Realization of blue spectrum in generalized Galileon super-inflation models

K. Nozari\textsuperscript{a,1} and S. Shafizadeh\textsuperscript{b,2}

\textsuperscript{a}Department of Physics, Faculty of Basic Sciences, University of Mazandaran, P. O. Box 47416-95447, Babolsar, IRAN
\textsuperscript{b}Department of Physics, Payame Noor University (PNU), P. O. Box 19395-3697, Tehran, Iran

Abstract

In the spirit of Galileon inflation and by considering some sorts of non-canonical kinetic terms in the action, we realize a stage of super-inflation leading to a blue-tilted tensor perturbation. We show also that addition of Galileon-like term to the action leads to avoidance of ghost instabilities in this setup.

Key Words: Inflation, G-Inflation, Cosmological Perturbation, Super-Inflation, Blue Spectrum

PACS: 98.80.Cq, 98.80.Es

1 INTRODUCTION

An important outcome of the inflationary scenario for very early stage of the universe evolution [1] is that it yields hypothetical framework for generation and growth of the scalar and tensor perturbations [2]. Such a framework can be tested via observations such as the CMB anisotropies, and could be used in essence as a tool to determine viability of different models of inflation [3].

Although many inflationary models predict observable gravitational wave background, primordial tensor perturbations (gravitational waves) construct a little contribution in the temperature perturbations (anisotropies) of the cosmic background radiation. Measurements of the CMB polarization can help in essence to detect these perturbations. The CMB polarization decomposes to orthogonal components, one of them is E-mode (curl-free) that is generated by density perturbations at recombination, hence is related to temperature anisotropies of the CMB radiation. Another one, the B-mode, is divergence-free and originates from the differential stretching of spacetime related to a background of initial gravitational waves [4]. Fortunately and finally the gravitational wave has been detected by the LIGO Scientific Collaboration and Virgo Collaboration in a context other than the CMB anisotropy (in the context of Binary Black Hole Merger) [5]. In this regard, we need some more theoretical advancements to see the role of tensor perturbation in cosmological structure formation. Since the tensor power spectrum depends only on the Hubble expansion rate during inflation, these power spectrum is think carefully as a straight investigation of the scale of inflation. In this regard, it is very often deduced that the tensor spectrum from vacuum fluctuations is always red-tilted. On the other hand, in standard inflation models, the hubble expansion rate reduces in a gradual way ($\dot{H} < 0$) and involves

\textsuperscript{1}knozari@umz.ac.ir
\textsuperscript{2}s.shafizadeh@tpnu.ac.ir
\( \epsilon > 0 \) where \( \epsilon \) is the slow-roll parameter. Therefore, we can conclude that the tensor spectral index of perturbations is always negative and leads to fluctuation modes with shorter wavelengths having lower magnitudes than those fluctuations with longer wavelength. Although the recent observational data are unable to determine the spectrum of tensor perturbation in CMB polarization, the possibility of being blue-tilted for inflationary perturbations is corresponding to the negative values of \( \epsilon \). This condition implies violation of the null energy condition that is concluded from \( \dot{H} > 0 \) [6,7,8].

This feature can be encoded in the notion of super-inflation. Loop quantum cosmology as an approach to super-inflation predicts an era of super-inflation by modified curvature sector of the Einstein-Hilbert action. These quantum modifications cause to achieve super-inflation epoch which happens during the early universe, independently of the shape of potential [9,10,11].

As another approach, it is possible also to realize an epoch of super-inflation via addition of non-canonical kinetic terms (in the presence of Galileon term) in the Lagrangian. Here, we suggest a simple mechanism to realize blue-tilted tensor spectrum in an era of super-inflation, with \( \dot{H} > 0 \), by considering some sorts of non-canonical kinetic terms in the Lagrangian. In fact, one advantage of super-inflation stage is that it can modify the spectral tilt by making it towards blue-tilted one before the conventional slow-roll inflation kicks in with a decreasing \( H(t) \), which is known to yield a red-tilted spectrum [12]. Prediction of the super-inflation epoch shows that \( H \) may grow during inflation when the null energy condition, \( \rho + P \geq 0 \) (with \( \rho \) the energy density and \( P \) the pressure), is violated.

