Numerical analysis of post-impact droplet deformation for direct-print

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Two-dimensional (2D) numerical investigations of droplet impacts on a solid surface and the consequent deformation have been performed. The application lies in the direct-print technology where droplets are used to create lines and instantly cured to maintain line dimension. The investigation focuses on the evolution of droplet shape at the initial stage after the impact for Newtonian fluids. More specifically, the investigation emphasized the time for an impacted droplet to start increasing its diameter after the initial compression due to impact. A computational model has been developed by utilizing an adaptive quadtree spatial discretization with piecewise-linear geometrical volume-of-fluid (VOF) for this multiphase problem. The continuum-surface-force method and the height-function (HF) were employed for estimating the surface tension and the interface curvature, respectively. The Gerris Flow Solver, an open-source finite-volume package, was used for developing the computational model. The investigation was performed for the governing parameters of the Froude number ($Fr$), Reynolds number ($Re$), and Weber number ($We$). The results are presented as the interface contour, spreading factor ($ξ$), and deformation ratio ($R_o$). The investigation shows that the results from the developed model have excellent agreement with the experimental results.

Keywords: 3D printing; direct-print; droplet impact and deformation; spreading factor; deformation ratio; adaptive meshes refinement

1. Introduction

Droplet impact and deformation is one of the fundamental studies in the area of manufacturing. Although this fundamental phenomenon has been studied for a long time, droplet impact and deformation is still an active field of research due to countless applications and the addition of various novel applications. Study of the droplet impact and deformation process for optimization and better manufacturing accuracy is required for state-of-the-art processes, including three-dimensional (3D) cell patterning for tissue engineering (Derby, 2008; Moon et al., 2010), hybrid biological–electronic device construction, and biochip-array fabrication (FitzGerald, Lamont, McConnell, & Benchikhel, 2001), as well as traditional manufacturing (e.g., high temperature surfaces with water spray for steel making). Material jetting is one of the manufacturing technologies where the extensive study of droplet formation, dispensing, impact and deformation is essential, as the process is designed to produce a continuous or drop-on-demand droplet at a high frequency. Inkjet is one of the most commonly used jetting technologies in the industry, for purposes as diverse as producing paper prints through to the fabrication of micro-lenses (Calvert, 2001). For polymer deposition processes such as the manufacturing of multicolor polymer light-emitting diode (PLED) displays and other polymer electronics, inkjet printing is considered one of the key technologies (Levi, 2001). Direct-print — a recently developed process of creating conductive patterns, complex structures, and tissue engineering scaffolds — uses jetting technologies, including inkjet printing and the continuous dispensing of filament. Direct-print is highly appreciated for processes such as laser-guided direct writing for applications in biotechnology (Odde & Renn, 1999), and complex functional materials (Lewis, 2006; Vatani, Lu, Lee, Kim, & Choi, 2013).

In the last few decades, the inkjet process has established itself as an essential class of technologies for many industries, including 3D shaping (Derby & Reis, 2003), flat panel display (Kobayashi et al., 2000), printed circuit boards (PCBs; Lopez, Diez, & Odriozola, 2007), semiconductor packaging, and DNA chip and biosensors (Li & Wong, 2001). Direct-print, having existed for decades, is emerging as an advanced form of manufacturing for sensors, antenna, flexible circuits and complex 3D structures (Engeberg, Vatani, & Choi, 2012; Ghafar-Zadeh, Sawan, & Therriault, 2007; Gratson, Xu, & Lewis, 2004; Lu, Vatani, & Choi, 2013; Pique et al., 2006; Vatani et al., 2013; Vatani, Engeberg, & Choi, 2014). Since direct-print is based on either a continuous or drop-on-demand droplet formation, a droplet study for impact and deformation...
plays a significant role for its development, as this technology is being used for manufacturing microscale and nanoscale high-precision devices.

One of the earliest studies of liquid droplet impact on a solid surface was reported about a century ago and was considered a pioneering work due to its relevance to many natural phenomena as well as industrial applications (Worthington, 1908). This work made significant progress in the understanding of the droplet impact process and has become a fundamental basis for impact studies on theoretical, computational, and experimental aspects. A more recent work made a significant contribution to the understanding of the droplet deformation process. The panoptic experimentation of Rioboo, Bauthier, Conti, Voué, and Coninck (2003) revealed the mechanism for the formation of a crown during the impact of a single droplet on a wetted surface. The dynamics of droplet impact were comprehensively reviewed, delineating many interesting phenomena such as splashing, spreading, receding, bouncing, and crown formation (Yarin, 2006). Much research has been accomplished to investigate the deformation process of water droplets due to their intensive applications, such as manufacturing and spray cooling processes. Wachters and Westerling (1996) performed an extensive experimental study on water droplet deformation, investigating the consequent volume decrease of water droplets after impact.

