On consistency types for lattice-based distributed programming languages

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Abstract
Distributed systems address an increasing demand for fast access to resources and provide fault tolerance for data. However, due to scalability requirements, software developers need to trade consistency for performance. For certain data, consistency guarantees may be weakened if application correctness is unaffected. In contrast, data flows from data with weak consistency to data with strong consistency requirements are problematic, since application correctness may be broken.

In this paper, we propose LatCalculus, a language and type system for distributed programming languages with replicated data types. There are two principal new features: first, the type system statically enforces noninterference between data types with weak consistency and data types with strong consistency; second, static encapsulation enables replication of object graphs with internal aliasing.

Keywords  consistency types, type system, distributed programming

1 Introduction
Distributed systems are popular and applied to many areas. Different applications have different requirements of system consistency, e.g., a bank service needs to ensure that users get globally consistent information wherever they are, while a Twitter-like micro-blogging service just needs to ensure the causal sequence of the messages (“tweets”) among related users. In the same application, consistency requirements may vary in different scenarios, e.g., an online shopping service needs to provide the information for the user to browse as fast as possible, while when the user checks out for payment, the service needs to ensure a safe and consistent transaction.

A lot of consideration is required when a developer fixes bugs or makes optimizations for the current project. Unlike a local application, the verification work for a distributed program involves more expert experience and knowledge. Holt et al. [Holt et al. 2016] show a common programming mistake caused by the misuse of consistency. They point out that a type checking method can be provided in order to avoid similar errors. They introduce the so-called Inconsistent, Performance-bound, Approximate (IPA) storage system which makes consistency properties explicit for distributed data and uses the type system to enforce consistency safety. However, their paper focuses on providing a programming model and typed API; no formal account of the type system is provided.

In this paper, we focus on the theoretical part of designing a type system for consistency types, and provide proofs of correctness properties such as type soundness. Importantly, we build on ideas from language-based information-flow security to enforce an essential noninterference property: in a well-typed program, weakly consistent data cannot flow into strongly consistent data.

This paper makes the following contributions:
• We introduce LCD\textsuperscript{1}, a higher-order static consistency-typed language with references. The language distinguishes different system behaviors between weak consistent and strongly consistent data. In addition, the type system guarantees noninterference, enabling the safe use of both weakly and strongly consistent data within the same program.
• We introduce LCD\textsuperscript{2}, a language that extends LCD\textsuperscript{1} with records and heap separation. By integrating elements of LA/CASA’s type system [Haller and Loiko 2016], namely encapsulated “boxes”, LCD\textsuperscript{2} enables safe propagation of reference graphs from clients to replicated servers under strong consistency.
• We prove type preservation for both LCD\textsuperscript{1} and LCD\textsuperscript{2}.

In addition, for LCD\textsuperscript{1} we provide a proof sketch for a progress theorem, and we state a noninterference theorem.

2 Motivation
A common program error that a distributed application developer might make is to mix the usage of data from different consistency levels. In the following example, we have a simplified shopping platform which is shown in Figure 1a. The \texttt{productName} function returns a consistent value from the storage and the \texttt{checkOut} function compares the number of products that the user ordered and the remaining product then proceed to the payment stage.

Later, the platform needs a real-time display for the user to see the remaining number of products before going to the
In the following work, we introduce a type system that checks the data flow before runtime to guarantee the non-interference between different consistency data.

3 Formalization: LCD

In this section, we introduce our lattice-based calculus for distributed programming (LCD) based on SSL-Ref, a higher-order static security-typed language with references. We make use of its natural property that the type system alone guarantees noninterference, so we can achieve the goal that there is no interference for different consistency types.

3.1 Available types and consistent types

In the distributed system, there are two main concerns: availability and consistency. CAP [Gilbert and Lynch 2002] theorem points out that these two properties can not be achieved at the same time, which becomes the main issue when designing a system. It is often the case that some data requires fast access while the other essential ones need to ensure consensus in the first place. It is natural to think of two types of data that are mainly used for the above two scenarios, so comes to available types and consistent types.

Well-known available types are conflict-free replicated data types [Preguiça et al. 2018] (CRDTs). They use the mathematical convergence property of the data to resolve concurrent conflicts for replicated systems. Cloud types [Burckhardt et al. 2012] are another example which is designed specifically for not-continuous network mobile applications. These data types have one thing in common: they have a quick response to users and we can assume the data can be immediately obtained, but the consistency level is weak and the consistency of the data is not guaranteed.

Consistent types reflex the idea that the distributed system should behave as a single machine. It is different than the local data types such as Int and String, but a more general idea for the system to distinguish that the data needs to be always consistent. In other words, if all the types in the system are defined as consistent types, then the system is strongly consistent and there is no difference between local types and distributed types.

Definition 1 (Partial order on consistency types). The partial order of consistency types is preserved:

\[
\cdot \leq \text{con} \quad \text{con} \leq \text{ava}
\]

3.2 Syntax

Figure 2 shows the syntax of our core language. Label \( \ell \) forms partial order \( \leq \) where \( \bot \) and \( \top \) denote the least and greatest labels. \( \cdot, \text{con} \) and \( \text{ava} \) refer to labels that are attached to local data types, consistent data types, available data types, respectively. Each value and type constructor is annotated
with label \( \ell \) to distinguish different operation behaviors. Function abstraction and arrow types are carrying a latent effect [Heintze and Riecke 1998], which restricts the consistency level of the values that might be written during the execution of the function. The two highlighted terms are not part of the surface syntax but only appear during evaluation. A reference location \( o \) is the evaluation result of a ref expression.

### 3.3 Static semantics

Figure 3 shows the type-and-effect system for LCD\(^1\). The typing judgement \( \Gamma; \Sigma; \ell_c :: t : \tau \) says that term \( t \) has type \( \tau \) under type environment \( \Gamma \), store typing \( \Sigma \) and effect \( \ell_c \). A type environment \( \Gamma \) is a finite function from variables to their types, a store typing \( \Sigma \) describes the store \( \mu \) in which some term \( t \) is evaluated, and a consistency label \( \ell_c \) denotes the consistency level of those references that a given term may allocate or mutate. The consistency effect prevents low-consistency computations from mutating the state of high-consistency data. A LCD\(^1\) source program \( t \) is well-typed if

\[ \vdash \vdash \vdash t : \tau. \]

