Towards a calculation of the halo mass function of a scalar field dark matter

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Abstract. In this work we are going to build the halo mass function for a scalar field dark matter model. This will be done with the aim of determine the minimum size of halos that our model predicts, in contrast with the LCDM model, which predicts an overabundance of substructures. We find that, for a scalar field mass $m = 10^{-22}$ eV and a Sharp-k window function, is possible to reproduce the suppression in the halo formation at small scales.

1. Introduction

The galaxy distribution on the Universe provides information about the formation and evolution of large scale structures. Several astronomical and cosmological surveys like BOSS \cite{1}, PLANCK \cite{2}, DES \cite{3}, have been exploring the content of matter and energy in the Universe for the last years, providing us with useful information to constraint theoretical models with higher precision. The commonly named LCDM model is so far the theoretical model that successfully describe several cosmological observations \cite{2}; nonetheless, it is well known that there are some tension between small scales observations and N-body simulations based on LCDM. As we will see later, LCDM model predicts an overabundance of galactic substructures that have not been observed. An alternative candidate that have been extensively studied in recent literature is the axion, which is represented by a scalar field $\phi$ endowed with a potential energy density $V(\phi)$. This particle was originally proposed to solve the strong CP problem in QCD \cite{4,5}, and also appears in some string theory scenarios \cite{6–11}. Our aim in this work is to show that an axion-like particle as scalar field dark matter (SFDM) is a good candidate to play the role of dark matter in the Universe, which means that this scalar field fits cosmological observations such as CMB anisotropies and Mass Power Spectrum (MPS) as good as CDM, and additionally predicts a cut-off in the sub-haloes production, which can be seen in the Halo Mass Function (HMF). Some reviews of this SFDM model can be found at \cite{12–14}.

We will be focused in the calculation of the HMF for a scalar field dark matter. Calculations of the HMF can be computed as has been shown in \cite{15–18}. Particularly in this work we are going to follow the procedure of reference \cite{15} for a scalar field with a quadratic potential

\begin{equation}
V(\phi) = \frac{1}{2} m^2 \phi^2,
\end{equation}
which has been considered in several cosmological scenarios for dark matter models in literature [19–22].

2. Power Spectra for a Scalar Field Dark Matter

The Einstein equations for a scalar field in a Friedmann-Robertson-Walker (FRW) spacetime with null spatial curvature are given by

\[
H^2 = \frac{\kappa^2}{3} \left( \sum_j \rho_j + \rho_\phi \right), \quad \dot{H} = -\frac{\kappa^2}{2} \left[ \sum_j (\rho_j + p_j) + (\rho_\phi + p_\phi) \right],
\]

(2a)

\[
\dot{\rho}_j = -3H(\rho_j + p_j), \quad \ddot{\phi} = -3H\dot{\phi} - m^2 \phi,
\]

(2b)

where \( \kappa^2 = 8\pi G \), a dot denotes derivative with respect to cosmic time \( t \), \( H \) is the Hubble parameter and \( \phi \) is the ALP that will be the source of dark matter. The scalar field energy density \( \rho_\phi \) and pressure \( p_\phi \) are obtained from the energy-momentum tensor of the scalar field, and are given by the canonical expressions:

\[
\rho_\phi = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} m^2 \phi^2, \quad p_\phi = \frac{1}{2} \dot{\phi}^2 - \frac{1}{2} m^2 \phi^2.
\]

(3)

In order to transform the Klein-Gordon (KG) equation (2b), we define a new set of variables based on [23,24] as follows

\[
\Omega^{1/2}_\phi \sin(\theta/2) = \frac{\kappa \dot{\phi}}{\sqrt{6H}}, \quad \Omega^{1/2}_\phi \cos(\theta/2) = \frac{\kappa m \phi}{\sqrt{6H}}, \quad y_1 = -\frac{2m}{H}.
\]

(4)

The KG equation can be written as a first order differential equations system as follows:

\[
\theta' = -3\sin \theta + y_1, \quad y'_1 = \frac{3}{2} (1 + w_{tot}) y_1, \quad \Omega'_\phi = 3(w_{tot} - w_\phi) \Omega_\phi,
\]

(5)

where a prime denotes derivative with respect to the number of e-foldings \( N \equiv \ln(a/a_i) \), with \( a \) the scale factor of the Universe and \( a_i \) its initial value. Now, we consider small perturbations around the background values of the FRW line element (in the synchronous gauge) as well as for the scalar field in the following form:

\[
ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j, \quad \phi(x,t) = \phi(t) + \varphi(x,t),
\]

(6)

where \( h_{ij} \) and \( \varphi \) are the metric and scalar field perturbations respectively. Then, the linearized KG equation reads [25–28]:

\[
\ddot{\varphi} = -3H \dot{\varphi} - \left( \frac{k^2}{a^2} + \frac{\partial^2 V(\phi)}{\partial \phi^2} \right) \varphi - \frac{1}{2} \dot{\phi} \ddot{h},
\]

(7)

where \( h \) is the trace of scalar metric perturbations, and \( k \) is a comoving wavenumber. Although functional dependence of the scalar field perturbation is not explicitly shown, note that Eq. (7) is written for a Fourier mode \( \varphi(k,t) \). Following the change of variables of [19,29], the Eq.(7) is described by the following system of equations

\[
\delta'_0 = [-3 \sin \theta - \omega(1 - \cos \theta)] \delta_1 + \omega \sin \theta \delta_0 - \frac{h'}{2}(1 - \cos \theta), \quad \delta'_1 = [-3 \cos \theta - \omega \sin \theta] \delta_1 + \omega(1 + \cos \theta) \delta_0 - \frac{h'}{2} \sin \theta,
\]

