Quantum repeater for continuous variable entanglement distribution

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Quantum repeaters have been proposed as a way of extending the reach of quantum communication. First generation approaches use entanglement swapping to connect entangled links along a long distance channel. Recently, there have been proposals for first generation quantum repeaters for continuous variables. In this paper, we present an improved continuous variable repeater scheme using fully deterministic optimal Gaussian entanglement swapping. Our scheme uses the noiseless linear amplifier for entanglement distillation. We show that even with the simplest configuration of the noiseless linear amplifier, our scheme beats the secret key rate over previous CV repeater schemes.

I. INTRODUCTION

The development of technologies according to the principles of quantum mechanics allows many promising real world applications. Under the umbrella term of quantum communication [1], these applications range from secure communication [2–4] and quantum state transfer [5], to enhanced quantum sensing [6–8] and computation [9, 10]. However, utilizing these technologies over long distances remains challenging due to fiber loss or free space attenuation. In classical communication, this problem is solved by having repeaters stationed at various points along the channel to amplify the signal. This solution, that has enabled classical communication to proceed, may not be employed for quantum communication as redundant copies of quantum information cannot be made due to the no-cloning theorem [11]. A more sophisticated solution is necessary if these issues are to be overcome and we are able to utilize the advantages of quantum communication over long distances.

One proposed solution has come in the form of a quantum repeater [12]. The first quantum repeater protocol from the late nineties used multiple rounds of entanglement swapping [13] in order to connect entangled pairs, and share entanglement between ends of a long distance channel. Entanglement purification [14] was also required to correct against building operation errors. Since this first proposal, there has been significant theoretical advancement on repeater protocols and experimental progress with repeater elements [15–17].

Currently, the majority of repeater proposals are for discrete variable (DV) encodings of quantum information [15–17], where information is encoded in a finite dimensional basis, such as the polarization of single photons. As an alternative, there are also continuous variable (CV) encodings of quantum information, where information is encoded in the quadrature amplitudes. Not only do continuous variable encodings of quantum information offer (in principle) easier state generation, manipulation and detection [18], they also offer the possibility of compatibility with existing infrastructure [19].

In the past few years, there have been three different proposals for the first generation of continuous variable quantum repeaters [20–22]. In this paper, we present an improvement upon one of these previous CV quantum repeaters, the protocol presented in Ref. [20]. Like our previous work in Ref. [20] and the CV repeater scheme from Ref. [22], our repeater uses the single quantum scissors (QS) to distill CV entangled states. Unlike Ref. [22], which uses non-deterministic non-Gaussian entanglement swapping, our CV quantum repeater uses fully deterministic Gaussian entanglement swapping. Our scheme surpasses a fundamental upper limit on quantum communication via direct transmission (the so-called PLOB bound) [23] for a total distance of 238 km.

This paper is arranged in the following way: in Sec. II we explain the structure of our CV repeater, in Sec. III we will present results comparing two alternative strategies for entanglement swapping and how performance of the repeater scales with distance. Finally, in Sec. IV we provide some future directions based on our findings and conclude.

II. CV QUANTUM REPEATER

First generation quantum repeaters are based on three core elements: entanglement distribution, entanglement swapping, and entanglement purification (or distillation) protocols (see Ref. [15] for a review). In the following section, we will give an overview of how each of these elements will be implemented in our repeater.

A. Entanglement distribution

Beginning with entanglement distribution, the entangled resource states used in our protocol are the Gaussian
Entanglement distribution

(a)

Entanglement distillation

(b)

Entanglement swapping

(c)

FIG. 1. Components of the CV quantum repeater (a) Entanglement distribution in the CV quantum repeater. One mode of a TMSV state is sent through a lossy channel to a neighboring repeater node. The other mode of the entangled state remains in the same node. (b) The CV repeater uses the NLA to distill entangled TMSV states. The simplest linear optics construction of the NLA is pictured here consisting of a single quantum scissor (QS). The input is combined with an ancilla photon which has passed through a beam-splitter of tunable ratio $\xi$, this is related to the gain of the NLA via $g = \sqrt{(1 - \xi)/\xi}$. The combined modes are detected and success is heralded when a single photon is detected at one output and none at the other. (c) Gaussian entanglement swapping protocol from Ref. [24]. Modes of two independent TMSV states are combined and input into a dual homodyne detection (dual HD). The results of the detection are sent in both directions to both output modes where displacements are performed accordingly.

