The optimal extended warranty length of durable-goods-based preventive maintenance behaviour

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The extended warranty is an important strategy for the manufacturers to expand their market share. In this work, we studied the optimal extended warranty length considering preventive maintenance behaviour, which mainly avoids the producers and consumers' moral hazard. Based on the profit function maximization, the optimal extended warranty length under different situations can be achieved. We conducted a simulation analysis to see the influence on preventive maintenance behaviour during the warranty period to the optimal extended warranty length.

Keywords: base warranty; extended warrant; preventive maintenance behaviour

1. Introduction

Product performances during the warranty period are determined by product characteristics and its use. From the manufacturer point of view, the consumers' abuse may lead to an increase in the total numbers of failures, which further generate more maintenance cost during the warranty period. If the maintenance cost is greater than the income derived, the manufacturer would not provide the extended warranty. Since the income derived and the extended warranty is fixed, the manufacturer should control the absolute numbers of failures to manage the maintenance cost. Preventive maintenance behaviour is acted, on the one hand, to guide the consumers to use the product correctly, and, on the other hand, to detect the quality of the products to reduce the total numbers of failures.

The relation between warranty and preventive maintenance behaviour is closeness. These include (i) the preventive maintenance behaviour models classes and policy (Jain & Maheshwari, 2006; Rai & Allada, 2006; Wu & Zuo, 2010), (ii) Preventive maintenance behaviour during the warranty period has an important impact on the warranty servicing cost (Chien, 2008; Kleyner & Sandborn, 2008; Maronick, 2007), (iii) the relation between preventive maintenance behaviour and age replacement (Chien, 2010; Jack & Iskandar, 2009; Jung & Park, 2010) and (iv) the optimal warranty policy (Wu & Li, 2007; Wu & Xie, 2008; Wua & Choub, 2009). Wu and Longhurst (2011) studied the lifecycle cost of a product protected by the extended warranty policies from the consumers' perspective. Jack and Murthy (2007) proposed that the method describing the degree of preventive maintenance behaviour is failure-rate reduction and age-reduction. Jun, Min, and Tsan (2011) studied a general periodic preventive maintenance behaviour policy. The research conclusions show that the time of the first preventive maintenance behaviour and its corresponding maintenance level determined the optimal preventive maintenance strategies for the consumers.

The paper integrates the base warranty and extended warranty breaking through the orthodox research separating the base warranty and extended warranty. Manufacturers carry preventive maintenance behaviour during the base warranty period and the extended warranty period to avoid the risk of increasing maintenance cost as the products' quality and the consumers' abuse. We seek to obtain the influence on preventive maintenance behaviour during the warranty period to the extended warranty length.

Due to difficulty of the empirical study of preventive maintenance and the extended warranty, at present, most scholars adopt the mathematical model method to study the topic on the extended warranty and preventive maintenance behaviour. Chien (2005) determined the optimal warranty period from the perspective of the seller and out-of-warranty replacement age from that of the buyer respectively, minimizing the corresponding cost functions by a concise numerical example. Lin, Wang, and Chin (2009) established a model-driven Decision Support System to determine the dynamic optimization of price, warranty length and production rate. Chang and Lin (2012) took the numerical examples to illustrate the influences of
the optimal extended warranty length. Therefore, this study establishes mathematical model and adopts the simulation analysis method to study the problem.

Preventive maintenance behaviour is quantized as the number of carrying preventive maintenance behaviour during the base warranty period and the extended warranty period. The numbers of failures during the warranty period can be expressed as the function of the number of carrying preventive maintenance behaviour. Combining the maintenance cost and the preventive maintenance behaviour cost, the profit function is constructed. Therefore, we can get the optimal extended warranty length under different situations by maximizing the profit function. The simulation analysis tells us the influence on preventive maintenance behaviour during the warranty period to the extended warranty length. Ultimately the influencing mechanism is helpful to increase consumers’ purchase of hardware products and expanded market share with indirect network effects. The rest of the paper is outlined as follows. The mathematical model is constructed based on some assumption, in Section 2. Section 3 elaborates the optimal extended warranty length under different situations. The numerical examples are analysed in Section 4. Finally, the conclusions are presented at the end of the paper.

2. Model formulation

Consider a new durable product sold with the base warranty period \( t_{bw} \) and the extended warranty period \( t_{ew} \). Assume that the relation between the base warranty and the extended warranty periods satisfies the following equation:

\[
t_{bw} = t_{ew} + kl, k = 1, 2, \ldots
\]  

(1)

where \( l \) means a fixed time length unit of the extended warranty, \( k \) presents purchasing the number of a fixed time length unit of the extended warranty.

