Committee Selection with Attribute Level Preferences

Venkateswara Rao Kagita\textsuperscript{1,*}, Arun K Pujari\textsuperscript{2,3}, Vineet Padmanabhan\textsuperscript{3} and Vikas Kumar\textsuperscript{2}

\textsuperscript{1}Mahindra École Centrale, Andhra Pradesh, India
\textsuperscript{2}Central University of Rajasthan, Rajasthan, India
\textsuperscript{3}University of Hyderabad, Andhra Pradesh, India

venkateswar.rao.kagita@gmail.com, akpujari@curaj.ac.in, vineetcs@uohyd.ernet.in, vikas007bca@gmail.com

Abstract

Approval ballot based committee formation is concerned with aggregating individual approvals of voters. Voters submit their approvals of candidates and these approvals are aggregated to arrive at the optimal committee of specified size. There are several aggregation techniques proposed in the literature and these techniques differ among themselves on the criterion function they optimize. Voters preferences for a candidate is based on his/her opinion on candidate suitability. We note that candidates have attributes that make him/her suitable or otherwise. Hence, it is relevant to approve attributes and select candidates who have the approved attributes. This paper addresses the committee selection problem when voters submit their approvals on attributes. Though attribute based preference is addressed in several contexts, committee selection problem with attribute approval has not been attempted earlier. We note that extending the theory of candidate approval to attribute approval in committee selection problem is not trivial. In this paper, we study different aspects of this problem and show that none of the existing aggregation rules satisfies Unanimity and Justified Representation when attribute based approvals are considered. We propose a new aggregation rule that satisfies both the above properties. We also present other analysis of committee selection problem with attribute approval.

1 Introduction

Committee selection by aggregating voters’ preferences is a fundamental problem of Social Choice Theory and has recently received considerable attention of AI community\textsuperscript{2,3,22,23}. Preference by approval ballots and preference by ranking are two important ways of expressing voters’ preferences. Approval ballot based method for committee selection is concerned with aggregating individual approvals of voters for selection of candidates. There are several aggregation techniques proposed in the literature and these techniques differ among themselves on the criterion function they optimize for candidate selection. Approval Voting maximizes approval counts; Satisfaction Approval Voting maximizes voters’ satisfaction and similarly, voter’s utility is optimized in Proportional Approval Voting. Minimax Approval Voting and Reweighted Approval Voting maximize different criteria and we elaborate these in the text. All these techniques assume that the voters approve individual candidates and these approvals are used to select a set of candidates to form a committee.

A voter prefers a candidate because of candidate’s suitability for the objective in hand. Candidates possess attributes that make a candidate suitable or otherwise. Instead of voters approving individual candidates, it is pertinent that voters express their preferences for attributes or characteristics of candidates. For instance, in order to form a cricket team, it is better to solicit approval on the attributes of player such as Run Tally Impact, Partnership-Building Impact, Pressure Impact, New Ball Impact etc., instead of seeking approval of individual players. Expressing preferences directly on attributes will lead to effective committee selection. Moreover, bribery, manipulability and bias can be controlled with attribute level preferences. The Constraint Approval Voting Systems \textsuperscript{5,7} utilizes candidates’ attributes for committee formation. They describe a candidate as a vector of attributes and select a committee \( W \) in such a way that some of the desired constraints over attributes is to be satisfied. For instance, Brams et al. \textsuperscript{5} represent a candidate with two attributes, ‘Region of the candidate’ and ‘Specialty’ and the constraint could be 10% of candidates in the committee should be from region A. However, the concept is completely different from our proposal in the sense that we deal with committee formation based on attribute approval. We also study different properties related to standard aggregation rules. Study of attribute-level preferences exists in various applications such as Food \textsuperscript{19,13,4}, Health Care \textsuperscript{21,11}, Housing \textsuperscript{10}, Farming \textsuperscript{16}, Airline Services \textsuperscript{11}, Technology Product Markets \textsuperscript{24}, Job \textsuperscript{12}, E-transactions \textsuperscript{9}, Travel \textsuperscript{14}. In this case, voters’ approval on attributes are required to be aggregated to determine a set of candidates for a committee. There has not been any attempt in this direction in the context of committee formation based on approval ballots.

