Pomeranchuk-Nematic instability in the presence of a weak magnetic field

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We analyze a two-dimensional Pomeranchuk-Nematic instability, trigger by the Landau parameter $F_2 < 0$, in the presence of a small magnetic field. Using Landau Fermi liquid theory in the isotropic phase, we analyze the collective modes near the quantum critical point $F_2 = -1, \omega_c = 0$ (where $\omega_c$ is the cyclotron frequency). We focus on the effects of parity symmetry breaking on the Fermi surface deformation. We show that, for studying the critical regime, the linear response approximation of the Landau-Silin equation is not sufficient and it is necessary to compute corrections at least of order $\omega_c^2$. Identifying the slowest oscillation mode in the disordered phase, we compute the phase diagram for the isotropic/nematic phase transition in terms of $F_2$ and $\omega_c$.

I. INTRODUCTION

The isotropic-nematic quantum phase transition was proposed as a possible mechanism to explain the anisotropic behavior of several strongly correlated systems. Some interesting examples are quantum Hall liquids, high $T_c$ superconductors and heavy fermions systems. An interesting review can be found in reference [1].

This transition can be understood as an instability of a Fermi surface under the influence of a strongly attractive two-body potential in the forward scattering channel, with d-wave symmetry (or equivalently, with angular momentum $\ell = 2$). From the point of view of Landau Fermi liquid theory, it is triggered by a Pomeranchuk instability produced by a large negative value of the Landau parameter $F_2$ in the charged sector. As a consequence of the transition, the Fermi surface is deformed, getting an ellipsoidal component. The Goldstone modes, related with rotational symmetry breaking, are dissipative over-damped excitations, characterized by dynamical exponent $z = 3$. The order parameter theory was developed using different techniques: mean field theory, multidimensional bosonization and Landau Fermi liquid theory. While the collective bosonic excitations are reasonably well understood, the fate of the fermionic spectrum is still under debate.

From an experimental point of view, the study of Fermi surface deformations can be performed by means of at least two independent techniques: ARPES and the observation of quantum oscillations, like for instance, the de Haas-van Alphen effect. The use of the latter resides in the ability to reconstruct Fermi surface shapes from the information contained in quantum oscillations of different observables, when an externally applied magnetic field is varied.

The application of a strong magnetic field suppresses any Pomeranchuk instability; since it opens a gap in the spectrum due to Landau level quantization. However, for small magnetic fields, the Landau levels form a dense set near the Fermi energy and strong attractive interactions mix all levels in a non-trivial way. Experimentally, nematic instability have been observed in the bilayer ruthenate compound Sr$_2$Ru$_2$O$_7$ at finite magnetic field, which suggests that a meta-magnetic quantum critical point can be reached by changing the direction of the applied magnetic field. Therefore, it is important to understand the critical behavior when the quantum critical point is reached by lowering the magnetic field.

With this motivation, we would like to present a study of a two-dimensional Pomeranchuk-nematic instability in the presence of a small magnetic field, applied perpendicular to the two-dimensional fermionic system. We have considered an isotropic and homogeneous charged Fermi liquid submitted to a small magnetic field, $k_BT \ll \omega_c \ll \epsilon_F$, where $\omega_c = eB/m^*$ is the cyclotron frequency and $\epsilon_F$ is the Fermi energy of the system. We have focused on a simplified model where only the attractive two-body $d$-wave interaction is present. Using a semi-classical approach, we have studied collective excitations of the fermionic system using the Landau-Silin equation. Studying the oscillatory slowest mode, we can compute the transition line where the isotropic phase gets unstable. The main result is presented in figure (1) where we depict the phase diagram for the nematic-Pomeranchuk instability. In this figure, the horizontal axis is the usual Landau control parameter $\alpha = 1 + F_2$ while the vertical axis is the adimensional magnetic field $(\omega_c/\epsilon_F)^2$. We observe a maximum value of the magnetic field above which no Pomeranchuk instability is possible. Moreover, we have observed a reentrant behavior of the isotropic phase for greater values of the interaction parameter. We have also analyzed the behavior of collective modes near the quantum critical point $(F_2 \to -1, \omega_c \to 0)$. Since the magnetic field breaks parity symmetry, the collective mode dynamics mixes symmetric as well as antisymmetric modes. Then, the Fermi surface deformation is not an ellipsoid but has a definite parity given by the direction of the magnetic field and the momentum $\mathbf{q}$ of the periodic perturbation.

The paper is structured as follows: in section II we briefly review the Landau theory of charged Fermi liq-
where the phase-space density at momentum $p$ and position $r$, $n(p,r)$ is the phase-space density at momentum $p$ and position $r$. $e$ is the quasi-particle charge and $f_{p+εA,p'+εA}(r-r')$ is the Landau amplitude characterizing finite range two-particle interactions. The Landau interaction function should depend on the electromagnetic vector potential to guarantee gauge invariance.

