Calculating the jet-quenching parameter in STU background

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Abstract

In this paper we use the AdS/CFT correspondence to compute the jet-quenching parameter in a $\mathcal{N} = 2$ thermal plasma. We consider the general three-charge black hole and discuss some special cases. We add a constant electric field to the background and find the effect of the electric field on the jet-quenching parameter. Also we include higher derivative terms and obtain the first-order correction for the jet-quenching parameter.

Keywords: AdS/CFT Correspondence; $\mathcal{N} = 2$ Supergravity; String Theory; QCD.

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1 Introduction

The AdS/CFT correspondence, proposed by Maldacena [1] and developed by Witten [2] and Gubser, et al. [3], is a relation between a supergravity theory in the \( d + 1 \)-dimensional anti-de Sitter (AdS) space and a conformal field theory (CFT) in the \( d \)-dimensional boundary of AdS space. The famous example of AdS/CFT correspondence is the relation between type IIB string theory in \( AdS_5 \times S^5 \) space and \( \mathcal{N} = 4 \) super Yang-Mills gauge theory on the 4-dimensional boundary of \( AdS_5 \) space. This duality gives us useful tools for studying QCD. An important problem in QCD, which is also interesting to experiment in the LHC and RHIC [4-7], is considering a moving heavy quark through the hot plasma [8]. This problem and related topics have been already studied by using the AdS/CFT correspondence [9-18]. There are also other thermodynamical quantities such as the free energy, the energy density, the heat capacity and the speed of sound which can be calculated and compared with RHIC data [4-7].

Another important property of the strongly-coupled plasma at RHIC is the ratio of the shear viscosity to the entropy density. The universality of the ratio of shear viscosity \( \eta \) to entropy density \( s \) [19, 20, 21, 22] for all gauge theories with Einstein gravity dual raised the tantalizing prospect of a connection between string theory and RHIC. The results were obtained for a class of gauge theories whose holographic duals are dictated by classical Einstein gravity. However, in the gravity side one can study higher curvature AdS models and show that the conjectured lower bound on the \( \frac{\eta}{s} \) can be violated [23].

Recently there have been many searches for less supersymmetric model than \( \mathcal{N} = 4 \), for example \( \mathcal{N} = 2 \) supergravity theory [24-31]. It is important to note that solutions of \( \mathcal{N} = 2 \) supergravity may be solutions of a supergravity theory with more supersymmetry such as \( \mathcal{N} = 4 \) and \( \mathcal{N} = 8 \). Also the AdS supergravity can obtained by gauging the \( U(1) \) group in \( \mathcal{N} = 2 \) supersymmetric algebra. We have Already studied the problem of the drag force of a moving quark and also quark-antiquark pair through the \( \mathcal{N} = 2 \) thermal plasma [28, 29, 30]. In the Ref. [28] we considered the same background with present paper and studied the drag force of moving quark through thermal plasma, we calculated the quasi-normal modes corresponding to the static string. Also we obtained the effect of the constant electromagnetic field on the energy-momentum current densities. Furthermore we considered the effect of higher derivative corrections. In the Ref. [28] we found that the problem of the drag force in \( \mathcal{N} = 2 \) supergravity theory at near-extremal limit is corresponding to the problem of the drag force in \( \mathcal{N} = 4 \) SYM theory.

In this paper we are going to obtain the jet-quenching parameter for the \( \mathcal{N} = 2 \) thermal plasma. Jet-quenching is a property of the quark-gluon plasma (QGP) and our knowledge about this parameter increases our understanding about the QGP. In that case the jet-quenching parameter obtained by calculating the expectation value of a closed light-like Wilson loop and using the dipole approximation [32]. Therefore one needs to describe the system in light-cone coordinates. In the description of the AdS/CFT correspondence the endpoint of both fundamental and Dirichlet strings under influence of non-zero NS NS B-field background corresponds to the moving quark with a constant electromagnetic field. Therefore it is interesting to add a constant \( B \) field [33] to the system and find the effect of
constant electric field on the jet-quenching parameter. In order to calculate this parameter in QCD one needs to use perturbation theory. But by using AdS/CFT correspondence the jet-quenching parameter calculated in non-perturbative quantum field theory. This calculations were already performed in the \( \mathcal{N} = 4 \) super Yang-Mills thermal plasma [34-39]. Also the effect of higher derivative corrections such as Gauss-Bonnet on the drag force and the jet-quenching parameter has been studied [39, 40]. It is shown that the jet quenching parameter is enhanced due to the Gauss-Bonnet corrections with positive \( \lambda_{GB} \), while \( \hat{q} \) decreases with negative \( \lambda_{GB} \).

