Decay of the Loschmidt echo in a time-dependent environment

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We study the decay rate of the Loschmidt echo or fidelity in a chaotic system under a time-dependent perturbation \( V(q, t) \) with typical strength \( h/\tau_v \). The perturbation represents the action of an uncontrolled environment interacting with the system, and is characterized by a correlation length \( \xi_0 \) and a correlation time \( \tau_0 \). For small perturbation strengths or rapid fluctuating perturbations, the Loschmidt echo decays exponentially with a rate predicted by the Fermi Golden Rule, \( 1/\tilde{\tau} = \tau_c/\tau_0^2 \), where typically \( \tau_c \sim \min[\tau_0, \xi_0/v] \) with \( v \) the particle velocity. Whenever the rate \( 1/\tilde{\tau} \) is larger than the Lyapunov exponent of the system, a perturbation independent Lyapunov decay regime arises. We also find that by speeding up the fluctuations (while keeping the perturbation strength fixed) the fidelity decay becomes slower, and hence, one can protect the system against decoherence.

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I. INTRODUCTION

The time-evolution of a quantum system is quite robust to changes of the system initial conditions, irrespective of the nature of the underlying dynamics. This is in deep contrast to classical evolution, particularly that of a chaotic system. In a seminal paper, Peres noticed that quantum time-evolution can be sensitive to the differences between chaotic and integrable dynamics in a peculiar set up: One needs to examine the overlap of identically prepared states, but evolved with slightly different Hamiltonians. This overlap, called Loschmidt echo (LE) or fidelity, measures the recovery obtained when a wave packet evolves for a time \( t \), followed by a backwards evolution with a perturbed Hamiltonian for the same time interval.

A considerable number of investigations has been devoted to study the interesting and intricate phenomena related to the LE, in particular the different regimes that arise depending on the perturbation strength. For very small perturbations, the LE is described by standard perturbation theory and a Gaussian decay is observed. For stronger perturbations, where perturbation theory breaks down, large phase fluctuations lead to an exponential decay of the LE described by the Fermi Golden Rule (FGR). For even stronger perturbations, but still weak in the classical sense, a semiclassical analysis yields an exponential LE decay that does not depend on the perturbation strength: The decay rate is determined by the Lyapunov exponent that characterizes the classical counterpart of the unperturbed system. The latter two cases are called the FGR and Lyapunov regimes respectively, where the LE decay rate is the minimum between the width of the local density of states (LDOS), as given by the FGR, and the Lyapunov exponent. These predictions were verified numerically in a number of systems. The theory is successful to the extend that, by analyzing the LE decay, the quantum evolution of a system can be used to quantitatively assess its classical Lyapunov exponent.

The theory was later extended to classically integrable systems, in which case a power law like decay is predicted. This result is still somewhat controversial, since as a rule integrable systems display non-generic features. In any event, these works indicate that the LE decay is very different whether the underlying classical system has a chaotic, integrable, or even mixed phase space.

Albeit this wealth of interesting results, so far the theory of the LE non-perturbative regime has only dealt with time independent perturbations. The most probable motivation for this restrictive choice can be traced back to the experiments that triggered the research on the LE problem. They studied the time reversal of many-spin dynamics, where the perturbation is simply a static part of the Hamiltonian.

Numerous physical situations call for an extension of the LE theory that accounts for a time-dependent perturbation. Let us explicit mention a few. Experimentally, a subsystem selected from a large spin system with many-body interactions can be represented as immersed in an external fluctuating potential – the same approximation holds whenever the uncontrolled degrees of freedom are those of an environment with complex dynamics. Formally, the current analytical description contrasts with numerical results observed in periodically kicked one-dimensional models, where the perturbation can be interpreted as time dependent. The need is further stressed by the relevance of the LE to quantum computation, decoherence in open systems, and mesoscopic physics. Indeed, the decay of the LE is related to the decay of quantum correlations and the quantum-classical correspondence, as can be shown using the Wigner function representation.

