Residual heavy quark and boson interactions: the role of the $Zb\bar{b}$ vertex

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Abstract

We establish the most general parametrization of the new physics tested by present precision measurements and possibly responsible for any deviation of the $Z \rightarrow b\bar{b}$ amplitude from its Standard Model result, under the assumption that it is CP symmetric and is induced by degrees of freedom which are too heavy to be directly produced at the future Colliders. This is achieved by writing the complete list of the $SU(3)_c \times SU(2) \times U(1)$ gauge invariant and CP symmetric $dim = 6$ operators, involving only quarks of the third family and/or bosons. The quark containing operators are divided into two classes, according to whether they involve the $t_R$ field or not. Each class contains 14 quark operators. We then proceed to derive the constraints from present precision measurements, on the first class of the 14 $t_R$ involving operators. We show that the $Zb\bar{b}$ vertex plays a fundamental role to discriminate not only between these operators, but also between this whole scheme and an alternative one like e.g. an MSSM description with light chargino and neutral Higgs.

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1 Introduction

A complete and rigorous investigation of the status of the Standard Model (SM) requires a critical analysis of its various sectors. As of today, this has been possible only for the fermionic sector, thanks to the impressive experiments that have been performed at LEP1, at SLC and at lower energy [1, 2, 3]. On the contrary the status of the bosonic sector (gauge and scalar boson interactions) is not yet empirically established to a convincing precision. Although a number of indirect tests concerning e.g. the triple gauge couplings already indicate that, there also, the deviation from SM cannot be dramatic [4, 5, 6], it is generally felt that more accurate tests at higher energy colliders are required in order to be able to state that we have really tested the theory.

As far as the fermionic sector is concerned, it is certainly true that the agreement with the SM predictions is amazingly good (up to a few permille) in the light fermionic part. The situation is slightly less triumphant in the heavy quark sector, where, as it has been exhaustively discussed in previous papers [2], the experimental value of $\Gamma_b$ (the Z width into $b\bar{b}$), shows a small discrepancy from the SM prediction, which increases with the top mass $m_t$ and reaches the $2\sigma$ level for $m_t$ values in the region of $175\text{GeV}$ [7]. In addition to this, the top quark interactions themselves are also to a large extent empirically unknown.

Within the SM, the most important difference between the $Zb\bar{b}$ vertex involved in the possibly ”rebel” $Z \to b\bar{b}$ width, and the light $Zf\bar{f}$ vertices, arises at the one loop level and has the form of a contribution proportional to $m_t^2$. Such a contribution appears in the $Zb\bar{b}$ vertex only and originates, in a $R_\xi$ gauge, from the Yukawa coupling of a charged would-be Goldstone boson with a ($t\bar{b}$) pair. Since the corresponding contribution to light fermion vertices is negligible, one suspects that a kind of link should somehow exist between the heaviness of (one of) the quarks of the third family and the possibility that the SM predictions for this family are ”slightly” inadequate. A similar inadequacy may apply to the bosonic sector as well.

In this spirit, we subscribe to the feeling that the fact that the $t$ quark and the ($W$, $Z$) pair are much heavier than the leptons or the other quarks is not casual, but rather deeply related to the scalar sector of the theory, on whose origin it might perhaps open one day an illuminating window. Thus, a kind of NP may exist, originating from the scalar sector, which could induce new interesting phenomena in the gauge boson, Higgs and top interactions, and which may have already been ”seen”, in the peculiarities of $Z \to b\bar{b}$. As far as the $Z \to b\bar{b}$ decay is concerned, this NP should appear in the form of contributions enhanced by some power of the heavy top quark mass.

One popular way of describing this kind of New Physics (NP) is that of assuming that it corresponds to an extension of the SM in which all extra new degrees of freedom appear at a scale $\Lambda$ that is much heavier than the electroweak scale; i.e. $\Lambda \gg v$. At present energies, the effects of NP are completely described by integrating out all these new heavy degrees of freedom using standard effective lagrangian techniques [8]. In this approach one has thoroughly examined until now only the possibility that this NP is entirely contained in the bosonic sector, where it has been satisfactorily described in terms of 11 independent $\dim = 6$ gauge invariant operators [6]. These purely bosonic
operators induce anomalous triple gauge boson couplings at the tree level \cite{5}, and at the
1-loop level they also affect the fermionic vertices. In particular two of these operators
also create at 1-loop $m_t^2$ corrections to the $Z \to b\bar{b}$ amplitude, which could provide an
explanation for the possible deviation of $\Gamma_b$ from its SM value \cite{9}.

With the exception of the very special case of the $Zt\bar{t}$ vertex considered in \cite{10}, anomalous
direct gauge boson-fermion interactions, possibly involving also the Higgs particle,
have been disregarded up to now. As stated above, the neglect of anomalous gauge boson
and fermion interactions appears well motivated for light fermions. It does not appear
justified though, in cases where a $t$ quark is participating, like $t$ and $b$ physics. A fortiori,
then, such anomalous interactions should be investigated, particularly also since they
can teach us something about $Z \to b\bar{b}$.

The aim of this paper is that of establishing a general description for the residual
NP interactions that may directly affect the quarks of the third family. Assuming that
the NP is CP symmetric and that it obeys $SU(3)_c \times SU(2) \times U(1)$ gauge invariance, we
classify all possible $\dim = 6$ operators that could be induced by it at the present low
energies. For purely bosonic operators, this has already been done \cite{5}. Here we establish
the operators involving quarks of the third family only, possibly together with gauge
and/or Higgs bosons. No light quarks (from the first two families) or leptons are allowed.
The complete set of the purely bosonic and the above ”third family” operators should
provide a full description of NP for energies lower than the threshold for the excitation
of the new degrees of freedom that may exist. After this classification, we investigate
what the existing experimental information on $Zb\bar{b}$ can teach us about these operators.