This paper is organized as follows, in section 2 we review the \( D = 5, \mathcal{N} = 2 \) supergravity known as STU model. Then in section 3, we calculate the jet-quenching parameter in the \( \mathcal{N} = 2 \) background. In section 4 we add a constant electric field to the system and discuss the effect of it on the jet-quenching parameter. Finally in section 5 we summarize our results.

## 2 STU background

In this section we introduce the three-charge non-extremal black hole solution in \( \mathcal{N} = 2 \) supergravity which is called the STU model [27, 41]. In this model, there are three real scalar fields \( X^i \) which is corresponding to three black hole charges. These fields satisfy in a condition as \( X^1 X^2 X^3 = 1 \). In this paper we discuss the special cases of the (I) one-charge black hole, \( q_1 = q, q_2 = q_3 = 0 \), (II) two-charge black hole, \( q_1 = q_2 = q, q_3 = 0 \), and (III) three-charge black hole, \( q_1 = q_2 = q_3 = q \). In the general case, the STU model is described by the following solution [41],

\[
ds^2 = -\frac{f}{\mathcal{H}^4} dt^2 + \mathcal{H}^2 \left( \frac{r^2}{R^2} d\vec{x}^2 + \frac{dr^2}{f} \right),
\]

where,

\[
f = 1 - \frac{\eta}{r^2} + \frac{r^2}{R^2} \mathcal{H}, \quad \mathcal{H} = \prod_{i=1}^{3} \left( 1 + \frac{q_i}{r^2} \right).
\]

In the equation (1) the parameter \( r \) is the radial coordinate along the black hole, so the boundary of the AdS space located on the brane (\( r \to \infty \)). In the equation (2) there is an overall factor \( q_i = \eta \sinh^2 \gamma_i \), where \( \eta \) is called non-extremality parameter and \( \gamma_i \) related to the electric charges of the black hole. Finally we should note that the above solution includes a three-dimensional sphere, but we assume that the motion is along \( \vec{x} \) directions.

The Hawking temperature of this model is given by,

\[
T = \frac{2 + \sum_{i=1}^{3} k_i - \prod_{i=1}^{3} k_i}{2 \prod_{i=1}^{3} (1 + k_i)^{\frac{1}{2}}} \frac{r_h}{\pi R^2},
\]

where \( k_i \equiv \frac{q_i}{r_h^2} \) and the constant \( R \) denotes the curvature of the AdS space-time. The radius \( r_h \) denotes the event horizon of the black hole. Previous studies of the jet-quenching parameter
focused on the near-extremal black hole in the \( \mathcal{N} = 4 \) super Yang-Mills theory. Here, we can see that at the \( \eta \to 0 \) limit, where there is near-extremal black hole, the equation (3) reduces to the Hawking temperature in \( \mathcal{N} = 4 \) super Yang-Mills theory, where \( T = \frac{r_h}{\pi R^2} \). Also the zero temperature limit obtained by putting \( r_h^2 = \frac{q_i^2}{k_i} \) or \( k_i = 2 \) in the equation (3).

As explained in the introduction, in order to obtain the jet-quenching parameter one needs to rewrite the metric (1) in the light-cone coordinates. Therefore one can introduce new coordinates as \( x^\pm = \frac{t \pm x_1}{\sqrt{2}} \) and rewrite the metric (1) in the following form,

\[
\begin{align*}
\text{ds}^2 &= \frac{1}{2} \left( \frac{\mathcal{H} \pi r^2}{R^2} - \frac{f}{\mathcal{H}^{\frac{1}{3}}} \right) \left( (dx^+)^2 + (dx^-)^2 \right) - \left( \frac{\mathcal{H} \pi r^2}{R^2} + \frac{f}{\mathcal{H}^{\frac{1}{3}}} \right) dx^+ dx^- \\
&\quad + \frac{\mathcal{H}^\frac{1}{3}}{2} \left( \frac{r^2}{R^2}(dx_2^2 + dx_3^2) + \frac{dr^2}{f} \right).
\end{align*}
\]

We need to obtain the lagrangian density by using the above metric and put in the following Nambu-Goto action,

\[
S = -\frac{1}{2\pi \alpha'} \int d\tau d\sigma \sqrt{g}.
\]

In the next section we obtain the jet-quenching parameter in the above background.

