Weak reactions with light nuclei - $^6\text{He}$ $\beta$-decay as a test case for the nuclear weak current

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Abstract

We present a microscopic calculation of the $^6\text{He}$ $\beta$-decay into the ground state of $^6\text{Li}$. To this end we use the impulse approximation to describe the nuclear weak current. The ground state wave functions are obtained from the solution of the nuclear 6-body problem. The nucleon-nucleon interaction is described via the J-matrix inverse scattering potential (JISP), and the nuclear problem is solved using the hyperspherical-harmonics approach. This approach results in numerical accuracy of about 2 per mil in the transition matrix element. Bearing in mind that the contribution of meson-exchange currents to the transition matrix element is about 5%, these results pave the way for accurate estimation of their effect.

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1. Introduction

The nuclear weak current contains a well established leading 1-body terms, which at low energy are known as the Fermi (F) and Gamow-Teller (GT) operators, and subleading two- and many-body currents, generally known as meson-exchange currents (MEC), which are a subject of vast contemporary research. The lightest nucleus that undergoes a $\beta$-decay is the triton. However, the theory of nuclear weak interaction cannot be checked
in the triton since its half-life is used to adjust a free parameter in the axial MEC. Therefore, the lightest nucleus that can provide a test to the theory is \( ^6\text{He} \). \(^6\text{He} \) \((J^π = 0^+) \) is an unstable nucleus, which undergoes a \( β \)-decay with a half-life \( \tau_{1/2} = 806.7 \pm 1.5 \) msec to the ground state of \(^6\text{Li} \) \((J^π = 1^+) \) \[1\].

So far, a microscopic calculation of \(^6\text{He} \) from its nucleonic degrees of freedom, failed to reproduce the \( β \)-decay rate. A comprehensive study, accomplished by Schiavilla and Wiringa \[2\], has used the realistic Argonne \( v18 \) (AV18) nucleon-nucleon (NN) potential, combined with the Urbana-IX (UIX) three-nucleon-force (3NF), to derive the nuclear wave functions, through the variational Monte-Carlo approach (VMC). The model used for the nuclear weak axial current includes one- and two-body operators. The two-body currents are phenomenological, with the strength of the leading two-body term – associated with \( Δ \)-isobar excitation of the nucleon – adjusted to reproduce the GT matrix element in triton \( β \)-decay. The calculated half-life of \(^6\text{He} \) is over-predicts the measured one by about 8%. An unexpected result of the calculation, was that two-body currents lead to a 1.7% increase in the value of the Gamow-Teller matrix element of \(^6\text{He} \), thus worsening the comparison with experiment. The authors of this paper have presumed that the origin of this discrepancy is either the in approximate character of the VMC wave functions, or in the model of the weak nuclear current.

The \( β \) decay rate of \(^6\text{He} \) is proportional to the square of the GT matrix element. The difference between the one-body contribution to the \(^6\text{He}-^6\text{Li} \) GT matrix element and the experimental value, \( 2.173 \pm 0.002 \), is of the order of few percent. A result which on the one hand is very satisfying, but on the other hand implies that numerical accuracy at a per mil level is required to assign the \(^6\text{He} \) \( β \)-decay as a test case for validating the MEC model.

The current contribution is dedicated to explore exactly that point. Having in mind the required level of convergence we use the JISP16 potential \[3\]. The JISP16 NN potential utilizes the J-matrix inverse scattering technique to construct a soft nuclear potential, formulated in the harmonic oscillator basis, that by construction reproduces the NN phase shifts up to pion threshold and the binding energies of the light nuclei \( A \leq 4 \).

