Freeze-out Conditions from Lattice QCD

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Chemical freeze-out condition in HIC

Initial state:

- QGP, hydro. expansion

Pre-equilibrium:

- Temperature: \( T_f, \mu_B^f, \mu_Q^f, \mu_S^f \)

Hadronization:

- Chemical freeze-out
  - no inelastic scattering, hadron abundances unchanged (freezes-out), chemical equilibration

Hadron resonance gas (HRG) model:

- Statistical / thermal / hadron resonance gas

By comparing hadron yields from HIC expt.

Thermalized, non-interacting hadrons & resonances
Chemical freeze-out condition in HIC

- **Initial state**
  - Chemical freeze-out conditions
  - Temperature $T_c$, chemical potential $\mu_c$

- **Pre-equilibrium**
  - Statistical / thermal / hadron resonance gas (HRG) model
  - Inelastic scattering
  - Hadron abundances unchanged (freezes-out)
  - Chemical equilibration

- **Hadronization**
  - Statistical / thermal / hadron resonance gas (HRG) model
  - Thermalized, non-interacting hadrons & resonances

- **Freeze-out**
  - No inelastic scattering
  - Hadron abundances unchanged (freezes-out)
  - Chemical equilibration

- **Time**

- **Model dependent connection, not from first principle QCD**

- **$\sqrt{s}$**

- By comparing hadron yields from HIC expt.
fluctuations of conserved charges

   net-proton, net-charge, ...

   characterize the
   chemical freeze-out condition

QCD critical point will be situated somewhere along this critical region
Freeze-out conditions from (L)QCD?

✓ how far is the critical region from the freeze-out line?

☆ if they are far enough, then locating QCD critical point in HIC experiments will be difficult

✓ to what extent the experimental observables are governed by non-critical QCD thermodynamics?

1) fix the freeze-out conditions from QCD
2) QCD calculations of observables at these freeze-out parameters
3) comparison with experiments

✓ do thermal models work perfectly for the freeze-out conditions at LHC? (ALICE, 2012)

freeze-out parameters \((T^f, \mu_B^f, \mu_Q^f, \mu_S^f)\) from LQCD
Strangeness and electric charge chemical potentials

1) strangeness neutrality: \[ \langle n_S \rangle = 0 \]

2) isospin asymmetry: \[ \langle n_Q \rangle = r \langle n_B \rangle \]

Expand these two relations in powers of \( \mu_B, \mu_Q, \mu_S \) around \( \mu_B = \mu_Q = \mu_S = 0 \):

\[ \mu_Q(T, \mu_B) = q_1(T)\mu_B + q_3(T)\mu_B^3 + \cdots \]
\[ \mu_S(T, \mu_B) = s_1(T)\mu_B + s_3(T)\mu_B^3 + \cdots \]

Assume: homogeneous system

\[ \langle n_S \rangle = 0 \]
\[ \langle n_Q \rangle = r \langle n_B \rangle \]

\[ \frac{\langle N_Q \rangle}{\langle N_B \rangle} = \frac{\langle N_p \rangle}{\langle N_p \rangle + \langle N_n \rangle} = r \]

Au-Au & Pb-Pb: \( r \approx 0.4 \)

\[ N_x = N_x - \bar{N}_x, \quad n_x = N_x / V \]
Strangeness and electric charge chemical potentials

$$\mu_Q(T, \mu_B) = q_1(T)\mu_B + q_3(T)\mu_B^3$$

$$\mu_S(T, \mu_B) = s_1(T)\mu_B + s_3(T)\mu_B^3$$

LO: continuum extrapolated

NLO: \(N_t = 6, 8\), small cut-off dependence, *continuum estimate*

NLO corrections < 10%

\(\mu_B / T \lesssim 1.3\)
Strangeness and electric charge chemical potentials

\[ \mu_Q(T, \mu_B) \]

\[ \mu_S(T, \mu_B) \]

5-15% deviations from HRG

BNL-BI, arXiv:1208.1220
Temperature and baryon chemical potential

All observables can now be obtained as function of two independent parameters $T, \mu_B$.

Hadron yields are inaccessible in LQCD. Comparison of 2 expt. measured ratios of cumulants of conserved charge fluctuations with LQCD calculations fixes 2 freeze-out parameters $T^f, \mu_B^f$.

