A compound control strategy for LCL-based three phase four leg shunt active power filter

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Abstract. Conventional single inductor filter which is applied to three-phase four-leg shunt active power filter hardly meet the requirements for high frequency harmonic suppression. LCL filter has advantage both in high dynamic response and high frequency harmonic suppression. Repetitive control can realize accurate tracking of multi-harmonics signal. However, the design of corrector for repetitive control is mostly complicated. This paper proposes a topology of three-phase four-leg shunt active power filter based on LCL filter and improve its stability through capacitive current feedback. Compound control strategy which includes PI control and repetitive control is adopted to improve the compensation effect of harmonic current. Corrector for repetitive control has simple structure and is easy to design. The conventional PI control and compound control are compared and analyzed in detail. Moreover, the difference of property between LCL filter and single inductor filter is compared and analyzed. Simulation results show the correctness and effectiveness of the designed LCL filter and the proposed compound control strategy. The proposed method provides a reference for making three-phase four-leg shunt active power filter more practical.

Keywords: LCL filter; capacitive current feedback; compound control strategy; four leg shunt active power filter.

1. Introduction
The three-phase four leg active power filter (APF) has the ability of harmonic and neutral current compensation, which is one of the important technical means to solve the harmonic problem of three-phase four wire system [1-2].

At present, single inductor filter is widely used in four leg APF, which requires high switching frequency and large inductance value of inverter to meet the requirements of filtering system harmonic and restraining inverter switching harmonic, and results in the increase of APF cost and the deterioration of dynamic performance of the system [3]. LCL filter has the characteristics of third-order low-pass filter, which has stronger attenuation effect on high-frequency harmonic current than L filter. There are many researches on LCL filter for three-phase three wire APF., A design method of LCL filter for three-phase three wire APF is proposed in [4-5], but the resistance used to suppress the resonance of LCL filter with capacitor branches in series will increase the power consumption of the system.
The notch filter can suppress the resonance [6], but it is difficult to meet the robustness requirements when the system impedance changes. If the capacitor current feedback technology is used to suppress resonance, the control is simple and less affected by the change of grid impedance [7]. However, there is little research on LCL filter applied to three-phase four leg APF.

In order to solve the problem of limited current bandwidth in the conventional compound current control algorithm, an improved compound current control algorithm is proposed. A state feedback pole assignment method is proposed in [8-9], but more feedback will increase the cost. The leading link is used to compensate the phase and the second-order filter is used to provide high-frequency attenuation in [10-11], which makes the phase frequency characteristics of the system more complex. This paper presents a topology of three-phase four leg active power filter using LCL filter to solve the problem of poor suppression ability of high frequency harmonic current of single inductor filter, and gives the calculation method of the filter element parameters.

The capacitor current feedback control is used to suppress the resonance of the filter to meet the robustness requirement of APF to the change of grid impedance. The composite control algorithm of PI + repetitive control is used to realize the steady-state compensation performance of APF system for harmonic current. The design of repetitive control corrector is simple and has good harmonic tracking compensation effect. The simulation results verify the correctness and effectiveness of the proposed filter design method and control strategy.

2. Three phase four leg APF topology with LCL filter

The circuit topology of three-phase four leg APF with LCL filter is shown in Figure 1. The capacitor can bypass the high frequency harmonic current, and the neutral line does not need filter inductor.

In the figure, \(e_a, e_b\) and \(e_c\) are the three-phase voltage of the system, \(L_s\) is the equivalent inductance of the system, \(L_1\) is the inverter side filter inductance of LCL filter, \(L_2\) is the grid side filter inductance, \(C\) is the filter capacitor; \(C_{dc}\) is the inverter DC bus capacitor, \(u_{dc}\) is the DC bus voltage; \(u_{an}, u_{bn}\) and \(u_{cn}\) are the inverter a, B and C point to N point voltage respectively.

