A new approach for load flow solution based on a two-step iterative technique

Abstract. A new approach for solving load flow problem in power system is developed. The presented method is based on a two-step predictor-corrector technique for solving nonlinear equation using the weight combination of mid-point, trapezoidal quadrature formula. The proposed method is applied on typical test systems. The bus voltages are represented by polar and rectangular coordinates system. The results presented show that the convergence of the proposed method is faster than the conventional Newton-Raphson method.

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1. Introduction
Load flow is the basic tool in many power system studies. It consists of computing the bus voltages and the power flow in each line of the transmission or distribution network for a pattern loading condition. The steady state performance of the system is modelled by a system of nonlinear algebraic equations. Several numerical techniques have been developed to solve the load flow problem [1-4]. Among them, the Newton-Raphson method is the most widely accepted one than other methods. Many research efforts have been done to improve Newton-Raphson method since the pioneering work of Tinney & Hart [2].

Stott [5] presented a starting scheme to improve the reliability of Newton-Raphson method. An algorithm has been presented in [6] to modify the Newton-Raphson method by including the second order terms from the Taylor series expansion of load flow equations. Researchers in [7] recommended an approach based on Newton method to treat the convergence difficulties arises from ill-conditioned system. An algorithm was developed to achieve a better decoupling of the Newton-Raphson method[8]. It also, provides starting values for load flow solutions. Several methodologies have been proposed to tackle the load flow problem for distribution system by a modified version of Newton method [9-10]. A faster and highly localized power flow solutions was obtained by exploiting the difference in convergence rate as suggested in [11]. An attempt to speed up the Newton process using generalized minimal residual was developed [12]. It has described a new preconditioning technique with a fast decoupled foundation. Schaffer and Tylavsky[13] have suggested a non-diverging Newton based power flow using polar form. A new formulation for the Newton-Raphson power flow based on current injection was developed in Reference [14]. A Newton-Raphson power flow for ill-conditioned large power system with embedded Flexible AC Transmission Systems was described in [15]. An improvement to the Newton-Raphson in rectangular coordinates has been developed in [16]. The state variables in the paper are the bus voltages and current injections, while both power and current...
mismatches are zeroed. A new pseudo load flow model was introduced in Reference [17]. The solution of the model can provide a robust starting for conventional Newton-Raphson method. Kulworawanichpong [18] suggested a simplified version of the Newton-Raphson method. The nonlinear current mismatch equations have been used instead of the power mismatch equations. Exploration the reliability of the distribution network load flow by using a novel search technique was given in [19].

In this paper a new algorithm for Newton-Raphson method is developed to speed up the iterative process. The nonlinear algebraic equations which describe the steady state performance are solved by a two-step predictor-corrector technique [20]. The proposed method used the weight combination of the midpoint and trapezoidal quadrature formula to solve the load flow equations. The structure of this paper is as follow: Section 2 review the two-step predictor-corrector method. The application of the above technique to the polar and rectangular form of the nonlinear power flow model is presented in Section 3 and 4, respectively. Section 5 show the results obtained by running the proposed algorithm on typical test systems. Conclusions are contained in Section 6.

2. Two-step predictor-corrector technique

For the following nonlinear algebraic equation:

\[ f(x) = 0 \]

Suppose that \( x \) be the simple zero of this equation. Equation (1) can be written as [20-21]:

\[ f(x) = f(x_k) + \int_{x_k}^{x} f'(t)dt \]  

(2)

The second term of equation (2) can be approximated with average of midpoint and Simpson quadrature formulas:

\[ \int_{x_k}^{x} f'(t)dt = \frac{x - x_k}{2} f'(\frac{x_k + x}{2}) + \frac{x - x_k}{12} \left[ f'(x_k) + 4f'(\frac{x_k + x}{2}) + f'(x) \right] \]  

(3)

Substituting (3) into (2) yields:

\[ f(x) = f(x_k) + \frac{x - x_k}{12} \left[ f'(x_k) + 10f'(\frac{x_k + x}{2}) + f'(x) \right] \]  

(4)

Since, \( f(x) = 0 \) then

\[ x = x_k + \frac{12f(x_k)}{f'(x_k) + 10f'(\frac{x_k + x}{2}) + f'(x)} \]  

(5)

The implementation of the above implicit can be carried out by the following two step formula:

With a guess value of the variable \( x_0 \), compute the approximate solution \( y_k \)

\[ y_k = x_k - \frac{f(x_k)}{f'(x_k)} \]  

