Anisotropic Power Law Inflation from Rolling Tachyons

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Abstract

We provide an explicit solution representing an anisotropic power law inflation within the framework of rolling tachyon model. This is generated by allowing a non-minimal coupling between the tachyon and the world-volume gauge field on non-BPS D3 brane. We also show that this solution is perturbatively stable.
If the origin of the anomalies in the WMAP data, namely, the observed power asymmetry \[1, 2\], the alignment of low-\(l\) CMB multipoles \[3, 4, 5, 6\], happens to be primordial, it will imply statistical anisotropy of the primordial curvature perturbation. How can one generate such anisotropy with-in the simplest scalar field models of inflation? Scalar field does not prefer any particular direction and hence it is hard to imagine that anisotropies can be created by scalars. This motivated many researchers to look into models of inflation with vector fields. In some of these models such as \[7\], vector fields do not satisfy dominant energy condition and thus evade the cosmic no-hair conjecture\[8\]. Though, these models provide examples of anisotropic inflation, they are plagued with instabilities \[9, 10\].

Recently, another model of anisotropic inflation was proposed in \[11\] and corresponding curvature perturbation power spectra was studied in \[12, 13\]. For further studies in this model, see \[14, 15\]. This model includes a non-minimal coupling between the gauge field and the inflaton. Such models typically lead to power law inflation rather than the deSitter ones. In particular, consider the scenario in \[16\]. It starts with an action of the form

\[
S = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{1}{4} f^2(\phi) F_{\mu\nu} F^{\mu\nu} \right],
\]

(1)

with \(V(\phi) = V_0 e^{\lambda \phi}\), \(f(\phi) = f_0 e^{\rho \phi}\). Here \(M_p\) is the four dimensional Planck mass and \(\lambda, \rho\) are two constants. Taking \(A_\mu = (0, A_x(t), 0, 0)\) and \(\phi = \phi(t)\), it is possible to consistently solve the Einstein, scalar and vector equations for metric of the form

\[
ds^2 = -dt^2 + t^p \left[ t^{-q} dx^2 + t^{q/2} (dy^2 + dx^2) \right],
\]

(2)

for some \(p >> 1\) and \(q > 0\). Further, an analysis of the dynamical system shows that the above power law solution in in fact an attractor for a large range of parameters\[4\].

Motivated by these developments, in this paper, we consider anisotropic inflation in models driven by rolling tachyons. A while back, in \[20, 21\], classical time dependent solution describing the decay process of an unstable D-brane in the open string theory was constructed. During this process, the tachyon field on the brane rolls down to the minimum of the potential. In these works, it was also pointed out that this rolling tachyon might have cosmological significance. Since then there are plethora of activities starting with \[22, 23, 24, 25, 26, 27\]. There are several reasons for that. Firstly, this model directly arises from string theory. Secondly, it promises potential applications in the inflationary scenario of our universe. However, rolling tachyon models also have their own problems. This is discussed, for example, in \[28\]. There, it was argued that tachyonic inflation, in general, can not result in a universe which is reasonably close to our observed one. More precisely, for the tachyonic potential \(V(T)\), that is expected to arise from string theory, there is an incompatibility between the slow role condition and the COBE normalisation of fluctuations. There are also few other problems associated with these models. It was however noted that some of these issues get cured if one works with a model of assisted tachyonic inflation\[5\]. Here, one considers \(N\) non-BPS D3 brane. In this system, there are two different kinds of open strings - one that stretches between two different branes and the others whose both ends are on the same brane. If the distance between the branes are larger than

\[\text{For other discussions on anisotropic inflations within this scenario, see for example } [17, 18, 19]\]

\[\text{† See } [29, 30] \text{ for models of assisted inflation.}\]
the string scale, one can neglect the first set of strings. Consequently, the theory reduces to a system of quite non-interacting tachyons, one on every brane. If number of tachyons is of the order of $10^{11}$, it seems that there is a compatibility between slow role condition and the COBE normalisation of fluctuations. However some of the other problems still remain [31].

