On Nonperturbative Solutions of Superstring Field Theory

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Abstract

Nonperturbative solutions to the nonlinear field equations in the NS sector of cubic as well as nonpolynomial superstring field theory can be obtained from a linear equation which includes a “spectral” parameter $\lambda$ and a coboundary operator $Q(\lambda)$. We borrow a simple ansatz from the dressing method (for generating solitons in integrable field theories) and show that classical superstring fields can be constructed from any string field $T$ subject merely to $Q(\lambda)T = 0$. Following the decay of the non-BPS D9 brane in IIA theory and shifting the background to the tachyon vacuum, we repeat the arguments in vacuum superstring field theory and outline how to compute classical solutions explicitly.

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1. Introduction. Sen’s conjectures (for a review see, e. g., [1, 2] and references therein) on tachyon condensation have sparked considerable activity in open string field theory. Even though there is no tachyon in the superstring spectrum (on flat spacetime) the decay of unstable non-BPS D9-branes in type IIA theory is due to the GSO(−) NS tachyon excitation. The dynamics of this transition from the D9-brane vacuum to the tachyon vacuum involves nonperturbative classical superstring configurations. Thus, the task is to solve the NS string field equations of motion, either in Witten’s cubic [3] or in Berkovits’ nonpolynomial [4] formulation. Although some progress has been made in this direction [5, 6, 7, 8] we still do not have an exact solution at hand as of today.

Some time ago it was shown by two of us [9] that Berkovits’ string field theory is integrable in the sense that its equation of motion derives from a system of linear equations. Clearly, one should take advantage of this fact and try to bring to application the powerful solution-generating technology available for integrable equations. The goal of this letter is to initiate such a program, based on the ideas presented in [10].

In analogy with gauge field theory, we write down a linear system for cubic as well as nonpolynomial open superstring field theory (in the NS sector) by introducing an auxiliary string field Ψ depending on a “spectral” parameter λ ∈ CP1. A single-pole ansatz for Ψ(λ) leads to a hermitian projector, whose building block is merely subject to a linear equation which can be solved in generality. From it all string fields can be reconstructed. Employing dressing transformations analogous to those in noncommutative field theories [11], we shift the background to the tachyon vacuum and propose a linear equation which governs classical vacuum superstring field theory. As a simple example, the supersliver [5, 12] is based on a trivial solution to this equation. Finally, we propose a strategy to reconstruct classical superstring fields from their building blocks in more detail by taking advantage of the Moyal formulation for superstring field theory.

2. Zero-curvature and linear equations for string fields. In cubic open bosonic string field theory [13], the equation of motion for the string field A has a zero-curvature form,

\[ F(A) = QA + A^2 = (Q + A)^2 = 0 \quad , \]  

where Q denotes the BRST operator (a nilpotent derivation) and Witten’s star product is implicit in all string field products. For any string field A one may look for solutions of the linear equation

\[ (Q + A) \Psi = 0 \quad (2) \]

on an auxiliary string field Ψ possibly carrying some internal indices. Equation (1) is the compatibility condition of the linear equation (2). If we let Ψ take values in the Chan-Paton group, then from (2) one may obtain solutions of (1) via \( A = \Psi Q \Psi^{-1} \) which are, however, pure gauge configurations. The cohomology of Q captures all other solutions.

This situation may change when a parametric dependence is introduced: Let \( (Q, A, \Psi) \rightarrow (Q(\lambda), A(\lambda), \Psi(\lambda)) \) with \( \lambda \in \mathbb{C}P^1 \). We demand \( Q(\lambda) \) and \( A(\lambda) \) to be linear in \( \lambda \),

\[ A(\lambda) = a + \lambda A \quad \text{and} \quad Q(\lambda) = \eta_0 + \lambda Q \quad \text{with} \quad \eta_0^2 = Q^2 = \eta_0 Q + Q \eta_0 = 0 \quad . \]  

In other words, we extend the string configuration space, thereby adding a second string field \( a \) and a second BRST-like operator \( \eta_0 \). This case arises for a one-parameter family of \( N = 2 \) superconformal

