The Control Method of Multivariable Time-delay Square System Containing Right Half Plane Zeros

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Abstract

Aiming at the multivariable system containing the time delay and right half plane zeros in the industrial process, the Smith decoupling compensation control method is proposed in this paper. By using the internal model method, the Smith estimated compensator is designed, the multiple time delays problem is solved and the robustness and anti-interference performance of the system is improved. The model order reduction for the generalized controlled object which has been decoupled is carried out by means of the suboptimal reduction algorithm and the design process of the controller is simplified. The simulation results show that the system possesses the good performance of set value tracking, disturbance rejection and the robustness of overcoming the parameter variation.

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1. Introduction

The non-minimum phase system is widely existed in the control engineering, such as the hydro turbine process, the rectifying column process, boiler drum level process. The right half plane (RHP) zeros exists in the non minimum phase system, as a result, undershoots which has a great effect on the industrial control can be shown in the step response of the system. Moreover, the non-minimum phase system with...
time delays is relatively difficult to control due to the simultaneous existence of right half plane zeros and
time delay.

In the document [1], the expected form of diagonal transfer function matrix of system is put forward, and then the decoupling device matrix which can be steadily realized is fitted using the nominal transfer function equation of system. However, owing to the setting of transfer function matrix in this method, the derivation and computation process of designing the decoupling device is complicated.

In this paper, a new method is provided aiming at the multivariable time delay system which contains the RHP zeros. The full static decoupling is achieved by making use of the static decoupling approach, and the model order reduction for the generalized controlled object which has been decoupled is carried out. In addition, the Smith estimated compensator is designed using the internal model control (IMC) and the decoupling device and controller are combined into the decoupling controller. By means of this method, the design process of the controller becomes simple. Moreover, this method can be extended to the controlled system which contains any order multi-input multi-output.

2. Multivariable control system structure

2.1. The equivalent of multivariable IMC and Smith estimated compensation control

Consider the following multivariable system with time delay and RHP zeros, the process transfer matrix is shown as:

\[ G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1m}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ g_{m1}(s) & g_{m2}(s) & \cdots & g_{mm}(s) \end{bmatrix} \]

where the transfer function from \( i \)th input to \( j \)th output is \( g_{ij}(s) = g_{ij-}(s)e^{-\theta_{ij}s} \), \( g_{ij-}(s) \) is minimum phase part, \( g_{ij+}(s) \) is non-minimum phase part with RHP zeros, and \( \theta_{ij} \) is the corresponding time delay of process transmission.

The basic structure of IMC is shown in Fig.1. Where \( G_{nc}(s) \) is the internal model controller, \( G_m(s) \) is the process object, and \( G_m(s) \) respectively represent the desired input, the control output, the actual output and the model output. \( d \) is unpredictable external disturbance.

![Fig.1 The structure of IMC](image1)

![Fig.2 Equivalent IMC of Smith control structure](image2)

The internal model controller \( G_{nc}(s) \) is equivalently divided into the part surrounded by the dotted line in Fig.2. Hence, the structure of IMC in Fig.1 translates into Smith estimated compensation structure in Fig.2. The relationship of the Smith controller \( G_c(s) \) and the internal model controller \( G_{nc}(s) \) is given by:

\[ G_c(s) = \frac{G_{nc}(s)}{1-G_m(s)G_{nc}(s)} \]
2.2. The relationship between the Smith estimated compensator and the internal model controller

In this paper, the IMC method is introduced to design Smith estimated compensator based on the structural equivalent between the Smith estimated compensation system and the IMC system. In order to ensure the controller realized, the transfer function of the internal model controller is taken as:

\[ G_{nc}(s) = \frac{G_{nc}^{-1}(s)f(s)}{1 - G_{nc}^{-1}(s)f(s)G_{m0}(s)} \]  

(3)

Where \( G_{nc}(s) \) is the minimum phase part of the process model, and \( f(s) \) is the low-pass filter which choose one type of filter. Its transfer function is expressed as \( f(s) = 1/(\lambda s + 1)^n (n = 1, 2, 3, \ldots, n) \) where \( \lambda \) is the filter time constant. Substituting Eq. (3) into Eq. (2):

\[ G_c(s) = \frac{G_{nc}^{-1}(s)f(s)}{1 - G_{nc}^{-1}(s)f(s)G_{m0}(s)} \]

