Flight Control of Air-breathing Hypersonic Vehicles Based on Disturbance Rejection Scheme

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ABSTRACT The flight dynamics of air-breathing hypersonic vehicles (AHVs) is highly nonlinear, multivariable coupling, and includes inertial uncertainties and external disturbances, which require strong robust and high accuracy controllers. Considering the inertial uncertainties and external disturbances, the flight control system of AHVs can be designed as the general control issue for an uncertain nonlinear system with mismatched disturbances. To achieve the target of disturbance rejection, this paper proposes direct and indirect disturbance rejection control approaches to investigate the flight control of AHVs. For different levels of system disturbance, the typical direct and indirect disturbance rejection control schemes are presented and compared with each other. Finally, simulation results illustrate the advantages and application limits of the direct and indirect disturbance rejection control approaches.

Keywords: Direct disturbance rejection control, Indirect disturbance rejection control, Air-breathing hypersonic vehicles.

INDEX TERMS Direct disturbance rejection control, Indirect disturbance rejection control, Air-breathing hypersonic vehicles.

I. INTRODUCTION

Air-breathing hypersonic vehicles (AHVs) have attracted lots of attentions for years due to its vast values in both civilian and military applications. Because of high speed and low cost, it has been considered as a feasible and affordable way for access to space. In recent years, although numerous efforts have been made to further its development and applications, flight control system design for AHV is still a challenging task due to its very high hypersonic speed which causes the vehicle to be very sensitive to changes in the flight conditions[1]. Furthermore, the flight dynamics of AHV have some unique characteristics, such as strong nonlinearity, strong couplings, large inertial uncertainties, and fast time-varying, which will bring great difficulties in flight controller design[2].

In the past few years, linearized-model based control methods have been employed to deal with the flight control problem of AHV, including techniques of small perturbation and linear parameter-varying (LPV). Based on linearized model of AHV, linear control methods are employed to design the closed-loop control systems, such as PID [3] pole placement[4], linear-quadratic regulator[5], gain scheduling[6], $H_\infty$ and $\mu$ synthesis[7]. Be considered as the most popular methods in classical linear control theory, PID has the merits of brief structure, reliability, and convenience for parameter tuning. Preller et al[3] had made use of conventional PID feedback to design the flight control system of an AHV. Pole placement is another classical method in linear control theories, and is applied widely in engineering applications. Thus based on LPV model of AHV, Fidani[8] employed pole placement method to design the tracking controller, and solved the tracking problem with parameter uncertainties. Additionally, Groves et al[9] used linear quadratic techniques with integral augmentation to deal with the inherent couplings between the propulsion system, and the airframe dynamics, and the presence of strong flexibility effects. On
the other hand, because the linearized-model based control methods could not reflect the AHVs’ nonlinearities, the nonlinear control approaches are investigated further, such as input/output feedback linearization [8], sliding mode control [9], high-order sliding mode control [10], back-stepping control [11], fuzzy control [12] etc. Especially in back-stepping controls, disturbance observers are commonly employed to estimate the system disturbances for the compensations of control inputs. In Ref. [13], a nonlinear Luenberger observer is constructed to estimate the unknown disturbance. Then a nonlinear composite control strategy is proposed to reject the flexible effects on pitch rate. In Ref. [14], a new nonlinear disturbance observer is designed based on hyperbolic function to estimate the model uncertainties and varying disturbances. In Ref. [15], the radial basis function neural network (RBFNN) is employed to approach the unknown functions with any desired accuracy. This method can be considered as an indirect observation of system uncertainty and external disturbance. In Ref. [16], the high order sliding mode disturbance observer (SMDO) is also applied to estimate the uncertainties to compensate the controllers and disturbance suppression.

