BEYOND THE STANDARD MODEL

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(I) Supersymmetry (general structure)
(II) Supersymmetry (phenomenology)
(III) Extra Dimensions
(IV) Strong dynamics, Little Higgs & more
Supersymmetry

• Evades Coleman-Mandula (Poincaré × internal group is the largest symmetry of the S-matrix)

• Relates particles with different spin ⇒ involves space-time transformations

3-d space 4-d space-time
translations/rotations Poincaré

• Special ultraviolet finiteness properties

Just a mathematical curiosity? Too beautiful to be ignored by nature? A solution in search of a problem

Quantum dimensions (not described by ordinary numbers)
New concept of space
Local susy contains gravity
In quantum theory, the vacuum is a busy place. Particle-antiparticle pairs can be produced out of nothing, borrowing an energy $E$ for a time $t \leq \hbar$.

Virtual particles are like ordinary particles, but have unusual mass-energy relations.

The Higgs field propagating in vacuum “feel” them with strength $E \Rightarrow \delta m_H \approx E_{\text{max}}$ (maximum energy of virtual particles).

If interacts with , after a while, we expect $E \approx T$. 
\[ \delta m_H \approx E_{\text{max}} \] What is the maximum energy?

\[ M_{\text{GUT}} = 10^{16} \text{ GeV? } M_{\text{Pl}} = 10^{19} \text{ GeV?} \]

Having \( M_W << M_{\text{Pl}} \) requires tuning up to 34th digit!

The “stability” of the hierarchy \( M_W / M_{\text{Pl}} \) requires an explanation

Higgs mass is “screened” at energies above \( m_H \Rightarrow \) new forces and new particles within LHC energy range

What is the new phenomenon?
A problem relevant for low-energy supersymmetry: hierarchy/naturalness

\[ \delta m_H^2 = \frac{3G_F}{4\sqrt{2}\pi^2} \left(2m_W^2 + m_Z^2 + m_H^2 - 4m_t^2\right) \Lambda^2 \approx -(0.2 \Lambda)^2 \]

\[ \delta m_H < 182 \text{ GeV} \text{ (95\% CL limit on SM Higgs)} \]

\[ \Rightarrow \Lambda < \text{TeV} \]

If susy is effective at the Fermi scale:

\[ \text{Higgs} \quad \text{top} \quad \text{stop} \quad = 0 \]

chiral symmetry \quad \Rightarrow \quad \tilde{m}_H = 0

supersymmetry \quad \Rightarrow \quad \tilde{m}_H = m_H \quad \Rightarrow \quad m_H = 0

The Fermi scale \((m_H)\) is induced only by susy breaking
Dynamical supersymmetry breaking

- If susy unbroken at tree-level, it remains unbroken to all orders in perturbation theory
- Non-perturbative effects can break susy with $m_S \sim e^{-1/\alpha} M_P$

\[ \text{Weak scale} \begin{cases} \text{• stable against quantum corrections} \\ \text{• naturally much smaller than } M_P \end{cases} \]

Supersymmetric SM with $m_S < \text{TeV}$ solves the Higgs naturalness problem

Can we construct a realistic theory?
Supersymmetry transformation corresponds to group element

$$G(x, \theta, \bar{\theta}) = e^{i(xP + \theta Q + \bar{\theta}\bar{Q})}$$

Translation

$$G(a, 0, 0) : (x, \theta, \bar{\theta}) \mapsto (x + a, \theta, \bar{\theta})$$

Susy

$$G(0, \xi, \bar{\xi}) : (x, \theta, \bar{\theta}) \mapsto (x + i\theta\sigma\bar{\xi} - i\xi\sigma\theta, \theta + \xi, \bar{\theta} + \bar{\xi})$$

In superspace differential operators represent action of generators

$$P_\mu \rightarrow -i\partial_\mu$$

$$Q_\alpha \rightarrow \frac{\partial}{\partial \theta^\alpha} - i\sigma_\mu^{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu$$

$$\bar{Q}^{\dot{\alpha}} \rightarrow \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha \sigma_\mu^{\mu\beta\dot{\alpha}}\varepsilon^{\beta\dot{\alpha}}\partial_\mu$$
Superfields

Function of superspace $\Phi(x, \theta, \bar{\theta})$  

Power series in $\theta$

$$\Phi(x, \theta, \bar{\theta}) = A(x) + \theta \phi(x) + \bar{\theta} \chi(x) + \theta \theta B(x) + \bar{\theta} \bar{\theta} C(x)$$

$$+ \theta \sigma_{\mu} \bar{\theta} V^\mu(x) + \theta \theta \bar{\theta} \lambda(x) + \bar{\theta} \bar{\theta} \theta \psi(x) + \theta \theta \bar{\theta} \theta D(x)$$

- finite number of component fields
- contains fields with different spin
- compact description of susy multiplets
- easy to write susy Lagrangians
General $\Phi$ is reducible. Additional constraints:

**CHIRAL SUPERFIELD**

$$\Phi = A(x) + \sqrt{2} \theta \psi(x) + i \theta \sigma^\mu \overline{\theta} \partial_\mu A(x) + \ldots$$

$$\overline{D}_\alpha \Phi = 0$$

$$\partial \partial F(x) - \frac{i}{\sqrt{2}} \partial \partial_{\mu} \psi(x) \sigma^{\mu} \overline{\theta} + \frac{1}{4} \partial \partial \overline{\theta} \overline{\theta} \overline{\partial}^{\mu} \overline{\partial}_{\mu} A(x)$$

Components:

$A(x)$ complex scalar field, $\psi(x)$ Weyl spinor, $F(x)$ auxiliary field

**SUSY ACTION FOR A CHIRAL SUPERFIELD**

$$\int d^4 x \ d^4 \theta \ \Phi^+ \Phi \quad \{ \text{Kinetic term for } A \text{ and } \psi \}$$

$$\int d^4 x \ d^2 \theta \ W(\Phi) \quad \{ \text{Superpotential: holomorphic function that defines interactions} \}$$

E.g.:

$$W = m \Phi^2 \quad \Rightarrow \quad L = -\frac{m}{2} (\psi \psi + \overline{\psi} \overline{\psi}) - m^2 A^+ A$$

$$W = \lambda \ \Phi^3 \quad \Rightarrow \quad L = -\lambda (\psi \psi A + \text{h.c.}) - \lambda^2 (A^+ A)^2$$

In general: no quadratic divergences in susy theory

$$\kappa = h^2 \text{ required for cancellation of } \Lambda^2$$
Global symmetry of superpotential can be made local ($\Lambda$ chiral)

if we introduce a vector superfield $V=V^+$ such that

$$\Phi^+\Phi \rightarrow \Phi^+ e^{i(\Lambda^- - \Lambda^+)} \Phi$$

is made invariant

$$L = \Phi^+ e^V \Phi \quad V \rightarrow V + i (\Lambda^+ - \Lambda)$$

When expanded in components, $V$ contains

- $\lambda(x)$ Weyl spinor, $V_\mu(x)$ vector field, $D(x)$ auxiliary field
- + gauge degrees of freedom
Chiral multiplet:
\[ A \quad \psi_\alpha \]
Vector multiplet:
\[ \lambda_\alpha \quad V_\mu \]
Gravity multiplet:
\[ G_{\mu\alpha} \quad g_{\mu\nu} \]
Supersymmetric Standard Model

