CALCULATIONS OF MESON MASS SPECTRA IN THE MODEL OF QUASI-INDEPENDENT QUARKS

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Abstract

A method of calculations of meson mass spectra in the framework of the model of quasi-independent quarks is discussed. Meson mass spectra evaluated with the help of the Dirac equation with the quasi-Coulombic and confinement potentials, as well as with the help of phenomenological mass formulae, are presented. Parameters of the quasi-independent quarks model, which are calculated on the basis of mass values of pseudoscalar and vector mesons, are given. Possible relations between parameters of the quasi-independent quarks model and constants of the Standard model of strong and electroweak interactions are considered.

1. Introduction

It is well known that the calculation of hadron mass spectra on the level of experimental data precision [1] still remains among the unsolved QCD problems due to some technical and conceptual difficulties, which are related mainly with the nonperturbative effects, such as confinement and spontaneous chiral symmetry breaking. For instance, although recently a considerable progress in lattice QCD was achieved, at present the calculations of hadron mass values on the basis of the first principles of QCD demand too much computation time especially for the light hadrons. So data interpretations and calculations of hadron characteristics are frequently carried out with the help of phenomenological models. Moreover the identification of hadrons bearing constituents, distinguishable from standard ones, now is the topical problem for strong interaction physics. For this purpose a detailed description of characteristics of standard hadrons is needed. Especially it concerns the evaluation of masses for ground and exited hadron states with light quarks and/or antiquarks.

One of the most interesting and simple hadron models is the relativistic model for quasi-independent quarks. The main statement of this model is that hadron’s properties can be described considering the hadron as a system of independent constituents (or quasi-independent ones with weak residual interactions), which move in some mean self-consistent field [2]. For the description of constituent’s motion the one-particle equation (Schrödinger, Dirac or Klein-Gordon-Fock equation) can be used. The confinement of color particle is available by using of a linear rising potential, for which the hypothesis of its flavour independence has been proposed [3, 4, 5]. The relativistic model for quasi-independent quarks has been applied with the particular potential entered in the Dirac equation for the description of meson properties in Refs. [6, 7]. The potential used is a generalization of Cornell potential and consist of a relativistic vector quasi-Coulomb potential and a relativistic scalar linear rising potential. In the framework of this model

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the hypothesis of universality of confinement potential offered in Refs. [3, 4, 5] has been confirmed for heavy and light quarks. The coefficient of linear rising scalar potential (string tension) has been found to be $\sigma = 0.20 \pm 0.01 GeV^2$. Below we present the results of calculations of mass spectra of mesons and model parameters, which may be related directly to constants of Standard Model. In order to obtain the consistent treatment of $1^{--}$ and $0^{--}$ mesons it is necessary to take into account spin-spin interaction between quark and antiquark as well as the values of the meson mean field energy [7]. Note that the account of these terms are most important in the case of the pseudoscalar mesons. We made a link of quasi-independent quark model with the constituent quark model and propose that the values of the meson mean field energy belong equidistant energy levels. We present also mass spectra of excited meson states with $I = 1$ consisting of $u-$, $d-$quarks and antiquarks with help of analytical mass formulae, which have the structure peculiar for mass formulae of the independent quark model. These formulae may be considered as the generalisations of the Chew-Frautschi and Nambu-Veneziano relations for the Regge trajectories of superlight mesons (mesons which consist from $u-$, $d-$quarks and antiquarks only). We compare results obtained with existing data and make predictions for mass values of unobserved meson states. Besides that, we consider possible structures of some mesons which are under discussion at present, such as scalar mesons and vector excited mesons.

The report is organized as follows. We briefly review the basic equations and the main statements of the model in Section 2. Here the Dirac equation in the external potential with the Lorentz scalar and vector parts is transformed into a form suitable for numerical calculations. The values of model parameters and meson masses are presented in Section 3. In Section 4 the phenomenological mass formulae for isovector radially and orbitally exited $qq'$-mesons are presented and the the results of calculations for mass spectra of exited mesons are displayed. In Section 5 the model of quasi-independent quarks is related to the constituent quarks model. We discuss the results obtained and the problems of identification of some meson states in vector and scalar channels in the last section.

2. Basic statements of the relativistic independent quark model

According to the main statement of the independent-quark model a hadron is considered as a system composed of a few non-interacting with each other directly, valency constituents (quarks, diquarks and constituent gluons) having the coordinates $r_i$, $i = 1, ..., N$, and moving in some mean field. We suppose that this field is a colour singlet confining field which is produced by the constituents and nonperturbative QCD vacuum and takes into account the effects of creation and annihilation of a sea of $q\bar{q}$ pairs as well. Furthermore, to simplify calculations it is assumed that this mean field is spherically symmetric and is a quasi-classical object possessing some energy $E_0$ inside a hadron. Each of $N$ constituents interacting with the spherical mean field gets the state with a definite value of its energy $E_i$, so that the hadron mass can be evaluated as

$$M_h = E_0 + E_1 + ... + E_N.$$  \hfill (1)

The wave function $\psi_i$ for any constituent is a solution of a single-particle equation with the mean field static potential $U(r_i)$. Angular dependence of the single-particle wave
functions in a stationary state for the spherically symmetric potential may be separated in a well-known manner, and in order to evaluate the constituent energies $E_i$ it is necessary to solve the radial equations with the model potentials for each constituent. Note that it is impossible to evaluate $E_0$ without additional assumptions, and this quantity take as a phenomenological parameter in the framework of the model.

According to these assumptions the mass $M_m$ of the $\bar{q}q$ - meson in the main approximation can be evaluated as

$$M_m = E_{0m} + E_1(n_1^r, j_1) + E_2(n_2^r, j_2).$$

(2)

where $E_i(n_i^r, j_i), i = 1, 2$, are the energy spectral functions or the mass terms for the $i$–th quark (antiquark) and represents the relativistic effective energy of the $i$–th quark/antiquark moving in the mean field inside the meson. Here $n_i^r$ and $j_i$ are the radial quantum number and the quantum number of the angular moment correspondingly for the $i$–th constituent.

