CONSTRaining THE PRIMORDIAL MAGNETIC FIELD FROM COSMIC MICROWave BACKGROUND
ANISOTROPIES AT HIGHER MULTipoLES

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ABSTRACT

The cosmological magnetic field is one of the important physical quantities that strongly affect the cosmic microwave background (CMB) power spectrum. Recent CMB observations have been extended to higher multipoles ($l \gtrsim 1000$), and they resultantly exhibit an excess power that is more than the standard model prediction in cosmological theory that best fits the Wilkinson Microwave Anisotropy Probe (WMAP) data at lower multipoles ($l \lesssim 900$). We calculate the CMB temperature anisotropies generated by the power-law magnetic field at the last scattering surface in order to remove the tension between theory and observation at higher multipoles and also to place an upper limit on the primordial magnetic field. In our present calculation we take account of the effect of the ionization ratio, without approximation. This effect is very crucial to precisely estimating the effect of the magnetic field on the CMB power spectrum. We consider effects of both the scalar and vector modes of the magnetic field on the CMB anisotropies, for which current data are known to be insensitive to the tensor mode, which we ignore in the present study. In order to constrain the primordial magnetic field, we evaluate the likelihood function of the WMAP data in a wide range of parameters of the magnetic field strength $|B|_l$ and the power-law spectral index $n_B$, along with six cosmological parameters in flat universe models, using the technique of the Markov chain Monte Carlo method. We find that the upper limit at the 2\,σ confidence level turns out to be $|B|_l \leq 3.9$\,nG at 1\,Mpc for any $n_B$-values, which is obtained by comparing the calculated result, including the Sunyaev-Zeldovich effect, with recent WMAP data of the CMB anisotropies.

Subject headings: cosmic microwave background — magnetic fields — methods: numerical

1.INTRODUCTION

Temperature and polarization anisotropies in the CMB provide very precise information on the physical processes in the early universe. However, recent new CMB data sets from the WMAP (Bennett et al. 2003), the Arcminute Cosmology Bolometer Array Receiver (ACBAR; Kuo et al. 2004), and the Cosmic Background Imager (CBI; Mason et al. 2003) have indicated a potential discrepancy between theory and observation at higher multipoles ($l \gtrsim 900$). The best-fit cosmological model of the WMAP data predicts a power spectrum that shows an appreciable departure from those observed in balloon and interferometer experiments. This discrepancy cannot be explained by taking account of tuned standard cosmological parameters.

One possible interpretation of an excess power at high multipoles is that it is a manifestation of the rescattering of CMB photons by hot electrons in clusters, also known as the Sunyaev-Zeldovich (SZ) effect (Sunyaev & Zeldovich 1980). Although there can be no doubt that some contribution from the SZ effect exists in the observed angular power spectrum, it has not yet been established conclusively that this is the only possible interpretation (Aghanim et al. 2001) of the small-scale power. Indeed, the best value of the matter fluctuation amplitude to fit the excess power at high multipoles is near the upper end of the range of the values deduced by the other independent method (Bond et al. 2002; Komatsu & Seljak 2002). In order to help toward a solution to this bothersome problem, it has recently been reported that the bump feature in the primordial spectrum gives a better explanation for both the CMB and the matter power spectra at small scales (Mathews et al. 2004).

The inhomogeneous cosmological magnetic field generated before the CMB last scattering epoch is also a possible candidate to reconcile the tension in such higher multipoles. It excites an Alfven-wave mode in the baryon-photon plasma in the early universe and induces small rotational velocity perturbations (Adams et al. 1996). Since the mode can survive on scales below those of the Silk damping during recombination (Jedamzik et al. 1998; Subramanian & Barrow 1998), it could be a new source of the CMB anisotropies in such small scales. Analytic expressions of temperature and polarization angular power spectra in rather larger angular scales ($l \lesssim 500$) based on the thin last scattering surface (LSS) approximation were derived for both vector and tensor modes (Mack et al. 2002). Subramanian & Barrow (2002) considered the vector perturbations in the opposite limit of smaller angular scales. However, a magnetic field that is too strong in the early universe may possibly conflict with the cosmological observations currently available. The combination of those studies and current observations places a rough bound on the strength of the primordial magnetic field $|B|_l < 1.0–10$\,nG.

