Thermal Quarkonium Mass Shift from Euclidean Correlators

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(Dated: March 20, 2019)

Brambilla, Escobedo, Soto, and Vairo have derived an effective description of quarkonium with two parameters; a momentum diffusion term and a real self-energy term. We point out that the real self-energy term can be expressed directly in terms of Euclidean electric field correlators along a Polyakov line. This quantity can be directly studied on the lattice without the need for analytical continuation. We show that existing Minkowski-space calculations of this correlator correspond with the known NLO Euclidean value of the relevant electric field two-point function.

Keywords: quarkonium, quark-gluon plasma, electric-field correlator

I. INTRODUCTION

Quarkonium (bound heavy quark-antiquark states) are an intriguing probe of the quark-gluon plasma [1]. Originally proposed by Matsui and Satz [2], the suppression of quarkonia has remained an active topic of experimental [3–8] and theoretical [9–15] investigation ever since. The central idea is that a thermal medium tends to break the spin-orbit interaction for quarkonia. At lowest order in pNRQCD, the shift is $\delta m \sim 3/2a_s$, where $a_s$ is the Minkowski time value, both for charmonium at the highest energies there are also important recombination effects from the many open charm quarks in the plasma [16–18]. Recently Brambilla, Escobedo, Soto, and Vairo have derived an effective description of quarkonium with two parameters: a real non-dissipative plasma effect which induces a mass shift in the heavy-quark bound states. At lowest order in pNRQCD, the shift is $\delta m = \frac{3}{2}a_s\gamma$ [10], where $\gamma$ is the following electric-field correlator

$$\gamma = \frac{g^2}{6N_c} \text{Im} \int_{-\infty}^{\infty} ds \langle T E^{a,i}(s,0) E^{a,i}(0,0) \rangle .$$

This correlator has received less attention in the literature, and as we will show, it will actually be much easier to determine via lattice QCD methods than the coefficient $\kappa$. Therefore we will focus in this paper on investigating the coefficient $\gamma$.

In the next section we will show how to analytically continue Eq. (2) to Euclidean time. This leads to a time-integral of a correlator of electric fields along a Polyakov loop. In the remainder of the paper we check this derivation by showing that the known next-to-leading order (NLO) value of $\gamma$, derived by Minkowski space methods in Ref. [10], correctly corresponds to the appropriate integral moment of the known NLO Euclidean correlator from Ref. [34]. This is a nice check of our continuation. However, to us the most interesting result is the possibility of a future nonperturbative determination of $\gamma$ on the lattice.

II. ANALYTIC CONTINUATION OF ELECTRIC-FIELD CORRELATOR

The analytic continuation of the electric-field correlator Eq. (2) is not trivially doable. However, we don’t consider this correlator directly, but instead use the heavy-quark current-current correlator

$$\int_{-\infty}^{\infty} dt \ e^{i\omega t} \int d^3x \ \langle [\hat{J}^{\mu}(t,x), \hat{J}^{\nu}(0,0)] \rangle ,$$

which Eq. (2) originates from. Here $\hat{J}^{\mu} = \bar{\psi} \gamma^\mu \psi$ is the heavy-quark field op-
The two-point functions are defined in the usual
\[ G^{AB}_{E}(\tau) = \sum_{\omega=1}^{\infty} \text{Tr} \left[ U(\beta, \tau) g_B E(\tau, 0) U(\tau, 0) g_B E(0, 0) \right] \] (4)
For the details of the continuation we refer the reader to the original paper \[29\].

