Models for predicting the suction of heaving compacted soils using geotechnical properties

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Abstract. Soil suction is the major property that controls the behaviour of unsaturated soils. Suction estimation is challenging both in the lab and the field. Various instruments to measure the suction have been developed with the recent technological advancements. Nonetheless, there are still limitations in regards to the reliability, cost, suction range, accessibility, scope of activity, and appropriateness for use either within the field or lab settings. The filter paper method is probably the simplest procedure to measure the suction for the entire range both in the field and the lab. Nevertheless, the procedure takes time. To alleviate the requirement for conducting this test, it becomes imperative to develop mathematical predictive models for soil suction. In this study, a detailed survey was carried out across the Free State province, South Africa, and sampling points identified. Samples were tested for their geotechnical properties. The influence of the geotechnical properties on soil suction was studied. Multivariate regression analysis was carried out utilizing MINITAB 18 program to develop the mathematical predictive models.

Keywords: Total suction, matric suction, regression analysis, geotechnical properties, filter paper.

1. Introduction

Soil suction is a free energy state of water inside the soil [1]. The total suction is formed of two components: the matric suction, represented by \( \psi_m \) and the osmotic suction, designated by \( \psi_0 \). Equation (1) gives the constitutive algebraic relation between the constituent’s elements of total suction. [2] investigated the unitary characterization of the matric suction as the free energy variation in a unit volume of water state to the free water state. The matric suction is observed as the key parameter determining unsaturated soils behaviour. It is the pressure difference between the pore air, designated by \( u_a \) and the pore water, designated by \( u_w \) as given in Equation (2).

\[
\psi_t = \psi_m + \psi_0 \\
\psi_m = u_a - u_w
\]  

The Filter paper method (FPM) is selected because it is probably the simplest technique to determine the soil suction both in the field and laboratory for the full range of interest for vapour transport, fluid, and other geotechnical applications [3]. Nevertheless, the FPM procedure is time-consuming. Moreover, soil compaction enhances shear resistance, bearing capacity, compressibility in the construction of a road, buildings, earth dams, and many other engineering structures. The main objective of this study is to develop semi-empirical models to predict the matric suction and the total suction of compacted heaving soils. To achieve the main objective, specific objectives such as determination of soil properties, investigation of the correlation between soil suction and soil properties, multivariate regression analysis, and the model validation are performed.

Practically all covered surfaces encountered in engineering practice are built close to the surface covering unsaturated soils that would most apparently remain unsaturated during the lifetime of the structure. When investigating unsaturated soils, the water content can be stated using soil suction. [4] developed a generalized
equation to predict the soil suction utilizing the volumetric water content, the percentage of clay and sand as independent variables. Besides, it can be observed that the values of the fitting parameters at the sand fraction are very small, which proves the negligible influence of this factor in the prediction of the soil suction. The correlation between the water content and the soil suction is continuous and non-linear from 10kPa to 1500kPa. The soil suction, denoted by \( \psi \); the volumetric water content \( \theta \); fitting parameters, denoted \((A, B)\).

\[
\psi = A \times \theta^B
\]  
(3)

\[
A = \exp[-4.396 - 0.0715 \times (%\text{clay}) - 4.88 \times 10^{-4} \times (%\text{sand})^2 - 4.285 \times 10^{-5}]
\]  
(4)

\[
B = -3.14 - 0.00222 \times (%\text{clay}) - 3.484 \times 10^{-4} \times (%\text{sand}) + 3.484 \times 10^{-4} \times (%\text{clay})
\]  
(5)

Utilizing this technique, the correlation between volumetric water content and the soil suction at a specific level can be determined. This method is advantageous from a practical point of view. However, the model cannot be utilized to obtain a soil suction value beyond 1500kPa. Hence, it would be convenient to propose a model that can predict soil suction beyond 1500kPa. [5] examined the soil-water characteristic curve (SWCC) inconstancy. Three distinct soils within a specific range of soils encountered in practice were utilized. It was discovered that the correlation between water content and soil suction can be estimated based on geotechnical properties. The estimation based on geotechnical properties is useful because of its low cost and simplicity. The SWCC in the predicted by particle size distribution-based algorithms and/or geotechnical properties was found to be small than that related to the operator. The fitting parameters Equations (7) to (14) were proposed utilizing the percent passing the # 200 sieve, designated by \((F)\) and the plasticity index, designated by \((PI)\) for the SWCC model proposed by [6] given in Equation (6). The volumetric water content at any suction, designated by \(\theta(\psi)\); the correction factor, denoted by \(C(\psi)\); any soil suction value, denoted by \(\psi\); the residual suction, denoted by \(C_0\); the saturated water content \(\theta_1\); fitting soil parameters for SWCC, denoted \((a, n, m)\).

