MAGNETOHYDRODYNAMIC WAVES IN TWO-DIMENSIONAL PROMINENCES EMBEDDED IN CORONAL ARCADES

J. Terradas, R. Soler, A. J. Díaz, R. Oliver, and J. L. Ballester
Departament de Física, Universitat de les Illes Balears, E-07122 Palma de Mallorca, Spain; jaume.terradas@uib.es
Received 2013 July 31; accepted 2013 September 23; published 2013 November 1

ABSTRACT
Solar prominence models used so far in the analysis of MHD waves in two-dimensional structures are quite elementary. In this work, we calculate numerically magnetohydrostatic models in two-dimensional configurations under the presence of gravity. Our interest is in models that connect the magnetic field to the photosphere and include an overlying arcade. The method used here is based on a relaxation process and requires solving the time-dependent nonlinear ideal MHD equations. Once a prominence model is obtained, we investigate the properties of MHD waves superimposed on the structure. We concentrate on motions purely two-dimensional, neglecting propagation in the ignorable direction. We demonstrate how, by using different numerical tools, we can determine the period of oscillation of stable waves. We find that vertical oscillations, linked to fast MHD waves, are always stable and have periods in the 4–10 minute range. Longitudinal oscillations, related to slow magnetoacoustic-gravity waves, have longer periods in the range of 28–40 minutes. These longitudinal oscillations are strongly influenced by the gravity force and become unstable for short magnetic arcades.

Key words: magnetic fields – magnetohydrodynamics (MHD) – Sun: corona

Online-only material: color figures

1. INTRODUCTION
It has been well known since the 1960s that solar prominences and filaments show oscillations (see Ramsey & Smith 1966; Hyder 1966). Many of these oscillatory motions have been classified as large-amplitude oscillations. A clear example of this kind of oscillation is the global motion found in winking filaments. These global oscillations are normally induced by nearby sub-flares or jets, Extreme ultraviolet Imaging Telescope (EIT) waves and Moreton waves. The reader is referred to Tripathi et al. (2009) for a review about observations of large-amplitude oscillations in prominences.

The MHD eigenmodes of oscillation of prominences in simple geometries as Cartesian slabs and cylindrical magnetic tubes have been studied in the past by many authors (see the reviews of Oliver & Ballester 2002; Oliver 2009; Mackay et al. 2010; Arregui et al. 2012). These studies have focused on small-amplitude oscillations using the linearized version of the MHD equations. Only recently, Blokland & Keppens (2011b) have attempted to understand localized MHD oscillations in more realistic configurations. The aim of our work is to study global oscillations using improved prominence models that are numerically constructed using a relaxation method. At this stage we are not interested on the internal fine structure of prominences and our focus is mainly on the global behavior and the possible link with winking filaments.

An issue that arises in complex magnetic topologies under the presence of gravity is stability. Here we conduct an MHD stability study that enables us to understand the stable/unstable nature of quiescent prominences. The stability analysis is not simple. One method to study the stability properties of the new equilibria is to compute the full ideal or resistive MHD spectrum by solving the linearized MHD equations. Another alternative is to consider the time-dependent problem by solving the nonlinear or the linearized MHD equations. This last approach, successfully used in the past in linear stability analysis of, for example, coronal arcades (see An et al. 1989), is adopted in the present work. There are other possibilities such as the variational or energy method (see Bernstein et al. 1958), which is based on the minimization of the second order change in the potential energy of the system when plasma elements are displaced from their equilibrium position. Another alternative is to use magneto-frictional methods (see Yang et al. 1986) based on the assumption that field lines move through a stationary medium. This method has been successfully used in the determination of the nonlinear force-free coronal field in response to the evolution of the photospheric magnetic field (see, e.g., Mackay & van Ballegooijen 2006, 2009).

It is worthwhile to mention that Galindo Trejo (1987) performed a detailed numerical stability analysis of two-dimensional (2D) prominence models based on known analytical magnetohydrostatic (MHS) solutions at that time (Kipphahn–Sclüter, Dungey, Menzel, and Lerche and Low models). However, the connection of the prominence magnetic field with the photospheric magnetic field was essentially missing in his analysis. In this work, we properly address this point, which turns to be very relevant regarding the stability of prominences. Later, de Bruyne & Hood (1993) demonstrated that the model of Low (1981) is unstable to localized disturbances and that the Hood & Anzer (1990) model is only stable for sufficiently low prominences.

In the literature, many prominence models have been proposed (see the review of Mackay et al. 2010). A popular model is the magnetic flux rope configuration. Using this configuration, Low & Zhang (2004) found analytical solutions using a polytropic model in a circular cylinder whose weight is supported by an external magnetic field. Later, Petrie et al. (2007) demonstrated how to numerically calculate MHS equilibria with properties close to realistic prominences, namely a cool, dense prominence surrounded by a cavity within a flux rope in a coronal environment. Blokland & Keppens (2011a) solved an extended Grad–Shafranov equation and, using a finite element-based code, were able to obtain numerical equilibria. Blokland & Keppens (2011b) used these equilibria to analyze
the continuous spectrum of modes of the structure. These authors focused on the modes of the core of the prominence rather than on the modes of the global structure since the 2D flux rope considered in their studies does not curve down and meet the photosphere (a three-dimensional (3D) model is required to fulfill this condition in flux rope models).

In this work, we avoid geometries with detached magnetic field lines, i.e., we study configurations with all field lines tied to the lower boundary. Detached models are usually considered in the study of 2D twisted flux rope prominence models. We prefer to concentrate on configurations that connect magnetic field lines to the photosphere. This can be also achieved considering twisted flux ropes in 3D and using, for example, toroidal geometries. Nevertheless, we think that it is more convenient to start with the investigation of the 2D problem rather than with the full 3D problem, which is also more complicated from the technical point of view. Additionally, we want to address the role of magnetic dips in the structure and also the dynamics of prominences. For this reason, we chose a topology that includes magnetic dips that are able to provide suitable conditions to support the cool plasma against gravity (see, e.g., Demoulin & Priest 1993; Aulanier et al. 2002; López Ariste et al. 2006; Mackay et al. 2010).

The purpose of this paper is first to construct prominence models by computing MHS solutions of the MHD equations. We seek prominence models that are bounded in the 2D plane and have a cool core with respect to the external coronal environment, meaning that the structure is non-isothermal. We focus our attention on models that describe both the prominence and the surrounding coronal environment under the presence of gravity. Here the prominence model is constructed using a relaxation process instead of the direct solution of the Grad–Shafranov equation. Mass is injected on an initial background equilibrium and the system is allowed to evolve toward a new equilibrium. We are not aiming to study the formation process itself. Instead, we are interested in the final MHS solution. The second goal of this work is to study the properties of MHD waves in the numerically generated prominence models. The time-dependent problem is solved numerically. An initial perturbation is introduced in the system and excites different kinds of oscillations that might be stable or unstable.

