Abstract

There are encouraging evidences for registration of excitations of quark oscillator levels in compressed 2-nucleon systems, which were hidden in two independently published experimental works for many years. Data obtained by EVA collaboration can be considered as the third possible confirmation of this conjecture. Moreover, they drop hints about existence of coherent dibaryons which may be interpreted as usual or generalized coherent states of the oscillator. These data may also be an explicit example of a physical process described by Landau and Peierls in their reasoning concerning the energy-time uncertainty relation. Two models of the coherent dibaryon creation are formulated. Phase transitions of baryon matter in few-nucleon systems are discussed.

Keywords: Resonances, two-nucleon system, quark matter, phase transitions

1. Introduction

It was suggested in [1, 2] to use a cumulative particle (CP) as a trigger for a detection of the multibaryon (MB) production. Our estimations showed that appearance of CP is a signature of “deep cooling” of a multiquark system, which brings it close to its ground state and allows it to have a narrow width thereby giving a chance to separate MB from a secondary particle background. Another method of deep cooling was described implicitly in [3]. In this case, events of n-p interactions with sufficient numbers of secondary pions were selected and dibaryons were supposed to be observed as peaks on the plot of yield versus effective masses of outgoing p-p pairs. These data may be described by a simple
formula $M_{2B} \approx M_{pp} + 10n$ MeV, where $n$ is a positive integer. Although not each such $n$ was matched with a dibaryon in [3], it was natural to assume that they exist too but were not seen against the background.

The possibility of deep cooling by CP was checked using data of paper [4] in which a hint on existence of an “excited state of deuteron” was noticed already in 1979. Our analysis showed [5, 6] that subtle contours of the dibaryon which was observed by WASA-at-COSY Collaboration [7] are seen indeed in a right place suggested in [4]. Hereafter these data are referred to as the Baldin group experiment (BGE) and data from [3] as the Troyan experiment (TE) data.

The main surprise waited for us in the range of two other peaks observed in BGE, which were never analyzed before but identified a priori with elastic $d$-$d$ and $d$-$N$ scattering. Calculations revealed this is definitely not so, and the peaks correspond very surprisingly to transitions between different dibaryons from TE [5, 6]. It will be shown below that an explanation of this unexpected result is connected with elucidation of a transverse momentum anomaly found out in data of EVA group [8] in paper [2]. All the facts in the aggregate will allow to assume a chance of experimental observation of phase transitions in two-nucleon systems.

2. Clarification from EVA experiment

We shall consider the total and relative momenta of intranuclear proton (IP) and neutron (IN) affiliated to a short range correlation (SRC), which are defined via momenta of IP and IN as follows:

$$p_{cm} = p_f + p_n, \quad p_{rel} = p_f - p_n.$$  

In the model of quasifree knockout (MQK), $p_f$ may be expressed via momenta of the incoming proton and the secondary registered ones in the following way [8]:

$$p_f = p_1 + p_2 - p_0.$$  

It is also assumed in MQK that IN leaves the target nucleus $C^{12}$ without essential changes of the momentum $p_n$ it had before the interaction.
It was shown in [8] that experimental data for longitudinal (along $p_0$) components of $p_{cm}$ and $p_{rel}$ are in a good agreement with MQK and SRC. Our analysis of the data confirmed this conclusion and gave the following estimations for the values under consideration (hereafter all momenta are in GeV/c):

$$\langle p_{cm}^z \rangle \approx 0, \quad \sigma_{cm}^z \approx 0.1, \quad \langle p_{rel}^z \rangle \approx 0.3, \quad \sigma_{rel}^z \approx 0.1. \quad (1)$$

Some additional work allowed us to obtain also similar estimations for the vertical, in the laboratory system, components of momenta and they turned out to be very different (see [2] for more details):

$$\langle p_{cm}^x \rangle \approx 0, \quad \sigma_{cm}^x \approx 0.6, \quad \langle p_{rel}^x \rangle \approx 0.6, \quad \sigma_{rel}^x \approx 0.2. \quad (2)$$

