The influence of the 3rd harmonic of the distorted primary current on the self-generation of the inductive current transformers

Michal Kaczmarek Senior Member IEEE, Ernest Stano
Institute of Mechatronics and Information Systems of the Lodz University of Technology, Stefanowskiego 22, 90-537 Lodz, Poland
Corresponding author: Michal Kaczmarek (e-mail: michal.kaczmarek@p.lodz.pl).

ABSTRACT This paper investigates for transformation of the distorted current the low order higher harmonic self-generation phenomenon of tested inductive current transformers. The influence of the main component and the 3rd harmonic of the distorted primary current on the RMS values of the 5th, 7th and 9th order self-generated higher harmonics is analyzed. This research indicate that the change of the RMS value of the 3rd harmonic of the distorted primary current causes variation of the lower order higher harmonics generated in to the secondary current. Therefore, this is a challenging phenomenon which have to be considered while compensating the values of current error and phase displacement of the inductive CTs during transformation of the distorted currents.

INDEX TERMS self-generation, distorted primary current, inductive current transformer, low order higher harmonics, current error, phase displacement.

I. INTRODUCTION

The current transformers (CT) with a magnetic core (inductive CT) are commonly used in the power grid to transform high values of currents into the acceptable level for measurement or protection equipment. These devices have gained their popularity due to the reliability and required accuracy of transformation ensured for the sinusoidal currents of frequency up to 50 Hz (60 Hz). Previous works show also their applicability for transformation of the distorted currents within the bandwidth up to 5 kHz [1]–[9]. Their wideband operation is required due to the low power quality and significant content of higher harmonics in current and voltages of the power grid [1], [4]–[7], [10]–[17]. Therefore, the power billing must be considered under the non-sinusoidal conditions [18]–[20]. Nowadays, there are no defined normative reference and test procedures for evaluation of the accuracy of inductive CTs for transformation of the distorted current. The standard IEC 61869-6:2017 applies only to the low power transformers [21], [22]. The main source of transformation errors of an inductive CT is the excitation current of the magnetic core. It should be emphasized that due to the non-linear shape of the magnetizing characteristic of the magnetic core, the transformation accuracy of the inductive CT depends on the value of the magnetic flux density and thus on the RMS values of the primary current harmonics and the load on the secondary winding [1], [3], [4], [19], [23]. The paper [5] presents analyses on the transformation accuracy of inductive CTs under real operating conditions in a distribution grid. According to the authors, the distortion of the primary current has little influence on their values of current error and phase displacement and may be neglected. However, in the article it is not considered the phenomenon of self-generation of the lower order higher harmonics (orders from 2nd to 10th) in to the secondary current of the inductive CT. By the concept of the self-generation it is understood in this paper that the CT introduces into the secondary current additional low order higher harmonics even if they are not present in the primary current. The analyses presented in the article [3] show that the values of current error and phase displacement of the transformation of the low order higher harmonics by inductive CTs depend significantly on their RMS values and the phase shift between them. The shape of the transformed primary current changes the value of the maximum magnetic flux density in the magnetic core. Therefore, it affects the values of the low-order harmonics self-generated in to the secondary current. Furthermore, during the evaluation of the accuracy of the inductive CT it is necessary to consider the phase shift between the generated and transformed harmonics. Obviously,
the value of the magnetic flux density in the magnetic core also depends on the value and load power factor of the secondary winding of the tested inductive CT. Therefore, its transformation accuracy may be determined only for a certain range and type of its load as well as the RMS value of the distorted primary current. The articles [24]–[27] present the methods of compensating the low order higher harmonics self-generated by the inductive CTs. These solutions do not consider the change in the value of the magnetic flux density in the magnetic core cause by the distortion of the transformed primary current including the influence of the 3rd higher harmonic. Therefore, all the changes in the values of the self-generated higher harmonics due to the non-linearity of the magnetization characteristics of the magnetic core are not compensated. However, the main influence on the level of self-generated higher harmonics by inductive CTs have the RMS values of the main component and the low order higher harmonics. The paper [28] gives a description of the method for determining the inductive CT properties evaluated in increased frequency range from 50 Hz to 1 kHz of transformed primary current based on digital signal processing using a measuring card with analog-to-digital converters. The comparison is made between voltages from resistive current shunts placed in the secondary circuits of the test CT and the reference transducer when they transform the same primary current. In the paper [1] measured values of the composite error were used to determine the accuracy of inductive CTs during transformation of distorted currents. In the papers [3], [6] the method was proposed to characterize the accuracy of inductive CTs for transformation of distorted currents composed of a fundamental component and one higher harmonic with adjustable phase angle. The values of current error and phase displacement are determined for a given harmonic during transformation of a non-sinusoidal current. In the studies [7], [29] the method of rated ampere turns was used to provide rated operating conditions for inductive CTs with a rated primary current of 300 A and 500 A. The study [8] presents a distorted current generator designed for testing inductive CTs. This device enables generation of a non-sinusoidal waveform with higher harmonic content similar to those occurring in typical distribution grids. Another method is to use a test system consisting of the wideband high-current transformer and the power source capable of generating distorted currents [30]–[32].

