Modeling the damage of composite materials subjected to fatigue loading

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Abstract. A two-parameter damage model based on residual strength, applied stress and cycle ratio is presented in this paper to predict progression of damage at different stress levels. Using the proposed model, damage factor progression was predicted for E-glass/vinyl ester laminate at three different stress levels. It is observed that proposed model predicts the damage progression closely even though it is not reporting the type of damage. Also, the damage is reported even at low cycles of loading because of anisotropic nature and inherent damages present in the composite materials.

1. Introduction

Composite materials are used extensively in automotive, aerospace and more recently in medical applications. Their high mechanical performances and desired qualities like good fatigue durability offer definite advantages compared with many traditional conventional isotropic materials. However, in comparison with traditional monolithic metals, damage mechanisms are complex due to anisotropy, heterogeneity and different types of defects.

In service, failure of parts results generally from a combination of conjoined factors that degrades the local characteristics of the material, in which fatigue can be considered as one of the main factors. Fatigue affects the weak links of structures and propagates the crack in the week lamina and joints that are used to transfer loads from one part of the structure to another [1]. The structural integrity of these components is therefore of great importance for the viability of the entire system. The factors that could cause fatigue fractures of composite part or structure are numerous such as the types of stresses, the environment of the material used i.e. temperature, contact with chemicals, humidity. In addition, it also depends on the type of reinforced fibres and laminate, as well as on the applied fatigue loading conditions. [2]. Similar to the isotropic materials, to assess the fatigue life of composites under cyclic loading, a damage model is required, in order to represent the damage and its accumulation during complex fatigue loading with a suitable damage function. Under a given frequency, temperature, moisture and specimen geometry; the fatigue damage function of a material, D, depends on the applied stress and the number of fatigue cycles [3], thus:

\[ D = D(n; \text{stress}) = D(n; N) \]

Where n is the number of fatigue cycles, N is the number of cycles to failure under an applied stress of r. In 1945, M.A. Miner popularized a rule that had first been proposed by A.Palmgren in 1924, the
Palmgren-Miner linear damage hypothesis, states that where there are $k$ different stress magnitudes in a spectrum, $S_i (1 \leq i \leq k)$, each contributing $n(S_i)$ cycles, then if $N(S_i)$ is the number of cycles to failure of a constant stress reversal $S_i$ (determined by uni-axial fatigue tests), failure occurs when $H$. However, it fails to recognize the probabilistic nature of fatigue and there is no simple way to relate life predicted by the rule with the characteristics of a probability distribution. In addition to the Palmgren – Miner linear damage hypothesis, Ellyn and El-Kadi [4] developed a fatigue failure criterion based on strain energy, which was independent of ply angle and loading condition. However, A.Rotem [1] suggested damage propagation increases with the increase in critical angle $\theta$. Consequently, fibre exhibit significant degradation of strength with the number of cycles. When composites are subjected to cyclic loading, an important effect called ‘load sequence effect’ influences the fatigue life and strength degradation of the composite material. The ‘load sequence effect’ is due to the difference between residual strength levels and the ‘memory effect’, where the strength/fatigue properties of the composite are influenced by accumulation of the previous load history, considering the matrix is of viscoelastic nature. On the other hand, there are many works of literature to prove that the fatigue behaviour of composites has been shown to be highly dependent on the stress ratio ($R$). Petermann and Schulte [5], observed that for a particular angle ply laminates of carbon-epoxy tested in tension-tension fatigue at high stress ratios and maximum stresses below the endurance limit and concluded that creep dominates damage propagations. In contrast to Petermann et.al, many authors reported that no creep is to be expected for $R = -1$ because of zero mean stress. Furthermore, Mandell and Meier [6] observed that for a given maximum stress in tension-tension loading the fatigue life increases with increasing $R$. In compression-compression loading, the increase of $R$ reduces the fatigue life of the composites. Allah et.al [7] observed that for a constant cyclic life the permissible initial stress amplitude decreases as the mean stress increases, from this assumption we can predict the fatigue life of composites. According to the literatures, the models available to predict the damage propagation are function of only number of cycles at applied stress to the number of cycles to failure. Damage factor can be predicted closely if it can be related to the residual or remaining strength of the material. Hence, a new damage model as a function of residual strength in addition to the cycle ratio is proposed in this paper.

