Canonical interpretation of the $\eta_2(1870)$

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Abstract

We argue that the mass, production, total decay width, and decay pattern of the $\eta_2(1870)$ do not appear to contradict with the picture of it being the conventional $2^1D_2$ $q\bar{q}$ state. The possibility of the $\eta_2(1870)$ being a mixture of the conventional $q\bar{q}$ and the hybrid is also discussed.

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1 Introduction

In $\gamma\gamma$ reactions, an isospin zero $2^+\pi^\pm$ resonance at about 1870 MeV in the $\eta\pi\pi$ channel was reported by the Crystal Ball Collaboration and the CELLO Collaboration[1, 2]. Subsequently, the Crystal Ball Collaboration presented a more detailed analysis of this resonance and its mass and width are determined to be about 1881 MeV and 221 MeV, respectively[3]. In $\bar{p}p$ annihilation, the Crystal Barrel Collaboration reported an isospin zero $2^+\pi^\pm$ resonance with a mass of about 1875 MeV and a width of about 200 MeV in the $f_2(1270)\eta$ channel[4]. In radiative $J/\psi$ decays, the BES Collaboration reported a definite $2^+\pi^\pm$ signal with a mass of about 1840 MeV and a width of about 170 MeV in the $\eta\pi\pi$ channel[5]. In central production, a similar $2^+\pi^\pm$ resonance was observed by the WA102 Collaboration in the $a_2(1320)\pi$, $f_2(1270)\eta$, and $a_0(980)\pi$ channels[6, 7, 8]. Using high statistics data on $\bar{p}p \rightarrow \eta\pi\pi\pi\pi$, Anisovich et al. confirmed the presence of an isospin zero $2^+\pi^\pm$ resonance with a mass of about 1860 MeV and a width of about 250 MeV in the $a_2(1320)\pi$, $f_2(1270)\eta$, and $a_0(980)\pi$ channels[9]. It has been established that these observations in different experiments refer to a single state $\eta_2(1870)[10, 11]$, although this state is stated to need confirmation[11]. The mass and width of the $\eta_2(1870)$ are quoted to be $1842 \pm 8$ MeV and $225 \pm 14$ MeV, respectively[11].

With the $\eta_2(1645)$ as the well-established $1^1D_2 q\bar{q}$ state[11], the $\eta_2(1870)$ looks like either a $1^1D_2 s\bar{s}$ [$\eta_2(1Ds\bar{s})$] or a $n\bar{n}$ hybrid [$\eta_2(Hn\bar{n})$] ($n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}$) based on its mass, because the observed mass of the $\eta_2(1870)$ just overlaps the Godfrey-Isgur quark model prediction of 1.89 GeV for the $\eta_2(1Ds\bar{s})$[12] and the flux-tube model prediction of $1.8 - 1.9$ GeV for the $\eta_2(Hn\bar{n})$[13]. Of course the strong $a_2(1320)\pi$ and $f_2(1270)\eta$ modes are not expected from $s\bar{s}$ and hence may imply large $n\bar{n} \leftrightarrow s\bar{s}$ flavor mixing in the $\eta_2(1870)$ if the $\eta_2(1870)$ is indeed a $1^1D_2 q\bar{q}$ state. The near degeneracy of the $\eta_2(1645)$ and $\pi_2(1670)$ suggests the ideal mixing in the $1^1D_2$ meson nonet, which disfavors the large flavor mixing in the $\eta_2(1870)$ qualitatively. Also, the calculations from the $^3P_0$ model quantitatively argue against assigning the $\eta_2(1870)$ to the $1^1D_2 n\bar{n} \leftrightarrow s\bar{s}$ mixed quark model state[14]. A feature of $\bar{p}p$ annihilation is that the well-known $s\bar{s}$ state such as the $f_2(1525)$ are produced very weakly, if at all[10]; the $\eta_2(1870)$, in contrast, is produced strongly, which makes the $\eta_2(1Ds\bar{s})$ interpretation for the $\eta_2(1870)$ unlikely.
Therefore, both the decay modes and production information for the $\eta_2(1870)$ do not favor it being the $\eta_2(1Dss)$. Apart from the $\eta_2(1870)$ mass, it dominantly decaying to the $a_2(1320)\pi$ and $f_2(1270)\eta$ is also in accord with the flux-tube model expectation for the $\eta_2(Hn\bar{n})$ where the preferred decay channels are to $P + S$-wave pairs[15, 16, 17]. In addition, the discovery of the $\eta_2(2030)$ in $\bar{p}p$ annihilation[9, 18] to some extent leaves the $\eta_2(1870)$ as an ‘extra’, i.e., non-$q\bar{q}$ state[9] since the $\eta_2(2030)$ looks like a natural candidate for the $\eta_2(1645)$’s first radial excitation from its mass which is close to the Godfrey-Isgur quark model prediction of 2.13 GeV[12] for the $2^1D_2$ $n\bar{n}$ state [$\eta_2(2Dn\bar{n})$]. So, the hybrid interpretation for the $\eta_2(1870)$ becomes a popular opinion[3, 9, 10, 14, 15, 16, 17, 19, 20]. Apart from the $\eta_2(1870)$, its companion $\pi_2(1880)$ was also regarded to be a viable non-exotic hybrid candidate[10, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24].