In order to compute a semi-classical evolution equation, we define the effective single-particle Hamiltonian

$$H_{\text{eff}}(p,r) = \frac{\delta E[n]}{\delta n(p,r)},$$

which generates the following time evolution equation:

$$\frac{\partial n(p,r,t)}{\partial t} = \{H_{\text{eff}}, n(p,r,t)\}_{PB} + I_{\text{coll}}[n(p,r,t)],$$

where $\{.,.\}_{PB}$ are Poisson brackets associated with the conjugate variables $r$ and $p$ and the effects of quasi-particle scattering are included in the collision integral $I_{\text{coll}}[n(p,r,t)]$.

By means of the Hamilton’s equations of motion $dr/dt = v_p H_{\text{eff}}(p,r,t)$ and $dp/dt = -\nabla_r H_{\text{eff}}(p,r,t)$, and using equations (2) and (3), it is obtained the so-called Landau-Silin kinetic equation:

$$\frac{\partial n(p,r,t)}{\partial t} + v(p,r,t) \cdot \nabla_r n(p,r,t) - (F(p,r,t)

+ \sum_p \int dr' f_{p+\epsilon A,p'+\epsilon A}(r-r') \nabla_{r'} n(p',r',t) \cdot \nabla_{r'} n(p,r,t)) = I_{\text{coll}}[n(p,r,t)],$$

where $F(p,r,t) = e[E(r,t) + v(p,r,t) \times B(r,t)]$ is the Lorentz force and $v(p,r,t) = \nabla_p H_{\text{eff}}$ is the quasi-particle velocity, including interactions. The Landau-Silin transport equation resembles the conventional classical Boltzmann equation. However, the effective Lorentz force $F(k,r,t)$ depends self-consistently on the quasi-particles distribution function $n(p,r,t)$. This evolution equation is the cornerstone of the present work.

In this paper we want to study the effect of an external magnetic field $B$, applied perpendicular to the plane of the system. We will assume that the cyclotron energy $\hbar \omega_c = eB/m^* \ll \varepsilon_F$. For simplicity through the paper we choose $\hbar \equiv 1$. In general, the scattering mechanisms described by the collision integral can be studied applying the relaxation-time $\tau$ approximation. We will consider that the typical collective mode frequencies are greater than the collision quasi-particle frequency. Of course, this is not true at criticality. However, to determine the position of the transition line, it is enough to consider $I_{\text{coll}} \to 0$. To set up the kinetic equation of the Fermi surface collectives modes, let us consider a constant isotropic equilibrium distribution $n^0(p)$ and a small perturbation $\delta n$, such that $n(p,r,t) = n^0(p) + \delta n(p,r,t)$.

In these conditions, the linear expansion of equation (4) in $\delta n$ provides the transport equation

$$\frac{\partial \delta n}{\partial t} + v_p^0 \cdot \nabla_r \delta n - e[v_p^0 \times B] \cdot \nabla r \delta n = 0,$$

where

$$\delta n(p,r,t) = \delta n(p,r,t) - \left(\frac{\partial n^0}{\partial \varepsilon_0}\right) \times \sum_p \int dr' f_{p+\epsilon A,p'+\epsilon A}(r-r') \delta n(p',r',t),$$

is the deviation from local equilibrium.

It is important to point out that equation (5) is linear in $\delta n_p$ however, is highly non-linear in the magnetic field.

II. SEMI-CLASSICAL APPROXIMATION

Following the standard Fermi liquid approach we start by writing down the energy functional for a two-dimensional system of spin-less quasi-particles of effective mass $m^*$, in an electromagnetic field defined by the vector potential $A = A(r,t),\quad E[n] = \sum_p \frac{(p + eA)^2}{2m^*} n(p,r) + \sum_{p,p'} \int drdr' f_{p+\epsilon A,p'+\epsilon A}(r-r') \times n(p,r)n(p',r) + \mathcal{O}(n^3),\quad (1)\quad$$

FIG. 1. Phase diagram for the Pomeranchuk instability $\ell = 2$ in the presence of a small magnetic field. The external part of the transition curve represents an isotropic Fermi liquid, while the inner part is an anisotropic liquid phase. The dimensional control parameters are $\alpha = 1 + F_2$ and $\omega_c/\epsilon_F$, where $\omega_c = eB/m^*$ is the cyclotron frequency and $\epsilon_F = v_F p_F$ is the Fermi energy. We have plotted equation (5) by fixing the interaction range $\kappa p_F = 10$.
since it enters the definition of the Landau interaction (equation (6)). Usually, to compute collective plasma modes in charged Fermi liquids this last contribution is neglected, ending with a true linear response theory. However, as we will show, this approximation is not consistent to study Pomeranchuk instabilities.

