Using bar model to solve word problems on profit, loss and discount

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Abstract. With Brunei’s participation in the PISA starting 2018, there is a need to improve students’ mathematics competencies, particularly their problem solving skills. The Bar Model is a distinctive problem-solving strategy introduced in Singapore and is one of the key to Singapore’s success in PISA 2012 Problem Solving. This action research examined the use of Bar Model to help fifty Year 9 students in one government school in Brunei in solving word problems involving profit, loss and discount. Paired-sample \(t\)-test revealed significant difference in the mean score at \(p < 0.05\) between the pre-test (\(M=4.76\)) and post-test (\(M=6.10\)) after intervention lessons. Despite the mean mark of post-test still lower than the passing mark, there is an overall increase in the number of students who improved in the post-test compared to the pre-test. In item-by-item analysis, improvements were observed in most questions; however questions on percentage calculation of profit, loss and discount still indicated small improvement, with two items decreased in the post-test performance. With more practice and guidance, particularly to overcome similar common errors done by students in the post-test, the Bar Model is a valuable problem-solving strategy in attempting word problem question, and help enhance students’ problem-solving competencies.

1. Introduction

Problem-solving is not entirely a new skill, however recently gained significance as a consequence of the rising concern for 21\(^{\text{st}}\) century learning. As modern society is getting more complex, students need to be able to transfer and apply the knowledge and skills they learned in school into real-life situations.

According to the Programme for International Student Assessment (PISA) results from PISA 2012 Problem Solving, Singapore scored the highest in the assessment for problem solving with the highest number of top-performing students in problem solving. The results are due to Singapore’s emphasis on problem solving in its mathematics curriculum, and the introduction of Bar Model [1]. As Brunei will officially participate PISA in 2018, there is a need to implement various teaching initiatives and strategies that will further improve students’ academic performance, particularly in mathematics [2]. This study aimed to investigate the effectiveness of Singapore’s Bar Model as a problem-solving strategy to help enhance Brunei Year 9 students’ problem solving skills.

Majority of teachers in Brunei are still practicing the drill-and-practice algorithmic method accompanied with the keyword strategy or rote memorisation, with little understanding, of the solution methods.
procedure (the formula strategy), to solve mathematical word problems. Some of these teachers are aware of the Bar Model; however, they fail to incorporate this method into their teaching strategies as they are not well-informed of its mechanisms, making them resort to familiar and comfortable formula strategy. With this in mind, this study not only intends to explore the effects of Bar Model on the performance of students, but also as a promising alternative to solve mathematical word problems at secondary level. This study was guided by this research question: How does the Bar Model affect Year 9 students’ performance in solving word problems involving profit, loss and discount?

2. Literature review

2.1. Solving word problems using diagrammatic representations

Four major components of cognitive process in problem solving are representing, planning/monitoring, executing and self-regulating [3]. This is aligned to the distinguished four-stage model for problem-solving proposed by Polya, which are: (i) Understand the problem, (ii) Devise a plan, (iii) Carry out the plan, (iv) Look back to check whether the answer is reasonable [4]. Representing involves a problem solver building a model which represents the situation described in the problem in order to understand the given problem, including the initial state, goal state and the legitimate operations in between the two states [3, 5, 6].

Problem representation is a useful method for improving the mathematics achievements of students with mathematics difficulties [7-10]. Twenty-five experimental and quasi-experimental studies on applying mathematical representation in solving problems to improve students’ performance with learning problems and those at risk of mathematics difficulties were reviewed and evaluated [9]. Results of the review suggest that mathematical representation is the best strategy, as also proven in another study [11]. For all students, the ability to think via mathematical representation is a vital and useful skill [9]. However, without highly effective instructions, representations may be counterproductive.

2.2. Singapore Bar Model

The Bar Model was introduced by the Singapore Ministry of Education to address students who were not attaining the basic numeracy skills and were not able to perform well in solving word problems, which was a national problem in the 1980s [1]. The Bar Model is a systematic method of representing word problems using pictorial rectangular “bars” to help students visualise the relationship before progressing to the abstract where numbers, notation and symbols are used [1, 12].

The revised Brunei’s Framework Mathematics Curriculum for Year 7 and Year 8 advocates the use of diagrams in the learning experience as pedagogy for some of the mathematics topics [13]. The Bar Model has been suggested as a problem solving strategy and part of the secondary mathematics textbooks for Year 7 and Year 8. More specifically, the Bar Model is included in the topic Profit, Loss and Discount in the secondary mathematics textbooks for Year 8, yet majority of secondary teachers chose to not adopt it in their teaching. For this study, the Bar Model introduced in the intervention lessons is adapted from the secondary mathematics textbook for Year 8 and the guide in Bar Model book [1].

