On the low temperature properties and specific anisotropy of pure anisotropically paired superconductors

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Abstract

Dependences of low temperature behavior and anisotropy of various physical quantities for pure unconventional superconductors upon a particular form of momentum direction dependence for the superconducting order parameter (within the framework of the same symmetry type of superconducting pairing) are considered. A special attention is drawn to the possibility of different multiplicities of the nodes of the order parameter under their fixed positions on the Fermi surface, which are governed by symmetry. The problem of an unambiguous identification of a type of superconducting pairing on the basis of corresponding experimental results is discussed. Quasiparticle density of states at low energy for both homogeneous and mixed states, the low temperature dependences of the specific heat, penetration depth and thermal conductivity, the I-V curves of SS and NS tunnel junctions at low voltages are examined. A specific anisotropy of the boundary conditions for unconventional superconducting order parameter near $T_c$ for the case of specular reflection from the boundary is also investigated.

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I. INTRODUCTION

The possibility of unconventional types of superconducting pairing in a number of heavy-fermion and high-\(T_c\) compounds, which follows from many experimental observations, results in much theoretical attention to a study of various physical properties of unconventional superconductors. If an unconventional superconducting order parameter \(\hat{\Delta}(\mathbf{p})\) vanishes at points or lines on the Fermi surface, low-energy quasiparticle excitations in the vicinities of these nodes may exist and are known to give rise to power laws for low temperature dependences of the specific heat \(C(T)\), the thermal conductivity \(\kappa(T)\) and some other quantities to be measured. Indices for these power laws depend not only on the dimension of nodes (that is whether there are points or lines of nodes) or, for example, on the strength of the impurity scattering. They differ also for the order parameters, which have zeroes of different orders at the nodes. Since always, even within the same symmetry type of pairing, there are basis functions which differ from each other by the multiplicities of nodes situated at the same lines and points on the Fermi surface, different indices may correspond to the same type of pairing. It seems to be quite important as different experimental consequences for superconductors with the same type of pairing become possible for different particular basis functions for the order parameter. For instance, let’s consider \((p_{x}^2 - p_{y}^2)\)-type of pairing in a tetragonal superconductor. For the basis function \((p_{x}^2 - p_{y}^2)\) and a spherical Fermi surface one gets at the Fermi surface \(\Delta = \Delta_0 \sin^2\theta \cos 2\varphi\). Near lines of nodes \(\varphi_{1,2} = \pm \pi/4\) and far from poles of the sphere this order parameter is proportional to \((\varphi - \varphi_{1,2})\) and near the intersections of lines at the poles one has \(\Delta \propto (\varphi - \varphi_{1,2})\theta^2\). Other basis functions for the same representation of a symmetry group \(D_{4h}\) and a spherical Fermi surface, for example, \(\Delta = \Delta_0 \sin^{2m} \theta \cos^{2n+1} 2\varphi\) \((n, m = 0, 1, 2\ldots)\) have obviously different orders of zeroes at the nodes and, consequently, one obtains different low-temperature behaviors of various physical quantities for these basis functions. The same arguments are applicable, of course, to the case of an arbitrary shape of the Fermi surface, for instance, to the cylindrical Fermi surface, when instead of the simplest representative for the order parameter \(\Delta = \Delta_0 \cos 2\varphi\) for the case of \((p_{x}^2 - p_{y}^2)\)-type of pairing, one can consider more complicated examples like \(\Delta = \Delta_0 \cos^{2n+1} 2\varphi\). Due to this fact an identification of the type of superconducting pairing (and frequently even a distinction between the contributions from lines and points of nodes) on the basis of experimental data for low temperature behavior of various quantities turns out to be ambiguous or at least more complicated as it is usually supposed. So, it is of importance to investigate the influence of the order parameter behavior near the nodes on some properties of unconventional superconductors.

Having in mind this circumstance we calculate below the quasiparticle density of states at low energy and low temperature dependences of the specific heat (for both homogeneous and mixed states), the penetration depth, the thermal conductivity and the I-V curves for SS and NS tunnel junctions at low voltages, permitting the order parameter to have zeroes of different orders at the nodes on a Fermi surface. We consider also within this context an anisotropy of boundary conditions for an unconventional superconducting order parameter near \(T_c\) for the case of specular reflection. Our results allow to carry out a qualitative and simple analysis of some experimental data, taking account of the possibilities mentioned above. We believe that such a consideration, which permits to discuss qualitatively experimental consequences for a wide set of the basis functions, is needed along with the detailed
quantitative (usually numerical) calculations of the effects for several currently preferable representatives from the set for the given compound. In particular, we discuss recent experimental data on the low temperature behavior of thermal conductivity for the heavy fermion superconductor $UPt_3$.\[1,2\]

II. DENSITY OF STATES AND SPECIFIC HEAT

If a superconducting order parameter doesn’t vanish anywhere on the Fermi surface (as it is in the particular case of an isotropic $s$-wave superconductor), the density of states $N(E)$ is equal to zero at low energies $E < \Delta_{\text{min}}$ and the electronic specific heat $C(T)$ falls off exponentially with the decrease of temperature $C \propto e^{-\Delta_{\text{min}}/T}$, since only a small number of quasiparticles are activated above the gap at $T \ll \Delta_{\text{min}}$. For an anisotropic pairing with nodes of the order parameter on the Fermi surface the low energy density of states is known to be nonzero. The contributions to $N(E)$ at low energy come from quite narrow vicinities of the nodes, they dominates also in $C(T)$ at low temperatures. Besides, in the case of unconventional superconductor the scattering on nonmagnetic impurities causes a substantial change in the low-energy density of states for various scattering strength and even at quite small impurity concentrations $[3,4]$. This underlies the low temperature behavior for various physical quantities including the specific heat. Below we consider only a pure unconventional superconductor which is relevant even to the case of resonance scattering at low impurity density, at least for the energies $\omega_c \lesssim E \ll T_c$ above a critical energy $\omega_c = (\Gamma \Delta_0)^{1/2}$ (where $\Gamma$ is the normal state scattering rate in the unitarity limit).

For a unitary gap matrix $\Delta(p)$ and a spherical Fermi surface the quasiparticle density of states in the case of pure superconductor is given by

$$N(E) = 2 \sum_p \delta(E - E_p) = N_F \int_{|\Delta(p)| < E} \frac{d\Omega}{4\pi} \frac{E}{\sqrt{E^2 - |\Delta(p)|^2}}, \quad (1)$$

where the integration is carried out over the momentum directions and $N_F$ is a density of states at the Fermi surface for normal metal.

