$B \to K_1 \gamma$ and tests of factorization for two-body non leptonic $B$ decays with axial-vector mesons

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The large branching ratio for $B \to K_1 \gamma$ recently measured at Belle implies a large $B \to K_1$ transition form factor and large branching ratios for non leptonic $B$ decays involving an axial-vector meson. In this paper we present an analysis of two-body $B$ decays with an axial-vector meson in the final state using naive factorization and the $B \to K_1$ form factors obtained from the measured radiative decays. We find that the predicted $B \to J/\psi K_1$ branching ratio is in agreement with experiment. We also suggest that the decay rates of $B \to K_1 \pi$, $B \to a_1 K$ and $B \to b_1 K$ could be used to test the factorization ansatz.

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I. INTRODUCTION

Our analysis is based on the recent announcement from the Belle collaboration concerning the first measurement of the branching ratio \( B \) for \( B \) decay into \( K_{1}(1270) \gamma \) \(^{1}\):  

\[
B(B^+ \to K_{1}^+(1270)\gamma) = (4.28 \pm 0.94 \pm 0.43) \times 10^{-5},
\]

together with an upper bound on \( K_{1}(1400) \):  

\[
B(B^+ \to K_{1}^+(1400)\gamma) < 1.44 \times 10^{-5} \text{ (at 90\% C.L.)}.
\]

These results should be compared to \( B \to K^* \gamma \). The decay fractions measured by CLEO \(^2\), BaBar \(^3\) and Belle \(^4\) Collaborations result in average branching ratios \( B(B^0 \to K^{*0}\gamma) = (4.17 \pm 0.23) \times 10^{-5} \) and \( B(B^+ \to K^{*+}\gamma) = (4.18 \pm 0.32) \times 10^{-5} \).

The large measured \( B(B^+ \to K_{1}^+(1270)\gamma) \) is a surprise since recent calculations \(^5\), \(^6\), \(^7\), \(^8\) predict a branching ratio smaller than the measured value by a factor \( \approx 4 \), though a previous calculation \(^9\) gives a larger branching ratio, in the range \((1-4) \times 10^{-5}\), not too far from the measured value. However the small tensor \( B \to K_{1} \) form factor for the radiative decays \( B \to K_{1}\gamma \) obtained by these recent calculations implies also a tiny branching ratio for non leptonic two-body \( B \) decays with axial-vector meson in the final state. Therefore one would expect a small branching ratio for \( B \to J/\psi K_{1} \). This is in contrast with the large measured value \(^10\) for this decay. This value is comparable with the \( B \to J/\psi K^* \) branching ratio, which implies that the form factors for the transitions \( B \to K_{1}(1270) \) and \( B \to K^* \) should be similar in size in order to explain the large branching ratios for both the radiative and the non leptonic \( B \to K_{1}(1270) \) decays. The aim of the present letter is to present arguments to show that this is indeed the case. We employ naive factorization and the heavy quark symmetry to relate the tensor form factor of the radiative transition to the form factors that describe non-leptonic decays. From the measured radiative decay rates as well as recent data on branching ratios and polarizations for \( B \to J/\psi K^* \) decays, we find that the predicted \( B(B \to J/\psi K_{1}(1270)) \) agrees with the experimental results.

From the data \(^11\), \(^12\) we also derive some straightforward predictions for a few non leptonic decay channels involving light strange or non-strange axial-vectors in the final state. This can be achieved by making use only of naive factorization and relations obtained from the Heavy Quark Effective Theory (HQET) \(^14\), \(^15\), \(^16\), \(^17\), \(^18\), and for a review \(^19\), where a similar approach was used to relate a number of decay channels of heavy mesons using the approximate symmetries of HQET.

