We consider models where moduli fields are not stabilized and play the role of quintessence. In order to evade gravitational tests, we investigate the possibility that moduli behave as chameleon fields. We find that, for realistic moduli superpotentials, the chameleon effect is not strong enough, implying that moduli quintessence models are gravitationally ruled out. More generally, we state a no-go theorem for quintessence in supergravity whereby models either behave like a pure cosmological constant or violate gravitational tests.

PACS numbers: 98.80.Cq, 98.70.Vc

I. INTRODUCTION

Dark energy and its properties is one of the most intriguing puzzles of present day theoretical physics. Indeed, there is convincing evidence, coming from SNIa supernovae [1], large scale structures of the universe [2, 3, 4] and the CMB anisotropies [5, 6] which leads to the existence of an acceleration of the universe expansion in the recent past. When interpreted within the realm of General Relativity, these results imply the existence of a pervading weakly interacting fluid with a negative equation of state and a dominant energy density. The simplest possibility is of course a pure cosmological constant. This has the advantage of both fitting the data and incorporating a mild modification of the Einstein equations. Now it happens that the value of the cosmological constant is so small compared to high energy physics scales that no proper explanation for such a fine–tuning has been found except maybe the anthropic principle [7] used in the context of a stringy landscape [8, 9]. This is all the more puzzling in view of the very diverse sources of radiative corrections in the standard model of particle physics and beyond.

A plausible alternative involves the presence of a scalar field akin to the inflaton of early universe cosmology and responsible for the tiny vacuum energy scale [10, 11, 12, 13, 14, 15, 16, 17]. These models of quintessence have nice features such as the presence of long time attractors (tracking fields) leading to a relative insensitivity to initial conditions [10]. In most cases, the quintessence runaway potentials lead to large values of the quintessence field now, of the order of the Planck mass. This immediately prompts the necessity of embedding such models in high energy physics where nearly Planck scale physics is taken into account. The most natural possibility is supergravity as it involves both supersymmetry and gravitational effects [18]. Moreover, superstring theories lead to supergravity models at low energy.

From the model building point of view, the quintessence field does not belong to the well-known sector of particles of the standard model. Therefore, one has to envisage a dark sector where this field lives and provide the corresponding Kähler, $K_{\text{quint}}$, and super potentials $W_{\text{quint}}$ in order to compute the quintessence scalar potential explicitly. Once a quintessence model has been built, one must also worry about the coupling to both matter and hidden sector supersymmetry breaking [19]. Indeed the rolling of the quintessence field can induce variations of constants such as the fine structure constants. Moreover the smallness of the mass of the quintessence field implies that its gravitational coupling to matter must be suppressed in order to comply with fifth force and equivalence principle violation experiments [20, 21].

The observable sector is fairly well-known and the hidden sector can be parameterized. Therefore, the main uncertainty comes from the dark sector, i.e. from the specific form chosen for $K_{\text{quint}}$ and $W_{\text{quint}}$. Recently, we have investigated this question for a class of models where the Kähler potential and the superpotential can be Taylor expanded or are given by polynomial functions of the (super) fields [22]. We have shown that this type of models, under the standard assumption of separate sectors (see also our conclusion), is in trouble as either they are uninteresting.
from the cosmological point of view (typically, in practice, they are equivalent to a cosmological constant) or they violate the bounds from gravity experiments (typically, they violate the bound on the fifth force and/or on the weak equivalence principle).

The aim of this paper is to study a general class of models, probably the most natural one from a string theory point of view \[23\], where the quintessence field is a moduli field (Kähler moduli). Technically, this means that $K_{\text{quint}}$ is taken to be a logarithm of the quintessence field \[23\]. Although the Kähler function is known, there is no specific standard choice for the superpotential which remains a free function. Therefore, we will derive model independent results and then discuss the various cases that have been envisaged in the literature for $W_{\text{quint}}$ (for instance, polynomial superpotentials and exponential ones à la KKLT \[24\]). We show that, for reasonable choices of $W_{\text{quint}}$, the corresponding models are also in trouble from the gravity experiments point of view. This last result is in fact more subtle than in the case of the first class of models treated in Ref. \[22\]. Indeed, contrary to the polynomial models, a chameleon mechanism \[25\] can be present in the no scale case and could be used to protect the quintessence field from gravity problems. However, unfortunately, we show that this mechanism is in fact not sufficiently efficient to save no scale quintessence in simple cases such as gaugino condensation and polynomial superpotentials.