In order to analytically continue \( \gamma \), we need to relate the imaginary part of \( \gamma \) with its analytic continuation \( \gamma(\tau) \). Therefore we want to remind the reader of the relation between the imaginary part of a two-point function of two hermitian operators \( A, B \) in real time with the zero Matsubara frequency limit of the corresponding Euclidean correlator, such that

\[ \text{Im} G^{AB}_{E}(\omega=0) = \text{Im} \int_0^\infty dt \, G^{AB}(t) = \int_0^\beta d\tau \, G^{AB}_{E}(\tau) = \tilde{G}_E(\omega_n=0). \] (5)

The two-point functions are defined in the usual way, \( G^{AB}(t) = i \text{Tr} \left\{ \hat{A}(t), \hat{B}(0) \right\} \) and \( G^{AB}_{E}(\tau) = \text{Tr} \left\{ \hat{A}(e^{i\tau}) B(0) \right\} \) with \( \hat{A} \equiv e^{-\beta H} \) the finite-temperature equilibrium density matrix. (Note that \( \int_0^\infty dt \, G^{AB}(t) \) is purely imaginary, since the commutator of Hermitian operators gives twice the imaginary part. Nevertheless, we take the imaginary part explicitly because the finite-frequency transform contains real and imaginary parts.)

We insert two complete sets of energy eigenstates in the definition of the two-point function such that the LHS of Eq. (5) becomes

\[ \int_0^\infty dt \sum_{n,m} \frac{2i}{Z} \text{Re} \left[ A_{nm} B_{nm} \right] e^{-\frac{\beta}{2}(E_n + E_m) t} \times \sinh \left( \frac{\beta(E_n - E_m)}{2} \right) e^{-i(E_n - E_m)n} \]
\[ = \sum_{n,m} \frac{2}{Z} \text{Re} \left[ A_{nm} B_{nm} \right] e^{-\frac{\beta}{2}(E_n + E_m)} \sinh \left( \frac{\beta(E_n - E_m)}{2} \right), \]
where we used the notation \( A_{nm} = \langle n | A(t=0) | m \rangle \).

Using the same procedure on the RHS of Eq. (5) yields

\[ \int_0^\beta d\tau \sum_{n,m} \frac{1}{Z} A_{nm} B_{nm} e^{-\beta E_m} e^{-(E_n - E_m)\tau} \]
\[ = \sum_{n,m} \frac{2}{Z} A_{nm} B_{nm} \sinh \left( \frac{\beta(E_n - E_m)}{2} \right). \]

So as long as \( A_{nm} B_{nm} \) has no imaginary part, which happens if \( A \) and \( B \) are Hermitian operators or if \( A = B^\dagger \), Eq. (5) is true. From this we conclude that the analytic continuation of the mass shift correlator \( \gamma \) is

\[ \gamma = -\int_0^\beta d\tau \, G^{HQ}_{E}(\tau), \] (6)

where the minus sign emerges from different factors of \( i \) in the definition of the color–electric field in real and imaginary time. The main result of this paper is therefore that the thermal effects on the mass shift \( \gamma \) can be determined by a nonperturbative calculation using the vacuum subtracted Euclidean color–electric correlator on the lattice.

To further clarify the need for vacuum subtraction, let us look at the Euclidean color–electric correlator at leading order \( O(g) \). It is obtained trivially by connecting the two chromoelectric fields with a gluon propagator, yielding \(29\)

\[ G^{HQ}_{E,LO}(\tau) = -\frac{g^2 C_F}{3} \int \frac{dk}{k} e^{ik\tau} (D-1) k^2 + k^2, \]

(7)

where \( T \sum_k \tilde{f}_k \), \( \tilde{f}_k \equiv \int dk/(2\pi)^d \) with \( D = d + 1 \) the dimension of spacetime and \( k_n \) the bosonic Matsubara frequency. One could immediately perform the \( \tau \) integration of Eq. (9), obtaining \( \beta \delta \delta_k \), at which point the \( \int_k \) integral would vanish in dimensional regularization (DR). So would, in this scheme, the vacuum contribution, where the \( \tau \) integrations runs from \(-\infty \) to \(+\infty \) and the Matsubara sum is replaced with an integral over a continuous Euclidean frequency \( k_4 \). However, to better illustrate the need for vacuum subtraction in other schemes, such as the lattice, let us instead perform first the Matsubara sum and then the \( \int_k \), which gives \(29\)

\[ G^{HQ}_{E,LO}(\tau) = g^2 C_F \pi^2 T^4 \left[ \frac{\cos^2(\pi T)}{\sin^2(\pi T)} + \frac{1}{3\sin^2(\pi T)} \right]. \]