two-mode squeezed vacuum (TMSV) state:

$$|\chi\rangle_{ab} = \sqrt{1 - \chi^2} \sum \chi^n |n\rangle_a |n\rangle_b,$$

(1)

where $0 < \chi < 1$ is the two-mode squeezing parameter. Distribution of these states (1), is performed asymmetrically (see Fig. 1a) with entangled states being generated at each node of the quantum repeater and then one mode of the entangled state is passed through a lossy channel through to the neighboring node. One mode of each of these entangled states would be decohered by loss from transmission through the channel while the other mode remains untouched in the same node.

B. Entanglement distillation

In our CV repeater, entanglement distribution in the repeater links is followed immediately by distillation on the entangled mode that has passed through the lossy channel. Entanglement distillation is a necessary component in first generation repeaters, needed to combat the decoherence effects from channel loss and entanglement swapping operations. In the scheme of Refs. [20, 22] and in this work, the Noiseless Linear Amplifier (NLA) [25] is used to distill the entangled states. When implemented with linear optics, the simplest NLA comprises of a single modified QS device [25]. The single QS implementation of the NLA has been demonstrated experimentally [26, 27], and more specifically entanglement distillation on TMSV states decohered by loss has been demonstrated with a similar device [28].

C. Entanglement swapping

Following entanglement distribution and distillation, our CV repeater will use deterministic Gaussian entanglement swapping [29] in order to connect the entangled repeater links. We employ the optimal Gaussian entanglement swapping protocol described in Ref. [24]. This involves sending classical signals to both ends of the channel and conducting displacements on both modes (see Fig. 1c). This is unlike other protocols (including CV teleportation) where classical communication and displacements are only performed on one mode. In this way, two pure Gaussian entangled states can be swapped and the resulting entangled state remains pure. In general, for any two Gaussian states, entanglement swapping in this way is optimal [24]. The use of the optimal Gaussian entanglement swapping scheme of Ref. [24] represents the main difference between this work and the work in Ref. [20] which used CV teleportation, and as we show in the following section, we observe a dramatic improvement in performance due to use of this swapping scheme.

III. RESULTS

A. Single-node repeater

The simplest implementation of our improved CV repeater protocol combing all the aforementioned elements,
is shown in Fig. 2. It is formed using a single repeater node in the center of the channel with NLAs implemented in their simplest configuration (consisting of a single quantum scissor). Entanglement distillation is performed by sending one mode of a TMSV state (1) through the channel between the single repeater node and ends of the channel. The mode of the entangled state that had passed through the lossy channel is then distilled using the single quantum scissor.

While the quantum scissor operation is non-deterministic, both entangled states are independent at this point in the protocol, therefore both quantum scissors can operate independently and simultaneously. When a quantum scissor heralds successful operation, we assume high quality quantum memories are available to store the distilled entanglement until the other quantum scissor is successful. After both entangled states have been distilled, they are then swapped by mixing the two modes at the repeater node and conducting a dual homodyne detection. The results of this detection are then sent to Alice and Bob and a displacement is performed on each mode based on the results of the detection which completes the entanglement swapping operation.

We investigated two configurations of this entanglement swapping scheme, both are pictured in Fig. 2. In Fig. 2(a), entanglement swapping is symmetric as both entangled sources remain at the central repeater node and are distributed to Alice and Bob where entanglement distillation is performed. In Fig. 2(b), the entangled sources are positioned with Alice and the repeater node, and are distributed to the node and Bob respectively.

