VORTICES IN PROTOPLANETARY DISKS

PATRICK GODON AND MARIO LIVIO

Space Telescope Science Institute, 3700 San Martin Drive, Baltimore, MD 21218; godon@stsci.edu; mlivio@stsci.edu

Received 1998 November 26; accepted 1999 May 6

ABSTRACT

We use a high-order accuracy spectral code to carry out two-dimensional, time-dependent numerical simulations of vortices in accretion disks. In particular, we examine the stability and the lifetime of vortices in circumstellar disks around young stellar objects. The results show that cyclonic vortices dissipate quickly, while anticyclonic vortices can survive in the flow for hundreds of orbits. When more than one vortex is present, the anticyclonic vortices interact through vorticity waves and merge together to form larger vortices. The exponential decay time \( \tau \) of anticyclonic vortices is of the order of 30–60 orbital periods for a viscosity parameter \( \alpha \approx 10^{-4} \) (and it increases to \( \tau \approx 315 \) for \( \alpha = 10^{-5} \)), which is sufficiently long to allow heavy dust particles to rapidly concentrate in the core of anticyclonic vortices in protoplanetary disks. This dust concentration increases the local density of centimeter-sized grains, thereby favoring the formation of larger scale objects that are then capable of efficiently triggering a state of gravitational instability. The relatively long-lived vortices discussed here may therefore play a key role in the formation process of giant planets.

Subject headings: accretion, accretion disks — circumstellar matter — hydrodynamics — planetary systems — stars: formation — stars: pre—main-sequence

1. INTRODUCTION

It is believed at present that the formation of the planets in the solar system took place via a progressive aggregation of dust grains in the primordial solar nebula. However, a mechanism for building planetesimals between the centimeter-sized grains (formed by agglomeration and sticking) and the meter-sized objects capable of triggering a state of gravitational instability is still lacking (see, e.g., Adams & Lin 1993 and the recent review of Beckwith, Henning, & Nakagawa 1999). Recently, it has been suggested (Barge & SOMMERIA 1995; Adams & Watkins 1995; Tanga et al. 1996) that heavy dust particles rapidly concentrate in the cores of anticyclonic vortices in the solar nebula, thus increasing the local density of centimeter-sized grains and favoring the formation of larger scale objects that are then capable of efficiently triggering gravitational instability. Gravitational instability is also enhanced because a vortex creates a larger local surface density in the disk. Consequently, even if giant planets form (as an alternative model suggests) initially because of gravitational instability in the gaseous disk (rather than by planetesimal agglomeration), vortices may still play an important role. The change in the Keplerian velocity of the flow in the disk caused by the anticyclonic motion in the vortex induces a net force toward the center of the vortex (the inverse happens in a cyclonic vortex). Consequently, within a few revolutions of the vortex around the protostar, the concentration of dust grains and the density in the anticyclonic vortex become much larger than outside of the vortex. Analytical estimates (Tanga et al. 1996) have shown that, depending on the unknown drag on the dust particles in the gas, the trapping time for dust in a small anticyclonic vortex (of size \( D/r \approx 0.01 \) or less) located at 5 AU (the distance of Jupiter from the Sun) is about \( 20 < \tau < 10^3 \) yr, i.e., between about 2 and 100 local periods of revolution. The mass that can accumulate in the core of the vortex on this timescale is of the order of \( 10^{-5} \) Earth masses (i.e., a planetesimal). Barge & SOMMERIA (1995) considered larger vortices (of size \( D \approx H \), where \( H \) is the disk half-thickness, and \( H/r \approx 0.06 \) at 5 AU) and obtained that on a timescale of the order of 500 orbits, a core mass of about 16 Earth masses can accumulate in the core of the vortex. Since small vortices are expected to merge into larger ones, these results should be regarded as complementary. The results of Tanga et al. (1996) could represent the early stages in planet formation (planetesimals), while the results of Barge & SOMMERIA (1995) could represent a later stage in the evolution (the formation of planets cores).

It is therefore important for the theory of planet formation to assess under which conditions vortices form in disks, whether or not they are stable, and how long they can survive in the disk.

