Calculate the Acoustic Parameters of Complex Media by Radon Transform

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Abstract. The propagation velocity of sound waves in complex media is not only the main basis for the processing and interpretation of acoustic data, but also the important data that reflects the structure of the medium. In this paper, Radon transform is used to calculate the acoustic parameters of the complex medium at different projection angles. The example shows that the complex medium reconstructed by this method is very consistent with the original image, which can reflect the complex medium The proposed method can be used to reconstruct the parameters of the complex medium and the reconstruction of the medium structure.

The basic principles of tomographic imaging were first applied to medical CT, radon transformation is the theoretical basis of tomographic imaging. In 1917, Australian mathematician Radon demonstrated the formula for reconstructing the two-D space distribution of the plane by projecting a plane of the object. His formula calls for a full cast of all possible lines along the plane for the Radon transform \cite{1}. At present, the radon transform and its inverse transform is an important research method in image processing, it is not necessary to know the details of the interior of the image, only using the projection value of the image can reverse the original image. Now, the Radon transform has become the primary tool for tomographic imaging and Remote sensing imaging \cite{2}. It has been successfully applied in medicine, radio astronomy and geophysics \cite{3-7}. This paper presents a method for calculating acoustic parameters of complex medium by using radon transform. Firstly, the acoustic parameter structure model is established, and the acoustic intensity parameters are obtained by Radon positive transformation. The inverse transformation of the Radon positive transformation is carried out to reconstruct the image, which can reflect the acoustic parameters of the complex medium. At the same time, this method can be applied to the medical, the current medical B-ultrasound is to the human body to send ultrasound, while receiving the internal organs of the reflection wave, and then complete the ultrasound imaging \cite{8}. We can use the method in this paper to pass the transmission wave through the human body, using radon transformation to simulate the reconstruction of human organs, which is beneficial to medical examination. Can also be applied to air detection, using electromagnetic waves through the haze to reverse the performance of different areas of haze concentration.

1. Radon Transform

The Radon transformation is a definite image function can be solved by the projection function of all the angles of incidence. The integral of a known function on the line becomes the classical Radon positive transformation, and the integral on the arc becomes the generalized Radon transformation, in other words the generalized Radon transformation is the implementation of the classical radon transformation.
Suppose a binary function \( f(x,z) \) is defined on the plane and smooth enough that it is zero except for a sufficiently large area, \( L \) is a straight line on the \( xOz \) surface, then the integral Radon positive transformation along a straight line \( L \) is defined as:

\[
Rf(\theta,p) = \int_L f(x,z)ds
\]  

(1)

Among them, \( ds \) is line length elements. \( f \) for the image, \( Rf \) projection of the image.

Any straight \( L \) line in the \( xOz \) plane can be expressed as:

\[
L: p = x\cos \theta + z\sin \theta
\]  

(2)

The unit normal vector of \( L \) is \((\cos \theta, \sin \theta)\), \( \theta \) is the angle between the normal of \( L \) and the positive direction of \( x \) axis, and \( p \) is the distance from the origin of coordinate to \( L \). Thus, any straight line \( L \) corresponds to a certain number of pairs \((p, \theta)\), which in turn determines a straight line within a unique \( xOz \) plane (for \( \theta \in [0,2\pi] \), the correspondence is one-to-one correspondence). Therefore, Radon positive transformation of \( f(x,z) \) can also be written as follows:

\[
Rf(\theta,p) = \int_{p=x\cos \theta + z\sin \theta} f(x,z)ds
\]  

(3)

The two-dimensional Radon inverse transform refers to the solution of \( f(x,z) \) Radon's transformation to find out the integrated function. Two-dimensional Radon inverse transform, also known as two-dimensional Radon inversion formula, the formula is as follows:

\[
f(x,z) = -\frac{1}{2\pi^2} \int_0^\pi \int_{-\infty}^{\infty} \frac{\partial}{\partial p} Rf(\theta,p) d\theta dp
\]  

(4)

2. Sonic Parameter Structure Model

The model of geological acoustic parameters is constructed in this paper. First, select a geological section (Figure 1) and turn it into a grayscale image (Figure 2).

