On the Comparison of Adam-Bashforth and Adam Moulton Methods for Non-Stiff Differential Equations

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Authors’ contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

This paper presents the comparison of the two Adams methods using extrapolation for the best method suitable for the approximation of the solutions. The two methods (Adams Moulton and Adams Bashforth) of step k = 3 to k = 4 are considered and their equations derived. The extrapolation points, order, error constant, stability regions were also derived for the steps. More importantly, the consistency and zero stability are also investigated and finally, the derived equations are used to solve some non-stiff differential equations for best in efficiency and accuracy.

Keywords: Adam-Bashforth; Adam Moulton; accuracy; stability region; consistency; zero stability.

1 Introduction

Most scientific and engineering problems such as the study of vibration, chemical reactions and elasticity are modeled mathematically using ordinary differential equations. It may be interesting to know that most of these problems do not have exact solutions [1-3].

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There are many methods for finding direct approximate solution to differential solution. These methods are referred to by a variety of different names including numerical methods, numerical integration or approximate solutions [4,5]. The numerical method for solving ordinary differential equation are method of integrating a system of first order differential equation since higher ordinary differential equation can be reduced to a set of first order differential equation [6,7].

Linear multi-step method is one of the ways of getting approximate solution to a certain differential equation when the exact or analytical can be derived or not. Linear multi-step method also increases efficiency Haire and Wannal [8] by using the information from several previous solution approximations. Examples of linear multi-step method are Adams-Bashforth method, Adams-Moulton method, Nytrum method and Milne-Simpson method. These methods perform well and are used widely by researchers to solve stiff and non-stiff differential equations.

In this work, we compare the Adams Moulton method of step k=3 and k = 4 with Adams Bashforth method of the same step for best in efficiency and implementation. We use Maple software to solve the problem and the results are presented in the tables.

2 Problem Definition and Methodology

The continuous and discrete form of the Adams Moulton and Adams Bashforth extrapolation points, are derived as follows:

\[
y(x) = \sum_{j=0}^{t} a_j(x) y(x_j) + h \sum_{j=0}^{m-1} \beta_j(x) f(x_j, \bar{y}(x_j))
\]  

(1)

Where \(a_j(x)\) and \(\beta_j(x)\) are the continuous coefficient define as follows:

\[
a_j(x) = \sum_{i=0}^{t+m-1} a_{j,i+1} x^i
\]  

(2)

and

\[
h\beta_j(x) = \sum_{i=0}^{t+m-1} a_{j,i+1} x^i
\]  

(3)

Also, we shall use DC = I

where I is the identity matrix of dimension (t+m) x (t+m) as used by Sirisena [9] to determine the coefficients \(a_j(x)\) and \(\beta_j(x)\).

D and C are derived as:

\[
D = \begin{bmatrix}
1 & X_n & X_n^2 & \cdots & X_n^t & \cdots & X_n^{t+m-1} \\
1 & X_{n+1} & X_{n+1}^2 & \cdots & X_{n+1}^t & \cdots & X_{n+1}^{t+m-1} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
1 & X_{n+t-1} & X_{n+t-1}^2 & \cdots & X_{n+t-1}^t & \cdots & X_{n+t-1}^{t+m-1} \\
0 & 1 & 2X_0 & \cdots & tX_0^{t-1} & \cdots & (t+m-1)tX_0^{t+m-2} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & 1 & 2X_{m-1} & \cdots & tX_{m-1}^{t-1} & \cdots & (t+m-1)tX_{m-1}^{t+m-2}
\end{bmatrix}
\]
Where \( m \) is the number of collocation points and \( t \) is the number of interpolation points and \( k \) is the step number. \( C \) is also of dimension \( (t + m) \times (t + m) \) where \( c \) is given as:

\[
C = \begin{bmatrix}
    a_{0,1} & a_{0,t} & \ldots & h_{0,2} & \ldots & h_{0,k-1} \\
    a_{1,0} & a_{1,t} & \ldots & h_{1,2} & \ldots & h_{1,k-1} \\
    \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
    a_{m,0} & a_{m,t} & \ldots & h_{m,2} & \ldots & h_{m,k-1} \\
\end{bmatrix}
\]

