We study the effect of baryonic matter and apparent dark matter on black hole shadow in Verlinde’s Emergent Gravity. To do so, we consider different baryonic mass functions and an optically-thin disk region described by a gas in a radial free fall around the black hole. Assuming that most of the baryonic matter in the galaxy is located near the galactic center, we use constant mass function as well as power law mass function for the baryonic matter to study the effect of apparent dark matter on the shadow and the corresponding intensity. We find that the surrounding matter increases the shadow radius while the effect becomes significant when the surrounding baryonic matter is comparable to the black hole mass. To this end, we show that the effect of constant mass function in Verlinde’s theory is similar to the global monopole-like metric in which the apparent dark matter is modeled as an isothermal dark matter profile. We also find the intensity of the electromagnetic flux radiation depending on the surrounding mass.

I. INTRODUCTION

Today’s astrophysical observations seem to suggest that the galaxies contain supermassive black holes (SMBHs) at their galactic centers. From general relativity, we know that black holes are characterized by event horizon at the boundary and a singularity at the center. The presence of event horizon means that you can enter them but never exit. The most compelling evidence that can be linked with supermassive black holes is at the center of our own galaxy. At the center of Milky Way galaxy, there is a black hole with mass four million times the mass of the sun. It has been shown that, black holes can be completely determined by the parameters: black hole mass, angular momentum and electric charge. In realistic astrophysical situations, around a supermassive black hole there is an accretion mass which is due to the fact that a black hole (BH) can capture the light received from nearby stars or accretion disks into bound orbits. Among other things, black holes are characterized by a photon sphere radius which consists of orbiting light rays. Furthermore, the light rays can be unstable/stable if the photon can fall/escape to infinity, respectively. The strongest evidences supporting the existence of black holes are the shadow images of the M87 galactic center black hole reported by the Event Horizon Telescope (EHT) [4, 5] and the detection of gravitational waves by LIGO [6].

Based on the properties of black hole shadow, one can test general relativity and different alternative theories of gravity [4]. It is thus important to note that the shadow images can shed light on many astrophysical problems, such as the accretion matter around black hole including the dark matter distribution in the galactic center. In this respect, not only the distortion in shadow images due to the BH mass/spin is important, but also the effect of surrounding matter can be significant. Furthermore, one can use different spherical accretion models to study the intensity of the electromagnetic radiation as seen by an observer at a far distance from the black hole. That being said, the shadow images and intensity of electromagnetic radiation are very important tools to test the existence of both black holes and other exotic objects such as wormholes or naked singularities [5–10].

Recently, Verlinde proposed emergent gravitational theory [11] according to which, dark matter can be viewed as an emergent manifestation of gravity. In this theory, the gravitational potential $\Phi(r)$ caused by enclosed baryonic mass distribution exceeds that of general relativity on galactic and larger scales. Furthermore, Verlinde argued that due to the contribution of baryonic mass to the gravitational potential, there exists an extra gravitational effect due to a volume law contribution to the entropy that is associated with positive dark energy in our universe. In this theory, the additional gravitational force can thus be understood as follows: the baryonic mass distribution reduces the entropy content of the universe (meaning that the total entropy associated with dark energy is maximal in a universe without matter due to the fact that it would be non-locally distributed over the entire space available), as a consequence of this removal of entropy due to matter, there is an elastic response of the underlying microscopic system. The important result here is that this effect has observational consequences on large scale structures as an additional gravitational force and provides an alternative way of describing the dark matter as an apparent dark matter distribution.

Verlinde’s theory was recently tested using weak gravitational lensing [12], radial acceleration relation [13], with early type galaxies [14], and galaxy cluster scales.
II. BLACK HOLE SURROUNDED BY MATTER

A. Model I

In a recent paper, Eric Verlinde [11] has proposed a novel emergent gravitational theory. The most important claim of the theory is that dark matter has no particle origin but instead is an emergent manifestation in modified gravity. Assuming spherical symmetry, Verlinde showed that

\[ \int_{0}^{r} \frac{M_B^2(r')dr'}{r'^2} = \frac{a_0 M_B(r) r}{6} \]  

