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Social equity-based distribution networks design for the COVID-19 vaccine
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A R T I C L E   I N F O

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A B S T R A C T

This study aims to investigate the role of social equity in vaccine distribution network design problems. Inspired by the current COVID-19 vaccine allocation in-country context, we capture social equity-based distribution by modeling three theories: Rawls’ theory, Sadr’s theory, and utilitarianism. We consider various social groups based on degree of urbanization, including inhabitants of cities, towns and suburbs, and rural areas. The distribution problem is subject to, on the one hand, demand-side uncertainty characterized by the daily contamination rate and its space–time propagation that anticipate the in-need population. On the other hand, supply-side uncertainty characterized by the stochastic arrival of vaccine doses for the supply period. To tackle this problem, we propose a novel bi-objective two-stage stochastic programming model using the sample average approximation (SAA) method. We also develop a lexicographic goal programming approach where the social equity objective is prioritized, thereafter reaching an efficiency level. Using publicly available data on COVID-19 in-country propagation and the case of two major provinces in Iran as example of middle-income country, we provide evidence of the benefits of considering social equity in a model-based decision-making approach. The findings suggest that the design solution produced by each social equity theory matches its essence in social science, differing considerably from the cost-based design solution. According to the general results, we can infer that each social equity theory has its own merits. Implementing Rawls’ theory brings about a greater coverage percentage in rural areas, while utilitarianism results in a higher allocation of vaccine doses to social groups compared to the Sadr and Rawls theories. Finally, Sadr’s theory outperforms Rawls’ in terms of both the allocation and cost perspective. These insights would help decision-makers leverage the right equity approach in the COVID-19 vaccine context, and be better prepared for any pandemic crisis that the future may unfold.

1. Introduction

1.1. Context and motivation

Since December 2019, COVID-19 emerged as one of the most critical threats to human survival. To date, it has severely damaged the global economy as well as the physical and psychological health of human beings (Thul and Powell, 2021). To face this emergency, regulatory authorities approved the rapid deployment of vaccines, including Pfizer/BioNTech, Moderna, Johnson & Johnson, AstraZeneca. Accordingly, many countries started their vaccination program in an emergency mode, not necessarily relying on a model-based decision-making approach to design the best distribution network for vaccine deployment. Similarly, the literature on vaccine distribution network design is limited. In fact, prior studies attempting to design a distribution network consider a single objective problem, namely cost minimization (Sadjadi et al., 2019; Lim et al., 2019). According to the World Health Organization (WHO), up to October 2021, 245 million individuals were infected with COVID-19 (WHO, 2021a), meaning that the number of COVID-19 infections far outweighs SARS or cholera. Fig. 1 shows the daily cases (a) and the cumulative cases (b) of COVID-19 worldwide (Ourworldindata, 2021). The propagation and criticality of COVID-19 cases are experienced differently in countries around the world depending on their location, population density, economic welfare, internal policies, and preparedness. The recent G20 report published in April 2022 quoted that the COVID-19 pandemic is far from over and the risk of a new Variant of Concern (VoC) emerging continues to be high due to intense transmission of the virus and low vaccination levels in many parts of the world (available at www.who.int/campaigns/vaccine-equity). Furthermore, the report shares figures on the low coverage of vaccination in low-income countries. Weintraub...
phenomenon is also referred to as the ripple effect (Ivanov and Dolgui, 2014). Such disruption propagation is more elaborate demand patterns than those commonly used in the daily contamination rate and its space–time propagation, similarly to large-scale disasters (Klibi and Martel, 2012b), clearly called for more elaborate demand patterns than those commonly used in the vaccine supply chain (Yadav et al., 2014). Such disruption propagation phenomenon is also referred to as the ripple effect (Ivanov and Dolgui, 2021). Mak et al. (2021) propose a SEIR (susceptible, exposed, infectious, recovered) model that predicts the infection trend. However, the rapid spread of the pandemic and the significant uncertainty on the demand side around vaccine doses play a major role in designing vaccine distribution. It is now well established that considering uncertainty improves the quality of the supply chain design (Klibi et al., 2010), and this work tackles the challenge to consider supply and demand uncertainty.

Many researchers have dedicated their studies to vaccine supply chain problems, as recently reviewed in Duijzer et al. (2017). From a vaccine distribution perspective, few studies cover the strategic decision-level (Sadjadi et al., 2019; Lim et al., 2019), as the majority deal with the tactical level (Chen et al., 2014; Dai et al., 2016; Lemmens et al., 2016). However, several aspects of the COVID-19 vaccination emergency and the dynamics of the spread of the pandemic make it strategically more challenging and appealing for novel modeling features. As underlined in Mak et al. (2021), COVID-19 vaccine distribution became the most crucial logistics challenge in the history of humankind.

First, a critical issue in several countries around the world is that access to COVID-19 vaccines is neither at the right level nor at the right time. Several shortfalls of supplies during the pandemic have been observed along the vaccine supply chain. This is mainly because the vaccines have not yet reached mass production, making their allocation challenging (Bertsimas et al., 2020; Duijzer et al., 2018). Mak et al. (2021) discuss the global impact of the limited supply of COVID-19 vaccines, proposing inventory policies for the case of two-dose deployment. Clearly, the shortage in the vaccine production level and the complexity of its global allocation give rise to critical uncertainty in the supply of doses at the country level, which was not the case in previous vaccine supply chains.

Second, when a novel pandemic breaks out, the vaccine doses needed are not known with certainty. Specifically for COVID-19, the magnitude of the spread was unprecedented, putting vaccine distribution networks under pressure. As discussed in Alam et al. (2021), vaccine supply chain analysts and healthcare experts needed to collaborate to establish appropriate vaccine distribution strategies to curtail COVID-19 infections. They also underlined that this latter is a complex propagation pattern considering population density and the availability and deployment speed of vaccines. Specifically, the uncertainty of the daily contamination rate and its space–time propagation, similarly to large-scale disasters (Klibi and Martel, 2012b), clearly called for more elaborate demand patterns than those commonly used in the vaccine supply chain (Yadav et al., 2014). Such disruption propagation phenomenon is also referred to as the ripple effect (Ivanov and Dolgui, 2021). Recently, Behbahani et al. (2019) proposed...
the mathematical formulation of several social equity theories and exemplified some of them in a stylized transportation network design model. Our paper builds on the work of Behbahani et al. (2019) regarding the modeling of social equity theories, and put it into the context of: a strategic distribution network design problem; a COVID-19 vaccine deployment problem under uncertainty; and a multi-objective decision-making approach. This is done with the aim to investigate and compare some of the well-known social equity theories for a COVID-19 vaccine distribution network, and provide insights on their impact on the network to design for any vaccination program of a variant of concern. To the best of our knowledge, no research explicitly considers social equity in a vaccine distribution network design model. We endeavor to fill this gap with our study.

1.2. Contributions

All the issues discussed above underline the complexity of the COVID-19 vaccine distribution design problem. In this study, we characterize the COVID-19 vaccine deployment context under demand and supply uncertainties, build a novel social equity-based stochastic distribution network design model, and then solve it using a multi-objective approach in the case of two major provinces in Iran, highly impacted by the COVID-19. Thereby, our paper makes three main contributions:

- To the best of our knowledge, we are the first to introduce the notion of social equity in vaccine distribution network design problems. We consider alternatively: utilitarianism, Rawls’ theory, and Sadr’s theory, where the latter is modeled based on the Gini index. Each social theory is then embedded in a bi-objective model considering social equity and cost-based efficiency objectives.

- We propose a scenario-based modeling approach that builds on: demand-side uncertainty characterized by the daily contamination rate and its space–time propagation by social group; supply-side uncertainty characterized by the stochastic arrival of vaccine doses for the supply period. This gives rise to a two-stage stochastic multi-period vaccine distribution network design model.

- We provide numerical evidence of the benefits of considering social equity in a multi-objective model-based decision-making approach, that is solved by a lexicographic method. Using publicly available COVID-19 data and the case of two major provinces in Iran, we compare the social equity theories and we test their sensitivity to key parameters’ variation. We also derive managerial insights to help decision-makers leverage the right equity approach for any potential pandemic.

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature and positions our study. Section 3 describes the context and proposes a mathematical model for COVID-19 vaccine distribution network deployment. The multi-objective solution approach is presented in Section 4. Section 5 presents the case study, discusses the numerical results and the managerial insights. Last, the conclusion and future research avenues are discussed in Section 6.

2. Literature review

In this section, we specifically review the literature on vaccine distribution networks and highlight the research gaps that emerge. We also review past work on social equity and its consideration in distribution networks. To start with, we notice that most of the body of literature on vaccine distribution networks take the basis on past work related to supply chain network design problems and we refer the reader to detailed reviews of Kilibi et al. (2010) and Govindan et al. (2017). Various modeling facets were reviewed in Melo et al. (2009) and Kilibi et al. (2010), which underline that several static-deterministic versions of the supply chain design models were proposed in the literature. The state of the art in the last decade considered multi-period and scenario-based stochastic versions of the supply chain network design model, as reviewed by Govindan et al. (2017). According to Govindan et al. (2017), only a few design models are multi-objective, but none incorporate social equity. For instance, multi-objective stochastic programming versions for supply chain network design considering risk (Azaron et al., 2008), service level (Ding et al., 2009), and green (Hasani et al., 2021) features could be found in the literature. Finally, recent work in multi-echelon distribution networks could be found in Ben Mohamed et al. (2020) and Janjevic et al. (2021), with a focus on last-mile operations.

2.1. Vaccine distribution networks

Vaccine distribution modeling approaches are separated into strategic models with a network focus and tactical models with an inventory focus. At the strategic level, Lim et al. (2019) redesign a vaccine supply chain network for low- and mid-income countries, focusing on the distribution center location problem, the allocation of vehicles in each flow, and the inventory problem. In fact, their primary purpose is optimizing allocation and location decisions in a vaccine supply chain network, and thus, they focus on the economical aspect of the vaccine distribution networks. Sadjadi et al. (2019) study a vaccine supply chain network under uncertainty and the perishability of vaccines, focusing on planning vaccine supply chain network location and capacity. Thul and Powell (2021) investigate allocation policy for testing kits and vaccines using a reinforcement learning method. They studied their model with two different scenarios, including a resource allocation scenario in all states of the United States and a nursing home scenario under extreme resource shortages in Nevada. Nevertheless, neither Lim et al. (2019) nor Sadjadi et al. (2019) consider social equity in their proposed models.

