$\alpha N\alpha c$ - Continuous And Contra$\alpha N\alpha c$-Continuous Mappings In Topological

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Abstract: The aim of this paper is to introduce a class of $\alpha N\alpha c$ - continuous mappings by using the concept of $\alpha N\alpha c$ - open sets in topological spaces like: $\alpha N\alpha c$, $\alpha N\alpha c$* and $\alpha N\alpha c$** - continuous mapping with some of their properties. Moreover, we studied a new kind of $\alpha N\alpha c$- continuous mappings which we called contra $\alpha N\alpha c$-continuous mappings with some of their applications.

Keywords: $N\alpha$–open; $\alpha N\alpha c$-open; $\alpha N\alpha c$-continuous mapping.

I.Introduction
The concept of mappings is very important. A new kind of mappings is given in this study which is named $\alpha N\alpha c$ – continuous and contra $\alpha N\alpha c$– continuous mappings. An $\alpha$ – open set was first studied in 1965 by O. Njasted see [1]. $\alpha$-open set was first studied in 2015 by N.A. Dawood and N.M.Ali see [2]. The idea of operation of mapping $\alpha$ on a space $X$ was introduced by Ogata for more details see [3]. The concepts of $N\alpha$-operation mapping and $\alpha N\alpha c$ - open sets were first studied in 2015 by N.A. Ali, see [4]. The idea of contra continuity in topological spaces was presented by Dontcher see [5]. Here, in this paper attempt has been made to employ the concept of $\alpha N\alpha c$-open sets to study some types of $\alpha N\alpha c$- continuous mappings like $\alpha N\alpha c$*, $\alpha N\alpha c$**-continuous mappings, and contra- $\alpha N\alpha c$ -continuous mappings. We shall mean in this study the space $X$ is a topological space or a space. On the other hand the interior and closure of a subset $A$ of $X$ we shall note by int $(A)$ , $\text{cl}(A)$ respectively.

II.Preliminary Concepts And Results
In this section we shall recall some definitions, propositions, and properties of $N\alpha$-open; $\alpha N\alpha$ - open and $\alpha N\alpha c$-open sets which we need to study the concept of $\alpha N\alpha c$ - continuity.

Definition (2.1): [1]
A set $A$ is an $\alpha$-open if $A \subseteq \text{int} (\text{cl} (\text{int} (A)))$, it's family is denoted by $\alpha O (X)$, it's complement is called $\alpha$ - closed and denoted by $\alpha C (X)$.
Definition (2.2): [2]
A set $A$ is $N_{α}$-open if $\exists$ nonempty $α$-open set $H$ s.t $\text{cl}(H) \subseteq A$, its complement is named $N_{α}$-closed set, the family of $N_{α}$-open ($N_{α}$-closed) is denoted by $N_{α}O(X)$, $(N_{α}C(X))$ respectively.

Definition (2.3): [2]
A set $A$ is $N_{α}$-clopen if $A$ is $N_{α}$-open and $N_{α}$-closed set.

Proposition (2.4): [2]
Every clopen set is $N_{α}$-open set.

The set $X$ is $N_{α}$-open in every topological space.

Definition (2.5): [4]
A mapping $ʎ: N_{α}O(X) \rightarrow P(X)$ is called $N_{α}$-operation on $N_{α}O(Χ)$ if $A \subseteq ʎ(A)$ for all nonempty $N_{α}$-open set $A$.

Remarks (2.6): [4]
(i) $ʎ$ in above definition is called an identity $N_{α}$-operation if $ʎ(A) = A$ for all $A \in N_{α}O(X)$
(ii) If $ʎ: N_{α}O(X) \rightarrow P(X)$ is any $N_{α}$-operation, then it is clear that $ʎ(X) = X, ʎ(Ø) = Ø$.
(iii) $ʎ$ is named $N_{α}$-regular operation if $\forall$ $N_{α}$-open sets $A, B$ containing $x \in Q \in N_{α}O(X)$ contains $x$ such that $ʎ(Q) \subseteq ʎ(A) \cap ʎ(B)$.

