Quark and lepton flavor model with leptoquarks in a modular $A_4$ symmetry

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Abstract We propose a quark-lepton model via leptoquarks and modular $A_4$ symmetry. Since the neutrino mass is induced at one-loop level mediated by down quarks as well as leptoquarks, we have to explain lepton and quark masses and mixings with a single modulus $\tau$. Here, we find predictions for lepton and quark sectors with unified modulus $\tau$, and show several constraints originating from leptoquarks.

1 Introduction

Since lepto-quark(LQ) bosons connect lepton and quark sectors, these models potentially explain several new physics beyond the standard model (SM); e.g., lepton(muon or electron) anomalous magnetic dipole moment ($\Delta a_\mu$) [1–6], $B$ meson decays such as $B \to s\mu\bar{\mu}$ [2,4,5,7–9] and $B \to c\ell\bar{\nu}_\ell(\ell = e, \mu, \tau)$ [4,8–10], $1$ and nonzero neutrino masses [6,9,11,14]. Especially, muon $\Delta a_\mu - 2$ anomaly is recently reported by E989 experiment at Fermilab combining BNL result [22], and its value is deviated from the SM by $4.2\sigma$ as follows:

$$\Delta a_\mu = (25.1 \pm 5.9) \times 10^{-10}. \quad (1.1)$$

Also, the LHCb collaboration [23] recently reported anomaly of rare $B$ meson decays of $b \to s\mu\bar{\mu}$ that is understood as violation of lepton universality. The updated result is given by

$$\frac{BR(B^+ \to K^+\mu^-\mu^+)}{BR(B^+ \to K^+e^-\negrightarrow)} = 0.846_{-0.039}^{+0.042} \pm 0.013 \quad (1.1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2), \quad (1.2)$$

where first(second) uncertainty is statistical(systematic) one and $q^2$ is the invariant mass squared for dilepton. In addition to the above phenomenologies, interestingly, the nonzero Majorana neutrino mass at one-loop can be realized without any additional symmetries by introducing appropriate LQs [12–14]. This may be natural realization of tiny neutrino mass model due to loop suppression.

Considering above issues, one finds that Yukawa flavor structure is also very important to explain them. Recently, powerful symmetries to restrict the number of parameters in Yukawa couplings, so called “modular flavor symmetries”, were proposed by authors in Refs. [24,25], in which they have applied modular originated non-Abelian discrete flavor symmetries to quark and lepton sectors. One remarkable advantage of applying this symmetries is that dimensionless couplings of model can be transformed into non-trivial representations under those symmetries, and all the dimensionless values are uniquely fixed once modulus is determined in fundamental region. We then do not need the scalar fields to obtain a predictive mass matrix. Along the line of this idea, a vast reference has recently appeared in the literature, e.g., $A_4$ [24,26–58], $S_3$ [59–64], $S_4$ [65–76], $A_5$ [70,77,78], double covering of $A_5$ [79–81], larger groups [82], multiple modular symmetries [83], and double covering of $A_4$ [84,85], $S_4$ [86,87], and the other types of groups [88–93] in which masses, mixing, and CP phases for the quark

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$^1$ The anomaly of $B \to c\ell\bar{\nu}_\ell$ processes are observed in experiments [15–21], and LQ model is one of the most promising explanations on this anomaly.

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and/or lepton have been predicted.\textsuperscript{2} Moreover, a systematic approach to understand the origin of CP transformations has been discussed in Ref. [102], and CP/flavor violation in models with modular symmetry was discussed in Refs. [56, 103–105], and a possible correction from Kähler potential was discussed in Ref. [106]. Furthermore, systematic analysis of the fixed points (stabilizers) has been discussed in Ref. [107].

