Zipf-Mandelbrot-Pareto model
for co-authorship popularity

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Abstract

Each co-author (CA) of any scientist can be given a rank \((r)\) of importance according to the number \((J)\) of joint publications which the authors have together. In this paper, the Zipf-Mandelbrot-Pareto law, i.e. \(J \propto 1/(\nu + r)^\zeta\) is shown to reproduce the empirical relationship between \(J\) and \(r\) and shown to be preferable to a mere power law, \(J \propto 1/r^\alpha\). The CA core value, i.e. the core number of CAs, is unaffected, of course. The demonstration is made on data for two authors, with a high number of joint publications, recently considered by Bougrine (2014) and for 7 authors, distinguishing between their "journal" and "proceedings" publications as suggested by Miskiewicz (2013). The rank-size statistics is discussed and the \(\alpha\) and \(\zeta\) exponents are compared. The correlation coefficient is much improved (\(\sim 0.99\), instead of 0.92). There are marked deviations of such a co-authorship popularity law depending on sub-fields. On one hand, this suggests an interpretation of the parameter \(\nu\). On the other hand, it suggests a novel model on the (likely time dependent) structural and publishing properties of research teams.

Thus, one can propose a scenario for how a research team is formed and grows. This is based on a hierarchy utility concept, justifying the empirical Zipf-Mandelbrot-Pareto law, assuming a simple form for the CA publication/cost ratio, \(c_r = c_0 \log_2(\nu + r)\). In conclusion, such a law and model can suggest practical applications on measures of research teams.

In Appendices, the frequency-size cumulative distribution function is discussed for two sub-fields, with other technicalities

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1 Introduction

One basic question in scientometrics concerns the number of relevant co-authors (CA) for the set of publications of some principal investigator (PI). This leads to examine the CA core index (Ausloos 2013), $m_a$, introduced on several grounds, including a statistical framework. Recall that the $m_a$-index is deduced from a plot of the number ($J$) of joint publications (NJP) of a PI with CAs ranked according to their rank ($r$) of importance; $r = 1$ being the most prolific CA with the PI. It has been found (Ausloos 2013) that

$$J(r) = a/r^\alpha,$$  \hspace{1cm} (1)

which holds best when there are enough $J$ and CA. It holds as well if the publication list is broken into specific types of publications, i.e. in sub-fields (Bougrine 2014) or peer-review journals and proceedings (Miskiewicz 2013). Miskiewicz (2013) has much discussed the exponent $\alpha$ value which turns out to be scientist dependent, whence is a valid criterion for team work assessment. However, neither Miskiewicz nor Ausloos have provided a statistical physics-like model or any argument for the findings.

In the present paper, such a model is presented as an adaptation of Mandelbrot model for text appraisal. In order to present arguments in such a favor, the Bougrine (2014) data is first reexamined along the 3-parameter Zipf-Mandelbrot-Pareto (ZMP) law (Zipf 1949, Mandelbrot 1960, Pareto 1896, West & Deering 1995)

$$J(r) = b/(\nu + r)^\zeta.$$  \hspace{1cm} (2)

Next, several other co-authorships are examined distinguishing between their publications in (peer review) journals or proceedings and in a few cases the research sub-fields.

Obviously, the ZMP distribution is a natural generalization of an inverse power law. The ZMP law leads to a curvature at low $r$ in a log-log plot (if $\nu$ is positive$^1$ and presents an asymptotic power law behavior at large $r$. Both $\alpha$ and $\zeta$ exponents must be greater than 1 for the distributions to be well-defined, greater than 2 for the mean to be finite, and greater than 3 for the variance to be finite. Note that the ZMP distribution has been applied in many contexts, e.g., in studies of scientific citations (Tsallis & Albuquerque 2000). In fact, another important aspect of the ZMP distribution is that it arises in the context of generalized statistical physics (Tsallis 1988).

Fitting the ZMP law is much more troublesome than fitting the Zipf hyperbolic law (Fairthorne 1969, Haitun 1982, Iszak 2006). Thus, a variant of the ZMP law, i.e. the 4-parameter relation

$$J(r) = c/(\eta + \lambda r)^\mu.$$  \hspace{1cm} (3)

$^1$Necessarily, $-1 \leq \nu$, since $r \geq 1$
has been also examined, for comparing the resulting precisions of the respective fits. Since nothing drastic has been found in the present case, some data and analysis is left in Appendix A for completeness.

