SU(7) SUSY GUTs with a natural intermediate scale

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Abstract

We investigate the SU(N) supersymmetric Grand Unified Theories with “custodial symmetry” mechanism to explain the doublet-triplet hierarchy. We show that in such type of SU(7) SUSY theory intermediate scale appears naturally and the correct value for $\sin^2 \theta_W$ is predicted via vector-like matter superfields splitting. The unification appears to be closed to $M_{Pl}$ for all the reasonable values of $\alpha_s$ and $M_{SUSY}$. Due to the large unification scale the baryon number violating $d = 5$ operator is suppressed in comparison with that in minimal SU(5) theory.

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1 Introduction

Perhaps the gauge hierarchy problem is main difficulty of the Grand Unified Theories (GUT). This problem can be stated into the following questions: a) why is the electro-weak scale stable against the radiative corrections? b) how do the Higgs doublets remain light, whereas their colour triplet partners must be superheavy in order to avoid fast proton decay?

As it is well-known [1] supersymmetry (SUSY) answers the first question, if its breaking scale is $M_{SUSY} \sim 1 \text{ TeV}$. This is one of the main motivations for the low energy SUSY.

Several attempts to answer the second question (which is known as DT splitting problem) were suggested in the literature. Possible solution to the DT splitting problem to be explored below could be due “custodial symmetry” mechanism. [2] (which is discussed Sect. 2.)

It is well known that the combined analysis, including the heavy and light threshold corrections, in the minimal $SU(5)$ GUT predicts a value of $\alpha_s \approx 0.126[3]$ when the superpartners masses are at the TeV scale. If the particle masses are at the 500 GeV scale the predicted value of $\alpha_s$ is $\approx 0.13$. On the other hand it is known that from Z-peak we have $\alpha_s = 0.118^{+0.004}_{-0.007}(exp) \pm 0.002(theor)$ [4]

So to reduce the predictions of $\alpha_s$ in the SUSY GUT one needs a high value of $M_{SUSY}$ which, on the other hand, is unnatural for the stability of the electro-weak scale.

Among the existing attempts to solve this problem we quote one the $SU(5)$ SUSY theory with missing partner mechanism with a scalar content $75; 50$ and $50 \bar{1}$ representation. As was shown [3] the heavy threshold effects coming from those multiplets at the GUT scale make possible to get a low value of $\alpha_s$ when $M_{SUSY} \leq 1 \text{ TeV}$.

Another attempt was proposed by Brahmachari and Mohapatra [5] in the SUSY $SO(10)$ theory with intermediate gauge ($G_I = SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$) symmetry at scale $10^{10} \div 10^{12} \text{ GeV}$.

In the present paper we propose an alternative scenario: in our case we have $SU(3)_C \otimes SU(3)_W \otimes U_1(1)$ intermediate scale which is independently motivated from the natural solution of the doublet-triplet splitting problem, and at the same time is an outcome of the theory in terms of the GUT and the week (or low energy SUSY breaking) scale. Using split-multiplet mechanism is possible to make unification for all the reasonable values of $\alpha_s$ and $M_{SUSY}$.

2 DT splitting mechanism

Consider a SUSY GUT based on $SU(6)$ gauge symmetry with a minimal set of Higgs superfields needed for the breaking of $SU(6)$ symmetry down to SM. These are adjoint 35-plet ($\Sigma$) and a pair of fundamental $(6 + \bar{6})$-plets ($H + \bar{H}$). The most general $SU(6)$ invariant renormalizable scalar superpotential of the Higgs superfields has the following
form:

\[ W = \frac{m}{2} Tr \Sigma^2 + \frac{h}{3} Tr \Sigma^3 + \lambda \HH \Sigma H + M \HH H \]  

(1)

where \( h \) and \( \lambda \) are the dimensionless constants, \( m \) and \( M \) are mass parameters (we suppose that \( M \gg m \)). One of the possible VEV configuration of this fields (in unbroken SUSY limit) is:

\[ < \Sigma > = \text{diag}[1 \ 1 \ 1 -1 -1 -1] \frac{M}{\lambda} + \text{diag}[2 \ 2 \ 3 -3 -3 0] \frac{m}{h} \]

\[ < \HH > = \langle H \rangle = [0 \ 0 \ 0 \ 0 \ 0 \ 1] \]  