At low temperatures $k_B T \ll \varepsilon_F$ the electron dynamics is confined to a small region around the Fermi surface. Then, it is more convenient to define $\delta n(\mathbf{p}, r, t) = -(\partial n_0^p/\partial \varepsilon_p) \nu_p(\mathbf{r}, t)$, where $\nu_p(\mathbf{r}, t)$ measures local Fermi surface deformation. Finally, Fourier transforming in the space variable $\mathbf{r}$, the kinetic equation (5) becomes,

$$\frac{\partial \nu_p(\mathbf{q}, t)}{\partial t} + (i \mathbf{v}_F^0 \cdot \mathbf{q} - e(\mathbf{v}_F^0 \times \mathbf{B}) \cdot \nabla_p) \times (\nu_p(\mathbf{q}, t) + \delta \varepsilon_p(\mathbf{q}, t)) = 0,$$

where $\delta \varepsilon_p(\mathbf{q}, t) = \frac{1}{V} \sum_{\mathbf{p}'} \frac{\partial n_0^p}{\partial \varepsilon_p} \int d\mathbf{r} d\mathbf{r}' e^{i\mathbf{q} \cdot \mathbf{r}'} \times f_{\mathbf{p+p'+A,A'}}(\mathbf{r} - \mathbf{r}') \nu_{\mathbf{p}'}(\mathbf{r}', t)$ describes the combined effect of interactions and magnetic field, being $V$ the space volume. Equations (7) and (8) are the starting point of our analysis. They describe the dynamics of Fermi surface deformations, given an initial condition $\nu^0(\mathbf{q}, 0)$, representing a small density fluctuation with wave-vector $\mathbf{q}$. In the next section we will set up our model and will study the Pomeranchuk instability in the nematic channel.

III. THE POMERANCHUK NEMATIC INSTABILITY

For simplicity we consider a two-dimensional circular Fermi surface. The interaction Landau function depends essentially on the angle between two Fermi momenta and can be expanded in Landau parameters as

$$f_{\mathbf{p+p'+A,A'}}(\mathbf{r} - \mathbf{r}') = \sum_{\ell} f_{\ell}(r)e^{i\ell \mathbf{r} \cdot \mathbf{p}/p F q},$$

where $\cos \varphi = \mathbf{p}_F \cdot \mathbf{p}'_F/p F q$. Moreover, we can expand the deformation of the Fermi surface in Fourier coefficients

$$\nu_p(\mathbf{q}, t) = \sum_{\ell} \nu_{\ell}(q, t)e^{i\ell \mathbf{q} \cdot \mathbf{p}/p F q}$$

where $\cos \theta = \mathbf{p}_F \cdot \mathbf{q}/p F q$.

To study the Pomeranchuk-nematic instability, it is sufficient to consider a simplified model defined by $f_2(r) \neq 0$, while $f_{\ell}(r) = 0$ for all $\ell \neq 2$. The presence of other interaction channels does not modify our results qualitatively, provided they are all stable. We will consider a short-ranged but non-local interaction $f_2(r)$, whose Fourier transform is non-linear.

$$f_2(q) = \frac{f_2}{1 + |F_2(\kappa q)|^2}.$$

where $F_2 = N(0)f_2$ is the usual adimensional Landau parameter with angular momentum $\ell = 2$ ($N(0)$ is the density of states at the Fermi surface), and $\kappa$ defines an effective interaction range $\xi = \sqrt{F_2}/\kappa$. Our approach is valid provided $p_F^{-1} \ll \xi \ll q^{-1}$ i.e., when the interaction range is much larger than the inter-particle distance, however shorter than the typical scale of long-ranged perturbations.

In the absence of a magnetic field, the collective dynamics of the Fermi surface, given by equation (7) with $A = 0$, reduces to

$$\frac{\partial \nu_{\ell}(q, t)}{\partial t} + \frac{iv_F q}{2} [\alpha_{\ell-1}\nu_{\ell-1}(q, t) + \alpha_{\ell+1}\nu_{\ell+1}(q, t)] = 0,$$

where we have defined $\alpha_1 = 1 + F_2$ and $F_2$ are adimensional Landau parameters. In our model $\alpha_2 = \alpha = 1 + F_2$ and $\alpha_{\ell} = 1$ for all $\ell \neq 2$.