Chandra and Avedisian (1991) performed an experimental investigation on the deformation of n-heptane over a hot surface at different temperatures. Computational modeling of water droplet deformation was performed by Songoro, Gjonaj, and Weiland (2012), while Hatta, Fujimoto, and Takuda (1995) performed an experimental and computational study of deformation of water and n-heptane droplets, concluding that a zero surface tension model was in accordance with the experimental results reported in Chandra and Avedisian (1991). Rioboo, Marengo, and Tropea (2002) experimentally investigated the time evolution of droplet deformation due to its impact on a dry surface. They investigated different Reynolds numbers ($Re = \rho u^r L_R/\mu^r$) and Weber numbers ($We = \rho u^r L_R/\sigma^r$) for glycerin, silicone oil and water, performing experiments on different solid surfaces of glass, coated glass, and polymer. The research they have carried out on droplet deformation is benchmarked as one of the most extensive works in the field. They divided the spreading factor plot into four distinctive phases: kinetic, spreading, relaxation and wetting. At the first phase of impact, the liquid is compressed for a sudden pressure increase and a reduction of volume takes place (Wachters & Westerling, 1966). Formation of a shock wave occurs in case of high impact velocities. At the end of the first phase, the spreading phase begins to become apparent, characterized by the formation of a radially expanding film. This is literally the spreading phase as the area of a droplet increases at this stage. The following phase, which is the relaxation phase, is characterized by either recoiling or stabilization of the radially expanding film of liquid. At the last stage, further wetting or equilibrium state is established.

For direct-print, the dispensed droplet needs to be cured by a light or heat source to create a predetermined pattern. For this reason, maintaining the desired dimensional accuracy and shape is crucial. For droplet impact, there is an initial stage of deformation where the maximum width of the droplet remains steady (later described in detail). To the authors’ best knowledge, very little experimental research has been done for droplet deformation analysis focusing on the initial period of the steady dimension (maximum width) of droplets, and there is a lack of computational work for the investigation of droplet deformation in terms of identifying the initial steady dimension as well. This initial stage of the steady dimension is a key factor of effective curing in the direct-print technology because curing of the dispensed droplet needs to be initiated and completed within this time period. And thus, the initiation and the intensity of curing depend on the small window of time, which is this initial period. In this present work, the droplet deformation has been numerically investigated up to the spreading stage (Rioboo et al., 2002) for different governing parameters, and results have been presented, as mentioned above.

2. Problem formulation

In the present study, a two-phase flow has been solved to investigate droplet impact and deformation over a solid surface. The deformation of a droplet was studied in terms of the interface contour, spreading factor ($\xi$), and deformation ratio ($R_t$). The spreading factor is the ratio of an instantaneous diameter to the initial diameter of the droplet. The dimension ratio is the ratio of the instantaneous diameter to the instantaneous height of the droplet. These parameters were studied for different values of Froude number ($Fr$), Reynolds number ($Re$) and Weber number ($We$). The density ratio ($\rho_r$) is the ratio of the liquid phase density ($\rho_L$) to the gaseous phase density ($\rho_G$), while the viscosity ratio ($\mu_r$) is the ratio of the liquid phase viscosity ($\mu_L$) to the gaseous phase viscosity ($\mu_G$). For all the cases, the density ratio and viscosity ratio of the liquid to the gaseous fluid were kept fixed at 500 and 25, respectively, except in the cases of a varying Reynolds number. For the case of a varying Reynolds number, the viscosity ratio varied as $2.5 \times 10^3 \leq \mu_r \leq 12.5$ for $10^3 \leq Re \leq 2 \times 10^3$, respectively. As the change of Froude number signifies the change of droplet velocity and for varying Reynolds number cases, the Froude number was kept constant and the viscosity ratio was varied to implement the variation of the Reynolds number. Figure 1 illustrates the schematic of the problem described above, and the definitions of instantaneous width ($D_t$) and height ($h_t$) are shown in Figure 2. The results are presented in terms of the interface contour, spreading factor and dimension ratio. Details of the
A two-dimensional (2D), two-phase flow case was solved to investigate the deformation of impacting droplets. The details of the geometry for the configurations considered are shown in Figure 1. The two-phase flow is governed by the partial differential equations, given by (i) the incompressible, variable-density, Navier-Stokes equations with surface tension, and (ii) an advection equation for volume fraction (interface tracking). These governing equations are defined as follows:

