Features of forward $\pi N$ scattering from a Reggeized model

Kook-Jin Kong and Byung-Geel Yi

Research Institute of Basic Science, Korea Aerospace University, Koyang, 10540, Korea

Charge exchange process $\pi^- p \to \pi^0 n$ and elastic scatterings $\pi^\pm p \to \pi^\mp p$ are investigated within the Regge framework where the relativistic Born amplitude is reggeized for the $t$-channel meson exchange. Charge exchange cross section is featured by the single $\rho$ exchange. Additional correction by Regge cuts, $p$-$f_2$ and $\rho$-Pomeron, agree with differential cross sections and new trajectory for the $\rho'(1450)$ exchange is attempted to reproduce polarization data. For the description of elastic scattering data up to pion momentum $P_{\text{lab}} \approx 250$ GeV/c, Pomeron exchange of the Donnachie-Landshoff type is newly constructed and applied in this work. Elastic cross section data are well reproduced with the dominance of $f_2$ and Pomeron exchanges in intermediate and high energies. Analysis of nucleon resonances is presented to test the validity of the present Regge framework below $W \leq 2$ GeV.

PACS numbers: 11.55.Jy, 13.75.Gx, 13.85.Dz, 14.20.Gk

I. INTRODUCTION

A $\pi N$ system is one of the fundamental objects to understand strong interaction with its origin from QCD. The $\pi N$ scattering near threshold offers a testing ground for the chiral dynamics of QCD in terms of soft pion interaction [1]. On the other hand, the rich structure of nucleon resonances $\Delta$ and $N^*$ in the $\pi N$ scattering below the reaction energy 2 GeV strongly supports the quark model prediction for the $\Delta$-baryon spectrum and their properties [2–4]. Over the resonances up to hundreds of GeV the reaction provides information on various meson exchanges and the nonresonant diffractive scattering that could be a manifestation of quarks and gluon degrees of freedom rather than hadronic degrees of freedom [5]. Therefore, though not listing a long history of theoretical development and experimental activities initiated by $\pi N$ scattering, the reaction should be regarded as an important source of our understanding the dynamics of QCD in the isospin symmetry sector.

Recently, Mathieu et al. [6] of JPAC studied $\pi N$ scattering to construct a new set of Regge amplitudes by matching the low energy partial wave analysis with a high energy data via the finite energy sum rule. Nys et al. of JPAC also analyzed world data of $K N$ charge exchange reactions with beam energy above 5 GeV/c [7]. Huang et al. [8] investigated $\pi N$ charge exchange scattering by using the Regge-cut model to provide high energy constraints above 2 GeV for the analysis of baryon resonances. The primary interest of these works is in the knowledge of $\pi N$ and $K N$ scatterings at high energy in order to provide a supplementary method for an extraction of properties of nucleon resonances in the low energy region. However, the information obtained from these analyses is less straightforward to current model calculations based on the effective Lagrangian approach such as the standard baryon pole model [9], because the residues in the $t$-channel helicity Regge poles fitted to empirical data in Refs. [6–8] cannot communicate with coupling strengths of hadron interactions in the Lagrangian formalism.

Therefore, it is desirable to investigate $\pi N$ scattering with hadron models that can utilize the effective Lagrangians for the description of the reaction beyond resonances up to the pion momentum $P_{\text{lab}} \approx 250$ GeV/c, the highest energy where data point exists. Unfortunately, however, there is no theory, or no model calculations at present available for such purpose.

In this paper, we investigate $\pi N$ charge exchange and elastic scatterings for the analysis of the reaction mechanism by the peripheral process. For doing this, we construct the Born amplitude to be reggeized for the meson exchange with our interest in establishing the reaction amplitude up to such high momentum with the interaction Lagrangians and the coupling constants sharing with other hadron reactions. Another issue to be addressed here is to provide the Pomeron exchange that could be well suited for the reaction amplitude thus constructed. The quark-Pomeron coupling picture is introduced to $\pi N$ elastic scattering similar to the case of photoproductions of neutral vector mesons $\pi N \to \gamma N$ [10–13]. Therefore, complementary to previous findings in Refs. [6–8], the result of this work may serve to a completion of our understanding the reaction mechanism of $\pi N$ scattering beyond resonances.

The paper is organized as follows. In Sec. II we begin with a statement of phenomenological features of charge exchange scattering $\pi^- p \to \pi^0 n$ to construct the Regge amplitude for the $t$-channel $\rho$ meson exchange. To account for the dip in the differential cross section we introduce Regge cuts [14], and to reproduce polarization asymmetry we consider a new trajectory $\rho'(1450)$. Sec. III follows the similar steps as in the previous section. Features of elastic scattering process $\pi^\pm p \to \pi^\mp p$ are introduced and the Regge pole amplitudes relevant to these reactions are fully constructed. The implementation of the Pomeron exchange from the quark picture...
II. CHARGE EXCHANGE SCATTERING

A. General features

By charge conservation and isospin symmetry the charge exchange reaction \( \pi^- p \rightarrow \pi^+ n \) allows only the single \( \rho \) exchange in the \( t \)-channel. Thus, it is natural to expect that differential cross sections would show up a dip structure at the nonsense wrong signature (NWSZ) of \( \rho \) trajectory \(-t \approx 0.5 \) (GeV/c)\(^2\) from \( \alpha_p(t) = 0 \). Moreover, polarization asymmetries in this process should appear vanishing, because the polarization asymmetry \( P(\theta) \) is defined as the interference between the spin non-flip and flip amplitudes, i.e.,

\[
P(\theta) = \frac{2 \text{Im}[M^+ M^{+\ast}]}{|M^+|^2 + |M^{+\ast}|^2},
\]

and the single \( \rho \) exchange which gives a dominant contribution to the spin flip amplitude cannot produce nontrivial phase interference itself between them. Therefore, the reaction needs more theoretical consideration such as the Regge cuts and other meson exchanges in the \( t \)-channel to reproduce the differential cross section data and polarization asymmetry at small angles.

B. Regge description

For the reaction \( \pi(k) + N(p) \rightarrow \pi(q) + N(p') \) process, we denote the incoming and outgoing pion momenta by \( k \) and \( q \), and the initial and final nucleon momenta by \( p \) and \( p' \), respectively. Then, conservation of 4-momentum requires \( k + p = q + p' \), and \( s = (k + p)^2 \), \( t = (q - k)^2 \), and \( u = (p' - k)^2 \) are the invariant mandelstam variables. The total energy \( W \) is related with the pion momentum in the laboratory system \( P_{\text{lab}} \) by the equation

\[
W = \sqrt{M^2 + m_\pi^2 + 2M\sqrt{P_{\text{lab}}^2 + m_\pi^2}}
\]

with \( M \) and \( m_\pi \) the nucleon and pion masses. The relevant expressions for the momenta in the Lab frame and c.m. frame are defined as

\[
P_{\text{lab}} = \sqrt{\left(\frac{s - M^2 - m_K^2}{2M}\right)^2 - m_K^2},
\]

\[
P_{\text{c.m.}} = \frac{1}{\sqrt{2s}} \sqrt{(s - (M + m_K)^2)(s - (M - m_K)^2)}.
\]

respectively.

