D-Brane Recoil and Supersymmetry Obstruction

A. Campbell–Smith and N.E. Mavromatos

Department of Physics, King’s College London, Strand, London WC2R 2LS

Abstract

We discuss a model in which our universe is pictured as a recoiling Dirichlet brane: we find that a proper treatment of the recoil leads naturally to supersymmetry obstruction on the four-dimensional world. An essential feature of our approach is the fact that the underlying worldsheet sigma model is non-critical, and the Liouville mode plays the rôlè of the target time. Also, the extra bulk dimensions are viewed as sigma model couplings, and as such have to be averaged by appropriate summation over worldsheet genera. The recoiling brane is in an excited state rather than its ground state, to which it relaxes asymptotically in time, restoring supersymmetry. We also find that the excitation energy, which is considered as the observable effective cosmological ‘constant’ on the brane, is naturally small and can accommodate upper bounds from observations.

1 Introduction

In this letter we present a model which describes a four-dimensional world with obstructed supersymmetry: the vacuum energy vanishes, but the particle spectrum does not respect supersymmetry. The four-dimensional world is pictured as a string soliton (Dirichlet brane) subject to emission/absorption events with closed string states or other solitonic objects; the resulting recoil is described by worldsheet logarithmic operators, Liouville-dressed to restore worldsheet conformal invariance. The Liouville-dressing furnishes the model with an extra target-space coordinate, the Liouville mode. We consider the D-brane soliton embedded in a six-dimensional target space: the four Neumann coordinates of the D-brane world-volume (with Euclidean signature), the Liouville mode and one further Dirichlet coordinate transverse to the brane. Identifying the Liouville mode with the target time and incorporating the effects of summing over worldsheet genera leads to an induced metric with Minkowskian signature describing the background of the excited, recoiling brane. The metric is that of a ‘thick’ four-dimensional brane-world and is consistent with supersymmetry obstruction for it has a deficit: this prevents the definition of a globally covariant spinor supercharge and leads to mass splittings in the excitation spectrum which do not preserve supersymmetry. The ‘thickness’ arises from integrating out the transverse coordinate, as is appropriate upon its proper identification as a coupling in the worldsheet sigma model. This is to be viewed as a dimensional reduction procedure and a natural method for compactifying extra transverse dimensions: the dimension is not completely integrated away, but leads to a ‘thickened’ four-dimensional world.

We compute the properties of our ‘thickened’ metric from the Einstein equations (in the presence of a dilaton scalar field related to the Liouville mode), which proves to be a non-trivial consistency check of our results. We find that the five-dimensional cosmological constant can be made to vanish, consistent with the picture of supersymmetry obstruction. This fits naturally with the idea that our world is a Dirichlet brane which is in an excited state as a result of recoil: asymptotically (in time) supersymmetry in the mass spectrum will be restored as the brane relaxes. While the underlying five-dimensional cosmological constant vanishes, an observer on the four-dimensional world will measure an effective cosmological constant arising from the excitation energy of the brane itself. We compute the recoil contribution to the excitation energy of the four-dimensional world and find that it is positive and decays as $1/t^2$ where $t$ is the target time.
time with origin at the scattering event causing the recoil. We also show that a one-loop matter contribution to the excitation energy on the four-dimensional world could partially cancel this energy and lead to a naturally small observed cosmological constant on the brane.

The model proposed here for the appearance of a ‘thick’ brane-world differs from previously proposed scenarios [14–20] in that the ‘thickening’ process appears dynamically from a non-critical (Liouville) string theory. This latter point of view was considered in reference [21] where, however, the Liouville mode was not identified with the target time. In the current model this identification is crucial for the relaxation mechanism described above.