Obviously, the aforementioned control approaches have made great contributions to increase the possibility and feasibility of AHV in engineering applications. However, these works mainly focus on the control algorithms and their theoretic analysis, but lack of systematic analysis for the control problems of AHV. Actually, the issue of tracking controller design for AHV can be considered as a typical control problem of nonlinear system with mismatched disturbances (include inertial uncertainties and external disturbances). The control task is not only to track the reference commands, but also to reject the system disturbances, in order to guarantee the desired design requirements and the system robustness. Therefore, based on the aforementioned analysis, this paper proposes different indirect and direct disturbance rejection control approaches for the flight control of AHVs with respect to different levels of system disturbance. Then two typical indirect and direct disturbance rejection control strategies are provided to design tracking controllers for the longitudinal dynamics of an AHV respectively. Finally, simulation results are presented to illustrate the control performance of the proposed control approaches.

The rest of this paper is organized as follows. In section II, the longitudinal dynamical model of an AHV is introduced. In section II and section IV, the indirect and direct disturbance rejection control methods are illustrated respectively. Finally, conclusions are included in section V.

II. Mathematical model of an AHV

The first principle model (FPM) of an air-breathing hypersonic vehicle considered in the paper is presented as Ref.[17]

\[
\begin{align*}
V &= \frac{T \cos \alpha - D}{m} - g \sin \gamma \\
\dot{h} &= V \sin \gamma \\
\dot{\gamma} &= \frac{T \sin \alpha + L}{mV} - \frac{g}{V} \cos \gamma \\
\dot{\alpha} &= Q - \dot{\gamma} \\
\dot{Q} &= \frac{M}{I_y} \\
\dot{\eta}_i &= -2 \zeta \omega_m \eta_i - \omega_m^2 \eta_i + N_i \quad i = 1, 2, 3
\end{align*}
\]

This model includes five rigid-body state variables \(x = [V, h, \gamma, \alpha, Q]^T\) and six flexible states \(\eta = [\eta_1, \eta_2, \eta_2, \eta_3, \eta_3, \eta_3]^T\). The selected outputs are velocity \(V\) and altitude \(h\), while the available control inputs are fuel equivalent ratio \(\phi\), elevator deflection \(\delta_e\), and canard deflection \(\delta_c\).

For the convenience of controller design, the aerodynamic Lift \(L\) and draft \(D\), the moment \(M\), the thrust \(T\), and the generalized forces \(N_i\) are approximated by curve fitted function as Ref. [18,19]

\[
\begin{align*}
T &\approx qS \left[ C_{T_d}(\alpha) \phi + C_T(\alpha) + C_{\eta} \eta \right] \\
L &\approx qSC_L(\alpha, \delta_e, \eta) \\
D &\approx qSC_D(\alpha, \delta_e, \eta) \\
M &\approx z_T T + qSC_{M}L(\alpha, \delta_e, \eta) \\
N_i &\approx qS \left[ N_i^{a} \alpha^2 + N_i^{n} \alpha + N_i^{e} \delta_e + N_i^{c} \delta_c + N_i^{n} \eta \right]
\end{align*}
\]

where \(\delta = [\delta_e, \delta_c]^T\), and

\[
\begin{align*}
C_{T_d}(\alpha) &= C_{T_d}^{a} \alpha^3 + C_{T_d}^{n} \alpha^2 + C_{T_d}^{e} \alpha + C_{T_d}^{c} \\
C_T(\alpha) &= C_{T}^{a} \alpha^3 + C_{T}^{n} \alpha^2 + C_{T}^{e} \alpha + C_{T}^{c} \\
C_{\phi}(\alpha, \delta_e, \eta) &= C_{\phi}^{a} \alpha^3 + C_{\phi}^{n} \alpha^2 + C_{\phi}^{e} \delta_e + C_{\phi}^{c} \eta \\
C_{\delta_e}(\alpha, \delta_e, \eta) &= C_{\delta_e}^{a} \alpha^3 + C_{\delta_e}^{n} \alpha^2 + C_{\delta_e}^{e} \delta_e + C_{\delta_e}^{c} \delta_c + C_{\delta_e}^{n} \eta \\
C_{\delta_c}(\alpha, \delta_e, \eta) &= C_{\delta_c}^{a} \alpha^3 + C_{\delta_c}^{n} \alpha^2 + C_{\delta_c}^{e} \delta_e + C_{\delta_c}^{c} \delta_c + C_{\delta_c}^{n} \eta \\
C_{\omega}(\alpha) &= \left[ C_{\omega}^{a} 0 0 0 0 \right]^{T}, \quad j = T, L, D, M \\
N_i^{a} &= \left[ N_i^{a} 0 0 \right]^{T}, \quad i = 1, 2, 3 \\
q &= 0.5 \rho(h) V^2, \quad \rho(h) = \rho_0 \exp\left[-(h-h_0)/h_T\right]
\end{align*}
\]