We can gain more physical insight by defining symmetric and antisymmetric variables,

$$\nu_{\ell}^+(q, t) = \frac{1}{2}[\nu_{\ell}(q, t) + \nu_{-\ell}(q, t)]$$

in terms of which, the Fermi surface deformations are parametrized as

$$\nu(\mathbf{q}, \theta, t) = \sum_{\ell=0}^{\infty} \nu_{\ell}^+(q, t) \cos(\ell \theta) + \sum_{\ell=1}^{\infty} \nu_{\ell}^-(q, t) \sin(\ell \theta).$$

Eliminating in equation (12) odd components in favor of even ones, we obtain the coupled oscillator equations:

$$\frac{\partial^2 \nu_{\ell}^+(q, t)}{\partial t^2} + \left(\frac{v_F q}{2}\right)^2 [A_{\ell} \nu_{\ell}^+(q, t) + C_{\ell-1} \nu_{\ell-2}^+(q, t) + C_{\ell+1} \nu_{\ell+2}^+(q, t)] = 0$$

with the adimensional coefficients,

$$A_{\ell} = \alpha \alpha_{\ell-1} + \alpha_{\ell+1},$$

$$C_{\ell} = \alpha_{\ell+1}\sqrt{\alpha_{\ell-1}}.$$  

It is clear from equation (15) that the even an odd components of $\ell$ are decoupled. The same happens with the symmetric and antisymmetric components. The physical reason for that is parity invariance. Hence, the $\nu_{\ell}^+$ mode is coupled with the even symmetric modes $\nu_0, \nu_2, \nu_4, \nu_6 \ldots$. Near $F_2 = -1$, or $\alpha \sim 0$, the $\nu_{\ell}^+$ mode oscillates with frequency

$$\omega_{\ell} = \sqrt{2\alpha \left(\frac{v_F q}{2}\right)^2},$$

while all the other modes essentially oscillate with $\omega_{\ell} \sim v_F q/\sqrt{2}$. Then, near $\alpha = 0, \omega_2 \ll \omega_{\ell}$ with $\ell \neq 2$ showing that, in time scales $\tau \sim (v_F q)^{-1}, \nu_{\ell}^+$ is a very slow mode, while all other rapid modes can be averaged to zero. Therefore, when $\alpha \to 0$, the Fermi surface has an essentially elliptic form during long periods of time. This is the onset of the Pomeranchuk-Nematic instability.
When a magnetic field is applied, parity, as well as time reversal symmetry are broken. Then, the symmetric and antisymmetric modes are no longer decoupled. In linear response theory, we can ignore the contribution of the magnetic field in equation (8), then, equation (7) can be simplify to,

\[
\frac{\partial \nu}{\partial t} + i \nu q \left[ \alpha_{\ell-1} \nu_{\ell-1} + \alpha_{\ell+1} \nu_{\ell+1} \right] + i t \alpha \nu \omega \nu_\ell = 0, \tag{18}
\]

where we have defined the cyclotron frequency \( \omega_c = eB/m^* \). Thus, the linear response correction to equation (12) is proportional to \( \alpha_c (\omega_c/v_F q) \), where \( \alpha_c = 1 + F_\ell \).

Since \( \alpha \sim 1 \) for stable modes \( (\ell \neq 2) \), this equation is suitable to study collective modes of the Fermi liquid in small magnetic fields. However, near the Pomeranchuk instability \( (\alpha_\ell \equiv \alpha \sim 0) \), \( (\omega_c/v_F q)^2 \) is of the same order of \( \alpha (\omega_c/v_F q) \) and cannot be ignored. To see this more clearly, we can compute the oscillation frequency of \( \nu_2^2 \), using equation (15), obtaining,

\[
\omega_2 \sim \sqrt{2 \alpha} \left( \frac{v_F q}{2} \right) \left[ 1 + 4 \alpha \left( \frac{\omega_c}{v_F q} \right)^2 + \ldots \right], \tag{19}
\]

where the ellipsis “…” means terms of order \( O(\alpha^2(\omega_c/v_F q)^4) \). Clearly, for small \( \alpha \ll 1 \), the frequency is approximately given by equation (17) without changing the behavior of the quantum critical point \( \alpha = 0 \).

Therefore, to study the transition line \( \omega_c(\alpha) \), we need to consider quadratic corrections in the magnetic field. To do this, we expand the Landau function \( f_2 \) in equation (8), keeping linear in the vector potential \( A \),

\[
f_{p+eA,p'+eA}(r-r') = f_2(r-r') \left( \frac{p \cdot p'}{p_F^2} \right)^2 + 2e \frac{f_2(r-r')}{p_F^2} \frac{(p \cdot p') [A(r) \cdot (p + p')]}. \tag{20}
\]

With this expression, equation (8) reduces to

\[
\delta \varepsilon_p(q,t) = \delta \varepsilon_p^0(q,t) + \delta \varepsilon_p^A(q,t), \tag{21}
\]

where the first term has no contribution from the magnetic field, and is given by

\[
\delta \varepsilon_p^0(q,t) = -i f_2 \sum_{p'} \left( \frac{\partial \varepsilon_p^0}{\partial p'} \right) \left( \frac{p \cdot p'}{p_F^2} \right)^2 \nu_{p'}(q,t), \tag{22}
\]

while the second term is linear in \( \omega_c \),

\[
\delta \varepsilon_p^A(q,t) = -2\pi \left( \frac{\omega_c}{v_F f_F} \right) \frac{(\kappa p_F)^2 F_2^2}{p_F^4 \nu(0)} \times \sum_{p'} \left( \frac{\partial \varepsilon_p^0}{\partial p'} \right) \left( p \cdot p' \right) [p + p'] \times q \nu_{p'}(q,t), \tag{23}
\]