![Fig.1 Topology of three phase four leg APF with LCL filter](image)

Figure 2 shows the single-phase equivalent circuit of three-phase four leg APF. Where \(e_s\) is the system phase voltage, \(u_i\) is the inverter output phase voltage, \(i_1\) is the inverter side output compensation current, \(i_2\) is the grid side output compensation current, \(i_c\) is the bypass capacitor current, \(u_c\) is the bypass capacitor branch voltage.

![Fig.2 Single phase equivalent circuit of three phase four leg APF](image)
3. Parameter design of LCL filter

The capacitive reactance value of capacitor C in LCL filter must be large enough, otherwise excessive reactive power will be generated. Generally, the reactive power is limited to no more than 5% of the system rated power [12]. Considering the voltage level of APF access system, the capacitance C is selected as 3 μF. The LCL filter will produce a resonant peak, the harmonic of this frequency will be amplified, and the resonant angular frequency can be given by equation (1):

$$\omega_{\text{res}} = \frac{L_1 + L_2 + L_S}{\sqrt{L_1(L_2 + L_s)}} C$$

(1)

Generally, the resonant frequency of LCL filter of grid connected inverter is between 10 times of power frequency and half of PWM switching frequency. Since the output of APF is harmonic current, it needs a large intermediate frequency bandwidth, and the resonant frequency should be close to half of PWM switching frequency. When the switching frequency $f_S = 10kHz$, the resonant frequency can be close to 5KHz. Since most of the high frequency current is bypassed by capacitor C, the harmonic of the inverter side current is mainly determined by $L_1$. Considering the influence of $L_1$ on the harmonic content and resonant frequency of the inverter side current, $L_1 = 1mH$ is selected in this paper. When the system equivalent reactance $L_S = 0.1mh$, the network side inductance $L_2 = 0.5mh$ can be calculated when the resonance frequency $f_{\text{res}} = 47Hz$.

4. Research on compound control strategy

According to the single-phase equivalent circuit of LCL filter, the following equation can be obtained:

$$u_i - u_c = L_1 \frac{di}{dt}$$

(2)

$$i_c = C \frac{du_c}{dt}$$

(3)

$$u_c - e_i = (L_2 + L_s) \frac{di_s}{dt}$$

(4)

The transfer function $G(s)$ from inverter output voltage $U_1$ to grid side output current $I_2$ is as follows:

$$G(s) = \frac{i_2}{u_1} = \frac{1}{(L_2 + L_s)C + (L_2 + L_s + L_1)s}$$

(5)

When $L_1 = 1mH$, $L_2 = 0.5mh$, $L_S = 0.1mh$, $C = 3\mu F$, the Bode diagram of $G(s)$ is as follows:
It can be seen from Fig. 3 that when there is no additional damping, there is a large resonant peak near 4800 Hz, there are three 0dB crossing points in $G(s)$, and the phase of the third crossing point is -270 degrees, so the LCL filter without additional damping is unstable.

The series resistance in the capacitor branch can suppress the resonance peak, but it will lead to additional loss and increase the impedance of the capacitor branch, thus the filtering effect becomes worse.

The capacitive current feedback inner loop can be used to increase the damping of the system, so as to suppress the resonance peak effectively. The outer loop of grid side current is corrected by PI controller to realize the direct control of grid connected current. At this time, the single-phase control block diagram of APF system is as follows:

$$G_0 = \frac{u}{i_c} = \frac{1}{cs}$$

(6)

$$G_i = \frac{i_c}{i} = \frac{(L_c + L_i)s}{1/cs + (L_i + L_c)s} = \frac{c(L_c + L_i)s^2}{1 + c(L_i + L_c)s^2}$$

(7)

$$G_2 = \frac{i_c}{i} = \frac{1/cs}{1/cs + (L_c + L_i)s} = \frac{1}{1 + c(L_c + L_i)s^2}$$

(8)

$$P(s) = k_p \frac{k_i}{s}$$

(9)
The damping current of the virtual inner loop capacitor can be suppressed by increasing the damping current of the virtual inner loop capacitor. After adding capacitor current feedback, the transfer function \( W(s) \) from input voltage \( u_i \) to output current \( i_2 \) is as follows:

\[
W(s) = \frac{1}{(L_1 + L_2 + L_s)s + K_R C (L_2 + L_s)s^2 + C_s (L_2 + L_s)s^3}
\]  

