Change of shape in the yrast sequence in $^{50}$Cr

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In shell model calculations for the yrast even spin states of $^{50}$Cr, the static quadrupole moments of the low spin states $2^+_1$ and $4^+_1$ are negative but those of the high spin states $10^+_1$, $12^+_1$ and $14^+_1$ are positive. Even spin states beyond the single $j$ shell limit, $J_{\text{max}}=14$, again have negative moments. While the $B(E2)$'s for the $J \rightarrow J-2$ transitions are strongest along the yrast path for $J \leq 8$, it is found that the transition from the second $J=10$ state to the first $J=8$ state ($10^+_2 \rightarrow 8^+_1$) is much stronger than the $10^+_1 \rightarrow 8^+_1$ transition. We also note that while the $16^+_1 \rightarrow 14^+_1$ transition is weak, the $16^+_2 \rightarrow 14^+_2$ is quite strong.

II. THE SINGLE J SHELL RESULTS FOR $^{50}$Cr

In the single $j$ shell ($f_{7/2}$) calculation for $^{50}$Cr, one can get states of all spins up to $J_{\text{max}}=14$. For $J=14$, there is a unique configuration in which the four protons couple to angular momentum $J_p=8$ and the six neutrons to $J_n=6$. All yrast even spin states up to $J=12$ have been identified experimentally. The energies of these even $J$ states from $J=2$ to $J=12$ as given in Ref. [1] are 0.783, 1.881, 3.167, 4.745, 6.341 and 7.614 MeV, respectively. The single $j$ MBZ results are 1.100, 2.173, 3.264, 4.982, 6.431 and 8.282 MeV. The overall agreement is quite good, considering the simplicity of the model. It should be noted that the MBZ and GF single $j$ results are much better than those of the “FPD6” interaction [2] used here (see Table I). The reason for this is that the “FPD6” interaction is designed to give good results in an extended $fp$ space calculation, rather than the $f_{7/2}$ space. In the older calculations, the act of taking matrix elements from experiment does succeed to a large extent in mocking up the effects of configuration mixing. However, we will soon see that even if this empirical procedure works for energies, it does not work for other quantities such as matrix elements of one-body operators, the quadrupole operator in particular.

In Table I we show the results of the quadrupole moments $Q$ and $B(E2)$'s for $J \rightarrow J-2$ transitions with the “FPD6” interaction [6]. We list separately the proton and neutron quadrupole moments $Q_p$ and $Q_n$ and the $E2$ amplitudes $A_p$ and $A_n$ such that

$$B(E2: J \rightarrow J-2) = (e_p A_p + e_n A_n)^2,$$

$$Q = e_p Q_p + e_n Q_n,$$

where $e_p$ and $e_n$ are the effective charges for protons and neutrons, which we take to be 1.5 and 0.5. We note that the $B(E2)$'s along the yrast path are quite strong but the quadrupole moments change sign as we go from low spin to high spin.

It is not difficult to understand the single $j$ results for the quadrupole moments. For the $f_{7/2}$ shell, the quadrupole moment of a single proton is negative:

$$Q(1p: j) = -\frac{2j-1}{2j+2}(r^2) = -3b^2.$$

The quadrupole moment for a proton hole is opposite in sign to that of a proton: $+3b^2$. Here $b = \sqrt{\hbar/(M\omega)}$ is
the characteristic length of the harmonic-oscillator basis. For two protons in the $j$ shell coupled to a total angular momentum of $J$, the quadrupole moment is

$$Q(2p : J) = \frac{2(2 J J | J J)}{(2j_0 0j | j j)} U(2j J j | j J)Q(1p). \tag{4}$$

Using $b = 2.0 \text{ fm}$, we get

$$Q(2p : J = 2) = 7.84 \text{ e fm}^2,$$

$$Q(2p : J = 4) = 1.25 \text{ e fm}^2,$$

$$Q(2p : J = 6) = -13.70 \text{ e fm}^2. \tag{5}$$

With four protons we are at midshell and the static quadrupole moment for any state of the $j^4$ configuration is zero.