The secret key rate of both configurations Fig. 2(a) and 2(b) are able to surpass the absolute maximum secret key rate for direct transmission (shown by the dashed, grey line on Fig. 3 and referred to as the PLOB bound [23]). In Appendix A, we give details on how we calculate the entangled output state of these protocols. Using these entangled states, we present the key rates in Fig. 3 calculated for an entanglement-based CV-QKD protocol [30], where Alice and Bob both perform heterodyne detection to their own entangled modes to obtain raw key. The key rate for the asymmetric entanglement swapping protocol is shown for reverse reconciliation, where Bob is the reference for reconciliation which is favorable in high loss regimes. Note that for the symmetric protocol, it does not matter which one is the reference for reconciliation. The secret key rate presented in Fig. 3 has been calculated from the covariance matrix (see details in Appendix A) and normalized by the success probability of the QS and the average number of steps needed to generate success in all distillation protocols [31]. Optimization of this normalized rate has been performed at each point over both gain of the QS's and strength of the TMSV state sources. Note that success probability of the QS decreases as gain is increased. The asymmetric
protocol achieves optimal performance for squeezing of 0.311 < $\chi_{\text{opt}}$ < 0.331 and the symmetric protocol optimizes for 0.075 < $\chi_{\text{opt}}$ < 0.087.

Fig. 3 shows that the asymmetric protocol outperforms the symmetric protocol and is able to surpass the direct transmission key rate at a shorter distance. From Fig. 3, this can be seen as 208km compared to 321km. The asymmetric and symmetric protocols surpass the PLOB bound for 264km and 377km respectively. This represents significant improvement upon single node operation of the CV repeater in Ref. [20] which uses CV teleportation and beats the PLOB bound for distances above 500km [32]. We emphasize these distances are total channel distances, meaning the point at which the asymmetric protocol beats direct transmission, 208km, corresponds to 104km of optic fiber between Alice and the node (and between Bob and the node).

B. Multi-node repeater with nested swapping

In order to use this CV quantum repeater over long distances, more nodes along the channel are required as well as more entanglement swapping operations to connect the entangled links. To illustrate how this would proceed, see Fig. 4 with four links of the repeater connected via three repeater nodes. The protocol in Fig. 4 is just two copies of the asymmetric entanglement swapping protocol in Fig. 2(b) connected via another Gaussian entanglement swapping. Because the asymmetric scheme outperformed the symmetric scheme at the single-node level, we will focus on asymmetric scheme in modeling operation with multiple repeater nodes.

For even longer distances and more repeater nodes, nesting proceeds in this way, where the output of two identical and independent copies of the protocol in Fig. 4 would be connected within another entanglement swapping operation. It is important to note that our repeater does not use nested entanglement distillation, meaning distillation occurs after entanglement distribution and not at any time after. Structuring the repeater in this way has an extremely favorable effect on the repeater rates, as entanglement distillation is the only non-deterministic element in our CV repeater protocol.

In Fig. 5, we present the secret key rate for higher numbers of repeater nodes showing how our CV repeater scales with distance. Calculations for these key rates proceed in the same way as the previous result in Fig 3 (please refer to Appendix B for details). This is the main result of the paper and represents orders of magnitude increases in performance upon the previous CV repeater protocol in Ref. [20]. Compared to the protocol of Furrier et al. in Ref. [21], our repeater outperforms results achieved using both non-Gaussian and Gaussian entanglement swapping and surpasses the PLOB bound for a shorter distance.

Like the result in Fig. 3, we show key rates in Fig. 5 that have been normalized by the success probability of the QS and average number of steps needed to generate successes in all QS’s [31]. Optimization has again been carried out at each point over the gain of the QS and strength of the squeezing in the TMSV source. Optimal squeezing ranges between $\chi = 0.405$ for 10km to $\chi = 0.070$ for 1600km, corresponding to between 0.61 to 3.73 dB of squeezing (see Appendix C).

As expected, the optimal QS gain increases with larger distances between repeater nodes. As an example, for the point highlighted on Fig. 5(b) at 264km with 16 repeater nodes, this corresponds to node spacing of about 16.5km. At this node spacing, the optimal amplitude gain of the QS is $g = 13.6$. In comparison, at 1600km with 16 repeater nodes (100km node spacing), the optimal QS gain is $g = 126$. We expect that for distances on the order of 1600km, rates could be improved by employing more repeater nodes, and the QS gain will likely optimize at a lower (and easier to experimentally realize) value.

