Multiparticle Dynamics in the LHC Era

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Using the theory talks at ISMD2010 as a guidance, I present a personal review of our current understanding of multiparticle interactions in QCD. For more clarity, I separately consider hard, semi-hard, and soft interactions, and I devote most of the space to those phenomena for which progress has been recently made from first principles. Also, priority is given to processes which are directly relevant for QCD studies at the LHC, notably to forward particle production and ultrarelativistic heavy ion collisions.

1 Introduction

The 2010 edition of the International Symposium on Multiparticle Dynamics has been a privileged one, in several aspects. First, it has marked the 40th anniversary of this Symposium — a respectable age which demonstrates the maturity and the tradition of this series of meetings, and also its capacity to continuously renew itself and adapt itself to the evolutions, and the revolutions, which marked the field of high-energy strong interactions over the last 40 years. Second, this edition has given us the opportunity and joy to celebrate the 60 anniversary of our colleague Eddie de Wolf, who was one of the pioneers of this field before becoming one of its pillars. Third, this edition has marked the entrance of ISMD in the LHC era. Indeed, this was the first meeting in this series after the LHC started operating, so the discussions at this meeting have naturally focused on the fresh LHC data and the first lessons that can be drawn from them. In particular, the results on the ‘ridge effect’ in p+p collisions at $\sqrt{s} = 7$ GeV by the CMS collaboration [1] have been opportunistically released during this meeting, thus making ISMD2010 the first international conference were these results have been presented and debated, within a special session which has extended quite late in the night.

The 40 years of existence of ISMD are also the years during which Quantum Chromodynamics emerged and gradually asserted itself as the fundamental theory of the strong interactions. The successive editions of ISMD have closely followed this evolution and at the same time preserved a specific identify via their preference for the original theme of this Symposium: the study of the ‘multiparticle’ (or many body) aspects of particle production in hadronic collisions at high energy. But the precise content of this general theme has continuously evolved, following the progress in our conceptual understanding (notably in the framework of QCD) and the rise in the energy of the accelerators.

Whereas for many years, the ‘multiparticle dynamics’ was almost exclusively associated with soft interactions among hadrons, which stay outside the realm of perturbative QCD and thus elude calculations from first principles, more recently this topic was extended to include semi-hard or even hard interactions among partons, thus opening the era of controlled calculations.
This is appropriate because the modern day accelerators — HERA, RHIC, the Tevatron and especially the LHC — give us access to the high-energy phase of QCD, which is characterized by high parton densities and thus relatively weak coupling, but also by important collective phenomena (at partonic level), which call for many-body methods. Such collective phenomena, which are most obvious in relation with the nucleus–nucleus collisions at RHIC and the LHC, are also present in lepton–proton deep inelastic scattering at HERA and in p+p scattering at the LHC, in the form of parton saturation in the hadron wavefunctions, or of multiparton interactions in the ‘underlying event’. Under the pressure of the experimental results, notably at RHIC, it became clear that multiparton phenomena are truly important at the present energies: they control the gross features of particle production (like single–particle spectra, rapidity distributions, multi–particle correlations) at moderate values of the transverse energy ($E_T \sim 1 \div 20$ GeV), and affect the reconstruction of very hard jets ($E_T \gtrsim 100$ GeV) in the context of the LHC. And they are most likely responsible for the ‘ridge effect’ alluded to above, as observed in both heavy ion (RHIC) and proton–proton (LHC) collisions. Thus, whether one is interested in QCD per se or merely as ‘background physics’, one cannot make economy of a thorough study of the multiparton dynamics. Fortunately, the theory has followed, or even anticipated, the experimental evolution and new formalisms, like the color glass condensate, have been developed from first principles to deal with the physics of high parton densities. These approaches and their consequences for the phenomenology have been discussed at length at this 40th edition of ISMD, with conclusions to be summarized below.

Another privileged playground for studying the dynamics in QCD at high parton densities is particle production at forward rapidities. The LHC has unprecedented capacities in that sense, due notably to the forward detectors at CMS, ATLAS and LHCb. The first respective results have not yet been released and our expectations for them are quite high, in view of the promising, previous, results at HERA (e+p) and RHIC (d+Au). Once they will become available, the LHC data should allow us to decisively test our theoretical understanding of parton evolution in perturbative QCD at high energy and in particular observe the so far elusive ‘BFKL Pomeron’ (possibly tamed by gluon saturation).

But however large the energy is, perturbative QCD cannot be the end of the story. Soft gluon interactions and confinement are important on large space–time separations and they introduce new correlations (e.g. in the process of hadronisation) which get imprinted on the final particle distribution. Soft interactions control observables like the total and elastic cross sections or rapidity gaps in diffractive processes, and they strongly influence the bulk features of the final state — albeit this influence becomes less and less important with increasing energy. The study of soft interactions and more generally of the non–perturbative aspects of high–energy scattering in QCD is one of the traditional central themes of ISMD and it received a corresponding attention at this edition as well. The discussions have been stimulated by the first LHC data, which show significant deviations with respect to the predictions of Monte–Carlo event generators. Such discrepancies reflect the inability of the current event generators to properly deal with the physics at, or beyond, the frontiers of pQCD, in particular with multiparticle interactions. Whereas on the short time, such discrepancies will likely be eliminated by new tunes (based on the LHC data) of the existing MC codes, on the long term they should be an incentive towards developing new types of event generators, which include more of our present understanding of QCD — in particular, the recent progress with the physics of high parton densities. However this poses serious challenges, to which I shall later return. Besides, there will always be the problem of the genuinely non–perturbative effects at soft momenta, for which it is difficult to foresee any progress from first principles.
Another QCD system where the multiparton dynamics and collective phenomena are undoubtedly important, is the deconfined phase of hadronic matter at finite-temperature, the *quark-gluon plasma* (QGP). The exploration of this phase started in the nucleus-nucleus collisions at SPS and RHIC and it will continue at the LHC. One of the surprises coming from RHIC and which seems to be confirmed by the first heavy ion data at the LHC (which became available a couple of months after ISMD2010), is that this plasma might be strongly coupled. This conclusion is still under debate, for reasons that I will later explain, but it anyway raises the question of the tools at our disposal for the study of strongly coupled systems. Lattice QCD, which is our unique first-principles tool in that sense, is giving us precious informations about the thermodynamics of the QGP, but it cannot accommodate the dynamics of the ephemeral phase produced in the intermediate stages of a heavy ion collision. Some help in that sense has arrived from a rather unexpected direction: string theory, or more precisely, the AdS/CFT correspondence which relates a gauge theory (which shares some similarity with QCD) at strong coupling to a string theory at weak coupling. This ‘duality’ is giving us some useful insight into the behaviour of a QGP at strong coupling, that has been also reviewed at this meeting.

In the discussion above, I have mentioned several types of transverse momentum scales, ‘soft’, ‘semi–hard’, and ‘hard’, without being more precise. This separation is pretty standard, but nevertheless let me explain what I mean. By ‘soft’ I refer to the QCD scale $\Lambda_{\text{QCD}} \sim 200 \text{ MeV}$ where physics is controlled by non-perturbative phenomena like confinement. By ‘semi–hard’ I mean a scale of the order of the saturation momentum $Q_s$, which is the typical transverse momentum of a gluon in the wavefunction of an energetic hadron (see Sec. 4 for details). This scale grows with the energy and the centrality of the collision, and with the atomic number (for a nucleus). In fact, at the LHC, $Q_s$ should be reasonably hard: from 2 to 5 GeV depending upon the total energy, the rapidity of the produced particles, and the nature of the hadron (proton or lead nucleus). For processes taking place at this scale, the coupling is relatively weak, so perturbative techniques still apply, but the density of the participating gluons is so high that collective phenomena cannot be neglected. The proper treatment of such phenomena requires resummations of the perturbation theory, that I shall describe later on. Finally, by ‘hard’ I mean transverse momenta much higher than $Q_s$ — say, of the order of the electroweak scale ($\sim 100 \text{ GeV}$) or higher. In this regime, which at the LHC is the most interesting one for searches of new physics, the parton distributions are dilute, the ‘higher twist’ (finite-density) effects are truly negligible, and the QCD coupling is very small ($\alpha_s(M_Z) \simeq 0.1$), so the traditional perturbative calculations apply. But one should not overlook the very recent results on ‘jet quenching’ in heavy ion collisions at the LHC, which suggest that medium effects can strongly affect even such a very hard jet with $E_T \gtrsim 100 \text{ GeV}$ \cite{2,3}. It will be interesting to see whether these data can be accommodated within perturbative QCD.

In what follows, I shall attempt a summary of the theory talks at ISMD2010, by using the above separation of scales as a guideline and following a path from ‘light to darkness’: from what is conceptually best understood — the hard sector — to the longstanding, but still unsolved (and always interesting) problem of soft interactions. Along the way, I will mark a long stop by the semi-hard sector, where significant progress has been realized in the recent years.

## 2 High $p_T$ interactions: the quest for precision

Hard processes in QCD can be accurately described within collinear factorization, by combining partonic cross-sections computed to some fixed order in perturbation theory — leading-order...
(LO), next-to-leading order (NLO), NNLO etc. — with parton distribution functions whose evolution is computed by solving DGLAP equations to the corresponding accuracy in pQCD. (A brief review and introduction to this session has been given by M. Grazzini [4].) As previously mentioned, such processes are the most interesting ones for searches of the physics beyond the Standard Model at the LHC. For that purpose, the corresponding rates must be known with a very good accuracy, at the NLO level at least. Indeed, in many channels, the new physics signals could lie in the tail of kinematic distributions and thus be hidden in broad distributions underneath Standard Model backgrounds. For instance, the observation of the Higgs via the production and decay chain \( pp \rightarrow t\bar{t}H^* \rightarrow t\bar{t}bb \) (a favored channel if the Higgs is relatively light) receives backgrounds from purely QCD processes with final state \( t\bar{t}bb \) or even \( t\bar{t}jj \), a couple of which are illustrated in Fig. 1 (left). The extraction of the signal then requires accurate predictions for the background processes, for which next-to-leading order (NLO) cross sections in perturbative QCD are crucial.

