Effective steady-state security regions of power systems considering voltage magnitude

Y Zhou¹, X L Yang¹, M Yang¹, Y Zhu², C Wang³, Y F Bo¹ and P J Du¹

¹Key Laboratory of Power Intelligent Dispatch and Control (Shandong University), Ministry of Education, Jinan 250061, China
²Economic & Technology Research Institute of State Grid Shandong Electric Power Company, Jinan 250021, China
³Pingdu Electrical Power Bureau of State Grid Shandong Electric Power Company, Pingdu 266700, China

E-mail: myang@sdu.edu.cn

Abstract. With high-level variable renewable energy integrating into power systems, the uncertainty significant challenges to the real-time dispatch (RTD). Meanwhile, the influence of voltage magnitude and reactive power on RTD always were ignored. Based on effective steady-state security regions of power systems and linear power flow model, a real-time dispatching method considering voltage magnitude and reactive power is proposed in this paper. This model has two goals, one is the security goal, the other is the economic goal, and priority of security objectives is higher than economy. Unlike previous real-time dispatch methods, all of the operation base point, reactive power output and participation factors of automatic generation control (AGC) generators are chosen as decision variables in the model. In order to ensure computational efficiency, the model is finally transformed into a deterministic linear programming (LP) problem by robust optimization and other methods. The accuracy and computational efficiency of the method are illustrated by modified IEEE 9-bus system and IEEE 118-bus system.

1. Introduction

With the integration of large-scale renewable energy into the power grid, the uncertainty brought by it has tremendous impact on the safe operation of power system [1]. In order to absorb more renewable new energy, we put forward higher requirements for real-time dispatch. As we all know, the current economic dispatch is based on the DC power flow method. The active and reactive power are completely decoupled and controlled by AGC and AVR respectively, which is completely adapted before. However, with the massive injection of new energy, could the influence of voltage magnitude and reactive power for active power flow on accuracy of power flow be completely ignored? Should voltage magnitude constraints be considered in real-time dispatch? This series of problems should be considered [2].

In the past few years, a lot of research has been done on the approximate linearization of OPF considering voltage magnitude and reactive power. Because the AC power flow model is a non-convex problem, it’s hard to solve. In order to linearize it, the key of its work is to realize decoupling of voltage and phase angle [3], and the error caused by decoupling is mainly in the process of simplifying $v_i v_j$ [2]. It can be divided into three types. One realizes decoupling of $v_i$ and $\theta$ [3,4]. It simplifies $v_i v_j$ to $v_i + v_j - 1$, the influence of line loss is considered and linearized in [4]. Other
regarding \( v^2 \) and \( v^2 \theta \) as independent variables [5], which simplifies \( v_j \) to \( v_j^2 \). Literatures [6,7] realize decoupling of \( v^2 \) and \( \theta \) by different methods, which simplifies \( v_j \) to a quadratic term. The accuracy of these three methods is also calculated and compared in detail in [2]. Among them, the third method has the highest accuracy, followed by the first one. All of the above research focus on deterministic optimal power flow models. Recently, through the second method mentioned above, an algorithm of linear power flow is proposed [3], which obtains a linear correspondence between active and reactive power and voltage phase angle, and this linear correspondence makes it possible to consider voltage when dealing with uncertain problems. According to the method described in [8], a distributed robust optimization model is proposed, the equation constraints at the nominal operation point are based on the complete AC power flow model, and the linear power flow model proposed in [3] was used to deal with the uncertainties. The method has high accuracy, but because of AC power flow model, the computational efficiency of this method needs to be improved.

Steady-state security regions was proposed very early [9,10]. Literature [11] proposed a LP model, which maximized steady-state security region. Literature [12] proposed a new way, which maximized the coincidence of static security region and nodal disturbance region formed by compensation control of all AGC units in system operation. All of these methods are based on traditional DC power flow, and they haven't consider the influence of reactive power and voltage magnitude.

Based on the above situation, a real-time dispatch considering reactive power and voltage magnitude is proposed in this paper. The method combines the advantages of the above two studies. Coordination decision-making is made on the operation base point, reactive power output and participation factors. The model has two objective functions. The security objective is the first priority, and economic optimum is realized on the premise under ensuring secure operation of power system. In order to solving the model, firstly, the uncertain optimization problem is transformed into deterministic optimization problem by robust method, and then the objective function is linearized. Finally, the whole model is transformed into LP problem, which ensures the computational efficiency of the solution. The advantages of the method are as follows: 1) Voltage magnitude and reactive power are taken into account when dealing with uncertain problems. 2) The model is finally transformed into deterministic LP problem.

