with a very small Mach number and small initial PUI pressure for upstream remain dominated by the thermal gas (Figs. (3b) and (3d)).

4. Comparison of the Pickup Ion Mediated Plasma Model with Observations

Voyager 2 and 1 are measuring the properties of the heliosphere boundaries and the VLISM. We numerically simulate the HTS and a shock in the VLISM using a PUI mediated plasma model and compare them with the observations.
Figure 1: Shock structure in the absence of the collisionless PUI or cosmic ray viscosity. Here $\gamma_g = 5/3$, and $P_{p1}/P_1 = 0.01$. Following [9] a gas sub-shock is typically added to connect the downstream and upstream states, as illustrated by the jump in $y$ to the downstream solution (green dashes). Red dashes correspond to the Rankine-Hugoniot condition downstream state, $y_\infty$. a: $M_{s1} = 14$ and $\gamma_p = 4/3$. A smooth transition exists between the upstream and downstream states. b: $M_{s1} = 2$ and $\gamma_p = 4/3$. Solutions are doubled-valued and a smooth transition is not possible. c: Same as Fig. (1a), except here $\gamma_p = 5/3$. d: Same as Fig. (1b), except here $\gamma_p = 5/3$.

4.1. The Heliospheric Termination Shock

In this section, we describe the structure of the HTS using our PUI mediated plasma model and compare the structure with Voyager 2 observations. The upstream temperature of the HTS was predicted by global MHD models to be $10^6$ K, but the Voyager 2 plasma instrument measured the temperature to be about 150,000-180,000 K. The PUI temperature upstream of the HTS is inferred by [16] and [17] to be $1.56 \times 10^6$ K. Although the PUI number density is 20% of the total proton densities, the PUI pressure is the dominant term and is much greater than the thermal gas pressure (i.e., $P_{g1} = 2.76 \times 10^{-16}$ Pa and $P_{p1} = 5.38 \times 10^{-15}$ Pa). Voyager 2 crossed the HTS multiple times and the properties of these crossing are determined by the observations ([2]). Here, we model the second HTS crossing, TS-2, which moved with a speed of about 90 km/s in the outward direction. Figures (4a) and (4b) are plots of the inverse compression ratio and the normalized PUI and thermal gas pressures as a function of normalized distance. From Fig. (4a), the HTS compression ratio is $r = \rho_2/\rho_1 \simeq 3$, which is a little greater than the value $r = B_2/B_1 \approx 2.38$, measured by [3] based on the observation that the shock was highly perpendicular. Fig. (4b) shows that the thermal gas experiences almost no heating at the shock (being heated adiabatically), with the incident ram pressure being converted almost entirely to PUI pressure.

Unnormalized plots of the PUI and thermal gas temperature profiles as a function of unnormalized distance are shown in Figures (4c) and (4d). The thermal gas downstream
temperature is about $1.27 \times 10^5$ K and the PUI temperature is about $1.82 \times 10^7$ K. These temperatures are consistent with the temperatures determined by [16] and from Voyager 2 observations ([2]). The total temperature downstream of the HTS, on the assumption that $n_p = 0.25 n_g$, is calculated to be about $3.69 \times 10^6$ K. This is consistent with the estimated total temperature downstream of the HTS by [16], which was about $3.4 \times 10^6$ K. The partition of the thermal gas internal energy in the heliosheath is about 2.5% and the PUIs form the dominant component with about 97.5% of the total internal energy. To determine the width of the HTS, we calculate the scattering time for PUIs in the vicinity of the HTS since the distance is normalized to the diffusion length scale. Voyager 2 observations of the HTS showed that the HTS upstream magnetic field was about 0.05 nT ([3]). The scattering time scale is taken to be $2 \times \Omega_g$, giving $4.17 \times 10^3$ s, and the diffusion coefficient is calculated to be $2 \times 10^{13}$ $m^2/s$. Thus the shock thickness of the PUI mediated HTS is about $3 \times 10^5$ km, consistent with observations of TS-2 that showed the shock width was $3 \times 10^5$ km ([2]).

The thermal gas Mach number through the shock (Fig. (5a)) shows that the flow remains supersonic with respect to the thermal gas downstream of the HTS. This is a consequence of the thermal gas remaining cold through the shock. Observations of the HTS showed that the small increase in the downstream thermal gas temperature did not correspond to a transition from a supersonic to subsonic flow through the HTS (see Fig. (4) in [2]). Figure (5b) shows the combined thermal gas and PUI Mach number through the HTS in which the sound speed is based on the long wavelength sound speed ($= \sqrt{a_g^2 + a_p^2}$, [5]). Here, because of the inclusion of PUIs, the transition from upstream to downstream does result in a supersonic to subsonic flow transition.
Here $\gamma_g = 5/3$, and $P_{p1}/P_1 = 0.01$. a: $M_{s1} = 13$ and $\gamma_p = 4/3$ (all the parameters are the same as Fig. 2a). b: $M_{s1} = 2$ and $\gamma_p = 4/3$ (all parameters are the same as Fig. 2b). c: Same as Fig. (3a), except here $\gamma_p = 5/3$. d: Same as Fig. (3b), except here $\gamma_p = 5/3$.

