Hadron formation from interaction among quarks

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Abstract

This paper deals with the hadronization process of quark system. A phenomological potential is introduced to describe the interaction between a quark pair. The potential depends on the color charge of those quarks and their relative distances. Those quarks move according to classical equations of motion. Due to the color interaction, coloring quarks are separated to form color neutral clusters which are supposed to be the hadrons.

1 Introduction

The study of particle production is an important subject in high-energy heavy-ion collisions. Physicists considered many mechanisms to describe the particle production processes, such as the string model [1] and the independent parton fragmentation model [2]. Both models can not explain the novel phenomena experimental observed at BNL/RHIC, such as an unexpectedly large $p/\pi$ ratio of about 1 at $p_T$ about 3 GeV/c [3], since they both predicted a very small $p/\pi$ ratio of about 0.2. In recent years, as a new approach to hadronization, the quark recombination model has been proposed [4], which can be applied in any $p_T$ region, and can solve the puzzles from RHIC, such as the unexpectedly high $p/\pi$ ratio and the constituent quark number scaling of the elliptic flow.

In the implementations of the quark recombination model as in [4], it is assumed that the hadronization takes zero time, thus the quark distributions does not change in the process. Then some analytical expressions for the spectra of the final state particles can be derived. However, since the yield of meson (baryon) is proportional to the square (cubic) of the quark density, it will be four (eight) time larger if the quark density is doubled. So the naive quark recombination model violates the unitarity in the hadronization. To shun this difficulty, a finite hadronization time is introduced in Refs. [5]. It is assumed there that the production rate for a species of meson is proportional to the product of densities of corresponding quark and antiquark. Similar for baryon production rate. Thus, the essence of the original quark recombination model is kept in the revised model. Because of a finite time for hadroinization, production of particles will reduce the number of quarks and thus influence later particle production. Therefore, all particle productions are correlated. In [6], only the yields of particles have been studied, with the production correlation fully considered. Frequently, one would like to learn the transverse spectra of produced particles. Such a job cannot be done analytically, and some Monte Carlo method must be used.

Another problem in the quark recombination model is about the interactions among quarks in quark-gluon-plasma (QGP) produced in ultrarelativistic heavy ion collisions. The QGP created at BNL/RHIC

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and LHC/ALICE is not a weakly coupling but strongly interacting matter. At the end of QGP evolution to the hadronization point, the interaction among quarks may even be stronger. Then it is important to ask whether or not such interactions have influence to the hadronization process.

This paper is a first step to solve the above two problems in the QGP hadronization. We will study hadron formation from a quark system with interactions by following the motion of quarks. We ask whether color neutral clusters can be formed from color interactions when quarks move according to classical equation of motion. The organization of this paper is as follows. In Sec. 2 we will discuss potential for quark interactions, and give a possible form of it. Then in Sec. 3 we present the numerical calculation details. Simulation results are shown in Sec. 4. The last section is for short discussions.

2 Potential for quark interaction

The interaction among quarks in hadronization should, in principle, be described by the basic theory, quantum chromodynamics (QCD). When hadronization is concerned, as in the case discussed in this paper, the color interactions among quarks cannot be calculated perturbatively, and many non-perturbative effects play role since hadronization process is a low momentum transfer process. Without first-principle guidance, one can use a phenomenological potential to describe the quark interaction, as done for bound state problems. Such a potential should depend on the colors carried by the interacting quarks. It has been shown that the potential corresponding to a single-gluon exchange is inversely proportional to the separation \( r \) between quarks when \( r \) is small. When \( r \) is large, a string may be formed between two quarks and the corresponding potential is \( \propto r \). Thus the potential is assumed as

\[
U_{ij} = c_{ij} a \left( r + \frac{b}{r} \right),
\]

