Strong magnetic field in a protoneutron star

Subrata Pal1, Debades Bandyopadhyay1, and Somenath Chakrabarty2
1Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Calcutta- 700 064, India
2Department of Physics, University of Kalyani, Kalyani 741235, India
and Inter-University Centre for Astronomy and Astrophysics, Post Bag 4, Ganeshkhind, Pune 411007, India

We investigate the influence of a strong interior magnetic field on the structure and composition of a protoneutron star allowing quark-hadron phase transition. In contrast to protoneutron stars with noninteracting quark phase, the stars with interacting quark phase have smaller maximum star masses than those of neutrino-free stars. The strong field makes the overall equation of state softer compared to the field-free case favoring the evolution of a protoneutron star more towards a neutron star than a black hole.

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The understanding of the final journey of a massive star, after its fuel has been exhausted is a challenging problem [1]. The outcome of it may be a supernova and the residue either be a neutron star or a black hole. The unique feature about supernova problem is that it involves all the forces of nature — strong, weak, electromagnetic and gravity. A massive star in its late stage of evolution undergoes gravitational core collapse as the core exceeds the Chandrasekhar mass. The subsequent core bounce occurs when the core density reaches nuclear matter value and above. A few milliseconds after the core bounce, the hot and lepton rich (neutrino-trapped) core settles into hydrostatic equilibrium [2]. The evolution of this protoneutron star into a neutron star or a black hole completes within a few tens of seconds.

Observations of pulsars predict large surface magnetic fields of $B_{m} \sim 10^{14}$ G [3]. The interior magnetic fields are a few orders of magnitude larger than the surface fields. In fact, the virial theorem [4] predicts large interior field of $\sim 10^{18}$ G or more [5]. One of the plausible explanations for such a large interior field is that the weak field of a progenitor is amplified because of flux conservation during the gravitational core collapse. The highly conducting core results in large ohmic diffusion time so that the field is frozen in the core, and consequently not manifested at the surface [6]. The energy of a charged particle changes significantly in the quantum limit if the magnetic field is comparable to or above a critical value $B_{m}^{c}(\eta)$, and the quantum effects are most pronounced when the particle moves in the lowest Landau level [7]. In addition to this large magnetic field embedded in the dense core of the (proto)neutron star, a transition from nuclear matter to a stable quark matter is also possible [8–11].

In this letter we investigate the effects of trapped neutrinos on the composition and structure of a protoneutron star in presence of a strong interior magnetic field and also allowing a hadron to quark phase transition in the star’s interior. The matter inside the protoneutron star is highly degenerate and the chemical potential of its constituents are a few hundreds of MeV. On the other hand, the central temperature of the star is a few tens of MeV. Therefore, neglecting finite temperature effect will have little influence on the gross properties of the star. However, the compositional changes caused by trapped neutrinos, which are primarily of the electron type $\nu_{e}$, may induce relatively larger changes in the maximum masses [3].

We describe the equilibrium conditions of the pure quark and baryonic matter and their mixed phase co-existing in a uniform background of electrons (e) and electron neutrinos ($\nu_{e}$) for neutrino-trapped matter in a uniform magnetic field $B_{m}$ along z axis. The neutrino-trapped pure quark phase consisting of $u$, $d$ and $s$ quarks interacting through one-gluon exchange in local charge neutral and $\beta$-equilibrium conditions is described by the bag model [12]. The interaction energy density due to one-gluon exchange term to order $g^{2}$ for each flavor with $B_{m} \neq 0$ in the zeroth Landau level, $\eta = 0$, is given by

$$\mathcal{E}^{f;\eta=0}_{I} = \frac{q_{f}B_{m}}{8\pi^{2}} \int_{-p_{F}^{f}}^{+p_{F}^{f}} dp_{z} \left[ U_{0}^{F} + \frac{p_{\nu\zeta} \pi^{2}}{\sqrt{p_{\nu\zeta}^{2} + m_{f}^{*2}}} U_{V}^{F} \right],$$

(1)

where $q_{f}$, $m_{f}^{*}$ and $p_{F}^{f}$ are the charge, effective mass and Fermi momentum of quark of $f$th flavor, and $p_{\nu\zeta} = p_{z}[1 + U_{V}^{F}/U_{0}^{F}]$. For the expressions of the Fock contributions, $U_{0}^{F}$ and $U_{V}^{F}$, to the single particle energy, we refer to Ref. [11]. The QCD coupling constant is defined by $\alpha_{c} = g^{2}/4\pi$. The general expression (for all Landau levels $\eta$) for the total kinetic energy of the quark phase in a magnetic field is

$$\mathcal{E}^{\eta}_{K} = \sum_{f=u,d,s} \frac{d_{f}q_{f}B_{m}}{4\pi^{2}} \sum_{\eta=0}^{\eta_{\text{max}}} g_{f} \Phi (\mu_{f,\eta}) + \frac{eB_{m}}{4\pi^{2}} \sum_{\eta=0}^{\eta_{\text{max}}} g_{e} \Phi (\mu_{e,\eta}) + \frac{e^{2}}{8\pi^{2}},$$