Definition (2.7): [4]
Suppose that $X$ is a topological space and $ʎ: N_{α}O(X) \rightarrow P(X)$ be $N_{α}$-operation defined on $N_{α}O(X)$, then $ʎ$ is a $N_{α}$-operation if $\forall x \in A \in N_{α}O(X)$ such that contains $x$ and $ʎ(Q) \subseteq A$, its complement is $N_{α}$-closed set, the family of $N_{α}$-open ($N_{α}$-closed) is denoted by $N_{α}O(X)$, $(N_{α}C(X))$ respectively.

Definition (2.8): [4]
A $N_{α}$-open set $A$ is $N_{α}$-open set if $\forall x \in A \in N_{α}O(X)$ such that $x \in F \subseteq A$ and its complement is called $N_{α}$-closed set, the family of all $N_{α}$-open ($N_{α}$-closed) sets is denoted by $N_{α}O(X)$, $(N_{α}C(X))$ respectively.

Remarks (2.9): [4]
(i) For any topological space $X$ with $N_{α}$-operation $ʎ$, the set $X$ and $Ø$ are $N_{α}$-open sets.
(ii) For any topological space $X$, we have $N_{α}O(X) \subseteq N_{α}O(X) \subseteq N_{α}O(X)$.

Theorem (2.10): [4]
(i) Let $\{A_{α}\}_{α \in I}$ be any collection of $N_{α}$-open sets, then $∪ A_{α}$ is $N_{α}$-open set $\forall \alpha \in I$. (ii) Suppose $A, B \in N_{α}O(X)$, then $A \cap B \in N_{α}O(X)$ such that $ʎ$ is $N_{α}$-regular operation.

Theorem (2.11): [2]
Suppose that $Y$ is a subspace of $X$ such that $A \subseteq Y \subseteq X$. Therefore:
(i) If $A \in N_{α}O(X)$ (NaO(Y)) respectively. Then $A \in N_{α}O(Y)$ (NaO(Y)) respectively.
(ii) If $A \in N_{α}O(Y)$ (NaO(Y)) respectively. Then $A \in N_{α}O(X)$ (NaO(Y)) respectively. where $Y$ is clopen set.

Theorem (2.12): [2]
Suppose that $X_{1}, X_{2}$ are topological spaces. Then $A_{1}, A_{2}$ are $N_{α}$-open sets in $X_{1}$, $X_{2}$ respectively if and only if $A_{1} \times A_{2}$ is $N_{α}$-open set in $X_{1} \times X_{2}$.
Theorem (2.13) [4]
Suppose that \(X_1, X_2\) are topological spaces. Then \(A_1, A_2\) are \(\aleph_{\alpha c}\) open sets in \(X_1, X_2\) respectively if and only if \(A_1 \times A_2\) is \(\aleph_{\alpha c}\) open set in \(X_1 \times X_2\).

Theorem (2.14) [4]
Suppose that \(Y\) is a subspace of \(X\) such that \(A \subseteq Y \subseteq X\). Then:

(i) If \(A \in \aleph_{\alpha c}O(X)\) (\(\aleph_{\alpha c}O(Y)\)) respectively, then \(A \in \aleph_{\alpha c}O(Y)\) (\(\aleph_{\alpha c}O(Y)\)) respectively.

(ii) If \(A \in \aleph_{\alpha c}O(X)\) (\(\aleph_{\alpha c}O(X)\)) respectively, then \(A \in \aleph_{\alpha c}O(Y)\) (\(\aleph_{\alpha c}O(Y)\)) respectively, where \(Y\) is open set in \(X\).

III. Some Kinds of \(\aleph_{\alpha c}\) – Continuity

In this section, the concept of \(\aleph_{\alpha c}\) open set will be used to define some kinds of \(\aleph_{\alpha c}\) - continuity like: \(\aleph_{\alpha c}\) – continuous, \(\aleph_{\alpha}^{*}\) – continuous and \(\aleph_{\alpha}^{**}\) – continuous mappings and the relations between these kinds of mappings with proofs or counter examples

Definition (3.1)
Suppose that \(X_1, X_2\) are topological spaces such that \(f : X_1 \to X_2\) is any mapping. Then \(f\) is named \(\aleph_{\alpha c}\) (\(\aleph_{\alpha c}^*\) (\(\aleph_{\alpha c}^{**}\) continuous) mapping if

\[
\forall A \in \aleph_{\alpha c}O(X) \quad \forall A \in \aleph_{\alpha c}O(Y) \quad \forall A \in \aleph_{\alpha c}O(X) \quad \forall A \in \aleph_{\alpha c}O(Y)
\]

then \(f^{-1}(A) \in \aleph_{\alpha c}O(Y)\) (\(\aleph_{\alpha c}O(Y)\)) respectively, \((\aleph_{\alpha c}O(X)\)) respectively.

