A model identification technique to characterize the low frequency behaviour of surrogate explosive materials

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Abstract. The mechanical response of energetic materials, especially those used in improvised explosive devices, is of great interest to improve understanding of how mechanical excitations may lead to improved detection or detonation. The materials are comprised of crystals embedded into a binder. Microstructural modelling can give insight into the interactions between the binder and the crystals and thus the mechanisms that may lead to material heating and but there needs to be validation of these models and they also require estimates of constituent material properties. Addressing these issues, nonlinear viscoelastic models of the low frequency behavior of a surrogate material-mass system undergoing base excitation have been constructed, and experimental data have been collected and used to estimate the order of components in the system model and the parameters in the model. The estimation technique is described and examples of its application to both simulated and experimental data are given. From the estimated system model the material properties are extracted. Material properties are estimated for a variety of materials and the effect of aging on the estimated material properties is shown.

1. Introduction
The focus of the research described in this paper is the estimation of the mechanical material properties of energetic materials, and the construction of models that can be used to predict the behavior of the material when incorporated in a system undergoing vibration excitation. Energetic materials are stored, transported and utilized in a wide variety of environments and there is a need to be able to predict their responses to vibration in these environments. Information on energetic materials’ properties is scarce in the existing literature. Typically the material properties published have come from experiments on small samples at very cold temperatures (less than 7mm and 196K) [1, 2]. The experiments reported have mostly been focused on determining elastic properties and there are very few studies on the dissipative mechanisms in these materials. While compression tests and split-Hopkinson pressure bar tests (with strain rates up to 5000 s⁻¹) have been conducted on these materials, these materials undergoing low frequency vibration (0 Hz - 1000 Hz) at room temperature has not been studied. Here, the response of much larger samples (up 7.6 cm diameter and height cylinders) incorporated into a mass-material system are examined at low strain rates (less than 13 s⁻¹). Experiments in this research were conducted at room temperature over frequency ranges from 10 Hz to 1000 Hz.
Energetic materials are comprised of granules embedded into a binder. In this research, surrogate energetic materials were used, and the volume fraction of crystals to binder was varied to produce test samples with 0% (binder only), 50% and 75% crystal volume. One goal of the research is to develop an understanding of how the crystals affect the macro-scale mechanical behavior of the surrogate materials. This modeling is complementary to micro-scale modeling of these types of materials and the experiments provide material properties and data for validation of micro-scale numerical models, see, e.g., [3]. To produce reliable estimates of the material properties as composition is changed, the materials age, environmental conditions vary, and the excitation changes, there is a need to develop models and parameter estimation techniques that are robust, so that a single model can be accurately predict system responses over a variety excitations. Here the parameters of the system model are estimated to identify properties of surrogates of energetic materials, however, the modeling and parameter estimation techniques may be applied to other materials (rubber-like, granular, plus others). Some of the structures examined are similar to those used to model the mechanical behavior of various types of polyurethane foams, see, e.g., [4–6].

A model was constructed of the mass-material system shown in Figure (1) where the mass is constrained to move uni-axially while undergoing base excitation. The identification results described in this paper are mainly focused on tests on a cylindrical HTPB-NH₄CL surrogate sample with a 50% crystal volume. The measured response is the acceleration of the mass attached to the material ($\ddot{z}$) and the excitation is provided by base excitation ($\ddot{y}$). The model input is the base acceleration and the response is the response of the mass relative to the base ($z$). The models examined consist of a nonlinear elastic component ($k(z)$), a viscous and/or a hysteretic damping component ($c(z, \dot{z})$) and a viscoelastic component ($\nu$) that is a convolution of a driving term ($f$) and a hereditary (sum of complex exponential terms) kernel.