Although the hybrid interpretation for the $\pi_2(1880)$ and $\eta_2(1870)$ has several attractive features, it is necessary to exhaust their possible conventional $q\bar{q}$ descriptions before resorting to more exotic interpretations. In fact, the observation of the $\pi_2(1880)$ in the $\rho\omega$ and the $f_2(1270)\pi$ $D$-wave channels strongly casts doubt over the hybrid interpretation for the $\pi_2(1880)$ since the $\rho\omega$ is expected to vanish and the $f_2(1270)\pi$ $D$-wave is strongly suppressed for the hybrid[17]. In our previous work[25], we argued that the experimental evidence for the $\pi_2(1880)$ is consistent with it being the conventional $2^1D_2$ meson rather than the $2^{-+}$ light hybrid by investigating its strong decay properties. If the $\pi_2(1880)$ can be described as the ordinary $2^1D_2$ meson, one natural question is whether its companion $\eta_2(1870)$ could also be the ordinary $2^1D_2$ meson or not. In this work, we shall discuss the possibility of the $\eta_2(1870)$ being the $\eta_2(2Dn\bar{n})$ from its mass, production, total width, and strong decay pattern.

The organization of this paper is as follows. In Sections 2-3, we discuss the mass and production properties of the $\eta_2(1870)$. In Sect. 4, after a brief review of the $^3P_0$ model and the flux-tube model used in this work, we present the partial decay widths of the $\eta_2(1870)$ as the $\eta_2(2Dn\bar{n})$ within these two models. The discussions and conclusion are given in Sections 5-6, respectively.
2 Mass

Godfrey-Isgur quark model predicted that the $\eta_2(2Dn\bar{n})$ mass is about 2.13 GeV\cite{12}, about 250 MeV higher than the $\eta_2(1870)$ mass. The $\eta_2(1870)$ therefore appears too light to be the $\eta_2(2Dn\bar{n})$ at first glance. However, it should be noted that the $a_1(1700)$ and $a_2(1700)$, both about 100-200 MeV lower in mass than the Godfrey-Isgur quark model anticipated\cite{12}, turn out the excellent candidates for radial excitations\cite{16, 17}, which indicates that Godfrey-Isgur quark model maybe overestimates the masses of the higher-$L$ radially excited mesons by about 100-200 MeV\cite{26}. So the $\eta_2(2Dn\bar{n})$ with a mass about 1.9 GeV is presumably not implausible. Also, the isovector state should act as a beacon for the mass scale of a meson nonet. If the $\pi_2(1880)$ can be identified as the isovector member of the $2^1D_2$ $q\bar{q}$ nonet\cite{25}, the $\eta_2(2Dn\bar{n})$ would be the orthogonal partner of the $\pi_2(1880)$ and one can naturally expect that the $\eta_2(2Dn\bar{n})$ degenerates with the $\pi_2(1880)$ in effective quark masses. The similar behavior also exists in the established $1^1D_2$ and $1^3D_3$ meson nonets\cite{11}. Recently, different approaches already consistently suggested that the $\eta_2(2Dn\bar{n})$ has a mass of about 1.9 GeV, close to the $\eta_2(1870)$ mass. For example, the Vijande-Fernandez-Valcarce quark model predicted $M_{\eta_2(2Dn\bar{n})} = 1.863$ GeV\cite{27}, the spectrum integral equation expected $M_{\eta_2(2Dn\bar{n})} = 1.937$ GeV\cite{28}, and the Mezoir-Gonzalez quark model found $M_{\eta_2(2Dn\bar{n})} = 1.913$ GeV\cite{29}. Therefore, the assignment of the $\eta_2(1870)$ as the $\eta_2(2Dn\bar{n})$ does not appear to be irrational based on its mass.

3 Production

For central production, Close and Kirk have found a kinematic filter that seems to suppress the well-established $q\bar{q}$ states when they are in $P$ and higher waves\cite{30}. Its essence is that the pattern of resonances produced in the central production process depends on $dp_T = |\vec{k}_{T1} - \vec{k}_{T2}|$, the vector difference of the transverse momentum recoil of the final state protons. It has been illustrated in several channels that for $dp_T$ large the $q\bar{q}$ states are prominent whereas for $dp_T$ small all the undisputed $q\bar{q}$ states are suppressed while the enigmatic states probably having more complex structures such as the $f_0(1500)$, $f_0(1700)$, and $f_0(980)$ survive\cite{31}. The application of this kinematic filter to the centrally produced $K\bar{K}\pi$ system, where the $f_1(1285)$ and $f_1(1420)$
have the same behavior as a function of the $dp_T$, successfully established the $f_1(1420)$ as the $^3P_1$ $q\bar{q}$ states[32]. At one time, the $f_1(1420)$ was interpreted as either a hybrid[33], a four quark state[34], or a $K^*K$ molecule[35].

In central production both the $\eta_2(1645)$ and the $\eta_2(1870)$ were clearly observed, furthermore they exhibit the same behavior as a function of the $dp_T$, appearing sharply when $dp_T > 0.5$ GeV, and vanishing as $dp_T \to 0$ GeV [6, 7] (see Table 2 of Ref.[6] and Table 2 of Ref.[7]), as do other well-established $q\bar{q}$ states such as the $f_1(1285)$ and $f_1(1420)$. This strongly suggests that the $\eta_2(1870)$ has the same dynamical structure as the $\eta_2(1645)$, namely the standard $2^{-+} q\bar{q}$.

As mentioned above, the production in $\bar{p}p$ annihilation process argues against the $s\bar{s}$ interpretation of the $\eta_2(1870)$. Therefore, with the $\eta_2(1645)$ as the well-established $1^1D_2 n\bar{n}$ state, the production properties of the $\eta_2(1870)$ are consistent with it being the $\eta_2(2Dn\bar{n})$.

4 Decay

4.1 The $^3P_0$ model and the flux-tube model

The $^3P_0$ model and the flux-tube model which are the standard models for strong decays at least for mesons in the initial state, have been widely used to evaluate the strong decays of hadrons[14, 16, 25, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45], since they give a good description of many of the observed decay amplitudes and partial widths of the hadrons. Below, we shall give the brief review of the two models employed in this work.