The preventive maintenance behaviour can be measured with the maintenance degree of a preventive maintenance behaviour \( x \), the time epochs for carrying out the first preventive maintenance behaviour \( t_1 \) and the marginal cost of carrying the preventive maintenance behaviour \( \rho \). Assuming the relation between the maintenance degree of a preventive maintenance behaviour \( x \) and the time epochs for carrying out preventive maintenance behaviour \( t_1 \) satisfies the following relation:

\[
t_1 = t_1 + (i - 1)x, i = 1, 2, \ldots
\]  

(2)

Let \( r(t) \) be the failure rate of the product. Assuming that the failure-rate function follows square function, the failure-rate function is

\[
r(t) = t^2
\]  

(3)

The expected total number of failures within the interval \((0, t)\) is

\[
R(t) = \int_0^\infty r(t) \, dt
\]  

(4)

The probability intensity follows \( r(t - ix) \), \( k \) is maximum when the extended warranty periods cover the life cycle. That is

\[
t_{ew} = t_d
\]  

(5)

Combing the Equation (1), the maximum \( k \) is

\[
k_{max} = \frac{t_d - t_{bw}}{l}
\]  

(6)

When consumers do not buy the extended warranty, the minimum \( k \) is

\[
k_{min} = 0
\]  

(7)

Therefore \( k \) is located during \([0, (t_d - t_{bw})/l]\).

2.1. Cost to manufacturer

The total cost to the manufacturer during the product sold includes two parts: the cost \( C_{bw} \) during the base warranty period and the cost \( C_{ew} \) during the extended warranty period. The cost during the warranty period mainly is the maintenance cost and preventive maintenance behaviour cost. The maintenance cost equals to each maintenance cost plus the total numbers of failures. Preventive maintenance behaviour cost is each preventive maintenance behaviour cost \( C_{pm}(x) \) plus the total number of preventive maintenance behaviour. Each preventive maintenance behaviour cost \( C_{pm}(x) \) is non-negative and non-decreasing function of maintenance degree \( x \).

\[
C_{pm}(x) \geq 0, C_{pm}'(x) \geq 0 \quad \text{for all} \quad x \geq 0
\]  

(8)

During the base warranty period, the numbers of failures can be expressed as

\[
\sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} r(t - ix) \, dt + \int_{t_n}^{t_{bw}} r(t - nx) \, dt
\]

\[
= \sum_{i=1}^{n} [R(t_i - ix) - R(t_{i-1} - ix)]
\]

\[
+ R(t_{bw} - nx) - R(t_n - nx)
\]

\[
= nR(t_1) + (n - 1)R(t_1 - x) + R(t_{bw} - nx) - R(t_n - nx)
\]  

(9)

The maintenance cost during the base warranty period equals to \( r_P \) plus the numbers of failures during the base warranty period, which is

\[
C_{bm} = r_p [nR(t_1) + (n - 1)R(t_1 - x)
\]

\[
+ R(t_{bw} - nx) - R(t_n - nx)]
\]  

(10)

The preventive maintenance behaviour cost during the base warranty period is equal to each preventive maintenance behaviour cost plus the numbers of preventive
maintenance behaviour during the base warranty period, which denotes

\[ C_{bp} = nC_{pm}(x) \]  

(11)

Let \( n \) denote the total number of preventive maintenance behaviour within base warranty period \((0, t_{bw})\). Combining Equations (10) and (11), we have the cost during the base warranty given by

\[ C_{bw} = C_{bm} + C_{bp} = r_p[nR(t_1) + (n - 1)R(t_1 - x) + R(t_{bw} - nx) - R(t_n - nx)] + nC_{pm}(x) \]  

(12)

During the extended warranty period, the numbers of failures can be expressed as

\[ \sum_{i=1}^{m} \int_{t_{bw} + i}^{t_{bw} + i + 1} r(t - ix)dt + \int_{t_n}^{t_{bw} + i} r(t - mx)dt \]

\[ = \sum_{i=1}^{m} [R(t_{bw} + i - i) - R(t_{bw} + i - ix)] + R(t_{bw} + kl - mx) - R(t_m - mx) \]

\[ = m[R[t_1 + (t_{bw} - 1)x] - R[t_1 + (t_{bw} - 2)x]] + R(t_{bw} + kl - mx) - R(t_m - mx) \]

