Strong total domination and weak total domination in Mycielski’s graphs

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Abstract
Let $G = (V,E)$ be a graph. A set $S \subseteq V$ is called a weak total dominating set (WTD-set) if each vertex $v \in V - S$ is adjacent to a vertex $u \in S$ with $\deg(v) > \deg(u)$ and every vertex in $S$ adjacent to a vertex in $S$. The weak total domination number, denoted by $\gamma_w(G)$, is minimum cardinality of a weak total dominating set. Analogously, a dominating set $S \subseteq V$ is called a strong total dominating set (STD-set) if each vertex $v \in V - S$ is dominated by some vertices $u \in S$ with $\deg(v) < \deg(u)$ and each vertex in $S$ adjacent to a vertex in $S$. The strong total domination number, denoted by $\gamma_s(G)$, is minimum cardinality of a strong dominating set. Weak total and strong total domination parameters were introduced by Chellali et al. and Akbari and Jafari Rad, respectively.

In this paper, we consider weak total and strong total domination of Mycielski’s Graph, denoted by $\mu(G)$. We also provide some upper and lower bound about weak total domination of Mycielski’s graph related with minimum and maximum degree number of a graph. In addition, the inequality about relationship between strong total domination of Mycielski’s graph $\mu(G)$ and underlying graph $G$, $\gamma_s(G) + 1 \leq \gamma_s(\mu(G)) \leq \gamma_s(G) + 2$, is obtained. Among other results, we characterize graphs $G$ achieving the lower bound $\gamma_s(G) + 1 = \gamma_s(\mu(G))$.

Keywords
Graph Theory, Strong Total Domination, Weak Total Domination, Mycielski’s Graph.

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1. Introduction

Let $G$ be $n$ order connected simple graphs. $V(G)$ and $E(G)$ are vertex and edge set of $G$, respectively. The open neighborhood of $v \in V$ is $N_G(v) = \{u \in V : uv \in E(G)\}$ and closed neighborhood of $v \in V$ is $N_G[v] = N_G(v) \cup \{v\}$. A vertex $v \in V(G)$ is an $S$-private neighbor of $u \in S$ if $N[v] \cap S = \{u\}$, while the $S$-private neighbor set of $u$, denoted by $pn[u,S]$, is the set of all $S$-private neighbors of $u$. If $v$ is a vertex of $V(G)$, then the degree of $v$ denoted by $\deg_G(v)$, is the cardinality of its open neighborhood. The maximum and minimum degree of a graph $G$ is denoted by $\Delta(G) = \Delta$ and $\delta(G) = \delta$, respectively.

A subset $S \subseteq V$ is a dominating set of $G$ is every vertex in $V - S$ has a neighbor in $S$ and the domination number of $G$, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set. For detailed information about domination parameters readers are referred to books [6, 7]. A dominating set that is independent is called an independent dominating set of $G$. The minimum cardinality of an independent dominating set is called independent domination number, $i(G)$. A dominating set that is connected is called a connected dominating set of $G$. The minimum cardinality of an connected dominating set is called connected domination number, $\gamma_c(G)$. A total dominating set, denoted by TD-set of $G$ with no isolated vertex is a set $S$ of vertices of $G$ and total domination number that is the minimum cardinality of a total dominating set denoted by $\gamma_t(G)$. Every graph without isolated vertices has
TD-set. Total domination was introduced by Cockayne et al. [4]. A dominating set \( S \subseteq V \) is called a weak dominating set (WD-set) if each vertex \( v \in V - S \) is dominated by some vertices \( u \in S \) with \( \deg(v) > \deg(u) \). The weak domination number, denoted by \( \gamma_w(G) \), is minimum cardinality of a weak dominating set. Similarly, a dominating set \( S \subseteq V \) is called a strong dominating set (SD-set) if each vertex \( v \in V - S \) is dominated by some vertices \( u \in S \) with \( \deg(v) < \deg(u) \). The strong domination number, denoted by \( \gamma_s(G) \), is minimum cardinality of a strong dominating set. The concept weak and strong domination number introduced by Sampathkumar and Pushpa Latha in [11]. A weak dominating set \( S \subseteq V \) induces a subgraph with no isolated vertex is called weak total dominating set (WTD-set). Total domination was introduced by Cockayne et al. [4].

