Searching for New Physics beyond the Standard Model in
Electric Dipole Moment

Takeshi Fukuyama

Department of Physics and R-GIRO,
Ritsumeikan University, Kusatsu, Shiga, 525-8577, Japan

*Electronic address: fukuyama@se.ritsumei.ac.jp
Abstract
This is the theoretical review of exploration of new physics beyond the Standard Model (SM) in electric dipole moment (EDM) in elementary particles, atoms, and molecule. EDM is very important CP violating phenomenon and sensitive to new physics.

Starting with the estimations of EDM of quarks-leptons in the SM, we explore new signals beyond the SM. However, these works drive us to wider fronteer where we search fundamental physics using atoms and molecules and vice versa.

Paramagnetic atoms and molecules have great enhancement factors on electron EDM. Diamagnetic atoms and molecules are very sensitive to nuclear P and T odd processes.

Thus EDM becomes the key word not only of New Physics but also of unprecedented fruitful collaboration among particle, atomic and molecular physics.

This review intends to help such collaboration over the wide range of physicists.

I. INTRODUCTION

This article is a review of the search of new physics beyond the Standard Model (SM) concentrating on electric dipole moments (EDM) of elementary particles like neutron, proton, leptons, quarks as well as atoms and molecules. The presence of EDM implies T-odd and P-odd interactions. So if it exists, it indicates the direct T noninvariance as well as CP violation if CPT invariance is assumed.

It is very important that these fundamental EDMs are enhanced in paramagnetic atoms ($d_{\text{atom}}$) and molecules ($d_{\text{molecule}}$) which have an unpaired electron. Also in diamagnetic atoms and molecules, proton and neutron EDMs appear via Schiff moment due to CP-violating hadron interactions.

The discovery of CP violation in $K_L^0 \rightarrow \pi^+\pi^-$ decay [1] in 1964 was an amazing event for the majority of theorists since the model at that time could not produce CP violation. The introduction of CP phase in the mixing matrix by Kobayashi-Maskawa [2] was 7 years after that, which becomes the unique origin of CP violation in the SM [3]. This CP phase opened the door to new frontiers in a vast range of physics, especially in B factories: Belle at KEKB (KEK) and BaBar at PEP-II (SLAC).
CP violation in B mesons is measured by observing the asymmetry

$$a_f(\tau) \equiv \frac{\Gamma(B^0(\tau) \to f) - \Gamma(B^0(\tau) \to \bar{f})}{\Gamma(B^0(\tau) \to f) + \Gamma(B^0(\tau) \to \bar{f})} = C_f \cos(\Delta m \tau) - S_f \sin(\Delta m \tau),$$

(1)

where

$$C_f \equiv \frac{1 - |\lambda|^2}{1 + |\lambda_f|^2}, \quad S_f \equiv \frac{2 \Im \lambda_f}{1 + |\lambda_f|^2}.$$  

(2)

Here $B = B_d^0 = |d\bar{b}>$ or $B = B_s^0 = |s\bar{b}>$, and $f$ is a CP eigenstate such as $J/\psi K_S$, $\pi^+ \pi^-$, $\rho K_S$. $\Im$ means an imaginary part. For $B_d^0 \to J/\psi K_s$, we have

$$\Im \lambda_{\psi K_s} = 0.734 \pm 0.054,$$

(3)

with world average. It may be more advantageous in searching for new physics to consider the SM loop suppressed process like $B^0 \to \phi K^0$ etc. However, these results seem to be consistent with the CKM mechanism.

As we will show, the EDM values predicted by the SM are very tiny because they appear first in three loops (quarks) and four loops (leptons) and are far smaller than the upper limit of the present and near future experiments. On the other hand, there are some physical phenomena which suggest new physics beyond the SM other than neutrino oscillation experiments. The anomalous muon magnetic moment, $a_\mu \equiv (g_\mu - 2)/2$ ($g$ is defined by (29)), is one such example

$$a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} = (26.1 \pm 8.0) \times 10^{-10},$$

(4)

corresponding to a $3.3\sigma$ discrepancy from the SM. There are also other indirect problems of the SM like the observed baryon asymmetry, $n_B/n_\gamma = 1 \times 10^{-10}$. Indeed, in the SM, CP violation is parametrized by the Jarlskog invariant, which is too tiny to produce this amount of asymmetry; we need other CP violating terms. The other implicit deficiencies of the SM are Dark Matter candidates and the hierarchy problem etc.

Under these situations, the EDM is very important since some models beyond the SM give rather marginal predictions on the electron and neutron EDMs on the upper bounds of ongoing experiments.

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1 In the broad sense, there is another CP phase called the $\theta$ term in the SM, playing an essential role in especially the EDM of diamagnetic atoms (see section (VC)).
So new models are required to recover all such discrepancies. Furthermore, it must reproduce much larger phenomena which the SM predicts beautifully like, for instance, flavour changing neutral currents (FCNCs) and other vast low energy physics phenomena.

Here we point out a peculiar property of the EDM:

As is well known, EDMs of elementary particles are enhanced in atoms and molecules. In this sense, the EDM provides an unprecedented strategy of using atoms and molecules for the search of fundamental properties of elementary particles.

Several review works on this subjects have been already published \[9\][10][11][12]. New features of this review is that it is written by the author who is studying new physics beyond the SM, and, therefore, emphasis is on this point. However, EDM studies drive us necessarily to a wide range of physics (and chemistry), particle physics, atomic and molecular physics. The great achievements are possible only by the collaboration of theoretical and experimental scientists over this wide range of fields. Under these situations, we try in this review to make a small but significant bridge between these wide communities of scientists.

Accordingly, we endeavor to give a self-complete concept of EDMs as far as possible, sometimes sacrificing the exhaustive citation of important references.

II. BASICS OF EDM

In this section we give the definitions and conventions used in this review, and basic formulae useful for the EDM.

A. Definitions and Conventions

Metric:

\[
g^{\mu\nu} \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1_{3\times3} \end{pmatrix}. \tag{5}\]

Pauli matrices and spin matrices:

\[
\sigma^1 \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{6}
\]

\[
S^i \equiv \frac{1}{2} \sigma^i. \tag{7}
\]
Gamma matrices:
\[
\gamma^0 \equiv \begin{pmatrix} 1_{2 \times 2} & 0 \\ 0 & -1_{2 \times 2} \end{pmatrix}, \quad \gamma^i \equiv \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 1_{2 \times 2} \\ 1_{2 \times 2} & 0 \end{pmatrix},
\]
\(8\)

Chirality projection:
\[
P_L \equiv \frac{1}{2}(1 - \gamma^5), \quad P_R \equiv \frac{1}{2}(1 + \gamma^5).
\]
\(9\)

Antisymmetric tensor:
\[
\sigma^{\mu\nu} \equiv \frac{1}{2} [\gamma^\mu, \gamma^\nu] \equiv \frac{1}{2} (\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu).
\]
\(10\)

The electromagnetic field tensor is
\[
F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu, \quad F^{0i} = -E^i, \quad F^{ij} = -\epsilon^{ijk}B^k, \text{ so } F^{12} = -B^3 \text{ cyclic.}
\]
\(11\)

The Cabbibo-Kobayashi-Maskawa matrix [2] and Jarlskog invariant [13]:
\[
V \equiv \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},
\]
\(12\)

\[
\begin{aligned}
\not{\gamma^\mu} P_L V \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \\
J_{CP} \equiv \left| \Im(V_{\alpha j}V_{\beta j}^*V_{\alpha k}V_{\beta k}^*) \right| = s_{12}s_{23}s_{13}c_{12}c_{23}s_{13}^2 \sin \delta.
\end{aligned}
\]
\(13\)

Here \(J_{CP}\) is the base independent CP phase called the Jarlskog parameter, appearing in CP violation processes via the Kobayashi-Maskawa mechanism. Hereafter, we denote the imaginary part (real part) of \(O\) by \(\Im(O)\) (\(\Re(O)\)). Apart from the EDM process discussed later, we also mention on neutrino oscillation processes,
\[
P(\nu_\beta \rightarrow \nu_\alpha) = \delta_{\alpha\beta} - 4 \sum_{j<k} U_{\alpha j}U_{\beta j}^*U_{\alpha k}^*U_{\beta k}^* \sin^2 \left( \frac{\Delta p_{jk}L}{2} \right) \\
+ 4i \sum_{j<k} U_{\alpha j}U_{\beta j}^*U_{\alpha k}^*U_{\beta k}^* \sin (\Delta p_{jk}L).
\]
\(15\)

Thus we can determine the CP odd term (the third term) by measuring both \(P(\nu_\beta \rightarrow \nu_\alpha)\) and \(P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha)\). T2K [14] found evidence of a nonzero \(\theta_{13}\) and recently the Daya Bay Collaboration [15] fixed it as
\[
\sin^2 2\theta_{13} = 0.092 \pm 0.016 \text{(stat)} \pm 0.005 \text{(syst)}.
\]
\(16\)

Therefore the above mentioned CP phase experiments have become crucial.
B. Effective Dipole Operator

A permanent EDM of the electron must lie along its spin, namely $d = d_e \sigma$.[16]

At tree level in the SM, a fermion $\psi$ of mass $m_\psi$ and electric charge $e$ (electron’s charge is $e = -|e|$) in the presence of electromagnetic field satisfies

$$ (\gamma(p - eA) - m) \psi = 0, \quad \bar{\psi} (\gamma(p + eA) + m) = 0. \quad (17) $$

However if we include loop corrections, the effective electromagnetic interaction Hamiltonian is given in general by

$$ V = e \bar{\psi} (p_2) \Gamma^\mu u_1 (p_1) A_\mu \equiv e J^\mu A_\mu(k) \quad (18) $$

with

$$ P \equiv p_1 + p_2, \quad k \equiv p_2 - p_1. \quad (19) $$

Here

$$ A^\mu = (\phi, A) \quad (20) $$

is a true vector and transforms as

$$ A^\mu \to (\phi, -A) \quad \text{under P, T transformation,} \quad (21) $$

whereas $J^\mu$ can be either a true or a pseudo vector.

First we consider the case where the two electron lines are external and the photon line internal. $J^\mu$ takes the general form

$$ J^\mu = F_1(k^2)(\bar{\psi}_2 u_1)P^\mu + F_2(k^2)\bar{\psi}_2 \gamma^\mu u_1 + F_3(k^2)(\bar{\psi}_2 u_1)k^\mu. \quad (22) $$

However, from gauge invariance, the current is conserved

$$ k_\mu J^\mu = 0 \quad (23) $$

and

$$ F_3(k^2) = 0. \quad (24) $$

Using Gordon’s decomposition for the bilinear form of a spinor of mass $m$,

$$ (\bar{\psi}_2 \sigma^{\mu\nu} u_1) k_\nu = -2m \bar{\psi}_2 \gamma^\mu u_1 + \bar{\psi}_2 u_1 P^\mu, \quad (25) $$
the interaction term is then given by

\[- e \bar{\psi} \gamma^\mu \psi A_\mu = - \frac{e}{2m} \bar{\psi} (i \partial^\mu - i \vec{\nabla}^\mu) \psi A_\mu - i \frac{e}{4m} \bar{\psi} \sigma^{\mu\nu} \psi F_{\mu\nu}. \tag{26}\]

We should note that

\[- i \frac{e}{4m} \bar{\psi} \sigma^{\mu\nu} \psi F_{\mu\nu} = \frac{e}{2m} \bar{\psi} \left[ \vec{\Sigma} \cdot \vec{B} - i \gamma_5 \vec{\Sigma} \cdot \vec{E} \right] \psi, \]

where

\[\Sigma \equiv \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix} . \tag{27}\]

Here off diagonal elements are suppressed by \(O(\nu/c)\) relative to diagonal ones. The magnetic dipole moment (MDM) is defined as the coefficient of \(\mathbf{B}\) in the above equation. However, we must consider quark condensate for hadron EDMs.

\[\mu_Q = Q \left( \frac{e}{2m} \right) \sigma, \quad \text{and} \quad \mu_u = -2 \mu_d, \tag{28}\]

where \(Q\) is the quark charge and \(\langle \bar{u}u \rangle = \langle \bar{d}d \rangle\) has been assumed (See (V C) for more detail). It is clear from (27) that the fermion has a magnetic dipole moment with \(g = 2\) at tree level in the SM (28), where

\[|\mu| = g \frac{e}{2m} \frac{1}{2}. \tag{29}\]

The second term is off-diagonal and there appears the additional P-odd \(\sigma^k p^k\) term in the product of the off diagonal element. The MDM and EDM of particles are defined in the rest frame and we hereafter neglect the off-diagonal element unless it is specified.  

On the other hand, for an axial vector current, the general form is

\[J^{5\mu} = G_1(k^2)(\bar{u}_2 \gamma_5 u_1) P^\mu + G_2(k^2)(\bar{u}_2 \gamma^\mu \gamma_5 u_1) + G_3(k^2)(\bar{u}_2 \gamma_5 u_1) k^\mu. \tag{30}\]

In this case, \(G_3\) survives due to chiral symmetry breaking. In weak interactions or higher loops in the SM or in new physics, the current includes both \(J^\mu\) and \(J^{5\mu}\) in general. Thus the following CP odd effective action appears,

\[- i \frac{e}{4m} \bar{\psi} \gamma^5 \sigma^{\mu\nu} \psi F_{\mu\nu} = \frac{e}{2m} \bar{\psi} \left[ i \vec{\Sigma} \cdot \vec{E} - \gamma_5 \vec{\Sigma} \cdot \vec{B} \right] \psi. \]

\[\text{This is true, especially for measuring EDM by spin precession as for most cases of neutral particles and atoms. It is not so serious for the measurements of EDMs of charged particles and neutral molecules.}\]
This will be discussed in more detail in connection with the EDM and MDM shortly.

Also we can consider another conserved current like the vector case

\[ a(k^2)(\gamma k k^\mu - k^2 \gamma^\mu)\gamma^5, \]  

which reduces to

\[ a(k^2)k^2 \sigma \]  

in the nonrelativistic limit.

This term is called an anapole term, which comes from the second term of

\[ A_i(r) = \int d^3 r' \frac{J_i(r')}{|r - r'|} \]  

in the expansion around \( r \), that is,

\[ A_i^{(2)}(r) = \left( \nabla_k \nabla_l \frac{1}{r} \right) T_{ikl} \]  

with

\[ T_{ikl} = \frac{1}{2} \int d^3 r' r'_k r'_l J_i(r'). \]  

At the loop level in the SM and/or models beyond the SM, the following effective interaction of gauge invariant form can be obtained:

\[ -i \bar{\psi} \left( A_L^{ij} P_L + A_R^{ij} P_R \right) \sigma^{\mu\nu} \psi_j F_{\mu\nu} \]

\[ \approx (A_L^{ij} + A_R^{ij}) \bar{\psi} \Sigma \cdot B \psi + i (A_R^{ij} - A_L^{ij}) \bar{\psi} \Sigma \cdot E \psi. \]

Here we have neglected off diagonal parts in the second equality.

For the electric and magnetic dipole moments, we take zero momentum of the photon. Then the imaginary part of the coefficients of the effective interaction vanish because of the optical theorem (imaginary part of the forward scattering amplitude is given by the sum of possible cuts of intermediate states). We find the anomalous magnetic dipole moment \( a_\psi \) and electric dipole moment \( d_\psi \) to be

\[ a_\psi = \frac{g - 2}{2} = -\frac{2m}{e} \Re(A_R^{ij} + A_L^{ij}), \]  

\[ d_\psi = 2 \Im(A_R^{ij} - A_L^{ij}). \]  

Note that \( A_L \) and \( A_R \) must include a fermion mass (\( m_\psi \) or a fermion mass in the loop) because the effective interaction \( \bar{\psi} \sigma^{\mu\nu} \psi \) changes the chirality which can be achieved by
adding a mass term in the fundamental Lagrangian. If one of the particles in the loop is
much heavier than the others, \( A_L \) and \( A_R \) are suppressed by the mass. Thus, for large \( A_L \)
and/or \( A_R \), it is preferred that masses of particles in the loop are similar to each other.

The effective interaction (36) also causes a \( \ell_i \rightarrow \ell_j \gamma \) decay where the decay rate is given by

\[
\Gamma(\ell_i \rightarrow \ell_j \gamma) = \frac{m^{3}_\ell}{4\pi} \left( |A_L^{ij}|^2 + |A_R^{ij}|^2 \right).
\]  

(39)

Thus EDM and MDM have opposite parities and different in the order of magnitude. However
they appears in parallel, and have some similarities also. One of them is their SU(6)
property \[17\] and will be discussed in Appendix A.

For an invariant electromagnetic field, EDM, MDM, anapole, and higher n-pole moments
appear as the multipole expansions of the Coulomb potential and vector potential. These
points are also discussed in Appendix B.

Quarks receive additional contributions, which will be discussed for diamagnetic atoms.
Here we list the results.

A strong CP violating term connected with the \( \theta \) vacuum (see Appendix G)

\[
L_{d=4} = \frac{g_2^2}{64\pi^2} \bar{q} G^a_{\mu\nu} G^a_{\rho\lambda} \epsilon^{\mu\nu\rho\lambda} \equiv \frac{g_2^2}{32\pi^2} \bar{q} G^a \cdot \tilde{G}^a.
\]  

(40)

In new physics beyond the SM, we have other P and T violating effective actions: the
chromoelectric dipole operator (cEDM)

\[
L_C = -\frac{i}{2} \bar{d}_q g_s \bar{q} G^{\gamma_5} T^a G_{\mu\nu} \equiv -\frac{i}{2} \bar{d}_q g_s \bar{q} G^{\gamma_5} q,
\]  

(41)

and the following dimension 6 operators,

\[
L_G = -\frac{1}{6} d_G f_{abc} G^{\phantom{a}a}_{\mu\rho} G^{\alpha b}_{\nu} G^{\phantom{c}c}_{\lambda\sigma} \epsilon^{\mu\nu\lambda\sigma} \equiv -\frac{1}{3} d_G f_{abc} G^{a} \tilde{G}^{b} \tilde{G}^{c},
\]  

(42)

the so-called Weinberg term \[18\], and

\[
L_{d=6} = \sum C_{ij}^{a} \bar{\psi}_i O_a \psi_j \bar{\psi}_j O_a \gamma_5 \psi_j.
\]  

(43)

Here \( \psi_i \) and \( \psi_j \) are leptons and/or nucleons. \( O_a \) are scalar, vector, and tensor gamma
matrices. We will explain the detailed physical implications in the diamagnetic atom in
section 6.
In the SM, weak interactions act with matter and gauge fields in the form,

\[ H_{\text{weak}} = \bar{\psi} P_L \Gamma_\mu \psi W^\mu \equiv J_\mu W^\mu. \] (44)

However, except for the top quark, fermion masses are small compared to weak bosons masses. They are described as the four-fermion coupling

\[ H_{\text{weak}} = J_\mu J^\mu. \] (45)

This is the case for tree diagrams. If you consider loop diagrams or new physics beyond the SM, we will encounter more general forms. We will discuss this in the Appendix C.

C. Experimental Bounds

We have no experimental signal of the EDM yet but have upper limits. They are as follows [19]. It is very impressive that recently we have a more precise upper limit of the electron EDM from molecule (YBF) than from atom (Tl).

\[ d_e \text{ from thallium atom } d(Tl) = (6.9 \pm 7.4) \times 10^{-28} \text{ e cm} \] (46)
\[ d_\mu = (3.7 \pm 3.4) \times 10^{-19} \text{ e cm} \] (47)
\[ d_n < 2.9 \times 10^{-26} \text{ e cm (90\%C.L.)} \] (48)
\[ d^{(199) Hg} < 3.1 \times 10^{-29} \text{ e cm (95\%C.L.)} \] (49)

\[ d_e \text{ from the molecule } d(YbF) = (-2.4 \pm 5.7_{\text{stat}} \pm 1.5_{\text{syst}}) \times 10^{-28} \text{ e cm} \] (50)

For reference, we give here the muon anomalous MDM, a non-null signal of new physics beyond the SM.

The deviations of the SM predictions from the experimental result are given by

\[ \Delta a_\mu[\tau] \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}}[\tau] = 14.8(8.2) \times 10^{-10}, \]
\[ \Delta a_\mu[e^+e^-] \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}}[e^+e^-] = 30.3(8.1) \times 10^{-10}. \] (51)

Here the hadronic contributions to \( a_\mu^{\text{SM}}[\tau] \) and \( a_\mu^{\text{SM}}[e^+e^-] \) were calculated [25] by using data of hadronic \( \tau \) decay and \( e^+e^- \) annihilation to hadrons, respectively. These values of \( \Delta a_\mu[\tau] \) and \( \Delta a_\mu[e^+e^-] \) correspond to 1.8\( \sigma \) and 3.7\( \sigma \) deviations from the SM predictions, respectively. EDMs and MDMs come from similar diagrams apart from CP transformation and the
differences of magnitudes in the MDM and EDM stem from the cancellation of diagrams and symmetry.

III. STANDARD MODEL

In this section we give the EDMs of quarks, hadrons, and leptons in the SM framework. Structure of matter multiplets in SM+(Dirac) neutrino is

\[
Q = \begin{pmatrix} u_1 & u_2 & u_3 \\ d_1 & d_2 & d_3 \end{pmatrix} \sim (3, 2, \frac{1}{6}),
\]

\[
u^c = (u^c_1, u^c_2, u^c_3) \sim (\frac{3}{6}, 1, -\frac{2}{3}),
\]

\[
d^c = (d^c_1, d^c_2, d^c_3) \sim (\frac{3}{6}, 1, \frac{1}{3}),
\]

\[
L = \begin{pmatrix} \nu \\ e \end{pmatrix} \sim (1, 1, -1),
\]

\[
e^c \sim (1, 1, 1),
\]

\[
\nu^c \sim (1, 1, 0).
\]

CP violation occurs as per Kobayashi-Maskawa mechanism, that is, CP phase in CKM mixing matrix for quarks or MNS matrix for leptons. Diagrammatically, it resembles with Lepton Flavour Violation (LFV) processes but for the former it is necessary to incorporate the non-zero Jarlskog parameter. Apart from the uncovered new phenomena in neutrino, the SM has the deficiency of baryon asymmetry. Jarlskog introduced \( A_{CP} \) defined by

\[
\det [M_u M_d^T M_d M_d^T] = i A_{CP}.
\]

Here \( M_u, M_d \) are up-type and down-type quark mass matrices. The observables are not these matrices but those which are invariant under rebasing and rephasing, that is, eigen values and CKM mixing matrix. By construction, \( A_{CP} \) is traceless and Hermitian, and characterizes the effect of CP violation. Its explicit form is

\[
\det A_{CP} = (m^2_t - m^2_e)(m^2_e - m^2_u)(m^2_e - m^2_u)(m^2_e - m^2_u)(m^2_e - m^2_u)(m^2_e - m^2_u)J_{CP},
\]

where \( J_{CP} \) is given by \( \text{(14)} \). As we will see the detail shortly, one-loop diagram (Fig. 1) gives zero contribution to the EDM. As for two loop diagrams (Fig. 2), using this \( J_{CP} \) and denoting by \( f \) the Green function of \( f \) flavoured fermion \( \text{(20)} \), the f quark EDM has the form
FIG. 1: The diagram for the EDM of $d$ quark at one loop level in the SM.

$$i \sum_{jkl} \Im(V_{jk}V_{lf}V_{jf}^*V_{lk}^*)f_{jklf} = \frac{1}{2} \Im(V_{jk}V_{lf}V_{jf}^*V_{lk}^*)f_{jkl} - lkJ.$$ \hspace{1cm} (55)

and $J_{CP} \approx 3 \times 10^{-5}$ is twice the area of unitary triangle $V_{ud}^*V_{ub} + V_{cd}^*V_{cb} + V_{td}^*V_{tb}$. One finds [27] that

$$n_B/n_\gamma \approx (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)/T_{12}^2 \times J_{CP} \approx 10^{-20}$$ \hspace{1cm} (56)

This falls short of the observed baryon asymmetry $n_B/n_\gamma \approx 10^{-10}$. We will show the detail loop by loop in the subsequent sections.

A. Quark EDM

For definiteness, we consider the EDM of $d$ quark. The advantage of $d$ in comparison with $u$ may be that $t$ can be in the loop with the weak interaction to avoid the GIM cancellation [28].

1. One Loop

In the one loop level (Fig. 1), elements of the CKM matrix appear as $|V_{qd}|^2$. Therefore, there is no EDM (imaginary part of coefficient) apparently.

2. Two Loop

At two loop level in the SM, two types of diagrams have an imaginary coefficient potentially. The diagrams are shown in Fig. 2. It is, however, shown that the imaginary part
FIG. 2: Diagrams for the EDM of $d$ quark at two loop level in the SM.

FIG. 3: Diagrams for the EDM of $d$ quark at three loop level in the SM.

of each diagram vanishes by the summation of contributions from all quarks of internal lines.

For the diagrams in Fig. 2 it is clear that $q''$ must not be $d$ quark because it gives $|V_{qq''}|^2|V_{qd}|^2$ of real value. By the same reason, $q'$ must be different from $q$.