There are several reasons why leading-order (LO) calculations are not accurate enough even though \( \alpha_s \) is quite small. LO results depend strongly upon the arbitrary renormalization and factorization scales (as introduced to define \( \alpha_s \) and the parton distributions), leading to a large uncertainty in the absolute value of the final result, which can be reduced only by going to NLO. This problem is even sharper for processes involving several scales like \( t\bar{t}H \), \( t\bar{t} + n\)-jets, \( W(\text{or } Z) + n\)-jets. Also, the shapes of distributions are first known at NLO. Moreover, at LO a jet is modeled by the evolution of a single parton, which is a very crude approximation; the situation can significantly be improved by including NLO corrections.

NLO calculations involve one-loop diagrams for virtual corrections (see the examples in Fig. 1 right), whose direct evaluation according to the familiar Feynman rules becomes prohibitively difficult when increasing the number of external legs. To cope with that, sophisticated methods have been developed which ‘automatize’ the NLO calculations, by decomposing the tensor one-loop diagrams into a basis set of scalar one-loop diagrams with up to 4 external legs, which can be numerically evaluated. In her talk at ISMD2010, M. Worek reported on a recent NLO calculation for the process \( pp \rightarrow t\bar{t}jj \) [5]. Whereas the absolute value of the NLO correction is relatively small (~10%), its main effect is to stabilize the result by reducing its sensitivity to the choice of a renormalization scale, from about 35% at LO to 10% at NLO. She has also mentioned the first ever calculation of a 2 \( \rightarrow \) 5 process at NLO: the process \( pp \rightarrow W^\pm + 4\)-jet has been recently computed (in a leading color approximation) via the unitarity method [6] (see Fig. 1 right).

Within the same session, R. Frederix addressed a set of three challenging data at the Tevatron, related to the top quark phenomenology, which exhibit a 2-\( \sigma \) deviation from the respective predictions of the Standard Model (as currently known). Such deviations could simply be sta-
tical fluctuations in the data, but one cannot exclude their potential to reflect new physics BSM \cite{7}. Unfortunately, the respective processes turn out to be very sensitive to NLO, or even NNLO, corrections (for instance, one of this observables, the forward–backward asymmetry in the top quark pair production\footnote{Very recently, the CDF Collaboration reported a 3.4-σ deviation from the SM for this quantity at large invariant mass for the \( t\bar{t} \) pair \cite{8}.}, first appears at NLO level in pQCD), as well as to uncertainties in the matching between partonic cross–sections and parton showers within MC event generators. Hence, one needs both more accurate data and more accurate pQCD calculations before drawing firm conclusions.

3 Forward physics: the quest for BFKL and saturation

Due to unprecedented experimental capabilities, the ‘forward physics’, \textit{i.e.} the study of particle production at forward or backward rapidities, very close to the collision axis, will be one of the highlights of the experiment program at the LHC \cite{9}. This sector too is important for new physics searches, e.g. for the Higgs production via vector boson fusion, of via double diffractive gluon–gluon fusion — one of the cleanest discovery channels envisaged so far. But at the same time this topics is very interesting for QCD \textit{per se}, in that it gives us access to the widest kinematical range for high–energy parton evolution in QCD and thus allows us to unveil and study phenomena like gluon saturation, multiple interactions, and the approach towards unitarity, which are at the heart of ISMD. A better understanding of such phenomena in the context of collider physics would be also beneficial for the cosmic ray experiments, in improving the modeling of the high–energy air showers: a fixed–target collision in the air with an incoming, cosmic, particle with \( E \sim 10^{17} \) eV corresponds to \( p+p \) collisions at the LHC energy \cite{10}.

The (pseudo)rapidity of a particle produced at an angle \( \theta \) with respect to the collision axis is \( \eta = -\ln \tan(\theta/2) \), so large rapidity (in absolute value) is tantamount to small angle, or relatively small transverse momentum \( k_\perp = E \sin \theta \) (for a given particle energy \( E \)). This is why forward physics was traditionally associated with soft particle production. However, the situation has changed with the advent of the modern–day accelerators, HERA, RHIC and especially the LHC, where the large center–of–mass energy makes it possible to produce hard particles at small angles and in particular study jet physics in the forward region.

For instance, the forward calorimeters at ATLAS and CMS can measure jets with \( k_\perp \geq 30 \) GeV up to rapidities as large as \( \eta \simeq 6 \). This looks like truly ‘hard’ physics, in the sense that the relevant values of the QCD coupling are small, but at the same time — due to the high energy and the forward kinematics (see below) — this explores the very low–\( x \) region (with \( x \) denoting the longitudinal momentum fraction of a parton) of the wavefunction of one of the incoming hadrons, where the gluon density is quite high even for such large transverse momenta and therefore usual perturbative techniques, like collinear factorization or the traditional notion of a ‘parton distribution’, fail to apply. This is the \textit{semi–hard} region, where an intrinsic transverse momentum scale, associated with the gluon density, emerges in the hadron wavefunction — the saturation momentum \( Q_s \) — and perturbation theory needs to be reorganized, even if the coupling is weak, to account for high energy radiative corrections (BFKL evolution) and ‘higher–twists’ effects (gluon saturation), which now become of order one.

In order to describe the essence of these resummations in a specific physical context, let me schematically address the problem of the production of a pair of jets in a hard scattering in hadron–hadron collisions (say, at the LHC). A diagram contributing to this process to LO
in pQCD is displayed in Fig. 2 left. Within collinear factorization, one assumes that the two partons which scatter with each other have negligible transverse momenta: \( p_{1\perp} = p_{2\perp} = 0 \). Then transverse momentum conservation requires that \( k_{1\perp} = -k_{2\perp} \), whereas the conservation of the energy and the longitudinal momentum determines the longitudinal momentum fractions \( x_1 \) and \( x_2 \) of the incoming partons as a function of the jets kinematic variables — their (equal) transverse momenta \( |k_{1\perp}| = |k_{2\perp}| = k_\perp \) and the (pseudo)rapidities \( \eta_1 \) and \( \eta_2 \):

\[
x_1 = \frac{k_{1\perp}}{\sqrt{s}} e^{\eta_1} + \frac{k_{2\perp}}{\sqrt{s}} e^{\eta_2}, \quad x_2 = \frac{k_{1\perp}}{\sqrt{s}} e^{-\eta_1} + \frac{k_{2\perp}}{\sqrt{s}} e^{-\eta_2}.
\]  

(1)

This yields the following estimate for the cross-section for di-jet production:

\[
\frac{d\sigma}{d\hat{k}_{1\perp} dk_{2\perp} d\eta_1 d\eta_2} = \sum_{ij} x_1 f_i(x_1, \mu^2) x_2 f_j(x_2, \mu^2) \delta^{(2)}(k_{1\perp} + k_{2\perp}) \frac{d\hat{\sigma}_{ij}}{dk_{2\perp}^4},
\]  

(2)

with \( d\hat{\sigma}/dk_{2\perp}^4 \propto \alpha^2/s^4 \) at high energy. Under these assumptions, a measurement of the azimuthal correlation \( d\sigma/d\Delta \phi \), with \( \Delta \phi \) the angle between \( k_{1\perp} \) and \( k_{2\perp} \), would yield a sharp peak at \( \Delta \phi = \pi \). This peak will get somewhat smeared after the inclusion of NLO corrections to the hard matrix element, but it will always be quite sharp within the context of collinear factorization, because the probability for emitting additional, hard, partons is small at weak coupling. However, this cannot be the physical reality at forward kinematics, as I explain now.

To be specific, I shall choose the situation where both \( \eta_1 \) and \( \eta_2 \) are positive and large, and relatively close to each other: \( \eta_1, \eta_2 \gg 4 \div 5 \), with \( |\eta_1 - \eta_2| \lesssim 1 \). This corresponds to the production of a pair of ‘forward di-jets’, a process that has been already measured in d+Au collisions at RHIC, with some interesting results \([17, 18, 19]\) that I shall later discuss. In this case, we have \( x_1 \gg x_2 \), so this process probes very asymmetric parton configurations. For p+p collisions with \( \sqrt{s} = 7 \) TeV and \( k_\perp = 35 \) GeV, we have \( x_1 \gtrsim 0.2 \) and \( x_2 \) between \( 3 \times 10^{-5} \) and \( 10^{-4} \). Thus, remarkably, the forward kinematics at the LHC allows us to probe very small values of \( x \) while staying in the pQCD-controlled regime at hard transverse momenta (unlike what happened at HERA, where small-\( x \) was accompanied by low \( Q^2 \)). For such small values of \( x_2 \), there is a large rapidity interval \( Y_2 \equiv \ln(1/x_2) \sim 10 \) for high-energy evolution via gluon
emission in the ‘target’ proton (the proton which moves oppositely to the two jets). Indeed, the differential probability for the emission of a ‘soft’ \( (x \ll 1) \) gluon from some other parton via bremsstrahlung reads \( (C_R \) is a color factor) \[
\frac{dP_{\text{Brem}}}{dx} \simeq \frac{\alpha_s C_R}{2\pi^2} \frac{d^2 k_\perp}{k_\perp^2} \frac{dx}{x},
\]
showing that there is a probability of order \( \alpha_s \) to emit a gluon per unit rapidity \( Y = \ln(1/x) \).

When \( \alpha_s Y \gtrsim 1 \), we have a large phase–space for soft gluon emission, leading to a rapid increase in the gluon distribution at small \( x \), via gluon cascades as that illustrated in Fig. 2 middle. Such a growth, which is clearly seen in the DIS results at HERA [20], can also be mimicked (at least over a limited interval in \( y \)) by the DGLAP evolution of the PDF’s; but this is not the proper way to describe the gluon evolution with decreasing \( x \), since there is no transverse momentum ordering in the successive emissions of soft gluons. Rather the transverse kinematics must be dealt with exactly, and this is what is done in the BFKL (Balitsky–Fadin–Kuraev–Lipatov) equation [22], which resums the radiative corrections of order \( (\alpha_s \ln 1/x)^n \) for any \( n \). This introduces some formal complications: one needs to use unintegrated parton distributions (uPDF), which also keep track of the parton transverse momentum \( k_\perp \) (in addition to its longitudinal fraction \( x \)), together with an appropriate factorization scheme, ‘the \( k_T \)–factorization’, which is known to hold to leading logarithmic accuracy (LLA) at high energy [21].