(2)

Obviously, with this solution the hierarchy of the symmetry breakings is the following: at the scale \( \sim M \) the gauge symmetry is broken down to \( SU(3)_C \otimes SU(3)_W \otimes U_1(1) \) by the 35-plet which develops a large VEV; then at the “geometrical scale” \( \sim \sqrt{\frac{Mm}{\lambda h}} \) the VEVs \( < H > = < \HH > \) break \( SU(3)_W \otimes U_1(1) \) to \( SU(2)_W \otimes U(1)_Y \). Obviously, in this case we have no light Higgs doublets.

Now let us consider an extension of the \( SU(6) \) symmetry to \( SU(6) \otimes SU(N)_{\text{cust}} \) \cite{2}, where we introduce \( N_6, \bar{N}_6 \) -plet superfields \( H_A \HH^A \) \((A = 1, 2...N)\) transforming as \( N, \bar{N} \) representations of \( SU(N)_{\text{cust}} \). We have the following superpotential

\[ W = \frac{m}{2} Tr \Sigma^2 + \frac{h}{3} Tr \Sigma^3 + \lambda \HH^A \Sigma H_A + M \HH^A H_A \]  

(3)

and we choose the solution for which a single pair \( (\HH^A + H_A) \) (say for \( A = 1 \)) develops a VEV. The VEVs of \( \Sigma \) and \( (\HH^1 + H_1) \) have the (2) form and relevant mass matrix for doublet (antidoublet) fragments from \( \Sigma \) and \( (\HH^A + H_A) \) is:

\[ \mathcal{D}_\Sigma \mathcal{D}_\HH \mathcal{D}_\HH^2 \cdots \mathcal{D}_\HH^N \]

\[ \begin{bmatrix}
D_{\Sigma} & \sqrt{ab} & 0 & \cdots & 0 \\
D_{H_1} & a & b & 0 & \cdots & 0 \\
D_{H_2} & \sqrt{ab} & b & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
D_{H_N} & 0 & 0 & 0 & \cdots & b
\end{bmatrix} \]  

(4)

where \( D(\mathcal{D}) \) stands for a doublet (antidoublet) from the suitable representation and

\[ a = -2 \frac{h}{\lambda} M + 2m \]

\[ b = -3 \frac{\lambda}{h} m \]  

(5)
The zero mass eigenstates of the above mass matrix
\[
G = [D_{\Sigma} \sqrt{b} + D_{H_1} \sqrt{a}] [b + a]^{-1/2}
\]
are eaten up Goldstone superfields. The orthogonal superpositions get the large masses \( \sim M \). As it is clear from eq.(4) the physical Higgs doublet from the \((H_A + H^A) (A \neq 1)\) automatically acquire small masses \((b = 3^{1/2}m)\).

So, we get \((N - 1)\) pairs of the physical Higgs doublets with masses \( \sim m \)

### 3 The model

As was shown in [6] in the case of \(SU(6) \otimes SU(2)_{\text{cus}}\)-symmetry we have no acceptable unification of the gauge constants. From the unification condition we get that “custodial” symmetry must be larger than \(SU(2)\). On the other hand the \(SU(6) \otimes SU(3)_{\text{cus}}\) symmetry case exhibits the problem in the fermion sector, namely for the top quark mass. This is because the masses of the “up” type quarks are induced from the following effective nonrenormalizable couplings (A,B,C is the indexes of \(SU(3)_{\text{cus}}\) symmetry)

\[ f \cdot \frac{1}{M_{20}} \cdot 15 \cdot 15 \cdot H^A \cdot H^B \cdot S^C \cdot \epsilon_{ABC} \]

