Probing QCD chiral cross over transition in heavy ion collisions with fluctuations

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Abstract: We argue that by measuring higher moments of the net proton number fluctuations in heavy ion collisions (HIC) one can probe the QCD chiral cross over transition experimentally. We discuss the properties of fluctuations of the net baryon number in the vicinity of the chiral crossover transition within the Polyakov loop extended quark-meson model at finite temperature and baryon density. The calculation includes non-perturbative dynamics implemented within the functional renormalization group approach. We find a clear signal for the chiral crossover transition in the fluctuations of the net baryon number. We address our theoretical findings to experimental data of STAR Collaboration on energy and centrality dependence of the net proton number fluctuations and their probability distributions in HIC.

Keywords: QCD phase transition and phase diagram • Chiral symmetry and charge fluctuations • Heavy ion collisions

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1. QCD phase diagram

Although, the important question addressed in QCD on the existence of a true 2nd order phase transition at finite chemical potential (critical point) has not been answered yet, nevertheless, there is recently an essential progress in the quantitative description of the QCD phase diagram. The lattice QCD (LQCD) provided a final value for the chiral cross over transition temperature at vanishing chemical potential [1]. The LQCD has also provided arguments that at small $\mu \simeq 0$ the chiral cross over line is the pseudo critical line of the 2nd order chiral phase transition belonging to the universality class of 3-dimensional, O(4) symmetric spin models [2]. Based on the universality arguments the LQCD has also provided the curvature of the chiral transition line, $T_c(\mu_c)$ [2]. These results confirm that, at least at small values of the baryon chemical potential $\mu$, the chiral cross over transition appears in the near vicinity to the chemical freezeout line [3] obtained from the analysis of particle yields measured in HIC.

The numerical coincidence of thermal parameters for the chiral transition and the freezeout conditions indicates that the hadron resonance gas partition function, which describes chemical equilibration of particle yields in HIC,

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should describe also the QCD thermodynamics up to a near vicinity to the transition to a quark gluon plasma phase. Indeed, the equation of state calculated on the lattice as well as other thermodynamical observables in the hadronic phase, were shown to be very well quantified by the hadron resonance gas (HRG) partition function [4]. Already at vanishing chemical potential, i.e. under conditions realized in the high energy runs at RHIC or LHC, the question arises to what extent a refined analysis of freeze-out conditions can establish the existence of a chiral phase transition. In this lecture we will argue that even at $\mu_B/T \simeq 0$ the net baryon number fluctuations and their higher moments can be used to identify the chiral cross over transition experimentally [5–8].

2. Charge fluctuations and the chiral cross over transition

Due to remnants of O(4) criticality related with the chiral phase transition observed in LQCD, the free energy \( f \) near the chiral phase transition temperature \( T_c \) may be represented in terms of singular \( f_s \) and regular contributions \( f_r \) as

\[
f(T, \mu_q, m_q) = f_s(T, \mu_q, m_q) + f_r(T, \mu_q, m_q),
\]

where in addition to the temperature \( T \) we also introduced explicit dependence on the light quark chemical potential, \( \mu_q = \mu_B/3 \), and the (degenerate) light quark masses \( m_q \equiv m_u = m_d \). The singular part of the free energy may be written as \([11]\)

\[
f_s(T, \mu_q, h) = h^{1+1/\delta} f_s(z), \quad z \equiv t/h^{1/\delta}
\]

with \( \beta, \delta \) are critical exponents of the 3-dimensional, O(4) universality class and \( t \equiv \frac{1}{T_0} \left( \frac{T - T_c}{T_c} + \kappa_q \left( \frac{\mu_q}{T_c} \right)^2 \right) \), \( h \equiv \frac{T_0}{T_0 - T_c} \). Here \( T_c \) is the phase transition temperature in the chiral limit and \( t_0, h_0 \) are non-universal scale parameters. The proportionality constant \( \kappa_q \simeq 0.06 \), has recently been determined from a scaling analysis in (2+1)-flavor QCD \([2]\). The scaling function \( f_s \) and its derivatives have recently been calculated using high precision Monte Carlo simulations of the 3-dimensional O(4) spin model \([12]\).

