On the universality of cross sections of hadron-hadron(nuclei) collisions at superhigh energies

L. Frankfurt  
School of Physics and Astronomy,  
Tel Aviv University, Tel Aviv, 69978 , Israel

M. Strikman  
Department of Physics, Pennsylvania State University,  
University Park, PA 16802, USA

M. Zhalov  
St. Petersburg Nuclear Physics Institute, Gatchina, 188300 Russia

Abstract

We analyze the pattern of the onset of complete absorption (the black limit) in the high energy hadron-hadron collisions. The black limit arises due to the hard and soft interaction dynamics as a function of the impact parameters $b$. Both hard and soft mechanisms provide universal dependence of the partial amplitude of the high energy elastic hadron-hadron scattering on the impact parameter $b$ and result in the radius of interaction proportional to $\ln(s/s_0)$. We find that with increase of the collision energies hard interactions lead to a faster increase of the impact parameter range where the partial wave amplitudes are approaching the unitarity limit. Consequently, we argue that at super high energies when the radius of hadronic interactions significantly exceeds static radii of the interacting hadrons(nuclei) the ratio of total cross sections of nucleon-nucleon, meson-nucleon, hadron-nucleus, nucleus-nucleus collisions becomes equal to one. The same universality is also expected for the structure functions of nuclei: $F_{2A}(x, Q^2)/F_{2N}(x, Q^2) \rightarrow 1$, at very small $x$, and for the ratio $\sigma_{\gamma A}/\sigma_{\gamma p}$ at superhigh energies. We analyze how accounting for the energy dependence of the interaction radii changes the geometry of hadron-nucleus and nucleus-nucleus collisions, the energy dependence of total, absorption and inelastic cross sections, the distribution over the number of wounded nucleons in proton-nucleus collisions and find that these effects are noticeable already for the LHC energies and even more so close to the Greisen-Zatsepin-Kuzmin limit.

1 Introduction

High energy behavior of the hadron-hadron interaction is a subject of the extensive theoretical studies [1] for almost fifty years. Still many long standing questions such as the universality of the hadronic and hadron-nucleus total cross sections [2] and the relative importance of hard and strong interactions as a function of the impact parameter remain open.
V. Gribov [3] demonstrated long ago that the \( t \) channel unitarity of the S-matrix combined with analytic properties of the scattering amplitude in momentum transfer space is inconsistent with the radius of a hadron being independent of the incident energy. An assumption that the single pole singularity in the angular momentum plane (a Pomeron pole) determines high-energy behavior of the elastic amplitude [4, 5] predicts an increase of the interaction radius with increase of energy. This phenomenon leads to increase of the slope parameter \( B_{pp} \) in the \( t \)-dependence of the \( pp \) elastic cross section which is by now well established experimentally -see [6] and references therein. Estimates within the Pomeron pole model show that the radius of interaction becomes comparable with the mean internucleon distances in nuclei already at the LHC energies and even exceeds these distances at energies corresponding to the Greisen-Zatsepin-Kuzmin (GZK) limit [7]. This leads to a gradual change of geometry in hadron-hadron, hadron-nucleus and nucleus-nucleus collisions. Also, it results in a gross difference between structure of the final states in central collisions (for example triggered by \( x_T \equiv 2p_\perp/\sqrt{s} \geq 0.01 \) dijet production) and peripheral collisions at the Tevatron energies and above [8].

Another fundamental property of the strong interaction is a rather rapid increase of the total hadron-hadron cross section with energy. Such a behavior is well described, for example, in the Donnachie-Landshoff model [9] where the intercept of the Pomeron trajectory is \( \alpha_{IP}(0) = 1.08 \). However, the hypothesis of the Pomeron exchange implies the existence of the Pomeron branch points in the plane of angular momentum which compete with Pomeron exchange [10]. Measurements of the total inelastic diffraction cross section in \( p\bar{p} \) scattering [11] discovered the important role of shadowing effects which can be described as due to the presence of the Pomeron branch points (for recent discussions of these issues see Refs. [12]). Account of the Pomeron branch points in the model with the value of \( \alpha_{IP}(0) = 1 > 0 \) predicted blackening of interaction at the range of impact parameters \( b \propto \ln(s/s_0) \) (see [13] and references therein). This corresponds to the Froissart-type behavior of the total cross section in the \( s \to \infty \) limit [14]. Experimentally, at energies of the Tevatron partial amplitude for \( pp \) scattering at impact parameter \( b = 0 \) reaches a value \( \sim 0.95 \). Thus both theory and experimental data indicate that the blackening of the soft interaction at particular fixed impact parameters, \( b \), until the unitarity limit is reached is one of the distinctive features of soft hadron-hadron dynamics at high energies.

On the other hand theoretical study of DIS in perturbative QCD (pQCD) [15] and experimental investigations at HERA [16] found that the total cross sections of DIS, being small, undergo a significantly faster increase with energy than the cross sections of soft hadron-hadron interactions. The significant cross section of diffraction observed in DIS at HERA [16] gives convincing evidence for the blackening of hard QCD interactions at small \( x \) [17]. In proton-proton collisions at collider energies, hard parton interactions are concentrated at significantly smaller impact parameters than generic inelastic interactions [8]. However, similar to the DIS case, these hard parton interactions rapidly increase with energy leading to blackening of interactions in the wide range of central impact parameters. Thus it is important to analyze the relative importance of soft and hard QCD interactions in the onset of the black body regime of high energy hadron collisions as a function of the impact parameter.

