CP symmetry and the strong interactions

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Abstract

I discuss several aspects of CP non-invariance in the strongly interacting theory of quarks and gluons. I use a simple effective Lagrangian technique to map out the region of quark masses where CP symmetry is spontaneously broken. I then turn to the possible explicit CP violation arising from a complex quark mass. After summarizing the definition of the renormalized theory as a limit, I argue that attempts to remove the CP violation by making the lightest quark mass vanish are not well defined. I close with some warnings for lattice simulations.

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I. INTRODUCTION

The SU(3) non-Abelian gauge theory of the strong interactions is quite remarkable in that, once an arbitrary overall scale is fixed, the only parameters are the quark masses. Using only a few pseudo-scalar meson masses to fix these parameters, the non-Abelian gauge theory describing quark confining dynamics is unique. It has been known for some time [1] that, as these parameters are varied from their physical values, exotic phenomena can occur, including spontaneous breakdown of CP symmetry.

The possibility of a spontaneous CP violation is most easily demonstrated in terms of an effective chiral Lagrangian. In Section II I will review this model for the strong interactions with three quarks, namely the up, down, and strange quarks. This lays the groundwork for the discussion in Section III of the CP violating phase. Section IV discusses how heavier states, most particularly the \( \eta' \) meson, enter without qualitatively changing the structure.

Included among the parameters of the strong interactions is a complex phase which, if present, explicitly violates CP symmetry. This parameter appears to be extremely small [2] since no such violation is seen phenomenologically. A puzzle for grand unification asks why is CP violation small for the strong interactions but not the weak [3]. It is sometimes suggested that a massless up quark would solve this problem, and I turn to this issue in section V. There I argue that the conventional view is incorrect. The concept of a single massless quark is renormalization scheme dependent and not physically meaningful. Indeed, due to non-perturbative effects, even the sign of a small up-quark mass is not uniquely defined. These effects are of higher order in the chiral expansion. But as long as the other quark masses are not identically zero, some ambiguity remains in the definition of the up-quark mass. This is likely not relevant for most phenomenological issues, but is unacceptable for solving something fundamental like the strong CP problem.

Because renormalization is required, the concept of an “underlying basic Lagrangian” does not exist in quantum field theory. Instead there are underlying symmetries, and the continuum theory is defined in terms of those and a few renormalized parameters. A single massless quark is not represented by any symmetry, and, because of confinement, its mass is not appropriate for a renormalized parameter. To elucidate these points, I review the non-perturbative definition of the continuum theory and the corresponding ambiguities in the quark mass. These issues remain even with the recently discovered chirally symmetric lattice fermions. Finally, Section VI contains some concluding remarks, including possible impacts of the CP violating structures for lattice
gauge simulations.

II. THE EFFECTIVE MODEL

A CP violating phase appears naturally in the simplest chiral sigma model of interacting pseudo-scalar mesons. In this section I review the basic model and the standard connections between the quark masses and the meson masses. Nothing in this section is new; I am setting the stage for later discussion.

To be specific, consider the three flavor theory with its approximate SU(3) symmetry. Using three flavors simplifies the discussion, although the CP violating phase can also be demonstrated for the two flavor theory following the discussion in [4]. I work with the familiar octet of light pseudo-scalar meson fields $\pi_\alpha$ with $\alpha = 1 \ldots 8$. In a standard way (see for example [5]) I consider an effective field theory defined in terms of the SU(3) valued group element

$$\Sigma = \exp(i\pi_\alpha \lambda_\alpha / f_\pi) \in SU(3). \quad (1)$$

Here the $\lambda_\alpha$ are the usual Gell-Mann matrices which generate the flavor group and $f_\pi$ is a dimensional constant with a phenomenological value of about 93 MeV. I follow the normalization convention that $\text{Tr} \lambda_\alpha \lambda_\beta = 2 \delta_{\alpha\beta}$. The neutral pion and the eta meson play a special role in the later discussion; they are the coefficients of the commuting generators

$$\hat{\lambda}_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (2)$$

and

$$\hat{\lambda}_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \quad (3)$$

respectively. In the chiral limit of vanishing quark masses, we model the interactions of the eight massless Goldstone bosons with the effective Lagrangian density

$$L_0 = \frac{f_\pi^2}{4} \text{Tr}(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma). \quad (4)$$

The non-linear constraint of $\Sigma$ onto the group SU(3) makes this theory non-renormalizable. It is to be understood only as the starting point for an expansion of particle interactions in powers
of their masses and momenta. Expanding Eq. (4) to second order in the meson fields gives the conventional kinetic terms for our eight mesons.

