The triple-pomeron regime and
the structure function of the pomeron
in the diffractive deep inelastic scattering
at very small $x$

N.N. Nikolaev$^{a,b}$, and B.G. Zakharov$^b$

$^a$IKP(Theorie), KFA Jülich, 5170 Jülich, Germany

$^b$L. D. Landau Institute for Theoretical Physics, GSP-1, 117940,
ul. Kosygina 2, Moscow V-334, Russia.

Abstract

We discuss the diffractive deep inelastic scattering at very small $x$ and derive the properties of the diffractive dissociation of virtual photons in the triple-pomeron regime. We demonstrate that the photon-pomeron interactions can be described by the partonic structure function, which satisfies the QCD evolution equations, and identify the valence and sea (anti)quark and the valence gluon structure functions of the pomeron. The gluon structure function of the pomeron
can be described by the constituent gluon wave function of the pomeron. We derive the leading
unitarization correction to the rising structure functions at small $x$ and conclude that the unita-
rized structure function satisfies the linear evolution equations. This result holds even when the
multipomeron exchanges are included.

Submitted to *Zeitschrift für Physik C* E-mail: kph154@zam001.zam.kfa-juelich.de
1 Introduction.

The pomeron (\(\Pi\)) remains one of the most mysterious objects in the high energy physics. Apart from the elastic scattering, the exchange by pomerons describes (Fig. 1) the diffraction dissociation of the projectile, which can be viewed as the projectile-pomeron interaction (Fig. 1c) [1]. In the diffractive lepton production at \(x = Q^2/(Q^2 + W^2) \ll 1\) one can think of the deep inelastic scattering (DIS) on the pomeron emitted by the target nucleon [2-8]. (Here \(W\) is the total energy in the photon-proton center of mass, \(W^2 = 2pq - Q^2\), where \(p\) and \(q\) are the 4-momenta of the proton and photon, and \(Q^2 = -q^2\) is the virtuality of the photon.) If the diffraction dissociation is dominated by the single pomeron exchange, which is a very strong assumption, and if the pomeron exchange can be treated as the factorizing particle exchange, which also is a very strong assumption, then one can introduce the operational definition of the (virtual) photon-pomeron cross section \(\sigma_{tot}(\gamma^{*}\Pi, Q^2, M^2)\) and of the structure function of the pomeron \(F_2^{(\Pi)}(x, Q^2)\) in terms of the differential cross section \(d\sigma_D/dtdM^2\) of the forward diffraction dissociation of virtual photons \(\gamma^{*} + p \rightarrow X + p\) (we follow the Regge-theory convention [1] with the substitution \(M^2 \rightarrow M^2 + Q^2\) which is natural for the DIS):

\[
\sigma_{tot}(\gamma^{*}\Pi, M^2) = \left. \frac{16\pi}{\sigma_{tot}(pp)}(M^2 + Q^2) \frac{d\sigma_D(\gamma^{*} + p \rightarrow X + p)}{dtdM^2} \right|_{t=0} \tag{1}
\]

and

\[
F_2^{(\Pi)}(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}\sigma_{tot}(\gamma^{*}\Pi, M^2) \tag{2}
\]

with the corresponding Bjorken variable

\[
x = \frac{Q^2}{Q^2 + M^2}. \tag{3}
\]

There has already been much work on the parton model phenomenology of the pomeron [2-10], but the definitive proof that the so-defined structure function of the pomeron satisfies the conventional Gribov-Lipatov-Dokshitzer-Altarelli-Parisi (GLDAP) QCD evolution equations [11-13] is as yet lacking. The definition (1) for \(\sigma_{tot}(\gamma^{*}\Pi)\) does implicitly assume that the pomeron has the intercept \(\alpha_{\Pi}(0) = 1\), i.e., the high energy cross sections are constant and the mass spectrum of excitation of large masses \(M\) has the \(1/M^2\) behavior,

\[
\left. \frac{1}{\sigma_{tot}(aN)} \frac{d\sigma_D(a + N \rightarrow X + N)}{dtdM^2} \right|_{t=0} = A_{3\Pi} \frac{1}{M^2}. \tag{4}
\]
If the factorization relations are valid, then $A_{3\mathcal{P}}$ is expected to be a universal dimensional constant independent of the projectile $a$. However, in QCD there are no a priori reasons for the factorization relations to hold, and there are indications to the contrary [7,8,14,15]. Furthermore, the factorization could strongly be violated by the absorption (unitarity) effects from the multiple pomeron exchanges in Figs. 1b,1d [6]. These multiple pomeron exchanges cast shadow on the reinterpretation of the diffraction dissociation in terms of the photon-pomeron interaction. Experimentally $\alpha_{\mathcal{P}}(0) > 1$ [16,17], the total cross sections are rising and the mass spectrum of the diffraction dissociation of protons exhibits slight deviations from the $1/M^2$ law [18]. Furthermore, the cross section of the diffraction dissociation of virtual photons was shown to be infrared sensitive [7-9,19]. The quantity related to the diffraction dissociation cross section - the unitarization (shadowing, absorption) correction to the structure functions at small $x$ - is also infrared sensitive [19-22]. Therefore, the possibility of introduction of the well-defined, and well-behaved in the sense of the QCD evolution, structure function of the pomeron and the issue of the infrared sensitivity of the diffraction dissociation cross section deserve further study.

In this paper we address the above problems in the framework the light-cone $s$-channel approach to the diffractive deep inelastic scattering at small $x = Q^2/(W^2 + Q^2)$ initiated in our previous publications [7,23,24] (for the related early work on the $s$-channel approach to the light-cone QED see Bjorken, Kogut and Soper [25]). Our strategy is to compute the high energy behavior of the total (virtual) photoabsorption cross section and of the diffraction dissociation cross section. We treat the diffractive $\gamma^* p$ scattering in terms of absorption of the light-cone partonic Fock components of the virtual photon on the target proton or nucleus. One can do so since at $x \ll 1$ the photon transforms into these partonic Fock components at large distances $\Delta z \sim \frac{1}{m_N x} \gg R_N, R_A$ (5) upstream the target nucleon (nucleus). As an illustration of the principal points of the light-cone formalism [24,7] consider interactions of the $q\bar{q}$ Fock state of the photon. Because of the condition (3) the transverse size $\vec{\rho}$ of the $q\bar{q}$ pair and the partition $z$ and $(1 - z)$ of the (light-cone) momentum of the photon between the quark and antiquark can be considered frozen in the scattering process. Therefore, one can introduce the spatial wave function of the light-cone $q\bar{q}$ Fock states $\Psi_{\gamma^*}(\vec{\rho}, z)$ and the dipole cross section $\sigma(\rho)$ such that the total photoabsorption cross section $\sigma_{T,L}(\gamma^* N, x, Q^2)$ for the (T) transverse and (L) longitudinal photons and the forward
diffraction dissociation cross section can be calculated as the conventional quantum-mechanical
expectation value [24,7]

\[ \sigma_{T,L}(\gamma^* N, x, Q^2) = \int_0^1 dz \int d^2\vec{\rho} |\Psi_{T,L}(z, \rho)|^2 \sigma(\rho) , \] (6)

\[ \frac{d\sigma_D(\gamma^* \rightarrow X)}{dt} \bigg|_{t=0} = \int dM^2 \frac{d\sigma_D(\gamma^* \rightarrow X)}{dt dM^2} \bigg|_{t=0} = \frac{1}{16\pi} \int_0^1 dz \int d^2\vec{\rho} |\Psi_{T,L}(z, \rho)|^2 \sigma(\rho)^2 . \] (7)

We emphasize that the factorization of the integrands in Eqs. (6,7) is exact, and corresponds to the exact diagonalization of the scattering matrix in the \((\vec{\rho}, z)\)-representation.

Interactions of the \(q\bar{q}\) Fock state of the photon give the driving terms of the structure function at small \(x\) and of the diffraction dissociation cross section. Specifically, Eq. (6) yields the photoabsorption cross section and the proton structure function \(F_2^{(N)}(x, Q^2)\) which are constant vs. \(x\). Eq. (7) yields the mass spectrum of the diffractively produced states \(d\sigma_D(\gamma^* \rightarrow X)/dM^2 \propto 1/(Q^2 + M^2)^2\) and can be associated with the ‘valence’ \(q\bar{q}\) component of the pomeron [7,23,24]. Both the rise of \(F_2^{(N)}(x, Q^2)\) towards small \(x\) and the triple-pomeron component of the mass spectrum in the diffraction dissociation of photons, which can be associated with the ‘sea’ \(q\bar{q}\) pairs and gluons in the pomeron, are generated by interactions of the higher, \(q\bar{q}g_1....g_n\) Fock states of the photon.

The subject of this paper is a generalization of the light-cone s-channel approach [24,7] to interactions of the higher Fock states of the photon. Our major findings can be summarized as follows: The diffraction dissociation cross section can indeed be factorized (Eq. (89)) into the flux of pomerons in the proton \(f_{IP}(y)/y\), where \(y = (M^2+Q^2)/(W^2+Q^2)\) is a fraction of proton’s (light-cone) momentum carried by the emitted pomeron, and the well-defined structure function of the pomeron \(F_2^{(IP)}(x/y, Q^2)\), which satisfies the conventional QCD evolution with \(Q^2\) (the definition (1) corresponds to the convention \(f_{IP}(y) = 1\)). This structure function and the flux of pomerons describe how the naive \(\propto 1/M^2\) mass spectrum (1) is modified by the rising hadronic cross sections and by the QCD evolution effects. The factorization (89) bears certain semblance to the usual Regge-theory factorization despite the fact that the Regge-theory factorization relations do not hold in deep inelastic scattering at small \(x\) and in our analysis we never assume nor use the Regge-theory factorization. We speak of the triple-pomeron regime just to pay a tribute to the fact that we mostly consider \(Q^2 \ll M^2 \ll W^2\), which in the Regge-theory would have been the triple-pomeron domain. The infrared sensitivity of the diffraction dissociation
cross section can be reabsorbed into the initial momentum distribution of the 'constituent' quarks, antiquarks and gluons in the pomeron, in close similarity to the conventional QCD evolution analysis of the proton structure function. We demonstrate that besides the 'valence' and 'sea' quark-antiquark components derived in [7] (for discussion of the valence $q\bar{q}$ component of the pomeron see also [5]), the pomeron has the 'valence' gluon component, and present the explicit derivation of the constituent gluon wave function of the pomeron. The absolute normalization of the pomeron structure function can be related to the triple-pomeron coupling $A_{3\text{IP}}(Q^2)$ Eq. (62), which gives the driving term of the diffraction dissociation mass spectrum (II). We shall confirm the earlier suggestion [26] that despite being dimensionfull, $A_{3\text{IP}}(Q^2) \propto GeV^{-2}$, this triple-pomeron coupling only has weak dependence on $Q^2$, although if $1/\sqrt{Q^2}$ were the only relevant scale in the DIS, then naively one could have expected $A_{3\text{IP}}(Q^2) \propto 1/Q^2$. The fact, that one can (approximately) relate the properties of the diffraction dissociation in the DIS and in the real photoabsorption, which here is proven rather than assumed, is interesting by itself. In [7] we gave a phenomenological estimate of the sea structure function of the pomeron and of the diffraction dissociation rate in terms of the triple-pomeron coupling $A_{3\text{IP}}(0)$ borrowed from the real photoproduction data [27]. The resulting predictions are in a good agreement with the first data on the diffraction dissociation of photons in DIS at HERA obtained recently by the ZEUS collaboration [28].

The diffraction dissociation of virtual photons and the unitarity (shadowing, absorption) corrections to the rising structure functions at small $x$ are two closely related phenomena. The GLDAP evolution equations predict [12] very rapid rise of the structure functions towards large $1/x$, which conflicts the $s$-channel unitarity [19]. The $s$-channel unitarization of the virtual photoabsorption cross section introduces the multiple-pomeron exchanges, which could lead to the departure of the $x$- and $Q^2$-dependence of structure function from predictions of the GLDAP evolution. We find that the unitarity corrections are large and persist at all $Q^2$. The unitarity correction is a nonlinear functional of the parton density and violates the conventional linear relationship between the photoabsorption cross section and the parton density. Our principle conclusion is that, nonetheless, the modified evolution equations retain their linear GLDAP form as distinct from the nonlinear equation suggested in [19] and discussed extensively in the literature during the past decade (for a recent review with many references see [22]).
diagonalization of the scattering matrix in the $(\vec{p}, z)$-representation greatly simplifies a discussion of the unitarization correction, as it allows to unambiguously identify the $s$-channel partial waves which must satisfy the unitarity bound.

This paper is organized as follows. In section 2 we review the light-cone $s$-channel approach to deep inelastic scattering starting with the diffractive interactions of the two-body $q\bar{q}$ Fock state of the photon [24,7]. In section 3 we study interactions of the 3-body $q\bar{q}g$ Fock state of the photon and derive the driving term of the triple-pomeron mass spectrum in the diffraction dissociation of virtual photons and the driving term of the sea structure function of the pomeron. In section 4 we apply our formalism to derivation of the rising structure function in the double-logarithmic limit. The subject of section 5 is the triple-pomeron regime of the diffraction dissociation of photons to higher orders in the perturbative QCD. Here we derive the structure function and the constituent gluon wave function of the pomeron. The latter absorbs the infrared-regularization dependence of the diffraction dissociation cross section. This completes a derivation of the valence and sea (anti)quark and the gluon structure functions of the pomeron, which are to be used as an input of the GLDAP evolution of the pomeron structure function. The factorization properties of the QCD pomeron and the flux of pomerons in the proton are discussed in section 6. In section 7 we discuss the unitarization of the rising structure functions of the proton at small $x$. We demonstrate that the unitarized structure functions of the proton still satisfy the linear GLDAP evolution equations, as distinct from the nonlinear Gribov-Levin-Ryskin (GLR) equations [19]. Remarkably, this conclusion holds even when the multiple pomeron exchanges are included. In this section we also present a brief phenomenology of the shadowing correction to the proton structure function and comment on the treacherous path to the interpretation of the shadowing in terms of the fusion of partons. In section 8 we comment on the relation between our scenario for deep inelastic scattering at small $x$ and the Lipatov’s work on the pomeron in QCD [29]. Our main results are summarized in section 9.

