Dynamics of the Inductive Single-Electron Transistor

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Using a classical equation of motion, dynamics of the phase is analyzed in the Inductive Single-Electron Transistor (L-SET) which is a promising new system suitable for quantum measurement with ultimate sensitivity and low back-action. In a regime of nonlinear dynamics, a shift of the oscillator resonant frequency is discovered which has a direct analogy to the switching of a dc-biased Josephson junction into voltage state. Results are reviewed for the predicted charge sensitivity, and it is shown that a performance challenging the best rf-SETs is foreseeable with the new device.

1 Introduction

The Radio-Frequency Single-Electron Transistor1 (rf-SET) has proven a suitable workhorse for high-frequency charge measurements. Basically, no other device has been able to track dynamic single-charge transport at MHz frequencies, a property relevant especially from the point of view of characterization and readout of superconducting qubits2. However, there are reasons to search for an alternative technology to overcome limitations principally due to the dissipative nature of the rf-SET 3; (1) it is not a truly quantum-limited detector, (2) relatively high power dissipation required for operation heat up the sample and surrounding qubits considerably, (3) it would be beneficial to be able to combine qubit and detector into a single device.

Purpose of the present paper is to present an intuitive picture of the operation a new kind of radio-frequency charge detector, the Inductive Single-Electron Transistor (L-SET), introduced recently by Sillanpää et al.4 both theoretically and experimentally. The L-SET is based on reactive readout of the Josephson inductance of a superconducting SET (SSET) in a resonator configuration. The schematics is ultimately a non-dissipative, high-bandwidth, and sensitive electrometer with the important property that it lacks the shot noise and thus excessive back-action inherent in dissipative detectors due to sequential tunneling.

2 The L-SET circuit

We use the circuit shown in Fig.1 where the SSET is coupled in parallel to an LC oscillator resonant at the frequency \( f_0 = 1/(2\pi)(LC)^{-1/2} \). The resonance of the whole system, \( f_p = 1/(2\pi)(L || L_J C)^{-1/2} > f_0 \) depends on the SSET effective Josephson inductance \( (L_J^*)^{-1} \), \( E_J = \partial^2 E(q_g, \phi)/\partial \phi^2 \) and critical current \( I_0^* \simeq 2eE_J^*/\hbar \) (not exact due non-sinusoidal current-phase relation) are due to the quantum band structure \( E(q_g, \phi) \), where \( \phi \) is the phase difference \( \phi = 2e/\hbar \int_{t_0}^{t} V(t)dt \) across the SSET. Since \( L_J^*(q_g) \) can have a substantial dependence on the gate charge \( q_g = C_g V_g \), the value of the resonant frequency \( f_p \) can be used for building a sensitive detector. Its bandwidth \( \Delta f \simeq f_0/Q_e \), where \( Q_e \) is the coupled quality factor, is typically in the range of tens of MHz.

3 Phase dynamics

Let us consider the SSET as a single Josephson junction which has the current-phase relation \( I_J \sim I_0^* \sin(\phi) \). Using the RCSJ-model, a classical equation of motion can be derived for the phase difference \( \phi \) across the resonator. The capacitance is due to the shunting \( C \) (Fig.1) and by
the resistor \( R \) we model the small residual dissipation. The resulting dynamics has an intuitive mechanical analog.

Current \( I \) in Fig. 1 is divided in the junction, \( R \) and \( C \),

\[
I = I_0^* \sin(\phi) + V/R + C\dot{V} = I_0^* \sin(\phi) + \hbar/(2eR)\dot{\phi} + C/h/(2e)\dot{\phi}. 
\]

The system is driven through the coupling capacitor \( C_c \). The originating current \( I_{ext} \) is divided, \( I_{ext} = I + I_{circ} \), where \( I_{circ} \) is the current circulating in the loop through the external inductor \( L \). It is simply related to \( \phi \) via the magnetic flux in \( L, \Phi = \Phi_0/2\pi\phi = LI_{circ} \).

Combining, we have classical non-linear equation of motion for internal dynamics of the phase \( \phi \)

\[
C h/2e\ddot{\phi} + \hbar/2eR\dot{\phi} + \Phi_0/2\pi L\phi + I_0^* \sin(\phi) = I_{ext},
\]

which is analogous to a mechanical particle moving in a sinusoidally modulated parabolic potential \( V(\phi) = \Phi_0^2/(8\pi^2L)\phi^2 - E^*_y \cos(\phi) \) and subject to a force \( I_{ext} \). Let us take a drive \( I_{ext} = I_e \cos(\omega t) \), and assume that the response is dominated by components at the drive angular frequency, \( \phi = x \sin(\omega t) + y \cos(\omega t) \). By keeping terms of the zeroth and the first harmonic, non-analytic equations are derived for the quadrature amplitudes \( x \) and \( y \),

\[
-M\omega^2 x - cw y + kx + 2I_0^* [J_1(x)J_0(y) + J_1(x)J_2(y)] = 0 \quad (2)
\]

\[
-M\omega^2 y + cw x + ky - I_e + 2I_0^* [J_0(x)J_1(y) + J_1(x)J_2(y)] = 0 \quad (3)
\]

where \( J_n \) are Bessel functions of the first kind and order \( n \), \( M = Ch/(2e) \), \( c = \hbar/(2eR) \), and \( k = \Phi_0/(2\pi L) \). In the absence of friction, \( R \to \infty \), the response is in phase with the drive, \( x = 0 \), and Eqs. 2, 3 simplify to

\[
-M\omega^2 y + ky - I_e + 2I_0^*J_1(y) = 0 \quad (4)
\]

which allows for a simple graphical interpretation.

