Off-shell unitarity and geometrical effects in deep–inelastic scattering and vector–meson electroproduction

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Abstract

Deep–inelastic scattering at low $x$ and elastic vector meson production are considered on the basis of the off-shell extension of the $s$–channel unitarity. We discuss behavior of the structure function $F_2(x, Q^2)$ at low $x$ and the total cross–section of virtual photon–proton scattering and obtain, in particular, the dependence $\sigma_{\gamma^*p}^{tot} \sim (W^2)^{\lambda(Q^2)}$ where exponent $\lambda(Q^2)$ is related to the interaction radius of a constituent quark. The energy dependence of the total cross–section of $\gamma^*\gamma^*$–interactions is calculated. The explicit mass dependence of the exponent in the power energy behavior of the vector meson production in the processes of virtual photon interactions with a proton $\gamma^*p \rightarrow Vp$ has been obtained. We also consider angular distributions at large momentum transfers.

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Introduction

Experimental data obtained at HERA [1] clearly demonstrated rising behavior of the structure function $F_2(x, Q^2)$ at small $x$ which is translated to the rising dependence of the total cross-section $\sigma_{\gamma^*p}^{tot}(W^2, Q^2)$ on center of mass energy $W^2$. This effect is consistent with various $W^2$-dependencies and has been treated in different ways, e.g. as a manifestation of hard BFKL Pomeron [2], a confirmation of the DGLAP evolution in perturbative QCD [3], a transient phenomena, i.e. preasymptotic effects [4] or as a true asymptotical dependence of the off-mass-shell scattering amplitude [5]. This list is far from being complete and other interpretations can be found, e.g. in the review papers [1, 6].

It is worth to note here that the essential point in the study of low-$x$ dynamics is that the space-time structure of the scattering at small values of $x$ involves large distances $l \sim 1/mx$ on the light-cone [7], and the region of $x \sim 0$ is sensitive to the nonperturbative contributions. Deep-inelastic scattering in this region turns out to be a coherent process where diffraction plays a major role and nonperturbative models such as Regge or vector dominance model can be competitive with perturbative QCD and successfully applied for description of the experimental data.

The strong experimentally observed rise of $\sigma_{\gamma^*p}^{tot}(W^2, Q^2)$ when

$$\sigma_{\gamma^*p}^{tot}(W^2, Q^2) \propto (W^2)^{\lambda(Q^2)}$$

with $\lambda(Q^2)$ rising with $Q^2$ from about 0.1 to 0.4 was considered to a somewhat extent as a surprising fact on the grounds of our knowledge of energy dependence of the total cross-sections in hadronic interactions, where $\lambda \sim 0.1$. The above comparison between photon-induced and hadron-induced interactions is quite legitimate since the photon is demonstrating its hadronlike nature for a long time. The apparent difference between the hadron and virtual photon total cross-sections however has no fundamental meaning since there is no Froissart–Martin bound in the case off-shell particles [5, 8]. Only under some additional assumptions this bound can be applied [2, 4].

The most common form of unitarity solution – the eikonal one – was generalized for the off-shell scattering in [5]. In this paper we consider off-shell extension of the $U$-matrix approach to the amplitude unitarization. It is shown that this approach along with the respective extension of the chiral quark model for the $U$-matrix [11] leads to (1), where the exponent $\lambda(Q^2)$ is related to the $Q^2$-dependent interaction radius of an off-shell constituent quark.

It is to be stressed here the importance of the effective interaction radius concept [12]. The study of the effective interaction radius dependence on the scattering variables seemed very useful for understanding of the relevant dynamics.
of high energy hadronic reactions [13, 14]. Now it is widely known that the respective geometrical considerations about interaction provide a deep insight in hadron dynamics and deep–inelastic scattering.

