Tracking Quantum Error Correction

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To implement fault-tolerant quantum computation with continuous variables, the Gottesman–Kitaev–Preskill (GKP) qubit has been recognized as an important technological element. We have proposed a method to reduce the required squeezing level to realize large scale quantum computation with the GKP qubit [arXiv: 1712.00294 (2017)], harnessing the virtue of analog information in the GKP qubits. In the present work, to reduce the number of qubits required for large scale quantum computation, we propose the tracking quantum error correction, where the logical-qubit level quantum error correction is partially substituted by the single-qubit level quantum error correction. In the proposed method, the analog quantum error correction is utilized to make the performances of the single-qubit level quantum error correction almost identical to those of the logical-qubit level quantum error correction in a practical noise level. The numerical results show that the proposed tracking quantum error correction reduces the number of qubits during a quantum error correction process by the reduction rate \((n - 1)/2n\) for \(n\)-cycles of the quantum error correction process. Hence, the proposed tracking quantum error correction has great advantage in reducing the required number of physical qubits, and will open a new way to bring up advantage of the GKP qubits in practical quantum computation.

I. INTRODUCTION

Quantum computation has a great deal of potential to efficiently solve some hard problems for conventional computers [1, 2]. Among the candidates for qubits, squeezed vacuum states in an optical system have shown great potential for large scale continuous variable quantum computation; in fact, more than one million-mode continuous variable cluster state has already been achieved in an experiment [3]. This ability of entanglement generation comes from the fact that squeezed vacuum states can be entangled with only beam splitter coupling through the time-domain multiplexing approach to miniaturize optical circuits [4, 5]. However, since quantum computation with continuous variables itself has an analog nature, it is difficult to handle the accumulation of analog errors caused, for example, by photon loss during quantum computation [6, 7]. This can be circumvented by digitizing continuous variables using an appropriate code, such as the Gottesman–Kitaev–Preskill (GKP) code [8], which are referred to as GKP qubits. Moreover, GKP qubits inherit the advantage of squeezed vacuum states on optical implementation; they can be entangled only by beam splitter coupling. Furthermore, we have proposed a high-threshold fault-tolerant quantum computation to alleviate the required squeezing level for fault-tolerant quantum computation to 9.8 dB [9], and have taken a step closer to the realization of large scale quantum computation. Hence, the GKP qubits will play an indispensable role in implementing fault-tolerant quantum computation with continuous variables.

In general, the quantum error correction (QEC) is repeatedly performed only by the logical-qubits during the quantum computation process. In large scale quantum computation, a large number of physical qubits are needed to obtain the highly accurate results of quantum computation. This required number of physical qubits is one issue that we should struggle with to implement large scale quantum computation. In this work, we propose a method to reduce the number of qubits required for the QEC during large scale quantum computation, where the logical-qubit level QEC is partially substituted by the single-qubit level QEC. The single-qubit level QEC [8] enables us to correct a displacement (deviation) error occurred in the single qubit by using a single ancilla qubit, unless the logical-qubit level error occurs. Since the single-qubit level QEC can not correct the qubit-level errors, the bit and phase flip errors, we just track the measurement outcomes in the single-qubit level QEC. Then, the QEC is performed with the help of a set of tracked measurement outcomes in the single-qubit level QEC to correct the qubit-level errors. Although the single-qubit level QEC can be also implemented by discrete variables, the tracking QEC in discrete variables can not work well as shown later in the numerical results. By contrast, in our method, since the analog QEC makes the performances of the single-qubit level QEC almost identical to those of the logical-qubit level QEC, the tracking QEC can work well. The numerical results show that the proposed method has a great advantage to reduce the required number of qubits, e.g. in the concatenated QEC with analog QEC proposed in Ref. [10].

The rest of the paper is organized as follows. In Sec. II, we briefly review the background knowledge regarding the GKP qubit. In Sec. III, we propose the method to reduce the number of qubits required for large scale quantum computation. In Sec. IV the numerical results show the superiority of the proposed method over the conventional methods. Section V is devoted to discussion and conclusion.

II. PRELIMINARIES

In this section, we review the some background knowledge regarding the GKP qubit, a noise model considered in this work, the single-qubit level QEC, and the analog QEC.

