Black Holes, Wormholes, and the Disappearance of Global Charge*

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One of the paradoxes associated with the theory of the formation and subsequent Hawking evaporation of a black hole is the disappearance of conserved global charges. It has long been known that metric fluctuations at short distances (wormholes) violate global-charge conservation; if global charges are apparently conserved at ordinary energies, it is only because wormhole-induced global-charge-violating terms in the low-energy effective Lagrangian are suppressed by large mass denominators. However, such suppressed interactions can become important at the high energy densities inside a collapsing star. We analyze this effect for a simple model of the black-hole singularity. (Our analysis is totally independent of any detailed theory of wormhole dynamics; in particular it does not depend on the wormhole theory of the vanishing of the cosmological constant.) We find that in general all charge is extinguished before the infalling matter crosses the singularity. No global charge appears in the outgoing Hawking radiation because it has all gone down the wormholes.

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1. Introduction

There are two long-standing problems with Hawking radiation: it contains too much entropy and it contains too few baryons[1]. The first is the problem of the disappearance of quantum coherence during the formation and subsequent evaporation of a black hole. This is a very deep and difficult problem and we shall say nothing about it here. The second is the problem of the disappearance of conserved global charges during the same process. This is the subject of this paper; we shall argue that this problem is neither deep nor difficult.

Let us begin by stating the problem: Consider the gravitational collapse of a star with a nonzero value of some strictly conserved global charge. For example, the global charge could be \( B - L \), and the star could be made of helium. In the late stages of collapse, the system begins to emit Hawking radiation. The Hawking radiation is formed outside the horizon and is totally insensitive to the presence of nonzero \( B - L \) in the collapsed star; it is just as likely to contain quanta with one sign of \( B - L \) as another. When the star finally evaporates altogether, the initial \( B - L \) has totally disappeared, despite the fact that at every stage the dynamics of the process has been governed by a Lagrangian that strictly conserves \( B - L \).

This problem doesn’t arise for (continuous or discrete[3]) gauge charges. These are detectable outside the horizon and can bias the Hawking radiation. For example, when a virtual electron-positron pair appears outside a black hole with positive electric charge, the black hole will attract the electron and repel the positron; thus a positively charged black hole tends to produce positively charged Hawking radiation.

Our proposed resolution of the problem rests on the fact that global charges are distinguished from gauge charges in another way; they are the charges that can disappear down a wormhole. A wormhole is a Euclidean field configuration in some field theory containing gravity, consisting of two asymptotically flat regions connected by a tube, or throat. Charge flowing down a wormhole represents a quantum tunneling event in which charge disappears from some small region of space-time and reappears in another region, or even in a totally disconnected universe[4]. From the viewpoint of a localized observer, the effect of the wormhole is a violation of charge conservation.

\[ ^1 \text{This problem can be avoided if the black hole does not evaporate altogether, but instead leaves behind a Planck-scale remnant[2]. However, the mechanism we shall present here works as well for remnants, where it is not needed, as it does for total evaporation, where it is.} \]
But if this is the case, if, because of wormholes, there are no conserved global charges, why are we tricked into believing that $B - L$, for example, is conserved? The standard answer is that this is an accidental symmetry in the sense of Weinberg. The exact gauge symmetries of the theory are such that all possible global-charge-violating interactions have mass dimension $d > 4$, and are therefore suppressed by factors of $M^{4-d}$, where $M$ is the scale of the symmetry breaking. (We work in units such that $\hbar = c = 1$.)

The prototypical example of this is the original grand unified theory, the $SU(5)$ model of Georgi and Glashow. In this theory, baryon-number-violating processes occur on the grand unified scale, but the only baryon-number-violating interactions one can construct have $d = 6$; thus baryon-number-violating amplitudes, like that for proton decay, are suppressed by a factor of the inverse square of the grand unification mass. In our case, the role of the grand unification mass is played by the wormhole scale, the inverse size of the dominant wormholes in the functional integral.