Besides the studies of deep–inelastic scattering at low \( x \) the interesting measurements of the characteristics of the elastic vector meson production were performed in the experiments H1 and ZEUS at HERA [15, 16]. It was shown that the integral cross section of the elastic vector meson production increases with energy in the way similar to the \( \sigma_{\gamma p}^{\text{tot}}(W^2, Q^2) \) dependence on \( W^2 \) [1]. It appeared that an increase of the vector meson electroproduction cross–section with energy is steeper for heavy vector mesons as well as in the case when the virtuality \( Q^2 \) is high. Discussion of such a behavior in various model approaches based on the nonperturbative hadron physics or perturbative QCD can be found, e.g. in the papers [6].

It was already mentioned that we use an approach based on the off–shell extension of the \( s \)–channel unitarity. Its application to the elastic vector meson production in the processes \( \gamma^* p \rightarrow V p \) allows in particular to consider angular dependence and mass effects in these processes. It appears that the obtained mass and \( Q^2 \) dependencies are in agreement with the experimentally observed trends. It is also valid for the angular distribution at large momentum transfers.

1 Off–shell unitarity

The extension of the \( U \)–matrix unitarization for the off-shell scattering was considered in [9]. It was supposed that the virtual photon fluctuates into a quark–antiquark pair \( q\bar{q} \) and this pair can be treated as an effective virtual vector meson state in the processes with small Bjorken \( x \). This effective virtual meson then interacts with a hadron. We considered a single effective vector meson field and used for the amplitudes of the processes

\[
V^* + h \rightarrow V^* + h, \quad V^* + h \rightarrow V + h \quad \text{and} \quad V + h \rightarrow V + h
\]

the notations \( F^{**}(s, t, Q^2) \), \( F^*(s, t, Q^2) \) and \( F(s, t) \) respectively, i. e. we denoted that way the amplitudes when both initial and final mesons are off mass shell, only initial meson is off mass shell and both mesons are on mass shell.

The extended unitarity relation for the amplitudes \( F^{**} \) and \( F^* \) has a similar structure as the equation for the on–shell amplitude \( F \) but relate the different amplitudes. In impact parameter representation at high energies it relates the amplitudes \( F^{**} \) and \( F^* \) in the following way

\[
\text{Im}F^{**}(s, b, Q^2) = |F^*(s, b, Q^2)|^2 + \eta^{**}(s, b, Q^2),
\]

3
where $\eta^{**}(s, b, Q^2)$ is the contribution to the unitarity of many–particle intermediate on–shell states. The function $\eta^{**}(s, b, Q^2)$ is the sum of the $n$–particle production cross–section in the process of the virtual meson interaction with a hadron $h$, i. e.

$$
\eta^{**}(s, b, Q^2) = \sum_n \sigma_n(s, b, Q^2).
$$

Similar relation exists for the functions $F^*$ and $F$. It is worth noting that the solution of the off–shell unitarity in the nonrelativistic case for a $K$–matrix representation was obtained for the first time in [17]. The solution of the off–shell unitarity can be written in the $U$–matrix form [9]:

$$
F^{**}(s, b, Q^2) = U^{**}(s, b, Q^2) + iU^*(s, b, Q^2)F^*(s, b, Q^2)
$$

$$
F^*(s, b, Q^2) = U^*(s, b, Q^2) + iU^*(s, b, Q^2)F(s, b).
$$

The solution of this system has a simple form when the following factorization relation is supposed to be valid

$$
[U^*(s, b, Q^2)]^2 - U^{**}(s, b, Q^2)U(s, b) = 0.
$$

Eq. (5) implies the following representation for the functions $U^{**}$ and $U^*$:

$$
U^{**}(s, b, Q^2) = \omega^2(s, b, Q^2)U(s, b)
$$

$$
U^*(s, b, Q^2) = \omega(s, b, Q^2)U(s, b).
$$

It is valid, e. g. in the Regge model with factorizable residues and the $Q^2$–independent trajectory. It is also valid in the off–shell extension of the chiral quark model for the $U$–matrix which we will consider further. Thus, we have for the amplitudes $F^*$ and $F^{**}$

$$
F^*(s, b, Q^2) = \frac{U^*(s, b, Q^2)}{1 - iU(s, b)} = \omega(s, b, Q^2)\frac{U(s, b)}{1 - iU(s, b)}
$$

$$
F^{**}(s, b, Q^2) = \frac{U^{**}(s, b, Q^2)}{1 - iU(s, b)} = \omega^2(s, b, Q^2)\frac{U(s, b)}{1 - iU(s, b)}
$$

and unitarity provides inequalities

$$
|F^*(s, b, Q^2)| \leq |\omega(s, b, Q^2)|, \quad |F^{**}(s, b, Q^2)| \leq |\omega^2(s, b, Q^2)|.
$$

It is worth noting that the above limitations are much less stringent than the limitation for the on–shell amplitude $|F(s, b)| \leq 1$.

