On the Other Five Unitarity Triangles

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Abstract

A comprehensive program of CP studies in heavy flavour decays has to go beyond observing large CP asymmetries in nonleptonic B decays and finding that the sum of the three angles of the unitarity triangle is consistent with 180°. There are many more correlations between observables encoded in the KM matrix; those can be expressed through five unitarity triangles in addition to the one usually considered. To test the completeness of the KM description one has to obtain a highly over-constrained data set sensitive to \( O(\lambda^2) \) effects with \( \lambda = \sin \theta_C \). Those fall into two categories: (i) Certain large angles agree to leading order only, yet differ in order \( \lambda^2 \) in a characteristic way. (ii) Two observable angles are – for reasons specific to the KM ansatz – \( O(\lambda^2) \) and \( O(\lambda^4) \) thus generating an asymmetry of a few percent and of about 0.1 \%, respectively. The former can be measured in \( B_s \rightarrow \psi \eta, \psi \phi \) without hadronic uncertainty, the latter in Cabibbo suppressed \( D \) decays. The intervention of New Physics could boost these effects by an order of magnitude. A special case is provided by \( D^+ \rightarrow K_{S,L} \pi^+ \) vs. \( D^- \rightarrow K_{S,L} \pi^- \). Finally, CP asymmetries involving \( D^0 - \bar{D}^0 \) oscillations could reach observable levels only due to New Physics.

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1 Overview

It has become customary to talk about the KM unitarity triangle as the one that represents the relation

\[ V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = \delta_{bd} = 0 \]  

(1)

This triangle plays a central role in B decays with the three terms in Eq.(1) controlling b \( \rightarrow \) u, b \( \rightarrow \) c transitions and \( B_d \rightarrow \bar{B}_d \) oscillations, respectively. It also has the important feature that its sides have comparable lengths, namely of order \( \lambda^3 \), where \( \lambda = \sin \theta_C \). Its three angles \( \phi_{1,2,3} \)

\[ \phi_1 = \pi - \arg \left( \frac{V_{tb}^* V_{td}}{-V_{ub}^* V_{ud}} \right), \quad \phi_2 = \arg \left( \frac{V_{tb}^* V_{td}}{-V_{ub}^* V_{ud}} \right), \quad \phi_3 = \arg \left( \frac{V_{ub}^* V_{ud}}{-V_{cb}^* V_{cd}} \right), \]  

(2)

are therefore naturally large – as are the CP asymmetries they generate in B decays. Beyond this qualitative observation the KM ansatz unequivocally states that these CP asymmetries are such that

\[ \phi_1 + \phi_2 + \phi_3 = 180^\circ \]  

(3)

holds. We anticipate that in the first round of measurements CP violation will be established in the beauty sector, presumably in \( B_d \rightarrow \psi K_S \); in a second round all three angles will be extracted with some degree of accuracy and Eq.(3) will be checked empirically. Yet – and this is the main message of this paper – a complete program has to move well beyond this stage both quantitatively and qualitatively:

- A dedicated effort has to be undertaken to determine the values of \( \phi_{1,2,3} \) as accurately as possible:
  - The only practical limitation on determining \( \phi_1 \) is of an experimental nature, and an accuracy of better than 5% appears attainable.
  - It should be possible in the long run to determine \( |V_{ub}/V_{cb}| \) and \( |V_{td}/V_{cb}| \) with 5% and 10% accuracy, respectively. It would allow us to construct the triangle of Eq.(1) with less than 10% uncertainty. Comparing the values measured for these angles with those inferred from the triangle then provides a highly sensitive probe of New Physics.
− Extracting $\phi_2$ and $\phi_3$ with similar precision will pose quite a challenge for theoretical and other reasons extensively discussed in the literature \[2, 3, 4\]. Whether ultimately the 5% accuracy level can be reached here is far from certain at the moment; we want to emphasize that it is a highly desirable goal deserving a dedicated effort.

− In that context we would like to sound the following note. The effective branching ratios for the interesting $B$ decays are small basically since so many channels are available. Once the nontrivial investment has been made to accumulate sufficient statistics in these modes one can turn this vice into a virtue: one can learn valuable lessons about the hadronization process by analyzing the multitude of channels driven by a mere handful of quark level transitions. We are optimistic that such studies will improve our theoretical understanding very significantly by trial and error.

• To obtain the desired accuracy one wants to combine the measurements on certain decay modes to enhance the statistics; inferring consistent values from different channels would also demonstrate that theoretical control has indeed been established.

• Yet a closer look at the weak parameters controlling the asymmetries in these modes reveal that they agree to leading order in $\lambda$ only; in higher orders they differ in a way that is very specific to the KM ansatz; a deviation reveals the intervention of New Physics.

• In some cases it is important to compare angles extracted from different asymmetries even if within the KM ansatz they have to coincide for all practical purposes. This represents a meaningful probe for New Physics in particular if one channel is dominated by a tree amplitude and the other by a Penguin amplitude \[5\].

A comprehensive program sensitive to such higher order effects has to be undertaken to probe the completeness of the KM description and to exploit the discovery potential to the fullest; it will have to include charm studies. This can be best discussed in terms of the other five triangle relations that follow from the $3 \times 3$ KM matrix being unitary \[6\].

While all six triangles possess the same area, their shapes are quite different. They can be grouped into three categories of two triangles each:

1. In addition to the $bd$ triangle of Eq.(1) there is another one where the lengths of all sides are of order $\lambda^3$: it is represented by the relation

$$V_{ud}^*V_{td} + V_{us}^*V_{ts} + V_{ub}^*V_{tb} = \delta_{tu} = 0 \quad \Rightarrow \quad tu \ \text{triangle}$$

To leading order in $\lambda$ – where $V_{ts} = -V_{cb}$ holds – it coincides with the first one given by Eq.(1).
2. Two triangles have a squashed appearance with the lengths of two sides of order $\lambda^2$ and the third one of order $\lambda^4$:

\[
v_{us}^* v_{ub} + v_{cs}^* v_{cb} = \delta_{bs} = 0 \quad \doteq \quad bs \text{ triangle} \quad (5)
\]
\[
v_{ts}^* v_{cd} + v_{cs}^* v_{cb} = \delta_{tc} = 0 \quad \doteq \quad tc \text{ triangle} \quad (6)
\]

3. The remaining two triangles are even more extreme with the lengths of two sides of order $\lambda$ and the third one of order $\lambda^5$:

\[
v_{ud}^* v_{us} + v_{cd}^* v_{cs} = \delta_{sd} = 0 \quad \doteq \quad sd \text{ triangle} \quad (7)
\]
\[
v_{ud}^* v_{cd} + v_{us}^* v_{cs} = \delta_{cu} = 0 \quad \doteq \quad cu \text{ triangle} \quad (8)
\]

Obviously we want to determine the fundamental KM parameters as precisely as possible in the hope that a future more comprehensive theory will explain them. As is well-known the KM matrix contains four \textit{independant} quantities in terms of which a host of observables is described. Thus there exist numerous correlations between these observables as expressed through the geometry of the six triangles. For example – as described below – to leading order in $\lambda$ one angle in four of the triangles coincides, yet differs in higher orders in a characteristic way. These higher order effects have to be studied when testing the completeness of the KM description thoroughly and sensitively as a probe for the presence of New Physics and the salient features of the latter. This will be achieved by measuring as many sides and angles of the six KM triangles as possible and as precisely as possible to obtain a highly overconstrained data set.

In this note we will

- show how the angles of the \textit{other} triangles can be interpreted,
- discuss how several of them can be measured which will
- yield a data set with highly overconstrained information on the KM parameters
- that probes the completeness of the KM description thoroughly.