### 3 Jet-quenching parameter

We begin with the general relation for the jet-quenching parameter \([34]\),

\[
\hat{q} \equiv 8\sqrt{2} S_I \frac{S_I}{L^- L^2},
\]

where \( S_I = S - S_0 \). Therefore, calculating the jet-quenching parameter reduces to obtain actions \( S \) and \( S_0 \). At the first, one should consider an open string whose endpoints lie on the brane. In the light-cone coordinates, the string may be described by \( r(\tau, \sigma) \). We use the static gauge where \( \tau = x^- \) and \( \sigma = x^2 \equiv y \), and all other coordinates considered as constants. In that case \(-\frac{L^-}{2} \leq y \leq \frac{L^-}{2} \), and \( L^- \leq x^- \leq 0 \), and because of \( L^- \gg L \) one can assume that the world-sheet is invariant along the \( x^- \) direction. Therefore the string may described by \( r(y) \), so the boundary condition is \( r(\pm \frac{L^-}{2}) = \infty \). In this configuration, the induced metric on the string fulfills obtained as the following,

\[
2g = \left( \frac{\mathcal{H}^\frac{2}{3} r^2}{R^2} - \frac{f}{\mathcal{H}^{\frac{1}{3}}} \right) \left( \frac{r^2}{R^2} + \frac{r'^2}{f} \right).
\]

Since the equation (7) is \( x^- \) dependent, one can integrate over \( x^- \) easily and then the Nambu-Goto action is given by,

\[
S = \sqrt{2L^-} \frac{\alpha'}{2} \int_0^{\frac{L^-}{2}} dy \sqrt{\left( \frac{\mathcal{H}^\frac{2}{3} r^2}{R^2} - \frac{f}{\mathcal{H}^{\frac{1}{3}}} \right) \left( \frac{r^2}{R^2} + \frac{1}{f} r'^2 \right)}.
\]
We can remove the $r'$ by using equation of motion. In that case, since the lagrangian density is time-dependent, one can write,

$$\mathcal{H} = \frac{\partial \mathcal{L}}{\partial r'} r' - \mathcal{L} = \text{Const.} \equiv E. \quad (9)$$

Therefore we can obtain the following relation,

$$r'^2 = \frac{f_{r'2}}{R^2 E^2} \left[ \frac{\mathcal{H}_{2}}{2R^2} \left( \frac{\mathcal{H}_{2} r^2}{R^2} - \frac{f}{\mathcal{H}_{2}} \right) r^2 - E^2 \right]. \quad (10)$$

The equation (10) has two poles where $r' = 0$. The main pole exist at the horizon, so it is clear that the equation (10) has a zero at the horizon where $f = 0$. In this case the string comes from infinity ($r(\frac{L}{2}) = \infty$) and touches the horizon and return to infinity ($r(-\frac{L}{2}) = \infty$).

The second pole of the equation (10) obtained by \( \frac{f_{r'2}}{\mathcal{H}_{2}} - \frac{\mathcal{H}_{2} r^4}{2R^2} + 2E^2 = 0 \). Ref. [42] show is that the string world sheet has one end at a Wilson line at the boundary with $Im[\ell] = 0$ and the other end at a Wilson line the boundary with $Im[\ell] = -i \epsilon$. The only way that the string worldsheet linking these two Wilson lines can meet is if the string worldsheet hangs down to the horizon. Therefore the only physical situation is the first case where the string touches the horizon. Also in our case, drawing the $r'^2$ in terms of $r$ tell us that the turning point of string should be $r_h$.

