A-dependence of $\phi$-meson production in $p+A$ collisions

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Abstract. A systematic analysis of the $A$-dependence of $\phi$-meson production in proton-nucleus collisions is presented. We apply different formalisms for the evaluation of the $\phi$-meson distortion in nuclei and discuss the theoretical uncertainties of the data analysis. The corresponding results are compared to theoretical predictions. We also discuss the interpretation of the extracted results with respect to different observables and provide relations between frequently used definitions. The perspectives of future experiments are evaluated and estimates based on our systematical study are given.

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1 Introduction

The modification of hadron properties in a nuclear environment remains one of the most mysterious problems in nuclear physics, see e.g. [12,13,14]. First experimental results on dilepton spectra from heavy ion collisions indicated a substantial modification of the spectra in the vicinity of the $\rho$-meson mass [8,9,10]. Very recent results from PHENIX for high-energy $Au+Au$ scattering [11] also indicated a significant enhancement of very low mass dielectrons within the energy range from 150 to 750 MeV. This modification of the high mass dilepton spectrum is a key question to understand the in-medium effects. For instance, the observation of a change of the $\phi$-meson spectral function would unambiguously clarify the role of in-medium effects. Unfortunately, the statistical accuracy of the recent data [11] is still not good enough to draw definite conclusions about the modification of the $\phi$-meson spectral function in heavy ion collisions.

On the other hand there is a certain belief [12,13,14] that the $\phi$-meson is almost not distorted in nuclear matter and thus can provide information about the early partonic stage of heavy ion collisions or the Quark Gluon Plasma (QGP). This means that $\phi$-meson production remains mostly unaffected by hadronic interactions and might serve as the perfect penetrating probe. Furthermore, it was proposed [12] that the formation of the QGP may be detected by enhanced $\phi$-meson production resulting from the absence of the Okubo-Zweig-Iizuka suppression that is quite substantial in elementary reactions [15,16,17]. In this case it is important that the distortion of the $\phi$-meson is almost negligible. The most recent results obtained by the STAR Collaboration [14] indicate some distortion of $\phi$-meson production in $Au+Au$ collisions, however, it is difficult to quantify the size of this effect.

Some indications of $\phi$-meson modification at normal nuclear densities have been reported from measurements involving photon [18] and proton beams [19,20,21,22,23]. The most remarkable observation in all these measurements is the anomalous $A$-dependence of $\phi$-meson production. The modification of the spectral function implies the change of the pole position and the width. When the effective in-medium width of the $\phi$-meson becomes larger, then its decay probability also increases, leading to a stronger distortion in nuclear matter. Thus one might expect [24,25,26,27,28,29] that the $A$-dependence indicates an in-medium modification of the imaginary part of the $\phi$-meson spectral function.

Our previous systematic analysis [29] of $\phi$-meson photoproduction from nuclear targets, however, showed that the observed $A$-dependence can be well understood by including the $\omega$-$\phi$ mixing in nuclei. We have investigated single and coupled-channel phenomena and reproduced the recent SPring-8 data [18] as well as the older Cornell results at higher energies [30].

Here we analyze the available data on $\phi$-meson production in proton-nucleus collisions. Most of the measurements were done at high energies. We provide the formalism for the data evaluation and give estimates for the theoretical uncertainties. Moreover, we review different formulas used in the analysis of the $A$-dependence at high energies and discuss their compatibility. We also collect the different theoretical predictions relevant to the $\phi$-meson distortion in nuclear matter. Finally, we hope that all this information allows for a systematic understanding of the problem and helps in planning future strategies for experiment analysis. Therefore, we provide estimates for
the data analysis currently carried out by the ANKE Collaboration at COSY [28].

2 Formalism

Recently, the eikonal formalism has been considered to extract the in-medium properties of hadrons from the $A$-dependence of their production on nuclei using photon and proton beams. The distortion of the produced hadrons at low nuclear matter densities can be expressed in terms of the so-called $\tau\rho$ approximation. Here $\rho$ is the nuclear density, while $t$ stands for the forward scattering amplitude. This amplitude is frequently referred to as $f(0)$ and we follow this convention. This amplitude in nuclear matter is not necessary equal to that in free space because it can be modified by the nuclear environment. Since this amplitude can be evaluated from the nuclear data, one might be able to extract the in-medium modification of the hadron interaction. The $\tau\rho$ approximation is well applicable at normal, but not high, nuclear densities and thus can provide a sensitive way for the nuclear data evaluation. See, e.g., Refs. [26,31,32,33] for evaluations using the $\tau\rho$ approximation in the eikonal formalism.

Originally this method was proposed to study the interactions of unstable hadrons with nucleons. Indeed, it is impossible to produce a beam of unstable hadrons, like $\omega$, $\rho$, $\phi$, etc. mesons and measure their scattering on a hydrogen target. However, a nucleus can be used as a source of unstable hadrons as well as a target for their interaction and detection. In that sense, the eikonal formalism explores the nucleus as a laboratory constructed at fm distances, i.e. some kind of “Fermilab”.

Furthermore, the eikonal formalism provides an unique opportunity to evaluate the scattering length for the interaction of an unstable hadron with a nucleus. Indeed by measuring the $A$-dependence of the produced particle with different energies one can extract the scattering amplitude as a function of energy. Interpolating this amplitude to threshold, one obtains the scattering length. This scattering length could also be modified by the nuclear medium and its extrapolation to free space requires, unfortunately, model dependent assumptions.

In many applications of the eikonal method the distortion of the produced hadron is factorized out of the full formalism in order to analyze the data. Furthermore, the interpretation of the evaluated results is given in terms of different variables. Thus it is difficult to compare different analyses and to link the simplified methods that were applied to the original one. For this reason, we briefly discuss the eikonal formalism for particle production processes and give the parameter set that can be extracted from the data analysis.

2.1 Eikonal approximation

The basic idea of the eikonal formalism is to express the interaction of a particle with a nucleus in terms of effective two-body interactions. To leading order, the total reaction amplitude can therefore be built up from a sum of amplitudes on a single nucleon. The corrections to the leading order come from multiple interactions. The eikonal approximation was proposed by Glauber [34,35] for coherent and incoherent scattering of hadrons from nuclei. The incident particle is assumed to interact independently with each target nucleon as it moves along a straight line trajectory through the nucleus.

In the original formulation the incident and final particles are identical. Nevertheless, the formalism can be applied to production processes, i.e. when incident and final particles are different. The model was extended by Formanek and Trefil [36,37] for the case of resonance production in proton-nucleus collisions and generalized by Berman and Drell [38,39] and Ross and Stodolsky [40] and Drell and Trefil [41,42] for vector meson photoproduction on nuclei. The best known extension and application of multi-scattering theory to the coherent and incoherent production of particles were given by Margolis [43,44] and Kölbl and Margolis [45].

In this section, we sketch the formalism of the eikonal approximation necessary to derive the $A$-dependence of incoherent particle production off nuclei. Our aim is to illustrate how the elementary amplitude on a quasi-free nucleon is related to the total production amplitude on a nucleus. This will clarify to which extent one can extract the elementary amplitude from the experimental data. In the following, we will also show how one can interpret this amplitude with respect to different nuclear phenomena.

We consider a two-body interaction at impact vector $b$ and describe the transition from the incident particle $i$ to a final state particle $f$ by the profile function $\Gamma_{if}(b)$ defined as

$$\Gamma_{if}(b) = \frac{1}{2\pi} \int d^2q f_{if}(q) e^{-iqb},$$

where $f_{if}$ is the elementary $i+N\rightarrow f+X$ transition amplitude at momentum transfer $q$, while $k$ is the projectile particle wave number. We also define profile functions $\Gamma_{ii}$ and $\Gamma_{ff}$ similar to Eq. (1) with $f_{ii}$ and $f_{ff}$ denoting the amplitudes for $i+N \rightarrow i+X$ and $f+N \rightarrow f+X$, respectively.

Using Huygen’s principle, the transition amplitude is given by the profile function as

$$f_{if}(q) = \frac{ik}{2\pi} \int d^2b \Gamma_{if}(b) e^{iqb}.$$

(2)

The distortion of particle $i$ is calculated under the assumption that each target nucleon $j$ fixed at transverse $(s_j)$ and longitudinal positions $(z_j)$ independently modifies the wave of particle $i$ passing through the target nucleon by the factor

$$1 - \Gamma_{ii}(b - s_j).$$

(3)

The distortion of the outgoing wave $f$ is defined in a similar way.

The nuclear profile function $\Gamma_{ij}^A(b)$ describes the transition from an initial nuclear state $|I\rangle$ to the final nuclear state $|F\rangle$ as

$$\langle F|\Gamma_{ij}^A(b)|I\rangle = \sum_l \langle F| \prod_{z_i < z_l} [1 - \Gamma_{ii}(b - s_j)]$$

$$x \Gamma_{ij}(b - s_l) \prod_{z_m > z_i} [1 - \Gamma_{ff}(b - s_m)] |I\rangle.$$  

(4)

This is an essential difference to cascade-like models that consider distortion on the basis of squared amplitudes. Our approach takes into account the interference between the distortion amplitudes and accounts for the quantum dynamics.
From this equation it is clear that the nuclear profile function accounts for the overall distortion of the incident particle \( i \) by nucleons in the target nucleus before the production of particle \( f \) at the positions \( z_j < z_i \), for the transition \( i+N \rightarrow f+X \) at \( z_i \), and for the distortion of the produced particle \( f \) at \( z_m > z_i \).

