Radiative feedback from ionized gas

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ABSTRACT
H\textsubscript{2} formation in metal-free gas occurs via the intermediate H\textsuperscript{−} or H\textsuperscript{2+} ions. Destruction of these ions by photodissociation therefore serves to suppress H\textsubscript{2} formation. In this paper, I highlight the fact that several processes that occur in ionized primordial gas produce photons energetic enough to photodissociate H\textsuperscript{−} or H\textsuperscript{2+} and outline how to compute the photodissociation rates produced by a particular distribution of ionized gas. I also show that there are circumstances of interest, such as during the growth of H\textsubscript{II} regions around the first stars, in which this previously overlooked form of radiative feedback is of considerable importance.

Key words: atomic processes – astrochemistry – galaxies: formation – cosmology: theory

1 INTRODUCTION
It has long been known that ro-vibrational line emission from molecular hydrogen, H\textsubscript{2}, is the dominant cooling process in gas of primordial composition at temperatures 200 < T < 10\textsuperscript{4} K (Saslaw & Zipoy 1967; Peebles & Dickel 1968; Matsuda, Sato, & Takeda 1969; Hirasawa 1969). Moreover, recent work has made it clear that H\textsubscript{2} cooling also plays a key role in the evolution of low-density metal-poor gas (Jappsen et al. 2007). In primordial gas and in low density metal-poor gas, H\textsubscript{2} formation on dust is ineffective, and H\textsubscript{2} forms primarily through the gas-phase reactions (McDowell 1963; Peebles & Dickel 1968)

\begin{align}
H + e^- &\rightarrow H^- + \gamma, \\
H^- + H &\rightarrow H_2 + e^-, \\
H^- + \gamma &\rightarrow H + e^-,
\end{align}

with a smaller contribution coming from (Saslaw & Zipoy 1967)

\begin{align}
H + H^+ &\rightarrow H_2^+ + \gamma, \\
H_2^+ + H &\rightarrow H_2 + H^+.
\end{align}

Although only a small fractional abundance of H\textsubscript{2}, of order 10\textsuperscript{−3}, is produced by these reactions, many authors have shown that this is more than sufficient to allow the gas to cool within a Hubble time, and to allow star formation to occur (see e.g. Tegmark et al. 1997; Bromm, Coppi, & Larson 2002; Abel, Bryan, & Norman 2002; Yoshida et al. 2003).

Once stars have formed, however, their radiation can interfere with the gas-phase production of H\textsubscript{2}. Most of the attention devoted to the study of so-called “radiative feedback” has focused on the ultraviolet radiation from massive stars, which can suppress further star formation by photoionizing atomic and molecular hydrogen (Haiman, Thoul & Loeb 1996; Kitayama et al. 2004; Whalen, Abel & Norman 2004; Whalen & Norman 2006; Abel, Wise & Bryan 2007) and by photodissociating H\textsubscript{2} (Haiman, Rees & Loeb 1997; Omukai & Nishi 1999; Haiman, Abel & Rees 2006; Glover & Brand 2001), with the latter process able to operate at far greater distances from the star, owing to the low opacity of the gas at frequencies redwards of the Lyman limit. On the other hand, radiative feedback due to the photodissociation of H\textsuperscript{−} and H\textsuperscript{2+}

\begin{align}
H^- + \gamma &\rightarrow H + e^-, \\
H_2^+ + \gamma &\rightarrow H + H^+,
\end{align}

has attracted little direct study. One reason for this is the speed of reactions 2 and 3 both have rate coefficients k ≈ 10\textsuperscript{−9} cm\textsuperscript{3} s\textsuperscript{−1}, and so for reactions 5 or 6 to be effective at suppressing H\textsubscript{2} formation, they must occur at a rapid rate. The other main reason for the comparative neglect of these processes is the nature of the radiation sources typically assumed in studies of radiative feedback. Work to date has focused on feedback from massive stars or from AGN. These sources are bright in the far-UV, and previous studies have found that feedback due to H\textsubscript{2} photodissociation by UV photons from these sources becomes effective long before feedback due to H\textsuperscript{−} or H\textsuperscript{2+} photodissociation by optical or near-IR photons from the same sources (see e.g. Machacek, Bryan & Abel 2001).

However, one issue that does not appear to have been previously considered in any great detail is that fact that once sources of ionization such as massive stars or AGN exist, the ionized gas that they produce will act as a secondary source of radiation. Radiative feedback due to the pho-
2 MODELLING EMISSION FROM IONIZED GAS

The radiation flux at a location \( x \) produced by emission from an ionized volume \( V \) is given by:

\[
F(\nu) = \frac{1}{4\pi} \int_V e^{-\tau(\nu, x', x)} \frac{n_e(x') \sum_i \gamma_i(\nu, x') n_i(x')}{|x' - x|^2} \, dx'.
\]