In order to argue in favor of a ZMP model for co-authorship popularity,

- two PI cases are here below examined: one is H.E. Stanley (HES): he has more than 1400 publications, is a guru of statistical mechanics; his group website distinguishes between activities in several research sub-fields. HES lists more than 500 CA. The other is M. Ausloos (MRA); he has published a little bit less than 600 papers in international journals or proceedings with reviewers; MRA has more than 300 CA. Both PIs have worked in different research fields, sometimes overlapping. Remarkable fits to the hyperbolic law, Eq.(1), are available (Miskiewicz 2013, Bougrine 2014) for both PIs, - at least in the central \( J(r) \) region. Deviations mainly exist in the extreme regions. Thus, it seems of interest to reexamine the data in a more parametrized way, along the ZMP law;

- several other authors (7) CA lists are also examined, beside HES and MRA, - distinguishing between their publications in journals or proceedings;

- finally, the largest publication lists, i.e. those of HES and MRA, can be broken into several research sub-fields; the ZMP law validity and fit parameter values are examined in such cases.

After this brief introduction serving as basic arguments, the \( J(r) \) data of the co-authorship features is rank-size analysed, in Sect. 2, for both MRA and HES, and for 7 other authors in Sect 2.1 and in Sect 2.2 along the above the simple empirical laws, Eqs.(1)-(2), distinguishing between the different types of publications mentioned here above.

In Sect. 3 some discussion on the statistical aspects of these illustrative cases are presented, stressing the parameter \( \nu \), beside the characteristic exponents, either for co-authored papers in journals or proceedings, Sect. 3.1 or according to envisaged sub-fields, Sect. 3.2. It is argued that the ZMP law should be further considered from a theoretical point of view., when examining so called king, vice-roy, queen and harem effects, in Sect. 3.3. Therefore, a novel model is presented on the structural and publishing properties of research teams, in Sect. 4. Thus, one can propose a (rather common sense, \( \sim \) Ockam’s razor mode) scenario for how a research team is formed around a PI and grows with preferred CAs. This is based on a hierarchy utility concept assuming a simple form for the CA publication/cost ratio.

Some summary and a short conclusion are found in Sect. 5.

As mentioned, practical considerations on fit precision through either Eq.(2) or Eq.(3) are found in Appendix A. A thorough discussion on generalizing the main text considerations to "sub-cores" of co-authors is found in Appendix B.

In fine, another type of display that the rank-size plots is of interest in order to remain in line with somewhat more conventional plots in bibliometrics, i.e.
2 The data, fits, and statistical analysis

The whole joint publication list for the two prolific PIs with many CAs, i.e. HES and MRA, has been first studied, - the data being fitted with the best power law and best ZMP fits. The parameter values are reported in Table 1. The fits are rather remarkably good, both for the power law and the ZMP laws, with an $R^2$ above 0.94. The exponents are close to 1.0, as expected. The parameter values and the regression fit coefficient $R^2$ as given for the different
cases have been rounded up to significant decimals.

Note that it is not surprising that all the numerical results slightly differ from those of Bougrine (2014). This is likely due to the type of algorithm used in these non-linear fits. Bougrine (private communication) used a classical Least Squares curve fit, on log-log plots. In the present study, the fit is a non-linear one that uses the Levenberg-Marquardt algorithm. However, there is no disagreement on the qualitative conclusions. In fact, as mentioned, fitting the ZMP law is much more troublesome than fitting the Zipf hyperbolic law (Fairthorne 1969, Haitun 1982, Iszak 2006). However, the error bars on the parameter values, calculated from several simulations with different initial conditions, are less than 10%, - usually less 5%. For completeness, note that the quality of the fits were examined through the statistics of the errors (Jarque & Bera 1980), i.e. whether the error distributions were relatively small and randomly distributed on the both side of the means. The worse cases occur when king and/or queen effects, see details below, are superposed to a power law tail, or when a strong exponential cut-off is present on the tail. Note also that fits in such cases necessarily imply large error bars on the amplitudes $a$, $b$, and $c$, since the number of CA having a given NJP can be large itself, the more so at high rank, when the number of publications with the PI, usually 1 or 2, is the same for many CAs.

It is well seen, on Fig.1, that a plot of the (whole) NJP data reveals a marked king effect both for MRA and HES, - even a king and vice-roy effect for HES. Thus, to emphasize the latter effect, the two lowest rank data points, the king and the vice-roy, have been removed before new fit attempts. The results are shown in Table 1 also, in HES* and MRA* lines. Observe that the major changes occur in the amplitudes, $\nu$ and $b$, - not in the fit exponents which rather indicate the asymptotic regime; note that $R^2$ goes from 0.92 to 0.99 for HES.