Let us begin with the scattering amplitude simply given by the single \( \rho \) exchange

\[
\mathcal{M}(\pi^- p \rightarrow \pi^0 n) = -\sqrt{2} M_\rho, \tag{4}
\]

and the Born amplitude in Fig. \( \Box \) relevant to the \( t \)-channel \( \rho \) meson exchange written as

\[
\mathcal{M}_\rho = \Gamma_\rho \bar{q}(q,k) \left\{ \rho_{\mu\nu}^\rho (Q) \frac{\Gamma_\nu}{t - m_\rho^2} \right\} \rho_{\nu NN}(p',p), \tag{5}
\]

with \( Q = q - k \) is the \( t \)-channel momentum transfer. The coupling vertices with the spin polarization \( \Pi^\rho_{\mu\nu} \) are expressed as

\[
\Gamma^\rho_{\rho\pi\pi}(q,k) = g_{\rho\pi\pi}(q + k)^\mu, \tag{6}
\]

\[
\Gamma^\rho_{\rho NN}(p',p) = \bar{u}(p') \left[ g_{\rho NN} \gamma^\nu + \frac{g_{\rho NN}^\mu}{4M} \left[ \gamma^\nu, Q \right] \right] u(p), \tag{7}
\]

\[
\Pi^\rho_{\mu\nu}(Q) = -g_\rho^\mu + Q^\mu Q^\nu/m_\rho^2, \tag{8}
\]

from the Lagrangians for \( \rho \pi \) and \( \rho NN \) interactions,

\[
\mathcal{L}_{\rho\pi\pi} = g_{\rho\pi\pi} \bar{q} \times \partial \times (\bar{q} \times \partial), \tag{9}
\]

\[
\mathcal{L}_{\rho NN} = N \left( g_{\rho NN}^\nu \gamma^\nu - i \frac{g_{\rho NN}}{2M} \sigma_{\mu\nu} \partial^\nu \right) (\tau_a) \rho^\dagger N. \tag{10}
\]

In order to describe the reaction at high energies up to tens of GeV, we make the above Born amplitude reggeized by replacing the Feynman propagator with the Regge one,

\[
\frac{1}{t - m_\rho^2} \rightarrow \mathcal{R}^\varphi(s,t), \tag{11}
\]

where the Regge propagator is written as

\[
\mathcal{R}^\varphi(s,t) = \frac{\pi a'_J \times \text{phase}}{\Gamma(\alpha_J(t) + 1 - J) \sin[\pi \alpha_J(t)]} \left( \frac{s}{s_0} \right)^{\alpha_J(t) - J} \tag{12}
\]

for the \( \varphi \) meson of spin-\( J \) and \( s_0 = 1 \) GeV\(^2\) collectively. The trajectory of spin-\( J \) meson is denoted by \( \alpha_J(t) \). The phase factor is, in general, taken to be \( \frac{1}{2}(-1)^J + e^{-i\pi \alpha_J(t)} \) for the exchange nondegenerate meson exchange.

From Eq. \( \Box \) the decay width is evaluated as

\[
\Gamma(\rho \rightarrow 3\pi) = g_{\rho\pi\pi}^2 k^3 \frac{6\pi m_\rho^3}{a_J(t) - J} \tag{13}
\]

which leads to \( g_{\rho\pi\pi} = 5.95 \) from \( \Gamma(\rho \rightarrow \pi^+ \pi^-) = 147.8 \) MeV reported in the Particle Data Group (PDG). For the \( \rho NN \) coupling constants, we use \( g_{\rho NN}^\mu = 2.6 \) throughout this work for the consistency with our previous works on photoproductions of hadrons. We choose \( g_{\rho NN} = 9.62 \) to be consistent with the vector meson dominance (VMD) for the anomalous magnetic moment \( \alpha_\rho = 3.7 \). However, the universality of \( \rho \) meson coupling constant is not exact between \( g_{\rho\pi\pi} \) and \( 2g_{\rho NN}^\mu \), the latter of which is estimated from the \( \rho \) meson decay width \( \Gamma(\rho \rightarrow e^+ e^-) \).
Given the coupling constants above we take the $\alpha_\rho(t) = 0.9t + 0.46$ from the Regge analyses of charged $\rho$ photoproductions \cite{17,18} together with the exchange nondegenerate phase for the $\rho$ Regge-pole to reproduce the differential cross section at $P_{\text{Lab}} = 20.8$ GeV/c. The result is presented in Fig. 2 by the dashed curve.

For comparison we show the (red) dotted curve resulting from the single $\rho$ exchange with $\kappa_\rho = 3.7$ and $0.9t + 0.46$ is given by the same trajectory $\rho - P$ Pomeron. Without any model parameters such as cutoff masses of form factors in this work, then, the model-dependence lies in the Regge trajectories and meson-baryon coupling constants chosen. Thus, by taking the advantage of the single $\rho$ dominance of the reaction, it is interesting to question what is the proper set for the $\rho$ meson coupling constants and trajectory. For comparison we show the (red) dotted curve resulting from the case of $\rho$ trajectory $0.8t + 0.55$ with $\kappa_\rho = 3.7$, and (blue) dash-dotted one from the $0.9t + 0.46$ with $\kappa_\rho = 6.2$. It is clear that both cross sections are overestimating experimental data. Moreover, the dip position at the NWSZ, $-t \approx 0.69$ (GeV/c$^2$) in the case of $0.8t + 0.55 = 0$ deviates from data. These findings in the $\pi^- p \rightarrow \pi^0 n$ support the validity of the $\alpha_\rho = 0.9t + 0.46$ with $\kappa_\rho = 3.7$ as much as in our previous study on the charged $\rho$ photoproductions \cite{17,18}.

- $\rho$-cuts

As discussed above we now find a way to fill up the deep dip in the differential cross section in Fig. 2. Similar to neutral pion photoproduction $\gamma p \rightarrow \pi^0 p$ \cite{14}, the $\rho$-cuts are introduced for this purpose, and the reaction amplitude in Eq. (11) is now extended to be \cite{13},

$$\mathcal{M}_\rho = \Gamma_{\rho\pi\pi}(q,k)\Pi_{\mu\nu}(Q)\Gamma_{\rho NN}(p',p) \times \left[ \mathcal{R}(s,t) + \sum_\varphi C_\varphi e^{d_\varphi t}e^{-i\rho(t)/2} \left( \frac{s}{s_0} \right)^{\alpha^\varphi(t)-1} \right], \quad (14)$$

where $\varphi = f_2$ and $P$ are the subsequent Regge pole exchanges following the $\rho$ exchange in the elastic cut, as shown in Fig. 1 (b). The cut parameters $C_\varphi$ and $d_\varphi$ represent the strength and range of the cut to be fitted to experimental data. The cut trajectory for $\rho-\varphi$ is given by a composite of two trajectories, i.e.,

$$\alpha^\varphi(t) = \frac{\alpha_\rho'}{\alpha_\rho' + \alpha_\varphi'} t + \left( \alpha_\rho(0) + \alpha_\varphi(0) - 1 \right) \quad (15)$$

with its slope and intercept consisting of each slope and intercept. Here we use the tensor meson $f_2$ trajectory $\alpha_{f_2}(t) = 0.9 t + 0.53$ sharing with the $\omega$ trajectory. The Pomeron trajectory is determined as $\alpha_P(t) = 0.12 t + 1.06$ by the fit of elastic scattering data at high momenta $P_{\text{Lab}} = 100$ and 200 GeV/c. We shall discuss this point in Sec. III later.

