Interferometric interpretation for the degree of polarization of classical optical beams

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Abstract
We introduce an interferometric interpretation for the degree of polarization as a quantity characterizing the ability of a light beam to generate polarization modulation when it interferes with itself. The result is confirmed experimentally in Young’s interferometer with beams of controlled degree of polarization and by comparing to a standard polarimetric measurement. The new interpretation is a consequence of the electromagnetic interference law that we formulate for stationary, quasi-monochromatic, partially polarized light beams in time domain. Our work provides fundamental insight into the role of polarization in electromagnetic coherence and interference.

Keywords: polarization, interference, coherence

1. Introduction

Polarization and intensity of light, characterized by the classic Stokes parameters [1], are fundamental physical concepts in optics [2–7]. Likewise, the degree of polarization that quantifies the fraction of energy in the fully polarized part of a partially polarized beam field is an important quantity introduced a long time ago [8]. Many natural light sources produce highly unpolarized light (low degree of polarization) while artificial, man-made emitters are often almost polarized (degree of polarization close to unity). Beams with equal degrees of polarization can, however, be distinguished by their polarization states, polarization dynamics.
[9, 10] and irreversibility properties [11]. Additionally, partially polarized beams may be classified into pure and impure types depending on whether or not the polarization state remains invariant in diffractive devices such as Young’s interferometer [12–14]. The notion of polarization purity is closely related to classical (nonquantum) entanglement between spatial coherence and polarization [15, 16]. It is further known that the degree of polarization also has an interpretation as a measure for the strength of correlation between the orthogonal electric field components of equal intensity at a single point [3, 8, 17], although this connection is often left without the attention it would deserve. The observation, in particular, suggests that in analogy to the two-point correlations or coherence which show up in interference, the one-point correlations represented by the degree of polarization could likewise be rendered visible in a proper interferometric arrangement.

In this work, we consider statistically stationary, quasi-monochromatic, partially polarized, partially spatially and temporally coherent optical beams in Young’s two-pinhole interference. We establish a time-domain electromagnetic interference law and demonstrate that the electromagnetic space–time coherence at the apertures may appear as the intensity and/or polarization state modulation on the observation screen. The variations are characterized by the traditional polarization (one-point) Stokes parameters whose modulation contrasts or visibilities are shown to be given by the magnitudes of the related normalized coherence (two-point) Stokes parameters at the apertures. This result explicitly illustrates the interplay between polarization and electromagnetic coherence and provides physical insight for the two-point Stokes parameters. We, in particular, then prove that when the beams at the pinholes are identical random fields, for instance as obtained by dividing the incident beam into two, the sum of visibilities pertaining to the polarization modulation gives directly the degree of polarization. This establishes a novel, interferometric interpretation for the degree of polarization as a quantity that measures a beam’s ability to produce polarization modulation when it interferes with itself. The result is a consequence of the electromagnetic interference law and we verify it experimentally. For this purpose, we show how the information on the polarization modulation is transferred by suitable wave plates to the intensity modulation prior to detection.

This paper is organized as follows. In section 2 we derive the time-domain electromagnetic interference law and introduce the interferometric interpretation for the degree of polarization. In section 3 we present a method to create a beam with desired degree of polarization and demonstrate how the polarization modulations occurring in interference are shifted to the intensity modulations. Section 4 then contains the experimental results that demonstrate the validity of the interferometric interpretation. In section 5 we summarize the main conclusions of the work.

2. Polarization in electromagnetic interference

A realization of a random, statistically stationary, quasi-monochromatic, electromagnetic beam field at a point \( \mathbf{r} \) and at time \( t \) is given by the column vector \( \mathbf{E}(\mathbf{r}, t) = [E_x(\mathbf{r}, t), E_y(\mathbf{r}, t)]^T \), where the superscript \( T \) denotes the transpose. The field may be non-uniformly polarized and it is taken to propagate along the \( z \) direction. At a single point the polarization characteristics of a partially polarized, partially spatially and temporally coherent beam are represented by the degree of polarization and the polarization state of the polarized part of the field. This information is contained in the \( 2 \times 2 \) polarization matrix or the equal-time coherence matrix
given as [2–4]
\[
J(r) = \left\langle E^*(r, t)E^T(r, t) \right\rangle,
\]
where the angular brackets stand for the time average (or ensemble average since the field is ergodic) and the asterisk denotes complex conjugation. The matrix \(J(r)\) is Hermitian, non-negative definite, and independent of time due to stationarity.

