Neutrino-less Double Beta Decay with Composite Neutrinos.

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Abstract

We study in detail the contribution of heavy composite Majorana neutrinos to neutrino-less double beta decay ($0\nu\beta\beta$). Our analysis confirms the result of a previous estimate by two of the authors. Excited neutrinos couple to the electroweak gauge bosons through a magnetic type effective Lagrangian. The relevant nuclear matrix element is related to matrix elements available in the literature and current bounds on the half-life of $0\nu\beta\beta$ are converted into bounds on the compositeness scale and/or the heavy neutrino mass. Our bounds are of the same order of magnitude as those available from accelerator experiments.

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I. INTRODUCTION

Neutrinoless double beta decay ($0\nu\beta\beta$), see Fig. 1, is certainly one of the more interesting non-accelerator processes that are presently being searched for. The interest in this process stems from the fact that its observation would undoubtedly signal lepton number violation, and at the same time would shed light onto the nature of the neutrino, one of the most elusive elementary particles. For these reasons it has received considerable attention both from the nuclear and the particle physics community [1].

In the standard model, $0\nu\beta\beta$ can only proceed if the neutrino is of Majorana type and has a non zero mass. A number of mechanisms studied in models beyond the standard electroweak theory [2], have verified that neutrino-less double beta decay is a very good probe of physics beyond the standard model. The experimental lower bound on the lifetime of the decay has been used to obtain constraints on the scale of new physics.

Recent work along these lines include: (i) an investigation of new super-symmetric contributions from R-parity violating MSSM [3] shows that constraints on parameters of the model from non-observation of $0\nu\beta\beta$ are stronger than those available from accelerator experiments; (ii) a detailed analysis of the contribution from left right symmetric models [4]; (iii) a study of the effective low energy charged current lepton quark interactions due to the exchange of heavy leptoquarks [5]. The phenomenology of Majoron models has also been studied in detail [6].

Panella and Srivastava [7] were the first to show that the compositeness scenario can give an additional contribution to $0\nu\beta\beta$ and derived bounds on the compositeness parameters from the non observation of $0\nu\beta\beta$. They explored phenomenologically the idea that the excited state of an ordinary neutrino might be a heavy Majorana neutral with a mass $M_N$ ranging from a few hundred GeV up to a TeV. However, the nuclear aspect of the calculation was treated only approximatively: time ordering of the hadronic charged current was neglected and an upper bound for the nuclear matrix element was used in deriving constraints on the compositeness parameters. Use of the HEIDELBERG-MOSCOW $\beta\beta$ experiment
lower bound \[T\] on the half-life of the decay \(^{76}\text{Ge} \rightarrow ^{76}\text{Se} + 2e^-\), yielded the following constraint \[\Lambda_C\] on the scale \((\Lambda_C)\), the heavy neutrino mass \((M_N)\), and the dimensionless coupling constant \(f\): 

\[
|f| \leq 3.9 \frac{\Lambda_C}{1\text{TeV}} \left(\frac{M_N}{1\text{TeV}}\right)^{1/2}. \tag{1}
\]

Apart from the obvious desire to improve on the above mentioned approximations the main motivation for the present work is twofold. Firstly, after the completion of the work of ref. \[7\], appeared a related work by Takasugi \[9\] who considered the same problem \((0\nu\beta\beta\) via the exchange of an heavy composite Majorana neutral) but arrived at quite different conclusions \[4, 9\]: 

\[
|f| \leq 5.8 \times 10^{-3} \frac{\Lambda_C}{1\text{TeV}} \left(\frac{1\text{TeV}}{M_N}\right)^{1/2}. \tag{2}
\]

In view of this discrepancy it is, of course, mandatory to investigate further the problem in order to understand if reliable constraints on the compositeness scale from \(0\nu\beta\beta\) are given by Eq. (1) or by Eq. (2).

Secondly, there is a recent claim by the CDF collaboration of a possible signal of compositeness in high energy proton-proton collisions \[10\]: the measured cross section for large transverse energy jets is significantly higher than predictions based on perturbative QCD calculations to order \(O(\alpha_s^3)\); a compositeness scale of \(\Lambda_C = 1.6\text{TeV}\) is suggested by the CDF study. Were this claim to withstand further data and analysis (such as angular distribution of dijets presently underway), the interest in new physics effects arising from a composite scenario will undoubtedly increase enormously. If so, low-energy processes, such as \(0\nu\beta\beta\), could play a complementary role and hence are worth further investigation.

In this paper we present a detailed analysis of the composite Majorana neutrino contribution to \(0\nu\beta\beta\); we show that: (i) the peculiarity of the dimension five effective coupling

\[1\] the numerical value used in ref. \[7\] was: \(T^{0\nu\beta\beta}_{1/2} \geq 5.1 \times 10^{24}\text{yr}\).

\[2\] the numerical value used in ref. \[9\] was: \(T^{0\nu\beta\beta}_{1/2} \geq 5.6 \times 10^{24}\text{yr}\).
(σ_{μν}) shows up in giving a larger than usual importance to the high-energy behaviour of the hadronic current correlation function; (ii) the nuclear matrix element is calculated exactly since it can be related to matrix elements already known; (iii) the results of Panella and Srivastava remain essentially unchanged; (iv) the calculation by Takasugi presented in ref. is not consistent, and its conclusions are not correct.

II. EFFECTIVE LAGRANGIANS FOR COMPOSITENESS

The idea that at an energy scale Λ_c, quarks and leptons might show an internal structure has been around for quite some time. Although many models describing quarks and leptons in terms of bound states of yet more fundamental entities (preons) have been proposed, so far, no consistent dynamical composite theory has been found. Phenomenologically however this idea can be probed by observing that one natural, model independent, consequence of compositeness is the existence of excited states of the ordinary fermions with masses at least of the order of the compositeness scale.

We review here, for the reader’s convenience, the effective interactions used in the literature to describe possible manifestations of lepton and quark substructure. A more detailed discussion of compositeness phenomenology can be found in refs. 12,13.

Effective couplings between the heavy and light leptons and quarks have been proposed, using weak isospin (I_W) and hyper-charge (Y) conservation. Within this model, it is assumed that the lightness of the ordinary leptons could be related to some global unbroken chiral symmetry which would produce massless bound states of preons in the absence of weak perturbations due to SU(2) x U(1) gauge and Higgs interactions. The large mass of the excited leptons arises from the unknown underlying dynamics and not from the Higgs mechanism.

Assuming that such states are grouped in SU(2) × U(1) multiplets, since light fermions have I_W = 0, 1/2 and electroweak gauge bosons have I_W = 0, 1, only multiplets with I_W ≤ 3/2 can be excited in the lowest order perturbation theory. Also, since none of the gauge
fields carry hyper-charge, a given excited multiplet can couple only to a light multiplet with the same $Y$.

