Avoiding the $H_c = 0$ Anomaly using FORC+ (expanded version of paper GG-05, MMM-Intermag 2019)

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In conventional FORC (First Order Reversal Curve) analysis of a magnetic system, reversible and low-coercivity irreversible materials are treated as being qualitatively different: the FORC distribution shows low-coercivity materials but completely hides reversible (zero-coercivity) ones. This distinction is artificial – as the coercivity approaches zero, the physical properties of an irreversible material change smoothly into those of a reversible material. We have developed a method (called FORC+, implemented in free software at http://MagVis.org) for displaying the reversible properties of a system (a reversible switching-field distribution, R-SFD) together with the irreversible ones (the usual FORC distribution), so that there is no sudden discontinuity in the display when the coercivity becomes zero. We will define a “FORC+ dataset” to include the usual FORC distribution, the R-SFD, the saturation magnetization, and what we will call the “lost hysteron distribution” (LHD) such that no information is lost – the original FORC curves can be exactly recovered from the FORC+ dataset. We also give some examples of the application of FORC+ to real data – it uses a novel complementary-color display that minimizes the need for smoothing. In systems which switch suddenly (thus having sharp structures in the FORC distribution) direct display of un-smoothed raw data allows visualization of sharp structures that would be washed out in a conventional smoothed FORC display.

This is an expanded version of paper GG-05, MMM-Intermag 2019, with a discrete derivation of the FORC distribution (Eq. [1]) and an additional example (Fig. [7]).

I. INTRODUCTION

The FORC method [1–4] for the characterization of magnetic systems has come into wide use in the past 15 years or so. It was originally designed for completely irreversible systems, modeled as a collection of Preisach hysterons, each of which has a rectangular hysteresis loop (Fig. [1]). As we lower the field from a large positive saturating value, it switches down at a field usually denoted by $H_R$ (the subscript R stands for “reversal”, for reasons that will become apparent later) and as we increase the field it switches back up at a field $H > H_R$. The Preisach distribution is the density of these hysterons in the $H - H_R$ plane.

The fundamental result behind the FORC idea is that this Preisach distribution can be obtained by measuring "first order reversal curves" – that is, by saturating the sample in the positive direction, decreasing the field to a reversal field $H_R$ (see Fig. [2]), then reversing $dH/dt$ from negative to positive and measuring the magnetization $m(H, H_R)$ as the field increases again past each value $H$. The distribution of hysterons is then given by

$$\rho(H, H_R) = -\frac{1}{2} \frac{\partial^2 m(H, H_R)}{\partial H \partial H_R}$$

(1)

It is surprisingly difficult to find a simple derivation of this result in the literature – we will give a discrete derivation in Sec. [III] below. Note that in a reversible system the moment $m(H, H_R)$ measured at a field $H$ is independent of the history, in particular independent of the reversal field $H_R$. Thus the derivative (Eq. [1]) vanishes – the FORC distribution gives no...
We need not save the I-SFD \((i.e., S_{irr})\) separately from the FORC distribution, since it can be obtained by integrating Eq. \[1\] with respect to \(H\) downward from saturation. However, the R-SFD is new information, and the FORC+ program displays it.

The FORC+ program\[6\] is driven by a script file, which can either read a magnetometer data file or create a set of synthetic FORC curves for a specified set of Preisach hysterons. In Fig. 3(a) we show the FORC+ display for a synthetic system of six hysterons of different coercivity but zero bias – this gives a positive Preisach density, which is displayed as an orange color, with primary color values \(RGB = (1, 0, 5, 0)\). This particular distribution has no negative density, but negative density is displayed in the complementary color \((0, 0, 5, 1)\). This is important because if there is noise (a mixture of equal amounts of positive and negative density) this will appear from a distance as a shade of grey, \(RGB = (x, x, x)\). If the grey is tinged with orange, that means there is a small net positive density, and it is not always necessary to do artificial smoothing to see this. The FORC curves are shown in the upper left, and below them the FORC distribution, with the same \(H\) scale. Each feature in the distribution appears exactly below the FORC curve that created it. The FORC distribution is repeated on the lower right, but rotated by 45° so the coercivity axis is vertical (and points downward). This is done so the R-SFD can be plotted upward as a bar graph, starting from the \(H_c = 0\) zero-coercivity line. This ensures a smooth transition as the coercivity of each hysterons decreases (Fig. 3(b), (c)) – it moves upward in the FORC distribution, and when each hysterons reaches \(H_c \approx 0\) it reappears at the same field value in the upper right display (R-SFD). In Fig. 3(c), all of the hysterons have become reversible. The horizontal axis is the bias field \(H_b\) in the lower display and \(H\) in the upper one, but these are equal along the \(H_c = 0\) line so they match smoothly. The display scale (number of orange pixels per emu of magnetic moment) has been chosen to be the same in the FORC distribution and in the bar graph of the R-SFD, so the total number of orange pixels remains the same as the system becomes reversible. It is common to over-interpret a very weak irreversible FORC distribution in a system that is primarily reversible, because the reversible part is invisible in an ordinary FORC display, and the display program sets the gain very high to normalize the weak signal – using FORC+, it will be clear that the R-SFD is dominant in such a case. It is also possible to turn up the gain to make the weaker signal visible (see Fig. 7 below), but this can be indicated on the display. In addition to the graphics window shown in Fig. 7 the FORC+ program also opens a text window with additional information about the system (for example, the reversible fraction, Eq. \[7\]).