Based on these preliminaries, in this paper we concentrate on the spectral index of primordial perturbations produced in a super-inflation stage. Our idea is that perturbations could give rise in a period of super-inflation during which the Hubble rate rapidly grows, instead of staying almost constant as it is the case throughout standard slow-roll inflation regime. Our purpose here is to study the possibility of blue tilted primordial perturbations in super-inflation regime by considering models of inflation containing running kinetic terms and non-canonical kinetic term in the action and in the spirit of G-inflation (see [13-17] for G-inflation). By using higher order kinetic terms we provide required conditions for rapidly growing of Hubble expansion parameter and creation of blue spectrum of perturbations without need to large contribution of potential of the scalar field. We show that incorporation of the Galileon-like term in this setup, which modifies the k-inflation, results in avoidance of ghost instability.

### 2 G-Inflation and background equations

We concentrate on a generalized G-inflation model with a scalar field. We phenomenologically consider a dimensionless scalar field \( \varphi \) with a action of the type

\[
S = \sum_{i=2}^{4} \int d^4x \sqrt{-g} \mathcal{L}_i .
\]  

Here \( g \) is the determinant of the metric \( g_{\mu\nu} \) and [18]

\[
\mathcal{L}_2 = K(\phi, X) ,
\]

\[
\mathcal{L}_3 = -G_3(\phi, X) \Box \phi ,
\]

\[
\mathcal{L}_4 = G_4R + G_4X \left[ (\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right] ,
\]

2
where $R$ is the Ricci tensor, $K$ and $G_i$ are arbitrary functions of $\phi$ and $X = -(\frac{1}{2})g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$, $G_i(\phi, X) = g_i(\phi) + h_i(\phi)X$ and $\Box \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu$ is the standard d’Alembertian operator. This theory was originally found by Horndeski [16,19] in a different form. In the recent form, it can be used as a framework to study the single-field inflation model. This G-inflation model contains k-inflation, the non-minimal coupling between the scalar field and gravity and also the non-minimal derivative coupling to the Einstein tensor. In this setup we remove higher derivatives that would in other respects appear in the field equations, and consider the G-inflation which includes the second derivatives of the scalar field. We consider inflation model with the following choices

\[ K(X, \phi) = K(\phi)X - V, \]  
\[ G_3 = -\gamma(\phi)X, \]  
\[ G_4 = \frac{1}{2}(m_{pl}^2 + \xi \phi^2) + \frac{1}{2\mu^2}X, \]

where $\gamma(\phi)$ is a dimensionless function of the scalar field. We first derive the background equations for the theories given by the action (1) on the flat Friedmann-Robertson-Walker (FRW) geometry with the scalar factor $a(t)$ of the form $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^idx^j$ where $t$ is the cosmic time. Taking variation of the action at first order with respect to the metric, leads to the Friedmann equations as

\[ H^2 = \frac{\rho_\phi}{3m_{pl}^2}, \quad \dot{H} = -\frac{\rho_\phi + P_\phi}{2m_{pl}^2}, \]

where the energy density and the pressure of the scalar field correspondingly can be written as follows:

\[ \rho_\phi = \frac{1}{2}\dot{\phi}^2 \left[ K(\phi) - 6\gamma H \dot{\phi} + \gamma \phi \dot{\phi}^2 - \frac{9}{\mu^2}H^2 - 12\dot{H} - 6\gamma H^2 \frac{\phi^2}{\dot{\phi}} \right] + V, \]
\[ P_\phi = \frac{1}{2}\dot{\phi}^2 \left[ K(\phi) + \gamma \phi \dot{\phi}^2 + 2\gamma \dot{\phi} - 4\dot{\phi} - \frac{3H^2 + 2\dot{H}}{\mu^2} \right] + 4\xi \]
\[ + 2\left( \frac{m_{pl}^2 + \xi \phi^2}{\phi^2} - 1 \right) \left( 3H^2 + 2\dot{H} \right) + 4\xi \left( \frac{\dot{\phi}}{\phi^2} + \frac{2H}{\phi} \right) - V \]