The term $E_{0m}$ contains the contribution from the mean field energy and the possible nonpotential corrections, which cannot be evaluated in the frame of mean field approximation. This term has a nonzero value only for some meson ground states. The terms $E_i(n_i^r, j_i), i = 1, 2$, which represent the energies of the constituents in the mean field, should be evaluated for quarks from the solution of the Dirac equation (for diquarks and constituent gluons from the solutions of Klein-Gordon-Fock equation):

$$\sqrt{\lambda_i + m_i^2} \psi_i(r_1) = [(\alpha_i p_1 + \beta_i (m_i + V_0) + V_1)] \psi_i(r_1),$$

(3)

with $E_i(n_i^r, j_i) = \sqrt{\lambda_i + m_i^2}, i = 1, 2$, $V_0(r) = \sigma r/2$ and $V_1(r) = -2\alpha_s/3r$, where the model parameters $\sigma$ and $\alpha_s$ have meanings of the string tension and the strong coupling constant at small distances, correspondingly. As it is seen from Eq.(3), the addition of some constant to the scalar linear confining potential $V_0(r)$ is equivalent to the addition of the same constant to the quark mass and vice versa, while the addition of some constant to the vector quasi-Coulombic potential $V_1(r)$ is equivalent to the energy shift of opposite sign.

It is well-known that the solutions of Eq.(3) with the total angular momentum $j$ and its projection $m$ can be represented as

$$\psi(r) \propto \begin{pmatrix} f(r) \Omega_m^{ji}(\mathbf{n}) \\ -ig(r)(\sigma \mathbf{n})\Omega_m^{ji}(\mathbf{n}) \end{pmatrix},$$

where $\mathbf{n} = r/r$, the subscript $i$ here and below is omitted. If $k = -\omega(j + 1/2)$, where $\omega$ is an eigenvalue of the space-parity operator, the system of the radial Dirac equations for the fermion in the mean field with the definite energy sign and spin projection reads

$$(\sqrt{\lambda + m^2} - V_0 - V_1 - m)f = -\frac{(1 - k)}{r}g - g',$$

$$(\sqrt{\lambda + m^2} + V_0 - V_1 + m)g = \frac{(1 + k)}{r}f + f'.$$

(4)
Using Eqs. (4) one can derive the second order equation for the "large" component \( f(r) \), and then making a substitution

\[ \phi(r) = rf(r) \left[ V_0(r) - V_1(r) + m + \sqrt{\lambda + m^2} \right]^{-1/2} \]

one comes on to the model radial equation for \( \phi(r) \) in the following form:

\[
\phi'' + \lambda \phi = [V_0^2 - V_1^2 + 2(mV_0 + \sqrt{\lambda + m^2}V_1) + \frac{k(k + 1)}{r^2} + \frac{3(V_0' - V_1')^2}{4(\sqrt{\lambda + m^2} - V_1 + V_0 + m)^2} \]
\[
+ \frac{k(V_0' - V_1')}{r(\sqrt{\lambda + m^2} - V_1 + V_0 + m)} - \frac{(V_0'' - V_1'')}{2(\sqrt{\lambda + m^2} - V_1 + V_0 + m)}] \phi \]  

(5)

The ground states of standard \( \bar{q}q' \) mesons consist of the S-wave \( 1^{--} \) and \( 0^{++} \) mesons and the most extensive set of precise data [1] for heavy mesons is related to the radially exited S-wave \( 1^{--} \) mesons. For this type of mesons \( k = 1 \) and the parameter \( \lambda \) entering Eq. (5) can be calculated with the help of the radial equation:

\[
\phi'' + \lambda \phi = \left\{ -\frac{4\alpha_s\sqrt{\lambda + m^2}}{3r} - \left( \frac{2\alpha_s}{3r} \right)^2 + m\sigma r + \right.
\]
\[
+ \left( \frac{\sigma r}{2} \right)^2 - \left[ \frac{\sigma r}{2} \left( m + \sqrt{\lambda + m^2} \right) + \frac{5\alpha_s\sigma}{6} - \frac{\alpha_s^2}{3r^2} \right] \bigg/ \frac{2\alpha_s}{3r^2} \bigg/ \frac{2\alpha_s}{3r^2} \bigg/ \frac{2\alpha_s}{3r^2} \bigg/ \frac{2\alpha_s}{3r^2} \bigg/ \frac{2\alpha_s}{3r^2} \right. \} \phi. \]

(6)

The right-hand side of Eq. (6) has a singularity at the origin, and when \( r \to 0 \) it behaves as \( 3/4r^2 - 4\alpha_s^2/9r^2 \). Therefore one should keep \( \alpha_s < 3/2 \) in order to prevent a fall at the origin. At the infinity \( r \to \infty \) the effective potential behaves as the oscillatory one and tends to \( \sigma^2 r^2/4 \). The form (6) of the effective potential partly clarify some varieties of the confining potential used in potential models. We see, that for light quarks the confining potential is mainly oscillatory, while for heavy quarks it is linearly rising in the main approximation.

3. Evaluations of model parameters and meson masses

The model equation (6) can be solved only by numerical methods. For calculating its eigenvalues the computer code algorithm, which was based on the Numerov three point recurrent relation [8], has been used jointly with the regularization procedure for the singular potential at \( r = 0 \), which is analogous to the procedure worked out, for instance, in Ref. [9]. The accuracy criterion for fitted parameters is the maximum value of the errors for evaluated hadron masses, which should not exceed typical experimental errors \( 30 \div 40 \) MeV. Thus, the parameter errors written below are our estimation of systematic errors of the model, which have been found in this manner using mesons mass spectra data [1].

