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To cite this article: P Huovinen 2017 J. Phys.: Conf. Ser. 798 012063

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P Huovinen
Institute for Theoretical Physics, University of Wrocław, 50-204 Wrocław, Poland
E-mail: pasi.huovinen@ift.uni.wroc.pl

Abstract. In ultrarelativistic collisions of heavy-ions at the RHIC and LHC colliders we have seen behaviour which can be interpreted as a formation of locally thermalized system expanding as a fluid. I briefly review the use of hydrodynamics to model the expansion of such a fluid, and what such modelling has taught us about the dissipative properties of QCD matter.

1. Fluid dynamics
Ultrarelativistic heavy-ion collisions aim to create strongly interacting matter—matter in a sense that the thermodynamical concepts like temperature and pressure apply. Therefore it is reasonable to use these concepts to describe the evolution and expansion of the collision system, i.e. to use fluid dynamics. In a baryon free environment, as expected at midrapidity of an ultrarelativistic collision, the equations of motion of fluid dynamics are the conservation laws for energy and momentum:

$$\partial_\mu T^{\mu\nu} = 0,$$

where

$$T^{\mu\nu} = (\epsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu},$$

and $\epsilon$ is energy density in the rest frame of the fluid, $P$ equilibrium pressure, $\Pi$ bulk pressure, $u^\mu$ is the fluid 4-velocity, $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ the metric tensor, and $\pi^{\mu\nu}$ the shear-stress tensor. These four equations contain eleven unknowns. To close the set of equations we need an equation of state (EoS) connecting equilibrium pressure to energy density, $P = P(\epsilon)$, and constitutive equations for bulk pressure and shear stress. A relativistic generalisation of Navier-Stokes equations, where the dissipative quantities are directly proportional to the gradients of flow velocity, leads to non-causal behaviour. Therefore heavy-ion collisions are modelled using so-called Israel-Stewart, a.k.a. transient, fluid dynamics where $\pi^{\mu\nu}$ and $\Pi$ are dynamical variables relaxing to their Navier-Stokes values on characteristic relaxation times $\tau_{\pi}$ and $\tau_{\Pi}$.

Once the equation of state and the constitutive equations are known, the expansion dynamics is uniquely defined, but the actual solution depends on the boundary conditions: The initial distribution of matter, and the criterion for the end of evolution. Fluid dynamics does not provide either of these, but they have to be supplied by other models. The end of evolution is usually taken to be a hypersurface of constant temperature or energy density, where the fluid is converted to particles (particlization). In pure hydrodynamical models all interactions are assumed to cease at this point and particle distributions freeze out. In so-called hybrid models particle ensembles formed at the end of fluid-dynamical evolution are fed into a hadron cascade describing the dilute hadronic stage.
2. Azimuthal anisotropies of final particle distribution

The particle production in the primary collisions is azimuthally isotropic, but the distribution of observed particles in A+A collisions is not. The anisotropy can be easily explained in terms of rescatterings of the produced particles: In a non-central collision the collision zone has an elongated shape. If a particle is heading to a direction where the collision zone is long, it has a larger probability to scatter and change its direction than a particle heading to a direction where the collision zone is short. Thus more particles end up in direction where the edge of the collision zone is near. Or, in a hydrodynamical language, the pressure gradient between the collision zone is near. This anisotropy is quantified in terms of Fourier expansion of the azimuthal distribution. The coefficients of this expansion \( v_n \), and the associated event angles \( \psi_n \), are defined as

\[
v_n = \langle \cos[n(\phi - \psi_n)] \rangle, \quad \text{and} \quad \psi_n = \frac{1}{n} \arctan \left( \frac{\langle p_T \sin(n\phi) \rangle}{\langle p_T \cos(n\phi) \rangle} \right).
\]

Of these coefficients \( v_1 \) is called directed, \( v_2 \) elliptic, and \( v_3 \) triangular flow. Elliptic flow of charged hadrons as a function of centrality was one of the first measurements at RHIC [1].

The measured elliptic flow was seen to be quite large and to increase with decreasing centrality, as expected if it has the described geometric origin. Thus there must be rescatterings among the particles formed in the collision, and an A+A collision is not just a sum of independent pp collisions. The measured values of elliptic flow were also very close to the hydrodynamically calculated values [2], which is a very strong indication of hydrodynamical behaviour of the matter.

3. \( \eta/s \) has very low minimum

What makes the anisotropy coefficients interesting observables is their sensitivity to the properties of the fluid—its equation of state and dissipative coefficients. The shear viscosity strongly reduces \( v_2 \) [3], and thus extracting the \( \eta/s \) ratio from the data is in principle easy: One needs to calculate the \( p_T \)-averaged \( v_2 \) of charged hadrons using various values of \( \eta/s \) and choose the value of \( \eta/s \) which reproduces the data. Unfortunately this approach is hampered by our ignorance of the initial state of the evolution. The values of \( v_2 \) calculated using non-zero value of \( \eta/s \) fit the data best [4], but the preferred value depends on how the initial state of hydrodynamic evolution is chosen: Whether one uses so-called MC-Glauber [5] or MC-KLN [6, 7, 8] model causes a factor of two difference in the preferred value (\( \eta/s \approx 0.08$–$0.16 \)) [4]. Furthermore, the approximations in the description of the late hadron gas stage in these calculations caused additional uncertainties, so it was estimated [9] that based on these results \( \eta/s < 5/4\pi \).