(8)

The integration of this object over the compactified time direction does not converge. As the integrand diverges as \( \tau^4 \) as \( \tau \to 0 \) and as \( (\beta - \tau)^{-4} \) as \( \tau \to \beta \). But this divergence is ultraviolet, as it comes about when the two \( E \) fields are brought together. It is thus equal to the behavior observed in vacuum, which can be easily obtained from the \( k_4 \) integration, leading to

\[ G^{HQ}_{E,LO}(\tau, T = 0) = \frac{g^2 C_F}{\pi^2 \tau^4}. \]

(9)

Hence, vacuum subtraction in a non-DR scheme takes the form

\[ \gamma = -2 \int_0^{\beta/2} d\tau \left[ G^{HQ}_{E,LO}(\tau) - G^{HQ}_{E,LO}(\tau, T = 0) \right] + 2 \int_0^{\beta/2} d\tau G^{HQ}_{E,LO}(\tau, T = 0) = 0 + O(g^4), \]

(10)

where we have exploited the symmetry of the thermal contribution at \( \tau = \beta/2 \) and that of the vacuum at \( \tau = 0 \).
Ref. [34] from the diagrams above, we notice that the only free contributions that vanish in any D
lighted, dimensional regularization does that automati-
correlator during the calculation. As previously high-
Ref. [34] and the contributing diagrams are shown in
perturbation theory up to next–to–leading order in
The Euclidean color–electric correlator was calculated
imaginary-time integration of the correlator in Eq. (4).

It is precisely a subtraction of this kind that would need
to be performed on the lattice: for all \( \tau < \beta/2 \) values,
one computes the difference between the correlator on
the thermal lattice and the vacuum lattice, and one then
subtracts the integral of the vacuum contribution over
\( \tau > \beta/2 \). In practice, due to the noisy denominator in
Eq. (4), it may be impossible to subtract \( \gamma \) at very low
temperature on the lattice; in practice a subtraction at
a temperature where thermal effects are expected to be
small should be sufficient.

At small separation, where the vacuum and thermal
correlators diverge but the difference stays finite, it may
be difficult to extract the difference with good statistical
power. However, we believe that, while the individual
short-distance values are sensitive to even small amounts
of gradient flow [35–37], the difference should not be.
This is supported by existing analytical studies [38], and
it would be useful to investigate this issue further. We
also refer to [39] for the perturbative \( O(\alpha_s) \) renormaliza-
tion of Eq. (4) in the lattice scheme.

III. NLO INTEGRATION OF EUCLIDEAN CORRELATOR

In this section, we validate our result in a perturba-
tive calculation at next–to–leading order and present the
imaginary-time integration of the correlator in Eq. (4).
The Euclidean color–electric correlator was calculated
in perturbation theory up to next–to–leading order in
Ref. [34] and the contributing diagrams are shown in
Fig. 1. Since we are interested in the thermal contribu-
tions to \( \gamma \), we subtract the vacuum contribution of the
correlator during the calculation. As previously high-
lighted, dimensional regularization does that automatic-
ly, so in the following we will not keep track of scale-
free contributions that vanish in any \( D \).

Using the integral expression of \( G_{ENLO}^{HQ}(\tau) \) obtained
in Ref. [34] from the diagrams above, we notice that the only
\( \tau \) dependence is in the Fourier transform. After applying
the Kronecker delta \( \beta \delta_{k_n} \) arising from the \( \tau \) integration, we obtain that
\[
\gamma_{LO} = - \int_0^\beta d\tau \frac{g^4 C_F}{3} \left[ \frac{1}{2} \left( 3 \right) \right]
\]
where we have introduced these sum integrals
\[
I_1, \quad \tilde{I}_1 = \int k \frac{1}{2} \left( \frac{1}{Q^2(K - Q)^2} \right)_{k_n = 0}^{k_n = 0},
\]
\[
I_2, \quad \tilde{I}_2 = \int k \frac{1}{2} \frac{q_n^2}{Q^2(K - Q)^2} \quad (k_n = 0),
\]
where \( K^2 = k_n^2 + k^2 \). The notation \( \{ Q \} \) represents the
fermionic Matsubara frequencies \( q_n = \pi T(2n + 1) \) for the
fermionic sum integrals, denoted by \( \tilde{I} \). These are directly
related to the bosonic ones via
\[
\sigma_f(T) = T \sum_{\{ q_n \}} f(q_n, \ldots) = 2\sigma_b(T/2) - \sigma_b(T).
\]