Our message consists of two main parts:

1. The goal should be to determine as many angles as accurately as possible. The benchmark here is the 5\% accuracy level.

2. A full program on heavy flavour decays has to include dedicated searches for CP asymmetries in charm decays. A 0.1\% sensitivity here would describe a complete program.
While it is not clear at present whether these benchmarks can be reached, they do not appear to be unrealistic.

The paper will be organized as follows: in Sect. 2 we describe how the angles of the KM triangles are in principle related to observables; in Sect. 3 and 4 we discuss in some detail two of those triangles, namely the \(bs\) triangle with \(B_s \rightarrow \psi \eta\) as its central transition and the \(cu\) triangle describing charm decays; in Sect. 5 we list gateways through which New Physics could enter before summarizing in Sect. 6.

## 2 Interpretation of the Six Triangles

### 2.1 Terminology and Notation

For numbering the angles of the various triangles we follow the same systematics as for the first triangle, Eq.\((1)\): in triangle \(mn\) we call \(\phi^{mn}_i\) the angle opposite the side from the \(i\)th family, i.e. \(V_{mi}V_{ni}^\ast\) or \(V_{im}V_{in}^\ast\). Then we rescale the triangles such that their base has unit length.

Direct \(\text{CP}\) asymmetries are described by \(\Delta B = 1\) or \(\Delta C = 1\) amplitudes alone. \(\text{CP}\) violation in \(B^0 - \bar{B}^0\) oscillations as probed in wrong-sign semileptonic \(B\) decays is controlled by a \(\Delta B = 2\) amplitude. \(\text{CP}\) asymmetries involving oscillations arise from the interplay of \(\Delta B = 1\&2\) or \(\Delta C = 1\&2\) transitions. As long as one studies such an asymmetry in a single channel only, its assignment to the \(\Delta B = 1\) or \(\Delta B = 2\) sector - in the first [second] case it would be referred to as direct [indirect] \(\text{CP}\) violation – is arbitrary, since a phase rotation in the quark fields would shift it from one to the other sector. Only if two \(B_d\) (or \(B_s\)) channels exhibit \(\text{CP}\) asymmetries that differ beyond their \(\text{CP}\) parities, has direct \(\text{CP}\) violation been established unequivocally.

Four classes of \(\Delta B = 1\) or \(\Delta C = 1\) amplitudes have to be distinguished with respect to the theoretical control one has over them and their potential to be affected by New Physics: decays that are

- dominated by a single tree level process,
- receiving contributions by two tree level reactions,
- being affected significantly by a Penguin process in addition to the tree level one, or
- dominated by Penguin reactions.

To make the unitarity constraints for three families explicit we will also employ the Wolfenstein expansion of the KM matrix up to order \(\lambda^4\) \([\lambda^6]\) for the real \[imaginary\] parts:

\[
V_{\text{CKM}} = \begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & \lambda & A\lambda^3 (\rho - i\eta + \frac{i}{2} \eta \lambda^2) \\
-\lambda & 1 - \frac{1}{2} \lambda^2 - i\eta A^2 \lambda^4 & A\lambda^2 (1 + i\eta \lambda^2) \\
A\lambda^3 (1 - \rho - i\eta) & -A\lambda^2 & 1 \\
\end{pmatrix} + \mathcal{O}(\lambda^6)
\]  

(9)
where
\[ \lambda \equiv \sin \theta_C \] (10)

### 2.2 On Determining Angles

1. The rescaled \( bd \) triangle is expressed through

\[ 1 + \frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}} V_{ub}^* V_{ud} + \frac{V_{cb}^* V_{cd}}{V_{sb}^* V_{cd}} = 0 \] (11)

As is well known, its angles \( \phi_1, \phi_2 \) and \( \phi_3 \) can be measured in \( B \) decays:

- \( \phi_1 \) can be determined through the time dependant \( CP \) asymmetry in \( B_d(t) \to \psi \pi \pi, D(\ast) \bar{D}(\ast) \) controlled by

\[ \text{Im} \left( \frac{q}{p F} \right) \equiv \text{Im} \left( \frac{q T(B_d \to f)}{p T(B_d \to f)} \right) . \] (12)

Up to tiny corrections of order \( \lambda^4 \) this can be achieved much more easily in \( B_d \to \psi K_S \) [9] where we find:

\[ \text{Im} \left( \frac{q}{p \rho \psi K_S} \right) = \text{Im} \left( \frac{V_{td}^* V_{tb} V_{tb}^* V_{cb}^*}{V_{tb}^* V_{td} V_{cs}^* V_{cb}} \right) = \sin 2 \phi_1 \]

\[ \simeq \frac{2\eta(1 - \rho)}{(1 - \rho)^2 + \eta^2} \left[ 1 - \lambda^2 \left( 1 - \rho - \frac{\eta^2}{1 - \rho} \right) \right] + O(\lambda^4) \] (13)

\( \phi_1 \) could be inferred also from the difference in \( B_d(t) \to D_{\pm} \pm \pi^\pm \pi^- \) vs. \( \bar{B}_d(t) \to D_{+[-]} \pm \pi^\pm \pi^- \) where \( D_{+[-]} \) denotes the \( CP \) even [odd] combination of \( D^0 \) and \( \bar{D}^0 \) [10].

\[ D_+ \to K^+ K^-, \pi^+ \pi^-, ... \]
\[ D_- \to K_S \pi^0, K_S \eta, ... . \] (14)

since

\[ \sin 2 \phi_1 = \text{Im} \left( \frac{V_{td}^* V_{tb} V_{cb}^* V_{ud}}{V_{tb}^* V_{td} V_{cs}^* V_{cb}} \right) + O(\lambda^4) \] (15)

holds. However it appears unlikely that such an analysis could yield a competitive extraction, and in any case we view it as quite unlikely that New Physics could intervene to induce a significant difference between the two values.

\*The discerning reader will have noticed that \( \frac{V_{td}^* V_{tb} V_{cb}^* V_{ud}}{V_{tb}^* V_{td} V_{cs}^* V_{cb}} \) is not invariant under phase rotations of the \( c \) and \( u \) fields. That does not pose a problem here since one analyses the transitions in terms of the \( CP \) eigenstates \( D_{\pm} \) rather than the flavour eigenstates \( D^0 \) and \( \bar{D}^0 \). An analogous situation holds arises for \( B_d \to \psi K_S \), see Eq.(13).
Here we can also illustrate our previous comment on whether an observed \( \text{CP} \) violation originated from \( \Delta B = 2 \) or \( \Delta B = 1 \) dynamics: the Wolfenstein parametrization, Eq.\((9)\), would suggest that the asymmetry in \( \bar{B}_d \to \psi K_S \) is due to \( \arg V_{td} \neq 0 \) which affects the \( \bar{B}_d - B_d \) oscillations; yet a phase rotation \((t, b) \to (t, b)e^{-i\phi_1} \) would make \( V_{ud}[V_{tb}] \) real with the large complex phase re-surfacing in \( V_{ub} \) which appears in the \( \Delta B = 1 \) amplitude.

• There is the intriguing possibility to extract \( \phi_1 \) from the asymmetry in \( \bar{B}_d(t) \to \phi K_S \) vs. \( B_d(t) \to \phi K_S \) \[1\]. The \( \Delta B = 1 \) transition amplitude is generated mainly by Penguin operators customarily expressed by \[8 \]
\[
T(\bar{B}_d \to \phi K_S) = V_{cb}V_{cs}^* (P_c - P_t) + V_{ub}V_{us}^* (P_u - P_t) \quad ; \quad (16)
\]
\( P_q \) contains the Penguin operator with an internal quark \( q \). It follows from the GIM mechanism as implemented through a unitary KM matrix that only the differences between the operators with different internal quarks enter. The contribution from \( P_c - P_t \) is leading with the one from \( P_u - P_t \) representing an \( \mathcal{O}(\lambda^2) \) correction.