In addition, conservation of the electro-magnetic current forces the transition coupling of heavy-to-light fermions to be of the magnetic moment type respect to any electroweak gauge bosons \[14\]. In fact, a $\gamma_\mu$ transition coupling between $e$ and $e^*$ mediated by the $\vec{W}^\mu$ and $B^\mu$ gauge fields, would result in an electro-magnetic current of the type $j_{e.m.}^\mu \approx \bar{\psi}_{e^*} \gamma^\mu \psi_e$ which would not be conserved due to the different masses of excited and ordinary fermions, (actually it is expected that $m_{e^*} \gg m_e$).

Let us here restrict to the first family and consider spin-1/2 excited states grouped in multiplets with $I_W = 1/2$ and $Y = -1$ (the so called homodoublet model \[13\]),

$$L = \begin{pmatrix} N \\ E \end{pmatrix}$$

which can couple to the light left-handed multiplet

$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} = \frac{1 - \gamma_5}{2} \begin{pmatrix} \nu \\ e \end{pmatrix}$$

through the gauge fields $\vec{W}^\mu$ and $B^\mu$. The relevant interaction can be written \[14\] in terms of two new independent coupling constants $f$ and $f'$:

$$\mathcal{L}_{\text{int}} = \frac{g f}{\Lambda_c} \bar{\ell}_L \sigma_{\mu\nu} \frac{-\gamma_5}{2} l_L \cdot \partial^\nu \vec{W}^\mu$$

$$+ \frac{g' f'}{\Lambda_c} \left( -\frac{1}{2} \bar{\ell}_L \sigma_{\mu\nu} l_L \right) \cdot \partial^\nu B^\mu + \text{h.c.}$$

where $\tau$ are the Pauli $SU(2)$ matrices, $g$ and $g'$ are the usual $SU(2)$ and $U(1)$ gauge coupling constants, and the factor of $-1/2$ in the second term is the hyper-charge of the $U(1)$ current. This effective Lagrangian has been widely used in the literature to predict production cross sections and decay rates of the excited particles \[13,15,16\].

The extension to quarks and strong interactions as well as to other multiplets and a detailed discussion of the spectroscopy of the excited particles can be found in the literature \[17\].

Here, let us write down explicitly the interaction Lagrangian describing the coupling of the heavy excited neutrino with the light electron, which will be slightly generalized in the
following section in order to discuss bounds on the compositeness effective couplings from low-energy, nuclear, double-beta decay:

\[ \mathcal{L}_{\text{eff}} = \left( \frac{g_f}{\sqrt{2} \Lambda_c} \right) \left\{ \left( \mathcal{N} \sigma^{\mu\nu} \frac{1 - \gamma_5}{2} e \right) \partial_\nu W^\mu_+ \right\} + \text{h.c.} \quad (6) \]

### III. NEUTRINO-LESS DOUBLE BETA DECAY (0νββ).

The transition amplitude of 0νββ decay is calculated according to the interaction Lagrangian:

\[ \mathcal{L}_{\text{int}} = \frac{g}{\sqrt{2} \Lambda_c} \left\{ \bar{\psi}_\nu(x) \sigma_{\mu\nu} (\eta_L L + \eta_R R) \psi_N(x) \partial_\mu W^{\mu(-)}(x) \right. \]
\[ \left. + \cos \theta_C J^h_\mu(x) W^{\mu(-)}(x) + \text{h.c.} \right\} \quad (7) \]

where we have generalized the interaction in Eq. (6) to include right-handed couplings (in order to allow comparison with other models than the homo-doublet one), although we will assume chirality conservation i.e. \((\eta_L, \eta_R) = (1, 0)\) or \((0, 1)\); \(R = (1 + \gamma_5)/2, L = (1 + \gamma_5)/2, \theta_C\) is the Cabibbo angle \((\cos \theta_C = 0.974)\) and \(J^h_\mu\) is the hadronic weak charged current.

We have:

\[ S_{fi} = (\cos \theta_C)^2 \left( \frac{g}{2\sqrt{2}} \right)^4 \left( \frac{f}{\Lambda_c} \right)^2 \left( \frac{1}{2} \right) \int \frac{d^4q}{(2\pi)^4} d^4x d^4y \exp\left[ -iq \cdot (x - y) \right] \times \]
\[ 4 \times \frac{1}{\sqrt{2}} (1 - P_{12}) \bar{\psi}(p_2) \sigma_{\mu\nu}(\eta_L L + \eta_R R) \frac{g + M_N}{q^2 - M_N^2} (\eta_L L + \eta_R R) \sigma_{\nu\rho} \psi^C(p_1) \times \]
\[ \exp[i(p_1 \cdot x + p_2 \cdot y)] (q - p_1)^\lambda (q + p_2)^\rho \frac{\langle F | T[J^h_\mu(x) J^h_\nu(y)] | I \rangle}{[(q - p_1)^2 - M_W^2][(q + p_2)^2 - M_W^2]} \quad (8) \]

where \((1 - P_{12})/\sqrt{2}\) is the antisymmetric operator due to the production of two identical fermions. Now let us change variables of integration:

\[
\begin{align*}
x &= z + \frac{u}{2} \\
y &= z - \frac{u}{2}
\end{align*}
\]
with \(d^4x d^4y = d^4z d^4u\). \quad (9)

In addition we make the ansatz that the hadronic current be given by the corresponding sum of the nucleonic charged current:
\[ J^h_\mu(x) = \sum_i J^{(i)}_\mu(x). \]  

This implies:

\[ \langle F|T[J^h_\mu(x) J^r_\mu(y)]|I\rangle = \exp[i(p_F - p_I) \cdot y] \langle F|T[J^h_\mu(x - y) J^r_\mu(0)]|I\rangle \]  

We have:

- \( p_1 \cdot x + p_2 \cdot y = (p_1 + p_2) \cdot z + (p_1 - p_2) \cdot u/2 \)
- \( (\eta_L L + \eta_R R)(g + M_N)(\eta_L L + \eta_R R) = (\eta^2_L L + \eta^2_R R)M_N \quad \text{if} \quad \eta_R \eta_L = 0. \)
- \( (g/(2\sqrt{2}))^4 = M^4_W G^2_F/2 \)

Thus we arrive at:

\[ S_{fi} = (\cos \theta_C)^2 G^2_F \left( \frac{f}{\Lambda_C} \right)^2 \int \frac{d^4 q}{(2\pi)^4} d^4 z d^4 u \exp(-i q \cdot u) \times \]
\[ M_N M^4_W \frac{1}{\sqrt{2}} (1 - P_{12}) \tilde{\psi}(p_2) \sigma_{\mu \lambda} \sigma_{\nu \rho} (\eta^2_L L + \eta^2_R R) \psi^C(p_1) \times \]
\[ \exp[i z \cdot (p_1 + p_2 + p_F - p_I)] \exp[i (u/2) \cdot (p_1 - p_2 - p_F + p_I)] \times \]
\[ (q - p_1)^\lambda (q + p_2)^\rho \frac{\langle F|T[J^r_\mu(u) J^r_\mu(0)]|I\rangle}{[(q - p_1)^2 - M^2_W][(q + p_2)^2 - M^2_W](q^2 - M^2_K)} \]  