This paper is mostly devoted to the derivation of the formalism, the numerical results for the unitarization of the proton structure function at small $x$ [30], for the (anti)quark and gluon distributions in the pomeron and the structure function of the pomeron [31], for the fusion of the nuclear partons and the nuclear shadowing [32] will be presented elsewhere.
2 Deep inelastic scattering in terms of the Fock states of the photon and the dipole cross section

2.1 The $q\bar{q}$ Fock states of the photon and sea quarks in the proton

We are interested in DIS at $x \ll 1$ where the structure functions are dominated by the scattering of photons on the sea quarks. The driving term of the sea is given by the perturbative QCD diagrams shown in Fig. 2. The same diagrams can be viewed as the scattering on the target proton of the $q\bar{q}$ Fock states of the photon. The principle finding of Ref. [24] is that the corresponding contribution to the photoabsorption cross section can be cast into the quantum-mechanical form (8). The wave functions of the $q\bar{q}$ fluctuations of the photon were derived in [24] and read (for the related discussion in the framework of QED see also Bjorken et al. [25])

$$|\Psi_T(z, \rho)|^2 = \frac{6 \alpha_{em}}{(2\pi)^2} \sum_{1}^{N_f} Z_f^2 \{ [z^2 + (1-z)^2] \epsilon^2 K_1(\epsilon \rho)^2 + m_f^2 K_0(\epsilon \rho)^2 \},$$

$$|\Psi_L(z, \rho)|^2 = \frac{6 \alpha_{em}}{(2\pi)^2} \sum_{1}^{N_f} 4Z_f^2 Q^2 z^2 (1-z)^2 K_0(\epsilon \rho)^2,$$

where $K_\nu(x)$ are the modified Bessel functions and

$$\epsilon^2 = z(1-z)Q^2 + m_f^2.$$  

In Eqs. (8)-(10) $m_f$ is the quark mass and $z$ is the Sudakov variable, i.e. the fraction of photon’s light-cone momentum $q_-$ carried by one of the quarks of the pair ($0 < z < 1$). In the diagrams of Fig. 2 the color-singlet $q\bar{q}$ interacts with the target nucleon via the Low-Nussinov [33] two-gluon exchange, which is the driving term of the QCD pomeron. The interaction cross section for the color dipole of size $\rho$ is given by [24]

$$\sigma(\rho) = \frac{16}{3} \alpha_S(\rho) \int \frac{d^2 \vec{k}}{(\vec{k}^2 + \mu_G^2)} \frac{V(k)}{\alpha_S(\vec{k}^2)}.$$

In (11) $\vec{k}$ is the transverse momentum of the exchanged gluons, the longitudinal momentum of the exchanged gluons is $\sim m_N x$ and is negligible at small $x$, $\mu_G \approx 1/R_c$ is some kind of effective mass of gluons introduced so that color forces do not propagate beyond the confinement radius $R_c \sim R_N$. The gluon–gluon–nucleon vertex function $V(\vec{k})$ is related to the two-quark form factor of the nucleon $G_2(\vec{k}_1, \vec{k}_2) = \langle N | \exp \left[ i(\vec{k}_1 \cdot \vec{r}_1 + \vec{k}_2 \cdot \vec{r}_2) \right] | N \rangle$ by

$$V(k) = 1 - G_2(\vec{k}, -\vec{k}) \approx 1 - F_{ch}(3\vec{k}^2),$$

where $F_{ch}(s)$ is the chiral form factor.
where $F_{ch}(q^2)$ is the charge form factor of the proton. $\alpha_S(k^2)$ is the running QCD coupling which we shall use both in the momentum, and the coordinate representations:

$$\alpha_S(k^2) = \frac{4\pi}{\beta_0 \log(k^2/\Lambda^2_{QCD})}, \quad \alpha_S(\rho) = \frac{4\pi}{\beta_0 \log(1/\Lambda^2_{QCD}\rho^2)}.$$  \hspace{1cm} (13)

### 2.2 Universality of the dipole cross section and infrared regularization

The salient feature of the dipole cross section (11) is its universality property [7,8]: $\sigma(\rho)$ depends only on the size $\rho$ of the $q\bar{q}$ color dipole. The dependence on $Q^2$ and the quark flavor is concentrated in the wave functions (8) and (9). The fundamental role of the color gauge invariance must be emphasized. By virtue of the color gauge invariance gluons with the wavelength $\lambda = 1/k > R_N$ decouple from the color-singlet nucleon. This decoupling is taken care of by the vertex function $V(k)$, which vanishes as $k \to 0$, and the size of the nucleon emerges as a natural infrared regularization: the dipole cross section (11) is infrared-finite even if $\mu_G = 0$. Similarly, the factor $[1 - \exp(i\vec{k}\vec{\rho})]$ takes care of the decoupling of gluons with $\lambda > \rho$ from the colour-singlet $q\bar{q}$ Fock state. As a result, at small $\rho$ the cross section $\sigma(\rho)$ is perturbatively calculable with its absolute normalization. For the nucleon target

$$\sigma(\rho) \approx \frac{4\pi}{3} \rho^2 \alpha_S(\rho) \int_0^{1/\rho^2} \frac{dk^2 k^2 V(k)}{(k^2 + \mu_G^2)^2} \alpha_S(k^2) = \frac{16\pi^2}{3\beta_0} \rho^2 \alpha_S(\rho) \log \left[ \frac{1}{\alpha_S(\rho)} \right] = C_N \rho^2 \alpha_S(\rho) L(\rho),$$

where

$$L(\rho) = \log \left[ \frac{1}{\alpha_S(\rho)} \right]$$

is the large parameter of the so-called Leading-Log Approximation (LLA) [11].

Another universal feature of $\sigma(\rho)$ is its saturation at $\rho > R_N, R_c$ because of the confinement [24]. This is a strong-coupling regime and in the regime of saturation $\sigma(\rho)$ depends on the infrared regularizations. (Following [34], we assume the freezing strong coupling $\alpha_S(\rho) = \alpha_S(R_f) \sim 1$ at $\rho > R_f$.) One natural infrared regularization - the size of the target proton - enters via the vertex function $V(k)$. The other two infrared regularizations are the effective confinement radius $R_c$ which enters via the effective gluon mass $\mu_G$ and the freezing point $R_f$ of the strong coupling $\alpha_S(R_f) \sim 1$. Evidently, by virtue of Eq. (13) the dependence on these infrared regularizations propagates into the proton structure function. Here we would like to emphasize that such an infrared sensitivity of $F_2^{(N)}(x, Q^2)$ is old news: in the conventional QCD–improved parton model this dependence is hidden in the parameterization of the input parton distributions at the low
factorization scale $Q_0^2$. In our light-cone s-channel approach we rather calculate the structure function in terms of the dipole cross section, reducing drastically the number of the infrared parameters. Furthermore, the crude test of the large-$\rho$ behavior of the dipole cross section $\sigma(\rho)$ is provided by the hadronic cross sections. For instance, the pion-nucleon total cross section can be evaluated as

$$\sigma_{\text{tot}}(\pi N) = \int_0^1 dz \int d^2 \vec{\rho} \, |\Psi_\pi(z, \rho)|^2 \sigma(\rho),$$

which roughly reproduces the observed value of $\sigma_{\text{tot}}(\pi N)$ at moderate energies [7,8,14,15,30]. Once the constraint (16) has been enforced, the predictions for DIS become to a large extent parameter-free. Such a minimal-regularization approach leads to a viable description of the absolute value of $F_2^{(N)}(x, Q^2)$ [23,24,30,35], of the longitudinal structure function $F_L(x, Q^2)$ [36] and of the nuclear shadowing in DIS [23,24,35], of the gluon distribution in the proton [37], of the excitation of charm in the muon and neutrino scattering [38] and roughly reproduces the total cross section of the real photoabsorption [7,8]. This minimal-regularization approach is not imperative, though, and all the results to be presented below could easily be reformulated in terms of more familiar parameterizations of the input parton distributions in the proton and pomeron at the expense of losing the predictive power.

2.3 Connection with the QCD evolution equations

At small $Q^2$ the transverse size of the $q\bar{q}$ Fock states of the photon shall be given by the Compton wavelength of the quark $1/m_f$. For the heavy flavors (charm,...) this is the perturbatively small size. For the light flavors it is natural to ask that also the quarks do not propagate beyond the confinement radius. With the natural choice $m_{u,d} \sim \mu_\pi \sim 1/R_c$ Eq. (8) with the wave function (8) reproduces the real photoabsorption cross section [7,8]. At larger $Q^2 \gg m_f^2, \mu^2, R_N^{-2}$ the conventional QCD-improved parton model description is recovered. Indeed, let us calculate the $Q^2$ dependence of the cross section (8). At large $Q^2$ the leading contribution to $\sigma_T(\gamma^* N)$ comes from the $K_1(\varepsilon \rho)^2$ term in Eq. (8). Making use of the properties of the modified Bessel functions, after the $z$-integration one can write

$$\sigma_T(\gamma^* N, x, Q^2) = \int_0^1 dz \int d^2 \vec{\rho} \, |\Psi_T(z, \rho)|^2 \sigma(\rho) \propto \frac{1}{Q^2} \int_{1/Q^2}^{1/m_f^2} d\rho^2 \frac{d^2}{\rho^2} \sigma(\rho) \propto \frac{1}{Q^2} \int_{1/Q^2}^{1/m_f^2} d\rho^2 \frac{d^2}{\rho^2} \alpha_S(\rho) \log \left[ \frac{1}{\alpha_S(\rho)} \right] \propto \frac{1}{Q^2} \frac{1}{2!} L(Q^2)^2. \quad (17)$$
We find the scaling cross section \( \propto 1/Q^2 \) times the LLA scaling violation factor, with one power of \( L(Q^2) = \log [1/\alpha_S(Q^2)] \) per QCD loop, which is a starting point of the derivation [11-13] of the QCD evolution equations. Notice, that the factor \( 1/Q^2 \) in Eq. (17), which provides the Bjorken scaling, comes from the probability of having the \( q\bar{q} \) fluctuation of the highly virtual photon. There is a finite, and also scaling, contribution to \( \sigma_T \) from the region of \( \rho^2 < 1/Q^2 \):

\[
\Delta \sigma_T(\rho^2 < 1/Q^2, x, Q^2) \propto \int_0^{1/Q^2} \frac{d\rho^2}{\rho^2} \sigma(\rho) \propto \int_0^{1/Q^2} d\rho^2 \alpha_S(\rho)L(\rho) \propto \frac{1}{Q^2}\alpha_S(Q^2)L(Q^2).
\]

(18)

This is the \( \sim \alpha_S(Q^2)/L(Q^2) \) correction to the LLA cross section (17). Notice the strong ordering in the LLA cross section:

\[
1/Q^2 < \rho^2 < 1/m_f^2.
\]

(19)

The QCD scaling violations are (logarithmically) dominated by \( \rho^2 \sim 1/Q^2 \). Similar analysis gives the longitudinal cross section

\[
\sigma_L(\gamma^* N, x, Q^2) = \int_0^1 dz \int d^2\bar{\rho} |\Psi_L(z, \rho)|^2 \sigma(\rho) \propto \frac{1}{Q^4} \int_{1/Q^2}^{1/m_f^2} \frac{d\rho^2}{\rho^4} \alpha_S(\rho) \log [1/\alpha_S(\rho)] \propto \frac{1}{Q^2}\alpha_S(Q^2)L(Q^2),
\]

(20)

which is completely dominated by \( \rho^2 \sim 1/Q^2 \) (for the more discussions on this point see [36]).

2.4 Diffraction excitation of the \( q\bar{q} \) Fock state of the photon and the 'valence' \( q\bar{q} \) component of the pomeron

The shape of the mass spectrum from the diffraction excitation of the \( q\bar{q} \) Fock state of the photon (Fig.3) can be estimated undoing the \( z \)-integration in Eq. (7). Firstly, we note that the diffraction dissociation cross section is dominated by large \( \rho^2 \sim R_c^2, m_f^{-2} \) [7]:

\[
\left. \frac{d\sigma_{D,T}(\gamma^* \rightarrow X)}{dt} \right|_{t=0} = \frac{1}{16\pi} \int_0^1 dz \int d^2\bar{\rho} |\Psi_T(z, \rho)|^2 \sigma(\rho)^2 \propto \frac{1}{Q^2} \int_{1/Q^2}^{1/m_f^2} \frac{d\rho^2}{\rho^4} \alpha_S(\rho) \propto \frac{1}{Q^2}\alpha_S(Q^2)L(Q^2).
\]

(21)

Secondly, the invariant mass squared of the \( q\bar{q} \) system equals

\[
M^2 = \frac{\vec{k}_q^2 + m_f^2}{z(1-z)},
\]

(22)

where \( \vec{k}_q \) is the transverse momentum of the (anti)quark of the pair. For the crude estimation of the mass spectrum at \( M^2 > Q^2 \) one can undo the \( z \) integration in (7,21) making use of
\[ k^2 \sim 1/\rho^2 \sim m_f^2, \text{ so that } M^2 \sim 1/z\rho^2 \text{ and} \]
\[ dz \sim \frac{1}{\rho^2 M^4}, \quad (23) \]

which gives (for the more detailed derivation see [7])
\[ \left. \frac{d\sigma_{D,T}(\gamma^* \to X)}{dM^2 dt} \right|_{t=0} \propto \frac{1}{(Q^2 + M^2)^2} \int^{1/m_f^2} d\rho^2 \propto \frac{1}{m_f^2 (Q^2 + M^2)^2} \quad (24) \]

Notice the strong flavor dependence of the diffraction dissociation cross section, Eqs. (21), (24).

For the excitation of heavy flavors the diffraction dissociation cross section is perturbatively calculable, for the excitation of light flavors it is evidently infrared-regularization dependent.

(We shall encounter such an infrared-regularization sensitivity of the diffraction dissociation cross section over and over again. We shall demonstrate that, however, this infrared sensitivity can be reabsorbed into the normalization of the input structure function of the pomeron, which by itself will be proven to satisfy the GLDAP evolution.) The corresponding contribution to the structure function of the pomeron has the form [7] reminiscent of the valence structure function of the proton:
\[ F_2^{(\text{IP})}(x, Q^2) = \frac{4Q^2}{\pi \alpha_{em} \sigma_{\text{tot}}(pN)} (M^2 + Q^2) \frac{d\sigma_D(\gamma^* \to q + \bar{q})}{dt \, dM^2} \bigg|_{t=0} \propto x(1-x) \quad (25) \]

(The convention (1) for the photon-pomeron cross section differs from that used in Ref. [7] by the factor \((M^2 + Q^2)/M^2\).) Therefore, the diffraction excitation of the \(q\bar{q}\) Fock state of the photon can be associated with DIS on the ‘valence’ \(q\bar{q}\) component of the pomeron (see also Ref. [5]).

### 2.5 Rising structure functions and higher Fock states of the photon

The diagrams of Fig. 2 can also be reinterpreted as the Bethe-Heitler production of the \(q\bar{q}\) pair by the photon-gluon fusion \(\gamma^* g \to q\bar{q}\). The conventional Weizsäcker-Williams formula for this Bethe-Heitler cross section reads
\[ \Delta \sigma_{\text{tot}}(\gamma^* N, W^2, Q^2) = \sum_{f=1}^{N_f} \int_x^1 dy \, g(y, Q^2) \sigma(\gamma^* g \to q_f\bar{q}_f, yW^2), \quad (26) \]

where \(g(y, Q^2)\) is the distribution function of the physical, transverse, gluons in the proton. By the kinematics of DIS
\[ y = \frac{(kq)}{(pq)} = \frac{(M^2 + Q^2)}{(W^2 + Q^2)}. \quad (27) \]
The diagram of Fig. 2 describes the driving term of the gluon distribution in the proton, which here is assumed to be entirely of the radiative origin ([35,37], see also [39]). At \( x \ll 1 \) we have [35,37]

\[
g(x, Q^2) = \frac{8}{x} \int_0^{Q^2} \frac{dk^2 k^2}{(k^2 + \mu_c^2)^2} V(k) \frac{\alpha_s(k^2)}{2\pi},
\]

(28)

Here it is worthwhile to emphasize, that the flux of soft gluons depends only on the color charge, but neither the spin nor the helicity, of the source of gluons. The vertex function \( V(\vec{k}) \) in the integrand of (28) is precisely the same as in eq. (11) and describes the destructive interference of the radiation of gluons by different quarks bound in the color-singlet nucleon. In terms of the mass \( M \) of the excited \( q\bar{q} \) pair, Eqs. (26),(28) give

\[
\Delta \sigma_{\text{tot}}(\gamma^* N, W^2, Q^2) \propto \int_{4m_f^2}^{W^2} \frac{dM^2}{Q^2 + M^2} \sigma(\gamma^* g \rightarrow q\bar{q}, M^2).
\]

(29)

Because of the spin-\( \frac{1}{2} \) exchange in the \( t \)-channel, the cross section of the photon-gluon fusion subprocess decreases at large \( M^2 \),

\[
\sigma(\gamma^* g \rightarrow q\bar{q}, M^2) \propto \frac{1}{Q^2 + M^2} \log \left( \frac{(Q^2 + M^2)}{4m_f^2} \right),
\]

(30)

the integral (29) converges at finite \( M^2 \) and gives the constant photoabsorption cross section at small \( x \). For the closely related reason, one finds the rapidly convergent mass spectrum (24).

