An alternative quality of life ranking on the basis of remittances

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Abstract

Remittances mean an important connection between people working abroad and their home countries. This paper considers them as a measure of preferences revealed by workers, underlying a ranking of countries around the world. We use the World Bank bilateral remittances data between 2010 and 2015 to compare European countries. The database contains international salaries and interpersonal transfers. The suggested least squares method makes the ranking invariant to country sizes and satisfies the property of bridge country independence. Our ranking reveals a crucial aspect of quality of life and may become an alternative to various composite indices.

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1 Introduction

Country rankings seem to be increasingly popular in economics and can often have huge impacts on politicians and government strategies. Yet most of them are composite indices, a construction which remains highly controversial due to the arbitrary selection of criteria and component weights (Ravallion, 2012a). While robustness check may provide a kind of remedy (Foster et al., 2013), there exists an alternative solution, that is, to apply a parameter-free algorithm on an appropriate dataset.

It is widely argued that people and their capabilities should be the ultimate criteria for assessing the development of a country, not economic indicators alone. One way to measure the perceptions of people is to observe their decisions on important questions of life such as working abroad. We use bilateral remittances for this purpose. A remittance
is a transfer of money by a foreign worker to an individual in their home country. Remittances constitute a significant part of international capital flows, especially in the case of labour-exporting countries. Using personal remittances to rank countries is not unique (see, for example, IndexMundi).

Our dataset contains estimates of bilateral remittances by the World Bank, based on migrant stocks, host country incomes, and origin country incomes (World Bank, 2017). They are not officially reported data since bilateral remittance flows are not readily available. The estimation uses the methodology of Ratha and Shaw (2007), who have devised a simple formula to allocate the recorded remittances received by each country to the source countries. This applies a remittance function assuming that the amount sent by an average worker increases with the migrant’s income but at a decreasing rate. Furthermore, in the case of migration to a country where the per capita income is lower than in the host country, the transfer is supposed to be at least as much as the per capita income of the origin country.

Consequently, there are some caveats attached to these estimates because: (1) the data on migrants in various destination countries are incomplete; (2) the incomes of migrants and the costs of living are proxied by per capita incomes in PPP terms; and (3) there is no way to capture remittances flowing through informal, unrecorded channels. Nevertheless, it seems to be the best database available on international remittances.

The paper is structured as follows. Section 2 outlines the theoretical background of our calculations. The alternative quality of life rankings are presented in Section 3, while Section 4 summarizes the main findings.

2 Methodology

We consider one unit of money transferred from a country to another country as the former is preferred over the latter by one voter, and use techniques from social choice theory to evaluate these "votes". Let us introduce the matrix $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ of bilateral remittances among $n$ countries such that $a_{ij}$ is the sum of transfers from country $i$ to country $j$ in the given period. It immediately determines the skew-symmetric results matrix $R = A - A^\top$ and the symmetric matches matrix $M = A + A^\top$.

Perhaps the simplest measure is to calculate the net remittances (the difference of total outflow and total inflow), denote it by $s_i = \sum_{j=1}^{n} r_{ij}$ for each country $i$. By dividing these amounts with the total remittance flow (the sum of total outflow and total inflow) $\sum_{j=1}^{n} m_{ij}$ of the country, one gets $p_i$.

Finally, the least squares method adjusts net remittances by taking the whole network of remittance flows into account. The least squares weight $q_i$ of country $i$ can be obtained as the solution of the following optimization problem:

$$\min_{q \in \mathbb{R}^n} \sum_{1 \leq i,j \leq n} m_{ij} \left( \frac{r_{ij}}{m_{ij}} - q_i + q_j \right)^2.$$  

(1)

The first order conditions of optimality lead to a linear equation for each country $i$:

$$\left( \sum_{j=1}^{n} m_{ij} \right) q_i - \sum_{j=1}^{n} m_{ij} q_j = s_i = \sum_{j=1}^{n} r_{ij}.$$  

(2)

After normalizing the weights by $\sum_{i=1}^{n} q_i = 0$, the solution of this system becomes unique if the countries are connected at least indirectly by transfers, that is, the multigraph of
bilateral remittances is connected (Čaklović and Kurdija, 2017). Our dataset has satisfied this condition every year.

All of the weight vectors above determine a ranking \( \succeq \) of the countries. For example, the ranking from the least squares method is given by \( q_i \geq q_j \iff i \succeq j \).