The integration over \( d^4 z \) gives the energy-momentum conservation and if we define:

- \( S_{fi} = (2\pi)^4 \delta^4(p_I - p_F - p_1 - p_2) T_{fi} \)
- \( G_{eff} = \cos \theta_C G_F (f/\Lambda_C) \)

we obtain:

\[ T_{fi} = \frac{G^2_{eff}}{\sqrt{2}} (1 - P_{12}) \int \frac{d^4 q}{(2\pi)^4} d^4 u \exp[-i(q - p_1) \cdot u] \times \]
\[ M_N M^4_W \tilde{\psi}(p_2) \sigma_{\mu \lambda} \sigma_{\nu \rho} (\eta^2_L L + \eta^2_R R) \psi^C(p_1) (q - p_1)^\lambda (q + p_2)^\rho \times \]
\[ \frac{\langle F|T[J^r_\mu(u) J^r_\mu(0)]|I\rangle}{[(q - p_1)^2 - M^2_W][(q + p_2)^2 - M^2_W](q^2 - M^2_K)} \]  

Next, we neglect \( p_1 \) and \( p_2 \) everywhere with respect to \( q \) except in the electronic wave functions. Using the identity:
\[(1 - P_{12}) \bar{\psi}(p_2) \sigma_{\mu \lambda} \sigma_{\nu \rho} (\eta^2_{\mu \lambda} L + \eta^2_{\nu \rho} R) \psi^C(p_1) = \bar{\psi}(p_2) \{ \sigma_{\mu \lambda}, \sigma_{\nu \rho} \} (\eta^2_{\mu \lambda} L + \eta^2_{\nu \rho} R) \psi^C(p_1) = 2 \bar{\psi}(p_2) (\eta_{\mu \rho} - \eta_{\mu \nu} \eta_{\lambda \nu} + i \gamma_5 \epsilon_{\mu \lambda \nu \rho}) (\eta^2_{\mu \lambda} L + \eta^2_{\nu \rho} R) \psi^C(p_1) \] (14)

we find:

\[
T_{fi} = \frac{2C^2_{\text{eff}}}{\sqrt{2}} \bar{\psi}(p_2) (\eta^2_{\mu \lambda} L + \eta^2_{\nu \rho} R) \psi^C(p_1) M_N M^4_W \int \frac{d^4q}{(2\pi)^4} \frac{(q^2 \eta^{\mu \nu} - q^\mu q^\nu) W_{\mu \nu}(q)}{(q^2 - M^2_W + i\epsilon)(q^2 - M^2_N + i\epsilon)},
\] (15)

where we have defined:

\[
W^{\mu \nu}(q) = \int d^4x \exp(-iq \cdot x) \langle F|T[J^\mu_h(x) J^\nu_h(0)]|I \rangle.
\] (16)

Eq. (15) gives:

\[
T_{fi} = \frac{2C^2_{\text{eff}}}{\sqrt{2}} \bar{\psi}(p_2) (\eta^2_{\mu \lambda} L + \eta^2_{\nu \rho} R) \psi^C(p_1) M_N M^4_W \int \frac{d^3q}{(2\pi)^3} \times \frac{dq_0 - q^i q^j [W_{ij} - \eta_{ij} (W_{00} + W_k^k)] + q^2 W_k^k}{2\pi \left( (q^2 - \omega^2_N + i\epsilon)^2 \right. \left. - (q^2 - \omega^2_W + i\epsilon)^2 \right)}.
\] (17)

where we have defined: \( \omega_{W,N} = \sqrt{q^2 + M^2_{W,N}} \) and terms in \( q_0 q_i \) have been dropped because they do not contribute to \( 0^+ \rightarrow 0^+ \) transitions.

Inserting a complete set of intermediate states one can cast \( W_{\mu \nu} \) in the form:

\[
W_{\mu \nu}(q) = (-i) \int d^3x \exp(iq \cdot x) \sum_X \left\{ \frac{\langle F|J^h(0)|X\rangle \langle X|J^h(x)|I \rangle}{q_0 - E_F + E_X - i\epsilon} + \frac{\langle F|J^h(0)|X\rangle \langle X|J^h(x)|I \rangle}{-q_0 - E_I + E_X - i\epsilon} \right\}
\] (18)

Using some known results on matrix elements of one particle operators, the quantity \( W_{\mu \nu}(q) \) can be readily evaluated. Note that the sum over intermediate states includes an integration over the continuous part of the spectrum, namely the center of mass momentum \( \mathbf{P} \): \( \sum_X \rightarrow (2\pi)^{-3} \int d^3\mathbf{P} \sum_n \). This integration can be carried out analytically.

The energy of a state \( |X\rangle \) is \( E_X = E_{\text{CM}}(\mathbf{P}) + \epsilon_n \), where \( E_{\text{CM}}(\mathbf{P}) \) is the energy of the center of mass translational motion and \( \epsilon_n \) is the excitation energy \( \text{[18]} \). \( E_{\text{CM}}(\mathbf{P}) = \sqrt{\mathbf{P}^2 + (m_p A)^2} \), with \( m_p \) the proton mass and \( A \) the mass number of the nucleus.

In the closure approximation (routinely applied in double beta calculations), the excitation energy $\epsilon_n$ of the intermediate state is replaced by an average value $\bar{\epsilon}_n$ and the sum over the discrete part of the intermediate states is performed.

The center of mass motion ($\mathbf{R} = (1/A) \sum_i \mathbf{r}_i$) can be separated out so that we have [18]:

$$\xi_i = \mathbf{r}_i - \mathbf{R} \quad \sum_i \xi_i = 0$$

(19)

In terms of the new coordinates ($\xi_i$) we have:

$$\psi(\mathbf{r}_1, \ldots, \mathbf{r}_A) = \exp (i \mathbf{P} \cdot \mathbf{R}) \Phi(\xi_1, \ldots, \xi_{A-1})$$

(20)

Using this for the calculation of one body operator matrix elements [18] one obtains (for states of definite center of mass momentum):

$$\langle F | O(\mathbf{r}) | X \rangle = \sum_i \langle \langle F | \exp[i (\mathbf{P}_X - \mathbf{P}_I) \cdot (\mathbf{r} - \xi_i)] O^{(i)}(\mathbf{P}_X - \mathbf{P}_I) | X \rangle \rangle$$

(21)

where we denote with $\langle \langle \ldots \rangle \rangle$ the matrix element in the space of the $A-1$ relative coordinates.

