Irregular Symmetrical Object Designed By Using Lambda Mu B-Spline Degree Four

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ABSTRACT. In Computer Aided Geometry Design (CAGD), B-splines curves are piecewise polynomial parametric curves that play an important role. CAGD involves the interpolation and approximation curves and surfaces. CAGD has been widely used which brings good impact of computers to industries in manufacturing. There are many improved methods in the B-spline curve such as extended cubic B-spline, trigonometric B-spline, quasi trigonometric B-spline, and λµ-B-spline. Each of the methods has its behaviour and advantage. In this paper, λµ-B-spline was used to be implemented in generating irregular symmetrical objects. λµ-B-spline has a shape parameter that can change the global shape by manipulating the value of the shape parameter. The bottle has been chosen as an irregular symmetrical object. The 2-dimensional symmetrical curves of Bottle design were formed by using λµ-B-spline degree 4. The curves designed are dependent on the shape parameter which can be adjusted. Then, the curves generated were revolved using the Sweep Surface method to form 3-dimensional objects. Every object has its volume and this research focused on the numerical method which was Simpson's 3/8 to compute the volume. The volumes obtained were compared to the actual volume to determine the best shape parameter used. The results show that the λµ-B-spline curve with a shape parameter of 1 is the best shape parameter in designing symmetrical irregular objects with the desired volume.

Keywords: Irregular symmetrical, simpson rule, sweep surface, λµ-B-spline

1. Introduction

In the past years, massive style and production industries are exploring economical Computer Aided Design (CAD) ways for the controlled generation of freeform curves and surface shapes. The use of systems like CAD has been widely used for the aim of making, modifying, analyzing, and optimizing any style [1]. Their target is to realize a mathematical model of an object to be utilized in its producing method [2]. A geometrical model of a form is extended to represent not solely its nominal dimensions but also tolerance info and surface specifications. Constraints are equivalent to the size of the technical drawings area unit applied to the structure, either by the designer or by an automatic dimension algorithmic rule. Within recent days, it was discovered that mathematics splines and polynomials are gaining interest in CAGD particularly in generating curves [3-5]. A 3-dimensional object can be formed by using λµ-B-spline such as vases and machinery parts [3-4]. In this study, the λµ-B-spline method was used. This method is the extension of interpolating B-spline by the singular blending technique. λµ-B-spline not only has local shape parameters but also global parameters. Besides inheriting their properties, λµ-B-spline curves has good performance in adjusting their local shapes by changing
multiple shape parameters [3, 4, 6]. The volume for each object is used to determine the equivalent size of the object formed with the actual object.

Bottle designs are chosen as an object of symmetrical irregular 3-dimensional. Numerous surfaces and shapes of bottles are designed up to the present day despite the high competition among the manufacturers. Throughout the assembly stages of bottles, the manufacturers and merchants should confront a demanding market and society until customer area unit is glad about the ultimate outcomes [7,8]. Bottles are often classified as aesthetic surfaces as they are in the main generated from 2-dimensional curves that are the components element in deciding the shapes and silhouettes of different industrial merchandise [9]. Furthermore, the shape of the bottles produced in the industry is being considered as most consumers prefer bottles with aesthetic designs but at the same time is easy to grip and user-friendly. Besides, the smoothness of the curves is vital particularly in designing bottles. To design curves, various methods can be implemented. As it is known by many, designing a bottle is not easy as the requirements of users must be considered as well. Although bottles are the object in this study, other symmetrical irregular objects are being commercialized in the market recently such as vases, skincare bottles, bowls and more using the same method.

In this paper, a 250ml bottle was chosen as the symmetrical irregular object. The curves of the bottles were formed by using \( \lambda \mu \)-B-spline with degree 4 with various shape parameter values. By applying the technique of revolution in the Sweep Surface method, various designs of 3-dimensional bottles can be generated. Volume for all objects formed was calculated by using Simpson's 3/8 rule with a particular subinterval. The design that best fit the actual volume was chosen as the best method to be used in designing a symmetrical irregular 3-dimensional object.

2. Methodology
The \( \lambda \mu \)-B-spline degree 4 with various shape parameter values was used to form 2-dimensional curves. Then, a 3-dimensional object was generated by using the revolution technique of the sweep surface method. Volume for each object was determined by using Simpson's 3/8 rule. Mathematica and Maple software were used in this calculation.