If \( C = (c_{ij}), i=j=1, D = (d_{ij}), I=j=1, \ldots, n \), and \( I = (e_{ij}), I = j=1, \ldots, n \)

### 3 Derivation of Adams Moulton and Adams Bashforth Methods

#### 3.1 Derivation of Adams Moulton explicit methods for Case \( k=3 \)

We consider first, the derivation of step \( k = 3 \), which has the general form

\[
y(x) = \alpha_1 y_{n+3} + h \beta_0(x) f_n \beta_1(x) f_{n+1} + \beta_2(x) f_{n+2} + \beta_3(x) f_{n+3} \tag{4}
\]

Using \( D \) above, we get

\[
D := \begin{bmatrix}
    0 & 1 & 2 x_n & 3 x_n^2 & 4 x_n^3 \\
    0 & 1 & 2 x_n + 2 h & 3 (x_n + h)^2 & 4 (x_n + h)^3 \\
    0 & 1 & 2 x_n + 4 h & 3 (x_n + 2 h)^2 & 4 (x_n + 2 h)^3 \\
    0 & 1 & 2 x_n + 6 h & 3 (x_n + 3 h)^2 & 4 (x_n + 3 h)^3 \\
\end{bmatrix}
\]

We use maple software to generate the matrix \( D \) above for case \( k=3 \). The inverse of the matrix \( D \) is obtained using maple software as:

\[
C := \begin{bmatrix}
    1 & \frac{(x_n + 2 h)}{h^3} (4 h^3 + 10 x_n h^2 + x_n^3 + 6 x_n^2 h) \\
\end{bmatrix}
\]
To obtain the continuous scheme of the method, we use C which is evaluated at the collocation point $x = x_{n+3}$ and obtain:

\[
y_{n+2} + \frac{1}{24} h f_n - \frac{5}{24} h f_{n+1} + \frac{19}{24} h f_{n+2} + \frac{3}{8} h f_{n+3}
\]

\[
y_{n+3} := y_{n+2} + \frac{1}{24} h f_n - \frac{5}{24} h f_{n+1} + \frac{19}{24} h f_{n+2} + \frac{3}{8} h f_{n+3}
\]

Extrapolating at $x = x_{n+4}$ yields

\[
y_{n+4} := y_{n+2} - \frac{1}{3} h f_n + \frac{4}{3} h f_{n+1} - \frac{5}{3} h f_{n+2} + \frac{8}{3} h f_{n+3}
\]

### 3.2 Derivation of Adam Moulton method for case k=4

The derivation of the Adams Moulton method of case k=4 uses the general form

\[
y(x) = a_1 y_{n+4} + b_1(x) f_n + b_2(x) f_{n+1} + b_3(x) f_{n+2} + b_4(x) f_{n+3}
\]
which results into the D matrix below:

\[
D := \begin{bmatrix}
1 & x_n + 3h & (x_n + 3h)^2 & (x_n + 3h)^3 & (x_n + 3h)^4 & (x_n + 3h)^5 \\
0 & 1 & 2x_n & 3x_n^2 & 4x_n^3 & 5x_n^4 \\
0 & 1 & 2x_n + 2h & 3(x_n + h)^2 & 4(x_n + h)^3 & 5(x_n + h)^4 \\
0 & 1 & 2x_n + 4h & 3(x_n + 2h)^2 & 4(x_n + 2h)^3 & 5(x_n + 2h)^4 \\
0 & 1 & 2x_n + 6h & 3(x_n + 3h)^2 & 4(x_n + 3h)^3 & 5(x_n + 3h)^4 \\
0 & 1 & 2x_n + 8h & 3(x_n + 4h)^2 & 4(x_n + 4h)^3 & 5(x_n + 4h)^4
\end{bmatrix}
\]

