Research Article

Solving Constrained Flow-Shop Scheduling Problem through Multistage Fuzzy Binding Approach with Fuzzy Due Dates

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This paper deals with constrained multistage machines flow-shop (FS) scheduling model in which processing times, job weights, and break-down machine time are characterized by fuzzy numbers that are piecewise as well as quadratic in nature. Avoiding to convert the model into its crisp, the closed interval approximation for the piecewise quadratic fuzzy numbers is incorporated. The suggested method leads a noncrossing optimal sequence to the considered problem and minimizes the total elapsed time under fuzziness. The proposed approach helps the decision maker to search for applicable solution related to real-world problems and minimizes the total fuzzy elapsed time. A numerical example is provided for the illustration of the suggested methodology.

1. Introduction

Scheduling contains the sequence of jobs following the resource as well as time constraints, with a specific objective. The job scheduling and controlling through a production is a significant role in any industrial manufacturing unit. The FS scheduling model is the simple version where all jobs are operated on all the machines in order, is one of the recent issues in the field of production control, and is to determine the job sequence on the machines to minimize the makespan. The scheduling model usually consists of three components: time of transportation, job weight, and machine time for the break down.

Job scheduling problems, normally, occur such as programs for running on a sequence using some computer operators and to order the jobs for processing in a plant of manufacturing. Numerous researchers studied various FS scheduling problem and job scheduling problems and proposed algorithms in the crisp environment [1, 2]. A new heuristic algorithm was introduced by Aggarwal et al. [3] for obtaining an optimal (near-optimal) sequence to bicriteria three-stage FS scheduling based on heuristic technique, which was further discussed by Patider et al. [4]. Abdullah and Abdolrazzaghi-Nezhad [5] developed an algorithm for solving theatrical models for fuzzy job-shop scheduling.

The FS scheduling model under the fuzzy processing time has been formulated by Ishibuchi et al. [6]. Afterwards, several researchers considered the machine sequence-dependent processing times. Ahonen and de Alvarenga [7] formulated and proposed a solution for the FS scheduling model, considering the recirculation and machine sequence-varying processing time. Qu et al. [8] proposed an algorithm to solve the no-wait FS scheduling problem based on the hormone modulation mechanism. Komaki et al. [9] introduced a consolidated survey of assembly FS models with their solution approach. Belabid et al. [10] proposed three methods for resolution of a permutation FS problem with independent setup time: mixed-integer LP model and two heuristics so as to minimize the maximum of job competition time.

In literature, authors, such as Zadeh [11] and Dubois and Prade [12], considered the FS problem with the
consideration transportation cost. Hnaien et al. [13] presented the makespan minimization problem by describing the two-machine FS under a constraint related to availability of the first machine. A two-stage multiprocessor FS scheduling problem was considered under the deterioration of maintenance in a cleaner production [14]. Khatami and Zegordi [15] suggested the flexible maintenance time intervals.

Yang et al. [16] studied the FS scheduling of many production lines for precast production. Toumi et al. [17] presented the branch-and-bound technique for the solution of blocking FS scheduling problem under the assumption of makespan criterion. Yu et al. [18] presented the iterative method for batching and scheduling problem for the minimization of total job tardiness in two-stage hybrid FS. Shahvari and Logendran [19] presented a comparison of hybrid algorithm for a batch scheduling problem in hybrid FS under the assumption of learning effect. They used a clustering-genetic algorithm-based technique.

A particular kind of FS problem is called the permutation FS scheduling problem, where the job processing order is the same for each subsequent step of the processing [20]. Over the years in literature, several authors studied the permutation FS problem. Damodaran et al. [21] proposed the particle swarm optimization procedure for solving the permutation FS. They considered the scheduling batch processing machines in the model. Some multiobjective methods were also suggested by many researchers. Li and Ma [22] presented an artificial bee colony algorithm for multiobjective permutation FS problem with sequence varying with setup times. Chaouch et al. [23] presented a modified method of ant colony optimization algorithm to determine the optimal scheduling for the distributed job shop problem. Khalifa [24] analyzed the single-machine preparation issue in a fuzzy date setting.

Several researchers studied the fuzzy methods for solving the permutation FS problem, for instance, Tirkolaee et al. [25], Sioud and Gagne [26], and Kumar [27]. Tirkolaee et al. [25] studied a multistage green capacitated arc routing problem with an application to urban services. They used the hybrid genetic algorithm. Sioud and Gagne [26] proposed a special type solution method based on the enhanced migrating birds to permutation FS problem with the assumption of sequence-dependent setup times. Goli et al. [28] proposed a FS scheduling problem with outsourcing option on subcontractors. They considered the just-in-time criteria in model formulation. Tirkolaee et al. [29] investigated the pollution-routing problem with cross-dock selection. They used the Pareto-based algorithm to deal with the multiobjective optimization problem. Afterwards, Khalifa and Kumar [30] proposed the fuzzy solution approach to fully neutrosophic linear programming problem. They also presented an application to stock portfolio selection. Very recently, Tirkolaee et al. [31] presented a FS scheduling problem with outsourcing option. They used fuzzy programming and artificial fish swarm algorithm. Goli et al. [32] investigated a fuzzy production scheduling model. They considered the automated guided vehicles as well as human factors.