This paper investigates the effect of the lower order higher harmonic self-generation on the harmonic transformation accuracy of the distorted currents by the tested inductive CTs. The influence of the 3rd harmonic present in the distorted primary current on the RMS values of the self-generated 5th, 7th and 9th order higher harmonics is determined and analysed. This research indicate that the change in the self-generation is caused by the change in the RMS value of the 3rd higher harmonics of the distorted primary currents. Therefore, this is the one of the main challenging phenomenon to consider in the compensation of current error and phase displacement of the inductive CTs for the transformation of distorted currents. The paper presents the frequency characteristics of transformation errors of tested CTs in the range from 50 Hz to 5 kHz. Selected inductive CTs with the rated current ratio of 100 A / 5 A and 300 A / 5 A are tested in conditions of the rated ampere turns by using the additional primary winding that is wounded through the window of their magnetic core. The number of its turns results from their rated current ratio. In the case of the 100 A / 5 A CT it was 20 turns, whereas for a 300 A / 5 A CT it was 60 turns. The measurements were carried out for 100% and 25% of their rated load of the secondary winding with the power factors equal to resistive 1 and inductive 0.8. The expanded uncertainties of determine the values of current error and phase displacement by the developed method and used measurement system are evaluated in accordance with the principles and recommendations of JCGM (Joined Committee for Guides in Metrology) [33].

The novelty of the paper concern:

- investigation of the influence of the RMS value of the 3rd higher harmonic on the level of the self-generation of others low order higher harmonics,
- evaluation of the influence of the load power factor of the secondary winding of the tested CT on the harmonic transformation accuracy of the distorted currents,
- calculation of the measurement uncertainty of the values of current error and phase displacement for the main and higher harmonics.

II. DEVELOPED METHOD AND USED MEASUREMENT SYSTEM

Testing of the transformation accuracy of the distorted current by the inductive CT of the pass-through type was carried out under its rated ampere turns conditions. The tested object is then winded with an additional primary winding which number of turns \( z_D \) corresponds to its rated current ratio in accordance with the formula:

\[
z_D = \frac{I_{2N}}{I_{1N}}
\]

\( I_{1N} \) – the RMS value of the rated current of the primary winding.

\( I_{2N} \) – the RMS value of the rated current of the secondary winding.

As a consequence, equivalent conditions as in normal state of operation of this CT are obtained as results from the equation:

\[
I_{1N} \cdot z_1 = I_{1AN} \cdot z_D
\]

\( I_{1AN} \) – the RMS value of the rated current of the additional primary winding of inductive CT under conditions of rated ampere turns.

\( z_1 \) – the number of turns of the primary winding.
When testing the accuracy of a current transformer under rated ampere turns condition it is not necessary to use the expensive high-current test systems to generate distorted currents and the reference primary current source [7], [22], [29]. The measurement system is shown in Figure 1.

\[ \Delta I_{hk} = \frac{\left| U_{D} \right|^2}{U_{R}} - 2 \frac{U_{D} U_{S}}{R_{S}} \cos \phi_{A} + \frac{U_{S}^2}{R_{S}} \cdot 100\% \] (3)

\[ U_{hk} \] - the RMS value of the hk voltage harmonic on current shunt \( R_{D} \),
\[ \phi_{hk} \] - the phase angle between hk voltage harmonic on current shunt \( R_{D} \) and the hk voltage harmonic on current shunt \( R_{S} \),
\[ U_{Sk} \] - the RMS value of the hk voltage harmonic on current shunt \( R_{S} \).

The values of the phase displacement of the hk harmonic of the distorted primary current are defined by the equation [3], [29]:

\[ \delta \phi_{hk} = \arcsin \left( \frac{1}{100\%} \left[ \frac{U_{D}}{U_{R}} \frac{U_{S}}{R_{S}} \cdot 100\% \right]^{-1} \left[ \left( \frac{U_{D}}{U_{R}} \right)^2 + \left( \frac{U_{D}}{R_{D}} \right)^2 - 2 \frac{U_{D} U_{S}}{R_{D}} \cos \phi_{A} + \frac{U_{S}^2}{R_{S}} \cdot 100\% \right]^{\frac{1}{2}} \right) \] (4)

The sign of the phase displacement is positive when the phase angle value \( \phi_{hk} \) is from 0° to 90° and 180° to 270°, while the value of the current error \( \Delta I_{hk} \) is positive. The sign of the phase displacement is also positive when the phase angle value \( \phi_{hk} \) is from 90° to 180° and 270° to 360°, while the value of the current error \( \Delta I_{hk} \) is negative. Therefore, the grounding point of the measurement system conditioning the direction of the current flow through the current shunt \( R_{S} \).

### III. TEST CONDITIONS

The first tested inductive CT has the rated current ratio of 300 A/5 A, a rated load of 5 VA and the accuracy class of 0.5 defined for sinusoidal current at 50 Hz. The second test object is inductive CT with a rated current ratio of 100 A/5 A, the accuracy class of 0.2 and the rated load of 2.5 VA. In the case of the first CT the additional primary winding of 60 turns was made, while for the second CT the additional primary winding of 20 turns was winded.

The tests were performed considering the change of the RMS values of the distorted primary currents in the range from 5% to 120% of the rated currents of the tested CTs.
Measurements were made for resistive and resistive-inductive loads of the secondary winding with the power factor of 0.8. Table I presents a summary of the analysed cases for which the tests of the transformation accuracy were performed.

