“Unlocking” the Ground: Increasing the Detectability of Buried Objects by Depositing Passive Superstrates

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Abstract—One of the main problems when trying to detect the position and other characteristics of a small inclusion into lossy Earth via external measurements is the object’s poor scattering response due to attenuation. Hence, increasing the scattered power generated by the inclusion when using not an active but a passive material is of great interest. To this direction, we examine a procedure of “unlocking” the ground by depositing a thin passive layer of conventional material atop of it. The first step is to significantly enhance the transmission into a lossy half-space, in the absence of the inclusion, by covering it with a passive slab. The redistribution of the fields into the slab and the infinite half-space, due to the interplay of waves between the interfaces, makes it possible to determine the thickness and permittivity of an optimal layer. The full boundary value problem (including the inclusion and the deposited superstrate) is solved semianalytically via integral equation techniques. Then, the scattered power of the buried inclusion is compared to the corresponding quantity when no additional layer is present. We report a substantial improvement in the detectability of the inclusion for several types of ground and burying depths by using conventional lossy materials. Implementation aspects in potential applications as well as possible future generalizations are also discussed. The developed technique may constitute an effective “configuration (structural) preprocessing” which may be used as a first step in the analysis of related problems before the application of an inverse scattering algorithm concerning the efficient processing of the scattering data.

Index Terms—Buried inclusion, detectability, integral equations, inverse scattering, mixing formulas.

I. INTRODUCTION

Finding a buried object inside the Earth is a very interesting problem with long history. It is motivated by important applications including detection of mines [1], [2], mineral deposits [3], [4], and unexploded ordnance (UXO) [5], [6] as well as scattering by buried pipes [7], [8], tree trunks [9], and surface and subsurface inhomogeneities (e.g., rocks) [10]. Various methods have been established for the localization and reconstruction of such objects. Characteristic representatives are methods employing primary fields generated by ground-penetrating radars (GPRs) using waves from a moving radar. The radar is located close to the ground, in order to scan and image the subsurface by recording the strength of the echo produced from the interaction between the impinging waves and the buried objects. GPRs also allow noninvasive diagnostics of the probed domain in a fast and simple way and are widely employed in civil engineering, shallow subsurface prospecting applications, and archaeology. Representative techniques as well as application domains are reviewed in [11] and [12]. The important characteristics of the GPR methods are the number of utilized frequencies and the type of the primary field (pulse or harmonic wave). More precisely, harmonic primary fields are employed in combination with single-frequency measurements in [13]–[17] and with multiple-frequency ones in [12] and [18]–[20]. Moreover, GPR pulses (obviously possessing continuous spectrum) are alternatively considered as primary excitations in several studies implementing GPR techniques (indicatively, we cite [21]–[24]). All of the aforementioned investigations [13]–[24] utilize multiple near-field receivers. We also note that plane waves have been also employed as primary fields for the detection and localization of buried objects (see, e.g., [10] and [25]–[27]).

Most of the methods described above rely mainly on the establishment of improved algorithms concerning the processing of the scattering data in order to make the location of the buried object more apparent. In this paper, we establish and implement a different approach which aims at increasing the detectability of a buried inclusion and, in this way, preparing the ground for a more effective implementation of such algorithms. In particular, we modify the structure of the considered configuration by depositing on the lossy Earth a suitable thin reciprocal and passive superstrate layer which assists both the primary (line-source) field’s penetration in the lossy Earth as well as the buried inclusion’s scattering response in the air region. Our main objective is to determine the proper permittivity and electric thickness of the superstrate in order to increase (make more feasible) the detectability of a cylindrical perfectly electric conducting (PEC) inclusion located at a certain distance from the surface of the ground.

We solve semianalytically the scattering problem via integral equation techniques. In particular, the associated boundary value problem is reformulated via a Fredholm integral equation for the current flowing on the surface of the PEC inclusion (see also the discussions in [28]). This integral equation is
subsequently solved by a semianalytical methodology providing high numerical stability and controllable accuracy. Then, we show that the choice of our optimal superstrate layer is not significantly affected by the size and depth of the inclusion, and therefore, it is actually the inclusion-free configuration that primarily dictates our choice.

Wet, medium dry, and very dry types of ground are considered, and for each of these types, specific realizable mixtures of ordinary/natural materials are reported, which accomplish the specific objectives of making the presence of the inclusion more visible, by maximizing the magnitude of transmitted wave into the lossy ground. It is worth to emphasize that we consider a passive superstrate layer in order to amplify the scattering response of the buried inclusion. We use a harmonic GPR primary field and utilize single-frequency far-field measurements. We present several numerical results demonstrating that it is indeed possible to amplify significantly the scattering response of the buried inclusion. We use a harmonic GPR primary field and utilizes single-frequency far-field measurements. We present several numerical results demonstrating that it is indeed possible to amplify significantly the scattering response of the inclusion by covering the ground with a suitable superstrate layer.

We note that the idea of modifying the effective properties of the ground in order to reduce background medium loss and provide improved detection of a buried target has been investigated to some extent in [29]–[31]. More precisely, in [29], a simple physical optics model was utilized for the analysis, while modifications of the ground by adding a large quantity of water along with an amount of liquid nitrogen were proposed. In [30], it was shown experimentally that an artificial dielectric, composed of an array of small insulated metal-coated plastic spheres and lossless uniform plastic spheres, can be placed over a chosen area and mitigate clutter effects due to ground surface roughness. The improved detection of a mine after covering a rough surface with a smoothed layer of appropriate sand was experimentally demonstrated in [31].

The developed technique in the present work actually constitutes an effective “configuration (structural) preprocessing” which may be used as a first step in the analysis of related problems. Practical limitations like the presence of noise and clutter as well as measurement sensitivity aspects are expected to be treated/remedied by algorithmic procedures which will follow as next steps based on already existing processing methods (like, e.g., the ones of [13]–[24]). The next step toward the practical implementation of the presented approach is to test how the existing algorithms cooperate with our method and evaluate possible weaknesses originating from the change of the configuration by adding the superstrate on the ground.

