Scaling laws for electron cold injection in the narrow collision pulse approximation

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Abstract. The latest advances in laser wakefield electron acceleration show a better beam quality, but much progress is still needed concerning the control and tunability of the electron beam. The recently proposed cold injection scheme offers a solution to this problem. It involves the use of two counter-propagating laser pulses to dephase a certain number of electrons into the wakefield of the main pulse, so that they are accelerated to high energies. As circular polarization is used, there is no stochastic heating and the injection process becomes much easier to model. We show that cold injection can be reduced to a one-dimensional problem in the case of a narrow collision pulse. The dephasing process can then be seen as competition between the longitudinal ponderomotive force of the main pulse and the stationary beatwave force arising from collision of the two circularly polarized lasers. This analysis leads to scaling laws for cold injection in the narrow collision pulse approximation and to a condition for its realization. Three-dimensional particle-in-cell simulations support both these scaling laws and the condition for cold injection to occur.

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## 1. Introduction

Since the first experimental demonstrations of quasi-monoenergetic, few-hundred-MeV electron acceleration a few years ago [1–3], laser- and plasma-based accelerators [4] have shown impressive improvements in both electron beam quality and stability of operation, thanks to higher-quality lasers [5], novel target designs [6] and new injection techniques [7–9]. Stable, well-collimated beams of a few-hundred-MeV electrons with moderate charge (10–50 pC) and sub-10% energy spread are now routine for experiments in many laboratories worldwide. However, electron beam quality and shot-to-shot stability remain an issue in pursuit of gigaelectronvolt (GeV) electron energy [10–12], which is a major concern for compact x-ray free electron laser (XFEL) and Compton x-ray source design [13–16]. It has long been recognized that a single-stage GeV-scale electron accelerator requires a petawatt-scale laser driver [17–19]. The performance of the single-stage accelerator critically depends on the process of electron injection from the ambient plasma. Indeed, a flexible and efficient injection mechanism is key for controlling the final beam emittance, energy, energy spread and charge.

Many current experiments (as well as planned near-future projects) operate in the blowout regime. The formation of an electron density bubble, trailing the laser driver is the hallmark of this regime. The laser ponderomotive force expels all electrons from the near-axis region, thus creating a bucket devoid of electrons (electron density bubble [20, 21]). Injection of electrons in the blowout regime occurs predominantly from the sheath and is very robust [18, 19, 21–23]. Properly optimized petawatt pulses are predicted to accelerate GeV electron beams with 1% energy spread in plasmas of $10^{17}$ cm$^{-3}$ electron density [19, 24]. Unfortunately, injection from the sheath in the bubble regime causes another major problem: significant emittance growth due to a large transverse momentum of injected electrons [22]. Normalized transverse emittance of a few tens of mm mrad [17, 19, 22, 24] has been calculated. However favorable this feature is for laser–plasma-based synchrotron radiation sources [25, 26], it is clearly unacceptable for compact XFEL and Compton x-ray sources [14, 15]. From this point of view, enforcing injection of electrons from the near-axis region (thus with negligible transverse momentum) is clearly favorable. Ionization-induced injection, based on the production of electrons (injection candidates) via optical field ionization of the high-Z gas inside the bubble [27–30], is a possible solution to inject electrons coming from the near-axis region.

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However, adjustment of the collection volume (managing the laser field distribution inside the bubble) is experimentally a very challenging task.

Control of the final beam energy can be achieved by controlling the injection location: an earlier injection means a longer acceleration length and a larger final energy. This control can be achieved by enforcing wavebreaking of the plasma wave in a density down-ramp [9, 31, 32]. With this scheme, it is possible to trigger injection at the precise location of a density gradient inside the gas jet, resulting in a stable, dark-current free electron beam [33]. The colliding pulse scheme [7, 34–36] is another alternative that has been used to control the injection location and demonstrated control over the final energy of the beam [8].

It was also shown, with the same scheme, that variation of the injection pulse intensity can enable control over the injected charge [37, 38]. However, in the presence of stochastic heating [39–41] and important wake inhibition, it can be difficult to obtain predictable control of the beam parameters.