In this paper, a novel method called multistage fuzzy binding for solving the problem under consideration in which jobs processing time, weights, and break-down machine are characterized as piecewise quadratic fuzzy numbers is proposed. Here, it is assumed that there is no power break up for dealing with break-down power as it has been assumed that the unit of production is still a small-scale one. The suggested method depends on the binding method applied by Pandian and Rajendran [33] which provides a noncrossing optimal sequence to the considered problem with the minimizing total fuzzy elapsed time.

The rest of the research work is organized as follows: the basic concept and arithmetic operations related to fuzzy numbers and their arithmetic operations are described in Section 2. Section 3 describes some of the assumptions and notations required in the proposed problem mathematical formulation. Section 4 formulates fuzzy constrained multistage FS scheduling problems. Section 5 proposes multi-stage fuzzy binding approach for obtaining a noncrossing optimal sequence. In Section 6, a numerical example to illustrate the methodology is introduced. Finally, some concluding remarks are reported in Section 7.

2. Preliminaries

This section introduces some of the basic concepts, and results related to fuzzy numbers, piecewise quadratic fuzzy numbers, and their arithmetic operations are recalled.

Definition 1 (see [34]). A piecewise quadratic fuzzy number (PQFN) is denoted by \( \tilde{a}_{PQ} = (a_1, a_2, a_3, a_4, a_5) \), where \( a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \) are real numbers, and is defined by if its membership function \( \mu_{a_{PQ}} \) as follows (as in Figure 1):

\[
\mu_{a_{PQ}} = \begin{cases} 
0, & x < a_1; \\
\frac{1}{2} \left( \frac{a_2 - a_1}{a_1} \right)^2 (x - a_1)^2, & a_1 \leq x \leq a_2; \\
\frac{1}{2} \left( \frac{a_3 - a_2}{a_2} \right)^2 (x - a_2)^2 + 1, & a_2 \leq x \leq a_3; \\
\frac{1}{2} \left( \frac{a_4 - a_3}{a_3} \right)^2 (x - a_3)^2 + 1, & a_3 \leq x \leq a_4; \\
\frac{1}{2} \left( \frac{a_5 - a_4}{a_4} \right)^2 (x - a_4)^2, & a_4 \leq x \leq a_5; \\
0, & x > a_5.
\end{cases}
\]

The interval of confidence at level \( \alpha \) for the PQFN is defined as follows:

\[
(\tilde{a}_{PQ})_\alpha = \left[ a_1 + 2 (a_2 - a_1) \alpha, a_5 - 2 (a_5 - a_4) \alpha \right]; \quad \forall \alpha \in [0, 1].
\]
Definition 2 (see [34]). An interval approximation \([A] = [(a_a)^L, (a_a)^U]\) of a PQFN \(\tilde{A}\) is called closed interval approximation if
\[
\begin{align*}
(a_a)^L &= \inf\{x \in \mathbb{R}: \mu_{\tilde{A}}^x \geq 0.5\}, \\
(a_a)^U &= \sup\{x \in \mathbb{R}: \mu_{\tilde{A}}^x \geq 0.5\}.
\end{align*}
\]

Definition 3 (see [35, 36]). An interval on \(\mathbb{R}\) is defined as
\[
A = [a^L, a^R] = \{a : a^L \leq a \leq a^R, a \in \mathbb{R}\},
\]
where \(a^L\) is the left limit and \(a^R\) is the right limit of \(A\).

Definition 4 (see [37]). The interval is also defined as
\[
A = a_c, a_W = \{a : a_c - a_W \leq a \leq a_c + a_W, a \in \mathbb{R}\},
\]
where \(a_c = (1/2)(a^L + a^R)\) is the center and \(a_W = (1/2)(a^R - a^L)\) is the width of \(A\).

Definition 5 The associated ordinal numbers of PQFN corresponding to the closed interval approximation \([A] = [(a_a)^L, (a_a)^U]\) are \(\tilde{A} = ((a_a)^L + (a_a)^U)/2\).

