Temperature effect analysis of pre-tension and deformation characteristics of planar membrane structure

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Abstract. In this paper, the planar membrane structure of an engineering example is analysed theoretically from the variation of pre-tension and membrane surface deformation with temperature. Then ANSYS software is used to analyse the pre-tension changes of the membrane structure and the membrane surface deformation with the uniform load under different temperatures (-50~70 °C) for the membrane structure. The trend image of pre-tension and membrane surface deformation with temperature was obtained, and the minimum value of pre-tension of the membrane structure was obtained according to the relevant regulations. According to the variation law of pre-tension, the pre-tension selection scheme for the installation of planar membrane structure is summarized, which provides some references for the determination of pre-tension in the installation of planar membrane structure.

1. Introduction
The pre-tension of the membrane structure is the key to ensure the shape and function of the membrane. The pre-tension determines the bearing capacity and appearance of the membrane structure. In the membrane structure engineering, the membrane structure is mostly a three-dimensional space structure, and can be subjected to secondary tension under a certain pre-tension of the initial installation, thereby effectively ensuring a large tension of the membrane structure. However, for a specific occasion using a two-dimensional planar membrane structure, it is difficult to achieve secondary tensioning. Once the pre-tension is too large and difficult to install, the influence of temperature change on the pre-tension is relatively large, so it is extremely important to find out the influence of temperature on the pre-tension.

At present, many scholars have done research on the effect of temperature on the properties of the membrane. Zhou Chuanzhi, Zhang Qilin did tensile tests on PVC membrane at different temperatures, found that with the increase of temperature, the tensile strength of PVC membrane decreased, and the elongation at break increased[1]; Zhang Yingying, Zhang Qilin did the tensile test of PTFE membrane at different temperatures showed that the tensile strength of PTFE membrane decreased with the increase of temperature, while the elongation at break and elastic modulus increased[2]; Mu Tong, Wu Minger did the tensile test of ETFE membrane at different temperatures showed that the yield strength, elastic modulus and tensile strength of ETFE membrane decreased with increasing temperature[3].

Most of the current researches have investigated the effect of temperature on the properties of the membrane, and little research has been done on the effect of temperature on the pretension of the installed membrane structure. The reason is because there is no unified temperature analysis theory at present, and theoretical analysis is difficult; the membrane structure is generally bulky and difficult to conduct outdoor environmental tests. Therefore, the article adopts ANSYS finite element analysis software to simulate the pre-tension at different temperatures in a convenient and reliable way, and obtains the relationship between pre-tension and temperature change, and the deformation of the
membrane surface at different temperatures under uniform load. Influence, summed up a method of selecting the pre-tension, which provides some reference for the design and installation of the membrane structure in the future.

2. Project example
There is a certain planar membrane structure, which adopts a rectangular frame as the skeleton of the membrane structure. The frame size is 3.15m×2.5m, which is surrounded by four flat steels. The geometric model is shown in Figure 1. The membrane is made of PTFE coated membrane. The flat steel material is Q235 ordinary carbon structural steel.

![Figure 1. Simplified membrane structure model](image)

The PTFE membrane is an orthotropic material, and the elastic modulus of the PTFE membrane changes with temperature. It is necessary to separately consider the change of the elastic modulus with temperature in different directions of the warp and weft of the membrane. The relationship between the elastic modulus of PTFE membrane and temperature is as shown in equations (1) and (2) \[2\]:

\[
E_1 = 2242 + 7.33t \\
E_2 = 2142 + 3.72t
\]

In the formula, \(E_1\) is a warp elastic modulus; \(E_2\) is a weft elastic modulus, and \(t\) is an ambient temperature.

