PQCD FACTORIZATION OF TWO-BODY $B$ DECAYS

Hsiang-nan Li
Institute of Physics, Academia Sinica, Taipei, Taiwan 115, ROC
Department of Physics, National Cheng-Kung University,
Tainan, Taiwan 701, ROC

abstract

I review the known approaches to two-body nonleptonic $B$ meson decays, including factorization assumption, modified factorization assumption, QCD factorization, and perturbative QCD factorization. Important phenomenological aspects of these approaches are emphasized.

1 Introduction

Two-body nonleptonic $B$ meson decays are a challenging subject for both theorists and experimentalists. These modes are complicated because of nonperturbative QCD dynamics, and important, because measurements of their CP violation reveal the information of the unitarity angles. In this talk I will review the known theoretical approaches to two-body nonleptonic $B$ meson decays, which include factorization assumption (FA), modified factorization assumption (MFA), QCD factorization (QCDF), and perturbative QCD (PQCD) factorization.

2 Factorization Assumption

The conventional approach to two-body nonleptonic $B$ meson decays is based on FA \cite{1}, in which nonfactorizable and annihilation contributions are neglected and final-state-interaction effects are assumed to be absent \cite{2}. Factorizable contributions are expressed as products of Wilson coefficients, meson decay constants, and hadronic form factors, which are then parametrized by models. For example, the amplitude of the decay $B \rightarrow \pi\pi$ can be written
as

\[ A(B \rightarrow \pi\pi) = C(\mu) \langle \pi\pi | O(\mu) | B \rangle , \]

\[ \approx C(\mu) f_{\pi} \langle \pi | (\bar{q}b)_{V-A} | B \rangle , \]  

(1)

where \( C \) is the Wilson coefficient, \( O \) the four-fermion operator, \( f_{\pi} \) the decay constant, \( (\bar{q}b)_{V-A} \) the \( V-A \) weak current, \( \mu \) the renormalization scale, and the matrix element in the second line the \( B \rightarrow \pi \) transition form factor \( F_{B\pi} \). Under the above approximation, FA is simple and provides qualitative estimation of branching ratios of various two-body nonleptonic \( B \) meson decays.

However, there exist several serious theoretical drawbacks in FA. First, FA breaks the scale independence of decay amplitudes, which are physical quantities. Meson decay constants and form factors, being measurable, are scale-independent, while Wilson coefficients are scale-dependent as indicated in Eq. (1). Hence, decay amplitudes, expressed as their products, become scale-dependent.

Second, nonfactorizable amplitudes are not always negligible. It has been known that the decay modes, whose factorizable contributions arise from the internal \( W \)-emission with the small Wilson coefficient \( a_2 = C_1 + C_2 / N_c \), are dominated by nonfactorizable contributions. The ratio of the \( B_d^0 \rightarrow D^{+} \pi^- \) and \( B^+ \rightarrow D^0 \pi^+ \) branching ratios and the \( B \rightarrow J/\psi K^{(*)} \) decays are the examples [3, 4].

Third, the evaluation of strong phases is ambiguous in FA. Strong phases are crucial, since they are related to CP asymmetries \( A_{CP} \) in two-body nonleptonic \( B \) meson decays:

\[ A_{CP} \propto \sin \delta \sin \phi , \]  

(2)

where \( \delta \) represents the strong phase. To extract the unitarity angle \( \phi \) from the data of \( A_{CP} \), \( \delta \) must be determined unambiguously. In FA strong phases arise from the Bander-Silverman-Soni (BSS) mechanism (the charm loop) [5], which are proportional to

\[ \int d\bar{u}u(1-u)\theta(u(1-u)q^2-m_c^2) , \]  

(3)

\( q \) being the external momentum flowing through the charm loop and \( m_c \) the charm quark mass. Since \( q^2 \) is unknown, the above integral has large uncertainty.
3 Modified Factorization Assumption

