Subharmonic transitions and Bloch-Siegert shift in electrically driven spin resonance

Judit Romhányi,1,2 Guido Burkard,3 and András Pályi2,4

1Leibniz-Institute for Solid State and Materials Research, IFW-Dresden, D-01171 Dresden, Germany
2Institute of Physics, Eötvös University, Budapest, Hungary
3Department of Physics, University of Konstanz, D-78457 Konstanz, Germany
4MTA-BME Condensed Matter Research Group, Budapest University of Technology and Economics, Budapest, Hungary

(Dated: April 24, 2015)

We theoretically study coherent subharmonic (multi-photon) transitions of a harmonically driven spin. We consider two cases: magnetic resonance (MR) with a misaligned, i.e., non-transversal driving field, and electrically driven spin resonance (EDSR) of an electron confined in a one-dimensional, parabolic quantum dot, subject to Rashba spin-orbit interaction. In the EDSR case, we focus on the limit where the orbital level spacing of the quantum dot is the greatest energy scale. Then, we apply time-dependent Schrieffer-Wolff perturbation theory to derive a time-dependent effective Hamiltonian without driving, which depends on the initial state of the spin is the ground state $|\uparrow\rangle$, and couples to the electron spin represented by the vector $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ of Pauli matrices.

A typical initial-value problem considered in MR is when the initial state of the spin is the ground state $\psi(t = 0) = |\uparrow\rangle$ of the Hamiltonian without driving, $-\frac{1}{2}g_B B \sigma_z$, and driving is switched on abruptly at $t = 0$.

In the case of weak driving $B_{ac} \ll B$, the rotating wave approximation (RWA) often provides a satisfactory description of the dynamics; using this approximation, one finds the following simple phenomenology. If the resonance condition $\hbar \omega = g_B B$ is fulfilled, the drive will induce complete Rabi oscillations resulting in a transition probability $P_\uparrow(t) = |\langle \downarrow | \psi(t) \rangle|^2 = \sin^2(\frac{1}{2} \Omega t)$, where $\Omega = \frac{1}{\hbar}g_B B_{ac}/h$ is called the Rabi frequency. Otherwise, i.e., in the case of a finite detuning $\delta = \omega - g_B B/h$ between the drive frequency and the resonance frequency, one finds incomplete Rabi oscillations with a $\delta$-dependent frequency $P_\uparrow(t) = P_{\uparrow}^{\text{max}} \sin^2(\frac{1}{2} \sqrt{\delta^2 + \Omega^2} t)$, with $P_{\uparrow}^{\text{max}} = \frac{g_B^2 B_{ac}^2}{\hbar^2} < 1$.

Still focusing on the weak-driving regime $B_{ac} \ll B$, one can go beyond the RWA, e.g., by numerical simulations or analytical techniques such as the Floquet perturbation theory. Then, a richer phenomenology is revealed, including (i) subharmonic or ‘multi-photon’ resonances, (ii) drive-strength-dependent shifts, a.k.a. Bloch-Siegert shifts $\Omega_{\text{BS}}(B_{ac})$ of the resonance frequencies, and (iii) Bloch-Siegert oscillations modulating the simple Rabi oscillations $\omega(t)$.

We restrict our attention to (i) and (ii) here.

(i) In the case of a transverse ac field, such as the example used in Eq. (1), odd subharmonic resonances appear. I.e., Rabi oscillations are obtained not only for the fundamental resonance $\omega \approx g_B B/h$, but also when $\omega \approx \frac{1}{N} g_B B/h$ with $N = 3, 5, 7, \ldots$. In the case of a misaligned, i.e., non-transversal, ac field, such as $B(t) = (0, -B_{ac} \cos \omega t, B - B_{ac} \sin \theta \cos \omega t)$ with $0 < \theta < \pi/2$, both even and odd subharmonics appear. The Rabi frequency $\Omega_{\text{res}}(N)$ at the $N$-photon subharmonic resonance is weaker than that of the fundamental one: $\Omega_{\text{res}}(N) \propto \frac{B_{ac}^N}{B^N(N-1)!}$. (ii) The resonance frequencies $\omega_{\text{res}}^{(N)}(\theta)$ (i.e., the drive frequencies where complete Rabi oscillations are induced) increase with increasing drive strength, by an amount that depends on $N$, and is proportional to $B_{ac}^2/B$.

In many situations, it is be more convenient to control spins using an ac electric field rather than an ac...
magnetic field. For example, if an electron spin is electrostatically confined in a quantum dot (QD), then an ac electric field can be easily created by applying an ac voltage component of the confinement gate electrodes. Along these lines, electrically driven spin resonance (EDSR) of individual electron spins was demonstrated in a variety of materials. As the ac electric field couples to the orbital degree of freedom of the electron and has no direct effect on the spin, a sufficiently strong coupling mechanism between the orbit and spin is required for EDSR. Such a coupling can be supplied by spin-orbit interaction, hyperfine interaction, or an inhomogeneous magnetic field.

Recent experimental and theoretical studies addressed subharmonic resonances in EDSR. One mechanism that leads to subharmonic resonances in EDSR is the appearance of higher harmonics $N\omega$ of the drive frequency $\omega$ in the induced orbital dynamics. In this case, the time-dependent effective magnetic fields caused by the orbital dynamics will also have components at frequency $N\omega$, leading to Rabi oscillations as $N\omega$ matches the Zeeman splitting. Higher harmonics in the orbital dynamics arise naturally if the confinement potential is anharmonic, or if the driving electric field is inhomogeneous. Subharmonic EDSR resonances can also arise in the presence of harmonic confinement and homogeneous ac electric field, if the gradient of the effective magnetic fields is inhomogeneous; this is the case, e.g., if the relative magnetic field is spatially localized or disordered. A third mechanism, able to cause strong subharmonics with large $N$, is provided by Landau-Zener dynamics in the vicinity of level anticrossings.

In this work, we theoretically describe the characteristics of the half-harmonic resonance in MR with a misaligned ac field, as well as in EDSR. First, we use Floquet perturbation theory to characterise the parameter dependence of the half-harmonic resonance frequency and the corresponding Rabi frequency in the case of MR; in particular, the BSS is calculated. As for EDSR, we study a model (see Fig. 1), where a single electron is parabolically confined in a one-dimensional (1D) quantum dot, and is subject to a dc magnetic field, an ac electric field, and spin-orbit interaction of Rashba type, the latter three being spatially homogeneous. We show that the half-harmonic resonance does arise in this model, despite the harmonic confinement and homogeneous ac electric field. In the perturbative regime of this model, i.e., when the orbital level spacing $\hbar\omega_0$ dominates over other energy scales, we analytically derive the parameter dependence of the half-harmonic resonance frequency and the corresponding Rabi frequency. This is achieved via a combination of time-dependent Schrieffer-Wolff perturbation theory (TDSW), which is used to obtain a $2\times2$ effective 'two-level' or 'qubit' Hamiltonian $\hat{H}_q$, and Floquet perturbation theory, applied to describe the qubit dynamics governed by $\hat{H}_q$. We find that both the fundamental and the half-harmonic resonance frequency changes upon increasing the strength of the driving electric field, in analogy with the BSS of MR. In contrast to MR, where the BSS is positive for the fundamental as well as all subharmonic resonances, we find a negative (positive) BSS for the fundamental (half-harmonic) EDSR resonance. Furthermore, for the fundamental resonance, the shift is small compared to the power broadening; in contrast, the shift and the broadening are comparable for the half-harmonic resonance. The analytical results are checked against the numerical solution of the time-dependent Schrödinger equation, and their qualitative interpretation is provided for simple limiting cases.

II. MAGNETIC RESONANCE WITH A MISALIGNED AC FIELD

In this Section, using Floquet perturbation theory, we derive and discuss the properties of the fundamental (single-photon) and half-harmonic (two-photon) resonances in MR, for the spin-1/2 case.
A. Problem formulation

We consider MR spin dynamics driven by a misaligned ac field. The Hamiltonian reads

$$\mathcal{H}(t) = -\frac{1}{2} \mathbf{B}(t) \cdot \mathbf{\sigma},$$

(2)

where the magnetic field has the form

$$\mathbf{B}(t) = \begin{pmatrix} 0 \\ \tilde{B}_{ac} \cos \theta \cos \omega t \\ \tilde{B}_{ac} \sin \theta \cos \omega t \end{pmatrix}.$$  

(3)

Here we introduced $\tilde{B} = g\mu_B B$ and $\tilde{B}_{ac} = g\mu_B B_{ac}$. Henceforth the parameters with tilde, e.g., $\tilde{B}$, have energy dimension. The Hamiltonian in Eq. (2) has four parameters: the strength of the static field $B$, the strength of the driving field $\tilde{B}_{ac}$, the frequency of the driving field $\omega$, and the misalignment angle $\theta$. Note that $\theta = 0$ corresponds to a transversal ac field, and $\theta = \pi/2$ corresponds to a longitudinal ac field. We consider the case of weak driving, $\tilde{B}_{ac} \ll B$.

In particular, we want to solve the initial-value problem described in Sec. I: the initial state is the ground state $|\downarrow\rangle$ of the Hamiltonian without driving i.e., $\psi(t=0) = |\downarrow\rangle$, driving is switched on abruptly at $t = 0$, and we are interested in the time evolution $\psi(t)$ of this state. We calculate the transition probability describing the time-dependent occupation of the excited state $|\uparrow\rangle$ at the fundamental and half-harmonic resonances, and from those we deduce the parameter dependencies of the resonance frequencies and the Rabi frequencies.

In the rest of this Section, we use Floquet perturbation theory to derive the results and to provide qualitative interpretations in simple limiting cases, such as the limits of transversal and longitudinal ac fields. Even though similar treatments can be found in the literature, we present a detailed discussion of the MR problem for the following reason. The MR problem is relatively simple as compared to the EDSR problem, which can be appreciated, e.g., by comparing the driven two-level Hamiltonians of Eqs. (2) and (30), respectively. Moreover, as we will show, the Floquet method and the qualitative interpretations we describe here for the MR problem can be carried over to the EDSR problem, once the $2 \times 2$ effective qubit Hamiltonian in Eq. (30) is obtained for the latter. This allows us to provide a rather compact description of the EDSR in the forthcoming Sections, by referencing this Section wherever possible.