The asymmetry is then controlled by
\[
\text{Im} \frac{V_{td}^*V_{tb}V_{cb}V_{cs}}{V_{tb}^*V_{td}V_{cs}^*V_{cb}} = \sin 2\phi_1 . \quad (17)
\]

The important point to remember here is that a Penguin operator constituting a quantum correction is sensitive to New Physics operating at high mass scales. Thus it is quite conceivable that the asymmetries in \( \bar{B}_d(t) \to \phi K_S \) and \( B_d \to \psi K_S \) would yield very different values for \( \phi_1 \). A limiting factor in the theoretical interpretation of such a probe is provided by the \( P_u - P_t \) contribution and the degree of computational control that can be established over it.

• \( \phi_2 \) can be extracted from \( B_d(t) \to \pi' s \):
\[
\sin 2\phi_2 = \text{Im} \frac{V_{td}^*V_{tb}V_{ub}V_{ud}}{V_{tb}^*V_{td}V_{ub}^*V_{ud}} \simeq \frac{2\eta}{[(1 - \rho)^2 + \eta^2][\rho^2 + \eta^2(1 - \lambda^2)]} ,
\]
\[
\left[ \rho - \rho^2 - \eta^2 + \lambda^2 \left( \rho^2 - \frac{1}{2}\rho^3 - \frac{1}{2}\rho + \frac{1}{2}\eta^2(1 - \rho) \right) \right] + \mathcal{O}(\lambda^4) \quad (18)
\]

While the intervention of Penguin transitions poses a serious problem here, we are optimistic that the various methods suggested to unfold the Penguin pollution will succeed at least in the long run \[8\]. Yet whether an accuracy of \( \lambda^2 \simeq 5\% \) or even better can be achieved is quite unclear at present.

• \( \phi_3 \) induces a direct \( \text{CP} \) asymmetry in \( B^+ \to D^0/\bar{D}^0 K^+ \) vs. \( B^- \to D^0/\bar{D}^0 K^- \) \[11, 13\]; \( D^0/\bar{D}^0 \) denotes a coherent superposition of \( D^0 \) and
\( \bar{D}^0 \). This situation arises when the neutral charm meson decays into a CP eigenstate: \( D^0 / \bar{D}^0 = D_\pm \). Measuring also \( B^\pm \to D^0 K^\pm, \bar{D}^0 K^\pm \) – i.e., when the neutral charm meson decays in a flavour specific manner – allows us the determine the relevant hadronic quantities and thus extract \( \phi_3 \) cleanly [12]. The asymmetry depends on

\[
\text{Im} \left( \frac{V_{cb}^* V_{ub}}{V_{us}^* V_{cb}} \right) = \text{Im} \left( \frac{V_{ud}^* V_{ub}}{-V_{cd}^* V_{cb}} \right) + \mathcal{O}(\lambda^4) = \sin \phi_3
\]

\[
= \eta \left[ 1 - \lambda^2 \left( \frac{1}{2} \right) \right] + \mathcal{O}(\lambda^4)
\]  

(19)

A probably more practical variant of this method is to rely on a final state that is common to \( D^0 \) and \( \bar{D}^0 \) decays due to the presence of a doubly Cabibbo suppressed transition: \( D^0 \to K^+ \pi^-, K^- \pi^+ \to \bar{D}^0 \) [14].

- An ‘oblique’ angle can be studied by comparing \[3, 13\]

\[
\overline{B}_d(t) \to D^{0(*)} K_S \ \text{vs.} \ \ B_d(t) \to \overline{D}^{0(*)} K_S ;
\]

(20)

their difference is controlled by \( \text{Im} \left( \frac{V_{td}^* V_{ub}^* V_{cs}^*}{V_{tb}^* V_{td} V_{cb} V_{us}^*} \right) \) with

\[
\text{Im} \left( \frac{V_{td}^* V_{ub}^* V_{cs}^*}{V_{tb}^* V_{td} V_{cb} V_{us}^*} \right) \approx \left| V_{ub} V_{cs} \right| \sin \left( \arg \left( \frac{V_{td}^* V_{ub}^*}{V_{cb} V_{td}} \right) \right) \approx \frac{V_{ub} V_{cs}}{V_{cb} V_{us}} \sin (\phi_1 - \phi_2)
\]

(21)

Analogously the difference in

\[
\overline{B}_d(t) \to \bar{D}^{0(*)} K_S \ \text{vs.} \ \ B_d(t) \to D^{0(*)} K_S ;
\]

(22)

is controlled by

\[
\text{Im} \left( \frac{V_{td}^* V_{ub}^* V_{cs}^*}{V_{tb}^* V_{td} V_{ub} V_{cs}^*} \right) \approx \left| V_{us} V_{cb} \right| \sin \left( \arg \left( \frac{V_{td}^* V_{ub}^*}{V_{cb} V_{td}} \right) \right) \approx \frac{V_{us} V_{cb}}{V_{ub} V_{cs}} \sin (\phi_1 - \phi_2)
\]

(23)

Comparing the two asymmetries in Eq.(20) and Eq.(22) allows to infer \( \sin (\phi_1 - \phi_2) \) cleanly.

That (at least) two angles can be observed in \( B_d \) transitions is not surprising since Eq.(11) represents the \( bd \) element of \( V_{KM}^T V_{KM} \). We will not add a superscript \( bd \) to these angles of the standard unitarity triangle.

Obviously only two of the three angles \( \phi_1, \phi_2, \phi_3 \) are independent of each other. In the Wolfenstein representation, Eq.(9), only \( V_{ub} \) and \( V_{cb} \) have large phases; then one would be tempted to argue that \( \phi_1 \approx -\arg V_{td} \) and \( \phi_3 \approx -\arg V_{ub} \) are elementary whereas \( \phi_2 \approx +\arg V_{td} + \arg V_{ub} \) is composite. Yet that would be fallacious: changing the phase of the \( b \) and \( t \) quark fields by \( \phi_3 \) and \( \phi_1 \), respectively, makes \( V_{ub} \) and \( V_{td} \) basically real; the large phases now surface.
in $V_{cb}$, $V_{tb}$ and $V_{ts}$. The values of $\phi_1$, $\phi_2$, $\phi_3$ do not change, of course, since they are phase invariant, yet their make-up does

$$\phi_1 \simeq \arg V_{tb} - \arg V_{cb}, \quad \phi_2 \simeq \pi - \arg V_{tb}, \quad \phi_3 \simeq \arg V_{cb},$$

(24)

which would make $\phi_{2,3}$ look elementary and $\phi_1$ composite - contrary to the appearance in the original parametrization! Thus the three angles have to be viewed on the same level - at least until one comes to understand their dynamical origin.

