Quantum-State Engineering of Multiple Trapped Ions for Center-of-Mass Mode*

ZENG Hao-Sheng,1,2 KUANG Le-Man,2 ZHU Xi-Wen1 and GAO Ke-Lin1
1Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Wuhan Institute of Physics and Mathematics, The Chinese Academy of Sciences, Wuhan 430071, China
2Department of Physics, Hunan Normal University, Changsha 410081, China

(Received January 2, 2001; Revised February 27, 2001)

Abstract We propose a scheme to generate a superposition of coherent states with arbitrary coefficients on a line in phase space for the center-of-mass vibrational mode of N ions by means of isolating all other spectator vibrational modes from the center-of-mass mode. It can be viewed as the generalization of previous methods for preparing motional states of one ion. For a large number of ions, only one cyclic operation enables one to generate such a superposition of many coherent states.

PACS numbers: 42.50.Vk
Key words: center-of-mass mode, coherent state

1 Introduction

In recent years, there has been much interest in the preparation of nonclassical quantum states, especially the engineering of arbitrary quantum states, in order to study fundamental properties of quantum mechanics. Many schemes have been proposed for the purpose of engineering various quantum states,[11–11] especially the superpositions of coherent states on a circle or superpositions of coherent states on a line in the field of cavity and trapped ions.[12–15] Because discrete superpositions of coherent states on a circle or on a line may approximate many quantum states,[12] such as number states, amplitude-squeezed states and quadrature squeezed states, which provides a new way for quantum-state engineering.

In previous papers on ion trap, one usually prepares the center-of-mass vibrational mode (COM mode) of one trapped ion to some quantum states, where only one vibrational mode has been involved. In this paper we shall consider the general case of N ions, where many other spectator vibrational modes are involved besides COM mode. We find that in our method one can still prepare the COM mode to a discrete superposition of coherent states on a line in phase space, and the contribution from the spectator vibrational modes can be neglected because of the very large off-resonant with them. We also find that for a large number of ions one needs only one cyclic operation to prepare a superposition of a number of coherent states of COM mode on a line, which may need many cycles for one ion. Thus the operation time for quantum state preparation can be greatly reduced, which is of importance for the experimental realization in the view of decoherence.

As the usual treatment for quantum computation, we employ the ground state |0⟩ and a particular long-lived metastable excited state |1⟩ of the trapped ion. The transition between these two internal states is dipole forbidden, but we can drive it by irradiating the ion with a pulsed laser that tuned to the frequency of the transition. In order to measure the internal state, we employ the third auxiliary electronic level |2⟩ of the ion[5,16,17] which has a strong electronic transition to the ground state |0⟩. By directing a laser resonant with the transition |0⟩ → |2⟩ on the ion, and then probing for the occurrence of fluorescence light, the electronic quantum state is projected either onto the ground or excited state, conditioned on the observation of fluorescence or of no-fluorescence event. Since any spontaneously emitted photon will disturb the motional quantum state via recoil effects, so we consider only those events where no fluorescence has been observed, i.e., the case that the internal state is projected onto the excited state |1⟩.

This paper is organized as follows: In Sec. 2 we introduce the laser-atom interaction mode which was first proposed by Sørensen and Mølmer. We consider the case in Lamb–Dicke limit and low-excitation regime, where we assumed that the two lasers performing on each ion are resonant with the red and blue sidebands for COM mode, so that we can use the approximation of RWA to omit the effects from all other spectator vibrational modes. In Sec. 3 we describe the method to generate arbitrary superpositions of coherent states of COM mode on a line in phase space for N ions. A summary is given in Sec. 4.