In section 2 we analyze the proton-proton elastic scattering amplitudes as a function of impact parameter \( b \). We argue that the black regime originates from hard interactions of leading partons in the nucleons with the small \( x \) gluon fields. We use this to estimate the rate of increase of the region in impact parameter space where the interaction is black and we find that it linearly increases with \( \ln s/s_0 \). We also give an alternative estimate based on the extrapolation of the soft Pomeron
dynamics which dominates at large $b$ to the region of smaller $b$ and find that two estimates give similar results at collider energies. We use these observations to argue in section 3 that a universal pattern of blackening of the interaction at fixed impact parameters leads to the universality of cross sections in the limit of superhigh energies:

$$\sigma_{tot}(h_1, h_2)/\sigma_{tot}(pp) \to 1.$$  \hspace{1cm} (1)

Here $h_i$ can be a hadron (nucleus). The same result should be valid for the structure functions of nuclei but at extremely small $x$:

$$F_{2A}(x, Q^2)/F_{2N}(x, Q^2) \to 1.$$  \hspace{1cm} (2)

In the string models, similar universality arises as the universality of the term in the cross section which increases linearly with energy - the "Pomeron" (loop contribution) related to the universality of the graviton interaction [18].

It is worth mentioning here that universality of cross sections of hadron-hadron, hadron-nucleus collisions at superhigh energies has been discussed long ago. V. Gribov suggested universality of cross sections of hadron-hadron, hadron-nucleus interactions [19] within the assumption that such cross sections should become constant at infinite energies. Later on, the universality of the Froissart limit for total cross sections of hadron-hadron, hadron-nucleus and nucleus-nucleus collisions was suggested within a particular generalized eikonal model of the supercritical Pomeron [20]. This model gives prescriptions for summing the unstable (divergent) series of terms $\propto s^{\alpha_I P - 1}$ due to multiPomeron exchanges which rapidly increase with energy because of the intercept of the bare supercritical Pomeron $\alpha_{IP}(0) > 1$. There exist also a number of eikonal models which combine elements of soft and hard dynamics for all impact parameters, see [21] and reference therein. However, universality and the increase of the radius of the hard interactions were not discussed in these models. Note also, that the eikonal models neglect contribution of the enhanced Pomeron diagrams which is rapidly increases with energy and at superhigh energies becomes comparable with eikonal diagrams [22].

Our approach assumes dominance of the Donnachie-Landshoff soft Pomeron exchange in peripheral collisions only. For the scattering at central impact parameters, where according to pre-QCD Reggeon Calculus strong interaction between the Pomerons is expected, our approach accounts for the large cross section of hard processes due to formation of large gluon densities with $p_\perp \approx 2 GeV$ and the consequent disappearance of soft elastic and diffractive processes at energies of LHC and above\(^1\). Such nontrivial interplay of hard and soft dynamics is absent in preQCD multiPomeron exchange models. Note here, that importance of hard interactions in the the Froissart limit in QCD follows from requirement of the self consistency [23].

We also analyze how the increase of the radius and strength of the nucleon-nucleon interaction influences cross sections of hadron-nucleus and nucleus-nucleus collisions at achievable energies. To visualize the role of these effects we use the formulae of the Glauber-Gribov model [24]\(^2\). We

\(^1\)Disappearance of diffractive processes for the scattering at central impact parameters has been observed recently at FNAL. Remember also that in the black limit elastic scattering occur at peripheral impact parameters only. In the realistic situation inelastic diffraction arises from the scattering at peripheral impact parameters where the interaction is grey.

\(^2\)V. Gribov has demonstrated that though the set of diagrams which contributes at high energies is different from that accounted in the original Glauber approach, the answer differs only due to the contribution of the inelastic diffraction - the inelastic shadowing corrections. Relative contribution of these corrections is rather modest and decreases at very high energies due to suppression of the inelastic diffraction.
calculate the total and absorption cross sections for pA collisions. The noticeable effects due to the blackening of the nucleon-nucleon interaction can already be observed at the LHC energies and above. The onset of black body limit leads to gradual weakening of the A-dependence of cross sections which ultimately results in the A-independent cross sections corresponding to the universality regime of Eq. [1.

We evaluated the effective energy dependent radius of a nucleus and estimated total cross sections for heavy ion collisions using the popular Bradt-Peters expression [25]. We compare this result to that obtained with more refined Glauber-like model of nucleus-nucleus interaction [26, 27]. Also we calculate the distribution over the number of inelastic collisions in nucleon-nucleus interaction [28] and find that accounting for the energy dependent radius of a nucleon-nucleon interaction leads to a significant change of distribution. Analysis of the possible role of this effect in the interpretation of the heavy ion collisions data in terms of the wounded nucleons already at energies of RHIC and above is beyond the scope of this paper.

2 Partial wave amplitudes for hadron-hadron collisions at ultrahigh energies

In this section we will use the properties of impact parameter representation of the elastic scattering amplitude,

\[ \Gamma(b, s) = \frac{1}{2i\pi k} \int d^2 q \exp[-i\mathbf{q} \cdot \mathbf{b}] f_{NN}(\mathbf{q}, s), \]  

(3)

to discuss basic features of the superhigh energy NN interactions. Here \( s \) is the invariant energy of \( NN \) scattering, and \( q \) is the two-dimensional transverse momentum.