For $^{50}\text{Cr}$, the simplest state is the one with the maximum possible $J$ in the single $j$ shell, namely, $J = 14$. The quadrupole moment for this state is

$$Q^{(50}\text{Cr} : J = 14) = e_pQ(4p : J = 8) + e_nQ(2n : J = 6) = 0 + e_n13.70 = 6.85 \text{ (e fm}^2). \tag{6}$$

This explains the $J = 14$ result in Table I. In the above equation, we use $2n$ to represent two neutron holes. The fact that for two identical particles (holes), the lower spins $J = 2$ and 4 have a positive (negative) $Q$ and the higher spin $J = 6$ has a negative (positive) $Q$, plus the fact that four identical nucleons have a zero $Q$ help explain why one gets corresponding results in $^{50}\text{Cr}$.

Let us consider one more example, the $J^\pi = 10^+_1$ state. The MBZ wave function is written as

$$|10^+_1\rangle_{\text{MBZ}} = \sum_{J_p,J_n} D^J(J_p, J_n|J_p, J_n), \tag{7}$$

where $D^J(J_p, J_n)$ is the probability amplitude that the protons couple to $J_p$ and the neutrons couple to $J_n$. The wave function in detail is

$$|10^+_1\rangle_{\text{MBZ}} = 0.42|6, 4|^{10} - 0.46|4, 6|^{10} + 0.71|4^*, 6|^{10} + \text{[small components]} \tag{8}.$$ 

In the above, the states $|4\rangle$ and $|4^*\rangle$ have seniorities 2 and 4 respectively. The fact that the six neutrons (or two neutron holes) have a high probability of coupling to angular momentum $J_n = 6$ accounts for the fact that for the $10^+_1$ state, the value of $Q_n$ in the single $j$ shell limit is large and positive. Again, in this limit, the four protons will have a zero quadrupole moment. However, as can be seen from the structure of the wave function, we can get off diagonal non-zero contributions to $Q_p$.

### III. ALLOWING NUCLEONS TO BE EXCITED FROM THE $F_7/2$ SHELL

In Tables II and III, we give results for $Q$ and $B(E2)$ obtained in extended space calculations in which a maximum number of $t$ nucleons ($t=1$ for Table II and $t=2$ for Table III) are allowed to be excited from the $F_{7/2}$ shell. In Table IV, results are shown for selected states obtained for $t > 2$. These results should be compared with those in Table I from the single $f_{7/2}$ shell model ($t=0$).

The first thing to notice is that the largest spin $J_{\text{max}}$ is 18 for $t=1$ and 20 for $t=2$ while it is 14 in the single $j$ shell ($t=0$). In a complete $fp$ space calculation ($t=10$), the largest even spin is 22 and the largest overall spin is 23.

Concerning the energy levels, we note that in the larger space calculation, they are more spread-out. For example, the $10^+_1$, $12^+_1$ and $14^+_1$ excitation energies in Table I are 3.50, 3.99 and 5.17 MeV respectively, but when up to two nucleons ($t=2$) are allowed to be excited (Table III), these energies become 5.45, 6.48, and 8.64 MeV respectively. There is a tendency for the spectrum to look more collective.

Note that the sign changes for the quadrupole moments of the $6^+_1$ and $8^+_1$ states as we go from $t=0$ to $t=1$, 2 and 3. The quadrupole moment $Q$ for the $6^+_1$ state is positive (7.62 e fm$^2$) in the single $j$ calculation; it changes sign and becomes -4.21 e fm$^2$ in the $t=1$ calculation; but in the $t=2$ calculation it again becomes positive (2.65 e fm$^2$): it undergoes another change of sign and becomes -8.11 e fm$^2$ in the $t=3$ calculation. In all these calculations, $Q$ has remained small in magnitude for the $6^+_1$ state. For the $8^+_1$ state, $Q$ starts with a small and positive number (1.97 e fm$^2$) in the single $j$ calculation and becomes negative for $t=1$ (-9.73 e fm$^2$) and becomes even more negative for $t=2$ (-13.37 e fm$^2$) and for $t=3$ (-20.72 e fm$^2$). The quadrupole moment for $16^+_1$ also starts out positive and becomes large and negative as the configuration space is enlarged. The $t=1$, 2 and 4 results for $16^+_1$ are 4.64, -0.26 and -9.88 e fm$^2$, respectively.

