Recovering physical parameters from galaxy spectra using MOPED

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\begin{abstract}
We derive physical parameters of galaxies from their observed spectrum, using MOPED, the optimized data compression algorithm of Heavens, Jimenez & Lahav (2000). Here we concentrate on parametrising galaxy properties, and apply the method to the NGC galaxies in Kennicutt’s spectral atlas. We focus on deriving the star formation history, metallicity and dust content of galaxies. The method is very fast, taking a few seconds of CPU time to estimate \( \sim 17 \) parameters, and so specially suited to study of large data sets, such as the Anglo-Australian 2 degree field galaxy survey and the Sloan Digital Sky Survey. Without the power of MOPED, the recovery of star formation histories in these surveys would be impractical. In the Kennicutt atlas, we find that for the spheroidals a small recent burst of star formation is required to provide the best fit to the spectrum. There is clearly a need for theoretical stellar atmosphere models with spectral resolution better than 1Å if we are to extract all the rich information that large redshift surveys contain in their galaxy spectra.
\end{abstract}

1 INTRODUCTION

Most of the information about the physical properties of galaxies comes from their electromagnetic spectrum. It is therefore of paramount importance to be able to extract as much physical information as possible from it. In principle, it is straightforward to determine physical parameters from an individual galaxy spectrum. The method consists of building synthetic stellar population models which cover a large enough range in the parameter space and then use a merit function (typically \( \chi^2 \)) to evaluate which suit of parameters fits the observed spectrum best.

There are two obvious limitations of the above method: first, the number of parameters that govern the spectrum of a galaxy may be very large and thus difficult to explore fully; secondly, in the case of ongoing large redshifts surveys which will provide us with about a million galaxy spectra, it will be computationally very expensive (and possibly intractable for redshift surveys like the 2dF and SDSS) to apply a brute-force \( \chi^2 \) analysis to each individual spectrum which itself may contain of the order of \( 10^3 \) data points.

A less obvious route to tackle the high computational requirement is to compress the original data set, giving more weight to those pixels in the spectrum that carry most information about a given parameter. In this paper we show how this can be done in an optimal way. It is worth remembering that data compression is commonly applied to galaxy spectra, either by the instrument, through the use of photometric filters, or in the interpretation, by concentrating on specific spectral features and ignoring others. Not surprisingly, this empirical data compression is not optimal since it is ad hoc. For example, the photometric \( B \) filter alone is not optimal to recover the age of a galaxy. On the other hand, more sophisticated and non-empirical methods have been proposed for extracting information from galaxy spectra, some of them as old as the Johnson’s filter system. Many of these are based on Principal Component Analysis (PCA) or wavelet decomposition (Murtagh & Heck 1987; Francis et al. 1992; Connolly et al. 1995; Folkes, Lahav & Maddox 1996; Galaz & de Lapparent 1998; Bromley et al. 1998; Glazebrook, Offer & Deelen 1998; Singh, Gulati & Gupta 1998; Connolly & Szalay 1999; Ronen, Aragon-Salamanca & Lahav 1999; Folkes et al. 1999). PCA projects galaxy spectra onto a small number of orthogonal components. The weighting of each component corresponds to its relative importance in the spectrum. However while these components appear to correlate reasonably well with physical properties of galaxies, their interpretation is difficult since they do not have known, specific physical properties – they can be amalgams of different properties. To interpret these components, we have to return to model spectra and compare them with the components (Ronen, Aragon-Salamanca & Lahav 1999).

This is a disadvantage of PCA since one important goal of the analysis is to study the evolution of the physical properties which dramatically affect galaxy spectra, such as the age, metallicity, star formation history or dust content. It is important to recognise that PCA can play an important role if there is no underlying model for how the data should behave. If such a model exists, then one can do better by using projections of the data which are designed to give the parameters of the model as accurately as possible.

An optimal parameter-extraction method, which we term MOPED\textsuperscript{[\textcopyright]} (Multiple Optimised Parameter Estimation and Data compression) was developed in Heavens, Jimenez & Lahav (2000). The purpose of this paper is to apply the MOPED method to a specific set of observations, to estimate their physical parameters and thus demonstrate the usefulness of the approach. We also want to demonstrate with a specific example the massive speed up factor that the method provides, making computationally-intensive problems into much more accessible ones. The outline of the paper is

\begin{itemize}
\item The MOPED algorithm has a patent pending
\end{itemize}
2 THE METHOD

The MOPED algorithm was presented in detail in Heavens, Jimenez & Lahav (2000). Here we briefly recall the main ingredients and steps of the method.

