Certain class of higher-dimensional simplicial complexes and universal C*-algebras

Saleh Omran

Abstract
In this article we introduce a universal C*-algebra associated to certain simplicial flag complexes. We denote it by $C^\sim_\Gamma^n$, it is a subalgebra of the noncommutative $n$-sphere which introduced by J.Cuntz. We present a technical lemma to determine the quotient of the skeleton filtration of a general universal C*-algebra associated to a simplicial flag complex. We examine the K-theory of this algebra. Moreover we prove that any such algebra divided by the ideal $I_2$ is commutative.

2000 AMS: 19 K 46

Keywords: Simplicial complexes; K-theory of C*-algebras; Universal C*-algebras

Introduction
In this section, we give a survey of some basic definitions and properties of the universal C*-algebra associated to a certain flag complex which we will use in the sequel. Such algebras in general was introduced first by Cuntz (2002) and studied by Omran (2005, 2013).

Definition 1. A simplicial complex $\Sigma$ consists of a set of vertices $V_\Sigma$ and a set of non-empty subsets of $V_\Sigma$, the simplexes in $\Sigma$, such that:

- If $s \in V_\Sigma$, then $\{s\} \in \Sigma$.
- If $F \subseteq \Sigma$ and $\emptyset \neq E \subseteq F$ then $E \in \Sigma$.

A simplicial complex $\Sigma$ is called flag or full, if it is determined by its 1-simplexes in the sense that $\{s_0, \ldots, s_n\} \in \Sigma \iff \{s_i, s_j\} \in \Sigma$ for all $0 \leq i < j \leq n$.

$\Sigma$ is called locally finite if every vertex of $\Sigma$ is contained in only finitely many simplexes of $\Sigma$, and finite-dimensional (of dimension $\leq n$) if it contains no simplexes with more than $n + 1$-vertices. For a simplicial complex $\Sigma$ one can define the topological space $|\Sigma|$ associated to this complex. It is called the “geometric realization” of the complex and can be defined as the space of maps $f : V_\Sigma \to [0,1]$ such that $\sum_{s \in V_\Sigma} f(s) = 1$ and $f(s_0) \cdots f(s_t) = 0$ whenever $\{s_0, \ldots, s_t\} \notin \Sigma$. If $\Sigma$ is locally finite, then $|\Sigma|$ is locally compact.

Let $\Sigma$ be a locally finite flag simplicial complex. Denote by $V_\Sigma$ the set of its vertices. Define $C_\Sigma$ as the universal C*-algebra with positive generators $h_s, s \in V$, satisfying the relations

$$h_{s_0}h_{s_1} \cdots h_{s_n} = 0 \text{ whenever } \{s_0, s_1, \ldots, s_n\} \notin V_\Sigma,$$

$$\sum_{s \in V_\Sigma} h_s^2 = h_s^2 \quad \forall t \in V_\Sigma.$$

Here the sum is finite, because $\Sigma$ is locally finite. $C_{\Sigma}^{ab}$ is the abelian version of the universal C*-algebra above, i.e. satisfying in addition $h_s^2 = h_s$ for all $s, t \in V_\Sigma$. Denote by $I_k$ the ideal in $C_\Sigma$ generated by products containing at least $n+1$ different generators. The filtration (of $I_k$) of $C_\Sigma$ is called the skeleton filtration.

Let

$$\Delta := \left\{ (s_0, \ldots, s_n) \in \mathbb{R}^{n+1} \mid 0 \leq s_i \leq 1, \sum_{i=1}^{n} s_i = 1 \right\}$$

be the standard $n$-simplex. Denote by $C_\Delta$ the associated universal C*-algebra with generators $h_s, s \in \{s_0, \ldots, s_n\}$, such that $h_i^2 \geq 0$ and $\sum_i h_i = 1$. Denote by $I_\Delta$ the ideal in $C_\Delta$ generated by products of generators containing all the $h_s, \; s = 0, \ldots, n$. For each $k$, denote by $I_k$ the ideal in $C_\Delta$ generated by all products of generators $h_s$ containing at least $k+1$ pairwise different generators. We also denote by $I_k^{ab}$ the image of $I_k$ in $C_\Delta^{ab}$. The algebra $C_\Delta$ and their
K-Theory was studied in details in (Omran and Gouda 2012). For any vertex \( t \) in \( \Delta \) there is a natural evaluation map \( \mathcal{C}_\Delta \rightarrow \mathbb{C} \) mapping the generators \( h_t \) to 1 and all the other generators to 0. The following propositions are due to Cuntz (2002).