Now, we shall discuss the relations between above kinds of mappings see the following Remarks:

Remarks (3.2)
(i) Every \(\aleph_{\alpha c}\) – continuous is \(\aleph_{\alpha c}^*\) – continuous mapping .

(ii) Every \(\aleph_{\alpha c}^*\) – continuous is \(\aleph_{\alpha c}^{**}\) – continuous mapping .

Proof: It follows from (Rem.(2.9(ii))). The converse of (Rem.(3.2))is not true in general see(3.3) :

Example(3.3)
Consider \(X = Y = \{a, b, c\}\), suppose \(f : X \to Y\) such that \(f(a) = f(c) = c, f(b) = b, \tau_x = \{X, \{a\}, \{b\}, \{a, b\}\}\)

\(\aleph_{\alpha c}O(X) = \{X, \{a\}, \{b\}, \emptyset\}\).

\(\aleph_{\alpha c}O(Y) = \{Y, \{a, c\}, \{b, c\}, \{c\}, \emptyset\}\).

\(\aleph_{\alpha c}O(Y) = \{Y, \{a, c\}, \{b, c\}, \{c\}, \emptyset\}\).

\(\aleph_{\alpha c}O(Y) = \{Y, \{a, c\}, \{b, c\}, \{c\}, \emptyset\}\).

\(\aleph_{\alpha c}O(Y) = \{Y, \{a, c\}, \{b, c\}, \{c\}, \emptyset\}\).

\(\aleph_{\alpha c}O(Y) = \{Y, \{a, c\}, \{b, c\}, \{c\}, \emptyset\}\).

Thus \(f\) is \(\aleph_{\alpha c}^{**}\) – continuous. But it is neither \(\aleph_{\alpha c}\) - continuous nor \(\aleph_{\alpha c}^*\) - continuous since \(f^{-1}(\{a, c\}) \notin \aleph_{\alpha c}O(X)\), also \(f^{-1}(\{c\}) = \{a, c\} \notin \aleph_{\alpha c}O(X)\).

We have the following diagram

\(\aleph_{\alpha c}\) – continuous \(\to \aleph_{\alpha c}^*\) – continuous \(\to \aleph_{\alpha c}^{**}\) – continuous
Theorem (3.4)
Suppose that $f: X \rightarrow Y$ is a mapping such that $X$ has $\mathcal{N}_\alpha$-regular operation. Therefore, the restriction mapping $f|_F$ of $\mathcal{N}_\alpha*(\mathcal{N}_\alpha**$-continuous) mapping $f$ to any subspace $F \subseteq X$ where $F \in \mathcal{N}_\alpha O(\mathcal{N}_\alpha O(\mathcal{N}_\alpha X))$ respectively is also $\mathcal{N}_\alpha*(\mathcal{N}_\alpha**$-continuous) respectively.

Proof: We shall prove only the case for $(\mathcal{N}_\alpha)$ and the other cases by the same way. Assume that $f|_F: F \rightarrow Y$ restriction mapping, assume that $B \in \mathcal{N}_\alpha O(Y)$, thus $f^{-1}(B) \in \mathcal{N}_\alpha O(X)$ also $F \in \mathcal{N}_\alpha O(X)$ since $\mathcal{X}$ is $\mathcal{N}_\alpha$-regular operation (by hypothesis) then by (Th. (2.10(iii))) $f^{-1}(B) \cap F \in \mathcal{N}_\alpha O(F)$ see (Th. (2.14)). Therefore $(f|_F(B))^{-1} \in \mathcal{N}_\alpha O(F)$, thus the proof is complete.