The main idea of the method is that, in practice, some of the data may tell us very little about the parameters we are trying to estimate, either through being very noisy, or through having no sensitivity to the parameters. So in principle, we may be able to throw away some data without losing much information about the parameters. It is obvious that simply throwing away some of the data is not in general optimal; it will usually lose information. On the other hand, by constructing linear combinations of the data we might do better and then we can throw away the linear combinations which tell us least. In fact one can do much better than this. Providing the noise has certain properties, one can reduce the size of the dataset down to a handful of linear combinations – one for each parameter – which contain as much information as possible about the parameters (star formation rates, metallicity etc.). These numbers are then used as the data set in a likelihood analysis.

In Heavens, Jimenez & Lahav (2000) an optimal and lossless method was found to calculate for multiple parameters (as is the case with galaxy spectra). The definition of lossless here is that the Fisher matrix at the maximum likelihood point (see Tegmark, Taylor & Heavens 1997) is the same whether we use the full dataset or the compressed version. The Fisher matrix gives a good estimate of the errors on the parameters, provided the likelihood surface is well described by a multivariate Gaussian near the peak. We find that the method is lossless provided that the noise is independent of the parameters. This is not exactly true for galaxy spectra, owing to the presence of a shot noise component from the source photons. However, the increase in parameter errors is very small in this case (see Heavens, Jimenez & Lahav 2000). The weights required are

\begin{equation}
\mathbf{b}_m = \frac{C^{-1} \mu_m}{\sqrt{\mu_m^T C^{-1} \mu_m}}
\end{equation}

and

\begin{equation}
\mathbf{b}_m = \frac{C^{-1} \mu_m - \sum_{q=1}^{m-1} (\mu_m^T \mathbf{b}_q) \mathbf{b}_q}{\sqrt{\mu_m^T C^{-1} \mu_m - \sum_{q=1}^{m-1} (\mu_m^T \mathbf{b}_q)^2}}
\end{equation}

where a comma denotes the partial derivative with respect to the parameter \( m \) and \( C \) is the covariance matrix with components \( C_{ij} \equiv \langle n_i n_j \rangle \). \( m \) runs from 1 to the number of parameters \( M \), and \( i \) and \( j \) from 1 to the size of the dataset (the number of flux measurements in a spectrum). To compute the weight vectors requires an initial guess of the parameters. We term this the fiducial model.

The dataset \( \{y_m\} \) is orthonormal: i.e. the \( y_m \) are uncorrelated, and of unit variance. The new likelihood is easy to compute (the \( y_m \) have mean \( \langle y_m \rangle = \mathbf{b}_m^T \mu \)), namely:

\begin{equation}
\ln \mathcal{L}(\theta) = \text{constant} - \sum_{m=1}^{M} \frac{(y_m - \langle y_m \rangle)^2}{2}.
\end{equation}

Further details are given in Heavens, Jimenez & Lahav (2000).

It is important to note that if the covariance matrix is known for a large dataset (e.g. a large galaxy redshift survey) or it does not change significantly from spectrum to spectrum, then the \( \langle y_m \rangle \) need be computed only once for the whole dataset, thus with massive speed up factors in computing the likelihood as will be shown in sections 3 and 4. Note that the \( y_m \) are only orthonormal if the fiducial model coincides with the correct one. In practice one finds that the recovered parameters are almost completely independent of the choice of fiducial model, but one can iterate if desired to improve the solution.

2.1 Estimating errors in the recovered parameters

If we have \( M \) parameters, we can get the marginal error on each of the parameters from the Fisher matrix

\begin{equation}
F_{ij} \equiv -\left( \frac{\partial^2 \ln \mathcal{L}}{\partial \theta_i \partial \theta_j} \right)
\end{equation}

where \( i \) and \( j \) run from 1 to \( M \) (Note the size of the dataset appears nowhere here - it is only relevant in computing \( \ln \mathcal{L} \)). The averaging is over many realisations.

