Research Article

Physical Aspects of Homogeneous-Heterogeneous Reactions on MHD Williamson Fluid Flow across a Nonlinear Stretching Curved Surface Together with Convective Boundary Conditions

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This article is concerned with the fluid mechanics of MHD steady 2D flow of Williamson fluid over a nonlinear stretching curved surface in conjunction with homogeneous-heterogeneous reactions with convective boundary conditions. An effective similarity transformation is considered that switches the nonlinear partial differential equations riveted to ordinary differential equations. The governing nonlinear coupled differential equations are solved by using MATLAB bvp4c code. The physical features of nondimensional Williamson fluid parameter $\lambda$, power-law stretching index $m$, curvature parameter $K$, Schmidt number $Sc$, magnetic field parameter $M$, Prandtl number $Pr$, homogeneous reaction strength $k_1$, heterogeneous reaction strength $k_2$, and Biot number $\gamma$ are presented through the graphs. The tabulated form of results is obtained for the skin friction coefficient. It is noted that both the homogeneous and heterogeneous reaction strengths reduced the concentration profile.

1. Introduction

The fluid is subdivided into two main categories: non-Newtonian fluid and Newtonian. One of its types of non-Newtonian fluid is a shear-thinning (pseudoplastic) fluid [1]. Pseudoplastic takes attention due to its large commercial applicability. Polymer solutions as well as molten polymers, complex fluids, and suspensions like nail polish, whipped cream, blood, ketchup, and paint are the industrial and everyday applications of pseudoplastic fluids. Gogarty [2] considered the porous media to study the rheological properties of pseudoplastic fluids. Researchers use different models to investigate the behaviour of non-Newtonian fluid like the Ellis model, Williamson model, cross model, Carreau model, and the power-law model, but the Williamson model for fluid flow takes more attention for the study of pseudoplastic fluid. Williamson [3] provides an experimentally verified model for the analysis of pseudoplastic fluids. In the last decade, several researchers [4–9] investigated the behaviour of pseudoplastic fluid by using the Williamson fluid model. Hayat et al. [10, 11] used the Homotopy analytical method to examine the impact of joule heating, thermal radiation, and Ohmic dissipation in the two-dimensional flow of Williamson fluid over a stretching surface.

From the last two decades, several investigators have focused on non-Newtonian fluid across nonlinear and linear stretching of a plate, flat surface, cylinder, or disk [12–16]. Flow across a curved surface is firstly introduced by Sajid et al. [17]. Later on, Abbas et al. [18] analyzed the heat transfer flow of MHD fluid across stretching curved surface. Ahmad et al. [19] examined the boundary layer flow across a curved surface embedded in a porous medium. Sanni et al. [20] investigated the flow of viscous fluid due to a nonlinear stretching curved surface. The effect of mass and heat transfer across a curve-shaped surface is numerically examined by Ramana et al. [21]. Saleh et al. [22] investigated the flow of unsteady micropolar fluid flow over a permeable curved stretching/shrinking surface. The transfer of heat and mass of an electrically conducting micropolar fluid with MHD effect across a curved stretching
of nonlinear index parameter \( m = 1/3 \), which provides an entirely similar Williamson fluid parameter. The impact of different parameters, i.e., curvature parameter \( K \), nondimensional Williamson fluid parameter \( \lambda \), Biot number \( \gamma \), magnetic field parameter \( M \), Schmidt number \( Sc \), Prandtl number \( Pr \), homogeneous reaction strength \( k_1 \), heterogeneous reaction strength \( k_2 \), power-law stretching index \( m \) on velocity, pressure, temperature, and concentration profiles, is presented through the graphs, whereas the results of skin friction coefficient and Nusselt number are depicted in a tabulated form.

