Four-Josephson-junction flux qubit with controllable energy gap

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Abstract.
We report the measurements of a double-loop four-Josephson-junction flux qubit. By varying
the magnetic flux in one of the two loops, the energy gap at the classical degeneracy point can
be controlled in situ. The basic operation of the system is illustrated by the appearance of a
qubit step on a two-dimensional flux map. The energy gap can be indirectly estimated from
the shape of the qubit step. The dependence of the shape of the qubit step on the control flux,
which is analyzed in terms of the step height and maximum slope, is in good agreement with
the simulation results, thereby implicitly indicating that the energy gap is varied from almost
zero, less than the thermal energy, to a value higher than 50 GHz.

1. Introduction
It is known that superconducting qubits are promising candidates for the implementation of
scalable quantum information processors [1]. Quantum manipulation of superconducting qubits
has been demonstrated in the charge, charge-phase, flux, and phase regimes. A qubit in the flux
regime has an advantage that it can be made insensitive to the randomly fluctuating charges in
the substrate. In particular, the three-Josephson-junction (3-JJ) flux qubit [2] has been studied
by many researchers. Recently, a tunable coupling scheme [3] and controlled-NOT quantum gate
operation [4] using a qubit pair have been demonstrated. It should be noted that the principle
of the four-Josephson-junction (4-JJ) flux qubit (e.g., as studied in Ref. [3]) is the same as that
of the 3-JJ flux qubit.

A major difficulty associated with the practical use of the 3-JJ flux qubit is that it is difficult
to reproduce its device parameters in fabrication. The circulating current $I_p$ in the classical
basis states and the energy gap $\Delta$ of the qubit at the classical degeneracy point are determined
by the sample parameters such as the Josephson energy $E_J$, charging energy $E_C$, and junction-
area ratio $\alpha$ of the Josephson junctions of the qubit [2]. All the parameters are subject to
variations in the fabrication process. Moreover, $\Delta$ has a strong dependence on these parameters.
Since the optimal manipulation of a qubit with minimum decoherence can be achieved at the
classical degeneracy point using resonant microwave pulses [5], adequate control over $\Delta$ can be
advantageous to the implementation of quantum operation schemes. It can also allow significant
flexibility in the manipulation of qubits. Mooij et al. have introduced the concept of a double-
loop 4-JJ flux qubit as a flux qubit with a controllable energy gap in situ. The 4-JJ flux qubit
is similar to the 3-JJ flux qubit except that in the 4-JJ qubit, the smallest junction in the 3-JJ

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qubit is replaced by a parallel circuit of two junctions [2]. It is important to note that the control
over two magnetic fluxes in the 4-JJ qubit allows one to perform any unitary transformation
of the qubit state [6]. Very recently, an efficient coupling scheme for the 4-JJ qubit has been
presented [7]. In this paper, we present the results of the measurements of a double-loop 4-JJ
flux qubit. The positions of the qubit step on a two-dimensional flux map and the dependence
of the shape of the qubit step on the control flux are compared with their theoretical predictions.

2. Experimental

Figure 1 shows a schematic diagram of the sample. The 4-JJ flux qubit is denoted by thick
lines. The circulating current in the qubit is measured by the readout DC-SQUID. Δ is
controlled by the control current. To increase the strength of their coupling to the qubit,
the DC-SQUID and the control line share a loop segment with the qubit. The sample is
fabricated by e-beam lithography and shadow deposition of Al. Two of the junctions have
an identical area S of approximately 0.03 (μm)² and the junction capacitance C, while the
other two junctions have a smaller area of βS, where β is approximately 0.7. The quantum
state of the 4-JJ qubit is manipulated through the magnetic frustration f₁ and f₂ in the two
loops; the magnetic frustration is the amount of magnetic flux in units of the superconducting
flux quantum Φ₀ = ℏ/2e. The areas of the qubit loops enclosing f₁ and f₂ are 75 (μm)² and
33 (μm)², respectively.

It is convenient to use \( \varphi_p = (\varphi_1 + \varphi_2)/2 \) and \( \varphi_m = (\varphi_1 - \varphi_2)/2 \) as the coordinates of the
system, where \( \varphi_1 \) and \( \varphi_2 \) are the phase differences of the two larger junctions. The Hamiltonian
of the system is

\[
H = \frac{P^2}{2M_p} + \frac{P^2}{2M_m} + E_J\{2 + 2\beta - 2\cos\varphi_p \cos\varphi_m - 2\beta\cos(\pi f_2) \cos(2\pi(f_1 + f_2/2) + 2\varphi_m)\},
\]

where the expressions for the momenta and the mass terms are shown in Ref.[8]. The term
\( \alpha(f_2) = 2\beta\cos(\pi f_2) \) plays the role of \( \alpha \) in the 3-JJ qubit. As \( \alpha(f_2) \) decreases, an increase should
be observed in the energy gap and the transition width of the persistent current due to the
decrease in the barrier height.

The sample was mounted in a copper cell, which was thermally anchored on the cold finger
of a dilution refrigerator with a base temperature of 20 mK. We measured the switching current
I_{sw} of the readout DC-SQUID as a function of the control current I_{cont} and the magnet current
I_{mag} that flows in the superconducting magnet used to apply a uniform magnetic field. We
note that f₁ and f₂ can be controlled independently by I_{cont} and I_{mag}. In the measurement
of the switching current, the SQUID bias current I_{bias} was swept using a triangular waveform
at 2 kHz and the average switching current was recorded. Twisted pairs of electrical leads for
I_{cont} and I_{bias} were carefully filtered. I_{cont} and I_{bias} were generated by current sources in a
symmetric mode, in which the voltages at the terminals were symmetric with respect to the
ground potential.