The range of temperature change is selected from -50 to 70°C. The elastic modulus of PTFE at different temperatures is calculated according to formulas (1) and (2) as shown in Table 1.

| Temperature /°C | elastic modulus /MPa |
|-----------------|-----------------------|
|                 | Warp      | Weft      |
| -50             | 1875.5    | 1956      |
| -40             | 1948.8    | 1993.2    |
| -30             | 2022.1    | 2030.4    |
| -20             | 2095.4    | 2067.6    |
| -10             | 2168.7    | 2104.8    |
| 0               | 2242      | 2142      |
| 10              | 2315.3    | 2179.2    |
| 20              | 2388.6    | 2216.4    |
| 30              | 2461.9    | 2253.6    |
| 40              | 2535.2    | 2290.8    |
| 50              | 2608.5    | 2328      |
| 60              | 2681.8    | 2365.2    |
| 70              | 2755.1    | 2402.4    |

It can be seen from Table 1 that in the temperature range of -50 to 70 °C, the warp modulus of PTFE membrane changed by 879.6 MPa, an increase of 46.9%; the latitudinal modulus of elasticity changed by 446.4 MPa, an increase of 22.8%. The temperature has a great influence on the elastic modulus in both the warp and weft directions. Therefore, the elastic modulus of the membrane should be considered by the temperature.
The average linear expansion coefficient of the PTFE membrane at -50 to 70 °C is $9 \times 10^{-5} \degree C^{-1}$[4]. The Q235 ordinary carbon structural steel is $1.2 \times 10^{-5} \degree C^{-1}$. It can be seen that the average linear expansion coefficient of steel is much smaller than that of the membrane. In addition, the planar membrane structure has a simple structure, and the frames are closely contacted after installation, and the size of a single frame is large. Therefore, when analysing such a membrane structure, ignore the effect of thermal deformation of the steel on the pretension.

3. Theoretical analysis

3.1 Relationship between pre-tension of membrane structure and temperature

In a certain direction, the formula for the average linear expansion coefficient is:

$$\alpha = \frac{1}{L} \times \frac{\Delta L}{\Delta t}$$  \hspace{1cm} (3)

Where, $\alpha$ is the average linear expansion coefficient; $L$ is the length of the membrane in a certain direction, m; $\Delta L$ is the amount of change in length, m; $\Delta t$ is the amount of temperature change, °C.

From the formula (3), the strain $\varepsilon$ of the membrane material can be derived when the temperature changes $\Delta t$:

$$\varepsilon = \frac{\Delta L}{L} = \alpha \times \Delta t$$  \hspace{1cm} (4)

The membrane is an elastic material within the tensile strength, and the stress-strain formula is:

$$\sigma = E\varepsilon$$  \hspace{1cm} (5)

Where, $\sigma$ is the stress applied to the membrane, MPa; $E$ is the modulus of elasticity of the membrane, MPa.

Then, when the temperature changes $\Delta t$, the amount of change in stress $\Delta \sigma$ is:

$$\Delta \sigma = \alpha \times \Delta t \times E$$  \hspace{1cm} (6)

It can be concluded that the residual pretension of the membrane is:

$$\sigma_1 = \sigma_0 - \alpha \times \Delta t \times E$$  \hspace{1cm} (7)

Where, $\sigma_1$ is residual pretension, MPa; $\sigma_0$ is initial pretension, MPa.

It can be seen from the formula (7) that the change in the pretension is related to the temperature, the average linear expansion coefficient, and the elastic modulus. In this case, the pretension change is only related to temperature due to the structure and material of the membrane structure. Since the higher the temperature, the larger the modulus of elasticity of the membrane, the higher the temperature of the high temperature zone than the low temperature zone changes under the same temperature change, so the speed of the pretension increase decreases as the temperature increases.

3.2 Relationship between membrane surface deformation and temperature under uniform load

According to the small deflection theoretical deformation formula of the planar membrane structure in the literature [5], the relationship expression of the membrane surface deflection is:

$$w = \frac{16q}{\pi^4 h \sigma_0 \left(\frac{1}{a^2} + \frac{1}{b^2}\right)}$$  \hspace{1cm} (8)

Where, $w$ is membrane surface deflection, m; $h$ is membrane thickness, m; $q$ is membrane surface load, MPa; $a$ is membrane length, mm; $b$ is membrane width, mm.

As the temperature changes, the pre-tension changes accordingly. Substituting equation (7) into equation (8) gives:

$$w = \frac{16q}{\pi^4 h \left(\sigma_0 - \alpha \times \Delta t \times E\right) \left(\frac{1}{a^2} + \frac{1}{b^2}\right)}$$  \hspace{1cm} (9)
It can be seen from formula (9) that the deformation of the membrane surface under uniform load is related to various factors. In this case, the deflection of the membrane surface is only related to the load and temperature. When the load is fixed, the higher the temperature, the faster the pre-tension is reduced, and the increase in the deflection of the membrane surface is accelerated.

