Robust Asynchronous Fuzzy Predictive Fault-Tolerant Tracking Control for Nonlinear Multiphase Batch Processes with Time-Varying Tracking Trajectories

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Robust asynchronous fuzzy predictive fault-tolerant tracking control for nonlinear multiphase batch processes with time-varying tracking trajectories

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Abstract: A T-S model-dependent robust asynchronous fuzzy predictive fault-tolerant tracking control scheme is developed for nonlinear multiphase batch processes with time-varying tracking trajectories and actuator faults. Firstly, considering the influence of the mismatch between the previous phase state and the current phase controller during the switching, a T-S fuzzy switching model including the match and mismatch case is established. Depending on the fuzzy switching model, a robust fuzzy predictive fault-tolerant tracking controller is designed by considering the situation whether the model rules correspond to the controller rules. Secondly, using the related theories and methods, the system stability conditions presented by the linear matrix inequality considering the above conditions are provided to guarantee the stability of the system. By solving these stability conditions, the T-S fuzzy control law gain of each phase, the minimum running time of each match case and the maximum running time of each mismatch case are obtained. Then, through the maximum running time, the strategy of the switching signal in advance is adopted to avoid the occurrence of asynchronous switching, so as to ensure that the system can operate stably. Finally, the simulation results verify that the designed controller is effective and feasible.

Keywords: unknown disturbances; actuator faults; time-varying tracking trajectories; nonlinear multiphase batch processes; robust fuzzy predictive control; asynchronous switching

1. Introduction

With the advancement of science and technology, the types of industrial products continue to increase, and the production methods become more and more complex, which makes nonlinear characteristics more common in the actual industrial production process. For industrial processes with nonlinear characteristics, the traditional linear controller is difficult to work. References [1,2] adopted a method based on the linearized model. However, the linear model cannot accurately describe the entire characteristics of the nonlinear system. After the nonlinear system is linearized, the linear model obtained cannot completely match the nonlinear system. References [3-6] treat the part of the linearized model that does not match the nonlinear system as uncertainties and combine with robust control to guarantee the system is stable. But as the system runs for a long time, the aging of equipment and other factors, it will bring more uncertainties, which will bring about the continuous decrease of the control performance of the controller. In order to further reduce the influence of model mismatch, references [7-9] used the T-S model combined with fuzzy control to design the controller of the nonlinear system. However, these researches [1-9] are for the continuous processes.

Batch processes are a kind of industrial production process with small scale characteristics, high added value and high control precision. In a diverse and individualized modern society, market competition is becoming increasingly fierce, and the time from design to delivery of new products is getting shorter and shorter. To this end, it puts forward higher requirements for the efficient and fast execution of the production process. Batch processes just meet this demand. However, how to make complex batch processes with high control accuracy and efficient, stable and safe operation has also attracted the attention of many researchers [10-14]. Especially for the study of nonlinear batch processes, it is more meaningful in actual industrial applications. At present, there have been some studies on nonlinear batch processes. Nagy and Braatz [15] raised a combine robust nonlinear model predictive control with the feedback control scheme for the batch processes with constraints and nonlinearities. Reference [16] raised an iterative learning control (ILC) method for the nonlinear batch processes with state delay. Reference [17] put forward a nonlinear monotonic convergence ILC method for batch processes. Reference [18] considers the impact of strong nonlinear in batch processes and brings forward a scheme combining ILC and predictive function control. It is a pity that the above studies [15-18] did not consider how to solve the problem of faults. In the actual production processes, due to the continuous operation of the equipment, faults are inevitable. The faults are generally divided into actuator faults, sensor faults and internal faults of system in which the most common type of fault is actuator faults. If the fault of the actuator cannot be corrected in time, it may cause the degradation of the system control performance, or even cause the system to crash. References [17,18] consider the fault problem in the controller design for
the nonlinear batch processes. Unfortunately, the above-mentioned references [15-18] are only for single-phase batch processes. In practice, multi-phase characteristics exist in most batch processes. As a result, the switching of the actuator is frequent, which is more prone to fault. At present, two main methods are applied to solve the problem of system fault. One is an active fault-tolerant control (FTC) method that combines fault diagnosis with FTC, and the other is passive FTC. Compared with the active FTC method, passive FTC does not require fault information and is easier to implement. The MPBP with asynchronous switching requires simpler and more reliable fault-tolerant mechanisms. Therefore, the passive FTC is more suitable for MPBP. In addition, the MPBP also need to consider the problem of switching between two adjacent phases. Peng et al. [19] proposed a robust switching predictive control algorithm using the synchronous switching way for MPBP. However, it does not consider that when two adjacent phases are switched in practice, the state and the controller may not be able to switch immediately due to system identification speed, system’s computing speed and other reasons. This condition will cause a mismatch between the system state and the controller. Reference [20] proposed an ILC method for asynchronous switching condition considering that the system state and the controller might not match when the switching occurs in MPBP. However, when the system has random and irregular disturbances or inconsistent batch length, the track performance of the ILC method will prodigiously decline. Moreover, the gain of the control law obtained by the ILC method is constant. As the system runs, the control performance of the controller will decrease. Synthesizing the analytical results, in the nonlinear MPBP, the actuator fault and asynchronous switching are urgent problems to be solved, and there is no research for such problems.

In the study of MPBP, the dwelling time of each phase is also crucial. At present, the most commonly used method for solving the dwelling time of each phase is the average dwell time method [21-23]. However, it is used to solve the average time of all of phases in a batch and the calculation accuracy is not high. In order to accurately obtain dwelling time per phase, scholars put forward the model-dependent average dwell time method [24-27], which takes into account the influence of parameter changes in each phase and can accurately calculate the residence time of each phase. Moreover, in the actual production processes, the tracking trajectories of the target often changes with the product demand. The set points of each phase of the MPBP are different. Therefore, the time-varying tracking trajectories are worthy of consideration in the study of MPBP. The problem of time-varying tracking trajectories has also received some attention [28-30]. As far as we know, it is rarely reported in MPBP.

In summary, this paper proposes a robust asynchronous fuzzy predictive fault-tolerant tracking control scheme for nonlinear MPBP with uncertainties, time-varying tracking trajectories and actuator faults. A T-S fuzzy switching model including the match and mismatch case is established, and the fuzzy FTC controllers are designed for different conditions. In the controller design, introducing fault factors and using the relevant methods and the theories, the sufficient conditions that can make the system stable are provided in form of linear matrix inequality (LMI). By solving stability conditions and combining the weights assigned by the fuzzy rules, we can get the minimum running time (Min-RT) of each phase in the match case, the maximum running time (Max-RT) in the mismatch case and the gain control law of per phase. On the one hand, the obtained controller in each phase has better tolerant performance and can effectively resist the influence of model mismatch after linearization. On the other hand, through Max-RT of the mismatch case, the strategy of the switching signal in advance is adopted to guarantee that the state and the controller are matched, which can avoid the occurrence of mismatch case.

The main contributions of this paper are as follows.

(1) Different from the synchronous switching method for the MPBP in reference [19], the proposed method achieves the smooth and stable switching and avoids state “Escape” condition.

(2) Different from using two-dimensional (2D) method in reference [20], the proposed method online solves and updates the control gain in real time rather than the fixed control gain. Moreover, the real-time update control gain also effectively solves the problems of random and irregular disturbances, inconsistent batch length in the system.

(3) Compared with existing researches on MPBP, the controller is designed from the T-S fuzzy switching model for the MPBP considering the non-linear characteristics, which can take into account the one-to-one correspondence and cross correspondence between the model rules and the controller rules.

(4) MPBP require a reliable, easy-to-implement fault tolerance mechanism. Compared with the complex active FTC methods, the proposed passive FTC methods that are more reliable and easier to implement are used in the controller design.