This theory is invariant under parity and charge conjugation. These operators generate the transformations

\[ P : \Sigma \to \Sigma^{-1} \]
\[ CP : \Sigma \to \Sigma^* \]

where the operation * refers to complex conjugation. The eight meson fields are pseudo-scalars. The neutral pion and the eta meson are both even under charge conjugation.

With massless quarks, the underlying quark-gluon theory has a chiral symmetry under

\[ \psi_L \to \psi_L g_L \]
\[ \psi_R \to \psi_R g_R. \]

Here \((g_L, g_R)\) is in \((SU(3) \times SU(3))\) and \(\psi_{L,R}\) represent the chiral components of the quark fields, with flavor indices understood. This symmetry is expected to be broken spontaneously to a vector \(SU(3)\) via a vacuum expectation value for \(\overline{\psi}_L \psi_R\). This motivates the sigma model through the identification

\[ \langle 0 | \overline{\psi}_L \psi_R | 0 \rangle \leftrightarrow v \Sigma. \]

The quantity \(v\), of dimension mass cubed, characterizes the strength of the spontaneous breaking of this symmetry. Thus our effective field transforms under the chiral symmetry as

\[ \Sigma \to g_L^\dagger \Sigma g_R. \]

Our initial Lagrangian density is the simplest non-trivial expression invariant under this symmetry.

The quark masses break the chiral symmetry explicitly. From the analogy in Eq. (7), these are introduced through a 3 by 3 mass matrix \(M\) appearing in a potential term added to the Lagrangian density

\[ L = L_0 - v \text{Re Tr}(\Sigma M). \]

Here \(v\) is the same dimensionful factor appearing in Eq. (7). The chiral symmetry of our starting theory shows the physical equivalence of a given mass matrix \(M\) with a rotated matrix \(g_R^\dagger M g_L\). Using this freedom we can put the mass matrix into a standard form. I will assume it is diagonal with increasing eigenvalues

\[ M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}. \]
representing the up, down, and strange quark masses. Note that this matrix has both singlet and octet parts under the vector flavor symmetry

\[ M = \frac{m_u + m_d + m_s}{3} + \frac{m_u - m_d}{2} \lambda_3 + \frac{m_u + m_d - 2m_s}{2\sqrt{3}} \lambda_8. \] (11)

In general the mass matrix can still be complex. The chiral symmetry allows us to move phases between the masses, but the determinant of \( M \) is invariant. Under charge conjugation the mass term would only be invariant if \( M = M^\ast \). If \( |M| \) is not real, then its phase is the famous CP violating parameter usually associated with topological structure in the gauge fields. For the moment I take all quark masses as real. Since I am looking for spontaneous CP violation, I consider the case where there is no explicit CP violation.

To lowest order the masses of the pseudo-scalar mesons appear on expanding the mass term quadratically in the meson fields. This generates an effective mass matrix for the eight mesons

\[ \mathcal{M}_{\alpha\beta} \propto \text{Re Tr} \, \lambda_\alpha \lambda_\beta M. \] (12)

The isospin-breaking up-down mass difference plays a crucial role in the later discussion. This gives this matrix an off diagonal piece mixing the \( \pi_0 \) and the \( \eta \)

\[ \mathcal{M}_{3,8} \propto m_u - m_d. \] (13)

The eigenvalues of this matrix give the standard mass relations

\[
\begin{align*}
m^2_{\pi_0} &\propto \frac{2}{3} \left( m_u + m_d + m_s - \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s} \right) \\
m^2_{\pi^+} &\propto m_u + m_d \\
m^2_{\pi^-} &\propto m_u + m_s \\
m^2_{K^+} &\propto m_u + m_s \\
m^2_{K^0} &\propto m_u + m_s \\
m^2_\eta &\propto \frac{2}{3} \left( m_u + m_d + m_s + \sqrt{m_u^2 + m_d^2 + m_s^2 - m_u m_d - m_u m_s - m_d m_s} \right).
\end{align*}
\] (14)

Here I label the mesons with their conventional names.