4.2. A shock in the VLISM

Voyager 1 is now measuring the properties of the VLISM. Burlaga et al. (2013) described the first in situ observations of a shock in the VLISM. The observed collisionless shock was very broad, weak, laminar, low beta, and quasi-perpendicular. Since this shock is $10^4$ times broader than shocks with the similar properties at 1 AU, there is no theoretical explanation to describe its structure. We use [6, 18] observations to numerically simulate a shock in the VLISM. Observations show that in the VLISM, the Alfvén speed dominates the sound speed and the shock wave appears to be very weak with a Mach number of about 1.4. Because the shock was highly perpendicular, we can assume that the total “thermal pressure” is given by $P_T = P_g + B^2/2\mu_0$.

As is well known (e.g., [19]) if the magnetic field configuration is perpendicular to the flow, the choice of $\gamma_g = 2$ reduces the MHD equations to a form identical to the hydrodynamic equations except that $P_T = P_g + B^2/2\mu_0$ and $B/\rho$ is constant. In view of the Voyager 1 observations ([6, 18]), we set $P_{T1} = P_{g1} + B_{1}^2/2\mu_0 = 8.28 \times 10^{-14}$ Pa in our gas dynamic equations and compute the structure and downstream properties of the interstellar shock. The PUI pressure in the VLISM was considered by [5] based on a multi-component treatment of the VLISM ([20]). This yields the ratio $P_{p1}/P_{T1} = 8.4 \times 10^{-3}$. Fig. (6) shows the properties of a VLISM shock which is weak. From Fig. (6a), we show that the compression ratio ($= 1/y$) is about 2.3, whereas [6] observed $B_2/B_1$ to be about 1.4. Fig (6b) shows that the shock is mediated by the combined thermal gas and magnetic field pressure, the total of which is the dominant component downstream of the shock. The fast magnetosonic Mach number through the shock shows that the flow changes from supersonic to subsonic (Fig. (6c)). Figure (6d) shows that the combined total thermal gas and PUI Mach number through the VLISM shock is a supersonic-subsonic transition. Here, one can see because PUIs do not contribute a large pressure through the

Figure 3: PUI and thermal gas pressure normalized to the thermal gas pressure far upstream. Here $\gamma_g = 5/3$, and $P_{p1}/P_1 = 0.01$. a: $M_{s1} = 13$ and $\gamma_p = 4/3$ (all the parameters are the same as Fig. 2a). b: $M_{s1} = 2$ and $\gamma_p = 4/3$ (all parameters are the same as Fig. 2b). c: Same as Fig. (3a), except here $\gamma_p = 5/3$. d: Same as Fig. (3b), except here $\gamma_p = 5/3$. 
Figure 4: Smoothed shock transition corresponding to HTS parameters when both the collisionless PUI heat flux and viscosity are included. Here $\gamma_g = 5/3$, $\gamma_p = 5/3$, $P_{p1}/P_1 = 19.5$, and $M_{s1} = 13.8$. a: Inverse compression ratio showing the smoothed shock. b: PUI and thermal gas pressure normalized to the thermal gas pressure far upstream. The HTS is mediated by energetic particles with almost all the upstream ram energy being converted to downstream PUI internal energy. c: The unnormalized thermal gas temperature through the shock as a function of unnormalized distance shows that thermal gas remains relatively cold. d: The unnormalized PUI temperature through the shock as a function of unnormalized distance shows that PUI are strongly heated at the HTS.

Figure 5: a: The thermal gas Mach number, $M_s \equiv u/a_g$, through the HTS. The flow remains supersonic with respect to the thermal gas sound speed even downstream of the HTS. b: The combined thermal gas and PUI Mach number, $M_c \equiv u/\sqrt{a_g^2 + a_p^2}$, through the shock. Relative to the combined sound speed, the flow is subsonic downstream of the HTS.
Figure 6: A smoothed shock transition corresponding to the VLISM parameters when both the collisionless PUI heat flux and viscosity is included. Here $\gamma_g = 2$, $\gamma_p = 5/3$, $P_{p1}/P_{T1} = 0.0084$, $M_{s1} = 2.24$, and $P_{T1} = P_{g1} + B_{1}^{2}/2\mu_0$. a: The inverse compression ratio showing the smoothed shock as a function of normalized position. b: The PUI pressure and the combined thermal gas and magnetic field pressure (i.e., total thermal gas pressure) normalized to $P_T$ far upstream. c: The total thermal gas Mach number through the shock shows the flow changes from supersonic to subsonic through the shock. d: The combined effective total thermal gas and PUI Mach number through the shock, showing a supersonic-subsonic transition.