Mathematical modeling in the frame of the intranuclear cascade model [9] did not reveal any visible influence of intranuclear scattering (see [2]) and then interaction between IP and IN was studied. It may be shown that in this case the formulas which were used in [8] for calculation of $p_{cm}$ and $p_{rel}$ by means of the external momenta $p_0$, $p_1$ and $p_2$ give

$$p_{cm} = p_f + p_n, \quad p_{rel} = p_f - p_n + 2\Delta p_f. \quad (3)$$

where $\Delta p_f$ is a momentum transfer from IN to IP. Thus, the difference between $\langle p_{rel}^x \rangle$ and $\langle p_{rel}^z \rangle$ may be, in principle, explained by the IP-IN interaction which became involved.

A reason for the difference between $\sigma_{cm}^z$ and $\sigma_{cm}^x$ is not so trivial. Whereas $\sigma_{cm}^z$ is well explained by the intranuclear Fermi motion [2], large value of $\sigma_{cm}^x$ is provided by an unobvious quantum effect firstly noted by L.D. Landau and R. Peierls [10, 11]. The fact is that an attempt to measure $p_{cm}^x$ in the EVA experiment exactly coincides with the well-known Landau-Peierls gedanken experiment which demonstrates an inevitable influence of a momentum measurement procedure on a final value of momentum if the process lasts a limited time.

\footnote{A possible reason for finiteness of the interaction time is the localization of projectile’s wave function in a restricted domain of space.}
\( \Delta t \). The corresponding momentum perturbation of SRC may be written in an explicit form as,

\[
p_{cm} = (p_f + \Delta_f) + (p_n + \Delta_n),
\]

where for a change of velocity of SRC, \( \Delta v = (\Delta_f + \Delta_n)/(m_f + m_n) \), we have \( \Delta v_z = 0 \), since \( z \)-component of the total momentum is not measured, and

\[
\Delta v_x = \frac{\hbar}{(\Delta p_x \Delta t)}.
\]

The last expression contains \( \Delta p_x \) which is a precision of measurement of \( p_{cm}^x \). It may be estimated as \( \sigma_{x}^{rel} = \langle |\Delta f_x - \Delta n_x| \rangle \). After substitution \( \langle \Delta v_x \rangle = \sigma_{x}^{cm} / (m_f + m_n) \) in (4), we find \( \Delta t \sim 10^{-23} \) s and \( \Delta E \sim \hbar / \Delta t \) = 66 MeV.

Now we can understand the reason for observation of excited deuteron levels in the Baldin group experiment. Indeed, due to Landau-Peierls uncertainty relation, masses of target and incident deuterons in the initial state were not fixed exactly and some of the excited dibaryon levels might show themselves in the peaks observed. Overwhelming contribution of projectile’s excited states as compared with target’s ones, which was established in [5, 6], may be a manifestation of the relativistic effect of time dilation that allows highly excited states to exist much longer in the incident deuteron (see section 5).

Kinematics of EVA experiment was designed to select events with

\[
|\langle p_f - p_n \rangle_x| < |\langle p_f - p_n \rangle_z|
\]

due to a preferable choice of those of them in which IP was rapidly moving forward in the same direction as the incident proton (cross-section dependence on \( \sqrt{s} \) described by the quark counting rules [12], [13] was used). Taking into account the last inequality and (1), (2), (3), we obtain \( 2 \langle |\Delta p_{fz}| \rangle > 0.3 \). At the same time, validity of MQK for \( z \)-direction, which was established by EVA collaboration [3], gives \( |\Delta p_{fz}| \approx 0 \). Significance and possible consequences of these two relations are discussed in next section.
3. Coherent dibaryons

