Fresh inflation and decoherence of super Hubble fluctuations

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Abstract

I study a stochastic approach to the recently introduced fresh inflation model for super Hubble scales. I find that the state loses its coherence at the end of the fresh inflationary period as a consequence of the damping of the interference function in the reduced density matrix. This fact should be a consequence of (a) the relative evolution of both the scale factor and the horizon and (b) the additional thermal and dissipative effects. This implies a relevant difference with respect to supercooled inflationary scenarios which require a very rapid expansion of the scale factor to give decoherence of super Hubble fluctuations.

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Introduction and review of fresh inflation: Most of inflationary models are based on the dynamics of a quantum field undergoing a phase transition [1]. The exponential expansion of the scale parameter gives a scale-invariant spectrum naturally. This is one of the many attractive features of the inflationary universe, particularly with regard to the galaxy formation problem. It arises from the fluctuations of the inflaton, the quantum field that induces inflation. This field can be semiclassically expanded in terms of its expectation value plus another field, which describes the quantum fluctuations. The standard slow-roll inflation model separates expansion and reheating into two distinguished time periods. It is first assumed that exponential expansion from inflation places the universe in a supercooled phase. Subsequently the universe in reheated. Two outcomes arise from such a scenario. First, the required density perturbations in this cold universe are left to be created by the quantum

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fluctuations of the inflaton. Second, the temperature cliff after expansion requires a temporally localized mechanism that rapidly sufficient distributes vacuum energy for reheating. So, the scalar field oscillates near the minimum of its effective potential and produces elementary particles. This process is completed when all the energy of the classical scalar field transfers to the thermal energy of elementary particles.

On the other hand, warm inflation takes into account separately the matter and radiation energy densities that are responsible for the fluctuations of temperature. In this scenario, the inflaton field interacts with other particles that are in a thermal bath with a mean temperature smaller than the grand unified theory (GUT) critical temperature \(T_{\text{GUT}}\). The problem with warm inflation is that, at the beginning of the universe, the thermal bath is unjustifiably introduced in the framework of chaotic initial conditions needed to give a natural beginning to the universe. In this sense, chaotic inflation provides a more successful and natural mechanism to describe the initial conditions in the universe. In the warm inflation scenario, slow-roll conditions are induced through dissipative damping with no requirement of an ultraflat potential. Furthermore, this scenario differs from standard inflation in that reheating is no longer required. In warm inflation, exponential expansion and thermal energy production occur together.

Very recently, a new model of inflation called fresh inflation was proposed. Fresh inflation can be viewed as a “unification” of both chaotic and warm inflation scenarios. Fresh inflation incorporates the following characteristics of chaotic and warm inflationary scenarios:

- As in chaotic inflation, the universe begins from an unstable primordial matter field perturbation with energy density nearly \(M_p^4\) \((M_p \approx 1.2 \times GeV\) is the Planckian mass\) and chaotic initial conditions. Furthermore, initially the universe is not thermalized so that the radiation energy density when inflation starts is zero \(\rho_r(t = t_0) = 0\). We understand the initial time to be the Planckian time \(G^{1/2}\). Later, the universe will describe a second-order phase transition. In other words, the inflaton rolls down towards the minimum of the potential.

- Particle production and heating occur together during the rapid expansion of the universe, so that the radiation energy density grows during fresh inflation \(\dot{\rho}_r > 0\). The interaction between the inflation field and the particles produced during inflation provides slow-rolling of the inflaton field. So, in the fresh inflationary model (as in warm inflation), the slow-roll conditions are physically well justified.

- The decay width of the produced particles grows with time. When the inflaton approaches the minimum of the potential, there is no oscillation of the inflaton around the minimum energetic configuration due to dissipation being too large \(\Gamma \gg H\) at the end of fresh inflation. Hence, the reheating period avoids fresh inflation (as in warm inflation).

