Hydrodynamic instabilities at decay flow of nematic liquid crystals through the plane capillary of a variable gap

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Abstract. In this paper we consider the possibility to study hydrodynamic instabilities (HDI) in a specific type of a decay flow existing at a rise of a nematic liquid crystal (NLC) in the flat capillary of a variable gap. In this case the capillary can be replaced by a number of parallel channels of different gaps with different instant values of a pressure gradient, which induces a flow. The last feature makes such flow to be different from the previously studied decay flow [1] with a constant value of a pressure gradient, applied to the channels of different thickness.

1. INTRODUCTION

Nematic liquid crystals (NLC) can be considered as anisotropic fluids with the same mechanism of translational molecular motions as in isotropic liquids, which is responsible for high fluidity and zero values of static shear elastic modules. The main difference from isotropic liquids arises due to long range orientational order described in terms of the director $\mathbf{n}$, which corresponds to the average directions of long molecular axes, and the scalar order parameter $S$ referred to the mean declination of molecules from the overall direction $n$. While order parameter $S$ is useful for a description of phase transition NLC – isotropic liquid, the unite vector $n$ plays the role of an additional hydrodynamic parameter, which has to be incorporated into hydrodynamic equations of nematic liquid crystals. The connection between translation motions, and a director field $\mathbf{n}(r, t)$ can be considered as fundamental physical property of liquid crystals. In particular, it results in non-Newtonian behavior and a number of specific hydrodynamic instabilities (HDI), arising in shear flows of nematic liquid crystals [2]. The realization of the given instability drastically depends on the initial hydrodynamic state, characterized by a space distribution of a velocity and orientation. For example, in the case of a simple shear flow of NLC layer of thickness $h$, confined between two plates, the initial homogeneous orientation, normal to the flow plane, becomes unstable at some critical value $s_c$ of the shear rate $s = v_0 / h$ ($v_0$ – the velocity of a relative motion of the plates). The similar instability arises also in a Poiseuille flow at a critical value $G_c$ of the pressure gradient $G = \frac{dP}{dx}$, which induces the flow of NLC confined between two motionless plates. Such instabilities was investigated in details both experimentally and theoretically [2]. At the same time, the initial orientation of NLC normal to the plates can be realized only in the absence of flows. So, the specific orientation instability (the escape of a director from the flow plane), arises in a Poiseuille flow at the initially non homogeneous orientation fields of the velocity and orientation [2,3]. It is important, that for both cases mentioned above, the critical values of a shear rate and a pressure gradient essentially depend on the thickness of LC layer. It provided effective usage of a wedge-like LC cell for visualization and study of the mentioned above escape of a director.
from the flow plane. In this case, the cell was considered as a number of capillaries of different thickness with the same instant value of a pressure gradient, slowly decreasing with time (decay flow).

The goal of this paper is to consider the alternative type of a decay flow, produced by a rise of NLC in vertically oriented wedge-like cell (see figure 1). The theoretical estimates for different initial geometries make possible to predict the dynamic of a capillary rise and time variations of local orientation of NLC layer, which can be used for explanation of the new experimental results, obtained at study of a capillary rise of NLC.

![Figure 1. Experimental geometry.](image)

2. **Decay flow and hydrodynamic instabilities in a wedge-shaped capillary**

At filling of a wedge-shaped cell such capillary with a liquid crystal one can observe a specific type of the Poiseuille flow with a permanently decreasing velocity. It allows us to make some analogy with the previously studied decaying Poiseuille flow in a wedge-shaped cell. At the same time, there are significant differences between these two cases. In particular, in the previously investigated case, a decay character of the flow resulted due to a decrease with time of the hydrostatic pressure drop applied to a wedge-shaped capillary of constant length previously filled with a liquid crystal. This geometry is equivalent to a set of variable-gap capillaries connected in a parallel scheme with the same pressure gradient applied to different capillaries. In our case the flow arises as a result of the motion of the LC on a previously dry surface, while the capillary pressure causing the flow depended on the local gap of the capillary. It led, in particular, to the dependence of the instant height of the meniscus on the coordinate y. In this connection, a corresponding modification of the previously considered theoretical models of linear and nonlinear phenomena in the decay Poiseuille flow of liquid crystals is necessary.

