Expert group information formalization based on Z-numbers

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Abstract. A model for formalizing expert group information with a given level of reliability was developed in the paper. To formalize the information Z-numbers were used, components of which are formalizations (fuzzy numbers) of terms of Full Orthogonal Semantic Scopes (linguistic variables with the properties of completeness and orthogonality). If experts evaluate a characteristic \( X \) of objects, then individual expert criteria for evaluating this characteristic are represented as a set of Z-numbers, the first components of which are formalizations of the terms of Full Orthogonal Semantic Scope with the title \( X \), and the second component are formalizations of the terms of Full Orthogonal Semantic Scope with the title "Reliability". The group expert criterion is also determined in the form of a set of Z-numbers, first components of which satisfy the Pareto condition. For Z-numbers, aggregation segments are determined, which are used to determine the distance between two Z-numbers and to recognize the reliability of the group expert criterion. Reliability recognition and identification with one of the linguistic terms (and, accordingly, with the fuzzy number formalizing this term) of the semantic scope "Reliability" is based on a comparative analysis of the distances between Z-number belonging to the group criterion and Z-numbers, the first component of which coincides with the first component of Z-number from the group criterion, and the second components are formalizations of the terms of semantic scope "Reliability". The reliability of expert group evaluation (the second component of Z-number from the group criterion) is determined by the linguistic value (and, accordingly the fuzzy number formalizing this value) that corresponds to the minimum distance.

1. Introduction

The problem of evaluating the reliability of information has always been relevant, but due to insufficient development of the mathematical apparatus for many years it was not solved properly. As a rule, there was a limited approach from the perspective of probability theory, but the data had crisp numerical values. When experts evaluated objects, processes, and their parameters, the data obtained could not always be represented in numerical form, since expert evaluations are usually linguistic or fuzzy.

In 2011, after defining a Z-number by prof. Lotfi Zadeh it became possible to properly evaluate the reliability of expert information, formalize and use it in solving problems of areas with the active participation of experts [1].

In papers [2-11], a fuzzy extension of the reliability of the data was considered as a fuzzy extension of a probability measure. In [3], the authors has proposed to operate over Z-numbers, converting them to ordinary fuzzy numbers, all operations on which are developed well enough. However, this
approach was criticized by the authors [4, 7, 15] because of the loss of information contained in the initial data.

In [12], it was proposed to operate over $Z$-numbers based on interval arithmetic. In [13], it was considered operations over continuous $Z$-numbers and using them to solve applied problems. Considering of $Z$-numbers with a probabilistic interpretation of their second components, was defined operations over such numbers using operations with ordinary fuzzy numbers and probability distributions. However, the main problem was that only fuzzy extensions of the probability values are known. Thus, in order to determine the result of an operation, it is necessary to somehow determine the initial probability distribution, the probability distribution of the operation result, and then the fuzzy extension of the probability value of the result in the form of a fuzzy number [5, 14].

It is not always possible to determine the law of probability distribution, unless some assumptions are made, assuming, for example, that the distribution law is normal [6]. This greatly simplifies the procedure of operation and allows you to get a result, but the question arises about the possible choice of another law of probability distribution and the validity of such a choice.

In [15], an approach was proposed based on probabilistic interpretation of reliability, a measure of specificity and horizontal membership functions [16-19]. However, when adding a large number of $Z$-numbers, the informational content of the result decreases sharply due to the expansion of the universal set and the increase in uncertainty.

The task of formalizing expert information with a certain level of reliability is practically not covered in publications. Only paper of authors [20] can be noted. The authors discussed discrete $Z$-numbers for which fuzzy reliability is considered as a value of the probability measure of the first fuzzy number (fuzzy expert evaluation). Based on this, operations over discrete $Z$-numbers are determined. The initial probability distributions of the first components of $Z$-numbers are determined based on the solution of the linear programming problem. In order to obtain an expert group opinion under $Z$-information, the authors developed aggregation operators for $Z$-numbers in the form of $Z$-T-norms operators and $Z$-T-conorms operators.