In this work we use the semiclassical approximation to
derive the LE decay in the presence of a time-dependent perturbation, generalizing the approach presented in Ref.\textsuperscript{3}. We show that the existence of a LE perturbation-independent regime is quite generic. For that purpose, instead of using a particular model, we use a statistical approach. We obtain a closed expression for the LE decay in the FGR regime using simple assumptions on the perturbation autocorrelation function. We conclude by discussing the different limits of our results and the seemingly strange feature that faster fluctuations of the perturbation or stronger chaos in the system lead to a slower decay of the Loschmidt echo.

II. LOSCHMIDT ECHO IN A TIME-DEPENDENT ENVIRONMENT

The object of interest, the Loschmidt echo, is defined as:

\[ M(t) = |\langle \psi_0 | U(t_0, t) U_0(t, t_0) | \psi_0 \rangle|^2, \]

where \( |\psi_0 \rangle \) is an arbitrary wave packet prepared at time \( t_0 \). For simplicity, and in line with Ref.\textsuperscript{3}, we choose the initial state \( |\psi_0 \rangle \) as a Gaussian wave-packet centered at an arbitrary point \( \mathbf{r}_0 \) with dispersion \( \sigma \) and initial momentum \( \mathbf{p}_0 \). This restricted choice can be relaxed by considering other kinds of localized states in phase space \textsuperscript{28, 29}, evolved states \textsuperscript{11}, and even eigenstates of \( H_0 \) \textsuperscript{4, 26}. In Eq. (1), \( U_0 \) is the standard time evolution operator, namely

\[ U_0(t_0, t_0) = T \exp \left( \frac{i}{\hbar} \int_{t_0}^{t} dt' H_0(t') \right), \]

where \( T \) is the time ordering operator, while

\[ U(t_0, t) = \mathcal{T} \exp \left( \frac{i}{\hbar} \int_{t}^{t_0} dt' H(t') \right), \]

with \( \mathcal{T} \) the inverse time ordering operator. Equation (1) is also viewed as the fidelity of two wave packets prepared at the same initial state and evolving forward in time under different Hamilton operators.

In general, time ordering makes the exact evaluation of \( M(t) \) for a time dependent Hamiltonian a daunting task. To circumvent this difficulty we employ the semiclassical approximation, in which time ordering is trivially accounted for by taking the time evolution of classical trajectories, as we detail in the sequel.

We consider the Hamiltonian \( H \) defined as

\[ H = H_0 + V(q, t), \]

where \( H_0 \) is a time independent Hamiltonian that displays chaotic motion in the classical limit and \( V(q, t) \) is the time-dependent perturbation potential or the system interaction with a complex environment.

The semiclassical propagator reads

\[ \langle \mathbf{q}' | U(t) | \mathbf{q} \rangle = \left( \frac{1}{2\pi\hbar} \right)^{d/2} \sum_{s(q', q, t)} C_{s}^{1/2} \times \exp \left( \frac{i}{\hbar} S_{s}(q', q, t) - \frac{i\pi}{2} \alpha_{s} \right), \]

where \( s \) is a classical path that spends a time \( t \) to travel from \( \mathbf{q} \) to \( \mathbf{q}' \), \( S_{s} \) is the action (Hamilton principal function), given by \( S_{s}(q', q, t) = \int_{0}^{t} d\tau L(q_{s}(\tau), q_{s}(\tau), \tau) \), \( \alpha_{s} \) is the number of conjugate points along \( s \), and \( C_{s} \) is the Jacobian of the phase-space transformation between \( \delta \mathbf{p}'(0) \) and \( \delta \mathbf{q}'(t) \) -- a density of classical paths.