where \(q, \rho, S, c\) and \(z_T\) denote the dynamic pressure, air density, reference area, aerodynamic chord, and thrust moment arm.

Furthermore, in order to eliminate the non-minimum phase behavior, the FPM is simplified which the canard is designed
to compensate the momentary loss of lift. Then the aerodynamic forces and thrust are expressed as follows

\[ T = qS \left[ C_L \left( \alpha \right) + C_D \left( \alpha \right) + \Delta C_T \right] = qS \left[ C_L + \Delta C_L \right] \]

\[ C_L \left( \alpha, \delta, \eta \right) = C_L^0 + \alpha \delta + \Delta C_L \]

\[ C_D \left( \alpha, \delta, \eta \right) = C_D^0 + \alpha \delta + \Delta C_D \]

\[ C_M \left( \alpha, \delta, \eta \right) = C_M^0 + \alpha \delta + \Delta C_M \]

\[ \Delta C_M = C_M^0 + k_n \Delta \tilde{v} \]

(4)

where \( \delta_c = -\left( C_L^c / C_L^0 \right) \delta \), and \( \Delta C_T \), \( \Delta C_L \), \( \Delta C_D \), and \( \Delta C_M \) are considered as model uncertainties and disturbances. More details of model simplification can be refereed in Ref. [20].

III. INDIRECT DISTURBANCE REJECTION CONTROL

The control task is to design a nonlinear robust controller for AHV to track the desired reference commands, as step-velocity and step-altitude signals \( V_{\text{ref}} \) and \( h_{\text{ref}} \). Consulting from the mathematical model of an AHV in section II, the tracking control task can be considered as a control issue for a typical uncertain nonlinear system with mismatched disturbances. The inertial uncertainties and external disturbances are lumped as the system disturbance. In order to achieve the target of system disturbance rejection, this section provides a type of indirect disturbance rejection control strategy. The indirect disturbance rejection control is named due to the reasons that this control strategy will not observe the precise information of system disturbance directly, while the system robustness is guaranteed by the control algorithms.

A. INPUT/OUTPUT FEEDBACK LINEARIZATION

If the engine dynamics are formulated as a second model as the form

\[ \dot{\phi} = -2\zeta_0 \omega_0 \phi - \omega_0^2 \phi + \omega_0^2 \phi_v \]

(5)

where \( \phi_v \) is the control input, then the longitudinal dynamics of AHV can be linearized completely by using the technique of input/output feedback linearization.

Differentiating 3 times of \( V \) and 4 times of \( h \) as follows:

\[ \dot{V} = f_1 \left( x \right) \]

\[ \ddot{V} = f_2 \left( x \right) \]

\[ \dddot{V} = f_3 \left( x \right) \]

\[ \dddot{h} = f_4 \left( x \right) \]

\[ \dddot{h} = V \sin \gamma + V \dot{\gamma} \cos \gamma \]

\[ \dddot{h} = \tilde{V} \sin \gamma + 2V \dot{\gamma} \cos \gamma - V \dddot{\gamma} \sin \gamma \]

\[ \dddot{h} = \dddot{h} \sin \gamma + 3V \dddot{\gamma} \cos \gamma - 3V^2 \dddot{\gamma} \sin \gamma + 3V \dddot{\gamma} \cos \gamma \]

\[ -3V^2 \dot{\gamma} \sin \gamma - V \dddot{\gamma} \cos \gamma + V \dddot{\gamma} \cos \gamma \]

