Pair-breaking in iron-pnictides

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(Dated: October 26, 2009)

The puzzling features of the slopes of the upper critical field at the critical temperature $T_c$, $H_{c2}(T_c) \propto T_c$, and of the specific heat jump $\Delta C \propto T_c^3$ of iron-pnictides are interpreted as caused by a strong pair-breaking.

PACS numbers: 74.20.-z, 74.20.Rp

I. INTRODUCTION

Newly discovered iron-pnictide superconductors have a number of uncommon properties. The subject of this paper are two such properties: (a) The specific heat jump $\Delta C$ is proportional to $T_c^3$ as demonstrated on “122” series of Ba(Fe$_{1-x}$Co$_x$)$_2$As$_2$ and Ba(Fe$_{1-x}$Ni$_x$)$_2$As$_2$. This behavior, according to Ref. 2, cannot be explained within the “realm of conventional BCS theory”. Similar behavior is recorded in 122 crystals with Ba substituted partially with K and with Fe substituted with Pd, Rh, and Co-Cu. (b) Slopes of the upper critical field $dH_{c2}/dT$ at $T_c$ are proportional to $T_c$ across both 1111 and 122 series.

It is shown below that both scalings can be understood within the weak-coupling BCS model provided a strong pair breaking is present in these materials. In fact, these features should also be present in conventional superconductors with magnetic impurities as discussed by Abrikosov and Gor’kov (AG) in their seminal work on the pair breaking for the nearly critical concentration of these impurities when $T_c \ll T_{c0}$, the critical temperature of clean material. AG had considered isotropic materials with a spherical Fermi surface and the s-wave order parameter constant along this surface. The symmetry of the order parameter in multi-band pnictides is not yet determined with certainty; however, many favor the $\pm s$ structure. The critical temperature in materials with a strongly anisotropic order parameter is suppressed not only by scattering breaking the time reversal symmetry (e.g., the spin-flip); in fact, any scattering reduces $T_c$. The term “pair-breaking” is used here in a broad sense for any process suppressing $T_c$. It is shown below that both features, $dH_{c2}/dT \propto T_c$ and $\Delta C \propto T_c^3$, follow from the assumption that the “pair-breaking in a broad sense” is strong.

Below, the linearized Ginzburg-Landau (GL) equation and the energy near $T_c$ are derived within the weak coupling scheme (that allows one to evaluate $dH_{c2}/dT$ and $\Delta C$ at $T_c$) for an arbitrary anisotropy of the order parameter $\Delta$ and of the Fermi surface in the presence of pair-breaking. The text is focussed on the situation when the average $\langle \Delta \rangle$ over the F-surface is close to zero that presumably is the case of pnictides. Comparison with the data available concludes the paper.

Perhaps, the simplest for our purpose is the Eilenberger quasiclassical formulation of the Gor’kov’s theory that holds for a general anisotropic F-surface and for any gap symmetry:

$$v \Pi f = 2\Delta g - 2\omega f + \frac{g}{\tau_-} \langle f \rangle - \frac{f}{\tau_+} \langle g \rangle,$$

$$-v \Pi^* f^+ = 2\Delta^* g - 2\omega f^+ + \frac{g}{\tau_-} \langle f^+ \rangle - \frac{f^+}{\tau_+} \langle g \rangle,$$

$$g^2 = 1 - ff^+,$$

$$\Delta(r, k_F) = 2\pi TN(0) \sum_{\omega > 0} \langle V(k_F, k'_F) f(v', r, \omega) \rangle,$$

Here, $v$ is the Fermi velocity, $\Pi = \nabla + 2\pi iA/\phi_0$, $\phi_0$ is the flux quantum, $\Delta(r, k_F)$ is the order parameter that in general depends on the position $k_F$ at the F-surface of other than the isotropic s-wave symmetry. The functions $f(r, v, \omega)$, $f^+$, and $g$ originate from Gor’kov’s Green’s functions integrated over the energy variable near the F-surface. Further, $N(0)$ is the total density of states at the Fermi level per one spin; the Matsubara frequencies $\omega = \pi T(2n + 1)$ with an integer $n$ and $h = k_B = 1$. The averages over the F-surface are shown as $\langle \ldots \rangle$.