In this work, I study decoherence of the state that describes super Hubble matter field fluctuations during fresh inflation. Decoherence of super Hubble fluctuations has been studied in the framework of supercooled inflation. It is well known that the evolution of the redefined coarse-grained field is described by a second-order stochastic equation. This topic
was studied in [7] for supercooled inflation and also in [8] for warm inflation. The effective Hamiltonian related to this stochastic equation can be described in a such way that the Schrödinger equation for the system can be written. The wave function that describes this system is \( \Psi(\chi_{cg}, t) \), where \( \chi_{cg} \) denotes the coordinate (i.e., the redefined coarse-grained field fluctuations). The effect of summing over unobservable degrees of freedom (ultraviolet sector) of small inhomogeneous modes reduces to a multiplication of the reduced density matrix by an interference term of the form \( e^{-\alpha(t)|\chi_{cg} - \chi'_{cg}|^2} \), where \( \alpha(t) \) is a time-dependent function and \( (\chi_{cg}, \chi'_{cg}) \) are two different configurations of the redefined coarse-grained field fluctuations. If the interaction of \( \chi_{cg} \) with the environment \( \chi_s \) (\( \chi_s \) takes into account only the short modes) is sufficiently strong to damp quantum interference, the system decoheres. The time evolution of the decoherence function as a consequence of damping of the interference function is the main feature to be analyzed in this paper.

I consider a Lagrangian for a \( \phi \)-scalar field minimally coupled to gravity, which also interacts with another \( \psi \)-scalar field by means of a Yukawa interaction, \[ L = -\sqrt{-g} \left[ \frac{R}{16\pi G} + \frac{1}{2} g^{\mu\nu} \phi,_{\mu} \phi,_{\nu} + V(\phi) \right] + L_{\text{int}}, \] where \( g^{\mu\nu} \) is the metric tensor, \( g \) is its determinant and \( R \) is the scalar curvature. The interaction Lagrangian is given by \( L_{\text{int}} \sim -g^2 \phi^2 \psi^2 \). Furthermore, the indices \( \mu, \nu \) take the values \( 0, \ldots, 3 \) and the gravitational constant is \( G = M_p^{-2} \) (where \( M_p = 1.2 \times 10^{19} \) GeV is the Planckian mass). The Einstein equations for a globally flat, isotropic, and homogeneous universe described by a Friedmann-Robertson-Walker metric \( ds^2 = -dt^2 + a^2(t)dr^2 \) are given by

\[ 3H^2 = 8\pi G \left[ \frac{\ddot{\phi}^2}{2} + V(\phi) + \rho_r \right], \]  
\[ 3H^2 + 2\dot{H} = -8\pi G \left[ \frac{\dot{\phi}^2}{2} - V(\phi) + \rho_r \right], \]

where \( H = \frac{\dot{a}}{a} \) is the Hubble parameter and \( a \) is the scale factor of the universe. The overdot denotes the derivative with respect to the time. On the other hand, if \( \delta = \dot{\rho}_r + 4H\rho_r \) describes the interaction between the inflaton and the bath, the equations of motion for \( \phi \) and \( \rho_r \) are

\[ \ddots + 3H \dot{\phi} + V'(\phi) + \frac{\delta}{\phi} = 0, \]
\[ \dot{\rho}_r + 4H\rho_r - \delta = 0. \]

As in a previous paper [3], I will consider a Yukawa interaction \( \delta = \Gamma(\theta) \phi^2 \), where \( \Gamma(\theta) = \frac{\phi^4 L}{192\pi} \theta \) and \( \theta \sim \rho_r^{1/4} \) is the temperature of the bath. Slow-roll conditions must be imposed to assure nearly de Sitter solutions for an amount of time, long enough to solve the flatness and horizon problems. If \( p_t = \frac{\phi^2}{2} + \frac{\rho_r}{3} - V(\phi) \) is the total pressure and \( \rho_t = \rho_r + \frac{\phi^2}{2} + V(\phi) \) is the total energy density, the parameter \( F = \frac{p_t + \rho_t}{\rho_t} \) which describes the evolution of the universe during inflation [10] is
\[ F = -\frac{2\dot{H}}{3H^2} = \frac{\dot{\phi}^2 + \frac{4}{3} \rho_r}{\rho_r + \frac{\dot{\phi}^2}{2} + V}. \] (6)

When fresh inflation starts (at \( t = G^{1/2} \)), the radiation energy density is zero, so that \( F \ll 1 \).

