Research on Landmarks of SLAM Based on Square Root Cubature Kalman Filter

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Abstract. Under the large noise, the system's observability is weak, which leads to the instability of the filtering algorithm and the slow convergence speed. The algorithm of simultaneous localization and mapping based on the square-root Cubature Kalman filter (SRCKF) with Spherical Simplex(SS) was proposed in this paper. It not only abandoned the intercept term of traditional filtering algorithm, but also avoided the large amount of computation in sigma point algorithm. The new algorithm was used in the navigation, and the simulation experiment results and data showed that the number of sampling points and the amount of calculation were reduced because of the hypersphere distribution sampling. At the same time, the square root decomposition guaranteed the non-negative of the matrix, and the algorithm had a good stability and a certain navigation precision.

1. Instruction

The basic idea of CKF is that using N sampling points to generate a new set of 2N points with the same weight on a unit spherical surface, and then the mean and variance of the state are obtained by the function of the state equation. The algorithm is a new independent algorithm and it is suitable for any nonlinear model because it does not need linear approximation. From the beginning to the present, the CKF algorithm has been cited to the nonlinear model of the carrier and its pose measurement and estimation in the navigation systems by many scholars [1-2], and a certain success has been achieved. The CKF algorithm is a generalization of Bayesian estimation based on Gauss's hypothesis. In the final analysis, it transforms the nonlinear filtering problem into an integral problem [3-4] for the product of nonlinear function and the probability density of Gauss.

In view of the problem of large initial error or large observation noise, in the series of Kalman and its extended filtering algorithms, many scholars have proposed square root Unscented Kalman filter (SRUKF). Here, the square root combined with the Cubature Kalman filter algorithm [5] was proposed in this paper, which was used to filter the system based on SLAM. Compared with the traditional CKF algorithm, the new square root Cubature Kalman algorithm has the following two advantages: ① The system covariance matrix is decomposed by square root, and the recursive estimation and updating of the algorithm is carried out. ② The algorithm uses square root decomposition to ensure the nonnegative of the matrix, maintains the stability of the system, and inhibits the filtering divergence effectively[6-7]. Whether UKF and its extension algorithm, CKF algorithm, or particle filter (PF) algorithm are inseparable from the selection of sampling points and the calculation of their characteristic parameters.
2. Square Root Cubature Kalman Filter Algorithm

The general nonlinear robot motion model.

\[
\begin{align*}
    x_{k} &= f(x_{k-1}, u_k + \delta u) \\
    z_k &= h(x_k) + \delta z
\end{align*}
\]  

(1)

Where, \(f(\cdot)\) and \(h(\cdot)\) are system nonlinear function and observation function. The system noise and the observed noise are \(\delta u \sim N(0, Q)\) and \(\delta z \sim N(0, R)\). \(u_k\) is the control information. The dimension of system is \(n_s\), and the dimension of system is \(n_m\).

Then the robot's state vector can be expressed as the robot's position and pose information and the extended state vector of the input control information at the next two moments, thus \(x_k = [s_k, u_k]^T\). The decomposition of covariance matrix \(P_k = \zeta_k \zeta_k^T\), and the extended square root covariance matrix \(S_k = \begin{bmatrix} \zeta_k & \sqrt{Q} \end{bmatrix}\).

The steps of the SRCKF algorithm are as follows:

2.1. Time Updates.

1. From the dimension \((n_s + n_m)\) to the dimension \(2(n_s + n_m)\) of the system state, the cubature points set calculation are made.

\[
\hat{x}_{k-1} = S_{k-1} \xi_i + \chi_{k-1}, \quad i = 1, 2, \ldots, 2(n_s + n_m) \quad j = 1, 2, \ldots, m
\]  

(2)

Here, \(\hat{x}_{k-1} = [\hat{x}_{k-1,1}, \hat{x}_{k-1,2}, \ldots, \hat{x}_{k-1,m}]^T\).