We want to focus here on properties of moments of net baryon number fluctuations, which are obtained from...
Figure 2. Left-hand figure: the ratio of quadratic fluctuations and mean net baryon number ($\sigma^2/M$), cubic to quadratic ($S_\sigma$) and quartic to quadratic ($\kappa_\sigma^2$) baryon number fluctuations calculated in the HRG model on the freeze-out curve [5] and compared to results obtained by the STAR Collaboration [14]. The dashed curves show the approximate \( \tanh(\mu_B/T) \) result for \( \kappa_\sigma^2 \) and \( S_\sigma \), respectively. Middle figure: The probability distributions, uncorrected for event-by-event counting efficiency, for the net proton number for different centralities taken by the STAR Collaboration in Au-Au collisions at \( \sqrt{s_{NN}} = 200 \) GeV [14]. The lines are the Skellam distributions calculated within HRG model [7]. Right-hand figure: Mean (M), variance (\( \sigma \)), skewness (S) and kurtosis (\( \kappa \)) of the net proton number calculated from the probability distributions shown in the middle figure.

Eq. 1 by taking derivatives with respect to \( \hat{\mu}_B = \mu_B/T \),

\[
\chi^B_n = \frac{1}{3^n} \frac{\partial^n f/T^4}{\partial \hat{\mu}_q^n} = \frac{1}{3^n} \frac{\partial^n f_r/T^4}{\partial \hat{\mu}_q^n} - \frac{1}{3^n} \frac{\partial^n f_s/T^4}{\partial \hat{\mu}_q^n} = \chi^B_{n,r} + \chi^B_{n,s} .
\]  

In the hadronic phase and away from transition temperature the regular part \( \chi^B_{n,r} \) should be well described by the hadron resonance gas partition function which will be a reference for critical fluctuations coming from the singular part \( \chi^B_{n,s} \). Thus, any deviations from the regular i.e. hadronic gas contribution could be an indication of criticality due to remnants of the chiral O(4) transition.

Higher order moments will become increasingly sensitive to the singular part of the free energy. From Eq. 2 it is apparent that these moments show a strong quark mass dependence in the vicinity of the critical temperature,

\[
\chi^B_{n,s} \sim \begin{cases} 
(2\kappa_\eta)^{n/2} h^{(2-a-n)/2\beta} f_s^{(n/2)}(z) & \text{for } \mu_q/T = 0, \text{ and } n \text{ even} \\
(2\kappa_\eta)^n \left( \frac{\mu_q}{T} \right)^{n(2-a-n)/2\beta} f_s^{(n)}(z) & \text{for } \mu_q/T > 0
\end{cases}
\]

where we used \( 2 - \alpha = \beta\delta(1+1/\delta) \). As \( \alpha = -0.2131(34) \) is negative in the 3-dimensional, O(4) universality class, the 4th order moments of the net baryon number fluctuations do not diverge yet in the chiral limit at the chiral transition temperature, \( z = 0 \). The first divergent moment is obtained for \( n = 6 \) if \( \mu_q/T = 0 \) and for \( n = 3 \) if \( \mu_q/T > 0 \).

In the hadron resonance gas (HRG) the regular part of the fluctuations \( \chi^B_{n,r} \) can be directly calculated from the thermodynamic pressure following Eq. 3. The HRG is a mixture of ideal gases of all particles and resonances, consequently the thermodynamic pressure exhibits a factorization of \( T \) and \( \mu_B/T \) dependence. Under the Boltzmann approximation the pressure in the HRG, \( P^{HRG}(T, \mu_B) \approx f(T) \cosh(\mu_B/T) \), where \( f(T) \) contains contributions
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from all baryons and baryonic resonances. With such $P^{HRG}$ there are particular properties of ratios of cumulants,

$$\frac{\chi_{2n,r}^B}{\chi_{2,r}^B}|_{HRG} = 1 \quad , \quad \frac{\chi_{(2n+1),r}^B}{\chi_{1,r}^B}|_{HRG} = 1 \quad , \quad \frac{\chi_{(2n+1),r}^B}{\chi_{2,r}^B}|_{HRG} \simeq \tanh(\mu_B/T) \quad , \quad \frac{\chi_{2n,r}^B}{\chi_{1,r}^B}|_{HRG} \simeq \coth(\mu_B/T), \quad (5)$$

which are independent of the number of baryons, their masses, degeneracy factors or decay widths.