\[
\theta(\psi) = C(\psi) \left[ \ln \left[ 1 + \left( \frac{\psi}{C_0} \right)^n \right] \right]^m = \left[ \ln \left[ 1 + \left( \frac{\psi}{C_0} \right)^n \right] \right]^m \left[ \ln \left[ \frac{\theta_s}{\theta_1} \right] \right] = \theta_s
\]  
(6)

When the soil exhibit a plasticity index greater than zero the equations used to predict the fitting parameters are as follows:

\[
a = 0.00364(F \times PI)^{3.35} + 4(F \times PI) + 11
\]  
(7)

\[
n = -2.313(F \times PI)^{0.14} + 5
\]  
(8)

\[
m = -0.0514(F \times PI)^{0.456} + 0.5
\]  
(9)

\[
h_0 = 32.44e^{0.016(F \times PI)}
\]  
(10)

For non-cohesive soils with a plasticity index equal to zero, the predictive fitting parameters are estimated based on D60 as follows.

\[
a = 0.8627(D60)^{-0.751}
\]  
(11)

\[
n = 7.5
\]  
(12)

\[
c = 0.1772 \times \ln(D60) + 0.7734
\]  
(13)

\[
h_0 = 1\frac{D60 + 9.7 \times e^{-T}}{a}
\]  
(14)

[7] provided a detailed characterization of the suction profile in a North Carolina residual soil and proposed an empirical model to predict soil suction based on basic soil indices. A field test site was chosen in Greensboro, North Carolina. Two major residual soil groups, high plasticity silt, and low plasticity silt were encountered at the site. An empirical model to predict residual soil suction is proposed to correlate the basic soil properties, i.e., grain size distribution, density, and specific gravity with the parameters of the SWCC model proposed by [6] given in Equation (6) was developed using multivariate analysis. The predicted models developed by [7] for each Fredlund-Xing \( a, n, \) and \( m \) parameter are

\[
a = 17.2 + \frac{1.89}{D_{60}} - \frac{0.363}{D_{30}} \times \frac{0.063}{D_{10}} + 2.5 \times D_{60}
\]  
(15)

\[
n = -0.105 - 0.018 \times P200 + 9.55 \times D_{60} + 0.012 \times D_{30} - 0.057 \times D_{10} + 9.55 \times \rho_d
\]  
(16)

\[
m = 11.24 + 0.0074 \times P200 - 0.075 \times 5\mu m - 2.665 \times G_s - 1.452 \times \rho_d
\]  
(17)
Predicted suction profiles give reasonable results, the proposed model provides much better fitness with a determination coefficient of $R^2 = 93\%$. The suction range of the soil profile was reported to be from 60kPa to 100 kPa. These models are used not to predict directly the soil suction of residual soil, rather to obtained the fitting parameters of the SWCC as proposed by [6]. The tensiometer was used as the suction measurement technique to obtain values for the validation of these models, the tensiometer suction measurement is within the range of 0 to 1500kPa. Hence, the fitting parameters will not be suitable for the suction values beyond 1500kPa. However, the study conducted by [8] on the correlation between the swelling stress and the suction of compacted unsaturated heaving soil across Free State in South Africa revealed that these soils display suction values within the range of 340,03kPa to 11283.21kPa. [9] carried out a study on the prediction of the soil suction using the soil colour recognition and developed a simple quick method to predict soil suction. The soil color and the water content are plotted together to establish the soil colour characteristic curve (SCCC), this curve is then combined with the SWCC as proposed by [10] to obtain a soil color-suction curve (SCSC) that is used to predict the suction. However, this method relies on the SWCC. Additionally, this method is computer-assisted, a software is required for colour recognition technique.

In general, the approach used to predict suction utilizing geotechnical properties is commonly founded on the prediction of the SWCC fitting parameters of the model developed by [6]. Nevertheless, suction prediction can be performed utilizing one of the three following methods:

**The first method** is based on the empirical correlation between basic soil properties and SWCC fitting parameters, this method commonly applied to many soils can easily be included in numerical modeling.

