A Conjugate-Cyclic-Autocorrelation Projection-Based Algorithm for Signal Parameter Estimation

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A new algorithm to estimate amplitude, delay, phase, and frequency offset of a received signal is presented. The frequency-offset estimation is performed by maximizing, with respect to the conjugate cycle frequency, the projection of the measured conjugate-cyclic-autocorrelation function of the received signal over the true conjugate second-order cyclic autocorrelation. It is shown that this estimator is mean-square consistent, for moderate values of the data-record length, outperforms a previously proposed frequency-offset estimator, and leads to mean-square consistent estimators of the remaining parameters.

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1. INTRODUCTION

Demodulation in digital communication systems requires knowledge of symbol timing, frequency offset, and phase shift of the received signal. Moreover, in several applications (e.g., power control) the knowledge of the amplitude of the received signal is also required.

Several blind (i.e., non data-aided) algorithms for estimating some of the parameters of interest have been proposed in the literature. In particular, some of them exploit the cyclostationarity properties exhibited by almost all modulated signals [1]. Cyclostationary signals have statistical functions such as the autocorrelation function, moments, and cumulants that are almost-periodic functions of time. The frequencies of the Fourier series expansion of such almost-periodic functions are called cycle frequencies and are related to parameters such as the carrier frequency and the baud rate. Unlike second-order stationary statistics, second-order cyclic statistics (e.g., the cyclic autocorrelation function and the conjugate-cyclic-autocorrelation function [1]) preserve phase information and, hence, are suitable for developing blind estimation algorithms.

Cyclostationarity-exploiting blind estimation algorithms for synchronization parameters have been proposed and analyzed in [2–10]. In particular, the carrier-frequency-offset (CFO) estimator proposed in [3, 5, 9], termed conjugate-cyclic-autocorrelation norm (CCAN), performs the maximization, with respect to the conjugate cycle frequency, of the $L^2$-norm of the conjugate-cyclic-autocorrelation function. In [3], it is shown that such an estimator is asymptotically Gaussian and mean-square consistent (i.e., the mean-square error approaches zero) with asymptotic variance $\Theta(N^{-3})$, where $N$ is the sample size.

The technique proposed in [5, 9] for the multiuser scenario, exploits the estimated frequency shifts to obtain the unknown conjugate cycle frequencies of the received signal. These conjugate cycle frequencies are then filled in cyclic statistic estimators that are used to estimate the remaining parameters (amplitudes, delays, and phases). Since it is well known that cyclic statistic estimators are very sensitive to errors in the cycle frequency values [1], a new CFO estimator, termed conjugate-cyclic-autocorrelation projection (CCAP) is proposed here for the single-user case. It is based on the maximization, with respect to the conjugate cycle frequency, of the projection of the measured conjugate-cyclic-autocorrelation function of the received signal over the true conjugate-cyclic autocorrelation. The amplitude, delay, and phase estimates are then obtained by exploiting the single-user version of the algorithm proposed in [5, 9]. This algorithm, for small or moderate values of the data-record length, outperforms the previously proposed CCAN method where the CFO estimation is obtained by maximizing with
respect to the conjugate cycle frequency the $L^2$-norm of
the conjugate-cyclic-autocorrelation function, that is, the
projection of the measured conjugate-cyclic-autocorrelation
function over itself (i.e., over a noisy reference). In the paper,
the asymptotic performance analysis of the CCAP method
is also derived. Specifically, it is shown that the CCAP CFO
estimator is asymptotically Gaussian and mean-square consis-
tent with asymptotic variance $O(N^{-3})$. Consequently, the
estimators of amplitude, delay, and phase are proved to be
in turn consistent. Moreover, simulations are carried out
to show that, for finite $N$, the CCAP CFO estimator vari-
can be smaller than that of the CCAN estimator. It is
worthwhile to emphasize that the considered algorithm is not
based on the usual assumption of white and/or Gaussian am-

titude. Instead, the considered algorithm is highly immune
against noise and interference, regardless of the extent of the
temporal and spectral overlap of the signals $x(n)$ and $w(n)$. Note that, even if the disturbance term $w(n)$ can contain, in general, both stationary noise and non-
stationary interference, the assumption (5) on $w(n)$ is mild.
In fact, it is verified provided that there is at least one (con-
jugate) cycle frequency of the user signal and its frequency-
shifted version that is different from the interference (conju-
gate) cycle frequencies. Moreover, the stationary component
of the noise term never gives contribution to the cyclic statis-
tics of $w(n)$.