These ideas lead to a novel picture of the fate of global charge during gravitational collapse. Let us imagine ourselves following infalling matter through the horizon on its way to the singularity. As the matter falls in, its charge density and energy density become very large, and the global-charge-violating interactions, negligible under ordinary conditions, start to become strong. When they are strong, they can drive the system towards chemical equilibrium, zero global-charge density. Of course, at the same time this is happening the matter is approaching the singularity.

Thus we have a race between equilibration and disaster. If disaster wins, if the matter reaches the singularity before the global-charge density is driven to zero, we have to confront strongly-coupled quantum gravity to find the fate of the global charge. If equilibration wins (and we shall argue it does) the matter streaming into the singularity carries no net global charge. In this case, there is no problem of disappearing global charge. No $B - L$ is seen outside the black hole because there is no $B - L$ inside the black hole. One way of phrasing things is to say that when the black hole evaporates utterly, no $B - L$ is released because all the initial $B - L$ was cooked away in the furnace of gravitational collapse. Another, totally equivalent way of phrasing things is to say no $B - L$ is released because all the initial $B - L$ has gone down wormholes.

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2 This scenario is thus a realization of Hawking’s suggestion that the missing charge goes down a wormhole—although in this case it’s not one big wormhole but many little ones. (Even earlier Zeldovich had suggested that the missing charge ended up on a disconnected universe.)
Wormholes are a controversial subject, so we should stress that our arguments do not depend in any substantial way on any detailed theory of wormhole dynamics. In particular, they are completely independent of the much-disputed wormhole theory of the cosmological constant. All we’ll need for our work is the statement that global-charge-violating interactions become strong at some large mass scale, $M$. For energy $E \ll M$, the leading effects of these interactions are represented by an operator of high ($> 4$) dimension, that is to say, by a charge-violating Lagrange density of the form

$$L_{CV} = M^{4-d} \times (\text{operator of order one}).$$

The most natural choice for $M$ would be the Planck mass, $M_P$. However, for most of our work we’ll choose $M$ to be a few orders of magnitude less than $M_P$. This is to keep ourselves honest; it keeps the physics we’re interested in well away from the regime of strongly-coupled quantum gravity. Most of our conclusions would formally go through even if $M$ were on the order of $M_P$, but our approximations would be totally unreliable, because the effects we compute would be of the same order of magnitude as the effects of gravitational fluctuations which we neglect.

To determine whether equilibration or disaster prevails, we must analyze the rate equations for the disappearance of global charge. In Sec. 2 we perform such an analysis for a very simple model of the singularity at the core of the collapsing star, a collapsing radiation-filled Friedman-Robertson-Walker universe. This model is inspired by the Oppenheimer-Snider solution for the gravitational collapse of a spherically symmetric dust cloud. Here the inside of the collapsing cloud is a section of a dust-filled collapsing Friedman-Robertson-Walker universe. We replace Oppenheimer and Snider’s dust by radiation (that is to say, by ultrarelativistic matter) because the physics we are interested in takes place on energy scales much larger than the relevant particle masses. We don’t claim that this is in any sense a good approximation to the generic black-hole singularity, or even the interior part of some Oppenheimer-Snider-like solution, but its homogeneity and isotropy makes it especially easy to analyze, and thus a good place to begin an investigation.

The result of our analysis is surprisingly simple: under the stated conditions, equilibration always wins. The disappearance of global charge need have nothing to do with strongly-coupled quantum gravity.

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3 For what it’s worth, in the wormhole theory of the cosmological constant it is possible to arrange matters such that the wormhole scale is indeed a few orders of magnitude less than $M_P$, without any fine-tuning of parameters. 

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Sec. 3 discusses both possible alternatives to wormholes and the question of to what extent we expect our results to survive if we weaken our assumptions. The analysis is still in progress, but, at the moment, things look good.

