Firing range optimization of a 155mm uni-modular charge howitzer by ETC technology

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Abstract. Electrothermal-chemical (ETC) launch technology can accurately control and enhance the ignition and combustion of solid propellants to improve the consistency and accuracy of ballistics. In this paper, a calculation model including the transient burning rate formula and six-degree-of-freedom rigid body trajectory equation is established to simulate both interior and exterior ballistics processes for a 155mm uni-modular charge howitzer. The simulation results show that the muzzle velocities are optimized via the ETC technology. The 155mm uni-modular charge ETC howitzer can effectively obtain the minimum range, increase the range overlap and maximum range, and improve the ballistic stability and consistency.

1. Introduction
In our previous work [1,2], the application advantages and prospects of electrothermal-chemical (ETC) launch technology in a 155mm uni-modular charge howitzer were discussed from the perspective of the technology development and actual applications, respectively [3–5]. Results show that the ETC launch technology can be combined with the uni-modular charge technology to apply in the 155mm uni-modular charge howitzer to advance the ballistic performance.

In this paper, a calculation model including the transient burning rate formula and six-degree-of-freedom rigid body trajectory equation is established to simulate both interior and exterior ballistics processes for a 155mm uni-modular charge howitzer. The firing range and range overlap are discussed to verify the advantages by ETC technology.

2. Calculation model
A calculation model including the transient burning rate formula [6] and six-degree-of-freedom rigid body trajectory equation [7] is established to simulate the ballistics processes for a 155mm uni-modular charge howitzer.
2.1. interior ballistics model

The propellant combustion process can be described as:

\[
\psi = \begin{cases} 
\chi Z (1 + \lambda Z + \mu Z^2) & (Z < 1) \\
\chi_s Z (1 + \lambda_s Z) & (1 \leq Z < Z_b) \\
1 & (Z \geq Z_b)
\end{cases}
\] (1)

\[
\frac{dZ}{dt} = \begin{cases} 
\frac{r}{\epsilon_1} & (Z < Z_b) \\
0 & (Z \geq Z_b)
\end{cases}
\] (2)

where, \(Z\) is the relative burned thickness of propellant. \(\chi, \chi_s, \lambda, \lambda_s, \mu\) are the shape function of propellant. \(Z_b\) is the relative burned thickness when the propellant begins to be regressive. \(r\) is the propellant burning rate. \(\epsilon_1\) is half of the propellant web thickness.

The dynamical equations of the projectile in the barrel are shown as:

\[
v = \frac{dl}{dt}, \quad Sp = \phi m \frac{dv}{dt}, \quad p[V + V_0 - \frac{\omega}{\rho_p} (1 - \psi) - \alpha \omega \psi] = f \omega \psi - \theta \phi mv^2 + \theta E_p
\] (3)

where, \(v\) is the velocity of projectile. \(l\) is the length of the barrel. \(S\) is the sectional area of the barrel. \(m\) is the projectile weight. \(p\) is the mean pressure in volume behind projectile. \(\phi\) is the coefficient of second work. \(\omega\) is the propellant charge weight. \(\rho_p\) is the density of the propellant. \(\alpha\) is the gas covolume of the propellant. \(E_p\) is the plasma energy injected into the chamber. And \(E_p = 0\) when in conventional ignition. \(V = Sl, \theta = k - 1, \) here \(k\) is the ratio of specific heat.

The propellants have been manufactured and tested in an improved closed bomb vessel into which electrical energy was discharged. The phenomenon of enhanced gas generation rates (EGGR) during the electrical discharge has been reported. Clive R. Woodley added an EGGR coefficient into the Vieilles law to simulate the effect of EGGR [8]. Dr. Yanjie Ni was introduced a transient burning rate formula of propellant including the influence of pressure gradient and an EGGR coefficient by electrical power. And this new formula is employed into our calculation model to instead of the geometric burning law.

