A Hyper-viscoelastic model for filled-rubber materials: experiments and simulations

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Abstract. Rubber has been widely addressed as anti-vibration components in long time service structure, time-dependent behavior is an undesirable problem contributing to the low acceptance by applications. In this paper, a hyper-viscoelastic model is established so that the tension behavior of rubber is simulated and analyzed. This model is constructed by a hyperelastic element in parallel with a viscoelastic model and Polynomial model, which was examined by uniaxial tension test, was proved validly to be hyperelastic. A fairly good agreement between the tension creep test and analytical results claimed that the Generalized Maxwell model with six variables is suitable for expression of hyper-viscoelastic method. In this paper, by combining the results of uniaxial tensile and tensile creep experiments, a hyper-viscoelastic constitutive model is established, and the experiment is simulated compared with the experimental results.

1. Introduction
Rubber-like materials have been widely used in various engineering applications due to their superior elastic and damping properties [1-4], however, the time-dependent behavior is an inevitable question when considering engineering design and applications on anti-vibration systems, which mainly reflected in creep and relaxation. When the rubber is subjected to a constant load for a long time, its deformation will increase with time, which is called creep [5]; when the rubber is deformed continuously for a long time, its stress will gradually decrease with the increase of time, which is known as stress relaxation. [5]. It is important to control the creep displacement of the rubber-like engineering application, so that it does not exceed its structural limit and to ensure its service life.

However, the long-term creep or relaxation experiments are expensive and time-consuming, the time temperature superposition principle (TTSP) is applied to construct the main creep or relaxation curves from short time creep or relaxation experiment [6-7]. Starkova [8] employs TTSP to build the creep master curves of three kinds of rubber, Bae [9] showed the stored energy and analogy between the temperature effect on creep compliance and relaxation modulus curves with the extension of TTSP, Bai [10] combined The Cross model with the TTSP to predict the long-term creep deformation. Similarly, time temperature stress superposition principle (TTSSP) [11] is also applied to construct master curve by taking both nonlinearity of viscoelastic behavior and by introducing a stress reduction function for creep calculation.

Viscoelasticity models were developed to predict the rubber creep and relaxation. Touati [12] presented a numerical method to predict the nonlinear temperature dependent stress relaxation from a given Schapery’s creep characterization, Sudduth [13] introduced a new unifying mathematical model to estimate creep, stress relaxation, and constant strain evaluations as required for a given application.
for a viscoelastic material, Oman [14] found a method to predict stress relaxation from known creep or vice versa which gives sufficiently accurate results over both primary and secondary creep regions.

This paper is mainly devoted to the prediction of uniaxial tension and tension creep results by using the short-term tension creep test results. The constitutive models containing hyperelasticity and viscoelasticity are described in Section 2. Next, experimental results for rectangular specimens is demonstrated in Section 3 and then the finite model of rectangular specimen is described in section 4. Finally, discussions on this investigation are presented in section 5.

2. Constitution models

The presented experiments revealed that the rubber exhibits such behaviors as large deformations and viscoelasticity. To simulate these behaviors, a hyperelastic model was added into the original viscoelastic model, such as Maxwell model or Kelvin model [6]. Compared with the origin generalized Maxwell or generalized Kelvin model, it can better describe the large deformation.

2.1. Hyperelastic model

The hyperelastic model is commonly used to describe the mechanical behavior of rubber-like materials, which is expressed by a strain energy density function $W$ depending on strain invariants $I_1, I_2, I_3$. Rubber is generally assumed to be incompressible material. Take $I_3 = 0$, that is, it does not contribute to the strain energy function. The most commonly used hyperelastic constitutive model is Neo-Hooke model, Polynomial model and Ogden model, and the expressions are listed below:

$$\text{Neo – Hooke} : W = C_{10}(I_1 - 3)$$

$$\text{Polynomial} : W = \sum_{i+j+k=1}^N C_{ij} I_1^{-i} I_2^{-j} I_3^{-k}$$

$$\text{Ogden} : W = \sum_{i=1}^N \mu_i (\lambda_i - 1) - \frac{\lambda_i}{2}$$

where $C$ represents the right Cauchy-Green stress.

