Electroweak Corrections to $e^+e^- \rightarrow 4$ fermions

A. Denner$^a$, S. Dittmaier$^b$, M. Roth$^b$, L.H. Wieders$^a$

$^a$Paul Scherrer Institut, Würenlingen und Villigen, CH-5232 Villigen PSI, Switzerland

$^b$Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), D-80805 München, Germany

The calculation of the full electroweak $O(\alpha)$ corrections to the charged-current four-fermion production processes $e^+e^- \rightarrow \nu_\tau \tau^- \mu^+ \mu^-$, $u \bar{d} \mu^+ \mu^-$, and $u \bar{d} \bar{s} \bar{c}$ is briefly reviewed. The calculation is performed using the complex-mass scheme for the gauge-boson resonances. The evaluation of the occurring one-loop tensor integrals, which include 5- and 6-point functions, requires new techniques. The effects of the complete $O(\alpha)$ corrections to the total cross section and to the production-angle distribution are discussed and compared to predictions based on the double-pole approximation, revealing that the latter approximation is not sufficient to fully exploit the potential of a future linear collider in an analysis of W-boson pairs at high energies.

1. INTRODUCTION

At LEP2, W-pair-mediated four-fermion (4f) production was experimentally explored with quite high precision (see Ref. [1] and references therein). For LEP2 accuracy, it was sufficient to include corrections in the so-called double-pole approximation (DPA), where only the leading term in an expansion about the poles in the two W-boson propagators is taken into account. Different versions of such a DPA have been used in the literature [2,3,4,5,6]. Although several Monte Carlo programs exist that include universal corrections, only two event generators, YFSWW [3,4] and RACOONWW [5,7,8], include non-universal corrections.

In the DPA approach, the W-pair cross section can be predicted within $\sim 0.5\% (0.7\%)$ in the energy range between 180 GeV (170 GeV) and $\sim 500$ GeV, which was sufficient for the LEP2 accuracy of $\sim 1\%$. In the threshold region ($\sqrt{s} < \sim 170$ GeV), the DPA is not reliable, and the best available prediction resulted from an improved Born approximation (IBA) based on leading universal corrections only, and thus possesses an intrinsic uncertainty of $\sim 2\%$.

At a future International $e^+e^-$ Linear Collider (ILC) [9,10,11], the accuracy of the cross-section measurement will be at the per-mille level, and the precision of the W-mass determination is expected to be $\sim 7$ MeV from a threshold scan of the total W-pair-production cross section [9,10]. For the cross-section prediction at threshold the theoretical uncertainty of the IBA is $\sim 2\%$ and thus definitely insufficient for the planned precision measurement of $M_W$ in a threshold scan.

Recently we have completed the first calculation of the complete $O(\alpha)$ corrections (improved by higher-order ISR) for $e^+e^- \rightarrow \nu_\tau \tau^- \mu^+ \mu^-$, $u \bar{d} \mu^+ \mu^-$, and $u \bar{d} \bar{s} \bar{c}$, which are relevant for W-pair production. We have presented results on total cross sections in Ref. [12] and on various differential distributions in Ref. [13]. The latter publication also contains technical details of the calculation, which is rather complicated. In the following we briefly describe the salient features of the calculation and present a selection of numerical results that are relevant for a future ILC.

2. METHOD OF CALCULATION

The actual calculation builds upon the RACOONWW approach [5], where real-photonic corrections are based on full matrix elements and virtual corrections are treated in DPA. Real and virtual corrections are combined either using two-
cutoff phase-space slicing or employing the dipole subtraction method \cite{15} for photon radiation.

In contrast to the DPA approach, the one-loop calculation of an $e^+e^- \rightarrow 4f$ process requires the evaluation of 5- and 6-point one-loop tensor integrals. We calculate the 6-point integrals by directly reducing them to six 5-point functions, as described in Refs. \cite{16,17}. The 5-point integrals are reduced to five 4-point functions following the methods of Refs. \cite{17,18}. Note that this reduction of 5- and 6-point integrals to 4-point integrals does not involve inverse Gram determinants composed of external momenta, which naturally occur in the Passarino–Veltman reduction \cite{19} of tensor to scalar integrals. The latter procedure leads to serious numerical problems when the Gram determinants become small.