A very recent paper of Ref. [108] finds a favorable fixed point
\[
\tau = (\text{domain of } PSL(A) \text{, } \omega)
\]
are motivated to consider lepton and quark sector together under the framework of modular flavor symmetry since we compactifications as well as investigating the probabilities of affect the leptons(quarks). In this sense, it would be a good tor via LQ, charge assignments for quarks(leptons) directly couplings. Since the quark sector connects to the lepton sector under 4 symmetry. In the quark sector, we assign
\[
\eta(\text{2,3 electric charges mix each other. Here, we parametrize their mixing matrices and mass eigenstates as follows:})
\]
where the subscript of the fields represents the electric charge, and \(u^+\) and \(\bar{z}\) are absorbed by the longitudinal component of the \(W^+\) and \(Z\) bosons, respectively. Due to the \(\mu\) term in Eq. (2.4), the charged components with 1/3 and 2/3 electric charges mix each other. Here, we parametrize their mixing matrices and mass eigenstates as follows:
\[
O_i = \begin{pmatrix} c_{a_i} & s_{a_i} \\ -s_{a_i} & c_{a_i} \end{pmatrix}, \quad (i = 1, 2).
\]
where their masses are denoted as \(m_{A_i}\) and \(m_{R_i}\), respectively. The interactions in terms of the mass eigenstates can be written as
\[
\begin{align*}
- L_Y^B & \approx m_{dij} \bar{d}_R i u_{L_j} + m_{dij} \bar{d}_R d_{L_j} + h.c., \\
- L_Y^C & \approx m_{\ell_{ij}} \bar{\ell}_R \ell_{L_j} + h.c., \\
- L_Y^{mix} & \approx f_{ij} \bar{d}_R v_{L_j} (c_{a_i} A_1^* + s_{a_i} B_1^*) \\
& + g_{ij} \bar{d}_R v_{L_j} (-s_{a_i} A_1 + c_{a_i} B_1) \\
& - f_{ij} \bar{d}_R \ell_{L_j} (c_{a_i} A_2 + s_{a_i} B_2) \\
& - g_{ij} \bar{d}_R \ell_{L_j} (-s_{a_i} A_1 + c_{a_i} B_1) \\
& - g_{ij} \bar{d}_R \ell_{L_j} \delta_{4/3} + g_{ij} \bar{u}_R v_{L_j} (-s_{a_i} A_2^* + c_{a_i} B_2^*) + h.c.,
\end{align*}
\]
where we define \(m_{a_{ij}} \equiv v_{m_{ij}}/\sqrt{2}\), \(m_{d_{ij}} \equiv v_{m_{ij}}/\sqrt{2}\), and \(m_{\ell_{ij}} \equiv v_{m_{ij}}/\sqrt{2}\).

The next task is to determine the matrices of \(\gamma^a, y^d, h, f, g\) via modular \(A_4\) symmetry. In the quark sector, we assign \(Q_L\) to be 3 and \(-2\), \(\bar{u}_R\) to be \(1, 1''\), \(-1\) and \(-4\), and \(d_R\) to be \(1, 1''\), \(1'\) and 0 under \(A_4\) and \(-k\), respectively. This assignment is the same as the one in Ref. [45], and it is already

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The up-type quark mass matrix is written as:

\[
y^u = \begin{pmatrix} a_u & 0 & 0 \\ 0 & a_c & 0 \\ 0 & 0 & a_t \end{pmatrix} \begin{pmatrix} f_1 & f_3 & f_2 \\ f_2 & f_1 & f_3 \\ f_3 & f_2 & f_1 \end{pmatrix} + \begin{pmatrix} g_{u1} & 0 & 0 \\ 0 & g_{u2} & 0 \\ 0 & 0 & g_{u3} \end{pmatrix} \begin{pmatrix} f'_1 & f'_3 & f'_2 \\ f'_2 & f'_1 & f'_3 \\ f'_3 & f'_2 & f'_1 \end{pmatrix},
\]

(2.12)

where \( Y_3^{(6)} = [f_1, f_2, f_3]^T \) and \( Y_3^{(6)} = [f'_1, f'_2, f'_3]^T \), \( g_{u1} = \alpha_u/\alpha_u, g_{u2} = \beta_u/\beta_u \) and \( g_{u3} = \gamma_u/\gamma_u \) are complex parameters, and \( a_u, a_c, \) and \( a_t \) can be used to fit the masses of up quarks. The explicit forms of \( f_1 \) and \( f'_1 \) are summarized in Appendix. Then \( m_u \) is diagonalized by two unitary matrices as \( D_u = V_{uR}^T m_u V_{uL} \), where \( D_u \equiv \text{diag}(m_u, m_c, m_t) \) is mass eigenvalues. Therefore, we find \( |D_u|^2 = V_{uR}^T m_u^2 V_{uL} \).