2 Logarithmic operators and D-brane recoil

Consider a string soliton (i.e. a Dirichlet brane) with four Neumann (longitudinal) coordinates $x^\mu$, $\mu \in \{0,1,2,3\}$ and one Dirichlet (transverse) coordinate $z$. Notice that from a target-space point of view a Euclidean time $x^0$ is included in the set of Neumann coordinates. As we shall see later on a Minkowskian signature will arise dynamically upon the identification of $x^0$ with the Liouville mode. A proper treatment of scattering events involving either open strings ending on the soliton or closed string states propagating in the bulk transverse dimension should include the resulting recoil of the solitonic object. Note that qualitatively the effects of these two types of scattering event are the same, for a closed string state impinging on the soliton will split into open string states ending on the brane. The recoil of the brane under these conditions is known [3] to be described in terms of a logarithmic conformal field theory [4–7] on the open string worldsheet, and encapsulates the effects of both virtual and on-shell scattering events. The pair of logarithmic operators describing the recoil are

$$C^\mu z = \varepsilon g^\mu z \int_{\partial \Sigma} \Theta_\varepsilon(x^\mu) \partial_\perp z,$$

$$D^\mu z = g^\mu z \int_{\partial \Sigma} x^\mu \Theta_\varepsilon(x^\mu) \partial_\perp z,$$ (2.1a, 2.1b)

where $\partial_\perp$ denotes the normal derivative on the worldsheet boundary $\partial \Sigma$. The couplings $g_C$, $g_D$ the recoil/folding of the D-brane soliton as a result of the scattering. In particular,

$$g_C^0 z = z_0, \quad g_D^0 z = U z$$ (2.2)

where $z_0$ is the origin of the scattering event in the $z$-direction and $U_z$ is the resulting velocity of the soliton in the $z$-direction and in general the other components of the two couplings quantify the folding in the longitudinal space. In all the above the parameter $\varepsilon$ is a regulating parameter for the regularized $\Theta$-function

$$\Theta_\varepsilon(x^\mu) = -i \int d\omega \frac{e^{i\omega x^\mu}}{\omega - i\varepsilon}.$$ (2.3)

This reproduces the normal Heaviside function in the limit $\varepsilon \to 0^+$. To ensure that the pair (2.1) satisfies the correct logarithmic algebra one must identify the regulating parameter with the worldsheet renormalization group scale $\ell \equiv \ln |L/a|$ (where $L$ and $a$ are respectively the infrared and ultraviolet worldsheet configuration space cutoffs)

$$\varepsilon^{-2} = 4 \ln \left| \frac{L}{a} \right|.$$ (2.4)

As shown in reference [3] the pair (2.1) are relevant operators in a worldsheet renormalization group sense with anomalous dimension $\Delta_C = \Delta_D = \Delta = -\varepsilon^2/2$. The resulting theory therefore requires Liouville dressing to restore conformal invariance on the worldsheet. As is well known, the Liouville dressing introduces an extra target-space coordinate $\phi$, the Liouville mode. The dressed operators are obtained by first rewriting the boundary operators (2.1) as bulk worldsheet operators
using Stokes’ theorem and then multiplying the integrand by \( \exp \delta \phi \) where \( \delta \) is the (worldsheet) gravitational anomalous dimension \([8, 9]\) which is given by
\[
\delta = -\frac{Q^2}{2} + \sqrt{\frac{Q^2}{4} - \Delta} = -\frac{Q^2}{2} + \sqrt{\frac{Q^2}{4} + \frac{\varepsilon^2}{2}}. \tag{2.5}
\]
In the above formula \( Q^2 \) denotes the central charge deficit of the non-critical theory resulting from the logarithmic deformations \((2.1)\). On general worldsheet renormalization group grounds, the central charge deficit \( Q^2 \) can be computed from the Zamolodchikov C-theorem \([22]\) as follows.

Consider the generic worldsheet deformation
\[
g^i \int_{\Sigma} V_i
\]
for vertex operators \( V_i \) with associated couplings \( g^i \). The Zamolodchikov metric in this case reads
\[
\Theta_{ij} = \lim_{w \to 0} |w|^4 \langle V_i(w)V_j(0) \rangle = \delta_{ij} + \mathcal{O}(g^2). \tag{2.6}
\]
The C-theorem then relates the central charge deficit to the worldsheet renormalization group \( \beta \)-functions for the couplings \( g^i \) as follows
\[
Q^2 \sim \int \beta^i \Theta_{ij} \beta^j \sim (\Delta_V g^i)^2 + \mathcal{O}(g^3). \tag{2.7}
\]
It can easily be checked that the second equality in equation \((2.6)\) is valid also for logarithmic operators (rewritten as bulk worldsheet operators by means of Stokes’ theorem), as follows directly from their operator product expansion \([3, 4]\).