(6)

Define \( x = \left[ V \ h \ \dot{V} \ \dot{h} \right]^T \), and \( \omega_1 = \delta f_1 \left( x \right) / \partial x \), \( \omega_2 = \delta f_2 \left( x \right) / \partial x \), \( \Omega_1 = \delta \omega_1 \left( x \right) / \partial x \), and \( \Omega_2 = \delta \omega_2 \left( x \right) / \partial x \). According to Eqs. (6)and (8), the output dynamics can be described by:

\[ \dot{h}^{\left( i \right)} = F \left( x \right) + G \left( x \right) u \]

(9)

The expressions of \( \omega_1 \), \( \omega_2 \), \( \Omega_1 \), \( \Omega_2 \), and \( F \), \( G \) are presented in Ref. [21].

If \( G \left( x \right) \) is nonsingular, Eq. (9)can be re-organized into a decoupled-integrator form:

\[ u = G \left( x \right)^{-1} \left[ v - F \left( x \right) \right], \ v = \left[ v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6 \right]^T \]

(10)

B. NOMINAL CONTROL LAW

By using the technique of input/output feedback linearization, the longitudinal dynamical model of AHV is transformed into two decoupled systems, as velocity and altitude sub-systems. Then the feedback control law based on nonlinear dynamical inverse (NDI) is established as Eq. (10), which includes outer-loop control depended on NDI and inner-loop control laws \( v \). The sketch map of the nominal control law is shown in Fig.1. In this section, the conventional higher-order sliding mode control will be employed to design the inner-loop control laws.

**FIGURE 1.** Sketch map of the nominal control law

The algorithm of arbitrary-order sliding model control was proposed by Levant in 2001 Ref.[22] Define \( p \) as a positive constant, and \( r \) as the relative degree of the sliding model face \( \left( p \geq r \right) \). Assuming

\[ N_{1r} = \left| \sigma \right|^{p\left( r-1 \right)} \]

\[ N_{1r} = \left( \left| \sigma \right|^{p\left( r-1 \right)} + \left| \sigma \right|^{p\left( r-2 \right)} + \ldots + \left| \sigma \right| \right)^{-\left( r-1 \right) /p} \]

\[ i = 1, \ldots, r-1 \]

\[ N_{1r-1} = \left( \left| \sigma \right|^{p\left( r-1 \right)} + \ldots + \left| \sigma \right|^{p\left( 2 \right)} \right)^{-1/p} \]

\[ \phi_{h_{1r}} = \sigma \]

\[ \phi_{h_{1r}} = \sigma \pm \beta_i N_{1r-1} \text{sign} \left( \sigma \right) \]

\[ i = 1, \ldots, r-1 \]

where \( \beta_1, \ldots, \beta_{r-1} \) are positive numbers, \( \sigma \) is the sliding mode variable.
Previous reports demonstrated that the following r-order sliding mode controller would guarantee an r-order sliding model that converges in finite time (the convergence time is a bounded function of initial conditions) [22], provided $\beta_1, \ldots, \beta_{r-1}$ and $k$ are positive:

$$u = -k \cdot \text{sign}(\phi_{r-1}, \sigma_1, \ldots, \sigma^{(r-1)})$$

(12)

where $\sigma_v = V - V_{ref}$, $\sigma_h = h - h_{ref}$, and

$$d = \{\alpha_1, \beta_{12}, p_r, \alpha_3, \beta_{32}, \beta_{33}, \rho, \phi\}$$

are controller design parameter vector. Then substituting the inner-loop control laws (12) into Eq.(10), the nominal control law $u$ is obtained.

