A NOTE ON NICOLAS-AUGUSTE TISSOT: AT THE ORIGIN OF QUASICONFORMAL MAPPINGS

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Abstract. Nicolas-Auguste Tissot (1824–1897) was a French mathematician and cartographer. He introduced a tool which became known among geographers under the name *Tissot indicatrix*, and which was widely used during the first half of the twentieth century in cartography. This is a graphical representation of a field of ellipses, indicating at each point of a geographical map the distortion of this map, both in direction and in magnitude. Each ellipse represented at a given point is the image of an infinitesimal circle in the domain of the map (generally speaking, a sphere representing the surface of the earth) by the projection that realizes the geographical map.

Tissot studied extensively, from a mathematical viewpoint, the distortion of mappings from the sphere onto the Euclidean plane, and he also developed a theory for the distortion of mappings between general surfaces. His ideas are close to those that are at the origin of the work on quasiconformal mappings that was developed several decades after him by Grötzsch, Lavrentieff, Ahlfors and Teichmüller.

Grötzsch, in his papers, mentions the work of Tissot, and in some of the drawings he made for his articles, the Tissot indicatrix is represented. Teichmüller mentions the name Tissot in a historical section in one of his fundamental papers in which he points out that quasiconformal mappings were initially used by geographers.

The name Tissot is missing from all the known historical reports on quasiconformal mappings. In the present article, we report on this work of Tissot, showing that the theory of quasiconformal mappings has a practical origin.

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1. Introduction

Darboux, starts his 1908 ICM talk whose title is *Les origines, les méthodes et les problèmes de la géométrie infinitésimale* (The origins, methods and problems of infinitesimal geometry) with the words: “Like many other branches of human knowledge, infinitesimal geometry was born in the study of practical problems,” and he goes on explaining how problems that arise...
in the drawing of geographical maps, that is, the representation of regions of the surface of the Earth on a Euclidean piece of paper, led to the most important developments in geometry made by Lagrange, Euler, Gauss and others.

The theory of quasiconformal mappings has its origin in the problems of drawing geographical maps. Teichmüller, in the last part of his paper *Extremale quasikonforme Abbildungen und quadratische Differentiale* (Extreme quasiconformal mappings and quadratic differentials), published in 1939 [35], which is the main paper in which he develops the theory that became known as *Teichmüller theory*, makes some comments on this origin, mentioning the work of the French mathematician and geographer Nicolas-Auguste Tissot (1824–1897). Grötzsch, in his paper *Über die Verzerrung bei nichtkonformen schlichten Abbildungen mehrfach zusammenhängender schlichter Bereiche* (On the distortion of non-conformal schlicht mappings of multiply-connected schlicht regions), published in 1930 [19], mentions several times the name Tissot, referring to the *Tissot indicatrix* which he represents in the pictures he drew for his article. The directions of the major and minor axes of this ellipse constitute an important element in some of his results. A geographical map is the image of a mapping—henceforth called a projection—from the surface of the Earth, considered as a sphere or spheroid, onto the Euclidean plane. The Tissot indicatrix is a device introduced by Tissot, who called it the *indicating ellipse* (ellipse indicatrice, which was used by geographers until the middle of the twentieth century. It is a field of ellipses drawn on the geographical map, each ellipse representing the image by the projection—assumed to be differentiable—of an infinitesimal circle at the corresponding point on the sphere (or spheroid) representing the surface of the Earth. Examples of Tissot indicatrices are given in Figure 1.

In Figure 2, we have reproduced drawings from a paper of Grötzsch in which he represents the Tissot indicatrix of the maps he uses.

Although the work of Tissot is closely related to the theory of quasiconformal mappings, his name is never mentioned in the historical surveys of this subject, and the references by Grötzsch and by Teichmüller to his work remained unnoticed. In this note, I will give a few indications on this work.

Before surveying the work of Tissot in §3, I will give, in §2, a short biographical note on him.

### 2. Biographical note on Tissot

Nicolas-Auguste Tissot was born in 1824, in Nancy, which was to become, 26 years later, the birthplace of Henri Poincaré (whom we shall mention soon). Tissot entered the École Polytechnique in 1841. He started by occupying a career in the Army and defended a doctoral thesis on November

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1. The expression “infinitesimal circle” means here, as is usual in the theory of quasiconformal mappings, a circle on the tangent space at a point. In practice, it is a circle on the surface which has a “tiny radius.” In the art of geographical map drawing, these circles, on the domain surfaces, are all supposed to have the same small size, so that the collection of relative sizes of the image ellipses becomes also a meaningful quantity.

2. The reader should note that the École Polytechnique was, and is still, a military school.
Figure 1. Four geographical maps on which the field of ellipses (Tissot indicatrix) are drawn. The maps are extracted from the book *Album of map projections* [33]. These are called, from left to right, top to bottom, the *stereographic* [33, p. 180], *Lagrange* [33, p. 180], *central cylindrical* [33, p. 30] and *equidistant conical* projections [33, p. 92]. The first two projections are conformal and not area-preserving. The last two are neither conformal nor area-preserving.