The scattering in the Born approximation is characterized by two scattering times, the transport scattering time $\tau$ responsible for conductivity in the normal state, and $\tau_m$ for spin-flip processes:

$$\frac{1}{\tau_+} = \frac{1}{\tau} + \frac{1}{\tau_m},$$

The strong scattering in unitary limit is not considered here. Commonly, the scattering is characterized by two parameters

$$\rho = \frac{1}{2\pi T_c \tau} \quad \text{and} \quad \rho_m = \frac{1}{2\pi T_c \tau_m},$$

or equivalently by $\rho_\pm = \rho \pm \rho_m$. This is of course a simplification; for multi-band F-surfaces one may need more parameters for various intra- and inter-band processes. This and other simplifying assumptions notwithstanding, the model employed is amenable for analytic work and may prove useful.

Long experience in dealing with pair-breaking effects has shown that the formal AG scheme in fact applies to various situations with different causes for the pair breaking, not necessarily the AG spin-flip scattering. In each particular situation, the parameter $\rho_m$ must be properly
defined. Without specifying the pair breaking mechanism in materials of interest here we apply below the AG approach to show that the pair breaking accounts for experimental data on slopes of $H_{c2}$ at $T_c$ and for quite unusual dependence of the specific heat jump on $T_c$.

Commonly, the effective coupling $V$ is assumed factorizable, $V(k_F, k_F') = V_0 \Omega(k_F) \Omega(k_F')$. One then looks for the order parameter in the form:

$$\Delta(r, T; k_F) = \Psi(r, T) \Omega(k_F).$$  \hspace{1cm} (7)

Our notation is motivated by the fact that so defined $\Psi(r, T)$ enters the Ginzburg-Landau (GL) theory near $T_c$. The function $\Omega(k_F)$, which describes the variation of $\Delta$ along the F-surface, is conveniently normalized:

$$\left\langle \Omega^2 \right\rangle = 1. \hspace{1cm} (8)$$

Then, the self-consistency equation (4) takes the form:

$$\Psi(r, T) = 2\pi T N(0)V_0 \sum_{\omega > 0} \frac{\omega \rho_{\omega}}{\omega} \left\langle \Omega(k_F) f(k_F, r, \omega) \right\rangle. \hspace{1cm} (9)$$

The assumption of a factorizable potential is quite restrictive as far as complicated F-surfaces and interactions are concerned. E.g., within a two-band scheme with four independent coupling constants $V_{ij}$, the factorizable model implies $V_{11}V_{22} - V_{12}V_{21} = 0$.

Instead of dealing with the effective microscopic electron-electron interaction $V$ and with the energy scale $\omega_D$, one can use within the weak coupling scheme the critical temperature $T_{c0}$ (of the hypothetic clean material) utilizing the identity

$$\frac{1}{N(0)V_0} = \ln \frac{T}{T_{c0}} + 2\pi T \sum_{\omega > 0} \frac{\omega \rho_{\omega}}{\omega}, \hspace{1cm} (10)$$

which is equivalent to the BCS relation $\Delta_0(0) = \pi T_{c0} \gamma \gamma = 2\omega_D \exp(-1/N(0)V_0) \gamma$ is the Euler constant. Substitute Eq. (10) in Eq. (9) and replace $\omega_D$ with infinity due to fast convergence:

$$\frac{\Psi}{2\pi T} \ln \frac{T_{c0}}{T} = \sum_{\omega > 0} \left( \frac{\omega}{\omega} - \left\langle \frac{\omega}{\omega} \right\rangle \right). \hspace{1cm} (11)$$

II. GL DOMAIN AND $T_c(\tau, \tau_m)$

Near $T_c$, $g = 1 - f f ^ + / 2$ and Eq. (1) reads:

$$\frac{1}{2} v \Pi f = \Delta - \omega f + \frac{\left\langle f \right\rangle}{2\tau_+} - \frac{ff^+}{2} \left( \Delta + \frac{\left\langle f \right\rangle}{2\tau_-} \right) + f f^+ f \left( \frac{\omega}{2\tau_+} \right). \hspace{1cm} (12)$$

Here,

$$\omega_+ = \omega + 1/2\tau_+, \hspace{1cm} (13)$$

and the terms on the RHS are arranged according to their order in powers of $\delta t = 1 - T / T_c$: the terms on the upper line are of the order $\delta^1$, whereas on the lower line $\sim \delta^3/2$. Note that on the LHS, $H f \sim f / \xi \sim \delta t$.

