Mass Bounds in Mirror-Fermion Models

LEE LIN

Department of Physics
National Chung Hsing University
Taichung 40227, Taiwan, ROC
E-mail: llin@phys.nchu.edu.tw

ABSTRACT

Numerical simulations are performed on different lattice sizes of chiral U(1) and SU(2) scalar-fermion models with explicit mirror pairs of fermions in the broken symmetry phase. Relevance of these models to the electroweak theory is discussed. Shift symmetry of the action is exploited to decouple the unwanted mirror-fermion. Upper and lower bounds on the renormalized scalar mass as a function of the renormalized fermion mass are obtained in the SU(2) version of the model. Our numerical data are found to be in qualitative agreement with one-loop results. No evidence for a nontrivial fixed point has been observed. The mirror-fermion models are likely to be trivial in the continuum limit. Our Monte Carlo data show that with a 180 GeV top quark, the Standard Model Higgs boson should have a mass between 100 and 800 GeV.

1. Introduction

It is believed that the only possible nonperturbative (i.e. strongly interacting) sectors in the electroweak theory of the minimal Standard Model are the Higgs and heavy fermion sectors. (Here heavy or light is meant to be compared to 100 GeV scale, the energy scale of the weak interaction.) If the electroweak theory is trivial in the infinite cutoff limit, then we can obtain upper and lower bounds on the Higgs mass as a function of the heavy fermion mass. Since those sectors can become strongly interacting, it is important that nonperturbative investigation be carried out. In this report, the nonperturbative method is Monte Carlo simulation on the lattice.

Because there are lots of bare parameters in the electroweak theory, we need to make some approximations before lattice calculations become feasible. The approximation we make is to ignore all the gauge couplings assuming that all gauge couplings produce perturbative effects. (Although QED has a Landau pole, its coupling at 100 GeV scale can still be safely ignored because Landau pole presumably appears at some astronomically high scale.) We call this the zeroth-order approximation. We also ignore all the light fermions. Therefore, the only fermion left is the top quark (and/or the fourth family fermions should they exist).

It is well known that naive fermion action on the lattice suffers from species doubling. The ways to handle fermion doublers are the Wilson fermion and the staggered fermion\(^1\). In this report, we take the approach of the Wilson fermion. However, the electroweak theory is a chiral gauge theory. Even after we ignore the gauge couplings, we still have global chiral symmetries left. The naive Wilson fermion
action breaks the chiral symmetry explicitly and is not good for the electroweak theory. In order to cure this, two models were proposed. One is the Smit-Swift model. The other is the mirror-fermion model. We will report on our lattice study of the electroweak theory using the mirror-fermion model in the zeroth-order approximation.

The global symmetry of the mirror-fermion model can be $U(1)$ or $SU(2)$. It is found that the $U(1)$ and $SU(2)$ versions have the same qualitative properties. Since $SU(2)$ symmetry is closer to the electroweak part of the real world, we will concentrate on the $SU(2)$ mirror-fermion model.

2. Lattice Action and Decoupling Limit

The lattice action of the mirror-fermion model is a sum of the $O(4)$ ($\cong SU(2)_L \otimes SU(2)_R$) symmetric pure scalar part $S_\phi$ and fermionic part $S_\Psi$:

$$ S = S_\phi + S_\Psi. $$

$\varphi_x$ is the $2 \otimes 2$ matrix scalar field, and $\Psi_x \equiv (\psi_x, \chi_x)$ stands for the mirror pair of fermion doublet fields (usually $\psi$ is the fermion doublet and $\chi$ the mirror fermion doublet). In the usual normalization conventions for numerical simulations we have

$$ S_\phi = \sum_x \left\{ \frac{1}{2} \mathrm{Tr} (\varphi_x^+ \varphi_x) + \lambda \left[ \frac{1}{2} \mathrm{Tr} (\varphi_x^+ \varphi_x) - 1 \right]^2 - \kappa \sum_{\mu=1}^4 \mathrm{Tr} (\varphi_x^{+\mu} \varphi_x) \right\}, $$

$$ S_\Psi = \sum_x \left\{ \mu_{\psi\chi} \left[ (\chi_x \psi_x) + (\bar{\psi}_x \chi_x) \right] 
- K \sum_{\mu=\pm1}^4 \left[ (\bar{\psi}_{x+\mu} \gamma_{\mu} \psi_x) + (\bar{\chi}_{x+\mu} \gamma_{\mu} \chi_x) \right] 
+ r \left( (\chi_{x+\mu} \psi_x) - (\chi_x \psi_x) \right) 
+ G_{\psi} \left[ (\bar{\psi}_{Rx} \varphi_{xL}^+ \psi_{Lx}) + (\bar{\psi}_{Lx} \varphi_{xR}^+ \psi_{Rx}) \right] 
+ G_{\chi} \left[ (\chi_{Rx} \varphi_{xL}^+ \chi_{Lx}) + (\chi_{Lx} \varphi_{xR}^+ \chi_{Rx}) \right] \right\}. $$