The electronic part of the specific heat may be written as

$$C(T) = 2 \int_0^\infty dE \left\{ N(E) \frac{E^2 \exp(E/T)}{T^2 (\exp(E/T) + 1)^2} + T \frac{dN(E)}{dT} \left[ \ln(1 + \exp(-E/T)) + \frac{E}{T(1 + \exp(E/T))} \right] \right\}. \quad (2)$$

One can easily estimate low energy and low temperature behavior of quantities $N(E)$ and $C(T)$ setting the particular form of the gap function only in the vicinities of nodes. For example, if the order parameter in the vicinities of lines of nodes takes the form $(n > 0)$

$$|\Delta(p)| = \Delta_0 |\varphi - \varphi_0|^n, \quad (3)$$

one easily gets the main contributions to $N(E), C(T)$:
\[ \frac{N(E)}{N_F} = 4h(n) (E/\Delta_0)^{1/n}, \quad (4) \]

\[ C(T) = N_F 4w(n) T (T/\Delta_0)^{1/n}. \quad (5) \]

Here the functions \( h(x), w(x) \) are given by

\[ h(x) = \frac{\Gamma(1/2x)}{8^{1/2} \pi x \Gamma((x+1)/2)}, \quad (6) \]

\[ w(x) = 2h(x) \left( 2 + \frac{1}{x} \right) \left( 1 - \frac{1}{2^{1+1/x}} \right) \Gamma(2 + 1/x) \zeta(2 + 1/x), \quad (7) \]

where \( \Gamma(x), \zeta(x) \) are the Gamma function and the Riemann Zeta function respectively.

Analogously, if near the lines of nodes one has \( |\Delta(p)| = \Delta_0 |\theta - \theta_0|^n \), \( n > 0 \)

the corresponding contributions to the density of states and specific heat are

\[ \frac{N(E)}{N_F} = \sin \theta_0 4\pi h(n) (E/\Delta_0)^{1/n}, \quad (9) \]

\[ C(T) = N_F 4\pi \sin \theta_0 w(n) T (T/\Delta_0)^{1/n}. \quad (10) \]

There are obviously other basis functions belonging to the same representation of the symmetry group \( D_{4h} \). So, let the order parameter near the nodes on the Fermi sphere within the small angle regions \(|\phi - \phi_0| \leq b, \quad \theta^2 \leq a \quad (a, b \ll 1) \) be

\[ |\Delta(p)| = \Delta_0 |\varphi - \varphi_0|^m \theta^{2n}, \quad (11) \]

where \( m, n \geq 0, \quad m + n > 0 \). Then one obtains from Eqs.\!(1), (2):

\[ \frac{N(E)}{N_F} = \begin{cases} (E/\Delta_0)^{1/m} \frac{1-\frac{m}{2}}{(1-\frac{m}{2})} h(m), & m > n \\ (E/\Delta_0)^{1/n} \frac{1}{n} (\ln(\Delta_0/E) + c) h(n), & m = n \\ (E/\Delta_0)^{1/n} \frac{b^{1-\frac{m}{2}}}{(1-\frac{m}{2})} h(n), & n > m, \end{cases} \quad (12) \]

\[ C(T) = \begin{cases} N_F T (T/\Delta_0)^{1/m} \frac{1-\frac{m}{2}}{(1-\frac{m}{2})} w(m), & m > n \\ N_F T (T/\Delta_0)^{1/n} \frac{1}{n} (\ln(\Delta_0/T) + c) w(n), & m = n \\ N_F T (T/\Delta_0)^{1/n} \frac{b^{1-\frac{m}{2}}}{(1-\frac{m}{2})} w(n), & n > m. \end{cases} \quad (13) \]
A role of logarithmic terms in Eqs. (12), (13) depends not only upon parameters $E/\Delta_0$, $T/\Delta_0$ but also on the magnitude of a constant $c$, which is formed by the contributions from the angular vicinity of the line of nodes, but not only very close to the pole. This constant may be calculated by making use of particular form of the basis function all over the Fermi sphere. For instance, in the case of $(p_x^2 - p_y^2)$-type of pairing with $\Delta = \Delta_0 \sin^2 \theta \cos 2\varphi$ one gets from (1)

$$\frac{N(E)}{N_F} = (A + \ln(\Delta_0/E)) \frac{E}{2\Delta_0},$$

where

$$A = \frac{2}{\pi} \int_0^1 dk \left[ K(k) - \frac{\ln k}{k} \left( \frac{E(k)}{1-k^2} - K(k) \right) \right] \simeq 1.38.$$  

Here $K(k)$ and $E(k)$ are the complete elliptic integrals of first and second type.

So, both terms in (14) may be of the same order and, as a rule, they both should be taken into account. Eq.(14) is in accordance with the second relation in (12), since the order parameter $\Delta$ near the nodes $\theta_0 = 0$, $\varphi_{1,2} = \pm \pi/4$ takes the same form as in (11) with $n = m = 1$, and the logarithmic term in (14) contains respective contributions from all four nodes (that is from two lines of nodes at two poles of a Fermi sphere). As in this example, the behavior of the order parameter of the form (11) usually corresponds to the intersection of two lines of nodes at the pole (each being of the form $|\Delta(p)| = \Delta_0 |\varphi - \varphi_0|^m$ near the line and far from the pole). One can easily see that under the condition $n \geq m$ the part coming from the point of intersection dominates or is at least of the same order as the total contribution to the quantities. Note, that in the particular case $m = 0$ Eqs.(12), (13) at $n > m$ correspond to the contributions from the point node which is described by Eq.(8) for $\theta_0 = 0$ and the substitution $n \rightarrow 2n$.

At last, for the order parameter which may be represented near the point node as follows

$$|\Delta(p)| = |\Delta_{01}(\varphi - \varphi_0)^m + \Delta_{02}(\theta - \theta_0)^n|, \quad m, n \geq 0, \quad m + n > 0,$$

one gets

$$\frac{N(E)}{N_F} \propto \sin \theta_0 E^{1/n+1/m}, \quad C(T) \propto \sin \theta_0 T^{1+1/n+1/m}.$$  

Summing up the contributions from all vicinities of the nodes of the order parameter, one finds total low energy density of states and low-temperature specific heat.

In discussing lines of nodes of the order parameter one usually considers zeroes of the first order at the nodes which correspond to relation (3) or (8) with the particular value $n = 1$. Then, according to the Eqs.(3), (10) one gets a quadratic low-temperature behavior for the heat capacity $C(T) \propto T^2$. Analogously, for the points of nodes it is frequently supposed $n = m = 1$ in (16) and then one obtains from (17) the well-known result $C(T) \propto T^3$. As it is seen from the more general relations (5), (10), (17), the possibility for distinguishing between the contributions from points and lines of nodes becomes substantially more restricted, and indices of the power laws are allowed to be fractional, even for integer values of $n, m$. For
instance, for an experimental determination whether there is a contribution from a cubic point of nodes \((m=n=3)\), when corresponding term in the specific heat is \(C_{\text{cp}} \propto T^{5/3}\), or from a first order line of nodes (when one has \(C_{\text{fl}} \propto T^2\), quite a high accuracy is needed as compared to the distinguishing between the heat capacity dependences \(C_{\text{fp}} \propto T^3\) and \(C_{\text{fl}} \propto T^2\). Note that a contribution to the specific heat from zeroes of second order at points (quadratic points) is proportional to \(T^2\) precisely as from the first order line of nodes. In the case of fractional values of \(n, m\) as well as for the dependences like \(|\theta - \theta_0|\) the question about the physical origin for the nonanalytical behavior of the order parameter near the nodes appears.

