Exotic pentaquark states with the $qqQQ\bar{Q}$ configuration

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In the framework of the color-magnetic interaction model, we have systematically calculated the mass splittings for the S-wave triply heavy pentaquark states with the configuration $qqQQ\bar{Q}$ ($Q = c, b, g = u, d, s$). Their masses are estimated and their stabilities are discussed according to possible rearrangement decay patterns. Our results indicate that there may exist several stable or narrow such states. We hope the present study can help experimentalists to search for exotic pentaquarks.

I. INTRODUCTION

The possible existence of mutiquark states beyond the ordinary hadrons were first proposed by M. Gell-Mann and G. Zweig [1, 2]. Nowadays, it is still an important and interesting topic to look for such states [3]. With the experimental progress in recent decades, we are able to find heavy quark multiquark candidates in various processes. In fact, experimentalists announced more exotic states in past years since the Belle Collaboration reported the observation of X(3872) in 2003 [4, 17], which provides good opportunities to study the nonperturbative color interactions. Some of the states have been considered as good tetraquark candidates [15-20].

Recently, the LHCb Collaboration reported the observation of new pentaquark states at the Rencontres de Moriond QCD conference [27, 28]. By analyzing the $J/\psi p$ invariant mass spectrum in the $\Lambda_c^0$ decay with updated data, a new pentaquark $P_c(4312)$ was discovered with 7.3$\sigma$ significance. Meanwhile, the analysis shows two narrow subpeaks with 5.4 significance, $P_c(4440)$ and $P_c(4457)$, for the previously reported $P_c(4450)$ [29, 30]. It is generally recognized that these three new states can be identified clearly as loosely bound $\Sigma_c D$ molecule with $I(J^P) = 1/2(1/2^-)$, $\Sigma_c D^*$ with $I(J^P) = 1/2(1/2^-)$, and $\Sigma_c D^*$ with $I(J^P) = 1/2(3/2^-)$, respectively, within the framework of one-boson-exchange (OBE) model [31-38].

Because of complicated interactions between the internal quarks, generally, it is hard to distinguish whether a hadron is a tightly bound tetraquark (pentaquark) state, a conventional meson (baryon), a molecular state, or a structure in others under various conditions. In understanding the internal structures of $P_c(4380)$ and $P_c(4450)$, many interpretations were proposed, such as the $\Sigma_c D^*$, $\Sigma_c^* D$, and $\Sigma_c^* D^*$ molecules [39-50].

compact pentaquark states [51-55], diquark-diquark-antiquark states, diquark-triquark states [56-64], $D$ solitons, and kinematical effects from the triangle singularity or due to $\chi_{c1p}$ rescattering [65-68].

The studies of more possible pentaquarks were also stimulated by the observation of $P_c$ states [69-73]. After the experimental confirmation of the doubly charmed baryon $\Xi_{c2}^+$ [74, 75], the multiquark states with two or more heavy quarks were studied in many works [69-73, 76]. For example, two possible triple-charm molecular pentaquarks $\Xi_{c2}^+ D_1$ and $\Xi_{c2}^+ D_2^*$ were considered in Ref. [77]. In this paper, we systematically study the mass splittings of compact pentaquark states with the $qqQQ\bar{Q}$ configuration ($q = u, d, s; Q = c, b$). If a heavy quark-antiquark pair forms an unflavored state, such pentaquarks look like excited $qqQ$ baryons. Otherwise, they are explicitly exotic states. At present, it is still not easy to dynamically solve the multi-body problem. Here, we use the color-magnetic interaction (CMI) model to calculate the mass splittings and investigate the mass spectrum of the $qqQQ\bar{Q}$ pentaquark states preliminarily. One may consult relevant studies with other methods in Refs. [78-80].

The Hamiltonian of the quark potential model consists of the one-gluon-exchange interaction part and nonperturbative scalar confining part, which was proposed by de Rujula, Georgi, and Glashow in Ref. [81]. For the ground state hadrons with the same quark content, such as $\Delta$ and $N$, their mass splitting is mainly determined by the color-magnetic interaction [82]. When the spatial contributions are encoded into effective quark masses and coupling parameters, the Hamiltonian can be written as the form containing just the quark mass term and the color-spin interaction term and one gets the CMI model. There are many studies about the mass spectrum for multiquark systems within this model [83-91]. The qualitative properties of the obtained spectra are helpful for us to search for relevant exotic states. In the early stage studies on the pentaquark properties, color-magnetic effects were intensively considered as the primary contribution in an attempt to explain the narrow hadronic resonances, too [102].

This paper is organized as follows. In Sec. II we introduce the CMI model and construct the flavor $\otimes$
color ⊗ spin wave functions for the $qqQ\bar{Q}$ pentaquark states. In Sec. [II], we calculate the relevant Hamiltonian elements and present the corresponding results. In Sec. [IV] we give numerical results for the masses of the pentaquark states, illustrate their possible rearrangement decay channels, and discuss the stability of the states. Finally, we present a summary in Sec. [VII] and an appendix in Sec. [VIII].

II. THE COLOR-MAGNETIC INTERACTION AND THE WAVE FUNCTIONS

The Hamiltonian of the CMI model has a simple form

$$H = \sum_i^5 M_i + H_{\text{CMI}},$$

$$H_{\text{CMI}} = -\sum_{i<j} C_{ij} \hat{x}_i \cdot \hat{x}_j \hat{\sigma}_i \cdot \hat{\sigma}_j.$$ 

Here, $M_i$ represents the effective quark mass for the $i$-th quark or antiquark and it takes account of effects from kinetic energy, color confinement, and other terms in realistic potential models. The effective constant $C_{ij}$ reflects the coupling strength between the $i$-th quark and the $j$-th quark, which depends on the quark masses and the spatial wave functions of the ground states. The Pauli matrix $\sigma_i$ and Gell-Mann matrix $\lambda_i(-\lambda_i^\dagger)$ act on the spin and color wave functions of the $i$-th quark (antiquark), respectively.

To calculate the required matrix elements, we construct the wave functions of the ground $qqQ\bar{Q}$ pentaquark states. They are the direct products of SU(3)$_c$ flavor wave function, SU(3)$_c$ color wave function, and SU(2)$_s$ spin wave function. Here, we treat the heavy quark/antiquark as a flavor singlet state instead of constructing the wave function with flavor SU(4)$_c$ symmetry. It is convenient to adopt the diquark-diquark-antiquark base in organizing the wave functions. The notation “diquark” only means two quarks and the meaning is different from that in the diquark model in Ref. [22], where the diquark is a strongly correlated quark-quark substructure with color=3 and spin=0. The constructed wave functions may also be used to study properties of the $qqQ\bar{Q}$ states in dynamical quark models.

In the SU(3)$_f$ flavor space, the $qqQ\bar{Q}$ states belong to the flavor symmetric 6$_f$ and antisymmetric 3$_f$ representations (Fig. 1), which is similar to the situation for part of the $QQqq$ states [22]. For the $nnQQ\bar{Q}$ ($n = u, d$) case, the isovector states ($I = 1$) and the isoscalar states ($I = 0$) do not mix since we do not consider isospin breaking effects. For the $nsQQ\bar{Q}$ case, the fact $m_n \neq m_s$ leads to SU(3)$_f$ breaking and thus the state mixing between 6$_f$ and 3$_f$. As a result, we need to consider four cases of states: $nnQQ\bar{Q}$ ($I = 1$), $nnQQ\bar{Q}$ ($I = 0$), $nsQQ\bar{Q}$ ($I = 1/2$), and $ssQQ\bar{Q}$ ($I = 0$). Note that the isovector and isoscalar $nnQQ\bar{Q}$ states are not degenerate since the Pauli principle has impacts.

In the color space, the wave functions can be analyzed with the SU(3)$_c$ group theory. The Young diagrams tell us that there are three color-singlet wave functions for the $qqQ\bar{Q}$ states. With the diquark-diquark-antiquark base, they are

$$\phi_1 \equiv \phi^{AA} = [(q_1 q_2)^3\bar{c}(Q_3 Q_4)^3\bar{c}\bar{Q}],$$

$$\phi_2 \equiv \phi^{AS} = [(q_1 q_2)^3\bar{c}(Q_3 Q_4)^6\bar{Q}],$$

$$\phi_3 \equiv \phi^{SA} = [(q_1 q_2)^6\bar{q}(Q_3 Q_4)^3\bar{Q}].$$

In the notation $[(q_1 q_2)^{color_1}(Q_3 Q_4)^{color_2}Q]$, the $color_1$ and $color_2$ stand for the color representations of the light diquark and heavy diquark, respectively. The $S$ ($A$) means “symmetric” (“antisymmetric”) with quark exchanges. The explicit wave functions are the same as those for the $QQqqQ$ states studied in Ref. [22].

One can also use the baryon-meson base $(qqQ-Q\bar{Q})$ or $(QQQ-qq\bar{Q})$ to construct the wave functions. The relevant decomposition is

$$(3c \otimes 3c \otimes 3c) \otimes (3c \otimes 3\bar{c}) = (1c \oplus 8c \oplus 8c \oplus 10c) \otimes (1c \oplus 8c) \rightarrow (1c \otimes 1c) \oplus (8c \otimes 8c) \oplus (8c \otimes 8c). \quad (1)$$

Ref. [22] adopted this base in studying the hidden-charm pentaquark states. Although the final Hamiltonians are different for these two bases, the eigenvalues and mass spectrum would be identical after diagonalization. However, the baryon-meson base is not suitable to the present systems since two pairs of identical quarks may exist in a state like $nncc\bar{Q}$.