After integrating over the center of mass motion, we obtain:

$$W_{\mu\nu}(q) = \frac{2i\Delta}{q_0^2 - \Delta^2 + i\epsilon} \langle \langle F | \sum_{kl} \exp(iq \cdot \xi_{kl}) j_{\mu}^{(k)}(-q) j_{\nu}^{(l)}(q) | I \rangle \rangle$$

(22)

(in the limit $\mathbf{p}_F \approx \mathbf{p}_I \approx 0$). We have also used the approximation $\epsilon_F \approx \epsilon_I$:

$$\Delta = \bar{\epsilon}_n - \epsilon_I + E_{CM}(q) - E_{CM}(0)$$

(23)

Since on the average $|q| \approx 40 \text{ MeV}$, we may conclude that the center of mass motion does not give any appreciable contribution to $\Delta$:

$$\Delta \approx \bar{\epsilon}_n - \epsilon_I \approx 10 \text{ MeV}$$

(24)

Next, we use the non-relativistic limit for the nuclear current [19] :

$$j_{\mu}^{(k)}(q) = f_A(q^2) \times \begin{cases} g_V \tau_+^{(k)} & \text{if } \mu = 0 \\ -g_A \tau_+(k) \sigma_i & \text{if } \mu = i \end{cases}$$

(25)

where $\sigma_k$ is the spin matrix of the $k$-th nucleon, and a nucleon form factor,
\[ f_A(q^2) = \frac{1}{(1 + q^2/m_A^2)^2} \]  

(26)

with \( m_A = 0.85 \) GeV, has been introduced to account for the finite size of the nucleon. The latter is known to give a sizable contribution for the heavy neutrino case.

Within the above approximations, we are thus led to:

\[ W_{0,0}(q) = \frac{2i\Delta}{q_0^2 - \Delta^2 + i\epsilon} g_V^2 f_A^2(q^2) \langle \langle F | \sum_{kl} \exp(iq \cdot r_{kl}) \tau^{(k)}_+ \tau^{(l)}_+ | I \rangle \rangle \]

\[ W_{i,j}(q) = \frac{2i\Delta}{q_0^2 - \Delta^2 + i\epsilon} g_A^2 f_A^2(q^2) \langle \langle F | \sum_{kl} \exp(iq \cdot r_{kl}) \tau^{(k)}_+ \tau^{(l)}_+ (\sigma_k)_i (\sigma_l)_j | I \rangle \rangle \]  

(27)

These expressions for \( W_{\mu\nu} \) are standard and the closure approximation has been routinely applied in double beta calculations \[6\].

### IV. DECAY FORMULAE AND NUCLEAR MATRIX ELEMENT

We can now proceed to calculate the \( 0\nu\beta\beta \) decay amplitude using the results of the previous section in Eq. \[17\].

Defining:

\[ I(q^2) = -2 \frac{\partial}{\partial \omega_W} J(q^2) \]

\[ J(q^2) = \int \frac{dq_0}{2\pi i} \left[ \frac{\Delta}{q_0^2 - \Delta^2 + i\epsilon} \right] \frac{1}{(q_0^2 - \omega_N^2 + i\epsilon)(q_0^2 - \omega_W^2 + i\epsilon)} \]

\[ I'(q^2) = -2 \frac{\partial}{\partial \omega_W} J'(q^2) \]

\[ J'(q^2) = \int \frac{dq_0}{2\pi i} \left[ \frac{\Delta}{q_0^2 - \Delta^2 + i\epsilon} \right] \frac{q_0^2}{(q_0^2 - \omega_N^2 + i\epsilon)(q_0^2 - \omega_W^2 + i\epsilon)} \]  

(28)

we can write:

\[ T_{fi} = \frac{2G_{eff}^2}{\sqrt{2}} \bar{\psi}(p_2) (\eta_L^2 L + \eta_R^2 R) \psi(p_1) M_N M_W^4 \int \frac{d^3 q}{(2\pi)^3} | \langle | I(q^2) \rangle \langle I' (q^2) | I \rangle | \times f_A^2(q^2) \]

\[ \left[ -g_A^2 \sigma_{(k)} \cdot \sigma_{(l)} I'(q^2) \right] | I \rangle \times f_A^2(q^2) \]  

(29)

The integrals \( J \) and \( J' \) are:
\[ J = - \frac{1}{2} \frac{\Delta + \omega_W + \omega_N}{\omega_W \omega_N (\omega_W + \omega_N)(\Delta + \omega_W)(\Delta + \omega_N)} \]

\[ J' = - \frac{1}{2} \frac{\Delta}{\omega_W + \omega_N)(\Delta + \omega_W)(\Delta + \omega_N)} \]  \hspace{1cm} (30)

We note that between the three scales \( \Delta, \omega_W \) and \( \omega_N \) involved in the problem we have the following ordering:

\[ \Delta \ll \omega_W \ll \omega_N \]  \hspace{1cm} (31)

Using Eq. \( (30) \) one finds:

\[ I \approx - \frac{1}{\omega_W^4 \omega_N^2} \left[ 1 + \frac{3}{2} \frac{\Delta}{\omega_W} + O \left( \frac{\Delta^2}{\omega_W^2} \right) \right] \]

\[ I' \approx - \frac{1}{\omega_W^4 \omega_N^2} \left[ - \frac{1}{2} + \frac{\Delta}{\omega_W} + O \left( \frac{\Delta^2}{\omega_W^2} \right) \right] \]  \hspace{1cm} (32)

Using these results into Eq. \( (29) \) we find:

\[ T_{fi} = \frac{2 G_{eff}}{\sqrt{2 M_N}} \bar{\psi}(p_2)(\eta_L^2 L + \eta_R^2 R)\psi^C(p_1) g_A^2 \times \]

\[ \langle \langle F \rangle \rangle \sum_{kl} \tau_+^{(k)} \tau_+^{(l)} \left\{ D_{ij}(r_{kl}) \left[ \sigma_i^{(k)} \sigma_j^{(l)} - \eta_{ij} \left( \frac{q_i^2}{q_A^2} - \sigma^{(k)} \cdot \sigma^{(l)} \right) \right] - \frac{1}{2} D(r_{kl})\sigma^{(k)} \cdot \sigma^{(l)} \right\} \vert I \rangle \} \]

where \( D_{ij}(r_{kl}) \) and \( D(r_{kl}) \) are given as:

\[ D_{ij}(r_{kl}) = \int \frac{d^3 q}{(2\pi)^3} \exp (i \mathbf{q} \cdot \mathbf{r}_{kl}) \frac{q_i q_j}{(1 + q^2/m_A^2)^4} \]

\[ D(r_{kl}) = \int \frac{d^3 q}{(2\pi)^3} \exp (i \mathbf{q} \cdot \mathbf{r}_{kl}) \frac{\Delta M_W}{(1 + q^2/m_A^2)^4} \]  \hspace{1cm} (34)

Explicitly :

\[ D_{ij}(r_{kl}) = \frac{m_A^4}{4\pi R_0} \frac{R_0}{r_{kl}} \left[ \delta_{ij} F_A(x_A) - \frac{(r_{kl})_i (r_{kl})_j}{r_{kl}^2} F_B(x_A) \right] \]

\[ D(r_{kl}) = \frac{\Delta M_W m_A^2}{4\pi R_0} \frac{R_0}{r_{kl}} F_N(x_A) \]  \hspace{1cm} (35)

where \( R_0 = r_0 A^{1/3} \) \( (r_0 = 1.1 fm) \), \( x_A = m_A r_{kl} \) and :
\[
F_A(x) = \frac{1}{48} \exp(-x) (x^2 + x)
\]
\[
F_B(x) = \frac{1}{48} \exp(-x) x^3
\]
\[
F_N(x) = \frac{1}{48} \exp(-x) (x^3 + 3x^2 + 3x)
\] (36)

