Does Current Data Prefer a Non-minimally Coupled Inflaton?

Lotfi Boubekeur,1,2 Elena Giusarma,3 Olga Mena,1 and Héctor Ramírez1

1Instituto de Física Corpuscular (IFIC), CSIC-Universitat de Valencia, Apartado de Correos 22085, E-46071, Spain.
2Laboratoire de Physique Mathématique et Subatomique (LPMS) Université de Constantine I, Constantine 25000, Algeria.
3Physics Department and INFN, Università di Roma “La Sapienza”, P.le Aldo Moro 2, 00185, Rome, Italy

We examine the impact of a non-minimal coupling of the inflaton to the Ricci scalar, \( \frac{1}{2} \xi R \phi^2 \), on the inflationary predictions. Such a non-minimal coupling is expected to be present in the inflaton Lagrangian on fairly general grounds. As a case study, we focus on the simplest inflationary model governed by the potential \( V \propto \phi^2 \), using the latest combined 2015 analysis of Planck and BICEP2/Keck Array. We find that, for all the data combinations used in this study, a small positive value of the coupling \( \xi \) is favoured at the 2\( \sigma \) level. When considering the cross-correlation polarization spectra from BICEP2/Keck Array and Planck, a value of \( r > 0 \) is found at 95\% CL.

PACS numbers: 98.70.Vc, 98.80.Cq, 98.80.Bp

Motivations. — Inflation provides the most theoretically attractive and observationally successful cosmological scenario able to generate the initial conditions of our universe, while solving the standard cosmological problems. Despite this remarkable success, the inflationary paradigm is still lacking firm observational confirmation. The picture that emerges from the latest data from Planck, including also the joint analysis of B-mode polarization measurements from the BICEP2 collaboration [1–4], is compatible with the inflationary paradigm. According to these observations, structure grows from Gaussian and adiabatic primordial perturbations. From the theoretical viewpoint, this picture is usually understood as the dynamics of a single new scalar degree of freedom, the inflaton, minimally coupled to Einstein gravity. However, the inflaton \( \phi \) is expected to have a non-minimal coupling to the Ricci scalar through the operator \( \frac{1}{2} \xi R \phi^2 \), where \( \xi \) is a dimensionless coupling. Indeed, successful reheating requires that the inflaton is coupled to the light degrees of freedom. Such couplings, though weak, will induce a non-trivial running for \( \xi \). Thus, even starting from a vanishing value of \( \xi \) (away from the conformal fixed point \( \xi = -1/6 \)) at some energy scale, a non-trivial non-minimal coupling will be generated radiatively at some other scale (see e.g. Ref. [5]). Therefore, it is important to study the impact of such a coupling on the inflationary predictions, especially in view of the latest Planck 2015 data.

Generically, for successful inflation, the inflaton should be very weakly coupled*. It follows that the magnitude of \( \xi \) is expected to be small. Yet, even with such a suppressed coupling, the inflationary predictions are significantly altered [6–13]. For instance, and as we will see, a small and positive \( \xi \) can enlarge considerably the space of phenomenologically acceptable scenarios (See also [14]). In this letter, we will focus on the simplest inflationary scenario with a potential \( V \propto \phi^2 \) [15], and a non-zero non-minimal coupling. According to the very recent Planck 2015 full mission results, the minimally-coupled version of this scenario (i.e. \( \xi = 0 \)) is ruled out at more than 99\% confidence level [2, 4], for 50 e-folds of inflation. Thus, before discarding it definitely from the range of theoretical possibilities, it is worthwhile to explore this scenario in all generality, given that, as explained earlier, the presence of non-minimal couplings in the inflaton Lagrangian is quite generic.

