Flavoured Soft Leptogenesis

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Abstract: We study the impact of flavour in “soft leptogenesis” (leptogenesis induced by soft supersymmetry breaking terms). We address the question of how flavour effects can affect the region of parameters in which successful soft leptogenesis induced by CP violation in the right-handed sneutrino mixing is possible. We find that for decays which occur in the intermediate to strong washout regimes for all flavours, the produced total $B - L$ asymmetry can be up to a factor $O(30)$ larger than the one predicted with flavour effects being neglected. This enhancement, permits slightly larger values of the required lepton violating soft bilinear term.

Keywords: Neutrino Physics, Beyond Standard Model.
1. Introduction

The discovery of neutrino oscillations makes leptogenesis a very attractive solution to the baryon asymmetry problem [1,2]. In the standard framework it is usually assumed that the tiny neutrino masses are generated via the (type I) seesaw mechanism [3] and thus the new singlet neutral leptons with heavy (lepton number violating) Majorana masses can produce dynamically a lepton asymmetry through out of equilibrium decay. Eventually, this lepton asymmetry is partially converted into a baryon asymmetry due to fast sphaleron processes.

For a hierarchical spectrum of right-handed neutrinos, successful leptogenesis requires generically quite heavy singlet neutrino masses [4], of order $M > 2.4(0.4) \times 10^9 \text{ GeV}$ for vanishing (thermal) initial neutrino densities [4, 5], although flavour effects [6–9] and/or extended scenarios [10, 11] may affect this limit. The stability of the hierarchy between this new scale and the electroweak one is natural in low-energy supersymmetry, but in the supersymmetric seesaw scenario there is some conflict between the gravitino bound on the reheat temperature and the thermal production of right-handed neutrinos [12]. This is so because in a high temperature plasma, gravitinos are copiously produced, and their late decay could modify the light nuclei abundances, contrary to observation. This sets an upper bound on the reheat temperature after inflation, $T_{RH} < 10^{8-10} \text{ GeV}$, which may be too low for the right-handed neutrinos to be thermally produced.

Once supersymmetry has been introduced, leptogenesis is induced also in singlet sneutrino decays. If supersymmetry is not broken, the order of magnitude of the asymmetry and the basic mechanism are the same as in the non-supersymmetric case. However, as shown in Refs. [13–15], supersymmetry-breaking terms can play an important role in the lepton asymmetry generated in sneutrino decays because they induce effects which are essentially different from the neutrino ones. In brief, soft supersymmetry-breaking terms involving the singlet sneutrinos remove the mass degeneracy between the two real sneutrino states of a single neutrino generation, and provide new sources of lepton number and CP violation.
As a consequence, the mixing between the two sneutrino states generates a CP asymmetry in the decay, which can be sizable for a certain range of parameters. In particular the asymmetry is large for a right-handed neutrino mass scale relatively low, in the range $10^5 - 10^8$ GeV, well below the reheat temperature limits, what solves the cosmological gravitino problem. Moreover, contrary to the traditional leptogenesis scenario, where at least two generations of right-handed neutrinos are required to generate a CP asymmetry in neutrino/sneutrino decays, in this new mechanism for leptogenesis the CP asymmetry in sneutrino decays is present even if a single generation is considered. This scenario has been termed “soft leptogenesis”, since the soft terms and not flavour physics provide the necessary mass splitting and CP-violating phase.

In general, soft leptogenesis induced by CP violation in mixing as discussed above has the drawback that in order to generate enough asymmetry the lepton-violating soft bilinear coupling has to be unconventionally small [13, 14]. Considering the possibility of CP violation also in decay and in the interference of mixing and decay of the sneutrinos [15], as well as extended scenarios [16, 17], may alleviate this problem.

In Refs. [13–15] soft leptogenesis was addressed within the ‘one-flavour’ approximation. This one-flavour approximation is rigorously correct only when the interactions mediated by charged lepton Yukawa couplings are out of equilibrium. This is not the case in soft leptogenesis since, as mentioned above, successful leptogenesis in this scenario requires a relatively low right-handed neutrino mass scale. Thus the characteristic $T$ is such that the rates of processes mediated by the $\tau$ and $\mu$ Yukawa couplings are not negligible implying that the effects of lepton flavours have to be taken into account.

The impact of flavour in thermal leptogenesis in the context of the standard see-saw leptogenesis has been recently investigated in much detail. [6, 7, 9, 11, 18, 19, 21, 23, 24]. The relevant Boltzmann Equations (BE) including flavour effects associated to the charged lepton Yukawa couplings were first introduced in Ref. [18]. Additional flavour effects associated to the light-to-heavy neutrino Yukawa couplings which are particularly relevant for the case of see-saw resonant leptogenesis were discussed in Ref. [19]. In Ref. [6, 7, 11] it was further analyzed how flavour effects can significantly affect the result for the final baryon asymmetry.

In this work we study the impact of flavour in soft leptogenesis. We address the question of how flavour effects can affect the region of parameters in which successful leptogenesis induced by CP violation in the right-handed sneutrino mixing is possible, and in particular their impact on the required value of the lepton-violating soft bilinear coupling. The outline of the paper is as follows. Section 2 revisits the soft leptogenesis scenario with CP violation in the mixing and we present the relevant BE describing the production of the lepton asymmetry in this scenario without including flavour effects. In Sec. 3 we discuss the way to include flavour-dependent processes associated with the lepton Yukawa couplings in this scenario. Finally in Sec. 4 we present our quantitative results.
2. Unflavoured Soft Leptogenesis

The supersymmetric see-saw model could be described by the superpotential:

\[ W = \frac{1}{2} M_{ij} N_i N_j + Y_{ij} \epsilon_{\alpha \beta} N_i L_i^\alpha H^\beta, \]

where \( i, j = 1, 2, 3 \) are flavour indices and \( N_i, L_i, H \) are the chiral superfields for the RH neutrinos, the left-handed (LH) lepton doublets and the Higgs doublets with \( \epsilon_{\alpha \beta} = -\epsilon_{\beta \alpha} \) and \( \epsilon_{12} = +1 \). The corresponding soft breaking terms involving the RH sneutrinos \( \tilde{N}_i \) are given by:

\[ \mathcal{L}_{\text{soft}} = -\tilde{m}_{ij}^2 \tilde{N}_i^* \tilde{N}_j - \left( A_{ij} Y_{ij} \epsilon_{\alpha \beta} \tilde{N}_i \tilde{\ell}_j^\alpha h^\beta + \frac{1}{2} B_{ij} M_{ij} \tilde{N}_i \tilde{N}_j + \text{h.c.} \right), \]

where \( \tilde{\ell}_i^T = (\tilde{\nu}_i, \tilde{\ell}_i^-) \) and \( h^T = (h^+, h^0) \) are the slepton and up-type Higgs doublets.

The Lagrangian for interaction terms involving RH sneutrinos \( \tilde{N}_i \) and RH neutrinos \( N_i \) in 4-component spinors is given by:

\[ \mathcal{L}_{\text{int}} = -Y_{ij} \epsilon_{\alpha \beta} \left( M_i \tilde{N}_i^* \tilde{\ell}_j^\alpha h^\beta + \tilde{h}^\beta P_L \tilde{\ell}_j^\alpha \tilde{N}_i + \tilde{h}^\beta P_L N_i \tilde{\ell}_j^\alpha + A N_i \tilde{\ell}_j^\alpha h^\beta \right) + \text{h.c.} \]

from (2.3) and (2.4), we can write down the Lagrangian in the mass basis as

\[ \mathcal{L}_{\text{int}} = -\frac{Y_{ij}}{\sqrt{2}} \epsilon_{\alpha \beta} \left\{ \tilde{N}_{+i} \left[ \tilde{h}^\beta P_L \tilde{\ell}_j^\alpha + (A_{ij} + M_i) \tilde{\ell}_j^\alpha h^\beta \right] \\
+ i \tilde{N}_{-i} \left[ \tilde{h}^\beta P_L \tilde{\ell}_j^\alpha + (A_{ij} - M_i) \tilde{\ell}_j^\alpha h^\beta \right] \right\} + \tilde{h}^\beta P_L N_i \tilde{\ell}_j^\alpha + \text{h.c.} \]

In what follows, we will consider a single generation of \( N \) and \( \tilde{N} \) which we label as 1. We also assume proportionality of soft trilinear terms and drop the flavour indices for the coefficients \( A \) and \( B \). As discussed in Refs. [13, 14], in this case, after superfield rotations the Lagrangians (2.1) and (2.2) have a unique independent physical CP violating phase:

\[ \phi = \text{arg}(AB^*) \]

which we chose to assign to \( A \).