Let $w_a(t)$ be circular (i.e., with zero conjugate corre-
lation function) and $x_a(t)$ noncircular and with conjugate cycl-
stationarity with period $Q T_s$. Thus, $w(n)$ is circular (i.e., its con-
jugate correlation function $r_{ww}(n,m) = E(w(n+m)w(n))$
is identically zero), and, moreover, $x(n)$ is noncircular and
exhibits conjugate cyclostationarity with period $Q$. Conse-
quently, $y(n)$ exhibits a conjugate correlation

$$r_{yy}(n,m) = \sum_{k=0}^{Q-1} r_{\beta}^a(k) A^2 e^{-j2\pi\beta k d} e^{j2\pi y m} e^{j2\pi (n+2\nu k) n}, \quad (6)$$

where $\beta_k \equiv k/Q$.

Let $y_2(n) \equiv [y(n-M)y(n), \ldots, y(n+M) y(n)]^T$ be the second-order lag product vector. The conjugate-cyclic-correlogram vector

$$r_{yy,N}^\beta \equiv [r_{yy}^{\beta}(M), \ldots, r_{yy}^{\beta}(M)]^T$$

at conjugate cycle frequency $\beta$, evaluated on the basis of the received signal observed over a finite interval of length $2N + 1$.

The proposed CCAP CFO estimator is

$$\hat{\omega}_N \equiv \arg \max_{\omega \in \Lambda_N} |f_N(\omega)|^2 \quad (8)$$

with

$$f_N(\omega) \equiv \sum_{m=-M}^{M} r_{yy,N}^{\beta}(m)e^{-j2\pi y m} r_{xx}^a(m)^* \quad (9)$$

where $\Lambda_N \equiv \{ \beta | -M \leq \beta \leq M \}$, $\delta_0 \equiv \min(\Delta \beta/2, \beta_i + \Delta \beta/2)$ with $\Delta \beta$ and the frequency shift satisfying the conditions $|\nu| \leq \Delta \beta/4$ and $\Delta \beta < 1/Q$.
The function $|f_N(\omega)|$ represents the magnitude of the projection of the conjugate-cyclic-autocorrelation function estimate $r_{yy,N}^{\beta_\omega+2\omega}(m)$ over its asymptotic ($N \to \infty$) expression obtained by setting $\beta = \beta_\omega + 2\omega = \bar{\beta}_\omega + 2\omega$ into (4) with $r_{yy}(m) \equiv 0$. Thus, in the limit for $N \to \infty$, $|f_N(\omega)|$ is nonzero only in correspondence of the discrete set of values of $\omega$ such that $\beta = \bar{\beta}_\omega + 2\omega$ are conjugate cycle frequencies of the signal $y(n)$. Consequently, it is nonzero only for $\omega = \nu$, provided that $r_{yy}(\tau) \equiv 0$ for $\beta \in I_0$. Thus, in the limit for $N \to \infty$, $|f_N(\nu)|$ exhibits a peak at $\nu = \nu$, and, for finite $N$, an estimate $\hat{\omega}_N$ of the frequency shift $\nu$ can be obtained by locating the maximum of the function $|f_N(\nu)|$ for $\nu \in I_0$.