IV. CONCLUSION

In summary, we have presented here a novel scheme for a CV repeater. We emphasize our approach here is different to that of Ref. [20] as we focus on distribution of CV entanglement, rather than preparing an improved channel. While the distributed entanglement can be used for many different quantum information applications, we show that when these entangled states are used for CV QKD, we are able to improve upon the rates achieved from previous CV repeaters in the literature [20, 21]. In our view, this improvement is attributed to the use of
the optimal (deterministic) Gaussian entanglement swapping protocol described in Ref. [24], because entanglement swapping in this way allows us to circumvent the need for nested entanglement distillation. The only probabilistic operation in our scheme is the single round of entanglement distillation. After establishing distilled entangled links between the repeater nodes, all subsequent swapping is deterministic.

While our analysis incorporates non-ideal reconciliation efficiency, it is highly idealized in all other senses. A remaining question to be answered would be how the performance of our CV repeater is affected by experimental inefficiencies including inefficient single photon sources in the NLAs, inefficient homodyne detection and imperfect quantum memories. Specifically with inefficient single photon sources, prior work has shown that this inefficiency causes a gain saturation effect thus limiting the actual achievable gain of the NLA [26, 27]. It is possible that operation of this repeater may be further optimized by use of a different distillation protocol. Consideration of how this scheme performs with different distillation protocols is left for future work. Never-the-less, the results we report here represent a significant and promising improvement upon previous CV repeater schemes.

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Appendix A: Single-node repeater

1. Symmetric entanglement swapping

In this section, we outline how to calculate the entangled output state of the symmetric single-node repeater protocol (Fig. 4(a) in the main text). Initial entanglement distribution begins with generating two independent two-mode Gaussian squeezed vacuum states of form:

\[ |\chi\rangle_{AC} = \sqrt{1 - \chi^2} \sum_{n=0}^{\infty} \chi^n |n\rangle_A |n\rangle_C \]  \hspace{1cm} (A1)

