Theory for the response of a superconducting kinetic inductance detector to an electromagnetic wave packet

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Abstract. Voltage pulses generated by local heating due to the irradiation of an electromagnetic (EM) wave packet in a superconducting current-biased kinetic inductance detector are theoretically investigated on the basis of our previous theory \cite{J. Phys. Conf. Ser. 1293, 012050}. The growth and decay of a hot spot created by an EM wave packet is described with the heat diffusion equation coupled with the time-dependent Ginzburg-Landau (TDGL) equation. Variation in the kinetic inductance of the current-biased stripline in the detector induced by a hot spot is explicitly derived. We clarify the characteristic feature in the shape of the voltage pulses.

1. Introduction

Radiation detectors with superconducting circuits have been widely utilized in various fields using radiations in recent years\cite{1, 2, 3, 4, 5}. Among them the one detecting variations in the kinetic inductance of the superconducting circuit caused by the absorption of incident radiations makes it possible to image the positions of the radiations in high spatial resolution and also works in a wide temperature range below $T_c$. To detect neutrons in such a superconducting kinetic inductance detector the one covered with a $^{10}$B layer, which absorbs neutrons well, has been intensively developed by Ishida’s group as a high performance neutron detector, which is called Current-Biased Kinetic Inductance Detector (CB-KID) \cite{6, 7, 8, 9, 10, 11, 12}. Recently, it has been demonstrated that clear neutron transmission images of various objects made of metals and also bio-materials can be obtained by using a CB-KID\cite{13}.

In our previous papers an electrodynamic theory was developed for the operation principle of CB-KID on the basis of the London theory\cite{14} and also the Ginzburg-Landau theory\cite{15}. We derived a basic equation for the voltage modes excited by a transient hot spot in the current-biased superconducting nanowire in a CB-KID. It was shown that a pair of voltage pulses with opposite polarities is created by a hot spot and these pulses propagate with a Swihart velocity\cite{16} toward the two nanowire’s ends.

In this paper we extend our previous theory for CB-KID to construct an electrodynamic theory for the response of a CB-KID without a $^{10}$B layer to electromagnetic waves. The purpose of this paper is to show that the CB-KID also works as a detector for electromagnetic waves.
Since CB-KID can get imaging of the radiations reaching the detector in a scale of $\mu$m, we investigate the response to a focused electromagnetic (EM) wave with small spatiotemporal dimensions such as an EM wave packet as the incident one. Explicit expression for an EM wave packet, which is given as a solution of the Maxwell equation in a vacuum, is presented. The incident EM wave packet heats the superconducting stripline in the detector with Joule-heat and creates a hot spot. The growth and decay of the hot spot is described in terms of the thermal diffusion equation and the TDGL equation. The shape of the voltage pulses reaching the two nanowire’s ends are explicitly derived. Our theory is applicable to the case of focused laser pulse irradiation.

2. Basic equation in superconducting kinetic inductance detectors

Consider a long superconducting stripline placed on a basal plane composed of a stack of superconducting and insulating layers as a model for the superconducting kinetic inductance detector such as CB-KID. Since this system has a structure similar to a S-I-S waveguide, an electromagnetic wave can propagate along the stripline, which can be regarded as a Swihart mode. In our previous papers we derived an equation for the superconducting phase difference between the stripline and the superconducting layer forming the basal plane in this system. It was shown that the equation can describe well the electrodynamic properties related to the operation principle of CB-KID such as the generation of voltage pulses by a transient hot spot that varies the kinetic inductance of the stripline.

