Determinations of $|V_{ub}|$ from Inclusive Semileptonic $B$ Decays with Reduced Model Dependence

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We report two novel determinations of $|V_{ub}|$ with reduced model dependence, based on measurements of the mass distribution of the hadronic system in semileptonic $B$ decays. Events are selected by fully reconstructing the decay of one $B$ meson and identifying a charged lepton from the decay of the other $B$ meson from $\Upsilon(4S) \to B\bar{B}$ events. In one approach, we combine the inclusive $B \to X_u\ell\nu$ rate, integrated up to a maximum hadronic mass $m_X < 1.67 \text{GeV}/c^2$, with a measurement of the inclusive $B \to X_s\gamma$ photon energy spectrum. We obtain $|V_{ub}| = (4.43 \pm 0.39_{\text{stat}} \pm 0.25_{\text{syst}} \pm 0.29_{\text{theo}}) \times 10^{-3}$. In another approach we measure the total $B \to X_u\ell\nu$ rate over the full phase space and find $|V_{ub}| = (3.84 \pm 0.70_{\text{stat}} \pm 0.30_{\text{syst}} \pm 0.10_{\text{theo}}) \times 10^{-3}$. 

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The measurement of the element $V_{ub}$ of the Cabibbo-Kobayashi-Maskawa quark-mixing matrix plays a critical role in testing the consistency of the Standard Model description of $CP$ violation. The uncertainties in existing measurements are dominantly due to uncertainties in the $b$-quark mass $m_b$ and the modeling of the Fermi motion of the $b$ quark inside the $B$ meson. In this paper, we present two techniques to extract $|V_{ub}|$ from inclusive $B \to X_u\ell\nu$ decays where these uncertainties are significantly reduced. Neither method has been previously implemented experimentally.

Leibovich, Low, and Rothstein (LLR) have presented a prescription to extract $|V_{ub}|$ with reduced model dependence from either the lepton energy or the hadronic mass $m_X$. A technique utilizing weight functions had been constructed ($\mathcal{B}$). The calculations of LLR are accurate up to corrections of order $\alpha_s^2$ and $(\Delta m_B/(\zeta m_b))^2$, where $\zeta$ is the experimental maximum hadronic mass up to which the $B \to X_u\ell\nu$ decay rate is determined and $\Lambda \approx \Lambda_{QCD}$. This method combines the hadronic mass spectrum, integrated below $\zeta$, with the high-energy end of the measured differential $B \to X_u\gamma$ photon energy spectrum via the calculations of LLR.

An alternative method ($\mathcal{B}$) to reduce the model dependence is to measure the $B \to X_u\ell\nu$ rate over the entire $m_X$ spectrum. Since no extrapolation is necessary to obtain the full rate, systematic uncertainties from $m_b$ and Fermi motion are much reduced. Perturbative corrections are known to order $\alpha_s^2$. We extract the $B \to X_u\ell\nu$ rate from the hadronic mass spectrum up to $\zeta = 2.5 \text{GeV}/c^2$ which corresponds to about 96% of the simulated hadronic mass spectrum.

The measurements presented here are based on a sample of 88.9 million $B\bar{B}$ pairs collected near the $\Upsilon(4S)$ resonance by the BABAR detector at the PEP-II asymmetric-energy $e^+e^-$ storage rings operating at SLAC. The analysis uses $\Upsilon(4S) \to B\bar{B}$ events in which one of the $B$ mesons decays hadronically and is fully reconstructed ($\mathcal{B}$) and the other decays semileptonically ($\mathcal{B}_{\ell\nu}$). To reconstruct a large sample of $B$ mesons, we follow the procedure described in Ref. 2 in which charged and neutral hadrons are combined with an exclusively reconstructed $D$ meson to obtain combinations with an energy consistent with a $B$ meson. While this approach results in a low overall event selection efficiency, it allows for the precise determination of the momentum, charge, and flavor of the $B_s$ candidates.

We use Monte Carlo (MC) simulations of the BABAR detector based on GEANT4 4 to optimize selection criteria and to determine signal efficiencies and background distributions. Charmless semileptonic $B \to X_u\ell\nu$ decays are simulated as a combination of resonant three-body decays ($X_u = \pi, \rho, \omega, \eta, \eta'$) 10, and decays to non-resonant hadronic final states $X_u$ for which the hadronization is performed by JETSET74 12. The effect of Fermi motion is implemented in the simulation using an exponential function with the parameters $m_0 = 4.79 \text{GeV}/c^2$ and $\lambda_1 = -0.24 \text{GeV}^2/c^4$. The simulation of the $B \to X_u\ell\nu$ background uses a Heavy Quark Effective Theory parameterization of form factors for $B \to D^{(*)}\ell\nu$ 14 and models for $B \to D\pi\ell\nu$ 15 and $B \to D^{(*)}\nu\ell\nu$ 16 decays.