The inverse of the above matrix for case \( k=4 \) is obtain as follows:

\[
C := \left[ 1 - \frac{1}{360} \frac{(x_n + 3h) (51 x_n h + 223 x_n^2 h^2 + 99 x_n^3 h + 12 x_n^4 - 153 h^4)}{h^4} \right]
\]

\[
\left[ 0, 1, \frac{24 h^4 + 50 x_n h^3 + 35 x_n^2 h^2 + 10 x_n^3 h + x_n^4}{h^4}, \frac{1}{6} \frac{x_n (24 h^3 + 26 x_n h^2 + 9 x_n^2 h + x_n^3)}{h^4}, \frac{1}{4} \frac{x_n (19 x_n h^2 + 8 x_n^2 h + x_n^3 + 12 h^3)}{h^4}, \frac{1}{6} \frac{x_n (7 x_n^2 h + x_n^3 + 14 x_n h^2 + 8 h^3)}{h^4}, \frac{1}{24} \frac{x_n (6 h^3 + 11 x_n h^2 + x_n^3 + 6 x_n^2 h)}{h^4} \right]
\]

\[
\left[ 0, -\frac{1}{24} \frac{25 h^3 + 35 x_n h^2 + 15 x_n^2 h + 2 x_n^3}{h^4}, \frac{1}{12} \frac{24 h^3 + 52 x_n h^2 + 27 x_n^2 h + 4 x_n^3}{h^4} \right]
\]
\[
\begin{align*}
&-\frac{1}{4} \left( 19 x_n^2 h^2 + 12 x_n^3 h + 2 x_n^3 + 6 h^3 \right), \\
&- \frac{1}{12} \left( 2 x_n^3 + 9 x_n^2 h + 11 x_n h^2 + 3 h^3 \right), \\
&\left[ 0, \frac{1}{72} 35 h^2 + 30 x_n h + 6 x_n^2 \right], \\
&\left[ 0, \frac{1}{18} 26 h^2 + 27 x_n h + 6 x_n^2 \right], \\
&\left[ 0, -\frac{1}{48} 5 h + 2 x_n \right], \\
&\left[ 0, -\frac{1}{24} 4 h + 4 x_n \right], \\
&\left[ 0, -\frac{1}{20} 120 h^4 \right], \\
&\left[ 0, -\frac{1}{30} 120 h^4 \right], \\
&\left[ 0, -\frac{1}{120} 120 h^4 \right], \\
&\left[ 0, -\frac{1}{30} 120 h^4 \right], \\
&\left[ 0, -\frac{1}{20} 120 h^4 \right], \\
&\left[ 0, -\frac{1}{12} 120 h^4 \right], \\
&\left[ 0, -\frac{1}{20} 120 h^4 \right], \\
&\left[ 0, -\frac{1}{30} 120 h^4 \right].
\end{align*}
\]

To obtain the continuous scheme of the method, we use C which is evaluated at the collocation point \( x = x_{n+4} \) to obtain:

\[
\begin{align*}
&y_{n+3} - \frac{19}{720} h f_n + \frac{53}{360} h f_{n+1} - \frac{11}{30} h f_{n+2} + \frac{323}{360} h f_{n+3} + \frac{251}{720} h f_{n+4} \\
&y_{n+4} := y_{n+3} - \frac{19}{720} h f_n + \frac{53}{360} h f_{n+1} - \frac{11}{30} h f_{n+2} + \frac{323}{360} h f_{n+3} + \frac{251}{720} h f_{n+4}
\end{align*}
\]

3.3 Derivation of Adams Bashforth method for k=3

The general form for Adams Bashforth method for order k = 3 is given by

\[
y(x) = \alpha_1 y_{n+3} + h(\beta_0(x) f_n + \beta_1(x) f_{n+1}) k=3 is given as \alpha_1 \beta_2(x) f_{n+2} + \beta_2(x) f_{n+3}
\]

such that the D matrix is given as:

\[
D = \begin{bmatrix}
1 & x_n + 2 h & (x_n + 2 h)^2 & (x_n + 2 h)^3 \\
0 & 1 & 2 x_n & 3 x_n^2 \\
0 & 1 & 2 x_n + 2 h & 3 (x_n + h)^2 \\
0 & 1 & 2 x_n + 4 h & 3 (x_n + 2 h)^2
\end{bmatrix}
\]

The inverse of the matrix D for k=3, Adams Bashforth method is obtain as

\[
C := \begin{bmatrix}
1, -\frac{1}{12} (x_n + 2 h) (2 h^2 + 5 x_n h + 2 x_n^2) & \frac{1}{2} (x_n + 2 h)^2 (-x_n + h) \\
\end{bmatrix},
\]