On the other hand, if the \( q\bar{q}g \) final state is produced in the photon-gluon fusion, then because of the spin-1 gluon exchange in the \( t \)-channel \( \sigma(\gamma^* g \rightarrow q\bar{q}g, M^2) \propto \text{const} \), which leads to the rising \( \propto \log(1/x) \) contribution to the photoabsorption cross section [41]. Notice, that excitation of the \( q\bar{q}g \) final state in the photon-gluon fusion can be reinterpreted as the scattering on the nucleon of the \( q\bar{q}g \) Fock state of the photon. Therefore, one has to study the effect of higher Fock states of the photon.

3 Interactions of the \( q\bar{q}g \) Fock state of the photon

3.1 Interaction cross section for the \( q\bar{q}g \) state

One can easily write down the interaction cross section for the color-singlet three-parton \( q\bar{q}g \) state using only the color gauge invariance considerations (the separation of partons in the impact-parameter space is shown in Fig. 4):

\[
\sigma_3(r, R, \rho) = \frac{9}{8} \left[ \sigma(R) + \sigma(\rho) \right] - \frac{1}{8} \sigma(r).
\]

(31)
Indeed, when separation of the quark and of the antiquark is small, \( r \ll \rho \approx R \), then the \( q\bar{q} \) pair will be indistinguishable from the pointlike colour-octet charge. In this limit

\[
\sigma_3(0, \rho, \rho) = \frac{9}{4}\sigma(\rho) \quad (32)
\]

where \( 9/4 \) is the familiar ratio of the octet and triplet strong couplings. In the opposite limiting cases of \( R = 0 \) or \( \rho = 0 \) the gluon and the (anti)quark with vanishing separation are indistinguishable from the pointlike (anti)quark and

\[
\sigma_3(r, 0, r) = \sigma_3(r, r, 0) = \sigma(r) \quad (33)
\]

The formal derivation goes as follows: In the integrand of the cross section (11), the two propagators \( 1/(\vec{k}^2 + \mu_G^2) \) correspond to the Fourier transforms \( U(\vec{k}) = \int d^3\vec{r} \exp(-i\vec{k}\vec{r}) \exp(-\mu_G r)/r \) and \( U(-\vec{k}) \) of the (infrared-regulated) gluonic Coulomb potential. If the color charge is located at the position \( \vec{c} \), then \( U(\vec{k}, \vec{c}) = U(\vec{k}) \exp(-i\vec{k}\vec{c}) \). Whenever the two exchanged gluons couple to the same parton, one gets the square of the corresponding strong charge. If the gluons couple to the two partons located at points \( \vec{r}_1 \) and \( \vec{r}_2 \), the corresponding contribution acquires the extra phase factor \( \exp[i\vec{k}(\vec{r}_2 - \vec{r}_1)] \). Precisely this is the origin of the factor \( [1 - \exp(i\vec{k}\vec{r})] \) in the integrand of eq. (11). The accurate calculation of the colour traces for the different couplings of the two exchanged gluons to the quark, antiquark and gluon of the \( q\bar{q}g \) Fock state leads precisely to the cross section (31).

If \( \Phi_1(\vec{r}, \vec{R}, \vec{\rho}, z, z_g) \) is the wave function of the \( q\bar{q}g \) Fock state, then the corresponding contribution to \( \sigma_{tot}(\gamma^*p) \) shall equal

\[
\Delta\sigma_{tot}(q\bar{q}g, x, Q^2) = \int dz d^2\vec{r} d^2\vec{\rho} |\Phi_1(\vec{r}, \vec{R}, \vec{\rho}, z, z_g)|^2 \sigma(r, R, \rho) \quad (34)
\]

The gluon of the \( q\bar{q}g \) Fock state is generated radiatively from the primary \( q\bar{q} \) Fock state (Fig.5), and this radiation simultaneously renormalizes the weight of the \( q\bar{q} \) component of the photon. If \( n_g(z, \vec{r}) \) is the number of gluons in the \( q\bar{q}g \) state with the \( q\bar{q} \) separation \( \vec{r} \) (we suppress the subscripts \( T \) and \( L \) in the \( |\Psi(z, r)|^2 \)) , defined by

\[
\int dz_g d^2\vec{\rho} |\Phi_1(\vec{r}, \vec{R}, \vec{\rho}, z, z_g)|^2 = n_g(z, \vec{r}) |\Psi(z, r)|^2 , \quad (35)
\]

then the wave function of the radiationless \( q\bar{q} \) component of the photon shall renormalize as

\[
|\Psi_{q\bar{q}}(z, r)|^2 = |\Psi(z, r)|^2 [1 - n_g(z, \vec{r})] \quad (36)
\]
It is convenient to introduce

$$\Delta \sigma(r, R, \rho) = \sigma_3(r, R, \rho) - \sigma(r) = \frac{9}{8} [\sigma(R) + \sigma(\rho) - \sigma(r)] , \quad (37)$$

which shows how much the interaction cross section of the $q\bar{q}g$ Fock state is different from the cross section for the $q\bar{q}$ state. Then, $\sigma_{tot}(\gamma^* N)$ from interactions of the $q\bar{q}$ and $q\bar{q}g$ Fock states of the photon takes the form

$$\sigma_{tot}(\gamma^* N, x, Q^2) = \int dzd^2 \vec{r} |\Psi(z, r)|^2 (1 - n_g(z, \vec{r})) \sigma(r) + \int dxd^2 \vec{r} dzg \vec{\rho} |\Phi_1(\vec{r}, \vec{R}, \vec{\rho}, z, \vec{z}_g)|^2 [\sigma(r) + \Delta \sigma(r, R, \rho)]$$

$$= \int dzd^2 \vec{r} |\Psi(z, r)|^2 \sigma(r) + \int dzd^2 \vec{r} dzg \vec{\rho} |\Phi_1(\vec{r}, \vec{R}, \vec{\rho}, z, \vec{z}_g)|^2 \Delta \sigma(r, R, \rho) . \quad (38)$$

Up to now we have manipulated the formally divergent quantity $n_g(z, \vec{r})$ as if it were finite. As a matter of fact, the renormalization (36) of the radiationless $q\bar{q}$ Fock state corresponds to the introduction of the so-called regularized splitting functions [13] in the GLDAP evolution equations, and takes care of the virtual radiative corrections (a very detailed discussion of the interplay of the virtual and real radiative corrections and of the emergence of the running strong coupling was given by Dokshitzer [12], see also the review paper [40], and needs not be repeated here). Of course, the final result for the physical cross section, the last line of eq. (38), does not contain any divergences.

### 3.2 Wave function of the $q\bar{q}g$ state and the rising photoabsorption cross section

We are interested in the $\propto \log(1/x)$ component of the increase of the photoabsorption cross section

$$\Delta \sigma_{tot}^{(1)}(\gamma^* N, x, Q^2) = \int dzd^2 \vec{r} dzg d^2 \vec{\rho} |\Phi_1(\vec{r}, \vec{R}, \vec{\rho}, z, \vec{z}_g)|^2 \Delta \sigma(r, R, \rho) . \quad (39)$$

This $\log(1/x)$ comes from the $dzg/zg$ integration in the domain $x < z_g < 1$ and we must concentrate on $z_g \ll 1$. One should not confuse the Sudakov variable $z_g$ which is the fraction of the light-cone momentum of the photon carried by the gluon, with $y$, which is the fraction of the light-cone momentum of the proton carried by the same gluon. The two quantities are related by

$$z_g y = x . \quad (40)$$
Only the diagrams of Fig. 6a in which the exchanged gluons couple to the gluon of the $q\bar{q}g$ Fock state give rise to $\Delta \sigma_{\text{tot}}(\gamma^*N, x, Q^2) \propto \log(1/x)$. The corresponding wave function can be reconstructed from the number of gluons $n_g$ in the $q\bar{q}g$ state, which on the one hand equals

$$n_g = \int dz_g d^2\vec{\rho} dz d^2\vec{r} |\Phi_1(\vec{r}, \vec{R}, \vec{\rho}, z, z_g)|^2$$

and, on the other hand, can be evaluated from the Weizsäcker-Williams formula (28)

$$n_g = \frac{2}{3} \cdot \frac{8}{\pi} \int \frac{dz_g}{z_g} \int dz d^2\vec{r} |\Psi(\vec{r}, z)|^2 \cdot \int \frac{d^2k_g k_g^2}{(k_g^2 + \mu_G^2)^2} \frac{\alpha_s(k_g^2)}{2\pi} \left[1 - \exp(i\vec{k}_g \vec{r})\right].$$

Here we have used the fact that for the $q\bar{q}$ source of gluons

$$V(\vec{k}_g) = \int dz d^2\vec{r} |\Psi(\vec{r}, z)|^2 \left[1 - \exp(i\vec{k}_g \vec{r})\right],$$

and the factor $2/3$ accounts for the 2 (anti)quarks in the $q\bar{q}$ state compared to the 3 quarks in the proton. Transformation of Eq. (42) into the configuration-space integral can easily be performed making use of [24]

$$\int \frac{d^2k_k k^2}{(k^2 + \mu_G^2)^2} = \int d^2\vec{\rho} \mu_G^2 K_1(\mu_G \rho) K_1(\mu_G R) R \rho,$$

which yields

$$n_g = \frac{4}{3\pi^2} \int \frac{dz_g}{z_g} d^2\vec{\rho} \int dz d^2\vec{r} |\Psi(\vec{r}, z)|^2 \frac{\alpha_s(r) \mu_G^2 |K_1(\mu_G \rho)|}{\rho} - K_1(\mu_G R) \frac{\vec{R}}{\rho},$$

so that

$$|\Phi_1(\vec{r}, \vec{R}, \vec{\rho}, z, z_g)|^2 = \frac{1}{z_g \frac{4}{3\pi^2}} |\Psi(z, r)|^2 \frac{\alpha_s(r) \mu_G^2 |K_1(\mu_G \rho)|}{\rho} - K_1(\mu_G R) \frac{\vec{R}}{\rho},$$

The wave function (17) has the $1/z_g$ behavior needed for the $\propto \log(1/x)$ rise of the cross section. The color gauge invariance property of the wave function (17) is noteworthy: because of cancellations of the color charges of the quark and antiquark in the color singlet state it vanishes when $(R - \rho) \to 0$. It counts only physical, transverse gluons. For those reasons and because of the related color gauge invariance properties of $\sigma(r, R, \rho)$, introduction of the infrared regularization and the modelling of the confinement by the effective mass of gluons exchanged in the $t$-channel does not conflict the color gauge invariance.
In the DIS the leading contribution to $\Delta\sigma_{tot}^{(1)}(\gamma^* N, x, Q^2)$ comes from the LLA ordering of sizes
\[
\frac{1}{Q^2} \ll r^2 \ll \rho^2 \ll R_N^2, \frac{1}{\mu_G^2}, \quad (48)
\]
when (here the angular averaging is understood)
\[
\mu_G^2 |K_1(\mu_GP)\vec{P}/\rho - K_1(\mu_GR)/R|^2 = \frac{r^2}{\rho^4}, \quad (49)
\]
which produces the factorized wave function
\[
|\Phi_1(r, R, \rho)|^2 = \frac{1}{z_g} |\Psi(z, r)|^2 \frac{4}{3\pi^2} \alpha_s(r) \frac{r^2}{\rho^4}. \quad (50)
\]
Naturally, the LLA wave function (50) does not depend on the infrared regularization parameter $\mu_G$. In the LLA
\[
\Delta\sigma(r, R, \rho) = \Sigma(\rho) = \frac{9}{4} \sigma(\rho) \quad (51)
\]
and the increase of the total cross section can be written as
\[
\Delta\sigma_{tot}(\gamma^* N, x, Q^2) = \int dz \frac{d^2T}{2} |\Psi(z, r)|^2 \alpha_s(r) r^2 \frac{4}{3\pi} \int_{z_g}^z \frac{dz_g}{z_g} \int_{r^2}^{\rho^2} \frac{d\rho^2}{\rho^4} \Sigma(\rho)
\]
\[
= \int dz \frac{d^2T}{2} |\Psi(z, r)|^2 \alpha_s(r) C_{N_f}^2 \frac{4}{3\pi} \log \left(\frac{z}{x}\right) \int_{r^2}^{\rho^2} \frac{d\rho^2}{\rho^4} 2^2 \alpha_s(\rho) L(\rho)
\]
\[
= \int dz \frac{d^2T}{2} |\Psi(z, r)|^2 \sigma(r) \frac{12}{\beta_0} \cdot \frac{1}{2!} L(r) \cdot \frac{1}{1!} \log \left(\frac{z}{x}\right) \propto \frac{1}{Q^2} L(Q) \log \left(\frac{1}{x}\right). \quad (52)
\]
Here we have made an explicit use of the small-$r$ behaviour of $\sigma(r)$, Eq. (14). This is the first instance when we encounter the expansion parameter of the Double-Leading-Logarithm Approximation (DLLA) [12,40,42]
\[
\xi(x, r) = \frac{12}{\beta_0} L(r) \log \left(\frac{1}{x}\right). \quad (53)
\]
We have one power of $L(Q)$ per QCD loop (which à posteriori justifies LLA ordering (48)) and one power of $\log(1/x)$ per gluon in the Fock state of the photon.

### 3.3 The triple-pomeron asymptotics of the mass spectrum of diffraction dissociation

Our starting point is the generic formula (9). Repeating the considerations of Section 3.2, we can write
\[
16\pi \frac{d\sigma_D(\gamma^* \rightarrow q\bar{q} + q\bar{q})}{dt} \bigg|_{t=0} = 16\pi \int dM^2 \frac{d\sigma_D(\gamma^* \rightarrow q\bar{q} + q\bar{q})}{dt dM^2} \bigg|_{t=0} =
\]
\[ \int dzd^2\vec{r} |\Psi(z, r)|^2[1 - n_g(\vec{r})]\sigma(r)^2 + \int dzd^2\vec{r}dz_gd^2\vec{p} |\Phi_1(\vec{r}, \vec{R}, \vec{p}, z, z_g)|^2[\sigma(r) + \Delta\sigma(r, R, \rho)]^2 = \int dzd^2\vec{r} |\Psi(z, r)|^2\sigma(r)^2 + \int dzd^2\vec{r}dz_gd^2\vec{p} |\Phi_1(\vec{r}, \vec{R}, \vec{p}, z, z_g)|^2[\Delta\sigma(r, R, \rho)^2 + 2\sigma(r)\Delta\sigma(r, R, \rho)]. \]  

The first term in the last line of Eq. (54) describes the diffraction excitation of the $q\bar{q}$ Fock states into the low masses $M^2 \sim Q^2$, see Eq. (24). The second term gives rise to the $1/M^2$ mass spectrum, which can be seen as follows: The invariant mass squared of the $q\bar{q}g$ state equals

\[ M^2 = \frac{m_f^2 + k_g^2}{z} + \frac{m_f^2 + k_q^2}{1 - z - z_g} + \frac{k_g^2}{z_g}, \]  

(55)

Anticipating the final results, we note that the leading contribution to the diffraction dissociation cross section comes from the slightly modified LLA ordering

\[ \frac{1}{Q^2} \ll r^2 \ll \rho^2 \sim R_N^2, \frac{1}{\mu_G}, \]  

(56)

i.e., from $Q^2 \gg k_q^2, k_q^2 \gg k_g^2 \sim \mu_G^2$. Therefore, the excitation of masses $M^2 \ll Q^2$ only comes from $z_g \ll z < 1$, and the $dz_g$ integration in (54) can easily be transformed into the $dM^2$ integration (see Eqs. (27, 40)):

\[ \frac{dM^2}{M^2 + Q^2} = \frac{dy}{y} = \frac{dz_g}{z_g}, \]  

(57)

where now $y$ is the fraction of proton’s momentum carried by the pomeron. The wave function (47) has precisely the needed $\propto 1/z_g$ behaviour and leads to (in view of (56) the term $\propto \sigma(r)\Sigma(r, R, \rho)$ in (54) can be neglected)

\[ \left. \frac{d\sigma_D}{dt dM^2} \right|_{t=0} = \frac{1}{M^2 + Q^2} \int dz d^2\vec{r} |\Psi(z, r)|^2\alpha_S(r)r^2C_N \]  

\[ \cdot \frac{1}{C_N} \cdot \frac{1}{16\pi} \cdot \frac{4}{3\pi} \cdot \int_{r^2}^{\infty} dp^2 \left[ \frac{\Sigma(p)}{p^2} \right]^2 F(\mu_Gp), \]  

(58)

where $F(\mu_Gp)$ is defined by the slight generalization of (49) to allow for $\mu_Gp \sim 1$:

\[ \mu_G^2|K_1(\mu_Gp)\frac{\vec{p}}{\rho} - K_1(\mu_GR)\frac{\vec{R}}{R}|^2 = \frac{r^2}{\rho^4}F(\mu_Gp). \]  

(59)

The form factor $F(x)$ satisfies $F(0) = 1$ and $F(x) \propto \exp(-2x)$ at $x > 1$.