II. \( B \to K_{1} \) RADIATIVE DECAYS AND THE MIXING ANGLE

The \( K_{1}(1270) \) and \( K_{1}(1400) \) are strange axial-vector resulting from a mixing of \( ^3P_l \) and \( ^1P_l \) states. Following PDG \(^20\), we denote by \( K_{1A} \) and \( K_{1B} \) the \( ^3P_l \) and \( ^1P_l \) states of \( K_{1} \). Thus we have

\[
\begin{align*}
K_{1}(1270) &= K_{1A} \sin \theta + K_{1B} \cos \theta, \\
K_{1}(1400) &= K_{1A} \cos \theta - K_{1B} \sin \theta.
\end{align*}
\]

The mixing angle \( \theta \) has been determined up to a fourfold ambiguity, see \(^8\) and, previously, \(^21\). The masses of \( K_{1A} \) and \( K_{1B} \), can be determined by the relations \(^21\)

\[
m_{K_{1A}}^2 = m_{K_{1}(1270)}^2 + m_{K_{1}(1400)}^2 - m_{K_{1B}}^2.
\]
\[ 2m_{K_{1B}}^2 = m_{b_1(1235)}^2 + m_{h_1(1380)}^2 . \]  

\( K_{1B} \) belongs to the same nonet as the states \( b_1(1235), h_1(1170) \) and \( h_1(1380); K_{1A}, a_1(1260), f_1(1285) \) and \( f_1(1400) \) are also in the same nonet, different from the previous one. Besides \( (5) \) we also have

\[ \cos 2\theta = \frac{m_{K_{1B}}^2 - m_{K_{1A}}^2}{m_{K_{1(1270)}}^2 - m_{K_{1(1400)}}^2} . \]  

Using \( (4) \) and \( (5) \) and restricting to \( 0 < \theta < 90^\circ \) we get only two solutions \( (2) \):

\[
\text{Sol.}[a]: \quad \theta = 32^\circ, \quad (m_{K_{1B}}, m_{K_{1A}}) = (1310, 1367) \text{ MeV} , \\
\text{Sol.}[b]: \quad \theta = 58^\circ, \quad (m_{K_{1B}}, m_{K_{1A}}) = (1367, 1310) \text{ MeV} .
\]

These results give a clue for understanding the Belle results. In fact, for any reasonable computational scheme the form factors \( T_1(0) \) that determine the radiative decays \( B \to K_{1A}\gamma \) and \( B \to K_{1B}\gamma \) should be almost identical. This is confirmed by the dynamical calculation of Ref. \( (3) \) that gives for this ratio

\[ \frac{T_1^{B\to K_{1B}}(0)}{T_1^{B\to K_{1A}}(0)} = 1.2 , \]  

where the form factor is defined by

\[
\langle K_1(p', \epsilon)|\bar{s}\gamma_{\mu}(1 + \gamma_5)q''b|B(p)\rangle = i\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^{*\rho}2T_1(q^2) + [\epsilon^{*\mu}(m_B^2 - m_{K_1}^2) - (\epsilon^{*} \cdot q)(p + p')_\mu]T_2(q^2) \\
+ [\epsilon^{* \mu}q_\mu - \frac{q^2}{m_B^2 - m_{K_1}^2}(p + p')_\mu]T_3(q^2) .
\]

Here \( T_1(0) = T_2(0) \), while \( T_3 \) does not contribute to the radiative decay. A similar definition holds also for \( B \to K^* \). We note that, from experiment,

\[ y = \frac{T_1^{B\to K_{1(1270)}}(0)}{T_1^{B\to K^*}(0)} = \sqrt{\left(\frac{m_B^2 - m_{B^*}^2}{m_B^2 - m_{K_1}^2}\right)^3 \frac{\mathcal{B}(B \to K_{1(1270)})}{\mathcal{B}(B \to K^*)}} \approx 1.06 . \]

As to \( K_1(1400) \) we get

\[ \frac{\mathcal{B}(B \to K_{1(1400)}\gamma)}{\mathcal{B}(B \to K_{1(1270)}\gamma)} = \left(\frac{m_B^2 - m_{K_1}^2(1400)}{m_B^2 - m_{K_1}^2(1270)}\right)^3 \left[ \frac{T_1^{B\to K_{1A}}(0) - \tan \theta T_1^{B\to K_{1B}}(0)}{T_1^{B\to K_{1B}}(0) + \tan \theta T_1^{B\to K_{1A}}(0)} \right]^2 . \]

Assuming the value \( (3) \) we can predict, from the Belle result \( (1) \) the value for \( \mathcal{B}(B \to K_{1(1400)}\gamma) \). The result is in Table \( 1 \). Both the solutions obtained are in agreement with the upper limit \( (2) \).

