Studies on the dynamics of impact interaction of the mechanoreological model under elastic plastic transformation of its mechanical system

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Abstract. Dynamic interaction of machine and equipment parts and elements is a widespread process. Therefore, simulation and analysis of these processes are relevant tasks. To study impact interaction, a mechanorheological viscoelastic plastic model was developed. The force in the beginning of plastic deformations is an important model parameter. However, previous researches were carried out under impact loading of the model. They assumed that the plastic deformation occurs along with the elastic one. However, impact processes are more sophisticated. At first, only elastic deformations occur. When stresses achieve a yield point, plastic deformations occur. Therefore, the influence of the force on the model impact interaction dynamics is of great interest. The results let us draw the following conclusions. At the loading stage, when a viscoelastic model transforms into a viscoelastic plastic model, plastic deformations decrease the impact force and rebound height. Due to plastic deformations, the surface becomes more flexible, the impact duration increases body braking acceleration and inertia decrease. It leads to the decrease in the dynamic load and rebound height. The research results can be used for further development of methods for calculating model parameters, enhancing impact process simulation accuracy and reliability.

1. Introduction

Dynamic interaction of machine and equipment parts and elements is a widespread process. Impact processes accompanied by high dynamic loading play a crucial role. Therefore, simulation and analysis of these processes are relevant tasks. Dynamic processes are studied by many researchers [1-5].

To study impact interaction processes, a more general mechanorheological viscoelastic plastic model was developed [6]. It includes a shift element which takes into account energy losses due to plastic deformations under impact interaction. To adapt the model to real dynamic processes, several researches were carried out [7-11].

The scheme of the viscoelastic plastic mechanorheological model describing impact interaction of a spherical body with a surface is presented in Figure 1. The model consists of two consecutive units: the viscoelastic unit $K_1 - C$ and elastic plastic unit $K_2 - f_2$. The resistive force of the viscoelastic deformation is calculated as $N_1 = F_{DIS} + F_{EL1}$; $F_{DIS} = C(y_1 - y_2)^{a1}(y_1 - y_2)^{a2}; F_{EL1} = K_1(y_1 - y_2)^{a1}$, where $y_1, y_2, y_1, y_2$ is the shift and mass speed $m_1$ and $m_2$; $K_1$ is the ratio of stiffness of an element of the viscoelastic unit; $C$ is the ratio of viscosity of an element of the viscoelastic unit.

The resistive force of the elastic plastic deformation is calculated as $N_2 = F_{PL} + F_{EL2}$; $F_{EL2} = K_2 y_2^{n2}; F_{PL} = f_2 y_2^{n3} + F_{ST}$, where $F_{ST}$ is the force corresponding to the occurrence of...
plastic deformation; $K_2$ is the ratio of stiffness of an elastic element of the elastic plastic unit; $f_2$ is the ratio of shift of the elastic plastic unit or the ratio of ductility.

The total body mass is concentrated in inertia element $m_1$, the mass of element $m_2$ is negligibly small ($m_2 \to 0$) and cannot influence the dynamics of mechanical system movement. It is used for mathematical description of the system with two second order differential equations.

The viscoelastic plastic model can work in the following way: at the initial stage of impact interaction, only elastic deformations occur, so the viscoelastic unit $K_1 - C$ is deformed. The unit describes elastic deformations and energy losses. The elastic plastic unit comes into operation when the dynamic force achieves the target value of force $F_{ST}$ corresponding to intensive development of plastic deformations. Varying the value of force $F_{ST}$, one can ensure viscoelastic or viscoelastic plastic interaction of mechanical systems when one or two units of the model come into operation.

When a dynamic force value is maximum $N_{MAX}$, the model starts unloading. At this stage, only the viscoelastic unit describing disappearance of elastic deformations comes into operation. The elastic plastic unit remains deformed as it characterizes plastic (residual) deformations.

All model elements have linear characteristics. The degree index is set in general terms ($a_1, a_2, n_1, n_2, n_3$). The approach expands model abilities and can be used in researches aimed to study the effects of non-linear characteristics of model elements on the dynamics of its impact interaction [8]. As for specific values of power indices, under impact interaction of the spherical body, the elastic component will be $n_1 = n_2 = 3/2$ [2-4], the plastic component can be proportional to the acting force ($n_3 = 1$) [2,3]. Unloading of materials follows the linear elasticity law [2].

