Scale-dependent bias from an inflationary bispectrum: the effect of a stochastic moving barrier

Matteo Biagetti & Vincent Desjacques
Département de Physique Théorique and Center for Astroparticle Physics (CAP), Université de Genève, 24 quai Ernest Ansermet, CH-1211 Genève, Switzerland

ABSTRACT

With the advent of large scale galaxy surveys, constraints on primordial non-Gaussianity (PNG) are expected to reach $O(f_{\text{NL}}) \sim 1$. In order to fully exploit the potential of these future surveys, a deep theoretical understanding of the signatures imprinted by PNG on the large scale structure of the Universe is necessary. In this paper, we explore the effect of a stochastic moving barrier on the amplitude of the non-Gaussian bias induced by local quadratic PNG. We show that, in the peak approach to halo clustering, the amplitude of the non-Gaussian bias will generally differ from the peak-background split prediction unless the barrier is flat and deterministic. For excursion set peaks with a square-root barrier, which reproduce reasonably well the linear bias $b_1$ and mass function $\bar{n}_h$ of SO haloes, the non-Gaussian bias amplitude is $\sim 40\%$ larger than the peak-background split expectation $\partial \ln \bar{n}_h / \partial \ln \sigma_8$ for haloes of mass $\sim 10^{13} M_\odot/h$ at $z = 0$. Furthermore, we argue that the effect of PNG on squeezed configurations of the halo bispectrum differs significantly from that predicted by standard local bias approaches. Our predictions can be easily confirmed, or invalidated, with N-body simulations.

Key words: cosmology: theory, large-scale structure of Universe, inflation

1 INTRODUCTION

Upcoming large scale structure surveys will take over the hunt for primordial non-Gaussianity (PNG) from CMB experiments. The recent (individual) limits on the nonlinear parameter $f_{\text{NL}}$ from measurements of galaxy clustering and the integrated Sachs-Wolfe (ISW) effect are already at the level of the CMB pre-Planck constraints, i.e. $\Delta f_{\text{NL}} \sim 80$ (Giannantonio et al. 2014, Ho et al. 2013; Leistedt, Peiris & Roth 2014). Forecasts for future Euclid-like galaxy surveys show that a measurement of the large scale galaxy power spectrum alone can constrain $f_{\text{NL}} \sim$ a few (e.g. Giannantonio et al. 2012; Camera, Santos & Maartens 2014; Ferramacho et al. 2014; de Putter & Dore 2014; Doré et al. 2014), whereas intensity mappings of the 21cm emission line of high-redshift galaxies could achieve $\Delta f_{\text{NL}} \sim 1$ (e.g. Camera et al. 2013).

One of the most powerful large scale structure probes of PNG to date is the galaxy/quasar power spectrum. In the original derivation of Dalal et al. (2008a), the amplitude $b_{\text{NG}}$ of the scale-dependent bias induced by a primordial bispectrum of the local shape (i.e. $f_{\text{NL}} \phi_2^2$) was found to be proportional to the first-order bias, i.e. $b_{\text{NG}} = \delta_c b_1$. Slosar et al. (2008) used the peak-background split argument (Kaiser 1984, Bardeen et al. 1986) to argue that the amplitude of the non-Gaussian bias is proportional to the logarithmic derivative of the halo mass function w.r.t. $\sigma_8$ (or any proxy for the normalisation amplitude), i.e. $b_{\text{NG}} = b_{\text{NG}}^{\text{pbs}}$, where

$$b_{\text{NG}}^{\text{pbs}} = \frac{\partial \ln \bar{n}_h}{\partial \ln \sigma_8}.$$  

Scoccimarro et al. (2012) generalised the peak-background split approach to include the non-Markovian (in the excursion set sense) and non-universality of the mass function. Nevertheless, they assumed that the first-crossing distribution is of the particular form $f(\delta_c, \sigma_8^2)$, i.e. a flat barrier. For $\nu_c \gg 1$, all these predictions converge to the high-peak result derived in Matarrese & Verde (2008). Here, $\nu_c = \delta_c / \sigma_0$ is the peak significance and $\sigma_0$ is the rms variance of the mass density field.