2.1 \( \lambda \mu \)-B-spline
A \( \lambda \mu \)-B-spline curve is defined as the quartic blending functions with two parameters \( \lambda \) and \( \mu \) and generated by four consecutive control points and designed using various parameter values. The basis function of \( \lambda \mu \)-B-spline with degree 4, are as follows [3, 4]:

\[
\begin{align*}
    b_0^4(t) &= \frac{1}{6}(1-\lambda,t)(1-t)^3 \\
    b_1^4(t) &= \frac{1}{6}(t-6,t^2+(3+\mu,t)k^3-\mu,t^4) \\
    b_2^4(t) &= \frac{1}{6}(t+(3+\lambda,t)+3(1-\lambda,t)k^2-3(1-\lambda,t)k^3-\lambda,t^4) \\
    b_3^4(t) &= \frac{1}{6}(1-\mu,t)(1-t)^3
\end{align*}
\]

(1)

The basis function of \( \lambda \mu \)-B-spline have the following properties [3, 4]:

a) Degeneracy: In the case where the shape control parameter, \( \lambda \) and \( \mu \) are equal to zero, the \( \lambda \mu \)-B basis functions are just the classical ones of the same degree.

b) Convex hull: When \( 2 \leq \lambda_i, -1 \geq \mu_i \) there are \( b_j^4(t) \geq 0 \ (j = 0, 1, 2, 3) \).
c) Partition of Unity: One has \[ \sum_{i=0}^{n} b_i^j(t) = 1 \]...

d) Symmetry: When \( \lambda_i = \mu_i \), \( b_i^j(t) \geq 0 \) \( (j = 0, 1, 2, 3) \) are symmetric, that is \( b_0^j(t) = b_3^j(1-t) \) and \( b_1^j(t) = b_2^j(1-t) \).

Given control points \( P_i \in \mathbb{R}^d \) \( (n = 2, 3; \ i = 0, 1, 2, \ldots, n) \) and knots \( u_0 < u_1 < \ldots < u_{n+1} \), the curves

\[
c_i(t; \lambda_i, \mu_i; t) = \sum_{j=0}^{n-3} P_{i+j-3} b_i^j(t), \quad t \in [0,1], \quad i = 3, 4, \ldots, n
\]

(2)
are call quartic \( \lambda \mu \)-B-spline curve segments where \( b_i^j(t) \) \( (j = 0, 1, 2, 3) \) are \( \lambda \mu \)-B-spline functions. All the curve segments make up the piecewise quartic blending spline curves with shape parameters \( \lambda_i \) and \( \mu_i \), defined as [3]:

\[
c_i(t; \lambda_i, \mu_i; u) = c_i(t; \lambda_i, \mu_i; \frac{u - u_i}{\Delta u_i}), \quad u \in [u_i, u_{i+1}] \subseteq [u_3, u_{n+1}]
\]

(3)
where \( \Delta u_i = u_{i+1} - u_i, \ i = 3, 4, \ldots, n \).

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**Figure 1.** The curve of \( \lambda \mu \)-B-spline degree 4 with different shape parameters.
Figure 1 shows a closed curve of $\lambda\mu$-B-spline degree 4 plotted using four control points $\{(0,0),(1,1),(4,1),(5,0)\}$ and different values of $\lambda$ and $\mu$. All the curves do not start and end at the control point. The shape parameter used in (a) is in range while (b) and (c) is out of range. Based on the properties of $\lambda\mu$-B-spline, the curve possesses a convex hull when $2 \leq \lambda, \mu \leq 1$. However, (b) shows the curve still possesses convex hull even though the value of $\lambda$ and $\mu$ are out of range. The distance of curve in (b) is closest to the control polygon compared to the curve in (a). The curve in (c) is not a convex hull.

2.2 Revolution technique by using Sweep Surface Method

A special case of a swept surface is known as a surface of revolution in which it is to be attained from rotating the space curve, also known as the profile of the surface along the axis of rotation. According to Salamon (2006), the rotation angle can be varied, whether complete $360^\circ$ or less. In 3-dimensions, a general revolution is completely specified by the axis of rotation, which is a vector as well as the rotation angle, which is a number. With $\theta$ being the rotational angle and $r$, being considered as the rotation axis (as a unit vector), it can be deduced that the rotation matrix $T(\theta)$ about $u$ is denoted by [10]:

$$
\begin{pmatrix}
  r_x^2 + \cos \theta (1-r_x^2) & r_x r_y (1-\cos \theta) + r_y \sin \theta & r_x r_z (1-\cos \theta) - r_z \sin \theta \\
  r_x r_y (1-\cos \theta) + r_y \sin \theta & r_y^2 + \cos \theta (1-r_y^2) & r_y r_z (1-\cos \theta) + r_z \sin \theta \\
  r_x r_z (1-\cos \theta) - r_z \sin \theta & r_y r_z (1-\cos \theta) + r_z \sin \theta & r_z^2 + \cos \theta (1-r_z^2)
\end{pmatrix}
$$