\textsuperscript{*}Contact Author
In this paper, we address the problem of committee selection wherein voters approve different attribute values. A candidate is represented by \(d\) attribute values which are drawn from a specified domain. Voters submit their ballots for each of \(d\) attributes. The objective is to select a committee \(W\) of \(k\) candidates given approval ballots of the voters. As mentioned earlier, though the problem is important in the context of committee selection, there has not been any study taken place in this direction. Extending the results of candidate-approval based committee selection to attribute-approval based method is not trivial. In this paper, we formally present the formulation of the problem and several analytical results of committee selection with approvals on attributes. We analyze standard aggregation rules and their properties. We show that two properties, namely Unanimity and Justified Representation are not satisfied by standard aggregation techniques while taking into consideration attribute based approvals. We propose here a new aggregation rule and show that this rule satisfies Unanimity as well as Justified Representation. In this process, we give a detailed analysis of Unanimity and Justified Representation.

The rest of the paper is structured as follows. In the next section, we formulate and show the committee selection problem with attribute level approval. The detail analysis on Unanimity and Justified Representation are presented in Section 3 and Section 4, respectively. We propose a new rule called Greedy Approval Voting (GAV) in Section 5. Section 6 provides conclusion and scope for future work.

## 2 Aggregation Rules

Let \(V = \{v_1, v_2, \ldots, v_n\}\) be the set of voters and \(C = \{c_1, c_2, \ldots, c_m\}\) be the set of candidates. Each candidate \(c_i, 1 \leq i \leq m\), is a \(d\)-dimensional attribute vector. The attribute value \(c_i[j]\) of candidate \(c_i\) on dimension \(j\) is from domain \(D^j, 1 \leq j \leq d\). Let \(C^j_i\) be the set of elements in \(D^j\) approved by \(v_i\). We use a notation \(V_i\) to denote set of voters who have approved an attribute-value \(a\). The goal is to select a committee \(W\) of size \(k\) given voters’ approvals over attributes i.e., \(C^j_i, \forall v_i \in V, j \in [1, d]\). Table 1 summarizes the notations used in this paper. There are different aggregating rules known in the context of approval voting with candidate-approval. We study below these rules in the context of attribute-approval committee selection.

**Table 1: Summary of notations**

| Notation | Description |
|----------|-------------|
| \(V\)    | Set of voters |
| \(C\)    | Set of candidates |
| \(c_i[j]\) | Attribute value of a candidate \(c_i\) on dimension \(j\) |
| \(V_{a}\) | Set of voters who approved an attribute value \(a\) |
| \(D^j\) | Set of domain values on dimension \(j\) |
| \(C^j_i\) | Set of attributes approved by a voter \(v_i\) on dimension \(j\) |
| \(v_i^*\) | Candidate with highest utility score |
| \(W\)    | Target committee |
| \(V_W\)  | Set of voters who approved at least one candidate in \(W\) |
| \(k\)    | Target committee size |
| \(n\)    | Number of voters |
| \(m\)    | Number of candidates |
| \(d\)    | Number of dimensions |

### Approval Voting (AV)

Approval Voting selects a committee \(W\) that maximizes \(\sum_{v_i \in V} \sum_{C^j_i} |W \cap C^j_i|\), where \(C^j_i\) is the set of candidates approved by a voter \(v_i\). When voters submit their approval on attributes, the approval voting score of a candidate \(c\) with respect to \(V\) is defined as \(AV(c, V) = \sum_{v_i \in V} \sum_{j=1}^{d} |W^j \cap C^j_i|\). The approval voting score of a committee \(W\) is \(AV(W, V) = \sum_{c \in W} AV(c, V)\). Hence, Approval Voting (AV) rule selects \(W\) with highest \(AV(W, V)\) which can be computed by maximizing \(\sum_{v_i \in V} \sum_{j=1}^{d} \frac{1}{|W^j \cap C^j_i|}\), where \(W^j\) is the set of attributes on dimension \(j\) of \(W\). Approval Voting rule can be computed in polynomial time to form a committee. Computing approval score for each attribute involves scanning \(n\) ballots and can be done in \(O(n^d)\), where \(n\) number of distinct attribute values. Approval score of a candidate can be computed in \(O(md)\) and identifying top-\(k\) candidates takes \(O(klog(m))\). Hence, AV for attribute-approval is of \(O(na^d)\).