2.1 Peer-review journals and proceedings

A reviewer of the first submitted version of this report pointed out that (I quote) "picking such singular authors allow a better statistic, ...". Indeed that is a relevant point. In fact, one tends in physics to look for large numbers, in view of finding some "universality", - but not many authors have so many papers as HES or MRA. For analysis consistency, I had taken the data in Bougrine (2014) publication. Nevertheless, it is of interest to look for less prolific authors, as pointed out by the reviewer. Thus, I have requested from reliable colleagues, having worked in statistical and/or condensed matter physics, some pertinent data. However, as an additional "constraint", I have asked such colleagues not to break their publication list into sub-fields as Bougrine (2024) did, but rather

\[ \text{The effect occurs when the data is upsurging at low rank and has been so called when examining city size by (Laherrère & Sornette 1998); it seems to have been emphasized first by Jefferson (1939), also when studying city sizes.} \]

\[ \text{The effect has been so called when examining co-authorship sizes size by Ausloos (2013); it occurs when the data is flattening at low rank} \]

\[ \text{that would have led to too few papers per field, and it would have been nonsense to do some meaningful fit thereafter.} \]
to break their publication list into "peer-review journals" and "peer-review (or not) proceedings", as Miskiewicz (2013) has investigated. No need to say that proceedings papers are often peer reviewed as well, but the distinction, as the one Miskiewicz proposed, is surely obvious to or acceptable by many.

Therefore, this leads to Table 1 and to many possible plots, - only a few are shown such that some emphasis is placed on major features or findings, Figs. 2, 3, and 4. Note that PIs are ranked according to NDCA rather than NJP, for better emphasis of PI popularity hierarchy than productivity, in Table 1.

In Fig. 2 a log-log scale display of the whole set of joint publications (NJP) number by various (3) authors (see insert: DS, KK, JMK, AP) with different production outputs and is shown with co-authors (CA) ranked by decreasing importance”. Note that $R^2 \simeq 0.98$, except for JMK (see Table 1). Observe also a king effect for JMK, but the data has not been further analyzed, as for HES and MRA; if done, i.e. removing the king, it should surely markedly improve the $R^2$ coefficient, actually 0.95.

In Fig. 3, co-authors are ranked on the x-axis in decreasing order of their number of publications, in proceedings, with the 2 two "main" PIs and 5 other PIs in statistical physics. It is seen that the ZMP law holds for such a type of publications, as well, - with marked king and queen effects.

In contrast, the display of Fig. 4 shows a comparison of various types of publications by one author (PC), pointing out to a marked king effect, whatever the publication type, and another comparison of publication types with co-authors (PVdB) where marked queen effect is seen. At this point, it is sufficient to know that PVdB is an experimentalist working in condensed matter.

Thus, the various types of behaviors have been illustrated for various types of publications, indicating the ZMP framework interest. The values of the fit parameters, reported in Table 1 can be discussed in Sect. 3.

2.2 Sub-field effects

Next, let it be recalled that the NJP analysis was suggested by Bougrine (2014) to be breakable into sub-fields: e.g., into 12 appropriate sub-fields, for HES, here called $s_i$, and into 8 sub-fields, in the case of MRA, here called $a_i$, - according to the web site of such authors. For coherence, the same set of sub-fields has been used here. Let it be recalled that for justified statistical purposes, Bougrine has also merged some $s_3$ and $s_6$ into some $s_{63}$ and some $a_4$ and $a_5$ into some $a_{54}$.

The same is done here; names of sub-fields are presently irrelevant. Two specific cases, i.e. $a_2$ and $a_7$ (they refer to work on Magnetic Materials and Superconductivity, respectively) will be examined in Appendix B and in Appendix C, as mentioned in the Introduction, i.e. for comparing the size-rank method with the size-frequency method.

The hyperbolic law, Eq. (1) has been studied on all $s_i$ and $a_i$ data; numerical results reported in Table 2. The law is often well obeyed, since $R^2 \in \{0.88, 0.99\}$, - except for $s_4$, in which case $R^2 \sim 0.787$. Observe that the cases with the "worse" exponent, i.e. $\alpha$ rather away from +1 in $s_9$ and $a_6$, correspond to the oldest sub-fields of investigations both for HES, i.e. phase
Figure 2: Log-log scale display of the number of all joint publications (NJP\(^{(w)}\)) by various authors (see insert: DS, KK, JMK, AP) with co-authors (CA) ranked by decreasing “importance”; best fits (color lines) are shown for the ZMP law, Eq. (2) transitions and critical phenomena, and for MRA, i.e., kinetic growth and spin models, - in fact, two closely related sub-fields! An open question is whether such a variation is due to time effects or to sub-field distinctions.

The 3 parameter ZMP (3-ZMP) law necessarily leads to a better \(R^2\), except for \(s_3\) and \(a_5\) which are almost unchanged with respect to the hyperbolic law. Recall that they belong to data sets with very low numbers of CAs and NJP. All other \(R^2\) values move close to 0.98 or above from \(\sim 0.92\) on average for the power law.

The values of the fit parameters will be discussed in Sect. 3.

