The bumblebee field excitations in a cosmological braneworld

L. A. Lessa¹, J. E. G. Silva²(a) and C. A. S. Almeida¹

¹ Universidade Federal do Ceará (UFC), Departamento de Física - Campus do Pici, Fortaleza, CE, C.P. 6030, 60455-760, Brazil
² Universidade Federal do Cariri (UFCA) - Av. Tenente Raimundo Rocha, Cidade Universitária, Juazeiro do Norte, Ceará, CEP 63048-080, Brazil

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Abstract – We investigate the effects of the spacetime curvature and extra dimensions on the excitations of the Lorentz violating bumblebee field $B_\mu$. By assuming the bumblebee field living in a five-dimensional $AdS_5$ bulk, we found an exponential suppression of the bumblebee self-interacting constant $\lambda$ and of the bumblebee vacuum expectation value (VEV) $b_M$ along the fifth extra dimension. The fluctuations of the bumblebee field upon the VEV can be decomposed into transverse and longitudinal modes with respect to $b_M$. For a spacelike $b_M$ along the extra dimension and assuming a thin FRW 3-brane embedded in the $AdS_5$, the transverse mode is localized on the brane. The bulk geometry leads to a propagating and unbound longitudinal mode along the extra dimension. On the brane, the cosmological expansion leads to the decay of the longitudinal mode in time, as $\Delta t \approx H^{-1}$.

Introduction. – In recent decades, the possible Lorentz violating (LV) effects steaming from Planck scale have been extensively studied. Some models in string theory [1], very special relativity [2], noncommutative spacetime [3] and loop quantum gravity [4], among others, allow violation of the Lorentz symmetry. A framework to explore Lorentz violating theories is provided by the Standard Model Extension (SME), wherein LV coefficients lead to violation of the particle Lorentz symmetry [5]. A mechanism for the local Lorentz violating is provided by a spontaneous symmetry breaking potential due to self-interacting tensor fields [6]. The vacuum expectation value (VEV) of these tensor fields yields to background tensor fields, which by coupling to the Standard Model (SM) fields violate the particle local Lorentz symmetry [6–8]. Moreover, the spontaneous Lorentz violation allows the LV terms in the Lagrangian to satisfy the Bianchi identi-
ties, a key property for the gravitational field [6].

A self-interacting vector field, the so-called bumblebee $B_M$ has a VEV $b_M$ which defines a privileged direction in spacetime [9]. In flat spacetimes, causality and stability features of this model were studied, both classically [10–12] and at the quantum level [13,14]. The spontaneous breaking of the Lorentz symmetry leads to the emergence of Nambu-Goldstone (NG) modes and massive modes [10]. For a quadratic potential, in the so-called Kostelecky-Samuel (KS) model in 3+1 dimensions, the fluctuations around the VEV $b_M$ yield to two transverse NG modes and one longitudinal massive mode [10]. Since only the transverse modes are propagating, the photon can be interpreted as a NG mode of the bumblebee field instead of an elementary particle [10,15,16].

In 3+1 curved spacetimes, the effects of the bumblebee VEV $b_M$, non-minimally coupled to the gravitational field, were studied for black holes [17–19], wormholes [20] and cosmology [21]. In higher dimensions, the bumblebee VEV modifies the Kaluza-Klein spectrum for bulk fields [22–24]. For a generalized bumblebee dynamics, an analysis of the fluctuations was performed in ref. [16].

In this work, we are interested in studying the propagation of the bumblebee fluctuations in curved spacetime. We consider the bumblebee living in a five-dimensional anti-de Sitter spacetime, $AdS_5$, with one spacelike extra dimension. Since $AdS_5$ is a maximally symmetric and conformal to a flat Minkowski spacetime, $AdS_5$ allows us to extend some results of the bumblebee field from flat to curved spacetimes. We show that the bulk curvature makes the bumblebee self-coupling constant $\lambda$ dependent on the spacelike extra dimension. Assuming two parallel 3-branes, this leads to an exponential suppression of $\lambda$, as in the Randall-Sundrum model [25,26]. Assuming a homogeneous and isotropic Friedmann-Robertson-Walker
(FRW) metric for a 3-brane embedded in $AdS_5$, for a spacelike VEV in the extra dimension the cosmological expansion produces a dissipative term for the longitudinal mode which decays in a rate $\Delta t \approx H^{-1}$, where $H = \frac{\dot{a}}{a}$ is the Hubble parameter and $a(t)$ is the scale factor. These results reveal that additional modes steaming from spontaneous symmetry breaking of the Lorentz symmetry in the early universe may be suppressed by the cosmological expansion. That seems an expected feature since spontaneous violation of Lorentz symmetry is believed to occur during early universe phase transitions [27–29].