In particular, the question of the possibility of using this type of flow to study the hydrodynamic instabilities of the orientational structure is of interest. These instabilities manifest themselves as a sharp change in the orientation structure at a certain threshold value of the pressure gradient \((\Delta P/\Delta x)_c\) or the average value of the velocity gradient (shear rate) \(sc\). In particular, the instability of the simplest type with the initial orientation, normal to the flow plane, is realized when the dimensionless Ericksen number

\[
Er = \frac{\Delta P}{\Delta x} \sqrt{\frac{\alpha_x \alpha_\delta}{K_{11}K_{22}}} \left(\frac{h}{2}\right)^3 \approx \overline{S_{av}} \sqrt{\frac{\alpha_x \alpha_\delta}{K_{11}K_{22}}} \left(\frac{h}{2}\right)^2
\]

(1)
increases up to the critical value \( Er = 17.3 \). In expression (1) \( \alpha \) and \( \alpha – \) the Leslie’s coefficients, \( K_i \) – the Frank’s modules of orientation elasticity, \( \eta \) – the shear viscosity coefficient (Miesovich viscosity), corresponding to the initial orientation, \( s \) – the average value of the velocity gradient.

From eq.(1) it follows a strong dependence (h²) of the threshold value \( s_c \) on the local thickness h. It is worthwhile to notice, that the same kind of functional dependence is valid for the mentioned above hydrodynamic instability connected with the escape of the director from the flow plane, which is realized at the initially normal orientation respectively to LC layer. This fact allows us to consider, at a qualitative level, the appearance of both types of instabilities for the wedge-shaped capillary in a similar way.

Taking into account equations (1), the generalized equation that determines the critical value \( s_c \) can be written in the form:

\[
 s_c^{\alpha} \left( \frac{h}{2} \right)^2 B = M
\]

(2)

where \( M \) is the critical value of the Eriksen number corresponding to the given geometry, \( B \) is the combination of the material parameters of the liquid crystal.

In particular, for the simplest geometry, the expressions for \( B \) and \( M \) are easily obtained by comparing formulas (1) and (2):

\[
 B = \left[ \frac{\alpha_2 \alpha_3}{(K_{11}K_{22})} \right]^{1/2}, M = Er_c = 17.5
\]

(3)

The instant value of \( s \) can be calculated from the average linear velocity \( v^0 \) of motion of the meniscus in the capillary.

To make such calculations we will consider a liquid crystal as a conventionally Newtonian liquid (Stewart) with the effective shear viscosity, independent on the velocity gradient. Such approximation holds for nematic liquid crystals with orientation, stabilized by strong magnetic (electric) fields or surfaces [2]. So, some results concerning to a rise of isotropic Newtonian liquids in the vertical capillaries of different cross sections and sizes [4] can be applied for a capillary flow of liquid crystals.

In particular, the well-known Poiseuille formula, describing the volumetric rate of a flow \( Q = dV / dt \) induced by a pressure difference \( \Delta P \) in a capillary of a constant gap with a length \( l \) is valid. For a plane capillary with a large value of the aspect ratio \( R= A/h \) \( (A \) and \( h \) – the width and the gap of a capillary) the Poiseuille equation is written as:

\[
 \frac{dV}{dt} = \frac{h^3A}{12\eta} \sum \frac{\Delta P}{l}
\]

(4)

where \( \sum \Delta P \) – the resulting pressure difference, which includes a hydrostatic pressure and a capillary pressure:

\[
 P_g = \rho gl
\]

(5)
\[ P_\sigma = \frac{2\sigma \cos \theta}{h} \]  

(6)

In these equations a density \( \rho \) and a shear viscosity \( \eta \) are the parameters describing bulk properties of a liquid, whereas a surface tension coefficient \( \sigma \) and a contact angle \( \theta \) are the parameters which characterize the interaction of a liquid with an air and a solid surfaces.

