I. INTRODUCTION

It may be noted that ordinary particles are described by a number of quantities, like mass and spin, and these quantities are not dynamically obtained in quantum field theories. However, in string theory, these can be obtained from the dynamics of world-sheet. This is because strings have an extended structure, and have their own internal dynamics associated with that internal structure. So, unlike particles in ordinary quantum field theory (which have no internal dynamics and are just points in space-time), the internal dynamics of a world-sheet of string theory are represented by a conformal field theory. Thus, such quantities in ordinary quantum field theory can be dynamically calculated using this conformal field theory describing the world-sheet dynamics of string theory. In string theory, there are background fields. These fields can interact with the strings, and change their oscillatory modes. The change in the oscillatory modes would also change the value of quantities, such as mass and spin, which can be obtained dynamically from the world-sheet dynamics. It is important to analyze the change in the value of such quantities due to the interaction of strings with background fields.

It is known that the change in a quantity due to the interaction of a system with some external potential can be obtained using two-point measurement scheme. In the two-point measurement scheme, a system interacts with a field, and this changes the initial eigenvalues of an operator to final eigenvalues. The difference between these two projective measurements of a quantity, made at the beginning and end of such a process, are then used to obtain the distribution function for the quantity. This difference is the difference between the initial and final eigenvalues of the operator, after the system has interacted with an external field. Such two projective measurements cannot be made on a system with an intrinsic Lorentz structure, due to problems like violation of locality and superluminal signals. However, these problems can be resolved by using the Ramsey scheme. The Ramsey scheme has been applied to quantum field theory.

In this scheme, first an auxiliary qubit is coupled to the system, and then the information about the system is transferred to the qubit. The information about the system is obtained by making measurements on the qubit. The effects of the evolution of the system by its interaction with an external field can also be probed using this qubit. This is done by preparing the qubit in a superposition of ground and excited states, and then transferring the information about the characteristic function of the distribution to the state of the qubit. This procedure can be used to obtain the information about the change in the eigenvalues of an operator, without making projective measurements. The Ramsey scheme has been used to obtain such a difference in quantum field theory. As the dynamics of the world-sheet of string theory is represented by a conformal field theory, in this letter, we will generalize the Ramsey scheme to string theory.

We will also analyze the flow of information on the world-sheet of strings. This will be done by using quantum fisher information. In fact, quantum fisher information can be used to analyze the behavior of those quantities, which are not represented by a quantum operator. As the difference of the initial and final values of a quantity, in a string theoretical process, cannot be represented by a quantum operator, we will use quantum fisher information to analyze its behavior. This quantum fisher information would be obtained from the distribution of the difference between the initial and final values of a given quantity. Here we point out that even though the original quantity can be represented by a quantum operator, the change in the quantity in a quantum process cannot be represented by any quantum operator. This is similar to the Hamiltonian being a quantum operator, but quantum work (which represents the change in the eigenvalues of the Hamiltonian in a quantum process) cannot be represented by any operator.
II. STRING THEORETICAL PROCESS

In this section, we will analyze string theoretical process in which a string interact with a background field. To analyze such a process, we will start from the free Polyakov action, which describes the world-sheet of bosonic string theory

\[ S = \frac{1}{4\pi\alpha'} \int d\sigma^2 \eta^{\alpha\beta} \partial_\alpha X_\mu \partial_\beta X^\mu. \]  

(1)

Here \( \alpha' \) is related to the length of string \( l_s^2 \). We can apply the Neumann boundary conditions for open strings, and write

\[ X^{\mu}(\tau, \sigma) = x^{\mu} + l_s \tau p^{\mu} + il_s \sum_{m \neq 0} \frac{1}{m} \alpha_m^{\mu} e^{-im\tau} \cos(m\sigma). \]  

(2)