Note that, for finite observation interval, the CCAP CFO estimator is expected to outperform the CCAN estimator. In fact, in [3, 9], the CCAN CFO estimate is obtained by maximizing the function $\omega \to ||r_{yy,N}^{\beta_\omega+2\omega}||^2$ which is the projection of the conjugate-cyclic-autocorrelation function estimate over itself. That is, for finite observation interval, the reference signal for the inner product (projection) in $||r_{yy,N}^{\beta_\omega+2\omega}||^2$ is a noisy version of that adopted in (9).

Once the frequency-shift estimate $\hat{\omega}_N$ has been obtained, the estimation of amplitude, delay, and phase can be performed by considering the single-user version of the algorithm proposed in [5, 9] for the multiuser scenario.

Let us assume now that $\alpha_\omega$ is a known nonzero cycle frequency of $x(n)$. Equation (3) (with $r_{xx}(m) \equiv 0$) suggests that the estimation of amplitude and time-delay parameters can be performed by minimizing with respect to $\gamma$ the function

$$g(y, y^*) \triangleq ||r_{yy,N}^{\nu} - yr_{xx, \nu} \odot a(\hat{\omega}_N)||^2.$$  \hspace{1cm} (10)

In fact, in the limit for $N \to \infty$ and for $\hat{\omega}_N = \nu$, it results that $g(y, y^*) = 0$ for

$$y = A^2e^{-j2\pi\alpha_\omega d}.$$  \hspace{1cm} (11)

For finite $N$, the value of $\gamma$ that minimizes $g(y, y^*)$ is given by

$$\gamma_{opt} = [r_{yy,N}^{\nu}]^T[r_{xx, \nu} \odot a(\hat{\omega}_N)]^* ||r_{xx, \nu}||^{-2}. \hspace{1cm} (12)$$

Thus, accounting for (11), the estimates of the phase $\phi$ and the arrival time $d$ are

$$\hat{\phi} = \sqrt{|\gamma_{opt}|},$$  \hspace{1cm} (13)

$$\hat{d} = -\frac{\gamma_{opt}}{2\pi\alpha_\omega}.$$  \hspace{1cm} (14)

respectively, where $\sqrt{[\cdot]}$ is the angle of a complex number.

Let us assume now that $\beta_\omega$ is a known conjugate cycle frequency of $x(n)$. Equation (4) (with $r_{yy}(\tau) \equiv 0$ for $\beta \in [\beta_\omega - \Delta\beta/2, \beta_\omega + \Delta\beta/2]$) suggests that the estimation of the phase $\phi$ can be performed by minimizing with respect to $\bar{\gamma}$ the function

$$h(\bar{\gamma}, \bar{\gamma}^*) \triangleq ||r_{yy,N}^{\beta_\omega+2\omega} - \bar{\gamma}r_{xx} \odot \bar{a}(\hat{\omega}_N)||^2.$$  \hspace{1cm} (15)

In fact, in the limit for $N \to \infty$ and for $\hat{\omega}_N = \nu$, it results that $h(\bar{\gamma}, \bar{\gamma}^*) = 0$ for

$$\bar{\gamma} = A^2e^{-j2\pi\beta_\omega d}e^{j2\phi}.$$  \hspace{1cm} (16)

For finite $N$, the value of $\bar{\gamma}$ that minimizes $h(\bar{\gamma}, \bar{\gamma}^*)$ is given by

$$\bar{\gamma}_{opt} = [r_{yy,N}^{\beta_\omega}]^T[r_{xx} \odot a(\hat{\omega}_N)]^* ||r_{xx}||^{-2}. \hspace{1cm} (17)$$

Thus, accounting for (11) and (16), it follows that the estimate of the phase $\phi$ is given by

$$\hat{\phi} = \frac{1}{2} \angle \left[ \frac{\gamma_{opt} e^{j2\pi(\beta_\omega - \alpha_\omega)\hat{d}}}{\gamma_{opt}} \right]. \hspace{1cm} (18)$$

It can be straightforwardly verified that the stationary points so determined for both the functions (10) and (15) are points of minimum.