To improve the theoretical approach to two-body nonleptonic $B$ meson decays, FA has been modified. It has been proposed to extract the $\mu$ dependence of the matrix element $\langle \pi\pi|O(\mu)|B\rangle$ before applying FA \[3, 4\]. The procedure is performed as follows:

$$A(B \to \pi\pi) = C(\mu)g(\mu)\langle \pi\pi|O|B\rangle_{\text{tree}},$$

$$\approx C_{\text{eff}}f_\pi\langle \pi|\bar{q}bV_{-A}|B\rangle,$$  \(4\)

where $g(\mu)$ represents the $\mu$ dependence of the matrix element and the effective Wilson coefficient $C_{\text{eff}} \equiv C(\mu)g(\mu)$ is scale-independent. The extraction of $g(\mu)$ involves the one-loop corrections to the four-quark vertex, which are, however, infrared divergent. If the infrared divergences are regulated by considering external quarks off-shell by $-p^2$, the decay amplitude becomes gauge-dependent. That is, $C_{\text{eff}}$ in fact depends on the infrared cutoff $-p^2$ and a gauge parameter $\lambda$. Therefore, the problem of the scale dependence in FA is not really solved in MFA, but just replaced by the one of gauge dependence.

In MFA nonfactorizable contributions are included via the parameter so-called effective color number $N_{c\text{eff}}$. For example, $a_2$ associated with the internal $W$-emission is written as

$$a_2 = C_1 + \frac{C_2}{N_{c\text{eff}}}.$$  \(5\)

By varying $N_{c\text{eff}}$, one can obtain better fit to the data of two-body nonleptonic $B$ meson decays. However, $N_{c\text{eff}}$ is obviously process-dependent, such that the predictive power of MFA is weak. The above prescription also implies that nonfactorizable contributions are real, an assumption which is certainly not general enough in the viewpoint of parametrization. In MFA strong phases still come from the BSS mechanism, which are ambiguous as explained in the previous section.

4 QCD Factorization

Recently, Beneke et al. proposed the QCDF approach to two-body nonleptonic $B$ meson decays \[8\], in which the above drawbacks of FA and MFA
can be resolved. The infrared divergences in the loop corrections to the four-fermion vertices are absorbed into the transition form factors, which are not calculable in perturbation theory. The external quarks then remain on-shell, and the problem of the scale dependence is resolved without breaking the gauge invariance. Hence, factorizable contribution in QCDF are treated in the same way as in MFA [see eq. (4)] but with different \( C_{\text{eff}} \).

Nonfactorizable contributions are calculated perturbatively in the heavy quark limit. In the \( B \to \pi \pi \) decays these contributions are written as the convolutions of hard amplitudes with meson distribution amplitudes \( \phi \) in momentum fractions of valence quarks,

\[
F^{B\pi} \otimes H^{(4)} \otimes \phi_{\pi 2} ,
\Phi^B \otimes H^{(6)} \otimes \phi_{\pi 1} \otimes \phi_{\pi 2} ,
\]

where \( H^{(4)} \) (\( H^{(6)} \)) represents a four-quark (six-quark) amplitude. The former collects the infrared finite piece of the corrections to the four-fermion vertices, and the latter corresponds to the pair of nonfactorizable diagrams with a hard gluon emitted from the spectator quark. However, in some cases the hard amplitudes contain end-point singularities, which are not smeared by the meson distribution amplitudes. To regulate these end-point singularities, cutoffs of the momentum fractions need to be introduced, which are complex in general parametrization \([9]\).

Annihilation diagrams have been neglected in FA. In QCDF these amplitudes are calculated in a similar way to the nonfactorizable ones. The end-point singularities still exist, and complex cutoffs must be introduced. These complex cutoffs could bring in large strong phases and large CP asymmetries in two-body nonleptonic \( B \) meson decays, since they are basically free parameters.