An \( \exp (+j2\pi ft) \) time dependence is assumed and suppressed throughout the analysis, where \( f \) is the operational frequency of the single-frequency radar.

II. BOUNDARY VALUE PROBLEM

The two-dimensional (2-D) geometrical configuration of the boundary value problem under consideration is depicted in Fig. 1(a). The utilized Cartesian coordinate system \((x, y, z)\) is also shown; the corresponding cylindrical coordinate system \((\rho, \phi, z)\) can be used interchangeably. A cylindrical inclusion of circular shape (with radius \( a \)) and PEC boundary is located at distance \( d \) into a half-space filled with a medium of relative complex permittivity \( \varepsilon_{r,2} \). This half-space is assumed to model the Earth, while the buried PEC cylinder is considered electrically small, i.e., \( k_0 h \ll 1 \), where \( k_0 = 2\pi f/c \) is the free-space wavenumber, with \( c = 1/\sqrt{\varepsilon_0\mu_0} \) as the speed of light (in vacuum with permittivity \( \varepsilon_0 \) and permeability \( \mu_0 \)). The objective of this study is to select the thickness \( h \) and the (possibly complex) relative permittivity \( \varepsilon_{r,1} \) of an electrically thin layer \((k_0 h \ll 1)\), deposited atop the Earth, in order for the detection of the inclusion’s location to become more feasible. The primary field is due to a moving line source carrying electric current \( I \) and located at \((x, y) = (x; d + h)\); its vertical position is fixed at \( y = d + h \), while its horizontal position \( x = \chi \) may vary. Such a primary excitation is expected to model practical implementations related to GPRs. The entire space has constant permeability \( \mu_0 \).

The configuration is illuminated by a \( z \)-polarized primary electric field due to a line source in free space (region #0). The field’s \( z \)-component is given by the Fourier integral [26]

\[
E_{0,\text{inc}}(x, y) = -\frac{jI k_0 s_0}{4\pi} \int_{-\infty}^{+\infty} \frac{e^{-g_0(\beta)|y-d-h|}}{g_0(\beta)} e^{-j\beta(x-x)} d\beta
\]  

Fig. 1. (a) Geometrical configuration of the structure under investigation: we deposit a dielectric layer of suitable characteristics (height \( h \) and permittivity \( \varepsilon_{r,1} \)) on the host half-space (permittivity \( \varepsilon_{r,2} \)) to increase the detectability of a buried inclusion at depth \( d \). The inclusion is considered as perfectly electrically conducting (PEC), has electrically small size \( a \), and is excited by an electric line source of current \( I \) positioned on the vacuum-superstrate surface at horizontal distance \( \chi \) from the inclusion. (b) Dielectric constant \( \varepsilon_{r,1} \) represented on its own complex plane when the host medium is the Earth for three different types of ground at UHF frequency range.
where $\zeta_0 = \sqrt{\mu_0/\varepsilon_0}$ is the free-space wave impedance and $g_0(\beta) = \sqrt{\beta^2 - k_0^2}$ is evaluated with positive real part and, in case it is zero, with positive imaginary part.

Due to the 2-D nature of the configuration and the incident field, all of the generated electric fields in each region will also be $z$-polarized and described in the sequel by their $z$-components.

The formulation of an integral representation of the scattered field by the configuration of Fig. 1(a) requires suitable analytic expressions of the following: 1) the fields induced on the homogeneous (without the PEC cylinder) structure, due to the primary cylindrical wave (1), and 2) the Green’s function of the homogeneous structure.

First, we solve the associated boundary value problem, in the absence of the cylindrical inclusion, and find that the primary cylindrical wave (1), and 2) the Green’s function of the homogeneous structure.

For the observation point $(x, y)$ lying in region #0, the Green’s function takes the form

$$G_0(x, y, X, Y) = \int_{-\infty}^{+\infty} \gamma_0(\beta) e^{-gy(\beta)(y-Y)} e^{-j\beta(x-X)} d\beta \quad (3)$$

where

$$\gamma_0(\beta) = \frac{j k_0 \zeta_0}{\pi} \sqrt{\beta^2 - k_0^2} \varepsilon_{r1}$$

with the radiation functions $g_1(\beta) = \sqrt{\beta^2 - k_0^2}$ and $g_2(\beta) = \sqrt{\beta^2 - k_0^2} e^{j2\beta}$ evaluated as $g_0(\beta)$.

Then, we consider the homogeneous structure excited by a 2-D infinite along $z$ electric line source, located at $(X, Y)$ in region #1. The Green’s function is the electric field generated at $(x, y)$ by this line source. For the observation vector $(x, y)$ in region #1, it is comprised of a primary (singular) term $G_1^{pr}$ and a secondary (smooth) term $G_1^{sec}$, respectively, given by

$$G_1^{pr}(x, y, X, Y) = -\frac{j}{4} H_0^{(2)}(k_0 \sqrt{x-X}^2 + (y-Y)^2)$$

$$G_1^{sec}(x, y, X, Y) = \int_{-\infty}^{+\infty} \gamma_1^{sec}(\beta) e^{g_1(\beta)(y+Y)} e^{-j\beta(x-X)} d\beta$$

where $H_0^{(2)}$ denotes the zeroth order and second kind cylindrical Hankel function, while

$$\gamma_1^{sec}(\beta) = e^{-2g_1(\beta)} \left[ \cosh (g_2(\beta)h) (g_1(\beta) - g_0(\beta)) g_2(\beta) + (g_0(\beta) g_1(\beta) - g_2(\beta)) \sinh (g_2(\beta)h) \right]$$

$$\times \left[ 4\pi g_1(\beta) \right]^{-1} \left[ \cosh (g_2(\beta)h) (g_1(\beta) + g_0(\beta)) g_2(\beta) + (g_0(\beta) g_1(\beta) + g_2(\beta)) \sinh (g_2(\beta)h) \right]^{-1}.$$
The scattered power in the upper half-space is obtained by using the method of stationary phase [33], yielding

\[
P_{sc} = 8\pi^4 (\varepsilon_0 \alpha)^2 k_0 \xi_0 |K|^2 \int_0^\pi |\gamma_0(\varepsilon_0 \cos \phi)|^2 \sin^2 \phi d\phi
\]

measured in Watt per meter of the z-axis.

