Wave dynamics and vortex formation under the impact of a spherical impulse on the boundary between gas and aqueous foam

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Abstract. The process of interaction of air shock-wave pulse and protective aqueous foam barrier in two-dimensional axisymmetric formulation using two-phase model of gas-liquid mixture including the laws of conservation of mass, momentum and energy for each phase is numerically investigated. The numerical implementation of the model is carried out using the twoPhaseEulerFoam solver of the OpenFOAM package. The results are presented in the form of spatial distributions of pressure fields, velocities and streamlines. The causes and dynamics of toroidal vortices formation in gas are investigated.

1. Introduction
The study of shock waves (SW) dynamics in the process of its interaction with aqueous foams is of great scientific and practical importance: the high compressibility of foam structures can significantly reduce the amplitude and velocity of the shock pulse, which makes it possible to use protective aqueous foam barriers to localize the after-effects of high-intensity explosions. In this regard, the study of damping properties of aqueous foam under dynamic impaction on the basis of mathematical and numerical modeling becomes relevant.

Analysis of the factors affecting the SW attenuation degree in aqueous foams was carried out in theoretical and experimental works [1–3]. In [4, 5] the features of one-dimensional plane shock waves dynamics in bubble media and foam structures are considered. Damping properties of aqueous foam were studied in the works [6–12]. In [6, 7] spherical explosion in gas and aqueous foam was simulated under conditions corresponding to experimental data of [8]. The obtained numerical solutions have a satisfactory agreement with the experiment. In [9–12] the dynamics of SW during the interaction of an air pressure spherical pulse and aqueous foam barrier in a two-dimensional axisymmetric approximation using the method of moving Lagrangian grids [9, 10] and solver compressibleMultiphaseInterFoam [11, 12] of OpenFOAM package [13] was studied. The reliability of the obtained solutions was estimated and the conditions for the toroidal vortices formation in the gas region, containing an aqueous foam barrier, were revealed.

This work was carried out using the software package OpenFOAM [13] and is a continuation of studies [10–12] on the investigation of two-dimensional wave flows in the gas region containing a layer of aqueous foam. Unlike [11, 12], this paper examines the impact of changing the initial liquid volume fraction of the foam on its damping properties and the dynamics of the vortex zones formation for longer-term processes.
2. Model equations

It is assumed that behind the front of strong shock wave the foam decays into monodisperse microdroplets of diameter \( d_0 = 30 \ \mu m \) [14] described by the gas-liquid mixture model. The system of governing equations of two-phase medium, according to the problem under study, includes the laws of conservation of mass, momentum and energy for each phase in accordance with single-pressure, two-speed, two-temperature approximations for a gas-liquid mixture in a two-dimensional axisymmetric formulation [15].

- Continuity equations for the phases

> \[
> \frac{\partial (\alpha_i \rho_i)}{\partial t} + \text{div}(\alpha_i \rho_i \vec{v}_i) = 0.
> \]  
> (1)

- Momentum equations for the phases

> \[
> \frac{\partial (\alpha_i \rho_i \vec{v}_i)}{\partial t} + \text{div}(\alpha_i \rho_i \vec{v}_i \vec{v}_i) = -\alpha_i \nabla p + \text{div}(\alpha_i \vec{\tau}_i) + \vec{F}_i,
> \]  
> (2)

where the viscous stress tensor \( \vec{\tau}_i \) has the form

\[
\vec{\tau}_i = \mu_i (\nabla \vec{v}_i + \nabla \vec{v}_i^T) - \frac{2}{3}(\mu_i \text{div} \vec{v}_i)I.
\]

The term \( \vec{F}_i \) determines the density of the interfacial forces and represents the sum of the interfacial drag force \( \vec{F}_{i,\text{drag}} \) and the virtual mass force \( \vec{F}_{i,\text{vm}} \) [15]:

\[
\vec{F}_i = \vec{F}_{i,\text{drag}} + \vec{F}_{i,\text{vm}},
\]

where

\[
\vec{F}_{i,\text{drag}} = \frac{3}{4} \alpha_2 C_D \frac{\rho_1}{d_0} (\vec{v}_i - \vec{v}_j) |\vec{v}_i - \vec{v}_j|,
\]

\[
\vec{F}_{i,\text{vm}} = 0.5 \alpha_2 \rho_1 \left( \frac{d_j \vec{v}_j}{dt} - \frac{d_i \vec{v}_i}{dt} \right).
\]