2. Model description of nonlinear MPBP
2.1 T-S fuzzy approximation model for MPBP

Considering a nonlinear MPBP with uncertainties and unknown disturbances, a T-S fuzzy switching model is considered as follows.

If \( Z_i(k) \) is \( M_i \), and \( Z_i(k) \) is \( M_i' \),...,and \( Z_i(k) \) is \( M_i'' \), then

\[
\begin{align*}
\dot{x}(k+1) &= A^{i_k}x(k) + B^{i_k}u(k) + \omega_k^{i_k}(k) \\
y(k) &= C^{i_k}x(k)
\end{align*}
\]

where \( k \) is time variable, \( x(k) \in \mathbb{R}^n \) is the system states variable, \( u(k) \in \mathbb{R}^m \) is the control input variable, \( y(k) \in \mathbb{R}^p \) is the system output variable and \( \omega_k^{i_k}(k) \) is unknown external disturbance. \( Z_i(k) \) is the premise variables, and \( Z_i(k) = x(k), Z_i(k) = x(k-1) \ldots \). \( M_i'(i = 1, 2, \ldots, l, h = 1, 2, \ldots, q) \) represents that the \( i \)th fuzzy rule consists of \( h \) fuzzy sets. \( \alpha(k) \) represents a
switching the signal, which satisfied $x(k): Z' \rightarrow p = 1, 2, 3 \ldots \ p$. $x(k) = p$ indicates that the system is in the $p$th phase. 

If the system has $p$ phases, then the system will go through the mismatch case of $p$-th phase, the match case of $p$-th phase, the mismatch case of $p$-th phase and the match case of $p$-th phase during operation. Therefore, the above-mentioned T-S fuzzy switching model in the $p$th phase are adapted as the following weighted linearization form.

$$
\begin{align*}
\dot{x^p}(k+1) &= \sum_{i=1}^{l} h_i(Z(k))(A^p_i(k)x^p(k)) \\
&\quad + \sum_{i=1}^{l} h_i(Z(k))B^p_iaw^p_i(k) + \sigma^p(k) \\
y^p(k) &= C^p_xx^p(k)
\end{align*}
\tag{2}
$$

where $h^p(x^p(k)) = \frac{M^p(Z^p(k))}{\sum_{i=1}^{l} M_i^p(Z^p(k))}$, and $M^p(Z^p(k))$ is the fuzzy criterion. $A^p(k) = A^p + \Delta A^p(k)$, where $A^p$ is the constant matrix, $\Delta A^p(k)$ is the uncertain matrix which is adapted as follows.

$$\Delta A^p(k) = N^p \Omega(k)H^p \tag{3}$$

$$\Omega^p(k) \Theta^p(k) \leq I \tag{4}$$

Eq. (2a) is the match case and Eq. (2b) is the mismatch case.

It’s worth noting that when the actuator fails, the system output cannot effectively track the set point. In order to deal with this condition, a controller with a fault factor can be expressed as follows.

$$u^p(k) = a^p + \alpha^p \sigma^p(k) \tag{5}$$

$$0 \leq \alpha^p \leq a^p \leq \tilde{a}^p \tag{6}$$

where $u^p(k)$ represents the control input of the system, $u^p(k)$ represents the partial fault input of the actuator, $\tilde{a}^p \leq I^p$ and $\tilde{a}^p \geq I^p$ are known matrices.

$$\beta^p = \frac{\tilde{a}^p + a^p}{2}$$

$$\beta^p = \frac{\tilde{a}^p - a^p}{a^p + \tilde{a}^p} \tag{7}$$

From Eq. (6) and Eq. (7), the following equation is adapted.

$$\alpha^p = (I^p + \alpha^p) \beta^p \tag{8}$$

yet

$$|\alpha^p| \leq \beta^p \leq I^p \tag{9}$$

Further, when the switching occurs, the system state of the previous phase may mismatch the system state of the current phase. However, a suitable state-transit matrix can usually be found to describe the relationship between two phases states. The following equation is adapted.

$$x^p(S^p+1) = \chi^p x^p(S^p) \tag{10}$$

where $\chi^p$ is called the state transition matrix.

When the switching is occurred, a switching signal is expressed as follows.

$$p = \begin{cases} 
    p + 1, & \text{if } \gamma_{(x^p+1)}(x(k)) < 0 \\
    p, & \text{other}
\end{cases} \tag{11}$$

where $\gamma_{(x^p+1)}(x(k)) < 0$ represent a switching situation.

Meanwhile, the switching time $s^p$ of the system satisfies:

$$S^p = \min \{ k > S^{(p-1)} | \gamma_{(x^p)}(x(k)) < 0 \} \tag{12}$$

Initial value: $S^0 = 0$

In this paper, an asynchronous switching problem is considered. Therefore, $s^{\text{eq}}$ and $s^{\text{acc}}$ represent the switching time between match and mismatch case.

The time series of a batch is shown as follows.

$$\begin{align*}
S^0 &\in \{(S^{11}, \ell(S^{11})), (S^{22}, \ell(S^{22})), (S^{33}, \ell(S^{33})), (S^{44}, \ell(S^{44})) \}
\end{align*} \tag{13}$$

where $S^{pp}$ is considered. Therefore, $S^{\text{eq}} - S^{\text{acc}} \geq \tau^p_0$ , $S^{pp} - S^{pp-1} \leq \tau^p_0$ , in which $\tau^p_0$ is the Min-RT of the match case and $\tau^p_0$ is the Max-RT of the mismatch case.

**Remark 1:** The asynchronous switching condition is a type of system-dependent, state-controller mismatch, and it should be addressed to control the multi-phase batch processes properly because this problem is often inevitable in operating the processes with phase transitions. Therefore, the asynchronous problem can be interpreted as a type of state-controller mismatch as it takes place in the situation where the controller uses the states of the past phase while the real phase of the system has been changed. The state-controller mismatch is defined as mismatch case. Correspondingly, the state-controller match is defined as match case. The reason for the mismatch of the state-controller is that the switching signal cannot be sent to the controller in time, causing the actual switching point (time) of the controller to be inconsistent with the state switching point (time). From Figure 1, the state switching point is inconsistent with the controller switching point, which causes a mismatch case in the red area.

![Fig 1](image-url)

**Fig 1.** The asynchronous switching condition of each phase in a batch
2.2 Establishment of extended T-S fuzzy switching model

Eq. (2) is rewritten as incremental model, and the output tracking error is expanded at the same time. Then, a new extended T-S fuzzy model is obtained.

\[
x^e(k+1) = \sum_{i=1}^l h_i(Z(k))A^e_i(k)x^e(k) + \sum_{i=1}^l h_i(Z(k))B^e_i(k)u(k) + C^e_i\omega^e(k) + I^e_i\Delta^e(k) + \frac{L_i^e}{2}\Delta^e(k+1)
\]

where

\[
\begin{align*}
A^e_i(k) &= A^e_i(k) + \Delta^e(k) \\
A^e_i(k+1) &= A^e_i(k) + \Delta^e(k) \\
E^e_i(k) &= E^e_i(k) + \Delta^e(k)
\end{align*}
\]

In this extended T-S fuzzy model, the tracking control laws for the match and mismatch case are obtained as follows.

\[
e^e(k) = e^e(0) + C^e_0\sum_{i=1}^l h_i(Z(k))A^e_i(k)\Delta^e(k) + B^e_i\Delta u^e(k)
\]

The relationship between the two adjacent phases system states for the extended model satisfies the below equation.

\[
\begin{align*}
\Delta x^e(S^p-1) & = \lambda^{p-1}x(S^p-1) \\
e^e(S^p-1) & = \begin{bmatrix}
\lambda^{p-1}x^e(S^p-1) + x^e(S^p-1) - r^p \\
\lambda^{p-1}e^e(S^p-1) + e^e(S^p-1) - r^p
\end{bmatrix}
\end{align*}
\]

where \(\lambda^{p-1}\) is the scaling factor.