The work is organized as follows. In the next section we present the bumblebee dynamics in five dimensions, obtain the equations of motion for the fluctuations and study the propagation of these modes. In the third section, we investigate the propagation of bumblebee fluctuations along the extra dimension. Furthermore, we study in the fourth section the effects of cosmic expansion on both transverse and longitudinal modes. Final remarks are summarized in the last section. Throughout the text, we adopt the capital Roman indices ($A,B,\ldots = 0,1,2,3,4$) for the spacetime indices of a $(3+1)$D hypersurface called 3-brane.

We start defining a 5D action for bumblebee field $B_M$ as [9,10]
\begin{equation}
S = \int d^5x \left[ - \frac{\alpha}{4} B^{MN} B_{MN} - \frac{\lambda}{2} (B^M B_M \pm b^2)^2 \right],
\end{equation}
where the field-strength tensor $B_{MN}$ of the bumblebee field $B_M$ is defined as $B_{MN} = \partial_M B_N - \partial_N B_M$ and $b^2 = g^{MN} b_M b_N$ is the norm of the bumblebee vacuum expectation value (VEV), $b_M$ is the bumblebee VEV which defines a privileged direction in spacetime. Moreover, $c = \sqrt{-g}$ is the determinant of the bulk metric in the five-dimensional spacetime whose interval is $ds_5^2 = g_{MN} dx^M dx^N$. We consider a fixed background spacetime, i.e., the spacetime is not modified by the bumblebee. In order to keep the bumblebee field with mass dimension one, we introduce the constant $\alpha$ with also mass dimension one, of which we will discuss the details later.

The quadratic potential chosen induces the spontaneous Lorentz violation. Assuming a mass dimension one bumblebee field, then $\lambda$ and $\alpha$ are positive and they have mass dimension one. Further, $b^2$ is a positive constant with squared mass dimension and the $\pm$ sign indicates if $b_M$ is spacelike or timelike. Moreover, the vacuum condition $V = 0$ implies the existence of a vacuum expectation value $\langle B_M \rangle = b_M$ of the form
\begin{equation}
g^{MN} b_M b_N = \pm b^2.
\end{equation}

In order to investigate the effects of spacetime curvature and extra dimensions on the bumblebee field, we adopt a special warped geometry of the form [25,26]
\begin{equation}
ds_5^2 = e^{-2cy} ds_{brane}^2 + dy^2,
\end{equation}
where $e^{-2cy}$ is the so-called warp factor of the Randall-Sundrum model, which depends only on the fifth dimension $y$. The constant $c$ has mass dimension one and it is related to the bulk curvature. For a flat 3-brane, i.e., $ds_{brane} = \eta_{\mu\nu} dx^\mu dx^\nu$, this metric describes an anti-de Sitter spacetime, $AdS_5$, which in the conformal coordinate $z = \frac{\sqrt{c}}{y}$ takes the form [25]
\begin{equation}
ds_5^2 = \frac{l^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2),
\end{equation}
where $l = 1/c$ is the $AdS$ radius and $z = le^{cy}$ is the conformal coordinate. The anti-de Sitter spacetime is a solution of the Einstein equation with a negative cosmological constant of the form $R_{MN} - \frac{2}{d} g_{MN} + \Lambda g_{MN} = 0$, with $\Lambda = -6c^2$. It is upon this symmetric background spacetime that we study the behaviour of the bumblebee fluctuations.

