Underwater Acoustic Intensity Field Reconstruction by Kriged Compressive Sensing

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ABSTRACT
This paper presents a novel Kriged Compressive Sensing (KCS) approach for the reconstruction of underwater acoustic intensity fields sampled by multiple gliders following sawtooth sampling patterns. Blank areas in between the sampling trajectories may cause unsatisfying reconstruction results. The KCS method leverages spatial statistical correlation properties of the acoustic intensity field being sampled to improve the compressive reconstruction process. Virtual data samples generated from a kriging method are inserted into the blank areas. We show that by using the virtual samples along with real samples, the acoustic intensity field can be reconstructed with higher accuracy when coherent spatial patterns exist. Corresponding algorithms are developed for both unweighted and weighted KCS methods. By distinguishing the virtual samples from real samples through weighting, the reconstruction results can be further improved. Simulation results show that both algorithms can improve the reconstruction results according to the PSNR and SSIM metrics. The methods are applied to process the ocean ambient noise data collected by the Sea-Wing acoustic gliders in the South China Sea.

KEYWORDS
Underwater Gliders, Underwater Acoustic Sensing, Compressive Sensing, Kriging

1 INTRODUCTION
Underwater communication and underwater target detection rely heavily on the knowledge of the ambient ocean acoustic fields e.g. the spatial distribution of acoustic energy. Acoustic simulation models \cite{5, 9} have played important roles in studying the ambient acoustic fields. However, the model simulated results are different from real measurements because the model input parameters, including the multi-scale ocean dynamics, sea surface shape, and bottom topography, can be hardly acquired exactly from the real environment. In this paper, we propose construction algorithms that generate the acoustic intensity field directly from measurements collected by mobile sensing platforms such as the underwater gliders equipped with acoustic transducers.

A collection of spatial Interpolation methods have been used to process experimental data \cite{15} collected in maritime domain. These methods rely on empirical regression models and suffers rough reconstruction results when sampling points are sparse. Geostatistical kriging estimators take spatial correlation structures into consideration, producing both interpolated mean field and also the field of variance \cite{17}. For underwater acoustic communication channels, kriging techniques have been applied to construct spatial maps for data collected by boats or robot swarms \cite{10, 22}. The limitation of kriging is that it does not prevent overfitting, where high frequency spatial fluctuations due to uncertainty in the measurements may affect the performance of the map construction. Underwater acoustic intensity fields generally demonstrate some coherent spatial patterns, suggesting the compressive sensing (CS) methods, which is effective in preventing overfitting in map construction \cite{6, 20}. If spatial patterns are coherent and consistent in the field, compressive sensing may also work well with under-sampled data \cite{18}. However, due to the sampling limitation induced by the trajectories of the mobile sensing platforms, the distribution of the sampling points are not homogeneous in the sampling space, hence the data collected may not be able to capture the coherent spatial patterns.
We propose kriged compressive sensing (KCS) that combines the kriging method and the compressive sensing method to process data collected along the trajectories of mobile sensing platforms. We first extract the spatial statistical properties of the field from the collected data samples by using the kriging methods. The data in the un-sampled blank area can then be generated through kriged interpolation. Some of these interpolated data are randomly selected as virtual samples. The collected data samples and the virtual samples are then processed by compressive sensing to construct the spatial map. The KCS can be viewed as a type of interpolated compressive sensing (ICS) methods, which have been proposed to improve the performance by supplementing additional information in the un-sampled areas. For example, for multi-slice magnetic resonance imaging (MRI), because a strong inter-slice correlation exists, an ICS method is proposed to utilize data of neighboring slices in the multi-slice MRI acquisition to further improve imaging speed or reduce sampling data size while preserving the image quality [7]. For seismic data reconstruction, an ICS method is proposed to reconstruct the observed data on an irregular grid to any specified nominal grid [13] to facilitate map construction. In this paper, the sampling of underwater acoustic intensity field is constrained by the trajectories of the mobile sensing platforms, we found that kriged interpolation will improve the quality of map construction. The virtual samples used by the KCS are generated from the real samples, hence they should not be treated equally as the real samples when processed by compressive sensing. Some evaluation criteria should be used for choosing the virtual samples. The weights of these virtual samples can be determined by the kriging data variance, resulted in a weighted KCS method that further improve the performance of map construction.

