Polarization control of attosecond pulses using bi-chromatic elliptically polarized laser

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Abstract
We study the high harmonic generation (HHG) using elliptically polarized two-color driving fields. The HHG via bi-chromatic counter-rotating laser fields is a promising source of circularly polarized ultrashort XUV radiation at the attosecond time scale. The ellipticity or the polarization of the attosecond pulses (APs) can be tweaked by modifying the emitted harmonics’ ellipticity, which can be controlled by varying the driver fields. A simple setup is used to control the polarization of the driving fields, which eventually changes the ellipticity of the APs. A well-defined scaling for the ellipticity of the AP as a function of the rotation angle of the quarter-wave plate is also deduced by solving the time-dependent Schrödinger equation in two dimensions. The scaling can further be explored to obtain the APs of the desired degree of polarization, ranging from linear to elliptical to circular polarization.

Keywords: attosecond pulse, polarization, high harmonic generation

(Some figures may appear in colour only in the online journal)

1. Introduction

The high harmonic generation (HHG) is a very promising source of coherent XUV and x-ray radiation beams with pulse duration in the attosecond regime. The celebrated three-step quasi-classical model describes the process of the HHG as the tunneling ionization of atomic electron followed by a free electronic motion under the driving laser field and eventually recombination with the parent ion. Thus, emitting a harmonic photon in the transition back to the ground state [1, 2]. The high coherence of HHG makes it a potential spectroscopic tool for unraveling various fast processes such as delay in photo-emission [3, 4], ultrafast molecular dynamics [5], charge migration in biologically relevant molecules, and many more, with unprecedented resolution [6]. Moreover, circularly polarized HHG (CP-HHG) offers unique opportunities in studying the chiral phenomena in general and has many applications in probing and characterizing the nanostructures and magnetic materials [7]. The CP-HHG is a remarkable probe to study chiral-sensitive light–matter interaction dynamics, such as ultrafast spin dynamics [8–11], x-ray magnetic circular dichroism [7, 11–15] and a few.

The field profile of the driving field plays a crucial role in determining the properties of the HHG. For example, the interaction of linearly polarized drivers with isotropic media generates harmonics with linear polarization [2], the sinc shaped fields are recently used to control the harmonic cutoff of the HHG [16], multicolor driver fields are routinely used to enhance the efficiency and cutoff of the emitted harmonics [17–19]. The ellipticity of the emitted harmonics and hence the associated attosecond pulses (APs) are valuable probes to reveal the dynamical symmetries of atoms and molecules and their evolution in time [20, 21]. However, one can not use the strongly elliptic driver for the generation of the highly elliptically polarized harmonics because the electron return to the parent ion is severely suppressed, thereby quenching the...
harmonic emission. On the contrary, the counter-rotating elliptically polarized driving pulses in the so-called ‘bicircular’ configuration are reported to circumvent the electron return problem and result in the elliptically or circularly polarized harmonics [22–24]. The generated harmonics spectrum consists of pairs of left- and right-rotating harmonics.

The HHG spectrum of circularly polarized harmonics with alternating helicity (the direction of rotation, clockwise or counter-clockwise) could only generate linearly polarized isolated ASPs or pulse train, with each subsequent pulse rotated by 120°. However, the magnitude of ellipticity of ASPs can be increased if the amplitude of particular harmonics say $3q + 1$ order ($q$ is an integer), is higher than the adjacent $3q + 2$ order harmonics across a range of spectral bandwidth. This can be achieved in several ways, such as by changing the intensity ratio between the two circular drivers [25, 26], using a generating medium with a non-zero magnetic quantum number ($m$) of the ground state [27], optimizing the phase-matching conditions [28, 29], choosing different frequency ratio of bicircular fields [30], and using custom fields [31, 32]. Recently the study of the control of high harmonic ellipticity through driver’s ellipticity is also reported [33]. This capability of generating high harmonic radiation and subsequently the APs with controlled polarization ellipticity is significant because it provides an elegant route to study response anisotropy in the matter at natural timescales. The optical response of a chiral molecule or crystal depends on the polarization of the interacting radiation, in this aspect, the tunable ellipticity of the emitted APs can serve as a scanning probe where only the polarization of the generated APs by varying the rotation angle of quarter-waveplate is deduced by solving the time-dependent Schrödinger equation (TDSE) in two-dimensional Cartesian grid. The scaling promises a very robust control over the polarization of the generated APs by varying the rotation angles of the quarter-wave plates.

