Nonmonotonic Temperature Dependence of the Hall Resistance for 2D Electron System in Si

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Weak field Hall resistance $\rho_{xy}(T)$ of the 2D electron system in Si was measured over the range of temperatures (1–35) K and densities, where the diagonal resistivity exhibits a “metallic” behavior. The $\rho_{xy}(T)$ dependence was found to be non-monotonic with a maximum at temperatures $T_{\text{max}} \sim 0.16T_F$. The $\rho_{xy}(T)$ variations in the low-temperature domain ($T < T_{\text{max}}$) agree qualitatively with the semiclassical model, that takes into account a broadening of the Fermi-distribution solely. The semiclassical result considerably exceeds an interaction-induced quantum correction. In the “high-temperature” domain ($T > T_{\text{max}}$), the $\rho_{xy}(T)$ dependence can be qualitatively explained in terms of either a semiclassical $T$-dependence of a transport time, or a thermal activation of carries from a localized band.

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Origin of the “metallic” type conduction ($d\rho/dT > 0$) in two-dimensional (2D) high-mobility electron system in Si remains in the focus of researcher’s interest for over a decade. Several models have been suggested to explain the metallic behavior: temperature-dependent screening of impurity potential [1], interaction-induced quantum corrections [2], macroscopic liquid and solid electron phase separation [3] etc. Though the theories [1–3] are substantially different from each other, all of them explain $\rho_{xx}(T)$ qualitatively or even quantitatively. Therefore, additional experimental data are needed for testing these theoretical models. Such data can be obtained, in particular, from Hall resistance measurements.

Hall resistance of 2D system in weak fields has been theoretically studied in Refs. [4–8]. As shown in Ref. [4], in the frameworks of the Fermi-liquid theory, Hall resistance is not renormalized and remains equal to that for the Fermi-gas as $T \to 0$. Zala, Narozhny and Aleiner [5] have calculated a quantum correction $\Delta \rho_{xy}$ to the Hall resistance due to electron-electron interaction for arbitrary $T \tau$ (hereafter we assume $\hbar = 1$, $k_B = 1$):

$$\frac{\Delta \rho_{xy}}{\rho_{xy}} = \frac{2^2(1 + n_C C(F_0^2))}{\pi^2 \sigma_D} \ln \left(1 + \frac{11\pi}{192} \frac{1}{T \tau} \right).$$

(1)

Here $\tau$ is the transport time, $n_C (= 15$ for two-valley system) is a number of triplet terms $C(F_0^2)$ which depend on a Fermi-liquid coupling constant $F_0^2$. The correction is small in a ballistic regime ($T \tau \gg 1$); in a diffusive regime ($T \tau \ll 1$) it is proportional to $\ln(T \tau)$ converging to the Altshuler-Aharanov result [7]:

$$\frac{\Delta \rho_{xy}}{\rho_{xy}} = 2\frac{\Delta \rho_{xx}}{\rho_{xx}}.$$  

(2)

In the frameworks of a semiclassical screening theory, Das Sarma and Hwang has suggested another mechanism for temperature dependence of the Hall coefficient [8]:

$$\rho_{xy} = \frac{H}{nec} < \tau^2 >,$$

(3)

where $\tau$ is averaged mainly over an interval $T$ around the Fermi energy. To the leading order in temperature the above relation depends only on smearing of the Fermi distribution and equals:

$$\frac{< \tau^2 >}{< \tau >^2} = 1 + \frac{\pi^2}{3}(T/T_F)^2.$$  

(4)

As temperature increases, the $\tau(T)$ dependence becomes of major importance and causes a non-monotonic behavior of $\rho_{xy}$ as a function of temperature. It is easy to see, that the $\rho_{xy}(T)$ dependences, predicted by the theories [5,8] differ qualitatively from each other. At the same time, both theories yield $\rho_{xx}(T)$ in a good agreement with each other as well as with the experimental data [9,10]. It is noteworthy, the quantum and semiclassical corrections to the Hall resistance do not exclude each other and seem to be taken into account simultaneously.

In the present paper, we report on measurements of the temperature dependence of the weak field Hall resistance and compare the data to the theories [5,8]. The measurements were carried out in the temperature range 1 to 35 K. The studied Si-MOS structure of a Hall bar geometry, 5mm×0.8mm, had a maximum mobility $\approx 25000 \text{cm}^2/\text{Vs}$ at $T = 0.3$ K. Magnetic field $B = 0.1$ T applied perpendicular to the 2D-plane was sufficiently high for measuring the Hall voltage. On the other hand, this field was small enough to ensure the inequality $\omega_c \tau \ll 1$ (in the present work $\omega_c \tau \leq 0.05$ for all measurements), as needed for the applicability of the theories [5,8] and for suppressing Shubnikov-de Haas (SdH) oscillations.

