CONSTRAINTS ON THE LONG-RANGE PROPERTIES OF GRAVITY FROM WEAK GRAVITATIONAL LENSING

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1. INTRODUCTION

Weak gravitational lensing provides a means of testing the long-range properties of gravity. Current measurements are consistent with standard Newtonian gravity and inconsistent with substantial modifications on megaparsec scales. The data allow long-range gravity to deviate from a $1/r$ potential only on scales where standard cosmology would use normal gravity, but be dominated by dark matter. Thus, abnormal gravity theories must introduce two fine-tuning scales (an inner scale to explain flat rotation curves and an outer scale to force a return to Newtonian gravity on large scales), and these scales must coincidentally match the scales produced by dark matter theory after evolving the universe for 10 billion years, starting from initial conditions that are exquisitely determined from the cosmic microwave background.

ABSTRACT

Weak gravitational lensing provides a means of testing the long-range properties of gravity in the mass distribution on large scales over a wide range of redshifts. As first pointed out by Blandford et al. (1991) and Miralda-Escude (1991), these effects are of the order of a few percent in adiabatic, cold dark matter models, making their observation challenging but feasible. Early predictions for the power spectrum of the shear and convergence were made by Kaiser (1992) on the basis of linear perturbation theory. Jain & Seljak (1997) estimated the effect of non-linearities in the density through analytic fitting formulae (Peacock & Dodds 1996) and showed that they substantially increase the power in the convergence below the degree scale. Because weak lensing can measure the matter-power spectrum without many of the problems of approaches based on the distributions of galaxies or clusters (e.g., bias), it may ultimately provide as clean a cosmological probe as the microwave background. Recently, several observational groups have reported convincing evidence of the effect (van Waerbeke et al. 2000, 2001; Bacon, Refregier, & Ellis 2000; Kaiser, Wilson, & Lupino 2001; Wittman et al. 2001; Rhodes, Refregier, & Groth 2001).

All these theoretical and observational studies are primarily motivated by standard theories of gravity and cosmology. Despite the tremendous overall success of these theories, there has been a recent resurgence of interest in nonstandard theories of gravity, largely motivated by the possibility that the standard paradigm has difficulty matching the observed rotation curves of galaxies and clusters on scales larger than the apparent distribution of matter (e.g., Krisher 1998; Walker 1994; Bekenstein & Sanders 1994; Zhytnikov & Nester 1994; Edery 1999; Kinney & Brisudova 2001; Uzan & Bernardeau 2001; Mortlock & Turner 2001), any longer ranged gravitational force, if it also affects photons, should have implications for gravitational lensing. In particular, it should profoundly affect the strength of weak-lensing shears on large scales. Many of the above authors, however, only consider gravitational lensing by isolated objects. To understand the effects on lensing of modifying gravity on large scales, it is necessary to use weak-lensing formalism, summing over the contributions from all density perturbations.

2. THE MODEL

We base our models on the discussion by Zhytnikov & Nester (1994) of modified gravity theories within the context of linearized relativity (see also Edery 1999). This framework provides a relativistic gravity model that automatically obeys the equivalence principle, and within which definite calculations can be made, while at the same time being as unrestrictive as possible. Further discussion of the experimental foundations for the assumptions can be found in Zhytnikov & Nester (1994), and also in Weinberg (1972), Misner, Thorne, & Wheeler (1973), and especially Will (1993, §§ 2 and 3).

For any such model, the important change in the formalism for the propagation of light through such a weak-field metric is to change the Poisson equation relating the density to the potential, whose derivative is used to determine the bend angle of photons. The angular power spectrum of the convergence, $\kappa$, can be written as an integral over the line of sight of the power spectrum of the density fluctuations (Kaiser 1992). For sources at a distance $D_s$,

$$l(l+1)C_l/(2\pi) = \frac{9\pi}{4l} (\Omega_m H_0^2 D_s^2) f^2 \int \frac{dD}{D_s} t^3(1-t)^2 \left[ \frac{\Delta^2_{\text{mass}}(k = l/D_s, a)}{a^2} \right] f^2(k = l/D_s) ,$$

where $t \equiv D/D_s$, $\Delta^2_{\text{mass}}(k) = k^3P(k)/2\pi^2$ is the contribution to the mass variance per logarithmic interval in physical wave-number, and $l(l+1)C_l/2\pi$ is the contribution to $\kappa^2_{\text{mass}}$ per logarithmic interval in angular wave-number (or, equivalently, multipole) $l$. The only change from the standard result is that the Poisson equation relating the potential to...
the density perturbations is modified from \( f(k) = 1 \) to a functional form determined by the Poisson equation of the modified theory of gravity. On small physical scales (large wavenumber \( k \)), \( f(k) = 1 \) is required to be consistent with the known properties of gravity.