For the choice of methodology, two axiomatic properties have been considered that the derived ranking should satisfy. First, it is required to be invariant to country sizes, that is, countries \( i \) and \( j \) should get the same rank if country \( j \) has a fixed proportion of transfers to and from every third country as country \( i \). Second, it should be independent of bridge country, namely, if a hypothetical world consists of two sets of countries connected only by a specific country called bridge country, then the relative rankings within each set of countries are not influenced by the remittances among the countries in the other set (González-Díaz et al., 2014).

Ranking by net remittances violates these properties. The ranking derived from vector \( p \) is invariant to country size but does not meet bridge country independence. The least squares method satisfies both axioms, therefore we suggest to apply this procedure.

Note that the least squares method is equivalent to the Potential Method (Čaklović and Kurdija, 2017), to the EKS (Éltető–Kőves–Szulc) method used for international price comparisons by the OECD (Éltető and Kőves, 1964; Szulc, 1964), and to the Logarithmic Least Squares Method defined in the framework of multiplicative pairwise comparison matrices (Bozóki et al., 2010; Bozóki and Tsyganok, 2017).

Its axiomatic properties are discussed by González-Díaz et al. (2014), while Csató (2015) gives a graph interpretation of the least squares method. Csató (2018a) and Csató (2018b) provide characterizations of the procedure on a more restricted domain. Finally, Csató and Rónyai (2016) present a potential failure of it, however, Csató (2018c) and Csató (2018d) prove that there is no hope to find a perfect ranking method in the case of such complex problems.

The least squares method has been recently applied for ranking the teams in a Swiss system chess team tournament (Csató, 2013, 2017), the Hungarian universities on the basis of applicants’ preferences (Csató, 2016; Csató and Tóth, 2018), the participating countries of the Eurovision Song Contest (Čaklović and Kurdija, 2017), as well as for the comparison of top historical players in Go (Chao et al., 2018) and tennis (Bozóki et al., 2016).

As an illustration, consider an example with four countries.

**Figure 1:** Remittances between the four countries in Example 2.1

![Diagram](image)

**Example 2.1.** Consider the four countries shown in Figure 1: the directed edges represent the direction of the remittances, and their weights correspond to the amount of the
transfer. The remittances, results, and matches matrices are as follows:

\[
A = \begin{bmatrix} 0 & 0 & 15 & 0 \\
0 & 0 & 30 & 0 \\
5 & 10 & 0 & 10 \\
0 & 0 & 10 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} 0 & 0 & 10 & 0 \\
0 & 0 & 20 & 0 \\
-10 & -20 & 0 & 0 \\
0 & 0 & 0 & 0 \end{bmatrix}, \quad \text{and} \quad M = \begin{bmatrix} 0 & 0 & 20 & 0 \\
0 & 0 & 40 & 0 \\
20 & 40 & 0 & 20 \\
0 & 0 & 20 & 0 \end{bmatrix}.
\]

Remittances to and from country \( B \) are the same as remittances to and from country \( A \), so size invariance implies the same rank for these countries. \( C \) is a bridge country between the sets \{\( A; B; C \)\} and \{\( C; D \)\}, that is, the relative ranking within the second set is determined only by the transfers between countries \( C \) and \( D \), and it makes no sense to rank one of them above the other.

Table 1: The weight vectors of Example 2.1

| Country | \( s(A) \) | \( p(A) \) | \( q(A) \) |
|---------|-----------|-----------|-----------|
| \( A \) | 10        | 1/2       | 1/4       |
| \( B \) | 20        | 1/2       | 1/4       |
| \( C \) | -30       | -3/8      | -1/4      |
| \( D \) | 0         | 0         | -1/4      |

The weights according to the three methods are shown in Table 1. The vector \( s \) of net remittances violate the two properties suggested above, the ratio of the net and total remittances \( p \) satisfies size invariance, but does not meet bridge country independence, while the least squares method \( q \) meets both requirements.

3 Some alternative quality of life rankings

In the following, the results of our calculations with the methodology suggested in Section 2 are presented.

3.1 The ranking in 2015

Table 2 presents the ranking of the European countries with these three methods based on the 2015 data such that all non-European countries are regarded as one entity. A lighter colour indicates a worse rank. The largest difference between the ranking from \( s \) and \( q \) is in the case of France. Net remittances place it to the 37th position, while the least squares method to the 17th. The reason is the size effect: France is one of the largest countries in Europe, so is natural that both inflow and outflow are huge (more than 20 billion USD), which implicates the difference is also great (−2482 million USD). In this case, Albania precedes France with net remittances of −852 million USD, but with almost five times higher inflow than outflow.