3. Three Loops

Fig. 3 shows three loops diagrams which contribute to $d$ quark EDM in the SM. The formula for the contribution is given in (30) as

$$
\frac{d_d}{e} \simeq \frac{m_dm_c^2\alpha_s G_F^2 J_{CP}}{108\pi^5} \left\{ \left( L_{bc}^2 - \frac{2}{3}L_{bc} + \frac{\pi^2}{3} \right) L_{Wb} + \frac{5}{8}L_{bc}^2 - \left( \frac{335}{36} + \frac{2}{3}\pi^2 \right) L_{bc} - \frac{1231}{108} + \frac{7}{8}\pi^2 + 8\zeta(3) \right\},
$$

where $L_{ab} \equiv \ln(m_a^2/m_b^2)$. It results in

$$
d_d \simeq -10^{-34} \text{ e cm}
$$
while the triple log approximation (taking only $L^3$ term) gives

$$d_d \simeq +10^{-34} \text{ e cm.}$$

(59)

B. Neutron EDM

The dominant contribution to the neutron EDM in the SM comes from "two loop" diagram in Fig. 4. The interaction on the left part of the loop is given by the phenomenological interaction hamiltonian

$$H = iG_F m_\pi^2 \pi_n (A + B \gamma^5) u_\Sigma \phi_\pi ,$$

(60)

$$A = -1.93, \ B = -0.65,$$

(61)

where $u_n$, $u_\Sigma$, and $\phi_\pi$ stand for wave functions of neutron, $\Sigma^-$ baryon ($dds$), and $\pi^+$, respectively. The interaction on the right part of the loop is so-called "strong penguin" whose effective operator is given by

$$H_{\text{pen}} = \frac{iG_F \alpha_s(m) \Delta}{12 \sqrt{2} \pi} s_{23} s_{13} c_{23} \sin \delta \ln \frac{m_t^2}{m_c^2} \bar{\phi} \gamma_\mu (1 - \gamma^5) \lambda^a d \sum_{q=u,d} \bar{q} \gamma_\mu \lambda^a q$$

(62)

Here $\Delta \approx 1.3$ arises due to strong interaction. Note that the $H_{\text{pen}}$ seems to be obtained for $m_c \simeq m_t < m_W$. With these interactions, the neutron EDM was estimated as

$$d_n = d_n^{\text{short}} + d_n^{\text{long}} \simeq 10^{-32} \text{ e cm}.$$  

(63)

Here the first is the contribution from Fig. 4 ($O(\alpha_s G_F^2) \approx 10^{-34} \text{ ecm}$) and the second from Fig. 4 [31].
If we incorporate the rephasing invariance of strange wave function, this value is modified to $1.4 \times 10^{-31} \leq |d_n| \leq 9.9 \times 10^{-33}$ e cm [32]. Recently there appeared new type of diagram which contribute to EDM in loopless diagram [33]. Thus some controversies are still left even in the naive SM scheme.

Furthermore, there are new CP violating five and six dimensional operators,

$$\mathcal{L}_{CPV} = \sum_q d_q \bar{q}(\sigma F)\gamma_5 q + \sum_q \bar{d}_q \bar{\tau}(\sigma G)\gamma_5 q + wGGG + ...$$  \hspace{1cm} (64)

The details of this contribution will be discussed in the section of diamagnetic atom.

The concrete and model independent calculations are expected in the lattice QCD. The EDM of neutron is estimated by lattice calculation (see Fig. 5). It is also possible to guess the rough estimate of hadron EDM using SU(6). The detailed discussion is given in Appendix A.

C. Lepton EDM

For definiteness, let us concentrate on electron. Similarly to quark EDM, one and two loop diagrams do not contribute to electron EDM. In order to avoid GIM cancelation
FIG. 6: Diagram for the EDM of $W$ boson at two loop in the SM.

\[ \left( \propto \left( m_i^2 - m_j^2 \right) / m_W^2 \right), \] CKM matrix is better to be used than Maki-Nakagawa-Sakata matrix (lepton mixing). Two $W$ bosons (at least) should attached to the electron line in order to use a quark loop. Then, the electron EDM is caused by the $W$ boson EDM. It was shown that the $W$ boson EDM vanishes at two loop level \cite{36}.

The two loop diagram is shown in Fig. 6.

$J_{CP}$ defined by (14) is antisymmetric under $j \leftrightarrow l$ (corresponding to side line’s quarks), whereas it is symmetric in Fig.6. Adding another loop with gluon (see Fig. 7), the $W$ boson EDM in three loop was estimated as

\[ d_W \simeq J_{CP} \left( \frac{1}{16\pi^2} \right)^2 \left( \frac{g^2}{8} \right)^2 \frac{\alpha_s}{4\pi} \frac{e}{2m_W} \simeq 8 \times 10^{-30} \text{ e cm} . \] (65)

FIG. 7: Three loop diagram which may give a nonzero contribution to the EDM of $W$ boson.
The electron EDM in four loop was estimated with \( d_W \) as
\[
de_e \simeq \frac{g^2}{32\pi^2} \frac{m_e}{m_W} d_W \simeq 8 \times 10^{-41} \text{e cm}.
\] (66)

IV. BEYOND THE STANDARD MODEL

In this section we will discuss models beyond the SM.

As we have shown in the previous section, the SM predicts rather smaller values of EDMs than the experimental upper limits by roughly ten orders of magnitude. However, we also know that CP violation in the SM is insufficient for baryon asymmetry in the real world. Also we have many direct signals of new physics beyond the SM like neutrino oscillations and muon g-2 etc. Even if we stand in the SM, we have new 4- and 6-dimensional CP violating effective actions like (40) to (43), which have never been discussed so much in the previous section. Then it is very natural to estimate how much such new physics or new models predict the EDMs. In this section we concentrate on new physics beyond the SM. As for the new 4- and 6-dimensional CP violating effective actions, we will discuss in the sections of diamagnetic atoms and molecules.

A. Minimal Supersymmetric Standard Model (MSSM)

In the MSSM, all particles have their SUSY partners; sfermions \( \tilde{f} \) (bosons) for fermions \( f \), Higgsinos (fermions) for Higgs bosons, gauginos (fermions) for gauge bosons. Also another Higgs doublet is added to the SM for recovering chiral anomaly free condition once broken by this doubling \[37\]. Yukawa coupling is given by
\[
W_{\text{MSSM}} = Y_u \bar{u}QH_u - Y_d \bar{d}QH_d - Y_e \bar{e}LH_d + \mu H_u H_d
\] (67)

SUSY is broken at \( O(1\text{TeV}) \) by soft SUSY breaking terms which retain hierarchy problem. MSSM is a minimally extended supersymmetric SM and we will consider below the constrained MSSM (cMSSM) and \( \nu \)MSSM including light neutrino masses. In general soft breaking terms are

\[
L_{SB}^{(1)} = -(m_{\tilde{q}})^2 \tilde{Q}_{Li} \tilde{Q}_{Lj} - (m_{\tilde{u}})^2 \tilde{u}_{Ri} \tilde{u}_{Rj} - (m_{\tilde{d}})^2 \tilde{d}_{Ri} \tilde{d}_{Rj} - (m_{\tilde{e}})^2 \tilde{L}_{Li} \tilde{L}_{Lj} \\
- (m_{\tilde{e}})^2 \tilde{\tilde{e}}_{Ri} \tilde{\tilde{e}}_{Rj} - \mu^2 H_u^\dagger H_u - \mu^2 H_d^\dagger H_d - \mu^2 S^* S - b(H_u H_d + \text{c.c.}) \\
- \left(A_{uij} \tilde{\tilde{Q}}_{Li} H_u - A_{dij} \tilde{\tilde{d}}_{Ri} \tilde{\tilde{Q}}_{Lj} H_d - A_{eij} \tilde{\tilde{e}}_{Ri} \tilde{\tilde{L}}_{Li} H_d + \text{c.c.}\right)
\] (68)
FIG. 8: Diagrams of SUSY contributions to the EDM of fermions. Gluino $\tilde{g}$ contributes only for quark EDM.

$$L_{SB}^{(2)} = -\frac{1}{2} \left( M_3 \tilde{g}^a \tilde{g}^a + M_2 \tilde{W}^a \tilde{W}^a + M_1 \tilde{B}^a \tilde{B}^a + c.c. \right)$$

These include many CP violating phases, in general.

For the loop correction to the Higgs masses, the problem on its quadratic divergence is cancelled by the loop of SUSY partners (different statistics with same coupling).

SUSY particles contribute to fermion EDM at one loop shown in Fig. 8. The neutralinos $\tilde{\chi}^0$ and charginos $\tilde{\chi}^\pm$ are mass eigenstates, and they are linear combinations of Higgsinos and gauginos of $SU(2)_L$ and $U(1)_Y$. The $d$ quark EDM from the diagram is estimated $[38]$ as

$$d_d/e = \frac{1}{2m} \left( -\frac{1}{3} \right) \frac{4}{3} \frac{\alpha_s}{\pi} v \Im \left( V_R^{d\dagger} A_d V_L^d \right)_{11} \frac{\mu m}{(M^2 - \mu^2)^2} \left( \frac{1}{2} + 3 \frac{\mu^2}{M^2 - \mu^2} - \frac{\mu^2(\mu^2 + 2M^2)}{(M^2 - \mu^2)^2} \ln \frac{M^2}{\mu^2} \right).$$

Here $v$ is the common vacuum expectation value of $H_u$ and $H_d$. $m$ and $M$ are masses of $d$ quark and the universal squark mass (by assumption), respectively. If we adopt $v \Im (V_R^{d\dagger} A_d V_L^d) \sim M^2$ with rough estimations of $M = 100$ GeV and maximal mixings, we obtain

$$d_d \sim 10^{-22} \text{ e cm}.$$  

The value is clearly in conflict with the experimental bound on $d_n$. The naive estimation was done with $M_\tilde{q} \simeq 100$ GeV and a sizable CP-violating phase $\sin \phi \sim 1$. Therefore, the contradiction can be resolved by small phase (approximate CP symmetry) and/or heavy masses of SUSY particles. However, we adopt not such fine tuning but the universal soft SUSY breaking (cMSSM). $V_{L(R)}$ is the unitary matrix which rotates left (right)-handed weak eigen states, and therefore if $A_{ij} \propto Y_{ij}$, $V_R^{d\dagger} A_d V_L^d$ is a real, diagonal matrix and the
imaginary parts of its matrix elements vanish. Thus the small EDM leads us to relations of
trilinear terms in (67) and (69)

\[ A_u = A_{u0} Y_u, \quad A_d = A_{d0} Y_d, \quad A_e = A_{e0} Y_e. \]  (72)

Also we accept the universal soft SUSY breaking which is realized by gravity or gauge mediated SUSY breaking.

\[ M_1, M_2, M_3 \sim m_{1/2}, \]  (73)
\[ m_Q, m_L, m_u, m_d, m_e, m_{H_u}, m_{H_d} \sim m_0. \]  (74)

CP-violating phases appear only in flavor off-diagonal parts of matrices (Hermitian Yukawa matrices) and the CP violating effect is suppressed by small mixings only due to RGE.

\[ \delta^q_{LL} = \frac{(m^2_{q})_{ij}}{m^2_q}, \quad \delta^u_{RR} = \frac{(m^2_{u})_{ij}}{m^2_u}, \quad \delta^d_{RR} = \frac{(m^2_{d})_{ij}}{m^2_d}, \]
\[ \delta^l_{LL} = \frac{(m^2_{l})_{ij}}{m^2_l}, \quad \delta^e_{RR} = \frac{(m^2_{e})_{ij}}{m^2_e}. \]  (75)

The diagrams for the flavored case are shown in Fig. 9. When both left- and right-handed squarks (sleptons) have mixings, they contribute to the EDM in the form:

\[ J^{(d)}_{LR} = \Im \{ \delta^d_{RR} y_d \delta^q_{LL} \} ii, \quad J^{(u)}_{LR} = \Im \{ \delta^u_{RR} y_u \delta^q_{LL} \} ii. \]  (76)

It was shown (see e.g. [39]) that \( d_d \) can be \( \sim 10^{-25}-10^{-26} \) e cm, and then SUSY parameters are constrained by hadronic EDM.
Also there are additional diagrams called Barr-Zee diagrams that contribute to the EDM beyond the one loop level (see Fig. 10).

Thus the MSSM gives an elegant backgroung but itself does not predict any definite relations between quarks and leptons including all observations in neutrino.

In other word, its predictions are not testified from many constraints from various already known observations. These observations must be complicatedly related in reliable models.

It is Grand Unified Theory (GUT) which fulfills these deficiencies

B. Minimal Supersymmetric SU(5) GUT

Here and hereafter, ”minimal” means the minimum number of Higgs fields with renormalizable Yukawa coupling.

In SU(5) model, matter multiplets in are classified to
We need two Higgs $5^*_H$ and $24_H$, and SU(5) breaks down to $SU(3)_c \times SU(2)_L \times U(1)_Y$ by
\[
24_H = \text{diag} \left( V, V, -\frac{3}{2}V, -\frac{3}{2}V \right).
\] (78)

Here 5 and 10 are broken to
\[
5 = (1, 2)(1/2) + (3, 1)(-1/3), \quad 10 = (1, 1)(1) + (\overline{3}, 1)(-2/3) + (3, 2)(1/6).
\] (79)

Then, $SU(3)_c \times SU(2)_L \times U(1)_Y$ breaks down to $SU(3) \times U(1)_Q$ via
\[
5^* = (0, 0, 0, 0, v/\sqrt{2}).
\] (80)

Yukawa coupling has the form,
\[
W = \frac{1}{4} f_{ij}^{u} 10_i 10_j 5^*_H + \sqrt{2} f_{ij}^d 10_i 5^*_j 5^*_H + f_{ij}^\nu 5^*_i 1_j 5_H + M_{ij} 1_i 1_j.
\] (81)

Here the products imply
\[
10_i 10_j 5^*_H = \epsilon_{abcde} 10_i^{ab} 10_j^{cd} 5^*_H
\]
\[
10_i 5^*_j 5^*_H = 10_i^{ab} 5^*_j \delta^{*b}_H \quad \text{etc.}
\] (82)

with $a, b = 1, \ldots, 5$. Then mass matrices have the following forms
\[
M^d = M^e = f^d v/\sqrt{2}, \quad M^u = f^u v/\sqrt{2}
\] (83)

at GUT scale. This gives nice $b - \tau$ unification. The disparity between their observed masses is supposed to be due to renormalization effect from $M_{GUT}$ to their mass shell. Unfortunately, we can not explain the disparities between the first and second families even if we take the renormalization effects since it predicts wrong relation
\[
m_d/m_s = m_e/m_\mu, \quad m_s/m_b = m_\mu/m_\tau.
\] (84)
It also predicts too fast proton decay \[43\]. Hadronic EDM in SUSY SU(5) was discussed in \[44\].

Flipped SU(5) changes

\[u^c \leftrightarrow d^c, \quad e^c \leftrightarrow \nu^c\]  

and, therefore, we obtain in place of \((83)\)

\[M_u = M_\nu.\]  

This does not lead to apparent pathology. Moreover, it is attractive from doublet-triplet splitting: Higgs super potential has the form

\[W_H = 10 \times 10 \times 5 + \overline{10} \times \overline{10} \times \overline{5},\]  

which give rise to triplet mass

\[
\langle (1, 1, 0)_{10}(3, 1; 1/3)_{10}(3, 1; -1/3)_5 + \langle (1, 1, 0)_{10}(3, 1; -1/3)_{10}(3, 1; 1/3)_5
\]

but has no doublet mass since 5+\[\overline{5}\] has no partner in 10+\[\overline{10}\] (the missing partner mechanism). This is a solution to the doublet-triplet problem without additional adjoint Higgs. However, flipped SU(5) drives us to unrenormalizable heavy Majorana neutrino mass term,

\[10, 10, \overline{10}_H \overline{10}_H / \Lambda\]  

for the seesaw mechanism.

The other approaches are to introduce unrenormalizable term \[45\],

\[W_Y = \epsilon_{abcde} \left( f_{1ij} 10^{ab}_{ij} 10^{cd}_{ij} 24^e_{Hf} 5^f_{H} + f^{2ij} 10^{ab}_{ij} 10^{cf}_{ij} 24^d_{Hf} 5^e_{H} \right) / \Lambda \]

or to add another Higgs in Yukawa coupling,

\[Y_{45} 5^*_i 10^*_j 45^*_H\]  

etc. Unfortunately in SU(5) model, right-handed heavy Majorana neutrino belongs to the singlet and we have no constraint on it. Usually it is assumed to be diagonal but there is no reason to justify it. The other undetermined parameters (like \(m_0, M_{\frac{1}{2}}, A_0, \tan\beta\)) crucially depend on this assumption.

These points are remedied in the case of renormalizable SO(10) GUT, which is discussed in the next subsection (cEDM and parity odd nuclear interaction).
C. Minimal Supersymmetric SO(10) GUT

In the SO(10) Grand Unified Theory \[46\], fermions belong to a multiplet of 16 representation as

\[
\psi \equiv (u^r_R, u^q_R, d^r_R, d^q_R, e_R, \nu_R, u^r_L, u^q_L, d^r_L, d^q_L, e_L, \nu_L)^T.
\] (92)

Note that the right-handed neutrino $\nu_R$ is included naturally.

So-called minimal renormalizable SO(10) model includes Higgs bosons of $10$ and $\overline{126}$ in Yukawa couplings. This is because

\[16 \times 16 = 10 + 120 + 126.\] (93)

In order to make singlet in Yukawa renormalizable coupling, therefore, Higgs can be $10, 120, \overline{126}$. \(^3\) One Yukawa coupling leads to the conclusion that the CKM mass matrix is unity and we needs at least (minimal) two Higgs, $10 + 120$ or $10 + \overline{126}$. We select the latter set. This is because

\[\overline{126} = (6, 1, 1) + (\overline{10}, 1, 3) + (10, 3, 1) + (15, 2, 2)\] (94)

under $SU(4)_c \times SU(2)_L \times SU(2)_R$ and the second and third terms play essential role in type I and type II seesaw, respectively. In its SUSY version \[48\], $\overline{126}$ is necessary to be added. Providing the Higgs VEVs, $H_u = v \sin \beta$ and $H_d = v \cos \beta$ with $v = 174\text{GeV}$, the quark and lepton mass matrices can be read off as

\[
M_u = c_{10}M_{10} + c_{126}M_{126}
\]
\[
M_d = M_{10} + M_{126}
\]
\[
M_D = c_{10}M_{10} - 3c_{126}M_{126}
\]
\[
M_e = M_{10} - 3M_{126}
\]
\[
M_T = c_TM_{126}
\]
\[
M_R = c_RM_{126},
\] (95)

where $M_u, M_d, M_D, M_e, M_T,$ and $M_R$ denote the up-type quark, down-type quark, Dirac neutrino, charged-lepton, left-handed Majorana, and right-handed Majorana neutrino mass.

\(^3\) If we relax the renormalizability, different SO(10) models are also possible \[47\]. However, in this case, we have much less predictivity.
FIG. 11: The predictions for the electron EDM $|d_e|$, the muon anomalous MDM $\delta a_\mu$ [51], and the decay branching ratio of $\mu \to e\gamma$ in the minimal SUSY SO(10) with respect to the universal gaugino mass $M_{1/2}$. Trilinear term $A_0$ is assumed to be zero except for the last panel. The last panel is added for the reference to see the behaviour of non zero $A_0$, where the branching ratios, $\text{Br}(\tau \to \mu\gamma)$ (top) and $\text{Br}(\mu \to e\gamma)$ (bottom) are given as functions of $A_0$ (GeV) for $m_0 = 600$ GeV and $M_{1/2} = 800$ GeV. All are cited from [48].

matrices, respectively. Note that all the quark and lepton mass matrices are characterized by only two basic mass matrices, $M_{10}$ and $M_{126}$, and four complex coefficients $c_{10}$, $c_{126}$, $c_T$ and $c_R$, which are defined as $M_{10} = Y_{10}\alpha^d v \cos \beta$, $M_{126} = Y_{126}\beta^d v \cos \beta$, $c_{10} = (\alpha^u/\alpha^d) \tan \beta$, $c_{126} = (\beta^u/\beta^d) \tan \beta$, $c_T = v_T/(\beta^d v \cos \beta)$ and $c_R = v_R/(\beta^d v \cos \beta)$, respectively. These are the mass matrix relations required by the minimal SO(10) model. The model is very predictive by virtue of the relation between quark Yukawa matrix, lepton Yukawa matrix, and neutrino Majorana matrix. Fig. [48] shows the prediction for the electron EDM $|d_e|$ in the minimal SUSY SO(10) with respect to the universal gaugino mass $M_{1/2}$. The muon EDM $|d_\mu|$ exists above $|d_e|$ by roughly a factor of $10^2$. The muon anomalous MDM $a_\mu$ and the LFV decay branching ratio of $\mu \to e\gamma$ are also predicted (see Fig. [51]). EDM and LFV are due to the essentially same diagrams, apart from the fact that the former (latter) comes
from diagonal (offdiagonal) part of sfermion mass matrix.\(^4\)

The effective Lagrangian relevant for the EDM, MDM, and the LFV processes (\(\ell_i \to \ell_j \gamma\)) is described in (36). \(R, L = (1 \pm \gamma_5)/2\) is the chirality projection

\[
\mathcal{L}_{\text{eff}} = -\frac{e}{2} m_{\ell_i} \bar{\ell}_j \sigma_{\mu\nu} F^{\mu\nu} (A^{ij}_L P_L + A^{ij}_R P_R) \ell_i ,
\]

where \(P_{L,R}\) are Left-Right projection operators, and \(A_{L,R}\) the photon-penguin couplings of 1-loop diagrams in which chargino-sneutrino and neutralino-charged slepton are propagating. It should be noted that we have changed the normalization of \(A_{L,R}\) from (36) by \(\frac{e m_{\ell_i}}{2}\). The explicit formulas of \(A_{L,R}\) etc. used in our analysis are summarized in [48] [49]. If the diagonal components of \(A_{L,R}\) have imaginary parts, the EDMs of the charged leptons are given by

\[
d_{\ell_i}/e = -m_{\ell_i} \Im(A^{ii}_L - A^{ii}_R) \quad (97)
\]

in the new normalization. The rate of the LFV decay of charged-leptons is given by

\[
\Gamma(\ell_i \to \ell_j \gamma) = \frac{e^2}{16\pi} m_{\ell_i}^5 \left( |A^{ij}_L|^2 + |A^{ij}_R|^2 \right) ,
\]

while the real diagonal components of \(A_{L,R}\) contribute to the anomalous magnetic moments of the charged-leptons such as

\[
\delta a^{\text{SUSY}}_{\ell_i} = \frac{g_{\ell_i} - 2}{2} = -m_{\ell_i}^2 \Re \left[ A^{ii}_L + A^{ii}_R \right] .
\]

In order to clarify the parameter dependence of the decay amplitude, we give here an approximate formula of the LFV decay rate [49],

\[
\Gamma(\ell_i \to \ell_j \gamma) \sim \frac{e^2}{16\pi} m_{\ell_i}^5 \times \frac{\alpha_2}{16\pi^2} \frac{\left| (\Delta m^2_{\ell_i})_{ij} \right|^2}{M_S^5} \tan^2 \beta ,
\]

where \(M_S\) is the average slepton mass at the electroweak scale, and \(\left( \Delta m^2_{\ell_i} \right)_{ij}\) is the slepton mass estimated in Eq. (101). We can see that the neutrino Dirac Yukawa coupling matrix plays the crucial role in calculations of the LFV processes. We use the neutrino Dirac Yukawa coupling matrix of Eq. (102) in our numerical calculations. In the leading-logarithmic approximation, the off-diagonal components (\(i \neq j\)) of the left-handed slepton mass matrix are estimated as

\[
\left( \Delta m^2_{\ell_i} \right)_{ij} \sim -\frac{3m_0^2 + A_0^2}{8\pi^2} (Y^i_{\nu} L Y^*_{\nu})_{ij} ,
\]

\(^4\) We have added the last panel with nonzero \(A_0\) reflecting the recent discovery of Higgs-like particle around 126 GeV by the LHC. See the last part of Discussion for more detail.
where the distinct thresholds of the right-handed Majorana neutrinos are taken into account by the matrix \( L = \log[M_G/M_R] \delta_{ij} \).

Unlike the muon MDM, quark and lepton EDMs have still null observation. This is of course due to tiny CP violation and due to the cancellation of diagrams where \( \gamma \) (gluon) couples with slepton (squark) and where it does with Higgsino (gluino) in Fig. [50].

If we consider gauge mediation scenario for SUSY breaking, \( A_0 \approx 0 \). In the basis where both of the charged-lepton and right-handed Majorana neutrino mass matrices are diagonal with real and positive eigenvalues, the neutrino Dirac Yukawa coupling matrix at the GUT scale is found to be

\[
Y_\nu = \begin{pmatrix}
-0.000135 - 0.00273i & 0.00113 + 0.0136i & 0.0339 + 0.0580i \\
0.00759 + 0.0119i & -0.0270 - 0.00419i & -0.272 - 0.175i \\
-0.0280 + 0.00397i & 0.0635 - 0.0119i & 0.491 - 0.526i
\end{pmatrix}.
\] (102)

Semi-leptonic LFV processes are discussed in [52].

Thus the EDM, MDM, lepton flavor violations etc. are all closely connected, which are expected to be explained universally by GUT. However, we do not adhere to any special model in this review, and will discuss more phenomenological models in the following subsections.