The rest of the paper is organized as follows. First, in section 2, a brief discussion of the optical setup for pulse synthesis is given, followed by the discussion of theoretical and computational aspects of laser–atom interactions. Next, numerical results of HHG by bi-chromatic counter-rotating driving fields are discussed in section 3 along with the polarization properties of the emitted harmonics and the generated APs. Finally, the concluding remarks are presented in section 4.

Figure 1. A schematic diagram of the optical setup is presented [37]. The BBO crystal is used to generate the second harmonic ($2\omega$) of the fundamental ($\omega$) field. BS is a beam splitter, while DM is a dichroic mirror working as a beam combiner. M1 and M2 are perfectly reflecting mirrors. A zero-order quarter-wave plate ($\lambda/4$) is placed in each arm to control the ellipticity of the fields. When both the plates are set at, $\phi_{\omega} = \phi_{2\omega} = 45^\circ$, the two fields are counter-rotating circularly polarized, and the total electric field has a shape similar to the equilateral triangle because of the intensity ratio of $\omega$ to $2\omega$ field is 4:1.

2. Numerical methods

We begin this section with the description of the optical setup, followed by a brief discussion of the theoretical and computational approach adopted to calculate harmonic generation in an atomic system subject to bicircular electric fields. In order to generate two-color counter-rotating pulses, the original linearly polarized laser beam is incident onto a beam splitter (BS) as shown in figure 1. The BS separates the beam into two pulses with a 4 : 1 intensity ratio. The weak pulse is directed onto a $\beta$-barium borate (BBO) crystal that generates the second harmonic ($2\omega$) pulse, while the pulse with higher intensity remains at the fundamental frequency ($\omega$). An achromatic zero-order quartz wave plate ($\lambda/4$) is placed in each arm to control the ellipticity of the fields. When both the plates are rotated by $45^\circ$, the outgoing pulses are circularly polarized, with the fundamental pulse being right circularly polarized and the second harmonic pulse being left circularly polarized. Finally, the counter-rotating pulses are combined on a dichroic mirror, yielding a rosette-shaped driving electric field. By rotating one of the $\lambda/4$ plates, the polarization of the emitted harmonics and eventually of the APs can be controlled. Hereinafter, the atomic system of units is used, unless stated otherwise, i.e., $|e| = \hbar = m_e = 1$.

We study the interaction of the laser pulse with a He atom by numerically solving the 2D TDSE under single-active-electron approximation. The TDSE in the length gauge is written as:

$$i\frac{\partial}{\partial t} \psi(\mathbf{r}, t) = \left[ -\frac{1}{2}\nabla^2 + V(\mathbf{r}) + \mathbf{r} \cdot \mathbf{E}(t) \right] \psi(\mathbf{r}, t),$$

where $\mathbf{E}(t)$ is the laser field and $\mathbf{r} \equiv (x, y)$ denotes the electron position in the two-dimensional $x$–$y$ plane. The atomic
Coulomb potential is modeled by the soft-core potential [26]:

\[ V(r) = -\frac{1}{\sqrt{r^2 + a_0}} \]  

(2)

where the soft-core parameter \( a_0 \) is dependent on the ionization potential of the atom under study. For He-atom, \( a_0 \approx 0.07 \) is considered such that the ground state energy (ionization potential) of the valence 1s orbital, \( E_{1s} \approx -0.904 \) a.u. (\( \sim 24.6 \) eV) is obtained, which is close to the experimental value of the first ionization potential of helium.