Inserting the results of Eq. (35) into Eq. (33), one can cast \( T_{fi} \) in the form:
\[
T_{fi} = \frac{2 G_{eff}^2}{\sqrt{2} M_N} \bar{\psi}(p_2)(\eta^2 L + \eta^2 R) \psi^C(p_1) \frac{g_A^2 m_A^4}{4\pi R_0} M_{FI}
\] (37)

where \( M_{FI} \) is:
\[
M_{FI} = \langle \langle F | \sum_{kl} \tau_+^{(k)} \tau_+^{(l)} \left( \frac{R_0}{r_{kl}} \right) \left\{ \left( \frac{g_V^2}{g_A^2} - \frac{2}{3} \sigma^{(k)} \cdot \sigma^{(l)} \right)(3F_A - F_B) \\
- \frac{1}{3} \left( \frac{\sigma^{(k)} \cdot r_{kl} \sigma^{(l)} \cdot r_{kl}}{r_{kl}^2} - \sigma^{(k)} \cdot \sigma^{(l)} \right) F_B - \frac{1}{2} \Delta M_W m_A^2 \sigma^{(k)} \cdot \sigma^{(l)} F_N \right\} | I \rangle \rangle
\] (38)

In Eq. (38) we have written \( M_{FI} \) in terms of the following known nuclear structure matrix elements [20]:
\[
M_{GT,N} = \langle \langle F | \sum_{kl} \tau_+^{(k)} \tau_+^{(l)} \sigma^{(k)} \cdot \sigma^{(l)} \left( \frac{R_0}{r_{kl}} \right) F_N(x_A) | I \rangle \rangle
\]
\[
M_{F,N} = \langle \langle F | \sum_{kl} \tau_+^{(k)} \tau_+^{(l)} \left( \frac{R_0}{r_{kl}} \right) F_N(x_A) | I \rangle \rangle
\]
\[
M_{GT'} = \langle \langle F | \sum_{kl} \tau_+^{(k)} \tau_+^{(l)} \sigma^{(k)} \cdot \sigma^{(l)} \left( \frac{R_0}{r_{kl}} \right) [3F_A(x_A) - F_B(x_A)] | I \rangle \rangle
\]
\[
M_{F'} = \langle \langle F | \sum_{kl} \tau_+^{(k)} \tau_+^{(l)} \left( \frac{R_0}{r_{kl}} \right) [3F_A(x_A) - F_B(x_A)] | I \rangle \rangle
\]
\[
M_{T'} = \langle \langle F | \sum_{kl} \tau_+^{(k)} \tau_+^{(l)} \left( \frac{R_0}{r_{kl}} \right) \left( 3 \sigma^{(k)} \cdot r_{kl} \sigma^{(l)} \cdot r_{kl} - \sigma^{(k)} \cdot \sigma^{(l)} \right) F_B(x_A) | I \rangle \rangle
\] (39)

The numerical values for the case of \(^{76}\text{Ge}\) quoted in ref. [20] are given in Table I for easy reference. We can decompose \( M_{FI} \) as:
\[
M_{FI} = M_{FI}^{(0)} + M_{FI}^{(\Delta)}
\]

3 Note that in ref. [20] a slightly different notation is used: \( F_4 = 3F_A - F_B \) and \( F_5 = F_B \).
\[ \mathcal{M}_{FI}^{(0)} = \frac{g^2_Y}{g^2_A} \mathcal{M}_{F'} - \frac{2}{3} \mathcal{M}_{GT'} - \frac{1}{3} \mathcal{M}_T, \]
\[ \mathcal{M}_{FI}^{(\Delta)} = -\frac{1}{2} \frac{\Delta m_N}{m_A^2} \mathcal{M}_{GT,N}. \]  

(40)

and finally obtain:

\[ \mathcal{M}_{FI}^{(0)} = +8.15 \times 10^{-3} \]
\[ \mathcal{M}_{FI}^{(\Delta)} = -6.27 \times 10^{-2} \]
\[ \mathcal{M}_{FI} = -5.45 \times 10^{-2}. \]  

(41)

Let us note that in Eq. (38), the part of the nuclear operator which is independent of \( \Delta \) coincides exactly with the result of ref. \([7]\). (Neglecting the time ordering in the hadronic current is equivalent to the limit \( \Delta \to 0 \).) Our result in Eq. (37) shows the expected scaling \( \sim 1/M_N \) with the composite neutrino mass \( M_N \); exchanging a very heavy Majorana neutrino one expects a factor \( M_N \times M_N^{-2} \) from the neutrino propagator and the exchange of heavier particles reduces the probability of the decay. This behaviour does not appear in the formulae of ref. \([9]\).

**V. DISCUSSION**

Calculation of the half-life of the decay from Eq. (37) is now straightforward. We could use the results of ref. \([7]\) but prefer instead to use the already well known phase space factors given in \([21]\) which take into account the distortion of the electron wave functions going beyond the Rosen-Primakoff approximation. The inverse half-life \( (T_{1/2} = \log 2 T) \) is given by:

\[ T^{-1}_{1/2} = \frac{1}{\log 2} \int \left| T_{FI} \right|^2 2\pi \delta(E_I - E_F - E_1 - E_2) \frac{d^3p_1}{(2\pi)^3 E_1} \frac{d^3p_2}{(2\pi)^3 E_2}. \]  

(42)

From Eq. (37) one has:

\( ^4 \)apart from a normalisation factor.
The electronic wave functions are given by:

$$\psi(p_{1,2}) = F_0(Z + 2, E_{1,2}) \, u(p_{1,2})$$  \hspace{1cm} (44)

where $F_0(Z + 2, E_{1,2})$ is the well known Fermi function describing the distortion of the electron wave in the Coulomb field of the nucleus. We have then with straightforward algebra:

$$\sum_{\text{electron spins}} |\bar{\psi}(p_2)(\eta_L^2 L + \eta_R^2 R)\psi^C(p_1)|^2 = F_0(Z + 2, E_1) \, F_0(Z + 2, E_2) \sum_{\text{electron spins}} |\bar{u}(p_2)(\eta_L^2 L + \eta_R^2 R)u^C(p_1)|^2 = F_0(Z + 2, E_1) \, F_0(Z + 2, E_2) (\eta_L^4 + \eta_R^4) \, 2 \, p_1 \cdot p_2$$ \hspace{1cm} (45)