Under the previous general assumptions, we find 28 independent ”third family” operators, which we classify in two classes. The first class contains 14 members which all
involve the $t_R$ field. Since it is precisely the $\bar{q}_L t_R \Phi$ combination which characterizes the
top mass in SM, it is natural to assume that the $t_R$ involving ”top” operators have a
”strong affinity” to the scalar sector and, therefore, some of them may get enhanced by
it. Incidentally, a similar ”strong affinity” also applies to (some of) the 11 purely bosonic
operators \cite{11}. On the opposite side, currents like e.g. $(\bar{q}_L \gamma^\mu q_L)$ have nothing to do with
the top quark mass. Consequently, the related operators are put in a second class, as we
feel that the possibility that they are enhanced by NP is rather remote.

Therefore, we end up with a picture where NP is described in terms of an Effective
Lagrangian containing the 14 ”top operators” of the first class and the 11 purely bosonic
ones mentioned above. Since the consequences of the purely ”bosonic” operators have
already been fully analysed, we concentrate in this paper on the 14 $t_R$ involving ones.
These operators induce anomalous effects in direct processes like e.g. top quark production
and decay, and also indirect effects in processes where a virtual top quark appears as
intermediate state.

The analysis of direct processes will require a clear and copious production of top
quarks which should be possible at future colliders like e.g. LHC, NLC or maybe the
Tevatron, after an important development program. Since this is not the most urgent
point, we leave it for the future, and we concentrate instead on the indirect processes
for which experimental results are presently available. We then find that existing data
can give useful constraints on some of the "top-operators," and provide an orientation on which operators one should retain in the future analysis of the direct processes.

In Sect.2 we give the full list of the 28 CP symmetric, $SU(3)_c \times SU(2) \times U(1)$ gauge invariant, $dim = 6$ "third family" operators of the first and second class. For completeness, we also give the 11 "bosonic" operators established in [5]. We then derive the constraints that can be obtained from the light fermionic sector using the LEP1, SLC and low energy experiments. They are of two different types. Firstly, those from the light fermionic processes (i.e. those not involving $b$ quarks), which are sensitive at 1-loop to top-operator contributions to the gauge boson self-energies. Using these, we calculate in Sect.3 the effects on the relevant $\epsilon_i$ parameters which establish constraints on four independent top-operators. Secondly, in Sect.4 we turn to the partial decay width $Z \to bb$ and to the $b$ asymmetry [12], which provide constrains on five top-operators, two of them belonging also to the group of the four ones just mentioned above and three new ones. We also find that two other "top operators" lead to anomalous magnetic moment $Z\bar{b}b$ and $\gamma \bar{b}b$ couplings, whose observable first order effects, however, are reduced by the factor $m_b/m_t$. Finally our conclusions and an outlook for the future are given in the last Sect.5.

2 Operators involving third family quarks or bosons

The complete list of the $dim = 6$, $SU(3)_c \times SU(2) \times U(1)$ invariant operators involving leptons, quarks, gauge bosons and scalar fields has been established in ref.[13]. Restricting to those operators involving quarks of the third family only, (i.e. either the left doublet $q_L \equiv (t, b)_L$ or the right singlets $t_R, b_R$), and bosons and imposing also CP invariance, we obtain the following set of operators classified in two classes. In class 1 we put the operators involving at least one $t_R$ field, while the remaining ones are put in class 2. The operators in each class are further divided into two groups; those containing four quark fields, and those including only two quark fields:

Class 1.

A1) Four-quark operators

\[ O_{qt} = (\bar{q}_L t_R)(\bar{t}_R q_L) \]  
\[ O_{qt}^{(8)} = (\bar{q}_L \gamma^\mu t_R)(\bar{t}_R \gamma^\mu q_L) \]  
\[ O_{tt} = \frac{1}{2} (\bar{t}_R \gamma^\mu t_R)(\bar{t}_R \gamma^\mu t_R) \]  
\[ O_{tb} = (\bar{t}_R \gamma^\mu t_R)(\bar{b}_R \gamma^\mu b_R) \]  
\[ O_{tb}^{(8)} = (\bar{t}_R \gamma^\mu \bar{t}_R)(\bar{b}_R \gamma^\mu \bar{b}_R) \]
\[ O_{qq} = (\bar{t}_R t_L)(\bar{b}_R b_L) + (\bar{t}_L t_R)(\bar{b}_L b_R) - (\bar{t}_R b_L)(\bar{b}_R t_L) - (\bar{b}_L t_R)(\bar{t}_L b_R), \tag{6} \]
\[ O_{qq}^{(8)} = (\bar{t}_R \vec{x} t_L)(\bar{b}_R \vec{x} b_L) + (\bar{t}_L \vec{x} t_R)(\bar{b}_L \vec{x} b_R) - (\bar{t}_R \vec{x} b_L)(\bar{b}_R \vec{x} t_L) - (\bar{b}_L \vec{x} t_R)(\bar{t}_L \vec{x} b_R). \tag{7} \]