4.1.1 The $^3P_0$ model of meson decay

The $^3P_0$ model, also known as the quark-pair creation model, was originally introduced by Micu[46] and further developed by Le Yaouanc et al.[36]. The main assumption of the $^3P_0$ model of meson decay is that strong decays take place via the creation of a $^3P_0$ quark-antiquark pair from the vacuum. The newly produced quark-antiquark pair $(q_3\bar{q}_4)$, together with the $q_1\bar{q}_2$ within the initial meson, regroups into two outgoing mesons in all possible quark rearrangement ways, which corresponds to the two decay diagrams as shown in Fig.1 for the meson decay process $A \to B + C$. 
Figure 1: The two possible diagrams contributing to $A \to B + C$ in the $^3P_0$ model.

The transition operator $T$ of the decay $A \to BC$ in the $^3P_0$ model is given by

$$T = -3\gamma \sum_m \langle 1m1 - m0 \rangle \int d^3\vec{p}_3 d^3\vec{p}_4 \delta^3(\vec{p}_3 + \vec{p}_4) \mathcal{Y}_1^m \left( \frac{\vec{p}_3 - \vec{p}_4}{2} \right) \chi_{11-m}^{34} \phi_0^{34} \omega_0^{34} b_3^4 (\vec{p}_3) d_4^4 (\vec{p}_4),$$

(1)

where $\gamma$ is a dimensionless parameter representing the probability of the quark-antiquark pair $q_3 \bar{q}_4$ with $J^{PC} = 0^{++}$ creation from the vacuum, and $\vec{p}_3$ and $\vec{p}_4$ are the momenta of the created quark $q_3$ and antiquark $\bar{q}_4$, respectively. $\phi_0^{34}$, $\omega_0^{34}$, and $\chi_{11-m}^{34}$ are the flavor, color, and spin wave functions of the $q_3 \bar{q}_4$, respectively. The solid harmonic polynomial $\mathcal{Y}_l^m(\vec{p}) \equiv |p|^l Y_l^m(\theta_p, \phi_p)$ reflects the momentum-space distribution of the $q_3 \bar{q}_4$.

For the meson wave function, we adopt the mock meson $|A(n_A^{2S_A+1} L_A J_A M_J_A)(\vec{P}_A)\rangle$ defined by[47]

$$|A(n_A^{2S_A+1} L_A J_A M_J_A)(\vec{P}_A)\rangle \equiv \sqrt{2E_A} \sum_{M_{LA},M_{SA}} \langle L_A M_{LA}, S_A M_{SA} | J_A M_{J_A} \rangle$$

$$\times \int d^3\vec{P}_A \psi_{n_A L_A M_{LA}}(\vec{P}_A) \chi_{S_A M_{SA}}^{12} \phi_A^{12} \omega_A^{12}$$

$$\times |q_1 \left( \frac{m_1}{m_1 + m_2} \vec{p}_A + \vec{p}_A \right) q_2 \left( \frac{m_2}{m_1 + m_2} \vec{p}_A - \vec{p}_A \right)\rangle,$$

(2)

where $m_1$ and $m_2$ are the masses of the quark $q_1$ with a momentum of $\vec{p}_1$ and the antiquark $\bar{q}_2$ with a momentum of $\vec{p}_2$, respectively. $n_A$ is the radial quantum number of the meson $A$ composed of $q_1 \bar{q}_2$. $S_A = \vec{s}_{q_1} + \vec{s}_{q_2}$, $J_A = \vec{L}_A + \vec{S}_A$, $\vec{s}_{q_1}$ ($\vec{s}_{q_2}$) is the spin of $q_1$ ($\bar{q}_2$), and $\vec{L}_A$ is the relative orbital angular momentum between $q_1$ and $\bar{q}_2$. $\vec{P}_A = \vec{p}_1 + \vec{p}_2$, $\vec{P}_A = \frac{m_2 \vec{p}_1 - m_1 \vec{p}_2}{m_1 + m_2}$.

$\langle L_A M_{LA}, S_A M_{SA} | J_A M_{J_A} \rangle$ is a Clebsch-Gordan coefficient, and $E_A$ is the total energy of the meson $A$. $\chi_{S_A M_{SA}}^{12}$, $\phi_A^{12}$, $\omega_A^{12}$, and $\psi_{n_A L_A M_{LA}}(\vec{P}_A)$ are the spin, flavor, color, and space wave functions of the meson $A$, respectively. The mock meson satisfies the normalization condition

$$\langle A(n_A^{2S_A+1} L_A J_A M_J_A)(\vec{P}_A)|A(n_A^{2S_A+1} L_A J_A M_J_A)(\vec{P}_A')\rangle = 2E_A \delta^3(\vec{P}_A - \vec{P}_A').$$

(3)

The $S$-matrix of the process $A \to BC$ is defined by

$$\langle BC|S|A\rangle = I - 2\pi i \delta(E_A - E_B - E_C) \langle BC|T|A\rangle,$$

(4)
with

\[ \langle BC|T|A \rangle = \delta^3(\vec{P}_A - \vec{P}_B - \vec{P}_C) \mathcal{M}^{M_A M_B M_C}_{(3P_0)}(\vec{P}), \]

where \( \mathcal{M}^{M_A M_B M_C}_{(3P_0)} \) is the helicity amplitude of \( A \to BC \). In the center of mass frame of meson \( A \), \( \mathcal{M}^{M_A M_B M_C}_{(3P_0)} \) can be written as

\[
\mathcal{M}^{M_A M_B M_C}_{(3P_0)}(\vec{P}) = \gamma \sqrt{SE_A E_B E_C} \sum_{M_{L_A}, M_{L_B}, M_{L_C}, M_{S_A}, M_{S_B}} \langle L_A M_{L_A} S_A M_{S_A}|J_A M_{J_A}\rangle \\
\times \langle L_B M_{L_B} S_B M_{S_B}|J_B M_{J_B}\rangle \langle L_C M_{L_C} S_C M_{S_C}|J_C M_{J_C}\rangle \\
\times \langle 1m1 - m|00\rangle \chi_{S_B M_{S_B}}^{14} \chi_{S_C M_{S_C}}^{32} \chi_{S_A M_{S_A}}^{12} \chi_{1-m}^{34} \\
\times \left[ f_1 I_{(3P_0)}(\vec{P}, m_1, m_2, m_3) \\
+ (-1)^{1+S_A+S_B+S_C} f_2 I_{(3P_0)}(-\vec{P}, m_2, m_1, m_3) \right],
\]

