Stimulated non-eigen electrostatic mode instability

Yao Zhao\textsuperscript{1,†}, Suming Weng\textsuperscript{2,3}, Zhengming Sheng\textsuperscript{2,3,4}, Jianqiang Zhu\textsuperscript{1,3}

\textsuperscript{1}Key Laboratory of High Power Laser and Physics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, China
\textsuperscript{2}Key Laboratory for Laser Plasmas (MoE), School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China
\textsuperscript{3}Collaborative Innovation Center of IFSA (CICIFSA), Shanghai Jiao Tong University, Shanghai 200240, China
\textsuperscript{4}SUPA, Department of Physics, University of Strathclyde, Glasgow G4 0NG, UK

E-mail: \textsuperscript{†}yaozhao@siom.ac.cn

Abstract. The development of stimulated non-eigen electrostatic mode instability (SNEMI) is studied theoretically and numerically. Different from general stimulated Raman scattering (SRS), the SNEMI can develop at density over the quarter critical density without including relativistic effect. To satisfy the matching conditions of frequency and wavenumber, the stimulated electrostatic mode has a constant frequency around half of the incident light frequency $\omega_0/2$, which is no longer the eigenmode of electron plasma wave $\omega_{pe}$. The phase velocity of non-eigen electrostatic wave approximates $0.58c$ with $c$ being the light velocity in vacuum. The concomitant light forms electromagnetic solitons or decays into electrostatic field as an evanescent wave in its propagation. With the threshold of triggering SNEMI, the density range for the absolute instability in inhomogeneous plasma can be obtained, which is proportional to the laser amplitude. Our theoretical model is validated by particle-in-cell simulations. The SNEMI may play a considerable role in the experiments of laser plasma interactions as long as the laser intensity is higher than $10^{15}$W/cm$^2$. 
1. Introduction

Laser plasma interactions (LPI) are widely associated with many applications, such as inertial confinement fusion (ICF) [1, 2, 3], radiation source [4], plasma optics [5, 6], and laboratory astrophysics [7, 8]. The concomitant parametric instabilities found in LPI are nonlinear processes which can greatly affect the outcome [9]. Generally, laser plasma instabilities [10, 11], especially stimulated Raman scattering (SRS), stimulated Brillouin scattering (SBS) and two-plasmon decay (TPD) instability, are mainly considered in ICF with the incident laser intensity less than $10^{15}$ W/cm$^2$ [12, 13, 14]. However, the laser intensity always in the order of $10^{16}$ or even $10^{17}$ W/cm$^2$ in shock ignition [15, 16, 17, 18], Brillouin amplification [19, 20], and the interactions of high power laser with matter [21, 22, 23]. Therefore, the parametric region close to the regime near the relativistic intensity needs to be explored in depth.

As one knows, SRS happens in the density not larger than the quarter critical density $n_e \leq 0.25n_c$ due to the decay of the scattering light in its propagation in the overcritical density medium [4, 10]. Relativistic intensity laser can reduce the effective electron plasma frequency, and therefore SRS may develop at $n_e > 0.25n_c$ [24]. In this work, we study the non-eigen electrostatic mode developed by intense lasers when $n_e > 0.25n_c$, which has nothing to do with the relativistic effect. Stimulated non-eigen electrostatic mode instability is mainly due to the fluid effect, which is a seed mode for the subsequent nonlinear fluid and kinetic instability [25]. With the triggering threshold of SNEMI, we obtain the density range for the development of absolute instability in inhomogeneous plasma [26]. The theoretical model is supported by particle-in-cell (PIC) simulations.

2. Theoretical analysis of stimulated non-eigen electrostatic mode instability

Stimulated non-eigen electrostatic mode instability (SNEMI) is a three-wave instability that a laser decays into an electrostatic wave with frequency not equal to the eigen electron plasma wave, and a light wave. Both frequencies are nearly half of the incident laser frequency. The mechanism of this instability can be described by the SRS dispersion relation at density $n_e > 0.25n_c$.