The transient burning rate formula [6] is shown as follow:

\[
r = u_1 \cdot P_{n_1} \left(1 + \frac{\alpha(t)n_1}{u_1^2 \rho p^2 n_1 + 1} \frac{dp}{dt}\right) (1 + \beta_e P_e)
\] (4)

where, \(u_1\) is the burn rate coefficient of the solid propellant. \(n_1\) is the burn rate index. \(\alpha(t)\) is the time variable function of pressure and flame structure. \(P_e\) is the electrical power(MW). \(\beta_e\) is the EGGR coefficient(MW\(^{-1}\)).

2.2. exterior ballistics model

A conventional six-degree-of-freedom rigid body trajectory model is introduced to simulate the exterior ballistic performance. To acquire a more precise describe of the motion of a spinning projectile, six degree of freedom flight dynamic equations are used in this paper. The detailed expression are as follows:

\[
\frac{dv}{dt} = \frac{1}{m} \left( -\frac{\rho v_r}{2} SC_{x_0} (v - w_{x_2}) - \frac{\rho v_r}{2} SC_{x_2} S_r^2 (v - w_{x_2}) + \frac{\rho S}{2} C_y v_r^2 \cos \delta_2 \cos \delta_1 \sin \delta_r \right)
\]

\[
- \frac{\rho S}{2} C_y v_{re} (v - w_{x_2}) - \frac{\rho v_r}{2} S c_z w_{z_2} \cos \delta_2 \sin \delta_1 \sin \delta_r + \frac{\rho v_r}{2} S c_z w_{z_2} \sin \delta_2 \sin \delta_r - mg \sin \theta_a \cos \psi_2
\]
\[
\frac{d\theta_a}{dt} = \frac{1}{m} \left( \rho v_r S c w_{y2} + \frac{\rho S c y}{2v \cos \psi_2} \left( v_r^2 \cos \delta_2 \sin \delta_1 + v_r \xi w_{y2} \right) - \frac{\rho v_r^2 S c^\prime y \delta N \cos \gamma_1}{2v \cos \psi_2} \right. \\
+ \left. \frac{\rho v_r S c z}{2v \cos \psi_2} \left( (v - w_{x2}) \sin \delta_2 + w_{z2} \cos \delta_2 \cos \delta_1 \right) - \frac{mg \cos \theta_a}{v \cos \psi_2} \right) \\
+ \frac{2\Omega E m v}{v \cos \psi_2} \left( \sin \psi_2 \cos \theta_a \cos \Lambda \cos \alpha_N + \sin \theta_a \sin \psi_2 \sin \Lambda + \cos \psi_2 \cos \Lambda \sin \alpha_N \right) \tag{6}
\]

\[
\frac{d\psi_2}{dt} = \frac{1}{m} \left( \rho v_r S c x w_{z2} + \frac{\rho S c z}{2v \cos \psi_2} \frac{1}{\sin \delta_r} \left( v_r^2 \sin \delta_2 + v_r \xi w_{z2} \right) - \frac{\rho v_r^2 S c^\prime y \delta N \sin \gamma_1}{2v} \right. \\
+ \left. \frac{\rho v_r S c z}{2v \cos \psi_2} \frac{1}{\sin \delta_r} \left( -w_{z2} \cos \delta_2 \cos \delta_1 \right) - \frac{\rho v_r S c}{2v \cos \psi_2} \frac{1}{\sin \delta_r} \left( v - w_{x2} \right) \cos \delta_2 \sin \delta_1 \right) \\
+ \left( \frac{1}{v} mg \sin \theta_a \sin \psi_2 + 2\Omega E m \left( \sin \Lambda \cos \theta_a - \cos \Lambda \sin \theta_a \cos \alpha_N \right) \right) \left( \frac{d\omega_e}{dt} = \frac{1}{C} \cdot \left( -\frac{\rho S d l}{2} m_x x \varphi_v + \frac{\rho v_r^2}{2} S l m_z w_\delta f \right) \right) \tag{7}
\]