Specifically, the \( k_T \)–factorized version of Eq. (2) reads
\[
\frac{d\sigma}{d^2 k_\perp \, d^2 p_\perp \, d\eta_1 \, d\eta_2} = \sum_{ij} \int \frac{d^2 p_{1\perp} \, d^2 p_{2\perp} \, \delta(2)(p_{2\perp} + p_{1\perp} - k_\perp - k_{2\perp})}{\hat{t}} \times \Phi_i(p_{1\perp}, x_1) \frac{d\sigma_{ij}}{d\hat{t}} \Phi_j(p_{2\perp}, x_2),
\]
where \( \hat{t} = (k_{1\perp} - p_{1\perp})^2 \) and \( \Phi_i(p_{\perp}, x) \) represents the unintegrated parton distribution (uPDF) for parton species \( i \), that is, the number of partons per unit transverse momentum per unit rapidity. The standard, ‘integrated’, PDF \( x f_i(x, \mu^2) \) is obtained by integrating \( \Phi_i(p_{\perp}, x) \) over \( p_{\perp} \) up to the factorization scale \( \mu \). In the context of the high–energy resummations, the use of uPDF’s is restricted to gluons, since the respective distribution is the only one to be amplified with decreasing \( x \). However, as recalled by I. Cherednikov [23] at ISMD2010, uPDF’s appear also in other contexts (often under the name of ‘transverse momentum dependent parton densities’, or TMD’s), like the study of spin asymmetries in semi–inclusive DIS, or the \( k_\perp \) distribution in the Drell–Yan process. At a formal level, they are defined as phase–space distributions (or Wigner functions), but in general this formal definition meets with ambiguities associated with overlapping ultraviolet divergences, or with the gauge–links required by gauge invariance [23]. Such difficulties disappear in the context of high–energy scattering (at least to the LLA of the \( k_T \)–factorization), where the would–be UV divergences associated with the rapidity are taken care by the BFKL evolution and the gauge–links are unambiguously identified as eikonal Wilson lines, to be discussed below.

The collinear factorization in Eq. (2) can be formally recovered from Eq. (4) by neglecting the sum \( p_{2\perp} + p_{1\perp} \) of the parton transverse momenta in the \( \delta \)–function inside the integrand. However, such an approximation would become incorrect for sufficiently small values of \( x \); BFKL equation predicts not only a rapid rise in the number of gluons with decreasing \( x \), but also the fact that the typical transverse momenta of these gluons can deviate quite strongly
from the starting value at large \( x \) — the more so, the smaller \( x \) is. This should not be a surprise: in the absence of any ordering in \( k_{\perp} \), the evolution proceeds as a random walk in the transverse momentum space, with the rapidity \( Y = \ln 1/x \) playing the role of an ‘evolution time’.

The most salient features of the BFKL evolution can be appreciated on the basis of its following, simplified, form:

\[
\frac{\partial \Phi_g (\rho, Y)}{\partial Y} \simeq \omega \alpha_s \Phi_g + \chi \alpha_s \rho^\gamma \Phi_g,
\]

(5)

where \( \omega \) and \( \chi \) are numerical constants (the ‘BFKL intercept’ and ‘diffusion coefficient’) and \( \rho = \ln (k^2_{\perp}/Q_0^2) \), with \( Q_0 \) the typical transverse momentum at the rapidity \( Y_0 \) at which one starts the evolution. The first term in the r.h.s. describes gluon multiplication and predicts an exponential increase with \( Y : \Phi_g \propto \exp (\omega \alpha_s Y) \). The second term describes diffusion in \( \rho \) and predicts that the typical transverse momenta of the emitted gluons can deviate from \( Q_0 \) according to \( |\ln (k^2_{\perp}/Q_0^2)| \lesssim \sqrt{\chi \alpha_s Y} \). Note that this diffusion in symmetric in \( \rho \) : the transverse momenta of the emitted gluons can be either harder or softer than the original \( Q_0 \).

If extrapolated to very large values of \( y \), these features become problematic: the rapid rise of the gluon distribution eventually leads to violations of the unitarity bounds for the scattering amplitudes; and the BFKL diffusion eventually enters the non–perturbative regime at soft momenta \( k_{\perp} \sim \Lambda_{QCD} \), where this whole approach becomes unreliable. I shall later argue that such problems are solved by gluon saturation within a consistent approach at weak coupling.

In particular, for the asymmetric situation \( x_1 \gg x_2 \) corresponding to the production of a pair of forward di–jets, the high–energy evolution is important only for the ‘target’ proton, for which \( Y_2 \sim 10 \). As for the ‘projectile’ proton (for which \( Y_1 \sim 1 \)), this can still be treated in the spirit of the collinear factorization. By neglecting \( p_{1\perp} \) next to \( p_{2\perp} \) in Eq. (4), using the \( \delta \)–function to integrate over \( p_{2\perp} \), and keeping only the gluon distribution in the target (since this is the only one to be enhanced at small \( x_2 \) ) one finds

\[
\frac{d\sigma}{d^2 k_{1\perp} d^2 k_{2\perp} d\eta_1 d\eta_2} \simeq \sum_i x_i f_i(x_1, \mu^2) \frac{\alpha_s^2}{k^4_{1\perp}} \Phi_g (k_{1\perp} + k_{2\perp}, x_2).
\]

(6)

This formula makes it clear that it is the BFKL evolution of the gluon distribution in the target which controls the energy and rapidity dependencies of the di–jet cross–section, as well the transverse momentum unbalance \( (k_{1\perp} + k_{2\perp} \neq 0) \) between the produced jets. Namely, the cross–section grows very fast with the COM energy, as the power \( s^{\alpha_s/2} \). For a fixed COM energy, it rises exponentially with the rapidities \( \eta_1 \simeq \eta_2 \) of the produced rapidites. Also, the distribution in \( k_{1\perp} + k_{2\perp} \) broadens rapidly with increasing \( \eta_i \), due to the BFKL diffusion.

Within the present approximations, the value of \( k_{1\perp} + k_{2\perp} \) in a given event is equal to minus the sum of the transverse momenta of the gluons radiated within the BFKL cascade (cf. Fig. 2 middle) and which are liberated in the final state. This also shows that the structure of the event is modified by the high energy evolution of the hadron wavefunctions.

Eq. (6) is based on the LLA at high energy, which is rather poor: not only the NLO corrections to the BFKL equation are known to be large \[24\] \[25\], but there are also important non–linear effects (gluon saturation) associated with the high gluon density in the target \[28\].

But before turning to a discussion of such effects, let me explain why the process described so far — the production of a pair of forward jets — is not necessarily optimal for a study of high–energy evolution at the LHC (although it was very useful in that sense at RHIC, as I shall later review). This discussion will also allow me to introduce a process which is better suited for the kinematics at the LHC: the Mueller–Navelet jets \[29\].
Specifically, the process in Eq. (6) involves two hard transverse momentum scales, $k_{1 \perp}$ and $k_{2 \perp}$, which in the context of the LHC cannot be smaller than 20 GeV (for the produced jets to be distinguishable from the hadronic background). These scales must be compared to the maximal transverse momentum that can be generated in the target proton wavefunction via the BFKL diffusion. More precisely, I shall shortly argue that the relevant comparison scale is the target saturation momentum, which grows with $1/x$ and for a proton at $x \sim 10^{-5}$ is expected (from analyses of the HERA data) to be $Q_s(x) \sim 1 \text{ GeV}$ [19]. This value is considerably smaller than the $k_\perp$ of the external jets, meaning that, first, the asymmetry expected on the basis of the BFKL evolution is rather tiny and thus difficult to observe and, second, the process also involves large collinear logarithms $\ln(k^2_\perp/Q^2_s)$ which need to be resummed in the parton distributions, on top of the high–energy logarithms $\ln(1/x^2)$.

The simultaneous presence of several, large, scales calling for different types of resummations is a generic feature of the forward physics, which often complicates the corresponding theoretical analysis. Yet, there exists a process for which this problem is less pronounced and which can create a larger asymmetry between the produced jets via the high–energy evolution. This is the production of a pair of forward–backward, or Mueller–Navelet (MN), jets [29]. In this case $\eta_1$ and $\eta_2$ are large but of opposite signs, say $\eta_1 > 0$ and $\eta_2 < 0$, so that both $x_1$ and $x_2$ are relatively large, $\sim 0.1$, and the collinear factorization in Eq. (2) is perfectly legitimate. In particular, the large collinear logarithms $\ln(k^2_\perp/Q^2_s)$ are absorbed in the standard way, in the PDFs. What is peculiar about this process is the large rapidity separation between the two jets, $Y \equiv \eta_1 - \eta_2$ with $Y \simeq 10$ at the LHC, which favors the BFKL evolution of the partonic cross–section $\hat{\sigma}$ (see Fig. 2 right). Note however that this evolution now refers to partons which are hard to start with, and not to protons. Accordingly, the typical momenta of the BFKL gluons are comparable to those of the final jets, thus permitting a large asymmetry between the measured momenta $k_{1 \perp}$ and $k_{2 \perp}$. But these final momenta are still comparable with each other, so there is no large collinear log $\ln(k^2_{1 \perp}/k^2_{2 \perp})$ to worry about. Thus, this process offers a clean set–up to test the BFKL evolution and is indeed under intense scrutiny at the LHC.

