HELAS and MadGraph/MadEvent with spin-2 particles

K. Hagiwara, J. Kanzaki, Q. Li, K. Mawatari

Abstract Fortran subroutines to calculate helicity amplitudes with massive spin-2 particles (massive gravitons), which couple to the standard model particles via the energy momentum tensor, are added to the HELAS (HELiCty Amplitude Subroutines) library. They are coded in such a way that arbitrary scattering amplitudes with one graviton production and its decays can be generated automatically by MadGraph and MadEvent, after slight modifications. All the codes have been tested carefully by making use of the invariance of the helicity amplitudes under the gauge and general coordinate transformations.

1 Introduction

The idea of extra space dimensions has attracted much attention in recent years, since it can give us a novel solution to the hierarchy problem, or an alternative explanation of the hierarchical difference between the Planck scale (\(M_{\text{Pl}} \sim 10^{19} \text{ GeV}\)) and the electroweak scale (\(m_W \sim 10^2 \text{ GeV}\)).

So far, there have been various extra dimension models, which can be divided into two major classes according to the geometry of the background space-time manifold. The first one includes the ADD (Arkani-Hamed, Dimopoulos, and Dvali) model [1–3] and its variants, which extend the dimension of the total space-time to \(D = 4 + \delta\), with a factorizable metric and large size of the compact extra dimensions (\(\gg 1/M_{\text{Pl}}\)). The second one includes the 5-dimensional RS (Randall and Sundrum) model [4, 5] and its variants, in which a warped metric is introduced along the 5-th dimension and the size of the extra dimension needs not to be much larger than the Planck length.

In both classes of extra dimension models, there appear Kaluza-Klein (KK) towers of massive spin-2 gravitons, which can interact with the standard model (SM) fields. The effective interaction Lagrangian is given by [6, 7]

\[
\mathcal{L}_{\text{int}} = -\frac{1}{\Lambda} \sum_n T(\tilde{n})_{\mu\nu} T_{\mu\nu},
\]

where \(T(\tilde{n})_{\mu\nu}\) is the \(\tilde{n}\)-th graviton KK modes, and \(\Lambda\) is the relevant coupling scale. In the ADD model we have

\[
\Lambda = \frac{M_{\text{Pl}}}{\sqrt{8\pi}} \sim 2.4 \times 10^{18} \text{ GeV},
\]

where \(M_{\text{Pl}}\) is the 4-dimensional reduced Planck scale, and in the RS model

\[
\Lambda = e^{-kr_c} \overline{M}_{\text{Pl}}
\]

is at the electroweak scale, where \(k\) is a scale of order of the Planck scale and \(r_c\) is the compactification radius.

In (1), \(T_{\mu\nu}\) is the energy-momentum tensor of the SM fields,

\[
T_{\mu\nu} = \left(-\eta_{\mu\nu} \mathcal{L}_{\text{SM}} + 2 \frac{\delta \mathcal{L}_{\text{SM}}}{g_{\mu\nu}}\right)_{g_{\mu\nu}=\eta_{\mu\nu}}
= T^H_{\mu\nu} + T^F_{\mu\nu} + T^g_{\mu\nu} + T^p_{\mu\nu} + T^Z_{\mu\nu} + T^W_{\mu\nu} + \cdots,
\]

where \(g_{\mu\nu}\) is the metric and \(\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1, -1)\) is the Minkowski value, and each energy-momentum tensor is:

\[
T^H_{\mu\nu} = -\eta_{\mu\nu} \left[\frac{1}{2} \frac{\partial^\rho}{\partial \rho} H \partial_\mu H - \frac{m_H^2}{2} H^2 - \frac{g_W m_Z^2}{4 m_W} H^3 \right.
\]

\[- \frac{g_W^2 m_Z^2}{32 m_W^2} H^4 - \frac{g_W m_F}{2 m_W} \bar{\psi} \psi H
\]

\[+ \frac{g_Z m_Z}{2} Z_{\mu} Z^\mu H + \frac{g_Z^2}{8} Z_{\mu} Z^\mu H^2
\]

\[+ \frac{g_Z}{2} Z_{\mu} Z^\mu H \right]^{\text{SM}}
\]

\]
\[ + g_W m_W W_\mu^+ W_-^\mu H + \frac{g_2^2}{4} W_\mu^+ W_-^\mu H^2 \]
\[ + \partial_\mu H \partial_\nu H + g_2 m_Z Z_\mu Z_\nu H + \frac{g_2^2}{4} Z_\mu Z_\nu H^2 \]
\[ + \left[ g_W m_W W_\mu^+ W_-^\mu H + \frac{g_2^2}{4} W_\mu^+ W_-^\mu H^2 \right. \]
\[ \left. + (\mu \leftrightarrow \nu) \right]. \quad (5a) \]
\[ \mathcal{T}_{\mu\nu}^F = -\eta_{\mu\nu} \left[ \bar{\psi} (i \gamma^\rho D_\rho - m_F) \psi - \frac{1}{2} \partial^\rho (\bar{\psi} i \gamma^\rho \psi) \right. \]
\[ - \left( \frac{g_W}{\sqrt{2}} \gamma_{ij} \bar{\psi}_{u_i} \gamma^\rho P_L \psi_{d_j} W_\rho^+ \right. \]
\[ + \left. \frac{g_W}{\sqrt{2}} U_{ij} \bar{\psi}_{u_i} \gamma^\rho P_L \psi_{d_j} W_\rho^+ + \text{h.c.} \right) \]
\[ + \left[ \frac{1}{2} \bar{\psi} i \gamma_\mu D_\nu \psi - \frac{i g_W}{4} \eta_{\mu\nu} \bar{\psi} \gamma_\nu \psi + (\mu \leftrightarrow \nu) \right] \]
\[ + \left. \left[ - \frac{g_W}{\sqrt{2}} V_{ij} \bar{\psi}_{u_i} \gamma_\mu P_L \psi_{d_j} W_\nu^+ \right. \right. \]
\[ - \frac{g_W}{\sqrt{2}} U_{ij} \bar{\psi}_{u_i} \gamma_\mu P_L \psi_{d_j} W_\nu^+ + \text{h.c.} + (\mu \leftrightarrow \nu) \right]. \quad (5b) \]
\[ \mathcal{T}_{\mu\nu}^g = -\eta_{\mu\nu} \left[ - \frac{1}{4} F^{a,\rho\sigma} F_{\rho\sigma}^a + \partial^\rho \partial^\sigma A_\sigma^a A_\rho^a - \frac{1}{2} (\partial^\rho A_\rho^a)^2 \right. \]
\[ - F_\mu^a \partial^\rho F_{\rho}^a + \partial_\mu \partial^\rho A_\rho^a A_\mu^a + \partial_\nu \partial^\rho A_\rho^a A_\nu^a, \quad (5c) \]
\[ \mathcal{T}_{\mu\nu}^\gamma = -\eta_{\mu\nu} \left[ - \frac{1}{4} F^{\gamma\rho\sigma} F_{\rho\sigma}^\gamma + \partial^\rho \partial^\sigma A_\sigma^\gamma A_\rho^\gamma - \frac{1}{2} (\partial^\rho A_\rho^\gamma)^2 \right. \]
\[ - F_\mu^\gamma \partial^\rho F_{\rho}^\gamma + \partial_\mu \partial^\rho A_\rho^\gamma A_\mu^\gamma, \quad (5d) \]
\[ \mathcal{T}_{\mu\nu}^Z = -\eta_{\mu\nu} \left[ - \frac{1}{4} Z^{\rho\sigma} Z_{\rho\sigma}^\gamma + m_Z^2 Z_{\rho}^\gamma Z_{\rho} \right. \]
\[ - Z_\mu^\gamma Z_{\nu}^\rho + m_Z^2 Z_\mu Z_\nu, \quad (5e) \]
\[ \mathcal{T}_{\mu\nu}^W = -\eta_{\mu\nu} \left[ - \frac{1}{2} W^{+\rho\sigma} W^-_{\rho\sigma} + m_W^2 W_\mu^+ W_\nu^- \right. \]
\[ \left. - [W_\mu^+ W_\nu^- - m_W^2 W_\mu^+ W_\nu^- + (\mu \leftrightarrow \nu)] \right]. \quad (5f) \]

with \( e = g_W \sin \theta_W = g_Z \sin \theta_W \cos \theta_W \) and the projection operator \( P_L = \frac{1}{2} (1 - \gamma_5) \). Here, the covariant derivative is
\[ D_\mu \equiv \partial_\mu + ig_Z T^a A_\mu^a + ie Q_f A_\mu \]
\[ + ig_Z (T^3 P_L - Q_f \sin^2 \theta_W) Z_\mu. \quad (6) \]