Analytical results (Adams & Watkins 1995, using a geostrophic approximation) have shown that many different types of vortex solutions are possible in circumstellar disks. However, Adams & Watkins (1995) used simplifying assumptions and they were unable to estimate the lifetime and stability of the vortices. To resolve the issue of the nonlinear behavior of vortices (e.g., vortex merging, scattering, growth, etc.), one needs to carry out detailed numerical simulations of the flow.

Recent simplified simulations of a disk (Bracco et al. 1998), which assume an incompressible flow and solve the shallow-water equations, indicate that two-dimensional (large-scale) vortices do not fragment into small vortices because of the inverse cascade of energy (characteristic of two-dimensional flows). Rather, the opposite happens: small vortices merge together to form and sustain a large vortex. It is believed, for example, that the Great Red Spot in the atmosphere of Jupiter is a steady solitary vortex also sustained by the merging of small vortices (Marcus 1993; see also Ingersoll 1990). In this case, however, the strong winds in Jupiter’s atmosphere are also partially feeding vorticity from the background flow into the vortex, and the decay time of the vortex is much longer.

The shear in Keplerian disks inhibits the formation of vortices larger than a given scale length \( L_\ast = v/(\Omega_c \delta) \), where \( v \) is the rotational velocity of the vortex (assumed to
be subsonic; otherwise the energy is dissipated through sound waves and shocks), and \( \Omega \) is the angular velocity in the disk. At the same time, the viscosity dissipates quickly vortices smaller than a viscous scale length \( L_v \). Consequently, only vortices of size \( L \) satisfying \( L_v < L < L_s \) can survive in the flow for many orbits before being dissipated. In the calculations of Bracco et al. (1998), positive density perturbation, counterrotating (anticyclonic) vortices were found to be long-lived, while cyclonic vortices dissipated very quickly. As noted above, however, Bracco et al. (1998) assumed a shallow-water approximation for the disk. These authors do not specify the value of the specific heats ratio \( \gamma \) that was used in the simulations (for the shallow-water equation to represent a polytropic thin disk, one has to assume \( \gamma = 2 \); see, e.g., Nauta & Tóth 1998). Most important, the Mach number of the Keplerian flow (or, equivalently, the thickness of the disk) is not defined. At present, it is not clear if incompressible results (using a shallow-water approximation) are valid for more realistic disks, i.e., for compressible (and potentially viscous) disks with a density profile \( \rho \propto r^{-15/8} \) and a temperature profile \( T \propto r^{-3/4} \)—the standard Shakura-Sunyaev disk model (Shakura & Sunyaev 1973).

The goal of this paper is to model vortices in disks using a more realistic approach. We conduct a numerical study of the stability and lifetime of vortices in a standard disk model of a protoplanetary nebula. For this purpose, we use a time-dependent, two-dimensional, high-order accuracy hydrodynamical compressible code, assuming a polytropic relation. Here it is important to stress the following. The vorticity (circulation) of the flow (with a velocity field \( u \)) is defined as

\[
\mathbf{w} = \nabla \times \mathbf{u} .
\]  

\( \text{(1)} \)

Taking the curl of the Navier-Stokes equations (see, e.g., Tassoul 1978), one obtains an equation for the vorticity

\[
\frac{\partial \mathbf{w}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{w} \propto \nabla \rho \times \nabla \mathbf{P} + \cdots .
\]  

\( \text{(2)} \)

The first term on the right-hand side of the equation is the source term for the vorticity. Therefore, vorticity can be generated by the nonalignment of \( \nabla \rho \) with \( \nabla \mathbf{P} \) (e.g., merid-
ional circulation in stars is generated by this term). The resultant growth of circulation owing to this term is also known as the baroclinic instability. Here, however, we do not address the question of the vortex generation process, since we are using a polytropic relation \( P = K \rho^\gamma \) (\( K \) is the polytropic constant, \( \gamma = 1 + 1/n \), and \( n \) is the polytropic index), and the source term vanishes. In this case, the vorticity (vorticity per unit mass) is conserved (Kelvin’s circulation theorem, see, e.g., Pedlosky 1987) and vortices cannot be generated.