In the spin space, the possible wave functions for the considered states in the diquark-diquark-antiquark base are

$$J^P = \frac{5}{2}^+ : \chi_1 = [(q_1 q_2)(Q_3 Q_4)\bar{Q}]_2^\frac{5}{2}.$$
Table I: The possible color-spin wave function bases.

- **$J^P = \frac{3}{2}^-$:**
  - $\phi_1 \chi_1 = [(q_1 q_2)_{(Q_3 Q_4)}_{1/2}^2 Q_{2/3}^2 \delta_{12}]$
  - $\phi_2 \chi_2 = [(q_1 q_2)_{(Q_3 Q_4)}_{1/2}^2 Q_{2/3}^2 \delta_{12} \delta_{34}]$
  - $\phi_3 \chi_3 = [(q_1 q_2)_{(Q_3 Q_4)}_{1/2}^2 Q_{2/3}^2 \delta_{12} \delta_{56}]$

- **$J^P = \frac{3}{2}^+$:**
  - $\phi_1 \chi_4 = [(q_1 q_2)_{(Q_3 Q_4)}_{1/2}^2 Q_{2/3}^2 \delta_{12} \delta_{34}]$
  - $\phi_2 \chi_5 = [(q_1 q_2)_{(Q_3 Q_4)}_{1/2}^2 Q_{2/3}^2 \delta_{12} \delta_{34}]$
  - $\phi_3 \chi_6 = [(q_1 q_2)_{(Q_3 Q_4)}_{1/2}^2 Q_{2/3}^2 \delta_{12} \delta_{34}]$

- **$J^P = \frac{1}{2}^-$:**
  - $\phi_1 \chi_7 = [(q_1 q_2)_{(Q_3 Q_4)}_{1/2}^2 Q_{2/3}^2 \delta_{12} \delta_{34}]$
  - $\phi_2 \chi_8 = [(q_1 q_2)_{(Q_3 Q_4)}_{1/2}^2 Q_{2/3}^2 \delta_{12} \delta_{34}]$
  - $\phi_3 \chi_9 = [(q_1 q_2)_{(Q_3 Q_4)}_{1/2}^2 Q_{2/3}^2 \delta_{12} \delta_{34}]$

- **$J^P = \frac{1}{2}^+$:**
  - $\phi_1 \chi_{10} = [(q_1 q_2)_{(Q_3 Q_4)}_{1/2}^2 Q_{2/3}^2 \delta_{12} \delta_{34}]$
  - $\phi_2 \chi_{11} = [(q_1 q_2)_{(Q_3 Q_4)}_{1/2}^2 Q_{2/3}^2 \delta_{12} \delta_{34}]$
  - $\phi_3 \chi_{12} = [(q_1 q_2)_{(Q_3 Q_4)}_{1/2}^2 Q_{2/3}^2 \delta_{12} \delta_{34}]

In the notation $[(q_1 q_2)_{\text{spin1}}(Q_3 Q_4)_{\text{spin2}} Q_{\text{spin3}}]_{\text{spin4}}$, spin1 and spin2 represent the spins of the light and heavy diquarks, respectively, spin3 represents the total spin of the four quarks, and spin4 represents the total spin of the pentaquark. The diquark is symmetric (antisymmetric) when spin1,2 is 1 (0).

Combining the spin and color wave functions together, we obtain thirty possible bases which are shown in Table I with the notation $[(q_1 q_2)_{\text{color1}}(Q_3 Q_4)_{\text{color2}} Q_{\text{color3}}]_{\text{color4}}$. Not all of them are allowed for a given set of quantum numbers. To reflect the constraint from the Pauli principle, we have inserted three symbols $\delta_{12}^2$, $\delta_{23}^2$, and $\delta_{34}$ in the wave functions. When the light diquark is symmetric (antisymmetric) in flavor space, $\delta_{12}^2 = 0$ (or $\delta_{12}^2 = 0$), otherwise $\delta_{12}^2 = 1$ (or $\delta_{12}^2 = 1$). When the two heavy quarks are the same, $\delta_{34} = 0$, otherwise $\delta_{34} = 1$. Considering all possible configurations, we need to analyze twelve $qqQQQ$ systems. They can be divided into six classes:

1. $nncbQ$ ($I = 1$), $nbb\bar{Q}$ ($I = 1$), $ssccQ$, $ssbb\bar{Q}$
2. $nccQ$ ($I = 0$), $nbb\bar{Q}$ ($I = 0$)
3. $nccQ$, $nbb\bar{Q}$
4. $nccQ$, $nbb\bar{Q}$
5. $nccQ$, $nbb\bar{Q}$
6. $nccQ$, $nbb\bar{Q}$

Each class has similar structures and the same CMI Hamiltonian expressions.

III. THE CMI HAMILTONIAN EXPRESSIONS

With the constructed wave functions, we can calculate CMI Hamiltonian matrix elements. To simplify the expressions, we define the combinations of the effective couplings shown in Table II.

For the pentaquark states without constraints from the Pauli principle, e.g. $nsbc\bar{Q}$, all the color-spin wave function bases in Table I are involved. In the Appendix, we show the obtained CMI matrices for the cases $J^P = 5/2^-$, $3/2^-$, and $1/2^-$ in Tables XII, XIII, and XIV, respectively. For the pentaquark states having constraints from the Pauli Principle, relevant matrices can be extracted from these tables. Here, we take the $nncbQ$ case as an example. When one considers the $I(J^P) = 1(5/2^-)$ state, one has $\delta_{12}^2 = 0$, $\delta_{23}^2 = 1$, and $\delta_{34} = 0$ and only the base $\phi_1 \chi_1$ is allowed. It is easy to read out the CMI
Hamiltonian from Table XIV
\[
\langle H_{\text{CMI}} \rangle = \frac{1}{3} \left[ 8\alpha + 2\zeta + 4(\nu + \lambda) \right].
\]

Similarly, when one considers the \( I(J^P) = 0(5/2^-) \) state, only the wave function base \( \phi_3 Y_1 \) is allowed because \( \delta_{12}^2 = 0, \delta_{13}^2 = 1, \) and \( \delta_{33}^2 = 0. \) The extracted CMI Hamiltonian from Table XIV is
\[
\langle H_{\text{CMI}} \rangle = \frac{1}{3} (4\gamma + 5\zeta - 2\lambda + 10\nu).
\]

IV. THE \( q\bar{q}Q\bar{Q} \) PENTAQUARK MASS SPECTRA

A. The determination of parameters and estimation strategy

Now, we determine the values of the seventeen coupling parameters \( (C_{12n}, C_{12s}, C_{1s}, C_{1n}, C_{2s}, C_{kb}, C_{bc}, C_{cc}, C_{bb}, C_{sc}, C_{sb}, C_{cs}, C_{bc}, C_{cb}, C_{bb}, C_{cb}, C_{bb}, \text{ and } C_{bb}) \) in order to estimate the pentaquark masses. Most of them can be extracted from the measured masses of the conventional hadrons (see Table XV). The related CMI expressions are
\[
H_{\text{CMI}}^{J=1}(q_1\bar{q}_2) = \frac{16}{3} C_{12},
\]
\[
H_{\text{CMI}}^{J=0}(q_1\bar{q}_2) = -16 C_{12},
\]
\[
H_{\text{CMI}}^{J=2}(q_1\bar{q}_2q_3) = \frac{8}{3} (C_{12} + C_{23} + C_{13}),
\]
\[
H_{\text{CMI}}^{J=4}(q_1\bar{q}_2q_3) = \frac{8}{3} \left( \frac{C_{12} - 2[C_{23} + C_{13}] \sqrt{3}[C_{23} - C_{13}]}{\sqrt{3}[C_{23} - C_{13}]} - 3C_{12} \right)
\]

where the two bases for the last matrix corresponds to the case of \( J_{q_1\bar{q}_2} = 1 \) and that of \( J_{q_1\bar{q}_2} = 0. \) We show the determined coupling parameters in Table XVI where \( C_{q\bar{q}} = C_{q\bar{q}} \) is implied. Further, we use the approximation \( C_{Q\bar{Q}} = 2/3C_{Q\bar{Q}} \) \( (C_{bb} = 2/3C_{bb} \approx 1.8 \text{ MeV}, \)
\]
\( C_{cc} = 2/3C_{cc} \approx 3.3 \text{ MeV}, \) and \( C_{bc} = 2/3C_{bc} \approx 2.0 \text{ MeV} \) because only one doubly heavy baryon \( \Xi_{cc} \) is observed in experiments. For the \( B_c^+ \) mass, it has not been observed yet and we take a theoretical result.