2. INITIAL CONFIGURATION AND SETUP

2.1. Background Equilibrium

The primary equilibrium is an isothermal stratified atmosphere permeated by a force-free magnetic field. Using a Cartesian coordinate system, with the \( z \) coordinate pointing in the vertical direction, the density profile is

\[
\rho = \rho_0 e^{-z/\Lambda},
\]

where \( \Lambda = c_0^2/\gamma g \) is the density scale height and \( \rho_0 \) is the coronal density value at the reference level \( z = 0 \) representing the photosphere or base of the corona. The sound speed, defined as \( \sqrt{\gamma p_0/\rho_0} \), takes a value of \( c_0 = 166 \text{ km s}^{-1} \) for a coronal temperature of \( 10^6 \text{ K} \). The gravity acceleration on the solar surface is \( g = 0.274 \text{ km s}^{-2} \) and for a monoatomic gas \( \gamma = 5/3 \). Hereafter, we choose a spatial reference length of \( H = 10^4 \text{ km} \) (the typical length of prominences), meaning that the density scale height is \( \Lambda \approx 6H \).

The initial potential force-free magnetic field considered in this work is based on superposition of arcade solutions.

The arcade configuration has the following magnetic field components:

\[
B_x(x, z) = B_0 \cos kx e^{-kz},
\]

\[
B_z(x, z) = -B_0 \sin kx e^{-kz},
\]

where \( B_0 \) is the magnetic field strength at the reference level. The parameter \( k \) is related to the lateral extension of the arcade \( (\pi/(2k)) \) and is also a measure of the vertical magnetic scale height. The \( B_y \) component is zero in the present work.

The magnetic field lines in the configuration given by Equations (2) and (3) do not have any dips because the magnetic structure is bipolar. Since we are interested in a configuration with dips for the reasons explained in the Introduction, we select a particular superposition of two magnetic arcades that mimics a quadrupolar configuration

\[
B_x(x, z) = B_1 \cos k_1 x e^{-k_1z} - B_2 \cos k_2 x e^{-k_2z},
\]

\[
B_z(x, z) = -B_1 \sin k_1 x e^{-k_1z} + B_2 \sin k_2 x e^{-k_2z}.
\]

The individual arcade solutions are quoted with the sub-indices 1 and 2. The width of the full structure is \( 2L \) and we select the following wavenumbers \( k_1 = \pi/(2L) \) and \( k_2 = 3\pi/(2L) \). The strength of the magnetic field at \( z = 0 \) of each arcade is \( B_1 (\geq 0) \) and \( B_2 (\geq 0) \). From the superposition of the two configurations, it is easy to show that at \( z = 0 \) the total magnetic field has a maximum value at \( x = \pm L \) of \( B_{\text{max}} = B_1 + B_2 \).

An example of the magnetic configuration for the case \( B_2 = B_1 \) is shown in Figure 1. At the center of the magnetic configuration \( (x = 0) \), there is an \( x \) point where the magnetic field in the \( xz \) plane is zero. In the example of Figure 1, the location of this point is at \( z = 0 \). In general, it can be shown that the height of the \( x \) point is

\[
z_X = \frac{1}{k_2 - k_1} \ln \frac{B_2}{B_1}.
\]

In this work we always impose that the \( x \) point is at the photospheric level meaning that \( B_2 = B_1 \).

Figure 1. Magnetic field lines based on a quadrupolar magnetic field. In this plot, \( B_2 = B_1, k_1 = \pi/(2L), \) and \( k_2 = 3\pi/(2L) \) have been used in Equations (4) and (5). Here \( L = 5 H, H \) being the reference length \( (10^4 \text{ km}) \). Solid curves correspond to the case without a dense prominence, while dashed curves are the new equilibrium structure after the dense material, representing a prominence, has been injected.
To obtain a model resembling a real prominence, a dense and cool plasma is required. In the present work we are not concerned about the actual physical process that provides mass to the prominence during its formation. Instead, here we are interested in finding an equilibrium configuration for the prominence and we do not care about the actual process that drives the formation (see Xia et al. 2011, 2012; Luna et al. 2012b for recent results about the formation process).

A simple way to generate the body of the prominence is to add mass at a given location in the preexisting magnetic configuration. We model the mass injection by artificially adding a source term in the continuity equation. This term has the following form

$$S = \hat{\alpha} \sin \left( \pi \frac{t}{t_m} \right) e^{-\left( (x/w_x)^2 - (z/z_0/w_z)^2 \right)}.$$ \hfill (7)

The parameter \( \hat{\alpha} \) represents the rate of mass injection and \( t_m \) is the total injection time. For \( t > t_m \), the source term is set to zero. The parameters \( w_x \) and \( w_z \) represent the characteristic spatial size of the source in the \( x \) and \( z \) directions respectively. The central point of the injection is located at \( x = 0 \) and \( z = z_0 \).

From Equation (7), it is straightforward to calculate the total mass injected in the system. Note that the configuration studied in this work is 2D and thus unbounded in the \( y \) direction. For this reason, it is more convenient to calculate the total mass of the prominence per unit length, \( M/L_y \). After the injection phase we have

$$M/L_y = \frac{2\hat{\alpha} t_m}{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-\left( (x/w_x)^2 - (z/z_0/w_z)^2 \right)} dx \, dz.$$ \hfill (8)

From observations it is possible to roughly estimate the total mass of a real prominence. The length along the main axis is usually a known parameter. For a real prominence it can be shown that \( M/L_y \sim 5 \times 10^5 \text{ kg km}^{-1} \). To get this value we have used a typical prominence density of \( 5 \times 10^{-2} \text{ kg km}^{-3} \), while for the spatial dimensions we chose that a width of \( 10^3 \text{ km} \), a height of \( 10^4 \text{ km} \), and a length (\( L_y \)) of around \( 10^4 \text{ km} \). In our computations we use the length \( H = 10^4 \text{ km} \), already introduced before, as the reference length. Another important parameter in our model is the size of the arcade (\( 2L_y \)) that provides the magnetic support. We assume, based on observations, that the typical length of the magnetic field lines are of the order of \( 10^3 \text{ km} \). For a given value of \( M/L_y \) and size of the prominence (\( w_x \) and \( w_z \)), the product of \( \hat{\alpha} \) and \( t_m \) is determined using Equation (8). From the practical point of view we fix the parameter \( t_m \) and calculate \( \hat{\alpha} \) for a given prominence mass and size. The reference time scale in this work is defined as \( t_A = H/c_{s0} \), which for \( H = 10^4 \text{ km} \) and \( c_{s0} = 166 \text{ km s}^{-1} \) is around 1 minute.

\section*{3. NUMERICALLY GENERATED MHS PROMINENCE MODELS}

\subsection*{3.1. Numerical Tools}

Using the initial background model, explained in Section 2.1, the injection of mass is performed employing the specific profile described in Section 2.2. The nonlinear ideal MHD equations are advanced in time numerically using the code MoLMHD (see...
Bona et al. 2009; Terradas et al. 2008 for details about the numerical method). The source term included in the continuity equation provides the mass required to generate a prominence model. It is important to mention that although the full nonlinear equations are solved, we only evolve in time perturbations on the background magnetic field (Powell et al. 1999). It turns out that this numerical technique is crucial to obtain new MHS models for low plasma-\(\beta\) problems.