Eq. (4) indeed illustrates that the \( A \)-dependence of particle production is entirely given by the distortion of both the projectile and the produced particles. In principle, Eq. (4) summarizes the physical content of the eikonal approximation.

The nuclear transition amplitude \( f_{ij}^A \) is now related to the nuclear profile function by Eq. (2). The cross section for production of the particle \( f \) in the \( i+A \) collision, summed over all nuclear final states \( |F \rangle \), is finally given as

\[
\frac{d\sigma^A}{dt} = \sum_F |f_{ij}^A|^2,
\]

where \( t=-q^2 \) is the four-momentum transfer squared.

The summation over the nuclear final states \( |F \rangle \) can be evaluated using closure and approximating the many body target wave function \( u_f(r_1, \ldots, r_A) \equiv |F \rangle \) by the product of single-particle density functions as

\[
|u_f(r_1, \ldots, r_A)|^2 = \prod_{i=1}^A \rho_A(r_i). \tag{6}
\]

One can assume further that

\[
\int d^2s \int_{-\infty}^{\infty} dz \Gamma_{ii}(b-s)\rho(s, z) \simeq \int d^2s \Gamma_{ii}(b-s) \times \int_{-\infty}^{\infty} dz \rho(b, z) = -f_{ii}(0) \frac{2\pi T(b)}{k A}, \tag{7}
\]

where \( f_{ii}(0) \) is forward \( i+N \rightarrow i+N \) scattering amplitude and the optical thickness function \( T(b) \) is defined as

\[
T(b) = A \int_{-\infty}^{+\infty} dz \rho_A(b, z), \tag{8}
\]

with \( r^2 = b^2 + z^2 \). Similar relations also hold for \( f+N \rightarrow f+N \) scattering. Furthermore, we use the relation

\[
\left[ 1 + f_{ii}(0) \frac{2\pi T(b)}{k A} \right]^A \simeq \exp \left[ \frac{2\pi f_{ii}(0) T(b)}{k} \right] \tag{9}
\]

in order to evaluate Eq. (5).

As a consequence, the \( A \)-dependence of the \( f \) particle production in \( i+A \) collisions becomes a function of the single-particle density function \( \rho_A \) and of the forward scattering amplitudes for the \( i+N \rightarrow i+N \) and \( f+N \rightarrow f+N \) processes. The transition amplitude for \( i+N \rightarrow f+N \) itself does not depend on \( A \) under the assumption that it does not depend on the Fermi motion, which in principle is different for different nuclei. This assumption is not satisfied for particle production off nuclei at energies below the reaction threshold in free space, where the Fermi motion is an essential part of the production process \( i+N \rightarrow f+N \).

The imaginary part of the forward \( f_{ii} \) scattering amplitude is given by the optical theorem, which is a straightforward consequence of \( S \)-matrix unitarity and

\[
\text{Im} f_{ii}(0) = \frac{k}{4\pi} \sigma_i, \tag{10}
\]

where \( \sigma_i \) is total cross section for the interaction of the particle \( i \) with a nucleon. Therefore, the forward scattering amplitude can be written as

\[
f_{ii}(0) = \frac{k}{4\pi} (i + \alpha_i) \sigma_i, \tag{11}
\]

where \( \alpha_i = \text{Re} f(0)/\text{Im} f(0) \) stands for the ratio of the real to the imaginary part of the forward scattering amplitude. Here one should note that \( \sigma_i \) and \( \alpha_i \) in nuclear matter are not necessary the same as in a free space. Eq. (10) explicitly illustrates why the total \( \sigma_f \) reaction cross section is used in the data evaluation. A similar relation holds for the final particle scattering. Since the ratio \( \alpha \) is a priori unknown, it is generally neglected and the differential cross section of Eq. (5) is finally given as

\[
\frac{d\sigma^A}{dt} = \frac{d\sigma^N}{dt} (N_A + \epsilon), \tag{12}
\]

where \( d\sigma^N/dt \) is the elementary cross section for \( f \) particle production in a collision of the particle \( i \) with a nucleon, while \( N_A \) is the effective number of target nucleons involved in the interaction,

\[
N_A = \frac{1}{\sigma_f - \sigma_i} \int d^2b \left[ e^{-\sigma_f T(b)} - e^{-\sigma_i T(b)} \right], \tag{13}
\]

The first term of Eq. (12) is the leading order term, which describes the production preceded and followed by the distortion of initial and final particles in the nucleus. Since the production is considered on a single target nucleon \( i+N \rightarrow f+X \) this process can be addressed as a direct production mechanism. If \( \sigma_i = \sigma_f = \sigma \) then Eq. (13) can be written as

\[
N_A = \int d^2b e^{-\sigma T(b)}, \tag{14}
\]

and for \( \sigma = 0 \) we have \( N_A = A \). So when initial and final particles are not distorted by the nucleus, the \( A \)-dependence of the production process is a linear function of the atomic mass number. In that case the nucleus is absolutely transparent, where the nuclear transparency is defined as

\[
T_R = N_A/A. \tag{15}
\]

The second term denoted \( \epsilon \) in Eq. (12) contains the corrections due to the scattering of the \( i \) and \( f \) particles out of the forward direction and contributions from multiple interactions. The next-to-leading order terms need to be controlled in order to perform the theoretical evaluation of the data. For this reason, we provide here the results of numerical calculations which can not be found explicitly in previous studies.

Note that \( \sigma_i \) can be converted back to the imaginary part of the forward scattering amplitude or other variables related to the total reaction cross section, as will be shown below.
The multiple scattering correction can easily be estimated under the assumption that the forward scattering amplitudes $f(q)$ are the same for $i+N\rightarrow i+N$ and $f+N\rightarrow f+N$ elastic scattering. By parameterizing $f(q)$ as

$$f(q) = f(0) \exp(-B q^2),$$

with the slope parameter $B$, the correction $\epsilon$ can be obtained as

$$\epsilon = \sum_{\nu=2}^{A} \frac{1}{\nu} \mu^\nu \exp[B q^2 (1 - \frac{1}{\nu})] \tilde{N}_\nu,$$

where

$$\mu = \frac{1 + \alpha^2}{16\pi B}.$$

The multiple scattering collision numbers $\tilde{N}_\nu$ are given by

$$\tilde{N}_\nu = \frac{1}{\nu!} \int d^2b \sigma^{\nu-1} e^{-\sigma T(b)} [T(b)]^\nu.$$

Furthermore, $\tilde{N}_1$ equals the effective collision number $N_A$ given by Eq. (13) and represents the leading term of the multiple collision series. Additional higher-order corrections given by $\epsilon$ in Eq. (12) are determined by contributions $\tilde{N}_\nu$ with $\nu>1$. Now $\mu$ can be estimated as the ratio of elastic to the total interaction cross section and its value can not exceed one.

Note that $\epsilon$ reflects the systematic uncertainty in the application of the given formalism. To estimate $\epsilon$ by Eq. (17) one needs to calculate $\tilde{N}_\nu$. The $\tilde{N}_\nu$ terms calculated by Eq. (19) for carbon and lead targets are listed in Tables 12 for different cross sections $\sigma$ and $\nu$. The calculations were performed with a nuclear density function $\rho_A$ taken as a Wood-Saxon distribution as

$$\rho_A(r) = \frac{\rho_0}{1 + \exp[(r-R)/d]},$$

using the density parameters.

$$R=1.28A^{1/3} - 0.76 + 0.8A^{-1/3}\, \text{fm}, \quad d=\sqrt{3}/\pi\, \text{fm}.$$

The multiple scattering terms given in Tables 12 should be compared to the leading term $\tilde{N}_1 = N_A$. It is clear that multiple scattering corrections to the $A$-dependence might be important when $\mu$ is close to one. In principle, the ratio $d\sigma^A/d\sigma^N$ could be different from the leading term $\tilde{N}_1 = N_A$ due to the $\epsilon$ contribution.

For instance, for $pp$ scattering at high energies the elastic cross section does not exceed $\pm 15\%$ of the total cross section, while at COSY energies this number accounts for $\pm 40\%$. This leads to large systematical uncertainties in the evaluation of the $f+N\rightarrow f+N$ forward scattering amplitude from the nuclear data collected at low energies.

Furthermore, $\tilde{N}_\nu$ cannot be considered as an effective number of multi-nucleon clusters in nuclei and should not be considered as an estimate for the production mechanisms involving the interaction of the particles with few nucleons. The multiple collision numbers $\tilde{N}_\nu$ are attributed to elastic scattering of initial ($i$) and final ($f$) particles before and after the production process $i+N\rightarrow f+N$ on a single nucleon.