TABLE III: Used masses of the conventional hadrons in units of MeV [99]. The adopted masses of the not-yet-observed doubly heavy baryons are taken from Ref. [100]. The values in parentheses are obtained with the parameters in Ref. [96].

| Mesons \((J = 0)\) | Mesons \((J = 1)\) | Baryons \((J = 
\frac{1}{2})\) | Baryons \((J = \frac{3}{2})\) |
|---|---|---|---|
| \( \pi \) | 139.6 | \( \rho \) | 775.3 | \( \nu \) | 1230.2 |
| \( \omega \) | 782.7 | \( \Xi \) | 1314.9 | \( \Xi^* \) | 1531.8 |
| \( \phi \) | 1019.5 | \( \Omega \) | 1672.5 | \( \Delta \) | 2286.5 |
| \( K \) | 493.7 | \( \Xi^* \) | 2577.9 | \( \Xi^* \) | 2578.4 |
| \( D \) | 1869.7 | \( \Xi^* \) | 2695.2 | \( \Omega^* \) | 2765.9 |
| \( D_s \) | 1968.3 | \( \Omega^* \) | 2518.4 | \( \Omega^* \) | 5832.1 |
| \( B \) | 5279.3 | \( \Omega^* \) | 5935.0 | \( \Xi^* \) | 5955.3 |
| \( B_s \) | 5366.9 | \| \| \| \|
| \( \eta_c \) | 2983.9 | \( \Psi \) | 3096.9 | \( \Xi_{cc} \) | 3621.4 |
| \( \epsilon_{cc} \) | 3557.4 | \( \Xi_{cc} \) | 3621.4 | \( \Omega_{cc} \) | 3802.4 |
| \( \eta_{bb} \) | 9399.0 | \( \Upsilon \) | 9460.3 | \( \Xi_{bb} \) | 10113.8 |
| \( \Omega_{bb} \) | 10193.0 | \( \Omega_{bb} \) | 10212.2 | \( \Omega_{bb} \) | 10212.2 |
| \( B_{bc} \) | 6275.1 | \( B_{bc}^* \) | 6331.0 | \( \Xi_{bc} \) | 6820.0 |
| \( \Xi_{bc} \) | 6845.9 | \( \Xi_{bc} \) | 6878.8 | \( \Xi_{bc} \) | 6920.0 |
| \( \Omega_{bc} \) | 6950.9 | \( \Omega_{bc} \) | 6983.4 | \( \Omega_{bc} \) | 6983.4 |

Using the mass formula \( M = \sum_i M_i + \langle H_{\text{CMI}} \rangle \) and the obtained parameters, one sees that the estimated masses of conventional hadrons are in general higher than the measured values, which is illustrated in Table XVII. The reason is that the adopted model and parameters could not account for the necessary attractions for all the hadrons. Overestimated masses with this approach were also obtained in various tetraquark and pentaquark states [88, 90]. To make a more reasonable estimation, we use the improved mass formula by replacing \( \sum_i M_i \) in Eq. (1) with \( M_{\text{ref}} - \langle H_{\text{CMI}} \rangle_{\text{ref}} \) where \( M_{\text{ref}} \) is a reference mass scale and \( \langle H_{\text{CMI}} \rangle \) is the corresponding CMI matrix ele-
obtained from several theoretical calculations. Since the observed doubly heavy baryons in the table, which were discussions, we also show some masses of the not-yet-

III. To understand the decay properties in the following of reference baryons and mesons have been given in Table

Although the pentaquark masses may be changed largely, mention. Then

\[ M = M_{\text{ref}} - \langle H_{\text{CMI}} \rangle_{\text{ref}} + \langle H_{\text{CMI}} \rangle. \] (4)

In the present study for pentaquark states, we choose the baryon-meson thresholds as the mass scales, where the reference baryon-meson system should have the same constituent quarks with a considered system. The attraction not incorporated in the original approach is somehow phenomenologically compensated in this procedure [88].

Before the detailed discussions about the \( qqQQ \) pentaquark states, we emphasize that our results are only rough estimations. They should be updated once a \( qqQQ \) pentaquark state is observed in future experiments and its mass can be chosen as a reference scale. Although the pentaquark masses may be changed largely, the mass splittings should not be affected significantly.

In the following parts, we only present the numerical values obtained with Eq. (4). Here, the involved masses of reference baryons and mesons have been given in Table III. To understand the decay properties in the following discussions, we also show some masses of the not-yet-observed doubly heavy baryons in the table, which were obtained from several theoretical calculations. Since the spin of the \( \Xi_{cc} \) observed by LHCb may be 1/2 or 3/2, we show results in both cases in Table III.

### B. The \( nncc, sscc, nnbb, \) and \( ssbb \) pentaquark states

Substituting the parameters into the CMI matrices and diagonalizing them, the pentaquark masses are obtained. Here, we present the masses with corresponding reference systems for the \( nncc, sscc, nnbb, \) and \( ssbb \) states in Table VI. In these systems, a state is explicitly exotic if the flavor of \( Q \) is different from the heavy quarks.

For the \( nncc \) states, there are two types of reference systems we can adopt, \((c\bar{c})(c\bar{c})\) and \((\bar{c}n)(c\bar{n})\). The mass \( M_{\Xi_{cc}} = 3621.4 \text{ MeV} \) measured by the LHCb Collaboration is used in the latter case. We assume that the spin of \( \Xi_{cc} \) is 1/2 although it has not been determined yet. If the spin is 3/2, the pentaquark masses estimated with the threshold relating to \( M_{\Xi_{cc}} \) would be shifted downward by 64 MeV according to Ref. [89], but the gaps are the same. As for the \( nnbb \), \( sscc \), and \( ssbb \) systems, we can similarly adopt two types of refer-

| Hadron | CMI | Hadron | CMI | Parameter |
|--------|-----|--------|-----|-----------|
| \( N \) | \(-8C_{nn}\) | \( \Delta \) | \( 8C_{nn} \) | \( C_{nn} = 18.4 \) |
| \( \Sigma \) | \( \frac{8}{3}C_{nn} - \frac{32}{3}C_{ns} \) | \( \Sigma \) | \( \frac{8}{3}C_{nn} + \frac{16}{3}C_{ns} \) | \( C_{ns} = 12.4 \) |
| \( \Xi^0 \) | \( \frac{8}{3}(C_{ss} - 4C_{ns}) \) | \( \Xi^0 \) | \( \frac{8}{3}(C_{ss} + C_{ns}) \) |
| \( \Omega \) | \( 8C_{ss} \) | \( C_{ss} = 6.5 \) |
| \( \pi^0 \) | \(-16C_{nn\bar{n}} \) | \( \rho \) | \( \frac{16}{3}C_{nn\bar{n}} \) | \( C_{nn\bar{n}} = 30.0 \) |
| \( K \) | \(-16C_{nn\bar{s}} \) | \( K^* \) | \( \frac{16}{3}C_{nn\bar{s}} \) | \( C_{nn\bar{s}} = 18.7 \) |
| \( D \) | \(-16C_{nn\bar{s}} \) | \( D^* \) | \( \frac{16}{3}C_{nn\bar{s}} \) | \( C_{nn\bar{s}} = 6.7 \) |
| \( D_s \) | \(-16C_{n\bar{s}\bar{s}} \) | \( D_s^* \) | \( \frac{16}{3}C_{n\bar{s}\bar{s}} \) | \( C_{ns\bar{s}} = 6.7 \) |
| \( B \) | \(-16C_{n\bar{n}\bar{n}} \) | \( B^* \) | \( \frac{16}{3}C_{n\bar{n}\bar{n}} \) | \( C_{bn} = 2.1 \) |
| \( B_s \) | \(-16C_{n\bar{n}\bar{s}} \) | \( B_s^* \) | \( \frac{16}{3}C_{n\bar{n}\bar{s}} \) | \( C_{bs} = 2.3 \) |
| \( B_c \) | \(-16C_{n\bar{s}\bar{s}} \) | \( B_c^* \) | \( \frac{16}{3}C_{n\bar{s}\bar{s}} \) | \( C_{bs} = 3.3 \) |
| \( \eta_c \) | \(-16C_{ee} \) | \( J/\psi \) | \( \frac{16}{3}C_{ee} \) | \( C_{ee} = 5.3 \) |
| \( \eta_b \) | \(-16C_{bb} \) | \( \Upsilon \) | \( \frac{16}{3}C_{bb} \) | \( C_{bb} = 2.9 \) |
| \( \Sigma_c \) | \( \frac{8}{3}C_{nn} - \frac{32}{3}C_{cn} \) | \( \Sigma^*_c \) | \( \frac{8}{3}C_{nn} + \frac{16}{3}C_{cn} \) | \( C_{cn} = 4.0 \) |
| \( \Xi'_c \) | \( \frac{8}{3}C_{ns} - \frac{16}{3}C_{cn} - \frac{16}{3}C_{cs} \) | \( \Xi^*_c \) | \( \frac{8}{3}C_{ns} + \frac{16}{3}C_{cn} + \frac{16}{3}C_{cs} \) | \( C_{cs} = 4.8 \) |
| \( \Sigma_b \) | \( \frac{8}{3}C_{nn} - \frac{32}{3}C_{bn} \) | \( \Sigma^*_b \) | \( \frac{8}{3}C_{nn} + \frac{16}{3}C_{bn} \) | \( C_{bn} = 1.3 \) |
| \( \Xi'_b \) | \( \frac{8}{3}C_{ns} - \frac{16}{3}C_{bn} - \frac{16}{3}C_{bs} \) | \( \Xi^*_b \) | \( \frac{8}{3}C_{ns} + \frac{16}{3}C_{bn} + \frac{16}{3}C_{bs} \) | \( C_{bs} = 1.2 \) |
en masse, with Eq. (4) should be further improved. If the adopted masses will change when one adopts different reference scale does not affect the mass splittings. However, the choice of reference scale does not affect the mass splittings. In showing the spectra in the figure form, we use the conventional hadrons in units of MeV. The thresholds for relevant rearrangement decay patterns are also displayed.