We look for the solution $f = f_1 + f_2 + \ldots$ where $f_1 \sim \delta^1$, $f_2 \sim \delta^2$. Hence, we have in the lowest order:

$$0 = \Delta - \omega f_1 + \frac{\left\langle f_1 \right\rangle}{2\tau_+}. \hspace{1cm} (14)$$

Taking the average over the Fermi surface we obtain

$$\left\langle f_1 \right\rangle = \frac{\Delta}{\omega_m}, \hspace{1cm} \omega_m = \omega + 1/\tau_m \hspace{1cm} (15)$$

(note the difference in definitions of $\omega_+$ and $\omega_m$). Hence:

$$f_1 = \frac{\Delta}{\omega_m} \left( \Delta + \frac{\langle \Delta \rangle}{2\tau_+ - \omega_m} \right). \hspace{1cm} (16)$$

Comparing terms of the order $\delta t$, we get

$$\langle f_2 \rangle = -\frac{\langle \omega \Pi f_1 \rangle}{\omega_m} = 0, \hspace{1cm} (17)$$

and

$$f_2 = -\frac{1}{\omega_m^2} \omega \Pi \left( \Delta + \frac{\langle \Delta \rangle}{2\tau_+ - \omega_m} \right). \hspace{1cm} (18)$$

Evaluation of higher order corrections for arbitrary $\Delta$ anisotropy is increasingly cumbersome unlike the case $\langle \Delta \rangle = 0$ for which one finds:

$$f_3 = -\frac{\Delta}{\omega_m^2} \left( \Delta^2 - \frac{\langle \Delta^2 \rangle}{2\tau_+ - \omega_m^2} \right). \hspace{1cm} (19)$$

The critical temperature of materials with anisotropic order parameter is suppressed by scattering. In zero field, all quantities are coordinate independent; besides, as $T \rightarrow T_c, g \rightarrow 1$. Therefore, we can utilize $f$ of Eq. (16) in the lowest order to obtain for $T_c$:

$$\frac{1}{2\pi T_c} \ln \frac{T_{c0}}{T_c} = \sum_{\omega > 0} \left( \frac{1}{\omega_+} - \frac{1}{\omega_-} - \frac{\langle \Omega \rangle^2}{2\omega_m^2 \omega_+ \omega_-} \right) \omega_m, \hspace{1cm} (20)$$

where the subscript $c$ is to denote that $\omega$’s are taken at $T_c$. This generalization of the well-known AG result gives the $T_c$ suppression for any (Born) scattering for arbitrary symmetry of the order parameter; it has originally been obtained by Openov. The sums here are expressed in terms of di-gamma functions:

$$\ln \frac{T_{c0}}{T_c} = \psi \left( \frac{1 + \rho^+}{2} \right) - \psi \left( \frac{1}{2} \right) - \langle \Omega \rangle^2 \left[ \psi \left( \frac{1 + \rho^+}{2} \right) - \psi \left( \frac{1}{2} + \rho_m \right) \right]. \hspace{1cm} (21)$$

If $T_c \rightarrow 0$, one can use asymptotic expansion $\psi(z) = \ln z - 1/2z$ for large arguments since $\rho, \rho_m \rightarrow \infty$. The
leading term then gives that $T_c = 0$ when scattering times satisfy the relation:
\[
\frac{1}{\tau_m} \left( \frac{\tau_m}{2\tau^+} \right)^{1-(\Omega)^2} = \frac{\Delta_0(0)}{2}.
\tag{22}
\]

Here, $\Delta_0(0) = \pi T_c e^{-\gamma}$ is the zero temperature gap of the (hypothetic) scattering free material. Clearly, this reduces to the AG critical rate $1/\tau_m = \Delta_0(0)/2$ for isotropic order parameters. If $\langle \Omega \rangle = 0$ (as, e.g., for the d-wave), we have the critical combined rate: $1/\tau^+ = \Delta_0(0)$.

