Ultracold Scattering Processes in Three-Atomic Helium Systems* †
E. A. Kolganova², A. K. Motovilov² and W. Sandhas³
²BLTP, JINR, Joliot-Curie 6, 141980 Dubna, Moscow Region, Russia
³Physikalisches Institut, Universität Bonn, Endenicher Allee 11-13, D-53115 Bonn, Germany

We review results on scattering observables for $^4\text{He}–^4\text{He}_2$ and $^3\text{He}–^4\text{He}_2$ collisions. We also study the effect of varying the coupling constant of the atom-atom interaction on the scattering length.

1. INTRODUCTION

Experimentally, $^4\text{He}$ dimers have been observed in 1993 by Luo et al. [1], and in 1994 by Schöllkopf and Toennies [2]. In the latter investigation the existence of $^4\text{He}$ trimers has also been demonstrated. Later on, Grisenti et al. [3] measured a bond length of $52 \pm 4 \, \text{Å}$ for $^4\text{He}_2$, which indicates that this dimer is the largest known diatomic molecular ground state. Based on this measurement they estimated a scattering length of $104^{+8}_{-18} \, \text{Å}$ and a dimer energy of $1.1^{+0.3}_{-0.2} \, \text{mK}$ [3]. Further investigations concerning $^4\text{He}$ trimers and tetr amers have been reported in [4], but with no results on size and binding energies. To the best of our knowledge, we are not aware of any experimental results on binding energy and size of the asymmetric helium trimer consisting of two $^4\text{He}$ and one $^3\text{He}$ atoms.

Many theoretical calculations of the helium three-atomic systems were performed in the past for various interatomic potentials. Variational, hyperspherical, and Faddeev-type techniques have been employed in this context [5]–[17]. It was found that the $^4\text{He}$ trimer has two bound states of total angular momentum zero: a ground state of about 126 mK and an excited state of about 2.28 mK. Experimentally this Efimov-type excited state has not yet been observed. It should be mentioned, however, that the year 2006 is noticeable due the first convincing experimental evidence for the Efimov effect in an ultracold gas of caesium atoms [18].

Due to the smaller mass of the $^3\text{He}$ atom, the $^3\text{He}–^4\text{He}$ system is unbound. Nevertheless, the $^3\text{He}^4\text{He}_2$ trimer exists, though with a binding energy of about 14 mK. In contrast to the symmetric case, there is no excited state in the asymmetric $^3\text{He}^4\text{He}_2$ system.

Phase shifts of $^4\text{He}–^3\text{He}_2$ elastic scattering at ultra-low energies have been calculated for the first time in [10, 11] both below and above the three-body threshold. An alternative $ab\ initio$ calculation was performed in [13], but only below the threshold. The only available results on the $^3\text{He}–^3\text{He}_2$ phase shifts were obtained in [16]. For completeness we notice that zero-range models are able to reproduce the $^4\text{He}–^4\text{He}_2$ scattering situation [19].

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In what follows we present our results on binding energies and scattering observables in the helium three-atomic systems obtained mainly with the LM2M2 potential by Aziz and Slaman [20].

2. RESULTS

In our calculations we employed the hard-core version of the Faddeev differential equations developed in [21]. Table 1 summarizes trimer binding energies and He–He$_2$ scattering lengths, calculated with the LM2M2 potential. The binding energies of the 4He trimer ground state ($E_{4\text{He}_3}^g$) and exited state ($E_{4\text{He}_3}^*$) are presented in the first two rows. These results demonstrate the good agreement between the different methods.

Table 1
Results for binding energies of the 4He$_3$ and 3He$_4$He$_2$ trimers and the He–He$_2$ scattering lengths.

|                  | Ref. [11] | [6]  | [7]  | [8]  | [9]  | [12] | [15] |
|------------------|-----------|------|------|------|------|------|------|
| $-E_{4\text{He}_3}$ (mK) | 125.9     | 125.2 | 125.5 | 126.41 | 126.39 | 126.4 | 126.39 |
| $-E_{4\text{He}_3}^*$ (mK) | 2.276$^b$ | 2.269 | 2.19  | 2.271 | 2.265 | 2.268 |
| $\ell_{\text{sc}}^{(4\text{He}^4\text{He}_2)}$ (Å) | 115.5$^c$ | 126.0 | 115.4$^d$ | 115.2 |
| $-E_{3\text{He}_4\text{He}_2}$ (mK) | 13.84$^e$ | 13.66 | 14.4$^f$ | 14.165 |
| $\ell_{\text{sc}}^{(3\text{He}^4\text{He}_2)}$ (Å) | 21.0$^e$ | 19.3$^f$ |

$^a$Calculations with maximal value $l_{\text{max}} = 4$ of the subsystem angular momentum. $^b$This value was rounded in [11]. $^c$Result of extrapolation, see [14]. $^d$Result from Ref. [13]. $^e$Result from Ref. [16]. $^f$Result from Ref. [17].

The third row shows values of the 4He–4He$_2$ scattering length. Notice that in Ref. [14] our previous calculations of [11] have been improved essentially by enlarging the grid parameters and the cut-off hyperradius. This table also contains the results by Blume and Greene [7], by Roudnev [13], and the most recent ones by Lazauskas and Carbonell [15]. The treatment of [7] is based on a combination of the Monte Carlo method and the hyperspherical adiabatic approach. Roudnev employs three-dimensional Faddeev differential equations. Lazauskas and Carbonell also use differential Faddeev equations, but with the hard-core boundary conditions of Ref. [21]. Our results agree rather well with these alternative calculations. For completeness we mention the model calculations of [19]. Being characterized by remarkable simplicity, they rely essentially on the binding energy obtained in *ab initio* calculations.