- Rule (T-Var), (T-Unit), (T-Bool) and (T-Location) type variables, unit values, boolean values, and reference locations. Besides the usual typing rules, the effect checking guarantees the current label \( \ell_c \) is lower than the type labels.
- Rule (T-Lat) types lattice values with effect checking.
- Rule (T-LatOp) types lattice meet and join operations which generate a new lattice value.
- Rule (T-RelOp) types relational operations between two lattice values, yielding a boolean result after partial order comparison.
- Rule (T-Lambda) type checks the function body with the latent label \( \ell' \) and the arrow type has the same label as the function abstraction.
- Rule (T-App) enforces consistency restrictions. The \( \leq \) relationships between types are defined as follows:

\[
\begin{array}{c c c}
T \in \{ \text{Bool}, \text{Unit}, \text{Lat} \} & \ell \leq \ell' & \ell \leq \ell'' \\
T\ell \leq T\ell' & \text{Ref}_\ell \tau \leq \text{Ref}_\ell' \tau
\end{array}
\]

\[
\begin{array}{c c c c c c c}
\ell_1' \leq \ell_1 & \ell_2 \leq \ell_2' & \ell_1 \leq \ell_1' & \ell_2' \leq \ell_2 \\
\ell_1 \rightarrow \ell_1' & \ell_2 \rightarrow \ell_2' & \ell_1 \ell_2 \rightarrow \ell_1' \ell_2'
\end{array}
\]

- Rule (T-If) checks the current type of the predicate and propagate the label to the type checking of each branch statement. In this way, this rule prevents explicit information flow from higher labels to lower labels. The \( \gamma \) between types only works for types which have the same raw type and returns a type which has the joined label. The complete definition can be found in Appendix A.1.

- Rule (T-Ref) requires the reference body to be lattice type when the label is "ava" and guarantees that the label of the reference body is the same as the label of the reference.
- Rule (T-Assign) ensures that the consistency level of the location and current consistency effect lower-bound the assigned value.
- Rule (T-deref) stamps the consistency level of the reference onto the resulting type.

### Difference between LCD\(^1\) and SSL\(_{\text{Ref}}\)

The following paragraph summarizes the difference between LCD\(^1\) and SSL\(_{\text{Ref}}\).

- LCD\(^1\) is using labels to annotate different consistency levels while SSL\(_{\text{Ref}}\) is using annotations for security checks.
- LCD\(^1\) does not support stamping on the types for references, which means the label of types within a reference cannot be upgraded. The reason is that the labels directly reflect the allocation of the data, and the upgrade of the labels needs to be supported by the structure of the distributed system, which is not straightforward enough.

### 3.4 Dynamic semantics

We formalize the dynamic semantics as a small-step operational semantics based on three judgements. \( t_1 | \mu_1 \mid b_1 \rightarrow t_2 | \mu_2 \mid b_2 \) and \( \langle t_1 | \mu_1 \mid b_1 \rangle \cup C | M_1 | S_1 \rightarrow \langle t_2 | \mu_2 \mid b_2 \rangle \cup C | M_2 | S_2 \). The first judgement says a term \( t_1 \), a local store \( \mu_1 \) and a message buffer \( b_1 \) reduce to \( t_2, \mu_2 \) and \( b_2 \), respectively. The second judgement reduces the whole cloud state which contains a sequence of clients, a multimset of messages \( M \) and a sequence of servers.

#### Local reduction

Figure 4 shows basic local reduction rules for LCD\(^1\). The reduction shapes like: \( t | \mu \rightarrow^t t' | \mu' \rightarrow^t \). We use evaluation context here for the reduction relations. Following a standard approach [Felleisen 1988], we write \( E[t] \) to represent the evaluation context \( E \) with term \( t \) in the hole.

\[
E ::= [ ] | [ ] \oplus \nu [ ] | \nu \text{ op } t [ ] | \nu \text{ op } \[
\]

All the expressions transfer the current program label \( \ell_c \) to subterms and there is no additional runtime checking for types and effects required.

Figure 4, 5 show the local reduction rules. \( b \) is used to model the practical network delay in eventual consistent systems. For local operations, \( b \) is not modified since there is no network communication requirement. Rule (E-avaRef) is for creating an available reference. Different from Rule (E-localRef), the generated location is stamped as label ava.
and since it is related with remote server but not require instant creation on the server side, a message shaped like "update(o, v, i, 0)" is stored in the buffer and will be processed later. Rule (E-avaDeref) and rule (E-avaAssign) operate "read" and "join" to available lattice types. Since the available data types guarantee weak consistency, rule (E-avaDeref) get the value directly from the local store and rule (E-avaAssign) first updates the local store and put a message shaped like "update(o, v, i, 0)" to the buffer for later propagation.

**Distributed reduction.** Figure 6 shows the distributed reduction rules which introduce a sequence of clients, a multiset of message $M$ and a set of servers into the judgment to represent a distributed application scenario. Each client contains a tuple $(t | \mu | b)$ and the server contains a store structure similar as $\mu$. Rule (E-Local) reveals the relationship between reduction relation $\rightarrow^t$ to $\rightarrow^t$. Rule (E-consRef) create a consistent reference and the generated location is stamped as label con. Since it is related with remote server and require synchronized creation on the server side, we simplified the semantics by executing the update on both local side and remote side in one step to show the strong consensus of this operation. Similarly, for rule (E-consDeref) and rule (E-consAssign), we update the remote side in one step to express the fact that to get a consistent result from the server side or to assign a consistent data type, all the servers need to synchronize and reach a consensus state before return a value to the client or finishing the consistent operation. However, the consensus algorithm of the server side is not expressed in our semantics since it is designed for
reasoning about source program instead of the underneath architecture.

Message Update\( [o, v, i, R] \) contains the update information \((o, v, i)\) and a set of receivers that are already received the message \((R)\). The following three rules in Figure 7 describe the message process.

- Rule \((E\text{-send})\) moves the message from buffer \(b\) to message set \(M\).
- Rule \((E\text{-process})\) updates a server that does not belong to \(R\) and adds a new message for further propagation. If the message contains a location \(o\) that does not exist on the server, then the server will create a new mapping, otherwise, the value on the same location will be updated.
- Rule \((E\text{-GC})\) removes the messages that are already received by all the servers.

3.5 Well-formed configuration

Figure 8 shows the well-formed configuration. These rules are essential for establishing subject reduction (Section 3.6). Rules \((WF\text{-store1-2})\) define the well typeness of the store with respect to a typeing context \(\Gamma\) and a store typing \(\Sigma\). Rule \((WF\text{-msg})\) specifies well-formedness of messages in \(\Gamma\) and \(\Sigma\). The remaining rules lift well-formedness to client configurations \((WF\text{-clientConfig})\), sets of clients \((WF\text{-client1-2})\), multisets of messages \((WF\text{-messages-emp})\), server configurations \((WF\text{-serverConfig})\), sets of servers \((WF\text{-server1-2})\), and configurations \((WF\text{-config})\), respectively.