The BSS mechanism also contributes to the strong phases. In QCDF, because of the introduction of meson distribution amplitudes, the external momentum flowing through the charm loop can be defined rigorously. Let the quark going into the pion emitted from the weak vertex carry the momentum fraction \( x_2 \). The quark going into the pion involved in the \( B \to \pi \) transition carries the momentum fraction \( x_3 \sim 1 \), since the transition form factor is assumed to be dominated by soft dynamics. The invariant mass \( q^2 \) appearing in Eq. (3) is then expressed as \( q^2 = x_2 M_B^2 \) unambiguously, \( M_B \) being the \( B \) meson mass.
The problems of FA and MFA are resolved in a different way in the PQCD approach. The infrared divergences in the vertex corrections are treated in the presence of the spectator quark \[10\]. Therefore, the leading-twist $B$ meson (pion) wave function can be defined, which absorbs the two-particle reducible infrared divergences on the $B$ meson (pion) side. The two-particle irreducible infrared divergences cancel between the pair of diagrams, for example, with the gluon emitted from the $b$ quark attaching the light quark and the spectator quark, which form the outgoing pion in the $B \rightarrow \pi$ transition. In this treatment the external quarks also remain on-shell, and the problem of the scale dependence is resolved without breaking gauge invariance.

In the PQCD picture the hard amplitudes for various topologies of diagrams, including factorizable, nonfactorizable and annihilation, are all six-quark amplitudes \[3,4,11\]. That is, the decay amplitudes are written as the convolutions in Eq. \[7\]. In PQCD the end-point singularities do not exist because of the inclusion of Sudakov effects \[12,13\], and the arbitrary cutoffs in QCDF are not necessary. Therefore, factorizable, nonfactorizable and annihilation amplitudes can be estimated in a more consistent way in PQCD than in QCDF. In QCDF factorizable contributions involve only four-quark amplitudes. As explained later, this difference will lead to different characteristic scales and different power counting rules in $1/m_b$, $m_b$ being the $b$ quark mass, for two-body nonleptonic $B$ meson decays in QCDF and in PQCD.

In PQCD strong phases mainly arise from the annihilation amplitudes, which are almost imaginary \[14,15,16\]. The detailed reason is referred to \[17\]. The strong phases are large, since they appear at the same order as the factorizable amplitudes. The BSS mechanism also contributes to the strong phases. In terms of the notation in the previous section, the invariant mass $q^2$ appearing in Eq. \[8\] is expressed as $q^2 = x_2 x_3 M_B^2$ unambiguously. However, compared to the annihilation contributions, the BSS mechanism is of next-to-leading order, and less important.
6 Sudakov Effects

If calculating the $B \to \pi$ form factor $F_{B\pi}$ at large recoil using the Brodsky-Lepage formalism [18, 19], a difficulty immediately occurs. The lowest-order diagram for the hard amplitude is proportional to $1/(x_1 x_3^2)$, $x_1$ being the momentum fraction associated with the spectator quark on the $B$ meson side. If the pion distribution amplitude vanishes like $x_3$ as $x_3 \to 0$ (in the leading-twist, i.e., twist-2 case), $F_{B\pi}$ is logarithmically divergent. If the pion distribution amplitude is a constant as $x_3 \to 0$ (in the next-to-leading-twist, i.e., twist-3 case), $F_{B\pi}$ even becomes linearly divergent. These end-point singularities have also appeared in the evaluation of the nonfactorizable and annihilation amplitudes in QCDF mentioned above.

In PQCD calculations small parton transverse momenta $k_T$ are included [12, 20], which smear the end-point singularities from small momentum fractions. Because of the inclusion of parton transverse momenta, double logarithms $\ln^2(Pb)$ are generated from the overlap of collinear and soft enhancements in radiative corrections to meson wave functions, where $P$ denotes the dominant light-cone component of a meson momentum, and $b$ is the variable conjugate to $k_T$. The resummation [21, 22] of these double logarithms leads to a Sudakov form factor $\exp[-s(P, b)]$, which suppresses the long-distance contributions from the large $b$ region with $b \sim 1/\Lambda$, $\Lambda \equiv M_B - m_b$ representing a soft scale. This suppression renders $k_T^2$ flowing into the hard amplitudes of order

$$k_T^2 \sim O(\Lambda M_B).$$

(8)

The off-shellness of internal particles then remain of $O(\Lambda M_B)$ even in the end-point region, and the singularities are removed. This mechanism is so-called Sudakov suppression.