In the absence of spin-flip scattering ($\tau_m \to \infty$) the LHS is zero and Eq. (22) has no solutions for $\tau$, i.e., $T_c$ does not turn zero for any $\tau$. However, a finite $\tau$ at which $T_c = 0$ does exist for any finite $\tau_m$. One can show that near the critical value $\tau_{c, cr}^2 = \Delta_0(0)/(\tau_m/2 (\Omega)^2$, the critical temperature behaves similarly to the AG gapless case, $T_c \propto (\tau_m - \tau_{c, cr}^2)^{1/2}$.

Combining Eqs. (11) and (20) one excludes the unphysical $T_c = 0$:
\[
\Psi \left( \frac{1}{2\pi T} \ln \frac{T}{T_c} \right) = \sum_{\omega > 0} \left( \frac{\Psi(\Omega)^2}{2t \omega_c^2 \omega_s} \right) - \langle \Omega f \rangle.
\tag{23}
\]

where $t = T/T_c$.

### III. THE CASE $T_c \ll T_c$

Situations of interested here are of $T_c$ strongly suppressed relative to $T_c$ (similar to the gapless superconductivity of AG, but not necessarily the same). It is convenient for this purpose to rearrange Eq. (23) by adding and subtracting $\Psi/\omega^+$ under the sum. We transform:
\[
2\pi T \sum_{\omega > 0} \left( \frac{1}{t \omega_c^2} - \frac{1}{\omega^+} \right) = \sum_{n=0}^{\infty} \left( \frac{1}{n+1/2+\rho^+/2} - \frac{1}{n+1/2+\rho^+/2t} \right) = \psi(\frac{\rho^+}{2t} + \frac{1}{2}) - \psi(\frac{\rho^+}{2} + \frac{1}{2}) = \ln t - 1 - t^2 \frac{6\rho^+}{t}. \tag{24}
\]

The parameter $\rho^+$ is large if $T_c \to 0$ and one can use large arguments asymptotics of the di-gamma functions. Combining Eqs. (23) and (24) we obtain the self-consistency equation in the form:
\[
\frac{\Psi(1-t^2)}{2\pi T t^2} = \sum_{\omega > 0} \left( \frac{\Psi(\Omega)^2}{2t \omega_c^2 \omega_s} \right) - \langle \Omega f \rangle.
\tag{25}
\]

### A. Linearized GL equation and the coherence length

The GL equations are obtained by utilizing smallness of $\Delta/\omega$ and of $v\Pi_k \Delta/\omega^2$ near $T_c$. Hence, one can use Eqs. (16), (18), and (19) for $f$ and the self-consistency equation. For the case of exclusively transport scattering ($\tau_m = \infty$), the GL equations have been derived in Ref. 13. It is done below taking a finite $\tau_m$.

To write the self consistency Eq. (25) near $T_c$ one has to express $\langle \Omega f \rangle$ with the help of Eq. (12). To this end, one applies $\langle \Omega f \rangle$ to (12) keeping terms up to the order $\delta t$:
\[
\langle \Omega f \rangle = \frac{\Psi(\Omega)^2}{2\tau - \omega^+} + \langle \Omega f \rangle + \langle \Omega(\Omega f) \rangle + \langle \Omega(\Omega f) \rangle.
\tag{26}
\]

and substitutes the result to Eq. (25):
\[
\frac{\Psi \delta t}{6\pi T t^2} = \sum_{\omega > 0} \left( \frac{\Psi(\Omega)^2}{2t \omega_c} \right) - \langle \Omega f \rangle \langle \Omega f \rangle + \langle \Omega(\Omega f) \rangle.
\tag{27}
\]

Since we are expanding in powers of $\sqrt{\delta t}$, the distinction between, e.g., $\omega_c$ and $\omega = \omega_c(1 - \delta t)$ is relevant.