The rest of the paper is organized as follows. In Section 2, KCS recovery for acoustic intensity field reconstruction based on spatial sampling is illustrated and then we introduce the weighted KCS method and algorithm. We also give two metrics to assess the performance of the proposed KCS methods. In Section 3, we present simulation results of the proposed KCS methods and discuss the sampling rate influence on reconstruction quality. In Section 4, we process the ocean ambient noise data collected by sea-wing acoustic gliders during the experiment conducted in the South China Sea.

2 METHODOLOGY

Mobile sensing platforms include autonomous underwater vehicles and underwater gliders that can host acoustic transducers on board to collect sound signals. The sampling patterns of these mobile sensing platforms are formed by continuous trajectories that cover a sampling area. Due to the motion constraints, these trajectories are not evenly distributed over space. For example, underwater acoustic gliders move along a sawtooth trajectory in the vertical plane, see figure 1. The acoustic intensity fields that vehicles measure often demonstrate spatial patterns due to interference enhancement or weakening of acoustic signals in local areas. These patterns are of great interest. Since some spatial locations are not visited by the mobile sensing platforms, it is quite challenging to capture these patterns from the data collected.

Figure 1: A sawtooth trajectory of underwater gliders in the vertical plane acquired from a experiment conducted in the South China Sea, on August 28, 2018.

2.1 Kriged Compressive Sensing

Compressive Sensing (CS) methods allow recovery from under-sampled measurements if the type of coherent patterns are known a priori. Suppose that we want to recover a $m \times n$ spatial field $f$. The prior knowledge that $f$ contains a coherent spatial pattern can be incorporated into a set of basis functions $\Psi$, so that $f$ can now be modeled as

$$x = \Psi f,$$

where coefficients $x$ is sparse. We further suppose that $x$ is a K-sparse signal where only $K$ elements of $x$ are nonzero. Here $K \ll N$, $N = m \times n$. There are many sparsity transforms $\Psi$ that can be chosen, such as the Fourier transform, wavelet transform, contourlet transform and others. The principle of selecting the sparsity transform is to minimize the sparsity $K$ of the coefficient matrix $x$. If $x$ is recovered, we can use the inverse transform of $\Psi$ to reconstruct the spatial field $f$.

Considering that the acoustic intensity field has smooth contours without noise, compressive sensing methods using a contourlet transform (CT) sparsity basis [14] is preferred by the reconstruction process. CT is composed of a double filter bank structure, including a Laplacian pyramid (LP) and a directional filter bank (DFB) [8]. Let the spatial field $f$ be the input image, the output after the LP stages is $J$ bandpass images $b_j, j = 1, 2, \ldots, J$ (in the fine-to-coarse order) and a lowpass image $a_f$. Specifically, the $j$-th level of the LP characterizes image approximation subspace $V_{j-1}$ at scale $2^j$ as two orthogonal subspaces $V_j$ and $W_j$.

$$V_{j-1} = V_j \oplus W_j,$$

The DFB is applied to the detail subspaces $W_j$. Each bandpass image $b_j$ is further decomposed by an $l_j$-level DFB into $2^{l_j}$ bandpass directional images $c_{j,k}^{(l_j)}, k = 0, 1, \ldots, 2^{l_j} - 1$. So we can briefly write the CT sparsity basis as

$$\Psi = D_{f_j}L_d,$$

where $D_{f_j}$ represents the $j$th directional filter.

The sampling matrix $\Phi \in \mathbb{R}^{M \times N}$ with $M \ll N$ has a single "1" in each row and up to a single "1" in each column. The spatial field
where \( y \in \mathbb{R}^M \) is a low-dimensional observation, \( \epsilon \in \mathbb{R}^M \) denotes measurement noises and \( \Lambda = \Phi \Psi^T \) is the sensing matrix that contains only the \( M \) sampled locations. Compressive sensing asserts that any \( K \)-sparse \( x \) could be perfectly recovered from \( y \) if \( A \) satisfies the restricted isometry property (RIP) [6].

