Constraining a $CP$-violating $WWV$ coupling from the $W^+W^-$ threshold cross section at LEP2

V. C. Spanos$^a$ and W. J. Stirling$^{a,b}$

$^a$ Department of Physics, University of Durham, Durham, DH1 3LE
$^b$ Department of Mathematical Sciences, University of Durham, Durham, DH1 3LE

Abstract

The most general form of the $WWZ$ and $WW\gamma$ interaction contains a $CP$-violating term which has the same threshold behaviour as the Standard Model $e^+e^- \rightarrow W^+W^-$ cross section. We calculate the cross section as a function of the corresponding anomalous coupling, and estimate the bounds which can be obtained from a measurement of the threshold cross section at LEP2. We show how the effect of the coupling is most pronounced in the angular distributions of the final-state fermions.
One of the most important physics goals of the LEP2 $e^+e^-$ collider ($\sqrt{s} \simeq 160 - 190$ GeV) is to measure or constrain anomalous couplings, in particular the trilinear couplings of the $W$, $Z$ and photon. In the SU(2)×U(1) Standard Model (SM) these couplings are completely specified, and so any deviations from the SM values would signal new physics. A very detailed and up-to-date study of how the various couplings can be measured at LEP2 can be found in Ref. [1]. The idea is to use the angular distributions of the decay products in $e^+e^- \rightarrow W^+W^- \rightarrow 4f$ production, which are sensitive to the form of the $WW\gamma$ and $WWZ$ vertices.

Another important measurement at LEP2 is the determination of the $W$ boson mass ($M_W$) [2]. Two methods have been discussed: the ‘direct reconstruction’ of the mass from the decay products, and the measurement of the total $W^+W^-$ cross section close to the threshold at $\sqrt{s} = 2M_W$, which is sensitive to $M_W$. Here one determines the $W$ mass from equating the SM prediction and the experimental measurement, with $M_W$ as a free parameter: $\sigma_{SM}(M_W \pm \Delta M_W) = \sigma_{exp} \pm \Delta \sigma_{exp}$. It has been shown [2, 3] that a collision energy of 161 GeV is optimal in this respect. An apparent drawback to this method is that it assumes that the theoretical cross section is correctly given by the SM. However it is very difficult to imagine any type of new physics corrections which would significantly alter the threshold cross section. The key point is that at threshold the total cross section is dominated by the $t$-channel neutrino exchange diagram (see Fig. 1 below). Contributions from $s$-channel photon and $Z$ exchange are suppressed by a relative factor $\beta^2 = 1 - 4M^2_W/s \ll 1$. This means that contributions from new physics processes such as $\gamma^*, Z^* \rightarrow XX$, or anomalous contributions to $\gamma^*, Z^* \rightarrow W^+W^-$ are heavily suppressed. Loop corrections involving new particles are also either very small, or else part of the renormalized $W$ propagator and therefore included in the definition of $M_W$ itself.

There is one important exception to this rule. The most general effective Lagrangian for the $WWV$ vertex ($V = Z, \gamma$) in $e^+e^- \rightarrow W^+W^-$ production contains a total of seven distinct couplings [4], see Eq. (1) below, each with different properties under discrete $C$, $P$ and $T$ transformations. All but one of these couplings are suppressed by a factor $\beta^2$ at threshold, including of course the SM couplings. The exception is the ‘$f^V_6$’ (in the notation of Ref. [4]) $CP$-violating coupling, which has the same threshold behaviour as the leading SM $\nu$-exchange contribution. The validity of the threshold cross section method for determining $M_W$ must therefore rely on the assumption that this coupling is either zero or very small.

In this letter we address the following question: if we assume that there is indeed a non-zero $f^V_6$ $CP$-violating $WWV$ coupling, what information on it can be obtained from a measurement of the threshold $W^+W^-$ cross section at LEP2? To do this we calculate the cross section as a function of $f^V_6$ and use the expected experimental precision to obtain

\[1\] There is a straightforward angular momentum argument for this, see for example Ref. [4].
an estimate for the uncertainty in its determination. Of course now one has to assume a value for $M_W$, for example from the direct measurements at $p\bar{p}$ colliders. We also study the effect of the anomalous coupling on several angular distributions.

