Dynamic Interference Behaviors of Arbitrary Multilayer Commingling Production in Heavy Oil Reservoirs with Water Flooding

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ABSTRACT: To obtain sustainable economical oil production and recovery of investment, some oil fields adopted the strategy of multilayer commingling production at an early stage. This leads to interlayer interference and losing part of the recoverable reserves. In this paper, dynamic interference behaviors of arbitrary multilayer commingling production in heavy oil reservoirs are analyzed. Based on the non-Darcy flow equation, the Buckley–Leverett equation, and the material balance equation, a mathematical model of arbitrary multilayer commingling production is obtained. Oil and water relative permeability, saturation, and bottom hole flow pressure microelement and the iteration method are employed to solve the mathematical model in the time domain. The new model is verified by comparing the results from the typical black oil model using the Darcy law. The sensitivity analysis of critical parameters on interference behaviors, such as permeability, oil viscosity, effective drainage boundary, and voidage replacement ratio, is carried out. The model obtained in this paper can be used for oil and liquid productivity analysis during the overall process of commingling production and extended to be applied in numerical experiments with different combinations of typical parameters as well.

1. INTRODUCTION

To pursue economical oil production, multilayer commingling was an efficient development strategy worldwide during low oil prices, especially in heavy oil reservoirs. But with an increase of oil prices, operators would like to pursue the economical recoverable reserves instead. Previously, many scholars1–6 studied the approach to improve oil productivity and recoverable reserves. They believed that for varying multilayer permeabilities, effective conformance control is expected to result in more efficient water flood by enhancing volumetric efficiency and potentially increasing the recovery by 5%. Much research has been done on interlayer interference, but most of it can only be applied to single-phase flow or some special water cut stage.7–9 Huang Shijun10 obtained a dynamic relationship of interference coefficients between fluid and oil productivity indexes for the multilayer commingling production of ordinary heavy oil reservoirs based on physical experiments. Jingchen11 revealed that the ultimate recovery factor of each layer with commingling injection was always lower than that being injected separately based on the physical experiment in low-permeability reservoirs. Farizal12 confirmed interlayer cross-flow if reservoir fluids were produced from commingling layers that have unequal initial pressures. Shen13 established a one-dimensional linear flow model and a planar radial flow model to evaluate seepage resistance, sweep efficiency, and recovery percent during commingling production.

As we can see from the literature review, the physical experiment can only simulate limited influence factors, such as

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reservoir permeability and oil viscosity. The model established by Shen was based on the balance assumption of injection and production and only analyzed the variation of permeability and viscosity. Numerical simulators being frequently used can only simulate special non-Darcy flow associated with the threshold pressure gradient (TPG) with an equivalent approach. In this paper, a mathematical model of the arbitrary multilayer commingling production in heavy oil reservoirs is developed without the considering of cross-flow. The mathematical model is solved by the iteration method. Different combinations of typical parameters can be analyzed with the proposed mathematical model, and the interlayer interference can be quantitatively characterized by the overall production process.

2. MODEL DEVELOPMENT

2.1. Physical Model and Assumptions. Dynamic interference behaviors in heavy oil reservoirs with arbitrary multilayer commingling production are considered in this paper, as shown in Figure 1. Before developing the mathematical model, some hypotheses are made to have a clearer derivation process:

1. The upper and lower boundaries of the formation are also impermeable, with a variable formation thickness hi.
2. Gravity and capillary force effects are ignored. Meanwhile, the seepage issue is deemed as an isothermal process.
3. The initial pressure of the reservoirs pi is variable with depth. The skin factor and threshold pressure gradient (TPG) effects are considered.

