Stable de Sitter Vacua from $\mathcal{N}=2$ Supergravity $^\dagger$

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Abstract

We find extrema of the potential of matter couplings to $\mathcal{N}=2$ supergravity that define de Sitter vacua and no tachyonic modes. There are three essential ingredients in our construction, namely non-Abelian non-compact gaugings, de Roo–Wagemans rotation angles and Fayet–Iliopoulos terms.

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1 Introduction

Recent cosmological observations lead to the conclusion that the cosmological constant is positive and give a confirmation of the idea of inflationary scenarios. Then, if string theory has to be able to provide realistic models for cosmology, it should admit de Sitter vacua. The de Sitter spaces are not as natural as anti-de Sitter ones in the context of supersymmetric theories. This fact can clearly be seen from algebraic considerations, and is illustrated in Table 1. The de Sitter superalgebras have typically a non-compact R-symmetry subalgebra, which leads to non-definite signs in the kinetic terms, and hence leads to the existence of ghosts. Therefore, de Sitter vacua can occur in physical supersymmetric theories only in a phase where supersymmetry is completely broken. This might even be welcome in view of the fact that supersymmetry breaking is anyhow necessary to make contact with reality.

Furthermore, it has been mentioned that de Sitter vacua are difficult to construct from higher dimensions, although this may not be completely excluded. In any case, de Sitter vacua have been found in 4-dimensional higher $\mathcal{N}$ supergravity models. In most cases, this was obtained by considering supergravities with a gauged non-compact group (see for $\mathcal{N} = 8$, for $\mathcal{N} = 4$, for $\mathcal{N} = 3$ and for $\mathcal{N} = 2$, and also the new possibilities for $\mathcal{N} = 8$ that were recently found). Such solutions have been reconsidered recently. However, it has been mentioned that such vacua have tachyons, and even that the negative masses of these tachyons often have a fixed ratio to the cosmological constant. Indeed, normalizing the scalars in the Lagrangian so that (for real scalars)

$$\mathcal{L} = \frac{1}{2} e \partial_\mu \phi \partial^\mu \phi - e V(\phi),$$

have typically a non-compact R-symmetry subalgebra, which leads to non-definite signs in the kinetic terms, and hence leads to the existence of ghosts. Therefore, de Sitter vacua can occur in physical supersymmetric theories only in a phase where supersymmetry is completely broken. This might even be welcome in view of the fact that supersymmetry breaking is anyhow necessary to make contact with reality.

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Table 1: Superalgebras with bosonic subalgebra a direct product of (anti) de Sitter algebra and R-symmetry.

| $\text{AdS}$ | superalgebra | R-symmetry |
|---|---|---|
| $D = 4$ | OSp($N|4$) | SO($N$) |
| $D = 5$ | SU($2,2|N$) | $N \neq 4$: SU($N$) $\times$ U(1) |
| | | $N = 4$: SU(4) |
| $D = 6$ | $F^2(4)$ | SU(2) |
| $D = 7$ | OSp($6,2|N$) | $N$ even: USp($N$) |

| $\text{dS}$ | superalgebra | R-symmetry |
|---|---|---|
| $D = 4$ | OSp($m^*|2,2$) | $m = 2$: SO(1,1) |
| | | $m = 4$: SU(1,1) $\times$ SU(2) |
| | | $m = 6$: SU(3,1) |
| | | $m = 8$: SO(6,2) |
| $D = 5$ | SU($4|2n$) | $n = 1$: SO(1,1) $\times$ SU(2) |
| | | $n = 2$: SO(5,1) |
| $D = 6$ | $F^1(4)$ | SU(2) |
many examples were found where at least one of the scalars has
\[ \frac{\partial}{\partial \phi} \frac{\partial}{\partial \phi} V = -2V. \] (1.2)
Therefore, the question still remained whether there are stable de Sitter vacua in \( N \geq 2 \) supergravity, and whether, in the case of an affirmative answer, they could be lifted to full fledged string theory. In the present paper, we give a positive answer to the first question for \( N = 2 \), and some preliminary arguments why the second question may also be answered affirmatively. Rigid \( N = 2 \) theories have already been used to construct inflation scenarios \[^3\]. Difficulties to generalize this construction to supergravities were identified. We thus make a first step to overcome these problems.

In section 2, we give the ingredients that will turn out to be necessary in the construction of \( N = 2 \) supergravities with stable de Sitter vacua. Three specific models are then discussed in section 3, and the masses in their corresponding de Sitter vacua are studied. In the first two models we find only positive mass fields. In the third model, which has non-trivial hypermultiplets, there are no negative masses, but there is a valley in the potential corresponding to zero-mass fields. In section 4, we discuss further steps that can be considered in order to lift our stable de Sitter vacua first to \( N = 4 \) theories and later to the field theory limit of superstrings. A first appendix gives explicit expressions for the geometry items appearing in the quaternionic-Kähler manifold that we consider. A second appendix gives a table with indices that are used throughout the paper.

2 Three ingredients

In this section, we show that in \( N = 2 \) supergravity we can obtain stable de Sitter vacua if we introduce three ingredients that turn out to be all equally necessary. The framework is provided by the coupling of vector multiplets based on the following choice for the special Kähler manifold:
\[ SK_n = ST[2,n] \equiv \frac{SU(1,1)}{U(1)} \times \frac{SO(2,n)}{SO(2) \times SO(n)}. \] (2.1)
The three essential ingredients are:

1. **Non compact gauging**, namely the gauge group \( G_{\text{gauge}} \) is a product of several factors and it involves the **non-compact simple factor** \( SO(2,1) \) times several other compact factors, including \( SO(3) \) or \( U(1) \) factors.

2. **de Roo–Wagemans symplectic angles** that magnetically rotate one gauge group with respect to another.

3. **Fayet–Iliopoulos (FI) terms** that are possible for either \( SO(3) \) or \( U(1) \) factors.

In this section, we present the construction of \( N = 2 \) supergravity models involving the three ingredients listed above.

\[^2\]The potentials for \( N \geq 2 \) are determined from gauging, while the one in \( N = 1 \) may originate from an ad-hoc superpotential, with no relation to higher dimensions or superstrings.
2.1 Special geometry items

The models we will consider fall into the general framework of matter-coupled supergravity that was extensively discussed in the literature using both superconformal tensor calculus methods \[11,31,32\] and the more direct geometric methods based on the rheonomic approach \[33, 34\]. Here we follow the notations and conventions of \[34\] where the most general form of an \( \mathcal{N} = 2 \) supergravity action was given for an arbitrary choice of the special Kähler manifold of the vector multiplets, of the quaternionic manifold for hypermultiplets and with general permissible gauging.

In order to construct the explicit form of our supergravity model, we need to begin with the symplectic sections of special geometry. Following the notations of \[34\], we write the holomorphic section as

\[
\Omega = \begin{pmatrix} X^A \\ F_A \end{pmatrix},
\]

(2.2)

where

\[
X^A(S, y) = \begin{pmatrix} \frac{1}{2} \left( 1 + y^2 \right) \\ \frac{1}{2} i (1 - y^2) \end{pmatrix}; \quad a = 1, \ldots, n,
\]

(2.3)

\[
F_A(S, y) = \begin{pmatrix} \frac{1}{2} S \left( 1 + y^2 \right) \\ \frac{1}{2} i S (1 - y^2) \end{pmatrix}; \quad y^2 = \sum_{a=1}^{n} (y^a)^2.
\]

In the above equations, the complex fields \( y^a \) are the Calabi–Vesentini coordinates for the homogeneous manifold \( \text{SO}(2n) / \text{SO}(2) \times \text{SO}(n) \), while the complex field \( S \) parametrizes the homogeneous space \( \text{SU}(1,1) / \text{U}(1) \) which is identified with the complex lower half-plane. Indeed, the positivity domain of the Lagrangian we are going to construct, implies

\[
\text{Im} S < 0.
\]

(2.4)

The Kähler potential is, by definition, identified as

\[
K = -\log \left( i \langle \Omega | \bar{\Omega} \rangle \right) = -\log \left[ i \left( \bar{X}^A F_A - F_A X^A \right) \right].
\]

(2.5)

The Kähler potential and metric associated with the above geometry are

\[
K = K_1 + K_2,
\]

\[
K_1 = -\log \left[ i \left( S - \bar{S} \right) \right], \quad K_2 = -\log \left[ \frac{1}{2} \left( 1 - 2 \bar{S} y^a + |y^a y^b|^2 \right) \right],
\]

\[
g_{S\bar{S}} = \frac{1}{(2 \text{Im } S)^2}, \quad g_{ab} = \frac{\partial}{\partial y^a} \frac{\partial}{\partial \bar{y}^b} K_2.
\]

(2.6)

The covariantly holomorphic section is then defined by the general formula

\[
V = \begin{pmatrix} L^A \\ M_\Sigma \end{pmatrix} \equiv e^{K/2} \Omega = e^{K/2} \begin{pmatrix} X^A \\ F_A \end{pmatrix},
\]

(2.7)

and satisfies the constraint

\[
1 = i \langle V | \bar{V} \rangle = i \left( \bar{L}^A M_A - \bar{M}_\Sigma L^\Sigma \right).
\]

(2.8)
2.2 Quaternionic-Kähler geometry items

The quaternionic-Kähler manifold with coordinates \( q^u \), (and \( u = 1, \ldots, 4 \dim QK \)) has a metric build from vielbein 1-forms \( V^{mt} = V^m_u dq^u \), with \( t = 1, \ldots, \dim QK \) and \( m = 1, \ldots, 4 \):

\[
h_{uv} dq^u dq^v = \frac{1}{2} \sum_{m,t} V^{mt} V^{mt}. \tag{2.9}
\]

These vielbeins and their inverse \( V^u_{mt} \) lead to the complex structures \((x = 1, 2, 3)\)

\[
J^x_{u v} = V^m_u J^x_m n V^v_n, \tag{2.10}
\]

where

\[
J_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \quad J_2 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad J_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \tag{2.11}
\]

such that the complex structures fulfill the quaternionic algebra

\[
J^x J^y = -\frac{1}{4} \delta^{xy} + \epsilon^{xyz} J^z. \tag{2.12}
\]

Triholomorphic Killing vectors \( k^v_{\Lambda} \) are related to moment maps \( P^x_{\Lambda} \) by

\[
2k^u_{\Lambda} \Omega^x_{uv} = \nabla_v P^x_{\Lambda} = \partial_v P^x_{\Lambda} + \epsilon^{xyz} \omega^y \partial_z P^x_{\Lambda}. \tag{2.13}
\]

They should further satisfy a relation (equivariance condition)

\[
2k^u_{\Lambda} k^v_{\Sigma} \Omega^x_{uv} - \epsilon^{xyz} P^y_{\Lambda} P^z_{\Sigma} = f^\Delta_{\Lambda \Sigma} P^x_{\Delta}, \tag{2.14}
\]

where the structure constants are defined by

\[
k^u_{\Lambda} \partial_u k^v_{\Sigma} - k^u_{\Sigma} \partial_u k^v_{\Lambda} = -f^\Delta_{\Lambda \Sigma} k^u_{\Delta}. \tag{2.15}
\]

2.3 Description of the 3 ingredients

Non-compact gauge groups. In all previous examples displaying de Sitter vacua, non-compact gauge groups were used. In \( \mathcal{N} = 2 \) tensor calculus, the appearance of just one compensating multiplet leads to the non-compact factor gauge group \( \text{SO}(2, 1) \) \([22]\). In order to gauge non-Abelian groups \( \mathcal{G}_{\text{gauge}} \), the special Kähler manifold of vector multiplets must be a homogeneous space \( \mathcal{G}/\mathcal{H} \), such that \( \mathcal{G}_{\text{gauge}} \subset \mathcal{G} \), since the gauge transformations must be continuous isometries of the scalar manifold. Furthermore, for consistency, if we call \( R \) the symplectic representation of \( \mathcal{G} \), to which the field strengths and their magnetic duals are assigned, then, under the reduction to \( \mathcal{G}_{\text{gauge}} \) we must have \( R \overset{\text{adj}}{\longrightarrow} \text{adj} + \text{adj} \). Going through the list of symmetric special manifolds \([35]\), and especially their symplectic embedings (see, e.g., table 2 of \([36]\)), we see that the only solution is the choice of the model \((2.1)\). In this case, the electric and magnetic field strengths are in the doublet representation of \( \text{Sl}(2, \mathbb{R}) \sim \text{SU}(1, 1) \) and in the \( n + 2 \) vector representation of \( \text{SO}(2, n) \). For any
compact group $G_{\text{compact}}$ of dimension $n - 1$, the group $\text{SO}(2,1) \times G_{\text{compact}}$ is naturally embedded in $\text{SO}(2,n)$ in such a way that the vector $n + 2 = 3 + \text{adj}(\text{compact})$.