The conditional error on \( \theta_i \) (fixing all other parameters) is just \( (F_{ii})^{-1/2} \) (no summation). This is not too relevant as we wish to estimate all the parameters simultaneously. More relevant then is the marginal error, which is

\begin{equation}
\sigma_i = \sqrt{F_{ii}^{-1}}.
\end{equation}

So the procedure to follow is to estimate \( F_{ij} \) from the likelihood surface near the peak, invert the \( M \times M \) Fisher matrix and use the diagonal components of the inverse matrix to assign marginal errors. All this assumes that the likelihood surface is well-approximated by a multivariate Gaussian, not just at the peak, but also far enough down the likelihood hill (so \( \ln \mathcal{L} \) drops by around unity).

Fisher matrix estimation

Assuming that the maximisation finds the maximum precisely, at position \( \theta_0 \) (an \( M \)-dimensional vector in parameter space), with value \( \ln \mathcal{L}_0 \), then a Taylor expansion of \( \ln \mathcal{L} \) around the maximum gives

\begin{equation}
\ln \mathcal{L}(\theta_0 + \Delta \theta) \approx \ln \mathcal{L}_0 + \frac{1}{2} \Delta \theta^T \frac{\partial^2 \ln \mathcal{L}}{\partial \theta_i \partial \theta_j} \Delta \theta
\end{equation}

where the summation convention is assumed. It will not apply for the rest of this section.

We estimate the diagonal components of the second derivatives by keeping all components of \( \theta \) constant apart from a single \( \theta_i \).

\begin{equation}
F_{ii} \approx -\frac{1}{\Delta \theta_i^2} \left[ \ln \mathcal{L}(\theta_0 + \Delta \theta_i e_i) + \ln \mathcal{L}(\theta_0 - \Delta \theta_i e_i) - 2 \ln \mathcal{L}_0 \right].
\end{equation}
The above procedure is not computationally expensive - for are estimated from This requires about 600 likelihood evaluations. The marginal errors we need 17 parameters, it takes about 1.5 hours of CPU time on an Athlon 1Ghz machine to find the minimum of the likelihood surface. With want to determine 17 parameters, it takes about 1.5 hours of CPU time on an Athlon 1Ghz machine to find the minimum of the likelihood surface. With for the SF in each bin are also given, showing good agreement.

Similarly, the off-diagonal terms are estimated from

\[
F_{ij} \simeq \frac{-1}{2\Delta \theta_i \Delta \theta_j} \left[ \ln L(\theta_0 + \Delta \theta_i, \theta_j) + \ln L(\theta_0 - \Delta \theta_i, \theta_j) - \ln L(\theta_0 + \Delta \theta_i, \theta_j) - \ln L(\theta_0 - \Delta \theta_i, \theta_j) \right].
\]

(8)

The above procedure is not computationally expensive - for \( M = 17 \) we need \( 17 \times 16/2 \) off-diagonal terms, and 17 diagonal terms. This requires about 600 likelihood evaluations. The marginal errors are estimated from \( \sigma_i \simeq 1/\sqrt{(F^{-1})_{ii}} \).

3 Determining Physical Parameters from Galaxy Spectra

3.1 The Problem

Our aim is to determine the star formation history, metallicity and dust evolution of the stellar population of a galaxy from its spectrum. We wish to do this for galaxy spectra that typically contain thousands of data points. Additionally, current large surveys contain on the order of \( 10^6 \) galaxies. It is computationally very expensive to compute a “brute-force” likelihood analysis with all the data points. For example, for a spectrum with \( 10^3 \) points for which we want to determine 17 parameters, it takes about 1.5 hours of CPU time on an Athlon 1Ghz machine to find the minimum of the likelihood surface. With \( 10^6 \) spectra, it would take about 170 years, and if the theoretical models change, it would take as long again. Clearly, the brute force approach is too slow by a factor of several hundred. As we show below, we can estimate 17 parameters of a galaxy with MOPED in about 20 seconds.