The paper is arranged as follows. In Sec. III we establish some general results relevant to the no-scale models. In particular, in sub-Sec. III A we calculate the quintessence potential for a general moduli superpotential and in sub-Sec. III B we give the corresponding soft terms in the observable sector. In sub-Sec. III C we study how the electroweak transition is affected by the no scale dark sector. Then, in Sec. III we briefly review the chameleon mechanism. In particular, in sub-Sec. III A we describe the thin shell phenomenon with, in sub-Sec. III B applications to the gaugino condensation case and in sub-Sec. III C to the polynomial case. In Sec. IV we present our conclusions and state a no-go theorem for the compatibility between quintessence in supergravity and gravity experiments.

II. NO SCALE QUINTESSENCE

A. The Scalar Potential

In this section we collect results related to the dynamics of Kähler moduli coming from string compactifications. In practice we only consider that there is a single moduli $Q$ which can be seen as the breathing mode of the compactification manifold. The reduction from 10 dimensions to 4 dimensions leads to a no-scale structure for the Kähler potential of the moduli. The Kähler potential is given by the following expression

$$K_{\text{quint}} = -\frac{3}{\kappa} \ln \left[ \kappa^{1/2} \left( Q + Q^\dagger \right) \right],$$

where $\kappa \equiv 8\pi/m_{pl}^2$. The moduli $Q$ has no potential and is a flat direction to all order in perturbation theory. In string theory, the validity of the supergravity approximation is guaranteed provided $\kappa^{1/2}Q \gg 1$, implying that the compactification manifold is larger than the string scale. A potential can be generated once non-perturbative effects are taken into account, this may lead to a superpotential

$$W_{\text{quint}} = W_{\text{quint}}(Q) \equiv M^3 \mathcal{W} \left( \kappa^{1/2}Q \right).$$

which will be discussed later. The advantage of the above writing is that it emphasizes the scale $M$ of the superpotential. The quantity $\mathcal{W}$ is dimensionless and of order one. Then, inserting the Kähler and the superpotentiales into the expression of the scalar potential, one gets

$$V_{\text{quint}}(Q) = \frac{\kappa^{1/2}}{\left[ \kappa^{1/2} (Q + Q^\dagger) \right]^2} \left( W \frac{\partial W}{\partial Q} + W^\dagger \frac{\partial W}{\partial Q} \right) + \frac{1}{3\kappa^{1/2} (Q + Q^\dagger)} \left| \frac{\partial W}{\partial Q} \right|^2.$$

The noscale property implies that the term in $-3|W|^2$ in the supergravity potential cancels. The kinetic terms of the moduli read $3|\partial Q|^2/(Q + Q^\dagger)^2$ implying that $Q$ is not a normalized field. The normalized field $q$ is given by

$$\kappa^{1/2}Q = \exp \left( -\sqrt{\frac{2}{3}} q \right).$$

where $q$ is a dimensionless scalar field.

As soon as a quintessence field has a runaway potential and leads to the present day acceleration of the universe expansion, its mass is tiny and may lead to gravitational problems. In order to minimize this problem, we assume
that the quintessence sector is only coupled gravitationally to the observable and hidden sectors \cite{19}. In some sense, this assumption is that of non triviality of the model. The corresponding situation can be described by the following Kähler and super potentials \cite{19}.

\[
K = K_{\text{quint}} + K_{\text{hid}} + K_{\text{obs}}, \quad W = W_{\text{quint}} + W_{\text{hid}} + W_{\text{obs}}.
\]

Now the observable sector is known since it comprises the fields of the Minimal Standard Supersymmetric Model (MSSM) \(\phi^a\) and the corresponding superpotential can be expressed as \cite{18}

\[
W_{\text{obs}} = \frac{1}{2}\mu_{ab}\phi^a\phi^b + \frac{1}{3}\lambda_{abc}\phi^a\phi^b\phi^c,
\]

where \(\mu_{ab}\) is a supersymmetric mass matrix and \(\lambda_{abc}\) the Yukawa couplings.

