Optimized state feedback control of quarter car active suspension system based on LMI algorithm

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Abstract. Suspension systems are applied in vehicles so as to improve the passengers ride comfort, vehicle stability and better road handling when the vehicle moves on roads with bumps and terrains. In this paper, Linear Matrix Inequality (LMI) and linear quadratic regulator (LQR) controllers were proposed and this was also compared with passive suspension system. Using nonlinear quarter car active suspension system model with hydraulic actuator to reduce the effect of bumpy road surfaces on vehicle and hence improve passenger ride comfort. Vehicle body suspension travel and wheel deflection were measured to determine the performance of the active suspension system. Mean absolute error (MAE) and integral square error (ISE) were used as the performance indexes, thus a MAE and ISE of passive 75.01%, LQR 82.65% and LMI 89.11% and passive 4.11x10⁻⁶, LQR 3.24x10⁻⁶ and LMI 2.19x10⁻⁶ respectively were recorded. Based on the simulation results and analysis, the LMI based control performs better in minimizing vibration effects and also guarantee better ride quality and vehicle stability when compared to the conventional LQR and the passive suspension system.

Keywords. vehicle, active suspension; ride comfort; passive suspension; terrains; LMI region; actuator; quarter car; bumps; passengers; vehicle stability

1. Introduction

Vehicle suspension system is a significant element in automotive system which plays key role in achieving the desired performances such as comfortable ride, road handling characteristics, suspension deflection and maximum actuator control force. It serves as a linkage between the passengers and the track along which the vehicle rides. The entire forces generated between the body of the vehicle and the track are transmitted through the suspension system. The vehicle’s body as well as the passengers are affected greatly as a result of poor ride quality and comfort, and hence, the need to design suspension systems capable of providing these constraints.

To achieve desired performances, vehicle suspension systems have been categorized into active, semi-active and passive suspension systems. Among these categories, active suspension system, characterized by components such as hydraulic, pneumatic and magnetorheological actuators (that produce the needed suspension force) have been reported as the most effective way of improving suspension performances, despite its high energy requirement, complexity and high cost [1-2]. The active suspension system has
Due to its importance in modern automobile systems, active control of vehicle suspension system has attracted significant interest among researchers in recent years. An approximation-free control technique is proposed in [3] to solve the issue of nonlinear actuator dynamics. The novel strategy neglects the need for function approximators (fuzzy logic systems and neural networks) which have heavy computational requirements is suitable for practical use. The effectiveness of the proposed simple but yet robust method is verified by simulations and then comparative study was done where the proposed method demonstrates improved performance over backstepping control strategy. In the study, the system closed-loop stability and performance requirements were proved and also both system transient and steady-state responses were investigated. Studies in [4] present a double loop PID control scheme for half vehicle active suspension force control with hydraulic actuator. Based on simulation results obtained, the proposed technique gives better performance when compared with passive system. However, as conventional control method, PID controller is known to have slow response and large overshoot. Fuzzy logic control of active suspension system is implemented in [5]. The proposed approach was validated on five degree-of-freedom half-vehicle with active suspension. Researchers in [6] looked into design of two separate robust controllers (state feedback and output feedback) based on H-infinity control strategy for active suspension system. Euler Lagrange dynamic equation was used to obtained an improved mathematical model of the vehicle including driver’s seat. The effectiveness of the proposed controllers in satisfying performance requirement was validated in simulations considering driver’s seat heave acceleration and the rotational acceleration around the vehicle’s centre of gravity.

Other control design techniques for vehicle active suspension systems include composite nonlinear feedback control approach as proposed in [7]. Adaptive control methods were deliberated in [8-9] considering the unknown actuator dynamics. Researchers in [10] study robust finite-time tracking control. Robust adaptive control is proposed in [11]. An intelligent controller based on fuzzy logic is implemented in [12]. In the study, the proposed controller is tested in simulation using full vehicle model with active suspension comprising of electro-hydraulic actuator and that the results demonstrate an improved performance over passive suspension system. Adaptive PID-sliding-mode controller is implemented in [1]. Output-feedback-based H-infinity controller is suggested in [2]. Linear quadratic regulator (LQR) and PID control methodologies are presented in [13] with application to quarter car model with active suspension. Studies in [14] considered a non-fragile H-infinity controller. Hybrid controller based on PI sliding mode approach is suggested in [15].