The traditional Stokes parameters, which we here call also the polarization Stokes parameters or the single-point Stokes parameters for reasons that will become clear shortly, provide a physically insightful way to treat the beam polarization. Such (real) quantities are defined as [2–4]
\[
S_0(r) = J_{xx}(r) + J_{yy}(r),
\]
\[
S_1(r) = J_{xx}(r) - J_{yy}(r),
\]
\[
S_2(r) = J_{xy}(r) + J_{yx}(r),
\]
\[
S_3(r) = i[J_{xy}(r) - J_{yx}(r)].
\]

Obviously \(S_0(r)\) is the intensity while the other three describe the polarization state of the polarized part of the field. The latter result follows from the fact that \(J(r)\) of an unpolarized light is proportional to the unit matrix for which the Stokes parameters \(S_n(r)\), with \(n \in (1, 2, 3)\), are zero. The parameters \(S_1(r)\), \(S_2(r)\) and \(S_3(r)\) describe, respectively, the intensity differences of the \(x\)- and \(y\)-polarized lights, the \(+45^\circ\) and \(−45^\circ\) polarized lights, and the right-hand circularly polarized and left-hand circularly polarized lights. Often the Stokes parameters are normalized with the intensity via
\[
s_n(r) = \frac{S_n(r)}{S_0(r)}, \quad n \in (0, \ldots, 3),
\]
for which \(s_0(r) = 1\), whereas \(|s_n(r)| \leq 1\), for \(n \in (1, 2, 3)\). The degree of polarization of the beam expressing the fraction of light energy contained in the polarized part is in terms of the normalized Stokes parameters written as [2–4]
\[
P(r) = \left[ \sum_{n=1}^{3} s_n^2(r) \right]^{1/2}.
\]

This quantity is bounded between \(0 \leq P(r) \leq 1\) with the lower and upper limits corresponding to fully unpolarized and fully polarized field at point \(r\), respectively.

Consider an electromagnetic version of Young’s interferometer in which a beam field is incident on an opaque screen \(A\) containing two pinholes at locations \(r_1\) and \(r_2\), as illustrated in figure 1. As is customary, the openings are assumed to be so large that their rims do not significantly alter the field at the holes and that the diffraction angles are small, but the holes are small enough to justify uniform polarization over them. The diffracted field is observed in the far zone on the screen \(B\) at a point \(r\) with distances \(R_1\) and \(R_2\) from \(r_1\) and \(r_2\), respectively.

The elements of the polarization matrix on \(B\) can be found by employing for both orthogonal field components the steps outlined for scalar fields in many textbooks [3, 4] or by following the electromagnetic frequency-domain analysis given in [18]. The field on \(B\) can be
written as
\[
E(r, t) = K_1 E(r_1, t - t_1) + K_2 E(r_2, t - t_2),
\]
where \(K_1\) and \(K_2\) are pure complex numbers specified by the geometry and the center wavelength, while \(E(r, t)\) and \(t_i = R_i/c\) are, respectively, the field in the aperture and the propagation time of light from the hole to the observation point, \(i \in (1, 2)\).