Definition 6 The associated ordinary (crisp) number corresponding to the PQFN \(\tilde{A}_{PQ}\) = \((a_1, a_2, a_3, a_4, a_5)\) is defined as
\[
\tilde{A}_{PQ} = \frac{a_1 + a_2 + 4a_3 + a_4 + a_5}{8}.
\]

Definition 7 (see [34]). Let \([A] = [(a_a)^L, (a_a)^U]\) and \([B] = [(b_b)^L, (b_b)^U]\) be two interval approximations of PQFN. Then, the arithmetic operations are defined as follows:

1. Addition: \([A] \oplus [B] = [(a_a)^L + (b_b)^L, (a_a)^U + (b_b)^U]\).
2. Subtraction: \([A] \ominus [B] = [(a_a)^L - (b_b)^L, (a_a)^U - (b_b)^U]\).
3. Scalar multiplication: \(a \cdot [A] = \{a(a_a)^L, a(a_a)^U\}\), \(a > 0\),
\[
\{a(a_a)^L, a(a_a)^U\}, \quad a < 0.
\]
4. Multiplication: \([A] \otimes [B] = \left[\min((a_a)^L + (b_b)^L, (a_a)^U + (b_b)^U), (a_a)^L + (b_b)^L, (a_a)^U + (b_b)^U)\right] \cup \left[\max((a_a)^L + (b_b)^L, (a_a)^U + (b_b)^U), (a_a)^L + (b_b)^L, (a_a)^U + (b_b)^U)\right].
\]
5. Division: \([A] \oslash [B] = \left[\min\left(\frac{(a_a)^L}{(b_b)^L}, \frac{(a_a)^U}{(b_b)^U}\right), \frac{(a_a)^L}{(b_b)^L}, \frac{(a_a)^U}{(b_b)^U}\right]\), \([B] > 0\).
6. Maximum: \([A] \vee [B] = [(a_a)^L \vee (b_b)^L, (a_a)^U \vee (b_b)^U]\).
7. Minimum: \([A] \wedge [B] = [(a_a)^L \wedge (b_b)^L, (a_a)^U \wedge (b_b)^U]\).

3. Notation and Assumptions

3.1. Notation. The following notations can be used in the proposed FS scheduling problem:

\(S_k\): sequence resulted by applying Johnson’s procedure, \(k = 1, 2, \ldots, m\).

\(i\): job \((i = 1, 2, \ldots, n)\).

\(M_j\): machine \(j \,(j = 1, 2, \ldots, m)\).

\(\tilde{M}_i\): quadratic piecewise fuzzy processing time of the \(i\)th job on machine \(M_i\) \((i = 1, 2, \ldots, m; j = 1, 2, \ldots, m)\).

\(\tilde{P}\): processes that require uninterrupted power supply and no break-down time are permitted.

\(\tilde{Q}\): processes that require power supply and break-down time are permitted.

\(\bar{M}\): processes that do not require power supply and may be continued during break-down time.

\(\bar{P}\): fuzzy performance measure \((i = 1, 2, \ldots, n), \bar{P} = ((\sum_{i=1}^{n} \bar{w}_i) \oplus \bar{f}_i))/((\sum_{i=1}^{n} \bar{w}_i))\).

\(\bar{f}_i\): flow time of the job \(i \,(i = 1, 2, \ldots, n)\).

\(\bar{w}_i\): fuzzy weights \((i = 1, 2, \ldots, n)\).

3.2. Assumptions. In this FS scheduling problem, the following assumptions are made:

(i) No passing is permitted.

(ii) All the jobs are available for processing at time zero.

(iii) All jobs are available at the beginning of scheduling time horizon.

(iv) The machines setup times are negligible.

(v) All jobs have deterministic processing times.

(vi) Due dates are PQF numbers.

(vii) Machine may be idle.

(viii) Processing times are independent of the schedule.
(ix) To feed a job on a second machine, it must be completed on the first machine.

(x) Each job has $m$ operations.

(xi) Each job must be completed once it is started.

4. Problem Statement

The aim of the problem is to minimize the total piecewise quadratic fuzzy elapsed time that is to find the optimal sequence of the jobs. Assume that job $i$ ($i = 1, 2, \ldots, n$) is to be processed on machine $j$ ($j = 1, 2, \ldots, m$) in the existence of specified rental policy. Let $M_{ij}$ ($i = 1, 2, \ldots, n; j = 1, 2, \ldots, m$) be the processing time of job $i$ on machine $j$ characterized by PCF numbers, which may be classified into three categories:

1. The processes require uninterrupted power supply, and no break-down is permitted (say, $P_1, P_2, \ldots$).
2. The processes require power supply, and break-down is permitted (say, $Q_1, Q_2, \ldots$).
3. The processes do not require power supply and can be continued during the break-down time. Let them be $M_1, M_2, \ldots$.

In addition, let job $i$ ($i = 1, n$) be assigned having fuzzy weights $w_i$ relative to the importance of performance in the sequence. The measure of the fuzzy performance is defined as

$$\bar{G} = \frac{\sum_{i=1}^{n} w_i \phi \bar{f}_i}{\sum_{i=1}^{n} w_i},$$

(9)

where $\bar{f}_i$ is the flow time of the $i$th job. Let the fuzzy break-down approximate interval be $[\bar{a}, \bar{b}]$. Our aim is to determine the optimal sequence of jobs to minimize the total fuzzy elapsed time. The problem can be illustrated as in Table 1.