(6)

\[ x_{k+1} = x_k + \frac{12f(x_k)}{f'(y_k) + 10f'(\frac{x_k + y_k}{2}) + f'(x)} \]  

(7)

3. Formulation of the proposed load flow algorithm in polar form

For N bus power system, the steady state performance of the system can be described by the following system of nonlinear algebraic equations in polar form [22]:

\[ P_k = \sum_{i=1}^{N} |V_k||V_i||Y_{ki}| \cos(y_{ki} + \delta_i - \delta_k) \]  

(8)

\[ Q_k = -\sum_{i=1}^{N} |V_k||V_i||Y_{ki}| \sin(y_{ki} + \delta_i - \delta_k) \]  

(9)
Where \( V_i = |V_i| \angle \delta_i \) is the magnitude and the angle of the bus voltage at bus \( i \).

The system of equations (8) & (9) can be written in vector notation as:

\[
\begin{align*}
    f(\delta, |V|) &= 0 \\
    g(\delta, |V|) &= 0
\end{align*}
\]

The following algorithm is proposed to solve the load flow equations in polar form by using predictor corrector technique:

1. Enter the line data, the bus data, voltage constraint, reactive power source constraint, maximum number of iterations, and tolerance in convergence.
2. Form the bus admittance matrix \( \mathbf{Y}_{BUS} \). The bus voltage magnitude of the load buses and the bus voltage angle of all buses are assigned guessed (starting) values (in this paper a flat start \( |V|_o=1.0 \text{ pu}, \delta_o=0 \) is assumed).
3. Set iteration account \( i=0 \).
   An approximate solution is predicted by the following formula for iteration:

\[
\begin{bmatrix}
    \delta p_i \\
    |Vp_i|
\end{bmatrix}
= \begin{bmatrix}
    \delta_i \\
    |Vi|
\end{bmatrix}
+ \begin{bmatrix}
    J_{f}\delta & J_{f}|V| \\
    J_{g}\delta & J_{g}|V|
\end{bmatrix}^{-1}
\begin{bmatrix}
    f_i \\
    g_i
\end{bmatrix}
\]

(12)

Where \( [\delta p_i \, |Vp_i|]^T \) is the vector of the predicted solution.

4. Defining the midpoint solution:

\[
\begin{bmatrix}
    \delta m_i \\
    |Vm_i|
\end{bmatrix}
= \frac{1}{2}
\begin{bmatrix}
    \delta_i + \delta p_i \\
    |V_i + |Vp_i|
\end{bmatrix}
\]

(13)

Where \( [\delta m_i \, |Vm_i|]^T \) is the vector of the midpoint solution.

5. The new form of the Jacobian matrix is calculated by the following formula:

\[
\begin{bmatrix}
    J_{Nf}\delta & J_{Nf}|V| \\
    J_{Ng}\delta & J_{Ng}|V|
\end{bmatrix}
= \begin{bmatrix}
    (J_{f}\delta & J_{f}|V|) + 10(J_{g}\delta m & J_{g}|Vm|) + (J_{f}\delta p & J_{f}|Vp|) \\
    (J_{g}\delta m & J_{g}|Vm|) + (J_{g}\delta p & J_{g}|Vp|)
\end{bmatrix}
\]

(14)

6. The corrected solution at the end of iteration \( i \) is given by the following equations:

\[
\begin{bmatrix}
    \delta i+1 \\
    |V|i+1
\end{bmatrix}
= \begin{bmatrix}
    \delta_i \\
    |Vi|
\end{bmatrix}
+ 12
\begin{bmatrix}
    J_{Nf}\delta & J_{Nf}|V| \\
    J_{Ng}\delta & J_{Ng}|V|
\end{bmatrix}^{-1}
\begin{bmatrix}
    f_i \\
    g_i
\end{bmatrix}
\]

(15)

7. Compute the vector of residuals \( [f_i \, g_i]^T \). If all the values are less than the prescribed tolerance, stop the iterations. Print the solutions (bus voltages and line flows). If the convergence criterion is not satisfied, advance the iteration account \( i=i+1 \), go to step 2 and repeat.

