Outward migration of a giant planet with a gravitationally unstable gap edge

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ABSTRACT

We present numerical simulations of disc-planet interactions where the planet opens a gravitationally unstable gap in an otherwise gravitationally stable disc. In our disc models, where the outer gap edge can be unstable to global spiral modes, we find that as we increase the surface density scale the gap becomes more unstable and the planet migrates outwards more rapidly. We show that the positive torque is provided by material brought into the planet’s coorbital region by the spiral arms. This material is expected to execute horseshoe turns upon approaching the planet and hence torque it. Our results suggest that standard type II migration, applicable to giant planets in non-self-gravitating viscous discs, is likely to be significantly modified in massive discs when gravitational instabilities associated with the gap occur.

Key words: planetary systems: formation — planetary systems: protoplanetary discs

1 INTRODUCTION

One of the common approximations in studies of disc-planet interactions is to adopt a non-self-gravitating disc. Thus, despite over three decades since the work of Goldreich & Tremaine (1979, 1980), only a relatively small number of studies have adopted self-gravitating disc models (Nelson & Benz 2003a,b; Pierens & Huré 2005; Baruteau & Masset 2008; Zhang et al. 2008; Avlilf & Bate 2010). Disc gravity was included in these works in order to obtain a self-consistent treatment of planetary migration. The discs remain gravitationally stable.

Recently, Baruteau et al. (2011) and Michael et al. (2011) performed numerical simulations of planetary migration in gravitationally unstable discs. These studies were motivated by the need to understand the fate of giant planets formed by disc instability (Durisen et al. 2007). Consequently, these authors consider massive discs which are gravitationally unstable without the presence of a planet because the surface density is high enough and the disc is sufficiently cool (see Toomre 1964).

However, there are other types of gravitational instabilities in discs. Of particular relevance to disc-planet interactions is gravitational instability associated with internal disc edges and grooves (Sellwood & Kahn 1991). As it is known that giant planets open annular gaps in discs (Lin & Papaloizou 1986), Meschiari & Laughlin (2008) suggested planetary gaps may become gravitationally unstable by analysing the stability of a prescribed disc profile. Instability was explicitly confirmed by Lin & Papaloizou (2011a) via linear and nonlinear calculations for gaps self-consistently opened by a planet. Lin & Papaloizou called these instabilities edge modes since they are associated with gap edges.

In this study we explore the consequence of edge modes on planetary migration using hydrodynamic simulations. We consider the specific configuration of a giant planet residing in a gap with unstable gap edges. Our specific aim is to understand how edge modes can modify the standard picture of planetary migration expected for giant planets. We therefore focus on a small set of simulations rather than a full parameter survey.

This paper is organised as follows. We describe our disc-planet models and numerical methods in §2. In §3 we discuss the expected stability properties of planetary gaps formed in our discs. We present migration results in §4. We find that the planet migrates outwards as the gap edge becomes increasingly unstable. We analyse a case in §5 to identify the required source of positive torque and explain how this can be attributed to edge modes. We conclude in §6 with a discussion of implications and limitations of our results.
2 MODEL SETUP

We consider a two-dimensional self-gravitating protoplanetary disc of mass $M_d$ orbiting a central star of mass $M_*$. We adopt polar coordinates $r = (r, \phi)$ centred on the star and a non-rotating reference frame. The governing hydrodynamic equations are given in Lin & Papaloizou (2011a). Units are such that $M_* = G = 1$, where $G$ is the gravitational constant.

