Inspecting the Process of Bank Credit Rating via Visual Analytics

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ABSTRACT

Bank credit rating classifies banks into different levels based on publicly disclosed and internal information, serving as an important input in financial risk management. However, domain experts have a vague idea of exploring and comparing different bank credit rating schemes. A loose connection between subjective and quantitative analysis and difficulties in determining appropriate indicator weights obscure understanding of bank credit ratings. Furthermore, existing models fail to consider bank types by just applying a unified indicator weight set to all banks. We propose RatingVis to assist experts in exploring and comparing different bank credit rating schemes. It supports interactively inferring indicator weights for banks by involving domain knowledge and considers bank types in the analysis loop. We conduct a case study with real-world bank data to verify the efficacy of RatingVis. Expert feedback suggests that our approach helps them better understand different rating schemes.

Index Terms: Human-centered computing—Visualization

1 INTRODUCTION

Bank credit rating classifies commercial banks into different levels based on the basis of the analysis of both the publicly disclosed information and part of the internal bank information according to a certain quantitative standard [20, 24, 25, 28]. These levels will be succinctly represented by a combination of numbers and letters, which is convenient for supervisory agencies to conduct an intuitive judgment on the risks of commercial banks and adopt corresponding supervisory measures [11, 20]. Bank credit rating methodologies have gone through several stages such as subjective judgments, rule-based scorecard analysis, and computational modeling [2, 9]. Most of these approaches leverage rule-based measures combined with automated methods [11, 26] for credit rating assistance. Although these approaches have demonstrated promising performances, domain experts still have the following concerns when directly applying the existing credit rating models to their business scenarios. (1) Obscure understanding. One purpose of the domain experts is to generate an internal bank rating that can be produced consistently by different analysts using the same information with high reliability. Therefore, the methodologies should be intuitive and explainable to benefit from comments and suggestions from the experts advancing the bank credit risk assessment methodologies. (2) Loose connection between subjective and quantitative analysis. Previous bank rating models are extremely biased towards a heavy quantitative approach that requires many quantitative indicators. Therefore, they are not suitable for those imperfect credit systems and banks with limited financial indicators [11, 20]. Meanwhile, domain experts...
have their knowledge in the bank rating decision-making process. Integrating domain knowledge and experience with quantitative indicators can improve bank credit rating reliability. (3) **Unified indicator weights.** Existing credit rating methods apply a unified indicator weight set to all bank entities; however, according to the experts, different types of banks should be assigned to different weights even for the same indicator [5]. To tackle these issues, we identify domain experts’ primary concerns regarding bank credit rating and propose a visual analytics system, namely, RatingVis to assist them in exploring and comparing different bank credit rating schemes. Our primary contributions can be summarized as follows: 1) we elicit design requirements of bank credit rating from the literature and an observational study; 2) we propose interactive schemes to infer indicator weights; 3) we design a visual analytics system to help explore and compare different rating schemes.

2 Related Work

Bank credit rating plays an important role in solving information asymmetry between depositors and banks [22]. For example, CAMEL rating system [31] evaluates the capital adequacy, asset quality, management, earnings, and liquidity of financial institutions, and adopts a five-level scoring system to rate the operation and management level of commercial banks. Moody’s bank rating system [3] focuses on asset quality evaluation by understanding the bank’s internal loan approval policies and procedures, internal risk rating methods, the means of managing asset quality and risk to obtain bank-related information and rating data. Standard & Poor’s rating [21] of banks takes into account eight factors such as economic and industry risks, capital, and returns. However, there is no universal global standard for rating. Some methods reply on different data indicators, making it impossible to generalize to other scenarios. Moreover, experts’ knowledge of the credit risk is missing in these rating models. Our work integrates domain knowledge into rating and combines quantitative analysis with qualitative evaluation.

Entities for rating are inherently multi-attribute data items and several visualization systems that provide interactive multi-attribute data item ranking were proposed [4, 17, 23, 30, 32]. ValueCharts [4] and LineUp [8] allow analysts to create customized rankings by clicking and dragging attributes with adjustable attribute weights. However, they assume that users are able to quantify the importance of particular attributes. To resolve this issue, researchers studied what weight sets lead to a specific ranking. Podium [30] allows users to drag the table rows to rank data based on their perception of data value. WeightLifter [19] facilitates the exploration of weight spaces, thereby better understand the sensitivity of a decision to weight changes. The unified attribute weights derived by the above methods are not applicable to our case since banks with various types should be assigned different weights even for the same indicator.