Note that, in order to avoid ambiguities in the estimates (14) and (18), the following relationships must hold: $|d| \leq 1/2|\alpha_\omega|$ and $|\phi| \leq \pi/2$. In [7] it is shown that, for an appropriate choice of the cycle frequency $\alpha_\omega$, the condition on the delay is not a restriction for the synchronization purpose. On the contrary, the condition on the phase leads to a phase ambiguity that can be resolved by using differential encoding.

### 3. ASYMMETRIC PERFORMANCE ANALYSIS OF THE CCAP CFO ESTIMATOR

In this section, the asymptotic performance analysis of the considered estimation algorithm is carried out. First partial results were presented in [2]. First, by following the guidelines given in [3], the CFO estimator is shown to be mean-square consistent with variance $O(N^{-3})$. Then, it is shown that such an asymptotic behavior allows to prove the consistency of the estimators of the remaining parameters.

Analytical nonasymptotic results of CFO estimators based on cyclic statistics are difficult to obtain due to the difficulty of obtaining analytic nonasymptotic results for the cyclic statistic estimators. In fact, even if analytical expressions for the bias and variance can be obtained for finite data-record-length estimators of cyclic temporal and spectral moments and cumulants, these expressions are extremely complicated. Moreover, only asymptotic results for the distribution function of the cyclic statistic estimators have been derived in the literature (see, e.g., [12] and references therein).

Let us consider the Taylor series expansion of the derivative of $|f_N(\omega)|^2$ with Lagrange residual term:

$$\frac{d}{d\omega} |f_N(\omega)|^2 \bigg|_{\omega = \hat{\omega}_N}$$

$$= \frac{d}{d\omega} |f_N(\omega)|^2 \bigg|_{\omega = \nu} + \frac{d^2}{d\omega^2} |f_N(\omega)|^2 \bigg|_{\omega = \hat{\omega}_N} (\hat{\omega}_N - \nu),$$  \hspace{1cm} (19)

where $\hat{\omega}_N = \nu + \eta_N(\hat{\omega}_N - \nu)$ and $\eta_N \in [0, 1]$. By following the guidelines in [3, 13], it can be shown that

$$\lim_{N \to \infty} N(\hat{\omega}_N - \nu) = 0 \quad \text{a.s.},$$  \hspace{1cm} (20)
and, hence,
\[
\lim_{N \to \infty} \tilde{\omega}_N = \nu \quad \text{a.s.} \tag{21}
\]

By setting \([d | f_N(\omega)]^2/d\omega|_{\omega = \tilde{\omega}_N} = 0\), it follows that
\[
(2N + 1)^{3/2}(\tilde{\omega}_N - \nu) = -\mathcal{A}_N^{-1} \mathcal{B}_N, \tag{22}
\]
where
\[
\mathcal{A}_N = (2N + 1)^{-2} \left| \frac{d^2}{d\omega^2} f_N(\omega) \right|^2|_{\omega = \tilde{\omega}_N} = 2(2N + 1)^{-2} \text{Re} \left\{ f''_N(\tilde{\omega}_N) f_N(\tilde{\omega}_N)^* \right\}
\]
\[
+ 2(2N + 1)^{-2} | f'_N(\tilde{\omega}_N) |^2,
\]
\[
\mathcal{B}_N = (2N + 1)^{-1/2} \frac{d}{d\omega} f_N(\omega) \left| \right.^2|_{\omega = \nu} = 2(2N + 1)^{-1/2} \text{Re} \left\{ f''_N(\nu) f_N(\nu)^* \right\} \tag{24}
\]
with \(f'_N(\omega)\) and \(f''_N(\omega)\) denoting the first-and the second-order derivative, respectively, of \(f_N(\omega)\).