In order to compare much more precisely the theoretical CMB anisotropies, calculated by including a primordial magnetic field, with observations, we need to perform numerical calculations of the fully linearized equations using a realistic recombination history of the universe. In particular, we first need to develop a numerical method to predict the theoretical spectrum for intermediate angular scales, in which the analytic approximation becomes inappropriate. The numerical study of only the scalar mode was introduced by Koh & Lee (2000), and they discussed the sensitivity of the CMB to the primordial magnetic field strength. Lewis (2004) has recently shown the characteristic numerical effects of a primordial magnetic field on the CMB in vector and tensor modes. However, there is no systematic study to put a constraint on the parameters of the primordial magnetic field by taking account of both scalar and vector modes simultaneously.
The purpose of this Letter is to explore more completely the effects of a primordial magnetic field on the CMB anisotropies and to place a new limit on the field strength, together with many other cosmological parameters using the Markov chain Monte Carlo (MCMC) method in the analysis numerically. The present calculation differs from all previous discussions in the following important ways. First, we take account of the effect of the ionization ratio, without approximation. Second, we consider the effect of the scalar mode of the magnetic field more precisely than the analysis of Koh & Lee (2000) did by introducing a damping wavenumber that is physically consistent with the magnetohydrodynamic (MHD) study of the damping Alfvén wave (Jedamzik et al. 1998; Mack et al. 2002). The effect of the scalar mode of the primordial magnetic field changes the CMB power spectrum significantly at larger scales for $l < 900$ (Koh & Lee 2000), while the effect of the vector mode, which we also consider in the present study, dominates at smaller scales. These effects are very crucial for quantitatively estimating the effects of the magnetic field on the CMB power spectrum over a wide range of multipoles $l$ for various scales. Third, it is possible for the first time to discuss the primordial magnetic field as a solution to removing the tension between theory and observation that is exhibited in smaller angular scales of the CMB anisotropies.

2. PRIMORDIAL STOCHASTIC MAGNETIC FIELD AND BARYON-PHOTON FLUID

Before recombination, Thomson scattering between photons and electrons and Coulomb interactions between electrons and baryons were sufficiently rapid that the photon-baryon system behaved as a single tightly coupled fluid. Since the trajectory of plasma particles is bent by the Lorentz force in the magnetic field, photons are indirectly influenced by the magnetic field through Thomson scattering.

Let us consider the primordial magnetic field created at some moment during the radiation-dominated epoch. The energy density of the magnetic field is treated as a first-order perturbation in a flat Friedmann-Robertson-Walker (FRW) background cosmology. Within the linear approximation, the magnetic field evolves as a stiff source, and therefore we can discard all back-reactions from the MHD fluid onto the field itself. Also, we assume that the conductivity of the primordial plasma in the early universe is infinite before decoupling, which is a very good approximation at the epochs that we are interested in. On this assumption we can decouple the time evolution of the magnetic field from its spatial dependence $B_\eta(x) = B_\eta(x)/a^2$ for a very large scale, where $\eta$ is the conformal time, $x$ is the spatial coordinate, and $a$ is the scale factor. We also assume that a background primordial magnetic field $B_0$ is statistically homogeneous, isotropic, and random. For such a class of magnetic field, the power spectrum can be taken as a power law $P(k) \propto k^n$ (Mack et al. 2002), where $k$ is the wavenumber in the Fourier space, and $n_B$ is the power-law spectral index of the primordial magnetic field and can be either negative or positive.

Evaluating the two-point correlation function of the electromagnetic stress-energy tensor of the vector mode, we can obtain the isotropic spectrum

$$|\Pi^{(1)}(k)|^2 \approx \frac{1}{4(2n_B + 3)} \left( \frac{(2\pi)^{2n_B+1}B_\Lambda^2}{2\Gamma([n_B + 3/2]k^{2n_B+3})} \right)^2 \times \left( k_{\Lambda}^{2n_B+3} + \frac{n_B}{n_B + 3} k_{\Lambda}^{2n_B+3} \right), \quad (1)$$

