Is the $\sigma(600)$ a Glueball?  
Two photon reactions can tell us

M.R. Pennington

Centre for Particle Theory, University of Durham, Durham DH1 3LE, U.K.

Abstract: Minkowski and Ochs have recently argued that the small two photon coupling of a conjectured $\sigma(600)$ is so small that it is likely to be a glueball. We ask whether this can be so or whether it is simply gauge invariance that produces the observed low mass suppression?

THE SCALAR GLUEBALL

QCD predicts that there should exist bound states of glue: states we know as glueballs [1]. All modellings of non-perturbative QCD, including lattice computations [2,3], indicate that the lightest glueball should be spinless. It is then not surprising that if we look at the Particle Data tables [4], there are more light isosinglet scalars than can fit into one $q\bar{q}$ nonet: $f_0(400 − 1200)$ (or $\sigma$), $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. But which one is the glueball?

Each state has its own protagonists. Lattice calculations expect the scalar glueball has a mass (in the quenched approximation) between 1600 and 1700 MeV [2,3]. Amsler and Close [5] claim the $f_0(1500)$, studied extensively (but not exclusively) in $pp$ annihilation with Crystal Barrel at LEAR, is the glueball, while others [3], even more vehemently, insist the scalar component of the $f_J(1710)$ [4] observed in $J/\psi$ radiative decays is gluish. Here we concentrate on the claim that it is the $\sigma(600)$ that is predominantly a glueball [6,7]. Though Minkowski and Ochs [6] are not the first to make such a proposal, it is the appearance of the $\sigma(600)$ in $\gamma\gamma$ reactions that is central to their claim, and so most appropriate for this meeting.

TWO PHOTON v. DI-PION PRODUCTION OF PIONS

In Fig. 1, we compare the cross-section for $\pi^+\pi^- \rightarrow \pi^0\pi^0$ from the very recently published results from the E852 experiment at Brookhaven [8] with the Crystal Ball data [9] on $\gamma\gamma \rightarrow \pi^0\pi^0$. Each is dominated by the spin two $q\bar{q}$ state, the $f_2(1270)$. Consequently, it is at this peak that these cross-sections (or rather the squares of the

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1Talk given at the 13th International Workshop on Photon-Photon Collisions (Photon2000), September 2000, Ambleside, U.K. — to appear in the Proceedings
moduli of the corresponding amplitudes) are normalised. Below 1 GeV we see the well-known enhancement in the $\pi\pi$ elastic cross-section, identified as the $\sigma(600)$, while in the $\gamma\gamma$ reaction little is seen. It is this smallness around 600 MeV that Minkowski and Ochs [6] claim points to the $\sigma$ being predominantly glue.

In the quenched approximation this is natural. Photons, of course, couple to electric charge. Consequently, a neutral $q\bar{q}$ meson can readily decay into photons. In contrast, a glueball can only decay radiatively through quark loops, Fig. 2. This means that, in the quenched approximation, a glueball doesn’t have photonic decays. In perturbative QCD, quark loops (Fig. 2) are suppressed by factors of $\alpha_s$. But are these arguments relevant at 600 MeV? Indeed, if the $\sigma(600)$ were to be a glueball, it would have to have large couplings to quarks, if its decay width is to be 300-500 MeV as experiment indicates (Fig. 1).

**Figure 1:** The modulus squared of the amplitudes for $\pi^+\pi^- \rightarrow \pi^0\pi^0$ and $\gamma\gamma \rightarrow \pi^0\pi^0$, deduced from BNL-E852 [8] and Crystal Ball [9] data, respectively.

**Figure 2:** Modulus squared of the amplitude for a $\bar{q}q$ state and a glueball, respectively, to decay to two photons — at lowest order in perturbative QCD.
Figure 3: Feynman diagram modelling of $\gamma\gamma \rightarrow \pi\pi$: the Born term plus direct channel $\sigma$ formation.

The idea that the $\gamma\gamma \rightarrow \pi^0\pi^0$ cross-section directly determines the $\sigma$ couplings to two photons requires a modelling of the $\gamma\gamma$ reaction. The simplest would be to imagine that the two photon amplitude is just described by the pion Born term plus the $\sigma$-resonance component, as in Fig. 3. Since the Born term does not contribute to the $\pi^0\pi^0$ channel, the cross-section in Fig. 1 then fixes the magnitude of this $\sigma$-component. But if this model makes any sense it must also explain the $\pi^+\pi^-$ cross-section [10] too. Its $S$-wave component [11], then determines the orientation of this $\sigma$-component. Moreover, unitarity requires that the phase of each $\gamma\gamma \rightarrow \pi\pi$ partial wave amplitude with definite isospin must equal the phase of the corresponding $\pi\pi$ elastic partial wave. As detailed in [12], this also fixes the orientation of the $\sigma$-component in the model of Fig. 3. These determinations don’t agree. This teaches us that the modelling shown in Fig. 3 is far too simplistic. There are in fact many other contributions that must be included, like those in Fig. 4, before we can be sure that unitarity is fulfilled and so extract resonance couplings in a meaningful way.