Non-minimally coupled Inflaton. — The dynamics of a non-minimally coupled scalar field \( \phi \) with a potential \( U(\phi) \) is governed, in the Jordan frame, by the following action\(^1\)

\[
S = \int d^4 x \sqrt{-g} \left[ \frac{M_P^2}{2} R + \frac{\xi}{2} R \phi^2 - \frac{1}{2} (\partial \phi)^2 - U(\phi) \right],
\]

where indices are contracted with the metric \( g_{\mu\nu} \), defined as \( dx^2 = -dt^2 + a^2(t) dx^2 \). Inflation can be conveniently studied in the Einstein frame, after performing a conformal transformation \( g^E_{\mu\nu} = \Omega(\phi) g_{\mu\nu} \), with \( \Omega = 1 + \xi \phi^2 / M_P^2 \) and canonically-normalizing the scalar field. Up to a total derivative, the action takes the familiar form

\[
S = \int d^4 x \sqrt{-g^E} \left( \frac{M_P^2}{2} R_E - \frac{1}{2} g^E_{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V[\varphi] \right),
\]

where now \( \varphi \) is the canonically-normalized inflaton, related to the original non-minimally coupled scalar field \( \phi \) through

\[
\left( \frac{d \varphi}{d \phi} \right)^2 = \frac{1}{\Omega} + \frac{3}{2} M_P^2 \left( \frac{\Omega'}{\Omega} \right)^2.
\]

* This requirement is also dictated by the non-detection of large primordial non-Gaussianities [3] and the soft breaking of the shift symmetry \( \phi \to \phi + c \), necessary to protect the flatness of the potential.

\(^1\) As usual, \( M_P = 1/\sqrt{8\pi G_N} \approx 2.43 \times 10^{18} \) GeV is the reduced Planck mass.
In terms of the original scalar field $\phi$, the physical potential takes the simple form

$$V[\phi(\phi)] = U(\phi)/\Omega^2(\phi).$$

(4)

In the following, as previously stated, we shall focus on the simplest inflationary model\(^1\), given by the quadratic potential $U(\phi) = \frac{1}{2}m^2\phi^2$, with a non-vanishing coupling $\xi$. In order to derive the primordial scalar and tensor perturbation spectra within the non-minimally coupled $\phi^2$ theory, we shall make use of the slow-roll parameters\(^2\):

$$\epsilon = \frac{M_P^2}{2} \left( \frac{V'}{V} \right)^2, \quad \eta = M_P^2 \frac{V''}{V}, \quad \xi_{SR} = M_P^4 \frac{V'_\phi V''}{V^2}. \quad \text{(5)}$$

It is straightforward to derive the expressions for the spectral index of the primordial scalar perturbations $n_s \equiv 1 + 2\eta - 6\epsilon$, its running $\alpha \equiv -\frac{d}{d\ln k} n_s \equiv -24\epsilon^3 + 16\eta - 2\xi_{SR}$, and the tensor-to-scalar ratio $r \equiv 16\epsilon$ from the above slow-roll parameters\(^3\).

Within the slow-roll approximation, it is straightforward to solve numerically the inflationary dynamics governed by the action Eq. (2). The number of e-folds is given by

$$N = \frac{1}{M_P} \int_{\phi_{end}}^{\phi_*} \frac{d\phi}{\sqrt{2\epsilon(\phi)}} = \frac{1}{M_P} \int_{\phi_{end}}^{\phi_*} \frac{d\phi}{\sqrt{2\epsilon(\phi)}} \left( \frac{d\phi}{d\epsilon} \right), \quad \text{(6)}$$