\[
\frac{d\omega_\eta}{dt} = \frac{1}{A} \cdot \left( \frac{\rho S l}{2} v_r m_z \frac{1}{\sin \delta_r} v_r \xi - \frac{\rho S d l}{2} v_r m_z w_\omega \omega_\xi - \frac{\rho S l}{2} m_y \frac{1}{\sin \delta_r} \omega_\xi v_r \eta - \frac{\rho v_r^2 S l m_z w_\delta M \sin \gamma_2}{2} \right) \\
- \frac{\omega_\xi \omega_\xi + \omega_\xi^2 \tan \varphi_2}{A} \tag{8}
\]

\[
\frac{d\omega_\zeta}{dt} = \frac{1}{A} \cdot \left( \frac{\rho S l}{2} v_r m_z \frac{1}{\sin \delta_r} v_r \eta - \frac{\rho S d l}{2} v_r m_z w_\omega \omega_\xi - \frac{\rho S l}{2} m_y \frac{1}{\sin \delta_r} \omega_\xi v_r \zeta + \frac{\rho v_r^2 S l m_z w_\delta M}{2} \right) \\
+ \frac{\omega_\xi \omega_\eta - \omega_\eta \omega_\zeta \tan \varphi_2}{A} \tag{9}
\]

\[
\frac{d\varphi_a}{dt} = \frac{\omega_\zeta}{\cos \varphi_2} \tag{10}
\]

\[
\frac{d\varphi_2}{dt} = -\omega_\eta \tag{11}
\]

\[
\frac{d\gamma}{dt} = \omega_\zeta - \omega_\zeta \tan \varphi_2 \tag{12}
\]

\[
\begin{align*}
\frac{dx}{dt} &= v \cos \psi_2 \cos \theta_a \\
\frac{dy}{dt} &= v \cos \psi_2 \sin \theta_a \\
\frac{dz}{dt} &= v \sin \psi_2 
\end{align*} \tag{13}
\]

The physical meanings of every variable in equation(5)-(14) and other unlisted equations all can be found in reference [7].
3. Results and discussion

The internal and external ballisitic performance of the 155mm uni-modular charge howitzer is analyzed by using the above calculation model. The main conditions for calculation are as follows:

- standard meteorological conditions;
- caliber 155mm;
- barrel length L/52;
- chamber volume 23L;
- HE projectile 45.5kg.

The uni-modular charge is composed of 37 holes triple-base propellant and 19 holes coated triple-base low temperature sensitivity propellant. The total mass of the uni-modular charge is 3.0kg.

The muzzle velocity classification of large-caliber artillery directly affects the firing command strategy, firing hit rate, ballistic maneuverability, effective firing time and firing feasibility of the artillery. In the design of ballistic trajectory, the firing accuracy of the whole system should be considered.

Firstly, conventional ignition condition is calculated as a criterion for our discussion. The conventional muzzle velocity classification is shown in table 1.

| Charge number | 1   | 2   | 3   | 4   | 5   | 6   |
|---------------|-----|-----|-----|-----|-----|-----|
| Muzzle velocity $v_0$ (m/s) | 340 | 490 | 620 | 740 | 880 | 1030 |

It is assumed that the input electric energy in the ETC launch is 400kJ [9]. The average bore pressure and projectile movement curves of each charge are shown in figure 1 and figure 2, respectively.

![Figure 1. Average bore pressure curves of each charge in ETC launch.](Figure1.png)

![Figure 2. Projectile movement curves of each charge in ETC launch.](Figure2.png)

Hence, the muzzle velocity classification of ETC launch are shown in table 2.
Table 2. ETC launch muzzle velocity classification.

| Charge number | 1   | 2   | 3   | 4   | 5   | 6   |
|---------------|-----|-----|-----|-----|-----|-----|
| Muzzle velocity $v_0$ (m/s) | 397 | 560 | 690 | 812 | 920 | 1038 |

The firing ranges and heights of the above two muzzle velocity classification of conventional and ETC launch at different firing angles are calculated, respectively. The specific calculation results are shown in the table 3 and table 4.

