\( (h_{11/2})^2 \) alignments in neutron-rich \(^{132}\)Ba with negative-parity pairs

Y. Lei (雷杨)\(^1\) and Z. Y. Xu (徐正宇)\(^2\)

\(^1\)Key Laboratory of Neutron Physics, Institute of Nuclear Physics and Chemistry, China Academy of Engineering Physics, Mianyang 621900, China

\(^2\)INPAC, Physics Department, Shanghai Jiao Tong University, Shanghai 200240, China

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The shell-model collective-pair truncation with negative-parity pairs is adopted to study the \((h_{11/2})^2\) alignment in \(^{132}\)Ba. The proton \((h_{11/2})^2\)-alignment state is predicted as an \(E \approx 4.6\) MeV and \(\tau \sim 0.5\) \(\mu\)s isomer with relatively strong E3 decay channels. The oblately deformed neutron \((h_{11/2})^2\) alignment in the yrast band and four negative-parity bands are confirmed, even although two of these negative-parity bands favor the prolate deformation, which directly manifests the \(\gamma\) un-stability of \(^{132}\)Ba.

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\(^{132}\)Ba is a typical \(\gamma\) unstable nucleus in the transitional region of \(Z \geq 50\) and \(N \leq 82\) with a large variety of co-existing nuclear shapes. Its negative-parity bands with \(I^+ = 5^-\) and \(6^-\) bandheads (bandhead energies: 2.12 and 2.358 MeV, respectively) maintain an oblate shape, while prolately deformed negative-parity bands with \(I^+ = 7^-\) and \(8^-\) bandheads (bandhead energies: 2.902 and 3.105 MeV, respectively) were also reported in the \(^{132}\)Ba level scheme \([1, 2]\). There four negative-parity bands here are denoted by “\(5^-\), \(6^-\), \(7^-\) and \(8^-\) bands” in sequence. In the yrast band, the \(\tau = 8.94(1)\) ns \(10^+\) isomer is assigned as the neutron \((h_{11/2})^2\)-alignment with an oblate shape, which has two \(h_{1/2}\) neutron holes rotate alignedly. On the other hand, the proton \((h_{11/2})^2\)-alignment with the prolate deformation is also expected in \(^{132}\)Ba (unfortunately unobserved yet), because the competition between proton and neutron \((h_{11/2})^2\) alignments are generally exhibited in this nuclear region \([3, 12]\).

There may exist strong electromagnetic transitions from the \((\pi h_{11/2})^2\)-alignment state to states in \(7^-\) and \(8^-\) bands, because initial and final states share a similar prolate shape. It’s desirable to theoretically estimate these transition rates before an experimental search for the \((\pi h_{11/2})^2\)-alignment in \(^{132}\)Ba.

The \((\nu h_{11/2})^2\)-alignment was also suggested in \(5^-\), \(6^-\), \(7^-\) and \(8^-\) bands according to their band irregularity around \(5^-\)~\(6\) MeV \([1, 2]\). However, the experimental evidence is not as solid as the \((\nu h_{11/2})^2\)-alignment in the yrast \(10^+\) isomer \([13, 14]\). It’s essential to confirm the \((\nu h_{11/2})^2\)-alignment in these negative-parity bands from a shell-model perspective.

The present work aims at studying the \((h_{11/2})^2\) alignment in both positive and negative-parity states of \(^{132}\)Ba within the shell-model framework. The model-space truncation is required to reduce the gigantic dimension of the shell-model description for \(^{132}\)Ba. It’s noteworthy that the collective-pair truncation of the Shell Model has been proved to be an efficient approach for the \((h_{11/2})^2\)-alignment description \([15, 17]\). In such truncation, collective pairs with spin 0\(h\) and 2\(h\) are normally employed to represent the low-lying collectivity \([18–24]\), which naturally provide \(\Delta I = 2\) band structures, as \(5^-\), \(6^-\), \(7^-\) and \(8^-\) bands of \(^{132}\)Ba behave. Recently, negative-parity pairs were introduced into the pair truncation \([22, 27]\), which enables a shell-model description of negative-parity states in a heavy even-nucleon system, e.g., negative-parity states of \(^{132}\)Ba discussed here. Thus, the collective-pair truncation with negative-parity pairs is adopted for our negative-parity-state related study on the \((h_{11/2})^2\)-alignment of \(^{132}\)Ba.