The scalar field equation of motion is given by variation of the action (1) with respect to $\phi(t)$

\[ \frac{1}{a^3} \frac{d}{dt}(a^3 J) = \mathcal{P}_\phi \]

where

\[ J = \dot{\phi} \left( K(\phi) + \frac{3}{\mu^2}H^2 \right) - 3\gamma H \dot{\phi} + \gamma \phi \dot{\phi}^3 \]

and

\[ \mathcal{P}_\phi = \frac{1}{2}K\phi \dot{\phi}^2 - V_\phi + \frac{1}{2}\dot{\phi}^2 \left( 2\gamma \phi \dot{\phi} + \gamma \phi \phi^2 \right) + 12\dot{\phi} \left( \frac{\dot{\phi}}{\phi^2} \right) \left( 2H^2 + \dot{H} \right) \]

Finally, by substituting equations (12) and (13) into the equation (11) we can derive the following equation of motion

\[ \ddot{\phi} \left( K - 6\gamma H \dot{\phi} + \frac{3}{\mu^2}H^2 + 2\gamma \phi \dot{\phi} \right) + 3H \dot{\phi} \left( K - 3\gamma H \dot{\phi} + \frac{3}{\mu^2}H^2 \right) \]
\[ + \frac{1}{2}\dot{\phi}^2 \left( K\phi - 6\gamma \dot{H} + \gamma \phi \phi^2 \right) - 6\phi \left( 2H^2 + \dot{H} \right) + V_\phi = 0 \]
Hence, we can derive the scalar field equation of state parameter as
\[
 w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2}{\dot{\phi}^2} \left[ \mathcal{K} + \gamma_\phi \dot{\phi}^2 + 2\gamma_\phi - 4\left(\frac{H\dot{\phi}}{\mu^2}\right) + 4\xi + 2\left(\frac{m_{pl}^2 + \xi\dot{\phi}^2}{\mu^2}\right) - 1 - \frac{1}{2\mu^2}(3H^2 + 2\dot{H}) + 4\xi\phi\left(\frac{\dot{\phi}^2}{\phi^2} + \frac{2H}{\phi}\right) \right] - V
\]
(15)
and the deceleration parameter as
\[
 q \equiv -1 - \frac{\ddot{H}}{H^2} = \frac{1}{2} + \frac{3}{2}w_\phi.
\]
(16)

Rapid roll condition during the super-inflation results in the following relation
\[
\frac{\dot{\phi}}{H} \approx -(m_{pl}^2 + \xi\phi^2) \frac{F}{V\left(\mathcal{K} - \frac{V}{U} + \sqrt{(\mathcal{K} + \frac{V}{U})^2 + 4\gamma F}\right)}
\]
(17)
where
\[
 F = \frac{V_\phi^2}{m_{pl}^2 + \xi(1 + 6\xi)\phi^2}, \quad U = \frac{V}{m_{pl}^2 + \xi\phi^2}.
\]
(18)