The disc occupies $r \in [r_1, r_o] = [1, 25]$ and its surface density $\Sigma$ is initialised to

$$\Sigma(r) = \Sigma_0 r^{-3/2} \left[1 - \frac{r_o}{r + H(r_o)}\right], \quad (1)$$

where $H(r)$ is the disc semi-thickness defined below, and the surface density scale $\Sigma_0$ is chosen by specifying the Keplerian Toomre $Q$ parameter at the outer boundary

$$Q_o = \frac{c_{iso} \Omega_k}{\pi G \Sigma r_o}, \quad (2)$$

where

$$\Omega_k = \frac{\sqrt{GM_*}}{r^3} \quad (3)$$

is the Keplerian orbital frequency, and

$$c_{iso} = H \Omega_k \quad (4)$$

is the sound-speed profile for a locally isothermal disc. We set $H(r) = hr$ and fix the aspect-ratio $h = 0.05$. The disc is also characterised by a uniform kinematic viscosity $\nu = 10^{-5}$ in dimensionless units. The initial azimuthal velocity $v_\phi$ is set from centrifugal balance with stellar disc gravity and pressure. The initial radial velocity is set to $v_r = 3\nu/2r$.

We introduce a planet of fixed mass $M_p = 2 \times 10^{-3} M_*$ on a circular orbit at $r = 10 = r_p(t = 0)$. This value of $M_p$ corresponds to 2 Jupiter masses if $M_* = M_{\odot}$, which opens a deep gap leading to type II migration in a typical non-self-gravitating viscous disc (Lin & Papaloizou 1986), provided no instabilities develop. In this standard picture, migration follows the viscous evolution of the gap.

2.1 Equation of state

The equation of state (EOS) is locally isothermal, so the vertically integrated pressure is $p = c^2 \Sigma$. Before introducing the planet, we set $c_s = c_{iso}$. When a planet is present, the sound-speed $c_s$ is prescribed as

$$c_s = \frac{h r \rho_{iso}}{[(h r)^{2/3} + (h \rho_{iso})^{2/3}]^{2/3} \sqrt{\Omega_{iso}^2 + \Omega_k^2}}, \quad (5)$$

where $\Omega_{iso}^2 = GM_p/d_p^2$, $d_p = \sqrt{(r - r_p)^2 + c_s^2}$ is the softened distance to the planet, $r_p$ being its vector position and $\epsilon_p$ is the softening length. The parameter $h_r$ controls the increase in sound-speed near the planet, relative to the $c_{iso}$ profile above, and is fixed to $h_r = 0.5$. The increase in sound-speed is $c_s/c_{iso} = 1.54$, 1.18 at $r_h$ and $2r_h$ away from the planet respectively, where $r_h = (M_p/3M_*)^{1/3} r_p$ is the Hill radius. Far away from the planet, the sound-speed becomes close to $c_{iso}$.

This EOS was proposed by Peplinski et al. (2008a) in a series of numerical simulations of type III migration in non-self-gravitating discs. The $c_{iso}$ profile, commonly used in disc-planet simulations, leads to mass accumulation near the planet and could lead to spurious torques from within the planet’s Hill radius. In a self-gravitating disc, it also causes the effective planetary mass $M'_p$, which is $M_p$ plus disc material gravitationally bound to the planet (Crida et al. 2004), to increase with the disc surface density scale (Lin & Papaloizou 2011b).

Equation 5 increases the temperature near $r_p$, thereby reducing the mass accumulation and effects above. Physically, disc material can be expected to heat up as it falls into the planet potential and provide a pressure buffer limiting further mass accumulation. The use of equation 5 in the present work is a simplified prescription for accounting for this. However, the gaps opened in a disc with $c_s = c_{iso}$ and with equation 5 are similar. The gap widths are identical and the gap depth is < 5% deeper in the case with $c_s = c_{iso}$.

2.2 Numerical methods

The hydrodynamic equations are integrated using the FARGO code (Masset 2000a,b, Baruteau & Masset 2008). The disc is divided into $N_r \times N_\phi = 1024 \times 2048$ zones in radius and azimuth, spaced logarithmically and uniformly respectively. The resolution is approximately 16 cells per $H$ (or 28 cells per $r_p$). The cells are nearly square ($\Delta r / \Delta \phi = 1.02$). An open boundary condition is applied at $r_1$ and a non-reflecting boundary condition at $r_o$ (as used by Zhang et al. (2008), see also Godon (1994)). Self-gravity is solved via a 2D Poisson integral, in which a softening length $\epsilon_p = 0.3H$ is set to prevent divergence and approximately account for the disc’s vertical thickness (Baruteau & Masset 2008).