It can be realized from formula (2) that \( q_i \) is close to the size-invariant ratio \( p_i = s_i/(\sum_{j=1}^n m_{ij}) \) if the weights of the countries connected to country \( i \) by remittances are close to the average weight of 0. On the other hand, \( q_i \) becomes higher (lower) than this ratio if country \( i \) is mainly connected to higher (lower) ranked countries by the transfers. Difference between the size-invariant ratio and the least squares method is the largest in the case of Croatia, Lithuania, and Slovenia. The main destinations for Croatia
Table 2: Ranking of European countries on the basis of remittances in 2015

| Country | s(A) | p(A) | q(A) | Country | s(A) | p(A) | q(A) |
|---------|------|------|------|---------|------|------|------|
| AL      | 25   | 37   | 35   | IS      | 18   | 21   | 19   |
| AT      | 10   | 13   | 16   | IT      | 5    | 9    | 9    |
| BA      | 32   | 41   | 41   | LT      | 28   | 39   | 31   |
| BE      | 39   | 24   | 22   | LU      | 23   | 19   | 21   |
| BG      | 30   | 40   | 39   | LV      | 27   | 34   | 28   |
| BY      | 14   | 14   | 13   | MD      | 29   | 35   | 33   |
| CH      | 4    | 3    | 2    | ME      | 22   | 38   | 40   |
| CY      | 13   | 8    | 6    | MK      | 20   | 28   | 30   |
| CZ      | 24   | 20   | 25   | MT      | 19   | 27   | 23   |
| DE      | 3    | 11   | 12   | NL      | 7    | 2    | 5    |
| DK      | 12   | 12   | 10   | NO      | 8    | 5    | 4    |
| EE      | 21   | 29   | 27   | PL      | 40   | 33   | 29   |
| ES      | 6    | 10   | 11   | PT      | 34   | 26   | 24   |
| FI      | 16   | 15   | 14   | RO      | 36   | 36   | 34   |
| FR      | 37   | 18   | 17   | RS      | 35   | 30   | 37   |
| GB      | 1    | 1    | 1    | RU      | 2    | 7    | 8    |
| GR      | 11   | 6    | 7    | SE      | 15   | 16   | 15   |
| HR      | 26   | 25   | 38   | SI      | 17   | 17   | 26   |
| HU      | 38   | 32   | 32   | SK      | 31   | 31   | 36   |
| IE      | 9    | 5    | 3    | UA      | 33   | 22   | 20   |
| Other   | 41   | 23   | 18   |

are Germany and Serbia, while it receives workers from Bosnia and Herzegovina, Serbia, and Slovenia, so Croatia is mainly connected to lower ranked countries. On the other hand, Lithuania is predominantly connected to some higher ranked countries (the United Kingdom, Russia), which implicates its better rank with \( q \).

3.2 The dynamics of country rankings in recent years

According to Table 3, the least squares ranking is relatively robust across the years and does not yield many unexpected results. The members of the Visegrád Group are around the 30th place, only the Czech Republic shows some improvement in the years 2013 and 2014.

However, there are some counterintuitive findings. Data problems are responsible for the decline in the performance of Iceland and Sweden. The top position of Cyprus can be probably explained by the significant role of its banks in international finance. The United Kingdom leads the ranking in certain years due to its liberal migration policy. We suppose that Russia gains from retaining connections of the Soviet era, as well as from the huge regional inequalities caused by the agglomerations of Moscow and Saint Petersburg.

3.3 Comparison with the Human Development Index (HDI)

The Human Development Index (HDI) is perhaps the most common measure of human development. It is a composite index of life expectancy, education, and per capita income
Table 3: Quality of life rankings by the least squares method