Let begin with following illustration about strong total and weak total dominations have been considered for some graphs.

Example 2.1. Let \( G \) be a connected graph with 6 vertices as in Figure 2 \( S_1 = \{v_2, v_3, v_5, v_6\} \) and \( S_2 = \{v_1, v_2, v_4, v_5\} \) are some WTD-set of \( G \) and \( S = \{v_1, v_2\} \) is a STD-set of \( G \). Besides these sets, we can also generate some other WTD and STD-sets. In addition, \( \gamma_{wt}(G) = 4 \) and any \( \gamma_{wt} - \text{set of } G \) must contain \( v_2 \) and \( v_6 \) of which degree smaller than their neighbors in the graph, \( \gamma_{st}(G) = 2 \) and any \( \gamma_{st} - \text{set of } G \) must contain \( v_1 \) with degree larger than their neighbors in the graph.

Theorem 2.2. [5] For any graph \( G \), \( \gamma(\mu(G)) = \gamma(G) + 1 \).

Theorem 2.3. [2] For any graph \( G \), \( \gamma_s(\mu(G)) = \gamma_s(G) + 1 \).

Theorem 2.4. [2] For any graph \( G \), \( \gamma_c(\mu(G)) + 1 \leq \gamma_c(\mu(G)) \leq 2\gamma_c(G) \).

Theorem 2.5. [5] Let \( G \) be a graph. Then \( \gamma(\mu(G)) = \gamma(G) + 1 \), \( \gamma_s(\mu(G)) = \gamma_s(G) + 1 \).

Theorem 2.6. [9] For any graph \( G \), \( i(\mu(G)) = i(G) + 1 \).

Theorem 2.7. [9] If \( \gamma_c(G) \geq 3 \) then, \( \gamma_c(\mu(G)) \leq \gamma_c(G) + 1 \).

Proposition 2.8. [3] For any graph \( G \) with no isolated vertices, \( \gamma_{wt}(G) \leq n + 1 - \Delta \).

Proposition 2.9. [1] For paths and cycles, \( \gamma_{st}(P_n) = \gamma_{st}(C_n) = \gamma_s(P_n) = \gamma_s(C_n) \).

Proposition 2.10. [1] For any graph \( G \) of order \( n \), maximum degree \( \Delta \) and with no isolated vertices, \( \gamma_{st}(G) \leq n + 1 - \Delta \).
3. Weak Total Domination of Mycielski’s Graph

In this section, we investigate some results about weak total domination number of Mycielski’s Graph. We initiate with special graph families and also, we characterize graph attaining some bounds about weak total domination of Mycielski’s graph.

Proposition 3.1. For special graphs:

a. $\gamma_{wt}(\mu(P_n)) = n + 3$, $n \geq 4$

b. $\gamma_{wt}(\mu(C_n)) = n + 1$, $n \geq 4$

c. $\gamma_{wt}(\mu(K_n)) = 3$, $n \geq 3$

d. $\gamma_{wt}(\mu(K_{1,n-1})) = 2n + 1$, $n \geq 3$

e. $\gamma_{wt}(\mu(W_{1,n-1})) = n + 1$, $n \geq 4$

f. $\gamma_{wt}(\mu(K_{m,n})) = m + n + 1$.