3.2 Choosing the Optimal Parametrisation

Which parametrisation should we choose in order to determine the star formation history (SFH) of a galaxy? The common procedure in the literature is to assume that the SFH proceeded as a decaying exponential law with one parameter, while more sophisticated modifications allow for the presence of a burst (or two) at a given time. Ideally one would prefer to avoid any assumptions about the form of the SFH. In fact, star formation in galaxies takes place in giant molecular clouds which are short lived (about \( 10^7 \) years) due to the explosion of SN. It is therefore clear that we could parametrise the SFH in a rather model–independent way by dividing time into bins of width smaller than \( 10^7 \) years using the height of each bin as the parameter. Of course, for galaxies as old as \( 10^{10} \) years this approach becomes useless, since one can hardly deal with over a thousand parameters. We therefore choose a different approach. We divide time into a fixed number of wider bins, and choose their positions and widths according to the following scheme. We consider bursts of star formation at the beginning and end of each time bin (at a fixed metallicity), and require that the maximum fractional difference in the final spectrum is the same for each bin. The maximum difference depends on how many bins we wish to consider.

Figure 1. (Solid line) Simulated spectrum with noise (\( S/N = 10 \)). This spectrum was created with 5 time bins, each of height indicated in the plot. The metallicity in each bin was kept constant at the solar value. The dashed line depicts the recovered spectrum using MOPED. The recovered values for the SF in each bin are also given, showing good agreement.

Figure 2. Same as Figure 1 but for a different SFH, in this case the agreement is also reasonable.

Figure 3. Same as Figure 1 but for a different SFH and arbitrary metallicity in the bins. With varying metallicity, higher signal-to-noise is required to obtain unambiguous correct recovery with this short, low-resolution spectrum.
6.33 and 14 Gyr (considering fixed solar metallicity). Further exploration with different metallicities showed that the bin boundaries differ by a small amount. In fact the bin boundaries are very close to equally spaced bins in logarithmic space – which for the above case are: 0.02, 0.05, 0.13, 0.33, 0.84, 2.15, 5.49 and 14.0 Gyr. For convenience, we choose equally-spaced logarithmic bin boundaries in what follows. Therefore, we are faced now with 8 parameters to determine the SFH. Note that we have not made any specific assumption about the actual shape of the SFH. Within this framework metallicity is extremely easy to parametrise since to each bin we simply assign an extra parameter which is the metallicity of the stars formed in that bin.

Another parameter to consider is the dust content of the galaxy. The importance of dust is being recognised (e.g. Jimenez et al. 2000), but modelling is difficult and currently much less sophisticated than the stellar population modelling. Since the purpose of this paper is only to illustrate the usefulness of the method, we will not develop very sophisticated models for the process of dust emission and absorption. We use the Calzetti (1997) parametrisation as sufficient to describe the major effect of dust absorption on the integrated light of galaxies. The Calzetti model depends only on one parameter: the amount of dust in the galaxy. We will therefore use this simple parameter to describe the global effect of dust. Note that as more realistic dust parametrisations are proposed (e.g. Jimenez et al. 2000; Charlot & Fall 2000), the search parameter space increases and data compression methods become increasingly necessary.

Emission lines are not naturally predicted in the spectrophotometric models. They could be included at the expense of a larger parameter space to explore. An alternative approach, which we follow here, is not to include the emission lines in the analysis at all. Thus we will not expect to fit the emission lines, but only the continuum and absorption lines. Similarly, we could include one (or more) parameters characterising the velocity dispersion of the galaxy. With the coarse spectral resolution of the models, we have chosen not to do this here.

3.3 Recovery of parameters of simulated spectra

In what follows, and for convenience, we will use the set of synthetic stellar population models developed in Jimenez et al. (1998). We emphasize that the choice of models is not crucial at any point in our argument. Furthermore, any suit of models can be used in principle. It is worth noting that given the low spectral resolution of the models (20 Å) we are rather limited in our capacity to extract information from the spectrum. Also, and since we are aiming (in this particular paper) to extract parameters from the Kennicutt (1992) atlas, we will limit the wavelength coverage between 3800 and 7000 Å. Note that this is very restrictive since with a larger spectral coverage and, more importantly, better spectral resolution of the models, we would be able to extract a larger number of parameters with a smaller error (see section 5).

We are now in a position to test our method. We can do this by building synthetic models with known star formation histories, metallicities and dust and then try the parameter recovery. Fig. 1 shows the model spectrum (solid black line) which has been constructed out of five bins with values: 0.4, 0.1, 0.0, 0.0 and 0.5 with fixed solar metallicity (no dust) in each bin and to which artificial Poisson noise has been added ($S/N = 10$). The dashed line shows the best recovered model with bin values corresponding to: 0.41, 0.10, 0.0, 0.0 and 0.49. The agreement is remarkable. Fig. 2 shows another example for constant metallicity, again the efficiency of the method is excellent.

