Performance analysis of power splitting SWIPT-enabled full duplex cooperative NOMA system with direct link

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Abstract
This paper considers a full duplex (FD) cooperative non-orthogonal multiple access system that employs power splitting relaying (PSR)-based simultaneous wireless information and power transfer (SWIPT) technique, that is, FD-PSR-NOMA system. A single cell network is considered consisting of a base station (BS) and two pre-paired users, where the near user is configured to act as FD relay to assist the BS for information delivery to the far user. It is assumed that the far user implements maximal ratio combining to combine the direct link signal arriving from the BS and the relayed signal from the near user, to exploit the diversity advantage. Analytical expressions are derived for the outage probabilities of the downlink users, system outage probability, and the throughput of the considered network under imperfect successive interference cancellation (i-SIC). Analytical expressions are also derived for the outage probabilities and throughput of half duplex (HD) cooperative NOMA system under PSR-based SWIPT, that is, HD-PSR-NOMA system. Further, the asymptotic outage probabilities and diversity orders are derived as well. The optimal power allocation factors at the BS that minimize the system outage probabilities of FD/HD-PSR-NOMA networks are determined. Extensive simulation results are provided to corroborate the analytical findings.

1 INTRODUCTION

The fifth generation (5G) wireless networks are envisioned to offer very high data rate, massive connectivity, low latency, and very high spectral efficiency [1]. Compared to the conventional orthogonal multiple access (OMA) scheme, non-orthogonal multiple access (NOMA) is a promising technique to improve the spectral efficiency. Accordingly, NOMA has recently been considered as an efficient multiple access technique for 5G wireless networks [1–3]. In downlink power domain NOMA, superposition coding is implemented by the base station (BS) to multiplex and transmit the messages intended for the users, in the same time-frequency resource block with distinct power levels. The receivers at the user equipment implement successive interference cancellation (SIC) to decode the messages. At the BS, power allocation is carried out based on the quality of the channel gains between users and the BS. In particular, users with weak channel gains (i.e. weak users) are allocated higher power, while users with strong channel gains are allocated lower power. This enhances the effective data rate of the weak users, which leads to improved fairness among the users. Further, since NOMA allows the BS to serve multiple downlink users simultaneously, it provides reduced latency as well [1–3].

To extend the coverage of the BS and to improve the reliability, cooperative relay based NOMA has been proposed, where the BS delivers messages to the far (i.e. cell-edge or weak) users with the assistance of near (i.e. cell-centric or strong) users [4]. However, if half duplex (HD) relay nodes are used for forwarding the messages from the BS to the far users, the spectral efficiency of the resulting HD-NOMA system significantly degrades, owing to the fact that two distinct non-overlapping time slots are needed to deliver messages to the far user [4]. Instead, full duplex (FD)-based NOMA system, where the near user is configured to operate in the FD mode, can achieve higher spectral efficiency, since the FD relay nodes are capable of performing simultaneous transmission and reception [5–7].
utilizing the same frequency resources. However, the disadvantage of FD-NOMA system is that the relay nodes experience very high self-interference (SI), that is, the loop back interference induced from the relay’s transmitter to its receiver section. Even though SI can be mitigated by efficient isolation/cancellation techniques, the relay node will continue to experience residual self-interference (RSI), which proportionally grows with the transmit power used at the relay [8, 9].

Recently, there has been tremendous upsurge of research activities on realizing simultaneous wireless information and power transfer (SWIPT), which is a promising solution for the design of energy-efficient wireless networks [10]. Generally, two protocols are used for realizing SWIPT, that is, power splitting relaying (PSR) and time switching relaying (TSR) [10]. When PSR is used, the received signal is split into two parts: one for information detection and the other for energy harvesting (EH). Under TSR, the total received RF signal is used for both information processing and EH, but in different time slots. By integrating FD relaying, spectral efficiency of NOMA system can be further improved, while the use of SWIPT will enhance the energy efficiency (EE) of the network. Thus, the integrated cooperative NOMA system consisting of FD relaying and SWIPT, that is, SWIPT-enabled FD-NOMA system, has very high practical relevance to meet the challenging requirements of 5G wireless networks. Motivated by these, the objective of this paper is to investigate and optimize the performance of SWIPT-enabled FD-NOMA networks under PSR, that is, we call it as FD-PSR-NOMA network.

Extensive research has been reported on the performance analysis of SWIPT-enabled HD-NOMA systems (see [11–27] and references therein). The outage and throughput performance of SWIPT enabled HD-NOMA system has been analyzed in [11] by modeling the locations of users using stochastic geometry based approach. Here, the near users are used as relays for assisting the BS to deliver messages to the far users. Two power allocation policies, namely NOMA with fixed power allocation and cognitive radio inspired NOMA, have been investigated in [12] for analyzing the outage performance of SWIPT enabled HD-NOMA system. The outage and ergodic rate performance of SWIPT enabled HD-NOMA system has been analyzed in [13] over Nakagami-m channels, by considering amplify-and-forward relaying scheme in the presence of channel estimation errors. The outage performance of SWIPT enabled HD-NOMA system has been investigated in [14] assuming Weibull fading channels, taking into account imperfect channel state information (i-CSI). The authors of [15] have explored the joint design of power allocation coefficients and the power splitting factor to enhance the outage performance of the near user in a SWIPT enabled HD-NOMA system. The authors of [16] and [17] have independently analyzed the outage performance of downlink users in SWIPT-enabled relay aided HD-NOMA system under partial relay selection scheme. The authors of [18] and [19] have independently analyzed the outage performance of SWIPT-enabled HD-NOMA system under PSR protocol.

In [20], the authors have evaluated the outage performance of SWIPT-enabled HD-NOMA system, where multiple antennas are used at the source and the destination terminals, while the relay node has been assumed to be a single-antenna device. The outage performance of SWIPT-enabled multi-user HD-NOMA system has been analyzed in [21]. The outage probability and the diversity order of the cell edge user has been analyzed in [22], for a two-user multiple-input single-output SWIPT-enabled HD-NOMA network. An efficient user-pairing method has been proposed in [23] considering a SWIPT-based HD-NOMA system, while the work in [24] has considered maximizing the throughput of the secondary network in SWIPT-enabled cognitive NOMA system. EE optimization for SWIPT-enabled HD-NOMA system has been considered independently by various authors in [25–27].

The detailed literature survey shows that, even though the SWIPT-enabled HD-NOMA systems outperform the conventional OMA systems, use of HD relaying degrades the spectral efficiency. On the other hand, the SWIPT-enabled FD-NOMA system can provide higher spectral efficiency, which is very essential to support data rate intensive applications in 5G wireless networks.

Recently, the performance of SWIPT-enabled downlink FD-NOMA system has been analyzed by a few researchers by considering that the BS employs the near user as a cooperative relay to deliver information to the far user [28–32]. The authors of [28] have considered the design of low complex algorithms to minimize the transmit power in SWIPT-enabled two-user downlink FD-NOMA system, with both full CSI as well as partial CSI. In [29], the authors have proposed an iterative algorithm to determine the jointly optimal power splitting ratio and transmit beamforming vectors to maximize the EE. The authors of [30] have proposed a suboptimal algorithm to maximize the data rate of the near user in SWIPT enabled FD-NOMA system, while satisfying the quality of service (QoS) of the far user. The outage probabilities of the users and the ergodic sum rate have been analyzed in [31], for a SWIPT-enabled FD-NOMA system, considering various instances of SI cancellation at the FD relay. In [32], the authors have evaluated outage and ergodic sum rate of SWIPT-enabled FD-NOMA system under TSR protocol, considering the mean RSI power to be a constant.

From the literature survey, it is clear that analytical models for the evaluation of outage probability and system outage performance of SWIPT-enabled FD-NOMA system under PSR protocol (i.e. FD-PSR-NOMA) by considering imperfect SIC (i-SIC) at the near user and maximal ratio combining (MRC) at the far user has not appeared in the literature so far. Further, the problem of determining the optimal power allocation (OPA) factor at the BS that minimizes the system outage probability of FD-PSR-NOMA network has not been attempted by the researchers previously. Motivated by the aforementioned facts, the major objectives of this paper are: (i) to analyze the outage and throughput performance of FD/HD-PSR-NOMA network, and (ii) to investigate the optimization of system outage performance of the considered networks.
1.1 Contributions

The major contributions of the paper are as follows:

- We consider a full-duplex cooperative NOMA system consisting of a BS and two downlink users, where the far user implements MRC to combine direct link signal from BS and relayed signal from the near user. We derive analytical expressions for the outage probabilities experienced by the users and the system outage probability in the presence of i-SIC. Further, we formulate expressions for the delay-limited system throughput as well. For comparison study, we formulate analytical expressions for the outage probabilities in HD-PSR-NOMA system as well. The impact of i-SIC factor and other system related parameters on outage and throughput performance of the considered systems are extensively analyzed.

- To obtain further insights on the outage probability performance, we derive analytical expressions for the asymptotic outage probabilities and determine the diversity order in terms of outage probabilities as well.

- Analytical expression for the OPA factor at the BS that minimizes the asymptotic system outage probability of FD/HD-PSR-NOMA network, assisted by the direct link is determined. Extensive numerical and simulation investigations are carried out to determine the sensitivity of the optimal quantity against various system parameters.

The rest of this paper is organized as follows: Section 2 describes the system model for FD/HD-PSR-NOMA network. Section 3 describes the outage probability analysis. Section 4 considers system outage probability minimization while Section 5 analyzes the delay-limited system throughput. The performance evaluation results are described in Section 6, while Section 7 describes the conclusions.

