THE THEORY OF COMBINATIONS FOR LAND PLOT EXCHANGE MODELLING IN THE COURSE OF LAND CONSOLIDATION

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Abstract. The paper is aimed at the promotion of voluntary land consolidation through the improvement of land plots exchange. The issue of the existing land plots boundaries adjustment in the course of voluntary land consolidation has been singled out. Possibilities, advantages and risks of land plots exchange without changing the existing boundaries as a constituent of land consolidation measures have been substantiated. The improvement of approaches to land plots exchange modelling without the existing boundaries adjustment has been suggested. Demands to the formation of consolidated land tenures as the result of exchange have been singled out. Theory of combinations has been applied to specify land plots exchange options. Calculation formulas for the number of the optimal consolidated agricultural land tenure placement options have been suggested. The calculations can be applied in the optimization and heuristic approaches to land reallocation and at land consolidation performance evaluation. Suggested approaches facilitate the implementation of heuristic methods especially in non-standard conditions, allow to increase the number of developers, especially involving experts with little experience. The results can be used for land plots lease optimization.

Keywords: agricultural land, land readjustment, peer land plots, land consolidation, land exchange, land mass, theory of combinations tasks.

Introduction

Land consolidation is one of the most efficient mechanisms of land plots spatial characteristics improvement, with the help of which the aims of sustainable development are achieved. Most Eastern European countries have their own experience of land consolidation (Hartvigsen, 2016; Sabates-Wheeler, 2002), which confirms the dependence of the respective measures effectiveness on the adaptation to local conditions. This is one of the reasons for the existence of a variety of methods and approaches to land consolidation (Food and Agriculture Organization [FAO], 2003; Thomas, 2006), of which the exchange method is of special importance. Land exchange as a separate approach (Fernández, 2009; Gollwitzer, 2012; Malashevskyi & Bugaienko, 2016) and as a constituent of voluntary land consolidation is widespread (Hartvigsen, 2015; Gedefaw et al., 2019; Sulonena et al., 2017).

The exchange method is the most reasonable for land plots of similar value within a separate land mass. For example, in Ukraine, most agricultural land tenures outside the inhabited localities were formed as the result of the parceling of solid collective and state-owned land masses at the transition to market relations and private land ownership. Normative monetary valuation of such land plots varies within the range of 10% with some exceptions connected with inheritance. Such land masses structure has been virtually unchanged due to the agricultural land market moratorium still in force in Ukraine (Malashevskyi et al., 2020).

In such circumstances, there is no sufficient technical justification of land plots exchange within a land mass. The exchange, subdivision and merger of land plots (Verkhovna Rada of Ukraine, 2003) are set out by the legislation of Ukraine (Verkhovna Rada of Ukraine, 2002, 1998), however, the exchange methodology has not been set out. In addition to that, it has not been specified, which exchange can be considered to be peer. Only for the state or communal property land, exchange within an agricultural land mass is set out provided the difference between the normative monetary values of such land plots is no more than 10 percent (Verkhovna Rada of Ukraine, 2002).

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Persons using 75% of the agricultural land mass have a right to exchange the land plot use right, however, there are no extra recommendations.

The issue of land consolidation is resolved using the optimization or heuristic method (Lemmen et al., 2012; Yimer, 2014). Significant attention is paid to the development and implementation of land plots reallocation optimization models (Bugaienko, 2018; Mihajlović et al., 2011; Yilmaz & Demir, 2015). It is worth mentioning that algorithms based on the combination of optimization and heuristic approach are recognized as the most effective. Land reallocation is carried out using the following algorithms: Automation of the Re-allotment Plan for Land Consolidation (ATOR) (the Netherlands), Allocation and Adjustment Model (AVL), TRANSFER etc. (Lemmen et al., 2012). It has been noticed at the implementation reviewing, the effectiveness of their implementation depends on the simplicity and understandability of calculations (Yimer, 2014).

Ukraine is one of the countries for which the voluntary land consolidation is recommended (FAO, 2017). The insufficient legal support of the compulsory land owners participation and poor readiness of the community for the compulsory land plots reallocation due to the lack of confidence of land owners in their land ownership right securing significantly influence the choice of the land consolidation approach in such circumstances.

In such circumstances, there is a demand for the most cost effective and easy to implement land consolidation measures preferably based on not advanced but field-tested land plots reallocation mechanisms, small project implementation time and especially the provision of all the possible and thoroughly substantiated consolidation options. The existing optimization and heuristic approaches should be supplemented with the substantiation of land plots swapping options aiming at design flexibility.

The goal of the research is to substantiate the application of the theory of combinations for land plot exchange modelling in the course of land consolidation.