Redundancies in these relations test the validity of the model. For example, comparing two expressions for the sum of the three quark masses

\[
\frac{2(m^2_{\pi^+} + m^2_{K^+} + m^2_{K^0})}{3(m^2_\eta + m^2_{\pi^0})} \sim 1.07
\] (15)

suggests the symmetry should be good to a few percent. Further ratios of meson masses then give estimates for the ratios of the quark masses \[5, 6, 7\]. For one such combination, look at

\[
\frac{m_u}{m_d} = \frac{m^2_{\pi^+} + m^2_{K^+} - m^2_{K^0}}{m^2_{\pi^+} - m^2_{K^+} + m^2_{K^0}} \sim 0.66.
\] (16)
This particular combination is polluted by electromagnetic effects; another combination partially cancels such while ignoring small $m_um_d/m_s$ corrections

$$\frac{m_u}{m_d} = \frac{2m^2_{\pi^0} - m^2_{\pi^+} + m^2_{K^0} - m^2_{K^+}}{m^2_{\pi^+} - m^2_{K^0} + m^2_{K^+}} \sim 0.55.$$  \hspace{1cm} (17)

Later I will comment on a third combination for this ratio. For the strange quark, one can take

$$\frac{2m_s}{m_u + m_d} = \frac{m^2_{K^+} + m^2_{K^0} - m^2_{\pi^+}}{m^2_{\pi^+}} \sim 26.$$  \hspace{1cm} (18)

**III. SPONTANEOUS CP VIOLATION**

So far all this is standard. Now I vary the quark masses and look for interesting phenomena. In particular, I want to find spontaneous breaking of the CP symmetry. Normally the $\Sigma$ field fluctuates around the identity in SU(3). However, for some values of the quark masses this ceases to be true. When the vacuum expectation of $\Sigma$ deviates from the identity, some of the meson fields acquire expectation values. As they are pseudo-scalars, this necessarily involves a breakdown of parity, as noted by Dashen [1].

To explore this possibility, I concentrate on the lightest meson from Eq. (14), the $\pi_0$. From Eq. (14) we can calculate the product of the $\pi_0$ and $\eta$ masses

$$m^2_{\pi_0} m^2_\eta \propto m_\pi m_d + m_u m_s + m_d m_s.$$  \hspace{1cm} (19)

Whenever

$$m_u = \frac{-m_d m_s}{m_u + m_d}$$  \hspace{1cm} (20)

the $\pi_0$ mass vanishes. For increasingly negative up-quark masses, our simple expansion around vanishing pseudo-scalar meson fields fails. The vacuum is no longer approximated by fluctuations of $\Sigma$ around the unit matrix; instead it fluctuates about an SU(3) matrix of form

$$\Sigma = \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & e^{-i\phi_1 - i\phi_2} \end{pmatrix}$$  \hspace{1cm} (21)

where the phases satisfy

$$m_u \sin(\phi_1) = m_d \sin(\phi_2) = -m_s \sin(\phi_1 + \phi_2).$$  \hspace{1cm} (22)
FIG. 1: The phase diagram of quark-gluon dynamics as a function of the two lightest quark masses. The shaded region exhibits spontaneous CP breaking. The diagonal lines with $m_u = \pm m_d$ trace where we have three degenerate pions due to isospin symmetry. The neutral pion mass vanishes on the boundary of the CP violating phase.

There are two minimum action solutions, differing by flipping the signs of these angles. The transition is a continuous one, with $\Sigma$ going smoothly to the identity as the boundary given by Eq. (20) is approached.

In the new vacuum the neutral pseudo-scalar meson fields acquire expectation values. As the neutral pion is CP odd, we spontaneously break this symmetry. This will have various experimental consequences, for example eta decay into two pions becomes allowed since a virtual third pion can be absorbed by the vacuum. Fig. (1) sketches the inferred phase diagram as a function of the up and down quark masses. Chiral rotations insure a symmetry under the flipping of the signs of both quark masses.

At first sight the appearance of the CP violating phase at negative up quark mass may seem surprising. Naively in perturbation theory the sign of a fermion mass can be rotated away by a redefinition $\psi \rightarrow \gamma \psi$. However this rotation is anomalous, making the sign of the quark mass observable. A more general complex phase in the mass would also have physical consequences, i.e. explicit CP violation. With real quark masses the underlying Lagrangian is CP invariant, but the above discussion shows that there exists a large region where the ground state spontaneously breaks this symmetry.
Vafa and Witten [8] argued on rather general conditions that CP could not be spontaneously broken in the strong interactions. However their argument makes positivity assumptions on the path integral measure. When a quark mass is negative, the fermion determinant need not be positive for all gauge configurations; in this case their assumptions fail.