where \( c_{ij} = c_{ji} \) is the color factor related quark \( i \) and quark \( j \), while \( a \) and \( b \) are two parameters of the model. If one chooses \( a > 0 \), then some properties of \( c_{ij} \) can be claimed. Since a quark and an anti-quark with the same color can combine to form a meson, they must attract one another, thus one may expect that for such a quark pair \( c_{ij} > 0 \). If a quark and an anti-quark do not carry the same color, \( c_{ij} < 0 \). For two quarks or two anti-quarks, a diquark can be formed if they carry different colors. Such a diquark is, in some sense, similar to an anti-quark, in interacting with a third quark. Then for two quarks or two anti-quarks, \( c_{ij} > 0 \) if they carry different colors. Otherwise, \( c_{ij} < 0 \). Now one can consider the interaction between a quark and a hadron. Since hadrons are color neutral, a quark would have no net force acting on a hadron if the hadron were a point particle. This condition can be satisfied if one chooses \( c_{rr} = -2c_{rb} = -2c_{rg} \). Similarly for other color combinations. Thus one can put all \( c_{ij} \) as a matrix

\[
C = Q \begin{pmatrix}
-2 & 1 & 1 \\
1 & -2 & 1 \\
1 & 1 & -2 
\end{pmatrix}
\]

with \( Q = -1 \) for a quark and an anti-quark, and otherwise \( Q = 1 \). The diagonal elements are for interaction between quarks with the same color, and the off-diagonal ones for those interactions with different colors. Then when two or three quarks move close enough to make a colorless cluster, the total force that act on an other quark is nearly zero, as shown in FIG. 1.

The next step is to fix values of parameters \( a \) and \( b \) in Eq. 1. While parameter \( a \) determines the strength of the total interaction between a pair of (anti) quarks, the parameter \( b \) tells the relative strength of the two interactions. Since the interactions among quarks are not considered in most event generators, one may impose that the sum of the interaction potentials is equal to zero, in order to conserve the total energy of the system. This additional requirement can be used to determine the value of parameter \( b \). It can be understood that the value of parameter \( a \) will be responsible for the spatial separation between quarks in a hadron. In this paper, we are only interested in whether color neutral clusters can be formed when color interactions are taken into account. Thus the value of \( a \) is not too relevant.
Figure 1: According to Eq. 2, the total force that a baryon acts on a quark (lower) or a anti-quark (upper) is about zero.

3 Numerical calculation for quark system evolution

In order to see the feasibility of our model, we deal with the hadronization of a quark system with 30 quark and antiquark pairs. It is assumed that the quarks have the same flavor. The generalization to include more flavors is straightforward. The total energy is assumed to be $E=60 \text{ GeV}$. We assume that they are taken from a large thermal equilibrium distribution

$$E = \sum_i \frac{gV}{\pi^2\hbar^3} \int f_i(T,p)|p^0|^2 dp,$$

$$N = \sum_i \frac{gV}{\pi^2\hbar^3} \int f_i(T,p)|p^2 dp,$$

(3)

where the summation runs over all flavors considered in the problem and $g = N_c N_s = 6$ is the degeneracy number for a quark. $f_i(T,p)$ is the distribution function for quarks of a specific flavor

$$f(T,p) = \frac{1}{1 + e^{p^0/T}},$$

(4)

with $p^0 = \sqrt{|\vec{p}|^2 + m_i^2}$. In the calculations, only $u, d$ and $s$ quark flavors are considered and the corresponding masses are chosen as 0.3GeV for $u$ and $d$ quarks, and 0.5GeV for $s$ quarks, very close to the one used in the constituent quark model. In fact, the main results obtained below have little dependence on the number of flavors used.

For given $E$ and $N$, a initial volume $V$ and the temperature $T$ can be obtained from Eq. (3). We put $N = 60$ and $E = 60\text{GeV}$ in this paper. Then the value for $V$ is about 8.0 fm$^3$ and that for $T$ is 0.28GeV. We assume that the quarks are uniformly distributed in a spherical space with volume $V$. Direct sampling according to Eq. (3) for momentum distribution is not easy. But one can do it in another way. One can assign to each quark a momentum with constant magnitude but a random direction to keep the total momentum zero. Then let the quarks undergo a certain number (for example 1000) elastic collisions inside
the spatial volume $V$ with periodic boundary conditions used, as described in [6]. One can check that the momentum distribution obtained is like the one in Eq. (3). Because of Eq. (4), the momentum distribution is spherically symmetric. Now we add the color property to each quark randomly. For convenience we use 1,2,3 to represent the colors, say, 1 for red, 2 for green, and 3 for blue, and -1,-2,-3 for the correspondingly anti-colors. To make the whole system color neutral, one can simultaneously assign the opposite colors to quark and antiquark for each pair. Potential of the whole system can calculate according to