3 Amplitudes and exponents. A discussion

After the statistical analysis based on the correlation coefficient, \(R^2\), let the exponent and amplitudes of the empirical laws be examined. first, reconsider

\(^5\) so does the 4 parameter ZMP (4-ZMP) law, see Appendix A
Table 1: Summary of fit parameters to NJP data corresponding to Figs. 2-4 for various PIs, distinguishing between their whole (w) publication list and their papers published in journals (j) or in proceedings (p); the parameter values correspond to the various formulae discussed in the text, Eqs. (1)-(2); the regression fit coefficient $R^2$ is given for the different cases; data has been rounded up to significant decimals; error bars $\leq 10\%$; NDCA : number of different CAs; $<\text{NCA}>$ : number of co-authors on average on a paper by the "main" author (PI); for HES* and MRA*, see text.
Table 2: Summary of fit parameters to NJP data grouped as for Figs. 5-8; the parameters correspond to the various formulae discussed in the text, Eqs. (1)-(2); \textit{q.pw.t} indicates a strong queen plus power law tail effect and \textit{exp.t} indicates a strong exponential tail cut-off, i.e. cases for which the scaling parameters have large error bars; the regression fit coefficient \(R^2\) is given for the different cases; data has been rounded up to significant decimals; error bars much below 10%
Figure 3: Log-log scale display of the number of joint publications ($NJP^{(p)}$) in "proceedings", by various authors (see insert) with co-authors ranked by decreasing "importance"; best fits (color lines) are shown for the ZMP law, Eq. (2).
Figure 4: Log-log scale display of the number of (various types of) joint publications (NJP) by (a) PVdB and (b) PC, with co-authors (CA) ranked by decreasing "importance"; best fits (color lines) are shown for the ZMP law, Eq. (2); note the "queen effect" in (a) and the "king effect" in (b).
the whole data for the two PIs, HES and MRA, displayed with the best power law and best ZMP fits on Fig. 1. The ZMP is visually much better performing, - the more so if the two lowest ranked data points are removed before a fit to Eqs. 1-2. The display emphasizes that both HES and MRA have at least two privileged CAs. This is also going to be emphasized through the display in Figs. 2-4 irrespective of the "very" or "less" plowed research sub-fields.

Observe that such a removal of two outliers, induce a huge change (almost a factor of 3) in the amplitude $b$ and in $\nu$, thereby strongly stressing a queen affect , - due tot he king and vice-roy of HES, and the king of MRA. These effects on $\zeta$ are mild, though in both cases increase $\zeta$, indicating a faster decay of the empirical law at high rank, - in other words, a weakening of the role of the rare CAs.

3.1 Journals

From Table 1, it appears that the range of amplitude can be large and exponents rather narrow. The former is due to the wide range in the number of publications of various PIs, but the latter indicates some stability thus a reasonable guess of the empirical law, - in fact, as should be stressed, whatever the type of publication. Most of the values of $\zeta$ are close to 1. The notable exceptions, pointing to non-universality in contrast indicate the interest of using such an exponent as a measure of the PI relationship with his/her team of CAs. Obviously, the greater the exponent $\zeta$ the shorter is the team size. The popularity is influenced by the number of rare CAs, thus by a widening of the NJP distribution, thus by a flattening of the tail, whence by a smaller $\zeta$ exponent.

Nevertheless, one should disregard the PIs who are in fact "not quite PIs", as exemplified by their small number of publications, and the value of $\nu$ being negative, i.e. PC and JM.

In so doing, the most popular PIs in the presently examined set are (in terms of the $\zeta$ exponent for the whole set of joint publications) : HES (0.83), JMK (0.87), MRA (0.97) and AP (0.98). It is thus very interesting to notice that neither DS (1.38) and KK (1.67) nor PVdB (1.44) are "popular". However, they have large "harem", - $\nu$ being quite large. This seems quite understandable due to the activities of the researchers.

Practically, it can be observed that HES presents a king and a vice-roy effect in NPJ(j). Indeed the list of publications of HES shows the high relevance of such two preferred CAS. However, HES presents rather a queen effect for NJP(p). Again, this is understandable on the same scientific and popularity ground. He is indeed one of the top masters and most popular in his field. QED.

3.2 Proceedings

Next, observe the data when broken into sub-fields as displayed and numerically fitted on Figs. 4 for these two PIs. The displayed data has been grouped both according to the size of NJP and the rank range for better visibility; thus in Fig. 5 one finds the $i = 1, 2, 4; 9, 11$ ("less prolific") sub-fields, and in Fig. 6...
the \( i = 5, 7, 8, 10, 12 \) ("more prolific") sub-fields for HES. The case of sub-fields \( i = 4 \) and \( 5 \), for MRA, and of sub-fields \( i = 3 \) and \( 6 \), for HES, are treated in the Appendix B, and displayed on Fig. 8 and compared to the regrouped data into \( a_{54} \) and \( s_{63} \).