2. Formulation of Problem

We examined the 2-dimensional steady, incompressible, fully developed magnetohydrodynamic Williamson fluid flow across a nonlinear stretching curved surface having radius \( H \). We consider \((r, s)\) coordinate system. The \( s \)-axis represents the flow direction, whereas the radial direction is taken along \( r \). The stretching of the curved surface is along the \( s \)-axis with velocity \( u = b_1 s^m \) where \( b_1 \) is the initial stretching rate. The variable magnetic field \( B = B_0 s^{-m-1} \) is applied in the radial direction. For \( h-h \) reactions, we consider two chemical species \( A \) and \( C \), respectively. Figure 1 shows the geometry of flow. In the case of Williamson fluid flow, Cauchy stress tensor is defined as \( \tau = -\rho I + \tau \) in which \( \tau \) represents extra stress tensor and defined as \( \tau = (\mu_{\infty} + (\mu_0 - \mu_{\infty})/(1 - \Gamma))A_1 \) where \( A_1 \Gamma, \mu_0, \) and \( \mu_{\infty} \) are the first Rivlin–Erickson tensor; positive time constant; limiting viscosities at zero and infinite shear stress rates, respectively; and \( \gamma \) is defined as \( \gamma = \sqrt{1/2\pi} \), whereas \( \pi = \text{trace}(A_1)^2 \). Here we consider the case in which \( \Gamma \gamma \) and \( \mu_{\infty} = 0 \). The homogeneous reaction for cubic autocatalysis with two chemical species \( A \) and \( C \) is represented by the equation below:

\[
A + 2C \rightarrow 3C, \quad \text{rate} = h_A a C^2. \tag{1}
\]

For cubic autocatalysis, the heterogeneous reaction on the catalyst surface is mathematically represented as follows:

\[
A \rightarrow C, \quad \text{rate} = h_A a, \tag{2}
\]

where \( h_A \) and \( h_C \) are the rate constants and \( a \) and \( c \) represent the concentrations for chemical species \( A \) and \( C \). Under these conditions, the governing boundary layer equations [9, 17–24, 26–32] are

\[
\frac{\partial}{\partial r} [(H + r)\nu] + H \frac{\partial u}{\partial s} = 0, \tag{3}
\]

\[
\frac{\partial p}{\partial s} = \frac{\rho}{H + r} \frac{\partial u^2}{\partial s}, \tag{4}
\]

\[
\rho \left( \frac{u}{r + H} \frac{\partial u}{\partial s} + \nu \frac{\partial u}{\partial r} + \frac{1}{r + H} \frac{\partial u}{\partial \nu} \right) = \left( \frac{H}{r + H} \right) \frac{\partial p}{\partial s} + \mu \left[ \sqrt{2} \Gamma \left( \frac{\partial u}{\partial r} - \frac{u}{H + r} + 1 \right) \right] \cdot \left[ \frac{\partial^2 u}{\partial r^2} + \left( \frac{1}{r + H} \right) \frac{\partial u}{\partial r} - \frac{1}{(r + H)^2} \right] - \sigma B^2 u, \tag{5}
\]
\[ l_f \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r + H} \frac{\partial T}{\partial r} \right) = \rho c_p \left( \nu \frac{\partial T}{\partial r} + \frac{H}{r + H} u \frac{\partial T}{\partial s} \right), \]

\[ D_A \left( \frac{\partial^2 a}{\partial r^2} + \frac{1}{r + H} \frac{\partial a}{\partial r} \right) - h_c a c^2 = \nu \frac{\partial a}{\partial r} + \left( \frac{R}{r + R} \right) u \frac{\partial a}{\partial s} \]

\[ D_c \left( \frac{\partial^2 c}{\partial r^2} + \frac{1}{r + H} \frac{\partial c}{\partial r} \right) + h_c a c^2 = \nu \frac{\partial c}{\partial r} + \left( \frac{H}{r + H} \right) u \frac{\partial c}{\partial s} \]

The accompanying boundary conditions are [20, 27]

\[ u = u_w = b_1 s^m, \]
\[ v = 0, \]
\[ -h_f \frac{\partial T}{\partial r} = l_f(T_f - T), \]
\[ D_A \frac{\partial a}{\partial r} = h_c a, \]
\[ D_c \frac{\partial c}{\partial r} = -h_c a, \]

For \( r = 0 \)
\[ u(s, r) \to 0, \]
\[ c \to 0, \]
\[ \frac{\partial u}{\partial r} \to 0, \]
\[ T \to T_\infty, \]
\[ a \to a_0, \]

for \( r \to \infty, \)

where \( u \) symbolizes the velocity component in the \( s \) direction and \( v \) describes the velocity component in the \( r \) direction. Furthermore, \( \rho \) represents the density, \( \nu \) and \( \sigma \) are the kinematic viscosity and the electrical conductivity, respectively.

