GAMMA-RAY BURST SYNTHETIC SPECTRA FROM COLLISIONLESS SHOCK PIC SIMULATIONS

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ABSTRACT

The radiation from afterglows of gamma-ray bursts is generated in the collisionless plasma shock interface between a relativistic outflow and a quiescent circum-burst medium. The two main ingredients responsible for the radiation are high-energy, non-thermal electrons and a strong magnetic field. In this Letter we present, for the first time, synthetic spectra extracted directly from first principles particle-in-cell simulations of relativistic collisionless plasma shocks. The spectra are generated by a numerical Fourier transformation of the electromagnetic field and the non-thermal particle acceleration. Both the electromagnetic field and the non-thermal particle acceleration are self-consistent products of the Weibel two-stream instability. We find that the radiation spectrum from a $T = 15$ shock simulation show great resemblance with observed GRB spectra—we compare specifically with that of GRB 000301C.

Subject headings: radiation, acceleration of particles, gamma rays: bursts, shock waves, instabilities, magnetic fields, plasmas

1. INTRODUCTION

Radiation from gamma-ray burst (GRB) afterglows is generally believed to be synchrotron radiation. The main ingredients in this type of radiation are a strong magnetic field and a power-law distribution of high-energy electrons. Multi-wavelength observations are more or less well fitted by hiding our ignorance about the microphysical details of these ingredients in the parameters: $\epsilon_B$ and $\epsilon_e$ describing the amount of the total shock energy that is deposited in magnetic field and electrons respectively, and $p$: the slope of the non-thermal electron distribution, $N(\gamma) \propto \gamma^{-p}$ where $\gamma = [1 - (\nu/c)^2]^{-1/2}$ is the relativistic Lorentz factor. However, the true origin and nature of these two ingredients remain unexplained.

Regarding the magnetic field, observations indicate that the typical value of the equipartition parameter in afterglows is $\epsilon_B = 0.0001 - 0.1$ (Panaitescu & Kumar 2002; Waxman 1997). This is many orders of magnitude larger than what can be achieved by shock compression of the interstellar medium. Even if the field is injected from the progenitor, it is hard to maintain such a high level of magnetic field in the shock region. (Gruzinov & Waxman 1999) and (Medvedev & Loeb 1999) suggested that the Weibel two-stream instability can generate a strong turbulent magnetic field in the shock region (Weibel 1959). This has been confirmed by numerical simulations (Frederiksen et al. 2004; Hededal 2005; Hededal et al. 2004; Hededal & Nissankawa 2005; Nissankawa et al. 2003; Silva et al. 2005). These simulations have shown that the two-stream instability generated magnetic field is highly turbulent and varies greatly through the shock region. In the non-linear stage of the instability, filaments merge and can become as strong as $\epsilon_B \sim 0.1$. Recently (Hededal et al. 2004) have further shown that the Weibel two-stream instability also involves non-thermal acceleration of the electrons. This new acceleration mechanism differs from Fermi acceleration in the sense that it is local and instantaneous. Electrons are trapped in the potential of the generated current filaments and oscillate with strong acceleration/ re-acceleration. The numerical experiments have also shown that the collisionless shock transition zone is at least of the order hundreds of ion skin-depths (Frederiksen et al. 2004; Hededal 2005; Hededal et al. 2004). This is considerably thicker than previously thought (Gruzinov 2001).

In this Letter we present a novel tool that allow us for the first time to extract radiation spectra directly from PIC experiments. In section 2 we present the radiation tool, in section 3 we present the simu...
we apply the tool to investigate the 3D jitter radiation spectrum, and in section 4 we present synthetic radiation spectra from a 3D relativistic shock simulation resembling a GRB afterglow collisionless shock. Finally, in Section 5 we give a brief discussion on the results and compare with observations.

2. SYNTHETIC SPECTRA FROM PIC-CODE SIMULATIONS

A particle-in-cell (PIC) code simulates plasma on a much more fundamental level than MHD. The code works from first principles, by solving the Maxwell equations with source terms for the electromagnetic fields, together with the relativistic equation of motion for a large number of charged particles. The electromagnetic fields are discretized on a three-dimensional grid whereas the particles are defined with continuous positions and momenta within the grid. The relativistic PIC code that we have used is described in Frederiksen et al. (2004) and Hededal et al. (2004). Recently, radiative cooling and synthetic spectra generation has been added to the code (Hededal 2005).