These may be the remnants from GUT or may be independent of GUT. For instance, the adjoint representation of SO(10), \( 45 \) is decomposed into

\[ 45 = (1, 3, 1) + (3, 1, 1) + (15, 1, 1) + (6, 2, 2) \] (103)

under \( SU(4)_c \times SU(2)_L \times SU(2)_R \) and leads to Left-Right symmetric model, \( g_L = g_R \). Also the \( 10 \) representation is decomposed into

\[ 10 = (1, 2, 2) + (6, 1, 1), \] (104)

which leads us to two Higgs \( SU(2)_L \) doublets under the SM. Also the \( 126 \) is

\[ 126 = (6, 1, 1) + (10, 3, 1) + (10, 1, 3) + (15, 2, 2). \] (105)

If \( (10, 1, 3) \) \((10, 3, 1)\) has vev, it gives type I (type II, or Higgs triplet Model) seesaw model.

In the following we consider these models independently of GUT principally.  

\[ ^5 \text{He et al. discussed neutron EDM in those models [53].} \]
D. Two Higgs Doublet Model

Most of models beyond the SM has some new Higgs bosons. As the simplest extension of the Higgs sector of the SM which has only one Higgs doublet $\phi_1$, another Higgs doublet $\phi_2$ is introduced in the Two Higgs Doublet Model [54]. There are several types of the model depending on which doublet couples with which fermion:

- **type I (SM-like)**: $\phi_1$ couples with all fermions
  - $\phi_2$ decouples with fermions
- **type II (MSSM-like)**: $\phi_1$ couples with down-type quarks and charged leptons
  - $\phi_2$ couples with up-type quarks
- **type III (general)**: both of Higgs doublets couple with all fermions
  - etc.

If CP-violating term exists in the Higgs potential, e.g. $(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2)$ with an imaginary coefficient, there appears the mixing between CP-even ($H^0$) and CP-odd ($A^0$) neutral Higgs bosons. Then these Higgs bosons can contribute to the EDM (see Fig. 12). The mixing between $H^0$ and $A^0$ provides also CP-violating electron-nucleon effective interactions $(\bar{e}i\gamma^5e\bar{N}N$, etc.) which will contribute to the atomic EDM.

Barger et al. [55] gave large $d_\mu$ close to the proposed experiments. It should be remarked
that their values are calculated in units of $\Im Z$ where

$$\langle \phi_2^0 \phi_1^0 \rangle_q = \sum_n \frac{\sqrt{2} G_F Z_n}{q^2 - m_{H_n}^2},$$

$$\langle \phi_2^0 \phi_1^0 \rangle_q = \sum_n \frac{\sqrt{2} G_F \tilde{Z}_n}{q^2 - m_{H_n}^2}$$

and it is probable that $|\Im Z| \approx 0$. Indeed, the masses of neutral and charged Higgses and phases are tightly constrained from $R_b \equiv \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{hadrons})}$, $\Gamma(b \to s\gamma)$, $B^0 - B$ mixing, $\rho$ parameter etc., and we should take those constraints into account.

E. Higgs Triplet Model

In the Higgs Triplet Model, we introduce a SU(2) triplet $Y = 2$ scalar as

$$\Delta \equiv \begin{pmatrix} \Delta^+ / \sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+ / \sqrt{2} \end{pmatrix}, \quad \mathcal{L}_{\text{triplet Yukawa}} = -h_{\alpha \beta} \overline{\ell}_\alpha i \sigma^2 \Delta P_L \ell_\beta + h.c.$$ (107)

This model generates neutrino masses without right-handed neutrinos with the triplet vacuum expectation value $v_\Delta$ which is given by the explicit breaking of the lepton number. This model is very predictive because of a clear relation

$$m_{\alpha \beta} = \sqrt{2} v_\Delta h_{\alpha \beta},$$ (108)

where $m_{\alpha \beta}$ denotes the Majorana mass matrix for neutrinos. There is no new interaction with quarks and no effect on quark EDM. Unfortunately, this model cannot give a large contribution to lepton EDM also because of the absence of the new interaction with right-handed fermions. For example, one loop diagram for electron has a factor of $|h_{ae}|^2$ (similarly to Fig. 1).

F. Left-Right Symmetric Model

Left-Right (LR) model is used in a variety of ways and needed to be clarified. If we consider it as a remnant of SO(10), $SO(10) \to SU(4)_c \times SU(2)_L \times SU(2)_R$, it satisfies at $v_{PS}$ energy scale

$$g_L = g_R$$ (109)
and this Pati-Salam (PS) model is unified at $M_{\text{GUT}}$ as

$$\frac{M_4}{\alpha_4} = \frac{M_{2L}}{\alpha_{2L}} = \frac{M_{2LR}}{\alpha_{2R}} = \frac{M_{1/2}}{\alpha_{\text{GUT}}}.$$  \hspace{1cm} (110)

Also mixing matrices for left-handed and right-handed fermions are the same. Of course these constraints are realized at $v_{PS}$ but start to be violated as the energy goes down to the SM scale by renormalization effects.

However, if we consider a model of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, we are free from the above constraints. For instance, in the framework of SO(10) GUT, $v_R$ is of order of $O(10^{12})$ GeV. However, if we go apart from GUT but still consider that the mixing matrix for right-handed quarks $V_R$ has a similar structure as that for left-handed quarks $V_L$, the lower limit of $M_{WR}$ is relaxed to $M_{WR} > 1.6$ TeV \cite{60}. Moreover we may go beyond the restriction to V-A and V+A interactions only and consider general form;

$$L_{\mu \rightarrow e\tau} = -\frac{4G_F}{\sqrt{2}} \left[ g_{RR}^S(\bar{\nu}_e)(\bar{\nu}_\mu \mu_R) + g_{RL}^S(\bar{\nu}_e)(\bar{\nu}_\mu \mu_L) 
+ g_{LR}^S(\bar{\nu}_e)(\bar{\nu}_\mu \mu_R) + g_{LL}^S(\bar{\nu}_e)(\bar{\nu}_\mu \mu_L) 
+ g_{RR}^V(\bar{\nu}_e \gamma^\mu \nu_R)(\bar{\nu}_\mu \mu_R) + g_{RL}^V(\bar{\nu}_e \gamma^\mu \nu_L)(\bar{\nu}_\mu \mu_L) 
+ g_{LR}^V(\bar{\nu}_e \gamma^\mu \nu_R)(\bar{\nu}_\mu \mu_R) + g_{LL}^V(\bar{\nu}_e \gamma^\mu \nu_L)(\bar{\nu}_\mu \mu_L) 
+ \frac{g_{RR}^T}{2}(\bar{\nu}_e \sigma^{\mu\nu} \nu_R)(\bar{\nu}_\mu \sigma_{\mu\nu} \mu_R) + \frac{g_{RL}^T}{2}(\bar{\nu}_e \sigma^{\mu\nu} \nu_R)(\bar{\nu}_\mu \sigma_{\mu\nu} \mu_R) + H.c. \right] \hspace{1cm} (111)
$$

The charge of $U(1)$ in the LR model has a clear meaning as the difference between the baryon number $B$ and the lepton number $L$ in contrast to the mysterious hypercharge $Y$ in the SM. Similarly to $SU(2)_L$ doublet in the SM, the right-handed fermions compose doublet of $SU(2)_R$ in the LR Model. Therefore, the right-handed neutrinos $\nu_R$ are introduced naturally as $SU(2)_R$ partners of right-handed charged leptons. After the spontaneous breaking of $SU(2)_R \otimes U(1)_{B-L}$ to $U(1)_Y$, the hypercharge is given by

$$Y/2 = I_{3R} + (B - L)/2. \hspace{1cm} (112)$$

Since electric charge is connected by

$$Q = I_{3L} + Y/2, \hspace{1cm} (113)$$

\cite{59} implies the charge quantization, which is one of great achievements of \cite{59}.

Since we require the parity symmetry to the theory, the gauge coupling of $SU(2)_R$ must be the same as the one of $SU(2)_L$: $g_2 \equiv g_{2L} = g_{2R}$. The Higgs field which gives the Yukawa
terms is a complex bidoublet of $SU(2)_L \otimes SU(2)_R$ with $B - L = 0$. The bidoublet field can be expressed as

$$
\Phi \equiv \begin{pmatrix}
\phi^0_1 & \phi^+_2 \\
\phi^-_1 & \phi^0_2
\end{pmatrix},
$$

which transform as $\Phi \rightarrow \Phi' = U_L \Phi U_R^\dagger$ under $SU(2)_L$ and $SU(2)_R$.

$$
\langle \Phi \rangle = \begin{pmatrix}
\kappa & 0 \\
0 & \kappa'
\end{pmatrix}
$$

with $\kappa \neq \kappa'$ gives rise to the breaking of L-R symmetry. However, $\Phi$ is neutral ($B-L=0$) and $U(1)_{B-L}$ is not broken. So we need another Higgs. Usually, two complex triplet fields ($\Delta_L$ for $SU(2)_L$ and $\Delta_R$ for $SU(2)_R$) with $B-L=2$ are also introduced to generate Majorana neutrino masses (see also the Higgs Triplet Model in Sect. [IV E])

$$
\Delta_L = (3, 1, 2) \quad \Delta_R = (1, 3, 2).
$$

Then, the gauge symmetry breaking proceeds as follows: first $\Delta^0_R$ acquires vev $v_R$, leading to $SU(2)_L \times U(1)_Y$ with [112], which furthermore breaks to $U(1)_Q$ by the vev of $\Phi$.

The triplet Yukawa coupling for $\Delta_R$ must be equal to the coupling for $\Delta_L$ because of the parity symmetry which is spontaneously broken by $v_R$, and we have

$$
v_R \gg \kappa, \kappa' \gg v_L.
$$

Thus the two Higgs doublets model (not of all but its measure part) and Higgs triplet model in the previous subsections are combined together in left-right model.

Their vevs $v_R$ and $v_L$ are different to each other and from $\kappa$ and $\kappa'$.

Figure 13 shows one of diagrams which contribute to the EDM of the electron. In a simple case where there is only one flavor of leptons, the electron EDM is estimated [61] as

$$
|d_e| < \begin{cases}
8.2 \times 10^{-27} \frac{|\text{Im}(m_D)|}{\text{MeV}} \text{ cm for } \left(\frac{m_R}{m_W}\right)^2 \gg 1, \\
3.3 \times 10^{-26} \frac{|\text{Im}(m_D)|}{\text{MeV}} \text{ cm for } \left(\frac{m_R}{m_W}\right)^2 \ll 1,
\end{cases}
$$

where $m_D$ denotes the Dirac mass of the neutrino ($m_D\nu_L\nu_R$) and $m_R$, $m_W = 80$ GeV are the masses of heavy right-handed neutrino and the light $W$ boson, respectively.

The contributions of Higgs bosons in the LR model are not significant [62].

Another important contribution of $W_R$ in CP violation may be the neutrinoless double beta decay [63].
FIG. 13: One of diagrams of gauge boson contributions to the electron EDM in LR symmetric model. Gauge bosons contribute to quark EDM also.

G. Fourth Family Model

We set the quarks of the fourth family \([64]\) as

\[
(t', b')^T.
\]

The mixing angles and CP violating phases for \(N\) families are given by

\[
N^2 - (2N - 1) = \frac{N(N - 1)}{2} + \frac{(N - 1)(N - 2)}{2},
\]

where the first term is mixing angles and the second CP phases (see Appendix D for Majorana fermion case).

If we consider 4-generation SM (SM4) \([26]\), we can construct new Jarlskog parameter in place of \([54]\)

\[
A_{(234)} = (m_t^2 - m_{t'}^2)(m_{t'}^2 - m_c^2)(m_c^2 - m_u^2)(m_u^2 - m_d^2)(m_d^2 - m_s^2)(m_s^2 - m_b^2)J_{(234)}.\]

If we take heavy quark masses \(t'\) and \(b'\) in the range of 300 to 600 GeV, \(A_{(234)}/T_{EW}^{12}\) can be of order \(n_B/n_\gamma\).

\[
|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9999(10).
\]

Thus SM4 enhances CP violation and therefore the EDM also.

We may consider inside the loop only \(t, t', b'\) heavy and identify as

\[
c = u = U, \quad d = s = b = D.
\]
Then two loop diagram of Fig.2 vanishes for u quark but survives for d,s quarks, giving
\[ d_d \approx \lambda^7 \frac{\alpha_s \alpha_W}{4\pi} \frac{16}{16\pi^2} G_F m_d \frac{m^2}{m_W^2} \approx 3 \times 10^{-32} \text{ ecm}, \] (124)
where \( \lambda \) is the Wolfenstein parameter, \( \lambda = |V_{us}| = 0.22 \). This is only two orders of magnitude larger than the SM result of section III A 3. However, if we consider the chromoelectric dipole moment of the s quark,
\[ \tilde{d}_s \approx \Im (V_{ts}^{*} V_{tb} V_{hb} V_{hs}) \frac{\alpha_s \alpha_W}{4\pi} \frac{1}{16\pi^2} G_F m_s \frac{m_t^2}{m_s^2} \approx \lambda^5 \alpha_s \frac{\alpha_W}{4\pi} \frac{1}{16\pi^2} G_F m_s \frac{m_t^2}{m_s^2}. \] (125)
Using the estimation of the relation between \( d_N \) and \( \tilde{d}_s \) by [65], we have [26]
\[ d_N \approx -\frac{1}{2} \tilde{d}_s \approx 5 \times 10^{-30} \text{ e cm}. \] (126)

H. Extra Dimensions

The motivations for extra dimensions are diverse for both SUSY and non-SUSY. There are many SUSY breaking scenarios. The extra dimension makes the geometrical SUSY breaking possible. The gauge supermultiplets propagate in the bulk, and we get gaugino mass
\[ M_a \approx \frac{\langle F \rangle}{M_5^2 R_5}, \] (127)
which is called gaugino mediation.

Even if the theory itself is CP invariant, it may be violated by extending the theory to extra dimensions. This is because the compactification of the extra dimensions does not respect the symmetry in general. The CP phases come either from the boundary condition of extra dimensions (One of Scherk-Scwarz mechanisms [66]) or vev of fifth gauge field (Hosotani mechanism [67]) [68] [69].

\[ L = i \psi \gamma^5 (\partial_N - ieA_N) \psi - M \bar{\psi} \psi. \] (128)
Inclusion of a torsion term results in a nonminimal term
\[ \kappa \bar{\psi} \sigma F \psi \] (129)
or the fermion mass term via Hosotani mechanism

$$\bar{\psi} (M + i\gamma_5 X_4) \psi$$

(130)

with

$$X_4 \equiv \int dy A_4.$$  

(131)

Here $y$ is the coordinate of the extra dimension. The rotation of mass term gives rise to

$$\kappa \bar{\psi} \sigma F \gamma_5 \psi.$$  

(132)

The concrete constraints from the observation are given, for instance,  

$$d_u = \frac{4}{3} d_d - \frac{1}{3} d_u = \frac{4}{3} d_d,$$

(133)

simply because they did not consider up quark and

$$d(KK) \sim -2.3 \times 10^{-23} (R m_W)^2 \text{ [e cm]}.$$  

(134)

Since $\frac{4}{3} d(KK)$ must be less than the experimental upper limit, we have

$$\frac{1}{R} > 33 m_W \simeq 2.6 [\text{TeV}].$$  

(135)

V. THE EDMS OF ATOMS

The origin of the difficulties of the measurement of electron EDM is due to the absence of resonance unlike the neutron. A possible way is to perform the resonance experiment on neutral atom and to interpret the result in terms of electron EDM or hadron EDM. These object has very tiny values and let’s consider the effect linear in the EDM. In the subsequent atomic and molecular experiments we treat an internal electric field $E_{int}$ induced by atom or molecule as well as an external electric field $E$. This $E$ induces the EDM $e r_i$, with the intrinsic $\sum_i \beta \sigma_i \equiv \sum_i d_i e$. Thus the total Hamiltonian is a sum of unperturbed P,T-even terms,

$$H_0 = \sum_i c \alpha \cdot p_i + \beta_i m e^2 + V_{nucl}(r_i) + \sum_{i<j} V_C(r_{ij}),$$  

(136)

and T,P-odd term

$$H_{PTV} = - \sum_i d_i e \cdot E_{int} - \sum_i d_i e \cdot E - e \sum_i r_i \cdot E.$$  

(137)
The last term of \( H_0 \) is a two-body interaction and can not be solved exactly. The first and third terms are \( P \)-odd and the second \( P \)-even. So the first and second order energy shifts are given by

\[
E^1_m = - \sum_i \langle m_0 | d^{\dagger}_i | m_0 \rangle \cdot E
\]  

and

\[
E^2_m = \sum_{n \neq m} \sum_i \left\{ \frac{\langle m_0 | d^{\dagger}_i \cdot E_{\text{int}} | n_0 \rangle \langle n_0 | e \cdot E | m_0 \rangle}{E^0_m - E^0_n} + \frac{\langle m_0 | e \cdot E | n_0 \rangle \langle n_0 | d^{\dagger}_i \cdot E_{\text{int}} | m_0 \rangle}{E^0_m - E^0_n} \right\}.
\]

(139)

Here \(| m_0 \rangle\) is an eigen state of \( H_0 \). It should be remarked that, as will be shown in \[333\], EDM appears as the coefficient of the energy shift linear in the external electric field. So

\[
E_m = E^1_m + E^2_m \equiv -d' \cdot E,
\]

(140)

where

\[
d' = \sum_i \langle m_0 | d^{\dagger}_i | m_0 \rangle - \sum_{n \neq m} \sum_i \left\{ \frac{\langle m_0 | d^{\dagger}_i \cdot E_{\text{int}} | n_0 \rangle \langle n_0 | e \cdot E | m_0 \rangle}{E^0_m - E^0_n} + \frac{\langle m_0 | e \cdot E | n_0 \rangle \langle n_0 | d^{\dagger}_i \cdot E_{\text{int}} | m_0 \rangle}{E^0_m - E^0_n} \right\}.
\]

(141)

However this \( d' \) vanishes as follows.

\[
\langle m_0 | e r^i \cdot E | n_0 \rangle \langle n_0 | d^{\dagger}_e \cdot E_{\text{int}} | m_0 \rangle = ie \langle m_0 | r_i \cdot E | n_0 \rangle \langle n_0 | d^{\dagger}_e \cdot p_i | m_0 \rangle (E^0_m - E^0_n).
\]

(142)

Using the commutation relation \([r_i, p_j] = i\delta_{ij}\), the second term of (141) cancels with the first term. This is the famous Sciffl's theorem \[7\]. Since the expectation value of

\[
\Sigma \cdot E_{\text{int}} = [\Sigma \cdot \nabla, H_0]
\]

(143)

does not contribute to a linear Stark effect, the residual EDM is becomes

\[
V_{\text{EDM}} = -d_e (\beta - 1) \Sigma \cdot E_{\text{int}}
\]

(144)

and the residual energy shift is

\[
\Delta E = -d_e \langle m_0 | (\beta - 1) \Sigma \cdot E | m_0 \rangle - 2d_e \sum_{n \neq m} \frac{\langle m_0 | r \cdot E | n_0 \rangle \langle n_0 | (\beta - 1) \Sigma \cdot E_{\text{int}} | m_0 \rangle}{E^0_m - E^0_n}
\]

\[
= -d_{\text{(atom)}} \cdot E.
\]

(145)
The first term has no enhancement factor unlike the the second term and is much smaller than the second term, and

\[
d_{\text{atom}} = -2d_e \sum_{n \neq m} \sum_i \frac{\langle m_0 | \mathbf{r}^i | n_0 \rangle \langle n_0 | (\beta - 1) \Sigma^i | m_0 \rangle}{E_m - E_n}. \tag{146}
\]

\(d_{\text{atom}}\) has a large value when these states are almost degenerate. However, this enhancement is reflected in quite different ways in paramagnetic atoms and diamagnetic atoms. Though (145) itself is rather universal, \(H_{PTV}\) is variant. One example is P,T-odd Nucleon-electron interaction like (see Appendix C for the detail)

\[
+ iG_S' \bar{N}N \zeta_5 L + iG_P' \bar{N}\gamma_5 NL.
\tag{147}
\]

There are other CP violating effective interactions (see the last part of section IIB). Another important interaction is due to Schiff moment. The origin of Schiff moment itself is not unique.

As we mentioned, in the nonrelativistic Hamiltonian for a system of particles of finite size, there is no interaction energy of first order in the EDM if there is no misalignment of charge and moment distribution. Schiff also indicated in [71] how this theorem is violated by relativistic (Breit equation \(O((v/c)^2)\)) and the misalignment (the Schiff moment), where \(v\) is the velocity of electron or nucleon. Prior to this discovery, Salpeter [72] indicated that radiative corrections of \(O((v/c)^3)\) enhances hydrogen EDM.

Sandars pointed out that relativistic effect of electron EDM in heavy alkali atom gives large atomic EDM [73].

Let us proceed to discuss in more detail for hydrogen-like atom. The EDM has, by definition, odd parity and naively vanishes between the states with same parity.

For nonrelativistic case, its energy levels are

\[
E = -\frac{mZ^2\alpha^2}{2n^2}. \tag{148}
\]

Here \(n\) is the principal quantum number, and this energy is degenerate \(n^2\)-ply, \(\sum_{l=0}^{n-1} (2l+1) = n^2\) (see Eq.(161)).

If we consider the relativistic effects (spin effects) the degenerate energy levels are split into \(n\) fine-structure components at different \(j\) [74]. Let us obtain the relativistic terms w.r.t. \(O(v/c)\) (see Appendix E for relativistic expansion and Appendix F for nonrelativistic approximation for more detail).
At the first order of \( v/c \), we obtain the Pauli equation

\[
i\hbar \frac{\partial \varphi}{\partial t} = H \varphi = \left[ \frac{1}{2m} \left( p - \frac{e}{c} A \right)^2 + e\Phi - \frac{e}{2mc} \sigma \cdot B \right] \varphi. \tag{149}\]

In further approximation of \( O((v/c)^2) \), we assume \( B = 0 \) (i.e. \( A = 0 \)), and we get

\[
H = \frac{p^2}{2m} + e\Phi - \frac{e^4}{8m^2c^2} \sigma \cdot [E \times p] - \frac{e^2}{8m^2c^2} \nabla \cdot E. \tag{150}\]

If \( E \) is centrally symmetric,

\[
E = -\frac{r}{r} \frac{d\Phi}{dr}. \tag{151}\]

The spin-orbit interaction (the fourth term) becomes

\[
V_{sl} = \frac{e^2}{4m^2c^2} \sigma \cdot [r \times p] \frac{d\Phi}{dr} = \frac{\hbar^2}{2m^2c^2} \frac{dU}{dr} \cdot s. \tag{152}\]

For many electron case of atomic number \( Z \),

\[
V_{sl} = \sum \alpha_a \mathbf{l}_a \cdot \mathbf{s}_a, \tag{153}\]

where

\[
\alpha_a = \frac{\hbar^2}{2m^2c^2r_a} \frac{dU(r_a)}{dr_a}, \tag{154}\]

\[
|U(r_a)| \approx \frac{Ze^2}{\alpha_B} \approx \frac{Z^2me^4}{\hbar^2}, \tag{155}\]

and, therefore,

\[
\alpha \approx Z^4 \left( \frac{e^2}{\hbar c} \right)^2 \frac{me^4}{\hbar^2}. \tag{156}\]

For given total \( \mathbf{L} \) and \( \mathbf{S} \), the averaged \( V_{SL} \) is

\[
V_{SL} = A \mathbf{S} \cdot \mathbf{L}, \tag{157}\]

\[
\mathbf{L} \cdot \mathbf{S} = \frac{1}{2} [J(J + 1) - L(L + 1) - S(S + 1)]. \tag{158}\]

Since the value of \( \mathbf{L} \) and \( \mathbf{S} \) are same for a multiplet, energy splitting is given by the Lande’s interval rule,

\[
\Delta E_{J,J+1} = AJ. \tag{159}\]

Then we obtain

\[
\begin{align*}
1s_{1/2} & \\
(2s_{1/2}, 2p_{1/2}), & 2p_{3/2} \\
(3s_{1/2}, 3p_{1/2}), & (3p_{3/2}, 3d_{3/2}), & 3d_{5/2}.
\end{align*} \tag{160}\]

36
The remaining degeneracy is removed by the hyperfine-structure components caused by the radiative correction (Lamb shift $[75]$). So using this hyperfine splitting, we obtain large atomic EDM $[72]$. 

To estimate atomic EDM we need two informations. One is that of atomic wave functions and another is that of $P$ (or $T$) violating interactions.

Atomic EDM is due to those of constituents, electrons and nucleons. For electron EDM it is very important that the atom has an unpaired electron, and electron EDM is proportional to $Z^3 [73]$ (For the review, see $[10][11]$). If there is no unpaired electron (diamagnetic atom), we can measure quark (or hadron) EDM. For proton EDM, nucleus has an unpaired proton. In this case polarized molecule takes an important role $[76]$.

We are dealing with many electrons system and the electron wave functions are not in general exact. In this case the expectation values of the EDM depends on the representation.

\[
< b | r | a > \equiv r_{ba} = \frac{1}{E_b - E_a} < b | H_0 r - r H_0 | a > , \tag{161}
\]

where $H_0 = -\frac{\hbar^2}{2m} \nabla^2 + V$. Inserting this into (161), we obtain

\[
r_{ba} = \frac{1}{E_b - E_a} < b | \nabla^2 r - r \nabla^2 | a >= -\frac{i}{m\omega_{ba}} p_{ba} \tag{162}
\]

\[
= \frac{1}{m\omega_{ba}^2} (\nabla V)_{ba} . \tag{163}
\]

These three representations are of course equivalent. However, if we use the approximate wave functions, these values are different in general. So we must be careful what is the origins of discrepancies, due to different approximations or to representations $[77]$.

