Properties of Valence Nucleon Distributions for Halo Nuclei

C.J. Lin, H.Q. Zhang, Z.H. Liu, Y.W. Wu, F. Yang, and M. Ruan

China Institute of Atomic Energy, P.O. Box 275(10), Beijing, 102413, China

Abstract

With the binding energies and configurations determined experimentally, the root-mean-square radii are calculated for a number of single-particle states by numerically solving the Schrödinger equations. By studying the relations between the radii and separation energies, the new scaling laws and necessary conditions for neutron halos and proton halos are established, respectively. Especially the existence region of true proton halos is pointed out. It is found that the effects of short-distance behaviours of valence nucleons at the edges of interaction potentials cannot be disregarded. Moreover, by means of the radii of interaction potentials, the contributions of outer parts are estimated as the criterions of halos.

Key words: Scaling laws for valence neutron and proton distributions in halo nuclei; Single-particle potential model; Criterion for halo occurrence.

PACS: 21.10.-k; 21.60.-n; 27.20.+n; 27.30.+t

The exotic properties of halo nuclei have been extensively investigated since this phenomena were found by Tanihata et al. [1,2]. So far, a number of light, unstable nuclei are experimentally found to be halo states, i.e., the one-neutron halo nuclei, $^{11}\text{Be}$ [3] and $^{10}\text{C}$ [4]; the two-neutron halo nuclei, $^6\text{He}$, $^{11}\text{Li}$ [1,2,5], $^{14}\text{Be}$ [6,7], and $^{17}\text{B}$ [6]; the four neutron structure, $^8\text{He}$ [1,2,8]; the proton halo nuclei, $^8\text{B}$ [9–11], $^{17}\text{F}$, $^{17}\text{Ne}$ [12], $^{20}\text{Mg}$ [13], and $^{26,27,28}\text{P}$ [14]; and the one-neutron halo structures in excited states of $^{12}\text{B}$ and $^{13}\text{C}$ [15,16]; and so on. Meanwhile, the general properties of halo nuclei have been gradually understood [17–20]. Nowadays, ones have recognized that in principle halo is a threshold effect when the state close to the continuum [18]. The necessary conditions to form halos are weak bound, low angular momentum, and short-range interaction. And these conditions are formulated as general scaling rules [19,20]. The general properties of halos in the above works, for example the scaling laws, are extracted by means of the square well potentials, because the Schrödinger equations have the analytic solutions. Fedorov et al. [19] found that some results of Gaussian and surface potentials obey
the scaling laws of square potentials. Moreover, Hansen et al. [18] drew two qualitative conclusions by studying the asymptotic behaviours of radial wavefunctions for two-body systems, i.e. (1) the states with \( l = 2 \) can not form the halo structures due to the affections of the centrifugal barriers; (2) the proton halos do not exist due to the affections of the long-range Coulomb potentials. Nevertheless, we often find that the experimental findings are not in agreement with these scaling laws. Of course, the asymptotic behaviours are the same for different potentials. But the short-distance behaviors of the valence nucleons at the edges of the interaction potentials should affect these conclusions. In other words, the results depend on the potential shapes. In this letter, we re-focus on the scaling laws of valence nucleon distributions: According to the experimental findings, adopt the more real shapes, such as Woods-Saxon forms, to get the more realistic scaling laws, and to reveal the affections of the short-distance behaviours.