Our calculation adopts a phenomenological shell-model Hamiltonian as in Ref. \([16]\):

\[
H = - \sum_{\sigma = \pi, \nu} \sum_{\nu J} \sum_{s = 0, 2} G_{\sigma}^* P_{\sigma J}^\dagger \cdot P_{\sigma J}^s \nonumber + \kappa_{\sigma} Q_{\sigma} \cdot Q_{\sigma} + \kappa_{\pi \nu} Q_{\pi} \cdot Q_{\nu},
\]

with

\[
P_{0J}^\dagger = \sum_a \frac{\gamma_{0J} q_{ab}}{2} (C_{ab}^J \cdot C_{ab}^J)^J, \quad P_{2J}^\dagger = \sum_{ab} q(ab) (C_{ab}^J \cdot C_{ab}^J)^2, \quad Q = \sum_{ab} q(ab) (C_{ab}^J \cdot C_{ab}^J)^2.
\]

In Eq. \([2]\), \(q(ab) = (q||r^2 Y^2||b)/r_0^2\), where \(r_0\) is the oscillator parameter, \(\sqrt{\hbar/(m\omega)}\). Hamiltonian parameters, i.e., \(G_{\sigma}^\dagger, \kappa_{\sigma}\), and \(\kappa_{\pi \nu}\) in Eq. \([1]\), are also taken from Ref. \([16]\) as listed in Table \(I\).

Our pair-truncated shell-model space for \(^{132}\)Ba is given by the coupling of three proton pairs and three (hole-like) neutron pairs in the 50–82 shell. These pairs can be

TABLE I. Hamiltonian parameters from Ref. \([16]\) in MeV.

| \(s_{1/2} \) | \( d_{3/2} \) | \( d_{5/2} \) | \( g_{7/2} \) | \( h_{11/2} \) |
|---------|---------|---------|---------|---------|
| \( \varepsilon_\pi \) | 2.990 | 2.708 | 0.962 | 0.000 | 2.793 |
| \( \varepsilon_\nu \) | 0.332 | 0.000 | 1.655 | 2.434 | 0.242 |
| \( \kappa_\pi \) | 0.130 | 0.130 | 0.266 | 0.045 | 0.006 |
| \( \kappa_{\pi \nu} \) | 0.329 | 0.000 | 1.655 | 2.434 | 0.242 |

* leiyang19850228@gmail.com
In previous pair-truncation calculations [15–17], the total angular momentum $I$ describes the $(\pi h \times \nu h)$ along with two $h - s$ are mainly constructed by two optimization structure coefficients, $\beta_{ab}$, to best describe the nuclear low-lying collectivity. Especially, $\beta_{ab}^\pi = 0^+$ corresponds to the Cooper-pair structure due to the strong nuclear pairing collectivity, and is determined following the principle of projected-particle-number BCS theory [28]. Other $\beta_{ab}^\pi$ with $I^\pi \neq 0^+$ is obtained by the diagonalization in the $\nu = 2$ broken-pair model space [29]. More details about the determination of $\beta_{ab}$ are described in Ref. [30].

In our calculation, four types of pairs are adopted:

1. Following previous pair-truncation calculations [18, 24], collective pairs with $I^\pi = 0^+$ and $2^+$ are introduced.

2. Bandheads of $5^-$ and $6^-$ bands may be given by coupling the $h_{11/2}$ neutron with the lowest positive-parity one-quasineutron configuration [1, 2]. Such coupling corresponds to the lowest $\nu = 2$ broken-pair state with $I^\pi = 5^-$ and $6^-$, i.e., $I^\pi = 5^-$ and $6^-$ neutron collective pairs from our $\beta_{ab}$ calculation with $I^\pi = 5^-$ and $6^-$. Thus, these two pairs are introduced to develop $5^-$ and $6^-$ bands, and denoted by $5^-$ and $6^-$ pairs, respectively.