(1)

with \( a_0 = c H_0 \), where \( H_0 = 2.36 \times 10^{-18} \) s\(^{-1} \) \( \simeq \sqrt{\Lambda/3} \) is the current Hubble parameter, \( \Lambda \) is the cosmological constant, and \( M_B(r) \) \( (M_D(r)) \) is the baryonic mass (dark mass) inside a sphere of radius \( r \). This equation describes the amount of apparent dark matter \( M_D(r) \) in terms of the amount of baryonic matter \( M_B(r) \) for spherically symmetrical case consisting of stars of mass, ionized gas of mass and neutral hydrogen of mass. Eq. (1) can be also rewritten as follows

\[ M_D^2(r) = \frac{a_0 r^2 d}{6} (r M_B(r)) \]  

(2)

We shall assume here that the metric describing the geometry near a black hole which is given by

\[ ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \]  

(3)

where

\[ F(r) = \left(1 - \frac{2M(r)}{r}\right). \]  

(4)

FIG. 1: Schematic representation of the galactic central region having a black hole and baryonic matter with mass \( M_B \).

At this point, it is important to note that we shall neglect the cosmological constant in Einstein field equations. We should, hence, obtain a Schwarzschild-like solution given by the above metric function. The total mass function can be considered as the sum of black hole mass, baryonic mass, and apparent dark matter

\[ M(r) = m + M_B(r) + M_D(r). \]  

(5)

In the subsequent section, we are going to consider a different mass to obtain the spacetime geometry around the black hole.

1. Case I

Let us now consider the simplest case of having constant baryonic mass shell around the black hole (see Fig. 1) with \( M_B(r) = \text{const.} \), then by means of Eq. (2) we find the apparent dark matter mass to be

\[ M_D(r) = r \sqrt{a_0 M _B}. \]  

(6)

If we now define \( v_0^2 = \sqrt{a_0 M _B} \), from metric (3), we find the following metric

\[ ds^2 = -\left(1 - \frac{2(m + M_B)}{r}\right) dt^2 \]  

(7)

\[ + \frac{dr^2}{\left(1 - \frac{2(m + M_B)}{r}\right) - 2v_0^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \]

It is interesting to note that this metric globally has non-trivial topology, named conical topology. Let us now rescale the coordinates

\[ t \rightarrow \frac{t}{\sqrt{1 - 2v_0^2}}, \]  

(8)
\[ r \to r \sqrt{1 - 2v_0^2}, \quad (9) \]

it follows that
\[
ds^2 = - \left( 1 - \frac{2(\tilde{m} + \tilde{M}_B)}{r} \right) dt^2 + \frac{dr^2}{1 - \frac{2(\tilde{m} + \tilde{M}_B)}{r}} + r^2(1 - 2v_0^2) \left( d\theta^2 + \sin^2 \theta d\phi^2 \right), \quad (10)\]

which is the exact form of global monopole metric. In the last metric, \( \tilde{m} \) and \( \tilde{M}_B \) represent the new masses for the black hole and the baryonic mass due to the rescaling of the coordinates, respectively. From a physical point of view, the last metric means that a collapsing matter will eventually form a black hole. However, in Verlinde’s theory there is a nontrivial global topology. Note that such a situation is similar to the global monopole metric originally found by [20]. Interestingly, the form of the last metric is in agreement to what has been shown by [21], where authors have used a different framework. They have solved Einstein’s field equations by considering a matter contribution, say, a star of constant density, which led to such nontrivial topology in large scale structures.

### 2. Case II

Let us proceed further by considering a power law profile for the distribution of baryonic matter around the black hole which is given by
\[
M_B(r) = M_B \left( \frac{r^3}{r + r_c} \right)^3. \quad (11)\]

Using Eq. (1) we find the apparent dark matter mass to be
\[
M_D(r) = \frac{r^2}{(r + r_c)^2} \sqrt{r a_M M_B (r + 4r_c)}, \quad (12)\]

where \( r_c \) is the core radius of the baryonic matter. Hence, the metric function can be written as
\[
F(r) = 1 - \frac{2m}{r} - \frac{2M_B r^2}{(r + r_c)^3} - \frac{2r v_0^2}{(r + r_c)^2} \sqrt{r (r + 4r_c)}. \quad (13)\]

In the special limit \( r_c = 0 \), we obtain the constant mass model. Moreover, if we take the limit \( r \to \infty \), we obtain
\[
\lim_{r \to \infty} F(r) = 1 - 2v_0^2. \quad (14)\]

The last result is expected due to the global monopole-like structure as we saw in the first example.