The above structure of cumulants ratios observed in the HRG will be modified if the singular part is included in Eq. 3. The $\chi_{n,s}^B$ are increasingly sensitive to the order of cumulants. Thus, ratios of cumulants with different $n$ should be strongly varying functions of $T$ and $\mu_B$ when approaching a chiral cross over transition. Consequently, the observed deviation from that expected in Eq. 5 could be considered as a signature of the singular part contribution to the overall fluctuations thus, also of the chiral cross over transition.

The generic structure of ratios of different cumulants near the chiral cross over transition is shown in Fig. 1. These ratios were calculated in the Polyakov loop extended quark-meson (PQM) model at the physical pion mass, applying the functional renormalization group (FRG) method. In FRG approach one includes quantum and thermal fluctuations which are needed to preserve the universal scaling behavior of physical quantities expected in the O(4) universality class. In the low temperature phase the ratios $\chi_{2n,r}^B/\chi_{2,r}^B = 1$ as expected in the hadron resonance gas from Eq. 5. For $n = 2$, the kurtosis $R_{4,2} = 9\chi_{4,r}^B/\chi_{2,r}^B$ is not affected by the chiral critical dynamics since there is no contribution of the singular part to the first four moments as seen in Eq. 4. The observed in Fig. 1 drop in $R_{4,2}$ is due to "statistical confinement" property of the PQM model [13]. Such behavior of kurtosis was first observed in LQCD calculations and was interpreted as being a signature of deconfinement in QCD [11].

The large deviations of $(R_{4,2}/9)$ from unity in hadronic phase, which are increasing with $\mu/T$, are due to a singular contribution to the $\chi_{4}^B$ and $\chi_{2}^B$ ratio (see Eqs. 3 and 4). With increasing order of cumulants and the value of the chemical potential their ratios are dominated by the singular part already deeply below the chiral cross over transition temperature. Such behavior could be observed in HIC if freezeout appears near the chiral transition.

Recently, the first data on charge fluctuations and higher order cumulants, identified through the net-proton fluctuations, were obtained by the STAR Collaboration in Au- Au collisions at several collision energies [14]. To explore possible signs of chiral criticality and a cross over transition, the STAR data on the first four moments are compared in Fig. 2 to HRG [5, 7] following Eq. 5. Different ratios of cumulants of the net proton number can be directly connected with measured mean ($M$), variance ($\sigma$), skewness ($S$) and kurtosis ($\kappa$) [5].

The basic properties of measured fluctuations and ratios of cumulants are consistent with that expected in the HRG model [5, 7]. This indicates, that moments of the net proton number are of thermal origin with respect to the grand canonical ensemble and that they freezeout along the same chemical freezeout line as particle yields, close to the chiral cross over line. However, already such first comparison of the HRG model with STAR data reveals that there are deviations [5, 7, 9]. This is seen in Fig. 2 on the level of different ratios of cumulants as well as by comparing directly the measured probability distributions with Skellam distribution expected in the HRG for the net proton number [7]. The HRG model, as seen in Fig. 2, results in a broader distribution than observed in data, particularly at the most central collisions. This implies deviations of data on different moments from the HRG
model which are increasing with centrality. Shrinking of measured widths of the net proton number distribution relative to the HRG results, seen in Fig. 2, is to be already expected due to deconfinement properties of QCD [7]. We have to stress, however, that the experimental net-proton distributions in Fig. 2 are not corrected for event-by-event proton/anti-proton counting efficiency. While the HRG lines in this figure used efficiency corrected mean multiplicities from the same experiment. If uncorrected data are used in the Skellam distribution, then the agreement of the HRG model and measure probability distributions is found to be much better [10].

From the model calculations in Fig. 1, as well as from Eqs. 3 and 4, it is clear that contributions of the O(4) singular part to fluctuations increase with the order of cumulants. Preliminary data of STAR Collaboration on $\chi_B^4/\chi_B^2$ ratio, taken in Au-Au collisions at the top RHIC energy, show a strong deviation of this ratio from the HRG model expectations with a rather moderate deviations of the lower order cumulant ratios. Recently, the STAR Collaboration has also observed that deviations in central Au-Au collisions of $\chi_B^4/\chi_B^2$ ratio from the HRG is non-monotonic in energy. Such behaviors of ratios of cumulants are to be expected due to remnants of criticality related with a chiral cross over transition. It is a further challenge to understand and quantify the observed energy and centrality dependence of these deviations from the HRG results.

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