(7)

where \( \tau(\nu, x', x) \) is the optical depth at frequency \( \nu \) between \( x \) and \( x' \), \( n_e \) is the number density of electrons, \( n_i \) is the number density of ions of species \( i \), \( \gamma_i \) is the emission coefficient for species \( i \) (with units erg cm\(^2\) s\(^{-1}\) Hz\(^{-1}\)), and we sum over all ionic species present in the gas. For clarity, in the simplified examples presented in this paper, I restrict my attention to emission from recombining \( H^+ \) ions. In normal circumstances this will produce the bulk of the emission, with a small additional contribution coming from helium, and with only negligible amounts of radiation coming from other ions. This simplification allows us to rewrite equation (7) as:

\[
F(\nu) = \frac{1}{4\pi} \int_V e^{-\tau(\nu, x', x)} \frac{n_e(x') n_{H^+}(x') \gamma_{H^+}(\nu, x')}{|x' - x|^2} \, dx'.
\]

(8)

where \( n_{H^+} \) is the number density of \( H^+ \) ions and where \( \gamma_{H^+} \) is the emission coefficient for emission from \( H^+ \). We can further simplify this equation by making the assumption that the temperature of the ionized gas is uniform. In this case \( \gamma_{H^+}(\nu) \) becomes independent of position and can be moved outside the integral:

\[
F(\nu) = \frac{\gamma_{H^+}(\nu)}{4\pi} \int_V e^{-\tau(\nu, x', x)} \frac{n_e(x') n_{H^+}(x')}{|x' - x|^2} \, dx'.
\]

(9)

A final simplification comes if we assume that the gas is optically thin. At the densities of interest in this work, this is a good approximation for most of the emission, as the continuum opacity of metal-free gas is very small (Lenzini, Chernoff & Salpeter 1991). The exceptions are the bound-free emission produced by recombination directly into the \( n = 1 \) ground state, and the Lyman series lines. As I discuss at greater length in the next section, this emission can be taken into account by assuming that case B recombination applies.

The assumption of optically thin gas allows us to rewrite Equation (9) as:

\[
F(\nu) = \gamma_{H^+}(\nu) I_V,
\]

(10)

where

\[
I_V = \frac{1}{4\pi} \int_V \frac{n_e(x') \gamma_{H^+}(x')}{|x' - x|^2} \, dx'.
\]

(11)

With these simplifications we have reduced the problem of computing the flux into two separate, simpler problems: that of computing \( \gamma_{H^+}(\nu) \) and that of evaluating the integral \( I_V \).

For a given distribution of ionized gas, numerical evaluation of the integral is trivial, and so the only remaining difficulty is the evaluation of \( \gamma_{H^+}(\nu) \). I discuss this in the next section.

2.1 Computing the emission coefficient

2.1.1 Bound-bound emission

In my treatment of the effects of recombination, I assume that Case B applies. In the classical Case B, the gas is optically thick in both the Lyman continuum and the Lyman series lines. Therefore, emission produced by recombination directly to the \( n = 1 \) ground state will be immediately re-absorbed, with the result that only recombinations to excited states with \( n \geq 2 \) actually result in a net decrease in the number of ionized hydrogen atoms. Every recombination to a state with \( n \geq 2 \) will be followed by a radiative cascade as the atom attempts to reach the ground state and some fraction of the photons produced during these cascades will be capable of photodissociating \( H^- \) or \( H_2^+ \).

The photodissociation threshold of \( H^- \) is 0.755 eV, and so \( H^- \) can be photodissociated by any photons in the Lyman series or Balmer series, and by most photons in the Paschen series (starting with Paschen-\( \beta \)). \( H_2^+ \), with its larger photodissociation threshold of 2.65 eV, can be dissociated by any Lyman series photon, and by most Balmer series photons (starting with H\( \gamma \)), but is not affected by Paschen series photons.

For the emissivities of the lines in the Balmer and Paschen series, I use the values computed for Case B by Storey & Hummer (1993), who tabulate frequency-averaged emissivities for transitions from all excited states up to \( n = 50 \). As the widths of these emission lines are much smaller than the frequency range over which the photodissociation cross-sections of \( H^- \) or \( H_2^+ \) vary substantially, accurate modelling of the line profiles is unnecessary and so for simplicity I model the lines as delta functions.

In Case B, all Lyman series photons more energetic than Lyman-\( \alpha \) are eventually degraded to Lyman-\( \alpha \) photons plus one or more lower energy photons. While the results of Storey & Hummer (1993) include these lower energy photons, they do not account for the eventual fate of the Lyman-\( \alpha \) photons. In the classical Case B, these photons
can never escape from the ionized region: the optical depth of the Lyman series lines is assumed to be infinite. In reality, the optical depth is finite and some photons can escape from the gas, although generally not before they have scattered multiple times. There is also a small chance that an excited hydrogen atom in the 2p state will reach the ground state by direct radiative decay to the 1s state, but rather by undergoing a collisional transition to the 2s state, followed by two-photon decay to the ground state. Over time, this process results in the net loss of a large number of Lyman-α photons from the ionized gas.

The relative importance of these two processes depends upon a number of factors, such as the density of the H II region and the Lyman-α optical depth of the gas. For simplicity, I consider in this paper only the limiting case in which two-photon decay dominates. In this case, the contribution to \( \gamma_{\text{th}}(\nu) \) arising from two-photon emission can be written as (Fernandez & Komatsu 2006):

\[
\gamma_{2\text{ph}}(\nu) = \frac{2h\nu}{\nu_{\text{Ly}a}} P(\nu/\nu_{\text{Ly}a})\alpha_B, \tag{12}
\]

where \( \nu_{\text{Ly}a} \) is the frequency at line center of the Lyman-α line, \( \alpha_B \) is the Case B recombination coefficient and \( P(y)dy \) is the normalized probability per two-photon decay of getting one photon in the interval \( dy = dv/\nu_{\text{Ly}a} \).