There are two main types of Bar Model, (i) the part-whole model and (ii) the comparison model [1]. Both types of Bar Model as visual representations, shown in Figure 1, allow students to have a clearer idea of how the known and unknown quantities are related so that student have better understanding of the word problem.
Figure 1. (a) Part-whole model and (b) comparison model.

Part-whole model, shown in Figure 1(a), is used to solve problems involving addition and subtraction. Instances where students can use the part-whole model include (i) given a part and a part, students add the parts to find the whole and (ii) given the whole and a part, students subtract the part from the whole to find the other part. Comparison model is used to compare two quantities, as shown in Figure 1(b). Like the part-whole model, comparison model is used to solve problems involving addition and subtraction and other complex comparison problem. The comparison model is particularly useful in solving ratio and percentage problems.

In this study, the comparison model is employed as word problems on profit, loss and discount that normally involve addition, subtraction, multiplication, division and computation with percentages. Figure 2(a) shows an example where the comparison model is used to find profit given the cost price and selling price, whilst Figure 2(b) is an example where the comparison model is used to find the percentage profit given only the profit and cost price.

Figure 2. Example of how the comparison model can be used to find profit given the cost price and selling price in (a) and to find the percentage profit given only the profit and cost price in (b).

Figure 3 shows a complex calculation where the comparison model can be used to find the cost price given only the selling price and percentage loss.

Figure 3. An example of how the comparison model can be used to find
the cost price given only the percentage loss (20%) and the selling price ($16).

3. Method
This study involved convenience sampling participants from two Year 9 classes in the Humanities stream, consisting of 50 mixed-ability co-ed students in one co-ed secondary school in the Brunei-Muara district. Quantitative data were collected from students’ numerical scores in the pre-test and post-test, presented and examined using descriptive statistics and graphical representation. Pre-test provided information on students’ prior knowledge and performance in solving word problems involving profit, loss and discount prior to the intervention lessons, whilst post-test examined whether there is an improvement in students’ scores after the intervention lessons. Paired-samples t-test analysed any significant difference between the means of the two tests using Statistical Package for the Social Sciences. SPSS version 20.0. Correct versus incorrect students’ responses of each item were compared to enhance the quantitative result. Qualitative data were collected and analysed from students’ pre-test and post-test answer scripts to identify any emerging themes to determine the correct and incorrect responses that affected and contributed to students’ performance in the post-test.

Both pre-test and post-test is utilised pencil-and-paper written test. Questions on profit, loss and discount in the written tests were adapted from Cambridge O-Level Mathematics D past examination papers and from SPN21 Mathematics Year 8 textbook. There are 15 questions in total and each question is allocated 1 mark, thus the total marks for all correct answers are 15 marks. The post-test contained same questions as in pre-test.

Before the actual study, reliability test of the written test was conducted to a class of Year 9 students which comprised of 20 students who were not involved in the study. Analysis of the reliability test using SPSS, produced a Pearson correlation of 0.580 which indicated that the written tests instruments had moderate reliability [14]. The items used in the test were also validated suitable for the intended participants by two experienced teachers, both with thirteen years of teaching experience at that level.

The intervention lessons for this study consisted of four lessons, focusing on introduction and application of Bar Model on word problem in each of the sub-topics. The lessons were designed in a sequence of increasing difficulties and hence students were expected to enhance their understanding in each lesson so that they will be able to proceed to the next. The lesson intervention lasted a total of 4 hours.

4. Results and discussion
As shown in Table 1, the overall mean score of the post-test is 6.10 which is 1.34 higher than the overall mean score of the pre-test of 4.76, despite both scores not reaching mean pass mark. In addition, based on the paired-sample t-test shown in Table 2, there was a significant difference in the mean scores between the pre-test and post-test at \( p < 0.05 \) level \( t (49) = -3.74, p = .00047 \).

|          | N  | Time | M   | SD  | Min. | Max. |
|----------|----|------|-----|-----|------|------|
| Overall  | 50 | Pre-test | 4.76 | 2.43 | 0    | 11   |
|          |    | Post-test | 6.10 | 2.66 | 2    | 14   |

Moreover, based on the Cohen effect size (.22), the intervention lessons with the Bar Model had a large impact on students’ performance in solving the word problems involving profit, loss and discount [15]. The minimum and maximum mark in pre-test is 0 and 11 respectively, whereas these values increased to 2 and 14 respectively in the post-test. This partially support, overall, there is an increase in the group performance, in terms of the minimum and maximum marks.