| Country | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | Country | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 |
|---------|------|------|------|------|------|------|---------|------|------|------|------|------|------|
| AL      | 38   | 38   | 37   | 37   | 36   | 35   | IS      | 1    | 1    | 2    | 19   | 21   | 19   |
| AT      | 18   | 18   | 20   | 12   | 13   | 16   | IT      | 11   | 10   | 12   | 8    | 8    | 9    |
| BA      | 41   | 41   | 41   | 41   | 41   | 41   | LT      | 36   | 36   | 35   | 32   | 31   | 31   |
| BE      | 22   | 22   | 22   | 26   | 25   | 22   | LU      | 23   | 23   | 23   | 23   | 23   | 21   |
| BG      | 39   | 39   | 39   | 39   | 39   | 39   | LV      | 24   | 24   | 25   | 20   | 22   | 28   |
| BY      | 15   | 15   | 15   | 14   | 13   | 13   | MD      | 34   | 34   | 34   | 31   | 30   | 33   |
| CH      | 8    | 8    | 8    | 3    | 2    | 2    | ME      | 37   | 37   | 38   | 40   | 40   | 40   |
| CY      | 5    | 4    | 4    | 2    | 7    | 6    | MK      | 32   | 32   | 31   | 36   | 35   | 30   |
| CZ      | 27   | 26   | 26   | 21   | 20   | 25   | MT      | 19   | 19   | 18   | 29   | 34   | 23   |
| DE      | 10   | 11   | 11   | 11   | 11   | 12   | NL      | 7    | 7    | 6    | 5    | 3    | 5    |
| DK      | 12   | 12   | 13   | 10   | 10   | 10   | NO      | 6    | 6    | 7    | 7    | 5    | 4    |
| EE      | 26   | 27   | 27   | 25   | 27   | 27   | PL      | 28   | 28   | 28   | 28   | 28   | 29   |
| ES      | 9    | 9    | 9    | 9    | 11   | 11   | PT      | 20   | 21   | 21   | 27   | 24   | 24   |
| FI      | 17   | 17   | 17   | 16   | 18   | 14   | RO      | 35   | 35   | 36   | 35   | 33   | 34   |
| FR      | 13   | 14   | 16   | 17   | 17   | 17   | RS      | 40   | 40   | 40   | 38   | 38   | 37   |
| GB      | 4    | 5    | 5    | 1    | 1    | 1    | RU      | 14   | 13   | 10   | 6    | 6    | 8    |
| GR      | 21   | 20   | 14   | 13   | 12   | 7    | SE      | 3    | 3    | 3    | 14   | 15   | 15   |
| HR      | 29   | 29   | 29   | 30   | 29   | 38   | SI      | 30   | 31   | 32   | 22   | 26   | 26   |
| HU      | 31   | 30   | 30   | 34   | 32   | 32   | SK      | 33   | 33   | 33   | 33   | 37   | 36   |
| IE      | 2    | 2    | 1    | 4    | 4    | 3    | UA      | 25   | 25   | 24   | 24   | 19   | 20   |
| Other   | 16   | 16   | 19   | 18   | 16   | 18   | Other   | 16   | 16   | 19   | 18   | 16   | 18   |

Figure 2: Comparison of the least squares and HDI rankings in 2015

(a) Least squares ranking from remittances

(b) HDI ranking

indicators. Despite its comprehensive use, HDI has got some criticism (Klugman et al., 2011; Ravallion, 2012b).

It is compared with our alternative quality of life ranking in Figure 2. A darker colour indicates a higher rank. The results are similar for most countries. While the HDI places Norway at the top, the UK remains in a leading position. However, Belarus, Russia, and Ukraine get a significantly better rank with our methodology than shown by the HDI. A possible reason is that we have restricted our analysis to the European countries, and handle all others as one entity. Nevertheless, this bias requires further research.
4 Conclusions

While it is clear that our ranking can be attacked from various aspects, the proposed methodology has some advantages as illustrated by its independence of arbitrary parameter choices and favourable axiomatic properties. It cannot immediately substitute other rankings, but we think that our remittances-based ranking reveals a crucial aspect of quality of life and may become an alternative to various composite indices. Thus this research will hopefully contribute to a better understanding of economic and social development.

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# Appendix

Table 4: List of the European countries and their abbreviations

| Country                  | Abbreviation | Country              | Abbreviation |
|--------------------------|--------------|----------------------|--------------|
| Albania                  | AL           | Lithuania            | LT           |
| Austria                  | AT           | Luxembourg           | LU           |
| Belarus                  | BY           | Macedonia, FYRM      | MK           |
| Belgium                  | BE           | Malta                | MT           |
| Bosnia and Herzegovina   | BA           | Moldova              | MD           |
| Bulgaria                 | BG           | Montenegro           | ME           |
| Croatia                  | HR           | Netherlands          | NL           |
| Cyprus                   | CY           | Norway               | NO           |
| Czech Republic           | CZ           | Poland               | PL           |
| Denmark                  | DK           | Portugal             | PT           |
| Estonia                  | EE           | Romania              | RO           |
| Finland                  | FI           | Russian Federation   | RU           |
| France                   | FR           | Serbia               | RS           |
| Germany                  | DE           | Slovak Republic      | SK           |
| Greece                   | GR           | Slovenia             | SI           |
| Hungary                  | HU           | Spain                | ES           |
| Iceland                  | IS           | Sweden               | SE           |
| Ireland                  | IE           | Switzerland          | CH           |
| Italy                    | IT           | United Kingdom       | GB           |
| Latvia                   | LV           | Ukraine              | UA           |