![Figure 1](image.png)

**Figure 1.** Experimental test-rig with surrogate test sample, and graphical representation of experimental set-up.

In previous studies, low order viscoelastic kernels were examined and only $f = z$ or $f = \dot{z}$ examined. Models estimated from a set of experimental data from one test were shown to predict responses reasonably well at the same excitation levels, but the estimated model parameters change for each excitation level, and when the parameters of a model were identified using data from multiple tests, the response predictions for individual tests are poor, particularly for materials with higher crystal content [7]. The focus in the research described in this paper is on the development of a more flexible system identification technique than was used previously. It was developed to be able to include different types of viscoelastic driving terms and much higher viscoelastic kernel orders in the model. The excitation range was also extended and random as well as swept sine excitations were used. Previously, a continuous-time approach was used for estimating model parameters [8, 9], but this method gets progressively more difficult to apply as viscoelastic model orders become higher and when viscoelastic driving terms are functions of system parameters.
Following, the model of the mass-material system is described in more detail along with the iterative parameter estimation technique that was developed. The results of applying the system identification technique when using simulated system responses corrupted with noise are shown, followed by the results of applying the technique to experimental data. The paper ends with some comments on future work. For more detailed explanations of the experimental procedures, experimental repeatability results, and changes in responses with material aging, refer to [7, 8].

2. System model, identification, and parameter estimation

The model is first introduced and then the system identification and parameter estimation process is described.

2.1. Mass-Material System Model

The mass-material system undergoing base excitation as shown in Figure (1) is modeled as a mass spring damper with a viscoelastic component. Recall that for a harmonic base excitation, the generic form of the equation of motion is of the form:

$$ m\ddot{z} + c(\dot{z}, z) + k(z) + v(t) = -m\ddot{y}, $$

where $m$ is the mass of the mid-plate plus a third of the sample mass, $z$ is the relative motion ($= x - y$), $k$ is the stiffness term, $c$ is the damping term, $v(t)$ is the viscoelastic term and $\ddot{y}$ is the base acceleration. The stiffness and damping terms are functions of the relative displacement and velocity and may be nonlinear. Many different damping and stiffness terms were considered, from a simple linear stiffness ($kz$) and viscous damping ($c\dot{z}$) to more complex hysteretic damping terms, e.g., $c\dot{z}|z|^\lambda$. When nonlinear stiffness models are considered, the stiffness term can be modeled as a $N^{th}$ order polynomial expression in $z$ ($\sum_{i=1}^{N} k_i z^i$), or the more complex Ogden’s hyperelastic model [7, 8]. For simplification of parameter estimation the first approach is used here. Using the sample geometry, the polynomial stiffness expression in $z$ may be formulated as a stress-strain relationship and converted back to the well-known constitutive Ogden model [10].

The viscoelastic term $v(t)$ was chosen to be a hereditary viscoelastic model. The general form of $v(t)$ is a convolution of the relaxation kernel $g(t)$ and a forcing function $f(t)$, it is of the form:

$$ v(t) = f(t) \ast g(t) = \int_{-\infty}^{t} f(\tau)g(t - \tau)d\tau = \int_{-\infty}^{t} f(\tau)\sum_{i=1}^{M} \beta_i e^{-\alpha_i(t-\tau)}d\tau, $$

where $M$ is the order of the viscoelastic kernel, and the viscoelastic parameters are the $\alpha_i$ and $\beta_i$, $i = 1, 2, ..., M$. Several forcing functions could be used, e.g., the relative velocity ($\dot{z}$), $\dot{z}|\dot{z}|^\lambda$, or $k(z)$.

Equation (2) can be reformulated as an $M^{th}$ order differential equation in $v$:

$$ \gamma_M \frac{d^M v}{dt^M} + \gamma_{M-1} \frac{d^{M-1} v}{dt^{M-1}} + ... + \gamma_0 v = \mu_{M-1} \frac{d^{M-1} f(t)}{dt^{M-1}} + ... + \mu_0 f(t), $$

where the $\mu_i$ and $\gamma_i$ are combinations of the $\alpha_i$ and $\beta_i$. Equation (1) can be rearranged to express $v$ as a function of $z$ and its derivatives. This equation can then be differentiated $M$ times and these expressions can be substituted into Equation (3) to generate an $(M + 2)^{th}$ order differential equation in $z$. This equation can be used in a continuous-time system identification approach where functions of the time histories of the relative displacement and base excitation and their derivatives are used directly to estimate the differential equation coefficients [11, 12]. The coefficients of the differential equation are straightforward to estimate, but the model parameters are nonlinear combinations of the nonlinear differential equation parameters. When the stiffness polynomial is high order, many terms are generated through the multiple differentiations and
there are signal conditioning challenges with the estimation matrix. The differentiation of the sampled signal $\ddot{z}$ is performed by using a digital filter differentiator, which introduces some errors and amplification of noise at higher frequencies is also a problem when differentiating. To address these problems and to enable us to examine a greater variety of models, an iterative approach to estimation of the system parameters was adopted.