Boundary conditions are treated using a decomposition in characteristic variables at the edge of the computational domain. The different fields are recalculated at the boundary by imposing conditions on the incoming fields. Line-tying conditions are applied at \(z = 0\), meaning that incoming fields are set to be equal to outgoing fields. For the rest of the domain, flow-through conditions are applied imposing that incoming fields are set to zero. Different sizes of the domain in the \(x\) and \(z\) directions have been considered but we have found that the results do not significantly depend on the extension of the computational domain. The use of conditions on the characteristic fields ensured minimal reflections from the lateral and top edges of the domain, and perfect reflection at the bottom of the computational box. Moreover, in order to obtain solutions that are close to the static stationary state for some specific cases, we have used the decomposition in characteristic variables to eliminate perturbations in the system. In particular, we have found that imposing flow-through conditions only on slow MHD modes and on the entropy mode helps the system to relax faster to the stationary state. In this case, line-tying conditions have been applied to fast and Alfvén MHD waves at \(z = 0\).

A linearized version of the code has been also used in the study of linear MHD waves. We have found that in this case it is not necessary to use decomposition in characteristic fields at the boundaries since simple reflection conditions at the photosphere and flow-through conditions at the lateral and top edges perform well.

The simulations have been carried out using grids of typically \(400 \times 400\) points in the \(x\) and \(z\) directions. We have found that the results converge if the resolution is raised above the previous numbers. The code \texttt{MoLMHD} runs in parallel using MPI and the computational facilities of the Solar Physics Group at UIB have been used to perform the simulations. The simulation time for a run with 32 processors is around 2 hours.

3.2. Properties of the Generated MHS Equilibria

We start by analyzing the results of the simulations, and we concentrate first on the maximum density in the system localized at the center of the prominence core. For the present case, the location of this maximum is not far from the center of the mass deposition (located at \(x = 0\) and \(z = z_0\)) but for configurations with higher \(\beta\) the maximum is located at lower heights. The results are plotted in Figure 3 (see continuous curve). The maximum density grows smoothly with time due to the source term in the continuity equation. This source term is set to zero for \(t > t_m\) (\(t_m = 20\tau_A\) in the present case). At this stage, \(\rho_{\text{max}}\) shows an almost constant value, suggesting that the system has reached or it is very close to a new equilibrium state. The maximum velocities, not shown here, are also rather small in the whole computational domain, typically of the order of \(0.5\) km s\(^{-1}\).

In the example shown in Figure 3 we have not eliminated the reflection of slow MHD waves at \(z = 0\). Therefore, oscillations in \(\rho_{\text{max}}\) after the injection phase are mainly due to the excitation of small-amplitude slow MHD waves. In Figure 3, the results of a case with \(t_m' = t_m/2\) and \(\hat{\alpha}' = 2\hat{\alpha}\) are also displayed (dashed line). According to Equation (8) the mass per unit length of the prominence is the same for the two simulations. From Figure 3, we see that now the oscillatory behavior has a larger amplitude, indicating a stronger back reaction of the system to the newly added mass. A fast mass injection produces a more complex relaxation of the configuration, since MHD waves with higher amplitudes are excited in the system. These oscillations are related to the eigenmodes of the configuration and will be studied later. As we are not intending to model the prominence formation itself, we choose the parameter \(\hat{\alpha}'\) in such a way that we get to the stationary state without large-amplitude oscillations involved in the transient phase.

In Figure 4, top panel, the 2D density distribution is plotted at a fixed time (\(t = 40\tau_A\)) in a reduced spatial domain of \(4 H \times 4 H\). The injected mass has been redistributed in the structure and the system is close to a new equilibrium, as Figure 3 already indicates. For this particular simulation, the total mass per unit length is around \(5.4 \times 10^5\) kg km\(^{-1}\) and it is of the same order as the reference value given in Section 2.2. During the relaxation process the gas pressure has changed and the Lorentz and gravitational forces have been adapted to the newly added mass. For comparison with the initial magnetic field, in Figure 1 we overplot the new magnetic field configuration (dashed curves). The deformation of magnetic field lines is clear. Due to the presence of the heavy prominence, the magnetic field has been pushed down mostly at the location of the enhanced density. The deformation of the magnetic structure is not big, but enough to support, together with the pressure force, the prominence. The dense material produces a depression in the Alfvén and sound speeds. Note that the generated prominence is bounded in the \(z\) direction from below and from above, a feature that is not easily implemented in theoretical models, with the exception of Fiedler & Hood (1992) who presented numerical examples of 2D quiescent prominences with normal polarity modeled by a cool isothermal slab of finite width and height.

The details about the distribution of the forces in the vertical direction when the system is close to an equilibrium are
Figure 4. Density distribution, red color scale, representing a prominence in the stationary state. Dashed curves are iso-contours of temperature while continuous curves are the magnetic field curves. In the top panel the configuration corresponds to the plasma-β distribution shown in Figure 2 (bottom panel), while in the bottom panel the plasma β is two times larger. (A color version of this figure is available in the online journal.)

Figure 5. Vertical component of the forces at x = 0 as a function of height at the initial (top panel) and close to the final state (bottom panel). The solid curve corresponds to the total Lorentz force, dashed curves represent the magnetic tension, and dot-dashed curves correspond to the magnetic pressure gradient. The gravitational force is plotted with dots and the pressure force with triple dot-dashed curves.

(around 20% in Figure 5). Just below the prominence, body tension and magnetic pressure forces are still larger than in the initial state, since the prominence has pushed down the magnetic configuration increasing the depth of the dips of the structure.

The distribution of the forces in the horizontal direction is represented in Figure 6. Initially (top panel), the total Lorentz force is zero and, since gas pressure is constant in the horizontal direction, the pressure gradient is also zero. Once the mass has been injected, the magnetic pressure gradient and the gas pressure gradient change inside the prominence in such a way that they balance each other.

The temperature at the center of the prominence after the mass injection is much lower than the initial temperature, while the gas pressure at the center of the prominence does not change much with respect to the initial value, around 14% (see the temperature and density distribution in Figure 7). This can be understood from the behavior of the gas pressure: if pressure is essentially constant at the center of the prominence, then the dense part of the prominence must be cooler than the light and hot coronal environment since $\rho_P T_P \approx \rho_c T_c$. The temperature profile is not imposed in the simulations and is self-adjusted during the injection phase. A cut of the density,
temperature, and pressure across $z = 2H$ is plotted in Figure 7 (top panel). In this simulation the density reaches a maximum value around 95 times the coronal density, while the temperature should have, according to the previous expression, a minimum value around 100 times lower than the coronal temperature. The exact value for the temperature minimum at the core of the prominence is 11,491 K, which is of the same order of the temperatures typically inferred from observations. In the bottom panel of Figure 7, the variation of density, temperature, and pressure is plotted as a function of height. The density enhancement representing the prominence is superimposed on the exponentially decreasing profile of the background model. The size of the prominence core and the corresponding PCTR is determined by the form of the source term and the parameters $w_x$ and $w_z$. Note that in the present model we ignore the effect of thermal conduction, which may have an important impact on the shape of the PCTR. Effects due to radiative processes are also neglected in the present model.

