Mapping dark matter and finding filaments: calibration of lensing analysis techniques on simulated data

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Accepted —. Received —; in original form 19 June 2020

ABSTRACT

We quantify the performance of mass mapping techniques on mock imaging and gravitational lensing data of galaxy clusters. The optimum method depends upon the scientific goal. We assess measurements of clusters’ radial density profiles, departures from sphericity, and their filamentary attachment to the cosmic web. We find that mass maps produced by direct (KS93) inversion of shear measurements are unbiased, and that their noise can be suppressed via filtering with MRLens. Forward-fitting techniques, such as LENSTOOL, suppress noise further, but at a cost of biased ellipticity in the cluster core and over-estimation of mass at large radii. Interestingly, current searches for filaments are noise-limited by the intrinsic shapes of weakly lensed galaxies, rather than by the projection of line-of-sight structures. Therefore, space-based or balloon-based imaging surveys that resolve a high density of lensed galaxies, could soon detect one or two filaments around most clusters.

Key words: galaxies: clusters: general — large-scale structure of Universe — gravitational lensing: weak — techniques: image processing

1 INTRODUCTION

The ΛCDM standard model of cosmology suggests that structures in the Universe formed hierarchically, via mergers of small over-densities in the early Universe into larger and larger objects (White & Rees 1978; Springel et al. 2005; Schaye et al. 2015). Thirteen billion years after the Big Bang, the largest objects are currently clusters of hundreds or thousands of galaxies. Because their growth has spanned the entire age of the Universe, and has depended upon the density of building material and its collapse under gravity, versus its disruption by supernovae, active galactic nuclei, and dark energy, measurements of the precise number and properties of clusters is a highly sensitive test of the standard cosmological model (e.g. Bahcall & Cen 1993; Bahcall & Bode 2003; Ho et al. 2006; Rozo et al. 2010; Weinberg et al. 2013; Jauzac et al. 2016; Schwinn et al. 2017; Mao et al. 2018; Fluri et al. 2019).

Gravitational lensing is particularly efficient at investigating clusters. The dense concentration of mass in a foreground galaxy cluster deflects light rays emitted by unrelated galaxies far in the background. Since adjacent light rays are almost coherently deflected, the shapes of those distant galaxies appear distorted, and typically stretched in such a way that their long axes make circular patterns around the cluster. Crucially, the deflection of light rays depends only upon the total projected mass distribution. Measurements of gravitational lensing are therefore uniquely sensitive to the distribution of invisible-but-dominant dark matter, and unbiased by the nature and dynamical state of ordinary matter (e.g. Hoekstra 2013; Massey et al. 2010; Kneib & Natarajan 2011; Treu & Ellis 2015; Kilbinger 2015; Bartelmann & Maturi 2017).

Ground-based observations of gravitational lensing by galaxy clusters have been successfully used to measure clusters’ average or bulk properties, such as mass (e.g. von der Linden et al. 2014; Umetsu et al. 2014; Okabe & Smith 2016; Medezinski et al. 2017; Sereno et al. 2017; Schrabback et al. 2018; McClintock et al. 2019; Miyatake et al. 2019; Rehmann et al. 2019; Umetsu et al. 2019; Herbonnet et al. 2019), and ellipticity (e.g. Evans & Bridle 2009; Oguri et al. 2010; Clampitt & Jain 2016; van Uitert et al. 2017; Shin et al. 2018; Umetsu et al. 2018; Chiu et al. 2018).

The CLASH survey (Cluster Lensing and Supernova Survey

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with Hubble; Postman et al. 2012) measured the mass and concentration of 25 clusters, by combining wide-field Subaru imaging with Hubble Space Telescope (HST) imaging of the cluster cores (Merten et al. 2015). However, ground-based observations have yielded only marginally significant detections of filaments (e.g. Clowe et al. 2006; Kaiser et al. 1998; Gray et al. 2002; Gavazzi et al. 2004; Dietrich et al. 2012; Martinet et al. 2016), whose dark matter density is too low (and the filaments too narrow to resolve).

Space-based imaging reveals the shapes of more background galaxies, and increases the S/N of lensing measurements in multiple resolution elements across an individual cluster. Thus the shape and morphology of individual mass distributions can be precisely mapped, without the need to average out features over a population of clusters. Space-based lensing reconstructions have resolved substructure near cluster cores (e.g. Merten et al. 2011; Natarajan et al. 2017); bimodality even in relatively distant clusters like the ‘Bullet Cluster’ (Bradac et al. 2006) or ‘El Gordo’ (Jee et al. 2014); and filaments in Abell 901/902 (Heymans et al. 2008) and MACS J0717+3745 (Jauzac et al. 2012). Nonetheless, these analyses remain rare because the ~3’ × 3’ field of view of HST’s Advanced Camera for Surveys (ACS) is smaller than a typical cluster’s angular size. Furthermore, both of HST’s contiguous surveys (GOODS and COSMOS) un luckily sampled regions of the Universe that are underdense at the z = 0.2–0.4 redshifts where lensing is most sensitive (Heymans et al. 2005; Massey et al. 2007a; Krolewski et al. 2018), so happen to contain few lensing clusters (Guzzo et al. 2007; Massey et al. 2007b). Until recently, only around one cluster, MS 0451-03, had a dedicated wide-field mosaic of contiguous HST imaging had been obtained (Moran et al. 2007).

There will soon be wide-field, space-resolution imaging taken around 6 more clusters through the HST/BUFFALO survey (Steinhardt et al. 2018), 200 more clusters from the balloon-borne telescope SuperBIT (Romualdez et al. 2016; Redmond et al. 2018), and 10,000 from Euclid (Laureijs et al. 2011). In the next decade, 40,000 clusters will be observed to even greater depth by WFIRST (Wide Field Infrared Survey Telescope; Spergel et al. 2013).

The intent of this work is to prepare for future observations, much as Van Waerbeke et al. (2013) calibrated mass mapping methods for the current generation of wide-field ground-based lensing surveys. We use mock space-based weak-lensing data to develop and quantify the performance of two different methods to map dark matter around galaxy clusters, to measure deviations from sphericity, and to search for filaments connecting it with the cosmic web. Where we must make decisions about general properties (e.g. distance, mass) of clusters that we simulate, we shall use MS 0451-03 as a template, so our predictions can be immediately tested on real observations (see our companion paper, Tam et al. submitted).

This paper is organised as follows. We summarise background theory in Section 2, and introduce the simulated data in Section 3. In the context of various scientific motivations, we describe weak-lensing mass mapping and analysis techniques in Section 4. We quantify their results in Section 5, and conclude in Section 6. Throughout the paper, we define angular diameter distances assuming a background cosmology with $\Omega_m = 0.287$, $\Omega_\Lambda = 0.713$, and $\Omega_k = 1 - \Omega_m - \Omega_\Lambda = 0$, $\gamma = \Omega_\Lambda/\Omega_0 = 0.731$, and $\kappa = H_0/100\text{km}\text{s}^{-1}\text{Mpc}^{-1} = 0.693$ (WMAP 9-year cosmology; Hinshaw et al. 2013). All magnitudes are quoted in the AB system.

## 2 Weak Gravitational Lensing Theory

### 2.1 Coherent deflection of light rays

Gravitational lensing is the deflection of light rays from a distant source, by massive objects along our line of sight. The apparent shape of the source becomes distorted when a bundle of light rays from it are coherently distorted. Because cosmological distances are so large, the 3D distribution of intervening mass can be conveniently represented (through the ‘thin lens’ approximation) as a 2D surface density, $\Sigma(R)$, where $R = (x, y)$ is the 2D angular position in the plane of the sky. A similar projection can be applied to obtain a 2D effective gravitational potential $\varphi(R)$. The angle through which light rays are deflected corresponds to spatial derivatives in the gravitational potential.