It is only possible to proceed analytically if we restrict ourselves to the regime of weak perturbations, in the sense that classical perturbation theory is applicable. More specifically, we approximate the action along a given trajectory \( s \) by

\[ S_{s}(t) \approx S_{s}^{0}(t) + \int_{0}^{t} dt' V(q_{s}(t'), t'), \]

where \( S_{s}^{0}(t) \) refers to the action corresponding to \( s \) obtained from \( H_0 \) and \( q_{s}(t) \) gives the particle position along the unperturbed trajectory \( s \) as a function of time. For chaotic systems, this approximation is accurate up to a time \( t_{cp} \) proportional to the logarithm of the strength of \( V \). In this sense, the perturbation is weak when \( t_{cp} \) becomes the largest time scale of the problem. This restriction does not preclude the perturbation to be quantum mechanically large, since the actions are measured in units of \( \hbar \). It has been observed that the classical perturbation approximation works, in general, surprisingly well even for times longer than \( t_{cp} \). This has been related to the structural stability of the manifold of trajectories in phase space \textsuperscript{30}. Even though individual trajectories are exponentially sensitive to perturbations, one can always find a “replacement” trajectory in the manifold that joins the points of interest for a given time interval \textsuperscript{28}.

Our calculation proceeds along the lines of Ref. \textsuperscript{3}, which we now briefly sketch. We assume that the wave packet \( |\mathbf{r}|\psi_0 \rangle \) is well localized, \( \xi > \sigma > \lambda_{dB} \), where \( \xi \) is a typical length of the perturbation (in Ref.\textsuperscript{3} the width of Gaussian impurities) and \( \lambda_{dB} \) is the de Broglie wavelength of the particle. Neglecting terms with a rapidly oscillating phase, one arrives at the semiclassical expression for the Loschmidt echo,

\[ M(t) \simeq \left( \frac{\alpha^{2}}{\pi\hbar^{2}} \right)^{d} \int d\mathbf{r} \sum_{s(\mathbf{r}, \mathbf{r}_0, t)} C_{s} \exp \left[ \frac{i}{\hbar} \Delta S_{s}(t) \right] \times \exp \left[ -\frac{\alpha^{2}}{\hbar^{2}} (\mathbf{p} - \mathbf{p}_0)^{2} \right], \]

where \( \Delta S_{s} \) is the action difference between trajectories evolved with \( H_0 \) and \( H \), and \( \mathbf{p} = -\partial S_{s}/\partial \mathbf{r} \big|_{\mathbf{r} = \mathbf{r}_0} \). All trajectories \( s \) start at \( \mathbf{r}_0 \), the position where the Gaussian
A. Non diagonal contributions to $\langle M(t) \rangle$

Let us first calculate the terms where the two trajectories lie far apart in phase space. Such contributions to $\langle M(t) \rangle$ are usually called non-diagonal (different trajectories), and read

$$\langle M^{nd}(t) \rangle \approx \left( \frac{\sigma^2}{\pi \hbar^2} \right)^d \int d\mathbf{r} \sum_{s(\mathbf{r}, \mathbf{p}, t)} C_s \left\langle \exp \left[ \frac{i}{\hbar} \Delta S_s(t) \right] \right\rangle \times \exp \left[ -\frac{\sigma^2}{\hbar^2} (\mathbf{p} - \mathbf{p}_0)^2 \right]^2,$$

(8)

where $\langle \ldots \rangle$ indicates that we average over the wave packet initial positions $\mathbf{r}_0$, as well as over an ensemble of perturbations.

We assume, as is customary for chaotic systems, that the actions for different paths are uncorrelated and Gaussian distributed. This leads to an enormous simplification, allowing us to write

$$\left\langle \exp \left[ \frac{i}{\hbar} \Delta S_s(t) \right] \right\rangle \approx \exp \left[ -\frac{1}{2\hbar^2} \langle [\Delta S_s(t)]^2 \rangle \right].$$

We remain with the task of evaluating the action variance

$$\langle [\Delta S_s(t)]^2 \rangle = \int_0^t dt' \int_0^t dt'' \langle V(q_s(t'), t') V(q_s(t'', t'')) \rangle.$$  

(10)

For that purpose we introduce an ensemble of perturbations $V$ to model the general features of the environment. We replace the phase space average $\langle \ldots \rangle$ by the ensemble average $\langle \ldots \rangle$, the equivalence between averages being supported by the ergodicity of the system. In order to keep our calculation as general as possible, we assume very little knowledge of the perturbation, requiring only that time and space correlations are independent, viz.