After straightforward developments the polarization Stokes parameters at the observation plane take on the forms
\[
S_n(r) = S_n^{(1)}(r) + S_n^{(2)}(r) + 2\left[S_0^{(1)}(r)S_0^{(2)}(r)\right]^{1/2} \left|\gamma_n(r_1, r_2, \tau)\right| \times \cos\left[\alpha_n(r_1, r_2, \tau) - \vec{k}(R_1 - R_2)\right], \quad n \in (0, ..., 3),
\]
where \(S_n^{(i)}\) is the Stokes parameter at \(B\) when only the pinhole at \(r_i\) is open. In addition, \(\tau = t_1 - t_2\) is the time difference in light propagation from the pinholes to \(r\), \(\vec{k}\) is the mean wave number, \(\alpha_n(r_1, r_2, \tau) = \arg[k_n(r_1, r_2, \tau)] + \vec{\omega}\tau\), where \(\arg[a]\) is the phase of complex number \(a\) and \(\vec{\omega}\) is the mean angular frequency. The quantity \(\alpha_n(r_1, r_2, \tau)\) is introduced to separate the spatially slowly and rapidly varying parts in \(\gamma_n(r_1, r_2, \tau)\) of a quasi-monochromatic light as will be discussed shortly. Moreover
\[
\gamma_n(r_1, r_2, \tau) = \frac{S_n(r_1, r_2, \tau)}{[S_0(r_1)S_0(r_2)]^{1/2}}, \quad n \in (0, ..., 3),
\]
where
\[
S_0(r_1, r_2, \tau) = \Gamma_{xx}(r_1, r_2, \tau) + \Gamma_{yy}(r_1, r_2, \tau),
\]
\[
S_1(r_1, r_2, \tau) = \Gamma_{xx}(r_1, r_2, \tau) - \Gamma_{yy}(r_1, r_2, \tau),
\]
\[
S_2(r_1, r_2, \tau) = \Gamma_{yy}(r_1, r_2, \tau) + \Gamma_{xx}(r_1, r_2, \tau),
\]
\[
S_3(r_1, r_2, \tau) = i\left[\Gamma_{yx}(r_1, r_2, \tau) - \Gamma_{xy}(r_1, r_2, \tau)\right],
\]
with
\[
\Gamma_{ij}(r_1, r_2, \tau) = \left\langle E_i^*(r_1, t)E_j(r_2, t + \tau) \right\rangle, \quad (i, j) \in (x, y),
\]
being the elements of the $2 \times 2$ electric mutual coherence matrix $
abla(\mathbf{r}_1, \mathbf{r}_2, \tau) = \langle \mathbf{E}^*(\mathbf{r}_1, t) \mathbf{E}^T(\mathbf{r}_2, t + \tau) \rangle$. The quantities $S_n(\mathbf{r}_1, \mathbf{r}_2, \tau)$ are the time-domain coherence or two-point Stokes parameters whose frequency-space analogues have previously been introduced and analyzed [18–20], and $\gamma_n(\mathbf{r}_1, \mathbf{r}_2, \tau)$ are the versions normalized with the intensities at the pinholes. Instead of using the elements of $\nabla(\mathbf{r}_1, \mathbf{r}_2, \tau)$ to describe electromagnetic coherence, the coherence Stokes parameters provide an alternative but equivalent set of four quantities for this purpose. The quantity $S_0(\mathbf{r}_1, \mathbf{r}_2, \tau)$ represents, with time difference $\tau$, the sum of the correlations pertaining to the electric $x$ and $y$ components at $\mathbf{r}_1$ and $\mathbf{r}_2$. In addition, $S_1(\mathbf{r}_1, \mathbf{r}_2, \tau)$ corresponds to the difference of those same correlations, while $S_2(\mathbf{r}_1, \mathbf{r}_2, \tau)$ and $S_3(\mathbf{r}_1, \mathbf{r}_2, \tau)$ describe, respectively, the correlation difference of the $+45^\circ$ and $-45^\circ$ polarized lights, and the right-hand circularly and left-hand circularly polarized lights at two space-time points [20]. In the case of $\tau = 0$ and $\mathbf{r}_1 = \mathbf{r}_2$ the coherence (two-point) Stokes parameters reduce to the polarization (one-point) Stokes parameters of equations (2)–(5), while the normalized counterparts coincide with those in (6).

We refer to expression (9) as the electromagnetic time-domain interference law. Since the observation plane is far from the screen $A$, the Stokes parameters $S_n^0(\mathbf{r}_1)$, $n \in (1, 2, 3)$ and $i \in (1, 2, 3)$, in (9) are slowly varying with position near the optical axis. Likewise, due to quasi-monochromacity $\gamma_n(\mathbf{r}_1, \mathbf{r}_2, \tau) \propto \exp(-i\omega\tau)$ and, consequently, the quantities $|y_n(\mathbf{r}_1, \mathbf{r}_2, \tau)|$ and $\alpha_n(\mathbf{r}_1, \mathbf{r}_2, \tau)$ are essentially constant over the region, where $|\tau|$ is much less than the coherence time of light. It follows, in complete analogy to the scalar case, where only the intensity is of interest, that all four polarization Stokes parameters can be modulated at screen $B$ and, due to the term $\vec{k}(R_1 - R_2) = \vec{\omega}\tau$ in equation (9), the variations exhibit sinusoidal fringes with the location determined by $\alpha_n(\mathbf{r}_1, \mathbf{r}_2, \tau)$. The modulation contrasts or visibilities of fringes can be quantified with the definition

$$V_n(\mathbf{r}) = \frac{\max [S_n(\mathbf{r})] - \min [S_n(\mathbf{r})]}{\max [S_0(\mathbf{r})] + \min [S_0(\mathbf{r})]}, \quad n \in (0, \ldots, 3),$$

where $\max$ and $\min$ denote the maximum and minimum near $\mathbf{r}$. When the intensities at the openings are the same [$S_0(\mathbf{r}_1) = S_0(\mathbf{r}_2)$], the visibilities assume the forms