\[ \nabla \cdot \bar{U} = 0; \]
\[ \rho \frac{\partial \bar{U}}{\partial t} + \rho \bar{U} \cdot \nabla \bar{U} = -\nabla P + \mu \nabla^2 \bar{U} + \rho g + \sigma \kappa \delta \bar{n}; \]
\[ \frac{\partial c}{\partial t} + (\bar{U} \cdot \nabla) c = 0. \]  

The Dirac distribution function \((\delta_i)\) in Equation (2) expresses the fact that the surface tension term is concentrated on the interface, where \(\sigma\) is the surface tension coefficient and \(\kappa\) and \(\bar{n}\) are the curvature and normal to the interface, respectively. For two-phase flows, the volume fraction \((c(x, t))\) of the fluid is used to define and separate the properties of two different fluids. The density and viscosity are evaluated by the volume-weighted formulae, as shown in Equations (4) and (5):

\[ \rho(\bar{c}) \equiv \bar{c} \rho_1 + (1 - \bar{c}) \rho_2, \]
\[ \mu(\bar{c}) \equiv \bar{c} \mu_1 + (1 - \bar{c}) \mu_2, \]

where \(\rho_1\) and \(\rho_2\) are the densities and \(\mu_1\) and \(\mu_2\) are the viscosities of the first and second fluids, respectively. Here, the field \((\bar{c})\) is either identical to \(c\) or is constructed by applying a smoothing spatial filter to \(c\) (Afkhami, 2007). In a scalar form, Equations (1–3) become Equations (6–10):

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0; \]
\[ \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \rho g_x + \sigma \kappa \delta \bar{n}_x; \]
\[ \rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial P}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho g_y + \sigma \kappa \delta \bar{n}_y; \]
\[ \frac{\partial c}{\partial t} + u \left( \frac{\partial c}{\partial x} + \frac{\partial c}{\partial y} \right) = 0; \]
\[ \frac{\partial c}{\partial t} + v \left( \frac{\partial c}{\partial x} + \frac{\partial c}{\partial y} \right) = 0. \]  

### Table 1. Dimensionless parameters for different cases.

| Case | \(Fr\) | \(Re\) | \(We\) |
|------|--------|--------|--------|
| 1    | 1.0, 2.5, 5.0, 7.5, 10, 15, 20 | 1000 | 20 |
| 2    | 10, 25, 50, 75, 100, 250, 500, 750, 1000, 1250, 1500, 1750, 2000 | 10.0 | 20.0 |
| 3    | 0.10, 0.25, 0.50, 0.75, 1.0, 2.5, 5.0, 7.5, 10.0, 15.0, 20.0, 25.0, 30.0 | 10.0 | 1000 |
where $L_R$, $U_R$ and $T_R$ are the reference length, velocities and time, respectively. $L_R$ corresponds to the diameter of the droplet, $U_R$ corresponds to the impact velocity of the droplet, and $u^*$ and $v^*$ are the dimensionless velocities in the $x^*$ and $y^*$ directions. Similarly, $g^x_*$ and $g^y_*$ are the components of gravitational acceleration in the $x^*$ and $y^*$ directions and $\rho^*$, $\mu^*$, $\sigma^*$, $\kappa^*$, $n^*$ and $P^*$ are the dimensionless density, viscosity, surface tension, surface curvature, normal vector and pressure, respectively, while $t^*$ is the dimensionless time. The following dimensionless parameters are defined using the above normalizing parameters:

$$Fr = \frac{u^*}{\sqrt{g^x_* L_R}} Re = \frac{\rho^* u^* L_R}{\mu^*}$$  and  $$We = \frac{\rho^* u^* L_R^2}{\sigma^*}.$$

