The long-standing problem of $^{176}$Lu/$^{176}$Hf branching: a new approach with full stellar evolutionary models

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1. Introduction

The s-process branching at mass number $A=176$ determines the abundances of the s-only isotopes $^{176}$Lu and $^{176}$Hf. This work is motivated by the discrepancy between the current theoretical expectations and solar-system abundances of the two isotopes involved in the branching. Asymptotic Giant Branch models have been calculated with the FRANEC evolutionary code, coupling a full nuclear reaction network with the equation describing the physical evolution of the stellar structure. With respect to previous studies, we can better follow the complex interplay between mixing and burning, which both strongly influence this branching. We show that a substantial reduction of the mixing velocity within the convective zone generated by a He-shell flash (or thermal pulse) may alleviate the discrepancy between the theoretical expectations and the observed solar abundances. Nonetheless, we reckon that a definitive solution of this problem has to be searched in the nuclear coupling scheme between the thermally populated levels in $^{176}$Lu.

1. Introduction

The s-process branching at mass number $A=176$ determines the abundances of the s-only isotopes $^{176}$Lu and $^{176}$Hf, which are produced by the reaction sequences

$$^{175}$Lu $(n,\gamma) \rightarrow^{176}$Lu and $^{175}$Lu $(n,\gamma) \rightarrow^{176}$Lu$^* \beta^- \rightarrow^{176}$Hf.

Due to its long half-life of $\sim$40 Gyr, $^{176}$Lu was initially considered as a potential cosmochronometer [1]. However, it turned out that the half-life is dramatically shortened at the temperatures of the s-process site in low mass AGB stars due to a coupling of the long-lived ground state with the short-lived isomer at 123 keV with $t_{1/2} = 3.68$ h. Because direct transitions between the two states are forbidden by K-selection rules, the coupling can only occur via thermal population of intermediate states at much higher excitation energies, in particular through a state at 839 keV (hereafter IS@839). Accordingly, the branching at $A=176$ has been interpreted as a sensitive s-process thermometer (see [2] and references therein) in the temperature range above $1.5 \times 10^8$ K, where the coupling between ground state and isomer becomes relevant, leading to a drastic reduction of the effective half-life of $^{176}$Lu and, therefore, to a larger $^{176}$Hf production.
Figure 1. Left Panel: Physical evolution of (top to bottom): inner border of the convective envelope, H-shell and He-shell of a star with \( M = 2M_\odot \) and \( Z = 10^{-2} \). Right Panel: Behavior of the surface \(^{176}\text{Lu}/^{150}\text{Sm}\) and \(^{176}\text{Hf}/^{150}\text{Sm}\) ratios, normalized to their solar abundances.

The main neutron source in low mass AGB stars (the major contributors to the s-process) is the \(^{13}\text{C}(\alpha, n)^{16}\text{O}\) reaction, which works under radiative conditions within the so-called \(^{13}\text{C}\)-pocket [3]. In this case, temperatures are too low (\( T < 1.5 \times 10^8 \) K) to couple \(^{176}\text{Lu}\) isomer and ground state and, therefore, they behave as two separate species. However, an additional minor neutron exposure comes from the \(^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}\) reaction, which is active during He shell flashes (Thermal Pulses, TPs). In this regime the maximum temperature at the bottom of the convective He shell reaches about \( 3 \times 10^8 \) K, where the population of the two states depends sensitively on the temperature and the neutron density. The probability that the s-process reaction flow passes through the neutron capture channel (to the detriment of the \( \beta^- \) decay channel) is described by the branching factor \( f_n = \lambda_n/(\lambda_n + \lambda_-) \). As a consequence of the coupling between ground state and isomer, the beta decay rate \( \lambda_- \) is therefore a complex function of the local temperature and neutron density [2,5]:

\[
\lambda_- = \lambda_n \cdot \frac{C_1 I_\sigma \exp[-C_2/K T] + (1 - B) \lambda_n}{I_\sigma \exp[-C_3/K T] + C_1 B}. \tag{1}
\]

Various symbols are defined as:

- \( \lambda_n = n_n v_T \langle \sigma \rangle \) is the neutron capture rate (\( n_n \) denotes the neutron density, \( v_T \) the mean thermal velocity and \( \langle \sigma \rangle \) the \((n, \gamma)\) cross section averaged over the stellar neutron spectrum);
- \( I_\sigma \sim V \times (1 - V)/\tau_i \) is the energy-integrated cross section (\( \tau_i \) is the underlying lifetime, while \( V \) is the decay branching of the IS@839 toward the isomer);
- \( B = 1-\text{IR} \) (IR being the isomeric ratio in the \(^{175}\text{Lu}(n, \gamma)^{176}\text{Lu}\) reaction);
- \( C_1, C_2 \) and \( C_3 \) are constant factors.

Basing on an updated set of nuclear input parameters\(^2\), [4] performed a sequence of post-process calculations and, by varying the two fundamental parameters \( I_\sigma \) and \( B \), found a \(^{176}\text{Lu}/^{176}\text{Hf}\) ratio compatible with the observed solar system abundances. This agreement was, however, challenged by new experimental data [5] leading to a lower lifetime of the intermediate state \( \tau_i \) of 6.6 ps compared to the previously adopted 80 ps and to an increase of \( B \) from 0.14 to 0.20. In the following the parameter sets of Refs. [4] and [5] are labelled HEIL and MOHR, respectively.

The stellar evolutionary code used to perform our theoretical calculations is described in Section 2 and the results are discussed in Section 3.

\(^1\) see formula 25 in [2].