The fact that susy is broken in an hidden sector modifies the shape of the quintessence potential. Another way to put it is that the susy breaking causes the appearance of soft terms in the dark sector and these soft terms are responsible for the modification of the quintessence potential. The new shape has been computed in Ref. \cite{19}. If we parametrise the hidden sector supersymmetry breaking in a model independent way, we have

\[
\kappa^{1/2}\langle z_i \rangle_{\text{min}} \sim a_i(Q), \quad \kappa\langle W_{\text{hid}} \rangle_{\text{min}} \sim M_s(Q), \quad \kappa^{1/2}\left\langle \frac{\partial W_{\text{hid}}}{\partial z_i} \right\rangle_{\text{min}} \sim c_i(Q)M_s(Q),
\]

where \(a_i\) and \(c_i\) are coefficients whose values depend on the detailed structure of the hidden sector. Notice that the coupling of the hidden sector to the quintessence sector implies that the vev’s of the hidden sector fields responsible for supersymmetry breaking can depend on the quintessence field. Taking into account the no scale shape of the Kähler potential, one finds

\[
V_{\text{DR}} = e^{\sum_i |a_i|^2} \kappa M^6 \left\{ \frac{1}{[\kappa^{1/2}(Q + Q^1)]^2} \left[ W_q \frac{\partial W_q}{\partial \langle \kappa^{1/2}Q \rangle} + W_q^\dagger \frac{\partial W_q^\dagger}{\partial \langle \kappa^{1/2}Q \rangle} \right] + \frac{1}{3\kappa^{1/2}(Q + Q^1)} \left[ \frac{\partial W_q}{\partial \langle \kappa^{1/2}Q \rangle} \right]^2 \right\} - M_s^3 M^3 \left[ \frac{\partial W_q^\dagger}{\partial \langle \kappa^{1/2}Q^1 \rangle} + \frac{\partial W_q}{\partial \langle \kappa^{1/2}Q^1 \rangle} \right] \sum_i |F_{z_i}|^2.
\]

where \(F_{z_i} = \langle e^{\kappa/2}(\partial\phi_i W + \kappa\partial\phi_i K) \rangle\). The dynamics of the quintessence field is determined by both the quintessence and hidden sectors. We also notice that, as expected, the correction coming from the hidden sector is proportional to the susy breaking mass \(M_s\).

### B. The Soft Terms

Let us now turn to the calculation of the soft terms in the observable sector. One usually obtains three types of terms. One is cubic in the fields while the others are quadratic. In the present situation, this property is clearly preserved. The new ingredient is that the soft terms become quintessence dependent quantities. Following Ref. \cite{19} and defining

\[
V_{\text{SUUGRA}} = \cdots + e^{\kappa K} V_{\text{susy}} + e^{\kappa K} A(Q)\lambda_{abc} \left( \phi_a \phi_b \phi_c + \phi_a^\dagger \phi_b^\dagger \phi_c^\dagger \right) + e^{\kappa K} B(Q)\mu_{ab} \left( \phi_a \phi_b + \phi_a^\dagger \phi_b^\dagger \right) + m_{ab}^2 \phi_a \phi_b^\dagger.
\]

where the soft terms are the terms which are not in \(V_{\text{susy}}\), one obtains for the \(Q\)-dependent coefficients \(A, B\) and \(m_{ab}\) in the noscale case

\[
A(Q) = M_s \left( 1 + \frac{1}{3} \sum_i |a_i|^2 + \frac{1}{3} \sum_i a_i c_i \right) + \kappa M^3 \left[ W_q^\dagger \left( 1 + \frac{1}{3} \sum_i |a_i|^2 \right) - \frac{1}{3}\kappa^{1/2}(Q + Q^1) \frac{\partial W_q}{\partial \langle \kappa^{1/2}Q \rangle} \right],
\]

\[
B(Q) = M_s \left( 1 + \frac{1}{2} \sum_i |a_i|^2 + \frac{1}{2} \sum_i a_i c_i \right) + \kappa M^3 \left[ W_q^\dagger \left( 1 + \frac{1}{2} \sum_i |a_i|^2 \right) - \frac{1}{2}\kappa^{1/2}(Q + Q^1) \frac{\partial W_q}{\partial \langle \kappa^{1/2}Q \rangle} \right],
\]

\[
m_{ab}^2(Q) = \frac{e^{\sum_i |a_i|^2}}{[\kappa^{1/2}(Q + Q^1)]^3} \left[ M_s^2 + \kappa M_s M^3 (W + W^\dagger) + \kappa^2 M^6 W W^\dagger \right] \delta_{ab}.
\]