From equation \((2.7)\) it is clear that for generic marginally relevant deformation operators, \( Q^2 \) is subleading in equation \((2.5)\), and as a result the gravitational anomalous dimension \((2.5)\) is \( \delta \approx \varepsilon \). We can confirm this result later as a non-trivial consistency check of our results.

### 3 The Induced Spacetime Metric

The Liouville-dressed worldsheet operators in our case read
\[
C^\mu_\nu^z_L = \varepsilon g^\mu_C \int \Sigma e^{\varepsilon \phi} \partial_\alpha (\Theta_\varepsilon (x^\mu) \partial^\alpha z), \quad \text{(no sum on } \mu), \tag{3.1a}
\]
\[
D^\mu_\nu^z_L = g^\mu_D \int \Sigma e^{\varepsilon \phi} \partial_\alpha (x^\mu \Theta_\varepsilon (x^\nu) \partial^\alpha z), \quad \text{(no sum on } \mu); \tag{3.1b}
\]
here \( \alpha \in \{1, 2\} \) is a worldsheet index. By partial integration these can be re-expressed as a sum of worldsheet bulk and boundary terms: the boundary terms describe the recoil excitations in a conformally invariant fashion, while the bulk operators
\[
- \int \Sigma e^{\varepsilon \phi} \left[ \varepsilon^2 g_C^{\mu z} + \varepsilon g_D^{\mu z} x^\mu \right] \Theta_\varepsilon (x^\mu) \partial_\alpha \phi \partial^\alpha z \tag{3.2}
\]
describe target-space metric deformation as a result of the recoiling D-brane \([1]\). The induced metric in the six-dimensional target space spanned by \( \{ \phi, x^\mu, z \} \) and far from and after the scattering event is
\[
G_{\phi \phi} = 1, \quad G_{\phi \mu} = G_{z \mu} = 0, \quad G_{\mu \nu} = -\delta_{\mu \nu}, \quad G_{zz} = -1, \quad G_{\phi z} = -\sum_{\mu} \left[ \varepsilon^2 g_C^{\mu z} + \varepsilon g_D^{\mu z} x^\mu \right]. \tag{3.3}
\]
Recall that the \( g_{C,D} \) are relevant couplings in a worldsheet sigma model: they can be made exactly marginal by a rescaling \([2, 22]\)
\[
g_{C,D} \rightarrow \varepsilon \tilde{g}_{C,D} \tag{3.4}
\]
where $\bar{g}$ is independent of $\epsilon$. As shown in reference [2], summation over worldsheet genera is equivalent to averaging (with Gaussian weight) over these couplings. In the limit $\epsilon \to 0^+$ the term involving $\bar{g}_C$ is subleading and can be dropped. The averaging procedure over the remaining coupling is performed around the classical value $\bar{g}_D = 0$ (corresponding to the classical static D-brane) and is defined through

$$\langle\langle \ldots \rangle\rangle \equiv \int d\bar{g}_D \frac{e^{-\bar{g}_D/\Gamma^2}}{\Gamma \sqrt{\pi}} \langle\ldots\rangle.$$ (3.5)

The resulting diagonalized [21] averaged line element is (for $x^\mu$ positive, i.e. away from and after the scattering event)

$$\langle\langle ds^2 \rangle\rangle = d\phi^2 - |1 - \alpha^2 z^2|\delta_{ij} dx^i dx^j - (1 + \alpha^2 x_\mu x^\mu) dz^2,$$ (3.6)

$$\alpha \equiv \frac{\epsilon^2 \Gamma}{2 \sqrt{2}}$$ (3.7)

The width $\Gamma$ is known to satisfy a Heisenberg uncertainty relation in the D-brane recoil picture [12]. In taking the absolute value in the line element above we imply a mirror extension around the origin in $z$, which we assume to be the initial position of the brane. An important feature of this line element is the appearance of a singularity at $z = \pm \alpha^{-1}$. For the purposes of this letter we will discard the $\alpha^2 x^2$ term in the fifth component of the metric since this introduces no additional singular structure and because $\alpha$ is small.