We introduce the following dimensionless variable transformations:

\[ \eta = \sqrt{\frac{b_1 s^{m-1}}{\nu}} r, \]
\[ v = \frac{R}{r + R} \sqrt{b_1 s^{m-1}} \left[ m - \frac{1}{2} \eta f' (\eta) + \frac{m + 1}{2} f (\eta) \right] \]
\[ u = b_1 s^m f' (\eta), \]
\[ p = \rho b_1^2 s^m P (\eta), \]
\[ K = \sqrt{\frac{b_1 s^{m-1}}{\nu}} H, \]
\[ \theta (\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \]
\[ a = a_0 f (\eta), \]
\[ c = c_0 h (\eta), \]

where \( f \) shows the dimensionless velocity, \( \eta \) shows the similarity variable, and \( p \) is the pressure.

Using equation (10) in equations (3)–(9), equation (3) satisfies identically, we get

\[ Pr (\eta) = \frac{f' (\eta)^2}{\eta + K}, \]

\[ P = \frac{1}{2Km(\eta + K)} \left[ (\eta + K)^2 f^{(3)} + (\eta + K) f'' + \frac{1}{2} K (m + 1)(\eta + K) f f'' - \frac{1}{2} \eta K (m - 1) f^{12} \right. \]
\[ \left. - \frac{1}{2} K (\eta + 2Km + \eta m) f^{12} - \frac{1}{2} (2 - f K (m + 1)) f' - M (\eta + K) f' \right. \]
\[ + \lambda \left( (\eta + K) f'' + \frac{f'^2}{\eta + K} (\eta + K)^2 f^{(3)} f' - (\eta + K) f^{(3)} f' - 2 f' f'' \right), \]
\[
\frac{1}{Pr} \left( \theta' + \frac{\theta}{K + \eta} \right) + \frac{K}{K + \eta} \left( \frac{m+1}{2} \right) f \theta' = 0, \tag{13}
\]

\[
\frac{\delta}{Sc} \left( h'' + \frac{h'}{K + \eta} \right) + \frac{K}{K + \eta} \left( \frac{m+1}{2} \right) f h' + k_1 \phi h^2 = 0, \tag{14}
\]

\[
f(0) = 0, \quad \phi'(0) = k_2 \phi(0), \quad \theta'(0) = -\gamma(1 - \theta(0)), \quad f'(0) = 1, \quad \delta h'(0) = k_2 \phi(0), \quad \theta(\infty) \longrightarrow 0, \quad f'(\infty) \longrightarrow 0, \quad \phi(\infty) \longrightarrow 1, \quad f''(\infty) \longrightarrow 0, \quad h(\infty) \longrightarrow 0. \tag{15}
\]

where \( \lambda = \sqrt{2b_1^2} T_s^{-1/2} \sqrt{\nu} \) represents the parameter for Williamson fluid; magnetic field parameter is expressed as \( M = B_0^2 b_1 / \nu \), \( Sc = \nu / D_A \) is the Schmidt number, \( Pr = \nu / \alpha \) is the Prandtl number, \( k_1 = a_2 \lambda b_1 \) is the heterogeneous reaction strength, \( \gamma = l_f / h_f \sqrt{\nu b_1} \) is the Biot number, \( \delta = D_C / D_A \) is the proportion of diffusion coefficients, and \( k_2 = h_J / D_A \sqrt{\nu b_1} \) is the heterogeneous reaction strength.