Note that the derivative couplings of the \( W \) bosons are written explicitly in (5b). Each field-strength tensor for the gauge bosons is
\[ F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_s f^{abc} A_\mu^b A_\nu^c, \]
\[ F_{\mu\nu}^\gamma = \partial_\mu A_\nu^\gamma - \partial_\nu A_\mu^\gamma + i e [W_\mu^+ W_\nu^- - (\mu \leftrightarrow \nu)], \]
\[ Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu + ig_W \cos \theta_W \]
\[ \times [W_\mu^+ W_\nu^- - (\mu \leftrightarrow \nu)], \quad (7) \]
\[ W_\mu^\pm = \partial_\mu W_\mu^\pm - \bar{c}_\mu W_\mu^\pm \]
\[ \mp ig_W [\sin \theta_W W_\mu^\pm A_\mu + \cos \theta_W W_\mu^\pm Z_\mu - (\mu \leftrightarrow \nu)]. \]

Notice that as in the standard HELAS package \[8\], we use the unitary gauge for the massive vector-boson propagators and the Feynman gauge for the massless ones.

In this paper, we present new HELAS subroutines \[8\] for the massive gravitons and their interactions based on the effective Lagrangian of (1), and implement them into MadGraph/MadEvent (MG/ME) \[9-11\].

The paper is organized as follows: In Sect. 2 we give the new HELAS subroutines. In Sect. 3 we describe how to implement amplitudes with a massive spin-2 graviton into MG/ME. In Sect. 4 we give sample numerical results. Section 5 contains a brief conclusion.

### 2 HELAS subroutines for spin-2 particles

In this section, we list the contents of all the new HELAS subroutines that are needed to evaluate massive spin-2 graviton production at hadron colliders in association with quark and gluon jets, and its decays into a pair of all the SM particles, or into arbitrary numbers of quarks and gluons.

To begin with, in Sect. 2.1 the subroutine to compute the external lines for a spin-2 tensor particle is presented. Next, in Sects. 2.2 to 2.7, the subroutines to compute the interactions of the tensor boson with the SM particles are explained. The new vertex subroutines are listed in Table 1. Those subroutines which we do not present in this paper are \( FFST, SSST \) and \( SSSST \) type vertices for the interactions with scalars; \( \gamma \gamma \gamma \gamma \) and \( \gamma \gamma \gamma \gamma \gamma \gamma \) for the interactions with the electroweak gauge bosons. These contribute \( e.g. \) to the graviton decays into three or more weakly interacting particles. We present the effective Lagrangian of (5a) to (5f) for completeness sake. Finally, we briefly mention how we test our new subroutines in Sect. 2.8.
Table 1 List of the new vertex subroutines in HELAS system

| Vertex | Inputs | Output | Subroutine |
|--------|--------|--------|------------|
| SST    | SST    | Amplitude | SSTXXX |
| ST     | S      | HSTXXX  |
| SS     | T      | USSXXX  |
| FFT    | FFT    | Amplitude | IOTXXX |
| FT     | F      | FTIXXX, FTOXXX |
| FF     | T      | UIOXXX  |
| VVT    | VVT    | Amplitude | VVTXXX |
| VT     | V      | JVTXXX  |
| VV     | T      | UVVXXX  |
| FFVT   | FFVT   | Amplitude | IOVTTXX |
| FVT    | F      | FVTXXX, FVTXXX |
| FFT    | V      | JIOTXXX |
| FFV    | T      | UIOVXX |
| VVVT   | VVVT   | Amplitude | VVTXXX |
| VVT    | V      | JVTXXX  |
| VVV    | T      | UVVXXX  |
| VVVVT  | GGGGT  | Amplitude | GGGGTXXX |
| GGGT   | G      | JGGGTXX |
| GGGG   | T      | UGGGX   |

2.1 Tensor wavefunction

2.1.1 TXXXXX

This subroutine computes the spin-2 tensor particle wavefunction; namely, $\epsilon^{\mu\nu}(p, \lambda)$ and $\epsilon^{\mu\nu}(p, \lambda)^*$, in terms of its four-momentum $p$ and helicity $\lambda$, and should be called as

```
CALL TXXXXX(P, TMASS, NHEL, NST , TC).
```

The input $P(0:3)$ is a real four-dimensional array which contains the four-momentum $p^\mu$ of the tensor boson, TMASS is its mass, NHEL $= \pm 2, \pm 1, 0$ specifies its helicity $\lambda$, and NST specifies whether the boson is in the final state ($\text{NST} = 1$) or in the initial state ($\text{NST} = -1$). The output $\text{TC}(18)$ is a complex 18-dimensional array, among which the first 16 components contain the wavefunction as

$$\text{TC}(4\mu + \nu + 1) = \mathcal{T}(\mu + 1, \nu + 1),$$

namely

$$\text{TC}(1) = \mathcal{T}(1, 1),$$
$$\text{TC}(2) = \mathcal{T}(1, 2),$$
$$\text{TC}(3) = \mathcal{T}(1, 3),$$
$$\text{TC}(4) = \mathcal{T}(1, 4).$$

The helicity states of the tensor boson can be expressed as

$$\epsilon^{\mu\nu}(p, \pm 2) = \epsilon^{\mu}(p, \pm)\epsilon^{\nu}(p, \pm),$$

$$\epsilon^{\mu\nu}(p, \pm 1) = \frac{1}{\sqrt{2}}[\epsilon^{\mu}(p, \pm)\epsilon^{\nu}(p, 0) + \epsilon^{\nu}(p, 0)\epsilon^{\mu}(p, \pm)],$$

$$\epsilon^{\mu\nu}(p, 0) = \frac{1}{\sqrt{6}}[\epsilon^{\mu}(p, +)\epsilon^{\nu}(p, -) + \epsilon^{\mu}(p, -)\epsilon^{\nu}(p, +) + 2\epsilon^{\nu}(p, 0)\epsilon^{\mu}(p, 0)],$$

by using the vector boson wavefunctions $\epsilon^{\mu}(p, \lambda)$ that obey the relation

$$J_-\epsilon^{\mu}(p, \lambda) = \epsilon^{\mu}(p, \lambda - 1),$$

where $J_- = J_x - i J_y$ is the $J_z$, lowering operator. The spin-1 vector wavefunction in the HELAS convention [8] satisfies this relation, and hence we simply use the HELAS code to obtain the tensor wavefunction. These tensor wavefunctions are traceless, transverse, orthogonal, and symmetric,

$$\epsilon(p, \lambda)\mu_\mu = 0, \quad p_\mu\epsilon^{\mu\nu}(p, \lambda) = p_\nu\epsilon^{\mu\nu}(p, \lambda) = 0,$$

and the completeness relation is

$$\sum_{\lambda=-2}^{+2} \epsilon^{\mu\nu}(p, \lambda)\epsilon^{\nu\beta}(p, \lambda)^* = B^{\mu\nu,\alpha\beta}(p)$$

with

$$B^{\mu\nu,\alpha\beta}(p) = \frac{1}{2}(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \eta^{\mu\nu}\eta^{\alpha\beta})$$

$$- \frac{1}{2m_T^2}(\eta^{\mu\alpha}p^\nu p^\beta + \eta^{\nu\beta}p^\mu p^\alpha + \eta^{\mu\beta}p^\nu p^\alpha$$

$$+ \eta^{\nu\alpha}p^\mu p^\beta)$$

$$+ \frac{1}{6}(\eta^{\mu\nu} + \frac{2}{m_T^2}p^\mu p^\nu)\eta^{\alpha\beta} + \frac{2}{m_T^2}p^\alpha p^\beta).$$

