Is quantum entanglement necessary for observing induced coherence between photons generated in separate biphoton sources?

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There is renewed interest on the concept of induced coherence between beams generated by separate biphoton sources, that has led to the demonstration of new schemes for imaging, spectroscopy, microscopy and optical coherence tomography (OCT). These schemes make use of continuous wave pumping in the low parametric gain regime, which reflects the frequency entanglement between the down-converted photons. However, is entanglement a requisite to observe induced coherence? Contrary to some claims, we will show that it is not. This might be an advantage for OCT applications. High axial resolution requires a large bandwidth and for continuous wave pumping this requires the use of short nonlinear crystals. This is detrimental since short crystals generate small photon fluxes. We show that the use of ultrashort pump pulses increases axial resolution even with long crystals.

I. INTRODUCTION

In 1991 Zou et al. [1, 2] demonstrated that beams generated at separate parametric down-converting sources can interfere. When two second-order nonlinear crystals (NLC$_1$ and NLC$_2$) are optically pumped by a coherent wave, pairs of signal and idler photons might emerge (signal $s_1$ and idler $i_1$ from NLC$_1$; signal $s_2$ and idler $i_2$ from NLC$_2$). In the regime of low parametric gain (weak pumping) paired photons are expected to be generated in one or the other crystal, with a very low probability that two pairs are generated simultaneously. If idler $i_1$ is injected into the second nonlinear crystal and the experimental configuration is arranged to make idlers $i_1$ and $i_2$ indistinguishable after NLC$_2$, the signal photons $s_1$ and $s_2$ will show rst-order coherence ($g^{(1)}_{s_1,s_2}(0) = 1$).

Interference is also present in the high parametric gain regime [3, 4]. There is an ongoing discussion [5, 6] about the quantum or classical nature of the coherence between signal beams in the low and high parametric gain regimes, more specifically about the role of stimulated emission in NLC$_2$ for observing coherence in both regimes. Here, we are interested in the low parametric gain regime, since this scenario allows to tailor, quantify and measure the degree of entanglement between down-converted photons straightforwardly.

Induced coherence in a system of two parametric downconverters is a particular case of a broader class of interferometers sometimes referred as nonlinear interferometers [7]. In general, both the signal and the idler generated in NLC$_1$ can be injected in nonlinear crystal NLC$_2$. The last few years has seen a surge of interest in using these interferometers for new schemes in imaging [8, 9], sensing [10], spectroscopy [11, 12], microscopy [13, 14] and optical coherence tomography [15–18]. From a practical point of view, the main advantage of these systems is that one can choose a wavelength for the idler beam that interacts with the sample and is never detected, and another wavelength for the signal beam to be detected that enhance photo-detection efficiency. They also can show better sensitivity than alternative schemes [19, 20].

Up to now all experiments, but one [15], are performed in the low parametric gain regime. In all these cases there is continuous wave pumping (central frequency $\omega_0^p$), and the bandwidth of the pump laser ($\delta_p$) is considerably smaller than the bandwidth of down-conversion ($\Delta_{dc}$).
This condition generates a high degree of entanglement between signal and idler beams. The entropy of entanglement [21] of the quantum state is large when the ratio between the bandwidth of down-conversion and the bandwidth of the pump beam is very large or very small, i.e., \( \Delta_{dc}/\delta_p \gg 1 \) or \( \Delta_{dc}/\delta_p \ll 1 \). One can thus ask if entanglement between signal and idler photons is a necessary condition to observe induced coherence in the low parametric gain regime. In [23] they showed that by introducing a differential delay between the idler photons \( i_1 \) and \( i_2 \) the visibility of interference between signals \( s_1 \) and \( s_2 \) changes. They attributed this observation to the quantum entanglement between signal and idler photons produced in down conversion. We will demonstrate below that the same effect happens even when there is no entanglement.

II. ROLE OF SIGNAL-IDLER ENTANGLEMENT FOR OBSERVING INDUCED COHERENCE

Figure 1 shows a scheme of an induced coherence experiment with two parametric down-converters (nonlinear crystals NLC1 and NLC2). In order to unveil the role of entanglement, we consider a pulsed laser that generates coherent light with a spectrum \( F(\Omega_p) \). The frequency of the pump is \( \omega_p = \omega_s^0 + \Omega_p \), with \( \omega_s^0 \) being the central frequency and \( \Omega_p \) the frequency deviation from the central frequency. A beam splitter divides the pump beam into two coherent sub-beams that pump the two nonlinear crystals. The two sub-beams travel distances \( z_{p_1} \) and \( z_{p_2} \) before reaching NLC1 and NLC2, respectively.