where we have chosen the symmetric gauge \( A = (1/2)r \times B \). This term depends on the interaction range \( (\kappa p_F)^2 \), then, for ultra-local interactions \( (\kappa = 0) \), it makes no contribution. On the other hand, the vectorial structure of equation (23) filters only contributions to the modes \( \nu_1, \nu_2 \). Therefore, Fourier transforming in \( p \) we find for these modes,

\[
\frac{\partial \nu_1}{\partial t} + i \left( \frac{v_F q}{2} \right) \left[ 1 - 2(1-\alpha)^2 \left( \frac{\omega_c}{v_F q} \right)^2 (\kappa q)^2 \right] \nu_0 + \alpha - 2(1-\alpha)^2 \left( \frac{\omega_c}{v_F q} \right)^2 (\kappa q)^2 \nu_2 + i \omega_c \nu_1 = 0 \tag{24}
\]

and

\[
\frac{\partial \nu_2}{\partial t} + i \left( \frac{v_F q}{2} \right) \left[ 1 - 4(1-\alpha)^2 \left( \frac{\omega_c}{v_F q} \right)^2 (\kappa q)^2 \right] \nu_1 + + i \left( \frac{v_F q}{2} \right) \nu_3 + 2i \alpha \omega_c \nu_2 = 0. \tag{25}
\]

The equations for the modes \( \nu_1 \) and \( \nu_2 \) are easily obtained from equations (24) and (25) by changing \( \ell \rightarrow -\ell \), and \( \omega_c \rightarrow -\omega_c \). The dynamical equations for the rest of the modes are simply given by equation (12). Then, building symmetric and antisymmetric mode combinations, and deriving the evolution equations to get a second order system, we get for \( \nu_2^+ \),

\[
\frac{\partial^2 \nu_2^+}{\partial t^2} + \Omega^2 \nu_2^+ + \left( \frac{v_F q}{2} \right) (\nu_0 + \nu_2^+) + \left( \frac{v_F q}{2} \right) \omega_c \left( 2 \nu_1^+ + 3 \nu_3^- \right) = 0 \tag{26}
\]

with

\[
\Omega^2 = 2(\nu_2^+)^2 \left[ 1 + 8\alpha \left( \frac{\omega_c}{v_F q} \right)^2 + \left( \frac{\kappa q}{2} \right)^2 (1-\alpha)^2 \omega_c^2. \tag{27}
\]

As we have anticipated, the magnetic field mixes symmetric and antisymmetric modes. The first contribution to the frequency in equation (27) comes from the linear response theory, while the last term, proportional to the interaction range \( q \kappa \ll 1 \), is the first “correction” coming from equation (23).

Near the transition line \( \Omega \rightarrow 0 \), \( \nu_0 \) and \( \nu_2^+ \) are very rapid and stable modes, while the coupling with the antisymmetric modes are very weak. Thus, they do not modify the transition qualitatively. In the next section we will study these couplings in more detail. Therefore, \( \nu_2^+ \) is unstable when \( \Omega = 0 \), leading to the condition line,

\[
\left( \frac{\omega_c}{v_F p_F} \right)^2 = -\frac{1}{8} \left( \frac{q}{p_F} \right)^2 \frac{\alpha}{\alpha^2 + (1-\alpha)^2 (2\kappa q)^2}. \tag{28}
\]

Near the quantum critical point, we can expand this expression in powers of \( \alpha \),

\[
\left( \frac{\omega_c}{v_F p_F} \right)^2 = -2 \left( \frac{1}{\kappa p_F} \right)^2 \alpha + O(\alpha^2), \tag{29}
\]

obtaining a linear critical region governed by the interaction range \( \kappa p_F \). Corrections of order \( \alpha^2 \) depend on
Thus, differently from the usual Pomeranchuk transition, small perturbations with different values of $q$ will contribute to the instability at different values of $\alpha$. On the other hand, the momentum perturbation is limited to the range $r_c^{-1} < q < \kappa^{-1}$. It is simple to show that the extremal line, necessary to build up the complete phase diagram is reached at $q = 1/\kappa$. Therefore, the transition line is given by

$$\left(\frac{\omega_c}{v_F p_F}\right)^2 = -2 \left(\frac{1}{\kappa p_F}\right)^2 \frac{\alpha}{16\alpha^2 + (1 - \alpha)^2},$$

where the only free parameter is the interaction range $\kappa p_F > 1$. We depict eq. (30) in figure (1). As expected, a magnetic field strongly reduces the phase space for Pomeranchuk instabilities. For small values of the magnetic field, the quantum critical point is shifted to greater attractive values of the interaction $\alpha < 0$, or $F_2 < -1$. Indeed, we observe a maximum value of the magnetic field

$$\left(\frac{\omega_c}{v_F p_F}\right)_{\text{max}} \sim 0.2 \left(\frac{1}{\kappa p_F}\right)^2$$

reached at $\alpha_{\text{max}} \sim -1/4$, above which, no Pomeranchuk instability is possible. Moreover, we observe a reentrant behavior of the disordered isotropic phase for greater values of the attractive interaction.

