Finite temperature $QED_3$ with light fermions

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Non-compact $QED_3$ is simulated both in the quenched and unquenched cases. In particular, we investigate the restoration of chiral symmetry at finite temperature. We also compute the zero temperature spectrum of the theory, including (in the quenched case) the dynamical fermion mass. From these two set of data, one can obtain estimates for the ratio of the mass gap to the critical temperature, of particular interest for applications to high-$T_c$ superconductivity.

1. Introduction

Over the last few years, $QED_3$ has attracted a lot of attention because of potential applications to models of high-$$T_c$$ superconductivity. However, at the present time, the phase diagram of the theory is still not known precisely. There have been several analytical studies but approximations are often drastic and different results are obtained depending on what assumptions are made. Hence the interest of trying to see if a lattice gauge theory simulation can sort out this situation.

To avoid any confusion, it is worth mentioning that we are considering here the 4-component formulation of $QED_3$ where the Dirac algebra is represented by the $4 \times 4$ matrices $\gamma_0$, $\gamma_1$ and $\gamma_2$. This formulation preserves parity and possesses an SU(2) group of chiral symmetries generated by $\gamma_3$, $\gamma_5$ and $\gamma_3 \gamma_5$. It also arises naturally as the continuum limit of the staggered fermion construction in (2+1) dimensions. If a mass term is spontaneously generated, the above SU(2) symmetry will break down to the U(1) generated by $\gamma_3 \gamma_5$ and there will appear two Goldstone bosons associated with the operators $\bar{\psi} \gamma_3 \psi$ and $\bar{\psi} \gamma_5 \psi$.

Before discussing the numerical simulation, it is useful to make a brief review of the existing analytical results. These are all based on approximate treatments of the Schwinger-Dyson equation for the fermion self-energy. The first approximation which was considered is the $1/N_f$ expansion ($e^2 N_f$ fixed) where $N_f$ is the number of fermion flavors. To first order, there is no vertex correction and the photon propagator becomes:

$$D(p) = 1/[p^2 + e^2 N_f p/8]$$

A numerical solution of the Schwinger-Dyson equation then gives for the critical number of flavors beyond which chiral symmetry is restored:

$$N_c = 3.28.$$  

This result has been criticized by Pennington and Walsh, who by considering a plausible expression for the interaction vertex arrive at the conclusion that chiral symmetry is always broken (however high $N_f$ can be). The finite temperature theory has also been investigated. In this case, the longitudinal photons are screened, but the transverse photons remain unscreened:

$$D_{00} = \frac{1}{q^2 + M^2} \quad D_{ij} = \frac{1}{q^2}$$

The transverse photons therefore give rise to severe infrared divergences and it is not known at present how this problem can be cured. Dorey and Mavromatos have assumed that transverse photons can be neglected in the Schwinger-Dyson equation and have computed the critical temperature in this case. They obtain in this way for $N_f = 2$ a ratio of the mass gap to the critical temperature which is of order 10 (much bigger than in BCS-like theories, hence the potential interest for models of high-$T_c$ superconductivity). Because of all the uncertainties in the above mentioned results, it would be very interesting to have lattice measurements of the relevant quantities. We will first discuss the techniques that we have at our
disposal to carry out such a project and will then present our current results for the chiral condensate (both at zero and non-zero temperature) and for the meson spectrum.

2. Symmetry Breaking on the lattice

When studying chiral symmetry on the lattice, we encounter from the start a fundamental problem: since the number of degrees of freedom is finite, a symmetry can not be spontaneously broken. Therefore $\langle \bar{\psi}\psi \rangle$ will tend to zero when $m \to 0$. However, one can hope that there will be two regimes in the dependence of $\langle \bar{\psi}\psi \rangle$ on $m$. In the intermediate mass range, $\langle \bar{\psi}\psi \rangle$ would be essentially independent of the size of the lattice, whereas for small masses, finite-size effects would show up and $\langle \bar{\psi}\psi \rangle$ would drop sharply to zero. If this is the case, then we can obtain an estimate of the infinite volume value of $\langle \bar{\psi}\psi \rangle$ by extrapolating from the intermediate mass range. Although this technique usually provides useful information, it may not be powerful enough to really identify a phase transition. However, further checks are available to confirm the presence or the absence of a phase transition. To be sure that chiral symmetry is really broken in the continuum, one has to show that $\langle \bar{\psi}\psi \rangle$ obeys the required scaling behaviour with respect to $\beta$. In 3 dimensions $\beta^2 \langle \bar{\psi}\psi \rangle$ (= continuum value of $\langle \bar{\psi}\psi \rangle$ in units of $e^4$) has to be constant. In practice, one is looking for a range of $\beta$ where there is a plateau in $\beta^2 \langle \bar{\psi}\psi \rangle$. Note however that finite size effects can significantly delay the appearance of such a plateau. In Fig. 1 for example it was shown that chiral symmetry is indeed broken in quenched QED3 but lattices of size ranging up to $(80)^3$ were necessary for that purpose. In the case where the extrapolation method seems to indicate the presence of a phase transition at finite $\beta$, we can assess its existence by using techniques borrowed from the “theory” of critical phenomena. At the critical value of the coupling, for example, it is expected that $\langle \bar{\psi}\psi \rangle$ will behave like $m^{1/\delta}$ (where $\delta$ is one of the critical exponents characterizing the transition).

