Could the $\tau$ be substantially different from $e$ and $\mu$ in the supersymmetric standard model?* 

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Abstract

R-parity stands as an ad hoc assumption in the most popular version of the supersymmetric standard model. More than fifteen years’ studies of R-parity violations have been restricted to various limiting scenarios. We illustrate how the single-VEV parametrization provides a workable framework to analyze the phenomenology of the complete theory of supersymmetry without R-parity. In our comprehensive study of various aspects of the resulting leptonic phenomenology at tree-level, we find that the physical $\tau$ lepton could actually bear substantial gaugino and higgsino components, making it very different from the $e$ and the $\mu$.

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1. INTRODUCTION

We are going to illustrate how the $\tau$ could be substantially different from $e$ and $\mu$ in the supersymmetric standard model. In a supersymmetric version of the standard model (SM), the leptons could assume a new identity unless an ad hoc symmetry called R-parity is imposed. We will show that R-parity is not theoretically well motivated, and the present experimental constraints from leptonic phenomenology allow significant violation of R-parity. In particular, the $\tau$ might contain substantial components in the gaugino and higgsino directions, making it very different from the $e$ and the $\mu$.

2. SUPERSYMMETRY AND R-PARITY

Supersymmetry is a symmetry between bosons and fermions. To produce a supersymmetric version of the SM, one has to be able to put its content in terms of superfields, which contain both bosonic and fermionic components. Each matter field is to be embedded into a chiral superfield containing a complex scalar and a Weyl fermion. It is then obvious that we need the $\hat{Q}_i$, $\hat{U}^c_i$, $\hat{D}^c_i$, and $\hat{E}^c_i$ superfields with three flavors, i.e. $i = 1$ to 3. Due to the holomorphic properties of the superpotential, separate superfields are needed to provide the Higgs scalars for the mass generation of the up- and down-sector quarks. For the former, a $\hat{H}_u$ is introduced. For the latter, a $\hat{H}_d$ as a superfield will be identical to a $\hat{L}$ from a leptonic doublet. However, the fermionic component of $\hat{H}_u$, the higgsino, will spoil the beautiful gauge anomaly cancellation of the SM fermion spectrum, unless a conjugate fermionic doublet is also added. Hence, four superfields (flavors) with the quantum number of $L$ or $H_d$ are needed. We denote them by $\hat{L}_\alpha$, $\alpha = 0$ to 3.

R-parity is defined by

$$R = (-1)^{3B+L+2S}$$

where $B$, $L$, and $S$ are the baryon number, the lepton number, and the spin of a superfield component respectively. As superfields, $\hat{Q}_i$, $\hat{U}^c_i$, $\hat{D}^c_i$, $\hat{E}^c_i$, and $\hat{L}_i$, taking as those three from the leptonic doublets, are odd, while $\hat{H}_u$ and $\hat{H}_d$ are even under R-parity. Componentwise, all SM particles have even R-parity while all the superpartners have odd R-parity. Hence, $\hat{H}_d$ and $\hat{L}_i$ no longer have the same quantum number. So, adding supersymmetry and R-parity is to first put in a symmetry between fermions and bosons and then put in another symmetry to distinguish the known fermions from the fermionic partners of the scalars, that we believe have to be there.

The most general superpotential admissible by

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the SM gauge symmetries can be written as
\[
W = \varepsilon_{ab}[\mu_a L^a_{\alpha} \hat{H}^b + h^a_{ik} \hat{Q}^a_{i} \hat{H}^b \hat{U}_k + \lambda'_{iak} \hat{Q}^a_{i} \hat{L}^b \hat{D}^c_k + 
\lambda_{abk} \hat{L}^a_{\alpha} \hat{E}^b_{\alpha k}] + \lambda''_{ijk} \hat{D}^i_{\alpha} \hat{D}^j_{\beta} \hat{U}_{\gamma k},
\]
where \((a, b)\) are SU(2) indices, \((i, j, k)\) are family (flavor) indices, \((\alpha, \beta)\) are (extended) flavor indices from 0 to 3, and \(\hat{L}_\alpha\)'s denote the four doublet superfields with \(Y = -1/2\). \(\lambda\) and \(\lambda''\) are antisymmetric in the first two indices as required by SU(2) and SU(3) product rules respectively. This is shown explicitly here only for SU(2), with \(\varepsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\). The SU(3) indices are suppressed. In the limit where \(\lambda_{ijk}, \lambda'_{ijk}, \lambda''_{ijk}\) and \(\mu_i\) all vanish, one recovers the R-parity conserving result (the minimal supersymmetric SM), with the following matching to the common notation of the latter:
\[
\begin{align*}
L_0 & \rightarrow \hat{H}_d \\
\lambda'_{0k} & \rightarrow h^d_{ik} \\
\lambda_{0k} & \rightarrow h^h_{ik}.
\end{align*}
\]