For the sake of simplicity, the two-body systems are considered and the single-particle potential model[19,21] is employed presently. In principle, it supposes that the nucleus is configurated by a core with a valence nucleon outside. In present work, the configuration of certain single-particle bound state is assigned by the experimental results, for example, the binding energy, angular momentum, spin, etc.. The Schrödinger equation of this state can be solved by the numerically analytic method. Then the radial wavefunction, the density distribution, and root-mean-square (rms) radius etc., can be calculated. The following points need to be mentioned: (1) Adopt the more real shape for the nuclear potential, i.e. Woods-Saxon shape, \( f(x) = [1 + exp(x/a)]^{-1} \) with \( x = r - r_0 A^{1/3} \); \( r_0 = 1.25 \) fm, and \( a = 0.65 \) fm; (2) For the cases of which valence nucleons are protons, the Coulomb potential of a uniformly charged sphere with radius \( r_{0C} A^{1/3} \) and \( r_{0C} = 1.30 \) fm is added in the interaction potential; (3) The spin-orbit-coupling potential with Thomas form \( \lambda = 25 \) [22] is also considered, because it is influential for some states with high angular momenta \((l > 2)\)and spins. Furthermore, the nuclear potential depth \( V \) is determined by the binding energy and the spin-orbit-coupling potential, \( V_{s.o.} \) is varied with \( V \), i.e. \( V_{s.o.} = (\lambda/45.2) V \). All the parameters mentioned above are commonly used as ‘standard parameters’. Sherr [21] pointed out that the predicted radii are insensitive to moderate changes of these parameters. It ascribes to the gross results of integrations of radial wavefunctions. Whereas the wavefunctions, describing the detail behaviours of the valence nucleons, are sensitive to some extent. Fig. 1 illustrates the single-particle wavefunctions for two bound states \( 2s_{1/2} \) and \( 1p_{1/2} \) of \(^{11}\)Be with Woods-Saxon form factors with the standard parameters (solid lines) and with the other parameters \((r_0 = 1.05 \text{ fm}, a = 0.45 \text{ fm}; \text{dashed lines})\), respectively. The results of the square well with \( r_0 = 1.25 \) fm (dotted lines) are also shown. The dash-dotted line indicates the position of the radius of interaction potential between core and valence nucleon.
For the neutron cases, the current scaling laws [18,19] of the square well are,

\[
\frac{< r_n^2 >}{R_{cn}^2} \simeq \begin{cases} 
10.44\text{MeV fm}^2/(S_n R_{cn}^2) & l = 0 \\
3.65\text{MeV}^{1/2} \text{fm}/(S_n R_{cn}^2)^{1/2} & l = 1 , \\
1.40 & l = 2 
\end{cases}
\]

(1)

where \( R_{cn} \) is the radius of interaction potential (core + nucleon), \( < r_n^2 > \) and \( S_n \) are the rms radius and separation energy of the valence neutron, respectively. According to the halo definition, i.e. the probability of finding the valence nucleon is larger than 50% outside the range of the interaction potential [18–20], these states with \( < r^2 > / R_{cn}^2 > 2 \) are the true halo states in strict meaning. The existence condition of neutron halo is derived from the above equation as yet. Obviously, no halo can be formed in the \( l = 2 \) state.

Using the single-particle potential model mentioned above, we calculated the rms radii of the valence neutrons for the neutron-rich nuclei from \(^4\text{He}\) to \(^{22}\text{C}\), including some excited states. All the states calculated are dominated by single-particle states and their configurations are determined by the experiments. As the results, the solid symbols shown in the Fig. 2 illustrate the \( < r_n^2 > / R_{cn}^2 \) versus \( S_n R_{cn}^2 \) for these states. Where the interaction potential radius is taken as, \( R_{cn} = 1.25(A_{\text{core}}^{1/3} + A_{\text{val}}^{1/3}) \), with \( A_{\text{core}} \) and \( A_{\text{val}} \) are the mass numbers of the core and valence nucleon, respectively. It is worthy of being pointed out that such \( R_{cn} \) definition is consistent in value with its in the Eq. 1, i.e. \( R_{cn}^2 = 5/3(< r^2 >_{\text{core}} + 4\text{fm}^2) \). For example, for the \(^{11}\text{Be}\) nucleus, \( < r^2 >_{\text{core}}^{1/2} = 2.37 \text{ fm} \) [21] then \( R_{cn} = 4.00 \text{ fm} \), and \( R_{cn} = 3.95 \text{ fm} \) in our case. In the Fig. 2, the solid lines are the fit results for the different orbit angular momenta. The forms of the fits are \( y = a/x^b \), where \( a \) and \( b \) are the parameters. So the scaling laws given by the Woods-Saxon shapes are,

\[
\frac{< r_n^2 >}{R_{cn}^2} \simeq \begin{cases} 
13.5/(S_n R_{cn}^2)^{0.667} & l = 0 \\
4.38/(S_n R_{cn}^2)^{0.446} & l = 1 , \\
1.43/(S_n R_{cn}^2)^{0.180} & l = 2 
\end{cases}
\]