Inspired by this work, cold injection for electron wakefield acceleration was presented recently [42]. This scheme is similar to the classical colliding pulse scheme [8], with the difference that it relies on a different mechanism and uses circularly polarized laser pulses. In this configuration, stochastic heating no longer exists [42]. However, the beatwave, created by the collision of the two circularly polarized lasers, is still able to dephase a certain number of near-axis electrons and to drop them directly into the wakefield of the main pulse with an extremely low momentum. These electrons then satisfy the sufficient conditions to be injected and accelerated to high energies, with good beam quality [43]. Having no stochastic heating, this scheme is also very promising in terms of tunability since it becomes possible to write simple scaling laws, as we demonstrate below.

In this paper, we focus on the cold injection process and present simple scaling laws based on a one-dimensional (1D) approach, allowing the determination of the volume of dephased electrons. This gives information about the shape of the electron beam, but also about the amount of cold-injected charge, and allows direct optical control of these parameters. Unlike previous published work, we focus here on cases where the collision pulse waist is much smaller than the main pulse waist (the narrow collision pulse assumption). This allows us to completely neglect any kind of injection due to transverse effects or wake inhibition, as described in [43]. After deriving the model in section 2, we test it using numerical particle-in-cell (PIC) simulations in section 3.

2. Model

Cold injection occurs when two counter-propagative, circularly polarized laser pulses collide in a plasma. The so-called main pulse, which carries much more energy than the collision pulse, excites behind it a very strong, nonlinear plasma wave, called the wakefield, which propagates at the group velocity of the laser. When the two laser pulses collide, a stationary beatwave arises and traps some electrons for the time of the collision, dephasing them before releasing them directly into the wakefield. These so-called cold-injected electrons will then be trapped inside the wakefield and accelerated to high energies. In this section, we express the typical characteristics of the cold-injected electrons as a function of the laser parameters only. In order to do so, we determine the volume of plasma over which electrons are going to be dephased by the beatwave, and assume that every electron in this volume is going to be injected. This assumption holds true if an electron having zero longitudinal momentum and located at the
back of the main pulse is injected eventually. This is the case if the laser and plasma parameters are close to the matched blowout regime and the normalized amplitude of the main laser pulse $a_0$ is greater than 2 [28]. Indeed, in a sufficiently nonlinear wakefield, an electron dephased up through the main pulse will be injected as it keeps a negligible longitudinal velocity, whereas electrons that have not been dephased reach this position with a large negative velocity and quickly drift backward in the wakefield. In all of the following, we also assume that the collision pulse transverse size is much lower than the main pulse transverse size (narrow collision pulse). It was shown in [43] that cold injection could be perturbed by wake inhibition [37]. In some cases, the dephased electrons are able to modify the accelerating structure and the wakefield. This can result either in fewer cold-injected electrons if the wakefield is inhibited too much or, in contrast, additional non-cold injection due to the damping of the positive front part of the wakefield. Indeed, some electrons that are not trapped in the beatwave can be decelerated less in the first half of the bubble if the field is strongly damped. Then, they may enter the accelerating part of the bubble with a higher velocity than usual, which can lead to their injection. The strength of wake inhibition scales with the ratio between the volume of the dephased electrons and the volume of the bubble [43]. By choosing a narrow collision pulse, we perturb only a small fraction of the whole bubble and minimize these potential alternative sources (or sinks) of injection. Cold injection is then the only possible injection process. Finally, we also assume that most of the cold-injected electrons are trapped in the beatwave during the whole period of the laser collision. The validity of this last assumption is discussed below (equation (19)) and in section 3 in the light of the results of numerical simulations. Thus, the model aims at evaluating only fully dephased cold-injected electrons and neglects partially dephased cold-injected electrons that are trapped inside the beatwave for only part of the collision time.

The basic idea of this model is that in order for a particle to be fully dephased, the beatwave force it feels must prevail over the ponderomotive force of the main pulse during the whole collision time. We first compare both force envelopes in a simple 1D approach. Then we consider what happens at a distance $r$ from the axis and finally we account for the spatial oscillation of the beatwave force.