Proof. a. Let $n \geq 5$ and $A = \{v_1, v_n, v'_1, v'_n\}$. Let $D$ be $\gamma_{wt}$-set of $\mu(P_n)$. Due to the degree of vertices in $A$, it must be $A \subseteq D$. However, $A$ only weakly dominates vertices $v_2, v'_2, v_{n-1}, v'_{n-1}$ in $\mu(P_n)$. Also, $\forall v_i$ and $v'_i$ in $\mu(P_n)$, $\deg(v_i) > \deg(v'_i)$ for $i \in \{2, \ldots, n-2\}$ and all vertices in $V(G)$ are disjoint. The vertices $v'_2, v'_{n-1}$ are weakly dominated by $A$, remaining disjoint vertices $v'_i, i \in \{3, \ldots, n-2\}$, is not dominated by $A$. In order to weakly dominates vertices of $P_n$ in $\mu(P_n)$, it is needed to add $n - 4$ vertices from $V(G)$ to $A$. Thus, $|A| = n = 4 + 4 = n$. All vertices in $\mu(P_n)$ are weakly dominated by $A$. To obtain $\gamma_{wt}$-set, it implies that $A \cup \{v_2, v_{n-1}, z\}$ or $A \cup \{v'_2, v'_{n-1}, z\}$ is not weakly dominated by $A$. Hence, we get $|D| = n + 3$ as desired.

de. For $G = K_{1,n-1} (n \geq 3)$, there are disjoint $2n - 2$ vertices in $\mu(K_{1,n-1})$ which have minimum degree also these vertices are adjacent to the vertex $v_1$ that has maximum degree. Hence, $\gamma_{wt}(\mu(K_{1,n-1})) = 2n - 1$.

e. For $G = W_{1,n-1} (n \geq 4)$, there are disjoint $n - 1$ vertices in $V(G)$ which have minimum degree and also these vertices adjacent to $z$. Hence, $\gamma_{wt}(\mu(W_{1,n-1}))$ weakly dominated by these $n$ vertices. Thus, $\gamma_{wt}(\mu(W_{1,n-1})) = n$.

f. For $G = K_{m,n}$. It is easy to see that there are disjoint $m + n$ vertices which have min $\{m, n\}$ and min $\{m, n\} + 1$ degree in $V(G')$. Hence, for a $\gamma_{wt}$-set of $\mu(K_{m,n})$ contains at least $m + n + 1$ vertices. Thus, $\gamma_{wt}(\mu(K_{m,n})) = m + n + 1$. □

Theorem 3.2. Let $G$ be an order graph with $\delta \geq 2$ then $3 \leq \gamma_{wt}(\mu(G)) \leq n + 1$.

Proof. For lower bound it is not possible to weakly total dominate $\mu(G)$ by two vertices. For upper bound, always possible to obtain a WTD-set such as $V(G') \cup \{z\}$ provided that $\delta \geq 2$.

□

Observation 1: Let $|\delta|$ be number of minimum degree vertices. According to definition of weak domination $|\delta| \leq \gamma_{wt}(G) \leq \gamma_{wt}(G)$.

Proposition 3.3. Let $G$ be an order graph $n \geq 3$, $\delta = 1$ then $n + |\delta| \leq \gamma_{wt}(\mu(G)) \leq n + 1 + |\delta|$ where $|\delta|$ is number of minimum degree vertices.

Proof. Let $D$ be WTD-set of $\mu(G)$. $D$ must be included all pendant vertices in $G$ denoted by $v_{\delta}$ for $1 \leq i \leq |\delta|$. All support vertices in $G$ in degree more than two in $\mu(G)$. According to form of $\mu(G)$, all vertices in $V(G')$ are disjoint vertices and also has degree less than all neighbors except $v_{\delta}$. Thus, $V(G') \cup \{v_{\delta}\}$ weakly dominates $G \cup G'$. In order to obtain a weakly total dominating set, $D$ can be included $\{\delta\}$. Thus $D$ may be $V(G') \cup \{z\} \cup \{v_{\delta}\}$ for all $i$. Hence, $\gamma_{wt}(\mu(G)) \leq n + 1 + |\delta|$. From the Observation 1, the vertices in $\{v_{\delta}\} i = 1, 2, \ldots, n$ and their copies in $G'$ may weakly dominate all vertices in $\mu(G)$. For a weakly total dominating set, $D$ must include $n - |\delta|$ vertices in $G'$ (or $G$). Hence, $2|\delta| + n - |\delta| \leq \gamma_{wt}(\mu(G))$. □
Proposition 3.4. Let $G$ be an order graph $\Delta = n - 1$, $\delta \geq 2$

$$n + 1 - |\Delta| \leq \gamma_{st}(\mu(G)) \leq n + 2 - |\Delta|$$

where $|\Delta|$ is the number of maximum degree vertices.