### III. \( K_1 \) Leptonic Decay Constant

\( K_1 \) leptonic decay constant can be derived from \( \tau \) decays. Let us denote by \( A \) a generic axial-vector meson, i.e one of the following states: \( K_{1A}, K_{1B}, a_1, b_1 \). We also denote by \( P, P(\epsilon) \) the pseudoscalar mesons, and we use the following definition for the matrix elements of weak currents:

\[ \langle 0| A_\mu | P(p) \rangle = i f_P p_\mu , \quad \langle A(\bar{\epsilon}, p)| A_\mu | 0 \rangle = f_A m_A \epsilon_\mu^* . \]

From the \( \tau \to K_1 \) data \( (20) \) we get

\[ f_{K_1(1270)} = 171 \text{ MeV} , \quad f_{K_1(1400)} = 126 \text{ MeV} \]
Using the mixing angle we derived and \(SU(3)\) symmetry we get

\[
\text{Sol.}[a] \quad (\theta = 32^0) : \quad (f_{h_1}, f_{a_1}) = (74, 215) \text{ MeV ,}
\]
\[
\text{Sol.}[b] \quad (\theta = 58^0) : \quad (f_{h_1}, f_{a_1}) = (-28, 223) \text{ MeV .}
\]

(13)

We note that these values might be useful to compute weak decays with non strange axial vector in the final state.

IV. \(B \to K_1J/\psi\)

For the decay \(B \to K^*J/\psi\) and \(B \to K_1J/\psi\) we have the experimental result reported in Table II and we may ask if they are compatible with the Belle result (1).

We use a simple scaling relations, based on HQET, which allows to relate the form factors for the transition \(B \to K^*\) via V-A current to those describing transitions by a tensor current. At large \(q^2\) it relates the \(A(q^2)\) and \(V_1(q^2)\) form factors defined by

\[
< A(\epsilon, p')|V^\mu - A^\mu|P(p) > = +i(m_P + m_A)\epsilon^\mu V_1(q^2) - i \frac{(\epsilon^* \cdot q)}{m_P + m_A} (p + p')^\mu V_2(q^2)
\]

\[
- i(\epsilon^* \cdot q) \frac{2m_A}{q^2} q^\nu [\bar{V}_3(q^2) - V_0(q^2)] - \frac{2A(q^2)}{m_P + m_A} \epsilon^{\nu\alpha\beta} \epsilon^{*}_\nu p_\alpha p_\beta
\]

(14)

where

\[
V_3(q^2) = \frac{m_A - m_P}{2m_A} V_2(q^2) + \frac{m_A + m_P}{2m_A} V_1(q^2)
\]

(15)

and \(V_3(0) = V_0(0)\), with \(T_1(q^2)\) in (5) as follows

\[
T_1(q^2) = \frac{q^2 + m_B^2 - m_{K_1}^2}{2m_B} \cdot \frac{A(q^2)}{m_B + m_{K_1}} - \frac{m_B + m_{K_1}}{2m_B} V_1(q^2).
\]

(16)

Moreover we assume that the effect of substituting \(K^*\) with \(K_1\) is identical in the radiative and in the non leptonic decay, in other words that each form factor for the \(B \to K_1\) transition is given by the corresponding form factor for \(B \to K^*\) multiplied by the same factor \(y\), once the change of parity between the two strange mesons is taken into account. On this basis we predict

\[
\frac{B(B \to K_1(1270)J/\psi)}{B(B \to K^*J/\psi)} \frac{B(B \to K_1^*(\gamma)\gamma)}{B(B \to K_1(1270)\gamma)} = \frac{p_{K_1}}{p_{K^*}} \left( \frac{m_B^2 - m_{K^*}^2}{m_B^2 - m_{K_1}^2} \right)^3 \left( x_\parallel + x_\perp \frac{p_{K_1}^2}{p_{K^*}^2} + x_L \frac{m_{K_1}^2}{m_{K^*}^2} \right)
\]

(17)