3 Background and Observational Study

Ensuring the quality of bank rating has attracted attention from many parties. The world-famous rating agencies such as Moody’s and Standard & Poor’s evaluate the financial and operational strength and resilience of banks based on operation-related indicators and generate the letter-grade credit. To understand the existing practice of bank rating, we worked with one bank rating expert (E.1, male, age: 32), one risk management expert (E.2, male, age: 35), one financial data analyst (E.3, female, age: 24), and one bank credit expert (E.4, male, age: 28), who attempted to address the rating problem by ranking. They selected a set of attributes and employed certain multi-criteria decision-making schemes such as Analytic Hierarchy Process (AHP) [1] and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [13] in Statistical Product and Service Solutions (SPSS) for ranking. Based on the distribution of certain indicators, they divided the ranking into several segments. The banks in a higher up segment are considered better than those in a lower segment. Nevertheless, several issues arise. First, the ranking relies on distance functions to obtain the pairwise similarity. However, different indicators have different numerical scales, thereby requiring normalization. Nevertheless, even the experts who are very familiar with the data have difficulty in estimating the attribute weights after normalization. “When we classify banks, we can tell the approximate level of banks by observing certain attributes.” However, the subjective perception of attribute importance is “difficult to quantify”, making it challenging to take this intuition to verify the appropriateness of distance functions. Second, banks are of different types. For example, banks in China are classified into “large state-owned commercial banks”, “joint-stock commercial banks”, “city commercial banks”, and “rural commercial banks”. Generally, the credit rating of the four categories of banks shows a decreasing trend as a whole, but some low-level banks may perform better than high-level banks. In existing bank ratings, experts apply the same weight to the same indicator of different types of banks. However, they realized that “the importance of the same indicator among different types of banks should be different.” For instance, banks with a high liquidity ratio of assets indicate a strong financing ability, e.g., large state-owned commercial banks have many means to obtain funds while the financing channels of city commercial banks are narrow. Therefore, experts should pay attention to the liquidity ratio of city commercial banks. To sum up, we should meet three requirements: R.1 Infer indicator weights by interaction. Conventionally, the experts rank banks by assigning weights to quantify attribute contributions, by which they cannot efficiently determine which and to what extent certain attributes are important since they only have a holistic understanding of the data. Therefore, they required that the indicator importance could be interactively inferred. R.2 Refine indicator weights for different types of banks. Applying uniform weights to all kinds of banks may result in an irrational credit rating result. As indicated in [5], ratings can communicate externally as well the overall riskiness of banks’ assets, and “different risk weights are assigned to assets with different ratings according to Basel capital rules”. The experts required an interactive mechanism to refine indicator weights that consider bank types; R.3 Facilitate comparison among rating schemes. The experts wished to preserve previous schemes for comparison so they can understand whether the adjustment leads to a better result. Thus, our approach should facilitate comparison among different schemes.

4 Back-End Engine

In this section, we derive indicator weights and constraints to generate ranking scores for all items, followed by three ranking schemes. **Modeling Ranking SVM.** We adopt Ranking SVM [12, 32] to derive a set of indicator weights on the basis of the rankings in the data table [30], which optimizes a SVM hyperplane to the ranking problem with pairwise constraints. Instead of a full set of data with labels, a limited set of pairs of data $d_i$ and $d_j$ and a label are available to derive whether $d_i$ is better or not. The input of the model is difference vectors for data pairs, e.g., $d_i - d_j$, and the output of the model is which point is better [12]. Specifically, Ranking SVM transfers the pair of $(d_i, d_j)$ and their relative ranks to a tuple: $d_i - d_j = 1$ if $d_i$ is preferred; otherwise $d_i - d_j = -1$. The generated model predicts which one is better given a pair of points. To avoid an empty result, all constraints are modeled as soft constraints instead of hard ones so that user interaction can always generate a set of attribute weights that model constraints as much as possible [12].

**Deriving Constraints.** To obtain SVM’s linear separator, the ranking problem is transferred into a binary classification problem. Particularly, the labeled data is generated for Ranking SVM using the data items with which the user has interacted, e.g., dragging to a new position and there are $k$ marked rows. Without loss of generality, for $k$ points $(d_1, \ldots, d_k)$ with indices $[l_1, \ldots, l_k]$, the set of all combinations of pairwise difference vectors are created as...
We propose using the entropy of the data to derive the attribute weight vector after experimental analysis.

Calculating Ranking Score. After transforming user interaction and learning the Ranking SVM model, a weight vector \( w \) is obtained for us to rank the data items. We calculate individual dot products of \( w \) with each data item to generate a rank score as: \( r(d_i) = w \cdot d_i = \sum_{j=1}^{m} w_j d_{ij} \) with the highest one corresponding to the top rank.