2 SYSTEM MODEL

The SWIPT-enabled FD-NOMA network shown in Figure 1 is considered, which consists of a BS and two pre-paired downlink NOMA users, that is, a near NOMA user \( U_1 \) and a far NOMA user \( U_2 \). To assist reliable information delivery to \( U_2 \), \( U_1 \) acts as a decode-and-forward (DF) based FD relay. Apart from the relayed signal from \( U_1 \), \( U_2 \) receives signal via the direct BS-\( U_2 \) link as well. Here, we focus on a two-user FD-PSR-NOMA scenario, since a large number of downlink NOMA receivers multiplexed in the power domain might not be feasible in practice, owing to the increased complexity of performing SIC at the receivers [1–3]. Since the near user \( U_1 \) is assumed to operate as FD relay, we assume it to have two antennas (i.e. separate antennas for implementing simultaneous transmission and reception). This enables \( U_1 \) to perform simultaneous transmission and reception in the same frequency band [33]. Further, we assume BS and \( U_2 \) to be single-antenna HD devices. The use of separate directional antennas for implementing simultaneous transmission and reception at the FD relay node \( U_1 \) can partially mitigate the effect of SI [34]. The considered communication system represents the downlink of single-cell wireless cellular network that employs user assisted relaying, where the near user is energy constrained and the BS can be the transmitter of a macro cell. Let \( b_{ij} \); \( i \in \{ 1, 2 \} \), \( j \in \{ 1, 2 \} \) be the fading channel coefficients corresponding to the link connecting nodes \( i \) and \( j \). Frequency flat block Rayleigh fading is assumed; thus, \( |b_{ij}|^2 \) have exponential PDF with \( E[|b_{ij}|^2] = \lambda_{ij} \).

The power domain NOMA signal transmitted by BS at time instant \( t \) is

\[
\mathbf{x}(t) = \sqrt{\alpha P_x x_1(t)} + \sqrt{(1-\alpha)P_x x_2(t)},
\]

where \( P_x \) is the total transmit power of BS; \( x_1 \) and \( x_2 \) are the messages for \( U_1 \) and \( U_2 \), respectively with \( E[|x_i|^2] = 1; \alpha \) is the power allocation factor for \( U_1 \) at BS; here \( 0 < \alpha < 0.5 \) so that higher power is allocated to the far user \( U_2 \) [4–6]. The signal received by \( U_1 \) is

\[
y_1(t) = b_{11} \sqrt{\alpha P_x x_1(t)} + b_{12} \sqrt{(1-\alpha)P_x x_2(t)} + n_1(t) + n_2(t),
\]

where \( n_1(t) \) is the receiver front-end noise, which is assumed to be Gaussian and \( s_1(t) \) is the RSI at \( U_1 \) arising due to FD operation. Similar to previous works (see [35, 36] and references therein), we assume RSI to be zero mean Gaussian random variable with variance \( I_{0,\text{PSR}} = E[|n_1(t)|^2] = \zeta (P_r)^\theta \), where \( P_r \) is the average transmit power of \( U_1 \); \( \zeta (0 < \zeta < 1) \) and \( \theta (0 < \theta < 1) \) are related to self interference cancellation technique used at \( U_1 \). This model for RSI justifies the experimental results reported in [37].

Notice that the FD node \( U_1 \) acts as energy harvesting (EH) relay as well under PSR scheme. Figure 2 shows the FD-PSR
scheme considered, where both EH and information transmissions are active for the entire transmission cycle [38]. Here, \( \rho \) portion of \( y_1(t) \) is used for EH, while \((1 - \rho)\) portion is used for information decoding (ID), where \( \rho \) \((0 < \rho < 1)\) is the PS factor. We assume that efficient techniques are used for SI cancellation at \( U_1 \), which are generally implemented in three stages; that is, passive suppression, analog domain cancellation and digital domain cancellation. With these, the SI can be suppressed to the level of receiver’s noise factor [8]. Accordingly, we assume the power harvested from the RSI component \( s_1(t) \) and the noise \( n_1(t) \) to be negligible, so that the total harvested power at \( U_1 \), \( P_1 = \eta \rho |h_{11}|^2 P_t \), where \( \eta \) \((0 < \eta < 1)\) is the energy harvesting efficiency. Accordingly, the mean RSI power, \( I_{O,PSR} = \xi (\eta P_s \rho \lambda_1)^2 \). The signal component available for ID at \( U_1 \) is given by

\[
\begin{align*}
\gamma_{1,ID}(t) &= \sqrt{1-\rho}|h_{11}|\sqrt{\alpha P_{S}|x_1(t)|^2} + h_{11}\sqrt{(1-\alpha)}|P_{S}|x_2(t) |s_1(t)| + n_1(t),
\end{align*}
\]

Here, \( n_1(t) = (\sqrt{1-\rho}) |y_1(t)| + n_2(t) \), where \( n_2(t) \) is the processing noise introduced by the receiver, which affects the SINR at \( U_1 \). We consider \( n_1(t) \) as AWGN of variance \( N_0 \).

### 2.1 | Signal to interference plus noise ratio

Here, the near user \( U_1 \) has to decode symbol \( x_2 \) from \( y_{1,ID}(t) \) firstly and then use SIC to decode its own symbol \( x_1 \), by canceling the known \( x_2 \) from \( y_{1,ID}(t) \). Imperfect SIC (i-SIC) will generate residual interference at \( U_1 \). Since \( U_1 \) operates as a DF relay in FD mode, it simultaneously forwards the symbol \( x_2 \) to \( U_2 \). The SINR corresponding to the decoding of \( x_2 \) and \( x_1 \) (\( \Gamma_{12}^{PSR} \) and \( \Gamma_{11}^{PSR} \) respectively) are given by

\[
\begin{align*}
\Gamma_{12}^{PSR} &= \frac{(1-\rho)|h_{11}|^2 \rho (1-\alpha)}{(1-\rho)|h_{11}|^2 \rho + \alpha + (1-\rho)|P_{O,PSR}^{12} + 1}, \\
\Gamma_{11}^{PSR} &= \frac{(1-\rho)|h_{11}|^2 \rho \alpha}{(1-\rho)|h_{11}|^2 \rho + \alpha + (1-\rho)|P_{O,PSR}^{11} + 1},
\end{align*}
\]

where \( \rho = \frac{\rho}{N_0} \), \( P_{O,PSR}^{12} = \frac{\rho_{o,PSR}}{N_0} \), \( \beta (0 < \beta < 1) \) is the i-SIC factor and \( \omega \) equals 1 for FD and 0 for HD based systems. Since the relaying link from \( U_1 \) to \( U_2 \) will suffer a finite time delay compared to the direct BS-\( U_2 \) link, we assume that signals from the relaying link and the direct link (DL) can be resolved and combined at \( U_2 \) using maximal ratio combining (MRC) [5]. The SINR at \( U_2 \) corresponding to the decoding of \( x_2 \) is given by \( \Gamma_{22}^{PSR,MRC} = |\beta h_{21}|^2 \rho + \Gamma_{22}^{PSR,DL} \); \( \Gamma_{22}^{PSR,DL} = \frac{(1-\alpha)|P_{o,PSR}^{22} + 1}{\alpha \rho |h_{21}|^2 + 1} \) where \( \rho = \frac{\rho_{o,PSR}}{N_0} \), \( \eta = \rho |h_{21}|^2 \).

### 3 | OUTAGE PROBABILITY ANALYSIS

Let \( R_1 \) and \( R_2 \) (expressed in bits per channel use), respectively, be target rates of the users \( U_1 \) and \( U_2 \), respectively. In FD-PSR-NOMA, the transmission of the symbols \( x_1 \) and \( x_2 \) to \( U_1 \) and \( U_2 \) will be completed within the time duration \( T \). Accordingly, the target SINRs for the successful decoding of \( x_1 \) and \( x_2 \) (i.e. \( u_1^{FD} \) and \( u_2^{FD} \), respectively) are given by \( u_1^{FD} = 2R_1 - 1 \) and \( u_2^{FD} = 2R_2 - 1 \). However, in HD-PSR-NOMA system (i.e. \( u_1^{HD} \) and \( u_2^{HD} \), respectively) are given by \( u_1^{HD} = 2R_1 - 1 \) and \( u_2^{HD} = 2R_2 - 1 \).

#### 3.1 | Outage probability experienced by the near user \( U_1 \)

The near user \( U_1 \) has to successfully decode \( x_2 \) and \( x_1 \); otherwise it will suffer outage. Thus, the outage probability experienced by \( U_1 \) in FD-PSR-NOMA system is determined as:

\[
\begin{align*}
P_{FD,PSR,\text{out},1} &= 1 - \Phi^{FD}(1-\rho|P_{O,PSR}^{12} + 1), \quad \text{where} \quad \Phi^{FD} = \max\{(u_2^{FD}((1-\alpha)|P_{O,PSR}^{12} + 1)), \frac{u_2^{FD}}{(1-\alpha)|P_{O,PSR}^{12} + 1}\}. \\
\text{Otherwise,} \quad P_{FD,PSR,\text{out},1} \text{ becomes unity.}
\end{align*}
\]

**Proof.** Setting \( \alpha = 1 \) in (4)-(5) and substituting for \( \Gamma_{12}^{PSR} \) and \( \Gamma_{11}^{PSR} \) in (6) and re-arranging, we get

\[
\begin{align*}
P_{FD,PSR,\text{out},1} &= 1 - \Phi^{FD}(1-\rho|P_{O,PSR}^{12} + 1), \quad \text{where} \quad \Phi^{FD} = \max\{(u_2^{FD}((1-\alpha)|P_{O,PSR}^{12} + 1)), \frac{u_2^{FD}}{(1-\alpha)|P_{O,PSR}^{12} + 1}\}. \\
\end{align*}
\]

Defining \( \Phi^{FD} \) as above and noting that \( |b_j|^2 \) have exponential PDF, \( P_{FD,PSR,\text{out},1} \) can be obtained as in (7) above. From (8), it can be seen that if either \( \alpha > \frac{1}{1+\bar{u}_2} \) or \( \alpha < \frac{\bar{u}_2^{FD}}{1+\bar{u}_2^{FD}} \), \( P_{FD,PSR,\text{out},1} \) will become unity.