When the function $\omega(s, b, Q^2)$ is real we can write down a simple expression for the inelastic overlap function $\eta^*(s, b, Q^2)$:

$$
\eta^*(s, b, Q^2) = \omega^2(s, b, Q^2)\frac{\text{Im}U(s, b)}{1 - iU(s, b)}
$$
2 Off–shell scattering in the \( U \)-matrix method

In this section we consider off–shell extension of the model for hadron scattering \([I]\), which is based on the ideas of chiral quark models. Valence quarks located in the central part of a hadron are supposed to scatter in a quasi-independent way by the effective field. In accordance with that we represent the basic dynamical quantity in the factorized form. In the case when the one of the hadrons (vector meson in our case) is off mass shell the off–shell \( U \)-matrix, i.e. \( U^{**}(s, b, Q^2) \) is represented as the product

\[
U^{**}(s, b, Q^2) = \prod_{i=1}^{n_{h}} \langle f_{Q_i}(s, b) \rangle \prod_{j=1}^{n_{V}} \langle f_{Q^*_j}(s, b, Q^2) \rangle. \tag{11}
\]

Factors \( \langle f_{Q}(s, b) \rangle \) and \( \langle f_{Q^*}(s, b, Q^2) \rangle \) correspond to the individual quark scattering amplitude smeared over transverse position of the constituent quark inside hadron and over fraction of longitudinal momentum of the initial hadron carried by this quark. Under the virtual constituent quarks we mean the ones composing the virtual meson. Factorization \([II]\) reflects the coherence in the valence quark scattering and may be considered as an effective implementation of constituent quarks’ confinement. The picture of hadron structure with the valence constituent quarks located in the central part and the surrounding condensate implies that the overlapping of hadron structures and interaction of the condensates occur at the first stage of the collision. Due to an excitation of the condensates, the quasiparticles, i.e. massive quarks arise. These quarks play role of scatterers. To estimate number of such quarks one could assume that part of hadron energy carried by the outer condensate clouds is being released in the overlap region to generate massive quarks. Then their number can be estimated by the quantity:

\[
\tilde{N}(s, b) \propto \frac{(1 - k_{Q}) \sqrt{s}}{m_{Q}} D_{c}^{h} \otimes D_{c}^{V}, \tag{12}
\]

where \( m_{Q} \) – constituent quark mass, \( k_{Q} \) – hadron energy fraction carried by the constituent valence quarks. Function \( D_{c}^{h} \) describes condensate distribution inside the hadron \( h \), and \( b \) is an impact parameter of the colliding hadron \( h \) and meson \( V \). Thus, \( \tilde{N}(s, b) \) quarks appear in addition to \( N = n_{h} + n_{V} \) valence quarks. Those quarks are transient ones: they are transformed back into the condensates of the final hadrons in elastic scattering. It should be noted that we use subscript \( Q \) to refer the constituent quark \( Q \) and the same letter \( Q \) is used to denote a virtuality \( Q^2 \). However, they enter formulas in a way excluding confusion.

