Brillouin propagation modes in optical lattices: interpretation in terms of nonconventional stochastic resonance

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We report the first direct observation of Brillouin-like propagation modes in a dissipative periodic optical lattice. This has been done by observing, in both theoretical and experimental work, a resonant behaviour of the spatial diffusion coefficient in the direction corresponding to the propagation mode with the phase velocity of the moving intensity modulation used to excite these propagation modes. Furthermore, we show theoretically that the amplitude of the Brillouin mode is a nonmonotonic function of the strength of the noise corresponding to the optical pumping, and discuss this behaviour in terms of nonconventional stochastic resonance.

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The last decade has witnessed dramatic progress in laser cooling techniques and nowadays in several laboratories around the world atoms are routinely trapped and cooled at very low temperatures and high densities [1]. Most of the current efforts within the cold atoms community are directed to reaching the regime of quantum degeneracy in both bosonic and fermionic samples, in order to investigate the properties of the quantum gases thus obtained, and realizing an atom laser, the matter wave analog of the laser. Cold atomic samples also constitute an ideal system for the study of complex nonlinear phenomena. This turns out to be especially true if the cold atoms are ordered by the light fields in periodic structures, so-called optical lattices [2,3]. These are obtained by the interference of two or more laser fields: the light imposes its order on the matter via the dipole force [4], creating a periodic structure of atoms.

Among the most significant studies of nonlinear dynamics in optical lattices, we recall here the observation of mechanical bistability in a strongly driven dissipative optical lattice [5] and the realization of the kicked rotor and corresponding detection of chaotic motion in a far detuned lattice [6]. Furthermore the macroscopic transport of atoms in an asymmetric optical lattice without the application of external forces has been observed [7]. This corresponds to the realization of an optical motor, i.e. a ratchet for cold atoms, a well-controlable model system for the molecular combustion motor [8]. Brillouin-like propagation modes of atoms in a dissipative optical lattice have also been theoretically studied to explain the nonlinear optical properties of optical lattices [9,10]. In this Letter, we report on the first direct observation of these modes. Furthermore we will discuss the propagation mechanism associated with these modes, completely different from the one encountered in dense fluids or solid media. Indeed in dilute optical lattices the interaction between the different atoms is completely negligible, therefore the mechanism for the propagation of atoms cannot be ascribed to any sound-wave-like mechanism. While a sound wave corresponds to a propagating density wave without a net transport of atoms, the Brillouin-like resonances analyzed in this work consist of a net motion of the atoms. In fact, the propagation of atoms through the lattice are determined here by the interaction with the light. The light fields determine both the potential wells where the atoms can oscillate, and whose vibrational frequency determines the velocity of the propagation modes, and the escape from the potential wells, which allows the propagation of atoms. We also show theoretically that the amplitude of the Brillouin mode is a nonmonotonic function of the strength of the noise corresponding to the optical pumping, and discuss this behaviour in terms of nonconventional stochastic resonance [11,12].

FIG. 1. Sketch of the experimental setup.

We consider a three dimensional (3D) linear near resonant optical lattice, as in previous work [13]. The periodic structure is determined by the interference of
four linearly polarized laser beams, arranged as in Fig. 1. This arrangement results in a periodic modulation of the light polarization and light intensity, which produces a periodic modulation of the light shifts of the different ground states of the atoms (optical potentials) 9. The optical pumping between the different atomic ground states combined with the spatial modulation of the light shifts leads then to the cooling of the atoms 14 and to their localization 13 at the minima of the optical potentials, thus producing a periodic array of atoms.

After the cooling phase, characterized by a significant reduction of the atomic kinetic temperature and by the creation of a periodic spatial order, the atoms keep interacting with the light undergoing optical pumping cycles. The optical pumping may transfer an atom from a potential well to a neighbouring one, giving rise to a variety of transport phenomena 13,16. Among these, there are modes which correspond to the propagation of atoms through the optical lattice in a given direction. They consist of a sequence in which one half oscillation in a potential well is followed by an optical pumping process to the neighbouring well, and so on. One can estimate their velocity by \( v_i = \lambda_i \Omega_i / (2\pi) \) where \( \lambda_i \) is the lattice constant and \( \Omega_i / (2\pi) \) the vibrational frequency in the \( i \)-direction 14. These modes were first identified through Monte-Carlo simulations in Ref. 10 and shown to produce resonance lines in the nonlinear optical response of optical lattices. However up to now no direct observation of these modes has been reported. This is achieved in the present work by observing a resonant behaviour of the spatial diffusion coefficient in the direction corresponding to the propagation mode with the phase velocity of the moving intensity modulation used to excite these propagation modes.

