A Generalized Bit Error Rate Evaluation for Index Modulation Based OFDM System

MAHMOUD ABDULLAHI, AIJUN CAO, ADNAN ZAFAR, PEI XIAO, and IBRAHIM HEMADEH

1Institute for Communication Systems (ICS), University of Surrey, Guildford, UK (e-mail: m.abdullahi, a.cao, p.xiao, i.hemadeh@surrey.ac.uk)
2WiSP Lab, Electrical Engineering Department, Institute of Space Technology (IST), Islamabad, Pakistan (e-mail: adnan.zafar@ist.edu.pk)

ABSTRACT Orthogonal frequency division multiplexing (OFDM-IM) is a multicarrier transmission technology that modulates information bits not just onto subcarriers by means of M-ary constellation mapping but also onto selected (active) subcarrier indices. Consequently, errors can occur in OFDM-IM systems indices in addition to the errors of M-ary symbols. This paper analyzes the error scenarios and derives mathematical expressions for the error performance based on the maximum likelihood (ML) detection. In evaluating the bit error rate (BER) in the additive white Gaussian noise (AWGN) channel, some assumptions are made and our analytical result show that the BER of OFDM-IM system is a weighted sum of exponential functions and Q-functions. Our general BER expression has been shown to be in excellent agreement with numerical simulation and proven to be accurate and can serve as a reference for the design and evaluation of any arbitrary size and configuration of OFDM-IM systems.

INDEX TERMS Index modulation, OFDM, maximum likelihood detection, probability density function, BER analysis

I. INTRODUCTION

OFDM is the most popular multicarrier transmission technology adopted in many recent broadband wireless standards such as 802.11a/g (Wi-Fi), 802.16 (WiMAX), Long Term Evolution (LTE), Digital Audio and Video Broadcasting (DAB and DVB). This is in order to satisfy the increasing demand for high data rate communications in frequency selective fading channels. The advantages of OFDM include its ability to mitigate Inter-Symbol Interference (ISI) caused by channel frequency selectivity [1], and its use of simple frequency domain equalizers at the receiver for efficient detection [2], [3]. However, OFDM suffers high Peak to Average Power Ratio (PAPR) which may severely impair system performance due to induced spectral regrowth and detection performance degradation. Furthermore, OFDM is very sensitive to timing and frequency synchronization errors [4], [5]. In [6], and motivated by the earlier works on parallel combinatorial spread spectrum [7], the parallel combinatorial OFDM (PC-OFDM) system was proposed for minimizing the PAPR as well as for improving the bandwidth and the bit error probability in AWGN channels. In another endeavour, inspired by the philosophy of Spatial Modulation (SM) [8], Subcarrier Index Modulation OFDM (SIM-OFDM) was proposed, where OFDM subcarriers are classified as active and inactive, with their indices exploited to implicitly convey information bits [9]. This concept’s primitive implementation allows binary information bits to control the selection of active and inactive subcarriers, where preliminary results showed that the system can provide significant improvement over the conventional OFDM in terms of BER performance, PAPR reduction and can also offer trade-off between spectral and energy efficiencies.

In recent years, 5G wireless systems have been conceptualised to provide unprecedented levels of spectral and energy efficiencies for high data rate and ubiquitous communications [10]. Accordingly, there has been a growing worldwide activity to develop new technologies that will support the 5G revolution. Consequently, SIM-OFDM system has been identified as one of the potential physical layer techniques towards 5G and beyond target realisation [11]–[13]. A plethora of research results have been published so far in the areas of design/enhancement of SIM-OFDM system, error perfor-
performance/capacity improvement and generalization/adaptation to different wireless environments. Even though the early SIM-OFDM system design has been shown to be capable of outperforming the classical OFDM and also offering trade-off between power saving and SNR improvement through implementation of power saving policy (PSP) and power redistribution policy (PRP) respectively, it has a limitation imposed by error propagation as a result of error in index bit detection [9]. In order to tackle this problem, [14], [15] proposed an Enhanced Subcarrier Index Modulation (ESIM) scheme, where subcarriers are grouped into clusters to localize and limit the error propagation. This scheme is also referred to as 'Orthogonal Frequency Division Multiplexing with Index Modulation' (OFDM-IM) [16] or 'Multi-Carrier Index Keying OFDM' (MCIK-OFDM) [17], [18] in the literature.

Some studies also aimed at improving the throughput of SIM-OFDM systems e.g., two generalised OFDM-IM structures were proposed (OFDM-GIM-I and OFDM-GIM-II) by modifying ESIM to enhance performance. More specifically, OFDM-GIM-I is capable of providing flexibility for active subcarriers selection and can transmit more bits per subblock compared to OFDM-IM, while OFDM-GIM-II is for improving the spectral efficiency by applying IM independently for in-phase and quadrature components of the complex data symbols [19]. Furthermore, multiple mode [20], dual mode [21] and generalized dual mode [22] OFDM-IM structures have also been proposed to improve spectral efficiency over the conventional OFDM-IM by conveying information through multiple distinguishable modes and their full permutation. Basically, additional bits are transmitted through indices of subcarriers modulated by the same constellation alphabet, and this can be improved by altering the number of subcarriers modulated by the same constellation mode in each subblock. Furthermore, MIMO and OFDM-IM were combined in [23], [24] to take advantage of the benefits of both techniques and it is demonstrated that MIMO-OFDM-IM improves the capacity and integrity of OFDM-IM system and provides a flexible tradeoff between the error performance and the spectral efficiency as well as achieves considerably better error performance than classical MIMO-OFDM.

Researchers have been conducting investigations to further establish the numerous advantages of SIM-OFDM over the conventional OFDM. For example, [25] showed sidelobe suppression and BER performance improvements in SIM-OFDM over OFDM when N-continuous signal processing is adopted. This implies that SIM-OFDM system is less prone to spectral leakages, which is a problem for multicarrier systems. Additionally, SIM-OFDM was found to be less sensitive to phase noise and doppler due to its robustness and tolerance more than the conventional OFDM [26]. Furthermore, [27] investigated the cell edge performance of FQAM, another IM based transmission system and found out that the noise plus interference signals which is a characteristic of the cell edge user, deviates from the Gaussian distribution and thus achieves higher capacity over OFDM at cell edge. [28] analysed OFDM-IM from an information theoretic perspective and proposed an optimal method for maximizing the energy efficiency (EE) of the system. In an attempt to continuously improve IM based OFDM, OFDM with Interleaved Subcarrier Index Modulation (OFDM-ISIM) has been proposed in [29], and result has shown that increasing the Average/Minimum Euclidean Distance (AMED) of elements within the sub blocks can substantially improve the BER performance. To further improve the distribution of AMED, a novel subcarrier allocation scheme based on optimal search algorithm was proposed in [30] which maximizes AMED among the clusters and simplified search algorithm which allocates subcarriers by sorting the channel gains have been presented. Owing to the degraded performance of OFDM-IM in severely fading environments, the diversity potential of OFDM-IM has been investigated in [31], [32] using Coordinated Interleaved OFDM-IM (CI-OFDM-IM)-where CI in conjunction with Space Time Block Codes Performance analysis based on the ML detection for both coded and uncoded systems show significant BER performance improvements over OFDM and OFDM-ISIM. Yet in another novel OFDM-IM scheme called repeated index modulation with coordinate interleaved OFDM (RIM-CI-OFDM) [33], performance analysis using both the optimal ML and low complexity ML detectors shows that higher reliability and flexibility in accurately detecting index and M-ary symbols can be achieved.