Encouraged by the above results we now consider a more general case where we allow the metallicity of each bin to take an arbitrary value. We also include dust as a free parameter. The first synthetic test case has got artificial Poisson noise added with $S/N = 10$. In this case the recovery of all 17 parameters is not optimal. The reason for this is that with such a low $S/N$ most of the information that is contained in the absorption features has disappeared: we are left only with a continuum. Given the fact of our limited wavelength range in this example (3800 to 7000 Å), it is almost impossible to break the “famous” age-metallicity degeneracy with only the continuum shape. This is in stark contrast with the first test case where the metallicity and dust where kept fixed, then it was possible with only the continuum to derive the heights in the bins. Therefore, we now increase the $S/N$ until we are able to recover the 17 parameters. This happens for $S/N = 20$ and is illustrated in Fig. 3. It can be seen that the parameter recovery is very good.

Given the success of the above tests we decided to apply the method to a real sample, but before doing it we turn our attention to the physical interpretation of the eigenvectors $b$.

3.4 Eigenvectors or where is the information?

We now concentrate our attention on the physical meaning of the eigenvectors computed from eq. (1) and (2). As shown in Heavens, Jimenez & Lahav (2000) the choice of the fiducial model is not important. One can always iterate, but this appears to be quite unnecessary. Figures 4 and 5 show the SFH and metallicity eigenvectors respectively as a function of wavelength in the range 3600 to 7000 Å for a model in which we have parametrised the SFH by using 8 bins. Figure 4 concerns the SFH eigenvectors for bins corresponding to: 0.02, 0.05, 0.13, 0.33, 0.84, 2.15, 5.49 and 14.0 Gyr (from top to bottom), while the 8 panels on Figure 5 are the corresponding metallicity eigenvectors. Not surprisingly, for young ages, the age eigenvector likes to weight most those pixels bluewards of 4500 Å, while those redwards of 4500 Å get very little weight (note that the eigenvector can always be multiplied by $-1$). As the population ages most of the information about the SFH becomes more distributed among wavelength, thus making it very difficult to design a narrow band filter which would capture most of the information. A similar situation occurs for the metallicity bins (see Figure 5) for young ages most of the information is concentrated on the bins bluewards of 5000 Å while for older ages the information becomes more equally distributed among different absorption features. Note that for recovering metallicity information the absorption features are more relevant than the continuum.

4 PHYSICAL PARAMETERS FOR THE SPECTRA IN THE KENNICUTT ATLAS

In this section we apply the method to the Kennicutt atlas (Kennicutt 1992). Although the Kennicutt sample is certainly small, about 50 galaxies, the $S/N$ of the spectra is very high. Furthermore, the largest redshift survey available at the moment, the 2dF, suffers from serious complications in calibrating photometrically the continuum of the spectra (Madwick, Lahav & Taylor 2000), thus, for the time being, we concentrate in illustrating the method on the small but significant atlas of observed spectra provided by Kennicutt (1992). We have re-binned the data to the same spectral res-
Recovering physical parameters from galaxy spectra using MOPED

Figure 4. From the top: eigenvectors $b_1$ to $b_8$ for the SFH. The fiducial model is the same for all the galaxies. It corresponds to equal star formation in each bin, solar metallicity and dust parameter=0.05.

Figure 5. Eigenvectors $b_9$ to $b_{16}$ for the metallicity.

olution as the models, with the new flux calculated as the mean flux unit wavelength in the bin. It is worth keeping in mind that the spectrophotometric calibration of the Kennicutt (1992) atlas has an error of 10% and that our models do not include emission lines in the spectra. It should be stressed that despite the excellent fits obtained to the Kennicutt (1992) atlas, one should not over-interpret the results since the 10% uncertainty in the spectrophotometric calibration affects the parameter determination. As stated above in the paper the physical parameters we are aiming at deriving are: the amount of star formation in the time bins chosen – here we chose 8, the metallicity in each bin and the global dust content, using the Calzetti (1997) formula for the dust extinction. This gives a total of 17 parameters for each galaxy. This is the most effective way to parametrise the SF since it does not depend on any previous knowledge about its shape, i.e. we are not assuming that it is a declining exponential or similar. Note that if we use the full dataset instead of the compressed data, we get the same results (although it takes much longer (see section 5)).