A. The GKP qubit

Gottesman, Kitaev, and Preskill proposed a method to encode a qubit in an oscillator’s \(q\) (position) and \(p\) (momentum)
quadratures to correct errors caused by a small deviation in the $q$ and $p$ quadratures \[8\]. This error correction of a small deviation can handle any error acting on the oscillator, even a superposition of displacements.

The basis of the GKP qubit is composed of a series of Gaussian peaks of width $\sigma$ and separation $\sqrt{\pi}$ embedded in a larger Gaussian envelope of width $1/\sigma$. Although in the case of infinite squeezing ($\sigma \to 0$) the GKP qubit bases becomes orthogonal, in the case of finite squeezing, the approximate code states are not orthogonal. The approximate code states $|\widetilde{0}\rangle$ and $|\widetilde{1}\rangle$ are defined as

$$
|\widetilde{0}\rangle \sim \sum_{l=-\infty}^{\infty} \int e^{-2\pi \sigma^2 l^2} e^{-l^2/2} |l\rangle dq,
$$

$$
|\widetilde{1}\rangle \sim \sum_{l=-\infty}^{\infty} \int e^{-\pi \sigma^2 (l^2+1)/2} e^{-(l^2+1)/\sqrt{\pi}} |l\rangle dq.
$$

In the case of the finite squeezing, there is a finite probability of misidentifying $|\widetilde{0}\rangle$ as $|\widetilde{1}\rangle$, and vice versa. Provided the magnitude of the true deviation is less than $\sqrt{\pi}/2$ from the peak value, the decision of the bit value from the measurement of the GKP qubit is correct. The probability $p_{\text{corr}}$ to identify the correct bit value is the area of a normalized Gaussian of a variance $\sigma^2$ that lies between $-\sqrt{\pi}/2$ and $\sqrt{\pi}/2$ \[11\]:

$$
p_{\text{corr}} = \frac{1}{\sqrt{2\pi \sigma^2}} \int_{-\infty}^{\infty} \exp(-x^2/2\sigma^2).
$$

In addition to the imperfection that originates from the finite squeezing of the initial states, we consider the degradation in the Gaussian quantum channel \[8, 12\], which leads to a displacement in the quadrature during the quantum computation. The channel is described by a superoperator $\zeta$ acting on a density operator $\rho$ as follows:

$$
\rho \to \zeta(\rho) = \frac{1}{\pi \sigma^2} \int d^2 \alpha e^{-|\alpha|^2/\pi} D(\alpha) \rho D(\alpha)^\dagger,
$$

where $D(\alpha)$ is a displacement operator in the phase space. The position $q$ and momentum $p$ are displaced by $D(\alpha)$ independently as

$$
q \to q + v, \quad p \to p + u,
$$

respectively, where $v$ and $u$ are real Gaussian random variables with mean zero and variance $\xi^2$. From Eq. 4, we see that the Gaussian quantum channel conserves the position of the Gaussian peaks in the probability density function on the measurement outcome of the GKP qubit, but increases the variance as

$$
\sigma^2 \to \sigma^2 + \xi^2,
$$

where the $\sigma^2$ is the variance before the Gaussian quantum channel. Therefore, in the next section, we evaluate the performance of a QEC method with a code capacity noise model, where the noise is parameterized by a single variance $\sigma^2$ that includes the squeezing level of the initial GKP qubit and the degradation via the Gaussian quantum channel.

B. The single-qubit level quantum error correction

In Ref. \[8\], the single-qubit level QEC has been proposed to correct a displacement (deviation) error derived from the finite squeezing of the GKP qubit or the Gaussian quantum channel. We here explain the single-qubit level QEC to correct the displacement error in the $p$ quadrature in detail. In this single-qubit level QEC in the $p$ quadrature, an additional single ancilla qubit is entangled with the data qubit by a CNOT gate, where the data qubit is the target qubit. The ancilla qubit is prepared in the state $|\widetilde{0}\rangle$ to prevent us from identifying the bit value of the data qubit. The CNOT gate, which corresponds to the operator $\exp(-i\hat{q}_p\hat{p}_D)$ for continuous variables, transforms

$$
\hat{q}_a \to \hat{q}_a, \\
\hat{p}_a \to \hat{p}_a - \hat{p}_D, \\
\hat{q}_D \to \hat{q}_D + \hat{q}_a, \\
\hat{p}_D \to \hat{p}_D.
$$

