Improved receiver autonomous integrity monitoring algorithm

Meina Li¹, Zhiuan Hao, Lei Zhang¹

¹Radar Research Laboratory of Beijing Institute of Technology, Key Laboratory of Electronic and Information Technology in Satellite Navigation (Beijing Institute of Technology), Ministry of Education, No. 5 Zhongguancun South Street, Haidian District, Beijing, People’s Republic of China
E-mail: 1294395452@qq.com

Abstract: The integrity monitoring algorithm guarantees that the receiver can get the correct positioning and speed, which has very important theoretical significance and practical application value. It is one of the most important development contents of the satellite navigation system. Based on the introduction of the traditional least square residual method, this paper compares the binomial fitting time-assisted receiver autonomous integrity monitoring (RAIM) algorithm and a grey model time-assisted RAIM algorithm and finds that the latter can well solve the clock error accumulation problem in the binomial fitting time-assisted RAIM algorithm. In the faulty satellite detection experiments with the same pseudo-range error, the minimum detectable pseudo-range error of the grey model time-assisted RAIM algorithm is 15 m, while the binomial fitting time-assisted RAIM algorithm is 28 m. Incidentally, the detection probability of a faulty satellite with 20 m pseudo-range error of the binomial fitting time-assisted RAIM algorithm is 10%, whereas the grey model time-assisted RAIM algorithm is 81%.

1 Introduction

Integrity is a key point of satellite navigation and positioning systems and satellite navigation receivers require the ability to monitor the integrity of the system. Receiver autonomous integrity monitoring (RAIM) uses the real-time observations to determine whether the satellite is faulty [1]. When we detect a faulty satellite, the receiver must have the ability to warn to ensure the normal operation of the system and find the faulty satellite at the same time.

Early satellite navigation and positioning technology is not very developed, there may be a lot of positioning problems, in order to solve these problems better, the study of receiver integrity starts from the beginning of the 20th century. As early as 1987, Kalafus proposed the concept of autonomous integrity monitoring at the Institute of Navigation meeting. During the 1980s and 1990s, Brown and Hwang [2] and Lee YC [3] started the research on RAIM algorithms. They proposed the snapshot algorithm and the filtering algorithm one after the other. Since then, the autonomous receiver integrity monitoring algorithm are developed on the basis of the two studies.

Due to the advantages of the snapshot algorithm are simple calculation and good real-time performance, it becomes the main trend of development of RAIM algorithm. It was later discovered that the geometrical distribution of space satellites would affect the availability of RAIM algorithms. In the late 1980s, Brown proposed the concept of radius protection [2]. The earliest research on RAIM algorithm performance was conducted in the GPS / GALILEO system by Ochieng.

The domestic research on RAIM algorithm starts late, but the overall development is fast. Mi Jinzhong studied the integrity of the satellite constellation of the GALILEO system, and divided the integrity of the satellite navigation system into three parts [4]: the space constellation part, the detection station part and the user receiver part. Sun Guangliang combined the snapshot algorithm with the Kalman filtering algorithm, and proposed a receiver autonomous integrity monitoring algorithm based on statistical theory and time-domain transform. This method not only reduced the number of visible satellites, but also improved the detection efficiency of the faulty satellite. Chen Xiaoping and Teng Yunlong [5] came up with a clock-assisted RAIM algorithm which ensured that a faulty satellite can be effectively detected when there were only four visible satellites. RAIM algorithm performance has a great relationship with the number of satellites and the amount of observations. While the satellite is shielding or in other harsh environment, RAIM algorithm and its availability were analysed by Guo Yubo. The performance indexes of RAIM algorithm under different environment were obtained. In recent years, the RAIM algorithm based on robust estimation, method of usability analysis and calculation of the protection level limits based on the change of elevation mask angle and error model parameters and the time-assisted autonomous integrity using Monte Carlo simulation emerged. The integrity monitoring algorithm is the guarantee that the receiver can achieve the normal positioning and speed regulation. Therefore, the research on the integrity has a very important theoretical significance and practical application value, which is also a crucial part for the development of satellite navigation systems.