The effective field theory Lagrangian on the D3 brane, in general, has a Born-Infeld form and is given by

$$\mathcal{L} = \sqrt{-g} \mathcal{L} = -V(T) \sqrt{-g} \sqrt{\det[\delta_{\mu \nu} + h(T)F_{\mu \nu} + \partial^\mu T \partial_\nu T]}.$$  \hspace{1cm} (3)

However, for non-interacting $N$-tachyon ($T$) assisted inflation, one works with the Lagrangian

$$\mathcal{L} = \sqrt{-g} \mathcal{L} = -NV(T) \sqrt{-g} \sqrt{\det[\delta_{\mu \nu} + h(T)F_{\mu \nu} + \partial^\mu T \partial_\nu T]}.$$  \hspace{1cm} (4)

Our humble aim in this paper is to show that besides usual inflationary expansion [24, 25], for a suitable choice of $V(T)$, this Lagrangian also provides us with an exact solution for anisotropic inflation. This happens if we allow for a suitable tachyon-gauge field coupling through the function $h(T)$ in (4). We further show that this solution is perturbatively stable.

We parametrise our metric as in [11]:

$$ds^2 = -dt^2 + e^{2\alpha(t) - 4\sigma(t)} dx^2 + e^{2\alpha(t) + 2\sigma(t)} (dy^2 + dz^2).$$  \hspace{1cm} (6)

It is easy to derive the equations of motion for $g_{\mu \nu}, A_\mu$ and $T$ that follow from the coupled action

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_p^2}{2} R + \mathcal{L} \right].$$  \hspace{1cm} (7)

Here $R$ is the usual four dimensional Ricci scalar. These are:

$$(\partial_t \alpha)^2 = (\partial_t \sigma)^2 + \frac{N}{3} \left[ 1 + \frac{1}{2} (\partial_t T)^2 \right] V + \frac{p^2}{2U} e^{-4\alpha - 4\sigma},$$

$$\partial_t^2 \sigma = -3 \partial_t \alpha \partial_t \sigma + \frac{Np^2}{3U} e^{-4\alpha - 4\sigma},$$

$$\partial_t^2 \alpha = -3 (\partial_t \alpha)^2 + NV + \frac{Np^2}{6U} e^{-4\alpha - 4\sigma},$$

$$V \partial_t T \left[ \partial_t^2 T + 3 \partial_t \alpha \partial_t T + \frac{1}{V} \frac{dV}{dT} \left( 1 + \frac{1}{2} (\partial_t T)^2 \right) \right] - \frac{p^2 \partial_t U}{2U^2} e^{-4\alpha - 4\sigma} = 0.$$  \hspace{1cm} (8)

\footnote{On general grounds, it is expected that the potential has a maximum near $T = 0$ and decays off exponentially at large $T$. We will work here with

$$V(T) = \frac{V_0}{T^2}, \text{ with } V_0 > 0.$$  \hspace{1cm} (5)

In spite of the divergence at $T = 0$, $V(T)$ mimics quite closely the expected tachyon potential with an advantage that it provides exact solution of a power law inflation.}

\footnote{In general, with the full contribution from the determinant in $L$ in the equations of motion, it is hard to solve the differential equations. We have approximated the determinant by expanding it in gauge field and scalar fields while keeping only the first terms both gauge field and scalar fields. We point out that in our way of writing, $h(T)$ has an overall $\alpha'$ dependence. We will discuss the range of the parameters later.}
In these equations, we have defined $U = h^2V$. Furthermore, we took $A_\mu(t) = (0, A_x(t), 0, 0)$. Using this, we integrated the gauge field equation of motion as

$$\partial_t A_x(t) = \frac{p}{U} e^{-\alpha-4\sigma}$$

and used this in (8). The parameter $p$ in (9) arises as an integration constant.

Out of the four equations in (8), only three equations are however independent. To see this, we differentiate the first and use the last equation to get

$$2\partial_t \alpha \partial^2_t \alpha = 2\partial_t \sigma \partial^2_t \sigma - NV \partial_t \alpha \partial^2_t T - \frac{2Np^2}{3U} e^{-4\alpha-4\sigma} \left( \partial_t \alpha + \partial_t \sigma \right).$$

Further using the first and second equation of (8), we can reduce (10) to the third of (8). Therefore, for now on, we will only consider the first, second and the fourth as independent equations of motion.