As regards the phase-space integration, we adopt the standard notation and define:

$$A_{0\nu} = \frac{(G_F \cos \theta_C g_A)^4 m_e^9}{64\pi^5}$$

$$G_{01} = \frac{A_{0\nu} \log 2 (m_e R_0)^2}{\int \frac{2 p_1 E_1 p_2 E_2}{m_e^5} \delta(E_I - E_F - E_1 - E_2) \times F_0(Z + 2, E_1) \, F_0(Z + 2, E_2) \, dE_1 \, dE_2}$$ \hspace{1cm} (46)

The phase-space integral in Eq. (46) is known in the literature and its value, which completely takes account of the Fermi functions, is $G_{01} = 6.4 \times 10^{-15} \text{yr}^{-1}$ (see ref. [21]). As for the half-life, we can write it as:

$$T_{1/2}^{-1} = \left( \frac{f}{\Lambda_c} \right)^4 \frac{m_A^4}{M_N^2} |\mathcal{M}_{FI}|^2 \frac{G_{01}}{m_e^2} (\eta_L^4 + \eta_R^4).$$ \hspace{1cm} (47)

Experimentally, $0\nu\beta\beta$ decay has never been observed so far and therefore we have available only lower bounds on the half-life $T_{1/2}^{\text{lower bound}}$:

$$T_{1/2} > T_{1/2}^{\text{lower bound}}.$$ \hspace{1cm} (48)

We can use the constraint in Eq. (48) into Eq. (47) in order to obtain constraints on the compositeness parameters:

$$\left| \frac{f}{\Lambda_c} \right| < M_N^{1/2} \left( \frac{m_e^2}{m_A^2} \right)^{1/4} \left[ G_{01} \, T_{1/2}^{\text{lower bound}} (\eta_L^4 + \eta_R^4)^{-1/4} \right] \frac{|\mathcal{M}_{FI}|^{1/2}}{G_{01}}$$ \hspace{1cm} (49)
Let us now consider the neutrino-less double beta decay:

\[ ^{76}\text{Ge} \rightarrow ^{76}\text{Se} + 2\ e^- \]  

(50)

for which the HEIDELBERG-MOSCOW $\beta\beta$ experiment provides the current lower bound on the half-life \[ T_{^{^{0\nu}\beta\beta}}^{1/2} > 7.4 \times 10^{24}\text{yr} \]  

(51)

Substituting this value and the other numerical constants ($\eta_L = 1$, $\eta_R = 0$) into Eq. \(49\) we obtain finally:

\[ |f| \leq 9.06 \frac{\Lambda_c}{1\text{TeV}} \left( \frac{M_N}{1\text{TeV}} \right)^{1/2}. \]  

(52)

Eqs. \(49\) & \(52\) are the central results of this work. They describe the constraints on compositeness parameters imposed by the non observation of neutrino-less double beta decay. A few comments are in order.

We have derived, according to the closure approximation, an expression for the nuclear matrix element of the form $\mathcal{M}_{FI} = \mathcal{M}^{(0)}_{FI} + \mathcal{M}^{(\Delta)}_{FI}$ where $\mathcal{M}^{(0)}_{FI}$ coincides with the matrix element given in ref. \([7,12]\), and $\mathcal{M}^{\Delta}_{FI}$ is the first order correction in powers of $\Delta$. Thus in the limit $\Delta \rightarrow 0$ we recover the previously published result \([7,12]\), obtained neglecting the time ordering of the hadronic charged current c.f. Eq. \(8\).

Another important improvement of the present work with respect to \([7,12]\) is that the matrix element has now been expressed in terms of known matrix elements \([20]\), so that while previously only a bound $|\mathcal{M}^{(0)}_{FI}| < 0.34$ could be given, we have now a definite number for $\mathcal{M}_{FI}$ c.f. Eq. \(41\). Thus while in \([7,12]\) only a most stringent bound for the compositeness parameters could be quoted, here we have derived the bounds imposed by $0\nu\beta\beta$ non observation. For definiteness we have shown in Fig. \(2\) the present bounds on $|f|$ for $(\Lambda_c = 1\text{TeV})$ as function of the heavy neutrino mass $M_N$. The bound (solid line) is compared with the most stringent bounds quoted in \([7,12]\). We can see that the true bound is, as expected, somewhat weaker than the previously published one. The same conclusion can be drawn from Fig. \(3\) where we give, as a function of $M_N$, the lower bound on $\Lambda_c$ for $(|f| = 1)$.
The fact that within the closure approximation we obtain an analytic expression of the nuclear operator identical to the previous one (c.f. [7,12]) up to a term linear in $\Delta$, which turns out to be numerically non negligible, gives us confidence on the correctness of our result, namely Eq. (40) and Eq. (49).

We believe that the analysis given by Takasugi [9], does not support his conclusion given in Eq. (2). That result is in contradiction with the physical expectation that exchanging an heavier Majorana neutral reduces the probability of the neutrino-less double beta decay and thus must impose weaker bounds on the remaining parameters. We give, in the appendix, a detailed discussion comparing our calculation with that of Takasugi which show explicitly why he obtained the wrong scaling behaviour.

A remark on the calculation of the matrix element is also due. While we think we have proved the correctness of the analytic expression for the nuclear operator involved in the $0\nu\beta\beta$ with a composite Majorana neutrino, there still remain some uncertainty on the actual value of the matrix element $M_{FI}$. As reported in ref. [20], $M_{F'}$ and $M_{GT'}$ are quite sensitive to the value of $m_A$, the cutoff parameter in the nucleon form factor, and $l_C$, the cutoff parameter of the short-range correlations between nucleons. This is due to the fact that the radial functions appearing in $M_{F'}$ and $M_{GT'}$, Eq.(39) are not positive definite resulting in a delicate cancellation within the radial integrals. This does not happen for $M_{T'}, M_{GT,N}$ and $M_{FN}$ which are rather stable in the region $650 \text{ MeV} < m_A < 1.5 \text{ GeV}$. The numerical values used here to get Eq. (41) refer to the values $m_A = 0.85 \text{ GeV}$ and $l_C = 0.7 \text{ fm}$.

The sensitivity of some of the nuclear matrix elements involved in this calculation together with the fact that $M_{FI}(\Delta)$ is also quite important simply indicates that due to the $\sigma_{\mu\nu}$ coupling appearing in our effective Lagrangian, nuclear physics aspects of the neutrino-less double beta decay calculation become more important. Further investigation is necessary. In particular, we believe that the term $I'$ in Eq. (28) containing $q_0^2$ in the numerator, gives a bigger weight to the high $q_0$ region and in order to properly account for the nuclear physics it might be necessary to go beyond the closure approximation.
Let us now compare our result in Eq. (52) with those from high energy experiments. The ZEUS and H1 collaborations (DESY) have recently published [23,24] results of a direct search of singly produced excited states in electron-proton collisions at HERA. They have studied the reaction \( ep \rightarrow l^*X \) with the subsequent decay \( l^* \rightarrow l'V \) where \( V = \gamma, Z, W \) (see Fig. 6). Upper limits for the quantity \( \sqrt{|c_{Vl^*e}|^2 + |d_{Vl^*e}|^2}/\Lambda_C \times Br^{1/2}(l^* \rightarrow l'V) \) are derived [23] as a function of the excited lepton mass and for the various decay channels. These experiments were sensitive up to 180 GeV for \( m_{\nu^*} \) and up to 250 GeV for \( m_{e^*} (m_{q^*}) \).