On the other hand, the down-type quark mass matrix is given as:

\[
y^d = \begin{pmatrix} a_d & 0 & 0 \\ 0 & a_s & 0 \\ 0 & 0 & a_b \end{pmatrix} \begin{pmatrix} y_1 & y_3 & y_2 \\ y_2 & y_1 & y_3 \\ y_3 & y_2 & y_1 \end{pmatrix},
\]

(2.13)

where \( a_d, a_s, \) and \( a_b \) can be used to fit the masses of down quarks, and \( Y_3^{(2)} = [y_1, y_2, y_3]^T \) in Appendix. Then \( m_d \) is diagonalized by two unitary matrices as \( D_d = V_{dR}^T m_d V_{dL} \), where \( D_d \equiv \text{diag}(m_d, m_s, m_b) \) is mass eigenvalues. Therefore, we find \( |D_d|^2 = V_{dR}^T m_d^2 V_{dL} \). Finally, we get the observable mixing matrix \( V_{CKM} \) as follows:

\[
V_{CKM} = V_{uL}^T V_{dL}.
\]

(2.14)

2.1 Lepton sector

Now let us move on the lepton sector. We assign \( L_L \) to be 3 and \(-2 \) and \( \bar{e}_R \) to be \( (1, 1', 0') \) and 0 under \( A_4 \) and \(-k \), respectively. Here, both of the leptoquark scalars are assigned to be true \( A_4 \) singlets with \(-2 \) modular weight. The assignments of \( A_4 \) and \(-k \) are also summarized in Tables 1 and known that allowed region [30]. Thus, we will work on the same \( \tau \) region of the lepton sector in our numerical analysis.

The charged-lepton sector after spontaneous symmetry breaking is given by

\[
-L_Y = \frac{\hbar}{\sqrt{2}} \ell_L e_R + \text{h.c.}
\]

(2.18)

Then \( m_\ell (\equiv v h/\sqrt{2}) \) is diagonalized by two unitary matrices as \( D_\ell = V_\ell^T m_\ell V_\ell \), where \( D_\ell \equiv \text{diag}(m_e, m_\mu, m_\tau) \) is mass eigenvalues. Therefore, we find \( |D_\ell|^2 = V_\ell^T m_\ell^2 V_\ell \). The active neutrino mass matrix \( m_\nu \) is given at one-loop level through the following interactions:

\[
-L_Y = F_{aj} d_{Rj} v_{Lj} (c_{a1} A_1 + s_{a1} B_1) - G_{aj} d_{Lj} v_{Lj} (s_{a1} A_1 + c_{a1} B_1),
\]

(2.19)

where \( F \equiv V_\ell^T f \) and \( G \equiv V_\ell^T g \) and \( d' \) is mass eigenstate. Then, the neutrino mass matrix in Fig. 1 is given at one-loop level as follows:

\[
(m_\nu)_{ij} = s_{2a1} \frac{3}{4(4\pi)^2} \left[ \frac{1}{m_{A_1}^2} \right] \times \sum_{i=1}^{3} \left[ F_{io} D_{oi} G_{oj} + G_{io} D_{oi} F_{oj} \right] F_i (r_{Ai}, r_{Di}).
\]

(2.20)

\[
F_i (r_1, r_2) = \frac{r_1 (r_2 - 1) \ln r_1 - r_2 (r_1 - 1) \ln r_2}{(r_1 - 1) (r_2 - 1) (r_1 - r_2)}. \quad (r_1 \neq 1),
\]