A general design guideline for OFDM-IM system has been presented in [34]. One of the important performance metrics in evaluating the superiority of OFDM-IM system over classical OFDM system is the BER performance. Most papers such as [15], [35],[40] have analysed the error performance for different detection schemes from the pairwise error probability (PEP) perspective and provided expressions for SER/IER/BER bounds. For example, [35] derived closed form expressions for PEP and symbol error probability (SEP) for hybrid detection/diversity reception in OFDM-IM based MIMO systems. [38] on the other hand provided a tight closed form expression for average SEP under channel state information (CSI) uncertainty and also investigated through asymptotic analysis, the achievable diversity, coding gain and impact of CSI uncertainty. [36], [39] investigated the BER for OFDM-IM systems using greedy detection. This is a low complexity detection method that makes detection of active subcarriers on the basis of their received power, which means the detection of active subcarrier does not depend on the detection of M-ary symbols. By deriving the BER from pairwise error probability, the expressions given can only be tight but not exact. Even though Maximum likelihood/Maximum posteriori (ML/MAP) detection are adjudged the optimal detection for most communication systems very few attempts have been made in analysing and providing exact BER expression. [37] provided a BER expression for the joint detection of OFDM-IM systems but results were far from exact as numerical simulations.
and theoretical results exhibit a gap up to 3dB at low SNR which reduces to 1dB as SNR increases. Although [14], [40] presented some insight into deriving the exact expressions for OFDM-IM BER, they limit the number of active subcarrier in a cluster to \(k = 1\) and showed that overall BER is the sum of errors due to each of index and M-ary modulation. To the best of the authors’ knowledge, no general expression has been given to evaluate the exact BER of any arbitrary OFDM-IM system configuration. Our paper aims to bridge this gap and derives a close form expression to evaluate the exact BER of OFDM-IM system in the AWGN channel. We present a generalized expression for evaluating the error due to misplacement of carrier indices for a maximum likelihood detector. Although our analysis is conducted for the AWGN channel, our analytical expression can be readily extended to evaluate the BER performance of a fading channel as well.

The main contributions in this work can be summarized as follows

1) Evaluation of error performance using decision regions in order to provide exact BER expressions.
2) Derivation and generalization of the expression for the ML detector.
3) Although the general expression is derived for the AWGN channel, it is shown that by averaging the instantaneous BERs over the SNR distribution, we can evaluate the performance of fading channels provided their distribution is known.

The rest of the paper is structured as follows. Section 2 presents an overview of the system model. Section 3 provides a brief overview of the optimum receiver (ML detector). Section 4 presents the analytical derivations of errors due to IM with a general expression as well as the overall bit error rate for both AWGN and Fading channels. Section 5 presents the numerical simulations to validate our derived expressions. Finally, Section 6 concludes the paper.

Notations
Vectors are denoted by lowercase letters with bar, and elements of vectors are denoted by just lowercase letters. \(\mathbb{E}\{A\}\) denotes the expectation operation of \(A\). \(\hat{B}\) represents the estimate of \(B\). \(f_n\) denotes the probability distribution function of \(n\) \(\binom{n}{k}\)

II. SYSTEM MODEL
The OFDM-IM transmitter depicted in Fig. 1 is considered where a single or multiple subcarrier selection is possible for each OFDM-IM sub-block. More specifically, the total number of subcarriers is partitioned into blocks of subcarriers, where each sub-block is the basic unit for bit modulation and denoted by (n,k,M-ary) system, with \(n\) being the number of subcarriers within a sub-block, \(k\) the number of active subcarriers out of \(n\) and M-ary is the digital modulation constellation size. The \(T\) input bits are divided with the aid of the bit splitter of Fig. 1 into Index bits \((P1)\) and M-ary bits \((P2)\).

In each sub-block \(P1 = \left\lfloor \log_2 \binom{n}{k} \right\rfloor\) bits are used by the index selector of Fig. 1 to activate \(k\) subcarriers out of \(n\), where \(\binom{n}{k}\) is a combination function and \(\lfloor . \rfloor\) is a floor function. Next, \(P2 = k\log_2 M\) bits are mapped into \(k\) number of symbols from the classical M-ary constellation to be transmitted over the \(k\) active subcarriers already selected by the index selector. As a result, each sub-block transmits \(P = P1 + P2\) bits. For example consider the (4,1, QPSK) system shown in Table 1, where the first two (2) bits are used by the index selector as follows: 00 \(\rightarrow\) 1, 01 \(\rightarrow\) 210 \(\rightarrow\) 311 \(\rightarrow\) 4. The last two bits are used by the M-ary mapper to choose from the QPSK symbols \(\{q_1, q_2, q_3, q_4\}\) as shown in the third column of the table. After the mappings, OFDM-IM system uses the new informations from both the index selector \(I_g = \{I\}^k\) and M-ary mapper \(x_g = \{x : q \rightarrow x\}^k\) to create sub blocks \(x_g \in S\). \(S\) is a set comprising all the possible messages \(s_i\), which the OFDM-IM modulator maps to. The \(x_g\) message which will be conveyed by the \(g^{th}\) sub block are then concatenated in the block creator to form the OFDM-IM frequency domain symbols to be transmitted. The frequency symbols are passed through an IDFT block to create an OFDM-IM time domain signal \(s(t)\). Then a parallel to serial converter outputs the time domain OFDM-IM signal for transmission after a cyclic prefix is appended to it.

At the receiver, a reverse process is applied. First, the cyclic prefix is removed then the received signal is passed through a serial to parallel which is forwarded to DFT block for reconstructing the frequency domain symbols. Next, the frequency symbols are fed into a block splitter to divide the OFDM-IM block into sub blocks for processing. By assuming perfect time and frequency synchronization, the frequency domain input output relationship of the OFDM-IM system can be expressed as

\[
y_g = H_g s_g + n_g,
\]

where \(y_g\) is the received signal cluster \(g\), \(H_g\) is the channel fading matrix whose diagonal elements \(|\text{diag}(h(1),...,h(n))|\) represent the channel frequency response of the sub-carriers. \(n_g\) is the AWGN noise vector for the \(g^{th}\) sub-block. \(x_g = [x_g(1), x_g(2), ..., x_g(n)]\) is the vector of transmitted symbols in which zeros and non-zero elements are present according to the indices of selected sub-carriers and modulation symbols.
The aforementioned ML detection process jointly detects both the index bits and the M-ary constellation bits. As a result, bit errors can occur in the detected message in three different ways as follows:

- bit errors resulting from erroneous detection of the indices of active subcarriers, while M-ary bits are correct,
- bit errors resulting from erroneous detection of the M-ary bits even though active carriers are correctly detected and
- bit errors occurring when both index and M-ary symbols are erroneously detected.