2. Cubature points are propagating through calculating the state equation and a priori estimate.

\[
\hat{x}_{k|k-1}^* = f(x_{k-1,v}, x_{k,u})
\]  

(3)

3. According to the cubature rule, it estimates the square root covariance of robot pose estimation.

\[
S_{ik|k-1} = qr(S_{ik|k-1}^{s}, S_{ik|k-1}), Q_{k-1} = S_{ik|k-1}^{s} S_{ik|k-1}^{T}
\]  

(4)

Using cubature transformation to approximate the three order state of a robot:

\[
\chi_{k|k-1,v}^j = \frac{1}{2(n_s + n_m)} \sum_{i=1}^{2(n_s + n_m)} \hat{x}_{k|k-1,v}^* - \hat{x}_{k|k-1,v}^j + Q_k
\]  

(5)

Here, \(qr\) means that the matrix is decomposed into lower triangular and upper triangular form, and the covariance prediction value

\[
P_{ik|k-1} = \frac{1}{2(n_s + n_m)} \sum_{i=1}^{2(n_s + n_m)} \hat{x}_{ik|k-1}^* (\hat{x}_{ik|k-1}^{T} - \hat{x}_{ik|k-1}^j + Q_k)
\]  

(6)

Error information is expressed in \(S_{ik|k-1}^{s}\), which can be pressed as follows:
\[
S_{ij}^{*} = \frac{1}{\sqrt{2(n_i + n_m)}} \begin{bmatrix}
Z_{ij}^{1,1} - \hat{Z}_{ij}^{1,1} \\
Z_{ij}^{1,2} - \hat{Z}_{ij}^{1,2} \\
\vdots \\
Z_{ij}^{2(n_i + n_m)} - \hat{Z}_{ij}^{2(n_i + n_m)}
\end{bmatrix}^T
\]  

(7)

2.2. Measurement Updates.

The observation equation of the characteristic point of \( i \) is expressed as a posteriori estimation formula.

\[
z_{i[k-1]}^j = h(x_{i[k-1],j}, \phi_{i[k-1],j}) + \delta \zeta = h(x_{i[k-1],j}, \mu_{i[k-1]}) + \delta \zeta
\]  

(8)

Here, \( x_{i[k-1],j} \) is the position and pose of robot in a priori estimation. \( \mu_{i[k-1]} \) is the estimated value of the feature point \( i \) at the \( k-1 \) moment. Observation noise is \( \delta \zeta \sim N(0, R) \).

1) Calculating the cubature point of \( i \).

\[
\hat{x}_{k-1}^j = \hat{\xi}_{k-1}^j + x_{i[k-1],j}, i = 1,2, \cdots, 2(n_i + n_m)
\]  

(9)

Where, \( \hat{x}_{k-1}^j = [x_{i[k-1],1}, \phi_{i[k-1],1}, \cdots, \phi_{i[k-1],m}] \). Here, \( x_{i[k-1],j} \) is the robot state of the cubature point of \( i \), and \( \phi_{i[k-1],j} \) is the \( j \)-th feature state of the cubature point of \( i \), \( j = 1,2, \cdots, m \).

2) Propagating cubature points.

\[
z_{i[k-1]}^{i,j} = h(x_{i[k-1],j}, \phi_{i[k-1],j})
\]  

(10)

3) According to the cubature rule, calculating observation prediction value at the \( k \) moment.

\[
\hat{z}_{i[k-1]} = \frac{1}{2n_s} \sum_{j=1}^{2n_s} z_{i[k-1]}^{i,j}
\]  

(11)

Calculating the observation and square root type of new information estimation error covariance.

\[
S_{z_{i[k-1]-1}} = qr \left[ \epsilon_{i[k-1]}^{i} \sqrt{R}^T \right]
\]  

(12)

Here, \( R = S_{R,R}^T S_{R,R}^T \), and new information error

\[
\epsilon_{i[k-1]}^{i} = \frac{1}{\sqrt{2n_s}} \left[z_{i[k-1]} - z_{i[k-1]}^j \right], i = 1,2, \cdots, 2n_s
\]  

(13)

4) Estimating square root of the covariance.

\[
P_{z_{i[k-1]-1}} = \zeta_{i[k-1]}^{i} \left( \epsilon_{i[k-1]}^{i} \right)^T
\]  