Neglecting supersymmetry breaking effects in the right sneutrino masses and in the vertex, the total singlet sneutrino decay width is given by

\[ \Gamma_{\tilde{N}_+} = \Gamma_{\tilde{N}_-} \equiv \Gamma_{\tilde{N}} = \frac{k \sum |M||Y_{1k}|^2}{4\pi}, \]

where

\[ \tilde{h}^\beta P_L \tilde{\ell}_j^\alpha + (A_{ij} - M_i) \tilde{\ell}_j^\alpha h^\beta \]
2.1 The CP asymmetry

As discussed in Ref. [14], when $\Gamma \gg \Delta M_{\pm} \equiv M_+ - M_-$, the two singlet sneutrino states are not well-separated particles. In this case, the result for the asymmetry depends on how the initial state is prepared. In what follows we will assume that the sneutrinos are in a thermal bath with a thermalization time $\Gamma^{-1}$ shorter than the typical oscillation times, $\Delta M_{\pm}^{-1}$, therefore coherence is lost and it is appropriate to compute the CP asymmetry in terms of the mass eigenstates Eq.(2.4).

As we will see below, the CP asymmetry produced in the decay of the state $\tilde{N}_i = \pm$ which enters into the BE is given by:

$$\epsilon_i = \frac{\sum_{a_k} \gamma(\tilde{N}_i \rightarrow a_k) - \gamma(\tilde{N}_i \rightarrow \bar{a}_k)}{\sum_{a_k} \gamma(\tilde{N}_i \rightarrow a_k) + \gamma(\tilde{N}_i \rightarrow \bar{a}_k)},$$

(2.9)

where $a_k \equiv s_k, f_k$ with $s_k = \tilde{\ell}_k h$ and $f_k = \ell_k \tilde{h}$ and we denote by $\gamma$ the thermal averaged rates. For convenience we also define the fermionic and scalar CP asymmetries in the decay of each $\tilde{N}_i$ as

$$\epsilon_{s_i} = \frac{\sum_k |\tilde{M}_i(\tilde{N}_i \rightarrow s_k)|^2 - |\tilde{M}_i(\tilde{N}_i \rightarrow \bar{s}_k)|^2}{\sum_k |\tilde{M}_i(\tilde{N}_i \rightarrow s_k)|^2 + |\tilde{M}_i(\tilde{N}_i \rightarrow \bar{s}_k)|^2},$$

(2.10)

$$\epsilon_{f_i} = \frac{\sum_k |\tilde{M}_i(\tilde{N}_i \rightarrow f_k)|^2 - |\tilde{M}_i(\tilde{N}_i \rightarrow \bar{f}_k)|^2}{\sum_k |\tilde{M}_i(\tilde{N}_i \rightarrow f_k)|^2 + |\tilde{M}_i(\tilde{N}_i \rightarrow \bar{f}_k)|^2}.$$  

(2.11)

Notice that $\epsilon_{s_i}$ and $\epsilon_{f_i}$ are defined in terms of decay amplitudes, without the phase-space factors which, as we will see, are crucial to obtain a non-vanishing CP asymmetry [13,14].

Neglecting supersymmetry breaking in vertices, the total asymmetry $\epsilon_i$ generated in the decay of the singlet sneutrino $\tilde{N}_i$ can then be written as:

$$\epsilon_i = \frac{\epsilon_{s_i} c_{s_i} + \epsilon_{f_i} c_{f_i}}{c_{s_i} + c_{f_i}},$$

(2.12)

where $c_{s_i}, c_{f_i}$ are the phase-space factors of the scalar and fermionic channels, respectively.

We compute the CP asymmetry following the effective field theory approach described in [28], which takes into account the CP violation due to mixing of nearly degenerate states by using resumed propagators for unstable (mass eigenstate) particles. The decay amplitude $\tilde{M}_i^a$ of the unstable external state $\tilde{N}_i$ defined in Eq. (2.4) into a final state $a$ is described by a superposition of amplitudes with stable final states:

$$\tilde{M}_i(\tilde{N}_i \rightarrow a) = \tilde{M}_i^a - \sum_{j \neq i} \tilde{M}_j^a \frac{i \Pi_{ij}}{M_i^2 - M_j^2 + i \Pi_{jj}},$$

(2.13)
where $M_i^a$ are the tree level decay amplitudes and $\Pi_{ij}$ are the absorptive parts of the two-point functions for $i, j = \pm$. The amplitude for the decay into the conjugate final state is obtained from (2.13) by the replacement $M_i^a \rightarrow M_i^{a*}$.

Neglecting supersymmetry breaking in vertices and keeping only the lowest order contribution in the soft terms we find the known result \[13, 14\]

\[ \epsilon_{s+} = \epsilon_{a-} = -\epsilon_{f+} = -\epsilon_{f-} \equiv \bar{\epsilon} = \frac{\text{Im}A}{M} \frac{4\Gamma B}{4B^2 + \Gamma^2}, \quad (2.14) \]

As long as we neglect the zero temperature lepton and slepton masses and small Yukawa couplings, the phase-space factors of the final states are flavour independent and they are the same for $i = \pm$. After including finite temperature effects in the approximation of decay at rest of the $\tilde{N}_\pm$ they are given by:

\[ c_{f+}(T) = c_{f-}(T) \equiv c_f(T) = (1 - x_\ell - x_h)\lambda(1, x_\ell, x_h) \left[ 1 - f_{\ell}^{eq} \right] \left[ 1 - f_{h}^{eq} \right] \quad (2.15) \]

\[ c_{s+}(T) = c_{s-}(T) \equiv c_s(T) = \lambda(1, x_h, x_\ell) \left[ 1 + f_{\ell}^{eq} \right] \left[ 1 + f_{h}^{eq} \right] \quad (2.16) \]

where

\[ f_{h,\ell}^{eq} = \frac{1}{\exp[E_{h,\ell}/T] - 1} \quad (2.17) \]

\[ f_{h,\ell}^{eq} = \frac{1}{\exp[E_{h,\ell}/T] + 1} \quad (2.18) \]

are the Boltzmann-Einstein and Fermi-Dirac equilibrium distributions, respectively, and

\[ E_{\ell,h} = \frac{M}{2}(1 + x_{\ell,h} - x_{\ell,h}), \quad E_{h,\ell} = \frac{M}{2}(1 + x_{h,\ell} - x_{h,\ell}), \quad \lambda(1, x, y) = \sqrt{(1 + x - y)^2 - 4x}, \quad x_a \equiv \frac{m_a(T)^2}{M^2} \quad (2.19) \]

The thermal masses for the relevant supersymmetric degrees of freedom are \[29\]:

\[ m_{\ell}^2(T) = 2m_h^2(T) = \left( \frac{3}{8}g_2^2 + \frac{1}{8}g_Y^2 + \frac{3}{4}\lambda_t^2 \right) T^2, \quad (2.20) \]

\[ m_{\ell}^2(T) = 2m_\ell^2(T) = \left( \frac{3}{8}g_2^2 + \frac{1}{8}g_Y^2 \right) T^2. \quad (2.21) \]

Here $g_2$ and $g_Y$ are gauge couplings and $\lambda_t$ is the top Yukawa, renormalized at the appropriate high-energy scale.