3. Design of fuzzy asynchronous switching controller

In this section, a fuzzy model predictive controller is designed. Firstly, the model (1) is used to predict the system state at the next moment, and an extended T-S fuzzy model combined with the predictive output tracking error which is established. Then, the controller designed by this model acts on the nonlinear multi-phase batch system. To clearly show the system’s running flow, the block diagram of the control flow is given.

3.1 Design of a fuzzy asynchronous switching control law

Depending on this extended T-S fuzzy switching model, the tracking control laws for the match and mismatch case are designed as follows.

\[
\Delta u^m(k) = K^m(x^m(k))\Delta x^m(k) + e^m(k)
\]

\[
\Delta u^m(k) = \sum_{i=1}^l h_i(Z(k))K^m_i\Delta x^m(k) + e^m(k)
\]

where \(K^m_i\) and \(K^m_{p+1}\) are the control law gains of the \(p\)th phase and the \(p+1\)th phase, respectively.

Because of Eq. (15) and \(y^{p+1} = 0\), Eq. (16b) is equivalent to Eq. (17) after being scaled.

\[
\Delta u^{p+1}(k) \leq K^m_{p+1}(\tilde{x}^{p+1})x^{p+1}(k)
\]

Substituting Eq. (16a) and (16b) into Eq. (14a) and (14b), the following closed-loop system state space models in the match and mismatch case are obtained:

\[
\begin{bmatrix}
\tilde{x}^r(k+1) \\
\Delta\tilde{x}^r(k+1)
\end{bmatrix} = \begin{bmatrix}
A^r(k) & B^r(k) \\
C^r & D^r
\end{bmatrix} \begin{bmatrix}
\tilde{x}^r(k) \\
\Delta\tilde{x}^r(k)
\end{bmatrix} + \begin{bmatrix}
E^r(k) \\
F^r(k)
\end{bmatrix}
\]

where \(\tilde{x}^r(k)\) is the system state.
$$\begin{align*}
\dot{x}(k+1) &= \sum_{i=1}^{n} \sum_{j=1}^{m} h_i(Z(k)) h_j(x(k)) \hat{A}^{ij}(k) \hat{x}(k) \\
&+ \hat{G}^{ij} \Delta \theta(k) + L_j^i \Delta \theta(k+1) \\
\Delta y(k) &= \hat{C}^{ij} \dot{x}(k) \\
z(k) &= e^*(k) = E^{ij} \hat{x}(k)
\end{align*}$$
(18)

where $\hat{A}^{ij}(k) = \hat{A}^{ij}(k) + \hat{B}^{ij} \Delta \theta(k) + \hat{K}^{ij} \Delta r(k)$

Remark 2: We consider two situations between the model rules and the controller rules. One is the one-to-one correspondence and another is cross correspondence, i.e., the case of $i=j$ and $i \neq j$. In Figure 3, when $i=j$, controller rules 1 corresponds model rules 1, controller rules 2 corresponds model rules 2 and so on. In Figure 4, when $i \neq j$, controller rules 1 corresponds model rules 2, controller rules 2, ..., or controller rules j. The corresponding model rules of the controller j rule are similar to the controller 1 rule.

![Figure 3. One-to-one correspondence (i=j)](image)

![Figure 4. Cross correspondence (i\neq j)](image)

### 3.2 Preparatory information

**Definition 1** [31] (Robust model predictive tracking control problem). For nonlinear MPBP system with uncertainty, the “min-max” optimization problem that minimizes the upper bound value of the objective function $J^*$ in the worst case is considered as follows:

$$\begin{align*}
\min_{\Delta \theta(k+1) \in S_0} \max_{\Delta \theta(k) \in S_0} J^*_p(k) &= \sum_{\tau=0}^{\infty} (\hat{x}(k+\tau) + \tau \hat{G}^{ij} \Delta \theta(k)) \hat{Q}^{ij} (\hat{x}(k+\tau) + \tau \hat{G}^{ij} \Delta \theta(k)) \\
&+ \Delta \theta(k+\tau) + \tau \hat{G}^{ij} \Delta \theta(k) R^{ij} \Delta \theta(k+\tau) \Delta \theta(k)
\end{align*}$$
(19)

where $Q^{ij} > 0$ is the tracking weight matrix and $R^{ij} > 0$ is the control weighting matrix. $x^*(k+\tau|k)$ and $u^*(k+\tau|k)$ represent predicted value of the system at the $k+\tau$ time.

**Definition 2** [32]. In MPBP, $k$ meets $k=0 \geq d = S_0^*$ for any discrete time. $N_0^*(d, O)$ represents the number of switching in a batch under match case. $T_r^*$ is the dwell time of the system during a single match case in a batch. In conclusion, $N_0^*(d, O) \leq N_0^* + S_0^*(d, O)/\tau_r^*$ can be obtained, where $N_0^*$ is chatter bound, $\tau_r^* > 0$.

Similarly, the system switching number in mismatch case meet $N_0^*(d, O) \leq N_0^* + S_0^*(d, O)/\tau_r^*$, where $N_0^*$ represents chatter bound, $\tau_r^* > 0$.

**Definition 3.** $\Delta \theta(k+1)$ represents the change of time-varying tracking trajectory. $\Delta \theta(k+1)$ that is treated as a bounded disturbance is affected only by actual production requirements. Therefore, we define an $H_\infty$ performance index for time-varying tracking trajectory, as $\|z^*\| < \sigma_r \|\Delta \theta(k+1)\|$. (20)

**Definition 4.** A quadratic function $V(x) = x^T P x$, $\theta > 0$ satisfies:

$$V(x(k+i+1|k)) = V(x(k+i|k)) \leq \sum_{i=0}^{n} \alpha_i V(x(k+i|k))$$

Under the condition of $V(x(\infty)) = 0$ or $x(\infty) = 0$ , from $i=0$ to $\infty$ on both sides of Eq. (19), it has

$$J(k) \leq V(x(k)|k) \leq \theta$$

where $\theta$ is the upper bound of $J(k)$.

**Lemma 1** [33]. $\zeta$ and $\zeta$ are given appropriate dimensions matrixes, where $\zeta$ and $\varphi$ are real matrixes, the below equation is obtained.

$$\begin{bmatrix}
-\zeta & \varphi^T \\
\varphi & -\zeta
\end{bmatrix} < 0$$

(22)

**Lemma 2** [34]. Suitable dimensions matrixes $D$, $F$, $E$ and $M$ are provided, and $M$ is symmetric. The following formula is available.

$$M + e^T D e + e F^T D^T < 0$$

where, for all matrices $F^T F \leq I$ meeting $F$ is hold, and there is only one constant $\varepsilon > 0$, the below equation is true:

$$M + e^T D D^T + \varepsilon E^T E < 0$$

(25)

**Lemma 3** [35]. To guarantee the asymptotic stability of the system at each phase, we can find a Lyapunov function $V'(x(k))$ satisfying the following conditions. Take the $p$th phase for example:

1) $V'(x(k)) \geq 0$ for $x(k) \in R^n$, and $x(k) \in R^n$.

$\leftrightarrow x(k);=0$.
2) $V^r(\chi(k))=0 \iff \|\chi(k)\|=0$.
3) For any bounded condition, $0<\Xi'<1$

\[ V^r(\chi(k+1)) \leq \Xi V^r(\chi(k)) \]

\[ (26) \]

**Lemma 5** [36]. Suppose a scalar $r^* > 0$ is true, an $H_u$ performance $\|z^r\| \leq r^* \|\tilde{z}\|$ can be obtained, which can guarantee the system is asymptotically stable under unknown disturbance $\tilde{w}(k)$.

