de Sitter entropy as entanglement

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Abstract

We describe connected timelike codim-2 extremal surfaces stretching between the future and the past boundaries in the static patch coordinatization of de Sitter space. These are analogous to rotated versions of certain surfaces in the AdS black hole. The existence of these surfaces via the dS/CFT framework suggests the speculation that dS\(_4\) is dual to two copies of ghost-like CFTs in a thermofield-double-type entangled state. In studies of entanglement in ghost systems and “ghost-spin” chains, we show that similar entangled states in two copies of ghost-spin ensembles always have positive norm and positive entanglement.

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1 On de Sitter entropy and dS/CFT

de Sitter space is fascinatingly known to possess entropy [1] (reviewed in [2]). In the static patch coordinatization of dS_{d+1} (Figure-1), the Northern/Southern diamonds N/S are static patches with time translation symmetries. de Sitter entropy is
\[ S_{dS_{d+1}} = \frac{l^{d-1}V_{d-1}}{4G_{d+1}} \xrightarrow{d\to 3} \frac{\pi l^2}{G_4} = S_{dS_4}, \]
the area of the cosmological horizon (size l), apparently stemming from degrees of freedom not accessible to observers in regions N/S, for whom the horizons are event horizons.

The natural boundary for dS is future/past timelike infinity I^\pm. It is thus of interest to understand if this entropy can be realized in gauge/gravity duality [3, 4, 5, 6] for dS, or dS/CFT [7, 8, 9], which conjectures dS to be dual to a hypothetical Euclidean non-unitary Conformal Field Theory that lives on I^+, with the dictionary Ψ_{dS} = Z_{CFT} [9]. Ψ_{dS} is the late-time Hartle-Hawking Wavefunction of the Universe with appropriate boundary conditions and Z_{CFT} the dual CFT partition function. The CFT\_d energy-momentum tensor T_{ij} 2-point correlators yield central charges C\_d \sim i^{1-d} \frac{l^{d-1}}{G_{d+1}}\, , effectively analytic continuations from the anti de Sitter case: e.g. for dS_4,
\[ Z_{CFT} = Ψ_{dS} \sim e^{iS_{cl}} \sim e^{-\int_k R_{dS}^2 k^4 \varphi_0 k^0 \varphi_0 + \ldots} \, , \quad \langle O_k O_{k'} \rangle \sim \frac{\delta^2 Z_{CFT}}{\delta \varphi_k \delta \varphi_k'}, \]
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(static) quantum systems with spatial subsystems on a constant time slice, entanglement entropy arises by tracing over the environment. The dual theory here however is Euclidean and spatial, with no intrinsic “time”. Operationally we could define some spatial isometry direction as boundary Euclidean time, then define a subsystem on this slice: this would lead to codim-2 bulk extremal surfaces stretching in the time direction, if they exist (all Euclidean time slices should be equivalent). From the boundary perspective, ghost-like CFTs as [9], [10], might suggest, are expected to have negative norm states/configurations, thus suggesting “negative entanglement”: it is interesting then to ask how a positive quantity like de Sitter entropy might arise.

2 Extremal surfaces

In AdS, surfaces starting at the boundary dip into the radial direction and exhibit turning points where they begin to return to the boundary. In dS, the boundary at $I^+$ is spatial: surfaces dip into the time direction giving a crucial minus sign that ensures that there is no real turning point where the surface starting at $I^+$ begins to turn back towards $I^+$. There are also complex extremal surfaces with turning points, which amount to analytic continuation from the AdS Ryu-Takayanagi surfaces [28]-[31]. While their interpretation is not entirely clear, in dS these have negative area, consistent with (2) for $dS_4/CFT_3$.

Since real surfaces starting at the future boundary $I^+$ sally forth into the bulk without returning, it is interesting to ask if they could instead end at the past boundary $I^-$ [23]. The bulk probability $\Psi^* \Psi_{dS}$ suggests two CFT copies: so such connected extremal surfaces stretching between $I^\pm$ are perhaps expected. Towards studying this, we recast $dS_{d+1}$ as

$$ds^2 = \frac{l^2}{\tau^2} \left( -\frac{d\tau^2}{1-\tau^2} + (1-\tau^2)dw^2 + d\Omega_{d-1}^2 \right), \quad (4)$$

$\tau$ is “bulk” time in the future/past universes $F/P$ with $0 \leq \tau \leq 1$. The horizons at $\tau = 1$ have area [11]. $N/S$ have $1 < \tau \leq \infty$ (with $w$ time). [Figure-1] From the $\tau \sim 0$ asymptotics, the boundary is seen to be Euclidean $R_w \times S^{d-1}$, while local patches are flat.