III. LOW ENERGY DENSITY OF STATES AND LOW-TEMPERATURE SPECIFIC HEAT FOR THE MIXED STATE

One of the most important qualitative features of an unconventional superconductor in a mixed state is that due to the existence of a supercurrent induced by a magnetic field, the quasiparticle states with negative energy (which are obviously occupied) appear for momentum directions within the vicinity of nodes of the order parameter, as it follows from the relation \(E(p) = \sqrt{\xi(p)^2 + |\Delta(p)|^2 + pv_s}\). The density of states for such a superconductor changes substantially at sufficiently low energies. In particular, the density of states at the Fermi surface differs from zero under these conditions and after spatial averaging over the mixed state is proportional in the simplest case to \(\sqrt{B}\), resulting in the characteristic magnetic field dependent term in the specific heat \(C \propto T \sqrt{B}\). Recently the term of such kind, which is specific for unconventional superconductors, has been observed experimentally for \(YBCO\). From these measurements a quite strong upper limit on the minimum gap value follows if an anisotropic \(s\)-wave superconductivity is assumed. We examine below the influence of the particular forms of basis functions, which belong to the same symmetry type of pairing, on the magnetic field dependence of the density of states and heat capacity (under the conditions similar to those considered in \([9]\)). It will be shown that an accurate experimental determination of the index value in this power law dependence permits to describe the behavior of the order parameter in the vicinity of nodes more carefully. The spatial distribution of the density of states around the vortex is also considered.

Let a strong type two pure superconductor with a cylindrical Fermi surface be at low temperature in a mixed state under the applied magnetic field, which satisfies the condition \(H_{c1} \lesssim B \ll H_{c2}\) and is directed along the principal cylindrical axis. For this inhomogeneous state the local density of states must obviously manifest a spatial dependence and, generally speaking, is formed by both a superconducting region outside the vortex cores and the quasiparticle states localized inside the cores. Since the total volume occupied by the vortex cores is proportional to the magnetic field, the contribution to the density of states from the states localized within the vortex cores is a linear function of \(B\) and the corresponding contribution to the specific heat \(\propto TB/H_{c2}\). The quasiparticles at distances \(r \gg \xi_0\) from the vortex core may be considered quasiclassically as having the energy

\[ E(p, r) = \sqrt{\xi(p)^2 + |\Delta(p)|^2 + pv_s(r)}, \quad (18) \]
which depends upon the distance from the vortex core along with the superfluid velocity (in the simplest case of circular supercurrents one has \( v_s = e_e K_1(r/\lambda)/2m_e\lambda \)). Then the corresponding density of states may be written as follows

\[
N(E, r) = 2 \int \frac{d^3p}{(2\pi)^3} \delta \left( \sqrt{\xi^2(p) + |\Delta(p)|^2} + pv_s(r) - E \right).
\] (19)

From the estimate \(|pv_s| \lesssim p_F/m_e r \ll v_F/\xi_0 \sim \Delta_0\) which is valid at distances \( r \gg \xi_0 \) it follows, that a nonzero contribution to the density of states for the zero quasiparticle energy appears only for unconventional superconducting pairing for momentum directions within the narrow vicinities of nodes of the order parameter \( \Delta(p) \) (\( \Delta_0 \) denotes throughout this paper the maximum value of the order parameter for the homogeneous superconducting state at low temperature and for any type of pairing). Due to this fact in considering the density of states at quite low energy one can describe the contributions from each line or point of nodes separately.

The location of a line of nodes of the order parameter, which is oriented parallel to the principal axis of the cylindrical Fermi surface, may be described by a constant polar angle \( \varphi_l \) of the cylindrical coordinate system, corresponding to the momentum direction \( p_{F,\perp}(l) \) to the line. In integrating in (19) one can put with a good accuracy \( pv_s = p_{F,\perp}(l)v_s \) due to a quite small parameter \( v_s/v_F \).

Suppose that the quasiparticle energy \( \xi(p) \) for the normal metal and the order parameter \( \Delta(p) \) near nodes depend upon the different momentum components – the magnitude of the momentum component \( p_r \) and the direction of the momentum \( p_{F,\perp} \) (which is perpendicular to the principal cylindrical axis) correspondingly. Then one obtains after integration over the energy \( \xi(p_r) \) \( (d\xi = v_F dp_r) \) the following contribution from the line of nodes

\[
N(E, r) = \frac{N_F}{\pi} \int_{\Omega_l} d\varphi \frac{|E - p_{F,\perp}(l)v_s| \Theta(E - p_{F,\perp}(l)v_s)}{\sqrt{(E - p_{F,\perp}(l)v_s)^2 - |\Delta(\varphi)|^2}}.
\] (20)

Here \( \Theta(x) \) is the step-like function, \( N_F \) is the density of states at the cylindrical Fermi surface for the normal metal and \( \Omega_l \) is the narrow angular region in the vicinity of the line of nodes, where the expression under the square root sign in (20) is positive at quite low energies in question.

If in the close vicinity of the nodes the order parameter takes the form

\[
|\Delta(\varphi)| = \Delta_0|\varphi - \varphi_l|^n, \quad n > 0,
\] (21)

one gets from (20) after the integration over \( \varphi \)

\[
N(E, r) = \frac{N_F \Gamma \left( \frac{1}{2n} \right)}{n \sqrt{\pi} \Gamma \left( \frac{1}{2} + \frac{1}{2n} \right)} \left( \frac{E - p_{F,\perp}(l)v_s(r)}{\Delta_0} \right)^{1/n} \Theta(E - p_{F,\perp}(l)v_s(r)).
\] (22)

According to this relation, the narrow angular region in the momentum space situated near the line of nodes on the Fermi surface, contributes to the local quasiclassical density of states at sufficiently low energy for quite a wide region of orientations in the coordinate space.
For instance, at zero energy the density of states formed by the line of nodes is nonzero for the whole spatial region defined by the condition $p_{F,\perp}(l)v_s(r) < 0$, though it naturally falls off with the increasing distance from the vortex core (along with the supercurrents induced by the magnetic field). So, in the presence of several lines of nodes the contributions are spatially superimposed and due to this fact the locations of a maximum value of the total density of states in the coordinate space (at the given distance $r$ from the vortex core) may substantially differ from the locations of corresponding maxima of the partial contributions from each separate line of nodes.

In the case of $(p_x^2 - p_y^2)$-pairing for the tetragonal superconductor with cylindrical Fermi surface, one can consider four similar lines of nodes for the superconducting order parameter, which are situated at the angles $\varphi_l = \pi/4 + (l-1)\pi/2$ $(l = 1, 2, 3, 4)$. Let the angle $\phi$ specifies the relative orientation of the vector $r$ and the crystalline axis $x$, and $v_s(r) = v_s(r)e_\phi$. Then the sum of all four terms of the form (22) results in the following spatial dependent density of states

$$N(E, r, \phi) = \frac{N_F \Gamma \left( \frac{1}{2n} \right)}{n \sqrt{\pi} \Gamma \left( \frac{1}{2} + \frac{1}{2n} \right)} \left( \frac{v_F K_1(r/\lambda)}{2\lambda\Delta_0} \right)^{1/n} \sum_l \left( \tilde{E} + \sin(\phi + \varphi_l) \right)^{1/n} \Theta \left( \tilde{E} + \sin(\phi + \varphi_l) \right),$$

(23)

where the dimensionless quantity $\tilde{E} = 2\lambda E/v_F K_1(r/\lambda)$ is introduced.