The initial state is obtained by the imaginary-time propagation method [38]. The TDSE is propagated on a 2D Cartesian grid using the time evolution operator \( U(t_0 + \Delta t, t_0) \) on the initial state wavefunction \( \psi_0(r, t_0) \):

\[ \psi(r, t_0 + \Delta t) \approx U(t_0 + \Delta t, t_0)\psi_0(r, t_0). \]  

(3)

The TDSE is solved numerically by adopting the split-operator technique [39]. A mask function,

\[ M_{abs}(r) = \frac{1}{1 + \exp[1.2|r - r_{abs}|]} \]  

(4)

is multiplied to the \( \psi(r, t) \) at each time step to avoid any non-physical reflections at the spatial grid boundaries. The time-dependent dipole acceleration \( a(t) \) is evaluated following the Ehrenfest theorem as [40]:

\[ a(t) = -\frac{\psi(r, t)|\nabla V(r) + E(t)|\psi(r, t)|}{\psi(r, t)|}. \]  

(5)

The harmonic spectra is then obtained by performing the Fourier transform of \( a(t) \), i.e.,

\[ S_n(\omega) = \frac{1}{\sqrt{2\pi}} \int a(t)e^{-i\omega t} dt = |a_n(\omega)|^2, \]  

(6)

where, \( n \) denotes the associated \( x \) or \( y \) components. To describe the polarization properties of the HHG, the intensity of the left- and right-rotating components can be obtained by

\[ D_\pm = |a_\pm(\omega)|^2, \]  

(7)

where \( a_\pm(\omega) = |a_\mp(\omega) \pm ia_\pm(\omega)|/\sqrt{2} \), with \( a_\pm(\omega) \) and \( a_\mp(\omega) \) are complex quantities. The ellipticity of the harmonics is calculated using the relation [41, 42],

\[ \varepsilon = \frac{|a_+(\omega)|^2 - |a_-(\omega)|^2}{|a_+(\omega)|^2 + |a_-(\omega)|^2}. \]  

(8)

The parameter \( \varepsilon \) varies in the interval from \(-1 \) to \(+1 \), and the sign of \( \varepsilon \) defines the helicity of the harmonics. The harmonics rotating in a counter-clockwise direction have positive helicity while those rotating in a clockwise direction have negative helicity. The temporal profile of an ASP is obtained by superposing several harmonics [43]:

\[ I_\omega(t) = |E_\omega(t)|^2 \equiv \sum_q a_{eq} \exp[-i\omega t]|^2, \]  

(9)

where \( q \) is the harmonic order and \( a_{eq} \) represents the inverse Fourier transformation given as:

\[ a_{eq} = \frac{1}{\sqrt{2}} \int a_e(t)\exp[-i\omega t]dt, \]  

(10)

and \( E_e(t) \) represents the electric field component along \( x \) and \( y \) direction. The bircircular field is obtained by combining two counter-rotating elliptically polarized laser fields at \( \lambda_1 = 800 \) nm (\( \omega \)-field) and \( \lambda_2 = 400 \) nm (\( 2\omega \)-field) wavelengths, respectively. The driving laser field in the \( x \)-\( y \) polarized plane is defined as:

\[ E(t) = f(t) \left[ E_1[\cos(\omega t + 2\phi_1)\hat{e}_x + \cos(\omega t)\hat{e}_y] + E_2[\cos(2\omega t)\hat{e}_x + \cos(2\omega t + 2\phi_2)\hat{e}_y] \right], \]  

(11)

with \( 0.5E_1 = E_2 \approx 0.05 \) a.u. (corresponding intensity \( I = 10^{14} \) W cm\(^{-2} \)). The temporal pulse envelope \( f(t) \) has a trapezoidal shape with two cycle raising and falling edges and five cycle plateau (in units of \( \omega \)-field). The angles \( \phi_1 \) and \( \phi_2 \) show the rotation of quarter-waveplates (refer figure 1), corresponding to \( \omega \) and \( 2\omega \) fields, respectively. When both the plates are set at \( \phi_1 = \phi_2 = 45^\circ \), the two fields are circularly polarized and the total electric field has trefoil rosette shape. The fundamental field (\( \omega \)) is rotating counter-clockwise, while the second harmonic field (\( 2\omega \)) is rotating in clockwise direction.