Abolishing \( P(\eta) \) from the equations (11) and (12) produces the following equation:

\[
f^{(4)} = \frac{2Km}{K + \eta - \lambda f' + (K + \eta) \lambda f''} \left[ \frac{(K (1 + m) (K + \eta) f - 2(K + \eta)) f'}{4Km(K + \eta)^3} + \frac{(3m - 1)f r^2}{4m(K + \eta)} + \frac{\lambda f r^2}{Km(K + \eta)^3} + \frac{f'}{2Km(K + \eta)} - \frac{(1 + m)f f''}{4m(K + \eta)} + \frac{(3m - 1)f f'''}{4m} - \frac{2\lambda f f' f''}{Km(K + \eta)^3} - \frac{4Km(K + \eta)^3 + (K + m)(K + \eta)^3 f}{Km(K + \eta)^3} \right] + \frac{\lambda f f^{(3)}}{Km(K + \eta)} \left[ -\frac{(K + \eta)\lambda f^{(3)}}{2Km} + \frac{\lambda f r^2}{Km(K + \eta)} - \frac{f f^{(3)}}{2Km} + M \left( \frac{f'}{2Km} + \frac{(K + \eta)f''}{2Km} \right) \right]. \tag{16}
\]

When \( D_A = D_C \), then \( \delta = 1 \), we have

\[
\phi(\eta) + h(\eta) = 1. \tag{17}
\]

Thus equations (14) and (15) take the form

\[
k_1 \phi(1 - \phi)^2 - \frac{1}{Sc} \left( \phi'' + \frac{\phi'}{K + \eta} \right) - \frac{K}{K + \eta} \left( \frac{m+1}{2} \right) f \phi' = 0. \tag{18}
\]

Along with boundary conditions

\[
\phi(\infty) \longrightarrow 1, \quad \phi'(0) = k_2 \psi(0). \tag{19}
\]

The local skin friction coefficient \( C_f \) and Nusselt number \( Nu \) are defined as

\[
C_f = \frac{\tau_w}{1/2(\rho u_w^2)}, \tag{20}
\]

\[
Nu = \frac{\sqrt{\text{q}_w}}{h_f(T_w - T_\infty)}.
\]

Shear stress \( \tau_w \) and heat flux \( q_w \) near to the surface are mathematically represented as
of Kumar et al. [38] and Sajid et al. [17] by fixing certain acceptable result in Table 1. We assumed selection and error control are built on the residual of the mesh points which formed the initial guess structure. Mesh the interval of integration from 0 to 4 and divided it into 30 continuous solution. HX he relative error tolerance considered in all discussions.

Using equation (10) in equation (21), equation (20) takes the form

\[
\sqrt{\text{Re}_f} C_f = 2 \left( f''(0) - \frac{1}{R} \right) \left[ 1 + \frac{\lambda}{2} \left( f''(0) - \frac{1}{R} \right) \right],
\]

where \( \text{Re}_s = u_w s/\nu \).

\[ \text{3. Solution Procedure} \]

The coupled nonlinear system of ODEs, (11), (13), (17), and (18), along with boundary conditions (16) and (19), are solved by using built-in MATLAB code bvp4c. In order to find out the solution of coupled ODEs, first of all, we rewrite equations (11) and (13), (17) and (18), with boundary conditions (16), (19) as an equivalent system of first-order differential equations by using the substitutions \( f = z_1, f' = z_2, f'' = z_3, f''' = z_4, P = z_5, \theta = z_6, \theta' = z_7, \phi = z_8, \) and \( \phi' = z_9 \). In the next step, we code these systems of first-order ODEs and the boundary conditions with function names "exlode" and "exlbc" in MATLAB. Furthermore, we choose the interval of integration from 0 to 4 and divided it into 30 mesh points which formed the initial guess structure. Mesh selection and error control are built on the residual of the continuous solution. The relative error tolerance considered in this study is \( 10^{-6} \). Finally, we call "bvp4c" function

\[
\text{sol} = \text{bvp4c (@exlode, @exlbc, solinit, options)}. \tag{23}
\]

The "deval" built-in MATLAB function is used to evaluate the solution at a specific point. The Comparison of skin friction coefficient from the present study with the work of Kumar et al. [38] and Sajid et al. [17] by fixing \( \lambda = K = M = k_1 = k_2 = y = Pr = Sc = 0 \) and \( m = 1 \) shows the acceptable result in Table 1. We assumed \( m = 1/3, M = 1.5, Wb = 0.2, Pr = 7.0, Sc = 0.6, k_1 = 0.5, k_2 = 0.5, g = 0.3, \) and \( K = 5 \) in all discussions.