As a particle undergoes acceleration it will emit radiation. In the PIC code we know this acceleration at each time step and hence we can reduce the energy of the particles in accordance with the Larmor radiation formula. The energy loss is highly dependent on the particles Lorentz factor and for relativistic particles, most of the radiated energy is beamed into a narrow cone with opening angle $1/\gamma$ around the momentum vector. Because of this relativistic beaming, we take an approximative approach and subtract the momentum lost to radiation along the velocity vector of the particle. We have tested this implementation of radiative cooling against the analytical result for a gyro-orbiting electron in a homogenous magnetic field and find excellent agreement (Hededal 2005).

Given the detailed information of particle positions, velocities, and accelerations it is possible to compute the emitted radiation spectrum. To do this, we start by adopting the expression from Jackson (1999) for the retarded electric field from a charged particle moving with instantaneous velocity $\beta = v/c$ under acceleration $\dot{\beta} = \ddot{v}/c$,

$$E = \frac{q}{4\pi\epsilon_0 c} \frac{\mathbf{n} \times \{ (\mathbf{n} - \beta) \times \dot{\beta} \}}{(1 - \mathbf{n} \cdot \beta)^2 R_{\text{ret}}}.$$  \hfill (1)

Here, $q$ is the unit charge, $c$ is the speed of light, $\epsilon_0$ is the vacuum electric permittivity, and $R$ is the distance to an observer. $\mathbf{n}$ is a unit vector that points from the particles retarded position towards the observer. In Eq. (1) we have neglected a "velocity term", which is justified when the typical length scale of the emitter is negligible compared to the distance to the observer (Jackson 1999). The radiation spectrum from an accelerated charge is then found by Fourier transforming the Poynting flux on Eq. (1) in the retarded time frame. This gives us the energy $d^2W$ radiated per unit solid angle $d\Omega$ and unit frequency $d\omega$.

$$\frac{d^2W}{d\Omega d\omega} = \frac{\mu_0 q c^2}{16\pi^3} \left| \int_{-\infty}^{\infty} \frac{\mathbf{n} \times \{ (\mathbf{n} - \beta) \times \dot{\beta} \}}{(1 - \mathbf{n} \cdot \beta)^2} e^{i\omega(t' - \mathbf{n} \cdot \mathbf{r}(t)/c)} dt' \right|^2.$$  \hfill (2)

This expression can only be analytically integrated under constrains on the morphology of the electromagnetic field. For example, in the case of a relativistic electron moving in a homogenous magnetic field with zero electric field, the integral reduces to the well known synchrotron case (Rybicki & Lightman 1979). In the general electromagnetic case, no analytical solution exists. However, from the PIC experiments we readily have positions, velocities and accelerations for the particles that define the plasma. With this information, we can numerically integrate Eq. (2) to obtain a synthetic radiation spectrum from each of the particles in a plasma. Finally we can add the spectrum of each particle into a total spectrum. Adding the spectrum from each particle linearly is valid as long as the phase of each contribution is completely uncorrelated to the others.

We have implemented Eq. (2) and tested several scenarios where an analytical solution exist. This includes synchrotron radiation, bremsstrahlung, and wiggler/undulator radiation. In all tests we find that our radiation tool can effectively reproduce the theoretical expected solutions. Figure 1 shows the synthetic spectrum (solid blue) of a single relativistic electron gyrating in a homogenous magnetic field. The envelope curve (dashed red) shows the analytical solution, which shows good agreement with the simulations. The inset in Fig. 1 shows that the peaks in the spectrum fall on the relativistic gyro-frequency and its higher harmonics as expected. For more details and tests, see Hededal (2005).

![Fig. 1](image)

3. EXAMPLE: 3D JITTER RADIATION

The PIC plasma simulation code, in combination with the radiation generation module, provides us with a powerful tool for testing various non-linear problems. In this section we examine the so-called jitter- or diffuse synchrotron radiation spectrum (Fleishman 2003; Medvedev 2000, 2005). This type of radiation is generated by relativistic particles that are deflected in a turbulent magnetic field, where the deflection angle is comparable to the relativistic beaming angle.