The quadrupole moments have remained negative for the lower spin states $2^+_1$ and $4^+_1$ and positive for the higher spin states $10^+_1$, $12^+_1$, and $14^+_1$ in different model spaces. For even higher spin states, $18^+_1$, $20^+_1$ and $22^+_1$ that only exist in the $t \geq 1$, $t \geq 2$, $t \geq 3$ spaces respectively, the quadrupole moments are again negative. The quadrupole moments increase in magnitude with $t$ in general, again indicating more collectivity in larger spaces.

It is interesting to note that the $B(E2)$’s for the $J \rightarrow J-2$ transitions get enhanced for $J^\pi = 2^+_1$, $4^+_1$, $6^+_1$ and $8^+_1$ but the transition from $10^+_1$ to $8^+_1$ gets reduced as we go to the larger space. This strange behavior motivated us to look at other transitions, in particular, the transition from the $10^+_1$ state to the $8^+_1$ state. While for $t=0$, 1, 2 and 3, the values of $B(E2)$ for $10^+_1 \rightarrow 8^+_1$ are 80.1, 40.7, 37.8, and 28.7 e$^2$ fm$^4$ respectively, the corresponding values for $10^+_2 \rightarrow 8^+_1$ are 25.1, 86.3, 95.3, and 140.0 e$^2$ fm$^4$ respectively. This suggests that we associate the $10^+_2$ state as a member of the “ground-state band” which includes $0^+_1$, $2^+_1$, $4^+_1$, $6^+_1$ and $8^+_1$. This is further supported by the fact that for $t=3$, the quadrupole moment of the $10^+_1$ state is large and positive (45.7 e fm$^2$) while that of
the $10^+_2$ state is small and negative ($-4.0 \text{e fm}^2$). The $8^+_1$ moment is also negative. We also note that although the quadrupole moment $Q$ for the $6^+_2$ state is small and sometimes positive (for $t = 0$ and 2), the $B(E2)$ for its transition to the $4^+_1$ state (whose quadrupole moment is large and negative) is strong and gets enhanced as we go from $t = 0$ to $t = 2$.

Of special interest is the transition $16^+_1$ to $14^+_1$. This is a transition from a state which cannot exist in the single $f_{7/2}$ model space to a state which has the highest possible angular momentum allowed in the single $f_{7/2}$ shell. The value of $B(E2)$ for this transition is persistently small: 4.8 $e^2 \text{fm}^4$ for $t=1$, 6.4 $e^2 \text{fm}^4$ for $t=2$ and 4.5 $e^2 \text{fm}^4$ for $t=4$. However, there is a very strong transition from the $16^+_1$ state to the second $14^+_1$ state, $B(E2) = 103.5 e^2 \text{fm}^4$ for $t=2$ and $B(E2) = 122.8 e^2 \text{fm}^4$ for $t=4$. Presumably the $14^+_1$ state is mainly of an $f_{10}^{10}$ character while the $16^+_1$ state, just like the $16^+_1$ and higher spin states, require excitations of outside the $f_{7/2}$ shell for its existence. The smallness of the $B(E2)$ for the transition $20^+_1 \rightarrow 18^+_1$ (and for $22^+_1 \rightarrow 20^+_1$) can be explained similarly by noting that $J = 18$ is the highest possible angular momentum for the $t=1$ model space while one has to allow two nucleons to be excited from the $f_{7/2}$ shell ($t=2$) to get $J=20$.