for $k < k_\Lambda$, to a very good approximation (Mack et al. 2002), where the vector mode Lorentz force, $L^{(1)}(k)$, is given by $L^{(1)}(k) = k\Pi^{(1)}(k)$, $B_\Lambda = |B_\Lambda|$ is the magnetic comoving mean field amplitude obtained by smoothing over a Gaussian sphere of comoving radius $\Lambda$, and $k_\Lambda = 2\pi/\lambda (\lambda = 1$ Mpc in this Letter). In this equation $k_\Lambda$ is the damping cutoff wavenumber in the magnetic power spectrum defined by

$$k_\Lambda = (1.7 \times 10^3)^{2(n_B+5)} \left( \frac{B_\Lambda}{10^{-8}} \right)^{-2(2n_B+5)} \times \left( \frac{k_\Lambda}{1 \text{ Mpc}^{-1}} \right)^{2(n_B+5)} G_b^{(2n_B+5)}, \quad (2)$$

where $h$ is the Hubble parameter in units of 100 km s$^{-1}$ Mpc$^{-1}$ (Jedamzik et al. 1998; Mack et al. 2002). Since the magnetic field source term $\Pi^{(1)}(k)$ depends on the magnetic field quadratically, the explicit time dependence of the magnetic stress is given by $\Pi^{(1)}(\eta, k) = \Pi^{(1)}(k)/a^4$. Also, the electromagnetic stress-energy tensor of the scalar mode is $\Pi^{(0)} = 2 \Pi^{(1)}$ (Koh & Lee 2000; Mack et al. 2002). In the previous studies the ionization ratio in the early universe was assumed to be a step function of time, i.e., $\chi_0 = 1$ before the LSS and $\chi_0 = 0$ after. However, we have to incorporate a correct recombination history in order to obtain a more accurate theoretical result. We accomplish this by using the numerical program RECFAST (Seager et al. 1999) in open use. Thus, we take account of the correct ionization ratio $\chi_0(\eta)$, and we can now rewrite $L^{(1)}(\eta, k) = k_\Lambda \chi_0(\eta)\Pi^{(1)}(k)$.

Combining Einstein equations with the fluid equations (Ma & Bertschinger 1995; Hu & White 1997; Seljak & Zaldarriaga 1996), we obtain evolution equations of scalar and vector perturbations. We evaluated the likelihood functions of WMAP data (Verde et al. 2003) in a wide range of parameters of the stochastic magnetic field, $B_\Lambda$ and $n_B$, with other cosmological parameters, $h$, $\Omega_m h^2$, $\Omega_b h^2$, $n_s$, $A_s$, and $\tau$ in flat universe models, where $\Omega_m h^2$ and $\Omega_b h^2$ are the baryon and cold dark matter densities, respectively, $n_s$ and $A_s$ are the spectral index and the amplitude of the primordial scalar fluctuation, respectively, and $\tau$ is the optical depth. To explore the parameter space, we make use of the Markov chain technique (Lewis & Bridle 2002). We also take account of the SZ effect in our analysis. For that, we follow an estimate of Komatsu & Seljak (2002), with $\sigma_v = 0.9$ (Spergel et al. 2003; Komatsu & Seljak 2002). Note that we consider linear perturbations in a flat FRW universe but that we do not consider the effect of gravity wave damping (Caprini & Durrer 2002), which would be relevant for the magnetic fields on superhorizon scales if they were generated before the epoch of big bang nucleosynthesis. Although this might alter the original spectrum of the primordial magnetic field by the time of the LSS in blue power spectral indices $-2 < n$ (Caprini & Durrer 2002), one can still allow it to represent the modified spectrum in the running power-law index.

3. RESULT AND DISCUSSIONS

Our study confirms that the effect of not only the vector mode but the scalar mode magnetic field plays an important role in solving the potential discrepancy between theory and observation of the CMB anisotropies at higher multipoles.