2.2. Governing Equation. According to non-Darcy theory, the governing equation of oil and water flow in heavy oil reservoirs with an arbitrary multilayer can be expressed as follows

\[ Q_{Li} = \sum_{i=1}^{n} Q_{Li} = \sum_{i=1}^{n} (P_{wfi} - P_{wfi} - G_i \times r_i) \times J_{Li} \]  

and with each layer production separately

\[ P_{wfi} \neq P_{wf2} \neq ... \neq P_{wfi} \]

where \(Q_{Li}\) is total daily liquid production of arbitrary multilayer commingling, m³/d; \(Q_{Li}\) is daily liquid production of layer i, m³/d; n is the total number of layers, dimensionless; \(P_{wfi}\) is the flowing bottom hole pressure (FBHP) of layer i, MPa; \(P_{wfi}\) is the total FBHP of multilayer commingling, MPa; \(G_i\) is the TPG of layer i, MPa/m; \(r_i\) is the effective drainage boundary of layer i, m; and \(J_{Li}\) is the liquid productivity index of layer i, m³/(MPa·d), according to the planar radial flow and can be expressed as

\[ J_{Li} = \frac{0.54287K_ih_i}{\ln(\frac{r_o}{r_i})} + S_i \] \hspace{1cm} (2)

and

\[ K_{roi} = K_{roi}(S_{wci})(1 - S_{wd})^n_{oi} \] \hspace{1cm} (3)

\[ K_{roi} = K_{roi}(S_{odi})S_{wd}^n_{oi} \] \hspace{1cm} (4)

where \(K_i\) is the permeability of layer i, \(10^{-3}\) m²; \(h_i\) is the thickness of layer i, m; \(K_{roi}\) is the relative permeability of oil for layer i, dimensionless; \(K_{roi}\) is the relative permeability of water for layer i, dimensionless; \(B_o\) is the volume factor of oil, dimensionless; \(B_w\) is the volume factor of water, dimensionless; \(\mu_o\) is the oil viscosity of layer i, mPa·s; \(\mu_w\) is the water viscosity of total layers, mPa·s; \(r_o\) is the well radius, m; \(S_i\) is the skin factor of layer i, dimensionless; \(S_{wci}\) is the irreducible water saturation of layer i, dimensionless; \(S_{odi}\) is the residual oil saturation of layer i, dimensionless; \(n_{oi}\) is the oil phase index of layer i, dimensionless; \(n_{wi}\) is the water phase index of layer i, dimensionless; \(S_{wd}\) is the dimensionless water saturation, which can be expressed as

\[ S_{wd} = \frac{S_w - S_{wci}}{1 - S_{wci}} \] \hspace{1cm} (5)

The water cut with the variance of saturation for each arbitrary layer can be expressed with the revised Buckley–Leverett equation
\[ f_{wi} = \frac{M_i S_{wi}^n}{M_i S_{wi}^n + (1 - S_{wi})^n} \]  

(6)

and

\[ M_i = \mu_i B_i K_i (S_{ori}) / \mu_i B_o K_o (S_{wc}) \]  

(7)

where \( f_{wi} \) is the water cut of layer \( i \), dimensionless and \( M_i \) is the water–oil mobility ratio of layer \( i \), dimensionless.

From eqs 1–7, we can only obtain the relationship between the daily liquid production, FBHP, and water cut with constant reservoir pressure for each layer produced separately. Usually, initial pressure is hard to be maintained due to variable drainage boundary, reservoir property, and unbalance water injection, and the recoverable reserves should be solved in the time domain. According to the material balance equation, the relationship between cumulative liquid production and average reservoir pressure can be given as

\[ \pi \rho_i h_i \phi_i (1 - S_{wc}) C_u \Delta P + \int_0^t Q_{oi} dt 
= \int_0^t Q_{Li} f_{wi} (S_{wi}) dt + \int_0^t Q_{Li} (1 - f_{wi} (S_{wi})) dt \]  

(8)

By integrating eq 8 and the voidage replacement ratio (VRR), the pressure variation of the arbitrary layer is

\[ \Delta P = \frac{\int_0^t (1 - I_{ki}) Q_{Li} dt}{\pi \rho_i h_i \phi_i (1 - S_{wc}) C_u} \]  

(9)