First, we summarize the derivation of the equation. Consider a CB-KID in which thickness of the stripline $s$ is much shorter than the London penetration depth $\lambda_L$, i.e., $s \ll \lambda_L$. In such a CB-KID it is convenient to define the phase difference between the stripline and the superconducting basal plane in the presence of an electromagnetic field as follows,

$$\theta(x, t) = \varphi_s(x, d, t) - \varphi_b(x, -D, t) - \frac{e^*}{\hbar c} \int_{-D}^{d} dz A_z(x, z, t),$$  \hspace{1cm} (1)

where $\varphi_s(x, d, t)$ and $\varphi_b(x, -D, t)$ stand for the superconducting phases, respectively, at a point on the lower surface of the stripline ($z = d$) and a point deeply inside the superconducting basal plane ($z = -D$), i.e., $D \gg \lambda_L$, and $A_z(x, z, t)$ is the $z$-component of the vector potential. Here, we take the coordinate axes as shown in Fig.1. Note that $d$ is equal to the thickness of the insulating layer inside the basal plane. It was shown that the phase difference $\theta(x, t)$ satisfies the equation[14],

$$\frac{\lambda_L}{d} + \frac{d}{d^2} \frac{e^*}{\hbar c} \frac{\partial^2 \theta(x, t)}{\partial x^2} - \frac{e^*}{\hbar c} \partial_x A_z^2(x, d, t) = \frac{e^*}{\hbar c} \partial_x A_z^2(x, d, t),$$  \hspace{1cm} (2)

![Figure 1. Cross section of the CB-KID perpendicular to the x-axis. The superconducting stripline on the basal plane extends in the x direction.](image-url)
where $A_x^x(x, d, t) = A_x(x, d, t) - (\hbar c/e^*) \partial_x \varphi_4$ is the $x$-component of the gauge-invariant vector potential at a point $x$ on the lower surface of the stripline ($z = d$). The vector potential in Eq.(2) is related to the superconducting current flowing inside the stripline ($d \leq z \leq d + s$) as

$$j_x(x, z, t) = \frac{e^2}{m^* c} |\Psi(x, z, t)|^2 A_x^x(x, z, t),$$

in the time-dependent Ginzburg-Landau (TDGL) theory. Here, $\Psi(x, z, t)$ is the superconducting order parameter. The vector potential $A_x^x(x, d, t)$ in Eq.(2) can be expressed in terms of the phase difference $\theta(x, t)$ by utilizing the Maxwell equations in the regions both the stripline and the insulating layer. In the case where the order parameter has weak spatiotemporal dependence expressed as

$$\Psi(x, z, t)/\Psi_0 = 1 + \psi(x, z, t),$$

with $|\psi(x, z, t)| \ll 1$ one can derive the equation with a source term from Eq.(2)[15],

$$v^{-2} \partial^2_\theta \theta(x, t) - \partial^2_\theta \theta(x, t) = F(x, t),$$

where $v$ is the Swihart velocity, $v = d^{1/2}[d + \lambda_L(1 + \coth(s/\lambda_L))^{-1} c^{-1/2}]$, and the source term $F(x, t)$ is given as

$$F(x, t) = (2\lambda_L/d) \coth(s/\lambda_L)(\epsilon/c^2) \partial^2_\theta \theta(x, t) \cdot \psi(x, d, t)$$

$$+ 2\lambda_L \coth(s/\lambda_L)[d + \lambda_L(1 + \coth(s/\lambda_L))^{-1} \partial_\theta \theta(x, t) \cdot \partial_x \psi(x, d, t)].$$

Note that the source term (6), which stems from the spatiotemporal variation of the order parameter $\psi(x, z, t)$, excites the phase oscillation modes that propagate along the stripline. Furthermore, since the voltage difference between the stripline and the basal plane can be obtained from the time derivative of the phase difference, i.e., $V(x, t) = (\hbar/e^*) \dot{\theta}(x, t)$, one understands that solutions of Eq.(5) can describe also the generation and transmission of voltage pulses in the CB-KID.