When substituting here $f = f_1 + f_2$ of Eqs. (16) and (18) note that $\langle \omega f \rangle$ $\Delta = 0$ because the angular dependence of $\Omega$ (the symmetry of $\Delta$) has nothing to do with that of the vector $\Pi$. We then obtain:
\[
\frac{\Psi \delta t}{6\pi T t^2} = \sum_{\omega > 0} \left( \frac{\Omega(\Omega f)^2}{2t \omega_c} \right) - \langle \Omega(\Omega f) \rangle + \langle \Omega(\Omega f) \rangle.
\tag{28}
\]

Note that the LHS and the term at the lower line of this equation are of the order $\delta t^{3/2}$; for this reason all $\omega$’s in this term are taken at $T_c$. Besides, the round brackets at the upper line of the RHS are easily shown to turn zero at $t = 1$. Expanding the bracketed expression in powers of $\delta t$ and keeping only the first term one obtains:
\[
A \Psi \delta t = -B_{ik} \Pi_i \Pi_k \Psi
\tag{29}
\]

with
\[
A = \frac{1}{6\pi T t^2} \left( \frac{\Omega}{2\tau - \omega^+} - \frac{\Omega^2}{2\tau - \omega^+} \right) \sum_{\omega > 0} \left[ \frac{\omega_x^2}{\omega_m \omega_c} + \frac{1}{2} \frac{\omega_m}{\omega_c} \right] - \frac{\Omega^2}{2\tau - \omega^+} \omega_c^2.
\tag{30}
\]

\[
B_{ik} = \frac{1}{4} \sum_{\omega > 0} \left( v_{ik} \left( \frac{\Omega^2}{2\omega_c^2} + \frac{\Omega(\Omega f)}{2\omega_c^2} \right) \right),
\tag{31}
\]

where all $\omega$’s are at $T_c$ and the subscript $c$ is omitted. This is, in fact, the linearized anisotropic GL equation
\[
-(\xi^2)_{ik} \Pi_i \Pi_k \Psi = \Psi.
\tag{32}
\]

with anisotropic coherence length given by
\[
(\xi^2)_{ik} = B_{ik}/A \delta t.
\tag{33}
\]
All sums in Eqs. (30) and (36) are expressed in terms of poly-gamma functions of large parameters $\rho_\pm$. Keeping the leading terms we obtain:

$$
A = \frac{1}{6\pi T_c \rho_+^2} - \frac{(\Omega)^2 (2\rho_+ - \rho_-)}{\pi T_c \rho_-} \ln \frac{\rho_+}{2\rho_m},
$$

(34)

$$
B_{ik} = \frac{(\Omega^2 \nu_i \nu_k) \tau^2_i}{2\pi T_c} + \frac{(\Omega) (\Omega \nu_i \nu_k) \tau^2_i}{2\pi T_c} \ln \frac{\rho_+}{2\rho_m} - \frac{\rho_- (2\rho_+ + \rho_-)}{2\rho_m^2}.
$$

(35)

B. Materials with $(\Omega) = 0$ near $T_c$

This corresponds, e.g., to the d-wave symmetry. Within a two-band model for iron-pnictides the order parameter has a $\pm s$ structure, so that $(\Delta) \ll |\Delta_{\text{max}}|$.\(^6\)

One then expects the model with the dirty case $H$ to be dominated by a strongly suppressed order parameter.

This result has been obtained in Ref. 11 for a clean $d$-wave with anisotropic order parameter. If $(\Omega) = 0$, $A$ and $B$ are simplified:

$$
A = \frac{1}{6\pi T_c \rho_+^2},
$$

(36)

$$
B_{ik} = \frac{(\Omega^2 \nu_i \nu_k) \tau^2_i}{2\pi T_c}.
$$

We then have:

$$
(\xi^2)_{ik} = \frac{3(\Omega^2 \nu_i \nu_k) \tau^2_i}{4\pi^2 T_c^2 \partial t}.
$$

(37)

For the d-wave order parameter and isotropic 2D Fermi surface, $\Omega = \sqrt{2} \cos 2\varphi$ and $(\Omega^2 \nu_i \nu_k) = \nu^2/2$:

$$
(\xi^2) = \frac{3\nu^2 \tau^2_i}{8\pi^2 T_c^2 \partial t}.
$$

(38)

This result has been obtained in Ref. 11 for a clean d-wave with a strongly suppressed $T_c$.