**Corollary 1.** The asymptotic outage probability \( P_{FD,PSR,\text{out},1} \), as \( \rho \to \infty \), can be obtained as:

\[
\begin{align*}
P_{FD,PSR,\text{out},1} \approx \Phi^{FD}(1-\rho|\eta \rho |h_{21}|^2 + 1),
\end{align*}
\]

\[
\begin{align*}
\Phi^{FD} = \frac{\Phi^{FD}(1-\rho|\eta \rho |h_{21}|^2 + 1)}{(1-\rho|\eta \rho |h_{21}|^2 + 1)}.
\end{align*}
\]
Proof. Consider the expression for $P_{out,1}^{FD,PSR}$ given by (7). Here we let $\rho_r \to \infty$ and by using the approximation $\lim_{x \to 0} e^{-x} \approx 1 - x$, we get (9). As $\rho_r \to \infty$, we find that $P_{out,1}^{FD,PSR}$ becomes a constant, independent of $\rho_r$, which shows that $U_1$ suffers an outage floor behaviour in the high SNR region. This is due to the presence of RSI in FD-PSR-NOMA system.

Corollary 2. When $0 < \alpha < \frac{1}{1 + \beta s_2}$ and $\frac{\beta s_2}{1 + \beta s_2} \rho_o < \alpha < 1$, outage probability experienced by $U_1$ in HD-PSR-NOMA system is given by:

$$P_{out,1}^{HD,PSR} = 1 - e^{-\phi^{HD}(1 - \rho)\rho_r\lambda_{12}}, \quad (10a)$$

where $\phi^{HD} = \max\{\frac{u_1^{HD}}{(1 - \alpha - \alpha\rho_o^{HD})}, \frac{u_2^{HD}}{(\alpha - \alpha\rho_o^{HD} + \beta s_1\beta s_2\rho_o)}\}$. The asymptotic outage probability of $U_1$ in HD-PSR-NOMA system can be obtained by setting $\rho_r \to \infty$ and using the approximation mentioned above, in (10a). Thus, we get

$$P_{out,1}^{HD,PSR,asy} \approx (1 - \rho)\rho_r\lambda_{12}. \quad (10b)$$

Notice that $P_{out,1}^{HD,PSR,asy}$ decreases as $\rho_r$ is increased. From (10b), it is clear that $U_1$ does not experience outage floor in HD-PSR-NOMA system, since RSI is absent in the system.

### 3.2 Outage probability experienced by the far user $U_2$

The outage probability experienced by the far user $U_2$ includes the following events: (i) symbol $x_2$ is successfully decoded at $U_1$, but not at $U_2$ after MRC or (ii) the received SNR at $U_1$ (i.e. via BS-$U_1$ link) and that at $U_2$ (i.e. via BS-$U_2$ link) are not enough for the successful decoding of $x_2$ at $U_1$ and $U_2$ respectively. Accordingly, $P_{out,2}^{FD,PSR}$ is given by

$$P_{out,2}^{FD,PSR} = P_r \left\{ \Gamma_{12}^{PSR} \geq u_2^{FD}, \Gamma_{22}^{PSR, MRC} < u_2^{FD} \right\} P_{out,2}^{FD,PSR,DL} \quad (11)$$

where

$$\tau_1 = \frac{u_2^{FD}}{\rho_r(1 - \alpha - \alpha u_2^{FD})}, \quad \tau_2 = \frac{u_2^{FD}((1 - \rho)\rho_o^{PSR,DL} + 1)}{(1 - \rho)\rho_r(1 - \alpha - \alpha u_2^{FD})}$$

and

$$\tau_{m} = \cos\left(\frac{(2m - 1)\pi}{2N}\right).$$

**Proposition 2.** Assuming $0 < \alpha < \frac{1}{1 + \beta s_2}$, $P_{out,2}^{FD,PSR}$ is given by (12).

When $\alpha > \frac{1}{1 + s_2}$, $P_{out,2}^{FD,PSR}$ will become unity.
where \( A(y) = \frac{(\alpha F D\alpha - 1 + \alpha)\rho y + u F D^2}{\eta \rho, (\alpha \rho, y + 1)} \); \( r_1 \) and \( r_2 \) are defined in (12).

Now \( B(y) \) in (14b) can be evaluated by utilizing Maclaurin series expansion for the term \( e^{-\frac{\tau y}{\lambda}} \), as follows:

\[
B(y) = \int_{x=r_2}^{\infty} e^{-\frac{x}{\lambda}} \sum_{n=0}^{\infty} \left( \frac{(-1)^n}{n!x^n} \right) dx, \tag{15a}
\]

\[
= \lambda_1 e^{-\frac{\tau y}{\lambda_1}} - \frac{A(y)}{\lambda_1} \Gamma \left( 0, \frac{\tau y}{\lambda_1} \right) + \sum_{n=2}^{\infty} \left( \frac{(-1)^n}{n!\lambda^{n-1}} \right) \left( \frac{\tau y}{\lambda_1} \right)^n,
\]

\[
x \left( -\frac{\tau y}{\lambda_1}, \frac{\tau y}{\lambda_1} \right) \right).
\tag{15b}
\]

Notice that (15b) is obtained by using (3.352.2) and (3.353.1) given in [39], where \( Ei(x) \) is the exponential integral. Now \( l_{01} \) in (14b) can be determined by utilizing (15b) in (14b), and it will have three terms. Let \( l_{01} = l_{01}^{(1)} + l_{01}^{(2)} + l_{01}^{(3)} \), which are determined as follows:

\[
l_{01}^{(1)} = e^{-\frac{\tau_0}{\lambda_2}} \int_{y=0}^{r_1} e^{-\frac{y}{\lambda_2}} dy = e^{-\frac{\tau_2}{\lambda_2}} \left( 1 - e^{-\frac{\tau_2}{\lambda_2}} \right), \tag{16}
\]

\[
l_{01}^{(2)} = -\frac{1}{\lambda_1 \lambda_2 \lambda_1} \Gamma \left( 0, \frac{\tau_2}{\lambda_1} \right) \times \int_{y=0}^{r_1} \left( \frac{u F D^2 \alpha - 1 + \alpha}{\eta \rho, (\alpha \rho, y + 1)} \right) e^{-\frac{y}{\lambda_2}} dy, \tag{17a}
\]

\[
= -\frac{\Gamma \left( 0, \frac{\tau_2}{\lambda_1} \right)}{\lambda_1 \lambda_2 \lambda_1} \left[ \left( \frac{u F D^2 \alpha - 1 + \alpha}{\eta \rho, (\alpha \rho, y + 1)} \right) \left( 1 - e^{-\frac{\tau_2}{\lambda_2}} \right) \right]
+ \frac{1 - \alpha}{\eta \rho, (\alpha \rho, y + 1)} \left[ Ei \left( -\frac{\tau_1}{\lambda_1}, \frac{1}{\alpha \rho, (\alpha \rho, y + 1)} \right) - Ei \left( -\frac{\tau_2}{\lambda_2}, \frac{1}{\alpha \rho, (\alpha \rho, y + 1)} \right) \right]. \tag{17b}
\]

Notice that (17b) is obtained by utilizing (3.352.1) in [39].

Now \( l_{01}^{(3)} \) is determined as:

\[
l_{01}^{(3)} = \frac{1}{\lambda_1 \lambda_2 \lambda_1} \sum_{n=2}^{\infty} \left( \frac{(-1)^n}{n!\lambda^{n-1}} \right) \left( \frac{\tau_2}{\lambda_1} \right)^n
\]

\[
\times \left( -\frac{\tau_2}{\lambda_1}, \frac{\tau_2}{\lambda_1} \right) \times \frac{1 - \alpha}{\eta \rho, (\alpha \rho, y + 1)} \left[ Ei \left( -\frac{\tau_1}{\lambda_1}, \frac{1}{\alpha \rho, (\alpha \rho, y + 1)} \right) - Ei \left( -\frac{\tau_2}{\lambda_2}, \frac{1}{\alpha \rho, (\alpha \rho, y + 1)} \right) \right]. \tag{17b}
\]

Since the evaluation of (18) is cumbersome, we use Gaussian Chebyshev approximation [40] to get the following expression for \( l_{01}^{(3)} \), that is,

\[
l_{01}^{(3)} = \frac{1}{\lambda_1 \lambda_2 \lambda_1} \sum_{n=2}^{\infty} \left( \frac{(-1)^n}{n!\lambda^{n-1}} \right) \left( \frac{\tau_2}{\lambda_1} \right)^n
\times \left[ -\frac{\tau_2}{\lambda_1}, \frac{\tau_2}{\lambda_1} \right] \times \frac{1 - \alpha}{\eta \rho, (\alpha \rho, y + 1)} \left[ Ei \left( -\frac{\tau_1}{\lambda_1}, \frac{1}{\alpha \rho, (\alpha \rho, y + 1)} \right) - Ei \left( -\frac{\tau_2}{\lambda_2}, \frac{1}{\alpha \rho, (\alpha \rho, y + 1)} \right) \right]. \tag{18}
\]

\[
N \times e^{-\frac{2\pi y}{\lambda_1}} \left( 1 - \eta \rho, (\alpha \rho, y + 1) \right), \tag{19}
\]

where \( N \) is the accuracy complexity trade-off parameter in the approximation. Substituting (16), (17b) and (19) in (14b) gives \( l_0 \). Now \( l_1 \) in (11) can be determined by substituting for \( \Gamma_{12}^{FR} \) and \( \Gamma_{22}^{FR} \) and re-arranging as follows:

\[
l_1 = P_1 \left\{ |b_{1}|^2 < \frac{\rho T_{FD}^2 (1 - \rho) \rho, (1 + \rho, (\alpha \rho, y + 1) + 1)}{\rho, (1 - \rho, (\alpha \rho, y + 1) + 1)} \right\}; \tag{20a}
\]

\[
|b_{2}|^2 < \frac{\rho T_{FD}^2 (1 - \rho, (\alpha \rho, y + 1) + 1)}{\rho, (1 - \rho, (\alpha \rho, y + 1) + 1)} \right\} \tag{20b}
\]

Notice that (19 b) follows under the assumption of independently faded links. Now \( P_{out, 2}^{FR, ISR} = l_0 + l_1 \) and is given by (12).

