Laughlin type wave function
for two-dimensional anyon fields
in a KMS-state *

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Abstract

The correlation functions of two-dimensional anyon fields in a KMS-state are studied. For $T = 0$ the $n$-particle wave functions of noncanonical fermions of level $\alpha$, $\alpha$ odd, are shown to be of Laughlin type of order $\alpha$. For $T > 0$ they are given by a simple finite-temperature generalization of Laughlin’s wave function. This relates the first and second quantized pictures of the fractional quantum Hall effect.

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The ingenious suggestion by Laughlin [1] of a wave function describing the fractional quantum Hall effect [2] is based on an empiric analysis of the approximate effective theory of this phenomenon. It is to be expected that a microscopic derivation of this function should rest upon a (second-quantized) picture of the edge excitations that form the corresponding chiral Luttinger liquid [3] — the quasiparticles and quasiholes characterized by fractional statistics [4] (see also [5] and references therein). The anyon fields constructed in [3,4,8] share most of the features of the above objects. Their exchange relations, the \(\alpha\)-commutators, show, however, a temperature dependence, so that finite-temperature effects in this construction manifest themselves not only globally but also locally. The important point is that their correlation functions do not factorize in the conventional manner. As we shall see, the wave function of an \(n\)-particle \(\alpha\)-anyon state is given by the \(\alpha\)-power of a Slater determinant and for zero temperature is of Laughlin type of order \(\alpha\).

In the discussion of Bose–Fermi duality at finite temperature there appear in a natural way field operators with exotic exchange relations, namely anyons [3,8]

\[
\Psi_\alpha(x) := \lim_{\varepsilon \to 0} n_\alpha(\varepsilon) \exp \left[ i 2\pi \sqrt{\alpha} \int_{-\infty}^{\infty} dy \phi_\varepsilon(x - y) j(y) \right], \quad \alpha \in \mathbb{R}^+, \tag{1}
\]

with \(n_\alpha\) — some renormalization parameter and \(\phi_\varepsilon(x)\) — an approximation to the Heaviside function

\[
\lim_{\varepsilon \to 0} \phi_\varepsilon(x) = \Theta(x), \quad \phi_\varepsilon(x) \in H_1,
\]

where \(H_1\) is the Sobolev space, \(H_1 = \{ f : f, f' \in L^2 \}\). It should be emphasized that these are one-dimensional objects in contrast to the “conventional” anyons, which exist in two spatial dimensions (compare [3]). Their anyonic nature is expressed through the exotic exchange relations they satisfy [4,8], which interpolate between bosons and fermions. An algebraic approach to fractional statistics in one dimension was introduced in [9], based on the analysis of the Heisenberg–Weyl algebra of observables for identical particles.

In our context, the anyon fields (1) appear as follows: If we start with bare fermion fields \(\{ \psi(x), \psi^*(y) \} = \delta(x - y)\), the smeared fields \(\psi_f := \int dx f(x) \psi(x)\) are bounded operators, \(\|\psi_f\|^2 = \int dx |f(x)|^2\), and form the canonical-anticommutation-relations (CAR) algebra \(\mathcal{A}\). The norm of \(\psi_f\) tends to infinity if \(f(x) \to \delta(x - x_0)\) and thus the current \(j(x_0) = \psi^*(x_0) \psi(x_0)\) cannot be reached as a norm limit. However, in a particular representation \(\pi\) the limit \(f(x) \to \delta(x - x_0)\) for the normal-ordered product \(\psi_f^* \psi_f\) can exist as a strong limit. This was shown to happen in [10] in the representation \(\pi_\beta\) given by the equilibrium (KMS) state for a temperature \(T = 1/\beta\). This means that if one enlarges \(\pi_\beta(\mathcal{A})\) by adding all strong limits, the ensuing algebra (denoted by \(\pi_\beta(\mathcal{A})''\)) will already contain the bosonic algebra \(\mathcal{A}_\circ\) spanned by the currents \(j(x)\). Its structure is the same for all non-negative temperatures \(0 \leq T < \infty\) but changes for \(\beta = 0\) or \(\beta < 0\). With these currents one can try to construct anyon fields by (1) but this works only by adding in addition to strong limits some ideal elements to form a bigger algebra \(\tilde{\mathcal{A}}_\circ\). The representation \(\pi_\beta\) extends naturally to it to give \(\tilde{\pi}_\beta(\tilde{\mathcal{A}}_\circ)\) where one might again include strong limits to get \(\tilde{\pi}_\beta(\tilde{\mathcal{A}}_\circ)''\). In this v. Neumann algebra Fermi fields like the ones proposed by Mandelstam [9] can be identified, thus giving rise to another Fermi algebra, CAR
(dressed). Thus the phenomenon of the Bose–Fermi duality can be interpreted as the existence of this particular chain of algebraic inclusions [6]