Finally, within an eikonal approximation, the $A$-dependence for direct particle production is given by total $i+N$ and $f+N$ cross sections and by the single density function $\rho_A$. Since $\sigma_i$ and $\sigma_f$ both depend on the type of particle, i.e. photon, pion, nucleon, etc., as well as on its kinetic energy, the $A$-dependence also is a function of those degrees of freedom. The eikonal approximation does not include the $A$-dependence of particle production due to the ejectile emission angle.

Now, the solid lines in Fig. 1 show the effective collision numbers $N_A$ calculated by Eq. (13) for carbon and lead nuclei as a function of $\sigma_i$. The results are shown for $\sigma_i$ = 10, 40 and 100 mb. The calculations indicate reasonable sensitivity to both $\sigma_i$ and $\sigma_f$.

The arrows in Fig. 1 indicate the $A^{2/3}$ dependence that is frequently assumed for inelastic hadron-nucleus reactions. The $A^{2/3}$ dependence is in general discussed in terms of the absorptive interaction of the incident particle at the nuclear surface. However, this is not an unique explanation as is illustrated by Fig. 1. The $A^{2/3}$ dependence might result from various combinations of $\sigma_i$ and $\sigma_f$, which reflect quite different physics.

The data analysis of the $A$-dependence of particle production from nuclei is frequently done in terms of an exponent $\alpha$.

The main advantage the eikonal approximation offers is that the multidimensional equations reduce to a differential equation in a single variable. This reduction into a single variable is the result of the straight line approximation involved.

### Table 1.
The multiple scattering collision numbers $\tilde{N}_\nu$ calculated from Eq. (19) for a $^{12}C$ target as a function of $\nu$ and total cross section $\sigma$. The $\nu=1$ term is the effective number of target nucleons involved in the interaction at leading order, while the terms for $\nu>1$ come from the multiple scattering correction $\epsilon$ of Eq. (12).

| $\sigma$ (mb) | $\nu=1$ | $\nu=2$ | $\nu=3$ | $\nu=4$ | $\nu=5$ |
|---------------|---------|---------|---------|---------|---------|
| 10            | 8.12    | 1.5     | .25     | .04     | .004    |
| 20            | 5.8     | 1.8     | .57     | .16     | .038    |
| 30            | 4.3     | 1.7     | .75     | .29     | .102    |
| 40            | 3.4     | 1.5     | .79     | .39     | .176    |
| 60            | 2.3     | 1.1     | .71     | .46     | .279    |
| 80            | 1.8     | 0.8     | .57     | .41     | .301    |
| 100           | 1.4     | 0.7     | .46     | .35     | .275    |

### Table 2.
The multiple scattering collision numbers $\tilde{N}_\nu$ calculated using Eq. (19) for a $^{207}Pb$ target as a function of $\nu$ and total cross section $\sigma$. The $\nu=1$ term is the first order effective collision number, while the $\nu>1$ terms are due to the multiple scattering correction $\epsilon$ of Eq. (12).

| $\sigma$ (mb) | $\nu=1$ | $\nu=2$ | $\nu=3$ | $\nu=4$ | $\nu=5$ | $\nu=6$ |
|---------------|---------|---------|---------|---------|---------|---------|
| 10            | 59.9    | 32.3    | 15.0    | 5.8     | 1.9     | 0.5     |
| 20            | 24.4    | 17.8    | 13.8    | 9.6     | 5.8     | 3.1     |
| 30            | 13.5    | 9.6     | 8.5     | 7.6     | 6.2     | 4.6     |
| 40            | 9.1     | 5.9     | 5.2     | 4.9     | 4.7     | 4.3     |
| 60            | 5.6     | 3.2     | 2.5     | 2.3     | 2.3     | 2.3     |
| 80            | 4.1     | 2.2     | 1.6     | 1.4     | 1.3     | 1.3     |
| 100           | 3.2     | 1.7     | 1.2     | 1.0     | 0.9     | 0.8     |
fitted to experimental results for a double differential cross section as

\[
\frac{d^2 \sigma}{d\Omega dT} = c A^\alpha,
\]

where \( c \) is a constant. In general, the exponent \( \alpha \) is evaluated from data collected at different kinematical conditions, such as emission angles \( \phi \), kinetic energies \( T \) of produced particles, incident beam energies and different kinds of projectile (\( i \)) and ejectile (\( f \)) particles. In the eikonal approximation, the variety of kinematical conditions can easily be classified by considering the \( A^\alpha \)-dependence as a function of the total \( \sigma_i \) and \( \sigma_f \) cross sections. However, one should not expect validity of the eikonal approximation at production angles away from the forward direction.

Furthermore, the data analysis in terms of the \( A^\alpha \) function introduces additional systematical uncertainties, which can be well understood by inspecting Fig. 1. We consider the following example: For \( \sigma_f = 10 \text{ mb} \) we obtain \( \alpha = 2/3 \), as is indicated by the arrows in Fig. 1 after fitting both \( ^{12}\text{C} \) and \( ^{207}\text{Pb} \) data. This \( A^\alpha \)-dependence corresponds to \( \sigma_i \approx 39 \text{ mb} \) for \( ^{12}\text{C} \), as is shown by the arrow in the upper panel of Fig. 1. At the same time the lower panel for a \( ^{207}\text{Pb} \) target shows that \( \alpha = 2/3 \) corresponds to \( \sigma_i \approx 22 \text{ mb} \). This large uncertainty in \( \sigma_i \) is reflected in the standard deviation of the \( \alpha \) slope and vice versa. To avoid this uncertainty in the theoretical analysis it is more useful to analyze the ratio of the production cross section measured with different targets.

Nevertheless, for completeness we evaluate Eq. (13) as a function of both \( \sigma_i \) and \( \sigma_f \) for various nuclei and fit the calculations by \( A^\alpha \) in order to determine the average slope \( \alpha \). This average slope \( \alpha \) is shown in Table 3 as a function of \( \sigma_i \) and \( \sigma_f \). The calculations were done for \( C, \text{Al, Cu, Ag, Au, and Pb} \) nuclei. (We do not indicate the calculations for \( \sigma_i = \sigma_f = 0 \) that obviously result in \( \alpha = 1 \).) The average slope parameters \( \alpha \) are afflicted with large uncertainties, e.g. \( \alpha = 0.35 \pm 0.18 \) for \( \sigma_i = 40 \text{ mb} \) and \( \sigma_f = 30 \text{ mb} \). For this reason, we discourage fitting cross sections for a single nucleus and recommend to analyze cross section ratios instead.

The eikonal approximation provides the \( A^{2/3} \)-dependence with \( 1 \leq \alpha \leq 0.3 \). As is shown in Table 3 the \( A^{2/3} \) dependence can be observed under various conditions given by \( \sigma_i \) and \( \sigma_f \). For instance, an \( A^{2/3} \)-dependence is expected for \( \sigma_i \approx 0 \) and \( \sigma_f \approx 50 \text{ mb} \), which might correspond to the photoproduction of \( \pi, \rho, \omega \) and other mesons. Finally, the \( A^{2/3} \)-dependence can not be addressed as only due to the interaction of the incident particle at the nuclear surface. It is also clear that an interpretation of the \( A^{1/3} \) dependence could not be given in an unambiguous way.

### Table 3. The average slope \( \alpha \) of the \( A^\alpha \)-dependence fitted to the effective collision numbers calculated from Eq. (13) for different cross sections \( \sigma_i \) and \( \sigma_f \). The calculations were done for \( C, \text{Al, Cu, Ag, Au, and Pb} \) nuclei.

| \( \sigma_f \) (mb) | \( \sigma_i \) (mb) | \( \sigma_f \) (mb) | \( \sigma_i \) (mb) |
|-------------------|-------------------|-------------------|-------------------|
| 10                | 0                 | 10                | 0                 |
| 20                | 10                | 20                | 10                |
| 30                | 20                | 30                | 20                |
| 40                | 30                | 40                | 30                |
| 50                | 40                | 50                | 40                |
| 60                | 50                | 60                | 50                |
| 70                | 60                | 70                | 60                |
| 80                | 70                | 80                | 70                |
| 90                | 80                | 90                | 80                |

### 2.2 Angular dependence

Since in the actual experiments the data are collected at some fixed angles or integrated over a certain angular interval it is important to estimate how much the \( A^\alpha \)-dependence is effected by the ejectile angle. Such an estimate can be done using a quasi-classical approximation as is shown below.

As a beam of particles \( i \) passes through the nucleus, its intensity is attenuated due to the scattering out of the beam direction. Since particles can be removed from the beam because of both elastic and inelastic interactions with the target nucleons, the attenuation is determined by the distortion cross section \( \sigma_i \).

The attenuation probability of an \( i \) particle passing through the nucleus at impact parameter \( b \) and longitudinal positions from \(-\infty \) to \( z \) is then given by

\[
S_i(b, z) = \exp \left[ -\sigma_i \int_{-\infty}^{z} dz' \rho_A(b, z') \right] = e^{-\sigma_i T_i(b)}, \tag{23}
\]

and \( T_i(b) \) can be considered as the linear nuclear density.

Thus Eq. (23) is a semi-classical description of particle \( i \) attenuation in matter and one might argue that in this case \( \sigma_i \) should be taken as an inelastic or absorption cross section.
rather than total reaction cross section. In that sense an emission of a particle out of the beam trajectory can be also considered a distortion. That is why \(\sigma_i\) is discussed as a distortion cross section.