Once these bit errors are evaluated, the expression for the average probability of bit error of the OFDM-IM sub block can then be derived. This relates to the overall probability of bit errors of the whole system since each sub block is independent and all have similar Gaussian distribution.

### A. PERFORMANCE ANALYSIS FOR AWGN CHANNEL

Considering the OFDM-IM received vector given in Eq. (1), the fading channel diagonal matrix $\mathbf{H}$ is a unity diagonal matrix in the case of AWGN channels which means we can ignore $\mathbf{H}$ in our analysis. Consequently, amplitude of the subcarriers at the receiver will be independent and have similar Gaussian distribution with mean $\mathcal{E}\{y(i)\} = \{0 \, \text{for} \, i = \alpha\}$ and variance $\bar{N}/2$, where $\alpha$ is the index of inactive subcarrier, $\alpha$ is the index of active subcarrier, $x \rightarrow q$ an element of the set of M-ary symbols transmitted in the active subcarrier. In order to derive the general expression for the bit error performance, a single active per sub-block OFDM-IM system is considered first, then using the same analogy, the analysis is extended to a multi-active OFDM-IM system for error performance generalization.

#### 1) $(n,1,M)$

In this OFDM-IM system configuration, $k = 1$ is the number of active subcarrier per cluster, $n$ is the number of subcarriers in each cluster and $M$ is the size of the digital modulation used to modulate the active subcarrier (e.g $M=4$ for QPSK). From the observed vector at the receiver $y_g = [y_1, \ldots, y_n]$, the ML detector makes an ML decision on $I_g$ considering the joint distribution of $I_g, x_g$ as shown in Eq.(4). Recall that all subcarriers are received with addition of AWGN and have similar Gaussian distribution. Figure 2 is an illustration of the distribution of the amplitudes of the subcarriers as a function of transmitted constellation symbol showing a decision region for the index detection. In the diagram above, $\times$ marks on the four quadrants of the cartesian plane are the constellation points of the digital modulation of the system, which in this example have four possible values, and $y(\alpha)$ is the highest received amplitude of the subcarriers within the sub-block. The remaining amplitudes are within the shaded area which is bounded by the detection decision region as shown. Assuming $y(\alpha)$ is the received amplitude of the active subcarrier, given that the $n-1$ amplitude values of the inactive subcarriers are located within the shaded region, the probability of correctly detecting the index bits of the OFDM-IM sub-block can be determined by taking the integral of the vector $y_g$ such that $y(\alpha) = x + n(\alpha)$ and $y(\alpha_i) = n(\alpha_i)$ respectively. Let us denote the probability of correctly detecting the index bits as $p(C)$ which can also be evaluated as the conditional probability that $|y(\alpha)| > |y(\alpha_i)|$.

#### III. OPTIMUM RECEIVER

The optimum receiver for this system is the one based on the Maximum A Posteriori Probability (MAP). Now given that each received OFDM-IM sub-block ($y_g$) contains both index information and M-ary symbols, the detector selects the message ($m_j$) with the joint maximum a posteriori probability given by

$$
\hat{m}_g = (\hat{I}_g, \hat{x}_g) = \arg \max_{(I, x)} P(s_i | y_g, H_g)),
$$

where $\hat{m}_g$ is the estimated message $m_j$ of the $g^{th}$ sub-block such that $m_j \rightarrow s_i \in S$. By employing bayes’ theory [41], Eq. (2) can be expressed as

$$
\hat{m}_g = \arg \max_{(I, x)} \left\{ \frac{P(y_g | s_i, H_g)P(s_i)}{P(y_g)} \right\}.
$$

If the OFDM-IM vectors ($s_i$) have equal joint A Priori Probability (AP), which is the case in the example shown in Table (1), the receiver becomes equivalent to the (ML) detector given by the minimization function below

$$
\hat{m}_g = \arg \min_{(I, x)} \|y_g - H_g s_i\|^2.
$$

#### IV. ERROR ANALYSIS

The aforementioned ML detection process jointly detects both the index bits and the M-ary constellation bits. As a result, bit errors can occur in the detected message in three different ways as follows:

- bit errors resulting from erroneous detection of the indices of active subcarriers, while M-ary bits are correct,
- bit errors resulting from erroneous detection of the M-ary bits even though active carriers are correctly detected and
- bit errors occurring when both index and M-ary symbols

| Input bits | Index selector | QPSK symbol | OFDM-IM symbol |
|------------|---------------|-------------|----------------|
| 0 0 0 0    | 0             | $q_1$       | $x_1 0 0 0$    |
| 0 0 0 1    | 1             | $q_2$       | $x_2 0 0 0$    |
| 0 0 1 1    | 2             | $q_3$       | $x_3 0 0 0$    |
| 0 0 1 0    | 3             | $q_4$       | $x_4 0 0 0$    |
| 0 1 0 0    | 4             | $q_5$       | $x_5 0 0 0$    |
| 0 1 1 0    |               | $q_6$       | $x_6 0 0 0$    |
| 1 0 0 0    |               | $q_7$       | $x_7 0 0 0$    |
| 1 0 1 0    |               | $q_8$       | $x_8 0 0 0$    |
| 1 1 0 0    |               | $q_9$       | $x_9 0 0 0$    |
| 1 1 1 0    |               | $q_{10}$    | $x_{10} 0 0 0$ |
| 1 1 1 1    |               | $q_{11}$    | $x_{11} 0 0 0$ |
| 1 1 1 1    |               | $q_{12}$    | $x_{12} 0 0 0$ |
| 1 1 1 1    |               | $q_{13}$    | $x_{13} 0 0 0$ |
| 1 1 1 0    |               | $q_{14}$    | $x_{14} 0 0 0$ |

[36x131] Different ways as follows: the index bits and the M-ary constellation bits. As a result, bit errors can occur in the detected message in three different ways as follows:

- bit errors resulting from erroneous detection of the indices of active subcarriers, while M-ary bits are correct,
- bit errors resulting from erroneous detection of the M-ary bits even though active carriers are correctly detected and
- bit errors occurring when both index and M-ary symbols are erroneously detected.

Once these bit errors are evaluated, the expression for the average probability of bit error of the OFDM-IM sub block can then be derived. This relates to the overall probability of bit errors of the whole system since each sub block is independent and all have similar Gaussian distribution.