### TABLE I

| Case | 1st harm. | 3rd harm. | from 4th to 100th harm. |
|------|-----------|-----------|-------------------------|
| I    |           |           |                         |
| II   | 5%, 20%, 100% 120% rated sinusoidal current | 3rd harm. equal to 15% of the fundamental component. | Single harm. (from 2nd to 100th) equal to 5% of the fundamental component. |
| III  |           | 3rd harm. equal to 30% of the fundamental component. | Single harm. (from 4th to 100th) equal to 5% of the fundamental component. |
| IV   |           | 3rd harm. equal to 45% of the fundamental component. |                                       |
| V    | 5.2%, 20.9%, 104.4%, 125.3% rated sinusoidal current | Single harm. (from 2nd to 100th) equal to 5% of the fundamental component. |                                       |

In the case I, the RMS value of the distorted current with respect to conditions of transformation of the sinusoidal current (5%, 20%, 100%, 120% according to the standard IEC 61869-2 [34]) is increased by adding the single higher harmonic of frequency from 100 Hz to 5 kHz with its value equal to 5% of the fundamental component of the sinusoidal current. In the cases II to IV, the distorted primary current consists of the fundamental component equal to 5%, 20%, 100%, 120% of the rated primary current of the tested inductive CTs and the 3rd harmonic is equal to 15%, 30% or 45% of the fundamental component, respectively. Moreover, it consists also the single higher harmonic of frequency from 200 Hz to 5 kHz with the value of 5% of the fundamental component. The case V concerns the transformation of the distorted current when the RMS value of its fundamental component is increased to achieve the same RMS values as in the case III after the primary current is increased by the 3rd harmonic. Additionally, a single higher harmonic of frequency from 100 Hz to 5 kHz with a value of 5% of the fundamental component of the distorted primary current is injected.

It should be noted that in the testing of the harmonics transformation accuracy for distorted currents, the value of the current error and phase displacement are determined for each single harmonic, not for the total value of the distorted current. The RMS value of the entire transformed current does not define the RMS values of its harmonics and their phase angles relative to the fundamental component. These parameters of primary waveform of tested inductive CT may affect the values of the transformation errors of the individual harmonics and the entire distorted current. In the standard IEC 61896-6 the limiting values of transformation errors of the low power transformers are also defined for individual harmonics of the distorted current [21]. During this research the phase angle between transformed higher harmonic and the fundamental component of the distorted current is chosen in order to obtain the maximum values of current error and phase displacement (the worst conditions).

### IV. SELF-GENERATION OF THE LOW ORDER HIGHER HARMONICS

The phenomenon of the self-generation of the higher harmonics in to the secondary current of the inductive CT results from non-linearity of the magnetizing characteristic of the magnetic core. Therefore, the RMS values of the self-generated harmonics mainly depend on position of the operating point of the tested CT on this curve. Their values will result from it shape and load of the secondary winding and RMS values of the low order harmonics of transformed distorted current. This phenomenon causes rapid increase of the values of current error and phase displacement determined for transformation of the low order higher harmonics. Their maximum values may be designated when the value of the phase angle of the transformed higher harmonic in relation to the fundamental component of the distorted primary current is continuously adjusted [9].

The transformation accuracy tests of inductive CTs were performed in the measuring system from Figure 1, using the rated ampere turns method. The Figure 2 shows for the tested inductive CT 300 A / 5 A the measured percentage values of the self-generated higher harmonics calculated in relation to the fundamental component $I_{ph\%}$. The tests were performed for the 100% and 25% of the rated resistive load of the secondary winding during transformation of the sinusoidal current with the values specified in Table 1 for the case I (without additional higher harmonics in the primary current).
The increase of the secondary winding load causes the increase of the percentage self-generated low order higher harmonics in to the secondary current. This results from the movement of the inductive CT operating point higher on the magnetization characteristic of the magnetic core (closer to the saturation point). Moreover, in this conditions for the highest value of the primary current the highest values of the self-generated 3rd, 5th and 7th harmonics are obtained. However, at 25% of the rated load the change of the rms value of the primary current only have slight influence on the level of the self-generated harmonics. The highest values of self-generated 3rd, 5th and 7th harmonics are obtained at 5% of the rated primary current. It results for the low values of the magnetic flux density from the non-linearity of the magnetization characteristic of the magnetic core of this inductive CT.

In order to illustrate the effect of the values of the self-generated low-order harmonics on the values of the current error and the phase displacement the frequency characteristics of the transformation accuracy are determined. In Figure 3 for the case I specified by Table 1 obtained results in the limited range of harmonic transformations from 1st to 20th are presented.

The percentage values of the self-generated higher harmonics during transformation of the sinusoidal current with the values specified for the case I for the inductive CT 300 A/5 A with the resistive load: (a) 25% and (b) 100% of its rated value

The measurements were made for 100% of the rated resistive load and four values of the primary current distorted by a single higher harmonic in accordance with the case I as specified in Table 1. Due to the phase angle of the transformed higher harmonic, in relation to the self-generated higher harmonic, the current error and phase displacement for a given primary current may obtain decreased (marked -) or increased (marked +) values. The maximum absolute values of the current error and phase displacement are determined for the worst condition of the phase angle.

The percentage values of the higher harmonics self-generated in to the secondary current of the inductive CT 100 A / 5 A for two resistive loads of its secondary winding equal to 100% and 25% of the rated value are determined. The obtained results are shown in Figure 4.
In the case of this tested CT the percentage values of the self-generated higher harmonics are the highest for 5% of rated current. This results from the fact that its operating point on the magnetization characteristic of its the magnetic core is close to the lower knee point.

In order to illustrate the influence of the values of the self-generated low order harmonics on the values of the current error and phase displacement the frequency characteristics are determined. In Figure 5 for the case I specified by Table 1 obtained results in the limited range of harmonic transformations from 1st to 20th are presented.

![Figure 5](image)

**FIGURE 5.** Impact of the self-generated in to the secondary current higher harmonics of 100 A/5 A CT in the tested case I for the rated resistive load on the values of: (a) current error, (b) phase displacement

The measurements were made for 100% of the rated resistive load and four values of the primary current distorted by a single higher harmonic in accordance with the case I as specified in Table 1. Due to the summation of the transformed and self-generated harmonics for a given primary current the current error and phase displacement may obtain decreased (marked -) or increased (marked +) values.

