Generator of periodic inertia force concentrated in one direction

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Abstract. The paper is devoted to the problem of design for generator of periodic mechanical force based on use of inertia forces. Suggested construction transforms rotational motion of a crank of special type into swinging motion of pair of unbalanced masses. This creates inertia force of a constant sign directed along certain axes. Analytic and graphic representations for dependences of periodic force on time are presented in the article revealing the harmonic type of the force. Given construction has wide set of parameters for control and adaptation of the produced force.

1. Introduction
Periodic dynamic action is widely used in different types of applications. The purpose of the devices which produce periodic force is to transmit energy to the media or objects. The main features of the systems which generate periodic forces is magnitude, dependence on time and special orientation. It must pointed out that usually the force produced by the traditional exciters works only during the part of the period. This is typical for the vibratory exciters based on the inertial forces of rotation of unbalanced masses [1]. The use of that type of exciters requires vibration protection of operator as well as the other parts of the machine. It plays special role for the exciters with high magnitudes of produced force [2],[3].

Mentioned drawbacks can be fixed by use of the planetary exciters of different designs implementing sprockets without rigid kinematic connection with the crown [4], [5], as well as with rigid connection which allows to take into account the force generated by the unbalanced masses of the sprockets [6],[7]. However the technical and scientific problem of development of highly directed periodic exciters. In [8] author attempted to develop design of the vibratory exciter with the produced force which is directed along chosen axis. Nevertheless the features of the construction and time dependence of the force put some restrictions on the applications of the device. At the first place that relates to the to the situation when purely harmonic view of the periodic force is required.

Problems which are solved in applications by the vibration exciters belong to the various fields of technics. They include problems of building and construction [9], metals [10], measurement systems [11], vibration screens [12] and vibration stands [13]. This makes the advances in technology of vibration exciters and development of the new constructions of periodic force generators persistently important.

2. Formulation of the problem
The problem of development of the theory and design of the device for generation of periodic force of highly directed type. The basic idea for that will be the approach of transformation of the rotational motion into swinging of massive object with the inertia force directed along chosen axes.

3. Theory
Theoretical approaches to the development of constructions of inertial [14], [15] and impact, usually crank type vibration exciters [16], [17], belong to the field of classical mechanics – mainly to the theory of motion and interaction of massive objects of different nature. Special interest of the researchers is usually attracted by the non-linear oscillators [18], [19], [20].
The given paper considers complex approach to the innovative design of periodic force generator. It combines the rotational motion with crank connected to the power drive with the swinging motion of heavy unbalanced masses as a source of inertia force and a special mechanism for their connection and energy transformation. This inertia force taking place into exciters with unbalanced masses is caused by centripetal acceleration and is defined as follows

\[ F_m = m\ddot{r} = m\dot{\psi}_r, \]  

(1)

where \( m \) - total mass of the body considered as point particle in the center of gravity, \( \dot{r} \) - radius vector of the center of gravity, \( \dot{\psi}_r \) - angular velocity of the center of gravity of the body. Condition of the positive value of the force (1) in respect to the coordinate axis with unit vector \( \hat{e}_\rho \)

\[ \text{sign}(F_m, \hat{e}_\rho) = |F_m| \cos(\psi) = \text{const}. \]  

(2)

The principal idea of the suggested construction is transformation of motion of the shaft connected with rotating crank into the reciprocal motion of the rack which is connected with gear attached to the massive unbalance bodies with ability to swind around fixed axes.

![Figure 1. Kinematic scheme of generator of periodic force](image)

Here we introduce following notations corresponding to the scheme of the device on Fig.1:

- \( \psi_1, \psi_2 \) - angular coordinate (deviations from the vertical) of the gravity centers of unbalanced masses 1 and 2 respectively, \( \psi_{10}, \psi_{20} \) - their initial phases counted counterclockwise from the positive direction of \( y \) axes;
- \( \phi \) - angle of rotation (angular coordinate) of crank 3, \( \phi_0 \) - initial phase counted counterclockwise from the positive direction of \( x \) axes;
- \( R_1, R_2 \) - distances between gravity centers of unbalanced masses 1 and 2 and axes of their rotation respectively;
- \( r_1, r_2 \) - gear radii of rack gears for unbalanced masses 1 and 2 respectively;
- \( m_1, m_2 \) - values of unbalanced masses;
- \( r \) - rotation radius of crank 3.