4. Simulation analysis

4.1 Relationship between pre-tension of membrane structure and temperature

In order to make the simulation analysis more accurate, we use ANSYS's own Design Modeler component to model, the modeling results are shown in Figure 2.

Figure 2. Membrane structure model

Since the modulus of elasticity of the membrane varies with temperature, it is simulated at different temperatures with different elastic moduli. According to Table 1, different elastic moduli were set, the thermal expansion coefficient of the PTFE membrane was set to $9 \times 10^{-5} \, ^\circ C^{-1}$.

An initial pre-tension of 18 MPa was applied to the membrane structure, the initial temperature was 20°C, and the frame around the membrane structure was set to a fixed constraint, as shown in Figure 3.

Figure 3. Temperature conditions and fixed constraints

Based on the ambient temperature at 20 °C, the pre-tension changes at different temperatures (-50~70 °C) were analyzed sequentially by changing the different temperatures and corresponding elastic moduli. The results are shown in Table 2.

| Temperature /°C | Pre-tension /MPa |
|----------------|-----------------|
| -50            | 35.2            |
| -40            | 33.2            |
| -30            | 31.0            |
| -20            | 28.7            |
| -10            | 26.2            |
| -0             | 23.6            |
| 10             | 20.9            |
| 20             | 18.0            |
| 30             | 15.0            |
| 40             | 11.8            |
| 50             | 8.5             |
| 60             | 5.0             |
| 70             | 1.4             |

It can be seen from Table 2 that the pre-tension decreases with increasing temperature. If the pre-tension is insufficient, the pre-tension is reduced to zero and the membrane surface is relaxed when the temperature is raised. Take the data from -50 to 70 °C to make a graph, as shown in Figure 4.

Figure 4. Pretension-temperature curve
It can be seen from Figure 4 that the temperature and the pre-tension are roughly in a quadratic curve. As the temperature increases, the pre-tension decreases rapidly, and the pre-tension from -50°C to -40°C decreases by 2 MPa, and from 60°C to 70°C decreased by 3.6 MPa, which is because the higher the temperature, the greater the modulus of elasticity of the membrane, according to formula (6), the change in pre-tension $\Delta \sigma$ is also greater.

Further, as is clear from the formula (6), the change in the pre-tension is independent of the initial pre-tension, so that the change in the pre-tension of the structure in the fixed temperature range is also fixed, as shown in Table 3.

| Temperature range /°C | Pre-tension change value /MPa |
|-----------------------|------------------------------|
| -50 ~ -40             | 2.0                          |
| -40 ~ -30             | 2.2                          |
| -30 ~ -20             | 2.3                          |
| -20 ~ -10             | 2.5                          |
| -10 ~ 0               | 2.6                          |
| 0 ~ 10                | 2.7                          |
| 10 ~ 20               | 2.9                          |
| 20 ~ 30               | 3.0                          |
| 30 ~ 40               | 3.2                          |
| 40 ~ 50               | 3.3                          |
| 50 ~ 60               | 3.5                          |
| 60 ~ 70               | 3.6                          |

Table 3 shows the variation of the pre-tension of this structure in different temperature intervals. Since the change of pre-tension in a certain temperature interval is fixed, only know the temperature and pre-tension at the time of installation, and then the temperature change can be introduced according to Table 3. The pre-tension, the data at different temperatures, and then fit the curve equation, so that the pre-tension at all temperatures can be obtained.

The initial temperature of this case is 20°C, and the initial pre-tension is 18 MPa. The image of Figure 4 approximates the quadratic curve and is fitted to the curve equation as follows:

$$F = 23.632 - 0.2673t - 0.0007t^2$$

Where, $F$ is the pre-tension and $t$ is the ambient temperature.

Different pre-tension equations with temperature can be derived from different initial conditions. This provides a reference for the installation of the membrane structure. During installation, the appropriate pre-tension should be selected according to the local temperature range of the year to avoid a series of problems caused by insufficient pre-tension.

4.2 Relationship between membrane surface deformation and temperature under uniform load

The membrane structure is inevitably subjected to loads such as wind, rain and snow in an outdoor environment, and the load that can be withstood is determined by the magnitude of the pre-tension. As the temperature changes, the pre-tension changes and the load carrying capacity changes.