- Quark: $u_L$, $d_L$, $u_R$, $d_R$
- Squark: $\tilde{u}_L$, $\tilde{d}_L$, $\tilde{u}_R$, $\tilde{d}_R$
- slepton: $\tilde{e}_L$, $\tilde{\nu}_L$, $\tilde{e}_R$
- Higgs doublet: $\tilde{H}_1$, $\tilde{H}_2$
- wino: $\tilde{\omega}^\pm$, $\tilde{\omega}^3$
- bino: $\tilde{b}$
- gluino: $\tilde{g}^A$
New particles, new interactions, but no new free parameters

Ex: SU(3) color interactions

- All vertices controlled by the SU(3) coupling
- There is a quartic scalar vertex
- Sparticles enter interactions in pairs:

\[
\text{sparticle parity } = (-1)^{\text{number of sparticles}} \quad \text{is conserved}
\]
1st problem: indirect new-physics effects

Any FT can be viewed as an effective theory below a UV cutoff

\[ L_{\text{eff}} = L^{d=4}(g, \lambda) + \frac{1}{\Lambda} L^{d=5} + \frac{1}{\Lambda^2} L^{d=6} + ... \]

\[ \Lambda \] has physical meaning: maximum energy at which the theory is valid. Beyond \( \Lambda \), new degrees of freedom

Higgs naturalness gives an upper bound on \( \Lambda \). However,

- B number \( \Rightarrow \frac{1}{\Lambda^2} qqqq l \) p - decay \( \Rightarrow \Lambda \geq 10^{15} \) GeV

- L number \( \Rightarrow \frac{1}{\Lambda} l l H H \) \( \nu \) mass \( \Rightarrow \Lambda \geq 10^{13} \) GeV

- individual L \( \Rightarrow \frac{1}{\Lambda^2} \bar{e} \sigma^{\mu\nu} \mu H F_{\mu\nu} \) \( \mu \rightarrow e \gamma \Rightarrow \Lambda \geq 10^8 \) GeV

- quark flavour \( \Rightarrow \frac{1}{\Lambda^2} \bar{s} \gamma^\mu d \bar{s} \gamma_\mu d \) \( \Delta m_K \Rightarrow \Lambda \geq 10^6 \) GeV

- LEP1,2 \( \Rightarrow \frac{1}{\Lambda^2} |H^+ D_\mu H|^2 \), \( \frac{1}{\Lambda^2} \bar{e} \gamma^\mu e \bar{l} \gamma_\mu l \) \( \Rightarrow \Lambda \geq 10^4 \) GeV
New theories at TeV are highly constrained

A first problem:  
\[ f = Q_L D_R^c H_1 + Q_L U_R^c H_2 + L_L E_R^c H_1 + 
U_R^c D_R^c D_R^c + Q_L D_R^c L_L + L_L L_L E_R^c + H_2 L_L \]

Violate B or L  
\[ \tau_p = \frac{1}{\lambda^4} \left( \frac{m_S}{\text{TeV}} \right)^4 \times 10^{-10} \text{ sec} \]

Usually one invokes R-parity (it could follow from gauge symmetry of underlying theory)

R-parity = + for SM particles, R-parity = – for susy particles

Important for phenomenology

- no tree-level virtual effects from susy
- susy particles only pair produced
- LSP stable (missing energy + dark matter)

Flavor gives powerful constraints on the theory
2\textsuperscript{nd} problem: supersymmetry breaking

Break susy, but keep UV behavior \(\Rightarrow\) soft breaking

\[ m_{\tilde{t}}^2 \neq m_t^2 \quad \rightarrow \quad \delta m_h^2 \propto (m_{\tilde{t}}^2 - m_t^2) \ln \Lambda \quad \text{Soft breaking} \]

\[ y_{\tilde{t}}^2 \neq y_t^2 \quad \rightarrow \quad \delta m_h^2 \propto (y_{\tilde{t}}^2 - y_t^2) \Lambda^2 \quad \text{Hard breaking} \]
$m_S$

$m_S \lambda \lambda$  gaugino mass

$m_S^2 \varphi^+ \varphi$  scalar mass

$m_S \varphi^3$  $A$ - term

- Soft susy breaking introduces a dimensionful parameter $m_S$
- Susy particles get masses of order $m_S$
- Susy mass terms are gauge invariant
- Treat soft terms as independent; later derive them from theory
- Different schemes make predictions for patterns of soft terms
Two robust features of low-energy susy: EW breaking & gauge coupling unification

**ELECTROWEAK SYMMETRY BREAKING**

Higgs potential

\[ V = m_1^2 |H_1^0|^2 + m_2^2 |H_2^0|^2 - m_3^2 (H_1^0 H_2^0 + \text{h.c.}) + \frac{g^2 + g'^2}{8} \left( |H_1^0|^2 - |H_2^0|^2 \right)^2 \]

- \( m_{1,2,3}^2 = O(m_S^2) \) determined by soft terms
- quartic fixed by supersymmetry

- Stability along \( H_1 = H_2 \) \( \Rightarrow m_1^2 + m_2^2 > 2 |m_3^2| \)
- EW breaking, origin unstable \( \Rightarrow m_1^2 m_2^2 < m_3^4 \)
EW breaking induced by quantum corrections

RG running:
- gauge effects
- Yukawa effects

- If $\lambda_t$ large enough $\Rightarrow SU(2) \times U(1)$ spontaneously broken
- If $\alpha_s$ large enough $\Rightarrow SU(3)$ unbroken
- Mass spectrum separation $m_2^2 < \text{weak susy} < \text{strong susy}$
HIGGS SECTOR

8 degrees of freedom  –  3 Goldstones = 5 degrees of freedom

2 scalars \((h^0, H^0)\), 1CP-odd scalar \((A^0)\), 1 charged \((H^\pm)\)

3 parameters \((m_{1,2,3}^2)\) – \(M_Z = 2\) free param. (often \(m_A\) and \(\tan\beta\))

\[
m_h \leq m_Z |\cos 2\beta|, \quad m_h < m_A < m_H, \quad m_{H^\pm}^2 = m_A^2 + m_W^2
\]

\[
m_{h,H}^2 = \frac{1}{2} \left[ m_A^2 + m_Z^2 + \sqrt{(m_A^2 - m_Z^2)^2 + 4 \sin^2 2\beta m_A^2 m_Z^2} \right]
\]

Large \(\tan\beta\): \(m_h\) 