In Fig. 2 the differential cross section shows the cut effect to fill up the dip by the $\rho$ exchange with parameters $C_{f_2} = 1.0$ GeV$^{-2}$, $d_{f_2} = 2$ GeV$^{-2}$ for $\rho - f_2$, and $C_\varphi = 0.1$ GeV$^{-2}$, $d_\varphi = 5$ GeV$^{-2}$ for $\rho - P$, respectively. Such an agreement with experimental data as can be seen in Fig. 2 appears repeatedly in differential cross sections at other momenta at the same level of quality.

Polarization asymmetry $P$ in the $\pi N$ scattering is the observable that could validate the accuracy of model predictions. As defined in Eq. (11), it arises via the interference between exchanges of different mesons. It is obvious that the $\rho$ cuts could give rise to no interference, though added, because they share the same interaction vertices with the single $\rho$ as in Eq. (14). Thus, we consider another isovector exchange in the $t$-channel to find that $\rho(1450)$ of the same quantum number $1^+(1^-)$ with the $\rho(775)$ but of the higher mass could be a candidate to produce nonvanishing polarization.

- Daughter trajectory $\rho'(1450)$
To induce the phase interference between two different Regge poles, we consider the daughter trajectory of ρ meson of higher mass ρ(1450) with the ρπ decay mode evident but not measured yet. (For distinction we denote ρ(1450) by ρ′.) Therefore, no information is available for the ρ′ coupling to π, and to nucleon either. We treat these coupling constants as parameters to fit to polarization data. The trajectory relevant to ρ′(1450) is calculated from the relativistic quark model in Ref. [22] to be α′(t) = t − 1.23, which is different from that of ρ(775). Thus, the amplitude in Eq. (4) is extended to include the amplitude $M_{ρ′}$ of the same form as in Eq. (5) in addition to Eq. (14). In the calculation we use both the ρ and ρ′ trajectories exchange non-degenerate. Since the intercept of ρ′ trajectory is very low, the ρ′ exchange gives no contribution significantly altering the differential cross section at high momentum as in Fig. 2. Nevertheless, the interference of phases between ρ′ and ρ-cuts could reproduce the polarization data to a good degree.

Figure 3 shows the polarization measured at $P_{Lab}=3.5$ GeV/c in the range 0.2 ≤ −t ≤ 1.8 (GeV/c)² where the solid curve is prediction by the full amplitude

$$-\sqrt{2}[ρ + ρ − cuts + ρ′(1450)].$$

At the choice of ρ′(1450) coupling constants, $G_{ρ′}^w = 40$ and $G_{ρ′}^p = −75$ with $G_{ρ′}^{pπ} = g_{ρ′ππ}g_{ρ′NN}^p$, $G_{ρ′}^{ππ} = g_{ρ′ππ}g_{ρ′NN}^π$, we obtain a quite good fit of the polarization data. Of course, adjusting these values leads to some change of the polarization in magnitude around −t ≈ 0.5 (GeV/c)², but not in shape, unless the signs of coupling constants are changed. The dotted curves are from the single ρ(775) showing null polarizations as discussed. The dashed curve results from the ρ(775) with ρ′(1450), but without ρ-cuts. Though dependent on the coupling constants $G_{ρ′}^w$ and $G_{ρ′}^p$, we appreciate highly the implication of the ρ-cuts and daughter ρ′ which result in an agreement with vanishing of the polarization at −t ≈ 0.7 (GeV/c)² and a slow increase as the −t becomes larger.

We present total cross section in Fig. 4 for comparison with experimental data from threshold up to W = 10 GeV. The single ρ exchange is given by the dotted curve and the case of ρ(775) + ρ′(1450) by the dashed curve, respectively. The cross section from the full amplitude $ρ + ρ′ + ρ-cuts$ is depicted by the solid curve.

III. ELASTIC SCATTERING PROCESS

A. General features

Elastic scatterings $π^±p → π^±p$ proceed via the s-channel Δ and $N^*$ excitations to show the prominent resonance structures around $W ≈ 1.2$ and 1.6 GeV, respectively. Over the resonances the peripheral scattering of the t-channel meson exchange dominates the reactions to gradually exhibit a slow increase as the reaction energy increases, i.e., the diffraction scattering which is a manifestation of the Pomeron exchange in hadron elastic reactions. Differential cross sections have smooth t-dependence and no dips there. The sign of nucleon spin polarization at the very small angle is consistent with the sign of pion charge in the $π^±p$ reactions.

B. Regge description

Previous studies on the $πN$ reaction are devoted to the analysis of total cross section which includes all the inelastic subprocesses in addition to the elastic one [6,23].
In the conventional approach where the residues are fitted to scattering data in the $t$-channel helicity Regge poles, the role of $\rho$ exchange was expected to account for the difference of total cross sections between the $\pi^+p$ and $\pi^-p$ reactions in addition to the Pomeron and the second vacuum exchange at high energy [22]. Recent models of JPAC [6, 7] improved the Regge amplitude to include tensor meson $f_2$ exchange instead of the second vacuum exchange.

In contrast to these works, however, the exclusive elastic reactions for $\pi^{\pm}p \to \pi^{\pm}p$ are the main topics of the present section to be investigated in the Reggeized Born term model as in previous section. The Lagrangian formulation of hadron interactions in the model requires those mesons that are decaying to $\pi\pi$ in the $t$-channel exchange. In this respect exchanges of scalar meson and vector meson $\omega$ are further included in the present work. The Pomeron exchange is viewed from the quark-Pomeron coupling picture and we construct a new amplitude, which is of Donnachie and Landshoff (DL) type rather than the original version by Pichowsky [6].

As advertised the reaction amplitude that contains all $\pi\pi$ exchange instead of the second vacuum exchange was expected to account for the difference of total cross sections between the $\pi\pi$ exchange and $\rho$ exchange. The simultaneous fitting of these experiments was made in the present section to be investigated in the Reggeized Born term model. Therefore, we favor to adopt the vector coupling scheme for $\omega$ and $\rho$ meson exchanges.

Scalar meson $\sigma$ exchange

The mass and full width of scalar meson $\sigma$ are reported to be $m_{\sigma} = 400 \sim 550$ MeV and $\Gamma_{\sigma} = 400 \sim 700$ MeV in the PDG. The scalar meson $\sigma$ is the lightest meson to exchange. So it could contribute to the threshold behavior of reaction cross sections for $\pi^{\pm}p$ elastic scattering.