Here \( p_0 \) and \( p \) – initial and current pressure. Coefficient \( C_D \) for Reynolds number

\[
Re = \frac{\rho_1 |\vec{v}_i - \vec{v}_j| d_0}{\mu_1}
\]

according to the Schiller-Naumann drag model [16] is written as

\[
C_D = \begin{cases} 
24(1 + 0.15 \text{Re}^{0.687}) & \text{Re} \leq 1000 \\
0.44 & \text{Re} > 1000 
\end{cases}
\]

- Energy equations for the phases

> \[
> \frac{\partial (\alpha_i \rho_i (e_i + K_i))}{\partial t} + \text{div}(\alpha_i \rho_i (e_i + K_i) \vec{v}_i) =
> -p \frac{\partial \alpha_i}{\partial t} - \text{div}(\alpha_i \vec{v}_i p) + \text{div}(\alpha_i \gamma_{i,\text{eff}} (\nabla h_i)) + K_{hi}(T_j - T_i),
> \]  
> (3)
where the effective thermal diffusivity for the $i$–th phase is
\[ \gamma_{i,\text{eff}} = \frac{c_{p,i}}{c_{V,i}} \gamma_i, \]
c$_{p,i}$, c$_{V,i}$, $\gamma_i$ – specific heat at constant pressure and constant volume, thermal diffusivity for the $i$–th phase.

The Ranz-Marshall model [7] is used to determine the heat transfer coefficient $K_{ht}$:
\[ K_{ht} = \frac{\kappa_1 N u}{d_2}, \quad N u = 2 + 0.6 Re^{1/2} Pr^{1/3}, \]

$Re$, $Nu$, $Pr$ — Reynolds, Nusselt and Prandtl numbers, respectively.

- Equations of state for gas and water:
  \[ \rho_i = p \psi_i + \rho_{i0}, \quad \rho_2 = p \psi_2, \quad \rho_2 = p \psi_2, \]
  where $\rho_{i0}$, $\psi_i = m_i/(RT_i)$ – the density of water under normal conditions and the compressibility for the $i$–th phase.

Equations (1-4) use the following notations: $\alpha_i$ – volume fraction, $\rho_i$ – density, $\vec{v}_i$ – velocity vector, $T_i$ – temperature, $\mu_i$ – dynamic viscosity, $e_i$ – internal energy, $K_i$ – kinetic energy, $h_i$ – enthalpy, $\kappa_i$ – thermal conductivity, $m_i$ – molar mass for the $i$–th phase; $i,j$ – designations of phases, for which numerical values 1, 2 correspond to liquid and gas, respectively; $I$ – unit tensor.

3. Problem statement and calculation results

Let’s consider a cylindrical domain of length $x = 1.2$ m and radius $y = 3$ m, with the conditions of symmetry on the axis $Ox$ and the plane $x = 0$. The domain is filled with gas ($0 \leq y \leq 3$ m, $0 \leq x < 1$ m) and contains aqueous foam layer of thickness 0.2 m ($0 \leq y \leq 3$ m, $1 \leq x \leq 1.2$ m) with initial liquid volume fraction $\alpha_{10} = 0.2$. The initial pressure pulse is the same as in the [10–12] and has the form:
\[ p(x,y) = p_0 + \Delta p e^{-(x^2+y^2)/a^2}, \]

where $\Delta p = 100$ MPa, $p_0 = 0.1$ MPa, $a = 0.15$ m. The region at the center of symmetry of radius 0.1 m has a no-slip wall boundary condition in order to increase the stability of numerical calculations.

The numerical solution of equations (1-4) is carried out using the twoPhaseEulerFoam solver based on the PIMPLE [13] algorithm. At the first stage of calculations the time step is selected depending on the Courant number. Then follows the second stage – predictor, during which the continuity (1), momentum (2) and energy (3) equations are being solved. At the stage of the corrector, the values of pressure and velocity components are adjusted. The predictor and corrector cycles run until the predetermined accuracy of the solution is reached.