For better visibility, the classical linear axes plots are limited to the region of interest, \( \text{NJP} \leq 40 \), and \( \text{NDCA} \leq 80 \) or less when convenient, i.e., where the \( m_\alpha \) index can be measured (Ausloos 2013), as suggested by the diagonal line indicating the core threshold. On such linear-linear axes the power law fits do not much indicate deviations. However, on the log-log plots, the ZMP fits are \( ll \) obviously better reproducing the empirical data. The queen and king effects are often well seen.

It is interesting to observe that \( \nu \) is negative in the \( a_5 \) and \( a_5 \) cases, and \( a_{54} \), though the latter case is one of merged fields. This implies a strong king effect for one CA of these sub-fields of MRA.

### 3.3 Remarks on the king, vice-roy, queen and harem effects

It is easily understood that the king and queen effects are at once seen on plots. Moreover, it has been observed that sometimes the low ranks contain more than one "equivalent (in terms of NJP) CA", thus one may propose the existence (or definition) of vice-roy(s), on one hand, and of a harem, on the other hand, effect. Numerically, this occurs respectively when

- \( \nu \to 0 \), and more drastically when \( \nu \leq 0, \to -1; \)
- when \( \nu \) is large.

Concerning the queen effect, it is best measured through the ratio \( \zeta/\nu \), as seen by taking the derivative of \( d\ln(J)/d\ln(r) \), i.e.

\[
\frac{d\ln(J)}{d\ln(r)} = -\frac{\zeta r}{\nu + r}
\]

which should be \( \leq 0 \), at small \( r \). The harem effect is quite obvious for DS (w) and DS (j), KK (w) and KK (j), and PVdB (all types). This is also remarkably true for the largest \( \nu \) cases, i.e. \( a_2 \) (see also Fig[7], \( s_4 \) and \( s_2 \). Those large \( \nu \) values indicate that both PIs have published many joint papers with a short set of CAs, - often the same ones on a large number of publications, and have thus "many queens". In fact, according to the sub-field definitions (Bougrine 2014), these sub-fields with many queens pertain to medical topics, for HES, and experimental work on materials, for MRA. It is understandable that a team effect, with kings and queens, are to be expected in such (transdisciplinary) domains, thereby stressing the intrinsic interest of the ZMP law, whence leading to suggest the following model.

Concerning the king and vice-roy effect, it has been discussed through a comparison of HES with HES* and MRA with MRA* in Table 3. The effect is
highly remarkable for JMK (j), and PC and JM, on all types of publications: they have all a few team co-leaders. It seems that such effects are strongly constraining the fits, when $\nu \leq 0.2$.

4 Model

In so doing, it seems reasonable to suggest a model of team publications, i.e. for a popular PI and his/her CAs, along the line of Mandelbrot’s model for quantitative linguistics (Mandelbrot 1960, Amati et al. 2002, Manin 2009), in fact similar to that of city formation (Madden 1956, Rosen & Resnick 1980, Glaeser 2008). Recall that Mandelbrot has first derived the Zipf’s law assuming that the optimization of the information/cost ratio is resulting from random character sequences, i.e. the “random typing model”. Mandelbrot’s model of texts assumes that the optimal language is one where each sequence of $n$ letters is as frequent as any other (all characters are equiprobable). Thus, the probability of any given word is exponential in length. Whence, each word’s frequency rank is asymptotically given by a power law. Nevertheless, note that it has been observed that the number of distinct words of the same length in a real language, is far from being exponential in length (Manin 2009).

In fact, Mandelbrot proposed that the language is optimal if it minimizes the average ratio of production cost to information content. This leads to Eq. (2). To do so, Mandelbrot defined the information content of a word to be the Shannon entropy, i.e. the negative logarithm of the word frequency in the text or language.

In the same spirit, one can assume to derive Eq. (1), in the present case, that the choice of CAs by a PI is at first a stochastic process, - as proposed by Hsu & Huang (2009). However, as ties developed or not with the PI, either the CAs become quasi happaxes, or maintain a strong tie. Moreover the CAs attract other CAs (or reject them, for whatever reason), whence organizing a CA utility network, with the PI as hub.