More recently, authors in [16] considered Adaptive backstepping control scheme based on fuzzy logic approach for a fourth-order active suspension system. Numerical simulation was used in demonstrating the effectiveness of the proposed technique on the closed-loop system. Sliding-mode control with consideration of actuator dynamics is presented in [17]. Lyapunov stability theory was employed to prove the closed-loop system stability and the performance of the proposed technique is verified by computer simulations. Studies in [18] sliding mode control is proposed for a half-vehicle with active suspension. In the study, half-vehicle vertical displacement is considered and performance of the proposed technique is tested using simulations with two increased Lyapunov surfaces. Neural network predictive controller is proposed for the control of a nonlinear quarter car active suspension system by [19]. In the suggested work, a proportional-Integral (PI) controller is designed to take care of nonlinearities introduced into the system by the hydraulic actuator with the PI controller gains been optimized by Simulated Annealing (SA) which has low computational nature. The performance of the suggested robust control scheme is evaluated in simulations. A novel technique based on cloud-aided adaptive backstepping method is proposed in [20]. Simulation studies were done using full-vehicle model with active suspension to verify the effectiveness of the novel scheme and a significant increase in system performance is recorded when compared with passive suspension.
In this paper, an improved state feedback controller using linear matrix inequalities (LMI) is proposed and this compared with linear quadratic regulator (LQR) using active and passive suspension system to reduce vehicle body oscillations due to irregular road excitations which have significant effect on passenger ride comfort and vehicle stability. Based on simulation performed to validate the effectiveness of the optimized LQR controller proposed.

The organization of the study is in the following manner. Details of the system dynamic model incorporating the active suspension system with hydraulic actuator is presented in section 2. LMI algorithms with LQR controller designed are given in section 3. Section 4 highlights the results and discussions. And finally, section 5 contains the conclusions and direction for future work.

2. The System Dynamic Model

Figure 1 shows the quarter car dynamic model with nonlinear hydraulic actuator.

![Figure 1. The System Dynamic Model with Nonlinear Hydraulic Actuator](image)

The dynamic expression is expressed as [3]:

\[
\begin{align*}
    m_s \ddot{z}_s + F_d(\dot{z}_s, \dot{z}_u) + F_s(z_s, z_u) &= F \\
    m_u \ddot{z}_u - F_d(\dot{z}_s, \dot{z}_u) - F_s(z_s, z_u) + F_t(z_u, z_r) + F_b(\dot{z}_u, \dot{z}_r) &= -F
\end{align*}
\]  

where; \( m_s \) is the sprung mass (mass of the vehicle chassis), \( m_u \) is the unsprung mass (mass of the wheel assembly), \( F_d \) is the damping force generated by the dampers, \( F_s \) is the spring force generated by the springs, \( F_t \) is the tire elastic force, \( F_b \) is the tire damping force, \( z_s \) is the sprung mass displacement, \( z_u \) is the unsprung mass displacement, \( z_r \) is the road input displacement and \( F \) is the force used in rejecting vehicle vertical motion as a result of road irregularities introduced into suspension system.

Equations (1) and (2) relate the motions of the tire, sprung and unsprung masses to the forces generated by the tire, spring and dampers.

The nonlinear hydraulic actuator model which is positioned parallel to spring and damper (see Figure 1) is clearly considered in this study. The force \( F \) produced by the hydraulic actuator is generally given as:

\[
    F = A_p \cdot P_L
\]

where; \( A_p \) is the piston area and \( P_L \) is the load pressure of the cylinder.
A four-way valve-piston, reported as the most widely used actuator [3], is considered in this study whose dynamic model is given as:

\[ \ddot{P}_L = \frac{4\beta_e}{V_t} [Q - C_{tp}P_L - A(\dot{z}_s - \dot{z}_u)] \]  \hspace{1cm} (4)

where; \( V_t \) is the total actuator volume, \( \beta_e \) is the effective bulk modulus, \( C_{tp} \) is the total piston leakage coefficient and \( Q \) is the hydraulic flow rates.