2.1. Envelopes

2.1.1. On-axis. The present model starts with the same 1D on-axis description as in [43]. The envelopes of the vector potentials of the two pulses are

$$\tilde{A}_0(x, t) = \frac{a_0}{\sqrt{2}} \exp \left[ -\left( \frac{t - x}{\tau_0} \right)^2 \right],$$

$$\tilde{A}_1(x, t) = \frac{a_1}{\sqrt{2}} \exp \left[ -\left( \frac{t + x}{\tau_1} \right)^2 \right].$$

$a_0$ and $a_1$ are, respectively, the normalized vector potentials of the main and collision pulses. $\tau_0$ and $\tau_1$ are the laser pulse durations normalized to $1/\omega_0$. The chosen frame implies that the two pulses fully overlap at $x = t = 0$. It is shown in [43] how the beatwave force arising during the interaction between the two circularly polarized laser pulses can be expressed as

$$F_b \simeq \frac{2}{\gamma} \tilde{A}_0 \tilde{A}_1 \sin(2\chi).$$
assuming that $\tau_0 \gg 1$ and $\tau_1 \gg 1$ (the laser pulses are made of, at least, a few cycles). Neglecting the collision pulse ponderomotive force leaves the following expression for the main pulse ponderomotive force:

$$F_p = -\frac{1}{2\gamma} \frac{\partial \tilde{A}_0}{\partial x} = \frac{2}{\gamma \tau_0^2} \tilde{A}_0^2 (x-t).$$

(4)

As in [43], we define $\chi$ as the ratio between the $F_b$ and $F_p$ envelopes

$$\chi = \frac{2\gamma \tilde{A}_0 \tilde{A}_1}{|F_p|} = \frac{\tau_0^2 \tilde{A}_1}{|x-t| \tilde{A}_0}. $$

(5)

We start by determining the region of space for which the beatwave envelope always prevails over the ponderomotive force or, to put it differently, the range of $x$ for which we have $\chi(x,t) > 1$ for any value of $t$. Let us define $\epsilon = x - t$ and restrain ourselves to the half-plane $\epsilon > 0$. The condition $\chi > 1$ then gives

$$x_{\text{bound}+}(\epsilon) < x < x_{\text{bound}+}(\epsilon),$$

with

$$x_{\text{bound}+} = \frac{\epsilon}{2} - L_{\text{bound}+}(\epsilon),$$

(7)

$$x_{\text{bound}+} = \frac{\epsilon}{2} + L_{\text{bound}+}(\epsilon),$$

(8)

$$L_{\text{bound}+}(\epsilon) = \frac{\tau_1}{2} \sqrt{\left(\frac{\epsilon}{\tau_0}\right)^2 + \ln \left(\frac{\tau_0^2 a_1}{a_0 \epsilon}\right)}. $$

(9)

The subscript + indicates that we are in the $\epsilon > 0$ half-plane. $\text{bound}$ and $\text{bound}$ stand for left and right boundaries. $L_{\text{bound}+}$ reaches its minimum when $\epsilon = \epsilon_{0+} = \frac{\tau_0}{\sqrt{2}}$. The value $\epsilon_{0+}$ is therefore a very good approximation of the value at which $x_{\text{bound}+}$ and $x_{\text{bound}+}$ reach, respectively, their maximum and minimum values (see figure 1). The same analysis in the $\epsilon < 0$ half-plane gives the symmetrical result $\epsilon_{0-} = -\epsilon_{0+}$ and $L_{\text{bound}-(\epsilon)} = L_{\text{bound}+}(-\epsilon)$. The boundaries for the particular cases $\epsilon = \epsilon_{0+}$ and $\epsilon = \epsilon_{0-}$ become

$$x_{\min+} = \frac{1}{2}(\epsilon_{0+} - L),$$

(10)

$$x_{\max+} = \frac{1}{2}(\epsilon_{0+} + L),$$

(11)

$$x_{\min-} = \frac{1}{2}(-\epsilon_{0+} - L),$$

(12)

$$x_{\max-} = \frac{1}{2}(-\epsilon_{0+} + L),$$

(13)

$$L = \frac{\tau_1}{\sqrt{2}} \sqrt{\alpha},$$

(14)

$$\alpha = 1 + 2 \ln \left(\frac{\sqrt{2}a_1 \tau_0}{a_0}\right). $$

(15)