One can also imagine that the choice of CAs, as time and research activity progress, has some cost or utility for the PI. He/she recognizes some interest in being associated, and publishing with one or another CA, - whatever the profound reasons. Such an optimization principle allows to demonstrate how the ”optimal state” can be achieved as a result of research evolution. The $\alpha$ exponent is close to 1, as in the case of language and of city population size optimization, i.e. a desirable situation in which the forces of concentration of coworkers balance those of competition by other external teams, called ”de-centralization” in city formation (Zipf 1949, Madden 1958, Glaeser 2008). In co-authorship popularity measures, as well, the exponent $\alpha$ (and $\mu$ or $\zeta$) can be interpreted as being close to 1, because of the ”same” type of balance between attraction and rejection of coworkers by the PI his/her king and queens. Thus,

\footnote{This has been recently examined considering pairs of leading CA through a binary scientific star concept (Ausloos 2014)}
Figure 5: Number of joint publications (NJP) for HES, with co-authors ranked by decreasing importance for his 5 "less prolific" sub-fields $s_i$; (see text for $i = 1, 2, 4, 9, 11$ definition): (a) in the vicinity of the co-author core measure; (b) log-log display of (a); best fits, i.e. power law in (a) and ZMP in (b), are given for the overall range; the co-author core can be easily deduced from the diagonal line position.
Figure 6: Number of joint publications (NJP) for HES, with co-authors ranked by decreasing importance for the 5 "most prolific" sub-fields $s_i$; see text for $i = 5, 7, 8, 10, 12$ definition; (a) in the vicinity of the co-author core measure (the co-author core can be easily deduced from the diagonal line position); (b) log-log display of (a); best fits, i.e. power law in (a) and ZMP in (b), are given for the overall range
Figure 7: Number of joint publications (NJP) for MRA, with co-authors ranked by decreasing importance for 6 different sub-fields $a_i$; see text for $i = 1, \ldots, 8$ definition; (a) in the vicinity of the co-author core measure; (b) log-log scale display of (a); the co-author core can be easily deduced from the diagonal line position; the best fits, i.e. power law in (a) and ZMP in (b), are given for the overall range.
Figure 8: Number of joint publications (NJP) with co-authors ranked by decreasing importance, in the case of sub-fields 4 and 5 (for MRA) and 3 and 6 (for HES), and their merging into a sub-field, i.e. 54 and 63 respectively; the best ZMP law fits are given
one considers detachment, beside preferential attachment, which by itself would lead to a much heavier tail exponent $\zeta$.

Thus, by similarity, along such a line of reasoning, consider the joint publication cost per CA to be

$$C = \sum p_r c_r$$

such that the "cost of a publication" with the CA at rank $r$ is $c_r$, and $p_r$ is the normalized value of $J_r$, normalization with respect to the whole histogram surface $J(r)$. Let the average information entropy per CA be also assumed to read

$$H = - \sum p_r \log_2(p_r).$$

It is easy, e.g. by the Lagrange multiplier method (Manin 2009), to show that the rank-frequency distribution $p_r$ will minimize the cost ratio $\hat{C} \equiv C/H$ and will have a ZMP law form if one assumes the individual cost function to be

$$c_r = c_0 \log_2(\nu + r),$$

which can be negative or positive. The cost is of course high when $r$ is large. Note that if $\nu = 0$, one recovers the Eq.(1) power law. Eq.(6) assumption (or ansatz) can be accepted if one considers that any research team has some hierarchical structure with a set of CA at different levels, the set increasing with the "distance" ($\sim r$) from the PI. For example, the CAs can be grouped according to a visual rule based on the behavior of $J(r)$ examined on linear or log-log plots. Practically, this is supplemented by trial and errors of local fits in moving windows of different sizes; the groups are deduced and confirmed, choosing them from the best $R^2$ among various realistic attempts. More complicated functions (Popescu et al. 2010, Voloshynovska 2011) could be used, but a detailed parameter interpretation for such function combinations is a challenging task and requests further study, much outside this report.

Nevertheless, the CAs can be seen as forming "circles" around the PI, as in e.g. the HES case, Fig. [10]. Note that this can be considered as the true PI community, - a visible college, since the CAs of CAs not involving the PI, have been removed from the start, by data construction. In this cluster, the PI has necessarily at least one link with some CA.

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7For simplicity of the writing, $r$ is taken as a continuous variable though it is manifestly a positive integer only.

8Benguigui and Blumenfeld-Lieberthal (2011) are perfectly right : (text adapted, but resulting from a quasi exact quotation) in order to be able to decide if Eq. (1) is (and Eqs. (2-3) are) verified or not, one has to fit the data to several functions and compare the results, using the same criterion. Naturally, it is not realistic to expect each $J(r)$ would be fitted to numerous formulas; thus, we ($\approx I$) propose to use a visual inspection in order to help decide which formulas might represent the data correctly. ... we ($\approx I$) trust the human mind and believe that a visual inspection can indeed give essential information; particularly it helps deciding if the studied system is homogeneous or not ... a simple visual inspection ... shows that the system (...) is not homogeneous. It can be divided into ... subsystems. This (...) emphasizes the need for a visual inspection of the rank-size relation of the real data on log-log scales. This gives the possibility to see (in the simple meaning of the word, see with the eye) if the points may be fitted with some mathematical function (not necessarily a straight line).
The cost of retrieving the CA at the $r$-th level can be, in a first approximation, assumed to be related to the NJP $J(r)$ of the CA, or to the rank, i.e. $\log_2(r)$. Therefore, the parameter $\nu$ can be understood to indicate that several "far ranked" CAs have to deal with some other "preferred CAs" of the PI in the joint publication process.