where $\epsilon(\phi) \equiv \frac{M_P^4}{8} |V'(\phi)/V(\phi)|^2$. Throughout this analysis, we fix $N = 60$. The inflationary predictions are depicted in Fig. 1, in the $(n_s, r)$ plane, for both positive and negative values of the coupling $\xi$. The case of $\xi = 0$ corresponds to the usual predictions of the chaotic inflationary scenario, with $n_s = 1 - 2/N \approx 0.967$ and $r = 8/N \approx 0.13$ for $N = 60$, and it is represented by a red circle. Notice that negative values of $\xi$ lead to a larger tensor-to-scalar ratio. Positive values of $\xi$, on the other hand, will reduce the tensor contribution, while also pushing $n_s$ significantly below scale invariance as $\xi$ increases. For instance, for $\xi > 0.002$, the scalar spectral index will always be smaller than the observationally preferred value $n_s \approx 0.96$. The predicted running of the spectral index $\alpha$ is shown in Fig. 2 as a function of the non-minimal coupling $\xi$. In general, negative (positive) values of $\xi$ lead to positive (negative) values of the running. Although the large positive values of the running shown in Fig. 2 are compatible with the recent Planck 2015 constraints \(^2\), $\alpha = -0.0065 \pm 0.0076$, they are nevertheless associated with values of the tensor-to-scalar ratio $r > 0.5$, which are excluded observationally. The red circle in Fig. 2 refers to the $\xi = 0$ case, corresponding to $\alpha = -2/N^2 \approx -0.00056$ for $N = 60$.

**Observational constraints on $\xi$.—** In this letter, we restrict our numerical fits to Cosmic Microwave Background (CMB) measurements. The inclusion of external data sets, such as Baryon Acoustic Oscillation measurements, or a Hubble constant prior from the HST team

\[\text{Parameter} \quad \text{Physical Meaning} \quad \text{Prior}\]

| Parameter | Physical Meaning | Prior |
|-----------|------------------|-------|
| $\omega_b \equiv \Omega_b h^2$ | Present baryon density. | $0.005 \rightarrow 0.1$ |
| $\omega_c \equiv \Omega_c h^2$ | Present Cold dark matter density. | $0.01 \rightarrow 0.99$ |
| $\Theta_s$ | $r_s/D_A(z_{dec})$ | $0.5 \rightarrow 10$ |
| $\tau$ | Reionization optical depth. | $0.01 \rightarrow 0.8$ |
| $n_s$ | Scalar spectral index. | $0.9 \rightarrow 1.1$ |
| $\ln(10^{10} A_s)$ | Primordial scalar amplitude. | $2.7 \rightarrow 4$ |
| $\xi$ | Non-minimal coupling. | $-0.002 \rightarrow 0.0005$ |

\(^a\) The parameter $\Theta_s$ is the ratio between the sound horizon $r_s$ and the angular diameter distance $D_A(z_{dec})$ at decoupling $z_{dec}$.

**TABLE I.** Uniform priors for the cosmological parameters considered in the present analysis.

\(^1\) A generalization to other interesting inflationary scenarios will be carried out elsewhere \([16]\).

\(^2\) Here, we use the notation $\xi_{SR}(\phi)$ to refer to the usual slow-roll parameter $\xi$, in order to avoid confusion with the non-minimal coupling to gravity $\xi$.

\(^3\) Notice that the expressions for both $n_s$ and $r$ are first-order in slow-roll, while $\alpha$ involves second order slow-roll terms. However, we have checked numerically that such second order corrections in slow-roll leave unchanged the constraints on the inflationary observables $(n_s, r)$. Therefore, higher order slow-roll corrections can be safely neglected.
will not affect the constraints presented in the following. Our data sets are the Planck temperature data (hereafter TT) [17–19], together with the low-$\ell$ WMAP 9-year polarization likelihood, that includes multipoles up to $\ell = 23$, see Ref. [20] (hereafter WP), and the recent multi-component likelihood of the joint analysis of BICEP2/Keck Array and Planck polarization maps (hereafter BKP), following the data selection and foreground parameters of the fiducial analysis presented in Ref. [1].** However variations of this fiducial model will not change significantly the results presented here.

These data sets are combined to constrain the cosmological model explored here, and described by the parameters\textsuperscript{11}:

$\{\omega_b, \omega_c, \Theta_s, \tau, \log[10^{10} A_s], \xi\}$

In Table I, we summarize the definition as well as the priors on these parameters. We use the Boltzmann code CAMB [21] and the cosmological parameters are extracted from the data described above by means of a Monte Carlo Markov Chain (MCMC) analysis based on the most recent version of cosmomc [22]. The constraints obtained on the non-minimal coupling $\xi$ are then translated into bounds on the usual inflationary parameters $n_s$, $r$ and $\alpha$.