### IV. THE ROTATIONAL MODEL

In the extreme rotational model, all the quadrupole moments and $B(E2)$'s in a rotational band are given in terms of a single intrinsic quadrupole moment $[3]$:

$$Q(J) = \frac{3K^2 - J(J+1)}{(J+1)(2J+3)} Q_K, \quad (9)$$

$$B(E2, J_1 \rightarrow J_2) = \frac{5}{16\pi} Q_K^2 (J_1 K 2 0 | J_2 K)^2. \quad (10)$$

The ratio $B(E2, J \rightarrow J-2)/B(E2, 2^+ \rightarrow 0^+)$ for a $K=0$ band would have the following values for $J=2$, 4, 6, 8 and 10: 1, 1.43, 1.62, 1.65 and 1.69. There is a steady but slow rise as $J$ increases. The corresponding ratios from Table I (single $f_{7/2}$ shell) for $J^\pi=2^+_1$, $4^+_1$, $6^+_1$, $8^+_1$ and $10^+_1$ are 1, 1.36, 1.09, 1.15 and 0.76. In the larger space calculation ($t=2$, Table III), for $J^\pi=2^+_1$, $4^+_1$, $6^+_1$, $8^+_1$ and $10^+_1$, the ratios are 1, 1.45, 0.95, 0.97 and 0.63. These calculated $B(E2)$'s do not obey a simple rotational formula with a constant $Q_0$. However, the $B(E2)$'s are still substantial so we can still speak of the $0^+_1$ to $8^+_1$ and $10^+_1$ states as being part of a band.

From the rotational formulae above, the ratio $B(E2, J \rightarrow J-2)/Q^2(J)$ has a value of 0.242 for $J=2$ and 0.215 for $J=4$. The $t=2$ results from Table III, using effective charges of $e_p=1.5$ and $e_n=0.5$, yields for the same ratio of 0.313 for $J=2$ and 0.269 for $J=4$. The agreement is not too bad. However, for $J \geq 6$ there are enormous deviations between the simple rotational model and the $t=2$ shell model so that it makes no sense to compare the two sets of results.

### V. VERY HIGH SPIN STATES – MORE ACCURATE CALCULATIONS

In Table IV, we present results for high spin states with as large as possible values of $t$ as we can handle. For $J^\pi=14^+_1$ and $16^+_1$, we have $t=4$; for $18^+_1$, $t=6$ and for $20^+_1$ and $22^+_1$, we have complete $fp$ shell calculations ($t=10$).

We see that for $14^+_1$, the $t=4$ results do not differ so much from the $t=2$ results. However, when we consider states with $J > 14$, which we recall do not occur in an $(f_{7/2}^4)(f_{5/2}^6)_n$ configuration, there is a large difference between the $t=2$ and the higher $t$ results. For example, when $t=2$, the values of $Q_n$ and $Q_f$ from Table III for $J^\pi=16^+_1$ are -1.37 and 3.58 $e^2 \text{fm}^2$. For $t=4$, the corresponding values, shown in Table IV, are -4.77 and -5.47 $e^2 \text{fm}^2$. For $20^+_1$, the $t=2$ values are -2.24 and -14.15 $e^2 \text{fm}^2$ but for $t=10$ (full $fp$ space) the values are -14.34 and -20.81 $e^2 \text{fm}^2$. For $J \geq 16$, it is clearly essential to do the calculation with high values of $t$. Fortunately, the number of configurations is less for very high $J$ than it is for low $J$ so that such high $t$ calculations can be done more readily. For example, to form a state with $J=22$, at least two protons and two neutrons have to be in the $f_{7/2}$ shell, therefore the $t=10$ calculation for this state is equivalent to the $t=6$ calculation.

