Numerical study on flow over stepped spillway using Lagrangian method

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Abstract. Flow over stepped spillway has been studied for centuries, due to its unstable and the characteristics of cavity, the simulation of this type of spillway flow is always difficult. Most of the early studies of flow over stepped spillway are based on experiment, while in the recent decades, numerical studies of flow over stepped spillway draw most of the researchers’ attentions due to its simplicity and efficiency. In this study, a new Lagrangian based particle method is introduced to reproduce the phenomenon of flow over stepped spillway, the inherent advantages of this particle based method provide a convincing free surface and velocity profiles compared with previous experimental data. The capacity of this new method is proved and it is anticipated to be an alternative tool of traditional mesh based method in environmental engineering field such as the simulation of flow over stepped spillway.

1. Introduction

Basically, most of the traditional numerical methods for simulating the flow over stepped spillway are Eulerian based mesh methods [1-3], which were also widely used around the world. During the last decades, a new type of numerical method in hydraulic simulation has been developed and the implementation of this new numerical method in the flow over stepped spillway simulation is attractive due to the inherent advantages of the mesh less particle based method, namely MPS (Moving Particle Semi-implicit) method [4-6].

The simulation of flow over stepped spillway is sensitive to the free surface tracking technology, which usually required special treatments in the traditional mesh based method to locate the free surface and improve the accuracy of the results [7,8]. However, in the mesh less particle based method such as MPS, locating the free surface becomes quite easier due to its inherent advantages, and thus, various flow phenomenon such as wave flow, bench flow, and moreover, the stepped spillway flow can be simulated well using MPS method[6,8]. Figure 1 shows the simulation of natural phenomenon using MPS method, the wave is represented by particles during the simulation.
The study of flow over stepped spillway has been popular along with the design of hydraulic structures such as dams. As an important type of spillway, the stepped spillway also draws people’s concern for centuries. When considering the stepped spillway, the height and length of each step constructed from the top to the bottom of the spillway will lead to different flow conditions, additionally, the different flow discharges, as well as the roughness of the stepped spillway, give rise to three flow regimes [4], namely, nappe flow, skimming flow and transition flow. Though the free surface and flow conditions are different in these three flow regimes, the MPS method is still considered as a good tool to simulate the flow over stepped spillway due to the characteristics of the free surface variation in the stepped spillway flows, such as shown in Figure 2.

2. Basis of MPS method
MPS method, as a typical mesh less particle based method, is originally established by Koshizuka in 1995 [2,9]. The fundamentals of MPS method are based on the spatial approximation within the simulation domain, which lead to a quite different simulation algorithm compared with most of the traditional mesh based method. The governing equations in MPS method for water flow simulation follows the continuity and momentum conservations, which are the two basic laws in numerical hydraulic studies, and are given as [8-10]:

$$\frac{\partial}{\partial t}(\rho u) + \nabla \cdot (\rho u u) = \nabla \cdot (\tau + f)$$

(1)
\[ \rho \frac{Du}{Dt} = -\nabla p + \mu \nabla^2 u + \rho f \]  

(2)

where \( u \) is the velocity vector, \( p \) is the pressure, \( \rho \) is the density of fluid, \( \mu \) is the dynamic viscosity and \( f \) is the body forces. In the Lagrangian frame, there is no convection term in the momentum equation, and the movement of particles is simply calculated by \( \frac{Dr}{Dt} = u \), with \( r \) being the position vector. Since the MPS method is a fully Lagrangian method, the discretization of the governing equations is also quite different from the traditional mesh based method, which is simply given by:

\[ \langle \nabla \phi \rangle_i = \frac{d}{n^0} \sum_{j \neq i} \frac{\phi_j - \phi_i}{R_{ij}^2} \]  

(3)

\[ \langle \nabla^2 \phi \rangle_i = \frac{2d}{\lambda n^0} \sum_{j \neq i} (\phi_j - \phi_i) W(R_{ij}, r_e) \]  

(4)

where \( n^0 \) is the initial particle density, \( d \) is the simulation dimension. As shown in Eq. (3) and Eq. (4), the gradient term and Laplacian term in governing equation are given by Lagrangian frame, and \( W(R_{ij}, r_e) \) is the kernel function, which is used to represent the spatial relationship between particles in MPS method. As shown in Figure 3, the spatial approximation of MPS method is based on the relationship among different particles used in the simulation.

