Heat Transfer of ERF in a Flat Channel Taking Dissipative Factor Into Account

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Abstract. The results of an investigation of rheophysical properties of a thermally stable dielectric controllable heat carrier — electrorheological fluid on the basis of hydrated aluminum oxide in the form of pseudo benit, are presented. Considering the obtained experimental data, we have received and discuss the numerical solution of a conjugated problem of hydrodynamics and heat transfer in a flat channel of ERF, the flow curve of which is described by a modified Buhlley-Herschel model. It is shown that the electrorheological effect results in the increase of an average over the channel length Nusselt number up to 17 %.

1. Introduction

Available models of hydrodynamics [1-4] and heat transfer [5-6] in narrow flat and circular channels are regarded to be difficult systems in the rheological aspect, as electrorheological fluids (ERF) are based on substantial assumptions relative to rheological and thermophysical properties of these fluids in respect of the dependence of these properties on the temperature and intensity of electric field. One of the main simplifications applied was to neglect the dissipation of energy into heat due to the internal friction. In a number of devices, operating on the basis of the electrorheological effect, the flux of electrorheological fluid (ERF) along a narrow circular or a flat channel is realized. Moreover, the dissipative heating of the whole system takes place. The change of the temperature results in the change of rheological properties of ERF. At high enough temperatures such devices can become inoperative. This investigation is dedicated to the problems of heat transfer of ERF in such conditions. The main aim of the paper is to evaluate the influence of the intensity of the electric field and the temperature on heat transfer at ERF flowing in the flat channel with and without consideration of the dissipative factor. The second aim of this paper is to analyze the possibilities of the intensification of heat transfer due to the electrorheological effect.

2. Statement of the problem of numerical modeling hydrodynamics and heat transfer of ERF in a narrow channel

In our approximation we consider that the channel length substantial excesses the width of its slot. It allows us to simplify the set of equations of motion due to the neglect of the summands that contain space derivatives of the transversal part of velocity. In practice, the possible supposition is also a supposition on the incompressibility of ERF and the limitation due to the consideration of only the laminar regime of flowing of such fluid. In the result, the equations of motion are reduced to one equation, identical to the boundary-layer equation. In such approximation the pressure depends only on the longitude coordinate and is constant across
the channel. In Cartesian reference system the set of nondimensionalized equations of mass and momentum conservation are changed into the following:

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0$$  \hfill (1)

$$\frac{\text{Re}}{\nu} \left( u'' \frac{\partial u'}{\partial x'} + v'' \frac{\partial u'}{\partial y'} \right) = -\frac{1}{\mu'} \frac{\partial P'}{\partial x'} + \frac{\partial^2 u'}{\partial y'^2}$$  \hfill (2)

$$u'' = u' - \frac{2}{\text{Re}} \frac{\partial v'}{\partial x'}, \quad v'' = v' - \frac{1}{\text{Re}} \frac{\partial v'}{\partial y'}$$  \hfill (3)

Where

$$u' = \frac{u}{U_0}, \quad v' = \frac{v}{U_0}, \quad x' = \frac{x}{H}, \quad y' = \frac{y}{H}, \quad P' = \frac{P \text{Re}}{\rho U_0^2}, \quad \nu' = \frac{\nu}{\nu_0}, \quad \rho' = \frac{\rho}{\rho_0},$$

where $H$ is the channel width, $P$ is pressure, $\rho$ is ERF density, $U_0$ is the mean velocity at section of channel. We use only the $x$ component of the Navier-Stokes equation because second equation is made approximation. Rheological properties of a thermally stable ERF were given for the thermal-and-hydraulic calculations. Rheological behavior of such medium is described rather well by the model of Bulkley–Herschel [7-10]:

$$\tau(E, T, \dot{\gamma}) = \tau_0(E, T) + K(E) \dot{\gamma}^{n(K)},$$  \hfill (4)

where $\tau_0$ is the yield stress, $\tau$ is the shear stress, $\dot{\gamma}$ is the shear rate, $K$ is the consistency parameter.

According to (4), sheer relative viscosity of ERF is described by the equation

$$\mu' = \frac{\tau_0}{\tau_{\text{ref}}} \frac{\tau'}{\dot{\gamma}'} \dot{\gamma}^{n-1},$$  \hfill (5)

where

$$\dot{\gamma}' = \frac{H}{U_0} \dot{\gamma} = \left[ \left( \frac{\partial u'}{\partial x'} \right)^2 + \left( \frac{\partial v'}{\partial y'} \right)^2 + \left( \frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'} \right)^2 \right]^{1/2}$$  \hfill (6)

$$\tau_0' = \frac{\tau_0}{\tau_{\infty}}, \quad \tau_0 = \tau_0(E = 0, T = T_0), \quad K' = \left( \tau_{\text{ref}} \right)^{n-1} \delta^n \mu', \quad \tau_{\text{ref}} = \frac{\mu_0 U_0}{H}, \quad \mu' = \frac{\mu}{\mu_0},$$  \hfill (7)

In (7) the parameter $\delta = 1 \text{Pa}^{1/n-1}$. It is introduced to comply with dimensionality rule.