Hence, using $ST[2,n]$, we can gauge a group of the following type

$$G_{\text{gauge}} = \text{SO}(2,1) \times G_1 \times \ldots \times G_r,$$

$$\dim(G_k) = d_k; \quad k = 1, \ldots, r,$$

(2.16)

where $G_k$ are compact factors that can, in particular, be $U(1)$ or $\text{SO}(3)$ factors. The condition on the dimensions $d_k$ is obviously

$$\sum_{k=1}^r d_k = n - 1.$$  

This is the first essential ingredient in our identification of theories admitting stable de Sitter vacua. We have introduced a gauge group with non-compact generators. For the $\text{SO}(2,1)$ Lie algebra we use the following normalization

$$[T_x, T_y] = e_0 \varepsilon_{xyz} T_z, \quad x, y, \ldots = 1, 2, 3;$$

$$[T_1, T_2] = -T_3; \quad [T_1, T_3] = -T_2, \quad [T_2, T_3] = T_1,$$

(2.17)

where $e_0$ denotes the coupling constant of this group. Calling collectively $t_\Lambda$ (with $\Lambda = 1, 2, \ldots, n+2$) the generators of the gauge group (2.16) the structure constants of the gauge Lie algebra are defined as follows:

$$[t_\Lambda, t_\Sigma] = f_{\Delta_\Lambda\Sigma} t_\Delta,$$

(2.18)

and the symplectic embedding of the adjoint representation of $G_{\text{gauge}}$ into the fundamental representation of the symplectic group $\text{Sp}(2n + 4, \mathbb{R})$ is realized by

$$G_{\text{gauge}} \ni t_\Lambda \mapsto T_\Lambda = \begin{pmatrix} t_\Lambda & 0 \\ 0 & -t_\Lambda^T \end{pmatrix} \in \text{Sp}(2n + 4, \mathbb{R}),$$

$$t_\Lambda^\Sigma \Gamma = f_{\Lambda\Sigma}^{\Delta} t_\Delta \Gamma.$$

(2.19)

Using (2.19) we can write the real potentials for the Killing vectors describing the infinitesimal action of the gauge group on the scalar fields. We set

$$P^0_\Lambda = \exp (\mathcal{K}) < \overline{\Omega} | T_\Lambda \Omega >,$$

(2.20)

and we have:

$$\delta z^\alpha = e^\Lambda k^\alpha_\Lambda (z); \quad k^\alpha_\Lambda (z) = i g^{\alpha\beta} \partial_\beta P^0_\Lambda,$$

(2.21)

where $z^\alpha = \{S, y^0, \vec{y}\}$ denotes the entire set of all $n + 1$ scalar fields. Applying (2.20) and (2.21) to the case of the $\text{SO}(2,1)$ Lie algebra, we obtain the following result for the Killing vectors:

$$\vec{k}_1 = e_0 \left[ -i \frac{1}{2} \left( 1 + (y^0)^2 - (\vec{y})^2 \right) \partial_0 - iy^0 \vec{y} \cdot \vec{\partial} \right],$$

$$\vec{k}_2 = e_0 \left[ \frac{1}{2} \left( 1 - (y^0)^2 + (\vec{y})^2 \right) \partial_0 - y^0 \vec{y} \cdot \vec{\partial} \right],$$

$$\vec{k}_3 = e_0 \left[ iy^0 \partial_0 + i \vec{y} \cdot \vec{\partial} \right].$$

(2.22)

Note also that the formula (2.21) for the Killing vector potential is symplectic invariant, so that any symplectic rotation of the section $\Omega$ does not affect the form of the Killing vector fields.

In the case where the compact part of the gauge group is just $\text{SO}(3)$, with Lie algebra normalized as follows:

$$[T_{x+3}, T_{y+z}] = e_1 \varepsilon_{xyz} T_{z+3}; \quad x, y, z, \ldots = 1, 2, 3,$$

(2.23)
and with $e_1$ denoting the associated coupling constant, the Killing vectors corresponding to these gauge group generators are

$$\vec{k}_{x+3} = e_1 \varepsilon_{xzw} y^z \frac{\partial}{\partial y^w}, \quad x, z, w = 1, 2, 3.$$  \hspace{1cm} (2.24)

de Roo – Wagemans angles. The second essential ingredient is the introduction of de Roo–Wagemans angles, which were introduced in $\mathcal{N} = 4$ supergravity in [20,38], and which parameterize a rotation of the relative embeddings of the $G_k$ groups inside $\text{Sp}(2(n + 2), \mathbb{R})$. These parameters are introduced through a symplectic non-perturbative rotation performed on the holomorphic section of the manifold prior to gauging. Different choices of the angles yield different gauged models with different physics. The de Roo–Wagemans rotation matrix has the following form:

$$\mathcal{R} = \begin{pmatrix} A & B \\ -B & A \end{pmatrix},$$

$$A = \begin{pmatrix} 1_3 & 0 & \ldots & 0 \\ 0 & \cos(\theta_1) & \bf{1}_{d_1} & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ldots & \cos(\theta_r) & \bf{1}_{d_r} \end{pmatrix},$$

$$B = \begin{pmatrix} 0 & 0 & \ldots & 0 \\ 0 & \sin(\theta_1) & \bf{1}_{d_1} & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ldots & \sin(\theta_r) & \bf{1}_{d_r} \end{pmatrix},$$ \hspace{1cm} (2.25)

the blocks being determined by the choice of the gauge group in (2.16). The symplectic section is rotated in the following way:

$$\Omega \rightarrow \Omega_R \equiv \mathcal{R} \cdot \Omega.$$ \hspace{1cm} (2.26)

The Kähler potential is clearly left invariant by the above transformation. In all calculations of the scalar potential, we have to use the symplectic section $\Omega_R$ rather than $\Omega$ and $V_R \equiv \exp[K] \Omega_R$ rather than $V$.

Fayet–Iliopoulos terms. Finally, the last ingredient we should introduce is the option of including also Fayet–Iliopoulos terms for the U(1) or the SO(3) factors that can appear in the compact part of the gauge group. This possibility is of crucial importance at the level of our analysis and is motivated by the following argument. In the absence of quaternionic scalars, the equivariance condition (2.14) for the triholomorphic moment maps reduces to

$$-\varepsilon^{xyz} \mathcal{P}_\Lambda^y \mathcal{P}_\Sigma^z = f^r_{\Lambda \Sigma} \mathcal{P}_r^x.$$ \hspace{1cm} (2.27)

In the case $G = \text{SO}(3)$, $f^r_{yz} = e \varepsilon_{xyz}$ ($e$ being the coupling constant) and the above condition is satisfied by setting

$$\mathcal{P}_\Sigma^x = \begin{cases} -e \delta^x_y & \text{for } \Sigma = 3 + y \\ 0 & \text{for } \Sigma = \text{otherwise.} \end{cases}$$ \hspace{1cm} (2.28)

\textsuperscript{3}We learned from S.J. Gates, Jr. that in the earlier paper [37] occur parameters $\varphi$ and $\theta_0$ in (3.1.1-2) and (3.2.1), respectively, which have the same effect.
For each abelian $U(1)$ generator $t_{\Lambda_\circ}$ included in the gauge algebra, (2.27) can instead be satisfied by setting:

$$P_\Sigma^x = \begin{cases} e\delta_3^x & \text{for } \Sigma = \Sigma_\circ \\ 0 & \text{for } \Sigma = \text{otherwise}. \end{cases}$$

In the context of conformal tensor calculus, the Fayet–Iliopoulos terms represent the transformation of the compensating hypermultiplet under a $U(1)$ or SO(3) gauge group [11].

### 2.4 General form of the scalar potential.

Having introduced the above ingredients, we can apply the general formula for the scalar potential of an $\mathcal{N} = 2$ supergravity that was derived in [34]. In order to write down such a formula, we still need to recall one more definition. Given the covariantly holomorphic section $V$ of special geometry (rotated in the style of de Roo–Wagemans or not), we name

$$f_\alpha^\Lambda \equiv (\partial_\alpha + \frac{1}{2}\partial_\alpha\mathcal{K}) L^\Lambda$$

the Kähler-covariant derivatives of the upper electric part and we introduce the positive-definite matrix

$$U^{\Lambda\Sigma} \equiv g^{\alpha\beta} f_\alpha^\Lambda f_\beta^\Sigma = -\frac{1}{2} (\text{Im}\mathcal{N})^{-1|\Lambda\Sigma} - L^\Lambda L^\Sigma,$$

where $\mathcal{N}_{\Lambda\Sigma}$ is the kinetic matrix of the vector fields.