Du et al. have studied the Sudakov effects in the evaluation of nonfactorizable amplitudes [23]. If equating these amplitudes with Sudakov suppression included to the parametrization in QCDF, it was observed that the corresponding cutoffs are located in the reasonable range proposed by Beneke et al. [4]. Sachrajda et al. have expressed an opposite opinion on the effect of Sudakov suppression in [24]. However, their conclusion was drawn based on a very sharp $B$ meson wave function, which is not favored by experimental data.
It is easy to understand the increase of $k_T^2$ from $O(\Lambda^2)$, carried by the valence quarks which just come out of the initial meson wave functions, to $O(\Lambda M_B)$, carried by the quarks which are involved in the hard weak decays. Consider the simple deeply inelastic scattering of a hadron. The transverse momentum $k_T$ carried by a parton, which just come out of the hadron distribution function, is initially small. After infinite many gluon radiations, $k_T$ becomes of $O(Q)$, when the parton is scattered by the highly virtual photon, where $Q$ is the large momentum transfer from the photon. The evolution of the hadron distribution function from the low scale to $Q$ is described by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation [25]. The mechanism of the DGLAP evolution in DIS is similar to that of the Sudakov evolution in exclusive $B$ meson decays. The difference is only that the former is the consequence of the single-logarithm resummation, while the latter is the consequence of the double-logarithm resummation.

Another support for considering $k_T$ in the analyses of exclusive $B$ meson decays can be found in [26]. It has been shown that the $k_T$ dependence appears as $\alpha_s \ln(1 + k_T/k^+)$ in radiative corrections, where $k^+$ ($k_T$) is the longitudinal (transverse) component of the light spectator quark momentum. Obviously, the $k_T$ dependence does not go away in the heavy quark limit, since both $k_T$ and $k^+$ are of $O(\Lambda)$.

7 Power counting

The power behaviors of various topologies of diagrams for two-body nonleptonic $B$ meson decays with the Sudakov effects taken into account has been discussed in details in [17]. The relative importance is summarized below:

$$\text{emission : annihilation : nonfactorizable} = 1 : \frac{2m_0}{M_B} : \frac{\bar{\Lambda}}{M_B},$$  \hspace{1cm} (9)

with $m_0$ being the chiral symmetry breaking scale. The scale $m_0$ appears because the annihilation contributions are dominated by those from the $(V-A)(V+A)$ penguin operators, which survive under helicity suppression. In the heavy quark limit the annihilation and nonfactorizable amplitudes are indeed power-suppressed compared to the factorizable emission ones. Therefore, the PQCD formalism for two-body charmless nonleptonic $B$ meson decays
coincides with the factorization approach as $M_B \to \infty$. However, for the physical value $M_B \sim 5$ GeV, the annihilation contributions are essential.

Note that all the above topologies are of the same order in $\alpha_s$ in PQCD. The nonfactorizable amplitudes are down by a power of $1/m_b$, because of the cancellation between a pair of nonfactorizable diagrams, though each of them is of the same power as the factorizable one. I emphasize that it is more appropriate to include the nonfactorizable contributions in a complete formalism. As stated in Sec. 2, the factorizable internal-$W$ emission contributions are strongly suppressed by the vanishing Wilson coefficient $a_2$ in the $B \to J/\psi K^{(*)}$ decays [4], so that nonfactorizable contributions become dominant. In the $B \to D\pi$ decays, there is no soft cancellation between a pair of nonfactorizable diagrams, and nonfactorizable contributions are significant [4].

In QCDF the factorizable and nonfactorizable amplitudes are of the same power in $1/m_b$, but the latter is of next-to-leading order in $\alpha_s$ compared to the former. Hence, QCDF approaches FA in the heavy quark limit in the sense of $\alpha_s \to 0$. Briefly speaking, QCDF and PQCD have different counting rules both in $\alpha_s$ and in $1/m_b$. The former approaches FA logarithmically ($\alpha_s \propto 1/\ln m_b \to 0$), while the latter does linearly ($1/m_b \to 0$).