The condition $\chi(x,t) > 1$ for any $t$ is therefore met in both half-planes when

$$\max(x_{\min+}, x_{\min-}) < x < \min(x_{\max+}, x_{\max-}).$$

(16)
Figure 1. $\chi$ on the laser axis, as a function of $x$ and $t$. In white regions, $\chi > 1$ and the envelope of the beatwave dominates. In gray regions, $\chi < 1$ and the ponderomotive force prevails. The short dashed line ($\epsilon = 0$) corresponds to the trajectory of the peak of the main pulse. Electrons located between $-x_{\text{lim}}$ and $x_{\text{lim}}$ are trapped in the beatwave and keep quasi-vertical trajectories in this $(x, t)$ plot during the whole pulse collision. For large absolute values of $\epsilon$, the beatwave looks dominant, but this is immaterial since both the ponderomotive force and the beatwave amplitudes tend to zero. The area of interest is limited to the narrow domain where collision occurs.

Electrons whose initial position is on-axis and between $-x_{\text{lim}}$ and $x_{\text{lim}}$ will experience a beatwave that has a greater envelope than the ponderomotive force at any timepoint during the pulse collision. They are most likely trapped in a beatwave bucket during the whole collision and will be fully dephased. Electrons initially located between $x_{\text{max}-}$ and $x_{\text{max}+}$ are going to be partially dephased and a few of them may eventually be injected, but they are not considered in this model (see figure 4). They can be neglected as long as $x_{\text{max}+} - x_{\text{max}-}$ is smaller than $x_{\text{max}-} - x_{\text{min}+}$, which can also be written as

$$2\tau_0/\sqrt{\alpha} \tau_1 < 1.$$  

2.1.2. Off-axis. We can extend this model to take into account the distance $r$ from axis assuming axial geometry of the laser intensity profiles. We simply introduce the waists $r_0$ and $r_1$ of the pulses so that $a_0$ becomes $a_0 \exp[-(r/r_0)^2]$ and $a_1$ becomes $a_1 \exp[-(r/r_1)^2]$. We obtain a generalized expression for $\chi$ and $x_{\text{lim}}$:

$$x_{\text{lim}}(r) = \frac{1}{2\sqrt{2}} \left( \tau_1 \sqrt{\alpha} - \tau_0 - 2\beta r^2 \right).$$  

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\[ \chi(x, r, t) = \chi(x, 0, t) \exp(-\beta r^2), \] 
(21)

with
\[ \beta = \frac{1}{r_1^2} - \frac{1}{r_0^2}. \] 
(22)

\( \chi(x, 0, t) \) is \( \chi \) on-axis as given by equation (5). We can therefore reconstruct the whole three-dimensional volume of plasma, \( V_d \), in which \( \chi > 1 \) over the whole collision time.

\[ V_d = \{(x, r) \mid x \in [-x_{\text{lim}}(r), x_{\text{lim}}(r)] \text{ and } r \in [0, r_d(x)] \}, \] 
(23)

with
\[ r_d(x) = \sqrt{\frac{\alpha}{2\beta}} \left( \frac{(2\sqrt{2}|x| + \tau_0)^2}{2\beta \tau_1^2} \right). \] 
(24)

Equation (24) is obtained by noting that \( x_{\text{lim}}(r_d(x)) = |x| \) or equivalently \( \chi(x, r_d, \epsilon_0 - x) = \chi(x, 0, \epsilon_0 - x) \exp(-\beta r_d^2) = 1 \). Therefore \( r_d(x) \) is the maximum distance from the axis at which \( \chi(x, r, t) > 1 \) at all times for a given \( x \).

