A Note on Bipolar Perfect Fuzzy Matching

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Abstract
The notion of matching in a fuzzy graph could be defined using the concept of effective edges and fractional matching. In this paper, we introduce the notion of bipolar fuzzy matching and bipolar perfect fuzzy matching of a bipolar fuzzy graph and prove some results.

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1. Introduction
In 1965, introduced the notion of fuzzy sets. It has been generalized by many researchers such as L-fuzzy sets by Gougan and Intuitionistic fuzzy sets by Atanasov. One such generalization is the concept of bipolar fuzzy sets by in 1994.
In 1975, introduced the notion of fuzzy graphs. In 2011, introduced the notion of bipolar fuzzy graphs. Using the concept of effective edges, defined matching in a fuzzy graph. Introduced the notion matching in a fuzzy graph using the concept of fractional matching.
In our earlier paper, we discussed the concept of perfect fuzzy matching on some fuzzy graphs. In, we defined intuitionistic perfect fuzzy matching and discussed some results. In this paper, we introduce the notion of bipolar fuzzy matching and bipolar perfect fuzzy matching of a bipolar fuzzy graph and we prove some results.

2. Preliminaries
In this section, we introduce some basic definitions that are required in the sequel.

Definition 1: A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma: V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$ with $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$, $\forall u,v \in V$, where $V$ is a finite nonempty set and $\wedge$ denote minimum.

Definition 2. Let $X$ be a non-empty set. A bipolar fuzzy set $B$ on $X$ is an object having the form: $B = \{(x, m^+(x), m^-(x) ) / x \in X \}$, where $m^+: X \rightarrow [0,1]$ and $m^-: X \rightarrow [-1,0]$ are mappings. For the sake of simplicity, we shall use the symbol $B = (m^+, m^-)$ for the bipolar fuzzy set.

Definition 3. A bipolar fuzzy graph with underlying set $(V, E)$ is defined to be the pair $G = (A, B)$ where $A = (m^+_A, m^-_A)$ is a bipolar fuzzy set on $V$ and $B = (m^+_B, m^-_B)$ is a bipolar fuzzy set on $E \subset V \times V$ such that $m^+_B(x, y) \leq \min\{m^+_A(x), m^+_A(y)\}$ and $m^-_B(x, y) \geq \max\{m^-_A(x), m^-_A(y)\}$ $\forall (x,y) \in E$.

Definition 4. A bipolar fuzzy graph $G = (A, B)$ is said to be strong if $m^+_B(x, y) = \min\{m^+_A(x), m^+_A(y)\}$ and $m^-_B(x, y) = \max\{m^-_A(x), m^-_A(y)\}$ $\forall (x,y) \in E$.

The graph $G$ is called a complete bipolar fuzzy graph if $m^+_B(u, v) = \min\{m^+_A(u), m^+_A(v)\}$ and $m^-_B(u, v) = \max\{m^-_A(u), m^-_A(v)\}$ $\forall u, v \in V$.

Definition 5. Let $G = (A, B)$ be a bipolar fuzzy graph where $A = (m^+_A, m^-_A)$ and $B = (m^+_B, m^-_B)$ be...
two bipolar fuzzy sets on a nonempty finite set $V$ and $E \subset V \times V$ respectively.

The positive degree of a vertex $u$ in $G$ is denoted by $d^+(u)$ and defined as $d^+(u) = \sum_{(u,v) \in E} m^+_2(u,v)$. The negative degree of a vertex $u$ in $G$ is denoted by $d^-(u)$ and defined as $d^-(u) = \sum_{(u,v) \in E} m^-_2(u,v)$. The degree of a vertex is $d(u) = (d^+(u), d^-(u))$.

If $d^+(u) = k_1$ and $d^-(u) = k_2$ for all $u \in V$ and $k_1, k_2$ are two real numbers, then the graph is called $(k_1, k_2)$-regular bipolar fuzzy graph.

**Definition 6:** Let $G = (\Sigma, \mu)$ be a fuzzy graph on $(V, E)$ where $V$ is the vertex set and $E$ is the set of edges with non-zero weights. A subset $M$ of $E$ is called a fuzzy matching if for each vertex $u \in V$,

$$\sum_{v \in V} \mu(u, v) \leq \sigma(u).$$

Further, a subset $M$ of $E$ is called a perfect fuzzy matching if for each vertex $u \in V$,

$$\sum_{v \in V} \mu(u, v) = \sigma(u).$$

### 3. Bipolar Fuzzy Matching

In this section, we introduce the notion of bipolar fuzzy matching and bipolar perfect fuzzy matching and prove some results.