(6)

with \( f_1 = \langle \phi_B^{14} \phi_C^{32} \phi_A^{12} \phi_0^{34} \rangle \) and \( f_2 = \langle \phi_B^{32} \phi_C^{14} \phi_A^{12} \phi_0^{34} \rangle \), corresponding to the contributions from Figs. 1 (a) and 1 (b), respectively, and

\[
I_{(3P_0)}(\vec{P}, m_1, m_2, m_3) = \int d^3\vec{p} \psi_n^* \psi_{nC} \left[ \frac{m_3}{m_1+m_3} \vec{P}_B + \vec{p} \right] \psi_{nC} \psi_{nA} \left[ \vec{P}_B + \vec{p} \right] \\
\times \psi_{nA} \psi_{nC} \psi_{nL} \left( \vec{P}_B + \vec{p} \right) \gamma_1^m (\vec{p}),
\]

(7)

where \( \vec{P} = \vec{P}_B = -\vec{P}_C, \vec{p} = \vec{p}_3, m_3 \) is the mass of the created quark \( q_3 \), and the \( \psi \)'s are the relative wave functions in momentum space.

The spin overlap in terms of Wigner’s 9j symbol can be given by

\[
\langle \chi_{S_B M_{S_B}}^{14} \chi_{S_C M_{S_C}}^{32} | \chi_{S_A M_{S_A}}^{12} \chi_{1-m}^{34} \rangle = \\
\sum_{S, M_S} \langle S_B M_{S_B} S_C M_{S_C}|SM_S\rangle \langle S_A M_{S_A} 1 - m|SM_S\rangle \\
\times (-1)^{S_C+1} \sqrt{3(2S_A + 1)(2S_B + 1)(2S_C + 1)} \left\{ \begin{array}{ccc} \frac{1}{2} & \frac{1}{2} & S_A \\ \frac{1}{2} & \frac{1}{2} & 1 \\ S_B & S_C & S \end{array} \right\}.
\]

(8)

In order to compare with the experiment conventionally, \( \mathcal{M}^{M_A M_B M_C}_{(3P_0)}(\vec{P}) \) can be converted into the partial amplitude by a recoupling calculation[48]

\[
\mathcal{M}^{S}_{(3P_0)}(\vec{P}) = \sum_{M_B, M_C} \langle LM_L S_M S|J_A M_{J_A}\rangle \langle J_B M_{J_B} J_C M_{J_C}|SM_S\rangle \\
\times \mathcal{M}^{M_B M_C}_{(3P_0)}(\vec{P}) \\
\times (-1)^{S_C+1} \sqrt{3(2S_A + 1)(2S_B + 1)(2S_C + 1)} \left\{ \begin{array}{ccc} \frac{1}{2} & \frac{1}{2} & S_A \\ \frac{1}{2} & \frac{1}{2} & 1 \\ S_B & S_C & S \end{array} \right\}.
\]

(9)
\[ \times \int d\Omega Y_{LML}^* M_{J_A}^{M_J} M_{J_B}^{M_J} M_{J_C}^{M_J} (\vec{P}). \]  

If we consider the relativistic phase space, the decay width \( \Gamma_{(3P_0)}(A \rightarrow BC) \) in terms of the partial wave amplitudes is

\[ \Gamma_{(3P_0)}(A \rightarrow BC) = \frac{\pi P}{4M_A^2} \sum_{LS} |M_{LS}^{(3P_0)}|^2. \]

Here \( P = \sqrt{(M_A^2 - (M_B + M_C)^2)(M_A^2 - (M_B - M_C)^2)} \), and \( M_A, M_B, \) and \( M_C \) are the masses of the meson \( A, B, \) and \( C \), respectively.

The simple harmonic oscillator (SHO) approximation for the meson space wave functions is used. This is typical of decay calculations and it has been demonstrated that using the more realistic space wave functions, such as those obtained from Coulomb, plus the linear potential model, does not change the results significantly\[41, 42, 43\]. Under the SHO wave function approximation, the partial amplitudes and partial widths for \( A \rightarrow BC \) can be calculated analytically based on relations (9) and (10), respectively.

In momentum space, the SHO wave function is

\[ \psi_{nLM}(\vec{p}) = R_{nL}^{SHO}(p) Y_{LM}(\Omega_p), \]

where the radial wave function is given by

\[ R_{nL}^{SHO}(p) = \frac{(-1)^n(-i)^L}{\beta^{3/2}} \sqrt{\frac{2n!}{\Gamma(n + L + \frac{3}{2})}} \left( \frac{p}{\beta} \right)^L e^{-p^2/(2\beta^2)} L_n^{L+1/2}(p^2/\beta^2). \]

Here \( \beta \) is the SHO wave function scale parameter, and \( L_n^{L+1/2}(p^2/\beta^2) \) is an associated Laguerre polynomial.