2.1 Small impact parameter behavior of the hadron collision amplitude

There are several generic features of \( \Gamma(b, s) \) at small impact parameters \( b \) which can be derived from unitarity of \( S \) matrix, from the current understanding of the spatial structure of the fast nucleon and dynamics of hard interactions. Indeed, let us consider nucleon-nucleon scattering at small impact parameters at large \( s \). An analysis, performed in Ref.[8], demonstrates that in this case the average transverse momenta of partons from nucleon become large after partons pass through the low \( x \) gluon fields of another nucleon. For example, at Tevatron energies quarks with \( x \geq 0.2 \) get average transverse momenta \( \geq 1 GeV/c \). If a leading quark gets a transverse momentum \( p_{\perp} \), the probability for the nucleon to remain intact is roughly given by the square of the nucleon form factor \( F_2^N(p_{\perp}^2) \). Since \( F_2^N(p_{\perp}^2) \leq 0.1 \) for \( p_{\perp} \geq 1 GeV/c \), the probability of survival averaged over \( p_{\perp} \) should be at most \( 1/2 \), provided average \( p_{\perp} \geq 1 GeV/c \). Since there are six leading quarks (plus a number of leading gluons) the survival probability for two nucleons on small impact parameter, \( |1 - \Gamma(b, s)|^2 \), should go as a high power of the survival probability for the case of one parton removal, \( \approx (1/2)^6 \). Consequently, \( |1 - \Gamma(b \sim 0, s)|^2 \) is close to 0 already at Tevatron energies.

Hence we conclude that

\[ \Gamma(b \sim 0, \sqrt{s} \geq 2 TeV) \approx 1. \]  

(4)

This evaluation is in a good agreement with the Tevatron data on elastic pp collisions and with their extrapolations to the LHC energies.
Since the probability of hard inelastic interactions at fixed impact parameter increases with energy at least as the gluon density at small $x$, $xG_N(x, Q^2) \propto x^{-n_h}$, the increase of this probability should be proportional to $s^{n_h}$ with $n_h \geq 0.2$. The HERA data suggest that taming of the interaction of small dipoles starts only when the probability of inelastic interaction becomes large enough ($\geq 1/2$). However for such probabilities of single parton interactions the multiparton dynamics insures the overall interaction to be practically black. Hence, the multiparton interactions will ensure a rapid (power law) onset of the regime of black interactions.

The analysis of the HERA data suggests [8, 29] that the transverse distribution of the hard partons (at the resolution scale $p_t$) in nucleons can be described as $\propto \exp(-m_h(x)b)$. Since the interaction amplitude of the hard high energy interaction is $\propto s^{n_h}$ we find that the range of $b \leq b_F$ where the interaction is completely absorptive should depend on energy as

$$b_F \approx \frac{n_h \ln \frac{s_0}{s}}{m_h(x)},$$

which corresponds to the Froissart limiting behavior. Note that Eq.5 is obtained in the limit of sufficiently high energies when $m_h(x)$ for $x$ resolved at the corresponding energy ($x \sim 4p_t^2/s$ where $p_t$ is hard scale) is much smaller than that for the fast partons.

Actually, a few uncertainties limit our accurate knowledge of the approaching to the black body limit due to the mechanism of hard interactions. First at all, there are the uncertainties in the $x$ dependence of the gluon densities at very small $x$ where one needs to take into account both $\ln(x_0/x)$, $\ln(Q^2/Q_0^2)$ effects (for the recent discussion see [30]). Also one has to account for the increase of the transverse spreading of the gluon distribution with decrease of $x$. The small $x$ evolution is likely to lead to increase of $n_h$ at very small $x$ and sufficient virtualities. The neglected smearing of the fast partons distribution leads to a decrease of effective mass parameter at preasymptotic energies and to a somewhat faster increase of $b_F$ with energy. Hence, both effects are likely increase the rate of the change of $b_F$ at extremely small $x$.

The value of $m_h$ and rate of its decreasing with energy for $x \leq 10^{-4}$ can be estimated based on the extrapolation of $m_h$ extracted from the HERA data for $J/\psi$ photoproduction which cover $x \geq 10^{-4}$ and correspond to $m_h(x = 10^{-4}) \sim 0.75 GeV$:

$$m_h(s) = 0.75 \left[1 - 0.027 \ln \left(\frac{s}{s_T}\right)\right].$$

The expected limiting value of $m_h$ is $2m_\pi$. Hence, with the reasonable value $n_h = 0.25$ we find that the true asymptotic is likely to be reached at fantastically high energies $s \approx 10^{22} GeV^2$. At these energies $m_h$ will be the same for any colliding hadrons build of light quarks. Note that in pQCD $n_h$ is determined by gluon distribution and, hence, will be also universal, the same for any colliding particles. In the case of hadrons with hidden heavy flavor, the onset of universality regime requires significantly larger energies.

### 2.2 Large impact parameter behavior of the hadron-hadron amplitude

To determine the behavior of the amplitude at large $b$ we can use arguments based on soft physics. At large $b$ only single Pomeron interactions are possible as all multiPomeron interactions have much smaller radius ($\sqrt{2}$ times smaller for the double Pomeron exchange, etc). Hence we can use here information from the analysis of the data on pp scattering at collider energies. Since
we are interested in the large \( b \) behavior of the amplitude which is determined by the properties of the scattering amplitude at small \( t \). we can use the simplest exponential parameterization of the \( t \) dependence of elastic scattering, leading to the proton-proton amplitude due to the Pomeron exchange:

\[
\Gamma_{P}(b, s) = \frac{\sigma(s)}{4\pi B_{P}(s)} \exp\left[-\frac{b^2}{2B_{P}(s)}\right].
\]  

(7)

We take the amplitude to be imaginary at high energies (the ratio of real part of amplitude to the imaginary one at high energies we consider here is practically constant and small \( \kappa \approx 0.1 \div 0.13 \)).