As string theory has a gauge degrees of freedom, we need to fix a gauge before quantizing it. This can be done using the BRST quantization in the conformal gauge \([23]\). This is done by adding a gauge fixing term (conformal gauge) and a ghost term to the Polyakov action. This new effective action obtained by the sum of the original Polyakov action with the gauge fixing and ghost term is invariant under BRST transformation. The physical states of the system are then constructed using the Noether charge corresponding to this symmetry of the effective action. This Noether change is called the BRST charge, \( Q \), and physical states satisfy \( Q|\psi >= 0 \). However, as the BRST quantization in conformal gauge is equivalent to light-cone quantization \([24, 25]\), we can use analyze string states in light-cone gauge \([26, 27]\). Furthermore, the covariant string theory, splits into the light-cone string theory and trivial excitations \([28]\). Thus, it is possible to analyze string theory in light-cone gauge \([26, 27]\). This finite field BRST transformation is not the symmetry of the path integral measure, and the generating functional in light-cone gauge is equivalent to light-cone quantization \([24, 25]\), we can use analyze string states in light-cone gauge \([26, 27]\). This finite field BRST transformation is not the symmetry of the path integral measure, and the Jacobian of transformation can be used to transform the generating functional from light-cone gauge to covariant gauge \([32, 39]\). In fact, it has also been demonstrated that finite field dependent BRST transformation can be used to obtain the generating functional in light-cone gauge from the generating functional conformal gauge \([40]\). Here again we observe that in the light-cone gauge the ghost sector decouples from the theory, and can be neglected in light-cone quantization \([31]\). Thus, it is possible to analyze string theory in light-cone gauge \([26, 27]\). This is similar to the situation in ordinary electrodynamics, where the ghost sector decouples from the theory, and can be neglected for perturbative calculations \([31]\). Thus, the string oscillations in the \( X^+ \) direction can be expressed in terms of other string oscillations, and need not be separately analyzed. Thus, in the light-cone gauge, only the string oscillations in the \( \{X^{i}_{1}\}_{i=1}^{24} \) direction have to be considered. The string algebra in these directions can be written as

\[ [\hat{\alpha}_{m}^{i}, \hat{\alpha}_{n}^{j}] = mn \delta_{i}^{j} \delta_{m+n,0}. \]  

(3)

This can be written as \( [\hat{\alpha}_{m}^{i}, \hat{\alpha}_{-m}^{j}] = m \) for \( n = -m \) and \( (\hat{\alpha}_{m}^{i})^\dagger = \hat{\alpha}_{-m}^{j} \) with \( \{i, j\} \), taking values in \( \{1, 2, 3, \ldots, 24\} \).

We define a vacuum as \( |0, p\rangle \) such that \( \hat{p}^{i}|0, p\rangle = p^{i}|0, p\rangle \). Using this vacuum, we can construct a state \( |k, 0\rangle \) from \( \alpha_k|0, p\rangle \). We start by constructing a string density matrix \( \hat{\rho} \) from these string states as

\[ \hat{\rho} = \mathcal{N} \sum_{l} \sum_{k} d_{k} d_{l}^{\dagger} |k; p\rangle \langle l; p|, \]  

(4)

\( \mathcal{N} = 1/\sum_{k} |d_{k}|^2 \) where \( \mathcal{N} \) is the normalization constant. This string density state evolves from \( \hat{\rho}(0) \) at time \( \tau = 0 \) to \( \rho(t) \) at time \( \tau = t \), such that

\[ \hat{\rho}(t) = U(t) \hat{\rho}(0) U^\dagger(t). \]  

(5)

Here \( U \) is a suitable unitary operator, which evolves the initial string density state to the final string density state \([11, 42]\). Such a unitary evolution occur due to the interaction of string states by a background field. Thus, for example, with a background field \([3, 6]\). The the Polyakov action with a background \( B^{\mu\nu}(X) \) field, gravitational field with metric \( G^{\mu\nu}(X) \), and dilaton field \( \Phi(X) \) (which couples to the two-dimensional world-sheet Ricci scalar \( R \)), can be written as

\[ S = \frac{1}{4\pi\alpha'} \int d\sigma^2 \left[ \eta^{\alpha\beta} \left( G^{\mu\nu}(X) + iB^{\mu\nu}(X) \right) \partial_\alpha X_\mu \partial_\beta X_\nu + \alpha' \Phi(X) R \right]. \]  