(8a) (8b)
where \( \omega = k^2 / k_J^2 = k^2 / H^2 a^2 y_1 \). With the solutions of Eqs. (8) is possible to build up cosmological observables such as CMB anisotropies and the MPS, which we show for the axion field and CDM in Figs. 1 and 2, for an axion mass \( m = 10^{-20}, 10^{-22}, 10^{-24}, 10^{-26} \)eV. The numerical solutions were obtained with an amended version of the CLASS code\(^1\), which is a Boltzmann code to solve the linear perturbations of the cosmological dynamics. Figure 1 shows a very good agreement between our model and observations from Planck satellite data at least for \( m \geq 10^{-24} \)eV. On the other hand, Fig. 2 shows the MPS for both CDM and SFDM model. In such figure can be noted that there is a cut-off at large values of \( k \) (small scales) which shifts to small wavenumber for smaller masses. In particular, we see that for \( m = 10^{-26} \)eV the axion is ruled out due to it is not in agreement with the observed MPS at linear regime.

3. Halo Mass Function

Now we are going to calculate the HMF for this dark matter model. Such function encodes the comoving number density of dark matter haloes as function of the halo mass, and it constitutes a representative cosmological probe of dark matter and dark energy. It can be use to constraint the value of the combined parameters \( \sigma_8 \) (power spectrum normalization) and \( \Omega_M \) (matter density parameter), and also to characterize the dark energy equation of state \( \omega_0 \) [31–33]. To accomplish this, it is necessary to use a window function, which is a filter function that fixes a cut-off at certain length scale \( r \). Following the recipe of [15] let us proceed as follows: first, we have to define the window function we are going to implement. In this work we will consider the Top-Hat \( W_{TH} \), which is a filter with spherical form in real space, and the Sharp-k \( W_{SK} \), which is defined as a Top-Hat function in Fourier space. The choice of those window functions is based on that the Top-Hat function is useful to work with the LCDM model, while the Sharp-k function does a better job with suppressed power spectra, which is the case for scalar field dark matter. An analysis using those window functions incorporating a mass dependent barrier, and

\(^1\) Such code can be found in https://github.com/lurena-lopez/class.FreeSF
Figure 2: MPS for SFDM with axion mass $m = 10^{-22}\text{eV}$. It can be noted that the cut-off at small scales (large wavenumbers) depends on the axion mass $m$. Data from BOSS [30] are indicated with red dots. See text for more discussion.

also including a comparison with numerical simulations can be found in [19]. Both window function mentioned above are given (in Fourier space) by

$$W_{TH}(kr) = \frac{3}{(kr)^3} [\sin(kr) - kr \cos(kr)], \quad W_{SK}(kr) = \Theta(1 - kr), \quad (9)$$

which encloses a mass given by $M = 4\pi \bar{\rho} r^3 / 3$ and $\bar{\rho} = 2.775 \times 10^{11}h^2M_{\odot}/\text{Mpc}^3$ is the critical density of the Universe. Now let us define the variance $S(r)$

$$S(r) = \int \frac{d^3k}{(2\pi)^3} P(k)W^2(kr). \quad (10)$$

The universal growth factor of perturbations is given by the following expression

$$D(a) = \frac{5\Omega_m}{2} H(a) \int \frac{da}{a^3H^3(a)}, \quad (11)$$

while the peak height of perturbations, the critical overdensity and the function for a spherical collapse model are given respectively by

$$\nu = \frac{\delta_c^2}{S(r)}, \quad \delta_c = 1.686 D(a), \quad f(\nu) = \sqrt{2\nu \pi} e^{-\nu/2}. \quad (12)$$

Finally, the HMF has the following expression

$$\frac{dn}{d\ln M} = -\frac{1}{2} \tilde{\rho} f(\nu) \frac{d\ln S}{d\ln M}. \quad (13)$$

The HMF is shown in Fig. 3 as function of halo mass. We note in such figure that the cut-off in the mass power spectrum (Fig. 2) is translated into a suppression of small scale structures.
Figure 3: HMF as function of halo mass. The fiducial LCDM model is shown in black line, while the axion case for a mass $m = 10^{-22}$eV is shown for a Top-Hat (blue solid line) and Sharp-k (dotted blue line) window functions. The suppression for the scalar field at small scales is better reproduced by the Sharp-k window function. See text for more discussion.

While numerical simulations are needed to build up the HMF, here we present a forecast for the HMF of our model, particularly for the axion mass $m = 10^{-22}$eV: our results show that for a suppressed power spectrum, the Sharp-k function seems to reproduce better the absence of substructures at small scales. This can be compared with the smaller galactic objects that have been observed so far in the Local Group, the so called dwarf spheroidal galaxies, whose total mass (stellar and DM masses) are of order of $10^7 - 10^8 M_{\odot}$ [34–36], i.e., the SFDM model predicts a lower bound for halo masses that is consistent with astrophysical observations.

4. Conclusions

In this work we obtain the MPS and the anisotropies of the cosmic microwave background for a dark matter model based on an ultra-light scalar field (axion-like particle). The cosmological implication of such model is evident in the structure formation: for axion with masses $m \leq 10^{-20}$eV there is a shift to small wavenumbers (large scales). This imply that there is a cut-off in the size of galactic structures. This can be seen in Fig. 3, which constitute the main result of this work: while the LCDM model predicts an overabundance of substructures at small masses, the SFDM shows a decrease in the number of halos for low values of halo mass. Particularly, using a Sharp-k window function we observe a suppression for halo masses, this is, the SFDM predicts a lower bound for the size of halo masses smaller than $4 \times 10^8 M_{\odot}$. It is important to note that this is a crude approximation, in the sense that full cosmological simulations of structure formation including the presence of baryons are needed to quantify in detail the amount of substructures[37,38]. This is work in progress that we expect to publish elsewhere.

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