AC measurements were performed at current $I_s = 20 \mu A$ and frequency 7.6 Hz with a lock-in amplifier, that detected both real and imaginary components of the signal. Hall voltage $V_y$ was measured for two opposite field
directions. The results were averaged to eliminate the admixture of $V_x$ to $V_y$. At temperatures $T < 1$ K, the two results, $\rho_{xy}(B)$ and $|\rho_{xy}(-B)|$, differed substantially due to a non-zero imaginary component in $V_y$. Therefore, we analyzed only the data for temperatures higher than 1 K. For each of the field directions, the sample was slowly heated, while the gate voltage $V_g$ (and hence the concentration $n$) periodically varied in a step-like fashion. Two components $\rho_{xx}$ and $\rho_{xy}$ were measured as functions of temperature for each $V_g$ value. A cycle of measurements took about 16 hours.

FIG. 1. a) Temperature dependence of a deviation of the Hall resistance from it’s classical value (dots). b) Temperature dependence of the diagonal component of conductivity (empty dots). Electron density is $n = 5.7 \times 10^{11}$ cm$^{-2}$. The insets show low-temperature domains of the $\rho_{xy}(T)$ and $\sigma_{xx}(T)$ dependences. Dashed lines are the calculated quantum corrections Eq. (1) [2]. Dash-dotted line is the semiclassical correction Eq. (4). Solid line is the thermal activation calculated according to Eq. (6).

Figures 1a and 2a show variations of the Hall resistance $\delta \rho_{xy}$ and the diagonal component of the conductivity as functions of temperature for electron density $5.7 \times 10^{11}$ cm$^{-2}$ and $11.7 \times 10^{11}$ cm$^{-2}$, respectively. The variation of the Hall resistance was calculated with respect to its classical value $H/(n_{SMH}e\hbar)$, where $n_{SMH}$ was determined from the frequency of SdH oscillations in a temperature range $T = 0.5 - 2$ K. There is a well defined maximum on the temperature dependences of $\delta \rho_{xy}$; this maximum moves to higher temperatures as the electron density increases. The maximum variation in $\rho_{xy}$ is about 3%. In contrast, $\sigma_{xx}$ decreases monotonically by a factor of five in the same temperature range (see Figs. 1b and 2b).

“Low-temperature” domain ($T \ll T_F$, $T \tau \lesssim 1$). The quantum corrections to the diagonal conductivity [2] quantitatively describe the low-temperature part of the $\sigma_{xx}(T)$ dependence with no adjusting parameters (dashed lines in Figs. 1b, 2b). For calculating this correction, we used independently determined $m^*$ and $F_0^*$ values [11].

The quantum correction to $\rho_{xy}$ was evaluated using Eq. (1) with the same $m^*$ and $F_0^*$ (see dashed lines in Figs. 1a and 2a): it appears to be an order of magnitude less than the experimentally measured $\delta \rho_{xy}(T)$.
At the same time, the semiclassical dependence Eq. (4) (dash-dotted line in Figs. 1a, 2a) does agree with the low-temperature part of the $\delta \rho_{xy}(T)$ dependence. This consistency points at the leading role of the semiclassical rather than quantum effects in $\rho_{xy}(T)$ for temperatures $T > 1$K. It should be noted that both theoretical curves were arbitrarily shifted vertically for the best coincidence with the experimental data. Such a shift is eligible because the absolute value of concentration has been determined from SdH oscillations with an accuracy of $\sim 1 - 2\%$.

**“High-temperature” domain** ($T \gtrsim 0.3 T_F$, $T \tau \gg 1$). The measured $\rho_{xy}(T)$ decreases as temperature grows. The quantum corrections [2] in this domain are negligible, hence the observed effect is presumably of a semiclassical origin. An analogous decaying $\rho_{xy}(T)$ dependence has been observed earlier for 2D electron system in Si [12] and for 2D hole system in GaAs [13,14]. This effect has been qualitatively explained in Ref. [8] on the basis of Eq. (3) by numerical calculation of the temperature-dependent $\tau$. This mechanism of the $\rho_{xy}(T)$-dependence might be of significance for our system as well. However, such a calculation would require a number of microscopic parameters of the disorder potential which are poorly-known and therefore should be used as adjustable ones.