(2.21)
Table 2 Charge assignments of the SM fermions under \(SU(3)_C \times SU(2)_L \times U(1)_Y \times A_4\) where \(k_I\) is the number of modular weight

| Fermions | \(Q_L\) | \(u_R\) | \(d_R\) | \(L_L\) | \(e_R\) |
|----------|---------|---------|---------|--------|--------|
| \(SU(3)_C\) | 3       | \(\tilde{3}\) | \(\tilde{3}\) | 1      | 1      |
| \(SU(2)_L\) | 2       | 1       | 1       | 2      | 1      |
| \(U(1)_Y\) | \(\frac{k_I}{3}\) | \(\frac{1}{3}\) | \(\frac{1}{3}\) | \(\frac{1}{2}\) | 1      |
| \(A_4\) | 3       | 1, 1', 1'' | 1, 1', 1'' | 3      | 1, 1', 1'' |
| \(-k_I\) | -2      | -4      | 0       | -2     | 0      |

3 Numerical analysis

Here, we perform numerical analysis. Before searching for allowed region, we fix some mass parameters as \(m_{A_2} = m_{A_1}\) and \(m_{B_3} = m_3 = m_{B_1}\), where we require degenerate masses for the components of \(\eta\) and \(\Delta\) to suppress the oblique parameters \(\Delta S\) and \(\Delta T\). Notice here that our theoretical parameters \(a_{\alpha,c,t}, \alpha_{d, t, b}, \alpha_{c, \beta, c_\ell}\) are used to determined the experimental masses for quarks and charged-leptons. Thus, only the following input parameters are randomly selected in the range of

\[
(m_{A_1}, m_{B_1}) \in [1, 100] \text{ TeV},
\]

\[
|\delta_{a_{\alpha, c, t}, \alpha_{d, t, b}, \alpha_{c, \beta, c_\ell}}| \leq \left[10^{-5}, 1.5\right]
\]

\[
(|a_\eta|, |b_\eta|, |c_\eta|, |a|, |b|, |c|, |b'|, |c'|) \leq \left[10^{-5}, 10\right]
\]

Above the range, we have numerical analysis in cases for quark and lepton, where experimental data in the quark sector should be within the range at 3\(\sigma\). While the one in the lepton sector is discussed in the range within 3\(\sigma\) (yellow dots) and 5\(\sigma\) (red dots) applying \(\chi^2\) analysis in NuFit 5.0.

3.1 NH

For NH case, we show our results of lepton sector in Figs. 2, 3, 4, 5. In Fig. 2, allowed value of \(\tau\) is shown where yellow(red) points present the values within 3(5)\(\sigma\). One finds that allowed space is rather localized. Especially, the region at nearby \(\tau \sim 1.75\) would be interesting since it is close to the fixed point that has a remnant \(Z_3\) symmetry. In Fig. 3, we demonstrate allowed region of \(\delta_{CP}\) in terms of \(\sum_{m_i} \sum_{m_i}\) is rather localized at 0.06-0.08 eV while whole the region is allowed for \(\delta_{CP}\). Moreover, almost all the points are within the cosmological constraint \(\sim 0.12\) eV [112]. In Fig. 4, we present allowed region of neutrinoless double beta decay \(m_{ee}\) in terms of the lightest neutrino mass \(m_1\). \(m_{ee}\) is allowed up to 0.025 eV while \(m_1\) is allowed up to 0.0025 eV. Moreover, allowed region of \(m_1\) is localized at around \(10^{-6}\) eV indicating tiny mass of the lightest neutrino mass. In Fig. 5, we
Fig. 2  Allowed value of $\tau$. Yellow and red points present the values within 3 and 5$\sigma$

Fig. 3  Allowed region of $\delta_{CP}$ in terms of $\sum m_i$

Fig. 4  Allowed region of the mass of neutrinoless double beta decay in terms of the lightest neutrino mass

Fig. 5  Allowed region of Majorana phases $\alpha_{21}$ in terms of $\alpha_{31}$