where $\hat{q}_D(\hat{p}_D)$ and $\hat{q}_a(\hat{p}_a)$ are the quadrature operators of the data and ancilla qubits in the position $q$ (momentum $p$), respectively. Regarding the deviation, the CNOT gate operation displaces the deviation of the $q$ and $p$ quadratures as

$$
\Delta_{q,a} \to \Delta_{q,a}, \\
\Delta_{p,a} \to \Delta_{p,a} - \Delta_{p,D}, \\
\Delta_{q,D} \to \Delta_{q,D} + \Delta_{q,a}, \\
\Delta_{p,D} \to \Delta_{p,D},
$$

where $\Delta_{q,D}(\Delta_{p,D})$ and $\Delta_{q,a}(\Delta_{p,a})$ are the true deviation values of the data and ancilla qubits in the position $q$ (momentum $p$), respectively. We assume that the deviations of the data qubit in the $q$ and $p$ quadratures are obeyed to the Gaussian distribution with variance $\sigma_{q,a}^2$ and $\sigma_{p,a}^2$, and the deviations of the data qubit in the $q$ and $p$ quadratures obey the Gaussian distribution with the variance $\sigma^2$. After the CNOT gate, we measure the ancilla qubit in the $p$ quadrature, and obtain the deviation of the ancilla qubit $\Delta_{mp,a}$ that obeys to the Gaussian distribution with the variance $\sigma_{mp,a}^2$. Then, we perform the displacement $|\Delta_{mp,a}|$ on the $p$ quadrature of the data qubit to correct by shifting back in the direction to minimize the deviation. If $|\Delta_{mp,a}|$ is less than $\sqrt{\pi}/2$, the true deviation value of the data qubit in the $p$ quadrature changes from $\Delta_{p,D}$ to $\Delta_{p,a}$ after the displacement operation, which displaces $\Delta_{p,D}$ by $\Delta_{mp,a}(=\Delta_{p,a} - \Delta_{p,D})$. On the other hand, if $|\Delta_{p,a} - \Delta_{p,D}|$ is more than $\sqrt{\pi}/2$, the bit error in the $p$ quadrature occurs after the displacement operation. To summarize, the single-qubit level QEC for the data qubit in the $p$ quadrature can reduce the variance of the data qubit in the $p$ quadrature from $\sigma_{p,D}^2$ to $\sigma^2$. The variance of the data qubit in the $q$ quadrature after the single-qubit level QEC increases from $\sigma_{q,D}^2 + \sigma^2$, since the true deviation $\Delta_{q,D}$ and $\Delta_{q,a}$ obey Gaussian distribution with the variance $\sigma_{D,q}^2$ and $\sigma^2$, respectively, where the $\Delta_{q,D}$ and $\Delta_{q,a}$ are the true deviation of the data qubit and the ancilla qubit, respectively. Similarly, the single-qubit level QEC in the $q$ quadrature can be performed.
quadratures become using bit value and the noise level of the correct decision is calculated by

\[
\Delta \quad \text{likelihood from only the binary information regardless of the value}
\]

of the tracking QEC (see also \[10\] for details of the amplitude of the true deviation is

\[
\| \Delta \|_E \quad \text{correction when the magnitude of the true deviation}
\]

\[
\frac{1}{2\pi\sigma^2} e^{-\frac{\Delta^2}{2\sigma^2}}.
\]

The likelihood of the incorrect decision is calculated by

\[
\frac{1}{2\pi\sigma^2} e^{-\frac{\Delta^2}{2\sigma^2}}.
\]

Strictly speaking, the likelihood function should be a periodic function including the sum of the Gaussian functions, considering that the GKP state is the superposition of the Gaussian states. Nevertheless, in this paper, the likelihood function is approximated by simple Gaussian functions given by Eqs. \[15\] and \[17\], since the tail of the Gaussian function second nearest to the measurement outcome is small enough to ignore. In the QEC, we can reduce the decision error on the entire code word by considering the likelihood of the joint event of multiple qubits to choose the most likely candidate. As a result, the analog QEC under the code capacity model can improve the QEC performance with a single block code without the concatenation such as the three-qubit flip code \[10\]. In the previously proposed digital QEC \[13\] \[14\] has been shown to improve the QEC performance with only the concatenated code.