The brief description of parameter fitting is as follows. When calculating the values of the model parameters we do not suppose \( a \) priori the validity of the hypothesis of flavor
independence for the confining potential. First of all the model parameters \( m, \alpha_s \) and \( \sigma \) for the \( b\bar{b} \) and \( c\bar{c} \) radial excitation of \( 1^{--} \) states were found. The fits were carried out for each meson family independently. The existing experimental data allowed to find these parameters as well as to prove the validity of the model. In this manner it was found that the value of \( \sigma \) is the same for the \( b\bar{b} \) and \( c\bar{c} \) states within the systematic errors of the model and equal to \( \sigma = (0.20 \pm 0.01) \text{GeV}^2 \).

Taking into account that there are no well established data for the higher radial excitations of \( 1^{--} \) light mesons, the \( \sigma \) value obtained was used for calculating \( m \) and \( \alpha_s \) for light quarks. Note that when doing the calculations we neglect the isotopic mass splitting of \( u \)- and \( d \)-quarks because it is beyond of the model accuracy. It should be noted that in the spin independent potential the constituent energies \( E_i(n_i^r, j_i) \), \( i = 1, 2 \), and \( E_{0m} \) in Eq.(2) represent the main contributions to the energy of the system due to the interaction of the constituents with the mean field and the mean field energy, respectively. However, it is obvious, that in the spin independent potential the constituent energies \( E_i(n_i^r, 1/2) \) are the same for the \( 1^{--} \) and \( 0^{+-} \) mesons with the identical radial quantum numbers and the mass differences can be the result of the spin-spin interaction and the different values of the \( E_{0m} \) terms.

In order to estimate the spin-spin interaction between quark and antiquark we take into account the following expression for the interaction of this type which has been used in the relativized quark model [10]:

\[
V_{s_1s_2} = \frac{32\pi\alpha_s s_1 s_2 \delta^3(r)}{9E_1E_2},
\]

where \( s_1 \) and \( s_2 \) are the spin operators and \( E_1 \) and \( E_2 \) are the energies of quark and antiquark, respectively. Evaluation of the expectation value of this operator in the first order of perturbation theory for the wave functions of quark and antiquark shows that the spin-spin interaction depends mainly on the \( E_1 \) and \( E_2 \), the total spin of quark-antiquark system and the strong coupling constant \( \alpha_s \). So one can use the modified mass formula for S-wave \( 1^{--} \) and \( 0^{+-} \) mesons, which takes into account spin-spin interaction between quark and antiquark, in the following form [7]:

\[
M_m = E_{0m} + E_1(n_1^r, 1/2) + E_2(n_2^r, 1/2) + 4 < s_1 s_2 >_{q_1q_2} V_{ss}
\]

Although in some cases for vector heavy mesons the contributions of spin-spin interaction are less than the model precision, nevertheless, for pseudoscalar mesons for all bound states of quark and antiquark, the spin-spin contributions should be taken into account. The magnitude of the spin-spin interaction constant \( V_{ss} \) is of the order of 100 MeV for light quarks and of the order of 10 MeV for heavy quarks, while the values of the \( E_{0m} \) parameter are: \( E_0(K) = -118\text{MeV} \), \( E_0(\eta_c) = -230\text{MeV} \), \( E_0(J/\Psi) = -125\text{MeV} \), \( E_0(\pi) = -236\text{MeV} \), \( E_0(\eta) = -600\text{MeV} \), \( E_0(\Upsilon) = -450\text{MeV} \), \( E_0(D_s) = -112\text{MeV} \), \( E_0(B_c) = -270\text{MeV} \). The values of \( E_{0m} \) for mesons, which were not pointed out above, within the model accuracy may be set to zero.

We obtain the following values for the coupling \( \alpha_s \) and the quark masses:

- \( \bar{m} = (0.007 \pm 0.005) \text{GeV} \), \( \alpha_s^m = 0.65 \pm 0.15 \),
- \( m_s = (0.14 \pm 0.03) \text{GeV} \), \( \alpha_s^s = 0.47 \pm 0.10 \),
- \( m_c = (1.28 \pm 0.05) \text{GeV} \), \( \alpha_s^c = 0.33 \pm 0.03 \),
- \( m_b = (4.60 \pm 0.10) \text{GeV} \), \( \alpha_s^b = 0.27 \pm 0.02 \).
Within the model accuracy all results for the well known experimental data on the $1^{−−}$ and $0^{−+}$ mesons, which are composed of the quark and antiquark with $u$, $d$, $s$, $c$- or $b$-flavours, are consistent with the presented above model parameters. The evaluated mass spectra of the pseudoscalar and vector mesons are displayed in the Table 1. Thus in the framework of the model the flavor independence of the confinement potential \cite{3,4,5} is confirmed as well as the accordance with the asymptotic freedom behaviour of the $\alpha_s$. One can see that the values of model parameters $m_i$ and $\alpha_s$ are not in contradiction with the values of Standard Model constants such as current quark masses and the strong interaction coupling constant $\alpha_s$.

4. Phenomenological mass formulae for isovector $\bar{q}q'-$mesons

In spite of QCD difficulties which exist in for evaluations of mass spectra of hadrons consisting of light quarks a number of relations among hadron masses have been obtained using symmetry or phenomenological considerations, such as the mass relations \cite{11} for Regge trajectories \cite{12} or the Gell-Mann-Okubo relation \cite{13,14}. According to the Regge trajectories approach a hadron with its spin $J$ and mass $M$ within some errors belongs to a straight trajectory on the $(M^2,J)$-plane with a slope $\alpha'$ and an intercept $\alpha_0$

$$J = \alpha_0 + \alpha'M^2$$

Some hadrons belong to trajectories, so called, daughter trajectories, which are roughly parallel to the main trajectory and are distinguished with different values of a radial quantum number $n^r$ or $n$, $n = n^r + 1$ \cite{15,16,17}.