The environmental load is simplified into a uniform load for analysis. Since the planar membrane structure is often used as the structure of a building roof, the load subjected to it is often rain and snow load. Since the rainwater can be discharged in time, the snow has a great influence on the planar membrane structure. Assuming that the maximum amount of snow in the area is 500mm and the average density of snow is 100kg/m², the pressure caused by snow can be calculated as 500Pa. Therefore, it can be considered that a uniform load of 500Pa is applied to the membrane structure as shown in Figure 5.
Under the initial conditions of temperature 20°C, pre-tension 18MPa and uniform load 500Pa, the deformation of the membrane surface at -50~70°C was analysed in turn, as shown in Table 4, according to the data in Table 4, the image is shown in Figure 6.

Table 4. Membrane deformation at different temperatures

| Temperature /°C | -50 | -40 | -30 | -20 | -10 | 0   | 10  | 20  | 30  | 40  | 50  | 60  | 70  |
|-----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Membrane deformation /mm | 12.8 | 13.5 | 14.4 | 15.5 | 16.9 | 18.7 | 21.0 | 24.0 | 28.2 | 34.2 | 42.8 | 54.3 | 68.0 |

Figure 6. Membrane deformation - temperature curve

It can be seen from Figure 6 that under uniform load, as the temperature increases, the deformation of the membrane surface gradually increases, and the growth rate becomes faster and faster, which is in accordance with formula (9).

According to the requirements of the Technical Specifications for Membrane Structures\cite{6}, when designed according to the normal use limit state, the relative normal displacement of the membrane surface in the structure should not exceed 1/75 of the maximum size of the membrane. The maximum size of this structure is 3.15m in the length direction, and the deformation of the membrane surface cannot exceed 42mm. In this case, the deformation of the membrane surface is determined by the temperature, so the temperature of the membrane surface deformation of 42mm can be simulated. As shown in Figure 7, when the temperature is 49.2°C, the membrane deformation is 42mm.
Figure 7. Membrane deformation at 49.2 °C

Substituting the temperature 49.2°C into the formula (10), the pre-tension was 8.79MPa. It can be seen that in the temperature change, the pre-tension should be maintained at least 8.79MPa to achieve the deformation of the membrane surface. If the temperature range of a region is -10~40°C, and the installation is carried out at -10°C, it is necessary to consider that the pre-tension will decrease by 14.4MPa at 40°C, and the pre-tension at 40 °C should be at least 8.79MPa. The pre-tension at -10°C is at least 23.19MPa.

In summary, the method of pre-tension selection during membrane structure installation can be summarized. Firstly, the pre-tension should be at least 8.79 MPa at the highest ambient temperature, and then the temperature at the installation to the maximum temperature can be calculated with reference to Table 3. In the reduction of the pre-tension, the two values are added together, which is the pre-tension required for installation. To accurately calculate the reduction of the pre-tension, it is best to re-simulate by the initial conditions, fit the curve equation, and substitute for the solution.

5. Conclusion

The paper analyses the pre-tension and membrane surface deformation of the planar membrane structure at different temperatures, and obtains the following conclusions:

(1) As the temperature increases, the pre-tension of the membrane structure decreases, and the decreasing speed gradually increases, which is roughly quadratic. In the same temperature range, the change in pretension is only related to the nature of the membrane itself, independent of the initial pretension.

(2) Under the uniform load of 500Pa, the deformation of the membrane surface increases with the increase of temperature, and the speed of increase gradually increases. The relationship between deformation and temperature is roughly quadratic. In the case, the pre-tension of the membrane structure should be at least 8.79 MPa to meet the requirements of membrane deformation.

(3) The pre-tension during installation can be determined according to the pre-tension, which is guaranteed at the highest temperature and the pre-tension, which is reduced when the temperature changes. The addition of the two values is the pre-tension during installation, which provides a reference for the installation of the membrane structure.

In summary, the pre-tension of the membrane structure should take into account the change of the ambient temperature during installation. According to different temperature changes of the four seasons and the temperature during installation, select the appropriate pre-tension to avoid the membrane surface slack or membrane surface deformation caused by temperature change being too large.

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