Ultimate Informational Capacity of a Volume Photosensitive Media

Yuriy I. Kuzmin† and Viktor M. Petrov†
Saint Petersburg State Polytechnical University, 29 Polytechnicheskaya Street, Saint Petersburg 195251 Russia
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The ultimate information capacity of a three-dimensional hologram for the case of an optimal use of the dynamic range of a storage medium, number of pages, the readout conditions is considered. The volume hologram is regarded as an object of the information theory. For the first time the formalism of the reciprocal lattice has been introduced in order to estimate the informational properties of the hologram. The diffraction-limited holographic recording is analyzed in the framework of the reciprocal lattice formalism. Calculations of the information capacity of a three-dimensional hologram involve analysis of a set of multiplexed holograms, each of which has a finite signal-to-noise ratio determined by the dynamic range of the holographic medium and the geometry of recording and readout. An optimal number of pages that provides a maximum information capacity at angular multiplexing is estimated.

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I. INTRODUCTION

Information capacity and throughput of different optical objects, including holograms, is of vital importance for optical information processing and storage systems. Here we analyze the ultimate information capacity of a three-dimensional (volume) hologram for the case of an optimal use of the dynamic range of the storage medium.

II. INFORMATION CAPACITY AND RECIPROCAL LATTICE FORMALISM

Information capacity is directly related to the physical properties of the optical media. It can be defined as the maximum amount of information that can be recorded and then read out with an arbitrarily small probability of error. According to the Kotelnikov-Shannon sampling theorem, the information recorded in a hologram is fully determined by $4A\Delta^2 W$ pixels, where $A$ is the hologram cross section, and $\Delta^2 W$ is the two-dimensional width of the spectrum of recorded spatial frequencies. The factor $4A\Delta^2 W$ is a two-dimensional analog of the Nyquist frequency. The upper limit of information capacity of a three-dimensional and two-dimensional hologram can be found by the analogy with the Shannon formula for the channel capacity of a communication link in the presence of white noise.

$$C_{3D} = 4A\Delta^2 W N \log_2 \left(1 + R_{3D} \left(\Delta^2 W, N\right)\right), \text{ bit} \quad (1)$$

$$C_{2D} = 4A\Delta^2 W \log_2 \left(1 + R_{2D} \left(\Delta^2 W\right)\right), \text{ bit} \quad (2)$$

where $N$ is the number of multiplexed holograms (pages), $R = P_s/P_n$ is the signal-to-noise ratio at readout of one pixel, $P_s$ is the upper boundary of the average power of the image-forming signal, and $P_n$ is the average power of optical noises.

Let us find the maximum number of pixels that can be recorded in a photosensitive media as a volume hologram in the case only diffraction limitations exist. We assume that the elementary holographic grating is a spatial distribution of the recorded physical parameter invariant with respect to the translation of the type.

$$T_{3D} = n_1 e_1 + n_2 e_2 + n_3 e_3, \quad \forall n_1 \in \mathbb{Z}, \forall n_2, n_3 \in \mathbb{R} \quad (3)$$

$$T_{2D} = n_1 e_1 + \nu e_2 + \nu_0 e_1, \quad \forall n_1 \in \mathbb{Z}, \forall \nu, \nu_0 \in \mathbb{R} \quad (4)$$

for the three- and two-dimensional holograms, respectively; where $e_i$ are the basis vectors of translation, $\nu_0$ is the grating plane coordinate, $\mathbb{Z}$ and $\mathbb{R}$ are the sets of integer and real numbers. In the three-dimensional space the vector of translation (3) describes the set of parallel planes $T_{3D} \in \mathbb{Z} \otimes \mathbb{R}^2$ (Fig. 1a), the vector in (4) describes the set of collinear and complanar lines $T_{2D} \in \mathbb{Z} \otimes \mathbb{R}$ (Fig. 1c).

The reciprocal lattices corresponding to translations (3) and (4) in the $k$-space are

$$Q_{3D} = m q_1, \quad \forall m^1 \in \mathbb{Z} \quad (5)$$

$$Q_{2D} = m^1 q_1 + \mu q_2 + \mu^3 q_3, \quad \forall m^1 \in \mathbb{Z}, \forall \mu, \mu^3 \in \mathbb{R} \quad (6)$$

where $\mu$ is the coordinate of the reciprocal lattice plane, $q_i$ are the basis vectors of the reciprocal lattice satisfying the orthogonality relation $e_i \cdot q_i = 2\pi \delta_i^j$, where $\delta_i^j$ is the Kronecker symbol.