Assume that the considered problem satisfies one or both the following conditions:

$$\min_{i} \bar{M}_{1i} \pm \max_{i} \bar{M}_{ij}, \quad j = 2, 3, \ldots, m - 1,$$

or/and

$$\min_{i} \bar{M}_{mj} \pm \max_{i} \bar{M}_{ij}, \quad j = 2, 3, \ldots, m - 1.$$

(10)

5. Proposed Approach

The steps of the approach are as follows:

Step 1: consider the piecewise quadratic fuzzy constrained multistage machines FS scheduling (PQFCMFSS) problem.

Step 2: convert the PQFCMFSS problem into the corresponding approximated closed-interval CMFSS problem.

Step 3: convert the CMFSS problem into a two-machine FS scheduling problem by introducing two fictitious machines $H_1$ and $H_2$ with

$$\bar{M}_{ij} = (a_{i1}, a_{i2}, a_{i3}, a_{i4}).$$

Table 1: Piecewise quadratic fuzzy processing times $\bar{M}_{ij} = (a_{i1}, a_{i2}, a_{i3}, a_{i4}).$.

| Job | Machines with PQF processing times | PCF weights of job |
|-----|-----------------------------------|--------------------|
|     | $M_1$ | $M_2$ | $M_3$ | $\ldots$ | $M_m$ | $\bar{w}_i$ |
| 1   | $\bar{M}_{i1}$ | $\bar{M}_{i2}$ | $\bar{M}_{i3}$ | $\ldots$ | $\bar{M}_{im}$ | $\bar{w}_1$ |
| $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\ldots$ | $\bar{M}_{nm}$ | $\bar{w}_n$ |

Here, $\bar{H}_1$ and $\bar{H}_2$ are the closed-interval processing time for job $i$ on machines $H_1$ and $H_2$, respectively.

Step 4: applying the method introduced by Pandian and Rajendran [33] to obtain the optimal sequence.

Step 5: identify the effect of break-down interval $[a^L, a^U]$, $[b^L, b^U]$ or $[\bar{a}, \bar{b}]$ on different jobs. If the affected jobs come under $\bar{M}_1, \bar{M}_2, \ldots$, there is no need to be modified and can be neglected.

Step 6: identify the modified processing time on different jobs under categories $P_1, P_2, \ldots$, and $Q_1, Q_2, \ldots$.

Step 7: modify the fuzzy processing time after categorizing the jobs as follows:

Let $[t^L, t^U]$ be the existing interval processing time and $[u^L, u^U]$ be a new interval processing time.

Let $[a^L, a^U]$ be interval processing time span begin and $[b^L, b^U]$ break-down time span interval end.

Let $[s^L, s^U]$ be interval existing processing time span begin and $[s^L, s^U]$ be existing interval processing time span end.

(i) Category 1: if the process is a continuous one not to be interrupted in any case as welding and forging, then add $([b^L, b^U] - [s^L, s^U])$ to the interval processing time $[t^L, t^U]$ to get $[u^L, u^U]$.

(ii) Category 2: if the process need not be a continuous one and is not affected by any interrupts such as packing, drilling, and threading, and then the existing interval processing time $[t^L, t^U]$ is converted to the new interval processing time $[u^L, u^U]$. There are two cases:

Case 1: if the break-down starts or/and stars and ends in between, $[b^L - a^U, b^U - a^L]$ is added to the interval processing time.

Case 2: if the break-down ends in between or/and stars before and ends after the interval processing time span, $[b^L - s^L, b^U - s^L]$ is added to the interval processing time.

Step 8: determine the minimum total elapsed time and the weighted men-flow for the FS scheduling problem.
6. Numerical Example

In this section, we solve a numerical example to illustrate the suggested approach.

Step 1: consider the following PQFCMFFS problem as in Table 2.

Consider the break-down interval is $[\hat{a}, \hat{b}] = [(29, 30, 31, 32, 33) \text{ to } (33, 34, 35, 36, 37)]$.

Step 2: use the approximated closed intervals corresponding to the piecewise quadratic fuzzy numbers as in Table 3.

$\text{Min} \ P_1 = [12, 14] \geq \text{Max } [[M_1], [Q_1], [Q_2]] = [[4, 6], [6, 8], [4, 6]] = [6, 8]$ is satisfied. Therefore, convert the problem into two machines problem.

Step 3: convert the problem into two machines problem as in Table 4.

Step 4: using the binding method introduced by Pandian and Rajendran [33]; the modified processing times are as in Table 5.

By applying Johnson’s algorithm, the PQF constrained multistage machines FS scheduling problem is given by the following sequence:

$$2 \rightarrow 5 \rightarrow 1 \rightarrow 3 \rightarrow 4.$$  \hspace{1cm} (12)

Hence, the PQF elapsed time is $(105, 106, 107, 108, 109)$.