From reviewing the literature, we observe that studies on vaccine distribution mainly focus on tactical decisions. For example, Chen et al. (2014) investigate polio vaccine distribution in developing countries, including resource constraints and inventory control policies in their model. However, they do not consider the effect of uncertainty on tactical decisions. Yarmand et al. (2014) study the vaccine allocation problem using mathematical modeling, designing a vaccine distribution network using two-stage stochastic programming. Moreover, many researchers believe that low production is a major problem in vaccine distribution, and delays in vaccine delivery exacerbate this problem. In this respect, Dai et al. (2016) study inventory problems in vaccine supply chain networks under uncertainty. The objective functions of their model are the maximization of the total benefit in on-time delivery and late-rebate. They develop a buyback-and-late-rebate contract to balance their objective functions. In another attempt, Lemmens et al. (2016) study inventory problems for the rotavirus vaccine, aiming to decrease inventory by integrating production capacity and production flow, thus reducing the lead time. Similarly, De-Carlovalho et al. (2019) investigate the vaccine inventory problem in a sustainable vaccine supply chain using the input data of a European company to validate their model. Yang et al. (2020) design a vaccine supply chain network for four countries in Africa using a data-driven framework, minimizing the inventory and maintenance costs of vaccine supply chains in countries with low- and middle-income. In light of the COVID-19 outbreak and the production and distribution constraints, prioritizing population segments is essential to vaccine allocation. Therefore, Bertsimas et al. (2020) develop a mathematical model for vaccine allocation using U.S. data during the COVID-19 outbreak. Although they attempt to study a vaccine allocation problem, they do not study the role of social equity in the vaccination program. Focusing on the inventory problem, Sinha et al. (2021) design a vaccine supply chain network to achieve herd immunity against COVID-19 using real-life data from an Indian vaccine producer to validate their model. Moreover, Balci et al. (2022) develop a deterministic single-objective model for equitable in-country COVID-19 vaccine allocation. The objective function of their model minimizes the weighted sum of the deviation from the equitable coverage levels.
et al., 2017).

Among these, Jeremy Bentham (1748–1832), John Stuart Mill (1806–1873), and Henry Sidgwick (1838–1900) are the most well-known theories. Numerous philosophers have highlighted utilitarianism over the years. Among these, Jeremy Bentham (1748–1832), John Stuart Mill (1806–1873), and Henry Sidgwick (1838–1900) are the most well-known contributors to this theory (Askari and Mirakhor, 2020). The theory is based on three principles: (1) human welfare is the basis of equity due to its inherent value; (2) equal respect is a value and should be practiced; and (3) the ethical judgment of an act should merely be based on its consequences, particularly on how to increase welfare (Pereira et al., 2017).

The term “equity” (also called justice or fairness) is defined in this regard is presented in Table 1.

### Table 1

| Authors (year) | Decisions | Objective(s) | Modeling features |
|---------------|-----------|--------------|-------------------|
| Chen et al. (2014) | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ |
| Yarmand et al. (2014) | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ |
| Dai et al. (2016) | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ |
| Lemmens et al. (2016) | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ |
| Sadjadi et al. (2019) | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ |
| Lim et al. (2019) | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ |
| De-Carvalho et al. (2019) | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ |
| Bertimas et al. (2020) | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ |
| Yang et al. (2020) | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ |
| Chen et al. (2020) | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ |
| Enayati and Ozaltun (2020) | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ |
| Rastegar et al. (2021) | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ |
| Sinha et al. (2021) | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ |
| Thul and Powell (2021) | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ |
| Balci et al. (2022) | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ | ✓ ✓ ✓ ✓ |

Current study | ✓ ✓ ✓ ✓ |

However, this work tackles a tactical allocation problem with pre-defined vaccines distribution centers and assumes that demand and supply levels are known. A review of the distribution network literature in this regard is presented in Table 1.

### 2.2. Social equity

In the early 21st century, the concept of social justice emerged as a new and critical principle in transportation planning (Manaugh et al., 2015). The term “equity” (also called justice or fairness) is defined as the distribution of benefits (Litman, 2002). Equity or fairness is vastly influential in decision-making on humanitarian activities (Gutjahr and Fischer, 2018). Generally, justice is a major issue in economies and can dramatically affect decision-making in humanitarian operations (Gutjahr and Fischer, 2018). Research on social problems is highly influenced by political and social considerations (Burns, 2018). Therefore, social equity and equal rights have been introduced in the operational research community as new directions for future studies (Johnson et al., 2018). The purpose of social justice is to ensure that neither a group nor an area is deprived of services and products. Given the limited production of COVID-19 vaccines, applying social equity in the vaccine distribution network can efficiently cut the number of the cases all around the world. Considering social justice is actually an ethical issue, since individuals have equal rights. Furthermore, studies suggest that ignoring an equitable approach can lead to anarchy of those who feel discriminated against (Gutjahr and Fischer, 2018). As previously discussed, since there is no universal measure for justice, the Gini coefficient and Theil index are two main measures used in studies (Mollanejad and Zhang, 2014). On the other hand, different justice theories, such as utilitarianism, intuitionism, egalitarianism, Rawls’ theory of justice, and Sadr’s theory of justice, provide various definitions for social equity (Pereira et al., 2017; Behbahani et al., 2019; Askari and Mirakhor, 2020). In the following, we review some of these well-known theories.

- **Utilitarianism**
  
  Numerous philosophers have highlighted utilitarianism over the years. Among these, Jeremy Bentham (1748–1832), John Stuart Mill (1806–1873), and Henry Sidgwick (1838–1900) are the most well-known contributors to this theory (Askari and Mirakhor, 2020). The theory is based on three principles: (1) human welfare is the basis of equity due to its inherent value; (2) equal respect is a value and should be practiced; and (3) the ethical judgment of an act should merely be based on its consequences, particularly on how to increase welfare (Pereira et al., 2017).

- **Rawls’ theory of justice**
  
  Rawls’ theory of justice is one of the main postulates in the social justice context. According to Rawls’ philosophy, social equity is a framework focused on the plight of society’s poorest (Askari and Mirakhor, 2020). In general, this theory is derived from egalitarianism (France-Mensah et al., 2019), which concerns allocating more distributable benefits to the poorest members of a society (Liu et al., 2019). Therefore, Rawls’ theory respects human dignity and seeks to lessen the gap between the poorest class of society and the other social classes. This theory of justice generally supports the gradual improvement of distributable benefits for the deprived classes of society (France-Mensah et al., 2019; Behbahani et al., 2019).

- **Sadr’s theory of justice**
  
  As Sadr’s theory postulates, social equity is the fair distribution of benefits among social groups (Reda, 2014). Accordingly, an equitable system allows people from all walks of life to gain wealth (Fahlevi, 2019). Sadr’s theory of justice also holds that the economic climate should be such that poverty and deprivation are eradicated alongside the maximization of total benefits. Sadr believed that achieving these two goals would create social balance among different classes in society, subsequently decreasing class differences (Behbahani et al., 2019). Generally, Sadr’s theory is based on two principles: social balance and mutual responsibility for support (Askari and Mirakhor, 2020). In this regard, Behbahani et al. (2019) develop a sub-model of Sadr’s theory. To achieve social balance, they set an upper bound for the Gini index, which is a criterion for inequity. Thus, inequity cannot exceed a predetermined upper bound using their proposed model, leading to social balance.

Although social equity is a new direction in operation research, several recent studies point out the effects of social justice on distribution networks. For instance, Gutjahr and Fischer (2018) study the impact of fairness in humanitarian logistics employing the Gini index. They consider budget constraints to approach real-world conditions, finding that minimizing the deprivation cost can bring about an unfair solution. Anaya-Arenas et al. (2018) design a humanitarian relief distribution network using mathematical modeling, considering fair distribution by minimizing the difference between unsatisfied demand for all demand zones. Enayati and Ozaltun (2020) attempt to optimally distribute the influenza vaccine among a heterogeneous population using the Gini coefficient to indicate inequity among subgroups of the population. The results indicate that the dynamic nature of virus transmission in each subgroup is more critical than the mere optimization of vaccine allocation. As the limited supply of COVID-19 vaccines is a major problem in curtailing the pandemic, Chen et al.
(2020) investigate how to fairly prioritize age groups for vaccination using the Gini index to measure fairness in their model. They find that if older people get vaccinated before younger groups, the spread of the pandemic can be better controlled in the first days of vaccination. As vaccine inventory problems can be very challenging, Rastegar et al. (2021) study the influenza vaccine inventory problem during the COVID-19 outbreak. Their aim is to equitably allocate influenza vaccines to healthcare providers, older people, and pregnant women in acute health conditions, maximizing the minimum delivery-to-demand ratio to provide justice. Table 2 presents the studies that apply equity to distribution networks. To sum up, no published research on social equity has considered the effect of uncertainty on decisions. Furthermore, most studies on social equity do not investigate the impact of social equity on strategic decisions.

3. Problem description and formulation

In this section, we first provide a description of the problem and present a mathematical model for a COVID-19 vaccine distribution network considering social equity. Finally, given the inherent uncertainty, we propose a scenario-based stochastic programming approach.

3.1. Problem definition

The scope of the decision to make: Following the production of the vaccine doses, national governments, with the support of local organizations and the collaboration of vaccine producers, have engaged in the deployment and sustainment phases of the supply chain to ensure the distribution of COVID-19 vaccines at the right place and at the right time. The current work focuses on the strategic phase of the network design and deployment that concerns the critical first weeks of the distribution process. The accessibility to vaccines depends on the network capabilities and the urgency of delivering the COVID-19 vaccine among social groups calls for an efficient distribution system. The outcome of the first vaccination phase is highly critical because it influences the subsequent phases (Yarmand et al., 2014). With this in mind, we cover in this study the strategic network design phase covering a horizon plan of several weeks (e.g., x months, a season, ...), decomposed into daily periods $t$ in $T$.

