Beyond skyrmions: Review and perspectives of alternative magnetic quasiparticles

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(Dated: May 5, 2020)

Magnetic skyrmions have attracted enormous research interest since their discovery a decade ago. Especially the non-trivial real-space topology of these nano-whirls leads to fundamentally interesting and technologically relevant effects – the skyrmion Hall effect of the texture and the topological Hall effect of the electrons. Furthermore, it grants skyrmions in a ferromagnetic surrounding great stability even at small sizes, making skyrmions aspirants to become the carriers of information in the future. Still, the utilization of skyrmions in spintronic devices has not been achieved yet, among others, due to shortcomings in their current-driven motion. In this review, we present recent trends in the field of topological spin textures that go beyond skyrmions. The majority of these objects can be considered the combination of multiple skyrmions or the skyrmion analogues in different magnetic surroundings, as well as three-dimensional generalizations. We classify the alternative magnetic quasiparticles – some of them observed experimentally, others theoretical predictions – and present the most relevant and auspicious advantages of this emerging field.

I. INTRODUCTION

Over the last decades, information technology has become eminently relevant for our everyday lives. The recent conquest of modern IT applications, such as streaming services and cloud storages, has further intensified the demand for energy efficient data storage and manipulation. While current electronic solutions struggle to keep up with Moore’s law, new spintronic proposals have been suggested and may become relevant in the near future.

One of the most auspicious and anticipated data storage devices is the racetrack memory. Originally proposed for utilizing domain walls as the carriers of information, the bits – encoded by the presence or absence of the magnetic object – are written, deleted, moved and read in a narrow track. This quasi-one-dimensional setup is stackable enabling the possibility for an innately three-dimensional data storage with drastically increased bit densities. This non-volatile concept without mechanically moving parts surpasses current random access memories (RAM) and hard disk drives (HDD) in terms of a lower energy consumption and faster access times.

Besides spintronics, topological matter is an aspiring research field which is why this review is concerned with non-collinear spin textures. The most prominent example is the magnetic skyrmion. This whirl-like nano-object was first observed a decade ago. Its topological protection gives it an enormous stability even at small sizes, which makes it a potential carrier of information in future data storage devices, such as the racetrack nanodevices.

Besides great stability, the topological properties of skyrmions induce emergent electrodynamics, namely the topological Hall effect and the skyrmion Hall effect. While the first – an additional contribution to the Hall effect of electrons in the presence of a topologically non-trivial spin texture – may become favorable for detecting skyrmions, the skyrmion Hall effect leads to a transverse deflection of skyrmions when they are driven by currents. This means that skyrmions are pushed towards the edge of the racetrack when a current is applied along the track, leading to pinning or even the loss of data. This is one of the reasons why no prototype of a skyrmion-based spintronic device exists today.

While there will be ongoing research for improving the applicability of magnetic skyrmions in spintronic devices, several alternative nano-objects have been predicted and observed during the last 6 years. Some of them promise even greater advantages compared to conventional skyrmions, which is why research in this direction will be drastically enhanced in the near future. In this review, we introduce and elaborate on these alternative magnetic quasiparticles. We establish a classification of the objects, explain methods to stabilize them, and compare their emergent electrodynamics to the case of conventional skyrmions.

We begin by elaborating on conventional magnetic skyrmions (Sec. II) to convey the differences to alternative magnetic quasiparticles later in the paper. We introduce how different types of skyrmions can be characterized topologically and geometrically (Sec. II A), explain various mechanisms for their stabilization (Sec. II B), and mediate the emergent electrodynamics (Sec. II C). Thereafter, we characterize and discuss the alternative magnetic quasiparticles (Sec. III). We distinguish three groups as visualized in Fig. 1: the fundamental excitations in ferromagnets (including different types of skyrmions; Fig. 1a), variations of these excitations (the combination of multiple excitations or excitations in other backgrounds than ferromagnets; Fig. 1b), and extensions (for example periodic arrays of magnetic objects or innately three-dimensional spin textures; Fig. 1c). First, we address the objects that are closely related to skyrmions (Sec. III A), namely skyrmions with an arbitrary helicity (Fig. 1b) and antiskyrmions (Fig. 2a). Thereafter, we consider combinations of skyrmions (Sec. III B), bimerons (Fig. 2c), biskyrmions (Fig. 2e), skyrmioniums (Fig. 2g), and antiferromagnetic skyrmions (Fig. 2i). Finally, we discuss chiral bobbers (Fig. 2d) and hopfions (Fig. 2h) as non-trivial three-dimensional continuations of skyrmions in Sec. III C. Other magnetic quasiparticles (shown in Fig. 2) are briefly addressed and put in context while discussing the above objects. We conclude this review in Sec. IV.
FIG. 1: Classification of observed and predicted spin textures. (a) The fundamental excitations are the building blocks for all textures. Skyrmions and merons of different types have been observed as the topologically non-trivial excitations in magnets. (b) These objects can be combined or considered in a different magnetic background to form new, distinct quasiparticles. (c) Also, the fundamental and derived objects can be continued as periodic or non-periodic arrangements in two dimensions or can be continued along the third dimension trivially or non-trivially. The objects discussed more thoroughly in this review are typeset bold.

II. MAGNETIC SKYRMIONS

Skyrmions have originally been predicted in the 1960’s in the context of particle physics. The British nuclear physicist Tony Skyrme proposed a field-theoretical description of interacting pions and showed that the particle-like solutions are fermionic while pions themselves are bosonic. These solitons, described by a non-linear sigma model, are the three-dimensional versions of what became known as skyrmions.

Today, skyrmions have been found in several fields of physics such as quantum Hall systems, Bose-Einstein condensates, liquid crystals, particle physics, string theory and, as considered here, in magnetism.

In this context, a skyrmion can be considered as a two-dimensional object (Fig. 1a), that is continued trivially along the third dimension (Fig. 1c). Such skyrmion tubes or skyrmion strings have been observed for the first time in 2009 in MnSi by reciprocal-space measurements and one year later using Lorentz transmission electron microscopy (LTEM).

The magnetic textures of these objects were in agreement with what had been predicted twenty years earlier. Magnetic skyrmions in a ferromagnetic medium are characterized by a continuously changing magnetization density which is oriented oppositely in its center compared to the surrounding leading to a non-trivial real-space topology. These objects can occur as periodic lattices, like in the above publications, or as individual particles.

The following discussion of conventional skyrmions is limited to their geometrical characterization, stabilizing mechanisms and emergent electrodynamics – all of which are a prerequisite for understanding the physics of the alternative magnetic quasiparticles. For a discussion of conventional skyrmions that goes beyond these points, we refer to one of the many review articles.
FIG. 2: Overview of the discussed topologically non-trivial spin textures. The first objects are different types of skyrmions, meaning skyrmions with various helicities and vortices: (a) antiskyrmion with a vorticity \( m = -1 \) and a topological charge of \( N_{Sk} = -1 \), (b) skyrmion with an intermediate helicity \( \gamma = \frac{\pi}{4} \) between Bloch and Néel type skyrmions characterized by \( N_{Sk} = 1 \), (c) higher-order skyrmion with \( N_{Sk} = 2 \). (d) Shows a magnetic bimeron consisting of two merons. Alternatively, it can be understood as a skyrmionic excitation in an in-plane magnetized medium, here characterized by \( N_{Sk} = -1 \). The middle row shows combinations of two skyrmions: (e) the biskyrmion with \( N_{Sk} = 2 \), (f) a skyrmionium with \( N_{Sk} = 0 \) and (g,h) ferrimagnetic and synthetic antiferromagnetic skyrmions for which the topological charges of the two subskyrmions compensate each other. The bottom row shows three-dimensional extensions of skyrmions: (i) skyrmion tubes (possibly with a varying helicity along the tube), (j) a chiral bobber as a discontinued skyrmion tube, (k) a pair of Bloch and anti-Bloch points constituting the building block of a three-dimensional crystal (hedgehog lattice), and (l) the hopfion.

A. Topology and characterization

The topological character of a skyrmion can be comprehended by a stereographic projection: A two-dimensional skyrmion (Fig. 3a) can be constructed by rearranging the magnetic moments of a three-dimensional hedgehog (also called Bloch point; see Fig. 3b), where all moments on a sphere point along the radial direction. This sphere is opened at the bottom and flattened to a disk without changing the moments’ orientations. The result is a topologically non-trivial magnetic object in two-dimensions.

In a continuous picture, where a skyrmion consists of a magnetization density \( m(r) \), this skyrmion cannot be transformed to a ferromagnetic state without discontinuous changes in the density. This is a manifestation of the non-trivial real-space topology quantified by the topological charge

\[
N_{Sk} = \int n_{Sk}(r) \, d^2r, \tag{1}
\]

which is as an integral over the topological charge density

\[
n_{Sk}(r) = \frac{1}{4\pi} \left[ \frac{\partial m(r)}{\partial x} \times \frac{\partial m(r)}{\partial y} \right]. \tag{2}
\]

The topological charge of a skyrmion can be determined more easily from its appearance due to the following transformation. One expresses the magnetization density in spherical coordinates with the azimuthal angle \( \theta \) and the polar angle \( \Phi \) and expresses the position vector in polar coordinates \( r = r(\cos \phi, \sin \phi) \). Exploiting the radial symmetry of the out-of-plane magnetization density \( \theta = \theta(r) \), the topological charge read

\[
N_{Sk} = \frac{1}{4\pi} \int_0^\infty \int_0^{2\pi} dr \, d\phi \, \frac{\partial \Phi(\phi)}{\partial \phi} \frac{\partial \theta(r)}{\partial r} \sin \theta(r)
= -\frac{1}{2} \cos \theta(0) \int_0^{2\pi} d\phi \frac{\partial \Phi(\phi)}{\partial \phi} \bigg|_{\phi=0}^\infty . \tag{3}
\]

The out-of-plane magnetization of a skyrmion is reversed comparing its center with its confinement. This is quantified
As an example, the skyrmion in Fig. 3a has a positive polarity $p = +1$ and vorticity $m = +1$ leading to a topological charge of $N_{\text{Sk}} = +1$. Since the in-plane component of the magnetization is always pointing along the radial direction, the helicity in this case is $\gamma = 0$. This type of skyrmion is called Néel skyrmion and is typically observed at interfaces. On the contrary, the skyrmions in MnSi (e.g. from the initial observation) are called Bloch skyrmions. There, the in-plane components of the magnetization density are oriented perpendicularly with respect to the position vector. This toroidal configuration is characterized by a helicity of $\gamma = \pm \pi/2$. In contrast to the polarity and the vorticity, the helicity is a continuous parameter allowing for skyrmions as intermediate states between Bloch and Néel skyrmions, as shown in Fig. 2b. Furthermore, the vorticity can in principle take any integer value constituting for example antiskyrmions for $m = -1$ (Fig. 2a) or higher-order (anti)skyrmions for $|m| > 1$ (Fig. 2c). Out of this manifold, Bloch[51], Néel skyrmions and skyrmions with an intermediate helicity[50] as well as antiskyrmions[51] have been observed experimentally. Higher-order skyrmions have been predicted.