$$V_n(\mathbf{r}) = |\gamma_n(\mathbf{r}_1, \mathbf{r}_2, \tau)|, \quad n \in (0, \ldots, 3),$$

where $\tau$ is determined by the distances of the observation point $\mathbf{r}$ from $\mathbf{r}_1$ and $\mathbf{r}_2$. The above equation implies that the visibility or contrast of modulation for the polarization Stokes parameter $S_n(\mathbf{r})$ is specified by the magnitude of the related normalized coherence Stokes parameter $\gamma_n(\mathbf{r}_1, \mathbf{r}_2, \tau)$ at the pinholes. We therefore conclude that the spatial and temporal coherence at the openings do not manifest themselves only as intensity fringes [21, 22] but lead also, or only, to polarization-state modulation on the observation plane. The sum of the squared visibilities gives the squared time-domain degree of electromagnetic coherence introduced in [23], i.e.

$$\gamma^2(\mathbf{r}_1, \mathbf{r}_2, \tau) = \frac{1}{2} \sum_{n=0}^3 V_n^2(\mathbf{r}) = \frac{\text{tr} [\nabla(\mathbf{r}_1, \mathbf{r}_2, \tau)\nabla(\mathbf{r}_2, \mathbf{r}_1, -\tau)]}{\text{tr} [\nabla(\mathbf{r}_1, \mathbf{r}_1, 0)] \text{tr} [\nabla(\mathbf{r}_2, \mathbf{r}_2, 0)]},$$

which is bounded as $0 \leq \gamma(\mathbf{r}_1, \mathbf{r}_2, \tau) \leq 1$. The factor $1/2$ in the middle is introduced to ensure the correct upper limit.
The possibility of polarization modulation is an essential feature in electromagnetic interference which naturally does not enter into considerations when the analysis is restricted within scalar framework. Equation (17) may also be regarded as an example of how electromagnetic coherence affects polarization, i.e., which part of the electric mutual coherence matrix contributes to a modulation of a certain polarization Stokes parameter. An opposite example demonstrating the influence of polarization onto electromagnetic coherence is found in the context of the electromagnetic van Cittert–Zernike theorem [24].

Assume next that the incident, partially polarized random beam is divided by a non-polarizing beam splitter into two identical replicas which are separately directed to the pinholes at \( r_1 \) and \( r_2 \). The fields at the holes are hence identical random processes corresponding to some position \( r_0 \) of the original undivided field. In this case, \( \mathbf{E}(r_1, t) = \mathbf{E}(r_2, t) = \mathbf{E}(r_0, t) \) and for regions close to the axis of interferometer (\( \tau \approx 0 \)) the coherence Stokes parameters of (11)–(14) reduce to the traditional polarization Stokes parameters in (2)–(5). Consequently, the visibilities of the Stokes-parameter modulations of (16) become identical with the normalized polarization Stokes parameters given in (6), i.e.

\[
V_n(r) = \left| s_n(r_0, 0) \right| = \begin{cases} 1, & \text{for } n = 0, \\ \left| s_n(r_0) \right|, & \text{for } n \in (1, 2, 3). \end{cases}
\]  

Notice that the visibilities characterizing the polarization state are different for different points \( r_0 \) of the original, generally non-uniformly polarized beam. However, since the fields at the apertures are identical random processes and hence in phase, the fringe patterns produced by the \( x \)- and \( y \)-polarized components are in the same position and the total intensity modulation occurs with maximal fringe visibility in all cases. Recalling equation (7), the degree of polarization of the incident beam can be written as

\[
P(r_0) = \left[ \sum_{n=1}^{3} V_n^2(r) \right]^{1/2}. \]  

This result establishes a new, interferometric interpretation for the degree of polarization of beam fields as an ability of a beam to generate polarization modulation when it is allowed to interfere with itself. It is a consequence (prediction) of the electromagnetic interference law of equation (9) and we present an experimental verification of this interpretation in later sections. The interferometric view implies that no polarization modulation takes place for an unpolarized beam, whereas for a fully polarized light the sum of modulations is maximal. In the latter case, the polarization state determines which of the Stokes parameters are modulated. We remark that the above interferometric interpretation is fundamentally different from that put forward very recently in [25], where the degree of polarization of classical light beams is expressed as a sum of purity and distinguishability with the latter given by a visibility in a Mach–Zehnder interferometer.

