Pulsar Coherent Radiation by Linear Acceleration Emission Mechanism

Jan Benáček, Patricio A. Muñoz, Jörg Bückner, and Axel Jessner

1 Center for Astronomy and Astrophysics, Technical University of Berlin, D-10623 Berlin, Germany
2 Max Planck Institute for Solar System Research, D-37077 Göttingen, Germany
3 Max-Planck-Institut für Radioastronomie, D-53121 Bonn, Germany

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ABSTRACT

Linear acceleration emission is one of the mechanisms proposed to explain the intense pulsar radio emissions. This mechanism is, however, not well understood because effects of collective plasma response and of nonlinear evolution on the resulting emission power have not been taken into account. We utilize 1D relativistic particle-in-cell simulations to derive the emission properties of two instabilities in neutron star magnetospheres: a relativistic beam (streaming) instability and interactions of plasma bunches/clouds. We found that the emission power by plasma bunch interactions exceeds the emission due to the streaming instability by eight orders of magnitude. The wave power generated by a plasma bunch interaction can be as large as $4.9 \times 10^{24}$ W. It alone can account for the total radio power emitted by typical pulsars ($10^{18} - 10^{22}$ W). The emission of the plasma bunch has a number of features of the observed pulsar radiation. Its spectrum is characterized by an almost flat profile for lower frequencies and a power-law with an index $\approx -2.5$ for higher frequencies. The angular width of the radiation decreases with increasing frequency. The generated wave power depends on the pulsar rotation angle. It can cause microsecond fluctuations in the observed intensity as it oscillates between positive and negative wave interference as a function of the emission angle.

Keywords: Pulsars — Fast radio bursts — Magnetospheric radio emissions — Plasma astrophysics — Neutron stars

1. INTRODUCTION

Pulsars are strongly magnetized neutron stars that emit coherent radio waves (Sturrock 1971; Ruderman & Sutherland 1975; Beskin et al. 1993). For more than fifty years, the properties of the relativistic plasma in their magnetospheres have been under discussions (Michel 2004; Beskin 2018) as well as the emission mechanisms behind their coherent radio waves (Weatherall 1997; Melrose & Gedalin 1999; Eilek & Hankins 2016; Melrose et al. 2020).

Most of the current pulsar radio emission models use the concept of Goldreich–Julian currents (Goldreich & Julian 1969) and sparking events that may occur in the magnetospheres of neutron stars (Ruderman & Sutherland 1975; Cheng & Ruderman 1977a,b; Buschauer & Benford 1977). They assume a strong electric field component ($E \approx 10^{12}$ V m$^{-1}$) directed parallel to the local magnetic field ($E \cdot B \neq 0$), formed in “gaps” on open magnetic field lines along which particles can escape the pulsar magnetospheres. If the current densities are small in the gaps, they do not fully screen off the convective electric fields, so particles can be accelerated to ultrarelativistic velocities. Particles with typical Lorentz factors $\gamma = 10^6 - 10^8$ form a “primary beam”. During their acceleration, they emit $\gamma$-ray photons that can propagate in the magnetosphere. In the strong pulsar magnetic fields, the photons decay into electron–positron pairs which form a “secondary beam.” The number of secondary beam particles is typically $10^3 - 10^5$ times higher than that of the primary particles. The Lorentz factors of the produced secondary particles are in the range $10^2 - 10^4$ (Arendt & Eilek 2002; Timokhin & Harding 2019).

The linear acceleration emission (LAE) is one of the mechanisms which was proposed to produce intense co-
The LAE approach may also apply to other plasma phenomena like to oscillations in large amplitude waves in pulsar magnetospheres (Akhiezer et al. 1975; Luo & Melrose 2008), pulsar pair cascades (Philippov et al. 2020; Cruz et al. 2021), or the interaction of plasma bunches in models of fast radio bursts (FRBs) (Lu & Kumar 2018; Zhang 2020).

This paper is structured as follows. We discuss the numerical properties and simulation setups in Section 2. In Section 3, we present the results of LAE simulations of the streaming instability and the bunch interaction model. We discuss the relevance and efficiency of the LAE mechanism for pulsar radio emissions. We draw out the final conclusions in Section 4. Details and the mathematical implementation of the LAE calculations are summarized in Appendix A.

2. METHODS

2.1. Linear Acceleration Emission

In the past, the computation of the plasma linear acceleration emission by PIC simulations required a significant amount of postprocessing of individual particles to obtain the emission power. Therefore, we decided to develop a specialized software toolkit that directly computes the emission power using the plasma currents and charge densities that are self-consistently obtained by the simulations. This approach is different from the “classical” approach of tracking and adding up the contributions of individual particles to the emission (e.g., Nishikawa et al. (2021)).

The energy radiated by the current density \( j(k, \omega) \) per unit spatial angle \( d\Omega \) and frequency \( d\omega \) is given by Melrose & McPhedran (1991, Eq. 16.8) in SI units

\[
\frac{dU(k, \omega)}{dk d\omega} = \frac{\omega^2}{16 \pi^3 \epsilon_0 c^3} |\mathbf{n} \times j(k, \omega)|^2, \tag{1}
\]

where \( \omega \) is the frequency, \( k \) is the wave vector, \( \mathbf{n} = k/k \) is the unit vector in the direction of the electromagnetic wave vector. The electric current density is represented by a Fourier transform over the whole space–time domain

\[
\mathbf{j}(k, \omega) = \int j(x, t)e^{i\omega t} e^{-i \mathbf{k} \cdot \mathbf{x}} \, dt \, d\mathbf{x}. \tag{2}
\]

In a 1D geometry, where particles move only along the \( x \) direction, the vectors simplify to \( \mathbf{x} = (x, y, z) = (x, 0, 0) \), \( j(x, t) = (j(x, t)\delta(y)\delta(z), 0, 0) \), where \( \delta \) is the Dirac delta distribution.
where \( d \) is the average radiated power assuming \( \omega, \theta \approx \omega, \gamma \). This relation was also used by Melrose & Luo (2009), and it is implicitly assumed by Reville & Kirk (2010). In the pulsar plasma, this relation may be applied for frequencies significantly higher than the relativistic plasma frequency. It applies when the emission region is surrounded by a diluted plasma, significantly less dense than in the emission region.

The resulting emitted energy is (in the 1D limit)

\[
\frac{dU(\omega, \theta)}{d\Omega d\omega} = \frac{\omega^2}{16\pi^3 c_0^3} \left| \sin^2(\theta) j \left( \frac{\omega}{c}, \omega, \theta \right) \right|^2 (4)
\]

where \( dP(\omega, \theta) = \lim_{\Delta t \to \infty} \frac{dU(\omega, \theta)}{\Delta t} \).

For the simulation grid-based current density in Fourier space \( j_{il}(\omega, k) \) (where \( i \) and \( l \) are grid indices corresponding to spatial and temporal coordinates), one can numerically calculate the average emission power in the plasma reference frame. The following question arises, how does one compute the emission power in the pulsar reference frame relativistically shifted by a Lorentz factor \( \gamma_s = (1 - \beta_s^2)^{-1/2} \)? One solution would be simulating the plasma directly in the pulsar reference frame. But that would mean running additional computationally expensive simulations for every change of \( \gamma_s \). Another possibility is to spatially shift the resulting grid data from the simulation with time and rescale the currents. Nevertheless, that requires grid interpolation to new positions and decreases the current precision.

