Production and anisotropic flow of thermal photons in most-central $\alpha$-clustered C+Au collisions at BNL Relativistic Heavy Ion Collider

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The presence of $\alpha$-clustered structure in the light nuclei produces different exotic shapes in nuclear structure studies at low energies. Recent phenomenological studies suggest that collision of heavy nuclei with $\alpha$-clustered carbon ($^{12}$C) at relativistic energies can lead to large initial state anisotropies. This is expected to impact the final momentum anisotropies of the produced particles significantly. The emission of electromagnetic radiations is considered to be more sensitive to the initial state as compared to hadronic observables and thus photon observables are expected to be affected by the initial clustered structure profoundly. In this work we estimate the production and anisotropic flow of photons from most-central collisions of triangular $\alpha$-clustered carbon and gold at $\sqrt{s_{NN}} = 200$ GeV at RHIC using an event-by-event hydrodynamic framework and compare the results with those obtained from unclustered carbon and gold collisions. We show that the thermal photon $v_2$ for most central collisions is significantly large for the clustered case compared to the case with unclustered carbon, whereas the elliptic flow parameter does not show much difference for the two cases. In addition, the ratio of anisotropic flow coefficients is found to be a potential observable to constrain the initial state produced in relativistic heavy-ion collisions and also to know more about the $\alpha$-clustered structure in carbon nucleus.

I. INTRODUCTION

Direct photon are considered as one of the cleanest probes to study the initial state and the evolution of the hot and dense matter produced in relativistic heavy-ion collisions. Various properties of the direct photon spectra and anisotropic flow parameters have been explored in detail in past two decades [11-10]. The most salient feature of the photon observables is their strong sensitivity to the initial conditions. Different collision geometries, initial-state fluctuations, inclusion of initial state nucleon shadowing as well as slight variation of initial parameters in the model calculation have been found to affect the anisotropic flow parameters of photons significantly [11-17]. Thus, the study of direct photons from relativistic nuclear collisions offers an excellent opportunity to explore the hot and dense initial stage of Quark-Gluon Plasma (QGP) produced in those collisions. However, it is important to mention that the inconsistency between the experimental photon anisotropic flow data and the results from theory calculations has been a subject of research for quite some time [18-19]. Realistic corrections in the initial conditions, upgraded hydrodynamical framework and modified rates of thermal photon production have been found to improve the theoretical calculation significantly [20-24].

In recent times, it has been shown that a large anisotropic flow of charged hadrons can appear even in small collision systems such as p + p, p + A, d + A, He + A etc. [25-27]. Significant initial-state anisotropies play a pivotal role in building up large final-state anisotropies for the small systems. Recent interesting studies have suggested that the geometric effects of $\alpha$-clustering in the light nuclei ($^7$B, $^{12}$C, $^{16}$O etc.) can also be realized in the realm of relativistic nuclear collisions [28-32]. Clustering has long been known to play a key role in understanding the structure of light nuclei. Over the past half century, complex clusters of light nuclei have been discovered, especially for the typical $3\alpha$-clustered and $4\alpha$-clustered structures in $^{12}$C and $^{16}$O [33-38]. Many theories, such as AMD [39], FMD [40], THSR [41, 42], and effective field-theory lattice calculations [43] attempt to explore possible geometries and even non-rigid structures of light nuclei. At present, the theoretical elucidation of the alpha structure in $^{12}$C and $^{16}$O is still a hard problem. Therefore, it is desirable to study the problem with some experimental probes [44-48]. Thus, experiments incorporating carbon and oxygen at the relativistic colliders can be useful to shed light on the clustered structures. Studies based on kinetic theory and hydrodynamic model simulations have shown definitive and significant effects of $\alpha$-clustering on the anisotropic flow of hadrons lately [28-32]. The effect of $\alpha$-clustering on the direct photon signals in C + Au collisions at 200A GeV at Relativistic Heavy Ion Collider (RHIC) was first studied in Ref. [17]. It has been shown that specific orientations of the $\alpha$-clustered C+Au collisions give rise to larger triangular flow ($v_3$) or elliptic flow ($v_2$) parameters consider-
ing a smooth initial energy density distribution \[17\]. In this work, we use a more realistic hydrodynamic framework to study the effect of clustered structure on different photon anisotropic flow \((v_1, v_2\) and \(v_3\)) observables in the most-central collision scenario.