B. Stabilizing mechanisms

Having discussed the different types of possible skyrmions mathematically, we will now address how these objects can be stabilized in ferromagnets. In such materials the magnetic moments $\{s_i\}$ are coupled via the exchange interaction

$$H_{\text{ex}} = -\frac{1}{2} J_{i,j} \sum_{i,j} s_i \cdot s_j.$$  \hspace{1cm} (9)

Typically, the nearest-neighbor interaction is dominant. In a ferromagnet it favors a parallel alignment ($J_{i,j} = J > 0$ for $(i,j)$ nearest neighbors; $J_{i,j} = 0$ otherwise) of the magnetic moments. In a continuous approximation, this term is expressed as

$$H_{\text{exchange}} = \int \sum_{i,j} J \left( \frac{\partial m_i}{\partial x_j} \right)^2 \, d^3r.$$ \hspace{1cm} (10)

In the continuous limit, a magnetic skyrmion would be stable due to the topological protection; it cannot be transformed into a uniform ferromagnet continuously, even though the ferromagnetic state would have the lower energy. However, since real skyrmions consist of magnetic moments and not a continuous density, this protection is not strict in nature bringing forth the necessity of additional stabilizing interactions.

In theory, skyrmions have been stabilized by frustrated exchange interactions, four-spin interactions, dipole-dipole interactions and Dzyaloshinskii-Moriya interactions[51]. In the following, we will focus on the latter two cases, since they are by far the most relevant mechanisms in nature.

_Dzyaloshinskii-Moriya interaction_. The Dzyaloshinskii-Moriya interaction[51,52] (DMI)

$$H_{\text{DMI}} = \frac{1}{2} \sum_{i,j} D_{i,j} \cdot (s_i \times s_j),$$ \hspace{1cm} (11)
is a chiral interaction responsible for the stability of most experimentally observed skyrmions. It is a correction due to spin-orbital coupling under a broken inversion symmetry and can be considered an antisymmetric exchange interaction, $D_{ij} = -D_{ji}$. The $D_{ij}$ are the Dzyaloshinskii-Moriya vectors, whose orientations account for the way the inversion symmetry is broken, satisfying the Moriya symmetry rules\textsuperscript{57}. For example, at an interface of a magnet and a heavy metal, like Co/Pt, the DMI vectors are oriented parallel to the interface plane, perpendicular to the bond of the Co atom\textsuperscript{58}. In a continuous approximation this is expressed as
\begin{equation}
H_{\text{interface}} = \int \tilde{D} \left( m_x \frac{\partial m_z}{\partial x} - m_z \frac{\partial m_x}{\partial x} 
+ m_y \frac{\partial m_z}{\partial y} - m_z \frac{\partial m_y}{\partial y} \right) d^3r. \tag{12}
\end{equation}

Such an interaction favors a canting of magnetic moments. As a consequence, Néel-type skyrmions may be stabilized, as first observed at the interface of Fe and Ir(111)\textsuperscript{53}. This type of DMI and the resulting skyrmions are well understood today. In multistack systems of magnetic and heavy metal materials, the effective DMI strength can be tuned\textsuperscript{59} changing the size and the stability of Néel skyrmions\textsuperscript{60}.

Different samples lead to different DMI vectors, since the effective DMI strength can be tuned\textsuperscript{59} changing the size and the stability of Néel skyrmions\textsuperscript{60}.

In B20 materials, such as MnSi, where the inversion symmetry is broken intrinsically, the DMI is expressed as
\begin{equation}
H_{\text{bulk}} = \int \tilde{D} \left( m_y \frac{\partial m_z}{\partial x} - m_z \frac{\partial m_y}{\partial x} + m_z \frac{\partial m_y}{\partial y} 
- m_x \frac{\partial m_z}{\partial y} + m_z \frac{\partial m_y}{\partial y} - m_y \frac{\partial m_x}{\partial y} \right) d^3r, \tag{14}
\end{equation}

leading to the stability of Bloch skyrmions\textsuperscript{61}.

In principle, lower symmetric lattice structures or nano-structuring allow to generate further types of DMI, leading to the stabilization of alternative topologically non-trivial excitations in magnets.

Dipole-dipole interaction. A second main mechanism for stabilizing skyrmions is the dipole-dipole interaction
\begin{equation}
H_{\text{dd},ij} = -\frac{\mu_0}{4\pi} \left( 3 \frac{(s_i \cdot r_{ij})(s_j \cdot r_{ij})}{r_{ij}^5} - s_i \cdot s_j \right). \tag{15}
\end{equation}

While the DMI typically stabilizes skyrmions smaller than a few hundred nanometers, the dipole-dipole interaction can stabilize objects of several micrometer in diameter\textsuperscript{62}. Typically, these objects have an almost ferromagnetic center surrounded by a narrow domain wall. They are sometimes labeled ‘bubble’ but are topologically equivalent to skyrmions, given that they have the same $N_{\text{Sk}}$. As a difference to the DMI, dipole-dipole interactions are achiral, meaning that they energetically favor Bloch skyrmions of both helicities $\gamma = \pm \pi/2$\textsuperscript{63}, allowing even for their coexistence\textsuperscript{64}.

C. Emergent electrodynamics

Magnetic skyrmions have successfully been generated and deleted in thin films (see reviews\textsuperscript{65,66} for different methods). In order to constitute an operating data storage device, they also have to be driven and read. In this section we will discuss the topological Hall effect and the current-driven motion of skyrmions. This will be the foundation for our discussion of alternative magnetic quasiparticles later in this review and will reveal why skyrmions may not be the optimal candidates for such spintronic devices.

Topological Hall effect. The topological Hall effect of electrons is considered the hallmark of the skyrmion phase\textsuperscript{11–13,17,18,64–69}. To measure it, an electric field $E$ is applied to the skyrmion host, and the current density $j$ is detected. According to Ohm’s law, $E = \rho j$, the Hall effect of electrons is characterized by the transverse resistivity tensor element $\rho_{xy}$. For a skyrmion crystal, it is commonly considered as three superimposed contributions\textsuperscript{42}:
\begin{equation}
\rho_{xy} = \rho_{HE} + \rho_{AHE} + \rho_{THE} \tag{16}
\end{equation}

that can be isolated due to their distinct proportionalities: the ordinary Hall contribution\textsuperscript{42} occurs due to an external magnetic field $B_{\text{ext}} \propto B_z$, the anomalous Hall contribution\textsuperscript{21} is due to spin-orbit coupling and, usually, a net magnetization $M_x \propto M_z$, and the topological Hall contribution appears due to the presence of skyrmions or other topologically non-trivial spin textures.

The presence of a skyrmion leads to the emergence of an additional contribution to the Hall effect for the following reason: If an electron hops between two sites and reorients its spin, the original transfer integral $t$ accumulates a complex phase factor\textsuperscript{43}. This factor is the analogue of the Peierls
phase, which characterizes the magnetic field in the ordinary Hall effect. In the skyrmionic case, it can also be related to an effective vector potential. In an adiabatic approximation, the corresponding field $B_{em}$ is called ‘emergent field’ and it is proportional to the topological charge density:

$$B_{em}(r) = 2\pi n_{Sk}(r) e_z.$$  \hspace{1cm} (17)

This fictitious field can be used to easily relate the transverse deflection and the generation of a Hall effect with the presence of skyrmions (Fig. 4). As long as the electron spin and the texture are coupled strongly, the topological Hall effect is proportional to the number of skyrmions in the sample $n_{Sk}(r) \propto \langle B_{em}, z \rangle \propto N_{Sk}$.

**Skyrmion Hall effect.** The non-trivial real-space topology of skyrmions also becomes apparent in the current-driven motion of the skyrmions themselves. One typically discusses two scenarios: the motion under spin-transfer torque (STT) where a spin-polarized current goes through the spin texture and reorients the magnetic moments of a skyrmion (Fig. 4), and the motion under spin-orbit torque (SOT) where a spin accumulation created by an electric current in the presence of spin-orbit interaction exerts a torque on the skyrmion texture. In both cases, the torque reorients the magnetic moments. This collective reorientation can be identified with a motion of the skyrmions. The SOT mechanism turns out to be more efficient, since electron spins and magnetic moments can have a large misalignment leading to larger torques. Still, it has been predicted and observed that the skyrmions do not move parallel to the current, but experience a transverse deflection in this case. This phenomenon is called the skyrmion Hall effect and originates in the topological charge of the magnetic objects.

For the given reasons, in this review we focus on the SOT setup and will only occasionally mention the spin-transfer torque. Typically, one considers a bilayer of a ferromagnet, potentially hosting skyrmions, and a heavy metal. The applied current mainly flows in the heavy metal, where the spin Hall effect generates a pure spin current along the perpendicular direction with spins oriented perpendicular to both currents. In a racetrack, the perpendicularly injected spins are pointing along the narrow width of the track.