### III. THE TRACKING QUANTUM ERROR CORRECTION

#### A. Logical-qubit level quantum error correction

To implement large scale quantum computation, a number of single (physical) qubits should be encoded into a logical qubit to correct errors on the logical qubit. Then, by using a fault-tolerant manner such as a concatenation, the failure probability of the logical-qubit level QEC can be reduced to an arbitrary value, if the error probability on a physical qubit is less than the threshold value, which varies on the QEC code. Since the logical-qubit level QEC is repeated during the quantum computation process, a large number of physical qubits are needed to obtain highly accurate results on the quantum computation. For example, for the Knill’s C4/C6 code \[15\], the required number of physical qubits to prepare a level \( l \) logical qubit and a Bell state are \( 4 \times 12^{l-1} \) and \( 16 \times 12^{l-1} \), respectively, where \( l \geq 1 \) is the concatenation level. Accordingly, this required number of physical qubits is one issue that we should struggle with to implement large scale quantum computation.

#### B. Tracking quantum error correction

In general, the QEC is repeatedly performed only in the logical-qubit level during the quantum computation process as shown in Fig. \[2\] (a). We propose a method to reduce the required number of qubits, which we call “tracking QEC”, because the logical-qubit level QEC is partially substituted by the single-qubit level QEC \[8\] whose measurement outcome is tracked in the repeated QEC process as shown in Fig. \[2\] (b).
In our method, we apply analog QEC \[10\] to the tracking QEC to improve the performance. Since the single-qubit level QEC can reduce the error probability as described in Sec. 11B and the number of qubits required for the single-qubit level QEC is less than that for the logical-qubit level one, the substitution will reduce the required number of qubits.

To provide an insight into our method, we focus on the tracking QEC with the two QECs cycle, where the QEC after the Gaussian quantum channel is repeated twice as shown in Fig. 2. As a specific example of a QEC code, we use the Knill’s $C_4/C_6$ code $[15]$, where the error correction is based on quantum teleportation (see also $[10]$ for details of analog QEC and the $C_4/C_6$ code). The quantum teleportation process refers the outcomes $M_p$ and $M_q$ of the Bell measurement on the encoded qubits, and determines the transformation of the qubits. We obtain the Bell measurement outcomes of bit values $m_{pi}$ and $m_{qi}$ for the $i$-th physical GKP qubit of the encoded data qubit and the encoded Bell state, respectively. In addition to bit values, we also obtain deviation values $\Delta_{pm}$ and $\Delta_{qm}$ for the $i$-th physical GKP qubit. In our method, the first and second QECs are performed by the single- and logical-qubit level QEC, respectively. Since the single-qubit level QEC cannot correct the qubit-level error, we just track the measurement outcomes in the first QEC. After the two QECs, we obtain a set of the likelihoods from two possible joint events: one is the correct decision, where no qubit-level error occurs in both QECs. In this case, both true deviation values of the first and second QECs, $|\Delta_1|$ and $|\Delta_2|$, are less than $\sqrt{\pi}/2$ or more than $\sqrt{\pi}/2$. When both true deviation values are less than $\sqrt{\pi}/2$, $|\Delta_1|$ and $|\Delta_2|$ are equal to $|\Delta_m|$ and $|\Delta_m|$, respectively. When both true deviation values are more than $\sqrt{\pi}/2$, $|\Delta_1|$ and $|\Delta_2|$ are equal to $\sqrt{\pi} - |\Delta_m|$ and $\sqrt{\pi} - |\Delta_m|$, respectively. The other is the incorrect decision, where the single error occurs in either of the two QECs. In this case, one of the two true deviation values of the first and second QECs is greater than $\sqrt{\pi}/2$, and satisfies $|\Delta_1| = |\Delta_m|$ and $|\Delta_2| = |\Delta_m|$, or $|\Delta_1| + |\Delta_1| = \sqrt{\pi}$ and $|\Delta_2| = |\Delta_m|$. Hence, the likelihoods for the correct decision without and with analog QEC are calculated by

$$F_{\text{corr}} = p_{\text{corr}}^2 + (1 - p_{\text{corr}})^2,$$

$$F_{\text{corr}}^\text{ana} = f(\langle |\Delta_m| \rangle f(|\Delta_m|)) + f(\sqrt{\pi} - |\Delta_m|) f(\sqrt{\pi} - |\Delta_m|),$$