2.2. The Iterative System Identification Process
The full iterative approach incorporates several steps including a continuous-time system identification, a Prony series model fit, either in the time or the frequency domain, and a model reduction method to select the dominant terms in the viscoelastic model. An overview of the parameter estimation process is shown in Figure 2 and the ten steps in the process are described in more detail below.

![Diagram of the parameter estimation process](image)

**Figure 2.** The steps in the parameter estimation.

**Step 1:** Import data from experiments: relative acceleration and base acceleration, $\ddot{z}, \dot{y}$.

**Step 2:** Generate the states of the model, $\ddot{z}, \ddot{y} \rightarrow \ddot{z}, \dot{z}$

**Step 3:** Assume an initial value for the viscoelastic term ($v(t)$).

**Step 4:** Estimate the stiffness and damping terms ($c, k_i$), by using the continuous system identification approach.

To demonstrate this process, consider a model with linear viscous damping, a $5^{th}$ order polynomial stiffness and an estimated viscoelastic term ($v(t)$), and then express Equation (1) to the following form:

$$c\ddot{z} + k_1 z + k_2 z^2 + k_3 z^3 + k_4 z^4 + k_5 z^5 = -m\ddot{y} - v(t) - m\ddot{z}. \quad (4)$$

Note that in the first iteration $v(t)$ is typically set to zero. Despite being nonlinear in $z$, Equation (4) is linear in parameters, which makes continuous-time system identification straightforward to implement. The signals $z, z^2, z^3, z^4, z^5, \dot{z}, \ddot{y}$ and the current estimate of $v$ are used with Equation...
(4) to form the matrix equation:

\[
\begin{bmatrix}
-m(\ddot{z}(1\Delta) - \ddot{y}(1\Delta)) - v(1\Delta) \\
-m(\ddot{z}(2\Delta) - \ddot{y}(2\Delta)) - v(2\Delta) \\
-m(\ddot{z}(N\Delta) - \ddot{y}(N\Delta)) - v(N\Delta)
\end{bmatrix}
= \begin{bmatrix}
\ddot{z}(1\Delta) \\
\ddot{z}(2\Delta) \\
\ddot{z}(N\Delta)
\end{bmatrix}
- \begin{bmatrix}
z(1\Delta) & z^2(1\Delta) & z^3(1\Delta) & z^4(1\Delta) & z^5(1\Delta) \\
z(2\Delta) & z^2(2\Delta) & z^3(2\Delta) & z^4(2\Delta) & z^5(2\Delta) \\
z(N\Delta) & z^2(N\Delta) & z^3(N\Delta) & z^4(N\Delta) & z^5(N\Delta)
\end{bmatrix}
\begin{bmatrix}
c \\
k_1 \\
k_2 \\
k_3 \\
k_4 \\
k_5
\end{bmatrix}
\]

(5)

which is written more simply as:

\[ q = [A]p, \]  

(6)

where \( \Delta \) is the time step. A challenge of applying the continuous time system identification approach is the need to derive the necessary time signals \((z, z^2, z^3, z^4, z^5, \dot{z}, \text{etc.})\) from the measured experimental acceleration. Care is taken to make sure the signals and matrix columns are conditioned appropriately by ensuring all the columns in \([A]\) and \(q\) were filtered equally due to their sensitivity of these transformations to measurement noise [13]. This equation is then solved in a least square sense.

**Step 5:** Update the estimate of the viscoelastic signal by using:

\[ v(t) = -m\ddot{y} - m\ddot{z} - c(\dot{z}, z) - k(z). \]

**Step 6:** Calculate viscoelastic kernel frequency response function, \(G(f)\) by using either \(F[v(t)]/F[f(t)]\) when signals are deterministic or \(S_{ff}/S_{ff}\) when signals are random. \(F[\cdot]\) denotes a Discrete Fourier Transform and \(S_{ab}\) denotes the cross spectral density between signal \(a\) and signal \(b\) estimated by using segment averaging with Hann windows, Discrete Fourier Transforms, and 50% overlapping of segments. If using time-domain estimation in the next step, the corresponding impulse response of the viscoelastic kernel is determined by applying an Inverse Discrete Fourier Transform to the estimated frequency response function.