Proton fluctuations (expt.) ?=? Baryon fluctuations (LQCD)

Asakawa-Kitazawa; Bzdak-Skokov

Safe to work with net electric charge fluctuations measured both in expt. and LQCD

Ratio of cumulants: cancels the unknown volume of the fireball
Temperature and baryon chemical potential

\[
\frac{M_Q(\sqrt{s})}{\sigma_Q(\sqrt{s})} = \frac{\langle N_Q \rangle}{\langle (\delta N_Q)^2 \rangle} = \frac{\chi^Q_1(T,\mu_B)}{\chi^Q_2(T,\mu_B)} = R^{Q,1}_{12}(T)\mu_B + R^{Q,3}_{12}(T)\mu_B^3 + \cdots = R^{Q}_{12}(T,\mu_B)
\]

\[
\frac{S_Q(\sqrt{s})\sigma^3_Q(\sqrt{s})}{M_Q(\sqrt{s})} = \frac{\langle (\delta N_Q)^3 \rangle}{\langle N_Q \rangle} = \frac{\chi^Q_3(T,\mu_B)}{\chi^Q_1(T,\mu_B)} = R^{Q,0}_{31}(T) + R^{Q,2}_{31}(T)\mu_B^2 + \cdots = R^{Q}_{31}(T,\mu_B)
\]

HIC

- mean: \(M_Q\)
- variance: \(\sigma^2_Q\)
- skewness: \(S_Q\)

\[\delta N_Q = N_Q - \langle N_Q \rangle\]

LQCD

generalized charge susceptibilities:

\[
\chi^Q_n(T,\tilde{\mu}) = \frac{1}{VT^3} \frac{\partial^n \ln \mathcal{Z}(T,\tilde{\mu})}{\partial (\mu_Q/T)^n}
\]

\(\mu_f^B\) fixes \(\mu_B\)

LO linear in \(\mu_B\)

\(T_f^\text{LO}\) fixes \(\mu_B\)

LO independent of \(\mu_B\)
Temperature and baryon chemical potential

\[ R_{12}^Q = \frac{M_Q}{\sigma_Q^2} \]

\[ R_{12}^Q(T, \mu_B) = R_{12}^{Q,1}(T)\mu_B + R_{12}^{Q,3}(T)\mu_B^3 \]

LO: continuum extrapolated

NLO: \( N_\tau = 6, 8 \)

NLO corrections < 10%

\[ \frac{\mu_B}{T} \lesssim 1.3 \]
Temperature and baryon chemical potential

\[ R_{31}^Q = S_Q \sigma_Q^3 / M_Q \]

LO: \( N_\tau = 6, 8 \)

NLO corrections: \( \leq 10\% , \mu_B / T \leq 1.3 \)

\[ R_{31}^Q(T, \mu_B) = R_{31}^{Q,0}(T) + R_{31}^{Q,2}(T) \mu_B^2 \]

large deviation from HRG for \( T > 155 \text{ MeV} \)
Temperature and baryon chemical potential

\[ R_{12}^Q = M_Q / \sigma_Q^2 \]

\[ R_{31}^Q = S_Q \sigma_Q^3 / M_Q \]

\[ \mu_B / T \approx 2 \sim 155 \]
\[ \sim 1.5 \sim 160 \]
\[ \approx 1 \approx 170 \]

\[ M_Q / \sigma_Q^2 \]
\[ \mu_B / T \]

\[ 0.01 - 0.02 \quad 0.1 - 0.2 \]
\[ 0.03 - 0.04 \quad 0.3 - 0.4 \]
\[ 0.05 - 0.08 \quad 0.5 - 0.7 \]

for: \( T_f \sim 160 \text{ MeV} \)

BNL-BI, arXiv:1208.1220
Thermodynamic consistency

all other cumulant ratios should be reproduced w/o fixing any further parameter, provided the system is in thermal & chemical equilibrium

\[
\frac{R^{Q}_{12}}{R^{B}_{12}} = \frac{M_{Q}/\sigma_{Q}^{2}}{M_{B}/\sigma_{B}^{2}}
\]
Systematic: estimates using HRG

series truncation: ~ 5%

continuum limit: ~ 5%

$m_u \neq m_d$: ~ 5%
Summary

chemical freeze-out conditions can be determined from first principle (lattice) QCD calculations by comparing with cumulants of net charge fluctuations currently being measured by STAR, PHENIX

controlled NLO Taylor expansion up to

\[ \mu_B \sim 200 \text{ MeV}, \quad \sqrt{s} \sim 19.6 \text{ GeV} \]

general agreement with thermal models within 15%