Now there is a growing interest in improving the accuracy of existing mass relations and obtaining these relations in the framework of the QCD or QCD inspired models. For instance, in Ref. \cite{18} on the basis of the renormalization group it has been shown that the intercept $\alpha_0$ cannot be calculated as a function of the coupling constant. In Ref. \cite{19} the expressions for the slope $\alpha'$ and the intercept $\alpha_0$ has been presented in terms of the string tension. On the base of finite energy QCD sum rules the following mass formulae have been obtained for radially excited $\rho-$mesons \cite{20} and $\pi-$mesons \cite{21}:

$${M^2(\rho^n) = M^2(\rho)(2n^r + 1),}$$  \hspace{1cm} (10)

$${M^2(\pi^n) = M^2(\pi')(n^r),}$$  \hspace{1cm} (11)

where $M^2(\rho)$ is the mass squared of the $\rho-$meson, $M^2(\pi')$ is the mass squared of the first radial excitation of $\pi-$meson. In the work \cite{22} more precise formulae for radially excited hadrons have been proposed. These formulae in general case can be written as

$${M^2_n = M^2_0 + \mu^2(n - 1)}$$  \hspace{1cm} (12)

and describe trajectories on the $(n,M^2)$-plane with different $M_0$ values and approximately the same $\mu$ for each trajectory similarly Chew-Frautschi plots \cite{9} on the $(M^2,J)$-plane and Nambu-Veneziano formulae for daughter Regge trajectories.

Interesting relations for orbital and radial exited mesons’ mass spectra have been obtained in the model of chromoelectric tubes \cite{23}. For mesons consisting from a massive quark and massless antiquark the mass spectrum describes with the formula:

$${M^2 = \pi\sigma(L + 2n + 3/2),}$$  \hspace{1cm} (13)
where $\sigma$ is the string tension, while for mesons consisting from two massless constituents the mass formula is

$$M^2 = 2\pi\sigma(L + 2n + 3/2).$$  \hfill (14)

Further on we restrict ourselves to those mesons which have isotopical spin values equal to 1, in order to bypass problems concerning unknown in some cases structure and mixing parameters for different mesons with the zero isotopical spin. We neglect mass splittings within isotopic multiplets, so systematical errors of the order of 10 MeV for the phenomenological scheme considered are without doubt admissible. Moreover, taking into account the existing experimental errors for meson masses we assume 30÷40 MeV as the typical absolute errors of the mass values evaluations, as in Section 3.

In the framework of relativistic independent quark model a mass formula for $\bar{q}q'$-mesons has a structure which differs from structures of mass formulae in other types of potential models. For instance, in Refs. [24] the following mass formula has been considered

$$M(n^{2S+1}L_J) = E_1(n^1_i, j_1, c, \kappa) + E_2(n^2_j, j_2, c, \kappa),$$ \hfill (15)

where the mass terms (mass or energy spectral functions) $E_i(n_i, j_i, c, \kappa), i = 1, 2,$ for a quark and an antiquark are defined as

$$E(n, j, c, \kappa) = \left\{ \begin{array}{c}
c + \kappa\sqrt{2n^r + L + j - 1/2}, \quad L + j - 1/2 = 2k, \quad k = 0, 1, ... \\
\kappa\sqrt{2n^r + L + j - 1/2}, \quad L + j - 1/2 = 2k + 1, \quad k = 0, 1, ...
\end{array} \right. \hfill (16)$$

In order to exclude superfluous meson states, the following selection rules for $\bar{q}q'$-mesons with given $J^{PC}$ values, quark masses $m_1$ and $m_2$ and quantum numbers $j_1$ and $j_2$ are used

$$j_1 = j_2 = J + 1/2, \quad if \quad J = L + S,
$$

$$j_1 = j_2 + 1 = J + 3/2, \quad if \quad J \neq L + S, \quad m_1 \leq m_2,$$ \hfill (17)

while the radial quantum numbers $n^1_i = n^2_j = n - 1$ for $n^{2S+1}L_J$ - state. The meson parity $P = (-1)^{L+S}$ and the eigenvalue $C$ of charge conjugation for the neutral meson of $q\bar{q}$ type is equal to $(-1)^{L+S}$, where the total spin $S = 0$ or 1.

Functions $E_i(n^r_i, j_i, c, \kappa)$, $i = 1, 2$, give the relativistic effective energies of the quark and antiquark moving in the mean field inside the meson, and include also an energy of the mean field and possible nonpotential corrections, which cannot be evaluated within the mean field approximation and according to the formula (2) should be taken into account with the help of the constants $E_{0m}$. Note that nonpotential corrections are the most important for mass spectra of light mesons [25, 26]. In general case the main part of mass term $E_i(n^r_i, j_i, c, \kappa)$, as it has been described in Section 2, are determined from the solution of the Dirac equation. However, for the superlight mesons, which consist of $u-$ and $d-$ quarks and antiquarks only, the phenomenological energy spectral functions $E_i(n_i, j_i, c, \kappa)$ in the form (16) are suitable within the assumed accuracy. When evaluating masses of unknown excited meson states we use the formulae written above together with the values of two parameters $c$ and $\kappa$, which have been obtained by fitting the mass values of experimentally detected meson states. The $c$ and $\kappa$ values obtained by this manner are $c = 69$ MeV, $\kappa = 382 \pm 4$ MeV [27]. The results obtained with $c = 69$ MeV, $\kappa = 385$ MeV are shown in the Table 2, where we present the mass values of orbital excitations of superlight meson states up to $L = 4$. 