By using equation (10) in (8) and also new definition of $B \equiv \frac{1}{E^2}$ one can rewrite the Nambu-Goto action in the following form,

$$S = \frac{L}{2\pi \alpha'} \int_{r_h}^{\infty} dr \sqrt{r \left( \frac{\mathcal{H}_{2} r^2}{R^2} - \frac{f}{\mathcal{H}_{2}} \right) + f R^2}. \quad (11)$$

For the low energy limit ($E \to 0$) we expand the equation (11) to leading order in $\frac{1}{E}$. This is reasonable since the determination of $\hat{q}$ demands the study of the small separation limit of $L$. Then at the first order of $\frac{1}{E}$ one can obtain,

$$S = \frac{L}{2\pi \alpha'} \int_{r_h}^{\infty} dr \sqrt{\frac{2\mathcal{H}_{2} r^2}{f} \left( \frac{\mathcal{H}_{2} r^2}{R^2} - \frac{f}{\mathcal{H}_{2}} \right) \left[ 1 + \frac{R^2}{(\mathcal{H}_{2} r^2 - \frac{f}{\mathcal{H}_{2}}) B r^2} \right]}. \quad (12)$$

Now, we are going to extract action $S_0$ which can be interpreted as the self energy of the isolated quark and the isolated antiquark. In that case by using the following relation [38],

$$S_0 = \frac{L}{\pi \alpha'} \int_{r_h}^{\infty} dr \sqrt{G_{rr} - G_{rr}}, \quad (13)$$

one can find,

$$S_0 = \frac{L}{2\pi \alpha'} \int_{r_h}^{\infty} dr \sqrt{\frac{2\mathcal{H}_{2} r^2}{f} \left( \frac{\mathcal{H}_{2} r^2}{R^2} - \frac{f}{\mathcal{H}_{2}} \right)}. \quad (14)$$
Then one can obtain easily,

\[ S_I = \frac{1}{\sqrt{B}} \frac{L}{2\pi\alpha'} \int_{r_h}^{\infty} dr \sqrt{\frac{2R^4}{(H^2r^2 - \frac{L}{\eta^2})f r^4}}. \]  

On the other hand, one can integrate equation (10) and obtain the following relation for infinitesimal \( \frac{1}{B} \),

\[ \frac{L}{2} = R^2 \int_{r_h}^{\infty} dr \frac{1}{\sqrt{\frac{B}{2}(\frac{H^2r^2}{R^2} - \frac{f}{\eta^2})f r^4}}. \]  

Therefore, by using relations (6), (15) and (16) we can specify the jet-quenching as the following,

\[ \hat{q} = \frac{(I(q))^{-1}}{\pi\alpha'}. \]  

where

\[ I(q) = R^2 \int_{r_h}^{\infty} \frac{dr}{\sqrt{\frac{H^2r^2}{R^2} - \frac{f}{\eta^2}} f r^4} \]  

Here, it is important to explain horizon structure of the STU solution. The \( f(r) = 0 \) from the equation (2) reduces to the following equation [27],

\[ r^6 + \mathcal{A}r^4 - \mathcal{B}r^2 + q_1q_2q_3 = 0, \]  

where \( \mathcal{A} = q_1 + q_2 + q_3 + R^2 \) and \( \mathcal{B} = \eta R^2 - q_1q_2 - q_2q_3 - q_1q_3 \). The equation (19) has two positive and one negative zero. Two positive roots interpreted as inner and outer horizon, so the horizon radius \( r_h \) denoted the outer horizon.

In order to obtain the explicit expression of the jet-quenching parameter we consider three special cases of one, two and three charged black hole.

### 3.1 One-charged black hole

In this case we set \( q_1 = q, q_2 = q_3 = 0 \). So, the integral (18) reduced to the following expression,

\[ I(q_1) = R^4 \int_{r_h}^{\infty} \sqrt{\frac{(1 + \frac{\eta^2}{r^2})^4}{(r^2 - \eta)(r^4 + (q + R^2)r^2 - \eta R^2)}} dr, \]  

where

\[ r_h^2 = \frac{1}{4}(-2q + 2\pi^2 R^4 T^2 + 2\sqrt{2q\pi^2 R^4 T^2 + \pi^4 R^8 T^4}). \]  

The relation (21) obtained from the Hawking temperature (3). On the other hand from the equation (19) one can obtain,

\[ r_h^2 = \frac{q + R^2}{2} \left[-1 + \sqrt{1 + \frac{4\eta R^2}{(q + R^2)^2}} \right]. \]
If the condition \( \frac{2r_0^2}{q^2 R^2} \gg 1 \) satisfied, then the rescaling \( \eta R^2 \equiv r_0^4 \) implies that \( r_h = r_0 \) and we recover the case of \( \mathcal{N} = 4 \) SYM plasma.