(2)

Compared with Eq. 1, it is immediately clear that they are quite different. In order to show the discrepancy, the scaling laws given by the square potentials are plotted as the dashed lines in the Fig. 2. Then the necessary conditions
for the formations of neutron halos are, 

\[
S_n R_{cn}^2 < \begin{cases} 
17.6 \text{MeV} \text{fm}^2 & l = 0 \\
5.79 \text{MeV} \text{fm}^2 & l = 1 \\
0.153 \text{MeV} \text{fm}^2 & l = 2 
\end{cases}
\] 

(3)

The actual conditions are more looser than those given by the square potentials. For example, there exist the possibilities of halo occurrences in the \( l = 2 \) states, although the conditions are too rigour. Without doubts, the conditions expressed by Eq. 3 have the realistic meanings for searching the unknown neutron halo states.

By introducing the Coulomb potentials, the density distributions of valence protons are calculated for some typical proton-rich nuclei, for example \(^8\text{B}, \, ^{17}\text{F}, \) and some proton-rich isotopes of \( Z=15, \, 16, \) etc., also including some excitation states with configuration ascertained. As same as the Fig. 2, the Fig. 3 shows the relations between the rms radii of valence protons, \(< r_p^2 >\) and the separation energies, \( S_p \). The solid symbols are calculation results. The solid lines are the fit results,

\[
\frac{< r_p^2 >}{R_{cn}^2} \simeq 1.73/(S_p R_{cn}^2)^{0.246} \quad l = 0, 1.
\] 

(4)

For the \( l = 2 \) states, the data points are too few to get a assurable fit, i.e. the dotted lines shown in the Fig. 3. So we do not discuss it here. It is found that the scaling laws are the same for the \( l = 0 \) and \( l = 1 \) states. We think it is just an accident, because of the affections by the long-range Coulomb potentials at the edges of the nuclei. Directly judging form Eq. 4, The necessary condition for the formation of proton halo is \( S_p R_{cn}^2 < 0.551 \text{ MeV} \). It should be noted that due to the existence of Coulomb barrier, some resonance states of which the energies are positive but lower than the barrier heights have enough surviving time and are able to form the halo structures. In other words, the proton halos could exist in these states. Hence, the actual necessary condition for the formation of proton halo is,

\[
-0.551/R_{cn}^2 \text{MeV} < E_p < V_B ,
\]

(5)

where \( E_p \) is the proton energy, \( V_B \) the height of Coulomb barrier. The shadow area in the Fig. 4 shows the region of proton halos occurrence. According to this condition, the existing findings figured as proton halos are in fact the proton skins. The location of \(^{17}\text{F}(2s_{1/2})\) state, the most close to the proton halos region, is also illustrated in the figure. The actual conditions and the occurrence region should be useful to find out the true proton halos.
Although the halo phenomenon is well studied, it is still confusion to judge that a nuclear state is a halo state or not experimentally. In general, when the distribution of a valence particle has a long tail, ones say that this nucleus has the halo structure. Such judgment is too ambiguous. Someone may think it is no meaning to make a strict definition for halo. But recent investigation indicates that because of the occurrence of neutron halo, there universally exists the overturn of the sequence of energy levels predicted by the shell model. It is well known that the shell model with harmonic-oscillator potential predicts that the $2s_{1/2}$ energy level is higher than the $1d_{5/2}$. But due to the halo state occurs in the $2s_{1/2}$ state, in fact its energy is lower than the $1d_{5/2}$. As an example, the first excited state of $^{13}$C is $2s_{1/2}$ state, not $1d_{5/2}$. Such overturn phenomenon is the signature of halo occurrence and results in the emergence of new magic number, $N=16$ [23]. So it is necessary to define a halo state. Clearly, the nucleon distributions outside the nuclear cores are important for halo structures. The contributions of the outer part can be estimated by [15,16,24]

$$D_\lambda = \left[ \int_{R_{cn}}^\infty r^{2\lambda} \Phi_{nlj}^2(r) dr \int_0^{\infty} r^{2\lambda} \Phi_{nlj}^2(r) dr \right]^{1/\lambda}$$