Tensor 4-point and 3-point integrals are reduced to scalar integrals with the Passarino–Veltman algorithm \cite{19} as long as no small Gram determinant appears in the reduction. If small Gram determinants occur, two alternative schemes are applied \cite{17}. One method makes use of expansions of the tensor coefficients about the limit of vanishing Gram determinants and possibly other kinematical determinants. In this way, again all tensor coefficients can be expressed in terms of the standard scalar functions. In the second, alternative method we evaluate a specific tensor coefficient, the integrand of which is logarithmic in Feynman parametrization, by numerical integration. Then the remaining coefficients as well as the standard scalar integral are algebraically derived from this coefficient.

As a further complication, also the evaluation of the three spinor chains corresponding to the three external fermion–antifermion pairs is non-trivial, because the chains are contracted with each other and/or with four-momenta in many different ways. There are $\mathcal{O}(10^3)$ different chains to calculate. We have worked out algorithms that algebraically reduce all these spinor chains to a few or, in an alternative method, to a minimal set of standard structures without introducing numerical problems. These algorithms are described in Ref. \cite{13} in detail.

The description of resonances in (standard) perturbation theory requires a Dyson summation of self-energy insertions in the resonant propagator in order to introduce the imaginary part provided by the finite decay width into the propagator denominator. This procedure in general violates gauge invariance, i.e. destroys Slavnov–Taylor or Ward identities and disturbs the cancellation of gauge-parameter dependences, because different perturbative orders are mixed (see, for instance, Ref. \cite{20} and references therein).

For our calculation we have generalized \cite{13} the so-called “complex-mass scheme”, which was introduced in Ref. \cite{7} for lowest-order calculations, to the one-loop level. In this approach the $W$- and $Z$-boson masses are consistently considered as complex quantities, defined as the locations of the propagator poles in the complex plane. To this end, bare real masses are split into complex renormalized masses and complex counterterms. Since the bare Lagrangian is not changed, double counting does not occur. Perturbative calculations can be performed as usual, only parameters and counterterms, in particular the electroweak mixing angle defined from the ratio of the $W$- and $Z$-boson masses, become complex. Since we only perform an analytic continuation of the parameters, all relations that follow from gauge invariance, such as Ward identities, remain valid. As a consequence the amplitudes are gauge independent, and unitarity cancellations are respected. Moreover, the on-shell renormalization scheme can straightforwardly be transferred to the complex-mass scheme \cite{13}.

The use of complex gauge-boson masses necessitates the consistent use of these complex masses also in loop integrals. The scalar master integrals are evaluated for complex masses using the methods and results of Refs. \cite{21,22,23}.

In order to prove the reliability of our results, we have carried out various checks, as described in detail in Ref. \cite{12}. We have checked the structure of the (UV, soft, and collinear) singularities, the matching between virtual and real corrections, and the gauge independence (by performing the calculation in the ’t Hooft–Feynman gauge and in the background-field gauge \cite{24}). The most convincing check for ourselves is the fact that we worked out the whole calculation in two independent ways, resulting in two independent computer
codes the results of which are in good agreement.