1. Land plots placement within the land mass after land consolidation

The placement of land plots of individual land owners within the land tenure of another land owner (Figure 1) is a widespread issue in Ukraine. Irrespective of that the land exchange is predefined by the legislation in effect, the examination of the actual use of land plots within the agricultural land masses has revealed the land plot exchange as a land tenure optimization tool is not widespread. At the same time, the interspersion of separate land plots into a land mass creates cultivation hardship and need for extra passages, etc.

The search of land consolidation options as a constituent of the respective projects (FAO, 2003) can be started with the statement of requirements to land plots placement within the land mass.

Demands to reallocated land plots spatial characteristics usually depend on the aims of consolidation (Malashevskyi et al., 2018) and are defined in the process of the respective projects implementation, especially at the formation of the environmental or infrastructure facilities land tenures (Thomas, 2012; Hendricks & Lisec, 2013; Rybicki, 2017). Agricultural land plots are most often involved in the process of reallocation, therefore, it is suggested to single out general demands defining the effectiveness of agricultural land as the production factor.

At the current stage, the practicability of the fragmented land plots merge and formation of a larger area space effective land tenure is sufficiently substantiated (FAO, 2003; Hartvigsen, 2016; Bullard, 2007).

As the result of land consolidation, space effective land tenures should be formed with access to each land plot. It is reasonable to place the consolidated land plots at the land mass boundary. Boundaries folding and acute angles should be avoided. The optimal land tenure should have a configuration close to a rectangle with the side ratio of no more than 1:4 or a rectangular trapezoid with the

Figure 1. Typical land plots placement within a land mass. Public Cadastral Map of Ukraine
acute angle of more than 60°. The longer sides of a land tenure in the form of a quadrangle should be parallel and placed along the boundary of the land mass. A land tenure should be placed considering the relief to begin with the long sides perpendicular to the slope direction. Placement on a slope of the same exposure is desirable. With such form and placement, the most effective land plots cultivation and land erosion control is achieved.

The abovementioned requirements are significantly influenced by the precondition if the existing land plots boundaries are kept in the process of land consolidation. The issue of boundaries demarcation is singled out among the technical aspects of land consolidation (Bullard, 2007). We consider land reallocation and land exchange without land plots boundaries adjustment as the key land consolidation instruments (Sonnenberg, 2002) according to the key aims of land consolidation (Figure 2).

According to the general definition of land readjustment (du Plessis, 2016), the respective measures envisage the development based on the land plots put together in units, which are "re-divided into plots and re-allocated to the landholders according to contribution size or value" in the process of land consolidation. Actually, some of land consolidation aims exclude the possibility of its implementation with keeping the existing land plots boundaries. Land readjustment has many advantages (Table 1), the most significant of which is the increased flexibility of design considerations. Such measures match the comprehensive land consolidation according to the FAO classification (FAO, 2003) and need the respective regulating
and promoting mechanisms which allow to alienate land plots especially in a compulsory way. At the same time, main costs of land management at land consolidation are caused by valuation methods and issues of boundaries demarcation (Thomas, 2006). Main costs are caused by the valuation method and the question, if boundaries have to be marked. 

The process of land consolidation at the reallocation is complicated due to the need for the adjusted boundaries approval by a number of land owners, ownership right legal formalization and the reallocated land plots boundaries demarcation at the terrain. As the result, the activities execution time and costs are increased. In such cases, land plots readjustment demands the alienation of land plots within the land mass (or its part) and their subsequent allocation. The latter is a risk for land owners especially in case it is not directly prescribed by legislation.

Land plots exchange without adjusting the land plots boundaries is effective at the voluntary initiatives of land owners as well as a constituent of projects aimed at land masses improvement. It is reasonable to carry out land plots reallocation without the existing land plots boundaries adjustment according to land consolidation types of individual of voluntary group land consolidation defined by FAO. To make a managerial decision at land consolidation by individual land owners or local communities, it is important to evaluate the possible exchange options considering demands on their placement within the land mass.

2. Land plots exchange options

At the peer land plots exchange within an agricultural land mass, it is suggested to calculate the readjustment options using the theory of combinations. The total number \( N \) of the possible land plots placement options within a land mass \( m \) which includes \( n \) land plots (given data \( n > m \)) (Figure 3) is \( A \) combinations:

\[
N = A^m_n = \frac{n!}{(n-m)!}. \tag{1}
\]

For land consolidation aims, land plots placement considering the demands on their spatial characteristics are of crucial significance (not the random placement). Let us define the possible consolidated land plots placement options at the land mass boundary. In this case, cultivation costs, expenses of transportation and additional costs of land plot access are reduced.