### 2. Optimization model

The model has two objective functions. With a clear priority, first of all, the model maximizes the goal of safe operation, then, achieves economic optimum. Security target maximizes the effective static security [12]. The definition of effective static security region is the coincidence of static security region and bus disturbance region formed by compensation control of all AGC units in system operation. Symbols with the index \( i \), \( j \) denote constants or variables at node \( i \), \( j \).

**Objective function:**

\[
\max Z = \sum_{j=1}^{N_p} \left( \min \left( \Delta \theta_j^{\max}, \Delta \theta_j^{\min} \right) + \min \left( \Delta d_j^{\max}, \Delta d_j^{\min} \right) \right)
\]

\[
\min \sum_{j=1}^{N_p} \left( c_i \cdot p_i + \bar{c}_i \cdot \Delta p_i^{\max} + \bar{c}_i \cdot \Delta p_i^{\min} \right)
\]

Where, \( \Delta \theta_j^{\max}, \Delta \theta_j^{\min} \) are the maximum and minimum of the load fluctuation, The setting of its value can affect the conservative degree of the model. \( \Delta d_j^{\max}, \Delta d_j^{\min} \) are maximum upward and downward fluctuation allowance. \( N_p \) are the number of the load and AGC units, \( c_i, \bar{c}_i \) are constant about the cost of generating electricity, \( p_i \) is the nominal operation point, \( \Delta p_i^{\max}, \Delta p_i^{\min} \) are maximum up and down capacity of AGC units.

**Constraints:**

- Equality constraints at the nominal operation point:
In the recent study of linear power flow (pf), a linear power flow which completely decouples the voltage amplitude and phase angle is proposed in [3]. As shown below, it is a completely linear corresponding relationship, and its accuracy has been proved. We choose them as the equality constraints at the base point of the proposed model.

\[ p = -B' \theta + Gv \]  \hspace{1cm} (3)
\[ q = -G \theta - Bv \]  \hspace{1cm} (4)

Where, \( p \), \( q \) are bus injected active and reactive powers; \( B' \), \( G \), \( B \) are constant matrix in [3], \( V \), \( \theta \) are voltage magnitudes and magnitude phase angles.

Constraints on the participation factors of AGC units:
According to its definition, its summation should be 1 and each of them should be greater than 0.

\[ \sum_{i=1}^{N} \alpha_i = 1 \]  \hspace{1cm} (5)
\[ \alpha_i \geq 0 \quad i = 1, 2, \ldots, N_a \]  \hspace{1cm} (6)

Where, \( \alpha_i \) is the participation factor of AGC unit \( i \).

Maximum up and down capacity of AGC units constraints:

\[ \Delta p^\text{max}_i = \alpha_i \sum_{j=1}^{N_a} \Delta p^\text{max}_j \quad i = 1, 2, \ldots, N_a \]  \hspace{1cm} (7)
\[ \Delta p^\text{max}_i = \alpha_i \sum_{j=1}^{N_a} \Delta p^\text{max}_j \quad i = 1, 2, \ldots, N_a \]  \hspace{1cm} (8)
\[ 0 \leq \Delta p^\text{max}_i \leq \Delta p^\text{max}_i \quad i = 1, 2, \ldots, N_a \]  \hspace{1cm} (9)
\[ 0 \leq \Delta p^\text{max}_i \leq \Delta p^\text{max}_i \quad i = 1, 2, \ldots, N_a \]  \hspace{1cm} (10)

Where, \( \Delta p^\text{max}_i \), \( \Delta p^\text{max}_i \) are upper and lower limits of maximum up and down capacity of AGC unit \( i \)

Active power output capacity constraints:

\[ p_i - \Delta p^\text{max}_i \geq p^\text{min}_i \quad i = 1, 2, \ldots, N_a \]  \hspace{1cm} (11)
\[ p_i + \Delta p^\text{max}_i \leq p^\text{max}_i \quad i = 1, 2, \ldots, N_a \]  \hspace{1cm} (12)

Where, \( p^\text{min}_i \), \( p^\text{max}_i \) are the minimum and maximum active output power of AGC unit \( i \).