Figure 4: Two of the additional contributions to the model amplitudes for $\gamma\gamma \rightarrow \pi\pi$ of Fig. 3 essential for ensuring the final state interaction theorem is satisfied.

Over ten years ago, David Morgan and I [13] worked out a procedure for ensuring that $\gamma\gamma \rightarrow \pi\pi$ amplitudes satisfy the constraints of analyticity and unitarity, as well obeying the Thomson limit of QED at low energies [14]. Applying this to all recent experimental data, Elena Boglione and I [11] have found that the $I = 0 \gamma\gamma \rightarrow \pi\pi$ $S$-wave cross-section has a quite different shape than that for $\pi\pi$ scattering, as shown in Fig. 4. Indeed, while the Born component dominates the $\gamma\gamma$ channel close to threshold, we see that any resonance contribution is pushed to higher mass away from 600 MeV. There must be some reason for this.
Figure 5: The $I = J = 0$ components of the cross-sections for $\pi\pi \rightarrow \pi\pi$ (dashed line) based on data analysed by Ochs [15] and by Bugg et al. [16], and for $\gamma\gamma \rightarrow \pi\pi$ (solid line) from the Amplitude Analysis by Boglione et al. [11] — dip solution.

GAUGE INVARIANCE

The simplest Lagrangian describing a scalar field $\phi$ coupling to two photons is

$$\mathcal{L}_I = g \phi \mathcal{F}^{\mu\nu} \mathcal{F}_{\mu\nu},$$

where $\mathcal{F}^{\mu\nu}$ is the electromagnetic tensor. The scalar’s radiative decay rate is proportional to $g^2$. Dimensional analysis requires $g$ to have dimension of $1/M$. So $g^2$ must be multiplied by a mass cubed to give the radiative width. Only a detailed non-perturbative calculation would tell us exactly what this mass scale is, but simple phenomenology indicates it must the mass of the scalar. This intuition is confirmed in the pseudoscalar sector. The dimensional analysis is the same [17]. In the quark model, the $\gamma\gamma$ couplings are related to the fourth power of the charges of the constituents (Fig. 2), but for the $\pi$, $\eta$ and $\eta'$ this has to be corrected by a factor of their mass cubed to bring agreement with experiment. Without this factor, the quark model predictions would be orders of magnitude wrong.

It is then natural to compare the cross-sections of Fig. 1 with the $\gamma\gamma \rightarrow \pi^0\pi^0$ one divided by a factor of $M_{\pi\pi}^3$, normalised to unity at the $f_2(1270)$. The result is displayed

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2Just as for the Higgs boson.

3Arguments from non-relativistic quark models would give other mass dependent factors too. While such models are applicable to heavy quark systems, it is not clear that they have much relevance to light quark systems, where, as stressed in [18], for instance, genuine bound state calculations, which are inevitably non-perturbative, are essential for meaningful results.
in Fig. 6. We see that the $\pi\pi$ and $\gamma\gamma$ cross-sections are now much more equal across the whole mass region below 1.5 GeV with no further suppression of the $\sigma$ at 600 MeV. Clearly the factor of mass cubed cannot be exact as $M_{\pi\pi}$ increases indefinitely. However, the phenomenology in the pseudoscalar sector indicates that the factor is indeed mass cubed, at least up to the $\eta'$ around 1 GeV, which is sufficient for this illustration.

We see that it is wholly a consequence of gauge invariance that the $\sigma$ contribution is suppressed at 600 MeV, just as the radiative width of the pion is reduced compared to the $\eta$ and $\eta'$. The $I = 0$ $S$-wave $\gamma\gamma \rightarrow \pi\pi$ cross-section is skewed to higher masses. The $f_0(400 - 1200)$ seen in Fig. 5 has a radiative width of about 4 keV, as discussed in [11]. This is just as expected for a scalar composed of non-strange quarks, i.e. a $(u\bar{u} + d\bar{d})$ state [19]. Thus the $\sigma(600)$ is not, as claimed by Minkowski and Ochs [6], the glueball. For this, we have to look above 1.4 GeV.

It is an important feature of the scalar sector and its relation to the vacuum that the glueball lies amongst the lightest $\bar{q}q$ isoscalars and inevitably mixes with these. In contrast, lattice calculators [1] compute the tensor glueball mass is above 2 GeV and so far above the lightest quark nonet, containing the $f_2(1270)$. Consequently, any mixing is likely to be less (ignoring the effect of radial excitations) and the glueball nearly naked [1]. While the $\xi(2230)$ [4,20] may be a candidate for such a state, it is essential to confirm its
existence and then to determine it does have spin 2, before speculating further about its composition [21].