Network structure and resources considered: In this context, a two-echelon distribution network must be designed. Fig. 2 depicts a generic schema of such a two-echelon network for COVID-19 vaccine distribution. It considers an implicit set of domestic and international sources, a set $I$ of potential DCs from which a choice must be made, and a predetermined set of ship-to points corresponding to social groups’ locations (e.g. vaccination centers, hospitals, ...). We notice that the set of potential DCs is predefined by public health authorities at some locations on the territory. At the first echelon, we assume that opened DCs receive batches of vaccine pallets from various sources, based on their requirements. As soon as the DCs receive supplies, either they unload pallets into cooling boxes or cross-dock the pallets/boxes for forwarding. At the second echelon, DCs plan their deliveries to social groups’ locations for the next day with respect to the transportation capacity. It is assumed that each DC can offer a distribution capacity from a set of alternative capacity levels $c$. Each level expresses a given space and equipment to acquire in order to ensure the desired DC throughput capability. We assume that an opening cost is associated with each potential DC and capacity level.

The specification and constraints of the vaccine: The distribution of vaccines follows a well-defined temperature-sensitive distribution system. Pfizer suggests a flexible and just-in-time vaccine supply chain (Pfizer, 2020). From the sourcing side, air cargo plays a key role to ensure international transportation that complies with international regulatory and manufacturers’ requirements, at controlled temperatures. Once vaccine batches enter the in-country distribution network, their handling and transportation must respect controlled temperatures to guarantee the quality of the product. As depicted in the right picture of Fig. 3, boxes in pallets arrive at the DC for unloading or cross-docking. In these DCs, advanced freezing and handling technology is needed, as illustrated in the left picture of Fig. 3. Such facilities are difficult to equip with cooling technology in a short delay, especially in low-income countries. For this reason, the number of potential DCs to consider could be to some extent limited in such a context. This issue should be compensated for by an efficient transport of COVID-19 vaccines based on the availability of a transportation fleet subject to temperature-sensitive conformity. A set of transportation vehicles $M$ is considered, with associated capacity and usage costs (fixed and variable).

The characterization of the social groups: The primary motive of our study is to investigate the role of social equity theories on the COVID-19 vaccine distribution network in countries with low- and middle-income. WHO (2021b) enjoins that all individuals worldwide should have access to safe and efficient vaccines as fast as possible. However, such distribution objective faces several challenges: (i) increased populations and crowded housing in large cities, (ii) dispersed populations and lack of health access in smaller cities, and (iii) poor logistics and health services and infrastructure in rural communities. For all these reasons, the characterization of social vulnerabilities is key to ensuring that COVID-19 vaccines are to be equitably allocated from DCs to the location of in-need social groups. The European Centre for Disease Prevention and Control (ECDC) links the risk groups to socially vulnerable populations due to their living conditions. For instance, the U.S. COVID-19 vaccination program considers a social vulnerability index (SVI) (Hughes et al., 2021), based on 14 social factors such as unemployment, lack of vehicle access, and crowded housing. The SVI provides a ranking of low, moderate, and high social vulnerability counties as depicted in the bottom level of Fig. 2 (see also Hughes et al., 2021). However, such a detailed assessment of social vulnerability is missing in most the low- and middle-income countries. Accordingly, this study takes three social groups into account, to adequately identify social vulnerabilities and ensure a fair distribution. The three categories are aligned to urbanization categories as shown in Table 3. According
to the latest urbanization categories and based on population density, human settlements are classified as a. cities, b. towns and suburbs, and c. rural areas (Dijkstra and Poelmann, 2014; Dijkstra et al., 2020). Table 3 provides details of the social groups and the way they are classified based on the degree of urbanization.

Table 3 provides details of the social groups and the way they are classified based on the degree of urbanization.

| Place of residence | Level of density       | Population          |
|--------------------|------------------------|---------------------|
| Cities             | High-density areas     | Population ≥ 50000  |
| Towns and suburbs  | Mid-density areas      | 5000 >Population ≥ 5000 |
| Rural areas        | Low-density areas      | 5000 >Population    |

The awareness of the conditions in each area is a prerequisite for decisions on equitable vaccine distribution when social theories are applied. Accordingly, each social group \( g \in \mathcal{G} \) is assumed to be of a specific type denoted \( o(g) \) where \( o = L, M, H \) such as \( G_o \subset \mathcal{G} \). In this case, we assume that the social groups are assessed with low vulnerability (L), moderate vulnerability (M), and high vulnerability (H), which defines their type \( o \) in a ranked way. For instance, the groups with high vulnerability (H) would correspond to the poorest social groups in several social equity theories. For a given social group \( g \), in addition to its type \( o(g) \), we identify the lower vulnerability level type by \( o^-(g) \) such as \( G_{o^-(g)} \) be the subset of groups with a type at a lower level than \( o(g) \) (i.e. for instance types L and M are lower than type H, or type M is lower than type H).
The uncertainty of the operating environment: Clearly, the network design problem considered in this work is a decision-making problem under uncertainty. One must consider demand-side uncertainty with the COVID-19 daily contamination rate and its space–time propagation from one social group to another. In addition, the unavailability of supplies is a major source of uncertainty, especially in low- and middle-income countries. Hence, one must characterize supply-side uncertainty in vaccine distribution networks with a compound Poisson process with income countries. Hence, one must characterize supply-side uncertainty from one social group to another. In addition, the unavailability of the COVID-19 daily contamination rate and its space–time propagation under uncertainty. One must consider demand-side uncertainty with the COVID-19 vaccine distribution network. The results can help decision-makers and practitioners select the best social equity theories. Table 4 summarizes the sets, parameters, and variables of our models.

3.2. Modeling approach

In this section, a mathematical model is presented for a COVID-19 vaccine distribution network considering alternatively three social equity theories. Table 4 summarizes the sets, parameters, and variables of our models.

3.2.1. Modeling social equity theories

In this study, we apply three well-known social equity theories to the COVID-19 vaccine distribution network. The results can help decision-makers and practitioners select the best social equity theory considering various factors, such as degree of urbanization and population share.

• Utilitarianism

As shown in Eq. (1), the objective function of the theory aims to maximize the allocation of COVID-19 vaccines to every social group.

\[
\max \sum_{s \in S} \sum_{t \in T} \sum_{g \in G} \sum_{o \in O} \rho_s Q_{igmst} \tag{1}
\]

• Rawls’ theory of justice

Eq. (2) shows that the number of vaccines allocated to the poorest social group (i.e. \( G_{LM} \) can be maximized according to Rawls’ theory of justice).

\[
\max \lambda_1 \sum_{s \in S} \sum_{t \in T} \sum_{g \in G} \sum_{o \in O} \rho_s Q_{igmst} + \lambda_2 \sum_{s \in S} \sum_{t \in T} \sum_{g \in G} \sum_{o \in O} \sum_{m \in M} \rho_s Q_{igmst} \tag{2}
\]

In Eq. (2), \( \lambda_1 \) and \( \lambda_2 \) are the weight coefficients. Since Rawls’ theory gives priority to the poorest social group, \( \lambda_1 > \lambda_2 \).

• Sadr’s theory of justice

Social justice comprises two primary principles, reciprocal responsibility and social balance among various classes of society. Sadr deemed that these principles result in social harmony and unity (Reda, 2014). Generally, Sadr’s theory of justice is based on social balance and the maximization of distributable benefits. Behbahani et al. (2019) propose a modeling approach to apply the tenets of Sadr’s theory of justice and illustrate it on a stylized static single-objective model. Eqs. (3)–(5) indicate vaccine allocation based on Sadr’s theory of justice where the objective function attempts to maximize the distributable benefits (i.e., vaccine doses) for every social group. In addition, \( \gamma_1 \) and \( \gamma_2 \) are the gap parameters, and \( \bar{Q} \) indicates the average of \( Q_{igmst} \). The calculation of \( \bar{Q} \) and values of \( \gamma_1 \) and \( \gamma_2 \) are detailed in Appendix A. These equations serve to decrease class differences and maximize total benefits. Specifically, the left-hand side of Eq. (5) indicates the Gini index, a measure to compute inequity. As explained in Section 2.2, Sadr’s theory posits that balance should be created among social groups. Thus, the left-hand side of Eq. (5) must be less than or equal to the gap parameter (\( \gamma_2 \)), which determines an upper bound for inequity among social groups. Eq. (5) shows that inequity must not exceed a pre-specified value.

\[
\max \sum_{s \in S} \sum_{t \in T} \sum_{g \in G} \sum_{o \in O} \rho_s Q_{igmst} \tag{3}
\]

\[\text{s.t.} \]

\[
Q_{igmst} \geq \gamma_1 Q_{igmst}, \quad \forall i \in I, g \in G, g' \in G_{o \neq o'}, m \in M, t \in T, s \in S. \tag{4}
\]
\[
\sum_{i \in T} \sum_{g \in G_{i}} (Q_{igm} - Q_{igm}) \leq 2 \gamma Z,
\quad \forall i \in T, s \in S,
\]

(5)

To linearize the absolute value in Eq. (5), we have to add an auxiliary positive variable \( \beta \). Then, we insert the term \((Q_{igm} - Q_{igm}) + 2\beta\) in Eq. (5) instead of using \((Q_{igm} - Q_{igm})\). Thereafter, we add Eqs. (6) and (7) to complete the linearization procedure.

\[
\sum_{i \in T} \sum_{g \in G_{i}} \sum_{m \in M_{i}} (Q_{igm} - Q_{igm} + \beta) \geq 0,
\quad \forall i \in T, s \in S,
\]

(6)

\[
\beta \geq 0,
\]

(7)

3.2.2. Distribution network design model

Eqs. (8)–(21) indicate the bi-objective mathematical model for the COVID-19 vaccine distribution network using Sadr's social equity theory. The two alternative social equity-based models are described at the end of the subsection. Eq. (8) maximizes the allocated vaccine doses to every social group. As mentioned above, even if it is not a priority, designing an efficient distribution network is crucial (Zhu and Ursavas, 2018). Especially in low- and middle-income countries, resources are limited during pandemics and their efficient use is desired. Therefore, to express the efficiency of the designed distribution network, we use the cost objective function. The network costs are minimized using Eq. (9), including the DCs opening costs, the costs of allocating transportation resources, and the vaccines delivery costs from DCs to the social groups.