Constraints on active power change rate of AGC units:

\[ -r_{u,i} \leq \Delta p_i \leq r_{u,i} \quad i = 1, 2, \ldots, N_a \]  \hspace{1cm} (13)

Where, \( p_i^0 \) is the original active output power of AGC before adjustment, \( r_{u,i} \), \( r_{u,i} \) are upper and lower limits of ramp rate of AGC unit \( i \).

Voltage magnitude constraints:
In [8], there has been a linear correspondence between voltage and active power fluctuations. Voltage variations are also related to reactive power, but assuming that the power factor of each bus remains unchanged, the relationship between reactive power and voltage can be expressed by linear correspondence of active power.
\[ v^\min_i \leq v_i \leq v^\max_i \]  
(14)  
\[ v^\min_{Li} \leq v_{Li} + \tilde{v}_{Li} \leq v^\max_{Li} \]  
(15)  
\[ \tilde{v}_L = (I^T \Delta d)A^v \alpha + B^v \Delta d \]  
(16)  

Where, \( v^\min_i, v^\max_i \) are minimum and maximum voltage magnitude of bus \( i \), \( v_i \) is voltage magnitude of bus \( i \), \( A^v, B^v \) are constant matrix in [8], index \( R, S, L \) represent respectively bus \( V \theta, PV, PQ \), \( \Delta d \) is disturbance of load, \( \tilde{v}_{Li} \) are voltage magnitude disturbance of bus \( i \).

Reactive power capacity constraints:

\[ q^\min_i \leq q_i + \Delta q_i \leq q^\max_i \]  
(17)  
\[ \Delta q_{Li} = \sigma \Delta d_{Li} \]  
(18)  
\[ \Delta q_{RSi} = (1^T \Delta d)A^q \alpha + B^q \Delta d \]  
(19)  

Where, \( \sigma \) is ratio of active and reactive power, \( q^\min_i, q^\max_i \) are minimum and maximum reactive power output of bus \( i \), \( q_i \) is reactive power of bus \( i \), \( \Delta q_i \) is reactive power disturbance of bus \( i \), \( A^q, B^q \) are constant matrix in [8].

Transmission constraints:

\[ -f^\max \leq \tilde{f} + f \leq f^\max \]  
(20)  
\[ \tilde{f} = (1^T \Delta d)A^f \alpha + B^f \Delta d \]  
(21)  
\[ f = gv - b\theta \]  
(22)  

Where, \( \tilde{f} \) are power flow disturbance, \( f^\min, f^\max \) are minimum and maximum power flow, \( f \) is power flow, \( A^f, B^f \) are constant matrix in [8], \( g, b \) are constant matrix in [3].

Formulas (1)-(22) above constitutes the real-time dispatch model mentioned above. Because of its non-deterministic nature, it is very difficult to solve. Next, it is transformed into a deterministic optimization model, which can be transformed into a simple LP problem to provide solution efficiency.

3. Solution approach

Solution of double objective function:

The optimization model has two objective functions with clear priority, so the following solution method is adopted. First, the optimal objective of the first function is obtained, then it is brought into the optimization of the next objective function.

\[ \sum_{i=1}^N (\min(\Delta d^\min_i, \Delta d^\max_i) + \min(\Delta d^\min_i, \Delta d^\max_i)) \geq Z^* \]  
(23)  

Where, \( Z^* \) is the optimal value obtained by solving the security objective function.

Solution of the first goal:

According to the treatment method in [12], we transform it into the following forms:
\[
\max Z = \sum_{i=1}^{N_a} \left( y_i^{up} + y_i^{dn} \right) 
\] (24)

\[
\begin{align*}
y_i^{up} &\leq \Delta d_i^{max} \\
y_i^{dn} &\leq \Delta d_i^{max} 
i = 1, 2, \ldots N_d
\end{align*}
\] (25)

\[
\begin{align*}
y_i^{dr} &\leq \Delta \tilde{d}_i^{max} \\
y_i^{dr} &\leq \Delta \tilde{d}_i^{max} 
i = 1, 2, \ldots N_d
\end{align*}
\] (26)

Where, \( y_i^{dn}, \ y_i^{up} \) are new introduced decision variables.