Normalizing the dimensional equations, non-dimensional governing Equations (11–15) can be found:

$$\frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = - \frac{\partial P^*}{\partial x^*} + \left( \frac{1}{Re} \right) \left( \frac{\partial^2 u^*}{\partial x^*^2} + \frac{\partial^2 u^*}{\partial y^*^2} \right) + \left( \frac{1}{Fr} \right) g^x_* + \left( \frac{1}{We} \right) \kappa^* \delta \rho^*_n;$$  (12)

$$\frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = - \frac{\partial P^*}{\partial y^*} + \left( \frac{1}{Re} \right) \left( \frac{\partial^2 v^*}{\partial x^*^2} + \frac{\partial^2 v^*}{\partial y^*^2} \right) + \left( \frac{1}{Fr} \right) g^y_* + \left( \frac{1}{We} \right) \kappa^* \delta \rho^*_n;$$  (13)

$$\frac{\partial c}{\partial t^*} + u^* \left( \frac{\partial c}{\partial x^*} + \frac{\partial c}{\partial y^*} \right) = 0;$$  (14)

$$\frac{\partial c}{\partial t^*} + v^* \left( \frac{\partial c}{\partial x^*} + \frac{\partial c}{\partial y^*} \right) = 0.$$  (15)

### 3.3. Initial condition

The computational domain for droplet deformation is initialized with zero velocity for air. The fluid volume is initialized with the velocity corresponding to the Froude number. The other material properties for air are defined by its properties. For solving the two-phase problem, the density ratio is kept as 500. The liquid phase viscosity is initialized with the corresponding viscosity of the Reynolds number. The gaseous viscosity is set to air viscosity. For each case, the droplet impact velocity is initialized from the Froude number. The velocity associated with the Froude number is used to calculate and initialize the liquid phase viscosity from the Reynolds number. Similarly, the surface tension coefficient is initialized from the Weber number. The velocity used for the calculation of the surface tension coefficient is the droplet impact velocity associated with the velocity from the Froude number for the case.

### 3.4. Boundary conditions

The computational domain considered for the problem is a square domain. The top and side walls are considered no slip boundaries and the bottom wall is considered to have a Navier-slip condition, i.e., the computational domain is considered as an impermeable box with a slip condition at its base. The Navier-slip condition was used to model the moving contact of the spreading droplet on the bottom wall. This is because the moving contact lines can be more accurately approximated with the implementation of a slip condition. The values for coefficients at the Navier-slip condition, $u_0$ and $\lambda$, are 0.0 and 0.05, respectively. The boundary condition described below is shown in Figure 1. Table 2 lists the boundary condition implemented for the simulation.

### 4. Numerical modeling

#### 4.1. Computational procedure

An open-source finite-volume code G Gerris Flow Solver (Popinet, 2003) was used to model and solve this two-phase fluid flow problem. As described above, the computational domain was taken as a square domain. The domain is spatially discretized with square finite volumes or cells organized hierarchically as a quadtree (Samet, 1990). The quadtree (first introduced by Finkel & Bentley, 1974) is a tree data structure that is used to partition a 2D space, dividing in half recursively in each direction. Thus, a parent cell is subdivided into four child cells. Each division is labeled as a level of refinement. For the undivided domain, the level of refinement is set to zero. With the increase to each level of refinement, each cell undergoes a division in half for each direction. For the level of refinement of $x$, the number of cells in each direction per unit length is $2^x$ and the grid spacing $\Delta x$ is $1/2^x$ (see Figure 3). With each increase in the level of refinement, each parent is subdivided into four child cells. This type of organizational structure was originally used for computer graphics and image processing – in particular, to find a certain pixel in a 2D image (Gonzalez & Wintz, 1987). Later, it was introduced for solving Euler equations for flows (Coirier, 1994; Khokhlov, 1998). Figure 4 shows the grid distribution for

| Table 2. Boundary conditions. |
|-------------------------------|
| (i) Top wall | $u^* = 0$ | $v^* = 0$ | $P^* = 0$ |
| (ii) Left wall | $u^* = 0$ | $v^* = 0$ | $P^* = 0$ |
| (iii) Right wall | $u^* = 0$ | $v^* = 0$ | $P^* = 0$ |
| (iv) Bottom wall | $u^* = u_0^* + \lambda (\delta u^*/\delta y^*)$ | $v^* = 0$ | $(\delta P^*/\delta n^*) = 0$ |
the computational domain. The initial discretization is $2^6$ and it goes up to $2^8$ near the interface with adaptive refinement. For the solution of the advection equation for volume fraction, a piecewise-linear geometrical volume-of-fluid (VOF) scheme, generalized for the quadtree spatial discretization, was used. Details of the numerical scheme are described by Scardovelli and Zaleski (1999) and Popinet (2003). The numerical formulations based on finite volumes are organized hierarchically as a quadtree and are described in Popinet (2009). The Gerris Flow Solver uses a dynamic adaptive mesh refinement (ARM) with the quadtree (2D) and octree (3D) discretization. This facilitates the solver to adaptively follow the small structures and features of the flow, which leads to an increased concentration of computational effort in the area where it is most needed. This dynamic adaptive mesh refinement makes the solver more efficient and less costly in terms of computational resources. Since the Gerris Flow Solver is implemented as a tree-based discretization for space, it is somewhat simpler, in a relative sense, to incorporate a fully flexible adaptive refinement strategy. Equation (16) describes an example of adaptive mesh refinement criteria based on the vorticity of the flow:

$$\frac{\Delta x \nabla \times \vec{U}}{\max \vec{U}} > \Gamma.$$  \hspace{1cm} (16)

In Equation (16), $\max \vec{U}$ is evaluated over the entire domain. $\Gamma$ is the threshold for the refinement criteria and it triggers the refinement process on and off when the criteria are fulfilled. For this particular example, it is interpreted as the maximum acceptable angular deviation. The computational cost associated with the implementation of this algorithm is relatively small compared to the cost of the solution of the Poisson solver. This dynamic adaptive mesh refinement algorithm requires less than 5% of the total cost (Popinet, 2003). This low computational resource requirement means that it can be applied at every time step, as the increase in overall computational cost is negligible.

For the first-stage refinement of cells, those cells which satisfy the refinement criterion undergo the refinement process. The neighboring cells are also refined—however, this depends on whether or not a refinement is needed. The refinement of neighboring cells may be required to satisfy imposed constraints due to applying the quadtree structure (Finkel & Bentley, 1974; Popinet, 2003). With the assumption of the flow being evolved slowly, the refinement is executed once per time step. At the second-stage refinement of cells, all the cells which do not satisfy the refinement criterion are coarsened. This process of refining and coarsening can be understood from Figure 4. The scheme that the Gerris Flow Solver uses follows a second-order
accurate temporal discretization (Popinet, 2009):

\[
\rho_{n+1/2} \left[ \frac{\bar{U}_{n+1} - \bar{U}_n}{\Delta t} + \bar{U}_{n+1/2} \cdot \bar{U}_{n+1/2} \right]
= -\nabla P_{n+1/2} + \mu_{n+1/2} \nabla^2 \cdot \left[ \bar{U}_n + \bar{U}_{n+1} \right] 
+ (\sigma \kappa \delta n)_{n+1/2}; 
\]

\[
\left[ \frac{c_{n+1/2} - c_{n-1/2}}{\Delta t} + \nabla \cdot (c_n \bar{U}_n) \right] = 0; 
\]

\[
\nabla \cdot \bar{U}_n = 0.
\]

This system is further simplified using a classical time-splitting projection method (Chorin, 1969):

\[
\rho_{n+1/2} \left[ \frac{\bar{U}_{n+1} - \bar{U}_n}{\Delta t} + \bar{U}_{n+1/2} \cdot \bar{U}_{n+1/2} \right]
= -\nabla P_{n+1/2} + \mu_{n+1/2} \nabla^2 \cdot \left[ \bar{U}_n + \bar{U}_{n+1} \right] 
+ (\sigma \kappa \delta n)_{n+1/2}; 
\]

\[
\left[ \frac{\bar{c}_{n+1/2} - \bar{c}_{n-1/2}}{\Delta t} + \nabla \cdot (\bar{c}_n \bar{U}_n) \right] = 0; 
\]

\[
\bar{U}_{n+1/2} = \bar{U}_{n+1} - \frac{\Delta t}{\rho_{n+1/2}} \nabla P_{n+1/2}; 
\]

\[
\nabla \cdot \bar{U}_{n+1} = 0.
\]

For the solution of the velocity field, a Poisson equation solution is also needed:

\[
\nabla \cdot \left[ \frac{\Delta t}{\rho_{n+1/2}} \nabla P_{n+1/2} \right] \bar{U} = \nabla \cdot \bar{U}_n.
\]

For the improvement of the convergence time, the momentum equation is rearranged as follows:

\[
\frac{\rho_{n+1/2}}{\Delta t} \bar{U}_{n+1} = \nabla^2 \cdot \left[ \mu_{n+1/2} \bar{U}_n \right] 
+ (\sigma \kappa \delta n)_{n+1/2} + \rho_{n+1/2} \times \left[ \frac{\bar{U}_n}{\Delta t} - \bar{U}_{n+1/2} \cdot \nabla \bar{U}_{n+1/2} \right]
\]