Firstly, we notice that the diffraction dissociation cross section depends on the infrared regularization, since the $\rho$ integration in (58) is essentially flat and is dominated by large $\rho \sim R_N, 1/\mu_G$. Then can take $\rho^2 = 0$ for the lower limit of integration, and $d\sigma_D/dt dM^2$ Eq. (58) factorizes into the $Q^2$-independent dimensional constant

\[ A_{3\Phi}^* = \frac{1}{C_N} \cdot \frac{1}{12\pi^2} \cdot \left( \frac{9}{4} \right)^2 \int dp^2 \left[ \frac{\sigma(p)}{p^2} \right]^2 F(\mu_Gp) \]  

(60)
and the cross section
\[ \sigma^*(Q^2) = \int dz d^2\vec{r} |\Psi(z, r)|^2 C_N r^2 \alpha_S(r), \tag{61} \]
which is nearly identical to \( \sigma_T(\gamma^* N, x, Q^2) \) Eq. (17), being short of \( L(r) \) in the integrand. Therefore, this driving term of the triple-pomeron component of the diffraction dissociation of virtual photons satisfies the approximate factorization reminiscent of factorization properties of the triple-pomeron diagram of the conventional Regge theory Fig. 7a,
\[ \frac{M^2 + Q^2}{\sigma_T(\gamma^* N, x, Q^2)} \cdot \frac{d\sigma_D(\gamma^* N \rightarrow X + N)}{dt dM^2} \bigg|_{t=0} = A_{3\text{IP}}(Q^2) = \frac{\sigma^*(Q^2)}{\sigma_T(\gamma^* N, x, Q^2)} A_{3\text{IP}}^* \tag{62} \]
To the considered lowest order in the perturbation theory, the quantity \( A_{3\text{IP}}^*(Q^2) \) does not depend on \( x \). \( A_{3\text{IP}}^*(Q^2) \) is the dimensionfull quantity, and as such it could have had a strong \( Q^2 \)-dependence, \( A_{3\text{IP}}^*(Q^2) \sim 1/Q^2 \) being a plausible guess if \( 1/\sqrt{Q^2} \) were the only scale relevant to deep inelastic scattering. The fact that \( \sigma^*(Q^2) \) and \( \sigma_{tot}(\gamma^* N, x, Q^2) \) are nearly identical, proves that this is not the case. Furthermore, the r.h.s of Eq. (62) has a very smooth extrapolation down to the real photoproduction limit \( Q^2 = 0 \), confirming the earlier suggestions [26,7] that \( A_{3\text{IP}}^*(Q^2) \) is close to \( A_{3\text{IP}}^*(0) \) as measured in the real photoproduction (the more detailed comparison of \( A_{3\text{IP}}^*(Q^2) \) with the \( A_{3\text{IP}}^* \) for the diffraction dissociation of protons, pions and the real photons will be presented elsewhere [43]). For the order of magnitude estimation of \( A_{3\text{IP}}^* \), in the dominant region of integration in (64) we can take \( F(\mu_G p) \sim \exp(-2\mu_G p) \), \( L(p) \sim 1 \) and \( \sigma(p)/p^2 \sim C_N \alpha_S(p = \frac{1}{2\mu_G}) \) with the result
\[ A_{3\text{IP}}^* \sim 27C_N \frac{\alpha_S(4\mu_G^2)^2}{256\pi^2} = \frac{\alpha_S(4\mu_G^2)^2}{8\mu_G^2}. \tag{63} \]
With \( \mu_G \sim 0.4 GeV \) this gives \( A_{3\text{IP}}^* \sim 0.3 \text{ (GeV)}^{-2} \). The experimental data on the diffraction dissociation of the real photons give \( A_{3\text{IP}}(0) \approx 0.16 \text{ (GeV)}^{-2} \) [27]. This dimensional coupling \( A_{3\text{IP}}^* \approx A_{3\text{IP}}^*(0) \) emerges as the principal normalization factor of the diffraction dissociation cross section, and Eq. (62) is a starting point of the derivation of the factorization representation [89] to all orders in the perturbation theory.

Combining Eq. (62) with the definition Eq. (23) we find the corresponding contribution to the structure function of the pomeron at \( x = Q^2/(Q^2 + M^2) \ll 1 \) of the form [7]
\[ F_2^{(\text{IP})}(x, Q^2) = \frac{4Q^2}{\pi \alpha_{em} \sigma_{tot}(pp)} \sigma^*(x, Q^2) A_{3\text{IP}}^* \approx \frac{16\pi A_{3\text{IP}}^*(0)}{\sigma_{tot}(pp)} F_2^{(N)}(x, Q^2) \approx 0.08 F_2^{(N)}(x, Q^2). \tag{64} \]
It describes DIS on the \( q\bar{q} \) 'sea' of the pomeron. The relationship (64) shows a deep connection between the triple-pomeron component of the mass spectrum and the sea structure function of
the pomeron. Notice the difference between the diffractive excitation of the $q\bar{q}$ state, Fig. 3, and of the $q\bar{q}g$ state, Fig. 7b: in the former the pomeron couples to (anti)quarks and the DIS probes the 'valence' $q\bar{q}$ structure of the pomeron, in the latter the pomeron couples to gluons, and the DIS probes the 'sea' of the pomeron, which can be treated as having been generated from the 'valence' (constituent) gluons of the pomeron. The diffraction dissociation cross section (58) and the pomeron structure function (64) are sensitive to the infrared regularization, and the normalization of both quantities contains the new dimensional parameter $A_{3I}^*$. The important result of the above analysis is that this dimensional parameter $A_{3I}^*$ can approximately be inferred from the real photoproduction data. Notice a close similarity between Eq. (60) for the normalization of the triple-pomeron mass spectrum and Eq. (21) for the normalization of the mass spectrum for the excitation of the $q\bar{q}$ state. However, whereas Eqs. (60,64) predict the flavor-independent relation between the proton and pomeron structure functions, the valence $q\bar{q}$ structure function of the pomeron has a strong flavor dependence [7]. Now we shall study how these conclusions shall change when higher order effects and QCD evolution are included.

4 Rising structure functions and higher order Fock states of the photon

Generalization of an analysis of Section 3 to interactions of the higher $q\bar{q}g_1...g_n$ Fock states of the photon is straightforward. The strong DLLA ordering of gluons

$$x \ll z_n \ll z_{n-1} \ll ... \ll z_1 \ll z < 1,$$

$$\frac{1}{Q^2} \ll r^2 \ll \rho_1^2 \ll .... \ll \rho_n^2 \ll R_c^2,$$

is required to have maximal possible powers of $\log(1/x)$ and $L(Q)$ [42,12]. To the DLLA the quark-loop insertions in the generalized ladder diagrams of Fig. 8 can be neglected. By virtue of the ordering of sizes (66) the $q\bar{q}g_1...g_n$ Fock state interacts like the colour-singlet octet-octet state, with the inner subsystem $q\bar{q}g_1...g_{n-1}$ acting like the pointlike colour-octet charge. Henceforth, the generalization of Eq. (51) is

$$\Delta\sigma(r, \rho_1, ...., \rho_n) = \Sigma(\rho_n) = \frac{9}{4}\sigma(\rho_n).$$

20
The DLLA wave function is a straightforward generalization of the wave function (50) for the \( qg \) Fock state:

\[
\left| \Psi(r, \rho_1, ..., \rho_n) \right|^2 = \left| \Psi(z, r) \right|^2 \cdot \frac{1}{z_1} \alpha_S(r) \frac{4}{3 \pi^2} \frac{r^2}{\rho_1^2} \cdot \frac{1}{z_2} \alpha_S(\rho_1) \frac{3}{\pi^2} \frac{\rho_1^2}{\rho_2^2} \cdot ... \cdot \frac{1}{z_n} \frac{3}{\pi^2} \alpha_S(\rho_{n-1}) \frac{\rho_{n-1}^2}{\rho_n^4}. \tag{68}
\]

Notice, that the first gluon is radiated by the triplet-antitriplet colour-singlet state. The subsequent gluons are radiated by the octet-octet colour-singlet state, which brings in the ratio \( 9/4 \) of the octet and triplet strong couplings. The corresponding increase of the total cross section equals (here we make an explicit use of eq. (14))

\[
\Delta \sigma_{tot}^{(n)}(\gamma^* N, x, Q^2) = \frac{4}{3\pi} \cdot \left( \frac{3}{\pi} \right)^{n-1} \int dz \, d^2r |\Psi(z, r)|^2 \alpha_S(r) r^2 \\
\cdot \int \frac{d\rho_1^2}{\rho_1^2} \frac{\alpha_S(\rho_1)}{\rho_1^2} \left( \int \frac{d\rho_2^2}{\rho_2^2} \frac{\alpha_S(\rho_2)}{\rho_2^2} \right) ... \left( \int \frac{d\rho_{n-1}^2}{\rho_{n-1}^2} \frac{\alpha_S(\rho_{n-1})}{\rho_{n-1}^2} \right) \cdot \int \frac{dz_1}{z_1} \frac{dz_2}{z_2} ... \int \frac{dz_n}{z_n} \\
= \left( \frac{3}{\pi} \right)^n \int dz \, d^2r |\Psi(z, r)|^2 \alpha_S(r) C_N r^2 \\
\cdot \int \frac{d\rho_1^2}{\rho_1^2} \frac{\alpha_S(\rho_1)}{\rho_1^2} \left( \int \frac{d\rho_2^2}{\rho_2^2} \frac{\alpha_S(\rho_2)}{\rho_2^2} \right) ... \left( \int \frac{d\rho_{n-1}^2}{\rho_{n-1}^2} \frac{\alpha_S(\rho_{n-1})}{\rho_{n-1}^2} \right) \cdot \int \frac{dz_1}{z_1} \frac{dz_2}{z_2} ... \int \frac{dz_n}{z_n} \\
= \int dz \, d^2r |\Psi(z, r)|^2 \alpha_S(r) C_N r^2 \left( \frac{12}{\beta_0} \right)^n \frac{1}{(n+1)!} \log \left( \frac{z}{x} \right)^n \\
= \int dz \, d^2r |\Psi(z, r)|^2 \sigma(r) \frac{\xi(x/z, r)^n}{(n+1)!n!}. \tag{69}
\]

Therefore, the total photoabsorption cross section can be represented as

\[
\sigma_{tot}(x, Q^2) = \int dz \, d^2r |\Psi(z, r)|^2 \sigma \left( \frac{x}{z}, r \right). \tag{70}
\]

where

\[
\sigma(x, r) = \sigma(r) \sum_{n=0}^{\infty} \frac{\xi(x, r)^n}{(n+1)!n!}, \tag{71}
\]

is the energy-dependent dipole cross section, which generalizes the Low-Nussinov pomeron to the DLLA pomeron.

The sum in (71) can be evaluated as (we neglect the slowly varying pre-exponential factor)

\[
\sum_{n=0}^{\infty} \frac{\xi^n}{(n+1)!n!} = \frac{\partial}{\partial \xi} \sum_{n=1}^{\infty} \frac{\xi^n}{(n!)^2} \propto \exp(2\sqrt{\xi}), \tag{72}
\]

leading to

\[
\sigma(x, r) \propto \sigma(r) \exp \sqrt{\frac{48}{\beta_0} \log \left[ \frac{1}{\alpha_S(r)} \right] \log \left( \frac{1}{x} \right)} \tag{73}
\]
$$F_2(\text{GLDAP}, x, Q^2) = \frac{Q^2}{4\pi\alpha_{em}} \sigma_{tot}(x, Q^2) \propto \exp \left[ \frac{48}{\beta_0} \log \left( \frac{1}{\alpha_s(Q^2)} \right) \log \left( \frac{1}{x} \right) \right].$$  \hfill (74)

The representation (71) for the photoabsorption cross section in terms of the DLLA dipole cross section (71) is a new result and is presented here for the first time. However, since the perturbative expansion (69) is completely equivalent to the GLDAP evolution equation for the structure function [11,12], the result (74) for the DLLA growth of the structure function is identical to the one derived from the GLDAP evolution equations [12,40], where it appeared as a rising density $g(\text{GLDAP}, x, Q^2)$ of gluons in the proton. In our light-cone s-channel approach it comes from interactions of the higher $q\bar{q}g_1...g_n$ Fock states of the photon and can be described in terms of the rising DLLA pomeron cross section (71) for the color dipole. As a matter of fact, it can easily be shown that in the DLLA [30]

$$\sigma(x, r) = \frac{\pi^2}{3} r^2 \alpha_s(r) x g(x, Q^2 = 1/r^2).$$  \hfill (75)

We note in passing, that the cross section (71,73,75) obviously does not satisfy the factorization relations usually assumed for the pomeron in the standard Regge phenomenology. The QCD evolution analysis of deep inelastic scattering needs not to assume any factorization of $F_2(x, Q^2)$.