| \( K \) | Ref. [17] | Ref. [38] | Present study |
|-------|----------|----------|---------------|
| 5     | 0.75763  | 0.74356  | 0.75763       |
| 10    | 0.87349  | 0.86435  | 0.87349       |
| 20    | 0.93561  | 0.92940  | 0.93561       |
| 30    | 0.95686  | 0.95212  | 0.95686       |
| 40    | 0.96759  | 0.96745  | 0.96759       |
| 50    | 0.79405  | 0.97403  | 0.79405       |
| 100   | 0.98704  | 0.98699  | 0.98704       |
| 200   | 0.99356  | 0.99356  | 0.99356       |
| 1000  | 0.99880  | 0.99880  | 0.99880       |

\[ \text{4. Results and Discussion} \]

The physical interpretation of the results obtained in the above section is presented here. The results of different physical parameters are analyzed graphically and presented in the tabular form. The skin friction \( -\sqrt{\text{Re}_f} C_f \) and Nusselt number \( -\theta(0) \) are presented through the table, and the impact of power-law stretching index \( m \), magnetic field parameter \( M \), the radius of curvature \( K \), Williamson fluid parameter \( \lambda \), Schmidt number \( Sc \), homogeneous reaction strength \( k_1 \), heterogeneous reaction strength \( k_2 \), Prandtl number \( Pr \), Biot number \( y \) on velocity \( f'(\eta) \), temperature \( \theta(\eta) \), pressure \( P(\eta) \), and concentration \( \phi(\eta) \) profiles is presented through the graphs.

Table 2 depicts the effect of \( m, M, y, K, Pr, \) and \( \lambda \) on \( -\sqrt{\text{Re}_f} C_f \) and \( -\theta(0) \). As we increase the value of \( m \), there is an increase in \( -\sqrt{\text{Re}_f} C_f \) and \( -\theta(0) \). The graph shows that increment in Williamson fluid parameter \( \lambda \) reduces the value of skin friction coefficient and Nusselt number because the collision of the fluid particles slows down. By the increase in the value of \( K \), \( -\sqrt{\text{Re}_f} C_f \) and \( -\theta(0) \) both decrease. The higher the value of \( M \), the higher the increase in the value of \( -\sqrt{\text{Re}_f} C_f \) and the decrease in the value of \( -\theta(0) \). By raising the value of \( Pr \) and \( y \), \( -\theta(0) \) is rising for both parameters, but \( -C_f \) remains unchanged because the velocity profile is independent of these parameters.

The effect of \( K \) on \( f'(\eta) \) is observed in Figure 2(a). The graph exhibits the obvious results that, by an increase in the value of \( K \), the radius of the surface enhances and hence boosts the velocity of the fluid. Figures 2(b)–2(d) display the changes of \( \theta(\eta), P(\eta), \) and \( \phi(\eta) \) profiles, for increasing radius of curvature \( K \). It is seen that there is a decrease in \( \theta(\eta), P(\eta), \) and \( \phi(\eta) \) for larger values of \( K \). This is because rising curvature makes the curved surface flat. The influence of \( M \) on \( f'(\eta), \theta(\eta), P(\eta), \) and \( \phi(\eta) \) is shown in Figures 3(a)–3(d). It is noted that fluid velocity, pressure, and concentration for higher values of \( M \) reduced. However, the temperature profile increases for greater values of \( M \). Practically, this effect is shown when the magnetic field is applied perpendicular to the flow direction, which creates resistance for the fluid flow. Figures 4(a)–4(d) illustrate the effect of nonlinearity parameter \( m \) on \( f'(\eta), \theta(\eta), P(\eta), \) and \( \phi(\eta) \) profiles. It is easy to discern that the velocity, temperature, and pressure decrease, whereas concentration profile increases by increasing \( m \) which is as expected practically.
Table 2: Variation of $-\sqrt{Re} C_f$ and $-\theta (0)$ for various values of physical parameters.