Consider a case in which a bias current $I$ is flowing in the stripline and a transient spatiotemporal variation emerges in the order parameter $\psi(x, t) \neq 0$. In this case Eq.(5) leads to the solution for $V(x, t)$ up to the first-order in $I$ as follows,

$$V(x, t) = -K \mathcal{L} v^2 \left\{ \int_{-\infty}^{x} dx' \partial_{x'} \psi(x', t') |_{t' = (x' - z^{(+)})/v} \right.$$

$$+ \int_{x}^{\infty} dx' \partial_{x'} \psi(x', t') |_{t' = (z^{(-)}/-x')/v} \left\} \cdot I, \right.$$  

where $K = (\lambda_L/c) \coth(s/\lambda_L)$, $z^{(\pm)} = x \mp vt$ and $\mathcal{L}$ is the inductance of the stripline. In the asymptotic regions, i.e., $x \to \infty$ ($|z^{(+)}/v| < \infty$) and $x \to -\infty$ ($|z^{(-)}/v| < \infty$), Eq.(7) describes a pair of voltage pulses propagating in the opposite directions with each other, i.e.,

$$V(x - vt) = -K \mathcal{L} v^2 \int_{-\infty}^{\infty} dx' \partial_{x'} \psi(x', t') |_{t' = (x' - z^{(+)}/v)} \cdot I,$$

for $x \to \infty$ and

$$V(x + vt) = -K \mathcal{L} v^2 \int_{-\infty}^{\infty} dx' \partial_{x'} \psi(x', t') |_{t' = (z^{(-)}/-x')/v} \cdot I,$$

for $x \to -\infty$. Note that Eqs.(8) and (9) read $V = dL_k/dt \cdot I$ with $L_k$ being the kinetic inductance of this system.
Let us next investigate the voltage pulses generated by a transient hot spot in the CB-KID on the basis of the above result. Consider a hot spot with a size \( w \) of a few \( \mu \)m, which is much larger than the superconducting coherence length \( \xi \) of the stripline, i.e., \( w \gg \xi \). Suppose that the temperature around the hot spot in the stripline is locally increased as \( T = T_0 \rightarrow T(r,t) = T_0 + T_1(r,t) \). To incorporate this local temperature variation into the GL theory we extend the temperature-dependent 2nd-order term in the GL free energy as follows,

\[
\alpha(T)\vert \Psi(r) \vert^2 \rightarrow \alpha(T(r,t))\vert \Psi(r,t) \vert^2,
\]

and expand the coefficient \( \alpha(T(r,t)) \) as \( \alpha(T(r,t)) \approx \alpha(T_0) + \alpha_1 T_1(r,t) \), assuming \( T_1(r,t) \ll T_c \). With this extension the order parameter acquires the spatiotemporal dependence, \( \Psi(r) = \Psi_0(T) \rightarrow \Psi(r,t) = \Psi_0(T)(1 + \psi(r,t)) \). For the normalized spatiotemporal variation \( \psi(r,t) \) one can impose the conditions, \( \vert \psi(r,t) \vert \ll 1 \) and \( \vert \nabla \psi \vert \ll \xi^{-1} \psi \). Here, the second condition is justified because \( \vert \nabla \psi \vert \sim w^{-1} \psi(r,t) \). Under these conditions we solve approximately the TDGL equation without a vector potential to obtain \( \psi(r,t) \). Note that the superconducting current up to the first order in \( A_x^\alpha \) is given in terms of the order parameter in the state without a vector potential (see Eq.(3) ). The solution for \( \psi(r,t) \) is, then, obtained as

\[
\psi(r,t) \approx -\tilde{\alpha}_1 \xi^2 (1 - e^{-(t-t_0)/\tau_{\text{qp}}}) T_1(r,t), \tag{10}
\]

with \( \tilde{\alpha}_1 = 2m^*\alpha_1/h^2 \). Here, \( \tau_{\text{qp}} \equiv \xi^2/D \) is the quasi-particle relaxation time and \( t_0 \) is the arrival time of the EM wave packet. The parameter \( D \) in \( \tau_{\text{qp}} \) is related to the quasi-particle damping constant \( \gamma \) in the TDGL equation as \( D^{-1} = 2m^*\gamma/h^2 \). In obtaining Eq.(10) we also assume that the time variation of \( T_1(r,t) \) is gentle, i.e., \( \vert \partial_t T_1(r,t) \vert \ll \tau_{\text{qp}}^{-1} T_1(r,t) \). Then, given the temperature variation \( T_1(r,t) \), one can calculate the voltages produced by a hot spot, using Eqs.(8), (9) and (10).

