Multi responses taguchi optimization using overlaid contour plot and desirability function

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Abstract. Optimization is the process of getting an optimal response. The taguchi design is an experimental design that is often used to get robust responses. In the multi-response Taguchi design, the optimization process is carried out by considering all responses simultaneously. Cases study used in this research is aluminum production cases with four treatments, namely tool rotational speed, welding speed, shoulder tool diameter and pin diameter. Each treatment was tried with three levels. Optimized responses are tensile strength, average microhardness and average grain size. The purpose of this research is determining the point of optimization using response surface method and desirability function. The optimization results obtained are tool rotational speed of 1100 RPM, Welding speed of 40 mm/min, Shoulder diameter of 21 mm, and pin diameter of 5.4 mm.

1. Introduction

Response optimization is an effort to get the treatment composition that produces an optimal response [1]. The response optimization process is often used in the manufacturing industry in the product quality engineering business. Quality products are obtained from experimental results involving statistical analysis. Product quality is judged by the performance or characteristics of the product. In general, product quality assessments not only consider one product characteristic but consider more than one quality indicator. The purpose of multi responses optimization is to obtain a treatment composition which is the optimal solution for all responses considered.

Response optimization analysis that is often used is Response Surface Methodology (RSM). RSM is a collection of mathematical and statistical techniques that are useful for modeling and analysis aimed at obtaining an optimal response [2]. RSM modeling can be done with a first order model that only involves the main influence of the experimental factor, or it can also be used a second order model that involves a quadratic influence and the interaction effect of the experimental factor. In its analysis RSM is supported by visual analysis using surface plots and contour plots. Surface plot is a three-dimensional plot that illustrates the relationship between two experimental factors and the observed response. It can be projected into a two-dimensional plot called a contour plot. Multi responses analysis with RSM is done with a visual approach using overlaid contour plot which is a contour plot buildup of each response.

Another approach to multi responses optimization analysis is done with the desirability function. It is a formal analysis done by transforming the response value to 0 to 1. There are individual desirability function is divided into three methods: the larger the better, smaller the better, and nominal the best. The use of these three methods depends on the purpose of optimizing each response. Larger the better...
is used when the desired optimization is maximizing response. Smaller the better is used when the desired optimization is minimizing response. While nominal the best is used when the optimization is done at a certain nominal target value. Each individual desirability function is compiled into a Composite desirability function by involving the desired weighting [1].

The case study used is the process of optimizing the response in the aluminum manufacturing industry . The quality of aluminum is considered from three response characteristics, namely tensile strength (Y1), average microhardness (Y2) and average grain size (Y3) . The factors tested were rotational speed (A), welding speed (B), shoulder tool diameter (C) and pin diameter (D), each tested with three levels [3]. In this research the purpose of optimization in these cases is to maximize response.

The problem that arises is how to optimize multi responses in the Taguchi design so that the treatment composition can be obtained that can optimize the overall response. This study aims to obtain the optimum point of treatment in the case of making aluminum using the response surface method and desirability function.

2. Literature Review
2.1. Response Surface Methodology (RSM)
In experiments involving quantitative factor levels, the optimization process can be done through RSM. RSM is one of the methods used to carry out the optimization process. In a single response experiment, the optimization process is done by modeling the response variable with the independent variable. The model can be form of a first order response surface model which is a linear model. If the first order model cannot describe the relationship between the independent variable and the response variable, the second order surface response model is used [2]. In a multi responses experiment the response surface optimization can be done visually using an overlaid contour plot.

RSM is one method that aims to optimize the response process. Basically RSM is the development of orthogonal polynomials. The optimization process is done by modeling and supported visually by surface plot and contour plot. Suppose an experiment with two factors (x1 and x2) aimed at maximizing response (y) then the shape of the relationship can be modeled in the following Equation 1.

\[ y = f(x_1, x_2) + \varepsilon \]  

with \( \varepsilon \) an error component of the response. The actual form of the relationship between the response variable and the independent variable is generally unknown. Estimation of the shape of the model is done to illustrate this relationship. We often estimate the model using low-level polynomials. If the first order polynomial model can describe well the relationship between the response variable and the independent variable, then the model used is as Equation 2.

\[ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \varepsilon \]  

In the model above there are only linear influences in the model. If the first order is not suitable, the second order model can be used (Equation 3).

\[ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{12} x_1 x_2 + \varepsilon \]  

In the second order model there are linear effects, quadratic influences and the influence of interactions between independent variables. In general, if there are \( k \) independent variables, the first order model can be written in the following Equation 4.

\[ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k + \varepsilon \]  

As for the model with the second order in general can be written with the following Equation 5 [1]

\[ y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i<j} \beta_{ij} x_i x_j + \varepsilon \]  

Response surface model obtained from tests lack-of-fit can be described visually through the surface plot and contour plot. Surface plot is a three-dimensional plot that illustrates the magnitude of the response for each treatment composition that is within the specified area boundary. The response surface plot illustrates how the pattern of response influences the experiment, whether maximum, minimum or saddle point [1]. Illustration of response surface is given in Figure 1.
2.2. Desirability Function

The desirability function is a geometric transformation from the response value to a value of 0 to 1 \((0 \leq d_i \leq 1)\). This value indicates the closeness of the response to the target. The response that is at the specified target interval has a value of zero to one desirability \((0 < d_i < 1)\). While the response is very close to the value the target has a value desirability of one \((d_i = 1)\). In contrast to responses that are beyond the specified target interval then the value desirability of it is zero \((d_i = 0)\). The function of \(d_i\) is a function of individual desirability. This function will form the function composite desirability which is the geometric mean of the function of individual desirability \([4]\).