By transforming the average variance of water saturation \( \overline{Swi} \) into cumulative water injection and production of each layer, eq 8 can be transformed into

\[ \pi \rho_i h_i \phi_i (1 - S_{wc}) C_u \Delta P + \int_{0}^{1-S_{wc}} \pi \rho_i h_i \phi_i (1 - S_{wc}) dS_w 
= \int_0^t Q_{Li} (1 - f_{wi} (S_{wi})) dt \]  

(10)

where \( I_{ki} \) is the VRR of layer \( i \), dimensionless; \( \phi_i \) is the porosity of layer \( i \), dimensionless; \( C_u \) is the comprehensive compressibility; \( 1/\text{MPa} \); \( \Delta P \) is the pressure variance of layer \( i \), MPa; and \( Q_{Li} \) is the daily water injection of layer \( i \), \( m^3/d \).

By combining eqs 1–10, we can obtain key parameters in the time domain with arbitrary multilayer commingling production, such as daily liquid and oil production, FBHP, reservoir pressure, water saturation, and water cut.

The interlayer interference coefficient can be defined as

\[ \varepsilon_L = \frac{\sum_{i=1}^{n} I_{Li} - I_{Lc}}{\sum_{i=1}^{n} I_{Li}} \]  

(11)

\[ \varepsilon_o = \frac{\sum_{i=1}^{n} I_{Li} \times (1 - f_{wi}) - I_{Lc} \times (1 - f_{wc})}{I_{Lc} \sum_{i=1}^{n} I_{Li} \times (1 - f_{wi})} \]  

(12)

where \( \varepsilon_L \) is the liquid productivity index (PI) interference coefficient, dimensionless; \( \varepsilon_o \) is the oil PI interference coefficient, dimensionless; \( f_{wc} \) is the total water cut of multilayer commingling production, dimensionless; and \( I_{Lc} \) is the liquid PI of multilayer commingling production, \( m^3/(\text{MPa} \cdot \text{d}) \).

For establishing \( I_{Lc} \), the total displacement pressure should be clarified. The utilization of the total displacement pressure can be expressed with liquid production volume-weighted average from each layer, which can be defined as

\[ I_{Lc} = \frac{\sum_{i=1}^{n} Q_{Li}}{\sum_{i=1}^{n} Q_{Li}} (1 - f_{wi} - f_{wc}) \]  

(13)

2.3. TPG Establishment of Heavy Oil. The existence of TPG has been approved by scholars with dynamic analysis and the physical experiment. The physical experiment of the threshold pressure gradient is carried out with a sand-pack flooding unit, which included pump, sand pack, pressure transducer, etc. The sand pack with a length of 60 cm and an inner diameter of 3.8 cm is filled with different meshes of quartz, with sand-pack permeability from 186 to 6698 mD. The experimental oil is a mixture of crude oil and kerosene, and the viscosity is between 10 and 450 mPa·s. The temperature should be maintained at 20 °C.

The relationship between TPG and reservoir mobility is shown in Figure 2. At an early stage, TPG declines sharply with the increase of mobility and becomes stable gradually until the mobility overpasses 25 mD/mPa·s. In total, we obtained 16 samples from the sand-pack experiment successfully, but two unrepresentative samples deviate from our analysis scope. There are varied relationships between TPG and mobility caused by various reservoir rock types and properties; meanwhile, quality control and screening among samples need to be implemented. In this paper, the TPG relationship can be expressed as

\[ G = 0.123 \times \left( \frac{k}{H_b} \right)^{0.65} \]  

(14)

3. MODEL SOLVING

The governing model we established on the basis of the flow equation, the material balance equation, and the revised Buckley–Leverett equation is difficult to be solved directly. However, the microelement of water saturation \( \Delta S_w \) and bottom hole flow pressure (BHFP) \( \Delta P_{BHFP} \) as a step length are used in the time domain by iterative calculation. The detailed solution flow is shown in Figure 3 where the variation can be set as 2–5% as the allowable deviation target. The model can be solved to obtain the dynamic interference of arbitrary multilayer commingling production such as productivity and...
recovery factors under various rock types, permeability, viscosity, VRR, and effective drainage boundary.