**B1) Two-quark operators.**

\[ O_{t1} = (\Phi^\dagger \Phi)(\bar{q}_L t_R \Phi + \bar{t}_R \Phi^\dagger q_L), \tag{8} \]
\[ O_{t2} = i \left[ \Phi^\dagger (D_\mu \Phi) - (D_\mu \Phi^\dagger) \Phi \right](\bar{t}_R \gamma^\mu t_R), \tag{9} \]
\[ O_{t3} = i \left( \Phi^\dagger D_\mu \Phi \right)(\bar{t}_R \gamma^\mu b_R) - i \left( D_\mu \Phi^\dagger \Phi \right)(\bar{b}_R \gamma^\mu t_R), \tag{10} \]
\[ O_{Dt} = (\bar{q}_L D_\mu t_R)D^\mu \Phi + D^\mu \Phi^\dagger (D_\mu t_R q_L), \tag{11} \]
\[ O_{W\Phi} = (\bar{q}_L \sigma^{\mu\nu} \tau^a t_R)\Phi \cdot \vec{W}^{\mu\nu} + \Phi^\dagger (\bar{t}_R \sigma^{\mu\nu} \tau^a q_L) \cdot \vec{W}^{\mu\nu}, \tag{12} \]
\[ O_{tB\Phi} = (\bar{q}_L \sigma^{\mu\nu} t_R)\Phi B^{\mu\nu} + \Phi^\dagger (\bar{t}_R \sigma^{\mu\nu} q_L)B^{\mu\nu}, \tag{13} \]
\[ O_{tG\Phi} = \left[ (\bar{q}_L \gamma^{\mu\nu} \lambda^a t_R)\Phi + \Phi^\dagger (\bar{t}_R \sigma^{\mu\nu} \lambda^a q_L) \right] G^{a}_{\mu\nu}. \tag{14} \]

**Class 2.**

**A2) Four quark operators**

\[ O_{qq}^{(1,1)} = \frac{1}{2} (\bar{q}_L \gamma_\mu q_L)(\bar{q}_L \gamma^\mu q_L), \tag{15} \]
\[ O_{qq}^{(1,3)} = \frac{1}{2} (\bar{q}_L \gamma_\mu \vec{\tau} q_L) \cdot (\bar{q}_L \gamma^\mu \vec{\tau} q_L), \tag{16} \]
\[ O_{qq}^{(8,1)} = \frac{1}{2} (\bar{q}_L \gamma_\mu \vec{x} q_L)(\bar{q}_L \gamma^\mu \vec{x} q_L), \tag{17} \]
\[ O_{qq}^{(8,3)} = \frac{1}{2} (\bar{q}_L \gamma_\mu \lambda^a \tau^j q_L)(\bar{q}_L \gamma^\mu \lambda^a \tau^j q_L), \tag{18} \]
\[ O_{bb}^{(1)} = \frac{1}{2} (\bar{b}_R \gamma_\mu b_R)(\bar{b}_R \gamma^\mu b_R), \tag{19} \]
\[ O_{qb}^{(1)} = (\bar{q}_L b_R)(\bar{b}_R q_L), \tag{20} \]
\[ O_{qb}^{(8)} = (\bar{q}_L \vec{x} b_R) \cdot (\bar{b}_R \vec{x} q_L). \tag{21} \]
B2) Two-quark operators.

\[ \mathcal{O}_{\Phi q}^{(1)} = i (\Phi^\dagger D_\mu \Phi)(\bar{q}_L \gamma^\mu q_L) - i (D_\mu \Phi^\dagger \Phi)(\bar{q}_L \gamma^\mu q_L) , \]  
\[ \mathcal{O}_{\Phi q}^{(2)} = i \left[ (\Phi^\dagger \gamma^\mu D_\mu \Phi) - (D_\mu \Phi^\dagger \gamma^\mu \Phi) \right] \cdot (\bar{q}_L \gamma^\mu q_L) , \]  
\[ \mathcal{O}_{\Phi b} = i \left[ (\Phi^\dagger D_\mu \Phi) - (D_\mu \Phi^\dagger \Phi) \right] (\bar{b}_R \gamma^\mu b_R) , \]  
\[ \mathcal{O}_{D b} = (\bar{q}_L D_\mu b_R) D^\mu \Phi + D^\mu \Phi^\dagger (\bar{D}_\mu b_R q_L) , \]  
\[ \mathcal{O}_{bW} = (\bar{q}_L \sigma^{\mu\nu} \gamma^\lambda b_R) \Phi \cdot \overrightarrow{W}_{\mu\nu} + \Phi^\dagger (\bar{b}_R \sigma^{\mu\nu} q_L) \cdot \overrightarrow{W}_{\mu\nu} , \]  
\[ \mathcal{O}_{bB} = (\bar{q}_L \sigma^{\mu\nu} b_R) \Phi B_{\mu\nu} + \Phi^\dagger (\bar{b}_R \sigma^{\mu\nu} q_L) B_{\mu\nu} , \]  
\[ \mathcal{O}_{bG} = (\bar{q}_L \sigma^{\mu\nu} \lambda^a b_R) \Phi G^a_{\mu\nu} + \Phi^\dagger (\bar{b}_R \sigma^{\mu\nu} \lambda^a q_L) G^a_{\mu\nu} , \]

where \( \lambda^a \) are the eight usual colour matrices.

In the preceding formulae the usual definitions

\[ \Phi = \left( \frac{1}{\sqrt{2}} (\phi^+ + \sqrt{2}(v + H + i \phi^0)) \right) , \]  
\[ D_\mu = (\partial_\mu + i g Y B_\mu + i g \gamma^\tau \cdot \overrightarrow{W}_{\mu} ) \]

are used where \( Y \) is the hypercharge of the field on which the covariant derivative acts and \( \gamma^\tau \) its isospin matrices.