With the normalization of the Lagrangian as

$$e^{-1} \mathcal{L} = g_{\alpha\beta} \nabla_\mu z^\alpha \nabla_\mu z^\beta + h_{uv} q^u \nabla^v q^u - V + \ldots,$$

the mass matrices for the special Kähler and quaternionic-Kähler parts are

$$(m^2)_\alpha^\beta = g^{\beta\beta} \partial_\alpha \partial_\beta V, \quad (m^2)_u^v = \frac{1}{2} h_{uv} \partial_u \partial_v V.$$

The scalar potential takes the form [11]

$$\mathcal{V} = \left( g_{\alpha\beta} k_\alpha^\Lambda k_\beta^\Sigma \bar{L}^\Lambda L^\Sigma \right) P^x_\Lambda P^x_\Sigma + (U^{\Lambda\Sigma} - 3\bar{L}^\Lambda L^\Sigma) P^x_\Lambda P^x_\Sigma,$$

and is the sum of three distinct contributions:

$$\mathcal{V}_1 = g_{\alpha\beta} k_\alpha^\Lambda k_\beta^\Sigma \bar{L}^\Lambda L^\Sigma = i \left( M_\Delta f_{\Lambda\Gamma} \bar{L}^\Lambda \right) \left( L^\Sigma f_{\Sigma\Pi} \Gamma \bar{L}^\Pi \right) + \text{h.c.},$$

$$\mathcal{V}_2 = 4 h_{uv} k_\alpha^\Lambda k_\beta^\Sigma \bar{L}^\Lambda L^\Sigma,$$

$$\mathcal{V}_3 = (U^{\Lambda\Sigma} - 3\bar{L}^\Lambda L^\Sigma) P^x_\Lambda P^x_\Sigma.$$

By their definition, the contributions $\mathcal{V}_{1,2}$ are positive definite and the only term that might involve negative contributions is $\mathcal{V}_3$. This can be understood from the fact that $\mathcal{V}_1$ and the first term constitute the square of the supersymmetry transformation of the gauginos (split in the SU(2) triplet and SU(2) singlet part), $\mathcal{V}_2$ is the square of the supersymmetry of the hyperinos, and the last term of $\mathcal{V}_3$ is the square of the gravitino supersymmetry. It is well known that the potential can be split in such a way, and that then only the gravitino contribution is negative definite. The term $\mathcal{V}_1$ differs from zero only if the gauge group is non-Abelian, as only then scalars of the vector
multiplets transform under $G_{\text{gauge}}$. Abelian factors are characterized by vanishing Killing vectors and do not contribute to $V_1$. In the term $V_2$, we have used $k^u_\Lambda$ that are the Killing vector fields describing the action of the gauge group on the quaternionic scalars pertaining to hypermultiplets. Hence $V_2$ is identically zero in the absence of hypermultiplets. Finally, the crucial term $V_3$ contains $\kappa^u_\Lambda$ that are the Killing vector fields describing the action of the gauge group on the quaternionic scalars. So $V_3$ describes contact interactions between the vector and the hyper scalars. Due to Fayet–Iliopoulos terms, the term $V_3$ can be non-zero also in the absence of hypers and then takes contributions only if we have SO(3) or U(1) factors.

2.5 Abelian gauging in special geometry lead to tachyonic vacua

To illustrate the difficulties for having stable de Sitter vacua, we give the eigenvectors that were found in [11, 14], having negative masses related to the value (1.2) as recently found in higher $N$ supergravities in [28].

For Abelian gaugings involving only vector multiplets, the relevant quantities are the contributions to the Fayet–Iliopoulos terms for each generator, as in (2.29), i.e.,

$$P^x_\Lambda = g_{\Lambda} \delta^x_3.$$  \hfill (2.36)

The potential again gets only contributions from $V_3$ in (2.35), and can now be rewritten as

$$V = g^{\alpha \bar{\alpha}} D_\alpha W D_{\bar{\alpha}} W - 3 |W|^2, \quad W \equiv g_\Lambda L^\Lambda. \hfill (2.37)$$

The basic relations of special geometry imply

$$D_\alpha D_\beta L^\Lambda = g_{\alpha \beta} L^\Lambda, \quad D_\alpha D_\beta L^\Lambda = C_{\alpha \beta \gamma} D^\gamma L^\Lambda,$$  \hfill (2.38)

where $C_{\alpha \beta \gamma}$ is the covariantly holomorphic symmetric 3-index tensor appearing in the fundamental curvature relation:

$$R_{\alpha \beta \gamma \delta} = 2 g_{\gamma \delta} g_{\bar{\alpha} \bar{\beta}} + 2 g_{\gamma \delta} g_{\alpha \beta} - 2 C_{\alpha \gamma \xi} C_{\beta \delta \xi} g^{\xi \bar{\epsilon}}. \hfill (2.39)$$

Applying (2.38) leads, as condition for extrema of the potential, to

$$\partial_\alpha V = C_{\alpha \beta \gamma} D^\beta W D^\gamma \bar{W} - 2 W D_\alpha W = 0. \hfill (2.40)$$

There are first the solutions with $D_\alpha W = 0$, which lead to the anti-de Sitter vacua. In other cases, we can use

$$2 \bar{W} |D_\alpha W|^2 = C_{\alpha \beta \gamma} D^\alpha \bar{W} D^\beta \bar{W} D^\gamma \bar{W} \hfill (2.41)$$

as expression for $W$ at the extremum. This allows us to write the vacuum value of the potential as

$$V = |D_\alpha W|^2 - 3 \frac{|C_{\alpha \beta \gamma} D^\alpha \bar{W} D^\beta \bar{W} D^\gamma \bar{W}|^2}{|D_\alpha W|^4}. \hfill (2.42)$$

The second derivative of the potential is for the holomorphic–holomorphic and the holomorphic–antiholomorphic parts

$$D_\alpha \partial_\beta V = (D_\alpha C_{\beta \gamma \delta}) D^\gamma \bar{W} D^\delta \bar{W},$$

$$\partial^\beta \partial_\alpha V = 2 C_{\alpha \gamma \delta} \bar{C}^{\beta \gamma \epsilon} D_\epsilon W D^\delta \bar{W} - 2 D^\beta \bar{W} D_\alpha W - 2 \delta^3_\alpha |W|^2. \hfill (2.43)$$
We get then for the trace with (2.41)\footnote{We use here $n$ for the number of vector multiplets. The remainder of the paper uses specifically the manifolds (2.1), which defines the meaning of $n$, and where the number of vector multiplets is, therefore, $n + 1$.}

\[ g^{\alpha\beta} \partial_\beta \partial_\alpha V = -2|D\alpha W|^2 - \frac{n}{2} \left( \frac{C_{\alpha\beta\gamma} D^\alpha \bar{W} D^\beta \bar{W} D^\gamma \bar{W}}{|D\alpha W|^4} \right)^2 + 2C_{\alpha\gamma\delta} \bar{C}^\alpha\gamma\epsilon \partial_\epsilon W D^\delta \bar{W} \]

\[ = -2 \left( \frac{1}{4} V_0 \right) - \frac{3 + n}{2} \left( \frac{C_{\alpha\beta\gamma} D^\alpha \bar{W} D^\beta \bar{W} D^\gamma \bar{W}}{|D\alpha W|^4} \right)^2 + 2C_{\alpha\gamma\delta} \bar{C}^\alpha\gamma\epsilon \partial_\epsilon W D^\delta \bar{W}. \] (2.44)

Note that for $n = 1$ (and thus all indices are the same), the last two terms cancel. This implies that the trace of the eigenvalues of the mass matrix is $-2$ in that case. It is a complex scalar; thus, there are two eigenvalues. The separate eigenvalues are dependent on the actual value of $C$. This is actually (6.22) of \cite{11}.

The result of \cite{14} that there is always an eigenvalue $-2V$ in the holomorphic–antiholomorphic derivative, is derived as follows. One multiplies the second equation of (2.43) with $D_\beta W$ and uses (2.40) and its complex conjugate. This leads to

\[ \partial_\beta \partial_\alpha V D_\beta W = -2V D_\alpha W. \] (2.45)

This shows that for Abelian gaugings with only vector multiplets, stable de Sitter vacua cannot exist, because we always have a complex tachyon with characteristic negative mass as in (2.45). Henceforth, the possibility of finding stable de Sitter vacua relies on the contribution $V_1$ coming from non-Abelian gaugings. In the following section, we prove that this can be sufficient to produce the desired result.

3 The three models

The three models that we are going to present are

- a model with 3 vector multiplets, in the manifold $\mathcal{ST}[2, 2]$, which, together with the graviphoton, are gauging $\text{SO}(2, 1) \times \text{U}(1)$, with a Fayet–Iliopoulos term for the U(1).
- a model with 5 vector multiplets, in the manifold $\mathcal{ST}[2, 4]$, which, together with the graviphoton, are gauging $\text{SO}(2, 1) \times \text{SO}(3)$, with a Fayet–Iliopoulos term for the SO(3); and
- the last model extended with 2 hypermultiplets with 8 real scalars in the coset $\frac{\text{SO}(4, 2)}{\text{SO}(4) \times \text{SO}(2)}$.

In the choice of the hypermultiplet sector, we made use of the fact that we can use the coset $\frac{\text{SO}(4, 2)}{\text{SO}(4) \times \text{SO}(2)}$ as well as a factor in the special Kähler manifold $\mathcal{ST}[2, 4]$ as for a quaternionic-Kähler manifold. Moreover, as we will discuss in the last section, such a choice makes a first step towards a generalization to $\mathcal{N} = 4$ supergravity and string theory. We will now discuss the three models consecutively.

3.1 $\text{SO}(2, 1) \times \text{U}(1)$ gauging

In this case, the Cartan–Killing metric on the group manifold, $\eta_{(2, 2)} = \text{diag}(+, +, -, -)$, naturally splits into the Cartan–Killing metric of the first non-Abelian non-compact factor, namely $\eta_{(2, 1)} = \text{diag}(+, +, -)$.
diag(+, +, −) of SO(2,1) plus η(1) = diag(−). In general, we denote by e₀ and e₁ the coupling constants of the non-compact and compact factors, respectively. Since U(1) is Abelian, e₁, rather than a coupling constant, is actually just the value of the Fayet–Iliopoulos parameter. The first three of the four Killing vectors kΛ corresponding to the generators of the gauge group SO(2,1) are given in (2.22), while the fourth one is zero. The scalar potential has the form

$$\mathcal{V}(S, \bar{S}, y, \bar{y}) = g_{\alpha \beta} k^\alpha_x k^\beta_y L^x L^y + \mathcal{V}_3,$$

where

$$\mathcal{V}_3 = (U^{\Lambda \Sigma} - 3L^\Lambda L^\Sigma) p^{x} p^{x}_{\Sigma} = e_1^2 (U^{44} - 3L^4 L^4).$$

In the first term of (3.1), only the terms with x = 1, 2, 3 contribute, due to the vanishing of the U(1) Killing vector. In the Fayet–Iliopoulos term of (3.2) instead, the only contribution comes from Λ = Σ = 4. This is so, because, for an Abelian group the constant moment map is

$$p^{x}_{\Sigma} = e_1 \delta^x_3 \delta_4 \delta_4.$$

Without de Roo–Wagemans rotation (a₁ = 0), the matrix appearing in the \(\mathcal{V}_3\) would take the simple form (see (9.58) of [34])

$$U^{\Lambda \Sigma} - 3L^\Lambda L^\Sigma = \frac{1}{2 \text{Im} S} \eta^{\Lambda \Sigma}_{(2, 2)},$$

and therefore the \(\mathcal{V}_3\) term would be

$$\mathcal{V}_3 = e_1^2 (U^{44} - 3L^4 L^4) = \frac{1}{2 \text{Im} S} e_1^2 \eta^{44}_{(2, 2)} = - \frac{1}{2 \text{Im} S} e_1^2 > 0,$$

where we have used the property \(\text{Im}(S) < 0\) required by \(\text{Im}(N) < 0\). This potential has obviously no extremum, and here the de Roo–Wagemans rotation (2.25) (with \(\theta = \theta_1\)) comes to rescue. Indeed, the effect of such a rotation amounts to a modular transformation of \(S\), so that

$$\mathcal{V}_3 = -e_1^2 \frac{1}{2 \text{Im} S} |\cos(\theta) - S \sin(\theta)|^2 > 0.$$

Let us now study the critical point of this potential. Explicitly, we obtain the following form:

$$\mathcal{V}_{SO(2,1) \times U(1)} = \mathcal{V}_3 + \mathcal{V}_1 = - \frac{1}{2 \text{Im} S} \left( e_1^2 |\cos \theta - S \sin \theta|^2 + e_0^2 \frac{P_2^+(y)}{P_2^-(y)} \right),$$

where \(P_2^+(y)\) are polynomial functions in the Calabi–Vesentini variables of the holomorphic degree specified by their index[^5] (here only 2, but will be higher in the next example)

$$P_2^\pm(y) = 1 - 2 y_0 y_0 \mp 2 y_1 y_1 + y_2^2 y^2.$$

The last term in (3.7) is another positive definite term that originates from the non-Abelian non-compact SO(2,1) gauging. Indeed, this term is just the norm of the Killing vectors.