The function of composite desirability is given in Equation \(6\) \([5][6]\).

\[
D = (d_1 \times d_2 \times \ldots \times d_k)^{1/k}
\]

where \(k\) is the number of responses measured. This composite desirability function will be optimized later. Based on its purpose, the desirability function can be categorized into three, namely: nominal-the-best (NB), larger-the-better (LB) and smaller-the-better (SB). If suppose \(T\) is the desired target value, \(L\) is the lower limit of the target, and \(U\) is the upper limit of the target \((L \leq T \leq U)\), then the shape of the desirability function of each of these categories is as follows \([2][7][8]\).

2.1.1. Larger-the-better (LB). Used to maximize response, has the form of individual desirability functions as in Equation \(7\).

\[
d = \begin{cases} 
0 & ; y \leq L \\ 
(y - L)^r & ; L \leq y \leq Y \\ 
1 & ; y > L
\end{cases}
\]

With a weight of \(r = 1\), the desirability function is linear. Whereas if \(r > 1\), then this is more said to be close to the response, but if \(0 < r < 1\) then it is said to be far from the expected response.

2.1.2. Smaller-the-better (SB). Used to minimize the response, the form of the desirability function is given in Equation \(8\).

\[
d = \begin{cases} 
1 & ; y \leq T \\ 
(u - y)^r & ; T \leq y \leq U \\ 
0 & ; y > U
\end{cases}
\]

2.1.3. Nominal-the-best (NB). Used for responses to target values, it has the form of individual desirability function as in equation \(9\).

\[
d = \begin{cases} 
0 & ; y < L \\ 
(y - L)^r & ; L \leq y \leq T \\ 
(y - L)^r & ; T \leq y \leq U \\ 
0 & ; y > U
\end{cases}
\]

The three desirability functions above can be illustrated by the desirability function graph in Figure 2.
a. Desirability Function for LB
b. Desirability Function for SB
c. Desirability Function for ND

Figure 2. Graphic desirability function for larger the better (a), smaller the better (b), and nominal the best (c), each for maximum, minimum and target response respectively.

The index of $r$ is a weighting which indicates the close emphasis on the response to the target value. Value $0 < r < 1$ indicate less emphasis on the target. The greater the value of $r$ the further the response value from the target. The value of $r = 1$ indicate the same importance to the target. At this value the desirability function is linear. The value of $r > 1$ indicate more emphasis on the target. The ideal condition is a high desirability value indicating a response value that is close to the target. [9][10].

3. Method
The optimization process using the response surface model and desirability function has a different approach. The response surface model uses a visual approach using overlaid contour plot, while the desirability function uses a function approach.

3.1. Optimization uses the response surface model
This method is done by forming the response surface model for each response, forming the surface plot and contour plot then overlapping all contour plots. The risk area of the contour plots is the optimum solution for all responses.

3.1.1. Determination of response surface models for each response
The first or second order response surface model is determined by the lack-of-fit model test. The lack-of-fit test is used to test whether a suitable model is a linear model or not. The hypothesis in this test is $H_0$: The model is linear, $H_1$: The model is not linear. The test statistic used is $F_{count} = KT \text{ lack-of-fit} / KT \text{ pure error}$. The rejection criterion for $H_0$ is rejecting $H_0$ if $F_{count} > F_{alpha}; db1; db2$. With db1 is db lack-of-fit and db2 is db pure error. The rejection criterion for $H0$ using P-value is reject $H0$ if P-value < $\alpha$. The general form following the model of response surface equation Equation 5 [2].

3.1.2. Performance of surface plot and contour plots for each response
Response surface models in each response are used to form the surface plot and contour plot as in Figure 1. In each surface plot and contour plot that is formed, the optimum point for each response.

3.1.3. Overlaid contour plot formation
Response surface models on the design done by forming the multi responses model of response surface of each response and then do an overlay on the contour plot. Overlaid contour plots formed provide clues as to which areas allow the optimization point to occur.

3.1.4. Determination of the optimum solution
Overlaid contour plot produces the optimum solution area that meets all the responses involved. The area of the solution is a slice of the optimum solution for each response. Determination of this optimum solution requires quite a trial and error process because the optimum solution is not necessarily directly visible from the resulting overlaid plot.
3.2. Optimization using desirability function

The optimization process using the desirability function is carried out with the following steps:

- Determine the value of individual desirability for each response using Equation 7 if the purpose of optimization is to maximize the response.
- Determine the total value of desirability using Equation 6
- Determine the main influence (average) of total desirability at each factor level
- Determine the optimum solution by taking the largest average total desirability on each factor.