(4)

The typical second-order plus time delay process model is written as:

\[ G_m(s) = \frac{b_m}{s^2 + a_m s + b_m} e^{-\theta_s s} \]

(5)

Taken the filter \( f(s) = 1/(\lambda s + 1)^2 \) into Eq. (4), the Smith estimated compensator based on the principle of IMC is obtained:

\[ G_c(s) = \frac{s^2 + a_m(s) + b_m}{b_m \lambda e(\lambda s + 2)} \]

(6)

3. Decoupling control system design

3.1. Decoupling controller design

In the design of internal model controller, the transfer function of the decoupled generalized process model based on process model transfer matrix \( G_m(s) \) is obtained:

\[ G_D(s) = G_m(s)[G_m(s) = 0]^{-1} \]

(7)

When system reaches steady state, \( G_D(0) \) should be unit diagonal matrix, which can attains complete static decoupling of the system. In other words, the diagonal elements \( g_{Di}(s) \) of \( G_D(s) \) equal to 1, the off-diagonal elements \( g_{Dij}(s), (i \neq j) \) equate to 0.

The decoupled element \( g_{Di}(s) \) is complex expression mixed with delay factor. It is difficult to carry out coprime decomposition, and it is not easy to design controller by IMC method. Therefore using sub-optimal reduction algorithm [3], \( g_{Di}(s) \) is approximately equivalent to a stable model \( \hat{g}_{Di}(s) \) which is the typical first order or second-order model plus time delay. According to IMC method, the decoupled n-loop SISO controller can be designed.

The approximate model \( \hat{g}_{Di}(s) \) is decomposed as \( \hat{g}_{Di}(s) = \hat{g}_{Di-}(s)\hat{g}_{Di+}(s) \). Where \( \hat{g}_{Di-}(s) \) is minimum phase part, \( \hat{g}_{Di+}(s) \) is non-minimum phase part. Substituting \( \hat{g}_{Di-}(s) \) into Eq. (4):

\[ G_{Di}(s) = \frac{\hat{g}_{Di-1}(s)f(s)}{1 - \hat{g}_{Di-1}(s)f(s)\hat{g}_{Di0}(s)} = \frac{f(s)}{\hat{g}_{Di-}[1 - f(s)]} \]

(8)
Then, the controller of the decoupled generalized process object is given by

\[
G_{dc}(s) = \text{diag}[G_{dc,1}(s), \cdots, G_{dc,m}(s), G_{dc,n}(s)]
\]  

(9)

Accordingly, Smith decoupling controller is written as:

\[
G_c(s) = [G_m(s = 0)]^{-1} \cdot G_{dc}(s)
\]  

(10)

3.2. Feedback filter design

The feedback filter can solve the unstable problem which is caused by model mismatch and external disturbances. It is given as follow:

\[
G_f(s) = \text{diag} \left[ \frac{1}{\alpha_1 s + 1}, \cdots, \frac{1}{\alpha_{n-1} s + 1}, \frac{1}{\alpha_m s + 1} \right]
\]  

(11)

Here, \( \alpha_i \) is only one adjustable parameter of each loop. The value of \( \alpha_i \) is taken as the half of time delay of the loop of process objects.

4. Simulation examples

Consider the following example, and the process transfer function matrix is given as:

\[
G = \begin{bmatrix}
\frac{1}{s + 2} e^{-2s} & \frac{-1}{s + 2} e^{-6s} \\
\frac{s - 0.5}{(s + 2)^2} e^{-3s} & \frac{(s - 0.5)^2}{2(s + 2)^3} e^{-8s}
\end{bmatrix}
\]  

(12)

This process has a RHP zeros at \( s = 0.5 \). Assume the model exactly matches the process, the inverse of steady-state gain matrix of the process model is calculated as:

\[
[G_m(s = 0)]^{-1} = \begin{bmatrix}-0.2857 & -9.1429 \\
-2.2857 & -9.1429
\end{bmatrix}
\]  

(13)