We decided to use a direct Lorentz transformation of the wave four-vector \( k^\mu = (\frac{1}{c} \omega, k) \) and the current density four-vector \( j^\mu = (c\rho(\omega, k), j(\omega, k)) \) to new (primed) pulsar coordinates \( k'^\mu = (\frac{1}{c} \omega', k') \) and \( j'^\mu = (c\rho'(\omega', k'), j'(\omega', k')) \),

\[
k'^\mu = \Lambda^\mu_\nu(\gamma_s) k^\nu, \quad j'^\mu = \Lambda^\mu_\nu(\gamma_s) j^\nu, (6)
\]

where \( \Lambda^\mu_\nu \) is the Lorentz transformation matrix. Then, we compute the emission power during the time interval \( \Delta t' \) in the pulsar reference frame as

\[
\frac{dP(\omega', \theta')}{d\Omega' d\omega'} = \frac{\omega'^2}{16\pi^3 c_0^3 c3\Delta t'} \left| \sin^2(\theta') j' \left( \frac{\omega'}{c}, \omega', \theta' \right) \right|^2. (7)
\]

Here we assume a negative value \( \beta_s \) for a plasma approaching the observer. In the results, we use the notation without primes. We distinguish between reference frames indicating the relativistic factor \( \gamma_s \).

More details about the numerical implementation, the limitations of this approach, and tests of the software toolkit are stated in Appendix A.

### 2.2. Particle-in-Cell Simulations

The LAE software toolkit (Appendix A) requires knowledge of space-time dependent values of the electric current and charge density in the pulsar plasma. To get these quantities, we carry out simulations using the fully-kinetic, electromagnetic, and explicit 1D3V version of the Particle-in-cell (PIC) code ACRONYM\(^1\) (“Another Code for Moving relativistic Objects, Now with Yee lattice and Macroparticles”) by Kilian et al. (2012). The code uses a Yee lattice (Yee 1966), a standard relativistic Boris particle push (Boris 1970), Esirkepov (2001) deposition scheme, and a recently developed and published “Weighting with time dependency” (WT4) fourth-order particle shape function (Lu et al. 2020). This shape function significantly decreases the numerical noise produced by relativistically moving particles in the form of the numerical Cerenkov radiation. The Cole-Kärkäinen CK5 solver (Kärkäinen & Gjonaj 2006) enhances the correct wave propagation at phase speeds close to the speed of light. Periodic boundary conditions and current smoothing are applied. The net-current is subtracted at each time step.

We run two types of simulations with similar initial conditions of as Benáček et al. (2021a,b) for the relativistic streaming instability and the interaction of plasma clouds/bunches with an initial drift speed between electrons and positrons.

\(^1\) [http://plasma.nerd2nerd.org](http://plasma.nerd2nerd.org)
The common setup of both simulations is the following. The simulation axis $x$ is located along the magnetic field line, neglecting its spatial curvature. We use a time step $\omega_p \Delta t = 0.025$ and normalized grid cell size $\Delta x/d_e = 0.05$, where $\omega_p$ is the plasma frequency and $d_e$ is the plasma skin depth. The current and charge densities are stored every 10th time step. The maximum frequency resolution is $4\omega_p$ in the plasma frame. The simulations initially have the same number of electrons and positrons in each grid cell.

The plasma frequencies are selected for both cases such that the maximum of the emission power is located in the frequency range $\omega = 10^8 - 10^9 \text{ rad s}^{-1}$ in the pulsar reference frame. This corresponds to typically $\omega_p$ such that the maximum of the emission power is located and positrons in each grid cell.

Simulations initially have the same number of electrons and positrons in each grid cell.

The plasma frequencies are selected for both cases such that the maximum of the emission power is located in the frequency range $\omega = 10^8 - 10^9 \text{ rad s}^{-1}$ in the pulsar reference frame. This corresponds to typically observed pulsar radiation frequencies, e.g., Bilous et al. (2016). In the plasma reference frame, we choose the plasma frequency $\omega_p = \sqrt{8} \times 10^6 \text{ rad s}^{-1}$ for the the relativistic beam simulation and $\omega_p = \sqrt{20} \times 10^7 \text{ rad s}^{-1}$ for the simulation of the bunch interaction. After the relativistic transformation of the corresponding plasma densities to the pulsar frame and taking into account a density scaling with the height in the magnetosphere (Goldreich & Julian 1969; Rahaman et al. 2020)

$$n(r) \approx 5.5 \times 10^5 \kappa \left( \frac{1 \text{ s}}{P} \right) \left( \frac{B}{10^{12} \text{ G}} \right) \left( \frac{R}{r} \right)^3 \text{[cm}^{-3}], \quad (8)$$

we can find the emission heights $r = 210 R$ and $r = 33 R$, respectively, where $\kappa = 10^3$ is the secondary plasma multiplicity factor, $P = 0.25 \text{ s}$ is the pulsar period, $B = 10^{12} \text{ G}$ is the pulsar surface magnetic field, $R = 10 \text{ km}$ is the neutron star radius, and $r$ is the height in the magnetosphere.

Both simulations are carried out for 140000 time steps. The simulation times are 15.6 $\mu$s for the relativistic beam simulation and 984 ns for the bunch interaction simulation, both in the pulsar reference frames. Assuming that the plasma reference frame moves at the velocity $\approx c$ along the magnetic field lines in the pulsar reference frame, the plasma propagates a distance $\sim 4.6 \text{ km}$ and $\sim 300 \text{ m}$, respectively, during the simulation time. In both cases, the distance is smaller than the light cylinder distance, even for a millisecond pulsar ($\sim 50 \text{ km}$).

The size of the analyzed region is 5.3 km for the relativistic beam instability and 2.42 km for the plasma bunch interaction. Assuming the Lorentz factor $\gamma_s = 500$ between the plasma and pulsar reference frames, they correspond to 10.6 m and 4.8 m in the pulsar reference frame. Therefore, we also neglect the effects of the curved magnetic fields on these scales.

| Parameter          | Simulations          | Streaming instability | Bunch interaction |
|--------------------|----------------------|-----------------------|-------------------|
| Time steps          | Simulation 1: 140000 | Simulation 2: 140000 |
| $L$ [\(\Delta x\)] | 100 000              | 720 000               |
| $\omega_p$ [rad s\(^{-1}\)] | $\omega_p = \sqrt{8} \times 10^6$ | $\omega_p = \sqrt{20} \times 10^7$ |
| $\rho$             | 0.05                 | 0.05                  |
| $n(x)$             | Uniform              | Bunch (Eq. 11)        |
| $n_0$ [PPC]        | 10 600               | 2000                  |

Table 1. Summary of the simulational parameters of the relativistic streaming instability and the plasma bunch interaction in the simulation reference frame: the number of time steps, the simulation lengths in grid cells, the grid cell size, the number of time steps, the plasma frequency, the initial inverse temperature, the initial density profile, and the initial number of particles per cell (PPC).

Though the lateral extent of the current in Equation 2 is assumed as the delta function, our approach is valid until $\lambda \gg D$, where $\lambda$ is the typical wavelength of emitted wave and $D$ is a real lateral extent of the current. The wavelengths are $\lambda \sim 670 \text{ m}$ for the streaming instability and $\lambda \sim 42 \text{ m}$ for the bunch interaction, both in the simulation reference frame, if we assume a wave emitted approximately at the plasma frequency in the perpendicular direction. Moreover, assuming the current profile as the delta function does not constrain the wave coherence. The waves interfere during their propagation, as follows from Equation 3. The only exception are waves with $\theta = 0$.