For the purpose of comparing our analysis of double-beta decay bounds on compositeness with the high energy bounds, we quote here the limit on the \( \nu^* \) coupling that the ZEUS collaboration has obtained at the highest accessible mass \( (m_{\nu^*} = 180 \text{ GeV}) \):

\[
\frac{\sqrt{|c_{W\nu^*e}|^2 + |d_{W\nu^*e}|^2}}{\Lambda_C} \times Br^{1/2}(\nu^* \rightarrow \nu W) \leq 5 \times 10^{-2} \text{GeV}^{-1}. \tag{53}
\]

Let us emphasise that these limits depend on the branching ratios of the decay channel chosen. For \( m_{\nu^*} = 180 \text{ GeV} \) (the highest accessible mass at the HERA experiments [23,24] with \( \Lambda_C = 1 \text{ TeV}, Br(\nu^* \rightarrow \nu W) = 0.61 \) [23] and \( |c_{W\nu^*e}| = |c_{W\nu^*e}| \) one has:

\[
|f| < 61. \quad \text{HERA} \tag{54}
\]

For the same values of \( m_{\nu^*} = M_N \) and \( \Lambda_C \) one obtains from the \( 0\nu\beta\beta \) constraint i.e. Eq. (52):

\[
|f| < 3.84 \quad 0\nu\beta\beta \tag{55}
\]

We can thus conclude that the bounds that can be derived from the low-energy neutrino-less double beta decay are roughly of the same order of magnitude as those obtained from the direct search of excited states in high energy experiments. We also note that, in contrast to bounds from the direct search of excited particles, our \( 0\nu\beta\beta \) constraints on \( \Lambda_C \) and \( |f| \) do not depend on any assumptions regarding the branching ratios of the decaying heavy particle.

Finally, let us conclude by recalling that with respect to the work of ref. [4,12], we have improved the calculation of the phase-space using the exact values of the integrals given in
ref. [21], thus removing the Rosen-Primakoff approximation for the Fermi functions. This however does not give appreciable changes in the numerical results as shown in Figures 2 and 3. The results appear stable.

APPENDIX: DETAILS OF CALCULATION

Here we show in some detail where the calculation of ref. [9] differs from ours and why the author of reference [9] got the wrong scaling behavior for Eq. (2). The discrepancy is that the author there used throughout an effective four-fermion interaction, i.e. an effective Fermi theory. This is clear in view of the absence of W gauge boson propagators in his formulae. The use of an effective four-fermion interaction of course amounts to let $M_W \to \infty$ after having introduced the effective Fermi coupling constant $G_F$. Normally this is a good approximation if there is no other mass scale comparable with or larger than $M_W$, as is the case, for instance, in typical low energy processes. However, in our problem, another mass scale enters the game, namely the heavy neutrino mass $M_N$ which we assume to be much greater than $M_W$. It is then inconsistent to let $M_W \to \infty$ while keeping $M_N$ finite. Nevertheless, this was done in ref. [9]. Since we are to evaluate effects due to the heavy mass $M_N$, we must include effects due to $M_W$. They can not be discarded a priori if we wish to discover the correct scaling behavior.

Takasugi’s Eq. (7) in ref. [9] reads:

$$S_{fi} = \frac{G_{eff}^2}{(2\pi)^4} \int dx dy \int dq M_N \exp \left[ \frac{-iq \cdot (x - y)}{q^2 - M_N^2} \right] \langle N_F | T(J^+_\nu(x)J^+_{\sigma}(y) | N_I \rangle \times$$

$$\frac{1}{\sqrt{2}} (1 - P_{12}) t^{\sigma}(E_1, E_2, q^0, q) \exp \left[ i(E_1 x^0 + E_2 y^0) \right]$$

$$t^{\sigma}(E_1, E_2, q^0, q) = \bar{\psi}_S(E_2) \sigma^{\mu\nu} \sigma^{\rho\sigma}(n^2_L, L + n^2_R R) \psi_S^C(E_1)(q_\mu - E_1 g_{\mu 0})(q_\rho + E_2 g_{\rho 0})$$

$$G_{eff} = \left( \frac{f}{\Lambda_c} \right) G_F \cos \theta_C \quad \text{(homodoublet model)}$$

In Eqs. (A1-A2) Takasugi is also neglecting the momenta of the outgoing electrons everywhere except in the electron wave function (which includes the relativistic Coulomb corrections of the nuclear field). Our Eq. (8) on the other hand may also be written as:
In order to compare Eq. (A3) with Eq. (A1) we neglect, at this stage, in Eq. (A3), the electron momenta everywhere but in the wave function (as opposed to section III where we did so after having extracted the four-dimensional delta-function of energy-momentum conservation). Hence Eq. (A3) reads:

\[
S_{fi} = i G_F^2 M_N \frac{1}{\sqrt{2}} (1 - P_{12}) \int d^4x \int d^4y \int \frac{d^4q}{(2\pi)^4} \exp \left[ -iq \cdot (x - y) \right] \times
\]

\[
M_W^4 \frac{(N_F \mid T(J^\mu_k(x)J^\nu_k(y) \mid N_I)}{[\Delta^2(q - p_1)^2 - M_W^2][q^2 - M_N^2]} \bar{\psi}(p_2)\sigma_{\mu\nu}\sigma_{\nu\rho}(\eta^2 \mathcal{L} + \eta^2 \mathcal{R})\psi^C(p_1) \times
\]

\[
\exp \left[ i(p_1 \cdot x + p_2 \cdot y) \right] (q - p_1)^2 (q + p_2)^2
\]

(A3)

Our result coincides with Eq. (A1) in the limit \( M_W \rightarrow \infty \). We have thus shown that our equations would coincide with those of ref. [9] if we were to adopt the Fermi’s effective theory.

Let us comment further on the calculation presented in ref. [9]. From Eq. (A1) Takasugi first performs the \( q^0 \) integration picking up contributions only from the Majorana neutrino pole. However, for us, keeping the W propagator is essential.