\[
\text{max} \quad Z_1 = \sum_{i \in T} \sum_{g \in G_{i}} \sum_{m \in M_{i}} \sum_{s \in S} \rho_{i} Q_{igm}
\]

\[
\min \quad Z_2 = \sum_{i \in T} \sum_{g \in G_{i}} \sum_{m \in M_{i}} \sum_{s \in S} \rho_{i} K_{igm} Q_{igm} + \sum_{i \in T} \sum_{g \in G_{i}} \sum_{m \in M_{i}} f_{iigm} V_{mt}
\]

\[
\text{s.t.} \quad Q_{igm} \geq \gamma z_{igm}, \quad \forall i \in I, g \in G_{i}, m \in M_{i}, t \in T, s \in S.
\]

(8)

\[
\sum_{i \in T} \sum_{g \in G_{i}} \sum_{m \in M_{i}} (Q_{igm} - Q_{igm} + 2\beta) \leq 2 \gamma Z,
\quad \forall i \in T, s \in S.
\]

(10)

\[
\sum_{i \in T} \sum_{g \in G_{i}} \sum_{m \in M_{i}} (Q_{igm} - Q_{igm} + \beta) \geq 0,
\quad \forall i \in T, s \in S.
\]

(11)

Eqs. (10) and (11) attempt to reduce social class differences as defined above in Sadr's theory of justice. Eq. (10) shows that the difference between allocated vaccine doses to \( g^{th} \) social group and \( g^{th} \) social group does not exceed the predetermined gap parameter. In addition, the left-side of Eq. (11) shows the Gini index, which reflects inequity. Hence, Eq. (11) imposes that inequity must not exceed the predetermined gap parameter of \( \gamma \). As explained above, Eq. (11) is coupled with Eqs. (12) and (22) to replace the original Eq. (5) and keep the mathematical model linear.

\[
\sum_{g \in G_{i}} \sum_{m \in M_{i}} Q_{igm} \leq \sum_{i \in T} c_{i} Y_{it}, \quad \forall i \in I, t \in T, s \in S.
\]

(13)

Eq. (13) concerns the capacity of DCs. Based on this equation, the additive capacity of each DC upper bounds all the COVID-19 vaccines transported from that center to social groups.

\[
X_{its} \geq \sum_{g \in G_{i}} \sum_{m \in M_{i}} Q_{igm}, \quad \forall i \in I, t \in T, s \in S,
\]

(14)

\[
\sum_{i \in T} X_{its} \leq X_{ist}, \quad \forall t \in T, s \in S.
\]

(15)

Eqs. (14) and (15) are the sourcing constraints. According to these constraints, vaccine doses are first transferred to the DCs and can then be allocated to social groups. We recall that (15) considers three incremental levels of capacity for each DC.

\[
\sum_{i \in T} \sum_{g \in G_{i}} \sum_{m \in M_{i}} Q_{igm} \leq \Phi_{igt}, \quad \forall g \in G_{i}, t \in T, s \in S,
\]

(16)

Eqs. (16) imposes that the vaccines allocated to each social group cannot exceed the population exposed to COVID-19 in that social group. Capacity of each transportation vehicle must be greater or equal to the transported COVID-19 vaccine doses from DCs to social groups, as shown in Eq. (17).

\[
\sum_{i \in T} \sum_{g \in G_{i}} \sum_{m \in M_{i}} Q_{igm} \geq \chi_{igm}, \quad \forall k \in K, m \in M_{t}, t \in T, s \in S.
\]

(17)

Eq. (18) indicates that \( v \) works as a lower bound for \( Q_{igm} \). This constraint ensures that at least the most vulnerable in the population are vaccinated in the first vaccination phase.

\[
\sum_{i \in T} \sum_{g \in G_{i}} \sum_{m \in M_{i}} \sum_{s \in S} Q_{igm} \geq v, \quad \forall s \in S,
\]

(18)

Eq. (19) denotes that variables \( Y_{it} \) are binary. Variables \( V_{mt} \) are positive integers, as imposed by Eq. (20). Eq. (21) and (22) are defined for continuous variables \( X_{its}, Q_{igm}, \) and \( \beta \), respectively.

\[
Y_{it} \in [0, 1], \quad \forall i \in I, t \in T.
\]

(19)

\[
V_{mt} \geq 0 \text{ and integer.} \quad \forall m \in M_{t}, t \in T.
\]

(20)

\[
Q_{igm}, X_{its} \geq 0, \quad \forall i \in I, g \in G_{i}, m \in M_{t}, t \in T, s \in S.
\]

(21)

\[
\beta \geq 0.
\]

(22)

To note is that to implement utilitarianism, Eqs. (8) is substituted with Eq. (1), and Eqs. (10)–(12) and (22) are eliminated. Also, to implement Rawls' social equity theory, Eqs. (8) is substituted with Eq. (2), and Eqs. (10)–(12) and (22) are omitted.

4. Solution approach

Clearly, solving the proposed multi-objective scenario-based stochastic programming model is challenging. To tackle such challenge, the sample average approximation (SAA) method and a multi-objective algorithm are developed. Hence, we first generate the adequate sample of scenarios using the Monte Carlo approach and the SAA method. Then, we employ a lexicographic goal programming approach to handle the multiple objectives considered in the scenario-based network design model.

4.1. Scenario building and generation

A well-known approach to manage the generation of plausible scenarios under known probability distribution is the Monte Carlo method (Shapiro, 2003). The Monte Carlo approach generates pseudorandom numbers \( \omega \), where \( \omega \) lies between 0 and 1. This approach utilizes the inverse cumulative distribution function of the random variables. In our case, a Monte Carlo procedure is developed through two main steps to produce plausible scenarios with demand and supply uncertainty.

In the first step, scenario building is implemented where we mimic daily COVID-19 contaminations based on the time–space dimensions. The approach is inspired from the concept of propagation or ripple effect studied in the literature (Klibi and Martel, 2012a; Gholami-Zanjani et al., 2021; Dolgui and Ivanov, 2021). The main idea is that a ripple effect arises when a disruption/disaster propagates across nodes or zones, instead of remaining isolated. To this end, a chronological list
\( \Psi \) is created. Hence, the attenuation probability of social group \( g \) being hit by COVID-19 (\( \varphi_{t,s} \)) is computed using the conditional probability in Eq. (23). In this equation, \( r \) denotes each region \( r \in R \), \( c_{t|g} \) is the conditional propagation probability in social group \( g \) when social group \( g' \) is hit by COVID-19, \( \sigma_{t|g} \) is the attenuation probability of region \( r \) to be hit by COVID-19, and \( \sigma_{t|g} \) is the attenuation probability in social group \( g \) if its containing region is hit by COVID-19. Thus, a social group \( g \) can be hit by COVID-19 directly when it is the centroid (i.e. main cluster) of the COVID-19 infection, or a social group \( g' \) could be hit due to COVID-19 propagating from another social group \( g'' \), which depends on the distance between social group \( g \) and \( g'' \). Thereafter, running the infection propagation test, pseudorandom numbers \( u'_i \) are generated uniformly on the interval \([0, 1]\), and the potential population exposed to COVID-19 infection \( \varphi_{t,s} \) is generated using a Poisson distribution. The number of daily contamination \( \varphi_{t,s} \) provides the expected needs for vaccination to be covered through the design model. On the supply side, we construct a chronological list of vaccine dose arrivals. Therefore, the arrivals are generated in the interval \([0, T]\) and mapped onto the corresponding time periods \( t \in T \). Finally, for every arrival in the chronological list \( T \), the quantity of received vaccine doses is generated. The Monte Carlo scenario building and generation procedure described above is detailed in Algorithm 1.

An illustrative example of the output of the procedure is provided in Fig. 4 showing the epidemic curves of COVID-19 for a given social group.

\[ x_{t} = \sum_{g' \in G} c_{t|g'} \times x_{t-1} + a_{g'|g} \times x_{t} \]  
\[ \text{(23)} \]

Algorithm 1 indicates the detailed sampling procedure for scenario generation and the outputs are the model parameter values, \( \varphi_{t,s} \), and \( x_{t} \), for each period under a given scenario. Running the Monte Carlo procedure \( N \) times creates a sample of equiprobable and independent scenarios \( S_N = \{ s_1, s_2, s_3, \ldots, s_N \} \), with \( p_s = \frac{1}{N} \). With a sample of \( N \) scenarios, the proposed two-stage scenario-based stochastic programing model is rewritten with the expected values Eqs. (24) and (25) as follows:

\[ \max \mathcal{c}_1 = \frac{1}{N} \left( \sum_{s \in S_N} \sum_{t \in T} \sum_{g' \in G} \sum_{m \in M} \sum_{q \in \Omega} O_{gpmst} \right) \]  
\[ \text{(24)} \]

\[ \min \mathcal{c}_2 = \frac{1}{N} \left( \sum_{s \in S_N} \sum_{t \in T} \sum_{g' \in G} \sum_{m \in M} \sum_{q \in \Omega} c_{t|g|m} O_{gpmst} \right) + \sum_{t \in T} \sum_{g \in G} a_{t|g} Y_{t|g} \]  
\[ + \sum_{t \in T} \sum_{g \in G} f_{t|g} V_{mt} \]  
\[ \text{s.t. (10)–(22) for all } s \in S_N. \]  
\[ \text{(25)} \]

As in many stochastic models the quality of the solution is highly impacted by the number and quality of the scenarios generated. A combination of the Monte Carlo sampling method (Shapiro, 2003) and the sample average approximation technique (SAA) (Shapiro et al., 2009) helps in finding a good trade-off in terms of the sufficient number of scenarios to consider in the model and the quality of the solution produced by the model. Thus an important parameter to calibrate for our stochastic model is the number \( N \) of scenarios to include in the optimization phase. First, the SAA consists in solving the problem with \( M \) independent samples of \( N \) scenarios generated with Algorithm 1. The average value among the \( M \) expected values, obtained from each model resolution with \( N \) alternative scenarios, gives a statistical lower bound. Second, we evaluate the obtained solutions based on the second-stage model (by fixing the first-stage decisions) by solving the resulting problem for \( N' = |S_{N'}| \gg N \) independent scenarios to get an upper bound on the optimal solution of the problem. Finally, a statistical

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Fig. 4. Epidemic curves of COVID-19 for a given social group using the Monte Carlo procedure.
optimality gap is computed for each obtained solution based on the difference between the lower and upper bounds.