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Using C we obtain the continuous scheme of the method which we evaluate at the collocation point $x = x_{n+3}$ to obtain:

$$y_{n+3} := \frac{23}{12} f_{n+2} h + y_{n+2} - \frac{4}{3} f_{n+1} h + \frac{5}{12} f_n h$$

Extrapolating at $x = x_{n+4}$ we get

$$y_{n+4} := \frac{7}{6} f_n h + y_{n+2} + \frac{19}{3} f_{n+2} h - \frac{20}{3} f_{n+1} h$$

### 3.4 Derivation of Adams Bashforth method for case k=4

The Adams Bashforth method for case k=4 have the general form

$$y(x) = \alpha_1 y_{n+4} + h(\beta_0(x)f_n + \beta_1(x)f_{n+1} + \beta_2(x)f_{n+2} + \beta_3(x)f_{n+3} + \beta_4(x)f_{n+4})$$

which gives the following matrix $D$

$$D := \begin{bmatrix}
1 & x_n + 3 h & (x_n + 3 h)^2 & (x_n + 3 h)^3 & (x_n + 3 h)^4 \\
0 & 1 & 2 x_n & 3 x_n^2 & 4 x_n^3 \\
0 & 1 & 2 x_n + 2 h & 3 (x_n + h)^2 & 4 (x_n + h)^3 \\
0 & 1 & 2 x_n + 4 h & 3 (x_n + 2 h)^2 & 4 (x_n + 2 h)^3 \\
0 & 1 & 2 x_n + 6 h & 3 (x_n + 3 h)^2 & 4 (x_n + 3 h)^3
\end{bmatrix}$$

The inverse of the matrix $D$ for Adams Bashforth of case k=4 is obtain as follows:

$$C := \begin{bmatrix}
1, & -\frac{1}{24} \frac{(x_n + 3 h) (3 h^3 + 7 x_n h^2 + x_n^3 + 5 x_n^2 h)}{h^3}
\end{bmatrix}$$
\[
\begin{align*}
&\frac{1}{24} (x_n + 3h) (-3 x_n h^2 - 11 x_n^2 h - 3 x_n^3 + 9 h^3), \\
&\frac{1}{24} (x_n + 3h) (7 x_n^2 h + 3 x_n^3 - 3 x_n h^2 + 9 h^3), \\
&\frac{1}{24} (x_n + 3h) (-x_n^3 - x_n^2 h - x_n h^2 + 3 h^3), \\
&\left[ 0, \frac{1}{6} \frac{6 h^3 + 11 x_n^2 h^2 + x_n^3 + 6 x_n^2 h}{h^3}, -\frac{1}{2} \frac{x_n (6 h^2 + 5 x_n h + x_n^2)}{h^3} \right], \\
&\left[ 0, \frac{1}{2} \frac{x_n (4 x_n h + x_n^2 + 3 h^2)}{h^3}, -\frac{1}{6} \frac{x_n (x_n^2 + 3 x_n h + 2 h^2)}{h^3} \right], \\
&\left[ 0, -\frac{1}{12} \frac{11 h^2 + 3 x_n^2 + 12 x_n h}{h^3}, \frac{1}{4} \frac{6 h^2 + 10 x_n h + 3 x_n^2}{h^3}, -\frac{1}{4} \frac{8 x_n h + 3 x_n^2 + 3 h^2}{h^3} \right], \\
&\left[ 0, \frac{1}{12} \frac{3 x_n^2 + 6 x_n h + 2 h^2}{h^3} \right], \\
&\left[ 0, \frac{1}{6} \frac{x_n + 2 h}{h^3}, -\frac{1}{6} \frac{5 h + 3 x_n}{h^3}, \frac{1}{6} \frac{4 h + 3 x_n}{h^3}, -\frac{1}{6} \frac{x_n + h}{h^3} \right], \\
&\left[ 0, -\frac{1}{24} \frac{1}{8} \frac{1}{h^3}, -\frac{1}{8} \frac{1}{h^3}, -\frac{1}{24} \frac{1}{h^3} \right].
\end{align*}
\]

the continuous scheme of the method evaluated at the collocation point $x = x_{n+4}$ is obtain as:

\[
y_{n+4} := y_{n+3} - \frac{3}{8} f_n h + \frac{37}{24} f_{n+1} h - \frac{59}{24} f_{n+2} h + \frac{55}{24} f_{n+3} h
\]