**Processing method of uncertainties in inequality constraints**

It can be seen that the constraints (17)-(22) contains uncertainties. In order to simplify the description, this paper only introduces the treatment of uncertainties in voltage constraints, reactive power constraints, power flow constraints and phase angle constraints, which are consistent with their treatment methods, and will not be repeated.

\[
\max \left\{ \sum_{j=1}^{N_l} (B_j^v + \sum_{i=1}^{N_a} A_j^v \cdot \alpha_i) \cdot \Delta \tilde{d}_j \right\} \leq v_i^{max} - v_i \quad l \in L 
\] (27)

Then, according to soyster’s robust processing method, new auxiliary decision variables are introduced to deal with the above formulas as follows.

\[
\begin{align*}
\sum_{j=1}^{N_l} &\left\{ (B_j^v + \sum_{i=1}^{N_a} A_j^v \cdot \alpha_i) (-\Delta \tilde{d}_j^{max}) + \lambda_{j}^{up} \right\} \leq v_i^{max} - v_i \\
\lambda_{j}^{up} &\geq (B_j^v + \sum_{i=1}^{N_a} A_j^v \cdot \alpha_i) (\Delta \tilde{d}_j^{max} + \Delta \tilde{d}_j^{max}) \\
\lambda_{j}^{up} &\geq 0, \quad j = 1, 2, \ldots N_d
\end{align*}
\] (28)

Where, \( \lambda_{j}^{dr}, \ \lambda_{j}^{up} \) are new introduced decision variables.

The accuracy of the method has been explained in the literature [12], so it will not be repeated here.

**4. Simulations**

In this section, the effectiveness of the proposed method is analyzed and validated by taking the modified IEEE-9-bus system as an example. The test calculation is based on the platform of MATLABR2014a, and the CPLEX solver is used to solve the problem. The computer is configured as Intel Core i5 6200U series CPU, the main frequency is 2.3 GHZ, and the memory is 8 G. Most of the information in the system can be referred to case 9.m in Matpower6.0, and other information is given by table 1.
Figure 1. Diagram of modified IEEE 9-bus system.

Table 1. Data parameters of modified the 9-bus system.

| AGC | cost | Ramp rate | Original active output power |
|-----|------|-----------|------------------------------|
| G1  | 0.7  | 0.1500    | 1.5                          |
| G2  | 0.65 | 0.1125    | 1.0                          |
| G3  | 1.35 | 0.0750    | 0.6                          |

* All data in the table are unitary values, in which the cost data is the three economic constants of generator generation in the model, in order to facilitate calculation and make it equal.

When the load fluctuation is set at 10%, the dispatching operation results are given in table 2. It can be seen that the model can optimize the base point, participation factors and reactive power synergistically. The running time of the program is 0.42540 s.

Table 2. Running results of modified the 9-bus system.

| G  | P   | Q    | α   |
|----|-----|------|-----|
| 1  | 1.36| 0.776| 0.4096 |
| 2  | 1.11| 1.309| 0.3543 |
| 3  | 0.675| 0.4  | 0.2362 |

The reference literature [3] has proved the accuracy of the method by some examples. Literature [8] illustrates the accuracy of the method through mathematical calculation. Here we use the IEEE 9-bus system verify it. RTDV is our proposed method. In order to facilitate verification, we set a deterministic model, that is, the load fluctuation is set to 0. AC curve was obtained by modified AC model based on Matpower6.0. In table 3 and figure 2, it can be seen that the method proposed has less error for voltage magnitude. In figure 3, we further prove that the accuracy of voltage is within a reasonable range by using the IEEE14-bus system and IEEE 118-bus system.

Table 3. Voltage magnitude comparison of modified IEEE 9-bus system.

| Bus | AC  | RTDV | error |
|-----|-----|------|-------|
| 4   | 0.989| 0.99575| -0.0068 |
| 5   | 0.977| 0.98673| -0.0097 |
| 6   | 1.004| 1.01031| -0.0063 |
| 7   | 0.987| 0.99565| -0.0086 |
| 8   | 0.997| 1.00605| -0.009 |
| 9   | 0.961| 0.9731| -0.0121 |
Figure 2. Voltage magnitude comparison of modified IEEE 9-bus system.

Figure 3. Voltage magnitude comparison of IEEE14-bus and IEEE 118-bus system.

In figure 4, the model proposed in this paper is compared with the model in reference [12]. The method in Document 12 is named RTD1. The method in this paper is named RTDV1 when \( v_{\text{max}} = 1.2 \), \( v_{\text{min}} = 0.85 \), and RTDV2 when \( v_{\text{max}} = 1.02 \), \( v_{\text{min}} = 0.97 \).