One can obtain with the help of the formulae (15), (16) and the selection rules (17) a great number of mass relations, which are fulfilled within the systematical errors of the phenomenological scheme considered. For instance, in the case of orbitally excitated vector mesons formulae (15) and (16) give the $\rho$−trajectory, while for radially excitated vector mesons they bring in the formula (10). In general case in the framework of this approach instead of the meson trajectories in $(M^2, J)$− and $(n, M^2)$−planes we have in $(L, n, M)$−space the following four series or trajectories for $n^{2s+1}L_J$− meson states.

There is a $(\pi\rho)$− series for $n^3L_{L-1}$−meson states with $P = (-1)^{L+1}$, $C = (-1)^L$. Mass values of the members of the $(\pi\rho)$− series are determined by the formula:

$$M_{n,L}^{\pi\rho} = c + \kappa \sqrt{2(n^r + L) - 1} + \kappa \sqrt{2(n^r + L)}$$  \hspace{1cm} (18)

There is a $\pi$− series for $n^1L_L$−meson states with $P = (-1)^{L+1}$, $C = (-1)^L$. Mass values of the members of the $\pi$− series are determined by the formula:

$$M_{n,L}^\pi = 2c + 2\kappa \sqrt{2(n^r + L)}$$  \hspace{1cm} (19)

There is a $(\rho\pi)$− series for $n^3L_L$−meson states with $P = (-1)^{L+1}$, $C = (-1)^L$. Mass values of the members of the $(\rho\pi)$− series are determined by the formula:

$$M_{n,L}^{\rho\pi} = c + \kappa \sqrt{2(n^r + L)} + \kappa \sqrt{2(n^r + L)} + 1$$  \hspace{1cm} (20)

There is a $\rho$− series for $n^3L_{L+1}$−meson states with $P = (-1)^{L+1}$, $C = (-1)^{L+1}$. Mass values of the members of $\rho$− series are determined by the formula:

$$M_{n,L}^\rho = 2\kappa \sqrt{2(n^r + L)} + 1$$  \hspace{1cm} (21)

It should be noted that only two phenomenological parameters enter in these formulae, which are in actual fact equal to half the pion mass and half the $\rho$−meson mass. With the help of these formulae for the series presented many mass relations which are not dependent on intrinsic quantum numbers $L$ and $n$ easily can be obtained. For instance, mass values of members of $\pi$− series, $(\rho\pi)$− series and $\rho$− series with the same $L$ obey the following mass relation:

$$2M_{n,L}^{\rho\pi}(n^3L_L) = M_{n,L}^\rho(n^3L_{L+1}) + M_{n,L}^\pi(n^3L_L)$$  \hspace{1cm} (22)

5. Link of model of quasi-independent quarks with the model of constituent quarks

As indicated above in order to improve the accuracy of calculations for meson mass spectra in the framework of quasi-independent quarks model an account of residual interactions’ contributions jointly with different values for mean field energy is needed. We find in Sec. 3 the values of mean field energy in combination with magnitudes of spin-spin interaction for quarks and antiquarks with different flavours inside the pseudoscalar and vector mesons. Further on we propose that these values are not simply some phenomenological constants, but represent in a rough approximation equidistant levels for mean field energy, which do not depend on quark flavours. We consider ordinary (non exotic) hadrons and relate the model of quasi-independent quarks to the constituent quark model.
It will be obtained that energies of fermionic constituents $E_i$ (quark mass terms) for meson ground states are equal to constituent quarks’ masses within uncertainties of model of quasi-independent quarks. Some complications arise when one treats exited states. In the model of quasi-independent quarks $E_i$ remains an exited energy of i-th constituent, while in the constituent quark model besides constituent quarks’ masses an additional contribution representing energy of excitation must be taken into consideration. For the pseudoscalar and vector mesons the spin-spin interaction may significantly account for a residual interaction in the ground states of $(\bar{q}q')$ system within the accuracy of the model and mesons’ masses can be evaluated by means of the formula (8) presented in Sec.3. It should be noted that in the framework of the constituent quark model one can find the similar formula [28, 29]:

$$M_m = m_0 + m_1 + m_2 + (s_1s_2)v_{ss},$$

(23)

where $m_1$, $m_2$ are masses of constituent quarks, $m_0$ is some additional phenomenological contribution. So we relate constituents’ energies $E_i, i = 1, 2$, with constituent quarks’ masses $m_i, i = 1, 2$. In what follows this assumption will be supported by numerical fit for meson masses. Formulae (8) and (23) which correspond each other may be named the Zeldovich-Sakharov formulae.

At present due to experimental uncertainties and mixing effects there is no only one solution for the set of parameters entering in Eq. (8). For instance, there is a well known difficulty related to the mass value of the $\pi^-$ meson. Below we give an admissible fit with minimal number of parameters for the mass values of pseudoscalar and vector mesons in the ground states, which can be considered as definite $(\bar{q}q')$ systems involving $u-, d-, s-, c-, b-$quarks and antiquarks. To do this, we use in an essential manner the results obtained in Sec.3. As indicated above in spite of the fact that the quark and antiquark energies $E_1$ and $E_2$ bring in a main contributions to meson masses in order to reduce relative uncertainties of evaluations up to $10^{-2}$ level for all mesons the additional contributions due to the $E_0$ and $V_{ss}$ are needed. Along similar lines $E_0$ and $V_{ss}$ contributions were considered in certain of approaches [28, 29, 221], however some ambiguities occur when $E_0$ and $V_{ss}$ are evaluated. When different fits for meson masses had been performed the case was adopted, such that the $V_{ss}$ value for $u-, d-$quarks and antiquarks was equal to $100 MeV$ and the $E_0$ value for $\rho-$meson was zero. In this case the $E_0$ value for $\pi-$meson is double that the $E_0$ value for $K$ meson: $E^K_0 = 2E^K_0$, where $E_0$ values are marked at the top with mesons’ names. The nonzero values of $E_0$ and $V_{ss}$ given in MeV units for different pseudoscalar and vector mesons are listed below.