In the weak-lensing regime, where deflection angles are small, the image distortions can be split into two dominant components. The first is an isotropic magnification, by a factor proportional to the projected density and known as ‘convergence’

$$\kappa(R) = \frac{\Sigma(R)}{\Sigma_c},$$

where the ‘critical density’

$$\Sigma_c = \frac{\Omega_m}{4\pi G} \frac{D_s}{D_{ls} D_{ls}},$$

depends upon the angular diameter distances from the observer to the lens, $D_l$, from the observer to the source, $D_s$, and from the lens to the source, $D_{ls}$. The lensing sensitivity function, $b(z_s, z_l) = D_{ls}/D_s$, describes the lensing strength as a function of the lens and source redshifts $(z_l, z_s)$. For a foreground galaxy with $z_s < z_l$, $b(z_l, z_s) = 0$. The second component of the distortion is a shear

$$\gamma = \gamma_1 + i\gamma_2 = |\gamma|e^{i\phi_s},$$

where the real component, $\gamma_1$, represents elongation along the $x$ direction, and the complex component, $\gamma_2$, represents elongation at 45°.

An observable quantity, ‘reduced shear’

$$g = \gamma_1,$$

can be measured from the apparent shapes of galaxies. In the weak-lensing regime, it is typically true that $|\kappa| < 1$, hence $g \approx \gamma$. For more information, see e.g. Bartelmann & Maturi (2017).

### 2.2 Analytic mass distributions

In several places throughout this paper, we will approximate a mass distribution using one of two parametric models. The models are usually described in circularly symmetric form, $\Sigma(R)$ or $\varphi(R)$, but can be made elliptical by a coordinate transformation

$$|R|^2 = (x^2 \cos^2 \phi + y^2 \sin^2 \phi) + (y^2 \cos^2 \phi - x^2 \sin^2 \phi)/q, \quad 0 \leq \phi \leq 2\pi,$$

where $q = 1 - \Omega_\Lambda/\Omega_0 = 0.269$, and $\Omega_k = 1 - \Omega_m - \Omega_\Lambda = 0$. These mass distributions are so large, the 3D distribution of mass can be conveniently represented (through the ‘thin lens’ approximation) as a 2D surface density, $\Sigma(R)$, where $R = (x, y)$ is the 2D angular position in the plane of the sky. A similar projection can be applied to obtain a 2D effective gravitational potential $\varphi(R)$. The angle through which light rays are deflected corresponds to spatial derivatives in the gravitational potential.

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2.2.1 tPIEMD profile

Massive elliptical galaxies are empirically observed to have an approximately isothermal density distribution ($ρ ∝ r^{-2}$), and total mass proportional to the velocity distribution of their stars, $σ$. This would have an inconvenient mathematical singularity at the centre, which is removed in the truncated Pseudo-Isothermal Elliptical Mass Distribution (tPIEMD; Kassiola & Kovner 1993; Limousin et al. 2005; Eliasdóttir et al. 2007)

$$\rho_{\text{tPIEMD}} = \frac{ρ_0}{1 + r^2/r_0^2(1 + r^2/r_1^2)}.$$  (6)

This has constant density $ρ_0 = \frac{σ^2}{2Gr_0 + r_1}$.

inside core radius $r_c$ and has finite integrated mass because of the truncation at radius $r_t$. The projected two-dimensional mass distribution is

$$\Sigma_{\text{tPIEMD}}(r) = \frac{σ^2}{2G} \frac{r_t}{r_t - r_c} \left( \frac{1}{\sqrt{r^2 + r_c^2}} - \frac{1}{\sqrt{r^2 + r_t^2}} \right).$$  (8)

2.2.2 NFW profile

Numerical simulations suggest that the distribution of dark matter in isolated haloes forms a Navarro-Frenk-White (NFW; Navarro et al. 1996, 1997) profile

$$\rho_{\text{NFW}} = \frac{ρ_s}{(r/r_s)(1 + (r/r_s))^2}$$  (9)

where $ρ_s$ and $r_s$ are a characteristic density and radius. For any given cosmology and cluster redshift, this model can also be parameterized in terms of a concentration $c_{200} = r_{200}/r_s$, where $r_{200}$ is the 3D radius within which the mean enclosed density is equal to 200 times the critical density $ρ_c$ of the Universe, and halo mass $M_{200} = (4π/3)200ρ_c r_{200}^3$. The projected two-dimensional mass distribution (Bartelmann 1996) is

$$\Sigma_{\text{NFW}} = 2ρ_s r_s F(x),$$  (10)

where $x = r/r_s$ and

$$F(x) = \begin{cases} \frac{1}{x^2} \left( 1 - \frac{2}{x^2} \arctan \frac{x}{\sqrt{x^2 - 1}} \right) & \text{if } x > 1 \\ \frac{1}{3} & \text{if } x = 1 \\ \frac{1}{x^2} \left( 1 - \frac{2}{x^2} \arctan \frac{1}{x} \right) & \text{if } x < 1 \end{cases}.$$  (11)

3 DATA

We use N-body particle data from the BAHAMAS suite of cosmological simulations (McCarthy et al. 2017, 2018).

| Cluster | $M_{\text{FOF}}(10^{14}M_\odot)$ | $M_{200}(10^{14}M_\odot)$ |
|---------|-------------------------------|--------------------------|
| Cluster 1 | 27.7                         | 17.3                     |
| Cluster 2 | 17.9                         | 15.0                     |
| Cluster 3 | 17.8                         | 17.7                     |
| Cluster 4 | 16.6                         | 14.6                     |
| Cluster 5 | 14.3                         | 9.7                      |
| Cluster 6 | 13.3                         | 11.0                     |
| Cluster 7 | 12.9                         | 8.9                      |
| Cluster 8 | 11.1                         | 4.0                      |
| Cluster 9 | 9.4                          | 8.2                      |
| Cluster 10 | 9.3                          | 5.7                      |

Table 1. Masses of the 10 most massive clusters in the BAHAMAS simulations, which we use as mock data for this study. Columns list the friends-of-friends masses $M_{\text{FOF}}$, and overdensity mass $M_{200}$. These were run with different background cosmologies and implementations of sub-grid galaxy formation physics, and designed to test the impact of baryonic physics on large-scale structure tests of cosmology. For this paper, we use the version with a WMAP 9-year (Hinshaw et al. 2013) cosmology, and sub-grid feedback model that is calibrated to produce a good match to the observed stellar mass function, X-ray luminosities and gas fractions of galaxy clusters. This simulation occupies a periodic cubic volume, 400 $h^{-1}$ Mpc on a side, with dark matter and (initial) baryon particle masses of 5.5 $×$ 10$^8$ $M_\odot$ and 1.1 $×$ 10$^9$ $M_\odot$, respectively.

3.1 Distribution of mass in clusters

We extract the ten most massive clusters from the $z = 0.5$ simulation snapshot. We first use the friends-of-friends algorithm (FOF; More et al. 2011) to identify all matter overdensities. For each FOF group, we calculate $r_{200}$ and $M_{200}$, the total mass enclosed within this sphere. For the ten most massive clusters, which have 4 $×$ 10$^{14}$ $M_\odot < M_{200} < 2 $×$ 10^{15} M_\odot$, we store the 3D distribution of dark matter, stars and gas.