In the whole paper, we use the SI system of units. The Gaussian CGS system of units (for current and charge densities) used by ACRONYM is converted into SI units. Table 1 shows a summary of the initial parameters of our simulations. For all analyses, we use the information obtained in the whole simulation boxes.

The particles of species $\mu$ are initialized with a 1D Maxwell-Jüttner velocity distribution (Jüttner 1911)

$$g_{\mu}(u) = \frac{1}{\gamma_\mu 2K_1(\rho_\mu)} n_\mu e^{-\rho_\mu \gamma_\mu \gamma(1-\beta \delta_\mu)}, \quad (9)$$

$$\beta = \frac{v}{c} = \frac{u}{c_\gamma}, \quad \beta \delta_\mu = \frac{v_\mu}{c} = \frac{u_\mu}{c_\gamma}, \quad (10)$$

where $n_\mu$ is the particle density, $\rho_\mu = m_e c^2 / k_B T_\mu$ is the inverse dimensionless temperature, $m_e$ is the electron mass, $c$ is the speed of light, $k_B$ is the Boltzmann constant, $T_\mu$ is the thermodynamic temperature, $K_1$ is the MacDonald function of the first order (modified Bessel function of the second kind), $\beta$ and $u$ are the particle velocities, $\beta \delta_\mu$ and $u_\mu$ are the species drift velocities, $\gamma$
and \( \gamma_{d\nu} \) are the corresponding Lorentz factors. The \( y \) and \( z \) spatial coordinates and associated velocity components are initially and also during the simulation zero because the evolution of the electromagnetic field vector has only non-zero components along the \( x \) axis. Physically this corresponds to the situation that any kinetic energy of a particle perpendicular to the magnetic field is radiated away by synchrotron radiation at timescales \( \ll \omega_p^{-1} \).

The \textit{relativistic beam instability} is carried out using four particle species: a background plasma composed by electrons and positrons with a density \( n_0 = 10^4 \) particles-per-cell (PPC) and a beam composed by electrons and positrons with a density \( n_1 = 600 \) PPC. The typical number of macro-particles per Debye length is \( 2 \times 10^5 \). The beam Lorentz factor is \( \gamma_b = 60 \). That corresponds to a beam-to-background density ratio \( \rho_b = n_1/(\gamma_b n_0) = 10^{-3} \). The inverse temperature of all species is \( \rho_\mu = 3.33 \). The simulation length is \( 10^5 \Delta (5000 d_\epsilon) \), and it is advanced for 140,000 time steps \( (\omega_p t = 3500) \). That allows a resolution of wavenumber \( \Delta k c/d_\epsilon = 1.3 \times 10^{-3} \) and a frequency resolution of \( \omega/\omega_p = 2.9 \times 10^{-4} \).

The \textit{plasma bunch interaction} is simulated by considering two particle species with opposite drift velocities: electrons with a drift velocity \( u_{d\epsilon}/c = 10 \) and positrons with a negative drift velocity \( u_{d\epsilon}/c = -10 \). Such drifts may be obtained by an acceleration of particles during the \( \gamma \)-rays decay in strong electric fields into electron–positron pairs (Rahaman et al. 2020). The inverse temperature of both species is \( \rho_\mu = 1 \) (Arendt & Eilek 2002). The initial density profile of the bunch describes the interaction of two consequently emitted bunches. We cover half of the leading and half of the trailing bunch. The density profile is

\[
n(x) = \begin{cases} 
0.1 \ n_0, & |x| \leq \frac{l}{2}, \\
n_0 \exp\left(-\left(\frac{1}{x_0}\right)^6 \right) & |x| > \frac{l}{2},
\end{cases}
\]

\[
x_0 = |\ln(0.1)|^{-\frac{1}{6}} \left(\frac{L - l}{2}\right) = 0.870 \cdot \frac{L - l}{2},
\]

where \( L = 720,000 \Delta \) (36,000 \( d_\epsilon \)) is the simulation length, \( l = L/30 \) is the distance between bunches, \( n_0 \) is the density in the bunch center, in this case represented by 2000 PPC, \( x_0 \) is chosen such that \( n(x) \) be a smooth function at \( x = \pm l/2 \). We assume that the initial plasma density is an even function. The typical number of macro-particles per the Debye length along the simulation box is \( \sim 200 - 2000 \). The simulation lasts 140,000 time steps \( (\omega_p t = 3500) \). The wavenumber resolution is \( \Delta k d_\epsilon = 1.7 \times 10^{-4} \) and in frequency \( \Delta \omega/\omega_p = 2.5 \times 10^{-4} \).

Figure 1 shows examples of the electric current and electrostatic charge densities in the Fourier space for both emission models in their plasma frames. The subluminal waves do not contribute to the emission flux, so they are set to zero. The currents and charges are distributed closer to \( \omega = 0, k = 0 \) for the plasma bunch interaction model compared to the relativistic beam instability model. Because a broad range of wave frequencies is excited by the simulation of plasma bunch interaction, no specific wave mode dominates. Generally, the emission parallel to the magnetic field direction \( (\theta \to 0) \) comes from the Fourier space regions close to the light lines (magenta dashed lines), \( \lim_{\theta \to 0} k(\theta) = \lim_{\theta \to 0} \frac{\pi}{2} \cos \theta = \frac{\pi}{2} \) (Equation 3). The emission perpendicular to magnetic field lines comes from Fourier space regions close to \( k \approx 0 \), because \( \lim_{\theta \to \frac{\pi}{2}} k(\theta) = \lim_{\theta \to \frac{\pi}{2}} \frac{\pi}{2} \cos \theta = 0 \).

3. RESULTS

We estimate the electromagnetic emission power of linearly accelerated particles by a streaming instability and by a plasma bunch interaction. We assume a transformation Lorentz factor between the plasma in 1D PIC simulations and the pulsar (observer) reference frames of \( \gamma_e = 500 \), which produces frequency peaks of emission at \( \sim 1 \) GHz for given plasma frequencies. As the radio emission is formed in open magnetic field lines, we assume that the waves associated with the instabilities move radially outwards, approximately in the direction away from the star. The results are presented in spherical coordinates \( (r, \theta, \varphi) \), where \( r \) is the distance, \( \theta \) is the polar angle, where \( \theta = 0 \) is along the local magnetic field, and \( \varphi \) is the azimuthal angle.