\(^2\) thus well constraining the value of \( \lambda_n \).
2. The FRANEC code

We revisit the Lu/Hf problem by using an up-to-date version of the FRANEC stellar evolutionary code [6,7,8]. Owing to the particular relevance for the present paper, our models have been calculated by fully coupling the physical evolution of the stellar structure with an extended nuclear reaction network involving about 500 isotopes (from H to Bi), linked by about 1000 reactions. Such a coupling allows for a proper treatment of all those situations when the nuclear burning time scales are comparable to those of convective mixing. The $^{176}\text{Lu}/^{176}\text{Hf}$ ratio, mainly determined during the freeze-out phase of each TP, can therefore be evaluated with a larger degree of accuracy than could be obtained in previous works. Models presented here have been calculated using the MOHR parameters.

3. Results

With the aim of reproducing the solar main s-process component (i.e. to obtain a flat distribution of the isotopes synthesized by the s-process only, see [9]), we follow the AGB phase of a star with initial mass $M = 2M_\odot$ and $Z = 1 \times 10^{-2}$ (see left panel of Fig. 1)$^3$. This model is characterized by 12 TPs followed by Third Dredge Up (TDU) episodes, which carry the synthesized s-process elements from the stellar interior to the surface. Since $^{176}\text{Lu}$ and $^{176}\text{Hf}$ are both shielded by their stable isobar $^{176}\text{Yb}$, we would expect for these isotopes the same overabundances (scaled to their solar value) as for $^{150}\text{Sm}$.$^4$ Instead, we obtain a strong overproduction of $^{176}\text{Lu}$ (2.40) and a strong underproduction of $^{176}\text{Hf}$ (0.55) with respect to $^{150}\text{Sm}$ (see right panel of Fig. 1), thus confirming previous results by [5], who obtained 1.80 and 0.61, respectively.$^5$

In order to understand the origin of this disagreement between stellar models and theoretical expectations we start a deeper analysis of this model focusing on the 9th TP followed by TDU. In left panel of Fig. 2 we report the behavior of the $\beta$-decay channel as a function of the three nuclear parameters ($\tau_i, B$ and $V$) in the convective shell. We confirm that major variations derive from the choice of $\tau_i$, pointing out that the maximum of the decay channel moves outward (increasing $M/M_\odot$ or stellar radius) with decreasing $\tau_i$. In contrast, the neutron capture channel has its maximum

$^3$ Note that a different metallicity or a different initial mass would imply minor changes to the results.

$^4$ This isotope is the best known case of an un-branched s-only isotope among the Rare Earth Elements.

$^5$ Note that differences with respect to $^{150}\text{Sm}$ up to 10% can be reasonably tolerated.
always at the base of the convective shell, where the neutron density attains its maximum value. This fact suggests that the $^{176}\text{Lu}/^{150}\text{Sm}$ and $^{176}\text{Hf}/^{150}\text{Sm}$ ratios could depend on the mixing efficiency, since the two channels reach their maximum in different regions of the convective shell. We test this hypothesis during the evolution of the 9th TP by artificially reducing the mixing velocity by a factor of 1000 with respect to the value estimated by means of the mixing length theory. The evolution of the intershell abundances of $^{176}\text{Lu}$ and $^{176}\text{Hf}$ normalized to the final abundance of $^{150}\text{Sm}$ are shown in right panel of Fig. 2. Solid curves refer to ratios obtained with the MOHR parameters, while dotted curves represent the ratios obtained with the same parameters but with reduced convective velocities. For comparison, we also run an additional model by using the HEIL parameters (dashed curves).

Right panel of Fig. 2 shows that, with respect to the MOHR results, a reduction of the convective velocities leads to a lower Lu production and to a lower Hf destruction (thus mimicking the HEIL results). In order to check the consistency of this test model, we look at the abundances of stable isotopes in the He-intershell at the end of the convective episode and find appreciable differences only for a restricted number of cases. Apart from the differences for $^{176}\text{Lu}$ and $^{176}\text{Hf}$, major changes occur for r-only isotopes (such as $^{110}\text{Pd}$, $^{150}\text{Nd}$, and $^{160}\text{Gd}$), which are enhanced up to a factor 2, for $^{96}\text{Zr}$ (enhanced by about 40%) and for the s-only $^{152}\text{Gd}$ (enhanced by about a factor 2, due to the temperature dependent $\beta$ decay of its unstable isobar $^{152}\text{Eu}$). From this chemical analysis, therefore, there seem to be no arguments against an extreme reduction of the convective velocities. Note that, even if convective velocities have been reduced for a factor 1000, we obtain the same $^{128}\text{Xe}/^{130}\text{Xe}$ ratio of the standard case, due to the accuracy characterizing the adopted temporal step (see [10] for details).

Nevertheless, the better agreement with theoretical expectations obtained with reduced convective velocities does not allow us to state that the problem of the $^{176}\text{Lu}/^{176}\text{Hf}$ ratio has been solved. Our doubts arise mainly from the fact that recent 3D hydrodynamical calculations of shell He flashes [11] tend to exclude that velocities within He-shell convective zones differ strongly from the ones obtained in the framework of the mixing length theory. While one could accept uncertainties of a factor 2-3 in the determination of convective velocities, we find it hard to justify differences up to a factor 1000. In our opinion, it is more likely that the solution to this problem has to be searched in a better comprehension of the nuclear coupling scheme between the populated levels in $^{176}\text{Lu}$. In this context, [12] has recently hypothesized that the rate of equilibration between the two isotopes could be further enhanced via $K$-mixing. This new analysis may lead to a strong revision of the nuclear parameters adopted in the present work.

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