$$E^K_0 = -118, E^{J/\psi}_0 = -125, E^{\eta_c}_0 = -230, E^{\eta_b}_0 = -600,$$

$$E^\Upsilon_T = -450, E^{D^*_s}_0 = -112, E^{D^*}_{0_c} = -270, V_{ss}^{K*} = 70, V_{ss}^{\phi} = 50,$$

$$V_{ss}^{D^*} = 27, V_{ss}^{D^*}_s = 5, V_{ss}^{J/\psi} = 3, V_{ss}^{B^*} = 3, V_{ss}^{B^*_s} = 2, V_{ss}^{D^*} = 1.3, V_{ss}^{\Upsilon_T} = 1$$

(24)

The uncertainties of determination of the $E_0$ and $V_{ss}$ values are of the order of several MeV. As is seen in the list of the $E_0$ values above all nonzero $E_0$ values for ground states
of pseudoscalar and vector mesons for all quark flavours are approximately multiples to the $E^K_0$. Thus we can accept that the $E_0$ values are varied according to the rule:

$$E_n^0 = -708 + nE^K_0,$$

(25)

where $n$ is a negative integer. To give also the values of constituents’ energies $E_i$ for $u-, d-, s-, c-, b-$-quarks and antiquarks (for two super light $u-$ and $d-$-quarks and antiquarks the mean value $E_N$ is presented):

$$E_N = 337 \pm 3, \ E_S = 485 \pm 8, \ E_C = 1610 \pm 15, \ E_B = 4952 \pm 20$$

(26)

Notice that within the model uncertainties the constituents’ energies for light quarks and antiquarks agree with the constituent quark masses obtained in Ref. [32]. Moreover if one take into account the difference between masses of U- and D-quarks obtained in this work, which is equal to 4 MeV, the masses of U- and D-quarks get the following values:

$$E_U = 335 \pm 2, \ E_D = 339 \pm 2$$

(27)

Masses of constituent quarks (26) and (27) can be used for the derivation of such important parameters of Standard Model as quark mixing angles in the Fritzsch-Scadron-Delbourgo-Rupp approach [31, 32, 33].

The model of quasi-independent quarks supplemented by the proposition about the equidistant discrete levels of mean field may be named the extended model of quasi-independent quarks. In the framework of this model it is possible to evaluate the pseudoscalar and vector mesons’ masses with uncertainties of order $10^{-2}$. From the results presented it may be inferred that the extended model of quasi-independent quarks is a workable generalization of the constituent quarks model.

6. Conclusions and discussion

It is highly plausible that the formation of constituent quarks take place in the nonperturbative region with the characteristic scales, which can be evaluated with the universal coefficient of a slope of a linear rising potential $\sigma = 0.20 \pm 0.01 GeV^2$. The scales’ values with dimensions of mass and length are equal to $\mu_C = 0.45 \pm 0.02 GeV$, $\lambda_C = 0.44 \pm 0.02 Fm$. As this takes place, $\mu_C$ defines typical magnitudes of transversal impulses $<p_T>$ for quarks-partons inside hadrons, while a radius of a perturbative region surrounded a current quark is equal to $r_C = \lambda_C/2 = 0.22 \pm 0.01 Fm$. The region $r > r_C$ is most likely to be the region of a formation for a constituent quark due to nonperturbative interactions. If one take into account typical sizes of hadrons, then a radius of a constituent quark is $0.25 \div 0.35 Fm$. The absolute value of $E^K_0$, which determinates the characteristic scale for mean field energies in the framework of the extended model of quasi-independent quarks can also be expressed in terms of $\mu_C$ as $\mu_C/4$.

There is a widespread opinion, that the confinement of quarks and gluons can be established rigorously in the QCD framework. Is this is the case, a exact dependence should take place, which relates $\Lambda_{QCD}$ to the characteristic parameter of strong interactions in the nonperturbative region – the string tension $\sigma$. However, in the case if the confinement cannot be demostrated in the QCD framework, there has to add the number of strong
interaction parameters and to introduce, at the least, one more parameter— the string tension. From this standpoint of some interest is investigations concerning with the generalization of the space-time Poincare symmetry of the QCD in the confinement region, for instance, to the inhomogeneous pseudounitary symmetry $IU(3, 1)$ \cite{34}, and in this case the universality of the confinement potential verified in the framework of the relativistic model of quasi independent quarks have appreciable significance.

In this report we considered the relativistic hadron model with quasi-independent quarks and presented the results of the evaluations of mass spectra of radially and orbitally exited meson states. It is important to compare evaluated meson masses with experimentally determined ones \cite{1}. As it is seen from Tables 1 and 2 there is an agreement between the evaluated and experimentally determined values on the level of $1 − 2\sigma$. For instance, let us compare with the data \cite{1} the mass relation (22) , which must obey for the mesons with $L \neq 0$ and $S = 1$. At present this relation can be checked only for the mesons with $L = 1$. After the substitution the experimental values of the masses for the $a_1(1260)$, $b_1(1235)$ and $a_2(1320)$ mesons, we obtain, that this relation is fulfilled with account of experimental uncertainties. Moreover, the mass formulae presented above permit to explain the degeneracy with respect to mass values for different $J^{PC}$ mesons.