Figure 2 shows the resulting average electromagnetic power per spatial angle unit and frequency unit as a function of frequency and emission angle for the two models obtained over the whole simulation time. The top row depicts the wave power in the plasma reference frame \( (\gamma_e = 1) \), the bottom row depicts the wave power after its transformation into the pulsar frame \( (\gamma_e = 500) \). In the plasma frame, the maximal emission power reaches \( 3 \times 10^{-7} \) W s rad\(^{-3} \) for the relativistic beam and \( 5 \) W s rad\(^{-3} \) for the plasma bunch. Relativistic effects strongly increase the emission power in the pulsar reference frame, since the total average emission power is \( P_{\text{tot}} \sim \gamma_e^6 \) (Griffiths 2017, Eq. 11.75). The maximal emission power reaches \( 4 \times 10^{11} \) W s rad\(^{-3} \) for the relativistic beam and \( 3 \times 10^{15} \) W s rad\(^{-3} \) for the plasma bunch, both in the pulsar reference frame. The total average electromagnetic emission power, \( P_{\text{tot}} = \int \int \int \sin(\theta) d\epsilon P(\omega, \theta) d\theta d\phi d\omega \), in the pulsar frame are \( 7.73 \times 10^{15} \) W for the relativistic beam and \( 1.21 \times 10^{24} \) W for the plasma bunch interaction, respectively.
Figure 1. Electric current density (top row) and electrostatic charge density (bottom row) as a function of frequency and wavenumber in the plasma reference frame. Both instabilities are selected in time interval $\omega_p t = 0 - 3500$. Subluminal waves, which do not contribute to the emission, are set to zero. Magenta dashed lines: the light lines $k = \pm \omega/c$. Positive wavenumbers correspond the direction towards the observer.

Figure 3 shows the average emission power as a function of the angle in the pulsar reference frame ($\gamma_s = 500$). Several time intervals are selected, each of them $\omega_p t = 1000$ long. The emission power per spatial angle unit and frequency unit is integrated over all frequencies, $dP_\theta = \int dP(\omega, \theta) d\omega$, normalized to the total emission power $P_{\text{tot}}$. For the relativistic beam, the power close to $\theta \approx 0$ is enhanced first at the time interval $\omega_p t = 1000 - 2000$. That corresponds to the region that is located close to the light line in the $\omega - k$ space, where initially most unstable superluminal waves line start to grow close to the light ($k \approx \omega/c \rightarrow \theta \approx 0$) (Benáček et al. 2021b). As the superluminal wave power also closer to $k = 0$ ($\theta \rightarrow \pi/2$ in the plasma frame) grows, the angular width of emission increases. In the case of the plasma bunch interaction, a sharp power maximum is reached at $\theta \approx 0.11\degree$ in the time interval $\omega_p t = 0 - 1000$. Then, it decreases and shifts to smaller angles, the emission angular widths become narrower and form power-law tails from the maxima to larger angles.

The average emission power in the pulsar reference frame ($\gamma_s = 500$) is presented as a function of the frequency in Figure 4. The emission power per frequency unit is the integral over all spatial angles, $dP_\omega = \int_{\Omega} \sin(\theta) dP(\omega, \theta) d\theta d\varphi$. The straight dashed lines in the figures correspond to power-law functions with an index $\alpha$. For the relativistic beam (left panel in Figure 4), the maximal intensity first grows at a frequency $3.3 \times 10^7$ rad s$^{-1}$ (corresponding approximately to the frequency of the subluminal waves) in the time interval $\omega_p t = 1000 - 2000$. Later, the emission maximum is enhanced, its maximum shifts and broadens to lower frequencies. The spectrum is characterized by a steep decrease of the wave power at a frequency $\sim 3.5 \times 10^9$ rad s$^{-1}$ at all times. See also this decrease in Figure 2c at angles $\theta \approx 0.02\degree$. We found that this frequency corresponds to the maximum frequency $\omega_{\text{max}}$ of superluminal waves in the simulation. $\omega_{\text{max}}$ is determined by the point in the $\omega - k$ domain where the L-mode branch crosses the light line, i.e., it goes from superluminal to subluminal mode (Rafat et al. 2019). Higher frequency wave modes ($\omega > \omega_{\text{max}}$) are subluminal, they do not generate electromagnetic waves in the 1D limit. At later times ($\omega_p t = 2000 - 3500$), the lower frequency part of the spectrum can be roughly approximated by a power-law function with index $\alpha \sim 3.3$.

Within the frequency interval $(3 - 75) \times 10^9$ rad s$^{-1}$ of the plasma bunch interaction, the emission spectrum
Figure 2. Average emission power per frequency and spatial angle units as a function of frequency and emission angle in the plasma reference frame ($\gamma_s = 1$, top row), and in the pulsar reference frame ($\gamma_s = 500$, bottom row) for the streaming instability (left column) and the interaction of plasma bunch (right column). Note the different scales of intensity and frequency.

Figure 3. Average emission power per spatial angle unit as a function of emission angle during selected time intervals. The emission power (see Figure 2c–d) is integrated over all spatial angles in the pulsar (observer) reference frame ($\gamma_s = 500$).

resolved frequency in the simulation, the resulting electromagnetic emissions occur up to that frequency.

Figure 5 depicts the evolution of the total emission power, $P_{\text{tot}}$, in the pulsar reference frame ($\gamma_s = 500$). The horizontal bars correspond to the time intervals for which the data were selected. Note that the initial emission power $\lesssim 10^{12}$ W of the relativistic beam instability broadens in time, going to the higher frequency part of the spectrum and developing power-laws. The corresponding specific power-law indices are $-2.9$, $-2.5$, $-2.5$, and $-2.7$ (for the time intervals in the order as presented in the figure). As superluminal modes are present (close to the light lines) at all frequencies up to the maximal

| $\omega_p t$ | $\omega_p t = 0 - 1000$ | $\omega_p t = 1000 - 2000$ | $\omega_p t = 2000 - 3000$ | $\omega_p t = 3000 - 3500$ |
|-------------|-----------------|-----------------|-----------------|-----------------|
| $dP / d\omega d\Omega$ | $10^{-11}$ | $10^{-10}$ | $10^{-9}$ | $10^{-8}$ |
Figure 4. Average emission power per frequency unit as a function of emission frequency during selected time intervals. The emission power (see Figure 2c–d) is integrated over all spatial angles in the pulsar (observer) reference frame ($\gamma_s = 500$). Straight dashed lines: power-law functions with indices $\alpha = 3.3$ and $\alpha = -2.5$.

Figure 5. Evolution of the total electromagnetic emission power and electromagnetic wave energy densities emitted per volume unit in the pulsar (observer) system ($\gamma_s = 500$). The time intervals are denoted by horizontal bars. The emission power is integrated over all frequencies and spatial angles.

(Figure 5a) is mostly due to the particle (current and charge density) noise in the simulations. This type of emission is, however, incoherent. Coherency is obtained in the later evolutionary stage when the coherent emission of the instability prevails. Starting at $\omega_pt \approx 1000$, the total emission power exponentially rises. It saturates at $5 \times 10^{19}$ W. The plasma bunch evolution (Figure 5b) starts at a total emission power $5 \times 10^{24}$ W, as the instability develops in times $\sim \omega_p^{-1}$. The incoherent emission by the noise is negligible in this bunch interaction case since the beginning of the simulation. Then its power decays and saturates at $\approx 8 \times 10^{23}$ W. Note that the energy densities are related to the local electromagnetic energy densities (scales on the right of figures) as $E_{EM} = P_{tot}/(eV)$, where $V$ is the emission volume in the pulsar frame.

The plasma bunch interaction emits significantly more powerful radiation than the relativistic beam instability. Therefore, we focus on predictions for observable quantities only for this bunch interaction case in Figures 6 and 7. Figure 6 shows the average emission spectrum obtained during the whole simulation time, $\omega_pt = 0−3500$, as a function of frequency. We fit the average emission spectrum by an empirical function that is often used for fitting of the observed pulsar spectra (Löhmer et al. 2008)

$$S(\omega) = \frac{S_0}{(1 + \omega^2\tau_e^2)^{\zeta+1}}.$$  \hspace{1cm} (13)

The obtained fit parameters are $S_0 = (1.8 \pm 0.3) \times 10^{14}$ W s rad$^{-1}$, $\tau_e = (0.3 \pm 0.2)$ ns, and $\zeta = (0.3 \pm 0.05)$. Although the fit function assumes average spectra over many pulsar periods, the estimated emission profile is produced by only one interacting bunch.

