Light-induced bound electron states in two-dimensional systems: Contribution to electron transport

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In two-dimensional (2D) electron systems, an off-resonant high-frequency circularly polarized electromagnetic field can induce the quasi-stationary bound electron states of repulsive scatterers. As a consequence, the resonant scattering of conduction electrons through the quasi-stationary states and the capture of conduction electrons by the states appear. The present theory describes the transport properties of 2D electron gas irradiated by a circularly polarized light, which are modified by these processes. Particularly, it is demonstrated that irradiation of 2D electron systems by the off-resonant field results in the quantum correction to conductivity of resonant kind.

I. INTRODUCTION

The control of physical properties of various quantum systems by an off-resonant high-frequency electromagnetic field became the important and established research area which resulted in many fundamental effects (see, e.g., Refs. 1–9). Since the frequency of the field lies far from characteristic resonant frequencies of an irradiated system (off-resonant field), the field can not be absorbed by electrons and only interacts non-resonantly with them (“dresses” them). As a result, the behavior of dressed electrons varies as a function of the dressing field. To clarify the field-induced features of electronic properties of low-dimensional systems, many works dedicated to the electromagnetic dressing of various nanostructures — including quantum rings10–13, quantum wells14–18, topological insulators19–21, graphene and related 2D materials22–41, etc — were published.

Among variety of phenomena in periodically driven systems, the effect of dynamical stabilization should be noted especially. Generally, the dynamical stabilization is the fundamental physical effect which consists in the stabilization of initially unstable systems by oscillating external actions (see, e.g., Ref. 4). Phenomenologically, this effect was known for a long time to any circus juggler who hold various objects in balance by vibrational movements of its hands. First correct description of the effect in the frame of classical Hamilton mechanics was done by P. L. Kapitza who suggested the simple mechanical model based on a pendulum42. The Kapitza pendulum is the system consisting of a point mass attached to a light inextensible rod connected to a vibrating suspension. In the case of fixed point of suspension, this model describes the conventional mathematical pendulum for which there are two points of equilibrium (the lower and the upper). The equilibrium of the pendulum in the upper point is unstable and any infinitesimal perturbation leads to loss of the equilibrium. However, in contravention of intuition, the upper (vertical) position of the pendulum can be steady in the case of fast oscillating suspension. Thus, the Kapitza pendulum acquires the quasi-stable equilibrium (the local minimum of its potential energy) at the upper point which corresponds to the maximum of its potential energy in the absence of oscillations. This effect is of universal nature and occurs in many different areas of physics. For example, an oscillating laser field can lead to stabilization of charged ionic systems whose components repulse each other according to the Coulomb law and, therefore, cannot exist as a whole without the field43. Despite the long prehistory of the effect of dynamical stabilization, its possible manifestations in nanostructures still wait for detailed study. To fill partially this gap in the theory, we analyzed recently the behavior of various repulsive potentials in nanostructures driven by an oscillating field in context of the dynamical stabilization44. Particularly, it was found that a circularly polarized electromagnetic field can induce the local minima of potential energy in the core of 2D repulsive potentials. As a consequence, the quasi-stationary electron states confined near the local minima appear. The present article is dedicated to theoretical analysis of the electron transport in 2D systems modified by the light-induced quasi-stationary electron states bound at repulsive scatterers.

The article is organized as follows. In Sec. II, we develop the theory of the light-induced quasi-stationary electron states bound at short-range scatterers modeled by the repulsive delta-potential. In Sec. III, the Boltzmann kinetic equation taking into account the scattering and capture of conduction electrons by the quasi-stationary bound states is solved. In Sec. IV, analysis of the conductivity of the irradiated 2D electron system is performed. The last two sections of the article contain conclusion and acknowledgments.