8 Penguin Enhancement

The leading factorizable contributions involve four-quark hard amplitudes in QCDF, but six-quark hard amplitudes in PQCD. This distinction also implies different characteristic scales in the two approaches: the former is characterized by $m_b$, while the latter is characterized by the virtuality of internal particles of order $\sqrt{\Lambda M_B} \sim 1.5$ GeV [14, 15, 16]. A six-quark hard amplitude must contain a hard gluon exchanged between the spectator quark and other quarks. The spectator quark in the $B$ meson, forming a soft cloud around the heavy $b$ quark, carries momentum of order $\Lambda$. The spectator quark on the pion side carries momentum of $O(M_B)$ in order to form the fast-moving pion with the light quark produced in the $b$ quark decay. Based on this reasoning, the hard gluon is off-shell by $O(\Lambda M_B)$. As explored in [27], this scale, characterizing heavy-to-light decays, is important for constructing a gauge-invariant $B$ meson wave function. The path-ordered exponential in the definition of the $B$ meson wave function will appear, only if the hard
scale is of $O(\Lambda M_B)$.

It has been known that to accommodate the $B \to K\pi$ and $\pi\pi$ data, penguin contributions must be large enough. In FA, MFA and QCDF one relies on chiral enhancement by increasing the mass $m_0$ to a large value $m_0 \sim 3-4$ GeV \cite{28}. Because of the renormalization-group evolution effect of the Wilson coefficients associated with the QCD penguin operators, the lower hard scale leads to dynamical penguin enhancement in PQCD. Whether dynamical enhancement or chiral enhancement is responsible for the large $B \to K\pi$ branching ratios can be tested by measuring the $B \to \phi K$ modes \cite{17,29}. In these modes penguin contributions dominate, such that their branching ratios are insensitive to the variation of the unitarity angle $\phi_3$. Because the $\phi$ meson is a vector meson, the mass $m_0$ is replaced by the physical mass $M_\phi \sim 1$ GeV, and chiral enhancement does not exist. If the branching ratios of the $B \to \phi K$ decays are around $4 \times 10^{-6}$ \cite{30,31}, chiral enhancement may be essential for the penguin-dominated decay modes. After including parametrized annihilation contributions in QCDF, the $B \to \phi K$ branching ratios reach around $7 \times 10^{-6}$ at most \cite{31}. If the branching ratios are around $10 \times 10^{-6}$ as predicted in PQCD \cite{17,32}, dynamical enhancement may be essential.

Recently, the charm penguin contributions \cite{33} have been proposed to be mechanism alternative to chiral enhancement and dynamical enhancement. It has been pointed out \cite{34} that contributions from intrinsic charms from the higher Fock states of the $B$ meson bound state may be also essential for the explanation of the branching ratios and CP asymmetries in the $B \to K\pi$ decays. These discussions indicate that penguin contributions to two-body nonleptonic $B$ meson decays need more thorough studies.

9 Conclusion

I have briefly reviewed the known approaches to two-body nonleptonic $B$ meson decays. Important aspects and phenomenological consequences of these approaches have been discussed. It is an urgent mission to construct a consistent and convincing approach to these decay modes. This requires continuous confrontation between theoretical and experimental progresses. After constructing such an approach, the Cabibbo-Kobayashi-Maskawa fit to the data of two-body nonleptonic $B$ meson decays, as performed in \cite{1}.
will make more sense.

Acknowledgments

I thank A. Ali, M. Beneke, S. Brodsky, G. Buchalla, M. Ciuchini, C.H. Chen, H.Y. Cheng, M. Diehl, L. Dixon, D.S. Du, T. Feldmann, R. Fleischer, S. Gardner, G. Hiller, T. Huang, Y.Y. Keum, G.P. Korchemsky, H. Lacker, C.D. Lu, J.P. Ma, H. Quinn, L. Roos, C.T. Sachrajda, A.I. Sanda, L. Silvestrini, G. Sterman, Z.T. Wei, Y.L. Wu, K.C. Yang, M.Z. Yang, Z.X. Zhang and members in the PQCD working group for useful discussions. The work was supported in part by the National Science Council of R.O.C. under the Grant No. NSC-89-2112-M-001-077, by the National Center for Theoretical Science of R.O.C., and by Grant-in Aid for Special Project Research (Physics of CP Violation) and by Grant-in Aid for Scientific Exchange from Ministry of Education, Science and Culture of Japan.