We can gain insight as to why the very high spin states have negative quadruple moments by looking at the largest spin, $J=23$. To form an $M=23$ state, we write down the following configuration:

**Neutrons**: $f_{7/2}^7, f_{5/2}^7, f_{7/2}^{10}, f_{5/2}^{10}, f_{5/2}^{12}$, $f_{5/2}^{14}, f_{5/2}^{16}$

**Protons**: $f_{7/2}^7, f_{5/2}^7, f_{7/2}^{10}, f_{5/2}^{10}, f_{5/2}^{12}, f_{5/2}^{14}, f_{5/2}^{16}$

Note that $M_n=13$ and $M_p=10$. The neutron configuration is unique. For the protons, there are two additional configurations with $M_p=10$ which can be obtained by replacing $f_{7/2}^{13,14}$ in the above by $f_{5/2}^{13,14}$ or $f_{3/2}^{13,14}$. We choose $f_{7/2}^{13,14}$ which has the lowest single particle energy for simplicity.

With the above wave function, the proton quadrupole moment $Q_p$ is just the sum of the four single particle moments; likewise for the neutrons. The results are: $Q_p = -4.714 b^2$ and $Q_n = -6 b^2$. With $b = 2 \text{fm}$, we get $Q_p = -18.86 \text{e fm}^2$ and $Q_n = -24 \text{e fm}^2$. The $t=10$ shell model result for $Q_n$ is exactly the same, as it should be since the neutron configuration is unique. For $Q_p$, the $t=10$ shell model result is $Q_p = -23.00 \text{e fm}^2$ due to configuration mixing.
VI. CLOSING REMARKS

We have shown that the shell model is useful for examining and uncovering non-trivial aspects of nuclear collectivity. By calculating static quadrupole moments in extended $fp$ spaces (Table III for $J \leq 4$ and Table IV for $J \geq 6$), we are able to find a change of shape in the yrast band from prolate at low $J$ ($J \leq 8$) to oblate at higher $J$ ($J = 14$ is the maximum spin allowed by the $f_{7/2}$ configuration) and another change of shape from oblate back to prolate for even higher $J$ ($J \geq 16$). By looking at $B(E2)$'s, it appears that the even spin states $0^+$, $2^+$ form a prolate band, while $10^+$, $12^+$, and $14^+$ belong to an oblate band. The prolate band member $J^\pi = 6^+$ has a smaller (in magnitude) quadrupole moment compared to other members of the band whose quadrupole moments are all large and negative.

The higher spins ($J \geq 16$) which are only present in the extended $fp$ space do not appear to be part of this oblate band because the transition $16^+_1 \rightarrow 14^+_2$ is weak and because their static quadrupole moments are negative while the members of the oblate band, $10^+_1$, $12^+_1$, and $14^+_2$ have (calculated) positive quadrupole moments. We note that while the E2 transition $18^+_1 \rightarrow 16^+_1$ is strong, the transitions $16^+_1 \rightarrow 14^+_2$, $20^+_1 \rightarrow 18^+_1$ and $22^+_1 \rightarrow 20^+_1$ (see Table IV) are weak. This can be attributed to the fact that the initial and final states in these weak transitions are dominated by different configurations: $(f_{7/2})^{10}$ for $J = 14$, $(f_{7/2})^8 (p_{3/2} f_{5/2})^2$ for $J = 16$ and 18, $(f_{7/2})^8 (p_{3/2} f_{5/2})^2$ for $J = 20$, and $(f_{7/2})^7 (p_{3/2} f_{5/2})^3$ for $J = 22$.

The most credit for the resurgence of interest in this region of the periodic table ($^{48}$Cr and environs) goes to E. Caurier and collaborators who not only developed a code which made it possible to handle large model spaces, but also made the important connection between the shell model and nuclear collectivity. With all the advances up to now, it still must be acknowledged that the convergence in terms of number of particles excited is rather slow. To make a precise connection between the shell model and nuclear collectivity is still not a trivial matter. New technologies for speeding up the shell model by the Caltech group look very promising.

The old problem of choosing the proper shell model basis to explain collective motion has not yet been properly solved for the $fp$ shell — we recall the early work of Bhatt and McGroery who showed that while SU(3) might work in the lower part of the $sd$ shell, this is not the case in the $fp$ shell. The culprit is the large $p_{3/2} - f_{7/2}$ splitting. Thus one can invent truncation schemes which appear to be clever but which don’t really work. Thus the large shell model approach, however distasteful it may appear to some people, seems to be the best way to go at the moment.