Firstly we introduce the nonrelativistic dispersion relation of SRS in cold plasma [10]

$$\omega_e^2 - \omega_{pe}^2 = \frac{\omega_{pe}^2 k_e^2 c^2 a_0^2}{4} \left( \frac{1}{D_{e+}} + \frac{1}{D_{e-}} \right),$$

where $D_{e\pm} = (\omega_e \pm \omega_0)^2 - (k_e \pm k_0)^2 c^2 - \omega_{pe}^2$, and $a_0$ is the laser normalized amplitude. The relation between $a_0$ and laser intensity $I$ is given by $a_0 = \sqrt{I(W/cm^2)[\lambda(\mu m)]^2/1.37 \times 10^{18}}$. $\omega_0$ and $k_0$ respectively are the frequency and wavenumber of incident light. The wavenumber of scattering light $k_{sc} = \sqrt{\omega_{sc}^2 - 2\omega_{pe}\omega_0}$ can be obtained from Eq. (1). Note that $k_{sc}$ is imaginary number when $\omega_{pe} > \omega_0/2$,
Figure 1. Numerical solutions of SRS dispersion equation at (a) the plasma density $n_e = 0.26 n_c$ and $n_e = 0.27 n_c$ with laser amplitude $a_0 = 0.09$, (b) the plasma density $n_e = 0.3 n_c$ with light amplitude $a_0 = 0.21$. The dotted line and continuous line are the imaginary part and the real part of the solutions, respectively.

Therefore SRS generally happens in the density $n_e \leq 0.25 n_c$. However, when the amplitude of incident laser $a_0$ larger than a threshold, stimulated non-eigen electrostatic mode will be developed.

The numerical solutions of Eq. (1) at density $n_e = 0.26 n_c$ and $n_e = 0.27 n_c$ under the same laser amplitude $a_0 = 0.09$ are shown in Fig. 1(a). We can see the almost equal frequency of the electrostatic waves under different density, i.e., the real part of $\omega_e$ is $\text{Re}(\omega_e) \approx 0.5 \omega_0$. Matching conditions are still satisfied for SNEMI, therefore the frequency of concomitant light is also $\text{Re}(\omega_s) \approx 0.5 \omega_0$ which can be obtained from the dispersion relation of light wave

$$
\omega_s^2 - k_s^2 c^2 - \omega_{pe}^2 = D_{s+} + D_{s-},
$$

where $D_{s\pm} = \omega_{pe}^2 (k_s \pm k_0)^2 c^2 a_0^2/4[(\omega_s \pm \omega_0)^2 - \omega_{pe}^2]$. The growth rate and instability region are reduced by the increasing of plasma density. From Fig. 1(b) we know that relativistic effect has little effect on SNEMI even at $a_0 = 0.21$.

Now we analytically solve Eq. (1) under $n_e > 0.25 n_c$. Writing $\omega_e = \omega_{er} + i \omega_{ei}$ where $\omega_{er}$ and $\omega_{ei}$ are the real and imaginary part of $\omega_e$, respectively. As discussed above, the wavenumber of scattering light is $k_s = 0$, and to keep the matching conditions, we set $k_e = k_0$. Considering the linear case $n_e \lesssim 0.35 n_c$, then the imaginary part of Eq. (1) can be simplified to

$$
(\omega_0^2 - 2 \omega_0 \omega_{er})(\omega_{er}^2 - \omega_{ei}^2 - 2 \omega_0 \omega_{er} + 3 \omega_0^2 - 3 \omega_{pe}^2) \left( \omega_{pe}^2 + \omega_{ei}^2 + \frac{\omega_{er} \omega_0 - \omega_{ei}^2}{2} \right) = 0.
$$

Equation (3) is satisfied for any $\omega_{pe}$ when $\omega_{er} = \omega_0/2$. Then, we have the dispersion relation of the non-eigen electrostatic mode

$$
\omega_{er}^2 = \frac{\omega_0^2}{4} = \frac{1}{4}(k_e c^2 + \omega_{pe}^2).
$$

The group velocity of non-eigen electrostatic wave is $v_g = \delta \omega_{er} / \delta k_e \approx 0$, therefore electrostatic solitons will be formed in the plasma.
Taking $\omega_{er} = \omega_0/2$ into the real part of Eq. (1), one obtains the growth rate of SNEMI

$$\omega_{ei} = \frac{1}{2} \sqrt{4 \omega_{pe}(\omega_0 - \omega_{pe}) + \omega_{pe}^2 a_0^2 k_0^2 c^2 / \omega_0^2 - \omega_{0}^2}. \tag{5}$$

The threshold $a_{th}$ for developing SNEMI can be obtained from $4 \omega_{pe}(\omega_0 - \omega_{pe}) + \omega_{pe}^2 a_{th}^2 k_0^2 c^2 / \omega_0^2 - \omega_{0}^2 \gtrsim 0$, i.e.,

$$a_{th} \gtrsim \frac{\omega_0 \sqrt{\omega_0^2 + 4(\omega_{pe}^2 - \omega_{pe}\omega_0)}}{\omega_{pe} k_0 c}. \tag{6}$$

Equation (6) indicates that $0.25n_c$ is the turning point between SRS and SNEMI, where the threshold $a_{th} = 0$.