3.6 Correctness properties

Type soundness of LCD\(^1\) follows from the following preservation and progress theorems.
\[
t | \mu | \frac{t}{b} \xrightarrow{\ell_c} t' | \mu' | b' \\
\{\langle t | \mu | b \rangle^i \} \cup C | M | S \xrightarrow{\ell_c} \{\langle t' | \mu' | b' \rangle^i \} \cup C | M | S \\
i \in \text{Ids} \quad o = (i, i) \text{ where } i \text{ fresh} \quad o \notin \text{dom}(\mu) \\
\mu' = \mu[\mu \mapsto v \land \ell_c \land \text{con}] \\
S' = \bigcup S' \text{ where } \forall S_r \in S, S_r' = S_r[\mu \mapsto v \land \ell_c \land \text{con}] \\
\{\langle \text{ref}_{\text{con}}; v | \mu | b \rangle^i \} \cup C | M | S \xrightarrow{\ell_c} \{\langle \text{unit}_{\text{con}} | \mu' | b \rangle^i \} \cup C | M | S' \\
S_r \in S \\
S_r(o) = v \\
\mu' = \mu[\mu \mapsto v \land \ell_c \land \text{con}] \\
S' = \bigcup S' \text{ where } \forall S_r \in S, S_r' = S_r[\mu \mapsto v \land \ell_c \land \text{con}] \\
\{\langle \text{ref}_{\text{con}}; v | \mu | b \rangle^i \} \cup C | M | S \xrightarrow{\ell_c} \{\langle \text{unit}_{\text{con}} | \mu' | b \rangle^i \} \cup C | M | S'
\]

**Figure 6. LCD^1:** Distributed reduction.

\[
b = m \cdot b' \\
M' = m \cup M \\
\{\langle t | \mu | b \rangle^i \} \cup C | M | S \xrightarrow{\ell_c} \{\langle t' | \mu' | b' \rangle^i \} \cup C | M' | S \\
M = \{\text{update}[o, v, i, R]\} \cup M'' \\
S = \{S_r\} \cup S'' \\
\ell \notin R \\
\text{if } o \notin S_r \text{ then } S_r' = S_r[o \mapsto v \land \ell_c] \\
\text{else } S_r' = S_r[o \mapsto v \land S_r(o) \land \ell_c] \\
S' = \{S_r'\} \cup S'' \\
M' = \{\text{update}[o, v, i, R \cup \{r\}]\} \cup M'' \\
C | M | S \xrightarrow{\ell_c} C | M' | S' \\
M = \{\text{update}[o, v, i, R \cup M']\} \\
R = \text{ids}(S) \\
C | M | S \xrightarrow{\ell_c} C | M' | S
\]

**Figure 7. LCD^1:** Message processing.

### 3.6.1 Preservation

**Theorem 1** (Preservation). 1. If \(\Gamma; \Sigma; \ell_c \vdash t : \tau, \Sigma \vdash \mu, \) \(\Sigma \vdash b\) and \(t | \mu | b \xrightarrow{\ell_c} t' | \mu' | b',\) then for some \(\Sigma' \supseteq \Sigma, \Gamma; \Sigma'; \ell_c \vdash t' : \tau'\) where \(\tau' \leq \tau, \Sigma' \vdash \mu'\) and \(\Sigma' \vdash b'.\)

2. If \(\Sigma \vdash C | M | S\) and \(C | M | S \xrightarrow{\ell_c} C' | M' | S'\) for some \(\ell_c,\) then \(\Sigma' \vdash C' | M' | S'\) for some \(\Sigma' \supseteq \Sigma.\)

**Proof.** Part 1: by induction on the derivation of \(t | \mu | b \xrightarrow{\ell_c} t' | \mu' | b'.\) Part 2: by induction on the derivation of \(\{\langle C \rangle^i\} \cup C | M | S \xrightarrow{\ell_c} \{\langle C' \rangle^i\} \cup C | M' | S'.\) See Appendix A.2.1 for the complete proof.

### 3.6.2 Progress

In the following lemma, we would like to consider a reduction relation \(\xrightarrow{\rightarrow}\) as the subset of \(\rightarrow\) relation excluding reduction rules (E-Process), so that we only consider the reduction on one single client.

**Lemma 1** (Single-client normalization). Let \(\Sigma \vdash C | M | S\) where \(C = \{\langle t | \mu | b \rangle^i\} \cup C', C' = \{\langle t' | \mu' | b' \rangle^i\} \cup C''.\) Then \(\forall t'_c \leq \ell_c, C | M | S \xrightarrow{\rightarrow} C' | M' | S'\) after a finite number of reduction steps, \(t'\) is a value, and some \(\Sigma' \supseteq \Sigma\) such that \(\Sigma' \vdash C' | M' | S'.\)

**Definition 2** (Eventual delivery). A message \(m\) in configuration \(\Sigma \vdash C | M | S,\) i.e., \(M = M' \cup \{m\},\) is eventually delivered, written \(ED(C, M, S)\) iff
\[ \emptyset \vdash \emptyset \quad \text{(WF-STORE1)} \]
\[ \text{dom}(\mu) = \text{dom}(\Sigma) \quad \text{∀} o \in \text{dom}(\mu) \exists ! \ell_c \in \Sigma : \ell_c \vdash \mu(o) : \tau : \tau = \Sigma(o) \]
\[ \Sigma \vdash \mu \quad \text{(WF-STORE2)} \]
\[ o \in \text{dom}(\Sigma) \quad i \in \text{Ids} \quad \emptyset \vdash d : \text{Lat} \quad \Sigma \vdash S \quad \Sigma \vdash \emptyset \quad \text{(WF-BUFFER-EMP)} \]
\[ \Sigma \vdash \text{update}[o, d, i, S] \quad (WF-MSG) \]
\[ \Sigma \vdash \mu \quad \Sigma \vdash b \quad \exists ! \ell_c, \Gamma : \Sigma : \ell_c \vdash t : \tau \quad \Sigma \vdash \langle t \mid \mu \mid b \rangle^i \quad \text{(WF-CLIENTConfig)} \]
\[ \Sigma \vdash \emptyset \quad \text{(WF-CLIENT1)} \]
\[ \Sigma \vdash o \in \text{dom}(\Sigma) \quad \exists ! \ell_c, \Gamma : \Sigma : \ell_c \vdash s(o) : \tau : \tau = \Sigma(o) \quad \Sigma \vdash (s|^i) \quad \Sigma \vdash S \quad \Sigma \vdash \{ (s|^i) \} \cup S \quad \text{(WF-SERVERConfig)} \]
\[ \Sigma \vdash C \quad \Sigma \vdash M \quad \Sigma \vdash S \quad \Sigma \vdash C \mid M \mid S \quad \text{(WF-CONFIG)} \]

**Figure 8.** Well-formedness

\[ \exists s' \in S' \cdot m = \text{update}[o, d, i, R] \land \Sigma \vdash m \Rightarrow C \mid M \mid \{ m \} \mid S \Downarrow^s C' \mid M' \mid S' \text{ where } S(o) \geq d. \]

**Theorem 2 (Finite progress).** Let \( \Sigma \vdash C \mid M \mid S \) such that \( ED(C, M, S) \), if \( C \mid M \mid S \Downarrow^s C' \mid M' \mid S' \) then \( ED(C', M', S') \).

