An Algorithm to find Superior Fitness on NK Landscapes under High Complexity: Muddling Through

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Abstract. Under high complexity—given by pervasive interdependence between constituent elements of a decision in an NK landscape—our algorithm obtains fitness superior to that reported in extant research. We distribute the decision elements comprising a decision into clusters. When a change in value of a decision element is considered, a forward move is explored if the aggregate fitness of the cluster members residing alongside the decision element is higher. The decision configuration with the highest fitness accomplished in the path is selected. Our algorithm obtains superior outcomes by enabling more extensive search, and allowing inspection of more distant decision configurations. We name this algorithm the muddling through algorithm, in memory of Charles Lindblom who spotted the efficacy of the process long before sophisticated computer simulations came into being.

Keywords. algorithm, complexity, fitness, interdependence, muddling through, NK model, policy making, public administration

“The most frequent and basic objection ...” (to muddling through in policy-making) “... is ... to the political practice of change only by increment ...” (i.e.) “to the incremental politics to which incremental analysis is nicely suited.”

—Charles Lindblom (1979: 520)
Introduction

Research in multiple fields—physics [1], evolutionary biology [2], management [3], organization studies [4] to name a few—motivate problems as one of finding high peaks on an NK model landscape. In the NK model, a decision contains \( N \) decision elements or nodes. Each node can take values of either “0” or “1”. A decision configuration is an instantiation of a decision, with all nodes filled with values of either “0” or “1”. The extent of Fitness of a decision configuration is computed by summing the fitness contribution of the \( N \) individual nodes making up a decision, and dividing the result by \( N \). The fitness contribution by an individual node is jointly dependent on its value (“0” or “1”) and the values in \( K \) other nodes with which the focal node shares a dependency. A matrix having \( 2^{(K+1)} \) rows and \( N \) columns—that we refer to as the Fitness Matrix—is populated with random draws from the Uniform Distribution, to enable calculation of the fitness contribution by a node.

The most common problem formulated through the NK model concerns finding decision configurations with the highest (or lowest) Fitness values. This is considered an NP-hard problem. This is because, as \( N \) increases, the number of feasible decision configurations increases exponentially. For example, if \( N = 16 \), it is necessary to examine \( 2^N = 65,536 \) decision configurations; for \( N = 20 \), it is necessary to examine over 1 million configurations.

Prior research on navigating the NK model in search of high fitness locations

Prior research has suggested several approaches to find high peaks in the NK landscape. In a majority of the approaches, search commences from a randomly chosen starting point or initial decision configuration, fashioned by populating an \( N \)-bit (decision) string with values “0” or “1” with equal probability (one-half).

(I) In the steepest_ascent approach, the fitness of all neighbors at a hamming distance of one unit from the initial decision configuration—i.e. differing in value in just one bit with respect to the initial decision configuration—is compared with the fitness of the initial decision configuration. A move is made to the neighbor having the highest fitness, if the fitness value is higher than fitness at current configuration. The process is repeated till a point is reached where there exist no hamming-one neighbor with a higher fitness.

(II) In the satisficing approach (also referred to as greedy approach and as local search), a move is made if a randomly-chosen hamming-one neighbor has fitness higher than the fitness at current configuration. As before, search terminates when it is not possible to find a hamming-one neighbor with higher fitness.
(III) Kaufman and colleagues [1] describe a *parallel_updating* approach to find peaks superior to those obtainable from the *steepest_ascent* or *satisficing* approaches. Decision elements are accorded a certain probability, $\tau$ ($0<\tau<1$) of flipping. In a given generation, all $N$ decision elements attempt to flip in parallel, with probability $\tau$. However, the nodes that are actually allowed to flip are the ones where overall higher fitness is accomplished if solely that node flipped, in a manner similar to the *satisficing* approach. The process is continued for a pre-specified number of generations, or till a point is reached when it is not possible to obtain higher fitness by flipping one node. The latter stopping criterion is identical to the stopping criterion of the *satisficing* approach.