**C. SIMULATION RESULTS AND ANALYSIS**

The indirect disturbance rejection control scheme for AHV has been validated in MATLAB environment. The simulations are carried out with two cases of aerodynamic coefficients, as the case of soft disturbance and the case of strong disturbance. The case of soft disturbance is assumed that the aerodynamic coefficients in Eq. (4) have perturbations of 10% with respect to nominal values as

$$|\Delta C_v| \leq 0.1 C_v, \quad |\Delta C_h| \leq 0.1 C_h, \quad |\Delta C_{\alpha}| \leq 0.1 C_{\alpha}, \quad |\Delta C_{\phi}| \leq 0.1 C_{\phi}. \quad (2)$$

While the case of strong disturbance is assumed that all the aerodynamic coefficients have the uncertainties of 30% with respect to nominal values, as $-30\% \leq \Delta \leq 30\%$. Additionally, the physical parameters of AHV are assumed as an additive variance $\Delta$ to the nominal values for control design. For illustration, the uncertain parameters are considered as the forms

$$\begin{align*}
    [m] &= m_0 (1 + \Delta m), \\
    [I_y] &= I_{y0} (1 + \Delta I), \\
    [S] &= S_0 (1 + \Delta S), \\
    [c] &= c_0 (1 + \Delta c), \\
    [z_f] &= z_{f0} (1 + \Delta z_f), \\
    [\rho] &= \rho_0 (1 + \Delta \rho),
\end{align*}$$

(14)

where $|\Delta m| \leq 3\%$, $|\Delta I| \leq 2\%$, $|\Delta S| \leq 2\%$, $|\Delta c| \leq 2\%$, $|\Delta z_f| \leq 2\%$, $|\Delta \rho| \leq 6\%$.

The vehicle is assumed to climb a maneuver from the initial conditions to the final trim condition, as shown in Table I. The reference commands are 100-ft/s step-velocity command and 2000-ft step-altitude command. The controller design parameters vector is chosen as $d = \{107.3, 1.3, 6.125, 1.0, 5.1, 3.12\}$.  

**TABLE I**

| Variable | Initial trim value | Final trim value |
|----------|--------------------|------------------|
| $V$      | 7710 ft/s          | 7810 ft/s        |
| $h$      | 85000 ft           | 87000 ft         |
| $\gamma$ | 0 deg              | 0 deg            |
| $\alpha$ | 1.7189 deg         | 1.4499 deg       |
| $Q$      | 0 deg/s            | 0 deg            |
| $\eta_1$ | 0.5083 ft·slug$^{1/2}$ | 0.4762 ft·slug$^{1/2}$ |
| $\eta_2$ | -0.0847 ft·slug$^{1/2}$ | -0.0486 ft·slug$^{1/2}$ |
| $\eta_3$ | -0.0130 ft·slug$^{1/2}$ | -0.0127 ft·slug$^{1/2}$ |
| $\delta_e$ | 5.2890 deg | 5.0476 deg |
| $\phi$ | 0.0907            | 0.0199           |

The closed-loop control system responses to the step-velocity and step-altitude commands with nominal parameters are shown in Fig.2 and Fig.3. As shown in the figures, the stable tracking in velocity and altitude channels could be achieved under the action of higher-order sliding mode control laws. Fig.4 and Fig.5 show random responses of closed-loop systems to the velocity and altitude instructions, respectively, assuming all uncertain parameters were homogeneously distributed, and their boundaries can be described by Eqs. (13)and(14). As can be seen, the closed-loop system presents good tracking performances in both velocity and altitude channels, despite observed parametric uncertainties. Simulation results suggested that the proposed indirect disturbance rejection control exhibits excellent robustness in the presence of soft disturbances.