**The second method** is related to the conceptual physical model with the relationship between grain size and SWCC, this method requires detailed data concerning particle size distribution and density, other soil data that may influence soil suction like the plasticity index is excluded in the models.

**The third method** is established by a direct relationship between the soil suction, volumetric water content, and basic soil properties. This method is selected in this study because it is convenient from a practical point since the prediction of the soil suction is done utilizing common geotechnical properties, and the semi-empirical model can be developed using multivariate regression analysis.

2. **Methods**

Soil samples were collected across the Free state province. Laboratory tests were performed to determine the properties of the soils according to laid down protocols and standards. Correlation between soil suction and other soil properties was conducted to assess the impact of soil properties on soil suction. Multivariate regression analysis was performed to select the relevant independent variables, the best fitting function, and obtained the predictive equations. The validation of these models was conducted by comparing the predicted values to the measured values, and by investigating the residuals errors.

2.1. **Sample location**

Soil samples were collected across Free State province, South Africa. Bloemfontein soils (BLS-A; BLS-B; BLS-C); Winburg soils (WBS-A; WBS-B; WBS-C); Welkom soils (WKS-A; WKS-B; WKS-C). The GPS coordinates are (BLS-A: 29°11′49.53″S; 26°12′ 52.55″E); (BLS-B: 29°08′04.40″S; 26°15′58.10″E); (BLS-C: 29°06′48.20″S; 26°10′56.70″E); (WBS-A: 28°30′43.5″ S; 27°00′12.8″E); (WBS-B: 28°30′ 59.8″S; 27°00′58.0″E); (WBS-C: 28°31′08.00″S; 27°00′22.00″E); (WKS-A: 27°57′51.8″S; 26°45′36.9″E); (WKS-B: 28°00′12.10″S; 26°43′52.30″E); (WKS-C: 27°58′15.10″S; 26°43′05.00″E).

2.2. **Laboratory test**

To assess physical and hydro-mechanical properties of the soils, laid down protocols and standards found in the literature were utilized: sieve analysis [11]; hydrometer analysis [12]; Atterberg limits [13]; free swell index [14]; Proctor compaction test [15]; Soil suction measurement using filter paper technique [16]; Specific gravity [17].

2.3. **Suction measurement**

The calibration curve in Equation (18) was established by a calibration process of the filter paper using a salt solution. The moisture content within the filter paper, designated by ($W_f$), was estimated using Equation (19). After, the calculated moisture content was inserted in Equation (18) to determine the soil suction. Compacted soil specimens were divided into two cylindrical parts with a width of 75mm and a depth of 35mm so that the specimen can be placed and removed from the glass jar easily for suction assessment. The suction assessment was performed using the Whatman No 42 type filter paper (Ashless circles 70mm diameter, Cat No 1442-070). Three filter papers (two protectives, and one for suction assessment with 70mm radius) were placed between these two surfaces using
tweezers for matric suction assessment. The two cylindrical specimens parts were joined using electrical tape and place into a glass jar. A plastic ring was placed on top of the soil specimen and the filter papers placed on top of the ring to measure the total suction. Glass jars were labeled, sealed, and placed into a temperature regulatory apparatus at 25±1°C for an equilibrium period of four weeks. The moisture tins were oven-dried at 105°C for overnight. Water content within the filter paper was measured using a 0.0001g readable balance. The water content in the filter paper, represented by \( W_f \); the mass of water in the filter paper, designated by \( M_w \); and the mass of the filter paper, denoted \( M_f \). The soil suction, denoted by \( \psi \) in kPa is given as:

\[
\log(\psi) = -0.0791 \times W_f + 5.313
\]

\[
W_f = \frac{M_w}{M_f} \times 100
\]

2.4. Free swell index

Two representatives oven-dried soil specimens of 10grams were sieved through 425-micron sieve. Each soil sample was poured in a graduated cylinder glass of 100ml capacity. One cylinder was filled up with kerosene, and another cylinder filled with distilled water up to 100ml mark. The volume of the specimen read from a cylinder containing distilled water, denoted by \( V_d \), the volume of the specimen read from a cylinder containing kerosene, denoted by \( V_k \). The free swell index is express as:

\[
FSI = \frac{V_d - V_k}{V_k} \times 100
\]

| Degree of expansion | Free swell index, % | Expansivity                           |
|---------------------|--------------------|---------------------------------------|
| Low                 | < 50               | Mixture of swelling and non-swelling  |
| Moderate            | 50-100             | swelling                              |
| High                | 100-200            | High swelling                         |
| Very high           | > 200              | Very High swelling                    |