The bosonic integrals are easily evaluated as
\[
I_1 = \frac{\Gamma^2(1 - d/2)\zeta(4 - 2d)}{8 \pi^{4-d}} \frac{d-3}{d-2} 0,
\]
\[
I_2 = \frac{(d-2)\Gamma^2(1 - d/2)\zeta(4 - 2d)}{16(d-3)\pi^{4-d}} \frac{d-3}{d-2} - \frac{\zeta(3)}{8\pi^2\beta^3},
\]
where we have used a Feynman parameter for \( I_2 \). The
fermionic counterparts, obtained from Eq. (14), are
\[
I_n \propto \frac{1}{\beta^{2d-3}} \to \tilde{I}_a = I_a \times (4^{2-d} - 1).
\]
Putting everything together, we find
\[
\gamma_{NLO} = -2\alpha_s^2 gT^3 \zeta(3) C_F \left( \frac{4}{3} N_c + N_f \right).
\]
This is the known result from the real–time calculation
of the mass shift in quarkonium given in Ref. [10]. Con-
sequently, we have shown that the imaginary time cor-
relator in Eq. (4) reproduces the leading-order result of
Eq. (2).

Interestingly, it is also possible to check the agreement
beyond LO. Our analysis has so far used unresummed
perturbation theory, which is appropriate when all mo-
menta are of order \( T \) and the Matsubara frequency \( q_n \)
is nonzero. Unlike \( \kappa \), which receives a contribution from
the \( gT \) scale at LO, \( \gamma \) does not. Inspection of Eqs. (9)
and (7) shows that \( k \sim gT \) contributes to \( \gamma \) at \( O(g^0) \).
This contribution is easily obtained by replacing Eq. (7)
with its resummed version. Since the \( \gamma \) integration forces
\( k_n = 0 \), it suffices to use Electrostatic QCD (EQCD) [40–
44], where \( \mathbf{E} = -i \nabla A_0 \). Then the temporal component of the
gauge field gets Debye-screened, yielding
\[
\gamma_{NLO} = \frac{g^2 C_F}{3} \int k \frac{k^2}{k^2 + m_D^2} \frac{\alpha_s C_F m_D^3}{3}.
\]

where $m_D^2 = g^2 T^2 (N_c/3 + N_f/6)$ is the Debye mass. Eq. (18) agrees with Eq. (87) of [10], recalling that $\text{Re} \, \delta V_\gamma(t) |^{10}_{11} = \gamma r^2/2$. This $\mathcal{O}(g^2)$ term is the only NLO contribution to $\gamma$; to the best of our knowledge, this was not observed in the previous literature.

IV. DISCUSSION AND CONCLUSIONS

In this paper, we showed that the coefficient $\gamma$, introduced by Brambilla, Escobedo, Soto and Vairo [14], and representing a thermal medium effect on the heavy quark bound state energies, can be re-expressed in terms of a Euclidean correlation function [4], which is highly amenable to a lattice determination. With the vacuum contributions removed, the time integral of the correlator, [5], should not suffer from divergences and the computational cost should be reasonable if smoothing techniques like gradient flow are employed. We confirmed that the LO results for $\gamma$, evaluated via real-time techniques in Ref. [10], agree with the Euclidean time-integration of the results of Ref. [34], which is a nontrivial check on our derivation. We also obtained the NLO correction to $\gamma$ in Eq. (18).