For the ncc̅ system, the I = 0 states have generally lower masses than the I = 1 states. The quantum numbers for both the lowest and the highest states are J/ψ = 1/2−. From the diagrams (b), (c), and (d) of Fig. 2 one sees similar features for the nbb̅, nbc̅, and ncc̅ systems.

As for the stability of the pentaquark states, their dominant decay modes should be related with the rearrangement mechanism. Now we move on to such decays. One has to consider the constraints from the angular momentum conservation, isospin conservation, parity conservation, and so on when discussing allowed decay channels. For convenience, we have marked the spin and isospin of the baryon-meson channels in the superscripts and subscripts of their symbols in Fig. 2 respectively. For the ssc̅Q and sbb̅Q states, only one isospin is possible and no label is given explicitly. Of course, whether the decay can happen or not is also kinematically constrained by the pentaquark mass which depends on models. In the following discussions, we assume that the obtained masses shown in the figures are all reasonable.

For the ncc̅ states, they look like excited ncc̅ baryons. Because only orbital or radial excitation energy cannot explain their high masses, the states once observed are good candidates of compact ncc̅ pentaquark states or hadronic molecules. To distinguish these two configurations, decay properties would be helpful. We here just discuss relevant rearrangement decay patterns. In the case of I(J/ψ) = 1(5/2−), the possible S-wave decay channels are Σcc/J/ψ and Ξcc/J/ψ. In the case of I(J/ψ) = 0(5/2−), the possible S-wave decay channel is only Ξcc/J/ψ. The I(J/ψ) = 0(5/2−) isoscalar pentaquark is a candidate of stable state. We mark it in Fig. 2a with a dagger. In the case of the I(J/ψ) = 1(3/2−), the possible S-wave channels are Σcc/J/ψ and Ξcc/J/ψ. In the case of I(J/ψ) = 0(3/2−), the possible S-wave channels are Σcc/J/ψ and Σcc/J/ψ. In the case of I(J/ψ) = 1(1/2−), the possible S-wave channels are Σcc/J/ψ and Ξcc/J/ψ. In the case of I(J/ψ) = 0(1/2−), the possible S-wave channels are Σcc/J/ψ and Ξcc/J/ψ. In the case of I(J/ψ) = 0(1/2−), the possible S-wave channels are Σcc/J/ψ and Ξcc/J/ψ. The observation of any one of the mentioned decay patterns could provide hints for the existence of a ncc̅ pentaquark state. Because the lowest I(J/ψ) = 0(1/2−) state is much lower than the Ξcc/J/ψ threshold, if an observed state in Λccηc (or Λccj/ψ) is around 5.4 GeV, this state would be more likely to be a compact pentaquark than a Ξcc/J/ψ molecule. If the spin of the observed Ξcc/J/ψ by LHCb is 3/2, Ξcc → Ξcc and the estimated pentaquark masses will be reduced by 64 MeV. The stability of the pentaquark states is not affected.
TABLE VI: The estimated masses for the $nnQQ\bar{Q}$ ($I = 1, 0$) and $ssQQ\bar{Q}$ systems in units of MeV. The values in the second column in each case are eigenvalues of the CMI Hamiltonian and those after this column are determined with the relevant reference systems.

| $nncc$ ($I = 1$) | $nncb$ ($I = 1$) | $nnbb$ ($I = 1$) |
|------------------|------------------|------------------|
| $J^P$ Eigenvalue ($\Sigma_1 J/\psi$) ($\Xi_{cc} D$) | $J^P$ Eigenvalue ($\Sigma_1 B^+$) ($\Xi_{cc} B$) | $J^P$ Eigenvalue ($\Sigma_1 B^-$) ($\Xi_{cc} D$) |
| $\frac{1}{2}^-$ | 100.5 | 5616.7 | 5732.7 |
| | 117.3 | 5633.5 | 5744.1 |
| | 81.2 | 5597.4 | 5708.0 |
| | 56.9 | 5573.2 | 5683.8 |
| | $-43.0$ | 5473.2 | 5583.8 |
| | 182.8 | 5699.1 | 5815.0 |
| $\frac{3}{2}^-$ | 104.4 | 5620.6 | 5736.5 |
| | 3.9 | 5520.1 | 5636.0 |
| | $-62.7$ | 5453.5 | 5569.5 |

| $sscc$ ($I = 1$) | $sscb$ |
|------------------|-------|
| $J^P$ Eigenvalue ($\Sigma_1 Y$) ($\Xi_{cc} B$) | $J^P$ Eigenvalue ($\Omega_1 J/\psi$) ($\Xi_{cc} D^+$) | $J^P$ Eigenvalue ($\Omega_1 B^+$) ($\Xi_{cc} Y$) |
| $\frac{1}{2}^-$ | 70.7 | 15291.6 | 15485.6 |
| | 83.9 | 15304.8 | 15498.9 |
| | 68.7 | 15289.7 | 15483.7 |
| | 57.7 | 15278.1 | 15472.2 |
| | 12.6 | 15233.5 | 15427.5 |
| | $107.8$ | 15328.8 | 15522.8 |
| | 79.8 | 15300.8 | 15494.8 |
| | 37.5 | 15258.5 | 15452.5 |
| | 6.8 | 15227.7 | 15421.8 |

| $sbbb$ |
|------------------|
| $J^P$ Eigenvalue ($\Omega_1 Y$) ($\Xi_{bb} D^+$) | $J^P$ Eigenvalue ($\Omega_1 B^+$) ($\Xi_{bb} Y$) |
| $\frac{1}{2}^-$ | 52.0 | 12421.8 | 12329.5 |
| | 73.6 | 12443.4 | 12350.1 |
| | 40.0 | 12409.8 | 12316.5 |
| | 14.9 | 12384.1 | 12291.4 |
| | $-68.9$ | 12300.8 | 12207.6 |
| | 97.6 | 12467.3 | 12374.1 |
| | 63.9 | 12433.7 | 12340.4 |
| | $-3.8$ | 12366.0 | 12272.7 |
| | $-67.7$ | 12302.1 | 12208.8 |

| $nncc$ ($I = 0$) | $nnbb$ ($I = 0$) |
|------------------|------------------|
| $J^P$ Eigenvalue ($\Sigma_1 J/\psi$) ($\Xi_{cc} D$) | $J^P$ Eigenvalue ($\Sigma_1 B^+$) ($\Xi_{cc} B$) |
| $\frac{1}{2}^-$ | 20.5 | 8796.0 | 8988.7 |
| | 11.5 | 8787.0 | 8979.7 |
| | $-45.4$ | 8730.0 | 8922.7 |
| | $-136.7$ | 8633.8 | 8831.5 |
| | $-19.4$ | 8756.1 | 8948.9 |
| | $-71.0$ | 8704.4 | 8897.2 |
| | $-121.9$ | 8653.6 | 8846.3 |

| $nncc$ ($I = 0$) | $nnbb$ ($I = 0$) |
|------------------|------------------|
| $J^P$ Eigenvalue ($\Sigma_1 B^+$) ($\Xi_{cc} D$) | $J^P$ Eigenvalue ($\Sigma_1 B^-$) ($\Xi_{cc} D$) |
| $\frac{1}{2}^-$ | 52.0 | 12421.8 | 12329.5 |
| | 73.6 | 12443.4 | 12350.1 |
| | 40.0 | 12409.8 | 12316.5 |
| | 14.9 | 12384.1 | 12291.4 |
| | $-68.9$ | 12300.8 | 12207.6 |
| | 97.6 | 12467.3 | 12374.1 |
| | 63.9 | 12433.7 | 12340.4 |
| | $-3.8$ | 12366.0 | 12272.7 |
| | $-67.7$ | 12302.1 | 12208.8 |
(a) $I = 1$ (solid) and $(I = 0)$ (dashed) $nncc\bar{c}$ states

(b) $I = 1$ (solid) and $I = 0$ (dashed) $nbb\bar{b}$ states

(c) $(I = 1)$ (solid) and $(I = 0)$ (dashed) $nbb\bar{c}$ states

(d) $I = 1$ (solid) and $I = 0$ (dashed) $nncc\bar{b}$ states

FIG. 2: Relative positions (units: MeV) for the $nncc\bar{c}$, $nbb\bar{b}$, $nbb\bar{c}$ and $nncc\bar{b}$ pentaquark states. The dotted lines indicate various baryon-meson thresholds. When the isospin (spin) of an initial pentaquark state is equal to a number in the subscript (superscript) of a baryon-meson state, its decay into that baryon-meson channel through S-wave is allowed by the isospin (angular momentum) conservation. We have adopted the masses estimated with the reference thresholds of (a) $\Xi_{cc}\bar{D}$, (b) $\Xi_{bb}\bar{B}$, (c) $\Sigma_{bb}\bar{B}_{c}$ and (d) $\Xi_{cc}\bar{B}$. The relatively stable states judged with the observed hadrons have been marked with a dagger.
Fig. 3: Relative positions (units: MeV) for the \( \Xi^* \) and \( \Xi^* \) pentaquark states. The dotted lines indicate various baryon-meson thresholds. When the spin of an initial pentaquark state is equal to a number in the superscript of a baryon-meson state, its decay into that baryon-meson channel through S-wave is allowed by the angular momentum conservation. We have adopted the masses estimated with the reference thresholds of (a) \( \Omega_{\Xi^*} \) and \( \Omega_{\Xi^*} \), (b) \( \Omega_{\Xi^*} \) and \( \Omega_{\Xi^*} \), (c) \( \Omega_{\Xi^*} \) and \( \Omega_{\Xi^*} \), and (d) \( \Omega_{\Xi^*} \) and \( \Omega_{\Xi^*} \).