### Satisfaction Approval Voting (SAV)

SAV selects a committee \(W\) that maximizes \(\sum_{v_i \in V} \sum_{|C^j_i|} \frac{1}{W^j \cap C^j_i|}\). In the case of attribute-approval voting, we define satisfaction score of \(v_i\) for \(W\) as \(SAV(W, v_i) = \frac{AV(W, v_i)}{\sum_{j=1}^{d} |W^j \cap C^j_i|}\), where \(W^j\) (resp. \(C^j_i\)) is the set of attributes on dimension \(j\) respectively. SAV selects the \(W\) that maximizes \(\sum_{v_i \in V} SAV(W, v_i)\). The complexity of SAV is the same as that of AV.

### Reweighted Approval Voting (RAV)

At every stage, RAV reweighs voter’s approval score of a candidate and then selects the candidate with highest approval score. We define reweighed score \(RAV(c, v_i)\) as \(r(v_i) \times \sum_{j=1}^{d} \frac{1}{|W^j \cap C^j_i|}\), where \(r(v_i) = \frac{1}{1 + AV(W, v_i)}\). At every stage, we start with \(W = \emptyset\) and at every stage we select a candidate \(c\) that maximizes \(\sum_{v_i \in V} RAV(c, v_i)\) till \(|W| = k\). RAV is a multi-stage AV, hence, score computation needs to be done \(k\) times. Hence, the overall computation required in RAV is \(O(na^dk)\).

### Proportional Approval Voting (PAV)

PAV was proposed by Forest Simmons in 2001 and is known to be NP-hard. The objective of PAV is to maximize the sum of voters’ utilities, where utility of voter \(v_j = 1 + \frac{1}{2} + \ldots + \frac{1}{|W\cap C^j_i|}\). With attribute-approval, PAV selects \(W \subseteq C\) of size \(k\) that maximizes \(\sum_{v_i \in V} u(AV(W, v_i))\), where \(u(p) = 1 + \frac{1}{2} + \ldots + \frac{1}{|p|}\).

### Minimax Approval Voting (MAV)

MAV selects a committee \(W\) that minimizes the maximum Hamming distance between \(W\) and voters’ approval ballots. Given attribute approvals, we define MAV-score of a committee \(W\) as \(MAV(W, v_i) = MAX(f(W, v_1), f(W, v_2), \ldots, f(W, v_n))\), where \(f(W, v_i) = MAX(d(W^j, C^j_i)_{j=1})\) and \(d(A^j, B^j) = |A^j \cap B^j| + |A^j \setminus B^j| + |B^j \setminus A^j|\). MAV returns a committee \(W\) with the lowest MAV-score. MAV is also known to be NP-hard problem.
Table 2: Summary of the properties satisfied by rules

|                | AV | SAV | PAV | RAV | MAV |
|----------------|----|-----|-----|-----|-----|
| Homogeneity    | ✓  | ✓   | ✓   | ✓   | ✓   |
| Consistency    | ✓  | ✓   | ✓   | ✓   | ✓   |
| Monotonicity   | ✓  | ✓   | ✓   | ✓   | ✓   |
| Committee Monotonicity | ✓ | ✓ | × | ✓ | ✓ |

2.1 Properties

In this section, we review some standard properties that are desired to be satisfied by multi-winner approval based rules. We omit all the trivial proofs. Table 2 summarizes different properties satisfied by different rules.

**Homogeneity** — A rule is said to satisfy Homogeneity property if it selects the same W independent of number of times voters’ ballot is replicated.

**Consistency** — A rule is said to satisfy consistency if it constructs the same committee W for voters lists V and V’, it also generates W’ with respect to the voters list V ∪ V’.

**Monotonicity** — A rule is monotonic if it satisfies the following two conditions, 1) If c ∈ W with respect to V then c ∈ W with respect to V_c[j] ← V_j[j] ∪ V_c, V_c[j] ∉ V_c[j], j ∈ [1, d] 2) If c ∉ W with respect to V then c ∉ W with respect to V_c[j] ← V_j[j] \ V_c, V_c[j] ∉ V_c[j], j ∈ [1, d].

**Committee Monotonicity** — Suppose W and W’ are the committees selected by rule R with |W| = k and |W’| = k + 1. The rule R is committee monotonic if W ⊂ W’.