5 The structure function of the pomeron and constituent gluons of the pomeron

Now we consider diffraction excitation of the higher order $q\bar{q}g_1...g_{n+2}$ Fock states of the photon (Fig. 7c). Large masses $M$ of the excited state

$$M^2 = \frac{m^2_f + k^2_q}{z_q} + \frac{m^2_f + k^2_{\bar{q}}}{z_{\bar{q}}} + \sum_{i=1}^{n+2} \frac{k^2_i}{z_i}$$  \hfill (76)

will be dominated by the softest gluon contribution. The fraction $y$ of proton’s momentum carried away by the pomeron is related to $z_{n+2}$ as (cf. Eq. (40))

$$z_{n+2} = \frac{x}{y}$$  \hfill (77)

and

$$\frac{dM^2}{M^2 + Q^2} = \frac{dy}{y} = \frac{dz_{n+2}}{z_{n+2}}.$$  \hfill (78)
Using the wave function (88), and repeating the considerations which have lead to eq. (58), we find the contribution of excitation of the \( qg \) Fock state to the mass spectrum of the diffraction dissociation and to the photon-pomeron cross section

\[
\Delta \sigma^{(n+2)}_{\text{tot}}(\gamma^* \mathbf{P}, Q^2, M^2) = \frac{16\pi}{\sigma_{\text{tot}}(pp)} (M^2 + Q^2) \left. \frac{d\sigma_D(\gamma^* \rightarrow q\bar{q}g_1...g_{n+2})}{dtdM^2} \right|_{t=0}
\]

\[
= \frac{1}{\sigma_{\text{tot}}(pp)} \cdot \left( \frac{4}{3\pi} \right) \cdot \left( \frac{3}{\pi} \right)^{n+1} \int dz \, d^2\vec{r} |\Psi(z, r)|^2 \alpha_S(r) r^2 
\cdot \int_{\rho_{n-1}^2}^{\rho_n^2} \frac{d\rho_{n+1}^2}{\rho_{n+1}^2} \alpha_S(\rho_{n+1}) \int_{\rho_{n+1}^2}^{\rho_{n+2}^2} \frac{d\rho_{n+2}^2}{\rho_{n+2}^2} \sum(\rho_{n+2})^2 F(\mu G \rho_{n+2}) 
\cdot \frac{1}{\rho_n^2} \alpha_S(\rho_n) \cdot \frac{81}{8\pi^4} \cdot \int \frac{d\rho_{n+1}^2}{\rho_{n+1}^2} \alpha_S(\rho_{n+1}) \cdot \int \frac{d\rho_{n+2}^2}{\rho_{n+2}^2} \frac{d\rho_{n+2}^2}{\rho_{n+2}^2} \alpha_S(\rho_{n+2})^2 F(\mu G \rho_{n+2}) 
\cdot \frac{\rho_n^2 \alpha_S(\rho_n)}{\rho_n^4} \cdot \frac{81}{8\pi^4} \cdot \int \frac{d\rho_{n+1}^2}{\rho_{n+1}^2} \alpha_S(\rho_{n+1}) \cdot \int \frac{d\rho_{n+2}^2}{\rho_{n+2}^2} \frac{d\rho_{n+2}^2}{\rho_{n+2}^2} \alpha_S(\rho_{n+2})^2 F(\mu G \rho_{n+2}) .
\]

(79)

Comparison with Eqs. (11), (68) shows that the last line of Eq. (79) can be reinterpreted as the dipole cross section for interaction with the pomeron treated as the two-gluon state with the wave function

\[
|\Psi_{\mathbf{P}}(z_g, \vec{r})|^2 = \frac{81}{8\pi^4} \cdot \frac{1}{\rho_n^2} \cdot \frac{1}{\sigma_{\text{tot}}(pp)} \left[ \frac{\sigma(r)}{r^2} \right]^2 F(\mu G r) ,
\]

(80)

where

\[
z_g = \frac{z_{n+2}}{z_{n+1}}
\]

(81)
is the fraction of pomeron’s momentum carried by the gluon.

Indeed, making use of Eq. (13) for the vertex function of the two-body system, the dipole cross section \( \sigma_{2g}(\rho) \) for the scattering on the gluon-gluon state can be written as

\[
\sigma_{2g}(\rho) = 2\pi \rho^2 \alpha_S(\rho) \int dz_g \int d^2\vec{r} |\Psi_{2g}(z_g, r)|^2 \int_0^{1/\rho^2} \frac{dk^2}{(k^2 + \mu_G^2)^2} \alpha_S(k^2) [1 - \exp(ik\vec{r})] 
\]

(82)
The factor 3/2 difference from Eqs. (11,14) is due to the ratio 9/4 of the gluon (octet) and the quark (triplet) strong couplings and the ratio 2/3 of the number of the constituents gluons in the pomeron and the number of the constituent quarks in the proton. The series of transformations of the integrand of (82),

\[
\sigma_{2g}(\rho) = 2\pi \rho^2 \alpha_S(\rho) \int dz_g \int d^2\vec{r} |\Psi_{2g}(z_g, r)|^2 \int_0^{1/\rho^2} \frac{dk^2}{k^2} \alpha_S(k^2) 
\]

(82)

\[
= 2\pi \rho^2 \alpha_S(\rho) \int dz_g \int d^2\vec{r} |\Psi_{2g}(z_g, r)|^2 \int_{\rho_1^2}^{r^2} \frac{d\rho_1^2}{\rho_1^2} \alpha_S(\rho_1)
\]

(82)
\[ = 2\pi \rho^2 \alpha_s(\rho) \int_{\rho^2} d\rho_1^2 \alpha_s(\rho_1) \int dz_g \int d^2\vec{r} |\Psi_{2g}(z_g, r)|^2 , \]  
and comparison with the last line of Eq. (79) complete the derivation of the wave function (80). This is one of the central results of the present paper.

The wave function \( \Psi_{2g}(z_g, r) \) gives the distribution of the 'valence' gluons \( g_{\text{IP}}(z_g) \) in the pomeron

\[ g_{\text{IP}}(z_G) = \int d^2\vec{r} |\Psi_{2g}(z_g, r)|^2 . \]  
The perturbative expansion (79) describes the QCD evolution of this 'valence' gluon distribution. The above derivation holds at \( z_g \ll 1 \), the region of \( z_g \sim 1 \) requires special consideration. Only \( z_g \ll 1 \) is important in the DLLA. The radius of the pomeron \( R_{\text{IP}} \sim R_N, 1/\mu_G \) and is controlled by both the form factor \( F(\mu_G r) \) and the behavior of \( \sigma(r) \) in the saturation regime. The absolute normalization of the flux of soft gluons in the pomeron is given by the familiar coupling \( A_3^{\ast \text{IP}} \):

\[ z_g g_{\text{IP}}(z_g) = \frac{81}{8\pi^3} \cdot \frac{1}{\sigma_{\text{tot}}(pp)} \int dr^2 \left[ \frac{\sigma(r)}{r^2} \right]^2 F(\mu_G r) = \frac{128\pi}{9} \cdot \frac{A_3^{\ast \text{IP}}}{\sigma_{\text{tot}}(pp)} \sim 0.06 . \]  
Extrapolation of (85) also to large \( z_g \) gives an estimate of the gluon momentum integral for the pomeron

\[ (x_g)_{\text{IP}} = \int_0^1 dx x g_{\text{IP}}(x) \approx \frac{128\pi}{9} \cdot \frac{A_3^{\ast \text{IP}}}{\sigma_{\text{tot}}(pp)} \sim 0.06 . \]  
The momentum integral for the 'valence' (anti)quarks of the pomeron, discussed in Section 2.4, was estimated in Ref. [7] with the result \( (x_v(q\bar{q}))_{\text{IP}} \sim 0.1 \). Eq. (64) gives a few per cent estimate for the momentum integral for the sea (anti)quarks. Here we just emphasize that the pomeron needs not be regarded as a particle and on the purely theoretical grounds there are no reasons why the momentum integrals for gluons and (anti)quarks in the pomeron must add to 100\% [5,7].

Henceforth, we have identified the three components of the input for the QCD evolution of the pomeron structure function: i) the valence quark-antiquark component with the structure function (24) (for the detailed analysis see Ref. [7]), ii) the valence gluon distribution with the structure function (85), iii) the sea (anti)quark distribution given by Eq. (64). All these input parton distributions are sensitive to the infrared regularization. There is nothing wrong with this sensitivity: the infrared sensitivity of the parton distributions is inherent to the QCD-improved parton model. In the conventional parton model phenomenology it is hidden in the parametrization of the parton densities at small factorization scale, which then is used as an
input in the QCD evolution analysis of the scaling violations. The important finding is that the absolute normalization of the sea and gluon distributions in the pomeron is determined by one and the same flavor-independent dimensional constant $A_{3IP}^*$ which must approximately be equal to the triple-pomeron coupling as measured in the real photoproduction. The normalization of the valence $q\bar{q}$ structure function of the pomeron is given by the very similar but flavor-dependent dimensional constant (cf. equations (21) and (58,60)). In the above DLLA analysis we omitted the quark loops in the ladder diagrams for the pomerons. These quark loops shall automatically be included in the GLDAP-evolution calculation of the structure function of the pomeron starting with the above described input parton distributions in the pomeron.

6 Flux of the QCD pomerons in the proton

To complete our analysis we must replace the Low-Nussinov two-gluon pomeron in the lower part of diagrams of Fig. 7c by the full QCD pomeron - the sum of the triple-ladder diagrams of Fig. 7d. This is done by replacing the dipole cross section $\sigma(\rho)$ by $\sigma(y,\rho)$ in the last line of Eq. (79), where $y$ is a fraction of the proton’s momentum carried by the pomeron. Then, the so calculated diffraction dissociation cross section will differ from that of Section 5 only by the $y$ dependent factor

$$f_{IP}(y) = \frac{\int d\rho^2 \left[\frac{\sigma(y,\rho)}{\rho^2}\right]^2 \mathcal{F}(\mu G\rho)}{\int d\rho^2 \left[\frac{\sigma(\rho)}{\rho^2}\right]^2 \mathcal{F}(\mu G\rho)}.$$  (87)

What is the proper interpretation of $f_{IP}(y)$?

We would like to preserve the most important result of the above analysis - the representation of the diffraction dissociation cross section through the GLDAP-evolving structure function of the pomeron. The scaling variable of the photon-pomeron scattering (8) equals $x_{IP} = Q^2/(Q^2 + M^2) = x/y$, so that

$$\frac{dM^2}{M^2 + Q^2} = \frac{dy}{y}.$$  (88)

With allowance for the factor $f_{IP}(y)$ Eq. (87) for the diffraction dissociation cross section can be written down in the factorized form

$$\left.\frac{d\sigma_D}{dt dy}\right|_{t=0} = \frac{\sigma_{tot}(pp)}{16\pi} \cdot \frac{4\pi\alpha_{em}}{Q^2} \cdot \frac{f_{IP}(y)}{y} \cdot F_2^{(IP)}\left(\frac{x}{y}, Q^2\right),$$  (89)

which has the conventional parton model representation with $f_{IP}(y)/y$ being the flux of pomerons treated as partons of the proton. In order not to introduce any spurious dependence on the
kinematical variables $x$ and $y$, the coefficient $\sigma_{\text{tot}}(pp)/16\pi$ in (89) must be taken constant, for instance fixing $\sigma_{\text{tot}}(pp) = 40 \text{ mb}$. Because the pomeron is not the particle with the well defined couplings (residues) and spin, and because in QCD the pomeron does not factorize, the regge-theory convention (1) is not unique. The coefficient $\sigma_{\text{tot}}(pp)/16\pi$ is the convention-dependent normalization constant for the correct dimensionality of the diffraction dissociation cross section in terms of the dimensionless structure function or vice versa. The absolute normalizations of the flux of pomerons and of the pomeron structure function too are the convention dependent ones: it is always the product of the two quantities which enters the experimentally observable cross sections. Eq. (89) shows how the QCD evolution effects and the rising dipole cross section $\sigma(x, \rho)$ modify the $1/M^2$ law (4) for the mass spectrum in the triple-pomeron region. The factorization (89) bears certain semblance of the usual Regge-theory factorization in the triple-Regge region. We emphasize that we have derived (89) neither assuming nor using any of the Regge-theory factorization relations.

The three pomerons in the triple-pomeron diagram are described by different QCD ladder diagrams. The top pomeron of Fig. 7d is in the LLA regime: the relevant sizes vary along the ladder from $\rho_{n+2}^2 \sim R_{\text{IP}}^2$ in the bottom cell of the ladder down to $r^2 \sim 1/Q^2$ in the top, quark-antiquark, cell of the ladder. For this reason we can introduce the structure function of the pomeron. By contrast, the exchanged pomerons in the lower part of Fig. 7d are the soft pomerons: since $\rho_{n+2}^2 \sim R_{\text{IP}}^2 \sim R_N^2$, the situation is reminiscent of the pomeron contribution to the typical hadronic cross section, see eq. (14). The predictive power of QCD for the hadronic total cross section is still very limited [44,19,17,45]. (We shall comment more on Lipatov’s work on the QCD pomeron [29] below.) The empirical observation is that the hadronic cross sections and the real photoabsorption cross section [46] have a very weak dependence on energy $\nu \sim m_N/x$, much weaker compared to a steep rise of the DIS structure functions with $1/x$. The dipole cross section (71) is consistent with this property: it is essentially flat vs. $1/x$ at large, hadronic, size $r$, and the smaller is the size $r$, the steeper is the rise of $\sigma(x, r)$ with $1/x$.

This rise has a certain impact on the radius of the pomeron. Namely, the replacement of $\sigma(r)$ by $\sigma(x, r)$ leads to the effective wave function of the pomeron

$$|\Psi_{\text{IP}}(y, z_G, r)|^2 = \frac{81}{8\pi^4} \cdot \frac{1}{z_G} \cdot \frac{1}{\sigma_{\text{tot}}(pp)} \left[ \frac{\sigma(y, r)}{r^2} \right]^2 \mathcal{F}(\mu_G r).$$

With the rising dipole cross section $\sigma(y, r)$ Eq. (4.7) the ratio $\sigma(y, r)/r^2$ will rise towards small
so that the effective radius of the pomeron \( R_{IP}(y) \) will decrease as \( y \to 0 \). The GLDAP evolution equations hold at \( R_i^2 Q^2 \gg 1 \), where \( R_i \) are the relevant hadronic radii. If one would like to formulate the input for the GLDAP evolution at certain fixed \( Q_0^2 \) then, because of the decreasing radius of the pomeron, the GLDAP applicability condition will be violated at very small values of \( y \approx y_c(Q_0^2) \) such that

\[
R_{IP}(y_c(Q^2)) Q^2 = 1. \tag{91}
\]

The factorization of the diffraction dissociation cross section into the flux of pomerons and the structure function of pomerons Eq. (89) will break down at \( y < y_c(Q^2) \). Our criterion (91) for the breaking of the GLDAP evolution is different from GLR criterion ([19], for the review and references see [22]) and its implications will be studied in the subsequent publications.

A brief comment on the target dependence is in order. In the Regge theory, in the approximation of exchange by the single factorizing pomeron,

\[
\frac{1}{\sigma_{tot}(pp)} \left. \frac{d\sigma_D(\gamma^* + p \to X + p)}{dtdM^2} \right|_{t=0} = \frac{1}{\sigma_{tot}(\pi\pi)} \left. \frac{d\sigma_D(\gamma^* + \pi \to X + \pi)}{dtdM^2} \right|_{t=0}. \tag{92}
\]

In our \( s \)-channel approach the target-dependence enters through the target-dependent dipole cross section (11), which is roughly proportional to the number of quarks in in the target hadron \( b \) (To the extent that the quark-quark separation in the proton and the antiquark-quark separation in the pion are similar, the Low-Nussinov cross section (11) reproduces the additive quark model [33,14,15]),):

\[
\frac{\sigma((q\bar{q})b, \rho)}{\sigma((q\bar{q})p, \rho)} \approx \frac{\sigma_{tot}(bp)}{\sigma_{tot}(pp)} \approx \frac{\sigma_{tot}(b\pi)}{\sigma_{tot}(p\pi)}, \tag{93}
\]

so that despite the lack of factorization, the QCD pomeron roughly satisfies the relation (92).