\(^1\)Formally \( A(\lambda) \) is a section of the bundle \( O(1) \) over \( \mathbb{C}P^1 \) with values in the string field Hilbert space \( \mathcal{H} \), and \( Q(\lambda) \) can be considered as an \( \text{End}(\mathcal{H}) \)-valued section of this bundle.
algebras embedded into a small $N=4$ algebra and their string field realizations [14, 15]. The extended zero-curvature condition

$$F(A(\lambda)) = (Q(\lambda) + A(\lambda))^2 = (\eta_0 a + a^2) + \lambda(\eta_0 A + Qa + \{A, a\}) + \lambda^2(QA + A^2) = 0 \quad (4)$$

is the compatibility condition of the associated linear equation

$$(Q(\lambda) + A(\lambda)) \Psi(\lambda) = 0 . \quad (5)$$

If $\Psi(\lambda)$ is group-valued, it follows that $a + \lambda A = \Psi(\lambda)(\eta_0 + \lambda Q) \Psi(\lambda)^{-1}$. As was shown in [9, 10], this equation yields nontrivial solutions to the equations of motion for $a$ and $A$.

Exploiting the gauge freedom in (4) allows one to gauge away $a$. Then, the ensuing equations,

$$\eta_0 A = 0 \quad \text{and} \quad QA + A^2 = 0 \quad (6)$$

are the (NS-sector) equations of motion in Witten’s cubic open superstring field theory in the zero picture [16, 17]: Bosonizing the fermionic reparametrization ghosts, $\gamma = \eta e^{i\phi}$ and $\beta = e^{-i\phi}\partial\xi$, we take $\eta_0$ above to be the zero mode of $\eta$, which indeed is nilpotent and anticommutes with $Q$. Then, the first equation in (6) simply denies any $\xi_0$ content in $A$ (originally defined in the large Hilbert space), and the second one is the field equation in the small Hilbert space. Of course, all fields are now NS-sector open superstring fields.

The system (6) may be reduced further. Since both $\eta_0$ and $Q$ have trivial cohomology in the large Hilbert space $\mathcal{H}$, we may either solve the first equation or alternatively the second one:

$$A = \eta_0 \Upsilon \implies Q \eta_0 \Upsilon + (\eta_0 \Upsilon)^2 = 0 , \quad (7)$$

$$A = e^{-\Phi}Qe^{\Phi} \implies \eta_0(e^{-\Phi}Qe^{\Phi}) = 0 . \quad (8)$$

Despite appearance, $A$ is not pure gauge (in the small Hilbert space) unless $\eta_0 e^{\Phi} = 0$ [7]. The second equation in (8) is precisely Berkovits’ nonpolynomial equation of motion for the NS string field $\Phi$.

All nonlinear superstring field equations, i.e. (6), (7) and (8), follow from the zero-curvature equation (4) (with $a=0$). Because both $Q$ and $\eta_0$ have empty cohomology in the large Hilbert space we can in fact construct all solutions from the associated linear system

$$(Q + \frac{1}{\lambda}\eta_0 + A) \Psi(\lambda) = 0 \quad (9)$$

for the string fields $A$ and $\Psi(\lambda)$. This equation is the key to generating classical superstring configurations.

Of course, one always has the “trivial” $\lambda$-independent solution

$$\Psi = e^{-A} \quad \text{with} \quad \partial_\lambda A = 0 \implies \eta_0 e^{-A} = 0 = (Q + A) e^{-A} \quad (10)$$

which leads to a pure gauge configuration $A_0 = e^{-A}Qe^A$. Since $\mathbb{C}P^1$ is compact, the $\lambda$ dependence of a nontrivial $\Psi(\lambda)$ cannot be holomorphic. Hence, we consider a meromorphic $\Psi(\lambda)$. If we require

\footnote{Note that $Q$ and $\eta_0$ act via (anti)commutator on world-sheet fields, or, equivalently, via contour integration of the respective currents.}

\footnote{For gauge theory the following goes back to Leznov and to Yang, respectively.}

\footnote{Formally $\Psi(\lambda)$ can be seen as an element of the space $\mathcal{H} \otimes \mathbb{C}[\lambda, \lambda^{-1}]$ carrying Chan-Paton labels.}