For \( \alpha_B \), I use the values tabulated by Storey & Hummer (1993), while for \( P(y) \) I use a fitting function taken from Fernandez & Komatsu (2000):

\[
P(y) = 1.307 - 2.627z^2 + 2.563z^4 - 51.69z^6, \tag{13}
\]

where \( z = y - 0.5 \).

### 2.1.2 Bound-free and free-free emission

The contribution to \( \gamma_{\text{th}}(\nu) \) from bound-free and free-free emission can be written as (Fernandez & Komatsu 2006):

\[
\gamma_{\text{bf+ff}} = \frac{6.84 \times 10^{-38}}{T^{1/2}} e^{-h\nu/kT} \left[ \bar{g}_{\text{ff}} + \sum_{n=2}^{\infty} \frac{x_n e^{x_n}}{n} g_{0,n}(n) \right] \tag{14}
\]

where \( T \) is the gas temperature, \( x_n = 1\text{Ryd}/(kT n^2) \), \( \bar{g}_{\text{ff}} \) is the thermally averaged Gaunt factor for free-free emission and \( g_{0,n}(n) \) is the Gaunt factor for free-bound emission from recombination into level \( n \). The value of this expression has been tabulated as a function of frequency and temperature by Ferland (1980). In Figure 1 I plot the value of \( \gamma_{\text{bf+ff}} \) as a function of photon energy for photons with \( 0.755 < h\nu < 13.6 \text{eV} \), assuming a gas temperature \( T = 10^4 \text{K} \). In the same figure, I also plot the two-photon contribution \( \gamma_{2\text{ph}} \). At low energies, close to the \( H^- \) photodissociation threshold, the bound-free and free-free contributions dominate, while at high energies, the two-photon contribution dominates. It is also plain from the figure that if we were to disregard the two-photon emission, and to instead assume that all of the Lyman-α photons produced by the ionized gas eventually escape by scattering into the wings of the line, we would nevertheless obtain photodissociation rates for \( H^- \) and \( H^+_2 \) of the same order of magnitude as those derived below.

![Figure 1](link)

**Figure 1.** Emission coefficients, plotted as a function of photon energy, for the bound-free and free-free emission (summed and plotted as solid line) and two-photon emission (dashed line) produced by an ionized gas of pure hydrogen with temperature \( T = 10^4 \text{K} \).

### 2.2 Evaluating the photodissociation rates

Given \( \gamma_{\text{th}}(\nu) \), it is easy to compute the photodissociation rates of \( H^- \) and \( H^+_2 \) as functions of the volume integral \( I_\nu \). The \( H^- \) photodissociation rate can be written as

\[
R_{\text{pd}, H^-} = \int_0^\infty \frac{\sigma_{H^-}(\nu) F(\nu)}{h\nu} d\nu, \tag{15}
\]

\[
= I_\nu \int_0^\infty \frac{\sigma_{H^-}(\nu) \gamma_{\text{th}}(\nu)}{h\nu} d\nu \tag{16}
\]

where the photodissociation cross section is (Wishart 1979)

\[
\sigma_{H^-}(\nu) = 7.928 \times 10^3 \frac{(\nu - \nu_{\text{th}})^{3/2}}{\nu^3}, \tag{17}
\]

at frequencies above the \( H^- \) photodissociation threshold \( \nu_{\text{th}} = 0.755 \text{eV} \). The \( H^+_2 \) photodissociation rate is given by a similar expression

\[
R_{\text{pd}, H^+_2} = \int_0^\infty \frac{\sigma_{H^+_2}(\nu) F(\nu)}{h\nu} d\nu, \tag{18}
\]

\[
= I_\nu \int_0^\infty \frac{\sigma_{H^+_2}(\nu) \gamma_{\text{th}}(\nu)}{h\nu} d\nu, \tag{19}
\]

where the photodissociation cross section at photon energies \( 11.27 \text{eV} > h\nu > h\nu_{\text{th}} \) is

\[
\sigma_{H^+_2}(\nu) = \text{dex} \left[ -40.97 + 82.01\eta - 93.22\eta^2 + 34.89\eta^3 \right], \tag{20}
\]

and at photon energies \( h\nu > 11.27 \text{eV} \) is

\[
\sigma_{H^+_2}(\nu) = \text{dex} \left[ -30.26 + 37.94\eta - 34.03\eta^2 + 8.89\eta^3 \right], \tag{21}
\]