For the \( \Xi_{bb} \) states shown in Fig. 2(c), they are explicitly exotic. Since the \( \Xi_{bb} \) states and the excited \( B_1^+ \) states have not yet been observed in experiments, we use the theoretical masses of \( B_1^+ \), \( \Xi_{bb} \), and \( \Xi_{bb}^* \) in Table III to check the pentaquark stability. Now, it is easy to see that the lowest-lying states with \( I(J^P) = 0(1/2^-) \) and \( I(J^P) = 0(3/2^-) \) are both stable. The situation for the \( nncb \) \( (\Xi_{bb}) \) states can be analyzed similar to the \( nncc \) \( (\Xi_{bb}) \) case, but now all of them are explicitly (implicitly) exotic.

For the \( sscc \), \( ssbb \), \( ssbc \), and \( ssbc \) states, their properties are similar to those of \( nncbQ \) \( (I = 1) \) and \( nnbcQ \) \( (I = 1) \). Here, we also use the theoretical masses of \( B_1^+ \), \( \Omega_{cc} \), \( \Omega_{bc} \), and \( \Omega_{bb} \) to discuss the possible decay channels. In the \( ssbc \) case shown in Fig. 3(a), any possible pentaquark is above their allowed rearrangement decay channels and thus there is no stable state. One does not find stable states in the \( ssbb \) and \( ssbc \) cases, either. In the \( ssbc \) system, the lowest-lying \( (J^P) = (3/2^-) \) pentaquark is slightly above its decay channel \( \Omega_{bb}D_1^- \). Probably it is not a broad state.

C. The \( nnbcQ \) and \( ssbcQ \) pentaquark states

All these \( nnbcQ \) and \( ssbcQ \) states are explicitly exotic. To estimate their masses, we can use three types of reference systems, \( (qqc)-(bQ) \), \( (qqb)-(cQ) \), and \( (qqb)-(qQ) \). We present the obtained masses in Table X and Table XI for the \( nnbcQ \) and \( ssbcQ \) states, respectively, where the theoretical masses \( M_{\Xi_{cc}^*} = 6820.0 \) MeV and \( M_{\Xi_{bc}^*} = 6920.0 \) MeV given in Table III are adopted. From the tables, the results with these three types of reference systems are slightly different.

In Fig. 4, we plot the relative positions for the \( nnbcQ \) and \( ssbcQ \) pentaquark states. The masses we use are obtained with the reference thresholds of \( \Xi_{bc}J/\psi \), \( \Xi_{bc}B \), \( \Omega_{bc}J/\psi \), and \( \Omega_{bc}B_1^+ \) channels for the \( nnbcQ \), \( nnbcQ \), \( ssbcQ \), and \( ssbcQ \) states, respectively. From the figure, 16 rearrangement decay channels are involved for the \( nnbcQ \) and \( nnbcQ \) states and 12 channels are involved for the \( ssbcQ \) and \( ssbcQ \) states.

We first check possible stable pentaquarks in the \( nnbcQ \) case. The lowest \( J^P = 1/2^- \) and \( J^P = 3/2^- \) states

\begin{align*}
\begin{array}{ccc}
1^- & 3^- & 5^- \\
1/2^- & 3/2^- & 5/2^- \\
\end{array}
\end{align*}

(a) \( sscc \) states

\begin{align*}
\begin{array}{ccc}
1^- & 3^- & 5^- \\
1/2^- & 3/2^- & 5/2^- \\
\end{array}
\end{align*}

(b) \( ssbb \) states

\begin{align*}
\begin{array}{ccc}
1^- & 3^- & 5^- \\
1/2^- & 3/2^- & 5/2^- \\
\end{array}
\end{align*}

(c) \( ssbc \) states

\begin{align*}
\begin{array}{ccc}
1^- & 3^- & 5^- \\
1/2^- & 3/2^- & 5/2^- \\
\end{array}
\end{align*}

(d) \( sscc \) states
TABLE VII: The estimated masses for the \(nnbc\bar{Q}\ (I = 1, 0)\) systems in units of MeV. The values in the second column in each case are eigenvalues of the CMI Hamiltonian and those after this column are determined with the relevant reference systems.

| \(nnbc(I = 1)\) | \(nnbc(I = 0)\) | \(nncc\bar{B}(I = 1)\) |
|------------------|------------------|----------------------|
| \(J^P\) | \(\Sigma_eB^-\) | \(\Sigma_bJ/\psi\) | \(\Xi_{bc}\bar{D}\) | \(\Sigma^e\ U\) | \(\Sigma_bB^+\) | \(\Xi_{bc}B\) | \(\Sigma^e\) | \(\Sigma_bB^+\) | \(\Xi_{bc}B\) |
| \(\frac{3}{2}^-\) | 91.3 | 8866.8 | 8936.1 | 8911.2 | 3\(^-\) | 85.9 | 11978.3 | 12189.9 | 12241.8 |
| 83.3 | 8858.8 | 8928.0 | 8903.1 | 71.3 | 11963.7 | 12175.3 | 12227.2 |
| 118.8 | 8894.3 | 8963.6 | 8938.7 | 94.1 | 11986.5 | 12198.1 | 12249.9 |
| 95.2 | 8870.7 | 8940.0 | 8915.1 | 82.9 | 11975.4 | 12187.0 | 12238.8 |
| 71.2 | 8846.7 | 8915.9 | 8891.0 | 70.4 | 11962.8 | 12174.4 | 12226.3 |
| \(\frac{5}{2}^-\) | 63.2 | 8838.7 | 8907.9 | 8883.0 | 3\(^-\) | 52.2 | 11944.6 | 12156.2 | 12208.0 |
| 33.1 | 8808.6 | 8877.8 | 8852.9 | 26.2 | 11918.6 | 12130.2 | 12182.0 |
| −2.7 | 8772.8 | 8842.0 | 8817.1 | 17.9 | 11910.3 | 12121.9 | 12173.8 |
| −54.1 | 8721.4 | 8790.7 | 8765.8 | −6.2 | 11886.2 | 12097.8 | 12149.6 |
| 160.0 | 8935.5 | 9004.8 | 8979.9 | 125.6 | 12018.0 | 12229.6 | 12281.4 |
| 121.5 | 8897.0 | 8966.3 | 8941.4 | 93.8 | 11986.2 | 12197.8 | 12249.6 |
| 93.7 | 8869.2 | 8938.4 | 8913.5 | 66.0 | 11958.4 | 12170.0 | 12221.8 |
| \(\frac{3}{2}^-\) | 51.9 | 8827.4 | 8896.6 | 8871.7 | 3\(^-\) | 53.4 | 11945.8 | 12157.4 | 12209.2 |
| 20.6 | 8796.1 | 8865.3 | 8840.4 | 34.5 | 11926.9 | 12138.5 | 12190.3 |
| 1.2 | 8776.7 | 8845.9 | 8821.0 | 17.0 | 11909.5 | 12121.0 | 12172.9 |
| −50.0 | 8725.5 | 8794.7 | 8769.8 | −8.3 | 11884.1 | 12095.7 | 12147.5 |
| −100.3 | 8675.2 | 8744.4 | 8719.5 | −57.0 | 11835.5 | 12047.0 | 12098.9 |
| \(nncc\bar{B}(I = 0)\) | \(nncc\bar{B}(I = 0)\) | \(nncc\bar{B}(I = 0)\) |
| \(J^P\) | \(\Sigma_eB^-\) | \(\Sigma_bJ/\psi\) | \(\Xi_{bc}\bar{D}\) | \(\Sigma^e\) | \(\Sigma_bB^+\) | \(\Xi_{bc}B\) | \(\Sigma^e\) | \(\Sigma_bB^+\) | \(\Xi_{bc}B\) |
| \(\frac{3}{2}^-\) | 37.4 | 8812.9 | 8882.1 | 8857.2 | 3\(^-\) | 8.3 | 11900.7 | 12112.3 | 12164.2 |
| 29.5 | 8805.0 | 8874.3 | 8849.4 | 2.6 | 11895.0 | 12106.6 | 12158.4 |
| 6.0 | 8781.5 | 8850.8 | 8825.9 | −21.7 | 11870.7 | 12082.3 | 12134.1 |
| \(\frac{3}{2}^-\) | −65.4 | 8710.1 | 8779.4 | 8754.5 | 3\(^-\) | −49.2 | 11843.2 | 12054.8 | 12106.6 |
| −121.9 | 8653.6 | 8722.9 | 8698.0 | −131.1 | 11761.3 | 11972.9 | 12024.8 |
| −173.1 | 8602.3 | 8671.6 | 8646.7 | −138.1 | 11754.3 | 11965.9 | 12017.7 |
| 14.9 | 8790.4 | 8859.6 | 8834.7 | −16.1 | 11876.3 | 12087.9 | 12139.8 |
| −66.1 | 8709.3 | 8778.6 | 8753.7 | −62.1 | 11830.3 | 12041.9 | 12093.8 |
| −79.2 | 8696.3 | 8765.5 | 8740.6 | −66.8 | 11825.6 | 12037.2 | 12089.0 |
| \(\frac{1}{2}^-\) | −129.0 | 8646.5 | 8715.7 | 8690.8 | 3\(^-\) | −123.4 | 11769.0 | 11980.6 | 12032.5 |
| −174.6 | 8609.0 | 8670.2 | 8645.3 | −146.5 | 11745.9 | 11957.5 | 12009.3 |
| −229.8 | 8545.7 | 8614.9 | 8590.0 | −190.8 | 11701.6 | 11913.2 | 11965.4 |
| −270.8 | 8504.7 | 8574.0 | 8549.1 | −216.1 | 11676.3 | 11887.9 | 11939.7 |
TABLE VIII: The estimated masses for the $ssbc\bar{Q}$ systems in units of MeV. The values in the second column in each case are eigenvalues of the CMI Hamiltonian and those after this column are determined with the relevant reference systems.