\[
\text{CAR(bare)} \subset \pi_\beta(\mathcal{A})'' \supset \mathcal{A}_c \subset \bar{\pi}_\beta(\bar{\mathcal{A}})_c'' \supset \text{CAR(dressed)}.
\]

The crucial ingredient needed at the first step is the appropriately chosen state. We choose the KMS-state which is unique for the shift (which for chiral fields is equivalent to the time development) over the CAR algebra. A limiting case would be to choose the Dirac vacuum by filling all negative energy levels in the Dirac sea. This is what has originally been done in the thirties [11, 12], in the attempts for constructing a neutrino theory of light and recovered later by Mattis and Lieb [13] in the context of the Luttinger model.

The essential result is the appearance of an anomalous (Schwinger) term in the quantum current commutator

\[
[j(x), j(x')] = -\frac{i}{2\pi} \delta'(x-x').
\]

For the smeared currents one gets

\[
[j_f, j_g] = \int_{-\infty}^{\infty} \frac{dp}{(2\pi)^2} p \tilde{f}(p) \tilde{g}(-p) = \frac{i}{2\pi} \int_{-\infty}^{\infty} df'(x)g(x) = i\sigma(f, g),
\]

\[\sigma(f, g)\] being the symplectic form on the current algebra \(\mathcal{A}_c\). However, an important detail might be overseen that way: symplectic structure (3) though formally independent on \(\beta\) (see also [14]), for \(\beta < 0\) changes its sign, \(\sigma \rightarrow -\sigma\), and for \(\beta = 0\) (the tracial state) becomes zero. Note that it is the parity \(P\) (which suffers a destruction on the passage from the CAR-algebra to the current algebra [13, 14]) that relates the states corresponding to positive and negative temperatures

\[
\omega_{-\beta} = \omega_\beta \circ P.
\]

However, the dressed fermions present in \(\bar{\pi}_\beta(\bar{\mathcal{A}})_c''\) are only a special type of anyons, defined by a particular value of the statistic parameter \(\alpha\), namely \(\alpha = 1\). In [8] the general anyonic field

\[
\Psi_\alpha(x) \simeq e^{i2\pi\sqrt{\alpha} \int_{-\infty}^{x} j(y) dy},
\]

has been represented as an operator valued distribution in a Hilbert space by exhibiting its \(n\)-point function in a \(\tau\)-KMS state \(\omega\). Thus completed, the rigorous construction of the anyonic fields proposed in [6, 7] allows for a detailed analysis of various properties of these interesting objects, in particular their thermal behaviour and its relevance for the corresponding correlation functions.

The field \(\Psi_\alpha(x)\) (4) is both infrared and ultraviolet singular. The infrared divergence amounts to the fact that admitting the (smeared) step function as a test function, one creates new elements in the field algebra which lead to orthogonal sectors in a larger Hilbert space

\[
\mathcal{H}_\beta = \oplus \mathcal{H}_\beta^n, \quad \mathcal{H}_\beta^n = \mathcal{A}_c \prod_{i=1}^{n} \Psi_\alpha(x_i)\Omega.
\]
The ultraviolet divergence is of another type: it does not lead out of $\pi_\beta$ if we smear $j(y)$ over a region of size $\eta$ to get $\Psi_{\alpha,\eta}$ and consider the renormalized field
\[
\lim_{\eta \to 0^+} c_\alpha(\eta) \int dx f(x) \Psi_{\alpha,\eta}(x) = \Psi_\alpha(f)
\] (6)
with a suitable $c_\alpha(\eta)$. This limit exists in a strong sense and $\Psi(f)$ has finite $n$-point functions.