Theorem (3.5)
Suppose that $f: X \rightarrow Y$ is a mapping. Assume $X = H \cup W$ whenever $H, W$ are disjoint clopen subsets of $X$. Therefore $f$ is $\mathcal{N}_\alpha*(\mathcal{N}_\alpha**$-continuous) mapping whenever the restriction mappings $f|_H, f|_W$ are $\mathcal{N}_\alpha*(\mathcal{N}_\alpha**$-continuous mappings) respectively.

Proof: We shall prove only the case of $(\mathcal{N}_\alpha)$, the other cases by the same way. Assume that $M \in \mathcal{N}_\alpha O(Y)$, we have $f^{-1}(M) = (f|_H)^{-1}(M) \cup (f|_W)^{-1}(M)$, where $(f|_H)^{-1}, (f|_W)^{-1} \in \mathcal{N}_\alpha O(H), \mathcal{N}_\alpha O(W)$ respectively. Thus $(f|_H)^{-1}, (f|_W)^{-1} \in \mathcal{N}_\alpha O(X)$ see (Th. (2.14)) → $f^{-1}(M) \in \mathcal{N}_\alpha O(X)$ see Th. (2.10(iii)).

Theorem (3.6)
Suppose that $f: X \rightarrow Y$ is a mapping, $f_1: f_1^{-1}(H) \rightarrow H$ defined by $f_1(x) = f(x)$, where $H$ is clopen set in $Y$. Therefore $f_1$ is $\mathcal{N}_\alpha*(\mathcal{N}_\alpha**$-continuous) mapping whenever $f$ is $\mathcal{N}_\alpha*(\mathcal{N}_\alpha**$-continuous) mapping respectively.

Proof: We shall prove only the case of $(\mathcal{N}_\alpha)$, the other cases by the same way. Assume that $H \in \mathcal{N}_\alpha O(H)$ since $H \in \mathcal{N}_\alpha O(Y)$ see (Th. (2.11)) hence $f_1^{-1}(H) \in \mathcal{N}_\alpha O(X)$ this implies $f_1^{-1} \in \mathcal{N}_\alpha O(f_1^{-1}(H))$ see (Th. (2.14)).

Theorem (3.7)
Assume that $f: X_1 \rightarrow X_2$ is any mapping, $X_2 \subseteq X_3$. Therefore $f: X_1 \rightarrow X_3$ is $\mathcal{N}_\alpha*(\mathcal{N}_\alpha**$-continuous) mapping whenever $f: X_1 \rightarrow X_2$ is $\mathcal{N}_\alpha*(\mathcal{N}_\alpha**$-continuous) mapping respectively.

Proof: We shall prove only the case for $\mathcal{N}_\alpha$ and the other cases by the same way. Assume that $M \in \mathcal{N}_\alpha O(X_1)$ then $M \in \mathcal{N}_\alpha O(X_3)$ see (Th. (2.11)) hence $f_1^{-1}(M) \in \mathcal{N}_\alpha O(X)$.

Theorem (3.8)
Suppose that $g: X \rightarrow X \times Y$ is a graph mapping of $f: X \rightarrow Y$ such that $(x, f(x)) \in X$. Therefore $f$ is $\mathcal{N}_\alpha*(\mathcal{N}_\alpha**$-continuous) mapping whenever $g$ is $\mathcal{N}_\alpha*(\mathcal{N}_\alpha**$-continuous) mapping respectively.

Proof: We shall prove only the first case. The other cases by the same way. Assume that $X \in \mathcal{N}_\alpha O(X)$ in every topological space see (propo. (2.4)), this implies $X \in \mathcal{N}_\alpha O(X)$ (see (Th. (2.12))) therefore $g^{-1}(X \times H) \in \mathcal{N}_\alpha O(X)$ but $g^{-1}(X \times H) = f^{-1}(H)$ thus $f^{-1}(H) \in \mathcal{N}_\alpha O(X)$. Therefore $f$ is $\mathcal{N}_\alpha$-continuous mapping.

