Effects of pressure distribution on parallel circular porous plates with combined effect of piezo-viscous dependency and non-Newtonian couple stress fluid

B Vijayakumar¹, Sundarammal Kesavan²

Department of Mathematics, SRM University, Kattankulathur, Tamilnadu-603203, India
vijayakumar.b@ktr.srmuniv.ac.in

Abstract: Piezo-viscous effect i.e., Viscosity-pressure dependency has an important part in the applications of fluid flows like fluid lubrication, microfluidics and geophysics. In this paper, the joint effects of piezo-viscous dependency and non-Newtonian couple stresses on the performance of circular porous plate’s squeeze film bearing have been studied. The results for pressure with various values of viscosity-pressure parameters are numerically calculated and compared with iso-viscous couple stress and Newtonian lubricants. Due to piezo-viscous effect, the pressure with piezo-viscous Non-Newtonian is significantly higher than the pressure with iso-viscous Newtonian and iso-viscous Non-Newtonian fluid.

1. Introduction
In the various field of engineering and technology, the squeeze film lubrications plays a vital role. Some of common examples of squeeze film lubrications are ball bearings, automotive engines, matching gears etc. Many researchers, especially, Lin, J.R. and Hung, C.R.[1-3] analyzed the characteristics for wide slider bearings with an exponential film profile and characteristics for MHD wide slider bearings with an exponential film profile. In recent years, the study of the effect of surface roughness on the lubrication of various bearing surfaces has attracted many researchers and many researchers have applied the micro-continuum theory of couple stress fluids to examine the squeeze film behaviors of various systems. Sundarammal and Ramaniah [4] have studied the consequence of bearing deformation on the characteristics of squeeze film between circular and rectangular plates and slider bearing. Lin JR. et.al [5] analyzed the characteristics of Squeeze-film between a sphere and a flat plate couple stress fluid model. A study of joint effects of non-Newtonian rheology and viscosity pressure dependence in the sphere-plate squeeze-film system was investigated by Rong-Fang Lu, Jaw-RenLin [6]. However, the researchers neglected the effects of pressure dependency on the fluid viscosity. Neminath Bujappa Naduvanamani, Siddangouda Appara, et al.[7] have investigated the pressure dependent viscosity effects on couple stress squeeze film lubrication between rough Parallel Plates.
Santhana Krishnan, N and Sundarammal.K[8] have analyzed effects of pressure distribution on squeeze film behavior in porous transversely triangular plates with couple stress fluid. Also Sundarammal,
K. and Santhana Krishnan.N [9-10] have studied effects of the surface roughness on squeeze film behavior in porous transversely triangular plates and parallel rectangular plates with couple stress fluid. The behavior of Squeeze film in porous transversely circular stepped plates with a couple stress fluids was studies by Sundarammal, K. and Santhana Krishnan.N and et.al [11].However, Naduvinamani N.B. et.al [12] have analyzed the effects of non-Newtonian couple stress on the Squeeze-film characteristics between a sphere and a flat plate with the couple stress fluid model. According to the study by Gould [13], the variation of viscosity with pressure is important especially in the high-pressure squeeze films. The performance of bearing with a pressure dependent viscosity considering the isothermal, incompressible flow of lubricant is of the form

\[ \mu = \mu_o e^{\beta P} \]  

Where \( \mu \) the coefficient of viscosity is, \( P \) is the pressure, \( \mu_o \) is the atmospheric viscosity pressure, and \( \beta \) is the viscosity pressure coefficient.

The effects of piezo-viscous dependency on squeeze film between circular plates of Couple Stress fluid model was investigated by U.P.Singh [14]. Since, in many practical applications such as braking mechanisms and lubricating joints, the squeeze-film parallel plates are important, a further study is therefore motivated. According to the Stokes micro-continuum theory of couple stress fluids together with the variation of viscosity with pressure, the combined effects of piezo-viscous dependency and non-Newtonian couple stresses in parallel circular pours plate squeeze-film characteristics are presented in this paper. The squeeze film pressure is obtained. Comparing with the iso-viscous Newtonian-lubricant case, the squeeze film characteristic of parallel circular pours plates are presented and discussed through the variation of the viscosity–pressure parameter and the non-Newtonian couple-stress parameter.

2. Mathematical formulation of the problem

In Fig. 1, a systematic diagram of squeeze film lubrication between porous circular plates with normal velocity \( \frac{dh}{dt} \) and approaching each other is shown.