One of the most difficult tasks in applying the VOF methods for surface-tension-driven flows is the accurate estimation of the surface tension term \((\sigma \kappa \delta n)_{n+1/2}\) at the discretized momentum (Equation (20)). The original continuum-surface-force approach of the work performed by Brackbill, Kothe, and Zemach (1992) proposes the following approximations, which have been considered in the scheme for calculating the surface tension term in Equation (25):

\[
\sigma \kappa \delta n \approx \sigma \kappa \nabla \cdot \bar{c} \kappa \approx \nabla \cdot \bar{n} \text{ with } \bar{n} \equiv \frac{\nabla \bar{c}}{|\nabla \bar{c}|}.
\]

Here, \( \bar{c} \) is either identical to \( c \) or a spatially-filtered value of \( c \). The estimation of the curvature of the interface is another difficult task when using VOF schemes. This inspires the use of alternatives, such as front-tracking schemes (Popinet & Zaleski, 1999; Shin, Abdel-Khalika, Darub, & Juricb, 2005), the level set method (Sussman, Smereka, & Osher, 1994) and the coupled VOF & level set method (Sussman & Puckett, 2000; Yang, James, Lowengrub, Zheng, & Cristini, 2006). Here, for the curvature estimation, the height-function (HF) curvature calculation has been utilized for the computational scheme, first proposed by Torrey, Cloutman, Mjolsness, and Hirt (1985). The estimated curvature from the HF curvature calculation resembles more practicality. The accuracy of the estimated curvature is comparable to results reported by Cummins, Francois, and Kothe (2005).

4.2. Grid independence

To test the grid independence of the presented results, a number of runs were performed for a test case at different levels of refinement to determine the optimum grid resolution. Figure 5 shows the representation of the droplet interface at each level of refinement considered. It shows that a level of refinement of 7, i.e., a mesh size \(1/2^7\) at each direction, or higher is good enough for an acceptable level of accuracy in the case of surface representation. Figure 6 illustrates the grid independence studies for the case of a single droplet impacting on a solid surface. The evolution of the width \((D)\) to height \((h)\) ratio, i.e., deformation ratio \((R_d)\) with time was plotted for different levels of refinement. For all the cases considered (see Figure 6), the initial level of refinement was \(4 (2^4)\) mesh per unit length and then for each case different adaptive levels of refinement were considered. The figure shows that an initial level of refinement 4 along with an adaptive of 7 or more provides very good results in terms grid independence. For the presented case, the initial and maximum adaptive levels of refinement are 6 and 8, respectively. This ascertains the acceptability of the chosen grid level.

4.3. Validation

The results obtained from the developed model were validated against the experimental results reported in Hatta...
5. Results and discussion

In the present study, a 2D, two-phase flow was simulated to investigate droplet impact and deformation over a solid surface. The deformation of droplets was studied in terms of the interface contour, spreading factor ($\xi$) and deformation ratio ($R_\delta$).