Both crystals have nonlinear susceptibility \( \chi^{(2)} \) and length \( L \). The nonlinear interaction generates signal and idler photons \( s_1 \) and \( i_1 \) in NLC1, and \( s_2 \) and \( i_2 \) in NLC2. The frequency of the signal and idler photons reads \( \omega_s = \omega_s^0 + \Omega_s \) and \( \omega_i = \omega_i^0 + \Omega_i \), where \( \omega_s^0 \) and \( \omega_i^0 \) are central frequencies and \( \Omega_s,i \) are frequency deviations from the corresponding central frequencies. The conditions \( \omega_p^0 = \omega_s^0 + \omega_i^0 \) and \( \Omega_p = \Omega_s + \Omega_i \) are satisfied.

The quantum operators \( a_{s_1,i_1} (\Omega_s) \) and \( a_{s_2,i_2} (\Omega_i) \) correspond to signal and idler modes at the output face of the corresponding nonlinear crystals. \( b_{s_1,i_1} (\Omega_s) \) and \( b_{s_2,i_2} (\Omega_i) \) designate the corresponding operators at the input face. In the low parametric gain regime, the Bogoliubov transformations that relate the input-output operators for NLC1 are [24] [25]:

\[
\begin{align*}
    a_{s_1} (\Omega_s) &= U_s(\Omega_s) b_{s_1} (\Omega_s) + \int d\Omega_i V_s(\Omega_s, \Omega_i) b_{i_1}^\dagger (\Omega_i), \\
    a_{i_1} (\Omega_i) &= U_i(\Omega_i) b_{i_1} (\Omega_i) + \int d\Omega_s V_i(\Omega_s, \Omega_i) b_{s_1}^\dagger (\Omega_s),
\end{align*}
\]

where \( V_s(\Omega_s, \Omega_i) = \exp [ik_s(\Omega_s)L], U_i(\Omega_i) = \exp [ik_i(\Omega_i)L] \) and

\[
\begin{align*}
    V_s(\Omega_s, \Omega_i) &= i(\sigma L) F_{p_1} (\Omega_s + \Omega_i) \text{sinc} \left( \frac{\Delta k L}{2} \right) \exp \left[ ik_s(\Omega_s)L \right], \\
    V_i(\Omega_s, \Omega_i) &= i(\sigma L) F_{p_1} (\Omega_s + \Omega_i) \text{sinc} \left( \frac{\Delta k L}{2} \right) \exp \left[ ik_i(\Omega_i)L \right].
\end{align*}
\]

The nonlinear coefficient \( \sigma \) is [21] [24] [25]

\[
\sigma = \left[ \frac{\hbar \omega_p^0 \omega_s^0 \omega_i^0 |\chi^{(2)}|^2 N_0}{16 \pi \epsilon_0 c^2 n_p n_s n_i A} \right]^{1/2}.
\]

where \( N_0 \) is the number of photons per pulse of the pump beam, \( A \) is the effective area of interaction in the nonlinear crystal and \( n_{p,s,i} \) are the refractive indexes at the central frequencies of all waves involved. The function \( F_{p_1} (\Omega_p) \)

\[
F_{p_1} (\Omega_p) = T_0^{1/2} \exp \left[ -\frac{\Omega_p^2 T_0^2}{2} \right] \exp [ik_p(\Omega_p)z_{p_1}],
\]

where we have assumed a Gaussian shape for the spectrum of the pump beam. The function \( F_p \) is normalized to 1. \( T_0 \) is the temporal width of the pump pulses. The wave-vector phase mismatch is \( \Delta k = k_p(\Omega_s + \Omega_i) - k_s(\Omega_s) - k_i(\Omega_i) \). If we expand in Taylor series to first order the wave-vectors as \( k_s(\Omega) = k_s^0 + N_s \Omega \) (\( N_{s,p,i} \) are inverse group velocities) and assume perfect phase matching at the central frequencies \( (k_p^0 = k_s^0 + k_i^0) \), we obtain \( \Delta k = D_s L_p - D_s L_i/2 \), where \( \Omega_s = \Omega_s - \Omega_i \), \( D_s = N_p - (N_s + N_i)/2 \) and \( D = N_i - N_s \).