At this point, no assumption has been made except, of course, the choice of the Kähler potential. However, it is clear that, in a realistic model, we always have \(M_s \gg \kappa M^3\) since the susy breaking scale is much larger than the
cosmological constant scale, typically $M_\phi \sim 1\text{ TeV}$ while $\kappa M^6 \sim (10^{-3}\text{eV})^4$. Now, the terms coming from $F_{z_i}$ in the scalar potential are of order $M^2/\kappa$ which is intolerably large compared to the cosmological scales. This is nothing but another manifestation of the cosmological constant problem which, again, is not solved in the framework of quintessence. This contribution must be taken to vanish and therefore $a_i = c_i = 0$. Interestingly enough, it turns out to be exactly the case when $W_{\text{hid}}$ is a constant \cite{22}. Therefore $M_s$ is constant, $A$ and $B$ are constant of the order of $M_c$, and
$$2B = -M_s + 3A,$$
while the mass $m_{ab}$ acquires a very simple $Q$-dependence given by
$$m_{ab} = \frac{M_s}{\kappa^{3/2} (Q + Q^1)} \delta_{ab}.$$  
(14)

It is interesting to compare the above results to those obtained in Ref. \cite{22} in the case of polynomial Kähler and superpotentials. The coefficients $A$ and $B$ were not constant but given by $A = M_s (1 + \kappa Q^2/3)$ and $B = M_s (1 + \kappa Q^2/2)$. We notice that, despite a different dependence in the quintessence field, $A$ and $B$ also satisfy Eq. \cite{13}. On the other hand, the dependence of the soft term $m_{ab}$ is the same as in Ref. \cite{22}, namely $m_{ab} \propto M_s \exp(\kappa K/2)$. In the SUGRA case this came from the fact that $(W_{\text{quint}}) = 0$ while in the no scale situation this originates from neglecting subdominant terms thanks to the relation $M_s \gg \kappa M^3$. However, since the Kähler potentials are different, the above relation leads to different $Q$-dependence for $m_{ab}$.

C. The Electro-Weak Transition in Presence of No-Scale Quintessence

We now consider the application of the previous results to the electroweak symmetry breaking since this is the way fermions in the standard model are given a mass. As is well-known, the potential in the Higgs sector which belongs to the observable sector is modified by the soft terms. Since these soft terms now depend on the quintessence field, the Higgs potential also becomes a $Q$-dependent quantity. In the MSSM, there are two SU(2)$_L$ Higgs doublets

$$H_u = \begin{pmatrix} H^+ \\ H^0_u \end{pmatrix}, \quad H_d = \begin{pmatrix} H^0_d \\ H^- \end{pmatrix},$$

(15)

that have opposite hypercharges, i.e. $Y_u = 1$ and $Y_d = -1$. The only term which is relevant in the superpotential is $W_{\text{obs}} = \mu H_u \cdot H_d + \cdots$. This term gives contribution to the globally susy term $V_{\text{susy}}$ via the F- and D-terms. Then, we have the contribution coming from the soft susy-breaking terms. There is a B-soft susy-breaking term coming from Eq. \cite{11} and a contribution from the soft masses, see Eq. \cite{12}. In order to evaluate the latter, one writes $m_{11} = m_{H_u} e^{\kappa K_{\text{quint}}}$, and $m_{22} = m_{H_d} e^{\kappa K_{\text{quint}}}$, where $m_{H_u} = m_{H_d} = m_{Q/2}$ at the GUT scale. This degeneracy is lifted by the renormalisation group evolution as necessary to obtain the radiative breaking of the electroweak symmetry \cite{26}. The total Higgs potential, taking $H^0_u$ and $H^0_d$ real since they have opposite hypercharges, reads

$$V^{\text{Higgs}} = e^{\kappa K_{\text{quint}}} \left[ \left| |\mu|^2 + m_{H_u}^2 \right| |H^0_u|^2 + \left| |\mu|^2 + m_{H_d}^2 \right| |H^0_d|^2 \right] - 2\mu B(Q) |H^0_u||H^0_d| + \frac{1}{8} (g^2 + g'^2) \left( |H^0_u|^2 - |H^0_d|^2 \right)^2. $$

(16)

The next step is to perform the minimization of the Higgs potential given by Eq. \cite{10}. In presence of dark energy, the minimum becomes $Q$-dependent and the particles of the standard model acquire a $Q$-dependent mass. Straightforward calculations give

$$e^{\kappa K_{\text{quint}}} \left( |\mu|^2 + m_{H_u}^2 \right) = \mu B(Q) \frac{e^{\kappa K_{\text{quint}}}}{\tan \beta} + \frac{m_{20}^2}{2} \cos(2\beta),$$