Step 5: the break-down interval $[(30, 31, 32, 33, 34), (34, 35, 36, 37, 38)]$ at affected jobs is listed in Table 6.

Step 6: we observe that $M_1$ for job 2: $(34, 34, 34, 34, 34)$ to $(35, 36, 37, 38, 39)$ is neglected.

Step 7: modify the processing time all in Table 7, except the one in Step 6, for job 2 and job 5, respectively.

$\tilde{O}_2: (30, 30, 30, 30, 30)$ to $(33, 34, 35, 36, 37)$ and $\tilde{P}_1: (17, 17, 17, 17) \text{ to } (30, 31, 32, 33, 34)$; the break down is started in between, and 3 is added to the PCF processing time. The new PQF processing times become

$\tilde{Q}_2 = (6, 7, 8, 9, 10)$,

$\tilde{P}_1 = (16, 17, 18, 19, 20)$.

Also, for job 2, and job 1, respectively, $\tilde{P}_2: (37, 37, 37, 37, 37)$ to $(44, 45, 46, 47, 48)$ and $\tilde{P}_1: (34, 34, 34, 34, 34)$ to $(46, 47, 48, 49, 50)$, the break down is end in between, and the processing time is started by adding 1 to the

PQF processing time. The new PQF processing time becomes

$\tilde{P}_2: (8, 9, 10, 11, 12), \hspace{1cm} (14)$

$\tilde{P}_1: (13, 14, 15, 16, 17).$

Based on Definition 6, Table 8 changes to Table 9 as follows.

It is obvious that the optimal sequence in fuzzy environment is

$$2 \rightarrow 5 \rightarrow 1 \rightarrow 3 \rightarrow 4.$$  \hspace{1cm} (15)

Accordingly, Table 9 changes to Table 10 as follows.
Table 6: Break-down effect on jobs.

| Jobs | Break down |
|------|------------|
| $i$  | $\bar{Q}_1$ $(30, 30, 30, 30)$ to $(33, 34, 35, 36, 37)$ | $\bar{P}_1$ $(17, 17, 17, 17, 17)$ to $(30, 30, 30, 30, 30)$ | $\bar{P}_1$ $(34, 34, 34, 34, 34)$ to $(46, 47, 48, 49, 50)$ |

Table 7: Piecewise quadratic fuzzy elapsed time.

| Job | Machines with PQF processing times | PCF weights of jobs $\bar{w}_i$ |
|-----|-----------------------------------|-------------------------------|
| $i$ | $\bar{P}_1$ | $\bar{M}_1$ | $\bar{Q}_1$ | $\bar{Q}_2$ | $\bar{P}_2$ |
| 2   | $(0, 0, 0, 0)$ to $(13, 14, 15, 16)$ | $(17, 17, 17, 17, 17)$ to $(22, 22, 22, 22, 22)$ to $(30, 30, 30, 30, 30)$ | $(33, 34, 35, 36, 37)$ | $(37, 37, 37, 37, 37)$ to $(44, 45, 46, 47, 48)$ |
| 5   | $(17, 17, 17, 17)$ to $(34, 34, 34, 34)$ to $(39, 39, 39, 39, 39)$ to $(46, 46, 46, 46, 46)$ to $(47, 47, 47, 47, 47)$ to $(52, 53, 54, 55, 56)$ |
| 1   | $(34, 34, 34, 34, 34)$ to $(50, 50, 50, 50, 50)$ to $(55, 55, 55, 55, 55)$ to $(64, 64, 64, 64, 64)$ to $(69, 69, 69, 69, 69)$ to $(72, 73, 74, 75, 76)$ |
| 3   | $(50, 50, 50, 50, 50)$ to $(65, 65, 65, 65, 65)$ to $(70, 70, 70, 70, 70)$ to $(75, 75, 75, 75, 75)$ to $(81, 81, 81, 81, 81)$ to $(84, 85, 86, 87, 88)$ |
| 4   | $(65, 65, 65, 65, 65)$ to $(88, 88, 88, 88, 88)$ to $(94, 94, 94, 94, 94)$ to $(100, 100, 100, 100, 100)$ to $(105, 106, 107, 108, 109)$ |

Table 8: Modified crisp break-down time of FS scheduling problem.