In order to illustrate the control capacities of indirect disturbance rejection control further, all the aerodynamic coefficients are enlarged to have the uncertainties of 30% with respect to nominal values, which is denoted as the case of strong disturbance. Then, the closed-loop system responses under the action of indirect disturbance rejection control are presented in Fig.6 As shown in the figures, the closed-loop system could not achieved stable tracking and convergence completely in both velocity and altitude channels. It means that the proposed indirect disturbance rejection control scheme cannot deal with the case of strong disturbance.
FIGURE 2. Velocity and altitude responses with nominal parameters

FIGURE 3. Elastic modes with nominal parameters

FIGURE 4. Stochastic response in velocity channel
IV. DIRECT DISTURBANCE REJECTION CONTROL

According to the aforementioned analysis, indirect disturbance rejection control achieves the tracking control target well for the case of soft disturbance, but not the case of strong disturbance. Therefore, direct disturbance rejection control is proposed to deal with the case of strong disturbances in this section. Compared to indirect disturbance rejection control, the core of the direct disturbance rejection control is to give the estimation of system disturbance, which is employed to compensate the nominal control inputs.

This section presents a typical direct disturbance rejection control scheme as uncertainty and disturbance estimator (UDE) based control strategy for the tracking control of AHVs. The block diagram of the UDE-based control strategy is presented in Fig.7. It is easy to denote that the rigid equations of motion can be decomposed into two subsystems, as the velocity subsystem and the altitude subsystem. According to the dynamics of AHV, we know that the velocity $V$ is mainly related to the fuel equivalence ratio $\phi$ and the altitude $h$ is mainly affected by the elevator deflection $e_d$. In the control laws, the virtual controls and their derivatives are estimated by nonlinear tracking differentiators (TDs) [23], which avoids the traditional problems of “explosion of complexity” and “circular construction problem”. While UDE is adopted to estimate the inertial system uncertainties and external disturbances for mismatched uncertainty and disturbance rejections.
A. MODEL TRANSFORMATION

To be convenient for the back-stepping design, the rigid dynamical equations of AHV (1) are rewritten as the nonlinear functions further

\[
\begin{align*}
\dot{V} &= f_V + g_V \phi + d_V \\
\dot{h} &= f_h + g_h \gamma + d_h \\
\dot{\gamma} &= f_\gamma + g_\gamma \alpha + d_\gamma \\
\dot{\alpha} &= f_\alpha + g_\alpha Q + d_\alpha \\
\dot{Q} &= f_Q + g_Q \delta_Q + d_Q
\end{align*}
\]  

(15)

where

\[
\begin{align*}
V &= qS[C_T (\alpha) \cos \alpha - C_D]m - g \sin \gamma \\
h &= V (\sin \gamma - \gamma) \\
\gamma &= qS[C_L' + C_T \sin \alpha] (mV) - g \cos \gamma/V \\
a &= -f \\
Q &= qS[z_T C_T + \bar{c} C_M]/1_{yy}
\end{align*}
\]

(16)

\[
\begin{align*}
g_V &= qS C_T \phi (\alpha) \cos \alpha/m \\
g_h &= V \\
g_\gamma &= qS C_L'/mV \\
g_\alpha &= 1 \\
g_Q &= qS \bar{c} C_M/m_{yy}
\end{align*}
\]

(17)

\[
\begin{align*}
d_V &= qS[\Delta C_T \cos \alpha - \Delta C_D]/m \\
d_h &= 0 \\
d_\gamma &= qS[\Delta C_T \sin \alpha + \Delta C_L]/(mV) \\
d_\alpha &= -d_\gamma \\
d_Q &= qS[z_T \Delta C_T + \bar{c} \Delta C_M]/1_{yy}
\end{align*}
\]  

(18)

B. REFERENCE MODEL

It can be clearly seen that, if the flexible effects and small parameter coefficients are ignored, the longitudinal dynamics of AHV can be completely linearized by using input/output feedback linearization technique, as shown in section III.A.

Therefore, the reference model is chosen as the longitudinal dynamics of AHV without uncertainties and disturbances. Then, the reference state vector and control input vector are chosen as \(x_m = [V_m, h_m]^T\) and \(u_m = [\phi, \delta_r]^T\) respectively. The control objective is to make the system track the reference velocity and altitude commands \(V_{ref}\) and \(h_{ref}\) asymptotically.