The decoupled generalized process object is:

\[
G_D(s) = G_m(s) \cdot [G_m(s = 0)]^{-1} = \begin{bmatrix} g_{D11}(s) & g_{D12}(s) \\
g_{D21}(s) & g_{D22}(s) \end{bmatrix}
\]  

\[
= \begin{bmatrix}
\frac{-0.2847}{s + 2} e^{-2s} + \frac{2.2857}{s + 2} e^{-6s} & \frac{-9.1429}{s + 2} e^{-2s} + \frac{9.1429}{s + 2} e^{-6s} \\
\frac{-0.2857(s - 0.5)}{(s + 2)^2} e^{-3s} + \frac{-2.2857(s - 0.5)^2}{2(s + 2)^3} e^{-8s} & \frac{-9.1429(s - 0.5)}{(s + 2)^2} e^{-3s} + \frac{-9.1429(s - 0.5)^2}{2(s + 2)^3} e^{-8s}
\end{bmatrix}
\]  

(14)

The system achieved complete static decoupling. The diagonal elements of \( G_D(s) \) in Eq. (14) is approximated with second-order plus time delay model using suboptimal reduction algorithm. The approximate model is respectively written as:

\[
\hat{g}_{D11}(s) = \frac{0.2875}{s^2 + 0.7541s + 0.2875} e^{-4.1s}, \quad \hat{g}_{D22}(s) = \frac{0.8072}{s^2 + 0.9625s + 0.8072} e^{-3.5s}
\]  

(15)
Internal model controller is designed according to approximate modes $\hat{g}_{D1}(s)$ and $\hat{g}_{D2}(s)$. The controller of the decoupled generalized process object is obtained as:

$$G_{ic}(s) = \begin{bmatrix} s^2 + 0.2875s + 0.7541 & 0.8072 & 0.022875 \\ 0.2875s(\lambda_s + 2) & 0 & 0.8072s(\lambda_2s + 2) \\ \end{bmatrix}$$

(16)

Filter time constants of internal model controller are taken as $\lambda_1 = 9$ and $\lambda_2 = 20$. Finally, Smith decoupling controller is:

$$G_c(s) = \begin{bmatrix} -0.2857s^2 - 0.2154s - 0.08214 & -9.1429s^2 - 8.8s - 7.38 \\ 23.2875s^2 + 1.15s & 322.88s^2 + 32.288s \\ -2.2857s^2 - 1.724s - 0.6571 & -9.1429s^2 - 8.8s - 7.38 \\ 23.2875s^2 + 1.15s & 322.88s^2 + 32.288s \\ \end{bmatrix}$$

(17)

When $r_1 = 1$ and $r_2 = 0$, servo and interactive responses for nominal system are shown in Fig.3. When $r_1 = 0$ and $r_2 = 1$, shown in Fig.4. The simulation results shows that the method in this paper has small overshoots, fast tracking and less interactivity.

When $t = 100s$, 2 input 2 output channels were respectively added to a unit step disturbance in d of Fig.2. The response is shown in Fig.5. It can be seen that this method has good anti-interference performance.

To verify the system robustness, the value of static gain and time delay of each element in Eq. (12) are decreased by 20%. Servo and interactive responses are respectively shown in Fig.6 and 7. Accordingly, all are increased by 20%. Servo and interactive responses are respectively shown in Fig.8 and 9. When process plant mismatches with process model, the method adopted in the paper shows strong robustness and good tracking performance.

In this paper, ISE performance index is adopted to measure the performance of the design method [4]. The smaller the ISE value, the better the performance. Table1 lists the ISE value of nominal system and perturbed system.
Table 1 ISE value of system

|                  | $y_1 - r_1$ | $y_2 - r_1$ | $y_1 - r_2$ | $y_2 - r_2$ | sum      |
|------------------|-------------|-------------|-------------|-------------|----------|
| Nominal system   | 16.6204     | 0.0015      | 4.3093      | 28.3333     | 49.2645  |
| Perturbed system (-20%) | 18.9076     | 0.0008      | 4.0256      | 33.5153     | 56.4493  |
| Perturbed system (+20%) | 15.8019     | 0.0029      | 5.7142      | 25.1721     | 46.6911  |

5. Conclusions

This paper proposes a new method using IMC to design Smith estimated compensator for multivariable time delay system with RHP zeros. This method has good robustness and less interference performance of the internal model controller and also dynamically compensates for defect caused by static decoupling. It overcomes the impact of model mismatch on system performance by model approximation using suboptimal reduction algorithm. The simulation results show that the method in the paper has better tracking performance, less interference and stronger robustness.

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