Although the initial magnetic field is potential the extra mass added to the system makes the configuration non-potential. In Figure 8, the current density in the $y$ direction is displayed. This variable peaks at the core of the prominence and reflects the fact
that now the magnetic field distribution has been modified. It is interesting to note that the distribution of the current is very similar to the density distribution (compare Figure 8 and the top panel of Figure 4).

A change in the strength of the magnetic field produces a different final equilibrium. In Figure 4’s bottom panel, density, temperature, and magnetic field distribution are plotted for a plasma $\beta$ two times larger than that of the case shown in the top panel. Now the deformation of the field inside the prominence is more pronounced since the Alfvén speed is $\sqrt{2}$ times smaller, meaning that tension and magnetic pressure forces are weaker. The injected mass is able to strongly modify the magnetic field configuration, producing a magnetic structure more curved at the core of the prominence. In fact, the center of the core of the prominence is located at a lower height in comparison with the upper panel of Figure 4. The density at the core of the prominence is also larger. The process of relaxation to this configuration lasts longer than that for the lower $\beta$ case. We have performed other experiments changing the plasma $\beta$ and found that for high values of this parameter, very strong shock waves are generated, indicating that the initial configuration is still far from an equilibrium state. It is easier to obtain a new equilibrium if the plasma $\beta$ is low. This is in agreement with the recent work of Hillier & van Ballegooijen (2013; see also An et al. 1988; Fiedler & Hood 1992).

Another relevant parameter in the models is the length of the arcade $(2L)$ in which the prominence is embedded. In Figure 9 the magnetic field lines crossing the center of the body of the prominence are plotted for two different values of $L$. In this example the large arcade is three times wider than the short arcade. We have considered that $L$ is in the range of $2$ to $9 \times 10^4$ km in this work. For each value of $L$ in this range, we have calculated the total length of the magnetic field line crossing the prominence center, $L_\text{fl}$. The relationship between $L$ and $L_\text{fl}$ is plotted in Figure 10 and will be used later.

In Figure 9, notice that the dip in the magnetic field lines is quite different in size for the two prominence models, and this can have important implications regarding stability, as we will discuss later. These two models have quite different lengths of the field lines and different variations of the equilibrium magnitudes. Alfvén and sound speeds are plotted in Figure 11 as a function of the $x$ coordinate for the two field lines represented in Figure 9. In this plot we see that for the narrow arcade, the Alfvén velocity variation is stronger than for the wide arcade, and that near the prominence body, characterized by the depression around $x = 0$, the Alfvén speed in the external medium is lower for the narrow arcade. On the other hand, the profile of the sound speed is quite similar for the two models. It is important to realize that the variation of the Alfvén speed along the field lines is due to the change in both the modulus of the magnetic field and density. In Figure 11, the Alfvén speed at the $z = 0$ level has been set to $v_{A0} = 10c_0$, i.e., $v_{A0} = 1666$ km s$^{-1}$. This value determines a strength of the magnetic field at the base of the corona of 10 G. Due to the variation with height of the quadrupolar magnetic configuration, the magnetic field strength decreases up to a value of around 2 G at the core of the prominence. This is a limitation of our model because to have higher values of the magnetic field at the prominence body requires a significant increase of the Alfvén speed, which is most likely unrealistic. Here we have considered that at most $v_{A0} = 20c_0$, and this leads to a value of 4 G at the prominence center.

Finally, it is worthwhile to mention that three different values for the total mass of the prominence have been considered in this
work. The values of $M/L$ are 5.4, 2.3, and $1.3 \times 10^5$ kg km$^{-3}$. This will allow us to analyze the effect of the total mass on the periods of oscillation of the different prominence models.

4. OSCILLATIONS IN THE NUMERICALLY GENERATED MODELS

The process of mass injection yields to the excitation of waves in the configuration. Some of these waves have a strong leaky character and leave the system quite quickly. However, there are other waves that are clearly associated with oscillations of the density enhancement and are analyzed here using the velocity field. A clear example of periodic oscillations is found in Figure 12. In this plot, the vertical component of the velocity is plotted at a fixed point near the center of the prominence. We observe a substantial variation of the amplitude, which is associated to the mass injection process, followed by short-period waves. This periodic oscillation is damped with time. In fact, vertical oscillations take place before the new equilibrium is reached, so that the prominence is still moving downward. In Figure 12, there is a small global negative velocity shift in the vertical velocity after the injection phase, meaning that the whole structure is not yet oscillating around the final equilibrium position, although it is quite close to it since the drift tends to zero for long times. For this reason, we think that, to better understand waves in the configuration, it is more convenient to study oscillations once the prominence is in equilibrium. Hence, we let the system evolve for long times until it relaxes to the stationary state. Then we have two possibilities: either we introduce a perturbation in the system using the full nonlinear MHD equation, or we simply focus on the linear problem by solving the linearized MHD equations around the final equilibrium. Here we adopt the last approach.

4.1. Vertical Oscillations

A particular prominence model is first selected. Using the linear code, a perturbation at the prominence body is introduced in the vertical direction at a given instant ($t = 0$). For simplicity the spatial dependence of the velocity perturbation is the same as in Equation (7). In Figure 13, we find the 2D distribution of the velocities at a given time of the evolution. The initial disturbance excites mainly fast MHD waves since the motion is dominated by the components normal to the magnetic field lines, while the parallel component is rather small. In a low plasma $\beta$, normal/parallel motions are typically related to fast/slow MHD waves. We have used the following relations to calculate the velocity components:

$$v_n = v_x \frac{B_z}{B} - v_z \frac{B_x}{B},$$

$$v_\parallel = v_x \frac{B_z}{B} + v_z \frac{B_x}{B}.$$ (9) (10)

The velocity field indicates that the prominence is oscillating as a whole, mostly in the vertical direction. The spatial distribution of $v_n$ has a maximum at the location of the prominence core, but it shows a tail extending in the $z$ direction.
Hereafter, we mostly focus on the period of oscillation. By performing a periodogram of the signal at the center to the prominence, we compute the dominant period of oscillation. This has been repeated for different equilibrium models. One of the parameters that has been changed is $L$ (half the length of the arc). In Figure 14, the period of the vertical mode is plotted as a function of $L$ (the relationship between $L$ and $L_\alpha$ is found in Figure 10). Other important parameters have been also changed, the different line styles correspond to the three different total masses of the prominence considered in this work, with thin lines corresponding to $v_{A0} = 10 \, c_{A0}$ and thick lines to $v_{A0} = 20 \, c_{A0}$, i.e., associated with values of the magnetic field at the prominence core of 2 and 4 G, respectively.