Taking into account the trivial expression

\[ dV = hAdl \]  

(7)

one can get from (4-6) the next differential equation, describing a motion of a contact line:

\[ \frac{dl}{dt} = \frac{\sigma h \cos \theta}{6\eta l} - \frac{h^2 \rho g}{12\eta} \]  

(8)

This equation can be considered as a simplified (and modified) version of well-known Washburn equation [4] proposed to describe the contact line motion in a capillary of a circular cross section.

Taking into account equations (1), (4) and (7), it is simple to obtain the connection between the averaged value of the velocity gradient and the velocity of the contact line motion \( (dl/dt) \):

\[ s^{av} = 3v^{av} / (h / 2) = 6h - 1(dl / dt) \]  

(9)

where \( v^{av} = Q / A_i h_i \), index \( i \) corresponds to the given elementary capillary of width \( A_i \) and thickness \( h_i \). Inserting value \( (dl/dt) \), defined by (8) into (9) results in the next expression:

\[ \left[ (\sigma \cos \Theta)/(\eta l_c) - (h \rho g)/(2\eta) \right] \cdot (h / 2) \cdot 2B = M \]  

(10)

Where \( l_c \) determines the critical value of the local height of the meniscus rise corresponding to the occurrence of the instability, \( \eta \) is the effective viscosity value corresponding to the given geometry. Using (10), we obtain the following equation for \( l_c \) as a function of the local thickness of the LC layer:

\[ l_c(h) = l_0(h) \left\{ 1 + \left( 8M \eta / (\rho g B h^3) \right) \right\}^{-1} \]  

(11)

where the dependence of the maximum lift height on the gap \( l_0(h) \) is determined by the well known expression:

\[ l_c = \frac{2\sigma \cos \theta}{\rho gh} \]  

(12)

obtained from the equilibrium condition:

\[ P\sigma = Pg \]  

(13)

The usage of the linear dependence of the local thickness on the coordinate \( y \)
\[ h = h_0 \left[ 1 + \left( \frac{\Delta h}{h_0} \right) \left( \frac{y}{A} \right) \right] \]  \quad (14)

(\Delta h = h_{\text{max}} - h_0, \; A \text{ is the cell size with respect to the } y\text{-coordinate}), \text{ makes possible to calculate the maximum height } l_0 \text{ of the meniscus’ rise and the critical height of the meniscus } l_c \text{ as a function of the } y \text{ coordinate. It is interesting to define the time } t_c \text{ needed for the meniscus to reach a critical height } l_c \text{ for different values of the local thickness } h, \text{ defined by a coordinate } y. \text{ The corresponding equation, derived from expression } (9), (11), \text{ reads as:}

\[ t_c = -\tau (h) \ln \left[ 1 - l_c (h) / l_0 (h) \right] \]  \quad (15)

where the characteristic decay time of the flow } \tau (h), \text{ defined as:

\[ \tau = \frac{24 \sigma \eta \cos \theta}{h^3 (\rho g)^2} = \frac{12 \eta l_0}{h^2 \rho g} \]  \quad (16)

enters into the solution of equation (8):

\[ y = \frac{l_0 - l}{l_0} \exp \left( \frac{l}{l_0} \right) = \exp \left( -\frac{t}{\tau} \right) \]  \quad (17),

which was obtained taking the time independent value of contact angle } \theta_c. \text{ Numerical calculations were performed for the instability of the simplest type mentioned above. In this case, equation } (11) \text{ can be written as:

\[ l_c = l_0 (h_0) \left[ 1 + \left( \frac{\Delta h}{h_0} \right) (y / L) \right] / \left[ 1 + \left( \frac{\Delta h}{h_0} \right) (y / A) \right] \right] \right] \right] \right] \right] \right] \]  \quad (18)