2.1.3. Conditions. A few conditions must be met for all the above equations to be valid. The first condition is imposed by equation (18): \( \alpha \) must be strictly positive for the square root to be defined. But as \( x_{\text{lim}} \) must be positive in order for solutions to exist, equation (18) imposes an even stronger condition on \( \alpha \) and we must have

\[ \alpha > \frac{\tau_0^2}{\tau_1^2}. \] 
(25)

Similarly, in order to have a well-defined \( r_d \), we need to have \( \beta > 0 \), which can also be written as

\[ r_1 < r_0. \] 
(26)

This last condition is always true because we restrict ourselves to narrow collision pulses.

2.2. Beatwave phase

2.2.1. Envelope modulation. In fact, not all electrons inside \( V_d \) are going to be fully trapped inside the beatwave. Trapping not only depends on the envelope, but also on the phase of the beatwave. It is stationary and its phase depends only on \( x \). Now that we have elucidated the properties of \( \chi \), we can consider the competition between the pondermotive force and the full beatwave force, including the oscillating part. The region in which we have \( \chi(x, r, t)|\sin(2x)| > 1 \) for any \( t \) should be the region in which the modulated beatwave prevails over the ponderomotive force at all times. It can be shown for a given \( x \) that \( \chi \) on-axis is minimum when \( \epsilon = \epsilon_0 \), so that the previous inequality implies that

\[ \chi_{\text{min}}(x) \exp(-\beta r_d^2)|\sin(2x)| > 1, \] 
(27)

where \( \chi_{\text{min}}(x) = \chi(x, r = 0, t = x - \epsilon_0) \). \( r_d \) is defined as the distance from the axis at which \( \chi_{\text{min}}(x) \exp(-\beta r_d^2) = 1 \) so equation (27) becomes

\[ \exp\left[-\beta \left(r^2 - r_d^2(x)\right)\right]|\sin(2x)| > 1, \] 
(28)
Figure 2. The gray and white regions are defined by $\chi_{\text{min}} \sin(2x)$, respectively lower and greater than 1. $r_p$ defines the border between those two regions and shows fast modulations inside the envelope defined by $r_d$. By definition of $r_{\text{lim}}$, the width of each white arch becomes lower than $\pi/4$ when $r < r_{\text{lim}}$. The blue dots are the points of coordinates $(m_i, r_{\text{lim}})$. which gives $r_p(x)$, the maximum distance from the axis at which the spatially modulated beatwave is always stronger than the ponderomotive force:

$$r < \sqrt{r_d^2(x) + \frac{\ln(|\sin(2x)|)}{\beta}} = r_p(x).$$ (29)

Note that $r_p$ is well defined only in regions where $|\sin(2x)| > \exp[-\beta r_d^2(x)]$. Using the following notation,

$$m_i = \frac{\pi}{4} + \frac{i\pi}{2}, \quad i \in \mathbb{Z},$$ (30)

$$r_i = r_d(m_i),$$ (31)

$$l_i = \frac{\pi}{4} - \arcsin(\exp(-\beta r_i^2)),$$ (32)

$$S_i = [m_i - l_i, m_i + l_i],$$ (33)

$r_p(x)$ is well defined for $x \in \bigcup_{i \in \mathbb{Z}} S_i$. Note that we neglect here the variations of $r_d$ on $S_i$. The validity of this assumption can be verified by examining figure 2, where $r_d$ remains almost constant whereas $r_p$ goes from 0 to 30 six times.
2.2.2. Oscillation inside the beatwave. A particle supposedly trapped in the beatwave will oscillate along the $x$-direction with an average normalized amplitude of approximately $\pi/2$. We assume that a particle will indeed be trapped and thus dephased if the beatwave force it experiences during this oscillation is stronger than the ponderomotive force during more than half of the average oscillation period. To put it differently, the arches defined by $r_p$ must have a width greater than $\pi/4$ (see figure 2). For each $i$, we define $r_{lim,i}$ as the maximum distance from the axis at which this assumption is true,

$$r_{lim,i} = r_p(m_i + \pi/8) = \sqrt{r_i^2 - \frac{\ln(2)}{2\beta}}. \quad (34)$$