### B. Model II

In this model, we are going to use a different way to construct the spacetime metric around the black hole.

![Fig. 2: Plots of the corresponding intensities using the infalling gas as seen by a distant observer in black hole spacetimes surrounded by baryonic and apparent dark matter.](image)

Let us start by writing the total acceleration on a test particle which is given by
\[
a = a_B + a_D, \quad (15)\]

where the effect of baryonic matter and apparent dark matter can be written as
\[
a_B = \frac{M_B}{r^2}, \quad a_{DM} = \frac{M_D}{r^2}. \quad (16)\]

One can now find tangential velocity of a test particle moving in the dark halo in spherically symmetrical spacetime using the well known relation
\[
v_{tg}^2(r) = \frac{M_B}{r} + \sqrt{a_M \frac{d}{dr}(r M_B)}, \quad (17)\]

where
\[
a_M = \frac{a_0}{6}, \quad (18)\]

and
\[
a_0 = 5.4 \times 10^{-10} \text{m/s}^2. \quad (19)\]
Let us now focus on a particular example by taking: $\beta_B = 3, \beta_D = 2$. In this way, it follows that

$$\tilde{\rho}_D = \frac{C}{r^2},$$

(31)

where $C$ is a constant. Similarly, we have an additional equation

$$\rho_D = \frac{1}{3} \frac{C}{r^2},$$

(32)

Taking the constant $C = 3\nu_0^2/4\pi$, we obtain

$$\rho_D = \frac{\nu_0^2}{4\pi r^2},$$

(33)

that is the well known isothermal sphere of dark matter profile in normal gravitational theory with the total dark matter mass inside a sphere of radius $r$ which is given by

$$M_D(r) = 4\pi \int_0^r \rho_D(r')r'^2dr' = \nu_0^2 r.$$  

(34)

Directly using equation (1), we find that the corresponding baryonic mass is equal to

$$M_B = \frac{\nu_0^4}{a_M}.$$  

(35)

In other words, we end up with a constant baryonic mass. If we now make use the equation for tangential velocity, it follows that

$$v_{tB}^2(r) = \frac{\nu_0^4}{a_M r} + \nu_0^2.$$  

(36)

On solving the last equation we find,

$$f(r) = \frac{r}{r_0} \nu_0^2 \exp\left(-\frac{2\nu_0^4}{a_M r}\right).$$  

(37)

By identifying,

$$\nu_0 = \sqrt{a_M M_B},$$  

(38)

we obtain the same expression as obtained in Eq. (23). We now consider black holes surrounded by apparent dark matter halo and baryonic matter. This space-time contribution can be obtained using corresponding energy-momentum tensors describing the total surrounding matter in Einstein field equation given by

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}.$$  

(39)

The space-time metric including black hole is thus given by

$$ds^2 = -(f(r) + F_1(r)) dr^2 + \frac{dr^2}{g(r) + G_1(r)} + r^2 d\Omega^2.$$  

(40)
FIG. 3: Images of shadows along with the corresponding intensities using the infalling gas for the black hole spacetime surrounded by baryonic and apparent dark matter. Top panel: We use metric function (7) in the Case I model. Down panel: We use metric function (13) in the Case II model.

where

\[ F(r) = f(r) + F_1(r), \quad G(r) = g(r) + G_1(r) \]  \hspace{1cm} (41)

where \( d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2 \). For any given dark matter density profile, we can obtain the corresponding space-time. In this way, it can be shown that \( F_1(r) = G_1(r) = -2m/r \) (see, for details [23]). Thus, in the the general black hole solution surrounded by matter with the assumption \( f(r) = g(r) \), it can be shown that

\[
F(r) = \left( \frac{r}{r_0} \right)^2 v_0^2 \exp \left( - \frac{2M_B}{r} \right) - \frac{2m}{r}. \]  \hspace{1cm} (42)

The black hole mass, for cosmological scales, can be neglected if we set \( r \to \infty \). Thus, we can write

\[
\lim_{r \to \infty} F(r) = \left( \frac{r}{r_0} \right)^2 v_0^2. \]  \hspace{1cm} (43)