The modulation scheme for the excitation of the propagation modes is completely analogous to the one used in previous investigations of the nonlinear optical response of optical lattices 13. An additional laser field linearly polarized along the \( y \)-axis is introduced with the \( z \)-axis as propagation direction. This probe field interferes with the copropagating lattice beams, creating an intensity modulation. The interference pattern consists of two propagating intensity waves moving with phase velocities \( \vec{v}_j = \vec{n}_j \delta / (\Delta \vec{k}_j) \) \( (j = 1, 2) \) with \( \vec{n}_j = \Delta \vec{k}_j / |\Delta \vec{k}_j| \), and \( \Delta \vec{k}_j = \vec{K}_j - \vec{K}_p \) the difference between the \( j \)-th lattice beam and the probe (\( p \)) wavevectors 14. Here \( \delta = \omega_p - \omega_L \) is the detuning between the probe (\( \omega_p \)) and the lattice (\( \omega_L \)) frequencies. According to the numerical simulations for the atomic trajectories presented in Ref. 14, for \( \delta = \pm \Omega_x \), the propagation modes along \( x \) are excited by the driving field, with the atoms effectively dragged by the moving intensity modulation 14. Intuitively, the dragging of atoms by the two propagating intensity modulations should result in an increase of the spatial diffusion coefficient \( D_x \) in the \( x \)-direction. Therefore it should be possible to detect these Brillouin propagation modes by monitoring \( D_x \) as a function of the detuning \( \delta \). The propagation modes are then revealed by a resonance in \( D_x \) around \( \delta = \pm \Omega_x \). We tested the validity of this reasoning with the help of semiclassical Monte-Carlo simulations 15. Taking advantage of the symmetry between the \( x \) and \( y \) directions (see Fig. 1), we restricted the atomic dynamics in the \( xOz \) plane. Our calculations are for a \( J_p = 1/2 \to J_x = 3/2 \) transition, as customary in numerical analysis of Sisyphus cooling, of an atom of mass \( M \). We expect our 2D calculations to reproduce the dependencies of the different quantities associated with the real 3D atomic dynamics to within a scaling factor corresponding to the difference in dimensionality 10. In the numerical simulations, we monitored the variance of the atomic position distribution at a given value of the probe field detuning. We verified that the spatial diffusion is normal, i.e. the atomic square displacements \( \langle \Delta x^2 \rangle \) and \( \langle \Delta z^2 \rangle \) increase linearly with time. Accordingly, we derived the spatial diffusion coefficients \( D_x \) and \( D_z \) by fitting the numerical data with \( \langle \Delta x^2 \rangle = 2D_x \Delta t \) \( (x = x, z) \). Results for the spatial diffusion coefficients as functions of the probe detuning \( \delta \) are shown in Fig. 2. Two narrow resonances, centered approximately at \( \delta = \pm \Omega_x \), appear clearly in the spectrum of the diffusion coefficient along the \( x \)-axis. In contrast, \( D_z \) does not show any resonant behaviour with the driving field detuning. This demonstrates the validity of the detection scheme based on the measurement of the diffusion coefficients.

![FIG. 2. Numerical results for the spatial diffusion coefficients in the \( x \) and \( z \) directions as functions of the probe field detuning. The lattice beam angle is \( \theta = 30^\circ \), the lattice detuning \( \Delta = -10 \Gamma \) and the light shift per beam \( \Delta_0 = -200 \omega_r \). Here \( \Gamma \) and \( \omega_r \) are the width of the excited state and the atomic recoil frequency, respectively. The amplitude of the probe beam is 0.4 times that of each lattice beam.](image-url)

In the experiment \(^85\)Rb atoms are cooled and trapped in a magneto-optical trap (MOT). The MOT laser beams and magnetic field are then suddenly turned off. Simultaneously the four lattice beams are turned on and after 10 ms of thermalization of the atoms in the lattice the probe laser field is introduced along the \( z \)-axis. We studied the
transport of atoms in the optical lattice by observing the atomic cloud expansion with a Charge Coupled Device (CCD) camera. The procedure to derive the diffusion coefficients has been described in detail in previous work [3], and we recall here only the basic idea. For a given detuning of the probe field we took images of the expanding cloud at different instants after the atoms have been loaded into the optical lattice. From the images of the atomic cloud we derived the atomic mean square displacement along the \( x \) - and \( z \)-axes.

![Graphs](image)

**FIG. 3.** Experimental results for the spatial diffusion coefficients in the \( x \) and \( z \) directions as functions of the probe field detuning. The experimental parameters are: lattice detuning \( \Delta/(2\pi) = -42 \text{ MHz} \), intensity per lattice beam \( I_L = 3.5 \text{ mW/cm}^2 \), lattice angle \( \theta = 30^\circ \). These parameters correspond to a vibrational frequency in the \( x \)-direction \( \Omega_x/(2\pi) \approx 55 \text{ kHz} \). A probe transmission spectrum is reported for comparison, \( T \) and \( T_0 \) being the intensity of the transmitted probe beam with and without the atomic cloud. The two resonances at \( \delta = \pm \Omega_x \) correspond to stimulated light scattering by the Brillouin propagation modes. The two resonances at larger detuning are Raman lines in the \( z \)-direction (\( \delta = \pm \Omega_z \)), which do not correspond to propagation modes. For the measurements of the diffusion coefficients, the probe beam intensity is \( I_p = 0.3 \text{ mW/cm}^2 \); in the transmission spectrum \( I_p = 0.1 \text{ mW/cm}^2 \).

