Self-organised localisation

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Critical phenomena

Phase transitions in the early universe (QCD, EW, inflation?, baryogenesis?)

Classical phase transitions: the phase changes as the temperature is varied.

Ferromagnet:
Quantum phase transitions: the phase changes as an external field is varied.

\[ V(\phi) = V_\phi + (\phi - \phi_c) \langle O \rangle \]

\[ \begin{cases} 
\langle O \rangle = 0 & \phi > \phi_c \\
\langle O \rangle \neq 0 & \phi < \phi_c 
\end{cases} \Rightarrow \quad V'(\phi) = V'_\phi + \langle O \rangle 
\]
discontinuous at \( \phi = \phi_c \)
**Ingredient 1:**
Some parameters of the microscopic theory are promoted to functions of one or more scalar fields.

\[ \mu \rightarrow \mu(\phi) \]

**Axion:** \[ \mathcal{L}_{\text{dim}=4} = \frac{g_s^2}{32\pi^2} \bar{\theta} G_\mu^a \tilde{G}^{\mu \nu, a} \]
\[ \bar{\theta} \rightarrow a \]

**Cosmological constant:** Abbott, Brown-Teitelboim, etc.

**Higgs mass:** relaxion, etc.
**Ingredient 2:**
Selection mechanism in the multiverse.

**Axion:** symmetry

**Cosmological constant:** anthropics (Weinberg)

**Higgs mass:** back-reaction from EW breaking (relaxon)

**Self-Organised Criticality (SOL):** criticality

(GFG, M. McCullough and T. You, JHEP 10, 093)
Ingredient 2:
Selection mechanism in the multiverse.

Self-Organised Criticality (SOL): criticality
(GFG, M. McCullough and T. You, JHEP 10, 093)
Stochastic approach (Vilenkin, Starobinsky, Linde, ...)

\[ P(\phi, t) \] distribution of volume occupied by \( \phi \) at time \( t \)

\[ \frac{\partial}{\partial \phi} \left[ \frac{\hbar}{8\pi^2} \frac{\partial (H^3 P)}{\partial \phi} + \frac{V' P}{3H} \right] + 3HP = \frac{\partial P}{\partial t} \]  

\( \text{(FPV)} \)  

classical term
Stochastic approach (Vilenkin, Starobinsky, Linde, ...)

$P(\phi, t)$ distribution of volume occupied by $\phi$ at time $t$

$$\frac{\partial}{\partial \phi} \left[ \frac{\hbar}{8\pi^2} \frac{\partial (H^3 P)}{\partial \phi} + \frac{V' P}{3H} \right] + 3HP = \frac{\partial P}{\partial t} \quad \text{(FPV)}$$

quantum term
Stochastic approach (Vilenkin, Starobinsky, Linde, ...)

$P(\phi, t)$ distribution of volume occupied by $\phi$ at time $t$

\[
\frac{\partial}{\partial \phi} \left[ \frac{\dot{h}}{8\pi^2} \frac{\partial (H^3 P)}{\partial \phi} + \frac{V'}{3H} P \right] + 3HP = \frac{\partial P}{\partial t}
\]  

(FPV)

volume term
Stochastic approach (Vilenkin, Starobinsky, Linde, ...)

$P(\phi,t)$ distribution of volume occupied by $\phi$ at time $t$

\[
\frac{\partial}{\partial \phi} \left[ \frac{\hbar}{8\pi^2} \frac{\partial (H^3 P)}{\partial \phi} + \frac{V' P}{3H} \right] + 3HP = \frac{\partial P}{\partial t} \quad \text{(FPV)}
\]

- **Quantum term**: $\hbar$
- **Classical term**: $1$
- **Volume term**: $1/M_p^2$

Classical mechanics $\rightarrow$ Quantum mechanics $\rightarrow$ SOL $\leftarrow$ General relativity $\leftarrow$ Critical phenomena
Fokker-Planck:
\[ \frac{\partial}{\partial \phi} \left[ \frac{\hbar}{8\pi^2} \frac{\partial (H^3 P_{\text{FP}})}{\partial \phi} + \frac{V' P_{\text{FP}}}{3H} \right] = \frac{\partial P_{\text{FP}}}{\partial t} \]