Proposition (3.9)
Assume that $f: X \rightarrow Y, g: Y \rightarrow Z$ are mappings. Therefore:

(i) The composition of tow $\mathcal{N}_\alpha*(\mathcal{N}_\alpha**$-continuous mappings) is also $\mathcal{N}_\alpha*(\mathcal{N}_\alpha**$-continuous mapping.
(ii) If \( f \) is \( \mathcal{N}\text{-}\text{nuc}^*\text{-continuous} \) and \( g \) is \( \mathcal{N}\text{-}\text{nuc}^*\text{-continuous} \), hence their composition is \( \mathcal{N}\text{-}\text{nuc}^*\text{-continuous} \).

Proof: By using Definition (3.1).

**iv. Contra \( \mathcal{N}\text{nc} \text{- Continuity}**

In this section the concept of \( \mathcal{N}\text{nc} \)- open set will be used to define a new class of \( \mathcal{N}\text{nc} \)- continuity which is called contra \( \mathcal{N}\text{nc} \)- continuous mapping. Some theorems will be proved.

**Definition (4.1)**
Assume that \( f : X \rightarrow Y \) is a mapping, then \( f \) is named contra \( \mathcal{N}\text{nc} \)- continuous if for all \( H \in \mathcal{N}_A(Y) \), then \( f^{-1}(H) \in \mathcal{N}_A(X) \).

Now we shall give equivalent theorems of Definition (4.1)

**Theorem (4.2)**
Assume that \( f : X \rightarrow Y \) is a mapping. Therefore \( f \) is contra \( \mathcal{N}\text{nc} \)- continuous if and only if for all \( F \in \mathcal{N}_A(Y) \) then \( f^{-1}(F) \in \mathcal{N}_A(X) \).

Proof: By using Definition (4.1).

**Theorem (4.3)**
Assume that \( f : X \rightarrow Y \) is a mapping. Therefore \( f \) is contra \( \mathcal{N}\text{nc} \)- continuous if and only if for all \( x \in X \) and for all \( F \in \mathcal{N}_A(Y) \) containing \( f(x) \) there exists \( G \in \mathcal{N}_A(X) \) such that \( x \in G \) and \( f(G) \subseteq F \).

Proof: Assume that \( f \) is contra \( \mathcal{N}\text{nc} \)- continuous, suppose \( x \in X \), \( F \in \mathcal{N}_A(Y) \), then \( f^{-1}(F) \in \mathcal{N}_A(X) \) where \( f^{-1}(F) \cap D \in \mathcal{N}_A(X) \) containing \( x \). Put \( G = f^{-1}(F) \), then \( x \in G \), \( f(G) \subseteq F \). Conversely obvious.

**Remark (4.3)**
The concepts of \( \mathcal{N}\text{nc} \)- continuous and contra \( \mathcal{N}\text{nc} \)- continuous are independent see the following Examples:-

**Example (1)**
Suppose \( X = \{a, b, c\} \) with \( \tau = \{X, \{a\}, \{b\}, \{c\}, \emptyset\} \), \( \langle \emptyset \rangle \) is identity \( \mathcal{N}\text{nc} \)- operation. Assume \( f : X \rightarrow X \) such that \( f(a) = c \), \( f(b) = a \), \( f(c) = b \). \( \mathcal{N}_A(X) = \langle \emptyset \rangle, \mathcal{N}_A\text{nc}(X) = \langle X, \{a\}, \{b\}, \{c\}, \emptyset \rangle \). It is clear that \( f \) is \( \mathcal{N}\text{nc} \)- continuous which is not contra \( \mathcal{N}\text{nc} \)- continuous.

**Example (2)**
Assume \( X = \{a, b, c, d\} \) with \( \tau = \{X, \{a\}, \{b\}, \{c\}, \emptyset\} \), \( \langle \emptyset \rangle \) is \( \mathcal{N}\text{nc} \)- operation defined by \( \langle \emptyset \rangle = \{c\} \text{ if } a \in X \) otherwise \( \emptyset \).

Conceder \( f : X \rightarrow X \) by \( f(a) = f(c) = f(d) = a, f(b) = c \). \( \mathcal{N}_A(X) = \langle X, \{a, c, d\}, f \rangle \), \( \mathcal{N}_A\text{nc}(X) = \langle X, \{a, c, d\}, f \rangle \).

It is easy to check that \( f \) is contra \( \mathcal{N}\text{nc} \)-continuous which is not \( \mathcal{N}\text{nc} \)- continuous since \( f^{-1}(\{a, b, c\}) = \{b\} \notin \mathcal{N}_A\text{nc}(X) \).