| $ssbc\bar{e}$ | $J^P$ | Eigenvalue | $(\Omega_c B_\pi^-)$ | $(\Omega_c J/\psi)$ | $(\Omega_c D)$ | $ssc\bar{b}$ | $J^P$ | Eigenvalue | $(\Omega_c T)$ | $(\Omega_c B_\pi^-)$ | $(\Omega_c B)$ |
|--------------|------|------------|----------------|----------------|----------------|--------------|------|------------|----------------|----------------|------------------|
| $\frac{3}{2}^-$ | 62.5 | 9119.5     | 9173.0         | 9085.2         |                |              | $\frac{3}{2}^-$ | 58.4         | 12232.3       | 12442.2         | 12439.7          |
|              | 51.9 | 9108.9     | 9162.4         | 9074.6         |                |              |              | 38.9         | 12212.8       | 12408.7         | 12420.2          |
| $\frac{3}{2}^-$ | 84.9 | 9141.9     | 9195.4         | 9107.6         |                |              | $\frac{3}{2}^-$ | 63.5         | 12237.4       | 12433.3         | 12444.8          |
|              | 58.4 | 9115.4     | 9168.9         | 9081.1         |                |              |              | 50.3         | 12224.2       | 12420.1         | 12431.6          |
| $\frac{3}{2}^-$ | 38.0 | 9095.0     | 9148.5         | 9060.7         |                |              | $\frac{3}{2}^-$ | 34.2         | 12208.1       | 12404.0         | 12415.5          |
|              | 33.9 | 9090.9     | 9144.4         | 9056.6         |                |              |              | 18.3         | 12192.2       | 12388.1         | 12399.6          |
| $\frac{3}{2}^-$ | −6.0 | 9051.0     | 9104.5         | 9016.7         |                | −7.4         | $\frac{3}{2}^-$ | 12166.5     | 12362.4         | 12373.9          |
|              | −31.3| 9025.7     | 9079.2         | 8991.4         |                | −16.7        |              | 12157.2     | 12353.0         | 12364.5          |
| $\frac{3}{2}^-$ | −91.2| 8965.7     | 9019.3         | 8931.4         | −42.7         |              |              | 12131.2     | 12327.1         | 12338.6          |
|              | 125.0| 9182.0     | 9235.6         | 9147.7         | 92.9          |              |              | 12266.8     | 12462.6         | 12474.1          |
| $\frac{3}{2}^-$ | 94.5 | 9147.4     | 9201.0         | 9113.1         | 63.9          |              |              | 12237.8     | 12433.7         | 12445.2          |
|              | 52.1 | 9109.1     | 9162.6         | 9074.8         | 23.5          |              |              | 12197.4     | 12393.3         | 12404.8          |
| $\frac{3}{2}^-$ | 11.7 | 9068.9     | 9122.2         | 9034.4         | 21.5          |              |              | 12195.4     | 12391.3         | 12402.8          |
|              | −8.9 | 9048.0     | 9101.6         | 9013.7         | −6.6          |              |              | 12167.3     | 12363.1         | 12374.6          |
| $\frac{3}{2}^-$ | −39.2| 9017.8     | 9071.3         | 8983.5         | −24.6         |              |              | 12149.3     | 12345.2         | 12356.7          |
|              | −90.1| 8966.9     | 9020.4         | 8932.6         | −45.5         |              |              | 12128.4     | 12324.3         | 12335.8          |
| $\frac{3}{2}^-$ | −138.0| 8919.0     | 8972.5         | 8884.7         | −96.2         |              |              | 12077.7     | 12273.6         | 12285.1          |
\[ \psi_2 \psi_5 \Xi \text{(angular momentum) conservation. We have adopted the masses estimated with the reference thresholds of (a) } \Sigma^b, \text{ (b) } Z_{bc}^b, \text{ (c) } \Omega\bar{b}, \text{ and (d) } \Omega_b B. \text{ The relatively stable states judged with the observed hadrons have been marked with a dagger.} \]

**FIG. 4:** Relative positions (units: MeV) for the \( nmbc \) and \( nncb \) states.

(a) \( I = 1 \) (solid) and \( I = 0 \) (dashed) \( nmbc \) states

(b) \( I = 1 \) (solid) and \( I = 0 \) (dashed) \( nncb \) states

(c) \( ssbc \) states

(d) \( sscb \) states
both have rearrangement decay channels and should not be very narrow. On the contrary, the lowest \( I(J^P) = 0(5/2^-) \) state is below the possible decay channel \( \Xi_b^* D_s^- \) and it is considered a relative state. Similarly, Fig. \[1\](b) tells us that the only possible stable \( nsbc \) pentaquark has the quantum numbers \( I(J^P) = 0(5/2^-) \) if the mass of \( \Xi_b^* \) is larger than \( 6870 \) MeV. Lastly, it seems that there is no stable \( ssbc \) or \( ssbc \) pentaquark state according to the diagrams (c) and (d) of Fig. \[4\]. Here, the possible stable pentaquarks in Fig. \[4\] have been marked with a dagger.

D. The \( nsccQ \) and \( nssb\bar{Q} \) pentaquark states

For the \( nsccQ \) and \( nssb\bar{Q} \) states, there are also three types of reference systems we can use to estimate the masses, \( (nsQ)-(Q\bar{Q}), (nQ\bar{Q})-(sQ), \) and \( (sQ\bar{Q})-(sQ) \). For example, we estimate the \( nscc \) masses with the thresholds of \( \Xi_s J/\psi, \Xi_s D_s^- \), and \( \Omega_s B \) channels. Similarly, we use the reference systems of \( \Xi_b^* B_s^- \), \( \Xi_b^* D_s^- \), and \( \Omega_b^* D \) to estimate the \( nsbc \) masses. In doing so, we adopt five theoretical masses, \( M_{\Omega_{bc}} = 3730.4 \) MeV, \( M_{\Omega_{db}} = 6920.0 \) MeV, \( M_{\Omega_{bb}} = 10193.0 \) MeV, \( M_{\Xi_{bc}} = 10993.0 \) MeV, and \( M_{\Xi_{bc}} = 6820.0 \) MeV given in Table \[1\]. The numerical results for the \( nsccQ \) and \( nssb\bar{Q} \) systems are presented in Table \[1\]. In the diagrams (a), (b), (c), and (d) of Fig. \[5\] we show the relative positions for the \( nscc, nssb, nsbc \), and \( nscc \) states, respectively. The adopted masses are obtained with the \( \Omega_{cc} \bar{D}, \Omega_{cc} B, \Omega_{bb} \bar{D}, \) and \( \Omega_{bb} B \) thresholds.

Of these states, the \( nscc \) and \( nssb\bar{Q} \) pentaquarks are explicitly exotic. The observation of such a state in future measurements will be an important finding, in particular when the state is narrow. From Table \[1\], the \( nscc \) pentaquark masses estimated with the \( (nsQ)-(Q\bar{Q}) \) type reference systems are lower than those with other type thresholds. Such states are easy to be identified as five-quark states because of their high masses, although they are implicitly exotic. The situation is different from the \( nscsQ \) or \( nsccQ \) states studied in Ref. \[88\]. It is not easy to distinguish such a pentaquark state from a 3q baryon once it is observed.

From Fig. \[5\] and the rough values of the doubly heavy 3q baryons in Table \[1\] it seems that no stable pentaquark states exist in the \( nscc, nssb, \) and \( nscc \) systems. However, according to the diagram (c) of Fig. \[5\] the lowest \( J^P = 1/2^- \), \( J^P = 3/2^- \), and \( J^P = 5/2^- \) states are below any possible rearrangement decay channels and they are possible stable. Of course, if its mass is underestimated, they may also decay into \( \Xi_{bb} D_s^- \) (and probably \( \Omega_{bb} \bar{D}, \Omega_{bb} \bar{D}, \) and \( \Omega_{bb} \bar{D} \)), respectively.

E. The \( nsbc\bar{Q} \) pentaquark states

The \( nsbc\bar{Q} \) states are implicitly exotic. Their wave functions are not constrained from the Pauli principle. The number of wave function bases for a pentaquark with given quantum numbers is bigger than that for other states. After diagonalizing the Hamiltonian, one gets numbers of possible pentaquark states. To estimate the \( nsbc \) (\( nsbc \)) masses, we use four different types of reference systems, \( \Xi_s B_s^- (\Xi_s \Upsilon), \Xi_b^* J/\psi (\Xi_b^* B_s^+), \Xi_{bc} D_s^- (\Xi_{bc} B_s^0), \) and \( \Omega_{bc} \bar{D} (\Omega_{bc} B) \). The numerical results are presented in Table \[X\] where we use two theoretical values of masses, \( M_{\Omega_{bc}} = 6920.0 \) MeV and \( M_{\Xi_{bc}} = 6820.0 \) MeV given in Table \[III\].