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2.2 SST vertex

The SST vertices are obtained from the interaction Lagrangian among the tensor and two scalar bosons:

\[ \mathcal{L}_{\text{SST}} = GT T^{\mu \nu} \left[ -\eta_{\mu \nu} \left( (\partial^\rho S^*)^\rho (\partial_\rho S^*) - m_S^2 S S^* \right) + (\partial_\rho S^*)^\rho (\partial^\rho S^*) + (\partial_\rho S^*)^\rho (\partial^\rho S^*) \right] \]  

(16a)

for the complex scalar field, or

\[ \mathcal{L}_{\text{SST}} = GT T^{\mu \nu} \left[ -\eta_{\mu \nu} \left( \frac{1}{2} \partial^\rho S^* \partial_\rho S^* - \frac{m_S^2}{2} S S^* \right) + \partial_\rho S^* \partial^\rho S^* \right] \]  

(16b)

for the real scalar field. Here,

\[ GT = \frac{-1}{\Lambda} \]

(17)

is the coupling constant.

2.2.1 SSTXXX

This subroutine computes the amplitude of the SST vertex from two Scalar boson wavefunctions and a Tensor boson wavefunction, and should be called as

\[ \text{CALL SSTXXX(S1, S2, TC, GT, SMASS, VERTEX).} \]

The inputs S1 (3) and S2 (3) are complex three-dimensional arrays which contain the wavefunctions of the Scalar bosons, S1 (1) and S2 (1), and their four-momenta as

\[ p_1^\mu = (\Re S1(2), \Re S1(3), \Im S1(3), \Im S1(2)) , \]
\[ p_2^\mu = (\Re S2(2), \Re S2(3), \Im S2(3), \Im S2(2)) . \]

The input TC (18) is a complex 18-dimensional array which contains the wavefunction of the Tensor boson, and its four-momentum; see the TXXXXX subroutine in Sect. 2.1.1, GT is the coupling constant in (17) in units of GeV\(^{-1}\), and SMASS is the scalar boson mass \( m_S \) in GeV units. Note that all the coupling constants in the latest HELAS library are defined as a complex number except for the 3-point and 4-point vector boson couplings. The output VERTEX is a complex number in units of GeV:

\[ \text{VERTEX} = GT T^{\mu \nu} (m_S^2 \eta_{\mu \nu} - C_{\mu \nu, \rho \sigma} p_1^\rho p_2^\sigma) S1(1) S2(1) , \]

(18)

where we used the notation

\[ T^{\mu \nu} = T(\mu + 1, \nu + 1) = TC(4 \mu + \nu + 1) \]

(19)

in (8) and (9), and

\[ C_{\mu \nu, \rho \sigma} = \eta_{\mu \rho} \eta_{\nu \sigma} + \eta_{\mu \sigma} \eta_{\nu \rho} - \eta_{\mu \nu} \eta_{\rho \sigma} . \]

(20)

2.2.2 HSTXXX

This subroutine computes an off-shell scalar current \( H \) made from the interactions of a Scalar boson and a Tensor boson by the SST vertex, and should be called as

\[ \text{CALL HSTXXX(TC, SC, GT, SMASS, SWIDTH, HST).} \]

The inputs TC (18) and SC (3) are the wavefunctions and momenta of the Tensor and Scalar bosons, respectively, and SWIDTH is the scalar boson width \( \Gamma_S \). The output HST (3) gives the off-shell scalar current multiplied by the scalar boson propagator, which is expressed as a complex three-dimensional array:

\[ \text{HST}(1) = i G T \frac{i}{q^2 - m_S^2 + i \mu \Gamma_S} T^{\mu \nu} \]
\[ \times (m_S^2 \eta_{\mu \nu} + C_{\mu \nu, \rho \sigma} p^\rho q^\sigma) S1(1) , \]

(21)

and

\[ \text{HST}(2) = TC(17) + SC(2) , \]

(22)
\[ \text{HST}(3) = TC(18) + SC(3) . \]

(23)

Here the momenta \( p \) and \( q \) are

\[ p^\mu = (\Re S C(2), \Re S C(3), \Im m S C(3), \Im m S C(2)) , \]
\[ q^\mu = (\Re H S T(2), \Re H S T(3), \Im m H S T(3), \Im m H S T(2)) . \]

2.2.3 USXXXX

This subroutine computes an off-shell tensor current \( U \) made from two flowing-out Scalar bosons by the SST vertex, and should be called as

\[ \text{CALL USXXXX(S1, S2, GT, SMASS, TMASS, TWIDTH, USS).} \]

The inputs TMASS and TWIDTH are the tensor boson mass and width, \( m_T \) and \( \Gamma_T \). The output USS (18) gives the off-shell tensor current multiplied by the tensor boson propagator, which is expressed as a complex 18-dimensional array:

\[ T^{\alpha \beta} = i G T \frac{i B^{\mu \nu, \alpha \beta}}{q^2 - m_T^2 + i \mu \Gamma_T} \]
\[ \times (m_S^2 \eta_{\mu \nu} - C_{\mu \nu, \rho \sigma} p_1^\rho p_2^\sigma) S1(1) S2(1) , \]

(24)

where we used the notation

\[ T^{\alpha \beta} = T(\alpha + 1, \beta + 1) . \]

(25)
whose components are assigned into the first 16 component of USS as in (8), and

\[ \text{USS}(17) = S1(2) + S2(2), \]  
(26)

\[ \text{USS}(18) = S1(3) + S2(3). \]  
(27)

Here, \( p_1 \) and \( p_2 \) are the momenta of the outgoing scalars, and \( q \) is that of the off-shell tensor boson given in (26) and (27) as

\[ q^\mu = (\Re \text{USS}(17), \Re \text{USS}(18), \Im \text{USS}(18)). \]

Although the effective Lagrangian of (1) does not dictates the off-shell behavior of the gravitons, we allow gravitons to propagate just once in the total amplitude where there are no external gravitons in the initial or final states. This is convenient when studying the correlated decays of the graviton production and its subsequent decays.

We may also note that the order of the GT couplings should be restricted to 1 when there is an external graviton, and 2 when a graviton is exchanged among the SM particles.

Before turning to the FFT vertex, it should be noticed here that the conventional factors of \( i \) in the vertices and those in the propagators are both included in the off-shell wavefunctions, such as (21) and (24) above, according to the HELAS convention. The HELAS amplitude, obtained by the vertices, such as (18), gives the contribution to the \( T \) matrix element without the factor of \( i \). See more details in the HELAS manual [8].