Let us next investigate the temperature variation of the hot spot, \( T_1(r,t) \). We assume that the local temperature variation in the stripline comes from the heat flows along the stripline and also into the basal plane. In this case, as shown in [15], \( T_1(r,t) \) obeys the linearized equation

\[
C_S \partial_t T_1(r,t) - \kappa_n (\partial_x^2 + \partial_y^2) T_1(r,t) + \zeta T_1(r,t) = q(r,t), \tag{11}
\]

in the case of \( T_1(r,t) \ll T_c \). Here, \( q(r,t) \) is the heat provided by the hot spot, \( C_S \) is the specific heat in the superconducting state, \( \kappa_n \) is the normal state thermal conductivity, and \( \zeta \) is the thermal relaxation rate. It is noted that the second and the third terms in Eq.(11) originates from the heat flows along the stripline and into the contacted basal plane, respectively. In the case of a long stripline (\(~a\) few m) with a narrow width of \( L \sim 1\mu \)m Eq.(11) may be converted to the one-dimensional form,

\[
C_S \partial_t \tilde{T}_1(x,t) - \kappa_n \partial_x^2 \tilde{T}_1(x,t) + \zeta \tilde{T}_1(x,t) = \tilde{q}(x,t), \tag{12}
\]

where \( \tilde{T}_1(x,t) \) and \( \tilde{q}(x,t) \) are defined as \( \tilde{T}_1(x,t) = L^{-1} \int_{-L/2}^{L/2} dy T_1(x,y,t) \) and \( \tilde{q}(x,t) = L^{-1} \int_{-L/2}^{L/2} dy q(x,y,t) \). Note that Eq.(12) yields the solution,

\[
\tilde{T}_1(x,t) = \frac{x_h}{C_S} \int_{-\infty}^{\infty} dx' \int_{-\infty}^{t} dt' \sqrt{\frac{\tau_z}{t-t'}} \exp\left[-\frac{t-t'}{\tau_z} - \frac{1}{2} \left( \frac{\tau_z}{t-t'} \right)^2 \right] \tilde{q}(x',t'), \tag{13}
\]

where \( \tau_z = C_S/\zeta \) and \( x_h = \sqrt{2\kappa_n/\zeta} \).
3. Hot spot created by an electromagnetic wave packet

Let us now calculate the voltage pulses generated by a hot spot which an EM wave packet incident vertically (|z-axis) on the stripline creates. As an example, we choose an EM wave packet made of Gaussian-distributed electromagnetic waves that are propagating in the z-direction and rotating around the z-axis. In Appendix A the electric field associated with such an EM wave packet is derived, which is given as

$$E(x, y, z, t) = \sqrt{\frac{\pi \rho_0}{2}} \eta^q E_0^q \cos(\eta z - vt) K_0\left(\sqrt{\frac{x^2 + (y - y_0)^2}{\mu}}\right) \exp\left(-\frac{1}{2\tau^2 (v^{-1}z - t)^2}\right) e_\eta,$$  \hspace{1cm} (14)

where \(\eta = \sqrt{(\nu/c)^2 + (1/\mu^2)}\), \(v_\eta = c^2 \eta/\nu\), and \(e_\eta = (\sin \theta, \cos \theta, 0)\) is the unit vector going around the z-axis in the cylindrical coordinates \((r, \theta, z)\). The function \(K_0(r)\) in Eq.(14) is the zeroth-order modified Bessel function of the second kind. This wave packet has a cigar-like shape. In obtaining the electric field (14) we utilized the frequency distribution function, \(\rho(\omega) = \rho_0 \exp(-\tau^2 (\omega - \nu)^2/2)\), where \(\nu\) and \(\tau^{-1}\) are, respectively, the central frequency and the standard deviation of the Gaussian-distributed electromagnetic waves. Note that the EM wave packet has the center at \((x, y, z) = (0, y_0, v^{-1}t)\), its expansion in the \(xy\)-plane is equal to \(2\mu\), and it moves with a group velocity \(v_\eta\) in the z-direction.