(14)

\[
\hat{\zeta}_{i[k-1]} = \frac{1}{\sqrt{2n_s}} \begin{bmatrix}
\hat{x}_{i[k-1]}^{1,1} - \hat{x}_{i[k-1],1,1} \\
\hat{x}_{i[k-1]}^{1,2} - \hat{x}_{i[k-1],1,2} \\
\vdots \\
\hat{x}_{i[k-1]}^{2(n_i + n_m)} - \hat{x}_{i[k-1],2(n_i + n_m)}
\end{bmatrix}^T
\]  

(15)
Kalman gain:

$$W_k = \left( P_{xz,k|k-1} / S_{zz,k|k-1}^T \right) S_{zz,k|k-1}$$  

(16)

So,

$$S_{k|k} = qr \left[ \zeta_{k|k-1} - W_k \epsilon_{k|k-1}, W_k S_{R,k} \right]$$  

(17)

3. Description of SSCKF Algorithm

The mean value of the $n$-dimensional stochastic state of $y$ is $\bar{y}$ and its covariance is $P_{yy}$. The non-linear function $x = f(y)$ applied to the random state vector $y$, which is mapped to $x$ with the dimension of $m$, and the mean and covariance are $\bar{x}$ and $P_{xx}$.

Hypersphere theory is that the random variable $x$ with dimension of $n+1$ and its mean value $\bar{x}$ which constitute a new Sigma sampling points with dimension of $n+2$. These sampling points are specific to the origin of its mean value, and the weights are distributed on the hypersphere with the mean center of the sphere. Then the probability density of these hyperspherical points are calculated to characterize properties of the state vector.

It is assumed that these $n+2$ sigma sampling points are centered at the origin of coordinates. The mean value of the state is zero, and the sampling points and feature weight values are calculated as follows:

1. Initializing weight, which is $0 \leq w_0 \leq 1$;
2. Calculating the weight of the sampling point of $i$.

$$w_i = \frac{(1 - w_0)}{(n+1)}, i = 1, 2, \ldots, n+1$$  

(18)

3. Initializing vector sequence.

$$\begin{bmatrix} e_0^1 & e_1^1 & e_2^1 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & \frac{1}{\sqrt{2w_i}} \end{bmatrix}^T$$  

(19)

4. Calculating the weight of the random variable sampling point of $i$ based on $m$ dimension.

$$e_i^m = \begin{cases} e_0^{m-1} & i = 1, 2, \ldots, m \\ -1 & i = m+1 \\ \frac{m}{\sqrt{m(m+1)w_i}} & i = m+1 \
\end{cases}$$  

(20)

Through a series of transformations mentioned above, we can get $n+2$ hypersphere sampling points $e_i^m$ about stochastic state variables $x$. Here, $i = 0, 1, 2, \ldots, n+1$. The distribution of hypersphere of the new sampling points can be expressed as follows:
\[ \chi^*_i = \bar{x} + \sqrt{P_{ss}} e^*_i, i = 0, 1, 2, \cdots n + 1 \]  

4. Description of SS-RCKF algorithm

Step 1. Recording the mean value \( \bar{x} \) and variance \( P_{ss,0} \) of the robot initial state, and setting the initial weight \( w_0 \).

Step 2. According to the state of system \( x_k \), it generates the \( n + 2 \)-dimensional sampling points \( x^*_{k-1} \) with equal weight and hypersphere distribution, which are used as the cubature points of the cubature Kalman filter, and the mean value \( \bar{\chi}_{k,i} \) of the cubature point set and its covariance \( P^*_{k-1} \) are obtained by using SSCKF.

Step 3. Calculating the weight \( w^*_i \) of the hypersphere cubature point.

Step 4. The new state vector under the distributed condition of SSCKF hypersphere is generated by formula (21).

Step 5. Repeating the above steps and resampling to get an ideal sampling points set.

Step 6. The new aspheric distribution sampling state is applied to the SRCKF algorithm, and the system time updating and forecasting updating are carried out based on the square root cubature Kalman filter algorithm.