In this paper, I will consider the parameter \( F \) as a constant. From the two equalities in eq. (6), one obtains the following equations:

\[ \dot{\phi}^2 \left( 1 - \frac{F}{2} \right) + \rho_r \left( \frac{4}{3} - F \right) - F V(\phi) = 0, \] (7)

\[ H = \frac{2}{3 \int F \, dt}. \] (8)

Furthermore, because of \( \dot{H} = H'\dot{\phi} \) (here the prime denotes the derivative with respect to the field), from the first equality in eq. (6) it is possible to obtain the equation that describes the evolution for \( \phi \),

\[ \dot{\phi} = -\frac{3H^2}{2H'} F, \] (9)

and replacing eq. (9) in eq. (7), the radiation energy density can be described as functions of \( V, H \) and \( F \)

\[ \rho_r = \left( \frac{3F}{4 - 3F} \right) V - \frac{27}{8} \left( \frac{H^2}{H'} \right)^2 \frac{F^2(2 - F)}{(4 - 3F)}. \] (10)

Finally, replacing eqs. (8) and (10) in eq. (2), the potential can be written as

\[ V(\phi) = \frac{3}{8\pi G} \left[ \left( \frac{4 - 3F}{4} \right) H^2 + \frac{3\pi G}{2} F^2 \left( \frac{H^2}{H'} \right)^2 \right]. \] (11)

Fresh inflation was proposed for a global group \( O(n) \) involving a single \( n \)-vector multiplet of scalar fields \( \phi_i \) \[11\], such that making \((\phi_i \phi_i)^{1/2} \equiv \phi \), the effective potential \( V_{\text{eff}}(\phi, \theta) = V(\phi) + \rho_r(\phi, \theta) \) can be written as

\[ V_{\text{eff}}(\phi, \theta) = \frac{M^2(\theta)}{2} \phi^2 + \frac{\lambda^2}{4} \phi^4, \] (12)

where \( M^2(\theta) = M^2(0) + \frac{(n+2)}{12} \lambda^2 \theta^2 \) and \( V(\phi) = \frac{M^2(0)}{2} \phi^2 + \frac{\lambda^2}{4} \phi^4 \). Furthermore, \( M^2(0) > 0 \) is the squared mass at zero temperature, which is given by \( M^2(0) \) plus renormalization counterterms in the potential \[ \frac{1}{2} M^2(0)(\phi_i \phi_i) + \frac{1}{4} \lambda^2 (\phi_i \phi_i)^2 \] \[12\]. I will take into account the case without symmetry breaking, \( M^2(\theta) > 0 \) for any temperature \( \theta \), so that the excitation spectrum consists of \( n \) bosons with mass \( M(\theta) \). Note that the effective potential (12) is invariant under \( \phi \to -\phi \) reflections and \( n \) is the number of created particles due to the interaction of \( \phi \) with the particles in the thermal bath, such that \[13\]

\[ (n + 2) = \frac{2\pi^2}{5\lambda^2} g_{\text{eff}}(\theta^2). \]
because the radiation energy density is given by \( \rho_r = \frac{\pi^2 g_{\text{eff}}}{60} \theta^4 \) (\( g_{\text{eff}} \) denotes the effective degrees of freedom of the particles and it is assumed that \( \psi \) has no self-interaction). A particular solution of eq. (11) is

\[
H(\phi) = 4\sqrt{\frac{\pi G}{3(4 - 3F)}} \mathcal{M}(0) \phi. \tag{14}
\]

From eq. (8), and due to \( H = \dot{a}/a \), one obtains the scale factor as a function of time

\[
a(t) \sim t^{\frac{2}{3F}}. \tag{15}
\]

Furthermore, the number of e-folds during fresh inflation is

\[
N(t) = \frac{2}{3F} \ln \left( \frac{t}{t_s} \right) \bigg|_{t_e} , \tag{16}
\]

which, due to the fact that \( \phi = \lambda^{-1} t^{-1} \), can be written as a function of \( \phi \): \( N(\phi) = \frac{2}{3F} \ln \left( \frac{1}{\lambda \phi} \right) \bigg|_{\phi_e} \), which grows as \( \phi \) decreases. Here, \( (t_s, t_e) \) are the starting and ending values of time and \( (\phi_s, \phi_e) \) are the starting and ending values of \( \phi \) (for \( t_e > t_s \) and \( \phi_e < \phi_s \)). Taking \( g_{\text{eff}} \approx 10^2 \) and \( \phi_e \approx 10^{-4} G^{-1/2} \) one obtains the number of created particles at the end of fresh inflation \( n_e \approx 4 \times 10^8 \).