In calculations, we assumed a gap ranging from } h_0 = 60 \mu m \text{ to } h_{\text{max}} = 100 \mu m (\Delta h = 40 \mu m) \text{ in a cell of width } A \text{ and used the material parameters } SCB \text{ [5]: } \alpha_s = -0.0812 \text{ Pa-s, } \alpha_3 = -0.0036 \text{ Pa-s, } K_{11} = 6.2 \times 10^{-12} \text{ N and } K_{22} = 3.9 \times 10^{-12} \text{ N, } \eta_1 = 0.0326 \text{ Pa-s, } \eta_2 = 0.0204 \text{ Pa-s, } \sigma = 0.0326 \text{ N/m}. \text{ The value of } \cos \theta = 0.820 \text{ was determined from the results of experimental measurements of the value } l_0 = 0.0532 \text{ m in a planar cell with a constant gap of } h=100 \mu m \text{ and a director perpendicular to the flow direction. The detailed of these experiments will be published elsewhere.}

**Table 1.** The characteristic times } \tau \text{ and } t_c \text{ for two values of the effective shear viscosity } \eta

| \ \ | \ \ \ \ \ \ | \ \ \ \ \ \ | \ \ \ \ \ \ | \ \ \ \ \ \ | \ \ \ \ \ \ |
|---|---|---|---|---|---|
| \ h (\mu m) \ | 60 | 70 | 80 | 90 | 100 |
| \ l_0 (cm) \ | 8.86 | 7.60 | 6.64 | 5.90 | 5.32 |
| \ l_c (cm) \ | 5.51 | 5.50 | 5.29 | 5.02 | 4.70 |
| \ \tau (s), \eta=\eta_1=0.0204 (Pa-s) \ | 600 | 378 | 253 | 177 | 127 |
| \ \tau (s), \eta=\eta_1=0.0326 (Pa-s) \ | 961 | 605 | 405 | 284 | 207 |
| \ t_c (s), \eta=\eta_1=0.0204 (Pa-s) \ | 210 | 212 | 200 | 188 | 162 |
| \ t_c (s), \eta=\eta_1=0.0326 (Pa-s) \ | 337 | 338 | 319 | 301 | 261 |
| \ \tau_n (s) \ | 3.6 | 4.9 | 6.4 | 8.1 | 10 |

The characteristic times } \tau \text{ and } t_c \text{ presented in Table 1 and were calculated for two values of the effective shear viscosity } \eta. \text{ It reflects a rather complicated scenario of the director variations under a decay flow. On the initial stage of a capillary rise the values of a pressure gradient are essentially.
higher than the critical values. It has to induce fast reorientation of LC from the initial state, characterized by a Mesovich viscosity $\eta_1$, towards the flow direction with corresponding viscosity $\eta_2$. After some time $t$, the flow decays to the critical value of a pressure gradient and backward relaxation to the initial state takes place. So, on practice, the effective viscosity can be varied in the range $\eta_2<\eta<\eta_3$ on different stages of a decay flow.

In this table we also present the calculated values of the characteristic time $\tau_n$, describing backward reorientation of LC director in the absence of external fields, defined as [6]:

$$
\tau_n = \left( \frac{\gamma_1 h^2}{\pi^2 K_{22}} \right)
$$

where $\gamma_1 = \alpha_1 - \alpha_2$ – the rotational viscosity. The values of $\tau_n$ are essentially lower than corresponding values of a decay time $\tau$. It means that a director follows the variations of the velocity film without phase shift after decreasing of the pressure gradient to the critical value, defined by equation (1). So, possible optical study of a director reorientation provides registration of the flow induced instability and experimental checking of theoretical estimates made above. The corresponding experiments are in a progress now.