We have considered the spatial simulation domain of \( \pm150 \) a.u. along both \( x \) and \( y \) directions. The value of parameter \( r_{abs} = \pm142 \) a.u. is considered. The spatial step \( \Delta x = \Delta y \approx 0.29 \) a.u. is used and the simulation time step \( \Delta t = 0.005 \) a.u. is considered, which is well within the criteria \( \Delta T < 0.5(\Delta x)^2 \). The convergence is tested with respect to the spatial grid as well as space and time steps. Our simulation utilizes widely used Armadillo library for linear algebra purpose [44].

3. Results and discussions

We start our discussion by describing the bircircular pulse scheme and the HHG by such pulses. The scheme combines two co-planar counter-rotating circularly polarized pulses at fundamental (\( \omega \)) and its second harmonic (\( 2\omega \)) frequencies. The total electric field of the two pulses traces a threefold rosette shape having symmetry with respect to a rotation of 120\(^\circ \). Upon interaction with the target medium, the electric field guides the tunnelled-out electron away from the parent ion and back again in every one third of the fundamental laser cycle (T) [45, 46]. In an isotropic and time-independent medium, this leads to a train of short XUV bursts emission, each with a linear polarization that rotates in space by 120\(^\circ \). In the time domain, the XUV burst is emitted every T/3 duration, totaling three bursts per cycle of the fundamental field. In the frequency domain, circularly polarized harmonics of order \( 3q + 1 \) and \( 3q + 2 \) are emitted and co-rotate with the fundamental and the second harmonic fields, respectively. The emission of harmonics corresponding to \( 3q \) orders are forbidden due to the threefold dynamical symmetry of the system [22, 47–50].
In figure 2(a), the Lissajous curve for the electric field amplitude of the bicircular counter-rotating driving field is shown. The driving field is obtained by setting the rotation angle \( \phi_1, \phi_2 \) of two quarter-waveplates at 45°. For the case of circularly polarized \( \omega - 2\omega \) fields of equal amplitude, the total electric field traces a three-lobed structure having symmetry with respect to a rotation of 120°. In our case, the total electric field has threefold dynamical symmetry. However, due to the amplitude ratio of 2 : 1 between the two fields \( \omega, 2\omega \), the bicircular field resembles equilateral triangles instead of the three-lobed structure. The total field co-rotates with the \( \omega \) field, i.e., counter-clockwise. In figure 2(b), we present the corresponding HHG spectrum consists of \( 3q + 1 \) order (red lines) right-handed and \( 3q + 2 \) order (blue lines) left-handed circularly polarized harmonics (refer equation (7)). Indeed, the \( 3q + 1(3q + 2) \) order harmonics correspond to the absorption of a net amount of \( q + 1 \) photons of \( \omega(2\omega) \) field and a net amount of \( q \) photons of \( 2\omega(\omega) \) field, thus co-rotates with the \( \omega(2\omega) \) field to fulfill the angular momentum conservation. On the other hand, \( 3q \) order harmonics correspond to the absorption of a net amount of \( q \) photons of each \( \omega \) and \( 2\omega \) fields, i.e., total \( 2q \) photons, preserving the parity of initial state and thus recombination by emitting a single photon of frequency \( 3q\omega \) is parity forbidden [46, 51, 52].