The interaction Lagrangians relevant to the coupling of $\sigma$ meson to hadrons are given by

\begin{equation}
\mathcal{L}_{S} = -\frac{1}{2}g_{\sigma\pi\pi}m_{\sigma}\vec{\sigma}\cdot\vec{\pi} - g_{\sigma NN}\sigma\bar{N}N, \tag{18}
\end{equation}

\begin{equation}
\mathcal{L}_{V} = \frac{f_{\sigma\pi\pi}}{2m_{\pi}}\sigma\partial_{\mu}\vec{\pi}\cdot\partial^{\mu}\vec{\pi} + g_{\sigma NN}\sigma\bar{N}N, \tag{19}
\end{equation}

where the scalar and vector couplings are considered for $\sigma\pi\pi$ coupling in the manner consistent with the scalar meson $\sigma$ as the two pion $s$-wave correlation. However, the uncertainty in the broad decay width makes our estimate of $\sigma\pi\pi$ coupling constant very model-dependent. A naive estimate of $g_{\sigma\pi\pi}$ by taking $\Gamma(\sigma \to \pi\pi) = 400$ MeV, for instance, from the decay width

\begin{equation}
\Gamma(\sigma \to \pi^+\pi^-) = \frac{2}{3}\Gamma(\sigma \to \pi\pi) = \frac{g_{\sigma\pi\pi}^2m_{\pi}^2k}{12\pi m_{\sigma}^2}, \tag{20}
\end{equation}

yields the value $g_{\sigma\pi\pi} = \pm 20.37$, which is larger than the value $7.91 \sim 16.54$ extracted from the $J/\psi$ decay [27, 28]. Here, the factor $2/3$ is taken into account for charged channels in the isospin space.

We now write the Born amplitude for $\sigma$ exchange as,

\begin{equation}
\mathcal{M}_{\sigma} = \frac{\Gamma_{\sigma NN}(p', p)}{\sqrt{t - m_{\sigma}^2}} \Gamma_{\sigma NN}(p', p), \tag{21}
\end{equation}

where the scalar and vector coupling vertices are given by

\begin{align}
\Gamma_{S}(q, k) &= g_{\sigma\pi\pi}m_{\pi}, \\
\Gamma_{V}(q, k) &= \frac{f_{\sigma\pi\pi}}{m_{\pi}}q \cdot k,
\end{align}

and

\begin{equation}
\Gamma_{\sigma NN}(p', p) = g_{\sigma NN}\bar{u}(p')u(p), \tag{24}
\end{equation}

for the $\sigma$-meson nucleon coupling vertex.

The $\sigma$ meson exchange as the two pion correlation in the $s$-state with a broad width was studied in the $\pi\pi \to NN$ reaction [29]. In the $\pi N$ scattering, as a result, the $t$-channel $\sigma$-pole derived from the dispersion relation is expressed as

\begin{equation}
\mathcal{M}_{\sigma} = \bar{u}(p')g_{\sigma}(t - m_{\sigma}^2) - \frac{2m_{\pi}^2}{t - m_{\sigma}^2}u(p), \tag{25}
\end{equation}

which corresponds to

\begin{equation}
g_{\sigma}(t) = \frac{g_{\sigma NN}g_{\rho\pi\pi}m_{\pi}}{2q \cdot k}, \tag{26}
\end{equation}

\begin{equation}
g_{\sigma}(t) = \frac{g_{\sigma NN}f_{\sigma\pi\pi}}{2m_{\pi}}, \tag{27}
\end{equation}

for the scalar and vector couplings in Eq. (21), respectively. These results imply that the $\sigma$-pole with the large decay width term, $1/(t - m_{\sigma}^2 + i\Gamma_{\sigma}m_{\sigma})$, in the pole model can be equivalently expressed as in Eq. (25) from dispersion relation. In Eqs. (26) and (27), while the latter term remains constant, the former has the energy dependence, $1/(q \cdot k)$, which is singular at threshold. Therefore, we favor to adopt the vector coupling scheme.
for $\sigma$ exchange in Eq. \((21)\) with the coupling constant $f_{\pi \pi}$ properly chosen to describe cross section data near threshold. In the calculation we use $g_{\rho NN} = 14.6$ for the consistency with photoproduction of neutral vector mesons \(^{12,13}\).

- Tensor meson $f_2$ exchange
  
  For the $f_2$ tensor meson exchange, we use the following interaction Lagrangian,
  
  \[ \mathcal{L}_{f_2\pi\pi} = \frac{2g_{f_2\pi\pi}}{m_{f_2}} \mu_{\pi\pi} \cdot \partial_{\mu} \partial_{\nu} f^{\mu\nu} \]  

  (28)

  for the $f_2\pi\pi$ coupling with $f^{\mu\nu}$ the spin-2 tensor meson field. This gives the coupling vertex
  
  \[ e_{\mu\nu} \Gamma_{f_2\pi\pi}^{\mu\nu} = \frac{g_{f_2\pi\pi}}{m_{f_2}} (k+q)^\mu (k+q)^\nu e_{\mu\nu} \]  

  (29)

  with $e_{\mu\nu}$ the spin-2 polarization tensor. The decay width for $f_2 \rightarrow \pi\pi$ is given by
  
  \[ \Gamma(f_2 \rightarrow \pi^+\pi^-) = \frac{2}{3} \Gamma(f_2 \rightarrow \pi\pi) = \frac{4g_{f_2\pi\pi}^2}{15\pi} \frac{p^5}{m_{f_2}^2} \]  

  (30)

  where $p = \sqrt{m_{f_2}^2/4 - m_{\pi}^2}$ is the momentum of $\pi$ meson.

  From the full width in the range, $121 \lesssim \Gamma(f_2) \lesssim 240$ MeV in PDG with the branching fraction $84.2\%$ for the $f_2 \rightarrow \pi\pi$, the coupling constant is estimated to be in the range $4.76 \lesssim g_{f_2\pi\pi} \lesssim 6.71$ in unit of $m_{f_2}^3$.

  The reaction amplitude for the $f_2$ exchange is written as
  
  \[ \mathcal{M}_{f_2} = \frac{\Gamma_{f_2\pi\pi}(q,k)}{t - m_{f_2}^2} \frac{\Gamma_{f_2NN}(p',p)}{\Gamma_{f_2NN}(p',p)} \]  

  (31)

  where the tensor meson-nucleon coupling vertex and the polarization tensor for spin-2 propagation are given by
  
  \[ \Gamma_{f_2NN}(p',p) = \bar{u}(p') \left[ \frac{2g_{f_2NN}^{(1)}}{M} (P^\alpha g^\beta + P^\beta g^\alpha) + \frac{4g_{f_2NN}^{(2)}}{M^2} P^\alpha P^\beta \right] u(p), \]  

  (32)

  and the spin projection operator for spin-2 particle
  
  \[ \Pi_{f_2}^{\mu\nu,\alpha\beta}(Q) = \frac{1}{2} (\tilde{g}^{\mu\alpha} g^{\nu\beta} + \tilde{g}^{\nu\alpha} g^{\mu\beta} - \frac{1}{3} \tilde{g}^{\mu\nu} g^{\alpha\beta}) \]  

  (33)

  with
  
  \[ \tilde{g}^{\mu\nu} = -g^{\mu\nu} + Q^\mu Q^\nu / m_{f_2}^2. \]  

  (34)

  The tensor-meson nucleon coupling constant extracted from the tensor meson dominance was $g_{f_2NN}^{(1)} = 2.12$ and $g_{f_2NN}^{(2)} \approx 0$. But the phenomenological information extracted from the dispersion relation as well as the partial wave analysis for $\pi N$ scattering suggested rather the scattered values for the $f_2NN$ coupling constants as discussed in Ref. \(20\), which showed

| meson | trajectory($\alpha_{f_2}$) | phase factor | $g_{\rho NN}$ | $g_{\omega NN}$ |
|-------|--------------------------|-------------|--------------|---------------|
| $\rho$ | 0.9 + 0.46               | (−1 + e$^{-i\alpha_{f_2}}$/2) | 5.95         | 2.6 (9.62)   |
| $\sigma$ | 0.7(t − m$^2_{\rho}$)      | (1 + e$^{-i\alpha_{f_2}}$/2) | 0.5          | 14.6         |
| $\omega$ | 0.9t + 0.44            | 1           | −0.18        | 15.6 (0)     |
| $f_2$ | 0.9t + 0.53          | 1           | 4.5          | 6.45 (0)     |

**TABLE I.** Listed are the physical constants and Regge trajectories with the corresponding phase factors for $\pi^\pm p \rightarrow \pi^\pm p$. The symbol $\varphi$ stands for $\sigma$, $\omega$, $f_2$ and $\rho$. For the $\sigma$ couplings $g_{\sigma NN}$ should be understood as the vector coupling constant $f_{\sigma NN}$.