Figure 7 shows intensity profiles for the bunch interaction as they would be seen by an observer at a distance $d = 1$ kpc for a pulsar rotating with a pe-
period $P = 0.25\, \text{s}$ for four different frequencies (200 MHz, 500 MHz, 1 GHz, and 5 GHz) at which are pulsars observed. The frequency bandwidth is $\Delta f = 20\, \text{MHz}$. We use the emission during the whole simulation time as in Figure 6. We further assume that the observer can see the emission regions from changing angles $\theta$ as the star rotates. When one bunch interaction ends, it is immediately replaced by the emission of the follow-up interacting bunches, which have the same emission properties. If at time $t = 0$ the emission angle corresponds to $0^\circ$, the emission is considered along the magnetic field at the moment when the center of the emission cone just crosses the observer. The observed intensity is the integrated average emission power estimated as follows, $I = \frac{dP_\theta}{d(\Delta f)}$. The time is related to the emission angles as $t = \theta P/(2\pi)$, where $P = 0.25\, \text{s}$ is the pulsar period. Because the emission is assumed to be symmetric with respect to the azimuthal axis in spherical coordinates, the intensity profiles are reversible in time (for $t < 0$ and for $t > 0$). We note that the numerical resolution of frequencies and wavenumbers gives the shortest timescale of intensity fluctuations. For the real pulsar plasma, the typical fluctuation time scales would, perhaps, be shorter.

Figure 6. Fit of the average emission power per frequency unit as a function of the emission frequency during the whole simulation time of the interacting plasma bunches ($\gamma_s = 500$). The fit is given by Equation 13. For the fit parameters, please see the text.

4. DISCUSSION

We calculated the pulsar radio wave emission power due to the linear acceleration process in a relativistic pair plasma by utilizing the collectively obtained current and charge densities of 1D kinetic PIC simulations. This way, for the first time, we estimated the electromagnetic emission properties of the linear acceleration pulsar emission. Our approach allows estimating the coherent emission power due to the collective plasma behavior, not based on adding up the emission of the individual particles. The electric fields, which accelerate the particles, directly result from plasma instabilities — the relativistic beam instability and the interaction of plasma bunches, which have been proposed as possible sources of pulsar radio waves.

We have found that the maximum of the estimated electromagnetic emission power emitted by a plasma bunch interaction ($\approx 4.9 \times 10^{24}\, \text{W}$) exceeds the maximum emission power caused by the streaming instability ($5.0 \times 10^{16}\, \text{W}$) by eight orders of magnitude. Moreover, every single one bunch interaction can account for the total pulsar radio emission power $10^{18} - 10^{22}\, \text{W}$ ($10^{25} - 10^{29}\, \text{erg s}^{-1}$). To get the total observed radio emission power $10^{22}\, \text{W}$, only one of such bunch interaction would be necessary. On the contrary, at least $2 \times (10^4 - 10^3)$ such emitting regions would be necessary to account for the observed emission power by the streaming instability.

The total emission flux depends on the properties of the plasma instabilities and on the intensities of the electrostatic superluminal L-mode waves, the solutions of the permittivity tensor component $\Lambda_{33} = 0$, see Rafat et al. (2019). For the streaming instability, the emission power depends on the formation of soliton-like superluminal waves, which are generated for inverse temperatures $\rho \geq 1.66$ and Lorentz factors $\gamma_s > 40$ (Benáček et al. 2021b). Otherwise, the intensities of superlumino- nous waves are weak. The emission power of the plasma bunch depends mostly on the drift speeds between particle species (Benáček et al. 2021a). Nevertheless, only for finite drift speeds, the energy of superluminal waves significantly exceeds the energy of subluminal waves. The emitted wave power increases nonlinearly with the drift speed. In a supplemental simulation where we kept the same parameters as the bunch simulation above but increased the drift speed from $u_d/c = 10$ to $u_d/c = 100$ ($\gamma_s \approx 100$), the total emission power increased from $\approx 4.9 \times 10^{24}\, \text{W}$ to $2.6 \times 10^{28}\, \text{W}$. We expect only a weak dependence of the superluminal wave energies on the plasma temperatures. The main reason is that the superluminal waves are generated by ambipolar diffusion at the bunch edges. The ambipolar diffusion is mostly influenced by a drift speed difference between the expanding particle species, while the plasma temperature represents only a correction term to this expansion for $\langle \gamma \rangle \ll \gamma_d \equiv (1 + u_d)^{\frac{1}{2}}$, where $\langle \rangle$ denotes an average over the particle velocity distribution.

For a maximum frequency of the emitted power $\omega_{\text{max}}$ in the plasma reference frame, the typical frequency in
the pulsar reference frame is not $\gamma_s^2 \omega_{\text{max}}$ in the pulsar frame (Eilek & Hankins 2016). The relation is given just by the relativistic transformation of the wave four-vector $k_{\mu}^\text{max} = (\frac{1}{c} \omega_{\text{max}}, k_{\text{max}}, 0, 0)$, where the emission frequency $\omega_{\text{max}}$ and wavenumber $k_{\text{max}}$ in the plasma frame relativistically transform into the frequency $\omega' = \gamma_s (\omega_{\text{max}} - \beta_s k_{\text{max}} c)$. For $\beta_s \to 1$ and $|k_{\text{max}} c| \ll \omega_{\text{max}}$, we may expect the typical observed frequency

$$\omega' \approx \gamma_s \omega_{\text{max}}, \tag{14}$$

for the high frequency part of the spectrum.

The bunch interaction case, for otherwise similar plasma parameters, has a significantly higher emission power than the streaming instability. Therefore, we compare its emission features with the observed pulsar radio waves (Kramer et al. 1997; Löhmer et al. 2008; Bilous et al. 2016; Jankowski et al. 2018):

1. The plasma bunch’s total emitted radio wave power exceeds the average observed pulsar radiation. However, the average observed intensity is measured over the whole pulsar period, while the power of bunch interaction is directed into a narrow cone. The calculated wave power depends on the relativistic factor, $P_{\text{tot}} \sim \gamma_s^6$ (Griffiths 2017). The relativistic factor is typically assumed in the range $\gamma_s = 10^2 - 10^4$. E.g., an decrease of the Lorentz factor from the lower limit of the interval $\gamma_s = 500$ to $\gamma_s = 100$ would decrease the total emitted wave power from $4.9 \times 10^{24}$ W to $3.4 \times 10^{19}$ W.

2. In the frequency range below $\omega \approx 10^{10}$ rad s$^{-1}$, the emission power spectrum flattens with time. Moreover, we found that the flat frequency range increases to higher frequencies for smaller bunch sizes as superluminous waves with higher frequencies are generated at plasma density gradients.