In addition to the above fermionic operators, NP induced by new heavy degrees of freedom, may also be hiding in purely bosonic \( \text{dim} = 6 \) operators. Provided CP invariance is imposed, this kind of NP is described by 11 independent \( \text{dim} = 6 \) purely bosonic operators first classified in [5]. For completeness we give them below as [11]:

\[ \overline{\mathcal{O}}_{DW} = 2 (D_\mu \overrightarrow{W}^{\mu\nu}) (D_\nu \overrightarrow{W}_{\nu}) , \]  
\[ \mathcal{O}_{DB} = (\partial_\mu B_{\nu\rho}) (\partial^\mu B^{\nu\rho}) , \]  
\[ \mathcal{O}_{BW} = \frac{1}{2} \Phi^\dagger B_{\mu\nu} \gamma^\nu \cdot \overrightarrow{W}_{\mu\nu} \Phi , \]  
\[ \mathcal{O}_{\Phi 1} = (D_\mu \Phi^\dagger \Phi) (\Phi^\dagger D_\mu \Phi) , \]  
\[ \mathcal{O}_{\Phi 2} = 4 \partial_\mu (\Phi^\dagger \Phi) \partial^\mu (\Phi^\dagger \Phi) , \]  
\[ \mathcal{O}_{\Phi 3} = 8 (\Phi^\dagger \Phi)^3 , \]  
\[ \mathcal{O}_{W} = \frac{1}{3!} \left( \overrightarrow{W}_{\mu} \times \overrightarrow{W}_{\nu} \right) \cdot \overrightarrow{W}_{\lambda} \mu , \]  
\[ \hat{\mathcal{O}}_{UW} = \frac{1}{2} (\Phi^\dagger \Phi) \overrightarrow{W}_{\mu\nu} \cdot \overrightarrow{W}_{\mu\nu} , \]  
\[ \hat{\mathcal{O}}_{UB} = 2 (\Phi^\dagger \Phi) B_{\mu\nu} B_{\mu\nu} , \]
\[ O_{W\Phi} = i (D_\mu \Phi)\dagger \rightarrow \cdot \overline{W}^{\mu\nu} (D_\nu \Phi), \] (40)

\[ O_{B\Phi} = i (D_\mu \Phi)\dagger B^{\mu\nu} (D_\nu \Phi). \] (41)

As mentioned in the previous section and provided CP invariance is assumed, NP is described in terms of an effective lagrangian containing the 14 fermionic operators of the first class given in (1-14), and the 11 bosonic operators in (31-41). We then define the effective lagrangian describing the corresponding residual interactions as

\[ \mathcal{L} = \sum_i \frac{f_i}{\Lambda^2} O_i, \] (42)

\( \Lambda \) being the NP scale and \( f_i \) the dimensionless coupling of the operator \( O_i \). The observable effects predicted by this lagrangian will be discussed in the following Sections. At this point we only note that it is convenient to remove from \( O_{t1} \) its tree level contribution to \( m_t \) by an appropriate renormalization of the top mass which leads to

\[ O_{t1} \rightarrow (\Phi^\dagger \Phi - \frac{v^2}{2})(\bar{q}_L t_R \tilde{\Phi} + \bar{t}_R \tilde{\Phi}^\dagger q_L). \] (43)

Similarly, a renormalization of the \( W \) and \( B \) fields leads to the substitutions

\[ \hat{O}_{UW} \rightarrow \frac{v^2}{2} O_{UW}, \] (44)

\[ \hat{O}_{UB} \rightarrow \frac{v^2}{2} O_{UB}, \] (45)

with the definitions

\[ O_{UW} \equiv \frac{1}{v^2} (\Phi^\dagger \Phi - \frac{v^2}{2}) \overline{W}^{\mu\nu} \cdot \overline{W}_{\mu\nu}, \] (46)

\[ O_{UB} \equiv \frac{4}{v^2} (\Phi^\dagger \Phi - \frac{v^2}{2}) B^{\mu\nu} B_{\mu\nu}. \] (47)

which remove the tree level contributions of these operators to the \( W_{\mu} \) and \( B_{\mu} \) kinetic energy.

### 3 Constraints from gauge boson self-energies and light fermions

The constraints on the couplings of the purely bosonic NP operators from the available experimental results (mainly) in the light fermionic sectors have already been derived in\[ \text{[5]} \]. For the ”non-blind” operators \( O_{DW}, O_{DB}, O_{\Phi 1} \) and \( O_{BW} \), these constraints are so strong that their relevance for NP is virtually excluded. Only the ”superblind” operators

\footnote{Note Table 1 in the second paper in this Ref.}
(\mathcal{O}_{\Phi_2}, \mathcal{O}_{\Phi_3}), the 5 "blind" operators (\mathcal{O}_{B\Phi}, \mathcal{O}_{W\Phi}, \mathcal{O}_{UB}, \mathcal{O}_{UW}, \mathcal{O}_{W} ) and of course the above 14 "top" operators have still a chance to describe an observable NP. The constraints on the purely bosonic blind operators from \(Z \to b\bar{b}\) have also been studied in [9], where it has been found that only \(\mathcal{O}_{B\Phi}\) and \(\mathcal{O}_{W\Phi}\) are sensitive to this process, since only these give a \(\ln \Lambda^2\) dependent contribution increasing with \(m_t\). We also note that unitarity considerations have also been applied to the five "blind" purely bosonic operators. They led to the conclusion that "unitarity" is as effective in constraining the "blind" couplings, as are present LEP1 measurements [14].

In this Section we give the constraints for the "top" operators of our first Class. These operators contribute to the light fermion processes only at the 1-loop level, giving universal oblique corrections to the gauge boson self-energies. In general, the relevant diagrams have the same topology as the SM ones; i.e. \(t\bar{t}\) loops for neutral currents and \(t\bar{b}\) loops for charged currents (in some cases tadpoles generated by 4-leg couplings may also appear). In the SM, these diagrams produce the well-known strong \(m_t^2\) contribution to \(\Delta \rho\). For the top operators listed in (1-14), contributions having a different \(m_t\) dependence may be generated. In the calculation, we only keep the divergent part of the leading \(m_t\) contribution. This is required for consistency with our effective lagrangian approach, where we restrict to \(\text{dim} = 6\) operators only.