We now look for an extremum. Equating to zero the \(S\) derivative, we obtain, consistent with the positivity of the kinetic term of the vector fields,

$$S = S_0 = \cot \theta - i \frac{e_0}{e_1 \sin \theta} \sqrt{\frac{P_2^+(y)}{P_2^-(y)}},$$

[^5]: We write from now on explicit components of the \(y^a\) variables with lower indices to distinguish them from squares of sums, . . . .
where we assumed that $e_0 e_1 > 0$. Inserting this extremum value of $S$ in the potential (3.7), we obtain

$$V_{\text{SO}(2,1)\times U(1)}|_{S=S_0} = e_0 e_1 \sin \theta \left( \frac{P_2^+(y)}{P_2^-(y)} \right) = (- \text{Im} S_0) e_1^2 \sin^2 \theta .$$

(3.10)

At the level of the above equation, where the extremum is only considered in the $S$-direction, we already reach the very relevant conclusion that the potential is strictly positive in the positivity domain of the Lagrangian ($\text{Im} S < 0$) and might vanish only at the boundary of moduli space. Therefore, we can have only de Sitter and no Minkowski vacua. Setting $y_0 = w \exp i \beta$ and $y_1 = \rho \exp i (\beta + \delta)$, the polynomials are

$$P_2^\pm = -2w^2 + w^4 + (\pm 1 + \rho^2)^2 + 2w^2 \rho^2 \cos 2\delta .$$

(3.11)

The resulting potential is illustrated in figure 1. It displays a valley along the direction of $w$, for $\rho = 0$. Note that for $\rho = 0$, we have $L^\Lambda P_2^\Lambda = 0$, which means that the gravitino shift (transformation of the gravitino under supersymmetry due to the scalars) is zero, as we will further discuss in the next model. This will, of course, also lead to a zero mode in the mass matrix.

![Figure 1: Potential (3.10) in units of $|e_0 e_1 \sin \theta|$ for $\delta = \pi/10$. For different values of $\delta$, the picture maintains the same shape.](image)

The critical values of the $y$ fields are

$$y_0 = \text{arbitrary} , \quad y_1 = 0 , \quad \text{i.e.} \quad \rho = 0 .$$

(3.12)

Therefore, the extremum value of the potential is at

$$V = V_0 = e_0 e_1 \sin \theta > 0 .$$

(3.13)
The matrix of second derivatives of the potential at the extremum, normalized using the inverse metric
\[ g^{S\bar{S}} = 4(\text{Im } S)^2, \quad g^{ab}\big|_0 = \frac{1}{2} \delta^{ab}(1 - w^2)^2, \] (3.14)
is the mass matrix
\[ \frac{\partial_a \partial^b \mathcal{V}}{\mathcal{V}} \big|_0 = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \partial_a \partial_\beta \mathcal{V} \big|_0 = 0. \] (3.15)
This displays two (complex) positive and one (complex) null eigenvalue. The zero modes are the two Goldstone bosons of the non-compact translations that become massive and are also the moduli of the flat direction displayed by the potential. We will explicitly show this in the next model, which has the same essential properties as the present toy model.

Thus, we find a stable de Sitter vacuum, with characteristic squared mass values for the scalars.

### 3.2 $SO(2, 1) \times SO(3)$ gauging

Let us now work out in detail the case of $\mathcal{N} = 2$ supergravity coupled to 5 vector multiplets whose scalar components span the manifold $ST[2, 4]$ choosing as gauge group $G_{\text{gauge}} = SO(2, 1) \times SO(3)$ and including a FI term associated with the second factor. In this case, the metric on the vector field representation $\eta_{(2,4)} = \text{diag}(+, +, -, -, -)$ naturally splits into the Cartan–Killing metrics of the two factor groups, namely $\eta_{(2,1)} = \text{diag}(+, +, -)$ and $\eta_{(3)} = \text{diag}(-, -, -)$. We denote by $e_0$ and $e_1$ the coupling constants of the non-compact and compact factor, respectively. The Killing vectors $k_\Lambda$ corresponding to the generators of the gauge group are therefore given by (2.22) and (2.24).

Due to the absence of hypermultiplets, $\mathcal{V}_2$ is still not contributing to the potential. The $\mathcal{V}_3$ term, instead, is
\[ \mathcal{V}_3 = (U^{\Lambda \Sigma} - 3L^\Lambda L^\Sigma) P_\Lambda^x P_\Sigma^x = e_1^2 \sum_{x=1}^3 \left(U^{(x+3)(x+3)} - 3L^{x+3}L^{x+3}\right) \]
\[ = \frac{1}{2\text{Im } S} e_1^2 |\cos \theta - S \sin \theta|^2 \sum_{x=1}^3 \eta^{(x+3)(x+3)} = -\frac{3}{2\text{Im } S} e_1^2 |\cos \theta - S \sin \theta|^2 > 0. \] (3.16)
As one notices, the only difference between the Abelian and non-Abelian case in the Fayet–Iliopoulos term is a factor 3. Each generator of SO(3) gives the same contribution as the $U(1)$ generator in the Abelian case.

Also in this case, the potential is positive definite since both $\mathcal{V}_1$ and $\mathcal{V}_3$ are positive definite. Their sum gives
\[ \mathcal{V} = -\frac{1}{2\text{Im } S} \left[ e_1^2 |\cos \theta - S \sin \theta|^2 \frac{P_4^{(1)}}{P_2^2} + e_0^2 \frac{P_4^{(0)}}{P_2^2} \right]. \] (3.17)
where $P_\ell(y, \bar{y})$ ($\ell = 2, 4$) are polynomials of holomorphic degree $\ell$ in $y$, whose important properties are
\[ P_2 = 1 - 2y\bar{y} + y^2\bar{y}^2, \]
\[ \partial_{y^a} P_\ell|_{y=0} = 0, \quad P_4^{(0)}|_{y=0} = 1, \quad P_4^{(1)}|_{y=0} = 3. \] (3.18)
Setting to zero the $S$-derivative of the scalar potential in (3.17), we obtain the critical value of $S$ in terms of the $y$ fields

$$S = S_0(y) = \cot \theta - i \frac{e_0}{e_1 \sin \theta} \left[ \frac{P_4^{(0)}(y)}{P_4^{(1)}(y)} \right].$$

(3.19)

Inserting this in the potential, reduces it to

$$V|_{S=S_0} = |e_0 e_1 \sin \theta| \sqrt{\frac{P_4^{(0)}(y) P_4^{(1)}(y)}{P_2^2(y)}}.$$  

(3.20)

With the properties (3.18), we conclude that the potential reaches an extremum at

$$z = \phi^{(0)} = \{S_0(0), y_0 = 0\}, \quad S_0(0) = \cot \theta - i \frac{1}{\sqrt{3}} \frac{e_0}{e_1 \sin \theta}. \quad (3.21)$$

At this extremum, the potential has the value

$$V_0 = V(\phi^{(0)}) = \sqrt{3} |e_0 e_1 \sin \theta| > 0.$$  

(3.22)

Hence this extremum defines again a de Sitter space.

**More detailed analysis of the potential.** The de Sitter vacuum that we displayed is the only possibility in the positivity domain of the Lagrangian. Just as in the first example, the extremum we have found is actually a point on a full line of extrema. This can be seen in 2 ways. Either by calculating the mass matrix and showing that it involves a zero mode, or, alternatively, the presence of a flat direction can be appreciated through a more detailed analysis of the potential. This involves a closer look at the structure of the polynomial functions $P_4^{(0,1)}(y, \bar{y})$ and $P_2(y, \bar{y})$ appearing in the final form (3.17) of the potential. This is what we do in this paragraph.

Since the potential is SO(3) invariant the best choice of variables are SO(3) invariants. We set

$$y_0 = w e^{i\beta}, \quad \bar{y} = \bar{v}_1 + i \bar{v}_2,$$

(3.23)

and expect that the polynomial functions entering the potential should depend only on

$$|\mathbf{v}_1|^2 \equiv \rho^2 \cos \phi, \quad |\mathbf{v}_2|^2 \equiv \rho^2 \sin \phi, \quad \mathbf{v}_1 \cdot \mathbf{v}_2 \equiv \rho^2 \sin \phi \cos \phi \cos \theta.$$  

(3.24)

Indeed this is what happens, and by means of a Mathematica program, one finds an explicit form of the $P_4$ polynomials, which is too lengthy to display here in full generality. Inserting the value (3.19) of the $S$ field into the derivatives of the potential $V$ with respect to $y^n$, the following equation should hold:

$$0 = F_a \equiv P_2 P_4^{(1)} \partial_a P_4^{(0)} + P_4^{(0)} \left( -4 P_4^{(1)} \partial_a P_2 + P_2 \partial_a P_4^{(1)} \right).$$

(3.25)

An SO(3)-invariant vacuum should occur at $\bar{y} = 0$ (i.e. $\rho = 0$). For this choice, the polynomials simplify dramatically and we obtain

$$P_0|_{\rho=0} = (-1 + w^2)^2, \quad P_4^{(0)}|_{\rho=0} = (-1 + w^2)^4, \quad P_4^{(1)}|_{\rho=0} = 3 (-1 + w^2)^4.$$  

(3.26)

Inserting this in (3.20), the potential becomes the constant $V_0$ of (3.22) independent of $w$ and $\beta$. In other words, we have an extremum for arbitrary values of $y_0$ as claimed.
The mass matrix. The factors of 3 and $\sqrt{3}$ that in this model are extra with respect to the first model, disappear in the final mass matrix, and we find

$$\frac{\partial_{\alpha} \partial^2 \mathcal{V}}{\mathcal{V}} \bigg|_0 = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1_3 \end{pmatrix}, \quad \partial_{\alpha} \partial_{\beta} \mathcal{V} \bigg|_0 = 0. \tag{3.27}$$

We thus find the same conclusions as in the previous model, and the shape of the potential is also similar to the one in figure 1.