The amplitudes \( \langle f_{Q}(s, b) \rangle \) and \( \langle f_{Q^*}(s, b, Q^2) \rangle \) describe elastic scattering of a single valence on-shell \( Q \) or the off–shell \( Q^* \) quarks with the effective field and
we use for the function $\langle f_Q(s, b) \rangle$ the following expression

$$\langle f_Q(s, b) \rangle = [\tilde{N}(s, b) + (N - 1)] V_Q(b)$$  \hspace{1cm} (13)

where $V_Q(b)$ has a simple form $V_Q(b) \propto g \exp(-m_Q b/\xi)$, which corresponds to the quark interaction radius $r_Q = \xi/m_Q$. The function $\langle f_Q^*(s, b, Q^2) \rangle$ is to be written as

$$\langle f_Q^*(s, b, Q^2) \rangle = [\tilde{N}(s, b) + (N - 1)] V_Q^*(b, Q^2).$$  \hspace{1cm} (14)

In the above relation

$$V_Q^*(b, Q^2) \propto g(Q^2) \exp(-m_Q b/\xi(Q^2))$$  \hspace{1cm} (15)

and this form corresponds to the virtual constituent quark interaction radius with effective field

$$r_Q^* = \xi(Q^2)/m_Q.$$  \hspace{1cm} (16)

The $b$–dependence of $\tilde{N}(s, b)$ is weak compared to the $b$–dependence of $V_Q$ or $V_Q^*$ [11] and therefore we have taken this function to be independent on the impact parameter $b$. Dependence on virtuality $Q^2$ comes through dependence of the intensity of the virtual constituent quark interaction $g(Q^2)$ and the parameter $\xi(Q^2)$, which determines the quark interaction radius (in the on-shell limit $g(Q^2) \to g$ and $\xi(Q^2) \to \xi$). According to these considerations the explicit functional forms for the generalized reaction matrices $U^*$ and $U^{**}$ can easily be written in the form of Eq. (6) with

$$\omega(s, b, Q^2) = \frac{\langle f_Q^*(s, b, Q^2) \rangle}{\langle f_Q(s, b) \rangle}. $$  \hspace{1cm} (17)

Note that Eqs. (5) and (6) imply that

$$\langle f_{Q^* \to Q}(s, b, Q^2) \rangle = [\langle f_Q^*(s, b, Q^2) \rangle \langle f_Q(s, b) \rangle]^{1/2}.$$

We consider the high–energy limit and for simplicity assume here that all the constituent quarks have equal masses and parameters $g$ and $\xi$ as well as $g(Q^2)$ and $\xi(Q^2)$ do not depend on quark flavor. We also assume for simplicity pure imaginary amplitudes. Then the functions $U, U^*$ and $U^{**}$ are the following

$$U(s, b) = ig^N \left( \frac{s}{m^2_Q} \right)^{N/2} \exp \left[ -\frac{m_Q N b}{\xi} \right].$$  \hspace{1cm} (18)
\[ U^*(s, b, Q^2) = \omega(b, Q^2)U(s, b), \quad U^{**}(s, b, Q^2) = \omega^2(b, Q^2)U(s, b) \]  \hspace{1cm} (19)

where the function \( \omega \) is an energy-independent one and has the following dependence on \( b \) and \( Q^2 \)

\[ \omega(b, Q^2) = \frac{g(Q^2)}{g} \exp \left[ -\frac{m_Q b}{\xi(Q^2)} \right] \]  \hspace{1cm} (20)

with

\[ \bar{\xi}(Q^2) = \frac{\xi \xi(Q^2)}{\xi - \xi(Q^2)}. \]  \hspace{1cm} (21)

### 3 Total cross–sections of \( \gamma^* p \) and \( \gamma^* \gamma^* \) interactions

It is obvious that for the on–shell particles \( \omega \to 1 \) and we arrive to the result obtained in [9] at large \( W^2 \)

\[ \sigma_{\gamma p}^{tot}(W^2) \propto \frac{\xi^2}{m_Q^2} \ln^2 \frac{W^2}{m^2_Q}, \]  \hspace{1cm} (22)

where the usual for deep–inelastic scattering notation \( W^2 \) instead of \( s \) is used. Similar result is valid also for the off mass shell particles when the interaction radius of virtual quark does not depend on \( Q^2 \) and is equal to the interaction radius of the on–shell quark, i.e. \( \xi(Q^2) \equiv \xi \). The behavior of the total cross–section at large \( W^2 \)