| Job | Machines with PQF processing times | PCF weights of jobs $\bar{w}_i$ |
|-----|-----------------------------------|-------------------------------|
| $i$ | $\bar{P}_1$ | $\bar{M}_1$ | $\bar{Q}_1$ | $\bar{Q}_2$ | $\bar{P}_2$ |
| 1   | $(13, 14, 15, 16, 17)$ | $(0, 1, 2, 3, 4)$ | $(5, 6, 7, 8, 9)$ | $(0, 1, 2, 3, 4)$ | $(3, 4, 5, 6, 7)$ |
| 2   | $(13, 14, 15, 16, 17)$ | $(0, 1, 2, 3, 4)$ | $(3, 4, 6, 7, 8)$ | $(6, 7, 8, 9, 10)$ | $(8, 9, 10, 11, 12)$ |
| 3   | $(11, 12, 13, 14, 15)$ | $(0, 1, 2, 3, 4)$ | $(1, 2, 3, 4, 5)$ | $(2, 3, 4, 5, 6)$ | $(3, 4, 5, 6, 7)$ |
| 4   | $(12, 13, 14, 15, 16)$ | $(3, 4, 5, 6, 7)$ | $(0, 1, 2, 3, 4)$ | $(0, 1, 2, 3, 4)$ | $(4, 5, 6, 7, 8)$ |
| 5   | $(16, 17, 18, 19, 20)$ | $(1, 2, 3, 4, 5)$ | $(3, 4, 5, 6, 7)$ | $(1, 2, 3, 4, 5)$ | $(5, 6, 7, 8, 9)$ |

Table 9: Modified PQF break-down time of FS scheduling problem.

| Job | Machines with PQF processing times | PCF weights of jobs $\bar{w}_i$ |
|-----|-----------------------------------|-------------------------------|
| $i$ | $\bar{P}_1$ | $\bar{M}_1$ | $\bar{Q}_1$ | $\bar{Q}_2$ | $\bar{P}_2$ |
| 1   | $15$ | $2$ | $7$ | $2$ | $5$ |
| 2   | $15$ | $2$ | $5.75$ | $8$ | $10$ |
| 3   | $13$ | $2$ | $3$ | $4$ | $5$ |
| 4   | $14$ | $2$ | $5$ | $2$ | $6$ |
| 5   | $18$ | $3$ | $5$ | $3$ | $7$ |

Table 10: PQF elapsed time of the scheduling problem.

| Job | Machines with PQF processing times | PCF weights of jobs $\bar{w}_i$ |
|-----|-----------------------------------|-------------------------------|
| $i$ | $\bar{P}_1$ | $\bar{M}_1$ | $\bar{Q}_1$ | $\bar{Q}_2$ | $\bar{P}_2$ |
| 2   | $(0, 0, 0, 0)$ to $(13, 14, 15, 16, 17)$ | $(17, 17, 17, 17, 17)$ to $(22, 22, 22, 22, 22)$ to $(30, 30, 30, 30, 30)$ to $(36, 37, 38, 39, 40)$ | $(40, 40, 40, 40, 40)$ to $(48, 49, 50, 51, 52)$ |
| 5   | $(17, 17, 17, 17, 17)$ to $(37, 37, 37, 37, 37)$ to $(39, 39, 39, 39, 39)$ to $(46, 46, 46, 46, 46)$ to $(47, 47, 47, 47, 47)$ to $(52, 53, 54, 55, 56)$ |
| 1   | $(33, 34, 35, 36, 37)$ | $(39, 41, 43, 45, 47)$ | $(42, 43, 44, 45, 46)$ | $(47, 47, 49, 50, 51)$ | $(69, 69, 69, 69, 69)$ |
| 3   | $(53, 54, 55, 56, 57)$ | $(50, 50, 50, 50, 50)$ to $(55, 55, 55, 55, 55)$ to $(64, 64, 64, 64, 64)$ to $(72, 73, 74, 75, 76)$ |
| 4   | $(61, 62, 63, 64, 65)$ to $(65, 65, 65, 65, 65)$ to $(70, 70, 70, 70, 70)$ to $(75, 75, 75, 75, 75)$ to $(81, 81, 81, 81, 81)$ to $(84, 85, 86, 87, 88)$ |
| 100 | $(77, 78, 79, 80, 81)$ to $(88, 88, 88, 88, 88)$ to $(94, 94, 94, 94, 94)$ to $(100, 100, 100, 100, 100)$ to $(105, 106, 107, 108, 109)$ |
Step 8. The total PQF elapsed time is (105, 106, 107, 108, 109), and hence, we have the following results:

- For job 2: \( f_2 = (48, 49, 50, 51, 52) \).
- For job 5: \( f_5 = (42, 43, 44, 45, 46) \).
- For job 1: \( f_1 = (35, 36, 37, 38, 39) \).
- For job 4: \( f_4 = (30, 31, 32, 33, 34) \).

Therefore, the closed interval weighted mean flow is \( ([315, 770]/[8, 19]) = ([315/19], (770/8)) \) hours.

The total PQF elapsed time and the weight flow by the proposed method is less than comparing to the ones obtained by Thangaraj and Rajendran [38]. All the calculations are entertained by MATLAB 2020a under Windows 10. The CPU frequency of the computer is 2.3 GHz, and the memory size is 8 GB.