Supersymmetry breaking. Let us now investigate the supersymmetry of the solution. The supersymmetry variations of the gravitinos and gauginos are

$$\delta \psi_{i\mu} = i S_{ij} \gamma^j \epsilon^j, \quad \delta \lambda^{i\alpha} = W^{i\alpha j} \epsilon^j, \tag{3.28}$$

where we have

$$S_{ij} = -\frac{1}{2} i \mathcal{P}_{\Lambda ij} \mathcal{L}^\Lambda, \quad W^{i\alpha j} = \varepsilon^{ij} k_\alpha^\Lambda \bar{L}^\Lambda - i \mathcal{P}_\Lambda^{ij} \mathcal{G}^{i\beta}, \quad \mathcal{P}_{\Lambda ij} \equiv (\sigma_x)^i_k \varepsilon_{kj} \mathcal{P}_\Lambda^k, \quad \mathcal{P}_\Lambda^{ij} \equiv (\mathcal{P}_{\Lambda ij})^* \tag{3.29}$$

Since at the extremum ($\bar{y} = 0$) we have $L^{x+3} = 0$, we find that the gravitino mass matrix $S_{ij}$ vanishes there. The gaugino shifts, instead, are

$$W^{S}^{i\alpha j} = 0, \quad W^{0}^{i\alpha j} = -\frac{e_0}{2 \sqrt{-\text{Im} S_0(0)}} \varepsilon^{ij}, \quad W^{x}^{i\alpha j} = -i \frac{e_1}{2 \sqrt{-\text{Im} S_0(0)}} \left( \cos \theta - S \sin \theta \right) (\sigma_x)^{ij}, \tag{3.30}$$

where the indices $S$, 0 and $x$ enumerate the basis of scalar fields $z^\alpha = \{ S, y_0, y_x \}$. Since the gaugino shift matrices (3.30) do not have any common zero eigenvalue, supersymmetry is broken to $\mathcal{N} = 0$.

Vector fields and BEH effect. The imaginary part of the period matrix has the form

$$\mathcal{N} - \bar{\mathcal{N}} = 2 i \text{Im} S \left( M_3 \begin{pmatrix} 0 & 0 \\ 0 & 1/\sin \theta \sin \theta \end{pmatrix} \right) \begin{pmatrix} 1_3 \end{pmatrix} \tag{3.31}$$

where the $3 \times 3$ matrix $M_3$ has been calculated by Mathematica and has all non-zero entries for $y_0 \neq 0$. Although we can calculate the effective vector field Lagrangian in a generic extremum $y_0 \neq 0$ by diagonalizing $M_3$, for the sake of simplicity we just consider $y_0 = 0$ where $M_3 = 1_3$. Then we can show that the two gauge vectors associated with the non-compact generators $T_{A=1,2}$ of $\text{SO}(2,1)$ acquire dynamical masses consistently with the Goldstone theorem. Indeed, these two generators are broken by our vacuum since by looking at the Killing vectors (2.22) we see that $y_0$ is not invariant under these two generators, and we rather have

$$\delta_{A=1} y_0 = -\frac{1}{2} i e_0, \quad \delta_{A=2} y_0 = \frac{1}{2} e_0. \tag{3.32}$$

Hence, the residual symmetry of all the de Sitter extrema that we have constructed is $\text{SO}(2) \times \text{SO}(3)$, where $\text{SO}(2)$ is the compact part of $\text{SO}(2,1)$. The masses of the vector fields arise from the kinetic...
term for the scalar fields $\nabla_\mu z^\alpha \nabla_\mu z^\beta g_{\alpha \beta}$ where $\nabla_\mu z^\alpha = \partial_\mu z^\alpha + A^A_\mu k_A^\alpha$. The mass term at the extremum $\phi(0)$ is

$$A^A_\mu A^{\Sigma \mu}(k_A^\alpha k_\Sigma^\beta g_{\alpha \beta})_{z=\phi(0)} = \frac{1}{2}e_0^2 \left( A^1_\mu A^{1|\mu} + A^2_\mu A^{2|\mu} \right), \quad (3.33)$$

where we used the extremum value $g_{ab} = 2\delta_{ab}$. Hence, the effective Lagrangian for the massive vector fields $A^0_\mu$ reads

$$\text{Im} S_0(0) \left( F^1_\mu F^{1|\mu} + F^2_\mu F^{2|\mu} \right) + \frac{1}{2}e_0^2 \left( A^1_\mu A^{1|\mu} + A^2_\mu A^{2|\mu} \right)$$

$$= -\frac{1}{4} \left( \tilde{F}^1_\mu \tilde{F}^{1|\mu} + \tilde{F}^2_\mu \tilde{F}^{2|\mu} \right) + \frac{1}{2}e_0^2 \left( \tilde{A}^1_\mu \tilde{A}^{1|\mu} + \tilde{A}^2_\mu \tilde{A}^{2|\mu} \right),$$

$$\mu^2 \equiv -\frac{e_0^2}{4 \text{Im} S_0(0)} = \frac{\sqrt{3} |\sin \theta| e_0 e_1}{4}, \quad (3.34)$$

where we have redefined $\tilde{A}_\mu = 2 \sqrt{-\text{Im} S_0(0)} A_\mu$. The kinetic term of the Goldstone boson $y_0$ can be absorbed by a gauge transformation on the broken gauge vector fields

$$A^1_\mu \rightarrow A^1_\mu + \frac{2}{e_0} \partial_\mu \text{Im} y_0, \quad A^2_\mu \rightarrow A^2_\mu - \frac{2}{e_0} \partial_\mu \text{Re} y_0. \quad (3.35)$$

In a vacuum where $y_0 \neq 0$, the three vectors $A^\Lambda_{\mu=0,1,2}$ are mixed together. Two linear combinations become massive while there is always one that remains massless.

### 3.3 SO(2,1) × SO(3) gauging with hypers

Let us now generalize our previous results to an $\mathcal{N} = 2$ model with 5 vector multiplets and 4 hypermultiplets. We shall consider the scalar fields in the vector- and hypermultiplets $S, y^a, q^u$ (with $a = 0, 1, 2, 3$ and $u = 1, \ldots, 8$) spanning the following product space:

$$\mathcal{M}_{\text{scal}} = \left[ \frac{\text{SU}(1,1)}{U(1)} \times \frac{\text{SO}(2,4)}{\text{SO}(2) \times \text{SO}(4)} \right] \times \left[ \frac{\text{SO}(4,2)}{\text{SO}(4) \times \text{SO}(2)} \right]. \quad (3.36)$$

The gauge group of our model is $G = \text{SO}(2,1) \times \text{SO}(3)$ which is embedded in the SO(2,4) subgroup of both the isometry group of the special Kähler and of the quaternionic manifold. We can choose for each factor whether this isometry is actually gauged (coupled to the vector multiplets) or not. As we had chosen $e_0, e_1$ as the coupling constants of SO(2,1) and SO(3), respectively, we now couple the quaternionic scalars with factors $r_0 e_0, r_1 e_1$ where $r_0$ and $r_1$ can be 0 or 1, indicating whether the corresponding isometry is gauged or not. As we will see, we need the coupling to SO(3), as this is replacing the FI term in the previous models, while for $r_0$ we can consider both choices $r_0 = 0$ and $r_0 = 1$.

It is convenient to choose the solvable parametrization of the quaternionic manifold [39,40]

$$\frac{\text{SO}(4,2)}{\text{SO}(4) \times \text{SO}(2)} \equiv \exp(\text{Solv})$$

$$\text{Solv} = \sum_{u=1}^{8} q^u T_u = a_1 E_{\epsilon_1 - \epsilon_2} + a_2 E_{\epsilon_1 + \epsilon_2} + \frac{a_3}{\sqrt{2}} (E_{\epsilon_1 + \epsilon_3} + E_{\epsilon_1 - \epsilon_3}) + i \frac{a_4}{\sqrt{2}} (E_{\epsilon_1 + \epsilon_3} - E_{\epsilon_1 - \epsilon_3}) +$$

$$+ \frac{b_1}{\sqrt{2}} (E_{\epsilon_2 + \epsilon_3} + E_{\epsilon_2 - \epsilon_3}) + i \frac{b_2}{\sqrt{2}} (E_{\epsilon_2 + \epsilon_3} - E_{\epsilon_2 - \epsilon_3}) + h_1 H_{\epsilon_1} + h_2 H_{\epsilon_2}. \quad (3.37)$$
We started from the $6 \times 6$ matrices in $\text{SO}(4, 2)$, using as Cartan subalgebra the matrices

$$
H_{\epsilon_1} = M^{35}, \quad H_{\epsilon_2} = M^{46}, \quad iH_{\epsilon_3} = M^{12}, \quad \text{where} \quad (M^{AB}) C_D = 2\eta^{C[A}[\delta_D^B],
$$

(3.38)

with $A = 1, \ldots, 6$ and the invariant metric of $\text{SO}(4, 2)$ is

$$
\eta = \eta^{(4,2)} = \text{diag}(+, +, +, +, -, -).
$$

The $E$-matrices in (3.37) are the $\text{SO}(4, 2)$ roots. The real form is such that the $\epsilon_3$ direction is imaginary, while the $\epsilon_1$ and $\epsilon_2$ are real. The generators that appear in (3.37) form the solvable algebra corresponding to this coset. These are generators with non-negative $\epsilon_1$-weight. The scalars with positive $\epsilon_1$-weight (the scalars $a_i$) are Peccei–Quinn scalars, which will not enter the metric $h_{uv}$.

The coset representative is defined as follows:

$$
\mathbb{L} = \sum_{i=1}^4 a_i T_i + \sum_{k=1}^2 b_k T_{k+4} + \sum_{k=1}^6 h_k H_k.
$$

(3.39)

It satisfies $\mathbb{L}^{-1} = \eta^{\mathbb{L} T}\eta$ and its explicit expression is given in [A.1]. This defines the vielbein 1-form, which was the fundamental quantity in section 2.2, as

$$
V^{mt} = (P_4 L^{-1} dL P_2)^{mt},
$$

(3.40)

where $P_4$ and $P_2$ are the projection matrices splitting the range $A = 1, \ldots, 6$ in $m = 1, \ldots, 4$ and 5,6 being re-labelled as $t = 1, 2$:

$$
P_4 = \begin{pmatrix} 1_4 & 0 \\ 0 & 0 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1_2 \end{pmatrix}.
$$

(3.41)

The expression is given explicitly in [A.4], and the corresponding metric, defined by (2.9), is given in (A.6). Observe that at the base point, where all the coordinates $q^u = 0$, the metric is just $h_{uv} = \frac{1}{2}\delta_{uv}$. Also the SU(2) curvature is defined from the vielbein using (2.12), and the connection 1-form in that equation can be written as

$$
\omega^x = \frac{1}{2} \varepsilon^{xy}_{\ m} n \left( P_4 L^{-1} dL P_4 \right)_{n} m.
$$

(3.42)

The isometries that are gauged by the vectors in the special Kähler sector, i.e., $\text{SO}(2, 1) \times \text{SO}(3)$, are acting on the 6 of $\text{SO}(4, 2)$. Due to the structure of the $\eta^{(4,2)}$, the SO(3) generators are chosen to act on the first three components, while the SO(2,1) act on the last three. See the explicit form of $t_\Lambda$ in [A.8]. The Killing vectors are then expressed as

$$
k^u_\Lambda V^{mt} = (P_4 L^{-1} t_\Lambda L P_2)^{mt}.
$$

(3.43)

The corresponding tri-holomorphic momentum maps, see (2.13), have the following form:

$$
P^x_\Lambda = \frac{1}{2} \varepsilon^{xy}_{\ m} n \left( P_4 L^{-1} t_\Lambda L P_4 \right)_{n} m.
$$

(3.44)

The explicit expressions of the Killing vectors and moment maps are quite complicated in this parametrization, but are simpler in another formalism.