Numerically, we draw graph of the jet-quenching parameter in terms of the black hole charge and the temperature in the Fig. 1 and Fig. 2 respectively. These plots show that the jet-quenching parameter of the \( \mathcal{N} = 2 \) theory is larger than the jet-quenching parameter of the \( \mathcal{N} = 4 \) theory. For example by choosing \( R^2 = \alpha' \sqrt{\lambda}, \alpha' = 0.5 (\alpha' = 0.25), \lambda = 6\pi, q = 10^6 \) and \( T = 300 MeV \) one can obtain, \( \hat{q} = 42 (\hat{q} = 18) \) GeV\(^2/\)fm. In that case the thermodynamical stability let us to choose \( q \leq 5 \times 10^6 \) for \( T = 300 MeV \). On the other hand, for the small black hole charge, by taking \( \alpha' = 0.5 \) and \( \lambda = 6\pi \) one can obtain \( \hat{q} = 37.5 \) GeV\(^2/\)fm. It means that the black hole charge increases the jet-quenching parameter. In order to obtain \( \hat{q} = 5 \) GeV\(^2/\)fm the corresponding temperature of the quark gluon plasma is 155 MeV, which is smaller than expected [43].

### 3.2 Two-charged black hole

In this case we set \( q_1 = q_2 = q, q_3 = 0 \). So, the integral (18) reduced to the following expression,

\[
I(q_{1,2}) = R^4 \int_{r_h}^{\infty} \sqrt{\frac{1 + \frac{q^2}{4\hat{q}^2}}{\rho(r^4 + (2q + R^2)r^2 - \eta R^2 + q^2)}} dr, \tag{23}
\]

where we defined \( \rho \equiv ((R^2 - 1)r^4 + (2qR^2 - R^2 - q)r^2 + R^2 q^2 + \eta R^2 - q^2) \), and,

\[
r_h = \pi R^2 T. \tag{24}
\]

The relation (24) obtained from the Hawking temperature (3). Numerically, we draw graph of the jet-quenching parameter in terms of the black hole charge and the temperature in the Fig. 1 and Fig. 2 respectively. These plots show that the jet-quenching parameter of the \( \mathcal{N} = 2 \) theory is larger than the jet-quenching parameter of the \( \mathcal{N} = 4 \) theory. Also we find that the jet-quenching parameter of the two-charged black hole is larger than the jet-quenching parameter of the one-charge black hole. For example by choosing \( R^2 = \alpha' \sqrt{\lambda}, \alpha' = 0.5, \lambda = 6\pi, q = 10^6 \) and \( T = 300 MeV \) one can obtain \( \hat{q} = 49 \) GeV\(^2/\)fm. In that case the thermodynamical stability let us to choose \( q \leq 3 \times 10^6 \) for \( T = 300 MeV \). If we consider small value of the black hole charge then find the same value of the jet-quenching parameter as the previous case, this point illustrated in the Fig. 2. Therefore, in order to obtain \( \hat{q} = 5 \) GeV\(^2/\)fm the corresponding temperature of the quark gluon plasma is 155 MeV for the small black hole charge.

### 3.3 Three-charged black hole

In this case we set \( q_1 = q_2 = q_3 = q \). So, the integral (18) reduced to the following expression,

\[
I(q_{1,2,3}) = R^4 \int_{r_h}^{\infty} \sqrt{\frac{r^2(r^2 + q)}{\rho(r^6 + (R^3 + 3q)r^4 + (3q^2 - \eta R^2)r^2 + q^2)}} dr, \tag{25}
\]
where we defined \( \rho \equiv ((R^2 - 1)r^6 + (3qR^2 - R^2 - 3q)r^4 + (3R^2q^2 + \eta R^2 - 3q^2)r^2 + (R^2 - 1)q^3) \), and \( r_h \) obtained from the Hawking temperature (3). Numerically, we give plot of the jet-quenching parameter in terms of the black hole charge and the temperature in the Fig. 1 and Fig. 2 respectively. These plots show that the jet-quenching parameter of the \( \mathcal{N} = 2 \) theory is larger than the jet-quenching parameter of the \( \mathcal{N} = 4 \) theory. Also we find that the jet-quenching parameter of the three-charged black hole is larger than the jet-quenching parameter of the one-charge and two-charged black holes. For example by choosing \( R^2 = \alpha' \sqrt{\lambda} \), \( \alpha' = 0.5 \), \( \lambda = 6\pi \), \( q = 10^6 \) and \( T = 300 \text{ MeV} \) one can obtain \( \hat{q} = 58 \text{ GeV}^2/\text{fm} \). In that case the thermodynamical stability let us to choose \( q \leq 2.5 \times 10^6 \) for \( T = 300 \text{MeV} \). If we consider small value of the black hole charge then find the same value of the jet-quenching parameter as the previous cases, this point illustrated in the Fig. 2. Therefore, in order to obtain \( \hat{q} = 5 \text{ GeV}^2/\text{fm} \) the corresponding temperature of the quark gluon plasma is 155 MeV for the small black hole charge.