### 3.3 Main Theorems

**Theorem 1.** For the given positive definite matrix $Q$, $Q^r$, $R^r$, $R^{r-1}$, the system (14) with $\Delta \theta^r(k+1)=0$ and $\tilde{w}=0$ is solvable, and if there are some scalars $0<\xi_r^*<1$, $\xi_r^* > 1$. $\theta^r>0$, $\theta^r-\xi_r^*$, a positive definite symmetric matrix $P_i^r \in \mathbb{R}^{n \times n^r}$, matrix $W_i^r \in \mathbb{R}^{n \times n_i^r}$, and unknown positive scalars $\varepsilon_i^r$, $\varepsilon_r^r$, $\theta^r>0$, $\mu_i^r > 1.0 < \mu_i^r < 1$ , such that the following LMI conditions hold.

\[ \begin{bmatrix}
I_i^r & I_i^r & I_i^r & I_i^r \\
-\mu_i^r I_i^r & 0 & 0 & 0 \\
-\mu_i^r I_i^r & 0 & 0 & 0 \\
-\mu_i^r I_i^r & 0 & 0 & 0 \\
-\mu_i^r I_i^r & 0 & 0 & 0
\end{bmatrix} < 0 \quad (27)
\]

\[ \begin{bmatrix}
I_i^r & I_i^r & I_i^r & I_i^r \\
-\mu_i^r I_i^r & 0 & 0 & 0 \\
-\mu_i^r I_i^r & 0 & 0 & 0 \\
-\mu_i^r I_i^r & 0 & 0 & 0 \\
-\mu_i^r I_i^r & 0 & 0 & 0
\end{bmatrix} < 0 \quad (28)
\]

\[ \begin{bmatrix}
I_i^r & I_i^r & I_i^r & I_i^r \\
-\mu_i^r I_i^r & 0 & 0 & 0 \\
-\mu_i^r I_i^r & 0 & 0 & 0 \\
-\mu_i^r I_i^r & 0 & 0 & 0 \\
-\mu_i^r I_i^r & 0 & 0 & 0
\end{bmatrix} < 0 \quad (29)
\]

\[ \begin{bmatrix}
I_i^r & I_i^r & I_i^r & I_i^r \\
-\mu_i^r I_i^r & 0 & 0 & 0 \\
-\mu_i^r I_i^r & 0 & 0 & 0 \\
-\mu_i^r I_i^r & 0 & 0 & 0 \\
-\mu_i^r I_i^r & 0 & 0 & 0
\end{bmatrix} < 0 \quad (30)
\]

\[ \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & -\mu_i^r I_i^r & 0 & 0 \\
0 & -\mu_i^r I_i^r & 0 & 0 \\
0 & -\mu_i^r I_i^r & 0 & 0 \\
0 & -\mu_i^r I_i^r & 0 & 0
\end{bmatrix} < 0 \quad (31)
\]

\[ \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & -\mu_i^r I_i^r & 0 & 0 \\
0 & -\mu_i^r I_i^r & 0 & 0 \\
0 & -\mu_i^r I_i^r & 0 & 0 \\
0 & -\mu_i^r I_i^r & 0 & 0
\end{bmatrix} < 0 \quad (32)
\]

where

\[ \begin{bmatrix}
P_i^r(Q_r^r) & 0 & 0 & 0 \\
0 & P_i^r(A_i^r + W_i^r B_i^r) & 0 & 0 \\
0 & 0 & P_i^r(H_i^r) & 0 \\
0 & 0 & 0 & P_i^r(H_i^r)
\end{bmatrix} < 0
\]

Proof 1. When $\Delta \theta^r(k+1)=0, \tilde{w}=0$ , in order to guarantee the MPBP system is asymptotically stable at each phase, the system state satisfies the below equation.

\[ V^r(\chi(k+1)|k) - V^r(\chi(k)|k) \leq -[(\chi^r(k+1)|k) Q^r\]

\[ (33) \]

According to Definition 1, under the condition of $\mathbb{V}^r(\infty)|0 = \mathbb{V}^r(\infty)|0 = 0$ from $t=0$ to $\infty$ on both sides of the Eq. (34), it has

\[ J^r(k) \leq \mathbb{V}_p^r(\chi(k)|k) \leq 0 \quad (35) \]
where $\theta^*$ is the upper bound of $J^*(k)$. Based on (34) and (35), the below equation is obtained:

$$ (\theta^*)^{-1} \Delta V(\bar{x}(k+1|k)) + (\theta^*)^{-1} \bar{J}_k \leq 0 \quad (36) $$

where $\bar{J}_k = \mathbb{E}[V'(k+1|k) \Delta \nu'(k+1|k)]^2$

Then, Eq. (36) is equivalent to

$$\bar{x}^T(k)[\sum_{i=1}^{n} b_i'^T \bar{A}^T(k) + \sum_{i=1}^{n} b_i'^T \bar{A}^T(k)] + \\
\left(\sum_{i=1}^{n} b_i'^T \bar{R}^T(k) \right) \bar{x}^T(k) + \bar{x}^T(k) \left(\sum_{i=1}^{n} b_i'^T \bar{R}^T(k) \right) \bar{x}(k)$$

According to Lemma 1, Eq. (38) can be obtained:

$$\bar{x}^T(k)[-\frac{1}{2} \sum_{i=1}^{n} b_i'^T \bar{A}^T(k) + \sum_{i=1}^{n} b_i'^T \bar{A}^T(k)] + \\
\left(\sum_{i=1}^{n} b_i'^T \bar{R}^T(k) \right) \bar{x}^T(k) + \bar{x}^T(k) \left(\sum_{i=1}^{n} b_i'^T \bar{R}^T(k) \right) \bar{x}(k)$$

When $i=j$, quoting Lemma 1 and Lemma 2, Eq. (39) can be obtained. When $i\neq j$, quoting Lemma 1 and Lemma 2, Eq. (40) can be obtained.
\[ V'(x_1(k)) \leq \alpha x^T \psi(x_1(T')) \]  

(45)

In summary, it can be ensured that the system based on condition (31) is exponentially stable.

To get the system invariant set, taking the maximum value of \( \xi(k) \), and setting \( \psi(x_1(k)) = \theta(x_1(k)) \), Eq. (32) can be got based on \( V(\xi(k)) \leq \bar{\xi}(k) \) if and only if \( \theta = \psi(x_1(k)) \) and Lemma 1.

To this end, Theorem 1 is proved completely.

Remark 3: In this paper, the asynchronous switching condition is undesirable. Therefore, obtaining the Max-RT of the former phase before the switching step, the strategy of the switching signal in advance is adopted to prevent asynchronous switching condition from occurring.