Differentiating the velocity \(V\) three times and the altitude \(h\) four times, we obtain

\[
\begin{align*}
\dot{V}^{(1)} &= F(x_m) + G(x_m)u_m \\
\ddot{h}^{(4)} &= F(x_m) + G(x_m)u_m
\end{align*}
\]  

(19)

Based on technique of feedback linearization and linear quadratic optimal theory, the dynamic-inversion-based linear quadratic regulators are designed for the reference model as

\[
u_m = G^{-1}(x_m)[-F(x_m) - K_{\bar{\psi}} \bar{\psi}] \quad (20)
\]

where \(K_{\bar{\psi}} > 0\) and \(K_{\bar{\alpha}} > 0\) are design parameters matrices derived from solving the Riccati equations, and

\[
\bar{\psi} = \\
\bar{\psi} - V_{ref} \quad \bar{\psi} = \bar{\psi} - V_{ref} \quad \bar{\psi} - h_{ref} \quad \bar{\psi} - h_{ref} \quad \bar{\psi} - h_{ref} \quad \bar{\psi} - h_{ref}
\]

Because the reference model is stable under the control of linear quadratic regulator (LQR), the tracking error dynamics are chosen as

\[
\dot{e}(t) = A_n e(t) = \text{diag} (-k_w, -k_a) e(t) \quad (21)
\]

where \(e(t) = x(t) - x_m(t)\) denotes the system tracking error, and \(A_n\) is selected to be Hurwitz stable.

C. UDE-BASED CONTROL LAW

Define the tracking errors of velocity \(V\), altitude \(h\), flight path angle \(\gamma\), angle of attack \(\alpha\), and pitch rate \(Q\), as \(\tilde{V} = V - V_m, \tilde{h} = h - h_m, \tilde{\gamma} = \gamma - \gamma_{cmd}, \tilde{\alpha} = \alpha - \alpha_{cmd}, \tilde{Q} = Q - Q_{cmd}\)

Differentiating (22) and invoking (15) yield

\[
\begin{align*}
\dot{\tilde{V}} &= \tilde{V} - V_{ref} = f_V + g_V \phi + d_V - V_m \\
\dot{\tilde{h}} &= \tilde{h} - h_{ref} = f_h + g_h \gamma + d_h - h_m \\
\dot{\tilde{\gamma}} &= \tilde{\gamma} - \gamma_{cmd} \approx f_\gamma + g_\gamma \alpha + d_\gamma - \gamma_{cmd} \\
\dot{\tilde{\alpha}} &= \tilde{\alpha} - \alpha_{cmd} = f_\alpha + g_\alpha Q + d_\alpha - \alpha_{cmd} \\
\dot{\tilde{Q}} &= \tilde{Q} - Q_{cmd} = f_Q + g_Q \delta_Q + d_Q - Q_{cmd}
\end{align*}
\]  

(23)

where \(k_i (i = V, h, \gamma, \alpha, Q) > 0\) are the error feedback coefficients. \(\gamma_{cmd}, \alpha_{cmd}, Q_{cmd}\) and their derivatives \(\dot{\gamma}_{cmd}, \dot{\alpha}_{cmd}, \dot{Q}_{cmd}\) are estimated by introducing three nonlinear tracking differentiators (NTDs) in Ref. [23] as

\[
\begin{align*}
\hat{z}_i &= z_i \\
\hat{Z}_i &= R_i^j \left[ -a_{ij} \sinh(b_{ij} (z_i - z_{ij})) - a_{ij} \sinh(b_{ij} z_i / R_i) \right] \\
&= i = 1, 2, 3
\end{align*}
\]  