Independently of these techniques based on measurements of the chiral condensate, one can also gain information about the phase diagram by considering the meson spectrum. One has the following alternative: Either there are parity doublets in the limit of massless fermions (and the $\pi$ and the $\sigma$ would be degenerate for example) or this is not the case and the mass of the pion satisfies the usual PCAC relation. Consequently for $m=0$, the ratio $M_\pi/M_\sigma$ should be a step function centered at $\beta = \beta_c$. If $m$ is not exactly zero then the function will be smoothed. This has recently been given a quantitative form by applying the hypothesis of correlation length scaling $\xi$.

3. The chiral condensate at $T=0$

We have extended the results of $\bar{\psi}\psi$ by measuring the chiral condensate on a $(12)^3$ lattice. Put together, these results indicate that finite size effects are strong, which makes the application of the techniques mentioned above rather difficult. But, if we try anyway, the data presented in fig. 1 and 2 seem to indicate the presence of a phase transition around $\beta = 0.2$ for $N_f = 4$ and around $\beta = 0.26$ for $N_f = 3$ (although the later is less compelling because several curves are compatible with linear behaviour). The critical exponents would be respectively $\delta = 3.7$ and 2.7 but the uncertainties are large. Clearly these results require some further confirmation either from measurements on bigger lattices or from some independent determination of $\delta$.

4. Spectrum computations

We used Kogut-Susskind fermions and computed the correlators between mesonic operators which are local in the lattice variables. Because of lack of space, only the results for the pion will be presented here. We have accumulated data for $N_f = 0$, 2 and 4 at values of the coupling ranging from $\beta = 0.2$ to $\beta = 0.4$ and for masses $m=0.0125$, 0.025 and 0.05. In the quenched case, chiral symmetry is clearly broken: we observe the behaviour expected from PCAC over a wide range of values of $\beta$ (Fig.3) and the finite-size effects can be investigated systematically. For $N_f = 2$ (Fig.4), at $\beta = 0.2$, we observe on a $(12)^2 \times 24$ lattice that chiral symmetry is broken. At $\beta = 0.4$,
strong finite-size effects are present but a comparison of our \((12)^2 \times 24\) and \((14)^2 \times 24\) results suggest that chiral symmetry will still be broken in the infinite volume limit. For \(N_f = 4\) (Fig.5), chiral symmetry is broken at \(\beta = 0.2\) but probably not at \(\beta = 0.4\) although finite size effects remain important even on a \((16)^2 \times 24\) lattice. If the above results are correct, they would confirm the analytical expectations based on a \(1/N_f\) expansions, namely that the critical value of \(\beta\) beyond which chiral symmetry is restored is around 3. At this point, it should be mentioned that in their lattice investigation of compact and non-compact QED3, Azcoiti and Luo [8] arrive at a rather different picture of the phase diagram than the one suggested here.

5. The chiral condensate at \(T \neq 0\)

Our measurements of \(\langle \bar{\psi} \psi \rangle\) at \(N_f = 2\) on \((12)^2 \times 4\) and \((8)^2 \times 4\) lattices are shown on Fig.6. Using the extrapolation method mentioned previously, we would conclude that chiral symmetry is broken at \(\beta = 0.2\) (Note that the extrapolated value of \(\langle \bar{\psi} \psi \rangle\) increases with the spatial size of the lattice) whereas it is not broken for \(\beta \geq 0.3\). However, one has to be very careful at this point. Indeed, if this is really indicative of a transition in the continuum, then we should see again the transition on lattices with \(N_s = 8\) and it should happen between \(\beta = 0.4\) and \(\beta = 0.6\). But nothing like this is observed on Fig.7, instead all the curves extrapolate smoothly to zero. In order to further test for the presence of a transition, we can plot \(\ln \langle \bar{\psi} \psi \rangle\) versus \(\ln m\) (Fig.8) and see if there is a curve which exhibits critical behaviour. Only \(\beta = 0.3\) would be compatible with a straight line provided that we eliminate the point at \(m = 0.00625\). The presence of this point in fact seems to indicate that \(\beta = 0.3\) is an upper bound for the value of the critical temperature. However, one should consider the possibility that this point is sensitive to finite-size effects (although for \(\beta \geq 0.4\) further runs on \((24)^2 \times 8\) lattices indicated that finite-size effects are small) and it is tempting to speculate that the actual transition is not far from \(\beta = 0.3\). If this were the case, the ratio of the critical temperature to the square of the electric charge would be close to \(T/e^2 = 0.0375\), almost an order of magnitude bigger than the result of Dorey and Mavromatos (0.0043)! Further runs at lower values of \(\beta\) are needed in order to resolve the ambiguity. Ideally one would like to obtain a curve with positive curvature on Fig.8 since it would give us a lower bound on the value of the critical temperature. Finally, measurements of screening length (analogous to meson spectrum calculations at zero temperature) could also provide useful information for locating the position of the phase transition. We hope to be able to report on this in the near future.

6. Conclusion

Our results seem to give some support to the following two statements:

1. At zero temperature, chiral symmetry is restored beyond some critical value \((N_c)\) of the number of flavors and \(N_c\) is close to 3.

2. For \(N_f = 2\), the temperature at which chiral symmetry is restored is much higher than the current analytical prediction \([\dagger]\).

However, as is clear from the discussion given in the text, further work is still needed in order to completely clarify the situation.

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