Several conclusions can be extracted from Figure 14. We see that the period of oscillation does not have a very strong dependence on $L_\alpha$. For small $L_\alpha$, the periods are slightly longer than for large $L_\alpha$, and in this last regime the period attains an almost constant value. We also see that, as expected, the period increases when the total mass of the prominence is raised. For example, for large $L_\alpha$, the differences in period between the lightest prominence (5 minutes) and the heaviest prominence (8 minutes) is only around 3 minutes for the case $v_{A0} = 10 \, c_{A0}$. Figure 14 also indicates that the periods for $v_{A0} = 20 \, c_{A0}$ are shorter than those for $v_{A0} = 10 \, c_{A0}$ by a factor of around two. These results suggest that there might be a simple relationship of the period with the different equilibrium parameters. To investigate this point, we compare the results of our simulations with simple analytical models whose explicit dependence of the period on the equilibrium parameters is known.

The first model we can use for the comparison is the infinite straight slab with a transverse constant magnetic field. If the prominence body has a length $L_p$ and the total length of the field lines is $L_T$, then for the situation $L_p \ll L_T$ the period is (see, e.g., Joarder & Roberts 1992; Díaz et al. 2010; Soler et al. 2010)

$$ P \simeq \frac{1}{\nu} \sqrt{(L_T - L_p) \nu}, $$

where $\nu$ is the typical speed in the prominence body. In the case of fast MHD oscillations, $\nu = v_{A0}$. We have calculated from the results of the simulations the Alfvén speed at the center of the prominence body and have also estimated $L_p$ and $L_T$ ($L_p \approx w_x$ and $L_T \approx L_\alpha$). Using these values, the period is calculated from Equation (11). The results are shown in Figure 15 with dotted curves. These curves have to be compared with the results of the period inferred from simulations shown in solid curves (thin curves correspond to $v_{A0} = 10 \, c_{A0}$ while thick curves represent $v_{A0} = 20 \, c_{A0}$). There is a difference of more than a factor of four between the simple analytical periods and the ones inferred from the solution of the full problem. This indicates that the simple slab model is not a good representation of our prominence model. Among other things, an infinite extension of the prominence in the vertical direction is assumed in Equation (11).

A way to improve the comparison is to evaluate the role of a finite height of the slab on the period of oscillation. To do this, we have to recall the results of Díaz et al. (2001), who studied the problem of fast MHD waves in a finite 2D slab representing a prominence thread. The magnetic field in their model is constant and density changes abruptly between the corona and the prominence. The model does not take into account the effect of gas pressure ($\beta = 0$) and gravity and curvature of the magnetic field are neglected. The authors derived a general dispersion relation based on an infinite system of homogeneous algebraic equations. For comparison purposes, we have used the method of Díaz et al. (2001) to calculate the eigenfrequencies of oscillation using the equilibrium values derived from our simulations, which include many effects that are still missing in the model of these authors. The results of these calculations are shown with dashed lines in Figure 15. We see a significant decrease of the period in comparison with the unbounded slab (around a factor of two). Notice that the behavior for large $L_\alpha$ for the finite slab is very similar to the profile of the numerically calculated periods for the full case. Therefore, the finite slab model incorporates an effect that improves the comparison with the full numerical case. Nevertheless, the differences with respect to the numerical simulations remain significant. One of the assumptions of the finite slab problem is that $\beta$ is zero, which
is not the case for the full problem. A simple way to assess the effect of finite $\beta$ is to compare the results for the two values of the Alfvén speed used in this work. Figure 15 shows that the difference between the periods of the finite slab model and the numerical results decreases as $\beta$ is decreased. For $v_{A0} = 10 \, c_s$, this ratio of the periods is typically 2.5, while for $v_{A0} = 20 \, c_s$, it is around 2.

The effect of curvature of the magnetic field can also play a role in the discrepancy between the finite slab model and the full problem. In this regard, it was shown by Díaz et al. (2006b) and Díaz (2006) that curved loops with cylindrical and elliptical geometries have periods for the fundamental vertical fast mode that are always smaller than that of the equivalent straight models. This is in agreement with the behavior found in Figure 15. Although the results of Díaz et al. (2006b) and Díaz (2006) are for a tube that is fully filled and here we consider a tube that is only partially filled by the prominence core, these theoretical works point out that curvature typically lowers the period in this sort of 2D configurations. It is interesting to point out that in curved configurations, most of the modes have a leaky character and the global fast mode studied here is not an exception. More details about this issue are given in Section 4.3.

So far, the analytical modes used in our comparison assume that the equilibrium magnitudes are piecewise constant along the magnetic field. This is certainly not true in our numerical prominence models (see, for example, Figure 11) and this might introduce also some changes on the period of oscillation. Unfortunately, the investigation of this effect is not straightforward since up until now the analytical works in this direction have been mainly focused on straight and fully filled cylinders representing coronal loops (see, e.g., Andries et al. 2005a, 2005b; Díaz et al. 2006a). In any case, it is known that, at least for the fundamental fast MHD mode, what really matters regarding the period of oscillation is the value of the equilibrium magnitudes around the center of the structure, while the changes around the footpoints are less important because of the line-tying boundary conditions.

Gravity has been ignored in the analysis of prominence oscillations in most of the previous analytical works. Here we have a simple way to check the role of the gravity force on fast MHD waves. Since the periods of oscillation are calculated using the linearized set of equations, we have compared the values of the periods when the gravity term is included and when it is set to zero. The differences are really small, of the order of 2% only, and this leads us to conclude that the effect of gravity on fast MHD modes in prominences is very small, at least for the range of parameters considered in this work.

Another question that we have addressed is whether the shape of the prominence has a strong influence on the period of the vertical oscillation. We have performed different experiments changing the prominence from vertical to horizontal, but keeping the same mass and geometrical aspect ratio. We have concentrated on models with the longest size of the field lines. The results of the simulations clearly indicate that the period of the vertical mode is essentially the same. Thus, the particular shape of the prominence seems not to be very important, regarding the period of vertical oscillations (at least in the Cartesian geometry studied here). We have also carried out the same experiment using the model of Díaz et al. (2001) and arrived at the same conclusion. This is an interesting result, that for fast MHD waves what really matters is the amount of mass of the prominence and not its particular geometrical shape. This conclusion might not be true in cylindrical geometry and also for the longitudinal harmonics in Cartesian geometry.

In summary, we deduce that the deviation of the actual period of vertical oscillation from the prediction of Equation (11) is mainly due to the effects of the finite vertical extent of the prominence and of curvature. Finally, it is important to mention that no hints of instability have been found when vertical oscillations are excited in the system. In principle, instability is possible in this configuration since at the bottom of the prominence, we have a situation of a heavy plasma on top of a light plasma under the presence of gravity (see Terradas et al. 2012 for an application of prominence threads). However, magnetic tension is sufficient to counteract gravity, making the interface stable. According to the dispersion relation of the interface (see, e.g., Chandrasekhar 1961) if perturbations are assumed to have a component along the y direction, then Rayleigh–Taylor instabilities would be enhanced since the gravitational term increases.