#### TABLE 1: 4,1,QPSK OFDM-IM Encoder

| Input bits | Index selector | QPSK symbol | OFDM-IM symbol |
|------------|---------------|-------------|----------------|
| 0 0 0 0    | 0             | $q_1$       | $x_1 0 0 0$    |
| 0 0 0 1    | 1             | $q_2$       | $x_2 0 0 0$    |
| 0 0 1 1    | 2             | $q_3$       | $x_3 0 0 0$    |
| 0 0 1 0    | 3             | $q_4$       | $x_4 0 0 0$    |
| 0 1 0 0    | 4             | $q_5$       | $x_5 0 0 0$    |
| 0 1 1 0    |               | $q_6$       | $x_6 0 0 0$    |
| 1 0 0 0    |               | $q_7$       | $x_7 0 0 0$    |
| 1 0 1 0    |               | $q_8$       | $x_8 0 0 0$    |
| 1 1 0 0    |               | $q_9$       | $x_9 0 0 0$    |
| 1 1 1 0    |               | $q_{10}$    | $x_{10} 0 0 0$ |
| 1 1 1 1    |               | $q_{11}$    | $x_{11} 0 0 0$ |
| 1 1 1 1    |               | $q_{12}$    | $x_{12} 0 0 0$ |
| 1 1 1 0    |               | $q_{13}$    | $x_{13} 0 0 0$ |

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∀\(\alpha\) and expressed as

\[
p(C) = P\left\{ |y(\alpha)| > 0, |y(\widehat{\alpha}_{i})|, ..., |y(\widehat{\alpha}_{i-1})| \right\},
\]

\[
|y(\widehat{\alpha}_{i})|, ..., |y(\widehat{\alpha}_{i-1})| \right\},
\]

\[
\left\{ \frac{1}{(2\pi)^{n/2} \sigma_{1} \cdots \sigma_{n}} \text{exp} \left( -\frac{\|y\|^{2}}{2} \right) \right\}, \quad (6)
\]

where \(d_{i} = |y(\alpha) - x(\alpha)|\) denotes the distance between the received amplitude and constellation symbol of the active carrier, which reduces to \(d_{i} = |y(\widehat{\alpha}_{i})|\) for the inactive carriers. The elements of \(y\) are complex variables hence the double integrals in Eq. (5). Substituting Eq. (6) in to Eq. (5), \(p(C_{1})\) becomes

\[
p(C) = \int_{0}^{2\pi} \int_{0}^{\pi} \left\{ \frac{1}{\pi N} \sum_{q=1}^{M} \frac{1}{\pi N} \right\} \times \left\{ \frac{1}{\pi N} \right\}
\]

\[
\text{exp} \left( -\frac{(y(\alpha) - \mu_{q})^{2}}{N} \right) \text{dadb} \right\} \times \left\{ \frac{1}{\pi N} \right\}
\]

\[
\exp \left( -\frac{(y(\alpha) - \mu_{q})^{2}}{N} \right) \text{dadb} \right\}, \quad (7)
\]

where \(\mu_{q}\) is the complex valued M-ary constellation symbol \(q\) on the cartesian coordinate. The multiple integral problem above which is based on cartesian constellation can be converted to a polar coordinate to simplify computation. The equivalent polar coordinate of the cartesian coordinate in Figure (2) is given in the Figure (3). With the amplitude of a received subcarrier being represented on a polar coordinate, the probability is computed by integrating over the entire area of the circle which gives

\[
p = \int_{0}^{2\pi} \int_{0}^{\pi} \left( \frac{1}{\pi N} \text{exp} \left( -\frac{r^{2}}{N} \right) \right) \times r \text{drd}\theta
\]

\[
= \frac{1}{\pi N} \left( -\frac{N}{2} \text{exp} \left( -\frac{r^{2}}{N} \right) \right)_{0}^{2\pi} \times 0
\]

\[
= 1 - \text{exp} \left( -\frac{r^{2}}{N} \right), \quad (8)
\]

Solving the inner integrals of Eq. (7) using Eq. (8), and then taking the limits of the amplitudes of the inactive subcarriers \(\lim_{i \to \infty} |y(\widehat{\alpha}_{i})| \to |y(\alpha)|\), ∀\(\alpha\), the \(p(C)\) of Eq. (7) can be expressed as

\[
p(C) = \int_{0}^{2\pi} \int_{0}^{\pi} \left\{ \frac{1}{\pi N} \sum_{q=1}^{M} \frac{1}{\pi N} \right\} \times \left\{ \frac{1}{\pi N} \right\}
\]

\[
\text{exp} \left( -\frac{(y(\alpha) - \mu_{q})^{2}}{N} \right) \text{dadb} \right\} \times \left\{ \frac{1}{\pi N} \right\}
\]

\[
\left( \frac{1}{(2\pi)^{n/2} \sigma_{1} \cdots \sigma_{n}} \text{exp} \left( -\frac{\|y\|^{2}}{2} \right) \right)_{n-1} \times \left( 1 - \text{exp} \left( -\frac{|y(\alpha)|^{2}}{N} \right) \right)
\]

\[
\times \left\{ \frac{1}{\pi N} \sum_{q=1}^{M} \left( \frac{1}{\pi N} \right) \exp \left( -\frac{(y(\alpha) - \mu_{q})^{2}}{N} \right) \text{dadb} \right\}. \quad (9)
\]
We can see in Eq. (9), that there is a binomial expansion problem, which can be solved by the binomial expansion theorem given by [42] as

\[(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k,\]  
\[\text{Eq. (10)}\]

and by substituting the binomial expansion expression into Eq. (9), we have

\[p(C) = \int \int \sum_{i=0}^{n-1} \binom{n}{i} \left( -\exp \left( \frac{|y(\alpha)|^2}{N} \right) \right)^i \times \left\{ \frac{1}{M} \sum_{q=1}^{M} \left( \frac{1}{\pi N} \right) \exp \left( -\frac{|y(\alpha) - \mu_q|^2}{N} \right) \right\} d\alpha d\beta.\]  
\[\text{Eq. (11)}\]

If \(k = 1\) and \(n = 2\), which is the case of a (2,1,M) OFDM-IM system, where only a single subcarrier is inactive in any carrier pair, the marginal probability of correctly detecting the index bits is given by (see appendix VI)

\[p(C_I) = 1 - \frac{1}{2M} \sum_{q=1}^{M} \exp \left( -\frac{|y(\alpha)|^2}{N} \right).\]  
\[\text{Eq. (12)}\]

In the case where \(k = 1\), \(n = 4\) for a (4,1,M) OFDM-IM system, (see appendix VI)

\[p(C_I) = 1 - \frac{3}{2M} \sum_{q=1}^{M} \exp \left( -\frac{9|y(\alpha)|^2}{2N} \right) + \frac{1}{M} \sum_{q=1}^{M} \left( \exp \left( -\frac{9|y(\alpha)|^2}{2N} \right) - \frac{1}{4M} \sum_{q=1}^{M} \exp \left( -\frac{9|y(\alpha)|^2}{2N} \right) \right)\]  
\[\text{Eq. (13)}\]

Given the expressions for (2,1,M) and (4,1,M) OFDM-IM systems in Eq. (12) and Eq. (13) respectively, it is evident that the marginal probability of correctly detecting the indices of the subcarriers is a sum of weighted exponentials, which weights are given by the coefficients of a binomial expansion represented by Pascal’s triangle shown in Table II. The expressions in Eq. (12) and (13) can be generalized for any arbitrary number of subcarriers with \(k = 1\) represented by (n,1,M) OFDM-IM systems as

\[p(C_I) = 1 - \frac{1}{nM} \sum_{q=1}^{M} \left\{ \sum_{i=1}^{K-2} \sum_{j=2}^{K-1} \sum_{m=3}^{K} (-1)^m A_n(m) \exp \left( -\frac{i(\mu_q)^2}{jN} \right) \right\},\]  
\[\text{Eq. (14)}\]

where \(A_n\) is a subset of \(K\) number of coefficients taken from the \(n^{th}\) row of the pascal’s triangle shown in Table 2, while \(i, j, m\) are counters, which relate to \(K\) coefficients.