The last two rows of Table 1 contain results for the binding energy of the asymmetric 3He$_4$He$_2$ trimer and for the 3He–4He$_2$ scattering length. The phase shifts $\delta(E)$ of 3He–4He$_2$ and 4He–4He$_2$ scattering are depicted in Fig. 1 where in both cases the normalization $\delta(0) = 0$ is used.

It is already widely accepted that the excited 4He trimer state is of Efimov nature. For the first time this was clearly shown by Cornelius and Glöckle [5]. A more detailed investigation has been made in [23] where the mechanism of emerging new Efimov states from resonances was studied. For a latest discussion of this subject see Ref. [15]. Table 2 which is borrowed partly from [23], demonstrates how a new Efimov level arises from a virtual state when the interatomic
Table 2
Dependence of the $^4\text{He}$ dimer and trimer energies (mK) and the $^4\text{He}$$-^4\text{He}$ and $^4\text{He}$$-^4\text{He}_2$ scattering lengths (Å) on the potential strength $\lambda$. The three-body results were obtained with a maximal value $l_{\text{max}} = 0$ of the subsystem angular momentum. The HFD-B potential from Ref. [22] was used.

| $\lambda$ | $\epsilon_d$ | $\epsilon_d - E^*$ | $\epsilon_d - E_{\text{virt}}$ | $\epsilon_d - E^{**}$ | $\ell_{\text{sc}}^{(1+2)}$ | $\ell_{\text{sc}}^{(1+1)}$ |
|-----------|-------------|-------------------|-----------------------------|------------------------|--------------------------|--------------------------|
| 1.30      | -199.45     | -                 | 1.831                       | -61                    | 11.4                     |
| 1.20      | -99.068     | -                 | 0.01552                     | -340                   | 14.7                     |
| 1.18      | -82.927     | -                 | 0.00058                     | -1783                  | 15.8                     |
| 1.17      | -75.367     | 0.0063            | -                           |                        |                          |
| 1.15      | -61.280     | 0.0737            | -                           | 256                    | 17.7                     |
| 1.10      | -32.222     | 0.4499            | -                           | 152                    | 23.1                     |
| 1.0       | -1.685      | 0.773             | -                           | 160                    | 88.6                     |
| 0.995     | -1.160      | 0.710             | -                           | 151                    | 106                     |
| 0.990     | -0.732      | 0.622             | -                           | 143                    | 132                     |
| 0.9875    | -0.555      | 0.222             | -                           | 125                    | 151                     |
| 0.985     | -0.402      | 0.518             | 0.097                       | 69                     | 177                     |
| 0.982     | -0.251      | 0.447             | 0.022                       | -75                    | 223                     |
| 0.980     | -0.170      | 0.396             | 0.009                       | -337                   | 271                     |
| 0.9775    | -0.091      | 0.328             | 0.003                       | -6972                  | 370                     |
| 0.975     | -0.036      | 0.259             | -0.002                      | 7120                   | 583                     |
| 0.973     | -0.010      | 0.204             | -0.006                      | 4260                   | 1092                    |

interaction in the $^4\text{He}_3$ is being weakened. To this end we multiply the original HFD-B potential of Ref. [22] by a factor $\lambda < 1$. Decreasing this coupling constant, there emerges, for $\lambda \approx 0.986$, a virtual state with energy $E_{\text{virt}}$ lying on the unphysical sheet. This energy, relative to the diatom energy $\epsilon_d$, is given in the lower half of column 4. When decreasing $\lambda$ further, this state turns at the value $\lambda \approx 0.976$ into the second excited state. Its energy $E^{**}$ relative to $\epsilon_d$ is shown at the bottom of the fifth column. When the second excited state emerges, the atom–diatom scattering length $\ell_{\text{sc}}^{(1+2)}$ changes its sign going through a pole, while the atom-atom scattering length $\ell_{\text{sc}}^{(1+1)}$ increases monotonically.

We have also studied the opposite case where $\lambda > 1$ is increasing. In this case the scattering length $\ell_{\text{sc}}^{(1+2)}$ is growing until $\lambda \approx 1.175$. There $\ell_{\text{sc}}^{(1+2)}$ becomes negative passing through a singularity and the excited state energy $E^{*}$ turns into a virtual level. The energy $E_{\text{virt}}$ of this state is shown for $\lambda \geq 1.18$ in the upper quarter of column 4.

In Fig. 2 we graphically display the behavior of the atom-atom and atom-diatom scattering lengths $\ell_{\text{sc}}^{(1+1)}$ and $\ell_{\text{sc}}^{(1+2)}$ shown in Table 2.

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Figure 1. S-wave He–He$_2$ phase shifts $\delta(E)$ obtained with the LM2M2 potential. The lower curve depicts the $^4$He–$^4$He$_2$ results of [11], the dots are the ones found in [13]. The upper curve represents the $^3$He–$^4$He$_2$ phase shifts of Ref. [16].

Figure 2. Dependence of the atom-atom scattering length $\ell_{sc}^{(1+1)}$ (Å) (solid curve) and atom-diatom scattering length $\ell_{sc}^{(1+2)}$ (Å) (dashed curve) on the factor $\lambda$.

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