As it follows from the Table 2, the mass value calculated for orbital excitation of the vector $1^{--}$ meson is equal to $1506 \text{MeV}$. So we expect that in the mass region between 1.3 GeV and 1.6 GeV two standard $\overline{q}q'$—meson resonances with $J^{PC} = 1^{--}$ and $I = 1$ exist, namely the first radial and orbital excitations mixing each another. However, it is possible that the more complicated situation should be considered when in the mass region $1.2 \div 1.8 \text{ GeV}$ the mixing of the standard $1^{--}$ mesons and the vector non $\overline{q}q'$—mesons take place as well. We considered above only possible mixing between the first radial and orbital excited standard $1^{--}$ mesons. The most complicated situation may arise, which is not disscussed here, when the mixing between two radial and orbital excitations or with vector cryptoexotic states can occur. These cases demand further investigation. For instance, in the framework of the well-known potential model \cite{10} the mass values of $2^{3}S_1$, $1^3D_1$ and $3^3S_1$ states lie considerably higher (at 1.45, 1.66, 2.00 GeV, correspondingly) than the predictions considered. Moreover, in Ref. \cite{35} the mass value 1486MeV has been evaluated the first radial excitation of $\rho$—meson. In Ref.\cite{36} the mass values for the radial exitations of $\rho$—meson, which are equal to 1.4, 1.8, 2.13 Gev, have been obtained. Some difficulties concerning with the identification of the excited states of $\rho$—meson are available in Ref.\cite{37}.

Another important problem is the interpretation of mesons discovered in scalar channel. The authors of Ref.\cite{22} found that in the region $\sim 1 \text{GeV}$ there are $P$—wave $\overline{q}q'$—mesons. The results of our evaluations (Table 2) support the existence of the $P$—wave $\overline{q}q'$—mesons with masses $\sim 980 \text{MeV}$. The complementary reasoning in favour of the existence of the $P$—wave $\overline{q}q'$—mesons with masses $\sim 980 \text{MeV}$ is the coincidence of the evaluated mass value for the first radial excitation of $a_0(980)$ meson with the mass value of the $a_0(1450)$ meson \cite{1}. This meson has been predicted for the first time in Ref. \cite{35} in the framework
of finite energy QCD sum rules. Although the mass values of the $a_0(980)$ and $f_0(980)$ mesons show the possible ideal mixing between them the problems with the intensities of different decay modes exist (in particular, the $K\bar{K}$ decay mode enhancement). The possible explanation of these facts now comprises the four-quark nature of the $a_0(980)$ and $f_0(980)$ mesons \cite{39, 40}. However, there is an accidental degeneracy of scalar mesons mass values with the value of $\bar{K}K$—threshold, which complicates the mechanism of an extraction of $a_0(980)$ and $f_0(980)$ decays characteristics \cite{41}. Moreover, it is plausible that the mixing of these mesons with the $\bar{K}K$—molecule arises \cite{42}. Taking into account the existing difficulties with identification of light mesons in the $1 \div 2$ GeV mass region we see that allowing for the mixing between two or three mesons with different structures but in some cases with about the same masses is admissible for an adequate data description in both scalar and vector channels.

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References

[1] W.-M. Yao et al. [Particle Data Group], J. Phys. G 33, 1 (2006).
[2] P.N. Bogoliubov, Ann. Inst. Henri Poincare 8, 163 (1967).
[3] A.E. Dorokhov, JINR R2-121159, Dubna, 1979.
[4] A. Martin, CERN TH-2741, Geneva, 1979.
[5] C. Quigg, J.L. Rosner, H.B. Thacker, FERMILAB 79-52 (THY), Batavia, 1979.
[6] V.V. Khruschev, V.I. Savrin, S.V. Semenov, Phys. Lett. B 525, 283 (2002).
[7] V.V. Khruschev, S.V. Semenov, Part. Nucl. Lett. 5, 5 (2002).
[8] B.V. Numerov, Proc. Centr. Astrophys. Observ. 2 (1933) 188.
[9] E. Buendia and R. Guardiola, J. Comput. Phys. 60 (1985) 561.
[10] S. Godfrey and N. Isgur, Phys. Rev. D32 (1985) 189.
[11] G.F. Chew, S.C. Frautschi, Phys. Rev. Lett. 8 (1962) 41.
[12] T. Regge, Nuovo Cim. 14 (1959) 951; 18 (1960) 947.
[13] M. Gell-Mann, prep. California Inst. of Technol. CTSL-20, 1961.
[14] S. Okubo, Prog. Theor. Phys. 27 (1962) 949.
[15] M. Toller in "Proc. of the Eight Nobel Symp. on Elemen. Particle Theory", N.Y. 1968.
[16] G. Veneziano in "Proc. of the Coral Gables Conf. on Fundament. Inter. at High Energy", N.Y. 1969.
[17] Y. Nambu in "Lect. at the 1970 Copenhagen Summer Symp.", 1970.
[18] V.A. Petrov, arXiv:hep-ph/0603103 (2006).
[19] A.M. Badalian, B.L.G. Bakker, Yu.A. Simonov, Phys. Rev. D66 (2002) 034026.
[20] N.V. Krasnikov, A.A. Pivovarov, Phys. Lett. B112 (1982) 397.
[21] A.L. Kataev, N.V. Krasnikov, A.A. Pivovarov, Phys. Lett. B123 (1983) 93.
[22] A.V. Anisovich, V.V. Anisovich, A.V. Sarantsev, Phys. Rev. D62 (2000) 051502.
[23] T.J. Allen and M.G. Olsson. Phys. Rev. D68 (2003) 054022.
[24] V.V. Khruschev, Yad. Fiz. 46 (1987) 219; ibid. 55 (1992) 773.
[25] A. T. Filippov, Yad. Fiz. 29 (1979) 1035.
[26] B. Diekmann, Phys. Rep. 159 (1988) 99.
[27] V.V. Khruschov, prep. IAE - 6359/2, Moscow, 2005; arXiv:hep-ph/0504077.
[28] Ya.B. Zeldovich, A.D. Sakharov, Yad. Fiz. 4 (1966) 395.
[29] A.D. Sakharov, ZHETF 78 (1980) 2112.
[30] S.M. Troshin, N.E. Tyurin, arXiv:hep-ph/0609248 (2006).
[31] H. Fritzsch, Phys. Lett. B 70, 436 (1977); B 73, 317 (1978).
[32] M.D. Scadron, R. Delbourgo, G. Rupp, J. Phys. G 32, 735 (2006).
[33] Yu.V. Gaponov, V.V. Khruschov, S.V. Semenov, arXiv:hep-ph/0612283 (2006).
[34] V.V. Khruschov, arXiv: hep-ph/0311346.
[35] D. Ebert, R.N. Faustov, V.O. Galkin, arXiv:hep-ph/0503238.
[36] A.A. Pivovarov, Yad. Fiz. 62 (1999) 2077.
[37] N.N. Achasov, A.A. Kozhevinikov, Yad. Fiz. 65 (2002) 158.
[38] S.G. Gorishny, A.L. Kataev and S.A. Larin, Phys. Lett. B135 (1984) 457.
[39] R.L. Jaffe, Phys. Rev. D15 (1977) 267, 281.
[40] N.N. Achasov, arXiv:hep-ph/0412155.
[41] V. Baru et al., arXiv:nucl-th/0410099.
[42] Yu.S. Kalashnikova et al., arXiv:hep-ph/0412340.
Table 1.
Evaluated masses of the $1^{--}$ and $0^{++}$ mesons in comparison with the data from Ref. [1].