Only four of the above "top" operators give a non vanishing NP contribution to either the \(\epsilon_1\) or \(\epsilon_3\) parameters conventionally defined in [15, 16]. All other "top" operators give no contribution to \(\epsilon_{1,3}\) and none of the operators contributes to \(\epsilon_2\). Thus, defining \(L \equiv \ln \Lambda^2/M_Z^2\), the only non vanishing results are:

\[
\epsilon_{1}^{(NP)}(t2) = -\frac{3m_t^2}{4\pi^2\Lambda^2}f_{t2}L = -0.011 f_{t2}
\]  \(\text{(48)}\)

from \(\mathcal{O}_{t2}\),

\[
\epsilon_{1}^{(NP)}(Dt) = -\frac{3gm_t^3}{16\pi^2\sqrt{2}M_W\Lambda^2}f_{Dt}L = -0.0028 f_{Dt}
\]  \(\text{(49)}\)

for \(\mathcal{O}_{Dt}\),

\[
\epsilon_{3}^{(NP)}(tW\Phi) = -\frac{5M_Wm_t}{4\pi^2\sqrt{2}\Lambda^2}f_{tW\Phi}L = -0.0060 f_{tW\Phi}
\]  \(\text{(50)}\)

for \(\mathcal{O}_{tW\Phi}\) and

\[
\epsilon_{3}^{(NP)}(tB\Phi) = -\frac{3c_WM_Wm_t}{4\pi^2\sqrt{2}s_W\Lambda^2}f_{tB\Phi}L = -0.0066 f_{tB\Phi}
\]  \(\text{(51)}\)

for \(\mathcal{O}_{tB\Phi}\). For the numerical results in (48-51) we have used \(m_t = 175\text{GeV}\) and \(\Lambda = 1\text{TeV}\), while \(s_W^2\) has been identified with \(s_0^2 \approx 0.231\) defined by \(s_0c_0 = \pi\alpha(M_Z)/(\sqrt{2}M_Z^2G_F)\) and describing the Weinberg angle including QED corrections only [15].

The present experimental knowledge from LEP1 and SLC is summarized e.g. in [18], where it is found that

\[
-3.2 \times 10^{-3} \lesssim \epsilon_{1}^{(NP)} \lesssim +3.2 \times 10^{-3} ,
\]  \(\text{(52)}\)

\[
-3.8 \times 10^{-3} \lesssim \epsilon_{3}^{(NP)} \lesssim +1.8 \times 10^{-3} ,
\]  \(\text{(53)}\)
provided $m_t$, $m_H$ are allowed to vary in the range $160 \lesssim m_t \lesssim 190\,GeV$ and $65\,GeV \lesssim m_H \lesssim 1\,TeV$. Comparing (52,53) with (48-51) one then gets
\begin{align}
-0.3 &\lesssim f_{t2} \lesssim +0.3, \\
-1.1 &\lesssim f_{Dt} \lesssim +1.1, \\
-0.27 &\lesssim f_{tW} \lesssim +0.47, \\
-0.27 &\lesssim f_{tB} \lesssim +0.43,
\end{align}
provided that each operator is considered separately, and that no cancellations among the contributions from different operators are taken into account.

4 Constraints from the $b\bar{b}$ observables

At 1-loop the top quark operators also affect the $Zb\bar{b}$ and $\gamma b\bar{b}$ couplings. In the SM case, the top and goldstone (in the $R_\xi$ gauge) exchange diagrams produce the well-known strong $m_t^2$ contribution. With our set of top operators one generates several new $m_t$ dependent contributions. Again, for each operator, we only retain the leading $m_t$ and $\ln \Lambda^2$ dependent contributions, and neglect quantities proportional to $m_b/M_Z$. Non-vanishing effects now arise only from the five four-quark operators $O_{qt}$, $O_{(8)qt}$, $O_{tb}$, $O_{qq}$, $O_{(8)qq}$, and from the two two-quark operators $O_{t2}$ and $O_{Dt}$. These operators give 3 different types of anomalous contributions. Namely, vector and axial vector couplings for $Zb\bar{b}$, and anomalous magnetic moment couplings for both $Zb\bar{b}$ and $\gamma b\bar{b}$. We normalize the vector and axial $Zb\bar{b}$ vertex (S-matrix elements) as
\begin{equation}
\frac{-ie}{2s_Wc_W}\gamma^\mu[g^Z_{Vb} + \delta g^Z_{Vb} - \gamma^5(g^Z_{Ab} + \delta g^Z_{Ab})],
\end{equation}
with $g^Z_{Vb} = (-1/2 + 2s_W^2/3)$, $g^Z_{Ab} = -1/2$, and the anomalous $Z$ and $\gamma$ magnetic moment couplings by
\begin{align}
\frac{e}{2s_Wc_Wm_t}(\sigma^{\mu\nu}q_\nu)\delta\kappa^Z, \\
\frac{e}{m_t}(\sigma^{\mu\nu}q_\nu)\delta\kappa^\gamma.
\end{align}

Turning now to the results, we start from the remark that the operators $O_{qt}$, $O_{(8)qt}$, $O_{t2}$ and $O_{Dt}$ give purely left-handed contributions to the anomalous $Zb\bar{b}$ coupling. These are written as
\begin{equation}
\delta g^Z_{Vb} = \delta g^Z_{Ab} = \frac{L}{32\pi^2\Lambda^2} \cdot \left[ (f_{qt} + 16f^{(8)qt}/3 - f_{t2})m_t^2 + \frac{5gf_{Dt}m_t^4}{2\sqrt{2}M_W^2} \right].
\end{equation}
\footnote{Note that charge conservation prohibits the appearance of anomalous vector and axial couplings for $\gamma$.}
On the contrary, the operator $\mathcal{O}_{tb}$ generates a pure right-handed NP contribution to $Zb\bar{b}$, which is given by

$$\delta g^Z_{Vb} = -\delta g^Z_{Ab} = -\frac{3f_{tb}m_t^2}{16\pi^2\Lambda^2}L. \quad (62)$$

