Quantum mechanix plus Newtonian gravity violates the universality of free fall

Matt Visser

School of Mathematics and Statistics
Victoria University of Wellington, PO Box 600,
Wellington 6140, New Zealand

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Classical point particles in Newtonian gravity obey, as they do in general relativity, the universality of free fall. However classical structured particles, (for instance with a mass quadrupole moment), need not obey the universality of free fall. Quantum mechanically, an elementary “point” particle (in the particle physics sense) can be described by a localized wave-packet, for which we can define a probability quadrupole moment. This probability quadrupole can, under plausible hypotheses, affect the universality of free fall. (So point-like elementary particles, in the particle physics sense, can and indeed must nevertheless have structure in the general relativistic sense once wave-packet effects are included.) This raises an important issue of principle, as possible quantum violations of the universality of free fall would fundamentally impact on our ideas of what “quantum gravity” might look like. I will present an estimate of the size of the effect, and discuss where if at all it might be measured.

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1. Introduction
While classical point particles in Newtonian gravity obey the universality of free fall, the situation is more subtle for quantum wave-packets, for which the finite size of the wave-packet implies the existence of a quantum probability quadrupole. This in addition to any classical mass quadrupole; and dominant whenever the classical mass quadrupole can be arranged to cancel out in the physics. Plausibly this quantum probability quadrupole will couple to gradients of tidal forces, (second derivatives of the local gravity, third derivatives of the Newtonian potential), in a well defined and calculable manner. I will explore this possibility, and estimate the size of the effect.

2. Quantum structure of wave-packets
Suppose we have an elementary quantum particle that is described by some localized and normalizable wave-packet. Then we can take the probability density to satisfy \( \int \rho(x) \, d^3x = 1 \). Define the centre of probability, and the spread of the wave-packet, by

\[
\bar{x}^i = \int \rho(x) \, x^i \, d^3x; \quad \sigma^2 = \int \rho(x) \, (x^i - \bar{x}^i)^2 \, d^3x. \tag{1}
\]

Now define the dimensionless probability quadrupole moment by

\[
Q^{ij} = \frac{1}{\sigma^2} \int \rho(x) \, (x^i - \bar{x}^i)(x^j - \bar{x}^j) \, d^3x. \tag{2}
\]

\[\text{a}\] Unfortunately “point like” to a particle physicist means something different than it does to a classical relativist. The particles of the standard model are “point like” down to at least \( 10^{-22} \) m, but can very easily have wave-packets at the Angstrom scale, some 12 orders of magnitude larger. It is this absolutely unavoidable wave-packet contribution to the probability quadrupole, (and hence the mass quadrupole), that I will focus on in this essay.

\[\text{b}\] See also a very recent attempt at formulating a quantum weak equivalence principle in terms of the Fisher information matrix.

\[\text{c}\] See also an earlier analysis in terms of quantum states that do not have a classical limit.

\[\text{d}\] There are also potential effects due to spin. Certainly in the classical limit a spinning test particle will have its angular momentum couple to the spacetime metric, thereby leading (via the Mathisson–Papapetrou–Dixon equations) to deviations from geodesic motion. It is less than clear whether one should then apply these classical arguments to quantum spin.
The trace of this probability quadrupole is automatically unity; implying that a traceless probability quadrupole can be defined by $Q^{ij} = Q^{ij} - \frac{1}{3} \delta^{ij}$. (Higher-order multi-pole moments can be added as desired.)

3. Gravitational force on a wave-packet

For a classical point particle located at position $x^i$ the Newtonian gravitational force is simply $F_i = -m \nabla_i \phi(x)$. For analyzing a quantum wave-packet we will have to make some assumptions.

One very natural assumption is that the net force is simply given by integrating over the wave-packet weighted by the normalized probability density $e_i = -\int \rho(x) \nabla_i \phi(x) \, d^3x$. (3)

This sort of assumption is very much in line with the quite standard ideas espoused in setting up the Schrödinger–Newton equation. To avoid this sort of result, one could for instance adopt a variant on Roger Penrose's ideas of a gravity-induced collapse of the wave function or a Diós i-like approach, or adopt a GRW variant, or possibly some variant of a Bohmian approach. In such a case $(F_{net})_i \to \langle F_{net} \rangle_i$ would presumably become an ensemble average over experimental outcomes. Be that as it may, for now I shall stay with the more-or-less standard interpretation of $(F_{net})_i$ as a net force on an uncollapsed wave-packet, and see where that leads.