Indeed, an alternative parametrization of the quaternionic coset manifold consists in using the analogue $\tilde{y}^a$ of the Calabi–Vesentini coordinates $y^a$ used to describe the $\text{SO}(2, 4)/\text{SO}(2) \times \text{SO}(4)$ factor in the special Kähler manifold. The relation between the $\tilde{y}^a$ coordinates and the solvable ones

\footnote{To check these expressions and the ones below, it is useful to remark that $\varepsilon^{xyz}_{\ m} n \varepsilon^{yp}_{\ q} = 2 \left[ \delta_{m[p} \varepsilon^{yz}_{\ q]} n - (m \leftrightarrow n) \right]$.}
is highly nonlinear and is formally discussed in appendix (A.2). Near the origin of the quaternionic manifold, the relation between the corresponding fluctuations can be linearized and will be used in the sequel for writing the mass eigenstates in two relevant cases. Some quantities are most conveniently expressed in one set of coordinates, the others in the other set. For instance, in terms of \( \tilde{y}^a \) the Killing vectors of the gauge group have the same expression as those acting on the special Kähler manifold provided one performs the obvious substitutions \( y^a \to \tilde{y}^a, e_0 \to r_0 e_0 \) and \( e_1 \to r_1 e_1 \). However, for the sake of constructing the scalar potential, for which one needs an explicit expression of the coset representative to work with, we find the solvable parametrization more convenient.

Now we have the ingredients to compute the scalar potential \( V \) given by (2.34). Although its expression is rather complicated, one can immediately verify that
\[
\partial_y V|_{y^a=q^a=0} = 0,
\]
\[
\partial_{\bar{q}} V|_{y^a=q^a=0} = 0,
\]
\[
V|_{y^a=q^a=0} = -\frac{1}{2\text{Im} S} \left[ 3 r_1^2 e_1^2 \left( \cos \theta - S \sin \theta \right)^2 + e_0^2 (1 + 2 r_0^2) \right].
\]  

The expression of the potential at the origin of the quaternionic manifold is the same as the one found in the previous models, see (3.7) and (3.17) at \( y = 0 \), except that if the SO(2,1) isometry is gauged, there is a rescaling in the SO(2,1) coupling constant: \( e_0^2 \to 3 e_0^2 \).

A main ingredient is the triholomorphic moment map. This is related to the complex structures acting in the upper 4 \( \times \) 4 part of the SO(4,2) matrices, see (3.44). The generators of SO(2,1) within the isometry group of the quaternionic manifold are zero in this upper 4 \( \times \) 4 block according to (A.8). On the other hand, those of SO(3) are chosen in this block. At the origin, where \( L = 1 \), it is seen immediately from comparing (2.11) and (A.8) that
\[
P_\lambda^{x=1,2,3}|_{q^a=0} = 0, \quad P_\lambda^{x=3+y}|_{q^a=0} = -r_1 e_1 \delta_y^x.
\]  

The above property allows us to obtain at \( q^a \equiv 0 \) the analogue of the SO(3) FI term, which in the model without hypermultiplets was introduced by hand. Here, this term appears if the SO(3) part of the isometry of the hypermultiplet manifold is gauged, i.e., \( r_1 = 1 \), and is again proportional to the SO(3) charge \( e_1 \).

The extremum of \( V \) corresponds to the point
\[
\phi^{(0)} = \left\{ \begin{array}{ll}
    y^a \equiv 0, \\
    q^a \equiv 0, \\
    S = S^{(0)} = \cot \theta - i \left[ \frac{e_0 \sqrt{1 + 2 r_0^2}}{\sqrt{3} r_1 e_1 \sin (\theta)} \right], \\
    \end{array} \right.
\]
\[
V|_0 = |3 (1 + 2 r_0^2) r_1 e_1 e_0 \sin (\theta)| > 0.
\]  

We see that \( r_1 = 0 \) leads to a singular value for \( S^{(0)} \), which corresponds to the remarks in the previous models that one needs a FI term. Thus we further restrict to \( r_1 = 1 \).

From the above analysis we may conclude that the potential has a dS critical point which is placed at the origin of the quaternionic manifold. Another property of the critical point \( \phi^{(0)} \) is that
\[
L^A P_A^{\bar{x}} = 0,
\]  

where we have used (3.46) and that \( L^A \propto \{1/2, i/2, 0, 0, 0, 0\} \) at \( y^a = 0 \).
Let us now compute the mass matrix for the scalar fields. This matrix is most easily expressed with respect to the fluctuation of the scalar fields around the vacuum \( (3.47) \), using for the quaternionic manifold the parametrization in terms of the \( \tilde{y}^a \) coordinates. The linear relation between the fluctuations \( \{\delta\tilde{y}^a\} \) and \( \{\delta a_i, \delta b_1, \delta b_2, \delta h_1, \delta h_2\} \) around the origin of the quaternionic manifold can be derived from the formal relations given in the appendix and are

\[
\begin{align*}
\delta \tilde{y}_0 &= \frac{i}{2 \sqrt{2}} (a_1 - a_2) + \frac{1}{2} h_2, \quad \delta \tilde{y}_1 = \frac{1}{2 \sqrt{2}} (a_1 + a_2) + \frac{i}{2} h_1, \\
\delta \tilde{y}_2 &= \frac{1}{2} (b_1 + i a_3), \quad \delta \tilde{y}_3 = \frac{1}{2} (b_2 + i a_4).
\end{align*}
\] (3.49)

We now split into two cases whether the SO(2, 1) part of the isometry group is gauged \( (r_0 = 1) \) or not \( (r_0 = 0) \).

\( r_0 = 0 \) : In this case, the scalar fluctuations in the special Kähler manifold and those in the quaternionic manifold do not mix. The eigenvalues of the mass matrix are

\[
\begin{align*}
\delta y_{0} &= \begin{pmatrix}
\frac{\partial \phi^{\alpha \beta}}{\partial y^\alpha} \\
\frac{\partial \phi^{\alpha \beta}}{\partial y^\beta}
\end{pmatrix} |_{0} = \begin{pmatrix}
2 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & \mathbf{1}_{3}
\end{pmatrix}; \quad \alpha, \beta \text{ over complex scalars in vector multiplets}
\end{align*}
\] (3.50)

Besides the complex Goldstone \( \delta y_{0} \) (recall that \( \langle y_{0} \rangle = 0 \) is the only vev which is not gauge invariant, in particular it transforms under the action of the non-compact isometries inside SO(2, 1)) we gain one more complex zero mode corresponding to \( \delta \tilde{y}_0 \) which survives the BEH mechanism as zero mode of the effective theory and is a singlet with respect to the residual gauge group SO(2) × SO(3).

\( r_0 = 1 \) : In this case, the eigenstates of the mass matrices at \( \phi^{(0)} \) are mixed states between the Calabi–Vesentini scalars of the vector multiplets and the corresponding ones in the hypermultiplets (here it is clear that the alternative parametrization is most suitable). As it can be easily checked from the Killing vectors \( (2.22) \) and their analogous expressions in the quaternionic coordinates \( \tilde{y}^a \), the generator of SO(2) ⊂ SO(2, 1) defines in the tangent space of the special Kähler and quaternionic-Kähler manifolds, at the points \( y^a = 0 \) and \( \tilde{y}^a = 0 \) respectively, the complex structures for \( y^a \) and \( \tilde{y}^a \). Indeed, its infinitesimal action on the fluctuations \( \delta y^a \) and \( \delta \tilde{y}^a \) around the corresponding origins is represented by the Killing vector \( k_3 \) and has the linear form

\[
\begin{align*}
\text{SO}(2) : \quad \left\{ \begin{array}{l}
\delta y^a \rightarrow \delta y^a + ek_3^a(\delta y) = \delta y^a + i\epsilon \delta y^a, \\
\delta \tilde{y}^a \rightarrow \delta \tilde{y}^a + ek_3^a(\delta \tilde{y}) = \delta \tilde{y}^a + i\epsilon \delta \tilde{y}^a.
\end{array} \right.
\end{align*}
\] (3.51)

Since this transformation is part of the SO(2) × SO(3) residual symmetry of our vacuum, we expect each term of the effective Lagrangian to be invariant under its action. The effective mass term that we find for the scalar fields can be expressed in terms of the variables \( Z^\sigma = \delta y^a \pm \delta \tilde{y}^a \) in the form

\[
(m_S)^2 \delta S \delta \tilde{S} + \sum_{\sigma=1}^{8} (m_{\sigma})^2 Z^\sigma \tilde{Z}^\sigma,
\] (3.52)

which is manifestly invariant under the residual SO(2) infinitesimal transformation \( Z^\sigma \rightarrow Z^\sigma + i\epsilon \delta Z^\sigma \). The generator of this SO(2) symmetry of the vacuum defines therefore a complex structure for the scalar fields in the effective theory deriving from \( y^a \) and \( \tilde{y}^a \).
Table 2: Mass square eigenvalues for $r_0 = 1$ in units of $V|_0$ and corresponding scalar fluctuations, indicating the number of (complex) scalars with the corresponding value and representation with respect to the residual gauge group $U(1) \times SO(3)$.

| $m^2 / V|_0$ | number | eigenstate   | $U(1) \times SO(3)$ rep. |
|------------|--------|-------------|-------------------------|
| 2          | 1      | $\delta S$  | (1,1)                   |
| 0          | 1      | $\delta y_0 + \delta \tilde{y}_0$ |             |
| $\frac{2}{3}$ | 1      | $\delta y_0 - \delta \tilde{y}_0$ | $(1_C, 1)$       |
| $\frac{4}{3}$ | 3      | $\delta \tilde{y} + \delta \tilde{y}$ | $(1_C, 3)$       |
| 0          | 3      | $\delta \tilde{y} - \delta \tilde{y}$ | $(1_C, 3)$       |

The states $\delta S$, $Z^\sigma$ and their masses $m_S$, $m_\sigma$ in units of $V|_0$ are listed in table 2. The complex zero mode $\delta y_0 + \delta \tilde{y}_0$ is the complex Goldstone boson associated with the broken non-compact $SO(2,1)$ transformations and therefore, its real and imaginary components are ‘eaten’ by the vector fields $A^1_\mu$, $A^2_\mu$ according to the BEH mechanism analogous to the one described in section 3.2. Besides this Goldstone zero mode, we have 3 more complex zero modes $\delta \tilde{y} - \delta \tilde{y}$, which survive the BEH mechanism and whose real and imaginary components transform in the $(2, 3)$ of the residual gauge group $SO(2) \times SO(3)$.

We thus find again that in both the cases the potential admits a stable dS vacuum, though there are now valleys in the scalar potential, which are reminiscent of the valleys that were present also in the rigid model with hypermultiplets of [30]. One may wonder whether quantum effects could lead to a sliding of the scalar vev. An indication of this could be given by the fact whether the valley ‘narrows’ or ‘broadens’ when one follows it. This is shown by the values of the masses in other vacua than (3.47). We did a perturbative analysis going to neighbouring points in the valley for the $r_0 = 0$ case and found that the masses do not change to the order that we considered.