The most important observation concerning the results of EVA is $2\langle |\Delta p_{fx}| \rangle > 0.3$ in combination with the quasifree knockout in the longitudinal direction. This definitely implies impossibility to consider IP-IN interaction as elastic scattering initiated by previous elastic projectile-IP interaction. Indeed, in a case like that IP would obtain a recoil momentum directed in the transversal plane and thus should necessarily transfer to IN some longitudinal momentum. But experimentally we see something strictly opposite: the whole of momentum transfer is confined to the transversal plane. Therefore IP-IN interaction should propagate through a very unusual intermediate state which has such a property. This conclusion is in a qualitative agreement with the observation following from TE and BGE about dimensionality of the 6-q excited oscillator. We shall consider intermediate states appearing in EVA experiment as 2-D coherent excitations of quantum oscillator because they can maintain relative distance and momentum of colliding particles with maximal accuracy permitted by the uncertainty relation, $\Delta p_i(t) \Delta x_i(t) \sim \hbar$, and transfer them safely in 2-D kinematics to outgoing particles, thereby keeping their emission in the transversal plane. Thus glueing two nucleons into a single 6-quark system together with strong excitation of its inner oscillators seems to be the most plausible explanation of the EVA experiment puzzle.

Before discussing Glauber’s coherent state for 2-D oscillator, it is convenient to define some vector notations:

\[
\alpha = (\alpha_1, \alpha_2), \quad a = (a_1, a_2), \quad [n] = (n_1, n_2), \quad [n]! = n_1!n_2!, \\
(a^\dagger)^{[n]} = (a^\dagger_1)^{n_1}(a^\dagger_2)^{n_2}, \quad ||[n]|| = (a^\dagger)^{[n]} |0\rangle / \sqrt{[n]!}.
\]

---

2Indeed, energy of the ground state of the oscillator should be equal to $\hbar\omega/2 + \hbar\omega/2$ according to TE and BGE. Thus it might consist of one degree of freedom oscillating in 2-D space or of two independent 1-D oscillators. The ground state was observed as dibaryon with mass 1.886 GeV/c$^2$ in [3] and may also be extracted from [4] as a particle X in processes:

\[X+d \rightarrow Y+d, \quad d+X \rightarrow d+d, \quad d+X \rightarrow X+d, \quad X+X \rightarrow X+d, \quad X+X \rightarrow Y+d\] (see [5, 6]).
The Glauber coherent state of 2-D oscillator may be written as a unitary transformation of the ground one,
\[ |\alpha\rangle = D(\alpha) |0\rangle, \quad D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a). \] (5)

This expression can be transformed to
\[ |\alpha\rangle = \exp(\alpha a^\dagger) |0\rangle = \exp\left(-|\alpha|^2/2\right) \sum_{|n|} \frac{\alpha^n}{\sqrt{|n|!}} |n\rangle. \] (6)

Actually, (6) corresponds to two independent 1-D oscillators swinging along \(x_1\)- and \(x_2\)-directions, and
\[ ||n|| \equiv a_1^{n_1} a_2^{-n_2} |0\rangle \otimes |0\rangle / \sqrt{n_1! n_2!}. \]

It is also possible to define operators describing excitations with negative and positive helicities,
\[ a_+ = (a_1 + ia_2)/\sqrt{2}, \quad a_- = (a_1 - ia_2)/\sqrt{2}, \]
accordingly. Corresponding basis vectors \[ ||n|| = a_1^{n_1} a_2^{-n_2} |0\rangle \otimes |0\rangle / \sqrt{n_1! n_2!}. \] are eigenvectors of operators
\[ L_3 = x_1 p_2 - p_1 x_2 = \hbar (a_1^\dagger a_- - a_1^\dagger a_+) = \hbar(n_- - n_+) \]
and
\[ H = \hbar \omega (n_1 + n_2 + 1) = \hbar \omega (n_+ + n_- + 1). \]

Thus, it is possible to define Glauber’s coherent states with nonzero projections of orbital momentum along \(x_3\)-axis of two different quarks belonging to the same 6-q system.

Probabilities of registration of dibaryon lying on \(n\)-th oscillator level are
\[ w_n = \exp(-|\alpha|^2) \sum_{n_i+n_j=n} \frac{|\alpha_i|^{2n_i} |\alpha_j|^{2n_j}}{n_i!n_j!}, \quad n = 0, 1, ... \]
Hereafter \(i = 1\) or \(+\) and \(j = 2\) or \(−\), correspondingly.

