Method of vibration reduction of synthesized systems or subsystems of machines

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Abstract. The paper presents the use of active synthesis methods in determining parameters of active and passive reduction of vibrations in mechanical systems. The presented method uses the synthesis method to determine the parameters of vibration reduction. The gist of the method is using dynamic properties, assumed at the beginning of the task, in the form of resonance frequencies and vibration amplitudes.

1. Introduction
Technological development, characterized by an increase in power and rotational speed of machines and devices drives, is inextricably linked with the growth in their dynamic activity. Shaping the dynamic properties of systems or subsystems of machines is a very important problem which should be considered by a designer already at the stage of concept and initial design phases [1, 2]. Obtaining the system of required dynamic properties (natural circular frequencies) with the ability to control them is a complex problem and it requires the use of network methods which allow for easy algorithmization and automatization of calculations [3 - 5]. In the present work is presented the algorithm that broadens earlier authors elaborations referred to the problem of shaping dynamic properties of systems or subsystems of machines using synthesis methods [6 - 15]. The presented method of synthesis of machine systems or subsystems, as discrete, longitudinal vibrating models, allow determining parameters and the structure of a system for the assumed in advance natural circular frequencies of a system and the required values of vibration amplitudes.

The paper presents the use of active synthesis methods in determining parameters of active and passive reduction of vibrations in mechanical systems [16 - 22]. The presented method uses the comparative synthesis method to determine the parameters of vibration reduction. The point of the method is utilizing dynamic properties, assumed at the beginning of the task, in the form of natural circular frequencies and vibration amplitudes. The vibration reduction method presented in this paper can be classified as one of the design methods used in designing vibrating discrete systems as sub-components of machines with desired dynamic properties. This work is a continuation of research on development of methods of synthesis of passive and active reduction of vibration in mechanical systems. This study presents the use of active synthetic methods to determine parameters of setting forces, reducing vibrations of selected natural circular frequencies. At the same time, it enables implementation of values of the forces in the system by means of passive elastic-damping components, as well as by combination of active and passive components in the system. Such
extension of the synthesis task gives the designer a great number of options for selecting optimal parameters of the designed system.

2. Theory of vibration reduction using the method of proportional parameters decomposition

Prior to the task of active or passive vibration reduction with synthesis methods it should be determined the desired dynamic properties of the analysed system and the forces reducing vibration in an analytical form. The method of determining the analytical form of the dynamic characteristic presented in the work consists in accepting the series of resonance frequencies and the anti-resonance ones (poles and zeros of the desired dynamic characteristic) in the form: \(\omega_{b1}, \omega_{b2}, ..., \omega_{bi}(i = 1, 2, 3, ..., n)\) – resonance frequencies, \(\omega_{z1}, \omega_{z2}, ..., \omega_{zj}(j = 1, 2, 3, ..., n)\) – anti-resonance frequencies, and free vibration frequency decrease coefficients in the form: \(h_{b1}, h_{b2}, ..., h_{bi}(i = 1, 2, 3, ..., n)\). The value of decrease coefficients of natural vibrations \(h_{b1}, h_{b2}, ..., h_{bi}\) are determined basing on the assumed desired values of amplitudes of vibration. The amplitudes are equal to the maximum deflection of the first inertial component, corresponding to the system response at the unit amplitude of the exciting force. A detailed description of determination of decrease in vibration frequency coefficients is shown in [6, 10]. Basing on such dynamic properties, characteristic functions are defined in the form of the slowness (mechanical impedance) \(U(s)\) or mobility (mechanical admittance) \(V(s)\), defined as:

\[
U(s) = \frac{1}{V(s)} = \frac{\dot{x}_1(s)}{F_1(s)}
\]

where: \(s\) – complex variable; \(\dot{x}_1(s)\) – Laplace transform of the velocity of the first synthesized inertial element, determined at zero initial conditions; \(F_1(s)\) – Laplace transform of the inducing force with respect to the first synthesized inertial element determined at zero initial conditions. In the case when the number of circular resonance frequencies \(l, \omega_{z1}\) is larger than the anti-resonance ones \(l, \omega_{zi}\) \((l, \omega_{bi} > l, \omega_{zi})\), the characteristic function adopts the following form the slowness of synthesized fixed systems:

\[
U(s) = \frac{1}{V(s)} = \frac{\prod_{i=1}^{n}(s^2 + 2h_{bi} + h_{bi}^2 + \omega_{bi}^2)}{s \prod_{j=1}^{n}(s^2 + \omega_{zj}^2)}
\]