We introduce three ranking schemes based on Ranking SVM: Scheme 1: Local Weight. We take original Ranking SVM and only consider the local information as the ranking scheme; Scheme 2: Global Weight. We generate a global weight sequence for all banks. When analysts drag the bank \( b_j \) to the position of \( k \), we sample a part of banks according to the proportion of each bank type in the original ranking from 1 to \( k \) and pair it with \( b_j \) to form a positive pair, i.e., the label of \( d_q - d_k \) \( (q \in [1, k-1]) \) is 1. Similarly, we sample banks to form a negative pair with \( b_j \) from \( k+1 \) to \( n \) according to the proportion of each bank type, i.e., the label of \( d_p - d_k \) \( (p \in [k+1, n]) \) is 1. In this way, we generate a general weight set suitable for different types of bank; Scheme 3: Type Weight. When generating pairwise training data, we take bank types into account and generate a set of weights for each bank type. Supposedly we have \( h = \{r_1, ..., r_i, ..., r_n \} \) that indicates bank \( b \) has \( m \) indicators and \( s(b) \) indicates the ranking score of bank \( b \) and we define \( \text{rankSeq} = c, b_1, ..., b_s, b_t, b_j \) as an increasing ranking order of banks, i.e., the number of banks is \( n \), among which \( s(b_1) > ... > s(b_t) > ... > s(b_j) \). We separate the ranking of different types of banks in \( \text{rankSeq} \) to form a ranking for each bank type. When analysts drag a bank \( b_j \) to the position of \( k \), we check the position of \( k \) of \( b_j \) in the ranking of each bank type and take 6 pairs near the position of \( k \), e.g., \( b_{k-1} \) is better than \( b_k \) and \( b_{k+1} \) is worse than \( b_k \). For each type, we generate a set of weights to calculate the ranking score of the banks of that type.

We adapt an entropy discretization method to transfer ranking to rating [7][16]. We first sort ranking scores and treat each score as a segmentation point, and then calculate the entropy of the left and right parts of each point. We consider the division with the minimum entropy as the first division. We repeat the above procedure until we obtain \( 2D \) projection by e.g., t-SNE [10][18][27][29][Fig. 1(A)]. Four subviews represent the projections corresponding to the attribute weight vectors from default, local, global, and type weight schemes. The colors indicate different ratings and we encode the bank asset size as the circle size. Analysts can lasso circles on any projection space and all identical ones will be connected via curves.

Ranking Comparison View. In previous schemes, different item and the dragged item form pairs of either positive or negative samples for training the Ranking SVM model, allowing analysts to understand what the impacts of different pairs on the ranking result are and how they affect the rating result (R.3). In [Fig. 1B], the ranking differences among schemes are represented by a parallel coordinate-like design, in which each axis represents a ranking scheme and each dot represents a bank. The black dot (dragged bank) is the item dragged by the analyst. Red dots indicate the negative samples while blue ones are the positive samples. We arrange the ranking sequentially (the smaller the better) from left to right and use lines to link all the identical banks across different schemes. For two adjacent ranking schemes, blue lines indicate an increasing ranking, and red lines indicate a decreasing ranking. We encode the bank types as background rectangles with different colors and discretize the ranking into 5 ratings. On the right side of [Fig. 1B], we use box plots to show indicator distribution of all positive and negative pairs. The color of each box plot represents one indicator. For one indicator, the left box plot indicates the indicator value distribution of negative pairs and the right box plot shows the indicator value distribution of positive pairs. The curve shows the attribute weight.

5 Front-End Visualization

We propose RatingVis to infer indicator weights by interactions and facilitate exploring and comparing bank credit rating schemes. Inspired by Lineup [8], we design Ranking Tabular View to help derive indicator weights from user interaction, evaluate the indicator contribution to the ranking, and support the comparison of different ranking schemes (R.1 – R.2). As shown in [Fig. 1C], we leverage a table to present the raw data. It displays the name of data items, rank, institution types, and the associated attributes. Analysts can perform a drag-and-drop operation to manually rank data based on their perception of the relative value of the data and domain knowledge. Green indicates the ranking is adjusted higher and red indicates the opposite adjustment; the deeper the color, the more the adjustment.