**Coarse-graining, interference and decoherence of the fluctuations:** Classical physics is characterized by the fact that one can assign a probability to each possible history of the system. In contrast, in quantum physics one must assign a complex probability amplitude to configuration variables, because there are no trajectories in quantum theory. When combined with the superposition principle, this implies the existence of quantum interference effects. On the other hand, it is an empirical fact that these effects are not seen at a macroscopic (or classical) level. One of the most popular ways of fixing this problem is by means of decoherence arguments, whose essence is the following: One can consider that the original system is part of a more complicated world and interacts with an “environment” formed by unobserved (or irrelevant) degrees of freedom. Then, under some circumstances, it is possible to show that the quantum interference effects on the system can be suppressed by the interaction with the environment. The system thus decoheres (it loses quantum coherence) and can be described as a statistical mixture of noninterfering branches [15]. Other possible descriptions involve the Schrödinger [16,17] or Wigner [14] pictures.

The equation of motion for the matter field fluctuations \( \delta \phi(\vec{x}, t) \) is

\[
\ddot{\delta \phi} + (3H + \Gamma) \dot{\delta \phi} - \frac{1}{a^2} \nabla^2 \delta \phi + V''(\phi) \delta \phi = 0, \tag{17}
\]

where \( V'' \) denotes the second derivative of the potential with respect to the field. This equation can be simplified by means of the map \( \chi = e^{3/2 \int (H + \Gamma/3)} \, dt \, \delta \phi \)

\[
\ddot{\chi}_k + \omega_k^2(t) \, \chi_k = 0, \tag{18}
\]

where I supposed a Fourier expansion for the scalar field \( \chi \) in terms of its modes \( \chi_k = e^{i \vec{k} \cdot \vec{x}} \xi_k(t) \). Furthermore, \( \omega_k^2(t) = a^{-2} [k^2 - k_0^2(t)] \) is the squared frequency for each mode with
wave number $k$, and $k_0(t)$ gives the time-dependent wave number that separates the infrared (IR) and ultraviolet (UV) sectors. The IR sector describes the large-scale fluctuations (or super Hubble fluctuations) and the UV sector takes into account the matter field fluctuations on sub Hubble scales ($k^2 \gg k_0^2$). The squared time-dependent wave number $k_0^2(t)$ is given by the expression

$$k_0^2 = a^2 \left[ \frac{9}{4} \left( \frac{H + \frac{1}{3} \dot{\Gamma}}{\dot{H} + \frac{1}{3} \Gamma} \right)^2 + 3 \left( \frac{\dot{H} + \frac{1}{3} \Gamma}{\dot{\Gamma}} \right) - V'' \right],$$

where $\Gamma = \frac{g_{\nu s}}{192 \pi} \theta$ and the temperature $\theta(t)$ is given by

$$\theta(t) = \frac{192 \pi}{g_{\nu s} \lambda^2} \left\{ M^2(0) \lambda^2 t + t^{-1} \left[ \lambda^2 (9F^2 - 18F + 8) + \mathcal{M}^2(0) \pi G (192F^2 - 72F^3 - 96) \right] \right\},$$

which, for late times, increases linearly with time. Note that $\theta(t)$ becomes zero at $t = G^{1/2}$ (we use Planckian units).

The redefined fluctuations $\chi$ can be decomposed in long-and short-wavelength components $\chi_{cg}$ and $\chi_s$, respectively. In our case, we are interested in the study of decoherence for super Hubble (cosmological) scales. So, the relevant degrees of freedom will be $k^2 \ll k_0^2$, which are incorporated in the coarse-grained field $\chi_{cg}$. The field $\chi_s$ takes into account the unobserved degrees of freedom (environment). The redefined coarse-grained field, can be written as a Fourier expansion in the $k$ space,

$$\chi_{cg}(\vec{x}, t) = \frac{1}{(2\pi)^3/2} \int d^3k \theta(\epsilon k_0 - k) \left[ a_k \chi_k(\vec{x}, t) + a_k^\dagger \chi_k^*(\vec{x}, t) \right],$$

where $\epsilon \ll 1$ is a dimensionless constant and $\chi_k(\vec{x}, t)$ are the modes of $\chi_{cg}$. Furthermore, the annihilation and creation operators $(a_k, a_k^\dagger)$, satisfy the commutations relations $[a_k, a_{k'}^\dagger] = \delta^{(3)}(k - k')$ and $[a_k, a_{k'}] = [a_k^\dagger, a_{k'}^\dagger] = 0$.