3. The power-law indices of the emission spectrum are between -2.9 and -2.5 for frequencies $\gtrsim 2 \times 10^9$ rad s$^{-1}$. This is less than the average observed pulsar spectral index $\approx -1.4$, but still in the observed range between -3.5 and 0 (Bilous et al. 2016). However, the discrepancy from the observed emission spectrum might be in the
case that the emission is generated at various plasma frequencies (plasma densities) and for various Lorentz factors $\gamma_s$ (bunch velocities in the observer frame). Moreover, we also carried out a supplemental simulation with a plasma bunch interaction for ten times shorter plasma bunch. The resulting total emitted power was $\approx 10$ times lower, but the maximum power-law index became $\alpha = -2.1$. In fact, shorter plasma bunches form stronger density gradients which cause the generation of smaller wavelengths and higher frequencies.

4. The electromagnetic waves are emitted omnidirectionally in the plasma frame. After the relativistic transformation into the pulsar frame, the emission narrows into an angle $\theta \sim 1/\gamma_s \approx 0.11^\circ$. Nonetheless, the emission width is significantly narrower than $\theta$ at frequencies $\gtrsim 8 \times 10^9$ rad s$^{-1}$ and wider at frequencies $\lesssim 8 \times 10^9$ rad s$^{-1}$. The cone angle decreases with increasing frequency in the whole frequency interval.

5. If the center of the emission cone $\theta \approx 0$ does not cross the observer, only low frequency waves can be observed. This effect is, for example, known for the main pulse of the Crab pulsar (Hankins et al. 2015). The upper frequency limit is given by the minimal angular distance from the cone center.

6. Submillisecond oscillations in the intensity of the waves (Figure 7) may be caused by positive and negative wave interference for specific angles $\theta$.

7. LAE implicitly provides linearly polarized waves. However, the exact polarization angle might depend on the geometry and change due to propagation effects.

Features 4–7 are similar also for the case of emissions due to the streaming instability.

While the emission properties may support strong temporal fluctuations on kinetic timescales, the long time average (e.g., over hours) over many plasma bunch interactions in different regions of the pulsar magnetosphere may remain stable for given pulsar observed properties (if the general magnetospheric parameters remain constant).

The width of the emission pulse of the plasma bunch interaction at high frequencies is too narrow in comparison with real pulses. However, the emission profile may be broadened by the effect that a pulse is formed by several simultaneously emitting bunches, assuming that individual emission regions radiate into a slightly different angle. If the emissions from all bunches occur into an angle $\Delta \theta$ that is larger than the emission width of one bunch at high frequencies but still smaller than the emission width of one bunch at low frequencies (e.g., $\Delta \theta = 0.2^\circ$ in our case), the resulting emission width at high frequencies can be significantly widened to the angular width $\Delta \theta$, while the effect on the low frequency emission, which has significantly broader angular width than $\Delta \theta$, will not be so strongly affected. As a result, this effect may produce a larger angular width of emission at high frequencies than the one shown in Figure 2d.

Most of the emission of the plasma bunch interaction occurs in the perpendicular direction to the magnetic field in the plasma reference frame. However, after a relativistic transformation, the emission is narrowed to a small angle. Assuming only one coherent emission region (e.g., an interacting bunch) is responsible for a given pulsar observation, the emission angular width directly indicates the transformation Lorentz factor of the emission source.

There is no direct relation between the frequency of emitted electromagnetic waves from the plasma bunch interaction and the emission height in the magnetosphere as the plasma bunch (small in comparison with the size of the magnetosphere) approximately radiates at all frequencies simultaneously. The plasma bunch is radiating mostly at its density gradient. The local current oscillation frequencies (emission frequencies), are frequencies of the superluminal electrostatic waves. The oscillation frequency may be characterized for $k = 0$ as $\omega \approx \omega_0 = (\gamma^{-3})\omega_p(x)$ (Rafat et al. 2019), where $\omega_p(x)$ is the local plasma frequency which varies along the bunch and $\langle \rangle$ denotes the average over the local particle velocity distribution function, $\omega_0 < \omega_p(x)$ for $\rho \sim 1$. If the emission is produced by the plasma bunch interaction, this conclusion invalidates the relation between the observed frequency of electromagnetic waves and the emission height in the pulsar magnetosphere, which is known as radius–frequency mapping. This is because all emission frequencies, whose intensities may strongly vary, are generated by the plasma bunch interaction region.

The LAE polarization properties obtained from the 1D simulations are 100 % linearly polarized along the magnetic field in the simulation reference frame. As the relativistic transformation is applied along the same axis, the polarization is still in the same direction. From the point of the observed emission cone, the polarization vector is always directed to the center of the emission cone, and it is independent of the emission frequency. If the emission cone center is crossing the observer (as the pulsar rotates), the polarization profile of the angle is a step function by an angle $180^\circ$. Otherwise, the transition is smoother. For more detailed polarization
properties, 2D or 3D simulations with the inclusion of geometrical propagation effects are necessary.

A direct comparison of our results with already published calculations of pulsar LAE is difficult. We have not found any other approach that calculates the same properties for the whole plasma, but only for particles characterized by “average” plasma properties (e.g., Melrose (1978); Melrose et al. (2009); Melrose & Luo (2009); Reville & Kirk (2010)). As we have shown by our simulations (Benáček et al. 2021a), the plasma properties significantly change along the emission region. The generated electric fields cannot be approximated by slabs of static electric fields, as used for the known analytical calculations of single particle emissions. Instead, electric fields in the simulations oscillate over a wide range of frequencies. Furthermore, the previous investigations were not compared with observations at all. Reville & Kirk (2010) provided calculations of the emission power as a function of frequency for a single charged particle. Their high frequency parts of spectra, however, do not show power-law dependence as it is known from observations.

Note further on that particle collective acceleration and oscillations in simulations are not influenced by the transverse electromagnetic emission because the wave components perpendicular to the magnetic field are suppressed in 1D approximations. However, in reality, particles are influenced by perpendicular wave fields. The emitted electromagnetic waves might, therefore, significantly decrease the particles’ kinetic energy and hinder the formation of instabilities for the high emission fluxes in both LAE models.

One can estimate over which time, $T$, the whole energy of the electrostatic waves, $E_{\text{tot}}$, which is only a part of the converted kinetic energy, is emitted with the maximal found emission power, $P_{\text{max}}$. In the pulsar reference frame, this time is $T = E_{\text{tot}}/P_{\text{max}} \sim 190 \mu s$ for the relativistic beam instability and $T \sim 300\,\text{ns}$ for the plasma bunch interaction. For comparison, the total simulation time of the streaming instability was 15.6 $\mu s$ and of the plasma bunch interaction 984 $\text{ns}$. Therefore, it can be expected that the emission process does not significantly influence the evolution of the relativistic beam instability. However, most of the electrostatic energy in the plasma bunch is emitted during the simulation time. Hence, our results must be cautiously interpreted for times $\omega_p t \gtrsim 1000$. Moreover, a question arises whether the plasma bunch does not radiate most of its energy before another emission mechanism (e.g., the relativistic plasma emission) has time to develop before it starts to radiate.

5. CONCLUSIONS

LAE is a promising mechanism that provides several of the observed features of pulsar radio signals. Hence, LAE should not be neglected in future studies of pulsar and FRBs emissions. Furthermore, in order to get more precise emission estimations, 2D or 3D fully-electromagnetic PIC simulations should be carried out. Consideration of transverse waves, their coherence, absorption, and propagation effects will provide even better insight into those processes.