However if \( \alpha > \frac{1}{1 + \frac{\rho T_{FD}^2}{\rho, (\alpha \rho, y + 1) + 1}} \), \( P_{out, 2}^{FR, ISR} \rightarrow 1 \), as is evident from (13a) and (20a).

\[\square\]

Corollary 3. As \( \rho, \rightarrow \infty \), the asymptotic outage probability of user 2, \( P_{out, 2}^{FR, ISR, \infty} \) is given by:

\[
P_{out, 2}^{FR, ISR, \infty} \approx \frac{\rho T_{FD}^2 (1 - \rho, (\alpha \rho, y + 1) + 1)}{\rho, (1 - \rho, (\alpha \rho, y + 1) + 1)} \eta \rho, (\alpha \rho, y + 1) + 1 \lambda_1 A_1 A_2} \]
Proof. Consider the definition of $P_{out,2}^{FD,PSR}$ given in (11), that is, $P_{out,2}^{FD,PSR} = I_0 + I_1$. Now the asymptotic value of $P_{out,2}^{FD,PSR}$, that is, $P_{out,2}^{FD,PSR,asy}$, can be obtained by setting $\rho_1 \rightarrow \infty$ and using the approximation $\lim_{\epsilon \rightarrow 0} e^{-\xi} \approx 1 - \xi$ in the expressions for $I_0$ and $I_1$. Thus, we write $P_{out,2}^{FD,PSR} = I_0^{asy} + I_1^{asy}$. Now $I_0^{asy}$ is determined starting with (14b), as follows, that is,

$$\frac{1}{\lambda_1 \lambda_2} \int_{x=t_2}^{\infty} \int_{y=s}^{\infty} \left(1 - \frac{\mu_{FD}^{2} \alpha - 1 + \alpha}{\eta \rho \lambda_{12}} \right) dx dy = \frac{1}{\lambda_1 \lambda_2} \int_{s}^{\infty} \int_{y=t_2}^{\infty} \left(1 - \frac{\mu_{FD}^{2} \alpha - 1 + \alpha}{\eta \rho \lambda_{12}} \right) dy dx$$

(22a)

$$= \left(1 - e^{-\frac{\mu_{FD}^{2} \alpha - 1 + \alpha}{\eta \rho \lambda_{12}}} \right) \left(1 - e^{-\frac{\mu_{FD}^{2} \alpha - 1 + \alpha}{\eta \rho \lambda_{12}}} \right)$$

(22b)

$$= \left(1 - e^{-\frac{\mu_{FD}^{2} \alpha - 1 + \alpha}{\eta \rho \lambda_{12}}} \right) \left(1 - e^{-\frac{\mu_{FD}^{2} \alpha - 1 + \alpha}{\eta \rho \lambda_{12}}} \right)$$

Notice that (22a) is obtained by using the tight lower bound for $-EI_i(-\infty)$, that is, $-EI_i(-\infty) \geq \frac{1}{1+K_x} e^{-\xi}$ and then letting $\rho_1 \rightarrow \infty$. Similarly $I_1^{asy}$ is determined from (20b) as follows, by using the approximation mentioned earlier as $\rho_1 \rightarrow \infty$:

$$I_1^{asy} = \left[ \left(1 - e^{-\frac{\mu_{FD}^{2} \alpha - 1 + \alpha}{\eta \rho \lambda_{12}}} \right) \left(1 - e^{-\frac{\mu_{FD}^{2} \alpha - 1 + \alpha}{\eta \rho \lambda_{12}}} \right) \right] \left(1 - e^{-\frac{\mu_{FD}^{2} \alpha - 1 + \alpha}{\eta \rho \lambda_{12}}} \right)$$

Now $P_{out,2}^{FD,PSR,asy} = I_0^{asy} + I_1^{asy}$; substituting (22d) and (23), we get (21). From (21), we observe that the asymptotic outage probability of $U_2$ in FD-PSR-NOMA system decreases as $\rho_1$ becomes higher; thus, we conclude that $U_2$ does not suffer outage floor in FD-PSR-NOMA system due to the presence of diversity provided by the direct link signal at $U_2$.

**Corollary 4.** When $0 < \alpha < \frac{1}{1+\rho_1}$, the outage probability experienced by $U_2$ in HD-PSR-NOMA ($P_{out,2}^{HD,PSR}$) network is given by (24).

By following similar procedure, we can observe that the asymptotic outage probability of $U_2$ in HD-PSR-NOMA (as $\rho_1 \rightarrow \infty$) is given by the following expression:

$$P_{out,2}^{HD,PSR} = \frac{\Gamma(0,\frac{\tau_3}{\lambda_1})}{\lambda_1 \lambda_2} \left[ \left(\frac{\mu_{FD}^{2} \alpha - 1 + \alpha}{\eta \rho \lambda_{12}} \right) \left(1 - e^{-\frac{\mu_{FD}^{2} \alpha - 1 + \alpha}{\eta \rho \lambda_{12}}} \right) \right]$$

(24)

where $\tau_3 = \frac{\mu_{FD}^{2} \alpha - 1 + \alpha}{\eta \rho \lambda_{12}}$ and $\tau_4 = \tau_3 (1 - \rho)$. Notice that the outage probability of $U_2$ in HD-PSR-NOMA system is proportional to $\rho_1^{-1}$, that is, $U_2$ does not suffer outage floor in HD-PSR-NOMA system, as well.

**3.3 System outage probability**

Here, we find the system outage probability, which includes any one or all of the following cases: (i) $U_1$ suffers outage; (ii) $U_2$ suffers outage; (iii) both $U_1$ and $U_2$ suffers outage. Accordingly,
this metric represents the effectiveness of the system to deliver message to both $U_1$ and $U_2$.

$$p_{FD,PSR}^{FD,PSR} = 1 - P_{FD} \left\{ \Gamma_{12}^{PSR} \geq \nu_{1}^{FD}, \Gamma_{11}^{PSR} \geq \nu_{1}^{FD} \right\},$$

$$\Gamma_{22}^{PSR,MRC} \geq \nu_{2}^{FD}, \Gamma_{22}^{PSR,DNL} \geq \nu_{2}^{FD} \right\}.$$  \hspace{1cm} (26)

where

$$\tau_{5} = \frac{\nu_{2}^{FD}}{\rho_{s}(1 - \alpha - \alpha \nu_{2}^{FD})} \text{ and } \tau_{6} = \frac{\phi^{FD}((1 - \rho)\rho \cdot \rho_{O,PSR} + 1)}{(1 - \rho)\rho},$$

**Proposition 3.** When $0 < \alpha < \frac{1}{1 + \nu_{2}^{FD}}$ and $\frac{\nu_{1}^{FD}}{1 + \nu_{2}^{FD}} < \alpha < 1$, $p_{FD,PSR}^{FD,PSR}$ is given by (27). Otherwise $p_{FD,PSR}^{FD,PSR}$ will become unity.

$$p_{FD,PSR}^{FD,PSR} = 1 - \left[ \frac{-\frac{\nu_{2}^{FD}}{\lambda_{2d}}}{\rho_{s}(1 - \alpha - \alpha \nu_{2}^{FD})} - \frac{\Gamma(0, \frac{\nu_{2}^{FD}}{\lambda_{2d}})}{\lambda_{11} \lambda_{12}} \right]$$

$$\times \left[ \frac{(\nu_{2}^{FD}, \alpha - 1 + \alpha)}{\eta \rho, \alpha} \right]$$

$$- \frac{1 - \alpha}{\eta \rho \cdot \alpha^{2}} \left[ Ei \left( -\frac{\tau_{5}}{\lambda_{2d}} - \frac{1}{\alpha \rho, \lambda_{2d}} \right) \right]$$

$$+ \frac{1}{\lambda_{11} \lambda_{12}} \sum_{n=2}^{\infty} \frac{(-1)^{n}}{n!} \left( \frac{1}{\lambda_{1d}} \right)^{n}$$

$$\times \left[ \frac{-\frac{\nu_{2}^{FD}}{\lambda_{2d}}}{(n - 1)!} \right] \sum_{i=1}^{n-1} \left( \frac{1}{\lambda_{1d}} \right)^{n-1}$$

$$\left[ \frac{-\frac{1}{\lambda_{1d}}}{(n - 1)!} \right] Ei \left( -\frac{\tau_{6}}{\lambda_{1d}} \right) \times d^{\eta} \sum_{k=0}^{\infty} C_{k} \left( \frac{k}{a} \right)^{a - k} d^{-n}$$

$$\times \pi^{2} \frac{(k + 1)}{\Gamma(n) \sin[\pi(k + 1 - n)]}$$

$$\times \left[ \frac{\frac{\nu_{2}^{FD}}{\lambda_{2d}}}{\lambda_{1d}} \right] \times \left[ \frac{\frac{\nu_{2}^{FD}}{\lambda_{2d}}}{\sin(\pi(k + 1)) \Gamma(1 - (k + 1))} \right]$$

$$- \left( \frac{\nu_{2}^{FD}}{\lambda_{2d}} \right) \frac{\nu_{2}^{FD}}{\sin(\pi(k + 1)) \Gamma(1 - n)} \right]$$

$$\leq \frac{\nu_{2}^{FD}}{\rho_{s}(1 - \alpha - \alpha \nu_{2}^{FD})} \right\}.$$  \hspace{1cm} (27)