The stochastic equation for the redefined coarse-grained field, $\ddot{\chi}_{cg} + \omega_k^2(t) \chi_{cg} + \xi_c(\vec{x}, t) = 0$, can be approximated to the zero-mode stochastic equation for very large scale fluctuations (i.e., for $k^2 \ll k_0^2$),

$$\ddot{\chi}_{cg} - \mu^2(t) \chi_{cg} + \xi_c(\vec{x}, t) = 0,$$

where $\xi_c = -\epsilon \left[ \frac{d}{dt} (\dot{k}_0 \eta) + 2\dot{k}_0 \kappa \right]$ is the effective noise and $(\eta, \kappa)$ are given by

$$\eta(\vec{x}, t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \delta(\epsilon k_0 - k) \left[ a_k \chi_k + h.c. \right],$$

$$\kappa(\vec{x}, t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \delta(\epsilon k_0 - k) \left[ a_k \dot{\chi}_k + h.c. \right].$$

This noise arises from (a) the inflow of short-wavelength modes produced by the relative evolution of both the horizon and the scale factor of the universe, and (b) the created particles during fresh inflation, which are thermalized.
The effective Hamiltonian for eq. (22) is
\[ H_{\text{eff}}(\chi_{cg}, t) = \frac{1}{2} P_{cg}^2 - \mu^2(t) \chi_{cg}^2 + \xi_c \chi_{cg}, \tag{25} \]
where \( P_{cg} \equiv \dot{\chi}_{cg} \). It is here that we depart from the Heisenberg to the Schrödinger picture. The derivative operator representation of the momentum ensures that the Heisenberg and the Schrödinger pictures reflect the same physics. Furthermore, \( \xi_c \) is an external classical force in the quantum Hamiltonian (25). With this assumption we can write the Schrödinger equation [17]
\[ i \frac{\partial}{\partial t} \Psi(\chi_{cg}, \xi_c, t) = -\frac{1}{2} \frac{\partial^2}{\partial \chi_{cg}^2} \Psi(\chi_{cg}, \xi_c, t) + \left( -\frac{\mu^2(t)}{2} \chi_{cg}^2 + \xi_c \chi_{cg} \right) \Psi(\chi_{cg}, \xi_c, t). \tag{26} \]
Here, \( \Psi(\chi_{cg}, \xi_c, t) \) is the wave function that describes the system in the \( \chi_{cg} \) representation. Notice that the states \( \Psi(\chi_{cg}, \xi_c, t) \) are fiducial because no pure states can be attributed to a quantum subsystem that is entangled with other subsystems. A method to solve this equation can be found in [12]. If the initial state is taken to be a harmonic-oscillator ground state and the external (stochastic) force \( \xi_c \) is set to zero, then, because of the time-dependent frequency (\( \omega^2 \equiv 0 = -\mu^2 \)), the solutions of the Schrödinger equation are squeezed states at zero momentum. In our case, the state becomes highly squeezed due to the accelerated expansion of the universe and the \( \phi - \psi \) Yukawa interaction, which introduces additional friction terms (proportional to \( \Gamma, \Gamma^2, \) and \( \dot{\Gamma} \) or \( \theta, \theta^2, \) and \( \dot{\theta} \)) in \( k_0^2(t) \) [see eq. (19)]. On the other hand, if the frequency is time-independent but the external force is nonzero, the solutions are coherent states [13]. As can be seen readily, equation (26) combines both situations.