B. Floquet method

The Floquet method allows one to find the solution of an initial-value problem of a periodically driven quantum system, described by the time-periodic Hamiltonian $H(t) = H(t + T)$. The period of the driving is denoted by $T$, and the corresponding (angular) frequency by $\omega = 2\pi/T$. The key ingredient of the method is the quantum-mechanical Floquet theorem, which guarantees that the Schrödinger equation $i\hbar \dot{\Psi}(t) = \mathcal{H}(t) \Psi(t)$ of a $d$-level system has $d$ solutions $\Psi_k(t)$ ($k = 1, \ldots, d$) that are themselves periodic with period $T$, apart from a phase factor. I.e., these special solutions have the form

$$|\psi_k(t)\rangle = e^{-iE_k T/\hbar} \sum_{l=1}^{d} \sum_{m=-\infty}^{\infty} c_{k,lm} e^{im\omega t} |\psi_l\rangle,$$

(4)

where $|\psi_l\rangle$ is an arbitrary basis of the Hilbert space. Note that the result of the double sum is a periodic function of $t$ with period $T$. In Eq. (4), the quantity $E_k$ and the coefficients $c_{k,lm}$ are a priori unknown; the former is called quasi-energy. Once these special solutions $|\psi_k(t)\rangle$ are found, they provide the propagator

$$U(t,0) = \sum_{k=1}^{d} |\Psi_k(t)\rangle \langle \Psi_k(0)|,$$

(5)

which in turn provides the solution of any initial-value problem via

$$\Psi(t) = U(t,0) \Psi(0).$$

(6)

The special solutions $\Psi_k(t)$ are found by using Eq. (4) as an Ansatz, substituting it to the Schrödinger equation, evaluating the scalar product of the equation with $\langle \psi_l \rangle$, multiplying the equation by $e^{-i\omega \delta m \omega t}$ and integrating the equation in time between $t = 0$ and $t = T$. This procedure yields the following eigenvalue equation for $E_k$:

$$\sum_{l=1}^{d} \sum_{m=-\infty}^{\infty} \mathcal{F}_{l,m'} \delta_{m,m'} c_{k,m} = E_k c_{k,m},$$

(7)

where

$$\mathcal{F}_{l,m} = \hbar \omega \delta_{l,m} + \sum_{n=-\infty}^{\infty} \langle \psi_l | H^{(n)} | \psi_n \rangle \delta_{m,n+1}$$

(8)

is the Floquet matrix or Floquet Hamiltonian, and we introduced the Fourier components $H^{(n)}$ of the Hamiltonian via

$$H(t) = \sum_{n=-\infty}^{\infty} H^{(n)} e^{in\omega t}.$$  

(9)

We call two eigenvalue-eigenvector pairs of $\mathcal{F}$ equivalent, if the two time-dependent solutions they generate via Eq. (4) are the same. Importantly, even though the number of eigenvalue-eigenvector pairs of $\mathcal{F}$ is infinite, they form only $d$ equivalence classes.

In summary, we have transformed the time-dependent Schrödinger equation of the periodically driven $d \times d$ Hamiltonian $\mathcal{H}(t)$ into the time-independent Schrödinger equation (7) of the infinite-dimensional Floquet Hamiltonian $\mathcal{F}$. To construct the special solutions (4), and thereby the solution of any initial-value problem via Eqs. (5) and (6), the quasi-energies $E_k$ and the corresponding eigenvectors $c_k$ should be found by solving the eigenvalue problem of the Floquet Hamiltonian $\mathcal{F}$. 
C. Perturbative description of the transition probability

After reviewing the Floquet method in general, we now apply this to the MR problem defined in Eq. (2). Here, \( d = 2 \) and we will use the notation \(|\alpha\rangle \equiv |\uparrow\rangle \equiv |\psi_1\rangle\) and \(|\beta\rangle \equiv |\downarrow\rangle \equiv |\psi_2\rangle\). Furthermore, the Fourier components of the Hamiltonian read

\[
\mathcal{H}^{(0)} = -\frac{1}{2}\tilde{B}\sigma_z, \quad (10a)
\]

\[
\mathcal{H}^{(\pm 1)} = \frac{1}{4}\tilde{B}_{ac}(\cos \theta \sigma_y + \sin \theta \sigma_z), \quad (10b)
\]

and the other Fourier components are zero.

First, consider the case when the drive frequency is close to the fundamental resonance, \( \hbar \omega \approx \tilde{B} \). Then, the diagonal elements of the Floquet Hamiltonian \( \mathcal{F} \) (Floquet levels) form pairs: \( \mathcal{F}_{\alpha m, \alpha m} = m\hbar \omega - \frac{1}{2}\tilde{B} \approx (m-1)\hbar \omega + \frac{1}{4}\tilde{B} = \mathcal{F}_{\beta m-1, \beta, m-1} \). The distance between different pairs is approximately \( \hbar \omega \approx \tilde{B} \), which is much larger than the energy scale \( \tilde{B}_{ac} \) characterising the off-diagonal elements of \( \mathcal{F} \). Therefore, the tools of quantum-mechanical perturbation theory can be used to provide an approximate solution of the eigenvalue problem of the Floquet Hamiltonian.

The structure of the Floquet Hamiltonian \( \mathcal{F} \) is visualised for the case \( \hbar \omega = \tilde{B} \) in the level diagram shown in Fig. 2. Horizontal lines represent the diagonal matrix elements \( \mathcal{F}_{lm,lm} \) of the Floquet Hamiltonian, their vertical positions correspond to their value, their colour (black, red) corresponds to their spin index \( \alpha, \beta \) and their horizontal position corresponds to their Floquet index \( m = \ldots, -1, 0, 1, 2, \ldots \). The vertical spacing of the Floquet levels is \( \hbar \omega = \tilde{B} \). The blue arrows represent the nonzero off-diagonal matrix elements of \( \mathcal{F} \), which are of the order of \( \tilde{B}_{ac} \) and hence small compared to the level spacing.

In the case \( \hbar \omega = \tilde{B} \) shown in Fig. 2, the Floquet levels form degenerate pairs. The pair formed by \( \mathcal{F}_{\beta, \beta-1} \) and \( \mathcal{F}_{\alpha, \alpha} \) is highlighted in Fig. 2 by the dashed green box. The subspace of this pair is weakly coupled to the other Floquet levels, and hence this coupling can be treated perturbatively using (time-independent) Schrieffer-Wolff perturbation theory\(^{39}\), a. k. a. quasi-degenerate perturbation theory\(^{40}\). This perturbative treatment is also applicable if there is a finite, although small detuning \( \delta = \omega - \tilde{B}/\hbar \ll \tilde{B}/\hbar \) from the resonance condition. The small dimensionless parameter characterising the strength of the perturbation is \( \epsilon = \tilde{B}_{ac}/\tilde{B} \).

In this case, the Floquet Hamiltonian reads

\[
\mathcal{F} = \begin{bmatrix}
\alpha_{-1} & \beta_{-1} & \alpha_0 & \beta_0 & \alpha_1 & \beta_1 \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
\alpha_{-1} & \ldots & -\frac{1}{2}\tilde{B} - \hbar \omega & 0 & \frac{1}{4}\tilde{B}_{ac}\sin \theta & -\frac{1}{4}\tilde{B}_{ac}\cos \theta & 0 & 0 & \ldots \\
\beta_{-1} & \ldots & 0 & \frac{1}{2}\tilde{B} - \hbar \omega & \frac{1}{4}\tilde{B}_{ac}\cos \theta & -\frac{1}{4}\tilde{B}_{ac}\sin \theta & 0 & 0 & \ldots \\
\alpha_0 & \ldots & \frac{1}{4}\tilde{B}_{ac}\sin \theta & -\frac{4}{3}\tilde{B}_{ac}\cos \theta & -\frac{1}{4}\tilde{B} & 0 & \frac{1}{4}\tilde{B}_{ac}\sin \theta & -\frac{1}{4}\tilde{B}_{ac}\cos \theta & \ldots \\
\beta_0 & \ldots & \frac{1}{4}\tilde{B}_{ac}\cos \theta & -\frac{4}{3}\tilde{B}_{ac}\sin \theta & 0 & \frac{1}{2}\tilde{B} & \frac{1}{4}\tilde{B}_{ac}\cos \theta & -\frac{1}{4}\tilde{B}_{ac}\sin \theta & \ldots \\
\alpha_1 & \ldots & 0 & 0 & \frac{1}{4}\tilde{B}_{ac}\sin \theta & -\frac{4}{3}\tilde{B}_{ac}\cos \theta & -\frac{1}{4}\tilde{B} + \hbar \omega & 0 & \ldots \\
\beta_1 & \ldots & 0 & 0 & \frac{1}{4}\tilde{B}_{ac}\cos \theta & -\frac{4}{3}\tilde{B}_{ac}\sin \theta & 0 & \frac{1}{2}\tilde{B} + \hbar \omega & \ldots \\
\end{bmatrix} \quad (11)
\]
1. Fundamental resonance within RWA