II. MODEL

Let us consider a 2D electron system in the \((x, y)\) plane irradiated by a circularly polarized electromagnetic wave propagating along the \(z\) axis with the vector potential \(\mathbf{A}(t) = (A_x, A_y) = (cE_0/\omega)(\sin \omega t, \cos \omega t, 0)\), where \(E_0\) is the electric field amplitude of the wave, and \(\omega\) is the wave frequency which lies far from characteristic resonant frequencies of the electron system (the off-resonant dressing field) [see Fig. 1a]. The physical properties of
the transformed Hamiltonian (1) reads
\[ H' = \hat{\mathbf{p}}^2 / 2m_e + U(\mathbf{r} - \mathbf{r}_0(t)), \]
(3)
where the radius vector \( \mathbf{r}_0(t) = (-r_0 \cos \omega t, r_0 \sin \omega t) \) describes the classical circular trajectory of electron movement in the circularly polarized field, and
\[ r_0 = |eE_0| / m_e \omega^2 \]
(4)
is the radius of the trajectory. Physically, the unitary transformation \[ T \]
corresponds to the transition from the laboratory reference frame to the rotating reference frame,
\[ \mathbf{r} \rightarrow \mathbf{r} - \mathbf{r}_0(t). \]
(5)
Expanding the oscillating potential in Eq. (3) into a Fourier series,
\[ U(\mathbf{r} - \mathbf{r}_0(t)) = U_0(\mathbf{r}) + \sum_{n=1}^{\infty} U_n(\mathbf{r}) e^{in\omega t} + \text{c.c.}, \]
(6)
the transformed Hamiltonian \[ H' \] can be rewritten as
\[ H' = \hat{\mathbf{p}}^2 / 2m_e + U_0(\mathbf{r}) + \sum_{n=1}^{\infty} U_n(\mathbf{r}) e^{in\omega t} + \text{c.c.}, \]
(7)
where
\[ U_0(\mathbf{r}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} U(\mathbf{r} - \mathbf{r}_0(t)) d(\omega t) \]
(8)
is the stationary part of the potential, which is responsible for the smooth motion of 2D electrons, and \( U_n(\mathbf{r}) \) are the Fourier coefficients of the oscillating potential. The stationary potential \[ U_0(\mathbf{r}) \] should be treated as a repulsive potential dressed by an oscillating field (dressed potential). The specific feature of the dressed potential \[ U_0(\mathbf{r}) \] is the local minimum existing near \( r = 0 \) if the field is strong enough\[ . \]
(see Fig. 1b). As a consequence of the local minimum, the domain of attraction takes place in the core of the repulsive potential. This attraction results in confinement of an electron at the repulsive potential and the bound electron state with the energy \( \varepsilon_s \) and the localized wave function \( \varphi_s \) appears. In the following, we will assume that the field frequency, \( \omega \), satisfies the two conditions: Firstly,
\[ \omega \tau_e \gg 1, \]
(9)
where \( \tau_e \) is the mean free time of conduction electrons; secondly, the field frequency lies far from resonant frequencies of the bound state. Under the first condition, scattering processes cannot destroy the bound state, whereas the second condition allows to neglect the effect of the oscillating terms of the Hamiltonian \[ \mathcal{H}_0 \] on the bound state. As a consequence, the electron dynamics of an electron confined at a scatterer can be described solely by the stationary part of the Hamiltonian\[ . \]
\[ \mathcal{H}_0 = \hat{\mathbf{p}}^2 / 2m_e + U_0(\mathbf{r}). \]
(10)
The field-induced local minimum of the dressed potential \( U_0(\mathbf{r}) \) forms the quantum well which separates the bound electron state inside the well, \( \varphi_s \), from the states of free conduction electrons outside the well, \( \psi_k = e^{ikr} \),
by the potential barrier between the turning points, \( r_a \) and \( r_b \) (see Fig. 1b). Physically, the electron tunneling through the barrier, \( \varphi_s \rightarrow \psi_s \), results in finite lifetime of the bound state and the broadening of its energy. To find the bound state energy \( \varepsilon \) and the energy broadening \( \Gamma \), we have solve the Schrödinger equation, \( \mathcal{H}_0 \psi = \varepsilon \psi \), with the Hamiltonian \( \mathcal{H}_0 \). Let us model the repulsive potential \( U(r) \) with the delta function,

\[
U(r) = u_0 \delta(r),
\]

where \( u_0 > 0 \) is the strength of the repulsive potential. Substituting the potential \( \mathcal{H}_0 \) into Eq. \( \mathcal{H}_0 \psi = \varepsilon \psi \), we arrive at the dressed delta-potential,

\[
U_0(r) = \frac{u_0 \delta(r - r_0)}{2\pi r_0}. \tag{12}
\]

Thus, the circularly polarized field turns the repulsive delta-potential \( \mathcal{H}_0 \) into the radial delta-potential barrier \( \mathcal{H}_0 \). As a consequence, quasi-stationary electron states confined inside the area banded by the radial barrier \( 0 < r < r_0 \) can exist. Substituting the dressed potential \( \mathcal{H}_0 \) into Eq. \( \mathcal{H}_0 \psi = \varepsilon \psi \), we arrive at the Hamiltonian

\[
\mathcal{H}_0 = -\frac{\hbar^2}{2m_e r} \left[ \frac{\partial}{\partial r} \left( \frac{\partial}{\partial r} \right) + \frac{1}{r} \frac{\partial^2}{\partial \varphi^2} \right] + \frac{u_0 \delta(r - r_0)}{2\pi r_0}, \tag{13}
\]

where \( \varphi \) is the azimuth angle in the 2D plane. The eigenfunction of the Hamiltonian, \( \psi \), which corresponds to an electron confined in the area \( 0 < r < r_0 \), must be finite at \( r = 0 \) and satisfy the condition \( \psi|_{r \rightarrow \infty} \propto e^{ikr} \), where \( k = \sqrt{2m_e \varepsilon}/\hbar \) is the electron wave vector. Therefore, the sought wave function can be written using the Bessel functions as

\[
\psi = e^{im\varphi} \left\{ A J_m(kr), 0 < r < r_0 \right\}, \tag{14}
\]

where \( m = 0, \pm 1, \pm 2, \ldots \) is the angular momentum, \( J_m(z) \) is the Bessel function of the first kind, \( H_m(z) = J_m(z) + iN_m(z) \) is the Bessel function of the third kind (the Hankel function of the first kind), \( N_m(z) \) is the Bessel function of the second kind (the Neumann function), and \( A, C \) are the constants. Substituting the wave function \( \psi \) into the Schrödinger equation with the Hamiltonian \( \mathcal{H}_0 \) and integrating it over \( r \) near \( r = r_0 \), we arrive at the continuity condition for electron current density at the radial delta-potential barrier,

\[
CH_m'(z) - A \left[ J_m'(z) + \frac{m_e u_0}{\hbar^2 kr_0} J_m(z) \right] = 0, \tag{15}
\]

where \( z = kr_0 \). As to the continuity condition for the electron wave function \( \psi \) at the radial barrier, it reads

\[
CH_m(z) - A J_m(z) = 0. \tag{16}
\]

The two homogeneous algebraic equations \( \mathcal{H} = 0 \) and \( \mathcal{H}_0 = 0 \) define the constants \( A \) and \( C \), whereas the secular equation arisen from them,

\[
H_m'(z) J_m(z) - H_m(z) \left[ J_m'(z) + \frac{m_e u_0}{\hbar^2 kr_0} J_m(z) \right] = 0, \tag{17}
\]

defines the total electron energy, \( \varepsilon \). To simplify Eq. \( \mathcal{H}_0 \), let us apply the known equalities, \( J_{\nu+1}(z) - J_{\nu-1}(z) = -2/\pi z \) and \( Z_{\nu}(z) = [Z_{\nu-1}(z) - Z_{\nu+1}(z)]/2 \), where \( Z_{\nu}(z) \) is any Bessel function (see, e.g., Ref. 50). Then Eq. \( \mathcal{H}_0 \) can be rewritten in the compact form as

\[
H_m(z) J_m(z) = i\alpha, \tag{18}
\]

where \( z = kr_0 \) and \( \alpha = 2\hbar^2/m_e u_0 \).

