The experimental-theoretical model of the jet HF induction discharge of atmospheric pressure

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Abstract. The paper considers the experimental-theoretical model devised to determine the regularities of the quasi-stationary electromagnetic field structure of the HFI discharge burning in the inductor of finite dimensions at atmospheric pressure.

1. Introduction
To manage effectively technological processes using the principle of high-frequency inductive gas heating and to optimize them, it is necessary to have reliable information on the structure of the HF-discharge and the distribution of main electromagnetic and thermal parameters in it as well. However, the lack of simple and trustworthy methods to calculate the operation of plasmatrons prevents the development of new plasma technologies.

Though existing methods of operational diagnostics of the HFI-discharge can provide information on the distribution of most of its electromagnetic and thermal parameters, it is actually difficult to maintain measurements within a single cycle when the discharge is burning and its characteristics do not change for a long time. It is due to the fact that usually different types of measurements require a different set of equipment and time for its installation and debugging.

In this regard, the task has become urgent to create an experimental-theoretical model that would require a minimum number of measured quantities, so that it would be possible to calculate all remaining characteristics of the discharge with a sufficient degree of accuracy.

2. Method of solution
Taking into account the harmonic nature of the change in magnetic and electric fields over time, the system of Maxwell equations in the case of a cylindrically symmetric quasi-stationary electromagnetic field of a high-frequency induction discharge of the finite length for the main field components can be represented in the following form [1-3]:

\[ \frac{\partial E_v}{\partial z} = -\frac{\omega}{c} H_L \sin \left( \varphi_{H_L} - \varphi_{E_v} \right); \]

\[ \frac{\partial \varphi_{E_v}}{\partial z} = \frac{\omega}{c} \frac{H_L}{E_v} \cos \left( \varphi_{H_L} - \varphi_{E_v} \right); \]
\[
\frac{\partial H_z}{\partial z} = -\frac{1}{r} \frac{\partial}{\partial r} \left( rH_r \right) \cos \left( \phi_{H_z} - \phi_{H_r} \right) - H_r \left( \frac{\partial}{\partial r} H_z \right) \sin \left( \phi_{H_z} - \phi_{H_r} \right),
\]
(1)

\[
\frac{\partial \phi_{H_r}}{\partial z} = \frac{1}{H_r} \left[ \frac{\partial}{\partial r} \left( rH_r \right) \sin \left( \phi_{H_z} - \phi_{H_r} \right) - H_r \left( \frac{\partial}{\partial r} \phi_{H_r} \right) \cos \left( \phi_{H_z} - \phi_{H_r} \right) \right],
\]

\[
\frac{\partial H_z}{\partial r} = \frac{\partial}{\partial r} \left( \cos \left( \phi_{H_z} - \phi_{H_r} \right) - H_z \left( \frac{\partial}{\partial r} \phi_{H_z} \right) \sin \left( \phi_{H_z} - \phi_{H_r} \right) \right),
\]

\[
\frac{\partial \phi_{H_z}}{\partial z} = \frac{1}{H_r} \left[ \frac{\partial}{\partial r} \left( rH_r \right) \sin \left( \phi_{H_z} - \phi_{H_r} \right) + H_r \left( \frac{\partial}{\partial r} \phi_{H_z} \right) \cos \left( \phi_{H_z} - \phi_{H_r} \right) \right] - \frac{4\pi}{c} \sigma E_{\phi} \cos \left( \phi_{H_z} - \phi_{E_{\phi}} \right),
\]

where \( H_z \) - is the amplitude of the longitudinal component of the magnetic field in the discharge, \( H_r \) - the amplitude of the radial component of the magnetic field in the discharge, \( E_{\phi} \) - the amplitude of the azimuthal component of its electric field, \( \phi_{H_z}, \phi_{H_r}, \phi_{E_{\phi}} \) phase angles of these components, respectively.

Analysis (1) shows that in this system of equations, the number of quantities characterizing the field is one unity greater than the number of equations themselves, i.e. the given system of equations is not closed. Therefore, if we consider one of the quantities characterizing the field as the given one, it is possible to obtain at the output of this system a set of various dependences consisting of electromagnetic quantities and conductivity in the discharge \( \sigma \). In its turn, using the found conductivity, one can also find the temperature field \( T(\sigma) \) in the discharge. Thus, the problem to choose the input quantity to solve the obtained system of equations arises.

