From $m_d = m_e$ to Realistic Mass Relations in Quark-Lepton Symmetric Models.

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Abstract

In recent years a new potential symmetry of fundamental particle physics has been investigated — discrete quark-lepton symmetry. When this symmetry is implemented, however, it often leads to either of the unrealistic predictions $m_u = m_e$ or $m_d = m_e$. This paper considers two possible ways models based on $m_d = m_e$ can be made realistic.

I. INTRODUCTION

There are many outstanding questions left unanswered by the standard model (SM) of particle physics. Various approaches have been used to address these problems, but one of the most common is the introduction of new symmetries to the Lagrangian, often gauge symmetries, and usually only unbroken above some threshold energy. Indeed, it is worth searching for possible new symmetries in order to see what simplifications or interesting new

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relations they may provide. One such new symmetry uncovered recently is quark-lepton symmetry \[1,2\]. This is a $Z_2$ discrete symmetry which seeks to put quarks and leptons on an equal footing. In order to do this, a new gauge group, the $SU(3)_l$ group of leptonic colour, needs to be introduced to parallel the standard quark colour group.

The introduction of this new symmetry provides a pleasing simplification of the particle spectrum, but can also entail an unrealistic equality between the masses of quarks and leptons. It is certainly possible to circumvent this unwanted equality while retaining the symmetry which initially lead to it, however a more intriguing possibility is to treat the mass equalities as a correct and promising starting point from which to obtain an explanation for at least some aspects of the observed mass spectrum. In this paper, we shall examine a couple of attempts to achieve just this aim, in the context of a left-right symmetric generalisation of the basic theory.

In Sec.II, the basic details of the quark-lepton-symmetric theory are reviewed, along with the left-right symmetric generalisation. Sec.III looks at the idea of completely broken leptonic colour in the framework of the left-right symmetric version of the theory. This is an extension of work done by one of the authors in a previous paper \[3\]. In Sec.IV a variant on the seesaw concept is implemented in order to obtain a suitable adjustment of the mass relations. Sec.V summarises and concludes the paper.

**II. REVIEW OF QUARK-LEPTON SYMMETRY**

In the basic implementation of quark-lepton symmetry, the SM gauge group is extended to $G_{ql} = SU(3)_l \otimes SU(3)_q \otimes SU(2)_L \otimes U(1)_X$ which is unbroken at energies of order 1 TeV or more. The particle fields are:

\[
\begin{align*}
Q_L & \sim (3,1,2)(\frac{1}{3}), \\
u_R & \sim (3,1,1)(\frac{4}{3}), \\
d_R & \sim (3,1,1)(-\frac{2}{3}), \\
F_L & \sim (1,3,2)(-\frac{1}{3}), \\
E_R & \sim (1,3,1)(-\frac{4}{3}), \\
N_R & \sim (1,3,1)(\frac{2}{3}),
\end{align*}
\]

(1)
and the Higgs fields are:

\[
\phi \sim (1, 1, 2)(1), \\
\chi_1 \sim (3, 1, 1)(\frac{2}{3}), \quad \chi_2 \sim (1, 3, 1)(-\frac{2}{3}).
\] (2)

The first stage of symmetry breaking, resulting from the generation of a vacuum expectation value (vev) \( \langle \chi \rangle \) for \( \chi_1 \), will reduce the gauge group factor \( SU(3)_l \otimes U(1)_X \) to \( SU(2)' \otimes U(1)_Y \), where

\[
Y = X + T_8/3,
\] (3)

where \( T_8 \) is the diagonal generator \( \text{diag}(-2, 1, 1) \) of leptonic colour. At this point the leptonic-colour triplets \( F_L, E_R \) and \( N_R \) will break up into the familiar lepton fields, and into \( SU(2)' \) doublet “liptons”, which will form into \( SU(2)' \)-singlet composite particles (since \( SU(2)' \) should be confining) that will decouple from the theory at energies below \( SU(3)_l \) unification. After this, the theory follows the same pattern as the SM, with \( \phi \) developing a vev and breaking \( SU(2)_L \otimes U(1)_Y \) down to \( U(1)_{em} \).