The next section presents the numerical modeling of the protoplanetary disk. The results are presented § 3 and a discussion follows.

2. ACCRETION DISK MODELINGS

The time-dependent equations (e.g., Tassoul 1978) were written in cylindrical coordinates \((r, \phi, z)\) and were further integrated in the vertical (i.e., \(z\)-) direction (e.g., Pringle 1981). We use a Fourier-Chebyshev spectral method of collocation (described in Godon 1997) to solve the equations of the disk. Spectral methods are global high-order accuracy methods (Gottlieb & Orszag 1977; Voigt, Gottlieb, & Hussaini 1984; Canuto et al. 1988). These methods are fast, accurate, and are frequently used to study turbulent flows (e.g., She, Jackson, & Orszag 1991) and interactions of vortices (Cho & Polvani 1996a). It is important to stress that spectral codes have very little numerical dissipation and that the only dissipation that occurs in the present simulations is caused by the \( \alpha \) viscosity introduced in the equations. All the details of the numerical method and the exact form of the equations can be found in Godon (1997). We therefore give only a brief overview of the modeling here.

The equations are solved in the plane of the disk \((r, \phi)\), with \(0 \leq \phi \leq 2\pi\) and \(R_0 \leq r \leq 2R_0\), where the inner radius of the computational domain, \(R_0\), has been normalized to unity. We use an alpha prescription (Shakura & Sunyaev 1973) for the viscosity law, \(\nu = \alpha c_s H = \alpha c_s^2/\Omega_K\), where \(c_s\) is the sound speed and \(H = c_s/\Omega_K\) is the vertical height of the disk. The unperturbed flow is Keplerian, and we assume a polytropic relation \(P = K \rho^{1 + 1/n}\) (\(n \) is the polytropic index, while the polytropic constant \(K\) is fixed by choosing \(H/r\) at the outer radial boundary). Both the inner and the outer

![Fig. 2.—Detailed view of an anticyclonic vortex in the disk, a few orbits later than shown in Fig. 1. The vorticity wings have folded up onto themselves owing to a Kelvin-Helmholtz instability (accompanied by the propagation of Rossby waves).]
radial boundaries are treated as free boundaries, i.e., with
nonreflective boundary conditions where the conditions are
imposed on the characteristics of the flow at the boundaries.

The Reynolds number in the flow is given by

$$R_e = \frac{L u}{v},$$

where $L$ is a characteristic dimension of the computational
domain, $u$ is the velocity of the flow (or more precisely, the
velocity change in the flow over a distance $L$), and $v$ is the
viscosity. Since we are solving for the entire disk, $L \approx r$, and
$u \approx v_K$, and the Reynolds number becomes

$$R_e = \alpha^{-1}(H/r)^{-2},$$

where we have substituted $v = \alpha H^2 \Omega_K$, since we are using
an $\alpha$ viscosity prescription. The Reynolds number in the
flow is very high (of the order of $10^4$ for the assumed
parameters).

The simulations were carried out without the use of spec-
tral filters and with a moderate resolution of $128 \times 128$
colloctions points. As noted above, the only dissipation in
the flow is caused by the $\alpha$ viscosity introduced in the equa-
tions (i.e., the Navier-Stokes equations).

