Redistribution of energy in a viscous heat-conductive medium during the interaction of a shock wave with a temperature layered plasma region

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Abstract. Experimental data on the creation of ionization unstable gas discharge plasma with large-scaled and small-scaled ionisation strata are presented. The redistribution of kinetic and internal energy between a stratified energy source and the area behind a shock wave is numerically studied on the base of the system of Navier-Stokes equations. Generation of the Richtmyer-Meshkov instabilities at many points is obtained which bend the shock wave front and, in some cases, completely destroy it (in density fields). The influence of heat conductivity on the obtained flow structures has been studied. For viscous heat conducting medium it was established the possibility to obtain local high energy zones behind the shock wave with values of energies several times higher than those for a non-stratified energy source with the same total energy. The values of energies in these zones were shown to be possibly controlled via the rarefaction parameter/temperature of a gas in the layers of a stratified energy source or by its geometry.

Introduction
Recently the problem of supersonic/hypersonic flow control by different types of energy deposition has gained wide interest in the field of aerospace engineering studies [1-3]. The first works on theoretical issues of non-stationary dynamics of the impact of an energy source on the supersonic flow structure over a body and changes in the aerodynamic drag of a surface were the studies in [4, 5]. An effect of significant rearrangement of the flow caused by energy sources in the form of a “heat spike” [4] and a longitudinal heat layer [5] and the productivity of them to reduce the aerodynamic drag of the body was established. The growth of a precursor and the formation of a circulation flow were set in the calculations [5]. In [6] the effects of a focused energy deposition in the external flow which cause an essential wave drag reduction were obtained for hypersonic flow. In the experimental research to control the supersonic flow near the body, discharge [7, 8], laser [9] and microwave [10] energy sources were proposed. The shock wave curvature was obtained during its passage through the discharge zone [7]. Gas-discharge effect on the shock wave and on the flow separation area was evaluated in [11].
In [12] the formation of a significantly inhomogeneous shock layer under the action of an energy source in the form of an extended heated channel was obtained, which is characterized by the presence of vortices, instabilities, and additional shock waves. In particular, a manifestation of the Richtmyer-Meshkov instability was obtained accompanied by the development of a vortex in front of the body. An extensive review of Richtmyer-Meshkov instability studies is contained in [13, 14]. Triple-shock configurations together with the additional vortices formed by slip streams (starting at their centers) during the precursor growth under the action of such energy sources in different gas media were investigated in [15, 16].

Recent works on research of the structure of shock waves in a plasma medium focus on combining energy sources or creating an inhomogeneous configuration of plasma regions. The blurring of a reflected shock wave in a boundary layer which was transformed by a structure of longitudinal plasma filaments formed by a high-frequency discharge has been obtained in the experiments [17]. The effect of a double laser pulse on the aerodynamic characteristics of a streamlined body was studied in [18, 19]. In [20] the efficiency of using combined energy sources was justified. Ionization instability in the discharge plasma was obtained in [21] and the effect of stratified energy sources on the shock wave front using the inviscid approach was investigated. In [22] the density and temperature fluctuations behind a shock wave front provided by the action of a thermally stratified energy source were studied for viscous heat conducting gas. It was shown the possibility of the formation of flow structures with considerably increased values of gas temperature via the creation of thermally stratified energy release before the shock wave.

This article focuses on the redistribution of different types of energy during a shock wave propagation in an area of thermally layered gas structure which is accompanied by the development of instabilities. This is necessary both to determine changes in the intensity of an initial shock wave and to identify the mechanisms of transformation of the source energy into thermal or acoustic energy of the medium behind the shock wave front. In addition, it is important for achievement of the efficient high-speed flow control and improvement of the control of flow parameters in jets and combustion chambers during the processes of mixing and ignition.

1. Experimental data on obtaining ionization unstable strata and effect of the striation region on the shock wave front

The studies were carried out in a direct chamber with square cross section at a pressure of 7 torr of air where a plane shock wave of Mach number M=5 entered. This shock wave was formed earlier in the shock tube. To study the interaction of the shock wave with an area of temperature-stratified medium in the chamber an ionization unstable discharge [23] was ignited before the wave arrived (within the time interval about a minute). In [21] it has been shown numerically that the model of thermally stratified energy source adequately describes the phenomena registered at the schlieren photographs during the interaction of a shock wave with the striation region which allowed us to assume the thermally stratified

![Figure 1. Photographs of ionization instable discharge plasma, a) large-scale strata, b) small-scale strata](image-url)
nature of the region of the ionization strata. The gas discharge current could vary which made it possible to change the repetition rate of the ionization striations - regions with an increased electron temperature and concentration and as a consequence with an increased gas temperature. As a result, using the discharge a medium was created in the region of the passage of the shock wave with alternation of the temperature zones in the transverse to velocity of the flow direction with the possibility of varying their width and quantity.