A. Relativistic Effects

The relativistic equation of atom with CP violating interaction ($\xi$ term) is

\[
\left[ \gamma_\mu \left( \partial_\mu - \frac{ie}{\hbar c} A_\mu \right) - \frac{imc}{\hbar} \right] u = \xi \frac{e}{4mc^2} \gamma_5 \gamma_\mu \gamma_\nu F_{\mu\nu} u . \tag{164}
\]

Electron EDM breaks the CP invariance and CP violating energy equation is

\[
(H_0 + H')u = Eu . \tag{165}
\]

Here $H_0$ is the Hamiltonian of the original single electron Dirac equation in the external field,

\[
H_0 = m\beta c^2 + \alpha \cdot (e p - eA) + e\phi , \tag{166}
\]

\[
\text{37}
\]
which leads to (136) in the static limit and $H'$ is CP violating interaction Hamiltonian of the right-hand side of (164) \cite{72}.

$$H' = \xi \frac{e \hbar}{2mc} \beta (\Sigma \cdot E + i \alpha \cdot B)$$

$$\approx -\xi \frac{e \hbar}{2mc} \beta \Sigma \cdot \nabla \phi,$$

(167)

where $\Sigma$ is defined by (27) and

$$\alpha \equiv \beta \gamma = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}.$$  \hspace{1cm} (168)

$\xi$ is dimensionless constant which measures the EDM in units of the Bohr magneton. Thus $\xi \ll 1$ implies that the EDM is small compared with $e$ times the Compton wavelength. The last approximation in (167) comes from the relativistic suppression due to the mixing of the upper half with the lower one. It should be remarked that $\sigma \cdot E$ is T-odd (T is time reversal operator) but $\alpha \cdot B$ (the suppressed term) T-even.

We consider a hydrogen-like atom with charge $Z$, where $E = \frac{Ze}{r^2}e_r$.

$$H' = -\xi Z \alpha r^{-2} s_r \text{ with } s_r = \sigma \cdot e_r / 2$$  \hspace{1cm} (169)

in atomic units $e = m = \hbar = 1$. The Hamiltonian of the above single electron system in the mean field approximation is generalized for many electrons system as \cite{73}

$$H'_0 = \sum_i \left[ \beta_i mc^2 + \alpha_i \cdot c_i + e\phi_i \right] + \sum_{j \neq k} \frac{1}{2} \left[ \frac{e^2}{r_{jk}} + B_{jk} \right].$$  \hspace{1cm} (170)

Here suffix indicates the quantity of i'th electron and $B_{jk}$ is the relativistic corrections. Taking the retard potential into consideration, Lagrangian can be described up to order $O((\frac{v}{c})^2)$ as

$$\frac{e^2}{r_{jk}} + B_{jk} \equiv \frac{e^2}{r_{jk}} \left[ 1 - \frac{1}{2} \left( v_j \cdot v_k + \frac{(v_j \cdot r_{jk})(v_k \cdot r_{jk})}{r_{jk}^2} \right) \right].$$  \hspace{1cm} (171)

Further higher order $\geq O \left( (\frac{v}{c})^3 \right)$ corrections come from photon emission (Breit interaction) \footnote{This term only contributes to the intrinsic EDM and we denote $H'$, distinguishing it from $H_{PTV}$ of (137).}.
and

$$U_{jk} = \frac{e^2}{r_{jk}} - \pi \left( \frac{e\hbar}{mc} \right)^2 \delta(r_{jk}) - \frac{e^2}{2m^2c^2r_{jk}} \left( p_j p_k + \frac{(r_{jk}p_j)(r_{jk}p_k)}{r_{jk}^2} \right)$$

$$+ \frac{e^2\hbar}{4m^2c^2r_{jk}^3} \left( -(\sigma_j + 2\sigma_k)[r_{jk}p_j + (\sigma_k + 2\sigma_j)r_{jk}p_k] \right)$$

$$+ \frac{1}{4} \left( \frac{e\hbar}{mc} \right)^2 \left( \frac{\sigma_j\sigma_k}{r_{jk}^3} - 3\frac{(\sigma_j r_{jk})(\sigma_k r_{jk})}{r_{jk}^5} - \frac{8\pi}{3} \sigma_j \sigma_k \delta(r_{jk}) \right).$$

The second line corresponds to spin-orbit interaction and the third spin-spin interaction.

If we incorporate the spin of nucleus, the degeneracy of $J$ is split (hyperfine structure),

$$V_{iJ} = a_i \cdot J,$$  \hspace{1cm} (173)

where $i$ and $J$ are the spin of nucleus and total angular momentum of electron envelope, respectively. However, in this hyperfine splitting dominant contribution comes from magnetic dipole and electric quadrupole and does not play important role in the EDM.

Thus the linear Stark appears as relativistic effects in duplicate meanings, i.e. $1 - \beta$ and $B_{jk}$ components. Many particle interaction effects are due to this relativistic effect as well as due to the other nonrelativistic excitation effects.

We proceed to the detailed calculation of single electron case (169). The operator $s_r$ commutes with $M^2$ and $M_z$, but has odd parity, and

$$< l = j \pm \frac{1}{2}, j, m | s_r | l = j \mp \frac{1}{2}, j, m > = \frac{1}{2}.$$  \hspace{1cm} (174)

Let the radial part of $u = r\chi_{nl}$. Then it satisfies

$$\left\{ \left[ 2\mu \phi - \frac{d^2}{dr^2} \right] + l(l + 1)r^{-2} - 2\mu E_{nl} \right\} \chi_{nl} = 0,$$  \hspace{1cm} (175)

where $\mu$ is the reduced mass of electron and set equal to unity in the subsequent equations. It follows from (174) that

$$< n, l_+, j, m | H'| n', l_-, j, m > = -\xi Z\alpha(2j + 1)^{-1}(E_n - E'_n) \int dr \chi_{nl_+}\chi_{nl_-}$$  \hspace{1cm} (176)

with $l_\pm = j \pm \frac{1}{2}$. Therefore, naively the first-order perturbation vanishes. The exception is discussed in section 5.2. The second-order perturbation energy is obtained by use of (176)

$$\Delta E_{n,l_\pm,j} = (\xi Z\alpha/(2j + 1))^2 \int dr \chi_{n,l_\pm} \left[ E_n + \frac{1}{2} \frac{d^2}{dr^2} + \frac{Z}{r} - \frac{1}{2} \frac{l_\pm(l_\pm + 1)}{r^2} \right] \chi_{nl_\pm}.$$  \hspace{1cm} (177)
Using
\[< r^{-2} > = \frac{Z^2}{n^3(l + 1/2)}, \quad (178)\]
we obtain
\[\Delta E_{n,l\pm,j} = \pm Z^4\xi^2n^{-3}(2j + 1)^{-1}(l_\pm + \frac{1}{2})^{-1}\alpha^2 Ry \quad (179)\]
with \(Ry = \frac{me^4}{2\hbar^2} = 13.6\text{eV} \).

**B. Peculiar Property of Paramagnetic Atom**

The atomic enhancement factor defined by
\[K \equiv \frac{d_{\text{atom}}}{d_e} \quad (180)\]
is given by
\[K = \sum_m \frac{4(Z\alpha)^3r_{m0}\hbar c}{(J + 1)\alpha_B\gamma(4\gamma^2 - 1)(N_0N_m)^3/2(E_m - E_0)} \quad (181)\]
for alkali atom. Here the sum is taken over the excited state \(m\), and \(N_0, N_m\) are effective principal quantum number defined in \((192)\). \(\gamma = \sqrt{(j + 1/2)^2 - Z^2\alpha^2}\) and \(r\) is electric dipole radial integral,
\[r_{nl,n'l'} = < n', l'|r|n, l - 1 > = \sqrt{l} \int_0^\infty R_{n',l-1}R_{nl}r^3 dr \quad (182)\]
in units of \(a_B = \frac{\hbar^2}{me^2}\) Bohr radius. We will derive \((181)\) (see Eq. \((195)\)). We start with the general relativistic arguments. For diamagnetic atoms the dominant contribution to atomic EDM comes from that of nucleus, which will be discussed later.

In hydrogen-like atom, states of different angular momenta \(l\) with fixed principal number \(n\) are degenerate in nonrelativistic approximation. The eigenfunctions with external field are the superposition of the field-free functions with different \(l\)-values, which gives the linear Stark effect. Let us write the Dirac spinor in the form
\[u_\pm = r^{-1} \begin{pmatrix} \chi_{2\pm}(r)\eta_{jl\pm}, & -i\chi_{1\pm}(r)\eta_{jl\pm} \end{pmatrix}^T. \quad (183)\]
\(\chi_i\) satisfy the following equations,
\[\begin{align*}
\frac{d\chi_1}{dr} - \kappa \frac{\chi_1}{r} &= \left[ \frac{mc}{\hbar} \left( 1 - \frac{E}{mc^2} \right) - \frac{Z}{r} \right] \chi_2, \\
\frac{d\chi_2}{dr} + \kappa \frac{\chi_2}{r} &= \left[ \frac{mc}{\hbar} \left( 1 + \frac{E}{mc^2} \right) + \frac{Z}{r} \right] \chi_1, \quad (184)
\end{align*}\]
\[ \kappa = \mp (j + \frac{1}{2}) \text{ for } j = l \pm \frac{1}{2}. \]  

Using (167) and (174), we obtain
\[ \langle n, j, l_+, m|H'|n, j, l_-, m \rangle = -\frac{1}{2} \xi Z \alpha \int_0^\infty dr r^{-2} (\chi_{2+} \chi_{2-} - \chi_{1+} \chi_{1-}). \]  

\( \chi_{2\pm} \) and \( \chi_{1\pm} \) are related to each other as (184) and we obtain
\[ 4 \langle l_+|H'|l_- \rangle = \xi Z \alpha^3 \int_0^\infty dr (D_+ \chi_+ + D_- \chi_-), \]  

where
\[ D_\pm = \frac{d}{dr} \pm \frac{j + 1/2}{r}. \]

The exact Dirac wave functions with given \( n, l, j \) are [74] [80]
\[ \chi_2 \frac{r}{r} = -\frac{\Gamma(2\gamma + n_r + 1)}{\Gamma(2\gamma + 1) \sqrt{n_r!}} \sqrt{\frac{1 + \epsilon}{4 N (N - \kappa)}} \left( \frac{2Z}{Na_B} \right)^{3/2} e^{-\frac{2Zr}{Na_B}} \left( \frac{2Zr}{Na_B} \right)^{\gamma-1} \times \]
\[ \times \left[ -n_r F \left( -n_r + 1, 2\gamma + 1, \frac{2Zr}{Na_B} \right) + (N - \kappa) F \left( -n_r, 2\gamma + 1, \frac{2Zr}{Na_B} \right) \right], \]

and
\[ \chi_1 \frac{r}{r} = -\frac{\Gamma(2\gamma + n_r + 1)}{\Gamma(2\gamma + 1) \sqrt{n_r!}} \sqrt{\frac{1 - \epsilon}{4 N (N - \kappa)}} \left( \frac{2Z}{Na_B} \right)^{3/2} e^{-\frac{2Zr}{Na_B}} \left( \frac{2Zr}{Na_B} \right)^{\gamma-1} \times \]
\[ \times \left[ n_r F \left( -n_r + 1, 2\gamma + 1, \frac{2Zr}{Na_B} \right) + (N - \kappa) F \left( -n_r, 2\gamma + 1, \frac{2Zr}{Na_B} \right) \right]. \]

Here \( F \) is the confluent hypergeometric function and \( n_r \) radial quantum number, the number of nodes of radial part of the wave function,
\[ n_r = \frac{\alpha Z \epsilon}{\sqrt{1 - \epsilon^2}} - \gamma, \quad n = n_r + |\kappa| \]

with \( \epsilon = \frac{E}{mc^2} \), and
\[ N = \sqrt{n^2 - 2n \kappa} \left( \sqrt{\kappa^2 - \alpha^2 Z^2} \right). \]

Here we have used
\[ 1 - \epsilon^2 = \frac{\alpha^2 Z^2}{N^2} \]

and normalized \( \lambda r = \frac{Z}{Na_B} \) as in the hypergeometric functions. \( N \) is called apparent principal quantum number and
\[ E_{nl} = -\frac{mZ^2 \alpha^2}{2N^2}. \]
Using these forms, (186) finally reads, for instance \[72\],

\[
\langle 2s_{1/2}|H'|2p_{1/2}\rangle = \frac{\xi Z^3 \alpha(\gamma - 1)}{2\gamma(2\gamma - 1)(\gamma + 1)(2\gamma + 1)^{1/2}} Ry, \tag{195}
\]

where \(\gamma\) takes the value \(\sqrt{1 - Z^2\alpha^2}\) in this case. For small \(Z\alpha\), it is reduced to

\[
\langle 2s_{1/2}|H'|2p_{1/2}\rangle = \xi Z^5 \alpha^3 \frac{8\sqrt{3}}{3} Ry. \tag{196}
\]

For more general case

\[
<j, l_+|H'|j, l_- >= -\frac{4(Z\alpha)^3}{\gamma(4\gamma^2 - 1)(N_sN_p)^{3/2}} Ry. \tag{197}
\]

For heavy alkali atom, for instance, cesium, \[9\]

\[
K(Cs) = d(Cs)/d_e = -\frac{16}{3} \frac{Z^3 \alpha^2 r(6s, 6p_{1/2})}{a_B \gamma(4\gamma^2 - 1)(N_sN_p)^{3/2}} \frac{Ry}{E(6p_{1/2}) - E(6s)} = 118. \tag{198}
\]

The radial integral is experimentally known \[81\]

\[
r(6s, 6p_{1/2}) = \int_0^\infty dr r^3 R_{60}(r)R_{61}(r) = 5.5a_B. \tag{199}
\]

Eq. (198) should be checked with the experimental result \[24\].

For Francium (Z=87, 7s→7p_{1/2}), \(K(Fr)\) is estimated as 873. However, as was stated in \[10\], (181) is not applicable for atoms with complex configurations and requires electrons’ correlations. Such calculations are performed in, for instance, \[82\] and \(K(Fr)\) is modified to 895.

There are some discrepancies on the estimate of enhancement factor of \(K\) of Thallium [Xe]4f^{14}5d^{10}6s^{2}6p^{1} \[83 \[84 \[85\]. The discrepancy seems to come from the starting assumptions. \[84\] considered that Thallium has three valence electrons, 6s^26p^1, whereas \[85\] considered it having one valence electron. If we adopt \[84\],

\[
d^{(205\text{Tl})} = -582(20)d_e \tag{200}
\]

or if we take \[85\],

\[
d^{(205\text{Tl})} = -466d_e \rightarrow d_e < 1.6 \times 10^{-27}\text{e cm}. \tag{201}
\]

In preparing this revised version, an interesting paper has just appeared \[86\] which asserts that this discrepancy disappears, converging to \(K = -573\).
Xe has closed electron shell of $5s^2\,5p^6$ but we may one electron of $5p$ state excited to $5p^5\,6s^1$, which resembles with that of Cs, [Xe]$6s^1$, whose enhancement factor was estimated to $K^{(133\text{Cs})} = 114$ or $120.53$.

As for $^{129}\text{Xe}$, the lowest excited state with a $6s$ electron has a enhancement value $K^{(129\text{Xe}^*)} = 120$ or $111$, and

$$d_e < 3.2 \times 10^{-24} \text{ cm.} \quad (202)$$

Using these results, Ellis et al. considered the maximal EDMs of nuclei.

C. Chiral Condensate

Before discussing diamagnetic atom, we will briefly resume QCD chiral dynamics. This is because hadronic matrix elements are described in terms of quark condensates by using operator product expansion.

Let us begin with the following effective action (see Appendix G for the implication of the effective action).

$$L = \overline{q}(i\gamma^\mu D_\mu - m)q - \frac{\alpha_s}{4\pi}G^{\mu\nu}\tilde{G}_{\mu\nu}, \quad (203)$$

where

$$D_\mu = \partial_\mu - ig_sA_\mu^a\lambda^a. \quad (204)$$

This action is invariant under $SU(3)_L \times SU(3)_R$ transformations in the limit of $m_u = m_d = m_s = 0$. That is,

$$Q_Lq = e^{i\alpha^a\lambda^a}q, \quad Q_Rq = e^{i\beta^a\lambda^a\gamma_5}q, \quad (205)$$

where $u, d, s$ quarks constitute $SU(3)$ group,

$$q = (u, d, s)^T \quad (206)$$

and we have the following conserved currents

$$j_{L,R}^{\mu\alpha} = \overline{q}_{L,R}\lambda^\alpha\gamma^\mu q_{L,R}. \quad (207)$$

Here $\lambda^a$ are the Gell-Mann’s $3 \times 3$ matrices and $q_L$ ($q_R$) are left-handed (right-handed) part of $q$.

$$j^{a\mu} = j_L^{a\mu} + j_R^{a\mu}, \quad (208)$$

$$j_5^{a\mu} = j_L^{a\mu} - j_R^{a\mu}. \quad (209)$$
So we have the conserved currents and conserved charges $Q_a$ and $Q_{5a}$. They satisfy the algebras

$$[Q_a, Q_b] = i f_{abc} Q_c, \quad [Q_{5a}, Q_b] = i f_{abc} Q_{5c}, \quad [Q_{5a}, Q_{5b}] = i f_{abc} Q_c. \quad (210)$$

However, this group is not exact and they are spontaneously broken to

$$Q_a |0\rangle = 0, \quad Q_{5a} |0\rangle \neq 0. \quad (211)$$

Thus there appear 8 pseudo Nambu-Goldstone bosons. Pseudo implies that the original chiral symmetry ($Q_5$ transformation) is not exact. It is broken by

$$H_{SB} = m_u \overline{u} u + m_d \overline{d} d + m_s \overline{s} s. \quad (212)$$

This can be rewritten as

$$H_{SB} = (m_u + m_d + m_s)(\overline{u} u + \overline{d} d + \overline{s} s)/3$$
$$+ (m_u - m_d)(\overline{u} u - \overline{d} d)/2$$
$$+ (2m_s - m_u - m_d)(2\overline{s} s - \overline{u} u - \overline{d} d)/6. \quad (213)$$

Here the first line is an SU(3) invariant, the second breaks isospin SU(2), and the third represents the deviation of s quark mass from the SU(3) symmetry.

$$M_{\pi}^2 = (m_u + m_d) B + O(m_q^2 \ln m_q), \quad (214)$$
$$M_{K^\pm}^2 = (m_u + m_s) B + O(m_q^2 \ln m_q), \quad (215)$$
$$M_{K^0}^2 = (m_d + m_s) B + O(m_q^2 \ln m_q). \quad (216)$$

Here $B = -\frac{2}{f_\pi^2} \langle 0|\overline{q} q|0\rangle$ with pion decay constant $f_\pi = 93$MeV, and we have used the chiral limit

$$f_\pi = f_K, \quad \langle 0|\overline{u} u|0\rangle = \langle 0|\overline{d} d|0\rangle = \langle 0|\overline{s} s|0\rangle. \quad (217)$$

Adler-Bell-Jackiw axial vector singlet current anomaly [93] and its non-Abelian version is

$$\partial_\mu j_5^\mu = 2i \sum_{q=u,d,s} m_q \overline{q} \gamma_5 q + \frac{N_f}{8\pi^2} \left( F \tilde{F} + G^a \tilde{G}^a \right). \quad (218)$$

with the number of flavour $N_F$. Since

$$\frac{N_f}{8\pi^2} G^a \tilde{G}^a = 2N_f \partial^\mu K_\mu \quad (219)$$
FIG. 14: Chiral anomaly induces $\pi^0 \to \gamma \gamma$ via $i f_0 \pi F \tilde{F}$ interaction.

with

$$K_\mu \equiv \frac{1}{16\pi^2} \epsilon_{\mu\nu\rho\sigma} A^\nu (\partial^\rho A^\sigma + \frac{1}{3} g f^{abc} A^\rho A^\sigma).$$

(220)

In the limit of $m_u = m_d = m_s = 0$, the axial current $j^5_\mu$ gets conserved again by replacing $j^5_\mu$ with

$$\tilde{j}^5_\mu = j^5_\mu - 2N_f K_\mu.$$  

(221)

Thus quark condensate is essential for chiral symmetry breaking. Nevertheless, anomalous term is crucial to the presence of $\pi^0 \to 2\gamma$. The neutral axial vector current gives the modified PCAC relation,

$$\partial^\mu j^0_5 = f_\pi m_\pi^2 \pi^0 + \frac{\alpha}{4\pi} F \tilde{F}.$$ 

(222)

We will come back to this problem in the next subsection. U(1) problem concerning QCD condensate is discussed in Appendix H.

D. Peculiar Property of Diamagnetic Atom

Diamagnetic atom has no unpaired electron, and the main contribution of atomic EDM comes from the misalignment between charge and the EDM distribution of nucleus. Thus hadronic part of the atomic EDM manifests itself through the Schiff moment [71] [94].

$$H_{atom} = H_{electron} + H_{nucleus} + \sum_{i=1}^{Z} (e\Phi(r_i) - e r_i \cdot E) - d_{nucleus} \cdot E,$$

(223)

where $r_i$ are the i’th electron coordinates and $d_{nucleus}$ is the nuclear EDM. $E$ means the external $E$. It should be remarked for diamagnetic atoms that the Stark term due to electron’ intrinsic spin term (the first term of (137)) is replaced by that of nucleon. $\Phi(r)$ is the nuclear electrostatic potential given by

$$\Phi(r) = \int \frac{\rho(x)d^3x}{|r-x|},$$

(224)
where $\rho(x)$ is the charge density of nucleus.

Here it is important to notice [95]

$$-i \left[ \sum_{i=1}^{Z} p_i, H_{atom} \right] = -e \sum_{i=1}^{Z} \nabla_i \Phi(r_i) + Ze E, \tag{225}$$

where $p_i$ are the momentum of atomic electrons, and the first term is the average electric field induced by atomic electrons. The expectation value of this commutator in the energy eigenstate vanishes and we may add

$$V = \langle d_{nucleus} \rangle \cdot E - \frac{1}{Z} \sum_{i=1}^{Z} \langle d_{nucleus} \rangle \cdot \nabla_i \Phi(r_i) \tag{226}$$

to $H_{atom}$ as far as we consider the expectation value. This implies we may change

$$- d_{nucleus} \cdot E \rightarrow -(d_{nucleus} - \langle d_{nucleus} \rangle) \cdot E. \tag{227}$$

So the expectation value is zero. This is another statement of the Schiff theorem. From the first term of (225), we should consider the interaction of atomic electrons with the nucleus,

$$\Phi(r_i) - \frac{1}{Z} \langle d_{nucleus} \rangle \cdot \nabla_i \Phi(r_i) \tag{228}$$

as the screened electrostatic potential. Therefore, the atomic EDM reads

$$d_{atom} = \sum_n \left\langle 0 \left| e \sum_i^Z r_i | n \right\rangle \langle n | e \sum_i^Z \left( \Phi(r_i) - \frac{1}{Z} \langle d_{nucleus} \rangle \cdot \nabla_i \Phi(r_i) \right) | 0 \right\rangle \frac{E_0 - E_n}{h.c.} \tag{229}$$

Using the charge distributions

$$\int \rho(x) d^3x = Z|e|, \quad \int x \rho(x) d^3x = \langle d_{nucleus} \rangle, \int x^2 \rho(x) d^3x = Z|e| \langle x^2 \rangle_{ch}, \int (x_k x_{k'} - \frac{1}{3} \delta_{kk'} x^2) \rho(x) d^3x = Z|e| \langle Q_{kk'} \rangle \quad \text{etc} \tag{230}$$

$$\left\langle 0_N \left| e \Phi(r) - \frac{1}{Z} \langle d_{nucleus} \rangle \cdot \nabla \Phi(r) \right| 0_N \right\rangle = -\frac{Ze^2}{|r|} + 4\pi e S \cdot \nabla \delta(r) + ... \tag{231}$$

Here ... indicates electric octupole and higher pole contributions, and $S$ is the famous Schiff moment [96], (The detailed derivation is given in Appendix B)

$$S_{ch}^{ch} = \frac{e}{10} \sum_{p=1}^{Z} \left( r_p^2 - \frac{5}{3} (r_p^2)_{ch} \right) r_p. \tag{232}$$

The $\langle Q_{kk'} \rangle$ vanishes for $^{199}$Hg, $^{129}$Xe, $^{225}$Ra.
There is another Schiff moment $S^{\text{nucl}}$ due to the misalignment between the charge distribution and the EDM distribution of nucleus, whose derivation is given in Appendix I.

Corresponding to these situations, we should consider (232) more generally

$$S = \frac{1}{10} \sum_{N}^{A} \sum_{i} e_{i} \left( (r_{N} + \rho_{i})^2 - \frac{5}{3} (r_{N}^2)_{ch} \right) (r_{N} + \rho_{i}).$$

Here $r_{N}$ is a $N$’th nucleon position and $\rho_{i}$ is the position of the $i$th charge inside the $N$’th nucleon, and

$$\sum_{i} e_{i} = e_{N}, \quad \sum_{i} e_{i} \rho_{i} = d_{N}.$$  \hspace{1cm} (234)

Retaining the terms up to linear in $\rho$, we have

$$S = S^{ch} + S^{\text{nucl}},$$

where $S^{ch}$ is given in (232) and

$$S^{\text{nucl}} = \frac{1}{6} \sum_{N}^{A} d_{N} (r_{N}^2 - \langle r^2 \rangle_{ch}) + \frac{1}{5} \sum_{N}^{A} \left( r_{N} (r_{N} \cdot d_{N}) - \frac{1}{3} d_{N} r_{N}^2 \right).$$

(236)

Usually $S^{\text{nucl}}$ is considered to be small compared with $S^{ch}$. The mean value of $S^{ch}$ is nonzero only in the presence of P- and T=odd nucleon-nucleon interactions.