The incident particle \(i\) interacts with target nucleon at a transverse \(b\) and longitudinal \(z\) coordinate and produces the final particle \(f\). Let us to consider that \(f\) is moving along the line fixed at an azimuthal angle \(\phi\) and polar angle \(\theta\) with respect to the incident particle beam direction. The attenuation probability of passing \(f\) through the nucleus in that case is given as

\[
S_f(b, z, \theta, \phi) = \frac{1}{2\pi} \exp\left[-\sigma_f \int d\xi \rho(|r_\xi|)\right],
\]  

(24)

where the integration is performed along the path of the produced particle \(f\) defined by

\[
r_\xi^2 = (b + \xi \cos \phi \sin \theta)^2 + (\xi \sin \phi \sin \theta)^2 + (z + \xi \cos \theta)^2.
\]  

(25)

Here we again assume that both initial and final particle move along the straight trajectories before and after the production vertex, but now the ejectile trajectory depends on \(f\) emission angles \(\phi\) and \(\theta\).

Finally, the effective collision number can be evaluated by integration over the nuclear volume as

\[
N_A = \int d^2 b \, d z \, \rho(b, z) \, S_i(b, z) \, S_f(b, z, \theta, \phi),
\]  

(26)

and now depends on the production angles. It is easy to show that Eq. (26) reduces to the eikonal formalism given by Eq. (13) after an integration over the azimuthal angle \(\phi\) at polar angle \(\theta=0^\circ\).

Now Fig. 2a) shows the effective collision numbers calculated from Eq. (26) for \(C\) and \(Pb\) nuclei and \(\sigma_i=40\) mb as a function of \(\sigma_f\). The solid lines indicate the results obtained for a final particle emission angle \(\theta=0^\circ\), while the dashed lines show the calculations for \(\theta=150^\circ\).

The calculations indicate a quite strong angular dependence of \(N_A\). Fig. 2b) shows the ratio of the effective collision numbers \(N_{Pb}/N_C\) as a function of \(\sigma_f\) for different angles: \(\theta=0^\circ\), \(45^\circ\) and \(150^\circ\). Note that the ratios are almost the same for small emission angles \(0 \leq \theta \leq 45^\circ\). But the ratio becomes large at \(\theta=150^\circ\).

In addition Fig. 2b) illustrates the uncertainty in the evaluation of \(\sigma_f\) from the ratios of data for \(f\) particle production from different nuclear targets. Namely, it is clear that for different emission angles the \(N_{Pb}/N_C\) ratios roughly saturate at \(\sigma_f \geq 40\) mb. This actually means that model is insensitive to the value of \(\sigma_f\) if it exceeds the limit of \(\approx 40\) mb.

For completeness, Fig. 3 illustrates the slope \(\alpha\) of the \(A^\alpha\)-dependence given by Eq. (22). The results are shown for the different production angles and as a function of \(\sigma_f\). The calculations were done with \(\sigma_i=40\) mb. Note that \(\alpha\) is saturated above \(\sigma_f \approx 40\) mb, while the slope varies significantly at \(\sigma_f \leq 20\) mb. The variation of \(\alpha\) with the emission angle is almost negligible at \(\theta \leq 45^\circ\). Because of the distortion of the incident particle the maximal value of \(\alpha\) is below one. The minimal slope is close to \(\approx 0.3\).

### 3 \(A^\alpha\)-dependence due to two-step production

Fig. 3 shows that, neglecting the distortion of the final particle, i.e., for \(\sigma_f=0\), one might expect the maximal value for the slope of the \(A^\alpha\)-dependence around \(\approx 0.7\), which is essentially driven by the distortion of the incident particle given by \(\sigma_i\) used in our calculations. Considering proton-nucleus interactions one can
use $\sigma_{t}=40$ mb in the range of proton energies from $\approx 3$ to $10^3$ GeV. However, many experiments \cite{19,47,48,49,21,22} done with high energy proton beams indicate a slope $\alpha \approx 1$. This is the case for $\phi$-meson production in $pA$ collisions.

This apparent discrepancy can be understood quantitatively in terms of multi-particle production at high energies. The possible scenario is production of many pions that interact inside the nucleus and produce $\phi$-mesons. Since the flux density of these pions or their multiplicity can be large and their energies are above the $\pi N \rightarrow \phi N$ threshold, the probability of this process could be larger than the probability of the direct production considered previously. Indeed the pions are distributed through the whole nucleus and thus the $\phi$-meson can be produced over the full volume of the target. Therefore one can expect that the $A$-dependence of the $\phi$-meson production in such a case is proportional to $A$, while neglecting the final distortion.

Quantitative estimates can be done for the two-step process. Assume that at some impact parameter $b$ and longitudinal point $z$ the incident energetic particle $i$ produces some intermediate state $j$. Due to the Lorentz boost, the $j$ particle is moving along the beam direction, i.e. at the same impact parameter $b$ and at some point $z$ produces the final particle $f$. The final particle is now moving along the path given by the polar angle $\theta$ and azimuthal angle $\phi$ of the emission.

Thus we have two sub-processes and this is called a two-step mechanism. The first one is $iN \rightarrow jN$ and the second one is $jN \rightarrow fN$ where the distortion of the $i$, $j$ and $f$ particles is taken into account. The probability of the first process can be derived in analogy to Eq. (23) and is given as

$$S_{ij}(b, z) = |f_{ij}|^2 \int_{-\infty}^{\tilde{z}} d\tilde{z} \rho_A(b, \tilde{z}) \exp \left[ -\sigma_i \int_0^{\tilde{z}} d\tilde{z}' \rho_A(b, \tilde{z}') - \sigma_j \int_{\tilde{z}}^{\infty} d\tilde{z}' \rho_A(b, \tilde{z}') \right],$$

(27)

where $f_{ij}$ is the amplitude of the $iN \rightarrow jN$ transition, while $\sigma_i$ and $\sigma_j$ account for the distortion of the $i$ and $j$ particle. The attenuation probability of passing $f$ through the nucleus is similar to Eq. (24).

The effective collision number for the two-step process is

$$N_A = \int d^2b dz \rho_A(b, z) S_{ij}(b, z) \cdot S_f(b, z, \theta, \phi),$$

(28)

where $S_f$ is given by Eq. (24). Note that for $S_{ij}=S_f=1$ the $A$-dependence is proportional to $A$.

Now Fig. 4 shows the slope $\alpha$ of the $A^\alpha$-dependence as a function of the effective cross section $\sigma_f$ for the different production angles. The calculations were done using Eq. (28) and the results are shown for the different production angles. Note that in the derivation of Eq. (29) it was assumed \cite{50} that $\sigma_f$ is small, so one can not seriously discuss the difference between the two-step formalism and the above estimates at large distortion cross sections.

Another simple evaluation of the $A$-dependence can be done using so-called $\rho L$ parameterization \cite{51}

$$N_A = A \exp \left[ -\sigma_f \langle \rho L \rangle \right],$$

(30)

where $\langle \rho L \rangle$ is the average amount of matter crossed by the final particle and

$$\langle \rho L \rangle = \frac{A - 1}{2A^2} \int d^2b |T(b)|^2.$$

Now if $\sigma_f$ is small one can use the following expression

$$N_A = A^\alpha, \quad \alpha = 1 - \sigma_f \langle \rho L \rangle / \ln A,$$

(32)

\footnote{Note that our normalization of the nuclear density is $A$, while in Refs. \cite{50,51} it is unity.}
which was extensively applied in the evaluation of the distortion of the charmonium cross section in nuclear matter.

Apparently the slope $\alpha$ can be calculated using only one target and as we found only slightly depends on $A$ unless one uses light targets. The dashed line in Fig. 4 shows the $\alpha$ obtained by this $\langle pL \rangle$ approximation using Eq. (32) and an $Ag$ target. As was mentioned before, the approximation is valid for small $\sigma_f$ and indeed is in rough agreement with Eq. (29) for $\sigma_f \leq 20 \text{mb}$.

To obtain experimental values for the slope $\alpha$ one needs the production cross sections measured for different nuclear targets $A$. It was found [51] in the evaluation of charmonium absorption, that the value of $\alpha$ extracted from a fit to a given data set depends on the nucleus used as the lightest target. Indeed the experiments that use heavy targets with hydrogen or deuterium systematically obtain large values of the slope $\alpha$.

### 4 $\phi$-meson production in $p+A$ collisions at high energies

As was mentioned previously the results on $A$-dependence of inclusive $\phi$-meson production from $p+A$ collisions at high beam energies indicate a large slope $\alpha \approx 1$. Here we shortly review the current status. Moreover, we evaluate the $\phi$-meson distortion cross section and collect the results in Table 4. Furthermore the interpretation of $\sigma_\phi$ is given in the next Section.

