Exploiting Electrostatic Sensors for Rotational Machinery Shaft Centerline Orbit and Vibration Monitoring

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Abstract. To realize a more economical measurement of rotational shaft centerline orbit and vibration, a shaft centerline orbit reconstruction method using a specifically designed electrostatic sensor is proposed in this paper. The sensor consists of 3 sensing electrodes, one shaft cover working as charge source, and a cylindrical shield covering the three electrodes. Based on electrostatic induction, the proposed sensor is sensitive with shaft movement so that the shaft centerline displacement and orbit can be reconstructed. The CAE software COMSOL Multiphysics is used for sensor modeling and simulation analysis. Experiment is carried out to test the performance of the proposed method which proves the feasibility of the novel sensor for shaft centerline orbit and vibration monitoring.

1. Introduction
Rotator shaft is a key component of rotational machineries which are widely used in various industrial process. Monitoring the condition of rotational machinery shaft is significant to most industrial areas. Generally, shaft vibration and centerline orbit include abundant information [1]. Traditional method is based on vibration and eddy current sensor [2], which has high cost and complex installation. Inspired by electrostatic sensor’s being used for machenical measurement [3-6], this paper develops a new method based on electrostatic sensors. The sensors’ signals can be used for vibration analysis and shaft centerline orbit reconstruction. The proposed method provides a new idea of vibration monitoring and a solution for some low-demanding circumstances. The following chapters will illustrate the method in aspects of sensing principle, sensor structure, and algorithm. An experiment is carried out for validation.

2. Sensing Principle and Sensor Structure

2.1. Sensor structure

As it is shown in Fig. 1, the sensor consists of three sensing electrodes, a shaft cover works as a constant charge source, and a glass sensor shell whose outer surface is covered by a copper sheet, grounded. The shaft cover is actually a cylinder capacitance connected to a button cell. When measuring, let the shaft insert throughout the sensor via the hole of the electrode support in the middle, as is presented in Fig. 1-(a). The key geometry parameters are presented in Fig. 1-(c). The three electrodes with radium $R_e (=3\text{mm})$ are fixed by two insulated electrode supports. The shaft and electrodes are enveloped in the bottomless cylinder glass sensor shell with radium $R (=48\text{mm})$. The
center distances between shaft and electrodes equal to \(d_i\) \((i=1, 2, 3, \text{ initial value is 5mm})\); The radium of the shaft equals to \(R_s\) (=15mm). Since, the copper sheet, the three electrodes, and the shaft approximately constitute an electrostatic isolated system, numbered 0-4, respectively.

![Figure 1](image1.png)

**Figure 1.** Structure of the sensor. (a) Overview; (b) Cross section; (c) Geometry parameters.

2.2. Sensing principle

A cylindrical coordinate system can be established whose origin is located at the bottom surface center of the shaft and the positive direction of \(z\)-axis is axially along the shaft, as shown in Fig. 2. \(\Omega\) (the air) is the solution domain with boundary \(\partial \Omega_i\), \(i=0, 1, \ldots, 4\). The solution domain does not include the electrodes and shaft entities but their surfaces.

![Figure 2](image2.png)

**Figure 2.** Coordinate system establishing

According to electrostatic theory [7], when connected to a DC voltage source, the outer surface of the shaft cover (a cylinder sheet capacitance) is charged uniformly. Therefore, the physical model of the measurement system can be described as following:

\[
\begin{cases}
\nabla^2 \varphi = 0, & \text{in } \Omega \\
-(\varepsilon_0 \nabla \varphi) \cdot \mathbf{n} = \text{cons.}, & \text{on } \partial \Omega_i
\end{cases}
\]

where \(\mathbf{n}\) is the outer normal vector of \(\partial \Omega_i\). It is obviously that (1) is a typical Laplacian function with Dirichlet boundary condition. Ignore the interaction of the sensing electrodes, and the induced charge \(Q_i\) \((i=1, 2, 3)\) of each electrode can be calculated by

\[
Q_i = -\int_{\partial \Omega_i} (\varepsilon_0 \nabla \varphi) \cdot \mathbf{n} dS
\]