Let us now continue the calculation from Eq. (A4), but following the method employed in ref. [9], deviating somewhat from that of the present work, but with the exception of postponing the \( q^0 \) integration. Expanding the T-product, inserting a complete set of intermediate states between the hadronic current operators, and integrating out \( x_0, y_0 \), after some algebra (exchanging \( x \leftrightarrow y, q \leftrightarrow -q, \mu \leftrightarrow \nu \) in the second term) Eq. (A5) yields:

\[
S_{fi} = 2\pi \delta(E_I - E_F - E_1 - E_2) R_{fi}
\]

\[
R_{fi} = \frac{2G_F^2}{\sqrt{2}} M_N M_W^4 \sum_X \int d^3x \int d^3y \int \frac{d^4q}{(2\pi)^4} \exp \left[ iq \cdot (x - y) \right] \langle N_F \mid T(J^\mu_k(x)J^\nu_k(y) \mid N_I) \times
\]

\[
1 \sqrt{2} (1 - P_{12}) \frac{t_{\mu\nu}(q; E_1, E_2)}{(q_0 + \Delta_X^{(1)} - i\epsilon)(q^2 - M_N^2 + i\epsilon)(q^2 - M_W^2 + i\epsilon)^2}
\]

(A5)
where \( \Delta_X^{(1)} = E_X - E_F - E_1 = E_X - 1/2(E_I + E_F) - 1/2(E_1 - E_2) \approx E_X - 1/2(M_I + M_F) \). As in section III of this work, \( E_X \) is replaced by an average excitation energy \( \langle E_X \rangle \) (closure approximation):

\[
\Delta_X^{(1,2)} \rightarrow \Delta = \langle E_X \rangle - \frac{1}{2}(M_I + M_F) \approx 10 \text{ Mev.} \tag{A6}
\]

Neglecting the electronic energies also in the numerator (in the tensor \( t_{\mu\nu} \)) and using the spinor identity given in Eq. (14) we obtain :

\[
R_{fi} = \frac{4G_{eff}^2}{\sqrt{2}} M_N M_W^4 \int d^3x d^3y \left( \frac{d^4q}{(2\pi)^4} \right) \exp \left[ i\mathbf{q} \cdot (\mathbf{x} - \mathbf{y}) \right] \langle N_F | J^\mu(\mathbf{x}) J^\nu(\mathbf{y}) | N_I \rangle \times
\]

\[
\eta_{\mu\nu} q^2 - q_\mu q_\nu
\]

\[
\frac{(q_0 + \Delta - i\epsilon)(q^2 - M_N^2 + i\epsilon)(q^2 - M_W^2 + i\epsilon)}{(q_0 + \Delta - i\epsilon)^2} \times \bar{\psi}(2)(\eta^2_L + \eta^2_R) \psi^C(1) \tag{A7}
\]

Now we make use of the non-relativistic impulse approximations for the hadronic charged current, just as in section III (inclusion of nuclear form factor etc.) and neglect terms in \( q_0 q_i \) since we only want to consider \( 0^+ \rightarrow 0^+ \) transitions. We find:

\[
R_{fi}(0^+ \rightarrow 0^+) = \frac{4G_{eff}^2}{\sqrt{2}} \bar{\psi}(2)(\eta^2_L + \eta^2_R) \psi^C(1) M_N M_W^4 \int \frac{d^3q}{(2\pi)^3} f_A^2(q^2) \times
\]

\[
\sum_{k,l} \langle N_F | \exp (i\mathbf{q} \cdot \mathbf{r}_{kl}) r_+^{(k)} r_+^{(l)} \left\{ -g_{\gamma}^2 q^2 \mathcal{I}(q^2) + g_A^2 \left[ -\eta^{ij} q^2 \right] \mathcal{J}(q^2) \sigma_i^{(k)} \sigma_j^{(l)} - g_A^2 \sigma^{(k)} \cdot \sigma^{(l)} \mathcal{J}'(q^2) \right\} | N_I \rangle \tag{A8}
\]

where we have defined:

\[
\mathcal{I}(q^2) = \frac{\partial}{\partial \omega_W^2} \mathcal{J}(q^2)
\]

\[
\mathcal{J}(q^2) = \frac{1}{2\pi} \int \frac{dq_0}{q_0 - \Delta + i\epsilon} \left[ \frac{1}{q_0^2 - \omega_N^2 + i\epsilon} \right] \frac{1}{q_0^2 - \omega_W^2 + i\epsilon}
\]

\[
\mathcal{J}'(q^2) = \frac{\partial}{\partial \omega_W^2} \mathcal{J}'(q^2)
\]

\[
\mathcal{J}'(q^2) = \frac{1}{2\pi} \int \frac{dq_0}{q_0 - \Delta + i\epsilon} \left[ \frac{1}{q_0^2 - \omega_N^2 + i\epsilon} \right] \frac{g_0^2}{q_0^2 - \omega_W^2 + i\epsilon} \tag{A9}
\]

with \( \omega_N \) and \( \omega_W \) are the same as in section III.

Eq. (A8) is found to be equivalent to Eq. (29) because of the identity:

\[
\int \frac{dq_0}{2\pi} \left[ \frac{1}{q_0 + \Delta - i\epsilon} \right] f(q_0^2) = -\frac{1}{2} \int \frac{dq_0}{2\pi i} \left[ \frac{2i\Delta}{q_0^2 - \Delta^2 + i\epsilon} \right] f(q_0^2) \tag{A10}
\]
which implies that the integrals $I, I'$ are proportional to $I, I'$ defined in Eq. (28), section IV, namely:

$$I = \left( \frac{i}{2} \right) I; \quad I' = \left( \frac{i}{2} \right) I'.$$

The fact Eq. (A8) coincides with Eq. (29) concludes our proof that including the W boson propagators in the calculation of ref. [9] gives the correct scaling behaviour as found in the present work (and also found in ref. [7]).
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| $\mathcal{M}_{GT,N}$ | $\mathcal{M}_{F,N}$ | $\mathcal{M}_{GT'}$ | $\mathcal{M}_{F'}$ | $\mathcal{M}_{T'}$ |
|---------------------|---------------------|---------------------|---------------------|---------------------|
| $1.13 \times 10^{-1}$ | $-4.07 \times 10^{-2}$ | $-7.70 \times 10^{-3}$ | $3.06 \times 10^{-3}$ | $-3.09 \times 10^{-3}$ |
FIGURES

FIG. 1. Neutrinoless double beta decay ($\Delta L = +2$ process) mediated by a heavy composite Majorana neutrino.

FIG. 2. New bounds on the parameter $|f|$ (for $\Lambda_C = 1$TeV) from $0\nu\beta\beta$ (solid line) compared with the estimate, based on a upper bound of the nuclear matrix element ($|\mathcal{M}_{FI}| < 0.34$), given in ref. [7,12] (dashed line). The dotted line is the calculation of ref. [7,12] augmented by the exact phase space calculation (no Rosen-Primakoff approximation for the Fermi functions).

FIG. 3. New bounds on the parameter $\Lambda_C$ (for $|f|=1$) from $0\nu\beta\beta$ (solid line) compared with the estimate given in ref. [7,12] (dashed line). The dotted line is the calculation of ref. [7,12] augmented by the exact phase space calculation (no Rosen-Primakoff approximation for the Fermi functions).
Figure 1

\[ (A,Z) \quad (A,Z+2) \]

\[ W^- \quad v^*_M \quad W^- \]

\[ n \quad p \quad n \quad p \]
Figure 2

Upper Bound on $|\Lambda_c|$

$\Lambda_c = 1 \text{ TeV}$

$M_N (\text{TeV})$
Figure 3

Lower Bound on $\Lambda_e$ (TeV) vs. $M_N$ (TeV)

$|f| = 1$