The reorientation of the magnetic moments can be modeled by the Landau-Lifshitz-Gilbert (LLG) equation

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times \mathbf{B}_{eff} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} + \mathbf{\tau}.$$ \hspace{1cm} (18)

It is written here in the micromagnetic formulation, where the magnetization density has been discretized in small volumes with a normalized magnetization $\mathbf{m}$ (the index has been dropped for simplcity). The first term on the right side describes the precession of each normalized magnetic moment around its space- and time-dependent effective magnetic field

$$\mathbf{B}_{eff} = -\frac{1}{M_s} \frac{\delta F}{\delta \mathbf{m}}.$$ \hspace{1cm} (19)

characterized by the free energy functional $F$ accounting for all magnetic interactions. $\gamma_c = \gamma/\mu_0 = 1.760 \times 10^{11} \text{ T}^{-1}\text{s}^{-1}$ quantifies the gyromagnetic ratio of an electron. The second term is the Gilbert damping quantified by the dimensionless parameter $\alpha$, leading to an alignment of the magnetic moment with its effective field after a certain propagation time without external perturbations. The last term is the torque $\mathbf{\tau}$. In the spin-orbit torque scenario it is given by the out-of-plane and in-plane torques. The out-of-plane torque term may change the shape of a skyrmion, but does not add a driving force to the effective equation of motion, which is why we consider only the in-plane torque term in the following. It reads

$$\mathbf{\tau} = \frac{\gamma_e h}{2e d_e M_s} \mathbf{\theta}_{SH}(\mathbf{m} \times \mathbf{s}) \times \mathbf{m},$$ \hspace{1cm} (20)

where $d_e$ is the thickness of the magnetic layer, $M_s$ is the saturation magnetization and $\mathbf{\theta}_{SH}$ is the spin-Hall angle describing the spin current $\mathbf{\theta}_{SH}$ with spin orientation $\mathbf{s}$.

Numerically solving this equation for a skyrmion in a ferromagnet in the presence of an applied current leads to a motion of the skyrmion partially along the current direction (along the track), but also partially towards the track’s edge. This transverse component is due to the topological charge of the skyrmion as can be found by considering the Thiele equation

$$b \mathbf{G} \times \mathbf{v} - b D_{\alpha\beta} \mathbf{v} - b J I_s = \nabla U(y).$$ \hspace{1cm} (21)

This effective equation of motion can be derived from the LLG equation by assuming a perfectly rigid shape of the magnetic object and by considering only its center coordinate (with velocity $\mathbf{v}$). The non-collinearity of the object is condensed in the gyroscopic vector $\mathbf{G}$, the dissipative tensor $D$ and the torque tensor $I$, which are calculated as follows (for a complete derivation see

$$\mathbf{G} = -4\pi N_{Sk} \mathbf{e}_z$$ \hspace{1cm} (22)

$$D_{ij} = \int \partial_i \mathbf{m}(\mathbf{r}) \cdot \partial_j \mathbf{m}(\mathbf{r}) d^2r,$$ \hspace{1cm} (23)

$$I_{ij} = \int [\partial_i \mathbf{m}(\mathbf{r}) \times \mathbf{m}(\mathbf{r})]_j d^2r.$$ \hspace{1cm} (24)

The non-collinear object is then condensed to a single point.

A circular Néel skyrmion for example is characterized by a topological charge of $N_{Sk} = \rho = \pm 1$, a diagonal dissipative tensor with only $D_{xx} = D_{yy} \neq 0$ and an antisymmetric torque tensor with only $I_{xy} = -I_{yx} \neq 0$. For injected spins $\mathbf{s} \parallel \pm \mathbf{y}$ this yields a skyrmion Hall angle of

$$\tan \theta_{Sk} = \tan \frac{\nu_d}{\nu_s} = -\frac{4\pi N_{Sk}}{\alpha D_{xx}}.$$ \hspace{1cm} (25)

For topological objects with more complicated $I$ and $D$ tensors (for example due to a broken rotational symmetry), this relation varies and can even allow for a vanishing skyrmion Hall effect, which can be seen as one main motivation for considering alternative magnetic quasiparticles.

**III. ALTERNATIVE MAGNETIC QUASIPARTICLES**

In this main section of the review, alternative magnetic quasiparticles are introduced and discussed concerning their...
perspective for spintron applications, namely for racetrack memories. In this regard, we will refer to the fundamentals established in the last section. We characterize the different types of objects related to skyrmions, address their stability and their emergent electrodynamics.

**Fundamental excitations in ferromagnets.** As presented in Fig. 1, we distinguish the fundamental excitations in ferromagnets from their variations and extensions. In panel a of Fig. 1 the fundamental excitations are shown. They can be separated into topologically trivial and non-trivial objects. The latter comprise merons and skyrmions of different varieties. As presented in the last section, topological excitations with a topological charge of ±1 have been found as Néel skyrmion, Bloch skyrmion, intermediate skyrmion and antiskyrmion (Fig. 2a). Furthermore, higher-order skyrmion and antiskyrmion (Fig. 2d) have been predicted. Most attractive application-wise are the intermediate skyrmions and antiskyrmions, since they can be moved without the occurrence of a skyrmion Hall effect as will be presented in Sec. III A.

**Variations of the topological excitations.** In Sec. IIIB, we discuss the combinations of skyrmions or merons to form new particles. Very promising are the combinations of two skyrmions with opposite topological charges: the antiferromagnetic skyrmion and the skyrmionium which are predicted to move without a skyrmion Hall effect.

As shown in Fig. 1b, some of the composite objects can ambivalently be considered as skyrmionic excitation in different magnetic backgrounds. The bimeron (Fig. 2b), for example, is on one side the combination of a meron and an antimeron, but also a skyrmion in a ferromagnet which is magnetized in-plane. Likewise, the antiferromagnetic skyrmion (Fig. 2c) is the combination of two ferromagnetic skyrmions with mutually reversed spins, but also is the fundamental topological excitation in a collinear antiferromagnet. The same holds for ferrimagnetic skyrmions (Fig. 2f) in ferrimagnets.

**Extensions of the topological excitations.** The fundamental objects as well as their variations can be extended in the sense that they can be arranged in a (pseudo-) two-dimensional system, or that they can be extended along the third spacial dimension, see Fig. 1c.

The two-dimensional arrangements are typically rather trivial: Periodic crystals of skyrmions, antiskyrmions, bimerons, biskyrmions, antiferromagnetic skyrmions and other particles can form. However, such an arrangement can also be highly non-periodic like in a skyrmion glass, or the arrangement can consist of multiple objects like for the meron-antimeron lattice, as observed recently.

However, more relevant for this review seem to be the three-dimensional extensions that are discussed in Sec. III C. Skyrmion tubes (Fig. 2a), chiral bobbers (Fig. 2b), Bloch anti-Bloch crystals (Fig. 2c), and hopfions (Fig. 2d) are the most prominent candidates.

### A. Different types of skyrmions

Skyrmions are characterized by the polarity $p$, vorticity $m$ and helicity $\gamma$. All types of skyrmions behave similar under STT but differently under SOT. For example, the missing rotational symmetry of antiskyrmions brings about an anisotropic skyrmion Hall effect, which is highly relevant for racetrack applications.

The vorticity can also have integer values larger than 1. This characterizes higher-order skyrmions and antiskyrmions with $|N_{sk}| > 1$. These objects exhibit a topological Hall effect and a skyrmion Hall effect in the STT scenario just like skyrmion, but their rotational symmetry is broken, similar to antiskyrmions.

Furthermore, in this class of magnetic quasiparticles the fundamental excitations in in-plane magnetized samples are worth to be mentioned: merons (or vortices) which are closely related to skyrmions. These objects are configured like skyrmions near their centers but the magnetic moments at the edges of the particles do not point into the opposite out-of-plane direction compared to the centers but along in-plane directions, which is often related to the magnetic anisotropy. The azimuthal angle changes only by $\pi/2$ giving these objects a polarity and a topological charge of $\pm 1/2$. These objects become relevant for forming non-trivial textures like the meron-antimeron crystal but are less relevant themselves, since they are always situated in a coplanar but non-collinear in-plane magnet.

In this section, we focus on skyrmions with an intermediate helicity and antiskyrmion. Due to their non-trivial topological charge, both objects exhibit a topological Hall effect and a skyrmion Hall effect in the STT scenario. However, in the SOT scenario the possibility to move these objects parallel to an applied current exists. In the following, we elaborate on this motion, the particles’ stability and other non-trivial observations.

#### 1. Skyrmions with arbitrary helicity

One main problem upon utilizing magnetic skyrmions in racetrack devices is the skyrmion Hall effect. A possible solution is the utilization of skyrmions with a helicity $\gamma$ different from that of Néel or Bloch skyrmions. In DMI systems this can in principle be achieved by considering a mix of interfacial and bulk DMI or by considering interfacial DMI in materials where the dipole-dipole interaction (favoring Bloch skyrmions) is considerable. Such objects have recently been observed using LTEM imaging.

In order to understand the skyrmion Hall effect, we analyze the tensors in the Thiele equation. For a positive polarity, skyrmions with an arbitrary helicity are characterized by a topological charge of $N_{sk} = +1$ leading to a gyroscopic vector of $G = -4\pi e_z$ independent of the skyrmion’s helicity. This independence holds also for the dissipative tensor which is diagonal with only $D_{xx} = D_{yy} \neq 0$. However, the torque tensor depends on the helicity $\gamma$, which implies that a SOT, characterized by spins $s \parallel y$ that are injected from the
perpendicular direction \( z \), drives skyrmions with different helicities differently, resulting in different skyrmion Hall angles and trajectories. In general, the torque tensor of a skyrmion has the shape of a rotation matrix \( R_\gamma \) around the \( z \) axis

\[
I = \begin{pmatrix}
\sin \gamma & \cos \gamma & 0 \\
-\cos \gamma & \sin \gamma & 0 \\
0 & 0 & 0 \\
\end{pmatrix} = R_\gamma(\gamma + \pi/2).
\] (26)

For a particular helicity \( \gamma \) (Fig. 5a,b), the transverse motion of a skyrmion due to the gyroscopic force is compensated by a component of the driving force (Fig. 5g) so that the skyrmion moves along the applied electric current. Likewise, there exists a helicity for which the skyrmion moves perpendicular towards the edge. Due to the recent observation of these objects, a verification of the theoretical predictions is highly anticipated.