Table 1. Descriptive statistics of overall results of the pre-test and post-test for the two classes.
Table 2. The results of the paired-samples T-test.

|       | M   | SD  | t    | df | Sig. (2-tailed) |
|-------|-----|-----|------|----|-----------------|
| Pre–Post | -1.340 | 2.528 | -3.748 | 49 | 0.00047         |

*Note. Statistical significance was set at the p < 0.05 level*

Figure 4 shows the line graphs for the overall number of students according to the marks obtained in the pre-test and post-test. Based on this, there is an overall increase in the number of students who showed improvement in individual marks in solving word problems involving profit, loss and discount after the intervention lessons with the Bar Model, indicated with overall slight shift to the right between the pre-test and post-test. Evidently, the Bar Model helped the students to perform better in solving the word problems involving profit, loss and discount as exhibited in their improved performances during the post-test in comparison to the pre-test, to some extent. The number of students passed the test (8 or above) slightly increased from seven in pre-test to eleven in post-test.

![Figure 4. Line graphs show students’ overall marks obtained in the pre-test and post-test.](image)

It is important to note that the Bar Model used in the intervention was useful, only to some extent, in helping students to solve word problems involving percentage profit, percentage loss and percentage discount. Figure 5 shows the number of correct responses from students in both classes for each items in the pre-test and post-test. The number of correct responses increased in almost all items in post-test compared to pre-test. Item 7, involving question on basic addition/subtraction, remained high in both. Both items 5 and 13 showed big drop, particularly in the former. Some items gained big correct response, but most increase remain low.

![Figure 5. Number of correct responses for each item in the pre-test and post-test.](image)
The changes in the number of responses for each item are summarised in Table 3, as well, the type of mathematical computation involved for each item. Application of Bar Model easily helped questions related to basic addition/subtraction calculation involving profit, loss and discount, whilst those with computation with percentage majority was facilitated by Bar Model, but remained low in overall responses.

**Table 3.** The change, from pre-test to post-test, in the number of correct responses.

| Change in correct responses | Items | Concept involved |
|-----------------------------|-------|------------------|
| Increased                   | 1, 2, 3, 8 and 12 | Addition/subtraction |
| Increased but still remained| 4, 6, 9, 10, 11, 14 and 1 | Computation with percentages |
| low                         | 7     | Addition/subtraction |
| No change                   | 5 and 13 | Computation with percentages |
| Decreased                   |       |                   |

Analysis of pre-test answer scripts revealed that students relied on the formula strategy to solve the word problems in which they either employed incorrectly or forgotten and subsequently did not attempt to solve the word problems. Based on students’ post-test answer scripts analysis, the Bar Model used in the intervention was effective in helping students to solve word problems which mainly involved addition and subtraction, especially word problems involving profit. The Bar Model allowed students to identify and organize the important information in the word problem. Students who labelled the bars correctly with the numerical information found in the word problem were able to identify the known and unknown quantities, as well as, understand the connection between the quantities and variables, allowing them to choose the correct operations to perform. Some of the students’ correct responses are shown in Figure 6.

Further analysis of the post-test answer scripts showed some common errors made by the students when employing the Bar Model. These common errors include mislabelling parts of the model, misapplication of mathematical operations, incomplete answer due to lack of conceptual and procedural knowledge. Evidence from the post-test also showed inaccuracies in drawing and labelling the Bar Model led some students to arrive to incorrect solutions. Surprisingly, partially correct Bar Models could help the student in solving the word problems. These errors were also consistent with previous findings for example research findings [16-17].

Word problems involving percentages were found to be the most difficult to solve among the students who participated in the study. Students commonly either could not recall or were uncertain with the procedural computation to derive the correct solution to the word problems involving percentages.
Figure 6. Responses from different students correctly attempting questions 1, 3, 5 and 7.

Students commonly have difficulties in representing the question with an accurate Bar Model, an example presented in Figure 7(a), also, difficulties in the procedural computation even after correct bar representation as illustrated in Figure 7(b). As a result, majority of the students chose not to attempt some of the word problems involving percentages in both the pre-test and post-test.