According to the linear parametric model of inhomogeneous plasma, the Rosenbluth gain saturation coefficient for convective instability is $G = 2 \pi \Gamma^2 / v_s v_p K'$, where $\Gamma$, $v_s$ and $v_p$ are instability growth rate, group velocity of scattering light and plasma wave, respectively. $K$ is the wavenumber mismatch for incident light, scattering light and plasma wave. The temporal growth of the convective instability will be gradually saturated out of the local resonant region due to the spatial mismatch. As it is known, convective instability transits to absolute instability when $K = 0$.

Based on the above discussions, the mismatching term of SNEMI is $K_{SNEMI} = k_0 - k_e - k_s = 0$ due to $k_e = k_0$ and $k_s = 0$ all the time. Therefore, SNEMI is an absolute instability in inhomogeneous plasma. The density range for absolute instability can be obtained from Eq. (5) when $\omega_{ei} \geq 0$. One finds that the absolute instability happens around $\omega_0/2$ with a finite range

$$\delta \omega / \omega_0 \approx 0.433 a_0, \tag{7}$$

dependent on laser amplitude is not large enough $a_0 \lesssim 10^{-2}$. Equation (7) indicates that the absolute instability region will be widened by the increase of laser amplitude.

In conclusion, different from general SRS, there is a threshold for developing SNEMI without considering damping. The stimulated electrostatic mode has an almost constant frequency around half of the incident light frequency $\omega_0/2$, which is no longer the eigenmode of the electron plasma wave $\omega_{pe}$. SNEMI is mainly a fluid instability mode at density $n_e > 0.25n_c$, which can trigger the subsequent nonlinear fluid and kinetic instabilities as a seed mode. The coandant light of SNEMI forms electromagnetic solitons or decays into electrostatic field as an evanescent wave in its propagation. The SNEMI threshold indicates an absolute instability region in inhomogeneous plasma.

3. Simulations for the stimulated non-eigen electrostatic mode instability

3.1. Comparison between analytical model and simulation results

To validate the analytical prediction of the threshold for SNEMI, we have performed several one-dimensional (1D) simulations by using the Klap code. The space and time given in the following are normalized by the laser wavelength in vacuum $\lambda$ and the
Figure 2. (a) Comparison of SNEMI threshold between analytical and simulation results. Distributions of the electrostatic wave in \((k_e, \omega_e)\) space obtained for the time window \([250, 350] \tau\) under (b) plasma density \(n_e = 0.26n_c\) and laser amplitude \(a_0 = 0.046\), and (c) plasma density \(n_e = 0.3n_c\), laser amplitude \(a_0 = 0.176\). (d) The wave-number distributions of electrostatic wave with \(n_e = 0.3n_c\) at \(t = 110\tau\).

laser period \(\tau\). A linearly-polarized semi-infinite pump lasers with a uniform amplitude is incident from the left boundary of the simulation box. In this subsection, only the fluid property of the instability is considered, therefore we set electron temperature \(T_e = 100\text{eV}\) with immobile ions.