If the sources have a range of redshifts, then one simply integrates the above expression over the redshift distribution of the sources. We assume throughout that

\[
\frac{dn}{dD} \propto D \exp \left[-\frac{(D/D_e)^4}{4}\right]
\]

and fix \( D_e \) by the requirement that \( \langle z_{\text{src}} \rangle = 1 \). In evaluating equation (1), we use the method of Peacock & Dodds (1996) to compute the nonlinear power spectrum as a function of scale factor. Throughout, we use the concordance cosmology of Ostriker & Steinhardt (1995), since it provides a reasonable fit to recent cosmic microwave background (CMB), weak-lensing, and large-scale structure data. For this choice of parameters, the lensing kernel peaks at \( z \approx 0.43 \) at a (comoving) angular diameter distance of \( 1150 \, h^{-1} \text{Mpc} \).

In our calculation, we only consider the propagation of rays through a known density distribution, and we model that known density distribution using a standard cosmological model viewed as a means to interpolate the evolution of structure with redshift. We do not attempt to self-consistently form the observed structures using the modified gravitational potential.\(^1\) If we assume that all theories must match the local density distribution, the only consequence of this assumption is that the evolution of structure implicit in equation (1) uses the standard growth rates rather than those of the modified gravity.

Examining the effects of modified gravity simply becomes a question of considering different structures for the function \( f(k) \). In four dimensions, the metric, being symmetric, contains 10 functions. The four constraints of energy-momentum conservation reduce the number of free functions to six. These six free functions can be decomposed under rotations into two scalar (density perturbations), two vector (vortical motions), and two tensor (gravity wave) modes. Within linearized theory, there are a number of propagating modes, which have the form of Yukawa (exponential) potentials:

\[
U(r; m) = G \int \frac{\rho(r')d^3r'}{|r - r'|} e^{-m|r-r'|}.
\]

Under a variety of reasonable assumptions, Zhytnikov & Nester (1994) conclude that the most general metric describes forces mediated by massive and massless scalar and tensor particles. We follow Zhytnikov & Nester (1994) in neglecting the vector modes; however, we allow arbitrary couplings for the scalar and tensor modes. In general relativity, in the weak-field limit,

\[
g_{00} = (1 + 2U), \quad g_{ij} = (1 + 2U)\delta_{ij},
\]

where \( U \) is the usual Newtonian potential. The metric of Zhytnikov & Nester (1994) has the same form, but with Yukawa potentials in addition to the Newtonian one.

For test particles with \( v \ll c \) or fluids with \( p \ll pc^2 \), only the time-time part of the metric is relevant, the contribution of the \( g_{ij} \) terms being suppressed by \( \mathcal{O}(r^2/c^2) \). However, for light, the bend angle due to the potential is actually the arithmetic mean of the coefficients in \( g_{00} \) and \( g_{ij} \). Although the extra scalar and tensor modes can enter into the space-time and time-time parts of the metric differently, we consider the one-parameter family of models in which these coefficients are equal. As Kinney & Brisudova (2001) discuss, the requirement that cluster-mass estimates from galaxy dynamics, pressure equilibrium of X-ray gas, and gravitational lensing agree means that any modified gravity law must affect photon propagation in roughly the same way as it affects particle orbits. A modified gravity that differentially affects particles and photons will almost always lead to these three cluster discrepancies among mass estimates.

Thus, in our model, in the weak-field limit, the propagation of light is the same as in standard general relativity, except that the potential is

\[
U(r) = (1 - x)U(r,0) + xU(r, m) + \ldots,
\]

where \( \ldots \) represents possible other terms of the same form as the second. We further simplify our calculation by considering only one correction term below. In such a theory, with one additional “field,” the function appearing in the estimate of the weak-lensing power spectrum is

\[
f(k) = (1 - x) + x \frac{k^2}{k^2 + m^2},
\]

where \( x = 0 \) for standard gravity, and \( x \approx -0.9 \) and \( m^{-1} \sim 50 \, \text{kpc} \) in order to produce flat rotation curves without dark matter (e.g., Sanders 1986). The corresponding potential for an object of mass \( M \) simplifies to the Newtonian result, \( -GM/r \), on small scales where \( mr \ll 1 \), and has a different effective coupling constant, \( -GM(1 - x)/r \), on large scales, \( Mr \gg 1 \).

