Estimation of pumpage from a riverside well at the lower reaches of the Songhua River

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Abstract. As a kind of vital water resources exploitation mode, riverside pumping has the important advantage of maintaining the stability of water supply and purifying surface water quality. It is of great significance to estimate the pumpage from a riverside well for the sustainable utilization of water resources in the area. In this article, the method of image and Girinskii’s potential function are used to derive the pumpage. A case study in the Jamusi City shows that the pumpage from a riverside well varies from 9000 to 15000 m³/d. The sensitivity of factors \( h_0, K, S_{\text{max}} \) and \( r_w \) is analysed, indicating that the influence degree from large to small is \( h_0, K, S_{\text{max}} \) and \( r_w \). Besides, \( h_0, K, S_{\text{max}} \) are far greater than \( r_w \). It is hoped that this paper can provide some theoretical reference for the rational utilization of groundwater in the areas near the rivers.

1. Introduction

Compared with direct exploitation of surface water, riverside pumping is advantageous because of the better quality of riverside groundwater due to water treatment through the underground media, and reliable quantities from river recharge [1]. So riverside pumping has been practiced in many areas over the world for a long time. However, how to determine the appropriate pumpage and ensure the quality of groundwater, is the focus of domestic and foreign scholars in recent years. Up to now, many scholars have paid much attention to the estimation of pumpage from a riverside well. The principle of water balance is firstly used to estimate the pumpage, where the method is simple, but it cannot accurately calculate the pumping rate from a riverside well. Numerical simulation is an effective method for analyzing surface water-groundwater interactions, but it will cost much time and data to construct a model. In most cases, analytical solutions provide a quick and draft estimation of pumpage from a riverside well [2].

Concerning with the analytical solutions, Theis obtained the first unsteady solution about riverside pumping problem in the form of an integral in 1941 [3]. In 1954, this same solution was rewritten by Glover and Balmer using the complimentary error function [4]. Then in 1965, Hantush [5] studied a problem which differed from the problem considered by Theis in 1941 only by the inclusion of a vertically semi-pervious material along the stream edge. In 1999, Hunt [6] studied an analytical model that considered the effects of streambed clogging and partial stream penetration. Then many scholars, such as Chen [7], Singh [8], Singh [9] and Zhang [10] studied the river-aquifer interaction. These solutions provide a theoretical basis for the calculation of pumpage and are applicable to the ideal or simple structure aquifer. Up to now, there are still some problems to be addressed, such as partially...
penetrated river, influences of semi-pervious beds, presence of unsaturated zone between river and groundwater. Therefore, the premise of applying analytical method is to make a reasonable generalization and comprehensive consideration about hydrogeological conditions.

The objective of this paper is to propose a new method for estimating pumpage using the method of image and Girinskii’s potential function, and then analyze the sensitivity of factors that impact the pumpage. A case study in the Jiamusi City, downstream of the Songhua River, is given to demonstrate the application of the analytical solutions.

2. Method

In most riverside areas, groundwater system may be under a dynamic steady state, due to the influences of river recharge, pumping well and rainfall infiltration. As shown in figure 1(a), the river can be considered as a constant-head linear boundary. It is also assumed that the phreatic aquifer is horizontal and has a uniform thickness, and the aquifer medium is homogeneous and isotropic. So the Girinskii’s potential function can be used to analyze the problem of groundwater flow and the potential of the unconfined aquifer is written as

\[ \varphi = \frac{Kh^2}{2} \]  

where \( \varphi \) represents the Girinskii’s potential; \( K \) denotes the hydraulic conductivity; \( h \) means the thickness of the saturated area in the unconfined aquifer.

![Figure 1. Schematic diagram of riverside pumping problem.](image)

For an unconfined aquifer, assume that the steady radial groundwater flow to a pumping well is horizontal, and the aquifer has uniform thickness. Then Girinskii’s potential can be described as follows

\[ Q = 2\pi r \frac{d\varphi}{dr} \]  

The general solution of equation (2) is

\[ \varphi = \frac{Q}{2\pi} \ln r + C \]  

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where \( Q \) is pumpage rate for the single well; \( C \) is a constant determined by boundary conditions; \( r \) is the distance away from any point to the pumping well.