(25)

where \(z_1 = \gamma_{cmd}, z_2 = \alpha_{cmd}, z_3 = Q_{cmd}\) are the estimations of NTDs. \(z_i (i = 1, 2, 3)\) are the input signals of NTDs. \(R_i > 0, a_{ij} > 0, b_{ij} > 0 \quad (i = 1, 2, 3; j = 1, 2)\) are design parameters. Consulting from Theory 2 of Ref. [24], we know that
the design parameters of the controller are chosen as \( k_V = 8 \), \( k_h = 2.5 \), \( k_\gamma = 1.5 \), \( k_\alpha = 10 \), \( k_Q = 4 \). The design parameters of NTDs are chosen as \( a_{i1} = 5 \), \( a_{i2} = 1 \), \( b_{i1} = b_{i2} = 2 \), \( R_i = 5 (i = 1, 2, 3) \). The control parameters of LQR for the reference model are design as \( K_{\hat{v}_i} = [1.546, 8.505, 9.455] \) and \( K_{\hat{v}_i} = [0.504, 4.477, 10.816, 8.267] \). Low-pass filter is designed for UDE to estimate the lumped uncertainty and disturbance. The methods to design low-pass filter can be referred to the Ref. [25, 26].

### Table II

**INITIAL AND FINAL CONDITIONS OF DIRECT DISTURBANCE REJECTION CONTROL**

| Variable | Initial trim value | Final trim value |
|----------|--------------------|------------------|
| \( V \)  | 7710 ft/s          | 7810 ft/s        |
| \( h \)  | 85000 ft           | 87000 ft         |
| \( \gamma \) | 0 deg            | 0 deg            |
| \( \alpha \) | 1.7189 deg       | 2.2380 deg       |
| \( Q \)  | 0 deg/s            | 0 deg/s          |
| \( \eta_1 \) | 0.5083 ft·slug\(^{1/2}\) | 0.504 ft·slug\(^{1/2}\) |
| \( \eta_2 \) | 0.0478 ft·slug\(^{1/2}\) | 0.0391 ft·slug\(^{1/2}\) |
| \( \eta_3 \) | 0.0130 ft·slug\(^{1/2}\) | 0.0095 ft·slug\(^{1/2}\) |
| \( \delta_t \) | 5.2890 deg       | 6.0875 deg       |
| \( \phi \) | 0.0907 deg        | 0.1279 deg       |

### D. SIMULATION RESULTS AND ANALYSIS

The simulation conditions are assigned as the same in section III.C. The vehicle is also assumed to climb a maneuver from the initial conditions to the final trim condition, as shown in Table II. The reference commands are 100-ft/s step-velocity command and 2000-ft step-altitude command. In simulations, the design parameters of the
The closed-loop system responses to a 100-ft/s step-velocity and 2000-ft step-altitude command are presented in Fig.8 and Fig.9. Fig.8 shows that the proposed UDE-base control strategy can provide excellent tracking performance in velocity and altitude channels, when the perturbations of aerodynamic coefficients increase from $\Delta = -30\%$ (Case I) to $\Delta = +30\%$ (Case II). And the velocity and altitude tracking errors converge to zero. The rigid modes $\alpha$, $Q$, elastic modes $\eta_1$, $\eta_2$, $\eta_3$ and control inputs $\phi$, $\delta_e$, demonstrated in the figures, are quite smooth and bounded rationally. The lumped uncertainty and disturbance estimations are shown in Fig.10.

V. CONCLUSIONS

This paper investigated the flight control problem of air-breathing hypersonic vehicles. The tracking control task can be considered as the control issue for a typical uncertain nonlinear system with mismatched disturbance. In order to achieve disturbance rejection, indirect and direct disturbance rejection control schemes are designed for different levels of system disturbance. For the case of soft disturbance, the indirect disturbance rejection control scheme could achieve the control objective well, while presents stable tracking convergence and excellent system robustness. For the case of strong disturbance, the direct disturbance rejection control scheme presents better tracking control capacities in both velocity and altitude channels. Compared to indirect disturbance rejection control, the direct disturbance rejection control scheme required precise estimations of the system disturbance to compensate the nominal control inputs, which leads to complexity of control system designing. Therefore, the tradeoffs between designing complexity and control performance should be taken into account in practical applications.
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