For a uniaxial material, the slope of the upper critical field along the $c$ direction near $T_c$ is given by

$$
\frac{dH_{c2,c}}{dT} = -\frac{2\pi \phi_0 k_B^2}{3\hbar^2 (\Omega^2 \nu_i \nu_k) \tau^2_i} T_c.
$$

(39)

(in common units). Although the scattering and pair-breaking parameters do not enter this result explicitly, they affect $H_{c2,c}$ and its slope via $T_c(\rho_+)$. One readily obtains for the other principal direction:

$$
\frac{dH_{c2,ab}}{dT} = -\frac{2\pi \phi_0 k_B^2}{3\hbar^2 (\Omega^2 \nu_i \nu_k) \tau^2_i} T_c.
$$

(40)

It is worth recalling that in isotropic materials with the standard s-wave order parameter the slope $H_{c2} \propto T_c$ in the clean limit (because $H_{c2} \propto 1/\xi^2 \propto T_c^2$) whereas for the dirty case $H_{c2} \propto T_c$ independent ($H_{c2} \propto 1/\xi \propto T_c$, $\xi$ is the mean-free path). The proportionality $H_{c2}$ to $T_c$ is a property of the AG gapless state. In our case, the result (39) is obtained for a strong pair-breaking in materials with anisotropic order parameter.

Note also that even without magnetic scatterers, in materials with $(\Omega) = 0$ and $\rho^+ \gg 1$, the superconductivity becomes “gapless” in a sense that the total density of states at the Fermi level is not zero. As in the AG case, if $T_c \to 0$, the superconductivity is weak at all temperatures, i.e., $f < \xi^2/2 = 1 - \Delta^2/2\Omega^2$ in the whole domain $0 < T < T_c$. Then the energy dependence of the total density of states $N(\epsilon) = N(0) \Re g(\Omega \to i\epsilon)$ reads:

$$
\frac{N(\epsilon)}{N(0)} = 1 - 2\Delta^2 \tau^2 + 1 - \frac{\eta^2}{(1 + \eta^2)^2}, \quad \eta = 2\tau_+ e^{-\epsilon_t}.
$$

(41)

Hence, at zero energy, $N(\epsilon)$ has a non-zero minimum, whereas the maximum of $N(\epsilon)$ is reached at $\epsilon_m = \sqrt{3}/2\tau_+$ (not at $\Delta$). Therefore, the ratio of the “apparent gap” $\epsilon_m$ to $T_c$ should vary as $1/T_c$. Since only the total density of states is non-zero, this does not exclude possibility to have gapped and gapless patches on the F-surface.

IV. THE SPECIFIC HEAT JUMP

Eilenberger equations (1) and (11) in zero field can be obtained minimizing the functional\(^7\)

$$
\mathcal{F} = N(0) \left[ \Psi^2 \ln \frac{T}{T_N} + 2\pi T \sum_{\omega > 0} \left( \frac{\Psi^2}{2\hbar \omega} - \left\langle I \right\rangle \right) \right].
$$

(42)

$$
I = 2\Delta f + 2\Delta \left( g - 1 \right) + \frac{f(f)}{2\tau} - \frac{\left( g(g) - 1 \right)}{2\tau^+}.
$$

(43)

The function $g$ here is an abbreviation for $\sqrt{1 - f^2}$. Taking account of the self-consistency equation (11), we obtain the energy difference between the normal and superconducting states:

$$
\frac{F_s - F_n}{2\pi T N(0)} = \left\langle \Delta f + 2\Delta (g - 1) + \frac{f(f)}{2\tau} + \frac{g(g) - 1}{2\tau^+} \right\rangle.
$$

(44)

One can check that this reduces to the known result for isotropic s-wave cases with or without pair breaking.\(^8\) This offers a straightforward way to calculate the specific heat near $T_c$. The calculation, in general, is tedious because one has to keep track of terms up to $\Delta^4 \propto B^2$. We consider only the case $(\Delta) = 0$.