(17)

$$e^{\kappa K_{\text{quint}}} \left( |\mu|^2 + m_{H_d}^2 \right) = \mu B(Q) e^{\kappa K_{\text{quint}}} \tan \beta - \frac{m_{20}^2}{2} \cos(2\beta),$$

(18)

where we have defined the Higgs vevs as $\langle H^0_u \rangle = v_u$, $\langle H^0_d \rangle = v_d$, $\tan \beta = v_u/v_d$, or $v_u = v \sin \beta$ and $v_d = v \cos \beta$ and $m_{20}$ as the gauge boson $Z^0$. Adding the two equations for the minimum, we obtain a quadratic equation determining $\tan \beta$. The solution can easily be found and reads

$$\tan \beta(Q) = \frac{2 |\mu|^2 + m_{H_u}^2(Q) + m_{H_d}^2(Q)}{2 \mu B(Q)} \left( 1 \pm \sqrt{1 - 4 \mu^2 B^2(Q) \left[ 2 |\mu|^2 + m_{H_u}^2(Q) + m_{H_d}^2(Q) \right]^{-2}} \right).$$

(19)
A priori, this equation is a transcendental equation determining \( \tan \beta \) as \( \tan \beta \) also appears in the right-hand-side of the above formula, more precisely in the Higgs masses. Indeed, the two loop expression for the renormalized Higgs masses gives \(^{27}\)

\[
m^2_{H_a}(Q) = m^2_{H_d}(Q) - 0.36 \left( 1 + \frac{1}{\tan^2 \beta} \right) \left\{ \left[ m^0_{3/2}(Q) \right]^2 \left( 1 - \frac{1}{2\pi} \right) + 8 \left[ m^0_{1/2}(Q) \right]^2 \right\}
\]
\[
+ \left(0.28 - \frac{0.72}{\tan^2 \beta}\right) \left[ A(Q) + 2m^0_{1/2}(Q)^2 \right],
\]

\[
m^2_{H_a}(Q) = \left[ m^0_{3/2}(Q) \right]^2 \left( 1 - \frac{0.15}{4\pi} \right) + \frac{1}{2} \left[ m^0_{1/2}(Q) \right]^2,
\]

where \( m^0_{1/2} \) is the gaugino mass at GUT scale. However, Eq. \(^{19}\) gives the leading order contribution of an expansion in \( 1/\tan^2 \beta \). As we have seen in the text, the noscale situation is such that \( A(Q) \) and \( B(Q) \) are constant in \( Q \) and, therefore, the Higgs mass given by Eqs. \(^{20}, \) \(^{21}\) and hence \( \tan \beta \) do not depend on \( Q \) in this particular case. Again, this is very different from the polynomial case where \( \tan \beta \) is a \( Q \)-dependent quantity, see Eq. (2.31) of Ref. \(^{22}\) for the exact formula.

From the equations \(^{17}\) and \(^{18}\), one can also deduce how the scale \( v \equiv \sqrt{v_u^2 + v_d^2} \) depends on the quintessence field. This leads to

\[
v(Q) = \frac{2e^{\kappa K_{\text{quint}}/2}}{\sqrt{g_u^2 + g_d^2}} \sqrt{|\mu|^2 + m^2_{H_u}} + O \left( \frac{1}{\tan^2 \beta} \right).
\]

Again, the noscale case is quite particular: the only \( Q \)-dependence is given by the factor \( \exp(\kappa K_{\text{quint}}/2) \) in front of the whole expression.

Then, finally, one has for the vevs of the two Higgs

\[
v_u(Q) = \frac{v(Q) \tan \beta(Q)}{\sqrt{1 + \tan^2 \beta(Q)}} = v(Q) + O \left( \frac{1}{\tan^2 \beta} \right),
\]

\[
v_d(Q) = \frac{v(Q)}{\sqrt{1 + \tan^2 \beta(Q)}} = \frac{v(Q) \tan \beta}{\tan \beta} + O \left( \frac{1}{\tan^2 \beta} \right),
\]

at leading order in \( 1/\tan^2 \beta \) (but if we insert the expression of \( v \), then \( v_u \) and \( v_d \) are only determined at first order in \( 1/\tan^2 \beta \)). This allows us to deduce the two kinds of fermion masses, depending on whether the fermions couple to \( H_u \) or \( H_d \)