The connection of the degree of polarization to interference is intuitive since the degree characterizes the correlation between orthogonal equal-intensity electric field components at a single space–time point and in traditional interference the two-point correlations are manifested in interference. Indeed, when the intensities of the field components are the same, a situation which can always be found by rotating the coordinate frame around the propagation direction, the degree of polarization is equal to the degree of correlation between the orthogonal components [3, 8]. We may thus establish a consistent view of various degree quantities of
optical coherence theory. It is well known that the scalar degree of coherence describes the visibility of intensity modulation in interference, or the strength of field correlations at two space-time points [3, 4]. The electromagnetic degree of coherence describes the visibility of intensity and polarization modulation in interference, or the strength of correlations of all electric field components at two space-time points [23, 26]. In a similar fashion the degree of polarization describes the visibility of polarization modulation when a field interferes with itself, or the strength of correlation of orthogonal field components at a single space-time point.

3. Measurement setup

3.1. Generation of partially polarized beam

An optical beam having a desired degree of polarization can be created with the delay-line setup depicted in figure 2. A linearly polarized beam from a He–Ne laser operating at the wavelength of 633 nm is incident on a polarizing beam splitter. The polarization direction of the incoming light is at an angle of +45° with respect to the x axis and hence the two output beams have equal intensities but one is x-polarized and the other is y-polarized. The y-polarized beam is delayed by \( \tau_d \), a time significantly exceeding the laser’s coherence time, leading to non-correlation between the beams. The delayed component is also subjected to a half-wave plate whose fast axis is at an angle of \( \theta \), where \( 0 \leq \theta \leq \pi/4 \), with respect to the y direction. The extreme cases thus correspond to orthogonal and parallel beams emerging from the two arms, respectively. Finally, the two beams are recombined with a non-polarizing beam splitter.

The total field after recombination is \( \mathbf{E}(\mathbf{r}, t) = \mathbf{E}_1(\mathbf{r}, t) + \mathbf{E}_2(\mathbf{r}, t) \), where

\[
\mathbf{E}_1(\mathbf{r}, t) = \frac{1}{2} A(\mathbf{r}, t) \hat{x},
\]

\[
\mathbf{E}_2(\mathbf{r}, t) = \frac{1}{2} A(\mathbf{r}, t + \tau_d) \sin 2\theta \hat{x} + \frac{1}{2} A(\mathbf{r}, t + \tau_d) \cos 2\theta \hat{y},
\]

are the fields from the two arms. In addition, \( A(\mathbf{r}, t) \) is the random scalar amplitude of the incident laser beam while \( \hat{x} \) and \( \hat{y} \) are unit vectors along the x and y directions, respectively. The factor 1/2 accounts for the fact that in two passages through the beam splitters a fraction 1/4 of the energy is retained. Making use of (7) together with the fact that \( \langle A^\dagger(\mathbf{r}, t) A(\mathbf{r}, t + \tau_d) \rangle = 0 \),
the degree of polarization for the total field \( E(r, t) \) takes on a position-independent form

\[
P = \sin 2\theta,
\]

where \( 0 \leq \theta \leq \pi/4 \). Thus, by varying the orientation of the wave plate the degree of polarization can be tuned to any desired value between zero and one. The polarization-state changes of the polarized part of the beam are not relevant for our work and they are not considered.

### 3.2. Detection of polarization modulation

According to the electromagnetic interference law of equation (9) all four Stokes parameters may be modulated in Young’s interferometer and, in view of (19), the modulation contrasts are specified by the related normalized coherence Stokes parameters at the pinholes. The visibility for \( S_0(r) \) is found by a direct intensity measurement. However, the modulations of \( S_1(r) \), \( S_2(r) \), and \( S_3(r) \) that characterize the polarization state must be converted into intensity modulation prior to detection. This is achieved by inserting suitable wave plates into the two pinholes [26] (see figure 3). The Jones matrix of a wave plate with the fast axis along the \( x \) direction is [2]

\[
T_C(\delta) = \begin{bmatrix} 1 & 0 \\ 0 & \exp(-i\delta) \end{bmatrix},
\]

where the phases \( \delta = \pi/2 \) and \( \delta = \pi \) correspond to a quarter-wave plate and a half-wave plate, respectively. The effect of a wave plate whose fast axis is at an angle \( \varphi \) with respect to the \( x \) direction (taken positive counterclockwise) is obtained by first rotating the field vector by \(-\varphi\), passing the field through a wave plate with the fast axis along the \( x \) direction, and then rotating the field back by the angle \( \varphi \). The matrix

\[
T_R(\varphi) = \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix}
\]

describes the rotation of a electric field vector by \( \varphi \).