2. \textit{Antiskyrmions}

Antiskyrmions are characterized by a vorticity of \( m = -1 \), i.e., the in-plane magnetization rotates oppositely to the position vector. Unlike skyrmions (irrespective of their helicity), these particles are not rotationally symmetric. For this reason, the helicity has a different meaning for antiskyrmions. It is not a global offset between the polar angle of the position vector \( \phi \) and the polar angle of the magnetization \( \Phi \), but distinguishes two axes along which the antiskyrmion has the profile of a Néel skyrmion with helicity 0 and \( \pi \), respectively (dashed lines in Fig. 5). The profile looks differently along other lines. For example, along the two bisectrices of these two axes the antiskyrmion has the same texture as a Bloch skyrmion with helicity \( \pm \gamma/2 \), respectively.

Geometrically, an antiskyrmion can be constructed from a skyrmion by rotating all spins of the skyrmion by \( 180^\circ \) around a distinguished in-plane axis – say \( y \). In this case, the \( x \) and \( z \) components of the magnetization change sign. The resulting antiskyrmion still has the same topological charge as the skyrmions since the polarity and the vorticity both change their signs. Furthermore, the rotation argument allows to also identify the DMI necessary to stabilize antiskyrmions: two of the DMI vectors need to change their signs. This leads to the anisotropic DMI [Eq. (13)] presented in an earlier section. Note again, that the DMI vectors are determined by the crystal symmetry: While Néel skyrmions arise at interfaces, where heavy metal atoms are located directly below the bond of two magnetic atoms, for antiskyrmions a layered system is required with heavy metal atoms above and below two different bonds. This is the case in some Heusler materials, and indeed antiskyrmion crystals have been observed in Mn\(_{1.4}\)P\(_{0.9}\)Pd\(_{0.1}\)Sn, a Heusler material with \( D_{2d} \) symmetry that exhibits this particular type of DMI\(^{[20]} \). The antiskyrmions in this material have been shown to have long lifetimes at room temperature\(^{[20]} \). Since an antiskyrmion can be understood to consist of Bloch and Néel parts, the observed LTEM contrast (Fig. 7) is unique. It exhibits two spots of high intensity and two spots of low intensity, corresponding to the different Bloch and Néel parts.

The change of the vorticity compared to skyrmions is highly relevant for the current-driven motion of antiskyrmions. The gyroscopic vector still points along \( z \) and the dissipative tensor \( D \) is still symmetric. However, the \( I \) tensor changes

\[
I = \begin{pmatrix}
-sin \gamma & -cos \gamma & 0 \\
-cos \gamma & sin \gamma & 0 \\
0 & 0 & 0 \\
\end{pmatrix}.
\] (27)

Like for the case of skyrmions with an arbitrary helicity, one can identify a particular helicity for which the skyrmion Hall effect is compensated for antiskyrmions. The important differ-
ence is that the helicity of antiskyrmions corresponds merely to a rotation in real space. Consequently, applying the current along a certain direction leads to an antiskyrmion motion parallel to the current, as presented in Ref. [56]. This allows to think of a racetrack, where the D_{2d} material is cut at an angle with respect to the high symmetry direction(108). In a rotated coordinate system, where one axis points along the current direction, this leads to a rotation of the effective micromagnetic DMI vectors and generates a rotated antiskyrmion, i.e., an antiskyrmion with a certain γ. If it is cut under the correct angle, this racetrack allows for the desired straight-line motion of the bits without any transverse deflection (Fig. 5.f).

Another particularly interesting feature of antiskyrmion systems is that their anisotropic DMI is in conflict with the ubiquitous dipole-dipole interaction. While the first interaction allows only for antiskyrmions, the dipole-dipole interaction energetically favors Bloch skyrmions. The coexistence of both topologically distinct nano-objects has recently been observed by LTEM measurements(101,102) and confirmed by micromagnetic simulations(103). Their distinct topological charges have later been confirmed by topological Hall measurements(103). The DMI-stabilized antiskyrmions may show a square-shaped deformation due to the perturbative effect of the dipole-dipole interaction. The Bloch parts are increased and the Néel parts shrink in order to minimize the dipole-dipole energy(103), as predicted in Ref. [104]. The dipole-dipole-mediated Bloch skyrmions, that come in two flavors (helicities γ = ±π/2), are elliptically deformed due to the DMI(103). This finding allows to generalize the concept of the racetrack device using both, antiskyrmions and skyrmions, as the bits of information(104,103) (Fig. 8b). Such devices would be insensitive to diffusion or interactions between the nano-objects, since non-periodic bit sequences are unproblematic in this case. The coexistence of skyrmions and antiskyrmions has also been predicted by frustrated exchange interactions(17) and for a very specific DMI tensor(23). However, these two cases remain to be observed experimentally.

B. Combination of skyrmions or merons

In this section we present alternative magnetic quasiparticles that are formed as the combination of two or more skyrmions or merons. We begin with the bimeron(23,24). It is the combination of two merons, which need to possess opposite vorticities in order to be geometrically compatible with each other and with the ferromagnetic background. Since also their polarities are mutually reversed, a bimeron is characterized by a topological charge of N_{Sk} = ±1. It can therefore be considered a skyrmion in an in-plane magnetized material. Due to their missing rotational symmetry, bimerons are highly relevant for racetrack applications similar to antiskyrmions and intermediate skyrmion as presented in the last section.

Biskyrmions on the other hand, are formed by two partially overlapping skyrmions giving the new texture a topological charge of N_{Sk} = ±2(25,26). In order to be compatible, this time the two Bloch skyrmions need to have opposite helicities instead of vorticities.

When instead two skyrmions with opposite polarities are combined, the new object has a vanishing topological charge. This makes skyrmionium(27,28) and antiferromagnetic skyrmion(29,30) the optimal candidates for spintronic applications. In the corresponding sections we discuss differences of these two objects, especially regarding the Hall effect of electrons.

Lastly, it is also worth mentioning that this list is non-exhaustive. For example, in synthetic antiferromagnets based on Co/Ru/Co films bimeronic topological defects (including both bi-vortices and bi-antivortices) living on domain walls were experimentally observed(105). Also, a pair of helix edges is considered the fundamental topological excitation in a helical phase(106).

1. Bimerons as in-plane skyrmions

The first object discussed in this category is the magnetic bimeron (Fig. 6). It was first predicted in 2017 as a bimeron crystal(29) and shortly after also as an individual particle(23).

As the name indicates, a bimeron consists of two merons – more precisely of a meron and an antimeron with mutually reversed out-of-plane magnetizations (bottom panel in Fig. 6a), i.e. opposite polarities. This gives both subparticles the same topological charge of either +1/2 or −1/2 and the bimeron a topological charge of ±1 (middle panel in Fig. 6a). In this sense, it can also be considered a skyrmionic excitation in an in-plane magnet(23). This becomes even more apparent from the following transformation: Starting from a conventional magnetic skyrmion, a rotation of all magnetic moments around a common in-plane axis by 90° in magnetization space results in a bimeron texture. Such rotations leave the topological charge invariant. A bimeron can therefore ambivalently be understood as a meron-antimeron pair with opposite polarities or as a skyrmion in an in-plane magnet(23).

Compared to the skyrmion, the bimeron is not characterized by an integer polarity, but the background magnetization can be rotated freely in the plane, mathematically speaking. This means that the class of bimerons has an additional continuous degree of freedom. A bimeron is characterized by two continuous angles instead of a discrete polarity and a continuous helicity. A possible characterization is that γ defines the angle of the connecting line between the two merons’ centers with respect to the x axis and α defines the orientation of the net magnetization (parallel to the surrounding) with respect to the x axis, as indicated in Fig. 5b.

Up to now, only individual bimerons have been observed experimentally that were generated in a Py film via local vortex imprinting from a Co disk(23) (Fig. 7a). Furthermore, bubbles have been observed in in-plane magnets possibly pointing towards the existence of topologically non-trivial spin textures in in-plane magnetized samples(107). Moreover, short skyrmion tubes were observed in MnSi whose cross-section along the tube direction has the profile of a bimeron(108). However, the three-dimensional continuation of a bimeron would be a bimeron tube which is not realized there. The observed objects are more similar to skyrmions than to bimerons.
The reason why only a handful of potential bimeron systems have been identified experimentally, may originate in the required DMI. One example generating one particular type of bimeron is given by\(^2\)

\[
H_{\text{bimeron}} = \int \left( \mu_x \frac{\partial m_x}{\partial x} - m_x \frac{\partial \mu_x}{\partial x} + m_y \frac{\partial m_y}{\partial y} - m_y \frac{\partial \mu_y}{\partial y} \right) \, d^3 r.
\]  

(28)

Since the DMI vectors are rotated around an in-plane axis compared to the interfacial DMI, the same must also apply to the texture. In this rather complicated setup, the DMI vectors along one direction are oriented in-plane, just like for interfacial and bulk DMI. However, along the other bond direction, the DMI vectors point out-of-plane, meaning that the inversion symmetry has to be broken in a particular way (an example is presented in Ref.\(^2\)). The required crystal structure lacks major symmetries, which is why most typical materials do not allow for a formation of bimerons. However, a similar DMI setup was recently calculated by first-principles calculations of Janus monolayers of the chromium trihalides Cr(I,Cl)\(_3\) and Cr(I,Br)\(_3\) for which bimerons have been stabilized in simulations\(^10\). Furthermore, it has been predicted that bimerons can be stabilized much more easily by frustrated exchange interactions. In contrast to the DMI, frustrated exchange interactions are achiral allowing to stabilize skyrmons, antiskyrmions, bimerons and even other objects likewise\(^8\).\(^9\).\(^10\).\(^11\).\(^12\).\(^13\) The only requirement for bimerons is that the external magnetic field has to be applied in the plane. Until now, topologically non-trivial spin textures with these properties stabilized by frustrated exchange interactions have never been observed. However, once this is the case, a rotation of the magnetic field will result in bimeron formation if the anisotropy is weak (or even better in the plane) and if the DMI is negligible. Without one of the two mechanisms bimerons can only exist as transition states that are unstable\(^11\).\(^11\).