At higher temperatures $T \sim T_F$ another mechanism of the $\rho_{xy}(T)$ dependence may come into play if the system contains localized electrons besides the delocalized ones. In this “two-band” model, the total electron concentration $n_{tot}$ is independent of temperature and given by the gate voltage. Due to thermal activation from the localized band, the density of the delocalized carriers becomes dependent on temperature. As temperature increases, a number of the delocalized electrons increases, therefore the Hall resistance decreases.

Let us consider a simple model of this phenomenon, which is schematically shown in Fig. 3. Let the electron system consists of the Fermi-gas of the delocalized particles and a localized band “tail” [15]. The latter has $n_{loc}^0$ states located in the vicinity of the bottom of the conduction band [16], with density of states being $n_{loc}^0 \delta(\varepsilon)$. The total electron concentration is temperature-independent:

$$n_{tot} = \frac{n_{loc}^0}{1 + e^{-\mu/T}} + \frac{2m}{\pi \hbar^2} T \ln(1 + e^{\mu/T}), \quad (5)$$

where $\mu$ is the chemical potential. Only the delocalized carriers contributes to the Hall resistance; their concentration $n_{del}$ is as follows

$$n_{del} = n_{tot} - \frac{n_{loc}^0}{1 + e^{-\mu/T}} \quad (6)$$

In the frameworks of this simple model, there are three parameters $m$, $n_{tot}$ and $n_{loc}^0$ which determine temperature dependences of $\rho_{xy}$ and, consequently, of $\rho_{xy}(T) = H/(n_{del} c)$. The first two parameters can be determined independently, whereas the last parameter is an adjustable one. In the studied range of carrier densities $n > 4 \times 10^{11}$cm$^{-2}$ the renormalization of the effective mass is insignificant [11], therefore we use a bare band mass $m = 0.205 m_e$. The variations of $\rho_{xy}$ are small (Fig. 1, 2), therefore $n_{tot} \approx n_{SdH}$. The remaining adjusting parameter $n_{loc}$ for simplicity was chosen independent of $n_{tot}$.

It appears that $n_{loc}^0 = 0.7 \times 10^{11}$cm$^{-2}$, as obtained from our fitting, provides a good agreement of the model with the “high-temperature” parts of $\rho_{xy}(T)$ data for various concentrations (see Figs. 1a and 2a). It should be emphasized that the results of calculations plotted in Figs. 1a and 2a were arbitrarily shifted vertically by 1-2% for the best coincidence with experimental data, similar to that it was done in the above comparison with Refs. [2,8]. It should be pointed out that $n_{loc}^0$ is approximately equal to $0.5 n_c$ for our 2D electron system, where $n_c \approx 1 \times 10^{11}$cm$^{-2}$ is the critical density of the metal-insulator transition (MIT) at $B = 0$ [17]. This $n_{loc}^0$ value qualitatively agrees with the localized band model [16] and with “a few electrons per ion scenario” of the MIT [18]. Indeed, for a 2D system with bare random potential, the electrons starts filling the conduction band only after the localized electrons have (nonlinearly) screened the potential fluctuations.

In conclusion, we observed a weak ($\sim 2\%$) and nonmonotonic temperature dependence of the Hall resistance for the 2D electron system in Si inversion layer, with a maximum at $T_{max} \approx (0.15 - 0.20) T_F$. In the same range of temperatures and concentrations, the diago-
nal resistivity exhibits strong and monotonically metallic-type behavior. The low-temperature ($T < T_{\text{max}}$) domain of the $\rho_{xy}(T)$ dependence agrees better with the simple semiclassical model [8] ($\propto (T/T_F)^2$), than with the interaction-induced quantum corrections [5]; this indicates that temperature in the studied domain ($T \tau > 0.4$) is too high to observe the quantum corrections. In a “high-temperature” domain ($T > T_{\text{max}}$), the $\rho_{xy}(T)$ dependence can be explained in terms of either a temperature-dependent impurity screening [8], or a thermal activation of the localized electrons to the conductivity band. All considered above models do not contradict each other and should possibly be taken into account simultaneously. It is noteworthy that if the concentration of localized carriers remains about the same down to lower densities, the localized band can play a significant role in the metal-insulator transition.

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