\(m_H\) decoupling region
**m_S is the seed of EW breaking**

EW breaking is related to susy breaking, $m_S \Rightarrow m_Z$

The quantum correction is negative and drives EW breaking

**Minimum of the potential**

$$m_Z^2 = \frac{2(m_1^2 - m_2^2 \tan^2 \beta)}{\tan^2 \beta - 1} \approx -2m_2^2$$

$$|2 \delta m_2^2| < \frac{m_Z^2}{\Delta} \Rightarrow \tilde{m}_t < 300 \text{ GeV} \left(\frac{10\%}{\Delta}\right)^{1/2}$$

$$m_h^2 \approx m_Z^2 + \frac{3}{2\pi^2} \lambda_t^4 v^2 \ln \left(\frac{\tilde{m}_t}{m_t}\right) > 114 \text{ GeV} \Rightarrow \tilde{m}_t > 1 \text{ TeV}$$

- $m_S$ plays the role of $\Lambda^2$ cutoff
- $m_S$ is the seed of EW breaking

**Tension with data**
“Natural” supersymmetry has already been ruled out
Connection susy breaking ⇔ EW breaking at the basis of low-energy supersymmetry

• Susy particle content dynamically determines EW breaking pattern
• Higgs interpreted as fundamental state, like $Q$ and $L$
• Higgs mass determined by susy properties and spectrum

After LEP, “natural” susy is ruled out

• Source of “mild” tuning (is it observable at LHC?)
• Missing principle?
GRAND UNIFICATION

- Fundamental symmetry principle to embed all gauge forces in a simple group

- Partial unification of matter and understanding of hypercharge quantization and anomaly cancellation

To allow for unification, we need to unify $g, g', g_S$ from effects of low-energy degrees of freedom (depends on the GUT structure only through threshold corrections)

\[
\frac{d g_i^{-2}}{d \ln Q} = \frac{b_i}{4 \pi}
\]

\[
\begin{align*}
    b_3 &= -7, \\
    b_2 &= -\frac{19}{6}, \\
    b_1 &= \frac{41}{6}
\end{align*}
\]

\[
\begin{align*}
    b_3 &= -3, \\
    b_2 &= 1, \\
    b_1 &= 11
\end{align*}
\]
QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.
3 equations, 2 unknowns $(\alpha_{GUT}, M_{GUT})$: predict $\alpha_S$ in terms of $\alpha$ and $\sin^2 \theta_W$

$\alpha_S^{\text{exp}} = 0.1176 \pm 0.0020$

- success of susy
- does not strongly depend on details of soft terms
- remarkable that $M_{GUT}$ is predicted below $M_P$ and above $\rho$-decay limit
THEORY OF SOFT TERMS

• Explain origin of supersymmetry breaking

• Compute soft terms

Similar to EW breaking problem

• Origin of EW breaking \( \Rightarrow \)

\[ V(H) = -m_H^2 |H|^2 + \lambda |H|^4 \]

• Compute EW breaking effects \( \Rightarrow \)

\[ L = D_\mu H^+ D^\mu H - \lambda H \bar{\psi} \psi \]

\( \text{Gauge boson mass} \quad \text{Fermion mass} \)
Invent a new sector which breaks supersymmetry
Couple the breaking sector to the SM superfields

But \[ \text{STr } M^2 = \sum_J (-1)^{2J} (2J + 1) M_J^2 = 0 \]

at tree level, with canonical kinetic terms

\text{sparticle} < \text{particle}

Squarks, sleptons, gauginos, higgsinos

What force mediates susy-breaking effects?
GRAVITY AS MEDIATOR
Gravity couples to all forms of energy
Assume no force stronger than gravity couples the two sectors
Susy breaking in hidden sector
\[ m_S = \frac{F_X}{M_P} \quad m_S = \text{TeV} \Rightarrow F_X^{1/2} = 10^{11} \text{ GeV} \]

ATTRACTIVE SCENARIO
- Gravity a feature of local supersymmetry
- Gravity plays a role in EW physics
- No need to introduce \textit{ad hoc} interactions
  - Lack of predictivity (10^2 parameters)

BUT
- Flavour problem

For simplicity, most analyses take universal \( m, M \) and \( A \)
Searching for supersymmetry at the LHC

• At a hadron collider, the total energy of the parton system is not known

• The initial momentum of the parton system in the transverse direction is zero

\[ E_T \] is a characteristic signal of supersymmetry

Background:
• \( \nu \) (mostly produced by W/Z or heavy quarks)
• incomplete solid angle coverage
• finite energy resolution of the detectors
• mismeasurement of jet energies
Colored particles have large cross sections at the LHC

\[ \sigma(\text{TeV} \tilde{g}) \approx \text{pb} \]

If MC tools for SM background are fully validated, if detector response is properly understood, then TeV susy particles can be discovered with low integrated luminosity

Already with 10 fb\(^{-1}\), parameter space is explored up to 1-2 TeV in gluino and squark masses

However, determining parameters and masses is a much more complicated issue
Many new particles in final states

Kinematics of the event cannot be fully reconstructed: unknown CM frame and pairs of particles carrying missing energy

Precise determination of masses and couplings is essential

- Confirm supersymmetric relations
- Understand pattern of supersymmetry breaking
  - Identify “unification” relations
  - Determine the DM mass
  - Reconstruct relic abundance
Susy mass (differences) from edges in invariant mass distributions

Consider the decay chain
\[ \tilde{q} \rightarrow q \tilde{\chi}_2^0 \quad \tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 \ell^+ \ell^- \quad \text{through } Z^0 \text{ or } \tilde{\ell} \text{ exchange} \]
max \( m^2(\ell^+ \ell^-) \) is obtained for \( \tilde{\chi}_1^0 \) and \( (\ell^+ \ell^-) \) at rest in \( \tilde{\chi}_2^0 \) ref frame
\[ \Rightarrow \max \left[ m(\ell^+ \ell^-) \right] = m_{\tilde{\chi}_2} - m_{\tilde{\chi}_1} \]

Consider two-body decay
\[ \tilde{q} \rightarrow q \tilde{\chi}_2^0 \quad \tilde{\chi}_2^0 \rightarrow \ell^+ \ell^- \quad \rightarrow \ell^+ \tilde{\chi}_1^0 \]
max \( m^2(\ell^+ \ell^-) \) is obtained for \( \ell^+ \) and \( \ell^- \) back to back in \( \tilde{\chi}_2^0 \) ref frame
\[ \Rightarrow \max \left[ m(\ell^+ \ell^-) \right] = m_{\tilde{\chi}_2} \sqrt{\left(1 - \frac{m_{\ell}^2}{m_{\tilde{\chi}_2}^2}\right)\left(1 - \frac{m_{\tilde{\chi}_1}^2}{m_{\ell}^2}\right)} \]

Repeating this technique along complicated chains and combining different channels, one can solve for (most) masses

Figure 9. Dilepton kinematic edge in \( \tilde{\chi}_2^0 \) decay (Atlas TDR).
This technique doesn’t exploit the kinematic constraints on $E_T$

\[ pp \rightarrow \tilde{g}\tilde{g} \rightarrow q qq \chi_1^0 \chi_1^0 \]

New techniques to derive all masses from kinematic distributions

Example:

$W$ “transverse mass” from $W \rightarrow l \nu$

\[ m_T^2 = m_\ell^2 + m_\nu^2 + 2(E_T^\ell E_T^\nu - \vec{p}_T^\ell \cdot \vec{p}_T^\nu) \leq m_W^2 \]