| $m$ | $M$ | $\lambda$ | $K$ | $Pr$ | $\gamma$ | $k_1$ | $k_2$ | $Sc$ | $-C_f Re^{1/2}$ | $-\theta (0)$ |
|-----|-----|-----------|-----|------|-------|------|------|------|---------------|-------------|
| 1/3 | 0.2 | 0.3       | 5   | 4    | 0.3   | 0.5  | 0.5  | 0.3  | 1.87306       | 0.23740     |
| 0.5 | ... | ...       | ... | ...  | ...   | ...  | ...  | ...  | 1.98728       | 0.24006     |
| 1   | ... | ...       | ... | ...  | ...   | ...  | ...  | ...  | 2.28650       | 0.24620     |
| ... | 0.2 | ...       | ... | ...  | ...   | ...  | ...  | ...  | 1.87306       | 0.23740     |
| ... | 0.3 | ...       | ... | ...  | ...   | ...  | ...  | ...  | 1.98957       | 0.23656     |
| ... | 0.4 | ...       | ... | ...  | ...   | ...  | ...  | ...  | 2.09402       | 0.23576     |
| ... | ... | 0.1       | ... | ...  | ...   | ...  | ...  | ...  | 2.00936       | 0.23786     |
| ... | ... | 0.2       | ... | ...  | ...   | ...  | ...  | ...  | 1.94315       | 0.23765     |
| ... | ... | 0.3       | ... | ...  | ...   | ...  | ...  | ...  | 1.87306       | 0.23740     |
| ... | ... | 1         | ... | ...  | ...   | ...  | ...  | ...  | 3.11378       | 0.23905     |
| ... | ... | 5         | ... | ...  | ...   | ...  | ...  | ...  | 1.87306       | 0.23740     |
| ... | ... | 10        | ... | ...  | ...   | ...  | ...  | ...  | 1.69968       | 0.23682     |
| ... | ... | ...       | 3   | ...  | ...   | ...  | ...  | ...  | 1.87306       | 0.22866     |
| ... | ... | ...       | 4   | ...  | ...   | ...  | ...  | ...  | 1.87306       | 0.23740     |
| ... | ... | ...       | 5   | ...  | ...   | ...  | ...  | ...  | 1.87306       | 0.24349     |
| ... | ... | ...       | 0.2 | ...  | ...   | ...  | ...  | ...  | 1.87306       | 0.17010     |
| ... | ... | ...       | 0.3 | ...  | ...   | ...  | ...  | ...  | 1.87306       | 0.23740     |
| ... | ... | ...       | 0.4 | ...  | ...   | ...  | ...  | ...  | 1.87306       | 0.29595     |

![Figure 2](a) ![Figure 2](b)

Figure 2: Continued.
Figure 2: Impacts of $K$ on (a) $f'(\eta)$, (b) $\theta(\eta)$, (c) $P(\eta)$, and (d) $\phi(\eta)$.

Figure 3: Continued.
Figure 3: Impacts of $M$ on (a) $f'(\eta)$, (b) $\theta(\eta)$, (c) $P(\eta)$, and (d) $\phi(\eta)$.

Figure 4: Continued.
Figure 5 portrays the impact of $\lambda$ on velocity and temperature profiles for different values of $\lambda$. We see that by raising the values of $\lambda$, the fluid velocity declines, as depicted in Figure 5(a). Figure 5(b) presents the effect of $\lambda$ on the temperature profile. It is easy to detect an increase in the temperature of the fluid particles for a better value of $\lambda$. HX_he Williamson fluid parameter $\lambda$ is defined as the proportion of relaxation time to retardation time. HX_he increasing values of Williamson fluid parameter $\lambda$ enhance the relaxation time; as a result of this, the fluid particles need additional time to reinstate their previous path.

Figure 6(a) shows the effect of $c$ on $\theta(\eta)$. For better values of $c$, the increment in the temperature profile is observed. Consequently, Biot number $c$ gives a simple index of the ratio of heat transfer resistance inside and at the surface of geometry. HX_he nature of temperature for different values of Pr is well portrayed in Figure 6(b). As thermal conductivity is inversely proportional to Pr, a decrement is
observed for higher values of Pr. Hence for conducting fluids, Pr is responsible for enhancing the cooling rate.