But in order to be conceptually meaningful and practically useful, the BFKL calculations must be extended to include NLO corrections and saturation effects. I have previously mentioned that the NLO corrections to the BFKL equation, i.e. the effects of order $\alpha_s(\alpha_s \ln 1/x)^n$ in perturbation theory, turn out to be very large [24] and in fact some effort was needed to render them meaningful in practice. This required a better understanding of the interplay between the high–energy radiative corrections and the collinear ones, leading to ‘RG–improved’ evolution equations which partially resums both types of effects [23]. These equations predict that the high–energy evolution is considerably slower than predicted by the LO BFKL equation: both the rise of the gluon distribution with $1/x$ and the transverse momentum diffusion are drastically reduced by the NLO corrections, but they are not completely washed out. (For instance, the NLO BFKL calculation of MN jets in [26] leads to results which are quite close to the respective predictions of the NLO DGLAP formalism.) In particular, the conceptual problems of the BFKL evolution are not cured by NLO corrections, but merely postponed to higher energies. Such energies, at which unitarity corrections become important even for relatively hard momenta, have been already reached by the modern day accelerators, as clearly shown by the need to include multiparticle interactions and ‘infrared cutoffs’ which rise with the energy (and thus mimic saturation; see below) in any Monte–Carlo code aiming at describing the exclusive final state at RHIC or the LHC.

A consistent description of such phenomena from first principles requires in particular the understanding of non–linear phenomena in parton evolution at small $x$ together with new
factorization schemes which go beyond the single–particle PDFs in such a way to capture the correlations associated with these non–linear phenomena. This is the topics of *gluon saturation* which met with important progress over the last years [28] and represented a main focus for two of the sessions of ISMD2010 — ‘Forward physics’ and ‘High–density QCD’ (see Sec. 4 below). To concisely describe this topics, let me refer to the cartoon of parton evolution depicted in Fig. 3 left. As shown in this figure, there are two main directions of evolution — the DGLAP evolution with increasing virtuality $Q^2$ (or transverse momentum $k_{\perp}^2$) and the BFKL evolution with increasing energy, or rapidity $Y = \ln 1/x$, which both lead to a rise in the number of partons, via parton branching, but with very different consequences.

By the uncertainty principle, a parton has a transverse area $1/Q^2$, so with increasing $Q^2$ the total area occupied by the partons decreases much faster than the rise $\propto \ln Q^2$ in the number of partons; accordingly, the DGLAP evolution leads to a partonic system which is more and more dilute. By contrast, when increasing $Y$, the transverse momenta vary only slowly and symmetrically (via BFKL diffusion), so the gluons emitted within the BFKL evolution are roughly of the same size, whereas their number grows exponentially with $Y$. So, clearly, this evolution produces a dense system of partons which overlap with each other. At high density, the coupling is weak, so the gluon interactions are suppressed by $\alpha_s \ll 1$, but they are enhanced by the gluon occupation number $n(k_{\perp}, Y)$, since a gluon can interact with all the other gluons that it overlaps with. Here $n$ is roughly the unintegrated gluon distribution per unit transverse area, $n \simeq d\Phi_g/d^2b_{\perp}$, and it grows very fast, $n \propto \exp(\omega_\alpha Y)$, within the BFKL approximation. Thus, despite of the weakness of the coupling, the system becomes fully non–linear when $Y$ is large enough for $n$ to become of order $1/\alpha_s$. When this happens, the gluons start to repeal each other, which inhibits the emission of new gluons, thus taming the BFKL rise of the gluon distribution. This is ‘gluon saturation’.

The theoretical description of this phenomenon within perturbative QCD is extremely complex, since the non–linear effects generate many–body correlations whose high–energy evolution is entangled. After pioneering work in the mid eighties [27], the pQCD description of this non–linear evolution has been constructed over the last 15 years, in the form of an effective theory for the small–$x$ gluons in the wavefunction of an energetic hadron: the *color glass condensate* (CGC) [28]. The ‘condensate’ refers to the strong, classical, color field describing a gluon configuration with large occupation numbers, while the ‘glass’ reminds that this configuration is

Figure 3: Left: A ‘phase–diagram’ for parton evolution in QCD; each colored blob represents a parton with transverse area $\Delta x_{\perp} \sim 1/Q^2$ and longitudinal momentum $k_z = xP_z$. Right: Gluon evolution in the presence of saturation, as probed via multiple scattering in DIS.
randomly generated via the high–energy evolution. Thus, to compute observables one needs to average over all the classical field configurations, with the ‘CGC weight function’ (a functional encoding the multi–gluon correlations at small \( x \)). The central equation of this effective theory is a functional renormalization group equation, known as ‘JIMWLK’ (from Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, and Kovner), which shows how the CGC weight function gets built via soft gluon emission in the background of a strong color field. This equation has been constructed in pQCD, to leading–log accuracy in the radiative corrections at high energy, but to all orders in the ‘higher–twist’ effects associated with the high gluon density. In the multi–color limit \( N_c \gg 1 \), it reduces to a single, non–linear, equation for the gluon occupation number — a non–linear generalization of the BFKL equation known as the Balitsky–Kovchegov (BK) equation, which has recently been extended to NLO accuracy \[30, 31\].

A cartoon version of BK equation, to be compared to the ‘BFKL equation’ (5), reads

\[
\frac{\partial n(\rho, Y)}{\partial Y} \simeq \omega \alpha_s n + \chi \alpha_s \rho \partial_\rho n - \alpha_s^2 n^2
\]

(7)

As compared to Eq. (5), this involves an additional, quadratic, term with a negative sign, which describes gluon recombination: when \( n \sim 1/\alpha_s \), the linear and non–linear terms in the r.h.s. cancel each other and then the gluon distribution stops rising. Due to the non–locality of this equation in \( \rho \equiv \ln(k_\perp^2/Q_0^2) \), the saturation domain progresses with \( Y \), towards larger and larger values of \( k_\perp \). This progression is characterized by the saturation momentum \( Q_s(Y) \) — the value of \( k_\perp \) below which one finds saturation at a given \( Y \). Namely, the BK equation implies

\[
n(k_\perp, Y) \approx \begin{cases} 
\frac{1}{\alpha_s} & \text{if } k_\perp \lesssim Q_s(Y), \\
\frac{1}{\alpha_s} \left( \frac{k_\perp^2}{Q_s^2} \right)^{\gamma_s} & \text{if } k_\perp \gg Q_s(Y),
\end{cases}
\]

(8)

where \( \gamma_s \simeq 0.63 \) and \( Q_s^2(Y) \simeq Q_0^2 \exp(\lambda_s(Y-Y_0)) \) with \( \lambda_s \simeq 0.3 \) at NLO accuracy \[32\].

The initial conditions \( Y_0 \) and \( Q_0 \) for the high–energy evolution depend upon the target. For a hadronic target, one generally takes \( Y_0 \approx 4 \) (or \( x_0 \approx 10^{-2} \)), which ensures that \( \alpha_s Y_0 \approx 1 \). The corresponding value of \( Q_0 \) is non–perturbative and it is fitted from the data. The fits to the DIS structure function at HERA using BK equation with running coupling yield \( Q_0^2 \simeq 0.2 \text{ GeV}^2 \) for a proton at \( x_0 = 10^{-2} \) \[19\]. For a heavy nucleus with atomic number \( A \gg 1 \) and for central collisions, we expect \( Q_0^2 \) to be enhanced by a factor \( A^{1/3} \) w.r.t. a nucleon, because of the correspondingly larger transverse density of valence quarks available for initiating the evolution \[28\]. When the evolution starts directly with a hard parton, so like for MN jets, \( Q_0 \) is the (hard) transverse momentum of that parton; but this becomes a ‘saturation scale’ only for a sufficiently large \( Y_0 \), such that the perturbative evolution of the original parton up to \( Y_0 \) be able to build a relatively dense system of gluons. This value \( Y_0 \) can be computed in pQCD, which predicts \( Y_0 \approx 8 \div 10 \) independently of \( Q_0 \) \[33\]. These estimates imply that the saturation momenta to be explored in the forward kinematics (\(|\eta| \gtrsim 5\)) at the LHC can be as large \( Q_s^2 \approx 2 \text{ GeV}^2 \) for protons, \( Q_s^2 \approx 6 \text{ GeV}^2 \) for lead nuclei, and \( Q_s^2 \approx 100 \text{ GeV}^2 \) for MN jets with \( Y \gtrsim 10 \). The last number is particularly striking: it shows that by focusing on the high–energy evolution a ‘hot spot’ inside a hadron (a parton with high \( Q^2 \) and large \( x \)), one can reach values of the saturation momentum which are much higher than the average saturation momentum in that hadron. But this comes with a price: a parton with large \( x \approx 0.1 \) and high
\( Q^2 \) represents a rare fluctuation in the proton evolution, hence the corresponding PDF \( f(x, Q^2) \) is very small.

Eq. (8) has some important consequences:

(i) It shows that the typical transverse momenta in the wavefunction of an energetic hadron are of order \( Q_s \) and thus they become hard for sufficiently large \( Y \). Hence, a weak–coupling approach is indeed appropriate for the study of the bulk part of the hadron wavefunction. Then the same is true for the bulk features (like multiplicities and single–particle spectra) of particle production in high–energy hadron–hadron collisions.

(ii) It exhibits geometric scaling: the occupation number \( n(k_\perp, Y) \) depends upon the two kinematical variables \( k_\perp \) and \( Y \) only via the ratio \( k_\perp^2/Q_s^2(Y) \). This property is very interesting in that it is a consequence of saturation which manifests itself outside the saturation region, at momenta \( k_\perp \gg Q_s(Y) \). This scaling transmits to the associated cross–sections and it has been observed indeed, over a wide kinematical range, in the HERA data for DIS at small \( x \leq 10^{-2} \)\(^2\). A similar scaling has been predicted for MN jets in the asymmetric regime where the momenta \( k_{1\perp} \) and \( k_{2\perp} \) of the final jets are different (but comparable) with each other and for sufficiently large rapidity separations \( Y \gtrsim 8 \)\(^2\).