The parameter,

\[
B_{P} = B_{0,P} + 2\alpha'_{P} \ln(s/s_{0})
\]

(8)

is the slope of the \( t \) dependence of the Pomeron exchange contribution into the amplitude of elastic hadron-hadron collision \([3, 5]\). The discovery of the diffraction cone shrinkage with increase of the energy in the elastic \( pp \) collisions confirmed experimentally the energy dependence of \( B_{P} \) with the values of parameters \( B_{0,P} \approx 8.5 \text{GeV}^{-2}, s_{0} = 1 \text{GeV}^{2} \) and \( \alpha'_{P} \approx 0.25 \text{GeV}^{-2} \). The Pomeron exchange hypothesis predicts \( B_{P}/B_{0,P} \approx 1.5 \) at the RHIC energies, \( B_{P}/B_{0,P} \approx 2 \) in the kinematics of the LHC and \( B_{P}/B_{0,P} \approx 2.7 \) when energy is close to the GZK limit. Thus separate analysis of peripheral and central collisions becomes appropriate at energies of LHC and above. While the Pomeron model predicts that at extremely high energies the partial amplitude at large impact parameters is determined by the Pomeron exchange \([31]\), the situation is much more complicated at smaller \( b \) because of shadowing effects (Pomeron branch points). This will lead to renormalization of the Pomeron parameters and to blackening of the interaction if \( \alpha_{P}(o) \geq 1 \) cf. \([13]\). So, the calculation of the partial wave amplitude at non-peripheral \( b \) is technically rather cumbersome. Instead, we assume (as discussed in the Introduction) that partial amplitudes for \( b \leq b_{F} \) are \( Im f(b \leq b_{F}, s) = 1 \) and at \( b \geq b_{F} \) are given by the dominant single Pomeron exchange\(^{3}\). Then, the minimal estimate of \( b_{F} \) can be obtained by requiring the continuity in the matching of these two regimes:

\[
\Gamma_{P}(b_{F}, s) = \frac{\sigma_{tot}^{pp}(s)}{4\pi B_{P}(s)} \exp\left[-\frac{b_{F}^2}{2B_{P}(s)}\right] = 1.
\]

For \( \sigma_{tot}^{pp}(s) \) we can use the Landshoff-Donnachie parameterization of the Pomeron contribution \( \sigma_{P}^{P}(s, s_{0}) = c \left[\frac{s_{0}}{s}\right]^{\alpha_{P}(0)-1} \) which provides a good description of the data in the region between ISR and Tevatron energies with parameter \( \alpha_{P}(0) = 1 = 0.0808 \). Hence, we get the following energy dependence from the condition that at the energies of Tevatron, \( s_{T} \), the partial wave of the \( NN \) amplitude at impact parameter \( b = 0 \) becomes black (\( \Gamma(b = 0, s_{T}) = 0.9 \div 0.95 \approx 1 \)):

\[
b_{F}^2 \approx (\alpha_{P}(0) - 1) \ln(s/s_{T})B_{P}.
\]

(9)

At \( s \rightarrow \infty \) the parameter \( b_{F} \) is universal-the same for any colliding particles:

\[
b_{F}^2 \approx 2\alpha'_{P}(\alpha_{P}(0) - 1) \ln^{2}(s/s_{T}).
\]

Another potentially important soft contribution to the partial wave amplitude may arise from the hadron scattering off meson tails. Using the dispersion representation of the amplitude over

\(^{3}\text{MultiPomeron cuts are decreasing with increase of } b \text{ much faster than Pomeron exchange } [31]. \text{ For example, if } \Gamma_{P}(b) \propto \exp(-\alpha b^2), n\text{-Pomeron cut is } \propto \exp(-\alpha n b^2).\)
momentum transfer, $t$, it is easy to obtain [32]:

$$\Gamma(b) = cs^{\alpha(\mu^2)^{-1}}\exp[-\mu b].$$

The natural expectation given the fast decrease of the amplitude with increase of $b$ is that $\mu$ is the minimal mass permitted in the channel with vacuum quantum numbers i.e $\mu = 2m_\pi$. Thus at energies when partial waves with fixed impact parameter $b$ become equal one we obtain:

$$b_F \approx 1/\mu(\alpha(\mu^2) - 1)\ln(s/s_0).$$

This value is smaller than that provided by the Pomeron exchange model,

$$b_F = \sqrt{B_{IP}(\alpha(0) - 1)\ln(s/s_T)},$$

at achievable energies but exceeds it at asymptotic energies. The leading term in $b_F$ is the same if the partial amplitude is less than one. It is easy to check that the combination of two types of the $b$ dependencies discussed in this subsection does not change the conclusion concerning the universal value for the leading term in $b_F$ at superhigh energies.

### 2.3 Matching of small $b$ and large $b$ behavior

We demonstrated above that the elastic amplitude is well constrained both at small and large impact parameters. To build a complete description we need to determine at what $b_F$ the two regimes match. Both soft and hard approximations give practically the same result for $b_F$ at energies in the range from the LHC up to the GZK limit - see Fig[1]. This is due to the presence of a large constant term in $B_{IP}$. Hence we find that there is very good consistency between the logic of matching starting from small $b$ and the logic of matching from large $b$. It means that there is a smooth transition from hard regime at small $b \leq b_F$ to soft regime at higher $b \geq b_F$ for the whole range of energies which maybe probed experimentally at colliders and in cosmic ray interactions near the GZK cutoff. At the same time we find that the asymptotic rate of the increase of $b_F$ is a factor of two larger in the hard matching approximation. This indicates that at very high energies, the hard mechanism component of the black interactions gives a dominant contribution to the cross sections. Also, both hard and soft dynamics of the hadron-hadron interaction predict the universal character of the $b_F$ at the superhigh energies.