Fig. 1 shows examples of the ionization-unstable gas discharge created in the experiment with large-scale strata (Fig. 1a) and small-scale strata (Fig. 1b). As shown by experimental schlieren photographs [21] large-scale stratification of the region of shock wave propagation leads to the distortion of its front, which bends acquiring an unstable shape. Small-scale inhomogeneities can lead to the complete disappearance of the shock wave structure at the schlieren photographs.

2. Modeling the interaction of a shock wave with a thermally stratified region

2.1. Methodology

Interaction of an initially plane shock wave with a stratified energy source is under consideration. Numerical simulations are based on the Navier-Stokes system of equations for viscous and heat conductive gas (air) [24, p. 329]. The state equation of the perfect gas is used with the “effective value” of the ratio of specific heats \( \gamma = 1.2 \). Such a value of \( \gamma \) corresponds to the degree of ionization of the gas medium of 0.00015 and the degree of non-equilibrium of 0.015 [25]. The problem is solved in non-dimensional variables, normalizing values for density and pressure, \( \rho_n = 0.01205 \text{ kg/m}^3 \) and \( p_n = 1013.25 \text{ Pa} \), and the Reynolds number \( Re = 9563 \) correspond to the experiments [21]. The dependence of dynamic viscosity \( \mu \) on temperature is approximated by the Sutherland law,

\[
\mu = T^{1.5} \frac{(1 + s_1)}{(T + s_1)},
\]

with the constant \( s_1 = 0.41 \) (120K). It was assumed that the coefficient of thermal conductivity \( k \) depends on temperature as

\[
k = T^{0.5}.
\]

Statement of the problem is presented schematically in Fig. 2. The effect of an energy source was modeled by a stationary region of heated gas layers in which the gas density was reduced relative to the initial undisturbed one (indicated by the index \( \infty \)), \( \rho_s = \alpha_s \rho_{\infty} \), \( \alpha_s < 1 \) and the pressure was equal to its initial value inside the shock tube (so the temperature was increased in the layers). Such an area was set at the initial time instant at some distance from the shock wave. Below the heated areas which model the action of energy sources are referred to as energy sources for shortness. The parameters of the plane shock wave were calculated from the Rankine–Hugoniot conditions. Horizontal tube walls were supposed to

![Figure 2. Statement of the problem (schematic)]
be cold with the value of undisturbed flow temperature. Additionally, the adhesion conditions and the condition of absence of the flows normal to the walls were set.

The numerical code was based on the conservative difference schemes of the second order of approximation [26]. For M=5 the TVD variant of the scheme was used, if necessary, with the limiter constructed guided by the Min-mod approach. There were 1000-2000 nodes on the transverse size of the tube (1.6*10^6 - 6.4*10^6 nodes in the calculation area, counting the central node).

Comparison of the calculations on two different grids is presented in Fig. 3a. Despite the fact that the grids differ significantly, it is clear seen that the main elements under the study (source layers, shock wave front, modified contact discontinuity front, mushroom structures, and instabilities near the front) appear in the calculations in a similar way. Behind the shock-wave structure, the fields are also almost identical. In addition, the calculations performed with different Reynolds numbers from the studied range give qualitatively similar pictures, but differ at the level of isochores even on fairly rough grids (Fig. 3b). Thus, the used numerical grids are sufficient to track the differences in Reynolds numbers and to give the adequate flow patterns for the study. In this research we use the grid with space steps $hx=hy=0.0005$.

In the simulations the fields of specific internal energy, $\varepsilon = p/(\rho(\gamma - 1))$, and volume density of kinetic energy, $E = \rho(\varepsilon + 0.5(u^2 + v^2))$ for the stratified energy source were investigated and compared with the fields of $\varepsilon$ and $E$ for a homogeneous non-stratified source with the same averaged values of the internal energy. In this case the values of total energies for stratified and non-stratified energy sources are equal, too (because the velocity components in the energy source are equal to zero). So, the advantages from the redistributing the source energy into layers are evaluated.

![Figure 3. Comparison of the calculations on two different grids (a), isochores, M=2, $\alpha_s=0.3$, $t=0.448$: blue – $hx=hy=0.001$, red – $hx=hy=0.0005$; and for different Re (b), $k=0$, $hx=hy=0.002$, $t=0.216$: blue – Re=9563; green – Re=5000](image)