The spectra for the \( nsbc\bar{Q} \) pentaquark states with the \( \Xi_b^* J/\psi \) or \( \Omega_{bc} B \) threshold are shown in Fig. \[6\]. From the figure and the masses given in Table \[III\] it is difficult to find stable pentaquarks in these systems. Only the lowest \( nsbc \) pentaquark is slightly above the \( \Xi_s B_s^- \) threshold and is possibly a state without broad width. Of course, the \( nsbc \) states can be searched for in the \( \Xi_s B_s^- \) or \( \Xi_s \Upsilon \) channel in future experiments. If such a state could be observed, its exotic nature can be easily identified, a situation different from the \( nsbc\bar{Q} \) case \[88\].

V. DISCUSSIONS AND SUMMARY

Recently, the observation of the \( P_c (4312), P_c (4440) \) and \( P_c (4457) \) at LHCb \[28\] gave us significant evidence for the existence of pentaquark states, which motivates us to study the ground compact \( qqQQ \) (\( q = u, d, s \) and \( Q = b, c \)) pentaquark states within the CMI model. In the considered pentaquark systems, the \( q\bar{q}bc \), \( q\bar{q}cb \) states are explicitly exotic and are easy to be identified. Other states can also be easily identified as exotic baryons because their large masses could not be understood without an excited \( QQ \) pair.

In this work, we have firstly constructed the flavor-color-spin wave functions for the \( qqQQ \) pentaquark states from the SU(3) and SU(2) symmetries and Pauli Principle. We extract the effective coupling constants from the mass splittings between conventional hadrons. Based on these, we systematically calculate the color-magnetic interaction for these pentaquark states and obtain the corresponding mass gaps. Then, various reference thresholds are used to estimate the masses of these states. Some theoretical results for the masses of the doubly heavy 3q baryons are adopted in our estimation. At last, we analyze the stability and possible rearrangement decay channels of the \( qqQQ \) pentaquark states.

We have shown the mass spectra and rearrangement decay patterns in the figure form. Following Figs. \[2\] \[3\] \[4\] \[5\] and \[6\] we can see ten stable states are possible which are also collected in Table. \[XI\] However, not all of them are really stable states. The reason is that the predicted pentaquarks in the current model may have mass deviations from the case they should be.

As a general feature, the high spin \( J^P = (5/2^-) \) pentaquark states should be usually narrow since they have many \( D \)-wave decay modes but one or two \( S \)-wave decay modes. This feature is similar to the \( QQQq \) and \( QQqq \)
TABLE IX: The estimated masses for the $nsccQ$ and $nsbbQ$ systems in units of MeV. The values in the second column in each case are eigenvalues of the CMII Hamiltonian and those after this column are determined with the relevant reference systems.

| $nscc\bar{c}$ | | | | $nscc\bar{b}$ | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $J^P$ | Eigenvalue | $(\Xi_cJ/\psi)$ | $(\Xi_cD^c_\pi)$ | $(\Omega_{cc}\bar{D})$ | $J^P$ | Eigenvalue | $(\Xi_cB^u_+)$ | $(\Xi_cB^u_0)$ | $(\Omega_{cc}B)$ |
| $\frac{3}{2}^-$ | 85.8 | 5746.2 | 5816.6 | 5835.5 | $\frac{3}{2}^-$ | 68.3 | 8988.0 | 9127.3 | 9154.1 |
| | 59.0 | 5719.4 | 5789.8 | 5808.7 | | 31.8 | 8951.5 | 9090.8 | 9117.5 |
| | 97.1 | 5757.5 | 5827.9 | 5846.8 | | 90.1 | 9009.8 | 9149.1 | 9175.8 |
| | 68.7 | 5729.1 | 5799.4 | 5818.4 | | 66.3 | 8986.0 | 9125.3 | 9152.0 |
| | 49.7 | 5710.1 | 5780.5 | 5799.4 | | 23.7 | 8943.4 | 9082.7 | 9109.4 |
| $\frac{1}{2}^-$ | 35.9 | 5696.3 | 5766.7 | 5785.6 | $\frac{3}{2}^-$ | 14.9 | 8934.6 | 9073.9 | 9100.6 |
| | −52.1 | 5608.3 | 5678.7 | 5697.6 | | −3.9 | 8915.8 | 9055.1 | 9081.8 |
| | −61.1 | 5599.3 | 5669.7 | 5688.6 | | −37.9 | 8881.8 | 9021.1 | 9047.8 |
| | −127.1 | 5533.3 | 5603.7 | 5622.6 | | −94.1 | 8825.5 | 8964.8 | 8991.6 |
| | 168.3 | 5828.7 | 5899.1 | 5918.0 | | 133.0 | 9052.7 | 9192.0 | 9218.8 |
| | 85.0 | 5745.4 | 5815.8 | 5834.7 | | 55.9 | 8975.5 | 9114.8 | 9141.6 |
| | 27.4 | 5687.8 | 5758.2 | 5777.1 | | 7.7 | 8927.4 | 9066.7 | 9093.4 |
| | −14.6 | 5645.8 | 5716.2 | 5735.1 | | −3.8 | 8915.9 | 9055.2 | 9081.9 |
| | −48.2 | 5612.2 | 5682.6 | 5701.5 | | −42.3 | 8877.4 | 9016.7 | 9043.4 |
| | −83.6 | 5576.9 | 5647.2 | 5666.2 | | −57.8 | 8861.9 | 9001.2 | 9027.9 |
| | −86.7 | 5573.7 | 5644.1 | 5663.0 | | −83.9 | 8835.8 | 8975.1 | 9001.8 |
| | −240.3 | 5420.1 | 5490.5 | 5509.4 | | −163.4 | 8756.3 | 8895.6 | 8922.3 |

| $nsbb\bar{c}$ | | | | $nsbb\bar{b}$ | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $J^P$ | Eigenvalue | $(\Xi_bQ)$ | $(\Xi_bD^c_\pi)$ | $(\Omega_{bb}\bar{D})$ | $J^P$ | Eigenvalue | $(\Xi_b\Upsilon)$ | $(\Xi_bB^u_0)$ | $(\Omega_{bb}B)$ |
| $\frac{3}{2}^-$ | 67.9 | 12311.0 | 12245.4 | 12245.8 | $\frac{3}{2}^-$ | 54.8 | 15414.9 | 15560.6 | 15568.7 |
| | 36.9 | 12280.0 | 12214.4 | 12214.8 | | 7.4 | 15367.5 | 15513.2 | 15521.3 |
| | 94.0 | 12337.2 | 12271.6 | 12271.9 | | 63.9 | 15424.0 | 15569.9 | 15577.8 |
| | 55.4 | 12298.6 | 12233.0 | 12233.3 | | 49.5 | 15409.6 | 15555.3 | 15563.4 |
| | 34.2 | 12277.3 | 12211.7 | 12212.1 | | 41.5 | 15401.6 | 15547.3 | 15555.4 |
| $\frac{1}{2}^-$ | 31.9 | 12275.1 | 12209.5 | 12209.8 | $\frac{1}{2}^-$ | 5.9 | 15366.0 | 15511.7 | 15519.8 |
| | −31.9 | 12211.3 | 12145.7 | 12146.0 | | −4.9 | 15355.2 | 15500.9 | 15509.0 |
| | −51.0 | 12192.1 | 12126.5 | 12126.9 | | −34.6 | 15325.5 | 15471.1 | 15479.3 |
| | −151.6 | 12091.6 | 12026.0 | 12026.3 | | −92.3 | 15267.8 | 15413.5 | 15421.6 |
| | 118.3 | 12361.4 | 12295.8 | 12296.2 | | 87.7 | 15447.8 | 15593.4 | 15601.6 |
| | 80.3 | 12323.5 | 12257.9 | 12258.2 | | 62.3 | 15422.4 | 15568.1 | 15576.2 |
| | 28.6 | 12271.8 | 12206.2 | 12206.5 | | 19.8 | 15379.9 | 15525.5 | 15533.7 |
| | 12.0 | 12255.1 | 12183.9 | 12189.9 | | −1.4 | 15358.7 | 15504.4 | 15512.5 |
| | −34.9 | 12208.3 | 12142.7 | 12143.0 | | −9.2 | 15350.9 | 15496.5 | 15504.7 |
| | −50.1 | 12193.0 | 12127.4 | 12127.8 | | −43.4 | 15316.7 | 15462.3 | 15470.5 |
| | −92.2 | 12151.0 | 12085.4 | 12085.7 | | −76.9 | 15283.2 | 15428.9 | 15437.0 |
| | −194.3 | 12048.9 | 11983.3 | 11983.6 | | −139.6 | 15220.5 | 15366.2 | 15374.3 |
We have adopted the masses estimated with the reference thresholds of (a) $\Omega_{cc}$, (b) $\Omega_{bb}$, (c) $\Omega_{cc}$, and (d) $\Omega_{bb}$ pentaquark states. The dotted lines indicate various baryon-meson thresholds. When the spin of an initial pentaquark state is equal to a number in the superscript of a baryon-meson state, its decay into that baryon-meson channel through S-wave is allowed by the angular momentum conservation. We have adopted the masses estimated with the reference thresholds of (a) $\Omega_{cc}$, (b) $\Omega_{bb}$, (c) $\Omega_{cc}$, and (d) $\Omega_{bb}$. The relatively stable states judged with the observed hadrons have been marked with a dagger.
Table X: The estimated masses for the \( nsbcQ \) systems in units of MeV. The values in the second column in each case are eigenvalues of the CMI Hamiltonian and those after this column are determined with the relevant reference systems.