Now Taylor-series expand around the centre of probability. We see

$$(F_{net})_i = -m \int \rho(x) \nabla_i \phi(x) \, d^3x.$$ (3)

The integrals are easy, the probability dipole contribution vanishing in the usual manner:

$$(F_{net})_i = -m \left\{ \nabla_i \phi(\bar{x}) + \sigma^2 \frac{Q^{jk}}{2} \nabla_i \nabla_j \nabla_k \phi(\bar{x}) + \ldots \right\}. (5)$$

The net acceleration is now:

$$a_{net}(i) = - \left\{ \nabla_i \phi(\bar{x}) + \sigma^2 \frac{Q^{jk}}{2} \nabla_i \nabla_j \nabla_k \phi(\bar{x}) + \ldots \right\}. (6)$$

*This is effectively an appeal to a minor variant of the Ehrenfest theorem, in the form $\langle F(x) \rangle = \langle -\nabla V(x) \rangle$. The analysis herein can be viewed as a refinement of the Ehrenfest theorem, wherein we construct an effective potential such that $\langle \nabla V(x) \rangle = \nabla V_{effective}(\bar{x}, \sigma, Q)$, with the effective potential dependent on the centre, spread, and shape of the wave-packet.*
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The bad news is that this is not universal — net acceleration depends both on the spread $\sigma$ and the shape $Q^{jk}$ of the wave-packet. Even for two wave-packets with the same centre of probability, acceleration differences will depend on differences in spread and shape.

$$\Delta(a_{net})_i = -\left\{ \frac{\Delta(\sigma^2 Q^{jk})}{2} \nabla_i \nabla_j \nabla_k \phi(\bar{x}) + \ldots \right\}. \quad (7)$$

One simplification is to split the quadrupole into trace and trace-free parts, so that

$$\Delta(a_{net})_i = \left\{ \nabla_i \phi(\bar{x}) + \frac{\sigma^2 Q^{jk}}{2} \nabla_i \nabla_j \nabla_k \phi(\bar{x}) + \frac{\sigma^2}{6} \nabla_i \nabla^2 \phi(\bar{x}) + \ldots \right\}. \quad (8)$$

In empty space Laplace’s equation $\nabla^2 \phi = 0$ implies that the last term drops out and

$$\Delta(a_{net})_i = \left\{ \nabla_i \phi(\bar{x}) + \frac{\sigma^2 Q^{jk}}{2} \nabla_i \nabla_j \nabla_k \phi(\bar{x}) + \ldots \right\} \quad (9)$$

So we need to estimate both the spread of the wave-packet and the trace-free part of the probability quadrupole. Note in particular that for any spherically symmetric wave-packet we have $Q^{jk} = 0$; then the quadrupole vanishes and the effect goes away. Furthermore, observe that this effect is not any usual notion of tidal effect — the usual tides are governed by $\nabla_i \nabla_j \phi(\bar{x})$ and are 2nd-order in gradients, and would have to do with internal stresses on the wave-packet — this effect is 3rd-order in the Newtonian potential gradient.

4. Classical point source with quantum probe

Consider now a classical point source for the externally imposed Newtonian potential $\phi = GM/r$. Then

$$\phi \propto \frac{1}{r}; \quad \nabla_i \phi \propto \frac{r_i}{r^2}; \quad \nabla_i \nabla_j \phi \propto \frac{\delta_{ij} r^2 - 3 r_i r_j}{r^5}; \quad (10)$$

and finally

$$\nabla_i \nabla_j \nabla_k \phi \propto -\frac{3(\delta_{ij} \hat{r}_k + \delta_{ik} \hat{r}_j + \delta_{jk} \hat{r}_i)}{r^5} + \frac{15 \hat{r}_i \hat{r}_j \hat{r}_k}{r^7}. \quad (11)$$