The last point raises the question of how to compute hadronic cross–sections in the presence of saturation. The \( k_T \)–factorization has the same limitations as the BFKL equation: it holds only in the dilute regime at sufficiently large momenta \( k_\perp \gg Q_s(Y) \). Indeed, if the gluon density becomes so high that the interactions among the gluons in the wavefunction start to be important, then an external probe which scatters off these gluons will undergo multiple interactions. So, one needs a factorization scheme capable to include these interactions. This is generally referred to as the CGC (or high-energy) factorization, although its specific form (when known !) depends upon the process at hand \(^{12}13,14,24,35,36\). For DIS, one speaks about dipole factorization: the virtual photon fluctuates into a quark–antiquark pair (a color dipole) which then multiply scatters off the gluons in the target (see Fig. 3 right). This multiple scattering is computed in the eikonal approximation, by associating one ‘Wilson line’ (a path–ordered exponential of the target gluon field) to each of the two quarks composing the dipole. Thus the DIS cross–section involves the product of two Wilson lines averaged over the gluon fields in the target with the CGC weight function. For the production of a pair of forward di–jets, the cross–section is computed by taking the modulus squared of the amplitude in Fig. 2 middle and involves four Wilson lines: two for the partonic jets in the direct amplitude and two for those in the complex conjugate amplitude.

One can now understand why the high–energy factorization cannot be universal (i.e. process–independent), although there is some systematics \(^{36}\) : according to their partonic content, different projectiles probe different correlation functions of the gluons in the target. Moreover, these correlations naturally occur as products of Wilson lines (one per parton) which encode multiple scattering. And the evolution equations at high energy, as derived from the functional JIMWLK equation, are most usefully written as coupled equations for products of Wilson lines — the Balitsky hierarchy. Besides, the whole formalism (the evolution equations and the various factorization schemes) is more conveniently written down in impact parameter space — that is, by using the transverse coordinates \( (x_\perp) \) instead of the transverse momenta \( (k_\perp) \). Indeed, as shown by the example of the Wilson lines, it is easier to deal with multiple interactions in impact parameter space, where they exponentiate at the level of \( S \)–matrix.

In principle, the JIMWLK equation can be solved exactly, via numerical methods; this

\(^2\)Namely, it holds within a finite window above \( Q_s \) whose with increases with \( Y \) via BFKL diffusion \(^{25}\).
The Fourier transform of the dipole amplitude can be viewed as a natural extension of the unintegrated gluon distribution (to which it reduces at $k_{\perp} \gg Q_s(Y)$) towards the saturation regime.
qualitatively described by CGC–inspired calculations [38], although a rigorous analysis (which would require computing the correlation of a product of four Wilson lines) is still lacking.

Figure 5: Di–hadron correlations as measured by RHIC (STAR) at forward (left) and central (right) rapidities. Left: the leading (trigger) and the subleading particle have rapidities \( 2 < \eta_{1,2} < 4 \). The ‘away’ (\( \Delta \phi = \pi \)) peak is visible for \( p+p \) collisions but is absent in \( d+Au \). The lines are theoretical calculations within the CGC framework [19, 38]. Right: all particles have \( \eta \approx 0 \) and \( k_\perp = 2 \div 6 \) GeV. The ‘away’ peak is clearly visible in \( p+p \) and \( d+Au \) collisions, but is absent in \( Au+Au \) collisions. This is ‘jet quenching’ by interactions in the medium.

All the above examples where the high–energy factorization is well under control refer to a dense–dilute scattering — the situation where the projectile is relatively dilute and can be described as a collection of a few partons, whereas the target is dense (possibly at saturation) and is described as a CGC —, and to a relatively simple final state — the inclusive production of a few partons. Some more complicated problems are the dense–dense scattering, e.g. the collision between two heavy ions to be discussed in the next section, and the description of exclusive final states in the presence of saturation and multiple scattering. In what follows, I would like to describe two methods for the construction of the final state in a high–energy scattering that have been presented at ISMD2010.

The first method, as described by H. Jung [39] and M. Deak [40], uses \( k_T \)–factorization together with a linear, BFKL–like evolution of the unintegrated gluon distributions to describe the hard scattering (say the production of a pair of jets) and identifies the ‘underlying event’ (the relatively soft radiation accompanying the hard jets) with the gluons from the partonic cascades involved in the scattering; these gluons are assumed to be liberated in the collision. For this description to be more realistic, it replaces the BFKL evolution with the CCFM one — a generalization of BFKL which takes into account the angular ordering along the partonic cascade. The angular ordering follows from quantum interference between successive emissions, which implies that any newly emitted gluon should make a larger angle with the collision axis than all the gluons emitted before it. (A similar property holds for the radiation produced via jet fragmentation in the final state; but in that case, the angles are decreasing from one emission to the next one, rather than increasing.) This whole scheme has been numerically implemented in the Monte–Carlo generator CASCADE, with results which satisfactorily describe the small–\( x \) phenomenology at HERA and which predict harder \( k_\perp \)–spectra for the forward jets at the LHC as compared to more standard event generators like PYTHIA. Note that there is no
saturation, nor multiple interactions, in this approach, although one could mimic saturation by introducing an energy–dependent infrared cutoff ('saturation boundary'), which ideally should be self–consistently computed within the CCFM evolution [41].

A very ambitious program, which aims at explicitly including saturation and multiple scattering together with the correlations induced by the high–energy evolution, has been presented by G. Gustafson [42]. This approach is based on an extension of Mueller’s dipole picture, which is a reformulation of the LO BFKL evolution, valid at large $N_c$, where gluons are replaced by color dipoles which multiply via $1 \rightarrow 2$ dipole splitting. By itself, this picture cannot accommodate saturation (there is no $2 \rightarrow 1$ dipole recombination in pQCD!), but it has the virtue to capture the fluctuations and correlations produced by the BFKL evolution in the dilute regime. Such fluctuations determine the cross–section for diffractive excitations in the Good–Walker picture. They may also be important for the evolution towards saturation, as suggested by the correspondence between the high–energy evolution in QCD and the reaction–diffusion process in statistical physics [43]. In fact, in a LO BFKL picture, these fluctuations are truly huge and rapidly growing with $Y$ [44, 45]. In real QCD, though, they are considerably tamed by saturation in the high–density regime and by the running of the coupling in the dilute regime [46]. So far, there is no first–principle theory for the QCD evolution including both fluctuations and saturation (and hence Pomeron loops), but some semi–heuristic approaches have been developed in that sense [47, 48]. In this approach, saturation is introduced via ‘dipole swing’ — a rule for color reconnections which prohibits the formation of large dipoles with transverse size $r \gtrsim 1/Q_s(Y)$. Also multiple sub–collisions are permitted in the interaction between two evolved dipole cascades, as illustrated in Fig. 6 (left). The results of the Monte–Carlo implementation of this model nicely illustrate the role of saturation and multiple interactions in suppressing fluctuations and removing unitarity violations, which would otherwise be substantial in p+p collisions at the LHC energies. The extension of the model to exclusive final states has been very recently given [49].

R. Enberg [50] has presented a new, ‘hard+soft’, model for diffractive deep inelastic scattering, which compares quite well with the respective HERA data for $Q^2 \geq 10$ GeV$^2$. The

Figure 6: Left: A collision between two evolved dipole cascades in the DCM [42]. One sees 3 subcollisions and a Pomeron loop (A) which opens via dipole splitting and closes via a dipole swing. Right: The ‘hard+soft’ model for diffractive DIS [54].

Note that the BK equation (7) is essentially the same as the FKPP equation describing ‘reaction–diffusion’ in the mean field approximation [43] (and refs. therein).
diffractive process is interpreted as the consequence of a hard scattering followed by a series of soft gluon exchanges, resummed in the eikonal approximation, resulting in an overall color singlet exchange (see Fig. 6 right). This resummation is similar to that performed by Wilson lines in the CGC formalism, so it would be interesting to clarify the interplay between these two approaches.

4 High density: the quest for the QGP

The CGC effective theory also provides the initial conditions for ultrarelativistic heavy ion collisions (HIC), as measured at RHIC and the LHC. By ‘initial conditions’ I mean the wavefunctions of the incoming nuclei (which at high energy are dominated by small–$x$ gluons) and the particle (actually, parton) production during the early stages of the collision, up to times $\tau_0 \simeq 1/Q_s \simeq 0.1 \div 0.2$ fm. Here $Q_s$ is the average saturation momentum in any of the two colliding nuclei, as probed by multiparticle production at central rapidities. Most of the particles produced in a A+A collision are ‘minijets’ with semi-hard transverse momenta $k_\perp \sim 1$ GeV, of the same order as $Q_s$ at the relevant values of $x$ (from $10^{-3}$ to $10^{-4}$). This is not a coincidence: these particles are either partons (mostly gluons) from the initial wavefunctions that have been liberated by the collision (via multiple short–range scattering among partons), or the products of their subsequent fragmentation. But after being liberated, the density of these partons within the interaction region is so high, and the size and lifetime of this dense partonic system are so large, that multiple scattering will play an important role in redistributing the energy and momentum and driving the system towards (local) thermal equilibrium. One can appreciate the density of this system either by estimating its energy density — one finds $\varepsilon \gtrsim 15$ GeV/fm$^3$ at the LHC, which is about 10 times larger than the density of nuclear matter (and 3 times larger than in Au+Au collisions at RHIC) —, or from the measured multiplicity in the final state — about 1600 particles per unit rapidity in central Pb+Pb collisions with $\sqrt{s_{NN}} = 2.76$ TeV at the LHC [51].

![Figure 7: The expected space–time picture for an ultrarelativistic heavy ion collision.](image)

The ‘standard scenario’ for the evolution of this dense partonic system until hadronization, as emerging from theoretical considerations amended by the experimental reality at RHIC, is illustrated in Fig. 7. There are still many zones of shadow in this scenario — notably, about
the precise time scales, the mechanism responsible for thermalization, and the nature of the quark–gluon plasma (QGP) which gets formed in this way (a ‘weakly interacting gas’ or a ‘nearly perfect fluid’?) — and many interesting observables whose theoretical interpretation is still under debate — like the ‘ridge effect’ or the strong ‘jet quenching’. In what follow I shall summarize the way how such open questions have been reflected in the talks and discussions at ISMD2010, by following the chronology of the HIC as illustrated in Fig. 4.