### 4.2. Longitudinal Waves

Slow magnetoacoustic–gravity modes are investigated in this section. In our model, the spectrum of slow–gravity modes contains a continuum of frequencies plus discrete modes. We first focus on discrete modes by introducing a horizontal perturbation exciting the whole prominence body. Since the motions are basically polarized along the magnetic field lines, we call these motions longitudinal. Snapshots at two different times of the velocity component parallel to the magnetic field, calculated using Equation (10), are found in Figure 16. The distribution of $v_\parallel$ shows a strong localization at the prominence body but also along the field lines that pass through the core and connect to the photosphere where line tying is imposed (not shown in Figure 16).

The period of oscillation of discrete slow magnetoacoustic–gravity waves, or for short, longitudinal modes, is plotted in Figure 17 and is longer than that of the transverse vertical modes. This is an expected result since we are in a low $\beta$ regime and therefore slow modes have lower frequencies (longer periods) than fast modes. Now the periods associated to the models with $v_{A0} = 20 \, c_s$, are a bit longer than those for $v_{A0} = 10 \, c_s$, but the difference is not 2 because for low $\beta$ slow modes, the characteristic velocity is essentially the sound speed, which does not change much for the two reference Alfvén velocities considered here. Note that the profile of the curves is different depending on the total mass of the prominence. Light prominences show an increase of the period with the length of the magnetic field line, while heavier prominences display the opposite behavior, the period is decreasing with $L_0$. Moreover, for the two heaviest prominences, the system becomes unstable to longitudinal oscillations (see dashed area in Figure 17) for values of $L_0$ smaller than a critical threshold. The unstable modes are characterized by an exponential increase with time of the amplitude of all the perturbed variables. Two examples of such behavior are plotted in Figure 18 for different lengths of the arcade. The physics behind this instability is basically that, if the dip is not big enough, the mass of the prominence is able to fall down by the effect of the gravity force along the field lines connecting to the photosphere. Thus the parameter $L_0$, and therefore $L_0$, plays a relevant role regarding the stability of the structure with respect to basically longitudinal motions, since it indirectly determines the size of the dip in the model. Figure 9 provides a good example, as the narrow arcade model is unstable (it falls in the unstable region plotted in Figure 14), while the prominence in the wider arcade is stable with respect to longitudinal motions. We have not performed a detailed study
of the growth rates since the instability is simply linked to the fact that the mass falls toward the base of the corona.

Note also in Figure 17 that heavy prominences have longer periods than light prominences. This is in agreement with the results of Zhang et al. (2013), who simulated the formation of a prominence and analyzed the periods of the corresponding slow modes using a one-dimensional model (see their Figures 3 and 4). The fact that the period increases when the mass is increased can be explained by the decrement of the sound speed in the prominence body. If gravity terms are not very important, then the motion of the prominence is governed by pressure forces and the frequency of oscillation is basically proportional to the internal sound speed. Therefore, heavy prominences, associated with low values of the sound speed, have longer periods than light prominences with higher values of the sound speed (if gas pressure is kept constant).

Concerning slow magnetoacoustic–gravity modes Luna & Karpen (2012) and Luna et al. (2012a) claim that longitudinal oscillations are mostly driven by gravity and have a period given by $P = \frac{2\pi}{\sqrt{R/g}}$, where $R$ is the radius of curvature of the dipped magnetic field. We have calculated the radius of curvature of different prominence models and have computed the corresponding period using the previous formula. The period from simulations together with the period due to gravity only are displayed in Figure 19 for the case $v_{A0} = 10e_0$. It is clear that the agreement between the two results is not good. The curves associated with the analytical expression show a completely different behavior with $L_f$. The analytical expression predicts that the heaviest prominence should have a shorter period than that of the lightest one, while the results from the simulations indicate the opposite dependence. These differences suggest that the identification of longitudinal motions as purely due to gravity is not appropriate, at least in the present configuration. The reason for the discrepancy is most likely due to the different assumptions made in the models. In Luna et al. (2012a), the variation of the magnetic field along field lines is neglected. Their model is isothermal and, more importantly, the radius of
curvature is constant. Since they consider prominence threads, the height of the plasma column is quite short.

We turn to the modes of the continuum. Goossens et al. (1985) found expressions for slow continua (and also Alfvén continua) in a general 2D magnetostatic equilibrium with invariance in the y direction (see their Equations (59) and (60)). These expressions were derived using a local orthogonal system of flux coordinates, which, for simplicity, are written here in terms of the distance along the field lines, denoted by $s$. For slow modes, purely polarized along the magnetic field lines for the situation $k_y = 0$, we have

$$\frac{d^2 \xi_s}{ds^2} + F(s) \frac{d \xi_s}{ds} + G(s, \omega) \xi_s = 0,$$

where

$$F(s) = 2 \frac{d c_T}{c_T} \frac{d \ln \rho}{ds} - \frac{d \ln B}{ds}, \quad (13)$$

$$G(s, \omega) = -\frac{1}{v_A} g_s \left( \frac{d \ln \rho}{ds} - \frac{g_s}{c_s^2} \right) + \frac{1}{c_T^2} \frac{d c^2}{ds^2} \left( \frac{c_s^2}{c_T^2} \right) g_s$$

$$+ \frac{1}{c_T^2} \left( g_s \frac{d \ln B}{ds} + \frac{d g_s}{ds} \right)$$

$$- \frac{d \ln B}{ds} \left( \frac{2 c_T}{c_s} \frac{d c_T}{c_s} + \frac{d \ln \rho}{ds} \right) - \frac{d^2 \ln B}{ds^2} + \frac{\omega^2}{c_T^2}, \quad (14)$$

Here $c_T$ is the tube or cusp speed, defined as $c_s v_A/\sqrt{c_s^2 + v_A^2}$, $g_s$ is the projection of gravity along the magnetic field line, and $\omega$ is the frequency that we want to calculate (perturbations of the form $e^{i \omega t}$ have been assumed). Note that the different terms involve derivatives of density, gravity, and magnetic field along the field lines.

Equation (12) must be complemented with appropriate boundary conditions, which are line tying at the two ends of the magnetic field lines. Solving this equation and determining the value of the eigenfrequency $\omega$ is not straightforward since our configuration does not have a simple geometry. The numerical procedure is the following, a particular footpoint is selected and the corresponding field line coordinates are calculated using the equilibrium that has been determined numerically. Once the coordinates of the field line are known, all the necessary variables are projected along this magnetic field line as a function of the distance $s$ along the field line. The last step is to numerically solve Equation (12) using the interpolated values.

In Figure 20, the computed frequency for slow MHD modes belonging to the continuum, $\omega_{cT}$, is represented as a function of the footpoint position in a fixed interval. The curve in dots in Figure 20 corresponds to the eigenmode calculations without a dense prominence and has been included for comparison purposes. These eigenmode computations show a good agreement with the results of the time-dependent problem, represented in Figure 20 with circles, when the initial excitation is localized on different magnetic surfaces, producing an efficient slow mode excitation. From the agreement between the time-dependent results and the eigenmode results, we conclude that our solutions to Equation (12) are correct. The inclusion in the arcade of a dense prominence produces the existence of a minimum in the cusp speed. It is interesting to note that the discrete slow modes have frequencies that are below this minimum. This is surprising since slow modes in homogeneous slabs (Edwin & Roberts 1982) have frequencies between internal cusp speed and the internal sound speed (under coronal conditions). This phenomena needs to be studied in more detail in future studies.