Let us first analyze the effects of bulk curvature effective action in $3+1$ dimensions and the corresponding effective constants $\lambda$ and $\alpha$. Suppose that the bumblebee field and its VEV have a dependence on the conformal extra dimension $z$ of the form $B_M = \tilde{B}_M(z^\mu) \tilde{Y}(z)$ and $b_M = \tilde{b}_M(z^\mu) \tilde{\Psi}(z)$. Thus, the VEV condition (2) leads to
\begin{equation}
b_M = (l/z) \tilde{b}_M(z^\mu),
\end{equation}
where $\tilde{b}_M \tilde{b}^M = \tilde{b}^2$ is constant with respect to the flat 5D Minkowski metric $\eta_{\mu\nu} dx^\mu dx^\nu = \eta_{\mu\nu} dx^\mu dx^\nu + dz^2$. Supposing that the bumblebee field decays as the VEV $b_M$, we obtain $B_M = (l/z) \tilde{B}_M(z^\mu)$.

Let us now consider two parallel and fixed 3-branes, one at the origin and the other at $y = L$, the well-known RS I model [25]. For $b_M = (l/z) \tilde{b}_M(z^\mu)$ and $B_M = (l/z) \tilde{B}_M(z^\mu)$, the potential term reads
\begin{equation}
S_V = -\frac{1}{2} \int dy \lambda e^{-4cy} \int d^4x \sqrt{-g_4} (\eta_{\mu\nu} \tilde{B}_\mu \tilde{B}_\nu \pm b^2)^2.
\end{equation}
Hence, we can define a $y$-dependent self-interacting coupling constant $\lambda(y)$ as
\begin{equation}
\lambda(y) = \lambda e^{-4cy}.
\end{equation}
The behaviour of the coupling constant with the extra dimension is shown in fig. 1.

For a given 3-brane, $y_0$ is fixed and, then, $\lambda(y_0)$ is the coupling constant on that 3-brane. Thus, the $AdS_5$ curvature in the RS I model yields to an exponential suppression of the bumblebee self-interaction constant between the 3-brane at $y = 0$ and at $y = L$. Accordingly, the bulk
Integrating the kinetic term along the extra dimension, we obtain the 3-brane located at $y = L$.

Performing the dimensional reduction on the kinetic term leads to

$$ S_K = -\frac{1}{4} \int (g^{-2\epsilon y} dy) \int d^4x \tilde{B}^{\mu\nu} \tilde{B}_{\mu\nu}. $$

(9)

Hence, $\alpha$ has the following dependence along the extra dimension:

$$ \alpha(y) = \alpha e^{-2\epsilon y}. $$

(10)

Therefore, the $\text{AdS}_5$ curvature reduces the effects of the bumblebee dynamics on the visible brane at $y = L$ on both the kinetic and the potential terms.

Another curved extra dimensional scenario is given by the RS II model, wherein there is only one 3-brane at the origin [26]. By integrating out the 5D potential term in the extra dimension yields

$$ S_V = -\frac{\lambda}{2} \int_0^\infty e^{-4\epsilon y} dy \int d^4x \sqrt{-g_4} g^{\mu\nu} \tilde{B}_\mu \tilde{B}_\nu + b^2)^2. $$

(11)

Thus, the effective $(3 + 1)$ coupling constant in the thin brane at the origin is given by

$$ \lambda_{\text{eff}} = \frac{1}{2\epsilon} \lambda. $$

(12)

Since $c$ has mass dimension one, for a five-dimensional bumblebee self-coupling constant $\lambda$ with mass dimension one, then $\lambda_{\text{eff}}$ is a dimensionless constant. For the $\alpha$, integrating the kinetic term along the extra dimension, we obtain

$$ S_K = -2\alpha \int_0^\infty (e^{-2\epsilon y} dy) \int d^4x B^{\mu\nu} B_{\mu\nu}. $$

(13)

Therefore, the relation between the five-dimensional $\alpha$ and the four-dimensional $\alpha_{\text{eff}}$ is given by

$$ \alpha_{\text{eff}} = \frac{\alpha}{2\epsilon}. $$

(14)

Once again, the effective $\alpha_{\text{eff}}$ on the brane is dimensionless and it depends not on the length of the extra dimension but on the $\text{AdS}_5$ spacetime curvature. A similar dimensional reduction result appears in the original RS II model for the gravitational constant and the electroweak scale [26].