The observation of bimerons is highly desirable due to their special emergent electrodynamics. Similar to what has been presented for antiskyrmions and skyrmions with an arbitrary helicity (see last section), bimerons have a reduced symmetry compared to Bloch or Néel skyrmions. While the dissipative tensor and the gyroscopic vector are the same as for skyrmions and antiskyrmions, the global spin rotation manifests in the torque tensor, which is not antisymmetric anymore\(^2\). Following the same argumentation as for the antiskyrmion, there exists a specific current direction for which bimerons move parallel to the current (specific \(\gamma, \alpha\) combinations), implying that specific bimerons can be used in racetracks as bits that move without skyrmion Hall effect\(^11\) as shown in Fig.\(^5\). In the STT scenario on the other hand, bimerons will always move under a skyrmion Hall effect, since the adiabatic torque transforms like the texture.

Furthermore, the bimeron is unique in terms of the topological Hall effect. Since a bimeron is an excitation in an in-plane magnet, the out-of-plane net magnetization vanishes and no anomalous Hall effect occurs. In a topological Hall measurement, the stabilizing field (typically along \(z\) for the skyrmion) is swept. For the bimeron this field has to point in-plane meaning that also no conventional Hall effect is measurable in the \(\rho_{xy}\) tensor element. For this reason, bimerons allow for a pure measurement of the topological Hall effect upon variation of the in-plane field, establishing the optimal playground to investigate topological Hall physics\(^2\).

As a final remark, we would like to mention that sometimes the term bimeron is also used for an elongated skyrmion (or short helix segments), where both ends of the skyrmion have topological charges of \(\pm 1/2\) and the center part is neutral\(^15\).\(^16\). This object is actually more similar to a skyrmion than to the bimeron in the sense of what has been discussed here.

2. Biskyrmions

The term biskyrmion is commonly used for two partially overlapping skyrmions (Fig.\(^6\)) first observed in 2014\(^2\). In order to be geometrically compatible (the magnetization between the two skyrmions must be steady), both skyrmions...
need to possess a helicity difference of π, meaning that the in-plane magnetizations of both skyrmions are mutually reversed.

Following from our explanations about the possible stabilizing mechanisms of skyrmions in Sec. II B, biskyrmions can hardly be stable when the DMI is large. This chiral interaction would prefer one particular skyrmion helicity leading to a discontinuous magnetization when two skyrmions would partially overlap (in that case, even two attractive skyrmions would always remain separated by a ferromagnetic area, like in Ref. [126]. On the contrary, the dipole-dipole interaction is achiral: it equally favors both types of Bloch skyrmions with helicities of γ = ±π/2, respectively [129, 130]. When these two skyrmions are partially superimposed, the in-plane magnetizations between the skyrmions’ centers point along the same direction, meaning that they are geometrically compatible. And indeed, it was found that already the short-range approximation of the dipole-dipole interaction leads to an attraction between two Bloch skyrmions with opposite helicities allowing to form magnetic biskyrmions [126]. Besides the stabilization of individual magnetic skyrmions, biskyrmion lattices have been stabilized by this mechanism in micromagnetic simulations as well [129].

Even before the theoretical prediction, biskyrmion lattices have been observed experimentally (Fig. 7c) in centrosymmetric materials [25, 26, 126] like the layered manganite La2−xSr1+xMnO3. In these materials the DMI is absent by symmetry, in agreement with the theoretically established stabilizing mechanism. Another possible origin for biskyrmion formation may be frustrated exchange interactions, as predicted in Ref. [126].

Regarding their emergent electrodynamics, biskyrmions behave similar to conventional skyrmions. Their topological charge of Nsk = ±2 (middle panel in Fig. 6b) leads to the emergence of a topological Hall effect and a skyrmion Hall effect in the STT scenario. The current-driven motion under SOT may turn out problematic, since the opposite helicities of the two Bloch skyrmions lead to sign reversed $I_{\text{Skyr}}$ tensors for the two subskyrmions. Consequently, they are driven along different directions eventually leading to the destruction of the biskyrmion. However, the lack of rotational symmetry — even in the $m_z$ component (bottom panel in Fig. 6b) — allows to utilize a rotation of biskyrmion for spintronic devices. As has been shown by micromagnetic simulations on a square lattice, the principle axis of a biskyrmion aligns with high-symmetry directions of the lattice (the second-nearest neighbor directions in this case), allowing in principle to switch between different configuration [53].

As a closing remark, it is worth mentioning that the initial observations of biskyrmion crystals by LTEM imaging are under debate right now. In two recent publications [129, 130], the authors showed that the unique Lorentz contrast can also arise due to a tilting effect. Tubes of topologically-trivial bubbles appear as skyrmion pairs with reversed in-plane magnetizations when viewed under an angle. Still, as explained, the stability of biskyrmions has recently been explained theoretically [82, 130, 135], so the observed textures may indeed be biskyrmions. Alternative experimental techniques have to be utilized to dispel remaining doubts on the existence of magnetic biskyrmions.

3. Skyrmioniums

Up to now, we have discussed fundamental and composite magnetic objects with a finite topological charge. If however two skyrmions with opposite topological charges are combined to a new particle, the topological charge vanishes. If both objects would have the same polarity, this would require the combination of objects with opposite vorticities, e.g. a skyrmion and an antiskyrmion. Although these objects have been found to coexist in D2d materials [61], a combination of both has not yet been observed.

If however both objects have the same vorticities but opposite polarities, they can even be stabilized by bulk or interfacial DMI — the interaction that is typically dominating in skyrmion hosts. For instance, it has been found that under certain conditions an increase of the DMI constant leads to the formation of a skyrmion inside of another skyrmion with mutually reversed spins [27] (Fig. 6c). This object is called skyrmionium or $2\pi$-skyrmion [130, 133, 134] since the azimuthal angle rotates by $2\pi$ (instead of $\pi$) when going from the object’s center to the confinement. The outer skyrmion has the shape of a ring and is characterized by the opposite topological charge compared to the inner skyrmion (middle panel in Fig. 6c).

Skyrmioniums have recently been observed in a thin ferromagnetic film on top of a topological insulator [27] and have been created by laser pulses [136]. Furthermore, their generation has been predicted by the perpendicular injection of spin currents [27, 136, 138] or via alternating the out-of-plane orientation of an external magnetic field [139].

The concept of the $2\pi$-skyrmion can be extended to $k\pi$-skyrmions, theoretically stable for certain parameters when the DMI is increased even further. Experimentally, such objects have been seen in nanodisks [139]. There, the texture is mainly stabilized by the confinement and the value of $k$ is determined by the disk radius. Such objects are also called target skyrmions due to the peculiar profile of the magnetization’s $z$ component (Fig. 5c). In these disks the value of $k$ is often not integer since the magnetic moments are not pointing perfectly out-of-plane at the confinement (Fig. 7f). Another texture related to $k\pi$-skyrmions are so called skyrmion bags [142, 143] where multiple skyrmions (instead of just one) are positioned next to each other inside of a skyrmion ring.

Due to the vanishing topological charge and the observation in experiments, magnetic skyrmioniums seem to be highly relevant for spintronic applications in the future. Their emergent electrodynamics will be discussed in the following. The trivial real-space topology on a global level leads to the absence of a skyrmion Hall effect [24]. For this reason, Néel-type skyrmioniums move without skyrmion Hall effect even in the SOT scenario. Still, the finite topological charges of the two subskyrmions become apparent: the skyrmionium deforms upon motion along the track. The inner skyrmion pushes towards the racetrack edge but is confined by the outer ring skyrmion that pushes transversally along the other direction. Both trans-
verse motions compensate each other on a global level but still this effect constitutes a barrier for the maximum driving current\textsuperscript{27,143}. When the current is too large, the transverse forces become too strong and the skyrmionium unzips\textsuperscript{27}. Consequently, both skyrmionium parts annihilate. Summarizing, the absence of the skyrmion Hall effect solves an important problem: a skyrmionium moves parallel to an applied current directly in the middle of a racetrack avoiding pinning at the edges. Still, the skyrmion Hall effect of the subsystems is problematic as the current density cannot be increased much higher than that of a conventional skyrmion system.

For the topological Hall effect one has to distinguish between global and local arguments as well. On a global level (measuring the Hall voltage of the charge accumulation on a mesoscopic length scale), the Hall effect is absent due to the vanishing topological charge. If, however, the detection is performed locally on the length scale of the skyrmionium size, one can find signatures of both subsystems. In Ref.\textsuperscript{[143]} it has been shown that the topological Hall effect of skyrmioniums in a racetrack exhibits a unique triple-peak feature originating from the alternating topological charges of the ring- and center-skyrmions. This effect can therefore be used to detect skyrmioniums electrically, in a similar way as presented for skyrmions\textsuperscript{[17,18]}.