Using first-order perturbation theory, the two non-equivalent eigenvalues and eigenvectors of $\mathcal{F}$ can be found approximately. For this we introduce $\mathcal{F}_0$ and $\mathcal{F}_1$ via $\mathcal{F} = \mathcal{F}_0 + \mathcal{F}_1$, where $\mathcal{F}_0$ is the diagonal component of $\mathcal{F}$, i.e., $(\mathcal{F}_0)_{lm,lm} = \delta_{l,m} \delta_{m,0} \mathcal{F}_{lm,lm}$ at $\omega = \tilde{B}/\hbar$, i.e. $\delta = 0$. First-order perturbation theory in $\mathcal{F}_1$ amounts to diagonalizing the $2 \times 2$ block highlighted in purple in Eq. (11) and Fig. 2. For future reference, we recast this $2 \times 2$ block to the form

$$\tilde{\mathcal{F}} = \begin{bmatrix} \epsilon_0 + \Delta & i\lambda \\ -i\lambda & \epsilon_0 - \Delta \end{bmatrix},$$

where $\epsilon_0 = -\frac{1}{2} (\tilde{B} + \hbar \delta)$, $\Delta = -\hbar \delta / 2$, and $\lambda = \frac{1}{4} \tilde{B}_{ac} \cos \theta$. The matrix $\tilde{\mathcal{F}}$ has eigenvalues

$$\tilde{E}_\pm = \epsilon_0 \pm \sqrt{\Delta^2 + \lambda^2},$$

and corresponding eigenvectors

$$\tilde{c}_\pm = N_\pm \begin{bmatrix} \frac{\Delta \pm \sqrt{\Delta^2 + \lambda^2}}{\lambda}, 1 \end{bmatrix},$$

where $N_\pm$ is a normalization constant. Note that instead of using the numerical index $k \in (1, 2)$ labelling the solutions (4), since Eqs (13) we use the values $k \in (+, -)$. The results (13) and (14) imply that the two non-equivalent approximate eigenvalue-eigenvector pairs of $\mathcal{F}$ are $(\tilde{E}_\pm, \tilde{c}_\pm)$, where

$$c_{\pm,lm} = \begin{cases} \tilde{c}_{\pm,1} & \text{if } (l, m) = (\beta, -1), \\ \tilde{c}_{\pm,2} & \text{if } (l, m) = (\alpha, 0), \\ 0 & \text{otherwise}. \end{cases}$$

This result allows us to construct the transition probability $P_{\beta \rightarrow \alpha}(t) = |\langle \beta | \Psi(t) \rangle|^2$ from the initial spin (ground) state $| \uparrow \rangle \equiv | \alpha \rangle$ to the excited state $| \downarrow \rangle \equiv | \beta \rangle$ via Eqs. (4), (5) and (6). A straightforward calculation yields

$$P_{\beta \rightarrow \alpha}(t) = \frac{\lambda^2}{\Delta^2 + \lambda^2} \sin^2 \left( \frac{1}{\hbar} \sqrt{\Delta^2 + \lambda^2} t \right).$$

According to Eq. (16), the spin makes complete Rabi oscillations if $\Delta = 0$, that is, $\delta = \omega - \tilde{B}/\hbar = 0$. Hence the single-photon resonance frequency is $\omega^{(1)}_{\text{res}} = \tilde{B}/\hbar$. The Rabi frequency upon resonant driving is $\Omega^{(1)}_{\text{res}} = 2\lambda/\hbar = \tilde{B}_{ac} \cos \theta$; i.e. only the transverse component of the ac field contributes to the Rabi frequency at the fundamental resonance. In fact, the result (16) is equivalent to the one obtained by neglecting the longitudinal ac field and making the RWA.

2. Fundamental resonance beyond the RWA: Bloch-Siegert shift of the resonance frequency

We also describe the corrections to $\omega^{(1)}_{\text{res}}$ and $\Omega^{(1)}_{\text{res}}$ beyond the RWA. To this end, we incorporate in the analysis the effect of those matrix elements of $\mathcal{F}_1$ that connect the two highlighted Floquet levels [see Eq. (11) and Fig. (2)] to the complementary subspace. This is done via a (time-independent) Schrieffer-Wolff transformation that is second order in $\mathcal{F}_1$. The resulting effective $2 \times 2$ Floquet Hamiltonian $\tilde{\mathcal{F}}$ has the form given in Eq. (12), with

$$\Delta = -\frac{1}{2} \hbar \delta + \frac{\tilde{B}_{ac}^2 \cos^2 \theta}{32B},$$

$$\lambda = \frac{\tilde{B}_{ac} \cos \theta}{4}.$$  

Recall that the eigenvalues and eigenvectors of $\tilde{\mathcal{F}}$ are given by Eqs. (13) and (14). From these, we conclude that the two non-equivalent approximate eigenvalue-eigenvector pairs of $\mathcal{F}$ are $(\tilde{E}_\pm, \tilde{c}_\pm)$, where

$$c_{\pm,lm} = \begin{cases} \tilde{c}_{\pm,1} + o(\epsilon^2) & \text{if } (l, m) = (\beta, -1), \\ \tilde{c}_{\pm,2} + o(\epsilon^2) & \text{if } (l, m) = (\alpha, 0), \\ o(\epsilon) & \text{otherwise}. \end{cases}$$

We neglect the perturbative corrections $\sim o(\epsilon)$ in the eigenvectors $\tilde{c}_\pm$, and this implies that the approximate transition probability is given by Eq. (16). Equation (16) predicts that complete Rabi oscillations are induced when $\Delta = 0$; solving Eq. 17a for $\omega$ (recall that $\delta = \omega - \tilde{B}/\hbar$) provides the resonance frequency:

$$\hbar \omega^{(1)}_{\text{res}} = \tilde{B} + \frac{\tilde{B}_{ac}^2 \cos^2 \theta}{16B}.$$  

The second term corresponds to the Bloch-Siegert shift of the resonance frequency: as the drive strength $\tilde{B}_{ac}$ is increased, the resonance frequency shifts upwards. This feature is further discussed in Sec. III D. Finally, the Rabi frequency at the fundamental resonance remains the same as in the RWA:

$$\hbar \Omega^{(1)}_{\text{res}} = 2\lambda = \frac{\tilde{B}_{ac}}{2} \cos \theta.$$
3. **Half-harmonic resonance**

Let us now consider the spin dynamics at half-harmonic resonance, when \( \hbar \omega \approx \frac{1}{2} B \). The level diagram visualising the Floquet Hamiltonian in the case \( \hbar \omega = \frac{1}{2} B \) is shown in Fig. 3. Again, we can identify degenerate pairs of Floquet levels, e.g., the pair \((F_{\beta,-1,\beta,-1}, F_{\alpha,1,\alpha,1})\) highlighted with the blue box in Fig. 3.

Note that in this case, there is no direct matrix element (blue arrow) connecting these two Floquet levels. This implies that by repeating the first-order perturbation theory (equivalent to RWA) done in Sec. II C 1, we would conclude that two-photon Rabi oscillations do not happen. However, this result is not correct; two-photon Rabi oscillations can happen. To see that, we perform a second-order Schrieffer-Wolff transformation on \( F \), as done in Sec. II C 2. Furthermore we use the appropriate notation \( \delta = \omega - \frac{B}{2} \). The obtained effective \( 2 \times 2 \) Floquet Hamiltonian \( \tilde{F} \) has the form given in Eq. (12), with

\[
\Delta = \hbar \delta - \frac{B^2 \cos^2 \theta}{6B}, \tag{21a}
\]

\[
\lambda = \frac{B^2 \sin 2\theta}{8B}. \tag{21b}
\]

Following the approach used in Sec. II C 2, and solving \( \Delta = 0 \) we find that the half-harmonic resonance frequency is

\[
\hbar \omega_{\text{res}}^{(2)} = \frac{B}{2} + \frac{B^2 \cos^2 \theta}{6B}. \tag{22}
\]

Substituting this in \( \hbar \Omega_{\text{res}}^{(2)} = 2\lambda \), the Rabi frequency at the half-harmonic resonance is obtained as

\[
\hbar \Omega_{\text{res}}^{(2)} = \frac{B^2 \sin 2\theta}{4B}. \tag{23}
\]

**D. Discussion**

Let us now discuss the main features of the results (19), (20), (22), and (23).

Consider first the fundamental resonance frequency \( \omega_{\text{res}}^{(1)} \) expressed in Eq. (19). The second term in Eq. (19) implies that \( \omega_{\text{res}}^{(1)} \) has a positive drive-strength-dependent correction \( B^2 / \hbar \) with respect to the nominal Zeeman splitting \( B \). This correction is known as the BSS, which can be regarded as a special case of the ac Stark shift.

Note that the parameter \( \lambda \) and hence the Rabi frequency \( \Omega_{\text{res}}^{(1)} \) sets the frequency broadening of the fundamental transition, as indicated by the prefactor \( \frac{1}{\sqrt{2\lambda}} \) on the right hand side of Eq. (16). According to Eq. (20), this power broadening of the fundamental resonance is greater by a factor of \( B / B_{\text{ac}} \) than the BSS.

Equation (19) also shows that the BSS is finite in the limit of purely transversal drive \( (\theta = 0) \), and vanishes in the limit of purely longitudinal drive \( (\theta = \pi/2) \). The respective Floquet level diagrams in Fig. 2a and b provide a straightforward interpretation: the BSS can be regarded as a consequence of coupling-induced repulsion between the Floquet levels. In Fig. 2a \( (\theta = 0) \), the Floquet level \( F_{\beta,-1,\beta,-1} \) is connected by a blue arrow (off-diagonal matrix elements of \( F \)) to the lower-lying Floquet level \( F_{\alpha,-2,\alpha,-2} \). The consequence of this coupling in second-order perturbation theory is level repulsion; i.e., the lower-lying Floquet level pushes \( F_{\beta,-1,\beta,-1} \) upwards. Similarly, \( F_{\alpha,0,0} \) is pushed downwards by its coupling to the higher-lying Floquet level \( F_{\beta,1,\beta,1} \). These second-order level shifts appear in Eq. (17a) as the last term, and give rise to a finite BSS. In contrast, each of the highlighted Floquet levels in Fig. 2b \( (\theta = \pi/2) \) is connected to one higher-lying and one lower-lying Floquet level, and the corresponding downward and upward level repulsions cancel each other, giving rise to a vanishing BSS in this case.