Both analytic models of halo collapse and numerical simulations support the fact that, at a given halo mass $M$, the linear threshold for halo collapse is not the deterministic constant $\delta_c$ predicted by spherical collapse (Bond & Myers 1996, Sheth, Mo & Tormen 2001; Desjacques 2008; Dalal et al. 2008b; Robertson et al. 2009). Owing to the tidal shear and other nonlinear effects, the
average linear threshold for collapse is a monotonically increasing function of decreasing halo mass. Furthermore, this collapse threshold fluctuates from halo to halo because it is strongly sensitive to the local density and shear configuration. To the best of our knowledge however, the effect of a moving barrier on the amplitude of non-Gaussian bias has thus far been discussed only in Afshordi & Tolley (2008) and Adshead et al. (2012). Afshordi & Tolley (2008) argued that the formula of Dalal et al. (2008a) remains valid if one substitutes \( \delta_{b1} \rightarrow \delta_{c} \beta_{B} \), where \( \delta_{c} \) is the threshold for ellipsoidal collapse. Adshead et al. (2012) investigated the effect of an ellipsoidal barrier on the non-Gaussian bias within the path integral approach to excursion set (see Maggiore & Riotto 2010a). They found that the non-Gaussian bias amplitude is generally different from \( \langle p_{c} \rangle \). Furthermore, this stochastic moving barrier as follows: each halo “sees” a moving barrier calculated in the peak approach. In the peak approach, the variable is parametrised by a random variable \( \beta \) where \( \beta_{c} \) is replaced by a generic moving barrier \( B(\sigma_{0}) = \delta_{c} + \beta \sigma_{0} \). We will consider the square-root stochastic barrier

\[
B(\sigma_{0}) = \delta_{c} + \beta \sigma_{0}
\]

where the stochastic variable \( \beta \) closely follows a lognormal distribution with mean \( \langle \beta \rangle = 0.5 \) and variance \( \text{Var}(\beta) = 0.25 \). This barrier furnishes a good description of the linearly extrapolated collapse threshold of SO (Spherical Overdensity) haloes identified with a constant overdensity \( \Delta_{c} = 200 \) relative to background (Robertson et al. 2008b).

Paranjape, Sheth & Desjacques (2013) interpreted this moving barrier as follows: each halo “sees” a moving barrier \( B = \delta_{c} + \beta \sigma_{0} \) with a value of \( \beta \) drawn from a lognormal distribution. Here, we will adopt the interpretation of Biagetti et al. (2014), which states that each halo “sees” a constant (flat) barrier with a height that varies on an object-by-object basis. Consequently, the first-crossing condition does not involve any derivative of \( B(\sigma_{0}) \) w.r.t. the halo mass, and we simply get

\[
B < \delta < B + \mu \Delta R_{T}
\]

where \( \mu = -d\delta/dR_{T} \) and \( R_{T} \) is the Top-hat radius of the Lagrangian patch which collapses to form a halo (see Bond 1986; Appel & Jones 1994, for early implementation of the first-crossing conditions). Consequently, the variable \( \mu \) will satisfy the constraint \( \mu > 0 \) rather than \( \mu > -dB/dR_{T} \).
in Paranjape, Sheth & Desjacques (2013). This has a very small impact on the predicted halo mass function and, at the same time, simplifies the effective peak bias expansion since they are no correlations induced by the barrier itself (but they may be present for actual halos).

The halo mass function predicted by the model is

$$\frac{d\bar{n}_h}{d\ln M} = \frac{\bar{\rho}}{M} f_{\text{ESP}}(\nu_c, R_T) \frac{d\log \nu_c}{d\log M}$$

$$= -\frac{1}{3} R_T \left( \frac{\gamma_{\nu} \nu_c}{\sigma_{\nu}} \right) V^{-1} f_{\text{ESP}}(\nu_c) ,$$

where $\sigma_{\nu}$ is the zeroth-order spectral moment of the linear density field smoothed with a Top-hat filter, $V$ is the cross-correlation between the $\nu$ and $\mu$ fields and $\nu$ is the Lagrangian volume of a halo. $f_{\text{ESP}}(\nu_c)$ is the multiplicity function of the so-called excursion set peaks (Paranjape, Sheth & Desjacques 2013, for details and notations).