Results are presented in the next 12 figures, where the best fit to the spectrum is drawn in the left panel and the corresponding SFH is plotted with horizontal error bars denoting the width of the bins and vertical error bars for the marginal uncertainty in
the height of the bin. The derived metallicity for each bin is also labeled in the right panel as well as the dust with corresponding errors.

The first result is somewhat surprising. We find that all galaxies classified as E/S0 (NGC3245, NGC3379, NGC3516, NGC4472, NGC4648, NGC4889, NGC5866, NGC6052) show a reasonable level of recent star formation activity although in all cases more than 50% of the stellar mass is older than 5 Gyr. NGC3245 (S0) shows recent star formation below 7% of the total mass as does NGC6052 (S0). NGC3379, NGC3516, NGC4472, NGC4648, NGC4889 and NGC5866 all show a significant (> 30% of the total mass) amount of star formation in the last 3 Gyr (Note NGC3516 has a Seyfert nucleus). Note also that the metallicities derived are very reasonable since the final mass-weighted metallicity for the above galaxies is about 1.5 times the solar value.

Note that these findings are in excellent agreement with the recent findings about the nature of spheroids by e.g. Dunlop et al. (1996); Spinrad et al. (1997); Menanteau, Abrahm & Ellis (2000); van Dokkum et al. (2000). Also significant is that the method is able to recognize current star formation, despite the fact of not having any emission lines included in the model. For example, for galaxies with slight Hα (like NGC1357, NGC3147, NGC3227, NGC3921 and NGC4750) the method always shows a significant level of recent star formation in the most recent bins, which agrees with the star formation rate that would be estimated using the Hα line itself (NB NGC3227 is complicated by the presence of a Seyfert component). As expected, galaxies classified as Sa to Sc show more recent bursts of star formation, but note that star formation activity does not correlate with the Hubble type. In other words, star formation history in galaxies does not proceed as a single exponential decaying law that changes according to the Hubble type in a monotonic manner but it is much more like a sequence of burst events. We now turn our attention to the metallicity evolution of the stars. The general trend is that high-metallicity stars are formed at an early epoch, the typical values being over the solar value. On the other hand the youngest bursts tend to have slightly sub-solar metallicities (about half the solar value). Note that this trend is in good agreement with predictions from infall models (e.g. Pagel (1997)). These models are motivated by the fact that if all the gas was available for consumption into stars from the very beginning (closed-box model), then a large number of low metallicity stars should be observed now. What is observed is that the metallicities of stars in galaxies (both disks and ellipticals) are gaussianly distributed. An obvious solution to this problem is not to allow all the gas to be accreted into the galaxy at the beginning but at a slower pace. As a result of this the late forming generations of stars will have a reservoir of fresh low metal gas which will decrease the metallicity that would be predicted by a closed-box model, which increases monotonically with time. Inspection of the figures in the appendix shows that this is not the case, but that late forming generations have a lower metallicity. Note also that where the star formation is effectively zero, the metallicity is not recovered accurately, as one might expect. We note also that dust values seem to be very reasonable.

5 DISCUSSION AND CONCLUSIONS

We have presented a fast and efficient algorithm (MOPED) to recover physical parameters from galaxy spectra. The algorithm is based on a data representation technique which discards most of the data set that does not contain any information with respect to a given parameter. By doing so we are able to speed up the parameter determination by a large factor: typically of the order of \( N/M = \) number of data points/number of parameters. We note here that if the noise matrix is non-diagonal, the speed-up is much larger \( \sim N^2/M \) (see Gupta & Heavens, in preparation). We have also applied the method to the Kennicutt (1992) atlas and derive star formation and metallicity histories for the NGC galaxies in the survey. We have found that most spheroids, albeit having at least 50% stars formed at ages older than 5 Gyr, have had recent star formation episodes. The quality of the spectral models does not allow us yet to determine if those are the consequence of recent mergers.