Although the secular equation \( \mathcal{H}_0 \) can be easily solved numerically, there is the important particular case of strong repulsive potential, \( \alpha \ll 1 \), when solution of this equation can be found analytically. Namely, let us seek roots of Eq. \( \mathcal{H}_0 \) as a power series, \( z_{nm} = \varepsilon_{nm} - i\Gamma_{nm}/2 \), and the corresponding wave functions \( \psi \). The energy of quasi-discrete electron level is

\[
\varepsilon_{nm} = \frac{\hbar^2 z_{nm}^2}{2m_e r_0^2} + \mathcal{O}(\alpha^3), \tag{19}
\]

the broadening of the energy level is

\[
\Gamma_{nm} = \frac{4\varepsilon_{nm}\alpha^2}{N_m(z_{nm})(J_{m+1}(z_{nm}) - J_{m-1}(z_{nm}))} + \mathcal{O}(\alpha^3), \tag{20}
\]

\( z_{nm} \) is the \( n \)th zero of the \( m \)th Bessel function of the first kind (i.e., \( J_m(z_{nm}) = 0 \)), and \( n = 1, 2, 3, \ldots \) is the principal quantum number. As expected, in the limiting case of \( u_0 \to \infty \), the energy broadening \( \Gamma_{nm} \) is zero and the quasi-stationary electron state \( \psi \) turns into the stationary one.

Generally, the discussed bound state exists if its energy broadening is small as compared with the characteristic distance between the neighbor energy levels. Therefore, we will restrict the following consideration by the ground bound state, \( \varphi_g(r) = \psi_{10}(r) \), which has the minimal broadening \( \Gamma_g = \Gamma_{10} \). It follows from Eqs. \( \varepsilon_{10} = 1 \) that the energy of the ground bound state and its broadening read

\[
\varepsilon_s = \frac{\hbar^2 z_{10}^2}{2m_e r_0^2}, \tag{21}
\]

\[
\Gamma_s = \frac{2\varepsilon_s \alpha^2}{N_{10}(z_{10})(J_{11}(z_{10}) - J_{10}(z_{10})), \tag{22}
\]

where \( z_{10} \approx 2.4 \) is the first zero of the zeroth Bessel function of the first kind. Correspondingly, the applicability condition of Eqs. \( \varepsilon_{10} = 1 \) can be written as

\[
\frac{\Gamma_s}{\varepsilon_s} \ll 1. \tag{23}
\]
The condition (23) can be satisfied if the scattering potential (11) is strong enough, i.e. \( \alpha = 2\hbar^2/m_e u_0 \ll 1 \).

### III. KINETIC EQUATION

The field-induced modification of a repulsive potential discussed above results in both capture of conduction electrons at the bound state and to scattering them through the quasi-discrete energy level \( \varepsilon_s \). Let us analyze the effect of these processes on electron transport under the stationary field \( E \) applied to the 2D system of the area \( S \). In the laboratory reference frame, the conventional Boltzmann kinetic equation (see, e.g., Ref. 51) can be written as

\[
\frac{\partial f_k}{\partial t} \bigg|_{\text{field}} + \frac{\partial f_k}{\partial t} \bigg|_{\text{scatter}} = 0, \tag{24}
\]

where \( f_k \) is the distribution function of conduction electrons with the wave vector \( k \),

\[
\frac{\partial f_k}{\partial t} \bigg|_{\text{field}} = -\frac{\partial f_k}{\partial k \cdot E}, \tag{25}
\]

is the field term describing the transport of conduction electrons under the weak stationary electric field \( E \),

\[
\frac{\partial f_k}{\partial t} \bigg|_{\text{scatter}} = \sum_{k'} [f_{k'} (1 - f_k) - f_k (1 - f_{k'})] w_{kk'}, \tag{26}
\]

is the term describing the scattering of conduction electrons, and \( w_{kk'} \) is the probability of electron scattering between the states \( k' \) and \( k \) per unit time. In the following, we will assume that the light-induced bound states of scatterers can capture only one electron per scatterer because of the Coulomb repulsion between electrons. Then the conservation law for the total number of electrons in the 2D system reads

\[
N_c + N_s f_s = N_0, \tag{27}
\]

where \( N_0 \) is the total number of conduction electrons in the 2D system in the absence of irradiation,

\[
N_c = 2 \sum_k f_k \tag{28}
\]

is the total number of conduction electrons in the irradiated 2D system, and \( N_s f_s \) is the total number of electrons captured by scatterers. Correspondingly, \( N_s \) is the total number of scatterers in the 2D system, and \( f_s \) is the distribution function of conduction electrons captured by the scatterers. In Eq. (28) and what follows, the sum symbol \( \sum_k \) denotes the summation over all electron states with different wave vectors \( k \), excluding the summation over the spin freedom degrees [see Eq. (A2) in Appendix].

