Master-Slave Locking of Optomechanical Oscillators Over A Long Distance

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Frequency-locking and other phenomena emerging from nonlinear interactions between mechanical oscillators are of scientific and technological importance. However, existing schemes to observe such behaviour are not scalable over distance. We demonstrate a scheme to couple two independent mechanical oscillators, separated in frequency by 80kHz and situated far from each other (3.2km), via light. Using light as the coupling medium enables this scheme to have low loss and be extended over long distances. This scheme is reversible and can be generalised for arbitrary network configurations.

Recent reports [5, 14, 15] on frequency-locking between micromechanical oscillators are of scientific and technological importance. However, existing schemes to observe such behaviour are not scalable over distance. We demonstrate a scheme to couple two independent mechanical oscillators, separated in frequency by 80kHz and situated far from each other (3.2km), via light. Using light as the coupling medium enables this scheme to have low loss and be extended over long distances. This scheme is reversible and can be generalised for arbitrary network configurations.

In this paper, we demonstrate a reconfigurable scheme to couple, via light, two independent micromechanical oscillators separated from each other by an effective path of 3.2km, in the master-slave configuration and show the ability to lock their oscillation frequencies. This coupling scheme is based on using light to send the information of the mechanical oscillations from the master oscillator to the slave oscillator. It is facilitated by the fact that each oscillator is an an optomechanical oscillator (OMO), consisting of co-localised optical resonances and mechanical resonances that are coupled to each other (Eqs. [1a][1b] [16]). The mechanical resonator can be modelled as a damped simple harmonic oscillator with position ‘x’, effective mass $m_{eff}$, frequency $\Omega_m$ and damping rate $\Gamma_m$. It is driven by its interaction with an optical force $F_{opt} = g_{om}|a|^2$, where $|a|^2$ is the energy in the optical cavity and $\omega$ is the laser frequency. $g_{om}$ indicates the strength of the interaction between optics and mechanics. The optical cavity can also be modelled as a damped oscillator, with a position-dependent frequency ($\omega_0 + g_{om}x$) and damping rate $\Gamma_{opt}$, and it is driven with a laser of power $|s|^2$ coupled to the cavity at the rate $\Gamma_{ex}$. The force on the mechanical resonator $F_{opt}$ can be controlled by changing the intracavity energy $|a|^2$, which is, in turn, affected by the laser power $|s|^2$. Any modulation of the laser power therefore couples to the mechanical resonator via the optical force $F_{opt}$ [17].

\[
\frac{da}{dt} = i((\omega - \omega_0) - g_{om}x)a - \Gamma_{opt}a + \sqrt{2\Gamma_{ex}}s \quad (1a)
\]

\[
\frac{d^2x}{dt^2} + \Gamma_m \frac{dx}{dt} + \Omega_x^2x = \frac{F_{opt}|a|}{m_{eff}} \quad (1b)
\]

The OMOs used for this demonstration each consist of two suspended Si$_3$N$_4$ microdisks stacked vertically (Figs. 1 (a), (b)). The optical and mechanical resonances under consideration are co-localised along the periphery of the structure. These structures are fabricated using e-beam lithography techniques [13]. The top and bottom Si$_3$N$_4$ disks are nominally 250nm and 220nm thick and have a radius of 20µm. These disks are separated from each other by a 170nm thick SiO$_2$ sacrificial spacer layer. This stack rests on a 4µm thick SiO$_2$ support layer. These layers are partially etched away to release the periphery of these disks. This suspended structure supports optical whispering-gallery modes that are overlap with the edges of the top and bottom disks (Fig. 1(a)) [14].
resonance frequency of this structure is strongly dependent on the separation between the two disks. Relative motion (represented by Eq. 1(b)) between the two disks changes the resonance frequency at the rate of $g_{om} = -2\pi \cdot 49\text{GHz/nm}$, as calculated from finite element simulations [14].

