Comments on “Towards Unambiguous Edge Bundling: Investigating Confluent Drawings for Network Visualization”

Jonathan X. Zheng, Samraat Pawar, and Dan F. M. Goodman

Abstract—Bach et al. [1] recently presented an algorithm for constructing confluent drawings, by leveraging power graph decomposition to generate an auxiliary routing graph. We identify two problems with their method and offer a single solution to solve both. We then recognize a limitation regarding planarity, and help to guide future research by introducing a new classification of ‘power-confluent’ drawing.

1 THE DEFINITION OF CONFLUENT DRAWING

Confluent drawing is a graph drawing technique that eliminates crossings by allowing edges to overlap, as long as the join is smooth. This is analogous to junctions on a train track that allow carriages to switch directions without stopping. For convenience, we recall the original definition (Dickerson et al. [2]) of a confluent drawing A for a graph G:

- There is a one-to-one mapping between the vertices in G and A, so that, for each vertex v ∈ V(G), there is a corresponding vertex v′ ∈ A, which has a unique point placement in the plane.
- There is an edge (v_i, v_j) in E(G) iff there is a locally-monotone curve (defined as having no self intersections or sharp turns) e′ connecting v_i′ and v_j′ in A.
- A is planar. That is, while locally-monotone curves in A can share overlapping portions, no two can cross.

Bach et al. [1] claim to construct such a drawing by converting a power graph decomposition (PGD) into an auxiliary routing graph (ARG). Then, for each adjacency, the graph-theoretic shortest path through the ARG is used as the sequence of control points for a spline.

We find that the combination of splines and shortest paths introduces problems not fully explored by the original authors. We solve these problems and then introduce a new subclass of confluent drawing which we call power-confluent drawing.

1.1 B-splines

We first identify issues regarding the use of B-splines (presumably of degree p = 3, although it is not specified) for interpolating control points. These were likely chosen because they satisfy the convex hull property (which prevents crossings at shared control points; see Jia et al. [3] for an example of poorly implemented splines that do not satisfy this property), and also offer local control (i.e. moving a control point only affects the surrounding p + 1 segments), which guarantees that splines that share enough intermediate control points will overlap. Local control is what makes it possible for drawings to be confluent; with the right ARG, it is possible for edges to share enough control points such that they produce the overlapping portions that denote a confluent drawing. Specifically, for two curves to be guaranteed to overlap they must share p or more control points (assuming a standard B-spline, see the supplemental material (SM), Section 1 for a proof).

There are two problems with using B-splines in this context. The first is that splines that share fewer than p control points will not overlap, but sharing even a single routing node should indicate a bundle (as in [1, Fig. 2]). The authors identified this, calling it the ‘crossing artifact’ [1, Fig. 4], and fix it by splitting routing nodes into two: one for incoming and one for outgoing edges. The intuition behind splitting is correct, as it introduces another shared control point that tightens the bundle to close the crossing, but their exact description contains an ambiguity in the context of undirected graphs, as it is not specified how to identify edges as incoming or outgoing. We resolve this ambiguity, and the following related problem, in Section 1.3.

The second problem is also caused by local control but has an opposite result: that splines will always overlap if they share p or more control points. Given the right ARG, splines may overlap so as to create the visual impression that extra edges, not in the original graph, exist (see the SM, Fig. 2 for an example). This violates the second condition in the confluent definition. We find that such an ARG can result from the PGD to ARG conversion, as explained in the following section.

1.2 Short-circuits

A PGD is an extension to the node-link diagram, that compresses the number of edges by grouping similar vertices together into power groups, and merging edges among group members that share the same target vertex into a single power edge instead. This is then converted into an ARG by (1) connecting the members of each power group to a routing node corresponding to the group, and (2) connecting pairs of routing nodes whose corresponding power groups are connected by power edges (see [1, § 3.1] for more detail). This conversion from a PGD to an ARG can cause problems
because (1) and (2) both result in routing edges in the ARG, which can cause a short-circuit effect (SM, Fig. 2).

This effect can be explained as follows. The structure of groups within a power graph can be represented as a tree, where groups are represented by branches and vertices by leaves. Trees are geometric (i.e., there exists a unique shortest path between any pair of vertices), but the short-circuits caused by power edges can invalidate this, to produce ambiguity either in the choice of path (if the shortest paths are equal) or in which edges exist at all, by routing edges in the wrong direction entirely.