The distribution function of captured electrons, \( f_s \), is defined by the balance equation for them,

\[
\sum_k W_{ks} f_s (1 - f_k) = \sum_k W_{sk} f_k (1 - f_s), \tag{29}
\]

where

\[
W_{sk} = W_{ks} = \frac{\hbar^2 \Gamma_s}{S m_e [(\varepsilon_k - \varepsilon_s)^2 + (\Gamma_s/2)^2]} \tag{30}
\]

is the probability of capture of a conduction electron with the wave vector \( k \) and the energy \( \varepsilon_k = \hbar^2 k^2/2m_e \) by a scatterer, which is derived in Appendix. Physically, the balance equation (29) means that the probability of electron transition from the bound state to the continuum of free conduction electrons is equal to the probability of the inverse process for any stationary distribution of electrons. Substituting Eq. (30) into Eq. (29) and keeping in mind Eqs. (A2) and (23), the distribution function of conduction electrons captured by scatterers reads

\[
f_s = \sum_k W_{sk} f_k. \tag{31}
\]

Since an oscillating field driving 2D electrons is assumed to satisfy the high-frequency condition \( \omega \), one can neglect the collisional absorption of the field by conduction electrons. Therefore, in the absence of the stationary field, \( E = 0 \), the electronic system is in the thermodynamic equilibrium and the distribution function is \( f_k = f^{(0)}(\varepsilon_k) \), where

\[
f^{(0)}(\varepsilon_k) = \frac{1}{\exp[(\varepsilon_k - \varepsilon_F)/T] + 1}
\]

is the Fermi-Dirac distribution function, \( \varepsilon_F \) is the Fermi energy, and \( T \) is the temperature. Substituting the distribution function \( f_k = f^{(0)}(\varepsilon_k) \) and the probability (30) into Eq. (27), we obtain the equilibrium distribution function of conduction electrons captured by the scatterers,

\[
f^{(0)}(\varepsilon_k) = \frac{1}{\pi} \int_0^\infty \frac{\Gamma_s/2 \, f^{(0)}(\varepsilon_k)d\varepsilon_k}{(\varepsilon_k - \varepsilon_s)^2 + (\Gamma_s/2)^2}. \tag{32}
\]

Certainly, in the limiting case of stationary bound electron state \( (\Gamma_s \to 0) \), the distribution function (32) turns into the Fermi-Dirac function, \( f_s^{(0)} = f^{(0)}(\varepsilon_s) \). Substituting the equilibrium distribution functions \( f_k = f^{(0)}(\varepsilon_k) \) and \( f_s = f^{(0)}(\varepsilon_s) \) into Eq. (27), we arrive at the equation defining the Fermi energy \( \varepsilon_F \),

\[
n_s f^{(0)}(\varepsilon_s) = n_c f^{(0)}(\varepsilon_k) \tag{33}
\]

where \( n_s = N_s/S \) is the density of scatterers, \( n_0 = N_c/S \) is the density of conduction electrons in the absence of irradiation, and

\[
n_c = \frac{2}{\pi} \sum_k f^{(0)}(\varepsilon_k) = -\frac{m_e T}{\pi \hbar^2} \ln \left( \frac{1}{1 + e^{\varepsilon_F/T}} \right). \tag{34}
\]
is the density of conduction electrons in the irradiated 2D system.

Assuming the thermodynamic equilibrium to be weakly perturbed by the stationary field $E$, the sought distribution functions can be written as $f_k = f^{(0)}(\epsilon_k) + \Delta f_k$ and $f_s = f_s^{(0)} + \Delta f_s$, where $\Delta f_k$ and $\Delta f_s$ are small nonequilibrium additions arisen from the field $E$. To find these additions, let us assume that the temperature is around zero in order to neglect the phonon scattering of conduction electrons. Then the scattering probability per unit time, $w_{\mathbf{k}\mathbf{k}'}$, can be written in the explicit form as

$$w_{\mathbf{k}\mathbf{k}'} = (1 - f_s)w^{(1)}_{\mathbf{k}\mathbf{k}'} + f_s w^{(2)}_{\mathbf{k}\mathbf{k}'} ,$$

where the first term,

$$w^{(1)}_{\mathbf{k}\mathbf{k}'} = \frac{2 \pi n_s}{\hbar} \gamma_{\mathbf{k}\mathbf{k}'} U_{\mathbf{k}\mathbf{k}'} \left[ \frac{\hbar^2}{m_e (\epsilon_k - \epsilon_s + i \Gamma_s / 2)} \right]^2 \delta(\epsilon_{k'} - \epsilon_k) ,$$

describes the scattering of conduction electrons by empty scatterers, and the second term,

$$w^{(2)}_{\mathbf{k}\mathbf{k}'} = \frac{2 \pi n_s}{\hbar} \gamma_{\mathbf{k}\mathbf{k}'} U_{\mathbf{k}\mathbf{k}'} w_{\mathbf{k}\mathbf{k}'} - \frac{\hbar^2}{m_e (\epsilon_k - \epsilon_s + i \Gamma_s / 2)} \left[ \frac{\hbar^2}{m_e (\epsilon_k - \epsilon_s + i \Gamma_s / 2)} \right]^2 \delta(\epsilon_{k'} - \epsilon_k) ,$$

describes the scattering of conduction electrons by scatterers containing captured electrons. Here $U_{\mathbf{k}\mathbf{k}'}$ is the matrix element of the Born scattering by the initial potential $U(\mathbf{r})$, $w_{\mathbf{k}\mathbf{k}'}$ is the matrix element of the Born scattering by the potential addition $u(\mathbf{r})$ produced by a captured electron, and $\gamma_{\mathbf{k}\mathbf{k}'} = J_0[2kr_0 \sin(\theta/2)]$ [see Appendix for details of derivation of the probabilities (36) and (37)].