\[
(1 - \delta_s)\|x\|^2_2 \leq \|\Lambda x\|^2_2 \leq (1 + \delta_s)\|x\|^2_2.
\]

For each integer \( s = 1, 2, \ldots \), the isometry constant \( \delta_s \geq 0 \) of a matrix \( A \) is defined as the smallest number to make \( A \) satisfies the RIP for all \( K \)-sparse signal \( x \). For example, if \( x \) is a column vector, we see that if any \( K \) columns of the matrix \( A \) are linearly independent, then the RIP is satisfied. Hence, we prefer to have \( M \) large and \( K \) small. Again, if \( x \) is a column vector, then each column of \( A \) represents the value of the basis function along the sampling trajectory. The RIP requires that the value of the the basis functions along different sampling trajectories be linearly independent. One way to satisfy this is to make the sampling trajectories to have different shapes and to be sufficiently spaced from each other.

In the case where the space is sparsely sampled, some randomly selected samples from a kriging generated field are added to the overall measurements. The semi-variogram generated by the kriging method results in a distance based correlations function among the measurements [10]. Kriging methods generate spatially interpolated mean and variance prediction at locations of interest according to measurements at known locations. The semi-variogram is estimated by

\[
y(h) = \frac{1}{2|N(h)|} \sum_{i=1}^{\lfloor N(h) \rfloor} |y(p_i) - y(p_i + h)|^2,
\]

where \( p_i \) (\( i \leq M \)) represents the \( i \)th sampling point, \( y(p_i) \) is the measurement at position \( p_i \), \( y(p_i + h) \) are all the measurements at a distance lag \( h \), and \( N(h) \) is the number of paired data \((y(p_i), y(p_i + h))\). Let \( b(p_i, p_j) \) be the distance between two sampling locations \( p_i \) and \( p_j \). The semi-variogram \( y(h) \) is then converted to a covariance function \( C(h(p_i, p_j)) = C(0) - y(h(p_i, p_j)) \), in which \( C(0) \) corresponds to the empirical self-correlation estimated from data.

Let \( p_0 \) be the point of interest where an interpolated measurements should be produced. Then we can estimate the weights \( \lambda_i \) paired with the measurements \( y(p_i) \) for \( i = 1, 2, \ldots, M \) for the interpolation. We solve the \( M \) weights \( \lambda_i \) and \( \nu \) from the following equation:

\[
\begin{bmatrix}
C(h(p_1, p_1)) & \ldots & C(h(p_1, p_M)) \\
\vdots & \ddots & \vdots \\
C(h(p_M, p_1)) & \ldots & C(h(p_M, p_M))
\end{bmatrix}
\begin{bmatrix}
\lambda_1 \\
\vdots \\
\lambda_M
\end{bmatrix}
= \begin{bmatrix}
C(h(p_1, p_0)) \\
\vdots \\
C(h(p_M, p_0))
\end{bmatrix}
\]

The kriged measurement at \( p_0 \) can then be calculated by

\[
y(p_0) = \sum_{i=1}^{M} \lambda_i y(p_i)
\]

and the variance of \( y(p_0) \) is

\[
\sigma^2 = C(0) - \sum_{i=1}^{M} \lambda_i C(\|p_i - p_0\|) - \nu.
\]

After we generate the kriged measurements, some of the locations with kriged measurements are randomly selected. These randomly sampled points are named virtual sampling points with values \( y_{\text{virtual}} \) as compared to the real samples \( y_{\text{real}} \). The virtual sampling points are only selected at the points with no real sampling values.

Then our objective is to solve the problem of formula (4) where \( y \) contains all \( y_{\text{real}} \) and \( y_{\text{virtual}} \). This is an underdetermined system. The \( K \)-sparse signal \( x \) can be reconstructed from \( y \) by solving a convex program

\[
\min \|\hat{x}\|_1 \text{ subject to } \|y - A\hat{x}\|_2 \leq \epsilon.
\]

A variety of reconstruction algorithms have been proposed to solve the optimization problem [19]. When we choose the solution algorithm, two considerations should be balanced, computational complexity and reconstruction quality. Typically, convex-programming approaches can reconstruct the field with high quality while the computational complexity is often large. Greedy algorithms and iterative thresholding algorithms [4] can significantly reduce computational complexity at the cost of lower reconstruction quality.