If we choose experimentally obtained data for the solution (1) as the input parameter for \( H_z \), two important factors should be taken into account. Firstly, in this case, there is no need to use additional dependencies to determine the field temperatures in the discharge since the change in the magnitude of the magnetic field over the discharge cross section is determined by the absorption of electromagnetic energy in the conducting layer of the gas. Therefore, ceteris paribus, electro-conductivity determines the rate of the magnetic field quantity change along the radius of the discharge. Due to this unambiguous dependence, it is not necessary to resort to additional equations and solve the problem within the framework of Maxwell equation system.

Secondly, to solve the system of equations (1), it is significant to specify boundary conditions apparent for all electromagnetic quantities except \( H_z \) and do not require additional experimental specification.

Based on the system of equations (1), conductivity can be calculated by the formula:

\[
\sigma(r, z) = -\frac{c}{4\pi} \left( \frac{\partial H_z}{\partial r} - \frac{\partial H_r}{\partial z} \cos \left( \phi_{H_z} - \phi_{H_r} \right) - H_r \left( \frac{\partial H_z}{\partial z} \right) \sin \left( \phi_{H_z} - \phi_{H_r} \right) \right) \frac{E_{\phi} \cos \left( \phi_{H_z} - \phi_{E_{\phi}} \right)}{(2)}
\]

If we approximate the amplitude of the longitudinal magnetic field in the discharge \( H_z \) by a smoothed cubic spline while solving the equation (2), then conductivity manifests a sharp divergence near the axis of the plasmoid and up to the distance of about one third of the plasma bunch radius. It is obvious that this phenomenon is a direct consequence of \( H_z(r) \) incorrect interpolation near zero and, therefore, in this case it is impossible to use the finite number of its experimentally measured values by cubic splines to solve the problem to reconstruct the amplitude \( H_z \) as a continuous function of the radial coordinate \( r \).
To find out true causes of erroneous conductivity calculations in the HFI-discharge, it is required to analytically study its structure and, especially, the behavior of all its main characteristics including $H_z^r(r)$ in the paraxial region of the plasmoid.

A two-dimensional mathematical model was constructed in works [4,5] to investigate the structure of the quasi-stationary electromagnetic field of the HFI discharge; it became a further development of the known model of constant conductivity of the HFI – J.J. Thomson’s discharge laborated for one-dimensional case [6].

Within the framework of the constructed model, relations were obtained to calculate the amplitude of the longitudinal component of the magnetic field $H^z_z(r, z)$, radial component of the magnetic field $H^r_r(r, z)$, and the azimuthal electric field $E^\phi( r, z)$, and phase angles of these quantities near the plasmoid axis at values $r$, close to zero as well:

$$H'^z_z(r, z) = H'^z_z(0,0)I_0(br)\cos(bz) \times \left\{ 1 + \frac{8\pi^2 \omega^2 \sigma^2}{b^4c^4} \left( \frac{2I_0(br) - \frac{b^2r^2}{4}}{I_0(br)} \right)^2 \right\} - \frac{16\pi^2 \omega^2 \sigma^2}{b^4c^4} \left( \frac{I_0(br) - \frac{b^2r^2}{4}}{I_0(br)} \right);$$

(3)

$$H'^r_r(r, z) = H'^r_r(0,0)I_1(br)\sin(bz) \times \left\{ 1 + \frac{8\pi^2 \omega^2 \sigma^2}{b^4c^4} \left( \frac{I_1(br) - \frac{br}{2}}{I_1(br)} \right)^2 \right\};$$

(4)

$$E'^\phi(r, z) = \frac{\omega}{bc} H'^z_z(0,0)I_1(br)\cos(bz) \times \left\{ 1 + \frac{8\pi^2 \omega^2 \sigma^2}{b^4c^4} \left( \frac{I_1(br) - \frac{br}{2}}{I_1(br)} \right)^2 \right\};$$

(5)

$$\varphi_{H^z_z}(r, z) = \frac{\pi}{2} + \frac{4\pi\omega \sigma}{b^2c^2} \left( \frac{I_0(br) - 1}{I_0(br)} \right);$$