7 Unitarization of the rising structure functions

7.1 Rising cross sections and the \( s \)-channel unitarity

The rising cross section \( \sigma(x, r) \) Eq. (71) conflicts the \( s \)-channel unitarity at sufficiently large \( 1/x \). The \( s \)-channel unitarity constraint is best formulated in the impact-parameter representation (the partial-wave expansion) and reads

\[
\Gamma(b) \leq 1. \tag{94}
\]
The profile function $\Gamma(b)$ is related to the elastic scattering amplitude $f(q)$ such that
\[
f(q) = 2i \int d^2b \Gamma(b) \exp(-i\vec{q}\vec{b}) = i\sigma_{tot} \exp\left(-\frac{1}{2}B_{el}q^2\right).
\] (95)
Here $B_{el}$ is the diffraction slope of the elastic scattering. The Gaussian parametrization (93) is viable for the purposes of the present discussion [47] and gives
\[
\Gamma(b) = \frac{\sigma_{tot}}{4\pi B_{el}} \exp\left(-\frac{b^2}{2B_{el}}\right).
\] (96)
The profile function of the bare pomeron exchange $\Gamma_0(b)$ defined for the rising cross section (71) will overshoot the $s$-channel unitarity bound at a sufficiently small $x$.

The are no unique prescriptions how to impose the $s$-channel unitarity constraint on the rising cross sections. The often used procedures are the eikonal [48,49]
\[
\Gamma(b) = 1 - \exp[-\Gamma_0(b)] = \sum_{\nu=1} \frac{(-1)^{\nu-1}}{\nu!} \Gamma_0(b)^\nu
\] (97)
and the $K$-matrix [50,44]
\[
\Gamma(b) = \frac{1}{1 + \Gamma_0(b)} = \sum_{\nu=1} (-1)^{\nu-1} \Gamma_0(b)^\nu
\] (98)
s-channel unitarizations. Both produce $\Gamma(b)$ which satisfies the unitarity bound (94). To the leading order in the $s$-channel unitarization, the unitarized profile function reads
\[
\Gamma(b) \approx \Gamma_0(b) - \frac{1}{2} \chi \Gamma_0(b)^2
\] (99)
with $\chi = 1$ for the eikonal unitarization, and $\chi = 2$ for the $K$-matrix unitarization. The eikonal unitarization is being routinely used in high-energy physics [51] and sums the $s$-channel iterations of the bare pomeron exchange (Fig. 9a) when only the elastic scattering intermediate states are included in the $s$-channel. Besides the elastic scattering states as in Fig. 9a, one must include the inelastic intermediate states of Fig. 9b, which correspond to the diffraction dissociation of the target nucleon. These inelastic intermediate states lead to an enhancement of the double and higher order rescattering terms in expansions (97,98) [52,49,53]. If one starts with the eikonal unitarization (which is an assumption), and includes the corrections for the diffraction dissociation of the target nucleons, then [53]
\[
\chi \approx 1 + \frac{\sigma_{D}(p \rightarrow X)}{\sigma_{el}(pp)} \sim 1.5.
\] (100)
In fact, the $K$-matrix prescription (98) was obtained in [44] starting with the eikonal unitarization of $\pi N$ scattering and including the inelastic intermediate states in the QCD inspired model of the diffraction dissociation of pions.
7.2 Shadowing correction to the proton structure function

Now we shall discuss the unitarization (shadowing, absorption) effects in DIS, taking full advantage of the diagonalization of the $S$-matrix in the $(\vec{\rho}, z)$-representation, which allows us to impose the $s$-channel unitarization on all the multiparton cross sections $\sigma_n(\vec{r}, \vec{\rho}_1, ..., \vec{\rho}_n)$ at all the values of $\vec{r}$ and $\vec{\rho}_i$. Although the unique $s$-channel unitarization procedure is lacking, we can still develop a sound phenomenology. We identify the cross section (71) which leads to the GLDAP-evolving structure function $F_2^{(N)}(GLDAP, x, Q^2)$ with the bare-pomeron exchange. The bare-pomeron structure function $F_2^{(N)}(GLDAP, x, Q^2)$ is the linear functional of the density of partons in the proton:

$$F_2^{(p)}(GLDAP, x, Q^2) = \frac{Q^2}{4\pi \alpha_{em}} \sigma_{tot}(GLDAP, x, Q^2)$$

The construction of the unitarized photoabsorption cross section goes as follows. For each Fock state we define the bare $\Gamma_0(b)$ and the unitarized $\Gamma(b)$ profile functions and the bare $\sigma_0$ and the unitarized cross section $\sigma^{(U)}$:

$$\sigma^{(U)} = 2 \int d^2\vec{b} \Gamma(b) = 2 \int d^2\vec{b} \Gamma_0(b) - 2 \int d^2\vec{b} [\Gamma_0(b) - \Gamma(b)] = \sigma_0 - \Delta \sigma^{(sh)}$$

Let us derive the shadowing (unitarity) correction to the scattering of the $q\bar{q}$ Fock state of the photon. To the leading order in $\Gamma_0(b)$, Eqs. (99),(96) give

$$\Delta \sigma^{(sh)}(\rho) \approx \chi \frac{\sigma(\rho)^2}{16\pi B(\rho)}$$

and the shadowing correction to the total photoabsorption cross section equals

$$\Delta \sigma^{(sh)}_{tot}(\gamma^* N, x, Q^2) = \int dz d^2\vec{r} |\Psi(z, \rho)|^2 \Delta \sigma^{(sh)}(\rho) \approx \chi \int dz d^2\vec{r} |\Psi(z, \rho)|^2 \frac{\sigma(\rho)^2}{16\pi B(\rho)}$$

$$= \chi \int dM^2 \frac{\sigma_{tot}(pp)}{M^2 + Q^2} \frac{\sigma_{tot}(\gamma^* P \to q + \bar{q})}{16\pi B_D(M^2)} .$$

Here $B_D(M^2)$ is the diffraction slope for the diffraction excitation of the mass $M$. The shadowing correction to the total photoabsorption cross section equals the diffraction dissociation cross section times the enhancement parameter $\chi \approx (1 - 2)$. The generalization of eq. (104) to
interactions of the higher Fock states of the photon is straightforward. Making use of eq. (89), we obtain the shadowing correction to the structure function of the proton

$$\Delta F_2^{(sh)}(x, Q^2) = \chi \sigma_{tot}(pp) \int_x^{y_m} \frac{dy}{y} f^{IP}(y) F_2^{IP}(\frac{x}{y}, Q^2) \frac{B_{3IP}}{B_D(M^2)}. \tag{105}$$

(The slope $B_{3IP}$ and the end-point $y_m$ of the pomeron distribution will be defined below.) Ignore for a minute the mass-dependence of the slope $B_D(M^2)$. Since $F_2^{IP}(x, Q^2)$ satisfies the GLDAP evolution, the convolution representation (105) implies that the shadowing correction to the proton structure function also satisfies the GLDAP evolution equations! Experimentally, in all the hadronic reactions and in the diffraction dissociation of real photons the slope $B_D(M^2)$ exhibits similar dependence on the excited mass $M$ [27]: in the triple-pomeron region the slope is constant to a good approximation,

$$B_D(M^2) = B_{3IP} \approx \frac{1}{2} B_{el}(hN), \tag{106}$$

whereas in the resonance excitation region

$$B_D(M^2) \sim B_{el}(hN). \tag{107}$$

In the DIS the counterpart of excitation of resonances is the excitation of the $q\bar{q}$ Fock states of the photon, for which we expect the slope (107), whereas for the higher Fock states and heavier masses the slope (106) is more appropriate. These assumptions can be tested at HERA. Consequently, as compared to the pomeron structure function as measured in the diffraction dissociation, in the shadowing structure function (105), the ‘valence’ $q\bar{q}$ component of the pomeron enters with the suppression factor $B_{3IP}/B_D(M^2) \approx 1/2$, which does not effect the QCD evolution properties. We conclude, that the unitarized structure function of the deep inelastic scattering

$$F_2^{(N)}(x, Q^2) = F_2^{(N)}(GLDAP, x, Q^2) - \Delta F_2^{(sh)}(x, Q^2) \tag{108}$$

satisfies the linear GLDAP evolution equation.

### 7.3 Brief phenomenology of the shadowing correction to the proton structure function

According to Eq. (104), the relative shadowing correction to the proton structure function equals the fraction $w_{DD}$ of DIS which goes via diffraction dissociation of photons times the enhancement
parameter $\chi$. In the diffraction dissociation events the proton changes its longitudinal momentum $p_L$ little, $\Delta p_L / p_L = y < y_m \lesssim 0.1$, and appears in the final state separated from the hadronic debris of the photon by the rapidity gap

$$\Delta \eta = \log \left( \frac{1}{y} \right)$$

The standard definition of the diffraction dissociation corresponds to $\Delta \eta \gtrsim \Delta \eta_{\text{min}} = 2 - 2.5$. The maximal value of the rapidity gap is $\eta_{\text{max}} = \log(1/x)$. The estimate of $w_{DD}$ is particularly easy when the pomeron and proton structure functions are approximately constant. In this case $f_{IP}(y) = 1$, the rapidity gap distribution is flat which is a signature of the triple-pomeron mechanism [1], and combining equations (64) and (104) we find [7]

$$w_{DD}(x) \approx \frac{A_{3\text{IP}}(0)}{B_{3\text{IP}}} \int_{\Delta \eta_{\text{min}}}^{\eta_{\text{max}}} d\Delta \eta \frac{A_{3\text{IP}}(0)}{B_{3\text{IP}}} = \frac{A_{3\text{IP}}(0)}{B_{3\text{IP}}} \log \left( \frac{y_m}{x} \right),$$

which is roughly $Q^2$-independent. Numerically, $A_{3\text{IP}}(0)/B_{3\text{IP}} \approx 0.03$, and in Ref. [7] we gave an estimate $w_{DD} \sim 0.15$ at $x \sim 10^{-3}$ and $Q^2 \sim 30 (\text{GeV/c})^2$. This prediction is consistent with the first determinations of $w_{DD}$ by the ZEUS collaboration at HERA [28].

For a somewhat more realistic evaluation of $w_{DD}$ let us assume that

$$f_{IP}(y) \sim \left( \frac{y_m}{y} \right)^{2\Delta},$$

where $\Delta = \alpha_{IP}(0) - 1 \sim 0.1$, as suggested by the pomeron phenomenology of the hadronic cross sections [16,17]. We also assume that at small $x$ the structure functions rise $\propto (1/x)\delta$ with the same exponent $\delta$ for the proton and pomeron. The analysis of Ref. [30] gives $\delta(Q^2) \sim 0.21$ at $Q^2 = 4(\text{GeV/c})^2$ and $\delta(Q^2) \sim 0.31$ at $Q^2 = 15(\text{GeV/c})^2$. The experience with the QCD evolution analysis suggests that the ratio $F_{2\text{IP}}(x,Q^2)/F_{2\text{P}}(x,Q^2)$ only will weakly change with $Q^2$, so that Eq. (64) can be used for the relative normalization of the proton and pomeron structure functions. Then,

$$w_{DD}(x) \approx \frac{A_{3\text{IP}}(0)}{B_{3\text{IP}}} \int_{\Delta \eta_{\text{min}}}^{\eta_{\text{max}}} d\Delta \eta \exp[-\gamma(\Delta \eta - \Delta \eta_{\text{min}})] = \frac{A_{3\text{IP}}(0)}{B_{3\text{IP}}} \cdot \frac{1}{\gamma} \left[ 1 - \left( \frac{x}{y_m} \right)^{\gamma} \right],$$

and to the extent that $\gamma = \delta - 2\Delta \ll 1$, and $\gamma(\eta_{\text{max}} - \Delta \eta_{\text{min}}) \ll 1$, we have still an approximately flat rapidity gap distribution and again obtain the estimate (110) for $w_{DD}$. Consequently, we predict rather large shadowing effect in the proton structure function

$$\frac{\Delta F_{2}^{(sh)}(x,Q^2)}{F_{2}^{(p)}(x,Q^2)} \approx \chi w_{DD}(x),$$

\[113\]
which persists at all $Q^2$. In the kinematical range of the DIS at HERA the shadowing effect can be as large as $\sim 30\%$. The more detailed phenomenology of the shadowing corrections is presented in Ref. [30].

7.4 Unitarization and shadowing correction to the parton densities

Since $F_2^{(sh)}(x, Q^2)$ satisfies the GLDAP evolution, the shadowing correction to the proton structure function can be reabsorbed into the modification of the parton densities in the protons. For instance, the shadowed density of gluons in the proton shall equal

$$g(x, Q_0^2) = g(\text{GLDAP}, x, Q_0^2) - \chi \frac{\sigma_{\text{tot}}(pp)}{16\pi B_{3\text{IP}}^2} \int_x^1 \frac{dy}{y^2} f_{3\text{IP}}(y) g_{3\text{IP}}\left(\frac{x}{y}\right).$$

Similarly, the valence and sea $q\bar{q}$ distributions in the pomeron will modify the sea quark distribution in the proton. The detailed phenomenology of the shadowing corrections to the parton distributions in the proton will be presented elsewhere [31]. Here we just notice that whereas the GLDAP defined parton distributions satisfy the momentum sum rule

$$\sum_{p=q,\bar{q},g} \langle x_p \rangle = \int_0^1 dx \left\{ g(\text{GLDAP}, x, Q^2) + \sum_i \left[ q_i(\text{GLDAP}, x, Q^2) + \bar{q}_i(\text{GLDAP}, x, Q^2) \right] \right\} = 1,$$

because of the shadowing correction this sum rule does not hold for the experimentally measured shadowed (unitarized) parton distributions. A crude estimate of violation of the momentum sum rule (115) is

$$\Delta \Sigma_x = 1 - \sum_{p=q,\bar{q},g} \langle x_p \rangle \sim y_m \frac{A_{3\text{IP}}^*}{B_{3\text{IP}}} \sim 0.003$$

With the $\approx 2\%$ systematic normalization errors in the most accurate measurements of $F_2^{(N)}(x, Q^2)$, presently the momentum sum rule can not be tested to better than $5\%$ [55]. The concept of the fusion (recombination) of partons must be used with much caution. For instance, the shadowing correction to the density of gluons Eq. (113) is not proportional to $g(x, Q^2)^2$ as it is often stated in the literature ([19-22], see below Section 7.6). Indeed, the shadowing term is proportional to $A_{3\text{IP}}^*$, the integrand of which is $\propto [\sigma(y, r)/r^2]^2 \propto [xg(x, Q_{3\text{IP}}^2 = 1/r^2)]^2$ and the integration is dominated by large hadronic values of $r \sim R_{3\text{IP}}$ and small virtualities of the fusing gluons $Q_{3\text{IP}}^2 \sim 1/R_{3\text{IP}}^2$ [23,24,35].

Because of the shadowing the parton distributions inferred from the GLDAP evolution analysis of the DIS structure functions will be different from the operator-product expan-
sion (OPE) defined parton distributions, which define the impulse approximation component \( F_2^{(N)}(GLDAP, x, Q^2) \) in Eq. (101). To this end, an analogy with the comparison of the electron-nucleus and proton-nucleus scattering is instructive: The elastic \( eA \) scattering is described by the sum of the impulse approximation diagrams of Fig. 10a and is a linear functional of the nuclear charge density. The \( eA \) scattering amplitude measures the charge of the nucleus which equals the sum of charges of its constituents (nucleons). Choosing an appropriate external field, one can study the whole sequence of the nuclear matrix elements which will be sensitive to the momentum distribution of nucleons in the nucleus. For instance, considering the scattering of the nucleus on the gravitational center one can derive the momentum sum rule that the constituent nucleons carry the total momentum of the nucleus [35]. Under the strong condition that the scattering in external fields is described by the impulse approximation, i.e., by the exchange of the single quantum of the external field, having measured the amplitudes of scattering in a variety of external fields one can reconstruct the momentum distribution of nucleons in the nucleus. One would recognize in the above the standard OPE definition of the parton densities (for instance, see the textbooks [56]). In the \( pA \) elastic scattering the impulse approximation amplitude \( f_A(\vec{q}) = Af_N(\vec{q})G_A(\vec{q}) \), where \( G_A(\vec{q}) \) is the body form factor of the nucleus, has the profile function \( \Gamma_A(b) \sim A^{1/3} \) which grossly overshoots the unitarity bound (94). Consequently, the \( pA \) scattering amplitude is subject to the large unitarization corrections (Fig. 10b) and is a nonlinear functional of the nuclear matter density.