Prior to proceed to main identification, that means the determination of the active force on the basis of the assumed dynamic properties, one should determine the structure and parameters of the system subjected to the active vibration reduction. The desired properties of the requested system are given in the form of a sequence of resonance frequencies \(\omega_{b1}, \omega_{b2}, ..., \omega_{bi}\). For the synthesis of fixed systems resonance frequencies are presented in the following form:

\[
\omega_{b1} = \omega_{b1}, \omega_{b2} = \omega_{z1}, \omega_{b3} = \omega_{z2}, ..., \omega_{b_{i-1}} = \omega_{zj}, \omega_{bi} = \omega_{bj} \quad (i, j = 1, 2, 3, ..., n)
\]

where: \(\omega_{b1}, \omega_{b2}, ..., \omega_{bi}\) – resonance frequencies presented in the form of poles, \(\omega_{z1}, \omega_{z2}, ..., \omega_{zj}\) – resonance frequencies presented in the form of zeroes of mobility.

The slowness function of analysed mechanical systems, undergoing active vibration reduction, takes the following form:

\[
U(s) = \frac{1}{V(s)} = \frac{\prod_{i=1}^{n}(s^2 + \omega_{bi}^2)}{s \prod_{j=1}^{n}(s^2 + \omega_{zj}^2)}
\]

As the result of the synthesis of the slowness function (4) by the characteristic decomposition of into a continued fraction it is obtained:
\[ U(s) = m_1 s + \frac{1}{s c_1 + \frac{1}{m_2 s + \frac{1}{\cdots + \frac{1}{s c_2 + \frac{1}{m_3 s + \frac{1}{\cdots + \frac{1}{s c_{n-1} + \frac{1}{m_n s + c_n}}}}}}}} = \frac{1}{s^{n+1} + c_1 s^n + \cdots + c_n}, \]  

(5)

where: \( m_1, m_2, \ldots, m_n \) – values of inertial components of the requested system, \( c_1, c_2, \ldots, c_n \) – values of elastic components of the identified system. The result of the performed synthesis, at this stage, is a mechanical system with a cascade structure (multi-degree-of-freedom) shown in figure 1. The dynamic characteristic in a form of slowness of the obtained system is corresponding with the slowness function subjected to the synthesis (2).

**Figure 1.** Mechanical system obtained in the result of slowness function decomposition (2) with active components of vibration reduction.

In the next step the control forces coefficients have been determined (other cases of the active synthesis are presented in [6 - 8, 11, 13]). On the basis of determined parameters of the identified structure of the system (figure 1) is created the stiffness matrix in the following form:

\[
Z(s) = \begin{bmatrix}
    m_1 s^2 + c_1 & -c_1 & \cdots & 0 \\
    -c_1 & m_2 s^2 + c_1 + c_2 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & m_n s^2 + c_{n-1} + c_n
\end{bmatrix},
\]

(6)

and the matrix of the active force:

\[
F(s) = \begin{bmatrix}
k_{1p} + k_{1v} s & -\left(k_{1p} + k_{1v} s\right) & \cdots & 0 \\
-\left(k_{1p} + k_{1v} s\right) & k_{1p} + k_{1v} s + k_{2p} + k_{2v} s & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & k_{n-1p} + k_{n-1v} s + k_{np} + k_{nv} s
\end{bmatrix},
\]

(7)

The rows of the matrix (7) correspond to the values of active forces and to the generalized point coordinates defining the position of the obtained inertial components, to which the requested force is applied. The proposed notation of forces arrangement relates to a system that is under influence of \( n \) active forces, in a number equal to the generalized coordinates of the obtained system in the result of the conducted identification.