![Figure 1. Systematic diagram of squeeze film bearing with porous circular plate](image)

\[ V = -\frac{dh}{dt} \]

\[ R \]

\[ h \]

\[ (r,u) \]
The lubricant is taken to be a Stokes couple stress fluid. Under the assumption, fluid inertia, body forces and body couples are negligible, in the following analysis we further assume that the viscosity varies with pressure only, the basic equations governing the lubricant velocity and pressure reduce to

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \bar{u}}{\partial r} \right) + \frac{\partial w}{\partial z} = 0 \]  
\[ \frac{\partial^2 \bar{u}}{\partial z^2} + \eta \frac{\partial^4 \bar{u}}{\partial r \partial z^3} = \frac{\partial p}{\partial r} \]  
\[ \frac{\partial p}{\partial z} = 0 \]

The above equations are solved under the boundary conditions

\[ \bar{u} = \frac{\partial^2 \bar{u}}{\partial z^2} = 0 \quad \text{and} \quad \bar{w} = 0 \quad \text{at} \quad \bar{z} = 0 \]  
\[ \bar{u} = \frac{\partial^2 \bar{u}}{\partial z^2} = 0 \quad \text{and} \quad \bar{w} = -\frac{\partial \bar{h}}{\partial r} \quad \text{at} \quad \bar{z} = \bar{h} \]

where \( \bar{u} \) and \( \bar{w} \) are the components of velocity in \( \bar{x} \) and \( \bar{z} \) directions, \( \bar{h} \) is the film thickness between the bearing plates, \( \mu \) is the viscosity, \( p \) is the pressure, \( \eta \) denotes the material constant accountable for non-Newtonian couple stress fluid.

Taking the dimensionless quantities as

\[ p = \frac{\bar{h} \bar{n}_0}{\mu_0 R^3 \left( \frac{dh}{d\bar{r}} \right)}, \quad r = \frac{\bar{r}}{R}, \quad \bar{z} = \frac{\bar{z}}{h_0}, \quad u = \frac{\bar{n}_0 \bar{h}}{R \left( \frac{dh}{d\bar{r}} \right)}, \quad w = \frac{\bar{w}}{\left( \frac{dh}{d\bar{r}} \right)}, \]

\[ \beta = \frac{\mu_0 R^3 \left( \frac{dh}{d\bar{r}} \right)}{h_0^3}, \quad h = \frac{h}{h_0} \quad \text{and} \quad \nu = \frac{1}{h_0} \sqrt{\frac{\eta}{\mu_0}} \]

The equation (2-4) takes the dimensionless form

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r (ru) + \frac{\partial w}{\partial z} \right) = 0 \]  
\[ \mu \frac{\partial^2 u}{\partial z^2} + \nu \frac{\partial^4 u}{\partial r \partial z^3} = \frac{\partial p}{\partial r} \]  
\[ \frac{\partial p}{\partial z} = 0 \]

and corresponding boundary conditions (5) and (6) takes the form
Solving equation (8) using the boundary conditions (10) and (11), the radial velocity profile is given by

\[
u = \frac{1}{2\mu} \frac{\partial }{\partial r} \left( z^2 - zh \right) + \nu \left[ \frac{Cosh \left( \frac{\sqrt{\mu}}{v} \left( z - \frac{h}{2} \right) \right)}{Cosh \left( \frac{\sqrt{\mu} h}{v} \right)} \right]
\]

In a porous medium, the flow of couple stress fluid is governed by the modified form of the Darcy’s law, which accounts for the polar effects:

\[
\overline{w}_0 = -\frac{\phi}{\mu(1-\beta)} \nabla p^* 
\]

where \( \overline{w}_0 = (u^*, w^*); u^*, w^* \) are the Darcy’s velocity components along \( x \) and \( z \) directions respectively, \( p^* \) denotes the pressure in the porous region, \( \beta = \frac{\eta}{\mu \phi} \) and \( \phi \) is the permeability of the porous medium.

The parameter \( \beta \) indicates the ratio of microstructure size of polar additives to the pore size of the porous medium. If \( \frac{1}{\sqrt{\eta}} \approx \sqrt{\phi} \) i.e., \( \beta \approx 1 \) then the Darcy flow through the porous matrix reduces due to microstructure additives present in the Newtonian fluid that block the pores of the porous region. When microstructure is very small compared to the porous size, i.e., \( \beta \ll 1 \) the polar additives percolate in to the porous matrix.

Integrating equation (7) with corresponding boundary conditions (10-11) using equations (7) and (12), the modified Reynolds equation is

\[
d\left( r f(h, \alpha, v, p) \frac{dp}{dr} \right) = -12r
\]

where \( f(h, \alpha, v, p) = h^3 e^{-\alpha p} - 12h v^2 e^{-2\alpha p} + 24 v^3 e^{-5\alpha p} \tan\left( \frac{h}{2v} e^{\alpha p} \right) \)

Since the modified Reynolds equations (13) is non linear in nature, perturbation technique is used to find the closed form solution for the pressure distribution.

