Guiding children in learning subtraction by using contextual strategy: as an attempt to develop students' number sense

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Abstract. Subtraction operations are basic number operations that have been introduced to students at the 2\textsuperscript{nd} grade of elementary school. Several studies have shown that this topic is difficult for students to understand and causes many problems in learning of numbers. Students' constraints in performing this subtraction operation usually occur when subtracting two numbers in tens, hundreds, or more or numbers containing 0. One of the procedures that teachers often teach students in solving the problem of subtracting two numbers is borrowing techniques. This technique is often seen as forced and very abstract to students. This study aims to describe how students thinking in performing subtraction on numbers through more contextual strategies that they understand. This study used a single-subject research design with multiple baselines across individuals \((n = 2)\) to evaluate the effectiveness of the measures performed. The intervention observed in this study was to use the number of subtraction technique with a different contextual strategy between the two subjects. The results indicate that the two contextual strategies used show a positive effect on understanding the number subtraction operation. The results also showed that subtraction without borrowing (not using formulas) made students more creative in solving the subtraction problem.

1. Introduction

Numbers and their operations are inseparable from mathematics and the activities we do every day. A good understanding of numbers and number operations affects student achievement in subsequent mathematics [1]–[4]. Therefore it is important for students in the early stages of mathematics to understand numbers and their operations well. A good understanding of numbers is closely related to number sense [3], [5]. Number sense or number sensitivity plays an essential role in solving mathematical problems [2]. It is possible to see number sense as a strong intuition about numbers and their interactions. As a result of exploring numbers, it progressively evolves, visualizes them in various contexts, and regulates them in ways that are not constrained by conventional algorithms [6]. Number sensitivity is needed by students in understanding and carrying out mathematical activities. However, several studies have shown that the exploration of
number sense in Indonesia's mathematics classrooms is still poorly trained and developed, so it is not surprising that students' number sense abilities are still low [7]–[9].

Reys et al [10] stated that number sense is the same as common sense in general, which is difficult to define, but at least can be explained by several characteristics: showing a good understanding of the concept of numbers and their operations, seeking to develop strategies that are useful in dealing with numbers and their operations, and will tend to use these insights in a flexible way to solve mathematical problems. Besides, a person with good number sense will be able to use his understanding of numbers to solve mathematical problems that are not limited by algorithms or traditional procedures [2], [5], [10]. The development of number sense needs to be a significant concern in introducing mathematics, especially at the elementary school level.

Subtraction is one of the primary operations that students learn at the beginning of the elementary school level. Doing subtraction is a part of our daily life, so conceptual mastery of this operation is crucial to be developed from an early age. We need to understand how to reduce to engage with society effectively when we use reduction when dealing with various everyday activities. Reys, et al. [10] explored the idea that subtraction and other basic mathematics such as addition provide the foundation for further computational-related work. However, so far, several studies have shown that this topic is still difficult for students to understand and causes many problems in learning numbers [3], [4], [11]–[13]. Among the difficulties experienced by students are problems with their informal approach [11] where students did not understand yet what the meaning of this operation [14], [15]; error patterns in computation (when using the algorithm) [16]; students try only followed the procedure without understanding the meaning of operation [15]; misconceptions in operations involving zeros [17]. Regarding the difficulties caused by using the algorithm or Standard Written Algorithm (SWA), many students experienced difficulties, especially in doing multi-digit number subtraction or numbers containing 0, especially with borrowing techniques [2], [12]. This situation then raises a debate: Should algorithms (SWTs) like borrowing techniques be taught to introduce subtraction operations in early mathematics classes? Or should it be removed and replaced with other techniques that are more rational for students? [16], [18].

The debate that occurs concerning the two questions above, accompanied by arguments that support each opinion. Groups that oppose standard written algorithms taught in the curriculum include the idea that the SWAs harm the number sense [19], negatively influence mental calculation [18], and several others. Meanwhile, people who defend SWAs argue that Students use SWAs more effectively and faster than mental computation [20], [21]. Students prefer SWA to mental computation [21] and several others. Fischer, through his comprehensive research, the title: Should we continue to teach standard written algorithms for the arithmetical operations? Concluded that a standard written algorithm (SWA), such as the columnar algorithm, which is taught by adding multiple procedures (e.g., multiplicity procedure), seems very effective. Fisher's research results are a bright point obtained scientifically to provide answers or views related to the above debate. Therefore, starting from Fisher's opinion, this research wants to see how teaching a standard written algorithm (SWA) on subtraction operations-borrowing techniques-combined with other techniques-non-borrowing techniques-can develop students' understanding of number subtraction operations.

The borrowing technique is the most common algorithm taught by teachers to complete number subtraction operations (especially multi-digit numbers). Many teachers are still teaching this part unreasonably to the students because it is too abstract and tends to provide a forced way, and does not offer a contextual strategy. This algorithm's giving is sometimes presented by the teacher without being accompanied by sufficient understanding of the concept of numbers, one of which is the concept of place value [4], [18] such as regrouping or trading. As a result, it is possible for students to succeed with the algorithm by relying on their memory of a series of procedures even without good understanding [16], [22]. However, when students forget the procedure, they will likely experience a failure to carry out the
subtraction operation. For example, the error lies when they subtract the number smaller than the number of the reducer or it contains 0. The procedure performed is correct, but the determination of the result of the subtraction is wrong. Forget to reduce the loan when subtracting units by units, tens by tens, and so on.