2. Experimental data

The experimental data are taken from Ref. [8] in which the material investigated is a psudoquasi-isotropic laminate manufactured by Northrop Grumman using vacuum assisted resin transfer molding (VARTM). The laminate consisted of 10 layers of Vetrotex 324 woven roving and Dow Derakane 510A vinyl-ester resin was used as the matrix. All tension-tension fatigue tests were performed with a stress ratio ($R$) of 0.1 and the operating frequency was maintained at 10 Hz. The static tensile strength of 20 specimens resulted in the median initial strength, of 334 MPa, is used as input for fatigue and residual strength tests. Residual strength experimental data tests were performed at five intervals during the life for each of three stress levels such as 0.52, 0.44 and 0.36 of the median ultimate tensile strength. Cycle counts corresponded to 10%, 30%, 50%, 60% and 70% of the target lifetimes for that stress level. The residual strength results are for E-glass/vinyl ester laminate taken from Ref. [8] are summarised in the tables 1-3.

Table: 1 Residual strength data summary for E-glass/vinyl ester laminate at 36% of ultimate tensile strength. Experimental data are taken from Ref. [8]

| S.No. | Cycles  | n/N  | Residual Strength |
|-------|---------|------|-------------------|
| 1     | 50000   | 0.13 | 270.5             |
| 2     | 150000  | 0.4  | 284.5             |
| 3     | 250000  | 0.66 | 277.5             |
| 4     | 300000  | 0.8  | 270.9             |
| 5     | 350000  | 0.93 | 268               |

Table: 2 Residual strength data summary for E-glass/vinyl ester laminate at 44%
of ultimate tensile strength. Experimental data are taken from Ref. [8]

| S.No. | Cycles | n/N | Residual Strength |
|-------|--------|-----|-------------------|
| 1     | 5000   | 0.11| 287.5             |
| 2     | 15000  | 0.34| 282.5             |
| 3     | 25000  | 0.57| 250.42            |
| 4     | 30000  | 0.68| 230               |
| 5     | 35000  | 0.8 | 222               |

**Table: 3** Residual strength data summary for E-glass/vinyl ester laminate at 52% of ultimate tensile strength. Experimental data are taken from Ref. [8]

| S.No. | Cycles | n/N | Residual Strength |
|-------|--------|-----|-------------------|
| 1     | 1000   | 0.09| 308.06            |
| 2     | 3000   | 0.27| 284.5             |
| 3     | 5000   | 0.45| 277.5             |
| 4     | 6000   | 0.54| 270.9             |
| 5     | 7000   | 0.63| 268               |

3. **Model description**

The strength and stiffness of any structure degrade when it is subjected to cyclic loading. The formation of damage when the material is subjected to cyclic loading and its progression plays a major role in understanding the fatigue behaviour [9]. Damage factor is used to predict the amount of damage initiated in the specimen when it is subjected to fatigue loading. Hwang and Han[10] proposed a damage model as a function of cycle ratio as shown in figure 1 which has a trend either A, B or C.

![Figure 1](attachment:image.png)

**Figure 1.** Damage as a function of cycle ratio and damage trends.

During fatigue loading, the damage progresses continuously which follows the power law [12] and the damage factor is given by the equation,

\[
\frac{dD}{dN} = \frac{A\sigma^C}{B\sigma^B - 1}
\]  

(1)

Where, \(D\) - Damage factor, \(N_f\) - Number of cycles to failure, \(\sigma_a\) - Applied stress, \(B, C\) – Constants.
Integrating the above equation and applying the boundary conditions,

If \( N=0, D=0 \) and \( N=N_f, D=1 \)

the damage factor (D) is given by

\[
D = \left[ \left( \frac{\sigma_a}{\sigma_r} \right)^C \left( \frac{N}{N_f} \right) \right]^{1/B}
\]  

(2)

Where, \( \sigma_a \) - Applied stress, \( \sigma_r \) - Residual strength, \( D \) - Damage factor, \( N_f \) - Number of cycles to failure, \( N \) - Number of loading cycles.