The comparison of the percentage values of the self-generated low order higher harmonics determined for transformation of the sinusoidal and the distorted primary current by the inductive CT 300 A / 5 A in the cases from I to V is shown in Figure 6. The tests were performed for the rated primary current and the secondary winding load equal to 25% (a) and 100% (b) of its rated resistive value.

![Figure 6](image)

**FIGURE 6.** The comparison of the percentage values of the self-generated higher harmonics for transformation by the inductive CT 300 A / 5 A of the distorted primary current in the cases from I to V with the resistive load: (a) 25% and (b) 100% of its rated value

Increase of the secondary winding load of this tested inductive CT results in increase of the values of the self-generated low order higher harmonics (as in the previously tested CT). During transformation of the sinusoidal current with the increased value of the fundamental component in the case V (Table 1) for the rated load of the secondary winding there is an increase in the values of the self-generated 3rd, 5th and 7th harmonics in relation to the all cases.

The comparison of the determined percentage values of the low order higher harmonics self-generated in to the secondary current of the CT 100 A / 5 A during transformation of the sinusoidal and the distorted primary current in the cases from I to V is presented in Figure 7. The tests were performed for the rated primary current and the secondary winding load equal to 25% (a) and 100% (b) of its rated resistive value.
(a) FIGURE 7. The comparison of the percentage values of the self-generated higher harmonics for transformation by the inductive CT 100 A / 5 A of the distorted primary current in the cases from I to V with the resistive load: (a) 25% and (b) 100% of its rated value

The CT 100 A / 5 A is characterized by noticeably smaller level of the self-distortion of the secondary current as a result of the non-linearity of the magnetic core’s magnetization characteristics than the CT 300 A / 5 A. Increase of the load of its secondary winding also for transformation of the distorted current causes increase of the percentage values of the self-generated lower order harmonics. The transformation of the sinusoidal current with increased value of the fundamental harmonic in case V causes increase the value of the self-generated 3rd, 5th and 7th harmonics in relation to all other considered cases.

Regarding both tested CTs, the increase of the RMS value of the injected 3rd harmonic in cases II to IV results in the increase of the self-generation of the 5th, 7th and 9th harmonics. Moreover, these values are different than as a result of the corresponding increase of the RMS value of the sinusoidal current according to case V. Therefore, it should be emphasized that the self-generation of the lower order higher harmonics in the secondary current of the inductive CT depends not only on the RMS value of the fundamental harmonic, but also on the RMS values of the lower order higher harmonics. This phenomenon is related to the change of value of the magnetic flux density in the magnetic core caused by the change of the fundamental component and the 3rd order higher harmonic of the transformed distorted current and it is inversely proportional to the frequency of the harmonics. It may be the main factor conditioning the transformation accuracy of the harmonics up to and including the 9th order. Higher frequency harmonics than 450 Hz, due to the fact that they order is above 9th, causes at least 9 times lower change of the value of the magnetic flux density in the magnetic core than the fundamental component and are negligible. The transformation accuracy of all harmonics in the investigated range up to 100th results from the value of the magnetic permeability and the active power losses in the magnetic core [1], [3].

V. THE COMPARISON OF THE FREQUENCY CHARACTERISTICS OF CURRENT ERROR AND PHASE DISPLACEMENT DETERMINED FOR DISTORTED CURRENTS

The graphs in Figure 8 show the frequency characteristics of the values of current error (a) and phase displacement (b) of the inductive CT 300 A / 5 A determined for a rated resistive load. The measurements were made in the conditions of the transformation of the distorted current with the rated rms value and the harmonic values of the distorted primary current in accordance with Table 1 for cases I and III. In the case V the RMS value of the fundamental component is increase to 104.4% of the rated sinusoidal current, while the higher harmonics values are equal to 5% of this value.

(a) FIGURE 8. The frequency characteristics of the values of current error (a) and phase displacement (b) of the inductive CT 300 A / 5 A for a rated resistive load in cases I, III and V

The applied 3rd harmonic in case III causes an increase in the values of current error and phase displacement in relation to
the case I, when single higher harmonics were transformed only with the fundamental component of the rated value. An analogous increase in the RMS value of the fundamental harmonic in the case V to an increase in the RMS value of the 3rd harmonic in the case III does not lead to an analogous increase in the values of current error and phase displacement. This is due to the fact that the change of the magnetic flux density in the magnetic core is proportional to the RMS value of the primary harmonic, but inversely proportional to its frequency. The graphs in Figure 3 show the frequency characteristics of the values of current error (a) and phase displacement (b) of the inductive CT 300 A / 5 A determined for the rated load with the resistance-inductive power factor equal to 0.8. The measurements were made in the conditions of the distorted current transformation for the same cases I, III and V as for previously tested CT.

The frequency characteristics of inductive CT 100 A / 5 A are also affected by the RMS values of the self-generated low order higher harmonics. The values of current error and phase displacement of harmonics of the 3rd, 5th, 7th and 9th orders are changed. The RMS values of applied 1st and 3rd harmonics in the distorted primary current does not affect the values of current error and phase displacement of higher harmonics transformation of orders from 11th to 100th. Their transformation accuracy results only from the values of the mutual inductance between CT’s windings and value of the active power losses in the magnetic core for a given frequency. Moreover, as frequency of higher harmonic increases its transformation accuracy becomes less dependent on its RMS value and the CT’s load of the secondary winding due to the fact that they cause reduced changes of the value of the magnetic flux density.