5. General Case
In this section, we consider the most general case in which we have included the dissipation terms (i.e., collisionless viscosity and heat flux) associated with both thermal gas and PUIs. Assuming the thermal gas is slightly non-Maxwellian, its distribution function introduces higher order corrections that yield both heat conduction and viscosity in the thermal gas system.

The 1D continuity, momentum and pressure equations governing the background thermal gas and non-thermal PUIs in this case are given now by

\[
\frac{\partial \rho}{\partial t} + \rho \frac{\partial U}{\partial x} = 0; \quad (24)
\]

\[
\rho \left( \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} \right) = - \frac{\partial P_g}{\partial x} - \frac{\partial P_p}{\partial x} + \frac{4}{3} \eta_p \frac{\partial^2 U}{\partial x^2} + \frac{4}{3} \eta_g \frac{\partial^2 U}{\partial x^2}; \quad (25)
\]

\[
\frac{\partial P_g}{\partial t} + U \frac{\partial P_g}{\partial x} + \gamma_g P_g \frac{\partial U}{\partial x} = \frac{1}{3} \frac{\partial}{\partial x} \left( K_g \frac{dP_g}{dx} \right) + \frac{4}{3} (\gamma_g - 1) \eta_g \left( \frac{\partial U}{\partial x} \right)^2; \quad (26)
\]

\[
\frac{\partial P_p}{\partial t} + U \frac{\partial P_p}{\partial x} + \gamma_p P_p \frac{\partial U}{\partial x} = \frac{1}{3} \frac{\partial}{\partial x} \left( K_p \frac{dP_p}{dx} \right) + \frac{4}{3} (\gamma_p - 1) \eta_p \left( \frac{\partial U}{\partial x} \right)^2. \quad (27)
\]

$\eta_g$ and $K_g$ represents the collisionless thermal gas viscosity and heat flux, respectively. The thermal gas does not obey the adiabatic thermal gas law and its entropy increases through the shock due to the dissipation terms associated with thermal gas in the system.
The numerical solution of the model for different values of the dissipation coefficients is shown in Fig. (7a)-(8d). The thermal gas and energetic particle heating depends on the choice of their specific values of viscosity and heat conduction. Fig. (7a) shows that when the thermal gas viscosity and heat conduction terms are absent, most of the energy is dissipated to energetic particles or PUIs rather than thermal gas and thus the shock is essentially controlled by energetic particles or PUIs for this moderately high gas Mach number shock. When \( \eta_p \gg \eta_g \) and \( K_p \gg K_g \), the solution almost looks like the case without \( \eta_p \) and \( K_p \) but the thermal gas is slightly hotter. A model shock with the properties \( \eta_p \approx \eta_g \) and \( K_p \gg K_g \) is still mediated by PUIs but the thermal gas is much more heated than the previous cases and \( P_{g2} \approx P_{p2} \) (Fig. 8c). The shock is mediated by the thermal gas and only very slightly mediated by PUIs when \( \eta_p \ll \eta_g \) and \( K_p \approx K_g \). The Earth’s bow shock is an example of a shock mediated by thermal gas. As is well-known, in the quasi-perpendicular regime, the dissipation process is due to reflected thermal ions. [2] compared one example of a planetary shock (Neptune’s bow shock) with the HTS data and showed that most of the energy was transferred to the thermal plasma at Neptune’s inbound bow shock.

![Figure 7: PUI and thermal gas pressure normalized to the thermal gas pressure far upstream.](image)

Here \( K_g, K_p > \eta_g, \eta_p; \gamma_g = 5/3; \gamma_p = 5/3; P_{p1}/P_{g1} = 0.01 \); and \( M_{s1} = 5 \). a: \( \eta_g = K_g = 0 \). b: \( \eta_p = 10\eta_g, K_p = 10K_g \). c: \( \eta_p = \eta_g, K_p = 10K_g \). d: \( K_g = K_p, \eta_g = 10\eta_p \).