The idler mode \( a_{i_2} \) traverses a distance \( z_2 \) before encountering a lossy sample characterized by reflectivity \( r(\Omega_i) \). The quantum operator transformation that describes this process is [26] [27]

\[
a_{i_2} (\Omega_i) \rightarrow r(\Omega_i) a_{i_2} (\Omega_i) \exp [ik_i(\Omega_i)z_2] + f(\Omega_i),
\]

where the operator \( f \) fulfills the commutation relationship \( [f(\Omega), f(\Omega')] = (1 - |r(\Omega)|^2) \delta(\Omega - \Omega') \).
The idler beam is injected into NLC$_2$ so that the operator $a_{s_2}$ that describes signal beam $s_2$ at the output face of NLC$_2$ is

$$a_{s_2}(\Omega_s) = U_s(\Omega_s)b_{s_2}(\Omega_s) + \int d\Omega_s V_{s_2}(\Omega_s, \Omega_i)f^\dagger(\Omega_i) \tag{8}$$

$$+ \int d\Omega_i r^\dagger(\Omega_i)V_{s_2}(\Omega_s, \Omega_i)U_i^\dagger(\Omega_i)\exp\left[-ik_i(\Omega_i)z_2\right]b_i^\dagger(\Omega_i),$$

where only terms up to first order in $\sigma L$ has been considered and the only terms that give a non-zero contribution in the calculation of the first-order correlation function. The expression of the function $V_{s_2}$ is analogous to the expression of $V_{s_1}$ in Eq. 3 with $F_{p_2} = F_p(\Omega_p)\exp\left[ik_p(\Omega_p)z_{p_2}\right]$.

Signal photon $s_1$ traverses a distance $z_1$ before detection, and signal photon $s_2$ traverses a distance $z_3$. The number of down-converted signal photons generated per pulse, $N_{s_1} = \int d\Omega a^\dagger_{s_1}(\Omega)a_{s_1}(\Omega)$ and $N_{s_2} = \int d\Omega a^\dagger_{s_2}(\Omega)a_{s_2}(\Omega)$ is

$$N_{s_1} = N_{s_2} = \frac{2\pi^2 L}{\sigma^2 L}. \tag{9}$$

It depends on the total number of pump photons per pulse, however it is independent of the shape of the pulse. This fact and that $N_{s_1} = N_{s_2}$ are characteristics of the low parametric gain regime.

We are interested in the normalized first-order correlation function $g_{s_1, s_2}^{(1)}$ between signal beams $s_1$ and $s_2$ that gives the visibility of the interference pattern detected after combining both signals in a beam splitter, i.e.,

$$g_{s_1, s_2}^{(1)} = \frac{1}{N_{s_1}^{1/2}N_{s_2}^{1/2}}\int d\Omega a^\dagger_{s_1}(\Omega)a_{s_2}(\Omega). \tag{10}$$

Let us first assume that there are no losses in the idler path ($r(\Omega) = 1$). Using Eqs. 1, 8 and 9 into Eq. 10 and taking into account the distances $z_1$ and $z_3$ that signal beams $s_1$ and $s_2$ propagate before combination in the beam splitter, the first-order correlation function can be written as

$$\left|g_{s_1, s_2}^{(1)}(T_1, T_2)\right| = \text{tri}\left(\frac{T_1}{DL}\right) \times \exp\left[-\frac{1}{16T_0^2}\left[1 - \frac{2D_+}{D}\right]T_1 + 2T_2\right]^2, \tag{11}\right.$$}



where $\text{tri}(\xi/2) = 1/\pi \int \sin^2(x)\exp(i\xi x)dx$ is the triangular function and

$$T_1 = \frac{z_3 - z_1 + z_2}{c} + NiL, \tag{12}$$

$$T_2 = \frac{z_{p_2} - z_{p_1} - z_2}{c} - NiL. \tag{13}$$

We assume that the condition $z_{p_2} = z_{p_1} + cNiL + z_2$ if fulfilled, so that $T_2 = 0$. In order to optimize pulsed parametric amplification in NLC$_2$ one needs to synchronize the time of arrival of pump and idler pulses to the nonlinear crystal [15].

FIG. 2. First-order correlation function as a function of the path delay $\Delta z$. We consider a nonlinear crystal with length $L = 5$ mm. The pump pulses have temporal widths: (a) $T_0 = 100$ ps; (b) $T_0 = 2$ ps; and (c) $T_0 = 100$ fs.