**Proposition 1.** (i) The evaluation map \( \mathcal{C}_\Delta \rightarrow \mathbb{C} \) defined above induces an isomorphism in K-theory. (ii) The surjective map \( \mathcal{I}_\Delta \rightarrow \mathcal{I}^b_\Delta \) induces an isomorphism in K-theory, where \( \mathcal{I}^b_\Delta \) is the abelianization of \( \mathcal{I}_\Delta \).

We can observe that \( I_k \) is the kernel of the evaluation map which define above so we can conclude that \( I_k \) is closed.

**Remark 1.** Let \( \Delta \) and \( \mathcal{I}_\Delta \subset \mathcal{C}_\Delta \) as above. Then \( K_n(\mathcal{I}_\Delta) \cong K_n(\mathbb{C}) \), \( *, = 0, 1 \), if the dimension \( n \) of \( \Delta \) is even and \( K_n(\mathcal{I}_\Delta) \cong K_n(\mathbb{C}(0,1)) \), \( *, = 0, 1 \), if the dimension \( n \) of \( \Delta \) is odd.

**Proposition 2.** Let \( \Sigma \) be a locally finite simplicial complex. Then \( \mathcal{C}^b_{\Sigma} \) is isomorphic to \( \mathcal{C}_0(|\Sigma|) \), the algebra of continuous functions vanishing at infinity on the geometric realization \( |\Sigma| \) of \( \Sigma \).

**Universal \( \mathcal{C}^* \)-algebras associated to certain complexes**

Universal \( \mathcal{C}^* \)-algebras is a \( \mathcal{C}^* \)-algebras generated by generators and relations. Many \( \mathcal{C}^* \)-algebras can be constructed in the form of universal \( \mathcal{C}^* \)-algebras an important example for universal \( \mathcal{C}^* \)-algebras is Cuntz algebras \( O_n \) the existence of this algebras and their K-theory was introduced by Cuntz (1981, 1984) more examples of universal \( \mathcal{C}^* \)-algebras can be found in (Cuntz 1993; Davidson 1996).

In the following, we introduce a general technical lemma to compute the quotient of the skeleton filtration for a general algebra associated to simplicial complex.

For a subset \( W \subset V_{\Sigma} \), let \( \Gamma \subset \Sigma \) be the subcomplex generated by \( W \) and let \( \mathcal{I}_{\Gamma} \) be the ideal in \( \mathcal{C}_\Gamma \) generated by products containing all generators of \( \mathcal{C}_\Gamma \).

**Lemma 1.** Let \( \mathcal{C}_{\Sigma} \) and \( \mathcal{C}_\Gamma \) as above, then we have

\[
I_k/I_{k+1} \cong \bigoplus_{W \subset V_{\Sigma}, |W| = k+1} \mathcal{I}_\Gamma
\]

**Proof.** \( \mathcal{C}_{\Sigma}/I_{k+1} \) is generated by the images \( \hat{h}_i, i \in V_{\Sigma} \) of the generators in the quotient.

Given a subset \( W \subset V_{\Sigma} \) with \( |W| = k + 1 \), let

\[
\mathcal{C}_\Gamma = \mathcal{C}^*([ \hat{h}_i|i \in W]) \subset \mathcal{C}_{\Sigma}/I_{k+1}.
\]

Let \( \mathcal{I}_{\Gamma} \) denote the ideal in \( \mathcal{C}_\Gamma \) generated by products containing all generators \( \hat{h}_i, i \in \Gamma \), and let \( \mathcal{B}_\Gamma \) denote its closure. If \( W \neq W' \), then \( \mathcal{B}_\Gamma \mathcal{B}_{\Gamma'} = 0 \), because the product of any two elements in \( \mathcal{B}_\Gamma \) and \( \mathcal{B}_{\Gamma'} \) contains products of more than \( k + 1 \)-different generators, which are equal to zero in the algebra \( \mathcal{C}_{\Sigma}/I_{k+1} \).