(6)
Here the background field will interact with the string for a time $T$, where $T$ would be defined in the target space. So, we can perturb this string with a background field, and write the resulting total world-sheet Hamiltonian as

$$\hat{H} = \hat{H}_0 + \hat{H}_I,$$

where $\hat{H}_0$ is the original free string Hamiltonian, and $\hat{H}_I$ is the interaction of the string fields with the background field. Here we have assumed that the is time dependent, and we can separated the temporal dependence of $\hat{H}_I(\tau)$ into $\lambda \chi(\tau)$. We can also extract a constant $\lambda$ from $\hat{H}_I$, from the interaction Hamiltonian, which would act as an effective coupling constant, and write

$$\hat{H}_I = \lambda \chi(\tau) \int d\sigma \mathcal{H}(\sigma).$$

Here we have chosen the gauge $\tau = T$, with $T$ being the time in target spacetime. With this choice of gauge, we can view $\chi(\tau)$ as a switching function, which turns the background fields on at $\tau > 0$ and off at $\tau < t$ on the world-sheet. Furthermore, and $\mathcal{H}(\sigma)$ will act as the smearing function representing the interaction. Now the interaction is turned on at time $\tau > 0$ and turned off at time $\tau = t$ such that $\hat{H}(0) = \hat{H}_0$ and $\hat{H}(t) = \hat{H}_0$ as $\hat{H}_I(0) = \hat{H}_I(t) = 0$. So, using this interaction Hamiltonian $\hat{H}$, we can write the unitary transformation $U$ as

$$\hat{U}(t) = \mathcal{T} \exp \left( -i \lambda \chi(\tau) \int d\sigma \mathcal{H}(\sigma) \right),$$

where $\mathcal{T}$ denotes time ordering between $0 < \tau < t$. Using the Dyson expansion, we can express this unitary operator in terms of the a series as

$$\hat{U}(\tau) = 1 + \hat{U}^1(\tau) + \hat{U}^2(\tau) + O(\lambda^3),$$

where $\hat{U}^1(\tau)$ and $\hat{U}^2(\tau)$ are the first and second order terms in the Dyson expansion.

We want to analyze the effect of such interaction on a quantity defined in string theory. This quantity can be a quantity like the square of the total mass $M^2$, or $\hat{J}$ of the string. It would be important to measure the change in such values. Usually, such a change would be expected to be measured, by first making an measurement at $\tau = 0$, then making the string interact with a field, and finally making another measurement at $\tau = t$. Even though this seems straightforward, it would in principle involve making two projective measurements on world-sheet. The problem here is that two such projective measurements cannot be made on a system with an Lorentz structure due to problems like superluminal signals [9, 10], and world-sheet of string has an intrinsic Lorentz structure build into it. However, such problems can be resolved for systems with a Lorentz structure by a scheme called the Ramsey scheme [12, 13]. The Ramsey scheme has already been applied to quantum field theories [14]. As the world-sheet of string theory is modeled by a conformal field theory, we propose that the change in the world-sheet operator $\hat{A}$ can be obtained by using a string theoretical generalization of the Ramsey scheme.

### III. Characteristic Function

We can construct the characteristic function for this process. As world-sheet has a Lorentz structure on it, we can use the Ramsey scheme to obtain information related to two world-sheet projective measurements. So, if $\vert A_i \rangle$ are the initial eigenvectors of the general world-sheet operator $\vert \hat{A}_i \rangle$ with eigenvalues $\hat{A}_i$. To obtain the information about change in the eigenvalues of $\hat{A}$ due to its interaction with the background field, we will now generalize Ramsey scheme [12, 13] to string theory. This will be done by first expressing the world-sheet operators using their eigenvectors and eigenvalues as

$$\hat{A}(0) = \sum_i A_i \vert A_i \rangle \langle A_i \vert, \quad \hat{A}(t) = \sum_j \hat{A}_j \vert \hat{A}_j \rangle \langle \hat{A}_j \vert.$$
We can write the probability for measuring eigenvalue \( A_i \) at time \( \tau = 0 \) as \( p_{ij}^0 \). The eigenvectors of \( \hat{A} \) evolves from \( |A_i\rangle \) to \( |A_j\rangle \) due to the interaction with the interaction with background field. As this evolution is captured by the unitary transformation \( \hat{U} \), we can also write the conditional probability for measuring eigenvalue \( \hat{A}_i \), at time \( \tau = t \), if the initial eigenvalue was \( \hat{A}_i \) at time \( \tau = 0 \) as \( p_{ij}^T = \langle \hat{A}_j | \hat{U} | A_i \rangle \). We can write the difference between these initial and the final eigenvalues of \( \hat{A} \) as

\[
\hat{A}_{i,j} = \hat{A}_j - \hat{A}_i. \tag{12}
\]