For the potential (5), there is an exact solution for the metric and other fields. These are given by

$$\alpha = \frac{NV_0}{2} \log t, \quad \sigma = 0, \quad T = \frac{2t}{\sqrt{3NV_0 - 2}}, \quad p = 0, \quad h(T) = 0.$$ (11)

Clearly, this leads to an isotropic inflationary power law expansion. The inflationary slow-roll parameters can be adjusted by tuning $NV_0$.

We now explicitly construct another simple solution, for the same potential, which provides us with an anisotropic accelerated expansion of the universe. To this end, let us define

$$\alpha = \alpha_0 \log t, \quad \sigma = \sigma_0 \log t, \quad h = t^\delta, \quad T = T_0 t.$$ (12)

We will now substitute these in (8) and solve for $\alpha_0, \sigma_0, \delta$ and $T_0$. Using (12) in (8), we get the following restrictions on the parameters

$$2 - \delta - 2\alpha_0 - 2\sigma_0 = 0,$$

$$\frac{Np^2T_0^2}{V_0} + NV_0 \left( 1 + \frac{2}{T_0^2} \right) - 6\alpha_0^2 + 6\sigma_0^2 = 0,$$

$$3(3\alpha_0 - 1)\sigma_0 - \frac{Np^2T_0^2}{V_0} = 0,$$

$$p^2 T_0^4(1 - \delta) + T_0^2 V_0^2(3\alpha_0 - 1) - 2V_0^2 = 0.$$ (13)

These equations can be further simplified by a replacement $p \rightarrow pV_0$.

$$2 - \delta - 2\alpha_0 - 2\sigma_0 = 0,$$

*For other discussions on anisotropic inflation, see [32]. The solution we construct here is new.*
\[ NV_0 \left[ p^2 T_0^2 + 1 + \frac{2}{T_0^2} \right] - 6 \alpha_0^2 + 6 \sigma_0^2 = 0, \]
\[ 3(3 \alpha_0 - 1) \sigma_0 - NV_0 p^2 T_0^2 = 0, \]
\[ p^2 T_0^4 (1 - \delta) + T_0^2 (3 \alpha_0 - 1) - 2 = 0. \]  

(14)

On substituting \( \sigma_0 \) from the third equation into the second, we get
\[ 2N^2 V_0^2 p^4 T_0^6 + 3(1 - 3 \alpha_0)^2 [2 + T_0^2 + p^2 T_0^4] NV_0 - 6 \alpha_0^2 T_0^2 = 0. \]  

(15)

Further using the first and the third into the fourth of (14), we have
\[ 2N^2 V_0 p^4 T_0^6 + 3(3 \alpha_0 - 1) [T_0^2 \{ p^2 T_0^2 (2 \alpha_0 - 1) + 3 \alpha_0 - 1 \} - 2] = 0. \]  

(16)

Now, given the parameters \( NV_0 \) and \( p \), (15) and (16) are two equations for two unknowns \( \alpha_0 \) and \( T_0 \). Though the above equations can be solved exactly for \( \alpha_0 \) and \( T_0 \), the solutions involve rather large expressions and non-illuminating. We, instead, prefer to plot the solutions.

![Figure 1](image)

**Figure 1:** In these figures, \( \log(\alpha_0) \) vs \( \log(p) \) has been plotted with different values of \( N \). \( N = 10^{10}, N = 10^{15} \) and \( N = 10^{18} \) in figure (a), (b) and (c) respectively. We have fixed \( V_0 = 1 \).

The variation of \( \alpha_0 \) with \( p \) for different \( NV_0 \) is shown in the \( \log_{10} \alpha_0 - \log_{10}p \) plot in figure (1). We see from (11), for isotropic inflation, \( \alpha_0 \) depends linearly on \( NV_0 \). Roughly, this continues
to happen even in the presence of anisotropy. From the figure, we note that the dependence of \( \alpha_0 \) on \( p \) increases substantially for large \( p \). A good measure of anisotropy is given by the quantity \( \dot{\sigma}/\dot{\alpha} \) which is equal to \( \sigma_0/\alpha_0 \) in our case. Figure (2) shows \( \log_{10}(\frac{\sigma_0}{\alpha_0}) - \log_{10} p \) plots for various \( NV_0 \). Indeed we see that a small amount of anisotropy persists during the inflation. Interestingly, anisotropy picks at a certain value of \( p \). This value, in turn, depends on \( NV_0 \).