Basically, Fusnot [2] defined algorithms as a structured set of procedures that can be used across a problem, regardless of the number - they have an essential place in mathematics. Likewise, algorithms on borrow and carry techniques are developed based on place values. It implies that the teaching of the borrowing algorithm on the subtraction operation must reinforce the concept of good place value. Therefore, this study aims to describe how students thinking or perform subtraction on numbers through more contextual strategies. The contextual strategy referred to here is to use borrowing techniques (by strengthening the concept of place value) and adding another strategy (like Fischer [18] suggested)- it is without borrowing strategy. These strategies are designed with attention to numbers' learning trajectory and their operations in the 2nd grade of elementary school. A series of activities related to counting and place value are given to students before and when the introduction to number operations is presented.

2. Method
This research used a single-subject research (SSR) design with multiple baselines across individuals to evaluate the measures performed [23], [24]. Fraenkel et al.[25] said that multiple baseline designs are often used to collect data about a single event on many subjects or measure the behavior of a subject in two or more different settings. In a SSR, researchers attempted to determine whether the intervention impacted participant behavior by observing individuals for long periods and recording it before and after the intervention [24]. In particular, this study was conducted to determine how students think and solve subtracting two numbers by implementing various interventions, namely the borrowing technique (with the strengthening of the place value concept) and the technique without borrowing. In this study, researchers collect data and analyses related to subtracting the number of counting done by the students. This research is presented descriptively because this study aims to describe students thinking in performing subtraction on numbers through more contextual strategies that they understand. This research is still a preliminary study for a more in-depth review of how to design addition and subtraction learning to develop students' number sense in elementary schools. Thus, the selection of the SSR method is considered appropriate for use in this study. According to Moras et al.[26], SSR can serve as the first step in a research process (the whole), giving an implication to whether it would be relevant to proceed with studies consisting of a more significant number of participants using a group design. Research participants were two children in the 2nd grade of elementary school children (7-8 years old). Behavior and way of students thinking in learning subtraction operations observed during the intervention take place. The observation was conducted on both participants for eight mounts (starting from learning place values to learning number subtraction operations). Data collection techniques using tests consisting of the multi-digit number operation task. In addition to the tests, it also uses interviews with class teachers to obtain more accurate data collection. Then, the technique of data analysis is done by checking the test results and analyzing student answers.

3. Results and Discussion
The intervention given to the subject of this study began with strengthening the concept of place value as the initial instruction before students were introduced to number operations. In general, the activities developed in this intervention are as illustrated in Figure 1 below.
A series of activities related to place values and counting experiences were given to the students until finally leading to two numbers' addition and subtraction operations. The concept of a number of the subtraction operation is presented by relating it to the concept of addition. Concrete activities in the form of elimination and differences, also linking them to various model representations as suggested by Fusnot [2] and Reys et al. [5] were given to students during the intervention.

Several results and findings were obtained from the two participants after a series of interventions were carried out. These findings are based on an analysis of the answers of the two participants and in-depth interviews of both. The first finding was that when given a subtraction of two numbers, for example, 42-7, naturally students tended to immediately eliminate 7 (objects) from 42 with the help of a model that the students made themselves (Figure 2). This situation illustrates that the concept of regrouping is used by students in the problem-solving process.

The second finding, the students can use the subtraction strategy without borrowing technique (Figure 3), but students often forget to add the numbers they save at the end of their calculations. However, using this method found that the students knew the relationship between 60 and 59 + 1, but it did not seem meaningful. Because in another question, they returned to solve the same problem by "removing" it by using...
their representation model. The teacher then used it to relate this subtraction operation to the place values concept that students already understood. This situation seems to be going well, where students seem to understand the teacher's procedures.

The strategy shown in Figure 4 is the borrowing technique (with the concept of regrouping and trading), which students use in determining the subtraction of three-digit numbers. However, even when making subtractions using the borrowing technique, students' tendency to reduce from the largest number always occurs. The result is the students looking confused to continue, even though they try to do it several times by using an understanding of the value of the place of numbers. This situation is under what was conveyed by Fusnot [2] and Reys [5]. Therefore, this constraint can be the third finding in solving number subtraction operations using borrowing techniques. Students are more flexible in manipulating numbers, even though this takes a little longer for students to think about it.

The observation result of student activities during learning often shows that students use all these manipulations in their minds more often than writing them directly on paper. However, when teachers ask how they get answers to these problems, they are a bit difficult to explain, or the teacher usually has to ask
them to write down what they think. This finding shows that there is a tendency if students prefer to use mental computation rather than represent it on paper or books.

4. Conclusion
The intervention (several activities were made related to the number) can encourage students to understand the subtraction with borrowing strategy (by strengthening the concept of place value) or without borrowing (multiple procedures) in a fair and meaningful understanding. Using the two contextual strategies used show a positive effect on engaging students to familiarize themselves with number sense like balance activity or regrouping (to tens) concept. However, doing subtraction without using an algorithm (not borrowing system) for 2nd grade of elementary school children requires guidance and scaffolding from the teacher because otherwise, this will be a challenge for students. Besides, though the technique without borrowing is taught takes longer time for the teacher, it also takes a long time for students to solve subtraction problems than using procedures (standard written algorithm). In addition, subtraction operations by emphasizing number sense, such as break numbers into other forms with the same value, will allow students' creativity to perform number operations and be more flexible in operating numbers. Students were not limited by traditional algorithms but by exploring the numbers for developing student number sense.

One of the limitations of this study is that the results obtained are still limited to a small number of participants, which can be used as suggestions and study material for further research on a larger sample size. As another suggestion, it is necessary to develop learning designs in learning number operations (especially subtraction operations), which can develop students' mathematical thinking and student number sense.

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