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TABLE I. The single $j$ results (i.e., $t=0$) for the excitation energies $E_x$ (in units of MeV) and the quadrupole moments $Q$ (in units of $e^2$fm$^2$) of the lowest one or two even $J$ states in $^{58}$Cr with the “FPD6” interaction \[^7\]. The $B$(E2)'s (in units of $e^2$fm$^4$) for the relatively strong E2 transitions from these states are also listed. The $B$(E2)'s for a few weak transitions are also given for the purpose of comparison. The effective charges $e_p = 1.5$ and $e_n = 0.5$ are used to compute $Q$ and $B$(E2). For other choices of the effective charges, $Q$ and $B$(E2) can be obtained from $Q_p$, $Q_n$ and $A_p$, $A_n$ shown in the table through: $Q = (e_p Q_p + e_n Q_n)$ and $B$(E2) = $(e_p A_p + e_n A_n)^2$. $J = 14$ is the maximum possible $J$ value in this model space.

| Initial State | $E_x$ | $Q_p$ | $Q_n$ | $Q$ | Final State | $A_p$ | $A_n$ | $B$(E2) |
|---------------|-------|-------|-------|-----|-------------|-------|-------|--------|
| $2^+_1$       | 0.913 | -3.24 | -5.86 | -7.79| $0^+_1$     | 4.58  | 4.02  | 78.8   |
| $4^+_1$       | 1.655 | -3.35 | -5.76 | -7.90| $2^+_1$     | 5.50  | 4.17  | 106.9  |
| $6^+_1$       | 2.107 | 5.34  | -0.78 | 7.62 | $4^+_1$     | 4.98  | 3.57  | 85.6   |
| $8^+_1$       | 2.954 | 1.89  | -1.72 | 1.97 | $6^+_1$     | 5.01  | 4.00  | 90.6   |
| $8^+_2$       | 3.573 | 6.72  | 4.62  | 12.39| $6^+_2$     | 0.29  | -0.29 | 0.1    |
| $10^+_1$      | 3.496 | 10.13 | 8.93  | 19.66| $8^+_1$     | 4.12  | 3.16  | 60.1   |
|               |       |       |       |      | $8^+_2$     | 3.26  | 1.00  | 29.1   |
| $10^+_2$      | 3.871 | 4.58  | 5.32  | 9.54 | $8^+_3$     | 2.55  | 2.39  | 25.1   |
| $12^+_1$      | 3.993 | 0.41  | 13.02 | 7.12 | $10^+_1$    | 3.83  | 1.97  | 45.3   |
| $12^+_2$      | 5.316 | 0.14  | -1.25 | -0.42| $10^+_2$    | 0.96  | 2.27  | 6.6    |
| $14^+_1$      | 5.166 | 0.00  | 13.70 | 6.85 | $12^+_1$    | 3.04  | 0.87  | 24.9   |

TABLE II. Same as Table I but for an extended model space in which one nucleon is allowed to leave the $f_{7/2}$ orbit and occupy the rest of the $fp$ shell (i.e., $t=1$). $J = 18$ is the maximum possible $J$ value in this model space.