### 3 Universality of cross sections at extremely large energies

In the previous section we argued that rate of the increase of the size of the region where $\Gamma(b, s) \approx 1$ should be followed by a rather steep drop of $\Gamma$ as given by single Pomeron exchange. Since the two scenarios of matching which we considered in the previous section give practically identical results for $10^3 \leq \sqrt{s} \leq 10^6 GeV$, we will consider here the soft matching dynamics which is obviously a more conservative way to estimate the approach to the unitarity limit.

Imposing the condition of complete absorption at fixed $b \leq b_F$ and using the Pomeron exchange formulae at larger $b$, we can build the partial amplitude so that it includes the continuity condition at the matching point

$$\Gamma_{NN}(b, s) = \Theta(b_F - b) + \exp\left[-(b - b_F)^2/2B_{IP}\right]\Theta(b - b_F).$$  \hspace{1cm} (10)
Figure 1: Energy dependence of the black body regime cutoff parameter $b_F$. Solid red line - estimate based on the soft matching approximation. Dashed blue line - prediction of the hard matching approximation. Here and in the following figures log s stands for $\log_{10}(s/1 \text{ GeV}^2)$. 
Since $b_F$ is universal at superhigh energies the only dependence on colliding particles is contained in the scale $1/2B_F$ for the impact parameter distribution.

With the amplitude in Eq (10) one can calculate the total cross section of the hadron-hadron interaction:

$$\sigma_{tot} = 2 \int \Gamma(b, s)db = 2\pi(b_F^2 + 2B_F).$$

(11)

The slope of the $t$ dependence of the elastic amplitude at $t = 0$ is given by the formula:

$$B = \frac{1}{\sigma_{tot}} \int \Gamma(b, s)b^2 db = \frac{(b_F^4/2 + b_F^22B_F + 4B_F^2)}{2(b_F^2 + 2B_F)}.$$ 

(12)

So,

$$\frac{\sigma}{4\pi B} = 2(1 - \frac{4B_F^2}{(b_F^2 + 2B_F)^2 + 4B_F^4}).$$

(13)

At accelerator energies where $2B_F \gg b_F^2$ we obtain relation: $\sigma/4\pi B \approx 1$. In contrast, at superhigh energies where $b_F^2 \gg 2B_F$, we obtain:

$$\sigma/4\pi B \approx 2(1 - 4B_F^2/b_F^4).$$

(14)

Thus, at superhigh energies where $2\alpha' \ln(s/s_0) \gg B_0F$ the cross section and the slope of the $t$ dependence at $t = 0$ become the same for all colliding particles. The memory of the nature of colliding particles is lost as a consequence of the blackening of the interaction and the ratio of total cross sections for any colliding particles should be equal to one:

$$\sigma_{tot}(h_1, h_2)/\sigma_{tot}(pp) \to 1.$$ 

(15)

The same universality of structure functions is expected for superhigh energies because the interaction and essential impact parameters are increasing with energy and as the consequence of the U matrix unitarity condition [33]:

$$F_{2A}(x, Q^2)/F_{2N}(x, Q^2) \to 1.$$ 

(16)

A similar prediction holds for the total cross section of a photon-nucleus scattering:

$$\sigma_{\gamma A}/\sigma_{\gamma p} \to 1.$$ 

(17)

4 Glauber model cross sections for pA and AA collisions

To visualize physical phenomena related to increase of the radius of a nucleon-nucleon interaction with energy and to learn whether the trend toward universality can be observed at the energies achievable at the accelerators or in cosmic ray we need to parameterize the energy dependence of the elementary pp cross section and the slope of t dependence for a wide range of energies. Up to the Tevatron energies ($s_T \approx 4 \cdot 10^6 GeV^2$) we have chosen the parameterization of the total pp cross sections in the form satisfying the Froissart theorem:

$$\sigma_{tot}(pp) = \sigma_0(1 + \epsilon \ln(s/s_0) + \epsilon^2/2\ln(s/s_0)^2)$$

(18)
with $\epsilon = 0.0808$. In the energy range where data are available this form is almost identical to the one suggested by Donnachie and Landshoff. The slope parameter $B_{pp}(t = 0)$ in this energy region is given by Pomeron exchange,

$$B_{pp}(t = 0) = B_0 + 0.5 \ln(s/1 \text{GeV}^2).$$

For higher energies we use as input the profile function $\Gamma(b, s)$ which we built in the previous section in order to calculate the total, elastic and inelastic cross sections and the slope parameter of the $NN$ amplitude. The matching of the cross section and $B_{NN}$ is determined by using $s = s_T$ as the reference point. The energy dependence of the elementary NN cross section and of the slope parameter $B_{NN}(s)$ is shown in Fig.2.

Now we can evaluate the total and absorption cross section for the proton-nucleus interaction at high energies in the conventional Gribov-Glauber model:

$$\sigma^{pA}_{tot}(s) = 2\Re \int_0^\infty \left[ 1 - \left[ 1 - \int \rho(z, r_t)\Gamma_{NN}(b - r_t, s)dz dr_t \right]^A \right] db,$$

and,

$$\sigma^{pA}_{abs}(s) = \int_0^\infty \left[ 1 - \left[ 1 - \frac{2\sigma_{pp}}{\sigma_{tot}} \int \rho(z, r_t)\Gamma_{NN}(b - r_t, s)dz dr_t \right]^2 \right] db.$$

Here $\rho(r) = A^{-1}\rho_A(r)$ is the single nucleon nuclear density normalized by the condition $\int \rho(r)dr = 1$. We calculated the nuclear density $\rho_A$ within the Hartree-Fock model with the effective Skyrme nucleon-nucleon interaction. Note that at intermediate energies this nuclear model provides a reasonable description of elastic proton and electron scattering off nuclei along the periodic table as well as the quasifree knockout of a nucleon in $(e,e'p)$ and $(p,2p)$ reactions without free parameters [34]. In the above formulae we neglected correlations between nucleons, single and double inelastic diffraction. This is legitimate because for central collisions, where these approximations look suspicious, the cross section is close to black limit and therefore independent on details of the model. For the peripheral collisions these effects are small.