Proof. Let $D$ be a WTD-set of $\mu(G)$.

- If $|\Delta| = 1$; Let $v'$ be the copy of the vertex $v$ that has degree $\Delta$ in $G$. $D$ must include the vertices $V(G') - \{v'\}$. Therefore, $S = V(G') - \{v'\} \cup \{z\}$ is a minimal WTD-set for $\mu(G)$. Thus, $\gamma_{st}(\mu(G)) = n - |\Delta| + 1$.

- If $|\Delta| \geq 2$; Let $D$ be set included all vertices in $V(G')$ except copy of $\Delta$-degree vertices. There are two cases:
  - Let, all vertices in $V(G')$ be weakly dominated by $D$. For totality, $\{z\}$ must be included by $D$. Thus, $|D| = n - |\Delta| + 1$.
  - Let, all vertices in $V(G)$ not be weakly dominated by $D$. Let $\{v\}$ be a copy of $\Delta$-degree vertex. Then, $D \cup \{v\}$ weakly dominates all vertices in $V(G)$. Hence, $D \cup \{v\} \cup \{z\}$ is a WTD-set of $\mu(G)$. We have $|D| = n - |\Delta| + 2$.

According to these two cases we obtain that $n + 1 - |\Delta| \leq \gamma_{st}(\mu(G)) \leq n + 2 - |\Delta|$. $\square$

4. Strong Total Domination of Mycielski’s Graph

In this section, we begin with some basic results about strong total domination complete bipartite graph $K_{n_1, \ldots, n_p}$ and the characterization of graph that have $\gamma_{st}(G) = 2$. Also, we derive some results about strong total domination of Mycielski’s graph. The bound $\gamma_{st}(G) + 1 \leq \gamma_{st}(\mu(G)) \leq \gamma_{st}(G) + 2$ has been presented. In addition, we characterize graphs $G$ holding the lower bound $\gamma_{st}(G) + 1 = \gamma_{st}(\mu(G))$.

Proposition 4.1. Let $G$ be an order graph such that at least one vertex has degree $(n - 1)$. Then

$$\gamma_{st}(G) = 2.$$ 

Proof. Let $v$ be a $\Delta = n - 1$ degree vertex of $G$. $G$ can be strongly total dominates by $v$ and its any neighbour. Thus $\gamma_{st}(G) = 2$. \square

Theorem 4.2. Let $G$ be an order graph,

$$\gamma_{st}(G) \geq \frac{n - |SS|}{\Delta}.$$ 

Proof. Let $S$ be a $\gamma_{st}$-set of $G$. Let $F$ be the set of all edges of $G$ that have one end vertex in $S$ the other in $V - S$. Also, a vertex in $SS(G)$ has degree at most $\Delta$. Therefore, $|F| \leq |SS| (\Delta - 1) + (|S| - |SS|) (\Delta - 2)$. In addition, a vertex can be dominated by more than one vertex then $|F| \geq n - |S|$. Using the inequalities the it is obtained that $|S| \geq \frac{n - |SS|}{\Delta - 1}$. \square

Proposition 4.3. Let $K_{m,n}$ be complete bipartite graph with $m + n$ vertices then $\gamma_{st}(K_{m,n}) = \begin{cases} 2, & \text{if } m = n \\ \min\{m,n\} + 1, & \text{otherwise} \end{cases}$