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its regularity for $\lambda \to 0$ and for $\lambda \to \infty$, then one may choose such a gauge that the asymptotics will relate $\Psi$ with the prepotentials $\Phi$ and $\Upsilon$ as follows:\footnote{\textit{I} denotes the identity string field.}

\begin{equation}
\Psi(\lambda) \longrightarrow \begin{cases}
\mathcal{I} - \lambda \Upsilon + \mathcal{O}(\lambda^2) & \text{for } \lambda \to 0 \\
e^{-\Phi} + \mathcal{O}(\frac{1}{\lambda}) & \text{for } \lambda \to \infty
\end{cases}.
\end{equation}

Clearly, $e^{-\Phi}$, $\Upsilon$, and $A = \Psi(\infty)Q\Psi(\infty)^{-1} = -\eta_0 \partial_\lambda \Psi(0)$ are computable once an appropriate $\Psi(\lambda)$ has been found.

\textbf{3. Single-pole ansatz and solutions.} Let us employ the linear system (9) to solve Witten’s or Berkovits’ superstring field equations (in the NS sector). In contrast to the non-parametric linear equation (2), the $\lambda$ dependence of (9) imposes two constraints on $\Psi(\lambda)$. Firstly, isolating $A$ in (9),

\begin{equation}
A = \Psi(\lambda)(Q + \frac{\eta_0}{\lambda})\Psi(\lambda)^{-1},
\end{equation}

we notice that the right-hand side must not depend on $\lambda$, hence all its poles must have vanishing residues. Although the above expression is pure gauge from the point of view of the $\lambda$-extended string configuration space, the string field $A$ is nontrivial on the small Hilbert space. A second condition follows from the reality of the string fields. To formulate it one must extend hermitian conjugation to an antilinear mapping (which we denote by a bar) on the $\mathbb{C}P^1$ family of $N=2$ superconformal algebras where it sends $Q \mapsto -\eta_0$ and $\eta_0 \mapsto Q$ but $\lambda \mapsto \bar{\lambda}$ \footnote{For more general multi-pole ansätze see \cite{10}.}. It can be shown that the reality condition requires

\begin{equation}
e^{-\Phi} = \Psi(\lambda)\Psi(-1/\bar{\lambda}).
\end{equation}

Again, the poles on the right-hand side must be removable.

The simplest nontrivial solution displays a single pole in $\lambda$,\footnote{\textit{I} denotes the identity string field.}

\begin{equation}
\Psi(\lambda) = \mathcal{I} - \frac{\lambda(1+\mu\bar{\mu})}{\lambda - \mu}P,
\end{equation}

whose location $\mu$ is a moduli parameter. $P$ is a $\lambda$-independent string field to be determined. Let us investigate for our ansatz (14) the consequences of (13) and (12), in that order. The residues of the $\lambda$-poles of $\mathcal{P}\mathcal{V}$ at $\lambda=\mu$ and $\lambda=-1/\bar{\mu}$ are proportional to $P(\mathcal{I}-\mathcal{P})$ and $(\mathcal{I}-P)\mathcal{P}$ (for $\mu \in \mathbb{C}P^1$ arbitrary and fixed), respectively, implying the projector property

\begin{equation}
P^2 = P = \mathcal{P}.
\end{equation}

This is achieved by parametrizing

\begin{equation}
P = T(\mathcal{T}T)^{-1}\mathcal{T},
\end{equation}

with some string field $T$. Similarly, the absence of poles in (12) yields

\begin{equation}
P(\mu Q + \eta_0)P = 0 \quad \text{and} \quad (\mathcal{I}-P)(Q-\bar{\mu}\eta_0)P = 0
\end{equation}

which are conjugate to one another. Since $PT = T$ by construction these equations are satisfied if

\begin{equation}
(Q - \bar{\mu}\eta_0)T = 0.
\end{equation}
It is important to note that $T$ is only subject to a linear equation and otherwise unconstrained. An obvious solution to (18) is

$$T = (Q - \mu \eta_0)W$$

for an arbitrary string field $W$. Every choice of $W$ or solution to (18) yields a classical Berkovits string field,