The result of our calculations is shown in Fig.3 as a ratio of the total proton-nucleus cross section to the total cross section of the $pp$ interaction as a function of the invariant energy $s$ for $p^{16O}$ and $p^{208Pb}$ collisions. We present the ratio calculated in the approach of soft dynamics blackening(solid red line) and in the hard regime (dashed blue line). The range between the two curves can be treated as a measure of uncertainty of our approximation. As we already discussed, both hard and soft regimes of blackening give close results up to the energies of the GZK limit. At higher energies, the hard mechanism leads to a faster approach to the universality regime. For comparison we also show the ratio found in the model neglecting the radius of the $NN$ interaction(dotted line).

In the energy range where blackening is still a correction, neglecting the radius of the interaction leads to a stronger decrease of the $\sigma_{tot}^{pA}/\sigma_{tot}^{pp}$ ratio in the case of light nuclei. This indicates a more important role of peripheral interactions in the case of light nuclei. Even so, the universal asymptotic is reached for light nuclei at extremely high energies, see Fig.4. The calculated absorption cross sections are shown in Fig.5.

Accounting for the energy dependence of the radius of hadron-hadron interaction in the energy domain where the $NN$ cross section becomes large and the radius of the interaction becomes
Figure 2: Energy dependence of the total cross section and the slope parameter of $NN$ interaction. Solid line - the cross section and the slope parameter as dictated by the soft dynamics matching. Dashed line - hard mechanism of the blackening of $NN$ interaction. Dotted line presents the slope parameter $B_P$ due to the pomeron exchange.
Figure 3: The dependence of $\sigma_{tot}(pA)/\sigma_{tot}(pp)$ on energy. Solid line - the ratio calculated with $NN$ amplitude dominated by soft dynamics, dashed line - hard mechanism of the blackening of interaction. Dotted line is the ratio calculated neglecting by the radius of interaction.
Figure 4: The ratio of the proton-oxygen cross section to the proton-proton one as a function of energy. Calculation in the Gribov-Glauber model demonstrating the onset of the universality regime.
Figure 5: The absorption cross section of pA interaction as a function of energy. Dashed line - calculation of cross section neglecting the radius of interaction.
comparable to the radius of the nucleus reveals new effects beyond those usually associated with Gribov-Glauber shadowing. Scattering from the nucleus edge and from the meson “halo” of a nucleus should lead to a decrease in the dependence of the cross section on atomic number as compared to nuclear shadowing effects. It is evident that at asymptotic energies, where the interaction is already black in the range of the impact parameters considerably exceeding the radius of nucleus, the dependence on the atomic number in the nucleon-nucleus collisions should completely disappear.

We also estimated how the energy dependence of the $NN$ interaction radius in the high energy domain will affect the total and inelastic nucleus-nucleus cross sections. These quantities are used in the study of the relativistic heavy ion collisions aimed to discover the new extreme state of the nuclear matter - the Quark-Gluon plasma. The calculation of the nucleus-nucleus cross sections in the Glauber-Gribov model is a rather complicated problem. Instead, for the rough estimates of the effect we used the generalization of the formulae of Bradt and Peters [25] for the total cross section of scattering of two heavy ions:

$$\sigma_{tot}^{AB} = 2\pi(R_{A}^{eff}(s) + R_{B}^{eff}(s) - c)^2.$$  \hspace{1cm} (22)

We use here the parameter $c = 0.8 \text{Fm}$ as found by Bradt and Peters, and we calculate the cross section of heavy ion collisions using the energy dependent nuclear radius $R_{eff}(s)$ determined from the calculated proton-nucleus total cross section,

$$R_{A}^{eff}(s) = \sqrt{\sigma_{tot}^{pA}(s)/2\pi}.$$  

The energy dependence of the effective nuclear radius for lead is shown in Fig.6. The energy dependence of the $PbPb$ total cross section is shown in Fig.7 where we also present the cross section given by the Bradt and Peters formula with the static nuclear radius(dotted line). The static nuclear radius was found from the calculated HF nuclear density (if one uses the empirical formula for the nuclear radius $R_{A} = r_{0}A^{2/3}$, the HF density of lead requires the value of $r_{0} = 1.156$). For comparison, we also calculated the total nucleus-nucleus cross section using the simplified Glauber approach expression [26, 27]

$$\sigma_{AB}^{tot} = 2R \int \int d\mathbf{b} \left[ 1 - \exp \left( -T_{AB}(\mathbf{b}) \right) \right],$$  \hspace{1cm} (23)

where

$$T_{AB}(\mathbf{b}) = \int d\mathbf{b}_{1} \int d\mathbf{b}_{2} \Gamma_{NN}(\mathbf{b}_{1} - \mathbf{b}_{2}) \int dz_{1} \rho_{A}(\mathbf{b}_{1}, z_{1}) \int dz_{2} \rho_{B}(\mathbf{b} - \mathbf{b}_{2}, z_{2}).$$  \hspace{1cm} (24)