3 Quadratic action for tensor and scalar perturbations

In this section we study tensor and scalar cosmological perturbations in generalized G-inflation by action expansion method. Let us first consider unitary gauge in which \(\delta \phi = 0\) and begin by writing
the perturbed metric in the Arnowitt-Deser-Misner (ADM) formalism as [20]
\[
ds^2 = -N^2dt^2 + h_{ij}(dx^i + N^idt)(dx^j + N^jdt)
\]
(19)
where \(N\) and \(N^i\) are the lapse and shift functions
\[
N = 1 + 2\Phi, \quad N_i = \delta_{ij}\partial^jB, \quad h_{ij} = a^2(t)e^{2\Psi}\delta_{ij}
\]
(20)
Here, \(\Phi, \Psi\) and \(B\) are scalar perturbations and \(h_{ij}\) is defined as tensor perturbation. Now we write the perturbed metric at the linear level as [21, 22, 23]
\[
ds^2 = -(1 + 2\Phi)dt^2 + 2a^2(t)B_{ij}dx^idt + a^2(t)(1 - 2\Psi)\delta_{ij}dx^i dx^j
\]
(21)
Expanding the action (1) up to the second order in the perturbations, we obtain the following result
\[
S_2 = \int dt d^3x a^3 \left\{ -3\left((m_{pl}^2 + \xi\phi^2) - \frac{X}{2\mu^2}\right)\Psi^2 + \frac{1}{a^2}\left[2\left((m_{pl}^2 + \xi\phi^2) - \frac{X}{2\mu^2}\right)\Psi - \left(\frac{6}{\mu^2}H X + 2H(m_{pl}^2 + \xi\phi^2)\right)\Phi B\right] - \frac{2}{a^2}\left((m_{pl}^2 + \xi\phi^2) - \frac{X}{2\mu^2}\right)\Phi\partial^2\Psi + \left(\frac{2}{\mu^2}H X + H(m_{pl}^2 + \xi\phi^2)\right)\Psi + \left(\frac{1}{\mu^2}(m_{pl}^2 + \xi\phi^2) + \frac{X}{\mu^2}\right)\Phi^2 + \right\},
\]
(22)
While the coefficients are quite complicated, the construction of the action (22) is similar to that expressed in Ref. [24]. By using the above second order action, we can find the momentum and Hamiltonian constrains as

\[
\Phi = \left( m^2_{pl} + \xi \phi^2 \right) - \frac{X}{\mu^2} \dot{\phi} (\gamma X + \xi \phi) \tag{23}
\]

\[
\frac{1}{a^2} \partial^2 B = \frac{X \left( \mathcal{K} - 12 \gamma \phi H + \frac{18}{\mu^2} \right) + 4 \gamma \phi X^2 - 3H^2 (m^2_{pl} + \xi \phi^2) - 6H \xi \phi \dot{\phi}}{\frac{3}{\mu^2} H X + H (m^2_{pl} + \xi \phi^2) + \dot{\phi} (\gamma X + \xi \phi)} \Phi \tag{24}
\]

By substituting Eq. (23) into Eq. (22), the second order action reduces to the following form

\[
S_2 = \int dt d^3 x a^3 U \left[ \dot{\Psi}^2 - \frac{c_s^2}{a^2} (\partial \Psi)^2 \right] \tag{25}
\]

where

\[
U = \left( m^2_{pl} + \xi \phi^2 \right)^2 \left[ \frac{X \left( \mathcal{K} - 12 \gamma \phi H + \frac{18}{\mu^2} \right) + 4 \gamma \phi X^2 - 3H^2 (m^2_{pl} + \xi \phi^2) - 6H \xi \phi \dot{\phi}}{\frac{3}{\mu^2} H X + H (m^2_{pl} + \xi \phi^2) + \dot{\phi} (\gamma X + \xi \phi)} \right]^2 + 3 \left( m^2_{pl} + \xi \phi^2 - \frac{X}{\mu^2} \right) \tag{26}
\]

and

\[
c_s^2 = \frac{\mathcal{K} + \frac{1}{\mu^2} (m^2_{pl} + \xi \phi^2)}{\mathcal{K} + (\frac{1}{\mu^2}) U + \sqrt{\left( \mathcal{K} + (\frac{1}{\mu^2}) U \right)^2 + 4\gamma F}} \tag{27}
\]

To avoid ghosts and Laplacian instabilities we require the following conditions

\[
U > 0, \quad c_s^2 \geq 0 \tag{28}
\]

The power spectrum of the curvature perturbations \( \Psi \), is given by

\[
P_{\Psi} = \frac{H^2}{8\pi^2 U c_s^2} \tag{29}
\]

The second order action for the tensor perturbations in this model can be written as

\[
S_T = \int dt d^3 x a^3 Q [h_{ij}^2 - \frac{c_s^2}{a^2} (\partial h_{ij})^2] \tag{30}
\]
where
\[ Q = \frac{(m_{pl}^2 + \xi \phi^2) - \frac{X}{2\mu^2}}{4}, \]  
\[ c_T^2 = \frac{(m_{pl}^2 + \xi \phi^2) + \frac{X}{\mu^2}}{(m_{pl}^2 + \xi \phi^2) - \frac{X}{2\mu^2}}. \]  
The power spectrum of primordial tensor perturbations is given by
\[ P_T = \frac{H^2}{2\pi^2 Q c_T^3}. \]  

4 Applications to concrete models of Super-Inflation

Now we consider parameter space of some specific models in this setup in order to study the blue spectrum of tensor and scalar modes and also to explore the conditions for super-inflation regime.