**Step 7:** Fit a higher than \(M\) order Prony Series model either to the frequency response function (frequency domain) or to the impulse response (time domain) and calculate energy contributions of each term.

The time domain approach requires fitting very high order models and then selecting terms based on, e.g., their energy contributions, to distinguish between "signal" terms and "noise terms." The frequency domain approach has the advantage of allowing the user to select particular regions of the frequency response function (e.g., in the region of excitation of the system) and avoid the parts of the frequency response likely to be corrupted by noise, thus much lower order models (but typically still larger than \(M\)) can be used in this case. The higher order Prony Series model has the same form as the viscoelastic kernel and can be expressed as:

\[ g(n\Delta) = \sum_{j=1}^{M_1} A_j e^{\gamma_j n\Delta}. \]  

(7)

The frequency response function of the viscoelastic kernel is thus of the form:

\[ G(e^{i2\pi f\Delta}) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_{M_1-1} z^{-(M_1-1)}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_{M_1} z^{-M_1}} \bigg|_{z=e^{i2\pi f\Delta}} = \sum_{j=1}^{M_1} \frac{A_j}{1 - e^{-\gamma j \Delta} e^{-i2\pi j f\Delta}}. \]  

(8)
When using the frequency domain approach, after the \( a_j \) and \( b_j \) have been estimated, a partial fraction expansion is performed to determine the \( A_j \) and the \( \gamma_j \).

**Step 8:** Reduce the viscoelastic model order by selecting the terms with the highest individual energy. When a term is complex, the energy contribution of the term together with the complex conjugate term is calculated. The term, or pair of terms, energy contribution is defined as the squared value of the component’s time response integrated from \( t_1 \) to \( t_2 \), where \( t_1 \) is usually close to the start of the estimated impulse response and \( t_2 \) is just before the signal enters the noise floor. The number of terms retained \((M)\) is currently not automatically determined by the algorithm, it is preset by the user. Future refinements of this step include using additional criteria for term selection, e.g., limiting the range of natural frequencies, and using goodness of fit criteria for automatic detection of the number of terms to be retained.

An example case of where ten terms were selected from an estimated \( M_1 = 250 \) model is illustrated in Figure 3. Here time-domain Prony Series modeling was used on a simulated impulse response with added noise. While the \( M_1 = 250 \) model does an excellent job at fitting the simulated impulse response in Figure 3a, most of the terms are fitting the noise structure. The reduced model order (Figure 3b) does a good job capturing the early characteristics of the impulse response.

**Figure 3.** Simulated impulse response with noise on signal (blue) with (a) the estimated Prony Series with order \( M_1 = 250 \) (red), and (b) the reduced model time response, \( M = 10 \), (green).

**Step 9:** Check convergence criteria. This is specified to be less than 0.1% change in the parameter estimates for the stiffness and damping terms, and less than 1% change in the estimated viscoelastic parameters.

**Step 10:** If conditions are not met, update the viscoelastic term estimate and return to Step 4 and repeat steps 4-9.

The iterative process continues until the stopping criteria are met. A maximum number of iterations of 100 is specified, but convergence is typically met before that number is reached, between 3 and 20 iterations.

3. Simulations

For the simulations, the sample rate was chosen to be the same as in the experiments \((f_s = 6000 \text{ samples per second})\), the viscoelastic kernel order, \( M = 10 \) (5 damped sine waves in the range of excitation), \( k(z) \) is a fifth order polynomial \( N=5 \), the damping term was \( c \dot{z} \), and the relative acceleration was also used as the viscoelastic term forcing function \((f(t) = \dot{z})\). The base acceleration \((\ddot{y})\) in the simulations was a random signal generated by passing white noise through
a band-pass filter with a passband 10 Hz to 1000 Hz. The excitation power spectral density level in the passband was set to be $1g^2/Hz$ and the excitation lasted for 100 seconds. This excitation was the one also used in the experiments reported later. In some simulations noise was added to the simulated relative acceleration signal. The exact and estimated coefficients for the noise-free case are listed in Table 1. The highest error is 3.18% for one of the damped sinewave amplitudes. The maximum error in the estimates of the frequencies of the damped sinewaves was less than or equat to 0.14% of their actual values.