![Graphs showing \( \log_{10}(\frac{\sigma_0}{\alpha_0}) - \log_{10} p \) for different values of \( NV_0 \).](attachment:image.png)

**Figure 2:** In all the figures, \( \log(\frac{\sigma_0}{\alpha_0}) \) vs \( \log(p) \) has been plotted with different values of \( N \). \( N = 10^{10}, N = 10^{15} \) and \( N = 10^{18} \) in figure (a), (b) and (c) respectively. We have fixed \( V_0 = 1 \)

Having found the anisotropic expanding geometry, we now provide a discussion on the validity of the assumption made in deriving the equations of motion (8). While writing down these equations, we expanded the determinant of the action and kept only leading terms in derivatives, both for the gauge field and for the scalar. While for the gauge field, this is perhaps justified as it is \( \alpha' \) suppressed. However, the reason for throwing away the higher derivative terms involving scalar requires some arguments. This is what we provide below.

Consider the Lagrangian for the matter part.

\[
L = -NV(T)\sqrt{\det \left[ \delta^{\mu}_{\nu} + h(T)F^{\mu}_{\nu} + \partial^{\mu}T\partial_{\nu}T \right]}.
\]  

(17)
Now the solution put in, it becomes
\[ L = -N \frac{V_0}{T_0^2 t^2} \sqrt{1 - \frac{p^2 T_0^2}{V_0^2} - T_0^2}. \]  
(18)

Therefore expanding square-root is valid when both \( \frac{p^2 T_0^2}{V_0} \) and \( T_0 \) are small compared to 1. After redefinition of our variable as \( p \rightarrow p V_0 \), we have \( p T_0^2 \) and \( T_0 \) which should be small. In the range of our parameters taken in above solution these conditions are satisfied. For example if we take the first case, that is \( N = 10^{10} \) and plot \( \log_{10}(p T_0^2) \) vs \( \log_{10}(p) \), we can see \( T_0 \) remains always small compared to 1.

**Figure 3:** Here we plot both \( \frac{\sigma_0}{\alpha_0} \) and \( p T_0^2 \) (dotted and solid lines respectively). One can see that the maximum value \( p T_0^2 \) reaches is of the order \( 10^{-5} \), while \( T_0^2 \) reaches \( 10^{-10} \) at the most.

It is important to check the stability of the proposed anisotropic solution and we do it now. For that, we re-write the equations of motion (8) in terms new variables
\[ X = \frac{\partial \sigma}{\partial \alpha}, \quad Y = \sqrt{NV} \frac{\partial T}{\partial \alpha}, \quad Z = \sqrt{NUe^{-\alpha+2\sigma}} \frac{\partial A}{\partial \alpha}. \]
(19)

It then follows that
\[ \frac{dX}{d\alpha} = X \left( 3(X^2 - 1) + \frac{Y^2}{2} + \frac{Z^2}{3} \right) + \frac{Z^2}{3}, \]
\[ \frac{dY}{d\alpha} = -\frac{Y^2}{\sqrt{NV}} - \frac{2}{\sqrt{NV}} \left( 3(X^2 - 1) + Z^2 \right) + Y \left( 3(X^2 - 1) + \frac{Y^2}{2} + \frac{Z^2}{3} \right) + \frac{Z^2 \partial T}{\sqrt{NV h}}, \]
\[ \frac{dZ}{d\alpha} = Z \left( -2 - 2X + \frac{Y^2}{2} + \frac{Z^2}{3} + 3X^2 - \frac{Y \partial T}{\sqrt{NV h}} + \frac{Y}{\sqrt{NV h}} \right), \]
(20)