Consider now the half-harmonic resonance. Equation (23) provides the corresponding Rabi frequency, and it indicates the existence of Rabi oscillations unless \( \theta = 0 \) or \( \theta = \pi/2 \). I.e., Rabi oscillations appear at half-harmonic excitation only if the transversal and longitudinal components of the driving field are both nonzero. The Floquet level diagrams shown in Fig. 3 provide a visual interpretation of this feature: Rabi oscillations arise if the blue arrows (off-diagonal matrix elements of \( F \)) draw at least one path between the two Floquet levels highlighted by the purple box, via virtual intermediate Floquet levels outside the box. In the special cases \( \theta = 0 \) and \( \theta = \pi/2 \) depicted in Figs. 3a and b, respectively, no such paths exist. However, there exist infinitely many such paths for \( 0 < \theta < \pi/2 \) (Fig. 3c), due to the coexistence of spin-conserving and spin-flip off-diagonal matrix elements. In particular, in our second-order Schrieffer-Wolff transformation leading to the result (23), the two two-step paths via \( F_{\alpha,0,\alpha,0} \) and \( F_{\beta,0,\beta,0} \) are incorporated.

In the case of the half-harmonic resonance, the relation between the power broadening and the BSS is qualitatively different from the case of the fundamental resonance. For the half-harmonic resonance, the power broadening is given by Eq. (23), whereas the BSS is given by the second term of Eq. (22), i.e., the two quantities are of the same order, both being \( B^2 / \hbar B_{\text{ac}} \). Hence we expect that for the half-harmonic resonance, the BSS is relatively easily resolvable experimentally, at least if the dissipative frequency scales are smaller than the power broadening.

Equation (22) also shows that the BSS is finite in the limit of purely transversal excitation \( (\theta = 0) \), and vanishes in the limit of purely longitudinal excitation \( (\theta = \pi/2) \). An interpretation completely analogous to the case of the fundamental resonance can be given based on the Floquet level diagrams in Fig. 3a and b.
III. ELECTRICALLY DRIVEN SPIN RESONANCE

A. The model

From now on, we describe EDSR mediated by spin-orbit interaction in a 1D parabolic quantum dot. The setup is shown in Fig. 1. The Hamiltonian

\[ \mathcal{H} = \mathcal{H}_0 + \mathcal{H}_E + \mathcal{H}_B + \mathcal{H}_{SO} \]  

includes the harmonic-oscillator Hamiltonian (\( \mathcal{H}_0 \)) consisting of the kinetic energy of the electron and the parabolic confinement potential, the ac electric potential arising from the driving electric field (\( \mathcal{H}_E \)), the static Zeeman effect caused by a homogeneous magnetic field (\( \mathcal{H}_B \)), and the spin-orbit term (\( \mathcal{H}_{SO} \)). The explicit forms of these terms, respectively, are as follows:

\[ \mathcal{H}_0 = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega_0^2 z^2 = \hbar \omega_0 \left( a^\dagger a + \frac{1}{2} \right), \]  

\[ \mathcal{H}_E = e\tilde{E}_{ac} \sin(\omega t) = \tilde{E}_{ac} \sin(\omega t)(a^\dagger + a), \]  

\[ \mathcal{H}_B = -\frac{1}{2}g^* \mu_B B \sigma_z = -\frac{1}{2}\tilde{B}\sigma_z \]  

\[ \mathcal{H}_{SO} = \alpha_p \mathbf{n}_{so} \cdot \sigma = i\tilde{\alpha}(a^\dagger - a)\mathbf{n}_{so} \cdot \sigma \]

Here, \( a \) and \( a^\dagger \) are the ladder operators of the harmonic oscillator Hamiltonian, and \( \mathbf{n}_{so} = (0, \cos \theta, \sin \theta) \) is the direction of the effective magnetic field arising from spin-orbit coupling. Furthermore, we defined

\[ \tilde{B} = g^* \mu_B B \]  

\[ \tilde{\alpha} = \alpha \sqrt{\frac{\hbar \omega_0}{2m}} \]  

\[ \tilde{E}_{ac} = e\tilde{E}_{ac} \sqrt{\frac{\hbar}{2m\omega_0}} \]

These quantities have the dimension of energy.

Note that we use the same notation \( \theta \) for two different quantities: \( \theta \) appears in Eq. (3) as the ac field misalignment angle in MR, and it also appears in this Section and in Fig. 1, as the angle characterising the direction of the spin-orbit term. We use the same notation for these quantities as they play very similar roles in the spin dynamics.

It is natural to represent the Hamiltonian terms (25a)–(25d) in the product basis of the orbital and spin degrees of freedom, \( \{|n_\sigma; \sigma| n = 0, 1, 2, \ldots; \sigma = \uparrow, \downarrow \rangle \} \), where \( n \) is the harmonic-oscillator orbital quantum number and \( \sigma \) is the spin quantum number with quantization along \( z \).

We will refer to the two lowest-energy eigenstates of our static Hamiltonian \( \mathcal{H}_0 + \mathcal{H}_B + \mathcal{H}_{SO} \) as the qubit basis states. The qubit basis state with the lower (higher) energy will be denoted by \( |G \rangle \) (\( |E \rangle \)).

The electron is initialized in state \( |G \rangle \) at \( t = 0 \). Our aim is to describe the time evolution of the state upon driving. In particular, we are interested in the time-dependent occupation probability \( P_E(t) \) of state \( |E \rangle \). It is expected that upon resonant driving \( \hbar \omega \approx \tilde{B} \), the dynamics resembles Rabi oscillations. Subharmonic (multi-photon or \( N \)-photon) resonances at \( \hbar \omega \approx \tilde{B}/N \) (where \( N = 1, 2, \ldots \)) are also expected. In this work we focus on the fundamental (single-photon, \( N = 1 \)) and half-harmonic (two-photon, \( N = 2 \)) resonances.

We aim at an analytical, perturbative description of spin transitions induced by the ac electric field. In particular, we calculate the resonance frequency and the Rabi frequency at resonant driving. We consider the parameter range where the energy scale \( \hbar \omega_0 \) of the confinement potential dominates the other four energy scales, the latter ones being assumed to be comparable in magnitude:

\[ \hbar \omega \sim \tilde{\alpha} \sim \tilde{E}_{ac} \sim \tilde{B} \ll \hbar \omega_0. \]  

This hierarchy of energy scales will allow for a perturbative description of the dynamics, with the small parameter \( \epsilon = 2\tilde{\alpha}/\hbar \omega_0 \sim \tilde{E}_{ac}/\hbar \omega_0 \sim \tilde{B}/\hbar \omega_0 \ll 1 \).

B. Effective qubit Hamiltonian

In the EDSR problem defined in Sec. III A, the hierarchy of the energy scales is given by Eq. (29). Because of this hierarchy, an effective time-dependent two-level Hamiltonian [see Eq. (30) below] can be derived for the qubit dynamics, using TDSW perturbation theory, which we outline in Appendix A. This qubit Hamiltonian can then be used to express the resonance frequencies \( \omega_{res}^{(1)} \) and \( \omega_{res}^{(2)} \), and the corresponding Rabi frequencies at these resonances, \( \Omega_{res}^{(1)} \) and \( \Omega_{res}^{(2)} \), corresponding to the fundamental and half-harmonic resonances, respectively [see Eqs. (33a), (33b), (33c), and (33d) below].

We use the orbital-spin product basis \( \{|n_\sigma; \sigma| n = 0, 1, 2, \ldots; \sigma = \uparrow, \downarrow \rangle \} \), as the starting point of TDSW, and take the two-dimensional subspace of \( |0_\uparrow \rangle \) and \( |0_\downarrow \rangle \) as the relevant subspace in TDSW. We carry out a fifth-order TDSW (in the small parameter \( \epsilon \)), which is expected to describe both the fundamental and the half-harmonic resonances. The TDSW procedure yields the effective qubit Hamiltonian

\[ \tilde{\mathcal{H}}_q \approx \tilde{\mathcal{H}}_q^{(0)} + \tilde{\mathcal{H}}_q^{(1)} + \tilde{\mathcal{H}}_q^{(2)} + \tilde{\mathcal{H}}_q^{(3)} + \tilde{\mathcal{H}}_q^{(4)} + \tilde{\mathcal{H}}_q^{(5)}, \]

where the six terms, representing terms from different orders in the perturbation, are listed below in Eq. (31). Note that the terms \( \tilde{\mathcal{H}}_q^{(0)} \), \( \tilde{\mathcal{H}}_q^{(2)} \), and \( \tilde{\mathcal{H}}_q^{(4)} \) are proportional to the \( 2 \times 2 \) unit matrix \( \mathbf{1}_2 \), therefore they do not influence the dynamics, and hence we disregard them in the forthcoming calculations; nevertheless we include them
here for completeness:

\[ \hat{H}_q^{(0)} = \frac{\hbar \omega_0}{2} \sigma_0 \]  
\[ \hat{H}_q^{(1)} = -\frac{\tilde{B}}{2} \sigma_3 \]  
\[ \hat{H}_q^{(2)} = -\frac{\tilde{\alpha}^2 + \tilde{E}_2}{\hbar \omega_0} \sin(\omega t) \sigma_0 \]  
\[ \hat{H}_q^{(3)} = -\frac{\tilde{B} \tilde{E}_2 \tilde{\alpha} \cos \theta}{\hbar^2 \omega_0^2} \sin(\omega t) \sigma_1 \]  
\[ \hat{H}_q^{(4)} = -\left(\frac{\tilde{B} \tilde{\alpha} \cos \theta}{\hbar^3 \omega_0^3}\right) \sigma_0 \]  
\[ \hat{H}_q^{(5)} = -\left(\hat{h}_x^{(5)} \sigma_x + \hat{h}_y^{(5)} \sigma_y + \hat{h}_z^{(5)} \sigma_z\right). \]

In Eq. (31f), we used

\[ \hat{h}_x^{(5)} = \frac{\tilde{B} \tilde{\alpha} \tilde{E}_2 \cos \theta}{\hbar^4 \omega_0^4} (2\tilde{\alpha}^2 - \tilde{B}^2) \sin(\omega t) \]  
\[ \hat{h}_y^{(5)} = \frac{\tilde{E}_2 \tilde{\alpha} \omega_0^3 \cos \theta}{\hbar^3 \omega_0^3} \cos(\omega t) \]  
\[ \hat{h}_z^{(5)} = \frac{\tilde{E}_2 \tilde{\alpha} \omega_0^3 \sin \theta}{\hbar^3 \omega_0^3} \cos(\omega t) \]  
\[ + \frac{\tilde{B} \tilde{\alpha}^2 \sin 2\theta}{2\hbar^4 \omega_0^4} (\tilde{B}^2 - \tilde{\alpha}^2 + \tilde{E}_2^2 \sin(\omega t)) \]  
\[ + \frac{\tilde{B} \tilde{\alpha}^2 \cos 2\theta}{\hbar^4 \omega_0^4} (\tilde{B}^2 - \tilde{\alpha}^2 + \tilde{E}_2^2 \sin(\omega t)). \]

Note that the upper index in, e.g., \( \hat{H}_q^{(3)} \) refers to the order of perturbation theory in which the terms appear.