3. NUMERICAL RESULTS

In all models presented here, we chose $H/r = 0.15$ to
match protoplanetary disks; however, similar results were
obtained using $H/r = 0.05$ and $H/r = 0.25$. The models
were first evolved in the radial dimension for an initial
dynamical relaxation phase (lasting several Keplerian orbits
at least). Then an axisymmetric disk was constructed from
the one-dimensional results, on top of which we introduced
an initial vorticity perturbation. The initial vorticity pertur-
bation had a Gaussian-like form and a constant rotational
velocity of the order of $\approx 0.2c_s$. In all the models we found
that cyclonic vortices dissipate very quickly, within about
one local Keplerian orbit, while anticyclonic disturbances
persist in the flow for a much longer period of time. It is
important to stress that in all the models, the anticyclonic
vortex, although rotating in the retrograde direction (in the
rotating frame of reference), has a rotation rate that is
slower than the local Keplerian flow. Consequently, in the
inertial frame of reference the vortex rotates in the prograde
direction, like a planet. As the models are evolved, the initial
vorticity perturbation is stretched and elongated by the
shear, within a few Keplerian orbits. The elongation of the
vorticity into a thin structure transfers entropy (the poten-
tial entropy is defined as the average of the square of the
potential vorticity) toward high wavenumbers. This
process is consistent with the direct cascade of entropy
characteristic of two-dimensional turbulence. It forms an
elongated negative (relative to the local flow) vorticity strip
in the direction of the shear, with an elongated vortex in the
middle of it (Fig. 1). Because of a Kelvin-Helmholtz insta-
bility, perturbations in a forming vorticity strip propagate
(along the strip) in the direction opposite to the shear. These
propagating waves are called Rossby waves in geophysics
(see, e.g., Hoskins, McIntyre, & Robertson 1985; in geo-
physics this instability is referred to as a shearing insta-
bility: e.g., see Haynes 1987; Marcus 1993, § 6.2). As a result
of this instability, the two bendings in the vorticity strip
(namely, the trailing and the leading vorticity waves of the
vortex) move in the direction opposite to the shear and fold
onto themselves. The extremities of the vorticity waves can
be again elongated by the shear and can undergo further
folding owing to the propagation of the Rossby waves. This
results in spiral vorticity arms around the vortex (Fig. 2).
The shape of the vortex thus formed does not change any
more during the remaining time of the simulation (tens of
orbits).

We also find that anticyclonic vortices act like overdense
regions in the disk, i.e., within about one orbit the density
increases by about 10% in the core of the vortex. We cannot
check, however, whether a cyclonic vortex decreases the
density in its core, since cyclonic vortices dissipate very
quickly.

3.1. Flat-Density Profile Models

In the first series of models, we chose a constant density
profile throughout the disk with a polytropic index $n = 3$. These
models are less realistic and are probably closer to
the models of Bracco et al. (1998), who used an incom-
pressible approximation (the shallow-water equation). We
ran four models with different values of the viscosity param-
eter $\alpha = 1 \times 10^{-4}$, $3 \times 10^{-4}$, $6 \times 10^{-4}$, and $1 \times 10^{-3}$. The
viscosity parameter was chosen so as to be consistent with
values inferred for protostellar disks (e.g., Bell et al. 1995).
The simulations were followed for up to a maximum time of
60 local Keplerian orbits of the vortex in the disk. We found
exponential decay times for the vortex of $\tau = 3.9$ periods for
$\alpha = 10^{-3}$ and $\tau = 32.4$ periods for $\alpha = 10^{-4}$ (see Fig. 3). In
all cases the decay was exponential.

3.2. Standard Disk Models

In an attempt to study the stability and decay times of
vortices in more realistic disks, we modeled a standard
Shakura-Sunyaev disk (Shakura & Sunyaev 1973) with
$\rho \propto r^{-15/8}$ and $T \propto r^{-3/4}$ (this was achieved by choosing a
polytropic index $n = 2.5$ together with an ideal gas equation of
state). In this model, we chose $\alpha = 10^{-4}$, and the simula-
tions were followed for up to 20 local Keplerian orbits of

![Graph](image-url)
As two vortices interact, they emit vorticity waves and eventually merge to form one large vortex. In this model the viscosity parameter was taken to be $\alpha = 10^{-4}$. For convenience, the vortices are shown roughly in the same orientation in the disk. The complete process of merging takes about five to ten orbits.

To gain further insight into the dynamics of vortices, we also ran two additional models (one with $\alpha = 10^{-4}$ and one with $\alpha = 10^{-3}$) where two vorticity perturbations were initially introduced in the flow. In the case of $\alpha = 10^{-4}$, the vortices interact by propagating vorticity waves; eventually, the two vortices merge together to form a single larger vortex (see Figs. 4a–4d). In the case of $\alpha = 10^{-3}$, the vortices do not interact (no vorticity waves were observed) and they dissipate quickly.