where \( \eta \) is the photon energy in Rydbergs, and where the photodissociation threshold energy \( h\nu_{\text{th}} = 2.65 \text{eV} = 0.195 \text{Ryd} \) (Dunn 1968).
The main contribution to H\textsc{ii} region temperatures which correspond to Balmer series photons. Photons with energies $\sim 6$ eV or above and hence in this case the energy which is always negligible. This difference is a consequence of the ionized gas is small and is significant even when emission is the dominant contribution when the temperatures in the range $3000 < T < 30000$ K, their values are $\langle T \rangle < 20000$ K. Two main points are worthy of note. Firstly, there is an obvious temperature dependence: the hotter the H\textsc{ii} region, the density profile is flat, indicating that the density of the ionized gas is almost constant. At this point, the evolution of the H\textsc{ii} region is dominated by the pressure-driven expansion of the ionized gas: as the gas expands, the recombination rate falls, and so more ionizing photons become available for ionizing previous neutral gas, expanding the amount of mass contained in the H\textsc{ii} region. If the density profile of the protogalactic gas is steep enough – it must fall off with radius more quickly than $\rho \propto r^{-3/2}$ (Franco, Tenorio-Tagle, & Bodenheimer 1990) – and if the massive star shines for long enough, then eventually the ionization front will undergo a transition back to a supersonic R-type front, and will soon thereafter break out of the protogalaxy. However, even if the source switches off before breakthrough occurs, most of the ionized gas will nevertheless escape from the protogalaxy, as the mean outward radial velocity of this material is typically several times higher than the escape velocity of the halo.

Although the strong Lyman-Werner band emission from the massive star will quickly dissociate diffuse H$_2$ gas throughout much of the halo (Omukai & Nishi 1994; Glover & Brand 2001), the presence of abundant free electrons in the partially ionized region ahead of the ionization front can catalyze H$_2$ formation in this region, and Susa & Umemura (2003) have shown that the shielding provided by this H$_2$ can allow H$_2$ to survive in dense cores elsewhere in the protogalaxy. However, these calculations did not take into account the possible effects of recombination emission from the ionized gas.

To compute the emission from a typical protogalactic H\textsc{ii} region, we assume, for simplicity, that it is spherically symmetric, with radius $R_i$ and that the gas within it has a uniform density $n_i$. Inspection of the results from detailed three-dimensional numerical simulations of H\textsc{ii} region growth in these protogalactic systems suggests that these should be reasonable approximations. In that case, the value of $I_V$ at a distance $D > R_i$ from the center of the H\textsc{ii} region is given by

$$I_V = n_i^2 R_i f(x_D),$$

### Table 1

| Region | H\textsc{ii} region temperature (K) |
|--------|------------------------------------|
|        | 5000     | 10000     | 20000     |
| H\textsuperscript{-} | Continuum | 3.49 | 1.78 | 1.01 |
|        | Line     | 23.8 | 3.44 | 0.720 |
|        | Total    | 27.3 | 5.21 | 1.73 |
| H\textsuperscript{+} | Continuum | 0.100 | 0.048 | 0.024 |
|        | Line     | $1.0 \times 10^{-10}$ | $3.0 \times 10^{-11}$ | $9.7 \times 10^{-12}$ |
|        | Total    | 0.100 | 0.048 | 0.024 |

The values of the frequency integrals in equations (19) and (20) are independent of the spatial distribution of the ionized gas, but do depend on its temperature. For gas temperatures in the range $3000 < T < 30000$ K, their values are fit to within 5% by the expressions:

$$R_{\text{pol,H}_{-}}/I_V = \text{dex}[\log(T/10^4 \, \text{K}) = -28.28 - 2.04 t_4 + 1.28 t_4^2] \text{ cm}^2 \text{s}^{-1}$$

and

$$R_{\text{pol,H}_{+}}/I_V = \text{dex}[-30.33 - 1.01 t_4 + 0.24 t_4^2] \text{ cm}^2 \text{s}^{-1},$$

where $t_4 = (T/10^4 \, \text{K})$. Table 1 shows the relative size of the line and continuum contributions for three different gas temperatures: $T = 5000$ K, $10^4$ K and $2 \times 10^4$ K. Two main points are worthy of note. Firstly, there is an obvious temperature dependence: the hotter the H\textsc{ii} region, the lower the photodissociative flux. Secondly, line emission is far more important in the case of H\textsuperscript{-} photodissociation than in the case of H\textsuperscript{+} photodissociation. In the former case, line emission is the dominant contribution when the temperature of the ionized gas is small and is significant even when $T$ is large. In the latter case, the contribution from line emission is almost negligible. This difference is a consequence of the fact that the H\textsuperscript{+} photodissociation cross-section is very small compared to the H\textsuperscript{-} photodissociation cross-section at the energies which correspond to Balmer series photons. The main contribution to H\textsuperscript{+} photodissociation comes from photons with energies $\sim 6$ eV or above and hence in this case the continuum processes dominate.

### 3.1 Expanding H\textsc{ii} region

The first simple scenario considered here involves emission from an expanding H\textsc{ii} region within a small protogalactic halo. Because the first stars to form in the Universe are predicted to be very massive (Abel, Bryan, & Norman 2002; Bromm, Coppi, & Larson 2002; Yoshida et al. 2006; Gao et al. 2006) and therefore to be emitters of large numbers of ionizing photons (Schaefer 2002), this scenario has been studied numerically by a number of authors (Whalen, Abel & Norman 2004; Whalen & Norman 2006; Alvarez, Bromm & Shapiro 2006; Johnson, Greif & Bromm 2006; Susa & Umemura 2006; Abel, Wise & Bryan 2007). This work has given us a good understanding of the basic chain of events. The nascent H\textsc{ii} region initially grows very rapidly, but recombinations in the dense gas surrounding the ionizing source quickly cause its growth to slow. After less than $10^7$ yr (assuming a 100 $M_{\odot}$ star), the ionization front bounding the H\textsc{ii} region has become a slow, D-type front that drives a shock ahead of itself into the dense neutral gas. At radii greater than the shock radius $r_s$, the density profile of the gas remains almost the same as before the switch-on of the ionizing source. Within the H\textsc{ii} region, on the other hand, the density profile is flat, indicating that the density of the ionized gas is almost constant. At this point, the evolution of the H\textsc{ii} region is dominated by the pressure-driven expansion of the ionized gas: as the gas expands, the recombination rate falls, and so more ionizing photons become available for ionizing previous neutral gas, expanding the amount of mass contained in the H\textsc{ii} region.