The scaling $\frac{l^{d-1}}{G_{d+1}}$ of de Sitter entropy suggests codimension-2 surfaces. Likewise, entanglement in the dual theory defined with respect to Euclidean time would suggest bulk surfaces on appropriately defined constant boundary Euclidean time slices of the bulk. Given $w$-translation symmetry and $S^{d-1}$ rotational invariance in [11], we restrict to a $w = const$ surface, or some $S^{d-1}$ equatorial plane (which are all equivalent). For the latter, we expect
extremal surfaces stretching from a subsystem \( \Delta w \times S^{d-2} \) at \( I^+ \) to an equivalent one at \( I^- \). Extremizing the area functional then gives the real surfaces \( w(\tau) \), with \( B^2 = \text{const} \),

\[
\dot{w}^2 \equiv (1 - \tau^2)^2 \left( \frac{dw}{d\tau} \right)^2 = \frac{B^2 \tau^{2d-2}}{1 - \tau^2 + B^2 \tau^{2d-2}},
\]

\( \dot{w} \) in (5) is the \( y \)-derivative, with \( y = \int \frac{d\tau}{1 - \tau^2} \) the “tortoise” coordinate, useful near the horizons. For any finite \( B^2 > 0 \), we have \( \dot{w} \to 0 \) near the boundary \( \tau \to 0 \), with \( \dot{w} < 1 \) for \( \tau < 1 \) (within \( F \)) and \( \dot{w} \to 1 \) as \( \tau \to 1 \). The turning point \( \tau_* \) is the “deepest” location to which the surface dips into in the bulk, before turning around: this is when

\[
|\dot{w}| \to \infty : \quad 1 - \tau_*^2 + B^2 \tau_*^{2d-2} = 0.
\]

Real \( \tau_*(B^2) \) arises only if \( \tau > 1 \) i.e. within \( N/S \). Overall this gives the smooth “hourglass”-like red curve in Figure-1 stretching from \( I^+ \) to \( I^- \), intersecting the horizons, turning around smoothly at \( \tau_* \) in \( N/S \). These are akin to rotated versions of the Hartman-Maldacena surfaces [32] in the \( AdS \) black hole (which itself resembles a rotation of (4)).

The limit \( B \to 0 \) gives \( \tau_* \to 1 \), the turning point approaching the bifurcation region (the red “hourglass neck” is pinching off). The width becomes \( \Delta w \sim \log \frac{2}{\tau_* - 1} \to \infty \) covering all \( I^\pm \) (on this equatorial plane). In this limit [23] the area becomes

\[
S \to \frac{2l^{d-1}V_{S^{d-2}}}{4G_{d+1}} \int_{\epsilon}^{1} \frac{d\tau}{\tau^{d-1}} \frac{1}{\sqrt{1 - \tau^2}} \to \frac{\pi l^2}{G_4} \frac{1}{\epsilon},
\]

scaling as de Sitter entropy. In the \( w = \text{const} \) slice, similar surfaces (tricky in general) can be shown to exist with the same area when the subregion covers all space.

3 Entanglement in ghost systems

In general, extremal surfaces appear tricky in de Sitter space, unlike \( AdS \): the surfaces here connecting \( I^\pm \) are thus interesting. These lie on some boundary Euclidean time slice (all of which are equivalent). As the boundary subregion approaches all of \( I^\pm \), they pass through the vicinity of the bifurcation region. The area law divergence scales as de Sitter entropy [7]. The existence of these surfaces suggests entanglement between the future and past boundaries (see also [33]). This cannot be entanglement in the usual sense, the dual CFTs being Euclidean. However boundary directions admitting translation symmetries can be taken as Euclidean time, leading to generalizations of entanglement.