**Proof.** Substituting for the SINRs in (26), and rearranging, we get

$$p_{FD,PSR}^{FD,PSR} = 1 - P_{FD} \left\{ \left| b_{1} \right|^{2} \geq \frac{\phi^{FD}((1 - \rho)\rho \cdot \rho_{O,PSR} + 1)}{(1 - \rho)\rho}, \right\}$$

$$\eta \rho \rho_{\delta} \left| b_{1} \right|^{2} \left| b_{1} \right|^{2} + \frac{(1 - \alpha)\rho \cdot \rho_{O,PSR} + 1)}{(1 - \rho)\rho}, \right\}$$

$$\left| b_{2} \right|^{2} \geq \frac{\nu_{2}^{FD}}{\rho_{s}(1 - \alpha - \alpha \nu_{2}^{FD})} \right\}.$$  \hspace{1cm} (28a)

$$= 1 - P_{FD} \left\{ \left| b_{1} \right|^{2} \geq \frac{\nu_{2}^{FD}}{\eta \rho \rho_{\delta} \left| b_{1} \right|^{2}} - \frac{\left| b_{1} \right|^{2}(1 - \alpha)\rho_{O,PSR} + 1)}{(1 - \rho)\rho}, \right\}$$

$$\left| b_{2} \right|^{2} \geq \tau_{5}; \left| b_{1} \right|^{2} \geq \tau_{6} \right\}$$  \hspace{1cm} (28b)

where $\tau_{5}$ and $\tau_{6}$ are defined in (27). Let $\left| b_{1} \right|^{2} = X, \left| b_{2} \right|^{2} = Y$ and $\left| b_{1} \right|^{2} = Z$; then $p_{FD,PSR}^{FD,PSR}$ becomes

$$p_{FD,PSR}^{FD,PSR} = 1 - P_{FD} \left\{ \left| b_{1} \right|^{2} \geq \frac{\nu_{2}^{FD}}{\eta \rho \rho_{\delta} \left( Y \rho, \alpha + 1 \right) \left( X \rho \right)}, \right\}$$

$$Y \geq \tau_{5}; X \geq \tau_{6} \right\}$$  \hspace{1cm} (29a)

$$= 1 - \frac{1}{\lambda_{1d} \lambda_{12}} \int_{\tau_{5}}^{\infty} \int_{\tau_{6}}^{\infty} e^{-\frac{(A_{y} \rho_{O,PSR})}{\lambda_{1d}}} \frac{d\tau_{5}}{\lambda_{1d}} dy.$$  \hspace{1cm} (29b)

where $A_{y}(y) = \frac{(\nu_{2}^{FD} \alpha + 2 - \alpha + \nu_{2}^{FD})}{\eta \rho, \rho_{\delta} + 1} - \tau_{5}$ and $\tau_{6}$ are defined in (27).

Now $B(y)$ in (29b) can be evaluated by utilizing Maclaurin series expansion for the term $e^{-\frac{A_{y}(y)}{\lambda_{1d}}}$.

$$B(y) = \int_{x=\tau_{6}}^{\infty} e^{-\frac{A_{y}(y)}{\lambda_{1d}} \sum_{n=0}^{\infty} (-1)^{n} \frac{\left( A_{y}(y) \right)^{n}}{n!} dx}, \hspace{1cm} (30a)$$

$$= \lambda_{1d} e^{\frac{\tau_{6}}{\lambda_{1d}}} - \frac{A_{y}(y)}{\lambda_{1d}} \Gamma \left(0, \frac{\tau_{6}}{\lambda_{1d}} \right) + \sum_{n=2}^{\infty} \left\{ \frac{(-1)^{n} \left( A_{y}(y) \right)^{n}}{n!} \right\}$$

$$\times \left[ e^{\frac{\tau_{5}}{\lambda_{1d}}} \sum_{n=1}^{\infty} \frac{(-1)^{n}}{(n - 1)! \lambda_{1d} \Gamma(1 - (n - 1))} \right]$$

$$\times Ei \left( -\frac{\tau_{6}}{\lambda_{1d}} \right).$$  \hspace{1cm} (30b)
Notice that (30b) is obtained by using (3.352.2) and (3.353.1) given in [39], where \( \text{Ei}(\cdot) \) is the exponential integral. Now \( f_0 \) in (29b) can be determined by utilizing (30b) in (29b), and it will have three terms. Let \( f_0 = f_{0}^{(1)} + f_{0}^{(2)} + f_{0}^{(3)} \), which are determined as follows:

\[
 f_{0}^{(1)} = e^{\frac{-\tau_6}{\lambda_1}} \int_{\tau_5}^{\infty} e^{-\frac{x}{\lambda_2}} \, dx = e^{\frac{-\tau_6}{\lambda_1}} e^{\frac{-\tau_5}{\lambda_2}},
\]

\[
 f_{0}^{(2)} = -\frac{1}{\lambda_1 \lambda_2} \int_{\tau_5}^{\infty} \frac{\left( n^2 \right) \alpha - 1 + \alpha}{\eta \rho \rho \eta \rho \alpha \rho \alpha + 1} e^{-\frac{x}{\lambda_2}} \, dx,
\]

\[
 f_{0}^{(3)} = \frac{1}{\lambda_1 \lambda_2} \sum_{n=2}^{\infty} \frac{(-1)^n \left( 1 \right)^a}{n!} \left[ e^{-\frac{\tau_6}{\lambda_1}} \int_{\tau_5}^{\infty} \frac{\left( n^2 \right) \alpha - 1 + \alpha}{\eta \rho \rho \eta \rho \alpha \rho \alpha + 1} e^{-\frac{x}{\lambda_2}} \, dx \right].
\]

Notice that (32b) is obtained by using (3.352.2) in [39],

\[
 \Gamma \left( n \right) = \frac{\left( n \right)^{n-1}}{\left( n - 1 \right)!} \text{Ei} \left( -\frac{\tau_5}{\lambda_1} - \frac{1}{\lambda_2} \right).
\]

After some mathematical manipulations and by using (3.383.5) in [39], we will get the following expression for \( f_{0}^{(3)} \):

\[
 f_{0}^{(3)} = \frac{1}{\lambda_1 \lambda_2} \sum_{n=2}^{\infty} \frac{(-1)^n \left( 1 \right)^a}{n!} \left[ e^{-\frac{\tau_6}{\lambda_1}} \int_{\tau_5}^{\infty} \frac{\left( n^2 \right) \alpha - 1 + \alpha}{\eta \rho \rho \eta \rho \alpha \rho \alpha + 1} e^{-\frac{x}{\lambda_2}} \, dx \right].
\]

where \( a = \left( n^2 \right) \alpha - 1 + \alpha \), \( b = \left( n^2 \right) \alpha - 1 + \alpha \), \( \lambda_5 = \eta \rho \rho \eta \rho \alpha \rho \alpha + 1 \), and \( L_n^a (z) \) is the Laguerre polynomial. Now \( f_0 \) is determined as \( f_0 = f_{0}^{(1)} + f_{0}^{(2)} + f_{0}^{(3)} \), where \( f_{0}^{(3)} \), \( f_{0}^{(2)} \), and \( f_{0}^{(3)} \) are given by (31), (32b) and (33b), respectively. Thus, the final expression for \( f_0 \) is

\[
 f_0 = e^{\frac{-\tau_5}{\lambda_1}} - \frac{\left( \left( n^2 \right) \alpha - 1 + \alpha \right)}{\eta \rho \rho \alpha \rho \alpha + 1} \left[ \left( \lambda_2 \right) e^{\frac{-\tau_6}{\lambda_1}} \right]
\]

\[
 + \frac{1}{\lambda_1 \lambda_2} \sum_{n=2}^{\infty} \frac{\left( n \right)^{n+1}}{n!} \text{Ei} \left( -\frac{\tau_5}{\lambda_1} - \frac{1}{\lambda_2} \right)
\]

\[
 \sum_{n=0}^{\infty} \frac{(-1)^n \left( 1 \right)^a}{n!} \left[ \left( \lambda_2 \right) e^{\frac{-\tau_6}{\lambda_1}} \right] \sum_{n=0}^{\infty} \frac{\left( n \right)^{n+1}}{n!} \text{Ei} \left( -\frac{\tau_5}{\lambda_1} - \frac{1}{\lambda_2} \right)
\]

\[
 \left( \frac{\pi^2}{\sin \left( \pi n \right)} \Gamma \left( 1 - (k + 1) \right) \right) - \left( \frac{\pi^2}{\sin \left( \pi n \right)} \Gamma \left( k + 1 \right) \right) \left( \frac{\pi^2}{\sin \left( \pi n \right)} \Gamma \left( 1 - (k + 1) \right) \right)
\]

Now \( \lambda_1^{\text{PSR,FD-P}} = 1 - f_0 \) and is given by (27).