The superpotential contains both B- and L-violating terms resulting in tree-level superparticle mediated proton decay. The experimental limit on the proton life-time then requires the product of the relevant couplings (\(\lambda\) and \(\lambda''\)) to be of the order \(10^{-27}\). While the only natural way to satisfy the stringent proton decay constraint is to forbid the process by a symmetry, R-parity is not the only option, nor is it necessarily the best one. For example, a baryon-parity enforcing \(\lambda'' = 0\) or other discrete symmetries can be used. The former option has an advantage over R-parity — it forbids, in addition to the dimension-4 B-violation terms (\(\lambda''\)), dangerous terms of dimension 5. These alternatives are usually much less restrictive; they allow quite a number of R-parity violating (RPV) terms in the Lagrangian for which there are interesting experimental constraints.

We consider it more interesting to adopt a purely phenomenological approach to study of the complete supersymmetric standard model without R-parity and analyze how the overall parameter space is restricted by the various constraints, as well as looking for potential experimental signals of R-parity violation.

3. THE SINGLE-VEV PARAMETRIZATION

The above suggestion sounds naively straightforward; however, its implementation demands a careful consideration. The large number of extra parameters involved makes the task difficult to manage. For instance, the tree-level color-single charged fermion mass matrix, in a generic flavor basis, is given by
\[
M_c = \begin{pmatrix} M_2 & \frac{v_{\tau}}{\sqrt{2}} & 0 \\ \frac{v_{\mu}}{\sqrt{2}} & \mu_0 & -\frac{h^h_{ik}}{\sqrt{2}} \\ 0 & \frac{h^h_{ik}}{\sqrt{2}} + 2\lambda_{ijk} & g_3 \end{pmatrix},
\]
where we have suppressed the last three rows and columns with indices \(j\) and \(k\) going from 1 to 3. Note the summation over \(i\); and that we have written the matrix in such a way as to distinguish the R-parity conserving and violating terms. The VEV’s are given by
\[
\begin{align*}
\frac{v_u}{\sqrt{2}} &= \langle \hat{H}_u \rangle, & \frac{v_e}{\sqrt{2}} &= \langle \hat{L}_i \rangle, \\
\text{and} & & \frac{v_\mu}{\sqrt{2}} &= \frac{v_\tau}{\sqrt{2}} = \langle \hat{L}_0 \rangle.
\end{align*}
\]
Remember the only knowledge we have about the matrix entries is that of the gauge couplings \(g_1\) and \(g_2\), the overall magnitude of the electroweak symmetry breaking VEV’s
\[
|v_u|^2 + \sum |v_i|^2 = v^2 = 246\text{GeV},\]
and that they have to yield the correct mass eigenvalues for the physical leptons (\(\ell\)), namely \(m_e, m_\mu,\) and \(m_{\tau}\). However, we need to have a good knowledge of the real nature of the \(\ell\)'s before we can study the experimental signatures of the heavier particles.

The problem is solved in the single-VEV parametrization, which is nothing more than writing the Lagrangian in the most convenient set of flavor bases. More details of the parametrization are discussed in Ref. Under the framework, the \(v_i\)'s and the off diagonal \(h^h_{ik}\)'s are set to zero. This is an optimal exploitation of the freedom in the choice of flavor bases. The above
mass matrix is then simplified to
\[
\mathcal{M}_c = \left( \begin{array}{ccccc}
  M_2 & \frac{\mu_1}{\sqrt{2}} & 0 & 0 & 0 \\
  \frac{\mu_1}{\sqrt{2}} & \mu_0 & 0 & 0 & 0 \\
  0 & \mu_1 & m_1 & 0 & 0 \\
  0 & \mu_2 & 0 & m_2 & 0 \\
  0 & \mu_3 & 0 & 0 & m_3 \\
\end{array} \right) .
\] (4)

Here, each \( m_i \) is a physical leptonic mass in the limit where the corresponding \( \mu_i \) is zero. In the general case, the correct value of the \( m_i \)'s can be determined, at least numerically, to guarantee acceptable eigenvalues of the \( \ell_i \) masses. Hence, the mass eigenstates and their exact nature, such as their gaugino and higgsino contents, can be found. In particular, our result shows that \( \mu_5 \), which characterizes the gaugino and higgsino contents of the \( \tau \), is not necessarily small. The chargino masses now also depend on \( \mu_5 \)'s, with interesting implications. An example is illustrated in Fig.1.