Based on Fig. 2(a), we know that the analytical solutions and simulation results are almost equal when density \(n_e \lesssim 0.28n_c\). However, the difference between them increases with plasma density. To explain this phenomenon, we plot the distribution of electrostatic wave in \((k_e, \omega_e)\) space, as shown in the following two figures. The threshold for developing SNEMI at density \(n_e = 0.26n_c\) is around \(a_0 = 0.045\). We can find a SNEMI mode at \(k_e c \approx 0.87\omega_0\) and \(\omega_e \approx 0.496\omega_0\) in Fig. 2(b). When plasma density increases to \(0.3n_c\), the analytical threshold is enhanced to \(a_0 = 0.208\). However, nonlinear mechanism should be considered at this high intensity. The intense pondermotive-force of laser will modulate the plasma density, and therefore develop SNEMI below the threshold. Therefore, SNEMI has been developed at \(a_0 = 0.176 < 0.208\) in the simulation as shown in Fig. 2(c). Note that a modulation mode can be
Figure 3. Distributions of the (a) electrostatic wave and (b) scattering light in \((k, \omega)\) space obtained for the time window \([550, 650]\)\(\tau\) under plasma density \(n_e = 0.27n_c\) and \(a_0 = 0.09\). (c) Spatial distribution of the ion density at \(t = 1000\)\(\tau\). (d) Energy distributions of electrons at different times. \(N_e\) is the relative electron number.

found at \(k_e c \sim 2k_0 c = 1.72\omega_0\) and \(\omega_e \sim 0\) in addition to the SNEMI mode. From Fig. 2(d) we can see that the density modulation is the major mode before SNEMI developing.

In a brief conclusion, the analytical prediction matches well with the simulation result at low laser intensity. Density modulation should be considered for the intense laser \(a_0 \gtrsim 0.2\), so that SNEMI can develop below the linear threshold.

3.2. Nonlinear evolution of stimulated non-eigen electrostatic mode instability

Here we study the nonlinear evolution of SNEMI. The length of the simulation box is 250\(\lambda\), where the plasma occupies a region from 50\(\lambda\) to 200\(\lambda\) with homogeneous density \(n_e(x) = 0.27n_c\). Ions are mobile with mass \(m_i = 1836m_e\) and charge \(Z = 1\). The initial electron and ion temperatures are \(T_{e0} = 3\) keV and \(T_{i0} = 1\) keV, respectively. The laser amplitude is \(a_0 = 0.09\) (the corresponding intensity is \(I_0 = 1.1 \times 10^{16}\) W/cm\(^2\) with \(\lambda = 1\mu m\)). We have taken 100 cells per wavelength and 50 particles per cell.

According to Eq. (3), we know that the linear growth time for SNEMI is around 263\(\tau\). After \(t = 550\tau\), the instability has entered into a strong nonlinear stage. A
spectral broadening can be found near the linear SNEMI mode in Fig. 3(a). A part of non-eigen electrostatic wave forms electrostatic solitons in its propagates in plasma due to its group velocity \( v_g \approx 0 \). Note that the eigenfrequency of plasma wave is larger than the frequency of scattering light. Therefore, the light with group velocity being zero \( (\omega_s = 0.5\omega_0) \) will be trapped in the plasma as shown in Fig. 3(b), which may cause laser energy deficit in ICF related experiments [30]. The trapped waves expel the ions to form electrostatic and electromagnetic cavities which strongly modulate the plasma density seen from Fig. 3(c). The density modulations subsequently affect the evolution of SNEMI, such as a part of lights escaping from plasma, and SRS developing in the modulated density lower than 0.25\( n_c \) [31]. The phase velocity of the electrostatic wave is around \( v_e \approx \omega_e/k_e = 0.585c \), which corresponds to the electron temperature \( T_e = 175\text{keV} \). Therefore, electrons are heated enormously at nonlinear stage \( t \gtrsim 700\tau \) as presented in Fig. 3(d).

3.3. Stimulated non-eigen electrostatic mode instability developed in inhomogeneous plasma

We have performed a simulation for the inhomogeneous plasma \( n_e = 0.253\exp(x/1000)n_c \) with density range \( [0.253,0.294]n_c \). We only consider SNEMI here, therefore the ions are immobile with a charge \( Z = 1 \). The initial electron temperature is \( T_{e0} = 3\text{keV} \). The driving laser is a linearly-polarized semi-infinite pump lasers with a uniform amplitude \( a_0 = 0.09 \). According to the theoretical threshold, the up-limit density for developing SNEMI is \( n_e \sim 0.27n_c \) under this laser intensity.

The spatial-temporal evolution of electrostatic wave is exhibited in Fig. 4(a). We find that a strong electrostatic wave develops at the front of plasma where plasma density less than 0.262\( n_c \). The depletion of laser energy leads to the threshold unsatisfied at \( x > 100\lambda \). Note that the spatial gradient has little effect on the development of SNEMI,
due to the phase matching of the three waves is always satisfied. The Fourier transform of Fig. 4(a) is presented in Fig. 4(b), where we can find an intense frequency shift near the linear SNIMI mode, due to the intense kinetic effects of the trapped particles.