Working in terms of unit vectors this becomes

$$\nabla_i \nabla_j \nabla_k \phi \propto -3(\delta_{ij} \hat{r}_k + \delta_{ik} \hat{r}_j + \delta_{jk} \hat{r}_i) + \frac{15 \hat{r}_i \hat{r}_j \hat{r}_k}{r^4}. \quad (12)$$

So if the gravitational field is generated by a classical point source then the acceleration of the wave-packet is

$$\Delta(a_{net})_i = -\frac{GM}{r^2} \left\{ \hat{r}_i + \frac{\sigma^2 Q^{jk}}{2r^2} \left[ -3(\delta_{ij} \hat{r}_k + \delta_{ik} \hat{r}_j + \delta_{jk} \hat{r}_i) + 15 \hat{r}_i \hat{r}_j \hat{r}_k \right] + \ldots \right\}, \quad (13)$$
which simplifies considerably

\[(a_{\text{net}})_i = -\frac{GM}{r^2} \left\{ \hat{r}_i \left[ 1 + \frac{15}{2} \frac{\sigma^2}{r^2} \{ \mathcal{Q}^{jk}_i \hat{r}_j \hat{r}_k \} \right] - 3 \frac{\sigma^2}{r^2} \mathcal{Q}^k_i \hat{r}_k + \ldots \right\}. \quad (14)\]

So the extra terms appearing in the net acceleration are of relative order \( \mathcal{O}(\sigma^2/r^2) \), multiplied by dimensionless factors of order unity, and unless one of the principal axes of the wave-packet is aligned with the vertical, do not necessarily seem to represent a “central force”. The “sideways” acceleration is perhaps a little less mysterious if one rewrites the acceleration as

\[(a_{\text{net}})_i = GM \nabla_i \left\{ \frac{1}{r} \left[ 1 + \frac{3}{2} \frac{\sigma^2}{r^2} \{ \mathcal{Q}^{jk}_i \hat{r}_j \hat{r}_k \} \right] + \ldots \right\}. \quad (15)\]

This is valid as long as the wave-packet is not appreciably evolving on the timescale \( \tau \) set by the experiment,

\[\tau \frac{\delta Q^{jk}}{Q^{jk}} \ll \frac{\delta Q^{jk}}{Q^{jk}}. \quad (16)\]

The net force is still a conservative potential force, but now with an “effective potential energy”

\[V_{\text{effective}} = -\frac{GMm}{r} \left[ 1 + \frac{3}{2} \frac{\sigma^2}{r^2} \{ \mathcal{Q}^{jk}_i \hat{r}_j \hat{r}_k \} \right] + \ldots. \quad (17)\]

5. Quantum source and quantum probe

If both the source and the probe are described by quantum wave-packets then both source and probe have independent centres of probability, and independent probability quadrupoles. Then, taking \( r \) to be the distance between the centres of probability, including the first quantum correction to the classical Newtonian potential implies

\[V_{\text{effective}} = -\frac{GMm}{r} \left[ 1 + \frac{3}{2} \frac{\sigma^2}{r^2} \{ \mathcal{Q}^{jk}_{\text{source}} \hat{r}_j \hat{r}_k \} \right] + \ldots. \quad (18)\]

So the analysis is symmetric under the interchange of source and probe. For the quantum \( N \)-body problem, let \( m_a \) be the mass, \( Q^{jk}_a \) the probability quadrupole, and \( \langle \hat{r}_{ab} \rangle \), the relative displacements between the centres of probability of the individual wave-packets. Then to leading order

\[V_{\text{effective}} = -\sum_{a \neq b} \frac{Gm_a m_b}{2 r_{ab}} \left[ 1 + \frac{3}{2} \left( \frac{\sigma^2}{r_{ab}} \mathcal{Q}^{jk}_{\text{source}} \hat{r}^+_j \hat{r}^+_k + \frac{\sigma^2}{r^2} \mathcal{Q}^{jk}_{\text{probe}} \hat{r}^-_j \hat{r}^-_k \right) \right] + \ldots. \quad (19)\]

This is nicely symmetric under the interchange of any two of the wave-packets.\(^\dagger\)