### 4.3. Alfvén Waves

In this work, we are not interested in the analysis of motions in the ignorable direction, which are related to the excitation of pure Alfvén waves. These modes lack of a global character and cannot produce coherent global motions of the prominence. However, we can derive useful information from the corresponding eigenmode calculations. If $k_y = 0$, we have the following second order differential equation (see Goossens et al. 1985; Oliver et al. 1993) for the modes of the Alfvén continuum

$$\frac{d^2 \xi_y}{ds^2} + \frac{d \ln B}{ds} \frac{d \xi_y}{ds} + \frac{\omega^2}{v_A^2} \xi_y = 0,$$

(A color version of this figure is available in the online journal.)
where $\omega$ is the eigenfrequency. Note that there is no dependence with gravity in this equation and that there is a term that accounts for the variation of the strength of the magnetic field along field lines. The computation of the Alfvén spectrum is useful in order to understand the damping of global transverse modes. From the numerical point of view, we proceed in a similar way as for the slow modes belonging to the continuum.

In Figure 20, the computed frequency for Alfvén modes, $\omega_A$, is represented as a function of the footpoint position. The Alfvén frequency is larger than that of the slow or cusp frequency, since we are in a situation where magnetic pressure dominates over gas pressure. The agreement between eigenmode calculations and the results of the time-dependent problem, using a localized perturbation on magnetic surfaces in $v_y$, is evident (compare circles with the dotted curve). Again the depression in frequency around the footpoint located at $x_0 = -4.2 H$ is due to the presence of the heavy prominence. The frequency of the discrete fundamental vertical mode determined from the linear time-dependent problem, $\omega_L$, is also plotted in Figure 20 as a horizontal dashed line. We see that, depending on the footpoint position, the frequency of the global vertical mode is above or below the local Alfvén frequency. If the frequency is above the Alfvén frequency, it means that the eigenfunction has an oscillatory behavior; while if it is below, its behavior is evanescent (see, e.g., Brady & Arber 2005; Brady et al. 2006; Verwichte et al. 2006; Rial et al. 2013). The fact that $\omega_L > \omega_A$ for $x_0 < -4.3 H$, which corresponds to magnetic field lines with apexes situated at higher heights as $x_0$, is decreased, means that the eigenfunction is not confined and it is oscillatory. This is an indication that the mode is unable to trap all the energy and has a leaky character, since its energy radiated away from the prominence body. The leaky character of the mode produces an attenuation of the amplitude with time, and this feature is already present in Figure 12. It is known that the inclusion of perpendicular wavenumber in the $y$ direction might significantly reduce this leakage. In addition, we have to bear in mind that the plasma $\beta$ changes significantly in the computational domain (see Figure 2) and there are regions where the sound speed is equal to the Alfvén speed ($\beta$ close to one). Under such conditions, the character of the modes can change due to mode conversion and contribute to the damping of the global modes. As the damping of oscillations is not the main topic of this paper, we have not investigated further the mechanism of mode conversion.

From the numerical perspective, both slow and Alfvén modes show a strong attenuation with time that is produced by numerical dissipation. As the modes of the continuum are localized on magnetic flux surfaces and a Cartesian grid is used in the simulations, it is difficult to capture the correct spatial structure of the modes, but nevertheless we still get periods, as Figure 20 indicates, that are quite reliable. A possible way to improve in this aspect is by using flux coordinates in the simulations. The reader is referred to Rial et al. (2013) for an example of such simulations in a curved magnetic field.

5. DISCUSSION AND CONCLUSIONS

A numerical method to obtain MHS solutions with some specific features that mimic real prominences has been presented. The method is based on the injection of mass through the continuity equation and on the relaxation of the system. The initial equilibrium is given an increase in gravitational energy by the increase of the density at a given location. Part of this energy is converted into an increment of the magnetic energy and internal energy, and also in an increase of kinetic energy. We have found that our numerical scheme is suitable to study the evolution of the system toward a situation that is close to a stationary state. The treatment of boundary conditions, based on a decomposition in characteristic fields, is the key part of the relaxation method since the energy excess is allowed to leave the system through the boundaries. By the time we were preparing this manuscript, we were aware of the work of Hillier & van Ballegooijen (2013) who have also used a relaxation method in the context of prominences. However, their study is based on flux rope structures and their relaxation technique uses over-dissipation to achieve a stationary state. Thus, the method used in our work, based on the decomposition in characteristics, is conceptually different.

Using the relaxation method, we have built a set of new prominence equilibrium models in 2D. These models include the connection of the prominence body with the photosphere, and contain a cool core that matches the internal temperature with the coronal temperature through a PCTR. The size of the prominence core and PCTR are determined by the choice of the spatial distribution of the mass source term. The gravity force is included in the numerically generated models.

It is clear that the method devised here to find MHS solutions should be extended to 3D geometries. In particular, a future application could be to use 3D magnetic field extrapolations of a real prominence to test the global support of the structure by injecting mass in the magnetic configuration and studying the time evolution. From the comparison of the model with observations, some conclusions about the magnetic field extrapolations could be extracted.

The second part of the work is about the analysis of MHD waves in the numerically generated models. The properties of the different types of MHD modes have been studied. Since we have restricted to oscillations without perpendicular propagation in the $y$ direction ($k_y = 0$), the different types of waves are easily identified because they are uncoupled. We have found that vertical oscillations, associated mainly with fast MHD waves, are always stable, at least for the equilibrium parameters considered in this work, when there is no perpendicular propagation. The obtained periods are typically in the 4–10 minute range. On the contrary, longitudinal oscillations, related to slow magnetoacoustic–gravity waves, have longer periods in the range of 28–40 minutes. These waves are strongly affected by gravity and can become unstable when short magnetic arcades are considered because they are unable to have significant dips that are the key to having stable magnetic configurations.

The two different groups of periods found in this work and related to fast and slow MHD waves are below the values of the reported periods from observations. For example, we can compare with the periods found in Table 1 of Tripathi et al. (2009) that are associated, in most of the cases, to motions of the whole structure. In that table we can distinguish that periods of vertical motions are typically in the range of 15–29 minutes, while longitudinal motions (along the filament axis) are in the range of 50–160 minutes. We think that more realistic configurations are necessary to improve the comparison and 3D models are the key point to achieve this. In this regard, the information inferred from the properties of MHD oscillations in prominence structures like the ones studied in this work may lead to the application of prominence seismology in the near future (see, for example, Arregui et al. 2012).

The periods of oscillation in our complex configurations have been compared with the periods predicted by simple models that miss many physical effects. Regarding fast MHD waves, we have found, for example, that the results of the infinite slab...
only provide an estimation of the order of magnitude of the period of oscillation. The effects of considering a prominence with a finite height improves the comparison. Gravity seems to be not important for vertical motions of the prominence, and interestingly, the geometrical shape of the prominence is not relevant. On the contrary, slow magnetoacoustic–gravity waves have a completely different behavior. They are strongly affected by the gravity force and can even become unstable. The existence of a continuous spectrum of slow MHD waves complicates the interpretation of the periods of oscillation since in general, there is a joint excitation of both discrete and continuum modes.