At first, let us apply the case in which the field has non-canonical form. We start from the action
\[ L_\phi = K(\phi, X), \]  
where \( X \equiv -\frac{1}{2} \nabla_\mu \phi \nabla^\nu \phi \) and
\[ K(\phi, X) = -V(\phi) + K(\phi)X, \ G_3 = 0, \ G_4 = \frac{1}{2} m_{pl}^2. \]  
The energy density and pressure of the scalar field are written respectively as follows
\[ \rho_\phi = K(\phi)X + V(\phi), \ P_\phi = K(\phi)X - V(\phi) \]  
With these quantities, it is easy to check that increment of the Hubble parameter with time during super-inflation stage yields the important result \((\rho_\phi + P_\phi) < 0\) which is violation of the null energy condition in this case. As a result, we are able to achieve \( K < 0 \) which means that \( K \) can be a negative function of the scalar field. One important point should be stressed here: in Ref. [25] the authors have shown that in potential-dominant generalized G-inflation models the blue tensor spectrum is prohibited in order to avoid instabilities. The assumptions made in our case seems to meet the postulates of Ref. [25]. However, this is not actually the case since the authors of [25] assume \( K(\phi) \) to be positive definite while here we consider the possibility of having negative \( K \) in super inflation stage. In fact, as has been shown in Refs. [7,10], in super inflation stage \( K \) can be negative. This is the main deference of our work with Ref. [25] in this respect. We note also that, an accurate description of universe during the phase of super-inflation can be acquire with a fast rolling scalar field as \( X \gg V(\phi) \). In other words, the dominated contribution is related to kinetic energy. In this case, the rapid roll condition can be provided so that \( KX \ll V(\phi), \) with \( K \) that is negative function of the scalar field. Also by taking time derivative of both sides of this condition we obtain \( V_\phi \gg (K_\phi X + K_\phi \dot{\phi}). \) Then we can derive the equation of motion for the scalar field as
\[ K\ddot{\phi} + 3K\dot{H}\dot{\phi} + K_\phi X + V_\phi = 0 \]  
By considering the condition \( V_\phi \gg (K_\phi X + K_\phi \dot{\phi}) \) the equation of motion is given by
\[ 3K\dot{H}\dot{\phi} + V_\phi \simeq 0 \]
so we achieve the result \( \dot{\phi} \simeq -\frac{V_{\phi}}{H} \). Then by using the second order action of the scalar and tensor perturbations, we can express spectral indexes in this model in terms of \( \phi \)-dependent functions as

\[
nt \simeq -2\epsilon \simeq -\frac{V_{\phi}^2}{K V}.
\] (39)

When we consider \( K < 0 \), this leads to \( n_T > 0 \). Hence the tensor spectrum can be blue tilted in this model of super-inflation. Also for \( n_s \) we find

\[
n_s - 1 \simeq -4\epsilon + \eta - \frac{j}{H J} = -\frac{3V_{\phi}^2}{K V^2} - \frac{V_{\phi}K_{\phi}}{VK^2} + \frac{2V_{\phi\phi}}{K V}.
\] (40)

For scalar modes, if \( \epsilon < 0 \) that is provided by \( K < 0 \), then \( n_s > 1 \) is possible in this case. To proceed further, we introduce ansatz for \( V(\phi) \) and \( K(\phi) \) as \( V(\phi) = \phi^n \) and \( K(\phi) = -\phi^{1/n} \) with \( n = 2 \).