3. $7^-$ and $8^-$ bands were supposed to be built on the coupling of rotationally aligned $h_{11/2}$ and $g_{7/2}$ protons [1, 2], which is represented by the non-collective $(\pi h_{11/2} \times \pi g_{7/2})^{I^\pi = 9^-}$ pair in this work.

4. We take the $(\pi h_{11/2} \times \pi h_{11/2})^{I^\pi = 10^+}$ pair to describe the $(\pi h_{11/2})^2$ alignment.

In previous pair-truncation calculations [15–17], the $(\nu h_{11/2})^{-2}$ alignment is represented by the $(\nu h_{11/2} \times \nu h_{11/2})^{I^\pi = 10^+}$ pair. However, we don’t introduce such pair, because the coupling of two $5^-$ and/or $6^-$ pairs with total angular momentum $I \geq 10h$ total angular momentum (denoted by the “$5^- \otimes 6^-$ coupling”) can also describe the $(\nu h_{11/2})^{-2}$ alignment. To demonstrate this point, we list structure coefficients of $5^-$ and $6^-$ pairs in Table III which suggests that the $5^- \otimes 6^-$ coupling are mainly constructed by two $s_{1/2}$ and/or $d_{3/2}$ neutrons along with two $h_{11/2}$ neutrons. These two $s_{1/2}$ and/or $d_{3/2}$ neutrons only provide little angular-momentum contribution, so that the $I \geq 10h$ total angular momentum of the $5^- \otimes 6^-$ coupling almost comes from the rest two $h_{11/2}$ neutrons. In other words, these two $h_{11/2}$ neutrons have to contribute $I \sim 10h$ angular momentum, which forces them to rotate alignedly. Thus, the $(\nu h_{11/2})^{-2}$ alignment emerges within the $5^- \otimes 6^-$ coupling.

### Table II. Structure coefficients, i.e., $\beta_{ab}^\pi$ defined in Eq. (3), of $5^-$ and $6^-$ pairs.

| $ab$ | $E^\pi = 5$ | $E^\pi = 6$ |
|------|-------------|-------------|
| $h_{11/2} \times s_{1/2}$ | +0.551 | +0.616 |
| $h_{11/2} \times d_{3/2}$ | -0.832 | +0.780 |
| $h_{11/2} \times g_{7/2}$ | +0.038 | +0.097 |
| $h_{11/2} \times g_{7/2}$ | -0.050 | +0.053 |

The electromagnetic transition operators adopted our calculation are

$$T(E2) = \sum_{\sigma = \pi, \nu} e_{\sigma} r_\sigma^2 \gamma_\sigma^{(2)}$$

$$T(E2) = \sum_{\sigma = \pi, \nu} e_{\sigma} r_\sigma^2 \gamma_\sigma^{(3)}$$

$$T(M1) = \frac{3}{4\sigma} \sum_{\sigma = \pi, \nu} g_\sigma l_\sigma + g_\sigma s_\sigma$$