In our consideration, the host medium with permittivity \(\varepsilon_{r1}\) is the lossy ground, the metallic (PEC) inclusion represents the buried inclusion, and the superstrate layer of permittivity \(\varepsilon_{r2}\) corresponds to a thin homogeneous layer deposited on the Earth’s surface. The applications related to buried objects in the ground usually utilize an operational UHF frequency \(f\) in the range from 500 MHz to 1 GHz. The variation of the complex dielectric permittivity \(\varepsilon_{r1}\) within the aforementioned frequency interval is negligible for all considered cases of very dry, medium dry, or wet ground (similar considerations are made in [34]). In particular, the following frequency variations hold for each case (see, e.g., [35]): 1) for very dry ground: \(\varepsilon_{r1}^{\text{dry}} \simeq 3 - j0.05(f/10^6)^{-0.4} \simeq 3 - j0.0036\); 2) for medium dry ground: \(\varepsilon_{r1}^{\text{medium}} \simeq 15 - j0.1(f/10^6)^{0.25} \simeq 15 - j0.52\); and 3) for wet ground: \(\varepsilon_{r1}^{\text{wet}} \simeq 30 - j3.2\), where \(f\) is measured in units of Hertz.

Note that the complex ground permittivities have been approximated by their average value over the UHF frequency range indicated previously. The validity of this approximation is demonstrated in Fig. 1(b), where the big markers (triangular, square, and circular) correspond to the mean values and the small dots correspond to the variations of the real and imaginary parts of \(\varepsilon_{r1}\) with respect to \(f\) for 500 Hz < \((f/10^6) < 1000\) Hz. It is evident that the adopted mean values of \(\varepsilon_{r1}\) approximate quite accurately all values of \(\varepsilon_{r1}\) in the entire UHF frequency range, and hence, we proceed in the sequel by using these mean values for each of the three considered cases of the ground. We note that it is possible to retain a frequency dependence of \(\varepsilon_{r1}\) in the integral representation (4); however, the difference is negligible since the frequency depends only the imaginary part and \(\text{Re}(\varepsilon_{r1}) > |\text{Im}(\varepsilon_{r1})|\). Furthermore, such a choice will make the presentation of the results of Section III less intuitive.

### III. Particularization and Approximation

The basic aim of the present study is to find the physical properties (permittivity \(\varepsilon_{r2}\) and size (thickness \(h\)) of the deposited superstrate atop the ground which aid the detection of the buried (into depth \(d\)) PEC cylinder of radius \(a\) from measurements of its response due to the moving scanning source (current \(I\) and distance \(\chi\)) excitation. In other words, we are searching for a layer that can “unlock” the ground in the sense that the presence of this specifically selected superstrate will make the two-layered configuration of the Earth together with the superstrate to have effective properties which will increase the scattered power due to the hidden target and hence reveal its presence to an external observer.

Obviously, the practical significance of the work would have been very limited if we confined our analysis to an inclusion of specific location and size; therefore, the proposed deposited layer should work for any type of inclusion inside the ground by substantially amplifying its scattered field. Such an amplification does not necessarily require an active medium; it can be achieved by a suitable redistribution of the fields into the superstrate layer and the half-space. The layer should enhance the transmission of the incident illumination into the ground in order for the PEC cylinder to get maximally excited, and at the same time, it should not block the scattered field from the inclusion so that it transmits back to free space. If one considers reciprocal superstrate (which is the case here), these two objectives are equivalent.

In order to find layers that perform this interplay between reflected and transmitted fields for the incident and the scattered components, we consider the inclusion-free structure excited by normally incident plane waves used for obtaining optimal solutions in an approximate way. (b) Typical variation of the transmission magnitude \(|T|\) on the complex \(\varepsilon_{r2}\) plane regardless of the ground permittivity \(\varepsilon_{r1}\). The gray “×” marks maximal \(|T|\). Note that it corresponds to real \(\varepsilon_{r2}\).
proportional to each other (due to reciprocity) and are defined as follows:

\[
\frac{T_0}{\sqrt{\varepsilon r_1}} = T_1 e^{-jk_0 h} = 4\sqrt{\varepsilon r_2} \left[ e^{jk_0 h} \sqrt{\varepsilon r_2} (1 + \sqrt{\varepsilon r_2}) (\sqrt{\varepsilon r_1} + \sqrt{\varepsilon r_2}) - e^{-jk_0 h} \sqrt{\varepsilon r_2} (1 - \sqrt{\varepsilon r_2}) (\sqrt{\varepsilon r_1} - \sqrt{\varepsilon r_2}) \right]^{-1} = T.
\]

The proportionality constant \(\sqrt{\varepsilon r_1}\) corresponds to the ratio of wave impedances between vacuum and host ground. For a fixed frequency \(f\) (fixed wavenumber \(k_0\)) and a fixed type of ground \(\varepsilon r_1\), the quantity \(|T|\) to be maximized is an exclusive function of a complex variable \(\varepsilon r_2\) and a positive real variable \(k_0 h\). In Fig. 2(b), we show a typical variation of function \(|T|\) on the complex plane of the layer’s permittivity \(\varepsilon r_2\) for a representative set of parameters (similar behavior is exhibited regardless of the permittivity \(\varepsilon r_1\)). One can observe that the maximal value of \(|T|\) (indicated by a gray ‘‘×’’) is achieved for a lossless superstrate (\(\text{Im}(\varepsilon r_2) = 0\)). That property cannot be considered to hold \(\text{a priori}\) for general layered systems, but for this specific configuration and for the considered ranges of the structure’s parameters, it has been checked to be indeed valid. Therefore, we can restrict the parametric space by adopting only real permittivities \(\varepsilon r_2\). Additionally, we avoid exotic metamaterials or \(\varepsilon\)-near-zero (ENZ) materials and hence consider that \(\varepsilon r_2 > 1\).