Besides the above properties, Unanimity and Justified Representation are very important properties that are desired to be satisfied by approval voting based rules. We study these two properties in subsequent sections.

3 Unanimity

Unanimity is agreement by all voters in the context of committee selection. If there exists a set of candidates who are unanimously approved by all voters then at least one of them should be present in the selected committee W. A rule is unanimous if, when ∩_{v ∈ V} C_i ≠ ∅, it selects a committee W such that ∩_{v ∈ V} C_i ∩ W ≠ ∅. Using the same principle, we define unanimity for attribute level preferences in two ways 1) Weak Unanimity — If ∃ j ∈ [1, d] with \bigcap_{v_i∈V} C^{j'}_i ∩ W^j' = ∅, where W^j is set of attributes of W on dimension j. 2) Strong Unanimity — \bigcap_{v_i∈V} C^{j'}_i ∩ W^j' = ∅ it holds \bigcap_{v_i∈V} C^{j'}_i∩ W^j' = ∅.

**Lemma 1.** There may not exist a committee that provides strong unanimity for k < d.

**Proof.** Consider two candidates c_1 = [a_1, b_1] and c_2 = [a_2, b_2], and k = 1. The approvals for each of these attributes are given as V_a = V_b = V and V_a = V_b = \{v_1\}. Selecting any one of these two candidates violates the strong unanimity property. One can assure a committee W that provides strong unanimity when k ≥ d.

It is to be noted that for d = 1 both weak and strong unanimity convey the same meaning. Other possible variant of unanimity is, if there exists multiple unanimous candidates, then all of them should be present in the committee W. We limit our study to the standard unanimity as defined at the starting of this section.

**Lemma 2.** Given voters’ attribute-approval, Approval Voting and Satisfaction Approval Voting do not satisfy weak unanimity for k ≥ 1 and d > 1. Approval Voting and Satisfaction Approval Voting satisfies unanimity for d = 1.

**Proof.** Let X_1 = \{v_1, v_2, \ldots, v_n/2\}, X_2 = V \ \{v_n\} and let V_c[1] = V \ (V_c[1] = X_1^d)_{j=1}^d, ((V_c[1] = X_2)_{m/2}^d)_{j=1}^d. AV or SAV selects a set W ⊆ C \ \{c_1\}, whereas c_1[1] is the only attribute which is unanimously approved by all voters and is not part of W. Hence, AV and SAV do not satisfy unanimity. For d = 1, if there exists an attribute which is unanimously approved by all voters then the corresponding candidate secures highest approval score and satisfaction approval score. Hence, AV and SAV satisfy unanimity for d = 1.

**Lemma 3.** In the context of attribute approval voting, Reweighted Approval Voting and Proportional Approval Voting do not satisfy weak unanimity for k ≥ 1 and d > 1. Reweighted Approval Voting and Proportional Approval Voting satisfies unanimity for d = 1.

**Proof.** Let X_1 = \{v_1, v_2, \ldots, v_n/2\}, X_2 = V \ \{v_n\} and X_3 = V \ \{v_1\}. V_c[1] = V \ (V_c[1] = X_1^d)_{j=1}^d, ((V_c[1] = X_2)_{m/2}^d)_{j=1}^d and ((V_c[1] = X_3)_{m/2}^d)_{j=1}^d. RAV or PAV selects a set W ⊆ C \ \{c_1\}, whereas c_1[1] is the only attribute which is unanimously approved by all voters and is not part of W. Hence, RAV and PAV do not satisfy unanimity for d > 1. When d = 1: Suppose W is the committee selected according to RAV (or, PAV) and has unanimous candidate (attribute) c. Replacing any candidate c' ∈ C \ W with c would not increase RAV-score (or, PAV-score) of W. Hence, RAV and PAV satisfy unanimity for d = 1.

**Lemma 4.** For attribute-approval voting, Minimax Approval Voting does not satisfy weak unanimity for k ≥ 1 and d > 1 and Minimax Approval Voting satisfies unanimity for d = 1.