**4.1.2 The flux-tube model of meson decay**

The flux-tube model is based on the strong-coupling Hamiltonian lattice formulation of QCD\[41\]. In the flux-tube model, a meson \( A \) consists of a quark \((q_1)\) and antiquark \((\bar{q}_2)\) connected by a tube of chromoelectric flux. Meson decay occurs when the flux-tube breaks at a point along its length, and a quark-antiquark pair \((q_3\bar{q}_4)\) is created from the vacuum to connect to the free ends of the flux-tubes, leaving a final state consisting of two mesons \( B \) and \( C \).
The flux-tube model of meson decay is similar to the \( ^3P_0 \) model, but extends the \( ^3P_0 \) model by considering the actual dynamics of the flux-tubes. This is done by including a flux-tube overlap function that represents the overlap of the flux-tube of the initial meson \( A \) with those of the two outgoing mesons \( B \) and \( C \). The flux-tube overlap function reflects the spatial dependence of the pair-creation amplitude. For the conventional \( q \bar{q} \) meson decay, the flux-tube overlap is usually chosen as the following form[41]

\[
\gamma(\vec{r}_A, \vec{y}) = \gamma_0 \exp \left( -\frac{1}{2} b \vec{y}_\perp^2 \right). \tag{13}
\]

Here \( \gamma_0 \) is the pair-creation constant, \( b \) is the string tension, \( \vec{y} \) is the pair (\( q_3 \bar{q}_4 \)) creation position, \( \vec{y}_\perp = -(\vec{y} \times \vec{r}_A) \times \vec{r}_A \), and \( \vec{r}_A \) is the antiquark-quark axes of meson \( A \) (see Fig. 2).

![Figure 2: \( A \to B + C \) in the flux-tube model.](image)

The expression for \( M_{(\tau)}^{M_A M_B M_C}(\vec{P}) \), the amplitude of \( A \to BC \) in the flux-tube model is the same as that of relation (6) except that the \( \gamma \) is replaced by the \( \gamma_0 \) and \( I^{(\gamma_0)}(\vec{P}, m_1, m_2, m_3) \) is replaced by

\[
I_{(\tau)}(\vec{P}, m_1, m_2, m_3) = -\frac{8}{(2\pi)^{3/2}} \int d^3\vec{r} \int d^3\vec{y} \psi^{*}_{n_B L_B M_B}(\vec{r}_B) \psi^{*}_{n_C L_C M_C}(\vec{r}_C) \\
\times Y_1^m \left( (\vec{P} + i \vec{y}_\perp) \psi_{n_AL_AM_A}(\vec{r}_A) \right) \exp \left( -\frac{1}{2} b \vec{y}_\perp^2 \right) \\
\times \exp \left( i \vec{P} \cdot (m_+ \vec{r} + m_- \vec{y}) \right), \tag{14}
\]

where \( \vec{r}_A = -2\vec{r} \), \( \vec{r}_B = -\vec{y} - \vec{r} \), and \( \vec{r}_C = \vec{y} - \vec{r} \) as shown in Fig. 2. \( m_+ = \frac{m_1}{m_1 + m_3} + \frac{m_2}{m_2 + m_3} \), \( m_- = \frac{m_1}{m_1 + m_3} - \frac{m_2}{m_2 + m_3} \), and the \( \psi \)'s are now the relative wave functions in position space.

As in the \( ^3P_0 \) model, the SHO wave function approximation for the meson space wave functions is taken. In position space, the SHO wave function is the Fourier transform of (11)

\[
\psi_{nLML}(\vec{r}) = R_{nL}^{\text{SHO}}(r) Y_{LM_L}(\Omega_r), \tag{15}
\]
where the radial wave function is given by

$$R_{nL}^{\text{SHO}}(r) = \beta^2 \frac{2n!}{\Gamma(n + L + \frac{3}{2})} (\beta r)^L e^{-\beta^2 r^2/2} L_n^L e^{-(\beta^2 r^2/2)}. \quad (16)$$

With these elements, the partial amplitudes and partial widths for $A \to BC$ in the flux-tube model can also be calculated analytically based on relations (9) and (10), respectively.

### 4.2 Decay properties of the $\eta_2(1870)$ as the $\eta_2(2Dn\bar{n})$

Under the SHO wave function approximation, the parameters used in this work involve the SHO wave function scale parameter $\beta$, the pair production strength parameter $\gamma$ in the $^3P_0$ model, the pair-creation constant $\gamma_0$ and the string tension $b$ in the flux-tube model, and the constituent quark mass $m_q$. In this work, we choose to follow the Refs.[39, 43, 44] and take $\gamma = 8.77$, $\beta_A = \beta_B = \beta_C = \beta = 0.4$ GeV, $\gamma_0 = 14.3$, $b = 0.18$ GeV$^2$, $m_u = m_d = 0.33$ GeV, and $m_s = 0.55$ GeV which are also the values used to evaluate the decays of the $\pi_2(1880)$[25].

The meson masses used to determine the phase space and final state momenta are $^2 M_\pi = 138$ MeV, $M_\eta = 548$ MeV, $M_K = 496$ MeV, $M_\rho = 776$ MeV, $M_\omega = 783$ MeV, $M_\phi = 1019$ MeV, $M_{K^*} = 894$ MeV, $M_{\pi(1300)} = 1300$ MeV, $M_{K^*(1410)} = 1414$ MeV, $M_{a_1(1260)} = 1230$ MeV, $M_{f_1(1285)} = 1282$ MeV, $M_{b_1(1235)} = 1230$ MeV, $M_{h_1(1170)} = 1170$ MeV, $M_{K_{1}(1270)} = 1272$ MeV, $M_{K_{1}(1400)} = 1403$ MeV, $M_{a_2(1320)} = 1318$ MeV, $M_{f_2(1270)} = 1275$ MeV, $M_{K^*_2(1430)} = 1429$ MeV, $M_{a_0(1450)} = 1474$ MeV, $M_{f_0(1370)} = 1370$ MeV, and $M_{K^*_0(1430)} = 1425$ MeV. The meson flavor wave functions follow the conventions of Ref.[14]. Based on (10), the numerical values of the partial decay widths of the $\eta_2(1870)$ as the $\eta_2(2Dn\bar{n})$ are listed in Table 1.