Once the shaft moves eccentrically, \(d_1\), \(d_2\), and \(d_3\) will change correspondently, causing the changing of \(Q_i\), which can be detected by the sensor conditioning circuit [8]. Let the shaft move horizontally and the shaft cover surface charge density be 1 C/m², the induced charge \(Q_i\) against the distance \(d_i\) is shown in Fig. 3. It can be seen from Fig. 3 that the induced charge on sensing electrodes is linear with
the distance in a relative small range (actually, the shaft vibration amplitude is usually small). Considering that (1) and (2) are linear differential equations, the linearity relation will not change if the surface charge density is changed:

\[ Q_i = K d_i \]  

In practice, \( k \) can be obtained by accurate calibration test or simulation. Since, the electrostatic sensor can be used for shaft centerline orbit and vibration monitoring.

3. Shaft Centerline Orbit and Vibration Monitoring Algorithm

Due to the electrostatic sensor is not only sensitive to a particular moving direction, the signals generated by the orthogonal electrodes cannot be used for centerline orbit synthesis directly. Therefore, a reconstruction method based on shaft center coordinate is proposed. The output of the sensor conditioning circuit is proportional to the induced charge

\[ U_{oi} = k_c Q_i \]  

where \( k_c \) is relative to circuit parameters. According to (3), the relation between \( U_{oi} \) and \( d_i \) is

\[ U_{oi} = k k_c d_i = K d_i \]  

On the interest plane, assume the coordinates of sensor center, electrode center, and shaft center are \((0,0), (x_i, y_i), \) and \((x_s, y_s),\) respectively. Considering (5), \( d_i \) can be expressed as

\[ \frac{1}{K^2} U_{oi}^2 = d_i^2 = X^T A X \]  

where

\[
X^T = (x_i, y_i, x_s, y_s) \\
A = \begin{bmatrix}
1 & 0 & -1 & 0 \\
0 & 1 & 0 & -1 \\
-1 & 0 & 1 & 0 \\
0 & -1 & 0 & 1
\end{bmatrix}
\]  

Solve (6) and the shaft center coordinate in one time can be obtained, which means the shaft centerline displacement information that used for orbit reconstruction. Moreover, it can be decomposed into two orthogonal movement for 2-D vibration analysis.

4. Experiment

4.1. Experiment rig

The experiment rig is shown in Fig. 4, mainly consisting of a motor-belt-bearing transmission system, a shaft to be measured, and a Variable-frequency Drive (VFD). Adjust the rotational speed to change the shaft vibration and centerline orbit.

![Figure 4. The test rig.](image)

4.2. Experiment result and discussion

Since an accurate calibration test for parameter \( k \) is hard to be carried out, this experiment uses normalized data. Thus, the amplitude of vibration waveform and centerline orbit do not represent the reality. According to Fig. 1-(b), the coordinates of electrodes centers are \((0.23), (11.5\sqrt{2},-11.5\sqrt{2}),\)
Control the rotational speed at 3600 r/min, and the displacement of shaft centerline and orbit are shown in Fig. 5 and 6, respectively.

Figure 5. Centerline displacement. Figure 6. Centerline orbits. Figure 7. Vibration spectrum.

Calculate the spectrum of x- and y-component of shaft centerline displacement, as is shown in Fig. 7. We can read from Fig. 7 that the vibration component with maximum value occurs at the frequency corresponding to rotational speed, which is consistent with mechanics theory. Fig. 5-7 show the success in centerline orbit reconstruction and vibration monitoring, validating the effectiveness of the proposed method.

5. Conclusion
The proposed electrostatic sensor and method have successfully realized normalized vibration and shaft centerline orbit reconstruction, which can be used for frequency domain analysis and fault diagnosis. This paper prefers providing researchers with a new instructive idea and this study needs to be further completed. The further research shall aim at the obtaining of $k$, so that the amplitude information of sensors’ signals can be used.

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