We setup PIC experiments with a periodic random magnetic field with a power spectrum that follows a power-law distribution in the Fourier domain $P_B(k) \propto k^\mu$ (Fig. 2). In this field we use the PIC code to trace an isotropic momentum distributions of electrons with a Lorentz factor $\gamma = 3$. The strength of the magnetic field is set such that the maximum deflection of the electrons is smaller than the relativistic beaming angle for all Fourier nodes.
GRB synthetic spectra

Fig. 2.— *Left column:* The Fourier structure of the turbulent magnetic field used in the jitter radiation experiments. Here we show three different values of the spectral power law index $\mu$: $\mu = -2$ (red noise, top), $\mu = 0$ (white noise, middle) and $\mu = 2$ (blue noise, bottom). *Right column:* Corresponding spatial 2D-slices of the magnetic field.

Fig. 3.— The jitter spectrum from a mono-energetic ensemble of electrons ($\gamma = 3$) moving in a turbulent magnetic field. The different graphs are for simulations with different values of $\mu$.

Fig. 4 shows the resulting radiation spectrum for different values of the slope $\mu$ of the magnetic Fourier decomposition $P_B(k) \propto k^\mu$. For $\mu < 0$, the high-energy part of the spectrum follows a power-law with a slope $\alpha \simeq \mu - 1$ independent of the electron energy. For $\mu > 0$, the flat spectrum continues to higher frequencies than for $\mu < 0$ and the spectrum has a hard cut-off at high frequencies. In all cases, the low energy part is flat and quite similar to the case of relativistic bremsstrahlung.

We observe that as the strength of the magnetic field is increased, so that the typical deflection angle of the electrons becomes larger than the relativistic beaming angle, the resulting spectrum converges through the wiggler domain to the synchrotron case [Attwood 2000].

4. RADIATION FROM COLLISIONLESS SHOCK

Our goal is to obtain spectra from the PIC shock experiments that may be compared directly with observations. Even though it is beyond the scope of this Letter, this will eventually allow us to put constraints on the conditions and physics of GRB afterglows such as GRB environments and jet structure.

We use the numerical PIC experiment studied in [Hededal et al. 2004]. Two colliding plasma populations are studied in the rest frame of one of the populations (downstream, e.g. representing the shocked ISM medium at the head of a GRB jet). In this frame of reference, a less dense population (upstream, representing the unshocked interstellar medium) is continuously injected at the left hand side boundary, with a relativistic velocity corresponding to a Lorentz factor $\Gamma = 15$. The two populations initially differ in density by a factor of 3. We use a computational box with $125 \times 125 \times 2000$ grid points and a total of $8 \times 10^6$ particles. The ion rest-frame plasma frequency in the downstream medium is $\omega_{pi} = 0.075$, rendering the box 150 ion skin depths long. The electron rest-frame plasma frequency is $\omega_{pe} = 0.3$ in order to resolve both the microphysics of the electrons. Hence, the ion-to-electron mass ratio is $m_i/m_e = 16$.

As the two plasma populations interpenetrate, we observe how the Weibel two-stream instability is excited and current filaments are formed [Frederiksen et al. 2004; Hededal et al. 2004; Medvedev & Loeb 1999; Nishikawa et al. 2003; Silva et al. 2003; Weibel 1959]. The current filaments induce a highly intermittent electromagnetic field with a strength of up to 10% of equipartition. The nature of the magnetic field is turbulent, with a power spectrum that follows a power-law in the Fourier domain [Frederiksen et al. 2004]. We note however, that unlike the random fields in section 3 there exists phase-correlations in the generated magnetic field that give the field a more coherent structure. The instability also directly leads to acceleration of electrons through a newly discovered mechanism [Hededal et al. 2004; Nishikawa et al. 2005b]. Electrons caught within the Debye cylinder surrounding the ion current filaments are strongly and repeatedly accelerated and decelerated. The acceleration is instantaneous and local, and thus differs substantially from recursive acceleration mechanisms such as Fermi acceleration. The resulting particle distribution function has a non-thermal high-energy component, correlated with the distribution of the generated electromagnetic filaments. We note in passing that a fraction of the ions are scattered back into the interstellar medium. Thus, ion Fermi acceleration may possibly take place in collisionless shocks.

