Inflation in Oscillating Universe

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Abstract

We make use of possible high energy correction to the Friedmann equation to implement the bounce and study the behavior of massive scalar field before and after bounce semianalytically and numerically. We find that the slow-roll inflation can be preceded by the kinetic dominated contraction. During this process, the field initially in the bottom of its potential can be driven by the anti-frictional force resulted from the contraction and roll up its potential hill, and when it rolls down after the bounce, it can driven a period inflation. The required e-folds number during the inflation limits the energy scale of bounce. Further unlike that expected, the field during the contraction can not be driven to arbitrary large value, even though the bounce occurs at Planck scale.

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I. INTRODUCTION

The idea of oscillating universe is ancient, in which the universe oscillates through many successive expansion/contraction cycles, which was proposed originally by Tolman \cite{1} in the 1930’s. Recently this idea has been awaken again in cyclic scenario \cite{2}, which is motivated by the string/M theory. The relevant dynamics with primordial perturbations can be described by an effective theory in which the separation of the branes in the extra dimensions is modeled as a scalar field. But the problems around bounce may remain. For contracting universe, it is hard to escape from a singularity in the frame of general relativity. Thus for the scenario of oscillating universe, one of difficulties of implementing it is how to obtain a non singular bounce. Several proposals have been discussed, such as using the negative energy density \cite{3, 4, 5} or curvature term \cite{6}, or some high order correction term in the action \cite{7, 8}.

During contraction of each cycles, the scalar field can be driven and roll up along its potential to large enough value for inflation to occur, which has been noticed in Ref. \cite{9} for closed universe, in which it has been shown that an oscillating universe certainly undergoes inflation after a finite number of cycles. During this process, a contracting phase dominated by kinetic energy followed by an inflation. Further, in Ref. \cite{10}, such scenario has been proposed to provide a possible explanation for the observed low CMB anisotropies on large angular scale. In this proposal, a kinetic dominated contracting phase is matched to an inflation phase. The power spectrum during the contraction $\sim k^{3-1}$, which lead to an intense suppression of CMB quadrupole, in the meantime the nearly scale-invariant spectrum from slow-roll inflation is recover on small scale. But a physical mechanism for bounce is not included. In this paper, we make use of possible high energy correction to the Friedmann Eq. \cite{15} to implement the bounce. We firstly focus the behavior of massive scalar field during such a bounce semianalytically and find that the slow-roll inflation can be preceded by the kinetic dominated contraction and the required e-folds number during the inflation after the bounce limits the energy scale of bounce. Finally we check and confirm these features numerically. Note that a similar scenario has been studied in string inspired frame \cite{13}, in which the Gauss-Bonnet term is used to construct a non singular bounce and the

\footnote{This is also the usual results of Pre Big Bang scenario \cite{11}, see Ref. \cite{12} for a review.}
superinflation regime before slow-roll inflation leads to a suppression of primordial spectrum. This has also a similarity to the case in loop quantum gravity \[14\]. The effects from loop quantum gravity can driven the inflaton to its potential hill and increase the parameter space of initial conditions for successful inflation. Furthermore, the oscillatory universe induced by the effects from loop quantum gravity was investigated in Ref. \[16\].

II. DYNAMICS OF MODEL

In this section we study the cosmological behavior of massive scalar field before and after a bounce phase semianalytically. We use the modified Friedmann equation

\[ h^2 = \frac{1}{3} \left( \rho_{\phi} - \frac{\rho_{\phi}^2}{\sigma} \right) \]  

(1)

to implement a realistic bounce, where $8\pi/m_p^2 = 1$ has been set, $\rho_{\phi}$ is the energy density of scalar field and $\sigma$ is the bounce scale, when $\rho_{\phi} = \sigma$, the universe bounces. Eq. \[11\] may be motivated in brane world scenario \[15\], see also \[17\], where our universe is embedded in high dimension space/time, see Ref. \[18\] for a recent introduction. Further it has been pointed out in Ref. \[19\] that due to the effects of the bulk and Israel matching conditions there might be some forms $h(\rho)$ which is not standard Friedmann-like.