On the basis of the matrix \( F(s) \) and the stiffness matrix \( Z(s) \) is determined the following polynomial:

\[
\det(Z(s) + F(s)) = A_{2n} s^{2n} + A_{2n-1} s^{2n-1} + A_{2n-2} s^{2n-2} + \cdots + A_1 s + A_0,
\]

(8)

In order to calculate the values of active forces the obtained polynomial should be divided by the factor \( A_{2n} \), and then compare to the polynomial characterizing the assumed dynamical properties in the form of resonance frequencies and decrease coefficients of chosen free vibration. The equation takes the following form:

\[
\det(Z(s) + F(s)) = \frac{A_{2n} s^{2n} + A_{2n-1} s^{2n-1} + A_{2n-2} s^{2n-2} + \cdots + A_1 s + A_0}{A_{2n}}.
\]
\[
\frac{\det(Z(s) + F(s))}{A_{2n}} = \prod_{i=1}^{n} \left( s^2 + 2h_{i1} s + h_{i1}^2 + \omega_{i1}^2 \right).
\] (9)

After comparison of coefficients occurring at the same polynomial powers from the equation (9) are determined the parameters of the active forces reducing vibration of the identified system. In the next step of the synthesis, using the presented method, it should be determined the coefficient of proportional decomposition \( \beta \) from the interval \((0,1)\). Depending on the desired symmetry of two-terminals in relation to the inertial component are determined next values:

- requested two-terminals of inertial type:

\[
\begin{cases}
  m_1 = m_3, \\
m_{21} = \beta m_2, m_{22} = m_2 - m_{21}, \\
m_{31} = \beta m_3, m_{32} = m_3 - m_{31}, \\
m_{n1} = \beta m_n, m_{n2} = m_n - m_{n1},
\end{cases}
\] (10)

- requested two-terminals of elastic type:

\[
\begin{cases}
  c_{11} = \beta c_1, c_{12} = c_1 - c_{11}, \\
c_{21} = \beta c_2, c_{22} = c_2 - c_{21}, \\
c_{n1} = \beta c_n, c_{n2} = c_n - c_{n1}.
\end{cases}
\] (11)

The values of requested parameters, determined in this way, are written as the sum of two continued fractions of the slowness function (4), satisfying the desired dynamic properties in the form of a sequence of accepted resonance frequencies.

\[
U(s) = m_1 s + \frac{1}{s c_{11}} + \frac{1}{m_{21} s + \frac{1}{s c_{21}} + \frac{1}{m_{31} s + \frac{1}{s c_{31}} \cdots + \frac{1}{s \frac{1}{m_{n1} s + \frac{1}{s c_{n1}}}}}}
\] (12)

While the values of active forces are determined in the form:

\[
\begin{cases}
  k_{11p} = \beta k_{1p} k_{12p} = k_{1p} - k_{11p}, k_{11v} = \beta k_{1v} k_{12v} = k_{1v} - k_{11v}, \\
k_{21p} = \beta k_{2p} k_{22p} = k_{2p} - k_{21p}, k_{21v} = \beta k_{2v} k_{22v} = k_{2v} - k_{21v}, \\
k_{n1p} = \beta k_{np} k_{n2p} = k_{np} - k_{n1p}, k_{n1v} = \beta k_{nv} k_{n2v} = k_{nv} - k_{n1v}.
\end{cases}
\] (13)

As the result of synthesis application, using the method of proportional decomposition of the parameters of dynamic characteristics in the form of slowness of fixed systems, it is obtained the system shown in figure 2, which dynamic properties are in accordance with the assumed characteristic (2).
3. Numerical example

In this chapter, a numerical example will be presented, showing how to designate the parameters and structure of a discrete system that meets pre-established dynamic properties in the form of resonance frequencies and amplitudes of vibrations:

\[
\begin{align*}
    \omega_{b_1} &= 121 \ \text{rad/s}, & \omega_{b_2} &= 175 \ \text{rad/s}, & \omega_{b_3} &= 330 \ \text{rad/s} & \rightarrow \text{resonance frequencies,} \\
    A_{b_1} &= 0.0004 \ \text{m}, & A_{b_2} &= 0.0004 \ \text{m} & \rightarrow \text{amplitudes.}
\end{align*}
\]  

(14)

Such a choice of dynamic properties, ignoring anti-resonance frequencies of the system, is associated with the use of proportional decomposition of parameters. Therefore, in the first step of the synthesis, a dynamic characteristic is built (dynamic flexibility), basing on the assumed resonance frequencies in the form of poles and zeroes of the characteristic (figure 3):

\[
Y(s) = \frac{s^2 + 175^2}{(s^2 + 121^2)(s^2 + 330^2)}.
\]

(15)

Figure 3. Dynamic characteristic of the system subjected to synthesis.