We expect that the inclusion of perpendicular propagation in the model can significantly change some of the properties of the MHD waves. First of all, it will produce the resonant damping of the global vertical mode with the modes of the Alfvén continuum. Second, it can have a strong effect on vertical modes since they may become Rayleigh–Taylor unstable (see Hillier et al. 2011, 2012 for recent results about 3D modeling). Nevertheless, the inclusion of shear and twist, ignored in the present work, might have an stabilizing effect. As far as we know, MHD stability analysis in theoretical prominence models of this type are very scarce. Thus, the problem of instabilities due to gravity, together with twist or shear, should be carefully examined in future studies using 3D models. This is an interesting problem since the dynamics of the instability could be exhaustively investigated and a more realistic comparison with the observations could be attempted.

J.T. acknowledges support from the Spanish Ministerio de Educación y Ciencia through a Ramón y Cajal grant. All the authors acknowledge the funding provided under the project AYA2011-22846 by the Spanish MICINN and FEDER Funds. A.J.D. acknowledges the financial support by the Spanish Ministry of Science through project AYA2010-18029. The financial support from CAIB through the “Grups Competitius” scheme is also acknowledged. The authors also thank M. Luna, G. Verth, and A. W. Hood for their comments and suggestions that helped to improve the paper. The comments of the anonymous referee are also acknowledged.

REFERENCES

An, C.-H., Bao, J. I., Wu, S. T., & Suess, S. T. 1988, SoPh, 115, 93
An, C.-H., Wu, S. T., & Suess, S. T. 1989, ApJ, 337, 989
Andries, J., Arregui, I., & Goossens, M. 2005a, ApJL, 624, L57
Andries, J., Goossens, M., Hollweg, J. V., Arregui, I., & Van Doorsselaere, T. 2005b, A&A, 430, 1109
Arregui, I., Oliver, R., & Ballester, J. L. 2012, LRSP, 9, 2

Aulanier, G., DeVore, C. R., & Antiochos, S. K. 2002, ApJL, 567, L97
Bernstein, I. B., Frieman, E. A., Kruskal, M. D., & Kulsrud, R. M. 1958, RSPSA, 244, 17
Blokland, J. W. S., & Keppens, R. 2011a, A&A, 532, A93
Blokland, J. W. S., & Keppens, R. 2011b, A&A, 532, A94
Bona, C., Bona-Casas, C., & Terradas, J. 2009, JCoPh, 228, 2266
Brady, C. S., & Arber, T. D. 2005, A&A, 438, 733
Brady, C. S., Verwichte, E., & Arber, T. D. 2006, A&A, 449, 389
Chandrasekhar, S. (ed.) 1961, Hydrodynamic and Hydromagnetic Stability

(Oxford: Clarendon)
de Brayne, P., & Hood, A. W. 1993, SoPh, 147, 97
Demoulin, P., & Priest, E. R. 1993, SoPh, 144, 283
Diaz, J. A. 2006, A&A, 456, 737
Diaz, J. A., Oliver, R., & Ballester, J. L. 2006a, ApJ, 645, 766
Diaz, J. A., Oliver, R., & Ballester, J. L. 2010, ApJL, 725, 1742
Diaz, J. A., Oliver, R., Erdélyi, R., & Ballester, J. L. 2001, A&A, 379, 1083
Diaz, A. J., Zaqa rashvili, T., & Roberts, B. 2006b, A&A, 455, 709
Edwin, P. M., & Roberts, B. 1982, SoPh, 76, 239
Fiedler, R. A. S., & Hood, A. W. 1992, SoPh, 141, 75
Galindo Trejo, J. 1987, SoPh, 108, 265
Goossens, M., Poedts, S., & Hermans, D. 1985, SoPh, 102, 51
Hillier, A., Berger, T., Isobe, H., & Shibata, K. 2012, ApJ, 746, 120
Hillier, A., Isobe, H., Shibata, K., & Berger, T. 2011, A&A, 536, L1
Hillier, A., & van Ballegooijen, A. 2013, ApJ, 766, 126
Hood, A. W., & Anzer, U. 1990, SoPh, 126, 117
Hyder, C. L. 1966, ZA, 63, 78
Joarder, P. S., & Roberts, B. 1992, A&A, 261, 625
López Ariste, A., Aulanier, G., Schmieder, B., & Sainz Dalda, A. 2006, A&A, 456, 725
Low, B. C. 1981, ApJ, 246, 538
Low, B. C., & Zhang, M. 2004, ApJ, 609, 1098
Luna, M., Diaz, A. J., & Karpen, J. 2012a, ApJ, 757, 98
Luna, M., & Karpen, J. 2012, ApJL, 750, L1
Luna, M., Karpen, J. T., & DeVore, C. R. 2012b, ApJ, 746, 30
Mackay, D. H., Karpen, J. T., Ballester, J. L., Schmieder, B., & Aulanier, G. 2010, SSRv, 151, 333
Mackay, D. H., & van Ballegooijen, A. A. 2006, ApJ, 641, 577
Mackay, D. H., & van Ballegooijen, A. A. 2009, SoPh, 260, 321
Oliver, R. 2009, SSRv, 149, 175
Oliver, R., & Ballester, J. L. 2002, SoPh, 206, 45
Oliver, R., Ballester, J. L., Hood, A. W., & Priest, E. R. 1993, A&A, 273, 647
Petrie, G. J. D., Blokland, J. W. S., & Keppens, R. 2007, ApJ, 665, 830
Powell, K. G., Roe, P. L., Linde, T. J., Gombosi, T. I., & de Zeeuw, D. L. 1999, JCoPh, 154, 284
Ramsey, H. E., & Smith, S. F. 1966, AJ, 71, 197
Rial, S., Arregui, I., Terradas, J., Oliver, R., & Ballester, J. L. 2013, ApJ, 763, 16
Soler, R., Arregui, I., Oliver, R., & Ballester, J. L. 2010, ApJ, 722, 1778
Terradas, J., Oliver, R., & Ballester, J. L. 2012, A&A, 541, A102
Terradas, J., Oliver, R., Ballester, J. L., & Keppens, R. 2008, ApJ, 675, 875
Tripathi, D., Isobe, H., & Jain, R. 2009, SSRv, 149, 283
Verwichte, E., Foullon, C., & Nakariakov, V. M. 2006, A&A, 446, 1139
Xia, C., Chen, P. F., & Keppens, R. 2012, ApJL, 748, L26
Xia, C., Chen, P. F., Keppens, R., & van Marle, A. J. 2011, ApJ, 737, 27
Yang, W. H., Sturrock, P. A., & Antiochos, S. K. 1986, ApJ, 309, 383
Zhang, Q. M., Chen, P. F., Xia, C., Keppens, R., & Ji, H. S. 2013, A&A, 554, A124