It is useful also to have explicit expressions for generalized coherent states \[14\] which may be observed experimentally as coherent dibaryons too. They
should be considered as excitations above different ground states of oscillator, which are possible now and determined by different representations of $su(1,1)$ algebra. In this case, quants of excitations are created and annihilated in pairs and that is described by three generators of $SU(1,1)$ group:

$$K = a_ia_j, \quad K^\dagger = a_i^\dagger a_j^\dagger, \quad K_0 = \frac{1}{2}\left(a_ia_i + a_j^\dagger a_j + 1\right),$$  \tag{7}

where

$$[K_0, K] = -K, \quad [K_0, K^\dagger] = K^\dagger, \quad [K, K^\dagger] = 2K_0.
$$

These coherent states are also defined as a unitary transformation of one of the ground states,

$$|\xi\rangle = D(\xi)|0\rangle_k = \left(1 - |\xi|^2\right)^k \exp(\xi K^\dagger) |0\rangle_k = \left(1 - |\xi|^2\right)^k \sum_{m=0}^{\infty} \left[\frac{\Gamma(m + 2k)}{m!\Gamma(2k)}\right]^{1/2} \zeta^m |k, k + m\rangle, \tag{8}

$$D(\xi) = \exp(\xi K^\dagger - \xi^* K), \tag{9}

$$\text{index } k = 1, 3/2, 2, 5/2, \ldots \text{ determines a ground oscillator state,}

$$\zeta = \tanh(|\xi|) \exp(i\psi), \quad \beta = 2 \ln \cosh(|\xi|) = -\ln \left(1 - |\xi|^2\right), \quad \gamma = -\zeta^*. \tag{10}

$$Energies of oscillator states are eigenvalues of the Hamiltonian,

$$H |k, k + m\rangle \equiv 2\hbar \omega K_0 |k, k + m\rangle = 2\hbar \omega (k + m) |k, k + m\rangle, \quad m = 0, 1, \ldots \tag{11}

$$and the probabilities of the dibaryon observation are

$$w_m = \left(1 - |\xi|^2\right)^{2k} \frac{\Gamma(m + 2k)}{m!\Gamma(2k)} |\zeta|^2m. \tag{12}

$$Taking into account \[3, 4\], we should choose $k = 1$. Then

$$w_m = \left(1 - |\xi|^2\right)^2 (m + 1)|\zeta|^{2m}, \tag{13}

$$where $|\zeta| < 1$, see \[10\].
4. Coherent dibaryons and phase transitions

There is a natural correlation between coherent states and phase transitions, which becomes apparent if we shall study metamorphoses of the oscillator Hamiltonian under unitary transformations $D(\alpha)$ and $D(\xi)$ defined above by (5) and (9). Good explanatory comments concerning calculations in this section can be found in [15].

Transformation of creation and annihilation operators corresponding to (5) is the following (in vector notations):

$$a \rightarrow a' = D^\dagger(\alpha) a D(\alpha) = a + \alpha,$$
$$a^\dagger \rightarrow a'^\dagger = D^\dagger(\alpha) a^\dagger D(\alpha) = a^\dagger + \alpha^*.$$

This alteration leads to a change of the oscillator Hamiltonian,

$$H = \hbar \omega (a^\dagger a + 1) \rightarrow H' = \hbar \omega (a'^\dagger a' + 1) = \hbar \omega (a^\dagger a + 1) + \hbar \omega \left[ |\alpha|^2 + (a^\dagger \alpha + a \alpha^*) \right].$$

Thus we see that the Hamiltonian gains two additional terms $\hbar \omega |\alpha|^2$ and $\hbar \omega (a^\dagger \alpha + a \alpha^*)$. The first of them may be interpreted as energy spent on creation of a complex field $\alpha$ (quantum condensate), the second one describes an interaction between oscillator excitations and this new field. In the general case, energy of the system varies due to the transformation (5) which therefore may be interpreted as describing a phase transition of the first order.