The two devices, when not coupled, oscillate at two distinct mechanical frequencies separated by 80kHz. In order to characterise the devices, light is coupled into each resonator with a tapered optical fiber. The transmission spectrum of the master OM resonator shows an optical resonance centered at $\sim 1565.22\text{nm}$ (Fig. 1(c)). Similarly, the slave OM resonator has an optical resonance centered at $\sim 1565.95\text{nm}$ (Fig. 1(c)). The splitting in the resonance is due to back-scattering induced lifting of degeneracy between the clockwise and counterclockwise propagating modes [18]. Thermal motion of the mechanical resonators modulates this transmission spectrum (Fig. 1(d)), which can be analysed with a spectrum analyser. The master is observed to have a mechanical resonance at 33.93MHz (Fig. 1(e)), with a linewidth of 16.39kHz, while the slave has a mechanical resonance centred at 32.82MHz (Fig. 1(e)), with a linewidth of 13.56kHz. When the optical resonances are excited with blue-detuned lasers ($\omega > \omega_0$), dynamical backaction [16] amplifies mechanical motion. As input power is increased, this mechanical gain increases, until it overcomes intrinsic mechanical damping. At this point, each resonator becomes a self-sustaining oscillator [16].

The master oscillates at 32.99MHz (Fig. 1(d)), and the slave oscillates independently at 32.91MHz (Fig. 1(d)), i.e. separated from the master by more than six times its natural mechanical linewidth. Note that the oscillation frequencies for the oscillators are centred at a frequency slightly higher than that for the thermal motion of the respective resonator, due to the optical-spring effect [16].

To demonstrate long-distance locking, we couple the two OMOs in a master-slave configuration, via a 3.2km long optical fiber, with an electro-optic modulator that is driven by the master OMO and that modulates the laser driving the slave OMO (Fig. 2, Eq. 2). Each OMO is pumped by an independent laser. The signal transmitted from the master OMO carries information about its position $x_{master}$. It travels through a 3.2km long delay line before it is detected with a high-speed detector. The output of this detector carries the radio frequency (RF) oscillations, which are a function of the mechanical displacement $x_{master}$ of the master. The slave laser drive $s_{slave}$ is modulated by this signal from the master (Eq. 2). The output of the slave OMO is detected with another high-speed detector and analysed with a spectrum analyser and an oscilloscope.

$$s_{slave} = s_{0, slave} + \gamma f(x_{master})$$  \hspace{1cm} (2)

The strength of coupling between the slave OMO and
Figure 2. Schematic of experimental setup to demonstrate master-slave locking. The two optomechanical (OM) resonators are driven by independent lasers. The optical signal from the master travels through 3.2km of fiber. The RF signal generated at the detector by the oscillations of the master modulate the laser driving the slave. The RF oscillations of the slave are analysed with a spectrum analyser and an oscilloscope.

As we increase the coupling strength, we show that the slave OMO transitions from oscillating independently to being frequency-locked to the master OMO. The coupling strength, determined by the modulation depth \( \gamma \) of the electro-optic modulator driven by the master-oscillator. A voltage-controlled variable gain amplifier provides a gain between -26dB and +35dB to the RF oscillations coming from the detector of the master OMO, and thereby controls the modulation depth. This is reflected in the power spectral density (PSD) of oscillation peak of the master OMO (\( H_{inj} \)) as seen in the light transmitted from the slave OMO (Fig. 3(a)).
Figure 3. (a) Spectrum of the power transmitted from the slave OMO for different injection ratios ($H_{\text{inj}}/H_{\text{slave}}$). (b) Numerical simulation of the power spectrum. (c), (d) Same as (a) and (b), respectively, only now measured by reversing the roles of master and slave.

Figure 4. Phase-portraits formed by the oscillation signals of the (a) free-running slave and (b) locked slave with the master oscillator, as measured with an oscilloscope, over more than 130 oscillation cycles. (Insets) Simulated phase-portraits
tune the coupling strength arbitrarily enables access to various regimes of nonlinear dynamics of such oscillator networks.

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