The calculation of particle production in the early stages of a heavy ion collision is particularly complex because of the need to account for two types of multiparton interactions: gluon saturation in the wavefunctions of the incoming nuclei and multiple subcollisions between partons in the two nuclei (see the right panel of Fig. 4). Within the CGC formalism, two strategies have been devised to perform such calculations, with different levels of rigor [28]:

(a) A heuristic extension of the $k_{\perp}$–factorization, which includes saturation effects within the ‘unintegrated gluon distributions’ of the two nuclei (obtained as solutions to the BK equation), but neglects the multiple scattering in A+A collisions. In principle, such a strategy is justified only for sufficiently large momenta $k_{\perp} \gg Q_s$. In practice, this has been quite successful in describing bulk features of the particle production like the multiplicity density $dN/d\eta$ at $\eta = 0$, which involve an integration over all values of $k_{\perp}$ [19]. Such calculations predict $dN/d\eta \sim Q_s^2 R_A^2$, and hence a power–law increase of the multiplicity with the center of mass energy: $dN/d\eta \sim s^{\lambda/2}$ with $\lambda \approx 0.25$. This is in agreement with the observed rise in the multiplicity when going from RHIC to the LHC [51]. Another interesting consequence of such calculations, as explained by G. Wolschin [52] at ISMD2010, is an increased baryon stopping due to saturation: the (large–$x$) valence quarks from one nucleus scatter off the high-density, small–$x$, gluons from the other one and thus get redistributed in rapidity; in particular, some of them are slowed down to smaller rapidities, which increases the net baryon density around $\eta = 0$ and, especially, pushes the fragmentation peaks towards smaller values of $\eta$. This effect should be strong enough to allow studies of saturation at the LHC based on the position of the fragmentation peaks.

(b) The CGC factorization [35, 28], in which the two nuclei prior to the collision are described as statistical ensembles of strong, classical, color fields, and the collision is represented by the solution to the classical Yang–Mills equations with initial conditions randomly chosen from these ensembles. This includes both saturation (via the CGC weight functions for the two nuclei) and multiple scattering (via the non–linear effects in the Yang–Mills equations) and is well adapted for the computation of sufficiently inclusive quantities like the single–particle spectra or the energy density and its correlations. But it has the drawback to require heavy numerical simulations, in the form of classical lattice calculations.

The non–equilibrium, dense, partonic matter produced in the early stages ($\tau_0 \sim 1/Q_s$) of a A+A collision is called the ‘Glasma’. The solutions to the Yang–Mills equation alluded to above show that the Glasma fields are longitudinal chromo-electric and chromo-magnetic fields with occupation numbers $\sim 1/\alpha_s$, that are screened at distances $1/Q_s$ in the transverse plane of the collision. As a consequence, the matter produced can be visualized (see Fig. 8) as comprising $R_A^2 Q_s^2$ color flux tubes of size $1/Q_s$, each producing $1/\alpha_s$ particles per unit rapidity.

The Glasma flux tubes carry topological charge ; the resulting, dynamical, topological transitions (‘sphalerons’) may result in observable metastable CP–violating domains. As explained by H. Warringa [53] at ISMD2010, such transitions could explain some puzzling charge correlations observed by STAR, via the ‘chiral magnetic effect’: under the action of the ultra strong magnetic fields ($B \sim 10^{18}$ Gauss) which are likely to be created in peripheral ultrarelativistic A+A collisions, the fluctuations in the topological charge can induce fluctuations in the electric charge density in the final state.
The Glasma flux tubes also generate $n$–particle long–range rapidity correlations and thus can naturally explain the ridge effect seen in di–hadron correlations in Au+Au collisions at RHIC [54] and also (with a much lower intensity, though) in p+p collisions at the LHC [1]. The ‘ridge’ is a two–particle correlation in the distribution of particles accompanying a jet which extends with nearly constant amplitude over several units in rapidity ($\Delta \eta \simeq 4$) and which is well collimated in the azimuthal separation $\Delta \varphi$ relative to the jet — so it looks like an extended mountain ridge in the $\Delta \eta$–$\Delta \varphi$ plane (see Fig. 9). This means that particles which propagate along very different directions with respect to the collision axis preserve nevertheless a common direction of motion (close to that of the triggered jet) in the transverse plane. By causality, such a correlation must have been induced at early times, when these particles — which rapidly separate from each other — were still causally connected (see the right panel of Fig. 8).
produced by the decay of the Glasma flux tubes \[55\]. These particles are correlated in rapidity over a range \(\Delta \eta \sim 1/\alpha_s\) since the flux tubes are uniform in \(\eta\) over that range. The angular collimation occurs because particles produced isotropically in a given flux tube are collimated by their subsequent radial, outward, flow \[56\]. This collimation effect is clearly demonstrated by the hydrodynamical calculations presented at ISMD2010 by R. Andrade \[57\] and Y. Hama \[58\]. In the case of p+p collision, where no flow is expected, a small collimation may be generated by the di–hadron production mechanism in the presence of saturation \[59\]. This would be in agreement with the experimental observation by the CMS \[1\] that the ridge is visible only in the high–multiplicity (central ?) events and only within a limited range in \(k_{\perp}\) (from 1 to 3 GeV), which is in the ballpark of the proton saturation momentum at the LHC.

The evolution of the Glasma into a thermalized Quark Gluon Plasma (QGP) is not yet fully understood, but some of the RHIC data — notably the collective motion known as elliptic flow — suggest that this happens very fast, on a time scale of one fermi/c. Hydrodynamical calculations were quite successful in explaining the elliptic flow seen at RHIC \[12, 14\] and predicting the one recently observed by the LHC \[60\], but to that aim they had to assume a very early thermalization time \(\tau_{\text{eq}} \lesssim 1\) fm/c and a relatively low viscosity. Explaining such a rapid thermalization from first principles is a challenge for the theory, and so is also the identification of new observables which would permit a more refined study of the approach towards thermalization in the data. In particular, we would like to know how fast is this plasma evolving towards isotropy from the Glasma initial conditions, which are highly anisotropic. To address such questions, W. Florkowski and collaborators \[62\] have developed a generalized hydrodynamical framework (ADHYDRO) which can accommodate highly anisotropic initial conditions and strong dissipation (proportional to the anisotropy). So far, calculations have been performed for the one–dimensional, longitudinal, expansion, with results which show the expected approach towards isotropy, on a time scale \(\tau_{\text{iso}}\) which is a parameter of the model. Within a related approach, P. Bozek \[63\] concluded that this isotropisation time should be very small, \(\tau_{\text{iso}} \lesssim 0.25\) fm/c, in order to reproduce the relatively large directed flow \(v_1\) seen in the RHIC data: increasing \(\tau_{\text{iso}}\) would rapidly kill \(v_1\). Hence, the directed flow is a very sensitive probe of thermalization, unlike the observables related to the transverse dynamics, like the \(k_{\perp}\) spectra or the elliptic flow \(v_2\), which are only little affected by the initial pressure anisotropy.

To be able to identify and study the QGP in HIC, one needs a good comprehension of its properties. For the case of a plasma in thermal equilibrium and with zero fermionic density, lattice QCD provides such a comprehension from first principles. In his review talk of this topic at ISMD2010, Z. Fodor \[64\] emphasized that, due to tremendous progress in numerical and computational techniques and to the strenuous efforts of continuously growing and re–concentrating collaborations, lattice QCD has finally reached the ‘productive phase’, where the continuum limit and the finite–size scaling are under control, and so is also the extrapolation to physical masses for the lightest quarks. This makes it possible to have a good control of the hadron spectrum with only few input parameters and, in particular, ‘predict’ the proton mass with high accuracy. The same progress allowed one to clarify the nature of the deconfinement ‘phase transition’ — which is actually a smooth cross–over, as demonstrated by the smooth behaviour of the Polyakov–line susceptibility in the thermodynamic limit \(V \to \infty\) (approached on the lattice via finite–size scaling) — and to solve a longstanding controversy concerning the value of the ‘critical’ temperature for deconfinement, for which a value \(T_c \approx 175\) MeV seems to be now widely accepted.

The lattice studies of the equation of state \[64\] also reveal that, after a rapid increase around \(T_c\), the energy \(\varepsilon\) (or entropy \(s\)) density of the QGP is very slowly approaching the Stefan–
Boltzmann limit, in such a way that \( s_{\text{QCD}}(T) \simeq (0.80 \div 0.85) s_{\text{SB}}(T) \) when \( T \) varies from 2 to 5 \( T_c \) (the temperature range of interest for HIC at RHIC and the LHC). Such a deviation of less than 20% from the ideal gas limit is small enough to suggest a weak–coupling behaviour, and it is indeed well accounted by calculations using ‘Hard Thermal Loop’ resummations of the perturbative expansion at finite temperature [65]. These calculations support the picture of the QGP as a weakly–interacting gas of ‘quasiparticles’, quarks and gluons, which are dressed by medium effects in a way that is computable in perturbation theory.

On the other hand, this value \( s_{\text{QCD}}/s_{\text{SB}} \approx 0.80 \) is also close to the value \( s_{\infty}/s_{\text{SB}} = 0.75 \) predicted by the AdS/CFT correspondence [66] for the strong coupling limit \( g^2 N_c \to \infty \) of \( N = 4 \) supersymmetric Yang–Mills theory — a ‘cousin’ of QCD which has the color gauge symmetry SU\((N_c)\), but also (four) supersymmetries, and which is conformal at quantum level (the coupling is fixed). Hence, by themselves, the lattice QCD results for the thermodynamics cannot exclude the possibility that the QGP be strongly coupled in this range of temperatures. Why would be such a possibility interesting? As earlier mentioned, a successful hydrodynamical description of the RHIC data for the elliptic flow requires a short thermalization time and a very low viscosity–to-entropy ratio \( \eta/s = 0.1 \div 0.2 \), two properties which are rather difficult to explain at weak coupling, but which become natural if the coupling is strong. (Recall, e.g., that in kinetic theory, the viscosity is proportional to the mean free path, which decreases with increasing coupling.) And indeed AdS/CFT calculations in gauge theories with a gravity dual, like \( N = 4 \) SYM, predict a small, universal, lower bound \( \eta/s = 1/4\pi \) in the strong coupling limit [67], which is compatible with the phenomenology at RHIC [68]. Although such calculations refer to conformal field theories, it still make sense to compare their results to the QGP phase of QCD, since from lattice QCD we know that the ‘trace anomaly’ (the violation of conformality by the running of the coupling) is quite small for temperatures \( T \geq 2T_c \).