For the arguments of hadronic EDM, we must express hadronic CP violating interactions in terms of those of (40) and (41). This will be discussed in the last part of this subsection (see (283)). They are described as

$$L_{\pi NN} \equiv g_{\pi NN}^{(0)} N \tau^{a} N \tau^{a} + g_{\pi NN}^{(1)} N N \pi^{0} + g_{\pi NN}^{(2)} (N N \tau^{3} N \pi^{0} - 3 N \tau^{3} N \pi^{0}).$$

(237)

Here $g_{\pi NN}^{(i)} (i = 0, 1, 2)$ are CP odd coupling constants, whereas we denote the CP even strong $\pi NN$ coupling constant as $G_{\pi NN} (= 13.5)$. The Schiff moment due to this coupling is calculated as follows. (237) gives rise to both $S^{ch}$ and $S^{\text{nucl}}$. P- and T-odd NN potential has the form via Fig.15. Using, for instance

$$\int \frac{d^3 q}{(2\pi)^3} i \sigma_a \cdot q \frac{e^{i \mathbf{q} \cdot \mathbf{r}}}{q^2 - m_{\pi}^2} = \sigma_a \cdot \nabla \frac{e^{-m_{\pi} r}}{4\pi r}$$

(238)

eq etc., its effective potential is given by

$$W(r_{a} - r_{b}) = \frac{G_{\pi NN} m_{\pi}^2}{8 \pi m_{N}} \left\{ \left[ g^{(0)} (\tau_{a} \cdot \tau_{b}) - \frac{g^{(1)}}{2} (\tau_{a}^z + \tau_{b}^z) + g^{(2)} (3 \tau_{a}^z \tau_{b}^z - \tau_{a} \cdot \tau_{b}) \right] (\sigma_{a} - \sigma_{b}) - \frac{g^{(1)}}{2} (\tau_{a}^z - \tau_{b}^z) (\sigma_{a} + \sigma_{b}) \right\} \cdot (r_{a} - r_{b}) \frac{\exp(-m_{\pi} |r_{a} - r_{b}|)}{m_{\pi} |r_{a} - r_{b}|^2} \left[ 1 + \frac{1}{m_{\pi} |r_{a} - r_{b}|} \right].$$

(239)
FIG. 15: One $g^{(i)}$ coupling induces effective CP-odd NN interaction, which give rise to $S^{ch}$.

Here we have suppressed the subscript $\pi NN$ in $g^{(i)}$. The EDM of $j$-th nucleon $d_j$ is generated via a diagram of Fig.16 and is given by and

$$d_j = \frac{eG_{\pi NN}}{4\pi^2 m_N} \ln \frac{m_N}{m_\pi} (g^{(0)} - g^{(2)}) \sigma_j \tau^z_j. \quad (240)$$

Given T and P-odd perturbation, let us calculate the Schiff moment using diagramatic technique [97] in

$$H = H_0 + H_{res}. \quad (241)$$

Here

$$H_0 = T + V_{00} + V_{11} \quad (242)$$

is unperturbative one-particle Hamiltonian and exactly solvable and

$$H_{res} = W + V_{22} + V_{13} + V_{31} + V_{04} + V_{40}. \quad (243)$$

$W$ is the pseudoscalar interaction (239) and $V$ the Skyrme interaction [98]. Subscripts $(ij)$ refer to the final and initial numbers of quasiparticles.

Let us assume that in the 0’th order approximation, the state is $\Phi_a = |\alpha\rangle$, and define $Q$ by

$$Q \equiv \sum_{\beta \neq \alpha} |\beta\rangle \langle \beta|. \quad (244)$$
Then perturbed wave function is given by

$$\Psi_a = \left(1 + \frac{Q}{\epsilon_a - H_0} H_{res} + \frac{Q}{\epsilon_a - H_0} H_{res} \frac{Q}{\epsilon_a - H_0} H_{res} + \ldots \right) \Phi_a. \quad (245)$$

This is the Brillouin-Wigner expansion and $\epsilon_a$ is the single quasiparticle energy of the valence nucleon.

So in the first order perturbation of $S^z$, we obtain

$$\langle \Psi_a | S^z | \Psi_a \rangle = N^{-1} \langle \Phi_a | \left[1 + H_{res} \left(\frac{Q}{\epsilon_a - H_0}\right) + \ldots \right] S^z \left[1 + \left(\frac{Q}{\epsilon_a - H_0}\right) H_{res} + \ldots \right] | \Phi_a \rangle. \quad (246)$$

The first-order (in $H_{res}$) quasiparticle (Goldstone) diagram is given in Fig. 17 [99].

Here the Goldstone diagram implies that

$$\alpha_r^a \langle r | - u | a \rangle = \frac{|\alpha_r^a \langle r | - u | a \rangle|}{\epsilon_a - \epsilon_r}. \quad (247)$$

Higher order quasiparticle calculations need some elaborate code and should be referred to [101], and we simply list the final results

$$\langle \Psi_a | S^z | \Psi_a \rangle \equiv S = (a_0 + b)G_{\pi NN}g^{(0)} + a_1 G_{\pi NN}g^{(1)} + (a_2 - b)G_{\pi NN}g^{(2)}, \quad (248)$$
TABLE I: Calculated coefficients $a_i$ and $b$ for $^{199}$Hg. The units are e fm$^3$. The last two references include the Skyrme interaction SkO'. Five results of Ban et.al. are due to Hartee-Fock and Hartree-Fock-Bogoliubov approximations. SLy4, SIII etc. indicate several Skyrme interactions.

|               | $a_0$   | $a_1$   | $a_2$   | $b$   |
|---------------|---------|---------|---------|-------|
| Dmitriev-Sen'kov 2003 [100] | -0.0004 | -0.055  | 0.009   | -     |
| de Jesus-Engels (averaged) [99] | 0.007   | 0.071   | 0.018   | -     |
| Ban et al [101] |         |         |         |       |
| SLy4(HF)       | -0.013  | 0.006   | 0.022   | -0.003|
| SIII(HF)       | -0.012  | -0.005  | 0.016   | -0.004|
| SV(HF)         | -0.009  | 0.0001  | 0.016   | -0.002|
| SLy(HFB)       | -0.013  | 0.006   | 0.024   | -0.007|
| SkM*(HFB)      | -0.041  | 0.027   | 0.069   | -0.013|

where the coefficients $a_i$ specify $S^{ch}$ and $b$ does $S^{nucl}$ defined by (235). The numerical results of $a_i$ and $b$ for $^{199}$Hg are given in Table I.

In this Table, the first two papers considered that EDM of nucleons $d_N$ is independent of $L_{\pi NN}$, whereas Ban considered $d_N$ is related as (240). The second and third papers incorporated collective modes based on the different approximation methods for nuclear structures but the results are still divergent. Thus it is a difficult task to precisely estimate the EDM of diamagnetic atom and to extract nucleon or quark EDMs due to lack of precise theory of nuclear structure. Bearing this point in mind, let us consider some cases. For $^{199}$Hg, numerical calculation is [102]

$$d^{(199\text{Hg})} = -2.8 \times 10^{-17} \left( \frac{S}{e\, fm^3} \right) \text{ e cm}. \quad (249)$$

In the case of $S = S^{\text{nucl}}$, the value of the Schiff moment of $d^{(199\text{Hg})}$ can be presented as a sum of proton and neutron EDMs [103]

$$S = s_p d_p + s_n d_n \quad (250)$$

with $s_p = 0.20 \pm 0.02 \, fm^2$ and $s_n = 1.895 \pm 0.035 \, fm^2$.

Combining the experimental value [104] (see more up-to-date data in [23])

$$d^{(199\text{Hg})} < 2.1 \times 10^{-28} \text{ e cm} \quad (251)$$
with (250), we obtain

\[ |d_p| < 3.8 \times 10^{-24} \text{ e cm} \quad |d_n| < 4.0 \times 10^{-25} \text{ e cm}. \] (252)

For \(^{129}\text{Xe}\) case, numerical calculation is [105]

\[ d^{(129)\text{Xe}} = 0.38 \times 10^{-17} \left( \frac{S}{e \text{ fm}^3} \right) \text{ e cm} \] (253)

The measurement is [106]

\[ d^{(129)\text{Xe}} = (-0.3 \pm 1.1) \times 10^{-26} \text{ e cm}. \] (254)

From (254) value, [107] obtained

\[ |d_p| \leq 4 \times 10^{-21} \text{ e cm} \quad |d_n| \leq 1 \times 10^{-21} \text{ e cm}. \] (255)

Lastly we comment on the deformed nucleus like Ra and Rn. When atomic weight \(A\) is in 150;\(A\);190 and \(A;\)220, nucleus becomes deformed and has the rotation energy levels, which enhances the Schiff moment by factor \(10^2 - 10^3\). For instance \(a_0 = 5.06, a_1 = 10.4, a_2 = -10.1\) for \(S^{(225)\text{Ra}}\) [108]. The classification of non-spherical nucleus is similar to that for a diatomic molecule consisting of like atoms (See Chapter VI). However, the energy levels of vibration and rotation are not so hierarchical as the molecule case.

General arguments for the CP violating four-fermion coupling are given in Appendices C and D.

1. cEDM and parity odd nuclear interaction

In this subsection we give a very short review of chiral symmetry and its breaking in strong interactions since it has many problems. Let us start with the conserved axial-vector current (CAC) hypothesis [109],

\[ \partial_\mu j_5^{\mu \alpha}(x) = 0. \] (256)

Of course CAC requires \(m_\pi = f_\pi = 0\) and is not realistic. However, it makes clear to understand how to break chiral symmetry. (256) leads us to

\[ \langle N' | j_5^{\mu \alpha} | N \rangle = \sqrt{\frac{m_N^2}{EE'}} F_A(t) \left[ i \not\! p \gamma_\mu \gamma_5 \tau_\alpha \frac{2}{2} u + 2 m_N \frac{q_1 \not\! p \gamma_\mu \gamma_5 \tau_\alpha}{q^2} \gamma_5 \frac{2}{2} u \right], \] (257)
FIG. 18: Nambu’s interpretation of pion dominance.

where \( F_A(0) = -g_A/g_V \). Nambu asserted \[110\] that \( 1/q^2 \) in the second term of (257) should be interpreted as

\[
\frac{1}{q^2} = \lim_{m_\pi \to 0} \frac{1}{q^2 - m_\pi^2}.
\]  

(258)

This corresponds to the diagram (Fig. 18), which can be written as

\[
G_{\pi NN} \bar{u} \gamma_5 \tau^\alpha u \frac{f_\pi q^\mu}{q^2 - m_\pi^2}.
\]  

(259)

Comparing (259) with (257) where (258) is inserted, we get

\[
f_\pi G_{\pi NN} = \frac{g_A}{g_V} m_N.
\]  

(260)

This is the Goldberger-Treiman’s relation \[111\]. Here use has been made of

\[
\langle 0 | j_\mu^\alpha(0) | \pi^\alpha \rangle = i \frac{f_\pi}{\sqrt{2\omega}} p_\mu.
\]  

(261)

\[
\langle N'| \partial_\mu j_5^{\mu\alpha} | N \rangle = -i \sqrt{\frac{m_N^2}{EE'}} F_A(t) \left[ -2m_N \bar{u} \gamma_5 \tau^\alpha \frac{\tau^\alpha}{2} u - 2m_N \frac{q^2}{q^2 - m_\pi^2} \bar{u} \gamma_5 \tau^\alpha \frac{\tau^\alpha}{2} u \right]
\]  

\[
= \sqrt{\frac{m_N^2}{EE'}} F_A(t) \left( \frac{m_N m_\pi^2}{q^2 - m_\pi^2} \right) \bar{u} \gamma_5 \gamma^\alpha u.
\]  

(262)

Substituting the equation of motion of \( \pi \),

\[
(\Box + m_\pi^2) \pi^\alpha = j^\alpha,
\]  

(263)

into (262), we obtain

\[
\langle N'| \pi^\alpha | N \rangle = - \langle N'| j^\alpha | N \rangle \approx -i \sqrt{\frac{m_N^2}{EE'}} G_{\pi NN} \bar{u} \gamma_5 \tau^\alpha u \frac{m_N}{q^2 - m_\pi^2}.
\]  

(264)

Assuming the matrix elements vary little between \( t = 0 \) and \( t = m_\pi^2 \), we obtain

\[
\langle N'| \partial_\mu j_5^{\mu\alpha}(0) | N \rangle \approx \frac{g_A}{g_V} \frac{m_N}{G_{\pi NN}} m_\pi^2 \langle N'| \pi^\alpha | N \rangle
\]  

\[
= f_\pi m_\pi^2 \langle N'| \pi^\alpha | N \rangle.
\]  

(265)
Thus we obtain the PCAC condition

$$\partial_\mu j_5^{\mu\alpha} = f_\pi m_\pi^2 \pi^\alpha$$  \hspace{1cm} (266)$$

and (222) with chiral anomaly.

Next, we proceed to discuss the path from the presence of the strong EDM of dimension 5 (cEDM and \(\theta\) term) to the effective CP-odd \(g^{(i)}\).

Let us write hadronic CP-violating operators like (40) and (41) etc. as \(O\) and consider the following two points correlation function 7.

$$M^\mu \equiv \int d^4x e^{-iqx} \langle N | T(j^{\mu\alpha}_5(x)O(0)) | N' \rangle. \hspace{1cm} (267)$$

From the definition of time ordered product, the right-handed side is rewritten

$$q_\mu M^\mu = -i \int d^4x e^{-iqx} \{ \langle N | T(\partial_\mu j^{\mu\alpha}_5(x)O(0)) | N' \rangle$$
$$-i\delta(x_0) \langle N | [j^{0\alpha}_5, O(0)] | N' \rangle \}. \hspace{1cm} (268)$$

Using (266) and LSZ reduction formula [113] in \(q \to 0\) limit, we obtain

$$q_\mu M^\mu = -f_\pi \langle \pi^\alpha N | O(0) | N' \rangle - i \langle N | [Q^\alpha_5(0), O(0)] | N' \rangle \hspace{1cm} (269)$$

or equivalently

$$\lim_{q \to 0} \sqrt{2\omega} \langle \pi^\alpha N | O(0) | N' \rangle = -i \frac{f_\pi}{\sqrt{\omega}} \langle N | [Q^\alpha_5(0), O(0)] | N' \rangle$$
$$- \lim_{q \to 0} \frac{q_\mu}{f_\pi} \int d^4xe^{iqx} \langle N | T(j^{\mu\alpha}_5(x)O(0)) | N' \rangle. \hspace{1cm} (270)$$

Substituting the concrete form of \(O(0)\) as (41) into the above equation and using

$$[Q^\alpha_5(0), q(0)] = it^\alpha \gamma_5 q(0) \hspace{1cm} (271)$$

with the generators of group of flavour \(t^\alpha\), we obtain [114], [115], [116]

$$\text{RHS of (270)} = \frac{1}{f_\pi} \langle N | \tilde{d}_u (g s \bar{u} G \sigma u - m_0^2 \bar{u} u) - \tilde{d}_d (g s \bar{d} G \sigma d - m_0^2 \bar{d} d) | N \rangle$$
$$+ \frac{m_s}{f_\pi} \left[ 2\bar{\theta} + m_0^2 \left( \frac{\tilde{d}_u}{m_u} + \frac{\tilde{d}_d}{m_d} + \frac{\tilde{d}_s}{m_s} \right) \right] \langle N | \bar{u} u - \bar{d} d | N \rangle \hspace{1cm} (272)$$

\(^7\) The general arguments on the operator expansion of T product of two currents are given in [112].
with
\begin{equation}
\frac{m^2}{m_u m_d + m_s m_d + m_s m_s} \approx \frac{m_u m_d}{m_u + m_d}, \quad m_0^2 = \frac{\langle 0 | g_s \bar{q} G \sigma q | 0 \rangle}{\langle \bar{q} q \rangle}.
\end{equation}

\(m_0^2\) is estimated as
\begin{equation}
m_0^2 \approx 0.8 \text{GeV}^2
\end{equation}
from QCD sum rule [117]. Here quantum corrections are also included. If we use the Peccei-Quinn mechanism [118], the second term of (272) vanishes in the following way [119].

\[L = \frac{\alpha_s}{8\pi} a G \tilde{G},\]

where \(a\) is axion field and \(G \tilde{G} = \frac{1}{2} \epsilon^{\mu \nu \alpha \beta} G_{\mu \nu}^b G_{\alpha \beta}^b\). When there exists cEDM, axion potential becomes
\[V_{\text{eff}}(a) = K_1 a + \frac{1}{2} K a^2.\]

Here
\begin{equation}
K \equiv -i \lim_{k \to 0} \int d^4 x e^{ikx} \langle 0 | T \left( \frac{\alpha_s}{8\pi} G \tilde{G}(x) \frac{\alpha_s}{8\pi} G \tilde{G}(0) \right) | 0 \rangle,
\end{equation}

\begin{equation}
K_1 \equiv -i \lim_{k \to 0} \int d^4 x e^{ikx} \langle 0 | T \left( \frac{\alpha_s}{8\pi} G \tilde{G}(x) \sum \frac{i}{2} \bar{d}_q g_s \bar{q} G \sigma \gamma_5 q(0) \right) | 0 \rangle.
\end{equation}

Eq. (276) is obtained by considering
\[L_{CPV} = \frac{\alpha_s}{8\pi} a G \tilde{G} + \frac{i}{2} \bar{q} g_s \bar{q} G \sigma \gamma_5 q\]
and performing path integral.

Next let us consider \(\partial_{\nu} j_5^{\nu \beta}(0)\) as an \(O(0)\) and use
\[\partial_{\nu} j_5^{\nu \beta} = i \bar{q} \gamma_5 \{t^\beta, M\} \gamma_5 q,\]
where \(M\) is the mass matrix of quarks. Then we easily obtain [120]
\begin{equation}
K = m_s \langle 0 | \bar{q} q | 0 \rangle,
\end{equation}

\begin{equation}
K_1 = \frac{1}{2} m_s \sum_{q=u,d,s} \frac{\bar{d}_q}{m_q} \langle 0 | g_s \bar{q} G \sigma q | 0 \rangle.
\end{equation}

So
\[\frac{\partial V_{\text{eff}}}{\partial a} = K_1 + Ka = 0\]
leads to vanishing of the second term of (272).
Finally we obtain \[116\]

\[
\begin{align*}
    g_{\pi NN}^{(0)} &= \frac{\tilde{d}_u + \tilde{d}_d}{f_\pi} \langle N | H_u - H_d | N \rangle \\
    g_{\pi NN}^{(1)} &= \frac{\tilde{d}_u - \tilde{d}_d}{f_\pi} \langle N | H_u + H_d | N \rangle.
\end{align*}
\] (283)

Here

\[ H_q = g_s \vec{\tau} \vec{G} \sigma q - m_0^2 \vec{q}. \] (284)

Thus \( g_{\pi NN}^{(2)} \) vanishes if we impose Peccei-Quinn symmetry. In the absence of Peccei-Quinn symmetry, there appears \( g_{\pi NN}^{(2)} \) \[121\]. The contribution of mixing of \( \eta \) with \( \pi \) was also considered in \[121\].

VI. THE EDMs OF MOLECULES

In this section we consider heteronuclear diatomic molecule which has permanent dipole moment. Polar paramagnetic molecules have stronger enhancement factors than paramagnetic atoms. Diamagnetic molecules are more sensitive to nuclear P,T violation than diamagnetic atoms.

There are many advantageous points in molecule \[122\]. Firstly, the polar molecule is polarized by a modest laboratory electric field \( E_{\text{lab}} \) but has a vast internal electric field \( E_{\text{int}} \). This implies the hugely enhanced stark effect and small fake magnetic field of \( \frac{v \times E_{\text{lab}}}{e} \) in comparison with atomic case. Secondly, there appears very small energy interval between nuclear rotation levels of opposite parity, which is roughly \( 10^{-3} \text{Ry} \) as will be discussed. Also \( g \)-factor can be very small etc.

In general the electric dipole moment \( D \) is defined by

\[
D = e \left( \sum_i Z_i \mathbf{R}_i - \sum_j \mathbf{r}_j \right),
\] (285)

where \( \mathbf{R}_i \) and \( \mathbf{r}_j \) are coordinates of nucleons and electrons composing molecule. For heteronuclear molecule

\[
D_a = \langle a | D | a \rangle \not= 0
\] (286)

and \( D \) has the permanent electric dipole moment.

So the behaviours of heteronuclear molecule and homonuclear molecule are different.
First we begin with diatomic molecule with total spin $S = 0$ case.

We first give the general rules of diatomic molecule.

In diatomic molecule, the field has axial symmetry along the two nuclei. Hence the projection of $\mathbf{L}$ (total orbital angular momentum of electrons) on this axis which is denoted by $\Lambda$ is conserved.

The motion of molecule is composed of the orbital motion of electrons, vibrations and rotation of nucleus, They interact complicatedly but their interactions are approximated as independent motions as the 0'th approximation (Born-Oppenheimer approximation)

$$\psi = \psi_e \psi_v \psi_r$$

and total energy is, therefore,

$$E = E_e + E_v + E_r. \quad (288)$$

They are electronic energy ($\approx Ry$), and vibration and rotation energies of nucleus, respectively. Let us consider the nuclear motions of diatomic molecule. First we begin with the case of total spin (mainly of electrons) $S = 0$. $E_v$ is considered as a harmonic oscillator and its energy are estimated from

$$M \omega_N^2 a^2 \approx \frac{\hbar^2}{m a^2} \equiv E_e. \quad (289)$$

Here $a$ is the distance between nucleus. $M$ and $m$ are the reduced mass of nuclei and electron mass, respectively. Therefore

$$E_v = \hbar \omega_N \approx \left( \frac{m}{M} \right)^{1/2} E_e. \quad (290)$$

Whereas the rotation energy is

$$E_r = B (\mathbf{K} - \mathbf{L})^2, \quad (291)$$

where $\mathbf{K}$ and $\mathbf{L}$ are total angular momentum of molecule and electron angular momentum, respectively, and

$$B(r) = \frac{\hbar^2}{2Mr^2} = \frac{\hbar^2}{2I}. \quad (292)$$

$\mathbf{K}$ and the axial component of $\mathbf{L}$ are conserved.

$$E_r = \frac{\hbar^2}{2I} l(l+1) \approx \frac{m}{M} J Ry. \quad (293)$$

This is much less than the atomic energy interval in general. We show in Fig. 19 the typical spectroscopies.
FIG. 19: The vibration \((v', v'')\) and rotation \((J', J'')\) terms of electron states A and B [122].

**Λ-doubling:**

In (291) \(K^2\) and \(L^2\) terms depend on \(|Λ|\), and \(K \cdot L \propto B^{2Λ} \approx (m/M)^{2Λ} [123].\)

Hence off diagonal parts are neglected, and \(+Λ\) and \(−Λ\) states are degenerate.

When we take the relativistic effect into consideration we have another coupling of Spin of electrons \(S\) (usually nucleon spin can be neglected) with orbital angular momentum of electrons \(L_e\) and of nucleons \(L_N\). The most important energy shift is \(A(r)L_e \cdot S\).

Selection rule in the electric dipole transition:

\[
|J' - J| \leq 1 \leq J + J' \\
+ \rightarrow -, - \rightarrow +
\]  

To obtain molecular spectra, we must consider the interactions among the above three terms; electron term \(E_e\), nuclear vibration \(E_v\), and rotation \(E_r\).

The interaction between \(E_e\) and \(E_r\) is especially important.

First we consider \(E_e\) for static nucleus. Unlike atomic case, conserved are not total orbital
angular momentum \( \textbf{L} \) and spin \( \textbf{S} \) of electrons but their projection to molecular axis

\[
\textbf{J}_z \equiv \Lambda + \Sigma = \Omega
\]

which takes the values over \( \Lambda + \Sigma, \Lambda + \Sigma - 1, \ldots, \Lambda - \Sigma \). These states are described as \( ^{2\Sigma+1}\Lambda\Omega \).

For example, \( ^2\Pi_{1/2} \), \( ^2\Pi_{3/2} \) for the states with \( \Lambda = 1, \Sigma = 1/2 \).

For atomic fine structure is given by (159), whereas the fine structure for diatomic molecule

\[
\Delta E = \frac{d\Lambda\Sigma}{d\Sigma} = AA = \text{const.}
\]

We call this relativistic interactions spin-axis interaction, which is composed of spin-orbit, spin-spin interactions, as well as the spin and orbital interactions with the rotation of molecule. Corresponding to the relative magnitudes of these interactions, we can classify molecule energy levels as follows [123] [124]. We define the magnitudes of interactions as follows.

\( LA \): the coupling of orbital angular momentum with the axis (the electric interaction between the two atoms in the molecule).

\( SA \): the coupling of spin angular momentum with the axis.

\( \Delta E_r \): the intervals between rotational levels.