**Proof.** We omit the generalized proof due to its complexity. Let C = \{c_1, c_2, c_3\}, c_1 = [a_{11}, a_{12}, a_{13}, a_{14}], c_2 = [a_{21}, a_{22}, a_{23}, a_{24}], c_3 = [a_{31}, a_{32}, a_{33}, a_{34}]. Consider 4 kinds of voters’ ballots(voters’ ballots for each attribute is separated by comma) i.e., \{a_{11}a_{21}a_{32}, a_{12}, a_{13}, a_{14}\}, \{a_{11}a_{22}a_{31}, a_{12}, a_{13}a_{23}a_{33}, a_{14}a_{24}\}, \{a_{11}a_{21}a_{31}, a_{22}, a_{23}a_{33}, a_{24}a_{34}\} and \{a_{11}, a_{22}, a_{32}, a_{23}a_{33}, a_{24}a_{34}\}. Let k = 1, MAV score of c_1 is 1, c_2 and c_3 is 2/3. MAV selects a W that minimizes MAV score, hence, it selects c_2 or c_3 whereas a_{11} is the only attribute approved by all voters and is neither part of c_2 nor c_3. Similarly, it is easy to show that for k > 1, MAV does not satisfy unanimity. When d = 1, if there exists an attribute which is approved by all voters then it has smaller MAV score than that of any other attribute. Thus MAV selects attribute which is unanimously approved by all voters.

**Proposition 1.** If a rule does not satisfy weak unanimity then it does not satisfy strong unanimity as well.
From the Lemmas \([2][4]\) and the Proposition\([1]\) it can be seen that none of the extended rules satisfy unanimity (both weak and strong). We propose a simple greedy based rule in Section 5, namely Greedy Approval Voting (GAV), that satisfies unanimity property.

4 Justified Representation

Justified representation is a crucial property that is desired to be satisfied by approval based rules. Justified Representation \([2]\), Extended Justified Representation \([2]\), Proportional Justified Representation \([?]\), Proportional Representation \([20]\) and Strong Proportional Representation \([20]\) are based on similar concepts. If there exists a sizeable group of voters with common preferences then the group should have representation in the committee. The minimum number of representatives to be selected from the group is based on the size of the group and the number of common approvals. Keeping this objective, we define justified representation for attribute approval voting in two ways, 1) Simple Justified Representation (SJR) and 2) Compound Justified Representation (CJR). For \(d = 1\) simple and compound justified representations are the same.

Definition 1. \(W\) provides simple justified representation if there does not exist a set of voters \(V' \subseteq V\) such that \((|V'| \geq \frac{n}{d}) \wedge (\bigcap_{i \in V'} C_i \neq \emptyset) \wedge (\bigcup_{i \in V'} C_i' \cap W') = \emptyset\), \(1 \leq j, j' \leq d\).

For \(k = 1\), any random candidate also satisfies simple justified representation unless there exists a candidate who has no approval on any attribute.

Lemma 5. Approval Voting does not satisfy simple justified representation for \(k \geq 2\).

Proof. Let \(C = \{c_1, c_2, \ldots, c_{k+2}\}\) be the set of candidates. Let \(X^1 = \{v_i, i \in [1, \frac{n(k-1)}{k}], X^2 = \{v_i, i \in [\frac{n(k-1)}{k} + 1, n]\}\) and \(X^3 = \{v_n\}\). Consider profiles \((V_{c_1[j]} = X^1)^{d}_{j=1}, (V_{c_2[j]} = X^2)^{d}_{j=1}\) and \((V_{c_2[j]} = X^3)^{d}_{j=1}, (V_{c_4[j]} = X^2)^{d}_{j=1}, (V_{c_2[j]} = X^3)^{d}_{j=1}, (V_{c_4[j]} = X^2)^{d}_{j=1}\), \(c_{k+1}\) or \(c_{k+2}\) is required to be present in \(W\) in order to satisfy simple justified representation whereas AV selects \(\{c_1, \ldots, c_k\}\).\(\square\)

Lemma 6. Proportional Approval Voting and Reweighted Approval Voting do not satisfy simple justified representation for \(k \geq 3\) and \(d \geq 2\) or \(d \geq 3\) and \(k \geq 2\). For \(k \leq 2\) and \(d \leq 2\), Proportional Approval Voting and Satisfaction Approval Voting satisfy simple justified representation.

