Anomalous properties of neutron resonances in Pt isotopes

M Krtička¹, P E Koehler², F Bečvár¹, J A Harvey², K H Guber³
¹Charles University, Faculty of Mathematics and Physics, Prague, Czech Republic
²Physics Division, Oak Ridge National Laboratory, Oak Ridge, USA
³Reactor and Nuclear Systems Division, Oak Ridge National Laboratory, Oak Ridge, USA
E-mail: krticka@ipnp.troja.mff.cuni.cz

Abstract. We obtained an unprecedentedly large number of s-wave neutron widths and total radiation widths through R-matrix analysis of neutron cross-section measurements on enriched Pt samples. Careful analysis of resonance parameters of even-even Pt targets was performed and a brief summary of findings is given.

1. Introduction

Neutron resonance parameters remain some of the most important information for testing the validity of predictions of the random matrix theory (RMT) [1], even more than half a century after these data served as the original stimulus for its creation. Today, RMT pervades the physics of almost all complex systems. In nuclear physics, predictions of RMT are expected to correctly describe fluctuation properties of nuclear levels at relatively high excitation such as near the neutron threshold in medium-weight and heavy nuclei.

Predictions of RMT implicitly yield that spacing of neutron resonances with the same spin and parity \( J^\pi \) follow so-called Wigner distribution [2] and reduced neutron widths, \( \Gamma_0^{\lambda n} \), of s-wave resonances \( \lambda \) follow a \( \chi^2 \) distribution with one degree of freedom, \( \nu = 1 \), which is known as the Porter-Thomas distribution (PTD) [3]. Today, the general consensus is that data on resonance spacing and \( \Gamma_0^{\lambda n} \) agree with these expectations. However, there are problems with both the data and analysis techniques used in probably the best test of the RMT predictions to date [4]. In addition, a few deviations from the PTD have been reported in the past, see references in [5]. Therefore, it is still worthwhile to perform new tests of predictions of RMT, especially considering that measurement and analysis techniques have improved considerably since the 1970’s when most of the data used in previous tests were taken.

To make reliable conclusions regarding the validity of the RMT predictions, it is important that the data set be as pure, complete, and large as possible. Perennial problems with neutron resonance data have been (i) contamination of s- by p-wave resonances and/or resonances of neighboring isotopes, (ii) obtaining enough resonances with known \( J^\pi \) to perform statistically meaningful tests, and (iii) missed resonances due to finite experimental threshold.

For our recent data on Pt isotopes measured at the Oak Ridge Electron Linear Accelerator (ORELA), problem (i) was minimized because Pt is near the peak of the s- and the minimum of the p-wave neutron strength functions \( (S_0/S_1 \approx 10) \), and because we made high resolution cross-section measurements on natural Pt as well as four samples enriched in \(^{192,194,195,196}\text{Pt}\).
Solving problem (ii) has been addressed in our case by very good sensitivity for observation of resonances up to neutron energies, \( E_n \), of several keV. If only even-even targets are considered in the analysis, all their s-wave resonances have \( J^\pi = 1/2^+ \), so that difficulty (ii) with the resonance spin assignment also disappears. Solving all three problems is facilitated by a novel approach of using an artificially imposed, energy-dependent threshold in the analysis.

2. **Experiment and data reduction**

Data from measurement of neutron capture and total cross sections for \(^{192,194,195,196,\text{nat}}\)Pt at the ORELA facility were exploited. During the experiment, cross sections were measured in the energy range from 10 eV up to several hundred keV.

Details of the measurements can be found in Ref. [6]. The ORELA was operated at a pulse rate of 525 Hz, a pulse width of 8 ns, and a power of 7-8 kW. Capture measurements were made at a source-to-sample distance of 40.12 m with a pair of C\(_8\)D\(_8\) detectors using the pulse-height-weighting technique, and were normalized via the saturated 4.9-eV resonance in the \(^{197}\text{Au}(n,\gamma)\) reaction. Total neutron cross sections were measured, at the same time on a separate flight path, via transmission using a \(^6\text{Li}\)-loaded glass scintillator at a source-to-detector distance of 79.83 m.

The multilevel, multichannel, \( R \)-matrix code SAMMY [7] was used to fit both our transmission and capture data and extract resonance parameters. Orbital angular momenta up to and including \( p \)-waves (\( \ell_n = 1 \)) were included in the analysis. Resonance energies \( E_\lambda \), total radiation widths \( \Gamma_\lambda \), and observables \( g_J \Gamma_\lambda / \sqrt{E_\lambda} \) were used in subsequent analyses described below. For even-even targets, s-wave resonances have \( g_J = g_{1/2} = 1 \), and hence, \( g_J \Gamma_\lambda = \Gamma_\lambda \). Overall, we determined parameters for 159, 413, 423, 258, and 11 resonances in energy range up to 5.0, 16.0, 7.5, 16.0, and 5.0 keV in \(^{192,194,195,196,\text{nat}}\)Pt systems, respectively.