**Proof.** See Appendix A.2.2. □

### 3.6.3 Noninterference

Noninterference property of the language states an observer with a lower-level label cannot distinguish the higher-level computations in a well-typed program. We use a common method using “logical equivalence” [Tse and Zdancewic 2005] for the proof.

The following corollary expresses an essential noninterference property enforced by LCD\(^1\).

**Theorem 3 (Noninterference).** If \( \Gamma ; \Sigma ; \ell_1 \vdash t : \tau \mid \tau \leq \tau \) and \( \Gamma ; \Sigma \vdash (\ell_1, \rho_1, \mu_1) \approx^k_{\ell_0} (\ell_2, \rho_2, \mu_2) \), then \( (\ell_1, \rho_1(t), \mu_1) \approx^k_{\ell_0} (\ell_2, \rho_2(t), \mu_2) : C(\tau) \) (\( k \geq 0, \Sigma \vdash \mu_i \))

**Proof sketch.** We can prove noninterference by proving that related substitutions preserve the logical equivalence. The definition of the logical equivalence relation \( \approx^k_{\ell} \) is analogous to the definition in [Toro et al. 2018]. □

### 4 Formalization: LCD\(^2\)

LCD\(^1\) prevents unexpected data flow, but it still has some limitations on efficiency. The section introduces LCD\(^2\) which extends LCD\(^1\) with box upgrade. We first explain the restrictions of LCD\(^1\) and then describe LCD\(^2\) as the solution.

#### 4.1 Motivation

Consider the following code:

1. \[ \text{let } x = \text{ref } \text{con} \text{ in} \]
2. \[ \text{let } y = \text{ref } \text{con} \text{ in} \]
3. \[ \text{let } z = \text{ref } \text{con} \text{ in} \]
4. \[ \text{...} \]

Using the semantics in LCD\(^1\), in Line 1 and 2, we generate consistent reference \( x, y \) and \( z \) for generating the reference graph, which requires three times synchronizations among the servers. It is acceptable because the example is rather small. When we have a graph which contains 10, 100, 1000 or even more nodes, the synchronization costs need to be reduced to make the language practical.

With this motivation, we would like to introduce LCD\(^2\) with the optimized upgradation operation box to solve the problem.
4.2 Extended syntax and semantics

**Syntax**
The intention of LCD² is to rewrite the example in Section 4.1 as the following:

1. `let x = ref. 3 in`
2. `let y = ref. x in`
3. `let z = ref. y in`
4. `let a = box_con(z) in`
5. ...

We see that the reference graph generation is made locally and thus a great amount of synchronization requirements is reduced which is particularly important for big graphs.

Figure 9 shows the syntactic extension of LCD² from LCD¹. First, we introduce a standard record type and record expressions, so that the program can be further developed as object-orient style. Then we introduce a box expression `box_lex(t)` for upgrading the type of expression `t` with label `ℓ`.

**Static semantics**
Figure 10 shows the added inference rules for the added typing terms and expressions of LCD². Rule (T-Record) calculates a least upper bound `ℓ` for upgrading the type of expression `t` with label `ℓ`.

**Dynamic semantics**
Evaluation context is extended as follows:

\[ E ::= \ldots | box([]) | [l_i] | i \in \ldots \]

Figure 11 shows the additional local reduction rules of LCD². We extend the configuration in LCD¹ to `⟨t | µ | b | oseq | root⟩`. `oseq` is a location sequence which keeps track of the reference connections, and `root` records the head of the graph. Rule (E-Box) upgrades the label of all the connected terms and makes a copy of the current local graph to the remote side. The highlighted part is where it requires synchronization. In this way, it reduces a lot of communications between the local client and the remote servers.

**Well formedness**
As an extension to LCD¹, here we only need to define the well formedness of location sequence, location root and the updated client configuration.

4.3 Soundness

Type soundness of LCD² follows from the following preservation and progress theorems.

**Theorem 4** (Preservation).
1. If \( Γ; Σ; \ell_c \vdash t : τ, \Sigma \vdash µ \),
   \( \Sigma \vdash b \land t | µ | b \xrightarrow{i} t' | \mu' | b', \) then for some \( \Sigma' \supseteq \Sigma \), \( Γ; Σ'; \ell_c \vdash t' : τ' \) where \( t' \leq τ, \Sigma' \vdash \mu' \) and \( \Sigma' \vdash b' \).
2. If \( Σ \vdash C | M \land S \land C | M | S \Rightarrow i C' | M' | S' \) for some \( \ell_c \), then \( Σ' \vdash C' | M' | S' \) for some \( Σ' \supseteq \Sigma \).

**Proof.** Part 1: Similar as LCD¹. Part 2: By induction on the derivation of \( C | M | S \Rightarrow i C' | M' | S' \). See Appendix B.1.1 for the complete proof.

**Theorem 5** (Finite progress). Let \( Σ; \ell_c \vdash C | M | S \) such that \( ED(C, M, S) \), if \( C | M | S \Rightarrow i C' | M' | S' \) then \( ED(C', M', S') \).

The proof of progress theorem for LCD² is the same as the one in LCD¹.

5 Case study: usage examples

**Replicated data types.** To apply the lattice type in practice, we can simply use Conflict-free Replicated Data Types (CRDTs) as a start. CRDTs are defined in a way that they naturally preserve lattice properties and the state of the data types monotonically grow so that it is suitable for merge operations. In order to make the following example explanation more clear, we here introduce a Positive-Negative Counter (PN-counter), which supports increment and decrement operations but still keeps the internal state increase monotonically.