Proof. Let $K_{m,n}$ be complete bipartite graph with partitions $V_1$ and $V_2$. If $m = n$ then degree of vertices are equal in each subset $V_1$ and $V_2$. Thus, a $\gamma_{st}$-set can be done as choosing a vertex from $V_1$ and $V_2$. Thus, $\gamma_{st}(K_{m,n}) = 2$. Without loss of generality, $m > n$ and $|V_1| = m$ and $|V_2| = n$. Therefore, degree of vertices in $V_2$ are equal and greater than degree of vertices in $V_1$. According to form of complete bipartite graph, all vertices in $V_2$ must be in $\gamma_{st}$-set of $K_{n,n}$. In order to totally strong dominates, a vertex from $V_1$ must be in $\gamma_{st}$-set. Hence, $\gamma_{st}(K_{m,n}) = n + 1 = \min\{m,n\} + 1$. \square

Proposition 4.4. For any integers $n_1 \geq \ldots \geq n_l \geq 1$,

$$\gamma_{st}(K_{n_1, n_2, \ldots, n_p}) = \min\{n_1, n_2, \ldots, n_p\} + 1.$$ 

Proof. Let $V_i$ be vertex set that included $n_i$ vertices. According to form of the graph there are $n_1$ vertex in $V_1$ that has maximum degree and adjacent to all other vertices in $V_i$, $i \neq 1$. Also, these disjoint vertices in $V_1$ must be included by any STD-set of $K_{n_1, n_2, \ldots, n_p}$. In order to totally dominates, it is needed a vertex in $V_i$, $i \neq 1$. Hence, $\gamma_{st}(K_{n_1, n_2, \ldots, n_p}) = n_1 + 1$. \square

Observation 2:[1] Support vertices always in the STD-set of graph.

Observation 3: Let $G$ be an order graph. If $\Delta < n - 1$ then the vertex $z$ must be in the $\gamma_{st}$-set of $\mu(G)$.

Proposition 4.5. Let $D$ be a $\gamma_{st}$-set of $G$ in $\mu(G)$ then $D$ strongly total dominates $V(G) \cup V(G')$ in $\mu(G)$.

Proof. Let $D$ be a $\gamma_{st}$-set of $G$ in $\mu(G)$. According to the form of $\mu(G)$, $N(D) = V(G) \cup V(G')$ where $N(D) = \bigcup_{v \in D} N(v)$. Since $\deg(v) \geq \deg(v')$, $v \in V(G)$ and $v' \in V(G')$, $D$ strongly total dominates $V(G) \cup V(G')$. \square

Proposition 4.6. For special graphs

a. $\gamma_{st}(\mu(P_n)) = \gamma_{st}(P_n) + 1$

b. $\gamma_{st}(\mu(C_n)) = \begin{cases} \gamma_{st}(C_n) + 2, & \text{if } n \equiv 0 \pmod{4} \\ \gamma_{st}(C_n) + 1, & \text{otherwise} \end{cases}$

c. $\gamma_{st}(\mu(K_n)) = 3$

d. $\gamma_{st}(\mu(K_{1,n-1})) = 3$

e. $\gamma_{st}(\mu(W_{1,n-1})) = 3$

f. $\gamma_{st}(\mu(K_{m,n})) = \begin{cases} 4 & \text{if } m = n \\ n + 2, & \text{otherwise} \end{cases}$ for $m \geq n$.

Proof. a. Let $D$ be the $\gamma_{st}$-set of $\mu(P_n)$ and $S$ be $\gamma_{st}$-set of $P_n$. From Observation 3, $z$ must be included by $D$ then $|S| < |D|$. From Proposition 4.5, vertices in $G$ and $G'$ are strongly total dominated by $S$. For totality, at least one vertex must be included by $D$. Let $v'$ be copy of support vertex $v$ of $P_n$. 1684
Thus, $D = (S - \{v\}) \cup \{v'\} \cup \{z\}$ is $\gamma_d$-set for $\mu(P_n)$ then $\gamma_d(\mu(G)) = |D| = \gamma_d(C_n) + 1$.