As we will see in Sec. 2.2 the contribution to the relevant BE for the lepton number scalar and fermion asymmetries can be factorized respectively as:

\[ \epsilon_s(T) \equiv \frac{\sum_k \gamma(\tilde{N}_\pm \rightarrow s_k) - \gamma(\tilde{N}_\pm \rightarrow \bar{s}_k)}{\sum_k \gamma(\tilde{N}_\pm \rightarrow a_k) + \gamma(\tilde{N}_\pm \rightarrow \bar{a}_k)} \equiv \bar{\epsilon} \frac{c_s(T)}{c_s(T) + c_f(T)}, \quad (2.23) \]
and
\[ \epsilon_f(T) \equiv \frac{\sum_k \gamma(\tilde{N}_\pm \to f) - \gamma(\tilde{N}_\pm \to \bar{f})}{\sum_{a_k,k} \gamma(\tilde{N}_\pm \to a_k) + \gamma(\tilde{N}_\pm \to \bar{a})} \equiv -\vec{\epsilon} \frac{c_f(T)}{c_s(T) + c_f(T)}. \tag{2.24} \]

The total CP asymmetry generated in the decay of any of the sneutrino \( \tilde{N}_\pm \) is then:
\[ \epsilon(T) = \vec{\epsilon} \frac{c_s(T) - c_f(T)}{c_s(T) + c_f(T)} \equiv \Delta_{BF}(T). \tag{2.25} \]

In this derivation we have neglected thermal corrections to the CP asymmetry from the loops, i.e., we have computed the imaginary part of the one-loop graphs using Cutkosky cutting rules at \( T = 0 \). These corrections are the same for scalar and fermionic decay channels, since only bosonic loops contribute to the wave-function diagrams in both cases, so they are not expected to introduce significant changes.

### 2.2 The Boltzmann Equations

We next write the relevant classical BE describing the decay, inverse decay and scattering processes involving the sneutrino states.

As mentioned above we assume that the sneutrinos are in a thermal bath with a thermalization time shorter than the oscillation time. Under this assumption the initial states can be taken as being the mass eigenstates in Eq. (2.4) and we write the corresponding equations for those states and the scalar and fermion lepton numbers. The CP fermionic and scalar asymmetries for each \( \tilde{N}_i \) defined at \( T = 0 \) are those given in Eq. (2.14).

The BE describing the evolution of the number density of particles in the plasma are:
\[
\frac{dn_X}{dt} + 3Hn_X = \sum_{j,l,m} \Lambda_{lm...}^{Xj...} \left[ f_l f_m \cdots (1 \pm f_X)(1 \pm f_j) \cdots W(lm \cdots \to Xj \cdots) - - f_X f_j \cdots (1 \pm f_l)(1 \pm f_m) \cdots W(Xj \cdots \to lm \cdots) \right]
\]

where,
\[ \Lambda_{lm...}^{Xj...} = \int \frac{d^3p_X}{(2\pi)^3 2E_X} \int \frac{d^3p_j}{(2\pi)^3 2E_j} \cdots \int \frac{d^3p_l}{(2\pi)^3 2E_l} \int \frac{d^3p_m}{(2\pi)^3 2E_m} \cdots , \]

and \( W(lm \cdots \to Xj \cdots) \) is the squared transition amplitude summed over initial and final spins. In what follows we will use the notation of Ref. [25]. We will assume that the Higgs and higgsino fields are in thermal equilibrium with distributions given in Eqs. (2.17) and (2.18) respectively. Strictly speaking this implies that we are not including all the effects associated with spectator processes [26, 27]. For the leptons and sleptons we assume that they are in kinetic equilibrium and we account for their asymmetries by introducing a chemical potential for the leptons, \( \mu_\ell \), and sleptons, \( \mu_{\tilde{\ell}} \):
\[
f_\ell = \frac{1}{e^{(E_\ell - \mu_\ell)/T} + 1}, \quad \bar{f}_\ell = \frac{1}{e^{(E_\ell - \mu_{\tilde{\ell}})/T} - 1}, \tag{2.26} \]
and the corresponding ones for the antiparticles with the exchange $\mu_\ell \to -\mu_\ell$ and $\mu_\tau \to -\mu_\tau$ respectively. Furthermore in order to eliminate the dependence in the expansion of the Universe we write the equations in terms of the abundances $Y_X$, where $Y_X = n_X / s$. Also for convenience we use the variable $z = M/T$.

We are interested in the evolution of sneutrinos $\tilde{Y}_{\tilde{N}_i}$ and the fermionic $Y_{L_f}$ and scalar $Y_{L_s}$ lepton numbers, defined as $Y_{L_f} = (Y_\ell - Y_\tau)$, $Y_{L_s} = (Y_\tau - Y_\tau)$. Moreover, in order to account for all the $\Delta L = 1$ terms we also need to consider the evolution of the right-handed neutrino $Y_N$.

Neglecting supersymmetry breaking effects in the right sneutrino masses and in the vertices, all the amplitudes for $N_+$ and $N_-$ are equal as well as their corresponding equilibrium number densities, $f_{\tilde{N}_+}^{eq} = f_{\tilde{N}_-}^{eq} \equiv f_{\tilde{N}}^{eq}$. So we can define a unique BE for $Y_{\tilde{N}_{\text{tot}}} \equiv Y_{\tilde{N}_+} + Y_{\tilde{N}_-}$. Thus, in total, in this unflavour case, we have a set of four BE.

The derivation of the factorization of the relevant CP asymmetries including the thermal effects is somehow lengthy but straightforward. In particular one has to use that at $O(\epsilon)$ we can neglect the difference between $f_{\tilde{N}_+}$ and $f_{\tilde{N}_-}$ in the definitions of the thermal average widths (see for example Ref. [16]). Many of the terms in the equations are equivalent to the ones given for example in Ref. [31].

Altogether we find:

$$sH \frac{dY_N}{dz} = - \left( \frac{Y_N}{Y_N^{eq}} - 1 \right) \left( \gamma_N + 4 \gamma_\ell^{(0)} + 4 \gamma_\ell^{(1)} + 4 \gamma_\ell^{(2)} + 3 \gamma_\ell^{(3)} + 4 \gamma_\ell^{(4)} \right) , \quad (2.27)$$

$$sH \frac{dY_{\tilde{N}_{\text{tot}}}}{dz} = - \left( \frac{Y_{\tilde{N}_{\text{tot}}}}{Y_{\til{N}}^{eq}} - 2 \right) \left( \gamma_{\til{N}} + 3 \gamma_\ell^{(3)} + 3 \gamma_\ell^{(4)} + 3 \gamma_\ell^{(5)} + 3 \gamma_\ell^{(6)} + 2 \gamma_\ell^{(7)} + 2 \gamma_\ell^{(8)} + 2 \gamma_\ell^{(9)} \right)$$

$$- \gamma_{\til{N}} Y_{L_f} \epsilon_f(T) + Y_{L_s} \epsilon_s(T), \quad (2.28)$$

$$sH \frac{dY_{L_f}}{dz} = \gamma_{\til{N}} \left[ \epsilon_f(T) \left( \frac{Y_{\til{N}_{\text{tot}}}}{Y_{\til{N}}^{eq}} - 2 \right) - \frac{Y_{L_f}}{Y_c^{eq}} \gamma_{\til{N}} \right]$$