Here the two fermion fields $\psi$ and $\chi$ are Wilson fermions, $K$ is the fermion hopping parameter, $r$ the Wilson-parameter, which will be fixed to $r = 1$ in the numerical simulations, and the indices $L, R$ denote, as usual, the chiral components of fermion fields. One can easily see that the above action has an explicit chiral symmetry in the sense that left- and right-handed components of the fermion fields can transform independently. However, the right-handed $\chi$-field has to transform exactly as the left-handed $\psi$-field, and left-handed $\chi$-component as the right-handed $\psi$-component. (So an off-diagonal bare fermion mass term can exist in the action.) Therefore, $\psi$ and $\chi$ are mirror partners of each other. In our simulations, we take normalization that fermion–mirror-fermion mixing mass is $\mu_{\psi\chi} = 1 - 8rK$. 
The fermionic part $S_{\Psi}$ is given here for a single mirror pair of fermions. Taking the adjoint transforms fermions to mirror fermions and vice versa, but as noted before, without $\text{SU}(3)_{\text{colour}} \otimes \text{U}(1)_{\text{hypercharge}}$ gauge couplings they are equivalent to each other.

All renormalized quantities can be defined. (See [details] for details.) Since the symmetry of the action does not exclude the mixing fermion mass term, we will have a nonzero renormalized fermion mixing mass $\mu_R$. The physical fermion states will be linear combinations of the renormalized $\psi$ and $\chi$ fields in general. One can always tune the bare parameters to have a zero renormalized mixing. It turns out that in most cases this tuning is rather difficult and time consuming in the simulation.

So far we have not discovered mirror-fermions in the experiments, we do not want to have them in the physical spectrum. There are different ways to decouple those unwanted fermions. We can tune the bare parameters such that mirror-fermion has its mass of the order of the cutoff scale. Hopefully this heavy mirror-fermion will decouple. The more elegant way is to take the advantage of the symmetry of the model and is explained below.

The action shown in eq.(2) is invariant under the following transformations of the fermion fields

$$\psi_x \rightarrow \psi_x + \omega, \quad \bar{\psi}_x \rightarrow \bar{\psi}_x + \bar{\omega} \quad \text{if} \quad G_{\psi} = 0 ;$$

$$\chi_x \rightarrow \chi_x + \epsilon, \quad \bar{\chi}_x \rightarrow \bar{\chi}_x + \bar{\epsilon} \quad \text{if} \quad G_{\chi} = 0$$

where $\omega$, $\epsilon$ do not depend on $x$. Due to the above symmetry (called shift symmetry from now on), one can easily derive the corresponding Ward identity and show that $G_{R\chi} = 0$ identically at $G_{\chi} = 0$. Furthermore, from the Ward identity for the fermion propagator, one can show that $\mu_R$ will vanish identically if we choose $K = 1/8$ in addition to $G_{\chi} = 0$. This analytic result tells us that at $G_{\chi} = 0$, $K = 1/8$, there is no mixing between the fermion and mirror-fermion, and the renormalized Yukawa coupling of the mirror-fermion is zero. Therefore, the mirror-fermion is massless and is not coupled to the scalar and fermion fields. In other words, at $G_{\chi} = 0$ and $K = 1/8$, the mirror-fermion behaves like the massless right-handed neutrino and is decoupled from the physical world. We call the above choice of the bare parameters the mirror-fermion decoupling limit.

At the energy scale of the weak interaction (around 100 GeV), all gauge couplings are weak. We therefore take the zeroth-order approximation by ignoring all gauge fields. We think that the only possible sources of nonperturbative physics in the electroweak theory are the Higgs and heavy fermion sectors. Notice that the shift symmetry of our mirror-fermion model is destroyed if the gauge fields are put in. Mirror-fermion will be coupled to the physical world via gauge interactions even if we set $G_{\chi} = 0$, $K = 1/8$. We simply assume that the zeroth-order approximation reported here serves as a good approximation to the electroweak theory without
worrying how to decouple the mirror-fermion in the full theory.

3. Vacuum Stability and the $\beta$-Functions

The upper bound on the Higgs mass comes from the notion of triviality $\mathbb{E}$. As for the lower bound, notions of triviality and vacuum stability are both required $\mathbb{F}$.