To verify the proposed model of the gas-droplet mixture, obtained by modifying the twoPhaseEulerFoam solver in accordance with the problem, calculations of the spherical shock pulse impact on the aqueous foam were carried out for the experimental conditions of [8]. In [8] a spherical explosion in dry aqueous foam with initial liquid volume fraction $\alpha_{10} = 0.0083$ was investigated. A comparative analysis of the calculated peak pressures trends with the experimental data of [8] and the general trend of experimental results on explosions in aqueous foams made by [18] showed their satisfactory agreement (see Figure 1).

The numerical solution of the problem of air shock pulse interaction with aqueous foam barrier is presented in Figures 2, 3 in the form of calculated pressure and velocity fields and
Figure 1. Peak pressures trends: 1 – results of calculations; 2, 3 – general trend for experimental values of the SW maximum pressures in aqueous foam [18] and gas [19], respectively; 4, 5 – the experimental points [8] of the SW maximum pressures in aqueous foam.

Streamlines at specified times. The layer of aqueous foam is marked in green. The maximum amplitude of the pressure pulse (5), initially equal to 100 MPa, due to the sphericity of the SW, decreases over time and when approaching the boundary of the foam layer (at \(t = 0.5\) ms) is \(\approx 1\) MPa. During the interaction of air SW with aqueous foam, the latter is compressed to \(\alpha_1 = 0.4\) (\(t = 3\) ms), what leads to a significant decrease in the velocity of SW propagation. Due to dissipative processes, the pressure pulse amplitude decreases by the time \(t = 3\) ms to 0.2 MPa at the foam layer boundary (see Figure 2). Near the axis of symmetry, starting from \(t = 1.2\) ms, a region of low pressures is formed, which expands over time. In this zone, a bending of streamlines is observed, leading to a formation of toroidal vortices. By the time \(t = 2.1\) ms, the first vortex is formed, which rotates counterclockwise [10] with the angular velocity of \(\omega_1 \approx 1400\) rad/s and moves over time to the axis of symmetry. At \(t = 2.1\) ms, a second toroidal vortex is formed that has the angular velocity \(\omega_2 \approx 600\) rad/s in the direction opposite to the first one (clockwise). At time \(t = 3\) ms, close to the foam layer appears a third vortex, rotating counterclockwise with the angular velocity \(\omega_3 \approx 250\) rad/s. Obtained results are in qualitative agreement with [10].

From the point of view of the authors, the main cause of vortex flows is the development of the so-called Richtmyer-Meshkov instability [20, 21], which occurs as a result of interaction of the SW with contacting media of different density and is accompanied by the bending of the gas-foam boundary.

4. Conclusion

The problem of air spherical shock pulse interaction with aqueous foam barrier, which is a continuation of [10–12] studies, is solved. Numerical solution of the problem is carried out in a two-dimensional axisymmetric approximation using the modified twoPhaseEulerFoam solver of the OpenFOAM package. Using Eulerian approach in the numerical implementation and taking into account the interfacial heat transfer and drag forces, more reliable results are obtained in comparison with [10]. The reliability of results is confirmed by satisfactory agreement with the solutions of similar problem by the method of moving Lagrangian grids [10] and experimental data of blast wave mitigation by dry aqueous foams [8]. Analysis of the results showed that the aqueous foam compaction under the influence of SW reduces the velocity of its front and blocks the SW propagation into the foam layer. The process of air shock-wave pulse reflection from the foam boundary leads to the bending of this boundary and formation of toroidal vortices in the gas, which are a consequence of the Richtmyer-Meshkov instability development [20, 21].
Figure 2. Distribution of pressure fields during an interaction of spherical SW in a gas with aqueous foam barrier at times $t = 1.9, 2.1, 2.5$ and $3$ ms, respectively. Layer of aqueous foam is marked in green.

Figure 3. Dynamics of velocity fields, streamlines, liquid fraction of the foam $\alpha_1$ and evolution of vortex zones formation during the interaction of a spherical SW in a gas with aqueous foam barrier at time moments $t = 1.9, 2.1, 2.5$ and $3$ ms, respectively.
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