Most recently the $A$-dependence of inclusive $\phi$-meson production off nuclei using a 920 GeV proton beam was measured with HERA-B detector at HERA storage ring [19]. This experiment was done with $C$, $Ti$ and $W$ targets and the $\phi \to K^+ K^-$ decay mode was used for the $\phi$-meson reconstruction. The data analysis shows the slope $\alpha = 0.96 \pm 0.02$. As discussed before only the multi-step mechanism can be an explanation of the HERA-B observation. Indeed the shaded box in Fig. 3 indicates the result from the HERA-B Collaboration, which in principle can be explained assuming $\sigma_\phi = 2.1 \pm 1.2 \text{mb}$. Here we use Eq. (29) as was done in the analysis of $J/\psi$ distortion.

A systematic study of $\phi$-meson production in $p+A$ collisions at a beam energy of 12 GeV was carried out by the KEK-PS E325 Collaboration [52] [23] [53] [54]. The slope $\alpha$ of the $A^\alpha$-dependence was evaluated using $C$ and $Cu$ targets. The most recent results [54] allows to investigate how $\alpha$ depends on the $\phi$-meson momentum as well as to obtain $\alpha$ for the $\phi \to K^+ K^-$ and $\phi \to e^+ e^-$ decay mode. It was found that $\alpha$ is statistically the same for these two different decay modes in the same kinematical region. Furthermore it turns out that $\alpha$ depends on reaction kinematics. Here we would like to make some comments.

The relevant kinematics for the evaluation of $A$-dependence is given by the final particle production angle. It is clear that this angle defines the path of the particle and therefore the amount of matter involved in the distortion. Eq. (29) is applicable at forward angles, while Eq. (28) can be used for large angles but accounts only for the two-step production mechanism. Nevertheless within such limitations one can realize from Fig. 4 that the angular dependence is essential for data evaluation.

Unfortunately, the KEK-PS E325 data are given either as a function of $\phi$-meson momentum alone [54] or as a function of rapidity and transverse momentum [23]. In our opinion, the ideal case is to fix forward angles and to extract the slope $\alpha$ for the different laboratory momenta of the produced $\phi$-mesons. That would give information about the momentum dependence of the distortion. In spite of that uncertainty in the analysis of the KEK-PS E325 data our results for the $\phi$-meson distortion cross section are summarized in the Table 4.

The $A$-dependence of the inclusive $\phi$-meson production in neutron-nucleus interactions at 30-70 GeV was studied by the BIS-2 Collaboration at the Serpukhov accelerator [20]. Here the $C$, $Al$ and $Cu$ targets were used and it was found that the slope $\alpha = 0.81 \pm 0.06$. That corresponds to a distortion cross section of $12 \pm 4 \text{mb}$.

In Ref. [21], the $A$-dependence of $\phi$-meson production by a 100 GeV/c proton beam was determined through the analysis of the data collected with $H_2$ and $Be$ targets. The measurements were done by the ACCMOR Collaboration at SPS. The $\phi \to K^+ K^-$ decay mode was used for the reconstruction. It was found that slope $\alpha = 0.96 \pm 0.04$. Moreover, it was argued [21] that the use of the $H_2$ target in general introduces additional systematic uncertainties, which are difficult to estimate. This result is close to the HERA-B observation.

The $A$-dependence of the inclusive $\phi$-meson production from beryllium and tantalum targets using a 120 GeV proton beam was studied with the NA11 spectrometer at CERN SPS [22]. The data analysis indicates that $\alpha = 0.86 \pm 0.02$. Applying Eq. (29) one can estimate $\sigma_\phi \approx 9 \pm 2 \text{mb}$.

Below we also list experiments that did not measure the $A$-dependence, but assume some values of $\alpha$ under certain assumptions in order to analyze the data.

Inclusive $\phi$-meson production off beryllium nuclei by 70 GeV/c protons was studied with the Sigma spectrometer [47] at the Serpukhov accelerator. For the evaluation of elementary $p N \to \phi X$ cross section it was assumed that the $A$-dependence of the nuclear cross section is proportional to $A^{0.7}$, as was measured for $K^- A$ interactions [48].

High statistics $\phi$-meson production from $p+Be$ collisions at beam momenta of 120 and 200 GeV/c was studied by the

| Experiment   | Ref.   | $\alpha$ | $\sigma_\phi$ (mb) |
|--------------|--------|----------|-------------------|
| HERA-B       | [19]   | 0.96±0.02| 2.1±1.2           |
| BIS-2        | [20]   | 0.81±0.06| 12±4             |
| ACCMOR       | [21]   | 0.96±0.04| 2.1±2            |
| NA 11        | [22]   | 0.86±0.02| 9±2              |
| KEK-PS E325  | [23]   |          |                   |
| $y$ = 0.9-1.1|        | 0.916±0.101±0.022| 4.9±4           |
| $y$ = 1.1-1.3|        | 1.050±0.101±0.02| 0±2.8            |
| $y$ = 1.3-1.5|        | 0.881±0.084±0.02| 7.2±5.8          |
| $y$ = 1.5-1.7|        | 0.780±0.119±0.019| 14±8.3         |
| $p_\perp$ = 0-0.25|    | 0.971±0.101±0.019| 1.7±7         |
| $p_\perp$ = 0.25-0.50| | 0.890±0.066±0.019| 6.7±4.9      |
| $p_\perp$ = 0.50-0.75| | 0.924±0.111±0.021| 4.4±4        |
ACCMOR Collaboration at SPS [49]. The data evaluation was done under the assumption that the $A$-dependence is proportional to $A^2$, which was motivated by the experimental results published in Refs. [21,22].

Finally the evaluated distortion cross sections are collected in Table 3. Unfortunately large uncertainties in the experimental results for $\phi$ produce large uncertainties in $\sigma_\phi$. In our opinion, the analysis of the ratios of $\phi$ production cross sections from different nuclei with respect to the $C$-target results might be less uncertain. In that case the systematical errors might cancel up to large extent. However, such an analysis requires measurements with many different nuclear targets, which is not the case for some experiments available now.

5 Interpretation of $\sigma_f$

5.1 Definitions

The interpretation of $\sigma_f$ is a general problem. Following our derivation given in Section 4, the eikonal formalism operates with the forward scattering amplitude $f(0)$ and $\sigma_f$ appears through the optical theorem. In that sense $\sigma_f$ is the total cross section for the interaction of a particle $f$ with a nucleon embedded in nuclear matter. In the classical derivation $\sigma_f$ is considered a distortion cross section. There is no conflict between these two definitions if we consider attenuation of the flux of the final particle due to all possible processes available in the nucleus. That might be general absorption, scattering out of the initial trajectory, decay of an unstable particle followed by the distortion of the decay products, interaction with few nucleon configurations and whatever one can assume. The total sum over all these possible processes is an effective total or distortion cross section.

As we emphasized previously $\sigma_f$ is not the free vacuum $f+N$ total cross section since it has to be extracted from the nuclear data and might be modified by in-medium effects. However it is always worthwhile to compare $\sigma_f$ with the free cross section if that is available. That comparison would show whether additional reaction channels were open in nuclear matter or whether reaction channels available in free space are blocked in the nucleus. For instance some transitions might be blocked due to the Pauli principle. In the analysis of charmonium properties in nuclear matter, the distortion cross section $\sigma_f$ is a standard variable generally used everywhere throughout the relevant discussions.

Since in some calculations not the distortion cross section but other variables are used we provide here some useful relations for the conversion. Let us first remind the reader that the complex forward scattering amplitude is related to the cross section by Eq. (11). At the same time the complex local potentials for the conversion. Let us first remind the reader that the complex local potential is given in terms of the complex forward scattering amplitude $f(0)$ as $\rho f(0)$ as [55,56,57,58]

\[
V = -2\pi \frac{m_N + m_f}{m_N m_f} \rho f(0),
\]

where $\rho$ is local nuclear density. The potential depends on $f$ due to the energy dependence of $f(0)$. It is possible to use a so-called in-medium collisional width $\Delta \Gamma$ and mass shift $\Delta m$ of the $f$ particle, which are [59,60,61,62]

\[
\Delta \Gamma = 4\pi \frac{m_N + m_f}{m_N m_f} \rho \ln f(0) = \frac{m_N + m_f}{m_N m_f} \rho k_f \sigma_f \quad (34)
\]

\[
\Delta m = -2\pi \frac{m_N + m_f}{m_N m_f} \rho \Re f(0), \quad (35)
\]

where $m_N$ is the nucleon mass and $k_f$ is the momentum of the final particle. In principle one can replace masses by total energies and discuss the $\Delta \Gamma$ and $\Delta m$ at high energy of the final particle. Note that the in-medium collisional width and mass shift are not invariants and can be changed by a Lorentz boost, so one should use these variables in the rest frame of the $f$ particle.

5.2 Estimates for $\sigma_\phi$ in vacuum

It is useful to compare distortion cross sections extracted from the nuclear data with its values in free space. That allows to inspect directly the possible in-medium modification of the $\sigma_\phi$. There are various well known methods to estimate the $\phi+N$ interaction cross section.