5.1. The effect of the Froude number

The effect of the Froude number for droplet deformation is presented in Figures 8 and 9. Figure 8 shows the interface contour for the droplet deformation for Fr values of 1.0, 5.0, 10.0 and 20.0. The interface contour for different values of Fr is presented at $t^* = 0.0443, 0.1772, 0.2544$ and 0.3909. Figure 9 is the plot for the spreading factor ($\xi$) against dimensionless time for various Froude numbers. From Figure 8, for all the cases, there is an initial compression phase after the impact. After this, the droplet starts to spread in the spreading phase. For lower values of Fr (such as $1.0 \leq Fr \leq 5.0$), there is no thin spreading film observed during the spreading phase. This is due to the lower inertial force compared to the gravitational force. The lower kinetic energy is absorbed during the initial compression after the impact. The aforementioned thin film is usually visible at the spreading phase for cases of high impact velocity. For $Fr = 1.0$, the circular droplet tends to take a hemispherical shape at $t^* = 0.1772$. With further deformation, the drop transforms into a dome-like shape at around $t^* = 0.3909$. For $Fr \geq 5$, the presence of a thin spreading film is observed during the spreading phase. Within this range of Froude number, $5.0 \leq Fr \leq 20.0$, the spreading film is thicker at lower values of Fr (e.g., $Fr = 5.0$) compared to higher values of Fr (e.g., $Fr = 20.0$). With increasing value of Fr, the spreading film gets thinner and the span of the spreading is higher at the same dimensionless time ($t^*$). This trend is obvious from Figure 9(a). During the spreading phase at a relatively higher value of Fr (such as $10.0 \leq Fr \leq 20.0$), the instantaneous height ($h_t$) seems to have magnitudes of the same order. However, the instantaneous width ($D_t$) has a greater magnitude at any dimensionless time with an increasing Froude number. Figure 10 illustrates the deformation ratio against the dimensionless time for various Froude numbers. It shows that the ratio of $D_t$ to $h_t$ is higher for higher values of Fr at any dimensionless time. The reason is, as the value of Fr increases, $D_t$ is higher for any certain dimensionless time and $h_t$ is of a similar order. This is why $R_\delta$ increases as the magnitude of Fr increases. Figure 9 shows that there is an initial period of steady maximum width for all values of Fr. This period is very significant for direct-print processes where the dispensed droplet needs to be cured by an energy source after dispensing. The curing needs to be started and finished within this initial steady period to maintain dimensional accuracy. The steady period reduces as the Froude number increases. For the range of $1.0 \leq Fr$
Figure 8. Droplet deformation for various values of Froude number ($Fr$) at $Re = 1000$, $We = 20.0$.  

Figure 9. Evolution of droplet deformation for various values of $Fr$ at $Re = 1000$, $We = 20.0$, for (a) spreading factor ($\xi$) and (b) deformation ratio ($R_\delta$).
Figure 10. Droplet deformation for various values of Reynolds number ($Re$) at $Fr = 10.0$, $We = 20.0$.

Figure 11. Evolution of droplet deformation for various values of $Re$ at $Fr = 10.0$, $We = 20.0$, for (a) spreading factor ($\xi$) and (b) deformation ratio ($R_{\delta}$).
≤ 20.0, this steady period varies in the dimensionless time ranging 0.8 ≤ t* ≤ 0.5.

5.2. The effect of the Reynolds number
The effect of the Reynolds number for droplet deformation is presented in Figures 10 and 11. Figure 10 shows the interface contour for the droplet deformation for Re values of 10, 100, 1000 and 2000. The interface contour for different values of Re is presented at t* = 0.0443, 0.1772, 0.3101 and 0.4429. For this case of varying Reynolds numbers, Fr and We are kept at 10.0 and 20.0, respectively, while keeping $\rho_r$ at 500. However, $\mu_r$ varies with the values of Re because the Froude number (= $u^*/\sqrt{g^*L_P}$) is kept fixed for this case. For the Froude number being kept as constant, the velocity is also constant. Therefore, for varying Re, the viscosity ratio is varied. This implies that the viscosity of the liquid phase is varied with fixed velocity ($u^*$) for varying values of Re. For the range of $10 \leq Re \leq 2 \times 10^3$, $\mu_R$ is in the range of $2.5 \times 10^3 \leq \mu_r \leq 12.5 \times 10^3$. For lower values of Re (≈ 10), the deformation trend is similar to that of lower Fr values (e.g., Fr = 1.0). The reason for not having any spreading phase is the dominance of viscosity. The higher viscosity prevents the droplets from forming a thin spreading film. For $10^2 \leq Re \leq 2 \times 10^3$, there is a visible spreading film in Figure 10. For $Re \leq 10^3$ at the spreading phase, there is a formation of rim at $t^*$ being as low as 0.1772. Figure 11(a) shows that the initial stage of steady dimension for the Re range considered is within $0.05 \leq t^* \leq 0.10$. With the increasing value of Re, the steady phase diminishes more rapidly. Figure 11(b) shows the deformation ratio for the range of Re considered. Figures 11(a) and 11(b) show that both the spreading factor ($\xi$) and the deformation ratio ($R_\delta$) increase more rapidly for $10 \leq Re \leq 5 \times 10^2$ compared to $10^3 \leq Re \leq 2 \times 10^3$.