Yet as long as this is kept in mind it is very convenient to express the angles through those KM parameters that exhibit a complex phase in the Wolfenstein expansion; this will be denoted by the superscript $W$:

$$\phi_1 = -\arg V_{td}^W - \arg V_{cb}^W, \quad \phi_2 = \pi + \arg V_{td}^W + \arg V_{ub}^W, \quad \phi_3 = -\arg V_{ub}^W + \arg V_{cb}^W$$

(25)

2. The rescaled $tu$ triangle

$$1 + \frac{V_{td}V_{us}^*}{V_{ts}V_{us}^*} + \frac{V_{tb}V_{ub}^*}{V_{ts}V_{us}^*} = 0$$

(26)

$$\phi_{1}^{tu} = \arg \frac{-V_{ub}^*V_{tb}}{V_{us}^*V_{ts}}, \quad \phi_{2}^{tu} = \arg \frac{-V_{ud}^*V_{td}}{V_{ub}^*V_{tb}}, \quad \phi_{3}^{tu} = \pi - \arg \frac{V_{ud}^*V_{td}}{V_{us}^*V_{ts}}$$

(27)

$$\phi_{3}^{tu} = -\arg V_{td}^W, \quad \phi_{2}^{tu} = \pi + \arg V_{td}^W + \arg V_{ub}^W, \quad \phi_{1}^{tu} = -\arg V_{ub}^W$$

(28)

coincides with the first one through order $\lambda$:

$$\phi_{3}^{tu} = \phi_1 + \mathcal{O}(\lambda^2), \quad \phi_{2}^{tu} = \phi_2, \quad \phi_{1}^{tu} = \phi_3 + \mathcal{O}(\lambda^2)$$

(29)

In order $\lambda^2$ the shapes of the $bd$ and $tu$ triangles differ since $\frac{V_{ub} + V_{ts}}{|V_{ts}|} \simeq \mathcal{i}\eta\lambda^2$. It would be instructive if one could measure the angles of the two triangles separately and check whether they agree to leading order and their $\mathcal{O}(\lambda^2)$ differences follow the pattern described by the KM matrix. This seems, however, to be quite unrealistic even beyond the question of statistical precision. For this triangle represents the $tu$ element of $V_{KM}^\dagger V_{KM}$. Its geometry could be probed directly in top hadron transitions; alas top quarks decay before they can hadronize [13].

There is one noteworthy exception, though: The angle $\phi_{3}^{tu}$ can be extracted from the CP asymmetry in $B_s(t) \to K_S p^0$. For the latter depends on

$$\text{Im} \frac{V_{ts}^*V_{tb}V_{us}^*V_{ud}}{V_{td}^*V_{ts}V_{ub}^*V_{ub}} = \sin 2\phi_{3}^{tu} \simeq \frac{2\rho\eta \left(1 - \frac{1}{2}\lambda^2\right)}{\rho^2 + \eta^2(1 - \lambda^2)}$$

(30)

$^1$Even if top hadrons like $T^0 = \bar{u}t$ existed, such prospects would presumably be quite academic due to slow $T^0 - \bar{T}^0$ oscillations etc.
\[ \tan \phi_{3}^{tu} = \frac{\eta}{\rho} \left(1 - \frac{\lambda^2}{2}\right). \]

In such an analysis one has to overcome the theoretical challenge of unfolding the (Cabibbo suppressed) Penguin contribution. Due to the anticipated tiny branching ratio this is unlikely to be a practical method for a precise measurement.

3. The \(bs\) triangle

\[1 + \frac{V_{cs} V_{cb}}{V_{ts} V_{tb}} + \frac{V_{us} V_{ub}}{V_{ts} V_{tb}} = 0\]

(32)
is qualitatively different as pointed out before: it is a squashed triangle with one angle \(-\phi_{1}^{bs}\) much smaller than the other two:

\[\phi_{1}^{bs} = \pi + \arg \left(\frac{V_{cs} V_{cb}}{V_{ts} V_{tb}}\right) \simeq \lambda^2 \eta \simeq 0.05 \cdot \eta\]

(33)

Since it is a novel angle, we denote it specially \cite{8}

\[\phi_{1}^{bs} \equiv \chi\]

(34)

As discussed in the next section \(\chi\) can be determined in \(B_s \rightarrow \psi \eta, \psi \phi\) \cite{9, 16, 17}.

With \(\phi_{1}^{bs}\) being so small, the other two angles are practically complementary:

\[\phi_{3}^{bs} \equiv \arg \left(\frac{V_{us} V_{ub}}{-V_{cs} V_{cb}}\right) \simeq \pi - \phi_{2}^{bs}, \quad \phi_{2}^{bs} = 2\pi - \arg \left(\frac{-V_{us} V_{ub}}{V_{ts} V_{tb}}\right)\]

(35)
or in our usual mnemonic

\[\phi_{1}^{bs} = -\arg V_{cs}^{W} + \arg V_{cb}^{W}, \quad \phi_{2}^{bs} = -\arg V_{ub}^{W}, \quad \phi_{3}^{bs} = \pi - \arg V_{cb}^{W} + \arg V_{ub}^{W} + \arg V_{cs}^{W}\]

(36)

The angle \(\phi_{1}^{bs}\) coincides with an angle of the \(tu\) triangle

\[\phi_{1}^{bs} = \phi_{2}^{tu},\]

(37)
since

\[\frac{V_{us} V_{ub}}{V_{ts} V_{tb}} = \frac{|V_{us}|^2 V_{ub}^{*} V_{ub}}{|V_{tb}|^2 V_{ts} V_{us}}\]

(38)

and with one of the \(bd\) triangle to leading order:

\[\phi_{2}^{bs} = \phi_{3} + \mathcal{O}(\lambda^2)\]

(39)

A more relevant observation is that \(\phi_{3}^{bs}\) can be extracted by comparing \(\bar{B}_s(t) \rightarrow D_s^+ K^-\) with \(B_s(t) \rightarrow D_s^- K^+\) \cite{2, 13} with the difference being controlled by

\[\text{Im} \left(\frac{V_{us} V_{ub} V_{cs}^{*} V_{cb}}{V_{tu} V_{ts} V_{us} V_{cb}}\right) \simeq \left|\frac{V_{ub} V_{cs}}{V_{us} V_{cb}}\right| \sin(\phi_{2}^{bs} + \phi_{1}^{bs}) = \left|\frac{V_{ub} V_{cs}}{V_{us} V_{cb}}\right| \sin \phi_{2}^{bs} + \mathcal{O}(\lambda^2)\]

(40)
Likewise we find that the difference between $\bar{B}_s(t) \to D^- K^+$ and $B_s(t) \to D^+ K^-$ is given by

$$\text{Im} \left( \frac{V_{ts}^* V_{cb}}{V_{tb} V_{cs}} \right) \sin(-\phi_{bs}^2 - \phi_{bs}^1) = - \left| \frac{V_{us} V_{cb}}{V_{ub} V_{cs}} \right| \sin \phi_{bs}^1 + O(\lambda^2)$$

(41)

4. The $tc$ triangle

$$1 + \frac{V_{tb} V_{cb}}{V_{ts} V_{cs}} + \frac{V_{td} V_{cd}}{V_{ts} V_{cs}} = 0$$

(42)

also has a small angle

$$\phi_{tc}^1 = \arg \left( \frac{V_{tb}^* V_{cb}}{V_{ts}^* V_{cs}} \right) - \pi = \arg \left( \frac{V_{us}^* V_{cb}}{V_{ub}^* V_{cs}} \right) = \phi_{bs}^1 = O(\lambda^2)$$

(43)

that coincides with the small angle of the $bs$ triangle and

$$\phi_{tc}^2 = \arg \left( \frac{V_{td}^* V_{cd}}{V_{ts}^* V_{cs}} \right), \quad \phi_{tc}^3 = 2\pi - \arg \left( \frac{V_{td}^* V_{cd}}{V_{ts}^* V_{cs}} \right) \simeq \pi - \phi_{tc}^1$$

(44)

$$\phi_{tc}^1 = -\arg V_{cs}^w + \arg V_{cb}^w, \quad \phi_{tc}^2 = -\arg V_{cb}^w - \arg V_{td}^w, \quad \phi_{tc}^3 = \pi + \arg V_{td}^w + \arg V_{cs}^w$$

(45)

Yet in the absence of $T_c = [t\bar{c}]$ mesons this triangle does not suggest new ways to measure $\phi_1$, $\phi_2$, $\phi_3$ or $\chi$.