Semileptonic $B\rightarrow X_{l}\nu$ candidates are identified by the presence of at least one electron or muon with momentum $p_{l} > 1 \text{GeV}/c$ in the $\mathcal{B}_{\ell\nu}$ rest frame. For charged $B_{s}$ candidates, we require the charge of the lepton to be consistent with a primary decay of a $B_{s}$. For neutral $B_{s}$ candidates, both charge-flavor combinations are retained and the average $B_{s0}\mathcal{B}_{\ell\nu}$ mixing rate 16 is used to determine the primary lepton yield. Electrons (muons) are identified 17 (Ref. 8), with a 92% (60–75%) average efficiency and a hadron misidentification rate ranging between 0.05% and 0.1% (1–3%).

The hadronic system $X_{l}$ in the $B \to X_{l}\ell\nu$ decays is reconstructed from charged tracks and energy depositions in the calorimeter that are not associated with the $B_{s}$ candidate or the identified lepton. The neutrino four-momentum $p_{\nu}$ is estimated from the missing momentum four-vector $p_{\text{miss}} = p_{T(4S)} - p_{B_s} - p_{X_l} - p_{\ell}$, where all momenta are measured in the laboratory frame and $p_{T(4S)}$...
consistent with a neutrino hypothesis, i.e. the resulting equality of the masses of the two leptons with $p_\ell^2 > 1$ GeV/c in the event, charge conservation ($Q_X + Q_\ell + Q_B = 0$), and a missing four-momentum consistent with a neutrino hypothesis, i.e., the mass of the hadronic system is improved by a kinematic fit that imposes four-momentum conservation, the mass of the hadronic system starts from the equation $[6]$ multiplying the photon energy spectrum of charged and neutral kaons (reconstructed as $K^0\rightarrow\pi^+\pi^-$ decays) in the decay products of the $B_s$. We suppress $\overline{B} \rightarrow D^+\pi^-$ backgrounds by partial reconstruction of charged and neutral $D^*$ mesons via identification of charged and neutral slow pions. The reconstruction of the mass of the hadronic system is improved by a kinematic fit that imposes four-momentum conservation, the equality of the masses of the two $B$ mesons, and $p_\ell^2 = 0$. The resulting $m_X$ resolution is $\sim 250$ MeV/c$^2$ on average.

The extraction of $|V_{ub}|/|V_{ts}|$ from the selected events starts from the equation

$$
\left| \frac{|V_{ub}|}{|V_{ts}|} \right| = \left\{ \begin{array}{ll}
6\alpha (1 + H_{\text{mix}} (C_7^{(0)})^2 / \pi I_0 (\zeta) + I_4 (\zeta)) \\
\delta R_u (\zeta)
\end{array} \right\} \times \delta R_u (\zeta),
$$

where $\delta R_u (\zeta)$ is the partial charmless semileptonic decay rate extracted from the number of $\overline{B} \rightarrow X_u \ell \overline{\nu}$ events up to a limit $\zeta$ in the $m_X$ spectrum. $H_{\text{mix}}$ accounts for interferences between electromagnetic penguin operator $O_7$ with $O_2$ and $O_8$ [13], and $C_7^{(0)}$ is the effective Wilson coefficient. The terms $I_0 (\zeta)$ and $I_4 (\zeta)$ are determined by multiplying the photon energy spectrum $d\gamma/dE_{\gamma}$ in $B \rightarrow X_s \gamma$ decays [13] with weight functions [13] and integrating. The weights are zero below a minimum photon energy $E_{\gamma}^{\text{min}} = m_B/2 - \zeta/4$.

In terms of measurable quantities, $\delta R_u (\zeta)$ is

$$
\delta R_u (\zeta) = \frac{N_u (\zeta) f (\zeta) B (\overline{B} \rightarrow X_u \ell \overline{\nu})}{N_{sl} \varepsilon_{u} (\zeta)} \times \frac{\varepsilon_{u}^{\text{sl}}}{\varepsilon_{\ell}^{\text{su}}} \times \frac{\varepsilon_{\ell}^{\text{rec}}}{\varepsilon_{\ell}^{\text{rec}}}. \quad (2)
$$