$$V(q, t) V(q', t') = \langle \mathbf{V} \rangle C_S(|q - q'|) C_T(|t - t'|).$$

(11)

The typical perturbation strength is $\langle \mathbf{V} \rangle^{1/2}$, and $\tau_V = \hbar / \langle \mathbf{V} \rangle^{1/2}$ is its associated time scale. The dimensionless functions $C_S$ and $C_T$ quantify the spatial and time correlations of the potential $V(q, t)$. We further require that $C_S$ or $C_T$ decay sufficiently fast, so that

$$\int_0^\infty dr r^{d-1} C_S(r) < \infty \quad \text{and} \quad \int_0^\infty dt C_T(t) < \infty.$$  

(12)

For chaotic systems this is a sensible assumption.

To guide the discussion, let us introduce the correlation length $\xi_0$ and the correlation time $\tau_0$ that characterize $C_S$ and $C_T$ respectively. Since the average $\langle \ldots \rangle$ is computed along the classical trajectories of the system, the asymptotic decay $\langle \ldots \rangle$ can be induced not only by the fluctuations of $V$, but also by the intrinsic chaotic dynamics of $H_0$. In general, $\xi_0$ and $\tau_0$ are given by the minimum between the natural scales of $V$ and $H_0$. For instance, when the perturbation is a static change in the mass tensor of a free particle bouncing off the walls of a billiard system, $\xi_0$ is solely given by the dynamics of $H_0$ and is equal to the mean free path between collisions $\xi_0 \approx \tau_0 = 1/\lambda$. Another example can be found in Refs. 4 and 5, where the effective scale $\tau_0$ is given by the kicking period of the unperturbed Hamiltonian – although the perturbation is a time independent change in the kicking strength. Hence, our results are valid not only for random perturbations, but also for static and periodic ones: the chaoticity of the underlying Hamiltonian alone can enforce conditions $<\partial \rangle$.

In the limit of $\tau_0 \gg 1/\lambda$ the perturbation is quasi-static and the results of Ref. 3 hold without further change. We are interested in the regime where the typical times of the perturbation are comparable to those of the system, $\tau_0 \lesssim 1/\lambda$.

Replacing space averages by ensemble averages $\langle \ldots \rangle$, we write Eq. (10) as

$$\langle \Delta S_s(t)^2 \rangle = \langle \mathbf{V} \rangle^2 \int_0^t dt \int_0^\infty d\tau \times C_R \left[ \left| \mathbf{q}_s(\tau - \frac{t}{2}) - \mathbf{q}_s(\tau + \frac{t}{2}) \right| \right] C_T(\tau),$$

where we considered times $t$ much larger than $\tau_0$ and $\xi_0/\nu$, which allows us to take the integral in $\tau$ from $-\infty$ to $+\infty$. Eq. (13) has two limiting regimes that are readily solved. In the first one, the spatial disorder has a much shorter scale than the temporal one: $\tau_0 \gg \xi_0/\nu = \xi_1$. In this case the decay of $M^{nd}(t)$ is dominated by the same exponent as the one found in Ref. 3

$$\langle \Delta S_s(t)^2 \rangle \approx \langle \mathbf{V} \rangle^2 \int_0^t dt \int_0^\infty d\tau C_S \left[ \left| \mathbf{q}_s(\tau - \frac{t}{2}) - \mathbf{q}_s(\tau + \frac{t}{2}) \right| \right] = \frac{t}{\tau_1} \hbar^2,$$

(14)

where $C_T(\tau)$ is assumed constant and $\tau_1$ is given by a FGR calculation

$$\frac{1}{\tau_1} = \frac{\tau_0}{\tau_V}.$$  

(15)

When $\tau_0 \ll \tau_1$, we deal the opposite regime, and

$$\langle \Delta S_s(t)^2 \rangle \approx \langle \mathbf{V} \rangle^2 \int_0^t dt \int_0^\infty d\tau C_T(\tau) = \frac{t}{\tau_2} \hbar^2,$$

(16)

with

$$\frac{1}{\tau_2} = \frac{\tau_0}{\tau_V}.$$  

(17)
Thus, in these two limits and complementary situations, the FGR exponent changes from being governed by the spatial to the temporal correlations of $V(q, t)$. The interesting “correlation crossover regime” – where neither the temporal nor the spatial correlation dominate – will be discussed shortly for a particular form of $C_S$ and $C_T$.