As regards the computation of the term \(\mathcal{A}_N\), let us observe that the second-order lag product vector \(y_2(n)\) can be decomposed into the sum of a periodic term (the conjugate correlation vector) and a residual term \(e(n)\) not containing any finite-strength additive sine wave component and generally satisfying some mixing conditions expressed in terms of the summability of its cumulants [3]:
\[
y_2(n) = \sum_{h=0}^{Q-1} \beta_h^2 \circ \alpha(n) e^{j2\pi(\beta_h + 2n)n} + e(n), \tag{25}
\]
where, for the purpose of CFO estimation error asymptotic analysis, without lack of generality, \(A = 1\), \(\varphi = 0\), and \(d = 0\) have been assumed.

By substituting (25) into (7) one has
\[
\hat{r}_{\beta_k + 2\omega}^{\beta_k + 2\omega} = \sum_{h=0}^{Q-1} \beta_h^2 \circ \alpha(n) \mathcal{D}_N(\beta_k + 2\omega - \beta_h - 2\nu) + s_N^{(0)}(\beta_k + 2\omega), \tag{26}
\]
where
\[
\mathcal{D}_N(\xi) = \frac{1}{2N + 1} \sum_{n=-N}^{N} e^{-j2\pi \xi n} = \frac{\sin \left( \pi \xi (2N + 1) \right)}{(2N + 1) \sin(\pi \xi)}. \tag{28}
\]
Moreover, by substituting (26) into (9), and accounting for (20), (21), and the results of Appendices A and B, it can be shown that
\[
\lim_{N \to \infty} f_N(\tilde{\omega}_N) = ||r_{\beta_k}||^2, \]
\[
\lim_{N \to \infty} (2N + 1)^{-1} f'_N(\tilde{\omega}_N) = 0, \tag{29}
\]
\[
\lim_{N \to \infty} (2N + 1)^{-2} f''_N(\tilde{\omega}_N) = -\frac{4\pi^2}{3} ||r_{\beta_k}||^2.
\]
Therefore, by substituting (29) into (23), this results in
\[
\lim_{N \to \infty} A_N = -\frac{8\pi^2}{3} ||r_{\beta_k}||^4. \tag{30}
\]

As regards the term \(\mathcal{B}_N\), accounting for (B.1) and the results of Appendix A, we have
\[
\lim_{N \to \infty} f_N(\nu) = ||r_{\beta_k}||^2, \]
\[
\lim_{N \to \infty} (2N + 1)^{-1/2} f''_N(\nu) = -\frac{4\pi}{3} ||r_{\beta_k}||^2 \mathbf{r}_{\beta_k}^* \tag{31}
\]
where
\[
\zeta \triangleq \lim_{N \to \infty} (2N + 1)^{1/2} s_N^{(1)}(\beta_k + 2\nu) \tag{32}
\]
is a zero-mean complex Gaussian vector whose covariance matrix can be determined accounting for the results of [3].

Therefore, by substituting (31) and (32) into (24), this results in
\[
\lim_{N \to \infty} B_N = -8\pi ||r_{\beta_k}||^2 \text{Re} \left\{ j \left[ \zeta \circ a^* (\nu) \right]^T (\mathbf{r}_{\beta_k})^* \right\}. \tag{33}
\]
Finally, by substituting (30) and (33) into (22) this results in
\[
\lim_{N \to \infty} (2N + 1)^{3/2}(\tilde{\omega}_N - \nu) = \mathcal{A}_N^{-1} \mathcal{B}_N = -\frac{3}{8\pi} ||r_{\beta_k}||^2 \text{Re} \left\{ j \left[ \zeta \circ a^* (\nu) \right]^T (\mathbf{r}_{\beta_k})^* \right\}. \tag{34}
\]
That is, the CFO estimation error is asymptotically Gaussian with zero mean and variance \(O(N^{-3})\). In [8, 9], it is shown that such an asymptotic behavior assures that the (conjugate-) cyclic-correllogram at the estimated (conjugate) cycle frequency \(\beta_k + 2\omega_{\nu}\) is a mean-square consistent estimate of the (conjugate-) cyclic-autocorrelation function at the actual cycle frequency \(\beta_k + 2\nu\). Consequently, since the parameters \(\gamma_{\text{opt}}\) and \(\gamma_{\text{opt}}\) are finite linear combinations of elements of the cyclic correlogram and the conjugate-cyclic-correlogram vectors, it follows that amplitude, delay, and phase estimators are in turn consistent.

