Parametric Method to Define Area of Allowable Configurations while Changing Position of Restricted Zones

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Abstract. The article presents the findings related to the development of the module for automatic collision detection of the manipulator with restricted zones for virtual motion modeling. It proposes the parametric method for specifying the area of allowable joint configurations. The authors study the cases when restricted zones are specified using the horizontal plane or front-projection planes. The joint coordinate space is specified by rectangular axes in the direction of which the angles defining the displacements in turning pairs are laid off. The authors present the results of modeling which enabled to develop a parametric method for specifying a set of cross-sections defining the shape and position of allowable configurations in different positions of a restricted zone. All joint points that define allowable configurations refer to the indicated sections. The area of allowable configurations is specified analytically by using several kinematic surfaces that limit it. A geometric analysis is developed based on the use of the area of allowable configurations characterizing the position of the manipulator and reported restricted zones. The paper presents numerical calculations related to virtual simulation of the manipulator path performed by the mobile robot Varan when using the developed algorithm and restricted zones. The obtained analytical dependencies allow us to define the area of allowable configurations, which is a knowledge pool to ensure the intelligent control of the manipulator path in a predefined environment. The use of the obtained region to synthesize a joint trajectory makes it possible to correct the manipulator path to foresee and eliminate deadlocks when synthesizing motions along the velocity vector.

1. Introduction

One of the most common emergencies that arise when an autonomous mobile robot operates is its collision with the environment [1-3]. To ensure the successful operation of autonomous mobile robots in complex environments with the least human intervention, it is necessary to improve the methods for analyzing the position of the robot mechanism in a known and unknown environment [4-6]. Therefore, there is a need to develop a module able to detect collisions with restricted zones automatically [7]. To reduce the calculation time required to specify the robot’s position, it was originally proposed in [8] to determine a sufficient collision condition, which corresponds to a certain condition when a path in the joint coordinate space does not intersect with allowable boundaries. Then the necessary condition is checked. All allowable joint configurations can be represented by \( n \)-dimensional geometric object \( \Lambda \), where \( n \) determines the number of joint coordinates. The authors propose a parametric method for specifying the area of allowable configurations \( \Lambda \), which is used for virtual simulation of manipulator path in the obstructed medium.
2. Task statement

Fig. 1 shows the kinematic diagram of the manipulator of the mobile robot Varan and the position of restricted zones $P_1$ and $P_2$. The low height value of the opening for the example under consideration in coordinate system $O_1$ is assumed as equal to $z_n = 500$ mm, and the height of the opening itself can be changed (for a test job this height is determined by parameter $z_p = 500$ mm), the minimal safe distance from the base of the manipulator to the obstacle is assumed to be $x_d = 1200$ mm (see Fig. 1). The lengths of the manipulator links are equal to $O_1O_2 = 900$mm, $O_2O_3 = 700$mm and $O_3O_4 = 500$mm respectively. The joint coordinates are labeled with $q_i$. In order to evaluate how the position of restricted zone $P_j$ affects the shape of the allowable area, it is necessary to consider two possible arrangements of restricted zones.

In the first case, the restricted zone is given in the form of a horizontal plane of level $\Delta$, and in the second – in the form of several front-projection planes that together indicate quadrangle $P_j$ in the front projection (see Fig. 1). The movements of the links will be studied in parallel front projections, given that $q_1 = 0$.

Obviously, the restricted zone, given by the horizontal plane of level $\Delta$ (see Fig. 1), creates the area of allowable configurations $\Lambda$ in joint coordinate space $L_q$ (see Fig. 2a,d,c,d). The configuration is considered allowable if it does not cross the restricted zones, satisfies upper and lower joint values as well as the specified positions of the output link center. In this respect, it can make sense to specify this area $\Lambda$ by several kinematic surfaces which are formed by straight lines or ellipses $l_i$ [8]. The shape parameters of the ellipse determined by the dimensions of large and small semiaxes vary in different sections when changed by joint coordinate $q_2$.

The images of regions $\Lambda$ for three different values of the parameter $z_{op}$ are shown in Fig. 2b,c,d. The labeling of position and shape parameters of one of ellipses $l_7$ (eight different lines $l_i$ are used to define cross-section boundaries) that generates one of the kinematic surfaces is shown in Fig. 3a.

![Figure 1. Kinematic diagram illustrating manipulator of mobile robot Varan and position of restricted zones](image-url)
A parametric method for specifying the area of allowable configurations Λ is proposed, which is used to create a knowledge pool on the past experience of synthesizing movements of the manipulator under various arbitrary values of parameters \( z_{\text{op}} \) and \( x_{\text{op}} \) (see Fig. 1).