To generate a 2D, pixellated convergence map, we follow the method of Robertson et al. (2018). In summary, we project the location of all simulation particles within 5$r_{200}$ of the centre of a cluster along a line of sight (here, the simulation $z$-axis). In a 25$×$25 Mpc (2048$×$2048 pixel) map centred on the most bound particle, we use an adaptive triangular shaped cloud scheme to smooth each particle’s mass over a kernel whose size depends on the 3D distance to that particle’s 32nd nearest neighbour. Resulting convergence maps are shown in figure 1, adopting the lens redshift $z_l = 0.55$ of galaxy cluster MS0451-03 as a concrete example, and source redshift $z_s = 0.97$ typical of $HST$ observations to single-orbit depth (Leauthaud et al. 2007). The masses of the clusters are listed in Table 1.

Before proceeding further, we identify 40 filaments in the ten projected mass maps, defined as radially extended regions with convergence 0.005 < $κ$ < 0.01, which is equivalent to a surface density of 1.7$×$10$^7$ < $Σ(M_\odot/kpc^2)$ < 3.4$×$10$^7$. These are indicated by white dashed lines in the bottom panel of figure 1.
Figure 1. Noise-free maps of the total mass distribution in the ten most massive clusters of the BAHAMAS simulations, projected along a randomly-oriented line of sight. Clusters have masses $M_{200}$ from $2 \times 10^{15} M_{\odot}$ (cluster 1) to $4 \times 10^{14} M_{\odot}$ (cluster 10), and are sorted in descending order of $M_{\text{FOF}}$, as in Table 1. Colours show the lensing convergence $\kappa$ (Top panel: linear scale; Bottom panel: logarithmic scale). Dotted white lines show filaments identified from the noise-free, projected mass distribution, above density thresholds defined in section 3.1. For reference, red lines indicate the field of view in which HST observations exist for real cluster MS 0451-03.

### 3.2 Distribution of all other mass along a line of sight

In addition to the mass of the galaxy cluster itself, we also account for large-scale structure (LSS) projected by chance along the same line of sight. This is a source of noise in the projected mass of the cluster, which is then added to the mock data in section 3.4.

To quantify the expected level of noise, we generate realisations of LSS along 1000 random lines of sight through the BAHAMAS simulation box. We then integrate the 3D mass along the line of sight, weighted by the lensing sensitivity function $\beta(z)$ with $\langle z_s \rangle = 0.97$, interpreting it as a mass distribution in a single lens plane at $z_l = 0.55$. For each realisation of LSS, we calculate an effective radial density profile, $\kappa(R)$. The mean of these realisations is (unsurprisingly) consistent with zero; we also calculate the rms scatter $\sigma_{\text{LSS}}$. In concentric annuli of width $\Delta R = 25''$, these are well-fit by

$$\sigma_{\text{LSS}}(R) = \frac{A}{\sqrt{R(\text{arcsec})}} + B,$$

with best-fit values for free parameters

$$A = 0.197 \pm 0.008, \quad B = 6.441 \pm 0.502.$$

We add this in quadrature to the statistical uncertainty on the reconstructed density profiles in Sect. 4.2. Note that it would also be possible to compute the full covariance matrix between LSS at different radii or in adjacent pixels of a mass map. Here we use only the diagonal elements, but in our companion paper (Tam et al., sub.), we fit to real observations using the full covariance matrix.

### 3.3 Mock near-IR imaging

To generate a mock catalogue of the cluster galaxies’ $K$-band magnitudes, we run SUBFIND algorithm (Springel et al., 2005).
on the particle distribution from the simulations, to identify individual galaxies. We sum their stellar masses, and convert these to $K$-band luminosity based on the relation presented by Arnouts et al. (2007) for the evolution of stellar mass to light ratio, $(M/L_K)$, with redshift for a sample of quiescent galaxies, and based on the Salpeter (1955) initial mass function. The power-law fitting function is defined as

$$\log_{10}(M/L_K) = az + b,$$

where the mass $M$ and luminosity $L_K$ are in units of $M_\odot$ and $L_\odot$, respectively. The best-fit value for parameters $a$ and $b$ from Arnouts et al. (2007) are

$$a = -0.18 \pm 0.04, \quad b = +0.07 \pm 0.04.$$ (15)

### 3.4 Mock weak-lensing shears

To generate mock weak-lensing observations, we convert the mass distributions into reduced shear. For the case with projected LSS, we sum the effective convergence from the cluster (section 3.1) and a random realisation of projected LSS (section 3.2). Since both convergence $\kappa(R)$ and shear $\gamma(R)$ fields are linear combinations of second derivatives of $\phi(R)$, it is possible to directly convert between their Fourier transforms $\hat{\kappa}(k)$ and $\hat{\gamma}(k)$

$$\hat{\gamma}_1(k) = \frac{k_1^2 - k_2^2}{k_1^2 + k_2^2} \hat{\kappa}(k)$$

$$\hat{\gamma}_2(k) = \frac{2k_1k_2}{k_1^2 + k_2^2} \hat{\kappa}(k),$$

where $k = (k_1, k_2)$ is the wave vector conjugate to $R$ (Kaiser & Squires 1993, hereafter KS93). To implement this in practice, we pixelate the fields within a $34\arcmin \times 34\arcmin$ ($2048 \times 2048$ pixel) grid, add zero padding to twice that linear size to mitigate boundary effects, then use discrete Fourier transforms. We finally use eq. 4 to convert shear $\gamma(R)$ into reduced shear $g(R)$.

We generate a mock shear catalogue by randomly placing source galaxies throughout the high-resolution pixelated shear field. Mimicking typical single-orbit depth HST observations, we sample 50 arcmin$^{-2}$ source galaxies. Note that we achieve a uniform density of background galaxies; in real observations, the number density of background galaxies is both clustered, and dips near the centre of a cluster because of obscuration by, and confusion with, its member galaxies. To each shear value, we add Gaussian random noise with width $\sigma_\gamma = 0.36$, representing each galaxy’s unknown intrinsic shape, plus uncertainty in shape measurement. This value matches that measured in HST measurements near MS0451-03 (Tam et al., sub.), and is consistent with that measured for faint galaxies in the HST COSMOS field (see figure 17 in Leauthaud et al. 2007). It is slightly larger than the intrinsic shape noise referenced elsewhere, because it also includes measurement noise.

## 4 METHODS

In this section, we describe several methods that have been used (or suggested) to analyse the distribution of mass in clusters. A common theme will be the suppression of noise — the two main sources of which are projected LSS, and galaxies’ intrinsic shapes. In particular, sophisticated non-linear noise-suppression techniques have been developed to map the 2D distribution of mass. Even for measurements that could be obtained directly from the shear field, it may therefore be efficient to first infer (and suppress noise in) a mass map, then to measure equivalent quantities from that.

### 4.1 Mass mapping

We start by exploring two frequently used methods to reconstruct the distribution of lensing mass: one frequentist, the second Bayesian. Where relevant, we adopt parameters in the methods that are typically used by their protagonists.

#### 4.1.1 Direct inversion with KS93+MRlens

Under the weak-lensing approximation $\mathbf{g} = \mathbf{\gamma}$, the KS93 Fourier space relation (see Sect. 3.4) can also be used to convert $\mathbf{\gamma}(\mathbf{R})$ into

$$\hat{\kappa}(k) = \frac{1}{2} \left( \frac{k_1^2 - k_2^2}{k_1^2 + k_2^2} \right) \hat{\gamma}_1(k) + \frac{1}{2} \left( \frac{2k_1k_2}{k_1^2 + k_2^2} \right) \hat{\gamma}_2(k).$$

This is a non-local mapping. In observations of the real Universe, any missing shear values (e.g. outside the survey boundary or behind bright stars) must be replaced via ‘inpainting’ (Pires et al. 2009; Raghunathan et al. 2019) to avoid suppressing the convergence signal inferred nearby. We avoid this effect by using a mock shear catalogue that is contiguous and covers a larger area ($34\arcmin \times 34\arcmin$) than the mosaicked HST imaging of MS0451-03. We bin the shear field $\mathbf{\gamma}(\mathbf{R})$ into 0.4$\arcmin$ pixels, add zero padding out to 105$\arcmin \times 105\arcmin$ (Merten et al. 2009; Umetsu et al. 2015), and implement eq. (18) using discrete Fourier transforms.