The results of the optimum solution obtained using this method is the composition of the level of factors that produce an optimum multivariate response

4. Result and Discussion

The analysis results obtained from the response surface model and desirability function are given as follows:

4.1. The results of the analysis use the response surface model

The results of the analysis of the determination of the response surface of the model obtained p-value lack-of fit test value is less than $\alpha = 0.05$ so that it was decided that the model is not linear. The response surface model for $Y_1$ is obtained as follows:

$$\hat{Y}_1 = 160.733 + 2.894A - 1.956B + 1.217C - 7.094A^2 - 7.311B^2 - 5.694C^2$$

Influence significant linear influence factor A, B, and C, as well as the influence of the square. The results of surface plots and contour plots are given in Figure 3. Factors C and D are fixed at level 3 for C and level 1 for D.

![Figure 3](image)

(a) Surface plot results for $Y_1$ (a), and contour plot for $Y_1$ (b). Darker colors indicate higher $Y_1$ values. The maximum value is obtained at factor A with a level approaching 3 and factor B with a factor level approaching 1.

The optimum solution at $Y_1$ is obtained at a factor A level 3, factor B level 1, factor C level 3 and factor D level 1. Response surface models for $Y_2$ are as follows:

$$\hat{Y}_2 = 83.692 + 3.638A - 2.376B + 1.144C - 4.839A^2 - 7.814B^2 - 6.529C^2 - 2.114D^2$$

The influence of significant factors on $Y_2$ is the linear effect of factors A, B, C and the quadratic effect of all factors. Using the response surface model, the surface plot and contour plot for $Y_2$ are obtained in Figure 4.
Figure 4. The surface plot results for Y2 (a), and contour plots for Y2 (b). Darker colors indicate higher Y2 values. At factor A approaching level 3 and factor B approaching level 1, the resulting Y2 value is maximum.

Thus the optimum solution at Y2 is found at factor A level 3, factor B level 1, factor C level 3 and factor D level 1. At Y3 the response surface model is obtained as follows:

\[
\hat{Y}_3 = 8.439 - 1.767A + 1.161B - 887D + 1.872A^2 + 3.559B^2 + 2.932C^2
\]

The linear effects of factors A, B and D are significant, as are the quadratic effects of factors A, B and C. Surface plot and contour plot for Y3 are given in Figure 5.

Figure 5. The surface plot results for Y3 (a), and contour plots for Y3 (b). Darker colors indicate higher Y3 values. The maximum value of Y3 is obtained at factor A which is close to 1 and factor B which is close to 3.

Thus the optimum solution at Y3 is at factor A level 1, factor B level 3, factor C level 3 and factor D level 1. In a separate optimization process for each response obtained a different optimum solution. Optimum at Y1 and Y2 was not optimal at Y3. Simultaneous optimization can be done by overlapping the three contour plots. The contourplot overlaid results are given in Figure 6.
Formation of overlaid contour plot is done by trial and error. Determination of target limits for Y1, Y2 and Y3 given will determine the optimum solution area. In Figure 6(a) the target boundary is not specific so that the optimum solution is obtained in the form of an optimum white area. When the target boundaries are made more specific, the optimum solution will also be more specific. In Figure 6(b) the optimum solution is obtained at factor A level 3, factor B level 1, factor C level 3 and factor D level 1. The optimum value obtained is Y1 at 152-154 MPa, Y2 at 74-76 Hv and Y3 at 10-12 μm.

The optimization process with the response surface model requires a fairly long trial and error process. The desirability function method can be used as an alternative for optimization. The purpose of optimization in the three responses is to maximize the response so that the desirability function used is a function in Equation 7. Composite desirability is obtained using Equation 6. With the help of the optimizer the optimum results are obtained in Figure 7.

The optimum solution obtained is factor A level 3, level 1 factor B, factor C level 3 and factor D optimum level 1. The value is 153.17 MPa Y1, Y2 and Y3 Hv at 75.76 at 11.94 μm. The results obtained are in line with those produced in the overlaid contour plot. The results of the optimum solution in the desirability function are very much determined by the lower limit and the target value used. In this study used a lower limit and target of 150-160 MPa for Y1, Y2 and 70-80 Hv for 15-12 μm to Y3. Different specification limits will produce different optimum solutions.
5. Conclusion
The optimization process using the response surface model separately for each response may produce a different optimum solution. In this case average grain size provides an optimum solution that is different from tensile strength and average microhardness. In simultaneous optimization using overlaid contour plot and desirability function, the optimum solution for all responses is the rotational speed factor at 1100 RPM, welding speed at 40 mm/min, shoulder tool diameter 21 mm and pin diameter at level 4.5 mm. The optimum value obtained using the overlaid contour plot is the tensile strength at 152-154 MPa, average microhardness at 74-76 Hv and the average grain size at 10-12 μm, is still a certain value interval. Desirability function at optimum values obtained for the tensile strength at 153.17 MPa, average microhardness at 75.76 Hv and the average grain size at 11.94 μm.

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