There is an important caveat to this approach. Neutron electric dipole moment data already rule out an anomalous $CP$-violating $WW\gamma$ coupling greater than $O(10^{-3})$ (in units of $e$) \[4\], unless one allows for fine tuning at this level between different contributions. If one further argues (see for example Ref. \[6\]) that any extension of the SM should respect the $SU(2)\times U(1)$ gauge symmetry, then the same order of magnitude limit should apply also to the $WWZ$ coupling, and there would be no observable effect at LEP2. We believe, however, that there is no substitute for a direct measurement. If one did discover a $CP$-violating $WWV$ coupling at LEP2, the theoretical implications would be immense. For this reason, we consider both non-zero $f_6^\gamma$ and $f_6^Z$ couplings in our study.

The most general $WWV$ vertex which exhausts all possible Lorentz structure for $W^+W^-$ production has the form \[4\]:

\[
g_{WWV}^{-1} \Gamma_V^{\alpha\beta\mu}(k_-,k_+,P) = f_1^V (k_- - k_+)^\mu g^{\alpha\beta} - \frac{f_2^V}{M_W^2} (k_- - k_+)^\mu P^\alpha P^\beta \\
+ f_3^V (P^\alpha g^{\beta\mu} - P^\beta g^{\mu\alpha}) + i f_4^V (P^\alpha g^{\beta\mu} + P^\beta g^{\mu\alpha}) \\
+ i f_5^V \epsilon^{\alpha\beta\mu\rho} (k_- - k_+)_\rho - f_6^V \epsilon^{\mu\alpha\beta\rho} P_\rho \\
- \frac{f_7^V}{M_W^2} (k_- - k_+)^\mu \epsilon^{\alpha\beta\rho\sigma} P_\rho (k_- - k_+)_\sigma ,
\]

where

\[
g_{WW\gamma} = -e \quad , \quad g_{WWZ} = -e \cot \theta_W .
\]

Here $k_\pm$ are the four-momenta of the outgoing $W^\pm$ and $P$ is the incoming four-momentum of the neutral boson $V = \gamma, Z$. The couplings $f_{1,2,3}^V$ in Eq. \[1\] are $C$ and $P$ conserving, while the remainder are $C$ and/or $P$ violating. In particular, the $f_6^V$ coupling is $C$ conserving, but $P$ violating. More importantly in the present context, it is the only coupling that gives a leading threshold behaviour. In what follows we set all the other $f_i^V$ to their SM values, i.e.

\[
f_1^V = \frac{1}{2} f_3^V = 1 , \quad f_2^V = f_4^V = f_5^V = f_7^V = 0 .
\]

At LEP2 energies, i.e. far above the $Z$ pole, the $f_6^\gamma$ and $f_6^Z$ contributions have a comparable effect on the total cross section and angular distributions. From a measurement with modest luminosity at a single threshold energy, it will therefore be very difficult to distinguish separate $f_6^\gamma$ and $f_6^Z$ contributions. In our numerical studies we therefore set $f_6^\gamma = f_6^Z \equiv f_6$. Using, for example, the spinor techniques of Ref. \[7\] it is straightforward
to compute the $e^+e^- \to W^+W^- \to 4f$ scattering amplitude including the anomalous $f_6$ contribution. The general form is

$$M_{WW} = M_\nu + M_V + M_{f_6},$$

where the contributions on the right-hand side correspond respectively to $t$-channel neutrino exchange, Standard Model $s$-channel $\gamma$ and $Z$ exchange, and the anomalous contribution. For positive helicity initial-state electrons the first of these is absent.

We begin by considering the total $e^+e^- \to W^+W^-$ cross section in the zero width (stable $W$) limit, at leading order. Unless otherwise stated, the numerical calculations described below use the same set of input parameters as the study of Ref. [3]. Where $M_W$ is needed as an input parameter, the recent world average value from direct measurements at the $p\bar{p}$ colliders,

$$M_W = 80.33 \pm 0.15 \text{ GeV},$$

is used.

Fig. 1 shows the contributions to the cross section corresponding to the decomposition of Eq. (4), with $f_6 = 1$ for illustration, as a function of the collider energy $\sqrt{s}$. Note that there is no interference between the SM and $f_6$ amplitudes in the total cross section. Just above threshold the dependence of the cross section on the $W$ velocity, $\beta = \sqrt{1 - 4M_W^2/s}$, can be parametrized as

$$\sigma = A\beta + B\beta^3 + \ldots.$$ 

The threshold behaviour discussed above is clearly evident: the anomalous contribution is $O(\beta)$, as for the $\nu$-exchange contribution and in contrast to the $O(\beta^3)$ behaviour of the SM $s$-channel contributions. For $f_6 = 1$, the ratio of $\Delta \sigma_{f_6}$ to $\sigma_{SM}$ is approximately 0.43 near threshold, in agreement with Ref. [9].