2) \((n,k,M)\) OFDM-IM system for a multi-active carrier OFDM-IM system, where \(k > 1\), and assuming index symbol \(I_g\) which corresponds to \(k\) number of active subcarriers, the detection is performed by the ML detector on the received vector \(y = [y(\alpha_1), ..., y(\alpha_n)]\), the probability of correctly distinguishing between active and inactive carriers is the probability that \(y(\alpha_1) > |y(\alpha_2)| > |y(\alpha_3)| > \ldots > |y(\alpha_n-1)| > |y(\alpha_n)|\), which can be expressed as:

\[p(C_I) = p\left( |y(\alpha_1)| > 0, |y(\alpha_2)| > 0, ..., |y(\alpha_n-k)| > 0 \right) \times ... \]

\[= \int \int \int ... \int f_y(y) s_i da(\alpha_{n-1}) db(\alpha_{n-1}) ... da(\alpha_1) db(\alpha_1) \times \ldots \times a(\alpha_1) b(\alpha_1) a(\alpha_2) b(\alpha_2) \ldots a(\alpha_n-k) b(\alpha_n-k)\]  
\[\text{Eq. (15)}\]
carriers, and so on for all the \( k \) active carriers. With the same 
mathematic manipulations used in solving (n,1,M), we obtain 
the expression below

\[
p(C_I) = 1 - \frac{1}{nM} \sum_{q=1}^{M} \left\{ \sum_{m=2}^{K} \sum_{j=1}^{K-1} \sum_{i=0}^{K-2} (-1)^m A_n(m) \right\} \\
\exp \left( - \frac{i(\mu_q)^2}{jN} \right) + \\
\frac{1}{(n-1)M} \sum_{q=1}^{M} \left\{ \sum_{m=3}^{K} \sum_{j=2}^{K-2} \sum_{i=1}^{K-2} (-1)^j A_{n-1}(m) \right\} \\
\exp \left( - \frac{i(\mu_q)^2}{jN} \right) + \ldots + \\
\frac{1}{(n-k+1)M} \sum_{q=1}^{M} \left\{ \sum_{m=3}^{K} \sum_{j=2}^{K-2} \sum_{i=1}^{K-2} (-1)^j A_{n-1}(m) \right\} \\
\exp \left( - \frac{i(\mu_q)^2}{jN} \right) \times \\
\frac{1}{(n-1)M} \sum_{q=1}^{M} \left\{ \sum_{m=3}^{K} \sum_{j=2}^{K-2} \sum_{i=1}^{K-2} (-1)^j A_{n-1}(m) \right\} \\
\exp \left( - \frac{i(\mu_q)^2}{jN} \right) \times \ldots \times \\
\frac{1}{(n-k+1)M} \sum_{q=1}^{M} \left\{ \sum_{m=3}^{K} \sum_{j=2}^{K-2} \sum_{i=1}^{K-2} (-1)^j A_{n-1}(m) \right\} \\
\exp \left( - \frac{i(\mu_q)^2}{jN} \right) \bigg].
\]

This expression can be generalized as

\[
p(C_I) = 1 - \frac{1}{(n-\beta)M} \sum_{q=1}^{M} \left\{ \sum_{m=3}^{K} \sum_{j=2}^{K-2} \sum_{i=1}^{K-2} (-1)^m A_{n-\beta}(m) \right\} \\
\exp \left( - \frac{i(\mu_q)^2}{jN} \right) + \\
k^{\sum_{\beta=0}^{k-1} \frac{1}{(n-\beta)M} \sum_{q=1}^{M} \left\{ \sum_{m=3}^{K} \sum_{j=2}^{K-2} \sum_{i=1}^{K-2} (-1)^m A_{n-\beta}(m) \right\} \\
\exp \left( - \frac{i(\mu_q)^2}{jN} \right) \bigg].
\] (17)

With these generalized expressions for the probability of 
correctly distinguishing between subcarriers in Eq. (14) 
and Eq. (17) for (n,1,M) and (n,k,M) OFDM-IM systems 
respectively, the probability of index detection error \( P(e_I) \)
can be deduced as

\[
p(e_I) = 1 - p(C_I)
\] (18)

Given the probability of index error \( p(e_I) \) in Eq. (18), we 
can express the index BER \( P_b(e_I) \) for a single active sub-
carrier in the same manner as the BER of M-ary orthogonal 
signaling system [43] as,

\[
p_b(e_I) = \frac{L/2}{L-1} p(e_I),
\] (19)

where \( L = 2^{\log_2(n_k)} \). For a more accurate \( p_b(e_I) \), we 
introduce \( k \) in Eq. (19) to give expression for the \( p_b(e_I) \) for 
a system with \( k \) active subcarriers, and Eq. (19) becomes

\[
p_b(e_I) = \frac{L/2}{(L-1)k} p(e_I).
\] (20)

Now that we have established the probability of bit error 
distinguishing between the active and inactive carriers, the 
average probability of index bit error in any given OFDM-IM 
sub block is given as

\[
p_b = p_b(e_I) \times P(I),
\] (21)

where \( P(I) \) is the apriori probability of Index bits given as

\[
p(I) = \frac{\log_2\left(\binom{n_k}{k}\right)}{\log_2\left(\binom{n}{k}\right) + k\log_2 M}.
\] (22)

Having derived the bits error in the index part of the OFDM-
IM symbol, we are left with the error that can occur in the 
M-ary symbol/symbols conveyed within the active subcar-
carrier/subcarriers either due to error in the index detection or 
due to the intrinsic error of the particular choice of M-ary 
modulation. Let us denote this conditional M-ary symbol 
error as \( p(e_M) \). Although the probability of bits error in 
detecting M-ary digital modulated signal is well established 
in the literature [44] and given below for QPSK and 16-
QAM, we need to take into account the errors due to index 
bits to determine the overall M-ary errors since they are 
caused by the same channel. Below are bits error expressions 
for QPSK and 16-QAM.

\[
p_b(e_{QPSK}) = Q\left\{ \sqrt{\frac{2E_b}{N_0}} \right\}
\]

\[
p_b(e_{16-QAM}) = \frac{1}{4} Q\left\{ \sqrt{\frac{36E_b}{5N_0}} \right\} + \frac{3}{4} Q\left\{ \sqrt{\frac{12E_b}{15N_0}} \right\}.
\] (23)

Lets denote the conditional probabilities of M-ary error as 
\( p(e_{M1}) \) and \( p(e_{M2}) \) being M-ary bits error when Index error 
occurrents and when index error does not occur respectively. 
When index error occurs, \( p(e_{M1}) \) will be given as the product 
of both errors occurring in the index bits as well as the M-
ary bits. But because the errors are caused by the same 
channel, we assume that given that error occurs in index bits, 
the probability of bit error in M-ary symbol will be given as 
\( \frac{1}{2} \), which means the bit is either correctly detected or
wrongly detected. Therefore, the average conditional probability \( p(e_{M1}) \) is given as
\[
p(e_{M1}) = \tilde{P}_b \times \frac{1}{2}.
\] (24)