![FIG. 5. Quark diagram for the Pomeron exchange in $\pi^\pm p$ elastic scatterings. Pseudoscalar coupling $\pi q\bar{q}g$ with the coupling constant $f_{qq}$ is assumed at the $\pi qq$ coupling vertex. Momenta for quark loops are denoted by $l$, $l + k$, and $l + q$. Quark loop of momentum $l + k$ is off mass-shell. Vector coupling $\gamma^5$ is taken for the couplings of the Pomeron-quark and Pomeron-proton currents.](image)

2.12 $\lesssim g_{f_2NN}^{(1)} \lesssim 7.93$ and $g_{f_2NN}^{(2)} \approx 0$. In those meson photoproductions involving the tensor meson exchange we used $g_{f_2NN}^{(1)} = 6.45$ and $g_{f_2NN}^{(2)} = 0$ \(30\) to agree with empirical data. For the elastic scattering of $\pi p \rightarrow \pi p$ reaction we resume these values for the sake of consistency, and make a list for the coupling constants and trajectories with the corresponding phase factors in Table I.

- Pomeron exchange

The quark-Pomeron coupling model, as depicted in Fig. 5, is based on the factorization of the exclusive $\pi N$ scattering amplitude in terms of the product of the $\pi \rightarrow q + \bar{q}$ fluctuation, the scattering of the $\bar{q}q$ system by the proton, and finally the $\bar{q}q$ hadronization into a pion. From the observation of total cross sections for $pp$, $\pi p$ and $Kp$ reactions at high energies, Donnachie and Landshoff stated that the Pomeron couples to the separate valence quark inside a hadron rather than to the hadron as a whole, and the strength of the Pomeron coupling to a hadron is determined by the radius of hadron. Therefore, assuming the quark-Pomeron coupling strength $F_h(t)\beta q^5$ with the hadron form factor $F_h(t)$ for its size, the Pomeron contribution to the $\pi N$
cross section can be simply written as \[10\]
\[
\frac{d\sigma}{dt} = \frac{1}{4\pi} \left[ 2\beta_q F_{\pi}(t) \right] (-i\alpha'_p s)\alpha_p(t)^{-1} [3\beta_q F_1(t)]^2
\]
with the nucleon isoscalar form factor and pion form factor given by
\[
F_1(t) = \frac{4M^2 - 2.8t}{(4M^2 - t)(1 - t/0.71\text{ GeV}^2)^2},
\]
\[
F_{\pi}(t) = (1 - t/\Lambda^2)^{-n},
\]
and the Regge-type propagator,
\[
\mathcal{R}^\pi(s,t) = (\alpha_p' s)^{\alpha_p(t)-1} e^{-i\frac{\pi}{2}[\alpha_p(t)-1]}.
\]
Here \(\alpha_p(t)\) is the Pomeron trajectory of the form
\[
\alpha_p(t) = \alpha'_p t + \alpha''_p.
\]

A more rigorous treatment of the Pomeron exchange in the \(\pi N\) elastic scattering can be found in Ref. \[5\], where the quark-meson coupling vertices in the incoming and outgoing states should be the Bethe-Salpeter amplitudes with the quark propagation arising from the Dyson-Schwinger equation for the bound state of the QCD. However, in the large momentum limit, the current quark propagation could be replace by the free quark (constituent quark) propagation with the constituent quark mass \(m_{u(d)} \approx 330\) and \(m_s \approx 490\) MeV. Hence, the on-shell approximation for the quark loops of \(l + q\) for the outgoing pion, while the quark loop of \(l+k\) considered as being off-shell with the hadron form factor at the Pomeron-\(\pi\pi\) vertex, is a good approximation to perform the loop-integral \[11\]. In this work we follow the on-shell approximation as the Donnachie-Landshoff ansatz \[31\] for vector meson photoproduction \[32\], and take the pseudoscalar coupling
\[
f_{\pi qq}g_{i}\gamma_5 q\pi
\]
for the \(\pi qq\) vertex with the coupling strength \(f_{\pi qq}\) in Fig. \[6\].

The on-shell approximation leads to the loop integral simple and the trace calculation in the loop results in the following expression
\[
\text{Tr}[(l + m_q)\gamma_\mu((l + k) + m_q)\gamma_\mu((l + q) + m_q)\gamma_5]
\]
\[
= -4l\cdot qk_\mu - 4l\cdot kq_\mu + 4k\cdot ql_\mu,
\]
\[
m_\mu^2(k_\mu + q_\mu).
\]
In the quark loop in Fig. \[5\] the two quarks in the outgoing pion state share the equal pion momentum, \(l = -q/2\), with the assumption that they are nearly on-shell. Then, the other quark loop of momentun \(l+k\) in the figure is off-shell and the propagator turns out to be
\[
\frac{1}{(l+k)^2 - m_q^2} = -\frac{2}{2m_q^2 - m_\mu^2/2 - t}
\]
with \(l = -q/2\).

For the \(\pi N\) elastic scattering, therefore, the Pomeron exchange is written as,
\[
\mathcal{M}_p = i2F_{\pi}(t)\beta_q \frac{2m_q^2 f_{\pi qq}^2}{2m_q^2 - m_\mu^2/2 - t} F_{\pi qq}(t)
\]
\[
\times 3F_1(t)\beta_q u(p')^{(k + q)}u(p)\mathcal{R}^\pi(s,t),
\]
where the \(F_{\pi}(t)\beta_q\gamma^\mu\) and \(F_1(t)\beta_q\gamma_\mu\) with \(\beta_u = 2.07\) GeV\(^{-1}\) and \(\beta_\mu = \beta_u\) are the Pomeron coupling to a quark in the pion and in the nucleon as discussed in Eq. \[35\]. The form factor \[31\]
\[
F_{\pi qq}(t) = \frac{2\mu_\pi^2}{2\mu_\pi^2 + 2m_q^2 - m_\mu^2/2 - t}
\]
is included to ensure the convergence of the off shell quark loop with the cutoff mass \(\mu_\pi^2 = 1.1\) GeV\(^2\) fixed to experimental data \[33\].