Figure 7(a) shows the effect of homogeneous parameter $k_1$ on concentration distribution $\phi(\eta)$. From this figure, we observe that the concentration profile decreases as there is a rise in $k_1$. Hence it is concluded that reaction rate dominates diffusion coefficients. Figure 7(b) shows the strength of a heterogeneous reaction $k_2$ on $\phi(\eta)$. The plot shows that the concentration of the fluid and associated boundary layer thickness is reduced for higher values of $k_2$. Figure 8 illustrates the effect of Sc on $\phi(\eta)$. Sc is the ratio of momentum to mass.
diffusivity. An increase in Sc means momentum diffusivity is dominated, resulting in an increment in the concentration profile.

5. Conclusions

In this article, we have modelled the MHD flow of Williamson fluid across a nonlinear stretching curved surface with h-h reactions and convective boundary conditions. The governing partial differential equations are adapted into ordinary differential equations by using suitable similarity transformation. The impact of involving parameters on pressure, velocity, concentration, and temperature profiles are examined. Some examples of this problem in biochemical science and engineering processes are blood flow, plasma flow, lubrication flow, wire drawing, continuous casting, metal extrusion, paper production, glass fibre production, hot rolling, crystal growing, etc. The main consequences are noted below:

(i) Williamson fluid parameter becomes globally similar byfixing \( m = 1/3 \).

(ii) Skin friction coefficient rises for \( m, M \), whereas it decreases for \( \lambda \) and \( K \).

(iii) For better values of \( m, M, \lambda \), \( -\theta(0) \) increases for \( m \) and decreases for \( M, \lambda \), and \( K \).

(iv) As we increase the values of \( K, m, M \), and \( \lambda \), \( f'(\eta) \) decreases for \( m, M \), and \( \lambda \) but increases for \( K \).

(v) As we increase \( m, K, \lambda, y \), and \( Pr \), \( \theta(\eta) \) decreases for \( K, m \), and \( Pr \), whereas it increases for \( M, \lambda \), and \( y \).

(vi) As we increase the values \( m, M, k_1, k_2 \) and \( K \), the concentration profile settles at lower values, whereas it settled as higher values for \( m \) and Sc.

(vii) Pressure decreases for higher values of \( m, M \), and \( K \).

Nomenclature

\[ A, C: \] Chemical species
\[ u, v: \] Velocity component \((\text{ms}^{-1})\)
\[ \tau_w: \] Wall shear stress
\[ \mu: \] Dynamic viscosity \((\text{kgm}^{-1}\text{s}^{-1})\)
\[ \nu: \] Kinematic viscosity \((\text{m}^2\text{s}^{-1})\)
\[ m: \] Power-law stretching index
\[ \lambda: \] Williamson fluid parameter
\[ h_{c}, h_{s}: \] Rate constant
\[ c_{p}: \] Specific heat capacity
\[ u_{w}: \] Velocity at surface
\[ \eta: \] Similarity variable
\[ H: \] Radius
\[ \theta: \] Dimensionless temperature
\[ Nu_{h}: \] Local Nusselt number
\[ K: \] Curvature parameter
\[ Pr: \] Prandtl number
\[ k_{1}: \] Homogeneous reaction parameter
\[ C_{f}: \] Skin friction coefficient
\[ f': \] Dimensionless velocity
\[ s, r: \] Coordinate axes
\[ a, c: \] Concentrations
\[ \rho: \] Fluid density
\[ p: \] Dimensional pressure
\[ q_{w}: \] Heat flux at the wall \((\text{Wm}^{-2})\)
\[ T_{\infty}: \] Ambient fluid temperature
\[ l_{c}, t: \] Convective coefficient
\[ T_{s}: \] Convective surface temperature
\[ b_{1}: \] Positive constant
\[ D_{A}, D_{C}: \] Diffusion coefficient
\[ k_{2}: \] Heterogeneous reaction parameter
\[ \delta: \] The ratio of the diffusion coefficient
\[ Sc: \] Schmidt number
\[ \phi(\eta): \] Dimensionless concentration
\[ \gamma: \] Biot number
\[ Re_{h}: \] Local Reynolds number
\[ h-h: \] Homogeneous and heterogeneous
\[ \eta: \] Thermal conductivity
\[ T: \] Temperature.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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