5. Using Image Processing To Realize The Map Feature Extraction in Slam

SLAM technology has been studied and tested by scholars from various countries. Through sonar sensor, terrain sensor, depth sensor, optical sensor and so on, some units have been collected aircraft wrecks which is obtained as shown in figure 1.

![Figure 1. Aircraft wreckage](image)

Using the SLAM processing method based on sonar image, the aircraft debris collected by the sensors is extracted and processed. In order to make a comparison, under the condition of adding Gauss noise, the level set segmentation algorithm is applied to extract the contour information of the wreckage, which is as shown in figure 2. Figure 3 is a direct extraction of the aircraft wreckage without additional noise.
In figure 2 and figure 3, when the aircraft debris contour was extracted, the horizontal set segmentation method was adopted on the basis of wavelet transform. First, the outline of the rough debris was marked with red lines, and manual program is used to find the contour automatically. Then binarization image processing were carried out and the area inside the contour was filled. Finally, the contour was segmented, and the effective feature objects was got. It could be analogous from the graph. As long as there are SLAM related sensors to collect all the object images in the current environment, we could get a clearer picture of the target feature and its contour features based on SLAM technology, so that in a certain environment, we could create an accurate feature map which contains all the effective targets.

6. Simulation Experiment and Analysis of Feature Position Estimation Based on SRCKF Algorithm
The simulation area is shown in figure 4.
The parameters of the motion state of the carrier are shown in table 1.

| Parameters                  | value    | Parameters                  | value    |
|-----------------------------|----------|-----------------------------|----------|
| Speed $\dot{x}_v$, $\dot{y}_v$ | 4m/s     | Control angular velocity error $\Delta \phi$ | 1.2°/s   |
| Maximum angular velocity $\phi$ | 8°/s     | Range error $w_r$           | 0.6m     |
| Control speed error $\Delta v$ | 0.16m/s | Angle measurement error $w_\theta$ | 0.8°     |

The initial state of the carrier was zero. The initial value of state covariance $P_{v0} = \text{diag} [3 \ 3 \ 0]^T$ and feature covariance $P_{m0} = \text{diag} [0.2 \ 0.2 \ 0]^T$. Sampling frequency of side scan sonar sensor $T = 0.01s$. System noise and observation noise were $w_r \sim N(0, 0.05)$ and $v_i \sim N(0, 0.1)$. This experiment used recursive model based on three filtering algorithms of the PF, EHF and SRCKF to locate pipe carrier under the path condition estimation analysis, and got the carrier positioning error in the x and y directions, which were showed in figure 5.
Figure 5. Positioning error of carrier under three kinds of filtering algorithms

Under the set of simulation conditions, the position error of the carrier based on the PF algorithm was the position error average after the particle iteration. The PF algorithm needed continuous particle screening, resampling and weight updating, and the posterior estimation model. It not only needed to estimate the path of the map and robot, but also had a quantitative estimate of the certainty of each estimator. For a given pipeline reference path, each of environmental feature location needed to be estimated independently based on the model. Each of the particles was the assumption path of the carrier, and the position estimation of the target object in the environment needed independent calculation, so when the new feature was added to the observation model, the particle set needed to be modified and updated, which made the particle weight updating more complicated and the position error large.

Under the simulated condition, when the carrier position was estimated, some sensor observation information was uncertain, and the computer needed to deal with the relationship between the old and new observation information. The EHF algorithm was robust to the partial uncertainty in the carrier position estimation model, which could reduce the motion estimation error of the carrier to a certain extent. According to the norm bounded system model, with the reasonable value of $\lambda$, both to keep the smaller variance and ensure good robustness and estimate, the estimation of the state filter would be close to the actual value with a small error and improved the navigation accuracy.

Compared with the two algorithms of PF and EHF, the location estimation of algorithm based on SS-SRCKF was the smallest. The algorithm used the hypersphere distribution sampling method to replace the traditional random volume point sampling of square root CKF, and the number of sampling points and the amount of calculation were reduced. At the same time, with which the non-negative of the matrix was guaranteed, and the stability of the algorithm was good.

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