2.5. Void ratio

The void ratio denoted by \( e \) was estimated using the soil parameters like the specific gravity, denoted by \( G_s \), the unit weight of water, denoted by \( \rho_w \), and the dry density of the soil, denoted by \( \gamma_d \), and calculate as

\[
e = G_s \times \frac{\rho_w}{\gamma_d} - 1
\]

2.6. Multivariate regression analysis

Regression analysis is performed using MINITAB 18 program. The soil suction is the dependent variable, and the independent variables are classified as follows: geotechnical index properties, expansive soil indexes. [18] stated that the prediction multi-linear model takes the form of Equation (22), the intercept, denoted by \( \beta_0 \), the regression coefficient, denoted by \( \beta_i \), the number of relevant soil parameters, denoted by \( n \), and the random error, denote by \( \epsilon \).

\[
Y = \beta_0 + \sum_{n=1}^{\infty} (\beta_i X_i + \epsilon)
\]

For the curve estimation procedure, regression statistics were performed for different regression models, including linear, logarithmic, inverse, quadratic, cubic and exponential models. It was found that linear function exhibited the strongest and most relevant choice.

3. Results and discussion

3.1. Material properties

The material properties of the soil samples are summarized in Table 2, these soils are fine-grained, more than 50% passing the No200 (0.075mm). The liquid limit range within these soil is > 50%. These soils exhibit high plasticity.
3.2. Swelling properties assessment
The free swell index results are presented in Table 3, it can be observed that WKS show a high expansivity, whereas, WBS and BLS displays a moderate expansivity. The clay type is swelling.

### Table 3. Free swell index test results

| Soil designation | FSI, % | Clay type | Soil expansivity |
|------------------|-------|-----------|-----------------|
| BLS-A            | 64.31 | Swelling  | moderate        |
| BLS-B            | 66.66 | Swelling  | moderate        |
| BLS-C            | 70.19 | Swelling  | moderate        |
| WBS-A            | 81.37 | Swelling  | moderate        |
| WBS-B            | 84.66 | Swelling  | moderate        |
| WBS-C            | 90.30 | Swelling  | moderate        |
| WKS-A            | 116.66| High swelling | High            |
| WKS-B            | 124.61| High swelling | High            |
| WKS-C            | 132.56| High swelling | High            |

3.3. Analysis of the correlation between the matric suction and geotechnical properties

3.3.1. Analysis of the correlation between the soil suction and initial water content
The correlation between soil suction and water content were investigated for each soil. Figures 1 and 2 shows the variation of total suction and matric suction respectively for water content for BLS-A, BLS-B, BLS-C. It can be observed that BLS-C displays a maximum total suction and matric suction values estimated at 6444.53kPa, and 5193.39kPa respectively. BLS-B exhibit a maximum total suction and matric suction values estimated at 6112.321kPa and 4925.68kPa respectively. BLS-A exhibit a maximum total suction and matric suction values estimated at 5883.92kPa and 4741.61kPa respectively. The correlation between the soil suctions and water content were investigated for each soil. Figures 3 and 4 shows the variation of the total suction and matric suction for the water content of WBS-A, WBS-B, WBS-C. It can be observed that WBS-C displays a maximum total suction and matric suction values estimated at 8239.036kPa and 6628.04kPa respectively. WBS-B exhibit a maximum total suction and matric suction values estimated at 7723.408kPa and 6213.234kPa respectively. WBS-A exhibit a maximum total suction and matric suction values estimated at 7439.15kPa and 5984.56kPa respectively. WKS-C exhibit a maximum total suction and matric suction values estimated at 11283.21kPa and 8745.48kPa respectively. WKS-B exhibit a maximum total suction and matric suction values estimated at 10621.01kPa and 8232.22kPa respectively. WKS-A exhibit a maximum total suction and matric suction values estimated at 11283.21kPa and 8745.48kPa respectively.
values estimated at 9926.183kPa and 7093.66kPa respectively. This can be justified by the percentage of clay within WKS-C, WKS-B, WKS-A estimated at 55.52%, 48.31%, and 40% respectively. These results are in line with the study conducted by [6] on SWCC of various type of soils which revealed that the soil with a higher fraction of clay exhibit a higher soil suction for the same water content, than the soil with the smaller fraction of clay. WKS exhibit the higher soil suction values whereas BLS displays the smaller suction values, and WBS the medium suction values.