The solution of pumping flow near a riverside can be obtained by the method of image. The method is to consider the influences of an image injection well and then substitute the actual effect influenced by the river [11] (figures 1(a) and 1(b)). The effects of pumping well and river can be superposed. Girinskii’s potential satisfies the Laplace equation. So the superposition rule can be used. Considering equations (1) and (3), one gets the solution

\[
h^2 = h_0^2 - \frac{Q}{\pi K} \ln \left( \frac{r}{r'} \right) \tag{4}
\]

\[
r' = \sqrt{(x + L)^2 + y^2} \tag{5}
\]

\[
r = \sqrt{(x - L)^2 + y^2} \tag{6}
\]

where \( r \) is the distance from the observation well \( p(x, y) \) to the pumping well; \( r' \) is the distance from the observation well \( p(x, y) \) to the image well; \( L \) is the distance between the river and well.

Equation (4) is the solution for a riverside well. When the observed location is in the pumping well, one can get the following equation (7).

\[
Q = \frac{\pi K}{\ln \left( \frac{r}{r'} \right)} \left( h_0^2 - (h_0 - s_{\text{max}})^2 \right) \tag{7}
\]

where \( h_0 \) is the thickness between the river surface and impervious base; \( r_w \) is the radius of the well; \( s_{\text{max}} \) is the maximum allowable drawdown.

3. Case studies and discussions

3.1. Study area

The Jiamusi City (129°55′~135°04′ E, 46°56′~48°30′ N) (figure 2) is located at the lower reaches of the Songhua River flood plain. The terrain is zonal and slightly inclined from the southwest to the northeast. Groundwater is mainly distributed in the Quaternary strata of the alluvial plain. The main groundwater type is pore phreatic water. The main components of groundwater recharge are rainfall infiltration and surface water recharge. The main discharge items are the exploitation of mining wells and discharge to the rivers. According to the hydrogeological characteristics and water abundance, the study area can be divided into two hydrogeological zones (I and II): (1) Zone I is mainly distributed in the area along the Songhua River, where the thickness of the aquifer in this area is 50~105 m, and the

![Figure 2. Map of study area.](image-url)
hydraulic conductivity is about 39~144 m/d. (2) Zone II is mainly distributed in the northwest and the thickness of the aquifer is 80~89 m. The hydraulic conductivity in Zone II is about 24~26 m/d.

In 2005, the total water consumption in the area was about 3.50×10^8 m³, whereas annual groundwater supply was about 0.70×10^8 m³, accounting for about 20% of the total water consumption. Meanwhile, the urban water supply sources in the Jiamusi city are from groundwater. Because of the natural supply source of the Songhua River, riverside pumping becomes the main form of groundwater exploration in the Jiamusi city. The two major groundwater sources (figure 2) in the Jiamusi city are located in the lower part of the Songhua River.

3.2. Change of pumpage with the distance from the river
According to the previous studies, the average aquifer thickness in this area is about 70 m, and the average hydraulic conductivity is about 30 m/d. In China, the allowable drawdown for a pumping well is usually set as 7 m. Most pumping wells are set in a range of about 3000 m away from the river. Based on equation (7), the change of pumpage with the distance from the river is shown in figure 3. It can be clearly seen that the pumpage from a well varies from 9000 to 15000 m³/d. With the increase of \( L \), \( Q \) will gradually reduce. Moreover, with the increase of \( L \), the degree of the influence of \( L \) on \( Q \) will decrease, mainly because groundwater system may obtain less recharge from the river when the distance away from the river is increased.

![Figure 3. Schematic diagram of the relationship between pumpage and the distance between well and river.](image)

For better understanding change of water table under the influences of different pumpage, figure 4 is drawn, which is obtained when \( L \) is 1000 m and the cross-section is set along the line from the pumping well to the imaging well. It can be seen that when water table increases inversely with the pumpage \( Q \) varying from 9000 m³/d to 15000 m³/d. The maximum difference of water table under the different pumpage is 3.47 m, and water table near the river is higher than that at the opposite side and the same distance from the well.

