Two stream instabilities in degenerate quantum plasmas

S. Son
18 Caleb Lane, Princeton, NJ 08540
(Dated: October 21, 2013)

The quantum mechanical effect on the plasma two-stream instability is studied based on the dielectric function approach. The analysis suggests that the degenerate plasma relevant to the inertial confinement fusion behaves differently from classical plasmas when the electron drift velocity is comparable to the Fermi velocity. For high wave vector comparable to the Fermi wave vector, the degenerate quantum plasma has larger regime of the two-stream instabilities than the classical plasma. A regime, where the plasma waves with the frequency larger than 1.5 times of the Langmuir wave frequency become unstable to the two-stream instabilities, is identified.

PACS numbers: 52.35.Qz, 52.40.Mj, 52.30.Ex

I. INTRODUCTION

Dense plasmas become important research subjects as plasmas with extremely high density are achieved in the laboratories and as our understanding of the astrophysical entities such as the white dwarf and super dense stars gets deeper. The two-stream instability in dense plasmas is especially important to understand since dense electron beams are common phenomenon in the inertial confinement fusion (ICF) and the violent astrophysical events in the dense astrophysics, involving the two-stream instabilities, are also being observed from the phenomena such as the gamma ray burst.

It becomes a challenge to understand the dense plasmas, as the quantum effects begin to modify the physical process at high density; a few physical processes deviating from the classical prediction have been identified. The main goal of this paper is to study the electron quantum effect on the plasma two-stream instabilities in dense quantum plasmas. There has been a general theoretical development in addressing the quantum effects on the two-stream instabilities by employing the fluid-type equation. In this paper, however, the author will utilize the more realistic Lindhard dielectric function and will focus on the completely degenerate electron plasma with the density from $n_e = 10^{24}/cc$ to $n_e = 10^{28}/cc$. Two cases are studied; the first case is one in which the plasma has two different group of electrons with different drift velocities and the second case is one in which the electrons have different velocity with the ions.

The findings are as follows. When the electron drift velocity is comparable to the Fermi velocity, the quantum plasma and classical plasma behave very differently. For high wave vector comparable to the Fermi vector, the two-stream instability regime from the quantum plasmas is larger than the classical plasmas with the temperature comparable to the Fermi energy. A regime is identified where the unstable Langmuir wave can have higher frequency than the Langmuir wave frequency by 1.5 times.

This paper is organized as follows. In Section II, the dielectric function approaches in the two-stream instability is introduced. In Section III, we consider the case when there are two different group of degenerate electron with different drift velocities. In Section IV, we consider the case when the degenerate electrons have different drift velocity with the ions. In Section V, the summary and discussion are provided.

II. DIELECTRIC FUNCTION FOR THE TWO STREAM INSTABILITY ANALYSIS.

The longitudinal dielectric function of a plasma is given as

$$\epsilon(k, \omega) = 1 + \frac{4\pi\epsilon^2}{k^2} \sum \chi_i.$$  \hspace{1cm} (1)

where the summation is over the group of particle species and $\chi_i$ is the particle susceptibility. Given the dielectric function $\epsilon(k, \omega)$, the analysis of the two-stream instability can be obtained by finding the root of $\epsilon(k, \omega)$ as a function of $\omega$. If the dielectric function has a root $\epsilon(k, \omega) = 0$ as a function of $\omega$ in the complex upper-half plane, the plasma is unstable to the two-stream instabilities. In classical plasmas, the susceptibility is given as

\begin{equation}
\end{equation}

\begin{equation}
\end{equation}
In this example, \( n_e = 10^{24} \text{cc}, T_e = 21 \text{eV} \), \( k \lambda_{de} = 0.75 \) \((k = 0.7k_F)\) and the drift velocity has the electron kinetic energy of 320 eV so that \( kv_0/\omega_{pe} \cong 4.11 \). The local maxima of the real part at \( \omega = 2.0 \omega_{pe} \) is larger than 0 and there is no instability in this wave vector.

The real part of the dielectric function \( \epsilon \) as a function of the frequency for the completely degenerate plasma and the classical plasmas. The x-axis is \( \omega/\omega_{pe} \) and the y-axis is \( Re[\epsilon] \).

For the classical plasmas, the instability begins to emerge when \( v_0/v_F \geq 3.5 \). For the degenerate plasmas, the instability begins to emerge when \( v_0/v_F \geq 2.75 \).