2 Laser-Atom Interaction

According to the consideration of Sørensen and Mølmer,[18,19] we consider a string of N ions trapped in a linear trap, which are strongly bounded in the y and

*The project supported by National Natural Science Foundation of China (19734006 and 69873015) and the Foundation of the Chinese Academy of Sciences
and the internal and motional state operators are defined

\[ H = H_0 + H_{\text{int}}, \]

\[ H_0 = \sum_{l=1}^{N} \nu_l (a_l^\dagger a_l + 1/2) + \omega \sum_{l} \sigma_{lz}/2, \]

\[ H_{\text{int}} = \sum_{l} \frac{\Omega_l}{2} \left( \sigma_{l+} e^{-i\omega_l t} + e^{i\omega_l t} \right) \]

where \( \nu_l \) and \( a_l^\dagger, a_l \) are the frequency and ladder operators of the \( l \)th mode, \( \omega \) is the energy difference between the ground state \( |0\rangle \) and the long-lived metastable excited state \( |1\rangle \) of the ion. For simplicity we further assume that the Rabi frequency for all ions participating in the operation is the same. \( \omega_1 = \omega + \delta \) and \( \omega_2 = \omega - \delta \) are the frequencies of two lasers addressing on each ion, and \( \delta \) is the detuning. The excursion of the \( i \)th ion in the \( l \)th mode is described by the Lamb–Dicke parameter \( \eta_{i,l} \) which may be represented as \( \eta_{i,l} = \eta \sqrt{N} b_l^i / \sqrt{\nu} \), where \( \eta \) and \( \nu \) refer to COM mode, and where \( b_l^i \) obeys the orthogonality conditions \( \sum_{i=1}^{N} b_l^{i}\dagger b_l^{i'} = \delta_{l,l'} \) and \( \sum_{l=1}^{N} b_l^{i}\dagger b_l^{i'} = \delta_{i,i'} \). For COM vibrational mode we have \( l = 1 \) and \( b_1^i = 1/\sqrt{N} \) for all ions. And \( l = 2 \) represents breathing mode which corresponds to each ion oscillating with an amplitude proportional to its equilibrium distance from the trap center, and \( l > 2 \) denote high-order normal modes. If all modes are suitably cooled and within the Lamb–Dicke regime, then we can expand the Hamiltonian up to the first order in \( \eta_{i,l} \), and in the interaction picture with respect to \( H_0 \), it becomes

\[ H_{\text{int}} = 4\Omega J_x \cos(\delta t) + \sum_{i=1}^{N} \Theta_i [x_i f_i(t) + p_i g_i(t)], \]

where

\[ f_i(t) = -2\sqrt{2} \eta \nu_l / \nu_i [\cos(\nu_i - \delta)t + \cos(\nu_i + \delta)t], \]

\[ g_i(t) = -2\sqrt{2} \eta \nu_l / \nu_i [\sin(\nu_i - \delta)t + \sin(\nu_i + \delta)t], \]

and the internal and motional state operators are defined by

\[ J_\alpha = \sum_{i} J_{i\alpha} = \sum_{i} \frac{\sigma_{i\alpha}}{2} \quad (\alpha = x, y, z), \]

\[ \Theta_i = \sqrt{N} \sum_{i=1}^{N} b_l^{i}\dagger j_{i\nu}, \]

\[ x_i = \frac{1}{\sqrt{2}} (a_i^\dagger + a_i^\prime), \quad p_i = \frac{i}{\sqrt{2}} (a_i^\dagger - a_i) \]

If we want to engineer the COM mode as a superposition of coherent states, then we can select the lasers performing on each ion to be resonant with the first red and blue sidebands for COM mode, i.e. \( \delta = \nu \). Choosing laser intensity \( \Omega \ll \delta \), we may neglect all the terms oscillating fast over time and only preserve the constant terms, so the Hamiltonian reads

\[ H_{\text{int}} = -2\sqrt{2} \eta \Omega J_x(a + a^\dagger). \]

Note that the spin operators for different ions commute, so the propagator for this Hamiltonian is

\[ U = \prod_{i=1}^{N} \frac{1}{2} \left( [D^+(\beta) + D(\beta)] - \sigma_{i\beta} [D^+(\beta) - D(\beta)] \right), \]

where \( \beta = i\eta \Omega t \) and \( D(\beta) = e^{\beta a^\dagger - \beta^* a} \).