In this context, the GLDAP approach corresponds to the impulse approximation and Eq. (101) gives the linear relationship between the Compton scattering amplitude and the parton densities. The shadowing term is an apparently nonlinear functional of the density of partons in the proton, but we have proven that this nonlinearity can be cast in the form of the renormalization of the parton densities, with retention of the linear GLDAP evolution properties.

### 7.5 The higher order unitarity corrections and fusion of partons

The higher order unitarity corrections, i.e., the multiple-pomeron exchanges Figs. 1b, 1d, 9, do technically give rise to the photon-multipomeron interactions, which cast shadow on the very definition of the photon-pomeron cross section and the pomerons structure function Eqs. (12). The remarkable observation is that one can still describe the diffraction dissociation cross section in
terms of the pomeron structure function and the factorization representation Eq. (88), and these \( \gamma^* (n_{IP}) \) interactions only do slightly modify \( f_{IP}(y) \) and the simple relationship (103) between the shadowing structure function and the pomeron structure function. Let us start with the unitarization of the diffraction dissociation cross section. The s-channel iterations of the QCD pomeron exchange to the left and to the right of the unitarity cut in Fig. 11 do separately sum up to the unitarized dipole cross section. For the \( q\bar{q} \) Fock state one must unitarize \( \sigma(y, r) \), for the \( q\bar{q}g_1...g_n \) Fock states one must unitarize \( \Sigma(y, r) = \frac{9}{4} \sigma(y, r) \). Barring the \( q\bar{q} \) state the flux of pomerons \( f_{IP}^{(D)}(y) \) which enters the diffraction dissociation cross section must be calculated with the substitution

\[
\sigma(y, r)^2 \rightarrow \left( \frac{4}{9} \right)^2 \Sigma^{(U)}(y, r)^2
\]

in the pomeron wave function (90), so that

\[
f_{IP}^{(D)}(y) = \left( \frac{4}{9} \right)^2 \cdot \frac{\int dr^2 \left[ \Sigma^{(U)}(y, r)/r^2 \right]^2 \mathcal{F}(\mu_G r)}{\int dr^2 \left[ \sigma(r)/r^2 \right]^2 \mathcal{F}(\mu_G r)}.
\]

Apart from this minor change, the perturbative QCD expansion (79) will retain its form.

Similarly, in the case of the shadowing correction, Fig. 12, the higher order unitarity corrections are accounted for by the substitution

\[
\sigma(y, r)^2 \rightarrow \frac{16\pi B_{3IP}}{\chi} \cdot \left( \frac{4}{9} \right)^2 \cdot \Delta \Sigma^{(sh)}(y, r) = \frac{16\pi B_{3IP}}{\chi} \cdot \left( \frac{4}{9} \right)^2 \cdot [\Sigma(y, r) - \Sigma^{(U)}(y, r)]
\]

so that \( \chi f_{IP}(y) \) in Eq. (103) shall be replaced by

\[
\chi f_{IP}^{(sh)}(y) = 16\pi B_{3IP} \cdot \left( \frac{4}{9} \right)^2 \cdot \frac{\int dr^2 \left[ \Sigma(y, r) - \Sigma^{(U)}(y, r) \right]/r^4 \mathcal{F}(\mu_G r)}{\int dr^2 \left[ \sigma(r)/r^2 \right]^2 \mathcal{F}(\mu_G r)}.
\]

Again, for the \( q\bar{q} \) state one must unitarize \( \sigma(y, r) \). The higher order unitarity corrections make the two fluxes \( f_{IP}^{(D)}(y) \) and \( f_{IP}^{(sh)}(y) \) slightly different both in the absolute normalization and in the \( y \)-dependence.

Equations (118) and (120) summ in a very compact form all the multiple-pomeron exchanges in the s-channel (Figs. 11, 12). The origin of this remarkable result is simple: the interaction cross section of the \( n \)--parton Fock state of the photon is dominated by the spatial extension of the softest gluon, which acts as a constituent gluon of the pomeron. This corresponds to the dominance of the multipomeron exchange diagrams of Fig. 13. Consequently, the unitarization affects only the normalization of the pomeron wave function but not the QCD evolution properties of the pomeron structure function.
The shadowing structure function can be reinterpreted in terms of the fusion of partons from the overlapping pomeron emissions by the same nucleon, which reduces the total density of partons. The fusion of partons from different nucleons of the nucleus was first introduced in 1975 [54] and remains a viable mechanism for the nuclear shadowing in deep inelastic scattering [23,24,35]. However, this interpretation must be taken with the grain of salt. The bare, GLDAP cross section $\sigma(U)(x,r)$ is a linear functional of the density of gluons, the unitarized cross sections $\Sigma(U)(x,r)$ contain the terms $\propto (-1)^{n+1}[xg(x,r)]^n$ which are sign-alternating. In more general terms, the multi-gluon exchange contribution is proportional to the many-gluon density matrix, the elements of which are not necessarily positive definite. Evidently, this quantum-mechanical property is missed in the probabilistic approach to fusion.

### 7.6 Unitarization and linear GLDAP versus nonlinear GLR evolution equations

There was much discussion of the unitarization of rising structure functions in the framework of the so-called Gribov-Levin-Ryskin (GLR) nonlinear evolution equation ([19], for the recent review with many references see [22]). Here we briefly comment on the origin of the nonlinear term in the GLR equation, following the standard derivation of the evolution equations [11-13]. (We only consider $x \ll 1$ of our interest.) One starts with evaluation of the derivative [12]

$$Q^2 \frac{\partial}{\partial Q^2} F_2(x,Q^2) \propto Q^2 \frac{\partial}{\partial Q^2} [Q^2 \sigma_{tot}(\gamma^N,x,Q^2)].$$

(121)

In Section 2 we have decomposed $Q^2 \sigma_{tot}(\gamma^N,x,Q^2)$ into the non-LLA component [18], which we can neglect, and the LLA component [17], in which all the explicit dependence on $Q^2$ is concentrated in the integration limit. This is the crucial point, since taking the derivative (121) and making use of Eq. (75) one obtains one of the small-$x$ GLDAP equations

$$Q^2 \frac{\partial}{\partial Q^2} [xq(x,Q^2)] \propto Q^2 \sigma(1/Q) \propto x\alpha_S(Q^2)g(x,Q^2),$$

(122)

in which the both r.h.s and l.h.s are evaluated at the same value of $Q^2$. Notice, that this property is a result of the singular behaviour of the integrand in Eq. (17).

To the contrary, the integrand of the leading shadowing correction (104) is a smooth function of $\rho$. Furthermore, it is dominated by the contribution from large $\rho \sim R_{IP}$. For this reason, it would be illegitimate to enforce the LLA limit of integration $\rho^2 > 1/Q^2$ in the shadowing correction (104). If, nonetheless, one proceeds so, then differentiation in the first line
of Eq. (104) will give
\[ Q^2 \frac{\partial}{\partial Q^2} [Q^2 \Delta \sigma^{(sh)}(\rho^2 > 1/Q^2, x, Q^2)] \propto \frac{1}{Q^2 B_{3IP}} [Q^2 \sigma(1/Q)]^2 \propto \frac{1}{Q^2 B_{3IP}} \alpha_S(Q^2)^2 [xg(x, Q^2)]^2 . \]  

(123)

The familiar form of the GLR nonlinear shadowing correction to the GLDAP equation for the density of gluons [19],
\[ Q^2 \frac{\partial}{\partial Q^2} [x \Delta g^{(sh)}(x, Q^2)] \propto \frac{\alpha_S(Q^2)^2}{Q^2 B_{3IP}} \int_x^{\infty} \frac{dy}{y} [yg(y, Q^2)]^2, \]

(124)

is different from our result (105, 89). Evidently, neglecting the contribution to the shadowing term from \( \rho^2 < 1/Q^2 \) and/or the \( \propto 1/Q^2 \) corrections to the leading form of the wave function in Eqs. (21, 104) can not be justified, which makes the GLR equation highly questionable. It is interesting to notice, that Mueller and Qiu [20] had already have expressed similar doubts on the validity of the GLR nonlinear equation (see also the recent preprint by Levin and Wüsthoff [21]). The GLR term is a part of the \( \propto 1/Q^2 \) corrections to the leading shadowing term given by our Eq. (105), see also the above discussion of the fusion of partons in Section 7.5.

8 Lipatov’s pomeron and diffractive deep inelastic scattering

Lipatov and his collaborators have shown [29, 42, 57] that the QCD pomeron of the Regge limit \( Q^2, r^2 = \text{const} \), \( 1/x \to \infty \) has the intercept
\[ \alpha_{IP} = 1 + \Delta_{IP} = 1 + \frac{12 \log 2}{\pi} \alpha_S \sim 1.5 . \]

(125)

which would have given a very rapidly rising total cross section
\[ \sigma_{tot} \propto \left( \frac{1}{x} \right)^{\Delta_{IP}} . \]

(126)

In our scenario of diffractive DIS we have assumed slow growth of the dipole cross section (71) with \( 1/x \) at large, hadronic, values of \( r \sim R_N \). Firstly, this is consistent with, and supported by, the lack of rapid rise of \( xg(x, Q^2) \) at small \( Q^2 \) (see Eq. (73)) found in the latest QCD analysis of the parton distributions in the proton [55]. Secondly, this assumption is perfectly consistent with the Lipatov’s theory of the pomeron.

Lipatov’s starting point is the lowest order QCD cross section for the scattering of the two color dipoles \( \vec{r}_1 \) and \( \vec{r}_2 \)
\[ \sigma_{dd}(\vec{r}_1, \vec{r}_2) = \frac{32}{9} \alpha_s^2 \int \frac{d^2 k}{(k^2 + \mu_c^2)^2} \left[ 1 - \exp(-ik\vec{r}_1) \right] \left[ 1 - \exp(ik\vec{r}_2) \right] . \]

(127)
In the frozen-\(\alpha_S\) approximation, the profile function of the dipole-dipole scattering has the form

\[
\Gamma_{dd}(\vec{r}_1, \vec{r}_2, \vec{b}) \propto \frac{1}{\mu_G^2} \left[ K_0(\mu_G|\vec{b} + \vec{s}|) + K_0(\mu_G|\vec{b} - \vec{s}|) + K_0(\mu_G|\vec{b} + \vec{\Delta}|) + K_0(\mu_G|\vec{b} - \vec{\Delta}|) \right] \ 
\]

where \(\vec{s} = (\vec{r}_1 + \vec{r}_2)/2\) and \(\vec{\Delta} = (\vec{r}_1 - \vec{r}_2)/2\). In the limit of

\[
b^2, r_1^2, r_2^2 \ll \frac{1}{\mu_G^2} \ 
\]

one can use \(K_0(x) \propto \log(x)\). Then, the dependence on \(\mu_G\) in Eq. (128) disappears and the dipole-dipole profile function acquires the conformally invariant form [29]

\[
\Gamma_{dd}(\vec{r}_1, \vec{r}_2, \vec{b}) \propto \log \left[ \frac{|\vec{b} + \vec{\Delta}|(|\vec{b} - \vec{\Delta}|)}{|\vec{b} + \vec{s}|(|\vec{b} - \vec{s}|)} \right] \log \left[ \frac{|\vec{b} + \vec{\Delta}|(|\vec{b} - \vec{\Delta}|)}{|\vec{r}_1||\vec{r}_2|} \right]. \ 
\]

It differs from (128) by terms which give vanishing contribution to the dipole-dipole cross section after the \(d^2\vec{b}\) integration. In the same conformal-invariant limit, also the wave functions of the many body Fock states will become the scale invariant functions, see Eq. (17). Because of this scale invariance no specific size ordering dominates in the higher order QCD diagrams. Using the powerful technique of the conformal field theories, Lipatov has found that with allowance for the running QCD coupling the QCD pomeron corresponds to the series of poles, which accumulate at \(j = 1\) in the complex angular momentum plane, with the intercept of the rightmost singularity given by eq. (125). Calculation of residues of these Lipatov’s poles is as yet lacking.

In the scenario of Ref. [44] the dominant, constant, component of the hadronic cross section comes from the \(j = 1\) cross section (11), whereas the contribution of poles with \(\Delta_{IP} > 0\) has a small residue. Indeed, the dipole cross section \(\sigma(\rho)\) is given by the expectation value of the dipole-dipole cross section over the target nucleon state,

\[
\sigma(r) = \frac{3}{2} \int d^3\vec{R}_1 d^3\vec{R}_2 |\Psi_N(\vec{R}_1, \vec{R}_2, \vec{R}_3)|^2 \sigma_{dd}(\vec{r}, \vec{R}_1 - \vec{R}_2), \ 
\]

and receives the dominant contribution from the separation of quarks in the proton in the nonscaling region of \((\vec{R}_1 - \vec{R}_2)^2 \sim R_N^2, 1/\mu_G^2\). If Lipatov’s conformal pomeron with the intercept (125) only is applicable in the scaling region of \((\vec{R}_1 - \vec{R}_2)^2 \ll R_N^2, 1/\mu_G^2\), then Eq. (131) gives a natural explanation for the small residue of the rightmost pole, corroborating the scenario [44].
(for the evaluation of the effective intercept of the pomeron in the hadronic scattering at finite $\mu_G$ see Ref. [43]).

Lipatov’s analysis and the intercept \textbf{[123]} with the running coupling $\alpha_s(t)$ are expected to be directly applicable to the hard elastic $pp, \bar{p}p$ scattering at $|t| \gg m^2$, $|t| \ll s$. The dominance of the rightmost pole of the pomeron would have implied

$$\frac{d\sigma_{el}(pp)}{dt} \propto s^{2(\alpha_s(t)-1)} \sim s^1.$$ \textbf{(132)}

The experimental lack of such a rapid growth of the differential cross section of elastic scattering with energy is another strong evidence for the small residue of poles with $\Delta_{IP} > 0$.

The DLLA pomeron and Lipatov’s pomeron summ (in a somewhat different kinematical regime) the gluon ladders with two gluons in the $t$-channel. They both share the unpleasant property of running into conflict with the $s$-channel unitarity, which shows that the initial DLLA and/or LLA selection of the subset of the perturbative QCD diagrams for the pomeron is unsatisfactory and open to a criticism. As a matter of fact, such a construction of the pomeron lacks self-consistency, since the sub-leading unitarization corrections do become larger than the ‘leading’ single-pomeron exchange. From the numerical point of view, the above presented phenomenology seems to be viable in the kinematical range of HERA: according to our analysis in Section 7.3, the unitarization effects are numerically large, but still significantly smaller than the leading GLDAP component of the structure function (for the more detailed phenomenology of the shadowing see Ref. [30]). More theoretical work on unitarization is badly needed. Some 16 years ago, Matinyan and Sedrakyan [58] have pointed out the potential significance of the multiparticle Regge singularities (Fig.14). In the recent very interesting paper [59], Bartels has shown that such a correlated four-gluon exchange leads to the new singularities in the complex angular momentum plane which are missed when the eikonal-like exchange by the two rightmost poles of the Lipatov’s pomeron is considered. The phenomenological implications of this result for the shadowing corrections to the proton structure functions are not yet clear, since the quantitative understanding of residues of Lipatov’s poles in the DIS regime is lacking. Our minimal-regularization approach, in which the radiative generation of the glue in protons ([35,37], see also [39]) is closely related to the slow rise of $\sigma(x,r)$ at large, hadronic, values of $r$, leads to a good parameter-free description of the proton structure function starting with the SLAC-NMC range of $x$ and $Q^2$ [23,24,35] and down to the kinematical range of HERA.
we have obtained [30] a good agreement with the HERA results on $F_2^{(p)}(x, Q^2)$ [60].