Here, $N_u (\zeta)$ is the number of reconstructed $\overline{B} \rightarrow X_u \ell \overline{\nu}$ events with $m_X < \zeta$, $f (\zeta)$ accounts for migration in and out of the region below $\zeta$ due to finite $m_X$ resolution, $B (\overline{B} \rightarrow X_u \ell \overline{\nu})$ is the total inclusive semileptonic branching fraction, and $\varepsilon_{u} (\zeta)$ is the efficiency for selecting $\overline{B} \rightarrow X_u \ell \overline{\nu}$ decays once a $\overline{B} \rightarrow X_u \ell \overline{\nu}$ decay has been identified with a hadronic mass below $\zeta$. $N_{sl}$ is the number of observed fully reconstructed $B$ meson decays with a charged lepton with momentum above 1 GeV/c, $\varepsilon_{u}^{\text{sl}}/\varepsilon_{\ell}^{\text{su}}$ corrects for the difference in the efficiency of the lepton momentum selection for $\overline{B} \rightarrow X_u \ell \overline{\nu}$ and $\overline{B} \rightarrow X_u \ell \overline{\nu}$ decays, and $\varepsilon_{\ell}^{\text{rec}}/\varepsilon_{\ell}^{\text{rec}}$ accounts for the difference in the efficiency of reconstructing a $B_\ell$ in events with a $\overline{B} \rightarrow X_u \ell \overline{\nu}$ and $\overline{B} \rightarrow X_u \ell \overline{\nu}$ decay. By measuring the ratio of $\overline{B} \rightarrow X_u \ell \overline{\nu}$ events to all semileptonic $B$ decays many systematic uncertainties cancel out.

We derive $N_u (\zeta)$ from the $m_X$ distribution with a binned $\chi^2$ fit to four components: data, $\overline{B} \rightarrow X_u \ell \overline{\nu}$ signal MC, $\overline{B} \rightarrow X_u \ell \overline{\nu}$ background MC, and a small MC background from other sources (misidentified leptons, $\overline{B} \rightarrow X_\tau \nu_\tau$, and charm decays), fixed relative to the $\overline{B} \rightarrow X_u \ell \overline{\nu}$ component. $N_u (\zeta)$ is determined after the subtraction of the fitted background contributions. For all four contributions, the combinatorial background is determined, separately in each bin of the $m_X$ distribution, with unbinned maximum likelihood fits to distributions of the beam energy-substituted mass $m_{\text{ES}} = \sqrt{s}/4 - p_B^2$ of the $B_\ell$ candidate, where $s$ is the $e^+e^-$ center-of-mass energy. The $m_{\text{ES}}$ fit uses an empirical description of the combinatorial background shape [14] with a signal shape [20] peaking at the $B$ meson mass. The combinatorial background varies from 5% (low $m_X$ bins) to 25% (high $m_X$ bins). The fitted $m_X$ distributions are shown in Fig. 1 before (a) and after (b) subtraction of backgrounds. The $m_X$ bins are 300 MeV/c$^2$ wide except that one bin is widened such that its upper edge is at $\zeta$.

We extract $N_{sl} = (3.253 \pm 0.024) \times 10^4$ from an unbinned maximum likelihood fit to the $m_{\text{ES}}$ distribution of all events with $p_\ell^2 > 1$ GeV/c. The efficiency corrections $\varepsilon_{\ell}^{\text{sl}}/\varepsilon_{\ell}^{\text{su}} = 0.82 \pm 0.02_{\text{stat}}$, as well as $\varepsilon_{u} (\zeta)$ and $f (\zeta)$ (see Table I) are derived from simulations, where we also find $\varepsilon_{\ell}^{\text{rec}}/\varepsilon_{\ell}^{\text{rec}}$ in agreement with one, assigning a 3% uncertainty.

We study three categories of systematic uncertainties in the determination of $|V_{ub}|$: uncertainties in the signal extraction, the simulation of physics processes, and the theoretical description. The quoted uncertainties have been determined for a value of $\zeta = 1.67$ GeV/c$^2$ where the total uncertainty on $|V_{ub}|$ is found to be minimal.

Experimental uncertainties in the signal extraction arise from imperfect description of data by the detector.
TABLE I: Quantities in Eq. (2) that depend on $\zeta$ and their statistical uncertainties. The LLR (full rate) technique is given in the first (second) column.

| $\zeta$ | $1.67 \text{ GeV}/c^2$ | $2.50 \text{ GeV}/c^2$ |
|---------|----------------|----------------|
| $f$     | $1.010 \pm 0.005$ | $0.998 \pm 0.002$ |
| $N_u$   | $120 \pm 17$     | $135 \pm 45$     |
| $\varepsilon_u$ | $0.231 \pm 0.005$ | $0.231 \pm 0.004$ |
| $\delta R_u \times 10^3$ | $1.43 \pm 0.21$ | $1.59 \pm 0.53$ |