4. Summary

In summary, we have shown theoretically and numerically that there is a new type of parametric instability, which is named as stimulated non-eigen electrostatic mode instability (SNEMI), occurring when the plasma density is over the quarter critical density. SNEMI is a three-wave instability that a laser decays into an electrostatic wave and a light wave, and the produced electrostatic wave has a constant frequency $\omega_0/2$ which is not the eigen electron plasma wave $\omega_{pe}$. The phase velocity of non-eigen electrostatic wave approximates $0.58c$ which correspond to the electron temperature $T_e = 175$keV. There is a certain laser intensity threshold for developing SNEMI according to the non-relativistic dispersion relation of SRS at density over $0.25n_c$. When laser amplitude $a_0 \gtrsim 0.2$, the intense pondermotive-force will modulate the plasma density, and therefore develop SNEMI below the linear threshold. The concomitant light forms electromagnetic solitons or decays into electrostatic field in its propagation as an evanescent wave. The density range for the development of absolute instability can be obtained from the growth rate of SNEMI, which is increased with the laser amplitude. Our theoretical model is validated by PIC simulations. The SNEMI may play a considerable role in the experiments of laser plasma interactions as long as the laser intensity higher than $10^{15}$W/cm$^2$.

5. Acknowledgement

This work was supported by the Natural Science Foundation of Shanghai (No. 19YF1453200) and the National Natural Science Foundation of China (Nos. 11775144 and 1172109).

References

[1] Campbell E M et al. 2017 Matter Radiat. Extrem. 2, 37
[2] Froula D H et al 2010 Phys. Plasmas 17, 056302
[3] Myatt J F et al 2014 Phys. Plasmas 21, 055501
[4] Liu C S, Tripathi V K and Eliasson B 2019 High-power laser-plasma interaction (Cambridge University Press)
[5] Lancia L et al 2016 Phys. Rev. Lett. 116, 075001
[6] Lehmann G and Spatschek K H et al 2013 Phys. Plasmas 20, 073112
[7] Drake R P 2006 High-energy-density physics: fundamentals, inertial fusion, and experimental astrophysics (Springer Science and Business Media)
[8] Falk K 2018 High Power Laser Sci. Eng. 6, e59
[9] Gibbon P 2004 Short pulse laser interactions with matter (World Scientific Publishing Company)
[10] Kruer W L 1988 The physics of laser plasma interactions (Addison-Wesley, New York)
[11] Montgomery D S 2016 Phys. Plasmas 23, 055601
[12] Craxton R S et al 2015 Phys. Plasmas 22, 110501
[13] Lindl J et al 2014 Phys. Plasmas 21, 020501
[14] Moody J et al 2001 Phys. Rev. Lett. 86, 2810
[15] Betti R and Hurricane O A 2016 Nature Phys. 12, 435
[16] Batani D et al. 2014 Nucl. Fusion 54, 054009
[17] Cristofoletti G 2019 High Power Laser Sci. Eng. 7, e51
[18] Klimo O et al. 2010 Plasma Phys. Control. Fusion 52, 055013
[19] Weber S et al 2005 Phys. Rev. Lett. 94, 055005
[20] Lancia L et al 2010 Phys. Rev. Lett. 104, 025001
[21] Rethfeld B et al. 2017 J. Phys. D: Appl. Phys. 50, 193001
[22] Price D F et al 1995 Phys. Rev. Lett. 75, 252
[23] George K M 2019 High Power Laser Sci. Eng. 7, e50
[24] Zhao Y et al 2014 Phys. Plasmas 21, 112114
[25] Ghizzo A et al 2006 Phys. Rev. E 74, 046407
[26] Zhao Y et al . 2019 Plasma Phys. Control. Fusion 61, 115008
[27] Rosenbluth M N 1972 Phys. Rev. Lett. 29, 565
[28] Liu C S et al 1974 Phys. Fluids 17, 1211
[29] Chen M et al 2008 Chin. J. Comput. Phys. 25, 43 (in Chinese)
[30] Zhao Y et al 2019 High Power Laser Sci. Eng. 7, e20
[31] Wu Charles F et al 2019 Acta Physica Sinica 68, 195202