The planet is introduced with zero mass at time $t = 20P_0$, where $P_0 = 2\pi/\Omega_k(r_p(t = 0))$ is the Keplerian period at the planet’s initial orbital radius. Its mass is then increased smoothly over 10$P_0$ to its final value $M_p$. The planet is then allowed to respond to disc gravitational forces for $t > 30P_0$. A standard fifth order Runge-Kutta scheme is used to integrate its equation of motion. The softening length for the planet potential is $\epsilon_p = 0.6H$.

3 GAP STABILITY

We first consider gap profiles in disc models with $Q_0 = 1.5, 1.7, 2.0$ to assess their stability. These correspond to Keplerian Toomre values at the planet’s initial radius of $Q_p = 2.77, 3.14, 3.70$ and total disc masses of $M_d/M_* = 0.080, 0.071, 0.060$, respectively. The discs are gravitationally stable to axisymmetric perturbations. For smooth radial profiles, they are also expected to be gravitationally stable to non-axisymmetric perturbations near $r_p$ because $Q_p > 1.5$.

The fundamental quantity for stability discussion in a structured barotropic disc is the vortensity (e.g. 1 Strictly speaking, our discs are not barotropic. However, the instabilities of interest are associated with an internal structure with characteristic thickness $H \ll r$, but the sound-speed varies on a global scale and can be approximated as constant, i.e. strictly isothermal and hence barotropic.
Papaloizou & Lin (1989; Lovelace et al. 1999)

\[ \eta = \frac{\kappa^2}{2\Omega \Sigma} \]  \tag{6}

where \( \kappa^2 = r^3 \frac{d(r^4 \Omega^2)}{dr} \) is square of the epicycle frequency and \( \Omega = \frac{v_\phi}{r} \) is the angular velocity. For gaps opened by a giant planet, the vortensity profile closely follows the Toomre \( Q \) profile when the latter is calculated with \( \kappa \) instead of \( \Omega \). Specifically, local extrema in \( \eta \) and \( Q \) occur approximately at the same radii (Lin & Papaloizou 2011a).

Lin & Papaloizou (2011a) have shown that planetary gaps in massive discs are unstable to global edge modes associated with \( \max(\eta) \) or \( \max(Q) \). For the adopted disc models, the background Keplerian Toomre parameter decreases with \( r \), i.e. the disc is more self-gravitating at larger radii. Thus edge modes are mostly associated with the outer disc \( (r > r_\text{p}) \), as found by Lin & Papaloizou (2011a).

Fig. 1 compares the relative surface density perturbation and the Toomre \( Q \) parameter for gaps in the above disc models. We expect edge modes to have corotation radius \( r_c \) at a vortensity maximum, or about \( 2r_h \) from the planet. This corresponds to a local minimum in relative surface density perturbation just inside the gap. Assuming the coorbital region of a giant planet is such that \( |r - r_h| \lesssim 2.5r_h \) (Artymowicz 2004; Paardekooper & Papaloizou 2009), \( r_c \) is just within the outer gap edge. This suggests the development of edge modes could bring over-densities into the coorbital region of the planet.

Edge modes also require coupling to the outer smooth disc via self-gravity, which is easier with lower \( Q_o \). The profiles here indicate edge modes will develop more readily with increasing disc mass and therefore have more significant effect on migration as \( Q_o \) is lowered.

Differences in gap profiles are also attributable to different effective planet masses, \( M'_p \). Without carefully choosing \( M_p \) and the EOS parameter \( \rho_p \), it is not possible to have exactly the same \( M'_p \) in discs of varying \( \Sigma_0 \). \( M'_p \) typically increases with increasing \( \Sigma_0 \), which promotes instability because gap edges become sharper. However, increasing surface density generally favours gravitational instability. Thus, planetary gaps in discs with decreasing \( Q_o \) are increasingly unstable, though perhaps more so than if \( M'_p \) is held fixed.