We find significant corrections to the cross section of heavy ion collisions calculated using the Bradt-Peters model with the static nuclear radii already at RHIC energies. We also compare in Fig.7 these estimates of the total cross section to the results of calculation within the simplified Glauber optical model approach. In the range of energies $10^{4} \leq s \leq 10^{8}$ the Bradt-Peters formula underestimates the cross section due to the assumption of the sharp edges of nuclei, hence neglecting the contribution of the interaction of surface nucleons. However, at energies, close to the GZK limit the Bradt-Peters cross sections with an account of the energy dependence of the interaction
Figure 6: Increase with energy of the effective nuclear radius in $pA$ collisions. Dotted line - neglecting the radius of interaction.
Figure 7: Cross section of nucleus-nucleus collisions. Dotted line - the Bradt-Peters cross section with static radii of nuclei; solid and short-dashed lines - the Bradt-Peters cross sections with energy dependent nuclear radii as provided by hard and soft dynamics mechanisms correspondingly; dash-dotted line - optical Glauber approach without accounting for the energy dependence of the radius of $NN$ interaction; long dashed line - optical Glauber model with accounting for the energy dependence of the interaction radius.
Figure 8: The average number of wounded nucleons in pA collision as a function of energy. Dotted line - calculation neglecting the energy dependence of the interaction radius.
Figure 9: The partial inelastic cross section as a function of the number of wounded nucleons. The yellow/light bar is the cross section calculated neglecting the interaction radius.
radius should be reasonable for the total cross section as well as for the absorption cross section which is important for the interpretation of cosmic ray data.

Now we want to demonstrate how accounting for the energy dependent radius of the $NN$ interaction will change the distribution over inelastic collisions with increasing energy. We calculated the average number of wounded nucleons in nucleon-nucleus collisions \[ \bar{\nu} = \frac{A \cdot \sigma_{NN}^{in}(s)}{\sigma_{pA}^{in}(s)}, \] as a function of energy. Here, the inelastic cross section is calculated using the expression \[ \sigma_{pA}^{in}(s) = \sum_{n=1}^{A} \sigma_n(s), \] where the partial cross sections are given by formula

\[ \sigma_n(s) = \frac{A!}{(A-1)!n!} \int db \left[ \sigma_{NN}^{in}(s)T(b, s) \right]^{n} \left[ 1 - \sigma_{NN}^{in}(s)T(b, s) \right]^{A-n}, \]

with the generalized nuclear width function

\[ T(b, s) = \frac{2}{\sigma_{NN}^{tot}(s)} \int dr_t \Gamma_{NN}(r_t - b, s) \int dz \rho(r_t, z). \]

The energy dependence of the average number of wounded nucleons calculated with and without taking into account the increase of the radius of the $NN$ interaction with energy is shown in Fig.8. The effect is still small, on the level of $10\%$, at collider energies. At the same time, at asymptotic energies where the interaction becomes black in a wide range of impact parameters $<\nu> |_{s\to\infty} = A$ as a result of the universality of the collision amplitudes. We also find that, though $<\nu>$ is weakly modified for collider energies, the partial cross sections are affected much more strongly, see Fig.9. In our calculations we neglected corrections due to inelastic diffraction, by the -momentum conservation law which are violated within the eikonal approximation. With the increase in radius of the nucleon-nucleon interaction the basic assumptions leading to Gribov- Glauber approximation, like the neglect of the simultaneous interactions with two nucleons which are located at different impact parameters, are violated since the radius of the $NN$ interaction becomes comparable to the internucleon distance.

The impact of these effects on the interpretation of the RHIC and future LHC data requires separate analysis which is beyond this paper.

5 Discussion and Summary

We demonstrated that the cross section of $NN$ collisions at impact parameters growing as $\ln(s/s_0)$ reaches the black limit due to the hard dynamics. As a result, we argue that all hadronic and nucleus cross sections become equal at ultra high energies. We analyzed the role played by this effect for the total and absorption cross sections of the proton-nucleus collisions in a wide energy range. The effects are of the order $10\% \div 20\%$ and, hence, should be taken into account in the future analyses
of the precision LHC data. They may also be relevant for interpretation of the cosmic ray data near the GZK limit. Dominance of hard dynamics at small impact parameters both in the elastic amplitude and in the structure of final states in the inelastic interactions at small impact parameters \[8\] together with onset of the universality of the cross sections can be considered as new signals for a presence of the phase transition in NN interactions at central impact parameters.

Acknowledgments

This work has been supported in part by the USDOE and GIF.
References

[1] V. N. Gribov, I. Y. Pomeranchuk and K. A. Ter-Martirosian, Phys. Lett. 9, 269 (1964).
   H. Cheng and T. T. Wu, Phys. Rev. Lett. 24, 1456 (1970).
   V. N. Gribov, V. D. Mur, I. Y. Kobzarev, L. B. Okun and V. S. Popov, Sov. J. Nucl. Phys. 13, 381 (1971) [Yad. Fiz. 13, 670 (1971)].
   U. Maor and S. Nussinov, Phys. Lett. B 46, 99 (1973).
   J. Pumplin, F. Henyey and G. L. Kane, Phys. Rev. D 10, 2918 (1974).
   V. N. Gribov and A. A. Migdal, Yad. Fiz. 8, 1213 (1968).

[2] E. M. Levin and L. L. Frankfurt, JETP Lett. 2, 65 (1965).
   P. Camillo, P. M. Fishbane and J. S. Trefil, Phys. Rev. D 10, 3022 (1974).

[3] V. N. Gribov, Nucl. Phys. 22, 249 (1961);
   V. N. Gribov, arXiv:hep-ph/9708424

[4] V. N. Gribov, B. L. Ioffe and I. Y. Pomeranchuk, Sov. J. Nucl. Phys. 2, 549 (1966) [Yad. Fiz. 2, 768 (1965)].

[5] G. F. Chew and S. C. Frautschi, Phys. Rev. Lett. 7, 394 (1961).
   G. F. Chew and S. C. Frautschi, Phys. Rev. Lett. 8, 41 (1962).

[6] M. M. Block and R. N. Cahn, Rev. Mod. Phys. 57, 563 (1985).