### 4. Antiferromagnetic skyrmions

The dynamic properties and the stability of single antiferromagnetic skyrmions have first been predicted in antiferromagnets\textsuperscript{[29,146]} and synthetic antiferromagnetic bilayers\textsuperscript{[147]}. Shortly after, they have been extended to two-sublattice antiferromagnetic skyrmion crystals\textsuperscript{[148]}. Like skyrmioniums, antiferromagnetic skyrmions can be understood as the combination of two skyrmions with mutually reversed spins. Therefore, they are characterized by a vanishing topological charge. However, in the present case, the subskyrmions are not spatially separated but intertwined. For this reason, the magnetization density vanishes locally and the Néel order parameter, the main order parameter for antiferromagnets, can be considered instead of the magnetization. Calculating the topological charge with this parameter gives ±1. Thus, the antiferromagnetic skyrmions are still skyrmions from the viewpoint of topology, but exhibit different dynamics compared to ferromagnetic skyrmions. These antiferromagnetic dynamics can also be described by the Thiele equation but by using the Néel order parameter\textsuperscript{[149]}.

Due to the compensated topological charge for the magnetization, an antiferromagnetic skyrmion moves without
skyrmion Hall effect\textsuperscript{[29,147]} similar to what has been discussed for the skyrmionium. However, the two subsystems are coupled much stronger and do not allow for a deformation due to a pairwise opposing transverse motion, as was the case for the skyrmionium. Typical for antiferromagnetic spin textures, the antiferromagnetic skyrmions can be propelled much faster by currents compared to conventional skyrmions. Velocities on the scale of kilometers per second have been simulated\textsuperscript{[29,147,150]} This makes antiferromagnetic skyrmions the ideal carriers of information for data storage devices. Additionally, it was theoretically shown that the diffusion constant\textsuperscript{[29]} of antiferromagnetic skyrmions is high in systems with small damping unlike for their ferromagnetic counterparts, thus showing a potential in driving them with temperature gradients. Furthermore, they do not exhibit stray fields, which potentially allows for a denser stacking of quasi one-dimensional racetracks upon building the three-dimensional storage device.

In terms of the required DMI, the stabilization of antiferromagnetic skyrmions is not problematic\textsuperscript{[29,131]}. Two skyrmions with mutually reversed spins (i.e. two skyrmions with opposite polarity and a helicity difference of $\pi$) are energetically favored by the same type of DMI. Furthermore, both skyrmions need to be coupled antiferromagnetically quite strongly, to accomplish the antiparallel alignment of the corresponding magnetic moments. And indeed, very recently, the bilayer-type antiferromagnetic skyrmions have been observed in synthetic antiferromagnets\textsuperscript{[29,131]} (Fig. 7e) at room temperature. In Ref. \textsuperscript{[31]} the small stray fields resulting from the bilayer setup have been detected using magnetic force microscopy (MFM). In Ref. \textsuperscript{[131]} the authors explain a method to prepare synthetic antiferromagnets with a tunable net moment. While they can achieve a complete compensated system, they deliberately prepared also a system with a small net moment to avoid static stray fields. In Ref. \textsuperscript{[131]} synthetic antiferromagnetic skyrmions have been realized\textsuperscript{[131]}. The resulting signal is the analogue of the conventional spin Hall effect, but originates in the non-collinearity of the spin texture. The topological spin Hall effect can most easily be comprehended if one assumes two electronically uncoupled subskyrmions (the results hold also for the coupled case): Due to the opposite spin alignment, the emergent fields of the two subskyrmions are oriented oppositely. This leads to a transverse deflection of the electrons in opposite directions. The two species of electrons align with their respective texture and can therefore be considered as ‘spin up’ and ‘spin down’ states, again due to the opposite spin alignment.

Summarizing, one can have a positive feeling about the utilization of antiferromagnetic skyrmions in spintronic devices in the future. Synthetic antiferromagnetic skyrmions have been observed and, just recently, the current-driven motion of synthetic antiferromagnetic skyrmions has been realized\textsuperscript{[131]}. Furthermore, single layer antiferromagnetic skyrmions have been predicted. The topological spin Hall effect may play an essential role for observing these objects despite completely compensated magnetizations, stray fields and topological charge densities of the magnetization. Moreover, even in antiferromagnetic insulators the skyrmions were predicted and can potentially be moved by an electrically created anisotropy gradient\textsuperscript{[147]} or by thermal gradients.

Signatures of the favorable emergent electrodynamics of antiferromagnetic skyrmions have also been seen for a similar object: the ferrimagnetic skyrmion\textsuperscript{[148,156]} (Fig. 2e). It consists of two coupled subskyrmions with mutually reversed spins, similar to the antiferromagnetic skyrmion. However, the magnetic moments have different magnitudes on the two sublattices leading to an uncompensated magnetization, which allowed for the detection of ferrimagnetic skyrmions in GdFeCo films by X-ray imaging\textsuperscript{[29]} (Fig. 7f). When these objects are driven by spin currents, there exists a critical temperature at which the skyrmion Hall effect is absent\textsuperscript{[148]}. At this temperature, the angular momentum is compensated, even though the magnetization is not, due to different gyromagnetic ratios for the magnetic moments in the different sublattices.\textsuperscript{[85]} Experimentally, this complete compensation of the skyrmion Hall effect still lacks observation, but a reduced skyrmion Hall angle of $\theta_{Sk} = 20^\circ$ has been observed at room temperature.\textsuperscript{[87]} Similar experiments will certainly be performed for antiferromagnetic skyrmions in the future. Moreover, it was recently experimentally observed that in ferrimagnetic insulators near the
compensation temperature domain walls driven by SOT can move at speeds reaching 6 km/s\cite{32}, thus hinting that the same speeds can be achieved as well by ferrimagnetic skyrmions in insulators in the future.

As a closing remark, we would like to mention that the idea to combine skyrmions on different sublattices has been generalized in several works. Three skyrmion crystals can for example be intertwined as stabilized by Monte Carlo simulations\cite{164,165,166}. However, these objects do not exhibit the advantages of antiferromagnetic skyrmions because the topological charge for the magnetization is finite in this case.

C. Three-dimensional objects

In the last main section we present three-dimensional solitons and extensions of skyrmions. We begin with the trivial case: two-dimensional skyrmions, antiskyrmions, bimerons and other objects are extended as tubes along the third dimension. Still, these tubes can show interesting interactions and can even have a varying helicity along the tube\cite{166}. We focus here on two more fundamental variations of such tubes.

First, we discuss chiral bobbers\cite{31,32} (Fig. 2). These are skyrmion tubes that end in a Bloch point (the hedgehog in Fig. 3b)\cite{164,165,166}, also shown in an experimental measurement in Fig. 2j. Interestingly, the chiral bobbers can appear in the same sample as skyrmion tubes. Both objects can be well distinguished experimentally, allowing to think of an improved racetrack storage device with both objects as the bits of information\cite{32} (Fig. 8b). On the other hand, skyrmion tubes can also bend and form closed loops called hopfions\cite{33} (Fig. 2l). These objects are unique from a fundamental point of view since they are characterized by a second topological invariant, the hopf number.

1. Chiral bobbers

As explained, the trivial continuation of skyrmions along the third dimension are skyrmion tubes. In MnSi for example, they penetrate the whole sample. However, materials are not always perfect, so it can happen that two skyrmion tubes merge to one\cite{166,167}. This decrease of topological charge along a tube’s cross section is caused by a Bloch point (Fig. 8b) with a singularity in its center. Similarly, a single skyrmion tube can end in or begin from a Bloch point\cite{165,166}. In this case, the resulting magnetic object is called chiral bobber\cite{31,32,166} (Fig. 2). It has been predicted\cite{32} and observed experimentally by LTEM measurements\cite{32} in B20 materials. Fig. 7 shows a schematic representation of a pair of two chiral bobbers, as well as an experimental measurement of a Bloch point in Ni_{50}Fe_{20}.

One unique characteristics of a chiral bobber is that the location of the Bloch point, with respect to the sample’s surface, is fixed\cite{31}. It depends only on the ratio of exchange to DMI and is for example independent of the sample thickness\cite{31}. Like a fishing bobber, the chiral bobber is ‘swimming’ always close to the confinement of the sample. For this reason, the chiral bobber must be seen as a distinct magnetic soliton despite its phenomenology as a discontinued skyrmion tube. Furthermore, this means that the volume which is occupied by a chiral bobber is fixed with respect to thickness variations, and so is the soliton’s energy. For a skyrmion tube on the other hand, the occupied volume and the energy scale linearly with the thickness. For this reason, skyrmions in these materials become unstable above a critical sample thickness which is where chiral bobbers are energetically preferred\cite{31}. It has been experimentally confirmed that chiral bobbers do not exist below a critical thickness\cite{32}.

In order to stabilize chiral bobbers, a specific experimental procedure had to be applied in Ref. 32. The magnetic field was rotated approximately 10° out of the normal direction and then the sample was magnetized and demagnetized a few times. After the nucleation of the chiral bobbers and skyrmion tubes, the field was rotated back to the conventional out-of-plane direction. A possible nucleation mechanism is the formation of chiral bobbers from edge dislocations of the spin spiral phase, similar to the formation of skyrmion tubes. Skyrmion tubes and chiral bobbers have been distinguished by the LTEM phase shift which is proportional to the occupied volume of the magnetic object, i.e., chiral bobbers exhibit a weaker intensity than skyrmion tube\cite{32}.

Even though a chiral bobber is not topologically protected (mathematically speaking the texture can be pushed out of the magnet via the confinement), it possesses a similar energy barrier compared to the skyrmion tube\cite{31}. This allows for a coexistence of skyrmion tubes and chiral bobbers allowing to think of an alternative racetrack storage device where the ‘1’ and ‘0’ bits are encoded by the presence of a skyrmion tube or a chiral bobber, respectively (Fig. 8b). This solves the problem that the bits do not have to be located at precise positions. Instead, information can be encoded also as an irregular sequence\cite{32}.

The two different objects can be read via their net magnetization or via their Hall signal. As has been shown numerically in Ref. 170, chiral bobbers exhibit an increasing Hall conductivity upon increasing the sample thickness, while this signal remains constant for skyrmion tubes.