4.2. Lexicographic goal programming

As discussed above, the vaccines distribution design problem is formulated as a bi-objective model. In this context, we attempt to attach priorities to the two objectives considered and to define a goal for each one of them. To this end, our solution approach builds on the lexicographic goal programming (LGP) method as it is suitable for multi-objective problems with established priorities. This method was first introduced by Ignizio (1983), who proposed the notion of prioritizing one goal over other goals for goal programming. The LGP method attempts to give priorities to various goals to diminish the undesirable deviation variables in a lexicographic order (Romero, 2001). For our bi-objective distribution network design model the LGP method can be conceptually formulated as follows:

\[
\text{Lex min } (d^1, d^2) \tag{26}
\]

s.t.

\[
\zeta_1 + d^1 - d^1_t = Z^*_1, \tag{26a}
\]

\[
\zeta_2 + d^2 - d^2_t = Z^*_2. \tag{26b}
\]

In Eq. (26), \(Z^*_1\) and \(Z^*_2\) refer to the goal obtained using a mono-objective optimization. In addition, \(d^1\) and \(d^2\) are respectively positive and negative deviations of each goal. Furthermore, \(\zeta_1\) and \(\zeta_2\) show the objective functions of the models that are prioritized in the LGP method. The LGP method is a two-phase problem-solving technique. First, the LGP method solves the first priority objective function; second, it fixes the solution of the first priority objective function, third, it solves the second priority objective function considering the fixed solution of the first priority objective function. Accordingly, we develop an optimization problem to calculate \(Z^*_1\). Therefore, a single-objective model is built wherein Eq. (24) is the only objective function subject to constraints (8)–(22). Then, \(Z^*_2\) is computed by constructing a single-objective model in which Eq. (25) attempts to minimize the distribution network costs considering Eqs. (8)–(22).

5. Numerical experiments

In this section, we present in Section 5.1, the case study built realistically to validate the modeling and solution approaches. The baseline and its extensions provide the set of instances that are used to present and discuss the findings from the numerical results in Section 5.2. Managerial insights are shared in Section 5.3 to provide decision-makers on how to select the right social equity theory considering costs, and discuss the findings from the numerical results in Section 5.2.

5.1. Case study

In this section, we present the case study built for the vaccine distribution network in two provinces in the west of Iran, namely Kermanshah and Kurdistan, and using real historical data from the COVID-19. Fig. 5 shows the locations of potential DCs in the case network. As can be seen, we consider 14 potential locations for DCs (\(|T| = 14\)), and three capacity levels for each DC (\(|C| = 3\)). The opening cost of each capacity level at DC ranges from 30 million Iranian rial (IRR) to 40 million IRR. As shown in Table 3, we classified the whole population into three social groups in these two provinces such that \(|Q| = 34\), obtaining eight communities in rural districts (\(|G_{Ruralareas}| = 8\), sixteen in towns and suburbs (\(|G_{Townsandsuburbs}| = 16\), and ten in cities (\(|G_{Cities}| = 10\)).

As discussed earlier, the planning horizon of this study covers the first vaccination phase consisting of 60 days (\(|T| = 60\)). Hence, in 60 vaccination days, three types of transportation vehicles (\(|M| = 3\)), namely lorries, trucks, and vans, are used to transport COVID-19 vaccine doses from DCs to social groups. Each of these vehicles has a specific capacity whereby the lorries, trucks, and vans can carry 8000, 6900, and 6500 vaccine doses on each trip, respectively. Also, we use a distance matrix containing the real distances between DCs and the location of social groups. Appendix B provides further details on the case study and the values of the parameters used in the various design models. Moreover, as explained, the plausible scenarios were generated using Monte Carlo scenario generation. When Algorithm 1 is applied, the generated level of the population infected with COVID-19 by period and social group (\(s_{gi}\)) is in the interval of 5 to 400 people. Thus, given the entire population of regions in the Kermanshah and Kurdistan provinces, the epidemic curves of COVID-19 are achieved, as shown in Fig. 4. In the same way, by applying Algorithm 1, the level of received doses (\(x_{gi}\)) is in the interval of 10,000 to 20,000 vaccine doses. The SAA procedure shows that the statistical gap of 0.48% can be obtained using a sample of 100 scenarios (\(|S| = 100\)), leading to a well-calibrated SAA model. Appendix A details the calibration of the number of scenario with the SAA method and also provides the value of the stochastic solution (VSS) for a typical instance of the model, that is quite high.

5.2. Numerical results

We start with a focus on the difference between design solutions produced by single and bi-objective models in Section 5.2.1. Next, we consider the comparison between design solutions produced by the three alternative bi-objective models in Section 5.2.2. Finally, we perform a sensitivity analysis on the arrival of sourcing in Section 5.2.3.

5.2.1. The impact of introducing a social equity objective

With three social equity theories applied to a bi-objective model and two single objective models, nine alternative design solutions are produced and compared hereafter. Table 5 provides the abbreviation associated to each design solution based on the social theory considered.
and the model solved. Table A.1 details the objective(s) and set of constraints associated to each of the nine mathematical models.

For each of the nine solutions, a detailed representation of the opening decisions and the capacity level in each opened DC is provided in Table C.1. Regarding utilitarianism, more DCs are opened in the USA solution compared to USC and UB. Specifically, all 14 DCs are opened by the USA solution. This result might stem from the USA solution model where neither cost nor distance affect the solution. Hence, the USA solution opens facilities as much as possible in order to increase the allocated vaccine doses. Fig. D.1 indicates the contrast between design solutions produced when the conflicting objectives are separately applied in a single-objective model. In addition, all three capacity levels become active in the USA solution. On the other hand, only one DC is opened by the USC solution. What stands out in Fig. D.1 is that only the DC in Sanandaj is opened, which is in the center of the distribution zone. Therefore, Sanandaj city is selected as the most cost-effective location for the DC in our case study. To note is that the USC solution opens only one capacity level of the Sanandaj DC. Furthermore, as shown, 12 DCs are opened by the UB solution, namely Kermanshah, Gilan Gharb, Sanandaj, Saghez, Kangavar, Javanrud, Marivan, Baneh, Dalahu, Bijar, Eslamabad-e Gharb, and Harsin.

Similar results emerged regarding Rawls’ theory. All the DCs are used to distribute vaccine doses via RSA. Since Sanandaj is in the center of the distribution zone, the RSC solution only opens the DC in this city. Lastly, DCs in 12 different locations, namely Kermanshah, Gilan Gharb, Sanandaj, Saghez, Kangavar, Javanrud, Marivan, Baneh, Paveh, Dalahu, Bijar, and Harsin, are opened by the RB solution. As to Sadr’s theory, we observe that the SSA and SB solutions open all 14 DCs. However, all three DC capacity levels are activated by the SSA solution, whereas only the first capacity level is activated by the SB solution. Similarly to USC and RSC, only the Sanandaj DC is opened by the SSC solution, implying that Sanandaj is the most cost-effective center. These results underline the tradeoffs actionable at the design level by the bi-objective models compared to cost-oriented and coverage-oriented single objective models.

The next part of the results concern the deployment of the vaccines distribution. Table 6 provides, for each design solution, the average number of transportation vehicles employed along the planning horizon. An interesting point is that when the decisions are made by simultaneously considering both cost-efficiency and the allocation maximization objectives, the decisions present a good compromise such as an efficient number of vehicles is used to reach the maximum allocation. As to utilitarianism, the highest average number of vehicles pertains to the USA solution. As detailed in Table 6, averagely, a hundred of each of the transportation vehicles are used daily during the planning horizon in the USA solution. In comparison, only one transportation vehicle is used averagely in the USC solution. In addition, the UB solution uses a total of 97 transportation vehicles. Specifically, 51 lorries, 29 vans, and 17 trucks are employed to deliver the vaccine doses in the UB solution. We find that the RSA solution, related to Rawls’ social theory, utilizes the highest number of vehicles during the planning horizon. Conversely, the RSC solution uses the lowest number of vehicles on average. Moreover, a total of 97 vehicles are used on average in the RB solution. Regarding Sadr’s theory, we obtained similar findings. First, the SSA solution uses the highest average number of vehicles. Second, the lowest average number of vehicles is produced in the SSC solution. Finally, the SB solution uses 49 lorries, 19 vans, and 18 trucks for a total of 86 vehicles. These results underline the impact of employing an LGP approach with prioritized objectives: maximum allocation first, then cost efficiency.