### 2.2 Weighted Kriged Compressive Sensing

Due to the estimation errors, the values of virtual samples will be different from real values if sampled at the virtual sampling points. For the more reliable sampling value at a virtual sampling point \((i, j)\), the corresponding weight \( \omega_{ij} \) should set to be larger. And the KCS reconstruction methods based on these weighted samples are named the weighted KCS.

Through the semi-variogram modeling, the kriging method provides measure of the reconstruction uncertainty which can be used to measure the reliability of the reconstructed field. We set the weight \( \omega_{ij} \) at a sampling point \((i, j)\), \( 1 \leq i \leq m, 1 \leq j \leq n \) as a function of the standard deviation \( \sigma_{ij} \) of the kriging generated field at the point \((i, j)\).

\[
\omega_{ij} = 1 - \frac{\sigma_{ij}}{\max_{i,j} \sigma_{ij}}.
\]

The denominator represents the maximum value of \( \sigma_{ij} \). This way, the weights \( \omega_{ij} \) for all virtual sampling points are limited to between \([0, 1]\).

After calculating the weights, we incorporate them into the proposed constrained optimization problem. A weighted constrained optimization problem is formulated as follows

\[
\min \|\hat{x}\|_1 \text{ subject to } \|w^T (y - A\hat{x})\|_2 \leq \epsilon.
\]

For virtual samples, the weights are calculated by the definition (10), while for real samples, the weights are all set to 1.

To solve the problem (11), as aforementioned, many algorithms have been proposed. Compared with constrained problem (9), the solver need to insert an additional term \( w \). Here, we choose projected Landweber (PL) algorithm, which is an iterative thresholding
algorithm, as the solution method because it is easy to be implemented. The specific implementation is presented in Algorithm 1. The “Thresholding” function sets \( \hat{x}^{(i)} \) to zero if its value is less than a small threshold, otherwise the value is unchanged.

**Algorithm 1** Weighted projected Landweber using a CT sparsity basis (WPL-CT) for spatial field reconstruction

**Input:** \( y, w, \Phi, \Psi \) and \( \delta \)
1. Initialize \( D^{(0)} = 0, f^{(0)} = \Phi^T y \)
2. Generate \( W = \text{Diag}(w) \)
3. repeat
   4. \( \hat{f}^{(i)} = f^{(i)} + \Phi^T W(y - \Phi f^{(i)}) \)
   5. \( \hat{x}^{(i)} = \Psi \hat{f}^{(i)} \)
   6. \( \hat{x}^{(i)} = \text{Thresholding}(\hat{x}^{(i)}) \)
   7. \( \bar{f}^{(i)} = \Psi^{-1} \hat{x}^{(i)} \)
   8. \( f^{(i+1)} = \bar{f}^{(i)} + \Phi^T W(y - \Phi \bar{f}^{(i)}) \)
   9. \( D^{(i+1)} = \| f^{(i+1)} - \bar{f}^{(i)} \|_2 \)
   10. \( i \leftarrow i + 1 \)
   11. until \( |D^{(i)} - D^{(i-1)}| < \delta \)
   12. \( f_{rec} = f^{(i)} \)

### 2.3 Performance Metrics

For compressive imaging [16], many metrics could be used to evaluate the performance of image recovery. In our paper, a 2D spatial field can be viewed as an image patterns along the sound propagation paths. So we can use the evaluation metrics for compressive imaging to judge the performance of vertical acoustic intensity field reconstruction. Among the many choices of metrics, the peak signal-to-noise ratio (PSNR) [12] and the structural similarity (SSIM) [21] index are most commonly adopted.