Motivated by (2) for \( \text{CFT}_{F,P} \) [9], [10], entanglement in ghost-like theories was studied [34] in toy 2-dim ghost-CFTs using the replica formulation, giving \( S < 0 \) for \( c < 0 \) ghost-CFTs under certain conditions, and in (possibly more illuminating) quantum mechanical toy
models of “ghost-spins” via reduced density matrices (RDMs). We define a ghost-spin as a 2-state spin variable with indefinite inner product $\gamma_{\alpha\beta}$ (akin to those in $bc$-ghost CFTs)

$$\langle \uparrow | \downarrow \rangle = 1 = \langle \downarrow | \uparrow \rangle , \quad \langle \uparrow | \uparrow \rangle = 0 = \langle \downarrow | \downarrow \rangle ,$$

unlike $\langle \uparrow | \uparrow \rangle = 1 = \langle \downarrow | \downarrow \rangle$ for a single spin. Redefined states $|\pm\rangle$ satisfy $\gamma_{\pm\pm} = \langle \pm | \pm \rangle = \pm 1$.

A two ghost-spin state then has norm

$$|\psi\rangle = \psi_{\alpha\beta} |\alpha\rangle |\beta\rangle : \quad \langle \psi | \psi \rangle = \gamma_{\alpha\kappa} \gamma_{\beta\lambda} \psi_{\alpha\beta} \psi_{\kappa\lambda}^* = |\psi^{++}|^2 + |\psi^{--}|^2 - |\psi^{+-}|^2 - |\psi^{-+}|^2 .$$

(9)

Thus although states $|\pm\rangle$ have negative norm, the state $|\pm\rangle |\pm\rangle$ has positive norm. For ghost-spin ensembles [35], generic states exhibit novel non-positive entanglement stemming from negative norm contributions. However “correlated” states such as $\psi^{++} |+\rangle |+\rangle + \psi^{--} |-\rangle |-\rangle$ entangling identical ghost-spins between two copies of ghost-spin ensembles can be shown to have positive norm, RDMs and entanglement. In [36], 1-dim ghost-spin chains with specific nearest-neighbour interactions were found to yield $bc$-ghost CFTs in continuum limits, suggesting that ghost-spins are perhaps microscopic building blocks of ghost-like non-unitary CFTs. Various $N$-level generalizations of these 2-level ghost-spins [37] also exhibit similar correlated states. Thinking thereby of appropriate 3-dim $N$-level ghost-spin systems as microscopic realizations in the universality class of ghost-CFT$^3$’s dual to $dS_4$ with $N \sim \frac{L^2}{G_4}$ finite albeit large, consider two ghost-CFT copies and a “correlated” state [23]

$$|\psi\rangle = \sum \psi_{i^n n^n}^{F} |i_n\rangle_F |i_n\rangle_P ,$$

(10)

entangling generic ghost-spin configurations $|i_n\rangle_P$ from CFT$_F$ at $I^+$ with identical ones $|i_n\rangle_P$ from CFT$_P$ at $I^-$, as the $I^\pm$-connecting surfaces suggest. These are entirely positive, the doubling cancelling the minus signs, giving positive RDM and entanglement. In toy examples of two $N$-level ghost-spin chain copies [37], the $N$ internal possibilities restricting to ground states gives (maximal) entanglement entropy scaling as $N \sim \frac{L^2}{G_4}$.

Bulk time evolution, mapping states at $I^-$ to $I^+$ [8], suggests the states (10) are unitarily equivalent to entangled states in two CFT$_F$ copies solely at $I^+$. While CFT$_P \equiv$ CFT$_F$ implies a single CFT, the state (10) is best regarded as a particular entangled slice in a doubled system. Thus (10) is akin to the thermofield double dual to the eternal AdS black hole [38]. This suggests the speculation that $dS_4$ is perhaps approximately dual to CFT$_F \times$ CFT$_P$ (or CFT$_F \times$ CFT$_F$) entangled as (10), $dS_4$ entropy arising as entanglement. Many issues arise of course (see the overview [39]): e.g. on regions $N, S$ (which are “behind the horizons” viewed from $dS$/CFT at $I^\pm$) and interior operators, akin to e.g. [40] for black holes.
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