Currently, the aggregation of expert criteria (opinions) under continuous $Z$-numbers has not been studied, not have general cases of $Z$-numbers been considered without a probabilistic restriction on their second components.

The purpose of this paper is to develop an aggregation model of individual expert criteria under $Z$-information based on continuous $Z$-numbers without restrictive requirements for reliability evaluation.

2. Basic concepts and definitions

A $Z$-number has been defined by prof. Lotfi Zadeh as a pair of fuzzy numbers $Z = (\tilde{A}, \tilde{B})$, where $\tilde{A}$ is a fuzzy value of a variable $X$ ($\mu_\tilde{A}(x) : X \rightarrow [0,1]$) and $\tilde{B}$ is a fuzzy measure of reliability $\tilde{B}(\mu_\tilde{B}(x) : [0,1] \rightarrow [0,1])$ [1].

Linguistic variable [21] is a Full Orthogonal Semantic Scope (FOSS), if according to [22], membership functions of their terms have the following properties: functions of all the terms equal to one at a point or at segment; functions increase to the left of the point or segment on which it takes the unit value, and decrease to the right of it; have a maximum of two break points of the first kind; the sum of all the functions at any point of universal set is equal to one.

In [23, 24], methods of constructing individual expert criteria based on statistical information or a direct survey of a single expert are described in detail. If a characteristic $X$ is qualitative, then the universal set $U$ is the segment $[0,1]$, if a characteristic $X$ is quantitative, then the universal set $U$ is the segment $[a,b]$. To construct the FOSS with a title $X$ and term-set $T(X) = \{X_1, X_2, ..., X_m\}$, an
expert determines for terms \( X_j, j = 1, m \) typical intervals \((a_i^1, a_i^2)\), for which membership functions equal to one. For some terms, points (one for each term) can be typical rather than intervals. Based on the obtained expert information, each term \( X_j, j = 1, m \) will be formalized using a fuzzy number \( \tilde{X}_j, j = 1, m \) with membership function \( \mu_j(x), j = 1, m \), which is traditionally determined by four parameters. For example, for trapezoidal number \( \tilde{A}_j (\mu_j(x) = (a_1, a_2, a_L, a_R)) \) parameters \( a_1, a_2 \) are abscissas of the apexes of the trapezoid upper bases, parameters \( a_L, a_R \) are the lengths of the left and right trapezoid wings correspondingly.

Then for a qualitative characteristic: \( \mu_i(x) = \left(0, a_i^2, 0, \frac{a_i^2 - a_i^1}{2}\right) \),

\[
\mu_i(x) = \left(\frac{a_i^1, a_i^2, a_i^2 - a_i^2, a_{i+1}^2 - a_i^1}{2}, l = 2, m-1, \mu_m(x) = \left(a_m^1, 1, a_m^2 - a_{m-1}^2, 0\right) \right).
\]

Then for a quantitative characteristic: \( \mu_i(x) = \left(a, x_i^2, 0, \frac{a_i^2 - a_i^1}{2}\right) \),

\[
\mu_i(x) = \left(\frac{a_i^1, a_i^2, a_i^2 - a_i^2, a_{i+1}^2 - a_i^1}{2}, l = 2, m-1, \mu_m(x) = \left(a_m^1, b, a_m^2 - a_{m-1}^2, 0\right) \right).
\]

An expert defines these intervals with some degree of reliability. The degree of reliability for different terms can be different, for example, for extreme terms (complete absence of manifestations of a characteristic and the complete presence of manifestations of a characteristic) an expert can give more reliable information than for medium ones, as this gives him less difficulty in determining. To determine the degree of reliability, the expert is invited to use a linguistic scale such as “Unlikely”, “Not very likely”, “Likely”, “Very likely”, “Extremely likely”.

A FOSS with the title “Reliability” is constructed similarly to the described above, and each linguistic value “Unlikely”, “Not very likely”, “Likely”, “Very likely”, “Extremely likely” is formalized using a fuzzy number \( \tilde{R}_i, i = 1,5 \) with a membership function \( \eta_i(x), i = 1,5 \).