Up to the forth order in $\Delta$ we have with the help of Eqs. (16) and (19):

$$
\frac{\Delta}{\omega_+} + \frac{\Delta}{2\omega_+} \left( \frac{\Delta^2}{2\tau_+ \omega_+} - \Delta^2 \right),
$$

(45)

$$
\frac{\Delta}{\omega_+} + 3\Delta^4 \omega_+^4 - \frac{\Delta^2 (\Delta^2)}{4\tau_+ \omega_+^2},
$$

(46)

where all $\omega$s are taken at $T_c$. Substituting these in the energy difference we obtain:

$$
\frac{F_s - F_n}{2\pi T N(0)} = \frac{\Psi^2}{4} \sum \left( \frac{\Omega^4}{\omega_+^2} - \frac{1}{2\tau_+ \omega_+^2} \right).
$$

(47)
For large $\rho_+$ one finds:

$$
\sum \left( \frac{(\Omega^4)}{\omega_+^4} - \frac{1}{2(\omega_+^4)} \right) \approx \frac{(3(\Omega^4) - 2)\tau^2}{3\pi T}.
$$

(48)

To complete the energy evaluation one needs $\Psi(T)$ which is obtained with the help of the self-consistency equation (27) and the expression (45) for $f$:

$$
\Psi^2 = \frac{4\pi^2 T_e^2(1 - t)}{3(\Omega^4) - 2}.
$$

(49)

Thus the energy difference between the normal and superconducting states reads:

$$
F_n - F_s = \frac{8\pi^4 N(0)\tau^2_+}{3h^2(3(\Omega^4) - 2)} k_B T_e^2 (T_c - T)^2
$$

(50)

in common units. The specific heat jump at $T_c$ follows:

$$
\Delta C = C_s - C_n = \frac{16\pi^4 k_B^2 N(0)\tau^2_+}{3h^2(3(\Omega^4) - 2)} T_c^3.
$$

(51)

Within a weak coupling scheme, this result in a more general form has been obtained in Ref. 12.

For the d-wave state on a cylindrical Fermi surface $\Omega = \sqrt{2} \cos^2 2\phi$ and $(\Omega^4) = 3/2$ this gives:

$$
\Delta C = \frac{32\pi^4 k_B N(0)\tau^2_+}{15h^2} T_c^3.
$$

(52)

V. DISCUSSION

Figure 1 is a compilation of data on the slopes $H'_{c2}$ for 1111 compounds with various dopants and, therefore, with various $T_c$'s. An approximate scaling $H'_{c2} \propto T_c$ is evident despite the fact that the compounds examined have $T_c$'s varying from 6 to 46 K. From this data one estimates the slope of $dH'_{c2}/dT_c$ as $\approx 0.2 T/K^2$. Then, the order of magnitude of the Fermi velocity follows from $|dH'_{c2}/dT_c| \sim \pi \phi_0 k_F^2 / h^2 v^2$ as $v \sim 10^7$ cm/s, a reasonable order that can be taken as yet another argument in favor of the picture presented.

In Fig. 2 the data for the 122 family are collected. The same approximate scaling is seen. A considerable scatter of the data points might be caused by variety of reasons: different criteria in extracting $H_{c2}$ from resistivity data, unavoidable uncertainties rooted in sample inhomogeneities in determination of $T_c$ and the slopes of $H_{c2}(T)$ near $T_c$, possible differences in Fermi velocities and the order parameter anisotropies, to name a few. Moreover, the model employing only two scattering parameters for multi-band iron-pnictides is a far-reaching simplification, so that one can expect the model to work qualitatively at best. Nevertheless, the observed scaling seems remarkably robust. One can take this as evidence in favor of a strong pair-breaking present in all compounds examined. It should be stressed again that for strongly anisotropic order parameters, $|\Delta| \approx 0$, the $T_c$ suppression (or the pair-breaking, which is the same) is caused by the combined effect of the transport and the spin-flip scattering.

