Data Compression with Prime Numbers

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Abstract

A compression algorithm is presented that uses the set of prime numbers. Sequences of numbers are correlated with the prime numbers, and labeled with the integers. The algorithm can be iterated on data sets, generating factors of doubles on the compression.
There are publicly available tens of million of prime numbers. The bit complexity of these sets of numbers can be reduced exponentially by associating an integer count to the sets of these numbers. Because there are an approximate $N/\ln(N)$ prime numbers below a number $N$, the complexity of the data set is reduced by an approximate $\ln(N)$.

This reduction in the complexity of the prime number data set can be used to compress any set of integers as well. By breaking the number up into bit sequences which are prime, and then labeling the numbers with the integers, a string of bits can be reduced in complexity by a $\ln(N)$ of the individual number sequences.

For example, consider the number $N=101113$. This number has the prime sequences of 101 or 113, which are the 26 and 27 the prime numbers. The two numbers could be registered by their indices, rather than the number sequences contained in $N$. The bit complexity is reduced to four digits, the 26 and 27, rather than the 6 digits of 101113. This reduction is not much for small numbers, but can be larger for prime number subsequences with ten or tens of digits. The relevant ratio is the fraction of the number of prime numbers below a number $N$, which decreases as the number $N$ increases.

The natural question is given a sequence of digits, what is the probability of finding a prime number contained in the subsequence. Clearly, a number with only an even number of digits would fail this test, but real datasets arent expected to be composed of only even number digits.

The chance of a random number $N$ of being prime is an approximate $1/\ln(N)$, which is the inverse of the number of digits. Checking a subsequence of a number with $N_d$ digits requires summing the probabilities. Checking the subsequences of a number with $P_d$ digits to $Q_d$ digits requires the sum,

$$\sum_{P_d}^{Q_d} \frac{1}{n} = \ln(Q_d/P_d) + C + O(P_d/Q_d) ,$$

which is always greater than unity if the numbers are chosen in a certain manner. Clearly, if $Q_d$ and $P_d$ are chosen as a large ration, the probability will eventually be unity.

The finite sums

$$\frac{1}{5} + \frac{1}{6} + \ldots + \frac{1}{9} \sim .75 ,$$
\[
\frac{1}{4} + \frac{1}{5} + \ldots + \frac{1}{9} > .99
\]  

are almost unity. The probability is unity to find a prime subsequence made of between 6 to 13 digits in a number \( N \) containing more than 13 digits. There are public databases of the first 15 million prime numbers, which consist of up to nine digits.

The examples in (3) indicate that the public databases could be used to break up numbers into sequences of numbers which are prime. The bit complexity in the reduction depends on the size of the sequences. The bit complexity of a prime number of the size of \( 10^9 \) is 30 and that of its index with a number of the order \( 10^7 \) is 23; the ratio is a naive estimate of the ‘worst’ case scenarios of using sets of 23 bits to label the prime number index versus the actual bit complexity of a nine digit number. This ratio is 1.3, which signals a 30 percent compression factor. An interesting aspect is that this algorithm can be iterated multiple times, with 30 percent factor in each iteration (three iterations is a factor of 2.2).

A more efficient algorithm is to use two units. The first states how many digits in the prime label, from 4 to 7, which has two bits. The second number specifies the prime number index. The advantage of this is that not all prime numbers have 9 digits (out of the exampled five to nine digits). The index with four digits requires 13 bits and the index with seven digits requires 23 bits. This should increase the compression factor to almost two, considering the distribution and probability of finding the prime number sequences. (This version is similar to minimizing the bit vacancies in a byte or series of bits are not required to specify the ‘color’ of the data in certain compression schemes \([1]\).)

The examples listed pertain to prime numbers with up to 9 digits, such as one billion. Asymptotically the larger the number of digits in the prime number, the larger the compression factor will be. Scanning for sequences with \( P_d \) to \( Q_d \) digits, with large numbers of digits is computationally intensive, but this will lead to larger compression factors. There is no bound to the compression factor given the distribution of primes \( N/\ln(N) \); this says something about the entropy of the information.

The iteration of the algorithm could easily produce compression factors of ten or so, given a front end for searching the prime sequences of the number \( N \). A database of the first 15 million primes would require a gigabyte of storage. Also, this compression algorithm can be incorporated with existing algorithms.
References

[1] Gordon Chalmers, *A Novel Data Compression*, physics/0510148