The graphs in Figure 4 show the frequency characteristics of the values of current error (a) and phase displacement (b) of the inductive CT 100 A / 5 A determined for the rated load. The measurements were made for cases I, III and V in accordance with the Table 1. In the cases I and III the RMS value of the fundamental component is equal to 100% of rated sinusoidal current. In the case III the RMS value of the 3rd harmonic is equal to 30% of the fundamental component. In the case V the RMS value of the fundamental component is increase to 104.4% of the rated sinusoidal current.

The change of the load power factor of the secondary winding of the tested CT from 1 to 0.8 increases the generation of the lower order higher harmonics as a result of the shifted operating point on the magnetization characteristic towards saturation. Therefore, during the transformation of the distorted current the percentage values of the generated harmonics of the 3rd, 5th, 7th and 9th orders increase in relation to the operating conditions of the CT with the resistive load.

The frequency characteristics of inductive CT 100 A / 5 A are also affected by the RMS values of the self-generated low order higher harmonics. The values of current error and phase displacement of harmonics of the 3rd, 5th, 7th and 9th orders are changed. The RMS values of applied 1st and 3rd harmonics in the distorted primary current does not affect the values of current error and phase displacement of higher harmonics transformation of orders from 11th to 100th. Their transformation accuracy results only from the values of the mutual inductance between CT’s windings and value of the active power losses in the magnetic core for a given frequency. Moreover, as frequency of higher harmonic increases its transformation accuracy becomes less dependent on its RMS value and the CT’s load of the secondary winding due to the fact that they cause reduced changes of the value of the magnetic flux density.
VI. EXPANDED MEASUREMENT UNCERTAINTY IN DETERMINING THE VALUES OF CURRENT ERROR AND PHASE DISPLACEMENT

The method of calculation and the dependencies defining the measurement uncertainties presented in this section were developed in accordance with the guidelines and recommendations of JCGM described in [33]. The uncertainty of the evaluation of the values of current error and phase displacement results from the accuracy of measurement of individual voltage harmonics and their corresponding phase angles by the digital power meter. Moreover, the tolerances of resistance and inductance of the used current shunts are considered. The calculations of expanded uncertainties of the values of current error and phase displacement were performed for the measurement method in the rated primary ampere turns conditions of the tested CTs. An analogical procedure for calculation of the expanded uncertainties for determine the values of voltage error and phase displacement of inductive voltage transformers is presented in the paper [11].

The phase angle between the current and the voltage of the current shunt resistor due to its inductance for a given \( h k \) harmonic of the current was determined from the equation:

\[
\delta_R = \arct g \left( \frac{2\pi f h k L}{R} \right) \tag{5}
\]

\( R \) – the resistance of the current shunt resistor determined for DC, \( L \) – the inductance of the current shunt resistor equal to 0.08 mH. The maximum percentage difference in the impedance of the current shunt resistor results from the manufacture tolerance of its resistivity specified for DC and the increase in the value of its reactance for a given \( h k \) harmonic of the current was determined in accordance with the following equation:

\[
\Delta Z_{RShk} = \Delta R + \sqrt{\Delta Z_{RShk}^2 + \Delta Z_{Shk}^2}, \text{ 100%} \tag{6}
\]

\( \Delta R \) – the tolerance of the current shunt resistance for DC defined by the manufacturer.

Calculated relative measurement uncertainties of the \( h k \) voltage harmonic RMS value \( Z_{RSRShk} \) and phase angle \( \phi_{RSRShk} \) results from usage of the current shunt resistors of values 0.1 Ω, 1 Ω and 10 Ω for the minimum and maximum values of the tested frequency range are equal to:

- 0.1 Ω (\( Z_{RSRShk} / \phi_{RSRShk} \)):
  - 50 Hz: ± 1%/± 0.015°
  - 5 kHz: ± 1.05%/±1.5°
- 1 Ω (\( Z_{RSRShk} / \phi_{RSRShk} \)):
  - 50 Hz: ± 0.02%/± 0.02°
  - 5 kHz: ± 0.02%/± 0.2°
- 10 Ω (\( Z_{RSRShk} / \phi_{RSRShk} \)):
  - 50 Hz: ± 0.02%/± 0.02°
  - 5 kHz: ± 0.02%/± 0.2°

Relative uncertainties of measurement of the \( h k \) voltage harmonic \( \psi_{UCWh} \) and phase angle \( \phi_{UCWh} \) by the used digital power meter determined by the manufacturer are equal to:

- 50 Hz: ± 0.3%/± 0.3°
- 5 kHz: ± 1.0%/±1.0°

The calculation of the expanded measurement uncertainty of the values of current error and phase displacement by the measurement system in which the accuracy tests are performed under rated ampere turns of CT requires to define the input quantities for equations (3) and (4):

- the RMS value of the \( h k \) voltage harmonic on current shunt \( R_S \),
- the RMS value of the \( h k \) voltage harmonic on current shunt \( R_D \),
- the phase angle between \( h k \) voltage harmonic on current shunt \( R_D \) and the \( h k \) voltage harmonic on current shunt \( R_S \).

The combined standard uncertainty is the positive square root of the combined variance given by equation:

\[
u^2(y) = \sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) \tag{7}\]

\( f \) – the function given by equation (3) or (4),
\( x_i \) – the each input estimate,
\( u(x_i) \) – the standard uncertainty,
\( \partial f \) – the partial derivatives.

The partial derivatives are equal to \( \Delta f / \Delta X_i \) evaluated at \( X_i = x_i \) and are often called sensitivity coefficients \( c_i \).

\[
\frac{\partial f}{\partial x_i} \approx \frac{\Delta f}{\Delta x_i} = c_i \tag{8}\]

\( \Delta f \) – the partial derivatives evaluated at \( X_i = x_i \).