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3 This has a similar form to the one used in Mack et al. (2002), which treats two kinds of Fourier transforms of different normalizations. Here we adopt the same normalization to the Fourier transform systematically in order to remove the uncertainty from the numerical calculations, as was pointed out by Lewis (2004).
We show the result of our MCMC analysis with WMAP data in the two-parameter ($|\mathbf{B}|$, $n_B$)-plane in Figure 1. Although the higher $l$ data from CBI (Mason et al. 2003), ACBAR (Kuo et al. 2004), and others may constrain the magnetic field strongly, we did not include those data in the present analysis because they have error bars that are too large. Therefore, the result is most sensitive to the WMAP data points at $500 \leq l \leq 900$. Shown in Figure 1 is the excluded region at the 2 $\sigma$ (95.4%) confidence level (CL) as bounded by the thick solid curve, but we could not find the lower boundary of the allowed region at the same 2 $\sigma$ (95.4%) CL. Note that we find a very shallow minimum of the reduced $\chi^2 \approx 1.08$ along the thick dashed line that is almost parallel to the upper 2 $\sigma$ boundary displayed as the thick solid line in Figure 1. We can thus obtain the strength of the primordial magnetic field $|\mathbf{B}| \leq 3.9$ nG (2 $\sigma$) at 1 Mpc for the power-law spectral index $n_B \sim 1.1$. This upper limit is the most reliable and newest one to the primordial magnetic field because we consider the effects of the CMB anisotropies, i.e., the effect of the ionization ratio, the SZ effect, and both scalar and vector mode effects of the magnetic field, for the first time, in the present estimate of $|\mathbf{B}|$ and $n_B$. In our numerical estimate of the magnetic field parameters, we continued the MCMC analysis until the other cosmological parameters converged toward the values listed in Table 1. The inferred parameter values are not very different from those of Spergel et al. (2003), but the important fact is that we cannot find obvious degeneracies of magnetic field parameters with other cosmological parameters. We understand this for the following reasons. The primordial magnetic field is constrained by the cutoff scale of the damping Alfvén wave as indicated by equation (2) (Jedamzik et al. 1998; Mack et al. 2002). Since the cutoff scale is small enough compared with the multipoles $l \ll 3000$ for current available data, which we are interested in, the manifestation of this effect is remarkably seen as a monotonically increasing contribution from both scalar and vector perturbations in the multipole region higher than $l \sim 500$ as far as $|\mathbf{B}|_s \leq 3.9$ nG. This sensitivity of the CMB power spectrum to the primordial magnetic field differs completely from those of the other cosmological parameters, which helps resolve the degeneracies among them.

There is, however, a strong degeneracy between the magnetic field strength $|\mathbf{B}|$ and the power spectral index $n_B$. At this moment, we cannot resolve this strong degeneracy, because the WMAP data are constrained to lower $l \leq 900$. The CBI and ACBAR data are in an interesting higher multipole region, but the error bars are too large to resolve the degeneracy. Precise data for higher $l$, where the effect of the primordial magnetic field is especially strong, are highly desirable.

Let us shortly discuss the consistency of our upper limit $|\mathbf{B}| \leq 3.9$ nG (2 $\sigma$) with the other observational constraint on the magnetic field of the cluster of galaxies. Many astronomical observations indicate that the magnetic field strength in the present-day cluster of galaxies is $\sim 0.1–1$ µG. Assuming isotropic collapse for cluster formation, one can straightforwardly estimate the magnetic field strength to be about 1–10 nG at the epoch of the LSS. If the field strength is extremely larger than this critical value of $\sim 1–10$ nG, we need some damping process, and if it is smaller than $\sim 1–10$ nG, on the other hand, we need an amplification process. We do not know of any viable physical process for damping the magnetic field, but there are several amplification processes proposed in the literature. We found a wide allowed region in the two-parameter ($|\mathbf{B}|$, $n_B$)-plane in Figure 1 for the primordial magnetic field that best fits the current CMB data. The parameter values satisfy the observational constraint from the cluster of galaxies, $|\mathbf{B}| \sim 1–10$ nG at the epoch of the LSS.

To summarize, we studied the CMB fluctuation power spectrum by taking account of the scalar and vector modes from the primordial magnetic field and the SZ effect. The likelihood analysis of the WMAP data indicates that the upper limit of the magnetic field strength is $|\mathbf{B}| \leq 3.9$ nG (2 $\sigma$) at 1 Mpc for any power spectral indices $n_B$ and reasonable values of the other cosmological parameters.

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