Let us consider now the two-sided-mean counterparts of the quantities defined in [3, (11) and (12)], that is,
\[
\mathcal{A}_N \triangleq (2N + 1)^{-1} \frac{d^2}{d\beta_k^2} \left| r_{\beta_k} \right|^2, \quad \beta = \beta_k, \tag{35}
\]
\[
\mathcal{B}_N \triangleq (2N + 1)^{-1/2} \frac{d}{d\beta_k} \left| r_{\beta_k} \right|^2, \quad \beta = \beta_k + 2\nu,
\]
where \(\tilde{\beta}_N = \beta_k + \eta_N (\bar{\beta}_N - \beta_k), \eta_N \in [0, 1], \) and
\[
\hat{\beta}_N = \arg \max_{\beta \in \mathbb{L}_0} \left| r_{\beta_k} \right|^2 \tag{36}
\]
with $J_b = (\beta_k - 1/2Q, \beta_k + 1/2Q)$. By using definition (32) in the results of [3] we get

$$\lim_{N \to \infty} \tilde{A}_N = \frac{-2\pi^2}{3} |r_{xx}|^2,$$

$$\lim_{N \to \infty} \tilde{B}_N = -4\pi \text{Re} \left\{ j[\xi \odot a^*(\nu)]^T (r_{xx}^k)^* \right\}. \quad (37)$$

Thus, the asymptotic errors of the CCAP CFO estimator $\hat{\omega}_N$ and of the CCAN CFO estimator $\hat{\theta}_N = (\hat{\beta}_N - \beta_k)/2$ have the same statistical characterization. In fact,

$$\lim_{N \to \infty} (2N + 1)^{3/2} (\hat{\theta}_N - \nu)$$

$$= \lim_{N \to \infty} \left\{ (2N + 1)^{3/2} \frac{1}{2} (\hat{\beta}_N - (\beta_k + 2\nu)) \right\}$$

$$= -\frac{1}{2} \lim_{N \to \infty} \tilde{A}_N^{-1} \tilde{B}_N \quad \text{(38)}$$

$$= -\frac{3}{\pi} ||r_{xx}||^2 \text{Re} \left\{ j[\xi \odot a^*(\nu)]^T (r_{xx}^k)^* \right\}$$

$$= \lim_{N \to \infty} (2N + 1)^{3/2} (\hat{\omega}_N - \nu).$$

In particular, the errors have the same asymptotic variance.

In the following section, however, simulation results are reported showing that for moderate values of $N$ the CCAP CFO estimator can outperform the CCAN estimator. Note that, since the (conjugate-) cyclic-autocorrelation estimate is highly sensitive to the errors in the cycle frequency knowledge [1], even a slight performance improvement in the frequency-shift estimate can lead to a significant performance enhancement of the (conjugate-) cyclic-autocorrelation estimate and, hence, of the remaining parameters.

4. SIMULATION RESULTS

In this section, simulation results are reported to corroborate the effectiveness of the theoretical results of Section 3.

In the experiments, the useful signal $x(n)$ is a binary pulse-amplitude-modulated (PAM) signal with full-duty cycle rectangular pulse with oversampling factor $Q = 4$ and $w(n)$ is complex circular stationary Gaussian noise.