3.3. Sensitivity analysis of key factors on pumpage
From the hydrogeological conditions, we know the range of \( K \) is 24~144 m/d and the range of \( h_0 \) is 50~105 m. According to empirical value, \( S_{\text{max}} \) and \( r_w \) should not be too large. The appropriate ranges of \( S_{\text{max}} \) and \( r_w \) are 1~22 m and 0.1~2.3 m. Based on equation (7), the relationships between \( Q \) and \( S_{\text{max}} \), \( Q \) and \( K \), \( Q \) and \( h_0 \), \( Q \) and \( r_w \) can be obtained and are shown in figures 5(a)-5(d). It can be easily seen from the following curves that with the increase of \( S_{\text{max}} \), \( K \), \( h_0 \) and \( r_w \), \( Q \) will also increase. \( K \) and \( h_0 \) are both positively proportional to \( Q \); \( S_{\text{max}} \) and \( Q \) present the relationship of quadratic function; Logarithmic function relationship exists between \( r_w \) and \( Q \).
Figure 4. Change of water table along x axis under different pumpage conditions.

Figure 5. Schematic diagram of the relationship between Q and the maximum allowable drawdown (a), the hydraulic conductivity (b), the thickness between the river surface and impervious base (c), the radius of the well (d).

The sensitivity of different parameters is analyzed when the distance between well and river is 1000 m. The sensitivity of each factor on pumpage is calculated using equation (8).

\[ J_i = \frac{\partial y}{\partial a_i} \quad (8) \]

Where \( J_i \) is the sensitivity coefficient which indicates the degree of influence on the output \( y \) when \( a_i \) changes; \( i \) is the parameter serial number; Because of the different units of different parameters, the formula 9 is obtained after the formula 8 is standardized.
\[ J_i = \frac{(\partial y/\partial \alpha_i)/\alpha_i}{(\partial y(\alpha_i + \Delta \alpha_i)/\alpha_i) - y(\alpha_i)/\alpha_i} \]

The sensitivity coefficients of \( S_{\text{max}}, K, h_0 \) and \( r_w \) are obtained according to formula 9 and the range of values is 0.08–1.07. Among them, the sensitivity coefficient of \( r_w \) is the smallest (0.08–0.13). The sensitivity coefficient of the other three parameters is basically same (\( J_{h_0} = 1.07–1.04; J_K = 1; J_{S_{\text{max}}} = 0.82–0.99 \)). So we know that the influence degree of \( h_0 \) is greatest followed by \( K, S_{\text{max}} \) and \( r_w \) and the effect of \( h_0, K \) and \( S_{\text{max}} \) on \( Q \) is much higher than that of \( r_w \).

4. Conclusions

The estimation of pumpage from a riverside well is very important for the sustainable development and utilization of regional groundwater in the area of riverside pumping. This study proposes an analytical solution of estimating pumpage using the method of image and Giriniskii’s potential. And then a case study in the Jiamusi City is used to analyze the single well pumpage. The result shows that the pumpage from a riverside well may vary from 9000 to 15000 m\(^3\)/d (\( S_{\text{max}} = 7 \text{ m}, r_w = 0.5 \text{ m} \)). The sensitivity of factors that impact the pumpage is analyzed. The influence degree of \( h_0 \) is greatest and \( r_w \) is smallest. But the effect of \( h_0, K \) and \( S_{\text{max}} \) on \( Q \) are basically equal and are much higher than that of \( r_w \).

The solutions can be well used to quickly estimate the pumpage at the downstream of the Songhua River. However, some assumptions are made in the derived analytical solutions, including fully-penetrated river, the horizontal, homogeneous and isotropic aquifer with a uniform thickness. So the method is only applicable for the draft and quick estimation of pumpage in an area. There is an important relationship between the maximum pumpage, the hydrogeological parameters and the design of the well, so it is suggested to carry out detailed investigation to identify the value of each hydrogeology properties to further refine the calculation in the field cases.

Acknowledgments

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