(where \(S^C\) is singlet under \(SU(6)\) symmetry and suppose \( < S^C > \sim M_{GUT} \)) that are though the generated by heavy particle exchange [10], transforming as \((20^A_1 + 20^A_3)\) and \((20^A_2 + 20^A_3)\) representations. If we assume that \((20^A_1 + 20^A_3)\) and \((20^A_2 + 20^A_3)\) multiplets (or one of them) have masses \(\sim < H >\), in this case the gauge couplings will enter in the nonperturbative regime before unification point. Increasing masses of \(20^A_1 + 20^A_3\) and \((20^A_2 + 20^A_3)\) multiplets, say \(\sim M_{GUT}\), will decrease the top mass unacceptably. We will show that this problem is solved in \(SU(7) \otimes SU(3)_{\text{cus}}\) symmetry case.

Let us choose the Higgs content of our \(SU(7) \otimes SU(3)_{\text{cus}}\) model as : \(\Sigma(48,1) + \overline{H}_A(7,3) + H^A(7^A,3) + \Phi(7,1) + \overline{\Phi}(7,1)\), where the brackets are indicated their transformation properties under the \(SU(7) \otimes SU(3)_{\text{cus}}\) group. We also introduce 3 pairs of \(SU(7)\)- triplet superfields \(S_i^A + S_i^A\) \((i = 1,2,3)\) and assume a \(Z_2\)-symmetry under which \((\Phi, \overline{\Phi}) = - (\Phi, \Phi)\).

Consider the most general \(SU(7) \otimes SU(3) \otimes Z_2\)-invariant renormalizable scalar superpotential \(SU(7)\) indexes are omitted)

\[
W = \frac{m_1}{2} Tr \Sigma^2 + \frac{h}{3} Tr \Sigma^3 + \lambda_1 \overline{H}_A \Sigma H^A + M_1 \overline{H}_A H^A + \lambda_2 \overline{\Phi} \Sigma \Phi + m_2 \overline{\Phi} \Phi + M_2 S_1^A S_1^A + + M_3 S_2^A S_2^A + + M_4 S_3^A S_3^A + + h_1 S_1^A S_7^B S_3^C \epsilon_{ABC} + + h_2 S_1^A S_7^B S_3^C \epsilon_{ABC} \]

(6)
where $\lambda_1$, $\lambda_2$, $h$, $h_1$ and $h_2$ are the dimensionless constants, $M_1$, $M_2$, $M_3$, $M_4$, $m_1$ and $m_2$ are the mass parameters. We assume that $m_1 \sim m_{3/2}$ and $m_2 \sim m_{3/2}$, $M_1, M_2, M_3$ and $M_4$ are of order of $M_{GUT}$. Note that introduction of the small mass term $m_1 Tr \Sigma^2$ and $m_2(\overline{\Phi}\Phi)$ in (6) is not necessary, since it could be generated after SUSY breaking. Namely, in the simplest version of the minimal $N=1$ supergravity SUSY violating terms lead to the well known scalar potential [7]

$$V = \left| \frac{\partial W}{\partial z_i} + \frac{m_{3/2}}{\lambda_1} z_i^* \right|^2 + m_{3/2}(A - 3)[W^* + W] + D - \text{terms}$$

(7)

It is clear, that if one puts $m_1 = 0$ $m_2 = 0$ in (6) the mass term $m_{3/2} Tr \Sigma^2$ and $m_{3/2} (\overline{\Phi}\Phi)$ will be automatically generated from (7).

Among the discretely degenerate SUSY minima of (6) the one of our interest is:

$$<\Sigma> = \text{diag}[1 1 1 - 1 - 1 - 1 0] \frac{M_1}{\lambda_1} + \text{diag}[1 1 1 - 1 - 1 0 - 1] \frac{m_2}{\lambda_2} + \text{diag}[2 2 2 - 3 - 3 0 0] \frac{m_1}{h}$$

$$<\overline{H}_A> = <H^A> = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} V_1$$