(6)

with $\lambda = 1, 2$, where $\Phi_{nlj}$ is the radial wavefunction of valence nucleon in the orbital $(nlj)$. The $D_1$ represents the probability of the valence nucleon outside the range of the interaction radius $R_{cn}$, and $D_2$ gives the contribution of the outer part to the rms radius. Here we suggest to use the strict halo definition, i.e. $D_1 > 50\%$. Moreover, $D_2$ are usually larger than 90% in the halo cases. It means that the uncertainty caused by the inner part is quite small. Very recently, Carstoiu et al. [24] calculated the density distribution of last proton in $^8$B and extracted its rms radius. They found that $D_1(2) = 0.29(0.85)$ for $R_N = 4$ fm whereas $D_1(2) = 0.64(0.90)$ for $R_N = 2.5$ fm. Taking the latter case, they judged that $^8$B is a proton halo nucleus. Obviously, $D_1(2)$ values vary with $R_N$. In order to avoid the arbitrariness, $R_N$ should be fixed to the radius of interaction potential, i.e. $R_N \equiv R_{cn}$. With the same procedure of Carstoiu et al., we gained $<r^2>^{1/2} = 4.5$ fm and $D_1(2) = 0.40(0.80)$ with $R_{cn} = 3.6$ fm. Thus, $^8$B is the nucleus with thick skin in nature.

With the experimental binding energies, angular momenta, spins, etc., the Schrödinger equations are numerically solved and the rms radii are calculated for a number of single-particle states by means of the single-particle potential model. In this model, the shapes of nuclear potentials are adopted Woods-Saxon form and the Coulomb and spin-orbit-coupling potentials are considered. The relations between the rms radii and separation energies are illustrated for different angular momenta. The new scaling laws and necessary conditions for neutron halos and proton halos are established, respectively. The actual conditions are more looser than those extracted by the square
well, due to the effects of short-distance behaviours of the valence nucleons at the edges of interaction potentials. Because of the existence of Coulomb barrier, some resonant states possibly have proton halo structures. The region of true proton halos existence is pointed out. Moreover, the contributions of outer parts are estimated by the radii of interaction potentials. The halo state can be strictly judged by this estimation.

This work was supported by the Major State Basic Research Development Program under Grant No. G200077400, and the National Natural Science Foundation of China under Grant Nos. 19875087, 10075077 and 10105016. One of us, C.J. Lin wish to thank to Prof. Z.Y. Ma and Prof. Z.X Li for many useful discussion.

References

[1] I. Tanihata, H. Hamagaki, O. Hashimoto, Y. Shida, N. Yoshikawa, K. Sugimoto, O. Yamakawa, T. Kobayashi, and N. Takahashi, Phys. Rev. Lett. 55 (1985) 2676.

[2] I. Tanihata, H. Hamagaki, O. Hashimoto, S. Nagamiya, Y. Shida, N. Yoshikawa, O. Yamakawa, K. Sugimoto, T. Kobayashi, D.E. Greiner, N. Takahashi, and Y. Nojiri, Phys. Lett. B 160 (1985) 380.

[3] M. Fukuda, T. Ichihara, N. Inabe, T. Kubo, H. Kumagai, T. Nakagawa, Y. Yano, I. Tanihata, M. Adachi, K. Asahi, M. Kouguchi, M. Ishihara, H. Sagawa, and S. Shimoura, Phys. Lett. B 268 (1991) 339.

[4] D. Bazin, B.A. Brown, J. Brown, M. Fauerbach, M. Hellstrom, S.E. Hirzebruch, J.H. Kelley, R.A. Kryger, D.J. Morrissey, R. Pfaff, C.F. Powell, B.M. Sherrill, and M. Thoennessen, Phys. Rev. Lett. 74 (1995) 3569.

[5] I. Tanihata, T. Kobayashi, T. Suzuki, K. Yoshida, S. Shimoura, K. Sugimoto, K. Matsuta, T. Minamisono, W. Christie, D. Olson, H. Wieman, Phys. Lett. B 287 (1992) 307.