Finally, $\mathcal{O}_{qq}$ and $\mathcal{O}^{(8)}_{qq}$ generate only anomalous magnetic moment-type couplings for both, $Z$ and $\gamma$. Using the definitions (59,60) we find

$$\delta \kappa^Z = -(f_{qq} + \frac{16}{3}f_{qq}^{(8)})\frac{m_t^2(1 - 8s_W^2/3)}{32\pi^2\Lambda^2}L, \quad (63)$$
$$\delta \kappa^\gamma = -(f_{qq} + \frac{16}{3}f_{qq}^{(8)})\frac{2m_t^2}{48\pi^2\Lambda^2}L. \quad (64)$$

The interesting thing about these anomalous magnetic couplings is that they have nothing to do with the $b$-quark mass $m_b$; i.e. they can exist even if $m_b$ vanishes. Their contribution to observable effects is however, to first order, proportional to $m_b/m_t$. This is easily understood because first order contributions could only arise from interference with the SM amplitude, which, being vector or axial, leads to $(b, \bar{b})$ pairs with opposite helicities, while the magnetic interactions induced by $\mathcal{O}_{qq}$ or $\mathcal{O}^{(8)}_{qq}$ want to give to $(b, \bar{b})$ the same helicity. Thus, in the $m_b \to 0$ limit there is no interference. We should also remark that the treatment of $\mathcal{O}_{qq}$ and $\mathcal{O}^{(8)}_{qq}$ to first order only is consistent with our approximation to neglect dim = 8 operators, which will inevitably arise in the divergent part of loops involving two dim = 6 ”top” operators.

We conclude therefore that seven of the 14 ”top” operators give NP contributions to $Z \to b\bar{b}$. These contributions, determined by (58 - 64), modify the partial width $\Gamma(Z \to b\bar{b}) \equiv \Gamma_b$ and the ”longitudinally polarized forward-backward asymmetry” $A_b$ defined at the $Z$ peak by

$$A_b \equiv \frac{\sigma(e^-_L \to bF) - \sigma(e^-_L \to bB) + \sigma(e^-_R \to bB) - \sigma(e^-_R \to bF)}{\sigma(e^-_L \to bF) + \sigma(e^-_L \to bB) + \sigma(e^-_R \to bB) + \sigma(e^-_R \to bF)} = \frac{\sigma(e^-_L \to bF) - \sigma(e^-_L \to bB)}{\sigma(e^-_L \to bF) + \sigma(e^-_L \to bB)} + \frac{\sigma(e^-_R \to bB) - \sigma(e^-_R \to bF)}{\sigma(e^-_R \to bB) + \sigma(e^-_R \to bF)}, \quad (65)$$

where the second line in (65) just follows by rotating the $Z$ spin by $180^\circ$ around an axis perpendicular to the beam direction. In [12], it has been shown that from these quantities one can measure two model independent parameters which are sensitive to the NP considered in the present work, namely

$$\frac{\Gamma_b}{\Gamma_s} \equiv 1 + \delta_{bw}, \quad (66)$$
$$\frac{A_b}{A_s} \equiv 1 + \eta_b. \quad (67)$$

The New Physics (NP) contributions to these parameters are

$$\delta_{bw}^{(NP)} = -\frac{4}{1 + v_d^2}[v_d\delta g^Z_{Vb} + \delta g^Z_{Ab} + 3v_d\frac{m_b}{m_t}\delta \kappa^Z] \quad (68)$$
\[ \eta^{(NP)}_b = -\frac{2(1 - v_d^2)}{v_d(1 + v_d^2)}[\delta g_Z^{bV} - v_d\delta g_Z^{Ab} +] - \frac{4(1 - 2v_d^2)m_b\delta\kappa^Z}{v_d(1 + v_d^2)m_t}, \quad (69) \]

where \( v_d = 1 - \frac{4}{3}s_0^2 \), and \( s_0^2 \approx 0.231 \) has already been defined immediately after (51). Using (61-64) we thus find

\[ \delta^{(NP)}_{bv} = -0.0021(f_{qt} + \frac{16}{3}f_{qt}^{(8)} - f_{t2}) - 0.0048f_{Dt} - 0.0023f_{tb} + 0.17 \times 10^{-4}(f_{qq} + \frac{16}{3}f_{qq}^{(8)}), \quad (70) \]

\[ \eta^{(NP)}_b = -0.00014(f_{qt} + \frac{16}{3}f_{qt}^{(8)} - f_{t2}) - 0.00036f_{Dt} + 0.0046f_{tb} + 5.4 \times 10^{-7}(f_{qq} + \frac{16}{3}f_{qq}^{(8)}), \quad (71) \]

where the same input parameters as in the preceding section have been used.