ACKNOWLEDGMENTS

We thank the Technische Universität Darmstadt and its Institut für Kernphysik, where this work was conducted and where JG was hosted during the early phase of this work. This work was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) Project number 315477589 TRR 211.

NOTE ADDED

As we were finalizing this paper, we became aware of the preprint “Transport coefficients from medium quarkonium dynamics” [15] by N. Brambilla, M. A. Escobedo, A. Vairo and P. Vander Griend. It proposes a way to determine $\gamma$ from the quarkonium spectral function reconstructed from lattice QCD [46]. We thank the authors for sharing their results with us prior to publication and for discussion.

[1] A. Andronic et al., Eur. Phys. J. C76, 107 (2016), arXiv:1506.03981 [nucl-ex].
[2] T. Matsui and H. Satz, Phys. Lett. B178, 416 (1986).
[3] A. Adare et al. (PHENIX), Phys. Rev. C91, 024913 (2015), arXiv:1404.2246 [nucl-ex].
[4] L. Adamczyk et al. (STAR), Phys. Rev. C90, 024906 (2014), arXiv:1310.3563 [nucl-ex].
[5] L. Adamczyk et al. (STAR), Phys. Lett. B735, 127 (2014), Erratum: Phys. Lett.B743,537(2015), arXiv:1312.3675 [nucl-ex].
[6] B. B. Abelev et al. (ALICE), Phys. Lett. B734, 314 (2014), arXiv:1311.0214 [nucl-ex].
[7] J. Adam et al. (ALICE), Phys. Lett. B766, 212 (2017), arXiv:1606.08197 [nucl-ex].
[8] Y. Khachatryan et al. (CMS), Phys. Lett. B770, 357 (2017), arXiv:1611.01510 [nucl-ex].
[9] S. Cao et al., (2018), arXiv:1809.07894 [nucl-th].
[10] N. Brambilla, J. Ghiglieri, A. Vairo, and P. Petreczky, Phys. Rev. D78, 014017 (2008), arXiv:0804.0993 [hep-ph].
[11] N. Brambilla et al., Eur. Phys. J. C71, 1534 (2011), arXiv:1010.5827 [hep-ph].
[12] A. Mocsy, P. Petreczky, and M. Strickland, Int. J. Mod. Phys. A28, 1340012 (2013), arXiv:1302.2180 [hep-ph].
[13] Y. Burnier, O. Kaczmarek, and A. Rothkopf, JHEP 12, 101 (2015), arXiv:1509.07366 [hep-ph].
[14] N. Brambilla, M. A. Escobedo, J. Soto, and A. Vairo, Phys. Rev. D96, 034021 (2017), arXiv:1612.07428 [hep-ph].
[15] N. Brambilla, M. A. Escobedo, J. Soto, and A. Vairo, Phys. Rev. D97, 074009 (2018), arXiv:1711.04515 [hep-ph].
[16] C. Young and E. Shuryak, Phys. Rev. C81, 034905 (2010), arXiv:0911.3080 [nucl-th].
[17] X. Zhao and R. Rapp, Nucl. Phys. A859, 114 (2011), arXiv:1102.2194 [hep-ph].
[18] J.-P. Blaizot, D. De Boni, P. Faccioli, and G. Garberoglio, Nucl. Phys. A946, 49 (2016), arXiv:1503.03857 [nucl-th].
[19] A. Pineda and J. Soto, Nucl.Phys.Proc.Suppl. 64, 428 (1998), arXiv:hep-ph/9707481 [hep-ph].
[20] N. Brambilla, A. Pineda, J. Soto, and A. Vairo, Nucl.Phys. B566, 275 (2000), arXiv:hep-ph/9907240 [hep-ph].
[21] N. Brambilla, A. Pineda, J. Soto, and A. Vairo, Rev.Mod.Phys. 77, 1423 (2005), arXiv:hep-ph/0410047 [hep-ph].
[22] J. Casalderrey-Solana and D. Teaney, Phys. Rev. D74, 085012 (2006), arXiv:hep-ph/0605199 [hep-ph].
[23] B. Svetitsky, Phys. Rev. D37, 2484 (1988).
[24] G. D. Moore and D. Teaney, Phys. Rev. C71, 064904 (2005), arXiv:hep-ph/0412346 [hep-ph].
[25] S. Caron-Huot and G. D. Moore, Phys. Rev. Lett. 100, 052301 (2008), arXiv:0708.4232 [hep-ph].
[26] S. Caron-Huot and G. D. Moore, JHEP 02, 081 (2008), arXiv:0801.2173 [hep-ph].
[27] X. Du, R. Rapp, and M. He, Phys. Rev. C96, 054901 (2017), arXiv:1706.08670 [hep-ph].
[28] C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz, and L. G. Yaffe, JHEP 07, 013 (2006), arXiv:hep-th/0605158 [hep-th].
[29] S. Caron-Huot, M. Laine, and G. D. Moore, JHEP 04, 053 (2009), arXiv:0901.1195 [hep-lat].
[30] H. T. Ding, A. Francis, O. Kaczmarek, F. Karsch, H. Satz, and W. Soldner, Quark matter. Proceed-
ings, 22nd International Conference on Ultra-Relativistic Nucleus-Nucleus Collisions, Quark Matter 2011, Annecy, France, May 23-28, 2011, J. Phys. G38, 124070 (2011), arXiv:1107.0311 [nucl-th].