Noise was suppressed in early incarnations of KS93 by convolving the mass distribution with a larger smoothing kernel whilst in Fourier space. We omit this step, and instead filter the final convergence map using the Multi-Resolution method for gravitational Lensing (MRLENS; Starck et al. 2006). This decomposes an image into multiscale starlet wavelets, and applies non-linear regularisation on each wavelet scale. It aims to retain statistically significant signal but suppress noise through an approach that, under the assumption of a multiscale entropy prior, optimises the False Discovery Ratio (FDR) of false detections to true detections. Starck et al. (2006) show that MRLENS outperforms Gaussian or Wiener filtering at this task, and Pires et al. (2010) demonstrate specifically that it improves the reconstruction of non-Gaussian structures like the distribution of mass in galaxy clusters. The software implementation has various free parameters: we use ten iterations during the filtering process, and decompose the noisy 2D convergence map into six wavelet scales, starting at $j = 3$. These have size $\theta = 2^j$ pixels. For a starlet wavelet (eq (11) of Leonard et al. 2012), the $j = 3$ (highest resolution) wavelet is a Mexican hat with full width at half maximum (FWHM) of 0.5$\arcmin$.

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1 We implement MRLENS using the 2017 June 26 version of software available from https://www.cosmostat.org/software/mrlens. Note that a 3D extension of this method has also been developed, known as GLIMPSE (Leonard et al. 2015).
For comparison to older analyses, we also repeat the analysis after smoothing and rebinning the shear field into larger, $1\arcmin$ pixels.

4.1.2 Forward fitting with Lenstool

We also use LENSTOOL\(^2\) (Jullo & Kneib 2009) to fit the reduced shear catalogues $g(R)$ with a sum of analytic mass distributions. The field of view considered is the same size as the mosaicked HST imaging around MS 0451-03. Jullo & Kneib (2009) advocate a mass model built of three components.

- **Cluster-scale halo:** For clusters that produce strong gravitational lensing, the observed positions of multiple images are typically used to pre-fit the smooth, large-scale distribution of mass (Kneib et al. 1996; Smith et al. 2005; Richard et al. 2011; Jauzac et al. 2015b). Like many clusters, our mock data do not include strong-lensing, so we omit this component. Note that our performance forecasts will therefore be conservative, because this information efficiently captures the broad features of a mass distribution in only a few parameters, and removes degeneracies between the remaining parameters that we shall fit (Jauzac et al. 2015a).

- **Cluster member galaxies:** We model the total mass of each galaxy in the cluster as a tPIEMD (Eq. 8). Following Jauzac et al. (2012), their core radii, truncation radii and velocity dispersions are scaled using empirical relations

\[
\begin{align*}
    r_c &= r^*_c \left( \frac{L}{L^*} \right)^\frac{1}{2}, \\
    r_t &= r^*_t \left( \frac{L}{L^*} \right)^{\frac{1}{2}}, \\
    \sigma &= \sigma^* \left( \frac{L}{L^*} \right)^{\frac{1}{2}},
\end{align*}
\]

where $r_c = 0.15 \text{kpc}$, $r_t = 58 \text{kpc}$ and $\sigma^* = 163.10 \text{km} \text{s}^{-1}$ for a typical galaxy with $K$-band magnitude $m^* = 18.699$ at $z = 0.55$. These scaling relations describe early-type cluster galaxies (Wyts et al. 2004), and assume a constant mass-to-light ratio for all cluster members.

- **Multi-scale, free-form grid:** We add a free-form (pixelated) mass distribution with spatially-varying resolution that is adapted to the cluster’s light distribution. Following Jullo & Kneib (2009, figure 1), we initialise a grid of points by drawing a large hexagon over the entire field of view, split into six equilateral triangles with side length = 115". If a single pixel inside any of these triangles extends a predefined light-surface-density threshold, we split that triangle into four smaller triangles. This refinement continues for six levels of recursion, until the brightest parts of the cluster are covered by the highest resolution grid with $r_c = 18"$. We extend this grid into the cluster centre, which is inevitably modelled at the highest resolution. At the centre of every triangle, we place a circular ($g = 1$) tPIEMD (Eq. 8), with core radius $r_c$ set to the side length of the triangle, truncation radius $r_t = 3r_c$, and velocity dispersion that is free to vary. This process represents a prior that light-traces-where-mass-is, rather than explicitly light-traces-mass.

We optimise free parameters in this model using the MASSINF Markov Chain Monte Carlo algorithm. The parameter space is highly dimensional, so to optimise the multiscale grid, we adopt the Gibbs approach (Jullo et al. 2007), whereby the most discrepant masses are adjusted during each step of the Markov Chain and as a prior, the initial number of RBFs to explore is set to be 2% (Jauzac et al. 2012; Jullo et al. 2014a). We apply a prior that the masses are all positive. This need not necessarily be true, since we are really fitting departures from the mean density of the Universe; for example, the convergence of the LSS is consistent with fluctuations around zero (Sect. 3.2). However, the prior is frequently used, and reasonable near a galaxy cluster. We then finally compute the marginalised mean convergence, and its 68% confidence limits.

4.2 Radial density profiles

Most analyses of galaxy clusters involve fitting models of an azimuthally-averaged density profile. Measuring density profiles is a key test of cosmological structure formation (e.g. the ‘splashback’ feature reveals a characteristic build-up of accreted mass, pausing at first apocentre after first core passage Diemer & Kravtsov 2014) and the nature of dark matter (Newman et al. 2013, 2015; Robertson et al. 2019). Because almost all clusters have irregular features, and approximately half are significantly unrelaxed (Smith et al. 2010), it is necessary to statistically combine the profiles of many clusters. This can be achieved by rescaling and averaging their density profiles in radial bins, or by fitting parametric models with radial (or elliptical) symmetry, then averaging the best-fit parameters.

We calculate the radial density profiles of each simulated cluster by azimuthally averaging the reconstructed density maps within linearly spaced annuli of fixed width $\Delta R = 25''$. For LENSTOOL reconstructions, we quote the statistical uncertainty in each annulus, $\sigma_{\text{dat}}$, determined during the MCMC sampling. When the signal from projected LSS is included, we add $\sigma_{\text{LS}}$, as detailed in Sect. 3.2, such that the total uncertainty error on the density profile, $\sigma^2_{\text{tot}} = \sigma^2_{\text{dat}} + \sigma^2_{\text{LS}}$.

4.3 Halo Shapes

On large scales, the accretion of matter from the surrounding large-scale environment plays a key role in determining the shape and orientation of cluster dark matter halos (Shaw et al. 2006). Halos are not necessarily self-similar (centric ellipsoids with the same orientation and ellipticity; Suto et al. 2016), but align with the infall direction of subhalos and surrounding filaments at large radii. Thus, the shape of galaxy clusters is a fundamental probe of the history of its mass accretion. Numerical simulations with collisionless dark matter predict cluster halos to be triaxial (Warren et al. 1992; Jing & Suto 2002). Allowing DM particles to self-interact isotropizes the orbits of dark matter particles, and makes the inner mass distribution more spherical. For a cross-section of 1 cm$^2$/g, the median minor-to-major axis ratio 100 kpc from the halo centre is ~0.8, compared with ~0.5 with CDM (Robertson et al. 2019).