Proof. Let us consider that \(C = \{c_1, c_2, \ldots, c_5\}\), \(n = 90\). Consider the following approvals ballots over attributes, \((V_{c_1[j]} = \{v_1, v_2, \ldots, v_35\})^{d}_{j=1}, (V_{c_2[j]} = \{v_21, \ldots, v_55\})^{d}_{j=1}, (V_{c_3[j]} = \{v_26, \ldots, v_60\})^{d}_{j=1}, (V_{c_4[j]} = \{v_91, v_92\})^{d}_{j=1}, (V_{c_5[j]} = \{v_61, \ldots, v_90\})^{d}_{j=1}\). Let us assume \(k = 3\) and \(d = 2\). PAV (or, RAV) selects \(\{c_1, c_2, c_3\}\) and ignores set of \(\frac{n}{d}\) voters who jointly approved \(c_4\) and \(c_5\). Hence, PAV and RAV do not satisfy simple justified representation. To extend the proof to \(k \geq 3\), we take \(k - 3\) additional candidates and \((k - 3) \times 30\) additional voters, and assign 30 unique votes to each new candidate (same set of voters for all the attributes of a candidate). In this case also PAV (or, RAV) ignores \(c_4\) and \(c_5\), hence, ignores a set of \(n/k\) voters who jointly approved for some attribute. For \(k = 2\) and \(d \geq 3\), the same generalization given in the above example works. Hence, PAV and SAV do not satisfy simple justified representation for \(k \geq 3\) and \(d \geq 2\) or \(d \geq 3\) and \(k \geq 2\).

For \(k = 2\) and \(d = 2\), RAV and PAV satisfy simple justified representation. In the case of RAV, after selecting the first candidate with the highest AV score, the candidate having \(\frac{n}{d}\) approvals for one of its attributes will have the highest score irrespective of number of approvals for the second attribute. Hence, no \(n/2\) voters are completely unrepresented if they jointly approve some attribute. Similar logic works for PAV. Hence, RAV and PAV satisfy simple justified representation for \(k = 2\) and \(d = 2\). When \(d = 1\), the analysis is same as given in \([2]\).\(\square\)

Lemma 7. Satisfaction Approval Voting and Minimax Approval Voting do not satisfy simple justified representation for \(k \geq 2\), \(d \geq 1\).

Proof. When \(d = 1\), attribute-approvals can be visualized as candidate-approvals. It is proved for candidate-approvals that MAV and SAV do not satisfy justified representation for \(k \geq 2\) \([2]\). For \(d > 1\), the same set of voters are repeated for all the attributes of a candidate then the proof follows. \(\square\)

Definition 2. \(W\) provides compound justified representation if there does not exist a set of voters \(V' \subseteq V\) such that \((|V'| \geq \frac{n}{d}) \wedge (\bigcap_{i \in V'} C_i \neq \emptyset) \wedge (\bigcup_{i \in V'} C_i' \cap W') = \emptyset\), \(1 \leq j, j' \leq d\).

Lemma 8. There may not exist a committee that provides compound justified representation for \(k \geq 3\).

Proof. Consider a set of candidates \(c_i = [a_i, b_i], i \in [1, m]\) with \(m = 6\). Voting approvals of attributes are given as \(V_{a1} = V_{a2} = \{v_1, v_2\}, V_{a2} = V_{a5} = \{v_3, v_4\}, V_{a3} = V_{a5} = \{v_5, v_6\}, V_{a1} = V_{a2} = V_{a3} = V_{a4} = V_{a5} = V_{a6} = \{v_1\}\). For \(k = 3\), none of the three candidate committees satisfy compound justified representation. One can assure a committee that satisfies CJR for \(k = 2\) if we assume that each attribute is approved by at least one voter. \(\square\)

Proposition 2. If a rule does not satisfies simple justified representation then it does not satisfies compound justified representation also.

It can be seen from Lemmas \([5][7]\) and Proposition\([2]\) that AV, SAV, RAV, PAV and MAV do not satisfy SJR as well as CJR. We adopt Greedy Approval Voting and extend it to attribute level in the next section that satisfies SJR. We show that the proposed GAV satisfies CJR based on some assumptions.
5 Attribute Level Greedy Approval Voting

Greedy Approval Voting (GAV) is a multi-step approach. It starts by setting $V' = V$ and $W = \emptyset$. At every step it selects a candidate $c_j$ having an attribute with the highest number of approvals with respect to $V'$ and add it to $W$. It then removes all voters who voted for at least one attribute of $c_j$ from $V'$. This process is repeated till $|W| = k$. In case if the voter list $V'$ is empty when $|W| = k$, set $V'$ to $V$. Once the voter list is empty, random selection of candidates would satisfy the weak unanimity and SJR properties but fails many other properties. GAV is similar to the greedy approach proposed in [2] when $d = 1$ and the selection is arbitrary. The main flow of GAV can be found in Algorithm 1.