2. Rate Equations in a Collapsing Universe

2.1. Preliminaries

The Einstein equation for a Robertson-Walker metric is

\[
\frac{\dot{R}^2}{R^2} = \frac{8\pi}{3M_P^2} \rho - \frac{k}{R^2}. \tag{2.1}
\]

where \( R \) is the scale factor, \( \rho \) the energy density, and \( k = 0, \pm 1 \). As always, this must be supplemented by an equation of state, giving the pressure as a function of the energy density. As explained in Sec. 1, we take the equation of state to be that of an ultrarelativistic gas, \( p = \rho/3 \). Then \( R \), for small \( t \), is proportional to \( \sqrt{|t|} \), where we have chosen the zero of time so the singularity, \( R = 0 \), occurs at \( t = 0 \).

It will be convenient for us to define an “energy length”, \( \ell \), by

\[
\rho = \ell^{-4}. \tag{2.2}
\]

Then, from the Einstein equation, for small \( t \),

\[
\ell \sim \sqrt{\frac{|t|}{M_P}}, \tag{2.3}
\]

where \( \sim \) denotes equality up to numerical constants. This is the only equation of general relativity we shall need in our work.

To keep things as simple as possible, we’ll speak in this section as if we were studying only one global charge, and as if the total charge was the difference between the number of particles (objects with global charge one) and antiparticles (objects with global charge minus one). The extension to many global charges and many types of particles carrying various amounts of the several charges is trivial.

We shall follow the time evolution of

\[
\hat{n} = j_0 \ell^3, \tag{2.4}
\]

4
where \( j_\mu \) is the current of the global charge. Because \( \ell/R \) is a constant of the motion, this is equivalent to following the global-charge density per unit comoving volume.

We’ll assume throughout our computations that our ultrarelativistic gas is locally in thermal equilibrium (although not of course in chemical equilibrium), so its local state is completely determined by \( \hat{n} \) and \( \ell \) (equivalently, by the temperature and the chemical potential). Even when \( \dot{R}/R \) is large, this is a safe assumption; for an ultrarelativistic gas, a state of thermal equilibrium stays a state of thermal equilibrium (with a blue-shifted temperature) under uniform contraction of the universe.

2.2. Lowest-Order Perturbation Theory in Both \( \mathcal{L}_{CV} \) and \( \hat{n} \)

The easiest situation to analyze is that in which the interaction is weak (so we can treat the rate of charge disappearance to lowest (second) order in the charge-violating interaction) and the difference between the number of particles and the number of antiparticles is small compared to their sum (so we can linearize the rate equation in \( \hat{n} \)). The rate equation is then determined completely, up to numerical multiplicative constants:

\[
\frac{d\hat{n}}{dt} \sim - \frac{\hat{n}}{\ell(M^2\ell^2)^{d-4}}. \tag{2.5}
\]

The power of \( M \) in this equation is determined by the fact that it is second order in \( \mathcal{L}_{CV} \); the power of \( \ell \) is then fixed by dimensional analysis. \( M_P \) does not appear because this is a purely microphysical equation, computed (in principal) by applying kinetic theory in locally inertial coordinates.

Since \( \ell \) is a proportional to a power of \( t \), integrating this with respect to \( t \) is (up to numerical constants), the same as multiplying by \( t \). Thus,

\[
\ln \hat{n} \sim - \frac{|t|}{\ell(M^2\ell^2)^{d-4}} + \text{constant} \tag{2.6}
\]

Using Eq. (2.3), we find

\[
\lim_{t \to 0} \hat{n} = 0 \quad \text{for } d > 4\frac{1}{2}. \tag{2.7}
\]

In particular, the global charge is always totally extinguished, vanishing more rapidly than any power of \( t \), for the interactions that interest us, \( d \geq 5 \).