### 3. A parametric method for specifying the area of allowable configurations

The area of restricted configurations given by fragments of kinematic surfaces with generators \( l_{7} \) and \( l_{8} \) will be denoted by \( \Omega_{7} \) and \( \Omega_{8} \) (see Fig. 2a). The method for specifying the area of allowable configurations was earlier proposed in [8], where regions \( \Omega_{1} - \Omega_{6} \) were also used. In this case, the region \( \Lambda \) is obtained on the basis of combining and subtraction operations of set theory with areas \( \Omega_{i} \). Inequalities defining the points belonging to areas \( \Omega_{7} \) and \( \Omega_{8} \) of space \( L_{q} \) and defining restricted configurations for this case look like:

\[
\left\{ q_{i} \sin \varphi \Omega^{7} + q_{i} \cos \varphi \Omega^{7} + m_{i}^{7} q_{i}^{2} + m_{i}^{7} q_{i} + m_{i}^{7} \right\}^{2} + \left\{ q_{i} \sin \varphi \Omega^{7} + q_{i} \cos \varphi \Omega^{7} + m_{i}^{7} q_{i}^{2} + m_{i}^{7} q_{i} + m_{i}^{7} \right\}^{2} - 1 \geq 0
\]

\[
\left\{ q_{i} \sin \varphi^{8} + q_{i} \cos \varphi^{8} + m_{i}^{8} q_{i}^{2} + m_{i}^{8} q_{i} + m_{i}^{8} \right\}^{2} + \left\{ q_{i} \sin \varphi^{8} + q_{i} \cos \varphi^{8} + m_{i}^{8} q_{i}^{2} + m_{i}^{8} q_{i} + m_{i}^{8} \right\}^{2} - 1 \geq 0
\]

Inequalities (1-2) are obtained on the basis of coordinate transformations when passing from the \( O_{x}O_{y}O_{z} \) system connected with the ellipse to the \( O_{x}O_{y}O_{z} \) system that sets the frame of section plane in the joint coordinate space \( L_{q} \) (see Fig. 3a). To determine the odds ratio (1) for the case when the restricted zone is given by the horizontal plane of the level \( \Lambda \), the dependences of the coefficients \( m_{i} \Omega = f_{i}(z_{\text{op}}) \), \( m_{i} \Omega = f_{i}(z_{\text{op}}) \), \( m_{i} \Omega = f_{i}(z_{\text{op}}) \), \( m_{i} \Omega = f_{i}(z_{\text{op}}) \), \( m_{i} \Omega = f_{i}(z_{\text{op}}) \), \( m_{i} \Omega = f_{i}(z_{\text{op}}) \),
of the inequality (1) will be represented in the form of interpolation cubic polynomials. To solve this problem, let us use polynomials that are clutched at node points. Let in interval \( [\gamma_{op}, \gamma_{op}^{l+1}] \) be given nodes \( m_{ij}^{O7} = \gamma_{op}^{l} < \gamma_{op}^{l+1} < \gamma_{op}^{l+2} < \gamma_{op}^{l+1} = \gamma_{op}^{l+1} \), and \( l+1 \) values of parameter \( m_{i}^{O7} \rightarrow m_{i}^{O7-1}, m_{i}^{O7-2}, \ldots, m_{i}^{O7-l}, m_{i}^{O7-l+1}, \ldots, m_{i}^{O7-l+1}, \) (which determines the value of coefficient \( m_{ij}^{O7} \) of the polynomial giving the value of coordinate \( m_{1}^{O7} \) of the center of ellipse \( l_7 \) (see Fig. 2a)) along the \( q_{3} \) axis. To solve this problem, let us use the cubic polynomial:

\[
m_{ij}^{O7} = l_{0}^{ij} + l_{1}^{ij} \gamma_{op} + l_{2}^{ij} (\gamma_{op})^{2} + l_{3}^{ij} (\gamma_{op})^{3}, \quad \gamma_{op} = (\gamma_{op}^{l}, \gamma_{op}^{l+1}), \quad l = 0, 1, \ldots, l-1. \tag{3}
\]

Using this method, it is possible to define the values of the coefficients \( l_{0}^{ij}, \ldots, l_{l-1}^{ij} \), in each of intervals \( (\gamma_{op}^{l}, \gamma_{op}^{l+1}) \). Thus, let us find the values of coefficients \( m_{ij}^{O7} = f_{ij}(\gamma_{op}), m_{ij}^{O7} = f_{ij}(\gamma_{op}), \ldots, m_{ij}^{O7} = f_{ij}(\gamma_{op}), a_{ij}^{O7} = f_{ij}(\gamma_{op}), b_{ij}^{O7} = f_{ij}(\gamma_{op}), \ldots \), ‘piece by piece’.