$m_W$ obtained from end-point of $m_T$

The end-point of the “gluino stranverse mass” has a kink structure when plotted as a function of the test LSP mass

The location of the kink corresponds to the physical $m_g$ and $m_\chi$ISR, finite resolution, background and finite width can smear end-points
$E_T$ may not be the discovery signature
(even in gravity mediation)

If the gravitino is the LSP:
Long-lived charged particle at the LHC ($\tau \rightarrow \tau \tilde{G}$)

Distinctive ToF and energy loss signatures

“Stoppers” in ATLAS/CMS caverns:

• Measure position and time of stopped $\tilde{\tau}$; time and energy of $\tau$
□ Reconstruct susy scale and gravitational coupling
GAUGE MEDIATION

Soft terms are generated by quantum effects at a scale $M \ll M_P$

$m_Z \quad M \quad \Lambda_F \quad M_P$

• If $M \ll \Lambda_F$, Yukawa is the only effective source of flavour breaking (MFV); flavour physics is decoupled (unlike sugra or technicolour)

• Soft terms are computable and theory is highly predictive

• Free from unknowns related to quantum gravity
BUILDING BLOCKS OF GAUGE MEDIATION

SUSY SM: observable sector with SM supermultiplets

SUSY: “hidden” sector with $<X> = M + \theta^2 F$

Messengers: gauge charged, heavy (real rep), preserve gauge unification (complete GUT multiplet)

Ex.:
\[ \Phi + \Phi = 5 + \bar{5} \text{ of } SU(5) \text{ with } f = X\Phi\Phi, \quad V = M^2 \left( |\phi|^2 + |\phi|^2 \right) + F(\phi\bar{\phi} + \text{h.c.}) \]

Parameters: $M, F, N$ (twice Dynkin index; $N=1$ for $5+\bar{5}$)
Gaugino mass at one loop, scalar masses at two loops:

\[ M_{\tilde{g}}(Q) = \frac{g^2(Q)}{16\pi^2} \frac{N}{M} F \]

\[ \tilde{m}_Q^2(M) = 2c \frac{g^4}{(16\pi^2)^2} \frac{N}{M^2} F^2 \]

\[ F/M \sim 10\text{-}100 \text{ TeV}, \text{ but } M \text{ arbitrary} \]

To dominate gravity and have no flavour problem

\[ \frac{F}{M_P} < 10^{-2} \frac{g^2}{16\pi^2} \frac{F}{M} \implies M < 10^{15} \text{ GeV} \]

From stability: \[ \sqrt{F} < M \implies M > 10 \text{–}100 \text{ TeV} \]

From perturbatativity up to the GUT scale: \[ N < 150/\ln\frac{M_{\text{GUT}}}{M} \]
• Theory is very predictive

• Gaugino masses are “GUT-related”, although they are not extrapolated to $M_{GUT}$

• Gaugino/scalar mass scales like $N^{1/2}$

• Large squark/slepton mass ratio and small $A$ do not help with tuning
Higgs mass is the strongest constraint: stop masses at several TeV
Crucial difference between gauge and gravity mediation

\[ m_{3/2} = \frac{F}{\sqrt{3} M_P} \Rightarrow \text{in gravity } m_{3/2} \approx m_S, \text{ in gauge } m_{3/2} \approx \left( \frac{\sqrt{F}}{100 \text{ TeV}} \right)^2 2 \text{ eV} \]

In gauge mediation, the gravitino is always the LSP

\[ L = -\frac{1}{F} J_Q^\mu \partial_\mu \tilde{G} = -\frac{1}{F} \left( \tilde{m}_\phi \bar{\psi}_L \psi + \frac{M_{\tilde{g}}}{4\sqrt{2}} \bar{\chi}^a \sigma^{\mu\nu} F_{\mu\nu}^a \right) \tilde{G} + \text{h.c.} \]

on mass shell

Goldberger-Treiman \textit{ino} relation

NLSP decays travelling an average distance

\[ \ell \approx \left( \frac{100 \text{ GeV}}{m_{\text{NLSP}}} \right)^5 \left( \frac{\sqrt{F}}{100 \text{ TeV}} \right)^4 \sqrt{\frac{E^2}{m_{\text{NLSP}}^2}} - 1 \quad 0.1 \text{ mm} \]

From microscopic to astronomical distances
\( \chi^0 \) or \( \tilde{\tau}_R \) are the NLSP (NLSP can be charged)

In gravity-mediation, “missing energy” is the signature

\[
\begin{align*}
\text{Susy particles} & \quad \rightarrow \quad \text{NLSP} \\
\chi^0 & \quad \rightarrow \quad \chi^0 \\
\tilde{\tau}_R & \quad \rightarrow \quad \tilde{\tau}_R
\end{align*}
\]

\( \sqrt{F} \leq 10^6 \text{ GeV} \)

\( \chi^0 \)

\( \sqrt{F} \geq 10^6 \text{ GeV} \)

\( \tilde{\tau}_R \)

\( \sqrt{F} \leq 10^6 \text{ GeV} \)

\( \tilde{\tau}_R \)

\( \sqrt{F} \geq 10^6 \text{ GeV} \)

Stable charged particle

Intermediate region very interesting

(vertex displacement; direct measurement of \( F \))
DARK MATTER

Indirect evidence for DM is solid

• rotational curves of galaxies
• weak gravitational lensing of distant galaxies
• velocity dispersion of galaxy satellites
• structure formation in N-body simulations

• Opportunity for particle physics
• Intriguing connection weak-scale physics ⇔ dark matter
The diagram illustrates the behavior of the comoving number density as a function of $m/T$ and time. As $T$ decreases relative to $M$, the density increases, indicated by the curves labeled $N_{eq}$ and $\langle \sigma v \rangle$. The images below show the density distribution for different temperature conditions:

- **$T \gg M$**: High density of particles.
- **$T \approx M$**: Moderate density of particles.
- **$T \ll M$**: Low density of particles.
Relic abundance

$$\Omega_\chi = \frac{m n_\infty}{\rho_c} = \frac{(4\pi)^2}{3} \sqrt{\frac{\pi}{45}} \frac{x_f g_S(\gamma)}{g_*^{1/2}} \frac{T^3_\gamma}{H_0^2 M_P^3 \sigma}$$

If \( \sigma = \frac{k}{128\pi m^2} \) \( \Rightarrow \) \( \Omega_\chi = \frac{0.22}{k} \left( \frac{m}{\text{TeV}} \right)^2 \)

Weak-scale particle candidate for DM

No parametric connection to the weak scale

Observation provides a link \( M_{DM} \leftrightarrow \langle H \rangle \)

Many BSM theories have a DM candidate

Susy has one of the most appealing
Supersymmetric Dark Matter

R-parity $\Rightarrow$ LSP stable
RG effects $\Rightarrow$ colour and electric neutral massive particle is LSP
Heavy isotopes exclude gluino, direct searches exclude sneutrino
Neutralino or gravitino are the best candidates