However, because of the presence of the extra discrete quark-lepton symmetry,

\[
Q_L \leftrightarrow F_L, \quad u_R \leftrightarrow E_R, \quad d_R \leftrightarrow N_R, \quad \phi \leftrightarrow \phi^C, \\
G_\mu \leftrightarrow G^\mu, \quad W^\mu \leftrightarrow W^\mu, \quad C^\mu \leftrightarrow -C^\mu,
\] (4)

the Yukawa mass terms in the Lagrangian will be constrained to be of the form

\[
\mathcal{L}_{\text{Yukawa}} = \lambda_1 (\overline{Q_L} u_R \phi^C + \overline{F_L} E_R \phi) + \lambda_2 (\overline{Q_L} d_R \phi + \overline{F_L} N_R \phi^C) + H.c.,
\] (5)

where \( \phi^C = i\tau_2 \phi \), thus resulting in tree level predictions for the quark and lepton masses:

\[
m_u = m_e, \quad m_d = m_\nu.
\] (6)

The seesaw mechanism [4] can be invoked to cure the second of these relations, by — for example — the introduction of new Higgs fields

\[
\Delta_1 \sim (\bar{6}, 1, 1)(\frac{4}{3}), \quad \Delta_2 \sim (1, \bar{6}, 1)(-\frac{4}{3}).
\] (7)
but the first remains, and is clearly not in good agreement with experiment.

There are two possible variants if left-right symmetry is simultaneously imposed on the theory [2]. In the first variant essentially the same model results, so that no new information relating to the promise, and problem, of the relations of Eq. (3) is obtained. In the second variant, however, the relations are switched, generating the different (if still problematic) relations

\[ m_u = m_\nu, \quad m_d = m_e. \]  

Again, the seesaw mechanism can be invoked to cure one of these relations, but not both.

The gauge group for the left-right symmetric version of the theory is \( G_{qlLR} = SU(3)_q \otimes SU(3)_l \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{X'} \) and the particle fields are:

\[
F_L \sim (3, 1, 2, 1)(-\frac{1}{3}), \quad F_R \sim (3, 1, 1, 2)(-\frac{1}{3}), \\
Q_L \sim (1, 3, 2, 1)(\frac{1}{3}), \quad Q_R \sim (1, 3, 1, 2)(\frac{1}{3}).
\]

The above mass relations, Eq. (8), then follow from the Lagrangian terms

\[
\mathcal{L}_{qlLR} = \lambda_1(\overline{F}_LF_R + \overline{Q}_LQ_R)\Phi + \lambda_2(\overline{F}_LF_R + \overline{Q}_LQ_R)^C\Phi + H.c.,
\]

where \( \Phi \sim (1, 1, 2, 2)(0) \) is the Higgs field. By contrast with \( \phi \) in Eq. (4), the electroweak Higgs \( \Phi \) is taken to transform into itself under the discrete quark-lepton symmetry.

**III. COMPLETELY BROKEN LEPTONIC COLOUR IN THE QLLR MODEL**

Clearly, the relations Eq. (3) and Eq. (8) above must be either circumvented or adjusted in some way. Previous work has looked at both such possibilities. Here, we shall extend the work done in [3] to the case of left-right symmetric leptonic colour. In the original formulation of quark-lepton symmetry, the leptonic colour group was only partially broken down to an \( SU(2)' \) subgroup [along with a \( U(1) \) factor]. In the approach of [3], the leptonic colour group is completely broken, leaving no confining subgroup. Obviously, this implies the existence of free “liptons” of comparable or higher mass to the standard quarks and
leptons. For an appropriate choice of the formula for the hypercharge, however, this apparent failing can instead become quite beneficial. The liptons will then possess the same quantum numbers under the SM group as the regular leptons, so that mass mixing will occur. It is possible to arrange the mass matrices so that they naturally lead to a lepton mass eigenstate suppressed with respect to its ql-symmetric quark partner, along with very large values for the other eigenstates, in a manner not dissimilar to the universal seesaw mechanism [5]. In [3], charged lepton masses had to be suppressed relative to up quark masses. In this paper, we will instead look at suppressing them relative to down quark masses. This may be a more natural approach for the second and third generations, since the $s/\mu$ and $b/\tau$ mass splittings are much smaller than $c/\mu$ and $t/\tau$, respectively.