Cumulative Damage model can also be defined as [12]

\[
D = 1 - \frac{\sigma_r}{\sigma_0}
\]

(3)

Substituting the above equation in the present damage model and rearranging gives,

\[
B = \log \left[ \left( \frac{\sigma_a}{\sigma_r} \right)^c \left( \frac{N}{N_f} \right) \right] 
\]

\[
\log \left[ 1 - \frac{\sigma_r}{\sigma_0} \right] 
\]

(4)

B and C values were calculated by plotting both terms by varying c values until the best fit obtained.

4. Results and Discussion

Damage factor is calculated from the Eqn.[2] by substituting the applied stress, residual strength, and the cycle ratio. The damage factor values at different stress levels are given in table 4-6. Figure 2-4 shows the damage trend for 32%, 44% and 52% of ultimate tensile strength respectively. P-M rule also presented in the figures for reference. At 36% of ultimate tensile strength damage progression follows the trend A and whereas it follows trend C for 44% and 52% of ultimate tensile strength. Damage has been reported even at 10% of the lifetime at all stress levels, which normally will not occur for metals and high stiff materials. This may be due to the inherent damage or flaw present in the constituent materials or due to manufacturing defects.

The damage factor reported at 10% of lifetime is 6.55%, 7.2% and 9.2% at 36%, 44% and 52% of ultimate tensile strength respectively. At 36% of ultimate tensile strength, damage grows slowly whereas in 44% and 52%, damage growth is slower during initial period but later it grows rapidly. Figure 5 shows the progression of damage at all stress levels throughout the lifetime. It is clear that even at 10% of the lifetime, the damage has been reported but it is almost same at all stress levels. In contrast when the percentage of lifetime increases, the behaviour of damage varies at all stress levels. At 52% of ultimate tensile strength, damage progression reached the maximum level at 70% of the lifetime.

Table: 4 Damage factor at 36% of UTS

| S.No | Cycles | n/N | Residual Strength | Damage Factor |
|------|--------|-----|------------------|---------------|
| 1    | 50000  | 0.13| 270.5            | 0.06          |
| 2    | 150000 | 0.4 | 284.5            | 0.59          |
### Table: 5 Damage factor at 44% of UTS

| S.No | Cycles  | n/N | Residual Strength | Damage Factor |
|------|---------|-----|-------------------|---------------|
| 1    | 5000    | 0.11| 287.5             | 0.07          |
| 2    | 15000   | 0.34| 282.5             | 0.49          |
| 3    | 25000   | 0.57| 250.42            | 0.76          |
| 4    | 30000   | 0.68| 230               | 0.85          |
| 5    | 35000   | 0.8 | 222               | 0.93          |

### Table: 6 Damage factor at 52% of UTS

| S.No | Cycles  | n/N | Residual Strength | Damage Factor |
|------|---------|-----|-------------------|---------------|
| 1    | 1000    | 0.09| 308.06            | 0.09          |
| 2    | 3000    | 0.27| 284.5             | 0.38          |
| 3    | 5000    | 0.45| 277.5             | 0.67          |
| 4    | 6000    | 0.54| 270.9             | 0.77          |
| 5    | 7000    | 0.63| 268               | 0.86          |

**Figure 2.** Damage progression at 36 % of ultimate tensile strength

**Figure 3.** Damage progression at 44 % of ultimate tensile strength.
5. Conclusions
A new damage model as a function of residual strength, applied stress and cycle ratio is proposed to predict the damage growth of composite. The model has been used to predict the damage progression at different stress levels. The damage factor follows trend C at low stress level and it has been changed to trend A when the stress level is increased. Damage has been reported even at low cycles of loading which may be due to inherent damage present in the composite materials. The model predicts the damage of the composites closely even though it does not report the types of damage. The model can be used effectively to predict the damage progression irrespective of the material used.

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