5.1. Numerical method
A relaxation method is used to solve the system of equations numerically. The general model includes thermal gas dissipation terms (i.e., collisionless heat conduction and viscosity). The boundary quantities can be calculated by solving the Rankine-Hugoniot conditions. The thermal gas does not obey the adiabatic law and \( P_g \gamma^\gamma_p \) is not constant, which means the entropy increases.
Here $K_g, K_p > \eta_g, \eta_p; \gamma_g = 5/3; \gamma_p = 5/3; P_{p1}/P_{g1} = 0.01, M_{s1} = 5 \eta_p = \eta_g, and K_p = 10K_g.$ (a)-(d) show the intermediate steps in the changing downstream pressures as the numerical solution converges to a stable shock structure.

through the shock. The mass, momentum, and energy flux equations in conservative form are

$$\rho U = \text{constant};$$

$$\rho U^2 + P_g + P_p - \frac{4}{3} \eta_p \frac{dU}{dx} - \frac{4}{3} \eta_g \frac{dU}{dx} = \text{constant};$$

$$\frac{1}{2} \rho U^3 + \frac{\gamma_g}{\gamma_g - 1} P_g U + \frac{\gamma_p}{\gamma_p - 1} P_p U - \frac{1}{3} \frac{K_p}{\gamma_p - 1} \frac{dP_p}{dx} - \frac{1}{3} \frac{K_g}{\gamma_g - 1} \frac{dP_g}{dx} - \frac{4}{3} \eta_p U \frac{dU}{dx}$$

Equations (28)-(30) show that two equations and three unknowns (i.e., the inverse compression ratio, thermal gas and PUI pressure) exist. The conservative form of equations can only specify the inverse compression ratio and total pressure ($P_g + P_p$), but it can not determine the ratio of PUI pressure and thermal gas pressure at the downstream state separately. Since the relaxation method needs the boundary conditions on the both sides, a numerical code has been written to determine $P_g$ and $P_p$ downstream (i.e., $P_{p2}$ and $P_{g2}$). In our numerical code, an initial guess is made for both $P_{p2}$ or $P_{g2}$ and then the system is iterated. The pressure profile is not stable for the downstream state because of the wrong guess, and for example in Fig. (8a), the downstream PUI pressure is greater than the initial guess. The downstream pressures are changed in every step subject to the constraint that $P_{g2} + P_{p2}$ satisfies equations (28)-(30) and the system is iterated until it reaches a stable downstream state. Figs. (8b) and (8c) show two intermediate steps as the downstream pressures changes, and Fig. (8d) is the final step where shock structure is stable.
6. Conclusions
All transitions from the upstream to the downstream state for a system that includes both PUI collisionless heat conduction and collisionless viscosity are smooth. Classically, in the absence of the collisionless energetic particle viscosity, a double-valued solution exists for some Mach number regimes. To eliminate this solution and reach a downstream state, a gas sub-shock was added to the shock waves [9, 10]. In fitting a gas sub-shock certain assumptions about the physics of the thermal gas and the sub-shock are made. In our PUI-mediated plasma model, we find that a combination of energetic particle viscosity and heat conduction smooths all the shocks and there is no need to introduce a gas sub-shock. In fitting a gas sub-shock, it is assumed that for relatively large Mach numbers the energetic particle heat conduction is not enough to smooth the shock and so there is an unspecified thermal dissipative process that acts only on the thermal gas. The downstream state of a combination of energetic particles heat conduction and a gas sub-shock can be different than the downstream state of an energetic particles viscous-heat conduction smoothed shock.

The observed HTS by Voyager 2 is mediated by PUIs, and despite being a perpendicular shock, reflected thermal protons do not provide a dissipation mechanism and the thermal gas remains relatively cold in crossing the HTS and they are only heated adiabatically. We numerically simulate the TS-2 and show that the thermal gas experiences very little heating at the shock (the downstream thermal plasma temperature $\approx 1.27 \times 10^5$ K) and almost all the internal energy is converted to PUIs, whose temperature increases to $\approx 1.82 \times 10^7$ K. When measured using the thermal gas sound speed, the flow of the model HTS remains supersonic downstream of the shock. However, when using the combined sound speed, accounting for both PUIs and thermal gas, the downstream Mach number is indeed subsonic. Based on our choice of the heat flux coefficient, the HTS thickness is about $3 \times 10^5$ km. All these results are consistent with Voyager 2 observations ([2, 16]). The HTS compression ratio is found to be about 3, which is a little stronger than inferred by [3]. Voyager 1 observed an interstellar shock ([6]) which was weak, laminar, low beta, and quasi-perpendicular. We use the observed shock data to numerically simulate a shock in the VLISM. The shock wave appears to be very weak with a Mach number at about 2.24 and a compression ratio about 2.3. The flow through the shock becomes subsonic downstream relative to the combined thermal gas and magnetic field pressure.

The general case considered different values of the dissipation coefficients associated with both the thermal gas and energetic particles (i.e., $\eta_p, \eta_g, K_p,$ and $K_g$). The choice of different values of the dissipation term yields either a thermal gas or energetic particle mediated shock. A large viscous thermal gas term yields a gas dynamic shock that is partially mediated by energetic particles. One example of this kind of shock is Neptune's inbound bow shock ([21]). An example of the shock mediated by energetic particles is the HTS.

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