(10)

When \( K_R = 10, 15 \) and \( 20 \), the Bode diagram can be drawn as follows:

**Fig.5** Bode plots of \( W(s) \)

It can be seen from the above figure that the resonant peak decreases gradually when the scale coefficient increases from small to large, but at the same time, the phase shift in the intermediate frequency region becomes larger. In order to ensure a better compensation effect for the harmonic current in the intermediate frequency region, the proportional coefficient should be properly compromised. In this paper, \( K_R \) is set to 15.

In this paper, we transform \( W(s) \) from zero order holder method to \( W(z) \) in Z domain. The discrete form of PI controller shown in equation (9) is \( P(z) \), Considering one beat control delay of digital control, the open-loop transfer function of current inner loop is:

\[
F(z) = W(z)P(z)\times z^{-1}
\]  

(11)

Increasing \( k_p \) can increase the open-loop gain in the low frequency band of the system, thus increasing the system bandwidth, but it will reduce the amplitude margin of the system. Increasing \( k_i \) can correct the phase lag in the middle and low frequency band and obtain higher current tracking accuracy, but it will also reduce the phase margin of the system. Figure 6 shows the closed-loop Bode diagram of the system with PI control in the current inner loop, When \( k_p = 4 \) and \( k_i = 4000 \), the system phase margin \( \gamma = 62.7^\circ \) and amplitude margin \( h = 5.6 \)dB;
It can be seen that there is an obvious lag in the phase of the current inner loop at about 500Hz, and the amplitude also has a certain attenuation. Increasing PI parameters can increase the bandwidth, but the effect is limited, and may lead to system instability, so it is difficult to guarantee the effect of harmonic current compensation by single PI control. It is an effective solution to find a compound control strategy which can compensate the harmonic current in high steady state.

Repetitive control is a kind of control strategy based on internal model theory, which can provide high gain for harmonic signal and realize no static error tracking for any harmonic signal in theory. In order to achieve high compensation accuracy, this paper introduces repetitive controller and PI controller to form a composite control system.

The block diagram of APF compound control system with repetitive control is shown in Figure 7.

\[
P(z) \text{is PI controller and } W(z) \text{ is transfer function of controlled object. Repetitive control consists of repetitive internal model, periodic delay element } Z^{-N} \text{ and corrector } C(z) \text{ is as follows:}
\]

\[
H(z) = \frac{1}{1 - Q(z)z^{-N}}
\]  

(12)

Where \( Q(z) \) is the attenuating filter, and the ideal integral in the repetitive internal model is usually changed to quasi integral, In this paper, we take \( Q(z) = 0.95 \) to sacrifice a certain steady-state accuracy for the stability of the system.

In the middle frequency band, the amplitude attenuation of LCL filter is large and needs to be corrected. Considering the delay caused by system signal detection and filtering, a certain phase compensation is needed. The corrector \( C(z) \) can be designed as a simple leading link \( z^4 \) and a
proportional link $K_c$. In order to ensure the stable open-loop gain in the middle and low frequency band, this paper takes $K_c=1.2$.

$$C(z) = K_c z^4$$  \hspace{1cm} (13)

In the high frequency band, due to the large phase lag, the amplitude needs to be attenuated to ensure the stability. In this paper, the capacitor current proportional negative feedback control is used in the current inner loop, which can effectively attenuate the amplitude of the high frequency band and ensure the stability of the system, therefore, there is no need to set the high frequency attenuation link separately.