There has been previous work on including reversible effects in the FORC formalism\[5\], some of which was aimed at incorporating the reversible material into the FORC distribution as a singular delta function, and some of which involved a separate plot of the reversible part, but none of these ensured that exactly the same scale was used for the reversible and irreversible displays.
III. INVERTIBILITY: NO INFORMATION LOSS

In this section we will construct the "FORC+ data set" which preserves all the information in the FORC curve measurement. An illustrative set of FORC measurements on a regular grid in the $H, H_R$ plane is shown in Fig. 4. Each black dot represents a single measurement of the magnetic moment $m(H, H_R)$, for a particular pair of fields ($H$ is the horizontal axis and $H_R$ the vertical one.) (Of course, no real measurement will give fields on an exactly regular grid. The FORC+ program operates on an arbitrary grid. However, it is often nearly regular, and it is very much easier to prove the results we prove below using a regular grid. The generalization to an arbitrary grid will be published elsewhere.) Each FORC curve is a horizontal string of black dots, starting at the reversal field $H = H_R$ and ending at some higher field. In the commonly used PM (Princeton Measurements) FORC protocol, the maximum field is determined by bounds on the bias field $H_b$ or the coercivity $H_c$, so the boundaries of the measured region are lines of constant $H_b$ (perpendicular to the $H_b$ axis, which is also shown) or constant $H_c$.

Fig. 5 shows a discrete derivation of Eq. 1, for a single hysteron in the yellow-shaded plaquette labeled $P$. Because all equations are linear, it follows that it is correct for an arbitrary distribution of hysterons. (Another discrete derivation, not using linearity in this way, has been given previously.\cite{7})

Fig. 6 shows the information (the "FORC+ data set") that is necessary to invert the FORC distribution calculation and recover the original data $m(H, H_R)$. The inset shows a single square plaquette, with four corners $A$, $B$, $C$, and $D$ where the moment $m$ has been measured. The discrete derivative $dm/dH$ is naturally defined at the blue line segment from $A$ to $B$. We will denote it (without the factor of $1/\delta$, for convenience) by $m_H = m(B) - m(A)$. Similarly, $m_{H_R}$ is the difference in the vertical direction, and $m_{H_R H}$ is the second difference, defined for each plaquette, and related to the FORC
We will graph these as part of the R-SFD function – if there are no lost hysterons above them, the system is completely reversible at these high fields. In addition to differences, we will need one actual value for the moment, which we take to be at the rightmost point labeled "mₗ". Then we alternately add horizontal and vertical differences to recover m at all the points on the upper right boundary. When we get to the top, we can come back down along the Hₛ axis, again alternately adding horizontal and vertical differences (recall that we computed the vertical differences everywhere by integrating mHRH from the right). Now we know m at the top of each column of dots, and can compute them downward along the column using the vertical differences.

Thus we have shown that the FORC+ data set consisting of (1) the FORC distribution mHRH, (2) the R-SFD, (3) the lost hysteron distribution LHD, and (4) the saturation moment mₛ allows us to exactly recover all the original measurements of m. The figures have been drawn for a particular choice of FORC lengths, but it is not hard to see that the argument works as long as the measured region in the H - Hₛ plane is convex (concave sections will cause problems).

IV. EXAMPLES

Figure 7 shows the FORC+ display produced by real data from a Princeton Measurements MicroMag 2900 AGM magnetometer, for a patterned perpendicular film. The FORC distribution (lower left) shows some noise – to see the orange area (positive density peak) we have turned up the gain so the center of the peak is oversaturated. But there is a clear peak, with a narrow distribution of bias field Hₛ and a broader distribution of coercivity Hₗ. Although the reversible fraction is 44%, we have multiplied the scale of the reversible SFD at the upper right by 4 so it can be seen more easily.
Figure 8 shows a very different type of system, an unpatterned film that switches suddenly by domain wall motion. This leads to a very sharp feature in the FORC distribution that could be very easily obscured by smoothing – it can be seen from the zoomed inset that the positive and negative parts of the "dipolar tail" are only one pixel wide ($\delta = 207 \text{ Oe}$) at places. This clearly demonstrates the utility of being able to view the FORC distribution without smoothing.

The FORC+ program has also been used for analysis of more complicated structures such as magnetic tunneling junctions (MTJs), in which the characteristic fingerprints of individual layers (such as the dipole tail) can be identified in the FORC distribution of a complex structure containing such a layer.

V. CONCLUSIONS

We have presented a new method for displaying FORC data that may offer some advantages over existing display options. It is a standalone Windows program, written in C++ and OpenGL, so that it should be portable to other operating systems, and does not require any proprietary platform or visualization package. No data manipulation is required of the user – one simply drags a raw data file from a VSM or AGM magnetometer to the executable. PM format files are supported in the current version, FORC+1.0, and others could easily be added. It uses a complementary-color scheme that minimizes the need for smoothing (although a polynomial-fit smoothing option is planned for a future version). The information displayed is as close as possible to the raw data, and we can prove that no information has been lost – the original FORC curves can be exactly recovered from the displayed information. The avoidance of smoothing facilitates the analysis of systems with very sharp features in the FORC distribution.

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