The fermion fields remain unchanged from those shown in Eq. (9). The Higgs content of the model is

$$\Phi \sim (1, 1, 2, 2)(0),$$
$$\chi_1 \sim (3, 1, 1)(\frac{2}{3}), \quad \chi_2 \sim (1, 3, 1)(-\frac{2}{3}),$$
$$\Delta_{1L} \sim (\bar{6}, 1, 3, 1)(\frac{2}{3}), \quad \Delta_{1R} \sim (\bar{6}, 1, 1, 3)(\frac{2}{3}),$$
$$\Delta_{2L} \sim (1, \bar{6}, 3, 1)(-\frac{2}{3}), \quad \Delta_{2R} \sim (1, \bar{6}, 1, 3)(-\frac{2}{3}).$$

The Yukawa Lagrangian for this model is

$$\mathcal{L} = \lambda_1(Q_LQ_R + F_LF_R)\Phi + \lambda_2(Q_LQ_R + F_LF_R)\Phi^C +$$
$$\lambda_3[(F_L)^C F_L\chi_1 + (Q_L)^C Q_L\chi_2 + (F_R)^C F_R\chi_1 + (Q_R)^C Q_R\chi_2] +$$
$$\lambda_4[(F_L)^C F_L\Delta_{1L} + (F_R)^C F_R\Delta_{1R} + (Q_L)^C Q_L\Delta_{2L} + (Q_R)^C Q_R\Delta_{2R}] + H.c. \quad (12)$$

The full symmetry of the model is then broken down via various vevs. These vevs are

$$\langle \Phi \rangle = \begin{pmatrix} 0 & f \\ f & 0 \end{pmatrix}, \quad (13)$$
$$\langle \chi_1 \rangle = \begin{pmatrix} \omega \\ 0 \\ 0 \end{pmatrix}, \quad (14)$$

5
and

\[
\langle \Delta_{1R} \rangle = \begin{pmatrix}
v_1 & 0 & v_2 \\
0 & 0 & 0 \\
v_2 & 0 & v_3 \\
0 & v_4 & 0 \\
v_4 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & v_5 & 0 \\
0 & 0 & 0 \\
\end{pmatrix}, \ t_{R3} = -1
\]

\[
\langle \chi_1 \rangle = \begin{pmatrix}
\end{pmatrix}, \ t_{R3} = 0
\]

\[
\langle \Delta_{1R} \rangle = \begin{pmatrix}
\end{pmatrix}, \ t_{R3} = +1,
\]

(15)

where \( t_{R3} \) is the diagonal generator of \( SU(2)_R \). The vevs \( \langle \chi_1 \rangle \) and \( \langle \Delta_{1R} \rangle \) will break the gauge group down from \( G_{q\ell LR} \) to \( G_{SM} \), breaking the left-right symmetry and completely breaking the leptonic colour group. The hypercharge \( Y \) is chosen to have the form

\[
Y = X + T_8/3 + T_3 + t_{R3}/2,
\]

(16)

where \( T_3 \) is the diagonal generator \( \text{diag}(0,1,-1) \) of \( SU(3)_L \). The vev \( \langle \Phi \rangle \) will then break the SM group down in the usual manner, so that the electromagnetic charge \( Q \) is given by

\[
Q = Y/2 + t_{L3},
\]

(17)

where \( t_{L3} \) is the diagonal generator of \( SU(2)_L \). If we now look at the components of the leptonic fields \( F_L \) and \( F_R \), we find

\[
Y(F_L) = Y \begin{pmatrix}
l_{1L} \\
(l_{2R})^C \\
l_{3L} \\
\end{pmatrix} = \begin{pmatrix}
-1 \\
+1 \\
-1 \\
\end{pmatrix} \sim \begin{pmatrix}
(v_{1L}, e_{1L}) \\
(e_{2R})^C, (v_{2R})^C \\
(v_{3L}, e_{3L}) \\
\end{pmatrix},
\]

(18)

\[
Y(F_R) = Y \begin{pmatrix}
l_{1R} \\
(l_{2L})^C \\
l_{3R} \\
\end{pmatrix} = \begin{pmatrix}
-1 \\
+1 \\
-1 \\
\end{pmatrix} \sim \begin{pmatrix}
(v_{1R}, e_{1R}) \\
(e_{2L})^C, (v_{2L})^C \\
(v_{3R}, e_{3R}) \\
\end{pmatrix}.
\]