Since the piezo-viscous parameter \( \alpha \), is small, the pressure \( p \) is given by

\[
p = p_0 + \alpha p_1 + \alpha^2 p_2 + \ldots \]

Using equation (15) in the modified Reynolds equation (13) and omitting the second and higher powers of \( \alpha \), we get
\[
\frac{d}{dr}\left( r \frac{dp_0}{dr} \right) = -\frac{12r}{f_0} \quad (16)
\]
\[
\frac{d}{dr}\left( r \frac{dp_1}{dr} \right) = -\frac{f_1}{f_0} \frac{d}{dr}\left( r p_0 \frac{dp_0}{dr} \right) \quad (17)
\]
Solving (16) and (17) using the appropriate boundary conditions
\[
\frac{dp}{dr} = 0 \quad \text{at} \quad r = 0 \quad (18)
\]
\[
p(0) = 0 \quad \text{at} \quad r = 1 \quad (19)
\]
The perturbed solution are given by
\[
p_0 = \frac{3}{f_0} \left(1 - r^2\right) \quad (20)
\]
\[
p_1 = \frac{9}{2} \frac{f_1}{f_0^3} \left(1 - r^2\right)^2 \quad (21)
\]
Where
\[
f_0 = h^3 - 12hv^2 + 24v^3 \tanh \frac{h}{2v} \quad (22)
\]
\[
f_1 = -h^3 + 30hv^2 - 60v^3 \tanh \frac{h}{2v} - 6hv^2 \tanh^2 \frac{h}{2v} \quad (23)
\]
3. **Results and Discussions**

The combined influence of non-Newtonian couple stresses and the piezo-viscous dependency on the squeeze film bearing characteristics of parallel circular porous plates have been predicted on the basis of Stokes couple stress fluid theory together with the exponential variation of viscosity with pressure. The piezo-viscous effect is analyzed with a viscosity-pressure parameter \( \alpha \) and the effects of couple stresses are analyzed using a dimensionless parameter, \( \nu = \frac{1}{R \sqrt{\mu}} \).

In Figure 2, the variation of Non-dimensional pressure \( p \) with respect to the Non-dimension radius \( r \) for film thickness \( h \) is shown. On comparison with the iso-viscous Newtonian case \( \nu = 0 \), the pressure with iso-viscous non-Newtonian couple stress is higher and it increases with increase of couple stress parameter \( \nu \) for each value of the radius. It is observed that for each value of couple stress parameter, the pressure with piezo-viscous Non-Newtonian is significantly higher than the pressure with iso-viscous Newtonian and iso-viscous Non-Newtonian fluid.

Figure 3 shows the Surface plot of three dimensional variation of Non dimensional pressure \( p \) with respect to the Non dimensional radius \( r \) for film thickness \( h = 0.05 \), piezo-viscous parameter \( \alpha = 0.02 \) and couple stress parameter \( \nu = 0.05, 0.15, 0.25 \) and 0.35. It is observed that for each value of couple stress parameter \( \nu \), the pressure with piezo-viscous Non-Newtonian is significantly higher than the pressure with lower values of piezo-viscous Non-Newtonian fluid.
Figure 2: Variation of dimensionless pressure with respect to dimensionless radius for iso-viscous Newtonian and piezo-viscous non-Newtonian lubricants.

Figure 3: Surface plot of three dimensional variation of dimensionless pressure with respect to dimensionless radius for different values of piezo-viscous non-Newtonian lubricants.
The variation of film pressure $p$ with respect to the dimensionless film thickness $h$ is shown in Figure 4. It is noticed that the film pressure with couple stress lubricants is more than that of Newtonian case. The pressure also increases with the increase of couple stress parameter $v$. Again, for each value of couple stress parameter, the pressure in piezo-viscous non-Newtonian fluid is higher than that in the iso-viscous Newtonian and iso-viscous non-Newtonian fluid.

In Figure 5, the Surface plot of three dimensional variation of film pressure $p$ with respect to the dimensionless film thickness $h$ is shown, for film thickness $r = 0.5$, piezo-viscous parameter $\alpha = 0.02$ and couple stress parameter $v = 0.05, 0.15, 0.25$ and $0.35$. It is noticed the film pressure with couple stress lubricants is higher than that in Newtonian case. The pressure also increases with the increase of couple stress parameter $v$. Again, for each value of couple stress parameter, the pressure in piezo-viscous non-Newtonian fluid is higher than that in the iso-viscous Newtonian and iso-viscous non-Newtonian fluid.
Figure 5: Surface plot of three dimensional variation of dimensionless pressure with respect to film thickness $h$ for different values of piezo-viscous non-Newtonian lubricants.

4 Conclusions
The analytical solution for pressure distribution is obtained using Perturbation method based on Stokes micro-continuum theory of couple stress fluids and the viscosity-pressure dependence. In the present theoretical analysis, following results have been drawn.

- The effect of piezo-viscous dependency as well as the couple stresses enhances the pressure significantly.

- A modest value of piezo-viscous parameter increases the pressure by nearly 15% in comparison with the iso-viscous case.

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