Grow-only counters which the state of the counter can only increase are the basic unit of LCD. There are two grow-only counters in PN-counter, one counts increments and one counts decrements. In the following example, we refer the lattice type as a PN-counter.
Data assignment. The first example is about data assignment. Recall the shopping platform implementation in Figure 1, we now use the following functional language style to show the usage of LCD. The original implementation in Figure 1a can be simplified as follows. In order to process a valid shopping, checkoutValidity compare the consistent storage and the user order number and returns a boolean value. reduceConNumTo then reassign the current storage.

```
let applestorage = @con ref @con 70 in
let customToBuy = (id1 = ref 30, id2 = ref 50) in
let realtimeDisplay = applestorage in
let reduceConNumTo =
  @con @con lam x: @con int.@con (applestorage := x) in
let checkoutValidity =
  @con @con lam x: int. x < (!applestorage) in
if (checkoutValidity !(customToBuy.id1))
  then reduceConNumTo @con 40
else @con unit
```

Information flow. Information flow is a very interesting study case here and we want to show that by using the type system, we can prevent the influence from the available data to the consistent data.

```
let x = @ava true in
let y = @con ref @con true in
if x then y := @con ref @con false
else @ava unit
```

The original code is straightforward is well-typed but when another programmer adds the realtimeDisplay function and changed the label for applestorage to ava for achieving high performance, in LCD this cannot be type checked. Thus, the problem of assigning an available value to a consistent value is prevented.
as con. In the control flow in Line 3, although the assignment for y is well-typed, since the label of x is ava, the program is still not well-typed and this prevents the influence from an ava labeled value to a operation labeled con.

1 let x = con {brand = con "BMW", model = con "s3"} in
2 let y = con {name = con "Alice", age = con "31" } in
3 let z = 
4 con {name = con "engineer", salary = con "22000"} in
5 let a = con {person = con ref con y, 
6 car = con ref con x, job = con ref con z} in
7 let b = con ref con a in
8 (!b).job :=
9 con {name = con "manager", salary = con "25000"}

Using box, we could reduce the data synchronization time and also simplify the code construction.

1 let x = (brand = "BMW", model = "s3") in
2 let y = {name = "Alice", age = "31" } in
3 let z = {name = "engineer", salary = "22000"} in
4 let a = {person = ref y, car = ref x, job = ref z} in
5 let b = con box (ref a) in
6 let z1 = {name = "manager", salary = "25000"} in
7 (!b).job := con box (z1)

6 Related Work

Consistency models. According to CAP theorem [Gilbert and Lynch 2002], consistency, availability and fault tolerance cannot be achieved at the same time for any distributed system. Therefore, there exist multiple consistency models such as eventual consistency [Burckhardt et al. 2013], causal consistency [Lamport 1978], read-after-write consistency [Lu et al. 2015], and sequential consistency [Lamport 1979]. They define different contracts between the system and the user. It is also common that a system provides a mixed level of consistency such as fork consistency [Li et al. 2004] [Mazieres and Shasha 2002], lazy replication [Ladin et al. 1992], and red-blue consistency [Li et al. 2012].

Consistency types. The concept of consistency types first appears in paper [Holt et al. 2016]. The idea is to keep track of the consistency level of the data and prevent unexpected information flow from an inconsistent object to a consistent object. ConSysT [Margara and Salvaneschi 2017] is another programming language that supports heterogeneous consistency specifications at the type level. However, the work is still incomplete: the authors only present the syntax of a core calculus without references, as well as subtyping rules. Neither dynamic semantics nor correctness properties are given.

Types for distributed systems. Conflict-free replicated data types (CRDTs) [Shapiro et al. 2011a] [Shapiro et al. 2011b] are designed to allow monotonically updating, and they are suitable for achieving eventual consistency. Cloud types [Burckhardt et al. 2012] are more general compared with CRDTs. They preserve availability by "read my own writes" and achieve eventual consistency using global sequence protocol whenever the network is available.

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A Appendix

A.1 Auxiliary functions

\begin{align*}
\text{label}(\tau) &= \ell \text{ if } \tau = \text{Bool}_\ell \lor \text{Unit}_\ell \lor \text{Lat}_\ell \lor \text{Ref}_\ell \tau' \lor \tau_1 \rightarrow_\ell \tau_2 \\
\text{containsRef}(\tau) &= \begin{cases} 
\text{true} & \text{if } \tau = \text{Ref}_\ell \tau' \lor \tau = \{i \in \text{for some } i \} \\
\text{false otherwise} & 
\end{cases}
\end{align*}

A.2 Security soundness Proof for LCD

A.2.1 Proof of Preservation theorem A.2.1

Lemma 2 (Substitution). If $\Gamma, x : \tau_1; \Sigma; \ell_c \vdash t : \tau$ and $\Gamma; \Sigma; \ell_c \vdash \nu : \tau_1'$ such that $\tau_1' \leq \tau_1$, then $\Gamma; \Sigma; \ell_c \vdash [\nu/x]t : \tau'$ such that $\tau' \leq \tau$.

Proof. By induction on the derivation of $\Gamma, x : \tau_1; \Sigma; \ell_c \vdash t : \tau$. \hfill \square

Theorem (Preservation). 1. If $\Gamma; \Sigma; \ell_c \vdash t : \tau, \Sigma \vdash \mu, \Sigma \vdash b$ and $t | \mu | b \rightarrow_\ell \tau' | \mu' | b'$, then for some $\Sigma' \supseteq \Sigma, \Gamma; \Sigma'; \ell_c \vdash t' : \tau'$ where $\tau' \leq \tau, \Sigma' \vdash \mu'$ and $\Sigma' \vdash b'$.

2. If $\Sigma \vdash C | M | S$ and $\Sigma \vdash C' | M' | S'$ for some $\ell_c$, then $\Sigma \vdash C' | M' | S'$ for some $\Sigma' \supseteq \Sigma$.

Proof. Part 1: by induction on the derivation of $t | \mu | b \rightarrow_\ell \tau' | \mu' | b'$ with case analysis of the last applied rule.

- Case (E-LatOp) $t = d_{1\ell_1} \oplus d_{2\ell_2}$.

1. By the assumptions,
   a. $\Gamma; \Sigma; \ell_c \vdash t : \tau$
   b. $\Sigma \vdash \mu$
   c. $\Sigma \vdash b$

2. $\Gamma; \Sigma; \ell_c \vdash d_{1\ell_1} : \text{Lat}_{\ell_1}$ (T-Lat) $\Gamma; \Sigma; \ell_c \vdash d_{2\ell_2} : \text{Lat}_{\ell_2}$ (T-Lat)

   $\Gamma; \Sigma; \ell_c \vdash d_{1\ell_1} \oplus d_{2\ell_2} : \text{Bool}_{\ell_1 \lor \ell_2}$ (T-LatOp)

3. $d = d_{1\ell_1} \oplus d_{2\ell_2} | \mu | b \rightarrow_\ell d_{1\ell_1} | \mu | b$ (E-LatOp)

4. By 3., T-LatOp, $\Gamma; \Sigma; \ell_c \vdash d_{1\ell_1} \oplus d_{2\ell_2} : \text{Bool}_{\ell_1 \lor \ell_2}$

- Case (E-RelOp) follows analogously.