2.2. Thermally homogeneously stratified energy source

The dynamics of density and pressure fields for the interaction of a thermally homogeneously stratified energy source with a plane shock wave of Mach 5 is presented in Fig. 4. Here in all source layers $\alpha_s=0.2$; time instants are indicated at the bottom of the pictures. These parameters correspond qualitatively to the situation of the complete disappearance of the shock fronts at the schlieren photographs [21]. One can see the curvature of the shock wave due to the boundary conditions on the horizontal walls (see, also, [7]) and the multiple generation of the Richtmyer-Meshkov instabilities which are manifested, in particular, in the density fields (Fig. 4, left row) (see, also, [12]). This may explain the effect of the disappearance of the shock fronts at the obtained schlieren photographs. The pressure field repeats the density one with the multiple generation of the Richtmyer-Meshkov instabilities (Fig. 4, right row). It should be noted that the structures of acoustic waves can be seen behind the shock wave front at $t=0.2$. The according redistributions of specific internal energy $\varepsilon$ and volume density of kinetic energy $E$ are shown in Fig. 5.
Figure 4. Dynamics of density (left row) and pressure (right row) during the interaction in a case of blurring the shock wave front, surface view, $M=5$, all $\alpha_s=0.2$

Figure 5. Dynamics of internal energy (left row) and kinetic energy (right row) during the interaction in a case of blurring the shock wave front, surface view, $M=5$, all $\alpha_s=0.2$

Fig. 6 presents the differences at the point $(x, y)$ between the fields of internal and volume kinetic energy for stratified and non-stratified energy sources, $\Delta \varepsilon(i, j) = \varepsilon(i, j) - \bar{\varepsilon}(i, j)$ and $\Delta \mathcal{E}(i, j) = \mathcal{E}(i, j) - \bar{\mathcal{E}}(i, j)$. The calculations show that at the local points behind the shock wave front
$\max(\Delta \varepsilon(i,j))/\bar{\varepsilon}(i,j)$ can achieve up to 105% in the forward structure (Fig. 6a) and $\max(\Delta \varepsilon(i,j))/\bar{\varepsilon}(i,j)$ can achieve 224% in the layered structure (yellow) (Fig. 6b) (here and below the fields for kinetic energy were analyzed only behind the shock wave front). It means that for the stratified source there exist the local zones of high energy behind the shock wave with the values which by several times exceed these values for non-stratified energy source with the same total energy.

Figure 6. Differences between the fields produced by stratified and non-stratified energy sources, M=5, all $\alpha_s=0.2$: a) - internal energy, b) - kinetic energy

Dynamics of the absolute maximum values over the whole calculation area of internal and kinetic energy, $\varepsilon_{\max}(t)$ and $E_{\max}(t)$, for stratified energy source (solid lines) and the according dependences $\bar{\varepsilon}_{\max}(t)$ and $\bar{E}_{\max}(t)$ for non-stratified energy source (dashed lines) for different $\alpha_s$ is presented in Fig. 7 (here the maxima are considered not at the same point). It can be seen that the increasing of $\varepsilon_{\max}$ in the case of a stratified energy source is stronger for smaller $\alpha_s$ (Fig. 7a). The maximum value of internal energy for the stratified energy source is 1.32 times greater than the according value for non-stratified one (for $\alpha_s=0.2$). The behavior of $E_{\max}(t)$ is more complicated and shows oscillations connected with the multiple manifestation of the Richtmyer-Meshkov instability (Fig. 7b). The maximum value of kinetic energy for the stratified energy source is 2.58 times greater than the according value for non-stratified one (for $\alpha_s=0.4$). This increase is connected with the growth of the instability peaks just behind the shock wave front.

Figure 7. Dynamics of $\varepsilon_{\max}$ (a) and $E_{\max}$ (b) during the interaction in a case of blurring the shock wave front for M=5: $\alpha_s=0.2$ (blue curve), $\alpha_s=0.3$ (green curve), $\alpha_s=0.4$ (red curve)

2.3. Thermally inhomogeneously stratified energy source

Experiment data showed that during the interaction of the shock wave with large-scaled strata the distortion of the shock wave front is taken place [21]. Here this situation is modelled by creating more heated central layers in the stratified energy source (Fig. 8). Differences between the fields of internal and volume kinetic energy for stratified and non-stratified energy sources at $t=0.18$ are presented in Fig. 9. The calculations show that at the local points behind the shock wave front $\max(\Delta \varepsilon(i,j))/\bar{\varepsilon}(i,j)$ can
achieve up to 305% in the forward structure (yellow) (Fig. 9a) and \( \max (\Delta E(i,j)) / \bar{E}(i,j) \) can achieve 140% in the structure behind the shock wave front (yellow) (Fig. 9b). Note, that the first result is connected with the fact that the heated area provided by the action of a stratified source comes to a given space point earlier than in the case of a non-stratified one.