The Fermi energy is given as \( E_F = 36.4 \times (n/n_{24})^{2/3} \text{eV} \) where \( n_{24} = 10^{24} \text{cc} \). The real part of \( h \) is given as

\[
h_r = \frac{1}{2} + \frac{1}{8 \pi^2} (1 - (z - u)^2) \log \left( \frac{|z - u + 1|}{|z - u - 1|} \right)
+ \frac{1}{8 \pi^2} (1 - (z + u)^2) \log \left( \frac{|z + u + 1|}{|z + u - 1|} \right)
\]

In the next two sections, we consider the two cases of two-stream instabilities. The first case is when there are two groups of electrons. Each group is the Maxwellian distribution (the degenerate Fermi distribution) with the same density and temperature but with different drift. The second case is when the Maxwellian (complete degenerate) electrons have different drift velocity with the ions. The ions are assumed to be the Maxwellian with the temperature comparable to the Fermi energy. In the first case, the dielectric function is given as

\[
\epsilon = 1 + (4\pi e^2/k^2)(\chi_e^C(\omega, k) + \chi_e^C(\omega - k \cdot v_0, k))
\]

or

\[
1 + (4\pi e^2/k^2)(\chi_e^C(\omega, k) + \chi_e^C(\omega - k \cdot v_0, k)),
\]

where \( v_0 \) is the drift velocity. In the second case, the dielectric function is given as

\[
\epsilon = 1 + (4\pi e^2/k^2)(\chi_e^C(\omega, k) + \chi_e^C(\omega - k \cdot v_0, k))
\]

or

\[
1 + (4\pi e^2/k^2)(\chi_e^Q(\omega, k) + \chi_e^C(\omega - k \cdot v_0, k)),
\]

III. WHEN THERE ARE TWO DIFFERENT GROUPS OF (DEGENERATE) ELECTRONS

In this section, we analyze Eq. (4). In the conventional classical two-stream instability, the threshold condition
for the two stream instabilities is that there should be a local maxima between $\omega_{pe} < \omega < kv_0$ and that local maxima should be less than 0. In Fig. 1, we plot the classical dielectric function as a function of $\omega$ for a particular $k$. The hump at $\omega \cong 1.3 \omega_{pe}$ is the local maxima and it is less than 0; the plasma Langmuir wave is unstable to the two stream instabilities of the classical plasmas.

In Fig. 2, we plot $\epsilon$ as a function of $\omega$ for a fixed $k$ in an example plasma. The classical plasma does not have the local maxima. The degenerate plasma has a local maxima at $\omega \cong 2 \omega_{pe}$ but is larger than zero. For this particular $k$, the plasma Langmuir wave is stable to the two-stream instabilities. In Fig. 3, the only difference in the physical parameter with Fig. 2 is the wave vector. In this case, the classical plasma does not have the local maxima and the degenerate plasma has the local maxima smaller than 0; the degenerate plasma (classical plasma) Langmuir wave is unstable (stable) to the two-stream instability.

For a fixed electron density and temperature (or completely degenerate), we plot $\epsilon$ as a function of $\omega$ and determine the critical $k_C(v_0)$ values for which the plasma becomes unstable to the two-stream instabilities. The necessary condition for the two-stream instabilities is $k < k_C(v_0)$. By varying the drift velocity $v_0$, we can determine the boundary of the two-stream instabilities. In Fig. 4, we do this for a particular plasma with $n_e = 10^{24}$/cc, $T_e = 21$ eV. The Fermi energy is $E_F = 36$ eV and we choose the electron temperature of the classical plasma as $T_e = 0.6 E_F$ since the average kinetic energy of the electron in the completely degenerate case is $0.6 E_F$. From Fig. 4, it can be concluded that the regime of the two-stream instability is larger in quantum prediction than classical prediction. As the drift velocity becomes larger than $v_0 > 3.5 v_F$, the difference between the quantum plasma and the classical plasma is small.

Similar analysis suggests that, for more dense plasma, the regime of the quantum deviation, where the degenerate plasma (classical plasma) is unstable (stable), widen further.

IV. WHEN THE (DEGENERATE) ELECTRONS HAVE DIFFERENT DRIFT VELOCITY FROM THE IONS

In this section, we analyze Eq. 1. We employ the same criteria that was used in Sec. II. In Fig. 5, we plot the real part of the dielectric function of the classical plasmas and the degenerate plasmas. The condition for the two-stream instability is the existence of the local maxima of the dielectric function that is lower than 0. In this section, we assume that the ion temperature is zero and the ion can be treated as the classical particle as the de Broglie wave length of the ion is smaller than any other length scale in the regime of our interest.