5.2.2. Comparative analysis of social equity-based solutions

Regarding the bi-objective solutions, i.e., UB, RB, and SB, various findings emerge. Fig. 6 illustrates the opening design solutions of UB, RB and SB. As can be seen, in the UB solution, twelve DCs are opened. Thanks to the proximity between Sahneh and Kangavar (20 miles), and Javanrud and Paveh (28 miles), the UB solution keeps the allocation at the highest level with an efficient vaccine distribution network of twelve DCs. Regarding the SB solution, Fig. 6 indicates that all DCs are opened. A reasonable explanation for this finding is that Sadr’s theory attempts to maximize distributable benefits and create balance among the various groups of the community. To establish a balance among classes in society, SB tends to cover more regions. Hence, in the SB solution, all fourteen DCs are opened so that a plausible percentage of social groups can be covered. Last, the RB solution opens twelve DCs to cover the whole region. In fact, in RB, only Sahneh and Eslamabad-e

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Table 5

| Number | Abbreviation | Single Objective | Bi-objective | Utilitarianism | Sadr’s theory | Rawls’ theory |
|--------|--------------|-----------------|-------------|---------------|--------------|---------------|
| 1      | USA          | –               | ✓           | ✓             | –            | –             |
| 2      | USC          | ✓               | –           | –             | ✓            | –             |
| 3      | UB           | –               | ✓           | –             | –            | ✓             |
| 4      | RSA          | –               | –           | ✓             | ✓            | –             |
| 5      | RSC          | ✓               | –           | –             | –            | ✓             |
| 6      | RB           | –               | –           | ✓             | –            | –             |
| 7      | SSA          | –               | ✓           | –             | –            | ✓             |
| 8      | SSC          | ✓               | –           | –             | ✓            | –             |
| 9      | SB           | –               | –           | ✓             | –            | ✓             |

Table 6

| Solutions          | Vehicles | Average number of vehicles |
|--------------------|----------|----------------------------|
| USA/SSA/RSA        | Lorry    | 100                        |
|                    | Van      | 100                        |
|                    | Truck    | 100                        |
|                    | Total    | 300                        |
| USC/SSC/RSC        | Lorry    | 0                          |
|                    | Van      | 0                          |
|                    | Truck    | 1                          |
|                    | Total    | 1                          |
| UB                 | Lorry    | 51                         |
|                    | Van      | 29                         |
|                    | Truck    | 17                         |
|                    | Total    | 97                         |
| SB                 | Lorry    | 49                         |
|                    | Van      | 19                         |
|                    | Truck    | 18                         |
|                    | Total    | 86                         |
| RB                 | Lorry    | 50                         |
|                    | Van      | 32                         |
|                    | Truck    | 15                         |
|                    | Total    | 97                         |
Gharb are not selected as locations for DCs. This result derives from the fact that Eslamabad-e Gharb city is on the fringes of the map, whereas two of its nearest cities, Kermanshah and Dalahu, have easier access to other regions, specifically rural districts.

Moreover, Fig. 7 shows closely the coverage percentage in (a) cities, (b) suburbs and towns, and (c) rural areas. To obtain the coverage percentage, we calculated the average $\frac{Q_{\text{immunized}}}{\text{population}}$ for each region in every model. As shown, regarding cities, the SB solution results in higher coverage. Furthermore, the RB and UB solutions bring about 94% and 93% coverage, respectively. As to suburbs and towns, the UB solution outperforms the SB and RB solutions. Indeed, UB, SB, and RB lead to 92%, 89%, and 90% coverage in suburbs and towns, respectively. Last, we observe that RB brings about 100% coverage in rural areas, while SB leads to 98% coverage. In addition, UB covers a lower percentage of infected people in rural areas compared to the SB and RB solutions. This result indicates that Rawls’ theory can be more beneficial for people in rural areas.

Furthermore, Fig. 8 shows the trade-off between the first objective function ($E[Q]$) and the second objective function ($E[\text{Cost}]$) produced by the UB, SB, and RB solutions. Throughout the rest of the paper, the values of the second objective function ($E[\text{Cost}]$) will be expressed in million IRR. As can be observed, the SB solution is the most cost-efficient. The reason that the SB solution has a lower cost compared to RB and UB may be due to the number of vehicles, as shown in Table 6. In other words, both the RB and UB solutions averagely utilize thirteen more vehicles daily compared to the SB solution. In addition, the findings reveal that each solution is congruent with the essence of its corresponding social equity theory. For instance, the UB solution has a higher $E[Q]$ than RB and SB. This result matches the utilitarianism attitude that attempts to maximize distributable benefits. Another insight that emerges from the analysis is that the SB solution leads to higher allocated vaccine doses than the RB solution. Moreover, as previously discussed, SB is a more cost-efficient solution than RB. Hence, we conclude that Sadr’s theory outperforms Rawls’ theory in terms of both cost-efficiency and allocation.

Regarding the deployment of the vaccines, interesting results are found in Table 6 based on the average number of vehicles. The most striking observation is that the SB solution averagely uses 86 vehicles, whereas the UB and RB solutions use on average 97 vehicles per day to deliver vaccine doses to social groups. This finding can be justified with the previously discussed results related to design decisions. As explained, all 14 DCs are opened by the SB solution, resulting in higher accessibility to social groups in terms of distance. Therefore, given the better access to social groups, fewer vehicles are utilized in SB than in UB and RB.

5.2.3. Sensitivity analysis on the arrival process of vaccines doses

In this section, we analyze the uncertainty of vaccines sourcing ($\chi_{t,s}$) in the designed COVID-19 vaccine distribution network. In Section 4.1, we used the Poisson distribution to reflect the inter-arrival rate of sources. In real-life situations, countries encounter supply unavailability for several reasons, including the negative effect of COVID-19 on
supply (Ivanov, 2020), limited production of COVID-19 vaccines, etc. To this end, we consider three different cases of supply sources arrival rate within the first vaccination phase, e.g., (i) an average rate of 1.46 arrivals, (ii) an average rate of 1.92 arrivals, and (iii) an average rate of 2.52 arrivals.

Here we only compare the solutions produced by bi-objective models. Table 7 indicates how supply uncertainty influences the number of opened DCs and the number of vehicles used. The fourth column of Table 7 shows the decision on capacity level, which is presented as a tuple: (the number of DCs opened at level 1, the number of DCs opened at level 2, the number of DCs opened at level 3, percentage of used capacity at opened DCs). The used capacity percentage is computed as the sum of available capacity at opened DCs with the selected level divided by the total available capacity from all potential DCs in the network. Looking closely at the findings reveals that as the average number of arrivals increases, all models tend to open more DCs to manage the sourcing. Similarly, the results reveal that the higher the average number of arrivals, the higher the number of vehicles. Indeed, as the number of arrivals increases, more vehicles are required to distribute vaccines among the social groups. Finally, when analyzing the used capacity column, we observe that although the network design tend to open a high number of DCs (sometimes all of them), this is often done at capacity level 1, which leaves sufficient flexibility in case of a much higher demand.

Moreover, Table 7 provides a clear account of the impact of supply uncertainty on the objective functions. As can be seen, the distribution network receives sources with a higher frequency, allocating more vaccine doses to the social groups. Therefore, a higher number of arrivals brings about an increase in $E[Q]$. In terms of cost-efficiency, the SB solution is the most cost-effective considering all three cases. However, we observe different results for the UB and RB solutions. In fact, the RB solution can be more costly than the others whenever the distribution network receives sources around 1.46 times on average over the horizon of the vaccination program. Therefore, the uncertainty of the arrival of sources is an influential factor in the performance of each social equity theory. Specifically, Rawls’ theory is the least efficient solution for the equitable vaccine allocation problem when the frequency of arrivals is considerably low. Finally, when the frequency of arrival increases, UB is the most costly solution.

5.2.4. Sensitivity analysis of network size

In this section, additional experiments are conducted in order to study the performance of the design solutions for larger size networks. To this end, the baseline network is expanded in two ways: case expansion 1 where only the number of DCs is increased to twenty; case expansion 2 where the number of potential DCs is increased to twenty and two additional social groups are appended. Table 8 provides a comparison of the performance of the three cases for the three bi-objective models (UB, SB, RB) with respect to the expected cost, the expected coverage, and the number of opened DCs. As shown in Table 8, when the number of potential DCs is increased (case expansion 1), the design solutions for the three models tend to improve the expected cost, either by adding more DCs (for UB and RB) or by swapping some DCs (SB). For instance, with SB solution the number of opened DCs remains the same but only 11 of the 14 DCs are similar. This could be explained by the fact that the new DCs proposed are better located (at a closer distance from some key social groups). In the second case expansion, with an increase in both the number of social groups as well as the number of DCs, we observe a similar behavior. In this case, UB and RB solutions show that the deployed capacity can absorb the additional demand from the novel social groups without more DC opening. SB solution opens an additional DC to manage the increase in social groups, which is congruent with the previous behavior of this equity approach. In general, the results show that the number of DCs stagnates at around 14 (with an exception of 15 DCs for SB in case expansion 2), which seems to be sufficient to provide adequate coverage to the social groups.
the higher the costs.

immunize the community. Clearly, the more allocated vaccine doses, increases

\[ \gamma \] obtained when SB model is subject to various

and an increase in the value of \[ \lambda \]

decrease by 15\%). This table shows the percentage change of objective

results in a higher \[ E \] and a reduction of \[ \lambda \].

functions, opened DCs, and deployed capacity when the inequity mea-

5.2.5. Sensitivity analysis of the inequity parameters impact on the network
design

In this section, additional experiments are conducted in order to

study the impact of the inequity parameters on the network design

solutions. This is conducted for Rawls’ theory and Sadr’s theory using RB

and SB design models, respectively, under the settings of the baseline

case. Table 9 provides the results of the solutions obtained when RB

model is subject to various \( \lambda_1 \) and \( \lambda_2 \) values (increase or decrease by

15\%). This table shows the percentage change in objective functions,

opened DCs, and deployed capacity. Since Rawls’ theory gives priority to the poorest (most vulnerable) social group when increasing the value of \( \lambda_1 \), Rawls’ theory shows better performance in terms of cost and
coverage. An explanation for this might be that with an increase of \( \lambda_2 \)

and a reduction of \( \lambda_2 \), more vaccine doses are allocated to the poorest

social group. So, the distribution network would focus on specific areas,
in this case, regions where the poorest people have settled. As a result,

Rawls’ theory opens fewer DCs compared to the baseline, leading to
cost reduction. In addition, findings indicate that an increase in \( \lambda_1 \)

results in a higher \( E[Q] \). Interestingly, these results are congruent with the

essence of Rawls’ theory since it tries to maximize welfare for the

poorest social group. However, a reduction in the value of \( \lambda_1 \) and an increase in the value of \( \lambda_2 \) weaken the performance of Rawls’

theory.