4. Formulation of the proposed load flow algorithm in rectangular form

For \( N \) bus power system, the steady state performance of the system can be described by the following system of nonlinear algebraic equations in rectangular form [22]:

\[
P_k = \sum_{i=1}^{N} e_k (G_{ki} e_i - B_{ki} f_i) + f_k (G_{ki} f_i + B_{ki} e_i)
\]

(16)
\[ Q_k = \sum_{i=1}^{N} f_k(G_{ki}e_i - B_{ki}f_i) - e_k(G_{ki}f_i + B_{ki}e_i) \]  
(17)

Where \( V_i = e_i + jf_i \) is the real and imaginary component of the bus voltage at bus \( i \).

The system of equations (16) & (17) can be written in vector notation as:

\[ h(e, f) = 0 \]  
(18)

\[ y(e, f) = 0 \]  
(19)

The following algorithm is proposed to solve the load flow equations in polar form by using predictor corrector technique:

1. Enter the line data, the bus data, voltage constraint, reactive power source constraint, maximum number of iterations, and tolerance in convergence.

2. Form the bus admittance matrix \( Y_{BUS} \). The bus voltage magnitude of the load buses and the bus voltage angle of all buses are assigned guessed (starting) values (in this paper a flat start \( e^a=1.0 \) pu, \( f^a=0 \) is assumed).

3. Set iteration account \( i=0 \).

4. An approximate solution is predicted by the following formula for iteration \( i \):

\[
\begin{bmatrix}
    e_{pi} \\
    f_{pi}
\end{bmatrix} = \begin{bmatrix}
    e_i \\
    f_i
\end{bmatrix} + \begin{bmatrix}
    J_{he} & J_{hf} \\
    J_{ye} & J_{yf}
\end{bmatrix}^{-1} \begin{bmatrix}
    h_i \\
    y_i
\end{bmatrix}
\]  
(20)

Where \( [e_{pi} f_{pi}]^T \) is the vector of the predicted solution.

5. Defining the midpoint solution:

\[
\begin{bmatrix}
    e_{mi} \\
    f_{mi}
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
    e_i + e_{pi} \\
    f_i + f_{pi}
\end{bmatrix}
\]  
(21)

Where \( [e_{mi} f_{mi}]^T \) is the vector of the midpoint solution.

6. The new form of the Jacobian matrix is calculated by the following formula:

\[
\begin{bmatrix}
    J_{he} & J_{hf} \\
    J_{ye} & J_{yf}
\end{bmatrix} = \begin{bmatrix}
    J_{he} & J_{hf} \\
    J_{ye} & J_{yf}
\end{bmatrix} + 10 \begin{bmatrix}
    J_{hem} & J_{hfm} \\
    J_{yem} & J_{yfm}
\end{bmatrix} + \begin{bmatrix}
    J_{hep} & J_{hfp} \\
    J_{yep} & J_{yfp}
\end{bmatrix}
\]  
(22)

7. The corrected solution at the end of iteration \( i \) is given by the following equations:

\[
\begin{bmatrix}
    e_{i+1} \\
    f_{i+1}
\end{bmatrix} = \begin{bmatrix}
    e_i \\
    f_i
\end{bmatrix} + 12 \begin{bmatrix}
    J_{he} & J_{hf} \\
    J_{ye} & J_{yf}
\end{bmatrix}^{-1} \begin{bmatrix}
    h_i \\
    y_i
\end{bmatrix}
\]  
(23)
8. Compute the vector of residuals \( \mathbf{h} = [h_i, y_i]^T \). If all the values are less than the prescribed tolerance, stop the iterations. Print the solutions (bus voltages and line flows). If the convergence criterion is not satisfied, advance the iteration account \( i=i+1 \), go to step 2 and repeat.

5. Results and discussion
The proposed method was coded with MATLAB R2009 platform, using Intel Core i7 1.8 GHz processor, 8GB RAM hp notebook computer. It has been tested on different power systems. The execution time is determined using MATLAB commands (TIC & TOC). The bus data and the line data of the IEEE 14 bus and IEEE 30 bus high voltage system are given in[23]. Application of the proposed method to the IEEE test systems shows a good match between the results obtained by the proposed approach compared to those obtained by conventional Newton-Raphson method (Tables 1-2). The number of iterations of the converged solutions and the execution times obtained by using the proposed method with those obtained by conventional methods are summarized in Table 3. For the 14-bus system, the solution of proposed method is converged with 3 iterations with total executable time of 0.0021 seconds, while the solution of the conventional method is converged with 4 iterations with a total executable time of 0.0025 seconds for the same tolerance (Tolerance=0.0001). Similarly, a reduction in the iterations numbers and the executable time is drawn from the results of other IEEE test systems. It is worth to note that the proposed method using rectangular coordinate of the load flow model is more efficient than the one using the polar form. The new approach has been applied to the 69-distribution low voltage system with high R/X ratio. The bus data and line data of this system are given in [24]. The load flow solutions by applying the conventional and the proposed method are given in Table 4. The solutions obtained are match up. Also, the proposed method was converged in relative short time with less iteration number. The results obtained in Table 2 are very encouraging. It reveals the pronounced effect of the proposed method in reducing the execution time.