(2)
magnetic field energy density and $B$ with the first term corresponds to those for quarks and the third term corresponds to that for the neutrinos. The total energy density of the pure quark phase is then $\mathcal{E} = \mathcal{E}_K^q + \int_0^{\pi} \mathcal{E}_I^q + \mathcal{E}_B + B$, where $\mathcal{E}_m = B_m^2/(8\pi)$ is the magnetic field energy density and $B$ is the bag constant. The pressure follows from the relation $P^q = \sum \mu f n_f + \sum \mu q n_q$, where $\mu f$ denotes the quark chemical potential, $n_f = (d_f q_f B_m/2\pi^2) \sum g_{\eta f q} (\mu_f^2 - m_f^2)^{1/2}$ is the quark density, and $l = (e, \nu_e)$. The electron density is $n_e = (e B_m/2\pi^2) \sum g_{\eta e q} (\mu_e^2 - m_e^2)^{1/2}$, and $\mu_e$ and $\mu_{\nu_e}$ are the chemical potentials for electrons and neutrinos. The charge neutrality condition, $Q^q = \sum q_f n_f - n_e = 0$, and the $\beta$-equilibrium conditions, $\mu_\alpha = \mu_q + \mu_e - \mu_{\nu_e} = \mu_\beta$, can be solved self-consistently together with the effective masses at a fixed baryon number density $n_B = (n_u + n_d + n_s)/3$ to obtain the equation of state (EOS) for the deconfined phase. For the ease of numerical computation we, however, add here the one-gluon exchange term perturbatively to energy density and pressure.

To describe the neutrino-trapped pure hadronic matter consisting of neutrons ($n$), protons ($p$), electrons ($e$) and electron neutrinos, we employ the linear $\sigma$-$\omega$-$\rho$ model of Ref. [11] in the relativistic Hartree approach. The Fock contribution to the hadron phase is quite small [14], and therefore neglected. The EOS for this phase is obtained by solving self-consistently the effective mass in conjunction with the charge neutrality and $\beta$-equilibrium conditions, $Q^h = n_p - n_e = 0$ and $\mu_n = \mu_p + \mu_e - \mu_{\nu_e}$ at a fixed baryon number density $n_B$. Here $n_i$ and $\mu_i$ denote the number density and chemical potential; the subscript $i$ refers to $n$, $p$, $e$ and $\nu_e$. The total energy density $\mathcal{E}^h$ is given in Ref. [11] and pressure $P^h$ in this phase are related by $P^h = \sum n_i \mu_i - \mathcal{E}^h$.

The mixed phase of hadrons and quarks comprising of two conserved charges, baryon number and electric charge is described following Glendenning [8]. The conditions of global charge neutrality and baryon number conservation are imposed through the relations $\chi Q^h + (1 - \chi)Q^q = 0$ and $n_B = \chi n_B^q + (1 - \chi) n_B^h$, where $\chi$ represents the fractional volume occupied by the hadron phase. Furthermore, the mixed phase satisfies the Gibbs’ phase rules: $\mu_p = 2\mu_n + \mu_q$ and $P^h = P^q$. The total energy density is $\mathcal{E} = \chi \mathcal{E}^h + (1 - \chi) \mathcal{E}^q$. The neutrino-free matter relations can be obtained [11] by putting $\mu_{\nu_e} = 0$ in the above expressions.

In the present calculation the values of the dimensionless coupling constants for $\sigma$, $\omega$ and $\rho$ mesons determined by reproducing the nuclear matter properties at a saturation density of $n_0 = 0.16$ fm$^{-3}$ are adopted from Ref. [8]. The current masses of $u$ and $d$ quarks are taken as $m_u = m_d = 5$ MeV and $m_s = 150$ MeV, and the QCD coupling constant is $\alpha_s = 0.2$. Because of trapping the numbers of leptons per baryon, $Y_{le} = Y_e + Y_{\nu_e}$, are conserved on dynamical time scale. Gravitational collapse calculations of massive stars indicate that, at the onset of trapping, $Y_{le} \approx 0.4$. We consider the bag constant $B = 250$ MeV fm$^{-3}$ which corresponds to the lower limit dictated by the requirement that, at low density, hadronic matter is the preferred phase. The magnetic field, like the baryon density, increases from the surface to the center of the star, and consequently the variation of the magnetic field $B_m$ with density $n_b$ is parametrized by the form