If the density profile of the protogalactic gas is steep enough – it must fall off with radius more quickly than $\rho \propto r^{-3/2}$ (Franco, Tenorio-Tagle, & Bodenheimer 1990) – and if the massive star shines for long enough, then eventually the ionization front will undergo a transition back to a supersonic R-type front, and will soon thereafter break out of the protogalaxy. However, even if the source switches off before breakthrough occurs, most of the ionized gas will nevertheless escape from the protogalaxy, as the mean outward radial velocity of this material is typically several times higher than the escape velocity of the halo.

Although the strong Lyman-Werner band emission from the massive star will quickly dissociate diffuse H$_2$ gas throughout much of the halo (Omukai & Nishi 1994; Glover & Brand 2001), the presence of abundant free electrons in the partially ionized region ahead of the ionization front can catalyze H$_2$ formation in this region, and Susa & Umemura (2003) have shown that the shielding provided by this H$_2$ can allow H$_2$ to survive in dense cores elsewhere in the protogalaxy. However, these calculations did not take into account the possible effects of recombination emission from the ionized gas.

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$$I_V = n_i^2 R_i f(x_D).$$

3 RESULTS

In order to establish whether or not the photodissociation of H\textsuperscript{-} and H\textsuperscript{+} by emission from ionized hydrogen is ever an important process, I compute in this section the amount of flux produced in several scenarios of cosmological interest, along with the size of the resulting photodissociation rates. As my aim in this paper is simply to produce estimates of these values with the correct order of magnitude, the model H\textsc{ii} regions considered here are all highly simplified compared to realistic systems. Nevertheless, the results of these simple models should be a good guide as to whether more detailed numerical modelling is justified.
where

\[ f(x_{D}) = \int_{-1}^{1} \frac{1}{3} \ln \left[ 1 + \frac{1 - x^2}{(x_{D} - x)^2} \right] \, dx, \tag{25} \]

and where \( x_{D} = D/R_{i} \). In deriving this expression I have assumed that the gas is fully ionized, with \( n_{H_{+}} = n_{e} = n_{i} \). The value of \( f(x_{D}) \) is plotted in Figure 2 as a function of \( x_{D} \). In the limit that \( x_{D} \to 0 \), we see that \( 1/n_{i} \to n_{i} R_{i} \), while for \( x_{D} = 1, \, 1/n_{i} \to 0.5 n_{i}^{2} R_{i} \) and in the limit of large \( D, \, 1/3 D^2 n_{i}^{2} R_{i} \).

If we assume that the temperature of the ionized gas is \( 2 \times 10^{4} \) K, then at a point just outside the \( \text{H} \, ii \) region, the \( \text{H}^{-} \) photodissociation rate is given by

\[ R_{\text{pd,H}^{-}} = 2.67 \times 10^{-11} \left( \frac{n_{i}}{1 \, \text{cm}^{-3}} \right)^{2} \left( \frac{R_{i}}{1 \, \text{pc}} \right) \, \text{s}^{-1}, \tag{26} \]

and the \( \text{H}^{+} \) photodissociation rate is given by

\[ R_{\text{pd,H}^{+}} = 3.76 \times 10^{-13} \left( \frac{n_{i}}{1 \, \text{cm}^{-3}} \right)^{2} \left( \frac{R_{i}}{1 \, \text{pc}} \right) \, \text{s}^{-1}. \tag{27} \]

For comparison, the rate at which \( \text{H}^{-} \) is destroyed by associative detachment (reaction 2) is given by

\[ R_{\text{c,H}^{-}} = 1.3 \times 10^{-9} n_{\text{H}} \, \text{s}^{-1}, \tag{28} \]

where \( n_{\text{H}} \) is the number density of atomic hydrogen, and where we have adopted the rate coefficient determined for this reaction by Schmeltekopf et al. (1967). As noted by Glover, Savin & Jappsen (2008), the rate of this reaction is uncertain by almost an order of magnitude, but for the initial order-of-magnitude study given in this paper, the use of a value from close to the middle of the range of uncertainty is probably justified. The rate at which \( \text{H}_{2}^{+} \) is destroyed by charge transfer (reaction 4) is given by (Karpas, Anicich & Hunten 1979)

\[ R_{\text{c,H}_{2}^{+}} = 6.4 \times 10^{-10} n_{\text{H}} \, \text{s}^{-1}. \tag{29} \]

In this particular example, the density of atomic gas just outside the \( \text{H} \, ii \) region will be similar to the density of the ionized gas within the \( \text{H} \, ii \) region, implying that \( n_{\text{H}} \approx n_{i} \).