Since at large distances from the vortex core the anisotropy of $N$ is small, we consider further only the distances $r \ll \lambda$. At the zero energy one gets from here $N \propto N_F(\xi_0/r)^{1/n} \left( |\cos(\phi + \pi/4)|^{1/n} + |\sin(\phi + \pi/4)|^{1/n} \right)$ and in the particular case $n = 1$ the spatial angular dependence of $N$ is simply $N \propto |\cos \phi|$ for $-\pi/4 < \phi + \pi m < \pi/4$ and $N \propto |\sin \phi|$ for $\pi/4 < \phi + \pi m < 3\pi/4$ $(m = 0, 1)$. Hence, the maximum value of the density of states at given $r$ lies at $\phi = 0, \pm \pi/2, \pi$ and minimum - at $\phi = \pm \pi/4, \pm 3\pi/4$. At the same time the maximum of the separated contribution from each line of nodes lies in the coordinate space at the direction, where the superfluid velocity is opposed to the momentum direction to the corresponding line on the Fermi surface (i.e. all four these maxima lie in the directions $\phi = \pm \pi/4, \pm 3\pi/4$). Furthermore, in the particular case $n = 1$ and for $\tilde{E} > 1$ (e.g. at the distances $r > (T_c/E)\xi_0$) all spatial dependence of the density of states (both from the distance and the direction) disappears and one gets from (23) $N \sim (E/\Delta_0)^{n-1}$. It follows from here that the greater the energy the smaller the spatial region where the anisotropy and inhomogeneity of the density of states manifest themselves. So, for $E \sim T_c$ one has the anisotropy only within the region $r < \xi_0$, where the local description based on the relation (18) for the quasiparticle energy, can’t be applied and more general approach is needed. Therefore, within the framework of the approach used above one may consider only the energies $E \ll T_c$. It is worth noting that in contrast to the particular case $n = 1$, for $n \neq 1$ the angular and distance dependences of the density of states at large distances don’t disappear completely though become quite weak. The spatial dependence of the density of states (23), including its fourfold symmetry, is accessible for studying to scanning tunneling microscopy experiments, which now have a high spatial resolution as a valuable complement to the high energy resolution of tunneling spectroscopy (see, for example, [11,12]).
As the density of states at zero energy is finite, then one gets from (2) the conventional (linear in \( T \)) expression for the low-temperature specific heat \( C(T) = \pi^2 T \tilde{N}(0) / 3 \). Due to the inhomogeneity of the state the spatial average of the density of states \( \tilde{N}(0) \) is presented here. For the magnetic fields within the interval \( H_{c1} \ll B \ll H_{c2} \) one substitutes \( \lambda / r \) instead of \( K_1(r / \lambda) \) in (22), (23) and then the spatial averaging over the region \( \xi_0 \ll r \ll R \) (where \( R \sim \xi_0 (H_{c2} / B)^{1/2} \) is the distance between vortices) gives rise to the following estimate

\[
\tilde{N}(0) = A_n N_F \left( \frac{v_F}{\Delta_0 \xi_0} \right)^{1/n} \left( \frac{B}{H_{c2}} \right)^{1/2n}.
\]  

Here \( A_n \) depends only upon the index \( n \). Note, that in forming \( \tilde{N}(0) \) just the distances \( r \gg \xi_0 \) are found to be important for \( n > 1/2 \). For these values of \( n \) the contribution to the specific heat \( \propto T (B / H_{c2})^{1/2n} \) is more essential, than the linear magnetic field dependence, coming from the interiors of the vortex cores.

According to (24), the index \( 1/2n \) of the power law for the magnetic field dependence of the low-temperature specific heat for the unconventional superconductor in the mixed state characterizes not only the existence of nodes of the order parameter on the Fermi surface themselves, but also the behavior of the order parameter in the vicinities of the nodes, for example, of a form (21).

**IV. PENETRATION DEPTH**

It is well known that deviation of the penetration depth at low temperatures from its zero temperature value is exponentially small in the case of a superconductor with finite gap and manifests a power law temperature dependence for a superconductor with nodes of the order parameter on the Fermi surface [15–19]. We are interested here in the indices of those power laws for temperatures \( (\Gamma \Delta_0)^{1/2} \approx T \ll \Delta_0 \) in the case of pure superconductors with unitary order parameters having various multiplicities of the nodes. As a starting point one can use the following expression for the eigenvalues of the penetration depth tensor:

\[
\frac{1}{\lambda_i^2} = \frac{4 \pi e^2}{c^2} \int \frac{d^2 S}{(2 \pi)^2 v_F^2} \sum_m \frac{|\Delta(\hat{p})|^2}{\left( (2m + 1)^2 \frac{\pi^2}{n^2} T^2 + |\Delta(\hat{p})|^2 \right)^{3/2}},
\]  

where index \( i \) denotes the direction of the supercurrent and \( v_{F,i} \) is the \( i \)-component of the quasiparticle velocity at the Fermi surface. Applying the Poisson formula to the sum in (25), we get for the deviation \( \delta \lambda_i(T) \) from \( \lambda_i(0) \) at \( T \ll T_c \):

\[
\frac{\delta \lambda_i(T)}{\lambda_i(0)} = - \int \frac{d^2 S}{v_F w_{F,i}^2} \sum_{m=1,2,\ldots} (-1)^m m K_1 \left( m \frac{|\Delta(\hat{p})|}{T} \right) \frac{|\Delta(\hat{p})|}{T},
\]  

where \( K_1(x) \) is the modified Bessel function.

For \( T \ll \Delta_0 \) the main contribution to the integral over the Fermi surface in the numerator comes from narrow regions near nodes of \( \Delta(\hat{p}) \). As in the previous section we consider a
tetragonal superconductor with cylindrical Fermi surface and the order parameter having four lines of nodes. Then it follows from (26) for the in-plane penetration depth

$$\frac{\delta \lambda_\parallel(T)}{\lambda_\parallel(0)} = -\int_0^{2\pi} \frac{d\varphi}{2\pi} \sum_{m=1,2,...} (-1)^m m K_1 \left( \frac{m|\Delta(\varphi)|}{T} \right) \frac{|\Delta(\varphi)|}{T}, \quad (27)$$

where the integration is carried out over the angle in the basal plane.

Assuming relation (21) for the order parameter in the vicinities of lines of nodes we get for the leading correction to the penetration depth at low temperatures

$$\frac{\delta \lambda_\parallel(T)}{\lambda_\parallel(0)} = \Omega(n) \left( \frac{T}{\Delta_0} \right)^{1/n}, \quad (28)$$

where

$$\Omega(n) = -\frac{2}{\pi n} 2^{1/n} \Gamma \left( \frac{2n+1}{2n} \right) \Gamma \left( \frac{1}{2n} \right) \sum_{m=1,2,...} (-1)^m m^{1/n}. $$

In the particular case $n = 1$ one obtains from here $\delta \lambda_\parallel(T)/\lambda_\parallel(0) = 2 \ln 2 (T/\Delta_0)$.

V. THERMAL CONDUCTIVITY

The investigation of the thermal conductivity in a superconducting state is an important experimental probe of the order parameter structure.