**Definition 7:** Let $G = (A, B)$ be a bipolar fuzzy graph where $A = (m^+_1, m^-_1)$ and $B = (m^+_2, m^-_2)$ be two bipolar fuzzy sets on a nonempty finite set $V$ and $E \subset V \times V$ respectively. If

$$\sum_{(u,v) \in M} m^+_2(u,v) \leq m^+_1(u)$$

and

$$\sum_{(u,v) \in M} m^-_2(u,v) \geq m^-_1(u), \forall u \in V,$$

then $M$ is said to be bipolar fuzzy matching in $G$.

$M$ is said to be bipolar perfect fuzzy matching if:

$$\sum_{(u,v) \in M} m^+_2(u,v) = m^+_1(u)$$

and

$$\sum_{(u,v) \in M} m^-_2(u,v) = m^-_1(u), \forall u \in V.$$

**Definition 8:** Let $G$ be a bipolar fuzzy graph on the underlying graph $(V, E)$. Let $M$ be a bipolar fuzzy matching for $G$. Then bipolar fuzzy matching number $\Gamma(G)$ is defined as

$$\Gamma(G) = (\sum_{(u,v) \in M} m^+_2(u,v), \sum_{(u,v) \in M} m^-_2(u,v)).$$

**Example 1:** Consider the following bipolar fuzzy graph $G$ on $(V, E)$ where $V = \{v_1, v_2, v_3\}$ and $E = \{e_1, e_2, e_3\}$.

E = \{(0.2, -0.3), (0.2, -0.5), (0.3, -0.2), (0.5, -0.7), (0.6, -0.9)\}

$E = \{e_1, e_2, e_3\}$ is a bipolar fuzzy matching for $G$. For $\sum_{(v_1, v_2) \in M} m^+_2(v_1, v_2) = m^+_2(v_1, v_2) + m^+_2(v_1, v_3) = 0.3 + 0.2 = 0.5 \leq 0.7 = m^+_1(v_1)$,

$\sum_{(v_2, v_3) \in M} m^-_2(v_2, v_3) = m^-_2(v_2, v_3) + m^-_2(v_2, v_1) = 0.3 + 0.2 = 0.6 = m^-_1(v_2)$,

$\sum_{(v_2, v_3) \in M} m^-_2(v_2, v_3) = m^-_2(v_2, v_3) + m^-_2(v_2, v_1) = 0.3 + 0.2 = 0.6 \geq 0.6 = m^-_1(v_2)$.

Here $\Gamma(G) = (0.5, 0.7)$.

**Example 2:** Consider the following bipolar fuzzy graph $G$ on $(V, E)$.

$E = \{(0.4, -0.2), (0.6, -0.4), (0.3, -0.3), (0.6, -0.7)\}$

$\sum_{(v_1, v_2) \in M} m^+_2(v_1, v_2) = m^+_2(v_1, v_2) + m^+_2(v_1, v_3) = 0.3 + 0.4 = 0.7 = m^+_1(v_1)$,

$\sum_{(v_2, v_3) \in M} m^-_2(v_2, v_3) = m^-_2(v_2, v_3) + m^-_2(v_2, v_1) = 0.3 + 0.4 = 0.6 = m^-_1(v_2)$,

$\sum_{(v_2, v_3) \in M} m^-_2(v_2, v_3) = m^-_2(v_2, v_3) + m^-_2(v_2, v_1) = 0.3 + 0.4 = 0.7 = m^-_1(v_2)$.

Thus $\{v_1, v_2, v_3\}$ is a bipolar perfect fuzzy matching.

Here, $\Gamma(G) = (0.6, 0.7)$. 

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Theorem 1: Let $G = (A, B)$ be a bipolar fuzzy graph on the cycle $(V, E)$. If \( m_1^+(u, v) = \text{constant} = k_1 \), (say) and \( m_2^-(u, v) = \text{constant} = k_2 \), (say), \( \forall (u, v) \in E \) and if \( m_1^+(u) = 2k_1 \) and \( m_2^-(u) = 2k_2 \) then \( M = E \) is a bipolar perfect fuzzy matching.