The results indicate that the exponential decay time of a vortex is inversely proportional to the viscosity (Fig. 5), and it can be very large indeed (in $\approx 30$–$60$ orbits the amplitude of the vortex decreases by a factor of $e$) for disks around
young stellar objects, where the viscosity might be low ($\alpha = 10^{-4}$). One expects the decay time to behave like $\tau \propto d^2/\nu$ (see also Bracco et al. 1998), where $d$ is the size of the vortex and $\nu$ is the viscosity in the flow. In principle, an inviscid model could have vortices that do not decay. Therefore, any decay that has been observed previously in inviscid numerical simulations (e.g., the shallow-water approximation of Bracco et al. 1998) was probably caused by numerical diffusion in the code (the hyperviscosity introduced by Bracco et al. 1998 in their model).

4. DISCUSSION

Accretion disks possess a very strong shear, which is normally believed to lead to a rapid destruction of any structures that form within it (e.g., the spiral waves obtained in a perturbed disk by Godon & Livio 1998 dissipate or exit the computational domain within about 10 orbits for $\alpha = 1 \times 10^{-4}$). However, we find that anticyclonic vortices are surprisingly long-lived, and they can survive for hundreds of orbits (the amplitude of the vortex decreases exponentially with a time constant of 60 orbits for $\alpha = 10^{-4}$ in disks around young stellar objects). These results are in agreement with other similar simulations of the decay of two-dimensional turbulence, e.g., the simulations of rotating shallow-water decaying turbulence on the surface of a sphere (see Cho & Polvani 1996a, 1996b for modeling of the Jovian atmosphere), where the only dissipation of the vorticity is due to the (hyper) viscosity. The size and the elongation of the vortices that form out of an initial vorticity perturbation increase with the viscosity. For the model to be self-consistent, the size of the vortices has to be larger than the thickness of the disk (to validate the two-dimensional assumption; otherwise the vortices are three-dimensional). We found that for an alpha viscosity of the order of $10^{-5}$ (with $H/r = 0.15$), the semimajor axis $a$ of the vortices is slightly smaller than $H$. However, when the viscosity is increased to $10^{-3}$, the semimajor axis becomes up to three times the thickness of the disk. In all cases, the semiminor axis $b$ remains smaller than $H$, while the elongation $(a/b)$ varies from about 4 (for the less viscous case) to about 10 (for the most viscous model). Tanga et al. (1996) and Barge & Sommeria (1995) solved numerically the equations of motion for dust particles in vortices and confirmed that dust particles concentrate inside vortices on a relatively short timescale. The time taken by a dust particle to reach the center of an anticyclonic vortex at a few AU ranges from a few orbits to a hundred orbits, depending on the exact value of the drag parameter. We have shown that in a standard disk model for a protoplanetary disk (a polytropic disk with $H/r = 0.15$), vortices can survive long enough to allow dust particles to concentrate in their core. For $\alpha = 10^{-3}$, only small vortices would form and they would not merge together. In this case (using the estimate of Tanga et al. 1996 for small vortices), one finds that the vortices would decay within about 10 orbits and the dust concentration in the core of the vortices would reach only $10^{-6}$ Earth masses (planetesimals, at 5 AU). For $\alpha = 10^{-4}$, small vortices would merge together to form larger vortices. The concentration of dust grains in the core of the larger vortices could reach about two Earth masses (within 100 revolutions and using the estimate of Barge & Sommeria 1995 at 5 AU). Therefore, the local density of centimeter-sized grains could be increased, thus favoring the formation of larger scale objects capable of efficiently triggering a state of gravitational instability. Our results therefore confirm earlier suspicions that were based on a simplified solution of shallow-water equations for an incompressible fluid.