3.3.2. Analysis of the correlation between matric suction, water content, and dry density

The correlation between the matric suction, initial water content and initial dry density at the optimum water content (OWC) is presented in Figure 7. It can be observed that the matric suction increases as initial water content increases. This can be explained by the reduction of the void as the initial water content increases within the soil. The trend line equation is given by $\psi = 75492 \times e^{-0.175 \times \omega_i}$ with a determination coefficient of $R^2 = 67.3\%$. Moreover, it can be noticed a reduction of matric suction as the initial dry density increases. This can be justified by the reduction of the void ratio upon increment of the energy of compaction. The exponential trendline is given as $\psi = 9.107 \times e^{-0.674 \times \rho_d}$ with a determination coefficient of $R^2 = 67.5\%$. Besides, Colour patterns are used to represent various range of matric suction values, light blue and blue represent the smaller values estimated at 672kPa, whereas the red represents the higher value of the matric suction evaluated at 2022kPa. Lastly, the surface plot displays a moderate positive correlation with some disparities. Hence, the initial water content and the initial dry density influences the matric suction. The local extremes observed in the surface plot in Figure 7 are caused by some discrepancies between the correlations, with a determination coefficients of $R^2 < 80\%$. 

![Figure 1. Total suction vs water content (BLS)](image1.png)

![Figure 2. Matric suction vs water content (BLS)](image2.png)

![Figure 3. Total suction vs water content (WBS)](image3.png)

![Figure 4. Matric suction vs water content (WBS)](image4.png)

![Figure 5. Total suction vs water content (WKS)](image5.png)

![Figure 6. Matric suction vs water content (WKS)](image6.png)
3.3.3. Analysis of the correlation between matric suction, clay content, and liquid limit

A three-dimensional representation of the correlation between the matric suction, clay content and liquid limit is shown in Figure 8. It can be observed a tendency of increase of matric suction values as clay content and the liquid limit increases. This can be explained by the presence of clay content inside the soil ranging from 30,40% to 55,25%. As the clay content increases, soil absorb much more water, as a result of that the liquid limit increases. The correlation between the matric suction and the clay content is an exponential function expressed as

\[ \psi_m = 198,69 \times e^{-0,0458 \times (\text{clay \%})} \]

with a determination coefficient of \( R^2 = 71,54 \). Furthermore, the correlation between the matric suction and the liquid limit is an exponential function given as \( \psi_m = 24,187 \times e^{0,0586 \times \text{LL}} \) with a determination coefficient of \( R^2 = 59,7\% \). The surface plot displays a positive moderate correlation with some disparities. Therefore, the clay content and the liquid limit impact the matric suction values. The local extremes observed in the surface plot in Figure 8 are caused by some discrepancies between the correlations, with a determination coefficients of \( R^2 < 80 \% \).

3.3.4. Analysis of the correlation between matric suction, specific gravity, and free swell index

To evaluate the interrelation between the matric suction, specific gravity, and free swell index, experimental values were plotted in Figure 9. It was observed that the matric suction increases as the specific gravity increases. An increment of specific gravity from 2,64 to 2,83 induces an increment of the matric suction values from 671,89kPa to 2223,23kPa. The trend line is an exponential function expressed as \( \psi_m = 6 \times 10^{-6} \times e^{0,986 \times \text{SG}} \) with a determination coefficient of \( R^2 = 68,6\% \). Moreover, the matric suction increases upon an increment of free swell index and exhibits an exponential relationship given by \( \psi_m = 311,69 \times e^{0,0152 \times \text{FSI}} \) with a determination coefficient of \( R^2 = 66\% \). An increase of the free swell index from 64,31% to 132,56% induces an increment of matric suction from 671,89kPa to 2021,814kPa. The surface plot displays a positive moderate correlation with some discrepancies. Therefore, the specific gravity and the free swell index influence the matric suction values. The local extremes noticed in the surface plot in Figure 9 are caused by some discrepancies between the correlations, with a determination coefficients of \( R^2 < 80 \% \).
3.3.5 Analysis of the correlation between matric suction, activity of clay, and void ratio

The correlation between the matric suction, the activity of clay, and the void ratio is presented in Figure 10. It can be noticed that the matric suction decreases slightly as the activity of the clay increases. This can be explained by the increment of the matric suction as the clay fraction increases. As a result of that, matric suction will decrease upon an increment of the activity of clay. The trendline equation is given by $\psi_m = 9461,6 \times e^{-1,704 \times A_C}$ with a determination coefficient of $R^2 = 15,8\%$. The surface plot displays a weak positive correlation with discrepancies.