$$+ \frac{Y_{L_f} - Y_{L_s}}{Y_c^{eq}} \gamma_{\text{MSSM}} . \quad (2.29)$$

$$sH \frac{dY_{L_s}}{dz} = \gamma_{\til{N}} \left[ \epsilon_s(T) \left( \frac{Y_{\til{N}_{\text{tot}}}}{Y_{\til{N}}^{eq}} - 2 \right) - \frac{Y_{L_s}}{Y_c^{eq}} \gamma_{\til{N}} \right]$$

$$- \frac{Y_{L_s}}{Y_c^{eq}} \left( \frac{1}{4} \gamma_N + \frac{1}{2} \gamma_N^{(2)} + \frac{1}{2} \gamma_N^{(4)} + \frac{1}{2} \gamma_N^{(5)} + \frac{1}{2} \gamma_N^{(6)} + \frac{1}{2} \gamma_N^{(7)} + \frac{1}{2} \gamma_N^{(8)} + \frac{1}{2} \gamma_N^{(9)} \right)$$

$$\gamma_{22} = \frac{Y_{L_f} - Y_{L_s}}{Y_c^{eq}} \gamma_{\text{MSSM}} . \quad (2.30)$$

\*However, some care has to be taken as the Eqs. in Ref. [31] are given in the weak basis for the $\tilde{N}$ while we give here the corresponding equations in the mass basis.
In the equations above, $Y_{eq}^{\gamma} \equiv \frac{15}{4 \pi^2 g_\ast^2}$ and $Y_{eq}^{\gamma}(T \gg M) = 90 \zeta(3)/(4 \pi^4 g_\ast^4)$, where $g_\ast$ is the total number of entropic degrees of freedom, $g_\ast^s = 228.75$ in the MSSM.

The different $\gamma$'s are the thermal widths for the following processes:

$$
\gamma_{\tilde{N}} = \gamma_{\tilde{N}}^j + \gamma_{\tilde{N}}^k = \gamma(\tilde{N}_\pm \leftrightarrow h\tilde{\ell}) + \gamma(\tilde{N}_\pm \leftrightarrow h\tilde{\ell}), \\
\gamma_{\tilde{N}}^{(3)} = \gamma(\tilde{N}_\pm \leftrightarrow \tilde{\ell}^* \tilde{u} \tilde{q}), \\
\gamma_{\tilde{N}}^{22} = \gamma(\tilde{N}_\pm \tilde{\ell} \leftrightarrow \tilde{u} \tilde{q}) = \gamma(\tilde{N}_\pm \tilde{q}^* \leftrightarrow \tilde{\ell}^* \tilde{u}) = \gamma(\tilde{N}_\pm \tilde{u}^* \leftrightarrow \tilde{\ell}^* \tilde{q}), \\
\gamma_{\tilde{N}}^{(5)} = \gamma(\tilde{N}_\pm \tilde{\ell} \leftrightarrow q \tilde{u}) = \gamma(\tilde{N}_\pm \tilde{\ell} \leftrightarrow q \tilde{u}), \\
\gamma_{\tilde{N}}^{(6)} = \gamma(\tilde{N}_\pm \tilde{\ell} \leftrightarrow q \tilde{q}) = \gamma(\tilde{N}_\pm \tilde{q}^* \leftrightarrow \tilde{\ell} \tilde{u}), \\
\gamma_{\tilde{N}}^{(7)} = \gamma(\tilde{N}_\pm \tilde{\ell} \leftrightarrow \tilde{\ell} \tilde{u}) = \gamma(\tilde{N}_\pm \tilde{u} \leftrightarrow \tilde{\ell} \tilde{q}), \\
\gamma_{\tilde{N}}^{(8)} = \gamma(\tilde{N}_\pm \tilde{\ell}^* \leftrightarrow \tilde{q} \tilde{u}), \\
\gamma_{\tilde{N}}^{(9)} = \gamma(\tilde{N}_\pm q \leftrightarrow \tilde{\ell} u) = \gamma(\tilde{N}_\pm \tilde{u} \leftrightarrow \tilde{\ell} \tilde{q}), \\
\gamma_N = \gamma(N \leftrightarrow \tilde{\ell} h) + \gamma(N \leftrightarrow \tilde{\ell}^* \tilde{h}), \\
\gamma_N^{(0)} = \gamma(N \tilde{\ell} \leftrightarrow q \tilde{u}) = \gamma(N \tilde{\ell} \leftrightarrow q \tilde{u}), \\
\gamma_N^{(1)} = \gamma(N \tilde{\ell} \leftrightarrow \tilde{\ell}^* \tilde{u}) = \gamma(N \leftrightarrow \tilde{\ell}^* \tilde{q}), \\
\gamma_N^{(2)} = \gamma(N \tilde{\ell} \leftrightarrow \tilde{\ell} \tilde{u}) = \gamma(N \tilde{\ell} \leftrightarrow \tilde{\ell} \tilde{q}) = \gamma(N \tilde{\ell} \leftrightarrow \tilde{\ell} \tilde{q}), \\
\gamma_N^{(3)} = \gamma(N \tilde{\ell} \leftrightarrow \tilde{q} \tilde{u}), \\
\gamma_N^{(4)} = \gamma(N \leftrightarrow \tilde{\ell} q) = \gamma(N \tilde{q} \leftrightarrow \tilde{\ell} \tilde{u}), \\
(2.31)
$$

where in all cases a sum over the CP conjugate final states is implicit.

We have included in Eqs. (2.27–2.30) the $\tilde{N}_\pm$ and $N$ decay and inverse decay processes as well as all the $\Delta L = 1$ scattering processes induced by the top Yukawa coupling. We ignore $\Delta L = 1$ scattering involving gauge bosons. We have accounted for the dominant CP asymmetry in the mixing as generated by the thermal effects in the $\tilde{N}_\pm$ two body decays but we have not included the possible CP violating effects induced by mixing in its three body decays or in its scattering processes. $\Delta L = 2$ processes involving the on-shell exchange of $N$ or $\tilde{N}_\pm$ are already accounted for by the decay and inverse decay processes. The $\Delta L = 2$ off-shell scattering processes involving the pole-subtracted s-channel and the u and t-channel, as well as the the L conserving processes from $N$ and $\tilde{N}$ pair creation and annihilation have not been included. The reaction rates for these processes are quartic in the Yukawa couplings, i.e., they involve factors $(YY^\dagger)^2$, and therefore can be safely neglected as long as the Yukawa couplings are much smaller than one, as it is the case.

The explicit expressions for the $\gamma$'s in Eq. (2.31) can be found, for example, in [31] for the case of Boltzmann-Maxwell distribution functions and neglecting Pauli-blocking and stimulated emission as well as the relative motion of the particles with respect to the plasma $^\dagger$. With these approximations, for example:

$$
\gamma_{\tilde{N}} = n_{eq}^{\tilde{N}} \frac{\Gamma_{\tilde{N}}}{\tilde{N}} \frac{K_1(z)}{K_2(z)}, \\
\gamma_N = n_{eq}^{\tilde{N}} \frac{\Gamma_{\tilde{N}}}{\tilde{N}} \frac{K_1(z)}{K_2(z)}, \\
(2.32)
$$

$^\dagger$Neglecting supersymmetry breaking effects in the right sneutrino masses and in the vertices, it can be shown that the thermal widths for the sneutrino mass eigenstates and weak eigenstates are the same.
where $K_{1,2}(z)$ are the modified Bessel function of the second kind of order 1 and 2 and $\Gamma_N = \Gamma_{\tilde{N}}$ are the zero temperature widths Eq. (2.8). In our calculation we keep the thermal masses and statistical factors on the CP asymmetries but we neglect them in the rest of the thermal widths, with the exception of the Higgs mass the in the $\Delta L = 1$ processes involving a Higgs boson exchange in the $t$-channel.