### 2.3 FFT vertex

The FFT vertices are obtained from the interaction Lagrangian among the tensor boson and two fermions:

\[
\mathcal{L}_{\text{FFT}} = 4GT T^{\mu
u} \left[ -\eta_{\mu\nu} \left( \bar{f}(i\gamma^\rho \partial_\rho - m_F)f \right) 
- \frac{1}{2} \theta^\rho \left( \bar{f}i\gamma_\rho f \right) 
+ \left\{ \frac{1}{2} \bar{f}i\gamma_\mu \partial_\mu f - \frac{1}{4} \partial_\mu (\bar{f}i\gamma_\rho f) + (\mu \leftrightarrow \nu) \right\} \right],
\]

where the coupling constant is

\[ \text{GT} = \text{GTF} = -1/(4A) \]  
(29)

### 2.3.1 IOTXXX

This subroutine computes the amplitude of the FFT vertex from a flowing-In fermion spinor, a flowing-Out fermion spinor and a Tensor boson wavefunction, and should be called as

CALL IOTXXX(FI, FO, TC, GT, FMASS, VERTEX).

The inputs FI (6) and FO (6) are complex six-dimensional arrays which contain the wavefunctions of the flowing-In and flowing-Out Fermions, and their four-momenta as

\[ p_1^\mu = (\Re \text{FI}(5), \Re \text{FI}(6), \Im \text{FI}(6), \Im \text{FI}(5)), \]
\[ p_2^\mu = (\Re \text{FO}(5), \Re \text{FO}(6), \Im \text{FO}(6), \Im \text{FO}(5)). \]

The input GT is the coupling constant in (29), and FMASS is the fermion mass \( m_F \). What we compute here is

\[ \text{VERTEX} = \text{GT}(\text{FO}) \left[ T^{\mu\nu} \left[ -\eta_{\mu\nu}(p_1 + p_2 - 2m_F) 
+ \gamma_\mu(p_1 + p_2)_\nu + (\mu \leftrightarrow \nu) \right] \right](\text{FI}), \]

where we use the notation

\[ (\text{FI}) = \begin{pmatrix} \text{FI}(1) \\ \text{FI}(2) \\ \text{FI}(3) \\ \text{FI}(4) \end{pmatrix}, \]

\[ (\text{FO}) = \begin{pmatrix} \text{FO}(1) \\ \text{FO}(2) \\ \text{FO}(3) \\ \text{FO}(4) \end{pmatrix}. \]

### 2.3.2 FTIXXX

This subroutine computes an off-shell Fermion wavefunction made from the interactions of a Tensor boson and a flowing-In fermion by the FFT vertex, and should be called as

CALL FTIXXX(FI, TC, GT, FMASS, FWIDTH, FTI).

The output FTI (6) gives the off-shell fermion wavefunction multiplied by the fermion propagator, which is expressed as a complex six-dimensional array:

\[ (\text{FTI}) = i\text{GT} \frac{i(q + m_F)}{q^2 - m_F^2 + im_F\Gamma_F} T^{\mu\nu} \left[ -\eta_{\mu\nu}(\beta + q - 2m_F) 
+ \gamma_\mu(p + q)_\nu + (\mu \leftrightarrow \nu) \right](\text{FI}), \]

where we use the notation

\[ (\text{FTI}) = \begin{pmatrix} \text{FTI}(1) \\ \text{FTI}(2) \\ \text{FTI}(3) \\ \text{FTI}(4) \end{pmatrix}, \]

and

\[ \text{FTI}(5) = \text{FI}(5) - \text{TC}(17), \]
\[ \text{FTI}(6) = \text{FI}(6) - \text{TC}(18). \]
Here the momenta \( p \) and \( q \) are
\[
p^\mu = (\Re \{\Pi\}, \Re \{\Sigma\}, \Im \{\Pi\}, \Im \{\Sigma\}),
\]
\[
q^\mu = (\Re \{\alpha\}, \Re \{\beta\}, \Im \{\alpha\}, \Im \{\beta\}).
\]

### 2.3.3 FTOXXX

This subroutine computes an off-shell Fermion wavefunction made from the interactions of a Tensor boson and a flowing-Out fermion by the FFT vertex, and should be called as

\[
\text{CALL FTOXXX(FO, TC, GT, FMASS, FWIDTH, FTO).}
\]

The output FTO(6) is a complex six-dimensional array:

\[
\text{FTO} = i \text{GT}(\text{FO}) T^{\mu \nu} \times \left[-\eta_{\mu \nu}(p + q - 2m_F) + \gamma_{\mu}(p + q) + (\mu \leftrightarrow \nu)\right] \times \frac{i(q + m_F)}{q^2 - m_F^2 + im_F \Gamma_F},
\]

where we use the notation

\[
\text{FTO} = (\text{FTO}(1), \text{FTO}(2), \text{FTO}(3), \text{FTO}(4)),
\]

and

\[
\text{FTO}(5) = \text{FO}(5) + \text{TC}(17),
\]

\[
\text{FTO}(6) = \text{FO}(6) + \text{TC}(18).
\]

Here the momenta \( p \) and \( q \) are
\[
p^\mu = (\Re \{\Pi\}, \Re \{\Sigma\}, \Im \{\Pi\}, \Im \{\Sigma\}),
\]
\[
q^\mu = (\Re \{\alpha\}, \Re \{\beta\}, \Im \{\alpha\}, \Im \{\beta\}).
\]

### 2.3.4 UIOXXX

This subroutine computes the bi-spinor tensor current \( U \) made from flowing-In and flowing-Out fermions by the FFT vertex, and should be called as

\[
\text{CALL UIOXXX(FI, FO, GT, FMASS, TMASS, FWIDTH, UIO).}
\]

The output UIO(18) is a complex 18-dimensional array:

\[
T^{\alpha \beta} = i \text{GT} \frac{i B^{\mu \nu, \alpha \beta}}{q^2 - m_F^2 + im_F \Gamma_F} \times (\text{FO}) \left[-\eta_{\mu \nu}(p_1 + p_2 - 2m_F) + \gamma_{\mu}(p_1 + p_2) + (\mu \leftrightarrow \nu)\right](\text{FI})
\]

for the first 16 components of UIO, and

\[
\text{UIO}(17) = -FI(5) + FO(5),
\]

\[
\text{UIO}(18) = -FI(6) + FO(6).
\]

Here, \( p_1 \) and \( p_2 \) are the momenta of the flowing-in and flowing-out fermions, respectively, and \( q \) is that of the tensor particle.

### 2.4 VVT vertex

The VVT vertices are obtained from the interaction Lagrangian among the tensor and two vector bosons:

\[
\mathcal{L}_{VVT} = GT T^{\mu \nu} \eta_{\mu \nu} \left\{ \frac{1}{2}(\partial^\rho V^{\sigma \rho} - \partial^\rho V^{\rho \sigma})^*(\partial_\rho V^\sigma) - \partial_\sigma V^\rho \right\} - m_V^2 V^\rho V^\rho - \xi^{-1} \eta_{\mu \nu} \left( \partial^\rho \partial_\sigma V^\rho V^\sigma + \frac{1}{2} \partial^\rho \partial_\rho V^\sigma V^\sigma \right)
\]

for the complex vector bosons, or \( W \) bosons, and

\[
\mathcal{L}_{VVT} = GT T^{\mu \nu} \eta_{\mu \nu} \left\{ \frac{1}{4}(\partial^\rho V^{\sigma \rho} - \partial^\rho V^{\rho \sigma})^* (\partial_\sigma V^\rho - \partial_\rho V^\sigma) - m_V^2 V^\rho V^\rho \right\} - (\partial_\mu V^\nu - \partial_\nu V^\mu) (\partial_\nu V^\rho - \partial_\rho V^\nu) + m_V^2 V^\mu V^\nu
\]

for the real ones, or gluons, photons, and \( Z \) bosons. Here the coupling constant is

\[
\text{GT} = GTV = -1/\Lambda.
\]

The \( \xi \) terms are the gauge-fixing terms, which vanish for massive vector bosons in the unitary gauge. For massless vector bosons we take \( \xi = 1 \) in the Feynman gauge.