5 Case Study

Adjusting rankings for some banks. The experts first observed that Beijing Rural Commercial Bank belonging to “Rural Commercial Bank” in Default Scheme ranks higher with the rating of 2 (Fig. 2A)). Bank of Communications belongs to “Large State-owned Commercial Bank” but has a lower rating of 2. From [Fig. 2B], the experts witnessed that although the asset size of Beijing Rural Commercial Bank is low (8,811), the provision coverage is high (1068.87%), and that is why it is relatively high in the ranking. On the contrary, although the asset size of Bank of Communications is relatively high (95,312), its provision coverage is much lower (173.13%), leading to a low ranking. The experts commented that according to the bank’s previous performance and their expertise,
the current ranking of Beijing Rural Commercial Bank is higher than expected, although it has excellent provision coverage, the asset size is worse than that of the banks ranked near it. Similarly, the ranking of Bank of Communications is also lower than expected. “After all, Bank of Communications is one of the six most famous state-owned banks in China,” said E.1, and “its asset size is strong”. The experts then manually adjusted the ranking of the two banks, “I will drag the bank to a ranking that meets my psychological expectation and also consider the bank type.” E.1 witnessed that the banks ranked 15 and 16 are “Rural Commercial Bank” which belong to the same bank type with Beijing Rural Commercial Bank. Meanwhile, E.1 thought that Beijing Rural Commercial Bank is better than them (banks at the position of 15 and 16) and is also better than Guangzhou Bank (at the position of 14) in terms of asset size. Therefore, E.1 decided to move Beijing Rural Commercial Bank from its current position of 5 to 13 and Bank of Communications from its current position of 8 to 5. They then clicked on the “Save Weight Scheme” button and the system generated a new weight set, which is applied to generate three new rankings. The adjusted result is shown in Fig. 1(C).

Comparing ranking schemes. The experts found that in Type Weight some banks with a rating of 2 are close to the banks with a rating of 1 but they are not very close in the ranking tabular view (Fig. 1A)). With curiosity, E.1 selected Beijing Rural Commercial Bank in Fig. 1(C), and observed the ranking differences in different ranking schemes and how indicator contribution affects the overall ranking in each scheme. In Fig. 3 the experts further identified that due to different schemes of sampling positive and negative pairs, the value distribution (the distance between the upper and lower edges of the box plot) in Local Weight and the height of the box plot are more concentrated than those in Global Weight, which makes sense since in Local Weight, both positive and negative samples are selected near the dragged bank, compared with the random selection of positive and negative samples in Global Weight. Particularly, in Local Weight, indicators such as capital adequacy ratio are more important while in Global Weight, indicators such as asset size, capital adequacy ratio, and capital profit ratio are emphasized.

Identifying critical indicators for ranking different types of banks. From Fig. 4 in Default Scheme, Beijing Rural Commercial Bank belongs to Rural Commercial Bank and it is located around large state-owned commercial banks, which is not reasonable. From Fig. 3 the experts confirmed that the indicator weights of large state-owned commercial banks mainly lie in asset size, while for rural commercial banks, the non-performing loans ratio matters. E.1 explained “smaller banks tend to lend more loans with risks, leading to a high non-performing loans ratio, while the risk management of large banks is good and lending is conservative. So non-performing loans ratio is smaller”. Thus, the non-performing loans ratio are not significant for distinguishing large state-owned commercial banks, leading a smaller weight. The experts further noticed that in the axis of Type Weight, Bank of Communications has a rating of 1, ranking the 5th and Beijing Rural Commercial Bank has a rating of 2 with ranking 21st; however, in Default Scheme, Bank of Communications has a rating of 2, ranking the 8th and Beijing Rural Commercial Bank has a rating of 2, ranking 5th. This observation demonstrates that the performance of Bank of Communications in Type Weight is improved while the performance of Beijing Rural Commercial Bank is worse, which is consistent with the adjustment by the experts, i.e., E.1 adjusted the ranking of Beijing Rural Commercial Bank to the 13th, although in Type Weight, the ranking is recommended as the 21st. The experts also observed that the ranking of Taizhou Bank varies among different schemes. As shown in Fig. 1(C), E.1 found that although the performance of Taizhou Bank in asset size is on average, it performs well in terms of provision coverage, asset profit ratio, and capital profit ratio, leading to overall good performance in Default Scheme (Fig. 4). Similarly, the rankings of Taizhou Bank in Local Weight and Global Weight are all highly ranked. However, E.1 thought the rankings of Taizhou Bank were seriously overestimated in the first three schemes and he was satisfied with the ranking in Type Weight. On the contrary, the experts identified the rankings of Ping’an Bank in the first three schemes were underrated (Fig. 4), but “it actually has a good credit rating,” said E.1. He commented that the rating in Type Weight (rating of 2) is more in line with his expectation, confirming his intuition that “the weights of different bank types should be treated separately.”

7 CONCLUSION AND FUTURE WORK
We propose RatingVis to facilitate domain experts exploring and comparing different bank credit rating schemes. A ranking tabular view deduces the indicator weights based on user interaction; a projection view visualizes the ranking data distributions generated by different ranking schemes, and a ranking comparison view compares the ranking schemes at an instance level. A case study verifies the efficacy of RatingVis. In the future, we will consider attributes with various types and explore more ranking schemes for comparison.

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