Multi-objective optimization design for airfoils with high lift-to-drag ratio based on geometric feature control

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Abstract. Airfoil is an important factor that affects the energy conversion efficiency of wind turbines. In this research, the airfoil is parameterized by the superposition of the mean camber line and the thickness distribution. This method can accurately control the geometric characteristic parameters. By integrating several expected targets into one target function, a unified target function is constructed. Finally, the non-dominated sorting genetic algorithm II (NSGA-II) is used to realise the multi-objective optimization design. In order to verify the reliability of this design theory, the traditional Clark Y airfoil was selected as a reference. Considering the aerodynamic characteristics at angles of attack of 0, 4, and 8 degrees for $Re = 1 \times 10^6$, two airfoils were designed and compared with the reference airfoil. The results show that the lift-to-drag ratio characteristics of the designed airfoils have been greatly improved, which fully confirms the reliability of the optimization design theory proposed in this study. This research will be significant for the design of wind turbine airfoils.

1. Introduction
Because of the dominant use of depleting fossil fuels, our planet is facing an environment crisis. The development of all forms of renewable and clean energy resources is needed and has been a rapid growth. As a new energy source with great potential, wind energy is clean and renewable. It is a permanent and large existence of local resources that can provide humans with long-term stable energy supply [1-4]. As one of the most mature and commercialized development prospects, the development of wind turbines is of great significance for ensuring energy security, adjusting the energy structure, reducing environmental pollution and achieving sustainable development.

As a high-tech achievement of aeronautical technology, the airfoil is widely used in the design of wind turbine blades and has been an important factor that affects the energy conversion efficiency of wind turbines [4-7]. Therefore, many scholars devote themselves to the research and development of wind turbine airfoils. By focusing on increasing the lift-to-drag ratios at the operating angles of attack (AOAs), Yang Z Q et al. [2] proposed a multi-AOA design optimization method for VAWT airfoil. This method is applied to optimize the airfoil NACA64618 and verified by simulation results. Yavuz T et al. [8] discussed the use of double-blade airfoils in the wind and hydrokinetic turbines. This research points out that double-blade hydrofoil may re-define the potentials of wind power and hydrokinetic power of the countries in positive manner. Mortazavi S M and Sobhgahi S [9] studied the performances of three different National Renewable Energy Laboratory airfoil families using a 2D incompressible unsteady computational fluid dynamics solver and exergy analysis. The result shows that the most exergetic effective flow condition can represent the lowest lift-to-drag ratio. The above
researches show that the lift-to-drag ratio characteristics of the airfoil play a vital role in the performance of wind turbines. This research has important significance for the improvement of energy conversion efficiency of wind turbines.

2. Parameterized design method of airfoil
The geometric figure of airfoil is given in Figure 1.

![Airfoil nomenclature](image)

\[ y = y_f \pm \frac{t}{2} y_t \]  \hspace{1cm} (1)

where \( y_f \) represents the function of the mean camber line; \( y_t \) denotes the function of the thickness distribution.

2.1. Function of the mean camber line
The design method of the mean camber line is shown in Figure 2. The mean camber line is composed of two elliptical arcs, which are cut from two ellipses with the same vertex.

![Mean camber line consisting of two elliptic arcs](image)

In Figure 2, coordinates (0, 0) and (1, 0) represent the leading edge and trailing edge, respectively. The coordinate \( (x_f, f) \) is the vertex of the two ellipse arcs. The function expression of the mean camber line is presented in Equation (2):
where $k_{f1}$ represents the shape factor of the front edge of the mean camber line, $k_{f1} \in (-1,0]$; $k_{f2}$ is the shape factor of the back edge of the mean camber line, $k_{f2} \in (-1,0]$. 

2.2. Thickness distribution function

The Joukowsky airfoil profile expression is shown as follows:

\[
y = \sqrt[4]{\frac{1}{64f^2}} + \frac{1}{8f^2} - x^2 - \frac{1}{8} + \frac{2\sqrt{3}}{9} f (1 - 2x) \sqrt{1 - 4x^2}.
\]

where $x_0$ denotes the abscissa; “+” is the upper surface; “-” indicates the lower surface.