7. Conclusions

In this research article, a new approach, namely, multistage fuzzy binding method has applied for solving the PQF constrained multistage FS scheduling problems, where the processing times and the jobs weight are characterized by PQF numbers. The advantage of the approach is that there is no risk for the decision maker, it is more applicable for real-world problems, it is easy and simple for understanding, and it is an important tool to the managers who are dealing with the flow-job problems so as to provide a noncrossing optimal sequence. The main findings are particularly useful for a fuzzy FS scheduling problem, while the processing times and the jobs weight are fuzzy parameters. Some practical implications and managerial insights can be drawn from this proposed study, under fuzzy due dates. In industry and business sector, the decision maker can apply to schedule the flow-shop of the machines in the workshop under fuzzy due dates. This would optimize the usages of the machines and hence the revenue of the company. For future research, the proposed problem may be extended by considering the stochastic random variable, for the processing times as well as the jobs weight.

Data Availability

The data used to support the findings of this research are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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References

[1] M. Mastrollilli and O. Svensson, “Hardness of approximating a flow and job shop scheduling problems,” Journal of the ACM, vol. 58, no. 5, pp. 20–32, 2011.
[2] N. B. Vahedi, P. Fatollahi, M. R. Tavakkoli, and R. Ramezanian, “An algorithm for flow shop scheduling problem with consideration of position- based learning effect and multiple availability constraints,” International Journal of Advanced Manufacturing Technology, vol. 73, no. 5–8, pp. 601–611, 2014.
[3] S. Aggarwal, D. Gupta, and S. Sharma, “Bi-criteria three stage fuzzy flow-shop scheduling with transportation time and job block criteria,” International Journal of Applied Operational Research, vol. 2, pp. 41–53, 2010.
[4] S. Patider, N. Kushwah, and A. Yadav, “A novel approach for job shop scheduling using particle swarm optimization,” International Journal of Digital Application and Contemporary Research, vol. 2, pp. 1–4, 2014.
[5] S. Abdullah and M. Abdolrazzaghi-Nezhad, “Fuzzy job-shop scheduling problems: a review,” Information Sciences, vol. 278, pp. 380–407, 2014.
[6] H. Ishibuchi, T. Murata, and K. H. Lee, “Formulation of fuzzy flow shop scheduling problem with fuzzy processing time,” in Proceedings of IEEE 5th International Fuzzy Systems, pp. 199–205, New Orleans, LA, USA, September 1996.
[7] H. Ahonen and A. G. de Alvarenga, “Scheduling flexible flow shop with recirculation and machine sequence-dependent processing times: formulation and solution procedures,” The International Journal of Advanced Manufacturing Technology, vol. 89, no. 1–4, pp. 765–777, 2017.
[8] C. Qu, F. Yanming, Z. Yi, and J. Tan, “Solutions to no-wait flow shop scheduling problem using the flower pollination algorithm based on the hormone modulation mechanism,” Complexity, vol. 2018, Article ID 1973604, 18 pages, 2018.
[9] G. M. Komaki, S. Sheikh, and B. Malakooti, “Flow shop scheduling problems with assembly operations: a review and new trends,” International Journal of Production Research, vol. 57, no. 10, pp. 2926–2955, 2019.
[10] J. Belabid, S. Aqil, and K. Allali, “Solving permutation flow shop scheduling problem with sequence-independent setup time,” Journal of Applied Mathematics, vol. 2020, Article ID 7132469, 11 pages, 2020.
[11] L. A. Zadeh, “Fuzzy sets,” Information and Control, vol. 8, no. 3, pp. 338–353, 1965.
[12] D. Dubois and H. Prade, Fuzzy Sets and Systems; Theory and Applications, Academic Press, Cambridge, MA, USA, 1980.
[13] F. Hnaien, F. Yalaoui, and A. Mhadhbi, “Makespan minimization on a two-machine flowshop with an availability constraint on the first machine,” International Journal of Production Economics, vol. 164, pp. 95–104, 2015.
[14] R.-H. Huang and S.-C. Yu, “Two-stage multiprocessor flow shop scheduling with deteriorating maintenance in cleaner production,” Journal of Cleaner Production, vol. 135, pp. 276–283, 2016.
[15] M. Khatami and S. H. Zegordi, “Coordinate production and maintenance scheduling problem with flexible maintenance time intervals,” Journal of Intelligent Manufacturing, vol. 28, no. 4, pp. 857–867, 2017.
[16] Z. Yang, Z. Ma, and S. Wu, “Optimized flowshop scheduling of multiple production lines for precast production,” Automation in Construction, vol. 72, no. 87, pp. 321–329, 2016.
[17] S. Toumi, B. Jarboui, M. Eddaly, and A. Rebai, “Branch-and-bound algorithm for solving blocking flowshop scheduling problems with makespan criterion,” International Journal of Mathematics in Operational Research, vol. 10, no. 1, pp. 34–48, 2017.
[18] J.-M. Yu, R. Huang, and D.-H. Lee, “Iterative algorithms for batching and scheduling to minimise the total job tardiness in two-stage hybrid flow shops,” International Journal of Production Research, vol. 55, no. 11, pp. 3266–3282, 2017.
[19] O. Shahvari and R. Logendran, “A comparison of two stage-based hybrid algorithms for a batch scheduling problem in hybrid flow shop with learning effect,” International Journal of Production Economics, vol. 195, pp. 227–248, 2018.