When convectional heat transfer along the channel and dissipation of hydraulic energy into the heat are taken into consideration, the energy equation is:

$$\frac{\partial \Theta}{\partial F_0} + \text{Pe} \frac{\partial \Theta}{\partial x'} u' = \frac{\partial}{\partial x'} \left( a \frac{\partial \Theta}{\partial x'} \right) + \frac{\partial}{\partial y'} \left( a \frac{\partial \Theta}{\partial y'} \right) + \Phi.$$  \hfill (8)

Non-dimensional complexes included in (8) are described by the following equations: $\text{Pe} = U_0 H / a_0$, $\Phi = \Pi \nu \dot{\gamma}^2$, $\Pi = v_0 U_0^2 / [a_0 (T_u - T_0)]$, where $a_0$ is a thermal conductivity at the temperature $T_0$ of ERF at the entrance of the channel, $v_0$ is a reference value of the apparent viscosity of ERF, $T_u$ is an average temperature
of the channel wall, $\Theta = (T - T_e)/(T_e - T_i)$ is a non-dimensional temperature. The initial condition is chosen as the condition of constant temperature in the channel. Further we assume that the temperature in the channel is the same as at the entrance of the channel. At the boundary $y = 0$ we use the heat transfer condition of the first kind (the boundary condition of H1):

$$\Theta(Fo = 0, x', y') = 0, \quad \Theta(Fo > 0, x' = 0, y') = 0, \quad \Theta(Fo > 0, x' > 0, y' = 0) = 1,$$

$$\frac{\partial \Theta}{\partial x'}|_{x' = L/H} = 0, \quad \frac{\partial \Theta}{\partial y'}|_{y' = 1} = 0,$$

where $L$ is the channel length. Local and integrated values of Nusselt number are determined according to expressions

$$\text{Nu} = -2 \frac{\partial \Theta}{\partial y'}|_{y' = 0}, \quad < \text{Nu} > = -\frac{2H}{L} \int_0^{L/H} \text{Nu}(x')dx'$$

The influence of the intensity of the electric field and temperature on Bulkley-Herschel model parameters is shown in figure 1, in which experimental data are displayed.

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Experimental dependencies of Bulkley-Herschel model parameters on the electric field intensity (left) and temperature (right). In right panel, curve 1 is for $E = 0.5, 2 - 1.0, 3 - 1.5, 4 - 2.0, 5 - 2.5$ kV/mm

These data were used to perform a computational experiment on quantitative evaluation of the effectiveness of heat transfer for developed ERF in a narrow circular slot (or a flat slot). In the numerics we find also the influence of the intensity of electric field on Nusselt number. The width of the slot was equal to 1 mm. The width of the plate was 100 mm. The length of the channel was varied from 0.25 m up to 1 m. The ERF temperature at the entrance was equal to $25^\circ$C. As the result of the solution of thermal-and-hydraulic problem the drop of the pressure along the channel, providing given mass velocity of ERF flowing, was determined.

3. **Numerical modeling and its results**

The hydrodynamic and heat problems (1-8) have been solved by the finite difference method. We have used the implicit scheme. The equations (4) – (6) describe interrelations between a non-dimensional shear rate and shear stress, and a non-dimensional apparent viscosity depending on non-dimensional shear rate. The expressions (7) give the procedure for nondimensionalizing. Evaluation of the impact of the electric field intensity on the mean and local (at the exit of the channel) value of Nusselt number and also thermal condition for heat transfer device is carried out in comparison for ERF and water. With this aim thermal-and-hydraulic calculation was performed at the boundary condition H1, mass rate of heat carrier (ERF or water) of 0.01 kg/s
and the value of heat stream averted from the channel wall of 10 kW/m². The result of such evaluation is shown in figure 2. The effect of electric field application is 10.4 % for the mean value of Nusselt number and 7.7% for the local value of Nusselt number at the channel exit (to the side of enhancement in comparison with the heat response coefficient without field). For ERF <Nu > is 17.1 % higher in comparison to water at application of electric filed intensity of 2.5 kV/mm, and local Nusselt number is 11.1% higher. Nevertheless, due to the approximately two times higher thermal-capacity and about five times higher thermal conductivity, water shows more effective cooling of the channel wall. We will note only that there was heat stabilization of the temperature profile (the local value of Nusselt number was put constant at the substantial distance from the exit of the channel, the flow was stationary and laminar) for the modeling conditions in both cases. The creeping type of flowing is characteristic for ERF at mass rate of 0.01 kg/s (Re<<1). Moreover the shear rate near the channel wall has been changed substantially depending on the value of electric field intensity (from 600 s⁻¹ without electric field, up to 2000 s⁻¹ at E = 2.5 kV/mm).

![Figure 2. The influence of the electric field intensity on Nusselt number. Curves 1 and 1′ mean the value of Nusselt number along the channel length; curves 2 and 2′, the local value of Nusselt number at the exit of the channel. The solid curves are for ERF, and the dash curves are for water.](image)

Dissipative heating of ERF was less than 0.1 °C. Dissipative heating of ERF has started to appear at mass rates more than 0.1 kg/s and was accompanied by the decrease of heat transfer coefficient by 8 %, and also by the increase of the average temperature in the exit cross section by more than 0.6 °C.

4. Conclusions
Electrorheological effect leads to a substantial intensification of heat transfer that should be considered at projecting devices based on it. The increase of nondimensionalized coefficient of heat, due to the increase of shear rate at the walls of the channel as a result of formation of a quasi solid core of flowing is not high enough for ERF to become a more effective heat carrier by an increase of the intensity of the electric field than water. Also by an increase of the intensity if the electric field the increase of hydraulic resistance and dissipative heating of ERF is observed. As the result, the increase of Nusselt number is changed by the decrease of this parameter.

5. References
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