![Figure 8. Distortion of the shock wave front under the action of stratified energy source M=5, \( \alpha_6=\alpha_7=\alpha_8=0.35 \), other \( \alpha_s=0.6 \): a) - internal energy; b) - kinetic energy](image)

![Figure 9. Differences between the fields produced by stratified and non-stratified energy sources, M=5, \( \alpha_6=\alpha_7=\alpha_8=0.35 \), other \( \alpha_s=0.6 \): a) - internal energy; b) - kinetic energy](image)

Dynamics of the absolute maximum energy values over the whole calculation area are presented in Fig. 10. One can see that the increase in \( E_{\text{max}} \) is stronger for smaller \( \alpha_s \) (Fig. 10a) and the defining parameters are the parameters of the central layers. The maximum value of internal energy for the stratified energy source is 2.08 times greater than the according value for non-stratified one (in the considered interval of the parameters). The behavior of \( E_{\text{max}} \) shows an oscillatory character connected with the manifestation of the Richtmyer-Meshkov instabilities (Fig. 10b). Here the maximum value of kinetic energy for the stratified energy source is 3.19 times greater than the according value for non-stratified one. In addition, we can control the behavior of these dependences by changing the temperature in the source layers.

2.4. A note on the influence of heat conductivity and boundary conditions

It should be noted that the effect of heat conductivity is essential in the considered phenomena because the flow is characterized by generating instabilities in a set of points which provide the flow structure with large values of space derivatives. As one can see from Fig. 11, the instabilities formed in a viscous flow with \( k=0 \) are numerous and highly pointed (Fig. 11a) while heat conductivity “cuts off” the amplitudes of these peaks and generally smooths the solution (Fig. 11b). However as shown by conducted calculations for large Mach numbers of the shock wave and small rarefaction parameters (or
high temperature) in the source layers the effects from the redistribution of the source energy into layers are noteworthy.

![Figure 10](attachment:image.png)

**Figure 10.** Dynamics of $\varepsilon_{\text{max}}$ (a) and $E_{\text{max}}$ (b) during the interaction in a case of distortion of the shock wave front, $M=5$: blue curve – $\alpha_6=\alpha_7=\alpha_8=0.1$, other $\alpha_s=0.3$; green curve – $\alpha_6=\alpha_7=\alpha_8=0.15$ other $\alpha_s=0.4$; red curve – $\alpha_6=\alpha_7=\alpha_8=0.35$, other $\alpha_s=0.6$

For wider layers in the energy source the generation of the Richtmyer-Meshkov instabilities is clearly seen via the development of mushroom structures (Fig. 12a). In Fig. 12b the dynamics of $e_{\text{max}}$ (solid lines) and $\bar{E}_{\text{max}}$ (dashed lines) are shown for stratified energy source with wider layers (blue curves) and with narrow layers (red curves). It is seen that the influence of heat conductivity is not so strong for wider layers compared with that for the narrow layers. This is connected with the fact that space gradients are smaller for wider layers.

It should be noted that the boundary conditions used at the horizontal boundaries are approximate and generate horizontal rarefaction waves to centre caused as parts of decays of discontinuities at the horizontal boundaries. Other parts of the decays (shock waves and contact discontinuities) do not arise due to the conditions on the walls. These discontinuities are associated with a discrepancy between the initial and boundary conditions in the region of the flow defined at the initial time moment behind the shock wave front. In the problem under consideration, the discharge area is limited in the direction tangential to the shock wave front. Therefore, in this direction, the temperature, density, and pressure decrease to their undisturbed values. In this case, the boundary conditions accepted in the work, strictly speaking, must be set at infinity. However, as was shown in [21], the boundary conditions used provide the curvature of the shock wave front which is adequate to that obtained in experiments. In addition, the dynamics of transverse structures far behind the shock wave front does not affect the results obtained.

### Conclusions

Experimental results on obtaining the regions of ionization instability in a gas discharge plasma and the effect of such regions on a plane shock wave are presented. The redistribution of specific internal and kinetic energy during the interaction of a plane shock wave with a thermally stratified energy source is
numerically studied. The calculations were performed for different temperatures in the source layers and different Mach numbers of the shock wave. It was shown that:

- Stratified energy sources are energetically more favorable than non-stratified ones and give possibilities to redistribute the energy behind the shock wave front in such a way that results in the distortion or complete disappearance of the shock fronts (in the density fields).
- Mechanisms of the obtained phenomena are connected with multiple generation of the Richtmyer-Meshkov instabilities.
- There exists the possibility of obtaining the local zones of high energy behind the shock wave with the values which by several times exceed these values for non-stratified energy source with the same total energy.
- The values of specific internal energy and volume kinetic energy in these zones can be controlled via the rarefaction parameter (or temperature of a gas) in the layers of a stratified energy source or by its geometry.

Figure 12. a) - Development of the Richtmyer-Meshkov instabilities in a case of wider layers, M=2, all $\alpha_s = 0.3$; b) - dynamics of $\epsilon_{\text{max}}$ and $\bar{\epsilon}_{\text{max}}$: narrow layers (red curves), wide layers (blue curves)

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