Therefore, \( \text{H}^{-} \) will be destroyed primarily by photodissociation, rather than associative detachment, if

\[ n_{i} \gtrsim 50 \left( \frac{R_{i}}{1 \, \text{pc}} \right)^{-1} \, \text{cm}^{-3}. \tag{30} \]

Similarly, photodissociation will dominate the destruction of \( \text{H}^{+} \) if

\[ n_{i} \gtrsim 2400 \left( \frac{R_{i}}{1 \, \text{pc}} \right)^{-1} \, \text{cm}^{-3}. \tag{31} \]

How do these numbers compare with the results of the numerical simulations of \( \text{H} \, ii \) region growth mentioned above? To take one particular example, consider the \( \text{H} \, ii \) region simulated by [Abel, Wise & Bryan (2007)]. At \( t = 0.1 \) Myr after the switch-on of the central ionizing source, the \( \text{H} \, ii \) region in their simulation has a mean density \( n_{i} \sim 500 \) cm\(^{-3}\) and a size \( R_{i} \sim 1 \, \text{pc} \). At a later time, \( t = 1 \) Myr, its size has grown to \( R_{i} \sim 20 \) pc, while its mean density has fallen to \( n_{i} \sim 10 \) cm\(^{-3}\).

In both cases, the diffuse flux from the \( \text{H} \, ii \) region is strong enough to dominate the destruction of \( \text{H}^{-} \) in the surrounding gas. It does not dominate the destruction of \( \text{H}^{+} \), but is important at the 10–20% level. Only once \( t = 2.7 \) Myr (at which point \( R_{i} \sim 50 \) pc and \( n_{i} \sim 1 \) cm\(^{-3}\)) does the diffuse flux decrease to a level at which it no longer dominates the destruction of \( \text{H}^{-} \). However, by this point in the evolution of the protogalaxy, the expansion of the central \( \text{H} \, ii \) region has accelerated almost all of the surrounding gas to velocities greater than the protogalactic escape velocity. Therefore, any \( \text{H}_{2} \) that does manage to form via \( \text{H}^{-} \) at \( t > 2.7 \) Myr will not be retained by the protogalaxy.

Although the preceding analysis is for gas located just outside the \( \text{H} \, ii \) region, \( \text{H}^{-} \) photodissociation will actually dominate out to much larger distances. This can be easily seen if we compare the dependence on radius of \( R_{\text{pd,H}^{-}} \) and \( R_{\text{c,H}^{-}} \). In the limit of large distances, \( R_{\text{pd,H}^{-}} \) scales with the distance \( D \) as \( R_{\text{pd,H}^{-}} \propto D^{-2} \), while at smaller distances the scaling is less steep. If the density distribution of the neutral gas is given by \( n \propto D^{-\alpha} \), then \( R_{\text{c,H}^{-}} \propto D^{-\alpha} \). Therefore, if \( \alpha > 2 \), then \( R_{\text{c,H}^{-}} \) will fall off more steeply with increasing radius than \( R_{\text{pd,H}^{-}} \). As simulations of population III star formation generally find that \( \alpha \simeq 2.2 \) (see e.g. Yoshida et al. 2006a), this implies that if \( \text{H}^{-} \) photodissociation dominates near the \( \text{H} \, ii \) region, it will also dominate throughout the remainder of the protogalaxy.

### 3.2 Neutral cloud embedded in large \( \text{H} \, ii \) region

The second scenario considered in this paper is that of a cloud of neutral hydrogen embedded in a large cosmological \( \text{H} \, ii \) region. This arrangement of ionized and neutral gas may be encountered within the earliest protogalaxies after the formation of the first stars if any clumps of gas exist within these protogalaxies that are dense enough to survive the passage of the expanding ionization front (I-front) without being immediately photoevaporated (Susa & Umemura 2004; Susa 2007). On larger scales, this arrangement can also be used to represent primordial minihalos that have been enveloped by a large intergalactic \( \text{H} \, ii \) region, but that have not yet been completely photoionized or photoevaporated (see e.g. Iiev, Shapiro & Raga 2005; Ahn & Shapiro 2007).

For simplicity, I assume that both the neutral cloud and
the surrounding H\textsc{ii} region are spherically symmetric, with radii \( R_d \) and \( R_i \) respectively. The cloud is assumed to have a power-law density profile
\[
n(R) = n_{c1} \left( \frac{R_d}{R} \right)^{\alpha}
\]  
with \( \alpha < 3 \), where \( n_{c1} \) is the density of the cloud at \( R = R_{c1} \), which can be written in terms of the cloud mass \( M_{c1} \) as
\[
n_{c1} = \frac{M_{c1}}{4\pi R_{c1}^3} \left( \frac{3 - \alpha}{\alpha} \right),
\]  
where \( m_H \) is the mass of a hydrogen atom. At early times, the outer edge of the cloud will be photoionized, but will not yet have had time to dynamically respond to the increased pressure, and so will maintain the same density profile as the neutral gas. The density of ionized gas at radii \( R_d > R > R_{c1} \), where \( R_{c1} \) is the innermost extent of the ionized region, is therefore given by Equation (32) above, while at radii \( R \geq R_d \) I take the ionized gas density, \( n_i \), to be constant. Finally, it is necessary to specify the distance \( D \) of the centre of the cloud from the centre of the H\textsc{ii} region.