**Bumblebee fluctuations.** Now let us analyze how the bumblebee fluctuations propagate on the 3-brane and along the extra dimension. By varying the action (1), we obtain the equations of motion [9, 10]

$$ D_N B^{NM} = J_B^M, $$

(15)

where $J_B^M$ arises from the bumblebee self-interaction and it is given by [9, 10]

$$ J_B^M = 2V'B^M. $$

(16)

The antisymmetry of the bumblebee field strength $B_{MN}$ implies a conservation law

$$ D_M J_B^M = 0. $$

(17)

Now consider the fluctuation about the bumblebee VEV, i.e.,

$$ B_M \approx b_M + \chi_M, $$

(18)

where $\langle B_M \rangle = b_M$ is the vacuum expectation value (VEV) for the bumblebee field. Thus, the equation of motion for the fluctuations is given by

$$ \Box \chi^N - D^N (D_M \chi^M) - R_T \chi^T + D_M b^{NM} \approx 4\lambda(\chi^M b_M)b^N, $$

(19)

where $\Box = D_M D^M = g^{MN} D_M D_N$ is the 5D D’Alembertian operator and $R_{MN} = R^P_{MPN}$ is the Ricci tensor in 5D. Equation (19) has a form similar to the fluctuations EoM in flat spacetime [10], except for the covariant derivatives, the coupling to the Ricci tensor and the varying VEV.

Unlike the flat spacetime, which allows us to define a constant background VEV, $\partial_M b_N = 0$, the curvature constrains the $b_M$ VEV. In fact, assuming a covariant constant $b_M$, i.e., $D_M b_N = 0$, leads to the constraint $b_M R^M_{NPQ} = 0$. This constraint means that the curvature vanishes in the direction of the background vector. If we adopt a less restrictive VEV definition, by assuming that the VEV norm $b^2 = g^{MN} b_M b_N$ is constant [8, 18, 19], the VEV satisfies

$$ (D_N b^M) b_M = 0. $$

(20)

Since the VEV defines a preferred direction in spacetime, we can decompose $\chi_M$ into transverse $A_M$ and longitudinal $\beta$ modes with respect to $b_M$ as [10]

$$ \chi_M = A_M + \beta b_M, $$

(21)

where by defining the projection operators $P_{MN}^M = \frac{b_M b_N}{b_M b^M}$ and $P_{MN}^L = g_{MN} - \frac{b_M b_N}{b_M b^M}$, we have $A_M = P_{MN}^L \chi^N$ and

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\[ \beta b_M = P_{M,N}X^N. \] As result, we have to \( A_M b^M = 0 \) and \( \bar{b}_M b^M = \mp 1 \), where \( \bar{b}_M = \frac{b_M}{\sqrt{\beta}} \). Using the decomposition (21), the smooth quadratic potential term at leading order becomes
\[ V \approx 2\lambda [(\hat{b}^4 a_\lambda)\beta]^2, \quad (22) \]
i.e., \( V(X) \neq 0 \), therefore the \( \beta \) is the longitudinal mode. For the linearized bumblebee current (16), i.e., \( J^4_b \approx 4\lambda(\hat{b}^4 a_\lambda)\beta b^M \), we have from eq. (17) the linearized conservation law
\[ D_M(\beta b^M) = 0, \quad (23) \]
where we assume that the VEV norm is constant, see (20). Using the decomposition (21), the condition \( b^2 \) constant, the conservation law eq. (23) and the condition \( A_M b_M = 0 \), we obtain the equation of motion for the longitudinal mode \( \beta \) in the form
\[ \Box \beta(\beta b^M b_M) - [R_N^M b_N^M b_M - (\Box b^M b_M)] + 4\lambda(\hat{b}^4 a_\lambda)b^M = 0, \quad (24) \]
where \( \Box = \partial^2 - 4\lambda(\hat{b}^4 a_\lambda) \). For the massless mode, i.e., \( m^2 = 0 \), eq. (25) leads to a solution
\[ \Gamma(z) = \Gamma_0 + \frac{c_1}{2}z^2, \quad (31) \]
where \( \Gamma_0 \) and \( c_1 \) are constants. Moreover, note that \( \Gamma(z) \) grows with \( z \), whereas \( A_\mu \) satisfies \( D_\mu F^\mu = 0 \). Thus, the bumblebee transverse mode propagates freely on the 3-brane and along the extra dimension. Like the Maxwell gauge field, the bumblebee transverse mode acquires a mass due to the dimensional reduction [30].