Figure 1. The scheme of the viscoelastic plastic model

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2. Purpose statement
When using theoretical models, it is important to adapt the simulation model to real dynamic (impact) processes. Model adequacy is assessed based on the coincidence of main parameters of the dynamic process determined using a model with experimental data. As for the impact process under study, these parameters are the impact interaction force variation regularity, impact time, body rebound height, and a deformation value. To calculate model parameters $K_1, K_2, f_2, C$ by experimental impact parameters, a special method was developed [12,13]. However, previous researches [7-14] were carried out for the case when $F_{ST}=0$, i.e. under the impact loading. They assumed that plastic deformations occur in the beginning of the impact process along with the elastic ones.

However, the impact process can be more sophisticated. At first, only elastic deformation occurs. When stresses achieve a yield point (when $N= F_{ST}$), plastic deformations described by the model unit $K_2 - f_2$ occurs. Therefore, the effects of a force value $F_{ST}$ on the model dynamics of impact interaction are of great interest. To study the process, researches were carried out.

3. Theory
Differential model movement equations are as follows:

$$m_1 \ddot{y}_1 + C_1(y_1 - y_2)^{a_1} (y_1 - y_2)^{a_2} + K_1(y_1 - y_2)^{n_1} = -m_1 g ;$$

$$m_2 \ddot{y}_2 + K_2 y_2^{n_2} + f_2 y_2^{n_3} + C_1(y_2 - y_1)^{a_1} (y_2 - y_1)^{a_2} + K_1(y_2 - y_1)^{n_1} = -m_2 g + F_{ST}.$$  

The mathematical model can be used to study the effects of viscoelastic and plastic properties of materials on the dynamics of impact interaction of a spherical body and a surface. The model served a basis for a special research program designed to calculate dynamic interaction parameters: impact time, impact interaction force, deformation value, speed and rebound height.

Figure 2 presents a diagram of changes in the impact interaction force (normal response force $N$) of a steel spherical body. Calculations are carried out using the program developed. The following parameters were used: $a_1 = 1, a_2 = 1, n_1 = 3/2, n_2 = 3/2, n_3 = 1, K_2 = 0.1K_1$. At the initial stage of contact interaction (segment $a-b$), elastic deformations occur. When normal response force $N$
achieves a target value of force $F_{ST}$, the elastic plastic unit comes into operation (segment $b-c$). When elastic deformations disappear, segment $c-d$ corresponds to the unloading stage (the viscoelastic unit operates). At point $d$, the body leaves the surface ($N = 0$).

**Figure 3.** Diagram of changes in force $N$

### 4. Analysis of research results

Two calculation variants were used (Figure 3). Variant 1: $F_{ST}=N_{\text{max}}$ (only the elastic deformation occurs). Variant 2: $F_{ST}=2N_{\text{max}}/3$. Variant 3: $F_{ST}=N_{\text{max}}/3$. Variant 4: $F_{ST}=0$ (the plastic deformation occurs since the beginning of the impact process). The following parameters of the impact interaction were analyzed: impact force $N$ and relative body rebound height $dh=h/2/h_1$ ($h_1$ is the drop height, $h_2$ is the rebound height). Model parameters $K_1, K_2, f_2, C$ varied within the prescribed limits.

Figure 3 presents diagrams of the normal response force $N$ under different values of $F_{ST}$ and $K_1$. The research results are presented in Figures 4 and 5. The following conclusions can be drawn from the results: at the loading stage, when a viscoelastic model transforms into a viscoelastic plastic model ($F_{ST}=N_{\text{max}} \rightarrow F_{ST}=0$), impact force $N_{\text{max}}$ and rebound height $dh$ decrease at different model parameters $K_1, K_2, f_2, C$. These changes are significant. The phenomenon occurs due to the fact that the surface becomes more ductile due to plastic deformations, the impact force increases, and the body braking acceleration and inertia decrease. It decreases the dynamic load (impact interaction force). Due to plastic deformations, the impact energy dissipates and rebound height decreases.

### 5. Conclusions

Thus, the research results for dynamic interaction of the mechanorheological model under the elastic plastic transformation of its mechanical system speak for significant changes in its behavior dynamics. Therefore, the model parameter $F_{ST}$ along with other parameters ($K_1, K_2, f_2, C$) are important control factors influencing the model behavior.

The practical application of the model can enhance accuracy and reliability of impact process simulation, and can be used for further development of methods for calculating physical and mechanical properties of materials. Knowledge of mechanical properties of materials is used for solving different research tasks by means of mathematical vibration and impact process simulation. The key task is to adapt the simulation model to real impact processes by parameter $F_{ST}$. To this end, appropriate methods should be developed. These tasks can be solved using the research results.
Figure 4. Changes in the normal response force
Figure 5. Changes in the rebound height
6. References

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