NEUTRALINO
Because of strong exp limits on supersymmetry, current eigenstates are nearly mass eigenstates:
Bino, Wino, Higgsino
\[ \langle \sigma_B v \rangle = \frac{3g^4 \tan^4 \theta_W r (1 + r^2)}{2\pi m_{\tilde{e}_R}^2 x (1 + r)^4}, \quad x \equiv \frac{M_1}{T}, \quad r \equiv \frac{M_1^2}{m_{\tilde{e}_R}^2}, \]

\[ \Omega_B h^2 = 1.3 \times 10^{-2} \left( \frac{m_{\tilde{e}_R}}{100 \text{ GeV}} \right)^2 \frac{(1 + r)^4}{r (1 + r^2)} \left( 1 + 0.07 \log \frac{\sqrt{r} 100 \text{ GeV}}{m_{\tilde{e}_R}} \right) \]

**HIGGSINO**

\[ \langle \sigma_{\text{eff}} v \rangle = \frac{g^4}{512 \pi \mu^2} \left( 21 + 3 \tan^2 \theta_W + 11 \tan^4 \theta_W \right) \]

\[ \Omega_{\tilde{H}} h^2 = 0.10 \left( \frac{\mu}{1 \text{ TeV}} \right)^2, \]

**WINO**

\[ \langle \sigma_{\text{eff}} v \rangle = \frac{3g^4}{16\pi M_2^2}, \]

\[ \Omega_{\tilde{W}} h^2 = 0.13 \left( \frac{M_2}{2.5 \text{ TeV}} \right)^2. \]
Neutralino: natural DM candidate for light supersymmetry

Quantitative difference after LEP & WMAP

Both $M_Z$ and $\Omega_{DM}$ can be reproduced by low-energy supersymmetry, but at the price of some tuning.

Unlucky circumstances or wrong track?
TO OBTAIN CORRECT $\chi$ RELIC ABUNDANCE

- Heavy susy spectrum: Higgsino (1 TeV) or Wino (2.5 TeV)
- Coannihilation Bino-stau (or light stop?)
- Nearly degenerate Bino-Higgsino or Bino-Wino
- S-channel resonance (heavy Higgs with mass $2m_\chi$)
- $T_{RH}$ close to $T_f$

All these possibilities have a very critical behavior with underlying parameters

- Decay into a lighter particle (e.g. gravitino)
How can we identify DM at the LHC?

Establishing the DM nature of new LHC discoveries will not be easy. We can rely on various hints

• If excess of missing energy is found, DM is the prime suspect
• Reconstructing the relic abundance (possible only for thermal relics and requires high precision; LHC + ILC?)
• Identify model-dependent features (heavy neutralinos, degenerate stau-neutralino, mixed states, $m_A = 2 m_\chi$)
• Compare with underground DM searches
SPACE DIMENSIONS AND UNIFICATION

Minkowski recognized special relativistic invariance of Maxwell’s eqs $\Rightarrow$ connection between unification of forces and number of dimensions

Electric & magnetic forces unified in 4D space-time

space-time $t, \vec{x} \rightarrow x^\mu = (t, \vec{x})$

EM potentials $\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$, $\vec{B} = \vec{\nabla} \times \vec{A} \rightarrow A^\mu = (\phi, \vec{A})$

EM fields $\vec{E}, \vec{B} \rightarrow F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix}$

current $\rho, \vec{J} \rightarrow J_\mu = (\rho, \vec{J})$

Maxwell's eqs $\rightarrow \partial_\mu F^{\mu\nu} = J^\nu$
UNIFICATION OF EM & GRAVITY

⇒ New dimensions?

1912: Gunnar Nordström proposes gravity theory with scalar field coupled to $T_{\mu}^{\mu}$

1914: he introduces a 5-dim $A_{\mu}$ to describe both EM & gravity

1919: mathematician Theodor Kaluza writes a 5-dim theory for EM & gravity. Sends it to Einstein who suggests publication 2 years later

1926: Oskar Klein rediscovers the theory, gives a geometrical interpretation and finds charge quantization

In the ‘80s the theory, known as Kaluza-Klein becomes popular with supergravity and strings
In General Relativity, metric $g_{\mu\nu}$ (4X4 symmetric tensor) dynamical variable describing space geometry (graviton)

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}$$

Dynamics described by Einstein action

$$S_G = \frac{1}{16\pi G_N} \int d^4 x \sqrt{-g} \ R(g)$$

- $G_N$ Newton’s constant
- $R$ curvature (function of the metric)
Consider GR in 5-dim

\[ \hat{S}_G = \frac{1}{16\pi \hat{G}_N} \int d^5 x \sqrt{-\hat{g}} \, R(\hat{g}) \]

Choose

\[ \hat{g}_{MN}(\hat{x}) = \begin{pmatrix} g_{\mu\nu} + \kappa^2 \phi A_\mu A_\nu & \kappa \phi A_\mu \\ \kappa \phi A_\nu & \phi \end{pmatrix}(\hat{x}) \]

Dynamical fields

\[ \hat{g}_{MN} \Leftrightarrow g_{\mu\nu}, A_\mu, \phi \]

Assume space is \( M_4 \times S_1 \)

- First considered as a mathematical trick
- It may have physical meaning
Extra dim is periodic or “compactified” \[ x_5 + 2\pi R = x_5 \]

All fields can be expanded in Fourier modes

\[
\varphi(\hat{x}) = \sum_{n=-\infty}^{+\infty} \frac{\varphi^{(n)}(x)}{\sqrt{2\pi R}} \exp\left(i \frac{n x_5}{R}\right)
\]

5-dim field ⇔ set of 4-dim fields: \( \varphi^{(n)}(x) \) Kaluza-Klein modes

Each \( \varphi^{(n)} \) has a fixed momentum \( p_5 = n/R \) along 5th dim

Extra dimensions

D-dim particle

4-d space

\[
E^2 = \vec{p}^2 + p_{\text{extra}}^2 + m^2
\]

KK mass

From KK mass spectrum we can measure the geometry of extra dimensions
Suppose typical energy $\ll 1/R \Rightarrow$ only zero-modes can be excited

Expand $S_G$ keeping only zero-modes and setting $\phi=1$

$$\hat{S}_G(\hat{g}_{MN}) = S_G(g^{(0)}_{\mu\nu}) + S_{EM}(A^{(0)}_{\mu})$$

To obtain correct normalization:

$$S_G \rightarrow \frac{1}{G_N} = \frac{1}{\hat{G}_N} = \frac{2\pi R}{\hat{G}_N}$$

$$S_{EM} \rightarrow \kappa = \sqrt{16\pi G_N}$$

Gravity & EM unified in higher-dim space: MIRACLE?
Gauge transformation has a geometrical meaning

\[ d\hat{s}^2 = \hat{g}_{MN}(\hat{x}) \, d\hat{x}^M \, d\hat{x}^N \quad \hat{g}_{MN}(\hat{x}) = \begin{pmatrix} g_{\mu\nu} + \kappa^2 \phi A_\mu A_\nu & \kappa \phi A_\mu \\ \kappa \phi A_\nu & \phi \end{pmatrix}(\hat{x}) \]