A strong–coupling scenario could also explain the relatively strong jet quenching observed in A+A collisions at RHIC [13,14] and the LHC [2,69,3]. This refers to the suppression of particle production with respect to ‘naive’ extrapolations from p+p collisions, as measured by the nuclear modification factor \( R_{A+A} \), or by the suppression of the ‘away’ jet in di–hadron correlations (see the right panel in Fig. 5). Such phenomena indicate that the deconfined medium created in the intermediate stages of a HIC is very opaque, including for relatively hard probes with \( k_\perp = 2 \div 20 \gev \). Within pQCD, the dominant mechanism for parton energy loss in the medium is gluon radiation stimulated by the scattering off the medium constituents [70]. But this mechanism seems unable to explain the strong suppression seen at RHIC, although definitive conclusions cannot be drawn, given the difficulty to perform realistic calculations. At strong coupling, AdS/CFT calculations [71,72,73] suggest that a new mechanism for energy loss should come into play — the medium–induced parton branching [71]. It is likely that all these mechanisms will coexist for realistic values of the coupling. Interestingly, the RHIC data for elliptic flow (which refer to relatively soft particles with \( k_\perp \simeq 1 \gev \)) and those for jet quenching (where \( k_\perp \simeq 5 \gev \) is semi–hard) can be simultaneously accommodated by models inspired by the respective AdS/CFT predictions at strong coupling, as explained by J. Noronha [74] at ISMD2010.

With the advent of the LHC, it became possible to perform calorimetric measurements of the energy loss for real jets (as opposed to leading particles). The first results in that sense [2,3] are already impressive: for central Pb+Pb collisions and for very hard jets with \( E_T \geq 100 \gev \), the coupling constant \( \alpha_s = g^2/4\pi \) in QCD can never become arbitrarily large, because of asymptotic freedom, but it can be of order one at scales of order \( \Lambda_{\text{QCD}} \) and this might lead to an effectively strong–coupling behaviour. In fact, for \( 2T_c < T < 5T_c \), \( g^2 N_c \simeq 7 \div 10 \) is indeed quite large.
one sees highly asymmetric dijet events, characterized by a large energy imbalance (a few dozens of GeV) between two back-to-back jets. This rises the challenge of understanding jet evolution (fragmentation and energy loss) in a dense medium — a topic addressed by T. Trainor at ISMD2010 [75]. It remains as an interesting open question whether these new results at the LHC can be fully explained by weak coupling calculations (as one may expect for such hard jets), or if there is still place for non-perturbative phenomena associated with the interactions between the radiation and the medium.

The QGP created in a heavy ion collision keeps expanding and hence it cools down, until the (local) density becomes so low that it must hadronize. The abundances of strange and non-strange mesons and baryons produced in heavy ion collisions in a wide range of collision energies are consistently described by statistical physics within the ‘hadron resonances gas’ model — that is, as an ideal gas of hadrons with ‘freeze-out’ temperatures and baryon chemical potentials that are a function of collision energy only. Moreover, the freeze-out temperature extracted at the highest RHIC energy, of about 170 MeV (at zero baryon chemical potential), is very close to the critical temperature for deconfinement from lattice QCD. In his contribution to ISMD2010, J. Cleymans [76] has reviewed the status of chemical equilibration in HIC at the freeze-out. In particular, he has emphasized that the net baryon density at the freeze-out, which is small for both very high (RHIC) and very low (FAIR) energies, should exhibit a maximum value $\rho_0 \simeq 0.15 \text{ fm}^{-3}$ corresponding to a freeze-out temperature $T \sim 140$ MeV.

5 Soft interactions: the quest for a theory

The physics of soft interactions at high energy — as relevant, e.g., for the calculation of the total, elastic, and diffractive cross-sections in hadron–hadron collisions, or for the description of the underlying event — has been the main scope of the early ISMD meetings and a main focus for all its subsequent editions over the last forty years. But in spite of undeniable progress, this remains the topics for which we have the less satisfactory understanding from first principles. There is, of course, a ‘good’ reason for that: this topics lies on the ‘dark’ (non-perturbative) side of QCD, and unlike other related topics like hadron spectroscopy it cannot be addressed via lattice calculations. The progress in this field can be associated with two main directions: (i) a shift in the borderline between Terra Incognita (the non-perturbative sector of QCD) and Mare Nostrum (the pQCD ‘sea’ that we know how to navigate over), and (ii) the development of new models, more sophisticated and better motivated, which heuristically describe some of the ‘dark regions’ of Terra Incognita.

With increasing energy, many of the phenomena that were traditionally associated with soft interactions are in fact controlled by the semi-hard ones. As discussed in the previous sections, the wavefunction of an energetic hadron is dominated by small-$x$ gluons with transverse momenta of the order of the saturation momentum, which is semi-hard ($Q_s \sim 1$ GeV) at the current energies. These gluons are liberated in a high-energy collision and they form the bulk of the ‘underlying event’ (UE) — the ensemble of radiation accompanying a hard parton–parton interaction and which are not directly associated with that interaction. After being liberated, the gluons can still evolve in the ‘final state’, via multiparticle interactions (which become important at high energies), fragmentation (radiation of new quanta), and hadronization. Some of these processes can still be treated in perturbation theory. And the genuinely soft ones, like hadronization, should not affect observables like the energy flow or the single-particle spectra (although they are of course essential for rapidity gaps and diffractive). Thus, besides the
hard scattering, there is a significant part of the UE which in principle can be described within pQCD. For the remaining, non–perturbative, part one has to resort on models involving free parameters, like the Lund string model for hadronization.

Such a hybrid description of the final state, combining perturbative and non–perturbative ingredients, has been implemented in Monte–Carlo (MC) event generators \[77\], which in practice are quite successful even though they do not fully reflect our current fundamental understanding (on the pQCD side). For instance, the non–linear physics associated with gluon saturation in the initial wavefunctions and with multiparticle interactions in the final state is not properly included, but only mimicked by the introduction of an energy–dependent cutoff \(p_0(\sqrt{s})\) (actually, an Ersatz of \(Q_s(x)\)) separating ‘hard’ and ‘soft’ interactions, and by the heuristic resummation of multiple scattering in the eikonal approximation. This treatment neglects coherence phenomena expected at high density and thus overestimates the cross–sections for the production of ‘minijets’. To compensate for that, the separation scale \(p_0(\sqrt{s})\) must be allowed to grow very fast with the energy and thus become even larger than the actual saturation scale \((p_0 \simeq 5 \text{ GeV at the LHC } \[75\])\). In principle, one could avoid such heuristic short–cuts and rely on the CGC formalism for a proper description of the transition region around \(Q_s\). In practice, however, this formalism seems difficult to reconcile with the conventional MC generators.

To understand this difficulty, one should remember that, in general, the predictions of quantum mechanics cannot be reproduced by a classical stochastic process, so like a Monte–Carlo. This only works under specific approximations, which neglect interference effects (or treat them only approximately) and whose applicability domains are limited and often mutually exclusive. Two examples in that sense are the collinear factorization at high \(Q^2\), which lies at the basis of most of the current MC generators \[77\], and the CGC factorization encompassing the physics of high gluon density to LLA. These two formalisms have very different mathematical structures, so it is hard to see how to merge them within a same event generator: collinear factorization is based on the notion of (integrated) parton distribution function and its DGLAP evolution, while the CGC formalism uses the language of classical color fields distributed with a functional weight function obeying JIMWLK. The Lund Dipole Cascade Model \[42\] provides a meaningful interpolation between the two, but it cannot accurately describe the high–\(Q^2\) regime which is so important for jet physics or BSM searches at the LHC. One can also envisage event generators based on the CGC approach \[28\], but once again they should not be accurate enough at high \(Q^2\) (and rather cumbersome in practice).

In view of this, it is important to keep developing the conventional MC event generators (Ariadne, Herwig, Hijing, Pythia, Sherpa etc \[77, 78\]), not only by successive ‘re-tunings’ of the free parameters, but also by (at least semi–heuristically) improving the treatment of the semi–hard sector, using guidance from the recent theoretical progress. The need for such an improvement is also demonstrated by the difficulty of the present generators to reproduce the soft and semi–hard parts of the underlying and minimum-bias events at the LHC, as reviewed at ISMD2010 by R. Field \[79\]. To quote Rick, “Pythia Tune DW describes the LHC distributions fairly well, but not perfectly”. More precisely, this particular Pythia tune which was calibrated to fit the UE data at CDF at 1.96 TeV is systematically underestimating the hadronic activity (in terms of either multiplicity or transverse energy) in p+p collisions at the LHC for the underlying events accompanying a leading jet (CMS) or a leading particle (ATLAS) with transverse momentum \(k_{\text{jet}}^\perp\) of a few GeV (say, below 10 GeV). The discrepancies become higher with increasing energy at a given \(k_{\text{jet}}^\perp\) (from 900 GeV to 7 TeV) and for decreasing \(k_{\text{jet}}^\perp\) at a given energy (see Fig. 10). Moreover, they persist (although to a lower degree) after re-tuning the MC on
Figure 10: CMS preliminary data (dots) for particle production in p+p collisions vs. the respective predictions of the Pythia DW tune (the continuous lines) [79]. Left: 2 different center-of-mass energies. Right: two different transverse resolutions (as determined by the transverse momentum of the leading jet).

the basis on the CMS data (‘Pythia Tune Z1’) [79]. This shows that, beyond merely re-tuning, we need a better description (like CGC-inspired models) for multiparticle interactions and the partonic distributions at small x and semi-hard $k_{\perp}$.