(13)

The maintenance cost during the extended warranty period equals to \( r_p \) plus the numbers of failures during the extended warranty period, which is

\[ C_{em} = r_p[mR[t_1 + (t_{bw} - 1)x] - R[t_1 + (t_{bw} - 2)x]] + R(t_{bw} + kl - mx) - R(t_m - mx)] \]  

(14)

The preventive maintenance behaviour cost in the extended warranty period is equal to each preventive maintenance behaviour cost plus the numbers of preventive maintenance behaviour during the extended warranty period, which denotes

\[ C_{ep} = mC_{pm}(x) \]  

(15)

Let \( m \) denote total number of preventive maintenance behaviour within extended warranty period \((t_{bw}, t_{ew})\). Combining Equations (14) and (15), we can get the cost during the extended warranty period given by

\[ C_{ew} = C_{ep} + C_{em} = r_p[mR[t_1 + (t_{bw} - 1)x] - R[t_1 + (t_{bw} - 2)x]] + R(t_{bw} + kl - mx) - R(t_m - mx)] + mC_{pm}(x) \]  

(16)

From Equations (12) and (16), we can obtain the expected total cost given by

\[ TC = C_{bw} + C_{ew} = r_p[nR(t_1) + (n - 1)R(t_1 - x) + R(t_{bw} - nx) - R(t_n - nx)] + nC_{pm}(x) \]

\[ + r_p[mR[t_1 + (t_{bw} - 1)x] - R[t_1 + (t_{bw} - 2)x]] + R(t_{bw} + kl - mx) - R(t_m - mx) + mC_{pm}(x) \]  

(17)

2.2. Profit to manufacturer

When the consumer purchased the product, the total income to manufacturer derives two aspects: the first is the selling price of the extended warranty \( C_E \), the second is the manufacturer’s total maintenance income. It equals to each maintenance income plus the absolute numbers of failures. After the extended warranty period expires, the consumer must pay maintenance cost \( r_c \) to the manufacturer when the product failed. The manufacturer must exclude the maintenance cost \( r_p \), then each maintenance income denotes \( r_c - r_p \). The total numbers of failures after the extended warranty period expires are

\[ \int_{t_{bw} + kl}^{t_d} r(t - (n + m)x)dt = R[t_d - (n + m)x] - R[t_{bw} + kl - (n + m)x] \]

(18)

The manufacturer’s total maintenance income is

\[ (r_c - r_p)[R[t_d - (n + m)x] - R[t_{bw} + kl - (n + m)x]] \]  

(19)

The manufacturer’s total income is

\[ C_E + (r_c - r_p)[R[t_d - (n + m)x] - R[t_{bw} + kl - (n + m)x]] \]  

(20)

Combining the total cost and income yield the total profit, which is

\[ TP = TR - TC = C_E + (r_c - r_p)[R[t_d - (n + m)x] - R[t_{bw} + kl - (n + m)x]] \]

\[ - R[t_{bw} + kl - (n + m)x] + r_p[nR(t_1) + (n - 1)R(t_1 - x) + R(t_{bw} - nx) - R(t_n - nx)] - (n + m)C_{pm}(x) \]

\[ - r_p[mR[t_1 + (t_{bw} - 1)x] - R[t_1 + (t_{bw} - 2)x]] - R(t_{bw} + kl - mx) - R(t_m - mx) + mC_{pm}(x) \]

(21)

3. Optimal policy

Our objective is to determine an optimal extended warranty period such that the expected total profit in Equation (13) is maximized. To assess the properties of the optimal extended warranty period, we take the first and second
partial derivatives of Equation (21) with respect to \( k \). We have

\[
\frac{\partial TP}{\partial k} = (r_c - r_p)[r(t_{bw} + k(l - (n + m)x))] - r_p[r(t_{bw} + kl - mx)]
\]

(22)

And

\[
\frac{\partial^2 TP}{\partial k^2} = r_p l^2 [r'[t_{bw} + k(l - (n + m)x)] - r'(t_{bw} + kl - mx)] - r_p l [r'[t_{bw} + kl - mx]]
\]

(23)

Analysing the first and second partial derivatives, if \((\partial^2 TP/\partial k^2) < 0\), \((r_p/r_c) < (r'[t_{bw} + k(l - (n + m)x)]/r'[t_{bw} + kl - mx])\), it is shown that \(\partial TP/\partial k\) is the decreasing function of \( k \).