Theorem 2. For the given positive definite matrix \( Q, Q_{i-1}, R_i, R_i^{-1}, \) the system (14) with \( \Delta\theta(k+1) = 0 \) and \( \theta \neq 0 \) is solvable, and if there are some scalars \( 0 < \varepsilon, \varepsilon \mu > 1, \theta^* > 0, \theta^{\mu^*} > 0 \), a positive definite symmetric matrix \( \bar{P} \in R^{(n+c)^2} \), matrix \( W_p \in R^{(n+c)^2} \), and unknown positive scalars \( \varepsilon, \varepsilon \mu > 1, \theta^* > 0, \mu^* > 0 \), such that the following LMI conditions hold.

\[
\begin{bmatrix}
V_{11}^{\bar{P}} & V_{12}^{\bar{P}} & V_{13}^{\bar{P}} & V_{14}^{\bar{P}} \\
V_{21}^{\bar{P}} & V_{22}^{\bar{P}} & V_{23}^{\bar{P}} & V_{24}^{\bar{P}} \\
V_{31}^{\bar{P}} & V_{32}^{\bar{P}} & V_{33}^{\bar{P}} & V_{34}^{\bar{P}} \\
V_{41}^{\bar{P}} & V_{42}^{\bar{P}} & V_{43}^{\bar{P}} & V_{44}^{\bar{P}}
\end{bmatrix} < 0
\]  

(46)

\[
\begin{bmatrix}
V_{11}^{P} & V_{12}^{P} & V_{13}^{P} & V_{14}^{P} \\
V_{21}^{P} & V_{22}^{P} & V_{23}^{P} & V_{24}^{P} \\
V_{31}^{P} & V_{32}^{P} & V_{33}^{P} & V_{34}^{P} \\
V_{41}^{P} & V_{42}^{P} & V_{43}^{P} & V_{44}^{P}
\end{bmatrix} < 0
\]  

(47)

\[
\begin{bmatrix}
V_{11}^{W} & V_{12}^{W} & V_{13}^{W} & V_{14}^{W} \\
V_{21}^{W} & V_{22}^{W} & V_{23}^{W} & V_{24}^{W} \\
V_{31}^{W} & V_{32}^{W} & V_{33}^{W} & V_{34}^{W} \\
V_{41}^{W} & V_{42}^{W} & V_{43}^{W} & V_{44}^{W}
\end{bmatrix} < 0
\]  

(48)

\[
\begin{bmatrix}
V_{11}^{V} & V_{12}^{V} & V_{13}^{V} & V_{14}^{V} \\
V_{21}^{V} & V_{22}^{V} & V_{23}^{V} & V_{24}^{V} \\
V_{31}^{V} & V_{32}^{V} & V_{33}^{V} & V_{34}^{V} \\
V_{41}^{V} & V_{42}^{V} & V_{43}^{V} & V_{44}^{V}
\end{bmatrix} < 0
\]  

(49)

\[
\begin{bmatrix}
V'_{x}((\bar{\xi}(k)) \leq \mu_{kk}^{x}V_{x}^{(n+c)}((\bar{\xi}(k)) \\
V'_{x}(\bar{\xi}(k)) \leq \mu_{kk}^{V}V_{x}^{(n+c)}((\bar{\xi}(k)) \\
V'_{x}(\bar{\xi}(k)) \leq \mu_{kk}^{y}V_{x}^{(n+c)}((\bar{\xi}(k)) \\
V'_{x}(\bar{\xi}(k)) \leq \mu_{kk}^{y(\n+c)}((\bar{\xi}(k))
\end{bmatrix} \leq 0
\]  

(50)

where

\[
\begin{bmatrix}
V_{11}^{A} & V_{12}^{A} & V_{13}^{A} & V_{14}^{A} \\
V_{21}^{A} & V_{22}^{A} & V_{23}^{A} & V_{24}^{A} \\
V_{31}^{A} & V_{32}^{A} & V_{33}^{A} & V_{34}^{A} \\
V_{41}^{A} & V_{42}^{A} & V_{43}^{A} & V_{44}^{A}
\end{bmatrix} < 0
\]  

(51)

\[
\begin{bmatrix}
V_{11}^{A} & V_{12}^{A} & V_{13}^{A} & V_{14}^{A} \\
V_{21}^{A} & V_{22}^{A} & V_{23}^{A} & V_{24}^{A} \\
V_{31}^{A} & V_{32}^{A} & V_{33}^{A} & V_{34}^{A} \\
V_{41}^{A} & V_{42}^{A} & V_{43}^{A} & V_{44}^{A}
\end{bmatrix} < 0
\]  

(52)

\[
\begin{bmatrix}
V_{11}^{A} & V_{12}^{A} & V_{13}^{A} & V_{14}^{A} \\
V_{21}^{A} & V_{22}^{A} & V_{23}^{A} & V_{24}^{A} \\
V_{31}^{A} & V_{32}^{A} & V_{33}^{A} & V_{34}^{A} \\
V_{41}^{A} & V_{42}^{A} & V_{43}^{A} & V_{44}^{A}
\end{bmatrix} < 0
\]
disturbances. When \( \bar{\omega} \neq 0 \), in order to guarantee that the system is stable, the H-infinity performance index is introduced. For any \( \delta(k) \in \mathbb{I}[0, \infty) \) with non-zero, as follow

\[
J_2^* \leq \sum_{i=1}^{n} \left[ (z^+(k))^T z^+(k) - (r^-)^T \bar{\omega}^+(k) \bar{\omega}^+(k) \right] + (\theta^-)^T \Delta V^-(\bar{x}^-(k)) + (\theta^-)^T J_1^*(k)
\]

(53)

According to Eq. (53), the below expression is expressed:

\[
(\theta^-)^T \Delta V(\bar{x}^+(k+t\bar{I}) + J_2^*(k) \leq 0
\]

(54)

Then, Eq. (54) is equivalent to

\[
\phi^T \begin{bmatrix}
-\xi^-_p(P^+_p)^{-1} & 0 \\
0 & -r^-
\end{bmatrix}
\begin{bmatrix}
\tilde{A}_1^+(\theta^-)^{-1} + \tilde{A}_2^+(\theta^-)^{-1} + \tilde{A}_3^+(\theta^-)^{-1} + \tilde{A}_4^+ + E^T E
\end{bmatrix}
\]

(55)

\[\phi = [X^+(k), X^+(k)]
\]

then \( \tilde{A}_1 = [Q^p]_1^{0} \).

Further, the following equation can be obtained.

\[
\begin{bmatrix}
-\xi^-_p(P^+_p)^{-1} & 0 \\
0 & -r^-
\end{bmatrix}
\begin{bmatrix}
\tilde{A}_1^+(\theta^-)^{-1} + \tilde{A}_2^+(\theta^-)^{-1} + \tilde{A}_3^+(\theta^-)^{-1} + \tilde{A}_4^+ + E^T E
\end{bmatrix}
\]

(56)

Theorem 2 is similar as Theorem 1, so we won’t repeatedly proof it. Theorem 2 is proved.

**Theorem 3.** For the given positive definite matrix \( Q^p, Q^p, R^p, R^p \), system (14) with \( \Delta \theta^+(k+1) \neq 0 \) and \( \bar{\omega} \neq 0 \) is solvable, if there are some scalars \( 0 < \xi^-_p < 1, \xi^-_p > 1, \theta^- > 0, \theta^- > 0, \) a positive definite symmetric matrix \( P^p \in \mathbb{R}^{n \times n} \), matrix \( W^p \in \mathbb{R}^{n \times n} \), and unknown positive scalars \( e^+_p, \varepsilon^+_p, \theta^- > 0, \mu^+_p > 1,0 < \mu^+_p < 1 \), such that the following LMI conditions hold.

\[
\begin{bmatrix}
\Lambda^p_{11} & \Lambda^p_{12} & \Lambda^p_{13} & \Lambda^p_{14} & \Lambda^p_{15} \\
* & \Lambda^p_{22} & 0 & 0 & 0 \\
* & * & \Lambda^p_{33} & 0 & 0 \\
* & * & * & \Lambda^p_{44} & 0 \\
* & * & * & * & \Lambda^p_{55}
\end{bmatrix} < 0
\]

(57)

\[
\begin{bmatrix}
\Lambda^p_{11} & \Lambda^p_{12} & \Lambda^p_{13} & \Lambda^p_{14} & \Lambda^p_{15} \\
* & \Lambda^p_{22} & 0 & 0 & 0 \\
* & * & \Lambda^p_{33} & 0 & 0 \\
* & * & * & \Lambda^p_{44} & 0 \\
* & * & * & * & \Lambda^p_{55}
\end{bmatrix} < 0
\]