\(^\dagger\)While the Pauli exclusion principle implies that the wavefunction for fermions is odd under particle (wave-packet) interchange \( \psi(x_a, x_b) = -\psi(x_a, x_b) \) the probability density \( \rho(x_a, x_b) = |\psi(x_a, x_b)|^2 = |\psi(x_b, x_a)|^2 = \rho(x_b, x_a) \) is even. So the effective potential is always even under wave-packet interchange.
6. Experimental estimates

Let us now consider some rough estimates regarding the experimental/observational situation:

- For a nano-scale wave-packet \((\sigma \sim 10^{-9} \text{ m})\) in the gravitational field of the Earth, (an idealized point Earth, \(r \sim 6.371 \times 10^6 \text{ m}\)), we have
  \[
  \frac{\sigma}{r} \sim 10^{-16}, \quad \left(\frac{\sigma}{r}\right)^2 \sim 10^{-32}. \quad (20)
  \]
  Now present-day Eötvös-type experiments are extremely good, but they are still nowhere near good enough to have any hope of seeing this wave-packet effect.

- For a typical laboratory-scale Cavendish experiment, \(r \sim 10 \text{ cm} = 10^{-1} \text{ m}\), so for a nano-scale wave-packet \((\sigma \sim 10^{-9} \text{ m})\) we have
  \[
  \frac{\sigma}{r} \sim 10^{-8}, \quad \left(\frac{\sigma}{r}\right)^2 \sim 10^{-16}. \quad (21)
  \]
  Detecting effects due to wave-packet structure still looks rather hopeless.

- Proposed meso-scale Cavendish experiments are aiming for sub-millimetre distance-scales, \(r \lesssim 1 \text{ mm} = 10^{-3} \text{ m}\), so for a nano-scale wave-packet \((\sigma \sim 10^{-9} \text{ m})\) we have
  \[
  \frac{\sigma}{r} \sim 10^{-6}, \quad \left(\frac{\sigma}{r}\right)^2 \sim 10^{-12}. \quad (22)
  \]
  For detecting a probability quadrupole, this still looks rather difficult.

- Only for a fully quantum Cavendish experiment, might one eventually hope to get wave-packet separations of order the wave-packet spread. In that situation the effects due to wave-packet structure would be of order unity; \(O(1)\). (So that the effects explored in this essay simply could not be ignored.) Achieving such sensitivity would be a very challenging experimental proposal.

The current proposal, looking for effects of the probability quadrupole associated with a wave-packet, is orthogonal to currently extant experiments:

- The COW experiment and its variants, look at quantum interference of a split particle-beam on scales of \(\sim 1 \text{ m}\); but one is not dealing with wave-packets \textit{per se}, and the COW experiments are insensitive to the probability quadrupole.
- The semi-classical Cavendish experiment reported by Rosi \textit{et al} is based on a gradiometer measuring acceleration differences between clumps of Rb atoms that are macroscopically separated on a scale \(\sim 33 \text{ cm}\). (The shape of the Rb clumps is unspecified, but if they are spherical, the entire quadrupole effect quietly vanishes.)
• Bouncing neutrons off the floor is logically orthogonal to the quantum quadrupole effect for a different reason: There one is interested in probing the wave-function (built out of linear combinations of Airy functions) directly; looking for quantization effects that do not depend on a multipole expansion.
• The Page–Geilker experiment is essentially a probe of the “collapse of the wave-function”, and strongly suggests that semi-classical quantum gravity is not the whole story, (at least when working on a macroscopic scale of some several metres).

7. Discussion

While certainly challenging, experimentally probing the ideas developed in this essay is not entirely impossible or implausible. Better, it is almost a no lose proposition — if the effect is looked for and seen, (with an experiment of suitable sensitivity), then the quantum violations of the universality of free fall are certainly telling us something fundamental concerning the gravity-quantum interface. Conversely if the effect is found to not be there, it is most likely telling us that gravity collapses the wave-function, a la Penrose–Diösi–GRW, since then integrating over the wave-packet to find the net force is not the appropriate thing to do. Either way, this would be a major step forward.

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