$$B_m(n_b/n_0) = B_{m0} + B_0 \left[1 - \exp\{-\beta(n_b/n_0)^\gamma\}\right],$$

where the parameters are chosen to be $\beta = 10^{-4}$ and $\gamma = 6$. The maximum field prevailing at the center of trapping is $\sim 0.1$ T for a neutron star of typical radius $R$ = 10 km and temperature $T = 10^9$ K, the characteristic diffusion time for such field, using the above estimate of Ref.
The phase boundaries, \( u_1 \) and \( u_2 \), and central densities \( u_c \) for neutrino-trapped (\( Y_{Le} = 0.4 \)) and neutrino-free (\( Y_{Le} = 0.4 \)) stars with maximum masses \( M_{\text{max}}/M_\odot \) with and without magnetic field that undergo a quark-hadron phase transition with interacting quark phase. The fraction of mass originating from the pure hadron phase is \( M_{\text{had}}/M_{\text{max}} \). The corresponding quantities with noninteracting quark phase are shown in parentheses. The variation of magnetic field with density \( n_b \) is given by Eq. (3) with \( \beta = 10^{-4}, \gamma = 6 \) for \( B_0 = 5 \times 10^{18} \) G. Calculations are performed for a mean field model of baryons and a bag model of quarks with bag constant of \( B = 250 \text{ MeV} \text{ fm}^{-3} \). The nuclear matter saturation density \( n_0 \) is 0.16 fm\(^{-3}\).

| \( Y_{Le} = 0.4 \) | \( B_m \) (G) | \( u_1 = n_1/n_0 \) | \( u_2 = n_2/n_0 \) | \( u_c = n_c/n_0 \) | \( M_{\text{max}}/M_\odot \) | \( M_{\text{had}}/M_{\text{max}} \) |
|-----------------|----------|----------------|----------------|----------|----------------|----------------|
| \( Y_{Le} = 0.4 \) | \( Y_{Le} = 0 \) | 0 | 8.167(6.541) | 28.067(19.390) | 11.442(11.102) | 1.677(1.664) | 0.816(0.698) |
| \( Y_{Le} = 0.4 \) | \( Y_{Le} = 0 \) | 0 | 4.107(3.329) | 25.941(17.681) | 7.152(7.754) | 1.707(1.610) | 0.748(0.584) |
| \( Y_{Le} = 0 \) | 10\(^8\) - 5 \times 10^{18} | 8.091(6.503) | 27.897(19.216) | 12.003(11.060) | 1.540(1.331) | 0.773(0.686) |
| \( Y_{Le} = 0 \) | 10\(^8\) - 5 \times 10^{18} | 4.110(3.322) | 25.754(17.530) | 6.202(6.304) | 1.553(1.487) | 0.859(0.718) |

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fraction of mass originates from the softer quark-hadron mixed phase in neutrino-free stars than neutrino-trapped matter. Consequently, $M_{\text{max}} = 1.610M_{\odot}$ for neutrino-free star with NQP is smaller than 1.664$M_{\odot}$ mass for protoneutron star. On the other hand, for stars with IQP (with $B_m = 0$), the pure hadronic part mostly governs the EOS (i.e. large $M_{\text{had}}/M_{\text{max}}$) for both neutrino-free and trapped matter. As a result the maximum mass here follows the trend of pure hadronic stars with $B_m = 0$, i.e. $M_{\text{max}}$ for $Y_{le} = 0.4$ is smaller than that for $Y_{\nu_e} = 0$. This also accounts for the fact that the masses of stars with IQP are larger than the masses of stars with the corresponding NQP matter.

In presence of magnetic field, the hadronic EOS becomes much softer compared to the stiffening caused in the quark EOS. Therefore, the overall EOS for the quark-hadron star with $B_m \neq 0$ turns out to be softer leading to smaller mass than that for the field-free star. Thus for $B_m \neq 0$, smaller the contribution to the EOS from stiff quark part of the mixed phase, smaller would be the mass. This, in fact, is revealed by stars with NQP for $Y_{\nu_e} = 0$ compared to $Y_{le} = 0.4$ case. The EOS of a star with IQP for $B_m \neq 0$ is primarily governed by the pure hadronic part, their mass is thus reminiscent of that for the pure hadronic stars with $B_m \neq 0$. Therefore, the trend of the masses from neutrino-free to neutrino-trapped star with magnetic field are same as that of field-free stars, both with interacting and noninteracting quark phase.