In order to compare our results with the case of \( \mathcal{N} = 4 \) SYM we also perform the following rescaling [41],

\[
\begin{align*}
    r &\rightarrow \lambda^\frac{1}{4} r, \\
    t &\rightarrow \frac{t}{\lambda^{\frac{1}{4}}}, \\
    \eta &\rightarrow \lambda \eta, \\
    q &\rightarrow \lambda^\frac{1}{2} q, \\
    \vec{x} &\rightarrow \frac{\vec{x}}{\lambda^\frac{1}{4}},
\end{align*}
\]

(26)
Figure 2: Plot of the jet-quenching parameter in terms of the temperature for small black hole charge. We fixed parameter as $\alpha' = 0.5$, $\lambda = 6\pi$. In that case three different cases of one, two, and three-charged black hole have similar manner.

Also we set $r_0^4 \equiv \eta R^2$, and taking $\lambda \to \infty$ yields us to the following result,

$$\hat{q} = \frac{r_0^2}{\pi \alpha' R^4} \left[ \int_{r_h}^{\infty} \frac{dr}{r^2 \sqrt{f}} \right]^{-1}, \quad (27)$$

where

$$f = H^3 - \frac{r_0^4}{r_h^4},$$

$$H = 1 + \frac{q}{r_h^2}, \quad (28)$$

which agree with the results of the Ref. [35], where the jet-quenching parameter calculated with the chemical potential. The horizon radius $r_0$ obtained for the case of zero-charge black hole. For the non-zero charge it is clear that the horizon radius decreases ($r_h < r_0$). In that case the relation between the chemical potential and black hole charge may be given by [35, 44],

$$\mu = \frac{\sqrt{2q(q + r_h^2)}}{r_h R^2}. \quad (29)$$

Therefore the $q = 0$ limit is equal to $\mu = 0$ limit and one can say that the jet-quenching parameter from the $\mathcal{N} = 2$ supergravity theory with zero chemical potential is equal to the jet-quenching parameter from the $\mathcal{N} = 4$ SYM theory.
4 Effect of constant electric field

In the description of the AdS/CFT correspondence the endpoint of both fundamental and Dirichlet strings under influence of non-zero NS NS B-field background corresponds to the moving quark with a constant electromagnetic field. In this section we would like to add a two form $F = edt \wedge dx_1$ as a constant electric field to the line element (1). Antisymmetric field $e$ is the constant electric field. Now we are going to obtain the effect of the constant electric field on the jet-quenching parameter. In that case the Nambu-Goto action is given by the square root of the following equation,

$$2g = \left( \frac{\mathcal{H}_+^2 r^2}{R^2} - \frac{f}{\mathcal{H}_+^2} + e \right) \left( \frac{r^2}{R^2} + \frac{r'^2}{f} \right).$$

(30)

Therefore one can obtain the jet-quenching parameter as the following,

$$\hat{q} = \frac{(I(q, e))^{-1}}{\pi \alpha'},$$

(31)

where $I(q, e) = R^2 \int_{r_h}^{\infty} \frac{dr}{\sqrt{(\frac{\mathcal{H}_+^2 r^2}{R^2} - \frac{f}{\mathcal{H}_+^2} + e) \mathcal{H}_+^2 fr^4}}$.