$$<\Phi> = <\Phi> = [0 0 0 0 0 0 1] V_2$$

$$<S_1^A> = \left( \frac{M_3^2 M_4}{h_2^3 h_1} \right)^{1/4} \quad <S_1^A> = \left( \frac{M_3^2 M_4}{h_2^3 h_1} \right)^{1/4}$$

$$<S_2^B> = \left( \frac{M_4^2 M_2}{h_3^3 h_1} \right)^{1/4} \quad <S_2^B> = \left( \frac{M_4^2 M_2}{h_3^3 h_1} \right)^{1/4}$$

$$<S_3^C> = \left( \frac{M_2^2 M_3}{h_2^3 h_1} \right)^{1/4} \quad <S_3^C> = \left( \frac{M_2^2 M_3}{h_2^3 h_1} \right)^{1/4}$$

(8)

where

$$V_1^2 = \frac{M_1}{\lambda_1} (6 \frac{m_1}{\lambda_1} + 2 \frac{m_2}{\lambda_2} h) + 5 \frac{m_1}{\lambda_1} \frac{m_2}{\lambda_2} + 6 \frac{m_1}{h} \frac{m_2}{\lambda_1} + \frac{h}{\lambda_2} \left( \frac{m_2}{\lambda_1} \right)^2$$

$$V_2^2 = \frac{(M_1)}{\lambda_1} (2 \frac{h}{\lambda_2} + \frac{M_1}{\lambda_1} (4 \frac{m_1}{\lambda_1} + \frac{m_2}{\lambda_2} h) + 6 \frac{m_1}{\lambda_2} \frac{m_2}{h} + \frac{m_2}{h} \frac{m_2}{\lambda_2})$$

In this vacuum the hierarchy of the symmetry breakings is the following: at the scale $M_{GUT}$ the gauge $SU(7)$ symmetry is broken down to $SU(3)_C \otimes SU(3)_W \otimes U_1(1)$ by the $\Sigma$ and $(\overline{\Phi}, \Phi)$ which develop the large ($\sim M_{GUT}$) VEVs, at the “geometrical scale”
\(\sim \sqrt{M_{\text{GUT}} m_{3/2}}\) the VEVs \(< H^A > = \langle \overline{H}_A \rangle\) break \(SU(3)_W \otimes U(1)_Y\) down to \(SU(2)_W \otimes U(1)_Y\). So, according to a general argument of Sec.2 in this case we have the two pairs of light Higgs doublets.

According to the standard scenario of \(SU(2)_W \otimes U(1)_Y\) breaking by radiative corrections [8] the main negative contribution to the one of the light Higgs [mass]\(^2\) comes from the loops with the matter fermions, corrections by the top quark exchange (due to its largest Yukawa coupling constant). The other ones do not have coupling with light matter fermions (as it is shown in section 4) so the flavour changing neutral processes are avoided.

4 Matter field sector

We place each generation of quark - lepton superfields we place in the set of anomaly free set of \(SU(7)\) representations. These are two \(\overline{21}\), \(\overline{21}'\) and three \(35\)'s \((A = 1, 2, 3\) and is the indexe of \(SU(3)_{\text{cust}}\) symmetry) respectively. Their decompositions under the \(SU(5) \otimes SU(2) \subset SU(7)\) subgroup are

\[
\overline{21} = (10, 1) + (5, 2) + (1, 1)
\]

\[
35^A = (10, 1)^A + (10, 2)^A + (5, 1)^A
\]

The masses of the quarks and leptons are generated by the following “Yukawa” couplings:

a) \(f_1 \cdot \overline{21} \cdot 35^A \cdot \overline{H}_A\) - for the “down” type quarks and leptons

b) \(f_2 \cdot 35^A \cdot 35^B \cdot H^C \cdot \epsilon_{ABC}\) - for the “up” type quarks

For the heavy multiplets we have the following “Yukawa” couplings.