In Fig. 1 we depict a comparison of the composition of neutrino-free matter (top panel) and neutrino-trapped matter (bottom panel) in a magnetic field of $B_0 = 5 \times 10^{18}$ G with IQP. In contrast to $B_m = 0$ case (not shown), the electron fraction, in particular, is enhanced in a magnetic field due to space phase modification. In the hadronic sector, the electron fraction increases because of neutrino trapping, which in turn increases the proton fraction due to charge neutrality condition and thereby reduces the neutron fraction due to baryon number conservation. On the other hand, $u$, $d$ and $s$ quarks themselves try to maintain charge neutrality, resulting in reduction of the electron fraction and consequently the neutrino fraction is enhanced in the mixed phase. Neutrino trapping also increases the quark abundance in comparison to the neutrino-free star.

The differences in the abundance of electron, in particular within the inner core of different stars investigated, would be manifested in the total number of $\nu_e$ escaping during deleptonization. The total number of $\nu_e$s emitted, $N_{\nu_e}$, from the inner core of mass 0.5$M_{\odot}$ during deleptonization from the initial value of $Y_{le} = 0.4$ is obtained by integrating $n_{\nu_e} = n_b Y_{\nu_e} = n_b (Y_{le} - Y_e)$ over this region of the neutrino-free star. For pure hadronic star, $N_{\nu_e}$ is found to be $9.416 \times 10^{55}$ at $B_m = 0$, while for $B_0 = 5 \times 10^{18}$ G it is $9.135 \times 10^{55}$. On the other hand, we obtain for stars with IQP(NQP), the value of $N_{\nu_e}$ to be $1.662 \times 10^{56}(1.994 \times 10^{56})$ for $B_m = 0$, and for $B_0 = 5 \times 10^{18}$G the corresponding values are $1.532 \times 10^{56}(1.801 \times 10^{56})$. The stars with quark-hadron phase transition emit more number of $\nu_e$s than that by the pure hadron stars. This is caused by the drop in the electron abundance in the mixed phase as quarks furnish net negative charge. The enhancement of electron fraction in magnetic field causes a decrease in $N_{\nu_e}$ from the corresponding field-free cases. The different scenarios may thus be discernible from the difference in the neutrino numbers.

Delayed neutrino emission and possible black hole formation in the context of SN 1987A have been much debated issues in recent years. So far there is no observation of a pulsar in it. Moreover, the fading away light curve leads one to think that it might have collapsed into a low mass black hole. Assuming that SN 1987A has gone to a black hole, Bethe and Brown estimated the gravitational mass for the compact object in it to be 1.56$M_{\odot}$ from Ni production. They argued that any compact object having mass larger than this limit would be a black hole. It has been counter argued that the surface magnetic field of a nascent neutron star is weak. It may take a few hundred years or more for the magnetic field trapped in the crust to reach the surface by ohmic diffusion which
would then increase the surface field so that the pulsar could be accessible to observation. In the present calculation, since the maximum masses of hadron stars increase when the trapped neutrinos leave, they will promptly collapse after core bounce into black holes once the mass of $1.56 M_\odot$ is reached, presuming Bethe-Brown limit for the maximum neutron star mass. Similar situation arises for stars with interacting quark phase (see Table I). In contrast, the maximum masses of stars with noninteracting quark phase decrease after deleptonization. Therefore, in the event that the maximum masses of these neutrino-free stars reach $1.56 M_\odot$, the stars will first explode, returning matter to the galaxy, and then collapse into low mass black holes [19]. We however find in our calculation that the maximum star masses both for neutrino-trapped and neutrino-free matter both for field-free stars. Therefore, the presence of a strong interior field favors the evolution of a protoneutron star more towards a neutron star than a black hole even if its mass increases after the trapped neutrinos leave.

Most of the dynamical supernova calculations [1,20,21] indicate that a successful prompt supernova explosion could be achieved by employing a soft EOS at densities larger than nuclear matter density $n_0$. The substantial softening of the EOS caused by the strong interior magnetic field (and also by quarks) may provide a viable prompt shock mechanism, and thereby merits consideration in dynamical simulations of explosions.

In summary, we have investigated the gross properties of a protoneutron star in presence of a strong magnetic field in the core with a quark-hadron phase transition. In contrast to the neutrino-trapped matter with noninteracting quark phase, the protoneutron star with interacting quark phase leads to smaller maximum star mass compared to neutrino-free case both with and without magnetic field. Besides the presence of quarks, the softening of the overall EOS caused by magnetic field has significant bearing on the evolution of the protoneutron star to a neutron star or a low mass black hole.

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