$\gamma_{MSSM}$ represent processes which transform leptons into scalar leptons and vice versa (for example $[e + e \leftrightarrow \tilde{e} + \tilde{e}]$). The rates for these reactions are larger than the ones in Eq. (2.31) because they do not involve the Yukawa couplings $Y_{ij}$. Consequently they enforce that $Y_{Lf} \approx Y_{Ls}$.

For $Y_{Lf} = Y_{Ls}$ we can combine the BE for $Y_{Lf}$ and $Y_{Ls}$ by defining

$$Y_{Ltot} \equiv Y_{Lf} + Y_{Ls},$$

which obeys the BE:

$$sH_d \frac{dY_{Ltot}}{dz} = \left[ \epsilon(T) \left( \frac{Y_{\tilde{N}Ltot}}{Y_{eq}^{\tilde{N}}} - 2 \right) - \frac{Y_{Ltot}}{2Y_{eq}^{\tilde{N}}} \right] \gamma_{\tilde{N}}$$

$$- \frac{Y_{Ltot}}{2Y_{eq}^{\tilde{N}}} \left( \frac{1}{4} \gamma_N + \frac{Y_{\tilde{N}Ltot}}{Y_{eq}^{\tilde{N}}} \gamma_{\tilde{N}}^2 + 2 \gamma_{\tilde{N}}^4 + 2 \gamma_{\tilde{N}}^6 + \frac{Y_N}{Y_{eq}^{N}} \gamma_N^2 + 2 \gamma_N^4 \right)$$

$$- \frac{Y_{Ltot}}{2Y_{eq}^{\tilde{N}}} \left( \frac{1}{4} \gamma_N + \gamma_{\tilde{N}}^2 + \frac{1}{2} \frac{Y_{\tilde{N}Ltot}}{Y_{eq}^{\tilde{N}}} \gamma_{\tilde{N}}^2 + 2 \gamma_{\tilde{N}}^4 + \frac{Y_N}{Y_{eq}^{N}} \gamma_N^2 + 2 \gamma_N^4 \right)$$

$$- \frac{Y_{Ltot}}{2Y_{eq}^{\tilde{N}}} \left( 2 + \frac{1}{2} \frac{Y_{\tilde{N}Ltot}}{Y_{eq}^{\tilde{N}}} \right) \gamma_{22}. \quad (2.34)$$

Also the second line in Eq. (2.28) can be written as $-\gamma_N \epsilon(T) \frac{Y_{Ltot}}{2Y_{eq}^{\tilde{N}}}$. So in total we are left with three BE for $Y_N$, $Y_{\tilde{N}Ltot}$, and $Y_{Ltot}$.

The final amount of $B - L$ asymmetry generated by the decay of the singlet sneutrino states assuming no pre-existing asymmetry can be parameterized as:

$$Y_{B-L}(z \to \infty) = -Y_{Ltot}(z \to \infty) = -2\eta \epsilon Y_{N}^{eq}(T >> M) \quad (2.35)$$

where $\epsilon$ is given in Eq.(2.14).

$\eta$ is a dilution factor which takes into account the possible inefficiency in the production of the singlet sneutrinos, the erasure of the generated asymmetry by $L$-violating scattering processes and the temperature dependence of the CP asymmetry and it is obtained by solving the array of BE above. Within our approximations for the thermal widths, $\eta$ depends on the values of the Yukawa couplings $(YY^\dagger)_{11}$ and the heavy mass $M$, with the dominant dependence arising in the combination

$$(YY^\dagger)_{11} v_u^2 \equiv m_{eff} M \quad (2.36)$$

The factor 2 in Eq. (2.35) arises from the fact that there are two right-handed sneutrino states while we have defined $Y_{\tilde{N}}^{eq}$ for one degree of freedom. Defined this way, $\eta$ has the standard normalization $\eta \to 1$ for perfect out of equilibrium decay.
Figure I: Efficiency factor $|\eta|$ as a function of $m_{\text{eff}}$ for $M = 10^7$ GeV and $\tan \beta = 30$. The two curves correspond to vanishing initial $\tilde{N}$ abundance (solid black curve) and thermal initial $\tilde{N}$ abundance, (dashed red curve).

where $v_u$ is the vacuum expectation value of the up-type Higgs doublet, $v_u = v \sin \beta$ ($v=174$ GeV). There is a residual dependence on $M$ due to the running of the top Yukawa coupling as well as the thermal effects included in $\Delta_{BF}$ although it is very mild.

In Fig. I we plot $|\eta|$ as a function of $m_{\text{eff}}$ for $M = 10^7$ GeV. Following Ref. [13, 29] we consider two different initial conditions for the sneutrino abundance. In one case, one assumes that the $\tilde{N}$ population is created by their Yukawa interactions with the thermal plasma, and set $Y_{\tilde{N}}(z \to 0) = 0$. The other case corresponds to an initial $\tilde{N}$ abundance equal to the thermal one, $Y_{\tilde{N}}^e(z \to 0) = Y_{\tilde{N}}^{eq}(z \to 0)$.

Our results show good agreement with those in Refs. [13–15]. In particular we reproduce that for zero initial conditions, $\eta$ can take both signs depending on the value of $m_{\text{eff}}$, thus it is possible to generate the right sign asymmetry with either sign of $\text{Im}A$. For thermal initial conditions, on the contrary, $\eta > 0$ and the right asymmetry can only be generated for $\text{Im}A > 0$. The plot is shown for $\tan \beta = 30$. But as long as $\tan \beta$ is not very close to one, the dominant dependence on $\tan \beta$ arises via $v_u$ as given in Eq. (2.36) and it is therefore very mild. For $\tan \beta \sim \mathcal{O}(1)$ there is also an additional (very weak) dependence due to the associated change in the top Yukawa coupling.

After conversion by the sphaleron transitions, the final baryon asymmetry is related

§The (in)dependence of the final asymmetry on the exact preparation of the initial state has been further explored in Ref. [32].
Figure II: $B, m_{\text{eff}}$ regions in which successful soft leptogenesis can be achieved. We take $|\text{Im}A| = 10^3$ GeV and $\tan \beta = 30$ and different values of $M$ as labeled in the figure. The two panels correspond to vanishing initial $\tilde{N}$ abundance (left) and thermal initial $\tilde{N}$ abundance, (right).

to the $B - L$ asymmetry by

$$Y_B = 24 + 4n_H Y_{B-L}(z \to \infty) = \frac{8}{23} Y_{B-L}(z \to \infty) \quad (2.37)$$

where $n_H$ is the number of Higgs doublets, which is taken to be $n_H = 2$ for the MSSM in the second equality.

This has to be compared with the WMAP measurements that in the ΛCDM model imply [33]:

$$Y_B = (8.78 \pm 0.24) \times 10^{-11} \quad (2.38)$$

We plot in Fig. II the range of parameters $B$ and $m_{\text{eff}}$ for which enough asymmetry is generated, $Y_B \geq 8.54 \times 10^{-11}$. We show the ranges for several values of $M$ and for the characteristic value of $|\text{Im}A| = 1$ TeV.

The figure illustrates our quantification of the known result that independently of the $\tilde{N}$ initial distributions, successful soft leptogenesis requires $M \lesssim 10^9$ GeV as well as $B \ll A$. Next we turn to the effect of flavour on these conclusions.