2 Receiver autonomous integrity monitoring

2.1 Least squares residual method

When the user receiver achieve positioning, a single system requires at least four pseudo-range observations, dual system requires at least five pseudo-range observations, and three systems need at least six pseudo-range observations [6]. The RAIM algorithm uses the redundant information of the satellite navigation receiver to positioning, for the single, double and triple system receivers need at least five, six and seven pseudo-range observations in order to achieve fault detection, respectively [7]. For single-system single-failure receivers, the linearised system pseudo-range observation equation is

\[ b = G \cdot \Delta x + \varepsilon, \]  

(1)

where \( b \in R^{N \times 1} \), \( N \geq 5 \) is the difference between the measured pseudo-range after error correction and the predicted pseudo-range, and \( G \in R^{N \times 4} \) represents the observation matrix calculated by \( N \) satellites:

\[
G = \begin{bmatrix}
\varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} & -1 \\
\varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} & -1 \\
\vdots & \vdots & \vdots & \vdots \\
\varepsilon_{N1} & \varepsilon_{N2} & \varepsilon_{N3} & -1
\end{bmatrix}.
\]  

(2)
\( \Delta x \in \mathbb{R}^{n \times 1} \) represents the matrix of the difference between the actual position and the predicted position of the user receiver and the amount of clock error of the receiver, including a total of three position components and the receiver clock deviation, which can be expressed as:

\[
\Delta x = [x \ y \ z \ \Delta t]^T.
\]

\( \epsilon \in \mathbb{R}^{n \times 1} \) represents the pseudo-range measurement error. The least squares method shows that the solution to the pseudo-range observation equation is

\[
\Delta \hat{x} = (G^T G)^{-1} G^T b.
\]

Pseudo-range residual vector \( \omega \) can be expressed as:

\[
\omega = b - G \cdot \Delta \hat{x} = b - G (G^T G)^{-1} G^T b = S \epsilon,
\]

where \( S = I - G (G^T G)^{-1} G^T \), the sum of square residuals of each satellite is

\[
\text{SSE} = \omega^T \omega.
\]

In practice, \( T_X = \sqrt{\text{SSE}/(N-4)} \) is usually used as a statistical measurement, and \( T_X = \sqrt{\text{SSE}/(N-4)} \) calculated in real time is compared with the detection threshold \( T_D \). If \( T_X > T_D \), the fault is detected and the user is warned.

### 2.2 Time-assisted RAIM algorithm

The predicted clock error obtained by the clock difference

Model [8] is taken into the pseudo-range equation to obtain:

\[
\begin{bmatrix}
G \\
0 \\
0 \\
\lambda
\end{bmatrix} \cdot \Delta x +
\begin{bmatrix}
\epsilon \\
\Delta \mathbf{b}
\end{bmatrix} =
\begin{bmatrix}
b \\
\mathbf{db}(t)
\end{bmatrix}
\]

where \( \lambda \) is the normalised parameter of the pseudo-range measurement error and the prediction error of the clock error, and \( \epsilon_i \) is the prediction error of the clock error with \( \sigma_i \) as the root mean square.

\[
\lambda = \frac{-\sigma}{\sigma_i}
\]

### 2.2.1 Binomial fitting time-assisted RAIM algorithm:

Based on previous experimental data, we can use the binomial fitting model to fit the variation of clock bias. Assuming that the navigation system is working properly and the user receiver's clock frequency drift is stable, then the clock model is

\[
b(t) = b_0 (t - t_0)^2 + b_1 (t - t_0) + b_0,
\]

where \( b_0, b_1, b_0 \) can be obtained by binomial fitting. If the receiver clock is stable, the root mean square error based on this fit model to predict the clock error is

\[
\sigma_b = \frac{\sigma_{\text{clock}}}{\sqrt{m - 3}}
\]

In the above formula, \( m \) is the number of statistically independent sampling points in all the sampled data; \( \sigma_{\text{clock}} \) is the error root mean square of the receiver clock error:

\[
\sigma_{\text{clock}} = \sigma \times \text{TDOP},
\]

where \( \sigma \) is the error root mean square of the pseudo-range measurement and TDOP is the time accuracy geometric factor.