The hydraulic flow rate can be determined as:

\[ Q = sgn[P_s - sgn(z_v)P_L]C_d\omega_3 z_v \sqrt{\frac{1}{\rho}} (|P_s - sgn(z_v)P_L|) \]  \hspace{1cm} (5)

where; \( P_s \) is the hydraulic supply pressure, \( \rho \) is the fluid density, \( C_d \) is the discharge coefficient, \( \omega \) is the spool valve area gradient, \( z_v \) is the spool valve displacement.

The hydraulic actuator valve displacement \( z_v \) can be controlled by voltage or current input \( u \) in corresponding to different forces required when in practical operations. The approximate servo valve system's linearized state model equations is expressed as [3]:

\[ \dot{z}_v = \frac{1}{\tau}(u - z_v) \]  \hspace{1cm} (6)

where; \( \tau \) = Time constant of the valve dynamic model and \( u \) = Voltage or current input.

As validated by Kim in [21], for small time constant \( \tau \), the spool valve dynamic model converges much faster than the actuator dynamics and therefore, equation (6) can be reduced to an algebraic equation given by \( u - z_v = 0 \). Hence, equation (4) can be rearranged as:

\[ F = -\beta F - aA^2(\dot{z}_s - \dot{z}_u) + A\gamma u \omega_3 \]  \hspace{1cm} (7)

where; \( \beta = aC_{tp} \), \( a = 4\beta_e/V_t \gamma = aC_d \omega \sqrt{\frac{1}{\rho}} \omega_3 = sgn[P_s - sgn(u)P_L] \sqrt{\frac{1}{\rho}} (|P_s - sgn(u)P_L|) \)  \hspace{1cm} (8)

The state variables defined for the derivation of the system’s linearized state-space model equations is as follows: \( x_1 = z_s, x_2 = \dot{z}_s, x_3 = z_u, x_4 = \dot{z}_u, x_5 = \mu \dot{P}_L \), where \( \mu \) is selected as a positive constant to scale the load pressure \( P_L \). Relating the dynamic equations given by (1), (2), (4), (5), (6) and (7) were obtained as follows:

\[ \dot{x}_1 = x_2 \]

\[ \dot{x}_2 = \frac{1}{m_s} \left( -F_d(x_2, x_4) - F_s(x_1, x_3) + \frac{A}{\mu} x_5 \right) + d_1 \]

\[ \dot{x}_3 = x_4 \]

\[ \dot{x}_4 = \frac{1}{m_u} \left( F_d(x_2, x_4) + F_s(x_1, x_3) - F_l(x_3, z_r) - F_b(x_4, \dot{z}_r) - \frac{A}{\mu} x_5 \right) + d_2 \]

\[ \dot{x}_5 = -\beta x_5 - \alpha A(x_2 - x_4) + \mu \gamma u \omega_3 \]  \hspace{1cm} (9)

The terms \( d_1 \) and \( d_2 \) indicate the modeling uncertainties, external disturbances and sensor noise effect. The parameters consider for the controller design are vehicle body acceleration, wheel deflection and suspension travel.
The numerical values of the quarter car active suspension and that of the nonlinear hydraulic actuator model used in the simulation studies are as presented in Tables 1 and 2 respectively.

**Table 1. Parameter Values for the Quarter Car Active Suspension Model**

| Parameters | Description | Values | Units |
|------------|-------------|--------|-------|
| $F_t$      | Elasticity of car tire | 190,000 | N/m   |
| $F_s$      | Spring stiffness       | 16,812  | N/m   |
| $M_s$      | Sprung mass            | 290     | Kg    |
| $M_u$      | Unsprung mass          | 59      | Kg    |
| $F_d$      | Damping coefficient    | 1000    | N/(m/s) |
| $F_b$      | Damping coefficient    | 1000    | N/(m/s) |

**Table 2. Parameter Values for the Hydraulic Actuator Model**

| Parameters | Description               | Values      | Units                        |
|------------|---------------------------|-------------|------------------------------|
| $\beta$   | Bulk Modulus              | 1           | s$^{-1}$                     |
| $\mu$     | Actuator parameter        | 1x10$^{-7}$ |                              |
| $\alpha$  | Actuator parameter        | 4.151x10$^{13}$ | N/m$^2$                     |
| $A$       | Piston cross-sectional Area | 3.35x10$^{-4}$ | m$^2$                        |
| $r$       | Actuator parameter        | 1.545x10$^8$ | N/m$^{52/5}$.Kg$^{1/2}$    |
| $P_s$     | Hydraulic supply pressure | 103x10$^5$  | Pa                           |

3. Control Approach

The controllers, Linear Matrix Inequality (LMI) and Linear Quadratic Regulator (LQR), proposed for the vehicle suspension system are designed in this section. The performance of these control schemes was compared on quarter car active and passive suspension system.