Fig. 6  The CP phase of quark $\delta$ versus (1, 3) component of CKM matrix. The red dashed lines represent 3$\sigma$ experimental bounds

Fig. 7  $|V_{ub}|$ versus $|V_{td}|$

In Fig. 6, we show the CP phase of quark $\delta$ in term of (1, 3) component of CKM matrix; $|V_{ub}|$, and find whole the region is allowed at 3$\sigma$ interval. In Fig. 7, we show $|V_{ub}|$ and $|V_{td}|$, and found that there is a weak linearly correlation between them. In Fig. 8, we show $|V_{cb}|$ and $|V_{ub}|$, and find that there is also a weak linear correlation between them.

Bench mark point for NH: We also give a benchmark point to satisfy the quark and lepton masses and mixings as well as phases in the left sides of Tables 3 and 4, where we extracted a value at nearby $\tau = 1.75i$. The corresponding lepton and neutrino mixings are given by
there is a weak linearly correlation between them. As for $|V_{cb}|$ and $|V_{ub}|$, we find that there is also a weak linear correlation between them.

**Bench mark point for IH**: We give two interesting benchmark points: $\tau \approx 1.06i$, $1.76i$ to satisfy the quark and lepton masses and mixings as well as phases in the center and right sides of Tables 3 and 4. The lepton and neutrino mixings are given by

$$\tau \approx 1.06i :$$

$$V_{\ell L} = \begin{bmatrix}
-0.65 + 0.0068i & 0.072 + 0.00024i & -0.25 + 0.00061i \\
-0.47 + 0.0067i & -0.64 + 0.0012i & -0.61 + 0.00072i \\
-0.60 + 0.0068i & -0.28 + 0.00098i & 0.75 + 0.000042i
\end{bmatrix},$$

$$V_{\nu L} = \begin{bmatrix}
-0.63 + 0.12i & 0.053 - 0.029i & -0.13 - 0.75i \\
0.990 + 0.11i & 0.14 + 0.098i & 0.015 - 0.089i \\
-0.75 + 0.10i & 0.12 + 0.097i & 0.068 + 0.63i
\end{bmatrix}. \tag{3.6}$$

$$\tau \approx 1.76i :$$

$$V_{\ell L} = \begin{bmatrix}
-0.21 - 0.00031i & 0.80 - 0.0015i & 0.56 + 0.00052i \\
0.63 - 0.0065i & -0.33 + 0.0026i & 0.70 + 0.0032i \\
0.75 - 0.0099i & 0.50 - 0.0010i & -0.44 - 0.0036i
\end{bmatrix}, \tag{3.7}$$

$$V_{\nu L} = \begin{bmatrix}
-0.010 + 0.016i & 0.063 - 0.098i & -0.87 + 0.48i \\
0.28 + 0.371i & 0.77 + 0.56i & -0.12 + 0.014i \\
0.53 + 0.80i & 0.25 + 0.13i & 0.016 + 0.0094i
\end{bmatrix}. \tag{3.8}$$

The quark mixings are given by

$$\tau \approx 1.06i :$$

$$V_{uL} = \begin{bmatrix}
-0.59 + 0.26i & -0.18 - 0.067i & -0.71 - 0.23i \\
-0.55 + 0.21i & -0.50 - 0.030i & 0.61 + 0.18i \\
-0.41 + 0.28i & 0.84 - 0.062i & 0.20 + 0.080i 
\end{bmatrix}, \tag{3.11}$$

$$V_{dL} = \begin{bmatrix}
-0.60 + 0.0068i & -0.28 + 0.00096i & -0.75 - 0.00044i \\
-0.47 + 0.0067i & -0.64 + 0.0011i & 0.61 - 0.00072i \\
-0.65 + 0.0068i & 0.72 + 0.00022i & 0.25 - 0.00061i
\end{bmatrix}. \tag{3.12}$$