The method presented in section 2.1 is good at estimating the local errors around the maximum. Unfortunately, the thing that usually happens is that the error estimation is mostly dominated by local maxima that are in distant regions of the parameter space, this is our case. In order to explore this we record all the maxima that the method finds starting from different initial random guesses (in this case 5000).† We then explore if some of these solutions are also allowed in the \( \chi^2 \) sense. We have done this for all the spectra in the Kennicutt (1992) atlas. Our findings are as follows: for systems with only absorption features the number of degenerate models is almost negligible, thus the best fitting model (which is plotted in the appendix) corresponds quite closely to an absolute maximum. On the other hand, for those spectra with strong emission lines degeneracies are more severe, to such extent that for NGC1569, NGC3310, NGC3690, NGC4194, NGC4449, NGC4485, NGC4631, NGC4670, NGC4775, NGC5996, NGC6052 and NGC6240 other acceptable fits that differ significantly from the best fit presented in the appendix exist. For example, an acceptable fit is also one where most of the star formation takes place in the youngest bins and the metallicity is about solar. This of course would be cured if we were able to have more detailed absorption features in the spectrum and especially more resolution, since this would constrain some of the models that now appear as plausible. Indeed, a larger wavelength coverage would also help. In any case, it is worth remembering that this degeneracy is generic of the present problem we are dealing with, and that using MOPED only facilitates the search in a large parameter space for such degeneracies and to understand the physics behind the problem.

The speed up factors obtained in the present analysis are quite remarkable. On a 1GHz Athlon machine it takes much less than a second for an initial starting guess to find a (possibly local) maximum of the likelihood. For 400 initial guesses it takes about 20 seconds of CPU time to find the global maximum. If one used the whole data set the same calculation would take about an hour and a half. Extrapolating these numbers one can see that for analysing a big dataset like the 2DF or SDSS, the time required using all of the data would be about 170 years, longer if higher-resolution models are used. This is obviously too long to be achieved, especially if one takes into account that the theoretical model may change,

† due to the fact that the Kennicutt (1992) sample is nearby, the slit of the spectrograph can only sample the inner regions of the galaxy, where the metallicity is higher.

‡ The space parameter is so large that one cannot compute the whole likelihood surface, instead we used a conjugate gradient method to find the maximum of the likelihood. In order to test that we had found the absolute maximum we repeated the process from new random initial guesses. We found that 400 of these guesses suffice to find the global maximum.
necessitating a full repeat of the analysis. With MOPED, it would take a few months on a single workstation, or much less by exploiting the trivially parallel nature of the problem. Finally, the rapid algorithm here has allowed us to explore more fully the likelihood surface, and we find that for galaxies with significant very recent star formation there is often more than one acceptable parameter fit. There is clearly a need for theoretical stellar atmospheric models with spectral resolution better than 1 Å if we are to extract all the rich information that large redshift surveys contain in their galaxy spectra.

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APPENDIX I

The following 12 figures show the best fitting model (thick solid line) for each spectrum (thin solid line) in the Kennicutt (1992) atlas (left panel) and the plot of the star formation history for the 8 age bins (right panel). The SFH is plotted with horizontal error bars denoting the width of the bins and vertical error bars for the marginal uncertainty in the height of the bins. Also shown for each model, is the value of the metallicity derived for each bin in solar units (left to right correspond to youngest to oldest bin) and the overall amount of extinction according to the Calzetti (1997) model. For those bins with zero star formation the metallicity is irrelevant and set to zero (and zero error). Note that there are five galaxies with Seyfert nuclei: NGC3227, 3516, 5548, 6764, and 7469.
| Galaxy       | Z/\(Z_\odot\) | Emission Lines | Absorption Lines |
|-------------|---------------|----------------|-----------------|
| NGC1275     | 0.01\pm0.05   | 0.46\pm0.10   | 0.0\pm0.0      |
| NGC1357     | 0.76\pm0.18   | 0.0\pm0.0     | 0.0\pm0.0      |
| NGC1569     | 0.07\pm0.10   | 0.0\pm0.0     | 0.0\pm0.0      |
| NGC1832     | 0.0\pm0.0     | 0.31\pm0.49   | 0.06\pm0.51    |

Figure 6.
Figure 7.
Figure 8.
Figure 9.
Figure 10.
Figure 11.
Figure 12.
Figure 13.
Figure 14.
Figure 15.
Figure 16.
Recovering physical parameters from galaxy spectra using MOPED

Figure 17.

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