\[
m^p_{u,a}(Q) = \lambda^p_{u,a} e^{\kappa K_{\text{quint}}/2} v_u(Q), \quad m^p_{d,a}(Q) = \lambda^p_{d,a} e^{\kappa K_{\text{quint}}/2} v_d(Q),
\]

where \( \lambda^p_{u,a} \) and \( \lambda^p_{d,a} \) are the Yukawa coupling of the particle \( \phi_a \) coupling either to \( H_u \) or \( H_d \). The masses pick up a \( \exp(\kappa K_{\text{quint}}/2) \) dependence from the expression of \( v(Q) \) and another factor \( \exp(\kappa K_{\text{quint}}/2) \) from the definition of the mass itself. As a result we have \( m \propto \exp(\kappa K_{\text{quint}}) \propto Q^{-3} \) in the no scale situation. This \( Q \) dependence is the same for particles of type “u” or “d” as \( \tan \beta \) is a constant. This leads us to the main result of the section: in no scale quintessence the behavior of the standard model particle masses is universal and given by

\[
m(Q) \propto \frac{1}{[\kappa^{1/2} (Q + Q^3)]^3} \propto e^{-\sqrt{\pi q}}.
\]

In the next section, we investigate the consequences of this dependence for gravity experiments.

III. GRAVITATIONAL TESTS AND CHAMELEONS

Let us now discuss the consequences of having \( Q \)-dependent masses. This can lead to strong constraints coming from gravitational experiments. Indeed, if the no-scale dark energy potentials obtained in the previous sections, see Eq. \(^{8}\) for the quantity \( V_{DE} \), are of the runaway type (otherwise, in general, one can show that the corresponding cosmological model is not interesting since it becomes equivalent to the case of the cosmological constant, for a specific example, see Ref. \(^{22}\)), then this implies that the moduli have a mass \( m_Q \sim H_0 \), i.e. of the order of the Hubble rate now.
This implies that the range of the force mediated by the quintessence field is large and, for instance, it induces a fifth force and/or a violation of the weak equivalence principle. In order to satisfy the constraints coming from fifth force experiments such as the recent Cassini spacecraft experiment, one must require that the Eddington (post-Newtonian) parameter $|\gamma - 1| \leq 5 \times 10^{-5}$, see Ref. [20]. If one defines the parameter $\alpha_{u,d}$ by

$$\alpha_{u,d}(Q) \equiv \left| \frac{d \ln m_u^0(q)}{dq} \right| ,$$

(27)

where the derivative is taken with respect to the normalized field $q$, then the difficulties are avoided by imposing that $\alpha_{u,d}^2 \leq 10^{-5}$ since one has $\gamma = 1 + 2\alpha_{u,d}^2$. In our case, Eq. (26) implies

$$\alpha_{u,d} = \sqrt{6} .$$

(28)

This result is valid for a gedanken experiment involving the gravitational effects on elementary particles. For macroscopic bodies, the effects can be more subtle and will be discussed later, see also Ref. [22]. Of course, the above result is in contradiction with the bounds on the existence of a fifth force and on the violation of the weak equivalence principle.

However, the above description is too naive because we have not taken into account the chameleon effect in the presence of matter which, in the framework used here, is necessarily present. Indeed, in the presence of surrounding matter like the atmosphere or the inter-planetary vacuum, the effective potential for the quintessence field is modified by matter and becomes

$$V_{\text{eff}}(Q) = V_{\text{DE}}(Q) + A(Q)\rho_{\text{mat}} ,$$

(29)

where $A(Q)$ is the coupling of the quintessence field to matter, i.e. the mass of matter is proportional to $\propto A(Q)$. This can lead to an effective minimum for the potential even though the Dark Energy potential is runaway. In our case, see Eq. (20), we have

$$V_{\text{eff}}(Q) = V_{\text{DE}}(Q) + \left( \frac{Q_0}{Q} \right)^3 \rho_{\text{mat}} = V_{\text{DE}}(q) + e^{-\sqrt{6}(q_0-q)}\rho_{\text{mat}} ,$$

(30)

where we have normalized the coupling to its present vacuum value when $Q = Q_0$. For runaway potentials, the effective potential possesses a minimum where

$$V''_{\text{DE}}(q_{\text{min}}) = \sqrt{6}e^{-\sqrt{6}(q_0-q_{\text{min}})}\rho_{\text{mat}} ,$$

(31)

and the mass at the minimum is

$$m_q^2 = \kappa \left[ V''_{\text{DE}}(q_{\text{min}}) + \sqrt{6}V'_{\text{DE}}(q_{\text{min}}) \right] ,$$

(32)

which is always of order $H_0$, i.e. an almost massless field. This would lead an observable fifth force if it were not for the possibility of a thin shell effect.