Figure 1 (right panel) shows the variation of the tensor spectral index \( n_T \) versus the scalar field. Obviously, in the presence of running kinetic term, the tensor spectral index is positive. So, the tensor spectrum can be blue tilted in super-inflation regime. The left panel of this figure shows the variation of scalar spectral index \( n_s \) versus scalar field. The spectral index can be larger than unity which shows the possibility of having blue spectrum in this case.

Note that with rapid roll condition in super-inflation regime, it is seen that \( w_\phi < -1 \). It shows \( \epsilon < 0 \) and we also achieve \( 0 < e_\phi^2 < 1 \). We expect a transition from the super-inflation regime to slow roll inflation in the infrared (IR) regime. On the other hand, increasing Hubble rate of expansion continues until \( \dot{H} \to 0 \). In fact, there is a reversal in the rate of \( H(t) \) then Hubble parameter starts to decrease in slow roll phase of the universe because of slowly rolling of the scalar field, for a review see [26]. It can be seen easily that

\[
\dot{H} \to 0 \Rightarrow \epsilon \to 0 \Rightarrow n_T \to 0
\] (41)
and this condition holds if $V_\phi \to 0$. Also, it leads to $n_s \to 1$, the scale invariance of the perturbation. We note that in this case there is ghost instability since $U < 0$ as can be seen in figure 2. About the equation of state parameter, we have

$$w_\phi = -1 + \frac{V_\phi^2}{3K} \quad c_s^2 \approx 1$$

that is corresponding to the super-inflation regime. It means $w_\phi < -1$ and also $\epsilon < 0$ in this case.

Figure 2 (left panel) shows the variation of the equation of state parameter versus the scalar field. In this class of inflationary models $w_\phi < -1$ but there is ghost instability in this setup (right panel of figure 2) as in k-inflation model.

In the next step, we consider a model with G-super inflation action in which it contains Galileon-like kinetic term. We limit our attention to the alternative to super inflation model with

$$K(X, \varphi) = \mathcal{K}(\phi)X - V, \quad G_3 = -\gamma(\phi)X, \quad G_4 = \frac{1}{2} m_{pl}^2,$$

where $\gamma(\varphi)$ is a negative dimensionless function of the scalar field. In this case Friedmann equations are written as follows

$$3m_{pl}^2 H^2 = X \left[ \mathcal{K}(\phi) - 6\gamma H \dot{\phi} + \gamma \phi \ddot{\phi}^2 \right] + V,$$

$$\dot{H} = -X \left[ \mathcal{K}(\phi) - 3\gamma H \dot{\phi} + \gamma \phi \ddot{\phi}^2 + \frac{1}{2} \gamma \dddot{\phi} \right]$$

During the super-inflation phase, $\dot{H} > 0$ which is corresponding to $(\rho_\phi + P_\phi) < 0$ to have fast roll expansion in this phase of the universe expansion. Also the energy density needs to be positive.
Therefore it is required from equations (43) and (44) together to have the following conditions

\[ K X < V(\phi), \quad |\gamma H \dot{\phi}| < V(\phi), \quad K(\phi) \ll 0. \]  

(46)

Using these conditions in equation of motion, we find

\[ \frac{\dot{\phi}}{H} \simeq \frac{1}{2\gamma V} \left( K - \sqrt{K^2 + 4\gamma V\phi} \right) \]  

(47)

Now, the spectral indexes of perturbations can be obtained as

\[ n_T \simeq \left( \frac{V_\phi}{V} \right)^2 \left( \frac{-2}{K + \sqrt{K^2 + 4\gamma V\phi}} \right) \]  

(48)

and

\[ n_s \simeq 1 - \frac{V_\phi}{V[K + \sqrt{K^2 + 4\gamma V\phi}]} \left[ \frac{V_\phi}{V} + \frac{\gamma_\phi}{\gamma} + 3V_{\phi\phi} - \left( \frac{K}{K + \sqrt{K^2 + 4\gamma V\phi}} \right)\phi - \frac{3}{2} \frac{K}{K + \sqrt{K^2 + 4\gamma V\phi}} \right] \]  

(49)

We consider the following functions

\[ K = -\phi^\frac{1}{4}, \quad \gamma = -\alpha\phi^n, \quad V(\phi) = \phi^2 \]  

(50)

where \( \alpha < 1 \) and we set \( n = 1 \) in numerical analysis.