Figure 3 shows the specific heat jump measured in a number of compounds and reported in Ref. 1. The “Ames scaling” $\Delta C \propto T_c^3$ suggested by Bud’ko, Ni, and Canfield is evident. Again, it is worth noting that only the combined effect $\rho_+$ enters the coefficient in front of $T_c^3$ of Eq. (51), so that the source of $T_c$ suppression is not necessarily the spin-flip AG pair-breaking. The ever present transport scattering suppresses $T_c$ as well, provided the

![FIG. 1: (Color online) The slopes of $H_{c2}(T)$ near $T_c$ (the absolute values) for a few 1111 compounds. The data for the first three compounds in the legend are taken from Ref. 14; the remaining three points are taken from Ref. 15. The two right-most points are for $H'_{c2,ab}$ of crystalline samples; the rest are for polycrystals, so that all points, in fact, reflect $H'_{c2,ab}$.](image1)

![FIG. 2: (Color online) The slopes $H_{c2}(T)$ near $T_c$ for a few 122 iron-pnictides. The data are taken from: RbFe$_2$As$_2$ – Ref. 17, KFe$_2$As$_2$ – Ref. 18, Ba$_{0.53}$K$_{0.47}$Fe$_2$As$_2$ – Ref. 16, the underdoped (ud) and overdoped (od) Ba(Fe$_{1-x}$Co$_x$)$_2$Fe$_2$As$_2$ – Ref. 3, Ba$_{0.6}$K$_{0.4}$Fe$_2$As$_2$ – Ref. 19, Sr$_{0.6}$K$_{0.4}$Fe$_2$As$_2$ – Ref. 20, Ba(Fe-Ni)$_2$Fe$_2$As$_2$ and Ba(Fe$_{1-x}$Co$_x$)$_2$Fe$_2$As$_2$ – Ref. 21.](image2)
order parameter is strongly anisotropic. This is presumably the case of iron-pnictides.

One may wonder why the scaling $H'_{c2} \propto T_c$ and $\Delta C \propto T^3$ seem to work across the whole class of iron pnictides for compounds with different couplings, F-surfaces etc. Clearly, the source of this scaling should be universal across the pnictides family of materials. The pair breaking is offered here as such an universal source.

As for the apparent simplicity of the model used one should have in mind the often overlooked strength of the weak-coupling scheme: the model is formulated in terms of the measured critical temperature $T_c$, in which the coupling constants and energy scales of the “glue bosons” are incorporated.

Having succeeded in describing the “Ames scalings” just discussed, one can venture to a prediction: according to Eq. (41), tunneling experiments are likely to show the ratio of the apparent gap (the maximum position of the total density of states) to $T_c$ varying as $1/T_c$ across the family of iron pnictides.

**Appendix A: Materials with $\langle \Omega \rangle \neq 0$ Near $T_c$**

Interestingly enough, the behavior of the $H_{c2}$ slopes as functions of $T_c$ turns out different if $\langle \Omega \rangle \neq 0$. To see this, consider the expression for the coefficient $A$ of Eq. (34). In terms of scattering times, it reads:

$$A = \frac{2 \pi T_c \tau^2}{3} \frac{\langle \Omega \rangle^2}{\pi T_c} \left( \frac{2\tau}{\tau_m} - 1 \right) \ln \frac{\tau}{2\tau_m}. \quad (A1)$$

Since all $\tau$’s are finite near the critical point where $T_c \to 0$, the term $\propto \langle \Omega \rangle^2$ is leading. Consider, e.g., a usual situation $\tau << \tau_m$:

$$A \approx \frac{\langle \Omega \rangle^2 \ln 2}{\pi T_c}. \quad (A2)$$

After simple algebra one obtains the slope of $H_{c2,e}$ at $T_c$:

$$\frac{dH_{c2,e}}{dT} = -\phi_0 \frac{\langle \Omega \rangle^2 \ln 4}{2 \pi \tau^2 T_c} \left( \frac{\langle \Omega \rangle^2}{\phi_0^2} + \frac{\langle \Omega \rangle^2}{\phi_0^2} \ln (2\tau_m/\tau e^{3/2}) \right). \quad (A3)$$

Thus, the slopes $H'_{c2} \propto 1/T_c$, the dependence opposite to that of the case $\langle \Omega \rangle = 0$.

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