Thus, expert evaluation criterion for the characteristic \( X \) is represented in the form of \( m \) \( Z \)-numbers \( Z_j = (\tilde{R}_j, \tilde{R}_j), j = 1, m \), the first components of which are fuzzy numbers \( \tilde{X}_j, j = 1, m \), and the second components of which are fuzzy numbers \( \tilde{R}_i, i = 1,5 \), \( \tilde{R}_j \) equal to one of \( \tilde{R}_i, i = 1,5 \).

\( \alpha \)-cut of \( \tilde{A} \) (\( \mu_A(x) = (a_1, a_2, a_L, a_R) \)) is \( A_\alpha \) such that:

\[
A_\alpha = \{ x \in R : \mu_A(x) \geq \alpha \} = [A_\alpha^1, A_\alpha^2] = [a - (1 - \alpha) a_1, a + (1 - \alpha) a_2], \alpha \in [0,1].
\]

\( \alpha \)-cuts are used to determine arithmetic operations with fuzzy numbers [23].

In [13], arithmetic operations with \( Z \)-numbers based on \( \alpha \)-cuts were defined. If \( Z_1 = (\tilde{A}_1, \tilde{B}_1) \), \( Z_2 = (\tilde{A}_2, \tilde{B}_2) \) and \( * \in \{+, -, \times, \div\} \) are the basic arithmetic operations, then

\[
Z_{12} = (A_{12}, \tilde{B}_{12}) = Z_1 * Z_2 = (A_{12}, \tilde{A}_{2a}, B_1 \times B_2), \text{ where } A_{1\alpha}, A_{2\alpha} \text{ are the } \alpha \text{-cuts of numbers } \tilde{A}_1, \tilde{A}_2 \text{ and } B_1 \times B_2 \text{ is the usual multiplication of fuzzy numbers } \tilde{B}_1, \tilde{B}_2 [25].
In [26], for a fuzzy triangular number \( \tilde{A} \) \((\mu_\lambda(x) = (a_l, a_m, a_r))\) the definition of a weighted point \( \Theta \) is given: \( \Theta = a + \frac{1}{6}(a_r - a_l) \). The authors of [27], using this definition, determined the aggregation segment \([\Theta_1, \Theta_2]\) for a number \( \tilde{A} \) \((\mu_\lambda(x) = (a_l, a_m, a_r))\), which is obtained as a union of weighted points of all the triangular numbers belonging to the number \( \tilde{A} \):

\[
\Theta_1 = \int_0^1 2a_l - \frac{(1-\alpha)}{2}a_r \, 2\alpha \, d\alpha = a_l - \frac{1}{6}a_r,
\]

\[
\Theta_2 = \int_0^1 2a_r + \frac{(1-\alpha)}{2}a_l \, 2\alpha \, d\alpha = a_r + \frac{1}{6}a_l.
\]

These segments will be used in the next section to determine the aggregation segments for \( Z \)-numbers and the distance between them.

3. Problem formulation and solution

Let us construct, according to individual expert criteria, a group expert criterion for evaluating a certain characteristic of objects in the form of a set of \( Z \)-numbers, the first components of which satisfy the optimal Pareto condition.

We denote by \( X^j, l = 1, k \) FOSSs of \( k \) experts with formalizations of linguistic values \( \tilde{X}^j, j = 1, m, l = 1, k \) \((\mu_\lambda^j(x) = (x_{lj}, x_{mj}, x_{rj}, x_{sj}))\), constructed according to the method described in the second section [28]. Methods of comparative analysis of individual expert criteria and their consistency are described in [23].

Let us consider a set of \( Z \)-numbers \( Z^j = (\tilde{X}^j, \tilde{R}^j), j = 1, m, l = 1, k \), which are evaluation criteria of \( k \) experts for the characteristic \( X \), \( \tilde{R}^j, i = 1, 5 \) (formalizations of linguistic values “Unlikely”, “Not very likely”, “Likely”, “Very likely”, “Extremely likely” of FOSS “Reliability”).