It is clear from Table 1 that the numerical results in the $^3P_0$ model are similar to those in the flux-tube model. The total width of the $\eta_2(2Dn\bar{n})$ at 1842 MeV is expected to be about 226 MeV in the $^3P_0$ model or about 246 MeV in the flux-tube model, both compatible with the $\eta_2(1870)$ width. The expected dominant decay modes are $a_2(1320)\pi$, $\rho\rho$, $f_2(1270)\eta$, $a_1(1260)\pi$, $\omega\omega$, and $K^*K$, in accord with the $\eta_2(1870)$ dominantly decaying to $a_2(1320)\pi$ and $f_2(1270)\eta$. It should be noted that the partial width of $\eta_2(2Dn\bar{n}) \to KK^*$ is expected to be

\[1\] Our value of $\gamma$ is higher than that used by Ref.[44] (0.505) by a factor of $\sqrt{96\pi}$, due to different field conventions, constant factor in $T$, etc. The calculated results of the widths are, of course, unaffected.

\[2\] We assume that the $a_0(1450)$, $f_0(1370)$, and $K^*_0(1430)$ are the ground scalar meson as Refs.[14, 16].
Table 1: Partial widths of the $\eta_2(1870)$ as the $\eta_2(2Dn\bar{n})$ in the $^3P_0$ model and the flux-tube model (in MeV). The initial state mass is set to 1842 MeV.

| Mode       | $\Gamma_{LS}$ in $^3P_0$ model | $\Gamma_{LS}$ in flux-tube model |
|------------|--------------------------------|----------------------------------|
| $K^*K$     | $\Gamma_{P1} = 13.95$          | $\Gamma_{P1} = 15.19$           |
|            | $\Gamma_{F1} = 3.75$           | $\Gamma_{F1} = 4.09$            |
| $\rho\rho$ | $\Gamma_{P1} = 43.00$          | $\Gamma_{P1} = 46.83$           |
|            | $\Gamma_{F1} = 9.16$           | $\Gamma_{F1} = 9.97$            |
| $\omega\omega$ | $\Gamma_{P1} = 14.23$   | $\Gamma_{P1} = 15.50$           |
|            | $\Gamma_{F1} = 2.66$           | $\Gamma_{F1} = 2.90$            |
| $K^*K^*$   | $\Gamma_{P1} = 2.08$           | $\Gamma_{P1} = 2.27$            |
|            | $\Gamma_{F1} = 0.01$           | $\Gamma_{F1} = 0.01$            |
| $K_1(1270)/K$ | $\Gamma_{D1} = 0.06$   | $\Gamma_{D1} = 0.07$            |
| $a_1(1260)\pi$ | $\Gamma_{D1} = 15.23$   | $\Gamma_{D1} = 16.59$           |
| $f_1(1285)\eta$ | $\Gamma_{D1} = 0.00$   | $\Gamma_{D1} = 0.00$            |
| $a_2(1320)\pi$ | $\Gamma_{S2} = 67.56$  | $\Gamma_{S2} = 73.58$           |
|            | $\Gamma_{D2} = 34.52$          | $\Gamma_{D2} = 37.59$           |
|            | $\Gamma_{G2} = 0.43$           | $\Gamma_{G2} = 0.47$            |
| $f_2(1270)\eta$ | $\Gamma_{S2} = 17.47$  | $\Gamma_{S2} = 19.02$           |
|            | $\Gamma_{D2} = 0.02$           | $\Gamma_{D2} = 0.02$            |
|            | $\Gamma_{G2} = 0.00$           | $\Gamma_{G2} = 0.00$            |
| $a_0(1450)\pi$ | $\Gamma_{D0} = 2.37$  | $\Gamma_{D0} = 2.58$            |
| $\Gamma$   | 226.50                        | 246.68                          |

large ($\Gamma(KK^*)/\Gamma(f_2(1270)\eta) \approx 1$), however, there is no indication of the $\eta_2(1870)$ in the WA102 data for the $KK\pi$[49], which suggests that the $\eta_2(1870)$ does not decay significantly to $KK^*$. This discrepancy could arise from the omission of the small $n\bar{n} \leftrightarrow s\bar{s}$ flavor mixing effect in the $\eta_2(1870)$. With $\eta_2(1870) \equiv \cos \theta n\bar{n} - \sin \theta s\bar{s}$, where $\theta$ is the mixing angle, in the $^3P_0$ model, the dependence of the predicted total width $\Gamma(\eta_2(1870))$ and the partial widths for the dominant decay modes on the mixing angle $\theta$ are shown in Fig. 3. (The results from the flux-tube model are very similar to those from the $^3P_0$ model). Fig. 3 indicates that the $\Gamma(KK^*)$ is very sensitive to the $\theta$. For small and negative $\theta$ ($\theta \approx -0.3 \sim -0.2$ radians), the total width and other dominant partial widths of the $\eta_2(1870)$ shown in Table 1 are not significantly changed.
but the $\Gamma(KK^*)$ would be small.

![Figure 3:](image)

Figure 3: In the $^3P_0$ model, the predicted $\Gamma(\eta_2(1870))$, $\Gamma(a_2(1320)\pi)$, $\Gamma(\rho\rho)$, $\Gamma(\omega\omega)$, $\Gamma(a_1(1260)\pi)$, $\Gamma(KK^*)$ and $\Gamma(f_2(1270)\eta)$ versus the mixing angle $\theta$.