This is particularly interesting because the right-hand side of Eq. (2) is scale-independent, has an amplitude given by $\bar{\rho}_N$, and is nonetheless equal to the effective bias expansion introduced in Desjacques (2013). In Fig. 2 some of the second order bias factors together with the resulting behaviour of $b_{NG}$ are shown for the constant barrier $B(\sigma_0) = \delta_c$ as a function of the peak significance $\nu_c$.

For this constant, deterministic barrier, Desjacques, Gong & Riotto (2013) demonstrated that the amplitude $b_{NG}$ of the non-Gaussian bias satisfies

$$b_{NG} = \delta_c b_1 = b_{NG}^{\text{bias}} ,$$

where $\bar{n}_h$ is the excursion set peaks mass function Eq. (4) and $b_N = b_{100}$ is the k-independent piece of the Nth-order Lagrangian, Gaussian bias (the usual Lagrangian bias parameters in the standard local bias model). They also showed that

$$b_N = \frac{(-1)^N \partial^N \bar{n}_h}{\bar{n}_h \partial \delta_c^N} .$$

in agreement with peak-background split predictions (Kaiser 1984). Under the approximation of a constant barrier, peak theory thus predicts that the amplitude of the non-Gaussian bias is equally given by the sum of quadratic bias factors Eq. (6), the original result $\delta_c b_1$ of Dalal et al. (2008a) or the peak-background split expectation $b_{NG}^{\text{bias}}$ obtained by Slosar et al. (2008). We tested this equivalence numerically and found that it indeed holds. The thin, indistinguishable curves in Fig. 2 show the various predictions. At this point, it is worth noticing that, although the excursion set peak mass function is not universal (it depends distinctly on $\delta_c$ and the spectral moments $\sigma_i$), the logarithmic derivative of $\bar{n}_h$ w.r.t. $\sigma_8$ is nonetheless equal to $\delta_c b_1$. This follows from the fact that the $\sigma_i$’s conspire to appear only in ratios such as $\gamma_i = \sigma_i^2 / (\sigma_0 \sigma_2)$ or in $\nu_c = \delta_c / \sigma_0$.

Thus far however, we have followed Desjacques, Gong & Riotto (2013) and assumed a constant barrier $B \equiv \delta_c$. How does the relation Eq. (7) change when we take into account the scatter and mass-dependence of the collapse barrier through the square-root stochastic barrier Eq. (2)? To answer this question, we have simply computed $b_{NG}$ and $b_1$ from the bias factors derived
from the ESP multiplicity function Eq. (6). We have also evaluated $b_{NG}$ numerically from the predicted halo mass function (we have again explicitly taken the numerical derivative of $n_b$ w.r.t. $\sigma_b$). The results are shown in Fig. 2 as the thick solid curves. They can be summarised as follows:

$$b_{NG} \neq \delta_n b_1 = b_{NG}.$$  \hspace{1cm} (9)

Eq. (9) is the main result of this paper. While $b_{NG}$ agrees with the two other quantities at the high mass end, where all the predictions converge towards the high-peak result of Matarrese & Verde (2008), it becomes increasingly larger as the halo mass decreases. For the lognormal distribution of $\beta$ adopted here, deviations are quite substantial. Namely, for $M = 10^{14}$ and $10^{15} h^{-1} M_\odot$, the predicted non-Gaussian bias amplitude $b_{NG}$ is $\sim 10\%$ and $\sim 40\%$ larger than the peak-background split amplitude $b_{BG}^{\text{biv}}$. Upon turning the scatter in $\beta$ on and off, we have found that the latter is driving the difference between $b_{NG}$ and $b_{BG}^{\text{biv}}$ for $\nu > 2$. At higher peak heights, the discrepancy originates mainly from the fact that the barrier is not flat.