The coefficients of inequalities (1-2) in accordance with the method described above are defined by the following sixteen dependencies:

\[
m_{ij}^{O7} = S_{0}^{ml} + S_{1}^{ml} \gamma_{op} + S_{2}^{ml} (\gamma_{op})^{2} + S_{3}^{ml} (\gamma_{op})^{3}, \quad m_{ij}^{O7-1} = S_{0}^{ml} + S_{1}^{ml} \gamma_{op} + S_{2}^{ml} (\gamma_{op})^{2} + S_{3}^{ml} (\gamma_{op})^{3}, \tag{4}
\]

\[
m_{ij}^{O7} = S_{0}^{m2} + S_{1}^{m2} \gamma_{op} + S_{2}^{m2} (\gamma_{op})^{2} + S_{3}^{m2} (\gamma_{op})^{3}, \tag{5}
\]

\[
a_{ij}^{O7} = S_{0}^{a2} + S_{1}^{a2} \gamma_{op} + S_{2}^{a2} (\gamma_{op})^{2} + S_{3}^{a2} (\gamma_{op})^{3}, \tag{6}
\]

\[
b_{ij}^{O7} = S_{0}^{b2} + S_{1}^{b2} \gamma_{op} + S_{2}^{b2} (\gamma_{op})^{2} + S_{3}^{b2} (\gamma_{op})^{3}. \tag{7}
\]

The calculated values of polynomials are determined on the basis of a set of node points resulting from computational experiments with different values of the parameter \( \gamma_{op} \). The obtained dependences (4-7) allow us to calculate the values of polynomials used in inequalities (1-2) for different tunnel heights or the value of parameter \( \gamma_{op} \). Similarly, the coefficients of inequality (2) can be determined to find the center of ellipse \( l_{8} \) and parameters of its shape (relations \( m_{ij}^{Q8} = f_{ij}(\gamma_{op}), m_{ij}^{Q8} = f_{ij}(\gamma_{op}), \ldots, m_{ij}^{Q8} = f_{ij}(\gamma_{op}), a_{ij}^{Q8} = f_{ij}(\gamma_{op}), \ldots, b_{ij}^{Q8} = f_{ij}(\gamma_{op}), \ldots \)).

Fig. 4a,b,c illustrates the results of calculating the coefficients of relations (4-8), and based on this graphical representation of functions \( m_{ij}^{Q7} = f_{ij}(q_{2}, \gamma_{op}), a_{ij}^{Q7} = f_{ij}(q_{2}, \gamma_{op}), \) and \( b_{ij}^{Q7} = f_{ij}(q_{2}, \gamma_{op}) \) determining the position and shape of ellipse \( l_{7} \) in different sections. Owing to the fact that the displacement of the center of the ellipse along the \( q_{4} \) axis is insignificant, function \( m_{ij}^{Q7} = f_{ij}(q_{2}, \gamma_{op}) \) is mapped on the graph close to the plane. Therefore, the paper does not provide the graph of this function.

Let us specify the centers of ellipses and the lengths of large and small semiaxes for the case when the restricted zone is confined by the front projection planes (which form the contour of rectangle \( P_{l} \) shown in Fig. 1), subject to values \( x_{op} \) and \( \gamma_{op} \). The computational results and calculations of the graphs of functions \( m_{ij}^{Q7} = f_{ij}(q_{2}, \gamma_{op}), a_{ij}^{Q7} = f_{ij}(q_{2}, \gamma_{op}), \) and \( b_{ij}^{Q7} = f_{ij}(q_{2}, \gamma_{op}) \) for three different values of parameter \( x_{op} \) are presented in Fig. 5a,b,c.
The analysis of the graphs presented in Fig. 5a,b,c shows the patterns of the change in the values of the parameters determining the position and shape of the fragments of ellipses used to define the cross-section boundaries of the area of allowable configurations $\Lambda$, from values $z_{op}$ and $x_{op}$ specifying the position of the restricted zone.