The decay dynamics of the $\eta_2(Hn\bar{n})$ with a mass of 1.8-2.0 GeV has been investigated in the flux-tube model[15, 16, 17]. We now shall compare the hybrid and quarkonium assignments for the $\eta_2(1870)$. Both assignments lead to the significant $a_2(1320)\pi$ and $f_2(1270)\eta$ signals, in accord with the experiment. The most characteristic decay modes are the $\rho\rho$ and $\omega\omega$, which are forbidden for the $\eta_2(Hn\bar{n})$ due to the selection rule while significant for the $\eta_2(2Dn\bar{n})$. Similar result follows for the $a_1(1260)\pi$. The $\rho\rho$ and $\omega\omega$ channels would be the strong discriminant between the hybrid and conventional meson for the $\eta_2(1870)$. Unfortunately, the experimental information on the $\rho\rho$ and $\omega\omega$ channels for the $\eta_2(1870)$ is not available. Also, the value of $R = \Gamma(a_2(1320)\pi)/\Gamma(f_2(1270)\eta)$ for the $\eta_2(2Dn\bar{n})$ is in fact different from that for the $\eta_2(Hn\bar{n})$. For example, at 1875 MeV, we predicted $R = 4$ for the $\eta_2(2Dn\bar{n})$ while Barnes et al. predicted $R = 8$ for the $\eta_2(Hn\bar{n})$[16]. Experimentally, the Crystal Barrel Collaboration gave $R = 4.1 \pm 2.3$[4], the WA102 Collaboration gave $R = 20.4 \pm 6.6$[7], and Anisovich et al. gave $R = 1.27 \pm 0.17$[9].

The world average value of $R$ quoted by PDG is $6 \pm 5$[11]3. Obviously, the uncertainty of the world average value for the $R$ is so large that we can not distinguish the $\eta_2(2Dn\bar{n})$ assignment from the $\eta_2(Hn\bar{n})$ interpretation for the $\eta_2(1870)$ based this ratio. The further confirmation of this ratio is needed. At present, the total width and the strong decay pattern of the $\eta_2(1870)$ do not exclude the possibility of it being in fact the $2^1D_2$ $qq$ state.

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3 The PDG does not quote the datum of 1.27 ± 0.17, and does not use this datum for averages, limits, etc.
5 Discussions

As mentioned in Sect.1, in $\bar{p}p$ annihilation the $I^G(J^{PC}) = 0^+(2^{-+})$ resonance called $\eta_2(2030)$ has been observed in the $a_2(1320)\pi$ and $f_2(1270)\eta$ channels [9, 18]. This state is listed as ‘Further state’ by PDG[11]. To some extent, the discovery of the $\eta_2(2030)$ leads to the conjecture of the $\eta_2(1870)$ being a hybrid because the $\eta_2(2030)$ looks like the first radial excitation of the $\eta_2(1645)$ based on its mass. We shall turn to the possibility of the $\eta_2(2030)$ being the $\eta_2(2Dn\bar{n})$ by investigating its decay dynamics. The partial widths of the $\eta_2(2030)$ as the $\eta_2(2Dn\bar{n})$ are estimated in the $^3P_0$ model and flux-tube model. The numerical results are listed in Table 2. The predictions from the $^3P_0$ model are similar to those from the flux-tube model. We find if the $\eta_2(2030)$ is the $\eta_2(2Dn\bar{n})$, its total width would be about 654 MeV or 709 MeV, far more than the experiment. Therefore, the $\eta_2(2Dn\bar{n})$ assignment for the $\eta_2(2030)$ seems unfavorable in the $^3P_0$ model and the flux-tube model. We also estimate the partial widths of the $\eta_2(2030)$ as the $I^G = 0, 3$ $1^1D_2\bar{n}n$ state [$\eta_2(3Dn\bar{n})$] in Table 2. If the $\eta_2(2030)$ is the $\eta_2(3Dn\bar{n})$, we find (1) its total width would be about 150 MeV or 162 MeV; (2) it dominantly decays to $a_2(1320)\pi$ D-wave; (3) $B(f_2\eta)/B(a_2\pi)_{L=2}$ is about 0.1. All these predictions are consistent with experiment [9, 18], and therefore the $\eta_2(2030)$ is more likely to be the candidate for the $\eta_2(3Dn\bar{n})$. This indicates that the presence of the $\eta_2(2030)$ does not contradict with the $\eta_2(2Dn\bar{n})$ interpretation for the $\eta_2(1870)$.

We note that the $\eta_2(1645)$, $\eta_2(1870)$, and $\eta_2(2030)^5$ approximately populate a common trajectory as shown in Fig. 4. The quasi-linear trajectories at the $(n, M^2)$-plots turned out to be able to described the light mesons with a good accuracy[50]. Fig. 4 therefore indicates that the $\eta_2(1870)$ and $\eta_2(2030)$ could be in fact the $2^1D_2$ and $3^1D_2$ $q\bar{q}$ states and the narrow level spacing between the $\eta_2(2Dn\bar{n})$ and $\eta_2(3Dn\bar{n})$ is not surprising in Regge phenomenology. More recently, Li and Chao[51, 52] have found that the coupled-channel and screening effects are important for the spectra of higher charmonia, and the masses of higher charmonia from the screened potential model or coupled-channel model are considerably lower than those from

\footnote{The one exception to this is that our predicted $B(a_2\pi)_{L=0}/B(a_2\pi)_{L=2}$ for the $\eta_2(2030)$ as the $\eta_2(3Dn\bar{n})$ is about 0.01, inconsistent with the experiment of about 0.74 ± 0.17[9].}

\footnote{The masses of these three states are 1617 MeV, 1842 MeV, and 2030 MeV, respectively[11].}
the naive quark model. The coupled-channel effect or the screening effect may be also a factor leading to the narrow level spacing between the \( \eta_2(2Dn\bar{n}) \) and \( \eta_2(3Dn\bar{n}) \). More theoretical investigations and more complete data on the light mesons are needed to clarify this issue.