| Bus code | NR conventional method | NR Proposed method |
|----------|------------------------|---------------------|
|          | Voltage magnitude (pu) | Angle (deg)        | Voltage magnitude (pu) | Angle (deg) |
| 1        | 1.06                   | 0.0                 | 1.06                   | 0.0         |
| 2        | 1.045                  | -5.014              | 1.045                  | -5.014      |
| 3        | 0.979                  | -12.494             | 0.979                  | -12.494     |
| 4        | 1.004                  | -10.244             | 1.004                  | -10.244     |
| 5        | 1.01                   | -8.754              | 1.01                   | -8.754      |
| 6        | 1.063                  | -14.434             | 1.063                  | -14.434     |
| 7        | 1.026                  | -13.389             | 1.026                  | -13.389     |
| 8        | 1.010                  | -13.389             | 1.010                  | -13.389     |
| 9        | 1.037                  | -15.025             | 1.037                  | -15.025     |
| 10       | 1.045                  | -15.449             | 1.045                  | -15.449     |
| 11       | 1.080                  | -15.798             | 1.080                  | -15.798     |
| 12       | 1.047                  | -15.307             | 1.047                  | -15.307     |
| 13       | 1.042                  | -15.374             | 1.042                  | -15.374     |
| 14       | 1.021                  | -16.204             | 1.021                  | -16.204     |
Table 2. The bus voltages of IEEE 30 bus system

| Bus code | NR conven. method Voltage magnitude (pu) | Angle (deg) | NR Proposed method Voltage magnitude (pu) | Angle (deg) |
|----------|-----------------------------------------|-------------|------------------------------------------|-------------|
|          | 1                                      | 1.06        | 0.0                                      | 1.06        |
|          | 2                                      | 1.043       | -5.497                                   | 1.043       |
|          | 3                                      | 1.022       | -8.004                                   | 1.022       |
|          | 4                                      | 1.013       | -9.661                                   | 1.013       |
|          | 5                                      | 1.012       | -11.398                                  | 1.012       |
|          | 6                                      | 1.010       | -12.115                                  | 1.010       |
|          | 7                                      | 1.003       | -13.150                                  | 1.003       |
|          | 8                                      | 1.051       | -14.434                                  | 1.051       |
|          | 9                                      | 1.044       | -16.024                                  | 1.044       |
|          | 10                                     | 1.082       | -14.434                                  | 1.082       |
|          | 11                                     | 1.057       | -15.302                                  | 1.057       |
|          | 12                                     | 1.071       | -15.302                                  | 1.071       |
|          | 13                                     | 1.042       | -16.191                                  | 1.042       |
|          | 14                                     | 1.038       | -16.278                                  | 1.038       |

Table 3. The number of iterations and the execution time for different method

| Method               | Number of iterations | Execution times |
|----------------------|----------------------|-----------------|
| Conventional NR (polar) | 4 5 3               | 0.00253 0.003140 0.007380 |
| Conventional NR (Rect.) | 5 4 3               | 0.002116 0.00280 0.006623 |
| Proposed NR (polar)     | 3 5 2               | 0.001945 0.00226 0.005440 |
| Proposed NR (Rect.)     | 4 3 2               | 0.001837 0.00213 0.005010 |
Conclusion

A new approach to solve load flow problem have been presented. The proposed algorithm is based on a two-step predictor-corrector technique for solving nonlinear equation using the weight combination of mid-point, trapezoidal quadrature formula. It was demonstrated that the load flow for both high voltage power system and distribution system with high R/X ratio can be successfully solved. The applications of the proposed algorithm on test systems are given in the paper. The results obtained verify the features of the new algorithm in terms of the execution time and the number of iterations for the system to be converged. The new approach is promising and can be extended to solve many other practical power system issues.