Besides, it can be noticed that the void ratio increases as the matric suction increases. This can be justified by the increment of the difference between the pore air pressure and pore water pressure. The trendline equation is given by $\psi_m = 73,7 \times e^{3,3822 \times \text{Void ratio}}$ with a determination coefficient of $R^2 = 69\%$. The scatter plotted data exhibits a moderate positive correlation with some discrepancies. The shape of the surface plot shows a reduction of the matric suction as the activity of clay increases and an increment of the matric suction as the void ratio increases. Finally, the local extremes observed in the surface plot in Figure 10 are caused by some discrepancies between the correlations, with a determination coefficients of $R^2 < 80\%$.

3.3.6 Analysis of the correlation between the total suction and geotechnical properties.

Figure 11 shows the correlation between matric suction values and total suction values. It can be observed that the total suction values increases as the matric suction values increases. The trendline equation is given by $\psi_t = 126,82 + 1,267 \times \psi_m$ with a determination coefficient of $R^2 = 99,6\%$. The scatter plotted data exhibits a strong positive correlation between total suction values and matric suction values. Moreover, a change in total suction is fundamentally equivalent to the variation of the matric suction [19]. Nonetheless, the contribution of matric suction to total suction is more significant compared to the osmotic suction. Therefore, the impact of the geotechnical properties on matric suction described in paragraph 3.3 is very similar to the influence of geotechnical properties on total suction.
4. Models development

4.1. Estimated models

To select the relevant parameters, and develop the predictive models, stepwise regression analysis was performed. [20] Pointed out that the stepwise multi regression method is commonly used to decrease the number of independent variables, and enhance the model accuracy. The soil suction is the dependent variable, and the other soil properties are independent variables. The correlation matrix A, matrix B, and matrix C are shown in appendix A, appendix B, and appendix C respectively. They are utilized to perform the multi-regression analysis using MINITAB 18 program. The mathematical predictive model for the matric suction given in Equation (23) is developed using the initial water content ($q_r$), initial dry density ($s_r$), clay content (%), liquid limit ($tt$), free swell index ($uvw$), specific gravity ($G_s$), void ratio ($e$), and the activity of clay ($A_c$) with respective coefficients of correlation $x_E, x_+,$ $x_C, x', x, x_L,$ $x_G,$ and the intercept $x_0$.

\begin{align}
[\psi_m] = -x_0 - x_1 \times W_1 - x_2 \times y_1 - x_3 \times \text{clay}(\%) - x_4 \times \log(LL) + x_5 \times \log(FSI) + x_6 \times \log(G_s) \\
+ x_7 \times 10^e + x_8 \times A_c
\end{align} \tag{23}

\begin{align}
[\psi_t] = -x_0 - x_1 \times W_1 + x_2 \times y_1 - x_3 \times \text{clay}(\%) - x_4 \times \log(LL) + x_5 \times \log(FSI) + x_6 \times \log(G_s) \\
+ x_7 \times 10^e + x_8 \times A_c
\end{align} \tag{24}

Where:

$\psi_t$= total suction, kPa,
$\psi_m$= matric suction, kPa,
$W_i =$ initial water content, %,
$\gamma_i =$ initial dry density, kN/m$^3$,
Clay(%) = clay content, %,
LL = liquid limit, %,
FSI = free swell index, %,
Gs = specific gravity
$e =$ void ratio
$A_C =$ activity of clay
$\lambda =$ Box-Cox transformation coefficient,
$R^2 =$ determination coefficient, %,
$\xi, \eta =$ intercepts,
$\xi_i, \eta_i =$ multi-regression coefficient, $i = 1, \ldots, 8$.

4.2. Models validation

Two mathematical predictive models to estimate the matric suction and the total suction were developed in this research work. The predicted suction profiles give a reasonable result, the proposed models provide much better fitness with a determination coefficients of $R^2 = 95.92\%$ and $R^2 = 95.78\%$ respectively for matric suction and total suction. Figure 12 shows that the matric suction predicted values are very accurate within the range of $0 < \psi_m \leq 7200$ kPa. However, for $\psi_m > 7200$ kPa, the model exhibits very small discrepancies. Furthermore, Figure 13 shows that the total suction predicted values are very accurate within the range of $0 < \psi_t \leq 8400$ kPa. Besides, for $\psi_t > 8400$ kPa, the model displays very small discrepancies.