$r_{lim,i}$ is well defined only when $r_i^2 > \frac{\ln(2)}{2\beta}$, which is the case if

$$|x| < \frac{\tau_1 \sqrt{\alpha - \ln(2)} - \tau_0}{4\sqrt{\ln(2)}} \equiv x_{\text{max}}. \quad (35)$$

Eventually, if we define $I$ as

$$I = \{i \in \mathbb{Z} \mid |m_i| < x_{\text{max}}\},$$

the total injected charge $Q$ will be

$$Q = \sum_{i \in I} \frac{\pi^2 r_{lim,i}^2}{2} n_e, \quad (37)$$

where $n_e$ is the plasma density. The injected volume of plasma is considered to be the whole volume inside $r_{lim,i}$. It is modeled as the discrete sum of cylinders of radius $r_{lim,i}$ and of height $\pi/2$.

2.2.3. Extra conditions. One more condition must be met in order for the above equations to remain valid. Equation (35) imposes $\alpha \geq \tau_0^2 \tau_1 + \ln(2)$, which makes equation (25) true as well. Eventually, the condition making cold injection possible in the frame of the narrow collision pulse assumption is reduced to

$$\alpha \geq \frac{\tau_0^2}{\tau_1^2} + \ln(2). \quad (38)$$

In the most likely case of $\tau_0 = \tau_1$, this leads to $a_1 > \frac{\omega_0}{\tau_0}$, which sets a threshold intensity for the collision pulse.

3. Simulations

Even though the description made in section 2 accounts for the distance from the axis, it remains fundamentally 1D, as the transverse motion of electrons is neglected. In this section, we verify that this approach is correct by running fully explicit electromagnetic (EM) 3D PIC simulations. We use the massively parallel quasi-cylindrical code CALDER-Circ [44], which is particularly well adapted to laser wakefield acceleration. The EM fields are discretized along the longitudinal and radial positions, and are decomposed into their poloidal modes. Only the first two modes are conserved, resulting in a cost equivalent to a few 2D Cartesian simulations. This is reasonable because the process never strays far from a cylindrical geometry, and it allows a large number
Table 1. Simulation parameters. $a_0$ and $a_1$ are the normalized vector potentials. Lengths are given in microns and times in femtoseconds. Bold face values are the only ones differing from simulation 1. The indicated density values are on-axis. The radial density profile is given by equations (39) and (40).

| Simulation | $a_0$ | $r_0$ | $a_1$ | $r_1$ | $n_{e0}/n_{crit}$ |
|------------|-------|-------|-------|-------|-------------------|
| 1          | 2.7   | 30    | 32.8  | 0.1   | 60                |
|            |       |       |       |       | $5 \times 10^{-4}$ |
| 2          | 2.7   | 30    | 32.8  | 0.1   | 120               |
|            |       |       |       |       | $5 \times 10^{-4}$ |
| 3          | 2.7   | 30    | 32.8  | 0.2   | 60                |
|            |       |       |       |       | $5 \times 10^{-4}$ |
| 4          | 2.7   | 30    | 32.8  | 0.03  | 60                |
|            |       |       |       |       | 25                |

Figure 3. Electron density plots during injection. Densities are normalized to the critical density. Panels from top left to bottom right are taken at time of pulse collision, electron dephasing, electron trapping and electron acceleration/focusing.

of macro-particles (40 per cell), a high resolution in the longitudinal direction ($dx = 0.125k_0^{-1}$, with $k_0 = \omega_0/c$ being the laser pulse wavenumber) and a low sampling noise. In the transverse direction, $dr = 2.5k_0^{-1}$, and the time step is $dt = 0.122\omega_0^{-1}$. The wavelength of the driver is $\lambda_0 = 2\pi k_0^{-1} = 1 \mu$m.