Before doing so, let us comment that, as pointed out in Refs. [13–15], these results indicate that in soft leptogenesis, the CP asymmetry is maximal when the parameter lie on the resonant condition $\Gamma = 2|B|$. In this case, the asymmetry is generated by the decays of two nearly mass-degenerate $\tilde{N}$. It has been recently discussed in Refs. [22] that for resonant scenarios, the use of quantum BE [22,34] (QBE) may be relevant. In particular it
has been shown that for standard see-saw resonant leptogenesis there are differences with
the classical treatment in the weak washout regime. There, however, no study on the
literature of the impact of the use of QBE for the case of soft leptogenesis and to discuss
those is beyond the scope of this paper. Thus in our work here we study the impact of
flavour in soft leptogenesis in the context of the classical BE as described above.

3. Flavour Effects

In the previous discussion flavour effects have been neglected. This is only justified when
the process of leptogenesis is completed at temperatures $T > 10^{12}$ GeV for which charged
lepton Yukawa processes are much slower than the processes involving $\tilde{N}$ and than the
expansion rate of the Universe.

However, as we have seen, soft leptogenesis is only effective enough for relatively light
right-handed sneutrino masses $M \lesssim 10^9$ GeV. Therefore in the relevant temperature win-
dow around $T \sim M$ processes mediated by the $\tau$ and the $\mu$ Yukawa couplings become faster.
As a consequence, the lepton states produced in the $\tilde{N}$ (and $N$) decay lose their coherent
between two subsequent $L$-violating interactions. So before they can re-scatter in reactions
involving $\tilde{N}$ and $N$ they are projected onto the flavour basis. In this case, the decay rates
and scattering processes involving the different flavours $l_k, \tilde{l}_k$ and anti-flavours $\bar{l}_k, \tilde{\bar{l}}_k$
have to be considered separately and we need to consider the BE for the single lepton-flavour
asymmetries.

To account for this effect we need to define the CP flavour asymmetries

$$\epsilon^k = \frac{\sum a_k \gamma(\tilde{N}_\pm \rightarrow a_k) - \gamma(\tilde{N}_\pm \rightarrow \bar{a}_k)}{\sum a_k \gamma(\tilde{N}_\pm \rightarrow a_k) + \gamma(\tilde{N}_\pm \rightarrow \bar{a}_k)}, \quad (3.1)$$

Taking into account that the Yukawa couplings can be chosen to be real the flavoured decay
rates verify (neglecting the zero temperature masses)

$$\gamma(\tilde{N}_\pm \rightarrow a_k) = K_k^0 \sum_k \gamma(\tilde{N}_\pm \rightarrow a_k)$$

$$\gamma(\tilde{N}_\pm \rightarrow \bar{a}_k) = K_k^0 \sum_k \gamma(\tilde{N}_\pm \rightarrow \bar{a}_k) \quad (3.2)$$

with projections $K_k^0$

$$K_k^0 = \frac{|Y_{1k}|^2}{\sum_k |Y_{1k}|^2} \quad (3.3)$$

so

$$\epsilon^k(T) \equiv \epsilon^k \Delta_{BF}(T) = K_k^0 \epsilon \Delta_{BF}(T) = K_k^0 \epsilon(T) \quad (3.4)$$

with $\epsilon(T)$ given in Eq. (2.25). We notice that because of the assumption of alignment
between the soft supersymmetry-breaking $A$ terms and the corresponding neutrino Yukawa
couplings, the only source of CP violation is a flavour independent phase. Therefore,
unlike in the case of see-saw leptogenesis induced by $N$ decay [6, 7], in this “minimal” soft leptogenesis scenario, it is not possible to have non-zero flavour asymmetries with a vanishing total CP asymmetry.

In Ref. [6] the relevant equations including flavour effects associated to the charged-lepton Yukawas were derived in the density operator approach. One can define a density matrix for the difference of lepton and antileptons such that $\rho_{kk} = Y_{L_k}$. As discussed in Ref. [6, 7] as long as we are in the regime in which a given set of the charged-lepton Yukawa interactions are out of equilibrium, one can restrict the general equation for the matrix density $\rho$ to a subset of equations for the flavour diagonal directions $\rho_{kk}$. In the transition regimes in which a given Yukawa interaction is approaching equilibrium the off-diagonal entries of the density matrix cannot be neglected [7, 21]. However, as we will see below, for the case of soft leptogenesis, this is never the case.

There are additional flavour effects associated to the neutrino Yukawa couplings as discussed in Ref. [19] such as those arising from processes mediated by $N_2$ and $\tilde{N}_2$ (and $N_3$ and $\tilde{N}_3$). These effects are particularly important in see-saw resonant leptogenesis in which right-handed neutrinos of different “generations” are close in mass. In leptogenesis with strong hierarchy among the masses of the different generations of right-handed neutrinos/sneutrinos (as we are assuming here) one can neglect the neutrino Yukawa couplings in most of the parameter space, because the charged-lepton Yukawa rates are faster at the temperatures when the asymmetry is produced. In what follows we will work under this assumption and neglect flavour effects associated to the neutrino Yukawas.

In writing the BE relevant in the regime in which flavours have to be considered, it is most appropriate to follow the evolution of $Y_{\Delta_k}$ where $\Delta_k = B - Y_{L_kf} - Y_{L_ks} \equiv B - Y_{L_k}^{tot}$. This is so because $\Delta_k$ is conserved by sphalerons and by other MSSM interactions. In particular, notice that the MSSM processes enforce the equality of fermionic and scalar lepton asymmetries of the same flavour. Hence, we can write down the flavoured BE for $Y_{\Delta_k}$

$$sHz \frac{dY_{\Delta_k}}{dz} = - \left\{ e^k(T) \left( \frac{Y_{\tilde{N}_tot}}{Y_{\tilde{N}}^{eq}} - 2 \right) \gamma_{\tilde{N}} - \sum_j A_{kj} \frac{Y_{\Delta_j}}{2Y_{\tilde{c}}^{eq}} \gamma_{\tilde{N}}^{(k)} \right. $$

$$- \sum_j A_{kj} \frac{Y_{\Delta_j}}{2Y_{\tilde{c}}^{eq}} \left( \frac{Y_{\tilde{N}_tot}^{(5)} Y_{\tilde{N}}^{(5)}}{Y_{\tilde{N}}^{eq}} \gamma_{\tilde{N}}^{(5)} + 2\gamma_{\tilde{N}}^{(6)} + 2\gamma_{\tilde{N}}^{(7)} + \frac{Y_{N}}{Y_{\tilde{N}}^{eq}} \gamma_{\tilde{N}}^{(3)} + 2\gamma_{\tilde{N}}^{(4)} \right)$$

$$+ \frac{1}{2} \gamma_{\tilde{N}}^{(2)} + \frac{1}{2} \gamma_{\tilde{N}}^{(8)} + 2\gamma_{\tilde{N}}^{(9)} + 2\gamma_{\tilde{N}}^{(10)} + 2\gamma_{\tilde{N}}^{(11)} + 2\gamma_{\tilde{N}}^{(12)}$$

$$- \left. \sum_j A_{kj} \frac{Y_{\Delta_j}}{2Y_{\tilde{c}}^{eq}} \left( 2 + \frac{1}{2} \frac{Y_{\tilde{N}_tot}}{Y_{\tilde{N}}^{eq}} \right) \gamma_{\tilde{N}}^{k} \right\}, \quad (3.5)$$

while the BE for $Y_N$ and $Y_{\tilde{N}_{total}}$ remain the same.
In Eq. (3.5) we have defined the flavoured thermal widths
\[ \gamma_{\tilde{N}}^K = K_k \gamma_{\tilde{N}}, \]
\[ \gamma_{t}^{(l)k} = K_k \gamma_{t}^{(l)} \] (3.6)

The value of \(A_{\alpha\beta}\) will depend on which processes are in thermal equilibrium when leptogenesis is taking place. For \(T < (1 + \tan^2 \beta) \times 10^9\)GeV where the processes mediated by all the three charged lepton (\(e, \mu, \tau\)) Yukawa couplings are in equilibrium i.e. they are faster than the processes involving \(\tilde{N}_\pm\), we have [20]

\[
A = \begin{pmatrix}
- \frac{93}{140} & \frac{6}{3} & \frac{6}{35} \\
\frac{3}{40} & - \frac{19}{30} & \frac{1}{30} \\
\frac{3}{40} & \frac{1}{30} & - \frac{19}{30}
\end{pmatrix}.
\] (3.8)

For \((1 + \tan^2 \beta) \times 10^9\)GeV < \(T < (1 + \tan^2 \beta) \times 10^{12}\)GeV where only flavours \(e + \mu\) and \(\tau\) are distinguishable, we have

\[
A = \begin{pmatrix}
- \frac{541}{761} & \frac{152}{494} \\
\frac{46}{761} & - \frac{152}{494} \\
\frac{152}{494} & \frac{761}{494}
\end{pmatrix}.
\] (3.9)

For \(T > (1 + \tan^2 \beta) \times 10^{12}\)GeV when all the flavours are indistinguishable i.e. the charged lepton Yukawa processes are much slower than the processes involving \(\tilde{N}_\pm\), we recover the unflavoured case where \(A = -1\).