The solution for the amplitude \( y \) is the intersection of the line \( z = (k - M\omega^2)y - I_e \), and the Bessel term \( z = -2I_0^*J_1(y) \). Slope of the line, \( k - M\omega^2 \), decreases with increasing drive frequency \( \omega \). At the resonance frequency \( \omega_R(I_e) \), the phase of the response shifts by \( 2\pi \), which in this picture corresponds to change of sign of the solution \( y \). While increasing the drive frequency (see Fig. 2), the solid lines a...d, plotted for increasing frequency but constant drive \( I_e = 0.5I_0^* \), rotate clockwise about the point \( z = -I_e \). At relatively low drive, the favored
solution which has the smallest absolute value and hence the lowest energy, changes sign at the frequency $\omega_R \simeq 1.19 \omega_0$ where the line $d$ tangents the Bessel function at the point marked with solid circle. We note that the line $d$ has a clearly negative slope, which corresponds to a relatively large value of $\omega_R$.

While increasing the drive, at the particular value $I_e \simeq 1.2 I_0^*$ the sign of the smallest solution changes when the line is horizontal, as depicted by the line $e$, which corresponds to a smaller frequency than in the previous case. At this $\omega$, the solution changes from $y \simeq 1.8$ (solid diamond) to $y = -\infty$, and approaches again zero (line $g$) at higher frequency. The infinity is due to absence of dissipation. At still higher drive, the resonance frequency does not change in this simplified picture. Note also that if the Bessel term due to the Josephson contribution $\to 0$, the resonance condition corresponds to a horizontal line as well.

We re-formulate the statements: At the "critical drive" $I_e \simeq 1.2 I_0^*$ the resonance frequency $\omega_R$ shifts from a high $\omega_R = 2\pi f_p$ which includes the Josephson-contribution, to the lower $\omega_R = 2\pi f_0$ which is determined only by $L$ and $C$.

This shift of resonance frequency of the L-SET oscillator at a certain "critical power" $P_c$ of the microwave drive is in a direct analogy to the switching of a DC-biased Josephson junction into voltage state. Of course, a similar phenomenon should take place equally well in a single junction than the SSET.

4 Experimental results

Results of two samples (see Table 1) are discussed. We focus here on the phase dynamics of sample 1. Since its $E_C$ was comparable to temperature, it did not work as a good electrometer, however, it was therefore rather close to a classical junction supposed in the calculations. Sample 2 (discussed in Ref. see also section) was a sensitive detector, with a gate shift of the resonance frequency of 15 MHz.

In Fig. (a) we plot the frequency response for sample 1 as a contour, a suitable repre-
sentation for the dynamics as a function of drive strength. The shift of resonance frequency from the plasma resonance at $f_p = 710$ MHz to the tank resonance at $f_0 = 610$ MHz is clearly visible at a critical power $P_c \simeq -102$ dBm. A prediction from our formalism which uses only independently determined parameter values, Fig. 3 (b), agrees qualitatively with the features of the experimental data (yet the resonance depth is different due to probably too small value of $C_c$).

5 Charge sensitivity

The bottleneck for detector sensitivity in the present schematics is the preamplifier having a noise temperature of 5 K. For sample 2 in the discussed operation mode of small plasma oscillations, we measured a charge sensitivity $s_q \simeq 2 \times 10^{-3}e/\sqrt{\text{Hz}}$. At higher drive which corresponds to several periods of the $\cos(\phi)$ Josephson term in the potential ("anharmonic mode"), such that the transduction is not due to tuning of $f_p$ but rather, due to tuning of non-linear phase dynamics, a better sensitivity of $1.4 \times 10^{-4}e/\sqrt{\text{Hz}}$ was achieved. Since the sensitivity is predicted to improve roughly as $(E_J/E_C)^{-1}$, the record numbers of the rf-SET, $s_q \sim 3 \times 10^{-6}e/\sqrt{\text{Hz}}$, should be beatable at experimentally accessible values of $E_J/E_C \sim 0.4$ in the anharmonic mode, or with $E_J/E_C \sim 0.15$ in the plasma oscillation mode, both using an Al SSET in the present experimental scheme.

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References

1. R. J. Schoelkopf et al., Science 280, 1238 (1998).
2. Y. Makhlin, G. Schönhals, and A. Shnirman, Rev. Mod. Phys. 73, 357, (2001).
3. A. B. Zorin, Phys. Rev. Lett. 86, 3388 (2001).
4. M. A. Sillanpää, L. Roschier, and P. J. Hakonen, cond-mat/0402045.
5. A similar effect was very recently observed in a classical junction by the Yale group, I. Siddiqi et al., cond-mat/0312553.
6. M. A. Sillanpää et al., to be published.