The motion equation of scalar field $\varphi$ with the mass $m$ is

\[ \ddot{\varphi} + 3h\dot{\varphi} + m^2 \varphi = 0 \]  

(2)

We assume that initially the universe is in contracting phase and the field $\varphi$ is in the bottom of its potential. Thus $\rho_{\phi}$ is small and the term $\rho_{\phi}^2$ in Eq. \[11\] is negligible. Some possible fluctuations will make $\varphi$ departure from its minimum and oscillate near $\varphi = 0$. Since in this case $m^2 \gg h^2$, i.e. the frequency of oscillation is much larger than the evolution rate of universe, thus after implementing the change of variable $\varphi = a^{-\frac{3}{2}}u$,

\[ \ddot{u} + m^2 u \simeq 0 \]  

(3)

can approximately be obtained, which can be solved as an oscillation

\[ u \simeq u_a \sin(mt) \]  

(4)

where $u_a$ is a constant amplitude. Thus we have

\[ \varphi = a^{-\frac{3}{2}}u_a \sin(mt) \]  

(5)
When taking the time average over many oscillations of field, \( < p_\varphi > \simeq 0 \) is obtained, which is similar to the case dominated by matter. Thus in this regime the universe contracts as \( a \sim t^{2/3} \) and the energy density of the field grows as

\[
\rho_\varphi \sim a^{-3}u_\alpha^2m^2 \tag{6}
\]

From (6), we can see that with the contraction of universe the energy density of scalar field increases. Thus when \( h^2 \sim \rho_\varphi \gtrsim m^2 \), the term relevant with \( h \) of Eq. (2) can no more be neglected. In this case, \( a^{-3}m^2 \gtrsim m^2 \), which can be reduced to \( \varphi_a^2 \gtrsim a^{-3}u_\alpha^2 \gtrsim 1 \). Thus as long as the oscillating amplitude of \( \varphi \) field is larger than 1, the oscillation of field will end and the field will roll up along its potential. Instead of Eq. (5),

\[
\ddot{\varphi} + 3h\dot{\varphi} \simeq 0. \tag{7}
\]

is satisfied. From Eq. (7), we have

\[
\dot{\varphi} \simeq \frac{c}{a^3} \tag{8}
\]

where \( c \) is the integral constant. Thus with the further contraction the universe will enter into the regime dominated by kinetic energy of field rapidly, i.e. \( \dot{\varphi}^2 \gg m^2\varphi^2 \), where \( p_\varphi \simeq \rho_\varphi \). This leads to \( \rho_\varphi \sim \dot{\varphi}^2 \sim 1/a^6 \), thus we have

\[
a^3 \simeq c\sqrt{\frac{3}{2}(t_s - t)} \tag{9}
\]

where \( t_s \) is the the integral constant, and

\[
\varphi \simeq \varphi_k + \sqrt{\frac{2}{3}} \ln \left( \frac{t_s - t_k}{t_s - t} \right) \tag{10}
\]

where \( \varphi_k \) and \( t_k \) are the value of field and time just entering into the regime dominated by kinetic energy, respectively.

When the kinetic energy of \( \varphi \) approaches \( \sigma \), we have to includes the \( \rho_\varphi^2 \) correction in Eq. (1). Thus instituting Eq. (8) into Eq. (1), we obtain

\[
6\left( \frac{\dot{a}}{a} \right)^2 = \frac{c^2}{a^{12}} \left( a^6 - \frac{c^2}{2\sigma} \right) \tag{11}
\]

whose solution can be obtained as follows

\[
a^3 = \sqrt{\frac{3c^2}{2}(t - t_b)^2 + \frac{c^2}{2\sigma}} \tag{12}
\]
where $t_b$ is the integral constant determined by $\sigma$ as well as the value of $\dot{\varphi}$ and $a$ at $t = t_k$. When $t = t_b$, the scale factor of the universe arrives at its minimum and after $t_b$ the universe bounces and expands.

For the universe dominated by stiff fluid with the same state equation $p = \rho$ as the case dominated by kinetic energy of scalar field, its evolution will symmetric before and after the bounce, and up to all time. But the evolution of the universe driven by massive scalar field is different from that with fluid. After the bounce, the field $\varphi$ will still roll up. However, since $h > 0$, the term $3h\dot{\varphi}$ of Eq. (2) serves as a damping term. Thus the roll-up motion of $\varphi$ will decay quickly. When the velocity of $\varphi$ arrives at 0, it reverses and rolls down along the potential. For the case of $\varphi \gtrsim 1$, we have $m^2 \lesssim h^2$. The field will enters the slow-roll regime in which the universe is dominated by the potential energy of the scalar field and

$$3h\ddot{\varphi} + m^2 \varphi \simeq 0,$$  (13)

is satisfied. In this case $p_\varphi \simeq -\rho_\varphi$ and the universe inflates. The motion of field $\varphi$ is given by

$$\varphi \simeq \varphi_r - \frac{2}{3}mt$$  (14)

where $\varphi_r$ is the value of field at reverse. When $m^2 \sim h^2$, i.e. $\varphi \sim 1$, the inflation ends and the field $\varphi$ will reenter into an oscillatory stage.