It is important to note a clear difference with the case of the usual Pomeranchuk instability. At zero magnetic field, below the critical point $\alpha = 0$, the isotropic Fermi liquid is unstable under non-homogeneous density perturbations characterized by a wave-vector $q$. Indeed, any value of $q$, no matter how small, will produce the phase transition. However, in the presence of a magnetic field, there is another length-scale given by the cyclotron radius $r_c = v_F/\omega_c$. This scale introduces an infrared cut-off for the relevant fluctuations that could trigger the phase transition. In other words, in the region below the transition line in figure (1), the isotropic Fermi liquid is unstable under density fluctuations in a typical length-scale $\kappa < q^{-1} < r_c$. In practice, $\kappa$ is a microscopic length and $r_c$ is very large, therefore the above restriction is not severe.

On the other hand, for $q^{-1} >> r_c$ there is no possible Pomeranchuk transition. This result is quite clear. In the regime $q^{-1} >> r_c$, the semi-classical approach is no more valid. It is necessary to treat the complete quantum problem, where the system is gapped due to Landau level quantization. This is the quantum Hall regime in which there is no Pomeranchuk instability. From an experimental point of view, fluctuations and in particular, the wave-vector $q$ are very difficult to control. However, any random inhomogeneous density fluctuation will contain components of $r_c^{-1} < q < \kappa^{-1}$ that, even with a very small amplitude, will trigger the anisotropic/isotropic phase transition. On the other hand, it is always possible to imagine (at least from a theoretical point of view) that one could induce a small density fluctuation by applying a modulated test field with a definite wave-vector $q$.

**IV. COLLECTIVE MODES NEAR THE QUANTUM CRITICAL POINT**

We would like to analyze the behavior of the stable oscillation modes of the Fermi surface near the quantum critical point ($\alpha = 0, \omega_c = 0$). We are interested in the regime, $\alpha \ll 1$, and $\omega_c \ll v_F q \ll v_F p_F$.

We will focus in the unstable model $\nu_2^\dagger$. This mode is directly coupled with $\nu_0, \nu_1^\dagger, \nu_2^\dagger$, through equation (20). The symmetric modes $\nu_0$ and $\nu_1^\dagger$ are stable modes and oscillate very rapidly near the quantum critical point. Therefore, if we are interested in time scales larger than $(v_F q)^{-1}$, we can average them to zero. The antisymmetric modes $\nu_2^\dagger$ and $\nu_3^\dagger$ couple with $\nu_2^\dagger$ through a magnetic field $\omega_c$, as a manifestation of parity symmetry breaking. Then, dismissing the symmetric couplings, $\nu_0, \nu_1^\dagger$, the remaining system $(\nu_2^\dagger, \nu_1^\dagger, \nu_3^\dagger)$ is a closed one. Defining the column vector $\nu = (\nu_2^\dagger, \nu_1^\dagger, \nu_3^\dagger)$, the collective modes satisfy

$$\frac{\partial^2 \nu(q, t)}{\partial t^2} + M \cdot \nu(q, t) = 0,$$

where the matrix $M$ takes the following form near the quantum critical point:

$$M = \begin{pmatrix}
\frac{2(\frac{v_F q}{\omega_c})^2}{\frac{\alpha}{\omega_c} + (\frac{v_F q}{\omega_c})^2} & \frac{(v_F q)}{\omega_c} & \frac{3(v_F q)}{\omega_c} \\
\frac{2\omega_c}{v_F q} \left[\frac{2(\frac{v_F q}{\omega_c})^2}{\frac{\alpha}{\omega_c} + (\frac{v_F q}{\omega_c})^2}\right] & \frac{(v_F q)}{\omega_c} & \frac{3(v_F q)}{\omega_c} \\
5\alpha \omega_c \left(\frac{v_F q}{\omega_c}\right) & \frac{\alpha}{\omega_c} + (\frac{v_F q}{\omega_c})^2 & \frac{3(\frac{v_F q}{\omega_c})^2}{\frac{\alpha}{\omega_c} + (\frac{v_F q}{\omega_c})^2}
\end{pmatrix}.$$

due to parity symmetry. Then, the $\nu_3^\dagger$ frequency coincides with equation (17). However, when approaching the quantum critical point lowering the magnetic field ($\alpha = 0, \omega_c \to 0$), the matrix $M_\epsilon = \lim_{\alpha \to 0} M$ takes the
form
\[
M_c = \begin{pmatrix}
\omega_c^2 \left( \frac{k F q}{2} \right)^2 & \left( \frac{v_F q}{2} \right) \omega_c & 3 \left( \frac{v_F q}{2} \right) \omega_c \\
\left( \frac{\omega_c}{v_F q} \right) \left( \frac{k F q}{2} \right)^2 & \omega_c^2 \left( 1 + \left( \frac{k F q}{2} \right)^2 \right) & \left( \frac{\omega_c}{v_F q} \right) \left( \frac{k F q}{2} \right)^2 \\
0 & 0 & \left( \frac{\omega_c}{v_F q} \right)^2 
\end{pmatrix}.
\]