The first terms under modulus in Eqs. (36)–(37) describes the conventional potential scattering, whereas the last term corresponds to the electron scattering through the quasi-discrete energy level $\epsilon_s$, and is physically identical to the Breit-Wigner equation for the resonant scattering. The probabilities (36)–(37) take into account the quantum interference of these two scattering processes, which can manifest itself in electron transport as features of the Fano kind. However, the interference term is small under the condition of small broadening, $\Gamma_s$, and, therefore, can be neglected at a first approximation. It should be noted also that the second terms under modulus in Eqs. (36)–(37) are of different signs. Physically, this originates from different intermediate states involved in the resonant scattering described by Eqs. (36) and (37): In the case of an empty scatterer the intermediate state corresponds to the scatterer containing a captured electron, whereas the intermediate state of a scatterer containing a captured electron corresponds to the empty scatterer.

Since the probabilities (36) and (37) describe the elastic scattering of conduction electrons with the isotropic energy spectrum $\epsilon_k = \hbar^2 k^2 / 2m_e$ and the scattering potentials $U(\mathbf{r})$ and $u(\mathbf{r})$ are assumed to be axially symmetric in the 2D plane, the total probability (35) can be easily rewritten as a function of the angle $\theta = \mathbf{k}\cdot\mathbf{k}'$ and the modulus of electron wave vector $k$. Therefore, it is convenient to write the scattering matrix elements as functions of these arguments, $U_{\mathbf{k}\mathbf{k}'} = U_k(\theta)$ and $w_{\mathbf{k}\mathbf{k}'} = w_k(\theta)$. Then the relaxation time approximation is applicable to solve the Boltzmann kinetic equation (24) and the sought nonequilibrium distribution functions read

$$\Delta f_k = \left[ -\frac{\partial f^{(0)}(\epsilon_k)}{\partial \epsilon_k} \right] e_k v_k \mathbf{E} ,$$

$$\Delta f_s = 0 ,$$

where $v_k = \partial_\epsilon \epsilon_k / \hbar$ is the velocity of conduction electron with the wave vector $k$, the transport relaxation time, $\tau_k$, is defined by the expression

$$\frac{1}{\tau_k} = \int_{-\pi}^{\pi} (1 - \cos \theta) w_k(\theta) d\theta ,$$

where the effective scattering probability per unit time, $w_k(\theta)$, reads

$$w_k(\theta) = \frac{n_s m_e}{2 \pi \hbar^2} \left[ (1 - f_s) J_0[2kr_0 \sin(\theta/2)] U_k(\theta) + \frac{\hbar^2}{m_e (\epsilon_k - \epsilon_s + i \Gamma_s / 2)} \right]^2 + f_s \left[ J_0[2kr_0 \sin(\theta/2)] U_k(\theta) + u_k(\theta) - \frac{\hbar^2}{m_e (\epsilon_k - \epsilon_s + i \Gamma_s / 2)} \right]^2 ,$$

Certainly, the distribution functions (38)–(39) satisfy the Boltzmann kinetic equation (24) and can be eas-
ily verified by direct substitution into this. Substituting Eqs. (41) and (42) into Eq. (38) and summatating it over all electronic states, we arrive at the conductivity of the 2D system

\[
\sigma = \frac{e^2 n_e \tau_F}{m_e},
\]

(43)

where \(\tau_F\) is the relaxation time for \(\varepsilon_k = \varepsilon_F\). Taking into account Eqs. (32) and (34), the density of conduction electrons, \(n_e\), is defined by the equation

\[
n_e = n_0 - \frac{n_s}{2} - \frac{n_s}{\pi} \tan^{-1} \left( \frac{2(\varepsilon_F - \varepsilon_s)}{1_s} \right),
\]

(44)

where \(\varepsilon_F = \pi h^2 n_e / m_e\) is the Fermi energy. In what follows, we will be to consider the physically relevant case of small density of scatterers, \(n_s \ll n_0\). Then the solution of Eq. (44) reads

\[
n_e = n_0 - \frac{n_s}{2} - \frac{n_s}{\pi} \tan^{-1} \left( \frac{2[\varepsilon_{F0} - \varepsilon_s]}{1_s} \right),
\]

(45)

where \(\varepsilon_{F0} = \pi h^2 n_0 / m_s\) is the Fermi energy in the absence of irradiation. Substituting Eq. (41) into Eq. (43) and keeping in mind that the broadening \(1_s\) is assumed to be very small, the resistivity of 2D system, \(\rho = 1/\sigma\), can be written as a sum,

\[
\rho = \rho_{2D} + \rho_Q,
\]

(46)

where the first term describes the resistivity of 2D system arisen from the usual potential scattering of conduction electrons, whereas the second term,

\[
\rho_Q = \frac{2n_s}{\pi n_e} \left( \frac{h}{e^2} \right) \left( \frac{1_s/2}{\varepsilon_F - \varepsilon_s} \right)^2 + \left( \frac{1_s/2}{\varepsilon_F - \varepsilon_s} \right)^2,
\]

(47)

is of purely quantum nature and describes the scattering of conduction electrons through the light-induced quasi-stationary bound states (the Breit-Wigner resonant scattering), where \(h/e^2\) is the resistivity quantum. It follows from (47) that the quantum resistivity \(\rho_Q\) depends resonantly on the Fermi energy \(\varepsilon_F\) with the resonant point \(\varepsilon_F = \varepsilon_s\) and the resonant amplitude is \(\rho_Q = (2n_s / \pi n_e) (h / e^2)\).