We fit an elliptical NFW mass distribution (eq. 10) to the 2D convergence maps reconstructed from

\[^{2}\text{We implement LENSTOOL using version 7.1 of the software available from https://projets.lam.fr/projects/Lenstool/wiki.}\]
KS93+MRLENS or LENSTOOL, with no noise, with shape noise, with LSS noise or both. The fit\(^3\) minimize the sum of the squared difference between the reconstructed surface mass density of each BAHAMAS simulated cluster and an elliptical NFW model, within a circle of radius \(R_{ap}\). We then vary \(R_{ap}\), to investigate changes between the cluster’s inner and outer halos. During the fits, we fix the centre of the NFW (to the location of the most bound particle) because it is degenerate with axis ratio. We adopt flat priors on 

\[ q \in [0.7, 1] \]

\[ \phi \in [0, 180] \]

\[ 0.1 \leq c_\text{200} \leq 8, \ 0 \leq \phi \leq 180 \] and \( 0.1 \leq q \leq 0.9 \), and neglect covariance between adjacent pixels. The uncertainties of \( q \) in this test can be under-estimated. However, it match those in observational data, as we add only one, fixed realisation of LSS along the line-of-sight associated with each cluster.

4.4 Searches for filaments

Dark matter and gas are accreted onto a cluster mainly through filaments that connect it to the ‘cosmic web’. Filaments are key transition regions in the evolution of galaxy morphology (Pandey & Bharadwaj 2006; Einasto et al. 2007; Nuza et al. 2014; Kutumla et al. 2017; Liu et al. 2019; Martizzi et al. 2019) and star formation (Crain et al. 2009; White et al. 2010; Alpaslan et al. 2015, 2016; Yuan et al. 2019).

Filaments are much lower density environments than a cluster, so appear in gravitational lensing observations with correspondingly lower signal-to-noise. While it is possible to search for filaments directly in shear data (Dietrich et al. 2005; Dietrich et al. 2012; Jauzac et al. 2012), we explore whether it is efficient to leverage the de-noising techniques developed for mass mapping, then to analyse the inferred convergence field.

4.4.1 Removing the smooth mass component

First, we subtract the smooth distribution of mass in the clusters, which would otherwise dominate the lower density contrast in the filaments.

We fit mock reduced shear data (with or without LSS and galaxy shape noise), using an elliptical NFW potential. This model has 6 free parameters: the coordinates of the centre of mass, \((x_c, y_c)\), the ellipticity, \(e = (1 - q^2)/(1 + q^2)\) where \(q\) is the axis ratio, the position angle, \(\phi\), the scale radius, \(r_s\), and the concentration, \(c\). We set flat priors on \(x_c, y_c\) within a \(15'' \times 15''\) box centred on the most bound particle, and flat priors on \(e \in [0.05, 0.7]\), \(\phi \in [0, 180]\), \(r_s \in [50, 1000]\) kpc, and \(c \in [0.5, 10]\). Note that we introduce ellipticity to this model via a coordinate transformation to the gravitational potential (rather than the mass, as in Sect. 2.2) because code to achieve this already exists within LENSTOOL.\(^4\). The smooth distribution of mass in most simulated clusters is well approximated by a single potential. However, we use two to fit bimodal clusters 1, 2 and 9, and three for cluster 3.

We then subtract the best-fit smooth halos from the convergence maps. Since the mass distribution of simulated clusters cannot be perfectly described by elliptical NFW potentials, small residuals are left near the cluster centre. Such residuals do not impact searches for filaments at much larger radii.

4.4.2 Aperture multipole moments

Schneider & Bartelmann (1997) first suggested looking for substructures or filaments using multipole moments of a convergence field within circular apertures. These are

\[
Q_n(R) = \int_0^\infty |R' - R|^n e^{i n \phi} \kappa(R') d^2R',
\]

(20)

where \( n \) is the order of the multipole, \((R, \phi)\) are polar coordinates, and \(U_n(R)\) is a radially symmetric weight function, for which Dietrich et al. (2005) suggested

\[
U_n(R) = \begin{cases} 1 - \left( \frac{R}{R_{\text{max}, n}} \right) ^2 & \text{for } R \leq R_{\text{max}, n} \\ 0 & \text{otherwise.} \end{cases}
\]

(21)

Eq. (20) can also be expressed in terms of shear measurements, which Dietrich et al. (2005) used to detect filament candidates in close pairs of clusters. Since modern mass reconstruction methods successfully suppress noise, we attempt instead to measure multiple moments directly from the pixelated convergence field

\[
Q_n(R) = A_{\text{pix}} \sum_{i=1}^{N_{\text{pix}}} R_i^n e^{i n \phi_i} \kappa(R_i),
\]

(22)

where \( N_{\text{pix}} \) is the total number of pixels inside the aperture and \( A_{\text{pix}} \) an area per pixel. For \( n > 0 \), \( Q_n \) is complex; we shall generally take its modulus, \|Q_n\|.

Multipole of different orders highlight different features in a mass distribution (see figure 2). Monopole moments \((n = 0)\) are the aperture mass or normalisation. Dipole moments \((n = 1)\) are the local gradient of a convergence field. They form ring-like structures around mass clumps. Quadrupole moments \((n = 2)\) are the locally-weighted curvature or Hessian of the convergence field. As Dietrich et al. (2005) explain using a toy model, linear overdensities with a lower mass on either side (i.e. filaments) have large quadrupole moments. However, regions between two substructures also have large quadrupole moments. To identify the former and suppress the latter, Mead et al. (2010) suggested combining multipole moments

\[
Q = a_0 Q_0 + a_1 Q_1 + a_2 Q_2 + ...
\]

(23)

where the constants, \( a_i \), can be adjusted to boost a signal of interest. We have tried different combinations and aperture sizes, and find that a choice of

\[
a_0 = -a_1 = 0.7 \quad \text{and} \quad a_2 = 1,
\]

(24)

\[
R_{\text{max}, 0} = 1' \quad \text{and} \quad R_{\text{max}, 1} = R_{\text{max}, 2} = 2'.
\]

(25)

typically highlights narrow filaments (see figure 3). The quadrupole term is sensitive to linearly extended mass distributions, and the rings that it adds around substructures are
Figure 2. An example of aperture multipole moments of various orders, which pick out different features of the noise-free mass distribution of one simulated cluster (Cluster 5, which happens to have several features in the plane of the sky). Moments are calculated after subtracting the large-scale smooth mass distribution. From left to right, panels show: (a) monopole, (b) dipole, (c) quadrupole moments and (d) the radial component of the quadrupole moment. For reference, black contours show the true mass distribution.

Figure 3. A combination of aperture multipole moments, $Q$ (equations 23–25), can be used to identify filamentary features in a mass map. Colours (Top panel: linear scale, Bottom panel: logarithmic scale) show $Q$ calculated from the true convergence map (without shape noise or LSS noise; black contours), after subtracting its best-fitting smooth component. Dotted lines reproduce the 40 filaments from figure 1. The 22 filaments successfully identified using $Q$ and the procedure described in section 4.4.3 are highlighted in magenta.
Figure 4. A combination of aperture multipole moments, \( Q \) (equations 23–25), can be used to identify features in a mass distribution with filamentary topology (see figure 3) and higher density than the background. Solid lines show the mean projected density (\( \kappa \)) inside a contour defined by \( Q_{\text{threshold}} \) for all 10 simulated clusters. The dotted line and shaded region show their mean and standard deviation. The normalisation of coefficients (24) is chosen so that \( \langle \kappa \rangle = Q_{\text{threshold}} \). The lower dashed line shows the mean convergence, weighted by the number of pixels that contain \( Q > Q_{\text{threshold}} \).

removed by the negative dipole term. The monopole term fills in the subtracted mass, and suppresses regions between two substructures but without mass.