Of course, these formulas are not to be trusted for very small \( t \), when the interaction becomes strong and lowest-order perturbation theory is inapplicable. We’ll take care of this regime shortly; meanwhile, it’s interesting to ask how much of the global charge has disappeared by the time the interaction does become strong. At \( \ell M \sim 1 \),

\[
\frac{|t|}{(M^2\ell^2)^{d-4}\ell} \sim \frac{M_P}{M}. \tag{2.8}
\]

Thus only \( \exp(-O(M_P/M)) \) of the original charge remains. If (as we have been assuming) \( M \) is a few orders of magnitude less than \( M_P \), this is not much charge at all.
2.3. Lowest-Order Perturbation Theory in $\mathcal{L}_{CV}$ but not in $\dot{n}$

Up to now we have assumed that the difference between the density of particles and of antiparticles was small compared to their sum. This assumption is certainly not true in the early stages of collapse, when the star contains only particles, and it is not obvious whether sufficient antiparticles to make it true are produced later on. Therefore in this subsection we eliminate this assumption.

We still assume that the charge-violating interaction is weak, so we can use second-order perturbation theory in $\mathcal{L}_{CV}$. Eq. (2.5) is replaced by

$$\frac{d\dot{n}}{dt} = -\frac{\dot{n}r(\dot{n})}{(M^2\ell^2)^{d-4\ell}},$$

where $r(\dot{n})$ is some dimensionless function of $\dot{n}$. We know from the previous analysis that $r$ is positive at zero. Because there is no equilibrium state other than $\dot{n} = 0$, $r$ must be positive everywhere.

Integrating this equation from some initial time $t_0$, we find

$$\int_{\dot{n}(t_0)}^{\dot{n}(t)} \frac{d\dot{n}}{\dot{n}r(\dot{n})} = -\int_{t_0}^{t} \frac{dt}{(M^2\ell^2)^{d-4\ell}}.$$  \hspace{1cm} (2.10)

For the cases of interest to us ($d \geq 5$), the right-hand side of this equation blows up as $t$ goes to zero. Therefore, so must the left-hand side. But since $r$ is everywhere positive, this can only happen if $\dot{n}$ goes to zero.

We have computed $r$ numerically and integrated Eq. (2.10) for a simple case, the theory of a single massless Dirac field with particle-number-violating interaction

$$\mathcal{L}_{CV} = M^{-2}(\bar{\psi}^c \psi)^2,$$  \hspace{1cm} (2.11)

where $\psi^c$ is the charge conjugate of $\psi$. In lowest-order perturbation theory, all the partial-wave amplitudes for this theory are proportional to $s$, the square of the center-of-mass energy. An easy calculation shows that the largest of these hits the unitarity bound at $s \approx 5M^2$. This gives us a rough measure of strong coupling: as long as the average value of $s$ is well below $5M^2$, we are well clear of the strong-coupling regime.

In our numerical computation, we chose the initial condition $\dot{n}(-\infty) = \sqrt{8/9\pi}$, the appropriate value for a cold gas of massless fermions with no antifermions. Our result is shown in the figure; we have chosen coordinates such that the linear (small $\dot{n}$) approximation (shown by the dashed line) is a straight line. The graph is indistinguishable from the linear approximation for the average value of $s/5M^2$ greater than $6(M/M_P)^{2/3}$; for $M$ a few orders of magnitude below $M_P$, this is well in the weak-coupling region. Note that even for earlier times, the linear approximation is a conservative estimate, in that it underestimates the rate of charge extinction.
2.4. The Strong Coupling Regime

We now have a good picture of what happens to global charge from the beginning of collapse to the time when $\ell M$ approaches one; at the end of this epoch, most of the global charge has been cooked away, rate equations linear in $\hat{n}$ are a good approximation, and the charge-violating interaction is about to become strong. We cannot go beyond this point with dynamical calculations, because we can only compute wormhole effects in weak-coupling approximations. Instead, we will resort to the old idea that when interactions become strong, cross sections become as large as they can be, consistent with the finite range of the interactions. This leads to constant cross-sections at high energies, possibly modified by slow (logarithmic) growth. This is in excellent agreement with experiment for the one strongly-interacting relativistic theory on which experiments have been performed, hadrodynamics.