From the results in Sec. 2 we see that in the relevant temperature window around \(T \sim M\) and for \(1 \leq \tan \beta \leq 30\) we are always in the regime when \(T < (1 + \tan^2 \beta) \times 10^9\)GeV thus we need to consider flavour effects associated to the three lepton flavours separately with \(A\) given in Eq. (3.8)

4. Results

We parametrize the asymmetry generated by the decay of the singlet sneutrino states in a given flavour as
\[ Y_{\Delta j}(z \to \infty) = -2\eta_j \bar{e}^j Y_{\tilde{N}}^{eq}(T >> M) \] (4.1)
where \(\bar{e}^j\) is defined in Eq.(3.4). Thus the final total asymmetry can be written as Eq. (2.35) where now
\[ \eta_{fla} = \sum_j \eta_j K_j^0 \] (4.2)

In Fig. III we plot \(|\eta_{fla}/\eta_0|\) as a function of \(m_{eff}\) for \(M = 10^7\) GeV and for \(K_1^0 = K_2^0 = K_3^0 = \frac{1}{3}\). We label \(\eta_0\) the corresponding efficiency factor without considering flavour effects. As seen in the figure for these values of the flavour projections, \(K_j^0\), and large \(m_{eff}\) (large washout region), flavour effects can make leptogenesis more efficient by up to a factor of the order 30. On the contrary flavour effects play no role for small \(m_{eff}\) (small washout). This can be easily understood by adding the equations for the three flavour asymmetries,
Figure III: Efficiency factor $|\eta/\eta_0|$ as a function of $m_{\text{eff}}$ for $M = 10^7$ GeV and $\tan \beta = 30$. The two curves correspond to vanishing initial $\tilde{N}$ abundance (solid black curve) and thermal initial $\tilde{N}$ abundance, (dashed red curve).

Eq. (3.5). We get an equation which can be written as:

$$s H z \frac{dY_{B-L}}{dz} = - \left\{ \epsilon(T) \left( \frac{Y_{\tilde{N}\text{tot}}}{Y_{\tilde{N}\text{eq}}} - 2 \right) \gamma_{\tilde{N}} - \sum_{k_j} A_{k_j} K_0^0 \frac{Y_{\Delta_j}}{2Y_{\tilde{N}\text{eq}}} W \right\}$$ (4.3)

where we have defined the washout term

$$W = \gamma_{\tilde{N}} + \frac{Y_{\tilde{N}\text{tot}}}{Y_{\tilde{N}\text{eq}}} \gamma^{(5)} + 2 \gamma^{(6)} + 2 \gamma^{(7)} + \frac{Y_{\tilde{N}}}{Y_{\tilde{N}\text{eq}}} \gamma^{(3)} + 2 \gamma^{(4)} + \frac{1}{2} \gamma_{\tilde{N}} + \gamma^{(2)}$$

$$+ \frac{1}{2} \frac{Y_{\tilde{N}\text{tot}}}{Y_{\tilde{N}\text{eq}}} \gamma^{(8)} + 2 \gamma^{(9)} + 2 \frac{Y_{\tilde{N}}}{Y_{\tilde{N}\text{eq}}} \gamma^{(10)} + 2 \gamma^{(1)} + 2 \gamma^{(2)} + \left( 2 + \frac{1}{2} \frac{Y_{\tilde{N}\text{tot}}}{Y_{\tilde{N}\text{eq}}} \right) \gamma^{(22)}$$ (4.4)

which can be directly compared with the unflavoured equation Eq. (2.34). We see that if we define $P_j = Y_{\Delta_j}/Y_{B-L}$, Eq. (4.3) is equivalent to Eq. (2.34) with

$$W \rightarrow -W \times \sum_{ij} A_{ij} K_j^0 P_i$$ (4.5)

Thus flavour effects are unimportant when the $W$ term in Eq. (4.3) is much smaller than the source term which happens when $m_{\text{eff}}$ is small enough (small washout regime).

We have verified that the equally distributed flavour composition $K_1^0 = K_2^0 = K_3^0 = 1/3$ (so all flavour are in the same washout regime) gives an almost maximum flavour effect.
for $m_{\text{eff}} \lesssim 10^{-2}$ eV. Conversely for $m_{\text{eff}} \gtrsim 10^{-2}$ eV values, flavour effects lead to larger $B - L$ for more asymmetric flavour compositions. In this case, the “optimum” flavour projection strongly depends on the value of $m_{\text{eff}}$.

We plot in Fig. IV the range of parameters $B$ and $m_{\text{eff}}$ for which enough asymmetry is generated, $Y_B \gtrsim 8.54 \times 10^{-11}$ for the the equally distributed flavour composition $K_1^0 = K_2^0 = K_3^0 = 1/3$. We show the ranges for several values of $M$ and for the characteristic value of $|\text{Im}A| = 1$ TeV. The dashed contours are the corresponding ones when flavour effects are not included. The figure illustrates to what extent flavour effects can affect the ranges of $B$ and $M$ for which successful soft leptogenesis can be achieved. This is more quantitatively displayed in Fig. V where we plot the asymmetry that can be achieved for a give value of $B$ (or $M$) maximized with respect to $m_{\text{eff}}$ and $M$ (or $B$) when flavour effects are included (for $K_1^0 = K_2^0 = K_3^0 = 1/3$) compared to the corresponding one when they are neglected. From the figure we read that successful soft-leptogenesis with (without) flavour effects considered requires $B \lesssim 8 \times 10^{-3}$ (3 $\times$ 10^{-3}) TeV and $M \lesssim 10^9$ (4 $\times$ 10^8) GeV for vanishing initial $\tilde{N}$ abundance and $B \lesssim 1.5 \times 10^{-2}$ (3 $\times$ 10^{-3}) TeV and $M \lesssim 3 \times 10^9$ (2 $\times$ 10^9) GeV for thermal initial $\tilde{N}$ abundance.

In summary, in this work we have studied the impact of flavour in soft leptogenesis. We have quantified to what extent flavour effects, which must be accounted for in the relevant
Figure V: Maximum baryon asymmetry be achieved as a function of $B$ (left) and $M$ (right). The solid (dashed) lines are for no flavour effects and vanishing (thermal) initial $\tilde{N}$ abundance. The dotted (dash-dotted) lines are the corresponding asymmetries after including flavour effects with $K_1^0 = K_2^0 = K_3^0 = 1/3$. The horizontal line correspond to the $1\sigma$ WMAP measurements in the $\Lambda$CDM model Eq. (2.38).

sneutrino mass range, can affect the region of parameters in which successful leptogenesis induced by CP violation in the right-handed sneutrino mixing is possible. We find that for decays which occur in the intermediate to strong washout regimes for all flavours, the produced total $B - L$ asymmetry can be up to a factor $O(30)$ larger than the one predicted with flavour effects being neglected. This enhancement, permits slightly larger values of the required lepton violating soft bilinear term $B$.