As already mentioned, for particular values of the statistic parameter $\alpha$ some special families of such renormalized field operators are distinguished: for odd $\alpha$’s we get fermions and for even $\alpha$’s — bosons. However, only the field $\Psi_1$ turns out to be a canonical Fermi field,
\[
[\Psi_1^*(x), \Psi_1(x')]_+ = \delta(x - x')
\]
with an $n$-point function of the familiar determinant form. $\Psi_2$ is a non-canonical Bose field, whose commutator is not a $c$-number
\[
[\Psi_2^*(x), \Psi_2(x')] \simeq \delta'(x - x') + ij(x)\delta(x - x').
\]
Similarly, the operator $\Psi_3$ describes a non-canonical (unbounded) Fermi field. For $\alpha \not\in \mathbb{Z}$ the anyonic commutator vanishes.

Investigation of anyonic field operators of the type (1),(4) represents by far not only an academic interest — such fields might become of importance in solid-state physics, in problems like quantum wires and fractional quantum Hall effect (FQHE). The relation between the objects there involved and the field operators (1) is rather obvious.

Thus, in quantum Hall fluids, the edge-excitation operators are identified with Wen’s fermions and have an exotic statistics depending on the filling fraction $\nu = \alpha^{-1}$. However, in the fermionic case — $\alpha = 2n + 1$, so for Laughlin’s states — one has to distinguish between fermions, corresponding to $n = 0$ and $n \neq 0$. As just mentioned, these fields, though locally anticommuting, are quite different: the former are canonical fields, while the latter are not and this difference shows up also in their thermal properties.

The current algebra $\mathcal{A}_c$ is defined for instance for $j_f$’s with $f \in C_0^\infty$, that is with functions which vanish for $x \to \pm \infty$. The anyons (4) are also Weyl operators but for which the smearing function is $f^\alpha_2(y) = 2\pi \sqrt{\alpha} \Theta(x - y)$. The structure of $\mathcal{A}_c$ is determined by the symplectic form $\sigma(f, g)$ (3) which is actually well defined for the Sobolev space, $\sigma(f, g) \to \sigma(\tilde{f}, \tilde{g})$, $\tilde{f}, \tilde{g} \in H_1$, $H_1 = \{ f : f, f' \in L^2 \}$. The state $\tilde{\omega}_\beta$ can be extended to $H_1$ as well, since $\tilde{\omega}_\beta(e^{ijf}) > 0$ for $f \in H_1$. Thus, the symplectic form (3) may be given a sense for functions that tend to a constant, however they cannot be reached as limits of functions from $C_0^\infty$. Let $\Phi_{x,\delta}$ be such a function, with $\delta$ being the infrared-regularization parameter. The point is then that $\sigma(\Phi_{x,\delta}, \Phi_{x',\delta'})$ depends on the order in which the limits $\delta, \delta' \to \infty$ are taken and only for $\delta = \delta' \to \infty$ we get the desired result $i \text{sgn}(x - x')$. Since this appears in the $c$-number part, in no representation can $j(\Phi_{x,\delta})$ converge strongly. Nevertheless, for functions with the same (nontrivial) asymptotics at, say, $x \to \infty$ and whose difference $\in \hbar$ (see below) one can succeed in getting the expectation values as limits.
Recall that for Weyl operators relation (2) is replaced by the multiplication law
\[
e^{ij(f)} e^{ij(g)} = e^{\frac{i}{\beta} \sigma(g,f)} e^{ij(f+g)}.
\] (7)
The \(\tau\)-KMS states are translation-invariant equilibrium states at an inverse temperature \(\beta\). On \(\mathcal{A}_c\) such a state is given by the two-point function
\[
\omega(j(f)j(g)) = \int dxdy \, K(x - y) f(x)g(y)
\]
with a kernel
\[
K(x - y) = -\lim_{\varepsilon \to 0^+} \frac{1}{(2\pi)^2 \text{sh}^2(x - y - i\varepsilon)}.
\] (8)
The expectation of the Weyl operators is given by
\[
\omega(e^{ij(f)}) = e^{-\frac{1}{2} \langle f|f \rangle},
\] (9)
where the scalar product \(\langle f|g \rangle\) defines the one-particle real Hilbert space \(h\) of the \(f\)'s. For consistency, it has to satisfy
\[
i\sigma(f, g) = (\langle g|f \rangle - \langle f|g \rangle).
\]
Eqs.(7),(9) imply
\[
\omega(e^{ij(f)}e^{ij(g)}) = \exp \left\{ -\frac{1}{2} \left[ \langle f|f \rangle + \langle g|g \rangle + 2 \langle f|g \rangle \right] \right\}.
\]
or generally
\[
\omega(\prod_k e^{ij(f_k)}) = \exp \left\{ -\frac{1}{2} \left[ \sum_k \langle f_k|f_k \rangle + 2 \sum_{k<m} \langle f_k|f_m \rangle \right] \right\}.
\] (10)
A trivial integration then yields for the \(\alpha\) two-point function
\[
\omega(\Psi^*_\alpha(x)\Psi_\alpha(y)) = \omega(e^{-ij(f^\alpha_{x,x})} e^{ij(f^\alpha_{y,y})}) = \left( \frac{i}{2\beta \text{sh} \frac{\pi(x-x'-i\varepsilon)}{\beta}} \right)^\alpha.
\] (11)
For all \(\alpha\)'s the two-point function (for \(x > x'\) and \(\beta = \pi\))
\[
\langle \Psi^*_\alpha(x)\Psi_\alpha(x') \rangle_\beta = \langle \Psi_\alpha(x)\Psi^*_\alpha(x') \rangle_\beta = \left( \frac{i}{2\pi \text{sh}(x - x')} \right)^\alpha =: S_\alpha(x - x')
\] (12)
has the desired properties