**B. Diagonal contributions to $\langle M(t) \rangle$**

Let us first explicitly write (17), namely

$$M(t) \simeq \left( \frac{\sigma^2}{\pi \hbar^2} \right)^d \int dq \int dq' \sum_{s,s',t,t',r_0} C_s C_{s'}$$

$$\times \left\langle \exp \left[ \frac{i}{\hbar} (\Delta S_s(t) - \Delta S_{s'}(t')) \right] \right\rangle$$

$$\times \exp \left[ -\frac{\sigma^2}{\hbar^2} \left( (q_s - p_0)^2 + (q_{s'} - p_0)^2 \right) \right].$$

(18)

and analyze the case where the trajectories $s$ and $s'$ remain close to each other. Now the action differences cannot be considered as uncorrelated, and we have to take into account the fluctuations in

$$\left\langle \exp \left[ \frac{i}{\hbar} (\Delta S_s(t) - \Delta S_{s'}(t')) \right] \right\rangle \simeq$$

$$\exp \left[ -\frac{1}{2\hbar^2} \left( [\Delta S_s(t) - \Delta S_{s'}(t')]^2 \right) \right].$$

(19)

In the same order of approximation of Eq. (10), we write

$$\Delta S_s(t) - \Delta S_{s'}(t) = \int_0^t dt' \left[ V(q_s(t'), t') - V(q_{s'}(t'), t') \right].$$

(20)

As the two trajectories remain close in coordinate space, we can expand $V(q_s(t), t)$ to first order around $s$ and obtain

$$\Delta S_s(t) - \Delta S_{s'}(t) \simeq \int_0^t dt' \nabla V(q_s(t'), t') \cdot [q_s(t') - q_{s'}(t')].$$

(21)

To calculate the action difference variance we turn our attention to the force correlation function, namely

$$C_V(|q - q'|, |t - t'|) \equiv \langle \nabla V(q, t) \cdot \nabla V(q', t') \rangle.$$  

(22)

As before, we introduce an ensemble of perturbations, and write

$$C_V(|q - q'|, |t - t'|) = \overline{\nabla C_T(|t - t'|) (\nabla q \cdot \nabla q')} C_S(|q - q'|),$$

(23)

such that $\langle \nabla q \cdot \nabla q' \rangle C_S(|q - q'|)$ decays sufficiently fast, in the sense defined by Eq. (12).

As time evolves, the separation between the coordinates $q_s(t)$ and $q_{s'}(t)$ grows as $e^{\lambda t}$, where $\lambda$ is the largest Lyapunov exponent of $H_0$. As a result, after some algebra, Eq. (19) gives

$$A = \sqrt{\frac{2}{\lambda_0}} \frac{1 - e^{-2\lambda t}}{2\lambda}.$$  

(24)

when $C_T$ dominates the decay of $C_V$, and

$$A = \sqrt{\frac{1 - e^{-2\lambda t}}{2\lambda}} \int dq \left[ 1 - d \frac{\partial C_S(q)}{\partial q} + \frac{\partial^2 C_S(q)}{\partial q^2} \right].$$

(25)

when $C_T$ decays slowly.

In summary, the main result of Ref. 3 holds, namely

$$M(t) = \overline{A} \exp(-\lambda t) + B \exp(-t/\tilde{\tau}),$$

(26)

where $\overline{A} = |m\sigma/(A^{1/2}t)|^d$, $\lambda$ is the classical Lyapunov exponent of the system and $1/\tilde{\tau}$ is given by Eq. (13). The exponential decay of the LE is dominated by the smallest between $1/\tilde{\tau}$ and $\lambda$, giving a crossover from FGR to Lyapunov decay as the perturbation strength increases.