![Figure 1. Minimum values of \( \mu_5 \) (in GeV) required to give both chargino masses above 90 GeV. \( (\mu_5 = \sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2}) \)](image)

4. A NEUTRINO MASS

Similarly, a simple form of the neutral fermion mass matrix is obtained:
\[
\mathcal{M}_\nu = \left( \begin{array}{ccccc}
  M_1 & 0 & 0 & 0 & 0 \\
  0 & M_2 & -\frac{\mu_1}{\sqrt{2}} & -\frac{\mu_1}{\sqrt{2}} & 0 \\
  0 & -\frac{\mu_1}{\sqrt{2}} & m_1 & 0 & 0 \\
  0 & -\frac{\mu_1}{\sqrt{2}} & 0 & m_2 & 0 \\
  0 & 0 & 0 & 0 & m_3 \\
\end{array} \right) .
\] (5)

Two neutrino eigenstates are left massless at the tree level, while the third one gains a mass through the RPV-couplings (\( \mu_5 \)'s) to the higgsino. In fact one can use a simple rotation to decouple the massless states. The remaining 5 × 5 mass matrix is then given by
\[
\mathcal{M}_\nu^{(5)} = \left( \begin{array}{ccccc}
  M_1 & 0 & 0 & 0 & 0 \\
  0 & M_2 & -\frac{\mu_1}{\sqrt{2}} & -\frac{\mu_1}{\sqrt{2}} & 0 \\
  0 & -\frac{\mu_1}{\sqrt{2}} & 0 & -\mu_0 & -\mu_5 \\
  0 & -\frac{\mu_1}{\sqrt{2}} & 0 & -\mu_0 & 0 \\
  0 & 0 & 0 & 0 & -\mu_5 \\
\end{array} \right) ,
\] (6)

where
\[
\mu_5 = \sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2} ;
\] (7)

and the corresponding massive neutrino state is given, in terms of the original neutrino basis, by
\[
|\nu_5\rangle = \frac{\mu_1}{\mu_5} |\psi_{L_1}\rangle + \frac{\mu_2}{\mu_5} |\psi_{L_2}\rangle + \frac{\mu_3}{\mu_5} |\psi_{L_3}\rangle ,
\] (8)
a mixture of all three of them in general. Adopting a “seesaw” approximation gives the neutrino mass
\[
|m_{\nu_5}| = \frac{\mu_5^2 v^2 \cos^2 \beta (x g_2^2 + g_1^2)}{\mu_0 [4 x M_2 \mu_0 - (x g_2^2 + g_1^2) v^2 \sin 2 \beta]} ,
\] (9)

where we have substituted \( v_d = v \cos \beta, v_u = v \sin \beta, \) and \( M_1 = x M_2 \). Note that for large \( \tan \beta, \cos \beta \) is a strong suppression factor. In order to look at potential large \( \mu_5 \) values, we can perform an alternative perturbative analysis, diagonalizing exactly the matrix without the EW-symmetry breaking terms, only to put them back as perturbation. This yields
\[
|m_{\nu_5}| = \frac{1}{4} \frac{\mu_5^2 v^2 \cos^2 \beta (x g_2^2 + g_1^2)}{(\mu_1^2 + \mu_2^2) x M_2} ,
\] (10)
giving
\[
\mu_5^2 < \frac{4 x M_2 M_2 m_{\nu_5}^b}{v^2 \cos^2 \beta (x g_2^2 + g_1^2) - 4 x M_2 m_{\nu_5}^b} ,
\] (11)

for an experimental neutrino mass bound \( m_{\nu_5}^b \).

As \( M_2 \) increases, the denominator above drops to zero, beyond which there is no bound on \( \mu_5 \).

The perturbative result in Eq (11) is borne out by exact numerical results from diagonalizing the neutral fermion mass matrix, as illustrated in Fig. 2, where the machine \( v_\nu \) bound of 18.2 MeV is and the generic neutrino mass bound of 149 MeV, from charged current data. The dependence of
the $\mu_i$ bounds on $\tan\beta$ is striking. For $\tan\beta = 45$, values of $\mu_i$ in the hundreds of GeV or beyond are not ruled out.