An asymmetrical shape in the transmission data could be used to assign \( \ell_n = 0 \) resonances [6, 8], see Fig. 1. However, there remained many weak resonances, most of which are \( p \) wave, for which we could not unambiguously determine the \( \ell_n \) value. As shown below, the potential problem posed by these resonances can be surmounted.

3. **Analyses of resonance parameters**

The need to use a threshold on observables \( g_J \Gamma_\lambda / \sqrt{E_\lambda} \), to guard against possible systematic errors due to instrumental effects and \( p \)-wave contamination, was realized very early on [3] in tests of theoretical predictions. Use of a \( E_\lambda \)-dependent s-wave threshold of the form \( T_0 = a_0 E_\lambda^{3/2} \), where \( a_0 \) is a constant factor, is a key improvement in our method compared to previous analyses for at least three reasons. First, \( p \)-wave contamination is eliminated equally effectively at all energies. Second, as shown in Fig. 1, our instrumental threshold follows very closely this same energy dependence, being at the same time substantially lower than \( T_0 \); thus, possible diffusiveness of the instrumental threshold can be surmounted equally effectively at all energies. Third, statistical precision of any analysis is maximized by allowing the largest number of s-wave resonances to be included.

Results from the analysis of the distribution of \( g_J \Gamma_\lambda \) in even-even Pt targets were already published [8]. Using the maximum-likelihood (ML) method and assuming that the distribution of reduced neutron widths is governed by a \( \chi^2 \) distribution with unknown number of degrees of freedom it was found that best estimates of \( \nu \) were \( \nu_{\text{exp}} = 0.57^{+0.16}_{-0.15}, 0.47^{+0.19}_{-0.18} \), and \( 0.60^{+0.28}_{-0.26} \) in \(^{192,194,196}\)Pt, respectively. This indicated a deviation from the PTD. But, as pointed out in [8], it is premature to draw from these values of \( \nu_{\text{exp}} \) a reliable conclusion.

To check the veracity of the ML results for \(^{192,194}\)Pt, we undertook additional analysis. First, fixing an expectation value \( E[\Gamma_\lambda] \) of \( \Gamma_\lambda \), we drew from the PTD and a uniform distribution of energies \( E_\lambda \) a random sample, consisting of \( n_0 \) pairs \([E_\lambda, \Gamma_\lambda] \), where \( n_0 \) is
the number of resonances above $T_0$. Then, with the aid of the ML analysis, we estimated $\nu$ for this sample. From a large number of samples for each value of $E[\Gamma^0_{\lambda n}]$ we constructed the empirical cumulative distribution function (CDF) of estimates $\hat{\nu}$ and determined a probability $p_\nu = P(\hat{\nu} > \hat{\nu}_{\text{exp}}|E[\Gamma^0_{\lambda n}])$ that a simulated value $\hat{\nu}$ is higher than the ML estimate $\hat{\nu}_{\text{exp}}$ deduced from experimental values of $\Gamma^0_{\lambda n}$. Given the value of $E[\Gamma^0_{\lambda n}]$, the probability $p_\nu$ represents the statistical significance at which the validity of the PTD can be rejected. Values of $p_\nu$ were very high ($\approx 99.5\%$ in cases of $^{192,194}\text{Pt}$ for the most probable values of $E[\Gamma^0_{\lambda n}]$), but varied considerably with $E[\Gamma^0_{\lambda n}]$. Therefore, we undertook further analysis to impose limits on $E[\Gamma^0_{\lambda n}]$. Simple statistics (named as $Z$ and $Z^2$ in [8]) similar to von Mises, Kolmogorov-Smirnov, or Anderson-Darling ones were adopted for this purpose; see [8] for details. Combining results from $p_\nu$, $Z$, and $Z^2$ for $^{192,194}\text{Pt}$ we arrived at the conclusion that PTD can be rejected with a statistical significance of at least $99.997\%$. Our $^{196}\text{Pt}$ data were not used in this analysis as the smaller number of observed resonances resulted in reduced statistical precision.

Our data allow also analysis of distribution of total radiation widths, $\Gamma_{\lambda\gamma}$, from individual resonances. Unfortunately, fluctuation properties of $\Gamma_{\lambda\gamma}$ are much more complicated than those of $\Gamma^0_{\lambda n}$ as the widths $\Gamma_{\lambda\gamma}$ are given by

$$
\Gamma_{\lambda\gamma} = \sum_f \sum_{XL} f_{XL}(E_\gamma) \xi^2_{XLf} E_\gamma^{(2L+1)} \rho(E, J, \pi),
$$

where $E_\gamma$ is $\gamma$-ray energy, $\rho(E, J, \pi)$ is the density of resonances (at excitation energy $E$) with spin $J$ and parity $\pi$, and $f_{XL}(E_\gamma)$ is radiative strength function for multipolarity $XL$. The sum runs over all final levels $f$ allowed by selection rules. Based on the predictions of RMT it is believed that $\xi^2_{XLf}$ is the random number from the PTD. The empirical CDF of measured widths $\Gamma_{\lambda\gamma}$ for $^{194}\text{Pt+n}$ system is shown in Fig. 2, see the thick solid line.