Figure 3 (right panel) shows the variation of the tensor spectral index \( n_T \) versus the scalar field. Obviously, in the presence of Galileon-like kinetic term, the tensor spectral index is positive. So, the tensor spectrum can be blue tilted in G-super-inflation regime within this setup. The left panel of this
Figure 4: Variation of the equation of state parameter (left panel) and ghost instability parameter ($\mathcal{U}$) (right panel) versus $\phi/m_{pl}$ for quadratic potential in G-super-inflation.

The figure shows the variation of the scalar spectral index $n_s$ versus the scalar field. We see that the scalar spectral index is blue tilted in this case. So, both scalar and tensor modes tilt toward realization of the blue spectrum in this setup. In this case the equation of state parameter and the sound speed squared are given by

$$w_\phi \simeq -1 + \frac{V_\phi}{6V_\phi^2(\mathcal{K} + \sqrt{\mathcal{K}^2 + 4\gamma V_\phi})}$$

and

$$c_s^2 \simeq \frac{2 - \frac{\mathcal{K}}{\mathcal{K} + \sqrt{\mathcal{K}^2 + 4\gamma V_\phi}}}{3\left(1 - \frac{\mathcal{K}}{\mathcal{K} + \sqrt{\mathcal{K}^2 + 4\gamma V_\phi}}\right)}.$$ 

Also in this case the ghost instability can be avoided if the following condition is satisfied

$$\mathcal{U} \simeq 3 + \frac{2\left[\left(\frac{\dot{\phi}}{\mathcal{H}}\right)^2(\mathcal{K} - 4\gamma V(\frac{\dot{\phi}}{\mathcal{H}}) + \frac{2\gamma V(\frac{\dot{\phi}}{\mathcal{H}})^2}{3}) - 6\right]}{3\left[1 + \frac{2V}{6}\left(\frac{\dot{\phi}}{\mathcal{H}}\right)^3\right]^2} > 0$$

Figure 4 (left panel) shows the variation of the equation of state parameter versus the scalar field. In the presence of Galileon-like kinetic term, the negative value of this parameter implies super inflation period. Figure 4 (right panel) shows the variation of instability parameter ($\mathcal{U}$) versus the scalar field. Enhancing the contribution of the kinetic energy in the action that contains non-canonical higher order kinetic term leads to avoidance of ghost instability in this setup.

Finally, let us consider a class of generalized G-inflation model where the potential is given by power law inflation. This setup corresponds to the action (1) with terms as given by equations (5)-(7) and also $\mathcal{K}(\phi) = -\phi^{1/n}$, $\gamma = -\alpha\phi^n$ with $n = 4$. Therefore, the condition to have rapid roll expansion
Figure 5: Variation of the scalar spectral index (left panel) and tensor spectral index (right panel) versus $\phi/m_{pl}$ for quadratic potential in generalized G-super-inflation.

in super-inflation stage is provided by

$$KX < V(\phi), \quad |\xi H \dot{\phi}| < V(\phi), \quad |\gamma H \dot{\phi}| < V(\phi), \quad K(\phi) \ll 0. \quad (54)$$

Using these conditions to calculate $(\dot{\phi}^2)$, the spectral index of perturbations can be determined by the following expression

$$n_s \simeq 1 - 6\epsilon - \frac{gF}{V(K + \left(\frac{1}{\mu^2}\right)U + \sqrt{(K + \left(\frac{1}{\mu^2}\right)U)^2 + 4\gamma F})} \left[\frac{\gamma\phi}{\gamma} - \frac{2(K\phi + \frac{1}{\mu^2}U\phi)}{K + \frac{1}{\mu^2}U}\right]$$

$$+ \frac{3gF\phi}{V(K + \left(\frac{1}{\mu^2}\right)U + \sqrt{(K + \left(\frac{1}{\mu^2}\right)U)^2 + 4\gamma F})}$$