It is worth noting from (66, 67) that the parameters \( \delta_{bv} \) and \( \eta_b \) are useful for any kind of coupling, while the parameter \( \epsilon_b \) defined in [15] applies only to the pure left-handed case for which it is given by \( \epsilon_b = -2\delta g_Z^{bV} = -2\delta g_Z^{Ab} \). Using (68, 69) we also notice for the NP contribution that the sign (and magnitude) of the ratio \( \eta^{(NP)}_b / \delta^{(NP)}_{bv} \) discriminates between the purely left handed or the magnetic anomalous contribution on the one side, and the purely right handed one induced by \( O_{tb} \). Indeed we find

\[ \eta^{(NP)}_b / \delta^{(NP)}_{bv} = \frac{(1 - v_d)^2}{2v_d} = 0.068 > 0 \quad (72) \]

for \( (O_{qt}, O_{qt}^{(8)}, O_{Dt}, O_{t2}) \), and

\[ \eta^{(NP)}_b / \delta^{(NP)}_{bv} = \frac{(1 - 2v_d^2)}{3v_d^2} = 0.03 > 0 \quad (73) \]

for \( (O_{qq}, O_{qq}^{(8)}) \), while the \( O_{tb} \) case gives

\[ \eta^{(NP)}_b / \delta^{(NP)}_{bv} = -\frac{(1 + v_d)^2}{2v_d} = -2.068 < 0 \quad (74) \]

This numerical difference between the predictions (74) and (72,73) could be essential in the search for the \( O_{tb} \) operator at SLC.

The results presently available on \( \Gamma_b \) alone from LEP [1, 2] and SLC [3], would lead to a difference between the experimental findings and the SM prediction:

\[ \delta^{(NP)}_{bV} = (+1.93 \pm 1.08) \times 10^{-2} \quad (75) \]

By comparing this with (69) one obtains the following one-standard deviation numerical constraints on the coupling constants of the contributing seven “top” operators, taken one by one:

\[ -15 \lesssim f_{qt} \lesssim -4 \quad , \quad (76) \]

\[ -3 \lesssim f_{qt}^{(8)} \lesssim -0.7 \quad , \quad (77) \]

\[ -6 \lesssim f_{Dt} \lesssim -2 \quad , \quad (78) \]
\[ +4 \lesssim f_{12} \lesssim +15 \]  
\[ -14 \lesssim f_{bb} \lesssim -4 \]  
\[ 0.5 \times 10^{+3} \lesssim f_{qq} \lesssim 2 \times 10^{+3} \]  
\[ 10^{+2} \lesssim f^{(8)}_{qq} \lesssim 4 \times 10^{+2} \]  

The very loose limit on \( f_{qq} \) and \( f^{(8)}_{qq} \) is due to the presence of the \( m_b/m_t \) factor in front of the magnetic coupling \( \delta \kappa^{Z} \) in (68,69). It corresponds to a \( \delta \kappa^{Z} \) value of the order of 0.1. One may wonder whether it could be possible to measure separately the magnetic \( \gamma b\overline{b} \) and \( Z b\overline{b} \) couplings by performing measurements outside the Z peak. The differential cross section for the process \( e^+e^- \rightarrow b\overline{b} \) going through photon and Z exchange, calculated at the tree level and neglecting for consistency quadratic terms in (\( \delta \kappa^{\gamma} \)) and (\( \delta \kappa^{Z} \)), is given by

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta_b}{4s} \left\{ Q_b^2 \left( 1 + \beta_b^2 \cos^2 \theta + \frac{4m_b^2}{s} \right) + \frac{8m_b}{m_t} Q_b \delta \kappa^{\gamma} \right. \\
+ \frac{s^2}{16s_W^4 c_W^2 |D_Z|^2} \left\{ (g_{Ve}^2 + g_{Ae}^2) \left( (g_{Ve}^2 + g_{Ab}^2)(1 + \beta_b^2 \cos^2 \theta) + (g_{Ve}^2 - g_{Ab}^2) \frac{4m_b^2}{s} \right) \right. \\
+ 8g_{Ve} g_{Ae} g_{Ve} g_{Ab} \beta_b \cos \theta + 8 \delta \kappa^{Z} \frac{m_b}{m_t} \left( (g_{Ve}^2 + g_{Ve}^2) g_{Ve} + 2g_{Ve} g_{Ae} g_{Ab} \beta_b \cos \theta \right) \\
\left. - \frac{s(s - M_Z^2)}{2s_W^2 c_W^2 |D_Z|^2} \right\} Q_b g_{Ve} g_{Ve} \left( 1 + \beta_b^2 \cos^2 \theta + \frac{4m_t^2}{s} \right) \\
+ 2Q_b g_{Ae} g_{Ab} \beta_b \cos \theta + 4m_b \left( g_{Ve} [Q_b \delta \kappa^{Z} + g_{Ve} \delta \kappa^{\gamma}] + g_{Ae} g_{Ab} \beta_b \delta \kappa^{\gamma} \cos \theta \right) \left\} \right. , \]  

where for \( Q_f \) is the fermion charge, \( g_{Ve} = t^{(3)}_f - 2Q_f s_W^2 \) , \( g_{Ae} = t^{(3)}_f \) , \( \beta_b = \sqrt{1 - 4m_b^2/s} \) is the b quark velocity and \( |D_Z|^2 = (s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2 \). We see from (83), that an accuracy of one percent below the Z peak would allow the determination of \( \delta \kappa^{\gamma} \) at the level of 0.1. This would mean roughly the same sensitivity to \( f_{qq} \) and \( f^{(8)}_{qq} \) as from Z peak experiments. Anomalous magnetic moment interactions have also been studied in [17].