Sensitivity coefficients describe how the output estimate varies with changes in the values of the input estimates. In order to determine the sensitivity coefficient \( c_i \), it is necessary to specify the initial conditions of the performed calculations. Therefore, the values of current error and phase displacement, regardless of the frequency, were assumed to be 0.1% and 0.1°, respectively. The next step is to determine, for a given configuration of the measuring system, the RMS values of the voltages of the current shunts and the values of the phase angle between them, for which the indicated values of current error and phase displacement are obtained. Then, it is possible to determine the maximum values of the expended measurement uncertainty for the assumed values of current error and phase displacement taking into consideration the relative uncertainty defined for the input quantities.

The calculated variance is defined by the following equation:

\[
u^2(y) = \sum_{i=1}^{n} [c_i \cdot u(x_i)]^2 \tag{9}\]

The standard uncertainty is defined by the following equation:

\[
u(x_i) = \frac{x_i}{p} \tag{10}\]

\( p \) – the assumed probability distribution.
A rectangular uniform probability distribution is assumed for all elements affecting the measurement uncertainty of current error and phase displacement of inductive CTs. Then, all results are equally probable.

Extended measurement uncertainty for the coverage factor $\chi = 2$, to ensure the level of confidence of about $p = 95\%$, is calculated in accordance with following equation:

$$U_{\Delta_{hk}} = \chi \cdot u_{\Delta_{hk}}$$ \hspace{1cm} (11)

$$U_{\delta_{\phi hk}} = \chi \cdot u_{\delta_{\phi hk}}$$ \hspace{1cm} (12)

The values of the current error and phase displacement depend on the phase shift $\varphi_{Ahk}$ between the $hk$ of the voltage harmonics $U_{RShk}$ and $U_{RDhk}$, so the evaluation of the maximum uncertainties of the determination of these errors required calculations for a various phase angles from 0° to 360°. The measurement uncertainties were determined for two frequencies: 50 Hz and 5 kHz. These are the limits of the accuracy test range of the inductive CTs.

Tables 2 and 3 show the uncertainty budget determined for the measurement system in Figure 1, in which the current error and phase displacement of the inductive CTs are determined under rated amperage turns. The calculations were performed for the distorted current of the additional primary winding with an RMS value of 5 A and a fundamental frequency of 50 Hz with 5% contribution of a single higher harmonic with a frequency of 5 kHz (case I in accordance with the Table 1).

| Source of uncertainty | Sensitivity coefficient ($\delta_{\phi hk}$) | Sensitivity coefficient ($\delta_{\phi hk}$) |
|-----------------------|-------------------------------------------|-------------------------------------------|
| $R_s$, impedance      | $0.035$                                   | $0.0001$                                  |
| $R_h$, impedance      | $0.003$                                   | $0.0004$                                  |

**TABLE II**

The uncertainty budget determined for evaluation of the values of current error in the measurement system in Figure 1 for distorted current of the additional primary winding with an RMS value of 5 A

| Source of uncertainty | Sensitivity coefficient ($\Delta_{\text{lim}}$) | Standard uncertainty $u_i(\Delta_{\text{lim}})$ [%] |
|-----------------------|------------------------------------------------|-----------------------------------------------|
| $50$ Hz               | $5$ kHz                                      | $50$ Hz                                       |
| $5$ kHz               | $0.066$                                      | $1.318$                                      |
|                       |                                               | $0.0002$                                     |
|                       |                                               | $0.0004$                                     |
| $R_s$, impedance      | $0.005$                                      | $0.008$                                      |
|                       |                                               | $0.0000$                                     |
|                       |                                               | $0.0073$                                     |
| $R_h$, impedance      | $19.98$                                      | $199.8$                                      |
|                       |                                               | $0.0000$                                     |
|                       |                                               | $0.0006$                                     |
| Measured value of the | $0.005$                                      | $0.004$                                      |
| voltage harmonic on   |                                               | $0.0001$                                     |
| current shunt $R_s$   |                                               |                                               |
| Measured value of the | $0.061$                                      | $0.965$                                      |
| voltage harmonic on   |                                               | $0.0005$                                     |
| current shunt $R_h$   | $0.005$                                      | $0.008$                                      |
|                       |                                               | $0.0010$                                     |
|                       |                                               | $0.00559$                                    |
| Measured value of the | $19.98$                                      | $199.8$                                      |
| voltage harmonic on   |                                               | $0.0005$                                     |
| current shunt $R_h$   | $0.005$                                      | $0.008$                                      |
|                       |                                               | $0.0010$                                     |
|                       |                                               | $0.00496$                                    |
| Combined standard uncertainty | $0.002$                                      | $0.008$                                      |

Extended uncertainty ($\chi=2$), 95% level of confidence $\pm 0.003 \pm 0.017$

**TABLE III**

The uncertainty budget determined for evaluation of the values of phase displacement in the measurement system in Figure 1 for distorted current of the additional primary winding with an RMS value of 5 A