When index bits are correctly detected, \( p(e_{M2}) \), which is also a conditional probability of M-ary bits error, can be computed as a product of the probability of correctly detecting the index bits and the probability of bits error of the M-ary symbols.
\[
p(e_{M2}) = (1 - p(e_1)) \times p_b(e_{M-ary}).
\] (25)

The average conditional probability \( p(\epsilon_{M2}) \) can be expressed as
\[
\bar{p}(e_{M2}) = p(e_{M2}) \times p(M),
\] (26)
where \( p(M) \) is the a priori probability of M-ary bits given as
\[
p(M) = \frac{k \log_2 M}{\log_2 M} + k \log_2 M.
\] (27)

Now that we have established the component errors likely to occur in any of the OFDM-IM sub block, the average bit error probability of the OFDM-IM system in AWGN can be expressed as the sum of all average bit errors, which is given as
\[
p_b(e_{AWGN}) = \bar{p}_b + \bar{p}(e_{M2}) + \bar{p}(e_{M2}).
\] (28)

By substituting Eqs. (19), (21), (24), (25) and (26) into Eq. (27), the generalized expression for the BER of OFDM-IM system in AWGN channel is given as
\[
p_b(e_{AWGN}) = p(e_1)\left[\frac{3n}{2}p(I) - p(e_{M-ary})p(M)\right] + p(e_{M-ary})p(M),
\] (29)
where \( \eta = \frac{L/2}{(L^2 - 1)} \). By substituting Eqs. (17) and (18) into Eq. (29), the generalized expression can be written as weighted sum of exponentials and Q-functions presented in Eq. 30, where \( \mu = \frac{1}{2} \) for \( k = 1 \) and \( \mu = 1 \) for \( k > 1 \).

From the generalized BER expression in Eq. (30), we can write out the closed form tight BER expression for any arbitrary OFDM-IM system configuration. For example, for (n,k,M-ary)=(2,1,16-QAM) system, the closed form tight BER expression is obtained as
\[
p_{AWGN}(E) = \left(\frac{5}{2}e^{-(\frac{\eta}{2})} + \frac{4}{3}e^{-(\frac{s}{2})} + \frac{3}{2}e^{-(\frac{\eta}{2})} + \frac{1}{2}e^{-(\frac{\eta}{2})}\right)
\times \left(\frac{5}{10} - \left(\frac{1}{5}Q\left(\sqrt{\frac{9\gamma}{5}}\right) + \frac{3}{5}Q\left(\sqrt{\frac{3\gamma}{15}}\right)\right)\right)
\times \left\{\frac{3}{5}Q\left(\sqrt{\frac{9\gamma}{5}}\right) + \frac{3}{5}Q\left(\sqrt{\frac{3\gamma}{15}}\right)\right\}.\] (31)

The closed form tight BER expression for (n,k,M-ary)=(4,2,4-QAM) system is also obtained as follows:
\[
p_{AWGN}(E) = \left(\frac{5}{2}e^{-(\frac{\eta}{2})} + \frac{4}{3}e^{-(\frac{s}{2})} + \frac{3}{2}e^{-(\frac{\eta}{2})} + \frac{1}{2}e^{-(\frac{\eta}{2})}\right)
\times \left(\frac{5}{10} - \left(\frac{1}{5}Q\left(\sqrt{\frac{9\gamma}{5}}\right) + \frac{3}{5}Q\left(\sqrt{\frac{3\gamma}{15}}\right)\right)\right)
\times \left\{\frac{3}{5}Q\left(\sqrt{\frac{9\gamma}{5}}\right) + \frac{3}{5}Q\left(\sqrt{\frac{3\gamma}{15}}\right)\right\}.\] (32)

Although we have considered (2,1,16-QAM) and (4,2,QPSK) as examples. The closed form tight expression can be obtained for combination of (n,1,M-ary) and (n,k,M-ary) systems from the generalized BER expression in Eq. (30).

B. PERFORMANCE ANALYSIS FOR FADING CHANNEL

The main difference between fading channels and AWGN is in the channel gains. While the AWGN has constant gain of \( \sqrt{\frac{N}{2}} \), the fading channel has a variable gain, which is random and characterized by a probability distribution function (PDF). Consequently, the average BER can be calculated by averaging the BER of instantaneous SNR over the distribution of SNR as:
\[
p_b = p_{AWGN}(E|\gamma)p_{\gamma}(\gamma)\delta\gamma
\] (33)
where \( \gamma \) is the instantaneous SNR given by \( \gamma = \frac{|h|^2E_s}{N_0} \) and \( h \) is the fading coefficient which is a random variable with a probability distribution function. Assuming that the envelope follows a Rayleigh distribution, the instantaneous power of the Rayleigh fading channel \( |h|^2 \) has a chi-square distribution with two degrees of freedom and can be expressed as
\[
f_{\gamma}(\gamma) = \frac{1}{\gamma} \exp\left(-\frac{\gamma}{\gamma}\right), \quad \gamma \geq 0,
\] (34)
where \( \tilde{\gamma} = E\{|h|^2\} = \frac{E_s}{N_0} \) is the average SNR of the system.

The closed form tight BER expressions for Rayleigh fading channel can be derived using Eq. (30). For example, consider a (n,1,M) system having \( n = 2, k = 1 \) and \( M = QPSK \), then the exact BER expression can be obtained as follows:

By evaluating the probability of bit error for (2,1,QPSK) in the AWGN channel using Eq. (30),
\[
p_{AWGN}(E|\gamma) = \frac{1}{3}\exp^{-\frac{\gamma}{2}} + \frac{2}{3}Q(\sqrt{\gamma}) - \frac{1}{3}Q(\sqrt{\gamma})\exp^{-\frac{\gamma}{2}}.
\] (35)

Substituting Eq. (35) and Eq. (34) into Eq. (33),
\[
p_b = \int_{0}^{\infty} \left(\frac{1}{3}\exp^{-\frac{\gamma}{2}} + \frac{2}{3}Q(\sqrt{\gamma}) - \frac{1}{3}Q(\sqrt{\gamma})\exp^{-\frac{\gamma}{2}}\right)
\times \left(\frac{1}{\gamma} \exp\left(-\frac{\gamma}{\gamma}\right)\right) d\gamma.
\] (36)
p_b = \frac{1}{\mu} \sum_{k=0}^{m-2} \left( \frac{1}{(n-\beta)M} \sum_{m=3}^{K} \sum_{j=2}^{K-1} \sum_{i=1}^{K-2} (-1)^{m} A_{n-\beta}(m) \exp \left( -\frac{i(p_{q})^2}{jN} \right) \right) + \prod_{\beta=0}^{k-1} \left( \frac{1}{(n-\beta)M} \sum_{m=3}^{K} \sum_{j=2}^{K-1} \sum_{i=1}^{K-2} (-1)^{m} A_{n-\beta}(m) \exp \left( -\frac{i(p_{q})^2}{jN} \right) \right) \times \left( \frac{3n}{2} p(M) + p_{1}(M-M_{arity})p(M) \right) + p_{2}(M-M_{arity})p(M). \quad (30)