(58)

\[
\begin{bmatrix}
\Pi^p_{11} & \Pi^p_{12} & \Pi^p_{13} & \Pi^p_{14} & \Pi^p_{15} \\
* & \Pi^p_{22} & 0 & 0 & 0 \\
* & * & \Pi^p_{33} & 0 & 0 \\
* & * & * & \Pi^p_{44} & 0 \\
* & * & * & * & \Pi^p_{55}
\end{bmatrix} < 0
\]

(59)

\[
\begin{bmatrix}
\Pi^p_{11} & \Pi^p_{12} & \Pi^p_{13} & \Pi^p_{14} & \Pi^p_{15} \\
* & \Pi^p_{22} & 0 & 0 & 0 \\
* & * & \Pi^p_{33} & 0 & 0 \\
* & * & * & \Pi^p_{44} & 0 \\
* & * & * & * & \Pi^p_{55}
\end{bmatrix} < 0
\]

(60)

\[
\begin{bmatrix}
V^p_{1}(\bar{x}^+(k)) \leq \mu^+_p V^p_{2}(\bar{x}^+(k)) \\
V^p_{3}(\bar{x}^+(k)) \leq \mu^+_p V^p_{4}(\bar{x}^+(k)) \\
V^p_{5}(\bar{x}^+(k)) \leq \mu^+_p V^p_{6}(\bar{x}^+(k)) \\
-1 \bar{x}^+(k | k) - \theta^-
\end{bmatrix} \leq 0
\]

(61)

(62)
where $\Lambda_{ii} = \Lambda_{ii}^p = \begin{bmatrix} -\sigma^2 p_i^p & 0 & 0 & 0 \\ 0 & -I^{(p)} & 0 & 0 \\ 0 & 0 & -\sigma^2 I^{(p)} & 0 \\ 0 & 0 & 0 & -I^{(p)} \end{bmatrix}$.

The performance index of the equation is obtained.

Theorem 3 considering time-varying tracking trajectories is based on Theorem 2. The proof of Theorem 3 is as follows:

**Proof.** The proof of Theorem 3 considering time-varying tracking trajectories is based on Theorem 2. The performance index of $H_2$ on time-varying tracking trajectory is introduced. For any $\Delta H_\theta(k+1)$, $\delta_\theta(k) \in l_1[0, \infty]$ with nonzero, the below equation is obtained.

$$
\begin{align*}
\Pi_{ii}^p &= \begin{bmatrix} -P_i^{p+1} + e_i^p N_i^p N_i^p & e_i^p B_i^p B_i^p \\
0 & 0 \\
0 & -I^{(p)} \\
0 & 0 
\end{bmatrix}, \\
\Pi_{ii}^p &= \begin{bmatrix} -P_i^{p+1} + e_i^p N_i^p N_i^p & e_i^p B_i^p B_i^p \\
0 & 0 \\
0 & -I^{(p)} \\
0 & 0 
\end{bmatrix}, \\
\Pi_{ii}^p &= \begin{bmatrix} -P_i^{p+1} + e_i^p N_i^p N_i^p & e_i^p B_i^p B_i^p \\
0 & 0 \\
0 & -I^{(p)} \\
0 & 0 
\end{bmatrix}, \\
\Pi_{ii}^p &= \begin{bmatrix} -P_i^{p+1} + e_i^p N_i^p N_i^p & e_i^p B_i^p B_i^p \\
0 & 0 \\
0 & -I^{(p)} \\
0 & 0 
\end{bmatrix}, \\
\Pi_{ii}^p &= \begin{bmatrix} -P_i^{p+1} + e_i^p N_i^p N_i^p & e_i^p B_i^p B_i^p \\
0 & 0 \\
0 & -I^{(p)} \\
0 & 0 
\end{bmatrix},
\end{align*}
$$

The Min-RT and the Max-RT can be obtained as below.

$$
\begin{align*}
\begin{cases}
\tau^*_p \geq \frac{\ln \mu_p'}{\ln \sigma_p'}, \\
\tau^*_p \leq \frac{\ln \mu_p'}{\ln \sigma_p'},
\end{cases}
\end{align*}
$$

(63)
\[ J^* \leq \sum_{i=1}^{10} (c_i^*(k)^2) z^*(k) - (\alpha^i) \Delta z^*(k) + (\phi^i) \Delta \phi^i(k) \]

\[ + (\zeta^i(k)^2) z^*(k) - (\sigma^i) \Delta \phi^i(k) + (\varphi^i) \Delta \varphi^i(k) \]

Therefore, Eq. (64) is equivalent to

\[ (\varphi^i)^{\Delta V} + (\phi^i)^{\Delta \varphi^i} + J^*(k) \leq 0 \quad (65) \]

Then, according to Lemma 1, the following equation is obtained.

\[ \varphi^i = \begin{bmatrix} -\gamma (P^i)^{-1} & 0 & 0 \\ 0 & -\gamma & 0 \\ 0 & 0 & -\gamma \end{bmatrix} \]

\[ \Lambda^i = (\phi^i)^{-1} \Lambda_i + (\phi^i)^{-1} \Lambda_2 + (\phi^i)^{-1} \Lambda_3 + 2 \cdot E^i \cdot E \]

where

\[ \varphi^i = \begin{bmatrix} \Delta^i(k) \\ \Delta (k+1) \end{bmatrix}, \Lambda_i = \sum_{i=1}^{10} h_i(x(k))(R^{i*}(k)^{-1} K^{i*} 0 0) \]

\[ \Lambda_2 = [(Q^{i*})^j 0 0], \lambda_4 = [0 0 0] \]

Further, the below equation can be obtained.

\[ \varphi^i = \begin{bmatrix} -\gamma (P^i)^{-1} & 0 & 0 \\ 0 & -\gamma & 0 \\ 0 & 0 & -\gamma \end{bmatrix} \]

\[ \Lambda^i = (\phi^i)^{-1} \Lambda_i + (\phi^i)^{-1} \Lambda_2 + (\phi^i)^{-1} \Lambda_3 + 2 \cdot E^i \cdot E \]

\[ \varphi^i = \begin{bmatrix} -\gamma (P^i)^{-1} & 0 & 0 \\ 0 & -\gamma & 0 \\ 0 & 0 & -\gamma \end{bmatrix} \]

\[ \Lambda^i = (\phi^i)^{-1} \Lambda_i + (\phi^i)^{-1} \Lambda_2 \]

\[ \Lambda^i = (\phi^i)^{-1} \Lambda_i + (\phi^i)^{-1} \Lambda_2 + (\phi^i)^{-1} \Lambda_3 \]

\[ (P^i)^{-1} \sum_{i=1}^{10} h_i(x(k)) (K^{i*}(k) G^i \lambda^i) \]

\[ + 2 \cdot E^i \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]

So far, the proof of the whole theorem is proved completely.

4. Simulation case

4.1 Simulation object

This paper takes a typical MPBP the injection phase and the holding pressure phase of the injection molding process [10] as an example to simulate.

Taking into account the nonlinear characteristics and fault of the two phases, the following input and output models are given.

Injection phase (IP):

\[ IV(k+1) = 0.9291IV(k) + 8.687 \cdot VO(k) + 0.03191IV(k-1) \]

\[ -5.617IV(k-1)(0.004IV(k)+0.8) \]

\[ NP(k+1) = NP(k) + 0.1054(0.004+0.9)(IV(k))^2 \]

Pressure holding phase (HP):

\[ NP(k+1) = 1.317NP(k) + 171.8z \cdot VO(k) - 0.3259NP(k-1) \]

\[ + 156.8VO(k-1)(0.001NP(k)+0.6) \]

where IV is the injection speed of the system, VO is the valve opening of the system, NP represents nozzle pressure of the system, and \( 0 \leq IV(k) \leq 50 \), \( 200 \leq NP(k) \leq 400 \).