- Case (E-Lambda) $t = (\lambda^\ell_{\ell'}x : \tau_11. \ t')_\ell \nu$.

1. By the assumptions
   a. $\Gamma, x : \tau_11; \Sigma; \ell_c \vdash t : \tau$
   b. $\Sigma \vdash \mu$
   c. $\Sigma \vdash b$

2. $\Gamma; \Sigma; \ell_c \vdash \nu : \tau_2$

   $\Gamma; \Sigma; \ell_c \vdash (\lambda^\ell_{\ell'}x : \tau_11. \ t')_{\ell : \tau_11 \rightarrow_\ell \tau_12} \ell \leq \ell' \ \tau_2 \leq \tau_1$ (T-App)
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3.

\( t' = [v/x]t \) \hspace{1cm} (E-Lambda)

\( (\lambda^\ell x : \tau.t)_{\ell} v | \mu | b \rightarrow^i t' | \mu | b \)

4. By 3., \( \tau'_{\ell v} \uparrow \ell \leq \tau_{12} \uparrow \ell \), the result holds.
   - Case (E-Iftrue) \( t = \text{if } t_1 \text{ then } t_2 \text{ else } t_2 \).

1. By the assumptions
   a. \( \Gamma; \Sigma; \ell \vdash t : \tau \)
   b. \( \Sigma \vdash \mu \)
   c. \( \Sigma \vdash b \)

2. \( \Gamma; \Sigma; \ell \vdash \text{true} : \text{Bool} \) \hspace{1cm} (T-Bool)
   \( \Gamma; \Sigma; \ell \vdash \text{if } t_1 \text{ then } t_2 \text{ else } t_2 : (\tau_1 \uparrow \tau_2) \uparrow \ell \) \hspace{1cm} (T-If)

   \( t' = \begin{cases} 
   t_1 \text{ if } t = \text{true} \\
   t_2 \text{ if } t = \text{false}
   \end{cases} \)

3. if \( t_\ell \) then \( t_1 \text{ else } t_2 \) \( | \mu | b \rightarrow^i t' | \mu | b \) \hspace{1cm} (E-If)

4. By 3., and \( \tau_1 \uparrow \ell \leq (\tau_1 \uparrow \tau_2) \uparrow \ell \), the result holds.
   - Case (E-Iffalse) follows analogously.
   - Case (E-LocalRef) \( t = \text{ref}^\tau v \).

1. By the assumptions
   a. \( \Gamma; \Sigma; \ell \vdash t : \tau \)
   b. \( \Sigma \vdash \mu \)
   c. \( \Sigma \vdash b \)

2. \( \Gamma; \Sigma; \ell \vdash \text{ref}^\tau v : \text{Ref}^\tau \) \hspace{1cm} (T-Ref)
   \( \tau' \leq \tau \quad \ell_c \leq \text{label}(\tau) \leq \cdot \)

3. Define \( \Sigma' = \Sigma, o : \tau \).

4. By 3., \( \text{dom}(\mu') = \text{dom}(\Sigma') \).

5. By 5., \( \Gamma; \Sigma' \vdash \mu' \)

6. By 6., \( \Sigma' (o) = \tau \)

7. \( \Sigma'(o) = \tau \) \hspace{1cm} (T-Location)

8. By 7., the result holds.
   - Case (E-avaDeref) \( t = o \).

1. By the assumptions
   a. \( \Gamma; \Sigma; \ell \vdash t : \tau \)
   b. \( \Sigma \vdash \mu \)
   c. \( \Sigma \vdash b \)

2. \( \Gamma; \Sigma; \ell \vdash \text{Ref}_\ell o : \text{Ref}_\ell \tau \) \hspace{1cm} (T-Deref)

3. \( \mu(o) = v \) \hspace{1cm} (E-localDeref)

4. By WF-store, \( \Gamma; \Sigma; \bullet \vdash \mu(o) : \tau' \text{ and } \tau' \leq \tau \).

5. By \( \tau' \uparrow \ell \leq \tau \uparrow \ell \), the result holds.
   - Case (E-avaDeref) follows analogously.
- Case (E-LocalAssign) $t = a \coloneqq v$.

1. By the assumptions
   a. $\Gamma; \Sigma; \ell_c \vdash t : \tau$
   b. $\Sigma \vdash \mu$
   c. $\Sigma \vdash b$

2. $\Gamma; \Sigma; \ell_c \vdash a_c : \text{Ref}_\tau$  
   \hspace{1cm} (T-Location)  
   $\Gamma; \Sigma; \ell_c \vdash \nu : \tau_2 \quad \tau_2 \leq \tau \quad \ell_c \vdash \nu \leq \text{label}(\tau)$  
   \hspace{1cm} (T-Assign)

\[
\begin{align*}
\mu' &= \mu[o \mapsto \nu \vdash \ell_c \vdash \cdot] \\
\ell_c' &= \ell_c \\
\end{align*}
\]

(E-LocalAssign)

3. $\Gamma; \Sigma \vdash b \mapsto \ell_c \vdash \mu' \vdash b$

- Case (E-avaRef) $t = \text{ref}_\tau^a v$.

1. By the assumptions
   a. $\text{dom}(\mu') = \text{dom}(\Sigma)$
   b. $\Gamma; \Sigma; \ell_c \vdash t : \tau$
   c. $\Sigma \vdash \mu$
   d. $\Sigma \vdash b$

2. $\Gamma; \Sigma; \ell_c \vdash \nu : \tau' \quad \tau' \leq \tau \quad \ell_c \vdash \text{label}(\tau) \leq \text{ava}$  
   \hspace{1cm} (T-Ref)

\[
i \in \text{lds} \quad \rho = (i, i) \text{ where } i \text{ fresh} \quad \mu' = \mu[o \mapsto \nu \vdash \ell_c \vdash \cdot] \quad b' = b \cdot \text{update}[o, \bot, i, 0]
\]

(E-AvaRef)

4. Define $\Sigma' = \Sigma, o : \tau$.

5. By 3., 4., $\text{dom}(\mu') = \text{dom}(\Sigma')$.

6. By 3., WF-Msg, WF-Buffer, $\Gamma; \Sigma \vdash b'$.

7. By 6., $\Sigma' \vdash \rho = (i, i)$.  
   \hspace{1cm} (T-Location)

8. By 7., the result holds.

- Case (E-avaAssign) $t = o_{ava} := v$.