If the distances between terms with different \( \Lambda \) are larger than both the intervals of fine structures \((2S+1)\) and rotational structures, they are further classified into

- Hund’s case a \( LA \gg SA \gg \Delta E_r \)

In this case, \( \Lambda, \Sigma, \Omega \) are well defined and electron state is expressed as \( ^{2\Sigma+1}\Lambda\Omega \). For
the $a \rightarrow a$ transition,

$$\Sigma' - \Sigma = 0, \quad \Omega' - \Omega = 0, \quad \pm 1 \quad (298)$$

$$\nu_L = \Delta T \gg \nu_s = \frac{\partial T}{\partial \Sigma} = A\Sigma \gg \nu_J = B_v(2J + 1) \quad (299)$$

$$U_J(r) = U(r) + A(r)\Omega + B(r)(J - L - S)^2. \quad (300)$$

Here the third term is a perturbation.

$L_e$ and $S$ precess around the internuclear axis $z$ implying that $\Lambda$ and $\Sigma$ are conserved quantum numbers. The total energy is described as

$$E = E_e + A_e\Omega + \hbar\omega(v + 1/2) + B_e\{J(J + 1) - 2\Omega^2\}. \quad (301)$$

In this review we are interested in the transition between parity odd rotation levels of the same electron term.

- **Hund’s case b** $LA \gg \Delta E_r \gg SA$

$\Sigma$ is not defined. Here the effect of the rotation of the molecule predominates over the multiple splitting and total angular momentum $J$ and the sum $K = L + N$ are conserved. In this case, $S$ is almost free from molecule (the vector $K + S$ precessing around $J$, and $\Sigma$ is not conserved)

$$\nu_K \gg \nu_s, \quad (302)$$

$$|K' - K| \leq 1 \leq K + K', \quad (303)$$

$$H_0 = H_e + BK^2 \quad (304)$$

with $K = \Lambda \hat{z} + N$ and $J = K + S$.

$$U_K(r) = U(r) + B(r)K(K + 1) + A(r)\Lambda\frac{(J - S)(J + S + 1)}{2K(K + 1)} \quad (305)$$

with

$$K = \Lambda, \Lambda + 1, \ldots \quad (306)$$

Here the third term is perturbation. The total energy is

$$E = U_e + \hbar \omega_e \left(\frac{1}{2}\right) + B_eK(K + 1) + A_e\Lambda\frac{(J - S)(J + S + 1)}{2K(K + 1)}. \quad (307)$$
Hund’s case c $SA \gg LA \gg \Delta E_r$

Only $\Omega$ is well defined. This is the case where the coupling of $\mathbf{L}$ with the axis is small compared with the spin-orbit coupling.

$$H_0 = H_e + H_{ls} + B\mathbf{J}^2.$$  \hfill (308)

Hund’s case d $\Delta E_r \gg LA \gg SA$

This is the case where the coupling of $\mathbf{L}$ with the axis is small in comparison with the intervals in $E_r$.

$$H_0 = H_e + B\mathbf{N}^2 - B(J^+l^- + J^-l^+).$$  \hfill (309)

Hund’s case e $SA \gg \Delta E_r \gg LA$.

A. Paramagnetic Molecule

As we will show, there are a variety of paramagnetic atoms, for instance, HgF, YbF, TIO whose electrons configurations are $^{70}$Yb=[Xe]$4f^{14}6s^2$, $^{80}$Hg=[Xe]$4f^{14}5d^{10}6s^2$, $^{81}$Tl=[Xe]$4f^{14}5d^{10}6s^26p^1$. The selection rules of transitions are

$$S' - S = 0,$$  \hfill (310)

$$\Lambda' - \Lambda = 0, \pm 1$$  \hfill (311)

$$\Sigma^+ \rightarrow \Sigma^+ , \Sigma^+ \rightarrow \Sigma^+ \text{ for } \Lambda = 0.$$  \hfill (312)

For BiS molecule $^{125}$, electron configuration of Bi is [Xe]$4f^{14}5d^{10}6s^26p^3$ and Bi$^{++}$ has one unpaired electron. The electric field of S leads to a mixing of parity odd states:

$$|\Omega\rangle = |1/2\rangle = a|s_{1/2}\rangle + b|p_{1/2}\rangle + c|p_{3/2}\rangle.$$  \hfill (313)

Here $\Omega = J_z = 1/2$. So

$$\langle \frac{1}{2}, \frac{1}{2} \rangle = -2ab\frac{4(Z\alpha)^2Z|\epsilon|d_e}{\gamma(4\gamma^2 - 1)a^2_B(N_sN_{p1/2})^{3/2}}.$$  \hfill (314)

For total $J$, angular momentum of nuclei takes two values, $N_1 = J+1/2$ and $N_2 = J+1/2$, so the characteristic energy splitting between P-odd states is

$$\Delta E_r = BN_2(N_2 + 1) - BN_1(N_1 + 1) = 2B(J + 1/2),$$  \hfill (315)
which is, for BiS, four to six orders of magnitude smaller than the case of heavy atom.

\[
d = \frac{2\omega d_M \langle \omega | H_d | \omega \rangle}{\Delta E_{J\eta}} \frac{J}{J(J+1)}
\]  

(316)

and

\[
K = \frac{d}{d_e} = 3 \times 10^7 \frac{(-1)^{J+1/2} \eta}{(J+1/2)(J+1)}.
\]  

(317)

The effective electric field on the valence electron is proportional to \(KE_{ind}\) for polar paramagnetic molecule. So it is very advantageous to measure molecular EDM.

Recently the most stringent upper limit of \(d_e\) was reported by using YbF \[24\]. Yb belongs to the rare-earth elements and its electron configuration is [Xe]+4f\(^{14}\)6s\(^2\) and Yb\(^+\) ion constitutes paramagnetic molecule. f electrons’ interaction with the axis of molecule is weakened by the deep position of the f electrons and classified as Hund’s c class. Their result is

\[
d_e = (-2.4 \pm 5.7_{stat} \pm 1.5_{sys}) \times 10^{-28} \text{e cm}
\]  

(318)

which sets the upper limit

\[
|d_e| < 10.5 \times 10^{-28} \text{ e cm}.
\]  

(319)

The other experiment using ThO \[126\] is also very interesting since a modest laboratory electric field \(E_{lab} \leq 100 \text{ V/cm}\) fully polarizes a ThO whose internal electric field \(E_{mol}\) is 100GV/cm. (The electron configuration of Thorium is Th=[Rn]6d\(^2\)7s\(^2\).) This gives another advantage for polar molecules. Furthermore, the triplet state \(^3\Delta_1\) of ThO gives the merit of g-factor cancellation (see Eq. \[336\]). Also for the other molecules we can expect g-factor cancellation, where g-factors are defined by the ratio of spin rotation energy \(H_E\) and \(\mu_B B_z\). Here \(H_E\) is given by

\[
H_E = \beta J^2 + \Delta S' \cdot J - Dn \cdot E.
\]  

(320)

Here \(\Delta\) is the \(\Omega\)-doubling constant and \(S'\) is the effective spin and \(S' = S\) for Hund’s case b. The detail of meanings of right-hand side is given in \[127\]. The expectation value of \(H_E\) crosses zero at a specific value of electric field and the molecule becomes insensitive to magnetic field at that point.

One of the problems for molecular EDM is the difficulty of laser cooling compared with atomic case. This may be solved by first cooling composite atoms and next combining them by the Feshbach resonance \[128\] and optical trap methods \[129\] \[130\]. The theoretical problem is to calculate matrix elements in Dirac-Coulomb + higher order approximation (see Appendix (J)).
B. Diamagnetic Molecule

We will consider TlF as an example of diamagnetic molecule. In searching for molecular EDM, we have two tasks. One is to derive \( d_{mol} \) from CP-odd elementary N-N and/or N-e interactions. Another is to deduce \( d_p \) and \( d_n \) from the observed \( d_{mol} \).

The electron configuration of Tl atom is \([\text{Xe}]4f^{14}5d^{10}6s^26p^1\) and Tl\(^+\) has a closed electron shell. Tl\(^+\) forms also incomplete shell \( 6s6p \) instead of \( 6s^2 \) [9],

\[
|\Omega\rangle = |6s, \Omega\rangle + \beta \left( -\frac{2\Omega}{\sqrt{3}}|6p_{1/2}, \Omega\rangle + \sqrt{\frac{2}{3}}|6p_{3/2}, \Omega\rangle \right) \quad (321)
\]

with

\[
\beta = \frac{2}{\sqrt{3}E_{6s} - E_{6p}} \frac{a^2r(6s,6p)}{r_1^2} = 0.27. \quad (322)
\]

Here \( \Omega = \pm 1/2 \), and \( r(6s,6p) \) is the radial integral defined by (182) whose value is 2.3. Using (F4) [9]

\[
\langle s_{1/2}\mid H\mid p_{1/2}\rangle = \frac{Gm^2\alpha^2}{\sqrt{2}\pi} \frac{Z^2R}{(N_sN_p)^{3/2}} Ry\{\gamma(Zk_{1p} + Nk_{1n}) - 4j \frac{2 + \gamma}{3} \langle k_{2p}\sum_p \sigma_p + k_{2n}\sum_n \sigma_n \rangle \}, \quad (323)
\]

where \( R \) is the relativistic factor

\[
R = \frac{4}{\Gamma^2(2\gamma + 1)} \left( \frac{a_B}{2Zr_0A^{4/3}} \right)^{2-2\gamma} \quad (324)
\]

with \( r_0 = 1.2\text{fm} \). As for the nuclear matrix element,

\[
\langle k_{2p}\sum_p \sigma_p + k_{2n}\sum_n \sigma_n \rangle \approx k_{2p} \frac{I}{T} \quad (325)
\]

since a valence proton in Tl atom is \( s_{1/2} \). Reference [12] goes further to get

\[
S(Tl) = -\frac{2\pi}{3}(r_q^2 - r_d^2)\rho_p, \quad (326)
\]

where \( r_q, r_d \) are defined by (117). From the experimental limit [131],

\[
S_{exp}(Tl) \leq 0.8 \times 10^{-8} \text{e fm}^3, \quad (327)
\]

we obtain

\[
d_p \leq 10^{-22} \text{e cm}. \quad (328)
\]
See (252) and (255) for diamagnetic atom. The numerical calculations were estimated along the following line of thoughts [132]: assuming Born-Oppenheimer approximation, total wave function of TlF is described as

$$\Psi = \psi_n(r_n)\psi_e(r_i)\psi_R(r_N, I).$$

(329)

Here $\psi_n(r_n)$ describes the motion of Tl nucleus, $\psi_e(r_i)$ does F nucleus and electrons with respect to the center of mass of Tl nucleus, and $\psi_R(r_N, I)$ the spin and motion of Tl nucleus.

Let us integrate over $\psi_e$ and take (135) into account. We obtain

$$\langle H_{edm} \rangle = D \langle \psi_R | a \cdot \sum_n \left( \frac{q_n}{Z} \sigma - \frac{d_n}{D} \right) \left( \int_{r_i=0}^{r_n} \psi_e^* \sum_i \frac{Y_{10}^i(\Theta, \Phi)}{r_i^2} \psi_e d^3 r_i \right) | \psi_R \psi_n \rangle,$$

(330)

where

$$D\sigma = \langle \psi_n | \sum_n d_n | \psi_n \rangle.$$

(331)

For the present approximation (B2), $\psi_e$ is given by

$$\psi_e = \Pi_i \psi_i(r_i) = \Pi_i \sum_l a_{l_i} Y_{l\text{im}}(\theta_i, \phi_i)$$

(332)

and so on.

Anyhow, analytical studies are restricted and we may need more elaborate numerical calculations as was done in the case of atomic structures or much more than that case. However, it is certain that unknown but very fruitful frontiers are expanding in front of this field. Many experiments are preparing or ongoing. In these situations, theoretical developments are strongly awaited.

VII. SUMMARIES AND DISCUSSION

We have explored the EDMs of quarks, leptons, hadrons, atoms, and molecules. First we studied the SM predictions on the EDM and showed that those are far from the present experimental upper limits. We have direct signals of new physics beyond the SM from neutrino oscillations and muon g-2, and many indirect ones like baryon asymmetry, DM etc.

Among them, the CP deficiency in baryon asymmetry $\eta \equiv \frac{n_B}{n_\gamma} \approx 10^{-10}$ is especially important for searching for new physics. Namely, we can not reproduce $\eta$ via CKM CP violating phase only even if we incorporate CP violation due to a $\theta$ term and other radiative corrections in the SM framework like $G\tilde{G}$ etc.
In order to estimate the deviation of phenomena from the SM, we have tried to estimate them first in the SM precisely, including the effects of the above mentioned extra terms.

Next we have explored many theories beyond the SM by focusing on the EDM of elementary particles.

The MSSM and two Higgs doublet model, for instance, give rather large values of EDMs. However, those values are mainly due to the ambiguities of the theories themselves. It is important to see whether such values are checked to be consistent with the other phenomena or not. We think those points are still very insufficient. More predictive models like the renormalizable minimal SO(10) GUT discussed in Section 4.3 often give more stringent values which are still several orders smaller than the present upper limits.

However, the situation is not so pessimistic. Some hope comes from unprecedented collaborations with atomic and molecular physics and elementary particles mainly via brilliant developments of laser physics. Most impressive is the new upper limit of the electron EDM from polarized molecule YbF. As for paramagnetic atoms, theoretical calculations have been developed and seems to be convergent. Whereas, for diamagnetic atoms there are still large discrepancies (Table I). Lattice QCD is very promising but it is not convergent in the limit of $m_{\pi} = 0$ (Fig.5). However, it is certain that these situations have been improved rapidly. The large parts of such progress have been and will be done by the collaboration of a wide field of physics and chemistry. The mutual close relationships among particle, atomic, and molecular physics require the wide range of studies over these regions.

We hope that this review gives some help for these difficult tasks.

This review is restricted in theoretical part and we have not discussed many excellent ideas on the experimental side. The latter is very attractive but is beyond the scope of this review simply due to the author’s ability. We only briefly explain the mechanism and a list of experiments though it is not exhaustive.

The procedure for the EDM measurement is as follows. First a static external electric field $\mathbf{E}$ is applied parallel to magnetic field $\mathbf{B}$. The energy splitting is measured as a spin precession frequency $\nu_{\uparrow\uparrow}$. Next we change $\mathbf{E}$ unti-parallel to $\mathbf{B}$ whose precession frequency is denoted by $\nu_{\uparrow\downarrow}$. Namely,

\begin{align}
\hbar \nu_{\uparrow\uparrow} &= 2\mu \cdot \mathbf{B} + 2d \cdot \mathbf{E} \\
\hbar \nu_{\uparrow\downarrow} &= 2\mu \cdot \mathbf{B} - 2d \cdot \mathbf{E}
\end{align}

(333)
and
\[ h\Delta \nu = 4d \cdot E. \] (334)

Its sensitivity is given by
\[ \delta d = \frac{h}{2eK} \frac{1}{E} \frac{1}{\sqrt{N\tau T}}. \] (335)

Here \( N \): number of sample, \( \tau \): coherence time, and \( T \): measuring time. \( K \) is an enhance factor for paramagnetic atoms and molecules given in (181) and \( K \propto Z^3\alpha^2 \). \( E \) is a magnitude of an internal electric field. So experiments try to get larger values of \( K, E, N, \tau, T \). We only list up ongoing and planned experiments (see Table 3). We have still more species, solids like \( GGG, Gd_2Ga_5O_{12}, Gd_3Fe_5O_{12}, PbTiO_3, Gd_3Ga_5O_{12}, \) solid He, liquid Xe (see Table nEDM Collaboration). Please refer to the corresponding sections for the terminologies in the experimental features. A few comments are in order. + signature at PbO molecule implies the parity under the mirror reflection (reflection under arbitrary plane including molecule axis) (see Eq. (312)). As for g-factor cancellation in molecular EDMs, ThO and the others’ cancellation mechanisms are different: the former is due to
\[ \mu = (\Lambda + g\Sigma)\mu_B \approx 0 \text{ for } \Lambda = 2 \] (336)
and the latters are due to Eq. (320).

This table is far from being exhaustive but reflects some prospect from a theoretical physicist.

For more detail see, for instance, ECT* Workshop: Violations of Discrete Symmetries in Atoms and Nuclei. Nov 15-19, 2010 [149].

We have not discussed about the EDMs of charged particles and ions. These are also very important and we have added short explanation on the storage ring in Appendix (K).

Finally we will give some comments on the recent results by the Cern Large Hadronic Collider (LHC). On July 2012 the LHC groups announced the discovery of Higgs-like particle around 126 GeV [151] [152]. This is not only the discovery of the last unknown particle in the SM but also gives the serious impact to the new physics beyond the SM, especially SUSY GUT. In this review we have estimated EDM value in the framework of SUSY GUT and given large EDM value relative to that of the SM. We briefly explain why 126 GeV Higgs mass is serious for GUT and especially SUSY GUT. First we explain the reason why 126
TABLE III: A list of ongoing and planned experiments searching for EDM. Superscript * indicates estimated sensitivity.

| Species | Group name | Features |
|---------|------------|----------|
| muon   | FNAL       | $10^{-21}$ e cm* (2015) |
|        | J-PARC     | $10^{-24}$ e cm* (2015) with spin frozen technique |
|        | PSI        | 3-4 orders below current limit* (spin frozen technique) |
| neutron (all $10^{-28}$ e cm*) | ILL (Grenoble) | $|d_n| < 2.9 \times 10^{-26}$ ecm (90% C.L.) [22] |
|        | ILL        | Squids as magnetometer [133] |
|        | PSI (Zurich) SNS(Oak Ridge) | Hg co-magnetometer and Cs gradiometer [134] |
|        | SNS        | $^3$He co-magnetometer and SQUIDS [135] |
|        | KEK-RCNP (Japan) | $^{129}$Xe co-magnetometer [136] |
| deuteron | KVI/BNL/COSY | $10^{-29}$ e cm* |
| $d_D$   | KVI/BNL/COSY | $10^{-29}$ e cm* |
| paramagnetic Atom | Amherst College | $d(Cs) = (-1.8 \pm 6.7 \pm 1.8) \times 10^{-24}$ ecm [137] |
|        | LBNL       | highly improved magnetic shielding [138] |
|        | Berkeley   | $d_e < 1.6 \times 10^{-27}$ (90% C.L.) e cm [20] |
| Fr      | CYRIC(Tohoku Univ.) | K(Fr)=895 EDM measurement starts on 2014 [139], |
| Ra      | KVI (Groningen) | magneto-optical trap [140] |
| $^{199}$Hg | Seattle | $d^{(199)Hg} < 3.1 \times 10^{-29}$ (95% C.L.) [23] |
| Ra      | Argonne/KVI | large enhancement $d(Ra)/d(Hg) \approx 10^{2-3}$ |
| Xe      | @nEDM Collaboration | polarized liquid Xe droplets |
|        | Tokyo Institute of Technology | artificial feedback mechanism [141] |
|        | Princeton  | liquid cell |
|        | Univ. Mainz | $d^{(129)Xe} \approx 10^{-30}$ e cm* |
| Rn/Xe   | Michgan   | $d^{(129)Xe} = (+0.7 \pm 3.3) \times 10^{-27}$ ecm [142] |
| Rn      | Rn EDM Collaboration | octupole enhancement of 400-600 |
| paramagnetic molecule | Hinds (Imperial College) et al. | the most stringent limit of $d_e$ [24] |
|        | ACME Collaboration | g-factor cancellation at $^3\Delta_1$ [126] |
|        | DeMille (Yale) et al. | g-factor cancellation at metastable $^3\Sigma_1^+$ [143] |
|        | Shafer-Ray (Oklahoma) et al. | g-factor cancellation at $^2\Pi_{1/2}$ [144] |
|        | Cornell group | trapped molecular ions in rotating electric field [145] |
|        | Hinds (Yale) et al. | same electron configuration as YbF |
|        | KVI | high $W_e$ parameter [146] |
|        | Aoki (Tokyo) et al. | ultra cold molecule/3D optical lattice [147] |
| diamagnetic molecule | Hinds (Yale) et al. | the measured $\Delta \nu = (1.4 \pm 2.4) \times 10^{-4}$ Hz [148] |
| YbHg   | Takahashi (Kyoto) et al. | ultra cold molecule/3D optical lattice |
FIG. 20: The catastrophic RGE behaviours of quartic coupling constant at the energy scale \( \Lambda \). The upper curve is the Landau pole where the coupling blows up at \( \Lambda \) and the lower curve is the point that the coupling becomes negative (vacuum is unstabilized). This diagram is cited from [155].

The RGE of the Higgs quartic coupling is given by [153]

\[
16\pi^2 \frac{d\lambda}{d\ln \mu} = 12\lambda^2 - \left( \frac{9}{5}g_1^2 + 9g_2^2 \right) \lambda + \frac{9}{4} \left( \frac{3}{25}g_1^4 + \frac{2}{5}g_1^2g_2^2 + g_2^2 \right) + 12Y_t^2\lambda - 12Y_t^4. \tag{337}
\]

Here the Higgs self coupling is

\[
V = \frac{1}{2} \lambda |\Phi^\dagger \Phi|^2 \tag{338}
\]

with

\[
m_h^2 = \lambda v^2, \quad \text{and} \quad < \Phi >= \frac{v}{\sqrt{2}}. \tag{339}
\]

If the SM is assumed to be valid to the energy scale \( \Lambda_{cut} = M_{Pl} = 2.44 \times 10^{18} \) GeV, it goes from the perturbative bound and vacuum stability bound [154] [155] that

\[
129 \text{GeV} \geq m_H \geq 170 \text{GeV} \tag{340}
\]

as depicted in Fig. 20. That is, if \( m_H \) was below 129 GeV, the coupling becomes negative and vacuum unstabilized. Whereas if \( m_H \) was above 170 GeV, the coupling blows up. \( m_H = 126 \)
GeV (very near to 129 DeV) implies that there exists some phase transition around the GUT scale.

As for the SUSY implication, for tree level Higgs mass satisfies the inequality

\[ m_h < M_Z |\cos(2\beta)|, \tag{341} \]

with \( M_Z = 91.2 \text{GeV} \), which is obviously wrong. One loop correction to \( m_h \) in CMSSM is

\[ m_h^2 \approx M_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \left[ \ln \frac{M_S^2}{m_t^2} + \frac{X_t^2}{M_S^2} \left( 1 - \frac{X_t^2}{12M_S^2} \right) \right], \tag{342} \]

where

\[ M_S = \sqrt{m_{\tilde{t}_1}m_{\tilde{t}_2}}, \quad X_t = A_t - \mu \cot \beta, \quad v = 174 \text{GeV} \tag{343} \]

with the trilinear Higgs-stop coupling constant \( A_t \). So 126 GeV indicates heavy stop masses and/or large \( X_t \) (left-right mixing). Experimental search of SUSY particles also gives large sfermion masses, larger than 1 TeV \[^{156,157}\]. One loop EDM due to the MSSM (Fig. 8) is proportional to \( O(M_S^{-2}) \) and heavy \( M_S \) reduces EDM. In Fig. 11 we have assumed \( A_0 = 0 \) since \( A_0 \) appears at two loop correction in the gauge mediation SUSY breaking and is suppressed. In order to preserve rather small \( M_S \) and still give large loop correction in (342), we may take rather large \( A_0 \). So we have added the last fourth panel with nonzero \( A_0 \) in Fig. 11. More detailed explanations will be given in the review paper of GUT prepared by us \[^{158}\].

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Appendix A: SU(6) and Dipole moments

Both magnetic dipole moment and electric dipole moment are proportional to \( eQ\sigma \).
So we can obtain the information of the ratio of \( d_p/d_n = \mu_p/\mu_n \) from SU(6) in the light quark (u,d,s) base \[17\] if CP violation in the EDM does not affect SU(3) symmetry. They are both represented as

\[ \langle 56|35|56 \rangle \]  

(A1)

Here baryons belong to 56-representation since irreducible representation of \( qqq = 56 \) and we use that dipole moments are the generator of SU(6).

\[
|p, 1/2\rangle = \frac{\sqrt{2}}{6} \left\{ |uud\rangle (2|++-\rangle - |+-+\rangle - |--+\rangle) \\
+|udu\rangle (2|++-\rangle - |--+\rangle - |+-+\rangle) \\
+|duu\rangle (2|--+\rangle - |+-+\rangle - |++-\rangle) \right\},
\]

(A2)

\[
Q\sigma_3|p, 1/2\rangle = \frac{\sqrt{2}}{6} \left\{ \frac{2}{3}|uud\rangle (2|++-\rangle - |+-+\rangle + |--+\rangle) \\
+\frac{2}{3}|udu\rangle (2|++-\rangle + |--+\rangle - |+-+\rangle) \\
-\frac{1}{3}|uud\rangle (2|--+\rangle - |+-+\rangle - |--+\rangle) \\
+\text{cyclic permutations} \right\},
\]

(A3)

\[
\langle p, 1/2|Q\sigma_3|p, 1/2 \rangle \\
= 3 \frac{2}{36} \left( \frac{2}{3}(4+1-1) + \frac{2}{3}(4-1+1) - \frac{1}{3}(-4+1+1) \right) = 1.
\]

(A4)

The corresponding neutron dipole moments are given by the exchange of \( u \leftrightarrow d \), resulting to \( \frac{2}{3} \leftrightarrow -\frac{1}{3} \). Therefore,

\[
\langle n, 1/2|Q\sigma_3|n, 1/2 \rangle \\
= 3 \frac{2}{36} \left( -\frac{1}{3}(4+1-1) - \frac{1}{3}(4-1+1) + \frac{2}{3}(-4+1+1) \right) = -\frac{2}{3},
\]

(A5)

and

\[
\frac{d_p}{d_n} = \frac{\mu_p}{\mu_n} = -\frac{3}{2}.
\]

(A6)
The experimental values of MDM of proton and neutron are \[19\]

\[
\mu_p = 2.792847356 \pm 0.000000023, \quad \mu_n = -1.91304273 \pm 0.00000045 \quad (A7)
\]

and the coincidence with SU(6) prediction is good up to quantum corrections. For the EDM, compare with the result of lattice calculations Fig 5.