**C. Correlation crossover**

In the regime where $\tau_0 \approx \tau_c$, one can only obtain further insight by assuming a specific form of the correlation functions. Although it is a less general result, one can still encompass a broad class of possible perturbations whose correlator decay in a particular way. We will consider the case where both $C_S$ and $C_T$ have Gaussian shapes,

$$V(q, t)V(q', t') = \frac{V^2}{\pi} \exp \left( -\frac{|q - q'|^2}{\xi^2} \right) \exp \left( -\frac{|t - t'|^2}{\tau_0^2} \right).$$

(27)

Under the assumption that $t$ is large compared to $\tau_0$ and $\tau_c$, we replace in Eq. (13) and Eq. (19), and obtain the decay rate for the FGR regime

$$\frac{1}{\tilde{\tau}} = \frac{\tau_c^{-2}}{\tau_0^{-2} + \tau_c^{-2}}.$$  

(28)

and the prefactor $A$ of the Lyapunov regime:

$$A = \left( \frac{\hbar^2}{v^2 \lambda_0} \right) \left( 1 - \frac{e^{-2\lambda t}}{\sqrt{\pi}} \right) \tau_c^4 \left( \frac{d - 1}{\tau_c^2} + \frac{d}{r_0^2 \tau_c^2} \right).$$  

(29)

When the temporal or spatial correlation dominate, we recover the previous limit

$$\frac{1}{\tilde{\tau}} \simeq \frac{\tau_c}{\tau_V} \text{ with } \tau_c = \min[\tau_0, \tau_c].$$  

(30)

As before, if the effective time scale $\tau_c$ becomes too short, the perturbation cancels itself out causing a very slow decay. This result is consistent with studies of time dependent errors in a quantum computer [20], where the dynamical decoupling to the environment was interpreted as a manifestation of the quantum Zeno effect [16, 32]. Notice that when $\tau_c$ is dominated by the dynamics of $H_0$, the fluctuations become faster for chaotic systems with a larger $\lambda$. [25]
III. CONCLUSIONS

We have extended the semiclassical theory of the Loschmidt echo to cope with time dependent perturbations. We expect our results to remain valid in more complex or analytically difficult cases, suitable only for numerical studies. Our treatment is sufficiently general as to describe the situations where the perturbation is the random effect of an uncontrolled environment on the system. The fluctuations we considered could arise either from an explicit time dependence of the perturbation potential, or from the ergodic nature of $H_0$. In the last case, the underlying chaotic dynamics mimics the randomness required for the decay of the correlation functions. Thus, our results should also apply to periodic or very simple oscillating perturbations.

We showed that the Loschmidt echo Lyapunov regime is barely affected by the time-dependence of the perturbation, except for prefactors: The decay is dominated by the system’s intrinsic dynamics of stretching and folding. In the FGR regime – when the non-diagonal terms dominate – the spatial and time scales of the perturbation compete with each other, and a simple behavior can be extracted when the relevant scales are far apart. In the intermediate regime, where the scales are comparable, using a simple (yet general) example we compute the decay rate of $M(t)$. The form of Eq. (30) stresses how fast fluctuations lead to self-cancellation of the interaction with the environment. In the case of the LE, a vanishing FGR exponent prevents the appearance of the perturbation independent Lyapunov regime. Surprisingly, this happens not only for rapidly fluctuating perturbations, but also by increasing the Lyapunov exponent. The slowing down of the FGR regime of decoherence – induced by fast fluctuations – was recently experimentally measured in NMR experiments, where a connection to the quantum Zeno effect was observed. It is interesting to recall that dynamical decoupling to the environment is what makes liquid NMR quantum computers possible (albeit small). The fast random movements of the molecules in the liquid average out the more difficult to control dipolar interactions present, e.g., in solids. Our work points to the importance of exploring dynamical alternatives to suppress quantum decoherence.

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