Figure 2. Two figures from Grötzsch’s paper *Über die Verzerrung bei nichtkonformen schlichten Abbildungen mehrfach zusammenhängender schlichter Bereiche* [19]. Grötzsch drew the Tissot indicatrices of his quasiconformal mappings. (In each drawing, the major and minor axes of the ellipses are shown.)
17, 1851; cf. [36]. On the cover page of his thesis, he is described as “Ex-
capitaine du Génie.” Tissot became later a professor at the famous Lycée
Saint-Louis in Paris, and at the same time examiner at the École Poly-
technique, in particular for the entrance exam. He eventually became an
assistant professor (répétiteur) in geodesy at the École Polytechnique.

After having published, in the period 1856–1858, several papers and
Comptes Rendus notes on cartography, in which he analyzed the distor-
tion of some known geographical maps (see [38], [39], [40]), Tissot started
developing his own theory, on which he published three notes, in the years
1859–1860, [41] [42] [43], and then a series of others in the years 1865–1880
[44] [45] [46] [47] [48] [49] [50] [51]. He then collected his results in the memoir [52],
published in 1881, in which he gives detailed proofs. In a note on p. 2 of
this memoir, Tissot declares that after he published his first Comptes Ren-
dus notes on the subject, the statements that he gave there without proof
were reproduced by A. Germain in his Traité des projections des cartes
géographiques [18] and by U. Dini in his memoir Sopra alcuni punti della
teoria delle superfici [12]. He notes that Germain and Dini gave their own
proofs of these statements, which are nevertheless more complicated than
those he had in mind and which he gives in the memoir [52]. He also writes
that Dini showed that the whole theory of curvature of surfaces may be
deduced from the general theory that he had developed himself. In fact,
Dini applied this theory to the representation of a surface on a sphere, using
Gauss’s methods. Tissot also says that his ideas were used in astronomy, by
Hervé Faye, in his Cours d’astronomie de l’École Polytechnique [16]. The
texts of the two Comptes rendus notes [43] and [40] of Tissot are reproduced
in the Germain’s treatise [18].

Besides his work on geographical maps, Tissot wrote several papers on ele-
mentary geometry. We mention incidentally that several preeminent French
mathematicians of the nineteenth and the beginning of the twentieth cen-
tury published papers on this topics. We mention Serret, Catalan, Laguerre,
Darboux, Hadamard and Lebesgue; see e.g. [44], [37], [53].

On the title page of Tissot’s memoir [52] (1881), the expression Exam-
inateur à l’École Polytechnique follows his name, as he was in charge of
the entrance examination. In his Éloge historique de Henri Poincaré [8],
Darboux relates the following episode about Tissot, examining Poincaré:

Before asking his questions to Poincaré, Mr. Tissot suspended the exam
during 45 minutes: we thought it was the time he needed to prepare
a sophisticated question. Mr. Tissot came back with a question of the
Second Book of Geometry. Poincaré drew a formless circle, he marked the
lines and the points indicated by the examiner, then, after wandering long
enough in front of the blackboard, with his eyes fixed on the ground, he
concluded loudly: “It all comes down to proving the equality \( \frac{AB}{CD} \).
This is a consequence of the theory of mutual polars, applied to the two
lines.” Mr. Tissot interrupted him: “Very good, Sir, but I want a more
elementary solution.” Poincaré started wandering again, this time not

\[3\] Poincaré entered the École Polytechnique in 1873. In the French system of oral examinations,
which is still in use, a student is given a question or a set of questions which he is asked to
prepare while another student (who had already been given some time to prepare his questions)
is explaining his solutions at the blackboard, in the same room. Thus, it is not unusual that at
such an examination, some students listen to the examinations of others.
in front of the blackboard, but in front of the table of the examiner, facing him, almost unconscious of his acts; then suddenly he developed a trigonometric solution. Mr. Tissot objected: “I would like you to stay in Elementary Geometry.” Almost immediately after that, the examiner of Elementary Geometry was given satisfaction. He warmly congratulated the examinee and announced that he deserves the highest grade.

Poincaré kept a positive memory of Tissot’s examinations. He expresses this in a letter to his mother sent on May 6, 1874, opposing them to the 10-minute examinations (known as “colles”) that he had to take regularly at the École Polytechnique and which he said are pitiful. He writes: “When I think about the exams of Tissot and others, I can not help but take pity of these 10 minutes little colles where one puts in danger his future with an expression which is more or less exact or a sentence which is more or less well crafted, and where a person is judged upon infinitesimal differences.”