4.4.3 Filament identification

To identify individual filaments, we search for spatially extended regions with \( Q \) above a threshold \( Q_{\text{threshold}} \). The normalisation of coefficients in eq. (24) conveniently ensures that regions inside a contour \( Q_{\text{threshold}} \) have mean convergence \( \langle \kappa \rangle \approx Q_{\text{threshold}} \) (figure 4). We identify as possible filaments any region with \( Q > Q_{\text{threshold}} \) in a contiguous area or multiple peaks with total area > 1.13 arcmin\(^2\), that is aligned within \( \sim 45^\circ \) of the radial direction to the cluster centre. Applied to noise-free data and using \( Q_{\text{threshold}} = 0.005 \), this recipe identifies 22 of the 40 filaments, all of which are real, i.e. 55% completeness (the number identified divided by the true number) and 100% purity (the number identified that are true divided by the number identified). The identified filaments are highlighted in magenta in figure 3.

4.4.4 Additional noise suppression strategies

Measurements of multipole moments will be more difficult in noisy data — especially for high \( n \) moments, where the diverging \( |R'| - R'|^n \) term is particularly sensitive to noise in \( \kappa \) near the aperture boundary. We shall explore three strategies to reduce noise. First, noise can be averaged away by enlarging the aperture. However, signal is also averaged away for a filter than is not matched to the size of the feature – and filaments are relatively narrow, even around clusters at low redshift. Second, negative noise peaks can be eliminated by forcing \( \kappa = \max(\kappa, 0) \). Negative convergence is physically possible, because convergence represents deviation from the mean cosmic density; but it is unlikely along the line-of-sight to even a low density structure, and probably noise rather than signal. Third, we could assume that all filaments extend radially away from the cluster, while noise is isotropic, and suppress quadrupole and dipole moments whose phases are tangential. We calculate

\[
Q_{n,\text{projected}} = |Q_n| \cos(\phi - \theta) \quad \text{with} \quad n = 1, 2
\]

where \( \theta \) is a phase angle of \( Q_n \). Figure 2(d) shows the projected quadrupole moments in the noise-free case, as an example.

5 RESULTS & DISCUSSION

To the ten simulated clusters presented in Sect. 3, we shall now apply the analysis methods described in Sect. 4. We compare the reconstructed convergence maps, radial density profiles and halo shapes, to the known, true distribution of mass. We then search for observable signatures of filaments extending from the clusters. For all these analyses, we quantify the impact of the two main sources of noise in weak lensing measurements: unrelated LSS projected by chance along the line of sight to the cluster (Sect. 3.2), and the intrinsic shapes of background galaxies (Sect. 3.4).

5.1 Mass mapping

We quantify the precision and accuracy of mass maps produced by KS93+MRLens (figure 5) and LENSTOOL (figure 6) by comparing them to the noise-free distributions of mass, \( \kappa_{\text{true}} \) (which includes only the mass of the cluster, not projected LSS). We first measure deviations from this truth, \( \kappa_{\text{res}} \equiv \kappa - \kappa_{\text{true}} \), to obtain the residual maps. For each map, we compute the noise level \( \sigma_{\kappa} \), defined as the root mean square (rms) deviation from the mean of \( \kappa_{\text{res}} \), over all pixels in a field of view equivalent in size to the HST observations of MS0451-03. We then average the performance of each method over all 10 clusters (table 2).

In observations of the real Universe, \( \sigma_{\kappa} \) cannot be calculated because there is no privileged knowledge of \( \kappa_{\text{true}} \). For comparison with observations, we therefore also measure \( \sigma_{\kappa, \text{obs}}^\alpha \), the rms deviation from the mean of \( \kappa \). We find values of \( \sigma_{\kappa, \text{obs}}^\alpha \) roughly consistent with \( \sigma_{\kappa} \) being added in quadrature to an irreducible component that is the rms deviation from the mean of \( \kappa_{\text{true}} \). 0.022 ± 0.0007 on average (0.027 for the five highest mass clusters, or 0.017 for the five lowest).

5.1.1 Direct inversion mass reconstruction

MRLens suppresses galaxy shape noise by a factor 3.8 (a factor 1.5 better than smoothing with 1’ pixels, and retaining higher spatial resolution). However, galaxy shapes still contribute more noise to the mass maps than physically real LSS noise. Spurious noise peaks are found in all regions of the field of view. Massive substructures with \( \kappa > 0.096 \) can be detected with \( S/N > 3 \).

Mass reconstructions using KS93+MRLens are statistically consistent with being unbiased. Both positive and negative noise fluctuations are produced, at all radii. The mean residual of maps with both sources of noise is \( \langle \kappa_{\text{res}} \rangle = \ldots \).
Figure 5. Projected mass maps of the 10 simulated clusters reconstructed using the KS93+MRLens direct inversion method, including different components of noise. Top panels: reconstruction with no noise. Second panels: including only shape noise from 50 background galaxies per square arcminute. Third panels: including only projected large-scale structure. Bottom panels: including both sources of noise simultaneously. Colour scales are identical for all panels. For reference, red lines indicate the field of view of the largest HST mosaic obtained around a massive galaxy cluster, MS0451-03.
Figure 6. Same as figure 5, but reconstructed using Lenstool.
Lenstool suppresses noise even further. Galaxy shape noise is an additional factor 2 lower than KS93+MRLens (averaged across the field of view) — with the similar level as the LSS noise.

The spatial distribution of noise is nonuniform. A Lenstool reconstruction has more freedom in regions with LSS noise. Because of the positive-definite constraint, this biases the inner profile slope that is often used to distinguish between cusps and cores. Lenstool is accurate in the cluster core, because its basis functions have a density profile that matches those of the simulated clusters. This is not affected by Lenstool’s positive-definite constraint, because the true mass distribution is very positive near the core. In the cluster outskirts, Lenstool strongly suppresses galaxy shape noise, and the reconstruction is dominated by LSS noise. Because of the positive-definite constraint, this is also potentially biased. The amplitude of LSS noise varies a great deal depending on environments along the line-of-sight LSS, but we typically find artificial boosts in inferred density of up to $\sigma_{\kappa,SS} = 4 \times 10^2 \, M_\odot / kpc^2$, at large projected radii, $R > 1000 \, kpc$. This effect must be taken into account when measuring properties at large radius (e.g. $M_{200}, \rho_{200}$, splashback radius). To mitigate against this, measurements of galaxy redshifts will be invaluable to disentangle structures connected to the cluster from those lying in the foreground or background.

5.2 Radial density profiles

We recover the clusters’ density profiles by azimuthally averaging the convergence maps (figure 7). The smoothing inherent to KS93+MRLens results in an underestimation of density in the cluster core, and an overestimate just outside. This biases the inner profile slope that is often used to distinguish between cusps and cores. Lenstool is accurate in the cluster core, because its basis functions have a density profile that matches those of the simulated clusters. This is not affected by Lenstool’s positive-definite constraint, because the true mass distribution is very positive near the core. In the cluster outskirts, Lenstool strongly suppresses galaxy shape noise, and the reconstruction is dominated by LSS noise. Because of the positive-definite constraint, this is also potentially biased. The amplitude of LSS noise varies a great deal depending on environments along the line-of-sight LSS, but we typically find artificial boosts in inferred density of up to $\sigma_{\kappa,SS} = 4 \times 10^2 \, M_\odot / kpc^2$, at large projected radii, $R > 1000 \, kpc$. This effect must be taken into account when measuring properties at large radius (e.g. $M_{200}, \rho_{200}$, splashback radius). To mitigate against this, measurements of galaxy redshifts will be invaluable to disentangle structures connected to the cluster from those lying in the foreground or background.