5. The $cu$ triangle

$$1 + \frac{V_{ub}^* V_{cb}}{V_{us} V_{cs}} + \frac{V_{ud} V_{cd}}{V_{us} V_{cs}} = 0$$

(46)

is even more extreme with a tiny angle

$$\phi_{cu}^3 = \arg \left( \frac{-V_{ud}^* V_{cd}}{V_{us}^* V_{cs}} \right) \simeq -\lambda^4 A^2 \eta \simeq -1.6 \cdot 10^{-3} \eta$$

(47)

Again, it is a novel angle and therefore we denote it specifically [8]:

$$\phi_{cu}^3 \equiv \chi'$$

(48)

Its value can be inferred from $\text{CP}$ asymmetries in $D$ decays, as described in Sect. [4].

As in previous cases the other angles practically coincide with angles already encountered, namely

$$\phi_{cu}^1 = \pi - \arg \left( \frac{V_{ub}^* V_{cb}}{V_{us}^* V_{cs}} \right) \simeq \pi - \phi_3 + O(\lambda^4), \quad \phi_{cu}^2 = \arg \left( \frac{-V_{ub}^* V_{cb}}{V_{ud} V_{cd}} \right) \simeq \pi - \phi_{cu}^1 \simeq \phi_3$$

$$\phi_{cu}^1 = \pi - \arg V_{cb}^w + \arg V_{ub}^w + \arg V_{cs}^w, \quad \phi_{cu}^2 = -\arg V_{ub}^w + \arg V_{cb}^w, \quad \phi_{cu}^3 = -\arg V_{cs}^w$$

(49)

(50)
6. The $sd$ triangle

\[ 1 + \frac{V_{us}^* V_{as}}{V_{cd} V_{cs}} + \frac{V_{td} V_{ts}}{V_{cd} V_{cs}} = 0 \tag{51} \]

has a very similar shape with

\[ \phi_{sd}^1 = 2\pi - \arg\left(\frac{-V_{td} V_{ts}}{V_{cd} V_{cs}}\right) \simeq \pi - \phi_{sd}^2 \]
\[ \phi_{sd}^2 = \arg\left(\frac{V_{td} V_{ts}}{-V_{td} V_{us}}\right) \]
\[ \phi_{sd}^3 = \arg\left(\frac{V_{us}^* V_{as}}{V_{cd} V_{cs}}\right) - \pi = \phi_{cu}^3 \tag{52} \]

\[ \phi_{sd}^1 = \pi + \arg V_{td}^W + \arg V_{cs}^W, \quad \phi_{sd}^2 = -\arg V_{td}^W \simeq \phi_1, \quad \phi_{sd}^3 = -\arg V_{cs}^W \tag{53} \]

Obviously it describes CP violation in the decays of strange hadrons.

To summarize this overview of the rescaled unitarity triangles:

- They fall into three groups of two members each:
  1. the lengths of all sides are of the same order of magnitude;
  2. one is down by $O(\lambda^2)$ relative to the other two;
  3. one is down by $O(\lambda^4)$.

- They can be described in terms of four independent angles; two can be picked from the first group, Eq. (51), and one each from the second and third groups, Eq. (52) and Eq. (53), respectively.

- To leading order in $\lambda$ both triangles in the first group have angles $\phi_1$, $\phi_2$ and $\phi_3$; the triangles in the second group have angles $\chi$, $\phi_1$, $\pi - \phi_1$ and $\chi$, $\phi_3$, $\pi - \phi_3$, respectively; in the third group $\chi'$, $\phi_1$, $\pi - \phi_1$ and $\chi'$, $\phi_3$, $\pi - \phi_3$, respectively. The four basic angles referred to above can then be taken as $\phi_1$, $\phi_3$, $\chi$ and $\chi'$ with $\chi$ and $\chi'$ being small and tiny, respectively.

- All six triangles exhibit a different shape once one goes beyond the leading order in $\lambda$.

- Information contained in the KM matrix is encoded in these six triangles in a highly overconstrained form. It would be desirable to determine the angles of all triangles with an accuracy of better than $O(\lambda^2)$. This, however, is not a realistic goal, also for systematic reasons: on top of theoretical uncertainties in evaluating hadronic matrix elements – we will face this problem in our later discussion – we cannot, even in principle, study transitions of top hadrons [15].

- Nevertheless the experimental information that can be inferred for sides and triangles is still considerably overconstrained.
In the subsequent discussion we will focus on the $bs$ triangle in general and on the angles $\chi$ and $\chi'$ in particular.

### 3 The $bs$ Triangle & $B_s \to \psi\eta, \psi\phi$

The most intriguing angle of the $bs$ triangle $-1 + \frac{V_{cs}^* V_{cb}}{V_{ts}^* V_{tb}} + \frac{V_{ts}^* V_{tb}}{V_{ts}^* V_{tb}} = 0$ is the Cabibbo suppressed quantity

$$\chi \equiv \arg(V_{cs}^* V_{cb}/V_{ts}^* V_{tb}) = \frac{1}{2} \cdot \eta \quad (54)$$

There are several important aspects to this angle:

- It can be measured through the CP asymmetry in $B_s(t) \to \psi\eta$ without hadronic pollution [9, 16]:

  $$\Gamma(B_s(t) \to \psi\eta) \propto e^{-t/\tau_{B_s}} \left(1 - \sin(\Delta m_s t) \cdot \text{Im} \frac{q}{p} \bar{\rho}(B_s \to \psi\eta)\right) \quad (55)$$

  $$\Gamma(\bar{B}_s(t) \to \psi\eta) \propto e^{-t/\tau_{B_s}} \left(1 + \sin(\Delta m_s t) \cdot \text{Im} \frac{q}{p} \bar{\rho}(B_s \to \psi\eta)\right), \quad (56)$$

  where

  $$\bar{\rho}(B_s \to \psi\eta) = \frac{T(\bar{B}_s \to \psi\eta)}{T(B_s \to \psi\eta)} \quad (57)$$

  and we find

  $$\text{Im} \frac{q}{p} \bar{\rho}(B_s \to \psi\eta) \simeq \text{Im} \frac{(V_{tb} V_{ts}^* V_{cs}^* V_{cb})^2}{|V_{tb} V_{ts}^* V_{cs}^* V_{cb}|^2} = -\sin 2\chi \simeq -\frac{1}{10} \eta \quad (58)$$

- This prediction is very reliable in terms of the parameter $\eta$. We also know for sure that it is small since Cabibbo suppressed by $\lambda^2$. Numerically one estimates at present

  $$\sin 2\chi \simeq (2 \div 5)\% \quad (59)$$

The smallness of this effect is specific to the KM ansatz: in $B_s \to \psi\eta$ and $B_s \to \bar{B}_s \to \psi\eta$ the leading contribution involves quarks of the second and third families only; yet then no asymmetry can arise on this level since the KM ansatz requires the interplay of three families (at least).

- On the other hand CP violation not connected to the family structure will not be reduced here. Comparing $B_s(t) \to \psi\eta$ and $\bar{B}_s(t) \to \psi\eta$ thus provides both a promising and a clean laboratory to search for a manifestation of New Physics.

Thus there are two aspects to probing $B_s \to \psi\eta$:...
• Any asymmetry in this channel that exceeds a few percent is a clear manifestation of New Physics. That means if an asymmetry is to become observable in the next few years, New Physics has to intervene. The KM prediction will be made more precise in the foreseeable future through data on $B_d \rightarrow \psi K_S$ and $|V_{ub}/V_{cb}|$.