Here (we use the BaBar data [22])

\[
x_\parallel = \frac{\Gamma_{K^*}(B \to K_1J/\psi)}{\Gamma_{K^*}(B \to K^*J/\psi)} = 0.24 \pm 0.04
\]

\[
x_\perp = \frac{\Gamma_{K^*}(B \to K_1J/\psi)}{\Gamma_{K^*}(B \to K^*J/\psi)} = 0.16 \pm 0.03
\]

\[
x_L = \frac{\Gamma_{K^*}(B \to K_1J/\psi)}{\Gamma_{K^*}(B \to K^*J/\psi)} = 0.60 \pm 0.04
\]

(18)

while \(p_{K^*}\) (resp. \(p_{K_1}\)) is the c.m momentum of \(K^*\) (resp. \(K_1\)) for the nonleptonic decay \(B \to K^*J/\psi\) (resp. \(B \to K_1J/\psi\)).
The r.h.s of eq. 18 has the numerical value r.h.s. = 0.64, while

\[
\text{l.h.s.} = \begin{cases} 0.94 & \text{(neutral mode)} \\ 1.30 & \text{(charged mode)} \end{cases}
\]

with experimental uncertainties of around 50\%. Thus we see that the experimental results for \( B \rightarrow K_1(1270) \gamma \) and \( B \rightarrow K_1(1270) J/\psi \) are compatible within the errors. We report in Table I our prediction. Similar arguments apply to the decay \( B \rightarrow K_1(1400) J/\psi \). Also these results can be found in Table I.

V. \( B \rightarrow K_1 \pi \)

For \( B \rightarrow K_1 \pi \) decays, if \( q_{K_1} \) and \( q_{K^*} \) are respectively the c.m. momenta of \( K_1 \) and \( K^* \) in the reactions \( B \rightarrow K_1 \pi \) and \( B \rightarrow K^* \pi \), one gets, using factorization:

\[
\frac{B(B^+ \rightarrow K_1^0 \pi^+)}{B(B^+ \rightarrow K^0 \pi^+)} = \frac{B(B^0 \rightarrow K_1^- \pi^-)}{B(B^0 \rightarrow K^* \pi^-)} = \frac{q_{K_1}}{q_{K^*}} \frac{m_{K^*}^2}{m_{K_1}^2} \left( \frac{f^{B \rightarrow \pi}_{K_1}(m_{K_1}) f_{K_1} m_{K_1}}{f^{B \rightarrow \pi}_{K^*}(m_{K^*}) f_{K^*} m_{K^*}} \right)^2.
\]

Here we use the form factor \( F_1 \) defined by

\[
(P'(p')|V_\mu|P(p)) = F_1(q^2) \left[ (p_\mu + p'_\mu) - \frac{m_{K_1}^2 - m_{K^*}^2}{q^2} q_\mu \right] + F_0(q^2) \frac{m_{K_1}^2 - m_{K^*}^2}{q^2} q_\mu
\]

and a simple pole formula, with a pole mass equal to \( m_{B^*} \), for the \( q^2 \) behavior of the \( f^{B \rightarrow \pi}_{K^*} \) form factor. The results obtained by 20 are reported in Table I and represent an interesting test of factorization. It is indeed quite possible that both \( B \rightarrow K^* \pi \) and \( B \rightarrow K_1 \pi \) decays take non-factorizable contributions from long distance operators formally suppressed in the \( m_b \) limit, see e.g. 23, or power corrections in QCD Factorization 24. In this case the predictions of the last four rows in Table I would get significant violations, pointing to non-factorizable contributions to the decay amplitude.

The reactions with a \( \pi^0 \) in the final state: \( B^+ \rightarrow K_1^+ \pi^0 \) and \( B^+ \rightarrow K_1^+ \pi^0 \) involve two form factors \( F_1 \) and \( V_0 \) and different combinations of Wilson coefficients and CKM matrix elements. As explained in the introduction the main purpose of this letter is to pick up a few decay channels involving light and strange axial-vector mesons in the final state whose rates can be predicted using only the Belle results 25, \( \tau \) decay rates and the factorization hypothesis. On this basis we skip these channels leaving a complete analysis to a future paper.