5.3 Halo shapes

Both mass reconstruction methods produce distributions that are rounder than the truth (figure 9), eNFW models fitted to the reconstructed mass maps (figure 5, 6) have a higher mean axis ratio $\langle q \rangle$ than models fitted to the true mass maps (figure 1). However, they successfully capture the decrease in $\langle q \rangle(R)$ at large radii that is seen in the true mass maps (reflecting a transition from dominant baryonic effects to the infall of structures along filaments; Suto et al. 2017). The orientation of most inner ($R = 650 \, kpc$) and outer ($R = 3 \, Mpc$) halos also remain aligned within $\Delta \phi \leq 10^\circ$, matching the true distributions (and also the simulations by Despali et al. 2017). Two exceptions to this are clusters 5 and 9, which have complex cores and $\Delta \phi = 17^\circ$ and $\Delta \phi = 15^\circ$. This likely indicates a transitory state during a major merger.

Using KS93+MRLens leads to inferred values of $\langle q \rangle$...
Figure 7. Surface mass density profiles for all 10 simulated clusters. Blue solid lines show the density profile calculated from the true mass distribution in Fig 1. Green solid lines are the density profiles of KS93+MRLENS reconstructed maps after adding shapes noise and LSS. Cyan, orange, and red lines show the results recovered by LENSTOOL including shape noise, projected LSS, and both shape noise and LSS, respectively. Error bars with line caps are statistical errors from the MCMC sample. Error bars with triangle caps are total errors which is the combination of statistical errors with the estimated noise from the projected LSS (eq. 12).
that are too high by about 6%. The level of bias is not significantly influenced by either source of noise in the shear catalogue (although adding noise increases scatter in individual measurements of \(q\) as expected). It is likely due to the isotropic blurring associated with pixelisation and MRLENS filtering.

Using LENSTOOL leads to inferred values of \(\langle q \rangle\) that are too high by 10% in the cluster core and 15% in the outskirts. The bias appears to be caused by two effects:

- The mass distribution is built from components that are all individually spherical. If the dominant halo in the cluster core is anomalously spherical (see clusters 4, 5, 8 or 10 in figure 8), it can bias the apparent axis ratio of the mass inside a circle by up to 10%, almost regardless of the size \(R_{ap}\) of that circle. Substructures far from the centre of the cluster look surprisingly uniform, but this does not affect measurements of the overall shape.

- The mass distribution is constrained to be positive definite. In the absence of noise, this has no effect. If we add galaxy shape noise, it is also relevant that the reconstructed mass distribution is higher resolution (has more freedom) along its major axis. The positive-definite bias in noise artefacts then exaggerates the major axis, reducing \(\langle q \rangle\) by ~5%. If we add LSS noise, \(\langle q \rangle\) increases by 8% because there is a larger area at close to zero convergence along the minor axis.

It is possible to mitigate the first effect by masking the cluster core. We successfully recover the true axis ratio when fitting an eNFW using to noise-free data inside an annulus \(35'' < R < R_{ap}\) (instead of a circle of radius \(R_{ap}\)). Fitting inside annuli also decorrelates measurements of \(\langle q \rangle\) at different radii, and steepens the apparent gradient in \(\langle q \rangle(R)\).

Note that the second effect still increases \(\langle q \rangle\) by ~6% in the presence of both sources of noise.

A different strategy to mitigate sphericity bias could be to pre-fit the axis ratio of central halos, then hold them fixed while the rest of the grid is constrained. A similar two-step process happens naturally in most combined analyses of strong plus weak lensing, where strong lensing information constrains a cluster core. This bias should therefore not affect LENSTOOL strong lensing analyses. However, it would be difficult to characterise statistical uncertainty in such analysis, because shear data would be used twice.

5.3.1 Comparison with previous studies

Previous work by simulators to measure the shape of cluster-scale halos split into two distinct conclusions. Hopkins et al. (2005) found that 2D cluster ellipticity increases with cluster-centric radius, in agreement with our results. However, they also found that the ellipticity is \(\epsilon \approx 0.05 \pm 0.04\) for the redshift range \(0 < z < 3\), which implies \(q = 0.64\) at the \(z = 0.55\) redshift of our simulated clusters. Similarly, Ho et al. (2006) found \(q \approx 0.616\) for halos with masses \(M > 10^{14} M_{\odot}\) at \(z = 0.55\) assuming \(\Omega_m = 0.3\), and \(\sigma_8 = 0.7\), and little dependence upon cosmological model. Both of these results are slightly rounder than our measurement of \(\langle q \rangle_{true} = 0.55 \pm 0.03\).

More recently, Despali et al. (2017) found that \(M \sim 10^{15} M_{\odot}/h\) halos in the SBARBINE N-body simulations had more elliptical shapes, with \(q \sim 0.55\). Suto et al. (2016) studied the probability distribution function (PDF) of \(q\) from projected density distributions without assumptions of self-similarity. Using their PDF fit formula for \(M_{vir}\) at \(z = 0.4\), we obtain \(q = 0.57 \pm 0.17\). These results match ours closely, and more recent independent analyses appear to be consistent.
Several observational studies of weak-lensing have attempted to measure cluster halo ellipticity. In the Sloan Digital Sky Survey (SDSS), Evans & Bridle (2009) found a mean projected axis ratio \( q = 0.48 \pm 0.04 \) in the redshift range \( 0.1 < z < 0.3 \). By directly fitting 2D shear-maps with eNFW models, Oguri et al. (2010) measured an mean projected axis ratio \( q = 0.54 \pm 0.04 \) for a sample of 18 X-ray luminous clusters in the redshift range \( 0.15 < z < 0.3 \). Shin et al. (2018) measured \( q = 0.56 \pm 0.09 \) for 10,428 SDSS clusters. These results are consistent with our measurement. Intriguingly, Umetsu et al. (2018) measured the median projected axis-ratio of 20 high-mass galaxy clusters in the HST-CLASH survey to be \( q = 0.67 \pm 0.07 \), within a scale of 2 Mpc\(^{-1}\). However, their measurement from the CLASH high-magnification subsample was \( q = 0.55 \pm 0.11 \), consistent with our results. This suggests a lensing selection bias towards halos that are more elliptical (in the plane of the sky as well as along a line of sight). In contrast, X-ray selected clusters tend to be relaxed clusters with rounder dark matter halo shapes. For clusters selected by the red sequence technique, it is more likely that they are elongated along the line of sight, causing an over-density of red galaxies in the projected sky-plane. Since our simulated cluster sample is selected by their high mass, with each halo projected along a random line-of-sight, we can only give the mass-selected mean halo shape. For direct comparison with observational data, future theoretical predictions will need to take the selection function of the observed sample into effect.

Other shape measurement techniques are possible. Studies using quadrupole estimators to quantify halo shape include Adhikari et al. (2015); Clampitt & Jain (2016); van Uitert et al. (2017); Shin et al. (2018). In particular, Clampitt & Jain (2016) developed a new estimator to measure the quadrupole weak-lensing signal from 70,000 SDSS Luminous Red Galaxies halos, and found a best-fit axis-ratio \( q \sim 0.78 \). Their analysis assumes that dark matter perfectly aligns with light, so one potential systematic in their study is the possibility of light and dark matter misalignment. The determination of the orientation of each lens-source pair could become inaccurate due to this misalignment, and result in the dilution of the final stacked signal of the halo ellipticity. Indeed, applying the misalignment distribution of Okamura et al. (2009) to their measurement, they obtain \( q \sim 0.6 \), consistent with our results.