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Table 4. The bus voltages of 69 bus distribution system

| Bus code | NR conventional method | NR Proposed method | Bus code | NR conventional method | NR Proposed method | Bus code | NR conventional method | NR Proposed method | Bus code | NR conventional method | NR Proposed method |
|----------|------------------------|-------------------|----------|------------------------|-------------------|----------|------------------------|-------------------|----------|------------------------|-------------------|
| 1        | 1.0                    | 1.0               | 25       | 0.9532                 | 0.9532            | 49       | 0.9244                 | 0.9244            | 64       | 0.9987                 | 0.9987            |
| 2        | 0.9999                 | 0.9999            | 26       | 0.9531                 | 0.9531            | 50       | 0.9194                 | 0.9194            | 65       | 0.9987                 | 0.9987            |
| 3        | 0.9998                 | 0.9998            | 27       | 0.9531                 | 0.9531            | 51       | 0.912                  | 0.912             | 66       | 0.9987                 | 0.9987            |
| 4        | 0.999                  | 0.999             | 28       | 0.9999                 | 0.9999            | 52       | 0.9117                 | 0.9117            | 67       | 0.9987                 | 0.9987            |
| 5        | 0.999                  | 0.999             | 29       | 0.9998                 | 0.9998            | 53       | 0.9113                 | 0.9113            | 68       | 0.9987                 | 0.9987            |
| 6        | 0.9899                 | 0.9899            | 30       | 0.9997                 | 0.9997            | 54       | 0.9094                 | 0.9094            | 69       | 0.9987                 | 0.9987            |
| 7        | 0.9805                 | 0.9805            | 31       | 0.9997                 | 0.9997            | 55       | 0.9088                 | 0.9088            |          |                        |                   |
| 8        | 0.9783                 | 0.9783            | 32       | 0.9996                 | 0.9996            | 56       | 0.9688                 | 0.9688            |          |                        |                   |
| 9        | 0.9771                 | 0.9771            | 33       | 0.9993                 | 0.9993            | 57       | 0.9688                 | 0.9688            |          |                        |                   |
| 10       | 0.972                  | 0.972             | 34       | 0.9989                 | 0.9989            | 58       | 0.9653                 | 0.9653            |          |                        |                   |
| 11       | 0.9689                 | 0.9689            | 35       | 0.9987                 | 0.9987            | 59       | 0.9653                 | 0.9653            |          |                        |                   |
| 12       | 0.9656                 | 0.9656            | 36       | 0.9998                 | 0.9998            | 60       | 0.9997                 | 0.9997            |          |                        |                   |
| 13       | 0.9625                 | 0.9625            | 37       | 0.9985                 | 0.9985            | 61       | 0.9995                 | 0.9995            |          |                        |                   |
| 14       | 0.9594                 | 0.9594            | 38       | 0.9947                 | 0.9947            | 62       | 0.9995                 | 0.9995            |          |                        |                   |
| 15       | 0.9564                 | 0.9564            | 39       | 0.9941                 | 0.9941            | 63       | 0.9995                 | 0.9995            |          |                        |                   |
| 16       | 0.9559                 | 0.9559            | 40       | 0.9999                 | 0.9999            | 64       | 0.9987                 | 0.9987            |          |                        |                   |
| 17       | 0.9549                 | 0.9549            | 41       | 0.9782                 | 0.9782            | 65       | 0.9984                 | 0.9984            |          |                        |                   |
| 18       | 0.9549                 | 0.9549            | 42       | 0.9781                 | 0.9781            | 66       | 0.9983                 | 0.9983            |          |                        |                   |
| 19       | 0.9544                 | 0.9544            | 43       | 0.9743                 | 0.9743            | 67       | 0.9983                 | 0.9983            |          |                        |                   |
| 20       | 0.9541                 | 0.9541            | 44       | 0.9711                 | 0.9711            | 68       | 0.9982                 | 0.9982            |          |                        |                   |
| 21       | 0.9536                 | 0.9536            | 45       | 0.9666                 | 0.9666            | 69       | 0.9982                 | 0.9982            |          |                        |                   |
| 22       | 0.9536                 | 0.9536            | 46       | 0.9622                 | 0.9622            |          |                        |                   |          |                        |                   |
| 23       | 0.9535                 | 0.9535            | 47       | 0.9397                 | 0.9397            |          |                        |                   |          |                        |                   |
| 24       | 0.9534                 | 0.9534            | 48       | 0.9287                 | 0.9287            |          |                        |                   |          |                        |                   |
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