Understanding the Core-Halo Relation of Quantum Wave Dark Matter, ψDM, from 3D Simulations

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We examine the nonlinear structure of gravitationally collapsed objects that form in our simulations of wavelike cold dark matter (ψDM), described by the Schrödinger-Poisson (SP) equation. A distinct gravitationally self-bound solitonic core is found at the center of every halo, with a profile quite different from cores modeled in the warm or self-interacting dark matter scenarios. Furthermore, we show that each solitonic core is surrounded by an extended halo composed of large fluctuating dark matter granules which modulate the halo density on a scale comparable to the diameter of the solitonic core. The scaling symmetry of the SP equation and the uncertainty principle tightly relate the core mass to the halo specific energy, which, in the context of cosmological structure formation, leads to a simple scaling between core mass (Mc) and halo mass (Mh), Mc ∝ a−1/3Mh1/3, where a is the cosmic scale factor. We verify this scaling relation by (i) examining the internal structure of a statistical sample of virialized halos that form in our 3D cosmological simulations, and by (ii) merging multiple solitons to create individual virialized objects. Sufficient simulation resolution is achieved by adaptive mesh refinement and graphic processing units acceleration. From this scaling relation, present dwarf satellite galaxies are predicted to have kpc sized cores and a minimum mass of ∼106 M⊙. Moreover, galaxies of 2×1012 M⊙ at z = 8 should have massive solitonic cores of ∼2×107 M⊙ within ∼60 pc. Such cores can provide a favorable local environment for funneling the gas that leads to the prompt formation of early stellar spheroids and quasars.

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Dark matter comprising very light bosons with a mass mφ ∼ 10^{-22} eV has been recognized as a viable means of suppressing low mass galaxies and providing cored profiles in dark matter dominated galaxies [1,2]. The relative deficiency of the observed number of low-mass galaxies is a major problem for standard cold dark matter (CDM) [3,5], for which a steeply rising mass function is predicted [6]. Furthermore, the dwarf spheroidal galaxies [7,17] and low surface brightness galaxies [18,19] are generally inferred to have large flat cores of dark matter, at odds with the singular cores required by standard CDM [20,21].

Light bosonic dark matter is assumed here to be nonthermally generated and in a Bose-Einstein condensate (BEC) state described by a single coherent wave function [1,22,25], which we term ψDM. Here solutions to both the missing-satellite and cusp-core problems arise from the uncertainty principle, leading to an effective quantum-mechanical stress tensor that suppresses small-scale structures below a Jeans scale. The Jeans scale evolves with the cosmic time slowly as a^{-1/4}, where a is the cosmic scale factor [1,25], thereby yielding a sharp break in the linear mass power spectrum. This expected behaviour has recently been demonstrated with the first cosmological simulations at sufficiently high resolution, capable of resolving the smallest galaxy halos forming in this context [20].

Warm dark matter (WDM) is also capable of suppressing small-scale linear power by free streaming [27], but it suffers from the \textit{Catch 22} problem [28], where the light particle mass required for creating a sufficiently large core (~1 kpc) would prevent the formation of dwarf galaxies in the first place. Collisional CDM does somewhat better in producing cores consistent with observations, but it cannot suppress the number of dwarf galaxies [29,30]. For these reasons, ψDM and scalar-field dark matter composed of extremely light particles have recently begun to attract attention as a viable contender for the long-sought dark matter (e.g., [26,31,32]).

Cosmic structures at high redshifts provide stringent tests for all alternative dark matter models attempting to solve the small-scale issues of CDM in the Local Group. For WDM a tension arises when requiring the relatively large cores of dwarf spheroidal galaxies without violating the small-scale power constrained by the Lyman-α forest [28,37,39]. For ψDM this problem may be less severe due to the sharper small-scale break in its linear power spectrum as compared to WDM [1,36]. The power spectrum is marginally consistent with the Lyman-α forest observations, while adding a small amount of CDM component (~10%) can certainly further relieve the tension [30]. High-z number counts provide another constraint for galaxies at 6 ≤ z ≤ 8 [40]. We notice that the ψDM power spectrum starts to deviate from CDM at
$k \sim 7 \; h \; \text{Mpc}^{-1}$, corresponding to a halo mass of $\sim 5 \times 10^9 \; M_\odot$. Above this mass scale the $\psi$DM galaxy number density should be close to CDM, and therefore consistent with the observational constraint \cite{40}. Larger $\psi$DM simulations with the addition of baryons will be invaluable for supporting these arguments.