• Even if no asymmetry is found above the KM expectation, it would be important to probe the region below that level as an essential self-consistency check on the completeness of the KM ansatz. For the three sides of the $bs$ triangle can be determined and also (hopefully) $\phi_3^{bs} \simeq \phi_3$ in addition to $\chi$. At this point (if not before) it becomes mandatory to deal with a technical complication: while the final state $\psi \eta$ is a pure CP eigenstate, $\psi \phi$ is not: for an S-[or D-]wave configuration it is CP even, for a P-wave it is odd. While the S-wave is expected to dominate due to kinematics, the P-wave will be present as well with an asymmetry of equal size, yet opposite sign! To avoid this (partial) compensation, which one might ill afford, one has to unfold the S- and P-wave components [18].

The angle $\chi$ controls also the CP asymmetry in semileptonic $B_s$ decays:

$$a_{SL}(B_s) = \frac{\Gamma(B_s(t) \rightarrow l^+ X) - \Gamma(B_s(t) \rightarrow l^- X)}{\Gamma(B_s(t) \rightarrow l^+ X) + \Gamma(B_s(t) \rightarrow l^- X)}$$

$$= \frac{|q|^2 - |p|^2}{|q|^2 + |p|^2} = \frac{\Delta \Gamma(B_s)}{\Delta M(B_s)} \sin \Phi(\Delta B = 2)$$

$$\sin \Phi(\Delta B = 2) \simeq F \cdot \frac{m_c^2}{m_b^2} \sin \chi$$  \hspace{1cm} (60)

where one estimates

$$F \sim 3$$  \hspace{1cm} (61)

and thus

$$a_{SL}(B_s) \sim O(10^{-4})$$  \hspace{1cm} (62)

This prediction lacks numerical precision – Eq.(61) represents a rough estimate only and $\Delta \Gamma(B_s)/\Delta M(B_s)$ is not known (yet) – and there is thus no realistic hope to extract $\sin \chi$ from it.

However the small value of $\sin \Phi(\Delta B = 2)$ is again very specific to the KM ansatz and New Physics could enhance it greatly to the 1 % level. In that case the asymmetry in $B_s \rightarrow \psi \eta$ would be likewise enhanced. Studying $a_{SL}(B_s)$ thus represents a back-up option in case $B_s \rightarrow \psi \eta$, $\psi \phi$ cannot be analyzed with the required sensitivity.
4 Unitarity Triangle for Charm Decays

The $cu$ triangle $1 + \frac{V_{ub}^* V_{cb}}{V_{us} V_{cs}} + \frac{V_{ud}^* V_{cd}}{V_{us} V_{cs}} = 0$ has a very extreme shape:

$$\left| \frac{V_{ud}^* V_{cd}}{V_{us} V_{cs}} \right| = 1 + \mathcal{O}(\lambda^4), \quad \left| \frac{V_{ub}^* V_{cb}}{V_{us} V_{cs}} \right| \sim \mathcal{O}(\lambda^4) \quad (63)$$

and a tiny angle\(^1\)

$$\chi' = \text{arg} \left( \frac{V_{ud}^* V_{cd}}{V_{us} V_{cs}} \right) \simeq A^2 \lambda^4 \eta \simeq 1.6 \cdot 10^{-3} \eta \quad (64)$$

First we list decay modes that would provide us with access to this new angle; then we will address the more complex issue of how well one might do in dealing with hadronic pollution to extract a numerical value or, alternatively, how we can reliably identify the intervention of New Physics.

In describing non-leptonic $D$ decays one can draw several different looking diagrams; they are usually referred to as spectator, Penguin and weak annihilation processes. On top of that one has to allow for prominent final state interactions since charm decays proceed in a region populated by many resonances; since the final state interactions mix those processes it makes little sense to treat them individually. Instead we will keep separate contributions controlled by different combinations of KM parameters while lumping all processes with the same KM dependence into one amplitude.

One has to distinguish between two categories, namely CP violation involving $D^0 - \bar{D}^0$ oscillations and direct CP violation. The former allows for an almost zero background search for New Physics since both $D^0 - \bar{D}^0$ oscillations as well as CP phases are predicted to be quite small within the KM ansatz; their combined effect is thus truly tiny. Therefore we will discuss the latter where the prospects for observing KM effects are significantly better (though not good).

For direct CP violation to become observable in integrated rates, two amplitudes with different weak and strong phases have to contribute. The former implies – within the KM ansatz – that one has to consider Cabibbo suppressed channels; the latter is typically satisfied when two different isospin amplitudes contribute.

4.1 $D \rightarrow \pi \pi, K \bar{K}$

With $D^\pm \rightarrow \pi^\pm \pi^0$ described by a single isospin amplitude, a CP asymmetry can arise only in $D^0 \rightarrow \pi^+ \pi^-$ [and a compensating one in $D^0 \rightarrow \pi^0 \pi^0$] where the final state can be $I = 0$ or $2$ or in $D^0 \rightarrow K^+ K^-$ with $I_f = 0, 1$. We have

$$T(D^0 \rightarrow \pi^+ \pi^-) \propto V_{cd} V_{ud}^* e^{i\delta_2} |T_2| + e^{i\delta_0} \left( V_{cd} V_{ud}^* |T_0| + V_{cs} V_{us}^* |\bar{T}_0| \right), \quad (65)$$

where $T_2 [T_0, \bar{T}_0]$ denote the transition amplitudes for $I = 2, 0$ final states with the CKM parameters and strong phase shifts $\delta_2 [\delta_0]$ factored out. In a naive diagrammatic representation $T_2$ is generated by the spectator process alone and $\bar{T}_0$ solely by
Penguin dynamics whereas $T_0$ receives contributions also from Penguin and weak annihilation transitions.

For the difference between CP conjugate rates one then finds

$$\frac{\Gamma(D^0 \rightarrow \pi^+\pi^-) - \Gamma(\bar{D}^0 \rightarrow \pi^+\pi^-)}{\Gamma(D^0 \rightarrow \pi^+\pi^-) + \Gamma(D^0 \rightarrow \pi^+\pi^-)} =$$

$$\frac{2 \sin(\delta_0 - \delta_2) \text{Im} \frac{V_{cs}V_{ud}^*}{V_{cd}V_{us}}}{1 + |r|^2 + |	ilde{r}|^2 - 2|r\tilde{r}| + 2\cos(\delta_0 - \delta_2)|r| - 2\cos(\delta_0 - \delta_2)|\tilde{r}|} \quad (66)$$

with

$$r \equiv \frac{T_0}{T_2}, \quad \tilde{r} \equiv \frac{\bar{T}_0}{T_2} \quad (67)$$

Eq.(66) shows explicitly that one needs nontrivial final state interactions – $\delta_0 - \delta_2 \neq 0$ – and a KM phase – $\text{Im} \frac{V_{cs}V_{ud}^*}{V_{cd}V_{us}} \sin \chi' \simeq -A^2\lambda^4\eta \leq 10^{-3} \neq 0$ – for such an asymmetry to become observable. From Eq.(64) we infer that one expects an asymmetry of order 0.1%. It is not inconceivable that such an effect might become observable in the future allowing us to probe the value of $\chi'$.

However this number can serve as an approximate guideline only since the actual value depends on the size of hadronic matrix elements contained in $r$ and $\tilde{r}$ which are notoriously hard to evaluate. Thus at present one could not rule out with certainty that a value as ”high” as 0.5% or so could be accommodated within the KM ansatz.