![Figure 7](image.png)

(a) Incorrect responses due to (a) wrongly linking the quantity to the Bar Model, and (b) incomplete answer.

5. Conclusion
Paired-sample t-test revealed significant difference in the mean score at \( p < 0.05 \) between the pre-test and post-test after intervention lessons. The overall mean score of the post-test is 6.10 which is 1.34 higher than the overall mean score of the pre-test of 4.76. With the t-test significant result and the Cohen effect size (.22), there was evidence that intervention lessons with the Bar Model had a significant and large impact on students’ performance in solving the word problems involving profit, loss and discount, despite still low mean mark of post-test (lower than passing mark of 8). There was an overall increase in the number of students who improved in the post-test compared to the pre-test, evident with increase marks of most students, in particular increase in number of students passing the test from seven to eleven.

When analysing the pre-test and post-test, most questions showed increase in number of correct responses, particularly those related to basic addition/subtraction calculation of profit, loss and discount. However questions on percentage calculation of profit, loss and discount still indicated small improvement, with two items decreased in the post-test correct responses. Analysis of the post-test scripts showed there are common errors made by the students when employing the Bar Model. As students are still novices in employing the Bar Model, some of the students’ difficulties are also reflected which may be due to their lack of mathematical understanding. However, overall, with more practice and guidance, the Bar Model could be a very helpful problem-solving tool that promotes students’ performance in solving mathematical word problems. Some previous local studies on the Bar Model also concluded that the Bar Model helped improve students’ performance in solving word problems [17-18].

6. Implications to learning and teaching
Bar Model has the potential to raise students’ understanding and performance in solving mathematical word problems. Teachers should consider employing the Bar Model in their classroom instruction to improve students’ performance in word problem, possibly of any mathematics topics, as it is aligned with Polya’s four-stage model for problem solving. With the Bar Model, students draw bar diagrams so that they can make sense of the word problem by organizing the given information in a visual form that may lead them toward solutions [19]. Students are able to plan, execute and check the reasonableness of their solution plan with the Bar Model. Using the Bar Model that utilises schematic
diagrams, it allows the students to move beyond rote memorization to developing deeper understanding of the mathematical problem and adopt flexible solution strategies [20-22].

The study also raised some problems encountered by the students in using the Bar Model. Due to the visual nature of the Bar Model, the models drawn by the students help to expose the nature of difficulties students faced with understanding word problems. Although it was not the intention of the study to investigate the issues associated with using the Bar Model, this side findings can help teachers to prepare on how to tackle these issues.

This study also revealed students’ gaps in conceptual and procedural knowledge on the topic of percentages. Percentage is introduced in Year 5 mathematics curriculum in Brunei and continuously revisited throughout higher levels. The percentage word problems in this study were only limited to the context of profit, loss and discount, and some students applied Bar Model successfully. Bar Model, or any instructions that involves visual representations, should be considered in teaching the topic on percentages, to improve students’ conceptual and procedural knowledge on those topics, as proven by a previous local Bar Model research [17].

Findings of the study provide useful information for mathematics teachers about the importance of using alternative approaches to help enhance the Year 9 students’ mathematics competencies and problem solving skills [23-26]. Teachers need to steer away from the traditional chalk-and-talk teaching that lacks in fostering meaningful learning or transferrable problem solving skills [27-30]. The game Sudoku can be used as tool to develop students’ pattern discovery skill which is highly valuable in problem solving task [31]. Visual approach by using graphic organizers as an innovative approach in handling word problems in school in Brunei was another successful strategy [32]. These teaching initiatives, not only will help cultivate students’ mathematics competencies, but also boost their confidence especially in solving mathematical problems.

7. Recommendations
Due to the small number of sample, the result of this study cannot be generalised to the Year 9 population of Brunei. Nevertheless, the findings of this study provide a promising ground in understanding the effectiveness of the Bar Model. The findings can be used as reference for further in-depth research into the Bar Model under new student profiles, problem types, and study topics.

Future studies should consider conducting more intensive and multifaceted intervention lessons focusing on students’ conceptual understanding of diagrams in the Bar Model, generating the diagrams, use of the diagrams as tools to express their mathematical ideas. In addition, future study should consider continued research on the long-term effectiveness of the Bar Model. Moreover, future study could be made exploring the problems encountered by the students associated with the use of Bar Model in solving word problems and how to confront those problems. Since the study only investigated the effectiveness of the Bar Model on one Year 9 mathematics topic, future research should consider investigating the use of Bar Model with different mathematics topics and grade levels.

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