\textbf{Corollary 5.} The asymptotic system outage probability of FD-PSR-NOMA system as \( \rho_1 \to \infty \) (i.e., \( \text{FD-PSR,FD-P} \)) is given by the following expression:

\[
 P_{\text{FD-PSR,FD-P}} = \frac{\left( n^2 \right) \alpha - 1 + \alpha}{\eta \rho \rho \eta \rho \alpha \rho \alpha + 1} + \Phi_{\text{FD-PSR,FD-P}} + \Phi_{\text{FD-PSR,FD-P}}
\]

\[
 \frac{\Phi_{\text{FD-PSR,FD-P}}}{\rho_1 (1 - \alpha - \alpha n_2) \lambda_2} + \frac{\Phi_{\text{FD-PSR,FD-P}}}{(1 - \alpha) \lambda_1}
\]

\[
 \frac{\Phi_{\text{FD-PSR,FD-P}}}{\rho_1 (1 - \alpha - \alpha n_2) \lambda_2} + \frac{\Phi_{\text{FD-PSR,FD-P}}}{(1 - \alpha) \lambda_1}
\]

\[
 \frac{\Phi_{\text{FD-PSR,FD-P}}}{\rho_1 (1 - \alpha - \alpha n_2) \lambda_2} + \frac{\Phi_{\text{FD-PSR,FD-P}}}{(1 - \alpha) \lambda_1}
\]

\[
 \left( \frac{\pi^2}{\sin \left( \pi n \right)} \Gamma \left( k + 1 \right) \right) \left( \frac{\pi^2}{\sin \left( \pi n \right)} \Gamma \left( 1 - (k + 1) \right) \right)
\]

\[
 \left( \frac{\pi^2}{\sin \left( \pi n \right)} \Gamma \left( k + 1 \right) \right) \left( \frac{\pi^2}{\sin \left( \pi n \right)} \Gamma \left( 1 - (k + 1) \right) \right)
\]
Assuming high SNR condition (i.e., $\rho_s \to \infty$) and utilizing the approximation $e^{-x} \approx 1 - x$ for $0 < x \ll 1$, we can get an approximation for $P_{\text{out,syr}}^{\text{HD,PSR}}$ as follows:

$$
P_{\text{out,syr}}^{\text{HD,PSR}} \approx 1 - \left[ \frac{1}{\lambda_1^2 \lambda_2^2} \int_{x=\tau_7}^{\infty} \int_{y=\tau_8}^{\infty} \left( 1 - \frac{u_2^{\text{HD}}}{\eta \rho \alpha \lambda_2^2} \right) e^{-\frac{x}{\lambda_1}} e^{-\frac{y}{\lambda_2}} dx dy \right],
$$

where

$$
P_{\text{out,syr}}^{\text{HD,PSR}} = 1 - \left[ \frac{1}{\lambda_1^2 \lambda_2^2} \int_{x=\tau_7}^{\infty} \int_{y=\tau_8}^{\infty} \left( 1 - \frac{u_2^{\text{HD}}}{\eta \rho \alpha \lambda_2^2} \right) e^{-\frac{x}{\lambda_1}} e^{-\frac{y}{\lambda_2}} dx dy \right] - \frac{\Gamma(0, \tau_7)}{\lambda_1 \lambda_2} \frac{1}{n!} \sum_{k=2}^{\infty} \frac{(-1)^{k-1} \left( \frac{1}{\lambda_1^2} \right)^k}{k!}.
$$

Using the relation, $-\text{Ei}(-\infty) \geq -e^{-\infty}$, (39) becomes:

$$
P_{\text{out,syr}}^{\text{HD,PSR}} \approx 1 - \frac{\frac{u_2^{\text{HD}}}{\eta \rho \alpha \lambda_2^2} - \frac{u_2^{\text{HD}}}{\eta \rho \alpha \lambda_2^2}}{\rho_s(1-\rho_s \alpha)} \frac{1}{\lambda_1^2 \lambda_2^2},
$$

where

$$
\tau_7 = \frac{u_2^{\text{HD}}}{\rho_s(1-\rho_s \alpha u_2^{\text{HD}})}, \quad \tau_8 = \frac{\phi^{\text{HD}}}{(1-\rho_s \alpha)}, \quad \gamma = (u_2^{\text{HD}} \alpha - 1 + \alpha) \rho_s, \quad f = \left( u_2^{\text{HD}} \alpha - 1 + \alpha \right) \rho_s \tau_7 + u_2^{\text{HD}}, \quad b = \eta \rho \alpha (\rho_s \alpha \tau_7 + 1).
$$

Substituting for $\rho_{O,PSR}$ with $\theta = 1$ and setting $\rho_s \to \infty$ in (40), we can get (35).

From (35), it is clear that $P_{\text{out,syr}}^{\text{HD,PSR}}$ decreases as $\rho_s$ is increased; however it becomes a constant (independent of $\rho_s$) when $\rho_s \to \infty$; thus, error floor behaviour is observed in system outage probability results in the high SNR region. Even though the far user shows improved performance in the high SNR region due to the presence of direct BS-$U_2$ link, the near user $U_1$ was shown outage floor in the high SNR region. This affects the overall performance of the system in the high SINR region.

\[ \square \]
From (42), it can be observed that the system outage probability is inversely related to $\rho_s$; thus as $\rho_s \to \infty$, system outage of HD-PSR-NOMA network reduces. This means that HD-PSR-NOMA network does not suffer outage floor in the high SNR region.

### 3.4 Analysis of diversity order

In this section, we find the diversity order of $U_1$ and $U_2$ in HD/FD-PSR-NOMA system in terms of asymptotic outage probabilities. We also find the diversity order in terms of system outage probability as well. The diversity order ($d$) is defined as:

$$d = -\lim_{\rho_s \to \infty} \frac{\log(P_{out}^{as}(\rho_s))}{\log(\rho_s)},$$  

where $P_{out}^{as}$ is the asymptotic outage probability.

The asymptotic outage probability of $U_1$ and $U_2$ in FD-PSR-NOMA systems was discussed respectively in sections 3.1 and 3.2 of this paper and are given by (9) and (21), respectively. Substituting (9) in (43), we get the diversity order of $U_1$ in the FD-PSR-NOMA system, $d_{1,FD,PSR} = 0$. Similarly, substituting (21) in (43) and simplifying, we get the diversity order of $U_2$ in the FD-PSR-NOMA system, $d_{2,FD,PSR} = 1$. Thus, we conclude that the near user $U_1$ in the FD-PSR-NOMA system experience the inherent zero diversity problem owing to the presence of RSI. This leads to error floor in the outage performance of $U_1$ in the high SNR region. The far user $U_2$ does not experience the error floor problem due to the presence of direct link, which leads to a better SINR at $U_2$. Accordingly, the diversity order of $U_2$ in FD-PSR-NOMA system is unity. For comparison, the asymptotic outage probability of $U_1$ and $U_2$ in HD-PSR-NOMA system are given by (10b) and (25), respectively. Substituting these expressions in (43), and simplifying, the diversity order of $U_1$ and $U_2$ in HD-PSR-NOMA system are, respectively, given by $d_{1,HD,PSR} = 1$ and $d_{2,HD,PSR} = 2$. Thus, outage floor behaviour is not observed in HD-PSR-NOMA system.

The asymptotic system outage probability of FD-PSR-NOMA and HD-PSR-NOMA networks are given by (35) and (42), respectively. Substituting these expressions in (43) and simplifying, we find that the diversity order in terms of system outage probability for FD-PSR-NOMA $d_{1,FD,PSR} = 0$ and that for HD-PSR-NOMA $d_{1,HD,PSR} = 1$. Thus, we conclude that the over all system outage probability results of FD-PSR-NOMA network shows an outage floor behaviour, owing to the presence of RSI at the relay node $U_1$. However, the equivalent HD-PSR-NOMA system does not exhibit the outage floor problem and the diversity order is unity.

### 4 OPA FACTOR THAT MINIMIZES SYSTEM OUTAGE PROBABILITY

In this section, we determine the optimal power allocation (OPA) factor that minimizes the system outage probability of FD/HD-PSR-NOMA systems.

**Proposition 4.** For a given $\rho$, the OPA factor $\alpha^*_{FD,PSR}$ that minimizes $P_{out,sys}^{FD,PSR}$ in FD-PSR-NOMA network is given by

$$\alpha^*_{FD,PSR} = \frac{n_{1,FD} + n_{1,FD}n_{2,FD}^\rho}{n_{1,FD} + n_{2,FD}^\rho + n_{1,FD}n_{2,FD} + n_{1,FD}n_{2,HD}^\rho}. \quad (44)$$

**Proof.** Refer Appendix A.

**Corollary 7.** For a given $\rho$, the OPA factor $\alpha^*_{HD,PSR}$ that minimizes $P_{out,sys}^{HD,PSR}$ in HD-PSR-NOMA network is given by

$$\alpha^*_{HD,PSR} = \frac{n_{1,HD} + n_{1,HD}n_{2,HD}^\rho}{n_{1,HD} + n_{2,HD}^\rho + n_{1,HD}n_{2,HD} + n_{1,HD}n_{2,HD}^\rho}. \quad (45)$$

### 5 ANALYSIS OF SYSTEM THROUGHPUT

In the delay-limited transmission mode, BS serves the users with constant target rates $R_1$ and $R_2$. Under this scenario, the system throughput depends on outage probabilities [5]. When $\alpha_1$ (or $\alpha_2$) is delivered successfully to $U_1$ (or $U_2$), the achieved rate is $R_1$ (or $R_2$). Thus, the throughput of FD-PSR-NOMA and HD-PSR-NOMA systems are determined as:

$$T_{FD,PSR} = R_1 \left(1 - P_{out,1}^{FD,PSR}\right) + R_2 \left(1 - P_{out,2}^{FD,PSR}\right), \quad (46a)$$

$$T_{HD,PSR} = R_1 \left(1 - P_{out,1}^{HD,PSR}\right) + R_2 \left(1 - P_{out,2}^{HD,PSR}\right). \quad (46b)$$

### 6 RESULTS AND DISCUSSIONS

The analytical results for outage probability and throughput are validated by extensive Monte-Carlo simulations considering a two-dimensional topology, where the locations of BS, $U_1$ and $U_2$ are selected as $(0,0)$, $(1.8,0.5)$ and $(3,0)$, respectively. The mean power gains $G_{ij} = \frac{d_{ij}^{-\eta}}{\sum_i d_{ij}^{-\eta}}$, where $G = 1$, $d_{ij}$ is the distance between nodes $i$ and $j$ and $\eta$ is the path loss exponent (equals 3 for all the links). Unless otherwise specified, we consider the energy harvesting efficiency ($\eta$) as 0.5. The analytical results for the outage probabilities of $U_1$ and $U_2$ in FD-PSR-NOMA system are obtained by using (7) and (12), respectively. We use (10a) and (24) to find the corresponding results for HD-PSR-NOMA system. The system outage probabilities are obtained using (27) and (41), respectively. Figures 3 and 4 compares the outage probability experienced by $U_1$ and $U_2$ against $\rho$. (Ph/N0).
Figure 4. However, in the high system compared to HD-PSR-NOMA system, as can be seen in Figure 3, outage probability for $\rho_s$ or $\theta$, an outage floor appears for $U_1$, since increase of either $\rho_s$ or $\theta$ increases the RSI at $U_1$. However, $U_2$ does not suffer outage floor due to the presence of direct BS-$U_2$ link.