3. On the work of Tissot on geographical maps

Tissot studied at the École Polytechnique, an engineering school where the students had a high level of mathematical training and at a period where the applications of the techniques of differential geometry to all the domains of science were an integral part of the curriculum. His work is part of a well-established tradition where mathematical tools are applied to the craft of map drawing. This tradition passes through the works of preeminent mathematicians such as Ptolemy [31, 3], Lambert [22, 23], Euler [13, 14, 15], Lagrange [20, 21], Gauss [17], Chebyshev [5, 6], Beltrami [2], Liouville (see the appendices to [26]), Bonnet [4], Darboux [9, 10, 11], and there are others. It was known since antiquity that there exist conformal (that is, angle-preserving) projections from the sphere to the Euclidean plane. But it was noticed that these projections distort other quantities (length, area, etc.), and the question was to find projections that realize a compromise between these various distortions. For instance, one question was to find the closest-to-conformal projection among the maps that are area-preserving. Hence, the idea of “closest-to-conformal” projection came naturally. Among the mathematicians who worked on such problems, Tissot came closest to the notion of quasiconformality.

\[ AB = CD. \]
Let us summarize a few of his results on this subject.

An important observation made by Tissot right at the beginning of his memoir \[52\] (p. 1) is that finding the most appropriate mode of projection depends on the shape of the region—and not only its size, that is, on the properties of its boundary. Finding maps of small “distorsion” (where, as we mentioned, this word has several possible meanings) was the aim of theoretical cartography. Tissot discovered that in order for the map to minimize an appropriately defined distortion, a certain function \(\lambda\), defined by setting

\[
d\sigma^2 = (1 + \lambda)^2 ds^2,
\]

must be minimized in some appropriate sense, where \(ds\) and \(d\sigma\) are the line elements at the source and the target surfaces respectively. The minimality of \(\lambda\) may mean, for example, that the value of the gradient of its square must be the smallest possible.

In fact, Tissot studied mappings between surfaces that are more general than those between subsets of the sphere and of the Euclidean plane. He started by noting that for a given mapping between two surfaces, there is, at each point of the domain, a pair of orthogonal directions that are sent to a pair of orthogonal directions on the image surface. Unless the mapping is angle-preserving at the given point, these pairs of orthogonal directions are unique. The orthogonal directions at the various points on the two surfaces define a pair of orthogonal foliations preserved by the mapping. Tissot calls the tangents to these foliations principal tangents at the given point. They correspond to the directions where the ratio of lengths of the corresponding infinitesimal line elements attains its greatest and smallest values.

Using the foliations defined by the principal tangents, Tissot gave a method for finding the image of an infinitely small figure drawn in the tangent plane of the first surface. In particular, for a differentiable mapping, the images of infinitesimal circles are ellipses. In this case, he gave a practical way of finding the major and minor axes of these ellipses, and he provided formulae for them. This is the theory of the Tissot indicatrix.

From the differential geometric point of view, the Tissot indicatrix gives information on the metric tensor obtained by pushing forward the metric of the sphere (or the spheroid) by the projection mapping.

We recall that in modern quasiconformal theory, an important parameter of a map is the quasiconformal dilatation at a point, defined as the ratio of the major axis to the minor axis of the infinitesimal ellipse which is the image of an infinitesimal circle by the map (assumed to be differentiable at the given point, so that its derivative sends circles centered at the origin in the tangent plane to ellipses). The Tissot indicatrix gives much more information than this quasiconformal dilatation, since it keeps track of (1) the direction of the great and small axes of the infinitesimal ellipse, and (2) the size of this ellipse, compared to that of the infinitesimal circle of which it is the image.

Darboux got interested in the work of Tissot on geography, and in particular, in a projection described in Chapter 2 of his memoir \[52\]. He wrote a paper on Tissot’s work \[11\] explaining more carefully some of his results. He writes: “[Tissot’s] exposition appeared to me a little bit confused, and
it seems to me that while we can stay in the same vein, we can follow the following method [...] 8

Tissot showed then how to construct mappings that have minimal distortion.

Tissot’s work was considered as very important by cartographers. The American cartographer, in his book *Flattening the earth: two thousand years of map projections* [34], published in 1997 and which is a reference in the subject, after presenting the existing books on cartography, writes: “Almost all of the detailed treatises presented one or two new projections, they basically discussed those existing previously, albeit with very thorough analysis. One scholar, however, proposed an analysis of distortion that has had a major impact on the work of many twentieth-century writers on map projections. This was Tissot [...]”

Modern cartographers are still interested in the theoretical work of Tissot, see [24].

We mentioned several preeminent mathematicians who before Tissot worked on the theory of geographical maps. From the more recent era, let me mention Milnor’s paper titled *A problem in cartography* [25], published in 1969. The reader interested in the theory of geographical maps developed by mathematicians is referred to the papers [29], [27] and [28] which also contain more on the work of Tissot.

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