4 Summary and conclusions: embedding into superstring theory?

In the present paper, we have shown that within the framework of standard matter-coupled $\mathcal{N} = 2$ supergravity, theories admitting stable de Sitter vacua do exist. We have shown two models with only positive mass fields and one which has also a flat valley in the moduli space of hypermultiplets, showing some similarities to a corresponding model in rigid supersymmetry [30]. We have emphasized that the catch to obtain such a positive result is the use of three equally essential ingredients, namely:

1. non-compact, non-Abelian gaugings;
2. de Roo–Wagemans angles corresponding to symplectic rotations of one simple gauge group factor with respect to another; and

3. the presence of Abelian or non-Abelian Fayet–Iliopoulos terms in the case of pure vector multiplet theories, alternatively coupling to hyper multiplets in such a way that one obtains an effective Fayet–Iliopoulos term produced by the hyper vev.

We have illustrated our positive result by analysing three models of increasing complexity by means of which we were able to show the role of the three ingredients in obtaining the final outcome. Essentially we can say that:

1. The use of non-compact gaugings contributes, into the formula of the scalar potential the positive definite term

\[ k^\alpha_\Lambda k^\beta_\Sigma g_{\alpha\beta} \bar{L}^\Lambda L^\Sigma, \quad (4.1) \]

which cannot be absorbed into a superpotential.

2. The de Roo–Wagemans angles are essential in order to introduce a non-trivial dilaton dependence of the scalar potential and hence to allow for extrema. Without such angles, the scalar potential would simply be

\[ V_{\text{scalar}} = \frac{1}{\text{Im} S} \times \text{function } V' \text{ of all other scalars.} \quad (4.2) \]

3. Finally, the Fayet–Iliopoulos term or the coupling to hypers contributes the source term that yields a finite value to the vev of the dilaton, once the de Roo–Wagemans angles are included.

The combination of these ingredients avoids the negative conclusions reached in (2.45). Important in that respect is that in the vacua the gravitino shifts \[ [S_{ij} \text{ in (B.28)}] \] which are zero due to the setting where \[ L^\Lambda \] is orthogonal to \[ P^\lambda x = 0. \] This removes the eigenvector that would generalize the one found in [14] and further discussed in section 2.3. This is the technical understanding of the mechanisms producing the de Sitter stable vacuum.

The next two questions which are intimately related are:

a. What is the physical relevance of the three ingredients quoted above?

b. How can our result be lifted to higher \( N \) supergravities and be embedded into superstring theory or M-theory?

To provide a first provisional answer to question [a], we emphasize that gauged supergravities in \( D = p + 2 \) dimensions emerge as the near-brane description of light bulk mode interactions in the geometry produced by p-brane configurations. This is fully understood for compact gaugings like the SO(6) gauging of \( N = 8 \) supergravity in \( D = 5 \), which emerges as the near-horizon description of the \( D3 \)-brane, or for the SO(8) gauging of \( N = 8 \) supergravity in \( D = 4 \), which is associated with the near horizon \( M2 \)-brane. This relation is much more poorly understood for non-compact gaugings, yet it is clear that also there one should be able to trace back the gauging to suitable brane constructions. In view of this, the de Roo–Wagemans rotation, which is a symplectic rotation turning part of the electric fields into magnetic ones, needs to be interpreted in terms of its action on candidate branes participating in the construction. Such an analysis is postponed to future publications and investigations. It is, however, worth mentioning here that the de Roo–Wagemans
angles were originally introduced in the context of $\mathcal{N} = 4$ supergravity and therefore do exist also in higher $\mathcal{N}$ theories. Indeed it has begun to be appreciated only recently that the classification of gauged supergravities can be extended if the electric group is modified, which is just what the de Roo–Wagemans rotation does. In particular, the recent results on new $\mathcal{N} = 8$ gaugings \cite{23, 24} pertain to such a scenario. In \cite{13}, the exhaustive classification of $\mathcal{N} = 8$ gaugings was obtained under the dogma that the electric group should be

$$G_{\text{electric}} = \text{SL}(8, \mathbb{R}) \subset \text{E}_{7(7)},$$

(4.3)

and it appears that within such classification no stable de Sitter vacuum is contained. If the dogma \cite{4.3} is removed, then new gaugings, as \cite{23, 24} have proven, are possible and the question is reopened whether stable de Sitter vacua could be present.

This brings the discussion to question \cite{b}, namely whether the successful $\mathcal{N} = 2$ models can be lifted to higher $\mathcal{N}$ and possibly interpreted within string theory. In this respect, the main observation is that the third most complex model, that including the hypermultiplets, is not just randomly chosen but it is a very specific one with a quite inspiring motherhood. To see this consider $\mathcal{N} = 4$ supergravity coupled to $n = n_1 + n_2$ vector multiplets. The scalar manifold is

$$M^{\mathcal{N}=4}_{\text{scalar}} = \frac{\text{SU}(1,1)}{\text{U}(1)} \times \frac{\text{SO}(6,n)}{\text{SO}(6) \times \text{SO}(n)}.$$  

(4.4)

Such a theory can be truncated to $\mathcal{N} = 2$ and a useful and consistent way to do it is by modding with respect to a discrete subgroup of the holonomy group $H_{\text{hol}} = \text{SO}(6) \times \text{SO}(n)$. For instance, if $\alpha$ denotes a square root of the identity ($\alpha^2 = 1$) one can embed a $\mathbb{Z}_2$ group into the holonomy group in the following way:

$$\mathbb{Z}_2 \ni \alpha \mapsto \begin{pmatrix} 1_{2 \times 2} & 0 \\ 0 & 1_{4 \times 4} \end{pmatrix} \otimes \begin{pmatrix} 1_{n_1 \times n_1} & 0 \\ 0 & 1_{n_4 \times n_4} \end{pmatrix} \in H_{\text{hol}}.$$  

(4.5)

There are two gravitinos that survive such a $\mathbb{Z}_2$ projection and the scalars that are $\mathbb{Z}_2$ singlets span the manifold

$$\begin{pmatrix} \text{SU}(1,1) \\ \text{U}(1) \end{pmatrix} \times \frac{\text{SO}(2,n_1)}{\text{SO}(2) \times \text{SO}(n_1)} \times \frac{\text{SO}(4,n_2)}{\text{SO}(4) \times \text{SO}(n_2)}.$$  

(4.6)

For $n_1 = 4$ and $n_2 = 2$, the above manifold just corresponds to the $\mathcal{N} = 2$ model with hypers studied in the present paper. Since $2 + 4 = 6$, this means that our successful model can be embedded into an $\mathcal{N} = 4$ supergravity with six vector multiplets and based on the scalar manifold

$$\mathcal{S}\mathcal{T}[6,6] = \frac{\text{SU}(1,1)}{\text{U}(1)} \times \frac{\text{SO}(6,6)}{\text{SO}(6) \times \text{SO}(6)}.$$  

(4.7)

As for the gauge group we can just choose

$$G^{\mathcal{N}=4}_{\text{gauge}} = \text{SO}(2,2) \times \text{SO}(4) \sim [\text{SO}(2,1) \times \text{SO}(3)]^2,$$

(4.8)

which has 12 generators in the fundamental of $\text{SO}(6,6)$ and contains two copies of the $\mathcal{N} = 2$ gauge group. Indeed the latter is just the diagonal subgroup. What remains to be proven and is left to a future publication is that the $\mathcal{N} = 4$ potential is extremum at a zero value of the additional scalars that are not $\mathbb{Z}_2$ singlets. If that is true, the embedding of our model into $\mathcal{N} = 4$ is perfect.
On the other hand, the scalar manifold \([4.7]\) is just the standard moduli space for the toroidal compactification of type IIA string theory on a \(T^6\) torus. Indeed, \(ST[6,6] \subset E_{7(7)}/SU(8)\) is just the submanifold of Neveu–Schwarz scalars in an \(\mathcal{N} = 8\) theory.

It follows from these observations that the prospects to reinterpret our stable de Sitter vacuum as a vacuum in a brane construction and within the framework of superstring theory are, at first sight, quite promising.

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A Quaternionic geometry items

In the main text, the essential ingredients of the quaternionic part of the scalar potential, namely the vielbein, the quaternionic metric $h_{ij}$, the triholomorphic moment maps $P^i_A$ and the triholomorphic Killing vectors $k^A_i$ have been defined through (3.40), (2.9), (3.44) and (3.43). In order to make such definitions explicit functions of the fields and workable by the reader one just needs to specify the explicit form of the involved matrices, namely the coset representative $L$, that defines the vielbeins. The latter determine the metric, which we will give explicitly and the complex structures $J^F$. For the gauging, we also need the group generators $t_A$. This is what we do in this appendix. For the sake of explicit calculations, the solvable parametrization is much simpler than any other parametrization and for this reason we use it, and make it explicit in section A.1. In section A.2 of this appendix, we discuss the relation between the solvable coordinates and the Calabi–Vesentini coordinates for the same manifold.

A.1 The solvable parametrization

Coset representative, vielbein and metric. The coset representative $L$ in solvable coordinates is given by a $6 \times 6$ matrix which, both for convenience and for further manipulations, can be written in block form as follows:

$$L_{\text{solv}} = \begin{pmatrix}
A_{\text{solv}} & B_{\text{solv}} \\
C_{\text{solv}} & D_{\text{solv}}
\end{pmatrix}_{4 \times 4 \times 2}$$

The explicit forms of the blocks can be displayed and are polynomial in the nilpotent fields $a_i$, $b_i$, while their dependence on the Cartan fields $h_i$ is via simple exponentials, namely

$$A_{\text{solv}} = \begin{pmatrix}
1 & 0 & 0 & a_4 + \sqrt{2} a_1 b_2 & a_3 + \sqrt{2} a_1 b_1 \\
0 & 1 & -a_3 e^{-h_1} & -b_2 e^{-h_2} & \cosh h_1 + c \\
-a_3 e^{-h_1} & -b_1 e^{-h_2} & \frac{1}{\sqrt{2}} e^{-h_1} & (-a_4 + a_2) \\
\end{pmatrix},$$

$$B_{\text{solv}} = \begin{pmatrix}
1 & 0 & 0 & a_4 + \sqrt{2} a_1 b_2 & a_3 + \sqrt{2} a_1 b_1 \\
0 & 1 & -a_3 e^{-h_1} & -b_2 e^{-h_2} & \cosh h_1 + c \\
-a_3 e^{-h_1} & -b_1 e^{-h_2} & \frac{1}{\sqrt{2}} e^{-h_1} & (-a_4 + a_2) \\
\end{pmatrix},$$

$$C_{\text{solv}}^T = \begin{pmatrix}
a_4 + \sqrt{2} a_1 b_2 & a_3 + \sqrt{2} a_1 b_1 & b_2 \\
\sinh h_1 + c & \frac{1}{\sqrt{2}} e^{-h_1} & \left\{ a_1 e^{h_2} + a_2 e^{-h_2} \right\} \\
\frac{1}{\sqrt{2}} e^{-h_1} & (-a_4 + a_2) \\
\end{pmatrix},$$

$$D_{\text{solv}} = \begin{pmatrix}
1 & 0 & 0 & a_4 + \sqrt{2} a_1 b_2 & a_3 + \sqrt{2} a_1 b_1 \\
0 & 1 & -a_3 e^{-h_1} & -b_2 e^{-h_2} & \cosh h_1 + c \\
-a_3 e^{-h_1} & -b_1 e^{-h_2} & \frac{1}{\sqrt{2}} e^{-h_1} & (-a_4 + a_2) \\
\end{pmatrix},$$

with

$$b^2 \equiv b_1^2 + b_2^2, \quad c \equiv \frac{1}{e^{h_1}} \left( 2 a_1 a_2 - a_3^2 - a_4^2 \right), \quad d \equiv e^{-h_2} (a_3 b_1 + a_4 b_2 + \frac{1}{\sqrt{2}} a_1 b^2).$$
These lead by (3.40) to the vielbein
\[ V^{mt} = \begin{pmatrix}
  e^{-h_1} \left( da_4 + \sqrt{2} b_2 da_1 \right) & e^{-h_2} db_2 \\
  e^{-h_1} \left( da_3 + \sqrt{2} b_1 da_1 \right) & e^{-h_2} db_1 \\
  \frac{1}{\sqrt{2}} e^{-h_2} \left( e^{h_1} da_1 + e^{-h_1} A_2 \right) & dh_2
\end{pmatrix}, \quad (A.4) \]

with
\[ A_2 = da_2 + b^2 da_1 + \sqrt{2}(b_1 da_3 + b_2 da_4). \quad (A.5) \]

