Skyrmionic textures in chiral magnets

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Abstract. In non-centrosymmetric magnets, the chiral Dzyaloshinskii-Moriya exchange stabilizes Skyrmion-strings as excitations which may condense into multiply modulated phases. Such extended Skyrmionic textures are determined by the stability of the localized solitonic Skyrmion cores and their geometrical incompatibility which frustrates regular space-filling. We present numerically exact solutions for Skyrmion lattices and formulate basic properties of the Skyrmionic states.

1. Introduction

During the last years the investigation of chiral modulations and Skyrmionic states, both in bulk non-centrosymmetric magnetic materials and nanomagnetic systems (see e.g. [1, 2, 3, 4, 5, 6] and bibliography in [2, 7]), has become a hot topic. Recently different specific modulated states have been discovered in non-centrosymmetric magnets and multiferroics including the manifestation of Skyrmionic states in the cubic helimagnet MnSi [4, 5, 6]. Such Skyrmionic states have been predicted two decades ago [8], and subsequent theoretical studies within the standard Dzyaloshinskii theory [9] have revealed a number of novel modulated states in magnetic materials with intrinsic and induced chirality [1, 2, 10, 11]. In this contribution, we outline main results on chiral Skyrmionic states in the low temperature region [10] and near the ordering temperature [2] derived within the standard phenomenological theory [9]. We also present new numerically exact solutions for Skyrmion lattices. We formulate basic properties of Skyrmionic states and elucidate physical mechanisms underlying their formation and stability.

2. Chiral flux-lines: the building blocks of Skyrmionic matter

According to Dzyaloshinskii [9], the magnetic energy density of a non-centrosymmetric ferromagnet can be written as

\[ w = A \sum_{i,j} (\partial_i M_j)^2 - \mathbf{M} \cdot \mathbf{H} + w_D(\mathbf{M}) + w_0(\mathbf{M}), \quad (1) \]

and includes the exchange stiffness with constant \( A \), the Zeeman energy, and chiral Dzyaloshinskii-Moriya (DM) coupling \( w_D \). \( w_0(\mathbf{M}) \) combines internal energy contributions independent on gradients of the magnetization.

Chiral modulations arise as a result of the competition between the exchange and DM interactions [9], and a sufficiently strong magnetic field compresses them into localized two-dimensional vortex-like structures (Fig. 1 (a)) [8, 10]. These chiral axisymmetric structures have...
a non-trivial topology and smooth non-singular cores with definite sizes. The DM couplings are described in continuum models of magnetic materials by so-called Lifshitz invariants, energy contributions linear in first spatial derivatives of the magnetization, \( \Lambda^{(k)}_{ij} = M_i \partial_k M_j - M_j \partial_k M_i \) [9]. For MnSi and other B20 compounds belonging to the crystallographic class T, \( w_D = D (\Lambda^{(z)}_{yx} + \Lambda^{(y)}_{xz} + \Lambda^{(x)}_{zy}) = D \mathbf{M} \cdot \mathbf{rot M} \) (where \( D \) is a Dzyaloshinskii constant). For uniaxial non-centrosymmetric classes the functionals \( w_D \) are given in Ref. [8].

For the sake of simplicity we include into model (1) only these basic interactions essential to stabilize Skyrmionic states, and neglect other less important energy contributions (as magnetic anisotropy, stray-fields, magneto-elastic coupling).

Figure 1. Cross-section through an isolated Skyrmion in ferromagnets with \( C_{nv} \) symmetry (a). Projections of the magnetization onto the basal plane in the Skyrmion core for ferromagnets with \( C_{nv} \) (b), \( D_n \) (c), and \( D_{2d} \) (d) symmetries. A hexagonal Skyrmion lattice for ferromagnets with \( C_{nv} \) symmetry (e). Evolution of the magnetization \( \theta(\rho/R_c) \) with increasing magnetic field (f) (adopted from Ref. [10]). Equilibrium size \( R_c \) of the Skyrmion cell as a function of an applied magnetic field \( (L_D = 4\pi A/D) \) (g). Inset shows the distribution of the magnetization in the Skyrmion lattice for \( H/H_D = 0 \) (I), 0.6 (II), 0.8 (III) \( (H_D = MD^2/(2A)) \).

For low temperatures \( M = \text{const} \), and the equations minimizing energy (1) with \( w_0 = 0 \) include solutions for axisymmetric localized structures \( \psi = \psi(\phi), \theta = \theta(\rho) \), where \( \mathbf{M} = M(\sin \theta \cos \psi; \sin \theta \sin \psi; \cos \theta) \) and the spatial variable \( \mathbf{r} = (\rho \cos \varphi; \rho \sin \varphi; z) \) is written in cylindrical coordinates (Fig. 1). The solutions \( \psi(\phi) \) are determined by crystal classes of the system [8]. Particularly, \( \psi = \phi \) for \( C_{nv} \) symmetry, \( \psi = \phi - \pi/2 \) for \( D_n \) and cubic T classes, and \( \psi = \pi/2 - \phi \) for \( D_{2d} \) symmetry (Fig. 1, (a-d)). The polar angle \( \theta(\rho) \) is derived from equation

\[
A \left( \frac{d^2 \theta}{d\rho^2} + \frac{1}{\rho} \frac{d \theta}{d\rho} - \frac{1}{\rho^2} \sin \theta \cos \theta \right) = \frac{D}{\rho} \sin^2 \theta - \frac{H}{2M} \sin \theta = 0, \tag{2}
\]

with the boundary conditions \( \theta(0) = \pi, \theta(\infty) = 0 \). This yields the solutions for isolated Skyrmions which are radially stable at well-defined sizes \( L \propto |D|/H \) [8] (Fig. 1 (f,g)). The chiral interactions play the crucial role to stabilize Skyrmions. In centrosymmetric systems \( (D = 0) \) such solutions are radially unstable and collapse spontaneously under the influence of the applied magnetic field or intrinsic short-range interactions [8, 10].