Thus we assume that for energies greater than $M$, the total cross-section for charge-violating scatterings is

$$\sigma_{CV} \sim M^{-2},$$

possibly times logarithms of the energy, which will not affect our argument. Here we've guessed that the range of the interactions is on the order of $1/M$, the only relevant length in the problem. If we underestimate the rate of extinction of global charge by only including the effects of two-body scatterings, we find, on the usual dimensional grounds, that

$$\frac{d\hat{n}}{dt} \sim -\frac{\hat{n}}{M^2 \ell^3}.$$  \hspace{1cm} (2.13)

This is identical to Eq. (2.5), second-order perturbation theory, for $d = 5$, (although, of course, the angular distribution of the outgoing particles is completely different), and, like it, it leads to total extinction of global charge.

3. Comments

We have found that, if there are wormhole-induced charge-violating interactions that are not observable at low energies because of the presence of inverse powers of the wormhole scale, then these interactions always remove all the global charge from a collapsing star, for a certain model of the collapse singularity. The obvious questions are: To what extent does our analysis depend on wormholes? To what extent does our analysis depend on our model of the singularity?
We can think of three mechanisms that might substitute for wormholes:

(1) Otherwise unspecified metric fluctuations at the Planck scale: All of the equations of Sec. 2 apply in this case, with $M$ set equal to $M_P$, and they lead to the same conclusion, the total extinction of the global charge. The problem here, as we remarked in the Introduction, is that in addition to the physics described in Sec. 2, there is the physics of strongly-coupled quantum gravity, which should be important at this scale, and which is totally ignored in Sec. 2, for the excellent reason that we don’t know anything about it. To phrase the same thought in another way, it’s silly to track global charge when the infalling matter is a Planck time away from the singularity; when you’re a Planck time away from the singularity, fluctuations in the metric are so large that concepts like “time” and “singularity” lose their meaning. Of course it’s always possible that someday, when strong quantum gravity effects can be treated reliably, it will turn out that the arguments of Sec. 2 are basically correct. However, as H. G. Wells said in another context, “If anything is possible, nothing is interesting.”

(2) Strings: It is essentially impossible to implement a continuous symmetry in string theory that is not a gauge symmetry. Thus strings are as much enemies of conserved global charges as wormholes. From this point on, the discussion is the same as that of the preceding paragraph.

(3) Grand unified theories: The absence of nongauged continuous symmetries in grand unified theories is as much a matter of fashion as of physics; nevertheless, grand unified theories do have one of the key feature of the theories we have been discussing, charge-violation suppressed by large mass denominators, and it is worth asking to what extent our analysis applies to them. The big difference is that the grand unified interactions never become strong; at the grand unification scale, $M_{GUT}$, they are weak (and renormalizable), and they stay weak at higher energies. This has only a mild effect on the arguments of Secs. 2.2 and 2.3. For example, in Eq. (2.4), the factor of $(1/M^2)^{d-4}$ is replaced by a factor of $\alpha_{GUT}(1/M_{GUT}^2)^{d-4}$ (assuming the underlying process is first order in the grand unified gauge coupling). This does not make a big difference; for reasonable choices of the parameters, most of the global charge is still extinguished. However, the last part of the extinction, discussed in Sec. 2.4, does not work; the mild high-energy behaviour allows the remaining global charge to reach the singularity.

As for the question of the singularity, there is a clear path to take. The mixmaster singularity is the outstanding candidate for a generic black-hole singularity. The mixmaster is neither isotropic nor homogeneous, and does not preserve thermal equilibrium;
thus it presents a more difficult problem than the cosmological singularity, but it is not beyond analysis. We have successfully analyzed the fundamental component of the mixmaster, the homogeneous Kasner universe, and have found, once again, total extinction of global charge. (Although the rate of extinction is quite different from the cosmological case; at the end, $\dot{n}$ only vanishes like a power of $t$.) We hope to extend our analysis to the full mixmaster and report on our results shortly.

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**Figure Caption**

The result of a numerical computation, for $L_{C V} = M^{-2} (\bar{\psi}^c \psi)^2$, of the charge per comoving volume in a collapsing universe. The system was started as a cold gas of particles (with no antiparticles) at $t = -\infty$. The dashed line is the linear approximation to the rate equation.
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