The $\phi+N$ cross section can be evaluated in the Vector Dominance Model from the $\gamma N \rightarrow \phi N$ reaction. Within VDM the hadron-like photon [63] is a superposition of all possible vector meson states. Therefore the $\gamma N \rightarrow \phi N$ reaction can be decomposed into the transition of the photon to a virtual vector meson $V$ followed by the elastic or inelastic vector meson scattering on the target nucleon and production of the final $\phi$-meson. The reaction amplitude is then written as [64,65]

\[
\hat{f}_{\gamma N \rightarrow \phi N} = \sum_V \frac{\sqrt{\pi \alpha}}{\gamma_V} \hat{f}_{VN \rightarrow \phi N}, \quad (36)
\]

where the summation is performed over vector meson states. Moreover, $\alpha$ is the fine structure constant, $\gamma_V$ is the photon coupling to the vector meson $V$ and $\hat{f}_{VN \rightarrow \phi N}$ is the amplitude for the $VN \rightarrow \phi N$ transition.

The coupling $\gamma_V$ is given by vector meson decay into a lepton pair [66]

\[
\Gamma(V \rightarrow l^+ l^-) = \frac{\pi \alpha^2}{3\gamma_V^2} \sqrt{m_V^2 - 4m_l^2} \left[ 1 + \frac{2m_l^2}{m_V^2} \right], \quad (37)
\]

where $m_V$ and $m_l$ are the masses of vector meson and lepton, respectively. Taking the di-electron decay widths [57], the photon couplings to the lightest vector mesons are

\[
\gamma_\rho = 2.51, \quad \gamma_\omega = 8.47, \quad \gamma_\phi = 6.69. \quad (38)
\]

Note that non-diagonal, i.e. $\rho N \rightarrow \phi N$ and $\omega N \rightarrow \phi N$ as well as diagonal $\phi N \rightarrow \phi N$ transitions contribute to the reaction amplitude of Eq. (36). VDM suggests that the virtual vector meson stemming from the photon becomes real through the four-

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[55,56,57,58] It is clear that the introduction of $\Delta \Gamma$ in the calculation described above requires an accurate definition of the nuclear density dependence rather then average estimate of Eq. (34).
momentum $t$ transferred to the nucleon, which in general requires the introduction of a form-factor at the interaction vertices \cite{68,69,70}. In many analyses \cite{64}, this form factor is neglected. Thus the VDM analysis of photoproduction data requires additional assumptions.

There are many precise data on $\phi$-meson photoproduction differential cross sections at energies close to the reaction threshold. These data can be used for evaluation of the $\phi N \rightarrow \phi N$ scattering amplitude squared applying the VDM as

$$
\frac{d\sigma_{\gamma N \rightarrow \phi N}}{dt} \propto \alpha \frac{\pi^2}{\gamma_\phi^2 q_\gamma^2} |f_{\phi N \rightarrow \phi N}|^2 .
$$

Now if the scattering is dominated by $s$-waves one could extract the $\phi N$ scattering length at threshold. It is related to the cross section as\footnote{In our normalization the scattering amplitude $f$ equals to the scattering length $a_{\phi N}$ at $q_\phi \rightarrow 0$.}

$$
\sigma_{\phi N} = 4\pi |f_{\phi N \rightarrow \phi N}|^2
$$

and is shown in Fig. 5 by open circles. Here we use the forward $\phi$-meson photoproduction cross section available near the reaction threshold \cite{71,72}. This scattering length can be compared with other theoretical predictions. For instance the estimate based on QCD sum rules \cite{73} provides a real $\phi N$ scattering length of $a_{\phi N} \approx -0.15$ fm that corresponds to a cross section of $\sigma_{\phi} \approx 2.8$ mb and seems to be in good agreement with the data evaluated by Eqs. \cite{39,40} under assumption of $s$-wave $\phi N$ scattering. However, note that the $\phi$-meson photoproduction differential cross sections are essentially anisotropic already at energies close to the threshold and can be well parameterized as \(d\sigma/dt \propto \exp(bt)\) with a slope $b \approx 3$ GeV$^{-2}$ and $t$ being the four momentum transfer squared \cite{71,72}. In that sense, the estimates shown by an open circles in Fig. 5 might not be correct and should be taken with a grain of salt in the evaluation of $\phi N$ scattering length.

The estimate \cite{74} based on an effective Lagrangian approach predicts the scattering length $a_{\phi N} \approx (-0.01+i0.08)$ fm that corresponds to $\sigma_{\phi} \approx 0.8$ mb.

Another estimate can be obtained from a QCD van der Waals potential calculation \cite{75}, which predicts a $\phi$-nucleon bound state. In that case the Born approximate scattering length is given by the potential as \cite{66,67}

$$
a_{\phi N} = 2\frac{m_N m_\phi}{m_N + m_\phi} \int_0^\infty dr r^2 V(r),
$$

where the potential was taken in the Yukawa form $V(r) = -\alpha \exp(-r\mu)/r$ with strength $\alpha = 1.25$ and range $\mu = 0.6$ GeV. These parameters were obtained for an attractive potential and result in $a_{\phi N} \approx 0.67$ fm. They correspond to $\sigma_{\phi} \approx 56$ mb at $q_\phi = 0$.

Furthermore, applying Eq. \cite{11} the $\gamma N \rightarrow \phi N$ differential cross section of Eq. \cite{39} at $t=0$ can be written as

$$
\left. \frac{d\sigma_{\gamma N \rightarrow \phi N}}{dt} \right|_{t=0} = \frac{\alpha}{16\gamma_\phi^2 q_\gamma^2} (1 + \alpha_\phi^2) \sigma_{\phi N}^2 .
$$

Since the ratio of the $\alpha_\phi$ and $\phi N$ total cross sections are unknown, one can extract from the photoproduction data only their combination, i.e. $\sigma_{\phi N} \times \sqrt{1 + \alpha_\phi^2}$, which is shown by closed circles in the Fig.\[5\] Here we use the data collected in Ref. \cite{29}. If one assumes that $\alpha_\phi = 0$ these results may be considered as the energy dependence of the $\phi N$ cross section. While for many processes the real part of the scattering amplitude vanishes at high energies, the ratio of real to imaginary part of the amplitude $\alpha$ is large at low energies and moreover substantially depends on the momentum of the scattered particle \cite{67}.

Just to illustrate such a possibility the dashed line in Fig. 5 shows the dependence

$$
\sigma_{\phi N} \times \sqrt{1 + \alpha_\phi^2} = 10 \text{ (mb)} \times \sqrt{1 + \frac{0.6 \text{ (GeV/c)}}{q_\phi^2}} .
$$

Again this might ensure that at high energies the $\phi N$ cross section approaches some value around 10 mb, but still does not provide a trustworthy estimate of $\sigma_{\phi N}$ close to threshold. At least it is not appropriate to estimate the real part of the forward scattering amplitude and to evaluate the in-medium mass shift using Eq. \cite{35}.

Within an additive quark model the $\phi N$ cross section is given as \cite{78}

$$
\sigma_{\phi N} = \sigma_{K^-N} + \sigma_{K^+N} - \sigma_{\pi^-N},
$$

where the elementary cross sections are taken at the same invariant collision energies. Since the $\phi N$ reaction threshold is $m_\phi + m_N \approx 1.96$ GeV one can safely use Regge parametrization for the continuum or non-resonant meson-nucleon scattering amplitudes \cite{79}. Now Eq. \cite{44} is shown by the solid line in
Fig. 5] and is in reasonable agreement with VDM results at high energies.

It is also worthwhile to show the estimate based on the dynamical study of the $\phi N$ bound state within the chiral SU(3) quark model. By solving a resonating group method based equation [80] it was found that the binding energy of the state might range from 1 to 9 MeV. With respect to the $s$-wave scattering length $a_{\phi N}$, the relation between the pole of the $S$-matrix and binding energy $\epsilon$ is given as

$$a_{\phi N} = \left( \frac{2m_N m_\phi}{m_N + m_\phi} \right)^{-1/2} \sigma, \quad (45)$$

so that the real part of the scattering length ranges from 2.1 to 6.3 fm. This scattering length is large compared to the other results. Note that this result is used in three-body calculations of the $\phi NN$ nuclear cluster binding energy [81].

5.3 Estimates for $\sigma_\phi$ in matter

Only some results evaluated from high energy proton-nucleus collisions (summarized in Table 4) are in agreement with the data shown in Fig. 5. The uncertainties of the KEK-PS data [23] are still too large to draw a definite conclusion right now. The results from BIS-2 [20] and NA-11 [22] are consistent with vacuum estimates at high energies.

However the results from HERA-B [19] and ACCMOR [21] indicate a substantially smaller $\phi$-meson distortion cross section. This observation is difficult to interpret since in high energy experiments, the $\phi$-mesons are produced with high momenta and should be almost blind to any in-medium modification.

The squares in Fig. 5 show the $\phi$-meson distortion cross section extracted in [29] from the data on $\phi$-photoproduction from nuclei [30,18]. As we already discussed, it is not necessary that these in-medium results are the same as $\sigma_{\phi N}$ in vacuum. However we observe reasonable agreement between nuclear results and those evaluated by VDM at high energies.

Substantial modification of slow $\phi$-mesons in nuclear matter was proposed in Ref. [74]. It was found that the mass of the $\phi$-meson almost does not change in matter, while the change of the width accounts for $\Delta \Gamma \approx 45$ MeV at normal nuclear density. Following Eq. (44) one can estimate the distortion cross section as $\sigma_\phi \approx 70$ mb for $k_\phi = 100$ MeV/c.