Out of the six terms in Eq. (30), \( \hat{H}_q^{(0)} \) and \( \hat{H}_q^{(1)} \) are simply the projected parts of \( \hat{H}_0 \) and \( \hat{H}_1 = \hat{H}_E + \hat{H}_B + \hat{H}_{SO} \), respectively. \( \hat{H}_q^{(2)} \) contains a static and a time-dependent second-order energy shift, due to the spin-orbit interaction and the ac electric field, respectively. \( \hat{H}_q^{(3)} \) has five terms. The first, second and fifth terms are spin-and-time-dependent, hence these all contribute to the qubit dynamics. The third and fourth terms are static; they describe the orbit-induced g-tensor renormalization. The fourth-order term \( \hat{H}_q^{(4)} \) of the qubit Hamiltonian, being diagonal, does not influence spin dynamics. The static parts of the fifth-order term \( \hat{H}_q^{(5)} \) describe higher-order g-tensor renormalisation (those proportional to \( \tilde{\alpha} \tilde{B} \)), or nonlinear Zeeman splitting (those proportional to \( \tilde{\alpha}^2 \tilde{B}^3 \)).

Already at this point, there are reasons to expect that in this EDSR model, a half-harmonic resonance occurs, and that the half-harmonic resonance frequency is driving-strength dependent: (i) The third-order effective Hamiltonian \( \hat{H}_q^{(3)} \) incorporates both longitudinal and transverse ac components, in analogy with the case of the misaligned-field MR discussed in Sec. II. (ii) The fifth-order effective Hamiltonian \( \hat{H}_q^{(5)} \) incorporates terms proportional to \( \tilde{E}_2^2 \sin^2 \omega t = \frac{1}{2} \tilde{E}_2^2 (1 - \cos 2\omega t) \); the longitudinal static part \( \propto \tilde{E}_2^2 \sigma_z \) can be interpreted as a drive-strength-dependent effective g-tensor renormalisation, which contributes to the BSS, whereas the dynamical part \( \propto \tilde{E}_2 \cos 2\omega t \sigma_y \) is expected to drive Rabi oscillations at half-harmonic excitation, i.e., when \( 2\hbar \omega \approx \tilde{B} \).

We note that the effective Hamiltonian in Eq. (31) fulfills the expectation that no spin transition occurs if the external B-field and the spin-orbit field are aligned, i.e., when \( \theta = \pi/2 \).

C. Floquet perturbation theory for EDSR

We apply Floquet perturbation theory, outlined in Sec. II C, to describe the fundamental and half-harmonic resonances. In particular, we derive the parameter dependence of the corresponding resonance frequencies \( \omega^{(1)}_{\text{res}} \) and \( \omega^{(2)}_{\text{res}} \), as well as the Rabi frequencies \( \Omega^{(1)}_{\text{res}} \) and \( \Omega^{(2)}_{\text{res}} \), at these two resonances, up to terms of the order of \( \tilde{B} \epsilon^4 \). There are two significant differences in the derivation of the EDSR results with respect to that of the MR results; we outline these differences in the following.

(1) The MR Hamiltonian (2) has a driving term that is proportional to \( \sin \omega t \). In contrast, the effective qubit Hamiltonian (30) we obtained for EDSR has \( \cos \omega t \) terms as well as second-harmonic terms proportional to \( \cos 2\omega t \). In practice, the latter fact implies that the Floquet matrix will contain off-diagonal matrix elements that connect Floquet levels with next-nearest-neighbor Floquet quantum numbers.

(2) In the EDSR case, we repeat the same second-order time-independent Schrieffer-Wolff transformation on the Floquet Hamiltonian \( \mathcal{H} \) that we applied in Secs. II C 2 and II C 3. The Floquet Hamiltonian itself contains terms of the order of \( \tilde{B} \), \( \tilde{B} \epsilon^2 \) and \( \tilde{B} \epsilon^3 \), since it is constructed from the effective qubit Hamiltonian that is itself the result of a finite-order perturbative calculation. When we separate the Floquet Hamiltonian to diagonal \( (\mathcal{F}_0) \) and off-diagonal \( (\mathcal{F}_1) \) components, and apply time-independent Schrieffer-Wolff transformation up to second order in \( \mathcal{F}_1 \), then the resulting \( 2 \times 2 \) effective Floquet Hamiltonian will involve higher-order terms, up to \( \tilde{B} \epsilon^8 \). As our original Hamiltonian was accurate only up to the \( \sim \tilde{B} \epsilon^4 \) terms, we drop from the effective Floquet Hamiltonian the terms that are of higher order than \( \sim \tilde{B} \epsilon^4 \).
D. Results

The results we obtain from Floquet perturbation theory are as follows:

\[
\hbar \omega_{\text{res}}^{(1)} = \tilde{B} - \frac{2\tilde{B} \hat{g}^2 \cos^2 \theta}{\hbar^2 \omega_0^2} \left[ 1 + \frac{2\tilde{B}^2 - 2\hat{g}^2(1 + \sin^2 \theta) + 2\tilde{E}_{\text{ac}}}{2\hbar^2 \omega_0^2} \right], \tag{33a}
\]

\[
\hbar \omega_{\text{res}}^{(1)} = 2 \tilde{B} E_{\text{ac}} \frac{\alpha \cos \theta}{\hbar^2 \omega_0^2} \left[ 1 + \frac{\tilde{B}^2 - 2\hat{g}^2}{\hbar^2 \omega_0^2} \right], \tag{33b}
\]

\[
\hbar \omega_{\text{res}}^{(2)} \approx \frac{1}{2} \tilde{B} - \frac{\tilde{B} \hat{g}^2 \cos^2 \theta}{(\hbar \omega_0)^2} + \frac{\tilde{B} \hat{g}^4 \cos^2 \theta (3 - \cos 2\theta)}{2(\hbar \omega_0)^4} + \frac{\tilde{B}^3 \hat{g}^2 \cos^2 \theta}{(\hbar \omega_0)^4} + \frac{2 \tilde{B} \hat{g}^2 \tilde{E}_{\text{ac}} \cos^2 \theta}{3(\hbar \omega_0)^4}, \tag{33c}
\]

\[
\hbar \Omega_{\text{res}}^{(2)} = \frac{\tilde{B} \hat{g}^2 \tilde{E}_{\text{ac}} \sin(2\theta)}{(\hbar \omega_0)^4}. \tag{33d}
\]

In the rest of this subsection we discuss these results and compare them to numerical results.

The terms describing the fundamental resonance frequency in Eq. (33a) are interpreted as nominal Zeeman splitting, g-tensor renormalisation, nonlinear Zeeman effect, higher-order g-tensor renormalisation, and BSS, respectively. We call the last term a BSS as it is a power-dependent correction to the resonance frequency, that is second order in the drive amplitude, hence analogous to the BSS in MR. Remarkably, the BSS in Eq. (33a) is a negative correction, whereas the BSS in MR is always positive. The last term of the half-harmonic resonance frequency [Eq. (33c)] is also interpreted as a BSS. Further similarities with the MR case: (i) For the fundamental resonance, the BSS is smaller (\(\sim B \epsilon^4\)) than the power broadening, the latter being given by \(\hbar \Omega_{\text{res}}^{(1)} \sim B \epsilon^2\). (ii) For the half-harmonic resonance, the BSS, being \(\sim B \epsilon^4\), is comparable to the power broadening, the latter being given by \(\hbar \Omega_{\text{res}}^{(2)} \sim B \epsilon^4\). (iii) The BSS for both the fundamental and the half-harmonic resonance is proportional to \(\cos^2 \theta\), i.e., it vanishes in the limit of purely longitudinal excitation, and finite for purely transversal excitation. These features can be explained by the argument provided in Sec. IID for the case of MR, applied to the effective qubit Hamiltonian (30).

Regarding the results (33a) and (33c) for the resonance frequencies, we note that their ratio is exactly two in the limit of vanishing driving power, i.e., \(\lim_{E_{\text{ac}} \to 0} \left( \frac{\omega_{\text{res}}^{(1)}}{\omega_{\text{res}}^{(2)}} \right) = 2\).

We have checked that the result (33b) for the fundamental Rabi frequency \(\Omega_{\text{res}}^{(1)}\) matches the corresponding result of Ref. 5; see Appendix B for details.

The analytical results are tested against numerically exact solutions of the time-dependent Schrödinger equation defined by the Hamiltonian \(\mathcal{H}\) in Eq. (24). The numerical results were obtained using the truncated Hilbert space spanned by the 8 lowest-energy eigenstates of \(\mathcal{H}_0 + \mathcal{H}_B\), corresponding to the 4 lowest-lying levels of the harmonic oscillator. We have checked that there was no visible change in the numerical results upon extending the Hilbert space with further, higher-lying orbitals.