Thermal conductivity for an isotropic s-wave superconductor is known to fall off exponentially at low temperatures due to the presence of a finite superconducting gap for all directions on the Fermi surface. For unconventional superconductors a temperature dependence of the electron thermal conductivity at low temperatures $T \ll \Delta_0$ is described by the power laws and proves to be associated with several factors. Besides the order parameter structure, it is substantially influenced by the strength of impurity scattering. For the thermal conduction components directed to the nodes of the order parameter one usually obtains in the case of weak scattering processes (Born approximation) a linear temperature dependence, as in the normal state. Other components vanish with the temperature according to the higher power laws. In the case of resonance impurity scattering the situation is more complicated and the region of low temperatures $T \ll \Delta_0$ is naturally divided into two parts. For temperatures $T \lesssim \omega_c = (\Gamma \Delta_0)^{1/2}$ the existence of the quasiparticle states bound to impurities is of importance. The correspondent thermal conductivity at those temperatures has also linear temperature dependence albeit with reduced density of states. By contrast, for the temperatures $\omega_c \lesssim T \ll \Delta_0$ one can disregard the bound states and then one obtains, generally speaking, the higher power laws for all the components of thermal conductivity. Since experiments have shown that in several heavy fermion superconductors the thermal conductivity vanishes with higher powers of $T$ at low temperatures (see, for example [20]) the theory for thermal conductivity in unconventional superconductors has been developed and examined in detail [21, 24, 25–31]. The experimental results for $UPt_3$ are in correspondence with the strong scattering from the impurities, close to the unitarity limit. Some physical discussion of the strength of the impurity scattering in $UPt_3$ is in [21, 22].
Recent experimental results for $UPt_3$ obtained in \cite{1,2} have drawn much attention \cite{28–31}, as they gave some further useful information about the order parameter structure in this heavy fermion hexagonal superconductor. In particular, it has been observed that the anisotropy ratio $\kappa_c/\kappa_b$ of the thermal conductivity components in $UPt_3$ does not vanish at low temperatures (down to $T_c/10$). The temperatures, at which one can disregard the influence of impurity bound states on the transport coefficients, depend upon the impurity concentration. It has been stressed earlier \cite{26} that for superconducting $UPt_3$ this influence usually may be neglected just down to $T_c/10$. It is confirmed by the absence of a linear term in the temperature dependence of the thermal conductivity observed in \cite{2} for temperatures down to $T_c/10$ as well.

Based on this fact below we give a simple qualitative examination of the thermal conductivity at low temperatures for pure unconventional superconductors at temperatures $\omega_c \lesssim T \ll \Delta_0$, separating the contributions from points and lines of nodes (and from the rest of the Fermi surface, when it is needed) and permitting the order parameter to have zeroes of different orders at the nodes. Such a consideration results in a simple condition for the realization of a finite anisotropy ratio $\kappa_c/\kappa_b$ at low temperatures. We believe that our simple consideration of the thermal conductivity at low temperatures $\omega_c \lesssim T \ll \Delta_0$, which permits to discuss qualitatively a wide number of basis functions for various types of pairing, complements in some aspects several recent quantitative investigations of this problem (see, for example, \cite{28,30,31}). On the basis of self-consistent description of the impurity scattering in unconventional superconductors, the numerical calculations are performed there for all temperatures and for several particular basis functions, which are considered now to be of special interest for the study of superconductivity in $UPt_3$ and in HTSC as well.

So, lets neglect the impurity bound states and consider only essentially anisotropic superconducting states which meet the condition $\sum_p \Delta(p)/(E^2 - E_p^2) = 0$. Then one has for the thermal conductivity of unconventional superconductor in the case of resonance scattering (see e.g. \cite{23,24})

$$\frac{\kappa_{ij}}{\kappa_N(T_c)} = \frac{18T}{\pi^2 T_c} \int_{0}^{\infty} dE \left( \frac{E}{T} \right)^2 \left[ - \frac{\partial n^0(E)}{\partial E} \right] \left| g(E) \right|^2 \text{Re} \frac{d}{dE} \int_{|\Delta(p)| \leq E} \frac{d\Omega_{p_i p_j}}{4\pi} \sqrt{E^2 - |\Delta(p)|^2} \frac{E}{E^2}. \quad (29)$$

Here $\kappa_N(T_c)$ is the thermal conductivity in the normal state at $T = T_c$, $n^0(E)$ is the equilibrium Fermi distribution function for excitations, and the function $g(E)$ is defined as follows

$$g(E) = \int \frac{d\Omega}{4\pi} \frac{E}{\sqrt{E^2 - |\Delta(p)|^2}}. \quad (30)$$

One can easily see that due to the factor $\partial n^0(E)/\partial E$ at low temperatures $T \ll \Delta_0$ only low energy behavior of the integrand in (29) is of interest (for $E \lesssim T$). Then only narrow regions near the nodes of the order parameter are essential for the integral over the momentum directions in (29) and for the real part of the function $g(E)$. But, generally speaking, it is not the case for the imaginary part of $g(E)$ at low energy (see (30)). Due to this fact and contrary to the case of specific heat, for the thermal conductivity at low temperatures $T \ll \Delta_0$ contributions from all the Fermi surface – not only from the narrow vicinities of nodes of the order parameter on the Fermi surface – may be of importance.
Nevertheless the contributions can be divorced from each other and specified, and in the case of zeroes of higher orders at the nodes only those narrow regions become dominating. Note that regions far from nodes, as it is seen from (30), result in the imaginary part of $g(E)$ at low energy which is simply a linear function of energy.

Let’s consider, for instance, the order parameter, which behaves near the line of nodes as $|\Delta(p)| = \Delta_0|\theta - \pi/2|^n$ ($|\theta - \pi/2| \ll 1, n > 0$) and near the point node as $|\Delta(p)| = \Delta_0\theta^m$ ($\theta \ll 1, m > 0$). Then one gets at low energy $Re g(E), Im g(E) \propto (E/\Delta_0)^{1/n}, (E/\Delta_0)^{2/m}$ respectively, but in the case of imaginary part of $g(E)$ this estimate is valid only if $n > 1, m > 2$. For values $m < 2$ and $n < 1$ one obtains for the main term $Im g(E) \propto E$ which results from regions lying far from the nodes of the order parameter on the Fermi surface. So, both $Re g(E)$ and $Im g(E)$ should be taken into account in considering the thermal conductivity at low temperatures. In the case $n > 1, m > 2$ only the behavior of the order parameter near nodes turns out to be essential; for $n < 1, m < 2$ contributions from all the Fermi surface are of importance. For specific and quite important particular values $n = 1$ and $m = 2$ one gets apart from the linear terms in $E$ for $Re g(E), Im g(E)$ also a contribution from the vicinity of nodes $Im g(E) \propto (E/\Delta_0)\ln(E/\Delta_0)$. As in the case of specific heat a pure logarithmic approximation is a very strong restriction on the magnitude of energy and usually logarithmic factors must be taken into consideration together with constants. The anisotropy of the thermal conduction is defined by the integral over momentum directions in (29) as well. As a result one gets for temperatures $\omega_c \lesssim \tilde{T} \ll \Delta_0$:

$$\kappa_{zz} = L_z \begin{cases} T^{1+4/n}, & n > 1 \\ T^5(\ln^2(T/\Delta_0) + l_1 \ln(T/\Delta_0) + l_2), & n = 1 \\ T^{2/n+3}, & n < 1 \end{cases}$$

$$+ P_z \begin{cases} T^{1+4/m}, & m > 2 \\ T^3(\ln^2(T/\Delta_0) + p_1 \ln(T/\Delta_0) + p_2), & m = 2 \\ T^{3}, & m < 2, \end{cases}$$

$$\kappa_{xx} = \kappa_{yy} = L_x \begin{cases} T^{1+2/n}, & n > 1 \\ T^3(\ln^2(T/\Delta_0) + l_1 \ln(T/\Delta_0) + l_2), & n = 1 \\ T^{3}, & n < 1 \end{cases}$$

$$+ P_z \begin{cases} T^{1+6/m}, & m > 2 \\ T^4(\ln^2(T/\Delta_0) + p_1 \ln(T/\Delta_0) + p_2), & m = 2 \\ T^{3+2/m}, & m < 2. \end{cases}$$

The coefficients $L, l_1, l_2$ and $P, p_1, p_2$ ($i = x, z$) correspond here to the contributions from the line of nodes and the point node respectively.