(32)

where $f$ and $H$ is given by the relation (2). In order to find effect of the constant electric field on the jet-quenching parameter we examine above integral for three different cases of one, two and three-charged black hole. Numerically, and under near boundary approximation, we draw graph of the jet-quenching parameter in terms of the constant electric field and find that the constant electric field increases the value of the jet-quenching parameter. In the Fig. 3 we draw the jet-quenching parameter in terms of the constant electric field for the large black hole charge.

As before, by rescaling (26) one can obtain,

$$\hat{q} = \frac{1}{\pi \alpha' R^4} \left[ \int_{r_h}^{\infty} \frac{dr}{r^4 \sqrt{f H(\frac{r^2}{R^2} + e)}} \right]^{-1},$$

(33)

where $f$ and $H$ is given by the relation (28). It is easy to show the effect of the constant electric field on the value of the jet-quenching parameter. The $q = 0$ limit of the equation (33) reduces to the following expression,

$$\hat{q}_0 = \frac{1}{\pi \alpha' R^4} \left[ \int_{r_0}^{\infty} \frac{dr}{\sqrt{(r^4 - r_0^4)(e r^4 + r_0^4)}} \right]^{-1}.$$

(34)

It is clear that the case of $e \to 0$ recovers the result of (27). We find that the effect of constant electric field is increasing the jet-quenching parameter, so it is in agreement with increasing the drag force in presence of constant electric field for ultra relativistic motion [30].
Figure 3: Plot of the jet-quenching parameter in terms of the constant electric field. We fixed our parameter as $\alpha' = 0.5$, $\lambda = 6\pi$, $q = 10^6$ and $T = 300$. The solid line represents the case of $q_1 = q, q_2 = q_3 = 0$. The dotted line represents the case of $q_1 = q_2 = q, q_3 = 0$. The dashed line represents the case of $q_1 = q_2 = q_3 = q$. It show that the jet-quenching parameter increased by the constant electric field.

5 Higher derivative correction

In this section, in absence of any external field, we would like to calculate the effect of higher derivative terms on the jet-quenching parameter. In this way we use results of the Ref. [45], where the first-order correction of the solution (2) is given by,

$$
\begin{align*}
f &= 1 - \frac{\eta}{r^2} + \frac{r^2}{R^2} (1 + \frac{q}{r^2})^3 + \frac{c}{24} \left[ \frac{\eta^2}{4r^6(1 + \frac{q}{r^2})} - \frac{8q(q + \eta)}{3R^2r^4} \right], \\
H &= 1 + \frac{q}{r^2} - \frac{c}{24} \left[ \frac{q(q + \eta)}{3r^2(r^2 + q)^2} \right].
\end{align*}
$$

(35)

where $c$ is small parameter of the higher derivative correction. We should note that the solution (35) obtained for the black hole with three equal charge ($q_1 = q_2 = q_3 = q$). In that case the jet-quenching parameter obtained as the following expression,

$$
\hat{q} = \frac{(I(q, c))^{-1}}{\pi \alpha'},
$$

(36)

where

$$
I(q, c) = \int_{r_H}^{\infty} \frac{dr}{\sqrt{(\frac{H^2r^2}{R^2} - \frac{f}{H})f r^4}},
$$

(37)
and $f$ and $H$ are given by the relation (35), also $r_h$ is given by the following equation [45],

\[
    r_H = r_h + \frac{cr_h}{24} \left(1 + \frac{q}{r_h^2}\right)^4 \left(3q^2 - 26qr_h^2 + 3r_h^4\right) \frac{1}{24R^4(1 + \frac{q}{r_h^2})}\left[\frac{(1 + \frac{q}{r_h^2})(q - 2r_h^2)}{R^2} - 1\right] - \frac{cr_h}{24} \left(1 + \frac{q}{r_h^2}\right)^2(13q - 3r_h^2) + 3R^2 \frac{r_h}{24R^2(1 + \frac{q}{r_h^2})}\frac{1}{(1 + \frac{q}{r_h^2})^2(q - 2r_h^2)} - 1,\quad (38)
\]

As before one can study near boundary behavior of the jet-quenching parameter and find that the higher derivative terms include at $O(cT^9)$. In that case we find that the higher derivative terms decrease the value of the jet-quenching parameter. So, for the fixed parameter such as $\alpha' = 0.5, \lambda = 6\pi, T = 300$ and small black hole charge, we obtain $c < 0.00021$ to have positive jet-quenching parameter. For example with the above fixed parameter and $c = 0.0001$ one can obtain $\hat{q} = 4.6 \text{ GeV}^2/\text{fm}$ which is approximately value of the jet-quenching parameter of the $\mathcal{N} = 4$ SYM theory. In order to obtain $\hat{q} = 5 \text{ GeV}^2/\text{fm}$ the corresponding higher derivative parameter should be $c \approx 97 \times 10^{-4}$ at $T = 300\text{MeV}$.