Here, $e_\sigma$, $g_\sigma$, and $g_\sigma$ (in units of $e$ and $\mu_N/\hbar$) are the effective charge, orbital and spin gyromagnetic ratios of valence nucleons, respectively. They are taken from Ref. [16] as $e_\pi = 2$, $e_\nu = -1$, $g_\pi = 5.58 \times 0.7$, $g_\nu = -3.82 \times 0.7$, $g_{\pi3} = 1.05$, and $g_{\nu4} = 0.05$. Fig. (a) presents the calculated $^{132}$Ba spectrum compared with the experimental one, including the yrast band and $5^-$, $6^-$, $7^-$, $8^-$ bands. A rough spectral consistency between our calculation and the experiment is achieved. However, calculated $7^-$ and $8^-$ bands based on the $(\pi h_{11/2} \times \pi g_{9/2})^{I^\pi = 9^-}$ pair are still observed to be systematically higher than those from experiments by $\sim 0.3$ MeV. This is because the single-particle energy of the $^{132}$Ba $h_{11/2}$ proton, i.e., the [550] $2^+$ level, is depressed by $\sim 0.3$ MeV relatively to the Fermi surface [32], due to the $\beta_2 \sim 0.12$ prolate deformation of $7^-$ and $8^-$ bands [33]. This effect shall also have direct impact on the prolately deformed $(\pi h_{11/2})^2$ alignment, which the shell-model framework can not intrinsically consider yet. Therefore, our calculated excitation energy of the $(\pi h_{11/2})^2$ alignment should be reduced by $\sim 0.6$ MeV as.
an amendment. By searching the \((\pi h_{11/2} \times \pi h_{11/2})^{I^=10^+}\) pair in output wave-functions, the lowest state with the \((\pi h_{11/2})^2\) alignment is located at \(E \sim 5.2\) MeV in our calculated spectrum, as demonstrated by the red \(I^\pi = 10^+\) level in Fig. 1(b). Thus, the \(^{132}\text{Ba}\) \((\pi h_{11/2})^2\) alignment energy should be around 4.6 MeV, which agrees with the systematics of the alignment energy in light Ba isotopes \(^{2}11\text{[11, 14]}.\)

To probe electromagnetic properties of the \(\sim 4.6\) MeV \((\pi h_{11/2})^2\)-alignment state, we calculate its E2, M1, E3 decay to observed levels. Resultant E2 and M1 decay rates to yrast \(8^+\), \(10^+\) and \(12^+\) states are all smaller than \(10^{-13}\) W.u. Such weak decays can be attributed to the parity conservation, which forbids E2 and M1 transition operators from scattering the negative-parity \(h_{11/2}\) nucleon to other positive-parity orbits of 50-82 major shell. In other words, any state related to the \((\pi h_{11/2})^2\) alignment by strong E2 or M1 transitions must be constructed with two \(h_{11/2}\) valence protons. However, yrast states, as well as most of low-lying positive-parity states in \(^{132}\text{Ba}\), have few valence \(h_{11/2}\) protons due to the large \(\pi h_{11/2}\) single-particle energy. Thus, strong E2 and M1 decays from the \((\pi h_{11/2})^2\) alignment to these low-lying positive levels are absent, which may partially explain the inaccessibility of the \((\pi h_{11/2})^2\) alignment in observed level scheme of \(^{132}\text{Ba}\). On the other hand, we obtain relatively strong E3 decays from the \((\pi h_{11/2})^2\) alignment to states of \(7^-\) and \(8^-\) bands as shown in Table III. Such strong E3 decays are expected, because this alignment shares the prolate deformation with \(7^-\) and \(8^-\) bands. Based on above calculated decay rates, the \((\pi h_{11/2})^2\)-alignment level is suggested to be a 0.51W.u. isomer with the main decay branch of \(E_\gamma \sim 1.3\) MeV to the \(I^\pi = 9^-\) level in the \(7^-\) band.

Now, let’s turn to the \((\nu h_{11/2})^-2\) alignment. The yrast \(I^\pi = 10^+\) isomer is the most typical \((h_{11/2})^-2\)-alignment observation in \(^{132}\text{Ba}\). We would like to revisit the yrast alignment before discussing \((\nu h_{11/2})^-2\) alignments in negative-parity bands. According to Fig. 1, our calculation spectroscopically reproduces the excitation energy of the yrast alignment. To present this alignment more explicitly, we additionally calculate and experimental associated energies \([E_I - E_{I-2}]\), reduced E2 transition rates \([B(E2, I \rightarrow I - 2)]\) and magnetic moments \([\mu]\) of the yrast band in Fig. 2. The backbending of \(E_I - E_{I-2}\), the sharp dropping down of \(B(E2, I \rightarrow I - 2)\) and the positive-to-negative reverse of \(\mu\) values at the \(I^\pi = 10^+\) isomer both experimentally and theoretically evident the intruding of the \((\nu h_{11/2})^-2\) alignment in the yrast band. To observe this alignment at the wave-function level, we also calculate the expectation value of \(5^-\) and \(6^-\) pair numbers, and plot the sum of \(5^-\) and \(6^-\) pair numbers in Fig. 2(d). Correspondingly to the \((\nu h_{11/2})^-2\) alignment at \(I = 10h\), two \(5^-\) and/or \(6^-\) pairs are suddenly excited as a representation of the \(5^- \otimes 6^-\) coupling. In other words, the \((\nu h_{11/2})^-2\) alignment indeed is induced by the \(5^- \otimes 6^-\) coupling. Therefore, we take the sudden increasing of \(5^-\) and \(6^-\) pair numbers by two as a signal of the \((\nu h_{11/2})^-2\) alignment.