In the first experiment, the sample standard deviation of the considered CFO estimators, evaluated on the basis of 500 Monte Carlo trials, is reported as a function of $\nu$. The SNR is fixed at $-10$ dB, where SNR is the ratio between the signal and noise powers. Thus, SNR = $\delta_b/(N_0Q)$, where $\delta_b$ is the per-bit energy and $N_0$ is the spectral density of the bandpass white noise. The two cases $\beta_k = 1/Q$ (Figure 1) and $\beta_k = 0$ (Figure 2) have been analyzed. In both cases it is evident that for $N$ sufficiently large both the CFO estimators exhibit a variance $\mathcal{O}(N^{-3})$ and, moreover, their asymptotic variance is the same and approaches the theoretical value given in [3]. The CCAP CFO estimator, however, outperforms the CCAN estimator for moderate values of $N$, especially in correspondence with the threshold values $N_t = 2^{12}$ (for $\beta_k = 1/Q$) and $N_s = 2^{10}$ (for $\beta_k = 0$). Such a result is in accordance with the fact that both methods perform the CFO estimation by maximizing a cost function which is the magnitude of the inner product of the vector $r_{yy,N}$ over a reference vector. In the CCAN method, however, the reference vector is a noisy version of that of CCAP.

In the second experiment, the sample root-mean-squared error (RMSE) of the considered CFO estimators, evaluated on the basis of 500 Monte Carlo trials, is reported as a function of SNR, with $N_s = 2^{12}$ for $\beta_k = 1/Q$ (Figure 3) and $N_s = 2^{10}$ for $\beta_k = 0$ (Figure 4). Also this experiment...
corroborates the usefulness of the proposed CCAP CFO estimator for moderate values of $N$ and low SNR values.

**APPENDICES**

### A. RESULTS ON $s_N^{(K)}(\alpha)$

Let us consider the vector function $s_N^{(K)}(\alpha)$ defined in (27). It can be easily shown that

$$
\frac{\mathrm{d}s_N^{(K)}(\alpha)}{\mathrm{d}\alpha} = -j2\pi(2N + 1)s_N^{(K+1)}(\alpha). \quad (A.1)
$$

Under appropriate mixing conditions expressed in terms of the summability of the cumulant of the vector process $e(n)$ this results in (see [3, Lemma 1])

$$
\lim_{N \to \infty} \sup_{\alpha \in [-1/2,1/2]} \|s_N^{(K)}(\alpha)\| = 0 \quad \text{a.s. \forall K}. \quad (A.2)
$$

Moreover, let $\{\xi_N\}_{N \in \mathbb{N}}$ be a real-valued sequence such that $\xi_N \in X$ with $X$ compact set contained in $[-1/2,1/2]$ and $\lim_{N \to \infty} \xi_N$ exists. Then

$$
\lim_{N \to \infty} \|s_N^{(K)}(\xi_N)\| = 0 \quad \text{a.s. \forall K}. \quad (A.3)
$$

### B. RESULTS ON $D_N(\xi)$

Let us consider the function $D_N(\xi)$ defined in (28) and denote by $D_N'(\xi)$ and $D_N''(\xi)$ its first- and second-order derivatives, respectively.

This results in

$$
\lim_{N \to \infty} (2N + 1)^{-1/2} D_N'(\xi) = 0 \quad \forall \xi. \quad (B.1)
$$

Let $\{\xi_N\}_{N \in \mathbb{N}}$ be a real-valued sequence such that $\xi_N \in X$ with $X$ compact set contained in $[-1/2,1/2]$.

If $\lim_{N \to \infty} \xi_N = 0$, and $\lim_{N \to \infty} N \xi_N = 0$, then

$$
\begin{align*}
\lim_{N \to \infty} D_N(\xi_N) &= 1, \\
\lim_{N \to \infty} (2N + 1)^{-1} D_N'(\xi_N) &= 0, \\
\lim_{N \to \infty} (2N + 1)^{-2} D_N''(\xi_N) &= -\frac{\pi^2}{3},
\end{align*} \quad (B.2)
$$

otherwise if $\lim_{N \to \infty} \xi_N \neq 0$, then

$$
\begin{align*}
\lim_{N \to \infty} D_N(\xi_N) &= 0, \\
\lim_{N \to \infty} (2N + 1)^{-1} D_N'(\xi_N) &= 0. \quad (B.3)
\end{align*}
$$

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