The solution for the Schrödinger equation (26) is given by [18]
\[ \Psi(\chi_{cg}, \xi_c, t) = \frac{1}{(2\pi)^{1/4} \Delta^{1/2}} e^{-\frac{1}{4 \Delta^2} [\chi_{cg} - \chi_{cl}]^2} e^{\frac{\chi_{cg}^2}{4 \Delta^2}} e^{\frac{2 \xi_c}{\Delta^2} \left[ \frac{\varphi^2}{\sigma^2} + \frac{R(t)}{\sigma^2} \right]} \times e^{\frac{\chi_{cg}^2}{2 \Delta^2} \left[ \frac{\varphi^2}{\sigma^2} - \frac{R(t)}{\sigma^2} \right]} e^{i \gamma(t)}, \tag{27} \]
where \( \gamma(t) \) is an arbitrary phase and, in our case, the time-dependent functions \( B \) are given by the zero modes \( \xi_0 \) of the redefined coarse-grained fluctuations \( \chi_{cg} \) [i.e., for \( B(t) = \xi_{k=0}(t) \equiv \xi_0(t) \)] [18]:
\[ R(t) = \int_0^t dt' \frac{1}{2 \xi_0^2(t')}, \tag{28} \]
\[ \Delta^2(t) = \frac{\xi_0^2(t)}{\sigma^2} \left[ \sigma^4 + R^2(t) \right], \tag{29} \]
where \( \sigma^2 = \Delta^2(t_0) \) is a constant and \( \Delta^2 \) gives the squared dispersion. The reduced density matrix related to the wave function \( \Psi(\chi_{cg}, \xi_c, t) \) for two different configurations of \( \chi_{cg} \) and \( \chi'_{cg} \) is [13]
\[ \rho_{\text{red}}(\chi_{cg}, \chi'_{cg}, t) = \int d\xi_c \rho_{\text{red}}(\chi_{cg}, \chi'_{cg}, \xi_c, t), \tag{30} \]
where \( \rho_{\text{red}}(\chi_{cg}, \chi'_{cg}, \xi_c, t) = \Psi^*(\chi_{cg}, \xi_c, t) \Psi(\chi'_{cg}, \xi_c, t) \) such that

\[
\rho_{\text{red}}(\chi_{cg}, \chi'_{cg}, t) = \int d\xi_c \rho_0(\chi_{cg}, \chi'_{cg}, \xi_c, t) \mathcal{I}(\chi_{cg}, \chi'_{cg}, t),
\]

and

\[
\rho_0(\chi_{cg}, \chi'_{cg}, t) = \frac{1}{(2\pi)^{1/2}} e^{-\frac{1}{4\Delta^2} \left[ (\chi_{cg} - \chi_{cl})^2 + (\chi'_{cg} - \chi_{cl})^2 \right]}
\times e^{-i \frac{1}{4\Delta^2} \Delta^2 \left( P_{cl} - \frac{\kappa(t) \chi_{cl}}{2\sigma^2} \right) [\chi_{cg} - \chi'_{cg}]}
\times e^{-i \frac{1}{4\Delta^2} \left( \frac{\kappa(t) \chi_{cl}}{2\sigma^2} \right) \chi_{cg} \chi'_{cg}}.
\]

The interference function \([14]\) that multiplies \( \rho_0 \) in eq. (31) is

\[
\mathcal{I}(\chi_{cg}, \chi'_{cg}, t) = e^{-i \sqrt{D} \chi_{cg} \chi'_{cg}}.
\]

where \( D(t) \) is the function that describes decoherence in the quantum state \( \Psi[16,18] \),

\[
D(t) = 1 - \frac{1}{2} \left( \frac{\xi_0 \Delta(t) + \mathcal{R}(t)}{2\Delta^2} \right)^2.
\]

(34)

(Note that in paper \([14]\) the authors named this function the “decoherence function,” but it describes the evolution of quantum interference.) The squared fluctuations \( \langle \chi_{cg}^2 \rangle \) and \( \langle P_{cg}^2 \rangle \) are

\[
\langle \chi_{cg}^2 \rangle = \chi_{cg}^2(t) + \Delta^2(t),
\]

and

\[
\langle P_{cg}^2 \rangle = P_{cl}^2 + \frac{1}{4\Delta^2(t)} + 2D(t),
\]

where \( \langle P_{cg} \rangle = P_{cl} \) and \( \langle \chi_{cg} \rangle = \chi_{cl} \). Notice we are dealing with the \( \chi_{cg} \) representation, so that \( P_{cg} \equiv -i \frac{\partial}{\partial \chi_{cg}} \). The second terms in eqs. (35) and (36) represent the quantum fluctuations which depend only on \( \mu(t) \). The third term in eq. (36) represents quantum fluctuations related to decoherence. In the case of a pure quantum state (i.e., in a coherent state) the function \( D(t) \) becomes zero.