Another quantity of new entry is the coupling constant \(f_{\pi qq}\) which is expected to obey the Goldberg-Treiman relation at the quark level as,
\[
\frac{f_{\pi qq}}{2m_q} = \frac{1}{2} \frac{3}{5} g_A.
\]
Given the nucleon axial charge \(g_A = 1.25\), pion decay constant \(f_\pi = 93.1\) MeV, and by using the quark mass \(m_q = 330\) MeV we determine \(f_{\pi qq} = 2.65\).

It is worth noting in Eq. \[12\] that the 
\[
t/(2 + k^2 - q^2/4 - m_q^2)
\]
becomes singular near \(-t \approx 0\) as \(k^2 - q^2 = m_\pi^2\) for the pion elastic scattering when \(m_\pi = 2m_q\) is assumed. For a better convergence of the quark loop in addition to the form factor \(F_{\pi qq}(t)\), therefore, we utilize the pion form factor \(F_{\pi}(t)\). Moreover, in order to adjust the range of the \(F_{\pi}(t)\) it is convenient to use the cutoff mass in Eq. \[37\] having an energy dependence as
\[
\Lambda(k) = \mu \left(W - W_{th}\right),
\]
where \(k\) is the incident pion momentum in the e.m. system, \(\mu\) is the parameter of mass unit, and \(W_{th}\) is the total energy at threshold.

Figure \[7\] shows the divergence of the Pomeron exchange depending on the quark mass, for instance, \(m_q = 140\) MeV taken without the form factor \(F_{\pi}(t)\). Dotted, dash-dotted, and dash-dot-dotted are the cases of Pomeron converging in the lower energy region due to the role of \(F_{\pi}(t)\) with the parameter \(\mu\) and power \(n\) as designated in the figure.

Figure \[7\] presents differential cross sections for \(\pi^+ p\) and \(\pi^- p\) elastic scatterings. In each reaction those cross sections at high momenta \(P_{lab} = 100\) and 200 GeV/c in the upper two panels are used to determine the Pomeron trajectory, while all the physical constants we take for \(f_{\pi qq}\), \(\beta_u\), \(\beta_\mu\), and \(\mu_\pi^2\) are fixed as before. Nevertheless, however, there is no criterion for what value we have to choose for the parameter \(\mu\) at present, because the existing data are insensitive to a change of \(\mu\). In this work we choose \(\mu = 1\)
FIG. 6. Dependence of Pomeron exchange on $F_+ (t)$. Given the physical pion mass $m_\pi$ and Pomeron trajectory in Eq. (47), the red dashed curve shows the divergence for the $m_\pi = 140$ MeV in the absence of $F_+ (t)$. $F_{+Q} \simeq 1.32$ is taken for a coincident with others for comparison. The rest of curves are resulting from the change of mass parameter $\mu$ and power $n$ with $m_\pi = 330$ MeV fixed.

FIG. 7. Differential cross sections $d\sigma / dt$ for $\pi^+ p$ (left) and $\pi^- p$ (right) elastic scattering at $P_{Lab} = 200, 100, 8.5,$ and 6 GeV/c, respectively. Dashed and dash-dotted curves in the lower two panels are the contributions of $f_2$ and Pomeron. Data at 100 and 200 GeV/c pion momenta are taken from Ref. [34] and data at 6.0 8.5 GeV/c are from Refs. [35, 36], respectively.

FIG. 8. Total cross section $\sigma$ for elastic reactions $\pi^+ p$ (blue) and $\pi^- p$ (red). Notations for $f_2$ and Pomeron are the same as in Fig. 7 for both processes. The difference between $\pi^+ p$ and $\pi^- p$ cross sections is due to the roles of $\rho + \omega$ exchanges. Dominance of $f_2+ Pomeron exchanges are apparent. World data are taken from PDG [37].

GeV and $n = 2$ for illustration purpose. Then, by leaving the slope and intercept of the Pomeron trajectory $\alpha_P (t)$ free parameters to fit to high energy data, we obtain a good agreement with the energy and $t$-dependence of the cross sections at the choice of

$$\alpha_P (t) = 0.12 t + 1.06$$

(47) for Eq. (49). We note that the slope in the $\pi N$ scattering is consistent with Ref. [8], but by the factor of 1/2 slower than that of the Pomeron $\alpha_P (t) = 0.25 t + 1.08$ fitted to total cross section of $\pi N$ reaction [26]. Note that the slope of the total cross section at high energies given as the energy to the power $\sim \sigma^{0.808}$ (thus, $\alpha(0) = 1.08$ by $\sigma \sim \sigma^{1.08}$) is far different from that of the elastic cross section of the present issue as can be seen in Fig. [8].

In Fig. [8] we present total elastic cross sections for $\pi^+ p$ and $\pi^- p$ where the contribution of the meson exchange as well as that of the Pomeron are shown. A few remarks are in order on the features of meson exchanges; The vector mesons $\rho$ and $\omega$ are responsible for the difference between $\pi^+ p$ and $\pi^- p$ cross sections, as shown from threshold up to $P_{Lab} \approx 2$ GeV/c. At high momenta, the exchanges of $f_2$ and Pomeron in the isoscalar channel are dominant over $\rho$ and $\omega$ so that the two cross sections coincide with each other, which should be distinguished from the difference between the total cross sections at high energy as in Ref. [6, 25, 31]. Thus, the reaction mechanisms of $\pi^\pm p$ elastic reactions are characterized by the dominance of the natural parity exchange in the isoscalar channel.

Polarization of target proton is the observable that could verify the accuracy of model predictions for the experimental measurement. To show the validity of the present model we present the polarization for $\pi^\pm p$ reactions in Fig. [9] at the intermediate and high momenta $P_{Lab} = 14$ and 100 GeV/c. In these results the mirror symmetry between $\pi^+ p$ and $\pi^- p$ which is the feature of the polarizations of opposite charges is well reproduced in any momentum range. In particular, polarizations are sensitive to the contribution of $f_2$ exchange with the coupling constant $g_{f_2NN}^{(2)} = 0$ for the better agreement with data.
IV. BARYON RESONANCES BELOW $W \leq 2$ GEV

In this section we present the nucleon resonances in the energy-dependence of cross section based on the $t$-channel exchanges as discussed in previous sections. More data from the angular distributions and spin polarizations could make improved the resonance parameters more precisely. The most updated analysis for the nucleon resonances can be found in the SAID programme with Ref. [40]. However, such a fine-tuning is beyond the scope of the present work and our aim here is to demonstrate how the Regge poles are well suited for the nucleon resonance of the Breit-Wigner form in the reaction amplitude,

$$M = (M_{\text{Regge}} + M_p) + M_R.$$  \hfill (48)