$$U = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} U_{ij},$$  \hspace{1cm} (5)

where $U_{ij}$ is calculated according to Eq. (1) with the fixed parameter $a = 0.85$ GeV/fm$^{-1}$, which is double that suggested in [7]. It should be mentioned that the value of parameter $a$ has no influence to the main results in this paper. The choice of this specific value for $a$ is just as a try. $b$ could be determined by requiring $U = 0$ at the the moment we assign colors to quarks. At any time $t$, we can calculate the force acted on quark $i$ from quark $j$

$$\vec{f}_{ij} = -\nabla U_{ij} = c_{ij}a \left( \frac{\vec{r}_{ij}}{r_{ij}} - \frac{b\vec{r}_{ij}}{r_{ij}^3} \right).$$  \hspace{1cm} (6)

The total force acted on quark $i$ from all other quarks is

$$\vec{F}_i = \sum_{j \neq i} \vec{f}_{ij}.$$  \hspace{1cm} (7)

After the system moves $dt$ forward, particle $i$ will move according to the classical dynamics, and its momentum and position are

$$\vec{p}_i(t + dt) = \vec{p}_i(t) + \vec{F}_i dt,$$
$$\vec{r}_i(t + dt) = \vec{r}_i(t) + \vec{v}_i dt,$$  \hspace{1cm} (8)

where $\vec{v}_i = \vec{p}_i/E_i$ and $E_i = \sqrt{|\vec{p}_i|^2 + m^2_i}$. The evolution of the system runs over a time period long enough to enable quark clusters well separated from one another.

## 4 Some results

Since interaction among quarks depends strongly on positions of quarks, analytical calculation for all quarks’ trajectories is impossible. Thus numerical calculation has to be done with a finite time step $\Delta t > 0$. As a result of the finiteness of time step $\Delta t$, numerical errors must occur, and the total energy of the system is, generally, not conserved exactly. In principle, only the $\Delta t \rightarrow dt \rightarrow 0$ the fluctuation could be eliminated. In our calculation, we let $\Delta t = 0.0001$ fm/c, and assume that it is small enough. The numerical results show that the energy is transformed between the potential and kinetic but the total energy is almost kept constant, as shown in FIG.2

With the method provided above, one can follow the evolution of our system. Actually, after a few dozens fermi/c both the potential and the kinetic energy of the system do not change anymore. That means that the system has been split into some clusters of color neutral and the interactions among those clusters is very weak. Thus one can say that hadrons or pre-hadrons has been formed then. In FIG.3, we show two instant pictures for the initial and final state distributions of the quarks in the space. From the final state picture many clusters with different size could be found clearly.

Numerically, one can judge a cluster as a meson or a baryon or multi-quark state easily by counting the quark numbers in the cluster. The species of the meson or baryon could be obtained by studying the composition of each cluster. Comparing the distribution of particles in configuration space before and after hadronization as shown in FIG.4, we found that each cluster is exactly a hadron.
Figure 2: Following the evolution of the system, the energy is transformed between the potential and kinetic.
Figure 3: Quark distributions for the initial and final state in the evolution of the system. Left part is for the initial distribution of quarks and right one for that after a time of 100 fermi/c.
Figure 4: Hadron spatial distribution obtained by replacing each cluster (in the left panel) by a hadron (in the right panel). Where a solid ball represents a particle, while a hollow one for an anti-particle. The size of the ball is used to distinguish meson and baryon. A bigger symbol is for a baryon.
5 Conclusion

The hadronization of a quark system is considered as a dynamical process. An interaction potential is introduced which depends on the colors and spatial separation of quarks. Quarks move according to classical equations of motion. It is found that various hadrons could be formed naturally by gathering coloring quarks to color neutral clusters. In the process, the total energy is conserved within a high accuracy. Thus this paper shows that such a method may be useful for studying more sophisticated hadronization processes in heavy ion collisions.

This work was supported in part by the National Natural Science Foundation of China under Grant Nos. 11075061 and 11221504, by the Ministry of Education of China under Grant No. 306022, and by the Programme of Introducing Talents of Discipline to Universities under Grant No. B08033, also by the Open innovation fund of the Ministry of Education of China under Grant No. QLPL2014P01.

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