### 3 Quantum-State Engineering for COM Mode

#### 3.1 The Case for One Ion (\( N = 1 \))

This case has already been discussed in Ref. [15]. For convenience of expression, we briefly repeat it here. We assume the initial internal state of the trapped ion to be in an arbitrary superposition of ground state \( |0\rangle \) and metastable excited state \( |1\rangle \) which can be realized by the application of a laser field resonant with the electronic transition, and the motional state to be in coherent state \( |\alpha\rangle \), so that the whole initial state of the ion is

\[ |\Psi(t_0)\rangle = \frac{1}{\sqrt{1 + |p_1|^2}} (|1\rangle + i p_1 |0\rangle) |\alpha\rangle, \]

where the parameter \( p_1 \) controls the weights of the two electronic levels, \( i \) denotes the imaginary unit.

We now illuminate the atom with two lasers discussed above for an interaction time \( t \), following by a measurement on the electronic state. With no fluorescence being detected, the internal quantum state is projected onto the excited state \( |1\rangle \), and the resulting conditioned vibrational quantum state at time \( t_1 \) reads

\[ |\Psi(t_1)\rangle \sim \frac{1}{2\sqrt{1 + |p_1|^2}} [(1 - p_1) D(\beta) + (1 + p_1) D(-\beta)] |\alpha\rangle. \]

Following the method in Ref. [15], we repeat this procedure \( n \) cycles with each cycle having the same iteration time and having controlling weights \( p_2, p_3, \ldots, p_n \) respectively. After the last detection, the motional state for COM mode is

\[ |\Psi(t_n)\rangle \sim \mathcal{R}_n \prod_{i=1}^{n} [(1 - p_i) D(\beta) + (1 + p_i) D(-\beta)] |\alpha\rangle \]
\[ = \mathbb{N}_n \sum_{k=0}^{n} C^k_n D[(2k - n) \beta] |\alpha\rangle \] (7)

with \( \mathbb{N}_n = \prod_{i=1}^{n} \frac{1}{2} (1 + |p_i|^2)^{-1/2} \), and

\[ C^k_n = \sum_{|\{k\}|i \in |\{n-k\}|} \prod_{j \in |\{n-k\}|} (1 + p_j) \prod_{i \in |\{k\}|} (1 + p_i) \], (8)

where \(|\{k\}|\) denotes a set that picking \( k \) numbers out of \( n \) natural numbers corresponding to \( n \) parameters \( p_i \), and \(|\{n-k\}|\) denotes a complementary set to \(|\{k\}|\). The first \( \prod \) represents the multiplication of \( k \) factors \( (1 - p_i) \) with \( i \) being the element of set \(|\{k\}|\), and the second \( \prod \) represents the multiplication of \( (n - k) \) factors \( (1 + p_j) \) with \( j \) being the element of set \(|\{n-k\}|\). The sum goes over all possible different sets \(|\{k\}|\) that formed by picking \( k \) numbers out of \( n \) natural numbers.

Equation (7) represents a discrete superposition of coherent states on a line in phase space with distance \( 2\beta \) between them and centered around the phase-space point of the initial coherent state \(|\alpha\rangle\). For the purpose of state preparation, we can determine all the controlling weight parameters \( p_i \) \((i = 1, \ldots, n)\) from known coefficients \( C^k_n(k = 0, \ldots, n)\) (See Ref. [15]).

### 3.2 The Case for Two Ions \((N = 2)\)

This case is usually viewed as the typical candidate used to implement the universal two-bit gate for quantum computation in an ion trap. In this case, the two ions are confined symmetrically in the two sides of the center of the linear trap along the axis. The external confining potential is symmetric about the center of the trap. Due to the Coulomb repulsive force between them, the two ions retain a certain distance. Similar to the above discussion, we prepare the initial state of the system of the two ions to be

\[ |\Psi(t_0)\rangle = \mathbb{N}_0 ([|1\rangle + ip_1|0\rangle)(|1\rangle_2 + ip_2|0\rangle_2)|\alpha\rangle \], (9)

where \( \mathbb{N}_0 = [(1 + |p_1|^2)(1 + |p_2|^2)]^{-1/2} \) with parameters \( p_1 \) and \( p_2 \) being the controlling weights of the electronic levels of the two ions respectively. Then we illuminate both two ions simultaneously with two lasers, i.e., performing the propagator (4) with \( N = 2 \) on the above initial state (9) for an interaction time \( t \), following by a measurement on the internal state of two ions. If we detect the two ions both in excited state \(|1\rangle\), then the conditioned state for the system reads