The paper is organized as follows. In the next section, we briefly discuss the initial parameters and the framework for the model calculation. In section III, we discuss the results of thermal photon spectra and anisotropic flow coefficients and finally, we summarize the results in section IV.

II. FRAMEWORK

In this study, we adapt a similar procedure used in Refs. \([17, 29]\) to prepare the initial conditions for \(\alpha\)-clustered C+Au collisions at 200\(A\) GeV at RHIC. We consider a triangle-shaped \(\alpha\)-clustered carbon where the vertices of 3 \(\alpha\)-clusters reside at 3 corners of an equilateral triangle. The side length (\(l\)) of the triangle is 3.05 fm and the radius of each cluster (\(r_\alpha\)) is 0.96 fm. The nuclear density distribution corresponding to each cluster follows:

\[
f_i(\vec{r}) = A \exp\left(-\frac{3}{2} \frac{(\vec{r} - \vec{r}_i)^2}{r_\alpha^2}\right),
\]

where, \(\vec{r}_i\) denotes the position of the center of \(i\)th cluster in carbon. For the unclustered case, we take a 2-parameter Wood-Saxon density profile in such a way that the root mean square radius of the unclustered carbon is similar to the clustered carbon (\(\sim 2.26\) fm).

A two-component Monte Carlo Glauber(MCG) model framework is used to distribute the initial entropy density on the transverse plane for each event where entropy density at any transverse coordinate \((x, y)\) is obtained using the following relation:

\[
s(x, y) = K \sum_{i,j=1}^{N_{part},N_{coll}} [\nu n_{coll}(x_i, y_i) F_i(x, y) + (1 - \nu) n_{part}(x_j, y_j) F_j(x, y)].
\]

In the above equation, \(n_{coll}\) and \(n_{part}\) denote the number of binary collision and participant sources at the \((x_i, y_i)\) and \((x_j, y_j)\) positions, respectively. The values of nucleon-nucleon inelastic cross section \((\sigma_{NN})\), hardness factor (\(\nu\)) and the normalization constant \((K)\) are taken as 42 mb, 0.145 and 81 fm\(^{-2}\), respectively. The function \(F_i(x, y)\) or \(F_j(x, y)\) is a normalized Gaussian distribution centering about the \(i\)th collision (or \(j\)th participant) source,

\[
F_{i,j}(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x-x_i)^2 + (y-y_j)^2}{2\sigma^2}},
\]

where the Gaussian smearing width (\(\sigma\)) of radius around each collision and participant source is considered as 0.4 fm \([17]\).

The initial conditions are subsequently evolved with a (2+1) dimensional longitudinally boost invariant ideal relativistic hydrodynamic framework \([49]\) to obtain the space-time evolution at mid-rapidity. The value of initial thermalization time is taken as \(\tau_0 = 0.17\) fm/c and the transverse components of initial flow velocities (i.e., \(v_x\) and \(v_y\)) are neglected. A lattice based equation of state \([54]\) is used in the hydrodynamic model and the constant freeze-out temperature \((T_f)\) is taken as 160 MeV.

The thermal photon production from individual events is estimated by integrating the emission rates (i.e., \(R = \frac{dN}{d^3p}\)) over entire space-time evolution:

\[
E \frac{dN}{d^3p} = \int R(E^\ast(x), T(x)) d^4x
\]

The \(T(x)\) in the above equation is the local temperature and \(E^\ast(x) = p^\mu u_\mu(x)\), where \(p^\mu\) represents the four-momentum of photon and \(u_\mu\) is the local 4-velocity of the flow field. We use the complete next-to-leading order emission rates from Refs. \([51, 52]\) to evaluate photon production from the QGP phase and the parameterized rates from Ref. \([53]\) for the hadronic sector.