By the conventional definition of the nonrelativistic scattering amplitude as in the Appendix A, we write the scattering amplitude as

$$M_R = \frac{8\pi W}{\sqrt{4M^2}} \sqrt{\frac{k}{q}} \left[ F(s, \theta) + i\sigma \cdot \hat{n} G(s, \theta) \right],$$ \hfill (49)

with $\hat{n} = \hat{k} \times \hat{q} / \sin \theta$, and consider the spin non-flip and flip amplitudes to be of the form, i.e., [41]

$$F(s, \theta) = \frac{1}{k} \sum R \frac{c_R (J_R + 1/2)}{\epsilon_R - i} e^{-d \epsilon_R} P_l (\cos \theta),$$ \hfill (50)

$$G(s, \theta) = \frac{1}{k} \sum R \frac{c_R (-1)^{J_R - l + 1/2}}{\epsilon_R - i} e^{-d \epsilon_R} \frac{dP_l (\cos \theta)}{d \cos \theta},$$ \hfill (51)

with the $d$ in the Gaussian type of the damping factor to adjust the width of the resonance. Here, $c_R = I_R X_R$ is a sort of the coupling strength of the resonance $R$ originating from the product of the Clebsch-Gordon coefficient for isospin and elasticity. $\epsilon_R = (M_R^2 - s)/M_R \Gamma_R$ is the $s$-channel pole with the mass and full width of the resonance $\Gamma_R$. $k$ and $\theta$ are the momentum and scattering angle.

| Process | Resonance | $M_R$ | $\Gamma_R$ | $c_R$ | $d$ |
|---------|-----------|------|-----------|------|-----|
| I       | $\Delta^0(1232)$ $P_{33}$ | 1232 | 125 | 0.5 | 0.3 |
|         | $N^*(1440)$ $P_{11}$ | 1440 | 400 | 0.6 | 0.7 |
|         | $N^*(1535)$ $S_{11}$ | 1510 | 150 | 0.4 | 0.5 |
|         | $N^*(1650)$ $S_{11}$ | 1650 | 125 | 0.7 | 0.4 |
|         | $N^*(1720)$ $P_{13}$ | 1720 | 250 | 0.3 | 0.4 |
|         | $\Delta^0(1905)$ $F_{35}$ | 1900 | 300 | 0.2 | 0.1 |
| II      | $\Delta^0(1232)$ $P_{33}$ | 1232 | 125 | 0.35 | 2.2 |
|         | $N^*(1440)$ $P_{11}$ | 1440 | 400 | 0.6 | 0.5 |
|         | $N^*(1535)$ $S_{11}$ | 1510 | 150 | 0.6 | 0.5 |
|         | $N^*(1650)$ $S_{11}$ | 1650 | 125 | 0.75 | 0.4 |
|         | $N^*(1720)$ $P_{13}$ | 1720 | 250 | 0.3 | 0.4 |
|         | $\Delta^0(1905)$ $F_{35}$ | 1900 | 300 | 0.2 | 1 |
| III     | $\Delta^+(1232)$ $P_{33}$ | 1235 | 120 | 2 | 0.03$^\text{(*)}$ |
|         | $\Delta^+(1905)$ $F_{35}$ | 1900 | 400 | 0.5 | 0.9 |

Figure 10 shows the total cross section for $\pi^- p \rightarrow \pi^- n$ in which case the $t$-channel meson exchanges in Eq. [10] constitutes a background contribution upon which nucleon resonances are mounting. Nucleon resonances $\Delta(1232)$, $N^*(1440)$, $N^*(1535)$, $N^*(1650)$, $N^*(1720)$, and $\Delta(1905)$ are introduced with their parameters fitted to the total cross section data as in Table III. Note that, unlike the Ref. [42], the contribution of the $t$-channel
exchanges in the present calculation are not passing through the average of the cross section on the energy interval below $W \approx 2$ GeV, and we expect that the problem of double-counting should be insignificant.

The resonance structures in $\pi^- p \to \pi^- p$, and $\pi^+ p \to \pi^+ p$ reactions are presented in Figs. 11 and 12. It is worth remarking that the scalar meson coupling constant $f_{\sigma \pi \pi} = -0.5$ is taken with its sign reversed, because it is advantageous to alleviate the problem of double-counting by reducing the contribution of the $\sigma$ meson which may overlap with resonance. On the other hand, we have to neglect the threshold divergence of the $\Delta$ pole in Figs. 11 and 12 which are not covered up by the Gaussian damping factor in the multipoles. In practice, it is the drawback of the present model calculation of the resonances, formulated as in Eqs. (50) and (51). Moreover, in the case of $\pi^+ p \to \pi^+ p$ reaction where we have to reproduce the $\Delta^{++}(1232)$ pole with such a wide width that amounts to $500 \sim 600$ MeV as can be seen in Fig. 12, the threshold divergence is even worse. In order to suppress the strong divergence near threshold, we apply the cutoff function in Eq. (37) for the $\Delta^{++}(1232)$ pole with $n = 1$ and $\mu = m_\pi$ in Eq. (41), in addition to the Gaussian damping factor.

V. SUMMARY AND CONCLUSIONS

In this work we have investigated $\pi^- p \to \pi^0 n$ charge exchange and $\pi^+ p \to \pi^+ p$ elastic reactions up to incident pion momentum $P_{\text{lab}} \approx 250$ GeV/c to provide a theoretical framework that could validate a consistency of the coupling strengths and forms of interaction Lagrangians between hadrons at low and high energies. For a description of the reaction at the Regge realm we utilizes the relativistic Born amplitude for the reggeization of the $t$-channel meson exchange. Through the reproduction of reaction cross sections the reaction mechanism by the $t$-channel meson and Pomeron exchanges are analyzed with the coupling constants for hadron interactions sharing with other hadron reactions, e.g., photoproductions of vector mesons.

A unique role of vector meson $\rho(775)$ in the charge exchange reaction is investigated. Given the single $\rho$ exchange with the deep dip at the NWSZ point $-t = 0.51$ GeV$^2$, the dip-filling mechanism for the differential cross section needs $\rho \rightarrow f_2$ and $\rho \rightarrow$ Pomeron cuts. In order to reproduce the spin polarization a second $\rho(1450)$ Regge-pole is called for with the trajectory predicted from the relativistic quark model, though the coupling constants of the $\rho(1450)$ are treated as free parameters. These theoretical entities yield the Regge description of the charge exchange process to a good degree.

The exchange of a soft Pomeron is newly constructed from the quark-Pomeron coupling picture and applied successfully for $\pi^+ p$ elastic scatterings with the trajectory quite different from that from the total cross sections for $\pi N$ reaction. The difference between the $\pi^+ p$ and $\pi^- p$ elastic cross sections is insignificant because of the minor roles of $\rho + \omega$ exchanges, while $f_2(1275)$ and Pomeron exchanges are dominant in the over all range of pion momentum. Polarizations are well reproduced with the mirror symmetry reflected between the two reactions of opposite charges.

Nucleon resonances below $W \leq 2$ GeV are reproduced in three channels, $\pi^- p \to \pi^0 n$, $\pi^+ p \to \pi^+ p$, and $\pi^- p \to \pi^- p$, and they are consistent with existing data within the masses, widths, and branching fractions reported in the PDG. These findings illustrate how the $t$-channel Regge poles in the present framework do well for the analysis of nucleon resonances in the low energy region as well as the description of the reactions at high energies.
ACKNOWLEDGMENTS

This work was supported by the National Research Foundation of Korea grant (Grant No. NRF-2017R1A2B4010117), and partially funded by (Grant No. NRF-2016K1A3A7A09005580).

Appendix A: Partial wave expansion for nucleon resonance

The scattering amplitude in the \( \pi N \) c.m. system is defined by

\[
\mathcal{M} = \chi^\dagger \left[ F(s, \theta) + i\sigma \cdot \hat{n} G(s, \theta) \right] \chi \quad (A1)
\]

with our convention for the normalization constant \( N = \sqrt{\frac{E + M}{2M}} \) for the Dirac spinor. Here \( \chi \) is the \( 2 \times 1 \) pauli spinor with spin and isospin indices understood.