Table 3. Firing ranges and heights of conventional launch muzzle velocity classification in different degrees. (Unit:m)

| No. | $v_0$ (m/s) | 20$^\circ$ | 45$^\circ$ | 48$^\circ$ | 50$^\circ$ | 70$^\circ$ |
|-----|-------------|------------|------------|------------|------------|------------|
| 1   | 340         | 6198/607   | 9023/2487  | 8947/2740  | 8848/2907  | 5837/4342  |
| 2   | 490         | 9212/1029  | 13196/3884 | 13115/4274 | 12997/4532 | 8727/6791  |
| 3   | 620         | 12155/1474 | 16891/5281 | 16875/5799 | 16710/6144 | 11382/9197 |
| 4   | 740         | 14937/1919 | 20270/6732 | 20337/7385 | 20284/7822 | 14195/11757|
| 5   | 880         | 18213/2463 | 24562/8617 | 24706/9464 | 24784/10033| 18351/15331|
| 6   | 1030        | 21730/3061 | 30298/10861| 31031/11987| 30929/12744| 24140/19913|

Table 4. Firing ranges and heights of ETC launch muzzle velocity classification in different degrees. (Unit:m)

| No. | $v_0$ (m/s) | 20$^\circ$ | 45$^\circ$ | 48$^\circ$ | 50$^\circ$ | 70$^\circ$ |
|-----|-------------|------------|------------|------------|------------|------------|
| 1   | 397         | 7283/750   | 10628/2996 | 10563/3300 | 10447/3502 | 6951/5250  |
| 2   | 560         | 10779/1263 | 15213/4614 | 15095/5071 | 14949/5375 | 10126/8046 |
| 3   | 690         | 13777/1731 | 18848/6109 | 18888/6703 | 18814/7099 | 12965/10645|
| 4   | 812         | 16627/2196 | 22402/7676 | 22516/8423 | 22486/8923 | 16225/13525|
| 5   | 920         | 19347/2653 | 26176/9310 | 26508/10236| 26690/10863| 20004/16698|
| 6   | 1038        | 21926/3094 | 30659/10987| 31226/12130| 31302/12898| 24489/20181|

The maximum range, minimum range and range overlap are calculated according to the above tables. According to the Chinese military standard, the minimum firing angle of the cannon is 20$^\circ$ and the maximum firing angle of the howitzer is 70$^\circ$. The external ballistics and the firing table compilation method stipulated that the range overlap $R_{RL}$ of each number of charges should be 4%.

The range overlap is defined as the following equation:

$$R_{RL} = \frac{X_{jmax} - X_{imin}}{X_{imin}}$$  \hspace{1cm} (15)
where, $X_{j_{max}}$ is the maximum range of No.$j$ charge. $X_{i_{min}}$ is the minimum range of No.$i$ charge. Normally, we have $j = i - 1$ and $j$ from 1 to 5.

Hence, the range parameters are calculated and shown in table 5 and table 6.

### Table 5. Ranges and range overlaps of conventional launch.

| No. | $v_0$ (m/s) | $X_{max}$ | $X_{min}$ | $R_{RL}$(%) |
|-----|-------------|-----------|-----------|-------------|
| 1   | 340         | –         | –         | –           |
| 2   | 490         | 13196     | 8727      | 16.0        |
| 3   | 620         | 16891     | 11382     | 19.0        |
| 4   | 740         | 20337     | 14195     | 11.7        |
| 5   | 880         | 24784     | 18213     | 14.1        |
| 6   | 1030        | 31031     | 21730     |             |