Let us now add a new variable

\[
dr' = \frac{dr}{\left( \frac{r}{r_0} \right)^2 v_0^2}. \]  \hspace{1cm} (44)

Solving the last integral, we can find the following relation

\[
r = r'(1 - v_0^2) \left( \frac{r_0}{1 - v_0^2} r' \right)^{\frac{v_0^2}{v_0^2 - 1}} \]  \hspace{1cm} (45)

If we consider a Taylor series around \( v_0 \), we find

\[
r = r' \left( 1 - v_0^2 \right) + r' v_0^2 \ln \left( \frac{r'}{r_0} \right) + .... \]  \hspace{1cm} (46)

At this point, we introduce a cut-off distance, therefore considering the topological nontrivial term only and scaling the time coordinate

\[
dt' = \left( \frac{r}{r_0} \right)^2 v_0^2 dt \]  \hspace{1cm} (47)

Taking into account the above equations, the new metric reduces to

\[
ds^2 = -dt'^2 + dr'^2 + r'^2 \left( 1 - v_0^2 \right)^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \]  \hspace{1cm} (48)

In fact, we can further write

\[
(1 - v_0^2)^2 \simeq 1 - 2v_0^2 + ... \]  \hspace{1cm} (49)

to obtain the argument in Eq. (10) in large cosmological scales. This reflects the conical nature of our metric, or to put in a simpler manner, our metric is not asymptotically flat and, as a result there is a deflection angle as follows

\[
\hat{\alpha} \simeq \pi v_0^2. \]  \hspace{1cm} (50)

This result is consistent with what was argued in our Model I, representing a global monopole-like metric at large distances.
III. BLACK HOLE SHADOW RADIUS

Here, we are interested in investigating the shadow of black hole solution surrounded by matter. To do so, we start from Hamilton-Jacobi method for null geodesics in the black hole spacetime written as \[ \frac{\partial S}{\partial \sigma} + H = 0, \]
\[ (51) \]
in which \( S \) is the Jacobi action and \( \sigma \) is some affine parameter along the geodesics. If we consider a photon along null geodesics in our spherically symmetrical spacetime surrounded by matter, one can show that the Hamiltonian can be written as \[ \frac{1}{2} \left[ -\frac{p_t^2}{f(r)} + F(r)p_r^2 + \frac{p_\phi^2}{r^2} \right] = 0. \]
\[ (52) \]

Due to the spacetime symmetries related to the coordinates \( t \) and \( \phi \), there are two constants of motion defined as follows
\[ p_t \equiv \frac{\partial H}{\partial \dot{t}} = -E. \]
\[ p_\phi \equiv \frac{\partial H}{\partial \dot{\phi}} = L. \]
\[ (53, 54) \]

In the last two equations, \( E \) and \( L \) are the energy and the angular momentum of the photon, respectively. Next, the circular and unstable orbits are related to the maximum value of effective potential in terms of the following conditions
\[ V_{\text{eff}}(r) \bigg|_{r=r_p} = 0, \quad \frac{\partial V_{\text{eff}}(r)}{\partial r} \bigg|_{r=r_p} = 0, \]
\[ (55) \]

Without going into details here, one can now show the following equation of motion
\[ \frac{dr}{d\phi} = \pm r \sqrt{f(r) \left[ r^2 f(R) - 1 \right]}. \]
\[ (56) \]

Let us consider a light ray sent from a static observer located at a position \( r_0 \) and transmitted with an angle \( \theta \) with respect to the radial direction. We, therefore, have
\[ \cot \theta = \sqrt{\frac{g_{rr}}{g_{\phi\phi}}} \frac{dr}{d\phi} \bigg|_{r=r_0}. \]
\[ (57) \]

Finally, the relation for shadow radius of the black hole as observed by a static observer at the position \( r_0 \) can be shown as
\[ r_s = r_0 \sin \theta = R \sqrt{\frac{f(r_0)}{f(R)}} \bigg|_{R=r_p}. \]
\[ (58) \]

where \( r_p \) represents the photon sphere radius. The apparent shape of a shadow as seen by the observer can be obtained by a stereographic projection in terms of celestial coordinates \( X \) and \( Y \) which are defined by
\[ X = \lim_{r_0 \to \infty} \left( -r_0^2 \sin \theta_0 \frac{d\phi}{dr} \bigg|_{(r_0, \theta_0)} \right), \]
\[ Y = \lim_{r_0 \to \infty} \left( r_0^2 \frac{d\theta}{dr} \bigg|_{(r_0, \theta_0)} \right). \]
\[ (59) \]