**Peak Signal-to-Noise Ratio.** PSNR estimates absolute errors and is formally defined as the ratio between the maximum possible power of a signal and the power of corrupting noise that distorts it. PSNR can be derived by the root-mean-square error (RMSE) as follows

\[
\text{PSNR} = 20 \times \log \left( \frac{\text{MAX}_f}{\text{RMSE}(f, f_{rec})} \right),
\]

where \( f_{rec} \in \mathbb{R}^{m \times n} \) is the reconstructed field, \( \text{MAX}_f \) is the maximum possible value of the field, and \( \text{RMSE} \) is defined as follows:

\[
\text{RMSE} = \sqrt{\frac{1}{m \times n} \sum_{i=1}^{m} \sum_{j=1}^{n} (f(i,j) - f_{rec}(i,j))^2}.
\]

**Structural Similarity Index.** SSIM is a perception-based model used for measuring the similarity between two images which is proven to be consistent with human eye perception. The SSIM formula is a combination of three comparison measurements between the images \( f \) and \( f_{rec} \), including luminance (\( l \)), contrast (\( c \)) and structure (\( s \)),

\[
\text{SSIM} = l(f, f_{rec}) \times c(f, f_{rec}) \times s(f, f_{rec}).
\]

The individual comparison functions are

\[
l(f, f_{rec}) = \frac{2\mu_f \mu_{f_{rec}} + c_1}{\mu_f^2 + \mu_{f_{rec}}^2 + c_1},
\]

\[
c(f, f_{rec}) = \frac{2\sigma_f \sigma_{f_{rec}} + c_2}{\sigma_f^2 + \sigma_{f_{rec}}^2 + c_2},
\]

\[
s(f, f_{rec}) = \frac{(\mu_f - \mu_{f_{rec}})^2 + c_3}{\mu_f^2 + \mu_{f_{rec}}^2 - \mu_f^2 + \mu_{f_{rec}}^2 + c_3},
\]

where \( \mu_f \) is the average of \( f \), \( \mu_{f_{rec}} \) is the average of \( f_{rec} \), \( \sigma_f^2 \) is the variance of \( f \), \( \sigma_{f_{rec}}^2 \) is the variance of \( f_{rec} \), \( c_3 = \sigma_f^2/2 \), \( c_1 = (k_1 L)^2 \) and \( c_2 = (k_2 L)^2 \) are two variables to stabilize the division with weak denominator where \( L \) is the dynamic range of the pixel-values.

### 3 SIMULATION STUDY

We simulate a fleet of underwater acoustic gliders that are deployed into a region in the Gulf of Mexico near the east coast of Mexico. The sound speed field in this region is calculated from the hydrological basic data acquired from HYCOM+NCODA Gulf of Mexico 1/25° analysis provided by the U.S. Naval Research Lab, Washington, DC, USA [3]. The bathymetry in this region is obtained from geospatial data and services provided by National Oceanic and Atmospheric Administration, Washington, DC, USA [2]. The acoustics toolbox we use to produce the acoustic field from environmental parameters is published in the Ocean Acoustics Library which is supported by the U.S Office of Naval Research, Arlington, VA, USA [1]. This region is chosen because the bathymetry and hydrological properties show high levels of spatial variation and data released in this region have a higher spatial resolution. We simulate a vertical 2D acoustic field within this region and use this simulated field as the ground truth. We then simulate the trajectories of multiple underwater acoustic gliders to sample this region along the sawtooth trajectories. Based on the simulated measurements taken by the gliders, the KCS and weighted KCS are used to construct the acoustic field. Through comparison with the ground truth, we demonstrate the performance of the proposed KCS and weighted KCS algorithms.

#### 3.1 Multiple Glider Sampling Simulation

We assume that underwater acoustic gliders can sample all the depth and range in this region along their sawtooth shaped trajectories. The sampling trajectories are shown by black lines in Fig. 2. We assume that these trajectories are straight lines without the influence of flow. We select one acoustic near-field area and one far-field area to perform the map construction. These two areas are marked by red squares in Fig. 2.

The specific results of reconstructed fields in Area I are shown in Fig. 3. After sampling the vertical acoustic field as shown in Fig. 3(a), we generate the kriged field from the real samples based on the resultant range semi-variogram in Fig. 3(c). Randomly select some virtual samples from the kriging estimated field, together with the real samples, we can utilize all the samples to reconstruct the intensity field. Fig. 3(e) is the field constructed by the unweighted KCS. By calculating the weights from the standard deviation of the kriged field in Fig. 3(d), we use the weighted KCS algorithm to construct the field in Fig. 3(f), whose two performance metrics, PSNR and SSIM, are the best. The same results are shown in Table 1. For both Area I and Area II, the proposed unweighted KCS and weighted KCS algorithms both perform better than CS or Kriging alone, and the weighted KCS method performs the best.
Figure 2: Simulated acoustic intensity field in the chosen region of the Gulf of Mexico. Black lines represent the sawtooth trajectories of underwater acoustic gliders. Two red squares marked areas that are randomly selected for reconstruction, in which Area I represents acoustic near field, and Area II stands for the far-field acoustic intensity.