Based on the latter assumptions, we can simplify our problem by maximizing a function \(|T|\) of two positive variables: \(h/\lambda_0 > 0\) and \(\varepsilon r_2 > 1\). We perform such a maximization operation for every single value of layer’s electrical thickness \(h/\lambda_0\) within a certain range, and in Fig. 3(a), we show the optimal values of the permittivity \(\varepsilon r_2\) for the three sorts of ground (\(\varepsilon r_1\) very dry, medium dry, and wet). In other words, we depict all of the combinations of electrical sizes \(h/\lambda_0\) and dielectric constants \(\varepsilon r_2\) which lead to maximal transmission \(|T|\) for each of the three ground type scenarios. Note the stability of the optimal material with respect to \(h/\lambda_0\) for the case of a very dry ground and the convergence of the best-case permittivities for the other two types of ground as \(h/\lambda_0\) gets larger. Furthermore, the larger is the dielectric constant \(\varepsilon r_1\) of the ground, the larger is the optimal dielectric constant \(\varepsilon r_2\) of the layer. From these “optimal pairs,” one should choose the one that gives the best transmission; such a selection is made in Fig. 3(b). Obviously, the optimal transmission is smaller the more lossy is the ground since the power lost during the travel and converted into heat becomes more substantial. The markers show the maxima of the three curves (one for each type) and indicate the electrical thicknesses \(h/\lambda_0\) which, combined with the respective \(\varepsilon r_2\) given by Fig. 3(a), give the optimal solutions for the deposited layer for each of the three considered scenarios (very dry, medium dry, and wet ground).

The maximal transmissions of Fig. 3(b) and the indicated three optimal parameters for the layer \((k_0 h_{\text{opt}}, \varepsilon r_{2,\text{opt}})\) correspond, according to our strategy described previously, to the inclusion-free configuration. In order to test the performance of

\[
p = \int_{-\lambda X}^{\lambda X} P_{sc}(\chi) d\chi, \quad p' = \int_{-\lambda X}^{\lambda X} P_{sc}'(\chi) d\chi
\]

the determined optimal superstrates when the buried inclusion is present, we go back to the complete structure of Fig. 1(a) and the formula of the scattered power (10). A meaningful indicator of how much more detectable the inclusion becomes in the presence of the layer is the ratio of a superposition of the scattered powers for all possible locations of the line source. In particular, we define the following two integrals corresponding to scattered powers in the presence and in the absence (\(\varepsilon r_2 = 1\)) of the deposited layer, respectively:

\[
p = \int_{-\lambda X}^{\lambda X} P_{sc}(\chi) d\chi, \quad p' = \int_{-\lambda X}^{\lambda X} P_{sc}'(\chi) d\chi
\]
where \( \mathcal{X} \) is a suitably large distance such that the evaluated integral values are stabilized. In Fig. 3(c), we show the ratios \( p/p' \) (for each type of ground) as functions of \( h/\lambda_0 \) for the permittivities indicated by Fig. 3(a). It is emphasized that (especially in the cases of medium dry and wet ground) the ratio of the scattered power integrals is substantial, which verifies that the superstrate layers, selected through the analysis of the inclusion-free structure of Fig. 2(a), are indeed effective in amplifying the scattering response from the inclusion. Even more importantly, the variations of \( p/p' \) are quite stable with respect to \( h/\lambda_0 \), and the maxima of the curves are exhibited at points close to the corresponding ones that the curves of \( |T| \) in Fig. 3(b) are maximized.

Therefore, it is verified that the strategy of simplifying the configuration (inclusion-free) and the excitation (plane wave) in order to extract the optimal dielectric constants \( \varepsilon_{r,2,\text{opt}} \) and electric thicknesses \( k_0h_{\text{opt}} \) is effective and yields satisfying results for the full problem of Fig. 1(a). For this reason, we are going to use the obtained solutions from this simplified problem in the following numerical results.

### IV. Numerical Results

#### A. Arbitrary Materials

The optimal dielectric constants \( \varepsilon_{r,2} \), corresponding to the results of Fig. 3(b) referring to the simplified model, are the following:

1. \( \varepsilon_{r,2,\text{opt}}^{\text{dry}} = 1.84 \) for very dry ground;
2. \( \varepsilon_{r,2,\text{opt}}^{\text{medium}} = 3.88 \) for medium dry ground;
3. \( \varepsilon_{r,2,\text{opt}}^{\text{wet}} = 5.52 \) for wet ground

while the corresponding electrical thicknesses \( k_0h_{\text{opt}} \) of the deposited layer are the following:

1. \( k_0h_{\text{opt}}^{\text{dry}} = 0.94 \) for very dry ground;
2. \( k_0h_{\text{opt}}^{\text{medium}} = 0.79 \) for medium dry ground;
3. \( k_0h_{\text{opt}}^{\text{wet}} = 0.66 \) for wet ground.

As referred previously, these optimal parameter values are derived by using the maximal \( |T| \) criterion (independent from depth or shape of the inclusion); the criterion of maximal \( p/p' \) is mainly used for verification.

Figs. 4(a)–6(c) depict the variations of the scattered fields and their powers for the optimal aforementioned superstrate’s parameters classified according to the considered type of ground. More precisely, first the very dry ground case is considered. Fig. 4(a) depicts the variations of the scattered power \( P_{sc} \) versus the horizontal position \( \chi \) of the primary line source when no superstrate is present and when the configuration is covered by the optimal superstrate layer. Moreover, Figs. 4(b) and (c) show the spatial variation of the magnitude of the scattered electric field \( E_{0,sc} \) in the air region with respect to the electrical coordinates \( x/\lambda_0, y/\lambda_0 \) when no superstrate is present and when the superstrate layer is the optimal one, respectively. From the symmetry of the distributions, it is obvious that Fig. 4(b) and (c) correspond to central excitation (\( \chi = 0 \)).