It follows from Eqs. (31), (32), that under the condition $m = 2n$ the main contribution to $\kappa_{zz}$ at low temperatures originates from the point node, while in the case of $\kappa_{xx}$ the contribution from the line of nodes dominates. Taking this fact into account one obtains in qualitative agreement with experimental results the finite (temperature independent) value for the anisotropy ratio $\kappa_{zz}/\kappa_{xx}$ at low temperatures under the condition $m = 2n$. It is worth noting that in the case of hexagonal superconductor an important particular example for
the type of pairing with \( m = 2n \) (and integer \( n, m \)) is a \( E_{2u} \)-representation (in the simplest case \( m = 2n = 2 \)). At the same time in the simplest case of \( E_{1g} \)-representation one should put \( n = m = 1 \) and then it follows from Eqs. (31), (32), that the ratio \( \kappa_{zz}/\kappa_{xx} \) manifests only feebly marked logarithmic temperature dependence at low temperatures, which possibly can not be determined now experimentally within the temperature interval \( \omega_c \lesssim T \ll \Delta_0 \).

From this qualitative discussion one can arrive at the conclusion, that the both representations \( E_{1g} \) and \( E_{2u} \) with the simplest particular basis functions do not contradict to the finite value of the thermal conductivity anisotropy ratio at temperatures \( \omega_c \lesssim T \ll \Delta_0 \). In more complicated cases the anisotropy ratio \( \kappa_{zz}/\kappa_{xx} \) in fact doesn’t depend upon temperature within the temperature region considered, if the indices \( n \) and \( m \) satisfy the condition \( m = 2n \). The presence of both line of nodes and point nodes is essential in order to form the observed temperature dependence of the thermal conductivity.

VI. QUASIPARTICLE TUNNELING AT LOW TEMPERATURE AND VOLTAGE

For isotropic s-wave superconductors a small number of quasiparticles activated above the gap at low temperatures \( T \ll \Delta_0 \) gives rise to the junction conductance which falls off exponentially with decreasing temperature. At zero temperature and for the externally applied voltage \( V < \Delta_0/e \) for NS junctions (and \( V < (\Delta_{0+} + \Delta_{0-})/e \) for SS junctions) the quasiparticle current across the tunnel junctions takes place only for superconductors with anisotropic order parameter. In the case of very small value of voltage \( (V \ll \Delta_0) \) the current occurs only for the superconductors with nodes of the order parameter on the Fermi surface. Below we show that respective I-V curves for these junctions essentially depend on the behavior of the order parameter in the vicinity of nodes, in particular, on the multiplicities of the nodes.

If the permanent voltage \( V \) is applied to the tunnel junction between two metals, then one gets the following expression for a dissipative contribution to the current across the junction in the lowest order in the junction transparency \( D(\hat{p}) \) \( (\hat{p} = p_F/|p_F|) \); see e. g. [32,33]:

\[
j_N = e \int_{v_x > 0} \left( \int \left[ \tanh \left( \frac{E}{2T} \right) - \tanh \left( \frac{E - eV}{2T} \right) \right] g_+(E - eV, \hat{p}_+) g_-(E, \hat{p}_-) dE \right) \\
\times v_x(\hat{p}_-) D(\hat{p}_-) \frac{d^2 S_-}{(2\pi)^3 v_F}.
\]

Here \( g_\pm(E, \hat{p}_\pm) \) are the normalized densities of states (for fixed both energy and the momentum direction) of two metals in the vicinity of a tunnel barrier, the index \(+(-)\) labels the right (left) half space with respect to the boundary plane, and \( v_x \) is the Fermi velocity component along the normal to the plane interface \( n \parallel Ox \). The integration in (33) is carried out over the part of the Fermi surface with \( v_x > 0 \). The relation between the incident and transmitted Fermi momenta ( that is between \( p_- \) and \( p_+ \) ) is as follows. The components parallel to the specular plane interface are equal to each other, while the values of their normal components are determined by the Fermi surfaces of corresponding metals. Naturally, in the particular case of identical superconductors with the spherical Fermi surfaces the total incident and transmitted momenta are equal to each other \( p_- = p_+ \). For an
anisotropically paired superconductor the tunneling density of states is essentially dependent upon the orientation of crystalline axes of the metal relative to the interface. Below we consider, for simplicity, only those particular crystal orientations, when the order parameter is not suppressed at the specularly reflecting interface as compared to its value in the depth of pure superconductor. These orientations are determined by the equation $\Delta(\hat{p}) = \Delta(\hat{p})$, where $\hat{p}$ (\hat{p}) denotes the direction of the incident (reflected) electron momentum (see e.g. [33]). Then the following expression for the density of states is valid

$$g_{\pm}(E, \hat{p}_\pm) = \frac{|E|\Theta(|E| - |\Delta_{\pm}(\hat{p}_\pm)|)}{\sqrt{E^2 - |\Delta_{\pm}(\hat{p}_\pm)|^2}}. \quad (34)$$

Under the condition $T \ll eV$ Eq. (33) is transformed after substitution (34) into (33) to the expression

$$j_N = 2e^2V \int_{v_x>0} \int_0^1 d\omega \frac{\omega(1-\omega)\Theta(\omega - |\Delta_-(\hat{p}_-)/eV|)\Theta(1-\omega - |\Delta_+(\hat{p}_+)/eV|)}{\sqrt{\omega^2 - |\Delta_-(\hat{p}_-)|^2}/(eV)^2(1-\omega)^2 - |\Delta_+(\hat{p}_+)/eV|^2}/(eV)^2$$

$$\times D(\hat{p}_-) v_x(\hat{p}_-) \frac{d^2S_-}{(2\pi)^3v_F}. \quad (35)$$

One can easily see that for SS junction in the case $|eV| \ll \Delta_{0\pm}$ the contribution to the integral in (35) comes entirely from the narrow vicinities of the directions to the common nodes of two order parameters, as if they were drawn on the same Fermi surface in taking account of the relative orientation of the superconductors. These directions are defined by the relation $|\Delta_-(\hat{p}_-)| = |\Delta_+(\hat{p}_+)| = 0$. Let the latter condition results in the momentum direction $\hat{p}_i$ which corresponds to a point of the intersection on the Fermi surface of two lines of nodes of the order parameters $\Delta_-$ and $\Delta_+$ respectively. We suppose that the behavior of the order parameters close to this point may be represented as follows

$$|\Delta_-(\hat{p})| = \Delta_{0-}|\gamma_-|^n, \quad |\Delta_+(\hat{p})| = \Delta_{0+}|\gamma_+|^m, \quad n, m > 0, \quad (36)$$

where $\gamma_{\mp}$ are the angles which are counted from $\hat{p}_i$ in the directions perpendicular to the lines of nodes of the corresponding order parameters $\Delta_{\mp}$. Substituting (36) into (35) we obtain in the particular case of spherical Fermi surface the leading order contribution to the quasiparticle current for $eV \ll \Delta_0$ :

$$j_N = a \frac{e^2p_F^2}{v_F|\sin \chi|} v_x(\hat{p}_i) D(\hat{p}_i)(eV)|eV/\Delta_0|^{\frac{1}{n} + \frac{1}{m}}. \quad (37)$$