In order to find the normal modes, we diagonalize \( M_c \) obtaining the eigen-values
\[
\lambda_1 = \left( \frac{k F q}{2} \right)^4 \omega_c^2, \quad \lambda_2 = \omega_c^2, \quad \lambda_3 = \left( \frac{v_F q}{2} \right)^2,
\]
with the corresponding eigen-vector matrix
\[
A = \begin{pmatrix}
1 & 2 \left( \frac{\omega_c}{v_F q} \right) \left( \frac{k F q}{2} \right)^2 & 0 \\
-2 \left( \frac{\omega_c}{v_F q} \right) \left( \frac{k F q}{2} \right)^2 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}.
\]

Thus, it is clear from equations (36) that the mode \( \nu_3^- \) decouples for \( \alpha = 0 \) and it is a rapid mode, oscillating with frequency \( v_F q/2 \). On the other hand, the modes \( \nu_1^+ \) and \( \nu_1^- \) are slow modes, coupled by the small quantities \( (\omega_c/v_F q) \ll 1 \) and \( (k F q/2)^2 \ll 1 \). The former is related with the cyclotron frequency that should be smaller than the frequency of a typical perturbation \( (v_F q)^{-1} \), while the latter is related with the interaction range that should be much smaller than the typical length of the Fermi surface perturbation \( q^{-1} \).

Therefore, very near the Pomeranchuk instability (\( \alpha = 0, \omega_c \ll v_F q \)), the Fermi surface fluctuates following the equation
\[
\delta k_F = \nu_1^+ \left\{ \cos(2\theta) + 2 \left( \frac{\omega_c}{v_F q} \right) \left( \frac{k F q}{2} \right)^2 \sin \theta \right\} \times \\
\times \cos \left[ \left( \frac{k F q}{2} \right)^2 \omega_c t + \varphi_1 \right] + \\
+ \nu_{1-1}^- \left\{ \sin(\theta) - 2 \left( \frac{\omega_c}{v_F q} \right) \left( \frac{k F q}{2} \right)^2 \cos(2\theta) \right\} \times \\
\times \cos \left[ \omega_c t + \varphi_2 \right] + \\
+ \nu_{1-3}^- \sin(\theta) \cos \left[ \left( \frac{v_F q}{2} \right) t + \varphi_3 \right],
\]
where \( \nu_1^+, \nu_{1-1}^- \) and \( \varphi_1, \varphi_2, \varphi_3 \) are the initial amplitudes and phases, respectively.

We see that there are two slow modes that oscillate with frequencies proportional to \( \omega_c \). The slowest mode \( \left( \lambda_1 \right. \text{ in equation (36)} \) is related with \( \nu_1^+ \), and it is responsible for the Pomeranchuk instability when \( \omega_c \rightarrow 0 \). On the other hand, the mode associated with the eigen-value \( \lambda_2 \) is related with the anti-symmetric mode \( \nu_1^- \). However, this mode is not unstable at the quantum critical point since, when \( \omega_c \rightarrow 0 \), not only its frequency goes to zero, but also its velocity \( \partial \nu_1^- / \partial t \rightarrow 0 \), implying a constant mode at the quantum critical point, decoupled from any other symmetric mode.

V. CONCLUSIONS

We have analyzed the behavior of a two-dimensional Fermi liquid submitted to an external magnetic field, near a Pomeranchuk instability triggered by the Landau parameter \( F_2 \) in the charged sector. We have considered a simple model in which the only interaction is given by the Landau parameter \( F_2 \). The presence of other interac-
tions does not modify the results qualitatively, provided they are all stable, i.e., distant from any other Pomeranchuk instability.

We have studied the Fermi surface stability, approaching the critical region from the isotropic phase, where the Landau theory of Fermi liquids can be used safely. We have studied collective modes using the semi-classical Landau-Silin equation. Usually, this equation was studied in the linear response approximation to analyze the Pomeranchuk instability this approximation is not sufficient in the linear response approximation to analyze Landau-Silin equation. Usually, this equation was studied non-perturbatively in the context of Landau theory of Fermi liquids, it is necessary to go beyond the linear approximation in $\delta n$ and to study the collision integral $I_{\text{coll}}$ in the Landau-Silin equation [1]. Conversely, it is possible to face this problem with other approaches like, for instance, non-perturbative calculations on specific fermionic models. We hope to report on this issue in the near future.