The lack of topological protection may become problematic for the current-driven motion of chiral bobbers. As micromagnetic simulations of the STT scenario revealed, chiral bobbers do not only move along the sample but also the Bloch point propagates towards the edge along the tube direction until the chiral bobber has disappeared\cite{167}. The SOT scenario remains to be investigated.

As a final remark, besides their presence in chiral bobbers, Bloch points can form three-dimensional Bloch anti-Bloch crystals with their counterparts\cite{167,168} (Fig. 2k). These crystals can be seen as the three-dimensional analogue of a skyrmion lattice. Recently, it has even been shown that in MnSi_{1-x}Ge\textsubscript{x} a topological transition between the skyrmion lattice and the this Bloch anti-Bloch lattice occurs\cite{158}. 

As a final remark, besides their presence in chiral bobbers, Bloch points can form three-dimensional Bloch anti-Bloch crystals with their counterparts\cite{167,168} (Fig. 2k). These crystals can be seen as the three-dimensional analogue of a skyrmion lattice. Recently, it has even been shown that in MnSi_{1-x}Ge\textsubscript{x} a topological transition between the skyrmion lattice and the this Bloch anti-Bloch lattice occurs\cite{158}.
A different way to transform a skyrmion tube into an innately three-dimensional object is to form a torus. These objects are known as hopfions (as in Fig. 2l) and have been introduced back in 1931. They are topologically characterized by the Hopf invariant

$$Q_H = -\int B_{em} \cdot A \, d^3 r,$$

where $B_{em}$ is the emergent field and $A$ is the corresponding vector potential fulfilling the condition $\nabla \times A = B_{em}$. In the simplest case, the cross-section texture is a bimeron with a topological charge of $\pm 1$ and the magnetization rotates once going around a circle of fixed radius around the hopfions center (as in Fig. 2l). This yields the Hopf number of $\pm 1$ as a product of the cross-section topological charge and the number of magnetization windings around the torus.

From a mathematical point of view, several types of hopfions can be constructed. For example, the cross section can have higher topological charges or the magnetization can rotate more than once going around a circle. Also, the Hopf number is increased by more complicated configurations, e.g., by linking multiple hopfions.

Hopfions have first been predicted in 1975 in a Skyrme-Faddeev model. Over the following years, hopfions have been found in hydrodynamics, electrodynamics, and other fields of physics, and recently they have been stabilized in micromagnetic simulations. The objects can be stabilized in chiral nanodisks due to the confinement and have even been stabilized without magnetic fields.

Their emergent electrodynamics are promising due to a globally compensated emergent field that is however finite locally (largest inside of the tube that forms the torus). This toroidal emergent field leads to the deflection of current electrons perpendicular to the hopfion plane. One half of the object leads to a positive Hall resistance while the other half gives a negative signal. While both signals cancel on a global level, a local measurement of a hopfion in a potential racetrack device yields a unique signature as simulated in Ref. Furthermore, the topological Hall signal depends on the orientation of the hopfion bringing about the possibility to switch electrical signals (Fig. 8c): the Hall voltage always emerges perpendicularly to the Hopfion plane. Therefore, it is expected to occur in a pure manner since it is characterized by a different Hall resistance tensor element than the anomalous and ordinary Hall effect.

Likewise, when driven by torques, the hopfions move along the current direction and do not experience a skyrmion Hall effect. The locally finite field leads however to a tilting of the hopfion plane, as was first predicted in Ref. and later confirmed by micromagnetic simulations. On the one hand, if this effect is kept small, hopfions are promising for racetrack storage applications. On the other hand, the tilting mechanism may even be utilized for other spintronic applications because it allows to switch between different hopfion configurations.

**IV. PERSPECTIVES AND CONCLUSION**

Summarizing this review, we have discussed the stability and the emergent electrodynamics of skyrmions and related alternative magnetic quasiparticles. We have classified the manifold of particles (Fig. 1) in fundamental excitations (topologically trivial and non-trivial), variations of these excitations and continuations. Out of all fundamental excitations, skyrmions with an arbitrary helicity and antiskyrmions have been discussed as technologically relevant objects. The variations comprise topological excitations in a different magnetic background and the combination of multiple subparticles. Here we have discussed in detail bimerons, biskyrmions, skyrnoniums, and antiferromagnetic skyrmions. ‘Continuation’ means that all of these objects can be arranged in two dimensions (periodically and non-periodically) and can be continued along the third dimension. For the innately three-dimensional objects, we have laid focus on chiral bobbers and hopfions.

Out of the presented objects (Fig. 2) antiferromagnetic skyrmion are often considered the optimal bits for spintronic...
applications. Their compensated magnetic texture allows to drive them by currents at enormously high velocities of up to several kilometers per second and the absence of the skyrmion Hall effect eradicates the problem of pinning of the bits at the edges. Furthermore, their local compensation of magnetization renders stray fields small allowing for a dense stacking of racetracks in a three-dimensional buildup.

Moreover, antiskyrmions seem to be highly promising. They are easy to detect and most experimental techniques working for the conventional skyrmions can be carried over. These objects can move parallel to an applied current allowing to consider antiskyrmions as the carriers of information in racetrack devices. One further problem that occurs whenever the '0' and '1' bits of information are constitued by the presence or absence of a magnetic quasiparticle is that the positions of the bits cannot be precisely fixed. On the long time scale the information may become falsified for example due to the repulsive interaction between the objects. A solution may be delivered by using two distinct objects to constitute the bits because then irregular sequences are not problematic. A promising example is using skyrmions and antiskyrmions in systems with $D_{2d}$ symmetry (Fig. 3). Also it is conceivable to use a three-dimensional approach with chiral bobbers and skyrmion tubes as bits of information (Fig. 5).

Furthermore, it seems worthwhile to look into other applications of topologically non-trivial quasiparticles. Conventional skyrmions have been considered for utility in logic devices, transistors, magnetic tunnel junctions, nanoscillator, as microwave devices, or magnonic device, neuromorphic application, as well as reservoir computing, and quantum computing. etc. Future will tell if the alternative quasiparticles are favorable also in these regards.

As a closing remark, we would like to mention that the presented characterization scheme is not complete. Multiple skyrmions can in principle be combined to form new objects; e.g., a quadskyrmion instead of a biskyrmion. Furthermore, one can combine multiple objects and change the magnetic background; antiferromagnetic versions of skyrmions, skyrmion bags and bimerons have already been predicted. Since antiferromagnetic skyrmions have been observed just recently, the desire arises to find also antiferromagnetic versions of other nano-objects.

Acknowledgments

This work is supported by SFB TRR 227 of Deutsche Forschungsgemeinschaft (DFG). O.A.T. acknowledges the support by the Australian Research Council (Grant No. DP200101027) and by the Cooperative Research Project Program at the Research Institute of Electrical Communication, Tohoku University.

