Matter temperature during cosmological recombination

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At early times atoms are coupled to Cosmic Microwave Background (CMB) photons through Compton scattering. In an expanding Universe matter would ‘like’ to cool as $T_M \propto (1 + z)^2$, i.e. faster than the radiation, which varies with redshift $z$ as $T_R \propto (1 + z)$. However, Compton coupling prevents the matter cooling this rapidly until the partial ionization of the atoms has fallen enough that the Compton heating timescale becomes long compared with the Hubble time. So although cosmological recombination is often referred to as ‘decoupling’, the huge photon bath prevents matter decoupling from the radiation until much later. The CMB ‘last scattering’ epoch is at $z \approx 1000$, while matter does not really start to cool adiabatically until $z \approx 300$.

Using the WMAP Markov Chains to account for the variation in the cosmological model, parameters we find that $T_M$ would be $(0.0215 \pm 0.0002)$ Kelvin, if there had been no additional sources of heat. This means that the asymptotic behaviour is the same as if the matter had instantaneously departed from the radiation at $1 + z = 2.725/0.0215 \approx 127$. Of course the growth of non-linear structure at $z \lesssim 20$ and subsequent feedback of gravitational and nuclear energy leads to intergalactic medium temperatures in today’s Universe which are much higher than the CMB temperature.

Although we found the uncertainty in today’s $T_M$ by considering the variation among currently acceptable cosmological model parameters, probably a bigger uncertainty lies in the actual physics of recombination at low redshift. Some simple algebra shows that $T_M \propto x_{e,t}^{2/5}$, where $x_{e,t}$ is the free electron fraction (normalized to hydrogen by $x_e \equiv n_e/m_H$) which ‘freezes out’ at low redshift. The additional uncertainty in $T_M$ due to $x_{e,t}$ is then expected to be of the order 2-4% (see for example Chiba, Rubino-Martín & Sunyaev 2007).

The explicit equation governing the kinetic temperature of the matter (here meaning electrons plus ions plus atoms, with dark matter being uncoupled) is given by equation (66) in Seager, Sasselov & Scott (2000). Ignoring the negligible atomic cooling processes (Bremsstrahlung, collisions, etc.) we have:

$$ (1 + z)T_M = \frac{(T_M - T_R)}{H t_C} + 2T_M, $$

where $H(z)$ is the Hubble parameter evaluated at epoch $z$ and

$$ t_C = \frac{3m_e c}{8\sigma_T a_R T_R^4} \frac{1 + f_{He} + x_e}{x_e}. $$

Here the prime denotes differentiation with respect to redshift, $\sigma_T$ is the Thomson cross-section, $a_R$ is the radiation constant ($= 8\pi^2 k^4/15c^3h^3$) and $f_{He}$ is the fractional abundance of helium by number (assumed ionized here for simplicity, but correctly dealt with in the full recombination codes).

Clearly $T_M$ is slightly below $T_R$ at early epochs, with the difference kept at just the right value for Compton heating to make the matter track the radiation. The small imbalance is also important because it produces a ‘Compton drag’ force on the matter particles. An estimate of this temperature difference appears to have been first been mentioned by Gamow (1949). He states (equation 20) without proof that

$$ \frac{T_R - T_M}{T_M} \simeq \frac{t(\text{years})}{10^{12}}. $$

Further discussion of this temperature difference is given by Weymann (1966), whose result is
where we have converted to our notation. This has the approximately right redshift dependence and ionization dependence, although differences in assumptions about the cosmological model make it difficult to compare the coefficient. Still, this result is essentially correct.

The difference between matter and radiation temperatures was also included in the textbook of Peebles (1971). He writes a version of equation (1) and states ‘Because the coefficient in the last term is so very large we get a good approximation to the solution by setting $T_M' = 0$ (converting to our notation), hence finding that

$$
\frac{T_k - T_M}{T_k} \approx \frac{60}{x_e} (1 + z)^{5/2},
$$

(4)

This is a good order of magnitude estimate, agreeing (in essence) with Weymann (1966), but it is worth pointing out that $T_M = T_R'$ would be a much better approximation than $T_M' = 0$.

Let us write $T_M = T_R - \epsilon$ at early times, with $\epsilon$ having the dimensions of temperature and fixing $T_R \propto (1 + z)$ at all times. Then the solution to equation (1) is simply

$$
\frac{\epsilon}{T_R} = H t_C.
$$

(6)

In the limit $x_e \to 1$ (and ignoring helium) this is half of the expression in Peebles (1971). We note that the same result is obtained in a rather different way in the Appendix of Hirata (2008).

With this approximation in hand we can write down an expression for the evolution of the matter temperature by differentiating equation (1):

$$
T_M' = \frac{T_R}{1 + z} + \epsilon \left\{ \frac{1 + f_{He}}{1 + f_{He} + x_e x_e} + \left[ \frac{3}{1 + z} - \frac{H'}{H} \right] \right\}.
$$

(7)

This expression is useful for improving the numerical accuracy of the solution to the matter temperature. The last two terms (in square brackets) are of similar magnitude and track each other at high redshift, hence can be combined. The second term depends on the derivative of the ionization fraction, and so contributes differently as a function of redshift. Together the derivative in equation (7) can be used to evolve the matter temperature to quite high accuracy until the departure of $T_M$ from $T_R$ stops being small.

In solving the coupled recombination equations one does not really need to follow $T_M$ explicitly at early times. In the commonly used code recfast (Seager, Sasselov & Scott 1999; 2000; Wong, Moss & Scott 2008) $T_M$ is set to $T_R$ until $H t_C$ reaches some predefined value, and the full equation (1) is switched on afterwards (typically at $z \approx 850$). This leads to a ‘glitch’ in the solution (pointed out by Fendt et al. 2008). We found that this glitch is easily removed by following equation (7) instead of just $T_M' = T_M/(1 + z)$ before the switch, and then solving the full equation (1) afterwards.

The results are shown in Fig. 1. The change in ionization fraction, roughly $0.2\%$ at $z = 850$, leads to a $\approx 0.2\%$ correction to the CMB power spectrum $C_{\ell}^B$ when using recfast in a Boltzmann code. It may seem that unnecessary calculations are being carried out by explicitly integrating the matter temperature at early times, but in fact the integrator is already so fast that there is negligible effect on the speed at which recfast runs.

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