The differential cross section is calculated by the equation

\[
\frac{d\sigma}{d\Omega} = \frac{q}{k} \left| \frac{\sqrt{4MM^*}}{8\pi W} \mathcal{M} \right|^2 = |F|^2 + \sin^2 \theta |G|^2 \quad (A2)
\]

The spin non-flip and flip parts of the scattering amplitude are expanded with the orbital momentum \( l \) and the total angular momentum \( J \),

\[
F(s, \theta) = \sum_{l=0} \left[ (l + 1) f_{l+} + l f_{l-} \right] P_l (\cos \theta), (A3)
\]

\[
G(s, \theta) = \sum_{l=1} \left[ f_{l+} - l f_{l-} \right] \frac{dP_l (\cos \theta)}{d\cos \theta}. (A4)
\]

Each partial wave of \( \alpha = l \pm \) is related to the phase shift by

\[
f_\alpha(s) = \frac{1}{2ik} \left( e^{2i\delta_\alpha} - 1 \right). (A5)
\]

The energy-dependence of the partial wave for the spin non-flip and flip amplitudes in Eqs. \( (A3) \) and \( (A4) \) are parameterized as in Eqs. \( (50) \) and \( (51) \).

[1] D. Siemens, V. Bernard, E. Epelbaum, A. M. Gasparian, H. Krebs, and Ulf-G. Meißner, Phys. Rev. C 96, 055205 (2017).
[2] R. A. Arndt, W. J. Briscoe, I. I. Strakovsky, and M. M. Pavan, Phys. Rev. C 69, 035213 (2004).
[3] R. A. Arndt, W. J. Briscoe, I. I. Strakovsky, and R. L. Workman, Phys. Rev. C 74, 045205 (2006).
[4] R. Koch and E. Pietarinen, Nucl. Phys. A 336, 331 (1980).
[5] M. A. Pichowsky and T.-S. H. Lee, Phys. Rev. D 56, 1644 (1997).
[6] V. Mathieu, I.V. Danilkin, C. Fernández-Ramírez, M.R. Pennington, D. Schott, A. P. Szczepaniak, and G. Fox, Phys. Rev. D 92, 074004 (2015).
[7] J. Nys, A. N. Hiller Blin, V. Mathieu, C. Fernández-Ramírez, A. Jackura, A. Pilloni, J. Ryckebusch, A. P. Szczepaniak, and G. Fox, arXiv:1806.01891 [hep-ph].
[8] F. Huang, A. Sibirtev, S. Krewald, C. Hanhart, J. Haidenbauer, and U.-G. Meißner, Eur. Phys. J. A 40, 77 (2009).
[9] C. T. Hung, S. N. Yang, and T.-S. H. Lee, J. Phys. G 20, 1531 (1994).
[10] A. Donnachie and P. V. Landshoff, Nucl. Phys. B 244, 322 (1984).
[11] J.-M. Laget and R. Mendez-Galain, Nucl. Phys. A 581, 397 (1995).
[12] B.-G. Yu, H. Kim, and K.-J. Kong, Phys. Rev. D 95, 014020 (2017).
[13] B.-G. Yu and K.-J. Kong, arXiv:1710.04511 [hep-ph].
[14] A. Donnachie and Yu. S. Kalashnikova, Phys. Rev. C 93, 025203 (2016).
[15] A. I. Titov and T.-S. H Lee, Phys. Rev. C 66, 015204 (2002).
[16] Y. Oh, A. I. Titov, and T.-S. H. Lee, Phys. Rev. C 63, 025201 (2001).
[17] B.-G. Yu and K.-J. Kong, Phys. Lett. B 765, 221 (2017).
[18] B.-G. Yu and K.-J. Kong, arXiv:1612.02071 [hep-ph].
[19] A. V. Barnes, D. J. Mellema, A. V. Tolstrepur, R. L. Walker, O. I. Dahl, R. A. Johnson, R. W. Kenney, and M. Pripleit, Phys. Rev. Lett. 37, 76 (1976).
[20] Y. Oh and T.-S. H. Lee, Phys. Rev. C 69, 025201 (2004).
[21] S. J. Brodsky, G. F. de Téramond, and H. Günter Dorsch, arXiv:1302.5390 [hep-ph].
[22] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Rev. D 79, 114029 (2009).
[23] D. Hill, P. Koehler, T. Novey, P. Rynes, B. Sandler, H. Spinka, A. Yokosawa, D. Eartly, K. Pretzl, G. Burleson, G. Hicks, C. Wilson, and W. Risk, Phys. Rev. Lett. 30, 239 (1973).
[24] W.D. Apel et al., Nucl. Phys. B 154, 189 (1979).
[25] R. J. N. Phillips and W. Ranita, Phys. Rev. 139, B1336 (1965).
[26] A. Donnachie and P. V. Landshoff, Phys. Lett. B 296, 227 (1992).
[27] A. V. Friesen, Yu. L. Kalinovsky, and V. D. Toneev, Phys. Part. Nucl. Lett. 9, 1 (2012).
[28] N. Wu, arXiv:hep-ex/0104050.
[29] C. Schütz, J. W. Durso, K. Holinde, and J. Speth, Phys. Rev. C 49, 2671 (1994).
[30] B.-G. Yu, T. K. Choi, W. Kim, Phys. Lett. B 701, 332 (2011).
[31] S. Donnachie, G. Dosch, P. Landshoff, and O. Nachtmann, Pomeron Physics and QCD, Cambridge University Press (2002).
[32] A. I. Titov, Y. Oh, S. N. Yang, and T. Morii, Phys. Rev. C 58, 2429 (1998).
[33] A. Donnachie and P. V. Landshoff, Phys. Lett. B 185, 403 (1987).
[34] C. W. Akerlof, R. Kotthaus, R. L. Loveless, D. I. Meyer, I. Ambats, W. T. Meyer, C. E. W. Ward, D. P. Eartly,
R. A. Lundy, S. M. Pruss, D. D. Yovanovitch, and D. R. Rust, Phys. Rev. D 14, 2864 (1976).

[35] I. Ambats, D. S. Ayres, R. Diebold, A. F. Greene, S. L. Kramer, A. Lesnik, D. R. Rust, C. E. W. Ward, A. B. Wicklund, and D. D. Yovanovitch, Phys. Rev. D 9, 1179 (1974).

[36] D. Harting et al., Nuovo Cim. 38, 60 (1965).

[37] C. Amsler et al. (Particle Data Group), Phys. Lett. B 667, 1 (2008), [http://pdg.lbl.gov/2009/hadronic-xsections/]

[38] M. Borghini et al., Phys. Lett. B 36, 493 (1971).

[39] I. P. Auer et al., Phys. Rev. Lett. 39, 313 (1977).

[40] R. L. Workman, R. A. Arndt, W. J. Briscoe, M. W. Paris, and I. I. Strakovsky, Phys. Rev. C 86, 035202 (2012).

[41] A. J. Lennox et al., Phys. Rev. D 11, 1777 (1975).

[42] R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968).