Approach to construction of mathematical models of geo-massif stress-deformed state under the influence of natural and technogenic forces

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Abstract. The paper presents the developed approach to the construction of mathematical models of the stress-strain state of a geo-massif under the influence of natural and technogenic forces. According to the proposed concept, the geological environment is considered as a nonlinear hierarchically organized multiscale system. The natural state of the geo-massif is formed as a result of the influence of gravitational, geotectonic forces and internal pressure of fluids, and the man-made impacts lead to a change in the natural stress field as a result of formation of disturbances in the integrity of rocks in the unloaded zones and stress concentrators around the areas. Description of the stressed-deformed state of the geo-massif is carried out through a hierarchical family of mathematical models, which are constructed in the form of boundary value problems of the theory of deformable solid. First, at the lower hierarchical level, the determining relationships are developed, in which the influence of natural forces is taken into account, and then the generalization of the obtained models for determining technogenic impacts is carried out at the upper level. The proposed approach allows the links in the resulting hierarchical structure of models to be built, which makes it possible to carry out studies of both the disturbed and the natural state of the geo-massif.

1. Introduction
Geological environment is a natural object with a complex structure, therefore its description is an actual task. Geomaterials and geo-environments demonstrate all the specific features of a complex structure and are considered as non-linear hierarchically organized multiscale systems, which include not only extended geo-environments, but all loaded solid bodies [1].

Technogenic impacts on the geo-environment lead to a change in its state, the description of which must be carried out not only taking into account the influence of natural forces, but also the conditions of rocks deformation that have changed as a result of a wide range of mining operations.

The construction of underground structures, such as mines, tunnels, bunkers, underground storages of water, oil, gas, requires the performance of mining works and subsequent arrangement of free space in accordance with the designation of the projected facility.

2. Methods of research
The methodology of describing complex systems was considered by H. Haken [2], who introduces three levels of description: micro-, meso- and macroscopic.
A continual, phenomenological description of the geo-environment is used at a mesoscopic and macroscopic level. The microscopic level corresponds to the description of the system at the level of elementary particles and atoms. However, it is difficult to describe the geo-environments on this scale due to the huge number of particles and the difference in scales. Therefore, at the micro level, it becomes necessary to introduce parameters that would allow the most significant features of behavior to be described at the meso level.

At the meso- and macrolevel the relationship between parameters is ensured by equations that take into account at the micro level not the processes themselves, but their manifestation and influence on the geo-environment under the influence of natural and technogenic forces. The meso- and macroscopic state of solid media and geomaterials has its own characteristic features, manifested in the deformations of geomaterials, which, depending on the acting stresses, can be elastic or plastic while maintaining continuity of the medium.

To describe the geomechanical state of geomaterials, a concept is proposed, according to which the geo-environment is considered as a hierarchical nonlinear system, subjected to the impact of natural and technogenic forces. It is proposed to implement this concept on the basis of a hierarchical approach, which is as follows:

- the object of limited sizes is allocated at the macro level;
- its decomposition into subjects of meso- and micro-level is carried out depending on the research tasks;
- the integral parameters characterizing the state of the natural object are determined;
- connections are established between the set parameters.

The description of the geo-environment through the hierarchical system allows general scenarios of the geomassive behavior to be formed, and also is the basis for determining of its stress-strain state by means of mathematical modeling.

The specific solution of the choice of the research object within the offered hierarchical approach depends on the considered subject area. If the task is to study the stress-strain state of the geo-massif under the impact of natural and man-made forces, we can confine ourselves to the meso level, considering it representative. In this case, the result of the decomposition of the geo-environment macro-volume of the at the meso-level is a natural or disrupted due to technogenic impacts geo-massif, consisting of blocks-rock layers and coal layers deformed under the action of applied forces. The state of the geomassif rocks at the micro level depends on the forces acting in it (figure 1).

The natural state, typical for the geomassif not affected by technogenic impacts, is formed during the geological time scales under natural conditions of occurrence of rocks (figure 1). There are theories in which, under certain conditions, the natural state of the undisturbed geomassif is described by the relations of the elasticity theory [3, 4].

In the disturbed state, the geomassif is under the influence of technogenic influences, which lead to the localization of deformations and the formation of the unloaded zones and stress concentrators around the areas of disturbance of rocks integrity, that significantly alter the natural state of the massif. In this case, irreversible deformations and the transition of rocks from the elastic to the elastoplastic, limit or out-of-limit states are observed, the description of which must be carried out by means of mathematical relation constructed on the basis of laws corresponding to the nonlinear character of rock deformation.

Due to the complexity in describing the geomechanical state of the rock massif formed in the result of combination of natural and man-made forces, the modeling of geo-massif stressed-deformed state should be based on the family of mathematical models that, because of the generality of connections and the hierarchical structure, should provide an opportunity to study both the disturbed and the natural state of the geo-massif.
The resolving system of nonlinear equations must, on the one hand, reflect the state of the rocks at the microlevel, and, on the other hand, at the mesolevel take into account the result of the work of the bulk forces typical to the macrolevel.

On the basis of the proposed concept, it is required to determine the main parameters of the geomassif characterizing its stress-strain state under the joint influence of natural and technogenic factors and to establish connections between them by means of equations.