### 2.4.1 VVTXXX

This subroutine computes the amplitude of the VVT vertex from two Vector boson polarization vectors and a Tensor wavefunction, and should be called as

\[
\text{CALL VVTXXX(V1, V2, TC, GT, FMASS, VERTEX).}
\]
The inputs $V_1(6)$ and $V_2(6)$ are complex six-dimensional arrays which contain the vector boson wavefunctions, and their momenta as

$$p_1^\mu = (\Re V_1(5), \Re V_1(6), \Im V_1(6), \Im V_1(5)), \quad p_2^\mu = (\Re V_2(5), \Re V_2(6), \Im V_2(6), \Im V_2(5)).$$

The input $GT$ is the coupling constant in (45), and $\text{VMASS}$ is the vector boson mass $m_V$. What we compute here is

$$\text{VERTEX} = GT T^{\mu \nu} \left[ (m_V^2 + p_1 \cdot p_2) C_{\mu \nu, \rho \sigma} + D_{\mu \nu, \rho \sigma} (p_1, p_2) + \xi^{-1} E_{\mu \nu, \rho \sigma} (p_1, p_2) \right] V_1^\rho V_2^\sigma,$$

where we use the notation

$$V_1^\mu = V_1(\mu + 1), \quad V_2^\mu = V_2(\mu + 1),$$

and

$$D_{\mu \nu, \rho \sigma} (p_1, p_2) = \eta_{\mu \nu} p_{1 \rho} p_{2 \sigma} - \left[ \eta_{\mu \sigma} p_{1 \nu} p_{2 \rho} + \eta_{\mu \rho} p_{1 \nu} p_{2 \sigma} - \eta_{\rho \sigma} p_{1 \nu} p_{2 \mu} - \eta_{\nu \sigma} p_{1 \rho} p_{2 \mu} + (\mu \leftrightarrow \nu) \right],$$

$$E_{\mu \nu, \rho \sigma} (p_1, p_2) = \eta_{\mu \nu} (p_{1 \rho} p_{1 \sigma} + p_{2 \rho} p_{2 \sigma} + p_{1 \rho} p_{2 \sigma}) - \left[ \eta_{\mu \sigma} p_{1 \rho} p_{1 \nu} + \eta_{\nu \sigma} p_{1 \rho} p_{2 \mu} + \eta_{\rho \sigma} p_{1 \nu} p_{2 \mu} + (\mu \leftrightarrow \nu) \right].$$

2.4.2 JVTXXX

This subroutine computes an off-shell tensor current $J$ made from the interactions of a vector boson and a tensor boson by the $\text{VVT}$ vertex, and should be called as

$$\text{CALL JVTXXX}(VC, TC, GT, \text{VMASS}, \text{TWIDTH}, JVT).$$

The input $VC(6)$ is the wavefunction and momentum of the vector boson. The output $JVT(6)$ gives the off-shell vector current multiplied by the vector boson propagator, which is expressed as a complex six-dimensional array:

$$JVT(\alpha + 1) = i GT \frac{i}{q^2 - m_V^2 + i m_V \Gamma_V} \left( -\eta^{\alpha \sigma} + \frac{q^\sigma q^\alpha}{m_V^2} \right) \times T^{\mu \nu} \left[ (m_V^2 - p \cdot q) C_{\mu \nu, \rho \sigma} + D_{\mu \nu, \rho \sigma} (p, -q) \right] V_1^\rho$$

for the massive vector boson, or

$$JVT(\alpha + 1) = i GT \frac{-i}{q^2} \eta^{\alpha \sigma} T^{\mu \nu} \left[ -(p \cdot q) C_{\mu \nu, \rho \sigma} + D_{\mu \nu, \rho \sigma} (p, -q) \right] V_1^\rho,$$

for the massless vector boson, and

$$JVT(5) = VC(5) + TC(17), \quad JVT(6) = VC(6) + TC(18).$$

Here the momenta $p$ and $q$ are

$$p^\mu = (\Re VC(5), \Re VC(6), \Im VC(6), \Im VC(5)), \quad q^\mu = (\Re JVT(5), \Re JVT(6), \Im JVT(6), \Im JVT(5)).$$

2.4.3 UVVXXX

This subroutine computes an off-shell tensor current $U$ made from two flowing-out vector bosons by the $\text{VVV}$ vertex, and should be called as

$$\text{CALL UVVXXX}(V_1, V_2, GT, \text{VMASS}, \text{TMASS}, \text{TWIDTH}, UVV).$$

The output $UVV(18)$ is a complex 18-dimensional array:

$$T^{\alpha \beta} = i GT \frac{i B^{\mu \nu, \alpha \beta}}{q^2 - m_V^2 + i m_V \Gamma_T} \times \left[ (m_V^2 + p_1 \cdot p_2) C_{\mu \nu, \rho \sigma} + D_{\mu \nu, \rho \sigma} (p_1, p_2) + \xi^{-1} E_{\mu \nu, \rho \sigma} (p_1, p_2) \right] V_1^\rho V_2^\sigma$$

for the first 16 components of $UVV$, and

$$UVV(17) = V_1(5) + V_2(5), \quad UVV(18) = V_1(6) + V_2(6).$$

Here, $p_1$ and $p_2$ are the momenta of the outgoing vector bosons, and $q$ is that of the tensor boson.

2.5 FFVT vertex

The $\text{FFVT}$ vertices are obtained from the interaction Lagrangian among the tensor boson, vector boson and two fermions:

$$\mathcal{L}_{\text{FFVT}} = -2 GT T^{\mu \nu \rho \sigma} (\eta_{\mu \nu} \eta_{\rho \sigma} - \eta_{\mu \rho} \eta_{\nu \sigma}) \times f \gamma^\rho \left[ GC(1) P_L + GC(2) P_R \right] f V^{\mu \sigma},$$

with the chiral-projection operator $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$. The coupling constant $GT$ is

$$GT = \text{GTFV} = -1/(2 \Lambda).$$
List of the coupling constants for each vertex. All the particles and the coupling constants are written in the $\hat{\mathbb{M}}$ notation. $y$ stands for a massive graviton, $f$ represents all possible fermions, and $v$ is the SM gauge bosons ($g, a, z, w$). GC is a SM coupling constant, while GT is a non-renormalizable coupling constant defined in each subroutine in Sect. 2.

| 3-point couplings | GT       |
|-------------------|----------|
| SST               | h        |
| FFT               | f        |
| VVT               | v        |

| 4-point couplings | GT   | GC   |
|-------------------|------|------|
| FFVT              | d    | a    | d    | y    | GTFV | GAD |
| v u l z           | v u l z | y    | GTFV | GAL |
| v u l z           | v u l z | y    | GTFV | GZU |
| v u l z           | v u l z | y    | GTFV | GZL |
| v v l z           | v v l z | y    | GTFV | GZN |
| FVT               | d    | u    | d    | w   | y    | GTFV | GWF |
| v l v l z         | v l v l z | y    | GTFV | GWF |
| v l v l z         | v l v l z | y    | GTFV | GWF |
| q q g g           | q q g g | y    | GTFV | GG  |

| 5-point couplings | GT   | GC   |
|-------------------|------|------|
| VVVVT             | g    | g    | g    | g    | y    | GTV  |

and GC(1) and GC(2) are the relevant FFV left and right coupling constants. The list of the coupling constants is shown in Table 2. For instance, in the case of the interaction with a gluon, the FFV couplings are

\[ \text{GC}(1) = \text{GC}(2) = -g_s, \]  

where the sign of the coupling is fixed by the HELAS convention [8].

### 2.5.1 IOVTXX

This subroutine computes the amplitude of the FFVT vertex from a flowing-In fermion spinor, a flowing-Out fermion spinor, a Vector boson polarization vector, and a Tensor boson wavefunction, and should be called as

CALL IOVTXX(FI, FO, VC, TC, GC, GT, VERTERX).