4. MODEL VERIFICATION

To verify our model of a directional well in the arbitrary multilayer commingling production in heavy oil reservoirs, a simplified case is adopted here to compare the results from different methods. The typical black oil model has been used worldwide as the base of the simulator, and the results are reliable. However, it is not easy to simulate with TPG and special non-Darcy flow. Thus, in this part, the dynamic behaviors of a directional well in the three-layer commingling production with and without TPG are used to verify our model. The drainage radius of the heavy oil reservoir is 300 m, the thickness is 20 m for each layer, and the oil viscosity is stable at 50 mPa·s. The permeabilities of the three layers are, respectively, 1000, 1500, and 2000 mD.

As can be seen from Figure 4, the calculation results of the dynamic behaviors of a directional well in the three-layer commingling production without TPG are in good agreement with the black oil model and our new method. The good agreement based on the two different methods reflects the correctness of our model. The rapid increases of water cut before and after TPG is taken into account, and additional pressure drop is needed to obtain the same liquid production target. The interference between layers leads to a more rapid decline in the oil production rate and lower recoverable reserves during commingling production.

Previously, Fetkovich approved negligible interference existence under ideal conditions with uniform formation pressure and without cross-flow. Furthermore, we verify our model under ideal conditions with uniform reservoir properties and a sustainable liquid production rate of 250 m³/d. It can be seen from Figure 5 that the oil productivity index (PI) of multilayer commingling production can match well with slicing production. The calibration result from a directional well in the three-layer commingling production also proves to be in agreement with Fetkovich.

5. RESULTS, DISCUSSION, AND APPLICATION

In this part, the dynamic interference behaviors of the three-layer commingling production in heavy oil reservoirs are analyzed. For the limitation of the paper length, we focus on...
the oil PI interference coefficient and the recovery factor through the whole process of water flooding caused by permeability, oil viscosity, VRR, and effective drainage boundary. The model can be extended to the arbitrary multilayer commingling production as needed. Detail parameters for different cases are shown in Table 1.

Table 1. Input Parameters for Model Analysis

| Model Parameters       | Value Designed for Different Cases of Sensitivity Analysis |
|------------------------|----------------------------------------------------------|
| Oil viscosity (mPa·s)  | 20–150 50 50 50                                        |
| Reservoir permeability (mD) | 2400 800–3200 2400 2400                                  |
| Drainage boundary (m)  | 300 300 200–300 300                                      |
| VRR (fraction)         | 1.0 1.0 1.0 0.94–1.0                                     |
| (n_f, n_w), dimensionless | (3,3) (3,3) (3,3) (3,3)                                 |
| Reservoir thickness (m) | 20 20 20 20                                            |
| Reservoir porosity (fraction) | 0.25 0.25 0.25 0.25                               |

5.1. Permeability. Interlayer interference inhibits the overall oil productivity during the whole commingling production period. The major interlayer interference regimes of the oil PI can be divided into three stages as follows: (I) the first stage is the weak inhibition period, where BHFP descends to match the liquid production target; (II) the second stage is the significantly intensified period after water breakthrough, where the interlayer interference accelerates and intensifies with the variation of the seepage resistance; and (III) the third stage is the stable inhibition period with a combined function of the seepage resistance and BHFP (Figure 6c).

With the increase of the permeability ratio, the oil PI and the recovery factor interference become more serious. For commingling with a permeability ratio of 4, the first interference stage is very short, which only sustains until a 0.07 pore volume (PV) of liquid production. The second stage lasts from 0.07 to 0.5 PV, where the oil PI interference coefficient is 0.4 and the recovery factor discrepancy reaches 0.1 compared to slicing production. During the third stage, the oil PI interference coefficient increases from 0.4 to 0.6, and the recovery factor discrepancy increases from 0.1 to 0.22 (Figure 6a,b).

As mentioned above, two mitigations can be carried out to enhance recoverable reserves, which include the conversion of commingling production to slicing before 0.5 PV and

![Graph](https://example.com/graph1.png)

(a) Oil PI interference coefficient as PV

![Graph](https://example.com/graph2.png)

(b) Oil recovery factor as PV

![Graph](https://example.com/graph3.png)

(c) Liquid production fraction and BHFP of each layer as WCT with commingling production

Figure 6. Dynamic interference behavior as PV with different combinations of permeability.
restriction of the permeability ratio as small as possible in the case of commingling production.