Table 1: Performance metrics of reconstructed fields under real sampling rate = 16.24% and overall sampling rate = 88.24%.

|          | Area | CS   | Kriging | Unweighted KCS | Weighted KCS |
|----------|------|------|---------|----------------|--------------|
| PSNR (dB)| I    | 22.88| 31.58   | 31.60          | 31.70        |
|          | II   | 19.90| 34.57   | 34.58          | 34.60        |
| SSIM     | I    | 0.711| 0.763   | 0.765          | 0.779        |
|          | II   | 0.693| 0.822   | 0.823          | 0.828        |

During all the simulations aforementioned, the dimensions of the selected Area I and Area II are all set to 128×128. By rearranging the measurements into pixels, the corresponding real sampling rate is 16.24% and the overall sampling rate (real sampling rate plus virtual sampling rate) is 88.24%. The next step is to explore the performance of the two KCS algorithms under the influence of different sampling rates, both real sampling rates and overall sampling rates. We will give more specific illustration in the next section.

3.2 Influence of Sampling Rate

If only the real samples are used for reconstruction, the reconstruction result will in general be better at a higher sampling rate. As shown in Fig. 4, the two separated points on each figure stand for the reconstruction performance for CS and kriging just based on real samples. The red stars are the results of CS and the black squares are the results of kriging. Both the values of PSNR and SSIM are increasing as the real sampling rate becomes larger. For the KCS algorithms, the virtual sampling rate have an influence on the reconstruction performance. We vary the overall sampling rate uniformly from the real sampling rate upto around 90%, and conduct the proposed unweighted and weighted KCS methods for the reconstruction.

The red lines in Fig. 4(a) to Fig. 4(c) show the PSNR performance curves of weighted KCS over overall sampling rates starting from different real sampling rates, among which the real sampling rate is the lowest for Fig. 4(a) and highest for Fig. 4(c). For all cases, the weighted KCS tends to perform stably near the largest overall sampling rate. SSIM variation curves in Fig. 4(d) to Fig. 4(f) can further support this observation. This property can be leveraged by practical implementation. We can set the value of the virtual sampling rate to make the overall sampling rate approaching to 90% to generate better reconstructed field.

On the other hand, the unweighted KCS does not give better performance when more virtual samples are added for construction. This is because as an interpolation method, kriging has an effect to smear the spatial pattern. The increased number of virtual samples can smear the spatial pattern that the CS algorithm tries to capture. This can be observed from Fig. 4(e) and Fig. 4(f). We can also see that kriging alone performs quite well at low sampling rates when the PSNR metric is used for evaluation. However, kriging alone does not perform very well when the SSIM metric is used for evaluation. This is also due to the smearing effect of kriging.
Figure 4: Reconstruction performance under different sampling rates for Area I. The real sampling rate from left column to right column is in turn 16.24%, 29.64% and 44.67%. (a)-(c) show PSNR values calculated from different reconstruction fields, and (d)-(f) show corresponding SSIM values. Red stars and black squares represent the results of CS and kriging estimation respectively. Red and blue lines illustrate the metric values of weighted and unweighted KCS, respectively, varying over overall sampling rates.

4 FIELD EXPERIMENT

Underwater gliders have been developed and applied for oceanographic research worldwide and proven as efficient long-distance, long-duration ocean sampling platforms. Because of the low self-noise characteristics of underwater gliders, acoustic underwater gliders have been deployed since 2006 [11]. The first sea-wing acoustic underwater glider was developed by the Shenyang Institute of Automation in 2017, equipped with an acoustic hydrophone [23]. Several sea trials have been performed where the sea-wing acoustic underwater gliders are mainly used to collect underwater environmental sound data. Fig. 5 shows the ambient sound sampled along the glider trajectories in an experiment conducted in the South China Sea in August 2018. Although the glider has low self noise, the oil pump work near the sea surface or the deepest working depth could disturb the measurements. Hence, we select one area marked by the red square from Fig. 5 avoiding the shallow and deep areas to apply the KCS for reconstruction.