The impact of energy harvesting efficiency ($\eta$) on the outage probabilities experienced by $U_1$ and $U_2$ is also shown in Figure 3. Recall that the mean RSI power at $U_1$ depends on the mean transmit power of the relay $U_1$ ($P_s$) and the parameter $\theta$ ($0 < \theta < 1$), which is related to the SI cancellation technique employed at $U_1$. Further, the transmit power of the relay node $U_1$ ($P_s$) is directly proportional to $\eta$. Higher values of $\eta$ will trigger the mean RSI power at $U_1$ to become higher, especially when $\theta$ is large. This leads to degradation of SINR over the BS-$U_2$ link. As a result, for larger values of $\theta$, the outage probabilities of both $U_1$ and $U_2$ increase as $\eta$ is increased. However, when $\theta$ is small (i.e. $\theta < 0.5$), the mean RSI power is not significant enough to affect the outage probabilities. In this case, even though higher $\eta$ leads to higher $P_s$, $U_2$ will experience lower outage probability, since the RSI effect is insignificant. On the other hand, outage probability of $U_1$ is insensitive to the variations of $\eta$, when $\theta$ is very small. To summarise, we observe that the impact of $\eta$ on the outage probabilities depend on the mean RSI power at $U_1$, which is strongly related to $\theta$.

Figure 5 shows the system outage probability in FD-PSR-NOMA and HD-PSR-NOMA system for distinct values of $\xi$ and $\theta$ (i.e. related to mean RSI power). As $\xi$ and $\theta$ are increased, the mean RSI power increases, causing significant increase of system outage probabilities of FD-PSR-NOMA network. For higher values of $\rho_s$ and $\theta$, an outage floor appears, since increase of either $\rho_s$ or $\theta$ increases the RSI at $U_1$. Further, $U_2$ does not suffer outage floor due to the presence of direct BS-$U_2$ link.

Figure 3 shows the system outage probability versus $\rho_s$ ($R_1 = 0.5$, $R_2 = 0.6$, $\eta = 5$, $\rho = 0.5$, $\rho = 0.3$, $\omega = 0.3$, $\xi = -3dB$, $\beta = 0.2$, $\beta = 0.9$) and $U_2$ experience lower outage probability in FD-PSR-NOMA system compared to HD-PSR-NOMA system, as can be seen in Figure 4. However, in the high $\rho_s$ region, HD system outperforms FD system, since increase of $\rho_s$ leads to increase of $\rho_s$ as well, which makes the RSI power at $U_1$ to be higher, triggering increase of outage probability. According to NOMA principle, the far user $U_2$ has to be allocated higher power at BS, that is, $\alpha < 0.5$; however, random selection of $\alpha$ leads to higher outage probability for $U_1$ compared to $U_2$, as can be seen in Figure 3 as well as Figure 4. Further, Figure 3 shows the impact of $\beta$ (i.e. i-SIC factor) and $\theta$ (i.e. related to mean RSI power) on outage probability in FD-PSR-NOMA system. As $\beta$ is increased, outage probability of $U_1$ becomes larger, due to the residual interference generated by i-SIC at $U_1$. However, outage probability of $U_2$ is insensitive to $\beta$, since SIC is not required for the detection of symbol $x_2$ at $U_2$. As $\theta$ is increased, the mean RSI power increases, causing significant increase of outage probability of both $U_1$ and $U_2$ in FD-PSR-NOMA system. For higher values of $\rho_s$ and $\theta$, an outage floor appears for $U_1$, since increase of either $\rho_s$ or $\theta$ increases the RSI at $U_1$. However, $U_2$ does not suffer outage floor due to the presence of direct BS-$U_2$ link.
network exhibits better performance in terms of system outage, while for higher $\rho$, HD-PSR-NOMA network experience lower system outage.

Figures 6 and 7, respectively, show the system outage probability results for FD-PSR-NOMA and HD-PSR-NOMA networks for distinct rates (i.e. $R_1$ and $R_2$) against the power allocation factor ($\alpha$). For both lower and higher values of $\alpha$, the system outage probability has been observed to increase as can be seen in Figures 6 and 7. When $\alpha$ is reduced, the probability of successful decoding of $x_1$ at $U_1$ reduces, which leads to increase of the system outage probability. When $\alpha$ becomes higher, $U_2$ suffers higher outage, which makes the system outage probability to increase. Optimal $\alpha$ values (i.e. $\alpha^*_{FD,PSR}$, $\alpha^*_{HD,PSR}$) exist that minimizes the system outage probability for both FD-PSR and HD-PSR systems, respectively. Figures 6 and 7, respectively, show the sensitivity of $\alpha^*_{FD,PSR}$ and $\alpha^*_{HD,PSR}$ against target rates $R_1$ and $R_2$. Increase of target rate $R_1$ (for fixed $R_2$), increases the target SINR for the decoding of $x_1$ at $U_1$, which leads to higher system outage. To minimize the system outage of FD-PSR-NOMA network, $\alpha^*_{FD,PSR}$ shall be increased, when $R_1$ becomes larger. When $R_2$ becomes larger (for fixed $R_1$), the target SINR for the decoding of $x_2$ increases, which makes the system outage to be higher. To minimize the system outage probability, $\alpha^*_{HD,PSR}$ shall be decreased as can be seen in Figures 6 and 7, respectively. Similar results are observed for HD-PSR-NOMA network as well. Figures 8 and 9 show the impact of $\alpha$ on system outage probability in FD-PSR-NOMA and HD-PSR-NOMA systems, respectively, for distinct $\beta$. When $\beta$ becomes larger, $P_{out}$ of $U_1$
TABLE 1  System outage minimization: $\alpha_{FD,PSR}$ and $\alpha_{HD,PSR}$ for distinct $(R_1, R_2)$ and $\beta = 0.8, \xi = -3 \text{ dB}, \rho = 0.3$.

| $\rho_s$ = 20 dB | $\rho_s$ = 25 dB |
|------------------|------------------|
| $R_1, R_2$      | $R_1, R_2$      |
| 0.3, 0.4        | 0.3, 0.4        |
| 0.4, 0.5        | 0.4, 0.6        |


d$^s$ = 20 dB  
d$^s$ = 25 dB

increases, which increases $P_{out,PSR}^{FD,PSR}$ as well. In this case, to minimize $P_{out,PSR}^{FD,PSR}$, $\alpha$ shall be increased; that is, larger values of $\beta$ lead to increase of $\alpha^{FD,PSR}$ and $\alpha^{HD,PSR}$ as well. Table 1 depicts $\alpha^{FD,PSR}$ and $\alpha^{HD,PSR}$ for distinct parameters. Figure 10 compares the system outage probability under OPA scheme against random power allocation (RPA) at the BS, where $\alpha$ values are chosen randomly. The results show that the system outage probability significantly decreases under the proposed OPA scheme. When $\rho_s = 30 \text{ dB}$, the proposed OPA leads to 50% reduction in system outage compared to RPA. Similar results are observed for HD-PSR-NOMA network as well. Thus, we conclude that intelligent selection of $\alpha$ at the BS can significantly improve the system outage performance of FD/HD-PSR-NOMA network.

In the low $\rho_s$ region, the throughput of FD-PSR-NOMA system is higher compared to HD-PSR-NOMA system; however, when $\rho_s$ becomes higher, the mean RSI power becomes larger, which leads to throughput degradation in FD-PSR-NOMA system. Furthermore, increase of $\beta$ leads to degradation of throughput, owing to the residual interference caused due to i-SIC.

Figures 12 and 13 show the throughput against PS factor $\rho$ for FD/HD-PSR-NOMA systems. Both lower and higher values of $\rho$ trigger decrease of the throughput. Smaller $\rho$ reduces the harvested power, which reduces the probability of successful decoding of the message at $U_2$. Larger $\rho$ reduces the probability of successful decoding of the message at $U_1$. An optimal $\rho^*$ exists that maximizes the throughput. The impact of target rates $R_1$ and $R_2$ on $\rho^*$ can be seen in Figure 12 for FD-PSR-NOMA system. Increase of $R_1$ (keeping $R_2$ constant) increases the threshold SINR for $U_1$. In this case, $\rho^*$ shall be reduced so as to ensure that more power is made available at $U_1$ to increase the probability of successful decoding. This implies that $\rho^*$ must be reduced, when $R_1$ becomes higher. The impact of $\rho_s$ and $\theta$ on $\rho^*$ can be observed in Figure 13. As mentioned before, when either $\rho_s$ or $\theta$ is increased, the mean RSI power becomes very high so that the SINR over the BS-$U_1$ link suffers, which leads to throughput reduction. To improve the throughput, $\rho^*$ shall be reduced so that more power is made available at $U_1$ for information decoding. Thus, $\rho^*$ for FD-PSR-NOMA system decreases,
maximizes the system throughput. The results have shown that numerical results were presented for the optimal PS factor that arbitrary selection of power allocation factor at the BS. Moreover, reduction of the system outage probability compared to arbitrary selection and opportunistic relay selection schemes. Further, we would like to analyze the performance of multiple input multiple output (MIMO)-NOMA systems integrated with SWIPT in the presence of i-SIC and i-CSI.