Table II shows the 95% CL constraints on the parameters $\xi$, $n_s$, $r$ and the running $\alpha$ arising from our numerical analyses using the two CMB data combinations used here. The preferred value of the non-minimal coupling $\xi$ from Planck TT plus WP measurements is positive and slightly larger than the mean value obtained when the cross-correlated polarized maps from BICEP2/Keck and Planck (BKP) experiments are included in the numerical analyses. This preference for a slightly larger $\xi$ (and consequently, smaller $r$) is clear from the one-dimensional posterior probability distribution of $\xi$ shown in the left panel of Fig. 3. The mean value of $\xi = 0.0028$ obtained from Planck TT plus WP data is translated into an upper 95% CL limit of the tensor-to-scalar-ratio $r < 0.09$, as can be seen from the right panel of Fig. 3. When considering BICEP2/Keck and Planck cross-spectra polarization data, the tensor-to-scalar ratio $r$ is found to be different from zero at the $2\sigma$ level, $r = 0.04^{+0.04}_{-0.03}$. Concerning the running of the spectral index, the two data combinations seem to have a preference for a small negative running $\alpha = -0.005$, associated to small values of $|\xi|$, as shown in Fig. 2. Figure 1 shows the 68% and 95% CL allowed regions in the $(n_s, r)$ plane, resulting from our MCMC analyses, together with the theoretical predictions. It is clear from these results that the combination of Planck TT plus WP data prefers slightly smaller values of both $n_s$ and $r$ than those obtained when the combined BKP likelihood is also included.

Let us now turn to future constraints on $\xi$. Future observations, as those expected from PIXIE [23], Euclid [24], COrE [25] and PRISM [26], could be able to reach an accuracy of $\sigma_\xi = \sigma_{n_s-1} = 10^{-3}$. With such precision, one could hope to test deviations from the quadratic potential [27], as the one studied here, by constructing quantities independent of $N$, up to leading $O(1/N^2)$ corrections. It is straightforward to get for our case

$$n_s - 1 + \frac{r}{4} = -20 \xi,$$

at leading order both in slow-roll and $\xi$. If it turns out that nature had chosen a very small value of $r$, future constraints on $\xi$ would be as strong as $\xi \lesssim 10^{-4}$: one order of magnitude stronger than the ones obtained in this analysis. Concerning the running $\alpha$, it is interesting to note that futuristic observations like SPHEREx [28] with a forecasted error of $\sigma_\alpha = 10^{-3}$, will be able to falsify the present scenario.

Finally, it is also interesting to explore the impact of the non-minimal coupling on the inflaton excursion. It is

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Parameter & Planck TT+WP & BKP+Planck TT+WP \\
\hline
$n_s$ & $0.958^{+0.010}_{-0.011}$ & $0.958^{+0.009}_{-0.011}$ \\
$r$ & $< 0.09$ & $0.04^{+0.04}_{-0.03}$ \\
$\xi$ & $0.0028^{+0.0023}_{-0.0025}$ & $0.0028^{+0.0023}_{-0.0022}$ \\
$\alpha \equiv \frac{dn_s}{d\ln k}$ & $-0.0005^{+0.0001}_{-0.0001}$ & $-0.0005^{+0.0001}_{-0.0001}$ \\
\hline
\end{tabular}
\caption{95% CL limits on the non-minimal coupling $\xi$ and on the inflationary parameters $n_s$, $r$ and $\alpha$ from the two possible CMB data combinations used in this study.}
\end{table}
well-know that large values of the tensor-to-scalar ratio $r$, as those found by previous BICEP2 measurements\textsuperscript{††} [29, 30] yield large inflaton excursions $\phi \gg M_P$ [31–34], which are hard to understand in the context of a consistent effective field theory. In particular, successful inflation requires that higher order non-renormalizable operators, which are expected to be naturally present in the inflationary potential, are sufficiently suppressed. A number of phenomenological studies have recently been devoted to address this problem [35–38]. In Fig. 4, we plot the excursion of both $\phi$ and $\varphi$, together with the corresponding tensor ratio $r$. It is clear that the excursion of the canonically-normalized inflaton $\varphi$ is lowered for positive values of $\xi$ i.e. $\Delta \varphi < \Delta \phi$. However, this decrease is rather mild and the excursion still takes on super-Planckian values for the phenomenologically acceptable values of $\xi$. Conversely, negative values of $\xi$ lead to an increase of the excursion of $\varphi$. Figure 4 also shows that super-Planckian values of both $\phi$ and $\varphi$ are still associated with large values of the tensor-to-scalar ratio $r$, in agreement with the Lyth bound [31]. Thus, when a small non-zero and positive value of the coupling $\xi$ is turned on, both the inflaton excursion and $r$ are slightly lowered, but without alleviating completely the super-Planckian excursion problem.