As well-known in the conventional Lindhard function [14], the degenerate plasma can support the collective Langmuir waves whose wave length is much smaller than the classical Debye length. The regime of the two stream instability is higher in the degenerate plasma than the classical plasmas due to this fact. In Fig. 6, we plot the threshold condition for the two-stream instability in the degenerate plasma and classical plasmas; the necessary condition is $k < k_C(v_0)$. For the classical plasmas, the regime of the instabilities is smaller than the degenerate plasma. For the plasma with the lower density, the difference between the classical plasma and the degenerate plasma is smaller than the example that we have shown.

The example in Fig. 5 is the case when the classical plasmas are stable regardless of the wave vector. As shown in the figure, the frequency of the unstable Langmuir wave is $1.4 \omega_{pe}$. Such high frequency wave cannot be sustained in the classical plasmas and can only be supported via the diffraction and degeneracy effect of the electrons. For the same plasma in Fig. 5, the unstable Langmuir wave frequency can reach $1.7 \omega_{pe}$.

V. SUMMARY AND CONCLUSION

In this paper, the author has analyzed the effect of the electron degeneracy and diffraction on the two-stream instability, focused on the plasmas relevant to the inertial confinement fusion and the astrophysical system ranging in density from $n_e = 10^{24}$/cc to $n_e = 10^{28}$/cc. Our approach is based on the Lindhard random phase approximation, which takes into accounts the quantum degeneracy and diffraction. We compare our prediction based on the Lindhard approaches with the classical plasma.

The analysis suggests that the quantum effect become more pronounced as the density gets higher and that the classical dielectric function describes, as accurately as the
Lindhard function, the two-stream instabilities when the drift velocity is higher than 3.5 times of the Fermi velocity. However, when the drift velocity is smaller than 3.5 times of the Fermi velocity, the quantum plasma and classical plasmas behave quite differently to the two-stream instability. Prominently, for the both cases we consider in Secs. II and III, the regime of the instability predicted by the Lindhard approach is considerably larger than the classical plasma as illustrated in Figs. (4) and (6). We also have shown that the unstable Langmuir wave due to the two-stream instability can have the frequency as large as $\omega_{pe}^{1/7}$, which is impossible in the classical plasmas. These findings in this paper can have major implications on the Backward Raman scattering [15–22] and the beam stopping by the dense background plasmas.

[1] M. Tabak, J. Hammer, M. Glinsky, W. Krueer, S. Wilks, J. Woodworth, E. Campbell, M. Perry, and R. Mason, Phys. Plasmas 1, 1626 (1994).
[2] S. Son, and N. J. Fisch, Phys. Rev. Lett. 95, 225002 (2005).
[3] S. Son, and N. Fisch, Phys. Lett. A 329, 76 (2004).
[4] S. P. D. Mangles et al., Nature 431, 535 (2004).
[5] M. Tatarakis et al., Phys. Rev. Lett. 90, 175001 (2003).
[6] T. P. Fleming, J. M. Stone, and J. F. Hawley, Astrophy. J. 530, 464 (2000).
[7] D. Biskamp, Phys. Fluids 29, 1520 (1986).
[8] G. Fishman, and C. A. Meegan, Annu. Rev. Astron. Astrophys. 33, 415 (1995).
[9] E. Nakar, Physics Report 442, 166 (2007).
[10] D. E. Innes, B. Inhester, W. I. Axford, and K. Wilhelm, Nature 386, 811 (1997).
[11] F. Hass, G. Manfredi, and M. R. Feix, Phys. Rev. E 62, 2763 (2000).
[12] G. Manfredi, and F. Hass, Phys. Rev. B 64, 075316 (075316).
[13] S. Son, and N. Fisch, Phys. Lett. A 356, 65 (2006).
[14] J. Lindhard, K. Dan. Vidensk. Sels. Mat. Fys. Medd 28, 8 (1954).
[15] S. Son, S. Ku, and S. J. Moon, Phys. of Plasmas 17, 114506 (2010).
[16] S. Son, and S. Ku, Phys. of Plasmas 17, 010703 (2010).
[17] S. Son, and Sung Joon Moon, Phys. of Plasmas 18, 084504 (2011).
[18] S. Son, S. Ku, and Sung Joon Moon, Phys. of Plasmas 17, 112709 (2010).
[19] S. Son, and Sung Joon Moon, Appl. Phys. Lett. 98, 081501 (2010).
[20] V. M. Malkin, G. Shvets, and N. J Fisch, Phys. Rev. Lett. 82, 4448 (1999).
[21] V. M. Malkin, N. J. Fisch, and J. S. Wurtele, Phys. Rev. E. 75, 026404 (2007).
[22] C. J. McKinstrie, and A. Simon, Phys. Fluids 29, 1959 (1986).