It is important to note, though, that this work has yet to address the problem of vortex formation. In addition to the baroclinic instability described in § 1, another potential way to generate vortices in disks around young stellar objects is through infall of rotating clumps of gas. It has been suggested that protostellar disks could grow from the accretion (or collapse) of rotating gas clouds (e.g., Cassen & Moosman 1981; Boss & Graham 1993; Graham 1994; Fiebig 1997). The clumps with the proper rotation vectors could then give rise to small vortices that would subsequently merge together.

Finally, we would like to mention that vortices can (in principle at least) have many other astrophysical applications. For example, interacting Rossby waves can result in radial angular momentum transport (e.g., Llewellyn Smith 1996). Vortices are also believed to be important in molecular cloud substructure formation in the Galactic disk (e.g., Chantry, Grappin, & Léorat 1993; Sasao 1973).

This work has been supported in part by NASA Grant NAG 5-6857 and by the Director Discretionary Research Fund at STScI.

REFERENCES

Adams, F. C., & Lin, D. N. C. 1993, in Protostars and Planets III, ed. E. H. Levy & J. I. Lunine (Tucson: Univ. Arizona Press), 721
Adams, F. C., & Watkins, R. 1995, ApJ, 451, 314
Barge, P., & Sommeria, J. 1995, A&A, 295, L1
Beckwith, S. V. W., Henning, T., & Nakagawa, Y. 1999, in Protostars and Planets IV, ed. V. G. Mannings, A. P. Boss, & S. S. Russell (Tucson: Univ. Arizona Press)
Bell, K. R., Lin, D. N. C., Hartmann, L. W., & Kenyon, S. J. 1995, ApJ, 444, 376

Boss, A. P., & Graham, J. A. 1993, Icarus, 106, 168
Bracco, A., Provenzale, A., Spiegel, E., & Yecko, P. 1998, in Proceedings of the Conference on Quasars and Accretion Disks, ed. A. Abramowicz (Cambridge Univ. Press), 254
Canuto, C., Hussaini, M. Y., Quarteroni, A., & Zang, T. A. 1988, Spectral Methods in Fluid Dynamics (New York: Springer)
Cassen, P., & Moosman, A. 1981, Icarus, 48, 353
Chantry, P., Grappin, R., & Léorat, J. 1993, A&A, 272, 555
Cho, J. Y. K., & Polvani, L. M. 1996a, Phys. Fluids, 8, 1531
Cho, J. Y. K., & Polvani, L. M. 1996b, Science, 273, 335
Fiebig, D. 1997, A&A, 327, 758
Godon, P. 1997, ApJ, 480, 329
Godon, P., & Livio, M. 1998, ApJ, submitted
Gottlieb, D., & Orszag, S. A. 1977, Numerical Analysis of Spectral Methods: Theory and Applications, NSF-CBMS Monograph 26 (Philadelphia: SIAM)
Graham, J. A. 1994, American Astronomical Society Meeting, 184, 44.07
Haynes, P. H. 1987, J. Fluid Mech., 175, 463
Hoskins, B., McIntyre, M., & Robertson, A. 1985, Q. J. R. Meteorol. Soc., 111, 877
Ingersoll, A. P. 1990, Science, 248, 308
Llewellyn Smith, S. G. 1996, Thesis, DAMTP, Cambridge University
Marcus, P. S. 1993, ARA&A, 31, 523
Nauta, M. D., & Tóth, G. 1998, A&A, 336, 791
Pedlosky, J. 1987, Geophysical Fluid Dynamics, 2d ed. (New York: Springer)
Pringle, J. E. 1981, ARA&A, 19, 137
Sasao, T. 1973, PASJ, 25, 1
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
She, Z. S., Jackson, E., & Orszag, S. A. 1991, Proc. R. Soc. London A 434 (1890), 101
Tanga, P., Babiano, A., Dubrulle, B., & Provenzale, A. 1996, Icarus, 121, 158
Tassoul, J. L. 1978, Theory of Rotating Stars (Princeton: Princeton Univ. Press)
Voigt, R. G., Gottlieb, D., & Hussaini, M. Y. 1984, Spectral Methods for Partial Differential Equations (Philadelphia: SIAM-CBMS)