The characteristic equation of the system is as follows:

$$D(z)=1+z^{-N}(C(z)W(z) - Q(z))$$  \hspace{1cm} (14)

According to the principle of small gain, a sufficient condition for the stability of the system [15,19] is derived:

$$E = |Q(z) - C(z)W(z)| < 1$$  \hspace{1cm} (15)

5. Simulation analysis

The APF simulation model as shown in Figure 1 is built. The A and B phase loads are resistive inductive load $Z$, and the C phase load is a single-phase rectifier full bridge load ($C_1, R$) with resistance and capacitance of parallel resistive inductive load $Z$ and series inductance $L$. The main parameters are shown in Table 1:

| parameter                     | numerical value |
|-------------------------------|-----------------|
| RMS of system phase voltage   | 220V            |
| System frequency              | 50Hz            |
| switching frequency           | 10000Hz         |
| sampling frequency $L_s$      | 20000Hz         |
| $L_1$                         | 0.1mH           |
| $L_2$                         | 1mH             |
| $C$                           | 0.5mH           |
| $U_{dc}$                      | 3µF             |
| $C_{dc}$                      | 600V            |
| $C_1$                         | 6000µF          |
| $Z$                           | 2200µF          |
| $R$                           | 22Ω             |
| $L$                           | 6mH             |
| $Z$                           | 10Ω+10mH        |

Figure 8 shows the three-phase current waveform of the system before compensation. Figure 9 and figure 10 show the current waveform of the system when APF adopts PI control and composite control respectively.
Table 2 and Table 3 show the analysis results of phase C current and neutral current respectively.

**Table 2 Analysis of system phase C current**

| working condition   | Fundamental RMS /A | 3rd harmonic content /% | 5th harmonic content /% | 7th harmonic content /% | Total THD /% |
|---------------------|---------------------|--------------------------|--------------------------|--------------------------|--------------|
| Not compensated     | 54.61               | 20.6                     | 5.1                      | 2.1                      | 21.5         |
| PI control          | 29.16               | 10.4                     | 5.7                      | 2.7                      | 12.9         |
| compound control    | 29.9                | 1.9                      | 0.7                      | 0.5                      | 3.2          |

**Table 3 Analysis of system neutral current**

| Neutral current     | Not compensated | PI control | compound control |
|---------------------|-----------------|------------|-----------------|
| RMS /A              | 35.9            | 4.2        | 2.1             |
Before compensation, the three-phase current of the system is seriously unbalanced, and the THD of the c-phase current is more than 20%. When PI control is adopted, the unbalanced state of three-phase current is improved, and the effective value of neutral current is reduced to about 4a, but the c-phase current still contains more 3rd, 5th and 7th harmonics, and there is still obvious distortion. After adopting the compound control, the 3rd, 5th and 7th harmonics of the c-phase current are further reduced obviously, the effective value of the neutral current is about 2a, and the thd value of the c-phase current is reduced to 3.2%. It shows that the compound control plays an important role in improving the compensation effect of APF.

Figure 11 shows the three-phase four leg APF topology with single inductor filter, $L_d=1.5\text{mH}$, $L_n=0.5\text{mH}$. Figure 12 shows the simulation results of system current with or without neutral line inductance $L_n$.

![Fig.11 Topology of three phase four leg APF with single inductance filter](image)

![Fig.12 System current waveform under compound control with single inductance filter](image)

(a) Without neutral inductor

(b) With neutral inductor
Table 4 shows the analysis of c-phase current results of single inductance filter and LCL filter with or without neutral line inductance $L_n$.

| working condition | Fundamental RMS /A | Nth harmonic content /% | Total THD /% |
|-------------------|--------------------|-------------------------|--------------|
|                   |                    | 150  | 250  | 1000 | 10000 |                   |
| Without $L_n$     | 30.50              | 2.1  | 0.5  | 5.1  | 6.7   |                   |
| With $L_n$        | 30.78              | 2.6  | 0.9  | 2.6  | 5.2   |                   |
| LCL               | 29.96              | 1.9  | 0.7  | 1.6  | 3.2   |                   |

In the case of single inductor filter without neutral line inductance $L_n$, the compensation effect of low order harmonics is equivalent to that of LCL filter, but the suppression ability of high frequency harmonics is weak. The THD value of harmonics at 10 kHz of PWM switching frequency reaches 5%. In the case of single inductor filter with neutral line inductance $L_n$, the suppression ability of high frequency harmonics is enhanced, and the THD value of harmonics at PWM switching frequency is reduced to about 2.6%. However, the compensation effect of APF system for low order harmonics becomes worse by increasing the filter inductance. The LCL filter has good ability of high frequency harmonic suppression and low order harmonic compensation.