It is clear that \( \mathcal{B}_\Gamma \subset I_k/I_{k+1} \) so that

\[
\bigoplus_{W \subset V_{\Sigma}, |W| = k+1} \mathcal{B}_\Gamma \subset I_k/I_{k+1}.
\]

Conversely, let \( x \in I_k/I_{k+1} \). Then there is a sequence \( (x_n) \) converging to \( x \), such that each \( x_n \) is a sum of monomials \( m_i \) in \( h_i \) containing at least \( k+1 \)-different generators. Then \( m_i \in \mathcal{B}_\Gamma \) for some \( W \) and

\[
x_n = \sum m_i \in \bigoplus_{W \subset V_{\Sigma}, |W| = k+1} \mathcal{B}_\Gamma.
\]

The space \( \bigoplus_{W \subset V_{\Sigma}, |W| = k+1} \mathcal{B}_\Gamma \) is closed, because it is a direct sum of closed ideals. It follows that

\[
I_k/I_{k+1} = \bigoplus_{W \subset V_{\Sigma}, |W| = k+1} \mathcal{B}_\Gamma.
\]

Let now

\[
\pi_{\mathcal{W}} : \mathcal{C}_{\Sigma} \rightarrow \mathcal{C}_\Gamma
\]

be the canonical evaluation map defined by

\[
\pi_{\mathcal{W}}(h_i) = \begin{cases} h'_i & \text{if } i \notin W, \\ 0 & \text{if } i \in W,
\end{cases}
\]

where \( h'_i \) denotes the generator in \( \mathcal{C}_\Gamma \) corresponding to the index \( i \) in \( W \), in other words

\[
\mathcal{C}_\Gamma = \mathcal{C}^*([ h'_i|i \in W])
\]

We prove that \( \pi_{\mathcal{W}}(I_{k+1}) = 0 \). Since polynomials of the form

\[
\sum \ldots h_{i_0} \ldots h_{i_j} \ldots h_{i_{k+1}} \ldots, \ i_0, \ldots, i_j, \ldots, i_{k+1} \ldots \in V_{\Sigma}
\]

are dense in \( I_{k+1} \), it is enough to show that \( \pi_{\mathcal{W}}(x) = 0 \) for each such polynomial \( x \). We have

\[
\pi_{\mathcal{W}}(x) = \sum \ldots \hat{h}_{i_0} \ldots \hat{h}_{i_j} \ldots \hat{h}'_{i_{k+1}} \ldots = 0,
\]

since there is at least one \( i_j \) which is not in \( W \). For this index \( \pi_{\mathcal{W}}(h_{i_j}) = 0 \). Thus \( \pi_{\mathcal{W}}(x) = 0 \). Therefore \( \pi_{\mathcal{W}} \) descends to a homomorphism

\[
\pi_{\mathcal{W}} : \mathcal{C}_{\Sigma}/I_{k+1} \rightarrow \mathcal{C}_\Gamma
\]

Now we show that \( \pi_{\mathcal{W}} \) is surjective as follows: Since \( \pi_{\mathcal{W}}(I_{k+1}) = 0 \), we have Ker \( \pi_{\mathcal{W}} \supset I_{k+1} \). It follows that the following diagram

\[
\mathcal{C}_{\Sigma} \rightarrow \mathcal{C}_\Gamma
\]

commutes and \( \pi_{\mathcal{W}}(\hat{h}_i) := \pi_{\mathcal{W}}(h_i) = h'_i, i \in W \) is well defined. This shows that \( \pi_{\mathcal{W}}(\mathcal{C}_{\Sigma}) \) is a closed subalgebra in

http://www.springerplus.com/content/3/1/258
and isomorphic to \( \pi_W(C_{\Sigma}/I_{k+1}) \). We have \( \pi_W(B_{\Gamma}) = I_{\Gamma} \). It is clear that \( \text{Ker} \pi_W \) is the ideal generated by \( h_i \) for \( i \) not in \( W \) and therefore \( \text{Ker} \pi_W \) is generated by \( h_i \) for \( i \) not in \( W \). This comes at once from the definitions of \( \pi_W(h_i) \) and \( \pi_W(h_i) \) above and the fact that both are equal. We conclude that \( B_{\Gamma} \cap \text{Ker} \pi_W = 0 \). This again implies that \( B_{\Gamma} ^2 \cap \text{Ker} \pi_W = 0 \). Moreover the following diagram is commutative:

\[
\begin{array}{ccc}
C_{\Sigma} & \longrightarrow & C_{\Gamma} \\
\cup & & \cup \\
B_{\Gamma} & \longrightarrow & I_{\Gamma} \\
\downarrow & & \uparrow \\
B_{\Gamma}/\text{Ker} \pi_W & & \\
\end{array}
\]

So, \( \pi_W(B_{\Gamma}) \) is dense and closed in \( I_{\Gamma} \). Therefore \( \pi_W : B_{\Gamma} \longrightarrow I_{\Gamma} \) is injective and surjective.