These assumptions allow us to write the volume integral \( I_V \) as
\[
I_V = \frac{n_{c1}^2 R_d}{2\alpha - 1} \left[ \left( \frac{R_d}{R_{c1}} \right)^{2\alpha-1} - 1 \right] + n_i^2 R_i f(x_D) - n_i^2 R_{c1},
\]  
where \( x_D = D/R_i \) and where \( f(x_D) \) is defined by Equation (28). The first term in this expression corresponds to the flux produced by the ionized outer edge of the cloud, while the second and third terms, taken together, correspond to the flux from the surrounding constant density H\textsc{ii} region.

We can explore the behaviour of this expression by considering a concrete example. Let us suppose that \( D = 0.5 R_i \), so that \( x_D = 0.5 \), and that \( R_{c1,1} = 0.5 R_{c1} \) and \( \alpha = 2 \). Then \( I_V \) becomes
\[
I_V = \frac{7}{3} n_{c1}^2 R_d + 0.91 n_i^2 R_i - n_i^2 R_{c1}.
\]  
If \( R_i \gg R_{c1} \), then the second term dominates, unless the density contrast between the cloud and the surrounding H\textsc{ii} region is large. The first term dominates only if \( n_{c1} > 0.6 (R_i/R_{c1})^{1/2} n_i \), while the third term is always negligible in these conditions. The required density contrast can be significantly reduced if we steepen the density profile of the cloud by increasing \( \alpha \), or increase the fraction of it which is already ionized by decreasing \( R_{c1,1} \). Altering \( D \), or on the other hand, has little effect on the required contrast.

To proceed with our example, let us suppose that the H\textsc{ii} region has a size \( R_i = 3 \) kpc and number density \( n_i = 10^{-5} \text{ cm}^{-3} \) (Johnson, Greif & Bromm 2006), that the cloud has a virial radius \( R_{c1} = 100 \) pc, and that \( n_{c1} = 0.2 \text{ cm}^{-3} \). In that case, the flux from the ionized edge of the cloud dominates, \( I_V \approx 2.9 \times 10^{19} \text{ cm}^{-5} \), and the resulting H\textsc{ii} and H\textsc{iii} destruction rates are \( R_{pd,H^-} = 5.0 \times 10^{-10} \text{ s}^{-1} \) and \( R_{pd,H^+} = 6.9 \times 10^{-12} \text{ s}^{-1} \) respectively. In comparison, \( R_{c,H^-} = 1.04 \times 10^{-9} \text{ s}^{-1} \) and \( R_{c,H^+} = 5.12 \times 10^{-10} \text{ s}^{-1} \) at \( R_{c1} \). Therefore, in this particular example the flux is of limited importance. If we were to assume that \( R_{c1,1} = 0.05 R_{c1} \), rather than \( R_{c1,1} = 0.5 R_{c1} \), however, then the photodissociation rates would increase by a factor of a thousand, while the collisional rates would increase by only a factor of a hundred. In that case, more H\textsc{ii} would be destroyed by photodissociation than by associative detachment. On the other hand, the photodissociation of H\textsc{ii} would still be of limited importance.

This simple example demonstrates that in this situation, radiative feedback due to the emission from the ionized gas is most effective during the final stages of the photoionization of the neutral cloud, when the highest density material is being photoionized. As we expect dynamical effects to have become important by this stage in the photoionization process (Ahn & Shapiro 2007), it is difficult to assess the ultimate importance of the feedback without performing more detailed calculations.

3.3 Recombining fossil H\textsc{ii} region

The final scenario examined here is that of a so-called ‘fossil’ H\textsc{ii} region (Oh & Haiman 2002), i.e. an H\textsc{ii} region in which the ionized source has switched off. For simplicity, I consider the evolution of the gas at the center of a spherically symmetric H\textsc{ii} region with constant density \( n_i \) and radius \( R_i \). If the gas is initially fully ionized, then \( I_V \) at the moment of switch-off is given by
\[
I_V = n_i^2 R_i.
\]  
If we again assume that the temperature of the ionized gas is \( 2 \times 10^4 \text{ K} \), then the flux produced by the ionized gas gives rise to photodissociation rates for H\textsc{ii} and H\textsc{iii} that are given by
\[
R_{pd,H^-} = 5.3 \times 10^{-8} \left( \frac{n_i}{1 \text{ cm}^{-3}} \right)^2 \left( \frac{R_i}{1 \text{ kpc}} \right) \text{ s}^{-1},
\]  
\[
R_{pd,H^+} = 7.4 \times 10^{-10} \left( \frac{n_i}{1 \text{ cm}^{-3}} \right)^2 \left( \frac{R_i}{1 \text{ kpc}} \right) \text{ s}^{-1}.
\]  
As time passes, however, the gas in the H\textsc{ii} region will recombine. Since \( I_V \propto x^2 \), where \( x \) is the fractional ionization of the gas, this means that both \( R_{pd,H^-} \) and \( R_{pd,H^+} \) will both decrease with time. If the temperature of the ionized gas were to be kept fixed, then these rates would also fall off as \( x^2 \). In fact, it is likely that some cooling of the gas will occur and so the effects of the decrease in \( x \) will be offset to some extent by an increase in the emission coefficient. Nevertheless, it is clear from Table 1 that changes in the emission coefficient will alter the rates by at most an order of magnitude, while a large decrease in \( x \) may alter them by many orders of magnitude. Therefore, consideration of the simplified case in which \( T \) is kept fixed serves to illustrate the basic behaviour of the gas without requiring one to model its thermal evolution.