Table 1. Exact and estimated frequency, amplitude, phase and decay of the viscoelastic terms derived from the simulations with no added noise.

| Frequency | Exact | Estimate | % Error | Amplitude | Exact | Estimate | % Error |
|-----------|-------|----------|---------|-----------|-------|----------|---------|
| 36        | 36.00 | 0.01%    |         | 20        | 19.88 | 0.59%    |         |
| 55        | 55.08 | 0.14%    |         | 15        | 5.98  | 0.33%    |         |
| 90        | 90.00 | 0.00%    |         | 10        | 5.96  | 0.39%    |         |
| 48        | 47.93 | 0.14%    |         | 12        | 5.93  | 0.33%    |         |
| 113       | 113.97| 0.85%    |         |           |       |          |         |
| Decay     |       |          |         | Phase     |       |          |         |
| -5        | -4.98 | 0.48%    |         | 3         | 2.97  | 0.87%    |         |
| -1        | -1.01 | 1.34%    |         | 6         | 6.12  | 1.92%    |         |
| -0.8      | -0.81 | 0.69%    |         | 8         | 7.90  | 1.27%    |         |
| -3        | -3.06 | 1.90%    |         | 7         | 7.01  | 0.21%    |         |
| -8        | -8.08 | 1.03%    |         | 0         | 0.00  | 0.00%    |         |

To examine the robustness of the models and sensitivity to noise, one hundred realizations of noise were generated with standard deviations of $n=0\sigma$ to $0.12\sigma$ in steps of $0.04\sigma$, where $\sigma$ is the average standard deviation of the system response calculated over 100 seconds. Order $M_1=20$ viscoelastic models were fit to the estimated viscoelastic kernel frequency response using frequency response data from 10 to 1000 Hz only. The mean and standard deviation of some of the normalized parameter estimates are shown in Figure (4). In the normalization, the true value was subtracted from the estimate and the result is scaled by the true value. As expected, as the noise level is increased, the standard deviation of the estimates increased. In some cases, e.g., $k_2$, the mean of the 100 estimates is also changing indicating some bias effects due to the noise. The bias is small for the estimates of $c$ and for the other parameters at all but the highest noise levels. One example of the estimated frequency response magnitudes of the viscoelastic kernel are plotted along with the true value in Figure (4(e)-(h)). The estimated model fit (red) is close to the noise free case (green) despite the noise level increasing. Monte Carlo simulations were also conducted for models with $f(t) = \ddot{z}|\dot{z}|^\lambda$, and the results are similar for the low noise cases but worsen as the noise levels increases.

4. Experiments
The system identification technique was applied to the experimental data collected from both harmonic and random base excitation tests on a TIRA vib 34kN uni-axial shaker. A brief description of the experimental set-up is described here, for a more details please refer to [7, 9].

The test-rig placed on the shaker is shown in Figure 1(a). Eight accelerometers were used to measure the acceleration of the mid-plate, shaker head and base-plate. Four were placed on the mid-plate, three on the base plate, and one on the shaker head. The acceleration signals were passed through a low-pass analog filter (Wavetek 852) with a cut-off frequency set at 600 Hz for the sweep tests and 1500 Hz for the random tests, and then sampled at 6000 samples/second. Since the system identification approach uses the relative motion ($\ddot{z} = \ddot{x} - \ddot{y}$) to estimate the
Figure 4. Results of Monte Carlo simulations where * denotes the normalized mean with +/- normalized standard deviation error-bars for (a) $k_1$, (b) $k_2$, (c) $k_3$, (d) $c$ for an Equation 4 model with $M=20$. (e)-(h) The estimated viscoelastic frequency response function magnitudes from Step 6: $\hat{G}(f) = S_{ff}/S_{ff}$ (blue); the estimated viscoelastic model frequency response magnitudes (red); and the actual viscoelastic frequency response magnitude (green). (e) $0\sigma$, (f) $0.04\sigma$, (g) $0.08\sigma$ and (h) $0.12\sigma$.

mass-material system parameters, the four mid-plate accelerometer signals were averaged to reduce effects due to noise and rocking motion, and the base-plate acceleration measurements were also averaged. The relative acceleration was calculated as the difference of these two signals.