\[ \sigma_{\gamma^* p}^{tot}(W^2) \propto \left[ \frac{g(Q^2)\xi}{g^2 m_Q} \right]^2 \ln^2 \frac{W^2}{m^2_Q}, \]  \hspace{1cm} (23)

corresponds to the result obtained in [9]. We consider further the off-shell scattering with \( \xi(Q^2) \neq \xi \) and it should be noted first that for the case when \( \xi(Q^2) \leq \xi \) the total cross–section would be energy-independent

\[ \sigma_{\gamma^* p}^{tot}(W^2) \propto \left[ \frac{g(Q^2)\xi}{g^2 \lambda(Q^2) m_Q} \right]^2 \]  in the asymptotic region. This scenario would mean that the experimentally observed rise of \( \sigma_{\gamma^* p}^{tot} \) is transient preasymptotic phenomena [4, 9]. It can be realized when we replace in the formula for the interaction radius of the on–shell constituent quark \( r_Q = \xi/m_Q \) the mass \( m_Q \) by the value \( m_Q^* = \sqrt{m_Q^2 + Q^2} \) in order to obtain the interaction radius of the off-shell constituent quark and write it down
as \( r_{Q^*} = \xi/m_{Q^*} \), or equivalently replace \( \xi(Q^2) \) by \( \xi(Q^2) = \xi m/Q^2 + Q^2 \).

The above option cannot be excluded in principle.

However, when \( \xi(Q^2) > \xi \) the situation is different and we have at large \( W^2 \)

\[
\sigma_{\gamma^*p}^{tot}(W^2, Q^2) \propto G(Q^2) \left( \frac{W^2}{m_Q^2} \right)^{\lambda(Q^2)} \ln \left( \frac{W^2}{m_Q^2} \right),
\]

(24)

where

\[
\lambda(Q^2) = \frac{\xi(Q^2) - \xi}{\xi(Q^2)}.
\]

(25)

We shall further concentrate on this we currently think the most interesting case.

All the above expressions for \( \sigma_{\gamma^*p}^{tot}(W^2) \) can be rewritten as the corresponding dependencies of \( F_2(x, Q^2) \) at small \( x \) according to the relation

\[
F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha} \sigma_{\gamma^*p}^{tot}(W^2),
\]

where \( x = Q^2/W^2 \).

In particular, (24) will appear in the form

\[
F_2(x, Q^2) \propto \tilde{G}(Q^2) \left( \frac{1}{x} \right)^{\lambda(Q^2)} \ln(1/x),
\]

(26)

It is interesting that the value and \( Q^2 \) dependence of the exponent \( \lambda(Q^2) \) is related to the interaction radius of the virtual constituent quark. The value of parameter \( \xi \) in the model is determined by the slope of the differential cross–section of elastic scattering at large \( t \) \[18\], i. e.

\[
\frac{d\sigma}{dt} \propto \exp \left[ -\frac{2\pi\xi}{m_Q N \sqrt{-t}} \right]
\]

(27)

and from the \( pp \)-experimental data it follows \( \xi = 2 - 2.5 \). The uncertainty is related to the ambiguity in the constituent quark mass value. Using for simplicity \( \xi = 2 \) and the data for \( \lambda(Q^2) \) obtained at HERA \[19\] we calculated the “experimental” \( Q^2 \)-dependence of the function \( \xi(Q^2) \):

\[
\xi(Q^2) = \frac{\xi}{1 - \lambda(Q^2)}.
\]

(28)

The results are represented in Fig. 1. It is clear that experiment leads to \( \xi(Q^2) \) rising with \( Q^2 \). This rise is slow and consistent with \( \ln Q^2 \) extrapolation. Thus,
assuming this dependence to be kept at higher $Q^2$ and using (25), we predict saturation in the $Q^2$–dependence of $\lambda(Q^2)$, i.e. at large $Q^2$ the flattening will take place.