[6] T. Suzuki, R. Kanungo, O. Bochkarev, L. Chulkov, D. Cortina, M. Fukuda, H. Geissel, M. Hellstrom, M. Ivanov, R. Janik, K. Kimura, T. Kobayashi, A.A. Korsheninnikov, G. Munzenberg, F. Nickel, A.A. Ogloblin, A. Ozawa, M. Pfutzner, V. Pribora, H. Simon, B. Sitar, P. Strmen, K. Sumiyoshi, K. Summerer, I. Tanihata, M. Winkler, and K. Yoshida, Nucl. Phys. A 658 (1999) 313.

[7] M. Labiche, N.A. Orr, F.M. Marques, J.C. Angelique, L. Axelsson, B. Benoit, U.C. Bergmann, M.J.G. Borge, W.N. Catford, S.P.G. Chappell, N.M. Clarke, G. Costa, N. Curtis, A. D’Arrigo, E.de Goes Brennand, O. Dorvaux, G. Fazio, M. Freer, B.R. Fulton, G. Giardina, S. Grevy, D. Guillemaud-Mueller, F. Hanappe, B. Heusch, K.L. Jones, B. Jonson, C. Le Brun, S. Leenhardt, M. Lewitowicz,
M.J. Lopez, K. Markenroth, A.C. Mueller, T. Nilsson, A. Ninane, G. Nyman, F. de Oliveira, I. Piqueras, K. Riisager, M.G. Saint Laurent, F. Sarazin, S.M. Singer, O. Sorlin, and L. Stuttge, Phys. Rev. Lett. 86 (2001) 600.

[8] M.V. Zhukov, A.A. Korsheninnikov, and M.H. Smedberg, Phys. Rev. C 50 (1994) R1.

[9] M.H. Smedberg, T. Baumann, T. Aumann, L. Axelsson, U. Bergmann, M.J.G. Borge, D. Cortina-Gil, L.M. Fraile, H. Geissel, L. Grigorenko, M. Hellstrom, M. Ivanov, N. Iwasa, R. Janik, B. Jonson, H. Lenske, K. Markenroth, G. Munzenberg, T. Nilsson, A. Richter, K. Riisager, C. Scheidenberger, G. Schrieder, W. Schwab, H. Simon, B. Sitar, P. Strmen, K. Summerer, M. Winkler, and M.V. Zhukov, Phys. Lett. B 452 (1999) 1.

[10] M. Fukuda, M. Mihara, T. Fukao, S. Fukuda, M. Ishihara, S. Ito, T. Kobayashi, K. Matsuta, T. Minamisono, S. Momota, T. Nakamura, Y. Nojiri, Y. Ogawa, T. Ohtsubo, T. Onishi, A. Ozawa, T. Suzuki, M. Tanigaki, I. Tanihata, and K. Yoshida, Nucl. Phys. A 656 (1999) 209.

[11] V. Guimaraes, J.J. Kolata, D. Peterson, P. Santi, R.H. White-Stevens, S.M. Vincent, F.D. Becchetti, M.Y. Lee, T.W. O’Donnell, D.A. Roberts, and J.A. Zimmerman, Phys. Rev. Lett. 84 (2000) 1862.

[12] A. Ozawa, T. Kobayashi, H. Sato, D. Hirata, I. Tanihata, O. Yamakawa, K. Omata, K. Sugimoto, D. Olson, W. Christie, and H. Wieman, Phys. Lett. B 334 (1994) 18.

[13] T. Suzuki, H. Geissel, O. Bochkarev, L. Chulkov, M. Golovkov, D. Hirata, H. Irnich, Z. Janas, H. Keller, T. Kobayashi, G. Kraus, G. Munzenberg, S. Neumaier, F. Nickel, A. Ozawa, A. Piechaczek, E. Roeckl, W. Schwab, K. Summerer, K. Yoshida, and I. Tanihata, Nucl. Phys. A 616 (1997) 286c.