\[ |\Psi(t_1)\rangle \sim \mathbb{N}_2 \prod_{i=1}^{2} ((1 - p_i)D(\beta) + (1 + p_i)D(-\beta)) |\alpha\rangle \] (10)

with \( \mathbb{N}_2 \) being the weight factors \( p_3 \) and \( p_4 \) for the two ions respectively, and repeat this procedure \( m \) cycles with each cycle having the same interaction time, then the motional state of COM mode for the system of two ions is

\[ |\Psi(t_m)\rangle \sim \mathbb{N}_2 m \prod_{i=1}^{2m} ((1 - p_i)D(\beta) + (1 + p_i)D(-\beta)) |\alpha\rangle \]

\[ = \mathbb{N}_2 m \sum_{k=0}^{2m} C^k_{2m} D(2(k - m)\beta)|\alpha\rangle \], (11)

where \( \mathbb{N}_2 m \) represents a discrete superposition of \( 2m \) coherent states on a line with required coefficients \( C^k_{2m} \). The sum goes over all possible different sets \(|\{k\}|\) that formed by picking \( k \) numbers out of \( 2m \) natural numbers.

From the above two equations we can see that if we let \( 2m = n \), then equations (11) and (12) are the same with Eqs (7) and (8) respectively. In other words, the form of the motional state by repeating \( m \) cycles for two ions is the same as that for one ion by repeating \( 2m \) cycles except for the difference between the motional states for one ion and two ions. Therefore the parameters \( p_i \) for two ions by repeating \( m \) cycles can be obtained from the parameters \( p_i \) for one ion by repeating \( 2m \) cycles which have the ready-made result in Ref. [15]. Thus we have prepared a superposition of \( 2m \) coherent states on a line with required coefficients for COM mode of two trapped ions by repeating \( m \) cycles. Note that this method can also be generalized to more ions.

### 3.3 The Case for \(N\) Ions

With \( N \) ions confined in the linear trap, the motion of each ion will be influenced by an overall harmonic potential due to the trap electrodes and by the Coulomb force exerted by all of the other ions. They consist of a multiple-ion-system. We usually describe this system in terms of COM mode and other high-order vibrational modes. But we find that in Sec. 2 we can only consider the contribution from COM mode, and neglect the effects from all other high-order vibrational modes due to the large off-resonant with these modes. Now we prepare the initial state of the system of \( N \) ions to be

\[ |\Psi(t_0)\rangle = \prod_{i=1}^{N} \frac{1}{\sqrt{1 + |p_i|^2}} (|1\rangle_i + ip_i|0\rangle_i)|\alpha\rangle \] (13)

with parameters \( p_i \) \((i = 0, \ldots, N)\) being the weight factors.

Then illuminate all \( N \) ions simultaneously with two lasers, i.e., performing the propagator (4) on the state (13), for an interaction time \( t \), following by a measurement on the internal state of all \( N \) ions. If we detect all
ions in excited state [1], then the conditioned motional state for \( N \) ions reads

\[
|\Psi(t_i)\rangle \sim \mathcal{N} \prod_{i=1}^{N} \{(1 - p_i)D(\beta) + (1 + p_i)D(-\beta)\}|\alpha\rangle
\]

\[
= \mathcal{N} \sum_{k=0}^{N} C_N^k D((2k - N)\beta)|\alpha\rangle.
\]

This superposition form of coherent states for \( N \) ions is completely the same as Eq. (7) for one ion if we set repeating times satisfying \( n = N \) in there. And the coefficients \( \mathcal{N} \) and \( C_N^k \) are also the same as Eq. (8) with \( n = N \). But the implications for these two cases are completely different: equation (7) represents a superposition of COM vibrational mode for one ion, however equation (14) represents a superposition of COM vibrational mode for \( N \) ions. When \( N \) is large, only one cyclic operation enables one to obtain a superposition of a number of coherent states on a line, which may need many cycles in the case for one ion. Owing to the fact that the operations on each ion can be performed simultaneously, so it can save much time in process of state-preparation. It is important for the experimental realization of quantum-state engineering in the view of decoherence. We also note that in our method the contribution from the spectator’s vibrational modes can be neglected because of the large off-resonant with them. It is worth while to note that we employed some conditions for our method. The conditions of low-excitation and resonant with sidebands can be met easily in practice, so the main problem in this method is the realization of cooling all vibrational modes to the Lamb–Dicke limit. But for less ions we may expectantly reach this limit, thus the present scheme might be experimentally feasible.