**Appendix B: Multipole expansions**

We will study the multipole expansions of electromagnetic potential \( A_\mu = (\phi, A) \) due to the charged system of finite size. The electric and magnetic fields are defined by

\[
E = -\frac{1}{c} \frac{\partial A}{\partial t} - \text{grad}\phi, \quad H = \text{rot}A. \quad (B1)
\]

Let us assume (as in the experimental environment) that the electromagnetic field is static, that is, the field is time independent. In such case, \( E \) (\( H \)) is determined only by \( A \) (\( \phi \)). Let us consider a stationary motion of charged particles where \( e_a \) charged particles are located at \( r_a \) and study how the observer at \( R \) feels vector potential \( A_\mu \).

\[
\phi(R) = \sum_a \frac{e_a(r_a)}{|R - r_a|}, \quad A(R) = \sum_a \frac{e_a v_a(r_a)}{|R - r_a|}. \quad (B2)
\]

Here we have neglected the retardation effect of fast particles. If we included it, charge distribution has a retarded time dependence and we should replace the arguments as,

\[
t \to t - \frac{|R - r_a|}{c}, \quad |R - r_a| \to |R - r_a| - \frac{v \cdot (R - r_a)}{c}. \quad (B3)
\]

If the scale \( R \gg r_a \), \( (B2) \) is expanded around \( R \),

\[
\phi = \frac{\sum_a e_a}{R} - \sum_e e_a(r_a \cdot \nabla) \frac{1}{R} + \sum e_a e_b r_a^i r_b^j \partial_i \partial_j \frac{1}{R} + ....
\]

Then \( \phi^{(l)} \) is given by

\[
\phi^{(l)} = \frac{1}{R^{l+1}} \sum_{m=-l}^{m=l} \sqrt{\frac{4\pi}{2l+1}} Q_{lm}(e^{(e)} Y_{lm}(\Theta, \Phi), \quad (B5)
\]

where

\[
Q_{lm}^{(e)} = \sum_a e_a r_a^l \sqrt{\frac{4\pi}{2l+1}} Y_{lm}(\theta_a, \varphi_a). \quad (B6)
\]
gives electric $2^l$-pole moment. The superscript $(e)$ indicates electric moment distinguishing magnetic counterpart (see (B23)). Its continuous representation is

$$Q_{lm}^{(e)} = \sqrt{\frac{4\pi}{2l+1}} \int d^3r \rho(r) r^l Y_{lm}(\frac{r}{R}).$$  \hspace{1cm} (B7)

This comes from

$$\frac{1}{|\mathbf{R} - \mathbf{r}|} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{r^l}{R^{l+1}} \frac{4\pi}{2l+1} Y_{lm}^*(\Theta, \Phi) Y_{lm}(\theta, \phi).$$  \hspace{1cm} (B8)

First few normalized spherical harmonics $Y_{lm}$ are

$$Y_{00} = 1/\sqrt{4\pi},$$

$$Y_{10} = i\sqrt{3/(4\pi)}\cos\theta, \hspace{0.5cm} Y_{1\pm1} = \mp i\sqrt{3/(8\pi)}\sin\theta e^{\pm i\phi},$$

$$Y_{20} = \sqrt{5/(16\pi)}(1 - 3\cos^2\theta),$$

$$Y_{2\pm1} = \pm \sqrt{15/(8\pi)}\cos\theta\sin\theta e^{\pm i\phi}, \hspace{0.5cm} Y_{2\pm2} = -\sqrt{15/(32\pi)}\sin^2\theta e^{\pm 2i\phi} \text{ etc.} \hspace{1cm} (B9)$$

For instance $Q_{lm}^{(e)}$ constitute electric dipole moment

$$Q_{10}^{(e)} = id_z, \hspace{0.5cm} Q_{1\pm1}^{(e)} = \mp i\sqrt{2}(dx \pm id_y). \hspace{1cm} (B10)$$

Analogously, vector potential $\mathbf{A}$ is expanded as

$$A_i(\mathbf{R}) = \int \frac{J_i(\mathbf{r})}{|\mathbf{R} - \mathbf{r}|} d^3r = A_i^{(0)} + A_i^{(1)} + A_i^{(2)} + ...$$  \hspace{1cm} (B11)

For instance $A_i^{(2)}$ is

$$A_i^{(2)} = \left( \nabla_k \nabla_l \frac{1}{R} \right) T_{ikl},$$  \hspace{1cm} (B12)

where

$$T_{ikl} = \frac{1}{2} \int d^3r r_k r_l J_i(r).$$  \hspace{1cm} (B13)

The identity

$$\int d^3r \nabla_m (r_i r_k r_l J_m) = 0$$  \hspace{1cm} (B14)

leads us to

$$\int d^3r (r_k r_l J_i + r_i r_k J_l + r_i r_k J_l) = 0,$$  \hspace{1cm} (B15)

where use has been made

$$\partial_m J_m = 0.$$  \hspace{1cm} (B16)
This identity gives 10 constrains. Since $T_{ikl}$ has $6 \times 3$ freedoms, $18 - 10 = 8$ physical freedoms remain. We will show that five of eight freedoms constitute the $M2$ moment and the remaining three the anapole moment. It goes from subtracting (B15) from (B13) that

$$T_{ikl} = -\frac{1}{3} \epsilon_{ikm} \epsilon_{mnr} \int d^3 r r_l r_n J_r = -\frac{1}{3} \epsilon_{ikm} \int d^3 r r_l M_m,$$

where

$$M_m = \epsilon_{mnr} r_n J_r.$$  \hfill (B18)

Dividing the $T_{ikl}$ of (B17) into symmetric and antisymmetric parts w.r.t. $l, m$, we obtain

Symmetric part of (B17) = $-\frac{1}{6} \epsilon_{ikm} M_{lm}$ \hfill (B19)

with

$$M_{lm} = \int d^3 r (r_l \epsilon_{mnr} + r_m \epsilon_{lnr}) r_n J_r.$$ \hfill (B20)

This gives magnetic quadrupole moment. Whereas,

$$\text{Anti-symmetric part of (B17)} = \frac{1}{6} \int d^3 r \left[ \delta_{il} (r_k (r_m J^m) - r^2 J_k) + \delta_{kl} (J_i r^2 - r_i (r_m J^m)) \right].$$ \hfill (B21)

Here we use the identity obtained from contracting (B15) w.r.t. $k$ and $l$

$$\int d^3 r (r^2 J_i + 2 r_i r_m J^m) = 0$$ \hfill (B22)

Then the anti-symmetric part becomes

$$\text{Anti-symmetric part of (B17)} = \frac{1}{4\pi} (\delta_{il} a_k - \delta_{kl} a_i),$$ \hfill (B23)

where

$$a_i = -\pi \int d^3 r r^2 J_i$$ \hfill (B24)

is called anapole moment.

General expression for magnetic photon corresponding to electric counterpart (B7) is

$$Q_{lm}^{(m)} = \frac{1}{l + 1} \sqrt{\frac{4\pi}{2l + 1}} \int d^3 r [r \times \nabla (r^l Y_{lm})]$$ \hfill (B25)

and called $2^l$-pole magnetic moment (For relativistic case $l$ is replaced by $j = |l + s|$).
Appendix C: C,P,T-transformations of Fermi coupling

We consider the four fermions (current-current) coupling. Here it is concerned with the transformation property of fermion but not with detailed dynamics, we consider it as $\bar{N}\hat{O}N\hat{O}'L$, where $N, L$ are spinors, and $\hat{O}$ and $\hat{O}'$ are combinations of gamma matrices. The most general forms are

$$G_S\bar{N}N\bar{L}L + G_P\bar{N}\gamma_5N\bar{L}\gamma_5L$$
$$+ G_V\bar{N}\gamma_\mu N\bar{L}\gamma^\mu\gamma_5L + G_T\bar{N}\sigma_\mu\nu N\bar{L}\sigma^{\mu\nu}L$$
$$+ G_{V'}\bar{N}\gamma_\mu N\bar{L}\gamma_\mu\gamma_5L$$
$$+ G_A\bar{N}\gamma_\mu\gamma_5N\bar{L}\gamma_\mu\gamma_5L$$
$$+ iG_{S'}\bar{N}N\bar{L}\gamma_5L$$
$$+ iG_{P'}\bar{N}\gamma_5N\bar{L}L + iG_{T'}\epsilon^{\mu\nu\rho\sigma}\bar{N}\sigma_\mu\nu N\bar{L}\sigma_{\rho\sigma}L.$$  \hfill (C1)

The first two lines constitute Lorentz scalars, the third line P-odd and the fourth line P,T-odd terms. Imaginary $i$ in the last line comes from the Hermiticity of action. The last term of the fourth line are also expressed as

$$\bar{N}\sigma_\mu\nu N\bar{L}\sigma^{\mu\nu}\gamma_5L \text{ or } \bar{N}\sigma_\mu\nu\gamma_5N\bar{L}\sigma^{\mu\nu}L$$  \hfill (C2)

since

$$\epsilon^{\mu\nu\rho\sigma}\gamma_\mu = -i\gamma_5\gamma^\nu\gamma^\rho\gamma^\sigma.$$  \hfill (C3)

C,P,T conjugations are defined by

$$C\psi(t, r) = \gamma^2\psi^*(t, r),$$  \hfill (C4)
$$P\psi(t, r) = i\gamma^0\psi(t, -r),$$  \hfill (C5)
$$T\psi(t, r) = i\gamma^3\gamma^1\psi^*(-t, r).$$  \hfill (C6)

The fourth line of (C1) is T-odd since

$$\bar{N}N \rightarrow -\bar{N}N, \quad \bar{L}\gamma_5L \rightarrow \bar{L}\gamma_5L \quad \text{etc.}$$  \hfill (C7)

under T-transformation.

Appendix D: CP phases in general L-R model and generation number

Type I (canonical) seesaw is composed of $N$ left-handed and $N$ right-handed neutrino. Right-handed neutrino has heavy Majorana mass term.

$$L_{\text{Yukawa}} = -\bar{\nu}_RM_D\nu_L - \frac{1}{2}\left\{\nu_L^cM_L\nu_L + \nu_R^cM_R\nu_R\right\} + \text{h.c.}$$  \hfill (D1)
This is described in terms of mass eigenvectors,

\[ L_M = \left( \frac{N(1), N(2)}{N(1)} \right) \left( \begin{array}{cc} U^{(1)} & U^{(2)} \\ V^{(1)*} & V^{(2)*} \end{array} \right)^T \left( \begin{array}{cc} M_L & M_D \\ M_D^T & M_R \end{array} \right) \left( \begin{array}{cc} U^{(1)} & U^{(2)} \\ V^{(1)*} & V^{(2)*} \end{array} \right) \left( \begin{array}{c} N^{(1)} \\ N^{(2)} \end{array} \right) + h.c. \] (D2)

\[ \nu_{\ell L} = \sum_{j=1}^{2N} U_{\ell j} N_{j L}, \quad \nu_{\ell R} = \sum_{j=1}^{2N} V_{\ell j} N_{j R}, \] (D3)

where \( \ell = 1, \ldots, N \).

So \( N \times 2N \) unitary matrices \( U \) and \( V \) are decomposed into \( N \times N \) matrices,

\[ U = (U^{(1)}, U^{(2)})^T, \quad V = (V^{(1)}, V^{(2)})^T. \] (D4)

In the SM \( m_{\nu_i} = 0 \) and there is no mixing in neutrino sector.

For Dirac neutrino case, \( U^{(2)} = V^{(2)} = 0 \). As we mentioned above, there exist \((N - 1)(N - 2)/2\) phases in this case.

For Majorana neutrino case, \( U^{(2)} = V^{(1)} = 0 \) when there exist both left-handed (L-type \( N_1, \ldots, N_N \)) and right-handed neutrino (R-type \( N_{N+1}, \ldots, N_{2N} \)). In this case \( 2N \times 2N \) unitary \( V^{(2)} = 0 \) is added for only left-handed Majorana neutrino case.

\[ H_W = \frac{G}{\sqrt{2}} \left[ j_{\ell a}^L j_{\ell a}^L + \lambda j_{\ell a}^R j_{\ell a}^R + \kappa \left( j_{\ell a}^L j_{\ell a}^R + j_{\ell a}^R j_{\ell a}^L \right) \right], \] (D5)

where

\[ j_{\ell a}^L = \sum_l \bar{\ell}(x) \gamma_a (1 - \gamma_5) \nu_{\ell L}(x), \] (D6)

\[ j_{\ell a}^R = \sum_l \bar{\ell}(x) \gamma_a (1 + \gamma_5) \nu_{\ell R}(x) \] (D7)

with \( \ell = e, \mu, \ldots \). Mass matrices has rebasing and rephasing symmetries, which does not change physics.

We start with \( N \) generation of quarks (Dirac fermions). \( N \times N \) unitary matrix has \( N^2 \) real numbers. Of these, \( 2N - 1 \) is absorbed by rephasing of \( 2N \) left-handed and right-handed quarks. An orthogonal \( N \times N \) orthogonal matrix has \( N(N - 1)/2 \) Euler angles. The remaining \((N - 1)(N - 2)/2\) is the number of phase parameters. Kobayashi-Maskawa predicted that there must at least three generations to incorporate CP phase in mass matrix \cite{2}. If we relax this arguments to include Majorana neutrino we can use only rephasing of
charged lepton in MNS mixing matrix whose freedom is $N$. Therefore, the number of phases in MNS matrix is $N^2 - N(N - 1)/2 - N = N(N - 1)/2$.

If we furthermore generalize the above arguments to include heavy right-handed neutrino \[63\],

$$\nu_{lL} = \sum_{j=1}^{2N} U_{lj} N_{jL}, \quad \nu_{lR} = \sum_{j=1}^{2N} V_{jl} N_{jR},$$  \hspace{1cm} (D8)

where $l = 1, ..., N$.

So $N \times 2N$ unitary maqtrices $U$ and $V$ are decomposed into $N \times N$ matrices,

$$U = (U^{(1)}, U^{(2)})^T, \quad V = (V^{(1)}, V^{(2)})^T.$$  \hspace{1cm} (D9)

In the SM $m_{\nu_l} = 0$ and there is no mixing in neutrino sector.

For Dirac neutrino case, $U^{(2)} = V^{(2)} = 0$. As we mentioned above, there exist $(N - 1)(N - 2)/2$ phases in this case.

**Appendix E: Expansion in power of 1/c**

Relativistic equation of fermion in an external electromagnetic foeld is

$$(\gamma(p - eA) + m) \psi = 0.$$  \hspace{1cm} (E1)

Let us study the relativistic effect as the deviation from the Schroedinger prescription which is obtained by expanding (E1) in power of $1/c$. For that purpose we must exclude $mc^2$ from energy, which implies to replace $\psi$ to $\psi'$

$$\psi = \psi' e^{-imc^2t/\hbar}.$$  \hspace{1cm} (E2)

and

$$\left(i\hbar \frac{\partial}{\partial t} + mc^2\right) \psi' = \left[c\sigma \cdot \left(p - \frac{e}{c}A\right) + \beta mc^2 + e\Phi\right] \psi'.$$  \hspace{1cm} (E3)

Substituting

$$\psi' = \begin{pmatrix} \phi' \\ \chi' \end{pmatrix}$$  \hspace{1cm} (E4)

into (E3), Dirac spinor is reduced to two component Weyl spinors\]

$$\left(i\hbar \frac{\partial}{\partial t} - e\Phi\right) \phi' = c\sigma \cdot \left(p - \frac{e}{c}A\right) \chi',$$  \hspace{1cm} (E5)

$$\left(i\hbar \frac{\partial}{\partial t} - e\phi + 2mc^2\right) \chi' = c\sigma \cdot \left(p - \frac{e}{c}A\right) \chi'.$$  \hspace{1cm} (E6)
Retaining only the term $2mc^2\chi'$ in the second equation, we obtain

$$\chi = \frac{1}{2mc} \sigma \cdot \left( p - \frac{e}{c} A \right) \phi. \quad (E7)$$

Substituting this into the first equation, we finally obtain the famous Pauli equation,

$$i\hbar \frac{\partial \phi}{\partial t} = \left[ \frac{1}{2m} \left( p - \frac{e}{c} A \right)^2 + e\Phi - \frac{e}{2mc} \sigma \cdot H \right] \phi. \quad (E8)$$

The current density is

$$j = c\psi^* \sigma \psi = c(\phi^* \sigma \chi + \chi^* \sigma \phi). \quad (E9)$$

Substituting $\chi$ of (E7) into it, we obtain

$$j = \frac{i\hbar}{2m} (\phi \nabla \phi^* - \phi^* \nabla \phi) - \frac{e}{mc} A \phi^* \phi + \frac{\hbar}{2m} \nabla \times (\phi^* \sigma \phi). \quad (E10)$$

In the presence of the EDM (see (31)), $j$ includes pseudo-vector part,

$$j_d = i d_N \nabla \times (\psi^* \gamma \psi), \quad (E11)$$

which in two components approximation is reduced to

$$j_d = \frac{d}{2m} \nabla \times [\phi' \sigma \times (p' + p) \phi], \quad (E12)$$

where $p$ and $p'$ are the momenta of $\phi$ and $\phi'$, respectively.

**Appendix F: Nonrelativistic approximation**

In the heavy nucleon limit, nucleon bilinear forms are approximated as

$$\bar{N}(x)\gamma_0 N(x) = \delta(r), \quad \bar{N}(x)\gamma N(x) = 0,$$

$$\bar{N}(x)\gamma_0\gamma_5 N(x) = 0, \quad \bar{N}(x)\gamma_5 N(x) = -\sigma_N \delta(r). \quad (F1)$$

We are interested in P-odd and T-odd weak interaction in the Fermi coupling between electron and nucleons. In the heavy nucleon limit (F1) these interactions are limited in the following forms,

$$H = \frac{G}{\sqrt{2}} \left( k_1 \bar{N}\gamma e\gamma_5 \sigma + k_2 \frac{1}{2} \epsilon^{\kappa \lambda \mu \nu} \bar{N}\sigma_{\kappa \lambda} N\sigma_{\mu \nu} e \right). \quad (F2)$$

In the nonrelativistic (heavy nucleon mass) limit it reduces to

$$H = i \frac{G}{\sqrt{2}} \delta(r) (k_1 \gamma_0 \gamma_5 + 4k_2 \sigma \cdot \gamma). \quad (F3)$$
Here you should consider $H$ is sandwiched by electron wave functions. For the case of a nucleus of charge $Z$ and mass number $A$, it gives

$$H = \frac{iG}{\sqrt{2}} \delta(r) \left[ (Zk_{1p} + Nk_{1n})\gamma_0\gamma_5 + 4 \left( k_{2p} \sum_p \sigma_p + k_{2n} \sum_n \sigma_n \right) \cdot \gamma \right], \quad (F4)$$

$$\langle s_{1/2} | H | p_{1/2} \rangle = g \frac{Z^2 R}{(N_s N_p)^{3/2}} Ry \left[ \gamma(Zk_{1p} + Nk_{1n}) - 8j \cdot \frac{2 + \gamma}{3} (k_{2p} \sum_p \sigma_p + k_{2n} \sum_n \sigma_n) + 8j \cdot (1 - \gamma) \langle k_{2p} \sum_p (n_p(\sigma_p \cdot n_p) - \frac{1}{3} \sigma_p) + k_{2n} \sum_n (n_n(\sigma_n \cdot n) - \frac{1}{3} \sigma_n) \rangle \right], \quad (F5)$$

The above arguments can be applied for both paramagnetic and diamagnetic atoms. Let us apply the above arguments to Cs, Tl, and Xe* atoms, corresponding to the arguments in Section 5.2 in the presence of $|F2\rangle$, The wave function for Cs is described as

$$\langle 6s_{1/2}, F \rangle = \langle 6s_{1/2}, F \rangle - 3.7 \times 10^{-11} [0.41k_{1p} + 0.59k_{1n} + 0.74 \times 10^{-2} (F(F + 1) - \frac{33}{2}) k_{2p}] |6p_{1/2}, F\rangle, \quad (F6)$$

and, therefore,

$$d(Cs) = e\langle 6s_{1/2}, F | z | 6s_{1/2}, F \rangle = -ea_B \times 1.34 \times 10^{-10} \times \left[ 0.41k_{1p} + 0.59k_{1n} + 0.74 \times 10^{-2} (F(F + 1) - \frac{33}{2}) k_{2p} \right]. \quad (F7)$$

Here $F$ is the total angular momentum of the atom. The observed value [159] is

$$d(Cs) = (-1.8 \pm 6.7 \pm 1.8) \times 10^{-24} \text{ e cm.} \quad (F8)$$

For Tl

$$d(Tl) = ea_B \cdot 0.96 \times 10^{-9} (0.4k_{1p} + 0.6k_{1n} - 2 \cdot 10^{-3}k_{2p}). \quad (F9)$$

For Xe*

$$d(Xe^*) = -1.3 \cdot 10^{-10} ea_B (0.41k_{1p} + 0.59k_{1n}). \quad (F10)$$

Appendix G: Strong CP violation

In the QCD world, the true vacuum is described by the $\theta$ vacuum,

$$|\theta\rangle \equiv \sum_n e^{-in\theta} |n\rangle, \quad (n = \text{integer}). \quad (G1)$$
\[ \langle \theta' | e^{-iHt} | \theta \rangle = \sum_{n,m} e^{im\theta'} e^{-in\theta} \langle m | e^{-iHt} | n \rangle \]
\[ = \sum_{m,n} e^{-i(n-m)\theta} e^{im(\theta'-\theta)} \int [dA]_{n-m} e^{i \int L d^4x} \]
\[ = \sum_{\nu} e^{-i\nu \theta} \int [dA]_{\nu} e^{i \int L d^4x} \]

Using that \( A_n \) gives
\[ n = \frac{1}{16\pi^2} \int d^4x \text{Tr} \left( G_{\mu\nu} \tilde{G}^{\mu\nu} \right) \tag{G3} \]
and substituting (G3) into (G2) we obtain
\[ \langle \theta' | e^{-iHt} | \theta \rangle = \sum_{\nu} \int [dA]_{\nu} e^{i \int L_{eff} d^4x} \]
with
\[ L_{eff} = L + \frac{\theta}{16\pi^2} \int d^4x \text{Tr} \left( G_{\mu\nu} \tilde{G}^{\mu\nu} \right) \tag{G5} \]

Appendix H: U(1) problem

\( \theta \) term in (203) comes from the fact that the vacuum in QCD is \( |\theta\rangle \), whereas \( G\tilde{G} \) term in (218) does from quantum anomaly, occurring irrelevant to Abelian and non Abelian. In this appendix we will show that these two terms are closely related and lead us to solve U(1) problem.