### 2.2.2 Grey model time-assisted RAIM algorithm:

Suppose the original clock sequence is

\[
T = [T(1), T(2), \ldots, T(L)].
\]

The procedure for establishing a grey model [9] for a sequence \( T \) is as follows:

i. Accumulate the sequence \( T \) to get the cumulative sequence \( T^{(1)} \):

\[
T^{(1)}(k) = \sum_{i=1}^{k} T(i), \; k = 1 \sim (L - 1).
\]

ii. Use the cumulative sequence \( T^{(1)} \) to establish the differential equation:

\[
\frac{dT^{(1)}}{dt} + aT^{(2)} = u.
\]

Solve the parameters \( a \) and \( u \) according to the least square method:

\[
[a \ u]^T = (M^T M)^{-1} M^T V.
\]

\[
V = [T(2) \ T(3) \ \cdots \ T(L - 1)]^T.
\]

 iii. Calculate the prediction sequence \( \tilde{T}^{(1)} \) of the cumulative sequence \( T^{(1)} \) according to the parameters \( a \) and \( u \):

\[
\tilde{T}^{(1)}(k + 1) = T^{(1)}(k) - \frac{a}{M} T^{(2)}(k) + \frac{u}{a}
\]

iv. Difference the prediction sequence \( \tilde{T}^{(1)} \) to obtain the prediction sequence of the sequence \( T \):

\[
\tilde{T}(k) = \tilde{T}^{(1)}(k) - \tilde{T}^{(1)}(k - 1), \; k \geq 2
\]

v. Calculate the predicted value \( \tilde{T}(L + 1) \) of the original clock sequence and the prediction error \( \delta_b^{(1)} \):

\[
\tilde{T}(L + 1) = T(L) + \tilde{T}(L)
\]

\[
\delta_b^{(1)} = \frac{1}{L} \sum_{k=1}^{L-2} (T(k) - \tilde{T}(k))^2
\]

### 3 Algorithm performance test results

In this paper, the least square residual method, the binomial fitting time-assisted RAIM algorithm and the grey model time-assisted RAIM algorithm are simulated. Fig. 1 shows the RAIM detection variable simulated by different algorithms. It can be seen that when there is no faulty satellite, the detection measure constructed by the least-squares residual method and the binomial fitting time-assisted algorithm is very close. The platform for performance testing of RAIM algorithms is a compatible interoperability receiver. System test processor is clocked at 456 MHz TMS320 C6748 DSP, and DSP uses the received ephemeris parameters, pseudo-range, carrier phase, Doppler and other information for subsequent message processing and receiver positioning.

In a short time, the binomial fitting time-assisted RAIM algorithm has a better prediction of the clock error, but as time
goes by, the clock error of binomial fitting increases fast and when the time approaches 1 min, it cannot be used for normal faulty satellite detection. Although the clock error in the grey model is larger, but it does not be influenced by the time, this algorithm has better detection efficiency which can be seen from Figs. 2 and 3.

Finally, the faulty satellite detection simulation results of the least square residual method, binomial fitting time-assisted RAIM algorithm and grey model time-assisted RAIM algorithm are shown in Fig. 4. When there is only one faulty satellite, it can be seen that the grey model time-assisted RAIM algorithm has the best detection efficiency with the smallest detectable error of 15 m, the binomial fitting of 20 m and the least square residual of 28 m. When there is one faulty satellite with the same 20 m pseudo-range error in this three methods, the binomial fitting method has a detection probability of 10% and the grey model has a detection probability of 81%.

4 Conclusion

The experimental results show that the grey model clock-assisted RAIM algorithm proposed in this paper can effectively solve the problem of clock error accumulation in the binomial fitting clock-assisted RAIM algorithm. In the same experimental environment, the detection efficiency of grey model clock-assisted RAIM algorithm is better, which verifies the validity and usability of the algorithm.

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6 References

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