[14] A. Navin, D. Bazin, B.A. Brown, B. Davids, G. Gervais, T. Glasmacher, K. Govaert, P.G. Hansen, M. Hellstrom, R.W. Ibbotson, V. Maddalena, B. Pritychenko, H. Scheit, B.M. Sherrill, M. Steiner, J.A. Tostevin, and J. Yurkon, Phys. Rev. Lett. 81 (1998) 5089.

[15] Z.H. Liu, C.J. Lin, H.Q. Zhang, Z.C. Li, J.S. Zhang, Y.W. Wu, F. Yang, M. Ruan, J.C. Liu, S.Y. Li, and Z.H. Peng, Phys. Rev. C 64 (2001) 034312.

[16] Lin Cheng-Jian, Liu Zu-Hua, Zhang Huan-Qiao, Wu Yue-Wei, Yang Feng, and Ruan Ming, Chin. Phys. Lett. 18 (2001) 1183 and 1446.

[17] P.G. Hansen and B. Jonson, Europhys. Lett. 4 (1987) 409.

[18] P.G. Hansen and A.S. Jensen, Annu. Rev. Nucl. Part. Sci. 45 (1995) 591.

[19] D.V. Fedorov, A.S. Jensen, and K. Riisager, Phys. Rev. C 49 (1994) 201; Phys. Rev. C 50 (1994) 2372; Phys. Lett. B 312 (1993) 1.

[20] A.S. Jensen, K. Riisager, Phys. Lett. B 480 (2000) 39.

[21] R. Sherr, Phys. Rev. C 54 (1996) 1177.
[22] G.R. Satchler, *Direct Nuclear Reactions*, Oxford University Press, Oxford OX2 6DP (1983).

[23] A. Ozawa, T. Kobayashi, T. Suzuki, K. Yoshida, and I. Tanihata, Phys. Rev. Lett. 84 (2000) 5493.

[24] F. Carstoiu, L. Trache, C.A. Gagliardi, R.E. Tribble, and A.M. Mukhamedzhanov, Phys. Rev. C 63 (2001) 054310.
Fig. 1. The single-particle wavefunctions for two bound states $2s_{1/2}$ and $1p_{1/2}$ of $^{11}$Be with different shapes. The radius of interaction potential is also indicated.

Fig. 2. The rms radii of valence neutrons vary with the separation energies for a number of states in light neutron-rich nuclei. The solid circles, squares and triangles represent the results of $l = 0, 1, 2$ states, respectively. The solid lines are the fit results with the origins of Woods-Saxon potentials and the dashed lines are the results extracted by square potentials.

Fig. 3. Same as the Fig. 2, but for the proton cases. The typical states of $^{17}$F($2s_{1/2}$), $^{8}$B($1p_{3/2}$) and $^{17}$F($1d_{5/2}$) are respectively pointed out.

Fig. 4. The shadow area illustrates the region of true proton halos occurrence, including the bound states and resonance states. The location of $^{17}$F($2s_{1/2}$) state is indicated.
$^{11}\text{Be} = ^{10}\text{Be} + n$

$0p, S_n = 0.184 \text{ MeV}$

$1s, S_n = 0.504 \text{ MeV}$

$WS (r_0 = 1.25\text{fm} \ a = 0.65\text{fm})$

$WS (r_0 = 1.05\text{fm} \ a = 0.45\text{fm})$

$SQ (r_0 = 1.25\text{fm})$
\[ l = 0 \]

\[ \langle r_n^2 \rangle / R_{cn}^2 \]

\[ S_n R_{cn}^2 \text{ (MeVfm}^2) \]

\( l = 1 \)

\( l = 2 \)

Woods-Saxon

Square
\begin{align*}
\langle r_p^2 \rangle / R_{cn}^2 &= 17F(2s_{1/2}) \\
l &= 0 \\
a) \\

\langle r_p^2 \rangle / R_{cn}^2 &= B(1p_{3/2}) \\
\langle r_p^2 \rangle / R_{cn}^2 &= 8B(1p_{3/2}) \\
l &= 1 \\
b) \\

\langle r_p^2 \rangle / R_{cn}^2 &= 17F(1d_{5/2}) \\
l &= 2 \\
c)
\end{align*}