We verified that the cloud expansion corresponds to normal diffusion and derived the diffusion coefficients \( D_x \) and \( D_z \). The procedure has been repeated for several different values of the detuning \( \delta \) of the probe field. Results for \( D_x \) and \( D_z \) as functions of \( \delta \) are shown in Fig. [3]. The probe transmission spectrum is also reported to allow the comparison of the position of the resonances in the spectrum of the diffusion coefficients and in that of the probe transmission. We observe two narrow resonances in the diffusion coefficient along the \( x \)-axis centered at \( \delta = \pm \Omega_x \). In contrast, no resonant behaviour of \( D_x \) with \( \delta \) is observed. This is in agreement with the numerical simulations and constitutes the first direct observation of Brillouin-like propagation modes in an optical lattice.

We turn now to the analysis of the mechanism behind these propagation modes. Brillouin-like propagation modes have been widely studied in condensed matter and dense fluids [2]. However, in the present case the mechanism associated with these modes is clearly of a different nature, as in dilute optical lattices the interaction between atoms is negligible and therefore soundwave-like propagation modes cannot be supported. On the contrary, in a dilute optical lattice the propagation of the atoms is determined by the synchronization of the oscillation within a potential well with the optical pumping from a well to a neighbouring one, as first identified in the numerical analysis of Ref. [4]. This dynamics can be interpreted in terms of noise-induced resonances: the probe field induces a large scale moving modulation of the periodic potential of the four-beam optical lattice, with the optical pumping constituting the noise source which allows transfer from a well to a neighbouring one. It is then natural to investigate the dependence of the amplitude of the Brillouin mode on the strength of the noise, i.e. on the optical pumping rate. We studied, via semiclassical Monte-Carlo simulations, the atomic cloud expansion for a given depth of the potential well at different values of the optical pumping rate \( \Gamma_p \), proportional to the rate \( \Gamma_{\text{esc}} \) of escape from the well [2]. This has been done by varying the lattice intensity \( I \) and detuning \( \Delta \) so as to keep the depth of the potential wells \( U_0 \propto I/\Delta \) constant while varying \( \Gamma_p \propto I/\Delta^2 \). The diffusion coefficient in the \( x \)-direction has been calculated both for a probe field at resonance (\( |\delta| = \Omega_x \)) and for a probe field far off-resonance (\( |\delta| \gg \Omega_x \)). The two diffusion coefficients will be indicated by \( D_x \) and \( D^0_x \) respectively. To characterize quantitatively the response of the atomic system to a noise strength variation, we introduce the enhancement factor \( \xi \) defined as

\[
\xi = \frac{D_x - D^0_x}{D^0_x}.
\]

Numerical results for the enhancement factor \( \xi \) as a function of the optical pumping rate at a given value of the potential well depth (i.e. for fixed light shift per beam \( \Delta_0 \)) are shown in Fig. [4]. At small pumping rates, \( \xi \) increases abruptly with \( \Gamma_p \); then a maximum is reached, corresponding to the synchronization of the oscillation of the atoms within a well with the escape from a well to the neighbouring one; finally at larger pumping rates this synchronization is lost and \( \xi \) decreases. This dependence
recalls the typical behaviour of stochastic resonance \cite{12}, with the noise enhancing the response of the atomic system to the weak moving modulation. It should be noted that the system analyzed here has one important peculiarity with respect to the model usually considered in the analysis of stochastic resonance. Stochastic resonance is in general understood as the noise-induced enhancement of a weak periodic signal with a frequency much smaller than the intrawell relaxation frequency within a single metastable state. In contrast, in the present case, the noise synchronizes precisely with the intrawell motion of the atoms. This corresponds to a nonconventional stochastic resonance scenario \cite{12}.

In summary, in this Letter we introduced a scheme for the detection of Brillouin propagation modes in optical lattices and we reported on their direct observation. Furthermore, we studied via Monte-Carlo simulations the behavior of the probe field, as a function of the optical pumping rate, for a given depth of the optical potential wells. Parameters for the calculations are: $\Delta_0 = -50\omega_r$ and $\theta = 30^\circ$. The two data sets correspond to different intensities of the probe beam. For comparison, we recall that in the experiment (Fig. 3) $\Delta_0 \approx -60\omega_r$ and $\Gamma_0 \approx 8.5\omega_r$.

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\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure4.pdf}
\caption{Numerical results for the enhancement factor $\xi$ as a function of the optical pumping rate, for a given depth of the optical potential wells. Parameters for the calculations are: $\Delta_0 = -50\omega_r$ and $\theta = 30^\circ$. The two data sets correspond to different intensities of the probe beam. For comparison, we recall that in the experiment (Fig. 3) $\Delta_0 \approx -60\omega_r$ and $\Gamma_0 \approx 8.5\omega_r$.}
\end{figure}

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