Keep only zero-modes:

\[ d\hat{s}^2 = g^{(0)}_{\mu\nu} \, dx^\mu \, dx^\nu + \phi^{(0)} \left( dx^5 + \kappa A^{(0)}_\mu \, dx^\mu \right)^2 \]

Invariant under local

\[
\begin{align*}
x^5 & \rightarrow x^5 - \kappa \Lambda \\
A^{(0)}_\mu & \rightarrow A^{(0)}_\mu + \partial_\mu \Lambda
\end{align*}
\]

(where \( g \) and \( \phi \) do not transform)

• Gauge transformation is balanced by a shift in 5th dimension

• EM Lagrangian uniquely determined by gauge invariance
CHARGE QUANTIZATION

Matter EM couplings fixed by 5-dim GR

Consider scalar field $\phi$

$$S = \int d^5 \hat{x} \sqrt{-\hat{g}} \hat{g}^{MN} \partial_M \phi \partial_N \phi$$

Expand in 4-D

$$S = \int dx_5 \sum_n \int d^4 x \sqrt{-g^{(0)}} \left[ \left( \partial^\mu - i \frac{n \kappa}{R} A^{(0)\mu} \right) \phi(n) \right]^2 - \frac{n^2}{R^2} \frac{\phi^{(n)2}}{\phi}$$

KK modes:

Each KK mode $n$ has: mass $n/R$ charge $n\kappa/R$

- charge quantization
- determination of fine-structure constant

$$\alpha = \frac{\kappa^2}{4\pi R^2} = \frac{4G_N}{R^2} \quad \Rightarrow \quad R = \sqrt{\frac{4G_N}{\alpha}} \approx 4 \times 10^{-31} \text{ m} = \left(5 \times 10^{17} \text{ GeV}\right)^{-1}$$

- new dynamics open up at Planckian distances

\[58\]
Not a theory of the real world

- $\phi = 1$ not consistent ($\phi$ dynamical field leads to inconsistencies: e.g. $F^{(0)}_{\mu\nu} F^{(0)\mu\nu} = 0$ from eqs of motion)
  - Charged states have masses of order $M_{Pl}$
  - Gauge group must be non-abelian (more dimensions?)

Nevertheless

- Interesting attempt to unify gravity and gauge interactions
- Geometrical meaning of gauge interactions
- Useful in the context of modern superstring theory
- Relevant for the hierarchy problem?
"At this point we notice that this equation is beautifully simplified if we assume that space-time has 92 dimensions."
Usual approach: fundamental theory at $M_{Pl}$, while $\Lambda_W$ is a derived quantity

Alternative: $\Lambda_W$ is fundamental scale, while $M_{Pl}$ is a derived effect

New approach requires

- extra spatial dimensions
- confinement of matter on subspaces

Natural setting in string theory $\Rightarrow$ Localization of gauge theories on defects (D-branes: end points of open strings)

We are confined in a 4-dim world, which is embedded in a higher-dim space where gravity can propagate
COMPUTE NEWTON CONSTANT

Einstein action in D dimensions

\[ S_E^D = \frac{1}{16\pi \hat{G}_N} \int d^D x \sqrt{-\hat{g}} R(\hat{g}) \]

Assume space \( R^4 \times S^{D-4} \): \( g_{\mu\nu} \) doesn’t depend on extra coordinates

Effective action for \( g_{\mu\nu} \)

\[ S_E = \frac{V_{D-4}}{16\pi \hat{G}_N} \int d^4 x \sqrt{-g} R(g) \]

\[ \Rightarrow \frac{1}{G_N} = \frac{V_{D-4}}{\hat{G}_N} \]

\[ \hat{G}_N = \frac{1}{M_D^{D-2}} \]

\[ V_{D-4} = R^{D-4} \]

\[ M_{Pl} = M_D \left(RM_D\right)^{\frac{D-4}{2}} \]
Suppose fundamental mass scale $M_D \sim \text{TeV}$

$$M_{Pl} = M_D \left( R M_D \right)^{\frac{D-4}{2}}$$

very large if $R$ is large (in units of $M_D^{-1}$)

Arkani-Hamed, Dimopoulos, Dvali

Radius of compactified space

$$R = \begin{cases} 
(5 \times 10^{-4} \text{ eV})^{-1} \approx 0.4 \text{ mm} & D - 4 = 2 \\
(20 \text{ keV})^{-1} \approx 10^{-5} \mu \text{m} & D - 4 = 4 \\
(7 \text{ MeV})^{-1} \approx 30 \text{ fm} & D - 4 = 6 
\end{cases}$$

- Smallness of $G_N/G_F$ related to largeness of $R M_D$
- Gravity is weak because it is diluted in a large space (small overlap with branes)
- Need dynamical explanation for $R M_D >> 1$
Gravitational interactions modified at small distances

\[ F_N(r) = G_N \frac{m_1 m_2}{r^2} \quad \text{at } r > R \]

At \( r < R \), space is \((3+\delta)\)-dimensional \((\delta=\text{D}-4)\)

\[ F_N(r) = \hat{G}_N^{(4+\delta)} \frac{m_1 m_2}{r^{2+\delta}} = \]

\[ = G_N R^\delta \frac{m_1 m_2}{r^{2+\delta}} \]

\[ V(r) = -G_N \frac{m_1 m_2}{r} \left[ 1 + \alpha \exp\left(-r/R\right) \right] \]

From SN emission and neutron-star heating:

\( M_D > 750 \) (35) TeV for \( \delta=2(3) \)

\( \lambda \)
Probing gravity at the LHC?

Gravitational wave
jet + $E_T$

Gravitational deflection
dijet

Black hole
multiparticle event

Gravitational phenomena into collider arena
Probability of producing a KK graviton

\[ \sigma(pp \rightarrow G^{(n)} \text{jet}) = \frac{\alpha_s}{\pi} G_N = 10^{-28} \text{ fb} \]

1 event \( \Rightarrow \) run LHC for \( 10^{16} \) \( t_U \)

Number of KK modes with mass less than \( E \) (use \( m=n/R \))

\[ \propto n^{D-4} \approx (ER)^{D-4} \approx \frac{E^{D-4}M_{Pl}^2}{M_D^{D-2}} \]

Inclusive cross section

\[ \sum_n \sigma(pp \rightarrow G^{(n)} \text{jet}) \approx \frac{\alpha_s E^{D-4}}{\pi M_D^{D-2}} \]

It does not depend on \( V_D \) (i.e. on the Planck mass)

Missing energy and jet with characteristic spectrum
\[ \sigma(p\bar{p} \rightarrow \text{jet} + E_T) \text{ [fb]} \]

\[ M_D \text{ [TeV]} \]

| \( \delta \) | Max \( M_D \) sensitivity \( \mathcal{L} = 100 \text{ fb}^{-1} \) & Max \( M_D \) sensitivity \( \mathcal{L} = 10 \text{ fb}^{-1} \) & Min \( M_D \) perturbativity |
|---|---|---|---|
| 2 | 8.5 TeV | 7.9 TeV | 3.8 TeV |
| 3 | 6.8 | 6.3 | 4.3 |
| 4 | 5.8 | 5.5 | 4.8 |
| 5 | 5.0 | 4.6 | 5.4 |
Contact interactions from graviton exchange