VI. \( B \rightarrow A_1 K \)

Also in this case we have some clear predictions based on factorization:

\[
\frac{B(B^+ \rightarrow a_1^+ K^0)}{B(B^+ \rightarrow \rho^+ K^0)} \approx \left( \frac{q_{a_1}}{q_\rho} \right)^3 \sin \theta \left( \frac{V_0^{B \rightarrow K_1(1270)}(m_{K_1})}{A_0^{B \rightarrow \rho}(m_{K_1})} \right)^2 + \cos \theta \left( \frac{V_0^{B \rightarrow K_1(1400)}(m_{K_1})}{A_0^{B \rightarrow \rho}(m_{K_1})} \right)^2 R_+ \tag{22}
\]

\[
\frac{B(B^+ \rightarrow b_1^+ K^0)}{B(B^+ \rightarrow \rho^+ K^0)} \approx \left( \frac{q_{b_1}}{q_\rho} \right)^3 \cos \theta \left( \frac{V_0^{B \rightarrow K_1(1270)}(m_{K_1})}{A_0^{B \rightarrow \rho}(m_{K_1})} \right)^2 - \sin \theta \left( \frac{V_0^{B \rightarrow K_1(1400)}(m_{K_1})}{A_0^{B \rightarrow \rho}(m_{K_1})} \right)^2 R_+ \tag{23}
\]
\[
\frac{B(B^0 \to a_1^- K^+)}{B(B^0 \to \rho^- K^+)}_{\text{fact.}} \approx \left( \frac{q_{a_1}}{q_{\rho}} \right)^3 \left( \sin \theta \sqrt{\frac{V_0^{B \to K_1(1270)}(m_K^2)}{A_0^{B \to \rho}(m_K^2)}} + \cos \theta \frac{V_0^{B \to K_1(1400)}(m_K^2)}{A_0^{B \to \rho}(m_K^2)} \right)^2 R_- \tag{24}
\]

\[
\frac{B(B^0 \to b_1^- K^+)}{B(B^0 \to \rho^- K^+)}_{\text{fact.}} \approx \left( \frac{q_{b_1}}{q_{\rho}} \right)^3 \left( \cos \theta \sqrt{\frac{V_0^{B \to K_1(1270)}(m_K^2)}{A_0^{B \to \rho}(m_K^2)}} - \sin \theta \frac{V_0^{B \to K_1(1400)}(m_K^2)}{A_0^{B \to \rho}(m_K^2)} \right)^2 R_- \tag{25}
\]

where the subscript means that we consider only factorizable contributions. \(V_0\) has been defined in [11] and, if \(|V\rangle\) is a vector meson state,

\[
<V(\epsilon'p')|V^\mu - A^\mu|P(p) > = -i(m_p + m_V)\epsilon^{*\mu} A_1(q^2) + i\frac{(\epsilon^* \cdot q)}{m_p + m_V} (p + p')^\mu A_2(q^2) + i(\epsilon^* \cdot q) \frac{2m_V}{q^2} q^\mu \left[A_3(q^2) - A_0(q^2)\right] + \frac{2V(q^2)}{m_p + m_A} \epsilon^{\mu\nu\alpha\beta} \epsilon^*_\alpha p_\alpha p'_\beta
\]

and

\[
A_3(q^2) = \frac{m_V - m_p}{2m_V} A_2(q^2) + \frac{m_V + m_p}{2m_V} A_1(q^2)
\]

with \(A_3(0) = A_0(0); q_{a_1}\) and \(q_{b_1}\) are the c.m. momenta of \(a_1\) and \(b_1\) respectively; the factors \(R_{\pm}\) are defined below.