In order that a quantum field theory is self-consistent, its energy spectrum has to be bounded from below. Or we say that its vacuum has to be stable. Since the minimum of the effective potential corresponds to the ground state (i.e. vacuum) of the theory, a self-consistent quantum field system should have an effective potential which is convex. Namely, as the field strength increases, the effective potential cannot go down to negative values indefinitely. In terms of the language of renormalization group, the running scalar coupling constant cannot go to negative values as we increase the energy scale $\mathbb{G}$.

What describe how the couplings change as the energy scale changes are the $\beta$-functions. The exact $\beta$-functions and effective potential are usually unknown. People rely on loop expansions to derive them. However, loop expansions can break down when the system becomes strongly interacting, we must go beyond perturbation theory. At the nonperturbative level, we find that the partition function of the Euclidean mirror-fermion model simply blows up when $\lambda$ is negative. If this is so, then the effective potential is not defined because the theory is not self-consistent. In order that the theory exists, $\lambda$ has to be positive. Then the partition function exists and the effective potential can be defined. Since the exact effective potential is defined via a Legendre transformation, its shape will be convex and has a minimum once it exists. Hence, at the nonperturbative level, once $\lambda$ is positive, the theory will have a stable vacuum.

If the theory is trivial in the continuum limit, then its exact $\beta$-functions will behave qualitatively like the one-loop ones. There we can see that at some large $G_\psi$ values, $\beta$-function of $g$ can turn negative. This shows that $g$ can become negative as we go to higher energy scales, thus spoiling the stability of the vacuum. In order that the vacuum state be stable, $g_R$ cannot be too small. Hence, we can obtain a vacuum stability lower bound on $g_R$ as a function of $G_{R\psi}$. Since the smallest value $\lambda$ can take without destroying vacuum stability is zero (or $0^+$), the lower bound on $g_R$ naturally corresponds to a very small but positive $\lambda^4$.

Thus in lattice study of the SU(2) mirror-fermion model, we can first decouple the unwanted mirror-fermion by setting $G_\chi = 0$, $K = 1/8$. If the mirror-fermion model is trivial in the continuum limit, Monte Carlo data on $g_R$ and $G_{R\psi}$ obtained at $\lambda = \infty$ and $\lambda = 0^+$ will give us upper and lower bounds on the Higgs mass as a function of the heavy fermion mass respectively. It goes without saying that both bounds are cutoff dependent.

We also carry out one-loop renormalized perturbation theory on the lattice and
calculate \( \beta \)-functions accordingly. We therefore can estimate the finite size effects and finite cutoff effects.

4. The Simulation and Data

The phase structure of the model was first explored. (See \[8\] for details.) We find that the phase boundary between the symmetric (PM) phase and the broken symmetry (FM) phase is a second-order phase transition line where the cutoff can be removed.

Numerical simulations were performed on \( 6^3 \cdot 12 \) and \( 8^3 \cdot 16 \) lattices at \( \lambda = 10^{-6} \) and \( \infty \). The small positive value of \( \lambda \) was chosen to mimic \( \lambda = 0^+ \). The Yukawa coupling \( G_\chi \) was kept at zero, the fermion hopping parameter \( K \) was kept near \( 1/8 \) on the finite lattice for exact decoupling of the mirror doublets \[9\]. Since it is the FM phase that is physically relevant, \( \kappa \) and \( G_\psi \) are tuned such that we approach criticality from within the FM phase and have a reasonable cutoff to reduce finite size effects. Since Ward identities give us some analytic results for renormalized quantities at \( G_\chi = 0 \), we put those results into our Fortran program to “guide” our Monte Carlo simulations. This helps to reduce the fluctuations.
5. Conclusions

Our numerical data are presented in Fig. 1. At all points of the simulation, we find that all fermion doublers are decoupled as expected.

It is seen that our Monte Carlo data are in qualitative agreement with one-loop perturbative results. This may not be so surprising since tree unitarity (which requires probability conservation) upper limit on $G^2_{R0}$ is $2\pi$. Below this value, the system should be weakly interacting. So far, no evidence for an ultra-violet stable fixed point has been found. Since all qualitative features of the one-loop $\beta$-functions are supported, our mirror-fermion models are likely to be trivial in the infinite cutoff limit. Assuming that the SU(2) mirror-fermion model is trivial in the continuum limit, we obtain, in zeroth-order approximation, upper and lower bounds on the Standard Model Higgs mass as a function of the heavy fermion mass. If the heavy fermion in our model is top quark with its mass around 180 $GeV$, then we claim at $\xi = 1.0$ that Standard Model Higgs has its mass between 100 and 800 $GeV$.

Although the observed qualitative behaviour is certainly consistent with the one-loop perturbative scenario implying the triviality of the continuum limit, one has to keep in mind that present simulations are done at relatively low cut-offs. In particular, the evolution of the couplings towards smaller values at decreasing cut-offs should be investigated in the future in order to make sure that the model is indeed trivial.

6. References

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Figure 1