In practice the stable $W$ approximation is inadequate in the threshold region. There are important contributions from finite width effects, initial-state radiation, Coulomb corrections etc. [2], all of which smear out the sharp rise from zero of the cross sections in Fig. 1. These effects are included in our calculation, exactly as described in Ref. [3].

Fig. 2 shows the total $WW$ cross section in the threshold region as a function of $\sqrt{s}$, for fixed $M_W = 80.33$ GeV and different $f_6$ values. Since there is no interference between the SM and $f_6$ amplitudes, the cross section depends quadratically on $f_6$ at a given energy. The curves are reminiscent of the behaviour of the threshold cross section on $M_W$ (see for example Fig. 4 of Ref. [3]), with one important difference: the sensitivity of the cross

---

2 Only diagrams with two resonant $W$ propagators are included.

3 The calculation of Ref. [8] assumed $f_6^2 = 0$, in which case the ratio is 0.29.
section to $f_6$ (as parametrized by the ratio of $\delta \sigma/\sigma$ to $\delta f_6/f_6$) is approximately independent of $\sqrt{s}$ in the threshold region. This contrasts with the corresponding sensitivity to $M_W$, which is maximal roughly 500 MeV above the nominal threshold at $\sqrt{s} = 2M_W$ [2, 3]. It is for this reason that the ‘threshold running’ of LEP2 will take place at the single collision energy $\sqrt{s} = 161$ GeV. In the remainder of this paper, therefore, we restrict our attention to this value.

Fig. 3 shows $\sigma_{WW}(161 \text{ GeV})$ as a function of $f_6$, for $M_W = 80.33 \pm 0.15$ GeV, the current world average. The expected quadratic behaviour is clearly evident. We can use this figure to estimate an approximate experimental error on $f_6$ from a total cross section measurement. In Ref. [2] it was estimated that for 4 experiments each with 50 pb$^{-1}$ total luminosity the error on the $W$ mass would be $\delta M_W = \pm 108$ MeV. This corresponds to $\delta \sigma/\sigma \approx 1/16$, indicated by the horizontal band in Fig. 3 centred on the SM prediction evaluated at the world average $M_W$. Fixing $M_W$ at this value gives $\delta f_6 = \pm 0.4$ for the same cross section uncertainty. If the current $\pm 150$ MeV error on $M_W$ is taken into account, this increases to $\delta f_6 = \pm 0.6$.

Information will also be available on the distribution of the $W^+W^-$ decay products, for example the angular distributions of the final-state leptons and jets. These can provide important additional constraints, particularly since in such distributions the interference between the SM and $CP$-violating amplitudes leads to a linear dependence on $f_6$. Given the relatively small statistics, the difficulty in reconstructing the quark momenta from the observed jets, and the presence of at least one energetic neutrino in more than half the events, only rather simple distributions are likely to be accessible in practice. Nevertheless, these can still be rather sensitive to a possible $CP$-violating contribution. As an example, Fig. 4(a) shows the (laboratory frame) polar angle distribution of the charged lepton $l^- = e^-, \mu^-, \tau^-$ from $W^- \rightarrow l^-\bar{\nu}_l$ decay. The curves show the SM and anomalous contributions for $f_6 = 1$. Notice the large negative contribution at small angles from the interference between the SM and anomalous amplitudes, which has the effect of producing a pronounced dip in the overall distribution. An important signature of the $CP$-violating properties of the anomalous interaction is the fact that the distribution in Fig. 4(a) is not symmetric under the interchange: $l^- \leftrightarrow l^+, \cos \theta \leftrightarrow -\cos \theta$. To illustrate this, we define the asymmetry

$$A = \frac{d\sigma/d\cos \theta(l^-, \cos \theta) - d\sigma/d\cos \theta(l^+, -\cos \theta)}{d\sigma/d\cos \theta(l^-, \cos \theta) + d\sigma/d\cos \theta(l^+, -\cos \theta)},$$