3.5 Analysis of the methods

3.5.1 Order and error constants

To find the order and error constants of the derived equations, the following equation is used:

\[
\begin{align*}
\zeta_q & := \frac{a_1 + 2^q a_2 + 3^q a_3 + 4^q a_4 + 5^q a_5 + 6^q a_6 + 7^q a_7 + 8^q a_8}{q!} \\
& - \sum_{i=2}^{q} \frac{1}{(q-1)!} \left( \frac{\beta_1 + 2^{(q-1)} \beta_2 + 3^{(q-1)} \beta_3 + 4^{(q-1)} \beta_4 + 5^{(q-1)} \beta_5 + 6^{(q-1)} \beta_6}{(q-1)!} \right)
\end{align*}
\]

Then we obtain below the error constant and order of the extrapolated equations from our derivations, using the error constant equation stated above as shown in the table:
Table 1. Order and error constant

|                    | Adams Bashforth New Explicit methods | Adams Moulton Explicit Methods |
|--------------------|--------------------------------------|--------------------------------|
| Order              | Error Constant                       | Order                          | Error Constant       |
| -                  | -                                    | -                              | -                   |
| $y_{n+3}$          | 3                                    | ⅓                              | 4                   | 1/3                 |
| $y_{n+4}$          | 4                                    | 3                              | 5                   | 29/90               |

3.5.2 Stability region of the new methods

Fig. 1. Stability region of Adams Bashforth explicit method K=3

Fig. 2. Stability region for Adams Bashforth New explicit method k=4
3.5.3 Zero stability and consistency (Adams Moulton explicit method)

i. \[ y_{n+3} = y_{n+1} - 2hf_{n+1} + 4hf_{n+1} \rho(\xi) = \xi^3 - \xi = 0 \Rightarrow \xi = 0, \xi^2 - 1 = 0, \xi = \pm 1 \Rightarrow |\xi| \leq 1 \]

ii. \[ y_{n+4} = y_{n+2} - \frac{7h}{3}f_{n+4} + \frac{20h}{3}f_{n+1} + \frac{19h}{3}f_{n+2} \rho(\xi) = \xi^4 - \xi^2 = \xi^2 (\xi^2 - 1) = 0 \Rightarrow \xi = \text{twice, } \xi^2 - 1 = 0 \Rightarrow \xi = \pm 1 \]

\[ \Rightarrow |\xi| = 1 \Rightarrow \xi = 0 \text{ four times on } \xi = \pm 1 \Rightarrow |\xi| = 1 \Rightarrow |\xi| \leq 1 \]
3.5.4 Zero stability and consistency (Adams Bashforth new explicit methods):

iii. \( y_{n+3} = y_{n+1} - 2hf_n + 4hf_{n+1} \rho(\xi) = \xi^3 - \xi = 0 \Rightarrow \xi = 0, \xi^2 - 1 = 0, \xi = \pm 1 \Rightarrow |\xi| \leq 1 \)

iv. \( y_{n+4} = y_{n+2} - \frac{2h}{3} f_n + \frac{20h}{3} f_{n+1} + \frac{19h}{3} f_{n+2} \)

v. \( \rho(\xi) = \xi^4 - \xi^2 = \xi^2 (\xi^2 - 1) = 0 \Rightarrow \xi = \text{twice}, \xi^2 - 1 = 0 \Rightarrow \xi = \pm 1 \Rightarrow |\xi| = 1 \)

vi. \( \xi = 0 \) four times on \( \xi = \pm 1 \Rightarrow |\xi| = 1 \Rightarrow |\xi| \leq 1 \)

since the derived equations satisfies the requirement for consistency and zero stability, then by Dalquist’s definition, the equations are convergent.

4 Numerical Examples

This section presents some numerical examples of the new explicit methods, applied on non-stiff initial value problems, to examine their performances

Example 1.