![Figure 12. Predicted matric suction vs experimental matric suction](image)

![Figure 13. Predicted total suction vs experimental total suction](image)

The investigation of residuals (errors) is significant in deciding on the suitability of the model. When the errors show any type of arrangement, the model isn't taking into consideration all the systematic data. For the best execution of the model, residuals have to be irregular, for example, they have to follow a typical distribution with zero mean, and a constant difference [21, 22]. [20] and [23] utilized ($R^2$) between experimental and predicted values to assess the model efficiency. All these methods were also utilized by [24] to verify the model efficiency. The histograms of matric suction and total suction residuals are shown respectively in Figures 14 and 15. The residuals are distributed typically with zero mean. It can be observed that the determination coefficients for Equations (23) and (24) are respectively $R^2 = 97.49\%$ and $R^2 = 97.52\%$, which are relevant in a statistical sense at 1% level of importance. Therefore, the developed models can be utilized to predict the matric suction and the total suction with accuracy.
5. Concluding remarks
The study revealed that the soil suction in a compacted heaving soil is influenced not only by the moisture content, but also by geotechnical properties indexes, and swelling parameters. As a result, the soil suction reduces as the initial dry density, and the activity of clay increases. Nonetheless, the soil suction values increase upon an increment of clay content, liquid limit, free swell index, specific gravity, and void ratio. Two semi-empirical models to predict the matric suction and the total suction of compacted heaving soils were developed. These models can be utilized to estimate the soil suction.

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7. Appendices

| Soil designation | PI (%) | LL(%) | FSI (%) | Clay (%) | Gs | Void ratio | A_c |
|------------------|--------|-------|---------|----------|----|------------|-----|
| BLS-A            | 36,82  | 58,98 | 64,31   | 30,40    | 2,64| 0,501      | 1,64|
| BLS-B            | 38,25  | 61,27 | 66,66   | 32,20    | 2,68| 0,561      | 1,7 |
| BLS-C            | 40,33  | 64,6  | 70,19   | 35,07    | 2,71| 0,598      | 1,79|
| WBS-A            | 42,48  | 63,78 | 81,37   | 34,03    | 2,73| 0,620      | 1,73|
| WBS-B            | 44,10  | 66,22 | 84,66   | 36,50    | 2,76| 0,657      | 1,8 |
| WBS-C            | 47,04  | 70,64 | 90,30   | 39,73    | 2,78| 0,689      | 1,92|
| WKS-A            | 49,87  | 69,45 | 116,66  | 40       | 2,73| 0,675      | 2,2 |
| WKS-B            | 53,36  | 74,31 | 124,61  | 48,31    | 2,78| 0,732      | 2,35|
| WKS-C            | 56,68  | 78,94 | 132,56  | 55,25    | 2,83| 0,768      | 2,5 |
## Appendix B Correlation matrix B

| Soil designation | Samples  | Initial water content | Total suction | Matric suction | Osmotic suction | Initial dry density |
|------------------|----------|-----------------------|---------------|----------------|----------------|--------------------|
|                  | WKS-A    | WKS-A1 | 15.93 | 9926,183 | 7093,666 | 2832,517 | 14.94 |
|                  | WKS-A    | WKS-A2 | 19.25 | 6922,321 | 5227,777 | 1694,544 | 15.48 |
|                  | WKS-A    | WKS-A3 | 23.37 | 4011,482 | 2986,456 | 1025,026 | 16.09 |
|                  | WKS-A    | WKS-A4 | 26.14 | 2475,62 | 1778,651 | 696,969 | 16.29 |
|                  | WKS-A    | WKS-A5 | 29.1 | 1397,745 | 890,47 | 507,275 | 15.85 |
|                  | WKS-B    | WKS-B1 | 17.2 | 10621,01 | 8232,22 | 2388,79 | 14.77 |
|                  | WKS-B    | WKS-B2 | 19.98 | 7406,88 | 5593,72 | 1813,16 | 15.2 |
|                  | WKS-B    | WKS-B3 | 23.05 | 4292,28 | 3195,51 | 1096,77 | 15.7 |
|                  | WKS-B    | WKS-B4 | 26.52 | 2648,91 | 1903,156 | 745,754 | 16.05 |
|                  | WKS-B    | WKS-B5 | 30.43 | 1495,59 | 952,8 | 542,79 | 15.41 |
|                  | WKS-C    | WKS-C1 | 18 | 11283,21 | 8745,48 | 2537,73 | 14.61 |
|                  | WKS-C    | WKS-C2 | 21.47 | 7868,687 | 5942,48 | 1926,207 | 15 |
|                  | WKS-C    | WKS-C3 | 24.56 | 4559,901 | 3394,741 | 1165,16 | 15.49 |
|                  | WKS-C    | WKS-C4 | 27.75 | 2814,07 | 2021,814 | 792,256 | 15.65 |
|                  | WKS-C    | WKS-C5 | 30 | 1588,83 | 1012,2 | 576,63 | 15.21 |
### Appendix C. Correlation matrix C