(7)

which vanishes in the SM. Fig. 4(b) shows $A$ as a function of $\cos \theta$ for $f_6 = 1$. The measurement of such distributions can be used to improve the precision of the $f_6$ determination, although obtaining a realistic, quantitative estimate will require a detailed detector simulation beyond the scope of the present work.
As a final example, Fig. 5 shows the distribution in the angle between the normals of
the two planes containing the fermions from each $W$ decay, i.e.
\[ \cos \varphi = \frac{(p_1 \times p_2) \cdot (p_3 \times p_4)}{|p_1||p_2||p_3||p_4|}, \quad (8) \]
where $p_1$ ($p_2$) labels the momentum of the fermion (antifermion) from the $W^-$, and $p_3$
($p_4$) labels the momentum of the antifermion (fermion) from the $W^+$. Since the relative
orientation with respect to the incoming leptons has been integrated out, there is no
interference between the SM and $CP$-violating amplitudes. The individual contributions
are evidently very different, however, with the SM ($CP$-violating) contribution preferring
the planes to be aligned (anti-aligned). Note that the definition in Eq. (8) assumes an
ideal situation where the fermions and antifermions can be identified in the $W$ decay
products. This could in principle be achieved for final state quarks jets by a jet-charge
analysis or by requiring a charm quark jet, but in practice the limited statistics are likely
to make this difficult.

In conclusion, we have studied the effect on the threshold $W^+W^-$ cross section at
LEP2 of a possible $CP$-violating $WWW$ interaction, parametrized by couplings $f_6^\gamma$ and
$f_6^Z$. Although there is indirect evidence that such couplings are very small, we believe
that a direct search is important, in view of the implications for the $W$ mass measure-
ment from the threshold cross section. We have estimated the likely precision on the
$f_6^V$ measurements, and studied several angular distributions which will provide further
information.

Acknowledgements

This work was supported in part by the EU under the Human Capital and Mobility
Network Program CHRX–CT93–0319.

References

[1] Report of the ‘Triple Gauge Boson Couplings’ Working Group, G. Gounaris, J.-
L. Kneur and D. Zeppenfeld (convenors), in Physics at LEP2, eds. G. Altarelli,
T. Sjöstrand and F. Zwirner, CERN Report 96-01 (1996), p. 525.

[2] Report of the ‘Determination of the Mass of the $W$ Boson’ Working Group, Z. Kunszt
and W.J. Stirling (convenors), in Physics at LEP2, eds. G. Altarelli, T. Sjöstrand
and F. Zwirner, CERN Report 96-01 (1996), p. 141.
[3] W.J. Stirling, Nucl. Phys. B456 (1995) 3.

[4] K. Hagiwara, R.D. Peccei, D. Zeppenfeld and K. Hikasa, Nucl. Phys. B282 (1987) 253.

[5] W.J. Marciano and A. Queijeiro, Phys. Rev. D33 (1986) 3449;
   D. Atwood, C.P. Burgess, C. Hamzaoui, B. Irwin and J.A. Robinson, Phys. Rev. D42 (1990) 3770;
   F. Boudjema, K. Hagiwara, C. Hamzaoui and K. Numata, Phys. Rev. D43 (1991) 2223.

[6] A. De Rújula, M.B. Gavela, O. Pène and F.J. Vegas, Nucl. Phys. B357 (1991) 311.

[7] R. Kleiss and W.J. Stirling, Nucl. Phys. B262 (1985) 235.

[8] D0 collaboration, U. Heintz, presented at Les Rencontres de Physique de la Vallée d’Aoste, La Thuile, 1996, preprint Fermilab-Conf-96/143-E (1996), and references therein.

[9] K. Hagiwara and D. Zeppenfeld, Phys. Lett. B196 (1987) 97.
Figure Captions

Fig. 1 Decomposition of the (on-shell) Born $e^+e^- \rightarrow W^+W^-$ cross section into its various SM components, together with the $f_6 = 1$ CP-violating contribution, as a function of $\sqrt{s}$.

Fig. 2 The total (off-shell) $e^+e^- \rightarrow W^+W^-$ cross section, including ISR and Coulomb corrections, for various values of the $f_6$ coupling, as a function of $\sqrt{s}$.

Fig. 3 The total $e^+e^- \rightarrow W^+W^-$ cross section at $\sqrt{s} = 161$ GeV, as a function of $f_6$. The solid line corresponds to $M_W = 80.33$ GeV, while the band defined by the short-dashed lines represents the experimental uncertainty of $\pm 0.15$ GeV. The horizontal band indicates a possible experimental measurement, as discussed in the text.

Fig. 4 Distributions in (a) the lepton ($l^-$) polar angle and (b) the $l^\pm$ forward-backward asymmetry defined in Eq. (7), at $\sqrt{s} = 161$ GeV, for $f_6 = 1$.

Fig. 5 Distribution in the angle between the normals to the planes of the $W^\pm$ decay products at $\sqrt{s} = 161$ GeV, for $f_6 = 1$. 
Figure 1
Figure 2
Figure 3
Figure 4 (a)
Figure 4 (b)
\[ \sqrt{s} = 161 \text{ GeV} \]

\[ f_6 = 1 \]

Figure 5