Algorithm 1: Greedy Approval Voting

\begin{algorithm}
\begin{algorithmic}
\State $W \leftarrow \emptyset$; $V' \leftarrow V$;
\State Let $V'_{[j]}$ be the set of voters approved for $c_{[j]}$ from $V'$;
\While {$|W| < k$}
\State $c_j \leftarrow \text{ARGMAX}_{c_j \in C} \{ |V'_{[j]}| \}_{j=1}^d$;
\State $W \leftarrow W \cup \{ c_j \}; C \leftarrow C \setminus \{ c_j \}$;
\State $V' \leftarrow V' \setminus \{ \cup_j V'_{c_{[j]}} \}$;  \textbf{if} $|V'| = 0$ \textbf{then} $V' \leftarrow V$;
\EndWhile
\end{algorithmic}
\end{algorithm}

Lemma 9. Greedy Approval Voting satisfies weak unanimity.

Proof. If there exists an attribute which is approved by all voters, GAV selects the corresponding candidate first. Hence, GAV satisfies weak unanimity.

Lemma 10. Greedy Approval Voting satisfies strong unanimity if ties are broken in favor of the candidates that provide strong unanimity.

Proof. We say that a voter is unrepresented on dimension $j$ if none of his/her approved attributes from the domain $D_j$ are present in $W$. We define the following tie-breaking rules,

1) If there exists multiple attributes with the same number of approvals, we select the one which is having the highest number of approvals according to the unrepresentative voters of the dimension where the attribute is present. 2) If multiple candidates have a unanimous attribute, we select the one with more number of unanimous attributes. Using the above two tie-breaking rules GAV selects a committee which is having at least one unanimous attribute on every dimension (if there exists such attribute on that dimension). Hence, GAV satisfies strong unanimity.

Lemma 11. Greedy Approval Voting satisfies simple justified representation.

Proof. A rule does not satisfy simple justified representation if it completely ignores a set of $\frac{n}{k}$ voters who jointly approved for some attribute. If we prove that GAV does not leave any $\frac{n}{k}$ voters who jointly approved for an attribute then we are done. GAV is a multi-stage approach and at every step it selects a candidate having an attribute with maximum number of approvals with respect to unrepresented voters. Even if there exist completely disjoint sets of voters, each of size $\frac{n}{k}$, GAV can cover all such voters in $k$ steps. Hence, GAV satisfies simple justified representation.

We could not find any polynomial time rule that satisfies compound justified representation. Given $W$ it is easy to see whether it satisfies CJR or not. However, checking every possible $W$ results in exponential behaviour. We consider each dimension separately and identify a set of attributes that satisfy justified representation for that dimension. This can be done in polynomial time using the proposed GAV. Let $J'$ be the set of attributes that satisfies justified representation for dimension $i$. Selection of a committee that satisfies CJR is polynomial if we assume that $J_1 \times J_2 \times \ldots \times J_d \subseteq C$. In addition to unanimity and justified representation, GAV satisfies other properties that are described in Section 2.1.

6 Conclusions and Discussion

The present work initiates a new direction of research namely, use of attribute approvals for a committee selection problem. We extended the existing rules for committee selection by candidate-approval voting to attribute-approval voting. We also analyzed the standard properties that are desired to be satisfied by these rules. When the rules are extended to attribute-space, most of these properties were violated. We proposed Greedy Approval Voting and gave a detailed analysis wherein properties like unanimity and justified representation are shown to be satisfied.

Preference by ranking is another way of expressing voters preferences apart from preference by approval ballots. Determining a committee of size $k$, given voters ranking over attributes is an interesting direction for future work. In recent years, ranking by pairwise preferences has gained attention due to its user friendliness and easiness. Forming a committee given pairwise preferences of voters over attributes is another interesting direction to pursue future research. Further, exploring the Pareto-optimal set of candidates and studying different properties of those sets is a good direction to pursue. We plan to investigate these aspects in the future.

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