Before turning to this question, it is worth commenting on the chameleon effect in the SUGRA case, see Ref. [22]. Since it is a natural consequence of the couplings between the observable and dark sector, the chameleon effect is also present in this model. However, it is hidden by the susy breaking term $m_{3/2}^2 Q^2$, where $m_{3/2}$ is the gravitino mass which largely dominates the term $A(Q)\rho_{\text{mat}}$. In the no scale case, thanks to the very particular form of the Kähler potential, the above susy breaking term is not present and a priori the chameleon effect can be efficient. In any case, in order to study whether no scale quintessence is ruled out or not because of the gravity experiments, it is mandatory to take into account the chameleon phenomenon correctly.

### A. The Thin Shell Mechanism

A theory, as described before in this article, where the particle mass depends on the quintessence field becomes a scalar tensor theory with the Lagrangian

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa} - \frac{1}{2} g_{\mu\nu} \partial_\mu q \partial_\nu q - V_{\text{DE}}(q) \right] + S_{\text{mat}}[\phi_a, A^2(q)g_{\mu\nu}] .$$

(33)
Then, the geodesic equation can be written as
\[\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\lambda} \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} + \alpha q \frac{\partial q}{\partial x^\mu} = 0,\] (34)
where \(\alpha_q \equiv \partial \ln A / \partial q\). In the above equation, the Christoffel symbols are those associated with the metric \(g_{\mu\nu}\). The last term, which represents the new force originating from the quintessence field, comes from the fact that the geodesic equation is established from the metric appearing in the matter Lagrangian. As is apparent from Eq. (33), this one is given by \(A^2(q) q_{\mu\nu}\) and the presence of the \(A^2(q)\) factor is responsible for the new term in Eq. (34). Analyzing this equation in the weak field regime, one finds that the acceleration felt by a test particle is given by
\[a = a_N - \alpha_q \frac{\partial q}{\partial r},\] (35)
where \(a_N\) is the usual Newtonian acceleration (assuming a spherical body, see below).

Let us now consider a situation where the gravitational experiments are performed on a body embedded in a surrounding medium. The body could be a small ball of metal in the atmosphere or a planet in the inter-planetary vacuum. The effective potential (29) is not the same inside the body and outside because \(\rho\) is different. The effective potential can be approximated by
\[V_{\text{eff}} \simeq \frac{1}{2} m_q^2 (q - q_{\text{min}})^2,\] (36)
where the minimum \(q_{\text{min}}\) is determined by \(\partial V_{\text{eff}} / \partial q = 0\) and the mass is \(\partial^2 V_{\text{eff}} / \partial q^2\) evaluated at \(q = q_{\text{min}}\). As already mentioned the minimum and the mass are different inside and outside the body. We denote by \(q_b\) and \(m_b\) the minimum and the mass in the body and by \(q_\infty\) and \(m_\infty\) the minimum and the mass of the effective potential outside the body. Then, the Klein-Gordon equation reads
\[\frac{d^2q}{dr^2} + 2 \frac{dq}{r dr} = \frac{\partial V_{\text{eff}}}{\partial q},\] (37)
where \(r\) is a radial coordinate. Of course, the field \(q\) should be continuous at \(r = R_b\) where \(R_b\) is the radius of the body. Notice that, in the Klein-Gordon equation, we have used canonical kinetic terms in accordance with the fact that \(q\) is a canonically normalized field. With an effective potential given by Eq. (36), the solution of Eq. (37) reads
\[q = q_{\text{min}} + \frac{A}{r} e^{-mr} + \frac{B}{r} e^{mr},\] (38)
where \(A\) and \(B\) are two arbitrary constant. Requiring that \(q\) remains bounded inside and outside the body and joining the interior and exterior solutions, one can determine the complete profile which can be expressed as
\[q_>(r) = q_b + \frac{R_b (q_\infty - q_b) (1 + m_\infty R_b)}{\sinh (m_b R_b) \left[ m_\infty R_b + m_b R_b \coth (m_b R_b) \right]} \frac{\sinh (m_b r)}{r}, \quad r \leq R_b,\] (39)
\[q_<(r) = q_\infty + R_b (q_b - q_\infty) \frac{m_b R_b \coth (m_b R_b) - 1}{m_\infty R_b + m_b R_b \coth (m_b R_b)} \frac{e^{-m_\infty (r-R_b)}}{r}, \quad r \geq R_b.\] (40)
A typical profile is represented in Fig. 1.