The differential anisotropic flow coefficients \((v_n(p_T))\) where \(n = 1, 2, \) and 3) for each event are obtained as:

\[
v_n(p_T) = \frac{\int_0^{2\pi} d\phi d\psi dN}{\int_0^{2\pi} d\phi dN} \frac{1}{p_T^4} \frac{dN}{dp_T dy d\phi d\psi},
\]

where, \(\phi\) is the azimuthal angle of particle’s momentum and \(\psi_n\) is the participant plane angle. We calculate the event-averaged final flow observables \(\langle v_n\rangle\), which we denote hereafter as \(v_n\) for simplicity) by using the following equation,

\[
\langle v_n(p_T) \rangle = \frac{\sum_{i=1}^{N_{events}} dN^{(i)}(x,y) v_n^{(i)}(p_T)}{\sum_{i=1}^{N_{events}} dN^{(i)}(x,y)},
\]

In the above equations, the superscript ‘\(i\)’ corresponds to the \(i\)th event. The participant plane angle is determined by:

\[
\psi_n = \frac{1}{n} \arctan \frac{\int dxdy r^2 \sin (n\Phi) \epsilon(x, y, \tau_0)}{\int dxdy r^2 \cos (n\Phi) \epsilon(x, y, \tau_0)} + \pi/n, \quad (T)
\]

The initial-state eccentricities \((\varepsilon_n)\) where \(n = 1, 2, \) and 3) for each event are obtained using the relation,

\[
\varepsilon_n = -\frac{\int dxdy r^2 \cos (n(\Phi - \psi_n)) \epsilon(x, y, \tau_0)}{\int dxdy r^2 \epsilon(x, y, \tau_0)},
\]

where \(\Phi\) and \(r\) are spatial azimuthal angle and the radial distance, and \(\epsilon\) is the energy density on the transverse plane.


III. PRODUCTION AND ANISOTROPIC FLOW OF THERMAL PHOTONS

In the present study, we focus on the most central collisions where more geometry-dominated effects are expected to be seen. As presently we do not have any knowledge about the minimum bias event distribution of charged hadrons from hydrodynamic simulations, we have chosen events with $N_{\text{part}} > 80$ for both the clustered and unclustered cases to understand the effects in most-central collision scenario. It should be noted that such an event selection criterion in the Glauber model calculations with GLISSANDO refers to almost similar centrality classes ($\approx 0 - 1\%$) for both the clustered and unclustered cases. To study the effect of initial clustered structure on photon observables, we have considered sufficiently large number of random events for both clustered and unclustered cases with the event selection criterion $N_{\text{part}} > 80$.

In Fig. 1 we show the $N_{\text{part}}$ dependent behavior of clustered to unclustered ratio of the event-averaged (over 500000 events) initial elliptic ($\langle \varepsilon_2^E \rangle / \langle \varepsilon_2^U \rangle$) and triangular eccentricities ($\langle \varepsilon_3^C \rangle / \langle \varepsilon_3^U \rangle$). We find that the ratio of elliptic eccentricities at various $N_{\text{part}}$ are slightly less than 1, whereas the the ratio for triangular eccentricities varies in the range 1.6 - 2.0 as a function of $N_{\text{part}}$. This clearly shows that the triangular anisotropies for the clustered case shows more than 50% rise than the unclustered case for most central collisions.

The event distribution of the $\alpha$-clustered C+Au and the unclustered C+Au collisions in the $v_n - \varepsilon_n$ plane is shown in Fig. 2. The integrated $v_n$ is obtained by integrating the differential thermal photon flow over the $p_T$ range 0.5 - 6.0 GeV. The linear correlation (Pearson correlation) coefficient $C_n(\varepsilon_n, v_n)$ for both clustered and unclustered cases is shown for a comparison. The correlation coefficient is defined as:

$$
C_n(\varepsilon_n, v_n) = \left( \frac{\langle (\varepsilon_n - \langle \varepsilon_n \rangle)(v_n - \langle v_n \rangle) \rangle}{\sigma_{\varepsilon_n} \sigma_{v_n}} \right),
$$

(9)

where the quantities without (with) angular bracket denote single event (event-averaged) values, $\sigma_{\varepsilon_n}$ and $\sigma_{v_n}$ are the standard deviations of $\varepsilon_n$ and $v_n$, respectively.