Q-function can be expressed in terms of complementary error function \( (erfc) \) which is given as

\[ Q(x) = \frac{1}{2} \text{erfc}\left(\frac{x}{\sqrt{2}}\right). \quad (37) \]

By substitution,

\[ p_b = \int_{a}^{\infty} \frac{1}{3\gamma} \exp^{-\gamma/a_{(\bar{\gamma}+1)}} d\gamma + \int_{b}^{\infty} \frac{1}{3\gamma} \exp^{-\gamma/b_{(\bar{\gamma}+1)}} d\gamma - \int_{c}^{\infty} \frac{1}{6\gamma} \exp^{-\gamma/c_{(\bar{\gamma}+1)}} d\gamma, \quad (38) \]

and integrating a, b and c, we have the following results

\[ a = \frac{2}{3(\bar{\gamma}+2)} \]
\[ b = \frac{1}{3} \left( 1 - \sqrt{\frac{\bar{\gamma}}{\bar{\gamma}+2}} \right) \]
\[ c = \frac{1}{6(\bar{\gamma}+2)} \left( 2 - \sqrt{\frac{2\bar{\gamma}}{\bar{\gamma}+1}} \right). \quad (39) \]

Substituting Eq. (39) into Eq. (35) yields

\[ p_b = \frac{2}{3(\bar{\gamma}+2)} + \frac{1}{3} \left( 1 - \sqrt{\frac{\bar{\gamma}}{\bar{\gamma}+2}} \right) - \frac{1}{6(\bar{\gamma}+2)} \left( 2 - \sqrt{\frac{2\bar{\gamma}}{\bar{\gamma}+1}} \right). \quad (40) \]

**V. RESULTS VALIDATION**

In this section, we validate our analytical expressions by comparing the BER results with numerical results of the ML detector. For the numerical simulations, we use OFDM-IM system with FFT size \( N = 64 \) and a CP length \( L = 12 \). The OFDM-IM system divides the total \( N \) subcarriers into \( G \) clusters of \( n \) subcarriers each, where \( n \in \{2,4\} \), \( k \in \{1,2,3\} \) and \( M \in \{4QAM,16QAM\} \). Each cluster of \( n \) subcarriers is received and detected independently using the ML criteria in the AWGN channel. For fading channel, we use the WI-FI channel model for the simulation of Rayleigh fading channel with 11 taps and a sufficient Monte-Carlo runs have been carried out to generate reliable error statistics.

Fig. 4 shows the BERs of both numerical simulation and analytical expressions for (2,1,16QAM), (4,1,4QAM) and (4,1,16QAM) systems in the AWGN channel. As seen in this figure, the results are accurate for a wide range of SNR, including the low SNR regions. Note that most of the OFDM-IM BER expressions in the literature depend on pair wise error analysis, which usually result in tight performance at high SNR region, our expressions model the closely tight BER performance across the whole SNR region. Fig 5 shows results for OFDM-IM systems with multi active subcarriers per sub-block. The results of our BER expressions also match very closely with the numerical simulations for up to a BER of \( 10^{-6} \). With this closely matching results, it is evident that when the decision boundaries over which errors can take place are properly established in deriving BER expressions, number of \( n \) or \( k \) will have a negligible effect on the accuracy of the expression. We also noticed that even though both index and M-ary errors contribute to the overall error of the system, index error affects the system more at low SNR where it is more prone to occur than at the high SNR region.
Even though the scope of our work is limited to AWGN channels, we show that our expression can be used to derive the expression for the performance of a fading channel. Fig. 6 depicts the BER performances (numerical and analytical) of a (2,1, QPSK) system in a Rayleigh fading channel with the ML detector. It can be seen to also closely match, which to a high degree of accuracy validates our generalized BER expression for OFDM-IM system with the ML detector in AWGN channel.

We evaluated the BER performance over fading channels by averaging the fading distribution across the entire length of the signals. An example of BER expression for (2,1, QPSK) in fading channel also shows close agreement with the numerical result.

**VI. CONCLUSION**

Using the decision region method, we derived an expression for the evaluation of BER of OFDM-IM system with ML detector in AWGN channel. Our general expression is a weighted sum and products of exponential functions, which gives a very tight performance, when compared to numerical simulations. By applying our expression to Rayleigh fading channel, closed form and tight expression for OFDM-IM systems in fading channels can be derived as well for the ML detector. Results for all the scenarios we tested are highly accurate across a wide range of SNR including the low SNR regions and up to a BER of 10^-6.

**APPENDIX**

**DERIVATION OF (12) AND (13)**

\[
p(C_l) = \frac{1}{2} \int_{-\infty}^{\infty} \left[ \exp \left( \frac{\alpha^2 + \beta^2}{\pi N} \right) \right] \left[ \exp \left( -\frac{(x - x_q)^2 + (y - y_q)^2}{N} \right) \right] \, dx \, dy,
\]

where \( \alpha, \beta \) are independent variables we can integrate them separately in both \( a \) and \( b \).

\[
a_x = \int_{-\infty}^{\infty} \exp \left( -\frac{(x - x_q)^2}{N} \right) \, dx,
\]

and

\[
b_y = \int_{-\infty}^{\infty} \exp \left( -\frac{(y - y_q)^2}{N} \right) \, dy.
\]
According to [45], special integral Gauss error function is given by

\[ erf(x) = \int \frac{2}{\sqrt{\pi}} \exp(-x^2) dx, \]  

(43)

By substitution, Eq. (42) becomes

\[ a_x = \sqrt{\frac{\pi}{N}} \exp\left[-\left(\frac{x-a}{\sqrt{N}}\right)^2\right], \]  

(44)

When we integrate \( a_y \) in the same manner as \( a_x \) and evaluating them together,

\[ a = \frac{\sqrt{\pi}N}{4} \int_{-\infty}^{\infty} \exp\left[-\left(\frac{x-a}{\sqrt{N}}\right)^2\right] dx, \]  

(45)

\[ b_x = \int_{-\infty}^{\infty} \exp\left[-\left(\frac{x-a}{\sqrt{N}}\right)^2\right] dx, \]  

(46)

Upon completing the squares for quadratic polynomial inside the exponential function, Eq. (46) becomes

\[ b_x = \int_{-\infty}^{\infty} \exp\left[-\left(\frac{\sqrt{2}x-a}{\sqrt{N}}\right)^2\right] dx, \]  

(47)

Substituting with equivalent special Gauss error function, Eq. (47) is equivalent to

\[ b_x = \frac{\sqrt{\pi}N}{2\sqrt{2}} \exp\left[-\left(\frac{x-a}{\sqrt{N}}\right)^2\right], \]  

(48)

Solving \( b_y \) in same manner and evaluating the overall \( b \), we have

\[ b = \frac{\pi N}{8} \int_{-\infty}^{\infty} \exp\left[-\left(\frac{2y-a}{\sqrt{N}}\right)^2\right] \exp\left[\frac{2y-a}{\sqrt{N}}\right] dy, \]  