As a consequence of the above lemma we have the following.

**Proposition 3.** Let \( \mathcal{C}_\Delta \) and \( I_k \) defined as above. Then we have an isomorphism

\[
I_k/I_{k+1} \cong \bigoplus_{\Delta} I_\Delta,
\]

where the sum is taken over all \( k \)-simplexes \( \Delta \) in \( \Sigma \).

**Proof.** As in the proof of lemma 1 above with \( \Sigma = \Delta \), we find that:

\[
I_k/I_{k+1} = \bigoplus_{\Delta} I_\Delta.
\]

In the following we study the \( C^* \)-algebras \( C_{\Gamma^n} \) associated to simplicial flag complexes \( \Gamma \) of a specific simple type. These simplicial complexes is a subcomplex of the “non-commutative spheres” in the sense of Cuntz work (Cuntz 2002). We determine the \( K \)-theory of \( C_{\Gamma^n} \) and also the \( K \)-theory of its skeleton filtration. The \( K \)-theory of \( C^* \)-algebras is a powerful tool for classifying \( C^* \)-algebras up to their Projections and unitaries , more details about \( K \)-theory of \( C^* \)-algebras found in the references (Blackadar 1986; Murphy 1990; Rørdam et al. 2000; Wegge-Olsen 1993).

We denote by \( \Gamma^n \) the simplicial complex with \( n + 2 \) vertices, given in the form

\[
V_{\Gamma^n} = \{0^+, 0^-, 1, \ldots, n\},
\]

and

\[
\Gamma^n = \{ \gamma \subset V_{\Gamma^n} | \{0^+, 0^-\} \nsubseteq \gamma \}. 
\]

Let

\[
C_{\Gamma^n} = C^*(h_0^-, h_0^+, h_1, h_2, \ldots, h_n | h_0^-, h_0^+) = 0, h_j \geq 0, \sum_i h_i = 1, \forall i
\]

be the universal \( C^* \)- algebra associated to \( \Gamma^n \). The existence of such algebras is due to Cuntz (2002). It is clear that for any element \( h_i \in C_{\Gamma^n} \), we have \( \|h_i\| \leq 1 \).

Denote by \( \mathcal{I} \) the natural ideal in \( C_{\Gamma^n} \) generated by products of generators containing all \( h_i, i \in V_{\Gamma^n} \). Then we have the skeleton filtration

\[
C_{\Gamma^n} = I_0 \supset I_1 \supset I_2 \supset \ldots \supset I_{n+1} := \mathcal{I}
\]

The aim of this section is to prove that the \( K \)-theory of the ideals \( \mathcal{I} \) in the algebras \( C_{\Gamma^n} \) is equal to zero. We have the following

**Lemma 2.** Let \( C_{\Gamma^n} \) be as above. Then \( C_{\Gamma^n} \) is homotopy equivalent to \( \mathbb{C} \).

**Proof.** Let \( \beta : \mathbb{C} \longrightarrow C_{\Gamma^n} \) be the natural homomorphism which sends 1 to 1\( C_{\Gamma^n} \). For a fixed \( i \in V_{\Gamma^n} \) such that \( i \neq 0^-, 0^+ \), define the homomorphism

\[
\alpha : C_{\Gamma^n} \longrightarrow \mathbb{C}
\]

by \( \alpha(h_i) = 1 \) and \( \alpha(h_j) = 0 \) for any \( j \neq i \). Notice that \( \alpha \circ \beta = \text{id}_\mathbb{C} \). Now define \( \varphi_2 : C_{\Gamma^n} \longrightarrow C_{\Gamma^n} \), \( h_i \longmapsto h_i + (1 - t)(\sum_{j \neq i} h_j), h_i \longmapsto t(h_j), j \in V_{\Gamma^n} \setminus \{i\} \). The elements \( \varphi_2(h_j), j \in V_{\Gamma^n} \), satisfy the same relations as the elements \( h_j \) in \( C_{\Gamma^n} \):

(i) \( \varphi_2(h_j) \geq 0 \)

(ii) \( \varphi_2 \left( \sum_j h_j \right) = \varphi_2(h_i) + \sum_{j \neq i} \varphi_2(h_j) = h_i + (1 - t) \left( \sum_{j \neq i} h_j \right) + t \left( \sum_{j \neq i} h_j \right) = h_i + \sum_{j \neq i} h_j \) for fixed \( i \)

(iii) \( \varphi_2(h_0^-) = t^2(h_0^- - h_0^+) = 0. \)

We note that \( \varphi_1 = \text{id}_{C_{\Gamma^n}} \) and \( \varphi_0 = \beta \circ \alpha \).