Fig. 4. (a) Scattered power \( P_{sc} \) versus the position \( \chi \) of the line source without any superstrate and with the optimal superstrate. (b) Contour plot of the magnitude of the scattered electric field \( |E_{0,sc}(x, y)| \) in the air region on the map of the Cartesian electrical coordinates \( (x/\lambda_0, y/\lambda_0) \) in the absence of the deposited layer (\( \chi = 0 \)). (c) Contour plot of the magnitude \( |E_{0,sc}(x, y)| \) on the map of \( (x/\lambda_0, y/\lambda_0) \) in the presence of the optimal layer (\( \chi = 0 \)).

B. Numerical Results

In this section, the numerical results of the proposed simplified configuration are presented. The numerical computations were performed using the FDTD method with a mesh size of \( \lambda_0/40 \). The optimal dielectric constants are \( \varepsilon_{r,2,\text{opt}} \), and the corresponding electrical thicknesses are \( k_0h_{\text{opt}} \). The results are shown in Figs. 4(a)–6(c).
the horizontal region with respect to $x/\lambda_0$ for which a larger scattered field is obtained shrinks as one moves from the very dry to the medium dry and finally to the wet ground. Hence, the location of the buried inclusion becomes most evident for the wet ground.

B. Practical Realization

The aforementioned optimal sets of superstrate’s parameters $(k_0 h_{opt}, \varepsilon_{r2, opt})$ have been obtained from a procedure that did not take into account practical realization limitations and constraints. As far as the thicknesses $k_0 h_{opt}$ are concerned, the actual implementation error is proportional to the sensitivity of the utilized length measurement device. Since we are talking about low radio frequencies, such an error is considered as negligible. However, it is difficult to fabricate a medium of specific dielectric constant $\varepsilon_{r2, opt}$ without suffering certain imperfections.

In this section, we are using specific natural/ordinary materials whose suitable mixtures can possess electromagnetic properties close to the ones dictated by the determined optimal permittivities. As a basic material, we use a very common granular substance: sand, whose dielectric constant $\varepsilon_{sand} \approx 3$ does not have an imaginary part and is dispersionless within the considered frequency range (see, e.g., [36]–[38, pp. 246–262]). By inspecting the permittivities $\varepsilon_{r2, opt}$ that we should effectively mimic, we need a sparser and a denser material than sand. For the sparser one, we can utilize polyurethane foam whose properties are identical to those of vacuum $\varepsilon_{foam} \approx 1$ (at
UHF band), and for a denser substance, we select 2-propanol, which is the least lossy of the common available materials for low radio frequencies. The permittivity of propanol varies according to the following law [39]:

\[
\varepsilon_{r_{\text{propanol}}} \approx 3.65 + \frac{17}{1 + 0.00028 j \left( \frac{f}{10^6} \right)^{0.966}}.
\]  

Our plan is to use sand as the host medium and proper inclusion of the two other substances (foam and propanol) so that the final mixture has effective permittivity similar to the optimal ones for each case. Certainly, there are many practical questions and difficulties to be answered in the operational use in the real world, e.g., the deposition of the utilized materials on real ground. These issues can be mitigated by, for example, putting our mixture layer on top of a very thin sheet of inert medium should be filled by small disks of foam, something which is quite challenging in terms of homogenization. That situation would be even harder if we used other shapes of foam inclusions with weaker effect [38].

As far as the cases of medium dry and wet ground are concerned, the optimal effective dielectric constants \(\varepsilon_{r_{\text{opt}}}\) are 3.88 and 5.52 can be directly obtained by mixing sand with 2-propanol (the procedure is quite straightforward; however, manpower in the form of mechanical assistants may be needed), and since the latter is liquid, simple weighted averages of the permittivities are quite accurate for the dielectric constant of the propanol-soaked sand

\[
\varepsilon_{r_{\text{opt}}} = \varepsilon_{r_{\text{opt}}} \varepsilon_{r_{\text{sand}}} + u_{\text{medium}} \varepsilon_{r_{\text{propanol}}}(f)
\]  

where \(u_{\text{medium}}, u_{\text{wet}}\) are the corresponding filling factors for the two medium dry and wet ground. Since the relations (16), (17) are dispersive, we select to equalize the real part of the aforementioned permittivities \(\varepsilon_{r_{\text{opt}}}, \varepsilon_{r_{\text{opt}}}(f)\) with the optimal ones \(\varepsilon_{r_{\text{opt}}}, \varepsilon_{r_{\text{opt}}}(f)\) only at the central frequency \(f = 750 \text{ MHz}\) of the considered spectral range. In particular, the proper coefficients \(u_{\text{medium}}, u_{\text{wet}},\) and the corresponding dispersive functions of the effective permittivity for the utilized materials are evaluated as follows:

\[
\text{Re} \left( \varepsilon_{r_{\text{opt}}}(f) \right) = \varepsilon_{r_{\text{opt}}}(f) \Rightarrow u_{\text{medium}} \approx 0.05 \Rightarrow \varepsilon_{r_{\text{opt}}}(f)
\]  

\[
\text{Re} \left( \varepsilon_{r_{\text{opt}}}(f) \right) = \varepsilon_{r_{\text{opt}}}(f) \Rightarrow u_{\text{wet}} \approx 0.15 \Rightarrow \varepsilon_{r_{\text{opt}}}(f)
\]  

Note that the \(u_{\text{medium}}, u_{\text{wet}}\) are quite small. If we used water instead of propanol, the respective filling factors would be even smaller (because \(\varepsilon_{r_{\text{water}}} \approx 81\)), which could jeopardize the isotropic nature of the mixture. Furthermore, water is more lossy than 2-propanol, which would deteriorate the performance of the (ideally lossless) layer in terms of amplifying the scatterer’s response.