(i) **Hermiticity:**

\[
S^*_\alpha(x) = S_\alpha(-x) \iff \langle \Psi^*_\alpha(x)\Psi_\alpha(x') \rangle^*_\beta = \langle \Psi_\alpha(x')\Psi^*_\alpha(x) \rangle_\beta;
\]
(ii) \( \alpha \)-commutativity:

\[
S_{\alpha}(-x) = e^{i\pi\alpha} S_{\alpha}(x) \iff \langle \Psi_{\alpha}(x') \Psi_{\alpha}^*(x) \rangle_{\beta} = e^{i\pi\alpha} \langle \Psi_{\alpha}^*(x) \Psi_{\alpha}(x') \rangle_{\beta};
\]

(iii) KMS-property:

\[
S_{\alpha}(x) = S_{\alpha}(-x + i\pi) \iff \langle \Psi_{\alpha}^*(x) \Psi_{\alpha}(x') \rangle_{\beta} = \langle \Psi_{\alpha}(x') \Psi_{\alpha}^*(x + i\pi) \rangle_{\beta}.
\]

For \( \alpha = 2 \) and an arbitrary temperature \( \beta^{-1} \) we get like for the \( j \)'s

\[
\langle \Psi_{2}^*(x) \Psi_{2}(x') \rangle_{\beta} = -\frac{1}{\left(\frac{2}{\beta} \text{sh} \frac{\pi(x-x'+i\varepsilon)}{\beta}\right)^2}, \tag{13}
\]

similarly, for \( \alpha = 3 \) we get a different kind of fermions

\[
\langle \Psi_{3}^*(x) \Psi_{3}(x') \rangle_{\beta} = -\frac{i}{\left(\frac{2}{\beta} \text{sh} \frac{\pi(x-x'+i\varepsilon)}{\beta}\right)^3}. \tag{14}
\]

These fields, though locally (anti)commuting, are not canonical and this becomes transparent by analysing temperature dependence and operator structure of their exchange relations. However, the Fermi fields \( \Psi_{2n+1} \) are similar to Wen’s fermions

\[
\langle \psi(z) \psi^\dagger(w) \rangle \sim \frac{1}{(z-w)^{2n+1}}
\]

that correspond to Laughlin’s plateaux in the theory of the FQHE (considered at a finite temperature), in which case these construction would provide a second-quantization picture of this phenomenon. For a detailed analysis of this relation we refer to [18].