The gray values of this gray scale are treated by rows, and the gray values of different ranges represent different geology and represent different velocities. The acoustic parameters selected in this paper are seismic waves, seismic waves are divided into transverse and longitudinal waves, and we select the reciprocal of the velocities of P-wave in several different major types of rock. In the experiment, the lowest wave velocities of seismic wave in clay, tight sandstone, limestone, metamorphic rock and granite were selected, respectively, 1200 m/s, 2500m/s, 3500m/s and 4500m/s. Then the reciprocal of the velocity is calculated, and the computed value is given to the image model of the acoustic parameter. When constructing the acoustic parameter image, because the assigned value is the reciprocal of the speed, so the numerical values are very small, the image display is basically black after the model is constructed directly, and no obvious changes can be seen. So after the assignment, we have to do the normalization, first of all, to get the maximum value in the acoustic parameter matrix, and then divide each number in the acoustic parameter matrix by this maximum, so that the data is 0-255 gray value, you can make the image normal display.

The sonic parameter structure model is thus obtained (Figure 3).
3. The Radon Forward Transform of Acoustic Structure Model is Carried out

Due to the lack of projective data in the simulation experiment, the acoustic intensity of different angles should be obtained by using Radon positive transformation. Radon transform is actually an image transformation: In the process of image processing, in order to quickly and effectively, we usually want to convert some form of image into the definition of other image space, and using the unique nature of these spaces, we can easily do some processing, the final transformation into the image space, so as to achieve the desired effect. The projection is essentially a line integral, and the summation of the image in a particular direction is called image projection. We take a two-dimensional function $G(X,Y)$ as an example to illustrate that the image in a certain direction of the line integral, is its image projection. So the projection of this function on the X-axis is the line integral in the vertical direction, naturally, the projection of the function on the y-axis, which is the line integral in the horizontal direction. Through these projections, we can determine some of the characteristics of the image, these characteristics are in the direction of determination, because of these properties, and we can apply this method to some processing, like image pattern recognition. The following is a projection image from the Radon positive transformation of the acoustic parametric structural model from 30°, 60° and 90° respectively. As Figure 4, Figure 5, figure 6 shows:
4. Inverse Transform Reconstruction Image

Radon inverse transform is applied to the projected image obtained after the transformation to reconstruct the image. Respectively using $10^\circ$, $5^\circ$, $3^\circ$ gradients, compared Figure 7, Figure 8, Figure 9, we can see that due to the minimum number of projections used to reconstruct the image, Figure 7 reconstructed the image effect is the worst. When the projection gradient is reduced to $5^\circ$, as shown in Figure 8, the effect of reconstructing the image is relatively good. When the projection gradient is reduced to $2^\circ$, as shown in Figure 9, it can be seen that the reconstructed image has the highest quality. From this we can see that there are many false points in the reconstructed image because the number of projections used to reconstruct the image in Figure 7 is too small. So in order not to happen, we can increase the number of projection angles after radon is transformed so that the reconstructed image can achieve a better result. So when the projection angle is more transformation, the better the reconstruction of the graphics. It can be seen that the reconstruction map of the $2^\circ$ gradient (Figure 9) and the constructed parameter model (Figure 3) are in good agreement with each other, which can well reflect the distribution of the acoustic parameters of the complex medium.
5. Conclusion
In this paper, we use Matlab software to construct a complex acoustic wave parameter structure model. Radon forward transform is used to project the energy intensity of the sound wave along different angles. At the same time, we compare the energy intensity of sound waves on $30^\circ$, $60^\circ$ and $90^\circ$ respectively, and then use the calculated results and parameters. Radon inverse transform algorithm uses three gradients of $10^\circ$, $5^\circ$ and $2^\circ$ respectively to reconstruct the image of the acoustic parametric structure model. It concludes that the more projection angles in the Radon transform, the better the reconstructed image effects. The example shows that the image reconstructed by this method is in good agreement with the original image, which can well reflect the distribution of acoustic parameters in complex media and can be fully applied to inversion of acoustic wave parameters.

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