Ashordi & Tolley (2008) advocated the replacement $\delta_{n} b_1 \to \delta_{n} b_1$ to account for the mass-dependence of the linear collapse threshold. We have found that substituting $\delta_{n}$ either by the mean barrier $\delta_{b} + \sigma_{b} (\beta)$ or by the square-root of $(\delta_{b} + \sigma_{b} (\beta))^{2}$ does improve the agreement with $b_{BG}$, yet the match is far from perfect, especially around $\nu \sim 1$. Furthermore, we do not select our peaks according to their formation history. Hence, this has nothing to do with the assembly bias effect pointed out in Skosz et al. (2008), for which the extended Press-Schechter formalism of Bond et al. (1991) furnishes a good description. Finally, Adshead et al. (2012) pointed out that $b_{NG}$ generally differs from $b_{BG}$ (but note that they did not discuss the validity of $b_{BG}^{\text{biv}}$). However, they found a much larger effect which did not lead us to our Fig. 3. Therefore, all this strongly suggests that $b_{BG}$ can only be written down as the sum of quadratic bias factors Eq. (9) as predicted by peak theory.

2.3 A closer look at the peak prediction

For simplicity, let us momentarily ignore the variable $\mu$ as it is not essential for understanding why a square-root stochastic barrier induces a difference between $b_{NG}$ and the peak-background split prediction (it is enough to retain the correlation between $\nu$ and $u$). In this case, the non-Gaussian bias amplitude takes the form

$$b_{NG} = \sigma_{b}^{2} b_{20} + 2 \sigma_{b}^{2} b_{11} + \sigma_{b}^{2} b_{20} + 2 \sigma_{b}^{2} \chi_{10} + 2 \sigma_{b}^{2} \chi_{01}.$$  \hspace{1cm} (10)

The peak bias factors $b_{ij}$ (associated with $\nu$ and $u$) and $\chi_{kl}$ (associated with the $\chi^{2}$-distributed variables) can all be computed by generalising the peak-background split argument to variables other than the density ($\delta$, Desjacques 2013). In particular, since the peak height $\nu(x)$ is correlated with $u(x)$ (at the same position $x$), we have

$$\sigma_{\nu}^{2} \sigma_{\beta}^{2} b_{ij} = \frac{1}{n_{pk}} \int d^{10} y \, n_{pk}(y) \, H_{ij}(\nu, u) \, P_{i}(y).$$  \hspace{1cm} (11)

Here, $n_{pk}(y)$ is the "localised" number density of BBKS peaks (as we momentarily ignore the first-crossing constraint), $y$ is a vector of 10 variables and $H_{ij}(\nu, u)$ are bivariate Hermite polynomials. When stochasticity in the barrier is taken into account, $n_{pk}(y)$ contains a multiplicative factor of $\delta_{j} (\nu(x))$ -- $\nu(x)$. In Eq. (11), the contribution $2 \sigma_{\nu}^{2} \chi_{10} + 2 \sigma_{\nu}^{2} \chi_{01}$ does not depend on the properties of the collapse barrier because the $\chi^{2}$-distributed variables do not correlate with $\nu(x)$ at a given position $x$. Therefore, we should focus on the piece proportional to $b_{ij}$.

On writing the bivariate Gaussian as

$$N(\nu, u) = \frac{\exp \left[ -\frac{\nu^{2} + u^{2} - 2 \nu u \sigma_{\nu} \sigma_{u}}{2(1 - \gamma_{b}^{2})} \right]}{2 \pi ^{1/2} (1 - \gamma_{u}^{2})},$$  \hspace{1cm} (12)

the sum $\sigma_{\nu}^{2} b_{20} + 2 \sigma_{\nu}^{2} b_{11} + \sigma_{\nu}^{2} b_{20}$ simplifies to (without writing down the integrals over $\beta$ and $u$)

$$\sigma_{\nu}^{2} b_{20} + 2 \sigma_{\nu}^{2} b_{11} + \sigma_{\nu}^{2} b_{20} 
\sim \left[ (\nu_{c} + \beta)^{2} + u^{2} - 2 \nu u (\nu_{c} + \beta) u - 2 \right] N(\nu_{c} + \beta, u),$$  \hspace{1cm} (13)