The above approach can be directly extended to the calculation of the $\gamma^*\gamma^*$ total cross–section. When $\xi(Q^2) > \xi$ the following behavior of the total cross–section at large $W^2$ will take place:

$$
\sigma_{\gamma^*\gamma^*}(W^2, Q^1_2, Q^2_2) \propto G(Q^1_1) G(Q^2_2) \left( \frac{W^2}{m_Q^2} \right)^{\lambda(Q^1_1) + \lambda(Q^2_2)} \ln \frac{W^2}{m_Q^2}. \quad (29)
$$

This strong energy dependence of the $\gamma^*\gamma^*$–total cross–section is consistent with LEP data [20].

4 Elastic vector meson production

As it was already mentioned we assumed that the virtual photon before the interaction with the proton fluctuates into the $\bar{q}q$ – pair and for simplicity we limited ourselves with light quarks under discussion of the total cross–section. The expression for the total cross-section is given by (24). The calculation of the the
elastic and inelastic cross-sections can also be directly performed in this approximation using (18), (19) and (20) and integrating over impact parameter (7) and (10). Then we obtain the following dependencies for the cross-sections of elastic scattering and inelastic interactions

\[ \sigma_{el}^{\gamma^* p}(W^2, Q^2) \propto G_e(Q^2) \left( \frac{W^2}{m_Q^2} \right)^{\lambda(Q^2)} \ln \frac{W^2}{m_Q^2} \]  

(30)

and

\[ \sigma_{inel}^{\gamma^* p}(W^2, Q^2) \propto G_i(Q^2) \left( \frac{W^2}{m_Q^2} \right)^{\lambda(Q^2)} \ln \frac{W^2}{m_Q^2} \]  

(31)

with the universal exponent \( \lambda(Q^2) \) given by the relation (25). The above relations mean that the ratios of elastic and inelastic cross-sections to the total one are approximately constant and do not depend on energy.

Now we consider elastic (exclusive) cross-sections both for light and heavy vector mesons production. We need to get rid of the light quark limitation and extend the above approach in order to include the quarks with the different masses. The inclusion, in particular, heavy vector meson production into this scheme is straightforward: the virtual photon fluctuates before the interaction with proton into the heavy quark–antiquark pair which constitutes the virtual heavy vector meson state. After the interaction with a proton this state turns out into the real heavy vector meson.

Integral exclusive (elastic) cross-section of vector meson production in the process \( \gamma^* p \rightarrow V p \) when the vector meson in the final state contains not necessarily light quarks can be calculated directly according to the above scheme and formulas of Section 2:

\[ \sigma_V^{\gamma^* p}(W^2, Q^2) \propto G_V(Q^2) \left( \frac{W^2}{m_Q^2} \right)^{\lambda_V(Q^2)} \ln \frac{W^2}{m_Q^2}, \]  

(32)

where

\[ \lambda_V(Q^2) = \lambda(Q^2) \frac{\bar{m}_Q}{\langle m_Q \rangle}. \]  

(33)

In (33) \( \bar{m}_Q \) denotes the mass of the constituent quarks from the vector meson and \( \langle m_Q \rangle \) is the mean constituent quark mass of system of the vector meson and proton system. Evidently \( \lambda_V(Q^2) = \lambda(Q^2) \) for the light vector mesons. This result is consistent with the most recent ZEUS data, but statistics is still limited [15]. In the case when the vector meson is very heavy, i.e. \( \bar{m}_Q \gg m_Q \) we have

\[ \lambda_V(Q^2) = \frac{5}{2} \lambda(Q^2). \]
We conclude that the respective cross-section rises faster than the corresponding cross-section of the light vector meson production, e.g. (33) results in

$$\lambda_{J/\Psi}(Q^2) \simeq 2\lambda(Q^2).$$

This is in a qualitative agreement with the recently observed trends in the HERA data [16].

5 Angular structure of elastic and proton–dissociative vector meson production

Besides the integral cross-section of elastic vector meson production it is interesting to consider angular distribution for these processes. Recently the first measurements of angular distributions were performed [21] and it was found that angular distribution in the light vector meson production is consistent with the power dependence $(-t)^{-3}$.

We apply the model described above for calculation of the differential cross-sections in elastic vector meson production using an analysis of the singularities of the amplitudes in the complex impact parameter plane developed in [18].