The sampling frequency of the on-board hydrophone was set to 8kHz. When we processed the recorded acoustic data, the frequency range of interest is between 200Hz and 2kHz. After bandpass filtering, the acoustic intensity was calculated by averaging the acoustic power of the received signal over a fix time interval of 2s. Each calculated acoustic intensity value was associated with a spatial location where the glider recorded the acoustic signal. Fig. 6 shows the acoustic intensity measurements within the selected area. We intended to reconstruct a 128 × 128 image. After the original measurements were rearranged to the grids, the sampling rate is 11.66%.
With the weighted KCS algorithm is better than kriging at higher performance of different algorithms. For the experimental result real sampling rate, especially when the PSNR is used as the metric. According to the simulation results shown in Fig. 4, the performance of the proposed KCS algorithm can not exceed the field constructed by weighted KCS. Also with 50% performance. The KCS performs consistently better than CS alone.

To show the comparison parameters among different reconstruction methods more clearly, we convert the MSE value to PSNR, defined as follows

\[ \text{PSNR}_{\text{Location}_\text{val}} = 20 \times \log \left( \frac{\text{MAX}_I}{\text{RMSE}(f, f_{\text{rec}})_{\text{Location}_\text{val}}} \right). \]

For performance validation, we conduct the ten-fold cross validation five times, and acquire the statistical result of the algorithm performance. The KCS performs consistently better than CS alone. With 50% probability, both proposed KCS algorithms perform better than kriging, as shown in Fig. 7(a), and Fig. 7(b) shows the field constructed by weighted KCS. Also with 50% probability, the performance of the proposed KCS algorithm can not exceed the kriging results, as shown in Fig. 7(c), and Fig. 7(d) shows the field constructed by weighted KCS.

The experimental results are consistent with the simulation results. The real sampling rate in the experiment is quite low. According to the simulation results shown in Fig. 4, the performance of the weighted KCS algorithm is better than kriging at higher real sampling rate, especially when the PSNR is used as the metric. For real experimental reconstruction, because we do not know the ground truth image, we cannot use the SSIM metric to evaluate the performance of different algorithms. For the experimental result with real sampling rate 10.49%, the PSNR value corresponding to weighted KCS algorithm may be lower than the results of kriging. This suggests that we may want to use other sound measuring method, such as towed hydrophone array to provide calibration for glider sampled data.

Another factor that should be considered is that the distribution of ocean ambient noise in spatial domain may not have strong coherent patterns. Hence the contourlet basis functions used by the weighted KCS may not be the best choice for map reconstruction. For the ambient noise field without specific patterns, directional filtering process used by the contourlet is not necessary. Other types of basis functions can be tested in the future.

5 CONCLUSION AND FUTURE WORK

This paper has demonstrated the feasibility of applying compressive sensing methods to the reconstruction of underwater acoustic intensity fields sampled by underwater gliders. A key contribution is to introduce the concept of KCS assisted by weighted virtual samples generated by the kriging method. We have demonstrated that the reconstruction quality can be improved with by using the weighted virtual samples. For further use of the weighted CS, the generation of interpolated field can be produced by other methods. We will carry out more experimental work on underwater gliders measuring different types of acoustic fields. We also plan to use other measuring methods to provide validation for the proposed algorithms.

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Figure 6: Acoustic intensity measurements within the selected area marked by red square in Fig. 5.
Figure 7: (a) and (c) show two typical reconstruction quality evaluated by PSNR over different sampling rate. Red stars are the performance of CS and black squares represent the performance of kriging. Lines in the figures illustrate the PSNR values varying over overall sampling rates. Red line shows performance of the weighted KCS and blue line shows performance of the unweighted KCS. (b) and (d) are the reconstructed fields by the weighted KCS method for which the real sampling rate is 10.49% and overall sampling rate is about 90%. (b) corresponds to (a) and (d) corresponds to (c).

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