| Initial State | $E_x$ | $Q_p$ | $Q_n$ | $Q$ | Final State | $A_p$ | $A_n$ | $B$(E2) |
|---------------|-------|-------|-------|-----|-------------|-------|-------|--------|
| $2^+_1$       | 0.726 | -9.74 | -11.14| -20.18| $0^+_1$     | 5.67  | 5.87  | 130.8  |
| $4^+_1$       | 1.555 | -12.36| -12.22| -24.64| $2^+_1$     | 7.05  | 5.92  | 183.4  |
| $6^+_1$       | 2.349 | -1.15 | -4.95 | -4.20 | $4^+_1$     | 6.22  | 5.72  | 148.6  |
| $8^+_1$       | 3.470 | -3.85 | -7.89 | -9.73 | $6^+_1$     | 6.13  | -6.04 | 38.2   |
| $8^+_2$       | 4.229 | 7.12  | 10.98 | 16.17 | $6^+_2$     | 0.56  | 0.28  | 1.0    |
| $10^+_1$      | 4.277 | 16.58 | 17.02 | 33.39 | $8^+_1$     | 3.13  | 3.37  | 40.7   |
|               |       |       |       |      | $8^+_2$     | 3.56  | 1.09  | 34.6   |
| $10^+_2$      | 4.807 | 2.54  | 2.36  | 4.99 | $8^+_3$     | 4.55  | 4.92  | 86.3   |
| $12^+_1$      | 5.191 | 3.45  | 17.27 | 13.81 | $10^+_1$    | 4.04  | 2.45  | 53.1   |
| $12^+_2$      | 6.825 | -2.10 | -1.42 | -3.86 | $10^+_2$    | 1.83  | 3.12  | 18.5   |
| $14^+_1$      | 6.973 | 1.78  | 18.39 | 11.87 | $12^+_1$    | 4.84  | 2.02  | 68.5   |
| $14^+_2$      | 10.493| -3.76 | 10.39 | -0.44 | $12^+_2$    | 0.28  | 0.88  | 0.7    |
| $16^+_1$      | 11.851| -0.20 | 9.88  | 4.64 | $14^+_1$    | 0.66  | 2.42  | 4.8    |
|               |       |       |       |      | $14^+_2$    | 0.77  | -0.07 | 1.3    |
| $18^+_1$      | 13.634| 0.00  | -1.71 | -0.86| $16^+_1$    | 2.33  | 3.42  | 27.1   |
TABLE III. Same as Table I but for an extended model space in which a maximum of two two nucleons are allowed to leave the $f_{7/2}$ orbit and occupy the rest of the $fp$ shell ($t=2$). $J = 20$ is the maximum possible $J$ value in this model space.

| Initial State | $E_x$ | $Q_p$ | $Q_n$ | $Q$  | Final State | $A_p$ | $A_n$ | $B(E2)$ |
|---------------|-------|-------|-------|------|-------------|-------|-------|---------|
| $2^+_1$       | 1.051 | -10.62| -12.14| -22.01| $0^+_1$     | 6.11  | 6.32  | 151.8  |
| $4^+_1$       | 2.061 | -14.32| -14.18| -28.56| $2^+_1$     | 7.63  | 6.76  | 219.6  |
| $6^+_1$       | 2.934 | 2.41  | -1.95 | 2.65  | $4^+_1$     | 6.11  | 5.67  | 144.1  |
| $8^+_1$       | 4.405 | -5.52 | -10.18| -13.37| $6^+_1$     | 6.04  | 6.19  | 147.6  |
| $8^+_2$       | 5.312 | 5.59  | 10.95 | 13.86 | $8^+_1$     | 1.33  | 0.52  | 5.1    |
| $10^+_1$      | 5.446 | 18.43 | 18.40 | 36.85 | $8^+_2$     | 2.98  | 3.36  | 37.8   |
| $10^+_2$      | 6.015 | 1.44  | 1.28  | 2.80  | $8^+_1$     | 4.68  | 5.50  | 95.3   |
| $12^+_1$      | 6.842 | 4.22  | 17.74 | 15.20 | $8^+_2$     | 0.03  | 1.67  | 0.6    |
| $12^+_2$      | 8.389 | -3.09 | -2.67 | -5.97 | $10^+_1$    | 3.70  | 2.42  | 45.8   |
| $14^+_1$      | 8.842 | 1.34  | 17.93 | 10.97 | $10^+_2$    | 1.66  | 3.05  | 16.2   |
| $14^+_2$      | 11.937| -0.53 | 4.01  | 1.21  | $10^+_1$    | 4.69  | 1.95  | 64.1   |
| $16^+_1$      | 13.883| -1.37 | 3.58  | -0.26 | $10^+_2$    | 0.62  | 1.77  | 3.3    |
| $18^+_1$      | 14.68 | -2.45 | -2.39 | -4.87 | $14^+_1$    | 0.90  | 2.34  | 6.4    |
| $20^+_1$      | 21.46 | -2.24 | -14.65| -10.69| $16^+_1$    | 3.85  | 5.03  | 68.6   |