In Equation (3), 1 and 2 represent the mean camber line and the thickness distribution of Joukowski airfoil, respectively. This study only takes the thickness distribution as the research object.

In order to move the leading edge to the coordinate origin, the following transformation is carried out, gives

\[
x_0 = x - \frac{1}{2}
\]

Through substituting Equation (4) into Equation (3), the thickness distribution expression of the Joukowski airfoil with the leading edge at the origin of coordinates is obtained, gives

\[
y_i = \frac{8\sqrt{3}}{9} \alpha x^{0.5} (1 - x)^{1.5}
\]

The form of Equation (5) is extended to Equation (6), gives

\[
y_i = \frac{8\sqrt{3}}{9} m x^k (1 - x)^d
\]

By solving $y'_i = 0$, the expressions of the maximum relative thickness of airfoil and the relative position of the maximum thickness for airfoil chord length are obtained as follows:

\[
t = y'_i \bigg|_{y'_i = 0} = \frac{8\sqrt{3}}{9} m \frac{k_i d^d}{(k_i + d)^{k_i + d}}
\]

\[
x'_i = \frac{k_i}{k_i + d}
\]

In order to control the thickness distribution conveniently, the thickness distribution of the airfoil is controlled separately with the relative position of the maximum thickness for airfoil chord length as the demarcation point, and the distribution equation of the airfoil thickness is obtained as follows:

\[
y_i = \begin{cases} 
\frac{t(k_{f1} + d_1)^{k_{f1} + d_1}}{k_{f1}^{k_{f1} + d_1}} x^{k_1}(1 - x)^{d_1}, & 0 \leq x < x_f \\
\frac{t(k_{f2} + d_2)^{k_{f2} + d_2}}{k_{f2}^{k_{f2} + d_2}} x^{k_2}(1 - x)^{d_2}, & x_f \leq x \leq 1
\end{cases}
\]
\[ d_i = k_{i1} \frac{1 - x_i}{x_i}, \quad d_i = k_{i2} \frac{1 - x_i}{x_i} \]

where \( k_{i1} \) represents the shape factor of the front edge of the thickness distribution, \( k_{i1} \in (0, \infty) \); \( k_{i2} \) is the shape factor of the back edge of the thickness distribution, \( k_{i2} \in (0, \infty) \).

3. Optimization design method

3.1. Calculation method of flow field

It is common to analyse airfoil geometries using computer programs that couple inviscid flow solutions with integral boundary-layer methods. Such programs include Mark Drela’s X-foil, Eppler’s PROFIL and RFOIL extended from X-foil. Furthermore, it is still prudent to predict the airfoil airflow by these commonly used theoretical methods. Because inaccurate prediction may make the design deviate from its aerodynamic optimization or even destroy the whole design. The X-foil program uses the combination of panel method and boundary layer integral formula to analyse potential flow around airfoil flow field [10]. The program, which ensures its convergence through the boundary layer displacement thickness and the iteration of the outflow field, is widely used to predict the aerodynamic performance of the airfoil at low Reynolds number. The accuracy of this program has been recognized by the vast number of scholars [11-13]. In this paper, the X-foil program is used to predict the aerodynamic performance of airfoil.

3.2. NSGA-II algorithm

In 2000, Professor K. Deb [14] developed a non-dominated sorting genetic algorithm II (NSGA-II) on the basis of the non-dominated sorting genetic algorithm (NSGA). The computational complexity of the non-dominated sorting algorithm is large, especially in the case of very large groups. Because identifying individuals that belong to the optimal non-dominant boundary needs to compare each solution with other solutions. However, NSGA-II uses an intelligent non-dominated sorting strategy to reduce the amount of computation required. For each solution, the dominating count is calculated: the solution of the dominating count 0 belongs to the optimal boundary. Then, the dominating count of all remaining dominating solutions is reduced by 1, and the solution of dominating count 0 is subdivided into sub optimal boundaries. Repeat this process until all the designs are sorted. This method has been used by many scholars in the work of multi-objective optimization. In this research, NSGA-II algorithm is applied to the multi-objective optimization design of airfoil.