1.3 A solution

The solution to these problems is to retain the structure of power groups as a tree, with all routing edges directed towards the leaves. We then add the power edges back, except as a special type of edge that is incoming at both ends (the purpose of which will soon be made clear). If we finish by discarding the root of the tree, we are left with the exact same ARG as before, except now with all routing edges explicitly directed (Fig. 1, third column). Note that this also means all adjacency information is now preserved in the ARG such that the original graph can be recovered.

To draw the adjacency edges back on top, we can now forgo any shortest path calculations. Instead, for each power edge, we perform a depth-first search for all child leaf nodes starting from both ends of the power edge, and concatenate the path to each leaf from one end to the reversed path to each leaf from the other end. Every concatenated path is then used as the sequence of control points for a spline. These paths are now guaranteed to be unique because the ARG is effectively a tree, due to power edges being incoming at both ends to prevent their traversal, and since only the one correct power edge is traversed for each spline, the short-circuit problem described in Section 1.2 is alleviated.

Imposing this directionality on the ARG also fixes the node splitting ambiguity in Section 1.1, guaranteeing that the split occurs in the correct direction. This works because every routing node can be seen as the boundary between two sides of a biclique which is replaced by a single bundled junction, and the explicit direction of routing edges now encodes the orientation of the bundle itself. This is likely what the original authors intended, but now with a more precise explanation with the exact directionality of routing edges clarified.

2 Planarity and power-confluent drawing

We now move to the third condition in the definition, which regards planarity. Unfortunately, the original method does not offer any guarantee of planarity due to the use of a force-directed method to lay out the graph. While the authors do recognize this, and make the distinction that their drawings are ‘non-planar confluent’, the loss of this condition means that almost any drawing, with or without curved edges, satisfies a now-trivial definition. However, the planarity condition can be relaxed in the context of finding a drawing that reduces the number of crossings, for example in layered confluent drawings [4]. Bach et al. [1] do not explicitly address this question, but in practice their approach can greatly reduce the number of crossings, making it a practical method to enhance readability.

Furthermore, it is easy to see that a planar ARG also leads to a planar drawing, as long as we are careful to avoid crossings at routing nodes (which is always possible because every routing node is simply a single bundled junction in the middle of a biclique). This means that the complete confluent definition can be satisfied if and only if the ARG is also planar, assuming no redundant routing edges without adjacencies routed through it. This naturally implies a new subset of confluent drawable graphs, much like the ∆-confluent [5] or strict confluent [6] subclasses, which can be found through a planar PGD to ARG conversion. We suggest naming such drawings as power-confluent.

Future directions could involve finding power-confluent drawings by constraining the PGD search algorithm to solutions with planar routing graphs, or investigating the complexity of determining whether a graph admits such a drawing at all.

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Supplemental Material for Comments on “Towards Unambiguous Edge Bundling: Investigating Confluent Drawings for Network Visualization”

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1. Node splitting

Here we give a more thorough explanation of issues with node-splitting, introduced in the main text Section 1.1. Figure 1 shows how the direction in which the node is split can strongly affect the resulting drawing.

Technically, when splitting we should produce $p - 1$ extra nodes, where $p$ is the degree of the spline. This is because at least $p$ shared control points are required to guarantee an overlap. The proof for this is simple: because of the local control property (each segment is affected by only the surrounding $p + 1$ control points) it means that at the exact point at which two segments join (known as a knot) the curve cannot simultaneously be affected by the two furthest control points that affect the two segments on either side. The remaining control points are therefore the last $p$ control points of the left segment, which overlap with the first $p$ control points of the right segment. Both tangents at this point are also guaranteed to be equal, through the same argument. This also assumes a uniform knot vector, which is defined as having segments spaced along even parametric intervals along the spline.

However in practice one split is often sufficient for edges to come close enough to look bundled, even for cubic splines (see Figure 1). Note that drawing a spline for every edge is not the only way to render the drawing, and also introduces a great deal of redundancy due to the large amount of overlap between edges. If a purely confluent drawing is desired, then it is only necessary to render each routing edge separately, as long as the junctions at routing nodes are bundled and oriented through the correct direction. Splitting nodes also adds unnecessary complexity to the ARG. Allowing bundles to be relaxed [1, Fig. 18] is, however, a useful option that rendering each edge does provide.

2. False edges

Here we show in Figure 2 how using graph-theoretic shortest paths through the ARG as control points for splines can lead to false edges being introduced, as explained in the main text Section 1.2. Note that only one power edge should ever be traversed for any given adjacency, which is guaranteed by the solution in the main text, Section 1.3. It may seem as if our counter-example is contrived and should not ever appear due to the redundant nested structure of power groups, but a similar pattern arises from the optimal decomposition of a clique, shown here in Figure 5.
3. Artifacts

The construction of an auxiliary routing graph (ARG) from a power graph decomposition (PGD) was recognized by the original authors to produce various artifacts [1, § 4.2]. In Figure 3 we illustrate the exact reasons behind these artifacts, specifically that densely connected components result in highly nested hierarchies of power groups.