Langevin:
\[ \frac{d\phi}{dt} + \frac{V'(\phi)}{3H} = \eta(t) \quad , \quad \langle \eta(t)\eta(t') \rangle = \frac{H^3}{4\pi^2} \delta(t - t') \]

Volume-weighted Fokker-Planck (FPV):
\[ \frac{\partial}{\partial \phi} \left[ \frac{\hbar}{8\pi^2} \frac{\partial (H^3 P)}{\partial \phi} + \frac{V' P}{3H} \right] + 3HP = \frac{\partial P}{\partial t} \]

each trajectory is weighed by \( e^{3Ht} \)

Probabilistic predictions in the multiverse?
GAUGE DEPENDENCE

\[ t \rightarrow t_\xi \quad \frac{dt_\xi}{dt} = \left( \frac{H}{H_0} \right)^{1-\xi} \quad 0 \leq \xi \leq 1 \]

\[
\begin{cases} 
\xi = 1 & \text{proper-time gauge} \\
\xi = 0 & \text{e-folding gauge}
\end{cases}
\]
MEASURE PROBLEM

Reheating surface: 3-volume hypersurface of all reheating events in spacetime.

Eternal inflation: the reheating surface is infinite and non-compact.

Steady-state solutions: $P(\phi, t) \xrightarrow{t \gg t_R} e^{K(t)} p(\phi)$
VALIDITY OF THE SEMICLASSICAL APPROXIMATION

\[ N < S_{dS} = \frac{8\pi^2 M_P^2}{\hbar H^2} \]

Arkani-Hamed \textit{et al}, 0704.1814
Creminelli \textit{et al}, 0802.1067
Dubovsky \textit{et al}, 0812.2246; 1111.1725

Does the semiclassical approach break down after this time?

Dvali \textit{et al}, 1312.4795; 1701.08776

SWAMPLAND CONJECTURES

Do super-Planckian field excursions, slow-roll inflation and eternal inflation live in the swampland?
PROBABILISTIC INTERPRETATION OF THE FPV EQUATION
FPV

\[
\frac{\partial}{\partial \phi} \left[ \frac{\hbar}{8\pi^2} \frac{\partial (H^3 P)}{\partial \phi} + \frac{V' P}{3H} \right] + 3HP = \frac{\partial P}{\partial t}
\]
LINEAR POTENTIAL

\[ V(\phi) = \frac{\phi}{f} \]
Steady-state solutions:

\[ P(\phi, t) \xrightarrow{t \gg t_R} e^{K(t)} p(\phi) \]
NEAR-CRITICALITY OF THE HIGGS SELF-COUPLING

![Diagram showing the near-criticality of the Higgs self-coupling with regions labeled Stability, Meta-stability, Instability, and Non-perturbativity. The horizontal axis represents the Higgs mass $M_h$ in GeV, and the vertical axis represents the top mass $M_t$ in GeV. There is a box indicating a specific region of interest.](image-url)
\[ V(\varphi, h) = \frac{M^4}{g_*^2} \omega(\varphi) + \frac{\lambda(\varphi, h)}{4} (h^2 - v^2)^2 \]
\[ V(\varphi, h) = \frac{M^4}{g_*^2} \omega(\varphi) + \frac{\lambda(\varphi, h)}{4} (h^2 - v^2)^2 \]
\[ V(\varphi, h) = \frac{M^4}{g_*^2} \omega(\varphi) + \frac{\lambda(\varphi, h)}{4} (h^2 - v^2)^2 \]

\[ \lambda(\varphi, M/g_*) = -g_*^2 \varphi, \quad \frac{d \lambda(\varphi, h)}{d \ln h^2} = \beta_\lambda(h) \]

IR Phase: \( \langle h \rangle = v \)

\[ V = \frac{M^4}{g_*^2} \omega(\varphi) \]

UV Phase: \( \langle h \rangle = \frac{\sqrt{2} c}{g_*} M \)

\[ V = \frac{M^4}{g_*^2} [\omega(\varphi) - c^4 \varphi] \]
SOL: at the end of inflation, there is a strong probabilistic preference for patches of the Universe where the Higgs self-coupling is near its critical value.