Unitary transformation $a \rightarrow a' = D^\dagger(\xi) a D(\xi)$ and $a^\dagger \rightarrow a'^\dagger = D^\dagger(\xi) a^\dagger D(\xi)$ with $D(\xi)$ described by (9) gives the Bogoliubov transformation for Bose operators [16],

$$a_i \rightarrow a'_i = a_i \cosh \xi + a_j^\dagger \sinh \xi, \quad a_i^\dagger \rightarrow (a'_i)^\dagger = a_i^\dagger \cosh \xi + a_j \sinh \xi,$$
$$a_j \rightarrow a'_j = a_j \cosh \xi + a_i^\dagger \sinh \xi, \quad a_j^\dagger \rightarrow (a'_j)^\dagger = a_j^\dagger \cosh \xi + a_i \sinh \xi.$$

Then the oscillator Hamiltonian takes the form:

$$H = \hbar \omega (a^\dagger a + 1) \rightarrow H' = \hbar \omega \left[ 2 \cosh(2\xi) K_0 + \sinh(2\xi) \left( K^\dagger + K \right) \right].$$

In the limit $\xi \rightarrow 0$, we have $H' = 2 \hbar \omega K_0$, in line with (11). Subject to this extreme condition, we can identify ground state $|0\rangle_{k=1}$ of $su(1,1)$ oscillator...
with state $|0\rangle \otimes |1\rangle$ of the usual 2-D oscillator, since it satisfies the necessary characteristic properties of $|0\rangle_{k=1}$:

$$K |0\rangle \otimes |1\rangle = 0, \quad K_0 |0\rangle \otimes |1\rangle = |0\rangle \otimes |1\rangle.$$  

So far as oscillator excitations are produced now by pairs, the spectrum doubles (phase transition of the second order). In the general case, when $\xi \neq 0$, relations (12) describe a first-order phase transition. It ends with nontrivial renormalization of spectrum and creation of a quantum condensate $\varphi = \sinh(2\xi)$ which interacts with pairs of oscillator excitations. This phase transition occurs after the minimal excitation of one of the oscillators, $|0\rangle \to |0\rangle_{k=1}$. Experiments \[3, 4\] allow existence of these processes with small values of $\varphi$.

There is a difference between phase transitions in macro- and microsystems. An amplitude $\alpha$ of coherent state $|\alpha\rangle$ may be interpreted as a macroscopic field only in the limit of large average number of excitations. Otherwise it seems that a possibility of observing superpositions of coherent states with different amplitudes may exist\[3\]. Then for Glauber’s coherent states, we should take instead of (6):

$$|C_i, C_j\rangle = \int d\mu_i d\mu_j C_i(\mu_i) C_j(\mu_j) \exp(\chi \mu_i a_i^{\dagger} + \chi \mu_j a_j^{\dagger} - \mu_i^2/2 - \mu_j^2/2) |0\rangle,$$

where $C_i(\mu_i)$ and $C_j(\mu_j)$ describe the superposition supposed, and $\alpha = \chi \mu_i$, $\chi = i$ or $-1$, $\mu$ is a real number. It is obvious,

$$|C_i, C_j\rangle = \tilde{f}_i(a_i^{\dagger}) \tilde{f}_j(a_j^{\dagger}) |0\rangle,$$

where $\tilde{f}_i$ is Fourier or Laplace transform of $C_i(\mu_i) \exp(-\mu_i^2/2)$. The generalized coherent states can be modified similarly using (8).

5. Conclusion

Although none of the discussed experimental papers \[3, 4, 8\] contains sufficient grounds for the definite conclusion about existence of oscillator excitations

\[3\] One special case of superposition of $N$ Glauber’s coherent states was considered in \[17\].
in 6-quark systems, interconnections between them appear striking. For example, let us estimate numbers of levels observed in [4] using the measurement time determined in [8]. We can hope for nearly the same value of the measurement time, though the measurement of momentum of p-n pair is performed by scattering accelerated deuteron in [4] rather than proton in [8]. Then the Landau-Peierls uncertainty relation tells us that we can observe 6-7 oscillator levels in target deuteron and, due to the relativistic effect of time dilation, 32 levels in incident one. In [5, 6], there were found 31 consecutive levels of the second type in agreement with this estimation and only 2 of the first type. It seems that the residual 4-5 of them were lost against a background of the dominant events.