In the spirit of reference [11] and in contrast to the approach of reference [21] we next identify the worldsheet zero-mode of the Liouville field $\phi$ with the target time $x^0 \equiv t$. In spite of the fact that $x^0$ was originally a Euclidean longitudinal coordinate, after this identification one obtains a line element with a Minkowskian signature:

$$\langle\langle ds^2 \rangle\rangle = \alpha^2 z^2 dt^2 - |1 - \alpha^2 z^2|\delta_{ij} dx^i dx^j - dz^2, \quad i, j \in \{1, 2, 3\}.$$ (3.8)

The identification of the Liouville zero-mode with the target time implies, on account of equation (2.4), that $\epsilon$ scales as

$$\epsilon \sim \frac{1}{\sqrt{t}}.$$ (3.9)

In what follows we shall check the consistency of this identification by demonstrating that the metric (3.8) satisfies Einstein’s equations in a five-dimensional target space with a scalar dilaton, $\varphi$, dependent only on the Liouville zero-mode [24]:

$$\varphi(t) \equiv \int^t dt' q(t') \approx M_s Q(t) t,$$ (3.10)

where $M_s$ is the string scale which we take to be $10^{19}$ GeV. In the above, $Q^2(t)$ is the running central charge deficit of the Liouville theory, whose $t$-dependence arises from equation (3.9). This is to be contrasted with standard non-critical string theory where $Q$ does not run [11, 10, 24] and the dilaton has a component linear in $Q t$.

The satisfaction of Einstein’s equations is equivalent to the restoration of conformal invariance on the worldsheet by the Liouville dressing procedure [24]. We will check this explicitly in section 4. Before doing so, we discuss the properties of the metric which we have derived.

The $(z, t)$ part of the metric (3.8) is flat, as can be seen from the transformations

$$(z, t) \rightarrow (u, v) = (z \cosh(at), z \sinh(at)) \quad (3.11)$$

in which coordinates the metric reads

$$\langle\langle ds^2 \rangle\rangle = du^2 - dv^2 - |1 - \alpha^2(u^2 - v^2)|\delta_{ij} dx^i dx^j.$$ (3.12)
This is simply a Rindler wedge (since \( u > |v| \)) space which is known to represent (for lines of constant \( z \)) the world-line of an accelerated observer with proper acceleration \( 1/z \). However, since our spacetime is only piecewise continuous the acceleration is only uniform for \( t > 0 \). Wedged spacetimes are known to break supersymmetry because they obstruct the definition of a global covariantly constant spinor supercharge.

The structure of this spacetime can be made more transparent by considering a Euclideanized compactified time coordinate, whence it becomes clear that the scale \( \alpha \) induces a conical deficit. In terms of the compactified time coordinate \( \tau = 2\pi it/\beta \) where \( \tau \in [0, 2\pi) \) and \( \beta \) the radius of compactification, the metric reads

\[
(ds_E^2) = -\frac{\alpha^2 \beta^2}{4\pi^2} z^2 d\tau^2 - d\tau^2 + |1 - \alpha^2 z^2| \delta_{ij} dx^i dx^j. \tag{3.13}
\]

From (3.13) we observe that the deficit is given by \((2\pi - \alpha\beta/2\pi)\). A spacetime with such a conical deficit is known to exhibit supersymmetry obstruction in which, although the vacuum state energy remains vanishing, supersymmetry is obstructed in the spectrum of massive excitations by splittings \( \delta m \propto \alpha \). Equivalently we can view this spacetime as being in thermal equilibrium with a bath of ‘temperature’ \( T = \beta^{-1} = \alpha/2\pi \) in which case the deficit disappears but the presence of a non-zero temperature gives rise to non-supersymmetric mass splittings in the excitation spectrum \( \delta m \propto T \) as before.