All we need to find the evolution of the decoherence function \([14]\) in fresh inflationary cosmology is to solve the equation for the zero mode function

\[
\ddot{\xi}_0(t) - \mu^2(t) \xi_0(t) = 0,
\]

and later calculate the functions \( \mathcal{R}(t) \) and \( \Delta(t) \). The squared parameter of mass \( \mu^2(t) = \frac{k^2}{a^2} \), in eq. (37) is

\[
\mu^2(t) = \frac{1}{(4 - 3F)^2} \left[ A_1 + A_2 \ t^2 + A_3 \ t^{-2} \right],
\]

where the constants \( A_1, A_2, \) and \( A_3 \) are given by
\[ A_1 = 9F \mathcal{M}^2(0) \left( \frac{F}{2} - 1 \right) + \frac{\mathcal{M}^4(0) \pi G}{\lambda^2} \left( 96F^2 - 36F^3 - 48 \right) + \mathcal{M}^2(0) \left( \frac{1}{F} - 1 + \frac{1}{\lambda^2} \right) + 1, \quad (39) \]

\[ A_2 = \frac{\mathcal{M}^4(0)}{4}, \quad (40) \]

\[ A_3 = \left\{ \frac{1}{(4 - 3F)^2} \left[ \frac{8}{F} - 9F^2 + 27F - 26 + 4\mathcal{M}^2(0) \right. \right. \]
\[ + \frac{\mathcal{M}^2(0) \pi G}{\lambda^2} \left( 72F^2 + 96 - 264F^2 - 96 \right) + \frac{1}{(4 - 3F)^2} \left[ \frac{81F^4}{4} - 81F^3 + 117F^2 \right. \]
\[ - 72F + 16 + \frac{\mathcal{M}^2(0) \pi G}{\lambda^2} \left[ 1512F^4 - 324F^5 + 336F^2 - 2016F^3 + 864F - 384 \right. \]
\[ + \frac{\mathcal{M}^2(0) \pi G}{\lambda^2} \left( 9216F^2 \left( F^2 - 1 \right) - 6912F^5 + 1296F^6 + 3456F^3 + 2304 \right) \left. \right] \]
\[ - 3 + \frac{1}{F} \left( \frac{1}{F} - 2 \right) \} \right. \}. \quad (41) \]

For late times, the squared parameter of mass can be approximated to
\[ \mu^2(t) \big|_{t \gg G^{1/2}} \simeq A_2 t^2. \quad (42) \]

The solution for the zero modes \( \xi_{k=0}(t) \), in eq. (37) is
\[ \xi_0(t) = c_1 \sqrt{t} I_\nu \left[ \frac{1}{2} \sqrt{A_2 t^2} \right] + c_2 \sqrt{t} K_\nu \left[ \frac{1}{2} \sqrt{A_2 t^2} \right], \quad (43) \]

where \( I_\nu \) and \( K_\nu \) are the Bessel functions, \( \nu = 1/4 \), and \((c_1, c_2)\) are arbitrary constants. For very large times (i.e., for \( t \gg 10^7 G^{1/2} \)), we can use the asymptotic expressions for the Bessel functions,
\[ I_\nu[x] \simeq \frac{e^x}{\sqrt{2\pi x}}, \quad (44) \]
\[ K_\nu[x] \simeq \frac{e^{-x}}{\sqrt{2\pi x}}, \quad (45) \]

so that the asymptotic zero mode \( \xi_0 \) can be approximated to
\[ \xi_0(t) \big|_{t \gg G^{1/2}} \propto \frac{1}{t^{1/2} \mathcal{M}(0)} e^{\frac{\mathcal{M}^2(0)}{2 t^2}}. \quad (46) \]

Hence, at the end of fresh inflation the function \( \mathcal{R}(t) \) becomes [see eq. (28)]
\[ \mathcal{R}(t) \sim e^{-\mathcal{M}^2(0) t^2}, \quad (47) \]

and the squared dispersion [see eq. (29)] is
\[ \Delta^2(t) \simeq \frac{1}{t \mathcal{M}^2(0)} e^{\mathcal{M}^2(0) t^2}. \quad (48) \]

Finally, the asymptotic evolution of the decoherence function \( \mathcal{D}(t) \) is given by [see eq. (34)]
\[ D(t) \sim M^2(0) t e^{-M^2(0)t^2}, \]  

which grows with time.