The possible existence of this phase was anticipated on the lattice some time ago by Aoki [10]. For the one flavor case he found this parity breaking phase with Wilson lattice gauge fermions. He went on to discuss also two flavors, finding both flavor and parity symmetry breaking. The latter case is now regarded as a lattice artifact of Wilson fermions. For a review of these issues see [11].

In conventional discussions of CP non-invariance in the strong interactions [9] appears a phase $e^{i\theta}$ appearing on tunneling between topologically distinct gauge field configurations. The famous U(1) anomaly formally allows us to move this phase into the determinant of the quark mass matrix. After rotating all phases into the up-quark mass, we see that our spontaneous breaking of CP is occurring at an angle $\theta = \pi$.

A crucial observation is that when the down quark mass is positive, the CP violating phase does not appear for up-quark masses greater than a non-zero minimum value. There exists a finite gap with $\theta = \pi$ without this symmetry breaking. The chiral model predicts a smooth behavior as the up-quark mass passes through zero. This is the main motivation for the discussion in section IV on the meaning of a vanishing up-quark mass. Indeed, from the effective Lagrangian point of view, the real and imaginary parts of the quark mass are independent parameters. The absence of experimental evidence for strong CP violation suggests that the imaginary part of the quark mass matrix vanishes, but says nothing about the real part.

An interesting special case occurs when the up and down quarks have the same magnitude but opposite sign for their masses, i.e. $m_u = -m_d$. In this situation it is illuminating to rotate the minus sign into the phase of the strange quark. Then the up and down quark are degenerate, and we have restored an exact vector $SU(2)$ flavor symmetry. The excitation spectrum will show three degenerate pions, but they will not be massless due to what might be thought of a vacuum condensate of eta particles.

IV. INCLUDING THE $\eta'$

The above discussion was entirely in terms of the pseudo-scalar mesons that become Goldstone bosons in the chiral limit. One might wonder how higher states can influence this phase structure.
Of particular concern is the $\eta'$ meson associated with the anomalous $U(1)$ symmetry present in the classical quark-gluon Lagrangian. Non-perturbative processes, including topologically non-trivial gauge field configurations, are well known to generate a mass for this particle. I will now argue that, while this state can shift masses due to mixing with the lighter mesons, it does not make a qualitative difference in the existence of a phase with spontaneous CP violation.

The easiest way to introduce the $\eta'$ into the effective theory is to promote the group element $\Sigma$ to an element of $U(3)$ via an overall phase factor. Thus I generalize Eq. (1) to

$$\Sigma = \exp \left( i \pi \alpha / f_\pi + i \sqrt{\frac{2}{3}} \eta' / f_\pi \right) \in U(3).$$

The factor $\sqrt{2/3}$ gives the $\eta'$ field the same normalization as the $\pi$ fields. Our starting kinetic Lagrangian in Eq. (4) would have this particle also be massless. One way to fix this deficiency is to mimic the anomaly with a term proportional to the determinant of $\Sigma$

$$L_0 = \frac{f_\pi^2}{4} \text{Tr}(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma) - C|\Sigma|.$$

The parameter $C$ parameterizes the strength of the anomaly in the $U(1)$ factor.

Now if we include the mass term exactly as before, additional mixing occurs between the $\eta'$, the $\pi_0$, and the $\eta$. The corresponding mixing matrix takes the form

$$\begin{pmatrix}
  m_u + m_d & \frac{m_u - m_d}{\sqrt{3}} & \frac{\sqrt{2}}{3} (m_u - m_d) \\
  \frac{m_u - m_d}{\sqrt{3}} & m_u + m_d + 4m_s & \frac{\sqrt{2}}{3} (m_u + m_d - 2m_s) \\
  \frac{\sqrt{2}}{3} (m_u - m_d) & \frac{\sqrt{2}}{3} (m_u + m_d - 2m_s) & 2m_a
\end{pmatrix},$$

(25)

where $m_a$ characterizes the contribution of the non-perturbative physics to the $\eta'$ mass. This should have a value of order the strong interaction scale; in particular, it should be large compared to at least the up and down quark masses. The two by two matrix in the upper left of this expression is exactly what is diagonalized to find the neutral pion and eta masses in Eq. (14).