respectively, where $p_{\text{corr}}$ is given by Eq. 3. The likelihoods for the incorrect decision without and with analog QEC are calculated by

$$F_{\text{in}} = 2(1 - p_{\text{corr}}) p_{\text{corr}},$$

$$F_{\text{in}}^\text{ana} = f(\langle |\Delta_m| \rangle f(\sqrt{\pi} - |\Delta_m|)) + f(\sqrt{\pi} - |\Delta_m|) f(|\Delta_m|),$$

respectively. By considering these likelihoods of the joint event and choosing the most likely candidate, we can reduce the decision error on the entire code word after the second logical-qubit level QEC. By contrast, the single-qubit level QEC does not work, and QEC is performed by two independently operating logical-level QECs. Although we focus on the tracking QEC with the GKP qubits, we note that the tracking QEC with the discrete variables can be also performed, where the likelihoods are given by Eqs. 17 and 19. In our method, we utilize analog QEC using Eqs. 18 and 20 to make the performances of the single-qubit level QEC almost identical to that of the logical-qubit level QEC as shown in the numerical calculations.

We here estimate the required number of physical qubits to implement the two QECs. In the single-qubit level QEC, each physical qubit composing a logical qubit requires two ancilla physical qubits to correct the small deviation in the $q$ and $p$ quadratures. Since the logical qubit is composed of the $4 \times 2^{l-1}$ physical qubits, the single- and the logical-qubit level QEC with the concatenation level $l$ ($l \geq 1$) consume $2 \times (4 \times 2^{l-1})$ and $16 \times 2^{l-1}$ physical qubits, respectively. In the case of the QEC process with the two-QEC cycle, the conventional and proposed methods consume the physical qubits of $2 \times (16 \times 2^{l-1})$ and $2 \times (4 \times 2^{l-1}) + 16 \times 2^{l-1}$, respectively. Hence, the proposed method for the two-QEC cycle reduces by $8 \times 2^{l-1}$ physical qubits with the concatenation level $l$. Similarly, $(n - 1) \times 8 \times 2^{l-1}$ physical qubits can be reduced by the proposed method with the $n$-QEC cycle, where the single- and logical-qubit level QECs are performed in the first $(n - 1)$-QECs and the $n$-th QEC, respectively. Finally, we describe the reduction rate of the number of physical qubits per $n$-QEC cycle. For the $n$-QEC cycle using $C_4/C_6$ code with the level $l$, the conventional logical-qubit level QEC and the tracking QEC consume $R_{\text{con}}^{(n,l)} = n \times 16 \times 2^{l-1}$ physical qubits and $R_{\text{pro}}^{(n,l)} = (n - 1) \times 8 \times 2^{l-1} + 16 \times 2^{l-1}$, respectively. Hence, the reduction rate for the $n$-QEC cycle is $(R_{\text{con}}^{(n,l)} - R_{\text{pro}}^{(n,l)}) / R_{\text{con}}^{(n,l)} = (n - 1)/2n$.

IV. NUMERICAL CALCULATIONS

To validate the effectiveness of our proposed method, we calculate the failure probability of the QEC and the number of
physical qubits required in the QEC using the Monte Carlo method. We examine the tracking QEC performance, taking the conventional logical-qubit level QEC without the analog QEC as a reference. We simulate the QEC after the Gaussian quantum channel is repeated twice as described in Sec. III. In this simulation, we use the Knill’s $C_4/C_6$ code \cite{15} for the concatenation and assume that the encoded data qubit, encoded Bell state, and the physical qubits are prepared perfectly, and the variance of the GKP qubits of the encoded data qubit is increased to $\sigma^2$ only by the Gaussian quantum channel.

In Fig. 3 the failure probabilities for the $q$ ($p$) quadrature up to level 5 of the concatenation are plotted as a function of the noise level as the standard deviation of the Gaussian quantum channel. The noise is given by the sum of the noise of the first and second QECs cycle, where the encoded state suffers from the same amount of the noise in the first and second QECs cycle. In Fig. 3 we plotted the failure probabilities (a) without the analog QEC and (b) with the analog QEC for the conventional two logical-qubit level QECs and the proposed method, respectively. As shown in Fig. 3 the conventional method without and with the analog QECs achieve the threshold of the standard deviation value $\sim 1.11$ and $\sim 1.21$, which are twice of those obtained for the single cycle QEC $\sim 0.555$ and $\sim 0.607$ in Ref. \cite{10}, respectively.