What we compute here is

\[ \text{VERTEX} = -\text{GT}(\text{FO})\left[T^{\mu\nu}(\eta_{\mu\nu}\eta_{\rho\sigma} - \epsilon_{\mu\nu,\rho\sigma})\times V^\rho V^\sigma(GC(1)P_L + GC(2)P_R)\right](\text{FI}). \]  

### 2.5.2 FVTIXX

This subroutine computes an off-shell fermion wavefunction by the FFVT vertex, and should be called as

CALL FVTIXX(FI, VC, TC, GC, GT, FMASS, FWIDTH, FVTI).

What we compute here is

\[
(FVTI) = -i\text{GT} \frac{i(q + m_F)}{q^2 - m_F^2 + im_F\Gamma_F} \times T^{\mu\nu}(\eta_{\mu\nu}\eta_{\rho\sigma} - \epsilon_{\mu\nu,\rho\sigma}) \times V^\rho V^\sigma[GC(1)P_L + GC(2)P_R](\text{FI})
\]  

for the first 4 components of FVTI (5), and

\[
\text{FVTI}(5) = \text{FI}(5) - \text{VC}(5) - \text{TC}(17),
\]

\[
\text{FVTI}(6) = \text{FI}(6) - \text{VC}(6) - \text{TC}(18).
\]

Here the momentum \( q \) is

\[
q^\mu = (\Re FVTI(5), \Re FVTI(6), \Im FVTI(5), \Im FVTI(6)).
\]

### 2.5.3 FVT0XX

This subroutine computes an off-shell fermion wavefunction by the FFVT vertex, and should be called as

CALL FVT0XX(FO, VC, TC, GC, GT, FMASS, FWIDTH, FVTO).

What we compute here is

\[
(FVTO) = -i\text{GT}(\text{FO})T^{\mu\nu}(\eta_{\mu\nu}\eta_{\rho\sigma} - \epsilon_{\mu\nu,\rho\sigma}) \times V^\rho V^\sigma[GC(1)P_L + GC(2)P_R] \times \frac{i(q + m_F)}{q^2 - m_F^2 + im_F\Gamma_F}
\]  

for the first 4 components of FVTO (5), and

\[
\text{FVTO}(5) = \text{FO}(5) + \text{VC}(5) + \text{TC}(17),
\]

\[
\text{FVTO}(6) = \text{FO}(6) + \text{VC}(6) + \text{TC}(18).
\]

Here the momentum \( q \) is

\[
q^\mu = (\Re FVTO(5), \Re FVTO(6), \Im FVTO(5), \Im FVTO(6)).
\]
2.5.4 JIOTXX

This subroutine computes an off-shell vector current by the FFVT vertex, and should be called as

CALL JIOTXX(FI, FO, TC, GC, GT, VMASS, VWIDTH, JIOT).

What we compute here is

\[
JIOT(\alpha + 1) = -iGT \frac{i}{q^2 - m_V^2 + i m_V \Gamma_V} \\
\times \left( -\eta^\rho_\alpha + \frac{q^\rho q^\alpha}{m_V} \right) \\
\times T^{\mu\nu}(\eta_{\mu\nu}\eta_{\rho\sigma} - C_{\mu\nu,\rho\sigma}) \\
\times (FO)[\gamma^\sigma (GC(1)P_L + GC(2)P_R)](FI)
\]  

(68)

for the massive vector boson, or

\[
JIOT(\alpha + 1) = -iGT \frac{-i}{q^2} \eta^\rho_\alpha T^{\mu\nu}(\eta_{\mu\nu}\eta_{\rho\sigma} - C_{\mu\nu,\rho\sigma}) \\
\times (FO)[\gamma^\sigma (GC(1)P_L + GC(2)P_R)](FI)
\]  

(69)

for the massless vector boson, and

\[
JIOT(5) = -FI(5) + FO(5) + TC(17),
\]  

(70)

\[
JIOT(6) = -FI(6) + FO(6) + TC(18).
\]  

(71)

Here \( q \) is the momentum of the vector boson.

2.5.5 UIOVXX

This subroutine computes an off-shell tensor current by the FFVT vertex, and should be called as

CALL UIOVXX(FI, FO, VC, GC, GT, TMASS, TWIDTH, Uiov).

What we compute here is

\[
T^{\alpha\beta} = -iGT \frac{i B^{\mu\nu,\alpha\beta}}{q^2 - m_V^2 + i m_V \Gamma_V} (\eta_{\mu\nu}\eta_{\rho\sigma} - C_{\mu\nu,\rho\sigma}) \\
\times (FO)[\gamma^\sigma (GC(1)P_L + GC(2)P_R)](FI)
\]  

(72)

for the first 16 components of UIOV(18), and

\[
UIOV(17) = -FI(5) + FO(5) + VC(5),
\]  

(73)

\[
UIOV(18) = -FI(6) + FO(6) + VC(6).
\]  

(74)

Here \( q \) is the momentum of the tensor boson.

2.6 VVVT vertex

The VVVT vertices are obtained from the interaction Lagrangian among the tensor and three vector bosons:

\[
L_{\text{VVVT}} = \frac{1}{2} \eta_{\mu\nu} (\partial_\rho V_\sigma^\alpha - \partial_\sigma V_\rho^\alpha) V^{\beta,\rho} V_\sigma^\beta \\
- (\partial_\rho V_\sigma^\alpha - \partial_\sigma V_\rho^\alpha) V^{\beta,\rho} V_\sigma^\beta \\
- (\partial_\rho V_\sigma^\alpha - \partial_\sigma V_\rho^\alpha) V^{\beta,\rho} V_\sigma^\beta
\]  

(75)

with the structure constant \( f^{abc} \) and the coupling constant, as in (45),

\[
GT = GTV = 1/\Lambda.
\]  

(76)

In this paper we concentrate on the interactions with gluons for the VVVT vertex, so in this case GC is the strong coupling constant,

\[
GC = g_s,
\]  

(77)

and \( f^{abc} \) is the structure constants of the group SU(3), which can be handled by the MG automatically. As in the original subroutines for the VVV vertex [8], the following subroutines, VVVTXX, JVVTTXX, and UVVXXX, can be used for the electroweak gauge bosons without any modifications.

2.6.1 VVVTTXX

This subroutine computes the amplitude of the VVVTT vertex from three Vector boson polarization vectors and a Tensor boson wavefunction, and should be called as

CALL VVVTTXXX(VA, VB, VC, TC, GC, GT, VERTEX).

What we compute here is

\[
\text{VERTEX} = -GTGC T^{\mu\nu} \left[ C_{\mu\nu,\rho\sigma} (p_a \lambda - p_b \lambda) \right. \\
+ C_{\mu\nu,\rho\sigma} (p_b \rho - p_c \rho) + C_{\mu\nu,\lambda\rho} (p_{\sigma} - p_{\alpha}) \\
+ F_{\mu\nu,\rho\sigma \lambda} (p_a, p_b, p_c) \biggr] V_\rho^\sigma V_\lambda^\alpha
\]  

(78)

with

\[
F_{\mu\nu,\rho\sigma \lambda} (p_a, p_b, p_c) = \eta_{\mu\rho} \eta_{\sigma\lambda} (p_b - p_c)_\nu + \eta_{\mu\rho} \eta_{\rho\lambda} (p_c - p_a)_\nu \\
+ \eta_{\mu\lambda} \eta_{\rho\sigma} (p_a - p_b)_\nu + (\mu \leftrightarrow \nu).
\]  

(79)

Here, the vector bosons (gluons in this paper) VA, VB, and VC have the momentum \( p_a, p_b, \) and \( p_c, \) and the color \( a, b, \) and \( c, \) respectively.
2.6.2 \textit{JVVTXX}

This subroutine computes an off-shell vector current by the VVVVT vertex, and should be called as

\texttt{CALL JVVTXX(VA, VB, TC, GC, GT, VMASS, WIDTH, JVVVT}).