The ‘thickened’ brane-world picture is obtained now by considering the effects of a proper sum over worldsheet genera which implies that the Dirichlet coordinate \( z \), as a coupling in a worldsheet sigma model, should also be averaged in analogy with equation (3.3). The Gaussian nature of this averaging implies that averages of odd powers of \( z \) will vanish. This picture will be incorporated at the level of a target-space effective action in the next section by considering a ‘stack’ of parallel D-branes in the \( z \)-direction each with their own excitation energy from the recoil. This ‘stacking’ is a purely formal construction describing a single physical brane fluctuating around \( z = 0 \) and should not be confused with the matrix D-brane structures of reference which involve open strings stretching between the branes. The ‘stack’ of D-branes here implies an explicit breaking of five-dimensional Lorentz symmetry; the four-dimensional subgroup of Lorentz rotations on the brane remains unbroken.

### 4 The Dynamics

From a standard low-energy sigma model point of view the gravitational part of the (order \( \alpha' \)) effective action in our five-dimensional spacetime is given by

\[
S = M_s^3 \int d^5 x \sqrt{-g} e^{-\gamma \varphi} \left[ R + \beta (\nabla \varphi)^2 - \Lambda \right] - M_s^2 \sum_i \int d^4 x \sqrt{-g^{(4)}(x, z_i)} e^{-\gamma \varphi} V(x, z_i). \tag{4.1}
\]

The interval of the \( z \)-integration is between the singularities at \( \pm \alpha^{-1} \). For convenience we have absorbed factors of \( M_s^{-2} \) into both \( \Lambda \) and \( V \). The Einstein frame corresponds to \( \gamma = 0 \) and \( \beta < 0 \). On the other hand, in the sigma-model frame \( \gamma > 0 \) and \( \beta > 0 \). As appropriate for a Liouville string theory, in what follows we shall work exclusively in the sigma-model frame. The sum in the last term in the action (4.1) describes the ‘stack’ of D-branes in the \( z \)-direction, each of which has a possible recoil excitation energy \( V(x, z_i) \). This should be contrasted with the orbifold construction of reference where there are only two D-branes. In our case there is a continuum representation of this sum in terms of an averaged ‘thickened’ D-brane:

\[
\sum_i \int d^4 x \sqrt{-g^{(4)}(x, z_i)} e^{-\gamma \varphi} V(x, z_i) \rightarrow M_s \int d^4 x \sqrt{-g^{(4)}(x, z)} e^{-\gamma \varphi} V(x, z); \tag{4.2}
\]

the continuum \( V(x, z) \) incorporates any non-trivial \( z \)-dependent measure appearing in the passage to the continuous form and the function \( V \) denotes the effective recoil excitation energy as measured
on the four-dimensional brane-world. Notice, however, that as a result of the ‘stack’ of D-branes the five-dimensional integral in equation (4.2) does not contain a dependence on the fifth component of the metric. This means that upon a variation with respect to the fifth component of the metric, this term will not contribute. With this in mind the equations of motion derived from the effective action (4.1), (4.2) are

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = - \frac{1}{2} g_{\mu\nu} [\Lambda + V] - \partial_\mu \varphi \partial_\nu \varphi + \frac{1}{2} g_{\mu\nu} (\partial^2 \varphi)^2, \quad \mu, \nu \in \{0, 1, 2, 3\}, \]  

(4.3a)

\[ R_{55} - \frac{1}{2} g_{55} R = - \frac{1}{2} g_{55} [\Lambda] - \partial_5 \varphi \partial_5 \varphi + \frac{1}{2} g_{55} (\partial^2 \varphi)^2, \]  

(4.3b)

and for the dilaton

\[ R - [\Lambda + V] + 2 (\nabla \varphi)^2 - 2 \nabla^2 \varphi = 0. \]  

(4.3c)

As mentioned in the previous section, summation over worldsheet genera requires an averaging procedure for the Dirichlet coordinate: \( \langle\langle z^2 \rangle\rangle_z = \sigma^2 \), which leads to the metric (recalling the identification (3.9))

\[ \langle\langle ds^2 \rangle\rangle = \frac{b^2 \sigma^2}{t^2} dt^2 - \left| 1 - \frac{b^2 \sigma^2}{t^2} \right| \delta_{ij} dx^i dx^j - dz^2, \]  