| Soil designation | Samples | Initial Water content % | Total suction kPa | Matric suction kPa | Osmotic suction kPa | Initial dry density kN/m³ |
|------------------|---------|-------------------------|-------------------|-------------------|--------------------|--------------------------|
| BLS-A            | BLS-A1  | 12.06                   | 5883.92           | 4741.62           | 1142.3             | 15.6                     |
|                  | BLS-A2  | 14.02                   | 4064.22           | 3327.18           | 737.04             | 16.2                     |
|                  | BLS-A3  | 17.03                   | 1923.09           | 1388.22           | 534.87             | 17.01                    |
|                  | BLS-A4  | 20.07                   | 1036.11           | 671.89            | 364.22             | 17.58                    |
|                  | BLS-A5  | 24.8                    | 340.034           | 200               | 140.034            | 17.2                     |
|                  | BLS-B1  | 12.25                   | 6112.321          | 4925.68           | 1186.641           | 15.15                    |
|                  | BLS-B2  | 15.32                   | 4221.982          | 3456.34           | 765.642            | 16.92                    |
| BLS-B            | BLS-B3  | 20.12                   | 1997.745          | 1442.11           | 555.635            | 16.98                    |
|                  | BLS-B4  | 22.61                   | 1076.324          | 697.98            | 378.344            | 17.16                    |
|                  | BLS-B5  | 25.5                    | 353.234           | 207.79            | 145.444            | 16.92                    |
|                  | BLS-C1  | 13.11                   | 6444.53           | 5193.39           | 1251.14            | 14.97                    |
|                  | BLS-C2  | 17.12                   | 4451.45           | 3644.19           | 807.26             | 15.98                    |
| BLS-C            | BLS-C3  | 19.98                   | 2106.33           | 1520.49           | 585.84             | 16.59                    |
|                  | BLS-C4  | 23                      | 1134.82           | 735.9             | 398.92             | 16.95                    |
|                  | BLS-C5  | 25.97                   | 372.43            | 219.08            | 153.35             | 16.6                     |
|                  | WBS-A1  | 14.98                   | 7439.15           | 5984.56           | 1454.59            | 15.6                     |
|                  | WBS-A2  | 17.5                    | 5410.66           | 4332.678          | 1077.982           | 16.05                    |
| WBS-A            | WBS-A3  | 21                      | 3580.89           | 2748.3            | 832.59             | 16.58                    |
|                  | WBS-A4  | 24.03                   | 1699.05           | 1199.35           | 499.7              | 16.85                    |
|                  | WBS-A5  | 28.06                   | 816.77            | 450.227           | 366.543            | 16.7                     |
|                  | WBS-B1  | 14.25                   | 7723.408          | 6213.234          | 1510.174           | 15.25                    |
|                  | WBS-B2  | 18.32                   | 5617.411          | 4498.234          | 1119.177           | 15.98                    |
| WBS-B            | WBS-B3  | 24.58                   | 3717.727          | 2853.32           | 864.407            | 16.71                    |
|                  | WBS-B4  | 26.21                   | 1763.982          | 1245.199          | 518.783            | 16.65                    |
|                  | WBS-B5  | 29.1                    | 847.98            | 467.431           | 380.549            | 16.32                    |
|                  | WBS-C1  | 15                      | 8239.036          | 6628.04           | 1610.996           | 15.19                    |
|                  | WBS-C2  | 20                      | 5992.43           | 4798.544          | 1193.886           | 15.87                    |
| WBS-C            | WBS-C3  | 23                      | 3965.92           | 3043.812          | 922.108            | 16.2                     |
|                  | WBS-C4  | 26.05                   | 1881.75           | 1328.33           | 553.42             | 16.45                    |
|                  | WBS-C5  | 29.97                   | 904.59            | 498.64            | 405.95             | 16.02                    |

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