Post processing of the signals includes the use of multiple digital filters (low pass, high pass, differentiator, and integrator) to calculate the relative displacement, velocity and instantaneous frequency and instantaneous amplitude (signal envelope). Here we will show the signal envelopes when comparing the responses on multiple days of testing and also to show the model fit results to swept-sine experimental data. Harmonic base excitations for the HTPB 50% sample were sinusoidal sweeps from 150 Hz to 330 Hz in 60 seconds (3 Hz/sec). The frequency ranges were chosen to include the mass-material system resonance and were conducted at 5g, 7.5g, and 10g excitation levels. Additionally, random base excitation tests were conducted on the HTPB 50% sample. These tests lasted for 100 seconds and the level of the power spectral density of the random excitation between 10 Hz and 1000 Hz was $0.1 \, g^2/Hz$.

In Figure (5a), the signal envelopes of the relative acceleration responses of the system with the HTPB 50% sample excited at 5g are plotted. Each color represents a different day of testing; the different color shades correspond to the 3 tests done on each day (30 minutes apart) from dark to light. The resonance of the system shifts from 220Hz on Day 1 to 260Hz on Day 120; this shift may be due to the material stiffening over time. The normalized response envelopes from a 5g and a 10g test conducted on the same day are shown in Figure (5b). The 10g normalized responses (red and cyan) are shifted to the left by about 10Hz. The location of the peak response shifting to the left could be due to a softening stiffness nonlinearity and/or a nonlinear damping
mechanism that produces higher damping as excitation levels increase.

Figure 5. Relative acceleration envelopes for the system with the HTPB 50% sample. (a) Responses when excited at 5g and experiments conducted over 120 days. (b) The normalized response of the system with the HTPB 50% sample for excitations at 5g and 10g and tests conducted on the same day. Swept sine excitation was from 150 Hz to 330 Hz in 60 seconds.

The system identification approach described in the previous sections was used to characterize the behavior of the system with the HTPB 50% sample. Two types of viscoelastic forcing functions were examined: either \( f(t) = \dot{z} \) or \( f(t) = \dot{z} |\dot{z}|^\lambda \).

4.1 Linear viscous damping and \( f(t) = \dot{z} \) and \( c(\dot{z}) = c\dot{z} \)

Estimation of lower order viscoelastic models \((M = 0\ or\ 1)\) has been investigated previously using the swept sine experimental data [7, 9]. It was found that including a single viscoelastic term improved the model fit over not having a viscoelastic term but the terms in the model changed with excitation level, and that using data from all forcing level tests (5g, 7.5g, and 10g) in the estimation produced poor results. This is illustrated in Figure 6(a)-(b).

Figure 6. Relative acceleration envelopes from measurements (blue) and from an \( M=1, N=5 \) estimated model (green). Model estimated from (a) a single data set from a 10g test, and (b) from multiple data sets form 5g, 7.5g, and 10g tests. Material in system is a HTPB 50% sample.

The system identification technique described above was used with a viscoelastic model order of \( M=20, \) and \( N=5 \). The damping term was \( c\dot{z} \) and the forcing term in the viscoelastic models was \( \dot{z} \). This is referred to as Model 4. Here the model was estimated using data from swept sine base excitation tests conducted on the same day with excitation levels of 5 g, 7.5 g, and 10 g, and the viscoelastic coefficients were estimated by fitting directly to the frequency response function. The frequency range that was used to estimate the viscoelastic coefficients is the region of excitation (150Hz-330Hz). The results are shown in Figure 7(a).
Figure 7. (a) Relative acceleration envelopes from experimental data (blue), and from an estimated model with $M=1$ (green) and $M=20$ (red). Responses to swept sine excitations at different levels (5g, 7.5g, and 10g) were used in the model estimation. The material was HTPB 50%. (b) Results of viscoelastic model estimation for the same model structure ($M=20$) but estimated using data from a random excitation test. Excitation range: 10Hz to 1000Hz, and excitation level: 0.1 $g^2$/Hz within this range. $|G(f)|$ estimated by using $S_{\hat{v}}/S_{ff}$ (blue) and $|G(f)|$ predicted from the estimated viscoelastic model (red).