(6)

$$\varphi_{H^r_r}(r, z) = \frac{\pi}{2} + \frac{4\pi\omega \sigma}{b^2c^2} \left( \frac{I_1(br) - \frac{br}{2}}{I_1(br)} \right);$$

(7)

$$\varphi_{E^\phi}(r, z) = \frac{4\pi\omega \sigma}{b^2c^2} \left( \frac{I_1(br) - \frac{br}{2}}{I_1(br)} \right).$$

(8)
In these relations the constant \( b = \frac{1}{L} \arccos \frac{H_z(0,L)}{H_z(0,0)} \), where \( L \) is the plasmoid length, \( I_0 \), \( I_1 \) is modified Bessel function of the zero and first order, respectively.

Experiments to measure magnetic field were conducted on a test stand based on a high-frequency plasmatron of 60 kW that operates at a frequency of 1.76 MHz, plasma-forming gas is air. To work in conditions of thermal discharge, a special water-cooled magnetic probe was used; it allows to have a long-term operation at high temperatures peculiar to this type of discharge.

Amplitude measurements of the longitudinal component of the magnetic field \( H_z \) were conducted at two charges of plasma-forming gas - 9 m\(^3\)/hour and 13 m\(^3\)/hour. As a result, radial distributions were obtained in different sections along \( z \) (Fig. 1) [7-9]; \( z = 0 \) denotes the plane of the plasmoid central section; the coordinate \( z \) is measured from this section downstream; magnetic probe step: \( r=0.4 \text{ cm}, z=1 \text{ cm} \).

![Fig. 1. Radial distribution \( H_z \) in different sections of the inductor at the expense of the plasma-forming gas: \(- - - - - Q_1=9 \text{ m}^3/\text{hour}; \quad - - - - - Q_2=13 \text{ m}^3/\text{hour}\)](image)

3. Results and conclusion
The system of equations (1) was solved numerically by a finite-difference method with respect to unknown quantities \( H_r, E_\varphi, \varphi_H, \varphi_E, \varphi_{E_\varphi} \) and \( \sigma \). In this case, the parameters of magnetic and electric fields in the near-beam axis of the plasmoid were approximated by means of the relations (3) - (8). We used amplitude measurements of the longitudinal component of the magnetic field \( H_z \) as an input parameter. Figures 2-5 present some results obtained by means of a numerical model in the form of conductivity dependency graphs in the discharge \( \sigma(r) \), eddy current density \( j = \sigma E_\varphi \), volume density of power included into the discharge \( W = \frac{1}{2} \sigma E_\varphi^2 \) and temperature distribution for three sections of the HFI discharge, starting from its central section downwards the stream for two different discharge rate of plasma-forming gas - 9 m\(^3\)/hour and 13 m\(^3\)/hour.
Fig. 2. Radial distribution of conductivity in different sections of the inductor at the discharge rate of the plasma-forming gas:
- $Q_1 = 9 \text{ m}^3/\text{hour}$; - - - - $Q_2 = 13 \text{ m}^3/\text{hour}$

Fig. 3. Radial density distribution of the eddy current in different sections of the inductor at the discharge rate of the plasma-forming gas:
- $Q_1 = 9 \text{ m}^3/\text{hour}$; - - - - $Q_2 = 13 \text{ m}^3/\text{hour}$

Fig. 4. Radial distribution of the volume density of the power included into the discharge in different sections of the inductor at the discharge rate of the plasma-forming gas:
- $Q_1 = 9 \text{ m}^3/\text{hour}$; - - - - $Q_2 = 13 \text{ m}^3/\text{hour}$

Fig. 5. Radial distribution of temperature in different sections of the inductor at the discharge rate of the plasma-forming gas:
- $Q_1 = 9 \text{ m}^3/\text{hour}$; - - - - $Q_2 = 13 \text{ m}^3/\text{hour}$

Conclusions
The combination of calculated and experimental research methods makes it possible to study the internal structure and properties of the high-frequency induction discharge of atmospheric pressure more profoundly, as well as to investigate the influence of the discharge rate of the plasma-forming gas on the distribution of its basic electromagnetic and thermal characteristics. Obtained results can be used to develop and optimize various types of power plants using the principle of high-frequency inductive gas heating.
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