To define reallocation options, let us scrutinize each of the outer rows of the land mass (Figure 4). Provided, the land mass comprises \( n_h \) land plots in length and \( n_l \) land plots in width:

\[
n_h \times n_l = n, \tag{2}
\]

where \( n \) is the number of land plots within the land mass. Initial data for calculation:

\[
1 < m \leq n_h; \tag{3}
\]

\[
1 < m \leq n_l. \tag{4}
\]

It is suggested to calculate the number of options \( N \) of \( m \) land plots at the land mass boundary using the combination \( C \) and the derangement \( P \) by the formula:

\[
N = 2(C_{n-h-m+1}^1 \times P_n) + 2(C_{n_l-m+1}^1 \times P_m). \tag{5}
\]

Then:

\[
N = 2\left(\frac{(n_h - m + 1)!}{(n_h - m + 1 - 1)!} \times m!\right) + 2\left(\frac{(n_l - m + 1)!}{(n_l - m + 1 - 1)!} \times m!\right); \tag{6}
\]

\[
N = 2m! \left(\frac{(n_h - m + 1)!}{(n_h - m - 1)!} + \frac{(n_l - m + 1)!}{(n_l - m)!}\right); \tag{7}
\]

\[
N = 2m!(n_h - m + 1 + n_l - m + 1); \tag{8}
\]

\[
N = 2m!(n_h + n_l - 2m + 2). \tag{9}
\]
Thus, there are $2(n_h + n_l - 2m + 2)$ placement options of $m$ consolidated land plots at the land mass boundary. At every option, there are $m!$ derangement options for the consolidated land plots. This number of derangements within a consolidated land mass predefines the possibility of the optimal selection of peer land plots at the exchange.

If $m = 1$, the reallotment options number is calculated by the formula:

$$N = 2C^2_{n_h} + 2C^2_{n_l}.$$  

$$N = 2 \left( \frac{n_h!}{(n_h-1)!} \right) + 2 \left( \frac{(n_l-2)!}{(n_l-3)!} \right);$$  

$$N = 2(n_h + n_l - 2).$$

Let us scrutinize the placement of the consolidated land plots at the land mass boundary, however, in the form of a rectangle consisting of $d \times p$ land plots (Figure 5):

$$d \times p = m,$$

where $m$ – is the general number of the reallocated land plots.

The case when the following condition is met, is scrutinized:

$$n_h \geq d,$$

$$n_l \geq p.$$

Let us calculate the placement options, provided that in the outer rows of the land mass which comprises $n_h \times n_l$ land plots, $d$ land plots are placed. The case when the following condition is met, is scrutinized:
$d \neq p. \tag{16}$

The total number of land plots reallocation options according to the specified conditions:

$$N = 2(C_{n_h - d + 1}^1 \times P_m) + 2(C_{n_l - d + 1}^1 \times P_m). \tag{17}$$

Then:

$$N = 2\left[ \frac{(n_h - d + 1)!}{(n_h - d + 1 - 1)!} \times (d \times p)! \right] + 2\left[ \frac{(n_l - d + 1)!}{(n_l - d + 1 - 1)!} \times (d \times p)! \right] \tag{18};$$

$$N = 2(n_h - d + 1)(d \times p)! + 2(n_l - d + 1)(d \times p)!; \tag{19}$$

$$N = 2(d \times p)!(n_h - d + 1 + n_l - d + 1); \tag{20}$$

$$N = 2(d \times p)!(n_h + n_l - d - 2). \tag{21}$$

Thus, there are $2(n_h + n_l - d + 2)$ placement options of the consolidated land plots at the land mass boundary. At every option, there are $(d \times p)!$ derangement options among the consolidated land plots.

On condition that $p = 1$, then $d = m$, so, formula (21) is converted into the formula (9).

Let us scrutinize the exchange option, when the consolidated land plots in the form of a rectangle consisting of $d \times p$ land plots are placed in the following way: $d$ land plots along the land mass side comprising $n_h$ land plots, $p$ land plots along the land mass side comprising $n_l$ land plots (Figure 6).

Then, at the fulfillment of conditions (14) and (15), the total number of land plots reallocation options is calculated by the formula:

$$N = 2(C_{n_h - d + 1}^1 \times P_m) + 2(C_{n_l - p - 1}^1 \times P_m); \tag{22}$$

$$N = 2\left( \frac{(n_h - d + 1)!}{(n_h - d + 1 - 1)!} \times (d \times p)! \right) + 2\left( \frac{(n_l - p - 1)!}{(n_l - p - 1 - 1)!} \times (d \times p)! \right); \tag{23}$$

$$N = 2(n_h - d + 1)(d \times p)! + 2(n_l - p - 1)(d \times p)!; \tag{24}$$

$$N = 2(d \times p)!(n_h - d + 1 + n_l - p - 1); \tag{25}$$

$$N = 2(d \times p)!(n_h + n_l - d - p). \tag{26}$$

Generally, $2(n_h + n_l - d - p)$ placement options of the consolidated land plots at the land mass boundary. At every option, there are $(d \times p)!$ derangement options among the consolidated land plots.