Increasing the model order from $M=1$ to $M=20$ improves the fit near resonance when using data sets from the three different excitation level tests, but there is now separation in the experimental and predicted response envelope below resonance (150Hz-180Hz). The results of the viscoelastic model estimation for the same model structure but with random base excitation are shown in Figure 7b. The prediction of the viscoelastic frequency response magnitude from the estimated model and the results from the $S_{\hat{v}}/S_{ff}$ estimation of the frequency response are very close. Here data from 10 - 1000 Hz was used in the viscoelastic model estimation.

4.2. Hysteretic damping $f(t) = \dot{z}|\dot{z}|^\lambda$ and $c(\dot{z})=c|\dot{z}|^\lambda$

Because the peak response of the normalized acceleration profile shifts to the left when the excitation level is increased, which may be due to nonlinear damping effects, the model was modified to include nonlinear hysteretic damping. Here we will consider the two cases, one where both the $c(\dot{z})$ and $f(t)$ are in the form of a hysteretic damping term $|\dot{z}|^{\lambda}$ (Model 6) and another (Model 7) where just the damping term is $|\dot{z}|^{\lambda}$ and the forcing function is $\dot{z}$. Here $\lambda$ is set to 1.8 based on some earlier investigations into including hysteretic damping terms into the model. As with Model 4, the system identification approach is applied with Model 6 and Model 7 and data from tests on HTPB 50% at multiple swept sine forcing levels (5g, 7.5g, and 10g). Again, the viscoelastic model is fitted directly to the frequency response function using only the data in the region of excitation (150Hz-330Hz). Model 7 has the same viscoelastic model structure as Model 4, and the converged viscoelastic model frequency response when random excitation data is used in the estimation is close to the frequency response estimated from $f$ and $\hat{v}$. For Model 6, where the viscoelastic model forcing function has the same structure as the hysteretic damping term, the $20^{th}$ order model was not as accurate. The results are shown in Figure 8.

All the system identification results from applying fitting a $M=20$ order viscoelastic model to the experimental data for multiple swept sine data sets are shown in Figure (9). The green line is the $M=1$ case shown in Figure 6; all the models with the higher order viscoelastic kernels perform better than this model close to resonance. In the 160-180 Hz region both Model 4 (red) and Model 6 (black) over-predict the response envelope and Model 7 (purple) under-predicts the response. Above resonance Model 6 also under-predicts the experimental response envelope, whereas Models 4 and 6 are close to the experimental response envelope. By changing $\lambda$ it may be possible to address some of these issues away from resonance, the value of 1.8 was chosen.
5. Concluding Comments

An iterative parameter estimation method has been developed to estimate the parameters in a combined high order nonlinear elastic and high order viscoelastic model. The estimation combines continuous-time estimation of the damping and nonlinear elastic terms and a Prony Series estimation of the viscoelastic model, which can be of arbitrary order. The estimation process also includes viscoelastic model order reduction to select true viscoelastic model terms from a set of terms that model noise and viscoelastic behavior. Simulations were used to examine the effects of noise on the parameter estimates. The iterative estimation technique was developed because lower order models yielded poor predictions of the responses of the system at different excitation levels, and the estimation method used for low order models became unwieldy when the viscoelastic model order increased and forcing functions became more complex. Here we either used swept sine or random base excitation, next steps include using both together in the estimation. Base excitation forcing levels ranged from 5g to 10g for the HTPB 0% and HTPB 50% sample, this corresponds to max strain-rate ranges of 6.1 s\(^{-1}\) to 12.3 s\(^{-1}\) and
1.4 $s^{-1}$ to 2.8 $s^{-1}$ respectively. This estimation method is now being used to examine the performance of different models in characterizing the mechanical behavior of surrogate explosive materials. There was little benefit seen when including the hysteretic damping terms instead of viscous damping terms in the model, although this may have been, in part, due to the exponent chosen in these investigations. Inclusion of higher order viscoelastic models improved prediction of the mass-material system response near resonance. While results are promising, there is clearly a need for further development of the estimation technique and the models of the surrogate explosive materials. Automation of viscoelastic model order selection including the introduction of additional criteria during the model reduction phase is one area of ongoing work. A greater variety of viscoelastic forcing functions are currently being investigated; these include different forms of the hysteretic damping term and also the nonlinear stiffness term. Despite the focus on materials used in improvised explosive devices, the experimental methodology, system identification approach and investigation into model structure presented here may be useful for other applications where viscoelastic materials are present.

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