In the present analysis of electron transport, we assumed the most general types of the scattering potential \(U(r)\) and 2D electron system. To proceed, we have to make some approximations. Let us restrict the following analysis by the case of short-range scatterers which are conventionally modeled in 2D systems by the delta-potential (see, e.g., Ref. 5). This corresponds, particularly, to a semiconductor quantum well doped by neutral atoms. Then we can apply Eqs. (21) and (22) to describe the energy of the quasi-stationary bound electron state, \(\varepsilon_s\), and its broadening, \(\Gamma_s\). Next, let us consider the 2D electron system in GaAs-based quantum well, where conduction electrons fill only the ground subband (the electron density is \(n_0 = 5 \cdot 10^{11} \text{ cm}^{-2}\) and the electron effective mass is \(m_e \approx 0.067 m_0\) (\(m_0\) is the electron mass in vacuum). In such modern quantum wells, the electron mobility is \(\mu = [e|\tau_e| / m_e] \sim 10^6 - 10^7 \text{ cm}^2 / \text{V} \cdot \text{s}\). Therefore, the condition \(\mu \tau_e \sim 10\) can be satisfied near the high-frequency border of the microwave range. For instance, we have \(\omega \tau_e \sim 10\) for the field frequencies around \(\nu = \omega / 2\pi = 100 \text{ GHz}\). Therefore, the photon energy of the dressing field can be chosen as \(\hbar \omega = 1 \text{ meV}\).

The dependence of the bound state energy, \(\varepsilon_s\), on the irradiation intensity, \(I = eE_0^2 / 4\pi\), is plotted in Fig. 2a, where the vertical arrow marks the resonant point \(\varepsilon_s = \varepsilon_F\). Far from the resonant point \(\varepsilon_s - \varepsilon_F \gg \Gamma_s\), the resonant term (47) can be neglected and the total resistivity (46) is \(\rho \approx \rho_{2D}\). On the left of the resonant point, the bound states are empty \((f_s \approx 0)\) and this resistivity reads \(\rho_{2D} = \rho_{2D}\), where

\[
\rho_{2D} = \frac{\rho_0}{2\pi} \int_{-\pi}^{\pi} (1 - \cos \theta) R_s^2 [2k_F R_0 \sin (\theta / 2)] d\theta,
\]

(48)

\(\rho_0\) is the resistivity of the considered 2D system in the absence of irradiation, and \(k_F = \sqrt{2\pi n_e} / \epsilon\) is the Fermi wave vector of conduction electrons. It should be noted that Eq. (48) exactly coincides with the equation derived and analyzed in Ref. 17 beyond the effect of light-induced quasi-stationary electron states. On the right of the resonant point \((f_s \approx 1)\), the bound states are filled by captured electrons and the resistivity is \(\rho_{2D} = \rho_{2D} + R[\mu(r)]\), where \(R[\mu(r)]\) is the functional depending on the scattering potential produced by a captured electron, \(\mu(r)\). Physically, the addition \(R[\mu(r)]\) to the resistivity (48) arises from the fact that the scatterers filled with captured electrons scatter conduction electrons more effective than the empty scatterers. It should be noted that the capture of conduction electrons by the light-induced bound states leads also to the decreasing of density of conduction electrons, \(n_e < n_0\) [see Eqs. (44)–(45)]. However, we have \(n_e \approx n_0\) under the condition \(n_s \ll n_0\) and, therefore, the contribution of decreasing density of conduction electrons to increasing resistivity is very small. Near the resonant point, we have to take into account the resistivity term (47). For typical parameters of the modern GaAs-based quantum well (the density of conduction electrons is \(n_0 \approx 5 \cdot 10^{11} \text{ cm}^{-2}\) and the electron mobility is \(\mu \approx 10^7 \text{ cm}^2 / \text{V} \cdot \text{s}\)), the resistivity of the system in the absence of irradiation is \(\rho_0 \approx 1 \Omega\), whereas the resistivity quantum is \(h/e^2 \approx 26 \text{ k}\Omega\). Since \(h/e^2 \gg \rho_0\), the resonant term (47) is dominant near the resonant point in
V. CONCLUSION

Irradiation of a 2D electron system by a circularly polarized off-resonant electromagnetic wave induces the quasi-stationary electron states confined at repulsive scatterers and immersed into the continuum of states of conduction electrons. These quasi-stationary bound electron states result in the corrections to conductivity of the irradiated 2D system through the two main mechanisms: The capture of conduction electrons by the bound states and the scattering of conduction electrons through these states. As a consequence, the corrections to resistivity of two kinds appear. The first of them is the non-resonant addition to the resistivity arisen from the increasing of the scattering of conduction electrons by the scatterers containing captured electrons. The second is the resonant addition to resistivity of purely quantum nature, which arises from the resonant Breit-Wigner scattering of conduction electrons through the quasi-stationary bound states (the resonant peak of the resistivity takes place when the Fermi energy of conduction electrons coincides with the quasi-discrete energy level of the bound state).

Within the model of short-range scatterers described by the repulsive delta-potentials, the resistivity of GaAs-based quantum well is studied in the broad ranges of irradiation intensity and scatterer density.

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Appendix A: Derivation of probabilities of electron transitions

The interaction between the bound electron state, $|s\rangle = \varphi_s(r)$, and the states of free conduction electrons, $|k\rangle = \sqrt{1/S}e^{ikr}$, arises from the electron tunneling through the potential barrier, $|s\rangle \to |k\rangle$ (see Fig. 1).