This should be compared to the full expression of the logarithmic derivative $\partial \ln n_{b} / \partial \ln \sigma_{b}$. The latter requires evaluating derivatives of the multiplicity function, which is an integral of the bivariate Gaussian $N(\nu_{c} + \beta, u)$ over $\beta$ and $u$ similar to Eq. (12). Therefore, the logarithmic derivative of the halo mass function w.r.t. $\sigma_{b}$ results in a term of the form

$$\frac{\partial}{\partial \sigma_{b}} N(\nu_{c} + \beta, u) = \frac{\partial}{\partial \nu} N(\nu_{c} + \beta, u) \frac{d\nu}{d\sigma_{b}} + \frac{\partial}{\partial u} N(\nu_{c} + \beta, u) \frac{du}{d\sigma_{b}},$$  \hspace{1cm} (14)

$$\frac{\partial}{\sigma_{b}} N(\nu_{c} + \beta, u) = \left[ (\nu_{c} + \beta - \nu_{u} u) \frac{d\nu_{u}}{d\sigma_{b}} + (u - \nu_{u} u) \frac{du}{d\sigma_{b}} \right] \frac{1}{1 - \gamma_{b}^{2}},$$  \hspace{1cm} (15)

where $\nu_{u}$ is the mean peak height $\nu_{c} + \beta$, with $\beta$ independent of $\sigma_{b}$ and $u \propto 1/\sigma_{b}$. Hence, $d\nu / d\sigma_{b} = - \nu / \sigma_{b}$ and $\partial / \partial \sigma_{b} = - \nu / \sigma_{b}$. Substituting these derivatives in the previous expression, we arrive at

$$\frac{\partial}{\partial \sigma_{b}} N(\nu_{c} + \beta, u) = \left[ (\nu_{c} + \beta - \nu_{u} u) \frac{d\nu_{u}}{d\sigma_{b}} + (u - \nu_{u} u) \frac{du}{d\sigma_{b}} \right] \frac{1}{1 - \gamma_{b}^{2}},$$  \hspace{1cm} (15)

We should now compare the square brackets in Eq. (15) with that of Eq. (11). We note that, in Eq. (11), there is an additional factor of $- 2$ inside the brackets which disappears when one takes into account the first-crossing constraint. So, the key difference is the fact that, for $\partial N / \partial \ln \sigma_{b}$, the square brackets reduce to $2 Q(\nu_{c} + \beta, u)$ as in Eq. (16) only if $\beta \ll \nu_{c}$, a condition which is only satisfied in the high peak limit $\nu_{c} \gg 1$. This is the reason why, in Fig. 2, $b_{NG}$ significantly differs from $b_{BG}^{\text{biv}}$ as $\nu_{c}$ decreases. We also note that Eqs. (13) and (15) will differ even in the absence of scatter in the moving barrier (i.e. $(\beta^{2}) = (\beta^{2})$).

The peak model and peak-background split predictions will agree for a moving barrier only if $d\nu / d\sigma_{b} = - (\nu_{c} + \beta) / \sigma_{b}$ or, equivalently, if $\beta \propto \sigma_{b}^{-1}$. This implies that the deviation from $\delta_{n}, \sigma_{n} \beta$, does not depend on $\sigma_{b}$. However, numerical simulations (Sheth & Tormen 2004; Robertson et al. 2003) clearly indicate that the scatter in the barrier increases with
decreasing halo mass and is approximately proportional to $\sigma_0$ (hence the designation square-root barrier). Therefore, we shall expect $b_{NG}$ \neq b_{NG}^{\text{bias}}$ for actual (SO) dark matter haloes if excursion set peak theory accurately describes their clustering properties.

### 3 The Squeezed Limit of the Galaxy Bispectrum

Retaining terms up to the fourth-point function and working within the usual local bias approximation $\delta_b(x) = b_0 \delta(x) + (1/2) b_2 \delta^2(x) + \ldots$, the halo bispectrum with primordial non-Gaussianity of the local type is given by [Jeong & Komatsu 2009; Sefusatti 2009; Jeong & Komatsu 2009]

$$B_b(k_1, k_2, k_3) = 2b_1^3 \left[ \left( \frac{\mathcal{M}(k)}{\mathcal{M}(k_1)\mathcal{M}(k_2)} + P_2(k_1, k_2) \right) \times P(k_1) P(k_2) + b_2^2 P(k_1) P(k_2) 
+ \frac{1}{2} b_1^2 b_2 \int \frac{d^3 q}{(2\pi)^3} T(q, k_1 - q, k_2, k_3) 
+ (2 \text{ cyc.}) \right] .$$