For the \( n \)-point function to get something finite for \( \delta \to \infty \) we have to take operators of the form

\[
\prod e^{\pm ij(f_{2k}^\alpha)} := \prod e^{ij(f_{2k}^\alpha)};
\]

where \( \sum_k s_k \partial_j f_{2k} = s_k \sum_k f_{2k}, s_k^\alpha = \pm \sqrt{\alpha} \). Since the individual expressions diverge with \( \delta \to \infty \), if the anyon contributions do not neutralize, i.e. if \( \sum_k s_k^\alpha \neq 0 \), the exponent as a whole diverges, so the expectation value (10) vanishes. If, on the other hand, \( \sum_k s_k^\alpha = 0 \), those terms that contain \( \delta \) can be combined in pairs to cancel and one thus remains in the limit \( \delta \to \infty \) (with the normalization factors \( c_\alpha \) (6) taken into account) with

\[
\omega(\Psi_{\alpha}(x_1) \ldots \Psi_{\alpha}(x_n) \Psi_{\alpha}(y_n) \ldots \Psi_{\alpha}(y_1)) = \prod_{k > l} (\text{sh}(x_k - x_l - i\varepsilon))^{\alpha} \prod_{k > l} (\text{sh}(y_k - y_l - i\varepsilon))^{\alpha} \prod_{k,l} (-2\pi i \text{sh}(x_k - y_l - i\varepsilon))^{\alpha}. \tag{15}
\]
For the case \( \alpha = 1 \) with the help of Cauchy’s determinant formula the \( n \)-point function (15) can be rewritten as
\[
\prod_{i > k} \text{sh}(x_i - x_k - i\varepsilon) \prod_{i > k} \text{sh}(y_i - y_k - i\varepsilon) \prod_{i,k} \text{sh}(x_i - y_k - i\varepsilon) = \text{Det} \frac{1}{\text{sh}(x_i - y_k - i\varepsilon)}.
\] (16)

We do not prove (16) but just remark that the two expressions have the same pole structure, homogeneity degree and symmetries. The state over the field algebra \( A_1 \) is quasifree, the fermion two-point function being given by
\[
\omega(\Psi_1^*(x)\Psi_1(y)) = \frac{i}{2\pi \text{sh}(x - y - i\varepsilon)}.
\]
It satisfies the KMS condition with respect to the shift for temperature \( \beta = \pi \). For arbitrary temperature by scaling arguments it follows
\[
\omega_\beta(\Psi_1^*(x)\Psi_1(y)) = \frac{i}{2\beta \text{sh}^{\frac{x - x' - i\varepsilon}{\beta}}}.
\] (17)

Since the \( \tau \)-KMS state over the CAR-algebra is unique, we thus recover the original free fermions.

Evidently, for \( \alpha \neq 1 \) the state is determined again by the two-point function but not in a way that corresponds to a truncation. Recall that an \( n \)-particle state is given by
\[
|n\rangle = \int \Psi^*(x_1) \ldots \Psi^*(x_n) |\Omega\rangle F(x_1, \ldots, x_n) dx_1 \ldots dx_n,
\]
its wave function being
\[
\phi(x_1, \ldots, x_n) := \langle \Omega | \Psi(x_1) \ldots \Psi(x_n) | n \rangle.
\]
\( |n\rangle \) is a Slater state if \( F(x_1, \ldots, x_n) = \prod_i f_i(x_i) \), for which \( \phi \) is a determinant if \( |\Omega\rangle \) is the Fock vacuum. We shall call a wave function \( \phi \) being of \textit{Laughlin type of order} \( \alpha \), if \( \phi \) is of the form \( \prod_{i > k} (x_i - x_k)^\alpha \prod_m \Phi(x_m) \), for \( 0 < |\Phi| < \infty \) and \( \alpha \) odd.