To find the visibility \( V_1(r) \) we place a half-wave plate in front of the pinhole \( r_2 \) with the fast axis parallel to the \( x \) axis, as shown in figure 3(a). The fields emanating from the pinholes are

\[
\begin{align*}
{\text{(a)}} & & \text{WP1} & & \text{r}_2 & & \text{r}_1 \\
{\text{(b)}} & & \pi/4 & & \text{WP1} & & \text{r}_2 & & \text{r}_1 \\
{\text{(c)}} & & \text{WP1} & & \text{r}_2 & & \text{r}_1 \\
\end{align*}
\]

**Figure 3.** Wave plates required to transfer the polarization modulation into the intensity modulation. WP1 and WP2 represent a half-wave plate and a quarter-wave plate, respectively. (a) For \( S_1(r) \) a half-wave plate with the fast axis along the \( x \) direction is put in front of the pinhole \( r_2 \). (b) The parameter \( S_2(r) \) is found as in (a) but the angle of the half-wave plate is \( \pi/4 \). (c) In the case of \( S_3(r) \), quarter-wave plates with the fast axes in the \( y \) direction are placed onto both pinholes and, in addition, a half-wave plate with the fast axis at an angle of \( \pi/4 \) to the \( x \) axis is inserted into \( r_2 \).
therefore given by
\begin{align}
E'(\mathbf{r}_1, t) &= E(\mathbf{r}_1, t), \\
E'(\mathbf{r}_2, t) &= T_C(\pi)E(\mathbf{r}_2, t).
\end{align}
\tag{26}
\tag{27}

Here and henceforth the primed and unprimed quantities refer to the fields with and without any additional wave plate, respectively. The normalized coherence Stokes parameter characterizing the intensity modulation takes on the form
\begin{align}
\gamma_0'(\mathbf{r}_1, \mathbf{r}_2, \tau) &= \frac{\left\langle E_x^*(\mathbf{r}_1, t)E_x'(\mathbf{r}_2, t + \tau) \right\rangle + \left\langle E_y^*(\mathbf{r}_1, t)E_y'(\mathbf{r}_2, t + \tau) \right\rangle}{\left[ S_0(\mathbf{r}_1)S_0(\mathbf{r}_2) \right]^{1/2}} \\
&= \frac{\left\langle E_x^*(\mathbf{r}_1, t)E_x'(\mathbf{r}_2, t + \tau) \right\rangle - \left\langle E_y^*(\mathbf{r}_1, t)E_y'(\mathbf{r}_2, t + \tau) \right\rangle}{\left[ S_0(\mathbf{r}_1)S_0(\mathbf{r}_2) \right]^{1/2}} \\
&= \gamma_1(\mathbf{r}_1, \mathbf{r}_2, \tau),
\end{align}
\tag{28}

where we used the fact that a wave plate does not modify the intensity. Recalling (17) the result above shows that the visibility of the intensity fringes is $V_0'(\mathbf{r}) = V_1(\mathbf{r})$.

In full analogy, $V_2(\mathbf{r})$ is found by inserting into $\mathbf{r}_2$ a half-wave plate whose fast axis makes an angle of $\pi/4$ with respect to the $x$ axis (see figure 3(b)). We thus write
\begin{align}
E'(\mathbf{r}_2, t) &= T_R(\pi/4)T_C(\pi)T_R(-\pi/4)E(\mathbf{r}_2, t),
\end{align}
\tag{29}

which leads to
\begin{align}
\gamma_0'(\mathbf{r}_1, \mathbf{r}_2, \tau) &= \frac{\left\langle E_x^*(\mathbf{r}_1, t)E_y'(\mathbf{r}_2, t + \tau) \right\rangle + \left\langle E_y^*(\mathbf{r}_1, t)E_x'(\mathbf{r}_2, t + \tau) \right\rangle}{\left[ S_0(\mathbf{r}_1)S_0(\mathbf{r}_2) \right]^{1/2}} \\
&= \gamma_2(\mathbf{r}_1, \mathbf{r}_2, \tau),
\end{align}
\tag{30}