Therefore from above discussion, we can see that before the bounce, the field initially in the bottom of its potential can be driven by roll up its potential hill and after the bounce it reverses and rolls down, and drives the inflation of universe.

III. NUMERICAL RESULTS

Then we study the above evolutive process numerically. We take $m = 0.001$ and $\sigma = 0.0001$, and $a(0) = 1$, $\varphi(0) = 0$ and $\dot{\varphi}(0) = 0.0001$ for initial values. The evolutions of $a$ and $h$ are plotted in Fig. 1. We can see distinctly that the bounce is implemented. In Fig. 2, we plot the evolution of field $\varphi$. Initially the field oscillates about the bottom of its potential many times, then the field $\varphi$ roll up along its potential rapidly. The field still rolls up after the bounce, but its velocity will approaches 0 quickly. Thus shortly it will reverse and roll down, and then enter into slow-roll regime. When the field decreases to $\sim 1$ linearly, the slow-roll ends and the field will oscillates about $\varphi = 0$ again. These numerical results
are consistent with our semianalytical discussions in last section. The modification scale $\sigma$ determines the energy scale of bounce, thus in some sense also determines the maximal value $\varphi_r$ which the $\varphi$ can roll up to, see Fig. 3 for numerical solutions with various $\sigma$. We can simply estimate the value of $\varphi_r$. When the field just enter into the stage dominated by kinetic energy, $\dot{\varphi}_k^2 \simeq m^2 \varphi_k^2$ can be satisfied approximately, and when the field is at the point of the bounce, its value is $\dot{\varphi}_b^2 \simeq \sigma$. Thus from Eqs. (8) and (9), we have

$$\left(\frac{t_s - t_b}{t_s - t_k}\right)^2 = \left(\frac{a_b}{a_k}\right)^6 = \left(\frac{\dot{\varphi}_k}{\dot{\varphi}_b}\right)^2 \simeq \frac{m^2 \varphi_k^2}{\sigma}$$

(15)

Instituting it into Eq. (10),

$$\varphi_r \lesssim \varphi_k + \sqrt{\frac{2}{3}} \ln \left(\frac{\sigma}{m^2 \varphi_k^2}\right)$$

(16)

can be obtained, where the second term has been doubled since during kinetic dominated regime the evolution of the field can be regarded approximately as symmetrical before and after the bounce, which can also be seen from Fig. 4. For a reasonable estimation, we take $\varphi_k \sim 1$, thus obtain $\varphi_r \lesssim 4$ for $\sigma = 0.0001$ and $\varphi_r \lesssim 10$ for $\sigma = 0.1$, which is compatible with the numerical results of Fig. 3. In this case there is a up-limit $\varphi_r \lesssim 11$ for $\sigma = 1$, in which the modification of Friedmann equation is at Planck scale. Therefore, from Eq. (16), for massive scalar field initially in the bottom of its potential with a small velocity from fluctuations, the maximal value that it can roll up to during the contraction is only dependent on its mass $m$ and the modification scale $\sigma$. The larger $\varphi_r$ is, the longer the time that the universe after the bounce inflates is.

Focusing on the chaotic inflation model [21], instead of $8\pi/m_p^2 = 1$ on whole paper, we take $m_p^2 = 1$ here. The e-folds number is given by

$$N \simeq 2\pi\varphi_r^2$$

(17)

and Eq. (16) is modified as

$$\varphi_r \lesssim \varphi_k + \frac{1}{12\pi} \ln \left(\frac{\sigma}{m^2 \varphi_k^2}\right)$$

(18)

For $N \gtrsim 60$, this requires $\varphi_r \gtrsim 3$. Considering $m \sim 10^{-6}$ required by observational primordial perturbations, the limit to $\sigma$ is given by $\sigma \gtrsim 10^{-8}$. Further for $\sigma = 1$ the maximal value that the field can roll up to is given by $\varphi_r \lesssim 1 + 12 \sqrt{\frac{1}{12\pi}} \ln 10 \approx 5.5$, which is suitable for a successful inflationary cosmology but is far away from eternal inflation [22].
FIG. 1: The scale factor \( \ln a \) (solid line) and the Hubble parameter \( h \) (dashing line) as a function of time. The inset is a zoom before and after the bounce. \( h \) has been multiplied by 1000.