(19)
If we now input the vevs $\langle \Phi \rangle$, $\langle \chi_1 \rangle$ and $\langle \Delta_{1R} \rangle$ into the Lagrangian, Eq. (12) above, we get, for the charged lepton sector,

$$(e^1_L, e^2_L, e^3_L) = \begin{pmatrix} m_d & 0 & 0 \\ M_4 & m_u & M_\omega \\ 0 & M_\omega & m_d \end{pmatrix} \begin{pmatrix} e_1^R \\ e_2^R \\ e_3^R \end{pmatrix},$$

where $M_4 = \lambda_4 v_4$ and $M_\omega = \lambda_3 \omega$. In general, each of the above entries is a $3 \times 3$ matrix in generation space. The three generational case is very complicated, but an idea of the effects on the mass eigenvalues can be seen by considering the matrix formed by taking into consideration only one generation. We find then, taking into account the fact that $v_4, \omega \gg m_u, m_d$, this matrix diagonalises, to lowest order, to

$$\text{diag}(M_\omega, \sqrt{M_\omega^2 + M_4^2}, m_e),$$

where

$$m_e \approx m_d \cos \phi \leq m_d,$$

and

$$\cos \phi = \frac{M_\omega}{\sqrt{M_\omega^2 + M_4^2}}.$$

Thus, within each generation, the charged lepton will be left with a suppressed mass with respect to its down quark partner. Furthermore, the size of this suppression is dependant on $M_4$ and $M_\omega$, which may vary from generation to generation, thus allowing for the correct level of suppression within each generation. This will be further complicated by any intergenerational mixing.

Next we turn to the neutrino fields. Here there are three matrices to look at. First, there is the Dirac-mass matrix between left- and right-handed neutrino fields:

$$(\nu^1_L, \nu^2_L, \nu^3_L) = \begin{pmatrix} m_u & 0 & 0 \\ M_4 & m_d & M_\omega \\ 0 & M_\omega & m_u \end{pmatrix} \begin{pmatrix} \nu_1^R \\ \nu_2^R \\ \nu_3^R \end{pmatrix}.$$
This matrix produces similar results to the charged lepton matrix, Eq. (20), generating two heavy neutrinos and one light neutrino, with a mass close to the corresponding up-quark mass, but suppressed by $\cos \phi$. The second neutrino mass matrix couples the right-handed neutrino fields to each other,

\[
\begin{pmatrix}
\nu_1^R, 
\nu_2^R, 
\nu_3^R
\end{pmatrix} = \begin{pmatrix}
\lambda_4 v_1 & 0 & \lambda_4 v_2 \\
0 & 0 & 0 \\
\lambda_4 v_2 & 0 & \lambda_4 v_3
\end{pmatrix} \begin{pmatrix}
\nu_1^R \\
\nu_2^R \\
\nu_3^R
\end{pmatrix},
\]

while the final matrix couples left-hand neutrino fields:

\[
\begin{pmatrix}
\nu_1^L, 
\nu_2^L, 
\nu_3^L
\end{pmatrix} = \begin{pmatrix}
0 & 0 & 0 \\
0 & \lambda_4 v_5 & 0 \\
0 & 0 & 0
\end{pmatrix} \begin{pmatrix}
\nu_1^L \\
\nu_2^L \\
\nu_3^L
\end{pmatrix}.
\]

These last two matrices provide heavy Majorana masses (with the mass scale set by the size of the vevs $v_1, v_2, v_3, v_5$) that will ensure all the physical neutrino mass eigenstates are heavy via the standard seesaw mechanism.

Thus far, we have largely ignored the effects of inter-generational mixing. It may be possible, however, that the very mass splitting we have achieved here between the down-quark and charged-lepton masses might follow from radiative corrections due to such inter-generational mixing. This was discussed by Foot and Lew in ref. [6]. Here, the mass splitting between the tauon and the bottom quark was addressed, with a view to showing that even such a large gap could be bridged by radiative corrections involving the much heavier top quark. The story becomes much more complicated for the lighter generations, but the same principles would apply. This would hopefully resolve the mass disparity satisfactorily without the need to invoke the more complicated symmetry breaking scenario used in this paper or to break the leptonic colour group down completely.