**Theorem (4.4)**
The restriction \( f_D \) of contra \( \mathcal{N}\text{nc} \)- continuous mapping \( f : X \rightarrow Y \) to any subspace \( D \subseteq X \) where \( D \in \mathcal{N}_A(X) \) is also contra \( \mathcal{N}\text{nc} \)- continuous.

Proof: Assume that \( M \in \mathcal{N}_A(Y) \), thus \( f^{-1}(M) \in \mathcal{N}_A\text{nc}(X) \), also \( D \in \mathcal{N}_A\text{nc}(X) \), hence \( f^{-1}(M) \cap D \in \mathcal{N}_A\text{nc}(X) \), so it is in \( D \) (see Th. (2.14)) but \( f(D) = f(D) \cap D \). Thus the proof is complete.
Theorem (4.5)
Assume that \( f: X \to Y \) is contra-\( \mathcal{N}_\alpha \)-continuous, then \( f_\ast f'(S) \) is also, where \( S \) is clopen subset of \( Y \).
Proof: Suppose \( W \in \mathcal{N}_\alpha O(Y) \) this implies \( W \in \mathcal{N}_\alpha C(X) \) where \( f'_W(W) \subseteq f'(S) \subseteq X \) hence \( f'(W) \in \mathcal{N}_\alpha C(f'(S)) \).

Theorem (4.6)
Assume that \( f: X \to Y \) is a mapping where \( X = H \cup F \) such that \( H, F \) are disjoint \( \mathcal{N}_\alpha \)-closed sets of \( X \). Therefore the restriction mappings \( f|_H \), \( f|_F \) of contra-\( \mathcal{N}_\alpha \)-continuous mapping \( f \) is also contra-\( \mathcal{N}_\alpha \)-continuous mappings.
Proof: Suppose \( G \in \mathcal{N}_\alpha O(Y) \), thus \( f^{-1}(G) \in \mathcal{N}_\alpha C(X) \) also \( H \in \mathcal{N}_\alpha C(X) \), hence \( f^{-1}(G) \cap H \) \( \subseteq f^{-1}(G) \cap \mathcal{N}_\alpha C(H) \) but \( (f^{-1}(G))^{-1} = f^{-1}(G) \cap H \). Thus \( f|_H \) is contra-\( \mathcal{N}_\alpha \)-continuous mapping.
The proof of \( f|_F \) by the same way.

Remark (4.7)
The composition of two contra-\( \mathcal{N}_\alpha \)-continuous mappings is not true in general see the next:
Example (3)
Assume that \( X = \{a, b, c\} \), \( \tau = \{X, \{a\}, \{b\}, \{c\}, \emptyset\} \) with an identity \( \mathcal{N}_\alpha \)-operation \( \lambda \) where \( \mathcal{N}_\alpha O(X) = \{X, \{a\}, \{b\}, \{c\}, \emptyset\} \) and \( \mathcal{N}_\alpha C(X) = \{X, \{a\}, \{b\}, \emptyset\} \).
Assume that \( Y = \{a, b, c, d\} = \mathcal{Z} \), \( \tau = \{Y, \{b\}, \{d\}, \{b, d\}, \emptyset\} \).
\( \mathcal{N}_\alpha O(Y) = \{Y, \{a\}, \{c\}, \{d\}, \emptyset\} \) and \( \mathcal{N}_\alpha C(Y) = \{Y, \{b\}, \{d\}, \emptyset\} \).
Suppose \( \mathcal{N}_\alpha \)-operation \( \lambda \) by \( \lambda(A) = \begin{cases} \{c, l(A)\} \quad & \text{if } d \not\in A \\ X \quad & \text{if } d \in A \end{cases} \)
\( \mathcal{N}_\alpha O(Y) = \mathcal{N}_\alpha C(Y) = \{Y, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\} \).
Suppose \( f:X \to Y \) by \( f(a) = a \), \( f(b) = f(c) = b \).
Assume that \( g:Y \to Z \) by \( g(a) = g(c) = g(d) = a \), \( g(b) = d \), \( \tau_Z = \{Z, \{a\}, \{b\}, \{a, b\}, \emptyset\} \).
\( \mathcal{N}_\alpha O(Z) = \{Z, \{b, c\}, \{a, c\}, \emptyset\} \) and \( \mathcal{N}_\alpha C(Z) = \{Z, \{a\}, \{b\}, \emptyset\} \), suppose \( \lambda \) by \( \lambda(A) = \begin{cases} \{c, l(A)\} \quad & \text{if } d \not\in A \\ Z \quad & \text{if } d \in A \end{cases} \)
\( \mathcal{N}_\alpha O(Z) = \mathcal{N}_\alpha C(Z) = \{Z, \{a, b\}, \{b, c\}, \{a, c\}, \emptyset\} \).
It is easy to check that \( f \) and \( g \) are contra-\( \mathcal{N}_\alpha \)-continuous, but \( g \circ f \) is not contra-\( \mathcal{N}_\alpha \)-continuous, since \( \{a\} \in \mathcal{N}_\alpha C(Z) \) but \( (g \circ f)(1) = a \notin \mathcal{N}_\alpha C(Z) \).
Now we shall give some applications of contra-\( \mathcal{N}_\alpha \)-continuous mapping. First we shall give some definitions which we need in our work.