1. By the assumptions
   a. $\Gamma; \Sigma; \ell_c \vdash t : \tau$
   b. $\Sigma \vdash \mu$
   c. $\Sigma \vdash b$

2. $\Gamma; \Sigma; \ell_c \vdash \text{Ref}_\tau^a$  
   \hspace{1cm} (T-Location)  
   $\Gamma; \Sigma; \ell_c \vdash \nu : \tau_2 \quad \tau_2 \leq \tau \quad \ell_c \vdash \text{ava} \leq \text{label}(\tau)$  
   \hspace{1cm} (T-Assign)

\[
\begin{align*}
w &= \mu(o) \\
\mu' &= \mu[o \mapsto (w \vdash \nu \vdash \ell_c \vdash \cdot) \vdash \cdot] \\
\end{align*}
\]

(E-AvaAssign)

3. $\Gamma; \Sigma \vdash b \mapsto \ell_c \vdash \mu' \vdash b$

4. By 3., WF-store,  
   a. $\text{dom}(\mu') = \text{dom}(\Sigma)$
b. \( \Gamma; \Sigma; \ell_c \vdash v : t_2 \) where \( t_2 \leq \tau \)
5. By 4., \( \Gamma; \Sigma; \ell_c \vdash v \; \forall \; \ell_r \; \forall \; \ell : t_2 \; \forall \; \ell_r \; \forall \; \ell \)
6. By 5., \( \ell_r \; \forall \; \ell \leq \ell_c \; \forall \; \ell \leq \text{label}(\tau) \)
7. By 6.,
   a. \( t_2 \; \forall \; \ell_r \; \forall \; \ell \leq \tau \)
   b. \( \Gamma; \Sigma; \vdash \mu' \)
8. By 3., WF-Msg, WF-Buffer, \( \Gamma; \Sigma; b' \).
9. By 8., E-Unit, the result holds.

Part 2: by induction on the derivation of \( \{(t \mid \mu \mid b)^i\} \cup C \mid M \mid S \xrightarrow{\ell_c} \{(t' \mid \mu' \mid b')^i\} \cup C \mid M \mid S \) with case analysis of the last applied rule.

- Case (E-Local)
  1. By the assumptions,
     a. \( \Sigma \vdash C \mid M \mid S \)
     b. \( C \mid M \mid S \xrightarrow{\ell_r} C' \mid M' \mid S' \) for some \( \ell_c \)
 2. 
   \[
   \{(t \mid \mu \mid b)^i\} \cup C' \mid M \mid S \xrightarrow{\ell_r} \{(t' \mid \mu' \mid b')^i\} \cup C'' \mid M \mid S
   \]
   \hspace{0.5cm} \text{(E-LOCAL)}

3. By 1.a) and WF-Config, \( \Gamma; \Sigma \vdash C \)
4. By 2., WF-Client2
   a. \( \Sigma \vdash \{t \mid \mu \mid b\}^i \)
   b. \( \Sigma \vdash C'' \)
   c. \( \Sigma \vdash C' \)
5. By 4.a), WF-ClientConfig
   a. \( \Sigma \vdash \mu \)
   b. \( \Sigma \vdash b \)
   c. \( \Gamma; \Sigma \vdash t : \tau \) for some \( \Gamma \)
6. By 2., 5., part 1
   a. \( \Gamma; \Sigma' \vdash t' : \tau \)
   b. \( \Gamma; \Sigma' \vdash \mu' \) for some \( \Sigma' \supseteq \Sigma \)
   c. \( \Gamma; \Sigma' \vdash b' \)
7. By 6., WF-ClientConfig, \( \Sigma' \vdash \{t' \mid \mu' \mid b'\}^i \)
8. By 4.c), 6., \( \Sigma'; \ell_c \vdash C' \mid M' \mid S' \).

- Case (E-consRef) \( t = \text{ref}_{\text{cons}}^\tau v \).
  1. By the assumptions
     a. \( \Sigma \vdash C \mid M \mid S \)
     b. \( C \mid M \mid S \xrightarrow{\ell_c} C' \mid M' \mid S' \) for some \( \ell_c \)
     \[
     i \in \text{Ids} \quad o = (i, i) \quad \text{where} \; t \text{ fresh} \quad o \notin \text{dom}(\mu) \]
     \[
     \mu' = \mu[o \mapsto v \; \forall \; \ell_c \; \forall \; \text{con}] \\
     S' = \cup S'_i \quad \text{where} \; \forall S_r \in S, S'_r = S_r[o \mapsto v] \]
 2. 
   \[
   \{(\text{ref}_{\text{cons}}^\tau v \mid \mu \mid b)^i\} \cup C'' \mid M \mid S \xrightarrow{\ell_r} \{(\text{ref}_{\text{cons}}^\tau v \mid \mu' \mid b')^i\} \cup C'' \mid M \mid S'
   \]
   \hspace{0.5cm} \text{(E-CONSRef)}

3. By 1.a) and WF-Config, \( \Gamma; \Sigma \vdash C \)
4. By 2., WF-Client2
   a. \( \Sigma \vdash \{t \mid \mu \mid b\}^i \)
   b. \( \Sigma \vdash C'' \)
   c. \( \Sigma \vdash C' \)
5. By 2., 4.a), WF-ClientConfig
   a. \( \Sigma \vdash \mu \)
   b. \( \Sigma \vdash b \)
   c. \( \Gamma; \Sigma \vdash t : \tau \) for some \( \Gamma \)
6. By 2., 5., part 1
   a. \( \Gamma; \Sigma' \vdash t' : \tau \)
   b. \( \Gamma; \Sigma' \vdash \mu' \) for some \( \Sigma' \supseteq \Sigma \)
   c. \( \Gamma; \Sigma' \vdash b' \)
7. By 2., WF-ServerConfig, \( \Sigma' \vdash S' \)
8. By 4.c),7., \( \ell_c \vdash C' | M | S' \)
   - Case (E-consDeref), (E-consAssign), (E-send), (E-process) follow analogously.

\[ \square \]

A.2.2 Proof of Theorem 2

Lemma 3 (Eventual population). Let \( \Sigma; \ell_c \vdash C | M | S \) and \( C = \{ (t | \mu | b)^{i} \} \cup \hat{C} \). Then \( \forall i \in Ids(C) : \{ (t | \mu | b \cdot m)^{i} \} \cup C' | M | S \rightarrow^{*} \{ (t | \mu | b)^{i} \} \cup C' | M \cup \{ m \} | S \) after a finite number of reduction steps.