As each width $\Gamma_{\lambda\gamma}$ is a sum of many terms, there is no simple prediction on distribution of $\Gamma_{\lambda\gamma}$. To check the compatibility of experimental distribution of $\Gamma_{\lambda\gamma}$ with RMT predictions we simulated a large number of $\Gamma_{\lambda\gamma}$ values using the DICEBOX [9] code for different models of $f_{XL}(E_\gamma)$ and $\rho(E, J, \pi)$. Statistics similar to $Z$ and $Z^2$ used in analysis of distribution of $\Gamma^0_{\lambda n}$ [8] were then used to check the agreement between experimental data and simulations.
Unfortunately, the distribution of $\Gamma_{\lambda\gamma}$ depends on adopted models of $f_{XL}(E_{\gamma})$ and $\rho(E, J, \pi)$. Employing the conventional models for these quantities [10] we found that the distribution of the simulated widths $\Gamma_{\lambda\gamma}$ is markedly narrower compared to that of the widths deduced from our $^{192,194,196}$Pt measurements. This disagreement makes it possible to reject the validity of the conventional models for $f_{XL}(E_{\gamma})$ and $\rho(E, J, \pi)$ [10] with high statistical confidence. The degree of disagreement is illustrated in Fig. 2, where the corridor delimited by thin curves represents the region covered by $10^3$ simulated CDFs for statistical variables $\Gamma_{\lambda\gamma}$.

There are several ways to reconcile the simulated CDF with the experimental one while maintaining the statistical approach: (i) random numbers $\xi_{XL\gamma}^2$ could be sampled from a distribution which differs from the PTD – a $\chi^2$ distribution with $\nu \sim 0.2 – 0.5$ instead of $\nu = 1$ is likely to be needed if this is the sole modification; (ii) $f_{XL}(E_{\gamma})$ could be strongly suppressed for small $E_{\gamma}$ – predictions with suppressed $f_{XL}(E_{\gamma})$ similar to that reproducing $\gamma$-ray spectra from $(n, \gamma)$ reactions on $^{197}$Au [11] is shown in Fig. 2, see the the corridor in gray; (iii) there could be abrupt changes in level density as a function of excitation energy $E$. As significant decrease of level density with $E$ at some interval of $E$ would be needed we guess that possibility (iii) is very unlikely.

Finally, using our data on $^{194}$Pt we tested the validity of the multivariate distribution (MVD) of resonance energies $E_{\lambda}$ predicted by the RMT. However, contrary to RMT predictions, we assumed that $\Gamma_{\lambda\nu}^0$ fluctuate according to $\chi^2$ with $\nu = 0.47$, in conformity what we found before [8]. As it is impossible to identify $J^\pi$ of weak resonances, a fraction of $s$-wave resonances must be controllably treated as missing resonances. Keeping this in mind, we simulated a large number of sequences of resonance energies $E_{\lambda}$ of $^{194}$Pt together with the corresponding widths $\Gamma_{\lambda\nu}^0$. As a next step, by imposing the condition of threshold $T_0$, from each such sequence we extracted a sequence of energies of the above-threshold resonances. Then, the MVD of the resulting energy sequences was compared with the sequence of energies $E_{\lambda}$ of experimentally observed above-threshold resonances. Using a statistic similar to $Z^2$, from this comparison we found that our experimental data on resonance energies can reject the validity of the RMT-based MVD of energies $E_{\lambda}$ only on a relatively low statistical significance level of about 95%. Nevertheless, we stress that our knowledge about the true distribution of $\Gamma_{\lambda\nu}^0$ is limited. For this reason the results of this outlined analysis should be taken with care.

Acknowledgement
This work was supported by the U.S. DOE under Contract No. DEAC05-00OR22725 with UT-Battelle, LLC, and by Czech Research Plans MSM-021620859 and INGO-LA08015.

References
[1] Guhr T et al., 1998, Phys. Rep. 299, 189
[2] Balian R, and Bloch C, 1970, Ann. Phys. (N.Y.) 60 401
[3] Porter CE and Thomas GE, 1956, Phys. Rev. 104, 483
[4] Haq, RU, Pandey A, and Bohigas O, 1982, Phys. Rev. Lett. 48 1086; Bohigas O, Haq RU, and Pandey A, 1983, in Nuclear Data for Science and Technology, edited by K. H. Bockhoff (D. Reidel, Dodrecht), p. 809
[5] Koehler PE, 2010, EPJ Web of Conferences 2, 05001
[6] Koehler PE et al., 2002, J. Nucl. Sci. and Tech., Suppl. 2, 546
[7] Larson N, 2008, Oak Ridge National Laboratory Technical Report No. ORNL/TM-9179/R8
[8] Koehler PE, Bečvář F, Krtištka M, Harvey JA, Guber KH, 2010, Phys. Rev. Lett. 105 072502
[9] Bečvář F, 1998, Nucl. Instrum. Methods Phys. Res. A 417, 434
[10] Capote R et al., 2009, Nucl. Data Sheets 110, 3107
[11] Krtištka M et al., 2006, AIP Conference Proceedings 831, 481