Since the integration goes over the variable $b^2$ rather than $b$ it is convenient to consider the complex plane of the variable $\beta$ where $\beta = b^2$ and analyze singularities in the $\beta$–plane. Using (7) we can write down the integral over the contour $C$ around a positive axis in the $\beta$–plane:

$$F^*(W^2, t, Q^2) = -i\frac{W^2}{2\pi^2} \int_C F^*(W^2, \beta, Q^2)K_0(\sqrt{t\beta})d\beta,$$

where $K_0$ is the modified Bessel function and the variable $W^2$ was used instead of the variable $s$. The contour $C$ can be closed at infinity and the value of the integral will be then determined by the singularities of the function $F^*(W^2, \beta, Q^2)$, where

$$F^*(W^2, \beta, Q^2) = \omega(\beta, Q^2)\frac{U(W^2, \beta)}{1 - iU(W^2, \beta)}$$

in a $\beta$–plane.

With explicit expressions for the functions $U$ and $\omega$ we conclude that the positions of the poles in the complex $\beta$–plane are

$$\beta_n(W^2) = \frac{\xi^2}{M^2}\left\{\ln \left[g^N \left(\frac{W^2}{mQ^2}\right)^{N/2}\right] + i\pi n\right\}^2, \quad n = \pm 1, \pm 3, \ldots.$$
where $M = \tilde{m}_Q n_V + m_Q n_h$. The location of the poles in the complex impact parameter plane does not depend on the virtuality $Q^2$. Besides the poles $F^*(W^2, \beta, Q^2)$ has a branching point at $\beta = 0$ and

$$
\text{disc } F^*(W^2, \beta, Q^2) =
$$

$$
\text{disc}[\omega(\beta, Q^2)U(W^2, \beta)] - iU(W^2, \beta + i0)U(W^2, \beta - i0)\text{disc }\omega(\beta, Q^2)
\left[1 - iU(W^2, \beta + i0)[1 - iU(W^2, \beta - i0)]\right],
$$

i.e.

$$
\text{disc } F^*(W^2, \beta, Q^2) \simeq i \text{disc }\omega(\beta, Q^2)
$$

since at $W^2 \to \infty$ the function $U(W^2, \beta) \to \infty$ at fixed $\beta$. The function $F^*(W^2, t, Q^2)$ can be then represented as a sum of poles and cut contributions, i.e.

$$
F^*(W^2, t, Q^2) = F^*_p(W^2, t, Q^2) + F^*_c(W^2, t, Q^2).
$$

The pole and cut contributions are decoupled dynamically when $W^2 \to \infty$. Contribution of the poles determines the amplitude $F^*(W^2, t, Q^2)$ in the region $|t|/W^2 \ll 1$. The amplitude in this region can be represented in a form of series:

$$
F^*(W^2, t, Q^2) \simeq iW^2(W^2)^{\lambda_V(Q^2)/2} \sum_{n=\pm 1, \pm 3, \ldots} \exp\left\{\frac{i\pi n}{\lambda_V(Q^2)}\right\} \sqrt{\beta_n} K_0(\sqrt{t\beta_n}).
$$

(35)

At moderate values of $-t$ when $-t \geq 1$ (GeV/c)$^2$ the amplitude (35) leads to the Orear type behavior of the differential cross–section which is similar to the Eq.(27) for the on–shell amplitude, i.e.

$$
\frac{d\sigma_V}{dt} \propto \exp\left(-\frac{2\pi \xi}{M} \sqrt{-t}\right).
$$

(36)

At small values of $-t$ the behavior of the differential cross–section is complicated, the oscillating factors $\exp\left\{\frac{i\pi n}{\lambda_V(Q^2)}\right\}$ which are absent in the on-shell scattering amplitude \[\text{[11]}\] play a role.