$$\tau \approx 1.76i :$$

$$V_{uL} = \begin{bmatrix}
-0.76 - 0.042i & 0.43 + 0.199 & -0.45 + 0.0188i \\
-0.62 + 0.015i & -0.33 - 0.052i & 0.71 + 0.026i \\
0.19 - 0.054i & 0.78 + 0.23i & 0.54 - 0.014i
\end{bmatrix}, \tag{3.13}$$

$$V_{dL} = \begin{bmatrix}
-0.64 + 0.0066i & -0.63 + 0.0011i & -0.44 - 0.0039i \\
-0.68 + 0.0064i & 0.21 - 0.00013i & 0.70 + 0.00037i \\
0.35 - 0.00023i & -0.75 + 0.0015i & 0.56 + 0.000089i
\end{bmatrix}. \tag{3.14}$$

### 4 Conclusions

We have proposed a LQ model to explain the masses and mixings for quark and lepton, introducing modular $A_4$ symmetry. Due to nature of LQ model that lepton/quark directly connects to the quark/lepton via LQ, a single modulus number has to be applied that leads to a good motivation towards unification of quark and lepton flavor in $A_4$ modular symme-
try. After giving an assignment for quark sector to reproduce the experimental results at 3σ interval, we have also constructed the lepton sector, where the neutrino mass matrix is induced at one-loop level running down quark sector, unified value of τ is used for quark and lepton. Then, we have performed numerical analysis to search for allowed region satisfying experimental measurements for both quark and lepton sector, depending on NH and IH. In case of NH, we have found rather wide allowed space within 3σ interval and obtained tendency of observables for quark and lepton. Especially, we have found allowed region at nearby τ = 1.75i that is close to the fixed point of τ = i∞. Thus, we have also shown a promising benchmark point at the nearby σ interval. Although

### Table 3 Numerical values of parameters and observables at the best fit points of NH and IH

| Lepton    | NH (τ ≈ 1.75i)                      | IH (τ ≈ 1.06i)                      | IH (τ ≈ 1.76i)                      |
|-----------|------------------------------------|------------------------------------|------------------------------------|
| τ         | -0.0000945 + 1.75i                 | -0.000689 + 1.06i                  | -0.000829 + 1.76i                  |
| $a_\eta$  | -0.23 - 1.4i                       | -0.31 + 0.013i                     | -4.1 + 4.3i                       |
| $b_\eta$  | -0.38 + 1.3i                       | -0.045 - 0.027i                    | -0.0014 + 0.0032i                 |
| $c_\eta$  | 0.0077 - 0.031i                     | 0.0014 - 0.000047i                 | -0.0023 + 0.0035i                 |
| $\alpha$  | 0.00016 + 0.00011i                 | 3.0 + 0.98i                        | 0.0085 + 0.025i                   |
| $b'$      | -0.017 + 0.0003i                    | 0.0014 + 0.00015i                  | -0.096 + 0.044i                   |
| $c'$      | -0.00300 - 0.000010i                | 0.0056 - 0.0021i                   | -1.8 + 4.5i                       |
| $\alpha_{m}A_{328}$ | 0.00014 - 0.000016i               | 0.00024 + 0.000016i                | -0.00011 + 0.00011i               |
| $\theta_{12}$ | 0.32                           | 0.28                              | 0.33                              |
| $\theta_{23}$ | 0.56                           | 0.46                              | 0.58                              |
| $\sin^2 \theta_{12}$ | 0.024                           | 0.024                             | 0.22                              |
| $\sin^2 \theta_{23}$ | 328°                          | 170°                              | 335°                              |
| $\delta_{CP}$ | 328°                          | 170°                              | 335°                              |
| $\sin^2 \theta_{13}$ | 0.024                           | 0.024                             | 0.22                              |
| $\sum m_i$ | 0.071eV                         | 0.11eV                            | 0.11 eV                           |
| $s_{\alpha_1}$ | 4.6 × 10^{-9}                 | 5.7 × 10^{-5}                     | 1.9 × 10^{-9}                     |
| $\langle m_{ee} \rangle$ | 3.2meV                        | 21meV                             | 19meV                             |
| $[m_{A_1}, m_{B_1}]$ | [18, 6.0] TeV                  | [37, 37] TeV                      | [33, 39] TeV                      |
| $\sqrt{\chi^2}$ | 2.9                               | 4.8                               | 4.5                               |