Angular coordinates \( \psi_1, \psi_2 \) of unbalanced masses functionally depend on angular coordinate of crank \( \phi \), which is the function of time \( t \).
\[\psi_1 = -\frac{r}{r_1} \sin(\phi) + \psi_{10},\]  
\[\psi_2 = -\frac{r}{r_2} \sin(\phi) + \psi_{20}.\]  

In case of steady rotation of the crank drive with the angular velocity \(\omega\) one has \(\phi = \omega t + \phi_0.\) 

Then the total inertial force of unbalanced masses that is applied to their axis of rotation looks like \(F_{in} = m_1R_1\dot{\psi}_1^2 + m_2R_2\dot{\psi}_2^2,\)  
and corresponding \(x\) and \(y\)-components of the force produced by the device are 
\[F_{inx} = m_1R_1\dot{\psi}_1^2 \sin(\psi_1) + m_2R_2\dot{\psi}_2^2 \sin(\psi_2),\]  
\[F_{iny} = m_1R_1\dot{\psi}_1^2 \cos(\psi_1) + m_2R_2\dot{\psi}_2^2 \cos(\psi_2).\]  

### 4. Results of numeric experiments

One of special but very interesting configurations of the device is the symmetric case which takes place under following conditions \(m_1 = m_2, \ r_1 = r_2, \ R_1 = R_2.\) Then if one assumes \(\psi_{10} = -\psi_{20}\) the \(x\)-forces acting on rotation axes of unbalanced masses 1 and 2 will compensate each other \(F_{inx} = 0,\)  
\[F_{iny} = 2mR_1\dot{\psi}_1^2 \cos(\psi_1).\]  

Kinematic condition of positive sign for \(y\) component of produced inertia force (8) is obvious \(\psi_1 \in (-\pi/2, \pi/2),\)  
and allows to find the corresponding relation between crank radius and gear radius of rack gear (taking into account (3)) \(2r < \pi r_1.\)  

This is important for the realisation of the device. In case of arbitrary time dependence of angle coordinate of crank rotation \(\phi(t)\) expression for the \(y\) component of produced inertia force looks as follows 
\[F_{iny}(t) = m \frac{Rr^2}{r_1^2} \rho(t)^2 \cos^2(\psi(t)) \cos\left(\frac{r}{r_1} \sin(\phi(t)) + \psi_{10}\right).\]  

In symmetric case and steady rotation of the crank (5) one obtains 
\[F_{iny}(t) = 2m \frac{Rr^2}{r_1^2} \rho^2 \cos^2(\omega t + \phi_0) \cos\left(\frac{r}{r_1} \sin(\omega t + \phi_0) + \psi_{10}\right)\]  

Figure 2 shows graphic representation og time dependence of inertial force (12) for one period of crank rotation under conditions \(\phi_0 = 0, \ \psi_{10} = 0, \ \frac{r}{r_1} = 0.5, \ \omega = 2\pi\)
5. Discussion

Periodic force produced by the device along with constant sign possesses great abilities for control, regulation and adaptation. This may be achieved by the correct choice of initial phases the unbalanced masses in respect to each other and angular coordinate of the crank and radius of the. It allows to control magnitude and direction of the force as well as relation between components of the force. The constant sign of the $y$-component of the force is clear from the graph on Fig. 2. Detailed analysis of the specific features of the work of suggested device must be performed using the dynamic properties of the system including real masses, dimensions and momentums. For that purpose dynamic analysis of the links between the elements of the system is necessary. One must take into account that crank 3 and rotating unbalanced masses 1 and 2 for the case of real device would’ve been mounted on the same base (frame). Since that the total force being generated by the system must include the part which drives rotating unbalanced masse. Exact form of that may be found using the equation of rotational dynamics

$$J \ddot{\psi} = M\,,$$  \hspace{1cm} (13)

where $J$ is inertial momentum of unbalanced mass (1 or 2), $M$ - angular mometum produced by the forces which cause rotation of unbalanced mass with angular acceleration $\dot{\psi}$. One can express the tangent force $F_t$ applied to the rack for unbalanced mass rotation. Taking into account notations introduced above and special features of construction one can obtain from (13)

$$F_t r_\tau = m R^2 \ddot{\psi}.$$  \hspace{1cm} (14)

This allows calculate the strength of dynamic link as

$$F_t = \frac{m R^2 \ddot{\psi}}{r_\tau}.$$  \hspace{1cm} (15)

Since that total force in $y$ direction which acts on the base of the device in symmetric case (10) can be defined as

$$F_{total} = 2 F_t + 2 F_m,$$  \hspace{1cm} (16)

Graphic representation of force (16) (rack and crank are considered inertialless) is given on Fig. 3 for the combination of parameters $\phi_0 = 0$, $\psi_{\omega0} = 0$, $R = 0.025m$, $m = 5kg$, $\omega = 100\pi$, $\frac{r_\tau}{r_\tau} = 0.5$, $r = 0.1m$. 

![Figure 2. Time dependence of periodic force generated by the device (for given set of parameters)](image-url)
6. Conclusion
The main feature of developed construction of vibratory exciter is that generated force is harmonic but has primarily constant sign. This fact may be used in constructions of different types of devices, for instance energy efficient designs of road rollers for soil compaction.

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