TABLE IV. Same as Table I but for larger model spaces where a maximum number of $t$ (given in the table) nucleons are allowed to be excited from the $f_{7/2}$ shell. Only states with $J \geq 6$ are calculated. $J = 23$ is the maximum possible $J$ value in the full $fp$ ($t=10$) space.

| Initial State | $t$ | $Q_p$ | $Q_n$ | $Q$   | Final State | $A_p$ | $A_n$ | $B(E2)$ |
|---------------|-----|-------|-------|-------|-------------|-------|-------|---------|
| $6^+_1$       | 3   | -3.37 | -6.10 | -8.11 | $6^+_1$     | 6.96  | 7.42  | 200.0   |
| $6^+_2$       | 3   | 19.04 | 14.80 | 35.95 | $6^+_1$     | 1.60  | 1.11  | 8.7     |
| $8^+_1$       | 3   | -9.37 | -13.33| -20.72| $8^+_2$     | 2.55  | 3.06  | 28.7    |
| $8^+_2$       | 3   | 5.41  | 11.54 | 13.89 | $8^+_1$     | 2.81  | 1.34  | 23.9    |
| $10^+_1$      | 3   | 22.74 | 23.18 | 45.70 | $10^+_2$    | 5.62  | 6.80  | 140.0   |
| $10^+_2$      | 3   | -1.92 | -2.19 | -3.98 | $10^+_1$    | -0.41 | 1.06  | 0.0     |
| $12^+_1$      | 3   | 5.83  | 19.64 | 18.56 | $12^+_1$    | 3.69  | 2.46  | 45.9    |
| $12^+_2$      | 3   | -5.47 | -3.78 | -10.09| $12^+_2$    | 2.03  | 3.26  | 21.9    |
| $14^+_1$      | 4   | 1.21  | 18.35 | 10.99 | $14^+_1$    | 4.58  | 2.35  | 64.6    |
| $14^+_2$      | 4   | -5.44 | -5.19 | -10.76| $14^+_2$    | 2.74  | 3.85  | 36.4    |
| $16^+_1$      | 4   | -4.76 | -5.47 | -9.88 | $16^+_2$    | 0.89  | 1.57  | 4.5     |
| $18^+_1$      | 6   | -4.90 | -5.22 | -9.96 | $18^+_1$    | 4.92  | 4.97  | 97.2    |
| $20^+_1$      | 10  | -14.34| -20.81| -31.92| $20^+_1$    | 0.60  | 1.03  | 2.0     |
| $22^+_1$      | 10  | -24.27| -27.28| -50.04| $20^+_1$    | 1.45  | 1.19  | 7.7     |

$^a$ The $t=4$ results for the $Q$'s of the $12^+_1$ state are: $Q_p = 6.27 e fm^2$, $Q_n = 19.88 e fm^2$ and $Q = 19.35 e fm^2$;

$^b$ The $t=4$ results for the $Q$'s of the $12^+_2$ state are: $Q_p = -8.25 e fm^2$, $Q_n = -5.58 e fm^2$ and $Q = -15.17 e fm^2$;

$^c$ The $t=6$ results for the $Q$'s of the $16^+_1$ state are: $Q_p = -5.45 e fm^2$, $Q_n = -6.72 e fm^2$ and $Q = -11.54 e fm^2$.

$^d$ The $t=10$ results for the $Q$'s of the $18^+_1$ state are: $Q_p = -4.90 e fm^2$, $Q_n = -5.23 e fm^2$ and $Q = -9.97 e fm^2$. 

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