It is a well known fact that factorization terms give small contribution to the decay rates \(B^+ \to \rho^+ K^0, B^+ \to \rho^- K^+\); for example, for the \(B^0 \to \rho^- K^+\) channel, the experimental result \(B(B^0 \to \rho^- K^+) = \left(7.3 \pm 0.8 \times 10^{-6}\right)\) is larger by one order of magnitude than theoretical predictions based on factorization [23, 26]. This is mainly due to the large cancellation between the penguin contributions appearing in the denominator of the two factors \(R_{\pm}\). These two factors differ by 1 because of the different parity of the vector and axial-vector mesons. The penguin operators \(O_6\) and \(O_8\) distinguish the two parities and therefore

\[
R_+ = \left( \frac{a_4 - a_{10}}{2} + \frac{(2a_6 - a_8) m_K^2}{(m_b - m_d)(m_s + m_u)} \right)^2,
\]

\[
R_- = \left( \frac{a_4 + a_{10}}{2} + \frac{2(a_6 + a_8) m_K^2}{(m_b - m_u)(m_s + m_u)} \right)^2.
\]

For numerical evaluation of these coefficients we take [27]: \(c_3 = 0.013, c_4 = -0.029, c_5 = 0.009, c_6 = -0.033, c_7/\alpha = 0.005, c_8/\alpha = 0.060, c_9/\alpha = -1.283, c_{10}/\alpha = 0.266, \) with \(a_i = c_i + \frac{G_{1-1}}{3}\) (i=even). The other two Wilson coefficients \(c_2 = 1.105, c_1 = -0.228\) are of no interest here. Moreover, for the current quark masses we use the values \(m_b = 4.6\, \text{GeV}, m_u = 4\, \text{MeV}, m_d = 8\, \text{MeV}, m_s = 0.150\, \text{GeV}.\) We get therefore

\[
R_+ \approx 160, \quad R_- \approx 80.
\]

Following the same procedure of Section [11] we evaluate the ratio of form factors as follows.

\[
\frac{V_0^{B \to K_1(1270)}(m_K^2)}{A_0^{B \to \rho}(m_K^2)} \approx \frac{V_0^{B \to K_1(0)}(0)}{A_0^{B \to \rho}(0)} = y \frac{m_K + m_B + m_{K_1} - (m_B - m_{K_1}) z}{m_{K_1} + m_B + m_K - (m_B - m_K) z}
\]

Here \(y\) is defined, for \(K_1(1270)\) by [9]; a similar expression holds for \(K_1(1400)\) and \(y_{K_1(1400)} = 0.14\) for \(\theta = 32^\circ\) and \(y_{K_1(1400)} = 0.35\) for \(\theta = 58^\circ\). The factor \(z\) is defined as

\[
z = \frac{A_2^{B \to \rho}(0)}{A_1^{B \to \rho}(0)} \approx \frac{A_2^{B \to \rho(0)}}{A_1^{B \to \rho(0)}}
\]
We take the value $z = 0.93$ intermediate between the value $z = 0.95$ predicted by light cone sum rules \cite{28} and $z = 0.95$ given by the BWS model \cite{29}. Although the phase space and the ratio of form factors act as suppressing factors, the big enhancement given by $R_\pm$ can produce very large predictions for the decays $B \to a_1, b_1 K$. As a matter of fact we get for the four ratios in eqns. (22)-(25) results of the order $\approx (59, 76, 29, 38)$ for the solution $\theta = 32^\circ$ and $\approx (147, 9, 73, 4)$ for the solution $\theta = 58^\circ$. This means that factorization terms give sizeable contributions to these decays, and especially to $B \to a_1^+ K^0$ for both values of the mixing angle. Our conclusion is that, in view of these results, two-body nonleptonic $B$ decays with a kaon and a light non-strange axial vector meson in the final state represent interesting decay channels with expected large branching ratios. Significant experimental deviations from the the abovementioned ratios would point to specific violations of the factorization model.