At large $-t$ the poles contributions is negligible and contribution from the cut at $\beta = 0$ is a dominating one. It appears that the function $F^*_c(W^2, t, Q^2)$ does not depend on energy and differential cross section depends on $t$ in a power-like way

$$
\frac{d\sigma_V}{dt} \simeq \tilde{G}(Q^2) \left(1 - \tilde{c}(Q^2)t\right)^{-3},
$$

(37)
which is in an agreement with the experimentally observed trends \cite{21, 22}. For large values of $-t$

$$-t \gg \hat{m}_Q^2/\xi^2(Q^2)$$

we have a simple $(-t)^{-3}$ dependence of the differential cross-section. This dependence significantly differs from the one in the on-shell scattering \cite{11} which approximates the quark counting rule \cite{23}, and this difference is in the large extent because of the off-shell unitarity role.

The ratio of differential cross-sections for the production of the different vector mesons $\frac{d\sigma_{V_1}}{dt}/d\sigma_{V_2}/dt$ does not depend on the variables $W^2$ and $t$ at large enough values of $-t$.

The production of the vector mesons accompanied by the proton dissociation into the state $Y$ with mass $M_Y$ can be calculated along the lines described in \cite{24} with account for non-zero virtuality. The extension is straightforward. Similar to the case of on–shell particles we have a suppression of the pole contribution at high energies. It is interesting to note that the normalized differential cross-section

$$\frac{1}{\sigma_0(W^2, M_Y^2, Q^2)} \frac{d\sigma}{dtdM_Y^2}$$

where $\sigma_0$ is the value of cross-section at $t = 0$ will exhibit a scaling behavior

$$\frac{1}{\sigma_0} \frac{d\sigma}{dtdM_Y^2} = \left(1 - \frac{4\xi^2t}{M^2(Q^2)}\right)^{-3}, \quad \text{(38)}$$

where the $\tilde{M}$ is the following combination

$$\tilde{M}(Q^2) = M_Y \left[1 - \frac{2\hat{m}_Q}{M_Y} \lambda(Q^2)\right]. \quad \text{(39)}$$

Note that $M(Q^2) \simeq M_Y$ at small values of $Q^2$ or when the value of $M_Y$ is large $M_Y \gg \hat{m}_Q$. The dependence \text{(38)} is in agreement with the experimentally observed dependencies in the proton–dissociative vector meson production at large values of $t$ \cite{21}.

6 Conclusion

We considered limitations the unitarity provides for the $\gamma^* p$–total cross-sections and geometrical effects in the model dependence of $\sigma_{\gamma^*p}^{\text{tot}}$. In particular, it was shown that the constituent quark’s interaction radius dependence on $Q^2$ can lead
to a nontrivial, asymptotical result: $\sigma_{\gamma^*p}^{\text{tot}} \sim (W^2)^{\lambda(Q^2)}$, where $\lambda(Q^2)$ will be saturated at large values of $Q^2$. This result is valid when the interaction radius of the virtual constituent quark is rising with virtuality $Q^2$. The reason for such rise might be of a dynamical nature and it could originate from the emission of the additional $q\bar{q}$–pairs in the nonperturbative structure of a constituent quark. In the present approach constituent quark consists of a current quark and the cloud of quark–antiquark pairs of the different flavors [11]. Available experimental data are consistent with the $\ln Q^2$–dependence of the radius of this cloud. The available experimental data for the structure functions at low values of $x$ continue to demonstrate the rising total cross-section of $\gamma^*p$–interactions and therefore we can consider it as a manifestation of the rising with virtuality interaction radius of a constituent quark. The steep energy increase of $\gamma^*\gamma^*$ total cross–section $(W^2)^{2\lambda(Q^2)}$ was also predicted.

We have also considered the elastic vector meson production processes in $\gamma^*p$–interactions. The mass and $Q^2$ dependencies of the integral cross–section of vector meson production are related to the dependence of the interaction radius of the constituent quark $Q$ on the respective quark mass $m_Q$ and virtuality $Q^2$. The behavior of the differential cross–sections at large $t$ is in the large extent determined by the off–shell unitarity effects. These predictions are in a qualitative agreement with the experimental data. The new experimental data will be essential for the discrimination of the model approaches and studies of the interplay between the non-perturbative and perturbative QCD regimes (cf. e.g. [19, 25]).

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