### Table 4 Numerical values of parameters and observables at the best fit points of NH and IH

| Quark    | NH (τ ≈ 1.75i)                      | IH (τ ≈ 1.06i)                      | IH (τ ≈ 1.76i)                      |
|----------|------------------------------------|------------------------------------|------------------------------------|
| τ        | -0.0000945 + 1.75i                 | -0.000689 + 1.06i                  | -0.000829 + 1.76i                  |
| $a_u$    | $1.3 \times 10^{-7}$              | $8.2 \times 10^{-8}$              | $1.1 \times 10^{-5}$              |
| $a_c$    | $4.6 \times 10^{-5}$              | $4.2 \times 10^{-5}$              | 0.0007                             |
| $a_l$    | 0.017                             | 0.017                             | 0.22                              |
| $s_{a_1}$ | 0.00066 + 0.0016i                 | -0.00013 + 0.013i                 | 0.57 - 0.35i                       |
| $s_{a_2}$ | 0.053 + 0.30i                     | 0.094 + 0.24i                     | -0.060 - 0.41i                     |
| $s_{a_3}$ | 0.11 + 0.0061i                    | 0.091 + 0.018i                    | 0.046 + 0.021i                     |
| $a_d$    | 0.00018                           | 0.00014                           | 0.00047                            |
| $a_s$    | $1.2 \times 10^{-5}$              | $8.7 \times 10^{-6}$              | $5.2 \times 10^{-5}$              |
| $a_b$    | 0.011                             | 0.011                             | 0.025                              |
| $|V_{us}|$ | 0.23                              | 0.22                              | 0.22                              |
| $|V_{cb}|$ | 0.033                             | 0.027                             | 0.042                              |
| $|V_{ub}|$ | 0.0031                            | 0.0020                            | 0.0039                             |
| $\delta_{CP}$ | 58°                              | 51°                               | 83°                               |
the number of allowed point is few, we have found all the allowed regions are localized at nearby $\tau = i, 1.76i$, both of which are nearby fixed points. We have shown them as benchmark points. These would be tested near future.

Before closing it is worthwhile mentioning on flavor physics and collider phenomenology of our model. On hadron collider experiments such as the LHC, lepto-quarks can be pair produced via strong interactions and lower limits of their masses are given as $O(1)$ TeV depending on its decay modes [113–115]. In addition the most striking signature would arise from the lepton flavor violating process at the LHC via lepto-quarks in the t-channel, $q\bar{q} \rightarrow \ell^+\ell^-$, with final states such as $e^\pm\mu^\mp, e^\pm\tau^\mp, \mu^\pm\tau^\mp$. Taking $g = f \approx 0.1$ and 1 TeV lepto-quark mass, we find 6 event rate for $d\bar{d} \rightarrow e^\pm\tau^\mp$, which is maximum, at the 13 TeV LHC with 300 fb$^{-1}$ luminosity; more discussion can be found in ref. [14]. For flavor physics, whenever considering radiative seesaw models, we have to consider lepton flavor violations, especially, $\mu \to e\gamma$. This gives the most stringent constraint on Yukawa couplings and masses of mediated particles which are lepto-quarks in our case. For our model, Yukawa couplings $f, g$ can be order 1, since the lepto-quark masses has to be larger than 1 TeV from the collider analysis at LHC. Thus, our parameters are totally safe from this constraint including other $\tau \rightarrow e(\mu)\gamma$ modes. Moreover from interactions associated with the Yukawa coupling $g$, we may find an interesting effects on $b \rightarrow s\mu\bar{\mu}$ anomaly, which can be characterized by Wilson coefficients $C_9 = -C_{10}$ of six dimensional effective Hamiltonian \[(\delta\bar{\nu}_\mu b)(\bar{\nu}_\mu^\prime b)\left((\bar{\nu}_\mu^\prime\nu_\mu)\right).\]