What happens to the SOL prediction during the thermal phase of the Universe?

$$\alpha^2 \beta > \left( \frac{\hbar H_0^4}{M_P H_{\text{now}} \Lambda^2} \right)^2 = \left( \frac{H_0}{2 \times 10^{-3} \text{ eV}} \right)^8 \Rightarrow Q^2 V \ & \text{eternal inflation}$$
Higgs naturalness: why is nature so close to the critical point?
HIGGS NATURALNESS

\[ V(\varphi, h) = \frac{M^4}{g_*^2} \omega(\varphi) - \frac{\varphi M^2 h^2}{2} + \frac{\lambda(h) h^4}{4} \]

scanning mass term

EW Phase: \( \langle h \rangle = 0 \)
\[ V = \frac{M^4}{g_*^2} \omega(\varphi) \]

IR Phase: \( \langle h \rangle = v \)
\[ V = \frac{M^4}{g_*^2} \left[ \omega(\varphi) - \frac{g_*^2}{4\lambda} \varphi^2 \right] \]

UV Phase: \( \langle h \rangle = \frac{\sqrt{2} c}{g_*} M \)
\[ V = \frac{M^4}{g_*^2} \left[ -\frac{c^4|\lambda_{UV}|}{g_*^2} + \omega(\varphi) - c^2 \varphi \right] \]
SOL prediction: \( v = e^{-\frac{3}{4}} \Lambda_I \)

\( \frac{v}{M^*} \sim \exp(-\lambda_{uv}/2\beta_\lambda) \)

natural hierarchy from dimensional transmutation

\[ \text{SM} \Rightarrow \]
weak doublet $\chi$ and a SM singlet $\psi$

(a) $\mathcal{L} = -y_{VL} \bar{\psi} \chi H_h + \text{h.c.}$

(b) $\mathcal{L} = -y_{VL} \bar{\psi} L H_h + \text{h.c.}$

Phenomenological SOL prediction: new matter that modifies $\beta_\lambda$ such that the theory is near-critical with respect to variations of the Higgs bilinear.
Parameters of a microscopic theory are functions of the apeiron.
SOL prediction: the distribution is peaked on phase $\nu$ at the point where the two phases are degenerate.
In equilibrium, $T$ in box A and B become equal.

In steady-state, the expansion rates in phase $h$ and $v$ become equal $\Rightarrow$ energy degeneracy.
Supersymmetry is a hidden feature of the theory to any observer, like us, who lives in phase $v$, and yet it determines parameters measurable in our vacuum.
REHEATING TEMPERATURE

\[ T_{\text{RH}} < c_{\xi}^{1/6} (\Lambda_{\text{CC}}^2 M_P)^{1/3} \approx c_{\xi}^{1/6} 25 \text{ MeV} \]

DARK-ENERGY EQUATION OF STATE

\[ w = \frac{P_{\phi}}{\rho_{\phi}} = -1 + \left( \frac{V_{\nu}'(0)}{3H_{\text{now}} \Lambda_{\text{CC}}^2} \right)^2 = -1 + \frac{c_{\xi}^2}{3} \]
TESTING SOL EXPERIMENTALLY?

SOL’s smoking gun is phase coexistence.

Near-criticality of the Higgs self-coupling

Higgs naturalness

New matter at the TeV makes the SM unstable under variations of the Higgs bilinear.

Cosmological constant

Dark energy EoS
CONCLUSIONS

• SOL is an approach radically different from the symmetry paradigm: critical points can become dynamical attractors during inflation and determine low-energy parameters.

  Single atom: energy?

  Gas in statistical equilibrium: probabilistic prediction.

  Single Universe: SM parameters?

  Multiverse in steady-state: probabilistic prediction.

• SOL can address some of the classical open questions in particle physics.
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