The energy dependence of the in-medium $\phi$-meson width was studied in Ref. [24]. While for $E_\phi = m_\phi$, the width is about 20 MeV at normal nuclear density, it increases up to 40 MeV at a $\phi$-meson energy of 1.1 GeV. So it is really changed by a factor of two over 80 MeV in energy [19]. This corresponds to a variation of the distortion cross section from $\approx 27$ to 15 mb and seems to be in agreement with the dashed line shown in the Fig. 5.

Furthermore, the energy dependence of the $\phi$-meson width at normal nuclear density was investigated in Ref. [25]. It was found that the in-medium width $\Delta \Gamma \approx 22-17$ MeV slightly varies with energy within the range $E_\phi = m_\phi$ to 1.2 GeV.

Fig. 6. The ratio of the effective collision numbers calculated for $^{64}$Cu and $^{12}$C nuclear targets as a function of $\phi$-meson production angle shown for the different $\sigma_\phi$ within the range from 0 to 40 mb with a step size of 5 mb.

6 Predictions for $\phi$-meson production in $p + A$ collisions at COSY energies

The COoler SYnchrotron (COSY) at Jülich provides an unique opportunity to study the $\phi$-meson distortion in nuclear matter at low energies. Our analysis indicates that even the vacuum $\phi N$ interaction is not well understood and different estimates illustrated in Fig. 5 are in substantial disagreement at $\phi$-meson energies below 2 GeV. Moreover, the available predictions [74,24,25] state that the in-medium modification of the $\phi$-meson width is substantial at low energies, although the real size of that change is not well established.

Such a situation requires precise measurements of $\phi$-meson production from $p + A$ collisions at low energies, as was proposed in Refs. [26,27]. A dedicated experiment on $\phi$-meson production from the proton interaction with $^{12}$C, $^{108}$Ag and $^{197}$Au targets at maximum COSY energies of 2.83 GeV was proposed by ANKE Collaboration [28]. Here we show the results for the $A$-dependence of the $\phi$-meson production in $pA$ collisions at few GeV energies. Note that at low energies the $\phi$-meson production due to multiple processes is suppressed due to the final particle multiplicities and large $\phi$-meson production threshold.

For the further calculations we fix $\sigma_\phi = 40$ mb, which stands for the average cross section for the interaction of the beam protons with target proton and neutron. Although the $pN$ interaction can be modified in nuclear matter one would not expect that this effect is significant for the protons with momenta above $\approx 1$ GeV/c.

We believe that the analysis of the ratios $R$ of the produced $\phi$-meson contains less theoretical uncertainties than the analysis of the differential cross section $d^2 \sigma / dT / d\Omega$ itself for each nuclear target $A$ or the slope $\alpha$, as was discussed previously.
Thus in the following we show our predictions for the ratio 

$$R = \frac{d^2\sigma_A}{dT d\cos \theta} \times \left( \frac{d^2\sigma_C}{dT d\cos \theta} \right)^{-1} = \frac{N_A}{N_C} \quad (46)$$

taken with respect to the carbon target. Here $T$ is the kinetic energy and $\theta$ is the emission angle of the produced $\phi$-meson, while $N_A$ is the effective collision number that was calculated for the different $\sigma_\phi$ and $\theta$. Moreover, the analysis of the ratios has additional advantages since systematic experimental uncertainties can be substantially reduced.

Once more we would like to emphasize that in the evaluation of an effective collision number $N_A$ we use an effective in-medium cross section $\sigma_f = \sigma_\phi$. This is not the cross section for the $\phi$-meson interaction with free nucleon. Moreover, as was discussed previously the multiple scattering series corrections $\epsilon$ given by Eq. (12) can not be isolated and thus the extracted $\sigma_\phi$ cross section contains such a multiple scattering contribution. Nevertheless it is of great importance to compare $\sigma_\phi$ evaluated from the nuclear data with the vacuum $\phi N$ cross section.

The calculations were done for the $^{12}C$, $^{64}Cu$, $^{108}Ag$ and $^{196}Au$ nuclear targets, which is in line with the targets proposed [28] for the measurements at COSY. Figs. [68] shows the calculated ratios as a function of the $\phi$-meson production angle. The lines indicate the results for different $\sigma_\phi$ given within the range from 0 to 40 mb with a step size of 5 mb.

The calculations indicate substantial angular dependence of the ratio. As we showed previously, the analysis of the $A$-dependence for the forward particle production is most preferable for several reasons. At forward angles, the reaction mechanism can be formulated within an eikonal basis and contains less theoretical uncertainties than the quasi-classical approximation. Furthermore, due to the scattering dynamics any possible multiple processes contribute less to the forward particle production.

Moreover, at forward angles the ratios indicate reasonable sensitivity to the distortion cross section when $\sigma_\phi$ stands below $\simeq 20$ MeV. At large angles and for large $\sigma_\phi$ the analysis requires very high precision data.

For completeness let us illustrate how to use the figures with the calculated ratios. We take as an example the ratio of the effective collision numbers from $^{108}Ag$ and $^{12}C$ nuclei shown in Fig. [7] Let us consider the production at forward angles, i.e. $\theta \leq 10^\circ$. If there is no distortion of the incident proton and final $\phi$-meson the ratio equals that given by the target mass numbers leading to $R=9$. Due to the distortion of the incident proton and neglecting the distortion of the produced $\phi$-meson one finds that $R=4.6$. That is the maximum value given by the direct $\phi$-meson production mechanism. The $A^{2/3}$ dependence corresponds to $R=4.3$ and leads to a $\phi$-meson distortion cross section of less than 5 mb. The $A^{1/3}$ dependence results in a ratio of $\simeq 2.1$ and corresponds to $\sigma_\phi > 40$ mb.

Now the question arises if the measured ratio is larger than $R=4.6$. This explicitly indicates the contribution from multiple or two-step processes, as is illustrated by Fig. [4] In that case the extraction of the $\phi$-meson distortion in nuclear matter is much more model-dependent. Then one could use additional kinematical constraints in order to isolate direct production mechanism, as was done for instance in Ref. [82].

Finally, we can estimate the $A$-dependence in case the distortion cross section is $\sigma_\phi = 10$ mb. Then one might expect at forward angles the ratio for $\phi$-meson production from $^{108}Ag$ and $^{12}C$ targets to be $R=3.2$ as is illustrated by Fig. [4]. This corresponds to a mass dependence of $\propto A^{0.52}$. This result is compatible with in-medium width of $\simeq 30$ MeV for an average $\phi$-meson momentum of 500 MeV/c.

![Fig. 7. The ratio of the effective collision numbers calculated for $^{108}Ag$ and $^{12}C$ targets as a function of the $\phi$-meson production angle shown for different $\sigma_\phi$ within the range from 0 to 40 mb with a step size of 5 mb.](image)

![Fig. 8. The ratio of the effective collision numbers calculated for $^{196}Au$ and $^{12}C$ targets as a function of $\phi$-meson production angle shown for the different $\sigma_\phi$ within the range from 0 to 40 mb with a step size of 5 mb.](image)
7 Conclusions

A systematic analysis of the A-dependence of φ-meson production in proton-nucleus collisions has been carried out. We discuss the application of an eikonal formalism, corrections due to multiple scattering and the extension to large angle production processes. Furthermore, the A-dependence due to two-step production mechanisms and multi-step processes are investigated in detail. We provide all formulas frequently used in the analysis of nuclear data and study their compatibility and conditions of applicability.

The φ-meson distortion cross section $\sigma_\phi$ was evaluated from the available nuclear data. It was found that different measurements result in different values of $\sigma_\phi$ ranging from 0 to 14 mb. Unfortunately, at present the uncertainties of the experimental results are too large to draw definite conclusions.

We also discuss an interpretation of the φ-meson distortion in nuclear matter and give the relation between various frequently used variables, such as in-medium width, distortion cross section and scattering length. Furthermore, we show the estimates for $\sigma_\phi$ in the vacuum obtained by VDM, the additive quark model, QCD sum rules and other theoretical frameworks available. Moreover, we collect predictions for the φ-meson modification in matter. While most of the estimates are in reasonable agreement with $\sigma_\phi$ $\approx$ 10 mb at φ-meson energies above 3 GeV there are very large uncertainties at lower energies.

To resolve the unsatisfactory current situation, we propose to study the A-dependence of φ-meson production from p + A collisions at COSY energies. We provide detailed calculations of the ratios of φ-meson production cross sections from different nuclear targets. Our results can directly be used for the evaluation of the $\sigma_\phi$ from such measurements.