There are potentially much stronger bounds on neutrino masses from cosmological considerations which however depend on the decay modes and other assumptions so that a neutrino mass above an MeV is not definitely ruled out. For much smaller neutrino masses, such as those fitting naturally fitting the oscillation scenario indicated by recent results from Super-Kamiokande\cite{7}, the $\mu_i$’s would have to be below the MeV scale unless much larger $\tan\beta$ value is assumed.

5. LEPTONIC PHENOMENOLOGY AND CONSTRAINTS

With the fermion mass matrices discussed above, we can elicit the properties of the mass eigenstates and their interactions in any region of the parameter space. The matrices are simple enough that effective perturbative approximations can be applied in the study of the leptons $\nu$’s and $\nu$’s, and exact numerical diagonalizations of both mass matrices can be performed. A rich implication in leptonic phenomenology results from nonzero $\mu_i$’s, which can also be used to place more experimental constraints on the admissible values of the latter.

A summary of all the phenomenological processes and corresponding experimental bounds we have investigated is given in Table 1, details of which we refer to our paper\cite{8}. Here we will only outline the major features. The first group of these constraints includes the three modified partial widths of $Z^0 \rightarrow \ell^+\ell^-$, in terms of their universality violation, their left-right asymmetries, and now nonvanishing off-diagonal $Z^0 \ell_i \ell_j$ couplings with the resulted decays of $\mu$ and $\tau$ into three $\ell$’s. Among them the $\mu \rightarrow 3\ell$ bound represents a particularly stringent constraint on the magnitude of the product $\mu_1\mu_2$. The second group of constraints is from the charged current interactions. Among them $R_{\pi\mu}$ and $R_{\pi\tau}$ are particularly important. The two refer to the ratio of $Br(\pi \rightarrow e\nu\nu)$ to $Br(\pi \rightarrow \mu\nu\nu)$ and that
Table 1
Summary of phenomenological constraints incorporated into our study: details of notations and sources of experimental bounds can be found in our paper.3

| Quantity | μi combo. constrained | Experimental bounds |
|----------|----------------------|---------------------|
| **Z0-coupling:** | | |
| Br(μ− → e−e+e−) | | 1.0 × 10⁻¹² |
| Br(τ− → e−e+e−) | | 2.9 × 10⁻⁶ |
| Br(τ− → μ−e+e−) | | 1.7 × 10⁻⁶ |
| Br(τ− → μ−e−e−) | | 1.5 × 10⁻⁶ |
| Br(τ− → e−μ+μ−) | | 1.8 × 10⁻⁶ |
| Br(τ− → μ−μ+μ−) | | 1.5 × 10⁻⁶ |
| Br(τ− → e±μ±μ−) | | 1.9 × 10⁻⁶ |
| Br(Z0 → e±μ±) | | 1.7 × 10⁻⁶ |
| Br(Z0 → e±τ±) | | 9.8 × 10⁻⁶ |
| Br(Z0 → μ±τ±) | | 1.2 × 10⁻⁵ |
| Br(Z0 → χ±τ±) | complicated | 1.0 × 10⁻⁵ |
| Br(Z0 → χ±χ±) | | 1.0 × 10⁻⁵ |
| | | |
| Ubr (e-μ universality) | μ2 − μ2 | (0.596 ± 0.37) × 10⁻³ |
| Ubr (e-τ universality) | μ2 − μ2 | (0.955 ± 0.49) × 10⁻³ |
| Ubr (μ-τ universality) | μ2 − μ2 | (1.55 ± 5.60) × 10⁻³ |
| ΔAεm (e-μ L-R asymmetry) | μ2 − μ2 + Rt. contrib. | (0.346 ± 2.54) × 10⁻² |
| ΔAετ (e-τ L-R asymmetry) | μ2 − μ2 + Rt. contrib. | 0.0043 ± 0.104 |
| ΔAεμ (τ-μ L-R asymmetry) | μ2 − μ2 + Rt. contrib. | 0.082 ± 0.25 |
| Γz (total Z0-width) | μ5 | 2.4948 ± 0.0075 GeV |
| ΓW (e) | μ5 | 500.1 ± 5.4 MeV |
| Br(Z0 → μj/τjχj; j ≠ 1) | μ5 | | 1.0 × 10⁻⁵ |
| W±-coupling: | | |
| Γμe (μ → eνν) | mν3 / μi ratio | 0.983 ± 0.111 |
| Γτe (τ → eνν) | mν3 / μi ratio | 0.979 ± 0.111 |
| Γτμ (τ → μνν) | mν3 / μi ratio | 0.954 ± 0.108 |
| Rπμ (τ decays) | mν3 / μi ratio and mν3 / μi (1.230 ± 0.012) × 10⁻⁴ |
| Rπτ (τ decays) | mν3 / μi ratio | 1.0265 ± 0.0222 |
| Rπτ (decays to e’s) | mν3 / μi ratio | 1.0038 ± 0.0219 |
| mν3 [12] ² [ββ]0ν | mν3 / μ3 and mν3 / μ3 | < 0.46 eV (only for mν3 < 10 MeV) |
| BEBC exp. | | |
| mass constraints: | | |
| ν3 mass | μ3 | < 18.2 MeV if ν3 = ντ |
| ν3 mass | μ3 | < 149 MeV if ν3 ≠ ντ |
| χ± mass | μ5 | > 70 GeV |
of \( Br(\mu \rightarrow e\nu\nu) \) to \( Br(\tau \rightarrow e\nu\nu) \) respectively. Depending on the ratio among the \( \mu_i \)'s, a constraint from either of the two quantities typically bounds the \( \mu_3 \) value a bit below that admissible by the 149 MeV neutrino mass bound. Related constraints from neutrinoless double beta decay and the BEBC beam dump experiment also have important roles to play. The former, when applicable, imposes the most stringent constraint on \( \mu_3 \). All the constraints taken together seem to prefer a strong hierarchy — \( \mu_1 \ll \mu_2 \ll \mu_3 \).