(55)

where $g = (m_{pl}^2 + \xi\phi^2)$, $F$ and $U$ are given by (18) and

$$n_t \simeq -2\epsilon \simeq \frac{-2\xi\phi F}{V(K + \left(\frac{1}{\mu^2}\right)U + \sqrt{(K + \left(\frac{1}{\mu^2}\right)U)^2 + 4\gamma F})} \left(1 + \frac{\phi F}{V}\right)$$

(56)

Furthermore, we can calculate the equation of state parameter for this case as

$$\omega_\phi \simeq -1 - \frac{\dot{H}}{H^2} = -1 + \frac{2\xi\phi F}{3V(K + \left(\frac{1}{\mu^2}\right)U + \sqrt{(K + \left(\frac{1}{\mu^2}\right)U)^2 + 4\gamma F})}.$$  

(57)

To proceed further, we consider the following functions for this generalized G-inflation model during the super-inflation stage

$$K = -\phi^{\frac{1}{n}}, \quad \gamma = \alpha\phi^{\frac{1}{n}}, \quad V(\phi) = \phi^4, \quad \xi \simeq 10^4.$$  

(58)

where $\alpha < 1$ and in numerical analysis we set $n = 4$. Figure 5 (right panel) shows the variation of the tensor spectral index $n_T$ versus the scalar field. In the presence of Galileon-like kinetic term the
Figure 6: Variation of the equation of state parameter (left panel) and ghost instability parameter $U$ (right panel) versus $\phi/m_{pl}$ for quadratic potential in generalized G-super-inflation.

tensor spectral index is positive. So, the tensor spectrum can be blue in super-inflation regime in this case. The left panel of this figure shows the variation of the scalar spectral index $n_s$ versus the scalar field. The spectral index is larger than unity and is blue tilted in this case.

Figure 6 (left panel) shows the variation of the equation of state parameter versus the scalar field. In the presence of Galileon-like kinetic term with non-minimal coupling between the scalar field and gravity and the non-minimal derivative coupling of the scalar field with Einstein tensor, the negative value of this parameter (equation of state parameter) implies the existence of super-inflation period in this setup. The right panel of this figure shows the variation of the instability parameter $U$ versus the scalar field. Once again, enhancing the kinetic energy in the action that contains non-canonical higher order kinetic terms leads to avoidance of ghost instability in this setup.

5 Conclusion

We have considered a generalized G-inflation model which contains non-canonical and Galileon-like kinetic terms. We have studied the early time cosmological dynamics of this setup during the super-inflation period. We have calculated the inflationary parameters and the primordial density and tensor perturbations with details. The analysis shows that the scalar spectral index in this setup can be larger than unity and the tensor spectral index is positive. Therefore, scalar and tensor modes can be blue-tilted in this setup. We have shown that presence of the Galileon-like term leads to modified k-inflation with possibility to avoid ghost instabilities. Since the tensor power spectrum depends only on the Hubble expansion rate, we can conclude that possibility of having blue spectrum of primordial perturbations is provided when the Hubble rate increases rapidly while the null energy condition is violated. We note that in Ref. [27], where the origin of an effective quantum-to-classical transition for perturbations is clearly explained, it has been shown that breaking of the null and weak energy conditions occurs only near the first Hubble radius crossing during inflation, and it is
The blue spectrum of gravitational wave perturbations is due to the specific background behavior ($\dot{H} > 0$) only, and not specifically due to the breaking of energy conditions that occurs for any spectrum of created perturbations. As has been indicated in [28], squeezed states as negative energy fluxes with negative pressure are responsible for this condition. Another possible effect of super-inflationary phase realizing blue spectrum can be provided by modified Friedmann equation to incorporate higher derivative extension of the Einstein’s GR in the action (see [11,29-33]). In summary, in addition to other approaches such as loop quantum cosmological considerations (see also [34] for another approach), it is possible also to realize blue spectrum of perturbation by taking a super-inflation stage into account. In this manner and via a generalized Galileon-like kinetic term in the action, it is possible to avoid ghost instability that are usually present in k-inflation models.

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