The calculations have been improved since [4] by a better treatment of the hadronic phase (see, e.g., [10]), but the same uncertainty remains. This uncertainty can be reduced by studying the higher flow coefficients (\( v_n \), \( n > 2 \)). Because of the fluctuations of the positions of nucleons in the nuclei, the initial collision region has an irregular shape which fluctuates event-by-event, see figure 1, and thus all the coefficients \( v_n \) are finite [12]. As illustrated in figure 2, the larger the \( n \), the more sensitive the coefficient \( v_n \) is to viscosity [14]. This provides a possibility to distinguish between different initialisations, and preliminary results for the \( p_T \)-dependence of \( v_2 \) and \( v_3 \) seem to favour the MC-Glauber initialisation [15].

On the other hand, in event-by-event studies it is not sufficient to reproduce only the average values of \( v_n \), but the fluctuations of the flow coefficients should be reproduced as well. The distributions of these fluctuations provide a way to constrain the fluctuation spectrum of initial state models independently of the dissipative properties of the fluid. As shown in figure 3, once the average \( v_n \) has been scaled out, the distributions of these fluctuations, i.e., \( (v_n - \langle v_n \rangle)/\langle v_n \rangle \) or \( v_n/\langle v_n \rangle \), are almost independent of viscosity. The independence extends to other details of
the evolution to such an extent, that the distributions of the fluctuations of initial anisotropies are good approximations of the measured distributions of $v_n$ [16], and thus it is sufficient to compare the fluctuations of initial shape, $\epsilon_n$, to the observed fluctuations of $v_n$. Neither MC-Glauber nor MC-KLN model seems to be able to reproduce the measured fluctuations [17], whereas the recent calculations using so-called IP-Glasma [18, 19] and EKRT [20] initialisations reproduce both the fluctuations and the average values of $v_2$, $v_3$ and $v_4$ [20, 21, 22], making these approaches very promising.

4. Temperature dependence of $\eta/s$

In the calculations discussed in the previous section the $\eta/s$-ratio was assumed to be constant. We know no fluid where the $\eta/s$-ratio would be temperature independent, and there are theoretical reasons to expect it to depend on temperature with a minimum around $T_c$ [23].

**Figure 1.** An example of the positions of interacting nuclei in MC-Glauber model. Figure is from [11], and reprinted with permission.

**Figure 2.** Ratio of the anisotropy coefficients of charged hadrons in viscous calculation to the coefficients in ideal fluid calculation [14]. Figure is from [13], courtesy to Bjoern Schenke.

**Figure 3.** Probability distributions: a) $P(\delta v_2)$ and $P(\delta \epsilon_2)$, and b) $P(\delta v_3)$ and $P(\delta \epsilon_3)$ in the 20–30 % centrality class with sBC Glauber model initialisation and two different values of $\eta/s$, $\eta/s = 0$ and $\eta/s = 0.16$. $\delta v_n = (v_n - \langle v_n \rangle)/\langle v_n \rangle$ and $\epsilon_n = (\epsilon_n - \langle \epsilon_n \rangle)/\langle \epsilon_n \rangle$. Figures are from [16].
Thus the temperature independent $\eta/s$ is only an effective viscosity, and its connection to the physical, temperature dependent, shear viscosity coefficient is unclear. What complicates the determination of the physical shear viscosity coefficient, is that the sensitivity of the anisotropies to dissipation varies during the evolution of the system. As studied in [24], and illustrated in figure 4, at RHIC ($\sqrt{s_{NN}} = 200$ GeV) $v_2$ is insensitive to the value of $\eta/s$ above $T_c$, but very sensitive to its minimum value around $T_c$, and to its value in the hadronic phase below $T_c$. At the lower LHC energy, $\sqrt{s_{NN}} = 2.76$ TeV, the shear viscosity in the plasma phase does affect the final $v_2$, but not more than the shear viscosity in the hadronic phase. It is only at the full LHC energy, $\sqrt{s_{NN}} = 5.5$ TeV, where the viscosity in the plasma phase dominates, and dissipation in the hadronic phase has only a minor effect. Note that a change of the minimum value of $\eta/s$ would clearly change $v_2(p_T)$ at all energies.

So far we have seen that calculations with constant $\eta/s$ require slightly larger value of $\eta/s$ at LHC ($\eta/s \approx 0.2$) than at RHIC ($\eta/s \approx 0.12$) [29, 30]. This is in line with the increase of $(\eta/s)(T)$ in high temperatures, but as shown in [29], one cannot uniquely constrain $(\eta/s)(T)$ by fitting the spectra and $v_2$ alone. We need further constraints to find the temperature dependence of $\eta/s$, and it looks like the correlations of the event planes, $\psi_n$, of different flow coefficients can provide such constraints [20]. The study of these coefficients is, however, still at its infancy,
and firm constraints to \((\eta/s)(T)\) are not yet available. Thus we can only say that the minimum value of the \(\eta/s\) ratio of strongly interacting matter is small, and in the vicinity of the postulated minimum of \(\eta/s = 1/4\pi\), but how small, and how it depends on temperature, is too early to say.

5. Further reading

A reader interested in the theory of hydrodynamics in ultrarelativistic heavy-ion collisions can find a good introduction in [31]. More general reviews about hydrodynamics and flow can be found in [32, 33, 34].

Acknowledgments

This contribution has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 665778 via the National Science Center, Poland, under grant Polonez DEC-2015/19/P/ST2/03333

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