$$e^{-\Phi} = \mathcal{I} - (1 + \mu \bar{\mu}) P \quad , \quad e^{\Phi} = \mathcal{I} - (1 + \frac{1}{\mu \bar{\mu}}) P$$

and, from $\lambda \to 0$,

$$A = -\frac{1 + \mu \bar{\mu}}{\mu} \eta_0 P \ .$$

4. **Shifting the background.** The form of the string field equations does not depend on the choice of background (termed “vacuum”). However, the explicit structure of the kinetic operator $Q$ is determined by this choice. For the open-string vacuum,

$$A_0 = 0 \ , \quad P_0 = 0 \ , \quad \Psi_0 = \mathcal{I} \ ,$$

one has the familiar BRST operator, $Q = Q_B$. Now, one may think of the solution $(\Psi, A)$ to (9) as the result of a “dressing map” [10]

$$\Psi_0 = \mathcal{I} \longrightarrow \Psi = \Psi(\lambda) \Psi_0 \quad \text{and} \quad A_0 = 0 \longrightarrow A = \text{Ad}_\Psi A_0$$

applied to a “seed solution” $(\Psi_0, A_0)$. This process can be iterated. Since any two classical superstring configurations are related by such a dressing transformation, a shift of the background $(\Psi_0, A_0)$ to a new reference configuration $(\Psi_1, A_1)$ is exactly of the same nature. The difference is only semantical.

We study the result of shifting the background by a dressing transformation according to

\[
\begin{align*}
\text{background:} & \quad \Psi_0 = \mathcal{I} \quad \longrightarrow \quad \Psi_1 \\
\text{deviation:} & \quad \Psi \quad \longrightarrow \quad \tilde{\Psi} \\
\text{and} & \quad A_0 = 0 \quad \longrightarrow \quad A = \text{Ad}_\Psi A_0 \quad (23)
\end{align*}
\]

where horizontal arrows represent the dressing map to the new background and vertical arrows turn on a deviation via dressing. Composing the two dressing transformations, the linear equation becomes $(\tilde{\Psi} = \Psi' \Psi_1$ and $\tilde{A} = A_1 + A')$

\[
0 = \left( Q + \frac{1}{\lambda} \eta_0 + \tilde{A} \right) \tilde{\Psi}
\]

\[
= \left[ Q \Psi' + A_1 \Psi' - \Psi' A_1 + \frac{1}{\lambda} \eta_0 \Psi' + (\tilde{A} - A_1) \Psi' \right] \Psi_1
\]

\[
= \left[ (Q' + \frac{1}{\lambda} \eta_0 + A') \Psi' \right] \Psi_1 \ , \quad (25)
\]

where we used $(Q + \frac{1}{\lambda} \eta_0) \Psi_1 = -A_1 \Psi_1$ and defined $Q' \Psi' := Q \Psi' + A_1 \Psi' - \Psi' A_1$. Hence, measuring our string fields from the new vacuum $A_1$, the relevant linear system,

\[
(Q' + \frac{1}{\lambda} \eta_0 + A') \Psi' = 0 \ , \quad (26)
\]
has the same form as the original (9), but $Q$ has changed into $Q'$. For the nonlinear string field equations the corresponding form invariance has been observed in [5], a fact almost trivial in our framework.

5. Tachyon vacuum superstring fields. Of special interest is the form of the theory around the tachyonic vacuum. Deviations from the tachyonic vacuum are governed by (26), and all equations pertaining to the open-string vacuum simply carry over (with primes added). However, this is not the whole story. As discussed in [18, 8], a new kinetic operator built entirely from ghosts can be “derived” via a redefinition of the new (tachyon vacuum) superstring fields,

\[ A' \mapsto \mathcal{U}_\varepsilon A' =: \hat{A} \quad \text{and} \quad \Psi' \mapsto \mathcal{U}_\varepsilon \Psi' =: \hat{\Psi} , \tag{27} \]

such that

\[ Q' \mapsto \mathcal{U}_\varepsilon Q' \mathcal{U}_\varepsilon^{-1} =: \hat{Q} \tag{28} \]