Through the fuzzy rules, the above process is established as a T-S fuzzy switching model:

(1) IP:

The premise variables of the IP are defined:

\[ Z^i_1(t) = 0.004x^i_1(k)+0.8, \quad 0 \leq IV(k) \leq 50 \]

\[ Z^i_2(t) = 0.004x^i_1(k)+1, \quad Z^i_1(t) = 0.004x^i_1(k)+1, \quad 0 \leq IV(k) \leq 50 \]

Then, the maximum and minimum ranges of \( Z^i_1(t) \) and \( Z^i_2(t) \) can be obtained, as shown in the following equation.

\[ \min Z^i_1(t) = 0.8, \quad \max Z^i_2(t) = 1.1, \]

\[ \min Z^i_2(t) = 1, \quad \max Z^i_1(t) = 1.2 \]

(70)
Then, the following fuzzy rules are established for the injection phase:

**Rule 1:** If \( Z_i(t) \in \text{NB} \) and \( Z_i(t) \in \text{NB} \), then
\[
x'(k+1) = A^{100}(k)x'(k) + B'aw'k(k) + \omega'(k)
\]

**Rule 2:** If \( Z_i(t) \in \text{NB} \) and \( Z_i(t) \in \text{ZE} \), then
\[
x'(k+1) = A^{102}(k)x'(k) + B'aw'k(k) + \omega'(k)
\]

**Rule 3:** If \( Z_i(t) \in \text{NB} \) and \( Z_i(t) \in \text{PO} \), then
\[
x'(k+1) = A^{103}(k)x'(k) + B'aw'k(k) + \omega'(k)
\]

**Rule 4:** If \( Z_i(t) \in \text{NS} \) and \( Z_i(t) \in \text{NE} \), then
\[
x'(k+1) = A^{104}(k)x'(k) + B'aw'k(k) + \omega'(k)
\]

**Rule 5:** If \( Z_i(t) \in \text{NS} \) and \( Z_i(t) \in \text{ZE} \), then
\[
x'(k+1) = A^{105}(k)x'(k) + B'aw'k(k) + \omega'(k)
\]

**Rule 6:** If \( Z_i(t) \in \text{NS} \) and \( Z_i(t) \in \text{PO} \), then
\[
x'(k+1) = A^{106}(k)x'(k) + B'aw'k(k) + \omega'(k)
\]

**Rule 7:** If \( Z_i(t) \in \text{PS} \) and \( Z_i(t) \in \text{NE} \), then
\[
x'(k+1) = A^{107}(k)x'(k) + B'aw'k(k) + \omega'(k)
\]

**Rule 8:** If \( Z_i(t) \in \text{PS} \) and \( Z_i(t) \in \text{ZE} \), then
\[
x'(k+1) = A^{108}(k)x'(k) + B'aw'k(k) + \omega'(k)
\]

**Rule 9:** If \( Z_i(t) \in \text{PS} \) and \( Z_i(t) \in \text{PO} \), then
\[
x'(k+1) = A^{109}(k)x'(k) + B'aw'k(k) + \omega'(k)
\]

**Rule 10:** If \( Z_i(t) \in \text{PB} \) and \( Z_i(t) \in \text{NE} \), then
\[
x'(k+1) = A^{110}(k)x'(k) + B'aw'k(k) + \omega'(k)
\]

**Rule 11:** If \( Z_i(t) \in \text{PB} \) and \( Z_i(t) \in \text{ZE} \), then
\[
x'(k+1) = A^{111}(k)x'(k) + B'aw'k(k) + \omega'(k)
\]

**Rule 12:** If \( Z_i(t) \in \text{PB} \) and \( Z_i(t) \in \text{PO} \), then
\[
x'(k+1) = A^{112}(k)x'(k) + B'aw'k(k) + \omega'(k)
\]

The triangle-shape grade of membership function of \( Z_i(t) \) and \( Z_i(t) \) are shown in Figure 1 and Figure 2, respectively.

Therefore, after defuzzification in the injection phase, it is transformed into the linear model:

\[
x'(k+1) = \sum_{i=1}^{12} h_i(Z(k))A_{ii}(k)x'(k) + \sum_{i=1}^{12} h_i(Z(k))B'aw'k(k) + \omega'(k)
\]

where \( A_{ii}(k) = A^{100}(k), A_{ii}(k) = A^{112}(k), i = 1, \ldots, 12 \).

\[
\begin{align*}
A_{101} &= \begin{bmatrix} 0.9291 & 0.8 & 0 \\ 0.03191 & 0 & 0 \end{bmatrix}, & A_{102} &= \begin{bmatrix} 0.9291 & 0.8 & 0 \\ 0.03191 & 0 & 0 \end{bmatrix}, \\
A_{103} &= \begin{bmatrix} 0.9291 & 0.8 & 0 \\ 0.03191 & 0 & 0 \end{bmatrix}, & A_{104} &= \begin{bmatrix} 0.9291 & 0.8 & 0 \\ 0.03191 & 0 & 0 \end{bmatrix}, \\
A_{105} &= \begin{bmatrix} 0.9291 & 0.8 & 0 \\ 0.03191 & 0 & 0 \end{bmatrix}, & A_{106} &= \begin{bmatrix} 0.9291 & 0.8 & 0 \\ 0.03191 & 0 & 0 \end{bmatrix}, \\
A_{107} &= \begin{bmatrix} 0.9291 & 0.8 & 0 \\ 0.03191 & 0 & 0 \end{bmatrix}, & A_{108} &= \begin{bmatrix} 0.9291 & 0.8 & 0 \\ 0.03191 & 0 & 0 \end{bmatrix}, \\
A_{109} &= \begin{bmatrix} 0.9291 & 0.8 & 0 \\ 0.03191 & 0 & 0 \end{bmatrix}, & A_{110} &= \begin{bmatrix} 0.9291 & 0.8 & 0 \\ 0.03191 & 0 & 0 \end{bmatrix}, \\
A_{111} &= \begin{bmatrix} 0.9291 & 0.8 & 0 \\ 0.03191 & 0 & 0 \end{bmatrix}, & A_{112} &= \begin{bmatrix} 0.9291 & 0.8 & 0 \\ 0.03191 & 0 & 0 \end{bmatrix}.
\end{align*}
\]

(2) HP:

The premise variables of the injection phase are defined: \( Z_i(t) = 0.001x'(k) + 0.6, Z_i(t) = 0.001x'(k) + 0.6 \).

Then, the maximum and minimum ranges of \( Z_i(t) \) and \( Z_i(t) \) can be obtained, as shown in the following equation.

\[
\begin{align*}
\min Z_i(t) & = 0.8, \max Z_i(t) = 1, \\
\min Z_i(t) & = 0.8, \max Z_i(t) = 1
\end{align*}
\]
Then, the following fuzzy rules are established for the injection phase:

Rule 1: If $Z_i^1(k) \in \text{NE}$ and $Z_i^2(k) \in \text{NE}$, then $\alpha(k+1) = \alpha(k) + B^2 \alpha(k) + \omega(k)$.

Rule 2: If $Z_i^1(k) \in \text{NE}$ and $Z_i^2(k) \in \text{ZE}$, then $\alpha(k+1) = \alpha(k) + B^2 \alpha(k) + \omega(k)$.

Rule 3: If $Z_i^1(k) \in \text{NE}$ and $Z_i^2(k) \in \text{PO}$, then $\alpha(k+1) = \alpha(k) + B^2 \alpha(k) + \omega(k)$.

Rule 4: If $Z_i^1(k) \in \text{ZE}$ and $Z_i^2(k) \in \text{NE}$, then $\alpha(k+1) = \alpha(k) + B^2 \alpha(k) + \omega(k)$.

Rule 5: If $Z_i^1(k) \in \text{ZE}$ and $Z_i^2(k) \in \text{ZE}$, then $\alpha(k+1) = \alpha(k) + B^2 \alpha(k) + \omega(k)$.