3.3. The establishment of the objective function

The general optimization problem can be described as follows:

Objective function: \( f(\bar{x}) \)

Constraint condition: \( g_i(\bar{x}) \leq 0 \quad i = 1, 2, \ldots, n \)

where \( f \) is the objective function; \( g_i \) donates the constraint condition; \( \bar{x} \) represents design variables.

For the multi-objective optimization problem of this research, multiple desired objectives are integrated into one objective function, gives

\[ f(\bar{x}) = \sum_{j=1}^{m} \omega_j f_{r,j}(\bar{x}) \quad j = 1, 2, \ldots, m \]  

(9)

where \( \omega_j \) is the weight of each indicator, and

\[ \sum_{j=1}^{m} \omega_j = 1, \quad j = 1, 2, \ldots, m \]

\( f_{r,j} \) represents the dimensionless objective function value obtained by dimensionless processing of the objective function of each index.
And then, the objective function is combined with the airfoil design method and the aerodynamic prediction program. Finally, the multi-objective optimization design of airfoil is realized by using the NSGA-II algorithm. The optimization design process is shown in Figure 3.

In order to verify the multi-objective optimization design theory proposed in this research, the lift-to-drag ratio of the design point that is a dimensionless parameter is selected as the dimensionless target function, as shown in Equation (10).

$$f_{r,j} = \frac{C_{l,j}}{C_{d,j}}$$

where $C_{l,j}$ represents the lift coefficient at each angle of attack; $C_{d,j}$ denotes the drag coefficient at each angle of attack.

As a traditional natural laminar flow airfoil, the Clark Y airfoil is used as a reference. Considering the aerodynamic characteristics at angles of attack of 0, 4 and 8 degrees for $Re = 1 \times 10^6$, two airfoils were designed, that is New Airfoil 1 and New Airfoil 2. For New Airfoil 1, the maximum relative thickness and the relative position of the maximum thickness for airfoil chord length are the same as that of the reference airfoil. Furthermore, each design parameter of New Airfoil 2 has a certain range of variation relative to the reference airfoil.

Because of the complexity of the flow at large angles of attack, it is not easy to get good aerodynamic performance by taking the same weight. If the weight at large angles of attack are increased appropriately, the improvement of aerodynamic performance will bring beneficial results. Therefore, the weights at angles of attack of 0, 4 and 8 degrees are 0.3, 0.2 and 0.5, respectively. The specific parameter settings are shown in Table 1 and Table 2.

| Table 1. Constraints of design variables and results for New Airfoil 1. |
|-----------------|-----------------|-----------------|
|                | Clark Y         | Constraints of design variables for New Airfoil 1 | Design results for New Airfoil 1 |
| $t$ (%)        | 11.71           | 11.71           | 11.71           |
| $x_t$ (%)      | 28.00           | [25.0, 55.0]    | 25.084           |
| $k_{f_1}$      | [0.15, 0.9]     | 0.572           |
| $k_{f_2}$      | [0.15, 0.6]     | 0.529           |
| $f$ (%)        | 3.43            | 3.43            | 3.43            |
| $x_f$ (%)      | 42.00           | [25.0, 55.0]    | 42.272           |
| $k_{f_1}$      | [-0.99, -0.2]   | -0.613          |
| $k_{f_2}$      | [-0.99, -0.2]   | -0.567          |
Table 2. Constraints of design variables and results of New Airfoil 2.

| Constraints of design variables for New Airfoil 2 | Design results for New Airfoil 2 |
|-----------------------------------------------|----------------------------------|
| Constraints of design variables for New Airfoil 2 | Design results for New Airfoil 2 |
|  | Clark Y | [10.50,12.88] | 12.604 |
|  | $t$ (%) | 11.71 | 25.187 |
|  | 28.00 | 0.572 |
|  | 25.00 | 0.48 |
|  | 3.43 | 3.612 |
|  | 42.00 | 40.625 |
|  | 0.48 |
|  | 0.15,0.9 |
|  | 0.48 |
|  | 0.15,0.6 |
|  | 0.48 |
|  | 0.15,0.6 |

4. Results and analysis

The design results of New Airfoil 1 and New Airfoil 2 are also given in Table 1 and Table 2. Compared with the reference airfoil, the relative position of the maximum thickness of the two designed airfoils is closer to the leading edge than that of the reference airfoil. And the relative position of the maximum curvature is near the 41% string length, which is similar to the reference airfoil.