Experimental results tell us $\Delta C_{9} \approx 70$ new physics contribution. To get this order, we need $g \approx 0.1$, supposing 1 TeV of lepto-quark mass. However since $\Delta C_{10}$ also contributes to the process of $b \rightarrow \mu\bar{\mu}$ that gives a constraint of $\Delta C_{10} \approx 0.1$. Thus, we would need to modify the model if one wants to get the sizable anomaly of $b \rightarrow s\mu\bar{\mu}$. Yukawa coupling $g$ (as well as $f$) also receives constraints from neutral meson mixings such as K-short and K-long at one-loop box diagram. Typically, $g(f)$ should be less than ~0.1 at 1 TeV of lepto-quark mass. We have a source of muon $g = 2$ from both the Yukawa couplings $g$ and $f$ at one-loop level. However, assuming $g = f \approx 0.1$ and 1 TeV lepto-quark mass, the value of muon $g = 2$ is at most $10^{-12}$ in our model that is far from the current experimental value $10^{-9}$. Thus, we need to improve this model such that it does not include chiral suppression. In conclusion, we need extension of the model to resolve flavor anomalies such as muon $g = 2$ and $b \rightarrow s\mu\bar{\mu}$.

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**Appendix**

The modular forms of weight 2, $Y_3(2) = [y_1, y_2, y_3]^T$, transforming as a triplet of $A_4$ is written in terms of Dedekind eta-function $\eta(\tau)$ and its derivative:

$$
\begin{align*}
\eta_1(\tau) &= \frac{i}{2\pi} \left( \frac{\eta'(\tau/3)}{\eta(\tau/3)} \right) + \frac{\eta'(\tau + 1/3)}{\eta((\tau + 1)/3)} \eta(\tau) \\
\eta_2(\tau) &= \frac{i}{\pi} \left( \frac{\eta'(\tau/3)}{\eta(\tau/3)} \right) + \omega^2 \frac{\eta'(\tau + 1/3)}{\eta((\tau + 1)/3)} + \omega \frac{\eta'(\tau + 2/3)}{\eta((\tau + 2)/3)} \\
\eta_3(\tau) &= \frac{i}{\pi} \left( \frac{\eta'(\tau/3)}{\eta(\tau/3)} \right) + \omega \frac{\eta'(\tau + 1/3)}{\eta((\tau + 1)/3)} + \omega^2 \frac{\eta'(\tau + 2/3)}{\eta((\tau + 2)/3)}
\end{align*}
$$

Then, any multiplets of higher weight are constructed by multiplication rules of $A_4$, and one finds the following:

$$
\begin{align*}
Y_4(4) &= y_1^2 + 2y_2y_3, \\
Y_3(4) &= \begin{bmatrix} y_1^2 \\ y_2^2 \\ y_3^2 \end{bmatrix} \\
Y_6(6) &= y_1^2 + y_2^2 + y_3^2 - 3y_1y_2y_3,
\end{align*}
$$

$$
\begin{align*}
Y_4(6) &= \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} y_1^2 + 2y_1y_2y_3 \\ y_2^2 + 2y_1y_2y_3 \\ y_3^2 + 2y_1y_2y_3 \end{bmatrix}, \\
Y_3(6) &= \begin{bmatrix} f_1^2 \\ f_2^2 \\ f_3^2 \end{bmatrix} = \begin{bmatrix} y_1^2 + 2y_2y_3 \\ y_2^2 + 2y_1y_2 \\ y_3^2 + 2y_1y_3 \end{bmatrix}, \\
Y_6(6) &= \begin{bmatrix} f_1^2 \\ f_2^2 \\ f_3^2 \end{bmatrix} = \begin{bmatrix} y_1^2 + 2y_2y_3 \\ y_2^2 + 2y_1y_2 \\ y_3^2 + 2y_1y_3 \end{bmatrix}.
\end{align*}
$$

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