What we compute here is

\begin{equation}
\text{JVVT}(\alpha + 1) = -iGT GC \frac{i}{q^2 - m_V^2 + i\Gamma_V} \left( -\eta^{\lambda\alpha} + \frac{q^\lambda q^\alpha}{m_V^2} \right) \\
\times T^{\mu\nu} \left[ C_{\mu\nu,\rho\sigma}(p_{\alpha\lambda} - p_{b\delta}) + C_{\mu\nu,\sigma\lambda}(p_{b\rho} + q_{\rho\mu}) \\
+ C_{\mu\nu,\lambda\rho}(-q_{\sigma} - p_{a\sigma}) + F_{\mu\nu,\rho\sigma\lambda}(p_{a\sigma}, p_{b}, -q) \right] V^\rho_a V^\sigma_b,
\end{equation}

(80)

for the massive vector boson, or

\begin{equation}
\text{JVVT}(\alpha + 1) = -iGT GC \frac{-i}{q^2} \eta^{\lambda\alpha} T^{\mu\nu} \left[ C_{\mu\nu,\rho\sigma}(p_{\alpha\lambda} - p_{b\delta}) \\
+ C_{\mu\nu,\sigma\lambda}(p_{b\rho} + q_{\rho\mu}) + C_{\mu\nu,\lambda\rho}(-q_{\sigma} - p_{a\sigma}) \\
+ F_{\mu\nu,\rho\sigma\lambda}(p_{a\sigma}, p_{b}, -q) \right] V^\rho_a V^\sigma_b,
\end{equation}

(81)

for the massless vector boson, and

\begin{equation}
\text{JVVT}(5) = VA(5) + VB(5) + TC(17),
\end{equation}

(82)

\begin{equation}
\text{JVVT}(6) = VA(6) + VB(6) + TC(18).
\end{equation}

(83)

Here the momenta \( p_a, p_b \) and \( q \) are

\begin{align*}
p^\mu_a &= (\Re e VA(5), \Re e VA(6), \Im m VA(6), \Im m VA(5)), \\
p^\mu_b &= (\Re e VB(5), \Re e VB(6), \Im m VB(6), \Im m VB(5)), \\
q^\mu &= (\Re e JVVT(5), \Re e JVVT(6), \Im m JVVT(6), \Im m JVVT(5)).
\end{align*}

Note that the off-shell gluon JVVT has the color \( c \).

2.6.3 \textit{UVVVXX}

This subroutine computes an off-shell tensor current by the VVVVT vertex, and should be called as

\texttt{CALL UVVVXX(VA, VB, VC, GC, GT, TMASS, WIDTH, UVVV}).

What we compute here is

\begin{equation}
T^{a\beta} = -iGT GC \frac{i B^{\mu\nu, a\beta}}{q^2 - m_V^2 + i\Gamma_V} \\
\times \left[ C_{\mu\nu,\rho\sigma}(p_{\alpha\lambda} - p_{b\delta}) \\
+ C_{\mu\nu,\sigma\lambda}(p_{b\rho} - p_{c\sigma}) + C_{\mu\nu,\lambda\rho}(p_{c\sigma} - p_{\alpha\sigma}) \\
+ F_{\mu\nu,\rho\sigma\lambda}(p_{a\sigma}, p_{b}, p_{c}) \right] V^\rho_a V^\sigma_b V^\lambda_c V^\delta_d
\end{equation}

(84)

for the first 16 components of UVVV(18), and

\begin{align*}
\text{UVVV(17)} &= VA(5) + VB(5) + VC(5), & (85) \\
\text{UVVV}(18) &= VA(6) + VB(6) + VC(6). & (86)
\end{align*}

Here \( p_a, p_b \) and \( p_c \) are the momenta of the outgoing vector bosons, and \( q \) is that of the tensor boson.

2.7 \textit{VVVVVT vertex}

The \( \text{VVVVVT} \) vertices are obtained from the interaction Lagrangian among the tensor and four vector bosons:

\begin{equation}
\mathcal{L}_{\text{VVVVVT}} = -GT GC^2 f_{abc} f_{ced} T^{\mu\nu\rho\sigma} \\
\times \left[ \frac{1}{4} \eta_{\mu\nu} V^a_{\rho\sigma} V^b_{\sigma\lambda} V^c_{\lambda\rho} V^d_{\rho\sigma} \\
- V^b_{\rho\sigma} V^c_{\lambda\rho} V^d_{\rho\sigma} \right]
\end{equation}

(87)

with the coupling constants \( GT = GTV \) in (45) and \( GC = g_s \) for the interactions with gluons.

We should note that the 5-point vertex cannot be generated by \( \mathcal{G} \), and thus we must add the following subroutines by hand, \texttt{GGGGTX}, \texttt{JGGGTX}, or \texttt{UGGGGX}, to the amplitudes which have the corresponding color structures.

2.7.1 \textit{GGGGTX}

This subroutine computes the portion of the amplitude of the \( \text{VVVVVT} \) vertex from four gluon polarization vectors and a Tensor boson wavefunction corresponding to the color structure \( f_{abc} f_{ced} \), and should be called as

\texttt{CALL GGGGTX(VA, VB, VC, GD, GC, GT, VERTEX)}.

The output is

\begin{equation}
\text{VERTEX} = -GT GC^2 T^{\mu\nu} G_{\mu\nu,\rho\lambda\delta} V^\rho_a V^\sigma_b V^\lambda_c V^\delta_d
\end{equation}

(88)

with

\begin{equation}
G_{\mu\nu,\rho\lambda\delta} = \eta_{\mu\nu} (\eta_{\rho\sigma} \eta_{\lambda\delta} - \eta_{\rho\delta} \eta_{\lambda\sigma}) \\
+ \left[ \eta_{\mu\rho} \eta_{\nu\delta} \eta_{\lambda\sigma} + \eta_{\mu\lambda} \eta_{\nu\sigma} \eta_{\rho\delta} - \eta_{\mu\rho} \eta_{\nu\sigma} \eta_{\lambda\delta} \\
- \eta_{\mu\lambda} \eta_{\nu\delta} \eta_{\rho\sigma} + (\mu \leftrightarrow \nu) \right].
\end{equation}

(89)
To obtain the complete amplitude, this subroutine must be called three times (once for each color structure) with the following permutations:

\begin{align*}
\text{CALL GGGGT}(\text{VA, VB, VC, TD, TC, GC, GT}, \text{VRTEX}1), \\
\text{CALL GGGGT}(\text{VA, VC, TD, VB, TC, GC, GT}, \text{VRTEX}2), \\
\text{CALL GGGGT}(\text{VA, TD, VB, VC, TC, GC, GT}, \text{VRTEX}3),
\end{align*}

corresponding to the color structure \( f^{abe} f^{cde}, f^{ace} f^{bde} \), and \( f^{ade} f^{bce} \), respectively.

### 2.7.2 JGGGT

This subroutine computes the portion of the off-shell gluon current by the \( VVVV \) vertex, corresponding to the color structure \( f^{abe} f^{cde} \), and should be called as

\begin{align*}
\text{CALL JGGGT}(\text{VA, VB, VC, TD, TC, GC, GT}, \text{JGGGT}).
\end{align*}

What we compute here is

\[ JGGGT(\alpha + 1) = -i GT \frac{G C^2}{q^2} \frac{1}{\eta_{a b}} \frac{1}{\sigma_{a b}} G^{\mu \nu, \rho \delta} V_{d a} V_{b c} V_{c d} V_{d e}. \]  

and

\[ JGGGT(5) = \text{VA}(5) + \text{VB}(5) + \text{VC}(5) + \text{TC}(17), \]  

\[ JGGGT(6) = \text{VA}(6) + \text{VB}(6) + \text{VC}(6) + \text{TC}(18). \]

Note that the off-shell gluon \( JGGGT \) has the color \( d \) and the momentum \( q \).