5 Conclusions

In this paper we have studied some of the New Physics signatures expected in the case where all the new degrees of freedom are too heavy to be directly produced at the Colliders in the foreseeable future. In such a case NP is predominantly described by \textit{dim} = 6 operators involving only standard model particles, including the usual Higgs doublet. Motivated by the overall picture implied by the amazing success of the SM in explaining
of the present precision measurements, we are led to a set of 39 $SU(3)_c \times SU(2) \times U(1)$ gauge invariant and CP symmetric operators. Eleven of these operators are purely bosonic and have been studied before, while the remaining 28 involve in addition quark fields of the third family. Among these 28 operators, there are 14 where the $t_R$ field appears, at least once. The motivation for singling out the quarks of the third family is supplied by the large top mass, which indicates a strong ”affinity” of these quarks to the Higgs sector. If we believe that a next possible step in particle physics is that of understanding the spontaneous breaking mechanism, then a good way to find some kind of new physics is that of looking whether any of these operators acquires an observable strength. In this respect it looks as if the $t_R$ involving operators, as well as the purely bosonic ones, are more likely to be enhanced by whatever NP is hidden in the scalar sector.

The above 14 ”top” operators should best be studied through their effects in top production at the future Colliders. Before doing this, though, we need to study what kind of hints on the expected strength of the various operators may be obtained from LEP1 and SLC. Thus in the present paper we have studied their effects on the gauge boson self energies and the $Z \rightarrow b \bar{b}$ decay. It turns out that five of these operators, namely $\mathcal{O}_{tt}$, $\mathcal{O}_{tb}^{(8)}$, $\mathcal{O}_{t1}$, $\mathcal{O}_{t3}$ and $\mathcal{O}_{tC\Phi}$, give no contribution to these quantities. Thus, present experimental knowledge provides no information on them. On the other hand, the remaining nine operators give non vanishing contributions to at least one of $\epsilon_1$, $\epsilon_3$ and the $Z \rightarrow b \bar{b}$ parameters $\eta_b^{(NP)}$ and $\delta_{b\nu}^{(NP)}$. The results are summarized in Table 1, where the blanks indicate no contribution from the corresponding operator.

| Operator | $\epsilon_1^{(NP)}$ | $\epsilon_3^{(NP)}$ | $\delta_{b\nu}^{(NP)}$ | $\eta_b^{(NP)} / \delta_{b\nu}^{(NP)}$ |
|----------|---------------------|---------------------|---------------------|---------------------|
| $\mathcal{O}_{qt}$ | $-2.1 \times 10^{-3} f_{qt}$ | $-2.1 \times 10^{-3} f_{qt}$ | 0.068 |
| $\mathcal{O}_{qt}^{(8)}$ | $-1.1 \times 10^{-2} f_{t2}$ | $-1.1 \times 10^{-2} f_{qt}^{(8)}$ | 0.068 |
| $\mathcal{O}_{t2}$ | $2.1 \times 10^{-3} f_{t2}$ | 0.068 |
| $\mathcal{O}_{Dt}$ | $-4.8 \times 10^{-3} f_{Dt}$ | $-4.8 \times 10^{-3} f_{Dt}$ | 0.068 |
| $\mathcal{O}_{qq}$ | $1.7 \times 10^{-5} f_{qq}$ | 0.03 |
| $\mathcal{O}_{qq}^{(8)}$ | $9.1 \times 10^{-5} f_{qq}^{(8)}$ | 0.03 |
| $\mathcal{O}_{tb}$ | $-2.3 \times 10^{-3} f_{tb}$ | $-2.3 \times 10^{-3} f_{tb}$ | -2.068 |
| $\mathcal{O}_{tW\Phi}$ | $-6.0 \times 10^{-3} f_{tW\Phi}$ | 0.03 |
| $\mathcal{O}_{tB\Phi}$ | $-6.6 \times 10^{-3} f_{tB\Phi}$ | -2.068 |

It should be noted that none of these operators contribute to $\epsilon_2$.

The most interesting result in Table 1 is given by its last column which indicates that the ratio $\eta_b^{(NP)} / \delta_{b\nu}^{(NP)}$ provides a very strong signature for discriminating between the left-handed, right-handed and the anomalous magnetic $Zb\bar{b}$ vertex. Note that if a single operator dominates, the ratio $\eta_b^{(NP)} / \delta_{b\nu}^{(NP)}$ is independent of the magnitude of its coupling and depends only on the nature of the induced $Zb\bar{b}$ vertex.

It should be stressed that the large and negative $\eta_b^{(NP)} / \delta_{b\nu}^{(NP)}$ ratio would be a rather peculiar signature of the $\mathcal{O}_{tb}$ operator. In practice, it would predict a two percent (neg-
tive) effect in $\eta_{b}^{(NP)}$ for a one percent positive effect in $\delta_{be}^{(NP)}$. This should be detectable at SLC at their expected final accuracy. Note that this effect would be of opposite sign (and larger in magnitude) than the corresponding prediction for the remaining operators $\mathcal{O}_{qt}, \mathcal{O}_{qt}^{(8)}, \mathcal{O}_{t2}, \mathcal{O}_{Dt}, \mathcal{O}_{qq}, \mathcal{O}_{qq}^{(8)}$ that contribute here. Note also that two of these operators, namely $\mathcal{O}_{t2}$ and $\mathcal{O}_{Dt}$ are (qualitatively at least) disfavoured by our analysis from the apparent inconsistency between their effects on $\epsilon_{1}^{(NP)}$ indicated in (54,55) and on $\delta_{be}^{(NP)}$ shown in (78,79).

Finally, it is more spectacular to remark, that the predicted ratio $\eta_{b}^{(NP)}/\delta_{be}^{(NP)}$ and the magnitude of $\eta_{b}^{(NP)}$ for the $\mathcal{O}_{tb}$ operator would be orthogonal to the expectations for the minimal supersymmetric SM. Here, in fact, the trend would be that of positive $\eta_{b}^{(NP)}$ (of order one percent) for positive $\delta_{be}^{(NP)}$. However, this prediction would be necessarily accompanied by the discovery of suitably light supersymmetric particles, like e.g. a light chargino and/or a light neutral Higgs.

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