3.1. LMI-Based Control Method

Positive state feedback based on pole-placement technique is the suggested control method. The system closed loop poles are located in an LMI region in order to control vehicle active suspension system.

3.2. Pole-Placement in LMI Region

The position of closed loop poles of a given linear dynamic system in a complex plane determines the stability and transient response of the system as expressed by equation (10):

$$\dot{X} = A_c X$$

If all poles of the linear dynamic system described by equation 9 lie in the left-hand side of the plane, the system is said to be asymptotically stable characterized in LMI terms by Lyapunov theorem stated as “the linear dynamic system described by equation 9 is said to be asymptotically stable if there exist a real symmetrical matrix $P$ satisfying the following LMI [3]”

$$A_c P + A_c A^T < 0, \quad P = P^T > 0$$
The stability conditions of the linear dynamic system presented in equation 10 is given by the LMIs expressed by equation 11. An LMI region is a subset of the complex plane which is represented by a LMI \[A_{LMI}\] [22]. The dynamic system transient performance may be improved by locating its poles in an LMI region. There are different forms of LMI regions including vertical strip, horizontal strip, circular and sector among other shapes. Figure 2 presents the LMI region used in study which is a combination of a circular and vertical strip.

![LMI Region](image)

Figure 2. LMI Region

If and only if there exist a symmetrical positive definite matrix \(P\) such that [3]:

\[
\begin{align*}
A_cP + PA_c^T + 2aP &< 0, \quad (12) \\
A_cP + PA_c^T + 2bP &> 0, \quad (13) \\
\begin{bmatrix}
-rP & cP + PA_c^T \\
cP + A_cP & -rP
\end{bmatrix} &< 0, \quad P = P^T > 0 \quad (14)
\end{align*}
\]

then the dynamic system represented by equation 9 will have all its poles lying in the LMI region shown in Figure 2.

The LMIs in (10) and (11) represents the vertical strip, while the LMI in (14) represents the circle centred at \((c \quad 0)\) with radius \(r > 0\).

3.3. The Proposed Controller

The control architecture of the control system involving the proposed controller is illustrated in figure 3. \(X\) is the state vector, \(U\) is the control input, \(r\) is the reference input vector, \(K\) is the controller gain vector and \(N\) is the reference input scaling factor vector.

![Proposed Control Architecture](image)

Figure 3. Proposed Control Architecture

Considering Figure 3, the control law can be formulated as:

\[
U = KX + Nr
\] (15)
Substituting the proposed control law of equation (15) in the linear model of the suspension system in equation (9) gives:

\[ \dot{X} = (A + BK)X + BNr \] (16)

\[ \dot{X} = A_cX + BNr \] (17)

At steady state:

\[ \dot{X} = 0 \] (18)

\[ X = X_{ss} \] (19)

Here, the control goal is to make the system states \( X \) track the reference inputs \( r \) at steady state. Thus:

\[ r = X_{ss} \] (20)

Substituting equations (18)-(20) in equation (16) gives:

\[ 0 = (A + BK)X_{ss} + BNX_{ss} \] (21)

Hence,

\[ BN = -(A + BK) \] (22)

Pre multiplying both sides of equation (22) by \( BT \) yields:

\[ B^TBN = -B^T(A + BK) \] (23)

Pre and post multiplying both sides of equation (23) by \( (B^TB)^{-1} \) gives:

\[ N = -(B^TB)^{-1}B^T(A + BK) \] (24)

From equations (16) and (17):

\[ A_c = (A + BK) \] (25)

Substituting equation (25) in (10)-(14) gives the following LMIs:

\[ AP + PA^T + BM + M^TB^T + 2aP < 0, \] (26)

\[ AP + PA^T + BM + M^TB^T + 2bP > 0, \] (27)

\[ \begin{bmatrix} -rP & cP + PA^T + M^TB^T \\ cP + AP + BM & -rP \end{bmatrix} < 0 \] (28)

\[ P = P^T > 0 \] (29)

where:

\[ K = MP^{-1} \] (30)
By solving the LMIs in (26)-(29), P and M can be found. The controller gains are obtained by substituting P and M in equation (30) as $K = 1.0e+04 \times [2.1328 \ -0.2492 \ -8.4414 \ -0.0974]$. 