To obtain the total spectrum from the accelerated particles in the Weibel-generated field, we have traced 20,000 particles from the PIC simulation described above. Placing the observer somewhere along the jet direction we have numerically integrated Eq. 2 for each of these particles.

Fig. 4 shows the radiated spectrum from the collisionless shock simulations described above. Here, we have rescaled the frequency into real space units and taken the relativistic...
fraction of the total internal energy that is deposited in magnetic field and electrons, respectively, and $p$ is the slope of the supposedly power-law electron momentum distribution function.

Since collisionless shocks are extremely non-linear and self-interacting systems, the only way to study their microphysics with any credence is through self-consistent, three-dimensional relativistic particle simulations. Using this approach we have previously explained the origin and nature of a strong electromagnetic field in the shock (Frederiksen et al. 2004) and been able to identify a new non-thermal electron acceleration mechanism (Hededal et al. 2004). These two ingredients are direct and unavoidable consequences of the Weibel two-stream instability.

In this Letter we have developed a novel tool that allows us to extract radiation spectra directly from the particle-in-cell experiments. Rather than applying the synchrotron approximation (assuming homogenous magnetic field and a single electron power-law distributed population) we trace a large number of electrons in the experiments, and calculate the exact radiation emitted from each electrons. The tool has been thoroughly tested and successfully reproduces spectra from synchrotron radiation, bremsstrahlung and undulator/wiggler radiation from small-angle deflections (Hededal 2005).

The tool has been utilized for a parametric study of 3D jitter radiation, where a population of electrons radiate as they move in a turbulent magnetic field whose power spectrum follows a power-law in the Fourier domain $P_n(k) \propto k^\alpha$. We have examined how the slope $\mu$ influence on the resulting radiation spectra. For $\mu < 0$, the high-energy part of the spectrum follows a power-law with a slope $\alpha \approx \mu - 1$ independent of the electron energy. For $\mu > 0$, the flat spectrum continues to higher frequencies than for $\mu < 0$ and the spectrum has a hard cut-off for high frequencies.

We have furthermore examined the radiation from a typical GRB afterglow collisionless shock that propagates with $\Gamma = 15$ through the interstellar medium. We find that the resulting radiation spectrum peaks just below $10^{12} \text{Hz}$. Above this frequency, the spectrum follows a power-law $F \propto \nu^{-\beta}$, with $\beta = 0.7$. Below the peak frequency, the spectrum follows a power-law $F \propto \nu^\alpha$ with $\alpha \approx 2/3$. This is steeper than the standard synchrotron value of $1/3$ and thus more compatible with observations. Both the slope and the peak is consistent with observations (e.g. Panaiteșcu 2001) who finds $\beta = 0.67 \pm 0.04$, a peak at $3 \times 10^{11} \text{Hz}$ and $\Gamma > 10$ for the afterglow of GRB 000301C after five days.

Finally, we would like to stress the following interesting point: If one hides all the real physical details of the magnetic field in the dimensionless parameters $\epsilon_B$, $\epsilon_e$, and $p$ the conclusion from standard synchrotron radiation theory would be that the slope of the electron distribution $N(\gamma) \propto \gamma^{-p}$ is found by solving $-\beta = -(p-1)/2 = -0.70 \rightarrow p = 2.4$ (which is consistent with standard analysis of the GRB 000301C afterglow; Panaiteșcu 2001). This would appear to be consistent with Fermi acceleration and the supposedly universal $p \approx 2.2 \pm 0.2$. But a closer look reveals that the acceleration mechanism is not Fermi acceleration but rather a natural consequence of the Weibel two-stream instability. The electrons are instantaneously accelerated and decelerated in the highly complicated electric and magnetic field near the ion current channels. Strong radiation is produced in this process. The electron distribution behind the radiation is a mixture of a thermal component for low energies and a power-law component with $p = 2.7$ for high energies (Hededal et al. 2004).
The paths of the electrons are more random than circular and the electron distribution function varies with shock depth, as does the magnetic topology and strength.

The outcome of the exercise is thus that it is indeed possible, from first principles, to produce synthetic spectra very similar to the ones that are observed in GRBs, and that the fit of such spectra with the standard model assumptions may be more or less fortuitous.

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