Definitions (4.8)
A space \( X \) is named:
(i) \( \mathcal{N}_\alpha \)-ultra (contra-\( \mathcal{N}_\alpha \)-ultra) - T2 space if for all \( x \neq y \) of \( X \) two disjoint \( \mathcal{N}_\alpha \)-open (\( \mathcal{N}_\alpha \)-closed) sets \( C, D \) respectively such that \( x \in C \), \( y \in D \).
(ii) \( \mathcal{N}_\alpha \)-ultra (contra-\( \mathcal{N}_\alpha \)-normal) space if for all pair nonempty disjoint \( \mathcal{N}_\alpha \)-clopen (\( \mathcal{N}_\alpha \)-closed) sets \( C, D \) can be separated by disjoint \( \mathcal{N}_\alpha \)-closed (\( \mathcal{N}_\alpha \)-open) sets respectively.
(iii) \( \mathcal{N}_\alpha \)-strongly closed (contra-\( \mathcal{N}_\alpha \)-compact) space if for all \( \mathcal{N}_\alpha \)-closed (\( \mathcal{N}_\alpha \)-open) cover of \( X \) respectively has finite subcover.

Theorem (4.9)
The inverse image of \( \mathcal{N}_\alpha \)-T2 space under injective contra-\( \mathcal{N}_\alpha \)-continuous mapping \( f \) is \( \mathcal{N}_\alpha \)-ultra T2 space.
Proof: Suppose \( x \neq y \) in \( X \), because \( f: X \to Y \) is injective this implies \( f(x) \neq f(y) \) in \( Y \), this implies \( f^{-1}(S) \) is also contra-\( \mathcal{N}_\alpha \)-continuous then \( f' \)
\(1(G), f^\prime(W) \in \mathcal{N}_{\alpha}C(X)\) s.t containing \(x, y\) and \(f^\prime(G) \cap f^\prime(W) = \varnothing\). Thus \(X\) is \(N_{\alpha_c}\) - ultra T2 space.

**Theorem (4.10)**
The inverse image of \(N_{\alpha}\) - ultra T2 space under injective contra \(N_{\alpha_c}\) - continuous mapping \(f\) is \(N_{\alpha_c}\) - normal space.

**Proof**: Assume \(x \neq y\) in \(X\), this implies \(f(x) \neq f(y)\) in \(Y\), because \(Y\) is \(N_{\alpha}\) - ultra T2 space, then \(\exists\) disjoint sets \(A, B \in \mathcal{N}_{\alpha}C(Y)\) sets s.t \(f(x) \in A, f(y) \in B\) because \(f\) is contra \(N_{\alpha_c}\) - continuous, thus by (Th. (4.3)) \(\exists C, D \in \mathcal{N}_{\alpha}O(X)\) s.t \(x \in C\), \(y \in D\) and \(f(C) \subseteq A, f(D) \subseteq B\), this implies \(f(C) \cap f(D) = \varnothing\). Thus \(f(C) \cap f(D) = \varnothing\) implies \(X\) is \(N_{\alpha_c}\) - T2 space.