In the following discussions we consider mass zero quark limit, and \( N_f = 3 \), up, down, strange quarks. Chiral invariant action has originally \( U_L(3) \otimes U_R(3) \) symmetry. If, as we have considered, QCD vacuum is quark condensate
\[ \langle \pi u \rangle = \langle \bar{d}d \rangle = \langle \pi s \rangle, \tag{H1} \]
action symmetry is reduced to the flavor symmetry U(3) and generates \( 3^2 \) NG bosons. They are \( \pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \eta_8, \) and \( \eta_0 \). Here the first eight particles constitute octet and the last a singlet. The observed mass eigen states, \( \eta \) and \( \eta' \) particles, are the linear combinations of \( \eta_8 \) and \( \eta_0 \), and their masses are \( m_\eta = 550 \text{MeV}, \ m_{\eta'} = 958 \text{MeV} \). Weinberg showed [160] that the observed \( m_{\eta'} \) is too heavy for predicted NG boson,
\[ m_{\eta'} \leq \sqrt{3}m_{\pi}. \tag{H2} \]
This is one of the U(1) problems. Another is concerned with \( \eta \rightarrow \pi^+\pi^-\pi^0 \) process.
Let us explain these problems \[161\]: The octet axial vector currents satisfy

\[ \partial^\mu J_{5\mu}^a = f_a m_a^2 \phi^a \quad (a = 1, \ldots, 8) \] (H3)

and

\[
\delta_{ab} m_a^2 f_a^2 = \frac{i}{m_b^2 - k^2} \frac{m_b^2 - k^2}{m_b^2} \left\{ i k_\nu \int d^4 x e^{-ikx} \langle 0 | T \left( \partial^\mu J_{5\mu}^a(0) \partial^\nu J_{5\nu}^b(x) \right) | 0 \rangle \right. \\
+ \left. \int d^4 x e^{-ikx} \langle 0 | \delta(x_0) \left[ \partial^\mu J_{5\mu}^a(0), J_{50}^b(x) \right] | 0 \rangle \right\}.
\] (H4)

In the low energy limit, if there is no massless pole, this reduces to

\[
\delta_{ab} m_a^2 f_a = i \int d^4 x \langle 0 | \delta(x^0) [J_{50}^b(x), \partial^\mu J_{5\mu}^a(0)] | 0 \rangle
\] (H5)

Whereas, isosinglet axial vector current constitute ABJ anomaly \[218\]. The isosinglet can be described as a sum of SU(3) octet and singlet,

\[
J_{5\mu} = \frac{1}{\sqrt{3}} J_{5\mu}^{(8)} + \sqrt{\frac{2}{3}} J_{5\mu}^{(0)}
\] (H6)

with

\[
J_{5\mu}^{(8)} = \frac{1}{\sqrt{3}} (\pi \gamma_\mu \gamma_5 u + m_d d_\mu \gamma_5 d - 2 \pi \gamma_\mu \gamma_5 s).
\] (H7)

\[
J_{5\mu}^{(0)} = \frac{\sqrt{2}}{3} (\pi \gamma_\mu \gamma_5 u + m_d d_\mu \gamma_5 d + \pi \gamma_\mu \gamma_5 s)
\] (H8)

Taking \[218\], \[220\], and \[221\] into considerations, we obtain the same equation for isosinglet case as \[H5\] by replacing \(J_{5\mu}\) with \(\tilde{J}_{5\mu}\),

\[
m_0^2 f_0^2 = \frac{m_0^2 - k^2}{m_0^2} \left\{ i k_\nu \int d^4 x e^{-ikx} \langle 0 | T \left( \partial^\mu \tilde{J}_{5\mu}^0(0) \tilde{J}_{5}\bar{\nu}(x) \right) | 0 \rangle \right. \\
+ \left. \int d^4 x e^{-ikx} \langle 0 | \delta(x_0) \left[ \partial^\mu \tilde{J}_{5\mu}^0(0), \tilde{J}_{50}(x) \right] | 0 \rangle \right\}.
\] (H9)

Here \(f_0\) is the isoscalar meson decay constant. If there is no zero mass pole like the octet cases, this relation is same as the octet case \[H5\] except for \(J_{5\mu}\) replaced by \(\tilde{J}_{5\mu}\), and we obtain

\[
m_0^2 f_0^2 = m_\pi^2 f_\pi^2.
\] (H10)
So, if SU(3) is good symmetry, it goes from (H6) and (H10) that

\[ f_0 \geq \frac{1}{\sqrt{3}} f_\pi, \quad \text{(H11)} \]

which directly leads to (H2). However if any massless particle couples to \( \tilde{J}_\mu^5 \), then first term of (H5) does not vanish and we can evade (H10) \[162\]. 't Hooft showed that this is indeed the case if we take \( \theta \) vacuum into consideration correctly \[163\]. Also Witten proposed a solution compatible with quark condensate \[164\]:

\[ m^2_{\eta^0} = 4N_c^2 \left( \frac{\partial^2 E_\theta}{\partial \theta^2} \right)_{\theta=0}, \quad \text{(H12)} \]

where

\[
\left( \frac{\partial^2 E_\theta}{\partial \theta^2} \right)_{\theta=0} = \frac{1}{N_c^2} \left( \frac{1}{16\pi^2} \right)^2 \int d^4x \langle T \left( Tr(G(x)\tilde{G}(x)) Tr(G(0)\tilde{G}(0)) \right) \rangle
\]

\[ (\pi^+\pi^-\pi^0|\eta) = \frac{m_u - m_d}{F_\pi m_q} \lim_{k \to 0} \langle \pi^+\pi^-|\partial^\mu J_{5\mu}(k)|\eta \rangle. \quad \text{(H13)} \]

The right-hand side vanishes due to momentum conservation. However, it is experimentally observed as \( \Gamma(\eta \to \pi^+\pi^-\pi^0) \approx 200\text{eV} \). This process is occurred via SU(2) violating operator \[165\]

\[ \mathcal{L} = \frac{1}{2}(m_u - m_d)(\bar{u}u - \bar{d}d) \quad \text{(H15)} \]

and

\[ \langle 3\pi|\mathcal{L}|\eta \rangle \to \frac{(m_u - m_d)A}{\sqrt{2}f_\pi^2}, \quad \text{(H16)} \]

where

\[
A \equiv \langle \pi\pi|(m_u \pi \gamma_5 u + m_d \bar{d} \gamma_5 d)|\eta \rangle \left\rangle = \frac{1}{2i} \langle \pi\pi|\partial^\mu \tilde{J}_\mu^5(0)|\eta \rangle. \quad \text{(H17)} \]

So this process is suppressed by axial vector current conservation even if \( m_u \neq m_d \). This is another U(1) problem.

**Appendix I: Schiff moment**

There are several origins for the Schiff moment. Here we discuss the Schiff moment induced by the nuclear EDM when the charge and the EDM distributions (\( \rho_q \) and \( \rho_d \), respectively) in the nucleus are different \[12\].
The interaction of the electron with the dipole moment of finite size nucleus is

\[ V_s = \int d^3r'[\rho_d(r') - \rho_q(r')] d_N(r') \cdot \nabla' \frac{-e}{|r - r'|} \]  
\[ = \frac{1}{2} e \int d^3r'[\rho_d(r') - \rho_q(r')] d_N(r') r'_n \nabla_l \nabla_m \nabla_n \frac{1}{r} \tag{I1} \]

Here we may assume [166]:

- \( \rho_q \) is spherically symmetric.
- \( d_N \) coincides with the EDM of a valence nucleon, \( d_N = d_{p,n} \sigma \).
- \( \rho_d \) is due to the valence nucleon.

Then

\[ V_s = \frac{1}{2} e d_{p,n} \int d^3r' 4\pi r'^2 \left[ \rho_d(r') \langle \sigma_l m_n n_n \rangle - \rho_q(r') \frac{1}{3} \delta_{mn} \langle \sigma_l \rangle \right] \nabla_l \nabla_m \nabla_n \frac{1}{r}, \tag{I2} \]

where \( n = r'/r \). Let us divide \( \nabla_l \nabla_m \nabla_n \) as

\[ \nabla_l \nabla_m \nabla_n = \left[ \nabla_l \nabla_m \nabla_n - \frac{1}{5} (\delta_{lm} \nabla_n + \delta_{mn} \nabla_l + \delta_{nl} \nabla_m) \Delta \right] + \frac{1}{5} (\delta_{lm} \nabla_n + \delta_{mn} \nabla_l + \delta_{nl} \nabla_m) \Delta. \tag{I4} \]

The first term corresponds to the electron interaction with the 2\(^3\)-pole moment of the nucleus.

The second term = \[ \left[ \rho_d(\sigma_l n_m n_n) - \rho_q \frac{1}{3} \delta_{mn} \langle \sigma_l \rangle \right] \frac{1}{5} (\delta_{lm} \nabla_n + \delta_{mn} \nabla_l + \delta_{nl} \nabla_m) \]

\[ = - \left[ \frac{1}{3} \rho_q \langle \sigma \rangle - \frac{1}{5} \rho_d \langle 2 \sigma \cdot n n + \sigma \rangle \right] \cdot \nabla \tag{I5} \]

Here we use \[ [166] \]

\[ \sigma \cdot n n = \frac{1}{3} \sigma - \frac{\sqrt{8\pi}}{3} [Y_2 \otimes \sigma]_{(1)} \]
\[ \sigma = \sqrt{4\pi} [Y_0 \otimes \sigma]_{(1)} \tag{I6} \]

\[ 2\sigma \cdot n n + \sigma = \sqrt{4\pi} \left( \frac{5}{3} [Y_0 \otimes \sigma]_{(1)} - \frac{2\sqrt{2}}{3} [Y_2 \otimes \sigma]_{(1)} \right) \]

where

\[ [Y_l \otimes \chi]_{j,m} = \sum_{m_l, m_s} \langle l, m_l; \frac{1}{2}, m_s | j, m \rangle Y_{l,m_l}(\theta, \phi) \chi_{m_s} \tag{I7} \]

\[ \text{8 The following arguments are indebted to discussions with T. Sato} \]
with the Clebsch-Gordan coefficient \( \langle l, m; 1, m | j, m \rangle \) related with 3j-symbol

\[
\langle k_1, q_1, k_2, q_2 | K, Q \rangle = (-1)^{k_1-k_2+Q} \sqrt{2K+1} \binom{k_1}{k_2} \binom{k_2}{q_1 q_2}^T \binom{K}{q_1 q_2 - Q}.
\]

The following equation is the Wigner-Eckart theorem (the definition of reduced matrix element) \( \langle || || \rangle \),

\[
\langle \kappa m | O_{JM} | \kappa m' \rangle = \frac{1}{\sqrt{2j+1}} \langle jmJM | jm' \rangle \langle \kappa || O_j || \kappa \rangle
\]

where \( \kappa \) is defined by (185) and

\[
| \kappa \rangle \equiv | [ Y_l(n) \otimes \chi ](j) \rangle.
\]

It should be noted that the reduced matrix element has no dependence on \( m, m', \) nor \( M \).

For \( J = 1 \) case

\[
\langle \kappa | \vec{O} | \kappa \rangle = \langle \kappa | \vec{J} | \kappa \rangle \frac{\langle \kappa || \vec{O} || \kappa \rangle}{\langle \kappa || \vec{J} || \kappa \rangle}.
\]

\[
\langle \kappa || \vec{J} || \kappa \rangle = \sqrt{J(J+1)(2J+1)}
\]

\[
\langle \kappa || [ Y_l \otimes \sigma ](1) || \kappa \rangle = 2|\kappa|(-1)^{|\kappa|} \langle j, \frac{1}{2}; j, -\frac{1}{2} | 1, 0 \rangle
\]

\[
\times \begin{cases} 
1 - 2\kappa & \text{for } l = 0 \\
\frac{\sqrt{3}}{2(1 + \kappa)} & \text{for } l = 2 \\
-\frac{\sqrt{6}}{\sqrt{2}(1 + \kappa)} & \text{for } l = 2.
\end{cases}
\]

Thus we obtain

\[
\left\{ \rho_d \langle \sigma_l m n | \sigma_l \rangle - \rho_q \frac{1}{3} \delta_{m n} \langle \sigma_l \rangle \right\} \frac{1}{5} \left[ \delta_{l m} \nabla_m + \delta_{m n} \nabla_l + \delta_{nl} \nabla_m \right]
\]

\[
= \left[ \frac{1}{3} \rho_q \left( \kappa - \frac{1}{2} \right) - \frac{1}{5} \rho_d \left( \kappa - \frac{3}{2} \right) \right] \frac{(j) \cdot \nabla}{j(j+1)}.
\]

So

\[
V_s = \frac{ed}{2} \rho_{p,n} \left[ r_q^2 \frac{1}{3} \left( \kappa - \frac{1}{2} \right) - r_d^2 \frac{1}{5} \left( \kappa - \frac{3}{2} \right) \right] \frac{(j) \cdot \nabla}{j(j+1)} 4\pi \delta(r),
\]

where the mean squared radii are defined by

\[
r_{q,d}^2 \equiv \int d^3r' r'^2 \rho_{q,d}(r').
\]
In this derivation we assumed that the nuclear charge is uniformly distributed over a sphere of radius \( r_0 = 1.2 \times 10^{-13} A^{1/3} \) cm, and \( r_q^2 = \frac{3}{5} r_0^2 \). Also we may assume \( r_d^2 = r_q^2 \). Then we get the final expression for the Schiff moment.

\[
S = d p_n r_0^2 \frac{4 \pi (\kappa + 1)j}{25 j(j + 1)}. \tag{I18}
\]

**Appendix J: Effective Hamiltonian in molecule**

We have said that there appears huge internal electric field \( E_{\text{int}} \) in polar molecule. Here we consider how to estimate \( E_{\text{int}} \). The Dirac-Coulomb Hamiltonian is

\[
H_0 = \sum_i \{c \alpha_i \cdot p_i + \beta_i mc^2 + V_{\text{nucl}}(r_i)\} + \sum_{i<j} \frac{1}{r_{ij}} \tag{J1}
\]

and P,T-odd perturbation (the intrinsic part of \( H_{PTV} \)) is

\[
H' = -d_e \sum_i \beta_i \sigma_i \cdot E_{i,\text{int}} \tag{J2}
\]

with

\[
E_{i,\text{int}} = -\nabla_i \left( V_{\text{nucl}}(r_i) + 2 \sum_{i>j} \frac{e^2}{r_{ij}} \right). \tag{J3}
\]

Here electric field is given by Eq.(B1) with \( \phi = \sum_i \{V_{\text{nucl}}(r_i)\} + \sum_{i<j} \frac{1}{r_{ij}} \}. We are considering a static field, \( \frac{1}{c} \frac{\partial A}{\partial t} = 0 \), and \( H' \) is represented as

\[
H' = d_e \sum_i [\beta_i \sigma_i \cdot \nabla_i, H - T]. \tag{J4}
\]

Here \( T \) is kinetic term of electron

\[
T = \sum_i \{c \alpha_i \cdot p_i + \beta_i mc^2\}. \tag{J5}
\]

The expectation value w.r.t. the eigen function of \( H_0 \) gives

\[
\langle \Psi | \sum_i [\beta_i \sigma_i \cdot \nabla_i, H] | \Psi \rangle = 0. \tag{J6}
\]

Whereas,

\[
\sum_i [\beta_i \sigma_i \cdot \nabla_i, T] = i \sum_i \sum_j \{[\beta_i \sigma_i \cdot p_i, \alpha_j \cdot p_j] + [\beta_i \sigma_i \cdot p_i, (\beta_j - 1) m_j c]\}. \tag{83}
\]
Here the second term vanishes and the first term gives
\[ i \sum_i \sum_j \left[ \beta_i \sigma_i \cdot p_i, \alpha_j \cdot p_j \right] = \begin{cases} \sum_i 2i\beta_i \gamma_5 p_i^2 & \text{for } i = j. \\ 0 & \text{for } i \neq j. \end{cases} \]
Thus \( H' \) of (J4) is rewritten as [167]
\[ H'_\text{eff} = -2icd_e \sum_i \beta_i \gamma_5 p_i^2 \] (J7)
and we obtain finally
\[ -2ic \langle \psi_0 | \beta \gamma_5 p^2 | \psi_0 \rangle = 4cp^2 \Im (\varphi^\dagger \chi). \] (J8)
The enhancement factor is given by
\[ K = \sum_n \frac{\langle \psi | -2ic \beta_i \gamma_5 p^2 | \phi_n \rangle \langle \phi_n | \sum_i e z_i | \psi \rangle}{E - E_n + h.c.} \] (J9)
So the detailed calculations are reduced to the electron wave functions in atoms and molecules. For molecular case, unfortunately, only \( H_2^+ \) can be solved in the Born-Oppenheimer approximation [168]. However, its perturbation expansion around atomic level is also interesting since this method is applicable to the other diatomic molecule [169]. For more detailed explanation for diatomic case, see [170].

Appendix K: Spin motion in storage ring

There are many ongoing and near future experiments measuring EDMs and anomalous MDMs of charged particles. There one of the most important equations is the following spin precession equation,
\[ \frac{d\sigma}{dt} = \Omega_s \times \sigma \] (K1)
where
\[ \Omega_s = -\frac{e}{m} \left[ \left( G + \frac{1}{\gamma} \right) H - \left( G + \frac{1}{\gamma + 1} \right) v \times E + \frac{n}{2} (v \times H + E) \right]. \] (K2)
(Notations will be explained shortly.) However, curiously enough, the explicit derivation of this equation has not been published [172]. There are several confusions on the interpretations of this equation. In this appendix we give an explicit derivation of this equation.

9 For spinor case the above equation was derived by Silenko [173]. We are greatly indebted to Silenko for the discussions of this appendix.
In relativistic theory, spin vector is not conserved. We must derive an equation of motion for the spin when the particle moves. For that purpose, it is convenient to introduce 4-pseudovector $a^\mu$ defined by

$$a^\mu = (0, \zeta), \quad p^\mu = (m, 0) \quad \text{(K3)}$$

in the rest frame. So in any frame

$$a^\mu p_\mu = 0, \quad a_\mu a^\mu = -\zeta^2. \quad \text{(K4)}$$

In a moving frame with velocity $v = p/\epsilon$, $a^\mu = (a^0, a)$ is given by

$$a = \zeta + \frac{p(\zeta \cdot p)}{m(\epsilon + m)}, \quad a^0 = \frac{a \cdot p}{\epsilon} = \frac{p \cdot \zeta}{m}, \quad a^2 = \zeta^2 + \frac{(p \cdot \zeta)^2}{m^2}. \quad \text{(K5)}$$

Using this 4-pseudovector $a^\mu$, relativistic spin motion in electromagnetic field is given by

$$\frac{da^\mu}{d\tau} = \alpha F^{\mu\nu} a_\nu + \beta u^\mu F^{\nu\lambda} u_\nu a_\lambda + \gamma F^{*\mu\nu} a_\nu + \delta u^\mu F^{*\nu\lambda} u_\nu a_\lambda \quad \text{(K6)}$$

with $F^{*\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$. Here $\alpha, \beta, \gamma, \delta$ are coefficients whose meanings are determined as follows. In the rest frame, (K6) becomes

$$\frac{da^i}{dt} = \frac{d\zeta^i}{dt} = \alpha F^{ij} \zeta_j + \gamma F^{*ij} \zeta_j = \alpha (\zeta \times H)^i + \gamma (E \times \zeta)^i. \quad \text{(K7)}$$

In nonrelativistic case, Hamiltonian is

$$H = H' - \mu \sigma \cdot H - d \sigma \cdot E, \quad \text{(K8)}$$

where $H'$ includes all terms independing of spin terms. The time variation of spin $s = \sigma/2$ is

$$\dot{s} = i(Hs - sH) = 2\mu s \times H + 2ds \times E. \quad \text{(K9)}$$

Comparing this equation with (K7), we obtain

$$\alpha = 2\mu, \quad \gamma = -2d. \quad \text{(K10)}$$

As for $\beta$ term, it goes from the equation of motion (up to P-odd term)

$$m \frac{du^\mu}{d\tau} = e F^{\mu\nu} u_\nu \quad \text{(K11)}$$

and from $a_\mu u^\mu = 0$ that

$$u_\mu \frac{da^\mu}{d\tau} = -a_\mu \frac{du^\mu}{d\tau} = \frac{e}{m} F^{\mu\nu} u_\mu a_\nu. \quad \text{(K12)}$$
On the other hand, multiplying $u_\mu$ on (K6) and taking $u_\mu u^\mu = 1$ into account, we obtain

$$u_\mu \frac{da_\mu}{d\tau} = (2\mu + \beta) F^{\mu\nu} u_\mu a_\nu + (-2d + \delta) F^{\nu\mu} u_\mu a_\nu$$

(K13)

and then

$$\beta = -2 \left( \mu - \frac{e}{2m} \right) \equiv -2\mu', \quad \delta = 2d.$$  

(K14)

Thus we obtain

$$\frac{da_\mu}{d\tau} = 2\mu F^{\mu\nu} a_\nu - 2\mu' u_\mu F^\nu_\nu a_\nu - 2d (F^{\nu\mu} a_\nu - u_\mu F^{\nu\nu} a_\nu).$$  

(K15)

This equation is the generalized Bargmann-Michel-Telegdi (BMT) equation [171]. The spatial part of this equation is described as

$$\frac{d\mathbf{a}}{dt} = \frac{2\mu m}{e} \mathbf{a} \times \mathbf{H} + \frac{2\mu m}{\epsilon} (\mathbf{a} \cdot \mathbf{v}) \mathbf{E} - \frac{2\mu' \epsilon}{m} \mathbf{v} \times (\mathbf{v} \cdot \mathbf{a}) \mathbf{H} + \frac{2\mu' \epsilon}{m} \mathbf{v} (\mathbf{a} \cdot \mathbf{v}) (\mathbf{v} \cdot \mathbf{E})$$

$$- \frac{2dm}{\epsilon} \left[ (\mathbf{a} \cdot \mathbf{v}) \mathbf{H} - \mathbf{a} \times \mathbf{E} + \gamma^2 \mathbf{v} \{- \mathbf{a} \cdot \mathbf{H} - \mathbf{v} \cdot (\mathbf{a} \times \mathbf{E}) + (\mathbf{a} \cdot \mathbf{v}) (\mathbf{v} \cdot \mathbf{H})\} \right].$$  

(K16)

We consider the time development of $\zeta$. Let us first consider in the absence of EDM. Using the equation of motion (K20) or equivalently its decompositions into spatial and temporal components,

$$\frac{dp}{dt} = e\mathbf{E} + e\mathbf{v} \times \mathbf{H}, \quad \frac{d\mathbf{v}}{dt} = e\mathbf{v} \cdot \mathbf{E},$$

(K17)

Eq. (K16) is described in terms of the rest frame spin $\zeta$ as

$$\frac{d\zeta}{dt} = \frac{2\mu m + 2\mu' (\epsilon - m)}{\epsilon} \zeta \times \mathbf{H} + \frac{2\mu' \epsilon}{\epsilon + m} (\mathbf{v} \cdot \mathbf{H}) (\mathbf{v} \times \zeta) + \frac{2\mu m + 2\mu' \epsilon}{\epsilon + m} \zeta \times (\mathbf{E} \times \mathbf{v})$$

$$= \frac{e}{2m} \left( g - 2 + \frac{2m}{\epsilon} \right) \zeta \times \mathbf{H} + \frac{e}{2m} (g - 2) \epsilon \frac{\epsilon}{\epsilon + m} (\mathbf{v} \cdot \mathbf{H}) \mathbf{v} \times \zeta + \frac{e}{2m} \left( g - \frac{2\epsilon}{\epsilon + m} \right) \zeta \times (\mathbf{E} \times \mathbf{v})$$

(K18)

with

$$\mu = \frac{g}{2} \frac{e}{2m}.$$  

(K19)

In order to obtain the spin precession, we must subtract the rotation of particles moving around the storage ring. It goes from (K17) that

$$\frac{dv}{dt} = \frac{e}{m^2} (\mathbf{E} + \mathbf{v} \times \mathbf{H} - \mathbf{v} (\mathbf{v} \cdot \mathbf{E})).$$  

(K20)

Hereafter we consider the experimental situation where

$$\mathbf{H} \cdot \mathbf{v} = 0, \quad \mathbf{E} \cdot \mathbf{v} = 0.$$  

(K21)
Then (K20) is rewritten as
\[ \frac{dv}{dt} = \Omega_p \times v, \] (K22)
where
\[ \Omega_p = \frac{e}{m\gamma} \left( \frac{v \times E}{v^2} - H \right). \] (K23)

Eq.(K18) is reduced to
\[ \frac{d\zeta}{dt} = \frac{e}{m} \left[ \left( G + \frac{1}{\gamma} \right) \zeta \times H + \left( G + \frac{1}{\gamma + 1} \right) \zeta \times (E \times v) \right] = \Omega_s \times \zeta \] (K24)
with
\[ G = \frac{g - 2}{2} \] (K25)
and
\[ \Omega_s \equiv -\frac{e}{m} \left[ \left( G + \frac{1}{\gamma} \right) H + \left( G + \frac{1}{\gamma + 1} \right) E \times v \right]. \] (K26)

Consequently we obtain
\[ \Omega(d = 0) = \Omega_s - \Omega_p = -\frac{e}{m} \left[ G \frac{H}{\gamma} + \left( G - \frac{1}{\gamma^2 - 1} \right) E \times v \right]. \] (K27)

In the presence of EDM \(d \neq 0\), it should be remarked that all effects of (K20) (CP-even) are imposed on the MDM part. The change of \(\frac{da}{dt}\) to \(\frac{d\zeta}{dt}\) in EDM case are obtaind by
\[ \frac{d\zeta}{dt} = \frac{da}{dt} - \frac{\gamma v}{\gamma + 1} \left( \frac{da}{dt} \cdot v \right). \] (K28)

Substituting (K16) into (K28), EDM part is given by
\[ \frac{da}{dt} = 2d \left[ \frac{1}{\gamma} (\zeta \times E) + \frac{\gamma}{\gamma + 1} (v \times E)(\zeta \cdot v) - (\zeta \cdot v)H + \gamma v(\zeta \cdot H) - \frac{\gamma^2}{\gamma + 1} v(v \cdot H)(\zeta \cdot v) + \gamma v(v \cdot (\zeta \times E)) \right] \] (K29)
and
\[ -\frac{\gamma v}{\gamma + 1} \left( \frac{da}{dt} \cdot v \right) = -2d \frac{\gamma v}{\gamma + 1} \left[ \gamma(v \cdot (\zeta \times E)) + \frac{\gamma^2 - 1}{\gamma} (\zeta \cdot H) - \gamma(\zeta \cdot v)(v \cdot H) \right]. \] (K30)

So \(\zeta\) spin precession due to EDM is given by
\[ \frac{d\zeta}{dt} = 2d \left[ \zeta \times E + \frac{\gamma}{\gamma + 1} (v \cdot E)(v \times \zeta) + \zeta \times (v \times H) \right]. \] (K31)
Finally we obtain $\Omega_s$ (K2) with both MDM and EDM and the subtracted rotation angular velocity,

$$\Omega(d \neq 0) = \Omega_s - \Omega_p = -\frac{e}{m} \left[ G\mathbf{H} + \left( G - \frac{1}{\gamma^2 - 1} \right) \mathbf{E} \times \mathbf{v} + \frac{1}{2} \eta (\mathbf{E} + \mathbf{v} \times \mathbf{H}) \right]$$  \hspace{1cm} (K32)

with

$$d = \frac{\eta}{2m}.$$ \hspace{1cm} (K33)

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