![Comparison between the designed airfoils and the Clark Y airfoil.](image)

Figure 4. Comparison between the designed airfoils and the Clark Y airfoil.

In Figure 4, the contrast between the designed airfoils and the reference airfoil is given. It can be seen that the relative thickness of the designed airfoils is slightly larger than the reference airfoil from the relative position of the maximum thickness to the leading edge, while the relative thickness gets slightly reduced from the relative position of the maximum thickness to the trailing edge.

![Aerodynamic performance comparison](image)

Figure 5. Aerodynamic performance comparison between designed airfoils and Clark Y airfoil for $Re=1.0 \times 10^6$. 

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Figure 5 shows the comparison of aerodynamic performance between designed airfoils and Clark Y airfoil for $Re=1.0\times10^6$. The lift coefficient curves shown in Figure 5.a indicate that the lift characteristics of New Airfoil 1 and New Airfoil 2 are significantly higher than that of the reference airfoil in the calculated range of angles of attack. And the lift characteristics of New Airfoil 2 are slightly better than that of New Airfoil 1. The reason is that New Airfoil 2 has two more degrees of freedom, which makes New Airfoil 2 have a larger design space. The lift-to-drag curves presented in Figure 5.b show that the maximum lift-to-drag ratios of New Airfoil 1 and New Airfoil 2 are 131.3 and 133.2 respectively, which are 14.25% and 15.93% higher than that of the reference airfoil. Furthermore, the range of high lift-to-drag ratio is also significantly increased. The polar curves of airfoil are also given in Figure 5.c. It can be seen that under the same lift coefficient, the drag coefficient of New Airfoil 1 and New Airfoil 2 is much smaller than that of the reference airfoil, and this phenomenon can be observed in nearly all the range of the lift coefficient.

The results show that not only the lift-to-drag ratio characteristics but also the polar curves of New Airfoil 1 and New Airfoil 2 have a significant improvement compared with the reference airfoil, which confirms the reliability of this design theory.

5. Conclusions
A multi-objective optimization design theory of airfoil is set up. This theory parameterizes the airfoil profile through the superposition of the mean camber line and the thickness distribution, which can accurately control the geometric characteristic parameters of airfoil. Then the aerodynamic performance of the airfoil is predicted by the X-foil program. In order to solve the multi-objective optimization problem, multiple desired objectives are integrated into one objective function. Finally, the multi-objective optimization design of airfoil is realized by using the non-dominated sorting genetic algorithm II (NSGA-II).

In order to verify the reliability of the design theory, a traditional natural laminar flow airfoil Clark Y was used as a reference. Two airfoils, New Airfoil 1 and New Airfoil 2, were designed by considering the aerodynamic characteristics at angles of attack of 0, 4, and 8 degrees for $Re = 1\times10^6$, and compared with the reference airfoil. The results show that, compared with the reference airfoil, the lift-to-drag ratio characteristics of the designed airfoils have been greatly improved and the range of high lift-to-drag ratio is also significantly increased, which fully confirms the reliability of the optimization design theory proposed in this research.

Appendices

| Symbol | Meaning |
|--------|---------|
| $y_f$  | function of the mean camber line |
| $y_t$  | function of the thickness distribution |
| $k_{f1}$ | the shape factor of the front edge of the mean camber line, $k_{f1} \in (-1,0]$ |
| $k_{f2}$ | the shape factor of the back edge of the mean camber line, $k_{f2} \in (-1,0]$ |
| $k_{t1}$ | the shape factor of the front edge of the thickness distribution, $k_{t1} \in (0,\infty)$ |
| $k_{t2}$ | the shape factor of the back edge of the thickness distribution, $k_{t2} \in (0,\infty)$ |
| $f_{ij}$ | the dimensionless objective function |
| $g_i$  | constraint condition |
| $\omega_j$ | weight of each indicator |
| $C_{li}$ | lift coefficient at each angle of attack |
| $C_{di}$ | drag coefficient at each angle of attack |
Acknowledgment
We gratefully acknowledge the financial support provided by National Natural Science Foundation of China (11772263) and Shaanxi Innovation Capability Support Plan (2017KG0200).

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