c) \(f_3 \frac{1}{M_{\text{GUT}}} \cdot \overline{21} \cdot 35^A \cdot \Phi \cdot S^I_A\)

d) \(f_4 \frac{1}{M_{\text{GUT}}} \cdot 35^A \cdot 35^B \cdot \Phi \cdot S^C_i \cdot \epsilon_{ABC}\)

e) \(f_5 \cdot 35^A \cdot 35^B \cdot H^C \epsilon_{ABC}\)

f) \(f_6 \cdot \overline{21} \cdot 35^A \cdot H_A\)

(c) and (d) are nonrenormalizable coupling and have to be understood be considered as effective operators. For example, this term can be easily generated trough the heavy (with mass \(\sim M_{\text{GUT}}\)) particle exchange [10], namely exchange of the representations of: \((35^A + \overline{35}^A)\) and \((21^A + \overline{21}^A)\) can do the job. The relevant couplings for the case (d) are

\[\kappa_1 35^A \cdot 35^{BC} \Phi \epsilon_{ABC} + M_{\text{GUT}} \overline{35}^B_{BC} \cdot 35^{BC} + \kappa_2 \overline{35}^B_{BC} \cdot 35^B \cdot S^C_i\]

for the case (c)

\[\kappa_2 35^A \cdot \overline{21}^A_{A} \Phi + M_{\text{GUT}} \overline{21}^A \cdot 21^A + \kappa_4 \overline{21}^A \cdot 21^A \cdot S^I_A\]
It is assumed that $\kappa_1, \kappa_2, \kappa_3$ and $\kappa_4$ are of order $\sim 1$

From the (c) and (d) couplings, the submultiplets: three pair of $(\overline{10}, 1) + (10, 1)$ and one pair $(5, 1) + (5, 1)$ acquire masses of order $M_{GUT}$. From the coupling (e) and (f) a single pair $(\overline{5}, 1) \cdot (5, 1)$ acquire masses of order $\sim \sqrt{\frac{m_3}{2}} M_{GUT}$.

Thus, we have per family: $(5, 1) + (\overline{5}, 1)$ submultiplets at the “geometrical scale”, and three pairs of the $(10, 1) + (\overline{10}, 1)$ and one pair of the $(5, 1) + (\overline{5}, 1)$ at the GUT scale.

5 Vector-like fermionic superfields

Besides the problem of the masses for quarks and leptons which could be solved in the above mentioned way there is a problem of unification. Because extension of the model with the intermediate scale the unification of the gauge couplings in general may be spoiled. So, we consider extension of the model, as to include not only chiral superfields but also vector-like fermionic superfields (VLFS).

Imagine that we have all possible antisymmetric and fundamental vector-like fermionic representations

$$\overline{7} + 7; \quad 21 + 21; \quad 3\overline{5} + 35$$

The following terms are possible in the “Yukawa” superpotential:

$$W_{VLFS} = m_7 \overline{7} \cdot 7 + \eta_1 \overline{7} \cdot \Sigma \cdot 7 + m_{21} 21 \cdot 21 + \eta_2 21 \cdot \Sigma \cdot 21 + \eta_3 \overline{35} \cdot \Sigma \cdot 35 + m_{35} \overline{35} \cdot 35 + \chi_1 21 \cdot 35 \cdot \Phi + \chi_2 3\overline{5} \cdot 21 \cdot \Phi + \chi_3 21 \cdot 7 \cdot \Phi$$

(9)

where $\eta_1, \eta_2, \eta_3, \chi_1, \chi_2, \chi_3, \chi_4$ are dimensionless constants and $m_7, m_{21}, m_{35}$ are mass parameters of order $m_{3/2}$. As was shown $\Sigma$s field VEV has the form:

$$<\Sigma> = \frac{M_{GUT}}{\lambda_1} \text{diag}[1 1 1 -1 -1 -1 0] + O(m_{3/2})$$

it is easy to check that $\eta$ terms can not generate masses of the following fragments