(4.4)

\[ b = \frac{\Gamma}{2 \sqrt{2}}. \]  

(4.5)

For notational convenience we have used the symbol \( \langle\langle \ldots \rangle\rangle \) to denote the combined Gaussian average over both logarithmic couplings \( \tilde{g}_C \) and \( \tilde{g}_D \). The components of the Ricci tensor and curvature scalar for this metric read

\[ R_{00} = \frac{3 b^2 \sigma^2}{1 - \frac{b^2 \sigma^2}{t^2}} t^6 \left[ (b^2 - 2) t^2 + b^2 \sigma^2 (1 - b^2) \right], \]  

(4.6)

\[ R_{ij} = \delta_{ij} \frac{(b^2 - 1)}{1 - \frac{b^2 \sigma^2}{t^2}} t^4 \left[ 3b^2 \sigma^2 - 2t^2 \right], \quad i, j \in \{1, 2, 3\}, \]  

(4.7)

\[ R_{55} = - \frac{3b^2}{1 - \frac{b^2 \sigma^2}{t^2}} t^2, \]  

(4.8)

\[ R = \frac{12 (b^2 - 1)}{1 - \frac{b^2 \sigma^2}{t^2}} t^2. \]  

(4.9)

We stress that the above components are computed from the averaged metric (4.4) and are not simply averaged versions of equations (4.3). This is essential for our dimensional reduction proposal, which should be viewed as an alternative to compactification: in our approach the fluctuations of the graviton field on the brane are \( z \)-independent as a result of the above average over the Dirichlet coordinate/coupling \( z \). As a result, Newton’s law on the four-dimensional world is unaffected in contrast to other proposals where \( z \) remains a fully-fledged coordinate \([19, 21, 8, 22]\). However, in our picture the effect of this Dirichlet coordinate is still felt in the ‘thickening’ of the recoiling brane and in the constraint coming from the fifth component of the Einstein equation (4.3c). It is understood that the above picture pertains strictly to the case of a stack of identical, parallel solitons; for the intersecting-brane case the rôle of the Dirichlet and Neumann coordinates are mixed and our construction fails.

To leading order in \( t \gg 1 \) the solutions of the Einstein equations (4.3) read

\[ q^2(t) = \frac{b^2 \sigma^2 (4 - b^2)}{t^2} > 0 \quad \text{for} \quad b^2 < 4, \]  

(4.10)

\[ \Lambda = \frac{(5b^2 - 8)}{t^2}, \quad V = \frac{(2b^4 + 4)}{t^2} > 0. \]  

(4.11)
From equations (4.10) and (5.10) we obtain the central charge deficit

\[ Q^2(t) = \frac{b^2 \sigma^2 (4 - b^2) (\ln t)^2}{t^2} \]  

(4.12)

so for \( b^2 < 4 \) the worldsheet sigma model is supercritical [24] and our identification of the Liouville mode with the target time is self-consistent. As anticipated in section 2, the central charge deficit is indeed subleading in equation (2.5), \( Q^2 \sim O(\epsilon^4 (\ln \epsilon)^2) \).

In the classical limit, \( \sigma \to 0 \) the dilaton equation of motion implies \( \mathcal{R} - \Lambda - V = 0 \) which puts a dynamical constraint on the width parameter, \( b^2 = 8/5 \). This constrains the cosmological term to vanish for a value of \( b \) compatible with the supercriticality of the worldsheet sigma model. If one demands continuity with the quantum case where \( \sigma \) is identified with the position uncertainty of the brane then we observe that this constraint can be preserved by the very natural choice \( \sigma = 5/\sqrt{96} M_s \simeq 1/2 M_s \) compatible with near-saturation of the position-momentum uncertainty relation for D-branes [24]. Note also that, as expected, the recoil excitation energy \( V \) is positive definite and that both \( Q(t) \) and \( V(t) \) relax so that asymptotically criticality and supersymmetry are restored [9].