\[
\frac{\partial TP}{\partial k} \Big|_{k=0} = r_p l [r(t_{bw} - (n + m)x) - r(t_{bw} - mx)] - r_p l [r(t_{bw} - (n + m)x)]
\]

\[
\frac{\partial TP}{\partial k} \Big|_{k=t_d - t_{bw}/l} > 0,
\]

that is

\[
\frac{r_p}{r_c} > \frac{r[t_{bw} - (n + m)x]}{r[t_{bw} - (n + m)x - r(t_{bw} - mx)]}
\]

\[
\frac{\partial TP}{\partial k} \Big|_{k=t_d - t_{bw}/l} > 0,
\]

that is

\[
\frac{r_p}{r_c} > \frac{r[t_d - (n + m)x]}{r[t_d - (n + m)x - r(t_d - mx)]}
\].

The profit expression is the increasing function of \( k \). Based on the profit maximum, the optimal extended warranty period is \( k = (t_d - t_{bw}/l) \). This proves the Lemma 1.

**Lemma 2** If

\[
\frac{r_p}{r_c} < \frac{r'[t_{bw} + kl - (n + m)x]}{r'[t_{bw} + kl - (n + m)x]} - r'(t_{bw} + kl - mx)
\]

and

\[
\frac{r_p}{r_c} < \frac{r[t_{bw} - (n + m)x]}{r[t_{bw} - (n + m)x - r(t_{bw} - mx)]}
\]

\[
\frac{r_p}{r_c} < \frac{r[t_d - (n + m)x]}{r[t_d - (n + m)x - r(t_d - mx)]}
\]

then \( k = 0 \).

If

\[
\frac{\partial TP}{\partial k} \Big|_{k=0} < 0,
\]

that is

\[
\frac{r_p}{r_c} < \frac{r[t_{bw} - (n + m)x]}{r[t_{bw} - (n + m)x - r(t_{bw} - mx)]}
\]

\[
\frac{\partial TP}{\partial k} \Big|_{k=t_d - t_{bw}/l} < 0,
\]

that is

\[
\frac{r_p}{r_c} < \frac{r[t_d - (n + m)x]}{r[t_d - (n + m)x - r(t_d - mx)]}
\].

The profit expression is the decreasing function of \( k \). Based on the profit maximum, the optimal extended warranty period is \( k = 0 \). This proves Lemma 2.

**Lemma 3** If

\[
\frac{r_p}{r_c} < \frac{r'[t_{bw} + kl - (n + m)x]}{r'[t_{bw} + kl - (n + m)x]} - r'(t_{bw} + kl - mx)
\]

and

\[
\frac{r_p}{r_c} > \frac{r[t_{bw} - (n + m)x]}{r[t_{bw} - (n + m)x - r(t_{bw} - mx)]}
\]

\[
\frac{r_p}{r_c} > \frac{r[t_d - (n + m)x]}{r[t_d - (n + m)x - r(t_d - mx)]}
\]

then there exists a unique solution \( k \in [0, (t_d - t_{bw}/l)] \).

If

\[
\frac{\partial TP}{\partial k} \Big|_{k=0} > 0,
\]

that is

\[
\frac{r_p}{r_c} > \frac{r[t_{bw} - (n + m)x]}{r[t_{bw} - (n + m)x - r(t_{bw} - mx)]}
\]

\[
\frac{\partial TP}{\partial k} \Big|_{k=t_d - t_{bw}/l} > 0,
\]

that is

\[
\frac{r_p}{r_c} > \frac{r[t_d - (n + m)x]}{r[t_d - (n + m)x - r(t_d - mx)]}
\].

The profit expression is not monotonous function of \( k \). Therefore, the optimal extended warranty period exists
between 0 and \((t_d - t_{bw})\). That is \(k \in [0, (t_d - t_{bw})/l]\). This proves Lemma 3.