Note that because of the anti-commutativity of the \( \Psi \)'s, the Slater determinant \( F = \text{Det} f_i(x_j) \) gives for fermions the same state as \( \prod_i f_i(x_i) \). If \( |\Omega\rangle \) is the vacuum then \( |n\rangle = 0 \) if for some \( f_k \), supp \( \tilde{f}_k \subset (0, -\infty) \). Furthermore, for functions \( f \) with supp \( \tilde{f}_k \subset (0, \infty) \) such that \( f(x) \) is analytic in the upper half-plane, the set \( \{ f(z) = (x - z)^{-1}, \text{Im } z < 0 \} \) is total, i.e. their linear combinations are dense. Therefore we get for \( \beta \to \infty \) up to a normalization factor
\[
\phi(x_1, \ldots, x_n) = \prod_{i > j} (x_i - x_j)^\alpha \int \frac{dy_1}{(y_1 - z_1)} \ldots \frac{dy_n}{(y_n - z_n)} \prod_{k,l} (y_k - y_l)^\alpha \prod_{k,l} (x_k - y_l + i\varepsilon)^\alpha = \prod_{l > j} (x_l - x_j)^\alpha \prod_{k,l} (z_k - z_l)^\alpha \prod_{k,l} (x_k - z_l + i\varepsilon)^\alpha,
\]
which is exactly the desired Laughlin-type wave function with

$$\Phi(x) = \prod_{l}(x - z_{l} + i\varepsilon)^{-\alpha}$$

with all required properties. Thus, for fermions of order $\alpha$ a Slater state in vacuum has Laughlin-type wave function of order $\alpha$ for a total set of $f$'s.

Obviously, for $\alpha = 1 \phi$ is (up to a constant factor) the Slater determinant $\text{Det} (x_{k}^{i}\Phi(x_{k}))$, for other $\alpha$'s it is the $\alpha$-power of such a determinant.

For finite temperature $T = \beta^{-1}$, $\pi(x_{l} - x_{k})$ is replaced by $\beta \text{sh}[\pi(x_{l} - x_{k})/\beta]$ and $\Phi(x)$ — by $\beta^{\alpha} \prod_{l=1}^{n} \text{sh}^{-\alpha}[\pi(x - z_{l} + i\varepsilon)/\beta]$. By pulling out $\prod_{l>k}(x_{l} - x_{k})^{\alpha}$ the rest gets a factor $\prod_{l>k} \text{sh}^{\alpha}[(x_{l} - x_{k})/\beta]/(x_{l} - x_{k})$ which is finite and symmetric but no longer a pointwise product. Thus the wave function is not exactly of the Laughlin type.

To summarize, the operators $\Psi_{\alpha}(x), \alpha \in \mathbb{R}^{+}$, constructed in [6, 7, 8] describe, depending on the value of the statistic parameter $\alpha$, a variety of fields — generally anyons but also (the integer classes) bosons and fermions. However, only the first-level fermions are canonical fields. This shows that local anticommutativity alone does not guarantee the uniqueness of the KMS-state, one needs in addition the CAR-relations. Thus $\Psi_{\alpha}$'s, $\alpha \in 2\mathbb{N} + 1$, describe an infinity of inequivalent fermions, characterized by temperature-dependent correlation functions and exchange relations. This dependence means a loss of the local normality of the representations corresponding to different temperatures, hence — (already local) observability of the temperature effects.

The wave functions for $T = 0$ of an $n$-particle state of noncanonical fermions of level $\alpha$ (so, for $\alpha$ odd) are of Laughlin type of order $\alpha$. For the first “extended” Fermi-class — $\Psi_{3}$, the original Laughlin’s form is obtained. For $T > 0$ the wave functions are of a similar form, which is a simple finite-temperature generalization of the zero-temperature case. Such a relation between our noncanonical Fermi fields and Wen’s fermions is another argument for their detailed analysis because of the possibility for a second-quantization picture of the fractional quantum Hall effect they provide. In particular, such anyon fields naturally appear in one of the two (1+1)-dimensional chiral theories whose tensor product gives the second quantization of the Hall Hamiltonian. They describe the macroscopic Hall current and the microscopic Larmor precession respectively [15].

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