In the next step, the resulting characteristic (15) is modified to the slowness function as:

\[
U(s) = \frac{1}{s Y(s)} = \frac{1}{s} \frac{s^2 + 121^2}{s(s^2 + 175^2)}.
\]

(16)
Such characteristic (16) allows to determine parameters and system structure using a synthesis method. In the presented example, in order to obtain the structure of the system (figure 4), the characteristic decomposition method was used (16) in the form of a continued fraction having the following form:

$$U(s) = m_1s + \frac{1}{s + \frac{1}{c_1}} + \frac{1}{m_2s + \frac{c_2}{s}}$$  \hspace{0.5cm} (17)

where \(m_1 = 1 \text{ kg}; m_2 = 6.9 \text{ kg}; c_1 = 92916 \text{ N/m}; c_2 = 118407.87 \text{ N/m}$$.

**Figure 4.** Mechanical system obtained using the synthesis method.

The next step in identification of the system characteristic, after determining the parameters of the mechanical system, is modification the equation (15) by introducing the parameters of decrease in resonance frequencies of the system.

$$Y_1(s) = \frac{(s^2 + 175^2)}{(s^2 + 2h_{b1}s + h_{b1}^2 + 121^2)(s^2 + 2h_{b2}s + h_{b2}^2 + 330^2)^2}$$ \hspace{0.5cm} (18)

where \(h_{b1}\) the value of decrease coefficient of free vibration frequency corresponding to \(\omega_{b1}\), and \(h_{b2}\) the value of decrease coefficient of free vibration frequency corresponding to \(\omega_{b2}\). In this example the coefficients values are:

$$h_{b1} = 1.75 \text{ rad/s},$$
$$h_{b2} = 3.14 \text{ rad/s}.$$

(19)

The graphic illustration of the dynamic characteristic (18), in the case of such determined values of decrease vibration coefficient is shown in figure 5.

In order to achieve the desired vibration amplitudes of resonance frequencies of the analysed system (figure 4), the action of additional forces in the system is assumed (figure 6) in the form of:

$$\begin{cases}
  u_1(t) = k_{1p}(x_1 - x_2) + k_{1v}(\dot{x}_1 - \dot{x}_2), \\
  u_2(t) = k_{2p}x_2 + k_{2v}\dot{x}_2,
\end{cases}$$  \hspace{0.5cm} (20)

where \(k_{1p}, k_{1v}, k_{2p}, k_{2v}\) – amplification of control forces dependent on displacement and velocity of inertial components of the analysed system.

In the next step, control forces coefficients were determined. On the basis of defined parameters of the identified the structure of the system (figure 6), the stiffness matrix was built in the following form:

$$Z(s) = \begin{bmatrix}
  m_1s^2 + c_1 & -c_1 \\
  -c_1 & m_2s^2 + c_1 + c_2
\end{bmatrix}$$  \hspace{0.5cm} (21)

and the active forces matrix in the following form:
Figure 5. Dynamic characteristics taking into account the free vibration frequency decrease parameters.

\[ F(s) = \begin{bmatrix} k_{1p} + k_{1v}s & -k_{1p} - k_{1v}s \\ -k_{1p} - k_{1v}s & k_{1p} + k_{1v}s + k_{2p} + k_{2v}s \end{bmatrix}. \] (22)

Figure 6. Mechanical system with active vibration reduction components.

Using the stiffness matrix \( Z(s) \) and force matrix \( F(s) \), the following polynomial is determined:

\[
\det(Z(s) + F(s)) = A_4 s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0,
\] (23)

where: \( A_4 = 6.9; A_3 = 7.9 k_{1p} + k_{2v}, A_2 = k_{1s}k_{2v} + 7.9k_{1p} + k_{2p} + 85244.27; A_1 = k_{2v}k_{1p} + k_{1v}k_{2p} + 118407.87k_{1v} + 92916k_{2v}; A_0 = 118407.87k_{1p} + 92916k_{2p} + k_{1p}k_{2p} + 11001985.92. \) In order to calculate the values of active forces, the resulting polynomial should be divided by the coefficient \( A_4 \), and then compared to the polynomial characterizing the assumed dynamic properties in the form of resonance frequencies and self-vibrations frequency decrease coefficients. The equation now becomes:

\[
\frac{\det(Z(s) + F(s))}{A_4} = (s^2 + 2h_{b1}s + h_{b1}^2 + 121^2)(s^2 + 2h_{b2}s + h_{b2}^2 + 330^2).
\] (24)