Let us perform the Kaluza-Klein (KK) decomposition for the transverse mode in the form
\[ A_\mu(x, z) = \hat{A}_\mu(x)\Gamma(z). \quad (28) \]
From eq. (25), the brane dependence of the transverse mode \( \hat{A}_\mu \) satisfies
\[ \frac{1}{\sqrt{-g_4}}\partial_\mu(\sqrt{-g_4} g_4^a \partial_\lambda \hat{F}_{a\lambda}) = m^2 \hat{A}^\nu, \quad (29) \]
whereas the extra dimension dependence is governed by
\[ \Gamma'' + \frac{1}{z} \Gamma' + m^2 \Gamma = 0, \quad (30) \]
where \( \hat{F}_{a\lambda} = \partial_\lambda \hat{A}_a - \partial_a \hat{A}_\lambda \) and the \( m \) is a constant called KK mass. The solution of eq. (30) is the Bessel functions of the first and second kind, respectively, given by \( \Gamma(z) = \Gamma_1 z J_1(mz) + \Gamma_2 z Y_1(mz) \), where \( \Gamma_1,2 \) are constants. Thus, like the gauge vector field, the bumblebee transverse mode acquires a mass due to the dimensional reduction [30].

For the longitudinal mode, by assuming the KK decomposition
\[ \beta(x, z) = \tilde{\beta}(x)Y(z), \quad (32) \]
eq (24) simplifies to
\[ \left[ D_4 D^4 + \left( \frac{4}{l^2} + \frac{4}{l^2} \beta \right) \right] \tilde{\beta} + 4\lambda(\hat{b}^4 a_\lambda)Y = b_4 D_4 D_\mu \hat{A}^\nu. \quad (33) \]
Considering the conservation law eq. (23) for the spacelike case, we find that
\[ \frac{\tilde{b}_z}{(l/z)^5} D_4 (l/z)^4 \Gamma = 0, \quad (34) \]
i.e., the solution is given by \( \Gamma(z) = \Gamma_0 (z/l)^4 \), where \( \Gamma_0 \) is a constant. Substituting \( Y(z) \) in the equation above, we find that
\[ D_\mu D^\mu \beta + 4\lambda(\hat{b}^4 a_\lambda) = \Gamma_0 (l/z)^4 \frac{\partial_\lambda(\hat{b}^4 a_\lambda)}{D_\mu \hat{A}^\nu}. \quad (35) \]

Consider a spacelike VEV pointing along the extra dimension. Then, in the conformal coordinates \( b_M \) has only a nonvanishing fifth component of the form
\[ b_M = (0, \bar{b}, \bar{b}(l/z)), \quad (27) \]
where \( \bar{b} \) is a constant that arises from the constant norm condition (2). The VEV choice in (27) has a vanishing field strength, i.e., \( b_{MN} = 0 \). In addition, this VEV choice constrains the transverse mode \( A^M \) to the 3-brane, i.e., \( A^4 = 0 \).
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$\Gamma = \Gamma_0 = \text{const,}$ the transverse and longitudinal modes decouple, hence the $U(1)$ symmetry is recovered. Another important point that we need to emphasize from eq. (35) is that due to the current conservation law (34), the longitudinal mode in the spacelike case did not generate Kaluza-Klein towers. An analysis of KK towers in presence of Lorentz-violating aether fields in space-time with extra dimensions was done in ref. [23].