In Fig. 7(a), we mark the (real) optimal lossless values of the permittivity \(\varepsilon_{r_{\text{opt}}}(f)\) for the three ground types (large markers) and the dielectric permittivity values depending on frequency \(f\) for \(500 \text{ Hz} < (f/10^4) < 1000 \text{ Hz}\) given by (18) and (19) of the actual mixtures (series of small dots). In the case of dry ground, the employed materials are lossless and nondispersive; thus, the actual solution is identical to the ideal one (and hence we do not examine it further). For the other two cases (medium dry and wet ground), the real part is very close to what should be for the entire frequency spectrum, while the imaginary part is moderate and dependent on \(f\). The good coincidence of the real parts is natural since the filling factors are selected by equalizing the real permittivities and the frequency range is relatively small; furthermore, the losses are larger in the wet-ground scenario since more (lossy) propanol is needed to approximate \(\varepsilon_{r_{\text{opt}}}\).

At this point, let us quantify the effect of these imperfections on the performance indicators of the proposed structures. In Fig. 7(b), we show the magnitude of the transmission coefficient \(|T|\) as a function of the operational frequency \(f\) for the two types of ground, when the actual materials are used. With dashed lines, we show the best transmission score for the optimal permittivities in each case. The quantity \(|T|\) possesses substantial values for the entire frequency spectrum which are close to the optimal values. It is remarkable that transmission is not maximized at the central frequency \(\tilde{f}\), for which the real part of the permittivity is exactly equal to \(\varepsilon_{r_{\text{opt}}}\), but close to it. This happens due to the fact that the real part of the permittivity does not change substantially with frequency, contrary to the imaginary part. In other words, at a frequency \(f = 700 \text{ MHz}\), smaller than \(\tilde{f}\), the real part of the effective dielectric constant.
may be very close to the optimal value, and the losses are more diminished, leading to higher transmissions. To test the proposed realizable structures, we evaluate also the criterion referred to the complete configuration (in the presence of the buried inclusion), namely, the ratio $p/p'$, as a function of the operational frequency $f$. The results are plotted in Fig. 7(c). One directly observes that the magnitude of the represented quantity is quite high for both scenarios of the ground. The difference from the optimal performance (indicated by dashed lines) is larger for the wet ground than in the case of medium dry ground. Furthermore, the maxima of the curves are exhibited at frequencies higher than $f$, which show the influence of the peripheral parameters (inclusion’s depth and point nature of the source) on the full problem.

In Fig. 8, we show the integral of the scattered power over all positions of the source, defined in (12), normalized by the corresponding quantity $p_0$ when the inclusion is located at the ground surface ($d = 0$), as a function of the electrical depth of the inclusion $d/\lambda_0$. Obviously, due to the presence of losses, the curves are downward sloping, indicating that the detection of the inclusion is more difficult the deeper it is buried. Furthermore, the rate of decrease is smaller, the less lossy is the ground, which is again attributed to the smaller attenuation the waves are subjected to. The graph of Fig. 8 has been produced for the optimal frequency $f = \bar{f}$, but due to the normalization constant $p_0$ which varies with $f$, the graphs are very similar (almost identical) for every frequency within our operational range (for the same type of ground).

C. Additional Test Cases and Discussions

We examine the validity and the performance of the developed methodology in some additional test cases. More specifically, we consider the scattering problem of a normally incident plane wave impinging on a buried inclusion of arbitrary shape at $d = 1.8\lambda_0$. Scattered field simulations are performed with COMSOL Multiphysics [40]; the consideration of the plane incident wave was selected for simplicity of the computations via COMSOL.

Figs. 9(a)–(c) depict the magnitude of the scattered electric field (namely, the difference between the total electric field and the background electric field; the latter is induced in the inclusion-free configuration composed of a three-layered dielectric medium) at all domains. It should be stressed that the field in the interior of the inclusion is nonzero because we represent only the scattered component which should be of equal magnitude and opposite with the background one (or better its analytic continuation in the area of the object) to give a zero outcome at the cross section of the scatterer as imposed by its PEC nature. We consider plane-wave normal illumination for the case of a wet ground and of operating frequency $f = 750$ MHz. Figs. 9(a)–(c) correspond, respectively, to an absent superstrate layer, to an optimal ideal superstrate, and to an optimal realizable superstrate, where the ideal parameters are computed by the techniques of Sections II and III, while...
Fig. 9. Magnitude of the scattered electric field at all domains for \( f = 750 \) MHz and a wet ground and (a) an absent, (b) an optimal ideal, and (c) an optimal realizable superstrate. The buried inclusion has an arbitrary shape and is located at \( d = 1.8\lambda_0 \). Normally incident plane-wave illumination.

the realizable parameters are computed by the procedure of Section IV-B. The significant increase in the scattered field’s values when using an optimal ideal layer is evident. Importantly, this increase still remains significant when using a realizable layer which approximates the parameters of the ideal layer. Moreover, it is worth to note that, although the optimal parameters of the superstrate layer were calculated via the analysis of the boundary value problem of Section II, involving a line-source primary field and an inclusion of circular shape (or even the scatterer-free problem), these parameters may also offer a significant increase in the scattered field in the problem considered in this section corresponding to plane wave incidence on a buried inclusion of arbitrary shape.

Next, we examine the variations in the achieved scattered field’s increase with respect to the changes of the operating frequency. Therefore, we depict in Figs. 10(a) and (b) the magnitude of the scattered electric field at all domains at \( f = 750 \) MHz for a medium dry ground and (a) an absent superstrate and (b) an optimal ideal superstrate. Figs. 10(c) and (d) depict the magnitude of the scattered electric field at \( f = 500 \) MHz and \( f = 1 \) GHz, respectively, where the parameters of the optimal superstrate layer are those computed for the same scattering problem at \( f = 750 \) MHz. It is observed that the superstrate’s parameters determined to achieve the enhanced scattered field’s values at \( f = 750 \) MHz also produce a substantial enhancement at \( f = 500 \) MHz and \( f = 1 \) GHz.