Here, $\mathcal{M}(k) \propto k^2$ is the transfer function between linear density and potential perturbations and $T$ is the matter bispectrum. We have also omitted the filtering kernels as they are not essential for the purpose of this discussion. In the squeezed configurations, the two dominant contributions are the first and fourth term in the right-hand side. The first is proportional to $f_{NL}$, whereas the fourth contains a contribution from the linearly evolved primordial trispectrum proportional to $f_{NL}^2$, and a cross-correlation between the primordial bispectrum and the nonlinearly evolved density field proportional to $f_{NL}$. For peaks, the analyses of [Desjacques 2013; Desjacques, Gong & Riotto 2013] and the correspondence with the Integrated Perturbation Theory (iPT) framework [Matsubara 2011, 2012], indicate that the fourth term shall be replaced by the more general expression

$$\frac{1}{2} c_1(k_2)c_1(k_3) \int \frac{d^3 q}{(2\pi)^3} c_2(q, k_1 - q) T(q, k_1 - q, k_2, k_3) ,$$

where the linear and quadratic Lagrangian peak bias parameters $c_n(k_1, \ldots, k_n)$ are given by

$$c_1(k) \equiv (b_{10} + b_{01} k^2)$$

and

$$c_2(k_1, k_2) \equiv \left\{ b_{20} + b_{11} (k_1^2 + k_2^2) + b_{02} k_1^2 k_2^2 - 2\chi_{10} (k_1 \cdot k_2) + \chi_{02} \left[ 3(k_1 \cdot k_2)^2 - k_1^2 k_2^2 \right] \right\} .$$

Here again, we have ignored the first-crossing constraint and omitted multiplicative factors of filtering kernels for sake of conciseness. Restricting ourselves to the contribution of the primordial trispectrum, terms of the form

$$f_{NL}^2 c_1(k_2)c_1(k_3) \mathcal{M}(k_2) \mathcal{M}(k_3) \left[ P_{\phi}(k_2) + P_{\phi}(k_3) \right] \times P_{\phi}(k_1) \int \frac{d^3 q}{(2\pi)^3} \mathcal{M}^2(q) c_2(q, -q) P_{\phi}(q) .$$

arise in the squeezed configurations $k_1 \to 0$. Since $\mathcal{M}^2(k) P_{\phi}(k) \equiv \mathcal{P}(k)$, where $P_{\phi}(k)$ is the power spectrum of the Gaussian part of the primordial curvature perturbation, the integral over $c_2(q, -q)$ simplifies to (after re-introducing the filtering kernels)

$$\int \frac{d^3 q}{(2\pi)^3} c_2(q, -q) \mathcal{P}(q) = \sigma_0^2 b_{20} + 2\sigma_0^2 b_{11} + \sigma_0^2 b_{02} + 2\sigma_1^2 \chi_{10} + 2\sigma_2^2 \chi_{01} \equiv b_{NG} .$$

Therefore, this suggests that some of the terms proportional to $\sigma_0^2 b_2$ in a calculation which assumes the standard local bias (e.g. Sefusatti 2009; Jeong & Komatsu 2009) are, in fact, proportional to $b_{NG}$. Since $b_{NG}$ is noticeably different than $\sigma_0^2 b_2$ (see Fig 1), this will of course have a large impact on the magnitude of the PNG signal and its dependence on halo mass. However, we stress again that, unless the barrier is flat and deterministic, $b_{NG}$ cannot be replaced by the peak-background split expression $b_{NG}^{\text{bias}}$. It will also be useful to compare the predictions of the peak approach with e.g. the models of [Baldauf, Seljak & Senatore 2011; Sefusatti, Crocce & Desjacques 2012], which are based on the multivariate bias scheme of [Giannantonio & Porciani 2010]. We leave all this for future work.