In Fig. 4, we plot the numerically computed time evolution of the occupation probability of the excited state \(|E\rangle\), for a finite range of the driving frequency in the vicinity of the ‘nominal’ half-harmonic resonance frequency \(\omega_0/B = 0.5\) (see caption for parameter values). The analytical result (33c) predicts complete Rabi oscillations at \(\omega = \omega_{\text{res}}^{(2)} = 0.4809\tilde{B}/\hbar\), and the Rabi frequency at this resonance is predicted by Eq. (33d) to be \(\Omega_{\text{res}}^{(2)} \approx \frac{1}{625}\tilde{B}\). These predictions are in line with the numerical data shown in Fig. 4. For a finite detuning from the resonance frequency, the Rabi oscillations become faster though incomplete (i.e., not reaching \(P_E = 1\)), leading to the characteristic chevron pattern known from magnetic resonance. The results of Fig. 4 therefore reveal simple Rabi dynamics at the half-harmonic resonance.

The density plot of Fig. 5 is a visual demonstration of the BSS, i.e., of that the resonance frequency increases with increasing drive strength. The figure shows the maximum \(P_E^{\text{max}}\) of the excited-state probability \(P_E(t)\) within a time span exceeding the Rabi period at the half-harmonic resonance, as a function of the amplitude \(E_{\text{ac}}\) of the ac electric field and the drive frequency \(\omega\). (See caption for parameters.) Therefore, vertical cuts of the
density plot correspond to resonance curves. The solid line represents the analytical result (33c) for the half-harmonic resonance frequency. The agreement between the analytical curve and the $P_{E}^{	ext{max}} \approx 1$ ridge of the numerical simulation reassures the validity and correspondence of the two approaches. Importantly, in Fig. 5, the BSS is comparable in magnitude to the power broadening, which makes the BSS relatively easily resolvable in experiments realizing the model we use.

A further question is how the BSS depends on the angle $\theta$ characterizing the direction of the spin-orbit interaction. This dependence is exemplified by Fig. 6, which shows $P_{E}^{	ext{max}}$ as a function of $\theta$ and the drive frequency. The latter is measured from the calculated fundamental resonance frequency $\omega_{\text{res}}^{(1)}$, see Eq. (33a). The solid line, showing good agreement with the centre of the bright $P_{E}^{	ext{max}} \approx 1$ region of the underlying density plot, shows the analytical result for the half-harmonic resonance frequency $\omega_{\text{res}}^{(2)}$ [Eq. (33c)].

IV. DISCUSSION AND CONCLUSIONS

(1) We provide a numerical example to estimate orders of magnitudes of the EDSR resonance shifts and Rabi frequencies. We take $B = 1.7$ T and an electronic $g$ factor of 2, yielding $\tilde{B} \approx 0.1$ meV. We also set $\tilde{\alpha} = 0.1$ meV and $E_{\text{ac}} = 0.1$ meV, whereas the orbital level spacing is chosen as $\hbar \omega_0 = 1$ meV. Then the order of magnitude of the Rabi frequency at the fundamental resonance is $\Omega_{\text{res}}^{(1)} \sim \tilde{B} \alpha / \hbar \approx 1.5 \times 10^{4} \frac{\text{s}^{-1}}{\text{meV}}$, corresponding to a spin-flip time of $\approx 4.3$ ns. For the half-harmonic resonance, $\Omega_{\text{res}}^{(2)} \sim \tilde{B} \alpha / \hbar \approx 1.5 \times 10^{7} \frac{\text{s}^{-1}}{\text{meV}}$, corresponding to a spin-flip time of $\approx 430$ ns. For both resonances, the BSS is comparable to the value of $\Omega_{\text{res}}^{(2)}$ estimated above.

(2) The results presented in this work describe a perturbative regime where spin-orbit interaction is assumed to be ‘weak’, in the sense that the spin-orbit energy scale in the QD is dominated by the QD level spacing, $\tilde{\alpha} \ll \hbar \omega_0$. In nanowire QD host materials such as InAs and InSb, spin-orbit interaction is known to be ‘strong’ in the sense that it creates a strong g-factor renormalisation, already in the bulk materials. A question arising from these facts is: are typical InAs and InSb nanowire QDs within the range of validity of our perturbative theory? One way to answer this question is via a comparison of the dependence of the fundamental EDSR resonance frequency obtained from the perturbative theory and from experiments. The experiments have found that the fundamental resonance frequency shows a similar angular dependence as the perturbative result Eq. (33a), i.e., for a magnetic field with a fixed magnitude, the resonance frequency is maximal if the magnetic field is aligned along a certain direction and minimal if it is aligned perpendicular to that direction. To be specific, we take the data given in the first row of Table I of Ref. 25, which indicates that the ratio of the minimal and maximal resonance frequencies in the considered case were $\approx 0.84$. Using the first two terms in Eq. (33a), we can identify that ratio with $1 - 2 \tilde{\alpha}^2 / \hbar^2 \omega_0^2$, yielding $\tilde{\alpha} / \hbar \omega_0 \approx 0.28$ for this particular InAs device. A similar analysis of the experimental data in Fig. 3c of Ref. 17.
results in an estimate $\alpha/h\omega_0 \approx 0.37$ for the measured InSb device. These estimates suggest that the InAs and InSb QDs are on the border between ‘weak’ and ‘strong’ spin-orbit interaction.

(3) To our knowledge, three experiments have reported subharmonic EDSR resonances in semiconductor nanowire QDs, where our model based on the Rashba-type spin-orbit interaction could be appropriate to describe the spin dynamics. The strong subharmonic resonances reported in Stehlík et al.\textsuperscript{26} are described by a theory developed for strongly driven double quantum dots\textsuperscript{34}. Faint half-harmonic resonances are visible in the data of Refs. 25 (see Fig. 2b therein) and 17 (see Fig. 2b therein). A quantitative experimental analysis exploring the parameter dependencies of the corresponding resonance and Rabi frequencies would allow for a comparison with our predictions.

(4) One of our conclusions was that the BSS of the fundamental EDSR resonance frequency has an anomalous, negative sign, see Eq. (33a). Here, we provide a simple physical picture explaining this result, using the unitary transformation applied in Ref. 42. For simplicity, we focus on the case when the spin-orbit field is perpendicular to the magnetic field, i.e., $\theta = 0$. Then, the unitary transformation $S$ of Ref. 42 (not to be confused with the generator of the Schrieffer-Wolff transformation in Appendix A) applied on our static Hamiltonian $\mathcal{H}_0 + \mathcal{H}_B + \mathcal{H}_{SO}$ eliminates the spin-orbit term and transforms the homogeneous magnetic field $H_B$ to an inhomogeneous, spiral-like magnetic field, $H_0' = S H B S^\dagger \propto \tilde{B} [\sigma_z \cos(z/\xi) - \sigma_x \sin(z/\xi)]$, where $\xi \equiv 1/\tilde{\alpha}$ is the spin-orbit length [see Eq. (2) of Ref. 42]. The driving electric field, incorporated in our model as $\mathcal{H}_E$, induces a spatial oscillation $z(t) = -A \sin \omega t$ of the electron’s center of mass with an amplitude $A \propto \tilde{E}_{ac}$. Inserting this time-dependent $z(t)$ to the above expression for $H_0'$, and expanding the terms up to second order in $A/\xi$, we find $H_0''(t) \propto \tilde{B} [\sigma_z \left(1 - \frac{A^2}{\xi^2} \sin^2 \omega t\right) + \sigma_x \frac{A}{\xi} \sin \omega t]$. That is, the time-averaged $z$ component of the time-dependent magnetic field in $H_0''(t)$ acquires a correction proportional to $-\tilde{B} \frac{A^2}{\xi^2} \propto -\tilde{B} \tilde{E}_{ac}^2 \tilde{\alpha}^2$. Notice that this correction is negative and has the same parameter dependence as the BSS in, i.e., the last term of, Eq. (33a).

In conclusion, we have studied the characteristics of EDSR in a 1D QD model with parabolic confinement, homogeneous Rashba spin-orbit interaction and homogeneous driving electric field. We demonstrated the existence of subharmonic (multi-photon) resonances in this model, and analysed the half-harmonic (two-photon) resonance in detail. We have analytically described the parameter dependence of the fundamental resonance frequency and the half-harmonic resonance frequency, and demonstrated that these resonance frequencies increase with increasing drive strength. This effect is analogous to the BSS in MR.

Our results describe a perturbative regime, where the orbital level spacing of the QD dominates the energy scales of the external magnetic field, spin-orbit interaction, and electrical drive. Therefore our results have direct experimental relevance for QDs with weak spin-orbit interaction. The results can also serve as benchmarks for numerical studies departing from the perturbative regime. The model we used here has only the minimal ingredients necessary to describe EDSR, which suggest that the subharmonic resonances and the BSS discussed here are generic features of electrically driven spin dynamics.

ACKNOWLEDGMENTS

We thank P. Scarlino and G. Széchenyi for useful discussions. We acknowledge funding from the EU Marie Curie Career Integration Grant CIG-293834 (Carbon-Qubits), the OTKA Grants PD 100373 and 106047, and the EU ERC Starting Grant CooPairEnt 258789. GB acknowledges funding from the Deutsche Forschungsgemeinschaft (DFG) within SFB767 and from the EU Marie Curie ITN S3NANO. AP is supported by the János Bolyai Scholarship of the Hungarian Academy of Sciences.

Appendix A: Time-dependent Schrieffer-Wolff perturbation theory

In this Appendix, we describe time-dependent Schrieffer-Wolff perturbation theory (TDSW), the method we use to derive the effective $2 \times 2$ time-dependent Hamiltonian (30) governing the dynamics of the qubit.

Let us first recall the basic idea of standard time-independent Schrieffer-Wolff (SW) perturbation the-
We consider a Hamiltonian $H = H_0 + H'$ where $H_0$ is diagonal and $H'$ is the perturbation. Furthermore, the basis states of $H$ can be divided into relevant A and irrelevant B subspaces that have well separated energy scales. A and B are weakly interacting, i.e. the matrix elements connecting them are small compared to the energy separation of the two subspaces. Ideally, we can introduce a unitary transformation $e^{-S}$ that brings $H$ into a block diagonal form $\tilde{H} = e^{-S}H e^S$ where the relevant and irrelevant subspaces are separated as illustrated in Fig. 7. However, in most of the cases we don’t know the explicit form of the transformation $e^{-S}$ so we have to construct it bit-by-bit until the elements connecting the two subsets vanish up to the desired order of perturbation. This is usually done by expanding $e^{-S}$ in a series and constructing the terms of different orders successively.