The boundary of the CP violating phase occurs where the determinant of this matrix vanishes. This modifies Eq. (19) to

$$m_\pi^2 m_\eta^2 m_{\eta'}^2 \propto m_a (m_u m_d + m_u m_s + m_d m_s) - m_u (m_d - m_s)^2 - m_d (m_u - m_s)^2 - m_s (m_u - m_d)^2.$$

The boundary shifts slightly from the earlier result, but still passes through the origin, leaving Fig. (1) qualitatively unchanged.
V. CAN THE UP QUARK BE MASSLESS?

A oft proposed solution to the strong CP problem \cite{12,13,14,15} asks whether \( m_u = 0 \). From the effective Lagrangian point of view, this appears to be an artificial setting of two parameters to zero, the real and imaginary parts of the quark mass. The earlier discussion shows that nothing special is expected to happen as the real part of the quark mass goes through zero. It is only the imaginary part that should vanish for CP to be a good symmetry, at least when the up-quark mass is larger than the value giving spontaneous breaking. But speculations on a vanishing up-quark mass continue, so it is interesting to ask if this can be given physical meaning. In this section I investigate precisely what is meant by a quark mass, and what \( m_u = 0 \) would mean. I will conclude that the question of whether \( m_u \) could vanish is ill posed. This is not relevant for most phenomenological purposes, but is unacceptable for solving something fundamental, like the strong CP problem. Ref. \cite{12} raises some of these issues, pursuing \( m_u = 0 \) anyway as an accidental symmetry.

The conventional description of this phenomenon changes variables from the complex quark mass to polar coordinates involving the magnitude of the up-quark mass and its phase. I argue below that non-perturbative effects give a scheme-dependent additive shift in the real part of the up-quark mass. Such a shift exposes the singular nature of this selection of coordinates. To emphasize the point, imagine changing variables to the magnitude and phase of \( (m_u - 4 \text{ MeV}) \). If the up-quark mass is 4 MeV, then in such coordinates the theory no longer depends on the phase. From this point of view \( m_u = 4 \text{ MeV} \) is an equally good solution to the strong CP issue as is \( m_u = 0 \).

While phenomenology, i.e. Eq. \cite{17}, seems to suggest that the up quark is not massless, there remains considerable freedom in extracting that ratio from the masses of the pseudo-scalar mesons. From Eq. \cite{14}, the sum of the squares of the \( \eta \) and \( \pi_0 \) masses should be proportional to the sum of the three quark masses. Subtracting off the neutral kaon mass should leave just the up quark. Thus motivated, look at

\[
\frac{m_u}{m_d} = \frac{3(m_\eta^2 + m_{\pi_0}^2)/2 - 2m_{K_0}^2}{m_{\pi^+}^2 - m_{K^+}^2 + m_{K_0}^2} \sim -0.8. \tag{27}
\]

Thus even the sign of the up-quark mass is ambiguous. This example is perhaps a bit extreme since it ignores the influence of mixing with the eta prime in the numerator. But it shows that there does exist a substantial phenomenological uncertainty in the quark masses. More formal attempts to extend the naive quark mass ratio estimates to higher orders in the chiral expansion have shown
fundamental ambiguities in the definition of the quark masses [5].

If two quark masses were to vanish simultaneously, then we would have exactly massless pions, Goldstone bosons for the resulting flavored chiral symmetry. In this case the concept of zero quark mass has definite physical consequences. But here I concentrate on whether the concept of only a single massless quark has any meaning. While I could carry along the baggage of the heavier quarks, let me simplify the discussion and consider the theory reduced to a single flavor of quark.

As stated in the introduction, because renormalization is required, the continuum concept of an “underlying basic Lagrangian” does not exist. The continuum theory is specified in terms of basic symmetries and a few renormalized parameters. Because of anomalies, a single massless quark does not correspond to any symmetry. In practice the definition of a field theory relies on a limiting process from a cutoff version. As the lattice is the only well understood non-perturbative cutoff, it provides the most natural framework for such a definition. But any regulator must accommodate the known chiral anomalies, and thus there must be chiral symmetry breaking terms in the cutoff theory. These chiral breaking effects come in many guises. With a Pauli-Villars scheme, there is a heavy regulator field. With dimensional regularization the anomaly is hidden in the fermionic measure. For Wilson lattice gauge theory there is the famous Wilson term. With domain wall fermions there is a residual mass from a finite fifth dimension. With overlap fermions things are hidden in a combination of the measure and a certain non-uniqueness of the operator. I will return to this last case shortly.