| Meson | $M_{\text{exp}}^{[1]}$ [MeV] | $M_{\text{th}}$ [MeV] | Meson | $M_{\text{exp}}^{[1]}$ [MeV] | $M_{\text{th}}$ [MeV] |
|-------|------------------|----------------|-------|------------------|----------------|
| $\rho$ | 775.5±0.4 | 740 | $B$ | 5279±0.8 | 5250 |
| $\rho'$ | 1459±11 | 1455 | $B_s$ | 5367.5±1.8 | 5370 |
| $\rho''$ | 1720±20 | 1730 | $B_c$ | 6286±5 | 6300 |
| $\phi$ | 1019.460±0.019 | 1010 | $\eta_b$ | 9300±40 | 9330 |
| $\phi'$ | 1680±20 | 1650 | $\pi'$ | 1300±100 | 1290 |
| $\phi''$ | - | 2050 | $K'$ | - | 1400 |
| $J/\psi$ | 3096.916±0.011 | 3060 | $D'$ | - | 2450 |
| $\psi'$ | 3686.093±0.034 | 3650 | $D_s'$ | - | 2560 |
| $\psi''$ | 4039±01 | 4070 | $\eta_c'$ | 3638±4 | 3600 |
| $\psi'''$ | 4421±4 | 4390 | $B''$ | - | 5650 |
| $\Upsilon$ | 9460.30±0.26 | 9470 | $B_{s'}$ | - | 5750 |
| $\Upsilon'$ | 10023.26±0.31 | 9990 | $B_{c'}$ | - | 6800 |
| $\Upsilon''$ | 10355.2±0.5 | 10325 | $\eta_{b'}$ | - | 9960 |
| $\Upsilon'''$ | 10579.4±1.2 | 10550 | $\pi''$ | 1812±14 | 1810 |
| $\Upsilon^{5S}$ | 10865±8 | 10830 | $K''$ | - | 1850 |
| $\Upsilon^{6S}$ | 11019±8 | 10985 | $D''$ | - | 2880 |
| $\pi$ | 138±3.1 | 120 | $D_s''$ | - | 2950 |
| $K$ | 495.65±0.02 | 500 | $\eta_{c''}$ | - | 3970 |
| $D$ | 1867.7±0.4 | 1850 | $B''$ | - | 6070 |
| $D_s$ | 1968.2±0.5 | 1990 | $B_{s''}$ | - | 6130 |
| $\eta_c$ | 2980.4±1.2 | 2990 | $B_{c''}$ | - | 7150 |

Table 2.
Evaluated masses in MeV for the ground states and the orbital excitations of the $\bar{q}q'$ – mesons in comparison with the data from Ref. [1].

| Meson | $J^{PC}$ | $M_{\text{exp}}(\text{MeV})^{[17]}$ | $M_{\text{ex}}(\text{MeV})$ | Meson | $J^{PC}$ | $M_{\text{ex}}(\text{MeV})^{[17]}$ | $M_{\text{ex}}(\text{MeV})$ |
|-------|---------|----------------|----------------|-------|---------|----------------|----------------|
| $\pi$ | 0$^{-+}$ | 138.039±0.004 | 138±30 | $\rho_3$ | 3$^{--}$ | 1688.8±2.1 | 1722±30 |
| $\rho$ | 1$^{-}$ | 775.5±0.4 | 770±30 | $a_{2}^{3}$ | 2$^{++}$ | - | 1873±30 |
| $a_0$ | 0$^{++}$ | 984.7±1.2 | 998±30 | $b_{4}^{3}$ | 3$^{--}$ | - | 2024±30 |
| $b_1$ | 1$^{-}$ | 1229.5±3.2 | 1227±30 | $a_{3}^{3}$ | 3$^{++}$ | - | 2031±30 |
| $a_1$ | 1$^{++}$ | 1230±40 | 1280±30 | $a_{4}$ | 4$^{++}$ | 2001±10 | 2037±30 |
| $a_2$ | 2$^{++}$ | 1318.3±0.6 | 1334±30 | $a_{4}^{3}$ | 3$^{--}$ | - | 2177±30 |
| $a_1^{2}$ | 1$^{-}$ | - | 1506±30 | $b_{4}^{1}$ | 4$^{-}$ | - | 2316±30 |
| $\pi_2$ | 2$^{-+}$ | 1672.4±3.2 | 1678±30 | $a_{4}^{3}$ | 4$^{--}$ | - | 2313±30 |
| $a_2^{2}$ | 2$^{-}$ | - | 1700±30 | $a_{5}^{4}$ | 5$^{--}$ | - | 2310±30 |