yields the proper zero-cohomology “vacuum” kinetic operator. The field redefinition (27) is induced by a world-sheet reparametrization which is singular for $\varepsilon \to 0$. As $\eta$ has conformal spin one, its zero mode $\eta_0$ is inert under the reparametrization. From now on, a hat indicates the presence of internal $2 \times 2$ Chan-Paton matrices distinguishing the GSO($\pm$) sectors, e. g.,

\[ \hat{A} = A_+ \otimes \sigma_3 + A_- \otimes i\sigma_2 \quad \text{(odd ghost number)} , \tag{29} \]

\[ \hat{\Phi} = \Phi_+ \otimes 1 + \Phi_- \otimes \sigma_1 \quad \text{(even ghost number)} . \tag{30} \]

The kinetic operator of this “vacuum superstring field theory” (VSSFT) is conjectured to have the form [6, 8]

\[ \hat{Q} = Q_{\text{odd}} \otimes \sigma_3 + Q_{\text{even}} \otimes i\sigma_2 \ , \tag{31} \]

where the subscript refers to the Grassmann parity and

\[ Q_{\text{odd}} = \frac{1}{4\pi i} \left[ c(i) - c(-i) \right] + \oint \frac{dz}{2\pi i} \gamma^2(z) , \tag{32} \]

\[ Q_{\text{even}} = \frac{1}{2\pi} \left[ \gamma(i) - \gamma(-i) \right] \Pi_+ + \frac{1}{2\pi} \left[ \gamma(i) + \gamma(-i) \right] \Pi_- \tag{33} \]

with projectors $\Pi_+$ and $\Pi_-$ onto the GSO($+$) and GSO($-$) sectors, respectively. These terms prevail in the limit $\varepsilon \to 0$. Consequently, the linear system for VSSFT reads

\[ (\hat{Q} + \frac{1}{\varepsilon} \hat{\eta}_0 + \hat{A}) \hat{\Psi}(\lambda) = 0 \tag{34} \]

where $\hat{\eta}_0 = \eta_0 \otimes \sigma_3$ and $\hat{\Phi} = \Phi_+ \otimes 1 + \Phi_- \otimes \sigma_1$. Again, solutions to Berkovits’ VSSFT or to the cubic VSSFT are obtained from (20) or (21) by firstly solving the linear equation (18) after replacing $Q \to \hat{Q}$ and secondly composing the projector via (16).

It is usually assumed that the D-brane solutions of VSSFT factorize into a ghost and a matter part, $\hat{A} = \hat{A}_g \otimes A_m$. Then, the cubic VSSFT equation,

\[ \hat{Q} \hat{A} + \hat{A}^2 = 0 \quad \text{with} \quad \hat{\eta}_0 \hat{A} = 0 \tag{35} \]

splits into

\[ A_m^2 = A_m \quad \text{and} \quad \hat{Q} \hat{A}_g + \hat{A}_g^2 = 0 \quad \text{with} \quad \hat{\eta}_0 \hat{A}_g = 0 \tag{36} \]

which turns $A_m$ into a projector. Within our single-pole ansatz (14), the full $\hat{A}$ is already proportional to a projector $\hat{P} = \hat{P}_g \otimes P_m$, hence we must simply factorize (21) and have

\[ A_m = P_m \quad \text{and} \quad \hat{A}_g = \frac{1 + \mu_0}{\mu} \hat{\eta}_0 \hat{P}_g \quad \text{with} \quad P_m^2 = P_m \quad \text{and} \quad \hat{P}_g^2 = \hat{P}_g . \tag{37} \]
Since $\hat{Q}$ is pure ghost the projector equation (17) factorizes, and (36) reduces to (37) plus
\[(\hat{I}_g - \hat{P}_g)(\hat{Q} - \hat{\mu}\hat{\eta}_0)\hat{P}_g = 0 , \]
which is solved by (we omit hats over $T_g$)
\[\hat{P}_g = T_g (\hat{T}_g T_g)^{-1} \quad \text{and} \quad (\hat{Q} - \hat{\mu}\hat{\eta}_0) T_g = 0 . \]