Suppose that an EM wave packet having the electric field (14) irradiates the stripline situated at \(z = 0\). In this incidence the EM wave packet heats the stripline. When the stripline has the optical conductivity \(\sigma(\nu)\), the quantity of heat supplied by the EM wave packet is given as

$$q(x, y, t) \sim \sigma(\nu) E^2(x, y, 0, t),$$  \hspace{1cm} (15)

where \(E^2\) is the time-averaged value of \(E^2\). Then, the heat source \(\bar{q}(x, t)\) in Eq.(12) is obtained as

$$\bar{q}(x, t) \sim \frac{Q_0}{L} \int_{-L/2}^{L/2} dy K_0\left(\sqrt{x^2 + (y - y_0)^2}/\mu\right)^2 \exp\left[-\frac{t^2}{\tau^2}\right],$$  \hspace{1cm} (16)

with \(Q_0 = (\pi \rho_0^2/4\tau^2) \sigma(\nu) E_0^q\). When Eq.(16) is substituted into the integrand \(\bar{q}(x, t)\) in Eq.(13), one can find the temperature variation of the hot spot, \(\bar{T}_1(x, t)\), after performing the integral in Eq.(13). Furthermore, given the \(\bar{T}_1(x, t)\), one can calculate the voltages induced by this hot spot from the integrals in Eqs.(8) and (9) in which the order parameter is given in Eq.(10). Let us now present numerical results for \(\bar{T}_1(x, t)\) and \(V(x + vt)\). Suppose that the parameters in our theory, \(\tau_{qp}, \tau_z, \tau, \nu, \mu, L\) and \(x_h\), take values in the regions of \(\tau_{qp} \sim 10^{-12}\) sec, \(\tau_z, \tau \sim 10^{-11}\) sec, \(\nu \sim 10^6\) cm/sec\(^{-1}\) and \(\mu, L, x_h \sim 10^{-3}\) cm. To carry out the calculations it is convenient to introduce the dimensionless quantities constituted from the above parameters, \(v\tau_{qp}/x_h, v\tau_z/x_h, x_h/\mu, \tau_z/\tau, L/\mu\). In the following we perform the calculations, choosing the parameter values as \(v\tau_{qp}/x_h = 5.0, v\tau_z/x_h = 10.0, x_h/\mu = 2.0, \tau_z/\tau = 1.5\) and \(L/\mu = 1.0\), which are in the parameter regions mentioned above.

In Fig.2 we present the numerical result for \(\bar{T}_1(x, t)\), which exhibits the growth and decay of a hot spot in the stripline. It is seen that the size of the hot spot is \(\sim 2x_h\) and its life time is \(\sim \tau_z\). Figure 3 represents the voltage pulse generated by this hot spot in the asymptotic region \(x \to -\infty\), i.e., \(V(x + vt)\). As seen in this figure, the voltage pulse shows oscillation from positive to negative values and its duration-time, i.e., the observed time-length of the voltage pulse, is much longer than \(x_h/\nu\). This result indicates that the voltage pulse observed at the end of the stripline in the CB-KID has a much wider duration-time than the lifetime of the hot spot and also the time-width of the incident EM wave packet. It is noted that we also have a voltage pulse propagating in the direction, \(x \to \infty\), i.e., \(V(x - vt)\), which has the opposite polarity, i.e., \(V(|x| - vt) = -V(x + vt)\) for \(x \to -\infty\). Then, one can read out the position and time of the EM wave packet incident on the stripline from the arrival times at two ends of the stripline. Finally,
we mention that the present theory is applicable to the analysis of focused laser pulse irradiation experiments in current-biased kinetic inductance detectors. Narukami et al. observed voltage signals by irradiating 20 ps laser pulses to a CB-KID [17]. The observed peak voltages depend linearly on the bias current in the mV region, though they are values amplified by about $10^2$ times, and the pulse width is $\sim 2.74$ns, which is much wider than that of the irradiated laser pulse. These experimental results are qualitatively consistent with our theoretical predictions.