QCD predicts that bound states of glue should exist. The scalar is the lightest. Surely soon we will know which of the many \( f_0 \)'s [4] it is. Two photon reactions tell us it is not the \( \sigma(600) \).

**ACKNOWLEDGEMENTS**

It is a pleasure to thank the organisers, particularly Alex Finch, for this de-lightful meeting in such a wonderful setting. I acknowledge support from the EU-TMR Programme, Contract No. CT98-0169, EuroDAΦNE.

**REFERENCES**

1. see, for instance, Pennington, M.R., *Proc. Workshop on Photon Interactions and the Photon Structure* (Lund, September 1998), eds G. Jarlskog, T. Sjöstrand, pub. Lund Univ, pp. 312-328.
2. Teper, M., *Proc. Int. Europhysics Conf. on High-Energy Physics (HEP 97)* (Jerusalem, Israel, August 1997) pp. 384-387; “Glueball masses and other physical properties of SU(N) gauge theories in D=(3+1): A review of lattice results for theorists”, hep-th/981217.
3. Sexton, J., Vaccarino, A., and Weingarten, D., *Phys. Rev. Lett.* 75, 4563-4566 (1995); Lee, W., and Weingarten, D., *Nucl. Phys. B (Proc. Suppl.*) 53, 236-238 (1997), 63, 194-196 (1998); Morningstar, C., and Peardon, M., *Nucl. Phys. B (Proc. Suppl.*) 53, 917-920 (1997).
4. Groom, D.E., et al. (PDG) *Eur. Phys. J.* C15, 1 (2000).
5. Amsler, C., and Close, F.E., *Phys. Lett.* B353, 385-390 (1995), *Phys. Rev.* D53, 295-311 (1996).
6. Minkowski, P., and Ochs, W., [hep-ph/9811518] *Eur. Phys. J.* C9, 283-312 (1999).
7. Kisslinger, L.S., Gardiner, J., and Vanderstraeten, C., *Phys. Lett.* B410, 1-5 (1997); Kisslinger, L.S., and Li, Z., *Phys. Lett.* B445, 271-273 (1999); see also, for instance, Narison, S., *Nucl. Phys.* B509, 312-356 (1998).
8. Gunter, J., et al., (E852 Collab.) [hep-ex/0001038]
9. Marsiske, H., et al., *Phys. Rev.* D41, 3324-3335 (1990); Bienlein, J.K., *Proc. IXth Int. Workshop on Photon-Photon Collisions* (San Diego, 1992) eds. D. Caldwell and H.P. Paar, pub. World Scientific, pp. 241-257.
10. Boyer, J., et al. (MarkII), *Phys. Rev.* D42, 1350-1367 (1990); Behrend, H.J., et al. (CELO), *Z. Phys.* C56, 381-390 (1992).
11. Boglione, M., and Pennington, M.R., *Eur. Phys. J.* C9, 11-29 (1999).
12. Pennington, M.R., *Proc. Workshop on Hadron Spectroscopy* (Frascati, March 1999), eds T. Bressani, A. Feliciello and A. Filippi, pub. INFN, 1999, pp. 95-114.
13. Morgan, D., and Pennington, M.R., *Z. Phys.* C37, 431-447 (1988); C39, 590 (1988); C48, 623-632 (1990).
14. Pennington, M.R., *DAΦNE Physics Handbook*, eds. L. Maiani, G. Pancheri and N. Paver, pub. INFN, Frascati, 1992, pp. 379-418; *Second DAΦNE Physics Handbook*, eds. L. Maiani, G. Pancheri and N. Paver, pub. INFN, Frascati, 1995, pp. 531-558.
15. Ochs, W., thesis submitted to the University of Munich (1974).
16. Bugg, D.V., Zou, B.S., and Sarantsev, A.V., *Nucl. Phys.* B471, 59-89 (1996).
17. Hayne, C., and Isgur, N., *Phys. Rev.* D25, 1944-1950 (1982);
   Schuler, G.A., Berends, F.A., and van Gulik, R., *Nucl. Phys.* B523, 423-438 (1998).
18. Pennington, M.R., *Proc. Int. Conf. on the Structure and Interactions of the Photon* (Freiburg, May 1999), ed. S. Söldner-Rembold, *Nucl. Phys. B (Proc. Suppl.)* 82, 291-299 (2000).
19. Li, Z.P., Close, F.E., and Barnes, T., *Phys. Rev.* D43, 2161-2170 (1991).
20. Baltrusaitis, R.M. et al. (Mark III), *Phys. Rev. Lett.* 56, 107-110 (1986);
    Bai, J.Z., et al. (BES), *Phys. Rev. Lett.* 76, 3502-3505 (1996).
21. Chao, K.T., *Commun. Theor. Phys.* 24, 373-376 (1995);
    Huang, T., et al., *Phys. Lett.* B380, 189-192 (1996); Paar, H.P., *Nucl. Phys. B (Proc. Suppl.)* 82, 337-343 (2000).