$$(3, 3) + (3, 3) \quad \text{from} \quad (\overline{35} + 35)$$

$$(\overline{3}, 3) + (3, 3) \quad \text{from} \quad (21 + 21)$$

(these are submultiplets under the $SU(3)_W \otimes SU(3)_C$ subgroup of $SU(7)$) the others acquire masses of order $\sim M_{GUT}$. The multiplets $(\overline{7} + 7)$ get mass of order $\sim M_{GUT}$. This mechanism is known as the split-multiplet mechanism and was proposed in ref [9]. From the $\chi$ terms the split fragments get masses $\sim \chi_i \cdot M_{GUT}$ and if we suppose that $\chi_i$ are in the $0.01 \div 0.007$ interval we have the split fragments in $M_{SPM} \sim 1.5 \cdot 10^{15} \div 5 \cdot 10^{15}$ GeV region which lead to the successful unification (see table 1) and this scale we denote as $M_{SPM}$.
6 Gauge coupling unification

Now, let us begin the renormalization group (RG) analysis of our model. The two-loop RG equations for the running gauge couplings of general effective \( G_1 \otimes G_2 \otimes \ldots \) gauge theory have the well known form

\[
\mu \frac{d}{d\mu} \alpha_i^{-1} = -\frac{1}{2\pi} \left( b_i + \frac{b_{ij}}{4\pi} \alpha_j + O(\alpha_i^2) \right)
\]

(10)

where \( \alpha_i(\mu) \) is the running gauge constant corresponding to \( G_i \) group, while \( b_i \) and \( b_{ij} \) are one and two-loop b-factors [11] respectively.

There are three energy regions in our case \( M_Z - M_{SUSY} (M_{SUSY} \sim m_{3/2}) \), \( M_{SUSY} - M_I \), and \( M_I - M_G \) where the b-factors are:

1) \( M_Z - M_{SUSY} \), SM region

\[
b_i^{SM} = \left( \begin{array}{ccc} 4 & -\frac{10}{3} & -7 \\ \frac{10}{3} & 0 & 5 \\ 1 & 1/2 & 0 \end{array} \right) + N_H \cdot \left( \begin{array}{ccc} \frac{1}{10} & \frac{1}{6} & 0 \\ \frac{9}{10} & \frac{9}{10} & 0 \\ 0 & 0 & 0 \end{array} \right)
\]

\[
b_{ij}^{SM} = \left( \begin{array}{ccc} \frac{10}{3} & \frac{9}{1} & \frac{44}{12} \\ \frac{11}{10} & \frac{9}{2} & -26 \end{array} \right) + N_H \cdot \left( \begin{array}{ccc} \frac{1}{10} & \frac{1}{6} & 0 \\ \frac{9}{10} & \frac{9}{10} & 0 \\ 0 & 0 & 0 \end{array} \right)
\]

(11)

Here \( N_H \) is the number of light Higgs doublets;

2) \( M_{SUSY} - M_I \), SUSY SM region

\[
b_i^{SUSY} = \left( \begin{array}{ccc} 6 & 0 & -3 \\ 0 & 6 & 0 \end{array} \right) \]

\[
b_{ij}^{SUSY} = \left( \begin{array}{ccc} \frac{38}{5} & \frac{18}{5} & \frac{88}{3} \\ \frac{6}{5} & 9 & 14 \end{array} \right) + N_H \cdot \left( \begin{array}{ccc} \frac{3}{10} & \frac{1}{2} & 0 \\ \frac{9}{10} & \frac{9}{10} & 0 \\ 0 & 0 & 0 \end{array} \right)
\]

(12)

3) \( M_I - M_G \) region

\[
b_i^{G} = \left( \begin{array}{ccc} 9 & 0 & 0 \\ 0 & 6 & 6 \\ 6 & 6 & 0 \end{array} \right) + N_H \cdot \left( \begin{array}{ccc} \frac{1}{3} & \frac{1}{2} & 0 \\ \frac{12}{6} & \frac{17}{3} & 0 \\ 0 & 0 & 0 \end{array} \right)
\]

\[
b_{ij}^{G} = \left( \begin{array}{ccc} 9 & 24 & 24 \\ 3 & 48 & 24 \\ 3 & 24 & 48 \end{array} \right) + N_H \cdot \left( \begin{array}{ccc} \frac{12}{6} & \frac{17}{3} & 0 \\ 0 & 0 & 0 \end{array} \right)
\]