Conclusions.— A small non-minimal coupling $\xi R \phi^2$ is expected to be present in the inflaton Lagrangian, and modifies the inflationary predictions in an interesting way. Focusing on the simplest quadratic potential scenario, and using the very recent joint analysis of BICEP2/Keck Array and Planck polarization maps, we found that a small, positive value of the coupling $\xi$ is always favoured at the $2\sigma$ level, see Table II. It would be interesting to see if upcoming $B$-modes measurements can reinforce or weaken the statistical significance of these findings. In particular, it would be crucial to discriminate between the presence of a non-minimal coupling in the theory and other departures from the quadratic approximation.

Acknowledgments. — OM is supported by PROM-
ETEO II/2014/050, by the Spanish Grant FPA2011-29678 of the MINECO and by PITN-GA-2011-28942-INVISIBLES. LB and HR acknowledge financial support from PROMETEO II/2014/050.

[1] P. A. R. Ade et al. [BICEP2 and Planck Collaborations], “A Joint Analysis of BICEP2/Keck Array and Planck Data,” [arXiv:1502.00612 [astro-ph.CO]].
[2] [Planck Collaboration], “Planck 2015 results. XIII. Cosmological parameters,” arXiv:1502.01589 [astro-ph.CO].
[3] P. A. R. Ade et al. [Planck Collaboration], Planck 2015 results. XVII. “Constraints on primordial non-Gaussianity,” arXiv:1502.01592 [astro-ph.CO].
[4] P. A. R. Ade et al. [Planck Collaboration], “Planck 2015. XX. Constraints on inflation,” arXiv:1502.02114 [astro-ph.CO].
[5] I. L. Buchbinder, S. D. Odintsov and I. L. Shapiro, Effective action in quantum gravity, Bristol, UK: IOP (1992) 413 pp.
[6] D. S. Salopek, J. R. Bond and J. M. Bardeen, Phys. Rev. D 40, 1753 (1989).
[7] T. Futamase and K. i. Maeda, Phys. Rev. D 39 (1989) 399.
[8] R. Fakir and W. G. Unruh, Phys. Rev. D 41, 1783 (1990).
[9] D. I. Kaiser, Phys. Rev. D 52, 4295 (1995) [astro-ph/9408044].
[10] E. Komatsu and T. Futamase, Phys. Rev. D 59, 064029 (1999) [astro-ph/9901127].
[11] M. P. Hertzberg, JHEP 1011, 023 (2010) [arXiv:1002.2905 [hep-ph]].
[12] N. Okada, M. U. Rehman and Q. Shafi, Phys. Rev. D 82 (2010) 043502 [arXiv:1005.5161 [hep-ph]].
[13] A. Linde, M. Noorbala and A. Westphal, JCAP 1103, 013 (2011) [arXiv:1101.2652 [hep-th]].
[14] S. Tsujikawa, J. Ohashi, S. Kuroyanagi and A. De Felice, Phys. Rev. D 88 (2013) 2, 023529 [arXiv:1305.3044 [astro-ph.CO]].
[15] A. D. Linde, Phys. Lett. B 129 (1983) 177.
[16] L. Boubekeur, E. Giusarma, O. Mena and H. Ramírez, In preparation.