Here $a$ is a numerical factor and $\chi$ is the angle between two lines of nodes in the point of their intersection. The angle $\chi$ is supposed to obey the condition $\chi \gg (eV/\Delta_0)^{1/n}$ ( for $m > n$). In the opposite limit the lines coincide and then one gets $j_N \propto V^{1+(1/n)}$. Summing up the contributions from all intersection points one finds

$$j_N = \frac{\tilde{a}}{R_N} V|eV/\Delta_0|^{\frac{1}{n} + \frac{1}{m}}, \quad (38)$$
where \( R_N \) is the normal-state resistance of a tunnel junction and the numerical factor \( \tilde{a} \) depends on the relative orientation of superconductors.

We see that the index of the power law (38) for the I-V curve in the case of SS junction at law voltage and temperature depends essentially upon the multiplicities of nodes of the order parameters. In the particular case \( n = m = 1 \) the result (38) is reduced to that obtained earlier in [33].

The quasiparticle current for the case of NS junction may be obtained by substituting \( \Delta_+ = 0 \) into (35) and integrating over \( \omega \)

\[
\hat{j}_N = 2e^2V \int_{v_x>0} \sqrt{1 - \frac{|\Delta(\hat{p})|^2}{(eV)^2}} \Theta \left( 1 - \frac{|\Delta(\hat{p})|}{eV} \right) D(\hat{p})v_x(\hat{p}) \frac{d^2S}{(2\pi)^3v_F}. \tag{39}
\]

If the externally applied voltage is small enough, so that \(|eV| \ll \Delta_0\), the contribution to \( j_N \) in (39) comes from narrow vicinity of nodes of the order parameter. One can easily verify that if the order parameter near the lines of nodes on the spherical Fermi surface has the form \(|\Delta(\hat{p})| = \Delta_0|\theta - \theta_0|^n\) or \(|\Delta(\hat{p})| = \Delta_0|\varphi - \varphi_0|^n\), it follows from (39) for the quasiparticle current across NS junction

\[
\hat{j}_N \propto e^2V|eV/\Delta_0|^{1/n}. \tag{40}
\]

Analogously, in the case of the order parameter behavior \(|\Delta(p)| = |\Delta_0|(|\theta - \theta_0|^n + \Delta_0|\varphi - \varphi_0|^m| \) close to the point node in the direction \( \hat{p}_0 \), one gets

\[
\hat{j}_N \propto \frac{V}{R_N} |eV/\Delta_0|^{\frac{n}{n} + \frac{1}{m}}. \tag{41}
\]

At last, for the order parameter behavior near the pole \(|\Delta(\hat{p})| = \Delta_0|\phi - \phi_0|^m\theta^{2n} \) the contribution to the quasiparticle current is given by

\[
\hat{j}_N \propto e^2D(\hat{p}_{pole})v_x(\hat{p}_{pole}) \begin{cases} 
V \left| \frac{eV}{\Delta_0} \right|^{1/m}, & m > n, \\
V \left| \frac{eV}{\Delta_0} \right|^{1/m} \left( \ln \left| \frac{\Delta_0}{eV} \right| + c \right), & m = n, \\
V \left| \frac{eV}{\Delta_0} \right|^{1/n}, & m < n.
\end{cases} \tag{42}
\]

The results (40) and (42) in the particular cases \( n = 1 \) and \( m = n = 1 \) are reduced to those obtained earlier in [33].

VII. ANISOTROPY OF THE BOUNDARY CONDITIONS FOR A SUPERCONDUCTING ORDER PARAMETER

Many aspects of the problem of boundary conditions for the order parameter of anisotropically paired superconductors have already been studied in the literature (see,
e.g. \[34,38,33,37\] and references therein). Below we discuss one characteristic feature of the boundary conditions near \( T_c \), which has been recently noticed in \[38\] for the case of specularly reflecting boundary. For simplicity, let’s consider a singlet pairing and a one-component anisotropic order parameter in the vicinity of the plane specular and fully reflecting boundary at \( x = 0 \). For any crystal orientation with respect to the interface the boundary condition may be written in the form \( \eta p'(0) = \eta(0) \). It turns out that with the aid of general symmetry arguments and, for example, irrespective of the shape of the Fermi surface of the given symmetry, one can unambiguously fix only those crystal orientations for which the parameter \( q \) is equal to zero and infinity. The value \( q = \infty \) corresponds to the boundary condition \( \eta'(0) = 0 \) coinciding with the condition for an isotropic s-wave superconductor at an impenetrable boundary, where the relation \( \eta(0) = \eta_\infty \) is valid. Here \( \eta_\infty \) is the value of the superconducting order parameter far from the boundary. For an anisotropically paired superconductor one has \( q = \infty \) only for those orientations of the boundary for which the relation \( \Delta(\hat p) = \Delta(\hat p) \) holds. The value \( q = 0 \) is realized under the condition \( \Delta(\hat p) = -\Delta(\hat p) \), in which case the boundary condition takes the form \( \eta(0) = 0 \) and the order parameter is fully suppressed at the interface. As earlier \( \hat p \) (\( \hat p \)) denotes here the direction of the incident (reflected) electron momentum. For the point symmetry group \( D_{4h} \) of the crystal there are five planes of symmetry, and the order parameter transforms according to one of the irreducible representations of this group. So, if the normal to the boundary is perpendicular to the symmetry plane and the character of the irreducible representation is \( 1 \) (-1), the boundary condition is \( \eta'(0) = 0 \) \( \eta(0) = 0 \).

For other orientations the detailed form of \( q \) as a function of crystal orientation relative to the boundary plane for the given type of pairing essentially depends on the particular form of the basis function \( \Delta(\hat p) \) in the depth of superconductor. Since the characteristic value for the parameter \( q \) is \( \xi(T) \) one can divide all orientations into two regions where \( q < \xi(T) \) and \( q > \xi(T) \) respectively. At first sight the characteristic scales for these two regions could be comparable on the order of values. However, it is not the case near \( T_c \) and may be valid only for low temperatures. E.g. for the particular basis function \( p_{x_0}^2 + p_{y_0}^2 \) (pertaining obviously to the pairing symmetry of the type \( \phi_0 - p_{y_0}^2 \) and the spherical Fermi surface, the values \( \eta(0) \) and \( \eta_\infty \) are found to be of the same order of magnitude (and hence \( \xi(T) \approx q \)) only for particular orientations of the normal to the boundary \( n \) within narrow angular intervals \( \Delta \phi_0 \sim \Delta \theta_0 \sim (\xi_0/\xi(T))^{1/2} \) around the crystal axes \( x_0, y_0, z_0 \). For other crystal orientations one has \( \eta(0) \approx \eta_\infty \xi_0/\xi(T) \approx \eta_\infty \). Thus, near \( T_c \) the d-wave superconducting order parameter proves to be strongly suppressed in the vicinity of the insulating barrier, except for the small part of orientations within the narrow angular intervals mentioned above. From this point of view the qualitative difference between the boundary conditions for specular and diffusive interfaces near \( T_c \) manifests only within those narrow angular intervals. Indeed, in the case of diffusive scattering at the interface one has \( q \approx \xi_0 \) irrespective of the orientations \[38\] and, hence, near \( T_c \) the parameter \( \eta(0) \approx \xi_0 / \xi(T) \approx \eta_\infty \) for all crystal orientations. So, near \( T_c \) narrow peaks \( \xi_0 \approx \xi(T) \approx q \) of the parameter \( q \), which occur in the case of specularly reflected boundaries, are simply cut if the boundary is diffusive.