4.2 $D^\pm \rightarrow K_{S,L}\pi^\pm$

The channels $D^+ \rightarrow K_S\pi^+$ as well as $D^+ \rightarrow K_L\pi^+$ can exhibit a CP asymmetry since they involve the interference between the Cabibbo allowed $D^+ \rightarrow \bar{K}^0\pi^+$ and doubly Cabibbo suppressed $D^+ \rightarrow K^0\pi^+$ channels \[20\]. There are actually two distinct sources for a CP asymmetry here, namely

- in the $\Delta C = 1$ sector we are primarily interested in and
- in $\Delta S = 2$ dynamics generating the $K_S$ (and $K_L$) mass eigenstate from the $\bar{K}^0$ and $K^0$ flavour eigenstates.

Ignoring the latter one finds for the transition amplitude

$$T(D^+ \rightarrow K_S\pi^+) = V_{cs}V_{ud}^* \left( \tilde{T}_1 + \frac{V_{cd}V_{us}^*}{V_{cs}V_{ud}^*} \tilde{T}_2 \right) \quad (68)$$

Since

$$\frac{V_{cd}V_{us}^*}{V_{cs}V_{ud}^*} = \frac{|V_{us}|^2}{|V_{ud}|^2} \cdot \frac{V_{cd}V_{ud}}{V_{us}V_{cs}} \quad (69)$$
we have

\[ \text{Im} \frac{V_{cd}V_{us}^*}{V_{cs}V_{ud}^*} = -\lambda^2 \chi' \simeq -\eta A^2 \lambda^6 \simeq -8 \cdot 10^{-5} \eta \]  

(70)

It seems unrealistic that such a tiny effect could ever be measured.

Within the KM ansatz the only observable effect is due to CP violation in the \( K^0 - \bar{K}^0 \) complex:

\[ \frac{\Gamma(D^+ \to K_S \pi^+) - \Gamma(D^- \to \bar{K}_S \bar{\pi}^-)}{\Gamma(D^+ \to K_S \pi^+) + \Gamma(D^- \to \bar{K}_S \bar{\pi}^-)} \simeq -2\text{Re} \epsilon_K \simeq -3.3 \cdot 10^{-3} \simeq \]

\[ \frac{\Gamma(D^+ \to K_L \pi^+) - \Gamma(D^- \to K_L \bar{\pi}^-)}{\Gamma(D^+ \to K_L \pi^+) + \Gamma(D^- \to K_L \bar{\pi}^-)} \]  

(71)

The real lesson to be learnt here is the following:

- Comparing \( D^+ \to K_{S,L} \pi^+ \) and \( D^- \to K_{S,L} \bar{\pi}^- \) provides an almost zero background probe for New Physics which could very conceivably enter through the doubly Cabibbo suppressed amplitude for \( D^+ \to K^0 \pi^+ \).[22]

- The intervention of New Physics can be distinguished against the effect driven by \( \epsilon_K \neq 0 \) through the size of the asymmetry and its relative sign in the \( K_L \pi^\pm \) and \( K_S \pi^\pm \) final states. For New Physics generates

\[ \frac{\Gamma(D^+ \to K_S \pi^+) - \Gamma(D^- \to K_S \bar{\pi}^-)}{\Gamma(D^+ \to K_S \pi^+) + \Gamma(D^- \to K_S \bar{\pi}^-)} = -\frac{\Gamma(D^+ \to K_L \pi^+) - \Gamma(D^- \to K_L \bar{\pi}^-)}{\Gamma(D^+ \to K_L \pi^+) + \Gamma(D^- \to K_L \bar{\pi}^-)} \]  

(72)

5 Gateways for New Physics

The corollary to testing the completeness of the KM description is to search for manifestations of New Physics. A comprehensive program analyzing \( B \) decays can also reveal salient features of that New Physics in addition to its existence.

New Physics is most likely to enter \( \Delta B = 2 \) and \( \Delta C = 2 \) dynamics driving \( B^0 - \bar{B}^0 \) and \( D^0 - \bar{D}^0 \) oscillations. Suppressed \( \Delta B = 1 \) and \( \Delta C = 1 \) decays are promising as well, in particular if the transition is dominated by a Penguin operator offering access to high mass scale dynamics.

We will illustrate this briefly through some examples:

- New Physics contributing significantly or even dominantly to \( B_s - \bar{B}_s \) oscillations could

\[ \text{At first sight it would seem that the quantities} \frac{V_{cd}V_{us}^*}{V_{cs}V_{ud}^*} \text{ or} \frac{V_{cd}V_{ud}^*}{V_{cs}V_{cs}^*} \text{ cannot be observables since they are not rephasing invariant. However the s and d phases can be absorbed into the} K \text{ state vectors – as it happens in the more familiar case of} \frac{V_{td}V_{td}^*}{V_{ub}V_{ub}^*} \text{ which controls the} CP \text{ asymmetry in} B_d \to \psi K_S. \]
lead to

\[ \phi_1 + \phi_2 + \phi_3 \neq 0, \quad (73) \]

- enhance the CP asymmetry in \( B_s(t) \rightarrow \psi \eta \) and in \( B_s \rightarrow l^- X \) by an order of magnitude even and
- cause a different value of \( \phi_3 \) to be extracted from \( B^\pm \rightarrow D_{\pm} K^\pm \) and \( B_s(t) \rightarrow D_s^* K^- \).

- If New Physics contributed to \( B_d - \bar{B}_d \) oscillations, the CP asymmetry measured in \( B_d \rightarrow \psi K_S \) would probably differ from the value of \( \phi_1 \) inferred from the \( bd \) triangle constructed through its sides.

- New Physics in the strong Penguin transition \( b \rightarrow sq\bar{q} \) could induce a significant difference in the values obtained for \( \phi_1 \) from the CP asymmetries in \( B_d \rightarrow \psi K_S \) and \( B_d \rightarrow \phi K_S \).

- New Physics could induce a time dependant CP asymmetry in \( D^0 \rightarrow K^+ K^- \), \( \pi^+ \pi^- \), \( K_S \pi^0 \), \( K_S \eta \) etc. of a few percent even and an order of magnitude larger in \( D^0 \rightarrow K^+ \pi^- \). The KM background is completely negligible.

- New Physics intervening in doubly Cabibbo suppressed channels could induce direct CP asymmetry in \( D^\pm \rightarrow K_{S,L} \pi^\pm \) of a few percent - again with insignificant KM background.

Some general comments should be made concerning \( \Delta C \neq 0 \) vs. \( \Delta B \neq 0 \) dynamics:

- A priori it is quite conceivable that qualitatively different forces drive the decays of down- and up-type quarks. More specifically, non-Standard-Model forces might exhibit a very different pattern for the two classes of quarks.

- The ‘background’ due to Standard Model forces is in general higher in the decays of up-type than down-type quarks since the former are KM allowed whereas the latter are KM forbidden transitions.

- Charm decays then offer not only the best, but probably a quite unique window onto this landscape: while on one hand nonstrange light-flavour hadrons do not allow for oscillations, doubly Cabibbo suppressed transitions and other rare decay, top hadrons on the other hand do not even form in a practical way.

- It is thus conceivable that even detailed studies of beauty decays might not reveal the intervention of New Physics, yet charm decays will!

\[ ^{\S} \text{This asymmetry should not be confused with a direct CP asymmetry.} \]
6 Summary and Outlook

Some of our findings are of a qualitative and others of a more quantitative nature:

- The six unitarity triangles (and the three weak universality relations) represent the information contained in the KM matrix in an immensely overconstrained form.
  - Fourteen out of the total of eighteen angles are naturally large; two are of order $\lambda^2$ and the two remaining ones of order $\lambda^4$. To leading order in $\lambda$ those fourteen large angles coincide into just two independent angles and their complements. In order $\lambda^2$ differences emerge between them. These findings are already apparent in the structure of the KM matrix, see Eq.(9).
  - In principle all angles of the six triangles could be measured through CP asymmetries in top, beauty, charm and strange weak decays.
  - In practice that is not possible for a variety of reasons: absence of top hadrons, tiny effective branching ratios, theoretical uncertainties in the size of hadronic matrix elements etc.
  - Yet the measurements that appear feasible will still provide us with a highly overconstrained data set that carries a high promise for revealing New Physics.