Using \( p_{ij}^0 \) and \( p_{ij}^T \), we obtain the probability associated with the occurrence of this difference \( \hat{A}_{i,j} \) as

\[
p_{i,j} = p_{ij}^0 p_{ij}^T = |\langle A_i | \rho | A_i \rangle| |\langle A_j | \hat{U} | A_i \rangle|^2. \tag{13}
\]

As we have the probabilities associated with the difference \( \hat{A}_{i,j} \), we can write the full probability distribution corresponding corresponding to it. This can be done by defining a distribution variable \( A \), which would correspond to different values of \( \hat{A}_{i,j} \), if there was no degeneracies. However, to account for degeneracies in the eigenvalues of \( \hat{A} \), we define the probability distribution as

\[
P(A) = \sum_{ij} p_{ij} \delta(A - \hat{A}_{i,j}). \tag{14}
\]

As the probability distribution for the variable \( A \) can be written as \( P(A) \), we can write the average value for \( A \) using \( P(A) \) as

\[
\bar{A} = \int \sum_{ij} p_{ij} \delta(A - \hat{A}_{i,j}) dA. \tag{15}
\]

Using the expression for \( p_{ij} \) and integrating this expression, we obtain

\[
\bar{A} = \sum_{ij} |\langle A_i | \rho | A_i \rangle| |\langle A_j | \hat{U} | A_i \rangle|^2 \left[ \hat{A}_j(\tau) - A_i(0) \right]. \tag{16}
\]

Now using \( [\dot{\rho}, \hat{A}] = 0 \), and \( [\hat{A}, \hat{U}^\dagger \hat{A} \hat{U}] = 0 \), we observe that

\[
\bar{A} = \sum_i p_i(\tau) A_i(\tau) - \sum_i p_i(0) A_i(0), \tag{17}
\]

where \( p_i(\tau) = \sum_j p_{ij}(\tau) \) and \( p_i(0) = \sum_j p_{ij}(0) \). We observe that \( \sum_i p_i(\tau) A_i(\tau) = tr[\rho(\tau)] \) and \( \sum_i p_i(0) A_i(0) = tr(\rho(0)) \), and so we can write

\[
\bar{A} = tr(\rho(\tau)) - tr(\rho(0)). \tag{18}
\]

We can define the corresponding characteristic function of \( A \) as

\[
\hat{P}(\mu) = \int P(A) e^{i\mu A} dA = \langle e^{i\mu A} \rangle. \tag{19}
\]

Here \( \mu \) is the parameter used in the Ramsey scheme. We first define an string auxiliary qubit, such that its ground state is \( |0\rangle \), and its excited state is \( |1\rangle \). This string auxiliary qubit is defined in such a way, that it is capable of transferring information from a string density matrix. This can be done by first coupling it to the string density state \( \rho(0) \). Initially both the string and qubit are in the ground state. So, we can write the product state of the system as

\[
\hat{\rho}_{\text{tot}} = \hat{\rho} \otimes \hat{\rho}_{\text{aux}}. \tag{20}
\]

After coupling the auxiliary qubit to the string density state, we apply a Hadamard operator to the qubit. Now the string interacts with the background field, and this evolves the total state of the system by a unitary operator \( \hat{U} \)

\[
\hat{C}(\mu) = \hat{U} e^{-i\mu \hat{H}(0)} \otimes |0\rangle \langle 0| + e^{-i\mu \hat{H}(t)} \hat{U} \otimes |1\rangle \langle 1|. \tag{21}
\]

Using this unitary operator, the string qubit state can be expressed as

\[
\hat{\rho}_{\text{aux}} = \text{Tr}_X [\hat{C}(\mu) \hat{\rho}_{\text{tot}} \hat{C}^\dagger(\mu)]. \tag{22}
\]
Here we have defined a string trace operation $T_{RX}$, which traces over the string states. After evolving the system, we apply a final Hadamard operation to the qubit state, and extract the information about the evolution of the system by the background field. This is done as we now have an expression for $\hat{\rho}_{\text{aux}}$, whose explicit form will depend on the precise form of the operator $\hat{A}$. This final expression for the string auxiliary qubit $\hat{\rho}_{\text{aux}}$, can then be compared to the general expression for $\hat{\rho}_{\text{aux}}$ as \[ \hat{\rho}_{\text{aux}} = \frac{1}{2} \left\{ 1 + \text{Re}[\hat{P}(\mu)] \hat{\sigma}_z + \text{Im}[\hat{P}(\mu)] \hat{\sigma}_y \right\}. \] (23)

This expression is general, and will also hold for this string theoretical process. Thus, we can obtain $\hat{P}(\mu)$ using a specific form of the string auxiliary qubit. So, this characteristic function is obtained from the string Ramsey scheme.