With the \( \eta_2(2030) \) being the \( \eta_2(3Dn\bar{n}) \), one can expect that the isovector member of the \( 3^1D_2 \) meson nonet would lie around 2030 MeV, which leads to that the \( \pi_2(2005) \) with a mass of 2005 \( \pm \) 15 MeV and a width of 200 \( \pm \) 40 MeV\(^5\) could be a good candidate for the \( I = 1, 3^1D_2 \) \( q\bar{q} \). We find if the \( \pi_2(2005) \) is the \( 3^1D_2 \) \( q\bar{q} \), its total width would be about 122 MeV in the \( 3^3P_0 \) model or 130 MeV in the flux-tube model, roughly consistent with the experiment.

Generally speaking, the pure \( \eta_2(2Dn\bar{n}) \) can mix with the pure \( \eta_2(Hn\bar{n}) \) to produce the physical state. We shall discuss the possibility of the \( \eta_2(1870) \) being a mixture of the \( \eta_2(2Dn\bar{n}) \) and \( \eta_2(Hn\bar{n}) \). Obviously, quantitative determination of its \( q\bar{q} \)-hybrid content should be essential to confirm or refute this possibility. The available decay information for the \( \eta_2(1870) \) is unfortunately not sufficient to do this\(^6\). However, we can qualitatively estimate the hybrid component of the \( \eta_2(1870) \) would be small if the \( \eta_2(1870) \) is really a mixture of the \( q\bar{q} \) and hybrid. As mentioned in Sect. 3, the fact of the \( \eta_2(1645) \) and \( \eta_2(1870) \) having the same behavior as a function of the \( dp_T \)\(^6\) strongly suggests the \( \eta_2(1870) \) having the same dynamical structure as the \( \eta_2(1645) \), which makes the substantial hybrid admixture in the \( \eta_2(1870) \) unlikely. The further experimental information of the \( \eta_2(1870) \) in the \( \rho\rho \) and \( \omega\omega \) channels would be crucial to shed light on this issue.

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\(^5\)Within the \( \eta_2(1870) \) being the mixture of the \( \eta_2(2Dn\bar{n}) \) and \( \eta_2(Hn\bar{n}) \), the measured partial widths of the \( \eta_2(1870) \) are needed to determine its hybrid-quarkonium content quantitatively.
Table 2: Partial widths of the $\eta_2(2030)$ as the $\eta_2(2Dn\bar{n})$ and $\eta_2(3Dn\bar{n})$ in the $^3P_0$ model and the flux-tube model (in MeV). The initial state mass is set to 2030 MeV.

| Mode         | $\eta_2(2Dn\bar{n})$ | $\eta_2(3Dn\bar{n})$ |
|--------------|-----------------------|-----------------------|
|              | $^3P_0$ model         | flux-tube model       | $^3P_0$ model         | flux-tube model       |
| $K^*K$       | 27.80                 | 30.27                 | 5.00                  | 5.44                  |
| $\rho\rho$   | 80.83                 | 88.02                 | 13.14                 | 14.31                 |
| $\omega\omega$| 26.42                 | 28.77                 | 4.28                  | 4.66                  |
| $K^*K^*$     | 14.22                 | 15.49                 | 2.88                  | 3.13                  |
| $K^*(1410)K$ | 6.32                  | 3.92                  | 5.38                  | 4.46                  |
| $K_1(1270)K$ | 1.68                  | 1.83                  | 0.36                  | 0.39                  |
| $K_1(1400)K$ | 0.59                  | 0.64                  | 0.14                  | 0.15                  |
| $b_1(1235)\rho$| 152.98                | 166.60                | 35.91                 | 39.11                 |
| $h_1(1170)\omega$ | 65.10                | 70.89                | 13.60                 | 14.81                 |
| $a_0(1450)\pi$| 16.82                 | 18.32                 | 9.26                  | 10.09                 |
| $f_0(1370)\eta$| 0.60                  | 0.65                  | 0.38                  | 0.41                  |
| $K_0^*(1430)K$| 0.59                  | 0.64                  | 0.32                  | 0.34                  |
| $a_1(1260)\pi$| 41.11                 | 44.77                 | 5.71                  | 6.22                  |
| $f_1(1285)\eta$| 1.43                  | 1.56                  | 0.29                  | 0.31                  |
| $a_2(1320)\pi$| 129.60                | 141.14                | 35.00                 | 38.11                 |
| $f_2(1270)\eta$| 23.47                 | 25.56                 | 4.21                  | 4.58                  |
| $K_2^*(1430)K$| 64.78                 | 70.54                 | 14.23                 | 15.49                 |
| $\Gamma$    | 654.34                | 709.61                | 150.09                | 162.01                |

Experiment: $\Gamma_{\eta_2(2030)} = 205 \pm 10 \pm 25[9]$ or $190 \pm 40[18]$

### 6 Summary and conclusion

From the mass, production, total width, and strong decay pattern of the $\eta_2(1870)$, we point out that the possibility of it being a canonical $2^1D_2$ $q\bar{q}$ state does exist. Also, the decay information for the $\eta_2(2030)$ is consistent with it being a $3^1D_2$ rather than $2^1D_2$ $q\bar{q}$ state, and the total width of the $\pi_2(2005)$ favors the argument that it could be the candidate for the isovector partner of the $\eta_2(2030)$. The possibility of the $\eta_2(1870)$ being a mixture of hybrid and $q\bar{q}$ might exist while the substantial hybrid admixture in this state seems unlikely. The further experimental information of the $\eta_2(1870)$ in the $\rho\rho$ and $\omega\omega$ channels is needed. We tend to conclude that the $\eta_2(1870)$ is the ordinary $2^1D_2$ $q\bar{q}$ state or the $2^1D_2$ $q\bar{q}$ with small hybrid
admixture, as does the $\pi_2 (1880)$.

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