(IV) Bauman and Siggelkow [4] describe an approach of *decreasing_parochialism* to attain superior peaks under high complexity, given by high interdependence (i.e. high $K$). They first select a subset of decision elements, say four out of $N$. Let us designate the size of the first subset as *init_size* (here *init_size* = 4). Moves are made by the *satisficing* approach till the highest feasible fitness contribution is attained from the *init_size*–sized group of decision elements. This is similar to solving a decision problem where $N = \text{init\_size}$ by the satisficing approach, with the understanding that fitness contributions by individual nodes will be impacted by all decision elements, i.e. not just by *init_size* elements. Thereafter they add one node, i.e. the decision problem becomes one with *init_size* + 1 nodes. The search process is repeated. This is done till size $N$ is reached.

**Algorithm design**

We call our algorithm the *muddling_through* approach. This is in honor of the decision-making process of the same name given by Lindblom [5], in the context of making policy decisions under high complexity. In our algorithm, at initialization, we distribute the set of $N$ decision elements into a number of clusters ($\text{num\_clusters} \geq 1$). A decision node is selected at random, for flipping. The move is accepted if the sum of fitness contributions of the co-members in the cluster where the focal node resides is higher than the corresponding value prior to the move; else, a different node is flipped and the calculation is repeated. The walk continues till resources—comprising the maximum number of moves permissible—exhaust, or when no hamming-one neighbors can be found where the fitness contribution of the corresponding cluster exceeds current contribution. The decision configuration having the highest fitness on the way is selected as the outcome of the search process. In effect, exploration of the landscape happens by myopic consideration of fitness contribution of a cluster, but moves are finalized only when a configuration having higher fitness is found.
Results

In Figure 1 we demonstrate that in a landscape with $N = 20$, the \textit{muddling\_through} approach obtains higher fitness outcomes compared to all the other alternatives we tested—\textit{steepest\_ascent}, \textit{satisficing}, \textit{parallel\_updating}, and \textit{decreasing\_parochialism}—for $K \geq 8$, i.e. under high complexity.

Figure 1. Comparison of \textit{Fitness} outcomes from alternative search algorithms

Note. For the algorithm “parallel update”, the maximum fitness obtained, given varying probability of nodes updating in parallel, is reported above.

In Figure 2 we plot the hamming distance between initial and final decision configurations. It shows that while the other algorithms fail to retain the ability to explore neighbors at a high hamming distance (say 6+) when complexity or interdependence goes up, \textit{muddling\_through} retains the ability to do so.

Figure 2. Comparison of \textit{hamming distance} between initial and final decision configurations, in alternative search algorithms
Note. For the algorithm “parallel update”, the hamming distance reported above corresponds to the fitness configuration at which maximum is fitness obtained, given varying probability of nodes updating in parallel.

**Figure 3** provides a comparison of the number of moves—flipping of decision nodes—across the algorithms. We observe that *muddling_through* is rather inefficient under low complexity ($K \leq 4$), since it uses much higher resources to accomplish results comparable to that from the other algorithms. At higher complexity, it comes into its own, given that other search algorithms terminate earlier, accomplishing lower fitness. All results shown are averages over experiments with 100 starting points per landscape, and further averaged over 100 landscapes. We use $N = 20$ and limit search to 10,000 steps in any given experiment. We use 4 clusters in *muddling_through*.

**Figure 3.** Average resource consumption in alternative search algorithms

![Average resource consumption in alternative search algorithms](image)

Note. For the algorithm “parallel update”, resource consumption reported above corresponds to the fitness configuration at which maximum is fitness obtained given varying probability of nodes updating in parallel

**Discussion**

In low complexity ($K \leq 4$), where there are fewer peaks, the discovery of a high fitness peak may be thwarted altogether, under search by *muddling_through*. The muddling through algorithm is blind to a subset of approach paths to fitness peaks. Non-accessing of a subset pathways to scarce fitness peaks results in attainment of lower fitness configuration in low complexity ($K \leq 4$). When $K$ is high ($K > 4$) peaks are more numerous in number. It is quite possible that some highly advantageous peaks exist, that cannot be reached by moving only to successively higher fitness points from the beginning, one step at a time. Since search by *muddling_through* permits getting on to lower fitness points, it is able to access paths to some advantageous peaks that are denied to search by other approaches.
The progress reported in this paper has wide application in a number of fields. Limiting ourselves to the field of policy decisions, we observe that radical change—or far-reaching adaptation—is more likely to materialize by the approach of incremental change or muddling through. This later finding confounds conventional wisdom in the policy field [6], but is in line with Lindblom’s claims [7] regarding the potency of his approach—as outlined in our opening quotation.

References

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