FIG. 2: The scale factor \( \ln a \) (solid line) and the field \( \varphi \) as a function of time.

FIG. 3: The field \( \varphi \) as a function of time for various \( \sigma \). From long dashing line to solid line, \( \sigma \) takes 0.0001, 0.001, 0.01, 0.1, respectively. Since \( m \) is taken same, initially the oscillation of fields has same frequency, which can be seen from Eq. 4.
FIG. 4: The potential energy (solid line) and the kinetic energy (dashing line) of the field as a function of time before and after the bounce.

FIG. 5: The scale factor $\ln a$ (solid line) and the field $\varphi$ (dashing line) as a function of time for an oscillating universe.

FIG. 6: The potential energy (solid line) and the kinetic energy (dashing line) of the field as a function of time for an oscillating universe. The time taken only includes two cycles.
FIG. 7: The scale factor $\ln a$ as a function of time for an oscillating universe. The long, short dashing and solid lines are the cases of $\frac{\sigma m^2}{m^2} = 50, 5, 2.5$, respectively. For $\frac{\sigma}{m^2} = 2.5$, from (16), we have $\varphi_r \lesssim 1$. In this case, inflation does not occur in each cycle, thus the amplitude of successive cycle does not increase.

We have found that the slow-roll inflation after the bounce can be preceded by the kinetic dominated contraction. Further we would like to check this feature in a controlled model of oscillating universe. We plot the evolutions of $(a, \varphi)$ and the (kinetic, potential) energy with time in Fig. 5 and Fig. 6, respectively. To show the oscillation of universe better, we take $m^2 = 0.2$, $\sigma = 50$ and $a(0) = 1, \varphi(0) = 0$ and $\dot{\varphi} = 1$ for initial values. We see that the maximum of expansion amplitude in each successive expansion/contraction cycle grows gradually. This asymmetry of cosmological evolution seems conflicted with intuition, since both the field equation (2) and Fridmann equation (11) are non dissipative and time reverse invariant. The reason of asymmetry has been analyzed in Ref. [9] for the case of closed universe. However, in fact this is not dependent on the curvature of universe, but the evolutive behavior of scalar field. For $\varphi \gtrsim 1$, during the expansion, the state equation $p_\varphi \simeq -\rho_\varphi$, thus the field is in slow-roll regime and the universe expands exponentially i.e. $a \sim e^{ht}$. However, during the contraction, the universe is dominated by kinetic energy of scalar field. In this case, $a \sim t^{1/3}$. Thus the time when the field rolls up to the value from which it will roll down into slow-roll inflationary regime is more shorter. It is the evolutive asymmetry of the field that leads to the growth of successive expansion/contraction cycles.

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$^2$ To realise a transition from expansion to contraction, we take the density of potential energy $< 0$ in the minimum. The details of cosmology with negative potential have been studied in Ref. [24].
When that $m^2$ is the same order as $\sigma$, from (10), we obtain $\varphi_r \lesssim 1$. In this case, the field can not roll up to slow-roll regime. Thus the asymmetry between the expansion driven by the potential of field and the contraction driven by the kinetic term of field will not exist. We can see from Fig. 7 that the amplitude of successive cycle does not increase in this case.

IV. CONCLUSION

We make use of possible high energy correction to the Friedmann equation to implement a bounce. We study the behavior of massive field and find that the slow-roll inflation can be preceded by the kinetic dominated contraction. During this process, the field initially in the bottom of its potential can be driven by the anti-frictional force resulted from the contraction and roll up its potential hill. We show that the required e-folds number during the inflation after the bounce limits the energy scale of bounce, for $N \gtrsim 60$, $\sigma \gtrsim 10^{-8}$ is required. Further unlike that expected, in our case the field can not arrive arbitrary large value, even though the bounce occurs at Planck scale, for $\sigma = 1$, we have $\varphi_r \simeq 5.5$. The reason is that during the contraction the kinetic energy of the field is much larger than its potential energy and the growth of the field value is only logarithmical. When the kinetic energy approaches $\sigma$, its potential energy is still far less, see Fig. 3. These features have been checked and confirmed numerically. Moreover, for possible oscillating universe model, we point out that the successive increasing of expansion amplitude of each cycle is actually depended on whether there exists inflation in each cycle. For the oscillating universe without inflation in its each cycle, its amplitude dose not increase with time.

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