One mode of each entangled state is distributed to both Alice and Bob, modeled by a pure-loss channel of trans-
The probability of success of this individual NLA can be described by the following operation [20]:
\[
\hat{U}_{BS} [\ket{n}_A \ket{0}_D] = \sum_{p=0}^{n} \sqrt{\binom{n}{p} \eta^{p/2} (1 - \eta)^{(n-p)/2}} \ket{p}_A \ket{n-p}_D
\]
(A2)
where mode \(D\) is an environment mode. The state becomes:
\[
\ket{\chi}_{AC} \rightarrow \sqrt{1 - \chi^2} \sum_{n=0}^{\infty} \sum_{p=0}^{n} \chi^n \ket{n}_C \sqrt{\binom{n}{p} \eta^{p/2} (1 - \eta)^{(n-p)/2}} \ket{p}_A \ket{n-p}_D
\]
(A3)
Entanglement distillation proceeds by acting an NLA on mode \(A\) with gain \(g\). The action of the NLA with a single quantum scissor can be described by the following truncation operator \(\hat{\Pi}\) [20]:
\[
\hat{T}_1 = \hat{\Pi}_1 g^\hat{a}
\]
(A4)
where the truncation operator \(\hat{\Pi}_1\) is defined as:
\[
\hat{\Pi}_1 = \frac{1}{\sqrt{g^2 + 1}} (\ket{0}_0 \bra{0} + \ket{1}_1 \bra{1})
\]
(A5)
After this operation, the \(|2\rangle\) and higher order photon terms in mode \(A\) are truncated and the state becomes:
\[
\ket{\psi}_{ACD} = \sqrt{\frac{1 - \chi^2}{g^2 + 1}} \left( \sum_{n=0}^{\infty} \chi^n (1 - \eta)^{n/2} \ket{0}_A \ket{n}_C \ket{n}_D \right) + g\eta \sqrt{\frac{1}{g^2 + 1}} \sum_{n=0}^{\infty} \chi^n \sqrt{m} (1 - \eta)^{(n-1)/2} \ket{1}_A \ket{n}_C \ket{n-1}_D
\]
(A6)
The probability of success of this individual NLA can be found via the norm of the un-normalised state (A6)
which is:
\[
P_s = \frac{(1 - x^2) \left( \chi^2 (\eta g^2 + \eta - 1) + 1 \right)}{(g^2 + 1) (\eta g^2 + \eta - 1)}
\]
(A7)
The final step in this single-node repeater protocol is the entanglement swapping operation. We use a second copy of the state (A6), with modes \(F\) and \(B\) distributed between the repeater node and Bob respectively, given by:
\[
\ket{\psi}_{BFE} = \sqrt{\frac{1 - \chi^2}{g^2 + 1}} \left( \sum_{m=0}^{\infty} \chi^m (1 - \eta)^{m/2} \ket{0}_B \ket{m}_E \ket{m}_F \right) + g\eta \sqrt{\frac{1}{g^2 + 1}} \sum_{m=0}^{\infty} \chi^m \sqrt{m} (1 - \eta)^{(m-1)/2} \ket{1}_B \ket{m}_E \ket{m-1}_F
\]
(A8)
where mode \(E\) is an environment mode. To summarize, in the symmetric version of this repeater scheme, modes \(F\) and \(C\) remain at the repeater node while modes \(A\) and \(B\) are distributed to Alice and Bob respectively. The identical modes \(F\) and \(C\) are combined and detected at the repeater node via a dual homodyne detection. To model this dual homodyne detection, we project modes \(F\) and \(C\) onto the eigenstate [33, 34]:
\[
\ket{\gamma}_{FC} = \frac{1}{\sqrt{\pi}} \sum_{n=0}^{\infty} \hat{D}_C (\gamma) \ket{n}_C \ket{n}_F
\]
(A9)
where \(\gamma\) corresponds to the measurement outcome of the dual homodyne detection. Optimal Gaussian entanglement swapping occurs for \(\gamma = 0\), however in the case where \(\gamma \neq 0\), displacements depending on the measurement outcome \(\gamma\) are performed on both remaining modes to complete entanglement swapping [24]. The state after swapping can be found via:
\[
\ket{\psi_{swap}}_{ABDE} = \bra{\gamma}_{FC} \ket{\psi}_{ACD} \otimes \ket{\psi}_{BFE}
\]
(A10)
From Eq. (A10), after tracing out the environment modes \(D\) and \(E\), we get the following un-normalized entangled state shared between Alice and Bob:
\[
\hat{\rho}_{AB} = \left( 1 - \chi^2 \right)^2 \frac{1}{\pi} \sum_{n=0}^{\infty} \chi^{4n} (1 - \eta)^{2n+1} \ket{0}_A \bra{0}_A \ket{0}_B \bra{0}_B + g^2 \eta \sum_{n=1}^{\infty} \chi^{4n} (1 - \eta)^{2n-1} \ket{1}_A \bra{1}_A \ket{0}_A \bra{0}_B + g^2 \eta \sum_{n=1}^{\infty} \chi^{4n} (1 - \eta)^{2n-2} \ket{1}_A \bra{1}_A \ket{1}_B \bra{0}_B + g^2 \eta \sum_{n=1}^{\infty} \chi^{4n} (1 - \eta)^{2n-3} \ket{0}_A \bra{1}_A \ket{1}_B \bra{0}_B
\]
(A11)
From the entangled output state (A11) shared between Alice and Bob, we are now in a position to calculate the secret key rate assuming collective attacks given by [35]:
\[
K = \beta I_{AB} - I_E
\]
(A12)
where $I_{AB}$ is the mutual information shared between Alice and Bob, $I_E$ is the Holevo bound representing the maximum amount of quantum information accessed by Eve, and $0 \leq \beta \leq 1$ is the reconciliation efficiency. We calculate the key rate from the covariance matrix of the entangled output state shared between Alice and Bob (A11). A two-mode Gaussian state has covariance matrix in standard form:

$$V = \begin{bmatrix} aI & cZ \\ cZ & bI \end{bmatrix}. \quad (A13)$$

With output state (A11), the covariance matrix elements $a$, $b$ and $c$ are given by:

$$a = b = \frac{(\eta - 1)^2 \chi^8 (\eta g^2 + \eta - 1) (3 \eta g^2 + \eta - 1) + \chi^4 (\eta (3 \eta g^4 - 4 (\eta - 1) g^2 - 2 \eta + 4) - 2) + 1}{(\eta - 1)^2 \chi^8 (\eta g^2 + \eta - 1)^2 + \chi^4 (\eta (g^2 - 2) (\eta + 2) - 2 \eta + 4) - 2 + 1}
$$

$$c = -\frac{2 \eta g^2 \chi^2 ((\eta - 1)^2 \chi^4 - 1)}{(\eta - 1)^2 \chi^8 (\eta g^2 + \eta - 1)^2 + \chi^4 (2 (g^2 + 2) \eta + (g^4 - 2 g^2 - 2) \eta^2 - 2) + 1} \quad (A14)$$