On the other hand, Table 10 provides the results of the solutions

obtained when SB model is subject to various \( \gamma_1 \) values (increase or
decrease by 15\%). This table shows the percentage change of objective

functions, opened DCs, and deployed capacity when the inequity mea-

sure parameter changes. An increase in the value of the gap parameter

(\( \gamma_1 \)) means mitigating the gap between social groups. As a matter of

fact, an increase in the value of \( \gamma_1 \) reduces the class difference. So,

the most striking result to emerge from this analysis is that with an

increment of social equity in Sadr’s theory, fewer vaccine doses need
to be allocated. In fact, in an equitable society, where people from

various social classes are treated in a fair manner, fewer sources are

utilized to develop herd immunity. Consequently, a higher value of \( \gamma_1 \)
brings about a reduction in costs. In contrast, a decline in the value of

the gap parameter \( \gamma_1 \) brings about a higher \( E[Q] \), which subsequently

increases \( E[Cost] \). These findings demonstrate that as the gap between

social groups increases, more costs and vaccine doses are spent to

immunize the community. Clearly, the more allocated vaccine doses,
the higher the costs.

Table 7
Effect of supply uncertainty on the number of opened DCs, transportation vehicles, and objective functions.

| Models | Average number of arrivals | Opened DCs | Used capacity per day | \[ E[Q] \] | \[ E[Cost] \] |
|--------|-----------------------------|------------|-----------------------|-----------|------------|
| UB     | 1.46                        | 11         | (11,0,0,24%)          | 95        | 9050       |
|        | 1.92                        | 12         | (12,0,0,26%)          | 97        | 12016      |
|        | 2.52                        | 14         | (0,0,14,100%)         | 300       | 15728      |
| SB     | 1.46                        | 11         | (11,0,0,23%)          | 84        | 8323       |
|        | 1.92                        | 14         | (14,0,0,30%)          | 86        | 11111      |
|        | 2.52                        | 14         | (14,0,0,30%)          | 89        | 14472      |
| RB     | 1.46                        | 10         | (10,0,0,23%)          | 94        | 3090       |
|        | 1.92                        | 12         | (12,0,0,24%)          | 97        | 4108       |
|        | 2.52                        | 14         | (14,0,0,30%)          | 100       | 5372       |

Table 8
Solutions in different network sizes.

| Case     | Size of network | UB  | SB  | RB  | UB  | SB  | RB  | UB  | SB  | RB  | UB  | SB  | RB  |
|----------|----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Baseline | 14             | 34  | 12016| 11111| 4108| 1199| 1043| 1192| 12  | 14  | 12  |
| Case 1   | 20             | 34  | 0%  | 0%  | 0%  | -1.7%| -0.7%| -0.3%| 14 | 11 | 13 |
| Case 2   | 20             | 36  | +2.2%| +1.8%| +2% | +2.2%| +4.2%| +0.7%| 14 | 15 | 13 |

Table 9
RB model sensitivity to the inequity parameters and their baseline settings.

| \( \gamma_1 \) | \( \gamma_2 \) | UB  | SB  | RB  | UB  | SB  | RB  |
|----------------|----------------|-----|-----|-----|-----|-----|-----|
| 0.8            | 0.2            | 4108| 1192| 12  | (12,0,0,24%)|
| +15%           | -15%           | +1.6%| -0.27%| 11 | (11,0,0,24%)|
| -15%           | +15%           | -1.58%| +0.77%| 14 | (14,0,0,30%)|

Table 10
SB model sensitivity to the inequity parameters and their baseline settings.

| \( \lambda_1 \) | UB  | SB  | RB  | UB  | SB  | RB  |
|----------------|-----|-----|-----|-----|-----|-----|
| 0.7            | 11111| 1043| 14  | (14,0,0,30%)|
| -15%           | +3.6%| +5.83%| 14 | (14,0,0,30%)|
| +15%           | -2.8%| -7.09%| 14 | (14,0,0,30%)|

5.3. Managerial insights

In line with our key findings, we provide managerial insights to help decision-makers with their pursuit of equitable and efficient design decisions in a vaccination context similar to the COVID-19.

• The importance of considering social equity theory in the decision-making approach

Social equity can serve as an appropriate framework to increase the general immunization level of populations, and more importantly, reduce health inequities among social classes (Weintraub et al., 2020). Notwithstanding the foregoing, decision-makers should be aware of the essence of social equity theories. Based on our results, utilitarianism is the most costly social equity theory. This finding is thoroughly in line with the principles of utilitarianism, recalling that utilitarianism tends to maximize welfare across all of society (Pereira et al., 2017). When governments find themselves in a precarious situation, as in the case of the COVID-19 pandemic, executing utilitarianism may be unaffordable in countries with low and middle-income. Furthermore, the results related to Sadr’s theory are in line with the theory’s intent.

As mentioned, Sadr’s theory of social equity attempts to create social balance among different classes of society and maximize welfare (Reda, 2014; Bebhabani et al., 2019). In a broader perspective, we find that Sadr’s theory opens more DCs compared to the other social equity theories to cover all social groups efficiently. We also observe that the solution produced by the
Sadr theory-based model is more cost-effective than the other models. Concerning welfare maximization, we find that Sadr’s theory tends to allocate a sufficiently high number of COVID-19 vaccine doses. Overall, Sadr’s theory can be the right choice for every country looking for balanced solutions. Rawls’ theory holds that more distributable benefits should be provided for the poorest class of society (Liu et al., 2019; Behbahani et al., 2019). Indeed, we find that the bi-objective model of Rawls’ theory covers more people in rural areas than any other model. Hence, Rawls’ theory can be the perfect choice for countries with a high population share in rural areas. Furthermore, findings reveal that Rawls’ theory performs better when more priority is given to the poorest group. In conclusion, according to the general results, we can infer that each social equity theory has its own merits. Therefore, managers and policy-makers should adopt a social equity theory that could suit their social vulnerability indexes and the country’s demographic profile, as well as their distribution capabilities. Generally, utilitarianism is an appropriate theory for high-income countries. In contrast, Sadr’s theory can be a better choice for middle-income countries since applying Sadr’s theory is not as costly as utilitarianism. Besides, given that Rawls’s theory covers more people in rural areas compared to other social equity theories, it is recommended that Rawls’ theory be adopted for countries with the highest population share in rural districts.

- The importance of designing the distribution network under demand and supply uncertainty.

Clearly the uncertainty generated by the COVID-19 pandemic is by far higher than any other regular vaccination program. Our findings suggest that supply uncertainty is a critical factor that can highly affect the decision-making process. During the COVID-19 pandemic, supply unavailability has been a significant problem for organizations (Ivanov, 2020). Accordingly, our study reveals that by receiving more vaccine doses from domestic and international sources, the distribution network acts better in terms of allocation. Moreover, as the amount of sources increases, decision-makers need to open a higher number of DCs and use more transportation capabilities to dispense COVID-19 vaccine doses among the social groups. Of course, opening more DCs and using more vehicles lead to higher costs. Above all, decision-makers should be aware that supply uncertainty can highly influence social equity theories. In fact, on most occasions, utilitarianism is the most costly theory, but in the case of a one-time only receipt of sources in the first vaccination phase, Rawls’ theory of justice carries the highest cost. On the other hand, COVID-19 also underlined the complexity to estimate the needs for vaccination. This calls for advanced modeling approaches of the daily contamination rate and its space-time propagation by social group, which clearly impacts the distribution pattern of vaccines doses, and thus influences the design solution.

### 6. Conclusions and future work

The equitable allocation of COVID-19 vaccines has become a significant issue. Therefore, this paper proposes a bi-objective model for a COVID-19 distribution network considering several well-known social equity theories. The first objective function of the model attempts to maximize allocation, while the second aims to minimize the total costs of the distribution network. This study considers three different social groups, e.g., inhabitants of a. cities, b. towns and suburbs, c. and rural areas, according to the degree of urbanization. Worth noting is that our study specifically focuses on the first vaccination phase. Moreover, we use a stochastic programming approach to handle the uncertainty in demand and supply. In addition, we use the Monte Carlo scenario generation method to reflect the effect of COVID-19 infection and supply uncertainty during the COVID-19 pandemic and employ an SAA approach to calibrate the number of scenarios. Next, we use the LGP method to prioritize the objectives of the model. Given the crucial importance of allocation, we give first priority to the allocation maximization objective function, and second priority to the cost objective function. Thereafter, we build a case study based on real-life data to empirically validate the model developed. Finally, the findings provide useful insights for decision-makers and practitioners. First, we show that the solution produced by the Sadr’s theory model tends to open more DCs to create a balance among social groups. Notwithstanding this, such solution uses lower transportation capability compared to the solutions produced by utilitarianism and Rawls’ theory models. Furthermore, Sadr’s theory acts better than utilitarianism in terms of coverage percentage. In fact, Sadr’s theory leads to 95% coverage in cities, 95% in suburbs and towns, and 98% in rural areas. Instead, utilitarianism brings about 93% coverage in cities, 92% in suburbs and towns, and 97% in rural areas. In addition, we find that Sadr’s theory acts better than Rawls’ theory in terms of coverage percentage in most areas except for rural areas. In fact, our findings reveal that Rawls’ theory results in 100% coverage in rural areas. Regarding the objective functions, utilitarianism outperforms both Sadr’s and Rawls’ theories in terms of allocation. In the cost perspective, Sadr’s theory acts better than utilitarianism and Rawls’ theory. Furthermore, we analyze the effect of supply uncertainty on the theories. First, as the number of arrivals of supplies increases, more DCs and more vehicles are required. Second, the analysis reveals that supply uncertainty can affect the solutions produced by the three alternative social equity theories. Although utilitarianism is likely the most costly theory on most occasions, when the distribution network receives a one-time only arrival of sources in the first vaccination phase, Rawls’ theory becomes the most high-cost theory.

Finally, this study opens several future research directions. First, the proposed stochastic model covers only the first vaccination phase concerned with deployment, whereas future studies could extend the modeling approach to cover the subsequent phases of the vaccination program. Second, we have not considered the human resources that must accompany the deployment of such a vaccination program. The case of COVID-19 has also shown the criticality of these resources and the sanitary challenges related to their operations. As a future research avenue, the network design model should consider the human capabilities and their fair allocation among opened DCs in order to guarantee the social equity of the vaccination. Third, based on the COVID-19 observation, the cold chain was a critical distribution challenge. Accordingly, the consideration of specific inventory-routing models integrating the constraints of the cold chain and fairness are needed to complement the current work. Lastly, alternative multi-objective methods, as well as other stochastic optimization techniques, could be developed for future models to deal with the multiplicity of objectives and uncertain parameters.