Since the tunneling is assumed to be weak, it can be described in the most general form by the tunnel Hamiltonian

$$\hat{H}_T = |s\rangle \varepsilon_s \langle s| + \sum_k |k\rangle \varepsilon_k \langle k| + \sum_k [|k\rangle \langle T_k| s\rangle + \text{H.c.}],$$

(A1)

where $T_k = \langle k| \hat{H}_T |s\rangle$ is the tunnel matrix element of the Hamiltonian between the localized and delocalized electron states. As to the summation over electron states with wave vectors $k = (k_x, k_y) = (k \cos \theta, k \sin \theta)$ in the 2D system of the area $S$, it is equal to the following integration:

$$\sum_k \to \frac{S}{(2\pi)^2} \int_0^\infty dk \int_0^{2\pi} d\theta.$$  

(A2)

The wave function satisfying the Schrödinger equation with the Hamiltonian (A1) can be written as $|\psi\rangle = a_s(t)e^{-i\varepsilon_s t/\hbar}|s\rangle + \sum_k a_k(t)e^{-i\varepsilon_k t/\hbar}|k\rangle$. It should be noted that a scatterer is assumed to be centered at $R = 0$, with wave vectors $k$.

FIG. 2: Electronic characteristics of a GaAs-based quantum well filled by conduction electrons with the density $n_0 = 5 \cdot 10^{11} \text{ cm}^{-2}$ and irradiated by a circularly polarized electromagnetic wave with the intensity $I$ and the photon energy $h\omega = 1 \text{ meV}$: (a) Dependence of the bound state energy, $\varepsilon_s$, on the irradiation intensity, $I$, where the horizontal blue line marks the Fermi energy, $\varepsilon_F$, and the red vertical arrow marks the resonant point, $\varepsilon_s = \varepsilon_F$; (b) Dependence of the quantum resistivity, $\rho_Q$, on the irradiation intensity, $I$, for the energy broadening $\Gamma_s = 0.1 \varepsilon_s$ and different scatterer densities, $n_s$. 

the broad range of scatterer density, $n_s$. Therefore, we have $\rho \approx \rho_Q$ there. The dependence of the quantum resistivity (17) on the irradiation intensity, $I$, is plotted in Fig. 2b for different scatterer densities, $n_s$. It should be noted that the resonance amplitude of the resistivity (17) is still large enough, $\rho_Q \sim 10^{-1} \Omega$, even if the scatterer density is very small, $n_s \sim 10^{-5} n_e$. Therefore, the quantum correction to the resistivity (17) can be detected in the state-of-the-art measurements in the broad range of scatterer densities.
where $\mathbf{R}$ is the radius vector of the scatterer position. In the most general case of $\mathbf{R} \neq 0$, the amplitudes $a_k$ should be multiplied with the phase factor $e^{i\mathbf{k}\cdot\mathbf{R}}$. Substituting this wave function into the Schrödinger equation, $i\hbar\partial_t|\psi\rangle = \hat{H}|\psi\rangle$, and taking into account the approximate orthogonality of the basic states, $\langle k|s \rangle = 0$, we arrive at the quantum dynamics equations for the expansion coefficients $a_s(t)$ and $a_k(t)$,

$$
i\hbar\dot{a}_s(t) = e^{i(\varepsilon_s-\varepsilon_k)t/\hbar}\sum_k T_k^* a_k(t),$$  \hspace{1cm} (A3)$$
i\hbar\dot{a}_k(t) = e^{i(\varepsilon_k-\varepsilon_s)t/\hbar} T_k a_s(t).$$  \hspace{1cm} (A4)

Let an electron be in the bound state at the time $t = 0$, i.e. $a_s(0) = 1$ and $a_k(0) = 0$. Then the integration of Eq. (A3) results in

$$a_k(t) = -\frac{iT_k}{\hbar}\int_0^t e^{i(\varepsilon_k-\varepsilon_s)t'/\hbar} a_s(t')dt'.$$  \hspace{1cm} (A5)

Since the considered system is axially symmetrical, the matrix element $T_k$ depends only on the electron energy, $\varepsilon_k = \hbar^2 k^2/2m$ and, therefore, can be denoted as $T_k = T_{\varepsilon_k}$. Substituting Eq. (A5) into Eq. (A3), we arrive at the expression

$$\dot{a}_s(t) = -\frac{1}{\hbar^2} \sum_k |T_{\varepsilon_k}|^2 \int_0^t e^{i(\varepsilon_s-\varepsilon_k)(t-t')/\hbar} a_s(t')dt',$$  \hspace{1cm} (A6)

which can be rewritten as

$$\dot{a}_s(t) = -\frac{Sm_{\text{e}}}{2\pi\hbar^2} \int_0^\infty d\varepsilon_k |T_{\varepsilon_k}|^2 \int_0^t e^{i(\varepsilon_s-\varepsilon_k)(t-t')/\hbar} a_s(t')dt'.$$  \hspace{1cm} (A7)

This is still an exact equation since we just replaced two differential equations (A3)–(A4) with one linear differential-integral equation (A6). Next we make the approximation. Namely, let us take into account that the tunneling between the states $|s\rangle$ and $|k\rangle$ is very weak. Then the quantity $|T_{\varepsilon_k}|^2$ varies little around $\varepsilon_k = \varepsilon_s$ for which the time integral in Eq. (A6) is not negligible. Physically, this means that the condition (A3) is assumed to be satisfied and, therefore, the energy of outgoing electron, $\varepsilon_k$, is near the energy of bound state, $\varepsilon_s$.