FIG. 2: $|V_{ub}|$ as a function of $\zeta$ with the LLR method (left) and for the determination with the full rate measurement (right). The error bars indicate the statistical uncertainty. They are correlated between the points and get larger for larger $\zeta$ due to larger background from $\bar{B} \to X_u \ell \bar{\nu}$. The total shaded area illustrates the theoretical uncertainty; the inner light shaded (yellow) area indicates the perturbative share of the uncertainty. The arrow indicates $\zeta = 1.67 \text{ GeV}/c^2$.

branching fractions of the resonant final states have been varied by $\pm 30\%$ ($\pi$, $\rho$), $\pm 40\%$ ($\omega$), and $\pm 100\%$ ($\eta$ and $\eta'$ simultaneously) resulting in uncertainties of $1.0\%$. An uncertainty of $0.7\%$ due to imperfect description of hadronization is determined from the change observed when we saturate the spectrum with the non-resonant component alone. We derive a $1.3\%$ uncertainty due to the imperfect modeling of the $K\bar{K}$ content in the $X_u$ system by varying the fraction of decays to $s\bar{s}$-pairs by $30\%$ for the non-resonant contribution [21]. Even though the extraction of $|V_{ub}|$ does not explicitly depend on a model for Fermi motion, there is still a residual dependency via the simulation of signal events. By varying the Fermi motion parameters $m_b$ and $\lambda_f$ within their respective uncertainties, taking correlations into account [13], we derive an uncertainty of $3.5\%$.

We calculate theoretical uncertainties in the weighting procedure including the calculation of all variables: $H^m_{\text{mix}}$, $\alpha_S$, and Wilson-coefficients. We vary $\alpha$ between $\alpha(m_b)$ and $\alpha(m_W)$ with a central value of $1/130.3$ and find an uncertainty of less than $1\%$. For perturbative effects, an uncertainty of $2.9\%$ is derived by varying the renormalization scale $\mu$ between $m_b/2$ and $2m_b$. Non-perturbative effects are expected to be of the order $(\Delta m_B/(|\zeta| m_b))^2$, where $\Lambda = 500 \text{ MeV}/c^2$ [22], resulting in an uncertainty of $5.4\%$. Theoretical uncertainties in the measurement via the full rate are taken from Ref. [22] to be $1.2\%$ (QCD) and $2.2\%$ (HEQ). Table II provides a summary of the uncertainties for $\zeta = 1.67 \text{ GeV}/c^2$ and for $\zeta = 2.5 \text{ GeV}/c^2$.

Finally, we present two different determinations of $|V_{ub}|$. First, using the weighting technique with the photon energy spectrum in $B \to X_u \gamma$ decays from Ref. [13], the hadronic mass spectrum up to a value of $\zeta = 1.67 \text{ GeV}/c^2$, we find $|V_{ub}|/|V_{ts}| = 0.107 \pm 0.009_{\text{stat}} \pm 0.006_{\text{syst}} \pm 0.007_{\text{theo}}$. If we assume the CKM matrix is unitary then $|V_{ts}| = |V_{cb}| \times (1 \pm O(1\%))$ and, taking $|V_{cb}|$
from Ref. [24], we derive
\[ |V_{ub}| = (4.43 \pm 0.38 \pm 0.25 \pm 0.29) \times 10^{-3}, \]
where the first error is the statistical uncertainty from $\bar{B} \to X_s \ell \bar{\nu}$ and from $B \to X_s \gamma$ added in quadrature, the second (third) is systematic (theoretical). Second, we determine $|V_{ub}|$ from a measurement of the full $m_X$ spectrum, i.e., up to a value of $\zeta = 2.5 \text{GeV}/c^2$, and find $|V_{ub}| = (3.84 \pm 0.70_{\text{stat}} \pm 0.30_{\text{syst}} \pm 0.10_{\text{theo}}) \times 10^{-3}$, using the average $B$ lifetime of $\tau_B = (1.604 \pm 0.012) \text{ps}$ [14, 25].

The weighting technique is expected to break down at low values of $\zeta$, since only a small fraction of the phase space is used. Figure 2 illustrates the dependence of the spectrum limit the sensitivity with which the behavior at high $\zeta$ can be probed.

The above results are consistent with previous measurements [2, 3] but have substantially smaller uncertainties from $m_b$ and the modeling of Fermi motion. Both techniques are based on theoretical calculations that are distinct from other calculations normally employed to extract $|V_{ub}|$ and, thus, provide a complementary determination of $|V_{ub}|$.

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