Even though the output entangled state is slightly non-Gaussian due to the QS operation and thus cannot be fully characterized by its covariance matrix, it is valid to use Gaussian key rate calculations as it overestimates Eve’s information [36–38]. Additionally, we have calculated the exact mutual information of the state shared between Alice and Bob and in the results presented, the difference is negligible (mutual information estimated from the covariance matrix is slightly lower than exact mutual information). This difference between exact mutual information and that calculated from the covariance matrix is negligible because the QS works best in regimes where input (and output) states have high fidelity with their Gaussian counterparts. This regime is of low average photon number input states or high-loss channels before the QS. We calculate the mutual information shared between Alice and Bob $I_{AB}$ for an entanglement-based protocol where Alice and Bob both conduct heterodyne detection on their entangled modes by [30, 35]:

$$I_{AB} = \log_2 \left( \frac{1 + a}{1 + a - \frac{c^2}{1 + b}} \right). \quad (A15)$$

We illustrate here how we calculate Eve’s information given Bob as the reference for reconciliation which is the case for reverse reconciliation, giving $I_E = I_{BE}$ and representing the mutual information between Bob and Eve (for direct reconciliation where Alice is the reference we would have $I_E = I_{AE}$). For the symmetric protocol, as Alice and Bob’s modes are identical, it does not matter who is the reference for reconciliation. Eve’s information $I_{BE}$ can be calculated via:

$$I_{BE} = S(E) - S(E|B) \quad (A16)$$

where $S(E)$ is the Von-Neumann entropy of Eve’s state before measurement and $S(E|B)$ is the Von-Neumann entropy of Eve’s state conditioned on Bob’s measurement outcome. $S(E)$ can be found by using the fact that Eve purifies Alice and Bob’s system, giving $S(E) = S(AB)$ which is defined as:

$$S(AB) = G \left( \frac{\nu_1 - 1}{2} \right) + G \left( \frac{\nu_2 - 1}{2} \right) \quad (A17)$$

where $\nu_1$ and $\nu_2$ are the symplectic eigenvalues of the covariance matrix $V$ and

$$G(x) = (1 + x) \log_2 (1 + x) - x \log_2 x. \quad (A18)$$

The symplectic eigenvalues $\nu_1$ and $\nu_2$ can be found via:

$$\nu_{1,2} = \sqrt{\frac{\Delta \pm \sqrt{\Delta^2 - 4 \det V}}{2}} \quad (A19)$$

where $\Delta = a^2 + b^2 - 2 c^2$. The Von-Neumann entropy of the conditional state $S(E|B)$ is a function of the symplectic eigenvalue of the conditional covariance matrix, $\nu_3 = a - \frac{c^2}{1 + b}$:

$$S(E|B) = G \left( \frac{\nu_3 - 1}{2} \right). \quad (A20)$$

Note that the key rates presented in Figs. 3 and 5 of the main text show the normalised secret key rate, i.e., the secret key rate (A12) normalized by the probability of success of the NLA (A7) and the average number of steps needed to generate success in all NLAs. This has been performed following the method outlined in Ref. [31].