After comparing the coefficients, standing at the same powers of polynomial of equation (24), the active forces parameters are determined, which reduce vibrations of the identified system. In the presented case, a six solutions of the equation were obtained: two real solutions and four composite solutions. Table 1 summarizes the results of solutions for real force amplification coefficients.

| Table 1. Value of amplification of control forces coefficients |
|-------------------------------------------------------------|
| 1\textsuperscript{st} solution | 2\textsuperscript{nd} solution |
| \( k_{1p} \) (N/m) | -77925.4172 | 8.9323 |
| \( k_{1v} \) (Ns/m) | 3.6596 | 4.8824 |
| \( k_{2p} \) (N/m) | 615699.0955 | 17.7342 |
| \( k_{2v} \) (Ns/m) | 38.5713 | 28.9107 |
The defined values of active forces, due to their configurations in the system, could also be obtained through both passive, elastic-damping components (figure 7a) or by hybrid structure of active-passive components (figure 7b).

![Figure 7](image)

**Figure 7. Mechanical system with components of vibration reduction**

a) passive b) passive-active.

Parameters of the passive system, in case of using the results for the second solution, take the following form:

\[
\begin{align*}
    c_{1p} &= c_1 + k_{1p} = 92925.69 \frac{N}{m}, \\
    b_{1v} &= k_{1v} = 4.88 \frac{Ns}{m}, \\
    c_{2p} &= c_2 + k_{2p} = 118431.01 \frac{N}{m}, \\
    b_{2p} &= k_{2p} = 28.91 \frac{Ns}{m}.
\end{align*}
\]  

(25)

When the parameters of the mechanical system are determined (figure 6), one can proceed to obtain the parameters and structure of the branched system. As mentioned in the introduction, the method of proportional decomposition of parameters is used to achieve the requested result. The coefficient of proportional decomposition of parameters, for this case, has been taken from the interval (0.1) and is:

\[
\beta = 0.75,
\]  

(26)

It was used to determine, with respect to the inertial component \(m_1\), the values of elastic and inertial two-terminals on the base of (10) and (11). They are listed in table 2.

| \(ij\) | \(m_i\) (kg) | \(c_{ij}\) (N/m) |
|---|---|---|
| 11 | 5.175 | 69687 |
| 12 | - | 23229 |
| 21 | - | 88805.9 |
| 22 | 1.725 | 29601.97 |

**Table 2. Values of elastic and inertial components**

Designated parameters describe the form of slowness (16) as the sum of continued fractions:

\[
U(s) = m_1s + \frac{1}{s} \frac{1}{c_{11} + \frac{c_{21}}{m_2s + \frac{c_{22}}{s}}}
\]  

(27)

Finally, as the result of using the synthesis method of proportional decomposition of parameters, the system shown in figure 8 is obtained.
Figure 8. Branched mechanical system obtained through the synthesis method.

The obtained values of vibration reduction in the mechanical system could be unambiguously attributed to the branched structure (figure 8). As in the case of the cascade, the system can be arranged with active components (figure 9a), passive components (figure 9b) or a hybrid of the former mentioned two types of components.

Adjusting forces in the case of the branched system, presented in figure 9a, take the form:

\[
\begin{align*}
  u_{11}(t) &= k_{11p}(x_1 - x_{21}) + k_{11v}(\dot{x}_1 - \dot{x}_{21}), \\
  u_{21}(t) &= k_{21p}x_{21} + k_{21v}\dot{x}_{21}, \\
  u_{12}(t) &= k_{12p}(x_1 - x_{22}) + k_{12v}(\dot{x}_1 - \dot{x}_{22}), \\
  u_{22}(t) &= k_{22p}x_{22} + k_{22v}\dot{x}_{22},
\end{align*}
\]