**TABLE III**

| Source of uncertainty | Sensitivity coefficient ($\delta_{\phi hk}$) | Sensitivity coefficient ($\delta_{\phi hk}$) |
|-----------------------|-------------------------------------------|-------------------------------------------|
| $50$ Hz               | $5$ kHz                                      | $50$ Hz                                       |
|                       | $0.035$                                      | $0.553$                                      |
|                       |                                               | $0.0001$                                     |
|                       |                                               | $0.0002$                                     |
| $R_s$, impedance      | $0.003$                                      | $0.005$                                      |
|                       |                                               | $0.0000$                                     |
|                       |                                               | $0.0004$                                     |
| $R_h$, impedance      | $11.45$                                      | $114.5$                                      |
|                       |                                               | $0.0000$                                     |
|                       |                                               | $0.0000$                                     |
| Measured value of the | $0.003$                                      | $0.553$                                      |
| voltage harmonic on   |                                               | $0.0003$                                     |
| current shunt $R_s$   |                                               | $0.0006$                                     |
| Measured value of the | $11.45$                                      | $114.5$                                      |
| voltage harmonic on   |                                               | $0.0003$                                     |
| current shunt $R_h$   | $0.003$                                      | $0.005$                                      |
|                       |                                               | $0.0006$                                     |
|                       |                                               | $0.0028$                                     |

Combined standard uncertainty $0.001$ $0.005$

The uncertainty budgets presented in Tables 2 and 3 include the calculated sensitivity coefficients and standard uncertainties for the indicated sources of uncertainty. The expanded measurement uncertainty of current error and phase displacement for the fundamental component and the higher harmonic of frequency 5 kHz are determined. Therefore, the value of current error determined by the developed method and measuring system is:

$\rightarrow (0.1 \pm 0.003)^\circ$ for 50 Hz and $(0.1 \pm 0.017)^\circ$ for 5 kHz. The value of phase displacement is:

$\rightarrow (0.1 \pm 0.002)^\circ$ for 50 Hz and $(0.1 \pm 0.01)^\circ$ for 5 kHz. Tables 4 and 5 show the uncertainty budget calculated for the measurement system in Figure 1 under analogous measurement conditions. The values of the current error and phase displacement of the inductive CT are determined for the distorted current of the additional primary winding with RMS value of 1 A.
TABLE IV
THE UNCERTAINTY BUDGET DETERMINED FOR EVALUATION OF THE VALUES OF CURRENT ERROR IN THE MEASUREMENT SYSTEM IN FIGURE 1 FOR DISTORTED CURRENT OF THE ADDITIONAL PRIMARY WINDING WITH AN RMS VALUE OF 1 A

| Source of uncertainty | Sensitivity coefficient (ΔI<sub>hk</sub>) | Standard uncertainty ui(ΔI<sub>hk</sub>) [%] |
|-----------------------|------------------------------------------|---------------------------------------------|
|                       | 50 Hz  | 5 kHz  | 50 Hz  | 5 kHz  |
| The value of current shunt R<sub>s</sub> impedance | \( \varphi_{RSh} \) | 0.748 | 7.478 | 0.0001 | 0.00009 |
|                       | \( \varphi_{RDh} \) | 0.004 | 0.004 | 0.0000 | 0.00005 |
| The value of current shunt R<sub>d</sub> impedance | \( \varphi_{RDh} \) | 99.98 | 999.8 | 0.0000 | 0.00003 |
|                       | \( \varphi_{RDh} \) | 0.004 | 0.004 | 0.0000 | 0.00005 |
| Measured value of the voltage harmonic on current shunt R<sub>s</sub> | \( \varphi_{U_{Ws}} \) | 0.241 | 2.413 | 0.0004 | 0.00139 |
|                       | \( \varphi_{U_{Ws}} \) | 0.004 | 0.004 | 0.0008 | 0.00248 |
| Measured value of the voltage harmonic on current shunt R<sub>d</sub> | \( \varphi_{U_{Wk}} \) | 99.98 | 1778.7 | 0.0004 | 0.00248 |
|                       | \( \varphi_{U_{Wk}} \) | 0.004 | 0.002 | 0.0008 | 0.00139 |
| Combined standard uncertainty |                                  | 0.001 | 0.004 |

Extended uncertainty (\( \chi = 2 \)), 95% level of confidence ±0.003 ±0.008

TABLE V
THE UNCERTAINTY BUDGET DETERMINED FOR EVALUATION OF THE VALUES OF PHASE DISPLACEMENT IN THE MEASUREMENT SYSTEM IN FIGURE 1 FOR DISTORTED CURRENT OF THE ADDITIONAL PRIMARY WINDING WITH AN RMS VALUE OF 1 A

| Source of uncertainty | Sensitivity coefficient (δϕ<sub>hk</sub>) | Sensitivity coefficient (δϕ<sub>hk</sub>) |
|-----------------------|------------------------------------------|------------------------------------------|
|                       | 50 Hz  | 5 kHz  | 50 Hz  | 5 kHz  |
| The value of current shunt R<sub>s</sub> impedance | \( \varphi_{Z_{RSh}} \) | 0.138 | 1.382 | 0.0000 | 0.0000 |
|                       | \( \varphi_{Z_{RSh}} \) | 0.002 | 0.002 | 0.0000 | 0.0000 |
| The value of current shunt R<sub>d</sub> impedance | \( \varphi_{Z_{RDh}} \) | 57.28 | 572.8 | 0.0000 | 0.0000 |
|                       | \( \varphi_{Z_{RDh}} \) | 0.002 | 0.002 | 0.0000 | 0.0000 |
| Measured value of the voltage harmonic on current shunt R<sub>s</sub> | \( \varphi_{U_{Ws}} \) | 0.138 | 1.383 | 0.0002 | 0.0008 |
|                       | \( \varphi_{U_{Ws}} \) | 0.002 | 0.002 | 0.0004 | 0.0014 |
| Measured value of the voltage harmonic on current shunt R<sub>d</sub> | \( \varphi_{U_{Wk}} \) | 57.28 | 1019.1 | 0.0002 | 0.0014 |
|                       | \( \varphi_{U_{Wk}} \) | 0.002 | 0.001 | 0.0004 | 0.0008 |
| Combined standard uncertainty |                                  | 0.001 | 0.002 |

Extended uncertainty (\( \chi = 2 \)), 95% level of confidence ±0.001 ±0.005

Considering the determined expanded uncertainties for the distorted current of the additional primary winding with an rms value of 1 A, the value of current error is:

→ (0.1 ± 0.003)% for 50 Hz and (0.1 ± 0.008)% for 5 kHz.