Again we use rescaling (26) and take $\lambda \rightarrow \infty$ limit, so we get,

\[
    \hat{q} = \frac{r_h^2}{\pi\alpha'R^4} \left[\int_{r_h}^\infty \frac{H}{\sqrt{f}} \frac{dr}{r^2}\right]^{-1},\quad (39)
\]

where

\[
    f = (1 + \frac{q}{r^2})^3 - \frac{r^4}{r^4} + \frac{cr_0^4}{24R^2r^4} \left[\frac{r_0^4}{4r^2(r^2 + q)} - \frac{8q}{3}\right],
\]

\[
    H = 1 + \frac{q}{r^2} - \frac{cq}{24R^2r^4(r^2 + q)},\quad (40)
\]

and radius $r_h$ is the root of the $f = 0$ from the relation (40). The equation (39) may be solved numerically, and explicit expression of the jet-quenching parameter can be obtained. But it is clear that the effect of higher derivative correction is to decrease the jet-quenching parameter. One can check this statement by taking $q = 0$ limit. In this limit the jet-quenching parameter derived as,

\[
    \hat{q}_0 = \frac{r_0^2}{\pi\alpha'R^4} \left[\int_{r_h}^\infty \frac{6R^2r^4}{96R^2r^4(r^4 - r_h^4) + cr_0^3 r} \frac{dr}{r}\right]^{-1},\quad (41)
\]

where,

\[
    r_h^4 = \frac{r_0^4}{2} \left(1 + \frac{1}{\sqrt{1 - \frac{c}{24R^2}}}\right).\quad (42)
\]

In that case it is necessary that $c < 24\alpha'\sqrt{\lambda}$. Comparing the equation (41) with the jet-quenching parameter of the $\mathcal{N} = 4$ SYM theory tell us that the effect of $c$ is decreasing the jet-quenching parameter.
6 Conclusion

One of the interesting properties of the strongly-coupled plasma at RHIC is jet quenching of partons produced with high transverse momentum. This parameter controls the description of relativistic partons and it is possible to employ the gauge/gravity duality and determine this quantity in the finite temperature gauge theories. In this paper we considered five dimensional $\mathcal{N} = 2$ thermal plasma and calculated the jet-quenching parameter in presence of the constant electric field, and higher derivative term. We found that the jet-quenching parameter of the charged black hole of the $\mathcal{N} = 2$ supergravity theory is larger than the $\mathcal{N} = 4$ SYM theory. We examine our solution for three special cases of one, two and three charged black hole. All cases yield to the same value of the jet-quenching parameter for the small black hole charge. However, thermodynamical stability allow to choose the black hole charge of order $10^6$. In that case, by choosing $\lambda = 6\pi$, $\alpha' = 0.5$ and $T = 300$MeV, we found $\hat{q} = 42, 49$ and $58$GeV$^2$/fm for one, two and three charged black hole respectively. These values of the jet-quenching parameter are far from experiments of RHIC with above fixed parameter (experimental data tell us that $5 \leq \hat{q} \leq 25$). There is no worry for this statement because the temperature of the $\mathcal{N} = 2$ supergravity theory should given smaller than the $\mathcal{N} = 4$ SYM theory. In that case with the temperature about 155MeV we obtained the jet-quenching parameter in the experimental range.

We studied the effect of the constant electric field and higher derivative correction on the jet-quenching parameter. We have shown that the effect of the constant electric field and the higher derivative correction is to increase and decrease the jet-quenching parameter respectively. By analyzing the near boundary behavior we found the increasing of the jet-quenching parameter due to the electric field and decreasing of it under the higher derivative is infinitesimal of order $O(\frac{e}{T^5})$ and $O(\frac{c}{T^9})$ respectively. We found that our results are agree with the results of the $\mathcal{N} = 4$ SYM plasma at $\eta \to 0$ ($q = 0$) limit or zero chemical potential and rescaling (26).

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