The first-order correlation function is the product of a triangular function of width $DL$ and a Gaussian function of width $T_0$, the temporal width of the pump pulses. Figure 2 plots the first-order correlation function as a function of $\Delta z = z_3 - z_1 + z_2 + cNiL$ for a crystal length $L = 5$ mm and three different pulse widths: $T_0 = 100$ ps, $T_0 = 2$ ps and $T_0 = 100$ fs. $\Delta z$ can be modified in an experiment by changing the path length difference $z_3 - z_1$. We have considered as example two MgO-doped LiNbO$_3$ crystals [28] pumped by a pulsed laser operating at $\lambda_0 = 532$ nm. The resulting type-0 signal and idler beams have wavelengths $\lambda_0^s = 810$ nm and $\lambda_0^i = 1550$ nm with $D = -263.50$ fs/mm and $D_+ = 780$ fs/mm.

In the limiting case of CW pumping ($T_0 \to \infty$), the shape of the first-order correlation function is dominated by the triangular function [see Fig. 2(a)], as it has been measured in many occasions [10]. As we decrease the temporal width of the pump pulses, the influence of the triangular and Gaussian functions on $g_{s_1, s_2}^{(1)}$ becomes comparable [Fig. 2(b)]. Finally, when $T_0 \ll DL$, the shape of the first-order correlation function is dominated by the Gaussian function [Fig. 2(c)].

Is entanglement between signal and idler photons relevant for observing induced coherence? Inspection of Fig. 2 shows that it is not, since for all values of $T_0$ and crystal length $L$, that correspond to quantum states with different degrees of entanglement, there is induced coherence. For the sake of clarity, let us be more specific. In the low parametric gain regime, the biphoton function

$$\Psi(\Omega_s, \Omega_i) = i\sigma LF(\Omega_s + \Omega_i)\text{sinc}\left[\frac{\Delta kL}{2}\right]\exp(i\sigma kL), \tag{14}\right.$$}

where $\sigma k = k_p(\Omega_s + \Omega_i) + k_s(\Omega_s) + k_i(\Omega_i)$, determines the nature of the correlations between the paired photons and the degree of entanglement between them [25]. If we can decompose $\Psi(\Omega_s, \Omega_i)$ into two functions that depend separately on the variables $\Omega_s$ and $\Omega_i$ the quantum state is non-entangled (separable).

For the sake of simplicity, let us consider $D_+ = 0$ and make the approximation $\text{sinc}(x) \sim \exp(-\alpha^2 x^2)$ with $\alpha = 0.455$ [29]. The normalized biphoton function derived
III. OPTICAL COHERENCE TOMOGRAPHY WITH LARGE BANDWIDTH AND HIGH PHOTON FLUX

OCT is a three-dimensional noninvasive optical imaging technique that permits cross-sectional and axial high-resolution tomographic imaging [31]. The axial and transverse resolutions of OCT are independent. To obtain information in the axial direction (along the beam propagation), OCT uses a source of light with short coherence length (large bandwidth) that allows optical sectioning of the sample.

Different OCT schemes that make use of biphoton sources have been proposed and demonstrated. In all these cases one photon of the pair probe the sample under investigation. To obtain information some schemes measure the second-order correlation function of signal and idler photons [32, 33], others measure the first-order correlation function of signal photons generated in different biphoton sources [15, 16] and other measure the flux of signal photons generated in an SU(1,1) nonlinear interferometer [9, 17].

Figures 2 and 3 demonstrate that one can observe induced coherence independently of the degree of entanglement between the signal and idler beams. This has an important consequence for the further development of optical coherence tomography based on nonlinear interferometers. Equation (9) shows that the photon flux generated increases with the nonlinear crystal length. However, for the case of CW pumping, the down-conversion bandwidth \(\Delta_{dc}\) goes as \(1/DL\). OCT with high axial resolution requires a large bandwidth. Therefore, for CW pumps high axial resolution implies the generation of low photon fluxes and so longer integration times to obtain high-quality images. This is detrimental for OCT applications.

The first-order correlation function is the measure of axial resolution in an OCT system. Equation (11) shows that one can obtain a narrow first-order correlation function, and thus high axial resolution, even for long nonlinear crystal by using an ultrashort pump pulse. Indeed, when the Gaussian function dominates the shape of \(g^{(1)}(\Omega_s, \Omega_i)\) for \(T_0 \ll DL\), one can achieve high axial resolution independently of the crystal length.