### 2.7.3 UGGGGX

This subroutine computes the portion of the off-shell tensor current by the \( VVVV \) vertex, corresponding to the color structure \( f^{abe} f^{cde} \), and should be called as

\begin{align*}
\text{CALL UGGGGX}(\text{VA, VB, VC, TD, TC, GC, GT}, \text{UGGGGX}).
\end{align*}

What we compute here is

\[ T^{\alpha \beta} = -i GT \frac{G C^2}{q^2} \frac{1}{T_T} \frac{1}{m_T} \frac{1}{\eta_{a b}} \frac{1}{\sigma_{a b}} \frac{1}{\rho_{a b}} \frac{1}{\delta_{a b}} G_{\mu \nu, \rho \delta} V_{d a} V_{b c} V_{c d} V_{d e}. \]  

for the first 16 components of \( \text{UGGGG} (18) \), and

\[ \text{UGGGG}(17) = \text{VA}(5) + \text{VB}(5) + \text{VC}(5) + \text{TD}(5), \]  

\[ \text{UGGGG}(18) = \text{VA}(6) + \text{VB}(6) + \text{VC}(6) + \text{TD}(6). \]

Here \( q \) is the momentum of the tensor boson.

### 2.8 Checking for the new HELAS subroutines

The new HELAS subroutines are tested by using the QCD gauge invariance and the general coordinate transformation invariance of the helicity amplitudes. In particular, we use the following processes:

\begin{align*}
q \bar{q} & \rightarrow g T & \text{for IOTXXX, FTIXXX, FTOXXX, IOVXX,} \\
gg & \rightarrow g T & \text{for VVTXXX, JVTXXX, VVVXXX,} \\
q \bar{q} & \rightarrow q \bar{q}(gg)T & \text{for FVTXXX, FTOXX, JIOTXX,} \\
gg & \rightarrow gg T & \text{for JVVXX, GGGGTX,} \\
q \bar{q}(gg) & \rightarrow T \rightarrow gg & \text{for UIOXXX, UVVXX,}
\end{align*}

More explicitly, we express the helicity amplitudes of the above processes as

\[ M_{\lambda T \lambda g} = T_{\mu \nu \rho \sigma} (p_T, \lambda_T)^* \epsilon^\rho(p_g, \lambda_g)^* \]  

with an external tensor and a gluon wavefunction. The identities,

\[ p_T / p_T^0 T_{\mu \nu \rho \sigma} (p_g, \lambda_g)^* = p_T / p_T^0 T_{\mu \nu \rho \sigma} (p_g, \lambda_g)^* = 0 \]

for the general coordinate transformation symmetry and

\[ p_T / p_T^0 T_{\mu \nu \rho \sigma} = 0 \]

for the SU(3) gauge invariance, test all the above subroutines thoroughly.

The subroutines \( \text{UIOXXX, UVVXX, JGGGTX and UGGGGX} \) have been checked in such a way that the above subroutines are used rewriting the helicity amplitudes of the processes

\begin{align*}
q \bar{q} & \rightarrow T \rightarrow q \bar{q}g(gg), \\
gg & \rightarrow gg T \rightarrow gg ZZ, \\
gg & \rightarrow gggg T.
\end{align*}

We test the agreement of the amplitudes between the expressions that use the above subroutines and those without them for all helicity combinations and at arbitrary Lorentz frame.

### 3 MadGraph/MadEvent implementing for spin-2 gravitons

In this section, we would like to describe how we implement spin-2 gravitons into \( \text{MG/ME} \). First, using the User Model framework in \( \text{MG} [11] \), we make our new model directories for both the \( \text{ADD and RS models, including the massive gravitons (particles.dat) and their interactions with the SM particles} \),

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T dependence of the total cross sections for the graviton productions with 1-jet (via the $gg$, $qg$ and $q\bar{q}$ channels) and 2-jets (via the $gg$, $qg$ and $qQ$ channels) in the ADD model at the LHC, where $q,Q=u,d,s,c$ (couplings.f and interactions.dat); see also Table 2. Then we insert all the new HELAS subroutines for spin-2 tensor bosons into the HELAS library in MG. Since the present MG does not handle external spin-2 particles, we further modify the codes in MG to tell it how to generate the SST, FFT and FFVT type of vertices and helicity amplitudes (for VVT and VVVVT type, it has already been done for the Higgs effective field theory (HEFT) model), and how to deal with the helicity of the spin-2 tensor bosons when they are external. Moreover, since MG can only generate Feynman diagrams with up to 4-point vertices, the amplitudes and their HELAS codes with the 5-point vertex, GGGGTX, JGGGTX, or UGGGGX, have been added by hand; see more details in Sect. 2.7.

Finally, we note that since in the ADD model the gravitons are densely populated and we should sum over their contributions by modifying the phase space integration in ME. In the ADD model, the spectrum of KK graviton modes can be treated as continuous for $\delta \leq 6$ [1–3], and the mass density function is given by [6]

$$\rho(m) = S_{6-1} \frac{M_{Pl}^2}{M_{Pl}^{6-2}} m^{5-1} \text{ with } S_{6-1} = \frac{2\pi^{5/2}}{\Gamma(\delta/2)},$$

where $M_{Pl}$ is the ADD model effective scale. Thus, we modify the phase space generating codes in ME to add one more random number for graviton mass generating and implement the above graviton mass integration.

4 Sample results

In this section, we present some sample numerical results for the graviton plus mono-jet and di-jet productions at the LHC, using the new HELAS subroutines and the modified MG/ME.

We use the following jet definition criteria

$$P_T > 20 \text{ GeV, } |\eta_j| < 5,$$

$$R_{jj} = \sqrt{\Delta \eta^2 + \Delta \phi^2} > 0.6,$$

where $\eta$ is the pseudorapidity of the jets and $\phi$ is the azimuthal angle around the beam direction, and further require

$$\eta_{j1} \cdot \eta_{j2} < 0, \quad |\eta_{j1} - \eta_{j2}| > 4.2.$$

The CTEQ6L1 parton distribution functions [14] are employed with the factorization scale chosen as $\mu_f = \min(P_T)$ of the jets which satisfy the above cuts. For the QCD coupling, we set it as the geometric mean value, $\alpha_s = \sqrt{\alpha_s(P_T^{j1}) \cdot \alpha_s(P_T^{j2})}$.

Figure 1 shows the $P_{T}\text{miss}$ distributions for the graviton productions with the mono-jet (via the $gg$, $qg$ and $q\bar{q}$ channels) and with the di-jet (via the $gg$, $qg$ and $qQ$ channels) in the ADD model at the LHC, with $\Lambda = 5 \text{ TeV}$ and $\delta = 4$. Note that the unitarity criterion $M_{Tn} < \Lambda$ is used. See more details in Ref. [13].

Figure 2 shows the $\phi_{jj}$ distributions for the first KK graviton plus di-jet productions in the RS model at the LHC, via the $gg$, $qg$ and $qQ$ channels ($q,Q = u,d,s,c$).
5 Summary

In this paper, we have added new HELAS subroutines to calculate helicity amplitudes with massive spin-2 particles (massive gravitons) to the HELAS library. They are coded in such a way that arbitrary scattering amplitudes with one graviton production and its decays can be generated automatically by MG and ME, after slight modifications. All the codes have been tested carefully by making use of the invariance of the helicity amplitudes under the gauge and general coordinate transformations.

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