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1. Eduardo Fradkin, Steven A. Kivelson, Michael J. Lawler, James P. Eisenstein, Andrew P. Mackenzie, Annual Reviews of Condensed Matter Physics 1, 153 (2010).
2. V. Oganesyan, S. A. Kivelson, and E. Fradkin, Phys. Rev. B 64, 195109 (2001).
3. D. G. Barci and L. E. Oxman, Phys. Rev. B 67, 205108 (2003).
4. Michael J. Lawler, Daniel G. Barci, Victoria Fernandez, Eduardo Fradkin, Luis Oxman, Phys. Rev. B 73, 085101 (2006).
5. Johan Nilsson, A. H. Castro Neto, Phys. Rev. B 72, 195104 (2005).
6. Max A. Metlitski and Subir Sachdev Phys. Rev. B 82, 075127 (2010).
7 Casper Drukier, Lorenz Bartosch, Aldo Isidori, and Peter Kopietz, Phys. Rev. B 85, 245120 (2012).
8 Photoemission in the High Tc Superconductors, J. C. Campo-ruzano, M. R. Norman, M. Randeria, Physics of Superconductors, Vol. II, ed. K. H. Bennemann and J. B. Ketterson (Springer, Berlin, 2004), p. 167-273.
9 R. A. Borzi, S. A. Grigera, J. Farrell, R. S. Perry, S. J. S. Lister, S. L. Lee, D. A. Tennant, Y. Maeno, A. P. Mackenzie, Science 23, (2006).
10 Hyeonjin Doh, Yong Baek Kim, and K. H. Ahn, Phys. Rev. Lett. 98, 126407 (2007), J. F. Mercure, S. K. Goh, E. C. T. O’Farrell, R. S. Perry, M. L. Sutherland, A. W. Rost, S. A. Grigera, R. A. Borzi, P. Gegenwart, and A. P. Mackenzie, Phys. Rev. Lett. 103, 176401 (2009) and references therein.
11 R. S. Perry, L. M. Galvin, S. A. Grigera, L. Capogna, A. J. Schofield, A. P. Mackenzie, M. Chiao, S. R. Julian, S. Ikeda, S. Nakatsuji, Y. Maeno, and C. Pfleiderer, Phys. Rev. Lett. 86, 2661 (2001).
12 R. A. Borzi, S. A. Grigera, J. Farrell, R. S. Perry, S. J. S. Lister, S. L. Lee, D. A. Tennant, Y. Maeno, and A. P. Mackenzie, Science 315, 214 (2007).
13 S. A. Grigera, R. S. Perry, A. J. Schofield, M. Chiao, S. R. Julian, G. G. Lonzarich, S. I. Ikeda, Y. Maeno, A. J. Millis, and A. P. Mackenzie, Science 294, 329 (2001).
14 L. D. Landau, Zh. Eksp. Teor. Fiz. 30, 1058 (1956) [Sov. Phys. JETP 3, 920 (1957)]; 32, 59 (1957) [5, 101 (1957)].
15 Silin V.P. A Zh. Eksp. Teor. Fiz., (1957), v.33, p.495 [Sov. Phys. JETP, (1957), v.6, p.387]; Zh. Eksp. Teor. Fiz., (1958), v.35, p.1243 [Sov. Phys. JETP, (1958), v.8, p.870].
16 G. Pines and P. Nozières, The Theory of Quantum Liquids (Perseus Books, Cambridge, MA, 1966).
17 T. K. Lee and J. J. Quinn, Phys. Rev. B 11, 2144 (1975); D. C. Marinescu and J. J. Quinn Phys. Rev. B 58, 15688 (1998); P. M. Platzmann and W. M. Walsh Jr., Phys. Rev. Lett. 19, 514 (1967); P. M. Platzmann and P. M. Wolff, Phys. Rev. Lett. 18, 280 (1967).
18 J. G. Fertig and M. B. Maple Solid State Commun. 15, 453 (1974).
19 E. Simanek, Phys. Rev. B 23, 5762 (1981).
20 P. E. Cladis, Phys. Rev. Lett. 35, 48 (1975).
21 N. H. Tinh, F. Hardouin, and C. Destrade, J. Physique, 43, 1127 (1982).
22 O. J. Indekeu and A. N. Berker, Phys. Rev. A. 33, 1158 (1986).
23 A. M. Manheimer, S. M. Bhagat, and H. S. Chem, Phys. Rev. B. 26, 456 (1982).
24 R. J. Vasconcelos dos Santos, F. C. Sa Barreto and S. Coutinho J. Phys. A: Math. Gen 23, 2563 (1990).
25 D. Reyes and M. A. Continentino, J. Phys.: Condens. Matter 19, 406203 (2007).