(13)

In addition we have to compute the contributions of split-fragments from \( (21 + 21) \) and \( (35 + 35) \)

\[
b_i^{SPM} = \left( \begin{array}{ccc} 0 & 6 & 6 \\ 0 & 6 & 6 \\ 0 & 6 & 6 \end{array} \right), \quad b_{ij}^{SPM} = \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 68 & 32 \\ 0 & 32 & 68 \end{array} \right)
\]

(14)
We assume that they begin to play the role only from the energy scale \( M_{SPM} \), where \( M_{SPM} \) is a free parameter which fixed from the unification condition:

\[
\alpha_G(M_G) = \alpha_1(M_G) = \alpha_2(M_G) = \alpha_3(M_{GUT})
\]

and the value of this scale we can explain from (9). The gauge couplings \( \alpha_3(\mu), \alpha_2(\mu), \alpha_1(\mu) \) correspond to \( SU(3)_c, SU(3)_W, U(1) \) gauge groups (in the \( M_I < \mu < M_G \) region) respectively. At the scale \( M_I \) they are related to \( \alpha_c(\mu), \alpha_W(\mu) \), and \( \alpha_Y(\mu) \) gauge couplings which correspond to \( SU(3)_s, SU(2)_W, U(1)_Y \) groups (in the \( M_Z < \mu < M_I \) region) respectively, by equations:

\[
\alpha_s(M_I) = \alpha_3(M_I), \quad \alpha_w(M_I) = \alpha_2(M_I), \\
\alpha_Y^{-1}(M_I) = \frac{4}{5}\alpha_1^{-1}(M_I) + \frac{1}{5}\alpha_2^{-1}(M_I)
\]

We have solved equations (13) numerically with b-factors (14), (15), (16), (17) and conditions (18), (19) using as input parameters [12]

\[
\alpha_s = 0.117 \pm 0.005 \\
sin^2\theta_W = 0.2319 \pm 0.0005 \\
\alpha_{EM}^{-1} = 127.9 \pm 0.02
\]

The result of computations for the low values of (17) for \( \alpha_s \) end low values of \( M_{SUSY} = 250GeV \) is presented in fig. 1. We have also plotted the flow of running gauge coupling constants in Table 1.

Note that the grand unification scale is close to \( M_{Pl} \). Such a large unification scale avoids the Standard SUSY GUT troubles with \( d = 5 \) operator induced baryon decay, since the Higgsino mass in our case is of the order of \( M_{H_c} \sim M_{G(SU(7))} \sim 10^{18} \) GeV (whereas in the standard SUSY GUT it is \( \sim 10^{16} \) GeV), so the proton lifetime is increased relatively to the standard SUSY GUT case by the factor \( \sim \frac{M_{G(SU(7))}^2}{M_{G(SU(5))}^2} \sim 10^{3+4} \) and no constraints on the SUSY parameter space are required.

## 7 Conclusions

We have studied the \( SU(7) \) SUSY GUT with the “custodial symmetry” mechanism for the explanation of the DT hierarchy, which naturally leads to existence of the intermediate \( G_I \) symmetry scale \( M_I \) in the desert between \( M_{SUSY} \) and \( M_G \).

To obtain the gauge coupling unification we have introduced an additional pair of light Higgs doublets and split the vector-like matter superfields.

As it is shown in this model it is possible to get unification of the gauge coupling constants for all the reasonable values of \( \alpha_s \) and \( M_{SUSY} \) and a correct value of \( \sin^2\theta_W \).
Since the unification scale is close to $M_{Pl}$, there is no problem with $d = 5$ operator induced baryon decay.

On the other hand, the introduction of four light doublets give the chance to obtain the correct value not only for $m_b/m_\tau$ but also for $m_s/m_\mu$ in the manner of ref.[13].