Figure 4. Variation curves of Z considering proportion of fluctuation.  
Figure 5. Expenses under different load fluctuation ranges.
It can be seen that the curve shapes of the two methods are approximately the same. Firstly, with the increase of load random fluctuation, the security region provided by the two methods is increasing, which shows that they have sufficient capacity to adapt to load fluctuation. However, with the increase of load fluctuation, the rate of curve increase decreases and remains unchanged until the end. When load fluctuation increases to a certain extent, all the spare capacity of AGC units has been put into operation, and the load fluctuation it could bear has reached the limit, so the size of the effective static safety zone will not change.

As can be seen from the figure 4, when the load fluctuation is less than 5%, the direction and size of the three curves are basically similar. This is because when the load fluctuation is less than 5%, the influence of voltage constraint and reactive power fluctuation on power flow constraint is not obvious. After 5%, contrast RTD and RTDV2, we could found that until the curve value remains unchanged, the proposed method has smaller security region than RTD method. This is because some constraints play a role, makes the set of adjustable units smaller. In finally, The Z of method 2 is the same as that of method 1 because the voltage amplitude constraint is relaxed and all spares can be transferred in the end. However, the Z value of method 3 is smaller than that of the former two, because the voltage amplitude is constrained tightly, so not all the spare inputs could be put into the end. The final difference is related to the setting of maximum and minimum value of voltage magnitude and reactive power constraints. The looser the conditions of voltage magnitude and reactive power constraints, the smaller the final difference.

Figure 5 shows that with the increase of load fluctuation, first, cost keeps increasing, then, it remains basically unchanged. Because all adjustment capacity has been exhausted with load fluctuation increasing. The results show that with the increase of load fluctuation, we need to spend more to ensure the safe operation of the system. But the regulation ability of the system is limited.

5. Conclusions
A linear real-time dispatch model considering voltage magnitude and reactive power is proposed in this paper. The operation base point, reactive power output and participation factors of AGC units are chosen as decision variables, which let the model take into account the interaction of active and reactive power. Compared with DC power flow model, the model in this paper has more accurate power flow expression and voltage expression with reasonable accuracy.

The modified IEEE 9-bus test system was used to confirm accuracy and rationality of the proposed model. It fully reflects the anti-jamming ability of power system in dealing with uncertainty, and provides a reasonable and effective real-time optimal dispatching method.

In future, the model proposed could be used to electricity markets, which make reactive power provided by generators be not only paid for as ancillary services.

Acknowledgments
This work was supported by a Science and Technology Project of State Grid Corporation of China (No.52060018000X)

References
[1] Gao Z et al 2017 Bidding strategy analysis of virtual power plant considering demand response and uncertainty of renewable energy IET Gener. Transm. Distrib. 11 3268-77
[2] Yang Z F et al 2017 Solving OPF using linear approximations: Fundamental analysis and numerical demonstration IET Gener. Transm. Distrib. 11 4115-25
[3] Yang J, Zhang N, Kang C et al 2016 A state-independent linear power flow model with accurate estimation of voltage magnitude IEEE Trans. Power Syst. 99 3607-17
[4] Akbari T and Bina M T 2016 Linear approximated formulation of AC optimal power flow using binary discretisation IET Gener. Transm. Distrib. 10 1117-23
[5] Fatemi S M, Abedi S, Gharahpetian G B et al 2014 Introducing a novel DC power flow method with reactive power considerations IEEE Trans. Power Syst. 30 1-12
[6] Yang Z, Zhong H, Bose A et al 2017 A linearized OPF model with reactive power and voltage magnitude: A pathway to improve the MW-Only DC OPF IEEE Trans. Power Syst. 2017 1
[7] Yang Z, Zhong H, Xia Q et al 2017 A novel network model for optimal power flow with reactive power and network losses Electr. Power Syst. Res. 144 63-71
[8] Duan C, Fang W, Jiang L et al 2017 Distributionally robust chance-constrained approximate AC-OPF with wasserstein metric arXiv:1706.05538.
[9] Wu F and Kumagai S et al 1982 Steady-State Security Regions of Power Systems IEEE Trans. Circuits Syst. Regul. Pap. 29 703-11
[10] Banakar M H and Galiana F D 1981 Power system security corridors concept and computation IEEE Trans. PAS 100 4524-32
[11] Zhu J, Fan R, Xu G et al 1998 Construction of maximal steady-state security regions of power systems using optimization method Electr. Power Syst. 44 101-5
[12] Yang M, Cheng F L, Han X S 2015 Real-time dispatch based on effective steady-state security regions of power system Proceedings of CSEE 35 1353-62