Previous theoretical work on $\psi$DM halos mainly focused on two aspects: (i) a stationary soliton profile with or without self-interaction (e.g., \cite{22, 23, 32}), or (ii) a Navarro-Frenk-White (NFW) profile \cite{21} with its inner cusp replaced by a flat core (e.g., \cite{1, 36}). In either case, the detailed connection between cores and halos in the fully nonlinear regime has not been addressed. This question can be best answered by simulations. The first attempt of three-dimensional simulations of the $\psi$DM structure formation has come to light only a few years ago \cite{25}, revealing complex interference fringes and a halo profile similar to NFW. This work however did not have a constant-density core introduced truncating as a self-bound mass clump superposed on the NFW profile \cite{25}, \cite{26} for an illustration: an important feature for the soliton profile is redshift-dependent. To see this, note that when $\rho \gg 1$ and $a = \text{const.}$, the SP equation remains unchanged under the transformation $(\tau, x, \psi, V) \rightarrow (\lambda^{-2}\tau, \lambda^{-1}x, \lambda^2\psi, \lambda^3V)$ for arbitrary $\lambda$. Having very high densities and forming in a short time compared with the Hubble time, all solitonic cores hence conform to this $\lambda$ scaling to a high accuracy. The relevant physical quantities scale as $(x_c, \rho_c, M_c, E_c) \rightarrow (\lambda^{-1}x_c, \lambda^4\rho_c, \lambda M_c, \lambda^3E_c)$, where $x_c$, $\rho_c$, $M_c$, and $E_c$ are the core radius, density, mass and energy, respectively. The soliton density profile can be well fit by \cite{26}

$$
\rho_c(x) = \frac{1.9 \; a^{-1}(m_\psi/10^{-23} \; \text{eV})^{-2}(x_c/kpc)^{-4}}{[1 + 9.1 \times 10^{-2}(x/x_c)^2]^{8/3}} \; M_\odot \; \text{pc}^{-3}.
$$

Here we define $x_c$ as the radius at which the density drops to one-half its peak value, and $M_c$ as the enclosed mass within $x_c$.

To address the core-halo connection, we conduct three structure formation simulations of different realizations with a spatial resolution up to 60 pc in a 2 Mpc comoving box. These runs begin at the matter-radiation equality around $z = 3,200$ and end at $z = 0$. Another simulation with a 40 Mpc box is conducted from $z = 3,200$ to $z = 8$ for probing the high-redshift galaxies. Our results verify that halos at different redshifts all contain self-similar solitonic cores. Density granules of about the same size as the solitonic core are apparent throughout the halos (see Fig. 2 in Ref. \cite{26} for an illustration): an important feature for the core-halo connection and will be explained later. The soliton profile is redshift-dependent. To see this, note that as long as $a$ can be regarded as a constant and the background density is negligible, the SP equation can be rewritten into a redshift-independent form by introducing a set of rescaled variables: $(\tau', x', \psi', V') \equiv (a^{-1/2} \tau, a^{1/4} x, \psi, a^{1/2} V)$. It follows that the soliton radius in the comoving (unprimed) coordinates scales as $a^{-1/4}$ for a fixed peak core density. Figure 1 shows the density profiles of typical halos in the simulations at five different epochs, $z = 12.0, 8.0, 2.2, 0.9$ and 0.0, in the unprimed coordinates. The agreements of the simulation data to both the $\lambda$ and $a$ scalings are excellent.

A question naturally arises concerning the relation between solitonic cores and their host halos. Aided by our structure formation simulations, we find all collapsed objects approximately follow a redshift-dependent core-halo mass relation,

$$
M_c \propto a^{-1/2} M_h^{1/3}.
$$

The halo virial mass is defined as $M_h \equiv$
FIG. 1: Density profiles of $\psi$DM halos. Dashed lines with various opened symbols show five examples at different redshifts between $12 \geq z \geq 0$. The DM density is normalized to the cosmic background density. A distinct core forms in every halo as a gravitationally self-bound object, satisfying the redshift-dependent soliton solution (solid lines) upon proper $\lambda$ scaling. Filled diamonds show an example from the soliton collision simulations renormalized to the comoving coordinates at $z = 0$. The same $z = 8$ halo in a CDM simulation (filled squares) fit by an NFW profile (dot-dashed line) is also shown for comparison.