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References

[1] E.Witten, Nucl. Phys. 188B (1981) 573;
N.Sakai, Z.Phys.C11 (1981) 153;
S.Dimopoulos and H.Georgi, Nucl.Phys.B 193 (1981) 150.
[2] G.Dvali, Phys.Lett. B324 (1994) 59.
[3] J.Ellis, E.Gardi, M.Karliner end M.A.Samuel, hep-ph/9607404.
[4] J.Bagger, K.Matchev and D.Pierce, Phys. Lett. 348B (1995) 443;
L.Clavevelli and P.Coulter, Phys. Rev. D51 (1994) 3913.
[5] B.Brahmachari and R.N.Mohapatra, Phys. Lett. 357B (1995) 566.
[6] A.Kobakhidze, hep-ph/9609359.
[7] R.Barbieri, S.Ferrara and C.A.Savoy, Phys.Lett. B119 (1982) 343;
P.Nath, R.Arnowitt and A.H.Chamseddine, Phys.Rev.Lett. 49 (1982) 970.
[8] K.Inoue, A.Kakuto, H.Kamatsu and S.Takashita, Prog. Theor. Phys. 68 (1982) 927
L.Alvarez-Gaume, J.Polchincki and M.B.Wise, Nucl. Phys. B221 (1983) 3441;
J.Ellis, J.S.Hagelin, D.V.Nanopoulos and K.Tamvakis, Phys. lett. 125B (1983) 275.
[9] J.L.Chkareuli, I.G.Gogoladze and A.B.Kobakhidze, Phys.Lett. B340 (1994) 63;
J.L.Chkareuli, I.G.Gogoladze and A.B.Kobakhidze, Phys.Lett. B440 (1995) 83.
[10] C.D.Froggatt and H.B.Nielsen Nucl. Phys. B147 (1979) 277;
Z.G.Berezhiani Phys. Lett. B129 (1983) 99; B150 (1985) 177;
S.Dimopoulos. Phys. Lett. B129 (1983) 417.
[11] D.R.T. Jones, Phys. Rev. D25 (1982) 581;  
M. Machacek and T. Vaughn Nucl. Phys. B222 (1983) 83; B236 (1984) 211; B249 (1985) 70.

[12] Review of Particle Properties, Phys. Rev. D54 (1996) 1.

[13] N.V. Krasnikov, Phys. Lett. B276 (1992) 127. N.V. Krasnikov, Fermilab preprint, FERMILAB-PUB-93/103-T.
Figure Captions

Fig.1: Gauge coupling unification with two-loop evolution for $M_{\text{SUSY}} = 250$ GeV, $\alpha_s = 0.112$, $\sin^2 \theta_W = 0.2314$
Table 1

| $\alpha_s$ | $M_{SUSY}, \text{GeV}$ | $M_I, \text{GeV}$ | $M_{SPM}, \text{GeV}$ | $M_G, \text{GeV}$ | $\alpha_G^{-1}$ |
|-----------|----------------------|------------------|---------------------|------------------|--------------|
| 0.112     | $2.5 \cdot 10^2$     | $9.2 \cdot 10^9$ | $4.47 \cdot 10^{15}$| $3.3 \cdot 10^{17}$| 12.7         |
| 0.117     | $2.5 \cdot 10^2$     | $1.27 \cdot 10^{10}$| $4.26 \cdot 10^{15}$| $6.46 \cdot 10^{17}$| 11.66        |
| 0.122     | $2.5 \cdot 10^2$     | $1.72 \cdot 10^{10}$| $4.17 \cdot 10^{15}$| $1.17 \cdot 10^{18}$| 10.73        |
| 0.112     | $10^3$                | $1.55 \cdot 10^{10}$| $3.09 \cdot 10^{15}$| $4.57 \cdot 10^{17}$| 14           |
| 0.117     | $10^3$                | $2.14 \cdot 10^{10}$| $2.95 \cdot 10^{15}$| $4.57 \cdot 10^{17}$| 13           |
| 0.122     | $10^3$                | $2.85 \cdot 10^{10}$| $2.88 \cdot 10^{15}$| $8.13 \cdot 10^{17}$| 12.1         |