The calculation of the instability threshold associated with the escape of a director from the flow plane for the initial homeotropic orientation is a much more complicated problem, the solution of which was performed for methoxybenzylidene-p-n-butylaniline (MBBA) by numerical methods both in the absence and in the presence of destabilizing electric fields [2]. It should be noted that in this case, the torque acting on the director from the flow is maximal and the shear flow can cause both primary and secondary instabilities at relatively small values of the pressure gradient (of the order of 30 Pa/cm at a thickness of the LC layer of 100 $\mu$m). The values of the pressure gradient in our experiments at the initial stage of capillary filling exceed by about an order of magnitude the given value. In this case, the flow can be considered sufficiently strong for the orientation of the entire sample at a small angle $\theta_\beta$, $tg^2 \theta_\beta = \alpha_1 / \alpha_2$.

For the values of the material parameters 5CB given above, we obtain $\theta_\beta = 11.8^0$. This, in turn, means that the effective value of the shear viscosity coefficient is close of the minimum Mesovich viscosity ($\eta_2$). Table 2 shows the results of calculating the parameters, which determine the dynamics of the meniscus’ motion, for $\eta_2$ and $\eta_3$. In this case, the value of $\cos \theta = 0.468$, obtained from the experimentally determined value of $l_0$, corresponds to the contact angle $\theta = 62.1^0$, which is significantly larger than those for the case of a planar orientation.

It is worthwhile to mention, that instabilities of higher orders are characterized by a very slow backward relaxation (up to some hours) which prevent the optical study of the primary instability [2]. The photos (in crossed polarizers) of the initially homeotropic wedge-like cell on the stage of a backward relaxation after filling with 5 CB confirm this conclusion. The situation can be partly improved by additional usage of stabilizing electric fields, which results in suppression of hydrodynamic instabilities and decreasing of the relaxation’s time of a director (see Table 2) defined as [2]:

$$
\tau_n = \left( \frac{\gamma_1 h^2}{\varepsilon_0 \Delta \varepsilon U^2 - \pi^2 K_{33}} \right)
$$

where $\Delta \varepsilon \approx 11$ – dielectric permittivity anisotropy, $U$ – applied voltage, $K_{33} = 8.2 \cdot 10^{-12}$ N – Frank’s module. The experiments of such kind are in a progress now.
Table 2. The characteristic times $\tau$ and $t_c$ for two values of the effective shear viscosity $\eta$

| $y/l$ | 0  | 0.25 | 0.5 | 0.75 | 1   |
|-------|----|------|-----|------|-----|
| $h$ (µm) | 60 | 70  | 80  | 90   | 100 |
| $l_0$ (cm) | 5.95 | 5.10 | 4.46 | 3.96 | 3.57 |
| $l_c$ (cm) | 3.70 | 3.69 | 3.55 | 3.37 | 3.15 |
| $\tau$ (s), $\eta=\eta_2=0.0204$ (Pa·s) | 403 | 254 | 170 | 119 | 87 |
| $\tau$ (s), $\eta=\eta_3=0.0326$ (Pa·s) | 645 | 406 | 272 | 191 | 139 |
| $t_c$ (s), $\eta=\eta_2=0.0204$ (Pa·s) | 141 | 142 | 134 | 126 | 109 |
| $t_c$ (s), $\eta=\eta_3=0.0326$ (Pa·s) | 226 | 227 | 214 | 202 | 175 |
| $\tau_n$ (s), $U=0$ (V) | 3.6 | 4.9 | 6.4 | 8.1 | 10 |
| $\tau_n$ (s), $U=5$ (V) | 0.14 | 0.20 | 0.26 | 0.32 | 0.4 |

3. CONCLUSION

In this paper the capillary decay flow of a nematic liquid crystal in a wedge-shape capillary was considered and applied to calculate the critical values of the height of a capillary rise and of the time of the rise, which corresponds to the hydrodynamic instabilities of two types. The explicit calculations for the simplest hydrodynamic instability were performed taking into account the characteristic time of a director motion and the decay time. It is shown, that the long living instabilities of higher order can prevent the observation of the primary instability, connected with an escape of a director from the flow plane. In this case the observations can be provided via additional usage of a stabilizing electric field.

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