Proof. Directly from applying rule (E-send).

\[ \square \]

Lemma 4 (Eventual apply). Let \( \Sigma; \ell_c \vdash C | M | S \) such that ED\( (C,M,S) \). If \( M = \{ \text{update} (o_{\text{ava}}, v, i, R) \} \cup M'' \) then \( C | M | S \rightarrow^{\ell_c} C | M' | S' \) and ED\( (C,M',S') \)

Proof Sketch. By rule (E-process)

1. \( C | M | S \rightarrow^{\ell_c} C | M' | S' \)
2. \( M = \{ \text{update} (o_{\text{ava}}, v, i, R) \} \cup M'' \)
3. \( S = \{ S_{\ell} \} \cup S'' \)
4. \( r \notin R \)
5. \( S' = S_{\ell} [ o \mapsto v \land \ell_{c} ] \)
6. \( S'' = \{ S'_{\ell} \} \cup S'' \)
7. \( M' = \{ \text{update} (o, v, i, R \cup \{ r \}) \} \cup M'' \)

- By 1.6), 1.7), we analyse the property of \( C | M' | S' \)

1. Case 1: \( M' = \{ \text{update} (o, v, i, R \cup \{ r \}) \} \cup M'' \) and \( R \cup \{ r \} = ids(S) \), by rule E-GC, the message is removed from the message set and all the servers receive message \( m \), which means ED\( (C,M',S') \).
2. Case 2: \( M' = \{ \text{update} (o, v, i, R \cup \{ r \}) \} \cup M'' \) and \( \exists n \notin R \cup \{ r \} \), by rule E-process, we can derive that
   a. \( S' = \{ S_{n} \} \cup S'' \)
   b. \( S'_{n} = S_{n} [ o \mapsto v \land \ell_{c} ] \)
   c. \( S_{\text{new}} = \{ S'_{n} \} \cup S'' \)
   Which shows ED\( (C,M',S') \) holds.

\[ \square \]

Theorem (Finite progress). Let \( \Sigma \vdash C | M | S \) such that ED\( (C,M,S) \) if \( C | M | S \rightarrow C' | M' | S' \) then ED\( (C',M',S') \).

Proof. Corollary of Lemmas 3 and 4.

\[ \square \]

B Appendix

B.1 Soundness Proof for LCD\(^{2}\)

B.1.1 Proof of Preservation theorem A.2.1

Theorem (Preservation). 1. If \( \Gamma; \Sigma; \ell_c \vdash t : \tau, \Sigma \vdash \mu, \Sigma \vdash b \) and \( t | \mu | b \rightarrow^{\ell_c} t' | \mu' | b' \), then for some \( \Sigma' \supseteq \Sigma, \Gamma; \Sigma'; \ell_c \vdash t' : \tau' \) where \( \tau' \subseteq \tau, \Sigma' \supseteq \mu' \) and \( \Sigma' \vdash b' \).
2. If \( \Sigma \vdash C | M | S \) and \( C | M | S \Rightarrow^{i} C' | M' | S' \) for some \( \ell_c \), then \( \Sigma' \vdash C' | M' | S' \) for some \( \Sigma' \supseteq \Sigma \).

Proof. By induction on the derivation of \( C | M | S \Rightarrow^{i} C' | M' | S' \) with case analysis of the last applied rule. As the extension of LCD\(^{1}\), here we only consider the additional rules.

- Case (T-LCD1)
  1. By the assumptions,
     a. \( \Sigma \vdash C | M | S \)
b. $C | M | S \Rightarrow^i C' | M' | S'$ for some $\ell_c$

$$\{ (t | \mu | b)^j \} \cup C | M | S \Rightarrow \{ (t' | \mu' | b')^j \} \cup C | M' | S'$$

(E-LCD1)

3. By 1.a) and WF-Config, $\Sigma \vdash C$.

4. By 2., WF-Client2,
   a. $\Sigma \vdash (t | \mu | b | oseq | root)$
   b. $\Sigma \vdash C''$
   c. $\Sigma \vdash C'$

5. By 4.a), WF-ClientConfig-LCD2,
   a. $\Sigma \vdash \mu$
   b. $\Sigma \vdash b$
   c. $\exists t: \tau; \Sigma \vdash t$
   d. $\Sigma \vdash oseq$
   e. $\Sigma \vdash root$

6. By 2., 5., preservation theorem for LCD$^i$,
   a. $\Sigma' \vdash t' : \tau$
   b. $\Sigma' \vdash \mu'$ for some $\Sigma \subset \Sigma'$
   c. $\Sigma' \vdash b'$

7. By 5., 6., WF-ClientConfig-LCD2, $\Sigma' \vdash (t' | \mu' | b' | oseq | root)^i$

8. 4.c) By 6., 7., $\Sigma' \vdash C'' | M' | S'$.

- Case (T-Box) $t = box_{con}(v)$

1. By the assumptions
   a. $\Gamma; \ell_c \vdash C | M | S$

2. $\forall \ell_r$ such that $\ell_r \leq \ell_c$, $C | M | S \Rightarrow^i C' | M' | S'$

   $$\begin{cases} 
   \text{if isLocation}(v) \text{ then } & v' = v \text{ } \forall \text{ con} \\
   \text{else } & v' = box_{con}(root) \\
   \end{cases} \forall S_r \in S, S'_r = S_r[\text{Graph}(oseq)] \mu' = \mu[\text{Graph}(oseq)]$$

   $$\forall S_r \in S, S'_r = S_r[\text{Graph}(oseq)] \mu' = \mu[\text{Graph}(oseq)]$$

   $$\{ (\text{box}_{con}(| \mu | b | oseq | root)^i) \} \cup C'' | M | S \Rightarrow^i \{ (v' | \mu' | b' | oseq | root')^i \} \cup C'' | M | S'$$

(E-Box)

3. By 2., WF-Client2-LCD2
   a. $\Sigma \vdash (t | \mu | b | oseq | root)^i$
   b. $\Sigma \vdash C''$
   c. $\Sigma \vdash C'$

4. By 2., 3.a), WF-clientConfig-LCD2,
   a. $\Sigma \vdash \mu$
   b. $\Sigma \vdash b$
   c. $\Sigma \vdash oseq$
   d. $\Sigma \vdash root$
   e. $\Gamma; \Sigma \vdash t : \tau$ for some $\Gamma$

5. By 2., 4., LCD1 preservation theorem
   a. $\Gamma; \Sigma' \vdash t' : \tau$
   b. $\Gamma; \Sigma' \vdash \mu'$ for some $\Sigma' \Gamma \Sigma$
   c. $\Gamma; \Sigma' \vdash b'$

6. By 5., WF-ClientConfig-LCD2, $\Sigma' \vdash (t' | \mu' | b' | oseq | root)^i$
7. By 3., WF-ServerConfig, $\Sigma' \vdash S'$
8. By 3.c),7., $\Sigma'; \ell_c \vdash C' \mid M' \mid S'$.