(4)

When the space curve is conveyed by the expression $P(u)$ in which the range is $0 \leq u \leq 1$, therefore the surface of revolution will consist of the form $P(u, \theta) = P(u)T(\theta)$, where $0 \leq u \leq 1$ and $0 \leq \theta \leq 2\pi$. With different values of $u$ and $\theta$, the movement will be along the curve and in a circular motion or arc about the axis of rotation, respectively.

If the angle of revolution is expressed as $w$ and revolved along the axis $r = (r_x, r_y, r_z)$, then it can be computed that the matrix revolution $T(w)$ is as follows in their respective axis, $x$, $y$ and $z$. Transformation $T(w)$ is a $3 \times 3$ matrix and the expression below is known as Matrix Revolution $T(w)$.

$$
T_x(w) = \begin{bmatrix}
\cos w & \sin w & 0 \\
-\sin w & \cos w & 0 \\
0 & 0 & 1
\end{bmatrix}
$$

(5)

$$
T_y(w) = \begin{bmatrix}
\cos w & 0 & \sin w \\
0 & 1 & 0 \\
-\sin w & 0 & 1
\end{bmatrix}
$$

(6)
The surface is obtained with the use of the following equation [10].

\[
T(w) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos w & \sin w \\ 0 & -\sin w & \cos w \end{bmatrix}
\]

(7)

where, \( P(t, w) = \) Revolution of Surface, \( P(t) = \) Cross Section and \( T(w) = \) Revolution of Matrix.

2.3 Simpson’s 3/8 rule

The computation of area and volume of irregular structures has become a subject of interest since it finds its application in several areas like agriculture and food processing, healthcare and many more. Simpson is one of the numerical methods used to approximate the definite integral. Simpson’s 3/8 rule is also known as the 3/8 Newton-Cotes formula. To be specific, Simpson's rule follows approximation for \( n+1 \) equally spaced subinterval where \( n \) is an even number. In this paper, Simpson’s 3/8 rule was used to determine the volume of the bottle's design. The Simpson method's approach would give the most accurate results. This is the reason why this method is widely used [11, 12].

The formula of Simpson’s 3/8 rule is as follows:

i. Formula to find area:

\[
A_{3/8} = \frac{3h}{8} (d_0 + 3d_1 + 3d_2 + 2d_3 + 3d_4 + 2d_5 + 3d_6 + 2d_7 + \ldots + d_n)
\]

(9)

ii. Formula to find volume:

\[
V_{3/8} = \frac{3h}{8} (A_0 + 3A_1 + 3A_2 + 2A_3 + 3A_4 + 3A_5 + 2A_6 + \ldots + A_n)
\]

(10)

Where \( d = \) common distance, \( A = \) area, \( h = \) step size \( n = 0, 1, 2, 3 \ldots \)

3. Result and Discussion

3.1 Bottle’s design in 2-dimensional and 3-dimensional

Various 3-dimensional bottle designs can be formed by revolving 2-dimensional \( \lambda \mu \)-B-spline degree 4 with different shape parameters \( \lambda \) and constant values for \( \mu \). The process of designing all the objects was implemented by using mathematical software Mathematica. The results were classified and analyzed to choose the best method in designing curves as well as 3-dimensional symmetrical irregular objects.
The figure above shows all the curves that were generated by using the $\lambda\mu$-B-spline method with shape parameter, $\lambda = 10, 2, 1, 0, -2, \text{ and } -7$. Control points were obtained from the actual object by mapping the object on graph paper. Then, the 3-dimensional bottle design was formed by revolving the curve at the x-axis by using the sweep surface method. It is clearly shown that curves and objects formed were smooth for $\lambda = 0, 1, \text{ and } 2$. The curve was also close to the control polygon. As every point on curve is located inside the control polygon, the curve possessed convex hull property. While for $\lambda = 10, -2, \text{ and } -7$, the curves and objects formed were not smooth. Tangled knots were visible on the bottle design when applying $\lambda = 10 \text{ and } -7$. Hence, it did not satisfy the shape needed. As for the distance between the curve and the control polygon, the curve generated by applying $\lambda = 1$ was the closest compared to the other five curves.