It is worth mentioning that the precluded harmonics at frequency \( 3q\omega \) correspond to the harmonic peaks at the integer multiple of summary frequency \( \omega + 2\omega = 3\omega \) and are not related to the third harmonic of the fundamental \( (\omega) \) field [53]. In figure 2(b), the positions of harmonic peak match well with those predicted by the selection rules as mentioned earlier. However, the selection rules consider the driving field components \( (\omega, 2\omega) \) to be perfectly monochromatic. In the actual physical conditions, the driving field has a finite extent in time; hence the harmonic peaks have a finite width, and their polarization will vary throughout the HHG spectrum. The driving field causes the electron ionization, acceleration, and recombination thrice in an optical cycle of \( \omega \) field. This dynamical symmetry of the driving field translates into the HHG spectrum, where the harmonics appear in pairs with opposite helicity and the \( 3q\omega \) harmonics are suppressed. It can be further seen from figure 2(b) that the intensities of the \( (3q + 1)\omega \) harmonics (red) are higher than the adjacent \( (3q + 2)\omega \) harmonics (blue) throughout the spectrum. As stated above, the \( (3q + 1)\omega \) harmonics correspond to the absorption of one extra photon of \( 2\omega \) field, while the \( (3q + 2)\omega \) harmonics corresponds to the absorption of one extra photon of \( 2\omega \) field. By taking the field amplitude ratio \( E_1 : E_2 = 2 : 1 \), the emission of \( (3q + 1)\omega \) harmonics is favored as can be seen in figure 2(b). This facet is earlier discussed and elaborated in references [51, 54–56].

In order to further understand the harmonic specific ellipticity, we have presented in figure 3(a) the ellipticity \( \varepsilon \) of individual harmonics (refer equation (8)) emitted in the HHG spectrum of He atom (refer figure 2(b)). It can be observed that the harmonics of order \( 3q + 1 \) (red square curve) have the same helicity (counter-clockwise) as the \( \omega \) field, and the harmonics of order \( 3q + 2 \) (blue circle curve) have the same helicity (clockwise) as the \( 2\omega \) field. However, the harmonics are not exhibiting perfect circular polarization \((-1 < \varepsilon < +1)\), and the magnitude of the ellipticity for both types of harmonics \( 3q + 1 \) and \( 3q + 2 \) is decreasing along with the increasing harmonic order. This deviation from perfect circular polarization shows that there are some temporal asymmetries present in the system. For example, the temporal asymmetries introduced by the rising and falling edges of driving field [52, 57], fast ionization of generating medium [57], and the excitation of the bound states and subsequent near-resonant emissions [32, 46]. Generally, the perfect circular polarization is expected if the \( x \) and \( y \) components of the harmonic radiation, i.e., \( S_x(\omega) \) and \( S_y(\omega) \) have equal contributions to the total HHG spectrum. In figure 3(b), the \( x \) and \( y \) components of the HHG signal are shown, and it can be seen that both the components are not well overlapped near the peaks of the harmonics, thus causing a deviation from the perfect circular polarization of the generated harmonics. Also, with increasing harmonic order, the contributions of the \( S_x(\omega) \) and \( S_y(\omega) \) components differ more and more near the harmonic peaks. This increasing difference alters the ellipticity and the polarization degree of the harmonics as observed in figure 3(a). One can see that the ellipticity of the 14th harmonic is much smaller than the ellipticity of the neighboring harmonics. It is also seen in figure 3(b)
that the intensity of this harmonic is significantly suppressed in the HHG spectrum. This suggests an onset of some other mechanism that is responsible for the suppression of the 14th harmonic and needs further investigation. We verified this suppression by solving the TDSE in velocity-gauge as well. One possible factor for this suppression could be the applied intensities of the fundamental (ω) and the second harmonic (2ω) driving fields. No such suppression in the intensity of the 14th harmonic is earlier reported in the HHG spectrum of helium [26, 51], though the applied intensities of ω − 2ω fields are different in their cases.