For a intergalactic H\textsc{ii} region with the same size and density as in the previous section (i.e. \( R_i = 3 \) kpc and \( n_i = 10^{-5} \text{ cm}^{-3} \)), we therefore have
\[
R_{pd,H^-} = 1.59 \times 10^{-13} x^2 \text{ s}^{-1},
\]  
\[
R_{pd,H^+} = 2.24 \times 10^{-15} x^2 \text{ s}^{-1}.
\]  
The destruction rate of H\textsc{ii} by associative detachment in the same conditions is given approximately by
\[
R_{c,H^-} = 1.3 \times 10^{-12} (1 - x) \text{ s}^{-1}.
\]  

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Clearly, $R_{\text{pd}, H^-} > R_{\text{c}, H^-}$ only if $(1 - x) \ll 1$. However, if the fractional ionization is as high as this, then $H^-$ will also be destroyed rapidly by mutual neutralization with protons $H^- + H^+ \rightarrow H + H$. (42)

The rate coefficient of this reaction is uncertain, possibly by as much as an order of magnitude (Glover, Savin, & Jappsen 2004). If we take the smallest of the values quoted in the literature, the rate of Dalgarno & Lepp (1987), then we find that the mutual neutralization rate in our example H II region is

$$R_{\text{mn}, H^-} = 7 \times 10^{-10} T^{-1/2} x s^{-1}. \tag{43}$$

This is comfortably larger than $R_{\text{pd}, H^-}$ for all temperatures in our range of interest. Therefore, in this particular scenario, $H^-$ photodissociation is unimportant. Comparison of the photodissociation rate of $H_2^+$ with the rate at which it is destroyed by charge transfer with $H$ (reaction 4) or by dissociative recombination

$$H_2^+ + e^- \rightarrow H + H, \tag{44}$$

leads to a similar conclusion regarding $H_2^+$. Can we avoid these conclusions by considering a larger H II region? If we suppose that we have somehow managed to create a fossil H II region with $R_I = 3$ Mpc, rather than 3 kpc, then it is easy to see that the resulting photodissociation rates would be a thousand times larger than in the case considered above. Even so, associative detachment still dominates the destruction of $H^-$ if $x < 0.1$. Similarly, photodissociation is unimportant in comparison to dissociative recombination or charge transfer for $x < 0.5$. Since $H_2$ formation in cooling, recombining gas becomes significant only once $x < 0.1$ (see e.g. Oh & Haiman 2002), it is clear that even in this somewhat unrealistic case, photodissociation of $H^-$ and $H_2^+$ is unimportant.

The other way in which to increase the importance of photodissociation is by increasing the density of the ionized gas. If, instead of a large intergalactic H II region, we consider a small interstellar H II region with $n_I = 100$ cm$^{-3}$ and $R_I = 10$ pc, then we find that

$$R_{\text{pd}, H^-} = 5.3 \times 10^{-6} x^2 s^{-1}, \tag{45}$$

$$R_{\text{pd}, H_2^+} = 7.4 \times 10^{-8} x^2 s^{-1}. \tag{46}$$

In this case, $R_{\text{pd}, H^-} > R_{\text{c}, H^-}$ only if $x > 0.15$, while $R_{\text{pd}, H_2^+} > R_{\text{c}, H_2^+}$ only if $x > 0.59$, so again the effects of photodissociation are unimportant.

4 CONCLUSIONS

The results from the simple models considered in the previous section demonstrate that in some circumstances, bound-free, free-free and two-photon emission from ionized gas can combine to produce a significant $H^-$ photodissociation rate. The effect is very sensitive to the density of the ionized gas, since both the recombination rate and the free-free emission rate scale as the square of the density. However, the required densities are not excessively high and there are a number of situations in which we may encounter them.

The particular examples considered here in which $H^-$ photodissociation proves to be important are the early growth of an H II region around a population III star, prior to its break-out from the confining protogalaxy (§3.1) and the late stages of the photoionization of a neutral cloud (or minihalo) by an external radiation source (§3.2). On the other hand, photodissociation of $H^-$ is not important within so-called ‘fossil’ H II regions, owing to the rapid decrease of the photodissociating flux associated with the decrease in the fractional ionization of the recombining gas.

The destruction of $H_2^+$ by emission from the ionized gas proves to be a less important effect, due to the much smaller photodissociation rate (compared to $H^-$) that results from the same amount of emission. The main reason for this is that the $H_2^+$ photodissociation cross-section is small (or zero) at the photon energies where much of the energy from the ionized gas is radiated, becoming significant only for photons with energies greater than about 6 eV. Nevertheless, even if $H_2^+$ survives where $H^-$ does not, the net effect is still a significant reduction in the $H_2$ formation rate, as $H_2$ formation via $H_2^+$ occurs much more slowly than $H_2$ formation via $H^-$. (Glover 2003).

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