Fig. 3(a) show that the tracking QEC degrades the threshold of the standard deviation by $\sim 0.17$ without the analog QEC. Fig. 3(b) show that the tracking QEC also degrades the threshold of the standard deviation by $\sim 0.07$ with analog QEC. However, the degradation with the analog QEC is smaller than that without the analog QEC. Furthermore, the gap of error probabilities between the conventional and proposed methods with the analog QEC narrows as the standard deviation of the Gaussian quantum channel decreases. In addition, it is also remarkable that the tracking QEC with the analog QEC in Fig. 3(b) suppresses errors more effectively than the conventional method without the analog QEC, and the analog QEC makes the performances of the single-qubit level QEC almost identical to that of the logical-qubit level QEC in a low noise level. These results show the virtue of use of analog information. On the basis of these results, we can conclude that the proposed method with analog QEC in the practical noise level can achieve efficient resource reduction by $8 \times 12^{-l-1}$ physical qubits with the concatenation level $l$ with only a small impact on the QEC performance, where the reduction rate for the 2-QEC cycle is 25%.

In the following, we consider admissible noise level of the Gaussian quantum channel for the tracking QEC. In practice, fault-tolerant quantum computation should be performed with a noise level smaller than the threshold value so as not to spend huge amounts of single qubits to prepare logical qubits with the required concatenation level $l$. To evaluate our proposed method, we assume that the single- and logical-qubit level QECs are performed with one-tenth of the threshold value according to Refs. \cite{16, 17}. For simplicity, we use the threshold value as the rate of the misidentifying the bit value of the GKP qubit. In the logical-qubit level QEC, the threshold of the noise level per cycle $\sim 0.555$ and $\sim 0.607$ for without and with analog QEC correspond to the error rate of the misidentifying the bit value $\sim 11.0\%$ and $\sim 14.3\%$, respectively, where the error rate of the misidentifying is obtained by $1 - p_{corr}$ given in Eq. (3). Therefore, we set the rate of the misidentifying the bit value as $\sim 1.1\%$ and $\sim 1.43\%$ which correspond to a noise level $\sim 0.346$ and $\sim 0.362$. As shown in Fig. 3(a), there is a gap of failure probabilities between the conventional and proposed method with the set noise level of $\sim 2 \times 0.346 = 0.692$ without the analog QEC. By contrast, the failure probabilities of the proposed method with the analog QEC is almost same as that of the conventional method with the set noise level $\sim 2 \times 0.362 = 0.724$ as shown in Fig. 3(b).

![Fig. 3. Simulation results for the failure probabilities of the two QECs for the $q$ ($p$) quadrature with the $C_4/C_6$ code using the conventional (blue line) and proposed method (red line). The results without the analog QEC (a) and with the analog QEC (b) are represented for the concatenated level 1 (solid), level 2 (dashed), level 3 (dashed-dotted), level 4 (open circles), and level 5 (filled circles), respectively.](image)

V. DISCUSSION AND CONCLUSION

In this work, we have proposed the tracking QEC with analog QEC to reduce the number of qubits required for large scale quantum computation, bringing up the advantages of the GKP qubits in practical quantum computation. In the proposed method, the single-qubit level QEC is combined with the standard logical-qubit level QEC, in a way that a part of the logical-qubit level QEC is substituted by the single-qubit level QEC during the quantum computation. Furthermore, we propose to apply the analog QEC to the tracking QEC to improve the QEC performance. The numerical results for the
two-QEC cycle showed that the proposed method with analog QEC reduces the required number of the qubits without degrading the QEC performance. The tracking QEC with analog QEC reduces the number of physical qubits required for the $C_4/C_6$ code by $8 \times 12^{l-1}$ with the concatenation level $l$, where the reduction rate for the 2-QEC cycle is 25%. Furthermore, it has been shown that the analog QEC makes the performances of the single-qubit level QEC almost identical to those of the logical-qubit level QEC under the condition of a practical noise level. To the best of our knowledge, this approach is the first practical attempt to utilize both the single- and standard logical-qubit level QECs to alleviate the requirement of the number of qubits. Hence, the proposed method has a great advantage in implementing fault-tolerant quantum computation with continuous variables and will open a new way to practical quantum computers.

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