Pseudo-stochastic signal characterization in wavelet-domain

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Abstract. In this paper we present the method for fast and accurate characterization of pseudo-stochastic signals, which contain a large number of similar but randomly-located fragments. This method allows estimating the statistical characteristics of pseudo-stochastic signal, and it is based on digital signal processing in wavelet-domain. Continuous wavelet transform and the criterion for wavelet scale power density are utilized. We are experimentally implementing this method for the purpose of sand granulometry, and we are estimating the statistical parameters of test sand fractions.

1. Introduction
Characterization of pseudo-stochastic signals, containing many similar but randomly-located fragments, is highly important problem of applied physics and signal processing. Images of granulated media, complex surfaces, and biological objects are common examples of such signals [1-13].

Development of novel techniques for pseudo-stochastic signal characterization is an important topic of applied physics. Fourier-domain signal processing does not perform well at solving this problem because of complete non-locality of Fourier transform kernels in spatial-domain. Signal processing in spatial-domain is commonly used for pseudo-stochastic signal analysis, including nonlinear filtering and morphological processing [14, 15]. Spatial-domain processing performs well at solving this problem; however, it exhibits relatively high computational complexity. Wavelet-domain processing of pseudo-stochastic signals is a perspective instrument for solving the specified problem because of high localization of wavelet transform kernels both in spatial-domain and frequency-domain [16,17].

This paper presents the wavelet-domain approach for characterizing the signals, consisting of a large number of randomly located fragments. We estimate the statistical characteristics for pseudo-stochastic signals using one-dimensional (1D) continuous wavelet transform and the criterion of wavelet-scale power density. We implement the proposed technique for the purpose of sand granulometry.
2. Estimation of pseudo-stochastic signal parameters

1D direct continuous wavelet transform of \( s(x) \) is defined with the projection of function on the wavelet transform basis:

\[
C(a, b) = \mathcal{W}[s(x)] = \langle s(x) \psi(a, b, x) \rangle = \int_{-\infty}^{+\infty} s(x) \psi(a, b, x) dx,
\]

\( \mathcal{W} : \mathbb{R} \rightarrow \mathbb{R}^2, \quad a, b \in \mathbb{R}, \quad a \neq 0 \) \hspace{1cm} (1)

where \( C(a, b) \) are coefficients of wavelet decomposition, and \( \psi(a, b, x) \) are wavelet decomposition kernels with \( a \)-scale and \( b \)-translation. All \( \psi(a, b, x) \)-kernels form a basis for signal projection in \( L^2(\mathbb{R}) \) wavelet-domain space. Kernels, \( \psi(a, b, x) \), are constructed by scaling and shifting of the mother wavelet, \( \psi(x) \):

\[
\psi(a, b, x) = |a|^{-1/2} \psi \left( \frac{x - b}{a} \right).
\] \hspace{1cm} (2)

Figure 1 shows schematic representation of proposed algorithm for characterizing the pseudo-stochastic signals. We are finding the statistical characteristics of randomly located similar fragments of signal, \( s(t) \), via post-processing of direct continuous wavelet decomposition [1]. We introduce the criterion of wavelet-scale power density, \( P(a) \), as an intermediate step of statistical characteristic calculation:

\[
P(a) = \frac{1}{a^2} \int_{-\infty}^{+\infty} |C(a, b)|^2 db.
\] \hspace{1cm} (3)

When \( s(x) \) contains a significant number of similar but randomly located fragments with the average size \( \bar{d} \), the \( P(a) \)-function exhibits a sharp peak centered at \( a_{\text{max}} \) with the full-width at half-maximum \( \Delta a \). If the signal contains several fractions of similar objects, randomized in space, several peaks appear in \( P(a) \). These peaks overlap in case of large standard deviation, \( \sigma_d \), or close average sizes of fraction elements, \( \bar{d} \).

In this paper, we are considering the problem of single fraction characterization, and we are utilizing \( P(a) \) for calculating the statistical characteristics of single fraction: the average
Figure 2. Digital images of sand with various average diameter of particle, $\bar{d}$: (a) corresponds to $\bar{d} \approx 0.45$ mm, (b) corresponds to $\bar{d} \approx 0.94$ mm, and (c) corresponds to $\bar{d} \approx 1.88$ mm.

fragment size, $\bar{d}$, and the standard deviation of fragment size, $\sigma_d$. These parameters depend on the parameters of peak in $P(a)$-function:

$$
\bar{d} = f_1(a_{\text{max}}), \quad \sigma_d = f_2(\Delta a),
$$

where $f_1(...), f_2(....)$ are some calibration functions.

3. Algorithm implementation for sand granulometry

We are implementing the proposed technique for studying the set of test sand fractions having calibrated grain diameters. Table 1 shows the sand fraction parameters ($\sigma_d \approx 1/6(d_{\text{max}} - d_{\text{min}})$), and figure 2 shows digital images of these sand fractions.