5 One-Loop Matter Contribution to the Excitation Energy

In our picture all the matter fields on the four-dimensional brane-world are viewed as excitations on the already excited (recoiling) D-brane. We can estimate their contribution to the excitation energy in our picture from the one-loop effective potential used to calculate the vacuum energy in standard four-dimensional (supersymmetric) field theory [33–36]:

\[ V_1 \ni \frac{M_{uv}^2}{32 \pi^2} \text{Str} M^2 \]  

(5.1)

where \( M_{uv} \) is the ultraviolet cutoff and

\[ \text{Str} M^n = \sum_i (-1)^{2J_i} (2J_i + 1) M_i^n, \]

whence

\[ \text{Str} M^2 \sim 2(M_0^2 - M_1^2). \]  

(5.2)

We noted in section 3 that the metric describing the background recoiling brane can be interpreted in terms of thermalization with temperature \( T = \alpha/2\pi \). With this interpretation the ‘thermal’ mass splittings required to evaluate the supertrace (5.2) can be computed from a thermal super-space formalism: the thermal mass modes for bosonic and fermionic excitations are respectively given by [28]

\[ M_0^2 = M_0^2 + 4\pi^2 n^2 T^2, \quad n \in \mathbb{Z}, \]
\[ M_1^2 = M_1^2 + (2n + 1)^2 \pi^2 T^2, \quad n \in \mathbb{Z}. \]

In the above, \( M_0 \) is the mass term which appears in the zero temperature Lagrangian density. Now the supertrace can be computed by summing (5.2) over the modes \( n \); regularizing the divergent sums using \( \zeta \)-function regularization the result is a sum over species living on the four-dimensional world:

\[ V_1 \ni -\sum_{i=1}^{N_s} \frac{b^2 M_{uv}^2}{8\pi^2} \zeta(-1, \frac{1}{2}) = \frac{b^2}{t^2} \frac{N_s M_{uv}^2}{8\pi^2} B_2\left(\frac{1}{2}\right) = \frac{N_s M_{uv}^2 b^2}{384\pi^2 t^2}, \]  

(5.3)
where $B_2(x) = x^2 - x + \frac{1}{6}$ is the second Bernoulli polynomial [37]. Comparison with the recoil contribution to $V$ in equation (4.11), shows that there is the possibility to achieve a naturally small total excitation energy $V$ if the ultraviolet cutoff has the not unnatural value

$$M_{uv} \sim \frac{20\pi}{\sqrt{N_s}} M_s. \quad (5.4)$$

Note that in the Standard Model $N_s \sim 40$. The resulting suppression allows us to choose the supersymmetry obstruction scale $\alpha$ in (3.7) to be of the order of a few TeV without generating an unacceptably large observable cosmological ‘constant’ (the rôle played by $V$) on the four-dimensional brane world, compatible with the astrophysical upper bound of $10^{-120}$ in Planck units. Note that global supersymmetry obstructed at scales of a few TeV provides a solution to the gauge hierarchy problem stable to higher order quantum corrections. In the case of supergravity theories, which describe the physics on brane-worlds, this conclusion is not always valid with the notable exceptions of effective theories derived from string models [36]. A detailed study of such issues in the context of the present model is beyond the scope of the current work.

6 Discussion

In this letter we have presented a mechanism whereby supersymmetry obstruction is realized dynamically in a recoiling D-brane framework. An essential feature of our approach is that dimensional reduction to four spacetime dimensions occurs as a result of an averaging procedure over Dirichlet coordinates in a worldsheet sigma model. As a result of this averaging procedure, the four-dimensional brane-world metric (4.4) can be recast as a standard Friedmann–Robertson–Walker metric by the rescaling $t \mapsto t_{FRW} = b \sigma \ln t$. In this frame the excitation energy $V$, the observable cosmological ‘constant’ on the four-dimensional world, relaxes as $\exp\{-2t_{FRW}/b\sigma\}$ and the dilaton (3.10) is linear in $t_{FRW}$: $\varphi = 12t_{FRW}/5$ [24]. Phenomenologically, in this framework, to ensure that the supersymmetry obstruction scale is of order a few TeV, one needs recoil events with temporal separation $t_{FRW} \sim 20M_s^{-1}$, which is a natural timescale for quantum gravity effects, and therefore consistent with the spacetime foam picture resulting from recoiling D-branes [38].

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