- Sensitive to UV physics
- d-wave contribution to scattering processes
- predictions for related processes
- Limits from Bhabha/di-\(\gamma\) at LEP and Drell-Yan/ di-\(\gamma\) at Tevatron: \(\Lambda_T > 1.2 - 1.4\) TeV
- Loop effect, but dim-6 vs. dim-8
- \(Y\) only dim-6 generated by pure gravity
- \(\Lambda_Y > 15 - 17\) TeV from LEP
G-emission is based on linearized gravity, valid at $s << M_D^2$

**TRANSPLANCKIAN REGIME**

Planck length

$$\lambda_p = \left( \frac{G_D \hbar}{c^3} \right)^{\frac{1}{\delta+2}}$$

quantum-gravity scale

Schwarzschild radius

$$R_S = \frac{1}{\sqrt{\pi}} \left[ \frac{8}{\delta + 2} \Gamma\left( \frac{\delta + 3}{2} \right) \right]^{\frac{1}{\delta+1}} \left( \frac{G_D \sqrt{s}}{c^3} \right)^{\frac{1}{\delta+1}}$$

classical

gravity

classical limit $$(\hbar \rightarrow 0): \quad R_S >> \lambda_p$$

transplanckian limit $$\left( \sqrt{s} >> M_D \right): \quad R_S >> \lambda_p$$

The transplanckian regime is described by classical physics (general relativity) $\Rightarrow$ independent test, crucial to verify gravitational nature of new physics
Gravitational scattering

Non-perturbative, but calculable for $b \gg R_S$
(weak gravitational field)

D-dim gravitational potential:

$$V(r) = \frac{G_D m M}{r^{\delta+1}} \quad D = 4 + \delta$$

Quantum-mechanical scattering phase of wave with angular momentum $mvb$

$$\delta_b = -\left(\frac{b_c}{b}\right)^\delta \quad b_c \approx \left(\frac{G_D m M}{v \hbar}\right)^{\frac{1}{\delta}}$$

$$\theta \approx \frac{\partial \delta_b}{\partial L} \approx \frac{b_c^\delta}{mvb^{\delta+1}} \quad \text{rel.} \quad \frac{G_D \sqrt{s}}{b^{\delta+1}}$$

$$\theta_E = \frac{4G_D \sqrt{s}}{b}$$
Gravitational scattering in extra dimensions: two-jet signal at the LHC

Diffractive pattern characterized by

$$b_c \approx \left(\frac{G_D s}{\hbar}\right)^\frac{1}{\delta}$$
b < R_S \quad \text{At } b < R_S, \text{ no longer calculable}

\textbf{Strong indications for black-hole formation}

BH with angular momentum, gauge quantum numbers, hairs
(multiple moments of the asymmetric distribution of gauge charges and energy-momentum)

Gravitational and gauge radiation during collapse
\Rightarrow \text{ spinning Kerr BH}

\sigma \sim \pi R_S^2 \quad 10 \text{ pb (for } M_{BH} = 6 \text{ TeV and } M_D = 1.5 \text{ TeV)}

Hawking radiation until Planck phase is reached
\quad T_H \sim R_S^{-1} \sim M_D (M_D / M_{BH})^{1/(\delta+1)}

Evaporation with \quad \tau \sim M_{BH}^{(\delta+3)/(\delta+1)} / M_D^{2(\delta+2)/(\delta+1)} \quad (10^{-26} \text{ s for } M_D = 1 \text{ TeV})

Characteristic events with large multiplicity \(<N> \sim M_{BH} / <E> \sim (M_{BH} / M_D)^{(\delta+2)/(\delta+1)})\) and typical energy \(<E> \sim T_H\)

Transplanckian condition \(M_{BH} \gg M_D\) ?
WARPED GRAVITY

A classical mechanism to make quanta softer

For time-indep. metrics with $g_{0\mu} = 0 \Rightarrow E|g_{00}|^{1/2}$ conserved.

(proper time $d\tau^2 = g_{00} dt^2$)

Schwarzschild metric $g_{00} = 1 - \frac{2G_NM}{r} \Rightarrow \frac{E_{\text{obs}} - E_{\text{em}}}{E_{\text{em}}} = \sqrt{|g_{00}|} - 1 = -\frac{G_NM}{r_{\text{em}}}$

On non-trivial metrics, we see far-away objects as red-shifted
GRAVITATIONAL RED-SHIFT

\[ ds^2 = e^{-2K|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \]

Masses on two branes related by

\[ \frac{m_{\pi R}}{m_0} = e^{-\pi R K} \]

Same result can be obtained by integrating \( S_E \) over \( y \)

\[ R \approx 10 K^{-1} \implies \frac{m_{\pi R}}{m_0} \approx \frac{M_Z}{M_{GUT}} \]
PHYSICAL INTERPRETATION

• Gravitational field configuration is non-trivial
• Gravity concentrated at $y=0$, while our world confined at $y=\pi R$
• Small overlap $\Rightarrow$ weakness of gravity

WARPED GRAVITY AT COLLIDERS

• KK masses $m_n = Kx_ne^{-\pi RK}[x_n$ roots of $J_1(x)]$ not equally spaced
• Characteristic mass $Ke^{-\pi RK} \sim \text{TeV}$
• KK couplings
  \[ L = -T^{\mu\nu} \left( \frac{G^{(0)}_{\mu\nu}}{M_{Pl}} + \sum_{n=1}^{\infty} \frac{G^{(n)}_{\mu\nu}}{\Lambda_\pi} \right) \]
  \[ \Lambda_\pi \equiv e^{-\pi RK} M_{Pl} \approx \text{TeV} \]
• KK gravitons have large mass gap and are “strongly” coupled
• Clean signal at the LHC from $G \rightarrow l^+l^-$
A SURPRISING TWIST

AdS/CFT correspondence relates 5-d gravity with negative cosmological constant to strongly-coupled 4-d conformal field theory.

- Warped gravity with SM fermions and gauge bosons in bulk and Higgs on brane
- Technicolor-like theory with slowly-running couplings in 4 dim

Theoretical developments in extra dimensions have much contributed to model building of 4-dim theories of electroweak breaking: susy anomaly mediation, susy gaugino mediation, Little Higgs, Higgs-gauge unification, composite Higgs, Higgsless, …
DUALITY

SM in warped extra dims ⇔ strongly-int’ing 4-d theory

KK excitations ⇔ “hadrons” of new strong force

Technicolor strikes back?

AdS/CFT

5-D warped gravity ⇔ large-N technicolor

⇒ Composite Higgs
What screens the Higgs mass?

- Boson: $\phi \rightarrow \phi + a$
  - No $m^2 \phi^2$
- Fermion: $\psi \rightarrow e^{ia\gamma_5} \psi$
  - No $m\bar{\psi}\psi$
- Vector: $A_\mu \rightarrow A_\mu + \partial_\mu a$
  - No $m^2 A_\mu A^\mu$

Spont. broken global symm.