Rule 6: If $Z_i^1(k) \in \text{ZE}$ and $Z_i^2(k) \in \text{PO}$, then $\alpha(k+1) = \alpha(k) + B^2 \alpha(k) + \omega(k)$.

Rule 7: If $Z_i^1(k) \in \text{PO}$ and $Z_i^2(k) \in \text{NE}$, then $\alpha(k+1) = \alpha(k) + B^2 \alpha(k) + \omega(k)$.

Rule 8: If $Z_i^1(k) \in \text{PO}$ and $Z_i^2(k) \in \text{ZE}$, then $\alpha(k+1) = \alpha(k) + B^2 \alpha(k) + \omega(k)$.

Rule 9: If $Z_i^1(k) \in \text{PO}$ and $Z_i^2(k) \in \text{PO}$, then $\alpha(k+1) = \alpha(k) + B^2 \alpha(k) + \omega(k)$.

Therefore, after defuzzification in the pressure holding phase, it is transformed into the linear model:

$$\begin{align*}
x(k+1) = & \sum_{i=1}^{n} h_i(Z(k)) A_i^2(k) x(k) + \\
& + \sum_{i=1}^{n} h_i(Z(k)) B_i^2 \alpha(k) + \omega(k)
\end{align*}
$$

The gain of control law after stabilization in the HP is

$$\begin{align*}
x^2(k+1) = & \sum_{i=1}^{n} h_i(Z(k)) A_i^2(k) x(k) + \\
& + \sum_{i=1}^{n} h_i(Z(k)) B_i^2 \alpha(k) + \omega(k)
\end{align*}
$$

where $A_i^2(k) = A_i^2 + \Lambda_i^2(k), \Lambda_i^2(k) = \Lambda_i^2, i = 1, 2, 3$,

$$
A_i^2 = \begin{bmatrix} 1.317 & 1 \\ -0.2607 & 0 \end{bmatrix}, B_i^2 = \begin{bmatrix} 1.317 & 1 \\ -0.293 & 0 \end{bmatrix},
A_i^2 = \begin{bmatrix} 1.317 & 1 \\ -0.3259 & 0 \end{bmatrix},
B_i^2 = \begin{bmatrix} 1.718 & 1 \\ -141.12 & 0 \end{bmatrix},
A_i^2 = \begin{bmatrix} 1.718 & 1 \\ -156.8 & 0 \end{bmatrix},
N^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},
\Delta A^2(k) = \Delta \Omega^2(k) H^2, \Omega^2(k) \Omega^2(k) \leq I,
H^2 = \begin{bmatrix} 0.0104 & 0 \\ -0.0304 & 0 \end{bmatrix}, \omega^2(k) = 0.1 \times \begin{bmatrix} \Omega^2(k) \\ \Omega^2(k) \end{bmatrix},
\Omega^2(k), \Omega^2(k), \Omega^2(k) \text{ are random number in [-1 1], a \in [0.9 1.1].}
$$

The tracking performance index is defined as follows.

$$D(k) = \begin{cases} \sqrt{e^2(k)} e(k) & 0 \leq k < T_i \\ \sqrt{e^2(k)} e(k) & T_i \leq k \leq T_i + T_2 \end{cases}
$$

4.2 Results

To effectively reflect the advantages of the proposed method, Reference [19] and the Reference [20] are used as comparison with Theorem 3, where Reference [19] is a synchronous switching control method, and Reference [20] is an asynchronous switching linear ILC method.

After a lot of repeated testing, the control parameters with excellent control effect are found by trial-and-error method. The parameters in the IP and the HP are chosen as $Q^2 = \text{diag}[5,3,1.5,1], R_i^2 = 0.1, Q_i^2 = \text{diag}[6,2.5,1], R_i^2 = 0.1$. The Max-RT under match case in the IP and the HP is $T^2 = 86s$ and $T^2 = 113s$, respectively. The Min-RT under mismatch case in the HP is $T^2 = 38s$. The gain of control law after stabilization in the IP is $K_i^2 = [-0.00874 -0.0952 0 -0.0476]$. The gain of control law after stabilization in the HP is $K_i^2 = [-0.00864 -0.0066 -0.0032]$.

Figure 9, Figure 10 and Figure 11 are the output tracking results for nonlinear MPBP with asynchronous switching, time-varying tracking trajectories and actuator fault. The range of fault is $\alpha^T \in [0.9 1.1]$.

Figure 9 shows the output response of the synchronous switching control method [19]. It can be seen from the Figure 9 that the system state appears “Escape” conditions during the switching process, and the control mode of synchronous switching cannot guarantee that the system is stable. Moreover, the impact of the fault on each phase is also very significant. Especially, the superimposed effect of asynchronous switching condition and fault make the “Escape” conditions worse.

Figure 10 is the system output response of the
Fig 9. Output response of synchronous switching method [19]

Fig 10. Output response of two-dimensional linear ILC control method [20]
two-dimensional linear ILC control method [20] in the 5th, 20th and 40th batches. From the Figure 10, the 2D ILC method requires system to run many batches in order to make it stable track the set point. When the products of different specifications are replaced, this control method requires the leaning of many production batches in order to produce qualified products, which causes unnecessary waste and reduces the economic benefits of the enterprise. Notably, when a fault occurs, the ILC method requires more batches running for learning to track the set point stably. In the face of individualized demands and MPBP of small batch production, the two-dimensional ILC method has great limitations.

Figure 11 shows the output response of the proposed control method. Since the linear asynchronous switching control method is aimed at the linear system, the linear model used cannot completely match the nonlinear system. In the injection molding processes, the model mismatch will cause a large overshoot in the system during the injection phase, which in turn makes the injected raw materials unevenly distributed, and ultimately reduces the quality of the product. This phenomenon can be easily found in Figure 9 and Figure 10. However, the proposed method shown in Figure 11 can not only stably track the time-varying tracking trajectories, but also avoid the overshoot caused by the model mismatch after the linearization of the nonlinear system. At the same time, the stable operation of the system can also be guaranteed when the switching is occurred.

In addition, the fault factor is introduced into the designed controller. As shown in Figure 11, although the output response of the proposed control method fluctuates, it can still track the set point. And this controller can ensure the effective operation of the system.

Fig. 12 Control input of the 40th batch of two-dimensional ILC controller [20]
frequent movement of the actuator in a short time can be avoided, the maintenance cost of the production equipment can be reduced and the production efficiency of the enterprise can be improved.

Figure 14 and Figure 15 show the tracking performance of ILC method and the proposed method, respectively. The average error of the control method proposed is 1.0016. The average error of the linear control method is 1.2211. Therefore, the control method of this paper has more excellent control performance.

5. Conclusion
Aiming at the nonlinear MPBP with time-varying tracking trajectories and actuator faults, a robust fuzzy predictive fault-tolerant tracking control method is developed. This method considers the MPBP with nonlinear characteristics and effectively reduces the adverse effect of model mismatching on the system control effect caused by model linearization. At the same time, by solving the system stability conditions given in the form of LMI, the Min-RT under match case and the Max-RT under mismatch case are obtained. Before the switching conditions are met, the strategy of the switching signal in advance is adopted according to the calculated Max-RT to guarantee that the previous phase state and the current phase controller are matched, thereby ensuring the stable operation of the system. Besides, the simulation results also show that the proposed method can make system is stable when the switching is occurred and effectively suppress the impact of state "Escape" on production equipment and product quality. In practical application, it can effectively reduce the consumption of raw materials and prolong the life cycle of production equipment, which is of great significance to the nonlinear injection molding processes.

Time-delay is a common problem in many industrial production processes. To this end, the time-varying time-delay problem will be considered in future studies based on the current result.

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Conflicts of interest statement
The authors have no relevant financial or non-financial interests to disclose.

Data availability statements
The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

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