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Table 2. Numerical results. Charges are given in picocoulombs. $q_{\text{full}}$ is the fully dephased cold-injected charge (green dots in figure 4) and $q_{\text{inj}}$ is the total injected charge (green and red dots in figure 4). In the absence of stochastic heating, transverse effects and wake inhibition, the slight difference between $q_{\text{full}}$ and $q_{\text{inj}}$ is mainly due to partially dephased injected electrons, except for simulation 4 where the violation of the narrow collision pulse assumption leads to wake inhibition and other forms of injection occur. $\Delta E/E$ is the energy spread full width at half maximum, $\epsilon$ is the transverse emittance $(1/(m_e c) \sqrt{\langle Y^2 \rangle \langle P_y^2 \rangle - \langle YP_y \rangle^2})$ given in mm mrad, $\theta$ is the divergence of the beam given in mrad, $\tau_b$ is the rms bunch length given in fs and $I_b$ is the beam peak current given in kA.

| Simulation | $q_{\text{full}}/q_{\text{inj}}$ (%) | $q_{\text{inj}}$ | $q_{\text{full}}$ | $q_{\text{model}}$ | $\Delta E/E$ (%) | $\epsilon$ | $\theta$ | $\tau_b$ | $I_b$ |
|------------|------------------------------------|-----------------|-----------------|-----------------|-----------------|-------|------|-------|------|
| 1          | 94                                 | 8.2             | 7.8             | 8.9             | 10.7            | 2.3   | 8.0  | 11.4  | 0.6  |
| 2          | 97                                 | 25.7            | 24.9            | 26.3            | 20.4            | 6.0   | 6.9  | 7.1   | 2.8  |
| 3          | 97                                 | 24.1            | 23.4            | 22.7            | 24.4            | 3.5   | 6.7  | 13.3  | 1.4  |
| 4          | 0                                  | 2.2             | 0               | 0               | –               | –     | –    | –     | –    |

Figure 4. Longitudinal momentum and position of all monitored macro-particles during the collision of the two pulses. Red and green particles are going to be injected into the wakefield, but only green particles are fully dephased cold-injected electrons. Red particles are labeled as ‘other injection’, meaning that they are injected either because of wake inhibition or, more likely, because of a partial but sufficient phase difference.

3.1. Testing the model

To validate the scenario described in section 2, we present a set of four simulations whose parameters are displayed in table 1. Simulation 1 is used as the reference. In simulation 2 we double the length of the collision pulse, and in simulation 3 we double its amplitude. In all the
Figure 5. Initial position of fully dephased cold-injected macro-particles compared to the model for simulation 1.

first three cases the ratio $r_1/r_0$ is around 0.15. Simulation 4 has a $a_1$ just below the threshold given by equation (38) and does not verify the narrow collision pulse assumption. The transverse density profile in all simulations mimics a capillary and is given by

$$n_e(r) = n_{e0} \left(1 + \frac{r^2}{R_c^2}\right), \quad \text{if} \quad r < 400 \, k_0^{-1} \simeq 64 \, \mu\text{m},$$

$$n_e(r) = n_{\text{max}}, \quad \text{if} \quad r \geq 400 \, k_0^{-1} \simeq 64 \, \mu\text{m},$$

where $n_{e0} = 1.5 \times 10^{-4} n_c$, $n_c$ is the critical density, $R_c = 106.9 k_0^{-1}$ and $n_{\text{max}} = 2.25 \times 10^{-3} n_c$. This profile helps guide the laser over long distances but has little influence on injection since the dephased volume is very close to the axis, and the density inside this volume only varies by about 8%. This is not going to significantly change the injection momentum threshold, also called the separatrix, and cannot prevent, or enhance, cold injection. Also, unlike self-injection, the cold injection process in the narrow collision pulse approximation is independent of the bubble dynamics and modifying it by using a capillary-shaped density profile is not going to have any influence. In order to understand precisely what is happening in the simulation and to be able to compare it with the model scenario, we monitor the complete trajectories of a randomly chosen 2% of all macro-particles that come closer than $150 \, k_0^{-1} \simeq 24 \, \mu\text{m}$ to the axis. This is wide enough to capture the whole bubble, as can be seen in figure 3. Any monitored macro-particle that ends up with a large longitudinal momentum is considered ‘injected’. Among the injected macro-particles, those whose longitudinal position varied by less than $\pi$, the beatwave period, during the whole laser interaction, are considered ‘fully dephased cold-injected’. In fact, as expected when equation (19) is true, a very few electrons are injected without being fully dephased in our cases (see figure 4 and table 2). The assumption in our model of neglecting partially dephased cold-injected electrons seems therefore to be both natural and convenient. Moreover, we can observe from simulation 4 of table 2 that no fully dephased cold
injection occurs when $a_t$ is below the threshold given by equation (38), but some other forms of injection may appear since the narrow collision pulse assumption is violated.