Giant planets can also induce fragmentation in self-gravitating discs, a phenomenon previously reported in SPH calculations (Armitage & Hansen 1999; Lufkin et al. 2004). Indeed, for a test run with \( Q_o = 1.3 \), we found the planetary wake, as well as the disc, fragments into clumps and no annular gap can be identified. In this case migration is expected to be strongly affected by clumps, instead of large-scale spiral arms which we will focus on below.

4 MIGRATION AND DISC STRUCTURE

The above discussion suggests that edge modes could lead to non-axisymmetric disturbances inside the gap. Torques may then originate from within the planet’s coorbital region. Because \( r_c \) is expected near the outer edge of the coorbital region, edge mode over-densities may undergo horseshoe turns upon approaching the planet. Furthermore, fluid elements just inside the separatrix will traverse close to the planet when executing U-turns, and this may provide significant torques. With this in mind, in this section we aim to identify the correlation between migration and gap evolution.

Fig. 2 shows the instantaneous orbital radius of the planet in the above disc models. Migration is non-monotonic and can be inwards or outwards on short timescales (\( \sim P_0 \)). However, with increasing disc mass, outward migration is favoured on timescales of a few 10’s of orbits. For \( Q_o = 1.5 \), \( r_p \) increases by 20% in only \( 70P_0 \). This is distinct from standard type II migration, which is inwards and occurs on much longer, viscous timescales (\( t_\nu = r^2/\nu = O(10^4 P_0) \) for our choice of \( \nu \)).

Fig. 3 shows the instantaneous disc-on-planet torques during the first \( 20P_0 \) after releasing the planet. Instabilities develop within this time frame and cause the torque to be positive or negative at a given instant. Up to about \( t = 40P_0 \),
the torques for \(Q_o = 1.5\) and \(Q_o = 1.7\) both show large and rapid oscillations in comparison to the \(Q_o = 2.0\) case. We shall see that this is related to edge modes developing for \(Q_o = 1.5, 1.7\) but not for \(Q_o = 2.0\).

4.1 Disruption of the outer gap edge

The behaviour of \(r_p(t)\) correlates with gap structure, particularly the outer gap edge. Fig. 4 shows the relative surface density perturbation when instabilities first develop. The least massive disc with \(Q_o = 2.0\) develops vortices rather than global edge modes (cf. \(Q_o = 1.7\)). This can be expected since edge modes require sufficiently strong self-gravity (Lin & Papaloizou 2011a).

In weakly or non-self-gravitating structured discs, unstable modes are associated with local vortensity minima (Lin & Papaloizou 2011b) and lead to vortex formation. The basic state (Fig. 4) shows that local min(\(Q\)), hence min(\(\eta\)), is located exterior to max(\(Q\)), and is most pronounced for \(Q_o = 2.0\). We did observe spiral arms to develop in \(Q_o = 2.0\) later on, but these probably resulted from the vortices perturbing the disc, rather than the linear edge mode instability. The important point with \(Q_o = 2.0\) is that instability leaves the outer gap edge intact and identifiable, unlike for more massive discs.

In Fig. 5 the \(Q_o = 1.7\) case develops a \(m = 2 - 3\) edge mode. Protrusion of the spiral arms into the gap makes the outer gap edge less well-defined. The surface density in the gap is on average higher in \(\phi > \phi_p\) than in \(\phi < \phi_p\). The edge mode spiral inside the gap and just upstream of the planet could provide a significant positive torque as the spiral pattern approaches the planet (from above in the figure). This is consistent with the large positive torque around the time of the chosen snapshot seen in Fig. 4. The outer gap edge for \(Q_o = 2.0\) is not as disrupted and this results in no secular increase in \(r_p\) for the simulated time for \(Q_o = 2.0\).

In Fig. 4 large-scale spirals can still be seen for \(Q_o = 1.5\). Weak fragmentation occurs without a collapse into clumps. Like \(Q_o = 1.7\), the gap is unclean with significant disruption to the outer gap edge. It is no longer a clear feature as for \(Q_o = 2.0\).