### Data availability

Data will be made available on request.

### Appendix A. Preprocessing and validation

First, a preprocessing step is required to calculate $\mathcal{Q}$. In fact, utilitarianism and Sadr’s theory of justice have the same allocation objective function. Hence, we first calculate $\mathcal{Q}$ as shown in Eq. (A.1) in the utilitarianism model. Then, we add the obtained value as $\mathcal{Q}$ in Sadr’s theory of justice model.

$$\mathcal{Q} = \sum_{i \in I} \sum_{g \in G} \sum_{m \in M} \sum_{t \in T} \sum_{s \in S} \mathcal{Q}_{igmst}$$

(A.1)

Also worth mentioning is that we calibrated the values of $y_1$ and $y_2$, setting $y_1$ and $y_2$ equal to 0.7 and 0.001, respectively. Table A.1 indicates how we obtained each model.
The sample average approximation technique (SAA) (Shapiro et al., 2009) requires an important validation step for stochastic models. The quality of the solution is highly impacted by the number and quality of the scenarios generated. An important parameter to calibrate for our stochastic model is the number $N$ of scenarios to include in the optimization phase. First, the SAA consists in solving the problem with $M$ independent samples of $N$ scenarios generated with Algorithm 1. The average value among the $|M|$ expected values, obtained from each model resolution with $N$ alternative scenarios, gives a statistical lower bound. Second, we evaluate the obtained solutions based on the second-stage model (by fixing the first-stage decisions) by solving the resulting problem for $N' = |S_N| \gg N$ independent scenarios to get an upper bound on the optimal solution of the problem. Finally, a statistical optimality gap is computed for each obtained solution based on the difference between the lower and upper bounds. After testing several sizes of $N$ scenarios, applying the procedure described above shows that the statistical gap of 0.48% can be obtained when using a sample of 100 scenarios ($|S| = 100$), leading to a well-calibrated SAA model.

Furthermore, the value of the stochastic solution (VSS) is a well-known method to evaluate the quality of the stochastic solution (Ben Mohamed et al., 2020). Therefore, we calculate the value of stochastic solution (VSS) for the USC model as a sample to validate the model. Suppose that $X^*(o)$ is the optimal solution from the USC under all scenarios $s \in S$. Thus, the optimal value of its objective function is called recourse problem ($RP$), which is computed as Eq. (A.2). After using the SAA approach, $S_N \subseteq S$, $\bar{X}(o)$ denotes the near-optimal solution gained by applying the SAA method. Then, the $RP$ solution with a sufficient number of scenarios is computed as Eq. (A.5).

$$RP = \mathbb{E}_S[q(X^*(o), S)], \quad (A.2)$$

$$\bar{RP} = \mathbb{E}_S[q(\bar{X}(o), S^N)]. \quad (A.3)$$

Next, all of the random variables are replaced with their corresponding expected values ($\mathbb{E}$) in order to compute the expected value ($EV$). So, the deterministic model ($EV$) is solved as shown in Eq. (A.4). Suppose that $\bar{X}(o)$ is the optimal solution obtained from solving Eq. (A.4), known as the expected value solution. Then, the evaluation of its expected value ($EEV$) is calculated as shown in Eq. (A.5). Finally, the estimate of VSS solution is computed using Eq. (A.6). Table A.2 provides information on the estimate of VSS, $EEV$, and $\bar{RP}$ solutions for the USC model.

$$EV = \min q(X(o), \mathbb{E}), \quad (A.4)$$

$$\mathbb{E}EV = \mathbb{E}_S[q(\bar{X}(o), S^N)], \quad (A.5)$$

$$\bar{VSS} = \mathbb{E}EV - \bar{RP}. \quad (A.6)$$

**Appendix B**

This appendix details the data gathered for the case study and the additional estimations considered in the baseline case described in Section 5.1. Recall that the baseline case in the current work considers two provinces in the west of Iran, namely Kermanshah and Kurdistan. In the current case study, 14 potential locations for DCs ($|I| = 14$) have been considered. As shown in Fig. 5 the potential locations for DCs are Saqqez, Baineh, Marivan, Bijar, Sanandaj, Paveh, Javanrud, Sahneh, Kangavar, Kermanshah, Dalahu, Harsin, Eslamabad-e Gharb, and Gilan Gharb. As can be observed, these fourteen potential locations are scattered all over the map. The mentioned locations are well-known cities in Kermanshah and Kurdistan provinces. In terms of logistics operations, these fourteen potential locations have better accessibility to the regions around them. Therefore, we selected the aforementioned potential locations for DCs due to better route connectivity with far and deprived areas. The opening cost of each capacity level at each DC is different. Since we consider a specific capacity for each capacity level at a DC, their corresponding opening cost ranges from 30 million IRR to 40 million IRR. By opening the DC capacity level, we do not mean the construction phase of a DC. By opening a capacity level at a DC, we mean the costs of providing requirements for vaccine distributions, including the costs of preparing pallets (as shown in Fig. 3), workforce, indoor logistics operations, and utility expenses.

Turning now to the interval of received sources and the COVID-19 propagation. As discussed earlier, the current study focuses on the first vaccination phase. The first vaccination phase co-occurred with the primary waves of the COVID-19 outbreak. During the spread of the first COVID-19 variants, people were infected daily. So, the space–time COVID-19 propagation was estimated using a Monte Carlo method. On the other hand, the interval of received sources is totally different from the COVID-19 propagation. Unlike the COVID-19 propagation rapidity, the time interval between received sources is sporadic. Unfortunately, in the first stage of the vaccination program, the shortage of COVID-19 vaccines, specifically in middle-income countries, is a tough challenge (Amit et al., 2022). Similarly, Iran, as a middle-income country, encountered a vaccine shortage during the first vaccination phase. More specifically, Iran received batches of COVID-19 vaccines once or twice a month. Since there is no accurate data on the interval of received sources, thus a Monte Carlo method is employed for estimating the interval of received sources based on a Poisson process. In the baseline case, the mean of the exponential distribution $\mu$ for the random variable $f_{t,x}$ follows a discrete uniform distribution on the interval $[10, 80]$. After running the scenario generation process, the interval of arrivals for received sources turned out to be 1.92 on average.

**Appendix C**

Table C.1 shows the active capacity levels in each DC resulting from the different solutions.

**Appendix D**

Fig. D.1 shows the locations of DCs resulting from the single-objective models.
Table C.1

Opened capacity level in each DC.

| DC             | Capacity level | Solutions |
|----------------|----------------|-----------|
|                |                | USA | USC | UB | RSA | RSC | RB | SSA | SSC | SB |
| Kermanshah    | 1st level      |     |     | ✓  |     |     |     |     |     |     |
|                | 2nd level      |     |     |     |     |     |     |     |     |     |
|                | 3rd level      | ✓   |     |     |     |     |     |     |     |     |
| Gilan Gharb   | 1st level      |     |     |     | ✓  |     |     |     |     |     |
|                | 2nd level      |     |     |     |     |     |     |     |     |     |
|                | 3rd level      |     |     | ✓  |     |     |     |     |     |     |
| Sanandaj      | 1st level      |     |     | ✓  |     |     |     |     |     |     |
|                | 2nd level      |     |     |     |     |     |     |     |     |     |
|                | 3rd level      |     |     |     | ✓  |     |     |     |     |     |
| Saghez        | 1st level      |     |     |     |     | ✓  |     |     |     |     |
|                | 2nd level      |     |     |     |     |     |     |     |     |     |
|                | 3rd level      |     |     | ✓  |     |     |     |     |     |     |
| Kangavar      | 1st level      |     |     |     | ✓  |     |     |     |     |     |
|                | 2nd level      |     |     |     |     |     |     |     |     |     |
|                | 3rd level      | ✓   |     |     |     |     |     |     |     |     |
| Javanrud       | 1st level      |     |     | ✓  |     |     |     |     |     |     |
|                | 2nd level      |     |     |     |     |     |     |     |     |     |
|                | 3rd level      |     |     |     | ✓  |     |     |     |     |     |
| Marivan        | 1st level      |     |     |     |     | ✓  |     |     |     |     |
|                | 2nd level      |     |     |     |     |     |     |     |     |     |
|                | 3rd level      | ✓   |     |     |     |     |     |     |     |     |
| Baneh          | 1st level      |     |     | ✓  |     |     |     |     |     |     |
|                | 2nd level      |     |     |     |     |     |     |     |     |     |
|                | 3rd level      |     |     | ✓  |     |     |     |     |     |     |
| Sahneh         | 1st level      |     |     |     |     |     | ✓  |     |     |     |
|                | 2nd level      |     |     |     |     |     |     |     |     |     |
|                | 3rd level      |     |     |     |     | ✓  |     |     |     |     |
| Paveh          | 1st level      |     |     |     |     |     | ✓  |     |     |     |
|                | 2nd level      |     |     |     |     |     |     |     |     |     |
|                | 3rd level      | ✓   |     |     |     |     |     |     |     |     |
| Dalahu         | 1st level      |     |     | ✓  |     |     |     |     |     |     |
|                | 2nd level      |     |     |     |     |     |     |     |     |     |
|                | 3rd level      | ✓   |     |     |     |     |     |     |     |     |
| Bijar          | 1st level      |     |     | ✓  |     |     |     |     |     |     |
|                | 2nd level      |     |     |     |     |     |     |     |     |     |
|                | 3rd level      | ✓   |     |     |     |     |     |     |     |     |
| Esalamabad-e Gharb | 1st level |     |     | ✓  |     |     |     |     |     |     |
|                | 2nd level      |     |     |     |     |     |     |     |     |     |
|                | 3rd level      | ✓   |     |     |     |     |     |     |     |     |
| Harsin         | 1st level      |     |     | ✓  |     |     |     |     |     |     |
|                | 2nd level      |     |     |     |     |     |     |     |     |     |
|                | 3rd level      | ✓   |     |     |     |     |     |     |     |     |

(1) Design-stage solution in the USA, SSA, and RSA models

(2) Design-stage solution in the USC, SSC, and RSC models

Fig. D.1. Opened DCs in the single-objective models.
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