In their paper, Foot and Lew did indeed show that radiative corrections could be of sufficient size to explain the mass difference between the tauon and the bottom quark. The precise size of these corrections, however, is dependant on unknown parameters, and could
vary over a large range of values. The sign of the corrections, also, is not easily fixed, so that it is possible that, far from helping the situation, the radiative corrections exacerbate the problem in some regions of parameter space. By contrast, the method described in this paper, at least for the case of no intergenerational mixing, clearly results in a suppression of the charged lepton masses.

**IV. SEESAW APPROACH**

In this second approach, we look more directly towards a seesaw-like method of suppressing the lepton masses. Again, the left-right-symmetric version of the basic theory is used. The idea is to invoke a seesaw-like mechanism to do for the electron what the standard seesaw idea does for the neutrino. To do this, singlet fermion fields are introduced which will pick up mass contributions from mixing with each other and with the regular leptonic fields, thus generating a mass matrix with the traditional seesaw form. Specifically, we have the fermion content shown in Eq. (9), in three copies - one for each generation - and in addition, one copy of the following new fermion fields:

$$\epsilon_L \sim (1, 1, 1, 1)(-2), \quad \epsilon_R \sim (1, 1, 1, 1)(-2)$$  \hspace{1cm} \text{(27)}$$

The Higgs content for this idea is as follows:

$$\Phi \sim (1, 1, 2, 2)(0),$$

$$\chi_1 \sim (3, 1, 1, 1)(\frac{2}{3}), \quad \chi_2 \sim (1, 3, 1, 1)(-\frac{2}{3}),$$

$$\Delta_{1L} \sim (\bar{6}, 1, 3, 1)(\frac{2}{3}), \quad \Delta_{1R} \sim (\bar{6}, 1, 1, 3)(\frac{2}{3}),$$

$$\Delta_{2L} \sim (1, \bar{6}, 3, 1)(-\frac{2}{3}), \quad \Delta_{2R} \sim (1, \bar{6}, 1, 3)(-\frac{2}{3}),$$

$$\eta_{1L} \sim (\bar{3}, 1, 2, 1)(-\frac{5}{3}), \quad \eta_{1R} \sim (\bar{3}, 1, 1, 2)(-\frac{5}{3}),$$

$$\eta_{2L} \sim (1, \bar{3}, 2, 1)(\frac{5}{3}), \quad \eta_{2R} \sim (1, \bar{3}, 1, 2)(\frac{5}{3}).$$

The Higgs and particle fields will couple according to the Yukawa mass Lagrangian

$$\mathcal{L}_{\text{Yuk}} = \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{liptons}} + \mathcal{L}_{\text{Majorana}} + \mathcal{L}_{\text{seesaw}},$$  \hspace{1cm} \text{(29)}$$
where

\[ \mathcal{L}_\text{Dirac} = \lambda_1 (F_L F_R + Q_L Q_R) \Phi + \lambda_2 (F_L F_R + Q_L Q_R) \Phi^C + \text{H.c.}, \]

\[ \mathcal{L}_\text{liptons} = \lambda_3 [(F_L)^C F_L \chi_1 + (F_R)^C F_R \chi_1 + (Q_L)^C Q_L \chi_2 + (Q_R)^C Q_R \chi_2] + \text{H.c.}, \]

\[ \mathcal{L}_\text{Majorana} = \lambda_4 [(F_L)^C F_L \Delta_{1L} + (F_R)^C F_R \Delta_{1R} + (Q_L)^C Q_L \Delta_{2L} + (Q_R)^C Q_R \Delta_{2R}] + \text{H.c.}, \]

\[ \mathcal{L}_\text{seesaw} = \lambda_5 (F_L \epsilon_R \eta_{1L} + \epsilon_L F_R \eta_{1R} + Q_L (\epsilon_L)^C \eta_{2L} + (\epsilon_R)^C Q_R \eta_{2R}) + M_{\epsilon \epsilon} \epsilon_L \epsilon_R + \text{H.c.}, \]

\[ (33) \]

where \( M_{\epsilon \epsilon} \) is a bare mass term [or alternatively, \( M_{\epsilon \epsilon} \) results from the vev of a Higgs field \( \sigma \sim (1,1,1,1)(0) \)].