2. Asymmetric entanglement swapping

In this section, we provide details on the calculation of the output state of the asymmetric single-node repeater protocol (Fig. 4(b) in the main text). Like the previous section, two copies of the state (A6) and (A8) are distributed between Alice, Bob and the repeater node. However, in the asymmetric entanglement swapping configuration, we change the location of one of the entangled
sources to be with Alice instead of the repeater node. Therefore, we have the following two entangled states distributed between the node and Alice and Bob:

\[
|\psi\rangle_{ACD} = \left(1 - \frac{\chi^2}{g^2 + 1}\right)^{\frac{1}{2}} \sum_{n=0}^{\infty} \chi^n (1 - \chi)^{n/2} |0\rangle_C |n\rangle_A |n\rangle_D \\
+ g \sqrt{n} \sum_{n=1}^{\infty} \sqrt{n} (1 - \chi)^{(n-1)/2} |1\rangle_C |n\rangle_A |n - 1\rangle_D \right)
\] (A21)

\[
|\psi\rangle_{BFE} = \left(1 - \frac{\chi^2}{g^2 + 1}\right)^{\frac{1}{2}} \left(\sum_{m=0}^{\infty} \chi^m (1 - \chi)^{m/2} |0\rangle_B |m\rangle_F |m\rangle_E + g \sqrt{n} \sum_{m=1}^{\infty} \sqrt{m} (1 - \chi)^{(m-1)/2} |1\rangle_B |m\rangle_F |m - 1\rangle_E \right)
\] (A22)

Note that in Eq.(A21) modes \(A\) and \(C\) have been switched relative to (A6). With these two entangled states (A21) and (A22), modes \(F\) and \(C\) are combined at the repeater node and a dual homodyne detection is performed. As in the previous section, this dual homodyne detection project modes \(F\) and \(C\) onto the eigenstate [33, 34]:

\[
|\gamma\rangle_{FC} = \frac{1}{\sqrt{\pi}} \sum_{n=0}^{\infty} \hat{D}_C(\gamma) |n\rangle_C |n\rangle_F
\] (A23)

Optimal Gaussian entanglement swapping occurs for \(\gamma = 0\) [24]. From Eq.(A10) using entangled states (A21) and (A22), after tracing out the environment modes, we get the following un-normalized entangled state shared between Alice and Bob:

\[
\hat{\rho}_{AB} = \left(1 - \frac{\chi^2}{1 + g^2}\right)^{\frac{2}{2}} \frac{1}{\pi} \left[ \sum_{n=0}^{\infty} \chi^{2n} (1 - \chi)^{n} |n\rangle_A |0\rangle_B \langle n\rangle_A \langle 0\rangle_B + (1 - \chi) g^2 \eta \chi \sum_{n=1}^{\infty} \chi^{2n} (1 - \chi)^{n - 1} |n\rangle_A |0\rangle_B \langle n\rangle_A \langle 0\rangle_B \\
+ g^4 \eta^2 \chi^2 \sum_{n=1}^{\infty} \chi^{2n+1} (1 - \chi)^{n} |1\rangle_A |1\rangle_B \langle n+1\rangle_A \langle 1\rangle_B + g^2 \eta \chi \sum_{n=0}^{\infty} \chi^{2n+1} (n + \chi) |n+1\rangle_A |0\rangle_B \langle n+1\rangle_A \langle 0\rangle_B \\
+ g^2 \eta \chi \sum_{n=1}^{\infty} \chi^{2n-1} (1 - \chi)^{n-1} |n\rangle_A |1\rangle_B \langle n-1\rangle_A \langle 0\rangle_B \right]
\] (A24)

The covariance matrix elements of the output state (A24) is given by:

\[
\begin{align*}
a &= -\frac{2}{g^2 \eta \chi^4 ((g^2 - 1) \eta + 1) + (\eta - 1) \chi^2 + 1} + \frac{4}{(\eta - 1) \chi^2 + 1} - 1 \\
b &= \frac{2 g^2 \eta^2 \chi^4}{g^2 \eta^4 ((g^2 - 1) \eta + 1) + (\eta - 1) \chi^2 + 1} + 1 \\
c &= \frac{2 g^2 \eta \chi^2}{g^2 \eta^2 ((g^2 - 1) \eta + 1) + (\eta - 1) \chi^2 + 1}
\end{align*}
\] (A25)

The secret key rate was then calculated using the reverse reconciliation method outlined in Section A.1. Note that for the asymmetric protocol, Alice and Bob’s output modes are not the same and using reverse reconciliation (where Bob is the reference for reconciliation) results in a higher key rate.