In order to show this effect, we consider a bilayer sample characterized by a reflectivity \(r(\Omega) = r_0 + r_1 \exp[i(\omega^0 + \Omega)\tau]\). The delay is \(\tau = 2d_0 n_0/\omega\) where \(d_0\) and \(n_0\) designate the thickness and refractive index, respectively, of the sample. The coefficient \(r_0\) is the Fresnel coefficient for the first layer, whereas \(r_1\) is the effective coefficient for the second layer, taking into account the transmissions from the primary interface. \(z_2\) is the distance traveled by the idler beam reflected from the first layer, while \(z_2 + 2n_0 d_0\) is the optical distance traveled by the idler beam reflected from the second layer.

The signal detected at one output port of the beam splitter is

\[
\Phi(\Omega_s, \Omega_i) = \left(\frac{\alpha T_0 DL}{\sqrt{2\pi}}\right)^{1/2} \exp\left[-\frac{(\Omega_s + \Omega_i)^2 T_0^2}{2}\right] \\
\times \exp\left[-\frac{\alpha^2(DL)^2}{16} \frac{(\Omega_s - \Omega_i)^2}{(\Omega_s + \Omega_i)^2}\right].
\]
where \( T'_1 = T_1 + \tau \) and \( T'_2 = T_2 - \tau \). \( T_1 \) and \( T_2 \) are given by Eqs. (12) and (13). We can choose \( z_{p2} = z_{p1} + cNcL + z_1 \).

Figure 4 shows the photon flux \( N \) as a function of \( \Delta z \) [Eq. (16)] for a 20 \( \mu \)m glass slab (refractive index \( n_0 = 1.5 \)) embedded between air (\( n_1 = 1 \)) and water (\( n_2 = 1.3 \)). We consider three scenarios. Fig. 4(a) considers a pump beam with \( T_0 = 100 \) ps (quasi CW) and a nonlinear crystal with \( L = 0.5 \) mm. The interferogram shows two maxima separated by 60 \( \mu \)m, which is equal to the sample’s optical path length \( c\tau \).

Figure 4(b) considers the same pulse duration but a crystal length \( L = 10 \) mm. In this case, the interferogram cannot resolve the thickness of the sample. There is not enough axial resolution to image the sample. Figure 4(c) considers the same crystal length \( L = 10 \) mm but now with \( T_0 = 100 \) fs. The interferogram recovers the two maxima, thereby resolving the layers of the sample. Interestingly, the two maxima are separated by 42 \( \mu \)m, which is smaller than the sample’s optical thickness. This result can be understood noticing that the peak of the interferogram when the shape of the first-order correlation function is dominated by the Gaussian function will take place for a value of \( T_1 \) [see Eq. (11)]

\[
1 - \frac{2D}{D} (T_1 + \tau) - 2\tau = 0, \\
\Rightarrow T_1 = \frac{D + 2D_+}{D - 2D_+}. \tag{17}
\]

Taking into account the values of \( D = -263 \) fs/mm and \( D_+ = 780 \) fs/mm considered, the factor \( (D + 2D_+)/(D - 2D_+) = -0.71 \) indicates that the separation between the two maxima corresponding to the two layers is \(-0.71 \times 60 \mu m \sim -42 \mu m \). This result is reminiscent of the fact that after reflection from the sample, we have two pulses separated a delay \( \tau \) that are injected in the second nonlinear crystal and both show certain delay with the pump beam pulse \( \Psi \). For a case with \( D_+ = 0 \) we would have again \( T_1 = \tau \) as in the quasi CW case.

Figure 4 also shows the signal-photon spectrum for each scenario given by \( S(\Omega_0) = \int d\Omega_1 |\Psi(\Omega_0, \Omega_1)|^2 \). Clearly the interferograms and spectra show the reciprocal relation between the spectral bandwidth of the photons and axial resolution.

IV. CONCLUSIONS

We have demonstrated that induced coherence in the low parametric gain regime can be observed independently of the degree of entanglement between signal and idler photons. We thus conclude that the induced coherence is not the result of the quantum entanglement between paired photons. Indeed, in the demonstration of the first OCT scheme based on parametric down-conversion in the high parametric gain regime, the bandwidth of the pump pulse and the bandwidth of down-conversion (0.36 nm) are made comparable due to the use of narrowband filters. However, in this regime of high parametric gain one cannot readily consider entanglement between signal and idler photons.

In the low parametric gain regime, the emission rate of signal-idler photons increase linearly with the length of the nonlinear crystal, regardless of the duration of the pump pulses. We have shown that in an OCT scheme based on induced coherence one can achieve high axial resolution and high photon emission rates by combining ultrashort pumping with millimeter-length crystals. Besides, the method maintains its salutary features, i.e., probing the sample with photons centered at the most appropriate wavelength while using the optimum wavelength for silicon-based photodetectors.

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