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References

1. T. Hatsuda and T. Kunihiro, Phys. Lett. B 185, 304 (1987).
2. V. Bernard, Ulf-G. Meißner and I. Zahed, Phys. Rev. Lett. 59, 966 (1987).
3. V. Bernard and Ulf-G. Meißner, Nucl. Phys. A 489, 647 (1988).
4. G.E. Brown and M. Rho, Phys. Rev. Lett. 66, 2720 (1991).
5. T. Hatsuda and S.H. Lee, Phys. Rev. C 46, 34 (1992).
6. G.-Q. Li, C.M. Ko and G.E. Brown, Nucl. Phys. A 606, 568 (1996) [arXiv:nucl-th/9608040].
7. K. Saito, K. Tsushima and A.W. Thomas, Prog. Part. Nucl. Phys. 58, 1 (2007) [arXiv:hep-ph/0506314].
8. G. Agakishiev et al. [CERES Collaboration], Phys. Rev. Lett. 75, 1272 (1995).
9. M. Masera et al. [HELIOS Collaboration], Nucl. Phys. A 590, 93C (1995).
10. G. Agakishiev et al. [CERES/NA45 Collaboration], Phys. Lett. B 422, 405 (1998) [arXiv:nucl-ex/9712008].
11. S. Afanasiev et al. [PHENIX Collaboration], [arXiv:nucl-ex/0706.3034].
12. A. Shor, Phys. Rev. Lett. 54, 1122 (1985).
13. J. Adams et al. [STAR Collaboration], Phys. Lett. B 612, 181 (2005) [arXiv:nucl-ex/0406003].
14. B.I. Abelev et al. [STAR Collaboration], Phys. Rev. Lett. 99, 112301 (2007) [arXiv:nucl-ex/0703033].
15. A. Sibirtsev and W. Cassing, Eur. Phys. J. A 7, 407 (2000) [arXiv:nucl-th/9907059].
16. A. Sibirtsev, Ulf-G. Meißner and A.W. Thomas, Phys. Rev. D 71, 094011 (2005) [arXiv:hep-ph/0503276].
17. A. Sibirtsev, J. Haidenbauer and Ulf-G. Meißner, Eur. Phys. J. A 27, 263 (2006) [arXiv:nucl-th/0512055].
18. T. Ishikawa et al., Phys. Lett. B 608, 215 (2005) [arXiv:nucl-ex/0411016].
19. I. Abt et al., Eur. Phys. J. C 50, 315 (2007); [arXiv:hep-ex/0606049].
20. A.N. Aleev et al., Czech. J. Phys. 42, 11 (1992).
21. C. Daum et al., Z. Phys. C 18, 1 (1983).
22. R. Bailey et al., Z. Phys. C 22, 125 (1984).
23. T. Tabaru et al., Phys. Rev. C 74, 025201 (2006).
24. E. Oset and A. Ramos, Nucl. Phys. A 679, 616 (2001) [arXiv:nucl-th/0005046].
25. D. Cabrera and M. J. Vicente Vacas, Phys.Rev. C 67, 045203 (2003) [arXiv:nucl-th/0205075].
26. V.K. Magas, L. Roca and E. Oset, Phys. Rev. C 71, 065202 (2005) [arXiv:nucl-th/0403067].
27. V.K. Magas, L. Roca and E. Oset, Nucl. Phys. A 755, 495 (2005) [arXiv:nucl-th/0412066].
28. M. Hartmann, Yu. Kiselev et al. [ANKE Collaboration], COSY Proposal 147 (2005).
29. A. Sibirtsev, H.-W. Hammer, Ulf-G. Meißner and A.W. Thomas, Eur. Phys. J. A 29, 209 (2006) [arXiv:nucl-th/0606044].
30. G. McCellan et al., Phys. Rev. Lett. 26, 1593 (1971).
31. P. Mühlich and U. Mosel, Nucl. Phys. A 765, 188 (2006) [arXiv:nucl-th/0510078].
32. P. Mühlich and U. Mosel, Nucl. Phys. A 773, 156 (2006) [arXiv:nucl-th/0602054].
33. M. Kaskulov, E. Hernandez and E. Oset, Eur. Phys. J. A 31, 245 (2007) [arXiv:nucl-th/0610007].
34. R.J. Glauber, Boulder Lectures in Theoretical Physics, 1, (1958).
35. R.J. Glauber, High Energy Physics and Nuclear Structure, ed. G. Alexander., North-Holland, Amsterdam, 311 (1967).
36. J. Formanek and J.S. Trefil, Nucl. Phys. B 3, 155 (1967).
37. J. Formanek and J.S. Trefil, Nucl. Phys. B 4, 165 (1968).
38. S.M. Berman and S.D. Drell, Phys. Rev. Lett. 11, 220 (1963).
39. S.M. Berman and S.D. Drell, Phys. Rev. 133, B791 (1964).
40. M. Ross and L. Stodolsky, Phys. Rev. 149, 1172 (1967).
41. S.D. Drell and J.S. Trefil, Phys. Rev. Lett. 16, 552 (1966).
42. S.D. Drell and J.S. Trefil, Phys. Rev. Lett. 16, 832 (1966).
43. B. Margolis, Phys. Lett. B 26, 524 (1968).
44. B. Margolis, Nucl. Phys. B 4, 433 (1968).
45. K.S. Kölbig and B. Margolis, Nucl. Phys. B 6, 85 (1968).
46. J. Knoll and J. Randrup, Nucl. Phys. A 324, 445 (1979).
47. Yu.M. Antipov et al., Phys. Lett. B 110, 326 (1982).
48. Yu.M. Antipov et al., Yad. Fiz. 28, 1299 (1978).
49. H. Dijkstra et al., Z. Phys. C 31, 375 (1986).
50. D. Kharzeev, C. Lourenco, M. Nardi, H. Satz, Z. Phys. C 74, 307 (1997) [arXiv:hep-ph/9612217].
51. B. Alessandro et al., Eur. Phys. J. C 33, 31 (2004).
52. K. Ozawa et al., Nucl. Phys. A 698, 535 (2002).
53. R. Muto et al., Phys. Rev. Lett. 98, 042501 (2007).
54. F. Sakuma et al., Phys. Rev. Lett. 98, 152302 (2007).
55. C.B. Dover and G.E. Walker, Phys. Rep. 89, 1 (1982).
56. G.P. Gopal et al., Nucl. Phys. B 119, 362 (1977).
57. A.S. Rosental and F. Tabakin, Phys. Rev. C 22, 711 (1980).
58. A. Sibirtsev and M.B. Voloshin, Phys. Rev. D 71, 076005 (2005) [arXiv:hep-ph/0502068]
59. W. Lenz, Z. Phys. 56, 778 (1929).
60. C.B. Dover, J. Hufner and R.H. Lemmer, Ann. Phys. 66, 248 (1971).
61. B. Friman, Acta Phys. Pol. B 29, 3195 (1998)
62. F. Klingl, T. Waas and W. Weise, Nucl. Phys. A 650, 299 (1999)
63. L. Stodolsky, Phys. Rev. Lett. 18, 135 (1967).
64. T.H. Bauer, R.D. Spital, D.R. Yennie and F.M. Pipkin, Rev. Mod. Phys. 50, 261 (1978).
65. E. Paul, Nucl. Phys. A 446, 203 (1985).
66. Y. Nambu and J.J. Sakurai, Phys. Rev. Lett. 8, 79 (1962).
67. W.-M. Yao et al., [Particle Data Group], J. Phys. G 33, 1 (2006).
68. J. Hufner and B.Z. Kopeliovich, Phys. Lett. B 426, 154 (1998); [arXiv:hep-ph/9712297].
69. A. Sibirtsev, K. Tsushima and A.W. Thomas, Phys. Rev. C 63, 044906 (2001); [arXiv:nucl-th/0005041].
70. A. Sibirtsev, S. Krewald and A.W. Thomas, J. Phys. G 30, 1427 (2004); [arXiv:nucl-th/0301082].
71. J. Barth et al., Eur. Phys. J. A 17, 269 (2003).
72. T. Mibe et al. [LEPS Collaboration], Phys. Rev. Lett. 95, 182001 (2005) [arXiv:nucl-ex/0506015].
73. Y. Koike and A. Hayashigaki, Prog. Theor. Phys. 98, 631 (1997). [arXiv:nucl-th/9609001]
74. F. Klingl, N. Kaiser and W. Weise, Nucl. Phys. A 624, 527 (1997) [arXiv:hep-ph/9704398].
75. H. Gao, T.S.H. Lee and V. Marinov, Phys. Rev. C 63, 022201 (2001) [arXiv:nucl-th/0010042].
76. S.J. Brodsky and G.A. Miller, Phys. Lett. B 412, 125 (1997) [arXiv:hep-ph/9707382].
77. S.J. Brodsky, I.A. Schmidt and G.F. de Teramond, Phys. Rev. Lett. 64, 1011 (1990).
78. H. J. Lipkin, Phys. Rev. Lett. 16, 1015 (1966).
79. J.R. Cudell et. al. [COMPLETE Collaboration], Phys. Rev. D 65, 074024 (2002) [arXiv:hep-ph/0107219].
80. F. Huang, Z.Y. Zhang and Y.W. Yu, Phys. Rev. C 73, 025207 (2006) [arXiv:nucl-th/0512079]
81. V.B. Belyaev, W. Sandhas (Bonn U.) , I.I. Shlyk, [arXiv:0707.4615]
82. V.V. Barmin et al. [DIANA Collaboration], Phys. Lett. B 464, 323 (1999).