4 Conclusion

In this paper we have shown that how an arbitrary superposition of equidistant coherent states can be created on a line in phase space for the COM mode of \( N \) ions. This approach can be viewed as the generalization of the previous methods for preparing motional states of one ion. After an initial preparation of internal state of every ion, two lasers are projected on each ion to entangle the vibrational modes with internal states. And then measuring the internal state of each ion, with no fluorescence for all ions one disentangles the vibrational modes and creates a superposition of ionic motion. By performing this procedure several times according to requirements, the desired superposition of displacing initial coherent state on a line is generated for the COM vibrational mode of \( N \) ions.

We note that for a large number of ions only one cyclic operation can obtain such a superposition of many coherent states for the COM vibrational mode. Because the operations on each ion can be performed simultaneously, the operation time for quantum-state preparation is greatly reduced. It is important for the experimental realization in the view of decoherence. We also note that in our method the contribution from the spectator’s vibrational modes can be neglected because of the large off-resonant with them. It is worth while to note that we employed some conditions for our method. The conditions of low-excitation and resonant with sidebands can be met easily in practice, so the main problem in this method is the realization of cooling all vibrational modes to the Lamb–Dicke limit. But for less ions we may expectantly reach this limit, thus the present scheme might be experimentally feasible.

References

[1] P. Filipowicz, J. Javanainen and P. Meystre, Phys. Rev. A34 (1986) 3077.
[2] A.S. Parks, P. Marte, P. Zoller and H.J. Kimble, Phys. Rev. Lett. 71 (1993) 3095.
[3] J.I. Cirac, R. Blatt, A.S. Parks and P. Zoller, Phys. Rev. Lett. 70 (1993) 762.
[4] J.I. Cirac, A.S. Parks, R. Blatt and P. Zoller, Phys. Rev. Lett. 70 (1993) 556.
[5] J.F. Poyatos, J.I. Cirac, R. Blatt and P. Zoller, Phys. Rev. A54 (1996) 1532.
[6] C.C. Gerry, Phys. Rev. A55 (1997) 2478.
[7] S.B. Zheng, Phys. Rev. A58 (1998) 761.
[8] S.B. Zheng, Phys. Lett. A245 (1998) 11.
[9] S.B. Zheng, Optics Communication 170 (1999) 67.
[10] E. Solano, P. Milman, R.L. de Matos Filho and N. Zagury, Phys. Rev. A62 (2000) 021401.
[11] C. Monre, D.M. Meekhof, B.E. King and D.J. Wineland, Science 272 (1996) 1131.
[12] J. Janszky, P. Domokos and P. Adam, Phys. Rev. A48 (1993) 2213; P. Domokos, J. Janszky and P. Adam, ibid. A50 (1994) 3340; S. Szabol, P. Adam, J. Janszky and P. Domokos, ibid. A53 (1996) 2698.
[13] S.B. Zheng and G.C. Guo, Quantum Semiclassic. Opt. 9 (1997) L45.
[14] S.B. Zheng, X.W. Zhu and M. Feng, Phys. Rev. A62 (2000) 033807.
[15] H. Moya-Cessa, S. Wallentowitz and W. Vogel, Phys. Rev. A59 (1999) 2920.
[16] S. Wallentowitz and W. Vogel, Phys. Rev. A54 (1996) 3322.
[17] R.L. de Matos Filho and W. Vogel, Phys. Rev. Lett. 76 (1996) 4520.
[18] A. Sørensen and K. Mølmer, Phys. Rev. Lett. 82 (1999) 1971; K. Mølmer and A. Sørensen, ibid. 82 (1999) 1835.
[19] A. Sørensen and K. Mølmer, Phys. Rev. A62 (2000) 022311.
[20] D.F.V. James, Appl. Phys. B: Lasers Opt. 66 (1998) 181.