It is worth noting that \( (r_0, \theta_0) \) are the position coordinates of the observer located at a far distance from the black hole. In the next section, we will consider a spherically symmetrical accretion model of infalling gas along with shadow images.
In this section, we consider a realistic and simple model of accretion flow surrounding the black hole; an optically thin, radiating accretion flow surrounding the object. The intensity of emitting region \( I_{\text{obs}} \) at the observed photon frequency \( \nu_{\text{obs}} \) at the point \((X, Y)\) of the observer’s image (usually measured in ergs \(^{-1}\) cm \(^{-2}\) str \(^{-1}\) Hz \(^{-1}\)) is given by [7]

\[
I_{\text{obs}}(\nu_{\text{obs}}, X, Y) = \int g^3 j(\nu_c) dl_{\text{prop}},
\]

where \( g = \nu_{\text{obs}} / \nu_c \) is the red-shift factor, \( \nu_c \) is the photon frequency as measured in the rest-frame of the emitter, \( j(\nu_c) \) is the emissivity per unit volume in the rest-frame of the emitter, and \( dl_{\text{prop}} = k_\alpha u_\alpha^\beta \) is the infinitesimal proper length as measured in the rest-frame of the emitter. The red-shift factor is evaluated from

\[
g = \frac{k_\alpha u_\alpha^\alpha}{k_\beta u_\beta^\beta},
\]

where \( k^\mu \) is the four-velocity of the photons, \( u^\alpha \) four-velocity of the accreting gas emitting the radiation, \( u^\alpha_{\text{obs}} = (1, 0, 0, 0) \) and \( \lambda \) is the affine parameter along the photon path \( \gamma \). Here, \( \gamma \) in the integral indicates that the integral has to be evaluated along the path of the photon (null geodesics). Here, we are considering a simplistic case of accreting gas where it is in radial free fall. For specific emissivity, we assume a simple model in which the emission is monochromatic with emitter’s-rest frame frequency \( \nu_* \) and the following power law profile.

\[
j(\nu_c) \propto \frac{\delta(\nu_c - \nu_*)}{\nu^2},
\]

where \( \delta \) is the Dirac delta function. Integrating the intensity over all the observed frequencies, we obtain the observed flux

\[
F_{\text{obs}}(X, Y) \propto - \int_\gamma \frac{g^3 k_\lambda}{\nu^2 k^\lambda} d\tau.
\]

In Fig. 2, we show the intensities of monopole-like metrics (7) and (13) for different values of the surrounding baryonic mass. For both metrics, we observe that with the increase in baryonic mass, the intensities decrease. In Fig. 3, we plot shadow images and the corresponding intensities using the infalling gas model as seen by a distant observer in a black hole spacetime surrounded by matter and apparent dark matter. It can be seen that in both cases, the shadow radius increases with the increase in the surrounding mass. In particular, the effect becomes significant when the surrounding mass is comparable to the black hole mass. We also note that the quantity \( r_c \) in Case II, is comparable to the black hole mass. On the other hand, in Fig. 4 and Fig.5, we show the intensities and shadow images for the black hole described by metric function (42) surrounded by dark matter using the constant mass function. Again, we observe that having a high energy density near the black hole affects the images as well as the intensity. That is, as the black hole mass increases, the shadow radius also increases. This shows that the shadow images can be used as an indirect tool to detect matter around black holes.

### V. Observational Constraints

In this section, we shall use the reported angular size of the black hole shadow in the M87 galactic center reported by EHT \( \theta_s = (42 \pm 3) \mu\text{as} \), along with the distance to M87 given by \( D = 16.8 \) Mpc, and the mass of M87 central object \( M = 6.5 \times 10^9 \) \( M_\odot \) to constrain the baryonic mass around the black hole M87. In order to constrain the surrounding baryonic mass \( M_B \), for simplicity, we are going to neglect the rotation. Next, the diameter of the shadow in units of mass \( d_{M87} \) is given by [24]

\[
d_{M87} = \frac{D \theta_s}{M_{87}} = 11.0 \pm 1.5.
\]