### 3.4. Linear Quadratic Regulator Controller

Linear Quadratic Regulator (LQR) control scheme is a full state-feedback controller which is model dependent. It is an optimal control technique that is very powerful in terms of input tracking, output and state regulations [23-25]. Figure 4 shows the schematic of LQR control system.

The quadratic performance index, which is minimized to achieve optimal control performance, is given in (19), herein expressed by equation (31),

$$ J = \int_0^\infty (x^T(t)Q(t)x(t) + u^T(t)Ru(t)) \, dt $$

where $Q$ is symmetrical positive semi-definite matrix and it affects the states of the system directly. The weighting matrix $R$ is symmetric positive definite that determines the systems control action.

The optimal linear feedback control law is given by:

$$ u = -Kx = -R^{-1}B^TPx(t) \quad (32) $$

where $K$ is the designed matrix gain obtain by tuning the parameters of $Q$ and $R$ matrices and $P$ is the solution of Riccati equation given in [20], as:

$$ A^TP + PA - PB^{-1}B^TP + Q = 0 \quad (33) $$

Thus, solving equation (33) the controller gains were obtained as $K = [2750 \ 9720 \ -206400 \ -8240]$. 

### 4. Results and Discussion

To verify the effectiveness of the suggested techniques, computer simulations were carried out and then performance comparison were made between LMI and LQR controllers with respect to the passive suspension system. The quarter car active suspension model described in equations 1 through 7 and the proposed controllers were simulated using MATLAB simulation software.

The characteristics nature of typical road disturbance input was formulated in the form:

$$ r(t) = \begin{cases} 
\frac{a(1 - \cos8\pi t)}{2}, & \text{if } 0.75 \leq t \leq 1 \text{ and } 3.5 \leq t \leq 3.75 \\
\text{otherwise}, & \text{otherwise}
\end{cases} $$
Figure 5 shows the typical road profile with two bumps of amplitudes 0.11m at 1s and 0.5m at 3.75s were used as input to the system.

![Figure 5. Typical Road Input](image1)

![Figure 6. Car Body Acceleration](image2)
In order to achieve the desired objective of better vehicle suspension system in improving passenger ride comfort and road handling capability, car body acceleration and wheel deflection are the two parameters considered in the simulation studies. For comparison purpose, suspension travel for LQR
and LMI controllers are presented in figure 8 for both active and passive suspension systems. The result for the vehicle body acceleration shown in figure 6 demonstrates the effectiveness of LMI in suppressing the vehicle vibration as compared to LQR and passive system, which guarantee better passenger ride comfort. Furthermore, as shown in figure 7, the suggested LMI controller produced minimum wheel deflection. The performances of the controllers summarized as in table 3, using the integral square error and mean absolute error performance indexing.

Table 3. Performances Indexes

| Controllers | ISE       | Suspension Reduction (MAE in %) |
|-------------|-----------|---------------------------------|
| Passive     | $4.11 \times 10^{-6}$ | 75.01                           |
| LQR         | $3.24 \times 10^{-6}$ | 82.65                           |
| LMI         | $2.19 \times 10^{-6}$ | 89.11                           |

5. Conclusion
In this paper, a comparative analysis of LQR and LMI controllers on nonlinear quarter car passive and active suspension system were proposed. MAE and ISE were used as the performance indexes, in which a Car body deflection reduction of passive $75.01\%$, LQR $82.65\%$ and LMI $89.11\%$ were recorded using MAE. And the effectiveness of the control schemes was confirmed by recording small errors using ISE, which were obtained as; passive $4.11 \times 10^{-6}$, LQR $3.24 \times 10^{-6}$ and LMI $2.19 \times 10^{-6}$. Simulations studies further proved that the LMI controller performs better in minimizing Car body deflection effects and also guarantee better ride quality and vehicle stability when compared to the conventional LQR and the passive suspension system. It was recommended that in the future, an experimental implementation of the proposed techniques and others robust control schemes should be look into.

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