In writing down the above set of equations, we have made extensive use of (8). Further, in terms of \( X, Y \) and \( Z \), the equations of motion (8) can be re-expressed simply as
\[ \frac{dX}{d\alpha} = \frac{dY}{d\alpha} = \frac{dZ}{d\alpha} = 0, \]
(21)
along with the energy conservation condition given in the first equation of (8)

\[- \frac{NV}{(\partial_t \alpha)^2} = 3(X^2 - 1) + \frac{Y^2}{2} + \frac{Z^2}{2}. \tag{22}\]

It can easily be checked that the anisotropic solution, constructed before, represents a point in (X, Y, Z) space

\[X_0 = \frac{\sigma_0}{\alpha_0}, \quad Y_0 = \frac{\sqrt{NV_0}}{\alpha_0}, \quad Z_0 = \frac{\sqrt{NV_0}}{\alpha_0} T_0 \alpha. \tag{23}\]

In writing down the above equations we have made the substitution \(p \rightarrow pV_0\) as before. We now wish to check the nature of the variations of coordinates around this point. Variations of (20), to linear order, around \((X_0, Y_0, Z_0)\) gives

\[
\begin{align*}
\frac{d(\delta X)}{d\alpha} &= (-3 + 9X_0^2 + \frac{Y_0^2}{2} + \frac{Z_0^2}{3})\delta X + X_0Y_0\delta Y + \frac{2Z_0}{3} \left( 1 + X_0 \right) \delta Z, \\
\frac{d(\delta Y)}{d\alpha} &= 6X_0 \left( \frac{-2}{\sqrt{NV_0}} + Y_0 \right) \delta X + \left( 3(X_0^2 - 1) + \frac{3Y_0^2}{2} + \frac{Z_0^2}{3} - \frac{2Y_0}{\sqrt{NV_0}} \right) \delta Y \\
&\quad + 2Z_0 \left( \frac{\delta}{\sqrt{NV_0}} - \frac{2}{\sqrt{NV_0}} + \frac{Y_0}{3} \right) \delta Z, \\
\frac{d(\delta Z)}{d\alpha} &= 2Z_0 \left( 3X_0 - 1 \right) \delta X + Z_0 \left( -\frac{\delta}{\sqrt{NV_0}} + \frac{1}{\sqrt{NV_0}} + Y_0 \right) \delta Y \\
&\quad + \left( -2 - 2X_0 + 3X_0^2 - \frac{Y_0^2}{\sqrt{NV_0}} + \frac{1}{\sqrt{NV_0}} + \frac{Y_0^2}{2} + Z_0^2 \right) \delta Z. \tag{24}\end{align*}
\]

This has the form \(A' = \Lambda A\) where \(A\) is a column matrix with entries \((\delta X, \delta Y, \delta Z)\) and \(\Lambda\) is a three by three matrix with entries written in terms of \((X_0, Y_0, Z_0)\) and \(NV_0, \delta\). The prime on \(A\) denotes derivative with respect to \(\alpha\). Our job is now to simply diagonalize \(\Lambda\) and study the nature of its eigen values. These eigen values can be computed and expressed solely in terms of \(NV_0, p\). However, expressions are very messy. Nevertheless, their natures can be understood from various plots. The real part of the three eigen values are plotted in figure (4). The plots are for \(N = 10^{10}\), with \(V_0 = 1\). Clearly one can see for certain range of \(p\) one of the eigenvalues is all most zero and then it becomes more negative. Real part of other two remain negative for given range of \(p\). Therefore, we conclude that our solution is perturbatively stable.

To summarize, we constructed an anisotropic power law inflationary solution in models where inflation is achieved by rolling tachyon. We further showed that the solution was perturbatively stable. Though simple tachyon driven inflationary model suffers from some observational problems, existence of stable anisotropic inflationary solution is encouraging. Existance of such solutions also indicates that the cosmic no heir conjecture \([8]\) may require appropriate modifications.
Figure 4: In these figures, real part of the eigenvalues of $\Lambda$ vs $\log(p)$ has been plotted for $N = 10^{10}$, with $V_0 = 1$. Figure (a), (b) and (c) are plots of three different eigenvalues. The graphs are actually smooth, the sharp points appear in the graph are because of high rate of change.

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