5 Conclusions

Most prolific scientists have joint publications on different subjects. Thus, co-authors might be specific to some research sub-field of a PI. It is thus if interest to examine, for such teams and leaders, whether the $j(r)$ relationship (Ausloos 2013) is obeyed when the research publications pertain to different sub-fields. Bougrine (2014) has broken the publication list of two such authors into sub-fields.

Observed irregularities have been thought to be due to different causes: publication inflation, proceedings counting, co-authorship inflation, and/or promotion of team size visibility. Other "intrinsic causes" might have arisen from the large productivity of the group based on a high turnover of young researchers, with $r \gg 1$, as well as a steady contribution from stable partners, with $r \simeq 1$.

Here, deviations have been searched through sub-field considerations. By using the Zip-Mandelbrot-Pareto function for the distribution $J(r)$, thus having one more parameter ($\nu$), it has been observed that the statistics on one hand is much improved. On the other hand, the introduction of the extra parameter, and the observation of its range of values, has suggested a model based on a co-author cost scheme, similar to the models of language usage and city population size distributions. One must admit that the model is a rather common sense model to anyone having worked with co-authors, in large or small teams.

However, the analysis has indicated the sensitivity of the sub-field notion, on one hand, and of the co-author distribution, on the other hand. Moreover the notion of cost and utility of co-authorship are introduced, but demand further elaboration; some work will be useful along such lines, for better quantification also.

Notice that the cost can be positive or negative according to the value of $\nu$; see Eq. (7). This observation can be considered to be already useful in order to imagine that one can be introducing selection and rewarding policies in the career of members of teams, along the co-author core measure. Technically, one could thus measure the relevant strength of a research group centered on some leader by combining the $h$-index and the $m_a$ measure in a three dimensional $(J, q, r)$ space taking into account the quotations $q$ of co-authored papers.

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Figure 9: Total number of joint publications (NJP) for HES and MRA, with co-authors ranked by decreasing importance allowing some visualization of major groups of CAs: (a) for low ranks, groups I, II, and III) and (b) for group IV; the best local power law fits are given as a guide to the eye
Figure 10: CAs of HES distributed in the I, II, III, and IV group according to their NJP
Table 3: Summary of fit parameters to NJP data corresponding to Fig.1; the parameters correspond to the various formulae discussed in the text, Eqs.(2)-(3); HES* and MRA* indicate that the two lowest rank data points corresponding to a strong king and vice-roy effect have been removed before fit attempts; the regression fit coefficient $R^2$ is given for the different cases; data has been rounded up to significant decimals.

|         | 4 param. ZMP, Eq.(3) | 3 param. ZMP, Eq.(2) | $R^2$ |
|---------|----------------------|----------------------|-------|
| NJP     | $\eta$ | $\lambda$ | $c$ | $\nu$ | $b$ | $\mu \equiv \zeta$ |       |
| HES (w) | 0.177 | 0.376 | 165.9 | 0.10 | 335.0 | 0.835 | 0.916 |
| HES* (w) | 1.741 | 0.236 | 230.6 | 7.384 | 1128.0 | 1.098 | 0.987 |
| MRA (w) | 0.523 | 0.558 | 161.7 | 0.876 | 276.25 | 0.975 | 0.988 |
| MRA* (w) | 1.547 | 0.266 | 206.7 | 4.149 | 723.3 | 1.203 | 0.988 |

Appendix A. ZMP fits with 3 or 4 free parameters

Using the 3-parameter free ZMP function, Eq.(2), for data fitting is much more troublesome than fitting with the Zipf hyperbolic law (Fairthorne 1969, Haitun 1982, Iszak 2006). Thus, a variant of the ZMP law, i.e. the 4-parameter relation Eq.(3) is sometimes proposed, since it allows for one more scaling parameter. It is often observed that the 4-ZMP has some advantage with respect to the 3-ZMP, from the point of view of the stability of the solutions of the non linear system of equations for the fit parameters. This is interpreted as due to the fact that the numerical values of the other parameters ($\mu, \eta, \lambda$, and the more so $c$) fall into more compact ranges. For examples, compare the amplitudes $c$ and $b$ for $s_2$ and $s_4$, respectively, in Tables 2 and 4 for the 4-ZMP and 3-ZMP fits.

However nothing drastic has been found in the present cases, as seen from Tables 3 and 5. Moreover, the meaning of $\nu$, in the 3-ZMP case seems more easily interpretable than the $\eta$ and $\lambda$ values in the 4-ZMP.