This implies that

\[
\varphi_0 = \beta \circ \alpha \sim \text{id}_{C_{\Gamma^n}}.
\]

This means that \( C_{\Gamma^n} \) is homotopy equivalent to \( \mathbb{C} \).

From the above lemma , we have \( K_*(C_{\Gamma^n}) = K_*(\mathbb{C}) \), for \( * = 0, 1 \).
Now we describe the subquotients of the skeleton filtration in \( C\Gamma^n \).

**Proposition 4.** In the C* -algebra \( C\Gamma^n \) one has

\[
I_k/I_{k+1} \cong \bigoplus_{\Delta} T_{\Delta}, \quad \bigoplus_{\gamma} T_{\gamma},
\]

where the sum is taken over all subcomplexes \( \Delta \) of \( \Gamma^n \) which are isomorphic to the standard k-simplex \( \Delta \) and over all subcomplexes \( \gamma \) of \( \Gamma^n \) which contain both vertices \( 0^+ \) and \( 0^- \) and the second sum is taken over every subcomplex \( \gamma \) which contains both vertices \( 0^+ \) and \( 0^- \) whose number of vertices is \( k + 1 \).

**Proof.** We use Lemma 1 above. For every \( W \subset V_{\Gamma^n} \) with \( |W| = k + 1 \), we have two cases. Either \( \{0^+, 0^-\} \) is a subset of \( W \), then \( \Gamma \) is a \( k \)-simplex, or \( \{0^+, 0^-\} \) is a subset of \( W \), then \( \Gamma \) is a subcomplex in \( \Gamma^n \) isomorphic to \( \gamma \). This proves our proposition.

**Lemma 3.** For the complex \( \Gamma^n \) with \( n + 2 \) vertices, \( C\Gamma^n/I_1 \) is commutative and isomorphic to \( \mathbb{C}^{n+2} \).

**Proof.** Let \( h_i \) denote the image of a generator \( h_i \) for \( C\Gamma^n \). One has the following relations:

\[
\sum_i h_i = 1, \quad h_i h_j = 0, \quad i \neq j.
\]

For every \( \hat{h}_i \) in \( C\Gamma^n/I_1 \) we have

\[
\hat{h}_i = \hat{h}_i \left( \sum_i \hat{h}_i \right) = \hat{h}_i^2.
\]

Hence \( C\Gamma^n/I_1 \) is generated by \( n + 2 \) different orthogonal projections and therefore \( C\Gamma^n/I_1 \cong \mathbb{C}^{n+2} \).

**Lemma 4.** \( I_1/I_2 \) in \( C\Gamma^n \) is isomorphic to \( I_1^{ab}/I_2^{ab} \) in \( C\Gamma^n \).

**Proof.** From the proposition 4 above, one has

\[
I_1/I_2 \cong \bigoplus_{\Delta^1} T_{\Delta^1},
\]

where \( \Delta^1 \) is 1-simplex, and

\[
I_1^{ab}/I_2^{ab} \cong \bigoplus_{\Delta^1} T_{\Delta^1}^{ab}.
\]

Since \( T_{\Delta^1} \subset C_{\Delta^1} \) is commutative because the generators of \( C_{\Delta^1} \) commute (since \( h_{\Delta^1} = 1 - h_{0^+} \)). We get

\[
T_{\Delta^1} \cong T_{\Delta^1}^{ab} \cong C_0(0, 1).
\]

**Lemma 5.** In \( C\Gamma^n \), we have \( K_0(I_1/I_2) = 0 \) and \( K_1(I_1/I_2) = \mathbb{Z}/(2n) \).

**Proof.** By applying above lemma, and proposition 4, we have

\[
I_1/I_2 \cong \bigoplus_{\Delta^1} T_{\Delta^1}.
\]

The sum contains \( \binom{n}{k} + 2n \) 1-simplexes, \( \Delta^1 \cong C_0(0, 1) \).

where \( K_0(C_0(0, 1)) = 0 \) and \( K_1(C_0(0, 1)) = \mathbb{Z} \).