It is worth mentioning that the formula (26) considers the case when $d = p$. If $d = p = 1$, then formula (26) is converted into the formula (12).

According to the formula (26), the placement of the consolidated land plots $d \times p$ has the same number of options, irrespective of the placement along a certain side of the land mass. I.e. the number of the consolidated mass placement options $d \times p$ is the same on Figures 7а and b.

It is taken into consideration at the placement substantiation, that the exchanged land plots should be adjacent, the consolidated land tenure should be close to square or rectangle with the side ratio up to 1:4, the front side should be longer.

Let us define the number of the consolidated land tenure formation combinations $N$ within a land mass with the possibility of the involvement of all land plots of the land mass (Figure 8).

![Figure 6. Land plots consolidation by the formation of a land tenure of the defined configuration and placement at the land mass boundary)
It is suggested to carry out the calculation by the formula:

\[ N = (n_l - p + 1) \times C_{n_l-d+1}^l \times P_{p+d}; \] (27)

\[ N = (n_l - p + 1) \times \left( \frac{(n_h - d + 1)!}{(n_h - d + 1 - 1)!} \times (d \times p)! \right); \] (28)

\[ N = (n_l - p + 1) \times (n_h - d + 1) \times (d \times p)! . \] (29)

Thus, there are \((n_l - p + 1)\times(n_h - d + 1)\) placement options of the consolidated land plots in the land mass. There are \((d \times p)!\) derangement options in every land tenure placement option.

3. Land plots exchange options practice

Let us scrutinize the formation of the consolidated land tenure with the area of 96 000 sq.m. as the result of land plots buying and selling and exchange. The formation of the consolidated land tenure can be carried out within two land masses. Land mass A comprises land plots with the approximate area of 12 000 sq.m. each, land mass B comprises land plots with the approximate area of 16 000 sq.m. each (Figure 9).

Let us calculate the possible consolidated land tenure formation options by the formula (26):

\[ N_A = 2(4 \times 2)! \left(12 + 16 - 4 - 2\right) . \] (30)
Thus, there are 44 consolidated land tenure formation options from eight land plots within a land mass, consisting of 192 land plots. It is worth mentioning that the consolidated land tenure formed from eight land plots having $8!$ (i.e. 40 320) derangement options within the consolidated land tenure at each placement option. Thus, there are 1 774 080 land plots placement options at the consolidated land tenure formation.

Let us carry out similar calculations by the formula (26) for land mass $B$:

$$N_A = 8!^{44}.$$  \hspace{1cm} (31)

$$N_B = 1774080.$$  \hspace{1cm} (32)

There are 54 consolidated land tenure formation options from six land plots within a land mass consisting of 156 land plots. It is worth mentioning that the consolidated land tenure includes six land plots having $6!$ (i.e. 720) derangement options within the consolidated land tenure at each placement option. Thus, there are 38 880 land plots placement options at the consolidated land tenure formation.

The suggested land plots reallocation options calculations are aimed at the support of the land consolidation decision-making, sustainable development at the heuristic approach and optimization approach evaluation, especially at the voluntary land consolidation.

Conclusions

The increase of land plots exchange effectiveness without boundaries adjustment is a necessary initial stage of land reallocation implementation and development of land consolidation strategy. It is demonstrated by the advantages of land plots exchange as compared to the reallocation at land consolidation on the land relations reforming stage. Land plots boundaries adjustment and demarcation and the following ownership right re-registration are the biggest risks at the voluntary land consolidation in Ukraine.

The practicability of land plots exchange without adjusting the existing boundaries according to various land consolidation aims has been analysed. It has been found that the fragmented land plots placement optimization within a land mass or a separate land plot within a consolidated land mass is the most prospective.

According to the research undertaken, a demand for the approaches specification to the search of the alternative land plots exchange options within land masses without adjusting the land plots boundaries has been specified. It is suggested to specify the possible land plots exchange options using the theory of combinations. The formation of a land tenure with the most preferable for the agricultural activity configuration and placement is in the gist of the substantiation.

At the development, the provision of general recommendations on the optimal land exchange for local communities and land owners is necessary based on the designer and expert experience. With this aim, general demands on the consolidated land tenure formation within a land mass have been singled out. Calculation formulas for consolidated land tenure placement options in typical cases have been suggested.
It is suggested to use the determination of land plots reallocation options number at the land plots reallocation heuristic and optimization method and as the optimization method accuracy evaluation. It is suggested to include the calculation approaches to the recommendation for local communities and land owners aiming at the voluntary agricultural land consolidation promotion.

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**Author contributions**

Mykola Malashevskyi was responsible for the design and development of the data analysis. Olena Malashevska was responsible for data collection, analysis and interpretation.

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