**Appendix B: Multi-node repeater with nested swapping**

As shown in Fig. 5 in the main text, for the single-node repeater, the asymmetric entanglement swapping protocol outperforms the symmetric protocol at the single node level. Therefore, we focus on the asymmetric entanglement swapping scheme when we model the performance of the CV repeater operating with multiple repeater nodes. We illustrate in this section, how we obtain the entangled output state after operation of the CV repeater using multiple nodes. As stated in the main text, in our CV repeater scheme, entanglement distillation occurs only after entanglement distribution. In multi-node operation, it is enough to swap multiple copies of the entangled output state (A24).

As an example, consider the four-link (three repeater node) scheme is pictured in Fig. 6 in the main text.
Entanglement distribution, distillation and swapping described in the previous section gives two copies of \((A24)\) distributed between Alice, Bob and the central repeater node:

\[
\rho_{AC} = \left(\frac{1 - \chi^2}{1 + g^2}\right)^2 \frac{1}{\pi} \sum_{n=0}^{\infty} \chi^{2n} (1 - \eta)^n |n\rangle_A \langle n|_A \otimes |0\rangle_C \langle 0|_C + (1 - \eta) g^2 \eta \chi^2 \sum_{n=1}^{\infty} \chi^{2n} n (1 - \eta)^{n-1} |n\rangle_A \langle n|_A \otimes |0\rangle_C \langle 0|_C + \eta g^2 \chi \sum_{n=0}^{\infty} \sqrt{n + 1} (1 - \eta)^n |n\rangle_A \langle n+1|_A \langle 1|_C \\
+ g^2 \eta \chi \sum_{n=1}^{\infty} \chi^{2n-1} \sqrt{n} (1 - \eta)^{n-1} |n\rangle_A \langle n-1|_A \langle 0|_C \right] \\
= \left(\frac{1 - \chi^2}{1 + g^2}\right)^2 \frac{1}{\pi} \sum_{n=0}^{\infty} \chi^{2n} (1 - \eta)^n |n\rangle_F \langle n|_F \otimes |0\rangle_B \langle 0|_B + (1 - \eta) g^2 \eta \chi^2 \sum_{n=1}^{\infty} \chi^{2n} n (1 - \eta)^{n-1} |n\rangle_F \langle n|_F \otimes |0\rangle_B \langle 0|_B + g^2 \eta \chi \sum_{n=0}^{\infty} \sqrt{n + 1} (1 - \eta)^n |n\rangle_F \langle n+1|_F \langle 1|_B \\
+ g^2 \eta \chi \sum_{n=1}^{\infty} \chi^{2n-1} \sqrt{n} (1 - \eta)^{n-1} |n\rangle_F \langle n-1|_F \langle 0|_B \right]
\]

(B1)

where we have used mode labels \(A\) and \(B\) to refer to modes held between Alice and Bob respectively. Modes \(C\) and \(F\) are held at the central repeater node and will be swapped. Again, this is modeled via projection on to the eigenstate \((A23)\) with \(\gamma = 0\). That is, the entangled state shared between Alice and Bob after the four link repeater scheme can be found using:

\[
\rho_{AB} = \langle \gamma |_{CF} \left[ \rho_{AC} \otimes \rho_{BF} \right] | \gamma \rangle_{CF}
\]

(B3)

To model the scheme operating with eight links, two copies of the four link output entangled state would be combined and swapped using (B3). For sixteen and higher links, calculations proceed in the same way.

Appendix C: Parameters for optimal operation

In Fig. 5 of the main text we show performance of the CV repeater measured by the normalised secret key rate per mode. In these results, optimization has been carried out at each data point over the squeezing of the TMSV states and the gain of the quantum scissor. In Fig. 6, we present the values of these parameters. We note two main features of this optimisation, namely the optimal squeezing \(\chi\) decreases and gain of the quantum scissor increases with increasing distance. As discussed in the main text, the value of optimal squeezing tends to \(\chi = 0.070\) as the distance goes to 1000km. At this same distance, the optimal gain is \(g = 126\).

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FIG. 6. Parameters for optimal operation of the CV quantum repeater (corresponding to the results shown in Fig. 5 of the main text). (a) Optimal squeezing of the TMSV entangled resource states. (b) Optimal gain of the QS.

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