Commonly in nonlinear field models Skyrmions arise due to specific invariants described by higher order spatial derivatives (so-called Skyrme mechanism). In condensed-matter systems...
there are no physical interactions providing such energy contributions. Hence, chiral couplings present the unique mechanism to stabilize Skyrmionic textures in ordered condensed matter systems described by a large class of nonlinear field models [2]. This singles out chiral condensed-matter systems with Lifshitz-type of invariants (including non-centrosymmetric magnets, multiferroics, and chiral liquid crystals) into a particular class of materials with Skyrmionic states. The Skyrmions in chiral magnets can be thought of as isolated filaments within spatially homogeneous phases. Isolated Skyrmions remind Abrikosov vortices in type II superconductors or thread-like textures in nematic liquid crystals [2]. Contrary to these defected patterns with singularity in the core, the distribution of the order parameter in Skyrmions is smooth (Fig. 1).

Figure 2. (Color online) A "half-Skyrmion" lattice cell (a). Profiles $\theta_i(\rho)$ and $M(\rho)$ (b) in a hexagonal cell for two different directions $i$ through the core (dashed-dotted line ($i = 1$) and dotted line ($i = 2$)) are plotted together with the corresponding profiles $\theta_0(\rho)$ and $M_0(\rho)$ for the circular cell approximation (solid line). Inset in (b) shows $\nu_i = |\theta_i - \theta_0|/\theta_0$ as functions of $\rho$. Contour plots of $M_z(x, y)$ (c,d) and $M(x, y)$ (e,f) for a square (c,e) and hexagonal (d,f) lattices. The results are derived for reduced parameters of model (1) near the ordering temperature: $\tilde{a} = a(T - T_C) A/D^2 = 0.23$, $\tilde{b} = b A/D^2 = 0.05$.

3. Skyrmion lattices versus helicoids

For strong DM interactions isolated Skyrmions condense into lattices (Figs. 1 (e), 2 (c-f)). These 2D modulated textures are alternatives to common one-dimensional (helical) modulations. In the model with a fixed magnetization modulus appropriate for low temperature the equilibrium parameters of the Skyrmion lattice have been calculated in circular cell approximation [10]. At zero field the equilibrium period of the lattice is close to the helicoid period ($2Rc \approx L_D$). Near the critical field $H_c = 0.8132$ the lattice transforms into a system of isolated Skyrmions by infinite growth of the period and localization of the core (Fig. 1 (f), (g)) [10].
Near the ordering temperature, the magnetization modulus becomes small ($M \ll 1$) and strongly depends on the applied field and temperature. A brute-force minimization of the functional (1) with $w_0(M) = a(T - T_c)M^2 + bM^4$ in zero field yields solutions for a ”half-Skyrmion” staggered structure and hexagonal lattice with radial variation of the magnetization modulus in the lattice cells (Fig. 2). [2]. The Skyrmion lattices are characterized by a strong variation of the cell sizes and transformation of their structures near cell boundaries. However, they preserve axisymmetric distribution of the magnetization near the cell center. This remarkable property is due to specific energetics of the Skyrmions. ”Double-twist” rotation of the magnetization near the Skyrmion core leads to larger energy reduction than in ”single-twisted” helical phases while edge areas of the cell have larger energy density than the helical states [2]. This explains the unusual axial symmetry of the cell cores and their stability. The condensation of Skyrmions creates spatially inhomogeneous twisted phases as a result of space-filling by these multi-dimensional solitonic objects.

4. Summary

Skyrmionic textures reveal common features imposed by the fundamental physical mechanisms underlying their stability. Summarizing the findings of previous papers [2, 8, 10] and results of this contribution we formulate basic properties of extended states built from chiral Skyrmion solutions. (i) In magnetic materials lacking inversion symmetry chiral DM interactions stabilize isolated Skyrmions, axisymmetric localized structures with a fixed rotation sense and definite shape and sizes. (ii) Strong DM coupling leads to the condensation of Skyrmions into lattices (Figs. 1, 2). The effective interactions between the Skyrmions are weak relative to energies determining their radial stability. Hence, in real materials, the detailed arrangement of Skyrmions into lattices and their stability strongly depend on additional magnetic couplings such as anisotropies, applied and dipolar stray fields, temperature, and static disorder. (iii) Skyrmions, both isolated and bounded, are characterized by axisymmetrical distribution of the magnetization in their cores (Fig. 2 (b-f)).

The main features of chiral Skyrmions highlighted in this paper provide a basis for detailed analysis of recent experimental findings [4, 5, 6] and allow to establish relations between the classical phenomenological model (1) and other theoretical approaches (see [2, 5] and bibliography of these papers).

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