It should be emphasized that the $R^2$ values are identical, up to the third decimal, for the 3- and 4-ZMP parameter law fits, see Table 4 except for $s_6$ and subsequently $s_{63}$, nevertheless found close to each other, as likely due to an exp.$t$ behavior pointing to a strong exponential tail cut-off, - in which cases the empirical laws can be hardly expected to hold. Thus, it is observed that $\mu \equiv \zeta$ in all cases, - i.e. the relevant conclusion.

Appendix B. On merging sub-fields

In order to investigate the effect of reduced size of data in considering sub-fields, Bougrine (2014) merged 2 sub-fields into a single one, both in the case of MRA and HES. For comparison, and completeness, ZMP and power law fits have been made on $a_4$ and $a_5$ merged into $a_{54}$ on one hand, and on $s_3$ and...
Table 4: Summary of fit parameters to NJP data grouped as for Figs. 5-8, according to different sub-fields of HES ($s_i$) and MRA ($a_i$); the fit parameter values correspond to the 4-ZMP form, Eq. (3); \textit{q.pw.t} indicates the presence of both a strong queen and power law tail effect, while \textit{exp.t} indicates a strong exponential tail cut-off, i.e. cases for which the scaling parameters have large error bars; the regression fit coefficient $R^2$ is given for the different cases; data has been rounded up to significant decimals; error bars $\leq 10\%$.
Table 5: Summary of fit parameter values to $a_2$ and $a_7$ frequency-size cumulative distribution function (CDF) data with notations explained in the text, - corresponding to Figs. 6 and 7, respectively; the parameters correspond to the various formulae discussed in the text, Eqs.(1)-(3); the regression fit coefficient $R^2$ is given for the different cases

| CDF | 4 param. ZMP | 3 param. ZMP | power law |
|-----|--------------|--------------|-----------|
|     | $\eta$ | $\lambda$ | $c$ | $\nu$ | $b$ | $\mu \equiv \zeta$ | $R^2$ | $a$ | $\alpha$ | $R^2$ |
| $a_2$ | 1.117 | 0.735 | 1.154 | 1.515 | 1.626 | 1.120 | 0.970 | 0.603 | 0.943 | 0.960 |
| $a_7$ | 7.127 | 4.408 | 7.759 | 1.597 | 1.584 | 1.143 | 0.996 | 0.898 | 1.032 | 0.932 |

$s_6$ merged into $s_{65}$ on the other hand. The parameters resulting from the fits are given in Table 2. The fits are displayed in Fig. 8. In such cases, with not many data points, the co-author core is low, and the effect of many CAs at rank $r \geq 4$ or 6 respectively is rather important. Thus, the instability of the fits with respect to initial conditions is due to the presence of a strong exponential cut-off superposed on the power law tail.

These features indicate the sensitivity of the sub-field definition, on one hand, and of the co-author distribution, on the other hand.

**Appendix C. On cumulative distribution functions (CDF)**

In Informetrics, one prefers to fit empirical data to some size-frequency functional form using a maximum likelihood fit, rather than making a least squares fit for the rank-frequency distribution. Indeed, one can also ask, as did Pareto (1896), how many times one can find an "event" greater than some size $y$, i.e. study the size-frequency relationship. Pareto found out that the cumulative distribution function (CDF) of such events follows an inverse power of $y$, or in other words, $P[Y > y] \sim y^{-\kappa}$. Thus, the (number or) frequency $f$ of such events of size $y$, (also) follows an inverse power of $y$.

Thus, for illustration, ZMP and power law fits have been made on two of MRA major sub-fields, i.e. $a_2$ and $a_7$. A log-log scale display of the number of joint publications (NJP) with co-authors ranked by decreasing importance and the corresponding CDF are shown in Figs. 11-12. Both the power law and ZMP law fits are shown for the all $r$ range. Note that the NJP data and fits are those seen in Fig. 7 with numerical values in Table 3.

The "queen effect" is well seen on the NJP data and fits, on Fig. 11 but not so much on the CDF. The "king effect" is well seen on the NJP data and fits, on Fig. 12 but the CDF shows a pronounced cut-off at high $r$. Therefore it would seem that the CDF is less pertinent to observe minute effects. This
Figure 11: Log-log scale display of the number of joint publications (NJP) on magnetic materials, for MRA, with co-authors, ranked by decreasing importance, and of the corresponding frequency-size cumulative distribution function (CDF); best fits, over the whole data range are shown for the power law and ZMP law.
Figure 12: Log-log scale display of the number of joint publications (NJP) on superconductivity, for MRA, with co-authors, ranked by decreasing importance, and of the corresponding frequency-size cumulative distribution function (CDF); best fits, over the whole data range are shown for the power law and ZMP law
is understandable since the CDF results from an integration scheme. However, again understandably, the CDF fits are much more stable.

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