Finally, it must be noted that the relevant field configuration for the super Hubble inflaton fluctuations during the inflationary stage is \( \delta \phi_{cg} = e^{-3/2 \int [H(t) + \Gamma(t)/3] dt} \chi_{cg} \). This means that the relevant decoherence function during inflation is

\[ D(\delta \phi_{cg}, t) \big|_{t \gg G_{1/2}} \simeq D(t) \big|_{t \gg G_{1/2}} t^{-2/F} e^{-\frac{M^2(0)}{2}t^2}, \]

where we have taken \( \Gamma(t) \simeq M^2(0)t \), which is a good approximation for late times. Hence, the decoherence function for \( \delta \phi_{cg} \) goes as

\[ D(\delta \phi_{cg}, t) \big|_{t \gg G_{1/2}} \sim t^{2(F-1)/F} e^{\frac{M^2(0)}{2}t^2}, \]

which also increases with time at the end of fresh inflation. Note that the decoherence function is maximized for large values of \( F \). In particular, when \( F > 1 \), the universe approaches a radiation-dominated regime (\( F = 4/3 \)). However, as was demonstrated in [5], to ensure slow-roll conditions one requires \( F < 0.55 \). Furthermore, the interference function \( I(\delta \phi_{cg}, \delta \phi'_{cg}, t) \) is given by

\[ I(\delta \phi_{cg}, \delta \phi'_{cg}, t) = e^{-\frac{i}{\sqrt{2}} \Gamma^2(\delta \phi_{cg} - \delta \phi'_{cg})^2}, \]

In the fresh inflationary model here considered gives

\[ I(\delta \phi_{cg}, \delta \phi'_{cg}, t) = \exp \left\{ -i \frac{t^{F+2}}{\sqrt{2} M(0)} e^{\frac{M^2(0)}{4}t^2} \left[ \delta \phi_{cg} - \delta \phi'_{cg} \right]^2 \right\}, \]

which decreases monotonically with time during the fresh inflationary period. Hence, during fresh inflation the quantum interference is damped very rapidly.

To summarize, in this work I studied decoherence of super Hubble fluctuations in the recently introduced fresh inflationary scenario. Decoherence occurs when there are no interference effects between alternative histories. In the context of inflationary cosmology, the system is the super Hubble fluctuations. This field takes into account the relevant degrees of freedom of the system (coarse-grained field). The irrelevant (or unobserved) degrees of freedom contain the modes of the ultraviolet sector (sub-Hubble sector). During the inflationary expansion of the universe, this sector works as an environment. Roughly speaking, one can say that the environment is “continuously measuring” the \( \delta \phi_{cg} \) variable and that this continuous measurement process suppresses quantum interference. It is a consequence of (a) the inflow of short-wavelength modes produced by the relative evolution of both the horizon and the scale factor of the universe (this effect is responsible for the squeezing of the state) and (b) additional thermal and dissipative effects. The latter are relevant to the dissipative (thermal) effects produced by the interaction between the inflaton field and other \( \psi \)-fields in the universe. Both facts produce decoherence of the coarse-grained field leading to the quantum-to-classical transition of the \( \delta \phi_{cg} \) field on super Hubble scales. This means that the stochastic description for super Hubble fluctuations in fresh inflation is consistent. This is the main difference with supercooled inflation (see [18]), in which the power of the expansion of the scale factor must be very large (i.e., requires \( p > 4.6 \) when \( a \sim t^p \)) in order for the system to lose its coherence during inflation. Notice we are not dealing with correlations between coordinates and momenta. In order to analyze whether a given wave function
predicts the existence of correlations between coordinates and momenta, many authors proposed to examine the peaks of the Wigner function associated with it [19]. However, this topic is not the subject of this work.

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