### Table 6. Ranges and range overlaps of ETC launch.

| No. | $v_0$ (m/s) | $X_{max}$ | $X_{min}$ | $R_{RL}$(%) |
|-----|-------------|-----------|-----------|-------------|
| 1   | 397         | 10628     | 6951      | 5.0         |
| 2   | 560         | 15213     | 10126     | 17.3        |
| 3   | 690         | 18888     | 12965     | 16.4        |
| 4   | 812         | 22516     | 16225     | 16.3        |
| 5   | 920         | 26690     | 19347     | 21.7        |
| 6   | 1038        | 31302     | 21926     |             |

**Figure 3.** Range of conventional launch and ETC launch.

**Figure 4.** Range overlap of conventional launch and ETC launch.

It is shown in figure 3 that the No.1 charge situation (red data as shown in table 3) must be abandoned in conventional launch because of the lower bore pressure, which cannot satisfy the...
firing safety request. According to the Chinese military standard, only 8.7km of No.2 charge can be accepted as the minimum range of the conventional launch. Its maximum range is 31.0km of No.6 charge. The range overlap of each charge number conform to the requirement of 4%. The ETC launch can increase the firing chamber pressure of No.1 charge. Thus, its minimum range can reach 7.0km of No.1 charge and the maximum range is 31.3km of No.6 charge. The range overlap of each charge number also conform to the requirement of 4%.

We introduced the average value and standard deviation to evaluate the range overlap of both conventional launch and ETC launch. Calculation results are shown in figure 4. Because the No.1 charge is abandoned in conventional launch. The average value and standard deviation are calculated based on the No.2 charge to No.6 charge in both conventional launch and ETC launch. It can be shown that the average value of the range overlap of conventional launch is 15.2% and the standard deviation of the range overlap of conventional launch is 3.08%. the average value of the range overlap of ETC launch is 17.9% and the standard deviation of the range overlap of ETC launch is 2.55%. The comparison shows that the ETC launch can achieve a smaller minimum range and a larger range overlap than the conventional launch, which facilitates the artillery firing command to have higher ballistic mobility and firepower flexibility.

4. Conclusion
Electrothermal-chemical (ETC) launch technology can accurately control and enhance the ignition and combustion of solid propellants to improve the consistency and accuracy of ballistics. A calculation model including the transient burning rate formula and six-degree-of-freedom rigid body trajectory equation is established to simulate both interior and exterior ballistics processes for a 155mm uni-modular charge howitzer in this paper.

The results show that the muzzle velocities are optimized via the ETC technology. The average range overlap is increased from 15.2% to 17.9% via input the electrical energy 400kJ. And it also has smaller standard deviation by ETC launch. The 155mm uni-modular charge ETC howitzer can effectively obtain the minimum range, increase the range overlap and maximum range.

In the future, this calculation model will be employed to discuss the regulation relationship between the input electrical energy and firing range overlaps after some minor corrections.

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References
[1] Jin Y, Ni Y, Li H and Li B 2016 Defence Technology 12 96–100
[2] Jin Y, Wan G, Yang C, Li H and Li B 2017 Applied analysis of electrothermal chemical launch technology in 155mm modular charge howitzer 30th International Symposium on Ballistics pp 561–566
[3] Aberg D, Hermansson P, Sattler A and Rakus D 2015 IEEE Transactions on Plasma Science 43 1316–1320
[4] Goodell B 2007 IEEE Transactions on Magnetics 43 456–459
[5] Dyvik J, Herbig J, Appleton R, O’Reilly J and Shin J 2007 IEEE Transactions on Magnetics 43 303–307
[6] Ni Y, Jin Y, Wan G, Yang C, Li H and Li B 2016 Defence Technology 12 81–85
[7] Han Z P 2014 Exterior Ballistics of Projectiles and Rockets (Beijing, China: Beijing Institute of Technology Press)
[8] Woodley C R and Billett S J 2001 IEEE Transactions on Magnetics 37 207–210
[9] Li Z, Zhang Y, Wu J, Jin Y and Li B 2015 Development and properties of a 500kj pulsed power supply 2015 IEEE Pulsed Power Conference (PPC) pp 1–4