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1. G. E. Moore, Electronics 38, 114 (1965).
2. M. M. Waldrop, Nature News 530, 144 (2016).
3. S. S. P. Parkin, Shiftable magnetic shift register and method of using the same (2004), US Patent 6,834,005.
4. S. S. P. Parkin, M. Hayashi, and L. Thomas, Science 320, 190 (2008).
5. S. S. P. Parkin and S.-H. Yang, Nature Nanotechnology 10, 195 (2015).
6. A. Bogdanov and D. Yablonskii, Zh. Eksp. Teor. Fiz 95, 182 (1989).
7. S. Mühlbauer, B. Binz, F. Jonietz, C. Pfleiderer, A. Rosch, A. Neubauer, R. Georgii, and P. Böni, Science 323, 915 (2009).
8. J. Sampaio, V. Cros, S. Rohart, A. Thiaville, and A. Fert, Nature Nanotechnology 8, 839 (2013).
9. A. Fert, V. Cros, and J. Sampaio, Nat. Nanotechnol. 8, 152 (2013).
10. G. Yu, P. Upadhyaya, Q. Shao, H. Wu, G. Yin, X. Li, C. He, W. Jiang, X. Han, P. K. Arimi, et al., Nano Letters 17, 261 (2017).
11. P. Bruno, V. Dugaev, and M. Taillefumier, Phys. Rev. Lett. 93, 096806 (2004).
12. A. Neubauer, C. Pfleiderer, B. Binz, A. Rosch, R. Ritz, P. Nikolowitz, and P. Böni, Phys. Rev. Lett. 102, 186602 (2009).
13. M. Lee, W. Kang, Y. Onose, Y. Tokura, and N. Ong, Phys. Rev. Lett. 102, 186601 (2009).
14. J. Zang, M. Mostovoy, J. H. Han, and N. Nagaosa, Phys. Rev. Lett. 107, 136804 (2011).
15. W. Jiang, X. Zhang, G. Yu, W. Zhang, X. Wang, M. B. Jungfleisch, J. E. Pearson, X. Cheng, O. Heinonen, K. L. Wang, et al., Nature Physics 13, 162 (2017).
16. K. Lützies, I. Lemesh, B. Krüger, P. Bassirian, L. Caretta, K. Richter, F. Büttner, K. Sato, O. A. Tretiakov, J. Förster, et al., Nature Physics 13, 170 (2017).
17. K. Hamamoto, M. Ezawa, and N. Nagaosa, Applied Physics Letters 108, 112401 (2016).
18. D. Maccariello, W. Legrand, N. Reyren, K. Garcia, K. Bouzehouane, S. Collin, V. Cros, and A. Fert, Nature Nanotechnology 13, 233 (2018).
19. T. Okubo, S. Chung, and H. Kawamura, Phys. Rev. Lett. 108, 017206 (2012).
20. J. A. Garlow, S. D. Pollard, M. Beleggia, T. Dutta, H. Yang, and Y. Zhu, Physical Review Letters 122, 237201 (2019).
21. A. K. Nayak, V. Kumar, T. Ma, P. Werner, E. Pippel, R. Sahoo, F. Damay, U. K. Rößler, C. Felser, and S. S. Parkin, Nature 548, 561 (2017).
22. Y. Kharkov, O. Sushkov, and M. Mostovoy, Phys. Rev. Lett. 119, 207201 (2017).
23. B. Göbel, A. Mook, J. Henk, I. Mertig, and O. A. Tretiakov, Phys. Rev. B 99, 060407 (2019).
24. N. Gao, S.-G. Je, M.-Y. Im, J. W. Choi, M. Yang, Q.-c. Li, T. Wang, S. Lee, H.-S. Han, K.-S. Lee, et al., Nature Communications 10, 5603 (2019).
25. X. Yu, Y. Tokunaga, Y. Kaneko, W. Zhang, K. Kimoto, Y. Matsui, Y. Taguchi, and Y. Tokura, Nature Communications 5, 3198 (2014).
26. B. Göbel, J. Henk, and I. Mertig, Scientific Reports 9, 9521 (2019).
27. X. Zhang, J. Xia, Y. Zhou, D. Wang, X. Liu, W. Zhao, and M. Ezawa, Physical Review B 94, 094420 (2016).
B. Göbel, A. Mook, J. Henk, and I. Mertig, Phys. Rev. B 96, 064046 (2017).
E. G. Tveten, A. Qaimzadeh, O. A. Tretiakov, and A. Brataas, Phys. Rev. Lett. 110, 127208 (2013).
C. Jin, C. Song, J. Wang, and Q. Liu, Applied Physics Letters 109, 182404 (2016).
P. F. Bessarab, D. Yudin, D. R. Gulevich, P. Wedray, M. Titov, and O. A. Tretiakov, Phys. Rev. B 99, 140411 (2019).
X. Liang, J. Xia, X. Zhang, M. Ezawa, O. A. Tretiakov, X. Liu, L. Qiu, G. Zhao, and Y. Zhou, arXiv:1909.10709 (2019).
N. Romming, C. Hanneken, M. Menzel, J. E. Bickel, B. Wolter, K. von Bergmann, A. Kubetzka, and R. Wiesendanger, Science 341, 636 (2013).
W. Jiang, P. Upadhyaya, W. Zhang, G. Yu, M. B. Jungfleisch, F. Y. Fradin, J. E. Pearson, Y. Tserkovnyak, K. L. Wang, O. Heinonen, et al., Science 349, 109 (2015).
P. M. Buhl, F. Freimuth, S. Blügel, and Y. Mokrousov, physica status solidi (RRL)-Rapid Research Letters 11, 1700007 (2017).
C. A. Akosa, O. Tretiakov, G. Tatara, and A. Manchon, Physical Review Letters 121, 097204 (2018).
L. Shen, J. Xia, G. Zhao, X. Zhang, M. Ezawa, O. A. Tretiakov, X. Liu, and Y. Zhou, Phys. Rev. B 98, 134448 (2018).
H.-A. Zhou, Y. Dong, T. Xu, K. Xu, L. Sánchez-Tejerina, L. Zhao, Y. Ba, P. Gargiani, M. Valvidares, Y. Zhao, et al., arXiv:1912.01775 (2019).
H. D. Rosales, D. C. Cabra, and P. Pujol, Physical Review B 92, 121419 (2015).
S. A. Díaz, J. Klinovaja, and D. Loss, Physical Review Letters 122, 187203 (2019).
Press release, Martin-Luther-Universität Halle-Wittenberg https://pressemitteilungen.pr.uni-halle.de/index.php?modus=pmanzeige&pm_id=3241 (4th of March 2020).
B. Göbel, C. A. Akosa, G. Tatara, and I. Mertig, Physical Review Research 2, 013135 (2020).
Y. Liu, W. Hou, X. Han, and J. Zang, arXiv preprint arXiv:2001.00417 (2020).
W. Legrand, J.-Y. Chauleau, D. Maccariello, N. Reyren, S. Collin, K. Bouzehouane, N. Jaouen, V. Cros, and A. Fert, Science Advances 6, eaat0415 (2020).
J. Wild, T. N. Meier, S. Pöllath, M. Kronsered, A. Bauer, A. Chacon, M. Halder, M. Schwalter, A. Rosenaar, J. Zweck, et al., Science Advances 3, ea1701704 (2017).
M. Birch, R. Takagi, S. Seki, M. Wilson, F. Kagawa, A. Štefančič, G. Balakrishnan, R. Fan, P. Steadman, C. Ottley, et al., Physical Review B 100, 014425 (2019).
F. Kagawa, H. Oike, W. Koshiba, A. Kikkawa, Y. Okamura, Y. Taguchi, N. Nagaosa, and Y. Tokura, Nature Communications 8, 1332 (2017).
W. Koshibae and N. Nagaosa, Scientific Reports 9, 5111 (2019).
A. S. Ahmed, J. Rowland, B. D. Esser, S. R. Dunsiger, D. W. McComb, M. Randera, and R. K. Kawakami, Physical Review Materials 2, 041401 (2018).
M. Redies, F. Lux, J.-P. Hanke, P. Buhl, G. Müller, N. Kiselev, S. Blügel, and Y. Mokrousov, Physical Review B 99, 140407 (2019).
T. Tanigaki, K. Shibata, N. Kanazawa, X. Yu, Y. Onose, H. S. Park, D. Shindo, and Y. Tokura, Nano Letters 15, 5438 (2015).
X.-X. Zhang, A. S. Mishchenko, G. De Filippis, and N. Nagaosa, Physical Review B 98, 174428 (2018).
M. Bornemann, S. Grytsiuk, P. F. Baumeister, M. dos Santos Dias, R. Zeller, S. Lounis, and S. Blügel, Journal of Physics: Condensed Matter 31, 485801 (2019).
Y. Fujihiro, N. Kanazawa, T. Nakajima, X. Yu, K. Ohishi, Y. Kawamura, K. Kakurai, T. Arima, H. Mitamura, A. Miyake, et al., Nature Communications 10, 1059 (2019).
H. Hopf, Math. Ann. 104, 637 (1931).
J. Whitehead, Proceedings of the National Academy of Sciences of the United States of America 33, 117 (1947).
F. Wilczek and A. Zee, Physical Review Letters 51, 2250 (1983).
F. N. Rybukov, N. S. Kiselev, A. B. Borisov, L. Döring, C. Melcher, and S. Blügel, arXiv preprint arXiv:2019.00250 (2019).
V. E. Korepin and L. D. Faddeev, Theoretical and Mathematical Physics 25, 1039 (1975).
E. A. Kuznetsov and A. V. Mikhailov, Physics Letters A 77, 37 (1980).
A. F. Raïada, Letters in Mathematical Physics 18, 97 (1989).
P. Sutcliffe, Physical Review B 76, 184439 (2007).
J.-S. B. Tai, I. I. Smalyukh, et al., Phys. Rev. Lett. 121, 187201 (2018).
P. Sutcliffe, Journal of Physics A: Mathematical and Theoretical 51, 375401 (2018).
W. Wang, A. Qaimzadeh, and A. Brataas, Physical Review Letters 123, 147203 (2019).
X. Zhang, Y. Zhou, M. Ezawa, G. Zhao, and W. Zhao, Scientific Reports 5, 11369 (2015).
Y. Zhou and M. Ezawa, Nature Communications 5, 4652 (2014).
S. Kasai, S. Sugimoto, Y. Nakatani, R. Ishikawa, and Y. K. Taka-hashi, Applied Physics Express 12, 083001 (2019).
N. Penthorn, X. Hao, Z. Wang, Y. Huai, and H. Jiang, Physical Review Letters 122, 257201 (2019).
L. Shen, J. Xia, G. Zhao, X. Zhang, M. Ezawa, O. A. Tretiakov, X. Liu, and Y. Zhou, Applied Physics Letters 114, 042402 (2019).
W. Wang, M. Beg, B. Zhang, W. Kuch, and H. Fangohr, Physical Review B 92, 204043 (2015).
X. Zhang, M. Ezawa, D. Xiao, G. Zhao, Y. Liu, and Y. Zhou, Nanotechnology 26, 225701 (2015).
Y. Huang, W. Kang, X. Zhang, Y. Zhou, and W. Zhao, Nanotechnology 28, 08LT02 (2017).
S. Li, W. Kang, Y. Huang, X. Zhang, Y. Zhou, and W. Zhao, Nanotechnology 28, 31LT01 (2017).
D. Przychynenko, M. Sitte, K. Litzius, B. Krüger, G. Bourianoff, M. Kläui, J. Sinova, and K. Everschor-Sitte, Physical Review Applied 9, 014034 (2018).
D. Pinna, G. Bourianoff, and K. Everschor-Sitte, arXiv preprint arXiv:1811.12623 (2018).
J. Závorka, F. Jakobs, D. Heinze, N. Keil, S. Kromin, S. Jaiswal, K. Litzius, G. Jakob, P. Virnau, D. Pinna, et al., Nature Nanotechnology 14, 658 (2019).
G. Yang, P. Stano, J. Klinovaja, and D. Loss, Physical Review B 93, 224505 (2016).
K. M. Hals, M. Schecter, and M. S. Rudner, Physical Review Letters 117, 017001 (2016).
M. M. M. Bhukta, A. Mishra, G. Pradhan, S. Mallick, B. B. Singh, and S. Bedanta, arXiv preprint: 1810.08262 (2018).
S. Obadero, Y. Yamane, C. Akosa, and G. Tatara, arXiv preprint: 1904.06870 (2019).
R. Fernandes, R. Lopes, and A. Pereira, Solid State Communications 290, 55 (2019).
L. Shen, J. Xia, X. Zhang, M. Ezawa, O. A. Tretiakov, X. Liu, G. Zhao, and Y. Zhou, Phys. Rev. Lett. 124, 037202 (2020).
X. Li, L. Shen, Y. Bai, X. Zhang, J. Xia, M. Ezawa, O. A. Tretiakov, X. Xu, M. Mruzkiewicz, M. Krawczyk, et al., arXiv: 2002.04387 (2020).