Solve the following problem

\[ y' = x - y, \quad \text{y}(0) = 0, \quad h = 0.1, \quad 0 \leq x \leq 1 \]

\[ y(x) = x + 1 + e^{-x} \]

using Adams Moulton and Adams Bashforth explicit Method

Solution

The results are summarize in Table 2 below:

| X   | Exact Solution | Explicit Adams Moulton | New Explicit Adams Bashforth |
|-----|----------------|-------------------------|-----------------------------|
| 0.0 | 0.00000000     | 0.00000000              | 0.00000000                  |
| 0.1 | 0.00487418     | 0.004837418             | 0.004837418                 |
| 0.2 | 0.018730753    | 0.018730753             | 0.18730753                  |
| 0.3 | 0.040818220    | 0.053477777             | 0.04081220                  |
| 0.5 | 0.106530659    | 0.103932536             | 0.023986143                 |
| 0.6 | 0.148811636    | 0.087168084             | 0.215214229                 |
| 0.4 | 0.196585303    | 0.192976084             | -0.052780229                |
| 0.8 | 0.249328964    | 0.249330734             | 0.403027339                 |
| 0.9 | 0.306569659    | 0.306571267             | 0.446717573                 |

Example 2

Solve the following

\[ y^2 = -y \]

\[ y(x) = e^{-x}, \quad \text{y}(0) = 1, \quad 0 \leq x \leq 1, \quad h = 0.1 \]

using Adam Moulton and Adams Bashforth Explicit method

Solution

Using Adams Moulton explicit and Adams Bashforth new explicit Methods, the result are summarize in Table 3 below:
Table 3. Results of example 2 using the Derived Explicit Methods

| X   | Exact Solution | Explicit Adams Moulton | New Explicit Adams Bashforth |
|-----|----------------|------------------------|-----------------------------|
| 0.0 | 1.000000000    | 1.000000000            | 1.000000000                 |
| 0.1 | 0.904837418    | 0.904837418            | 0.904837418                 |
| 0.2 | 0.818730753    | 0.818730753            | 0.818730753                 |
| 0.3 | 0.74081822     | 0.74081822             | 0.74081822                  |
| 0.4 | 0.670320046    | 0.471765048            | 0.67092888                  |
| 0.5 | 0.606530659    | 0.53604304             | 0.606325112                 |
| 0.6 | 0.548811636    | 0.559106238            | 0.548625653                 |
| 0.7 | 0.496585303    | 0.438868246            | 0.496417017                 |
| 0.8 | 0.449328964    | 0.397104407            | 0.449386137                 |
| 0.9 | 0.406569659    | 0.359314909            | 0.406431879                 |
| 1.0 | 0.367879441    | 0.325121591            | 0.36775479                  |

Example 3

Solve the following

\[ y'' = -8(y-x) + 1, \ y(0) = 2, \ h = 0.2, \ \text{0} \leq x \leq 2 \]

Giving the exact solution is \( y(x) = x + 2e^{-8x} \)

Solution:

Using the new Adams Moulton explicit and the Adam Bashforth new explicit methods, the results are presented in Table 4 below:

Table 4. Results of example 3 using the derived Explicit Methods

| X   | Exact Solution | Explicit Adams Moulton | New Explicit Adams Bashforth |
|-----|----------------|------------------------|-----------------------------|
| 0.0 | 2.000000000    | 2.000000000            | 2.000000000                 |
| 0.2 | 0.603793036    | 9.583654937            | 0.603793036                 |
| 0.4 | 0.4181524408   | 0.474914869            | 0.817523081                 |
| 0.6 | 0.616459494    | 0.61437073             | 0.141991824                 |
| 0.8 | 0.803323114    | 0.080280611            | 1.460029255                 |
| 1.0 | 1.000670925    | 1.000538289            | 1.474491255                 |
| 1.2 | 1.200135457    | 1.20010511             | 1.19920843                  |
| 1.4 | 1.400027348    | 1.330638466            | 1.400012409                 |
| 1.6 | 1.600005522    | 1.693625009            | 1.739118981                 |
| 1.8 | 1.800001115    | 1.800023111            | 1.80000916                  |
| 2.0 | 2.000000225    | 2.000003548            | 2.000003998                 |