9 Conclusions and discussion of results

Our principle conclusion is that the diffraction dissociation of virtual photons in DIS can be described as the DIS on pomeron with the well-defined and GLDAP-evolving structure function. Our analysis completes the proof of the parton model phenomenology of the pomeron put forward some 8 years ago by Ingelman and Schlein [2]. Furthermore, we have shown that such a description persists beyond the single-pomeron exchange approximation. Our new result is that we have identified the valence $q\bar{q}$, the valence glue and the sea $q\bar{q}$ parton distributions in the pomeron, which are to be used as an input in the QCD evolution of the pomeron structure function. We have found that the normalization of the valence glue and sea in the pomeron is fixed by the single dimensional coupling $A_{3\mathbf{IP}}^\ast$, which is sensitive to the infrared regularization. Our principle finding is that this coupling $A_{3\mathbf{IP}}^\ast$ (and the corresponding triple-pomeron coupling $A_{3\mathbf{IP}}(Q^2)$ which we have shown only weakly depends on $Q^2$) must be approximately equal to the triple-pomeron coupling $A_{3\mathbf{IP}}(0)$ as measured in the diffraction dissociation of the real photons [27]. This approximate equality $A_{3\mathbf{IP}}^\ast \approx A_{3\mathbf{IP}}(0)$ was conjectured long ago [26] and has been a basis of the successful phenomenology of the nuclear shadowing in DIS [23,24,35]. This equality also was used in the prediction [7] of the rate of the diffraction dissociation in DIS, which is in good agreement with the first data by the ZEUS collaboration [46]. Important implication of separation of the infrared-sensitive input structure function of the pomeron from the hard QCD evolution effects is that the jet activity in the DIS on the pomeron must be similar to that in the DIS on the proton.

We have derived the unitarity (shadowing) correction to the proton structure function at small $x$ and have demonstrated, that the unitarized structure function satisfies the conventional, linear, GLDAP evolution equations. We emphasize the intrinsic simplicity of our light-cone $s$-channel formalism used in this derivation. Firstly, our formalism implements in a very simple way the color gauge invariance constraints. Secondly, exact factorization of the photoabsorption cross section into the wave function and into the (multiparticle) dipole cross section allows an easy identification of the partial waves of the dipole cross section as an object of the $s$-channel unitarization. Thirdly, we took full advantage of the diagonalization of the scattering matrix
as a function of the transverse separation and longitudinal momenta of partons in the multi-parton Fock states of the photon. This enabled us to easily impose the $s$-channel unitarization on the total cross sections of all the multiparticle Fock states of the photon. This also enabled us to identify the constituent gluon wave function of the pomeron, which gives a very economic description of the shadowing process in terms of the single parameter $A_{3IP}^*$, which is under the good control as it is related to the triple-pomeron coupling $A_{3IP}(0)$ known from the real photoproduction experiments. We have shown how the multipomeron exchanges in the shadowing structure function and in the diffraction dissociation can be summed in a very compact form which only renormalizes the effective flux of pomerons in the proton.

Applications of the above formalism to the hadronic scattering problem will be presented elsewhere [43].

Acknowledgements

One of the authors (NNN) would like to thank V.V.Anisovich and L.G.Dakhno for criticism and L.N.Lipatov for numerous discussions on the pomeron and deep inelastic scattering during the past years. We are grateful to V.Barone, M.Genovese, E.Predazzi and V.R.Zoller for useful comments and discussions.
References

[1] K.A.Ter-Martirosyan, *Phys.Lett.* **B44** (1973) 179; A.B.Kaidalov and K.A.Ter-Martirosyan, *Nucl.Phys.* **B75** (1974) 471.

[2] G.Ingelman and P.Schlein, *Phys. Lett.* **B152** (1985) 256.

[3] H.Fritzch and K.H.Streng, *Phys.Lett.* **B164** (1985) 391.

[4] E.L.Berger, J.C.Collins, D.E.Soper and G.Sterman, *Nucl.Phys.* **B286** (1987) 704.

[5] A.Donnachie and P.V.Landshoff, *Phys.Lett.* **B191** (1987) 309; *Nucl.Phys.* **B303** (1988) 634.

[6] V.V.Abramovskii and R.G.Betman, *Sov.J.Nucl.Phys.* **49** (1989) 1205.

[7] N.N. Nikolaev and B.G. Zakharov, *Z. Phys.* **C53** (1992) 331.

[8] N.N.Nikolaev, in *Diffractive and Elastic Scattering*. Proc. Int. Conf. on Elastic and Diffractive Scattering, La Biodola, Isola d'Elba, Italy, 22-25 May 1991; *Nucl.Phys.(Proc. Suppl.)* **B25** (1992) 152.

[9] J.Bartels and G.Ingelman, *Phys.Lett.* **B235** (1990) 175.

[10] G.Inlelman and K.Prytz, *Z.Phys.* **C58** (1993) 285.

[11] V.N. Gribov and L.N. Lipatov, *Sov. J. Nucl. Phys.* **15** (1972) 438; L.N. Lipatov, *Sov. J. Nucl. Phys.* **20** (1974) 181.

[12] Yu.L. Dokshitser, *Sov. Phys. JETP* **46** (1977) 641.

[13] G. Altarelli and G. Parisi, *Nucl. Phys.* **B126** (1977) 298.

[14] E.M.Levin and M.G.Ryskin, *Sov.J.Nucl.Phys.* **34** (1981) 619.

[15] A.S.Pak, N.O.Sadykov, A.V.Tarasov, *Sov.J.Nucl.Phys.* **42** (1985) 619.

[16] P.E.Volkovitski, A.M.Lapidus, V.I.Lisin and K.A.Ter-Martirosyan, *Sov.J.Nucl.Phys.* **24** (1976) 648; A.Capella and J.Kaplan, *Phys.Lett.* **B52** (1974) 448.

[17] A.Donnachie and P.V.L.Landshoff, *Phys.Lett.* **B296** (1992) 227.
[18] E710 Collaboration: N.A. Amos et al., Phys. Lett. B301 (1993) 313.

[19] L.V. Gribov, E.M. Levin and M.G. Ryskin, Phys. Reports C100 (1983) 1.

[20] A.H. Mueller and J. Qiu, Nucl. Phys. B268 (1986) 427.

[21] E.M. Levin and M. Wüsthoff, DESY 92-166 (1992).

[22] B. Badelek, K. Charchula, M. Krawszyk and J. Kwiecinski, Rev. Mod. Phys. 64 (1992) 927.

[23] N.N. Nikolaev and B.G. Zakharov, Phys. Lett. B260 (1991) 414.

[24] N.N. Nikolaev and B.G. Zakharov, Z. Phys. C49 (1991) 607.

[25] J.D. Bjorken, J. Kogut and D.E. Soper, Phys. Rev. D3 (1971) 1382.

[26] N.N. Nikolaev, EMC effect and quark degrees of freedom in nuclei: facts and fancy. Oxford Univ. preprint OU-TP 58/84 (1984); Also in: Multiquark Interactions and Quantum Chromodynamics. Proc. VII Intern. Seminar on Problems of High Energy Physics, 19-26 June 1984, Dubna, USSR.

[27] T.J. Chapin et al., Phys. Rev. D31 (1985) 17.

[28] ZEUS Collaboration: M. Derrick et al., DESY 93-093, July 1993.

[29] L.N. Lipatov, Sov. Phys. JETP 63 (1986) 904; L.N. Lipatov, Pomeron in Quantum Chromodynamics. In: Perturbative Quantum Chromodynamics, editor A.H. Mueller, World Scientific, 1989.

[30] V. Barone, M. Genovese, N.N. Nikolaev, E. Predazzi and B.G. Zakharov, Torino preprint DFTT 28/93, June 1993.

[31] V. Barone, M. Genovese, N.N. Nikolaev, E. Predazzi and B.G. Zakharov, paper in preparation.

[32] V. Barone, M. Genovese, N.N. Nikolaev, E. Predazzi and B.G. Zakharov, paper in preparation.

[33] F.E. Low, Phys. Rev. D12 (1975) 163; S. Nussinov, Phys. Rev. Lett. 34 (1975) 1286; J.F. Gunion and D. Soper, Phys. Rev. D15 (1977) 2617.
[34] V.N.Gribov, Lund preprint LU TP 91-7 (1991).

[35] V. Barone, M. Genovese, N.N. Nikolaev, E. Predazzi and B.G. Zakharov, Z. Phys. C58 (1993) 541.

[36] V. Barone, M. Genovese, N.N. Nikolaev, E. Predazzi and B.G. Zakharov, Phys. Lett. B304 (1993) 176.

[37] V. Barone, M. Genovese, N.N. Nikolaev, E. Predazzi and B.G. Zakharov, Int. J. Mod. Phys. A8 (1993) 2779.

[38] V. Barone, M. Genovese, N.N. Nikolaev, E. Predazzi and B.G. Zakharov, Phys. Lett. B268 (1991) 279; Torino preprint DFTT 39/93, July 1993.

[39] N.N.Nikolaev, University of Torkyo preprint INS-Rep.-539 (1985); M.Glueck, E.Reya and V.Vogt, Phys.Lett. 306 (1993) 391.

[40] Yu.L.Dokshitzer, D.I.Dyakonov and S.I.Troyan, Phys.Rep. C58 (1980) 265.

[41] G.V.Frolov, V.N.Gribov and L.N.Lipatov, Phys.Lett. 31 (1970) 34; L.N.Lipatov and G.V.Frolov, Sov.J.Nucl.Phys. 13 (1971) 333.

[42] E.A.Kuraev, L.N.Lipatov and V.S.Fadin, Sov.Phys. JETP 44 (1976) 443; 45 (1977) 199.

[43] N.N.Nikolaev, B.G.Zakharov and V.R.Zoller, paper in preparation.

[44] B.Z.Kopeliovich, N.N.Nikolaev and I.K.Potashnikova, Phys. Rev. D39 (1989) 769.

[45] E.M.Levin and M.G.Ryskin, Phys.Rep. C189 (1990) 267.

[46] ZEUS Collaboration: M.Derrick et al., Phys.Lett. B293 (1992) 465; H1 Collaboration: T.Ahmed et al., Phys.Lett. B299 374.

[47] B.G.Zakharov, Sov.J.Nucl.Phys. 49 (1989) 860.

[48] R.C.Arnold, Phys.Rev. 153 (1967) 1523.

[49] K.A.Ter-Martirosyan, Sov.J.Nucl.Phys. 10 (1970) 600.

[50] R.C.Arnold, Phys.Rev. B136 (1964) 1388.
[51] T.T.Chou and C.N.Yang, Phys.Rev. D19 (1979) 3268; Phys.Lett. B123 (1983) 457; C.Bourelly, J.Soffer and T.T.Wu, Phys.Lett. B121 (1983) 284; Z.Phys. C37 (1988) 369.

[52] V.N.Gribov, Sov.Phys. JETP 26 (1968) 414.

[53] A.B.Kaidalov, Sov.J.Nucl.Phys. 13 (1971) 226; Phys.Rep. 50 (1979) 157.

[54] N.N. Nikolaev and V.I. Zakharov. Phys. Lett. B55 (1975) 397; V.I. Zakharov and N.N. Nikolaev. Sov. J. Nucl. Phys. 21 (1975) 227;

[55] NMC Collaboration: M.Arneodo et al., Phys.Lett. B309 (1993) 222.

[56] R.G.Roberts, The structure of the proton. Deep inelastic scattering. Cambridge University Press, 1990; Yu.L.Dokshitzer, V.A.Khoze, A.H.Mueller and S.I.Troyan, Basics of Perturbative QCD. Editions Frontieres, 1991.

[57] Ya.Ya.Balitsky and L.N.Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822.

[58] S.G.Matinyan and A.G.Sedrakyan, JETP Lett. 23 (1976) 538; 24 (1976) 214; Sov.J.Nucl.Phys. 24 (1976) 440; 25 (1977) 475.

[59] J.Bartels, Phys.Lett. B298 (1993) 204.

[60] H1 Collaboration: C.Vallé, talk at the XXVIIIth Rencontres de Moriond, Les Arcs, France, March 1993, preprint DESY 93-077, June 1993; ZEUS Collaboration: M.Krzyzanowsky, talk at the International Conf. on Elastic and Diffractive Scattering, Brown Univ., Providence, June 1993.
Figure captions

Fig.1 - The single- and multiple-pomeron exchange contributions to the \((a,b)\) elastic scattering and \((c,d)\) the diffraction dissociation amplitudes.

Fig.2 - The lowest order QCD diagrams for interaction of the \(q\bar{q}\) Fock state of the photon with the target nucleon. In all the figures the wavy, solid and dashed lines are for the photon, (anti)quarks and gluons, respectively.

Fig.3 - One of the 16 lowest order QCD diagrams for the inclusive cross section of the forward diffraction dissociation of the \(q\bar{q}\) Fock state of the photon. The vertical dashed lines shows the unitarity cut corresponding to the diffractively excited state.

Fig.4 - The spatial structure of the \(q\bar{q}g\) Fock state in the impact-parameter plane.

Fig.5 - Scattering of the \(q\bar{q}g\) Fock state of the photon on the nucleon by interaction of its radiatively generated gluon.

Fig.6 - Different couplings of the exchanged gluons to the color-octer \(q\bar{q}\) pair and the gluon of the \(q\bar{q}g\) Fock state of the photon.

Fig.7 The triple-pomeron diagrams for the diffraction dissociation of virtual photons in the deep inelastic scattering:

[a] - The triple-pomeron diagram which describes the \(\propto 1/M^2\) component of the mass spectrum in the triple-Regge phenomenology of diffraction dissociation.

[b] - The driving term of the triple-pomeron mass spectrum in QCD - the diffraction excitation of the \(q\bar{q}g\) Fock state of the photon.

[c] - Diffraction excitation of the many-particle Fock states in the Low-Nussinov approximation for the exchanged pomerons.

[d] - The same as (c) with the exchange by the full QCD pomerons.

Fig.8 - Gluon-ladder representation of the DLLA pomeron in QCD.

Fig.9 - \(s\) channel iteration of the pomeron exchange:

[a] - In the approximation of elastic intermediate states.
[b] - Contribution of the inelastic intermediate states (diffraction dissociation of the target) to the s-channel iteration of the pomeron exchange.

Fig.10 - The archetype operator product expansion:

[a] - The impulse approximation diagram for the electron-nucleus scattering which is a linear probe of the nuclear charge distribution.

[b] - Multiple scattering diagrams which unitarize the proton-nucleus elastic scattering amplitude.

Fig.11 - Absorption (unitarization) corrections to the diffraction excitation of the $q\bar{q}g_1...g_n$ Fock state of the photon. The vertical dashed line shows the unitarity cut.

Fig.12 - Unitarization of the scattering amplitude for the $q\bar{q}g_1...g_n$ Fock state of the photon by the s-channel iteration of the QCD pomeron exchange.

Fig.13 - The dominant multipomeron interactions in the deep inelastic scattering.

Fig.14 - The multiparticle Reggeons.