**IV. FISHER INFORMATION**

We can use this characteristic function to analyze the behavior of physical quantities due to such an interaction with background field. It can be used to explicitly obtain the value of this average difference $\bar{A}$ as

$$\bar{A} = i{d \over d\mu} \hat{P}(\mu).$$  (24)

So, basically we start from initial world-sheet string states. Then we perturb this system, and so this perturbation changes the string states. Then we obtain the difference between the initial and final eigenvalues of $A$ using Ramsey scheme \[12\, 13\]. Now if $A$ does not contain any information about string states, then this average difference will vanish for such a string theoretical process, $\bar{A} = 0$.

The fisher information is used for a quantity, which is not represented by a quantum operator \[15\, 18\]. As the difference of two quantities during a string theoretical process cannot be represented by a quantum operator, we will use fisher information to analyze it. We can associate a parameter, say $\mu$ with the change in a physical quantity $A$. Here $A$ is a quantity defined on world-sheet such as the change in the mass square or spin of strings during a string theoretical process. We can then use this parameter to obtain the fisher information associated with the change in that physical quantity, during a string theoretical process. If there is no fisher information, then the change in the quantity cannot have a physical meaning. However, if there is a change in the fisher information, then we can assume that the change in the physical quantity will have a physical meaning. We can write the fisher information associated with $A$ as

$$F = \int P(A) \left| {\partial \over \partial \mu} \log P(A) \right|^2 dA. \quad (25)$$

To analyze string theoretical processes on world-sheet, we need to analyze the change of string states on the world-sheet. This change of string states can be measured by measuring the change in a quantity on the world-sheet, if that quantity contains information about string states of the world-sheet. Thus, we would start by defining a quantity on the world-sheet, represented by an operator $A$, such that it contains information about world-sheet string states. However, if the quantum fisher information does not vanish (and so operator $A$ contains information about string states), then we can analyze a string theoretical process by measuring the change in the value of $A$ during such a process.

**V. CONCLUSION**

It is possible to obtain quantities, like mass and spin dynamically from the world-sheet dynamics of strings. It is also possible for strings to interact with background fields. In this letter, we have investigated how such an interaction will change the a physical quantity. This has been done by constructing the characteristic function for such a string theoretical process. We have argued that such a characteristic function cannot be constructed using the usual two point measurements, as the world-sheet has an intrinsic Lorentz structure. We have also resolved this problem by generalizing the Ramsey scheme to world-sheet. This has been done by coupling an string auxiliary qubit to a string density state. After the first Hadamard operation, this combined system is evolved due to the interaction of a background field. Then a second Hadamard gate is applied, and the information about the process is extracted from the qubit. This is done without making two point measurements. It is used to obtain the characteristic function for such a process. Finally, using the characteristic function, we also obtain fisher information for the difference between the initial and final values of a quantity, in a string theoretical process.
It would be interesting to apply this formalism to specific operators defined on the world-sheet of string theory, such as $M^2$ or $J$. Then using a specific perturbation by a background field, we can explicitly obtain the characteristic function for such operators. This can then be used to analyze a change in their value, after the string has interacted with a background field. It would be interesting to analyze the consequence of such string theoretical processes. It is also possible to generalize this analysis to thermal states. This can be done by using KMS state instead of vacuum state.