The lattice regulator involves introducing a dimensionful parameter, the lattice spacing \( a \). This feature is not special to the lattice. The scale anomaly is responsible for masses of hadrons such as the proton and glueballs, even in the massless quark limit. For such physics, any complete regulator must introduce a scale.

The renormalization process tunes all relevant bare parameters as a function of the cutoff to fix a set of renormalized quantities. In the case of the strong interactions, the bare gauge coupling is driven to zero by asymptotic freedom. Its cutoff dependence is absorbed into an overall scale in a well known way via the phenomenon of dimensional transmutation [16]. The only other parameters of the strong interactions are the quark masses. For these one inputs a few particle masses to finally determine the continuum theory uniquely. For the three flavor theory the most natural observables to fix these parameters are the masses of the pseudo-scalar mesons.

In the one flavor theory there are no Goldstone bosons, but massive mesons and baryons should exist. I need some physical parameter with which to carry out the renormalization of the quark
FIG. 2: Defining the continuum limit. For one-flavor strong interactions I consider the ratio of the lightest boson to lightest baryon masses as my renormalized parameter. With the cutoff in place, we flow towards the origin along a curve of constant renormalized quantity. Below the contour where this ratio vanishes lies the region of spontaneous CP violation.

As both are expected to be stable, this precludes any ambiguity from particle widths. Calling the lightest boson the \( \eta \) and the baryon \( p \), I define

\[
r = \frac{m_\eta}{m_p}.
\]  

(28)

I expect to be able to adjust this parameter via the quark mass, which should be tuned to give the desired value. It should be possible to give this ratio any value throughout the range from \( r = 0 \) at the boundary of the above CP violating phase to \( 2/3 \) in the heavy quark limit.

With a cutoff in place, I can in principle determine this ratio given any values for the bare quark mass and bare coupling. For pedagogy, let me assume a lattice regulator and trade these parameters for the lattice spacing \( a \) and the quark mass in lattice units, \( m_q a \). Both of these quantities go to zero in the continuum limit. The renormalization prescription is to select a desired value of \( r \) and follow the contour with this value in the \((a, m_q)\) plane towards the origin. This process is sketched in Fig. (2). Perturbative divergences in the bare quark mass appear in the fact that these contours approach the origin with zero slope.

When the cutoff is in place, we expect prescription dependent artifacts. In particular, the pre-
cise locations of the constant \( r \) contours will depend on details of the formulation. Holding the bare quark mass at zero will cross a variety of \( r \) contours, with none obviously favored as the origin is approached. Different cutoff schemes will give different continuum limits for \( m_u = 0 \). Alternatively, \( m_u = 0 \) in one scheme could give the same continuum limit as some other scheme with a non-vanishing \( m_u \). A non-perturbative additive shift in the up-quark mass distinguishes the two schemes. Asking that the up-quark mass vanishes is unphysical.

With two or more degenerate flavors there will be one special contour where the lightest meson does represent a Goldstone boson. With the Wilson fermion formulation, the quark mass axis is represented by the hopping parameter. As this particular cutoff explicitly breaks chiral symmetry, the critical hopping parameter, where the meson mass vanishes, is renormalized away from its value in the continuum limit.

Recently there has been considerable progress with lattice fermion formulations that preserve a remnant of exact chiral symmetry \[11\]. With such, the two flavor theory will have the \( r = 0 \) contour naturally preserved as the \( m_q = 0 \) axis. The crucial point is that this is not true for the one flavor theory. Dynamics generates a mass for the pseudo-scalar meson; thus, the \( m_q = 0 \) axis will cut through various finite values of \( r \). An interesting question is whether, as we take the lattice spacing to zero along this axis, some physical value of \( r \) will be picked out as special and corresponding to vanishing quark mass. That this is unlikely follows from the non-uniqueness of these chiral lattice operators. For example, the overlap operator \[17\] is constructed by a projection process from the conventional Wilson lattice operator. The latter has a mass parameter which is to be chosen in a particular domain. On changing this parameter, the massless Dirac operator still satisfies the Ginsparg-Wilson relation \[18\], but this condition does not guarantee that the contours of constant \( r \) in Fig. 2 will not shift. Thus the horizontal axis is not expected to select one contour as special. Again, holding \( m_u = 0 \) is not expected to give a unique continuum theory.