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Software: PIC-code ACRONYM, Python

APPENDIX

A. LINEAR ACCELERATION EMISSION SOFTWARE TOOLKIT

A.1. Implementation Details and Limits

There are several notes with respect to the numerical implementation of the above described approaches:

- Before computing discrete Fourier transforms of current and charge densities, a cosine window filter is applied to space–time array $j_{il}(x, t)$ to avoid strong aliasing. That filter suppresses the influence of waves close to the edge
of the simulation box and highlights the waves in the center of the box. Without the filter, aliases, which are usually present as strong lines from low to high frequencies at specific wavenumbers, would significantly change the structure of results, and the emission power would artificially increase.

- The maximum resolved frequency is given by the storage interval $\Delta t$ of electromagnetic field data in the PIC code. This frequency (together with the wavenumber) then scales to the maximum frequency, $\omega_{\text{max}} = 2\pi/\Delta t$, in the plasma frame. We choose a storage time step short enough to cover well all frequencies presented in this paper.

- For the relativistic beam instability simulation, a weak alias appears in currents at multiple of the plasma frequency as a horizontal superluminal mode in the plasma reference frame. That cannot be avoided either by increasing the size of the simulation domain, the number of particles, or the order of the particle shape function. Thus, it is also present in some emission properties. But it does not significantly influence the eventually estimated total flux because its intensity is approximately four orders lower than that of the main superluminal mode.

- Although only the electric current density is present in Equation 7; the electric charge density is also needed for the correct construction of the relativistic transformation in Equations 6, $j' = \gamma_s (j - \beta_s \rho c)$. Both grid cell quantities are obtained self-consistently by the simulations.

- If electric currents are obtained from PIC simulation instead of individual particles, the calculation profits from the full information obtained by high-order shape function of the macro-particle.

Several effects can hinder wave generation and propagation. We assume that the emission frequency is higher than the absorption edge frequency of the surrounding plasma. The absorption effects do not significantly restrain the emission of electromagnetic waves. This is an advantage of the plasma bunch interaction model over the relativistic beam instability mechanism because the bunch density is higher than the density of the surrounding medium. Another possibility is that the electromagnetic waves are already initially absorbed. When the absorbing plasma reaches an arbitrary saturation level, its dispersion properties, however, may change. Thus, the absorption may decrease when the saturation energy level is reached. We also neglect spatial profiles perpendicular to the magnetic field. The emission is simultaneously provided by several sources; the emitted waves might mutually cancel out by interference. However, we can assume that the typical emission angle width of one region is significantly smaller than the typical observed half-width pulse profile of the main pulse. Hence, each emission source may emit without interference with others into different angles given by its position in the magnetosphere. Thus, while the observed flux at a given time may come from only one single coherent source, the total observed pulse profile is formed by several emission sources.

A.2. Tests of Implementation

We tested the software toolkit in a one-particle approximation and for an oscillating current wave with an arbitrary spatial size. We assumed that an electron oscillates with frequency $\omega_0 = 1 \text{ rad s}^{-1}$ for non-relativistic to relativistic drift velocities. Specifically, the size of the particle orbit is negligible in comparison with the size of one grid cell. The used drift velocity corresponds to the average particle velocity over many orbits in the observer reference frame. Thus, it reminds an oscillating dipole. As follows from Equation 1, there is no feedback of particle emission on its motion. We superposed the currents onto a grid (charge densities are zero) and input them into the LAE toolkit instead of the simulation currents. The emission power of such a point oscillating particle with frequency $\omega_0$ that has a drift velocity $\beta_s$ can be analytically estimated as (Griffiths 2017, Chapter 11)

$$\frac{dP_\theta}{d\Omega} = \frac{\omega_0^2 q^2 \beta_s^2 \sin^2 \theta}{16 \pi^4 c \epsilon_0 (1 + \beta_s \cos \theta)^5},$$  \(A1\)

where $q$ is the charge of the particle and $\epsilon_0$ is the permittivity of vacuum. The resulting electromagnetic powers obtained by the LAE toolkit are in good agreement with the analytical solutions of emission power as a function of emission angle Equation $A1$ (see Figure 8). They manifest a systematic error of $\approx 5\%$. We also tested a particle with a spatial length (current wave) represented by a Gaussian shape function. The emission angle narrows with increasing the particle length, and it approaches a delta distribution for an emission length going to infinity.
Analytical solution

\[ \frac{dP}{d\Omega} \frac{\partial}{\partial \omega} \theta \left[ \text{rad}^{-2} \right] \]

Figure 8. Test of the LAE toolkit values computed for three selected relativistic transformations \( \gamma_s = 1, 5, \) and 100. Top row: Emission powers as a function of emission angle normalized to their total emission power \( P_{\text{tot}} \). The analytical solution is represented by Equation A1. Bottom row: Corresponding relative errors. We assume a point electron oscillating with frequency \( \omega_0 = 1 \text{rad s}^{-1} \) and zero drift velocity that is superposed on a space–time grid of the size 6400 \( \times \) 6400 grid cells, and it is inserted as an input into the LAE toolkit. The computed frequency–angle grid sizes of the power as a function of frequency and emission angle are 400 \( \times \) 1800 (a), 9600 \( \times \) 1800 (b), and 120,000 \( \times \) 1800 (c). The angular resolution is 0.1\(^\circ\). The frequency resolutions are 5 \( \times \) 10\(^{-2}\) rad s\(^{-1}\), 1.25 \( \times \) 10\(^{-3}\) rad s\(^{-1}\), and 2.5 \( \times \) 10\(^{-3}\) rad s\(^{-1}\).

Figure 9 shows an example of how the electron emission can be represented in dependence on frequency, wavenumber, emission angle, and emission power into a unit spatial angle. Two reference frames are considered: the plasma frame, most of the power is represented by wave \( C \) in the plasma frame, and most of the power is represented by wave \( D \) in the relativistic reference frame. These wave representations and transformations are useful for an intuitive understanding of our results.

REFERENCES

Akhiezer, A. I., Akhiezer, I. A., Polovin, R. V., Sitenko, A. G., & Stepanov, K. N. 1975, Oxford Pergamon Press International Series on Natural Philosophy, 1

Arendt, P. N., & Eilek, J. A. 2002, ApJ, 581, 451, doi: 10.1086/344133

Benáček, J., Muñoz, P. A., & Büchner, J. 2021a, arXiv e-prints, arXiv:2106.13525.

https://arxiv.org/abs/2106.13525

Benáček, J., Muñoz, P. A., Manthei, A. C., & Büchner, J. 2021b, ApJ, 915, 127, doi: 10.3847/1538-4357/ac0338

Beskin, V. S. 2018, Uspekhi Fiz. Nauk, 188, 377, doi: 10.3367/UFNr.2017.10.038216

Beskin, V. S., Gurevich, S. V., & Istomin, Y. N. 1993, Physics of the pulsar magnetosphere (Cambridge University Press)

Bilous, A. V., Kondratiev, V. I., Kramer, M., et al. 2016, A&A, 591, A134, doi: 10.1051/0004-6361/201527702

Boris, J. P. 1970, in Proceedings of the Fourth Conference on the Numerical Simulation of Plasmas, Washington DC, ed. J. Boris (Naval Research Laboratory), 3–67