3. NUMERICAL RESULTS

The precisely defined input for the numerical results presented in the following can be found in Refs. [12,13]. Figure 1 depicts the total cross section for the energy ranges of LEP2 and of the high-energy phase of a future ILC, focusing on the semileptonic final state $e^+e^-\rightarrow u\bar{d}\mu^+\nu_\mu$. The respective figures for the relative corrections $\delta$ to the leptonic (shown in Ref. [12]) and hadronic final states look almost identical, up to an offset resulting from the missing or additional QCD corrections. Specifically, the upper plots show the absolute prediction for the cross section including the full $O(\alpha)$ corrections and improvements from higher-order ISR. The lower plots compare the relative corrections as obtained from the full $O(\alpha)$ calculation, from an IBA, and from the DPA. The IBA [8] implemented in RACOONWW is based on universal corrections only and includes solely the contributions of the CC03 diagrams. The DPA of RACOONWW comprises also non-universal corrections [5] and goes beyond a pure pole approximation in two respects. The real-photonic corrections are based on the full $e^+e^-\rightarrow 4f + \gamma$ matrix elements, and the Coulomb singularity is included for off-shell W bosons. Further details can be found in Ref. [5].

A comparison between the DPA and the predictions based on the full $O(\alpha)$ corrections reveals differences in the relative corrections $\delta$ of $\lesssim 0.5\%$ (0.7\%) for CM energies ranging from $\sqrt{s} \approx 170$ GeV to 300 GeV (500 GeV). This is in agreement with the expected reliability of the DPA, as discussed in Refs. [15,20]. At higher energies, the deviations increase and reach 1–2\% at $\sqrt{s} = 1–2$ TeV. In the threshold region ($\sqrt{s} \lesssim 170$ GeV), as expected, the DPA also becomes worse w.r.t. the full one-loop calculation, because the naive error estimate of $(\alpha/\pi)(\Gamma_W/M_W)$ times some numerical safety factor of $O(1–10)$ for the corrections missing in the DPA has to be replaced by $(\alpha/\pi)(\Gamma_W/\sqrt{s} - 2M_W)$ and thus becomes large. In view of that, the DPA is even surprisingly good near threshold. For CM energies below 170 GeV the LEP2 cross section analysis was based on approximations like the shown IBA, which follows the full one-loop corrections even below the threshold at $\sqrt{s} = 2M_W$ within an accuracy of about 2\%, as expected in Ref. [8]. More results on total cross sections, including numbers on leptonic, semileptonic, and hadronic final states, can be found in Ref. [12].

The distribution in the cosine of the W production angle is shown in Figure 2 for the process $e^+e^- \rightarrow u\bar{d}\mu^+\nu_\mu$ at $\sqrt{s} = 500$ GeV. Further distributions, also for $\sqrt{s} = 200$ GeV, are presented in Ref. [13]. For the W-production-angle distribution the full $O(\alpha)$ calculation and the DPA agree within $\sim 1\%$ for LEP2 energies (see Fig. 12 of Ref. [13]), but at 500 GeV the difference of the corrections in DPA and the complete $O(\alpha)$ corrections rises from $\sim 1\%$ to about $\sim 2.5\%$ with increasing scattering angle (see inset in r.h.s. of Figure 2). Note that such a distortion of the shape of the angular distribution can be a signal for anomalous triple gauge-boson couplings.

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Figure 1. Absolute cross section $\sigma$ (upper plots) and relative corrections $\delta$ (lower plots) to the total cross section without cuts for $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$ obtained from the IBA, DPA, and the full $\mathcal{O}(\alpha)$ calculation (ee4f). All predictions are improved by higher-order ISR.

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$$\frac{d\sigma}{d\cos\theta_{W^+}} \text{ [fb]} \quad \sqrt{s} = 500 \text{ GeV}$$

$$e^+e^- \to u\bar{d}\mu\bar{\nu}_\mu \quad \text{Born}$$

$$e^+e^- \to u\bar{d}\mu\bar{\nu}_\mu \quad \text{ee4f}$$

Figure 2. Distribution in the cosine of the $W^+$ production angle with respect to the $e^+$ beam (r.h.s.) and the corresponding corrections (lower row) at $\sqrt{s} = 500$ GeV for $e^+e^- \to u\bar{d}\mu\bar{\nu}_\mu$. The inset plot shows the relative difference $\Delta$ between the full $\mathcal{O}(\alpha)$ corrections and those in DPA. (Taken from Ref. [13].)

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