To calculate the average of $v_n$, we use the integrated photon yield as weight, whereas, the total deposited energy on the transverse plane has been considered as weight for calculating average of initial state anisotropy. We see that the correlation coefficient between $v_2$ and $\varepsilon_2$ for the clustered and unclustered cases is similar (about 0.65). On the other hand, we see a stronger linear correlation (coefficient $\sim 0.82$) in $v_3 - \varepsilon_3$ plane for the clustered carbon compared to the unclustered case (coefficient $\sim 0.65$).

In Fig. 3, we show a comparison of the event-averaged thermal photon spectra from $\alpha$-clustered and unclustered C+Au collisions. The spectra for both the cases are found to be close to each other. In the region $p_T > 4$ GeV, we observe a slight excess of photon production for the clustered case in comparison to the unclustered case which perhaps occurs due to the presence of initial
flow anisotropy for the clustered and unclustered cases is found to be close to each other which is consistent with the average initial elliptic eccentricities obtained for the respective cases. We see a slightly larger elliptic flow at $p_T \sim 2$ GeV for the unclustered case in comparison to the clustered case.

However, $v_3$ for the clustered case is found to be twice as large as the same obtained for the unclustered case. It is to be noted that the orientation averaged triangular flow parameter for the clustered case is still significantly large and similar to the $v_3(p_T)$ obtained from most-central ($b \approx 0$ fm) collisions of $\alpha$-clustered carbon with Au nucleus at an orientation angle $\theta = \pi/4$ (see Fig. 5(a) of Ref. [17]) considering smooth initial density distribution. Additionally, such large triangular flow anisotropy is also comparable to the direct photon $v_3$ data obtained for 20 – 40% Au+Au collisions at RHIC. It is to be noted that a larger initial thermalization time $\tau_0$ or a smaller freeze-out temperature $(T_f)$ would further increase the value of thermal photon $v_3$ as discussed in the Ref. [17]. These results clearly state that photon $v_3$ in relativistic nuclear collisions can efficiently reflect the initial state triangular anisotropy associated with the triangular $\alpha$-cluster structure in carbon nucleus. It is also well known that a significant contribution of the prompt photons appears in the region $p_T > 3$ GeV [17] in the direct photon spectrum and it dominates over the thermal radiation in that $p_T$ range. However, the prompt photons do not contribute directly to the anisotropic flow. These non-thermal photons only dilute the flow parameters in the larger $p_T$ region by adding extra weight in the denominator of Eq. 5. One can still expect to get a large direct photon $v_3^{dir}(p_T)$ after including the non-thermal prompt contribution in the calculation [17].

In a recent study, it has been emphasized that the ratio of anisotropic flow coefficients ($v_n/v_m$) can mini-
mize the uncertainties arising due to the non-thermal contributions [55] in the photon anisotropic flow calculation. The ratio is shown to be a potential observable to probe the thermal phase contribution in the direct photon anisotropic flow. Although the individual photon flow parameters ($v_2$, $v_3$) found to underestimate the experimental $v_n$ data at RHIC, the photon $v_2/v_3$ is found to be close to the PHENIX data in the $p_T$ region $2-3.5$ GeV, which is believed to be dominated by the thermal radiation. The ratio is also found to be sensitive to the initial conditions of the model calculation in different $p_T$ regions compared to the individual flow parameters. The ratio of directed flow parameter with photon $v_2$ (or $v_3$) (along with the individual photon anisotropic flow parameters) provides additional information to constrain the initial parameters of the model calculation. We show the ratio of thermal photon $v_2$ to $v_3$ as a function of $p_T$ in Fig. 5. The dashed line shows the ratio for the unclustered case which is found to be about 2 at $p_T \sim 1$ GeV and above $p_T > 3$ GeV the ratio gets closer to 1. However, for the clustered case the ratio (solid line) is found to be smaller than 1 in the region $p_T > 1$ GeV, which in turn indicates a significantly larger thermal photon $v_3$ compared to the thermal photon $v_2$. We show the ratio of PHENIX direct photon $v_2$ and $v_3$ data from $0-20\%$ Au+Au collisions at RHIC (solid circles) on the same plot for a comparison [18]. The ratio for the central Au+Au collisions is found to be closer to the result from unclustered C+Au collisions whereas the ratio from clustered case is significantly smaller in the thermal ($2-4$ GeV) $p_T$ range. Thus, experimental determination of photon $v_2/v_3$ ratio from C+Au collisions can be an important observable to identify the clustered structure in carbon nucleus.