In the nonpolynomial formulation, a different ansatz, $\hat{\Phi} = \hat{\Phi}_g \otimes \Phi_m$ with $\Phi_m^2 = \Phi_m$, was advocated by Mariño and Schiappa [5]. It allows one to factorize Berkovits’ equation (8) since one implies $\Phi_m$ which is solved by (we omit hats over $T$)
\[N = 4 \text{ superconformal algebra}, \text{ such a coboundary operator (in the case of the open string vacuum) was proposed initially in [14, 15].} \]

6. Ghost picture modification. As it stands, the linear equations (34) and (39) face a problem due to the ghost picture degeneracy of the NSR superstring. If our string fields are to carry a definite picture charge, they must reside in the zero-picture sector. Since $\eta_0$ lowers the picture charge by one unit, the above-mentioned coboundary operator is not homogeneous in picture. Therefore, from (34) or (39) one concludes that any string field, including $\hat{A}$ and $T_g$, must in general be an infinite sum over all picture sectors. Obviously, any such field may be expanded into a formal series $T_g = \sum_{n \in \mathbb{Z}} (\hat{\mu}^{-n} T_n$, where $T_n$ carries picture number $n$. From (39) then we obtain the recursion relations $\eta_0 T_{n+1} = -\hat{Q} T_n$. If we want to maintain Berkovits’ original proposal that all string fields have picture number zero (e. g., $T_{n\neq0} = 0$) then only the trivial solutions of (34) with $\hat{Q} T_0 = 0 \Rightarrow \hat{Q} \hat{P}_g = 0 \Rightarrow \hat{\eta}_0 \hat{P}_g$ and therefore $\hat{A} = 0$. The supersliver [5, 12] is gauge equivalent to this vacuum [7].

To obtain nontrivial solutions, we have two possibilities: Either we admit string fields inhomogeneous in picture, or we modify our linear equation. In the following we shall pursue the second option and restrict all string fields to the zero picture. The obvious cure then is to introduce a picture-raising multiplier, $\eta_0 \rightarrow \hat{X}(i) \eta_0$. This is admissible as long as $\hat{X}(i)$ commutes with both $\eta_0$ and $\hat{Q}$ and can be pulled through the star product.\(^9\) We propose to take $\hat{X}(i) := \{ \hat{Q}, \hat{\xi}(i) \}$, i. e. the picture-raising operator $\hat{X}$ of VSSFT evaluated at the string midpoint.\(^10\) With this modification, our master linear equation becomes
\[(\hat{Q} + \hat{X}(i) \hat{\eta}_0 + \hat{A}) \hat{\Psi}(\lambda) = 0 , \]

\(^{9}\)Any midpoint insertion of conformal spin zero commutes with Witten’s star product, as can be seen by its definition in terms of correlation functions of the disk.

\(^{10}\)Due to the explicit form (31)–(33) of the kinetic operator, $\hat{X}(i)$ consists of Grassmann-even and -odd parts. The Grassmann-even part simply reads $-\partial(bg^2\Psi)(i) - b \partial b \Psi^2(i)$; the Grassmann-odd part has to be regularized due to the pole in the OPE of $\gamma$ with $\xi$. Around the open-string vacuum, we may simply take $X(i) = \{Q, \xi(i)\}$.
and all subsequent equations continue to hold after the obvious insertions of $\hat{\mathcal{X}}(i)$. In particular, the ghost picture modification changes Berkovits’ string field equation (8) to

$$\hat{\mathcal{X}}(i) \hat{\eta}_0 (e^{-\hat{\Phi}} \hat{\mathcal{Q}} e^{\hat{\Phi}}) = 0 .$$

Any solution $\hat{\mathcal{A}}$ in the form of (21) will, however, automatically be annihilated by $\hat{\eta}_0$ so that it fulfills also Berkovits’ equation of motion without $\hat{\mathcal{X}}(i)$. Note that the action will remain unchanged; we use $\hat{\mathcal{X}}(i)$ only as a means to solve our linear equations.