We use the 1D wavelet decomposition of certain image row, $s(x) \equiv s(x, y = y')$, instead of 2D wavelet decomposition of entire image, $s(x, y)$, in order to reduce the computational complexity. Each row represents the cross-section of the object textures, $y = y'$, and contains complete information about the inhomogeneities of pseudo-stochastic object. Moreover, 1D approximation is much faster than the 2D approach, allowing us real-time data processing. Figure 3 shows the result of direct continuous wavelet transform (1) of image rows. The difference between wavelet spectra can be observed directly from the result of the wavelet decomposition, but this it is difficult to estimate this difference without data post-processing.

We calculate the 1D criterion of wavelet-scale power density (3) for all the wavelet spectrums (figure 3) of sand samples, and figure 4 shows the results. The curves of figure 4 exhibit sharp maximum centered at $a_{\text{max}}$ and having the full-width at half-maximum $\Delta a$. We are estimating $a_{\text{max}}$ and $\Delta a$ for each peak; Table 2 shows the results. On the step of system calibration (we study various test samples of priory characterized sand fractions for this purpose) we estimate the following polynomial approximation of calibration functions, $\bar{d} = f_1(a_{\text{max}})$ and $\sigma_d = f_2(\Delta a)$:

$$
\bar{d} = -8.104 + 0.916a_{\text{max}} - 0.021a_{\text{max}}^2, \\
\sigma_d = 0.247 - 0.041\Delta a + 0.018\Delta a^2.
$$

Table 1. Priory parameters of sand fractions.

| #  | $\bar{d}$, mm | $d_{\text{min}}$...$d_{\text{max}}$, mm |
|----|---------------|-----------------------------------------|
| 1  | 0.45          | 0.35...0.55                             |
| 2  | 0.94          | 0.63...1.25                             |
| 3  | 1.88          | 1.26...2.50                             |
Figure 3. 1D continuous wavelet transform of appropriate rows of digital images (figure 2): (a) corresponds to $d \approx 0.45$ mm sand fraction, (b) corresponds to $d \approx 0.94$ mm sand fraction, and (c) corresponds to $d \approx 1.88$ mm sand fraction. Daubechies mother wavelet $'db8'$ is applied for wavelet decomposition.

Nonlinearity of (5) occurs due to the large number of factors, including aberrations of optical system, strongly correlated noises, overlapping of separate grains in digital images, and decay of optical modulation transfer function with an increase of spatial frequency of image textures.

Table 2 shows results of estimation of sand characteristics, $d^\prime$ and $\sigma^\prime_d$, using equation (5). We

Figure 4. Wavelet-scale power density criterion, $P(a)$, calculated for various sand fractions: (1) corresponds to $d \approx 0.45$ mm sand fraction, (2) corresponds to $d \approx 0.94$ mm sand fraction, and (3) corresponds to $d \approx 1.88$ mm sand fraction.
Table 2. Estimated parameters of sand fractions.

| #  | $\bar{d}$, mm | $a_{max}$, arb. units | $\bar{d}'$, mm | $\delta_{\bar{d}}$ |
|----|----------------|------------------------|----------------|-------------------|
| 1  | 0.450          | 13.25                  | 0.430          | 0.05              |
| 2  | 0.940          | 15.00                  | 0.946          | 0.01              |
| 3  | 1.880          | 19.75                  | 1.979          | 0.01              |

$\langle \delta_{\bar{d}} \rangle = 0.04$

| #  | $\sigma_d$, mm | $\Delta a$, arb. units | $\sigma_d'$, mm | $\delta_{\sigma}$ |
|----|----------------|------------------------|-----------------|-------------------|
| 1  | 0.033          | 13.50                  | 0.028           | 0.15              |
| 2  | 0.103          | 18.25                  | 0.105           | 0.02              |
| 3  | 0.207          | 21.59                  | 0.206           | 0.01              |

$\langle \delta_{\sigma} \rangle = 0.11$

are also estimating the relative errors of statistic parameter determination

$$
\delta_{\bar{d}_i} = (\bar{d}'_i - \bar{d}_i)/\bar{d}_i, \quad \delta_{\sigma_i} = (\sigma'_d - \sigma_d_i)/\sigma_d_i,
$$

$$
\langle \delta_{\bar{d}} \rangle = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \delta_{\bar{d}_i}^2}, \quad \langle \delta_{\sigma} \rangle = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \delta_{\sigma_i}^2},
$$

(6)

where $i$ is the sample index, and $N = 3$ stands for the total number of samples. The accuracy of the statistical parameter estimation is much higher for $2^{nd}$ and $3^{rd}$ samples compared to the $1^{st}$ one due to the limitations of the imaging system resolution. The higher the contrast in sand images we observe, the higher the accuracy of statistics reconstruction we achieve.

We could implement real-time signal processing with a repetition rate of algorithm implementation higher than 50 Hz since 1D continuous wavelet transform is applied. The repetition rate can be significantly raised by increasing the camera frame rate and via the parallel processing of pseudo-stochastic data. This would allow implementation of the method in real-time technological applications, such as remote sensing in a fast flow of microparticles.

4. Conclusion

We have proposed fast and precise method for characterizing the pseudo-stochastic signals based on digital signal processing in wavelet-domain. We have implemented this technique for processing the digital images of various sand fractions as a representative example.

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