[1] S. Hellerman and I. Swanson, Phys. Rev. Lett. 114, 111601 (2015).
[2] F. Rojas and C. B. Thorn, Phys. Rev. D 84, 026006 (2011).
[3] A. E. Lawrence and E. J. Martinec, Class. Quant. Grav. 13, 63-96 (1996).
[4] I. Bars and K. Sfetsos, Phys. Rev. D 46, 4510-4519 (1992).
[5] J. T. Liu and R. Minasian, Nucl. Phys. B 874, 413-470 (2013).
[6] M. Schnabl, JHEP 11, 031 (2000).
[7] S. Suomela, P. Solinas, J. P. Pekola, J. Ankerhold and T. Ala-Nissila, Phys. Rev. B 90, 094304 (2014).
[8] F. W. J. Hekking and J. P. Pekola, Phys. Rev. Lett. 111, 093602 (2013).
[9] M. Redhead, Found. Phys. 25, 123 (1995).
[10] D. M. T. Benincasa, L. Borsten, M. Buck, and F. Dowker, Class. Quantum Grav. 31, 075007 (2014).
[11] R. D. Sorkin, [gr-qc/9302018] (1993).
[12] R. Dorner, S. R. Clark, L. Heaney, R. Fazio, J. Goold and V. Vedral, Phys. Rev. Lett. 110, 230601 (2013).
[13] L. Mazzola, G. D. Chiara and M. Paternostro, Int. J. Quantum Inf. 12, 1461007 (2014).
[14] A. Ortega, E. McKay, A. M. Alhambra and E. Martín-Martínez, Phys. Rev. Lett. 122, 24, 240604 (2019).
[15] J. Liu, H. Yuan, X-M. Lu and X. Wang, J. Phys. A. Math. Theor. 53, 023001 (2020).
[16] A. T. Rezakhani, M. Hassan and S. Alipour, Phys. Rev. Lett. 109, 190404 (2012).
[17] Y. Yang, Phys. Rev. Lett. 123, 110501 (2019).
[18] B. M. Escher, R. Dorner, R. Clark, L. Heaney, R. Fazio, J. Goold and V. Vedral, Phys. Rev. Lett. 109, 190404 (2012).
[19] A. del Campo, I. L. Egusquiza, M. B. Plenio and S. F. Huelga Phys. Rev. Lett. 110, 050403 (2013).
[20] Erik Lucero, M. Hofheinz, M. Ansmann, Radoslaw C. Bialczak, N. Katz, Matthew Neeley, A. D. O’Connell, H. Wang, A. N. Cleland and John M. Martinis Phys. Rev. Lett. 100, 247001 (2008).
[21] L. Rigovacca, A. Farace, L. A. M. Souza, A. De Pasquale, V. Giovannetti and G. Adesso, Phys. Rev. A 95, 052331 (2017).
[22] R. Nichols, P. Liuuzzo-Scorpo, P. A. Knott and G. Adesso, Phys. Rev. A 98, 012114 (2018).
[23] J. Bischoff, S. V. Ketov and O. Lechtenfeld, Nucl. Phys. B 438, 373 (1995).
[24] N. Ishibashi and K. Murakami, JHEP 06, 087 (2016).
[25] N. Ishibashi and K. Murakami, JHEP 09, 053 (2013).
[26] E. Smith, Nucl. Phys. B 382, 229-241 (1992).
[27] D. Tong, [arXiv:0908.0333 [hep-th]].
[28] H. Matsumaga, JHEP 04, 143 (2019).
[29] P. P. Srivastava and S. J. Brodsky, Phys. Rev. D 64, 045006 (2001).
[30] R. Bentin, [arXiv:hep-th/0310094 [hep-th]].
[31] R. P. Malik, Mod. Phys. Lett. A 16, 477 (2001).
[32] S. Upadhyay, Phys. Lett. B 740, 341 (2015).
[33] S. Upadhyay, Phys. Rev. D 92, 065027 (2015).
[34] S. Upadhyay and B. P. Mandal, Eur. Phys. J. C 81, 270 (2021).
[35] S. Upadhyay, Int. J. Mod. Phys. A 30, 1550150 (2015).
[36] S. Upadhyay and B. P. Mandal, Phys. Lett. B 744, 231 (2015).
[37] S. Upadhyay, Annals Phys. 340, 110 (2014).
[38] S. Upadhyay, EPL 105, 21001 (2014).
[39] S. Upadhyay, EPL 104, 61001 (2014).
[40] V. K. Pandey and B. P. Mandal, EPL 122, 21002 (2018).
[41] S. Vinjanampathy and J. Anders, Contemp. Phys. 57, 545 (2016).
[42] M. Esposito, U. Harbola, and S. Mukamel, Rev. Mod. Phys. 81, 1665 (2009).