Figure 3. Instantaneous disc-on-planet torques. We have made this plot more comprehensible by multiplying the torques by a factor \(f = 1 - \exp(-|r - r_p|^2/r_p^2)\). This reduces local contributions from the Hill sphere but does not affect the important feature — rapid oscillatory torques — or their behaviour as a function of \(Q_o\). Despite tapering, instabilities still have significant impact on disc-planet torques.

Figure 4. Lin and Papaloizou

Figure 5. Running-time averages of gap properties: the dimensionless outer gap depth (top) and the gap asymmetry in units of Hill radius (bottom). These quantities are defined in

4.2 Gap evolution

In this section we use the following prescription to examine the evolution of the gap structure. We first calculate the azimuthally averaged relative surface density perturbation,

\[ \delta \Sigma(r) = \frac{\langle \Sigma - \Sigma(t = 0) \rangle}{\Sigma(t = 0)} . \tag{7} \]

We define the outer gap edge as \(r_{e,\text{out}} > r_p\) such that \(\delta \Sigma(r_{e,\text{out}}) = 0\), and similarly for the inner gap edge \(r_{e,\text{in}} < r_p\). The outer gap depth is defined as \(\delta \Sigma\) averaged over \(r \in [r_p, r_{e,\text{out}}]\). We also define the outer gap width, in units of the Hill radius, as \(w_{\text{out}} = |r_{e,\text{out}} - r_p|/r_h\) and the inner gap width as \(w_{\text{in}} = |r_{e,\text{in}} - r_p|/r_h\). The gap asymmetry is \(w_{\text{out}} - w_{\text{in}}\). Running time-averaged plots of these quantities are shown in Fig. 5.

Edge modes have a gap-filling effect. The case \(Q_o = 1.5\), for which the planet immediately migrates outwards, has the smallest magnitude of gap depth throughout. The \(Q_o = 2.0\) case has the deepest gap and is non-migrating over the simulation timescale. In the \(Q_o = 1.7\) case, the magnitude of gap depth decreases relative to that for \(Q_o = 2.0\) after about \(t = 80P_o\). Note that this corresponds to outward migration for \(Q_o = 1.7\). These observations suggest that material brought into the gap by edge modes, is responsible for outward migration.
Migration with unstable gaps

We have defined gap asymmetry as the distance from the planet to the outer gap edge minus the distance between the planet and the inner gap edge. It follows that the more negative the gap asymmetry is, the closer the planet is to the outer gap edge. Fig. 5 shows the planet is located closer to the outer gap edge in discs with lower $Q_o$ than in discs with higher $Q_o$. Furthermore, the non-migrating $Q_o = 2.0$ case reaches a constant asymmetry, whereas in the outwards-migrating cases asymmetry decreases with time.

Recall that edge mode spiral arms are associated with the outer gap edge. The trend above suggests that the planet is moving outer gap edge material inwards via horseshoe turns (this would also be consistent with shallower gaps with decreasing $Q_o$). This provides a positive torque on the planet. If on average this effect dominates over sources of negative torques, e.g. Lindblad torques or coincidence of an edge mode spiral arm with the outer planetary wake (Lin & Papaloizou 2011a), then the planet should on average migrate outwards, i.e. closer to the outer gap edge.

At this point, we found the surface density contrast ahead and behind the planet conforms to outward type III migration (Masset & Papaloizou 2003; Pepliński et al. 2008b). This may have resulted from the fact that edge modes in our discs supply over-densities ahead of the planet (see below), providing the initial condition, or kick, for outward runaway migration.

Once the planet is in type III migration, edge modes become irrelevant. This situation differs from migration sustained by edge mode spirals associated with the gap edge, where the planet can still be seen to reside in an annular gap. Henceforth we shall focus on the time frame in which this configuration holds ($t < \sim 130P_0$).