In order to seek the optimal extended warranty length during \([0, (t_d - t_{bw})/l]\), we let Equation (22) be zero,

\[
\frac{\partial \text{TP}}{\partial k} = (r_c - r_p)l[r(t_{bw} + kl - (n + m)x)]
- r_p[r(t_{bw} + kl - mx)] = 0
\]  

That is,

\[
\frac{r_c}{r_p} = 1 + \frac{r(t_{bw} + kl - mx)}{r(t_{bw} + kl - (n + m)x)}
\]  

Combing specific failure-rate function, Equation (25) will become

\[
\frac{r_c}{r_p} = 1 + \frac{(t_{bw} + kl - mx)^2}{(t_{bw} + kl - (n + m)x)^2}.
\]

4. Numerical examples

4.1. Example 1

Example 1 mainly analyses the influence of preventive maintenance behaviour during the base warranty to the optimal extended warranty length. When analysing the influence of preventive maintenance behaviour during the base warranty to the optimal extended warranty length, the paper assumes that preventive maintenance behaviour during the extended warranty period is fixed. The value of preventive maintenance behaviour during the extended warranty period is from 0.5 to 2.5, and the corresponding step is 0.04. The total number of the value of preventive maintenance behaviour during the extended warranty period is 51.

The other parameters in the simulation are as follows:

\(x = 1, \quad l = 0.5, \quad t_1 = 0, \quad r_c = 100, \quad r_p = 40, \quad n = 1, \quad t_{bw} = 1\)

The simulation result is observed in Figure 1. Specially, preventive maintenance behaviour during the base warranty period positively influences the optimal extended warranty length. The increasing extent keeps steady.

4.2. Example 2

Example 2 mainly analyses the influence of preventive maintenance behaviour during the extended warranty period to the optimal extended warranty length. When analysing the influence of preventive maintenance behaviour during the extended warranty period to the optimal extended warranty length, we assume that preventive maintenance behaviour during the base warranty period is fixed. The value of preventive maintenance behaviour during the extended warranty period is from 0.5 to 2.5, and the corresponding step is 0.04. The total number of the value of preventive maintenance behaviour during the extended warranty period is 51.

The other parameters in the simulation are as follows:

\(x = 1, \quad l = 0.5, \quad t_1 = 0, \quad r_c = 100, \quad r_p = 40, \quad n = 1, \quad t_{bw} = 1\)

The simulation result is seen from Figure 2. Specially, preventive maintenance behaviour during the extended warranty period positively influences the optimal extended warranty length. The increasing extent keeps steady.

Figures 1 and 2 describe the properties of the extended warranty optimal length influenced preventive maintenance behaviour during the base warranty period and extended warranty period. From Figures 1 and 2, we can see that preventive maintenance behaviour during the base warranty period and extended warranty period positively influences the optimal extended warranty length. The differences embody two aspects: firstly, the influence timeliness of preventive maintenance behaviour during the extended warranty period is ahead of that during the base warranty period. Secondly, the influence degree of preventive maintenance behaviour during the base warranty period is greater than during the extended warranty period. The conclusions are helpful for manufacturers to
make the optimal extended warranty length. On the one hand, the manufacturers should notice the positive influence relation of preventive maintenance behaviour during the base warranty period and the extended warranty period to the extended warranty optimal length. On the other hand, as the influence degree of preventive maintenance behaviour during the base warranty period is greater, the manufacturers should pay attention to preventive maintenance behaviour during the base warranty period making the optimal extended warranty length.

5. Conclusion
The manufacturers optimize the extended warranty to increase the consumers’ purchase of hardware products and expand market share with indirect network effects. The manufacturers carry preventive maintenance behaviour during the base warranty period and the extended warranty period to avoid the risk of increasing the warranty cost as the products’ quality and the consumers’ abuse, reducing the failure rate.

Therefore, the paper breaks the past method for studying the extended warranty period which combines the relation between the failure rate and the maintenance cost. Preventive maintenance behaviour is embedded in the conventional profit function to control the maintenance cost during the extended warranty period from the perspective of in-process control. Preventive maintenance behaviour is divided into the BWPM behaviour and EWPM behaviour. The paper analyses the influence relation on BWPM behaviour and EWPM behaviour to the extended warranty length and compares the influence difference to propose the new mechanism of designing the extended warranty length for the manufacturers.

This study constructs the model introducing preventive maintenance behaviour to get the optimal extended warranty length under different situations by maximizing the profit function. The simulation analysis can be done to observe the influence on preventive maintenance behaviour during the base warranty period and the extended warranty period to the optimal extended warranty length. The following conclusions can be obtained: firstly, preventive maintenance behaviour during the base warranty period and the extended warranty period positively influences the optimal extended warranty length. Secondly, the influence degree of preventive maintenance behaviour during the base warranty period is later and greater than during the extended warranty period.

Disclosure statement
No potential conflict of interest was reported by the authors.