Another traditional topic of ISMD, which has been also present at the 2010 edition, concerns the attempts to model the low $k_{\perp}$ part of the multiparticle production. Some of these models are merely fits with a heuristic physical interpretation. For instance, A. Rostovtsev [80] has shown that a combination (somewhat reminiscent of the photon spectrum in the Solar flares) of a thermal-like exponential and an inverse power law can describe the low–$k_{\perp}$ part of the spectrum within a wide range of energies and for various processes (p+p, $\gamma + \gamma$, DIS, A+A). G. Wilk [81] emphasized that a power–law spectrum is not necessarily a signal of perturbative–like, hard QCD, behaviour: a power distribution of the Tsallis type can be also generated via fluctuations in the (local) temperature associated with a thermal distribution. Moreover fluctuations in temperature ($T$) are equivalent to fluctuations in the interaction volume ($V$) provided the total energy $E$ is kept constant, since $E \sim VT^4$. W. Ochs [82] has argued that the zero momentum limit of the spectra should be independent of the energy since dominated by the bremsstrahlung of soft gluons which cannot resolve the partonic structure of the final state, but only the overall color charge of the primary sources. V. Abramovsky [83] used a model combining valence quarks and string hadronization to argue that particle production should be different in p+p vs. p+p collisions, a difference that could be observed by comparing high–multiplicity events at the LHC and the SPS (at the common energy $\sqrt{s} = 900$ GeV). S. Todorova [84] has described a refinement of the Lund string model in terms of a helix string which can describe the ‘bump’ observed in the LEP (DELPHI) data at $k_{\perp} \sim 0.5$ GeV. H. Grönvist [85] proposed to measure the elastic p+p cross–section at the LHC by tagging the forward bremsstrahlung photons with the Zero Degree Calorimeter at the CMS. Such a measurement could also be used to check the relative alignment of the ZDCs and of the Roman Pot detectors.

Besides the single particle spectra, the correlations in the multiparticle production at soft momenta represent a challenge for the theorists and the topics of one of the traditional sessions of ISMD. In his introductory talk to this session, A. Bialas [86] made some remarks on the possible origin of four important classes of correlations: (i) the negative binomial distribution
in the multiplicity, which reflects the particle production by a superposition of various sources, possibly grouped into ‘clans’; (ii) the forward–backward correlations in rapidity, which give information about the early stages of the collision; (iii) the balance functions for pairs of charged particles, as measured by STAR \[87\], which are narrow in rapidity (and narrower in central A+A collisions as compared to peripheral ones, or to p+p), thus suggesting that charges are created in the late stages of the collisions, just before the freeze-out, and (iv) the HBT correlations, for which the descriptions based on the Lund string picture \[88\] and respectively on intermittency \[89\] can be related to each other by assuming a fluctuating string tension. T. Csorgo \[90\] proposed the measurement of the mass of the $\eta'$ meson as a probe of chiral symmetry restoration in A+A collisions. The expected reduction of this mass in the medium should lead to an increase of the abundance of $\eta'$ in the final state, which in turn can be measured using Bose–Einstein correlations. The respective analysis of the RHIC data suggests indeed an in–medium reduction of the $\eta'$ mass by at least 200 MeV \[90\].

6 Conclusions

The 2010 edition has marked the entrance of the ISMD series of conferences in the LHC era. It has demonstrated the vitality of the original theme of this meeting — the physics of multiparticle interactions — which nowadays lies at the heart of the QCD physics at the LHC. It has also demonstrated the capacity of this meeting to permanently renew itself by integrating new themes — notably, the hard and semi–hard partonic interactions —, thus following and stimulating the progress on both theoretical and experimental sides. All these themes have met with important developments over the recent years, that I tried to succinctly summarize here, with emphasis on the contributions presented at ISMD2010. All these themes need further progress in order to match the exigences of the present day high–energy experiments. The ISMD series offers a privileged, almost unique, opportunity for fruitful exchanges and cross-fertilizations between these various themes. Forty years after, the International Symposium on Multiparticle Dynamics is more than ever at the center of the scientific debate. ISMD, Happy Anniversary!

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References

[1] V. Khachatryan et al. [CMS Collaboration], JHEP 1009 (2010) 091 [arXiv:1009.4122 [hep-ex]].
[2] G. Aad et al. [ATLAS Collaboration], arXiv:1011.6182 [hep-ex].
[3] The CMS Collaboration, arXiv:1102.1957 [nucl-ex].
[4] M. Grazzini, these proceedings.
[5] M. Worek, these proceedings.
[6] C. F. Berger et al., Phys. Rev. Lett. 106 (2011) 092001 [arXiv:1009.2338 [hep-ph]].
[7] R. Frederix, these proceedings.
[8] T. Aaltonen et al. [CDF Collaboration], arXiv:1101.0034 [hep-ex].
[9] S.V. Chekanov, Experimental Summary of ISMD2010, these proceedings.
[10] G. Rodriguez, these proceedings.
[11] I. Arsene et al. [BRAHMS Collaboration], Nucl. Phys. A 757 (2005) 1 [arXiv:nucl-ex/0410002].
[12] B. B. Back et al. [PHOBOS Collaboration], Nucl. Phys. A 757 (2005) 28 [arXiv:nucl-ex/0410022].
[13] J. Adams et al. [STAR Collaboration], Nucl. Phys. A 757 (2005) 102 [arXiv:nucl-ex/0501009].
[14] K. Adcox et al. [PHENIX Collaboration], Nucl. Phys. A 757 (2005) 184 [arXiv:nucl-ex/0410003].
[15] J. Adams et al. [STAR Collaboration], Phys. Rev. Lett. 97 (2006) 152302 [arXiv:nucl-ex/0602011].
[16] I. Arsene et al. [BRAHMS Collaboration], Phys. Rev. Lett. 93 (2004) 242303 [arXiv:nucl-ex/0403005].
[17] J. Adams et al. [STAR Collaboration], Phys. Rev. Lett. 97 (2006) 152302 [arXiv:nucl-ex/0602011].
[18] E. Braidot for the STAR collaboration, arXiv:1005.2378.
[19] C. Marquet, these proceedings.
[20] A. Cooper-Sarkar, these proceedings.
[21] F. Hautmann, these proceedings.
[22] L.N. Lipatov, Sov. J. Nucl. Phys. 23 (1976) 338; E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Zh. Eksp. Teor. Fiz 72, 3 (1977); Ya.Ya. Balitsky, L.N. Lipatov, Sov. J. Nucl. Phys. 28 (1978) 822.
[23] I. Cherednikov, these proceedings.
[24] V.S. Fadin and L.N. Lipatov, Phys. Lett. B429 (1998) 127; G. Camici and M. Ciafaloni, Phys. Lett. B430 (1998) 349.
[25] G.P. Salam, JHEP 9807 (1998) 19; M. Ciafaloni, D. Colferai, Phys. Lett. B452 (1999) 372; M. Ciafaloni, D. Colferai, and G.P. Salam, Phys. Rev. D60 (1999) 114036.
[26] D. Colferai, F. Schwennsen, L. Szymanowski and S. Walon, JHEP 1012 (2010) 026 [arXiv:1002.1365 [hep-ph]].
[27] L.V. Gribov, E.M. Levin, and M.G. Ryskin, Phys. Rept. 100 (1983) 1; A.H. Mueller and J. Qiu, Nucl. Phys. B268 (1986) 427.
[28] F. Gelis, E. Iancu, J. Jalilian-Marian, and R. Venugopalan, “The Color Glass Condensate”, arXiv:1002.0333 [hep-ph].
[29] A. H. Mueller and N. Navelet, Nucl. Phys. B 282 (1987) 727.
[30] Y. V. Kovchegov and H. Weigert, Nucl. Phys. A 784 (2007) 188 [arXiv:hep-ph/0609090]. I. Balitsky, Phys. Rev. D 75 (2007) 014001 [arXiv:hep-ph/0609105].
[31] I. Balitsky and G. A. Chirilli, Phys. Rev. D 77 (2008) 014019 [arXiv:0710.4330 [hep-ph]].
[32] D. N. Triantafyllopoulos, Nucl. Phys. B648 (2003) 293-316. [hep-ph/0209121].
[33] E. Iancu, M. S. Kugeratski, D. Triantafyllopoulos, Nucl. Phys. A808 (2008) 95. [arXiv:0802.0343 [hep-ph]].
[34] A. M. Stasto, K. J. Golec-Biernat, J. Kwiecinski, Phys. Rev. Lett. 86 (2001) 596-599. [hep-ph/0007192].
[35] F. Gelis, T. Lappi and R. Venugopalan, Phys. Rev. D 78 (2008) 054019 [arXiv:0804.2630 [hep-ph]].
[36] F. Dominguez, C. Marquet, B. -W. Xiao, F. Yuan, arXiv:1101.0715 [hep-ph].
[37] K. Rummukainen and H. Weigert, Nucl. Phys. A 739 (2004) 183 [arXiv:hep-ph/0309306].
[38] J. L. Albacete and C. Marquet, Phys. Rev. Lett. 105 (2010) 162301 [arXiv:1005.4065 [hep-ph]].
[39] H. Jung, these proceedings.
[40] M. Deak, these proceedings.
[41] E. Avsar and E. Iancu, Nucl. Phys. A 829 (2009) 31-75 [arXiv:0906.2683 [hep-ph]].
[42] G. Gustafson, these proceedings.
[43] S. Munier and R. Peschanski, Phys. Rev. Lett. 91 (2003) 232001; E. Iancu, A.H. Mueller and S. Munier, Phys. Lett. B606 (2005) 342.
[76] J. Cleymans, these proceedings.
[77] A. Buckley et al., arXiv:1101.2599 [hep-ph].
[78] W. T. Deng, X. N. Wang and R. Xu, Phys. Rev. C 83 (2011) 014915 arXiv:1008.1841 [hep-ph].
[79] R. Field, these proceedings.
[80] A. Rostovtsev, these proceedings.
[81] G. Wilk, these proceedings.
[82] W. Ochs, these proceedings.
[83] V. Abramovsky, these proceedings.
[84] S. Todorova, these proceedings.
[85] H. Grönsqvist, these proceedings.
[86] A. Bialas, these proceedings.
[87] M. M. Aggarwal et al. [STAR Collaboration], Phys. Rev. C 82 (2010) 024905 arXiv:1005.2307 [nucl-ex].
[88] T. Csorgo, S. Hegyi, T. Novak and W. A. Zajc, Acta Phys. Polon. B 36 (2005) 329 arXiv:hep-ph/0412243.
[89] A. Bialas, Acta Phys. Pol. B 23 (1992) 561; AIP Conf. Proc. 828 (2006) 409.
[90] T. Csorgo, these proceedings.