### 4 Conclusion

The peak-background split has become the standard lore in analytic models of large scale structure. However, our findings raise concerns about its validity when it comes to the non-Gaussian bias of actual dark matter haloes. Our analysis builds on peak theory, which furnishes a good fit to the mass function and linear bias of SO haloes, and suggests that the peak-background split gives the wrong answer when the barrier is moving and stochastic. The latter is a reasonable description of the scatter plots $\sigma_0 - B(\sigma_0)$ constructed from numerical simulations. Based on our findings, we predict that

- The non-Gaussian bias amplitude $b_{NG}$ of SO haloes is not given by the standard “peak-background split” expression, i.e.

$$b_{NG} \neq b_{NG}^{\text{bias}} .$$

The fractional departure is expected to increase with decreasing halo mass in the proportions shown in Fig 2.

In light of the model assumptions, this inequality strictly applies to dark matter haloes closely related to an initial density peak, which is approximately the case for $M \gtrsim M_{\text{c}}$ ([Ludlow & Porciani 2011]). Notwithstanding this, it will be very instructive to consider also haloes with $M \sim M_{\text{c}}$. If the simulations turn out to support $b_{NG} = b_{NG}^{\text{bias}}$ even for massive haloes, then this would imply that either the peak approach is wrong or that moving stochastic barrier are not properly implemented in this framework.

We stress that our prediction is strictly valid for SO haloes only since the excursion set peak model used in the present analysis was calibrated with SO haloes identified with a fixed overdensity $\Delta_c = 200$ relative to the background. For FoF haloes for instance, the amplitude of non-Gaussian bias is suppressed relative to $\delta, b_1$. We believe that
this is also related to the mass-dependence and stochasticity of barrier. Nevertheless, we will postpone a more detailed analysis to future work since such a discussion is beyond the scope of this work.

Our analysis has also revealed that \( \delta_b \approx b_{NG}^{obs} \) even though the excursion set peak mass function is not universal. As we have shown, this follows from the fact that, in peak theory, \( \bar{n}_b \) depends only on ratios of the spectral moments \( \sigma_l \) in addition to \( \nu_0 = \delta_l/\sigma_0 \). Note, however, that this equality may hold only for square-root barriers. Furthermore, the functional dependence \( \bar{n}_b(\nu_c, \gamma_{\nu_c}, \ldots) \) may be very peculiar to the peak approach. Hence, it is unclear whether the clustering of actual dark matter haloes satisfies \( \delta_b \approx b_{NG}^{obs} \). Nevertheless, it will be very instructive to also test this relation with N-body simulations.

Finally, Desjacques, Gong & Riotto (2013); Desjacques et al. (2010) also showed that, when the first-crossing condition is included, the scale-independent piece of the linear, Lagrangian peak bias satisfies

\[
\delta_b = -\frac{1}{\bar{n}_b} \frac{d\bar{n}_b}{d\delta},
\]

which truly follows from a peak-background split \( \delta = \delta_b + \delta_l \) (Kaiser 1984). This relation has already been tested successfully in N-body simulations (Tobias Baldauf, private communication). It is interesting that the peak approach predicts it from first principles (see Schmidt, Jeong & Desjacques 2013 for another justification), in the sense that \( b_{NG} \) is independently obtained from a calculation of the peak correlation function whereas the right-hand side was obtained by explicitly taking the derivative of \( \bar{n}_b \) w.r.t. \( \delta \). Overall, measuring separately \( b_{NG} \), \( \partial \bar{n}_b / \partial \delta_{NG} \) and \( \delta_b \) will help constraining the shape of the collapse barrier.

To conclude, we note that, if \( b_{NG} > b_{NG}^{obs} \approx \delta_b \), then this may at least partly explain the results of Desjacques, Seljak & Iliev (2010); Hamaus, Seljak & Desjacques (2011), who measured a significant increase in the amplitude of the non-Gaussian bias for SO haloes with evolved linear bias \( b_{NG}^{obs} \approx 2 \). If all this turns out to be correct, then our current forecasts for measurements of \( f_{NL} \) from the non-Gaussian halo bias may be in need of revision. Ongoing work is aimed at testing these predictions with numerical simulations, in an attempt to (in)validate peak theory. Finally, we also stress that these considerations apply to any tracer of the large-scale structure whose distribution can effectively be represented by a stochastic moving barrier.

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