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References

[1] M. Fukugita and T. Yanagida, Phys. Lett. B174 (1986) 45

[2] S. Davidson, E. Nardi and Y. Nir, arXiv:0802.2962 [hep-ph].

[3] P. Minkowski, Phys. Lett. B 67, 421 (1977); M. Gell-Mann, P. Ramond and R. Slansky, Proceedings of the Supergravity Stony Brook Workshop, New York, 1979, eds. P. Van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam); T. Yanagida, Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe, Tsukuba, Japan
1979 (eds. A. Sawada and A. Sugamoto, KEK Report No. 79-18, Tsukuba); R. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).

[4] S. Davidson and A. Ibarra, Phys. Lett. B 535 (2002) 25 [arXiv:hep-ph/0202239].

[5] W. Buchmuller, P. Di Bari and M. Plumacher, Nucl. Phys. B 643 (2002) 367 [arXiv:hep-ph/0205349]; J. R. Ellis and M. Raidal, Nucl. Phys. B 643 (2002) 229 [arXiv:hep-ph/0206174].

[6] A. Abada, S. Davidson, A. Ibarra, F. X. Josse-Michaux, M. Losada and A. Riotto, arXiv:hep-ph/0605281; A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada and A. Riotto, JCAP 0604 (2006) 004 [arXiv:hep-ph/0601083];

[7] E. Nardi, Y. Nir, E. Roulet and J. Racker, JHEP 0601, 164 (2006) [arXiv:hep-ph/0601084];

[8] S. Blanchet and P. Di Bari, arXiv:hep-ph/0607330.

[9] O. Vives, Phys. Rev. D 73 (2006) 073006 [arXiv:hep-ph/0512160].

[10] E. Ma, N. Sahu and U. Sarkar, J. Phys. G 32, L65 (2006)

[11] P. Di Bari, Nucl. Phys. B 727 (2005) 318 [arXiv:hep-ph/0502082].

[12] M. Y. Khlopov and A. D. Linde, Phys. Lett. B 138 (1984) 265; J. R. Ellis, J. E. Kim and D. V. Nanopoulos, Phys. Lett. B 145 (1984) 181; J. R. Ellis, D. V. Nanopoulos and S. Sarkar, Nucl. Phys. B 259 (1985) 175; T. Moroi, H. Murayama and M. Yamaguchi, Phys. Lett. B 303 (1993) 289; M. Kawasaki, K. Kohri and T. Moroi, Phys. Lett. B 625 (2005) 7; For a recent discussion, see: K. Kohri, T. Moroi and A. Yotsuyanagi, Phys. Rev. D 73 (2006) 123511

[13] Y. Grossman, T. Kashti, Y. Nir and E. Roulet, Phys. Rev. Lett. 91 (2003) 251801 [arXiv:hep-ph/0307081];

[14] G. D’Ambrosio, G. F. Giudice and M. Raidal, Phys. Lett. B 575, 75 (2003) [arXiv:hep-ph/0308031].

[15] Y. Grossman, T. Kashti, Y. Nir and E. Roulet, JHEP 0411 (2004) 080 [arXiv:hep-ph/0407063].

[16] J. Garayoa, M. C. Gonzalez-Garcia and N. Rius, JHEP 0702 (2007) 021 [arXiv:hep-ph/0611311].

[17] G. D’Ambrosio, T. Hambye, A. Hektor, M. Raidal and A. Rossi, Phys. Lett. B 604 (2004) 199 [arXiv:hep-ph/0407312]; M. C. Chen and K. T. Mahanthappa, Phys. Rev. D 70, 113013 (2004) [arXiv:hep-ph/0409096]; Y. Grossman, R. Kitano and H. Murayama, JHEP 0506, 058 (2005) [arXiv:hep-ph/0504160]; E. J. Chun and S. Scopel, Phys. Lett. B 636, 278 (2006) [arXiv:hep-ph/0510170]; A. D. Medina and C. E. M. Wagner, JHEP 0612, 037 (2006) [arXiv:hep-ph/0609052]; E. J. Chun and L. Velasco-Sevilla, JHEP 0708, 075 (2007) [arXiv:hep-ph/0702039].

[18] R. Barbieri, P. Creminelli, A. Strumia and N. Tetradis, Nucl. Phys. B 575 (2000) 61 [arXiv:hep-ph/9911315].

[19] A. Pilaftsis and T. E. J. Underwood, Phys. Rev. D 72 (2005) 113001 [arXiv:hep-ph/0506107]; A. Pilaftsis, Phys. Rev. Lett. 95, 081602 (2005) [arXiv:hep-ph/0408103].

[20] S. Antusch, S. F. King, and A. Riotto, JCAP 0611 [arXiv:hep-ph/0609038].

[21] A. De Simone and A. Riotto, JCAP 0702 (2007) 005 [arXiv:hep-ph/0611357].
[22] A. De Simone and A. Riotto, JCAP 0708 (2007) 002 [arXiv:hep-ph/0703175]; A. De Simone and A. Riotto, JCAP 0708 (2007) 013 [arXiv:0705.2183 [hep-ph]].

[23] V. Cirigliano, A. De Simone, G. Isidori, I. Masina and A. Riotto, JCAP 0801 (2008) 004 [arXiv:0711.0778 [hep-ph]].

[24] T. Endoh, T. Morozumi and Z. h. Xiong, Prog. Theor. Phys. 111, 123 (2004) [arXiv:hep-ph/0308276]; T. Fujihara, S. Kaneko, S. Kang, D. Kimura, T. Morozumi and M. Tanimoto, Phys. Rev. D 72, 016006 (2005) [arXiv:hep-ph/0505076]; S. Pascoli, S. T. Petcov and A. Riotto, arXiv:hep-ph/0609125; G. C. Branco, R. Gonzalez Felipe and F. R. Joaquim, Phys. Lett. B 645 (2007) 432 [arXiv:hep-ph/0609297]; S. Antusch and A. M. Teixeira, arXiv:hep-ph/0611232; S. Pascoli, S. T. Petcov and A. Riotto, arXiv:hep-ph/0611338; S. Blanchet, P. Di Bari and G. G. Raffelt, arXiv:hep-ph/0611337.

[25] E. W. Kolb and S. Wolfram, Nucl. Phys. B 172, 224 (1980) [Erratum-ibid. B 195, 542 (1982)].

[26] W. Buchmuller and M. Plumacher, Phys. Lett. B 511, 74 (2001) [arXiv:hep-ph/0104189].

[27] E. Nardi, Y. Nir, J. Racker and E. Roulet, JHEP 0601, 068 (2006) [arXiv:hep-ph/0512052].

[28] A. Pilaftsis, Phys. Rev. D56 (1997) 5431 [arXiv:hep-ph/9707235]

[29] G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, Nucl. Phys. B 685 (2004) 89 [arXiv:hep-ph/0310123].

[30] A. Pilaftsis and T. E. J. Underwood, Nucl. Phys. B 692 (2004) 303 [arXiv:hep-ph/0309342].

[31] M. Plümacher, Nucl. Phys. B 530 207-246(1998) [arXiv:hep-ph/9704231].

[32] O. Bahat-Treidel and Z. Surujon, arXiv:0710.3905 [hep-ph].

[33] J. Dunkley et al. [WMAP Collaboration], arXiv:0803.0586 [astro-ph].

[34] W. Buchmuller and S. Fredenhagen, Phys. Lett. B 483, 217 (2000) [arXiv:hep-ph/0004145].