b. Let $S$ be $\gamma_d$-set of $C_n$. If $n \neq 0 \pmod 4$ then there exist a vertex $v \in S$ such that $pn[v,S] = \emptyset$. Thus, $(S - \{v\}) \cup \{v'\} \cup \{z\}$ is $\gamma_d$-set of $\mu(G)$ where $v'$ is copy of $v$ in $\mu(G)$. Therefore, if $n \neq 0 \pmod 4$, we have $\gamma_d(C_n) = \gamma_d(C_n) + 1$. If $n = 0 \pmod 4$ then for all $v \in S$, $pn[v,S] \neq \emptyset$. Due to the form of $\mu(G)$, $V(G)$ is not strongly dominated by a vertex $v' \in V(G)$. Thus, $S \cup \{v'\} \cup \{z\}$ is $\gamma_d$-set of $\mu(G)$. Therefore, if $n = 0 \pmod 4$, we have $\gamma_d(C_n) = \gamma_d(C_n) + 2$.

c.d.e. Let $D$ be $\gamma_d$-set of $\mu(G)$ and $v$ be a vertex in $G$ and $\deg v = n - 1$. From Observation 2 and 3, $v$ and $z$ are contained by $D$. All vertices in $\mu(G)$ are strongly dominated by $\{v\} \cup \{z\}$. For totality at least one vertex from $G'$ must be included by $D$. Hence, $D = \{v\} \cup \{z\} \cup \{v_i\}$. Finally we have, $\gamma_d(\mu(G)) = 3$.

d. Let $m = n$, from Proposition 4.3, $\gamma_d(K_{m,m}) = 2$. Let $S$ and $D$ be a $\gamma_d$-set of $K_{m,m}$ and $\mu(K_{m,m})$, respectively. It is easily to see that $D = S \cup \{u\} \cup \{v\}$. Hence, $\gamma_d(\mu(K_{m,m})) = 4$. Let $m > n$ then due to the degree of vertices, $n$ disjoint vertices in $K_{m,n}$ must be included. For $\gamma_d$-set of $\mu(G)$ it is needed two more vertices such that $v' \in V(G)$ and $z$. Therefore $\gamma_d(\mu(K_{m,n})) = n + 2$ for $m > n$.

**Theorem 4.7.** Let $G$ be an order graph then $\gamma_d(G) + 1 \leq \gamma_d(\mu(G)) \leq \gamma_d(G) + 2$.

**Proof.** Let $S$ be a $\gamma_d$-set of $G$. From Proposition 4.5, vertices in $G$ and $G'$ are strongly total dominated by $S$. All vertices in $\mu(G)$ is also strongly dominated by $S \cup \{z\}$. It is known that for any graph $G$: $\gamma_d(G) \leq \gamma_d(G)$. The strong total domination number of $\mu(G)$ is at least $\gamma_d(G) + 1$. Therefore, $\gamma_d(G) + 1 \leq \gamma_d(\mu(G))$. Let $y_1$ be a vertex in $G'$. It is obvious that $S \cup \{y_1\} \cup \{z\}$ is a $\gamma_d$-set of $\mu(G)$. Thus, $\gamma_d(\mu(G)) \leq \gamma_d(G) + 2$.

**Theorem 4.8.** Let $G$ be an order graph, $\Delta < n - 1$ and $\delta = 1$. Then $\gamma_d(\mu(G)) = \gamma_d(G) + 1$.

**Proof.** Let $S$ and $D$ be a $\gamma_d$-set of $G$ and $\mu(G)$, respectively, and $x_i$ be support vertex then from Observation 2, $x_i$ is contained by $S$. Also, from Observation 3, $z$ is included by $D$. Any vertex from $G'$ ensures totality. Therefore, $S \cup \{x_i'\} \cup \{z\}$ is any $\gamma_d$-set of $\mu(G)$, where $x_i'$ is copy of $x_i$. It is obvious that degree of the pendant vertex, denoted by $u$, less than degree of $x_i'$ in Mycielski’s Graph. Since $N_{\mu(G)}(u) = \{x_i, x_i'\}$, $D$ is not included both $x_i$ and $x_i'$. Thus, the lower bound of Theorem 4.7 can be obtained as $D = (S - \{x_i\}) \cup \{x_i'\} \cup \{z\}$. Hence, $\gamma_d(\mu(G)) = \gamma_d(G) + 1$.