Table 2 shows very good agreement between the injected charges given by the model and those given by the simulations. The difference is less than 10% in all cases. As far as the shape of the bunch is concerned, figure 5 confirms that most of the fully dephased cold-injected particles are initially inside the volume defined by $r_{lim}$. In particular, the length and radius of the fully dephased volume are reasonably well described. The poor energy spread has several explanations. Firstly, the simulations end not long after injection, and the beam energies are of the order of 200 MeV. Energy spreads and divergences could be strongly reduced if acceleration up to the dephasing length was achieved, reaching energies of several GeV [42]. Secondly, the bunch shape and the beam loading effect are not optimized, so the bunch front and its rear see quite different accelerating fields, thus increasing the absolute energy spread during propagation. However, as can be seen in table 2, the beam current and duration evolve when the collision pulse parameters are changed. Therefore, it should be possible to optimize the beam loading effect and improve the energy spread by tuning these parameters. Divergence and emittance are given as an indication but, for the reasons previously mentioned, they are not typical of an optimized cold-injected electron beam after a long acceleration. Energy spread and divergence are expected to drop significantly during the propagation, but the normalized transverse emittance is likely to be conserved and is already comparable with the best emittances.
that have been measured in laser wakefield accelerators [45]. Nevertheless, the present work is focused on the scaling of the injected electrons, and optimization of the beam quality is left for future work.

3.2. More information from the simulation

It is worth noting in figure 5 that even though a few fully dephased cold-injected macro-particles are initially outside the volume defined by \(r_{\text{lim}}\), none of them are located beyond \(r_{d}\). Figures 6 and 7 confirm that most electrons within the volume \(V_{d}\), defined in equation (23), are trapped in the beatwave buckets but are not necessarily injected. Figure 6 shows that the dynamics of non-injected particles is mostly driven by the ponderomotive force of the pump laser. Particles that are at the front of the pulse are pushed forward and those at the back of the pulse are pushed backward. The local proportion of particles influenced by the ponderomotive force is all the higher when the beatwave is weak. To show this, we represent by a black line in figure 7 the time at which particles should be pushed by the forward ponderomotive force. It shows that, as time advances, the forward ponderomotive force is opposed to a stronger and stronger beatwave amplitude. This explains why many particles are pushed forward in the early stages of the collision, but very few later. The opposite is the case for the rear ponderomotive force: very few particles are pushed backward early, but almost all of them are kicked out of the beatwave later when its amplitude becomes much smaller.
4. Conclusion

Cold injection is a complex process that depends on many parameters, as explained previously [42, 43]. By using a narrow collision pulse, it becomes possible to neglect all transverse effects and wake inhibition. This reduces the cold injection to an essentially 1D problem. Additionally, if one also assumes $2\tau_0/\sqrt{\alpha\tau_1} < 1$, partially dephased injected particles become only a small fraction of the total injected particles and it can be assumed that injected particles all result from trapping and dephasing in the beatwave during the whole collision time. As a consequence, it becomes possible to formulate a simple scaling law for the total injected charge and the shape of the injected volume, making these parameters tunable. Control over transverse extension and length, in particular, is of interest for future x-ray sources using betatron radiation [13, 26, 46]. The length of the volume will directly influence the duration of the x-ray source, and its width the amplitude of the betatron oscillations. In these respects, beam tunability may be as important an achievement as beam quality, and cold injection is promising for both.

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