It was established that the proposed OPA leads to significant BS that minimizes the system outage probability was derived. Also, we would like to analyze the performance of multiple NOMA systems (FD/HD-PSR-NOMA system). A single-cell optimization of power splitting relaying based full/half duplex scenarios was considered consisting of a BS and two down link users with distinct channel conditions (i.e. near and far users). Under the assumption of i-SIC at the near user and MRC at the far user, analytical expressions were derived for (i) outage probabilities experienced by the users, (ii) system outage probability, and (iii) system throughput. The asymptotic outage probability and the diversity order performance of the users were also analyzed. Analytical expression for the OPA factor at the BS that minimizes the system outage probability was derived. It was established that the proposed OPA leads to significant reduction of the system outage probability compared to arbitrary selection of power allocation factor at the BS. Moreover, numerical results were presented for the optimal PS factor that maximizes the system throughput. The results have shown that efficient selection of power allocation factor at the BS and the PS factor at the relay are crucial for improving the performance of FD/HD-PSR-NOMA systems. As part of the future work, we plan to extend the analysis to SWIPT-enabled relay-aided NOMA systems in the presence of i-SIC and i-CSI under partial relay selection and opportunistic relay selection schemes.

TABLE 2 Throughput maximization: $\rho_{FD}$ and $\rho_{HD}$ for distinct ($R_1$, $R_2$) ($\theta = 0.9, \zeta = -3 \text{ dB}$) $\beta = 0.3$ $\beta = 0.1$

| $R_1$, $R_2$ | $\rho_{FD}$ | $\rho_{HD}$ | $\rho_{FD}$ | $\rho_{HD}$ |
|-------------|-------------|-------------|-------------|-------------|
| 0.3,0.4     | 0.1600      | 0.3100      | 0.4,0.3     | 0.4,0.3     |
| 0.4,0.3     | 0.3100      | 0.3100      | 0.4,0.3     | 0.4,0.3     |
| 0.4,0.4     | 0.3100      | 0.3100      | 0.4,0.3     | 0.4,0.3     |

when either $\rho$ or $\beta$ is increased. Table 2 depicts optimal $\rho$ for distinct parameters. Thus, our results show that proper tuning of $\rho$ can improve the throughputs of FD/HD-PSR-NOMA systems.

7 CONCLUSION

The focus of this paper was on performance analysis and optimization of power splitting relaying based full/half duplex NOMA systems (FD/HD-PSR-NOMA system). A single-cell scenario was considered consisting of a BS and two down link users with distinct channel conditions (i.e. near and far users). Under the assumption of i-SIC at the near user and MRC at the far user, analytical expressions were derived for (i) outage probabilities experienced by the users, (ii) system outage probability, and (iii) system throughput. The asymptotic outage probability and the diversity order performance of the users were also analyzed. Analytical expression for the OPA factor at the BS that minimizes the system outage probability was derived. It was established that the proposed OPA leads to significant reduction of the system outage probability compared to arbitrary selection of power allocation factor at the BS. Moreover, numerical results were presented for the optimal PS factor that maximizes the system throughput. The results have shown that

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Appendix A

Derivation of (44): Consider the asymptotic expression \( \rho^{FD,ISR}_{out,sys} \) given in (35), where

\[
\phi^{FD} = \max\left\{ \frac{u_2^{FD}}{1 - \alpha - u_2^{FD}}, \frac{u_1^{FD}}{\alpha - u_1^{FD} + \beta u_1^{FD}} \right\}
\]

Corresponding to this, we have two distinct cases for (35) as discussed below:

**Case (i):**

\[
\frac{u_2^{FD}}{1 - \alpha - u_2^{FD}} > \frac{u_1^{FD}}{\alpha - u_1^{FD} + \beta u_1^{FD}}
\]

The above implies

\[
\frac{u_1^{FD}}{1 + \beta u_1^{FD}} < \alpha < 1.
\]

From proposition 1, we have

\[
0 < \alpha < \frac{1}{1 + u_2^{FD}}
\]

and

\[
\beta u_1^{FD} < \alpha < 1.
\]

Combining these conditions, we have

\[
\max\left\{ \frac{u_1^{FD}}{1 + \beta u_1^{FD}}, \frac{u_1^{FD}}{1 + \beta u_1^{FD}} \right\} < \frac{u_1^{FD}}{1 + \beta u_1^{FD}} < \frac{\beta u_1^{FD}}{1 + \beta u_1^{FD}} < \alpha < 1.
\]

Since

\[
\frac{u_1^{FD}}{1 + \beta u_1^{FD}} = \frac{u_1^{FD}}{u_1^{FD} + u_2^{FD} + \beta u_1^{FD}} < \alpha < 1,
\]

the previous condition is equivalent to

\[
\frac{u_1^{FD}}{u_1^{FD} + u_2^{FD} + \beta u_1^{FD}} < \alpha < 1.
\]

Substituting the value of \( \phi^{FD} \) corresponding to case (i) above, in (35), \( \rho^{FD,ISR}_{out,sys} \) becomes,

\[
\rho^{FD,ISR}_{out,sys} \cong \frac{u_2^{FD}}{\rho \left( 1 - \alpha - u_2^{FD} \right)} \frac{\alpha}{\lambda_2} + \frac{u_2^{FD}}{\eta \rho \left( 1 - \alpha - u_2^{FD} \right)}
\]

Given

\[
\rho_1^{FD} \left( 1 - \alpha - u_2^{FD} \right)^2 + (1 - \rho) \rho_1^{FD} (1 - \alpha - u_2^{FD}) \lambda_1
\]

\[
\frac{1}{\eta \rho \left( 1 - \alpha - u_2^{FD} \right)} \frac{\alpha + 1 + \alpha}{\eta \rho \left( 1 - \alpha - u_2^{FD} \right)}
\]

Differentiating (A.1) w.r.t. \( \alpha \) and after conducting extensive numerical investigations, we observe that \( \frac{\partial \rho^{FD,ISR}_{out,sys}}{\partial \alpha} > 0 \), for the range of \( \alpha \) considered, that is, \( \rho^{FD,ISR}_{out,sys} \) monotonically increases, when

\[
\frac{u_1^{FD}}{u_1^{FD} + u_2^{FD} + \beta u_1^{FD}} < \alpha < 1.
\]

**Case (ii):**

\[
\alpha - u_2^{FD} < \frac{u_1^{FD}}{1 - \alpha - u_2^{FD}} < \frac{u_1^{FD}}{\alpha - u_1^{FD} + \beta u_1^{FD}}.
\]
In this case, \( 0 < \alpha < \frac{u_{1}^{FD} + u_{2}^{FD} \beta}{u_{1}^{FD} + u_{2}^{FD} \beta} \). However, since \( 0 < \alpha < \frac{1}{1 + u_{2}^{FD}} \) and \( \frac{\beta_{1}^{FD}}{1 + \beta_{1}^{FD}} < \alpha < 1 \), the previous condition implies that \( \frac{\beta_{1}^{FD}}{1 + \beta_{1}^{FD}} < \alpha < \min\left(\frac{u_{1}^{FD} + u_{2}^{FD} \beta}{u_{1}^{FD} + u_{2}^{FD} \beta}, \frac{1}{1 + u_{2}^{FD}}\right) \). However, since \( \frac{u_{1}^{FD} + u_{2}^{FD} \beta}{1 + u_{2}^{FD}} < \frac{1}{1 + u_{2}^{FD}} \), the previous condition is equivalent to \( \frac{u_{1}^{FD} + u_{2}^{FD} \beta}{1 + u_{2}^{FD}} < \alpha < \frac{u_{1}^{FD} + u_{2}^{FD} \beta}{u_{1}^{FD} + u_{2}^{FD} + u_{1}^{FD} \beta} \). Substituting the value of \( \phi^{FD} \) corresponding to case (ii) in (35), \( p^{FD,PSR}_{out,sys} \) becomes

\[
p^{FD,PSR}_{out,sys} \approx \frac{u_{2}^{FD}}{\rho_{1}^{FD}} \left( 1 - \alpha - u_{2}^{FD} \alpha \right) \lambda_{2} + \frac{u_{1}^{FD} \delta \eta \rho}{\rho_{1}^{FD}} \left( \alpha - u_{1}^{FD} \beta + u_{1}^{FD} \beta \alpha \right) \lambda_{2} + \frac{u_{1}^{FD}}{\rho_{1}^{FD}} \left( 1 - \rho_{1}^{FD} \left( \alpha - u_{1}^{FD} \beta + u_{1}^{FD} \beta \alpha \right) \lambda_{1} \right)
\]

Differentiating (A.2) w.r.t. \( \alpha \), we observe, from numerical investigations, that \( \frac{\partial p^{FD,PSR}_{out,sys}}{\partial \alpha} < 0 \) for the range of \( \alpha \) considered, that is, \( p^{FD,PSR}_{out,sys} \) monotonically decreases, when \( \frac{\beta_{1}^{FD}}{1 + \beta_{1}^{FD}} < \alpha < \frac{u_{1}^{FD} + u_{2}^{FD} \beta}{u_{1}^{FD} + u_{2}^{FD} + u_{1}^{FD} \beta} \). Thus, it can be seen that \( p^{FD,PSR}_{out,sys} \) is a monotonically decreasing function of \( \alpha \) for \( \frac{\beta_{1}^{FD}}{1 + \beta_{1}^{FD}} < \alpha < \frac{u_{1}^{FD} + u_{2}^{FD} \beta}{u_{1}^{FD} + u_{2}^{FD} + u_{1}^{FD} \beta} \) and monotonically increasing function of \( \alpha \) for \( \frac{u_{1}^{FD} + u_{2}^{FD} \beta}{u_{1}^{FD} + u_{2}^{FD} + u_{1}^{FD} \beta} < \alpha < \frac{1}{1 + u_{2}^{FD}} \). Thus, the OPA factor that minimizes \( p^{FD,PSR}_{out,sys} \) is given by \( \alpha^{FD,PSR}_{out,sys} = \frac{u_{1}^{FD} + u_{2}^{FD} \beta}{u_{1}^{FD} + u_{2}^{FD} + u_{1}^{FD} \beta} \). The proposition 4 is thus proved.