### 5.4 Searches for filaments

In the presence of galaxy shape noise and LSS noise, maps of our combination of aperture multipole moments \( Q \) have lower signal-to-noise than maps of convergence \( \kappa \) (figure 10; given the noise level, we show them only in linear scale, not logarithmic). We quantify the noise level by defining \( \sigma_Q \) as the standard deviation of all pixels in the final \( Q \) map. Despite our attempt to eliminate isolated substructures from the \( Q \) maps by combining different multipole moments, clusters 1, 2 and 5 contain sufficiently massive substructures to induce higher \( Q \) than lower-density filaments. Following the methodology in section 4.4.3, we then search for filaments as extended regions with \( Q > 3 \sigma_Q \) (illustrated in figure 10) or \( Q > 4 \sigma_Q \). Results for both are listed in table 3.

In the default Lenstool mass reconstructions, we find \( \langle \sigma_Q \rangle = 0.011 \) and, with \( Q_{\text{threshold}} = 3 \sigma_Q \) we identify 17 of the 40 filaments (42.5\% completeness), plus 5 false positive detections (77.3\% purity). Increasing the detection threshold to \( 4 \sigma_Q \) removes all but one false detection, but finds only 12 real filaments.

Identifying filaments in the noisier KS93+MRLens mass reconstructions is much more difficult. To obtain useful results, we need to apply all three denoising strategies presented in Sect. 4.4.4. We enlarge the apertures to \( R_{\text{max}} = 2' \), \( R_{\text{max}} = 2'' \), \( R_{\text{max}} = 3'' \), and we replace negative convergence by zeros; and we project all quadrupole and dipole moments in the radial direction. In combination, these strategies reduce \( \langle \sigma_Q \rangle \) from 0.11 to 0.06. Filament identification statistics after this noise suppression are listed in table 3. At \( 3 \sigma_Q \) detection threshold, we identify 15 of the 40 filaments (37.5\% completeness), but also 21 false positive detections (41.7\% purity).

Most of the false-positive filament detections are caused by galaxy shape noise. Repeating the KS93+MRLens analysis with only shape noise yields a \( Q \) map with \( \sigma_Q = 0.058 \); with only LSS noise, it is \( \sigma_Q = 0.033 \). Because shape noise is apparently so dominant, we also investigate the effect of different survey strategies on the success of filament identification. We simulate ground-based observations, which typically resolve the shapes of only 20 galaxies arcmin\(^{-2}\), and extremely deep space-based observations that resolve
Figure 10. Results for the filament search around 10 simulated clusters. Colours show a linear combination of aperture multipole moments $Q$, calculated from the mass maps after subtracting their best-fit smooth component. Dotted lines show true filaments, reproduced from figure 1; those identified successfully (with $Q_{\text{threshold}} = 3\sigma_Q$, see section 4.4.3) are highlighted in magenta. Solid lines show false positive detections. The top and second panel use mass maps created by LENSTOOL (including shape noise and LSS), with $50\,\text{arcmin}^{-2}$ and $100\,\text{arcmin}^{-2}$ source galaxies, respectively. The third and bottom panel show the phase-projected version of the filter applied to the positive-only KS93+MRLens mass map (with a different colour scale to the top two panels). In all panels, red contours show $Q = 3\sigma_Q$ and $4\sigma_Q$, and black contours show the true mass distribution.
that samples the shear field only along the lines of sight to galaxies.

We find that MRLENS is particularly efficient at suppressing noise owing to the diverse intrinsic shapes of background galaxies, whilst retaining signal from statistically significant structures on all scales. In a typical cluster field, it reduces total noise $\sigma_\kappa$ from $0.088 \pm 0.001$ to $0.026 \pm 0.001$. The KS93+MRLENS method will be appropriate for use on stacked observations of a large number of galaxy clusters. However, it has no knowledge of cluster physics, and its noise suppression via smoothing softens the inferred central density profile. At large projected radii, $R > 1$ Mpc, noise in the map of an individual cluster becomes dominated by unrelated structures at different redshifts, projected along adjacent lines of sight.

LENSTOOL incorporates physical knowledge of galaxy clusters by imposing strong priors on the distribution of mass. For example, it preserves central cusps. The method is more aggressive in denoising the reconstructed convergence field, achieving $\sigma_\kappa = 0.015 \pm 0.004$. By adjusting the grid’s adaptive resolution, it is also possible to suppress the spurious signal from unrelated, isolated structures at different redshifts, once they have been identified via multiband photometry or spectroscopy. We find that this method is well-suited to reconstructions of individual clusters, or measurements of low signal-to-noise quantities, such as filaments.

In its standard configuration however, we find that LENSTOOL biases a mass reconstruction at large distances from the centre of a cluster, by imposing a prior that the projected density everywhere in a field of view must be positive (relative to the mean density in the Universe). This bias will need to be managed carefully when statistical errors are reduced by averaging over a population of clusters: perhaps by reconfiguring the Bayesian optimisation engine. The standard configuration of LENSTOOL also forces the mass distribution in every grid point to be spherically symmetric. In a purely weak-lensing analysis, this leads to spuriously spherical cluster cores, even when the global mass distribution is well modelled. This issue is automatically solved and irrelevant if strong gravitational lensing information is available, and used to pre-fit the axis ratio of the core. In this weak lensing-only study, we adopt a simple solution by masking the central $R < 35''$ regions of a weak-lensing-only reconstruction. This avoids modelling the central spherical core for halo shape measurement.

Based on the performance of these two methods, for an individual cluster, or measurements of highly nonlinear quantities such as filament detection, LENSTOOL is well-suited to applications that require as precise a reconstruction as possible. However, for high-precision analyses that stack many clusters, it would be necessary to drop LENSTOOL’s positive definite constraint to reduce bias of mass overestimation. By contrast, KS93+MRLENS retains a higher level of noise, but the positive and negative fluctuations are preserved in a manner which can reduce bias in stacked measurements.

We also develop a filter to search for filaments and measure their orientation. The low density of filaments leads to low signal-to-noise in reconstructed maps, and they can rarely be stacked usefully. To retain their individual signal whilst suppressing noise, we construct a linear combination of multipole moments. We explore two further strategies: (1)
filtering on the orientations (complex phases) of higher-order moments, exploiting the prior knowledge that filaments typically extend radially out from cluster halos, and (2) replacing with the mean density of the Universe those regions inferred to have (negative) less density, which are more likely to be noise than regions inferred to have (positive) higher density. We find that it will be impossible to detect individual filaments using data from ground-based telescopes, and remains challenging with current space-based (HST) data. However, we find that the dominant source of noise relevant to filament detection comes from lensed galaxies’ intrinsic shapes. Deeper observations with the next generation of space-based telescopes will resolve more background galaxies, and efficiently beat down this noise. Our filtering method successfully finds 45% of filaments with projected density $\Sigma > 1.7 \times 10^7 M_\odot/kpc^2$ (with a false detection rate $<20\%$), when applied to mock observations at the depth of possible future surveys.

ACKNOWLEDGEMENTS
We would like to thank anonymous referee for giving useful comments and improving our manuscript. We are grateful to Ian McCarthy for sharing his BAHAMAS simulation data, and supporting its interpretation. SIT is supported by Van Mildert College Trust PhD Scholarship. RM is supported by the UK Science and Technology Facilities Council (grant number ST/P000541/1). AR is supported by a Royal Society University Research Fellowship.

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DATA AVAILABILITY
The simulation data underlying this article are available from https://www.astro.ljmu.ac.uk/~igm/BAHAMAS/.

MNRAS 000, 1–19 (0000)