(28)

where \(k_{11p}, k_{11v}, k_{21p}, k_{21v}, k_{12p}, k_{12v}, k_{22p}, k_{22v}\) – assumed force amplification coefficients in the system, determined using the method of proportional decomposition of parameters. Using (13), the values of force amplification coefficients were determined. They are listed in table 3.

| Table 3. Value of amplification of control forces coefficients |
|---------------------------------------------------------------|
| \(k_{11p}\) (N/m) | 1st solution | \(k_{11v}\) (Ns/m) | 2nd solution | \(k_{12p}\) (N/m) | \(k_{12v}\) (Ns/m) | \(k_{21p}\) (N/m) | \(k_{21v}\) (Ns/m) | \(k_{22p}\) (N/m) | \(k_{22v}\) (Ns/m) |
|-------------------|--------------|-------------------|--------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| -58443.5496       | -58443.5496  | 2.7446            | 2.7446       | -19481.1832       | -19481.1832       | 0.9149            | 0.9149            | 461778.7945       | 461778.7945       |
| 6.6993            | 6.6993       | 3.6618            | 3.6618       | 2.2331            | 2.2331            | 1.2206            | 1.2206            | 13.3006           | 13.3006           |
| -58443.5496       | -58443.5496  | 2.7446            | 2.7446       | -19481.1832       | -19481.1832       | 0.9149            | 0.9149            | 461778.7945       | 461778.7945       |
| 6.6993            | 6.6993       | 3.6618            | 3.6618       | 2.2331            | 2.2331            | 1.2206            | 1.2206            | 13.3006           | 13.3006           |
| 28.9289           | 28.9289      | 21.6830           | 21.6830      | 153926.2648       | 153926.2648       | 9.6430            | 9.6430            | 7.2277            | 7.2277            |
In the case of the system illustrated in figure 9b, using the second solution in table 2, the passive vibration reduction components take the following values:

\[
\begin{align*}
    c_{11p} &= c_{11} + k_{11p} = 69694.27 \frac{N}{m}, \quad b_{11p} = k_{11v} = 3.66 \frac{N_s}{m}, \\
    c_{12p} &= c_{12} + k_{12p} = 23231.42 \frac{N}{m}, \quad b_{12p} = k_{12v} = 1.22 \frac{N_s}{m}, \\
    c_{21p} &= c_{21} + k_{21p} = 88823.26 \frac{N}{m}, \quad b_{21p} = k_{21v} = 21.68 \frac{N_s}{m}, \\
    c_{22p} &= c_{22} + k_{22p} = 29607.76 \frac{N}{m}, \quad b_{22p} = k_{22v} = 7.23 \frac{N_s}{m}.
\end{align*}
\]  

(29)

The determined parameters of active or passive vibration reduction allow to reduce vibration to the desired vibration amplitude of selected resonance frequencies. Using professional Matlab/Simulink software, calculations were performed to validate the results of passive and active vibration reduction of analysed systems (figure 9a and 9b) at the desired frequency spectrum. In the Matlab/Simulink was created the system structure shown in figure 10, allowing for passive and active reduction of vibrations.

![Figure 10. The structure of passive and active vibration reduction system in Matlab/Simulink.](image)

During the simulation, the periodic force with a unit amplitude and circular frequency corresponding to the first (figure 11) and third (figure 12) resonance frequency of free vibration of the system was assumed as the forcing signal. In addition, it was assumed that in the case of active vibration reduction, the control force is activated after 1 second. The generated time characteristics of the reduced systems, with respect to the predetermined amplitude, confirm the correctness of the synthesis method in reducing vibration due to dynamic properties of a mechanical system.

4. Conclusions

The paper presents the use of active synthesis as the method of determining the parameters and structure of the designed mechanical system. Particular attention was paid to the task of vibration reducing to desired values of amplitudes. For this purpose the active synthesis approach was used, allowing the designation of a number of adjustment forces in the system. This is the extension of the
traditional approach of the method of active synthesis of mechanical systems. Moreover, such approach of adjustment forces allows to replace them with passive elastic-damping components, or to implement hybrid, passive-active structures. Such approach to the method of synthesis allows a designer of mechanical systems for higher flexibility in choosing both the type of a system structure, and the components which reduce vibration according to the assumed characteristic.

Figure 11. Displacement of the first inertial component, taking into account passive reduction.

Figure 12. Displacement of the first inertial component, taking into account active reduction.

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