A great advantage of SW with respect to conventional perturbation theory is that here we don’t need to distinguish between the degenerate and non-degenerate cases.

Now we introduce the time-dependent SW perturbation theory as a natural extension of the time-independent case. Similar approaches have been applied for particular problems in Refs. 43–45; here, we provide a general description of the method, which we utilised in the main text for deriving the effective qubit Hamiltonian (30) of the EDSR problem.

Consider the time-dependent Hamiltonian $H(t) = H_0 + H'(t)$, where the perturbation is divided into a block-diagonal and block-off-diagonal part $H'(t) = H_1 + H_2$ as shown in Fig 7.

In our problem (see Sec. III A), $H_1 = H_B$ and $H_2(t) = H_E(t) + H_{SO}$. Note that there $H_1$ happens to be a time-independent perturbation, but the treatment outlined here is readily applicable to a time-dependent block-diagonal perturbation as well.

Similarly to the SW we successively build the unitary transformation $U(t) = e^{-S(t)}$ that separates the subspaces A and B, but here the matrix $S(t)$ is now time-dependent. Note that any unitary transformation can be written in this form, and the matrix $S(t)$ should be anti-Hermitian to ensure the unitary character of $U(t)$. The matrix $S(t)$ is chosen to be block-off-diagonal (see Fig 7). Note also that because of the weakness of the inter-subspace coupling, the unitary transformation $U(t)$ is close to unity, and hence $S(t)$ is small and can be expressed as a power series with respect the perturbing terms.

The transformation of time-dependent Schrödinger equation $-\hbar \frac{\partial}{\partial t} \psi(t) + H(t)\psi(t) = 0$ with the above $U(t)$ is canonical, i.e., it preserves the form of the time evolution equation. The transformed wave function and Hamiltonian read as:

\begin{equation}
\tilde{\psi}(t) = e^{-S(t)}\psi(t),
\end{equation}

\begin{equation}
\tilde{H}(t) = e^{-S(t)}H(t)e^{S(t)} + i\hbar \frac{\partial e^{-S(t)}}{\partial t} e^{S(t)}. \tag{A2}
\end{equation}

From now on, we might suppress the time argument and denote time derivatives such as $\frac{\partial}{\partial t} \psi$ as $\dot{\psi}$.

Starting from Eq. (A2), we utilize the power series of the exponential function. The second term in Eq. (A2) is the heart of the time-dependent SW transformation; in the time-independent case this term vanishes as $S$ is time-independent. The expansion of the first term in Eq. (A2) is known from SW formalism, therefore we do not discuss it here. The explicit form of the second term, after expanding the exponential function, has the following form:

\begin{equation}
\frac{\partial e^{-S}}{\partial t} e^S = \left[ \frac{\partial}{\partial t} \left( -S + \frac{1}{2!} S^2 - \frac{1}{3!} S^3 + \ldots \right) \right] \left( I + S + \frac{1}{2!} S^2 + \frac{1}{3!} S^3 + \ldots \right) \times \left( I + S + \frac{1}{2!} S^2 + \frac{1}{3!} S^3 + \ldots \right) = -[\dot{S}, S]^{(0)} - \frac{1}{2!} [\dot{S}, S]^{(1)} - \frac{1}{3!} [\dot{S}, S]^{(2)} \ldots = -\sum_{j=0}^{\infty} \frac{1}{(j+1)!} [\dot{S}, S]^{(j)}. \tag{A3}
\end{equation}

The transformed Hamiltonian then equals to

\begin{equation}
\tilde{H} = \sum_{j=0}^{\infty} \frac{1}{j!} [H, S]^{(j)} - i\hbar \sum_{j=0}^{\infty} \frac{1}{(j+1)!} [\dot{S}, S]^{(j)}. \tag{A4}
\end{equation}

with $[H, S]^{(n+1)} = [H, S]^{(n)} S$ and $[H, S]^{(0)} = H$. Note that the second term in (A4) is new with respect to time-independent SW, and it is a consequence of the time dependence of the Hamiltonian and therefore that of the matrix $S$. Considering a time-independent Hamiltonian the second term vanishes and we are left with the well-known SW transformation.

We now exploit the block-off-diagonal property of $S$ in order to separate the block-off-diagonal and block-
The diagonal parts of the transformed Hamiltonian:

\[ \tilde{H}_{\text{off-diag}} = \sum_{j=0}^{\infty} \frac{1}{(2j+1)!} [H_0 + H_1, S]^{(2j+1)} \]
\[ + \sum_{j=0}^{\infty} \frac{1}{(2j)!} [H_2, S]^{(2j)} \]
\[ - i\hbar \sum_{j=0}^{\infty} \frac{1}{(2j+1)!} [\dot{S}, S]^{(2j)} , \] \tag{A5}

\[ \tilde{H}_{\text{diag}} = \sum_{j=0}^{\infty} \frac{1}{(2j)!} [H_0 + H_1, S]^{(2j)} \]
\[ + \sum_{j=0}^{\infty} \frac{1}{(2j)!} [H_2, S]^{(2j+1)} \]
\[ - i\hbar \sum_{j=0}^{\infty} \frac{1}{(2j+2)!} [\dot{S}, S]^{(2j+1)} . \] \tag{A6}

Then, \( S \) is determined by solving

\[ \tilde{H}_{\text{off-diag}} = 0 . \] \tag{A7}

The effective (now block-diagonal) Hamiltonian becomes \( \tilde{H} = \tilde{H}_{\text{diag}} \). Note that \( \tilde{H} \) as well as the term ‘effective Hamiltonian’ is also used to describe the block of \( \tilde{H} \) corresponding to the relevant subspace.

So far no approximation has been made; now we make use of the smallness of the perturbation. Following the approach of time-independent SW perturbation theory, we aim at solving Eq. (A7) via expanding \( S \) as a power series in the perturbation,

\[ S = S_1 + S_2 + S_3 + \ldots , \] \tag{A8}

where \( S_j \) represents an operator of \( j \)th order in the perturbation. Recall that in TDSW, \( S \) is time dependent, and its time derivative appears in its defining equation (A7) as well as in the effective Hamiltonian (A6). Therefore, to separate the terms of different order in perturbing parameter in Eq. (A7), it is necessary to make an a priori assumption on the frequency scale characterizing the magnitude of \( S_j \). As the drive frequency is \( \omega \), expectedly the frequency characterizing the time evolution of all \( S_j \) - s will be \( \sim \omega \), hence we assume \( \dot{S}_j \sim \omega S_j \). In the EDSR problem defined in Sec. III A, the relevant subspace is the subspace of the ground-state orbital spanned by \( |\uparrow_1 \rangle \) and \( |\downarrow_1 \rangle \). Furthermore, the energy scales of the drive frequency, drive strength, Zeeman splitting and spin-orbit coupling are much lower than the splitting between the oscillator levels \( \sim \omega_0 \), and all of them are treated as perturbation. This implies that \( \dot{S}_j \) is of the order of \( (j + 1) \) in perturbation.

Obviously, after solving Eq. (A7) with this assumption, we need to check if the obtained \( S_j \) functions are consistent with our assumption above.

From the order-by-order expansion of Eq. (A7), we obtain the following hierarchy of simple algebraic equations for the \( S_j \) matrices:

\[ [H_0, S_1] = -\mathcal{H}_2 . \] \tag{A9a}
\[ [H_0, S_2] = -[H_1, S_1] + i\hbar \dot{S}_1 , \] \tag{A9b}
\[ [H_0, S_3] = -[H_1, S_2] - \frac{1}{3} [H_2, S_1] + i\hbar \dot{S}_2 , \] \tag{A9c}
\[ [H_0, S_4] = -[H_1, S_3] - \frac{1}{3} [H_2, S_1, S_2] \]
\[ - \frac{1}{3} [[H_2, S_2], S_1] + i\hbar \dot{S}_3 , \] \tag{A9d}

\[ \vdots \]

Once the first equation (A9a) is solved for \( S_1(t) \), the solution can be inserted to (A9a) which then forms an algebraic equation for \( S_2(t) \), etc. Note that since we work in the eigenbasis of \( H_0 \), the above procedure simplifies to subsequently solving single-variable linear equations, which is a trivial analytical task, well suited for symbolic computation.

After obtaining the \( S_j \) matrices and inserting them into Eq. (A6), we have an order-by-order expansion \( \tilde{H} = \tilde{H}_{\text{diag}} = \sum_{j=0}^{\infty} \tilde{H}^{(n)} \), where

\[ \tilde{H}^{(0)} = H_0 \] \tag{A10a}
\[ \tilde{H}^{(1)} = H_1 \] \tag{A10b}
\[ \tilde{H}^{(2)} = [H_2, S_1] + \frac{1}{2} [H_0, S_1]^{(2)} \] \tag{A10c}
\[ \tilde{H}^{(3)} = [H_2, S_2] + \frac{1}{2} [H_1, S_1]^{(2)} + \frac{1}{2} [[H_0, S_1], S_2] \]
\[ + \frac{1}{2} [[H_0, S_2], S_1] - i\hbar \frac{1}{2} [\dot{S}_1, S_1] \] \tag{A10d}

\[ \vdots \]

With the use of Eqs. (A9) we can further simplify Eqs. (A10):

\[ \tilde{H}^{(0)} = H_0 \] \tag{A11a}
\[ \tilde{H}^{(1)} = H_1 \] \tag{A11b}
\[ \tilde{H}^{(2)} = \frac{1}{2} [H_2, S_1] \] \tag{A11c}
\[ \tilde{H}^{(3)} = \frac{1}{2} [H_2, S_2] \] \tag{A11d}
\[ \tilde{H}^{(4)} = \frac{1}{24} [H_2, S_3] - \frac{1}{4} [H_2, S_1]^{(3)} \] \tag{A11e}
\[ \tilde{H}^{(5)} = \frac{1}{24} [H_2, S_4] - \frac{1}{4} [[H_2, S_1], S_2] \]
\[ - \frac{1}{4} [[H_2, S_2], S_1] - \frac{1}{4} [[H_2, S_2], S_1] \] \tag{A11f}

\[ \vdots \]
Finally, we need to check the consistency of our assumption for the time evolution of $S_j$ with the actual solution we obtained for $S_j$ using that assumption. From (A9a), $S_1$ inherits harmonic time-dependence from $H_2$ with frequency $\omega$. This implies that the time derivative is $\dot{S}_1 \sim \omega S_1$, as assumed. From Eq. (A9b), the matrix $S_2$ might contain frequency components at $\omega$, as well as at zero frequency and $2\omega$ (if $H_1$ is time-dependent with frequency $\omega$); nevertheless, the $\dot{S}_2 \sim \omega S_2$ relation still holds, etc.