and hence $V_0'(\mathbf{r}) = V_2(\mathbf{r})$. The visibility $V_3(\mathbf{r})$ is accessed by putting quarter-wave plates onto both pinholes with the fast axes parallel to the $y$ direction and, into the aperture at $\mathbf{r}_2$, also a half-wave plate with the fast axis at an angle $\pi/4$ to the $x$ direction. The arrangement is explicitly depicted in figure 3(c) (note the order of the elements). The fields at the apertures are written as
\begin{align}
E'(\mathbf{r}_1, t) &= T_R(\pi/2)T_C(\pi/2)T_R(-\pi/2)E(\mathbf{r}_1, t), \\
E'(\mathbf{r}_2, t) &= \left[ T_R(\pi/4)T_C(\pi)T_R(-\pi/4) \right] \\
&\quad \times \left[ T_R(\pi/2)T_C(\pi/2)T_R(-\pi/2) \right]E(\mathbf{r}_2, t),
\end{align}
\tag{31}
\tag{32}

indicating that
\begin{align}
\gamma_0'(\mathbf{r}_1, \mathbf{r}_2, \tau) &= \frac{i\left[ \left\langle E_x^*(\mathbf{r}_1, t)E_y'(\mathbf{r}_2, t + \tau) \right\rangle - \left\langle E_y^*(\mathbf{r}_1, t)E_x'(\mathbf{r}_2, t + \tau) \right\rangle \right]}{\left[ S_0(\mathbf{r}_1)S_0(\mathbf{r}_2) \right]^{1/2}} \\
&= -\gamma_3(\mathbf{r}_1, \mathbf{r}_2, \tau),
\end{align}
\tag{32}

thus leading to $V_0'(\mathbf{r}) = V_3(\mathbf{r})$. The minus sign in equation (33) appears as a half-period shift in the fringe pattern but does not affect the visibility.
The above discussion demonstrates how the Stokes-parameter modulations that would appear on the observation screen in Young’s interferometer are transferred into intensity modulations. In this work, we are interested in the polarization modulation when a beam interferes with itself. In order to find $V_1(r)$, $V_2(r)$, and $V_3(r)$ in this case, we employ the setup shown in figure 4 to insert the wave plates in front of the pinholes. The field $E(r, t)$ created with the delay-line system of figure 2 is divided into two parts by a non-polarizing beam splitter. The two beams are subject to appropriate wave plates as discussed above. Finally the beams are made parallel with suitable spatial separation and directed to the pinholes of Young’s interferometer.

4. Experimental results

We next demonstrate experimentally the interferometric interpretation for the degree of polarization as stated in equation (20). The optical beam is created with the delay-line setup shown in figure 2, where the degree of polarization is adjusted by rotating the wave plate according to (23). The beam is subsequently divided into two equal parts and directed into the pinholes of Young’s interferometer, as depicted in figure 4. The apertures are circular with ~30 μm diameter and their center-to-center distance is 2 mm. This geometry ensures that the beam at one pinhole does not significantly affect the field at the other hole. Information on the polarization-state modulation is transferred into the intensity modulation by inserting suitable wave plates onto the apertures (see figures 3 and 4).

Consider first a fully unpolarized beam created from a completely polarized He–Ne laser (wavelength 633 nm) with a half-wave plate at an angle $\theta = 0$ in the upper arm. The beams in both arms are adjusted in such a way that their intensities are equal on recombination. Non-polarization is confirmed by employing a conventional polarimetric system that consists of a linear polarizer and a quarter-wave plate ([3] section 10.9, [4] section 6.2). The same method is used later to obtain the references also for the other values of the degree of polarization. A CCD detector (pixel width 7 μm) is utilized to observe the intensity and polarization distributions over the beam area. As an example, figure 5 shows the distributions of the intensity and the degree of polarization on the beam cross section. The field is clearly almost uniformly unpolarized in regions, where the intensity is high. The beam is divided into
two identical parts which are directed onto the pinholes so that the intensity of light over them is maximal.

Figure 6 illustrates, for an unpolarized beam, the Stokes-parameter distributions in Young’s interferometer transferred into intensity variations as described in section 3.2. Near the optical axis, corresponding to the centers of the curves, we can identify the maximum and minimum values of all the parameters $S_n$, $n \in (0, ..., 3)$. These values are indicated in the figure by the red and blue horizontal dashed lines, respectively, and they are listed in table 1 together with the corresponding modulation contrasts $V_n$, $n \in (0, ..., 3)$, given by equation (16). The visibility of the intensity fringes, $V_0 \approx 0.943$, is near unity as it should be independently of the degree of polarization (recall equation (19)). The visibilities of the three other Stokes parameters that characterize the polarization state, viz, $V_1 \approx 0.016$, $V_2 \approx 0.028$, and $V_3 \approx 0.049$, are close to zero. According to (20), these values give for the degree of polarization the result $P \approx 0.059$, which is close to $P \approx 0.044$ acquired by the traditional detection with a wave plate and a polarizer. The above procedure is repeated for a partially polarized beam with $P = 0.5$ which according to (23) is obtained when $\theta = 15^\circ$ for the delay-line wave plate. Figure 7 shows the Stokes-parameter modulations in this case and table 1 lists the minimum and maximum values at the axis of interferometer, as well as the corresponding visibilities. The resulting interferometric degree of polarization is $P \approx 0.508$. Again, this closely matches the reference value of $P = 0.515$ found by traditional means.