[31] A. Francis, O. Kaczmarek, M. Laine, and J. Langelage, Proceedings, 29th International Symposium on Lattice field theory (Lattice 2011): Squaw Valley, Lake Tahoe, USA, July 10-16, 2011, PoS LATTICE2011, 202 (2011) arXiv:1109.3941 [hep-lat]

[32] D. Banerjee, S. Datta, R. Gavai, and P. Majumdar, Phys. Rev. D85, 014510 (2012) arXiv:1109.5738 [hep-lat]

[33] A. Francis, O. Kaczmarek, M. Laine, T. Neuhaus, and H. Ohno, Phys. Rev. D92, 116003 (2015) arXiv:1508.04543 [hep-lat]

[34] Y. Burnier, M. Laine, J. Langelage, and L. Mether, JHEP 08, 094 (2010) arXiv:1006.0867 [hep-ph]

[35] R. Narayanan and H. Neuberger, JHEP 03, 064 (2006) arXiv:hep-th/0601210 [hep-th]

[36] M. Lüscher, Commun. Math. Phys. 293, 899 (2010) arXiv:0907.5491 [hep-lat]

[37] M. Lüscher, JHEP 08, 071 (2010) [Erratum: JHEP03,092(2014)] arXiv:1006.4518 [hep-lat]

[38] A. M. Eller and G. D. Moore, Phys. Rev. D97, 114507 (2018) arXiv:1802.04562 [hep-lat]

[39] C. Christensen and M. Laine, Phys. Lett. B755, 316 (2016) arXiv:1601.01573 [hep-lat]

[40] E. Braaten, Phys.Rev.Lett. 74, 2164 (1995) arXiv:hep-ph/9409434 [hep-ph]

[41] E. Braaten and A. Nieto, Phys.Rev. D51, 6990 (1995) arXiv:hep-ph/9501375 [hep-ph]

[42] E. Braaten and A. Nieto, Phys.Rev. D53, 3421 (1996) arXiv:hep-ph/9510408 [hep-ph]

[43] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, Nucl.Phys. B458, 90 (1996) arXiv:hep-ph/9508379 [hep-ph]

[44] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, Nucl.Phys. B503, 357 (1997) arXiv:hep-ph/9704416 [hep-ph]

[45] N. Brambilla, M. A. Escobedo, A. Vairo, and P. Vander Griend, (2019), TUM-EFT preprint 122/18.

[46] S. Kim, P. Petreczky, and A. Rothkopf, JHEP 11, 088 (2018) arXiv:1808.08781 [hep-lat].