We will define a group expert criterion (opinion) in the form of a set of \( Z \)-numbers \( Z_j = (\tilde{Y}_j, \tilde{R}_j), j = 1, m, \) where \( \tilde{Y}_j, j = 1, m \) are fuzzy numbers with unknown membership functions \( \mu_j(x) = (x_{lj}, x_{mj}, x_{rj}, x_{sj}) \), \( \tilde{R}_j, j = 1, m \) equals to one of \( \tilde{R}, i = 1, 5 \).

In the theory of expert evaluation for group opinion, the optimal Pareto condition is formulated [22]. This condition means that if \( X = F(X^1, X^2, ..., X^k) \) is a group opinion, which is a function of individual expert opinions \( X^1, X^2, ..., X^k \), then \( \bigcup_{j=1}^{k} X^j \subseteq X \subseteq \bigcup_{j=1}^{k} X^j \).

Let us construct the first components of the group expert criterion with optimal Pareto condition or

\[\min_x \left( \mu^1_j(x), \mu^2_j(x), ..., \mu^k_j(x) \right) \leq \mu(x) \leq \max_x \left( \mu^1_j(x), \mu^2_j(x), ..., \mu^k_j(x) \right), \forall j = 1, m, \forall x \in [0,1].\]

Let us define parameters \( x_{lj}, x_{mj}, x_{rj}, x_{sj} \) of membership functions from the condition:

\[
F = \sum_{j=1}^{m} \sum_{l=1}^{k} \omega_j \left[ (x_{lj} - x_{lj})^2 + (x_{mj} - x_{mj})^2 + (x_{rj} - x_{rj})^2 + (x_{sj} - x_{sj})^2 \right] \rightarrow \min,
\]

where \( \omega_j, l = 1, k \) - weighted coefficients of \( X^j, l = 1, k \).

Unknown parameters are determined from system of normal equations. We obtain solutions:
\[ x_{ij} = \sum_{l=1}^{k} \omega_i x_{il}^j, \quad x_{l} = \sum_{i=1}^{k} \omega_i x_{il}^j, \quad x_{ij} = \sum_{l=1}^{k} \omega_i x_{il}^j, \quad x_{ij} = \sum_{l=1}^{k} \omega_i x_{il}^j, \quad j = \overline{1,m}. \]

It is easy to prove that these solutions satisfy Pareto condition. As

\[
\min_x \left( \mu_1^j (x), \mu_2^j (x), \ldots, \mu_k^j (x) \right) = \sum_{i=1}^{k} \omega_i \left( \min_x \left( \mu_1^i (x), \mu_2^i (x), \ldots, \mu_k^i (x) \right) \right) \leq \mu_j (x) = \sum_{i=1}^{k} \omega_i \mu_i^j (x) \leq \sum_{i=1}^{k} \omega_i \left( \max_x \left( \mu_1^i (x), \mu_2^i (x), \ldots, \mu_k^i (x) \right) \right) = \max_x \left( \mu_1^j (x), \mu_2^j (x), \ldots, \mu_k^j (x) \right) \quad \forall x \in [0,1], \ j = \overline{1,m},
\]

then

\[
\min_x \left( \mu_1^j (x), \mu_2^j (x), \ldots, \mu_k^j (x) \right) \leq \mu_j (x) \leq \max_x \left( \mu_1^j (x), \mu_2^j (x), \ldots, \mu_k^j (x) \right) \quad \forall x \in [0,1], \ j = \overline{1,m}.
\]

To determine the weighted coefficients \( \omega_i, l = \overline{1,k} \) it is proposed to use the Fishburn scale [22]:

\[
\omega_i = \frac{2(k-l+1)}{k(k+1)}, \quad l = \overline{1,k}.
\]

Thus, we define \( Z \)-numbers \( Z_j = (\bar{X}_j, \bar{R}_j), \ j = \overline{1,m} \) of the group expert criterion as follows:

\[
Z_j = \omega_1 Z_1^j + \omega_2 Z_2^j + \ldots + \omega_k Z_k^j, \ j = \overline{1,m}.
\]

Since the first components of these \( m \) \( Z \)-numbers are determined, it is necessary to determine their second components or the reliability of the first components. To do this, we need to determine the aggregation segment for a \( Z \)-number.