We shall give the following definition which we need in the following theorem

**Definition (4.11)**
Assume that \(f: X \rightarrow Y\) is a mapping then \(f\) is named \(N^*\) - closed mapping if for all \(G \in \mathcal{N}_{\alpha}C(X)\), then \(f(G) \in \mathcal{N}_{\alpha}C(Y)\).

**Theorem (4.12)**
The inverse image of \(N_{\alpha}\) - ultra normal space under injective \(N^*\) - closed contra \(\mathcal{N}_{\alpha_c}\) - continuous mapping \(f\) is \(N_{\alpha_c}\) - normal space.

**Proof**: Suppose disjoint sets \(F_1, F_2 \in \mathcal{N}_{\alpha}C(X)\), because \(f\) is \(N^*\) - closed mapping, this implies \(f(F_1), f(F_2) \in \mathcal{N}_{\alpha}C(Y)\), because \(Y\) is \(N_{\alpha}\) - ultra normal space then we have two disjoint \(N_{\alpha}\) - clopen sets \(B_1, B_2\) of \(Y\) s.t \(f(F_1) \subseteq B_1, f(F_2) \subseteq B_2\) hence \(F_1 \subseteq f^{-1}(B_1)\) and \(F_2 \subseteq f^{-1}(B_2)\) because \(f\) is injective contra \(\mathcal{N}_{\alpha_c}\) - continuous mapping, this implies \(f^{-1}(B_1), f^{-1}(B_2)\) are disjoint \(\mathcal{N}_{\alpha_c}\) - open sets. Thus the proof is complete.

**Theorem (4.13)**
Let \(f: X \rightarrow Y\) be a surjective contra \(\mathcal{N}_{\alpha_c}\) - continuous mapping. Then if \(X\) is \(\mathcal{N}_{\alpha_c}\) - compact space, then \(Y\) is \(N_{\alpha}\) - strongly closed space.

**Proof**: Let \(\{V_i : i \in I\}\) be \(\mathcal{N}_{\alpha_c}\) - closed cover of \(Y\) since \(f\) is contra \(\mathcal{N}_{\alpha_c}\) - continuous mapping, thus \(\{f^{-1}(V_i) : i \in I\}\) is \(\mathcal{N}_{\alpha_c}\) - open cover of \(X\), but \(X\) is \(\mathcal{N}_{\alpha_c}\) - compact space, thus \(X\) has a finite subcover.

\[X = \bigcup \{f^{-1}(V_i) : i \in I_0\}\]

Thus \(f(X) = \bigcup \{f^{-1}(V_i) : i \in I_0\}\) since \(f\) is surjective. Thus \(Y = \bigcup \{V_i : i \in I_0\}\) \(Y\) is \(\mathcal{N}_{\alpha}\) - strongly closed space.

**V. Future Work**

(i) We can use the concept of \(\mathcal{N}_{\alpha_c}\) - open sets to study new kind of \(\mathcal{N}_{\alpha_c}\) - compact spaces.

(ii) We can use the concept of \(\mathcal{N}_{\alpha_c}\) - open sets to study some types of separation axioms.

**References**

[1] O. Njastad , On some classes of Nearly open sets , Pacific J. Math. , 15 (3) , 1965 , pp. 961 – 970 .

[2] N. A. Dawood , N. M. A. Al-Tabatabai , No- open sets and No- Regularity In Topological Spaces International J. of Advanced Scientific and Technical Research 5,(3), (2015) , pp. 87 – 96 .

[3] H. Ogata,, Operation on Topological Spaces and Associated Topology,,Math. Japonica 3, 6, (1999), pp. 175 – 184.

[4] N. M. Al- Tabatabai "\(\mathcal{N}_{\alpha_c}\) - Open sets And \(\mathcal{N}_{\alpha_c}\) – Separation Axioms In Topological Spaces” International J. of Advanced Scientific and Technical Research 5 (6) September (2015) pp. 83 – 92.

[5] J. Dontcher , Contra continuous and Strongly S - closed Spaces . International J. Math. Math.Sci. 19 (1996) pp. 303 – 310.