Chiral symmetry

LITTLE HIGGS

SUPERSYMMETRY

HIGGS-GAUGE UNIF.

TECHNICOLOR

HIGGSLESS

EXTRA DIMENSIONS

Dynamical EW breaking

Delayed unitarity violat.

Fundamental scale at TeV

Symmetry

Dynamics

- Very fertile field of research
- Different proposals not mutually excluded
It is a problem of naturalness, not of consistency!

Necessary tuning \( \frac{M_Z^2}{\Lambda^2} \rightarrow \frac{M_Z^2}{M_{\text{GUT}}^2} \approx 10^{-28} \)

Cancellation of
- electron self-energy
- \( \pi^+-\pi^0 \) mass difference
- \( K_L-K_S \) mass difference
- gauge anomaly
- cosmological constant

Existence of
- positron
- \( \rho \)
- charm
- top

CAVEAT
EMPTOR

10^{-3} \text{ eV}??
HIGGS AS PSEUDOGOLDSTONE BOSON

\[ \Phi = \frac{\rho + f}{\sqrt{2}} e^{i\theta/f} \]  
\[ \langle \Phi \rangle = f \]  
\[ \Phi \rightarrow e^{ia}\Phi : \begin{cases} 
\rho \rightarrow \rho \\
\theta \rightarrow \theta + a
\end{cases} \]

Non-linearly realized symmetry \( h \rightarrow h + a \) forbids \( m^2 h^2 \)

Gauge, Yukawa and self-interaction are non-derivative couplings ⇒ Violate global symmetry and introduce quadratic divergences

Top sector

No fine-tuning

\[ |\delta m^2_H| < (200 \text{ GeV})^2 \]  
\[ \Lambda_{NP} < 600 \text{ GeV} \]

If the scale of New Physics is so low, why do LEP data work so well?
A less ambitious programme: solving the little hierarchy

**New physics**

**Little Higgs**

**Composite Higgs**

**Higgsless**

**Energy**

1 TeV

10 TeV

**Bounds on Λ [TeV]**

\[ L = \pm \frac{1}{\Lambda^2} O \]

**Strong dynamics**

**QuickTime™ and a TIFF (Uncompressed) decompressor are needed to see this picture.**
LITTLE HIGGS

Explain only little hierarchy

One loop \( \delta m^2_H = \frac{G_F}{\pi^2} m^2_{SM} \Lambda^2_{SM} \Rightarrow \Lambda_{SM} < \frac{\pi}{\sqrt{G_F}} \approx \text{TeV} \)

At \( \Lambda_{SM} \) new physics cancels one-loop power divergences

Two loops \( \delta m^2_H = \frac{G_F^2}{\pi^4} m^4_{SM} \Lambda^2 \Rightarrow \Lambda \approx \frac{\pi^2}{G_F m_{SM}} \approx 10 \text{TeV} \approx \Lambda_{LH} \)

“Collective breaking”: many (approximate) global symmetries preserve massless Goldstone boson
Realistic models are rather elaborate

Effectively, new particles at the scale $f$ cancel (same-spin) SM one-loop divergences with couplings related by symmetry

Typical spectrum:

Vectorlike charge $2/3$ quark

Gauge bosons EW triplet + singlet

Scalars (triplets ?)
New states have naturally mass \( \sim \frac{\alpha}{4\pi} \Lambda_{NP}^2 \equiv e^2 f^2 \)

\( \sim 1 \text{TeV} \)

New states cut-off quadratically divergent contributions to \( m_H \)

**Ex.: littlest Higgs model**

\( H \in SU(5)/SO(5) \)

\[
\frac{\Lambda^2}{16\pi^2} \left( 1 + \lambda_i^2 \right)
\]

\[
+ \lambda_T^2
\]

\[
-2 \frac{\lambda_T m_T}{f}
\]

Log term: analogous to effect of stop stops in supersymmetry

Severe bounds from LEP data
• Discover new states (T, W', Z', …)

• Verify cancellation of quadratic divergences

\[
\frac{m_T}{f} = \frac{\lambda_t^2 + \lambda_T^2}{2\lambda_T}
\]

\[
\begin{cases}
  f & \text{from heavy gauge-boson masses} \\
  m_T & \text{from T pair-production} \\
  \lambda_T & \text{we cannot measure TThh vertex} \\
  & \text{(only model-dependent tests possible)}
\end{cases}
\]
$f$ and $g_H$ from DY of new gauge bosons

Production rate and BR into leptons in region favoured by LEP ($g_H > g_W$)

Can be seen up to $Z_H$ mass of 3 TeV

$M_T$ from $T$ production can be measured up to 2.5 TeV
\[ \Gamma(T \rightarrow bW) = 2\Gamma(T \rightarrow tZ) = 2\Gamma(T \rightarrow th) \propto \lambda_T^2 \]

Measure T width?

In order to precisely extract \( \lambda_T \) from measured cross section, we must control b-quark partonic density.

Possible to test cancellation with 10% accuracy for \( m_T < 2.5 \text{ TeV} \) and \( m_Z < 3 \text{ TeV} \).
Concept of symmetry central in modern physics

Now fundamental and familiar concept, but hard to accept in the beginning

Ex.: Earth’s motion does not affect $c$

Lorentz tried to derive it from EM

Einstein postulates $c$ is constant (invariance under velocity changes of observer)

Einstein simply postulates what we have deduced, with some difficulty and not always satisfactorily, from the fundamental equations of the electromagnetic field
General relativity deeply rooted in symmetry

SM: great success of symmetry principle

Impose $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1) \implies$ determine particle dynamics of strong, weak and EM forces

Will symmetries completely determine the properties of the “final theory”?

Or new principles are needed to go beyond our present understanding?
Breaking of naturalness would require new principles

• the “final theory” is a complex phenomenon with IR/UV interplay

• some of the particle-physics parameters are “environmental”
A different point of view

Vacuum structure of string theory

~ $10^{500}$ vacua

(N d.o.f in M config. make $M^N$)

Expansion faster than bubble propagation

Big bang $\Rightarrow$ universe expanding like an inflating balloon

Unfolding picture of a fractal universe $\Rightarrow$ multiverse
Not a unique “final” theory with parameters = $O(1) \times$ allowed by symmetry but a statistical distribution

In which vacuum do we live? Determined by “environmental selection”

$\Lambda$

- Large and positive $\Rightarrow$ blows structures apart
- Large and negative $\Rightarrow$ crunches the Universe too soon

Is the weak scale determined by “selection”? Are fermion masses determined by “selection”?

Will these ideas impact our approach to the final theory?

The LHC will address this question!

SPLIT SUPERSYMMETRY abandons the hierarchy problem, but uses unification & DM
CONCLUSIONS

LHC will soon begin operation:

Unveiling the mechanism of EW breaking

Higgs?
Unconventional Higgs?
Alternative dynamics?

If Higgs is found,

New physics at EW scale curing the UV sensitivity? (many theoretical options, none of which is free from tuning)
New principle in particle physics?