![Kernel function W(R_\text{ij}, r_e)](image)

Figure 3. Spatial distribution of particles in MPS method.

3. Model configuration and boundary conditions

In this study, a self-developed MPS model is configured based on the experimental study provided by Chanson and Toombes [4], the flow over stepped spillway experiment is conducted in a 4.2m long and 0.5m wide channel, the slope angle of the stepped spillway equals to 15.9 degree and the height of each step equals to 0.1m, the flow discharge \( q = 0.26 \text{m}^3/\text{s} \) is used as the inflow boundary condition in both experiment and MPS numerical model, and in MPS model, the inflow zone is 0.5m longer than the experimental setup in order to stabilize the inflow velocity. The configuration of both experiment and numerical study of stepped spillway flow is shown as:

As the basic simulation elements, all the fluid characteristics and properties, such as density, viscosity, velocity and pressure, are assigned to the particles during the simulation. Moreover, the relationship of particles used in MPS method are represented by the kernel function, which is the mimic function of Dirac delta function used in MPS method in order to represent point mass or point charge in the entire integration area inside the searching radius \( r_e \) as also shown in Figure 3. With the development of MPS method, several kernel functions are created for the purpose of different flow simulations [11]. Table 1 shows some famous kernel functions used in MPS method.
Table 1. Some kernel functions used in MPS method.

| Type | Kernel function formulation | Reference |
|------|-----------------------------|-----------|
| 1    | $W(R_{ij}, r_e) = \begin{cases} e^{-\frac{R_{ij}}{\alpha r_e}} & 0 \leq R_{ij} \leq r_e \\ 0 & r_e < R_{ij} \end{cases}$ | Koshizuka and Oka, 1996 [9] |
|      | $\begin{cases} \frac{2}{3} - 4\left(\frac{R_{ij}}{r_e}\right)^2 + 4\left(\frac{R_{ij}}{r_e}\right)^3 & 0 \leq R_{ij} \leq \frac{r_e}{2} \\ 0 & r_e < R_{ij} \end{cases}$ | Koshizuka and Oka, 1996 [9] |
| 2    | $W(R_{ij}, r_e) = \begin{cases} 4\left(\frac{R_{ij}}{r_e}\right)^2 - 4\left(\frac{R_{ij}}{r_e}\right) - 4\left(\frac{R_{ij}}{3 r_e}\right)^3 & \frac{r_e}{2} < R_{ij} \leq r_e \\ 0 & r_e < R_{ij} \end{cases}$ | Fu and Jin, 2014 [10] |
| 3    | $W(R_{ij}, r_e) = \begin{cases} \left(1 - \frac{R_{ij}}{r_e}\right)^3 & 0 \leq R_{ij} \leq r_e \\ 0 & r_e < R_{ij} \end{cases}$ | Fu and Jin, 2014 [10] |

Figure 4. Configuration of experimental and numerical study
(a) Experiment (Chanson and Toombes 2007), (b) MPS model.

The configuration of MPS simulation is similar with the experiment provided by previous study as shown in Figure 4, although the channel length increased to 5m in order to stabilize the outflow zone in MPS simulation. However, the boundary conditions used in MPS simulation are quite different from the traditional mesh based method, in total three boundary conditions are used in this study, which are solid boundary, inflow and outflow boundary and free surface boundary. In this study, firstly, a simple bounce boundary is used to represent the solid boundary in MPS method, secondary, a recycle boundary is introduced to define the inflow and outflow boundary in the MPS simulation, moreover, a simple free surface boundary based on the particle number density $n_i < \beta n^0$, where $\beta = 0.8-0.99$, is introduced to locate the free surface particles during the MPS simulation.
4. Case study: MPS simulation of flow over stepped spillway
Due to the inherent advantage of MPS method itself, flow over stepped spillway has been successfully simulated and the flow characteristics including the free surface profiles and velocity profiles are both reproduced well in the numerical results.

![Figure 5](image_url)

**Figure 5.** MPS simulation results (a) Initial (b) Early stage (c) Mid stage (d) Late stage.
As shown in Figure 5, the flow over stepped spillway has been reproduced well using MPS method, different stages have been represented at different time steps in MPS result. For instance, at the early stage, as shown in Figure 5(b), the MPS result shows a clear stepped flow characteristics from the first step to the fifth step, which also reveals the development of the flow over stepped spillway from the initial condition. While at the mid stage of the simulation as shown in Figure 5(c), all the steps have been filled with the water flow and the cavity areas are also represented at different steps, which are usually difficult to obtain in traditional mesh based method. And later on, when the simulation reaches the late stage, the free surface and cavity areas become similar to the experimental data, the comparison of free surface profile between experimental data and MPS result is given in Figure 6.