![Figure 2. Spatial variation of the temperature $T_1(x,t)$ at several times.](image)

![Figure 3. Normalized voltage pulse $V(x+vt)/V_0$ generated by a hot spot in the asymptotic region, $x \to -\infty$. Here, $V_0 = KL^2\dot{\alpha}_1\xi^2Q_0I$.](image)

4. Summary

In this paper we have developed a theory for the response of a CB-KID without a $^{10}$B layer to an EM wave packet on the basis of our previous theory for the electrodynamics of CB-KID. We considered a hot spot formed by the absorption of an EM wave packet in the superconducting stripline. The temperature variation of the hot spot was explicitly calculated by using the heat diffusion equation in the superconducting state. It was shown that the hot spot causes a variation of the kinetic inductance of the stripline and generates a pair of voltage pulses with opposite polarities when it is current-biased. Our theory is applicable to the study of the generation of voltage pulses in kinetic inductance detectors with a S-I-S structure on which a bunch of EM waves (photons), e.g. a focused laser pulse, is irradiated.
**Appendix A**

Let us construct a pulsed electromagnetic (EM) wave packet propagating in a vacuum. Suppose that the EM wave packet has a cigar-like shape and is propagating in the $z$-direction. Consider an oscillatory electric field with a frequency $\omega$ which is going around the $z$-axis,

$$E(x, y, z, t) = E_0(r, z, t)e_\theta = E_\omega^0(r, z)e^{-i\omega t}e_\theta,$$

where $e_\theta = (-\sin \theta, \cos \theta, 0)$ is one of the unit vectors in the cylindrical coordinates $(r, \theta, z)$. The component $E_\omega^0(r, z)$ in Eq.(17) can be determined by the Maxwell equation in a vacuum as follows,

$$E_0^\theta(r, z, t) = E_0^\theta K_0(r/\mu)e^{i\sqrt{\omega^2/c^2 + 1/\mu^2}z}e^{-\omega t},$$

where $K_0(r/\mu)$ is the zeroth-order modified Bessel function of the second kind and $E_0^\theta$ and $\mu$ are arbitrary parameters. Note that the electric field given by Eq.(18) is confined in a region $r = \sqrt{x^2 + y^2} < \mu$ around $x = y = 0$ in the $xy$-plane.

Using Eq.(18), one can construct a wave-packet solution as

$$E(r, z, t) = E_0^\theta K_0(r/\mu) \int_0^\infty d\omega \rho(\omega) \exp\left[i\sqrt{\omega^2/c^2 + 1/\mu^2}z - i\omega t\right]e_\theta,$$

where $\rho(\omega)$ is a frequency distribution function for the EM field forming the wave packet. We assume a Gaussian-type distribution function having the center at $\omega = \nu$ for $\rho(\omega)$, i.e.,

$$\rho(\omega) = \rho_0 e^{-\tau^2(\omega-\nu)^2/2},$$

where $\tau^{-1}$ gives the standard deviation of the Gaussian distribution. Then, substituting Eq.(20) into Eq.(19) and utilizing the approximation,

$$\exp\left[i\sqrt{\omega^2/c^2 + 1/\mu^2}z - i\omega t\right] \simeq e^{inz - ivt} e^{i(\omega-\nu)(v_g^{-1}z-t)},$$

with

$$\eta = \sqrt{\frac{\nu^2}{c^2} + \frac{1}{\mu^2}}, \quad v_g = \frac{c^2}{\nu},$$

one can obtain

$$E(r, z, t) \simeq \sqrt{\frac{\pi \rho_0}{2\tau}} E_0^\theta e^{inz - ivt} K_0(r/\mu) \exp\left[-\frac{1}{2\tau^2}(v_g^{-1}z - t)^2\right]e_\theta,$$

(23)

Note that the wave packet (23) has a finite duration time of $\sim \tau$ and $v_g$ is understood to be the group velocity of the EM wave packet.

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