| \( nsbc \) | \( J^P \) | Eigenvalue | \( (\Xi'_c B^-) \) | \( (\Xi'_s J/\psi) \) | \( (\Xi'_b D^-) \) | \( (\Omega'_b D) \) | \( nsbc \) | \( J^P \) | Eigenvalue | \( (\Xi'_c T) \) | \( (\Xi'_b B^+) \) | \( (\Omega'_b B') \) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| \( 5/2^- \) | 101.1 | 9018.6 | 9085.1 | 9045.5 | 9055.6 | 78.1 | 12112.5 | 12321.3 | 12350.6 | 12368.5 |
| \( 7/2^- \) | 16.4 | 8933.9 | 9000.3 | 8960.8 | 8970.9 | 8.9 | 12043.3 | 12252.1 | 12281.4 | 12299.4 |
| \( 7/2^- \) | 142.2 | 9059.7 | 9126.1 | 9086.6 | 9096.7 | 108.9 | 12143.2 | 12352.0 | 12381.4 | 12399.3 |
| \( 5/2^- \) | 105.7 | 9023.1 | 9089.6 | 9050.0 | 9060.1 | 78.6 | 12113.0 | 12321.8 | 12351.1 | 12369.1 |
| \( 7/2^- \) | 73.0 | 8990.5 | 9056.9 | 9017.3 | 9027.5 | 44.2 | 12078.6 | 12287.4 | 12316.7 | 12334.7 |
| \( 7/2^- \) | 31.9 | 8949.4 | 9015.8 | 8976.2 | 8986.4 | 37.7 | 12072.1 | 12280.9 | 12310.3 | 12328.2 |
| \( 5/2^- \) | 26.2 | 8943.6 | 9010.1 | 8970.5 | 8980.6 | 14.0 | 12048.4 | 12257.1 | 12286.5 | 12304.4 |
| \( 5/2^- \) | 5.7 | 8923.2 | 8989.6 | 8950.0 | 8960.2 | -3.4 | 12301.0 | 12239.8 | 12269.1 | 12287.1 |
| \( 5/2^- \) | -18.9 | 8989.6 | 8950.0 | 8925.4 | 8935.6 | -4.2 | 12030.2 | 12238.9 | 12268.3 | 12286.2 |
| \( 1/2^- \) | -36.0 | 8881.5 | 8947.9 | 8908.3 | 8918.5 | -26.6 | 12007.8 | 12216.6 | 12246.0 | 12263.9 |
| \( 3/2^- \) | -54.8 | 8862.7 | 8929.1 | 8889.5 | 8899.7 | -50.7 | 11983.7 | 12192.5 | 12221.9 | 12239.8 |
| \( 1/2^- \) | -70.0 | 8847.5 | 8913.9 | 8874.3 | 8884.5 | -53.9 | 11980.5 | 12189.2 | 12218.6 | 12236.5 |
| \( 3/2^- \) | -88.5 | 8828.9 | 8895.4 | 8855.8 | 8865.9 | -76.4 | 11958.0 | 12166.8 | 12196.1 | 12214.0 |
| \( 1/2^- \) | -119.1 | 8798.4 | 8864.8 | 8825.2 | 8835.4 | -81.8 | 11952.6 | 12161.4 | 12190.7 | 12208.6 |
| \( 3/2^- \) | -138.6 | 8778.8 | 8845.3 | 8805.7 | 8815.8 | -101.1 | 11933.3 | 12142.1 | 12171.4 | 12189.4 |
| \( 1/2^- \) | -192.6 | 8724.9 | 8791.3 | 8751.7 | 8761.9 | -148.0 | 11886.4 | 12095.1 | 12124.5 | 12142.4 |
| \( 3/2^- \) | -238.5 | 8679.0 | 8745.5 | 8705.9 | 8716.0 | -172.7 | 11861.7 | 12070.5 | 12099.8 | 12117.7 |
FIG. 6: Relative positions (units: MeV) for the \( nsc\bar{c}\) and \( ns\bar{c}b \) pentaquark states. The dotted lines indicate various baryon-meson thresholds. When the spin of an initial pentaquark state is equal to a number in the superscript of a baryon-meson state, its decay into that baryon-meson channel through S-wave is allowed by the angular momentum conservation. We have adopted the masses estimated with the reference thresholds of (a) \( \Xi \) and (b) \( \Omega \).

VI. ACKNOWLEDGMENTS

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VII. APPENDIX

In this appendix, we show the CMI Hamiltonian matrix elements with \( J^P = 5/2^- \), \( J^P = 3/2^- \), and \( J^P = 1/2^- \) in Tables XII, XIII and XIV respectively.
TABLE XII: The $H_{\text{CMI}}$ matrix elements for the case $J^P = \frac{1}{2}^\pm$.

| Elements | Bases | $\phi_1 \chi_1$ | $\phi_2 \chi_1$ | $\phi_3 \chi_1$ |
|----------|-------|----------------|----------------|----------------|
| $\phi_1 \chi_1$ | $\frac{1}{2}[\alpha + 2\zeta + 4(\nu + \lambda)]$ | $\sqrt{2}\eta - 2\sqrt{2}\mu$ | $-\sqrt{2}\theta - 2\sqrt{2}\xi$ |
| $\phi_2 \chi_1$ | $\sqrt{2}\eta - 2\sqrt{2}\mu$ | $\frac{1}{2}(-4\beta + 5\zeta + 10\lambda - 2\nu)$ | $-\kappa$ |
| $\phi_3 \chi_1$ | $-\sqrt{2}\theta - 2\sqrt{2}\xi$ | $-\kappa$ | $\frac{1}{4}(4\gamma + 5\zeta - 2\lambda + 10\nu)$ |

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### TABLE XIII: The $H_{CMI}$ matrix elements for the case $J^P = \frac{3}{2}^-$.  

| Elements | Bases | $\phi_1\chi_2$ | $\phi_1\chi_3$ | $\phi_1\chi_4$ | $\phi_1\chi_5$ | $\phi_2\chi_2$ | $\phi_2\chi_3$ | $\phi_2\chi_4$ | $\phi_2\chi_5$ | $\phi_3\chi_2$ | $\phi_3\chi_3$ | $\phi_3\chi_4$ | $\phi_3\chi_5$ |
|----------|-------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\phi_1\chi_2$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $2\phi_1^{4\phi}$ | $\sqrt{3}(\eta + 3\xi)$ | $-\sqrt{3}\mu$ | $\sqrt{3}(\eta + 3\xi)$ | $-\sqrt{3}\mu$ | $2\sqrt{5}\nu$ | $0$ | $\sqrt{2}(\eta + 3\mu)$ | $\sqrt{10}\mu$ | $0$ | $2\sqrt{15}\lambda$ |
| $\phi_1\chi_3$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $2\phi_1^{4\phi}$ | $\sqrt{3}(\eta + 3\xi)$ | $-\sqrt{3}\mu$ | $\sqrt{3}(\eta + 3\xi)$ | $-\sqrt{3}\mu$ | $2\sqrt{5}\nu$ | $0$ | $\sqrt{2}(\eta + 3\mu)$ | $\sqrt{10}\mu$ | $0$ | $-2\lambda$ |
| $\phi_1\chi_4$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $2\phi_1^{4\phi}$ | $\sqrt{3}(\eta + 3\xi)$ | $-\sqrt{3}\mu$ | $\sqrt{3}(\eta + 3\xi)$ | $-\sqrt{3}\mu$ | $2\sqrt{5}\nu$ | $0$ | $\sqrt{2}(\eta + 3\mu)$ | $\sqrt{10}\mu$ | $0$ | $-2\lambda$ |
| $\phi_1\chi_5$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $2\phi_1^{4\phi}$ | $\sqrt{3}(\eta + 3\xi)$ | $-\sqrt{3}\mu$ | $\sqrt{3}(\eta + 3\xi)$ | $-\sqrt{3}\mu$ | $2\sqrt{5}\nu$ | $0$ | $\sqrt{2}(\eta + 3\mu)$ | $\sqrt{10}\mu$ | $0$ | $-2\lambda$ |

### TABLE XIV: The $H_{CMI}$ matrix elements for the case $J^P = \frac{1}{2}^-$.  

| Elements | Bases | $\phi_1\chi_6$ | $\phi_1\chi_7$ | $\phi_1\chi_8$ | $\phi_1\chi_9$ | $\phi_1\chi_{10}$ | $\phi_2\chi_6$ | $\phi_2\chi_7$ | $\phi_2\chi_8$ | $\phi_2\chi_9$ | $\phi_3\chi_6$ | $\phi_3\chi_7$ | $\phi_3\chi_8$ | $\phi_3\chi_{10}$ |
|----------|-------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\phi_1\chi_6$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ |
| $\phi_1\chi_7$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ |
| $\phi_1\chi_8$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ | $\frac{1}{2}(\beta + \gamma\phi)$ | $\frac{1}{2}(\beta - \gamma\phi)$ |

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