In conclusion, TDSW allows for obtaining an effective time-dependent Hamiltonian for the relevant subspace. The procedure is to evaluate the transformation matrices $S_j$ up to the desired order via solving Eq. (A9), and substituting the resulting $S_j$ matrices into Eq. (A11).

**Appendix B: Rabi frequency of the fundamental resonance: relation to the results of Ref. 5**

EDSR in a QD in a two-dimensional electron gas due to Rashba and Dresselhaus spin-orbit interactions has been described by Golovach, Borhani and Loss (GBL) in Ref. 5. Therein, the Rabi frequency of the fundamental resonance as a function of system parameters (magnetic field strength, magnetic field direction, spin-orbit interaction strengths and ac electric field amplitude and direction) has been calculated. Even though the dimensionality and the spin-orbit Hamiltonian in the model of GBL differs from our model, the calculated Rabi frequencies can be compared after a special case of the model of GBL has been reduced to one dimension. Here we show that after this dimension reduction our result for the fundamental Rabi frequency equals that of GBL.

In the model of GBL, the 2DEG lies in the $x$-$y$ plane. We consider the special case when the confinement potential is parabolic and has a cylindrical symmetry, the Dresselhaus coupling vanishes, $\beta = 0$, the B-field is in the $y-z$ plane, and the E-field is along the $x$ axis. Furthermore, we project the Hamiltonian on the $y$-ground-state orbital of the harmonic oscillator, yielding

$$H_{\text{GBL}} = \frac{p_x^2}{2m} + \frac{1}{2} m \omega_0^2 x^2 + \alpha p_x \sigma_y + \frac{1}{2} \theta^* \mu_B B \cdot \sigma + e E_{ac} x \sin \omega t$$  \hspace{1cm} (B1)

For simplicity, we focus on the special case $B = (0, 0, B)$ from now on. Then, the Hamiltonian in Eq. (B1) is equivalent to our Hamiltonian $H$ at $\theta = 0$.

To deduce the Rabi frequency calculated by GBL for the special case above, we start from their Eqs. (13) and (14), where they provide the time-dependent part of the effective qubit Hamiltonian as

$$H_{\text{GBL}} = \frac{1}{2} h(t) \cdot \sigma,$$  \hspace{1cm} (B2)

where

$$h(t) = 2 \mu_B B \times \Omega(t).$$  \hspace{1cm} (B3)

A straightforward calculation shows that

$$\frac{1}{2} h(t) = \frac{\alpha E_{ac} \sigma^* \mu_B B}{\hbar \omega_0^2} \sin(\omega t) e_x$$  \hspace{1cm} (B4)

$$= 2 \frac{\tilde{\alpha} E_{ac} B}{\hbar^2 \omega_0^3} \sin(\omega t) e_x$$  \hspace{1cm} (B5)

Note that this effective ac magnetic field is perpendicular to the static magnetic field, which is applied in the $z$ direction. The Rabi frequency due to this ac magnetic field at the fundamental resonance frequency reads

$$\Omega_{\text{res, GBL}}^{(1)} = 2 \frac{\tilde{\alpha} E_{ac} B}{\hbar^2 \omega_0^3},$$  \hspace{1cm} (B6)

which is identical to our result in Eq. (33b), if the latter is evaluated at $\theta = 0$ and terms above third order are dropped.

---

1. F. Bloch and A. Siegert, *Phys. Rev.* **57**, 522 (1940).
2. J. H. Shirley, *Phys. Rev.* **138**, B979 (1965).
3. F. H. L. Koppens, C. Buizert, K. J. Tielrooij, I. T. Vink, K. C. Nowack, T. Meunier, L. P. Kouwenhoven, and L. M. K. Vandersypen, *Nature* **442**, 766 (2006).
4. M. Veldhorst, J. C. C. Hwang, C. H. Yang, A. W. Leenstra, B. de Ronde, J. P. Dekhollain, J. T. Muhonen, F. E. Hudson, K. M. Itoh, A. Morello, and A. S. Dzurak, ArXiv e-prints (2014), arXiv:1407.1950 [cond-mat.mes-hall].
5. V. N. Golovach, M. Borhani, and D. Loss, *Phys. Rev. B* **74**, 165319 (2006).
6. C. Fidant, A. S. Sorensen, and K. Flensberg, *Phys. Rev. Lett.* **97**, 240501 (2006).
7. Y. Tokura, W. G. van der Wiel, T. Obata, and S. Tarucha, *Phys. Rev. Lett.* **96**, 047202 (2006).
8. E. I. Rashba, *Phys. Rev. B* **78**, 195302 (2008).
9. J. D. Walls, *Phys. Rev. B* **76**, 195307 (2007).
10. D. V. Khomitsky, L. V. Gulyaev, and E. Y. Sherman, *Phys. Rev. B* **85**, 125312 (2012).
11. R. Li, J. Q. You, C. P. Sun, and F. Nori, *Phys. Rev. Lett.* **111**, 086805 (2013).
12. Y. Kato, R. C. Myers, D. C. Driscoll, A. C. Gossard, J. Levy, and D. D. Awschalom, *Science* **299**, 1201 (2003).
13. K. C. Nowack, F. H. L. Koppens, Y. V. Nazarov, and L. M. K. Vandersypen, *Science* **318**, 1430 (2007).
14. M. Pioro-Ladriere, T. Obata, Y. Tokura, Y.-S. Shin, T. Kubo, K. Yoshida, T. Taniyama, and S. Tarucha, *Nat. Phys.* **4**, 776 (2008).
15. E. A. Laird, C. Barthel, E. I. Rashba, C. M. Marcus, M. P. Hanson, and A. C. Gossard, *Phys. Rev. Lett.* **99**, 246601 (2007).
16. S. Nadj-Perge, S. M. Frolov, E. P. A. M. Bakkers, and L. P. Kouwenhoven, *Nature* **468**, 1084 (2010).
17. S. Nadj-Perge, V. S. Friebig, J. W. G. van den Berg, K. Zuo, S. R. Plissard, E. P. A. M. Bakkers, S. M. Frolov, and L. P. Kouwenhoven,
18. F. Pei, E. A. Laird, G. A. Steele, and L. P. Kouwenhoven, Nat. Nanotech. 7, 630 (2012).
19. V. S. Pribyag, S. Nadj-Perge, S. M. Frolov, J. W. G. van den Berg, I. van Weperen, S. R. Plissard, E. P. A. M. Bakkers, and L. P. Kouwenhoven, Nat Nano 8, 170 (2013).
20. E. A. Laird, F. Pei, and L. P. Kouwenhoven, Nat. Nanotech. 8, 565 (2013).
21. E. Kawakami, P. Scarlino, D. R. Ward, F. R. Braakman, D. E. Savage, M. G. Lagally, M. Friesen, S. N. Coppensmith, M. A. Eriksson, and L. M. K. Vandersypen, Nat Nano 8, 170 (2013).
22. E. A. Laird, C. Barthel, E. I. Rashba, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Semiconductor Science and Technology 24, 064004 (2009).
23. J. Stehlik, M. D. Schroer, M. Jung, and J. R. Petta, Phys. Rev. Lett. 107, 176811 (2011).
24. M. D. Schroer, K. D. Petersson, M. Jung, and J. R. Petta, Phys. Rev. Lett. 107, 176811 (2011).
25. J. Stehlik, M. D. Schroer, M. Z. Maialle, M. H. Degani, and J. R. Petta, Phys. Rev. Lett. 112, 227601 (2014).
26. A. De, C. E. Pryor, and M. E. Flatté, Phys. Rev. Lett. 102, 017603 (2009).
27. J. Pingenot, C. E. Pryor, and M. E. Flatté, Phys. Rev. B 84, 195403 (2011).
28. E. I. Rashba, Phys. Rev. B 84, 241305 (2011).
29. M. P. Nowak, B. Szafran, and F. M. Peeters, Phys. Rev. B 86, 125428 (2012).
30. E. N. Osika, B. Szafran, and M. P. Nowak, Phys. Rev. B 88, 165302 (2013).
31. V. S. Pribyag, S. Nadj-Perge, S. M. Frolov, J. W. G. van den Berg, I. van Weperen, S. R. Plissard, E. P. A. M. Bakkers, and L. P. Kouwenhoven, Nat Nano 8, 170 (2013).
32. E. A. Laird, F. Pei, and L. P. Kouwenhoven, Nat. Nanotech. 8, 565 (2013).
33. F. Forster, M. Mühlbacher, D. Schuh, W. Wegscheider, and S. Ludwig, ArXiv:1503.01938 (unpublished).
34. E. A. Laird, C. Barthel, E. I. Rashba, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Semiconductor Science and Technology 24, 064004 (2009).
35. Y. Goldin and Y. Avishai, Phys. Rev. B 61, 16750 (2000).
36. L. S. Levitov and E. I. Rashba, Phys. Rev. B 67, 115324 (2003).
37. F. Schwabl, Advanced Quantum Mechanics, Chapter 9. (Springer, 2008).