(49)

By substituting \( a \) and \( b \) into Eq. (41), \( p(C_1) \) for a (2,1,M) OFDM-IM system is given as

\[ p(C_1) = \frac{1}{M \pi N} \sum_{q=1}^{M} \left( \alpha + b \right) \]  

(50)

To derive Eq. (13) which is for the (4,1,M) OFDM-IM system, the marginal probability of correctly detecting the index symbol is given as

\[ P(C_1) = \int_0^\infty \int_{-\infty}^{\infty} \left( \frac{3}{2} \left( - \exp\left[\frac{x^2+y^2}{N}\right] \right) + \frac{3}{4} \left( - \exp\left[\frac{x^2+y^2}{N}\right] \right) \right) \]  

(51)

Segmenting Eq. (51) like we did in Eq. (41) into

\[ \alpha = \int_0^\infty \int_{-\infty}^{\infty} \exp\left(-\left(\frac{x-a}{\sqrt{N}}\right)^2 + \left(\frac{y-a}{\sqrt{N}}\right)^2\right) dx \]  

(52)

When we apply integration in the same way as Eq. (45) and Eq. (49) to Eq. (52), \( a \), \( b \), \( c \) and \( d \) can be expressed in a closed and compact form as
$$a = \pi N$$
$$b = \frac{3}{2} \pi N \exp \left(-\frac{\mu_2}{2N}\right)$$
$$c = \pi N \exp \left(-\frac{2\mu_2}{3N}\right)$$
$$d = \frac{1}{\pi} \pi N \exp \left(-\frac{3\mu_2}{4N}\right),$$

(53)

In the same way as Eq. (50), the marginal probability of correctly distinguishing between the carriers is given by

$$p(C_1) = \frac{1}{M \pi N} \sum_{q=1}^{M} \left( a + b + c + d \right)$$

$$= \frac{1}{M \pi N} \sum_{q=1}^{M} \left( \pi N - \frac{3\pi N}{2} \exp \left(-\frac{2}{\pi N}\right) \right)$$

$$\pi N \exp \left(-\frac{3\pi N}{2} \exp \left(-\frac{3\pi N}{2}\right)\right)$$

$$= 1 - \frac{3}{2M} \sum_{q=1}^{M} \left( \exp \left(-\frac{2}{\pi N}\right) \right)$$

$$\frac{1}{M} \sum_{q=1}^{M} \exp \left(-\frac{2}{\pi N}\right) - \frac{1}{4M} \sum_{q=1}^{M} \exp \left(-\frac{3\pi N}{2}\right)$$

(54)

REFERENCES

[1] G. Zhang, M. De Leenheer, A. Morea, and B. Mukherjee, “A survey on OFDM-based elastic core optical networking,” IEEE Communications Surveys & Tutorials, vol. 15, no. 1, pp. 65–87, 2012.

[2] J. Mao, M. A. Abdullahi, P. Xiao, and A. Cao, “A low complexity 256QAM soft demapper for 5G mobile systems,” in 2016 European Conference on Networks and Communications (EuCNC). IEEE, 2016, pp. 16–21.

[3] M. Abdullahi and P. Xiao, “Performance analysis and soft demapping for coded MIMO-OFDM systems,” in International Symposium on Wireless Communication Systems (ISWCS). IEEE, 2016, pp. 242–246.

[4] T. Hwang, C. Yang, G. Wu, S. Li, and G. Y. Li, “OFDM and its wireless applications: A survey,” IEEE transactions on Vehicular Technology, vol. 58, no. 4, pp. 1673–1694, 2008.

[5] T. Jiang and Y. Wu, “An overview: Peak-to-average power ratio reduction techniques for OFDM signals,” IEEE Transactions on broadcasting, vol. 54, no. 2, pp. 257–268, 2008.

[6] P. K. Frenger and N. A. B. Svensson, “Parallel combinatory OFDM signaling,” IEEE transactions on communications, vol. 47, no. 4, pp. 558–567, 1999.

[7] S. Sasaki, J. Zhu, and G. Marubayashi, “Performance of parallel combinatorial spread spectrum multiple access communication systems,” in IEEE International Symposium on Personal, Indoor and Mobile Radio Communications. IEEE, 1991, pp. 204–208.

[8] R. Y. Mesleh, H. Haas, S. Sinanovic, C. W. Ahn, and S. Yun, “Spatial modulation,” IEEE Transactions on vehicular technology, vol. 57, no. 4, pp. 2228–2241, 2008.

[9] R. Abu-Alhiga and H. Haas, “Subcarrier-index modulation OFDM,” in 2009 IEEE 20th International Symposium on Personal, Indoor and Mobile Radio Communications. IEEE, 2009, pp. 177–181.

[10] C.-X. Wang, F. Haider, X. Gao, X.-H. You, Y. Yang, D. Yuan, H. M. Aggoune, H. Haas, S. Fletcher, and E. Hepsaydir, “Cellular architecture and key technologies for 5G wireless communication networks,” IEEE communications magazine, vol. 52, no. 2, pp. 122–130, 2014.

[11] E. Basar, M. Wen, R. Mesleh, M. Di Renzo, Y. Xiao, and H. Haas, “Index modulation techniques for next-generation wireless networks,” IEEE Access, vol. 5, pp. 16693–16746, 2017.
[37] Y. Ko, “A tight upper bound on bit error rate of joint OFDM and multi-carrier index keying,” IEEE Communications Letters, vol. 18, no. 10, pp. 1763–1766, 2014.

[38] T. Van Luong and Y. Ko, “Impact of CSI uncertainty on MCIK-OFDM: Tight closed-form symbol error probability analysis,” IEEE Transactions on Vehicular Technology, vol. 67, no. 2, pp. 1272–1279, 2017.

[39] ——, “The BER analysis of MRC-aided greedy detection for OFDM-IM in presence of uncertain CSI,” IEEE Wireless Communications Letters, vol. 7, no. 4, pp. 566–569, 2018.

[40] Q. He and A. Schmeink, “A Better Decision Rule for OFDM with Subcarrier Index Modulation,” in WSA 2017; 21th International ITG Workshop on Smart Antennas. VDE, 2017, pp. 1–4.

[41] M. K. Simon, S. M. Hinedi, and W. C. Lindsey, Digital communication techniques: signal design and detection. Prentice Hall PTR, 1995.

[42] J. G. Proakis and M. Salehi, Digital communications. McGraw-hill New York, 2001, vol. 4.

[43] J. F. Paris, E. Martos-Naya, and U. Fernández-Plazaola, “Exact BER analysis of M-ary orthogonal signaling with MRC over Ricean fading channels,” International Journal of Communication Systems, vol. 21, no. 4, pp. 447–452, 2008.

[44] S. Bernard, “Digital communications: fundamentals and applications,” Prentice Hall, USA, 2001.

[45] G. R. Cooper and C. D. McGillem, Probabilistic Methods of Signal and System Analysis (The Oxford Series in Electrical and Computer Engineering). Oxford University Press, 1995.