5.2. Oil Viscosity. With the increase of the oil viscosity ratio from 3 to 7.5, the oil PI and recovery factor interference becomes more intense but is less sensible than caused by permeability. For a viscosity ratio of 7.5, the first stage of interlayer interference is much shorter than that caused by permeability ratios and can even be ignored. The second stage lasts from 0 to 0.4 PV, where the oil PI interference coefficient increases up to 0.5 and the recovery factor discrepancy reaches 0.1 compared to slicing (Figure 7a). During the third stage, both the oil PI interference coefficient and the recovery factor remain stable with the increase of PV, and little improvement of the recovery factor is obtained by the decrease of the

![Figure 7. Dynamic interference behavior as PV with different combinations of oil viscosity.](image1)

![Figure 8. Dynamic interference behavior as PV with different combinations of the drainage boundary.](image2)

![Figure 9. Dynamic interference behavior as PV with different combinations of VRR.](image3)
The development stage can be divided into three periods. First, the implementation of the commingling production strategy with an inverse nine-spot pattern due to ultralow oil prices from 1993 to 2009, which obtained 16% recovery of STOIIP. Second, integratedly infilling the wells by converting to a five-spot pattern and shortening the well spacing from 450 to 350 m from 2009 to 2016, which eventually obtained 30% recovery of STOIIP. Third, slicing production was planned to solve the interlayer conflict caused by heterogeneous properties nowadays. A pilot including nine producers and five injectors was chosen to verify the availability of slicing, where six wells were infilled for slicing (Figure 10a). The multilayer permeability ratio within the well group was approximately 4.4, and the oil viscosity was close to 50 mPa·s. The average tested oil PI interference coefficient was 0.45–0.5 with 80% water cut (0.6–0.8 PV), which was consistent with our model (Figure 6a).

After slicing, the average displacement pressure increased sharply from 4.5 to 6.8 MPa, which led to the additional oil production from reservoirs with low permeability, and daily oil production increased from 55 to 67 m³/d. Furthermore, water cut decreased from 80 to 71% and sustained for almost 3 years (Figure 10b). The production performance was obviously improved after slicing.

6. CONCLUSIONS

A mathematical model is presented to analyze the dynamic interference of multilayer commingling production in heavy oil reservoirs with water flooding. The solution procedure is established by the microelement of water saturation and the bottom hole flow pressure approach; the type curves of the oil PI interference coefficient and the recovery factor are plotted and discussed. Based on the above analysis, the following conclusions are drawn

1. The iteration approach with the microelement of water saturation and BHFP can be applied to solve the mathematical model of a directional well in heavy oil reservoirs with multilayer commingling production, which was verified by comparing with the typical black oil model.
2. The major three oil PI interference regimes are identified, which are the weak inhibition period, the significantly intensified period, and the stable inhibition period.
3. The dynamic interference behaviors caused by oil viscosity, permeability, drainage boundary, and VRR are obtained and analyzed. Meanwhile, the type curves of the oil PI interference coefficient and the recovery factor are plotted.
4. The VRR ratio mainly determines the weak inhibition period, while the oil viscosity ratio, permeability ratio, and drainage radius ratio determine the significantly intensified period during commingling production and injection.
5. The model we obtained can be used for oil and liquid productivity analysis during the overall process of

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(a) Well map of slicing pilot

(b) Daily oil rate and WCT before and after slicing

Figure 10. Pilot performance comparison before and after slicing.
development and extended to be applied in numerical experiments with different combinations of typical parameters as well.

ASSOCIATED CONTENT

1 Supporting Information

The Supporting Information is available free of charge at https://pubs.acs.org/doi/10.1021/acsomega.0c06154.

Solving modules of the above model and cases designed (ZIP)

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Notes
The authors declare no competing financial interest.

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