The \((\nu h_{11/2})^-2\) alignment in \(5^-\) and \(6^-\) bands was proposed according to the band irregularity around \(I = 14h\) in References [1, 2]. In Fig. 3(a), level spacings

| \(I^\pi\) | \(B(E3)\) (W.u.) | \(J^\pi\) | \(B(E3)\) (W.u.) |
|-------|-----------------|-------|-----------------|
| \(7^-\) | 0.109           | \(9^-\) | 0.001           |

TABLE III. E3 transition rates (in W.u.) from the \((\pi h_{11/2})^2\) alignment to states in \(7^-\) and \(8^-\) bands. The superscript "*" labels the main decay branch.

FIG. 2. \(E_I - E_{I-2}\) (a), \(B(E2, I \rightarrow I - 2)\) (b) and \(\mu\) values (c), as well as the sum of \(5^-\) and \(6^-\) pair numbers (d), of yrast states in \(^{132}\text{Ba}\) from experiments (Exp) [31] and our calculation (Cal). The yrast \((\nu h_{11/2})^-2\) alignment is highlighted.

FIG. 3. (Color online) \(E_I - E_{I-2}\) (a) and the sum of \(5^-\) and \(6^-\) pair numbers (b) in \(5^-\) and \(6^-\) bands. All the data in Plane (a) is from Fig. 1. The grey zone highlights the Stockholm-band crossing.
(E_{l+2} - E_l) also occur a sudden decrease around I = 14\hbar, and more implicitly demonstrate this band irregularity in both experimental and calculated level schemes. We present the sum of 5^− and 6^− pair numbers in Fig. 3(b) correspondingly. Below I = 14\hbar, only one 5^− or 6^− pair exists in 5^− and 6^− bands as expected by Ref. [1][2]; beyond this point, two more 5^− and/or 6^− pairs, i.e., the 5^− \otimes 6^− coupling, are excited. The (\nu h_{1/2})^−2 alignment in 5^− and 6^− bands is confirmed.

The (\nu h_{1/2})^−2 alignment is also expected in 7^− and 8^− bands around I = 16\hbar [2], even though the spectral evidence was insufficient. We plot the sum of 5^− and 6^− pair numbers in 7^− and 8^− bands in Fig. 4 where the 5^− \otimes 6^− coupling is observed beyond I = 16\hbar. Thus, we confirm the (\nu h_{1/2})^−2 alignment with oblate shape in 7^− and 8^− bands, although these two bands favor the prolate deformation.

To summarize, we adopt the pair-truncation of the Shell Model with negative-parity pairs to describe the \langle h_{1/2} \rangle^2 alignment in ^{132}\text{Ba}. Our calculation is strictly consistent with experiments. In the calculated level scheme, the (\pi h_{1/2})^2 alignment is predicted to be an E \sim 4.6\text{MeV} and \tau \sim 0.5\mu\text{s} isomer with relatively strong E3 transitions to 7^− and 8^− bands, which requires further experimental verifications. (\nu h_{1/2})^−2-\text{alignment observations in both yrast band and negative-parity bands are also well reproduced by the 5^− \otimes 6^− coupling, which is suggested to be another representation of the (\nu h_{1/2})^2-\text{alignment configuration. This oblately deformed alignment in prolate-favor 7^− and 8^− bands typically evidences the γ unstability of ^{132}\text{Ba}.}

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