[17] P. A. R. Ade et al. [Planck Collaboration], arXiv:1303.5062 [astro-ph.CO].
[18] P. A. R. Ade et al. [Planck Collaboration], arXiv:1303.5075 [astro-ph.CO].
[19] P. A. R. Ade et al. [Planck Collaboration], arXiv:1303.5077 [astro-ph.CO].
[20] C. L. Bennett et al. [WMAP Collaboration], Astrophys. J. Suppl. 208, 20 (2013) [arXiv:1212.5225 [astro-ph.CO]].
[21] A. Lewis, A. Challinor and A. Lasenby, Astrophys. J. 538, 473 (2000) [arXiv:astro-ph/9911177].
[22] A. Lewis and S. Bridle, Phys. Rev. D 66, 103511 (2002) [arXiv:astro-ph/0205436].
[23] A. Kogut, D. J. Fixsen, D. T. Chuss, J. Dotson, E. Dwek, M. Halpern, G. F. Hinshaw and S. M. Meyer et al., JCAP 1107 (2011) 025 [arXiv:1105.2044 [astro-ph.CO]].
[24] R. Laureijs et al. [EUCLID Collaboration], “Euclid Definition Study Report,” arXiv:1110.3193 [astro-ph.CO].
[25] F. R. Bouchet et al. [CoRE Collaboration], “CoRE (Cosmic Origins Explorer) A White Paper,” arXiv:1102.2181 [astro-ph.CO].
[26] P. Andre et al. [PRISM Collaboration], “PRISM (Polarized Radiation Imaging and Spectroscopy Mission): A White Paper on the Ultimate Polarimetric Spectro-Imaging of the Microwave and Far-Infrared Sky,” arXiv:1306.2259 [astro-ph.CO].
[27] P. Creminelli, D. López Nacir, M. Simonović, G. Trevisan and M. Zaldarriaga, Phys. Rev. Lett. 112 (2014) 24, 21303 [arXiv:1404.1065 [astro-ph.CO]].
[28] O. Doré, J. Bock, P. Capak, R. de Putter, T. Eifler, C. Hirata, P. Korngut and E. Krause et al., “SPHEREx: An All-Sky Spectral Survey,” arXiv:1412.4872 [astro-ph.CO].
[29] P. A. R. Ade et al. [BICEP2 Collaboration], Phys. Rev. Lett. 112 (2014) 241101. [arXiv:1403.3985 [astro-ph.CO]].
[30] P. A. R. Ade et al. [BICEP2 Collaboration], Astrophys. J. 792 (2014) 62 [arXiv:1403.4302 [astro-ph.CO]].
[31] D. H. Lyth, Phys. Rev. Lett. 78 (1997) 1961 [hep-ph/9605387].
[32] L. Boubekeur, Phys. Rev. D 87 (2013) 6, 061301 [arXiv:1208.0210 [astro-ph.CO]].
[33] G. Efstathiou and K. J. Mack, JCAP 0505 (2005) 008 [astro-ph/0503360].
[34] L. Verde, H. Peiris and R. Jimenez, JCAP 0601 (2006) 019 [astro-ph/0506036].
[35] J. Garcia-Bellido, D. Roest, M. Scalisi and I. Zavala, JCAP 1409, 006 (2014) [arXiv:1405.7399 [hep-th]].
[36] G. Barenboim and O. Vives, arXiv:1405.6498 [hep-ph].
[37] J. Garcia-Bellido, D. Roest, M. Scalisi and I. Zavala, Phys. Rev. D 90, no. 12, 123539 (2014) [arXiv:1408.6839 [hep-th]].
[38] L. Boubekeur, E. Giusarma, O. Mena and H. Ramírez, arXiv:1411.7237 [astro-ph.CO].