7. Towards explicit solutions. In order to extract the physical properties of classical VSSFT configurations, e. g., a D-brane interpretation or the role of our moduli parameter $\mu$, it is desirable to construct solutions to the field equations in a more explicit manner. In keeping with the paradigm of matter-ghost factorization (see (36)) we are asked to solve eq. (39) with $\hat{\mathcal{X}}(i)$ inserted. Because $\hat{\mathcal{Q}} - \bar{\mu} \hat{\mathcal{X}}(i) \hat{\eta}_0$ can be “inverted” the general solution of VSSFT may be constructed from

$$\mathcal{T}_g = (\hat{\mathcal{Q}} - \bar{\mu} \hat{\mathcal{X}}(i) \hat{\eta}_0) \hat{\mathcal{W}}_g$$

for an arbitrary ghost string field $\hat{\mathcal{W}}_g$.

For cubic VSSFT, the $\varepsilon_r$ expansion of [8] can be reproduced in this framework. In particular, since the leading term of $\hat{\mathcal{Q}} - \bar{\mu} \hat{\mathcal{X}}(i) \hat{\eta}_0$ is identical to $Q_{GRSZ} \otimes \sigma_3$ [18], the lowest order in $\varepsilon_r$ involves only the “natural” Grassmann assignments of all quantities.

Certain special solutions can be seen directly. When $\bar{\mu} = 1$, for instance, one may employ the picture-lowering operator $\hat{\mathcal{Y}}(i)$ to write $\mathcal{T}_g = \hat{\mathcal{Y}}(i) \hat{\xi}(i) \hat{\Xi}_g$ where $\hat{\mathcal{Q}} \hat{\Xi}_g = 0 = \hat{\eta}_0 \hat{\Xi}_g$. At leading order in $\varepsilon_r$ we may identify $\hat{\Xi}_g$ with the ghost supersliver $\Xi_g \otimes 1$.

In any case, the main difficulty arises in the composition of $\hat{\mathcal{P}}_g$ from a given $\mathcal{T}_g$ since Witten’s star product is implicit in (39). In order to circumvent this technical obstacle we propose to make use of the (discrete [19] or continuous [20]) Moyal formulation of Witten’s star product. In such a situation, the Moyal-Weyl map can be inferred to encode the non(anti)commutativity into Heisenberg or Clifford algebras, which are represented in auxiliary Fock spaces. The advantage of this (auxiliary) operator formulation is its calculational ease. As an example, the basic projector for a single Moyal pair can be expressed as follows:

$$[a,a^\dagger] = 1 \implies |0\rangle \langle 0| = : e^{-a^\dagger a} : = 1 - a^\dagger (aa^\dagger)^{-1} a ,$$

$$\{c,c^\dagger\} = 1 \implies |0\rangle \langle 0| = : e^{-c^\dagger c} : = 1 - c^\dagger c = 1 - c^\dagger (c c^\dagger)^{-1} c ,$$

displaying a simple connection between the Gaussian form and the “fractional” form (cf. (16)) of a projector. Of course, for the application to VSSFT infinite tensor products of Heisenberg and Clifford algebras have to be considered [19, 20, 21, 22]. However, (45) and (46) suggest the possibility to take $\mathcal{T}_g$ not to be an operator but a state $|\mathcal{T}_g\rangle$ in the auxiliary Fock space. This would be in tune with the construction of noncommutative abelian solitons [11]. Finally, a direct comparison with results in the conventional string oscillator basis requires the reverse basis transformation to be applied to the string field configurations constructed in the Moyal basis.

In closing, we should like to stress that we have reduced the problem of solving the superstring field equations (in cubic or nonpolynomial form) to the easier task of considering a linear equation,

11Our $Q_{odd}$ in (32) has a different $\varepsilon_r$-dependence (coinciding with that of [6]), but this is irrelevant here.

12Such Fock spaces are not to be confused with the string oscillator Fock space.

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whose solution $T$ then serves as a building block for the string field configuration. Although demonstrated here with the simplest (single-pole) ansatz for the auxiliary string field $\Psi(\lambda)$, this strategy generalizes to the universal (multi-pole) case [10]. Projectors emerge naturally only in the single-pole setup while $T$ (rather a collection of such) continues to play the decisive role. The formalism is ideally suited to handle the superposition of solitonic objects in integrable systems. We therefore expect it to yield multi-brane configurations automatically.

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