5.3. The effect of the Weber number
The effect of the Weber number for droplet deformation is presented in Figures 12 and 13. The interface contour of

![Figure 12](image-url)
the droplet deformation for \( We \) values of 0.1, 5.0, 15.0 and 30.0 is presented in Figure 12. The interface contour for different values of \( We \) is presented at \( t^* = 0.0044, 0.3101, 0.6201 \) and 0.8859. For this case of varying Weber numbers, \( Fr \) and \( Re \) are kept at 10 and 1000, respectively. For these cases, at all values of \( We \) the characteristic spreading film can be seen. For a lower value of \( We (= 0.1) \), the spreading film observed is relatively thicker and the deformation rate is lower. This is because the lower value of Weber number corresponds to higher surface tension, which hinders the deformation rate and causes it to become slower. This higher surface tension force also maintains a lower surface area and this causes a thicker spreading film at a lower Weber number. For \( We \geq 5 \), there is a slight rim formation at the end of the spreading front. This happens due to interaction between higher kinetic energy and higher surface tension force. At the range of \( We \), the surface tension force is strong and tends to retract the spreading. The fluid near the free surface tends to retract to maintain a lower surface area and creates the rim at the spreading front. As the value of \( We \) increases, the spreading profile of the surface contour seems to change from a more spherical to a more flat shape at any \( t^* \). This is obvious, as a higher value of \( We \) corresponds to lower surface tension. Figure 13(a) shows that the initial steady period is the same order for all values of \( We \). The range for this steady dimension is within 0.05 \( \leq t^* \leq 0.06 \). Figure 13(b) shows that the deformation ratio increases almost linearly after the initial steady phase. Both Figures 13(a) and 13(b) show that the spreading factor (\( \xi \)) and the deformation ratio (\( R_\delta \)) vary only slightly for the wide range of the value considered for the Weber number.

6. Conclusion

In this present study, a 2D, two-phase fluid flow problem of droplet impact and deformation was studied for varying Froude numbers, Reynolds numbers and Weber numbers, keeping the density ratio and viscosity ratio constant, except for the case of the varying Reynolds number. For the case of the varying Reynolds number, the viscosity ratio was varied, i.e., the liquid phase density was varied to implement the variation of Reynolds numbers for a constant droplet velocity. The results obtained from computation are as follows:

- An increase in Froude number results in a reduction in the initial steady spreading factor period.
- The duration of the initial steady spreading factor period ranges between dimensionless time 0.08 \( \geq t^* \geq 0.05 \) for an increasing trend of Froude number.
- An increase in Reynolds number also results in a reduction in the initial steady spreading factor period.
- For the case of a varying Reynolds number, the duration of the initial steady spreading factor period ranges between dimensionless time 0.1 \( \geq t^* \geq 0.05 \) with an increase in Reynolds number.
- For the case of a varying Reynolds number, at higher values of Reynolds number such as \( Re \leq 10^3 \), rim formation occurs at the ends of the spreading film, while at Reynolds numbers of \( 2 \times 10^3 \), formation of satellite droplets, i.e., splashing, occurs.
- For the case of a varying Weber number, the initial steady spreading factor period does not vary with the variation of the considered range of 0.1 \( \leq We \leq 30 \).
- For all values of Weber number, i.e., \( 0.1 \leq We \leq 30 \), the initial steady spreading factor period ranges between dimensionless time 0.06 \( \geq t^* \geq 0.05 \).
- With a constant Froude number, Reynolds number, density ratio and viscosity ratio, an increase in Weber number yields in a very small degree of
variation in the spreading factor for any dimensionless time.

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Nomenclature
- $c$ volume fraction field
- $c^*$ filtered volume fraction field
- $D$ droplet diameter
- $F$ force
- $Fr$ Froude number
- $g$ gravitational acceleration
- $h$ droplet height
- $HF$ height function
- $L$ length
- $n$ normal vector
- $P$ pressure
- $Re$ Reynolds number
- $Rs$ deformation ratio
- $t$ time
- $U$ velocity vector
- $u, v$ velocity in $x$ and $y$ directions
- $x, y$ Cartesian coordinates
- $VOF$ volume of fluid
- $W$ width
- $We$ Weber number

Superscript
- * dimensionless parameter

Greek Symbols
- $\delta$ deformation
- $\Delta$ difference
- $\kappa$ surface curvature
- $\lambda$ Navier-slip coefficient
- $\mu$ viscosity
- $\bar{\kappa}$ spreading slip coefficient
- $\rho$ density
- $\sigma$ surface tension

Superscript
- $g$ gravitational
- $G$ gaseous
- $L$ liquid
- $\sigma$ initial
- $U$ velocity vector
- $r$ ratio
- $R$ reference
- $s$ surface
- $st$ surface tension
- $t$ instantaneous
- ** projected

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