V. CONCLUDING REMARKS, APPLICABILITY ASPECTS, AND FUTURE STUDIES

The traditional problem of detecting a buried inclusion into lossy Earth was considered. In this paper, we did not make any changes in the measuring process, but we did change the configuration of the problem. In particular, we have deposited a passive superstrate on the surface of the Earth to redistribute the fields into all of the regions (air, superstrate, and ground) so that the transmission of the incident illumination is maximized. Using rigorous integral equation formulation, we have showed that the scattered power due to the inclusion is significantly enhanced when an optimal superstrate layer is used, and, importantly, this power enhancement is not severely downgraded when conventional realizable materials are employed.

The implementation of the proposed method is expected to be quite inexpensive because realizing the permittivity values of the optimum superstrate layer resulting by using mixing formulas and mixing sand/water/propanol (according to the discussions of Section IV-B) does not require the use of any costly materials. Moreover, since the superstrate layer needs only to be of centimeters to ten centimeters thick, the amount of the utilized materials is also reasonable. If very large areas are needed to be probed with the proposed method, then the experimental procedure might become time-consuming. Still, this cost is in a way only a very “low-level” expense, requiring no skilled workforce or technical expertise, just mixing and subsequently deploying very inexpensive materials. In general, the labor needed for the production, transportation, and deposition of the mixture superstrate is a “low-cost” labor and certainly does not require technologically efficient realizations.
Another issue is the fact that the determined parameters of the optimal superstrate depend, in general, on the variations of the ground’s parameters. However, it is expected that the values of the superstrate’s parameters obtained by the developed method are not so sensitive in the variations of the other quantities of the problem (in the sense that, when a characteristic of the configuration varies, then beneficial influence in the scattered field is still observed). A partial evidence of this fact was already reported in Section IV-C, where it was shown that a substantial change in the operating frequency does not at all deteriorate the achieved field enhancement. In any case, if in an experimental implementation it is anticipated that the ground’s characteristics are variable, then the optimal superstrate’s parameters can be easily controllable by changing the mixing ratios of the utilized two-component mixture.

The proposed method is noninvasive and thus can be very useful in many applications where nondestructive remote sensing methods are required. Hence, our approach would be perfectly suited to those applications where one cannot or may not mechanically go below the surface of the ground, i.e., cannot dig in (or even if one could dig, that would be much slower than an electromagnetic scanning method). Some indicative applications to this direction are all kinds of civilian applications of GPRs, like nondestructive testing of structures and pavements, locating utility lines or buried structures, tunnels, etc. Other areas could be archaeological searches and mapping as well as environmental applications, like characterizing the pollution state of ground or even perhaps for gold nugget prospecting in riverbeds. In particular, concerning the detection of land mines or UXO, some special precautions need to be taken in the practical implementation of the proposed approach. For
example, some type of remote control of setting the optimized layer on top of sites, where there is an indication that land mines or UXO may be located, could prove useful. Moreover, envi-
ronmental consequences after the utilized material is removed should be taken into account in cases that the ground is to be subsequently exploited. For example, if the ground surface is on a farming field and it is of importance not to jeopardize the continuation of cultivation, then the effect of the material mixtures on the ground could be examined, and in some cases, other substances could be sought. Nevertheless, in other applica-
tions, like archaeology which may take place in deserts, there would certainly be no harm, and problems of deploying sand mixtures or polluting agricultural areas would perhaps not be so important. When it comes to demining applications, then there is of course a tradeoff: winning is more (discovering and getting rid of mines) than losing (environmental concerns due to the utilized mixture).

Several interesting future extensions of the presented method are feasible. A rough surface ground could be considered, which would model properly bouldered terrains. However, since we are working at low microwave frequencies, the surface roughness can be, in most cases, safely considered quite small so that the assumption of a flat ground surface can accurately approximate realistic situations. Indeed, for a moderately rough ground surface, the relative rms surface height deviation is small compared to the operating wavelength. Moreover, non-
PEC buried inclusions could be tested in order to find how much the “unlocking” effect is deteriorated compared to the PEC case. It is expected that the method would work with similar accuracy and also produce a considerable amplification of the scattered wave. This is because our approach is based on the inclusion-free structure, and as shown in detail in Section III, the superstrate layers, selected through the analysis of the inclusion-free structure, are indeed very effective in amplifying the scattering response from the inclusion. The 2-D solution, obtained in this paper, is expected to be quite accurate in realistic applications when the intention is to try to detect elongated objects which can be approximated as 2-D in the transverse plane; in such a case, some a priori knowledge of the direction the long axis of the object is pointing is required. The extension of the method to 3-D scattering is complicated but feasible. The dyadic Green’s function has to be considered and subsequently expanded into Fourier integrals involving the vector eigenfunctions in the corresponding coordinate system (e.g., spherical or spheroidal system; in the spherical one, these are the vector spherical wave functions M and N). For an electrically small scatterer, the mean value theorem for vector triple integrals can be invoked in order to extend the procedure, leading to the derivation of the approximate expression (9) of the present 2-D analysis.