Error of example 1

| X   | Error of Adams Moulton extrapolation | Error of Adams Bashforth extrapolation |
|-----|-------------------------------------|---------------------------------------|
| 0.1 | 0.00                                | 0.00                                  |
| 0.2 | 0.00                                | 0.00                                  |
| 0.3 | 1.33 x 10^{-2}                      | 0.00                                  |
| 0.4 | 4.03 x 10^{-2}                      | 6.40 x 10^{-1}                       |
| 0.5 | 2.60 x 10^{-3}                      | 8.25 x 10^{-2}                       |
| 0.6 | 6.16 x 10^{-2}                      | 6.64 x 10^{-2}                       |
| 0.7 | 3.61 x 10^{-3}                      | 2.49 x 10^{-3}                       |
| 0.8 | 1.77 x 10^{-6}                      | 1.54 x 10^{-1}                       |
| 0.9 | 1.61 x 10^{-6}                      | 1.40 x 10^{-3}                       |
| 1.0 | 2.00 x 10^{-3}                      | 2.10 x 10^{-3}                       |
The result in Table 4 reveals that Adams Moulton explicit method performed better than Adams Bashforth new explicit method.

Errors of example 2

| X   | Error Of Adams Moulton Explicit Methods | Error of Adams Bashfort New Explicit Method |
|-----|----------------------------------------|--------------------------------------------|
| 0.1 | 0.00                                   | 0.00                                       |
| 0.2 | 0.00                                   | 0.00                                       |
| 0.3 | 1.33 x 10^-2                           | 0.00                                       |
| 0.4 | 1.98 x 10^-1                           | 2.27 x 10^-4                              |
| 0.5 | 7.05 x 10^-2                           | 2.06 x 10^-4                              |
| 0.6 | 1.03 x 10^-2                           | 1.86 x 10^-4                              |
| 0.7 | 5.77 x 10^-2                           | 1.68 x 10^-4                              |
| 0.8 | 5.22 x 10^-2                           | 5.32 x 10^-4                              |
| 0.9 | 4.73 x 10^-2                           | 1.38 x 10^-4                              |
| 1.0 | 4.28 x 10^-2                           | 1.25 x 10^-4                              |

The result reveals that the Adams Bashforth extrapolation performed better than the new Adams Moulton Extrapolation

The absolute error using the two method is displayed on Table 5 for example 3

Table 5. Errors of example 3

| X   | Error of Adams Moulton Explicit Method | Error of Adams Bashfort New Explicit Method |
|-----|----------------------------------------|--------------------------------------------|
| 0.2 | 2.01 x 10^-2                           | 0.00                                       |
| 0.4 | 6.61 x 10^-3                           | 3.36 x 10^-1                              |
| 0.6 | 2.09 x 10^-3                           | 4.74 x 10^-1                              |
| 0.8 | 5.17 x 10^-4                           | 6.57 x 10^-1                              |
| 1.0 | 1.33 x 10^-4                           | 4.74 x 10^-1                              |
| 1.2 | 3.03 x 10^-4                           | 2.15 x 10^-4                              |
| 1.4 | 6.94 x 10^-2                           | 1.49 x 10^-5                              |
| 1.6 | 9.36 x 10^-2                           | 1.39 x 10^-1                              |
| 1.8 | 2.31 x 10^-5                           | 1.99 x 10^-7                              |
| 2.0 | 3.54 x 10^-5                           | 1.75 x 10^-8                              |

The result reveals that the Adams Moulton explicit method performed better than the new Adams Bashforth explicit method.

5 Discussion

From the numerical examples considered to justify the methods, it is observed that both methods are suitable for solving non-stiff problems and mostly from the Tables 3 and 5 that the Adams Moulton explicit method performs better than the new Adam Bashfort method.

6 Conclusion

In this work, the Adams Moulton and Adams Bashfort extrapolation methods were formulated for steps k = 3; and 4: The methods yielded a class of explicit methods and both methods were apply to solve non-stiff initial value problems. The results obtained show that both methods are efficient and suitable for such problems.

Competing Interests

Authors have declared that no competing interests exist.
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