To see that this non-universality is indeed expected, consider that the dynamics of the one flavor case generates a mass gap in the \( \eta \) channel. This means that the eigenvalues of the Dirac operator important to low energy physics are are not near the origin, but dynamically driven a finite distance away. Changing the projection procedure to generate the overlap operator will modify the size of this gap, changing the \( \eta \) mass and the ratio \( r \).

One might attempt to define \( m_u = 0 \) as the point where the topological susceptibility vanishes. For this purpose, the overlap operator provides a natural definition of the gauge field topology. With this prescription a vanishing topological susceptibility is synonymous with vanishing quark
FIG. 3: Non-perturbative classical gauge configurations can generate an effective mass term for the up quark. The magnitude of this mass is proportional to the product of the heavier quark masses. The precise value, however, is scheme and scale dependent.

mass for that particular overlap operator. As the result depends on the operator chosen, with one flavor the susceptibility is no more physical than the quark mass. As a rather abstract concept, the topological susceptibility is not directly measurable in scattering processes for physical particles.

This non-perturbative ambiguity in the quark mass carries over to the explicitly CP violating case where the mass is complex with phase $\theta$. The above discussion shows that even the sign of the up quark mass is ambiguous, indicating that different schemes can have an ambiguity of $\theta$ between 0 and $\pi$, a particularly severe example. For other values of $\theta$, to fix the continuum theory uniquely we need to introduce another renormalized quantity. For example, this could be a three meson coupling or the electric dipole moment of a baryon. Beyond the lowest order in the chiral expansion, the precise dependence of the renormalized parameter on $\theta$ is scheme-dependent.

While the use of the lattice provides a framework for precise discussion, the source of the additive shift in the up-quark mass follows qualitatively from non-perturbative classical gauge configurations, i.e. “pseudoparticles” or “instantons” \[19\]. As shown some time ago by ’t Hooft \[20\], these configurations generate an effective multi-fermion vertex where all flavors of quark flip their spin. If we take this vertex and tie together the massive quark lines with mass terms, then the resulting process generates an effective mass term for the light quark. The strength of this term is proportional to the product of the masses of the more massive quarks. This process is illustrated in Fig. 3. As the strength of the effective interaction is scheme and scale dependent, so is the resulting light quark mass.
VI. FINAL REMARKS

While I have been exploring rather unphysical regions in parameter space, these observations do raise some issues for practical lattice calculations of hadronic physics. Current simulations are done at relatively heavy values for the quark masses. This is because the known fermion algorithms tend to converge rather slowly at light quark masses. Extrapolations by several tens of MeV are needed to reach physical quark masses, and these extrapolations tend to be made in the context of chiral perturbation theory. The presence of a CP violating phase quite near the physical values for the quark masses indicates a strong variation in the vacuum state with a rather small change in the up-quark mass; indeed, less than a 10 MeV change in the traditionally determined up-quark mass can drastically change the low energy spectrum. Most simulations consider degenerate quarks, and chiral extrapolations so far have been quite successful. But some quantities, namely certain baryonic properties [21], do seem to require rather strong variations as the chiral limit is approached. These effects and the strong dependence on the up-quark mass may be related.

Another issue is the validity of current simulation algorithms with non-degenerate quarks. With an even number of degenerate flavors the fermion determinant is positive and can contribute to a measure for Monte Carlo simulations. With light non-degenerate quarks the positivity of this determinant is not guaranteed. Indeed, the CP violation can occur only when the fermions contribute large phases to the path integral. Current algorithms for dealing with non-degenerate quarks [22] take a root of the determinant with multiple flavors. In this process any possible phases are ignored. Such an algorithm is incapable of seeing any of the CP violating phenomena discussed here. This point may not be too serious in practice since the up and down quarks are nearly degenerate and the strange quark is fairly heavy. But these issues should serve as a warning that things might not work as well as we want.

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contribution, or allow others to do so, for U.S. Government purposes.

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