2.2. Viscoelastic model

Taking generalized Kelvin model as example, several Kelvin models are connected in parallel, which represented instantaneous elasticity, delayed elasticity with various retardation times, stress relaxation with various relaxation times and also viscous flow. Considering that the total strain is the sum of the creep strain of each individual Kelvin model, the creep strain under constant stress $\sigma_0$ has the following form:

Figure 1. Hyperelastic and viscoelastic models.
(4)

\[ \varepsilon(t) = \sigma_0 \left[ J_0 + \sum_{i=1}^{n} J_i \left( 1 - e^{-\frac{\eta_i}{\tau_i}} \right) \right] \]

Where \( J_i = 1/E_i \) is the compliance, and \( \tau_i = \eta_i/E_i \) is the relaxation time.

3. Experiment and parameters identification

Two types of rubber specimens, rectangular specimens and cylinder specimens, were selected for the experiment. As shown in Figure 2, the rectangular specimen size was 35mm × 5mm × 2mm, and uniaxial tension creep test was conducted, the test part was 25mm, and 5mm at both ends was used for clamping the specimen.

![Figure 2. Rectangular specimens.](image)

3.1. Experiment on rectangular specimen

The temperature was maintained at room temperature (23°C). Repeated cyclic stretching was carried out on the specimen before the experiment to ensure that the stress was within the range of linear viscoelasticity during the experiment, and the effect of Mullins effect was removed [15-16]. Figure 4(a) shows the results of the uniaxial tension test, plotted as nominal stress versus nominal strain, which was carried out on rectangular specimens at the rate of 100mm/min. The stress \( \sigma = 1.245\text{MPa} \), \( 1.66\text{MPa} \), \( 2.07\text{MPa} \), \( 2.49\text{MPa} \) were carried out on the creep experiment after the initial mechanical loading procedure. The whole experiments, contained uniaxial and creep test, were lasted for 3600 s. Due to the influence of transient stretching, data from 30s to 3600s were selected for the creep test results, and regarded the 30th second as the starting time of the creep experiment. It can be seen from Figure 3(b) that in the beginning, the slope of the curve was maximum in the creep response. After that period, the gradient of the creep curve changed gradually to a lower value as time progressed.
3.2. Material parameters identification

In uniaxial experiment, the elongation ratio of rubber samples in three directions can be expressed as $\lambda_1 = \lambda = 1 + \epsilon$, $\lambda_2 = \lambda_3 = \lambda^{1/2}$. Before and after the uniaxial experiment, the uniaxial stress of the rubber sample can be written as:

$$\sigma = \frac{P}{S_1} = \frac{P}{S_0} (1 + \epsilon)$$  \hspace{1cm} (5)

The large deformation method is used to conduct on the experimental data of uniaxial tension and compression, and the stress-strain relationship is shown in the Figure 4.

![Figure 4](image)

**Figure 4.** Tension experiment data with large deformation method.

The TTSSP was applied to deal with the data of tension creep experiment. When temperature remains constant, TTSSP reduced to TTSP, and creep acceleration experiment is conducted in the form of stress, its expression is written as Eq. (6).

$$\log \phi_\sigma = -\frac{C_1 (\sigma - \sigma_0)}{C_3 + (\sigma - \sigma_0)}$$  \hspace{1cm} (6)

where $C_1$ and $C_3$ are the material constant, $\phi_\sigma$ is the stress shift factor.

$\sigma = 1.245$MPa was adopted as the reference stress, and creep compliance curve of other stress in Figure 3(b) is horizontally shifted, and the distance of shifted is the corresponding stress shift factor. The main creep compliance curve obtained is shown in Figure 5. It can be seen that time span of the main curve increased to $1.24 \times 10^7$s, which is significant increased compared with the experiment time span. Namely, short-term creep test data can be used to predict long-term creep performance, providing accelerated characterization method for rubber compression creep test. Figure 6 shows the variation of...
shift factor with stress, the constants $C_1$ and $C_3$ in Eq. (6) are listed in Tab.2 based on the least square method.