Figure 1 shows the anisotropy spectrum predicted for a range of models. If we limit the range of gravity \( x > 0 \), then the shear fluctuations on large angular scales are suppressed, and if we extend the range they are enhanced. This should be a generic feature of any modification to the long-range force law. To obtain limits on the parameters in our model, we calculate the rms shear expected in Gaussian windows with FWHM of \( 5' \) and \( 10' \) as a function of \( x \) and \( m \) (Fig. 2). These predictions are consistent with the rms shear measured on these scales by van Waerbeke et al. (2001), but only for models with parameters close to those of standard gravity. We can minimize the model dependence of the result by examining the ratio of the power at \( 5' \) and \( 10' \), since this largely removes any dependence of the result on the matter density and the normalization of the power spectrum. In Figure 3, we see that the data are consistent with standard gravity and a broad range of alternate theories. These theories are acceptable because our alternate gravity model has a \( 1/r \) potential on large scales, so that when the \( 5' \) scale corresponds to a physical scale larger than \( m^{-1} \), the change in the coupling constant \( x \) is degenerate with a change in the enclosed mass. For sources with a mean redshift of unity, the \( 5' \) scale corresponds to a length scale at the peak of the lensing kernel of approximately \( 1 \, h^{-1} \text{Mpc} \).

\(^1\) In the model described below, a linear fluctuation analysis suggests that long-wavelength modes would grow more slowly than the standard model would predict. Thus, neglect of this effect is conservative if we start from an initially scale-invariant spectrum.
Theories that do not return to a $1/r$ potential on large scales are relatively easy to rule out (see Walker 1994). Assuming that the bend angle of light remains proportional to the gradient of the projected gravitational potential, such theories predict that random lines of sight would be highly sheared and (de)magnified, in contradiction to observations. This problem can be traced to the lack of degeneracy between renormalizing the mass and adjusting the coupling constant. For example, ignoring the Kinney & Brisudova (2001) Ansatz for permissible forms of alternate gravity, we could use the force law

$$\frac{-\phi(r)}{GM} = -\frac{1}{r^2} - \frac{\exp(-mr)}{rr_0},$$

which is Keplerian for $r \ll r_0$ and $r \gg m^{-1}$ but is a $1/r$ force law, producing a flat rotation curve, in between. The potential corresponding to this force law is

$$\phi/GM = -1/r + \text{Ei}[-mr],$$

where $\text{Ei}[x]$ is the exponential integral. The corresponding kernel for the weak-lensing integral is

$$f(k) = 1 - \frac{km - (k^2 + m^2)\tan^{-1}(k/m)}{r_0 k (k^2 + m^2)}.$$

Figure 4 shows the angular power spectrum in this model for a range of scales $r_0$ and a large outer cutoff $m^{-1} = 50 h^{-1}$ Mpc. Compared to normal gravity, the modified theo-

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2 For example, for a log $r$ potential and a Poisson distribution of lenses, the convergence, $\kappa$, of a source at $D_s$ (assumed to be much larger than the scale $r_0$, beyond which gravity is log $r$) is $\kappa \approx \pi a_0 (mr_0^2 D_s) \gg 1$ for any reasonable source density $n$. 

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larger scales, the models must return to the $r^{-2}$ force law of normal gravity in order to be consistent with measurements.

In standard cosmological models, once we postulate the existence of dark matter, the inner and outer scales appear naturally. On small scales, the cooling of the baryons concentrates the baryons relative to the dark matter and renders them luminous and detectable. Thus, normal matter combined with normal gravity naturally explain dynamics on scales $\lesssim 10 h^{-1}$ kpc. On intermediate scales, dark matter provides an additional source of density, which can be explained by an abnormal gravitational theory using only visible baryons as sources. On large scales, the universe returns to homogeneity, and the special properties of the $1/r^2$ force law make the weak-lensing power slowly diminish. Abnormal, longer ranged theories lose the cancellation properties of the $1/r^2$ force law on large scales, despite the increasing homogeneity of the density on these scales, leading to enormous enhancements in the strength of the weak-lensing shear. Such strong shears are in gross disagreement with even the first generation of weak-lensing measurements on these scales (van Waerbeke et al. 2000, 2001; Bacon et al. 2000; Kaiser et al. 2001; Wittman et al. 2000; Maoli et al. 2001; Rhodes et al. 2001). Thus, abnormal gravity theories must introduce two fine-tuning scales (an inner scale to explain flat rotation curves and an outer scale to force a return to Newtonian gravity on large scales), and these scales must coincidently match the scales produced by dark matter theory after evolving the universe for 10 billion years, starting from initial conditions that are exquisitely determined from the cosmic microwave background.

Finally, although we lack a formalism for estimating weak lensing in nonpotential theories, such as modified Newtonian dynamics, Mortlock & Turner (2001) have emphasized that weak-lensing results should be generic, since they require only that photons and particles have similar responses to gravitational fields. This similarity of behavior is observed on the relevant scales (megaparsec) through the near equivalence of weak-lensing, dynamical, and X-ray determinations of cluster masses (Kinney & Brisudova 2001).

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