We will do it based on the definition of the aggregation segment for ordinary fuzzy number, given in [27] in the form of an aggregation segment of the product of the first and second component of a \( Z \)-number. If \( Z = (\bar{X}, \bar{R}), \ \mu_X = (x_1, x_2, x_L, x_R), \ \mu_R = (r_1, r_2, r_L, r_R) \), then the aggregation segment \([\delta_1, \delta_2]\) for this \( Z \)-number we determine such as:

\[
\delta_1 = r_1 \left( x_1 - \frac{1}{6} x_L \right) - r_2 \left( \frac{1}{6} x_1 - \frac{1}{12} x_L \right), \quad \delta_2 = r_2 \left( x_2 + \frac{1}{6} x_R \right) + r_L \left( \frac{1}{6} x_2 + \frac{11}{12} x_R \right).
\]

Define the distance between two \( Z \)-numbers \( Z_1 \) and \( Z_2 \) with aggregation segments \([\delta_1^1, \delta_2^1]\), \([\delta_1^2, \delta_2^2]\) accordingly:

\[
\rho(Z_1, Z_2) = \sqrt{(\delta_1^1 - \delta_1^2)^2 + (\delta_2^1 - \delta_2^2)^2}.
\]

Define the aggregation segment \([\beta_1, \beta_2]\) for the \( Z_j = \omega_1 Z_1^j + \omega_2 Z_2^j + \ldots + \omega_k Z_k^j, \ j = \overline{1,m} \) and the aggregation segments \([\alpha_1, \alpha_2]\) for the products \( \bar{R}_i \times (\omega_1 X_1^i + \omega_2 X_2^i + \ldots + \omega_k X_k^i) = \bar{Z}_i, \ i = \overline{1,5} \), where \( \bar{R}_i, i = \overline{1,5} \) are formalizations of linguistic values “Unlikely”, “Not very likely”, “Likely”, “Very likely”, “Extremely likely” of expert information reliability.

Let define

\[
\rho(Z_j, \bar{Z}_i) = \sqrt{(\beta_1^j - \alpha_1^i)^2 + (\beta_2^j - \alpha_2^i)^2}, \quad i = \overline{1,5}.
\]
If \( \rho(Z_j, \bar{Z}_p) = \min_i \rho(Z_j, \bar{Z}_i) \), then the reliability of the \( \bar{Y}_j \) (the first component of \( Z \)-number \( Z_j, j = 1, m \) from group expert criterion) is fuzzy number \( \bar{R}_j \) and accordingly \( p \)-th term of linguistic variable “Reliability”.

4. Conclusion
The paper is devoted to the construction of a group expert criterion for evaluating a certain characteristic under \( Z \)-information. Individual expert criteria are presented in the form of a set of \( Z \)-numbers, the first components of which are fuzzy numbers - formalizations of terms of semantic scopes, constructed for each expert on the basis of a direct survey. The second components of \( Z \)-numbers are formalizations of terms of the semantic space "Reliability" and accordingly fuzzy evaluations of the first components.

The group expert criterion is presented as a set of \( Z \)-numbers. The first components of these numbers are determined based on the Pareto condition. The second components of \( Z \)-numbers are determined on the basis of recognition and identification of the reliability of expert information with one of the linguistic terms (and, accordingly, with a fuzzy number that formalizes it) of the semantic scope "Reliability". For this, aggregation segments are defined for \( Z \)-numbers. Based on these segments, the distance between two \( Z \)-numbers is determined.

The second components of \( Z \)-numbers from the group expert criterion are determined on the basis of a comparative analysis of the distances between \( Z \)-number belonging to the group expert criterion and \( Z \)-numbers, the first component of which coincides with the first component of \( Z \)-number from the group criterion, and the second components are formalizations of the terms of the semantic scope “Reliability”. The reliability of group expert evaluation is determined by the linguistic value of the semantic scope "Reliability", which corresponds to the minimum distance.

Considering the practical absence of models for formalizing of group expert criteria under \( Z \)-information, the model proposed in this paper opens up new possibilities for formalizing and processing group expert information, taking into account its reliability.

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