3. Results and discussion

According to the results of studies [5, 6, etc.], it is established that the natural state of geomassif is formed as a result of the influence of gravitational, geotectonic forces and fluid pressures, which, depending on the physical and mechanical properties of the rocks, are balanced by stresses arising.

As a result of technogenic influences, the change of the natural stress field in the zones of influence of mine workings is accompanied by the change in the laws of rocks deformation of and their transition from elastic to elastic-plastic, limit and out-of-limit states [4, 6, etc.]. Rocks have unequal tensile strengths under uniaxial compression or tension, so their resistance to applied forces must be taken into account when determining the stress-strain state of the geomassif [7, 8].

The technogenic impact on the geomassif leads to a redistribution of the stress field in relation to its natural state. Therefore, in order to build a family of mathematical models of the stress-strain state of geomassif under joint influence of natural and man-made forces, it is first necessary to develop determining relationships that take into account the effect of natural forces, and then generalize the resulting models to determine the technogenic impacts.

When constructing a family of mathematical models of the massif stress-strain state, a hierarchical approach is proposed, which provides technology for developing models according to the principle from the bottom to the top in the order of taking into account natural and technogenic factors.

Mathematical models of the geomassif stress-strain state are constructed in the form of boundary-value problems of the theory of deformable solid. To establish relations between the parameters of
models, while preserving the continuity of the geomassif, it is possible to use the equilibrium equations, Cauchy relations, the boundary conditions that determine the relations.

At the lower hierarchical level, the mathematical models are designed to investigate the deformation of geomassif rocks, the stress-strain state of which is determined by the action of natural forces – gravitational, geotectonic and internal pressure of natural gas. The hierarchical structure of the family of developed mathematical models of the geomassif stress-strain state is shown in figure 2.

The upper level of the hierarchy of mathematical models

Determining relations of the plasticity theory

- Polymorphism of defining relations
- The method of taking into account the different resistance of rocks under tension or compression

The lower level of the hierarchy of mathematical models

- The defining relations of the theory of elasticity
- Equilibrium equations
  - Volume force vector
  - Joint action
    - Natural gas internal pressure
    - Gravitational forces
- Options of boundary conditions
  - Geotectonic stress field
  - Gravitational stress field
- Geometric equations

Figure 2. Hierarchical structure of the family of mathematical models of the geomassif stress-strain state.

The law of rocks deformation under the combined effect of gravity forces and natural gas pressure must be taken into account in the defining relations of the mathematical model, and the acting forces – in the equations of equilibrium.

Since the effect of the surrounding strata on the studied section of geomassif leads to the formation of a gravitational or geotectonic stress field in it, it is necessary in the mathematical model of the lower level to provide the possibility to determine the ratio of the vertical and horizontal components of the stress tensor.

In order to take into account the external influences on the section of geomassif under consideration, it is required to make a transition from external forces acting on the surface to internal forces near the boundary, by developing or modifying the boundary conditions of the boundary-value problem that connect the effects of external and internal forces. Therefore, it is necessary to define boundary conditions for the boundary value problem that correspond to the gravitational or geotectonic stress fields (figure 2).

Thus, mathematical models of the geomassif stress-strain state of the lower-level hierarchy should provide for the mechanisms of accounting the effect of the joint action of gravitational, geotectonic forces and internal gas pressure in one boundary-value problem.

The combined effect of natural and man-made forces on the rock massif leads to the formation of a new geomechanical state, for the study of which the mathematical models of the geo-massif stress-strain state of the upper hierarchical level are developed.
Numerous studies [4, 6, 9, etc.] have established that under technogenic impact on the geo-massif in the mining zone, redistribution of stresses relative to the natural state of the geo-massif and formation of zones of tensile and contraction of rocks around the gob are observed. Therefore, in the models of the upper hierarchical level it is necessary to provide in the determining ratios the mechanism of taking into account different rocks resistance to the tensile and contraction forces (figure 2).

It should be noted that underground mining of a coal seam leads to non-linear deformation of rocks, which under the influence of natural and man-made forces transfer from elastic to elastic-plastic, limit or out-of-limit states. Therefore, in the top-level models it is necessary to take into account the laws of rock deformation, which are characteristic for the corresponding zones of mining work, and to develop a method for their connection by developing an approach to joint use of defining relations of theories of elasticity and plasticity in one boundary-value problem.

To construct such a class of inhomogeneous boundary value problems with a system of nonlinear equations, it is necessary that the defining relations of mathematical models of deformation of upper-level rocks have polymorphism for transition to the defining relations of low-level models. Since static equations and geometric equations in the theory of plasticity and elasticity have the same form, the inheritance of the properties of models between levels of hierarchy is ensured by the general structure of differential equations of the developed mathematical models that can be constructed both in two and in a three-dimensional setting.

The developed concept is implemented in the family of mathematical models of the geo-massif stress-strain state presented in [10-15].

4. Conclusion

Thus, the concept of investigation of the geo-massive stress-strain state under the influence of natural and technogenic forces was proposed. According to it the parameters determining the geomechanical state of the rock massif and the mechanisms of their interconnection are established in the process of decomposition of the geo-environment through a hierarchical approach.

The description of the geo-massive stress-strain state is carried out using different laws of rock deformation in the zones of influence of mine workings, which requires the integration of the theory of elasticity describing reversible deformations in the undisturbed geo-massif and the theory of plasticity allowing inelastic deformations arising from technogenic impacts to be taken into account.

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