The gauge group is then broken down via non-zero vevs \( \langle \Phi \rangle, \langle \chi_1 \rangle, \langle \Delta_{1R} \rangle \) and \( \langle \eta_{1R} \rangle \) according to the hierarchy

\[ SU(3)_l \otimes SU(3)_q \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_X \]

\[ \downarrow \]

\[ \langle \chi_1 \rangle, \langle \Delta_{1R} \rangle, \langle \eta_{1R} \rangle \]

\[ SU(3)_q \otimes SU(2)_L \otimes U(1)_Y \]

\[ \downarrow \]

\[ \langle \Phi \rangle \]

\[ SU(3)_q \otimes U(1)_{em}. \]

From \( \mathcal{L}_\text{Dirac} \) we get the basic quark-lepton mass equalities, Eq. (3). \( \mathcal{L}_\text{liptons} \) will lead to heavy masses for the liptons. \( \mathcal{L}_\text{Majorana} \) will provide the usual neutrino seesaw mechanism.

Finally, \( \mathcal{L}_\text{seesaw} \) will generate mass terms involving the new fermion fields \( \epsilon_L, \epsilon_R \). Combining all the mass terms involving the \( \epsilon \) and charged lepton fields, we obtain a matrix of the form

\[
\begin{pmatrix}
m_d & 0 & 0 & 0 \\
0 & m_s & 0 & 0 \\
0 & 0 & m_b & 0 \\
\lambda_5 \langle \eta_{1R} \rangle & \lambda_5 \langle \eta_{1R} \rangle & \lambda_5 \langle \eta_{1R} \rangle & M_{\epsilon \epsilon}
\end{pmatrix}.
\]

(35)

If we assume \( \lambda_5 \langle \eta_{1R} \rangle \) is of order 200 GeV, and that \( M_{\epsilon \epsilon} \) is of order 5 GeV, then upon diagonalisation, we find three mass eigenvalues suppressed with respect to the input quark masses, and one very heavy eigenvalue.
A numerical analysis was performed to see if realistic levels of suppression could be achieved. It was found that suitable results were easy to obtain, with very accurate values obtainable if one allows for a slight generational hierarchy in $\lambda_5$ (which is a $3 \times 3$ matrix in generation space). For example, for the following input values:

\[
\begin{pmatrix}
0.01 & 0 & 0 & 0 \\
0 & 0.2 & 0 & 0 \\
0 & 0 & 5 & 0 \\
100 & 150 & 500 & 5
\end{pmatrix},
\]

we obtain, upon diagonalisation, the eigenvalues

\[
m_e = 0.5\text{MeV}, \quad m_\mu = 110\text{MeV}, \quad m_\tau = 1.7\text{GeV}, \quad M = 530\text{GeV},
\]

which is clearly a good match with experiment.

V. CONCLUSION

Although it provides a pleasing simplification of the fermion fields, a discrete quark-lepton symmetry can introduce an unrealistic mass relation between the charged leptons and either the up-quark fields Eq. (6) or the down-quark fields Eq. (8) (in the case of a left-right symmetric version of the theory). Rather than simply circumventing this mass relation, in this paper we have treated the relation as a sign post towards a greater understanding of the fermion mass spectrum.

After summarising the basic framework of quark-lepton symmetry, we discussed two ideas by which the mass relations Eq. (8) might lead to explanations for the major trends in the masses of the fermions within each family.

The first of these efforts was an extension of the method used in \[3\], in which the leptonic colour group is completely broken down by the various Higgs field vevs, to consider the case of the left-right symmetric version of the theory. There it was shown that the method would successfully suppress the masses of the charged fermion fields with respect to those of the
down quark fields in the absence of intergenerational mixing. In comparison, the use of radiative corrections to split these masses (investigated in \cite{3}), does not necessarily lift the degeneracy in the right direction.

In the second proposal, the addition of one generation of vector-like charged lepton fields was shown, via a see-saw like mechanism, to also suppress the charged lepton masses with respect to their quark counterparts. A numerical analysis showed that realistic levels of suppression are possible with this method.

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