The directed flow of thermal photons from C+Au collisions is shown as a function of $p_T$ in Fig. 6(a). The photon $v_1$ for both clustered and unclustered cases is found to be close to each other. This observation is consistent with the directed initial eccentricities obtained for the two cases. We show the ratio of the directed flow of thermal photons with triangular flow in Fig. 6(b). The ratio is seen to be different in the region $p_T < 2$ GeV where the ratio for the clustered case is found to be larger than the unclustered case. In the region $p_T > 2$ GeV an opposite behavior is observed, however, the difference between the two is relatively smaller. The $v_1/v_3$ has been shown to be more sensitive to the hadronic phase unlike $v_2/v_3$ of photons [55]. Thus, ratio of photon $v_n$ can also be valuable to know about the freeze-out temperature of the evolving fireball.

Therefore, this study overall provides a qualitative understanding of thermal photon production and anisotropic flow from most-central $\alpha$-clustered C+Au collisions at RHIC. A more realistic calculation requires a 3+1 dimensional viscous hydrodynamic framework which, however, is expected to deliver a similar qualitative nature of photon flow observables as obtained from the present study.

IV. SUMMARY

We calculate the production and anisotropic flow of thermal photons from most-central $\alpha$-clustered C and Au collisions at RHIC using an event-by-event hydrodynamic framework and compare the results with those obtained from unclustered carbon and gold collisions. The slope of the thermal photon spectrum from the clustered carbon is found to be similar to the same obtained from the unclustered carbon. However, the clustered structure affects the initial triangular eccentricity as well as the triangular flow parameter of photons significantly compared to the unclustered carbon and gold collisions. Although we find a similar thermal photon $v_2$ for both clustered and unclustered carbon collisions, the thermal photon $v_3$ for the clustered case is found to be almost twice as large as the unclustered case.

In addition, we show that the ratio of photon anisotropic flow parameters ($v_2/v_3$) can be a useful observable to recognize the clustered structure in carbon nucleus. The $v_2/v_3$ ratio is found to be strongly sensitive
to the α-cluster structure and is significantly suppressed compared to the unclustered case. We also show that directed flow parameter of photons can be a potential observable along with the elliptic and triangular flow parameters to constrain the initial state produced in heavy ion collision. We conclude that photon flow observables are potential probes to detect the α-cluster structure in carbon nucleus and experimental determination of the anisotropic flow parameters from C+Au collisions can be useful to study the initial state in relativistic heavy ion collisions.

Acknowledgments

PD very much appreciates the generous help from Dr. Han-Sheng Wang and thanks the Fudan cluster computer facilities at VECC. GM thanks Prof. Bo Zhou for helpful discussions. This work is supported by the National Natural Science Foundation of China under Grants No. 12150410303 , No.12147101, No. 11890714, No. 11835002, No. 11961131011, No. 11421505, the Strategic Priority Research Program of Chinese Academy of Sciences under Grant No. XDB34030000, and the Guangdong Major Project of Basic and Applied Basic Research under Grant No. 2020B0301030008 (G. M. and P. D.).
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