Fig. 4 shows the outer disc ($r > r_p$) is more unstable than the inner disc ($r < r_p$). Large-scale, coherent spirals are maintained throughout the simulation. The gap is unclean with material brought into it by the edge modes. Over-densities may lie within the planet’s coorbital region ($2.5r_h \geq r - r_p > 0$). Such material is expected to execute inward horseshoe turns and exert a positive torque on the planet.

The above effect causes the planet to move outwards, towards the outer gap edge where the surface density is higher. The planet can then interact with that material by moving it through inward horseshoe turns, providing further positive torque. The torque magnitude should also increase with the migration speed. These effects can result in a positive feedback akin to that seen in classic type III migration (Masset & Papaloizou 2003), which is consistent with the faster-than-linear increase in $a$ for $t < 130P_0$.

This interaction is local and depends primarily on the surface density, so the background Toomre $Q$ parameter, which decreases with radius on a global scale, is not expected to be of direct relevance in this feedback mechanism apart from its dependence on the surface density.
Figure 6. Orbital evolution of the planet in the $Q_o = 1.7$ disc, in terms of Keplerian semi-major axis $a$ (solid) and eccentricity $e$ (dotted). These have been calculated assuming Keplerian ellipses without accounting for the disc potential.

Figure 7. Overall evolution of the $Q_o = 1.7$ case. The logarithmic relative surface density perturbation $\log [\Sigma/\Sigma(t=0)]$ is shown.

5.1 Surface density asymmetry

In Fig. 8 we plot two interaction events between an edge mode spiral arm and the planet. In the first, we see that the spiral wave disturbance extends into within $2r_h$ upstream of the planet. This causes a surface density asymmetry ahead and behind the planet, the configuration here corresponding to positive coorbital torque. This is more apparent in the second event: over-density builds up just ahead of the planet as material undergoes inward horseshoe turns. The fluid shocks while executing the U-turn, since giant planets induce shocks close to their orbital radii (Lin & Papaloizou 2010). A stable gap would have been cleared of material by a Jovian mass planet and such an over-density ahead of the planet would not exist.

It is important to note that we have chosen the snapshots in Fig. 8 where the orbital radius $r_p$ increases, to demonstrate how edge modes can provide a positive torque. However, not every passage of the spiral arm increases $r_p$, as implied by the non-monotonic migration. An example is shown in Fig. 9. Since the outer planetary wake is associated with negative Lindblad torques, surface density perturbations due to edge modes may enhance it when the two overlap.

The overall outward migration implies positive torques produced by edge modes are on average more significant. This may not be surprising since the positive torque comes from material crossing $r_p$. This is also the mechanism for type III migration (Masset & Papaloizou 2003) which can be much faster than migration due to Lindblad torques.
Because the outer disc is more unstable, the corotation radius \( r_c \) of edge modes lies beyond \( r_p \) so its pattern speed is smaller than the planet’s rotation. Thus, over-densities associated with corotation approach the planet from upstream, but because \( r_c \) is actually within the planet’s coorbital region, the associated fluid elements are expected to execute inward horseshoe turns upon approaching the planet. This provides a positive torque on the planet, and if the edge mode amplitude is large enough, it may reverse the usual tendency for inward migration.

The picture outlined above is consistent with the fact that edge modes are have corotation at vortensity maxima. A giant planet can induce shocks very close to its orbital radius and vortensity is generated as fluid particles execute horseshoe turns across the shock. That is, vortensity maxima are associated with fluid on horseshoe orbits. So when non-axisymmetric disturbances associated with vortensity rings develop (i.e. edge modes), the associated over-densities can be expected to execute horseshoe turns.

### 5.3 Torque distribution

The analysis above suggests that torques from within the gap are responsible for the gradual increase in \( r_p \) or \( a \). Fig. 11 compares torque densities in the fiducial case to the non-migrating case. The torques have been averaged over 30 \( P_0 \) at two time intervals.