### 3.2 Approximation volume

The volumes of each object designed by using $\lambda\mu$-B-spline were calculated by using numerical approaches which was Simpson’s 3/8 according to the subintervals chosen which were 6 and 18. The results are tabulated as follows.

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**Figure 2.** Result of designing 2-dimensional curve by using $\lambda\mu$-B-spline method and bottle design in 3-dimensional
Table 1. Approximation of volume for Simpson’s 3/8 Method for symmetrical irregular objects designed by $\lambda\mu$-B-Spline

| Shape parameter | Sub-interval, m | Approximated volume $(cm^3)$ |
|-----------------|-----------------|-----------------------------|
| $\lambda$       | $\mu$           |                             |
| -7              | 0               | 6                           | 197.4698 |
|                 |                 | 18                          | 219.9295 |
| -2              | 0               | 6                           | 187.1884 |
|                 |                 | 18                          | 221.0563 |
| 0               | 0               | 18                          | 190.5302 |
|                 |                 | 6                           | 204.7851 |
| 1               | 0               | 18                          | 223.5728 |
|                 |                 | 6                           | 191.0062 |
| 2               | 0               | 18                          | 220.6216 |
| 10              | 0               | 6                           | 203.0126 |
|                 |                 | 18                          | 218.9957 |

Based on the result, it can be extracted from the tables that all shape parameters were suitable to be used, however not all would compute a volume closest to the actual one. From the table 1 above, it can be observed that for this method of designing curves, $\lambda\mu$-B-spline, the shape parameter value which possessed volumes closest to the actual one was 1. From analyzing the results obtained, it can be summarized that the $\lambda\mu$-B-spline method held the theoretical hypothesis in which the higher the number of subintervals, the more accurate the results will be [9].

For each method and each shape parameter, the percentage error could be defined by comparing the actual data with the approximated data. The shape parameter with the smallest percentage relative error would determine the best shape parameter in generating symmetrical irregular objects with the desired volume. Table 2 below shows the percentage error when all the cases were considered.

Table 2. Percentage of error for volume approximation for Simpson’s 3/8 method

| Shape parameter | Sub-interval, m | Percentage of error (%) |
|-----------------|-----------------|-------------------------|
| $\lambda$       | $\mu$           |                         |
| -7              | 0               | 6                       | 21.01 |
|                 |                 | 18                      | 12.03 |
| -2              | 0               | 6                       | 25.12 |
|                 |                 | 18                      | 11.58 |
| 0               | 0               | 18                      | 11.35 |
|                 |                 | 6                       | 18.09 |
| 1               | 0               | 18                      | 10.57 |
|                 |                 | 6                       | 23.60 |
| 2               | 0               | 18                      | 11.75 |
|                 |                 | 6                       | 18.79 |
| 10              | 0               | 18                      | 12.40 |

Based on the table 2 above, the percentage error decreased as the intervals increased from 6 to 18. For the results with subinterval 18, it can be concluded that the best shape parameter with the smallest value was 1 which was 10.57%, while for the result with subinterval 6, also the smallest value was shape parameter of 1 with 18.09%.
4. Conclusion
Various shapes of bottle design can be formed by revolving 2-dimensional $\lambda\mu$-B-spline curves degree 4 with different values of shape parameter such as $\lambda = 10, 2, 1, 0, -2, and -7$ using sweep surface method. The volume for each bottle design could be determined by using Simpson’s 3/8. The subintervals that have been considered to approximate the volume were 6 and 18. The relative error was also calculated to determine which object had its volume closest to the actual one.

A comparison was made for all the shape parameters that have been implemented including the difference that each curve, designed by using $\lambda\mu$-B-spline, possessed. The behaviours are discussed in detail based on each shape parameter value used. The objects were also observed in terms of the designs and smoothness. The results from the approximation of volume play a vital role as it helps determine the best method in designing symmetrical irregular objects. From the results obtained after the execution of the coding, the behaviour of each curve was observed in terms of the smoothness and the distance between the curves and the control polygon. Thus, the first objective is to design a 2-dimensional irregular curve by using $\lambda\mu$-B-spline method was satisfied. The best value of shape parameter used was $\lambda = 1$ when $\mu = 0$.

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