6. Conclusion
In this paper, the topology of LCL filter for three-phase four leg APF system is designed, and the principle and method of filter parameter tuning are described in detail. Compared with the traditional single inductor filter, the proposed LCL filter has strong harmonic suppression ability for high frequency harmonics and better compensation effect for low order harmonics, and has excellent performance and engineering practical value. At the same time, a compound controller combining repetitive control with PI control is proposed. The structure of the controller is simple and the corrector is easy to design, which makes the three-phase four leg APF system have high steady-state compensation accuracy for harmonic current and overcomes the problem of insufficient bandwidth of traditional PI control.

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References
[1] A. Zabihian, A.Y. Varjani, H. Ghoreishy, Using SHM-PWAM modulation technique for three-phase four-leg active power filter, 2014 22nd Iranian Conference on Electrical Engineering (ICEE), 2014, pp. 648–653.
[2] A. V. Barval, P. R. Bhavsa, Design and simulation of four-Leg based three phase four-wire shunt active power filter, 2018 International Conference on Communication, Information & Computing Technology (ICCICT), 2018.
[3] M. Bouzidi, S. Barkat, Backstepping-Direct power control of three-level four-leg shunt active power filter, 2018 International Conference on Communications and Electrical Engineering (ICCEE), 2018.
[4] I. I. Abdalla, K. S. Rama Rao, N. Perumal, Three-phase four-leg shunt active power filter to compensate harmonics and reactive power, 2011 IEEE Symposium on Computers & Informatics, 2011, pp. 495–500.
[5] S. S. Seyedalipour, H.A. Aalami, A. Barzegar, A novel control technique for stable operation of four-leg shunt active power filters in electrical grids, 2017 Conference on Electrical Power Distribution Networks Conference (EPDC), 2017, pp. 175–181.
[6] S. Vahid, H. Rastegar, S.H. Fathi, M. Jedari, A comprehensive comparison between three different
control strategies for four-leg active power filters, 2016 4th International Symposium on Environmental Friendly Energies and Applications (EFEA), 2016.

[7] M. Malinowski, W. Szczygieł, M. P. Kazmierkowski, et al. Simple sensorless active damping solution for three-phase PWM rectifier with LCL filter [J]. Industrial Electronics Society. 2005. 32nd Annual Conference of IEEE, 2005: 5.

[8] Xu Zhiying, Xu Aiguo Xie Shaojun. Dual-loop grid current control technique for grid-connected inverter using an LCL filter. Proceedings of the CSEE, 2009, 29(27): 36-39.

[9] Qiu Zhiling, Yang Enxing Kong Jie. Current loop control approach for LCL-based shunt active power filter [J]. Proceedings of the CSEE, 2009, 29(18): 15-18.

[10] Liu Fei Zou Yunping, Li Hui. The repetitive control algorithm based current waveform correction for voltage source inverters [J]. Proceedings of the CSEE, 2005, 25(19): 58-63.

[11] Liu Fei Zha Xiaoming. Research on grid-connected strategy combining pole-assignment and repetitive control in three-phase photovoltaic system [J], Transactions of China Electronic technical Society, 2008, 23(12): 130-136.

[12] B Ciprian, G. Gelu, M. Toader, F. Grigore, Comparison of three phase 4-leg shunt active power filter algorithms, International Aegean Conference on Electrical Machines and Power Electronics and Electromotion, Joint Conference, 2011, pp. 478-483.

[13] I. I. Abdalla, K. S. Rama Rao, N. Perumal, Harmonics mitigation and power factor correction with a modern three-phase four-leg shunt active power filter, 2010 IEEE International Conference on Power and Energy, 2010, pp. 156-161.