$$(4\pi x_{vir}^3/3)\zeta(z)\rho_{m0},$$

where $x_{vir}$ is the comoving virial radius and $\zeta(z) \equiv (18\pi^2 + 82(\Omega_m(z) - 1) - 39(\Omega_m(z) - 1)^2)/\Omega_m(z) \sim 350$ ($180$) at $z = 0$ ($z \geq 1$). Note that this definition of virial mass is the same as that for CDM. This is because once an object exceeds the Jeans mass on its way to collapse, the dynamics is almost identical to the cold collapse, for which the Eikonal approximation of wave dynamics to particle dynamics holds until virialization takes place. Figure 2 shows this scaling relation over three orders of magnitude in halo mass from $10^8$ to $10^{11} M_\odot$. We demonstrate the redshift evolution by showing coalescence of the core-halo mass relations of halos at five different epochs between $10 > z > 0$ as well as the evolutionary trajectory of a single halo. The deviation of the core mass from Eq. (4) is less than a factor of two.

To understand this core-halo mass relation, we further conduct a set of controlled numerical experiments, where multiple solitons are initially placed randomly with zero velocity and start to merge until the systems relax. Solitons are chosen as a convenient initial condition for their stability. Here we assume $a = const.$ and zero background density. We would like to know whether the core-halo configuration still persists in a different setting from cosmological structure formation, and if so, we want to ascertain what factors determine the soliton scale among the infinite number of self-similar solutions. Intuitively, one expects that the final relaxed state should lose the memory of its initial configuration and thus depends only on the globally conserved quantities, namely, the total mass $M$ and energy $E$ (assuming there is no net angular momentum). We conduct 29 runs in total with different initial conditions of various $M$ and $E$. For the same $M$ and $E$, we repeat runs with different realizations, including different initial soliton numbers ranging from 4 to 128, different soliton sizes and initial positions. Figure 3 shows one example of the soliton collision simulations. The AMR scheme is again adopted in order to achieve sufficient resolution everywhere; in particular, we ensure that every soliton is well resolved with at least $\sim 10^4$ cells and verify that $M$ and $E$ remain conserved with at most a few percent error in all simulations.

The resulting relaxed structures that form in these soliton collision experiments are always found to consist of a halo and a solitonic core (see Fig. 1 and panel (d) of Fig. 3), similar to the results of cosmological simulations. The core profiles satisfy the $\lambda$ scaling and the halo profiles are close to NFW. This result establishes that the core-halo configuration is a generic structure of $\psi$DM in virialized gravitational equilibrium.

More importantly, as shown in Fig. 4, the core mass
The smallest halo should be close to a single isolated soliton, with a wide core and a steeper outer gradient. Our definition of core mass, $M(r < r_c)$, makes up about...
25% of the total soliton mass. Thus by taking $M_c = M_\odot/4$ in Eq. (8) we readily obtain a minimum halo mass $M_{\text{min}}(z) = a^{-3/4}(\zeta(z)/\zeta(0))^{1/4}M_{\text{min},0} \sim 3 \times 10^8 M_\odot$ at $z = 8$, consistent with Fig. 2.

Finally, we conclude this Letter by a conjecture regarding the possible consequences of the early formation of the dense solitonic cores. A present-day galaxy with a typical halo mass of $2 \times 10^{12} M_\odot$ will have $M_c \sim 5 \times 10^8 M_\odot$ and $r_c \sim 160$ pc. For a high-redshift galaxy with the same halo mass, its core mass and gravitational acceleration near the core, $M_c/r_c^2$, will be enhanced by a factor of $a^{-1/2}$ and $a^{-3/2}$, respectively. This much greater gravitational force may quickly attract a large amount of gas into a small central region, thereby creating an ultra-dense gas favorable for major starbursts and formation of supermassive black holes. For example, a galaxy of $2 \times 10^{12} M_\odot$ forming at $z = 8$ has a core mass $\sim 2 \times 10^9 M_\odot$ in $\sim 60$ pc radius and it captures at least $4 \times 10^9 M_\odot$ gas if the baryon fraction at the core is the same as or above the cosmic mean. If furthermore the gas temperature maintains near the Lyman-$\alpha$ onset temperature, 10 eV, this radius is only a factor of two greater than the 30 pc thermal Jeans length of the gas. Such a solitonic core can certainly help the prompt formation of quasars appearing as early as $z = 7$ [15].

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