- **Feet** are straight lines caused by two nodes being connected in the lowest level of a hierarchy. This can be seen once in Figure 3, top, between the two leftmost vertices, and four times in Figure 3, bottom, between the pairs of vertices in each corner.
- **Fractals** are a tree-shaped structure of bundles, which can mistakenly imply a tree-like topology in the graph (Figure 3, bottom). These are caused by the deep and regular hierarchy of power groups that often results from the PGD search algorithm [2].
- **Loops and S-curves** are caused by the large number of routing nodes a spline may have to traverse when there are many groups in the decomposition. This can be seen in Figure 3, bottom, where edges between nodes on the left and right sides have multiple routing nodes to traverse and therefore bend around before they reach their destination. The use of a force-directed method can also exacerbate this effect, as they often offer no guarantee on angular resolution and are known to become tangled when the graph is too densely connected.

4. Similar methods and classification

In the case that the PGD to ARG conversion is not constrained to producing a planar ARG, i.e. the drawing is not power-confuent as described in the main text Section 2, we suggest that the algorithm should be classified alongside other edge bundling methods. This is because it is misleading to claim to have a method that can produce a confluent drawing for any graph, since there are certain graphs for which it is known that this is impossible (Figure 4). As mentioned in the main text, relaxing the planarity restriction also makes the overall definition trivial.

Considering it as an edge bundling method would make clearer the link to existing methods that also generate an ARG. For example, hierarchical information has been used to produce a tree as the ARG, most notably by Holten [4] using metadata, and also Jia et al. [5] and recently Zheng et al. [6] using hierarchical clustering algorithms. Spatial proximity between edges has also been considered, with MINGLE by Gansner et al. [7] and Metro Bundling (MB) by Pupyrev et al. [8] being two techniques that do not route based on topology, but are instead formulated as an optimization problem on a predetermined layout. MINGLE tries to ‘reduce ink’ in its ARG through an agglomerative method, while MB is based on avoiding label boundaries whilst minimizing routing cost, a metric that includes multiple criteria including ink and path lengths.

We recommend classifying all such methods together within the wider context of ARG generation. This both highlights the contribution of Bach et al. [1] as unique in its construction of an ARG from a PGD, and will serve to better contextualize future work within the field. The concept of using the topology of the graph itself to generate an ARG is also an important contribution from Bach et al. [1], and future work may be directed towards finding more optimal solutions in this context. Note that with our solution to the various ambiguities in the main text Section 1.3, the algorithm is also special in that there is a guarantee that no edges will erroneously appear to exist.

We must also note that the definition of confluent drawing itself is somewhat confusing. The original definition of locally-monotone curves states that the curve may not contain any point with left and right tangents that form an angle \( \angle \leq 90^\circ \), which means that obtuse-angled kinks in a line are permitted. However, this seems to rule out the possibility of crossings completely, since any point at which lines cross is now interpreted as a junction, even if the tangents do not match. Later definitions [9] drop this locally-monotone requirement, instead opting for defining lines as smooth, now requiring tangents at junctions to match. This seems to be the more intuitive and widely-used definition.

References

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Figure 4. This is a simple example of a graph that does not admit a confluent drawing, known as the Petersen graph. The reason it is non-confluent is because the graph $K_{2,2}$ (shown in Figure 1) is the smallest graph for which tracks can overlap to remove a crossing; since the Petersen graph both has no such subgraph, and is non-planar, it is also non-confluent [10].

Figure 5. A full example of Figure 2, without any redundant power groups. Top: the power graph decomposition (PGD) of the graph. Bottom left: a standard node-link diagram layout. Bottom middle: the Auxiliary Routing Graph (ARG) that results from the PGD. Bottom right: the resulting confluent drawing, where a short-circuit from the vertical power edge connected to $b$ causes the edge $\{a, b\}$ to be routed the wrong way, making the edge $\{c, b\}$ erroneously appear to exist due to an overlap with the edge $\{c, d\}$. The edge $\{a, e\}$ also has two equal-length shortest paths through the ARG, either through the line shown or downwards through $b$, albeit this specific case can easily be prevented by only allowing routing nodes to be used as intermediate spline control points, as the original authors appear to have done in [1, Fig. 2(c)] for the edge $\{u, w\}$.

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