In the $k$-space the reciprocal lattice (5) is a set of equidistant points $Q_{3D} \in \mathbb{Z}$ (Fig. 1b) whereas the reciprocal lattice (6) is a set of collinear and complanar lines.
III. DYNAMIC RANGE, WORD LENGTH AND NUMBER OF MULTIPLEXED PAGES

The maximum number of pages recorded in a three-dimensional hologram at angular multiplexing can be calculated by summation over all vectors of the reciprocal lattice sitting on the Ewald sphere (i.e., over all the wave vectors of the recorded holographic gratings): \( \max(N) = 2k/\Delta k = 4L/\lambda \), where \( \Delta k = \pi/L \) is the minimum uncertainty of the wave vector, and \( L \) is the hologram thickness. Taking into account the expression (7), it is easy to find the maximum number of pixels that potentially could be recorded at all pages of a three-dimensional hologram in the case of an unlimited dynamic range

\[
\sup (4A\Delta^2W) \max(N) = \frac{32\pi AL}{\lambda^3} \tag{8}
\]

Estimates of the type of "volume"/\( \lambda^3 \) are often given for the ultimate information capacity of a hologram [2,5]; but an unjustified assumption of information storage in the form of stored elements of volume ("voxels") is frequently made in this case, and the two-dimensionality of the spectrum of spatial frequencies of the recorded image is ignored. The number of multiplexed holograms is determined by the finite dynamic range of the holographic medium on which the signal-to-noise ratio depends. This is the reason, while the estimation [5] practically is unachievable. At multiplexing, the information capacity does not grow in \( N \) times, as it could be concluded from a shallow analysis of expressions (1) and (8) that did not take into account the dependence \( R_{3D} = R_{3D}(N) \).

Let us consider now, how the number of pages affects the information capacity. In the case of a two-dimensional hologram the entire dynamic range is used to code each pixel with the maximum word length. For a three-dimensional hologram an exchange of the word length on the number of pages in the limits of the same dynamic range is possible. An increase in the number of pages is achieved by decreasing \( R_{3D} \) up to the word length of one bit per pixel. Let us show that there is an optimal number of pages at which the information capacity is the highest. The number of multiplexed holograms can be presented in the form \( N = \sqrt{P_s(1)/\sqrt{P_s(N)}} \), where \( P_s(1) \) is the maximum signal power for recording only one page for using the entire dynamic range, \( P_s(N) \) is the signal power for recording one of the \( N \) multiplexed pages. Now we can relate \( R_{3D}(N) \) and \( R_{3D}(1) = \max N \) \( R_{3D}(N) \) as

\[
R_{3D}(N) = \frac{P_s(N)}{P_n} = \frac{P_s(1)}{N^2P_n} = \frac{P_{3D}(1)}{N^2}
\]

If \( R_{3D}(1) >> N^2 \) the expression (11) acquires the form

\[
C_{3D}(N) = NC_{3D}(1) - 8A\Delta^2WN\log_2N \tag{9}
\]
This function has a maximum at

\[ N_0 = 2 \left( \frac{C_{3D}(1)}{8A\Delta^2W} - \frac{1}{\ln 2} \right) \]

Therefore, there is an optimal number of pages \( N_{opt} = \text{entier}(N_0) \) above which the information capacity will decrease because of a reduction in the signal-to-noise ratio \( R_{3D}(N) \).

It is interesting to note that in the case of a sufficiently high \( R_{3D}(1) \) the information capacity \( C_{3D}(N) \) of the three-dimensional hologram in which \( N \) pages are recorded is lower than the information capacity \( NC_{3D}(1) \) of the set consisting of \( N \) holograms in each of which only one page is recorded, all other conditions being equal. As follows from Eq. (9), the difference in the information capacity per one pixel is described by the function

\[ L(N) \equiv \frac{NC_{3D}(1) - C_{3D}(N)}{4A\Delta^2W} = 2N \log_2 N, \text{ bit} \]

The numerical calculations are shown in Fig. 2.

IV. CONCLUSION

To summarize, the hologram was regarded in our study as an object of the information theory. Calculations of the information capacity of a three-dimensional hologram involved analysis of the set of multiplexed holograms, each of which had a finite signal-to-noise ratio determined by the dynamic range of the storage medium. The problem of an optimal use of the dynamic range at angular multiplexing was solved. Analysis of diffraction-limited holographic information recording was carried out in the framework of the reciprocal lattice formalism, which allowed us to use such a basic property of an optical image as two-dimensionality of the spectrum of its spatial frequencies to a full extent.

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