It would be tempting to extend the present analysis to the case of $B$ transitions to other orbitally excited $K$ mesons. For example two decay modes with $K_2^*(1430)$ in the final state have been measured: $B \to K_2^*(1430)\gamma$ and $B \to K_2^*(1430)J/\psi$. The radiative transitions $B \to K_2^*(1430)$ have been investigated by some authors. In Ref. \cite{31} HQET is used and the strange quark is treated as heavy, which is however a rather crude approximation. As a result, these authors predict $\mathcal{B}(B \to K_2^*(1430)\gamma)/\mathcal{B}(B \to K_1(1270)\gamma) = 3$, which is at odds with the data, though the predicted $B \to K_2^*(1430)\gamma$ branching ratio is in agreement with experiments. On the contrary in \cite{31} the $s$ quark is considered light. Also this relativistic quark model reproduces correctly the $B \to K_2^*(1430)\gamma$ decay mode, but predicts a too small branching ratio for $B \to K_1(1270)\gamma$. This again brings up the problem with the small predicted radiative branching ratio involving $K_1(1270)$ state, the motivation for the present work. There are also more recent calculations \cite{8} with results in agreement with experiment for the $B \to K_2^*(1430)\gamma$ branching ratios, obtained by various techniques such as light cone sum rules or covariant relativistic quark models as given in the Table V of \cite{8}. An analysis of the $B \to K_2^*(1430)J/\psi$ decay mode is performed in \cite{32}. These authors use the ISGW2 quark model \cite{33,34} and find results that are however sensitive to the model-dependent form factors. Tests of factorization would be therefore desirable also for these channels. However it must be said that the extension of the present study of $B \to K_1$ transitions to the $B \to K_2^*(1430)$ decay modes cannot be immediate. To study $B \to K_1$ transitions we have used data from $B \to K^*(892)$ transitions and made some further hypotheses, based on the chiral similarity between $1^-$ and $1^+$ states (see the discussion on the $B \to K_1 J/\psi$ channel presented above). For $B \to K_2^*(1430)$ we are in a less favorable situation and some further assumption has to be made. We plan to come back to this issue in the future.

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TABLE I: Theoretical branching ratios and comparison with experiment; [a] and [b] refer to the two possible values of the mixing angle between the $K_{1A}$ and $K_{1B}$ states, $\theta = 32^\circ$ and $\theta = 58^\circ$ respectively. The experimental values for the processes in lines 1, 2, 3, 5, 9 and 10 are used as inputs.

| Process | $B$ (theory) | exp. |
|---------|--------------|------|
| $B^+ \to K^*+\gamma$ | input | $(4.18 \pm 0.31) \times 10^{-5}$ (av. of [2, 3]) |
| $B^0 \to K^0\gamma\gamma$ | input | $(4.17 \pm 0.23) \times 10^{-5}$ (av. of [2, 3]) |
| $B^+ \to K_1^+(1270)\gamma$ | input | $(4.28 \pm 0.94 \pm 0.43) \times 10^{-5}$ [25] |
| $B^+ \to K_1^+(1400)\gamma$ | $7.7 \times 10^{-7}$ [a] | $< 1.44 \times 10^{-5}$ [25] |
| $B^+ \to K_1^+(1400)\gamma$ | $4.4 \times 10^{-6}$ [b] | |
| $B^+ \to K^+ J/\psi$ | input | $(1.35 \pm 0.10) \times 10^{-3}$ [20] |
| $B^+ \to K_1^+(1270) J/\psi$ | $0.89 \times 10^{-3}$ | $(1.8 \pm 0.5) \times 10^{-3}$ [?] |
| $B^0 \to K_1^0(1270) J/\psi$ | $0.89 \times 10^{-3}$ | $(1.3 \pm 0.5) \times 10^{-3}$ [?] |
| $B^+ \to K_1^0(1400) J/\psi$ | $1.4 \times 10^{-5}$ [a] | $< 5 \times 10^{-4}$ [20] |
| $B^+ \to K_1^0(1400) J/\psi$ | $8.1 \times 10^{-5}$ [b] | |
| $B^+ \to K^*0\pi^+$ | input | $(1.9^{+0.6}_{-0.8}) \times 10^{-5}$[20] |
| $B^0 \to K^+\pi^-$ | input | $(1.6^{+0.6}_{-0.5}) \times 10^{-5}$[20] |
| $B^+ \to K_1^0(1270)\pi^+$ | $1.0 \times 10^{-5}$ | == |
| $B^0 \to K_1^+(1270)\pi^-$ | $0.85 \times 10^{-5}$ | == |
| $B^+ \to K_1^0(1400)\pi^+$ | $0.54 \times 10^{-5}$ | $< 2.6 \times 10^{-4}$ [20] |
| $B^0 \to K_1^+(1400)\pi^-$ | $0.46 \times 10^{-5}$ | $< 1.1 \times 10^{-3}$ [20] |

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