Other future research steps would be to test how the achieved enhancement in the scattering response of the buried inclusion is translated into improvement of the detecting inclusion accu-

uracy (related to position, size, and texture) of the standard methods. It is also interesting to test how the addition of a super-

\textit{References}

[1] A. C. Dubey and R. L. Barnard, Eds., Detection and Remediation Tech-
nologies for Mines and Minelike Targets, vol. 3079. Bellingham, WA: SPIE, 1997.
[2] J. M. Bourgeois and G. S. Smith, “A complete electromagnetic simulation of the separated-aperture sensor for detecting buried land mines,” IEEE Trans. Antennas Propag., vol. 46, no. 10, pp. 1419–1426, Oct. 1998.
[3] S. Constable and L. J. Senka, “An introduction to marine controlled-

source electromagnetic methods for hydrocarbon exploration,” Geophysics, vol. 72, no. 2, pp. WA3–WA12, Mar./Apr. 2007.
[4] S. D. Khan and S. Jacobson, “Remote sensing and geochemistry for detecting hydrocarbon microseepages,” Geol. Soc. Amer. Bull., vol. 120, no. 1/2, pp. 96–105, Jan./Feb. 2008.
[5] H. Huang and I. J. Won, “Automated anomaly picking from broad-

band electromagnetic data in UXO survey,” Geophysics, vol. 68, no. 6, pp. 1070–1078, Nov./Dec. 2003.
[6] H. Huang, B. SanFilippo, A. Oren, and I. J. Won, “Coaxial coil towed EMI sensor array for UXO detection and characterization,” J. Appl. Geophys., vol. 61, no. 3/4, pp. 217–226, Mar. 2007.
[7] L. Peters, Jr., J. J. Daniels, and J. D. Young, ‘‘Ground penetrating radar as a subsurface environmental sensing tool,’’ Proc. IEEE, vol. 82, no. 12, pp. 1802–1822, Dec. 1994.
[8] C. Liu and L. C. Shen, “Numerical simulation of subsurface radar for detecting buried pipes,” IEEE Trans. Geosci. Remote Sens., vol. 29, no. 5, pp. 795–798, Sep. 1991.
[9] Y. C. Lin and K. Sarabandi, “Electromagnetic scattering model for a tree trunk above a tilted ground plane,” IEEE Trans. Geosci. Remote Sens., vol. 33, no. 4, pp. 1063–1070, Jul. 1995.
[10] J. He, T. Yu, N. Geng, and L. Carin, ‘‘Method of moments analysis of electromagnetic scattering from a general three-dimensional dielectric target embedded in a multilayered medium,’’ Radio Sci., vol. 35, no. 2, pp. 305–313, Mar./Apr. 2000.
[11] D. J. Daniels, Subsurface Penetrating Radar. London, U.K.: IEE Press, 1996, ch. 1.
[12] L. Crocco and F. Soldovieri, ‘‘GPR prospecting in a layered medium via microwave tomography,’’ Annu. Geophys., vol. 46, no. 3, pp. 559–572, Jun. 2003.
[13] T. J. Cui, W. C. Chew, A. A. Aydiner, and S. Chen, ‘‘Inverse scatter-
ing of two-dimensional dielectric objects buried in a lossy Earth using the distorted born iterative method,’’ IEEE Trans. Geosci. Remote Sens., vol. 39, no. 2, pp. 339–346, Feb. 2001.
[14] T. J. Cui, W. C. Chew, A. A. Aydiner, and Y. H. Zhang, ‘‘Fast-forward solvers for the low-frequency detection of buried dielectric objects,’’ IEEE Trans. Geosci. Remote Sens., vol. 41, no. 9, pp. 2026–2036, Sep. 2003.
[15] L.-P. Song and Q. H. Liu, ‘‘Fast three-dimensional electromagnetic non-
linear inversion in layered media with a novel scattering approximation,’’ Inv. Probl., vol. 20, no. 6, pp. S171–S194, Nov. 2004.
[16] L.-P. Song, Q. H. Liu, F. Li, and Z. Q. Zhang, ‘‘Reconstruction of three-
dimensional objects in layered media: Numerical experiments,’’ IEEE Trans. Antennas Propag., vol. 53, no. 4, pp. 1556–1561, Apr. 2005.
[17] A. Savin et al., ‘‘Increasing the probability of detection and evalu-
ation of buried metallic objects by data fusion GPR-low frequency electromagnetic sensor array,’’ in Proc. 4th Eur.-Amer. Workshop Rel. Non-Destructive Eval., Berlin, Germany, 2009, pp. 1–8.
[18] C. Yu et al., ‘‘Microwave imaging in layered media: 3-D image recon-
struction from experimental data,’’ IEEE Trans. Antennas Propag., vol. 58, no. 2, pp. 440–448, Feb. 2010.
[19] T. B. Hansen and P. M. Johansen, ‘‘Inversion scheme for ground penetra-
ting radar that takes into account the planar air–soil interface,’’ IEEE Trans. Geosci. Remote Sens., vol. 38, no. 1, pp. 496–506, Jan. 2000.
[20] P. Meincke, ‘‘Linear GPR inversion for lossy soil and a planar air–soil interface,’’ IEEE Trans. Geosci. Remote Sens., vol. 39, no. 12, pp. 2713–2721, Dec. 2001.
[21] H. Brunzell, ‘‘Detection of shallowly buried objects using impulse radar,’’ IEEE Trans. Geosci. Remote Sens., vol. 37, no. 2, pp. 875–886, Mar. 1999.
[22] W. Al-Nuaimy et al., ‘‘Automatic detection of buried utilities and solid objects with GPR using neural networks and pattern recognition,’’ J. Appl. Geophys., vol. 43, no. 2–4, pp. 157–165, Mar. 2000.
[23] D. J. Daniels, P. Vanheeghe, E. Duflos, and M. Davy, ‘‘An abrupt change detec-
tion algorithm for buried landmines localization,’’ IEEE Trans. Geosci. Remote Sens., vol. 44, no. 2, pp. 260–272, Feb. 2006.
[24] E. Pasolli, F. Melgani, and M. Donelli, ‘‘Automatic analysis of GPR images: A pattern-recognition approach,’’ IEEE Trans. Geosci. Remote Sens., vol. 47, no. 7, pp. 2206–2217, Jul. 2009.
[25] P. C. Chaumet, K. Beikebir, and R. Lenczer, ‘‘Three-dimensional optical imaging in layered media,’’ Opt. Exp., vol. 14, no. 8, pp. 3415–3426, Apr. 2006.
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