The value of phase displacement is:

→ (0.1 ± 0.001)° for 50 Hz and (0.1 ± 0.005)° for 5 kHz.

The expanded measurement uncertainties of current error and phase displacement, for a given measurement system configuration (1 A or 5 A), do not depend on the RMS values of the primary current harmonics. This is due to the assumption of constant standard uncertainty for a given uncertainty source. However, these values increase linearly with the determined values of current error and phase displacement.

VII. DIsCUSSION

The transformation accuracy of all harmonics in the tested range (up to the 100th harmonic) is influenced by the values of the magnetic permeability and the active power losses in the magnetic core [1], [3]. Decrease of the value of the magnetic permeability and increase of the value of the active power losses with an increase in the frequency of the transformed harmonic leads to an increase in the RMS value of the reactive and active components of the magnetic core excitation current, and consequently to an increase of the values of the current error and the phase displacement. The self-generation of the lower order harmonics in to the secondary current of the inductive CT may be the main factor determining the accuracy of the harmonic transformation up to their order 9th. It should be noted that as the load of the secondary winding of inductive CT increase the values of the self-generated higher harmonics also increase. This is a result of the inductive CT's operating point shifting higher on the magnetization curve closer to the saturation point. High value of the primary current at rated load typically causes higher values of the self-generated low order harmonics. Different value of the phase angle of the transformed low order higher harmonic in relation to the self-generated higher harmonic results in different values of the current error and phase displacement. Therefore, these values for transformation of a given higher harmonic of the distorted primary current are reduced or increased in relation to the value resulting from the active power losses and the magnetic permeability obtained at a given frequency of the transformed harmonic. In order to determine the transformation accuracy of the harmonic in the worst condition, the maximum values of the current error and phase displacement are determined. In Figure 11 the block diagram summarizing the contribution of the research presented in the article is shown.
VIII. CONCLUSION

This article shows that the increase of the RMS value of the 3rd harmonic, due to the change in the shape of the resultant magnetization characteristic for distorted excitation current, causes an increase in the values of the self-generated low order higher harmonics. However, its influence is 3-times smaller than caused by the main component, because the increase in the magnetic flux density in the magnetic core is also 3-times smaller than caused by the fundamental harmonic. The RMS value of the fundamental component of the distorted primary current also significantly affects the values of the low order higher harmonics self-generated in the secondary current by the inductive CT. The content of the lower order harmonics in the distorted primary current have no influence on the values of current error and phase displacement of transformation of the higher harmonics of orders from 11th up to 100th. It should be emphasized that the application of a resistive-inductive load as a result of an increase in the magnetizing voltage of the magnetic core leads to an increase in the self-generation of low order higher harmonics compared to the operating conditions of the inductive CT with a resistive load. In the case of inductive CTs the change in the values of the self-generated low order higher harmonics in to the secondary current with the RMS values of the fundamental component and the 3rd higher harmonic is the main challenging task in terms of the successful compensation of the values of current error and phase displacement for the transformation of distorted currents.

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He is a member of the IEC TC38 (Instrument Transformers) work groups: JWG 5 for Electronic Current Transformers (<1000 Vac), JWG 8 for Additional Requirements for Current Transformers and Voltage Dividers and Digital Interface for Instrument Transformers, WG 49 Instrument Transformers for Low Voltage Applications (<1000 Vac) and WG 55 Uncertainty Evaluation in the Calibration of Instrument Transformers, MT 48 Revision of IEC 61869-1: Instrument Transformers – General Requirements.

ERNEST STANO is a member of the Technical Body No. 81 for Instrument Transformers and Low Power Transformers in The Polish Committee for Standardization. He received M. Sc. degree in 2018 at Lodz University of Technology, Poland in electrical engineering with specialization electric power engineering. He works as a research assistant in the Institute of Mechatronics and Information Systems of the Lodz University of Technology. His work concerns evaluation and analysis of instrument transformers and voltage dividers accuracy of transformation of distorted current/voltage and increased frequency signals. This includes determination of their impact on accuracy of energy metering and PQ measurements. He designs MV wideband current, voltage transformers and dividers. In EMC his work concerns conductive and radiated disturbances transformation through instrument transformers and the influence of such interferences on the accuracy of the measuring systems.

MICHAL KACZMAREK is the chairman of the Technical Body No. 81 for Instrument Transformers and Low Power Transformers in The Polish Committee for Standardization. He received Ph. D. degree in 2009 and Sc.D. degree in 2018 at Lodz University of Technology, Poland in electrical engineering with specialization in electromagnetism and instrument transformers. He works as an associate professor in the Institute of Mechatronics and Information Systems of the Lodz University of Technology. His area of experience is evaluation and analysis of instrument transformers and voltage dividers operation during transformation of distorted current/voltage and increased frequency signals. This includes determination of their impact on accuracy of energy metering and PQ measurements. He designs MV wideband current, voltage transformers and dividers. In EMC his work concerns conductive and radiated disturbances transformation through instrument transformers and the influence of such interferences on the accuracy of the measuring systems.