**Lemma 6.** \( C\Gamma^n/I_2 \) is a commutative C* -algebra.

**Proof.** Consider the extension

\[
0 \rightarrow I_1/I_2 \rightarrow C\Gamma^n/I_2 \rightarrow C\Gamma^n/I_1 \rightarrow 0
\]

and the analogous extension for the abelianized algebras.

The extensions above induce the following commutative diagram:

\[
\begin{array}{ccc}
0 & \rightarrow & I_1/I_2 \\
\downarrow & & \downarrow \\
I_1^{ab}/I_2^{ab} & \rightarrow & C\Gamma^n/I_1 \\
\downarrow & & \downarrow \\
I_1^{ab}/I_2^{ab} & \rightarrow & C\Gamma^n/I_2 \\
\end{array}
\]

We have from 3 isomorphisms \( C\Gamma^n/I_1 \cong C_{\Gamma^n}^{ab}/I_1^{ab} \cong \mathbb{C}^{n+2} \) and from 4 that \( I_1/I_2 \cong I_2^{ab}/I_2^{ab} \), so

\[
C\Gamma^n/I_2 \cong \mathbb{C}_{\Gamma^n}^{ab}/I_2^{ab}.
\]

**Lemma 7.** C* -algebra \( C\Gamma_1 \) is commutative and \( K_*(I_2) = 0 \), \( * = 0, 1 \) where \( I_2 \) is an ideal in \( C\Gamma_1 \) defined as in the above.

**Proof.** \( C\Gamma_1 \) is generated by three positive generators, \( h_0, h_0^+, h_1 \). Consider the product of two generators, say \( h_1 h_0^+ \). We have that \( 1, h_0, h_1, h_0^+, h_1 \) commute with \( h_0^- \), therefore also \( h_1 = 1 - h_0^+ - h_0^- \).

By a similar computation we can show that \( h_0^+ \) and \( h_1 \) commute. This implies that \( C\Gamma_1 \) is commutative. Therefore \( I_2 = 0 \) in \( C\Gamma_1 \). Then, at once \( K_*(I_2) = 0 \).

**Competing interests**
The author declare that he has no competing interests.

Received: 20 December 2013  Accepted: 17 March 2014
Published: 21 May 2014

**References**
- Blackadar B (1986) K-theory for operator algebras. MSRI Publ. 5, Cambridge University Press
- Cuntz J (1981) K-theory for certain C* -algebras. Ann Math 113:181–197
- Cuntz J (1984) K-theory and C* -algebras. Proc. Conf. on K-theory (Bielefeld,1982). Springer Lecture Notes in Math. 1046:55–79
- Cuntz J (1993) A survey of some aspects of noncommutative geometry. Jahresber D Dt Math-Verein 95:60–84
- Cuntz J (2002) Non-commutative simplicial complexes and Baum-Connes-conjecture. GAFA, Geom Func Anal 12: 307–329
- Davidson KR (1996) C* -algebras by example. Fields Institute monographs, Amer. Math. Soc, Providence
- Murphy GJ (1990) C* -algebras and operator theory. Academic Press
- Omran S (2005) C* -algebras associated with higher-dimensional noncommutative simplicial complexes and their K-theory. Dissertation, Münster: Univ. Münster-Germany
Omran S (2013) C∗-algebras associated noncommutative circle and their K-theory. Aust J Math Anal Appl 10(1): 1–8. Article 8
Omran S, Gouda GhY (2012) On the K-theory of C∗-algebras associated with \(n\)-Simplexes. Int J Math Anal 6(17): 847–855
Rørdam M, Larsen F, Laustsen N (2000) An introduction to K-theory for C∗-algebras. London Mathematical Society Student Text 49, Cambridge University Press
Wegge-Olsen NE (1993) K-theory and C∗-algebras. Oxford University Press, New York

doi:10.1186/2193-1801-3-258
Cite this article as: Omran: Certain class of higher-dimensional simplicial complexes and universal C∗-algebras. SpringerPlus 2014 3:258.

Submit your manuscript to a SpringerOpen journal and benefit from:

- Convenient online submission
- Rigorous peer review
- Immediate publication on acceptance
- Open access: articles freely available online
- High visibility within the field
- Retaining the copyright to your article

Submit your next manuscript at http://springeropen.com