For \( t \in [50, 80]P_0 \) radial plots for both cases show a positive torque at \( r_p \). By \( t \in [80, 110]P_0 \) this torque has diminished for \( Q_o = 2.0 \). This is expected for Jovian planets as they open a clean gap leaving little material near \( r_p \) to torque the planet. However, for \( Q_o = 1.7 \), this torque is maintained (and even slightly increased) by the edge modes because they bring material into the co-orbital region. Note that the planet in the \( Q_o = 1.7 \) case is migrating outwards for \( t \in [80, 110]P_0 \), so migration itself may also contribute to sustaining the torque.

This torque is caused by over-density ahead in comparison to that behind the planet, i.e. material flowing radially inwards across the planet’s orbital radius, as seen in Fig. 8. However, unlike the single scattering events by vortices or spirals described in Lin & Papaloizou (2010, 2011), which dominate most of the migration, here one has to average over many spiral-planet interaction events to see the migration.

### 6 SUMMARY AND DISCUSSION

We have performed self-gravitating disc-planet simulations to see the effect of large-scale spiral modes associated with a gap opened by a giant planet on its migration. In our disc models, edge modes lead to outward migration over timescales of a few tens of initial orbital periods. This contrasts to standard type II inwards migration on viscous timescales (Lin & Papaloizou 1986). This difference demonstrates that gap instabilities significantly affect planetary migration in massive discs.

It is important to note that we have specifically considered unstable planetary gaps. This differs from disc models employed by Baruteau et al. (2011) and Michael et al. (2011), which are at least twice as massive as our fiducial case and develop gravitational instabilities without a planet.
Furthermore, the planet does not open a gap in their models. By contrast, gravitational activity in our discs are entirely due to the planet opening a gap. The instability then back-reacts on planet migration.

Baruteau et al. (2011) argued that a single giant planet in a massive, gravito-turbulent disc effectively undergoes type I migration, which resulted in rapid inwards migration in their models. This again contrasts to our less massive discs, which are not gravito-turbulent and the interaction we consider is that between a planet and large-scale spiral waves associated with gap edges.

In the planet’s frame, an edge mode spiral, with corotation at a vortensity maximum exterior to \( r_p \), approaches the planet from \( \phi > \phi_p \). However, the disturbance extends into the planet’s coorbital region, so there are associated over-densities that execute inward horseshoe turns, which provide a positive torque. The resulting outward migration means that this positive torque must on average exceed any negative torques.

In order to sustain this positive torque, edge modes need to be sustained to supply material to the planet for interaction. This in turn requires the existence of vortensity maxima, despite the development of edge modes destroying them. However, they are easily regenerated by giant planets because they induce shocks which act as a source of vortensity (Lin & Papaloizou 2010).

Planetary migration observed here closely resembles classic type III migration because it relies on torques from the coorbital region and the accelerated increase in semi-major axis indicates a positive feedback (Masset & Papaloizou 2003; Peplinski et al. 2008b). Accelerated migration may be expected because edge modes move the planet toward regions of higher surface density (the gap edges) so more material enters the planet’s co-orbital region. However, other outcomes are also conceivable (see §6.3).

The main difference from the type III migration originally described by Masset & Papaloizou is that the flow across the planet’s orbit is non-smooth, because edge modes provide distinct fluid blobs, rather than a continuous flow across \( r_p \). Our results above may be interpreted as a discontinuous runaway type III migration.

Also, where type III migration usually applies to partial gaps and therefore Saturn-mass planets, the Jovian mass planet used here resides in a much deeper gap. Radial mass flux across the gap is still possible, despite the gap-opening effect of Jovian planets, because the unstable edge modes protrude the gap edge.

Our understanding of the effect of edge modes is based on the geometry of the gap structure. That is, the existence of a vortensity maximum (equivalently \( Q \)-maximum) close to the gap edge, and its association with global spiral modes. Thus the mechanism we have identified, that edge modes bring material to the planet for interaction, should be a robust phenomenon for planetary gaps in massive discs.