Buschauer, R., & Benford, G. 1977, MNRAS, 179, 99, doi: 10.1093/mnras/179.2.99

Cheng, A. F., & Ruderman, M. A. 1977a, ApJ, 212, 800, doi: 10.1086/155105

—. 1977b, ApJ, 214, 598, doi: 10.1086/155285

Cocke, W. J. 1973, ApJ, 184, 291, doi: 10.1086/152326
Figure 9. An example of how arbitrary charged particle can radiate in the plasma frame and in the relativistically shifted frame when it approaches an observer. We study five arbitrary emitted waves as a function of the frequency, wavenumber, emission angle, and emission power (per unit spatial angle). The particle oscillates at frequency \( \omega_0 = 1 \) rad s\(^{-1}\) and has the elementary charge. \( A, B, \ldots, E \) denote five types of emitted waves (blue crosses) with the same frequency \( \omega_0 \) and five wavenumbers \( k c \approx -0.966, -0.707, 0, 0.707, 0.966 \) rad s\(^{-1}\), both in the plasma reference frame. Top row: The plasma reference frame. Bottom row: The relativistically shifted reference frame with \( \gamma_s = 5 \). Left column: Positions of waves in \( \omega - k \) space. Middle column: Positions as a function of frequency and their emission angle. Right column: Position denoted by arrows at the analytical emission function. Black dashed line: The light line \( \omega = kc \). Grey dash-dotted line: Frequencies \( \omega_0 \). Black line: Analytically estimated electromagnetic emission by oscillating particle from Equation 7.

Cruz, F., Grismayer, T., & Silva, L. O. 2021, ApJ, 908, 149, doi: 10.3847/1538-4357/abd2c0
Eilek, J., & Hankins, T. 2016, J. Plasma Phys., 82, 635820302, doi: 10.1017/S002237781600043X
Esirkepov, T. 2001, Computer Physics Communications, 135, 144, doi: https://doi.org/10.1016/S0010-4655(00)00228-9
Goldreich, P., & Julian, W. H. 1969, ApJ, 157, 869, doi: 10.1086/150119
Griffiths, D. J. 2017, Introduction to Electrodynamics, 4th edn. (Cambridge University Press), 620, doi: 10.1017/9781108333511
Hankins, T. H., Jones, G., & Eilek, J. A. 2015, The Astrophysical Journal, 802, 130, doi: 10.1088/0004-637X/802/2/130
Jankowski, F., van Straten, W., Keane, E. F., et al. 2018, Monthly Notices of the Royal Astronomical Society, 473, 4436–4458, doi: 10.1093/mnras/stx2476
Jüttner, F. 1911, Ann. Phys., 339, 856, doi: 10.1002/andp.19113390503
Kärkkäinen, M., & Gjonaj, E. 2006, Proc. International Computational Accelerator Physics Conference, 35
Kilian, P., Burkart, T., & Spanier, F. 2012, in High Perform. Comput. Sci. Eng. ‘11, ed. W. E. Nagel, D. B. Kröner, & M. M. Resch (Berlin, Heidelberg: Springer Berlin Heidelberg), 5–13, doi: 10.1007/978-3-642-23869-7_1
Kramer, M., Jessner, A., Doroshenko, O., & Wielebinski, R. 1997, ApJ, 488, 364, doi: 10.1086/304706
Kroll, N. M., & McMullin, W. A. 1979, ApJ, 231, 425
Löhrner, O., Jessner, A., Kramer, M., Wielebinski, R., & Maron, O. 2008, A&A, 480, 623, doi: 10.1051/0004-6361:20066806
Lu, W., & Kumar, P. 2018, MNRAS, 477, 2470, doi: 10.1093/mnras/sty716
Lu, Y., Kilian, P., Guo, F., Li, H., & Liang, E. 2020, Journal of Computational Physics, 413, 109388, doi: 10.1016/j.jcp.2020.109388
Luo, Q., & Melrose, D. 2008, MNRAS, 387, 1291, doi: 10.1111/j.1365-2966.2008.13321.x
Manthei, A. C., Benáček, J., Muñoz, P. A., & Büchner, J. 2021, A&A, 649, A145, doi: 10.1051/0004-6361/202039907
Melrose, D. B. 1978, ApJ, 225, 557, doi: 10.1086/156516
Melrose, D. B. 2017, Reviews of Modern Plasma Physics, 1, 5, doi: 10.1007/s41614-017-0007-0
Melrose, D. B., & Gedalin, M. E. 1999, ApJ, 521, 351, doi: 10.1086/307539
Melrose, D. B., & Luo, Q. 2009, ApJ, 698, 124, doi: 10.1088/0004-637X/698/1/124
Melrose, D. B., & McPhedran, R. C. 1991, Electromagnetic Processes in Dispersive Media (Cambridge University Press), 432
Melrose, D. B., Rafat, M. Z., & Luo, Q. 2009, ApJ, 698, 115, doi: 10.1088/0004-637X/698/1/115
Melrose, D. B., Rafat, M. Z., & Mastrano, A. 2020, MNRAS, 500, 4530, doi: 10.1093/mnras/staa3324
Michel, F. C. 2004, Advances in Space Research, 33, 542, doi: 10.1016/j.asr.2003.06.019
Nishikawa, K., Duţan, I., Köhn, C., & Mizuno, Y. 2021, Living Reviews in Computational Astrophysics, 7, 1, doi: 10.1007/s41115-021-00012-0
Philippov, A., Timokhin, A., & Spitkovsky, A. 2020, PhRvL, 124, 245101, doi: 10.1103/PhysRevLett.124.245101
Rafat, M. Z., Melrose, D. B., & Mastrano, A. 2019, J. Plasma Phys., 85, 905850305, doi: 10.1017/S0004-6361/202039907
Rahaman, S. M., Mitra, D., & Melikidze, G. I. 2020, MNRAS, 497, 3953, doi: 10.1093/mnras/staa2280
Reville, B., & Kirk, J. G. 2010, ApJ, 715, 186, doi: 10.1088/0004-637X/715/1/186
Rowe, E. T. 1992a, Australian Journal of Physics, 45, 1, doi: 10.1071/PH920001
Rowe, E. T. 1992b, Australian Journal of Physics, 45, 21, doi: 10.1071/PH920021
Ruderman, M. A., & Sutherland, P. G. 1975, ApJ, 196, 51, doi: 10.1086/153393
Sturrock, P. A. 1971, ApJ, 164, 529, doi: 10.1086/150865
Timokhin, A. N. 2010, MNRAS, 408, 2092, doi: 10.1111/j.1365-2966.2010.17286.x
Timokhin, A. N., & Arons, J. 2013, MNRAS, 429, 20, doi: 10.1093/mnras/sts298
Timokhin, A. N., & Harding, A. K. 2019, The Astrophysical Journal, 871, 12, doi: 10.3847/1365-2966/2010.17286.x
Usov, V. V. 1987, ApJ, 320, 333, doi: 10.1086/165546
Usov, V. V. 2002, in Neutron Stars, Pulsars, and Supernova Remnants, ed. W. Becker, H. Lesch, & J. Trümper, 240. https://arxiv.org/abs/astro-ph/0204402
Weatherall, J. C. 1994, ApJ, 428, 261, doi: 10.1086/174237
Weatherall, J. C. 1997, ApJ, 483, 402, doi: 10.1086/304222
Yee, K. S. 1966, IEEE Trans. Antennas Propag., 14, 302, doi: 10.1109/TAP.1966.1138693
Zhang, B. 2020, Nature, 587, 45, doi: 10.1038/s41586-020-2828-1