Finally, let us explore the effects of the brane cosmological expansion on the dynamics of the massive mode. Assuming that $\tilde{\beta} = \tilde{\beta}(t)$ in $m^2 = 0$, eq. (35) leads to

$$\ddot{\tilde{\beta}} + 3H \dot{\tilde{\beta}} + 4\lambda \tilde{\beta} \approx 0,$$

where the dot is the derivative with respect to time and $H = \frac{\dot{a}(t)}{a(t)}$ is the Hubble factor. We can see again that the terms of the kinetic part of $\beta$ that depend on $z$ cancel out with the mass terms due to the constraint (23). Note also that the cosmological expansion produces a dissipative term proportional to $3H$. For an accelerated de Sitter phase, i.e., $a(t) \propto e^{Ht}$, the solution of eq. (36) is given by

$$\tilde{\beta} = \beta_0 e^{-\frac{1}{2} (3H_0 + \sqrt{9H_0^2 - 16\Lambda h^2})},$$

where $\beta_0$ and $H_0$ are constants. Assuming $H_0 \approx 10^{16}$ GeV (inflation era), the longitudinal mode decays in a damping time $\Delta t \approx 10^{-10}$ (GeV)$^{-1}$, corresponding to a cosmic time of $10^{-38}$ seconds. For $m^2_\beta = \lambda \beta^2 \sim H_0^2$, the longitudinal mode has the same order as the GUT scale and it decays exponentially in time. For $m^2_\beta < H_0^2$, $\beta$ decays exponentially. On the other hand, for $m^2_\beta > \frac{9}{4\pi} H_0^2$, the massive mode exhibits a damped oscillation with frequency $\omega_\beta = \sqrt{16m^2_\beta - 9H_0^2}$.

Final remarks and perspectives. – We investigated how the curvature of spacetime and the extra dimensions modify the fluctuations of a self-interacting vector field $B_M$ that undergoes a spontaneous Lorentz symmetry breaking, the so-called bumblebee field. We considered the bumblebee field living in a five-dimensional bulk, with a spacelike fifth extra dimension and a warped geometry with a bulk cosmological constant.

Assuming two parallel thin 3-branes embedded in an $AdS_5$ bulk (RS I model), the curved spacetime leads to an exponential suppression of the self-interacting coupling constant $\lambda$ between the branes and the bumblebee VEV $b_M$ also decays with the extra dimension. Therefore, the $AdS_5$ curvature of RS I model might explain the yet unobserved Lorentz violating effects as a result of the suppression along the extra dimension. The parameter $\alpha$, which was included in order to keep the bumblebee field with mass dimension one, turns out to be equivalent to a specific dilaton field configuration. Since the dilaton field arises due to the conformal symmetry of the string world surface, an extension of the present work analysing the possible conformal invariance and its respective couplings to the bumblebee field seems promising.

We also studied the propagation of the fluctuations of the bumblebee field $\chi_M = B_M - b_M$ upon the VEV $b_M$. By assuming the VEV $b_M$ along the fifth dimension, the transverse mode $A_M$ has no component on the extra dimension. Unlike the bumblebee field in flat spacetime [9], the curvature and the varying VEV $b_M$ turn the transverse NG $A_M$ and longitudinal $\beta$ modes into highly coupled modes. Assuming the Kaluza-Klein (KK) decomposition for the modes, we find a KK mass tower for the transverse mode. The longitudinal mode only acquired a Lorentz violating mass, $m^2_\beta = \lambda \beta^2$, for the current conservation law prevents $\beta$ to acquire KK masses.

Along the thin 3-brane, the cosmological evolution of the brane also modified the longitudinal mode propagation. Assuming a FRW 3-brane embedded in the $AdS_5$ bulk, the brane curvature due to the cosmological expansion leads to a dissipative term proportional to the Hubble constant $H = \dot{a}/a$. For a de Sitter accelerated expansion, the time decay is proportional to $1/H_0$. Thus, the cosmic expansion suppresses the longitudinal mode $\beta$ leaving only the NG mode $A_M$ in late times. Therefore, if the spontaneous violation of the Lorentz symmetry occurred in the early universe, the inflationary period may have strongly suppressed the effects of the longitudinal mode. Hence, this cosmological suppression might explain the yet unobserved longitudinal mode. This result suggests further analysis on the effects of combined bumblebee, gravity and matter fluctuation effects in the early universe.

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