![Figure 5](image1.png)  
**Figure 5.** Creep compliance master curve at $\sigma = 1.245$MPa.

![Figure 6](image2.png)  
**Figure 6.** Creep compliance curve shift factor.

4. Finite element model

For engineering applications, compression is the most widely used, thus, it is necessary to identify whether the model mentioned above can be utilized or not. The uniaxial and creep tension experiments have been fitted with Origin based on the constitutive models described above, then the tension mechanics were simulated with validation, at last the compressive characteristics were calculated by the finite element method (in this paper, Abaqus simulation was adopted).

For rectangular rubber specimen, the analysis was force controlled, and was applied on one side of the model, the other side was fixed completely. The geometry of the specimen was discretized into 8-noded hexahedral hybrid elements (C3D8H), as shown in Figure 7.

Two steps were carried out in the finite element analysis. The first step is the uniaxial tension analysis using Mooney-Rivlin model, which is time independent. The second step is the time dependent part, creep and relaxation analysis, with Prony series. The time period of the analysis was set to 3600 seconds, which is convenient to compare with the experimental results.

The constitutive equations of the hyper-viscoelastic model involve the identification of parameters summarised in Tab.1 and Tab.2. Three different hyperelastic model were selected to simulate the uniaxial tension behavior, six for the three Maxwell contributions $G_1$, $\tau_1$; $G_2$, $\tau_2$; $G_3$ and $\tau_3$, with two shift factor $C_1$ and $C_3$.

![Figure 7](image3.png)  
**Figure 7.** Finite element model.

| Table 1. Hyperelastic parameter identification. |
|------------------------------------------------|
| **Mooney-Rivlin** | **Polynomial (N=2)** | **Ogden (N=2)** |
| $C_{10}=0.391209949$ | $C_{10}=0.232527113$ | $\mu_1=0.120454629$ |
| $C_{01}=0.540631834$ | $C_{01}=0.915868070$ | $a_1=3.49571931$ |
| $C_{20}=8.154665293\times10^{-3}$ | $C_{20}=6.367671239\times10^{-3}$ | $\mu_2=2.09564395$ |
| $C_{11}=-0.10327374$ | $C_{12}=0.10327374$ | $a_2=-0.59854925$ |
Table 2. Viscoelastic parameter identification.

| Modulus/MPa | Relaxation time/s | Shift factor |
|-------------|-------------------|--------------|
| G_1=6.95130×10^{-2} | τ_1=94.578 | C_1=13.11991 |
| G_2=0.12180 | τ_2=595.34 | C_3=3.43317 |
| G_3=0.34146 | τ_3=16538 | |

5. Results and discussions

For the static analysis, the stress-strain curves were obtained from simulation and experiment results, as Figure 8 shows. It can be seen that polynomial(n=2) model fitted the data better than Mooney-Rivlin model and Ogden(n=2) model, which means, in hyper-viscoelastic model, polynomial(n=2) model can better and more accurately describe the mechanical behavior of uniaxial tension.

The creep comparison between the experiments and simulations is plotted in Figure 9, which demonstrated that the predicted creep curve is close to the measured data. All three models can well realize the uniaxial stretching process, compared with the experimental results, the maximum errors of the three models were 10.14%, 10.94% and 14.49%, respectively. This comparison on the rectangular specimen indicated that the proposed approach can produce a good result for the creep.

![Figure 8. Simulation results of uniaxial tension.](image)

![Figure 9. Simulation results of tension creep.](image)

6. Conclusions

This article presents an engineering approach to evaluate the time-dependent loading response for rubber products. A hyperelastic spring parallel to a Maxwell element was used as rheological model of the materials. The model parameters were fitted according to the experimental data and the appropriate constitutive model of uniaxial tension and tension creep. It has also been established that the time dependent response of industrial products can be predicted based on the responses from simple experiment, like uniaxial tension and short-term tension creep, which could make significant savings on both cost and time for industries.

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