The discussion of the tree-level neutrino mass above also serves to illustrate our approach and the some major features of the result for all the other processes. The most interesting one is the very dramatic \( \tan\beta \) dependence. The RPV effects on the leptonic sector are suppressed by \( \cos^2\beta \). The factor comes into all the nonstandard parts of \( Z^0 \ell_i \ell_j \) couplings and into the charged current constraints through the neutrino mass. The feature is easy to understand intuitively. The properties of the leptons are changed as a result of their mixing, through the \( \mu_i \)'s, with the fourth doublet \( L_0 \). However, the first order effect of such a flavor mixing is unobservable in the mass eigenstates. The observable effects of the mixing, and hence R-parity violation, comes in as a result of the difference between the \( L_i \)'s and the \( L_0 \), which it is really a higgsino — it couples through the VEV of its scalar partner to the gaugino. This is where the factors of \( \cos^2\beta \) come into the game.

6. CONCLUSIONS

We have presented the right framework for the phenomenological studies of the complete theory of supersymmetry without R-parity where no \textit{a priori} assumptions needed to be imposed. Under the single-VEV paramatization framework, the trilinear RPV couplings, \( \lambda, \lambda', \) and \( \lambda'' \), \textit{do not} involve in tree level mass matrices. The full effect of any form of R-parity violation is characterized there by the three bilinear couplings \( \mu_i \). The latter could give rise to a rich leptonic phenomenology even when analysis is restricted only to the tree-level. We cannot do much more than giving an outline of the full result here. Interested readers are referred to our detailed report on the topic.

Of particular relevance here, a nonzero \( \mu_3 \) represents the gaugino and higgsino content of the physical \( \tau \) lepton. While nonzero \( \mu_i \) and \( \mu_2 \) characterize the corresponding properties of the physical \( e \) and \( \mu \), the experimental constraints on their admissible magnitude are much stronger, particularly stringent in the case of \( \mu_1 \). The constraint on \( \mu_3 \) is quite weak in many cases, particularly for intermediate to large values of \( \tan\beta \). In case the \( \mu_3 \) happens to be substantial, it would make the \( \tau \) substantially different from \( e \) and \( \mu \), which is a very interesting scenario.

We quote some illustrative numbers here, for \( M_2 = \mu_0 = 200 \text{ GeV} \): at \( \tan\beta = 10 \), \( \mu_3 \), with a dominating \( \mu_{12} \) and a minor \( \mu_{23} \) contribution, can go to 101 GeV; at \( \tan\beta = 2 \), 19 GeV; both giving \( m_{\nu_e} \) around 110 MeV and \( Br(\tau \rightarrow \mu e e) \sim 10^{-10} \), with deviations of \( Z^0 \) to \( \tau \) width and \( \tau \) L-R asymmetry at \( > 1 \% \) level for the latter case.

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