References

[1] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C 34, 103 (1987); Z. Phys. C 29, 637 (1985).
[2] C.H. Chen and H-n. Li, Phys. Rev. D 63, 014003 (2001), and references therein.
[3] C.H. Chang and H-n. Li, Phys. Rev. D 55, 5577 (1997).
[4] T.W. Yeh and H-n. Li, Phys. Rev. D 56, 1615 (1997).
[5] M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. 43, 242 (1979).
[6] A. Ali and C. Greub, Phys. Rev. D 57, 2996 (1998).
[7] H.Y. Cheng and B. Tseng, Phys. Rev. D 58, 094005 (1998).
[8] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999); Nucl. Phys. B 591, 313 (2000).
[9] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Nucl. Phys. B 606, 245 (2001).
[10] H.Y. Cheng, H-n. Li, and K.C. Yang, *Phys. Rev.* D **60**, 094005 (1999).

[11] C.Y. Wu, T.W. Yeh, and H-n. Li, *Phys. Rev.* D **53**, 4982 (1996).

[12] H-n. Li and H.L. Yu, *Phys. Rev. Lett.* **74**, 4388 (1995); *Phys. Lett.* B **353**, 301 (1995); *Phys. Rev.* D **53**, 2480 (1996).

[13] H-n. Li, hep-ph/0102013.

[14] Y.Y. Keum, H-n. Li and A.I. Sanda, *Phys. Lett.* B **504**, 6 (2001); *Phys. Rev.* D **63**, 054008 (2001).

[15] C. D. Lü, K. Ukai, and M. Z. Yang, *Phys. Rev.* D **63**, 074009 (2001).

[16] Y.Y. Keum and H-n. Li, *Phys. Rev.* D **63**, 074006 (2001).

[17] C.H. Chen, Y.Y. Keum, and H-n. Li, [hep-ph/0107165](http://arxiv.org/abs/hep-ph/0107165), to appear in *Phys. Rev.* D.

[18] G.P. Lepage and S.J. Brodsky, *Phys. Rev.* D **22**, 2157 (1980).

[19] A. Szczepaniak, E.M. Henley, and S. Brodsky, *Phys. Lett.* B **243**, 287 (1990).

[20] H-n. Li and G. Sterman, *Nucl. Phys.* B **381**, 129 (1992).

[21] J.C. Collins and D.E. Soper, *Nucl. Phys.* B **193**, 381 (1981).

[22] J. Botts and G. Sterman, *Nucl. Phys.* B **225**, 62 (1989).

[23] D. Du, C. Huang, Z. Wei, and M. Yang, hep-ph/0107320.

[24] S. Descotes-Genon and C.T. Sachrajda, hep-ph/0109260.

[25] V.N. Gribov and L.N. Lipatov, *Sov. J. Nucl. Phys.* 15, 428 (1972); G. Altarelli and G. Parisi, *Nucl. Phys.* B **126**, 298 (1977); Yu.L. Dokshitzer, *Sov. Phys. JETP* 46, 641 (1977).

[26] G.P. Korchemsky, D. Pirjol, and T.M. Yan, *Phys. Rev.* D **61**, 114510 (2000).

[27] H-n. Li, *Phys. Rev.* D **64**, 014019 (2001).
[28] N.G. Deshpande, X.G. He, W.S. Hou and, S. Pakvasa, *Phys. Rev. Lett.* **82**, 2240 (1999); W.S. Hou, J.G. Smith, and F. Würthwein, hep-ex/9910014.

[29] H-n. Li, hep-ph/0103305, talk presented at the 4th International Workshop on B Physics and CP Violation (BCP4), Ise-shima, Japan, Feb. 2001.

[30] X.G. He, J.P. Ma, and C.Y. Wu, *Phys. Rev. D* **63**, 094004 (2001).

[31] H.Y. Cheng and K.C. Yang, *Phys. Rev. D* **64**, 074004 (2001).

[32] S. Mishima, hep-ph/0107206.

[33] M. Ciuchini *et al.*, *Phys. Lett. B* **515**, 33 (2001); hep-ph/0110022.

[34] S. Brodsky and S. Gardner, hep-ph/0108121.