- CP studies in $B$ decays will at first proceed in two stages that have been discussed extensively in the literature:
  - The first task will be to establish the existence of CP violation in $B$ decays, most likely through observing $\sin 2\phi_1 \neq 0$. To increase the size of the available sample one will put together different channels driven by the same quark level transition, namely $B_d \rightarrow \psi K_S, \psi K_L, D \bar{D}$ etc.
  - The next task will be to extract $\phi_2$ and $\phi_3$ where one has to face also the theoretical challenge of having to deal with more than one transition operator and its hadronic matrix elements.
    * As far as $\phi_2$ is concerned one will presumably tackle this problem by studying several channels driven by the same quark level transition, namely $B \rightarrow \pi's$.
    * For $\phi_3$ on the other hand one will analyze different quark level transitions. Their CP asymmetries depend on angles that to leading order in $\lambda$ coincide with $\phi_3$. It will be crucial to see whether the different transitions yield consistent values of $\phi_3$.

- Yet a comprehensive program has to push tests of the completeness of the KM description considerably further:
Ultimately the goal has to be to go after the $\mathcal{O}(\lambda^2) \sim 5\%$ differences predicted by the KM ansatz between different angles that agree to leading order. A significant deviation from those predictions would reveal New Physics. The most promising case is provided by comparing the CP asymmetries in $B^+ \to D_\pm K^+$, $B_s(t) \to K_S \rho^0$ and $B_s(t) \to D_s^+ K^-$:

$$
\begin{align*}
B^+ \to D_\pm K^+ & \quad \Rightarrow \quad \tan \phi_3 = \frac{\eta}{\rho} \left[ 1 - \frac{1}{2} \lambda^2 \left( 1 - 2 \rho - \frac{2 \eta^2}{\rho} \right) \right] \\
B_s(t) \to K_S \rho^0 & \quad \Rightarrow \quad \tan \phi_3^{tu} = \frac{\eta}{\rho} \left[ 1 - \frac{1}{2} \lambda^2 \right] \\
B_s(t) \to D_s^+ K^- & \quad \Rightarrow \quad \tan \phi_2^{bs} = \tan \phi_3
\end{align*}
$$

Rather than search for differences predicted to be small between two large angles one can undertake to measure novel angles that are small in the KM description.

Such an angle enters in the $bs$ triangle denoted by $\chi \sim \mathcal{O}(\lambda^2)$. Its value can be extracted in a theoretically clean way from $B_s \to \psi \eta$ and $B_s \to \psi \phi$ where in the latter case one has to separate the contributions from even and odd angular momentum partial waves. The CP asymmetry is reliably predicted to be parametrically

$$
\sin 2\chi \simeq 2\lambda^2 \eta
$$

which numerically translates into $2 \div 5 \%$ at present.

* Future data on $B_d \to \psi K_S$ vs. $\bar{B}_d \to \psi K_S$ in particular will make the KM prediction of Eq. (73) more precise numerically.

* Observing an asymmetry above the expected value establishes the presence of New Physics.

* Every effort should be made to measure an asymmetry in $B_s \to \psi \eta$ even if its size appears to be consistent with the KM expectation. For with $\chi$ and $|V_{ub}/V_{cb}|$ one can construct the rescaled $bs$ triangle and read off $\phi_3^{bs}$; the latter can then be compared with the value of $\phi_3$ extracted from other transitions. From a significant numerical difference between the two one can infer New Physics.

It has to be stressed that these $\mathcal{O}(\lambda^2)$ effects could be considerably larger due to New Physics.

Another new angle enters in the $cu$ triangle, namely $\chi'$, which is very small:

$$
\chi' \simeq A^2 \lambda^4 \eta \sim 10^{-3}
$$

It will give rise to CP asymmetries in $D^0 \to \pi^+ \pi^-$, $K^* K$, $\rho \pi$ etc. with a characteristic scale of 0.1 %. The main theoretical uncertainty originates

---

*This does not mean it is a promising case, though.
from the relative size of the various hadronic matrix elements including their phase shifts. On general grounds one actually expects considerable variations in the size of the observable CP asymmetries in the different channels around the 0.1 % level. At present values of 0.5 % – and possibly even 1 % – appear conceivable without a clear need of New Physics. Three comments can elucidate the situation:

* A dedicated experimental effort should be made to study charm decays with a sensitivity level of 0.1 % for CP asymmetries. Finding a signal is a fundamental discovery irrespective of its theoretical interpretation.
* Establishing a signal above the 1 % level provides strong evidence for New Physics.
* Detailed theoretical engineering – namely describing a host of well-measured D decay channels – might enable us to establish whether an observed asymmetry is still consistent with KM expectation or reveals New Physics [21].

– The related quantity

\[ \text{Im} \frac{V_{cd}V_{us}^*}{V_{cs}V_{ud}^*} = -\lambda^2 \chi' \approx -\eta A^2 \lambda^6 \approx -\text{few} \times 10^{-5} \]  

(77)

can in principle be probed in \( D^\pm \to K_{S,L} \pi^\pm \) decays. In practice it is so small that there is no realistic hope to ever extract it. On the other hand that means that any CP asymmetry observed here over the well-known and precisely predicted one due to \( \epsilon_K \) is safely ascribed to New Physics.

To state it in a nutshell: before an experimental program on \( B^0 - \bar{B}^0 \) oscillations and CP violation can be called complete it must meet at least the following benchmarks:

* \( \phi_1, \phi_2 \) and \( \phi_3 \) have been extracted with a relative accuracy of better than 5%.

* The values of \( \phi_3 \) inferred from different B transitions have been compared and it has been analyzed whether the \( \lambda^2 \) corrections predicted by the KM scheme can be identified. While it is not clear whether this is a realistic goal, it has to be attempted nevertheless.

* It is absolutely mandatory to measure \( \chi \) in \( B_s \to \psi \eta, \psi \phi \) with a sensitivity on the percent level which carries a high potential to reveal even a subtle intervention of New Physics.

* The angle \( \chi' \) induces direct CP asymmetries in Cabibbo suppressed nonleptonic D decays on about the 0.1 % level. Every effort should be made to acquire the experimental sensitivity to observe such effects. Establishing whether an observed effect indeed is consistent with KM or requires New Physics will be a highly challenging task unless the signal is an order of magnitude larger. Yet it represents an important benchmark nevertheless.
Figure 1: The unitarity triangle for B decays.

- Finally one has to search for an asymmetry in $D^\pm \to K_{S,L} \pi^\pm$ decays. Irrespective of the origin of CP violation an asymmetry has to arise there as described by the observable $\epsilon_K$; KM $\Delta C = 1$ dynamics creates an asymmetry described by $\lambda^2 \chi' \leq 10^{-4}$ which in all likelihood is too small to be ever observed. Such studies thus represent zero background searches for New Physics – analogous to probing for CP asymmetries in the decay time evolutions in $D^0(t) \to \pi^+\pi^-$, $K^+K^-$, $K^+\pi^-$ transitions which involve $D^0 - \bar{D}^0$ oscillations.

- The discovery potential for New Physics in heavy flavour Dynamics has not been exhausted, unless a comprehensive and dedicated program in charm decays has been pursued!

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