Figure 1. Schematic diagram of the 4-JJ
qubit. The qubit, denoted by thick lines,
is coupled to a control line and readout
SQUID. The crosses represent Josephson
junctions. The areas of the larger junctions
and the ratio of the areas for the 4-JJ qubit
are approximately \( S = 0.03 \, \mu m^2 \) and \( \beta = 0.7 \), respectively.
3. Experimental results and discussion

In the dependence of $I_{sw}$ on $I_{mag}$ for various $I_{cont}$ values, we observed qubit steps, which reflect the reversal of the circulating current in the qubit. The qubit steps appeared in limited ranges of $I_{cont}$, and the positions of the qubit steps changed linearly with $I_{cont}$ in these ranges. The positions of the observed qubit step in the $I_{cont}$-$I_{mag}$ plane are summarized in Fig. 2.

The position of a qubit step is easily determined in view of the effective temperature $T_{eff}$. Near the lattice point where $(f_1, f_2) = (N_1 + 1/2, N_2)$, the qubit step appears on a line segment satisfying $f_1 + f_2/2 = N_1 + (N_2 + 1)/2 = 0, \pm 1/2, \pm 1, \pm 3/2, \cdots$ in the $f_1$-$f_2$ plane. The regular pattern shown in Fig. 2 is in complete agreement with the theoretical predictions stated above. The range of $f_2$ for the occurrence of the qubit step is approximately $|f_2 - N_2| < 0.33$.

We now discuss the change in the shape of the qubit step for variation in $f_2$. The energy eigenvalues and circulating current of the qubit were calculated using the one-dimensional approximation of the Hamiltonian. We examine the shape of the qubit step as a function of the effective flux $f = f_1 + f_2/2$ with $f_2$ constant in terms of the step height and maximum slope. When the tight-binding approximation holds, the maximum slope at $f = 1/2$ is

$$
\frac{\partial < I >}{\partial f} = \frac{f_2^2 \Phi_0}{\Delta/2} \tanh \left( \frac{\Delta}{2k_B T_{eff}} \right),
$$

where $< I >$ is the thermal average of the circulating current at the effective temperature $T_{eff}$. It should be noted that both $\Delta$ and $I_p$ depend on $f_2$, because of the dependence of $\alpha(f_2)$ on $f_2$. As $\alpha(f_2)$ decreases, $\Delta$ increases exponentially, while $I_p$ decreases slowly.

The observed values of the step height and maximum slope are shown in Figs. 3(a) and 3(b), respectively, in terms of the current in the qubit. As $f_2$ decreases below $-0.3$, the decrease in the maximum slope is exponential, while that in the step height is nearly linear. These changes are explained by the theory described above and are a manifestation of the exponential increase in the energy gap with decreasing $\alpha(f_2)$. The dashed lines in the figures are a theoretical fit to the data. The parameters used in the calculation are $E_J = 350$ GHz, $E_3/E_C = 15$, $\beta = 0.7$, and $T_{eff} = 100$ mK. The values of $E_C$ and $\beta$ are consistent with the values expected for the observed junction areas. The agreement between the values attained by measurement and theory is very good with the exception of the values of the saturated maximum slope. The saturated maximum slope, which is approximately 50 $\mu$A, is an order of magnitude smaller than the value achieved by calculation. The small slope may be attributed to the noise in the control current. Since the control line is attached directly to the qubit as shown in Fig. 1, the current noise may cause a

![Figure 2. Circles representing the position of the qubit step in the $I_{cont}$-$I_{mag}$ plane. The flux map for $f_1$ and $f_2$ is overlapped. The position of the qubit step is in agreement with the position obtained by the theory described in the text.](image)
phase fluctuation in the junction in the qubit. We find that current noise of the order of 10 nA leads to the observed broadening of the qubit step. This noise would be suppressed in a future design of the sample, in which the strength of the coupling of the control line and the readout SQUID to the qubit are much smaller.

Our calculation indicates that for the sample we studied here \( \Delta \) increases exponentially from a value less than 300 MHz, which is less than the thermal energy, to a value higher than 50 GHz as \( \alpha(f_2) \) decreases. We intend to measure \( \Delta \) by microwave spectroscopy [9] in a future study.

![Figure 3](image.png)

**Figure 3.** Dependence of the (a) step height and (b) maximum slope of the qubit step on the control flux \( f_2 \). These values are plotted in terms of the circulating current in the qubit. The dashed lines represent theoretical results for \( E_J = 350 \) GHz, \( E_J/E_C = 15 \), \( \beta = 0.7 \), and \( T_{\text{eff}} = 100 \) mK.

4. Conclusions

In conclusion, we have demonstrated the basic operation of a double-loop 4-JJ flux qubit. The magnetic fluxes in the two loops are controlled independently using a control current and an external magnet. The position of the qubit step on the two-dimensional flux map agrees with the theoretical prediction. The change in the shape of the qubit step as a function of control flux is also in good agreement with the simulation results and implicitly indicates that the energy gap is controlled over a wide range of over 50 GHz. The controllability and scalability of this scheme allows efficient quantum manipulation of many qubits, which may be connected to a resonator state.

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