Proof: Since only two edges are incident with each vertex for a cycle, for any vertex \( u \in V \),
\[
\sum_{(u,v) \in E} m_2^-(u,v) = m_2^+(u,v) + m_2^-(u,w),
\]
where \( v, w \in V \).
\[
= k_1 + k_2 = 2k_1 = m_1^+(u)
\]
\[
= k_2 = m_1^-(u).
\]
Therefore \( E \) is a bipolar perfect fuzzy matching in \( G \).

The converse of the above theorem need not be true. This can be seen from the following example.

Example 3: Consider the following bipolar fuzzy graph \( G \) on \( (V, E) \).

Here \( E \) is bipolar perfect fuzzy matching in \( G \) but the conditions of the above theorem are not satisfied.

Theorem 2: Let \( G = (A, B) \) be a bipolar fuzzy graph on a complete graph \( (V, E) \). If \( m_1^+(u) = \text{constant} = k_1 \), (say) and \( m_2^-(u) = \text{constant} = k_2 \), (say), \( \forall (u, v) \in E \) and if \( m_1^+(u) = \frac{k_1}{n} \) and \( m_2^-(u, v) = \frac{k_2}{n} \) (say), \( \forall (u, v) \) on the cycle \( C_n \) and \( m_1^+(u,v) = \frac{k_1 - 2k_2}{n-3} \) and \( m_2^-(u,v) = \frac{k_2 - 2k_1}{n-3} \), \( \forall \) edges not on the cycle \( C_n \) then \( M = E \) is a bipolar perfect fuzzy matching.

Proof: Let \( G = (A, B) \) be a bipolar fuzzy graph on a complete graph \( K_n \) on \( (V, E) \).

For any complete fuzzy graph \( K_n \), two edges are incident with each vertex of the cycle and remaining \( (n-3) \) edges are incident with the interior vertices.

Hence,
\[
\sum_{(u,v) \in E} m_2^-(u,v) = 2k_3 + (n-3)\left(\frac{k_2 - 2k_1}{n-3}\right)
\]
\[
= 2k_3 + k_3 - 2k_3 = k_1 = m_1^+(u), \text{for each vertex } u.
\]
\[
\sum_{(u,v) \in E} m_2^-(u,v) = 2k_4 + (n-3)\left(\frac{k_2 - 2k_1}{n-3}\right)
\]
\[
= 2k_4 + k_4 - 2k_4 = k_2 = m_1^+(u), \text{for each vertex } u.
\]

Therefore \( E \) is a bipolar perfect fuzzy matching in \( G \).

The converse of the above theorem need not be true.

This can be seen from the following:

Example 4.
Here E is a bipolar perfect fuzzy matching. But the conditions of the above theorem are not true.

The following theorem establishes that a strong regular bipolar fuzzy graph need not have a bipolar perfect fuzzy matching in G.

**Theorem 4:** Let G = (A, B) be a strong regular bipolar fuzzy graph on (V, E), with each vertex is of degree at least two. Then E is not a bipolar perfect fuzzy matching for G.

**Proof:**

Suppose E is a bipolar perfect fuzzy matching for G. Then

\[ \sum_{(u, v) \in E} m_1^+(u, v) = m_1^-(u), \forall u \in V \]

and

\[ \sum_{(u, v) \in E} m_2^+(u, v) = m_2^-(u), \forall u \in V. \]

Since G is regular, \( \sum_{u \in V} m_2^+(u, v) = \text{constant} = k_1 \) (say) and \( \sum_{u \in V} m_2^-(u, v) = \text{constant} = k_2 \) (say).

Therefore, \( m_1^+(u) = k_1 \) and \( m_1^-(u) = k_2, \forall u \in V. \)

Since each vertex is of degree atleast two, \( m_2^+(u, v) < k_1. \)

Therefore, \( \min\{m_1^+(u), m_1^+(v)\} < k_1 \), since G is strong.

\[ \min\{k_1, k_1\} < k_1 \Rightarrow k_1 < k_1 \]

a contradiction.

Thus E is not a bipolar perfect fuzzy matching for G.

### 4. Conclusion

In this paper we have derived a sufficient condition for a bipolar fuzzy graph on a cycle or a complete graph or a star graph to have a bipolar perfect fuzzy matching. This could be extended for fractional fuzzy matching.

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