Figure 8 shows the behavior of the degree of polarization as a function of $\theta$ that specifies the orientation of the wave plate in the delay line. The curves correspond to the theoretical result of (23) (black solid line), traditional measurements with a wave plate and a polarizer (red triangles), and the interferometric measurements introduced in this work (blue dots). In the experiments, $P(\theta)$ of the incoming beam is increased from zero to unity with the step of 0.10 (approximately). We see that the values found through interferometric detection are in good agreement with those obtained theoretically or by the conventional approach. This is especially true for $P$ values between 0.1 and 0.7. In practise it may be difficult to get zero and unit visibilities and hence the interferometric results could slightly differ from the others at the edges. In particular, the incident beam is only approximately uniformly (partially) polarized and it is possible that the fields at the small apertures may not correspond to exactly the same spatial point of the beam. In addition, the non-idealities of optical components (beam splitters, wave
plates, mirrors) may create extra intensity and phase differences for beams traveling to different pinholes. However, the former have only a small effect on the visibilities, while to compensate the latter we employed Babinet–Soleil compensator in front of one of the pinholes. In the experiments the compensator was adjusted to produce minimum visibility cases. The intensities are in arbitrary units and the width of the detection area is 2.1 cm.

**Table 1.** Nearby minimum and maximum values of the Stokes parameters and the related visibilities measured at the axis of Young’s interferometer. The data are for an unpolarized beam ($P = 0$) and for a partially polarized beam ($P = 0.5$). In the case of $S_1$, $S_2$, and $S_3$ the modulations are transformed into $S_0$ variations.

| Stokes parameter | Min  | Max  | Visibility | Min  | Max  | Visibility |
|------------------|------|------|------------|------|------|------------|
| $S_0$            | 0.0175 | 0.5974 | 0.943      | 0.0184 | 0.5686 | 0.937      |
| $S_1$            | 0.2983 | 0.3079 | 0.016      | 0.2256 | 0.3606 | 0.230      |
| $S_2$            | 0.2920 | 0.3165 | 0.028      | 0.2162 | 0.3715 | 0.265      |
| $S_3$            | 0.2865 | 0.3165 | 0.049      | 0.1864 | 0.4020 | 0.367      |

**Figure 6.** Illustration of the Stokes-parameter modulations in Young’s interferometer in the case of identical unpolarized beams at the apertures. In (a) the intensity ($S_0$) modulation is shown, whereas (b)–(d) represent, in terms of intensity, the variations of $S_1$, $S_2$, and $S_3$, respectively. The blue and red horizontal dashed lines indicate the minimum and maximum intensity, respectively, in the immediate vicinity of the optical axis. The insets show magnifications of the fringes at the axis for the low-visibility cases. The intensities are in arbitrary units and the width of the detection area is 2.1 cm.
Figure 7. Illustration of the Stokes parameters in Young’s interferometer when identical beams with the degree of polarization of 0.5 are incident onto the pinholes. In (a) the intensity ($S_0$) modulation is shown, whereas (b)–(d) represent, in terms of intensity, the variations of $S_1$, $S_2$, and $S_3$, respectively. The blue and red horizontal dashed lines indicate the minimum and maximum intensity, respectively, in the immediate vicinity of the optical axis. The intensities are in arbitrary units and the width of the detection area is 2.1 cm.

Figure 8. Degree of polarization $P$ as a function of the half-wave plate’s orientation angle $0 \leq \theta \leq 45^\circ$ in the delay line: theoretical curve of equation (23) (solid black line), traditional polarimetric measurements (red triangles), and results obtained from the interferometric measurements (blue dots).
5. Conclusions

We showed theoretically and experimentally that the degree of polarization of optical beams can be considered as an ability of light to produce polarization modulation in self-interference. This is a new interpretation of the degree of polarization and it follows directly from the electromagnetic interference law. The formulation and experiments in this work are in the time domain, but analogous results are naturally found in the frequency domain for the spectral degree of polarization [27–29]. The results of this research illustrate and demonstrate experimentally the role of partial polarization in electromagnetic coherence and interference and are obviously important for many applications dealing with physical optics.

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