### 6.1 Comparison to previous work

Migration here differs to scattering by an edge mode spiral arm described by Lin & Papaloizou (2011a), in which a single interaction increases the orbital radius of a Saturn mass planet by 40\% over a timescale of only \( 4P_0 \), moving it out of its gap (see their Fig. 20). By contrast, the 2 Jupiter mass planet simulated here migrates outwards by \( 20\% \) over a few 10’s of \( P_0 \), during most of which it remains in a gap, and there are multiple encounters with edge mode spiral arms.

In both cases the torque comes from material crossing the planet’s orbital radius, but interaction occurs more readily with the less massive Saturn mass planet since its gap-opening effect is weaker. Furthermore, the equation of state employed in the preliminary simulation of Lin & Papaloizou (2011a) allows high surface densities near the planet, potentially increasing the corotational torque magnitudes.

Note that if the planet is first allowed to move after edge modes develop, then the initial migration direction would depend on the relative position of the edge mode spiral with respect to the planet. This was the case in Lin & Papaloizou (2011a), the planet being released when a spiral arm is just upstream.

However, for the models discussed in this paper, edge modes do not develop before planet release (\( t = 30P_0 \)). This can be seen by the well-defined max(\( Q \)) , equivalently vortensity maximum, in the gap profiles in Fig. 1. The onset of edge modes would have destroyed this local maximum (Lin & Papaloizou 2011a, Fig. 14).

### 6.2 Implications

A consequence of the above is that formation of a clean gap is expected to become increasingly difficult in massive discs. Planetary gaps correspond to existence of vortensity maxima, but these become unstable to edge modes in massive discs and instability tends to fill the gap. Therefore the standard picture and formulae for type II migration should not
be applied to gap opening planets in massive protoplanetary discs.

We remark that migration is not only relevant to planets in protoplanetary discs. Analogous interactions have been discussed in the context of stars in black hole accretion discs (Kocsis et al. 2011, McKernan et al. 2011). Self-gravity could be important in outer regions of these discs, so our results may also be relevant to these situations.

6.3 Outstanding issues

Our focus in this paper was to identify the mechanism by which edge modes may torque up the planet. We have thus considered one particular disc model and only varied one parameter, the surface density scale. Although the simulation timescales were sufficient for our purpose, these are short compared to disc lifetimes.

We expect inter-dependencies between planet, disc properties and migration for the specific situation of a planet interacting with a gravitationally unstable gap, which was induced by it in the first place. In our fiducial simulation the planet eventually goes into classic type III migration. However, if the planet moves out and leaves its gap with a slow enough migration speed, it may open a new gap which develops edge modes and induces further outward migration, or even fragmentation. It is also conceivable for some disc parameters, the positive torque from edge modes are sufficiently small, so that it may be balanced by inwards type II migration on longer timescales. Possible long term outcomes need to be explored in a parameter study.

It should be noted that our adopted disc model is biased towards outward migration because the Toomre $Q$ decreases outwards (approximately like $r^{-1/2}$) so edge modes associated with the outer disc develop preferentially. This is consistent with the expectation that typical discs become more self-gravitating with increasing radius. However, if the disc model was such that the inner disc is more unstable, then edge modes should promote inward migration. And if independent edge modes of comparable amplitude develop on either side of the gap there could be little migration overall.

Significant torques originate from material close to the planet, so numerical treatment of the Hill sphere is a potential issue. This concern also applies to standard type III migration, but we remark that the adopted equation of state was originally designed for numerical studies of type III migration (Pepliński et al. 2008a). Furthermore, the fully self-gravitating discs considered here are what is required for accurate simulations of type III migration (Crida et al. 2009). As a check we have also performed lower resolution simulations which also resulted in outward migration.

This EOS mimics heating near the planet but is not a quantitative model for the true thermodynamics. If this EOS under-estimates the heating, then the positive torque could be over-estimated and vice versa. On the other hand, edge mode corotation resides outside the Hill radius but still inside the gap. Thus the numerical treatment within the Hill radius does not affect existence of edge modes and their associated over-densities should affect torques originating from the coorbital region in the way described in this paper.

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