It is of interest to consider whether the choice of a different basis function pertaining to the same symmetry type of pairing can result near \( T_c \) in the expansion of the angular regions where the parameter \( q \) has large values \( \xi(T) \approx q \). Below we show that it is not possible for quite a wide set of the bases functions. In particular, for the bases functions with higher
multiplicities of the nodes on the Fermi surface the angular regions with $\xi(T) \lesssim q$ prove to be more narrow, not wider.

Microscopic consideration based, in particular, on some variational procedure, leads to the following expression for the parameter $q$ near $T_c$ [33,34,37]:

$$q = \xi_0 \left( \frac{\pi^3}{336\zeta(3)} \int_{v_x>0} |\psi(\hat{p}) + \psi(\hat{p})|^2 \hat{v}_x^2 d^2S + \frac{7\zeta(3)}{4\pi^3} \frac{(\int_{v_x>0} |\psi(\hat{p}) + \psi(\hat{p})|^2 \hat{v}_x^2 d^2S)^2}{\int_{v_x>0} |\psi(\hat{p}) - \psi(\hat{p})|^2 \hat{v}_x^2 d^2S} \right)$$

$$\times \left( \int |\psi(\hat{p})|^2 \hat{v}_x^2 d^2S \right)^{-1}. \quad (43)$$

Here integration over the Fermi surface is confined to its part with $v_x > 0$ and the following notations are introduced: $\hat{v}_x = v_{Fx}/v_F$, $\xi_0 = v_F/T_c$. The superconducting order parameter for the inhomogeneous state near $T_c$ is supposed to be represented in the conventional form $\Delta(\hat{p}, r) = \psi(\hat{p}) \eta(r)$.

For isotropic superconductors the basis function $\psi$ doesn’t depend upon the momentum at all and one gets from (43) the equality $q = \infty$. For anisotropically paired superconductors there are, as a rule, only several isolated and governed by the symmetry crystal orientations relative to the boundary for which $q = \infty$ and, hence, $\psi(\hat{p}) = \psi(\hat{p})$ for all $\hat{p}$. The crystal orientation with respect to the boundary may be characterized by the unit vector $n$ which describes the orientation of the normal to the boundary relative to the crystalline axes $x_0, y_0, z_0$. Denoting by $\psi$ the orientation corresponding to the value $q = \infty$, we are interested now in the characteristic scale of the angular regions around the direction $n_0$ where $\xi(T) \lesssim q$, i.e. the parameter $q$ has quite a large value yet.

It follows from Eq.(43), that the relation $\xi(T) \lesssim q$ is valid under the condition

$$\frac{\xi(T)}{\xi_0} \lesssim \frac{\int_{v_x>0} |\psi(\hat{p}) + \psi(\hat{p})|^2 \hat{v}_x^2 d^2S}{\int_{v_x>0} |\psi(\hat{p}) - \psi(\hat{p})|^2 \hat{v}_x^2 d^2S}. \quad (44)$$

Since in the case $n = n_0$ one gets $\psi(\hat{p}) = \psi(\hat{p})$ (in the particular case when the boundary coincides with the symmetry plane of the Fermi surface, one has $\hat{p} = \hat{p} - 2(\hat{p}n)n$), in the narrow vicinity of this direction one obtains $\psi(\hat{p}) \approx \psi(\hat{p}) + (\delta n \nabla n)\psi(\hat{p})|_{n=n_0}$. Here $\delta n = n - n_0$ and one has $(n, n_0) \approx |\delta n|$ for the angle between vectors $n$ and $n_0$. Substituting these relations into (44), we obtain the following restriction for the angle $(n, n_0)$:

$$(n, n_0) \lesssim \left( \frac{\xi_0}{\xi(T)} \right)^{1/2} \frac{(\int_{v_x>0} |\psi(\hat{p})|^2 \hat{v}_x^2 d^2S)^{1/2}}{(\int_{v_x>0} |(e_{\delta n} \nabla n)\psi(\hat{p})|_{n=n_0}|^2 \hat{v}_x^2 d^2S)^{1/2}}, \quad (45)$$

where $e_{\delta n}$ is the unit vector along the direction $\delta n$.

We see that the appearance of the small parameter $(\xi_0/\xi(T))^{1/2}$ is a quite general characteristic feature for the estimation of the angle $(n, n_0)$, which is independent upon the particular form of the basis functions. In order to estimate the other factor by which the first one is multiplied in (45), let’s consider the following set of the basis functions for the tetragonal unconventional superconductors

$$\psi(\hat{p}) = \Delta_0 \tanh \left\{ \mu (p_{x0}^2 - p_{y0}^2)^n \right\}, \quad n = 1, 2, \ldots, \quad (46)$$
where $\mu$ and $n$ are the parameters. All functions with odd values of $n$ belong evidently to the same type of pairing as the function $(\hat{p}_{x0}^2 - \hat{p}_{y0}^2)$. Analogously, the functions with even values of $n$ pertain to the unit representation of the point tetragonal group $D_{4h}$ (then respective nodes of the order parameter, of course, are accidental and not associated with the symmetry type of pairing).

The substitution of (46) into (45) results in the following estimation for the angular intervals around $n_0$, where $\xi(T) \lesssim q$:

$$\Delta \varphi_0, \Delta \theta_0 \sim \frac{1}{\nu n} \left(\frac{\xi_0}{\xi(T)}\right)^{1/2},$$

(47)

where $\nu = \max(1, \mu)$.

It follows from here that the angular intervals can’t be essentially more than $(\xi_0/\xi(T))^{1/2}$, but may be more narrow, for instance, in the case of higher multiplicities of the nodes of the order parameter.

**VIII. CONCLUSIONS**

Density of states at low energy, the specific heat in homogeneous and mixed states at low temperatures, the low temperature behavior of the thermal conductivity, the penetration depth and I-V curves for the quasiparticle current at low voltage have been examined for pure anisotropically paired superconductors having various multiplicities of the nodes of the order parameter. A specific anisotropy of the boundary conditions for unconventional superconducting order parameter near $T_c$ for the case of specular reflection from the boundary was also investigated. Since the multiplicities may be different within the same symmetry type of pairing, the problem of an unambiguous identification of a type of superconducting pairing on the basis of corresponding experimental results becomes more complicated and should be considered taking into account the results obtained above.

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