As a consequence, one can make the replacement $|T_{\varepsilon_k}| \to |T_{\varepsilon_s}|$ and replace the lower limit in the $\varepsilon_k$ integration with $-\infty$. Since this integration results in the delta function,

$$\int_{-\infty}^\infty e^{i(\varepsilon_s-\varepsilon_k)(t-t')/\hbar} d\varepsilon_k = 2\pi\hbar \delta(t-t'),$$

Eq. (A6) takes the form

$$\dot{a}_s(t) = -\frac{\Gamma_s}{2\hbar} a_s(t),$$  \hspace{1cm} (A8)

where

$$\Gamma_s = \frac{Sm_{\text{e}}}{\hbar^2} |T_{\varepsilon_s}|^2$$  \hspace{1cm} (A8)

is the energy broadening of the bound electron state. It follows from Eq. (A8) that

$$a_s(t) = e^{-\Gamma_s t/2\hbar}.$$  \hspace{1cm} (A9)

Now, we can calculate the probabilities $W_{ks}$ and $w_{kk'}$ which appear in the Boltzmann kinetic equation [see Eqs. (20) and (31)]. Substituting Eq. (A9) into Eq. (A5), the amplitude of emission of free electron with the wave vector $k$ from the bound state during the time $t$ reads

$$a_k(t) = -T_{\varepsilon_s} \frac{e^{i(\varepsilon_k-\varepsilon_s)t/\hbar-\Gamma_s t/2\hbar} - 1}{\varepsilon_k - \varepsilon_s + i\Gamma_s/2}.$$  \hspace{1cm} (A10)

Making the replacement $T_{\varepsilon_s} \to T_{\varepsilon_s}$, the total amplitude of electron transition from the state $|s\rangle$ to the state $|k\rangle$ is

$$a_k(\infty) = \frac{T_{\varepsilon_s}}{\varepsilon_k - \varepsilon_s + i\Gamma_s/2}.$$  \hspace{1cm} (A11)

Taking into account the reversibility of electron transitions and using Eq. (A8), the total probability of capture of a conduction electron with the wave vector $k$ by a scatterer, $W_{sk} = W_{ks} = |a_k(\infty)|^2$, can be written as

$$W_{sk} = \frac{\hbar\Gamma_s}{Sm_{\text{e}}[(\varepsilon_k - \varepsilon_s)^2 + (\Gamma_s/2)^2]}.$$  \hspace{1cm} (A12)

To describe the scattering of conduction electrons in the most general form, let us add the scattering Hamiltonian,

$$\hat{H}_S = \sum_{k'} \sum_{k''} |k''\rangle\langle k'|U(r - r_0[t])|k''\rangle/|k'|, \hspace{1cm} (A13)$$

to the tunnel Hamiltonian (A11), where $U(r - r_0[t])$ is the oscillating potential [9]. Then the probability of electron scattering per unit time between the states $|k'\rangle$ and $|k\rangle$, which takes into account both tunnel transitions arisen from the Hamiltonian (A11) and the potential scattering induced by the Hamiltonian (A13), is defined in the lowest order of the conventional perturbation theory as

$$w_{kk'} = \frac{2\pi}{\hbar} \left| \sum_{j=1}^{N_S} \left[ \frac{\langle k'|\hat{H}_T|s\rangle \langle s|\hat{H}_T|k'\rangle}{\varepsilon_k - \varepsilon_s + i\Gamma_s/2} + \langle k|U_0(r)|k'\rangle \right] + e^{i(k'-k)\mathbf{R}_j} \right| \delta(\varepsilon_{k'} - \varepsilon_k), \hspace{1cm} (A14)$$

where the summation index $j$ numerates different scatterers, and $U_0(r)$ is the time-averaged oscillating potential [8]. The first term in the square brackets of Eq. (A14) corresponds to the tunnel electron scattering through the quasi-discrete energy level $\varepsilon_s$. As to the second term, it
describes the elastic scattering of conduction electrons by the oscillating potential \( \Gamma \) and reads

\[
(k|U_0(r)|k') = \frac{\gamma_{kk'} \Gamma_s}{\hbar} U_{kk'},
\]
(A15)

where \( U_{kk'} = (e^{ikr}|U(r)|e^{ik'r}) \) is the matrix element of the Born scattering for the initial potential \( U(r) \), the Bessel-function factor,

\[
\gamma_{kk'} = J_0(2kr_0 \sin(\theta/2)),
\]

arises from oscillations of the scattering potential \( \Gamma \), and \( \theta = k - k' \) is the angle between the wave vectors of incident and scattered electron waves. Taking into account the random arrangement of scatterers and using Eq. (A8), the summation over the index \( j \) in Eq. (A14) results in

\[
w_{kk'} = \frac{2\pi n_s}{\hbar} \gamma_{kk'} U_{kk'} + \frac{\hbar^2 \Gamma_s}{m_e(\epsilon_k - \epsilon_s + i\Gamma_s/2)} \delta(\epsilon_{k'} - \epsilon_k). \quad (A16)
\]

It should be noted that the probabilities discussed above are derived in the rotating reference frame \( \xi \), where the scattering potential \( \Gamma \) oscillates. However, they have the same form in the laboratory reference frame, where the kinetic Boltzmann equation depending on these probabilities is written (see Section III). Generally, such an invariance follows from the fact that an unitary transformation of the Hamiltonian to the new reference frame does not change matrix elements of physical quantities derived with using the Hamiltonian. In the particular case of the considered system, this invariance can be proved by a direct calculation of the matrix element \( \langle A15 \rangle \) in the laboratory reference frame, where the oscillating potential \( \Gamma \) turns into the stationary potential \( U \) but the plane electron waves \( e^{ikr} \) and \( e^{ik'r} \) should be replaced with the corresponding Floquet functions [the eigenfunctions of the first term of the Hamiltonian \( \Gamma \)]. As expected, the calculation results in the same matrix element \( \langle A15 \rangle \). Details of the calculation can be found in Appendix of Ref. \( 15 \), where the potential scattering of conduction electrons in 2D systems driven by an oscillating field was analyzed within the Floquet theory beyond the effect of field-induced quasi-stationary electron states.

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