![Figure 6](image_url)

**Figure 6.** Free surface profile comparison between experimental data and MPS result.
The free surface comparison between experimental data and MPS result in Figure 6 confirms the capacity of MPS method in reproducing a high accuracy free surface profile during the simulation of flow over stepped spillway. It actually attributes to the Lagrangian frame of MPS method, which makes this method a high capability of free surface tracking. Since the free surface profile of MPS simulation agrees well with experimental data, the velocity comparisons are also executed in this study in order to further confirm the capacity of MPS method in flow over stepped spillway simulation. Hereafter, as indicated in the experimental study by Chanson and Toombees (2007), the velocity profiles in the seventh step and eighth step are selected for the velocity comparison between experimental data and MPS result.

![Figure 7. Velocity comparison between experimental data and MPS result.](image)

The velocity comparison between experimental data and MPS result in Figure 7 shows some differences, the MPS simulated velocity is slower than the experimental data either on the seventh step and eighth step. The largest differences between MPS result and experimental data in the velocity comparison on these two steps occurs at y = 0.74m on the eighth step, which is about 30% difference (\(u = 0.242\text{m/s} \) in experiment and \(u = 0.175\text{m/s} \) in MPS). Due to the fact that the flow over stepped spillway is a high unstable flow, thus, the flow velocities on each step are also highly unstable, therefore, the time-averaged MPS results still show discrepancies compared with the experimental data, the velocity profiles, in total, are acceptable in this study.

5. Conclusions
Flow over stepped spillway is simulated by a new and robust Lagrangian based particle method in this study, both the free surface and velocity profiles are reproduced well in MPS method compared with previous experimental study. The good simulation results obtained by MPS method indicate that this new Lagrangian based method is a strong tool and a good alternative of the traditional mesh based method in the field of environmental engineering. Due to it is a young and robust method, the efficiency, accuracy and stability of MPS method become key considerations in the future.

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Reference

[1] Bombardelli F A, Meireles I and Matos J 2011 Laboratory measurements and multi-block numerical simulations of the mean flow and turbulence in the non-aerated skimming flow region of steep stepped spillways *Environmental Fluid Mechanics* **11**(3) 263-288

[2] Valero D and Bung D B 2015 Hybrid Investigations of Air Transport Processes in Moderately Sloped Stepped Spillway Flows *Proc. 36th IAHR World Congress* 28 June – 3 July 2015 Hague Netherlands

[3] Valero D and García-Bartual R 2016 Calibration of an air entrainment model for CFD spillway applications *Advances in Hydroinformatics* Springer Singapore 571-582

[4] Chanson H and Toombes L 2007 Hydraulics of stepped chutes: The transition flow *Journal of Hydraulic Research* **45**(1) 140-141

[5] Fu L and Jin Y C 2013 A mesh-free method boundary condition technique in open channel flow simulation *Journal of Hydraulic Research* **51**(2) 174–185

[6] Shakibaeinia A and Jin Y C 2011 A mesh-free particle model for simulation of mobile-bed dam break *Advances in Water Resources* **34**(6) 794-807

[7] Oertel M and Bung D B 2012 Initial stage of two-dimensional dam-break waves: laboratory versus VOF *Journal of Hydraulic Research* **50**(1) 89-97

[8] Fu L and Jin Y C 2011 Modeling flow around cylinders using mesh-free particle method 20th Canadian Hydrotechnical Conference 14-17 June 2011 Ottawa Ontario Canada CSCE

[9] Koshizuka S and Oka Y 1996 Moving particle semi-implicit method for fragmentation of incompressible fluid *Nuclear Science and Engineering* **123**(3) 421-434

[10] Fu L and Jin Y C 2014 Simulating velocity distribution of dam breaks with the particle method *Journal of Hydraulic Engineering* **101**(6) 7900 0000915 04014048

[11] Kondo M and Koshizuka S 2011 Improvement of stability in moving particle semi-implicit method *International Journal for Numerical Methods in Fluids* **65**(6) 638-654