[20] J. Deng and L. Wang, “A competitive memetic algorithm for multi-objective distributed permutation flow shop scheduling problem,” Swarm and Evolutionary Computation, vol. 32, pp. 121–131, 2017.

[21] P. Damodaran, A. G. Rao, and S. Mestry, “Particle swarm optimization for scheduling batch processing machines in a permutation flowshop,” The International Journal of Advanced Manufacturing Technology, vol. 64, no. 5–8, pp. 989–1000, 2013.

[22] X. Li and S. Ma, “Multiobjective discrete artificial bee colony algorithm for multiobjective permutation flow shop scheduling problem with sequence dependent setup times,” IEEE Transactions on Engineering Management, vol. 64, no. 2, pp. 149–165, 2017.

[23] I. Chaouch, O. B. Driss, and K. Ghedira, “A modified ant colony optimization algorithm for the distributed job shop scheduling problem,” Procedia Computer Science, vol. 112, pp. 296–305, 2017.

[24] H. A. Khalifa, “On single machine scheduling problem with distinct due dates under fuzzy environment,” International Journal of Supply and Operations Management, vol. 7, no. 3, pp. 272–278, 2020.

[25] E. B. Tirkolaee, A. A. R. Hosseinabadi, M. Soltani, A. K. Sangaiah, and J. Wang, “A hybrid genetic algorithm for multi-trip green capacitated arc routing problem in the scope of urban services,” Sustainability, vol. 10, no. 5, pp. 1–21, 2018.

[26] A. Sioud and C. Gagné, “Enhanced migrating birds optimization algorithm for the permutation flow shop problem with sequence dependent setup times,” European Journal of Operational Research, vol. 264, no. 1, pp. 66–73, 2018.

[27] P. Kumar, “An inventory planning problem for time-varying linear demand and parabolic holding cost with salvage value,” Croatian Operational Research Review, vol. 10, no. 2, pp. 187–199, 2019.

[28] A. Goli, E. Babaee Tirkolaee, and M. Soltani, “A robust just-in-time flow shop scheduling problem with outsourcing option on subcontractors,” Production & Manufacturing Research, vol. 7, no. 1, pp. 294–315, 2019.

[29] E. B. Tirkolaee, A. Goli, A. Faridnia, M. Soltani, and G.-W. Weber, “Multi-objective optimization for the reliable pollution-routing problem with cross-dock selection using pareto-based algorithms,” Journal of Cleaner Production, vol. 276, p. 122927, 2020.

[30] H. A. E.-W. Khalifa and P. Kumar, “Solving fully neutrosophic linear programming problem with application to stock portfolio selection,” Croatian Operational Research Review, vol. 11, no. 2, pp. 165–176, 2020.

[31] E. B. Tirkolaee, A. Alireza Goli, and G. W. Weber, “Fuzzy mathematical programming and self-adaptive artificial fish swarm algorithm for just-in-time energy-aware flow shop scheduling problem with outsourcing option,” IEEE Transactions on Fuzzy Systems, vol. 28, no. 11, pp. 2772–2783, 2020.

[32] A. Goli, E. Babaee Tirkolaee, and N. S. Aydin, “Fuzzy integrated cell formation and production scheduling considering automated guided vehicles and human factors,” IEEE Transactions on Fuzzy Systems, 2021.

[33] P. Pandian and P. Rajendran, “Solving constrained flow-shop scheduling problems with three machines,” International Journal of Contemporary Mathematical Sciences, vol. 5, no. 19, pp. 921–929, 2010.

[34] S. Jain, “Close interval approximation of piecewise quadratic fuzzy numbers for fuzzy fractional program,” Iranian Journal of Operations Research, vol. 2, no. 1, pp. 77–88, 2010.

[35] M. Sakawa and H. Yano, “Interactive decision making for multiobjective nonlinear programming problems with fuzzy parameters,” Fuzzy Sets and Systems, vol. 29, no. 3, pp. 315–326, 1989.

[36] E. R. Moore, Methods and Applications of Interval Analysis, Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 1979.

[37] H. Ishibuchi and H. Tanaka, “Multi objective programming in optimization of the interval objective function,” European Journal of Operational Research, vol. 48, pp. 219–225, 1999.

[38] M. Thangaraj and P. Rajendran, “Solving constrained multi-stage machines flow-shop scheduling problems in fuzzy environment,” International Journal of Applied Engineering Research, vol. 11, no. 1, pp. 521–528, 2016.