**Figure 4.** Graphs of functions: a) $\Omega^a = f(z_{op})$; b) $\Omega^b = f(z_{op})$; c) $\Omega^c = f(z_{op})$.
4. Simulation of manipulator path using allowable areas

The main task of virtual control over the manipulating mechanism is to calculate the gain vector of joint coordinates at each step of calculations [9]. This vector is determined by the following relation:

\[ Q_N = Q_M + \sum_{i=1}^{P} k_i mQ_1. \]  

where \( Q_M \) is the vector specifying point \( M^0 \in \Gamma^0 \) corresponding to the motion miniaturization criterion. Point \( M^0 \) defines the center of the frame connected with \( p \)-plane \( \Gamma^0 \) determined by linear equations that specify the relation between the joint increments and those of the output link [10]; \( k_i \) are the coordinates of point \( N^0 \) in \( p \)-plane \( \Gamma^0 \) (each point \( N^0 \) has a corresponding instantaneous state of the manipulator). Superscript \( ^0 \) determines the reference of geometric objects to the joint incremental space; \( m \) is the length of a single frame interval of \( p \)-plane \( \Gamma^0 \); \( Q_1 \) are reference vectors of the frame axes; \( p \) is the dimension of \( p \)-plane \( \Gamma^0 \).

For each time point, it is necessary to determine the mutual position of a current design configuration of the manipulator with respect to the restricted zones. The position of the configuration, relative to the restricted zones, will be determined using area allowable configurations \( \Lambda \). To overcome deadlocks when synthesizing motions by velocity vector, let us use scale \( h \) of vector presentation connecting points \( B_i (q_{i_1}^{t_1}, q_{i_2}^{t_1}, q_{i_3}^{t_1}) \) and \( B_{i+1} (q_{i_1}^{t_1}, q_{i_2}^{t_1}, q_{i_3}^{t_1}) \) in the \( L_q \) space at different steps of calculations (where \( t \) specifies current parameter, \( q_{i_1}^{t_1}, q_{i_2}^{t_1}, q_{i_3}^{t_1} \) are coordinates of point \( B_i \) setting the current configuration).

The algorithm to synthesize the joint path is presented in Fig. 6. In this figure the following notations are accepted, namely: 1 – Beginning; 2 – Specification of points \( A_i \) and \( A_t \) that define the initial and target position of the center of the output link in the intervals of the given trajectory from point \( A_1 \) to point \( A_2 \) (see Fig. 7); 3 – Synthesis of motion from point \( A_i \) to target point \( A_t \) by the motion miniaturization criterion; 4 – Determination of points \( B_i \) and \( B_{i+1} \) in the joint coordinate space specifying configurations at different steps of calculations using the motion miniaturization criterion. Definition of vector \( L_q \) connecting points \( B_i \) and \( B_{i+1} \) in space \( Q \). Computation of vector \( L_{qh} \) taking into account the scale of mapping \( h \). Determination of points \( B_i \) where vector \( L_{qh} \) meets with boundary surfaces of region \( \Lambda \) or with generators \( l_i \). The component of vector \( L_{qh} \) along the \( q_2 \) axis specifies the cross-section and position of kinematic surfaces forming \( l_i \); 5 – \( B_i = \text{null} \); 6 – Change of the position of point \( B_{i+1} \) and direction of vector \( L_{qh} \) by changing parameter \( k \) of dependence (8); 7 – Change in joint values \( q_i = q_i + \Delta q_i \); 8 – Target point reached; 9 – Derivation of joint calculated values \( q_i \); 10 – End.

![Figure 5. Graphs of functions: a - \( f_i (x_{op}, z_{op}) \); b - \( f_i (x_{op}, z_{op}) \); c - \( f_i (x_{op}, z_{op}) \).](image-url)
5. Results

Based on the results of theoretical studies the paper solves the test problem related to the displacement of the output link center from point $A_1$ to point $A_2$ in the presence of restricted zones $P_1$ and $P_2$ (see Fig. 7). To ensure the displacement, the center of the output link must first be moved upward to a certain height. Then it should be moved along the corresponding horizontal trajectory and lower it down to align with target point $A_3$. The joint values were based on the parametric method for specifying the area of allowable configurations $A$. The results of the virtual simulation of the manipulator path of the mobile robot are shown in Fig. 7.

6. Conclusions

The time necessary to calculate the test task using the developed method is reduced by an order of magnitude as compared with the method in which the intersection of primitives specifying links of the manipulator and the restricted zones is determined for each iteration.

The developed method for parametrical specification of the area of allowable configurations is more universal and determines the positions of configurations satisfying the given requirements more precisely. The new method for specifying the area of allowable configurations enables one to reduce time necessary for analyzing information about the position of the manipulator and restricted zones in virtual path simulation.

Analytical dependencies can be used in knowledge pools on the past experience of the synthesis of movements to provide intelligent control of movement of manipulators of mobile robots in complex environments.

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