[7] K. Greisen, Phys. Rev. Lett. 16, 748 (1966).
   G. T. Zatsepin and V. A. Kuzmin, JETP Lett. 4, 78 (1966) [Pisma Zh. Eksp. Teor. Fiz. 4, 114 (1966)].

[8] L. Frankfurt, M. Strikman and C. Weiss, Phys. Rev. D 69, 114010 (2004) [arXiv:hep-ph/0311231].

[9] A. Donnachie and P. V. Landshoff, Phys. Lett. B 296, 227 (1992) [arXiv:hep-ph/9209205];
   arXiv:hep-ph/9703366

[10] V. N. Gribov, I. Y. Pomeranchuk and K. A. Ter-Martirosian, Phys. Rev. 139, B184 (1965).
   V. A. Abramovsky, V. N. Gribov and O. V. Kancheli, Yad. Fiz. 18, 595 (1973) [Sov. J. Nucl. Phys. 18, 308 (1974)].

[11] K. Goulianos, arXiv:hep-ph/0407035.

[12] A. B. Kaidalov, V. A. Khoze, A. D. Martin and M. G. Ryskin, Acta Phys. Polon. B 34, 3163 (2003) [arXiv:hep-ph/0303111].
   A. B. Kaidalov, Phys. Rept. 50, 157 (1979).
[13] A. A. Migdal, A. M. Polyakov and K. A. Ter-Martirosian, Sov. Phys. JETP 40, 420 (1975) [Zh. Eksp. Teor. Fiz. 67, 848 (1974)].
A. M. Polyakov, A. A. Migdal and K. Ter-Martirosian, Sov. Phys. JETP 41, 406 (1975) [Zh. Eksp. Teor. Fiz. 68, 817 (1975)].

[14] W. Heisenberg, Z. Phys. 133, 65, (1952); M. Froissart, Phys. Rev. 123, 1053 (1961).

[15] D. J. Gross and F. Wilczek, Phys. Rev. D 8, 3633 (1973); Phys. Rev. D 9, 980 (1974).

[16] H. Abramowicz and A. Caldwell, Rev. Mod. Phys. 71, 1275 (1999) [arXiv:hep-ex/9903037].

[17] L. Frankfurt and M. Strikman, Nucl. Phys. Proc. Suppl. 79, 671 (1999) [arXiv:hep-ph/9907221].

[18] E. V. Gedalin, E. G. Gurvich Sov. Journ. Yad. Phys. 24, 187 (1976)

[19] V. N. Gribov, Sov. J. Nucl. Phys. 17, 313 (1973) [Yad. Fiz. 17, 603 (1973)].

[20] K. A. Ter-Martirosian, Nucl. Phys. A 477, 696 (1988).

[21] M. M. Block, E. M. Gregores, F. Halzen and G. Pancheri, Phys. Rev. D 60, 054024 (1999) [arXiv:hep-ph/9809403].

[22] M. S. Dubovikov and K. A. Ter-Martirosian, Nucl. Phys. B 124, 163 (1977).

[23] O. V. Kancheli, arXiv:hep-ph/0008299

[24] R. J. Glauber, in High energy Physics and Nuclear Structure, ed. by G. Alexander (North-Holland, Amsterdam, 1967), pp. 311;

V. N. Gribov, Sov. J. Nucl. Phys. 9, 369 (1969) [Yad. Fiz. 9, 640 (1969)].
R. J. Glauber, G. Matthiae, Nucl. Phys. B21, (1970), 135;
V. Franco, G. Varma, Phys. Rev. C18, (1978), 349.

[25] H. L. Bradt, B. Peters, Phys. Rev. 77 (1950) 54;
Yu. M. Shabelski, Acta Phys. Polonica, B10, (1979), 1049.

[26] H. Sato, Y. Okuhara, Phys. Rev. C34, (1986), 2171;
G. F. Bertsch, B. A. Brown, H. Sagawa, Phys. Rev. C39, (1989), 1154.

[27] A. Capella, A. Kaidalov and J. Tran Thanh Van, Heavy Ion Phys. 9, 169 (1999) [arXiv:hep-ph/9903244].

[28] A. Bialas, M. Bleszynski and W. Czyz, Nucl. Phys. B 111, 461 (1976).
L. Bertocchi, D. Treleani, J. Phys. G: Nucl. Phys., vol. 3, (1977), 147.
[29] L. Frankfurt and M. Strikman, Phys. Rev. D 66, 031502 (2002) [arXiv:hep-ph/0205223].

[30] G. Altarelli, R. D. Ball and S. Forte, Nucl. Phys. B 674, 459 (2003) [arXiv:hep-ph/0306156];
    M. Ciafaloni, D. Colferai, G. P. Salam and A. M. Stasto, Phys. Rev. D 68, 114003 (2003)
    [arXiv:hep-ph/0307188].

[31] V. N. Gribov, Sov. Phys. JETP 26, 414 (1968) [Zh. Eksp. Teor. Fiz. 53, 654 (1967)].

[32] V. N. Gribov, “Lectures On The Theory Of Complex Momenta,” KHFTI-PREPRINT-70-47.

[33] L. Frankfurt, V. Guzey, M. McDermott and M. Strikman, Phys. Rev. Lett. 87, 192301 (2001)
    [arXiv:hep-ph/0104154].

[34] S. L. Belostotsky et al., "Proceedings, Modern development in nuclear physics.", ed. by
    Sushkov, Novosibirsk, 1987, pp 191-210.

    L. Frankfurt, M. Strikman and M. Zhalov, Phys. Lett. B 503, 73 (2001)
    [arXiv:hep-ph/0011088].