We use the Hyperspherical-Harmonics (HH) expansion to solve the Schrödinger equation. The HH functions constitute a general basis for expanding the wave functions of an \( A \)-body system \[4,5\]. In the HH method, the translational invariant wave-function is written as

\[
Ψ = \sum_{[K]n} C_{[K]n} R_n(\rho) Y_{[K]}(Ω, s, t) \tag{1}
\]

where \( ρ \) is the hyperradius, and \( R_n(\rho) \) are a complete set of basis functions symmetric under particle permutations as \( \rho^2 = \frac{1}{3A} \sum_{i,j} (r_i - r_j)^2 \). The hyperangle, \( Ω \), is a set of \( 3A - 4 \) angles, and \( Y_{[K]}(Ω, s, t) \) are a complete set of antisymmetric basis functions in the Hilbert space of spin, isospin and hyperangles. The functions \( Y_{[K]}(Ω, s, t) \) are characterized by a set of quantum numbers \([K]\) \[6,7\], and possess definite angular momentum, isospin, and parity quantum numbers. They are eigenfunctions of the hyperspherical, or generalized, angular momentum operator \( \hat{K}^2 \), \( \hat{K}^2 Y_{[K]}(Ω, s, t) = K(K + 3A - 5) Y_{[K]}(Ω, s, t) \).

Our results for the ground state properties of \(^6\text{He} \) and \(^6\text{Li} \), are presented in table \[1\]. The energies and rms matter radii are given as a function of \( K_{\text{max}} \) – the limiting value of \( K \) in the HH expansion. As we have used the bare interaction in our calculations
the results are variational. Using the formula $E(K_{\text{max}}) = E_{\infty} + A/K_{\text{max}}^\alpha$ for $K_{\text{max}} \geq 8$ we have extrapolated the binding energies to the limit $K_{\text{max}} \rightarrow \infty$. It can be seen that the extrapolated binding energies are rather close to the experimental values. In the last column of the table we present our $^6\text{He}-^6\text{Li}$ GT transition matrix element in the impulse approximation, i.e. at the 1-body level. It can be seen that the convergence pattern of the matrix element is not regular. Extrapolating its value using the expression $GT(K_{\text{max}}) = GT_{\infty} + B/K_{\text{max}}^\beta$ for $K_{\text{max}} \geq 6$, we get $GT_{\infty} = 2.228(3)$. This result is in accordance with the values $GT = 2.28$ for AV8'/TM'(99) and $GT = 2.30$ for AV8' obtained by Navratil and Ormand [8], $GT = 2.28$ for the N3LO NN-force of Navratil and Caurier [9], $GT = 2.25$ for AV18/UIX of Schiavilla and Wiringa [2], and $GT = 2.16-2.21$ for AV18/IL2 by Pervin et al. [10]. Moreover, it can be seen that our accuracy estimating the GT matrix element is in the level of few per mil. Such an accuracy enables us to disentangle numerics from physics and utilizes the $^6\text{He}$ $\beta$-decay as a test ground for axial MEC models.

Table 1

The $^6\text{He}, ^6\text{Li}$ binding energies, rms matter radius, and GT matrix element in the impulse approximation.

| $K_{\text{max}}$ | B.E.($^6\text{He}$) | B.E.($^6\text{Li}$) | $r_{\text{rms}}(^6\text{He})$ | $r_{\text{rms}}(^6\text{Li})$ | $\langle \text{GT} \rangle$ |
|------------------|-------------------|-------------------|------------------|------------------|------------------|
| 4                | 18.367            | 19.392            | 1.840            | 1.859            | 2.263            |
| 6                | 24.103            | 26.124            | 1.902            | 1.909            | 2.247            |
| 8                | 26.392            | 28.854            | 1.979            | 1.984            | 2.234            |
| 10               | 27.560            | 30.156            | 2.051            | 2.051            | 2.232            |
| 12               | 28.112            | 30.797            | 2.112            | 2.110            | 2.229            |
| 14               | 28.424            |                   |                  |                  |                  |
| $\infty$         |                   |                   |                  |                  | 2.165            |

Shirokov et al. [3] 28.32(28) 31.00(31)

Exp. 29.269 31.995 2.18 2.09 2.170

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