We now discuss the effect of the ellipticity of 2ω driver on the polarization properties of the generated harmonic spectrum. Figure 4 presents the HHG spectra of helium for different ellipticities of the second harmonic (2ω) beam. The fundamental (ω) beam is kept right (counter-clockwise) circularly polarized, that is, the rotation angle of 800 nm quarter-waveplate is fixed at φω = 45°. The rotation angle of 400 nm quarter-waveplate is scanned from 25° to 45° and the harmonic spectrum at φ2ω = 25°, 30°, 35°, and 40° are shown in figures 4(a)–(d), respectively. The harmonic spectrum corresponding to φ2ω = 45° has already presented in figure 2(b).

Besides the rotation angle φ2ω, all the remaining pulse parameters are kept similar to the case shown in figure 2(a). In the HHG spectrum, red lines indicate D+ harmonic components, which co-rotate with the ω field (counter-clockwise), while the blue lines indicate D− harmonic component, which co-rotates with the 2ω field (clockwise). From figure 4, we can see that the forbidden 3qω harmonics surfaced, and their intensity increases with the decrease in the ellipticity of the second-harmonic field. The appearance of 3qω order harmonics is related to breaking the dynamical symmetry in the system. For 3qω harmonics, the blue lines dominate in the below-threshold energy harmonics (for He, threshold energy 16 a.u.) while the red lines dominate in the above-threshold energy harmonics [32, 51, 58]. It should also be noted in figure 4 that the intensity of high harmonics generated for φω = 45°, φ2ω < 45° cases, are comparable to the intensities of harmonics emitted for the case of bicircular field, i.e., φω = φ2ω = 45° (refer figure 2(b)).

So far, we have discussed how the ellipticity of 2ω driver affects the generated harmonic spectrum, specifically, the appearance of 3qω order harmonics and a decrease in the circularity of individual harmonic peaks is observed. These effects translate into the ellipticity of generated APs. In figures 5(a)–(d), we show the Lissajous curve for the total electric field of the attosecond pulse train (APT) computed by taking the inverse Fourier transform of the corresponding harmonic spectra (refer figures 2(b) and 4(b)–(d), and equation (9)). A band of harmonics from 13th to 29th order are filtered out for the construction of the pulse train. There are three XUV bursts per laser cycle (T = 110.32 a.u.) of the fundamental field. Due to the electric field amplitude ratio E1 : E2 = 2 : 1 of the fundamental and the second harmonic beam, the bursts from the APT are highly elliptically polarized and co-rotate with the fundamental driver
The ellipticity of APT is calculated by integrating the two counter-rotating components $E_x(t)$ and $E_y(t)$ of the time dependent APT electric field over a time interval (refer equation (9)). The ellipticity is then defined as \[ \chi = \frac{|E_x|^2 - |E_y|^2}{|E_x|^2 + |E_y|^2}, \]
where parameters $\alpha$, $\beta$, and $\gamma$ are fitting constants. These fitting parameter for H, He and Ne (pseudo) atom are summarized in table 1. Here, by ‘pseudo’ we mean that the soft-core potential parameter is tweaked such that the spherically symmetric ground state energy is the same as ionization potential of the Ne atom ($\sim 0.792$ a.u. ($\sim 21.6$ eV) is considered.

### Table 1. The summary of the scaling parameters is presented.

| Atom    | $\alpha$  | $\beta$  | $\gamma$ | Filtered harmonics |
|---------|-----------|----------|----------|--------------------|
| H       | 0.703     | 0.545    | 0.146    | 13–20              |
| He      | 0.917     | 0.598    | 0.363    | 13–29              |
| Ne (pseudo) | 1.046    | 0.250    | 0.077    | 16–29              |

The calculated APT ellipticity $\chi$ can be given as:

\[ \chi = \alpha/(1 + \beta \exp(-\gamma(\phi_{2\omega} - 35))), \]

where parameters $\alpha$, $\beta$, and $\gamma$ are fitting constants. These fitting parameter for H, He and Ne (pseudo) atom are summarized in table 1. Here, by ‘pseudo’ we mean that the soft-core potential parameter is tweaked such that the spherically symmetric ground state energy is the same as ionization potential of the Ne atom ($\sim 0.792$ a.u. ($\sim 21.6$ eV) is considered.