We are further going to study the following separate cases:

#### A. Model I

In this model we have two specific cases; Case I described by metric (7), and Case II described by metric function (13). Within 1\( \sigma \) confidence, we have the interval \( 9.5 \leq d_{M87} \leq 12.5 \), whereas within 2\( \sigma \) uncertainties, we have \( 8 \leq d_{M87} \leq 14 \). In Figs. 6-7, we show the regions of parameter space of the diameter of the shadow for the Case I and Case II, respectively. Within 2\( \sigma \) confidence, we find the upper bound for the surrounding baryonic mass to be \( M_B \leq 3.5 \). On the other hand, for Case II, within 2\( \sigma \) confidence, we find the upper bound for the surrounding baryonic mass to be \( M_B \leq 6.4 \).

#### B. Model II

Finally, let us consider Model II, and see the effect of baryonic mass on the shadow. In Fig. 8, we show the regions of parameter space of diameter of the shadow for the Model II. Within 2\( \sigma \) confidence, we find the upper bound for the surrounding baryonic mass to be \( M_B \leq 3.85 \). From all these plots, we conclude that the effect becomes significant when the surrounding baryonic matter is comparable to the black hole mass. We should point out that our analysis is based on the assumption that most of the baryonic matter is located near the
FIG. 5: Images of shadows along with corresponding intensities using infalling gas as seen by a distant observer in a black hole spacetime described by the metric function (42). Upper panel: We chose $M_B = 0.1, M_B = 0.5$ and $M_B = 1$. Lower panel: We chose $M_B = 1.1, M_B = 1.5$ and $M_B = 2$.

FIG. 6: The regions of parameter space of the diameter of the black hole shadow for the Case I model within $1\sigma$ and $2\sigma$ uncertainties, respectively.

FIG. 7: The regions of parameter space of the diameter of the black hole shadow for the Case II model within $1\sigma$ and $2\sigma$ uncertainties, respectively.

VI. CONCLUSIONS

In this paper, we studied the shadow images of black holes in Verlinde’s Emergent Gravity. Toward this purpose, we considered a black hole surrounded by baryonic mass and an optically-thin gas medium in radial free fall. In order to study the influence of surrounding matter on the photon sphere, we assumed that most of the baryonic matter in the galaxy is located near the galac-
FIG. 8: The regions of parameter space of the diameter of the black hole shadow for the Model 2 within $1\sigma$ and $2\sigma$ uncertainties, respectively.

In particular, we used two different models to construct the spacetime metric near the black hole. In the first toy model, we considered the simplest case, namely a constant mass function for the baryonic mass, then we extended our analysis by assuming a power law mass function for the baryonic matter. In both cases, we studied not only the effect of baryonic mass but also the effect of apparent dark matter on the shadow and the corresponding intensities. We have shown that, the surrounding matter increases the shadow radius while the effect becomes significant when the surrounding baryonic matter is comparable to the black hole mass. It is also shown that intensity of the electromagnetic flux radiation observed by distant observer decreases with the increase in mass. This of course can be explained by the fact that as the black hole spacetime gets distorted by the extra effect coming from the baryonic and apparent dark matter, the number of photons captured by the black hole increases. As a result, we end up with a smaller value for the intensity at a large distance. It is important to note that the effect of surrounding mass is strong when it surrounding mass is comparable to the black hole mass(constraining plots). In addition to that, the constant $r_c$ in the power law model, should be of black hole mass order, that means the surrounding matter should be mostly located at the galactic center near the black hole. We studied the influence of baryonic/apparent dark matter on the electromagnetic radiation emitted from spherical accretion medium which was assumed to be an optically-thin region surrounding the black hole. In the second model, we used the tangential velocity of the test particle to construct the spacetime metric having a black hole at the center. Similarly, we found that increasing the surrounding baryonic mass increases the shadow radius. In particular, it is shown that the effect of constant mass function in Verlinde’s theory is similar to the global monopole-like metric in which the apparent dark matter is modeled as an isothermal dark matter profile in ordinary general relativity. In that sense, the shadow images of a black hole surrounded by a constant baryonic matter and apparent dark matter in Verlinde’s theory, are almost indistinguishable from shadow images obtained in ordinary general relativity having dark matter described as an isothermal sphere.

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