**Proposition 4.9.** Let $G$ be an order graph such that at least one vertex has degree $(n - 1)$. Then $\gamma_d(\mu(G)) = 3$.

**Proof.** Similar proof can be done as shown in case c, d, e of Proposition 4.6. \qed

**Remark 4.10.** It can be said from Propositions 4.9, if graph contain at least one vertex of degree $n - 1$ then $\gamma_d(\mu(G)) = \gamma_d(G) + 1$.

**Theorem 4.11.** Let $G$ be a graph and $S$ be a $\gamma_d$-set of $G$. If at least one vertex $u \in S$ such that $pn[u,S] = \emptyset$ then $\gamma_d(\mu(G)) = \gamma_d(G) + 1$.

**Proof.** Let $u'$ be a vertex in $G'$ that is copy of $u$. If $pn[u,S] = \emptyset$, then this means that $u$ is not private neighborhood of any vertex in $G$. The vertex $u$ is contained by $S$ due to totality. From Proposition 4.5, $(S - \{u\}) \cup \{u'\}$ strongly total dominates $V(G) \cup V(G')$. Therefore, $(S - \{u\}) \cup \{u'\} \cup \{z\}$ is a $\gamma_d$-set of $\mu(G)$. Hence, $\gamma_d(\mu(G)) = \gamma_d(G) + 1$.

Let $SS(G)$ be the set of all vertices $v \in V(G)$ such that $\deg(v) > \deg(u)$ for every $u \in N(v)$. $SS(G)$ may be empty. If $SS(G) \neq \emptyset$ then it is included by every STD set of $G$.

## 5. Conclusion

Mycielski present a construction that increase chromatic number [10]. This construction has been taken attention and beside chromatic number, there have been many results about various parameters of Mycielski’s graphs in literature. However, there exist considerably fewer research on strong total and weak total domination numbers. In this paper, strong total domination and weak total domination of Mycielski’s graph was investigated and also the strong and weak total domination numbers of Mycielski’s graph, $\mu(G)$, associated with strong and weak total domination of underlying graph, $G$. This provides a good starting point for discussion and further research. Future research on strong total and weak total domination number might extend the explanations of some graph operations.

## References

[1] M.H. Akhbari and N. Jafari Rad, Bounds on weak and strong total domination number in graphs, *Electronic Journal of Graph Theory and Applications*, 4(2016), 111-118.

[2] S. Balamurgan, M. Anitha and N. Anbazhagan, Various domination parameters in Mycielski’s graphs, *International Journal of Pure and Applied Mathematics*, 119(2018), 203-211.

[3] M. Chellali and N. Jafari Rad, Weak total domination in graphs, *Utilitas Mathematica*, 94(2014), 221-236.

[4] E.J. Cockayne, R.M. Dawes, S.T. Hedetniemi, Total Domination in graphs, *Networks*, 10(1980), 211-219.

[5] D.C. Fisher, P.A. McKenna, E.D. Boyer, Hamiltonicity, diameter, domination, packing, and biclique partitions of Mycielski’s graph, *Discrete Applied Mathematics*, 84(1998), 93-105.
[6] T.W. Haynes, S.T. Hedetniemi, P.J. Slater (Eds.) Fundamentals of Domination in Graphs, Marcel Dekker, Inc., New York, 1998.

[7] T.W. Haynes, S.T. Hedetniemi, P.J. Slater (Eds.) Domination in Graphs: Advanced Topics, Marcel Dekker, Inc., New York, 1998.

[8] N. Jafari Rad, On the complexity of strong and weak total domination in graphs, Australasian Journal of Combinatorics, 65(2016), 53-58.

[9] D.A. Mojdeh and N. Jafari Rad, On Domination and its Forcing in Mycielski’s Graphs, Scientia Iranica, 15(2008), 218-222.

[10] J. Mycielski, Sur le coloriage des graphs, Colloq. Math., 3(1955), 161-162.

[11] E. Sampathkumar and L. Pushpa Latha, Strong, weak domination and domination balance in graphs, Discrete Math., 161(1996), 235-242.

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