Furthermore, we would like to stress that the simple scaling provided the opportunity to fine-tune the polarization of generated ASPs and pulse train, all the way from linear through elliptical to circular, by simply changing the rotation angle of the quarter-waveplate.

![Figure 5](image1.png)

**Figure 5.** APTs generated by taking an energy window of 13th–29th harmonics for different rotation angle ($\phi_{2\omega}$) of 400 nm quarter-waveplate (a) $\phi_{2\omega} = 30^\circ$, (b) $\phi_{2\omega} = 35^\circ$, (c) $\phi_{2\omega} = 40^\circ$, and (d) $\phi_{2\omega} = 45^\circ$. The ellipticity ($\chi$) of the generated APTs is also presented. The electric-field strengths $E_x$ and $E_y$ are given in arbitrary units.

![Figure 6](image2.png)

**Figure 6.** The calculated APT ellipticity $\chi$ (purple circle) for different rotation angle ($\phi_{2\omega}$) of the 400 nm quarter-waveplate. The calculated $\chi$ is labeled with the corresponding values. The fitting curve (equation (12)) of the ellipticity scaling with $\phi_{2\omega}$ is depicted as purple solid line. Please refer table 1 for the fitting parameters used for He atom.
is not obligatory. On the contrary, the harmonics emitted during the HHG process carry the signature of the path the electron progressed through during its excursion in the continuum. Thus, the ellipticity of driving fields significantly affects the polarization properties of the emitted harmonics irrespective of the generating medium.

4. Summary and concluding remarks

In summary, we have theoretically investigated the HHG and the generation of APs from helium (and similar atomic species) using the bi-chromatic counter-rotating elliptically polarized laser fields. The dependence of harmonic polarization state on the laser pulse parameters offers an opportunity to shape the polarization properties of the emitted APs. It is a well known fact that in the HHG by bi-chromatic counter-rotating laser pulse with \( \phi_{\omega} = \phi_{2\omega} = 45^\circ \) (refer equation (11)), the 3\( \omega \) harmonics are parity forbidden (refer figure 2). The ellipticity of the individual harmonic in such cases would depend on the intensity ratio of \( \omega \) and 2\( \omega \) fields [60] as can also be seen from figure 3. Furthermore, the ellipticity of the \( \omega \) field is kept constant by fixing the polarization angle \( \phi_{\omega} = 45^\circ \), and the \( \phi_{2\omega} \) is varied. The harmonic spectra for different polarization angle \( \phi_{2\omega} \) is presented in figure 4, and the respective changes in the harmonic ellipticity are discussed. The spectra in figure 4 is filtered from the 13th to 29th harmonic and superposed to construct the respective APs (refer figure 5). The ellipticity of the generated APs can be easily calculated and are presented in figure 6.

From the experimental perspective, the proposed study has the following merits. The polarization or the ellipticity of the APs can be tuned without disturbing the filtering assembly (13th to 29th harmonic components), and without changing the intensity ratio of the \( \omega \) and 2\( \omega \) driver fields. Furthermore, this study is carried out by solving the \textit{two-dimensional} TDSE in an \textit{ab initio} manner, which takes into account the dispersion of the wavefunction, and any, if and mimics a realistic situation. This study helps us to generate APs of varying degrees of polarization, all the way from linear to circular, in a very simple and feasible experimental setup. The fine control over the APT polarization has very profound importance in various applications, such as ultrafast chiral recognition via photoelectron circular dichroism, ultrafast XUV magnetization, and spin dynamics, to name a few. Further analysis on the ellipticity scaling we reserve for the future.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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