In this paper we have inverted the conventional logic of utilization of canonical transformations in theory of phase transitions. Usually they are applied for diagonalization of a model Hamiltonian and estimation of its spectrum. Here we assumed that the oscillator spectrum and coherent states were indeed disclosed in [3, 4, 8] for search of non-diagonalized Hamiltonian which might generate the states hypothetically detected.

Relying on the facts discussed above, it is reasonable to assume that phase transitions of baryon matter can be observed at light nuclei interactions as well as at heavy ions collisions usually considered. As far as this important assumption is based on the results of only 3 experimental papers [3, 4, 8], they should be verified once again with an accuracy maximum possible to enhance reliability of the assertion. In any case, the surest method to recognize what nature really knows is to ask nature itself about that.

\footnote{For a quantum measurement of the required type to be carried out it is sufficient that an information about stopping power of secondary particles to be stored in an environment. A person may neither participate in making measuring apparatus nor in observing results. According to R. Feynman, it is quite enough that “nature knows” [18].}
Acknowledgment

I thank my co-authors J. Pribiš and V. Filinova which assisted me with numerical calculations in an early stage of the study. I dedicate this paper to M.Z. Yuriev for many valuable discussions.

References

[1] B.F. Kostenko and J. Pribiš, Phys. Atomic Nuclei, 75 (2012) 888.

[2] B. Kostenko, J. Pribiš, V. Filinova, PoS (Baldin ISHEPP XXI) 105.

[3] Yu.A. Troian, Phys. Part. Nucl. 24 (1993) 294.

[4] A.M. Baldin, V.K. Bondarev, A.N. Manyatovskij, N.S. Moroz, Yu.A. Panebrattsev, A.A. Povtorejko, S.V. Rikhvitskij, V.S. Stavinskij, A.N. Khrenov, Differential elastic proton-proton, nucleon-deuteron and deuteron-deuteron scatterings at big transfer momenta, JINR 1-12397, 1979 and online version at inis.iaea.org.

[5] B. Kostenko and J. Pribiš, PoS (Baldin ISHEPP XXII) 122.

[6] B. F. Kostenko and J. Pribiš, arXiv:1503.04956.

[7] WASA-at-COSY Collaboration, P. Adlarson et al., Phys. Rev. Lett., 106 (2011) 242302.

[8] A. Tang, J. W. Watson, J. Aclander, J. Alster, G. Asryan et al., Phys. Rev. Lett., 90 (2003) 042301.

[9] V.S. Barashenkov, B.F. Kostenko, A. M. Zadorogny, Nucl. Phys. A 338 (1980) 413.

[10] L.D. Landau, R. Peierls, Zs. Phys. 69 (1931) 56.

[11] L.D. Landau, E.M. Lifshitz, A course of theoretical physics III. Quantum mechanics, (Pergamon Press, Oxford, U.K., 1977).
[12] V.A. Matveev, R.M. Muradian, A.N. Tavkhelidze, Lett. Nuovo Cimento, 7 (1973), 719.

[13] S.J. Brodsky, G.R. Farrar, Phys. Rev. Lett. 31 (1973) 1153.

[14] A. Perelomov, *Generalized Coherent States and Their Applications*, (Springer, Berlin, 1986).

[15] N.N. Bogoliubov, D.V. Shirkov, *Quantum Fields*, (Nauka, Moscow, 1982).

[16] N.N. Bogoliubov, J. Phys. (USSR) 11 (1947) 23.

[17] Z. Bialynicka-Birula, Phys. Rev. 173 (1968) 1207.

[18] R.P. Feynman, R.B. Leighton, M. Sands, *The Feynman Lectures on Physics*, Vol.3, (Addison-Wesley, London, 1965).