The vielbein defines the metric by (2.9). We obtain
\[ 2ds^2 = e^{-2(h_1+h_2)} a_{ij} da_i da_j + e^{-2h_2} [(db_1)^2 + (db_2)^2] + (dh_1)^2 + (dh_2)^2, \]
\[ a_{ij} = \begin{pmatrix}
  (e^{2h_2} + b^2)^2 & b^2 & \sqrt{2} b_1 \left( e^{2h_2} + b^2 \right) & \sqrt{2} b_2 \left( e^{2h_2} + b^2 \right) \\
  b^2 & \sqrt{2} b_1 & e^{2h_2} + 2b_1^2 & 2b_1 b_2 \\
  \sqrt{2} b_1 \left( e^{2h_2} + b^2 \right) & \sqrt{2} b_2 & e^{2h_2} + 2b_2^2 & 2b_2 b_2
\end{pmatrix}. \quad (A.6) \]

The group generators. The generators of \( \text{SO}(2,1) \times \text{SO}(3) \subset \text{SO}(2,4) \) are the 6 \( \times \) 6 matrices already spelt out in (2.19). Indeed, the quaternionic manifold is just a copy of the submanifold \( \text{SO}(2,4)/\text{SO}(2) \times \text{SO}(4) \) of \( ST[2,4] \) and therefore the generators of the isometry algebra are the same. One has just to be careful with the fact that for the use in the vector multiplet sector the 6-dimensional representation of \( \text{SO}(2,4) \) was written in the basis where the 2 \( \times \) 2 block is the first while the 4 \( \times \) 4 block is the last. For the use in the quaternionic case, the same matrices have to be written in a basis where the 4 \( \times \) 4 block is instead the first. Hence it suffices to take the matrices (2.19) and do the permutation of axis:
\[ \{1,2,3,4,5,6\} \Rightarrow \{6,5,4,3,2,1\}. \quad (A.7) \]
Explicitly, we obtain

\[
\begin{align*}
t_1 &= e_0 
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 \\
0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}
, \\
t_2 &= e_0 
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}
, \\
t_3 &= e_0 
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
, \\
t_4 &= e_1 
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
, \\
t_5 &= e_1 
\begin{pmatrix}
0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
, \\
t_6 &= e_1 
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
.
\end{align*}
\]

Rather than giving the other relevant items in terms of the solvable coordinates it is more interesting to explore the relation between the latter and the Calabi–Vesentini coordinates for the same manifold which we have already used in the vector multiplet sector.

### A.2 Transformation from the solvable to the Calabi–Vesentini coordinates

The coset manifold

\[
M_{2,4} \equiv \frac{\text{SO}(2, n)}{\text{SO}(2) \times \text{SO}(4)}
\]

is remarkable in that it can be alternatively seen as a complex manifold of the series

\[
\frac{\text{SO}(2, n)}{\text{SO}(2) \times \text{SO}(n)},
\]

or a quaternionic manifold of the series

\[
\frac{\text{SO}(4, m)}{\text{SO}(4) \times \text{SO}(m)}.
\]

The double interpretation implies that although it is quaternionic, yet it admits a description in terms of the Calabi–Vesentini complex coordinates already used in the case of vector multiplets. In this section, we elaborate the coordinate transformation from the solvable basis to the Calabi–Vesentini basis.

Our starting point is provided by equations (C.1)–(C.4) of [34]. It follows from there that if

\[
L_{\text{CV}} = \begin{pmatrix}
A_{\text{CV}} & B_{\text{CV}} \\
C_{\text{CV}} & D_{\text{CV}}
\end{pmatrix}
\]

(A.12)
is the coset representative in the CV basis, the upper part of the symplectic section

$$X^A = \begin{pmatrix} 1 & \frac{1}{2} (1 + y^2) \\ \frac{1}{2} i (1 - y^2) \\ y^a \end{pmatrix}$$  \hspace{1cm} (A.13)$$

is related to the matrix blocks in the following way. Let

$$S = \begin{pmatrix} I_{4 \times 4} & 0 & 0 \\ 0 & I_{4 \times 2} & 0 \\ 0 & 0 & I_{2 \times 2} \end{pmatrix}, \quad \text{where} \quad T = \frac{1}{\sqrt{2}} \begin{pmatrix} i & -i \\ 1 & 1 \end{pmatrix}.$$  \hspace{1cm} (A.14)$$

Defining

$$\hat{L}_{\text{CV}} = S^{-1} L_{\text{CV}} S = \begin{pmatrix} \hat{A}_{\text{CV}} & \hat{B}_{\text{CV}} \\ 0 & D_{\text{CV}} \end{pmatrix},$$  \hspace{1cm} (A.15)$$

one has

$$\hat{B}_{\text{CV}} = \exp[\kappa_2/2] \begin{pmatrix} y^a \\ \bar{y}^a \end{pmatrix}, \quad \hat{D}_{\text{CV}} = \frac{1}{\sqrt{2}} \exp[\kappa_2/2] \begin{pmatrix} 1 & y^2 \\ \bar{y}^2 & 1 \end{pmatrix},$$  \hspace{1cm} (A.16)$$

where \(y^2 = y^a y^a\) and \(\bar{y}^2 = \bar{y}^a \bar{y}^a\) and \(\kappa_2\) is the Kähler potential as given in Eq. (2.6). Consider next the following \(2 \times 4\) matrix:

$$\mathbb{P} \equiv \hat{B} \cdot \hat{D}^{-1} \equiv (Y^a, \bar{Y}^a).$$  \hspace{1cm} (A.17)$$

By explicit calculation we obtain

$$Y^a = \sqrt{2} \frac{1}{1 - |y^2|^2} \left( y^a - y^2 \bar{y}^a \right).$$  \hspace{1cm} (A.18)$$

This relation can be inverted by means of the following formula:

$$y^a = \frac{1}{\sqrt{2}} \left( Y^a + t \bar{Y}^a \right), \quad \text{with} \quad t = \frac{(1 - \bar{Y} \cdot Y) - \sqrt{(1 - \bar{Y} \cdot Y)^2 - |Y \cdot \bar{Y}|^2}}{Y \cdot \bar{Y}}. \hspace{1cm} (A.19)$$

Why do we consider the matrix \(\mathbb{P}\) defined in (A.17)? The reason is simple. By construction it is a projective invariant that is independent from the choice of the coset representative out of which it is constructed. It depends only on the equivalence class, namely on the point of the coset manifold. Indeed, since the subgroup \(\text{SO}(4) \times \text{SO}(2)\) is block diagonal, under a transformation

$$L \mapsto L' = L \begin{pmatrix} H_4 & 0 \\ 0 & H_2 \end{pmatrix},$$  \hspace{1cm} (A.20)$$

the matrix \(\mathbb{P}\) remains invariant. Hence, although the coset representatives calculated in the solvable and in the CV parametrizations are different choices of representatives in the same equivalence classes, we can safely identify

$$\mathbb{P}_{\text{sol}} = \mathbb{P}_{\text{CV}}.$$  \hspace{1cm} (A.21)$$

Equation (A.21) combined with (A.19) provides the desired coordinate transformation expressing the Calabi–Vesentini coordinates in terms of the solvable ones. It suffices to set

$$y^a = \frac{1}{\sqrt{2}} \left( Y^a_{\text{sol}} + t_{\text{sol}} \bar{Y}^a_{\text{sol}} \right). \hspace{1cm} (A.22)$$
where in the r.h.s. the $Y^a$ and $t(Y)$ are calculated from the solvable coset representative.

It turns out that the $Y^a$ are lengthy polynomial functions of the solvable parameters and that the invariant $t$ is very much complicated although completely explicit. So, as it might be expected, the transformation

$$y^a = y^a(a_i, b_1, b_2, h_1, h_2)$$

(A.23)
is highly nonlinear and quite involved. Yet, near the origin of the coset manifold, namely for very small fields, the transformation [A.23] linearizes and becomes fairly simple

$$y^a \simeq \frac{1}{2} \begin{pmatrix} i a_4 + b_2 \\ i a_3 + b_1 \\ \frac{1}{\sqrt{2}}(a_1 + a_2) + i h_1 \\ \frac{1}{\sqrt{2}}(a_1 - a_2) + h_2 \end{pmatrix}.$$  

(A.24)

These $y^a$ are the $\tilde{y}^a$ in the main text, giving the alternative parametrization of the quaternionic-Kähler manifold.

## B Indices

Table 3: *Indices in this paper, and their ranges, where $\dim SK$ stands for the (complex) dimension of the special Kähler manifold, and $\dim QK$ for the (quaternionic) dimension of the quaternionic-Kähler manifold.*

| index and range | meaning |
|-----------------|---------|
| $\mu = 0, \ldots, 3$ | spacetime |
| $i = 1, 2$ | supersymmetry extension |
| $\Lambda = 1, \ldots, \dim SK + 1$ | gauge group |
| $x = 1, 2, 3$ | $SO(3)$ or $SO(2, 1)$ |
| $\alpha, \bar{\alpha} = 1, \ldots, \dim SK$ | complex scalars in $SK$ |
| $a = 0, \ldots, n - 1$ | complex scalars in $SO(2, n)$ \(\times SO(n)\) (sometimes split in $y^0$ and $\tilde{y}$) |
| $A = 1, \ldots, 6$ | $SO(4, 2)$ fundamental representation |
| $u = 1, \ldots, 4 \dim QK$ | real scalars in $QK$ |
| $t = 1, \ldots, \dim QK$ | quaternions |
| $m = 1, \ldots, 4$ | quaternionic components |
| $i = 1, \ldots, 4$ | Pececi-Quinn scalars in the solvable parametrization of $QK$ (in section 3.3 only) |
| $\sigma = 1, \ldots, 8$ | complex scalars $Z^\sigma = \delta y^a \pm \delta \tilde{y}^a$ in the effective theory around the dS vacuum (in section 3.3 only) |

Table 3 shows the indices used, their range and meaning.

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