We are now in a position to estimate the acceleration caused by the quintessence field. Assuming, as is always the case in practice, that \(m_b \gg m_\infty, m_b R_b \gg 1\), one has
\[\frac{\partial q_>(r)}{\partial r} \simeq \frac{R_b}{r^2} (q_\infty - q_b),\] (41)
from which we deduce that the acceleration felt by a test particle is given by
\[a = -\frac{G m_b}{r^2} \left[ 1 + \frac{\alpha_q (q_\infty - q_b)}{\Phi_N} \right],\] (42)
where \(\Phi_N = G m_b / R_b\) is the Newtonian potential at the surface of the body. Therefore, the theory is compatible with gravity tests if
\[\frac{\alpha_q (q_\infty - q_b)}{\Phi_N} \ll 1.\] (43)
FIG. 1: Profile of the canonically normalized quintessence field inside and outside a spherical body according to Eqs. (39) and (40). As explained in the text, \( R_b \) is the radius of the body and \( q_b \) is the value of the quintessence field inside the body.

We see that the gravity tests are not sensitive to \( \alpha_q \) but to the combination \( \alpha_q (q_\infty - q_b) / \Phi_N \). Hence, even if \( \alpha_q \) is quite large, if the new factor \( (q_\infty - q_b) / \Phi_N \) is small then the model can be compatible. This is the thin shell effect.

In our case, as \( \alpha_q = \sqrt{6} \), this implies that the moduli fields must be small in order to satisfy the thin shell property. In general, the Newton potential is very small, implying that the moduli field \( q \) must be small too. This strongly depends on the shape of the potential and, therefore, on the superpotential in the moduli sector. In the following we will give two examples which do not lead to a thin shell. These examples have a well-motivated superpotential. In non-generic cases, no general obstruction to the existence of a thin shell exists and, therefore, one may find moduli superpotential leading both to quintessence and a thin-shell.

B. Gaugino Condensation and Quintessence

In order to go further, and to perform a quantitative calculation, one must specify the dark energy potential which requires an explicit form for the superpotential.

In string compactifications, on top of the \( \mathbb{K} \)ähler moduli there are complex structure moduli and the string dilaton. These fields can be stabilized once fluxes have been introduced. This leads to a superpotential for the complex structure moduli and the dilaton. The complex structure moduli and the dilaton lead to a supersymmetric vacuum where they are fixed and the superpotential becomes a constant. We are thus left with the \( \mathbb{K} \)ähler moduli as a flat direction. Once \( D7 \) branes are introduced in the setting, non-perturbative gauge dynamics such as gaugino condensation implies that a superpotential for the \( \mathbb{K} \)ähler moduli is generated. On the whole the dynamics of the \( \mathbb{K} \)ähler moduli are governed by the following superpotential \[28\]

\[
W = M^3 \left[ w_0 + c \exp \left( -\beta \kappa^{1/2} Q \right) \right],
\]

where \( w_0, c \) and \( \beta \) are free and positive dimensionless constants. It is immediate to find that the potential \( V_{\text{quint}} \) reads

\[
V_{\text{quint}}(Q) = \frac{\kappa M^6 c^2 \beta}{2 (\kappa^{1/2} Q)^2} e^{-\beta \kappa^{1/2} Q} \left[ \frac{w_0}{c} + e^{-\beta \kappa^{1/2} Q} \left( \frac{\beta}{3} \kappa^{1/2} Q + 1 \right) \right].
\]

Then, one should take into account the corrections coming from the susy breaking terms. Using Eq. \[30\], one arrives at

\[
V_{\text{DE}}(Q) = \frac{\kappa M^6 c^2 \beta}{2 (\kappa^{1/2} Q)^2} e^{-\beta \kappa^{1/2} Q} \left[ \frac{w_0}{c} + \frac{M_s}{c \kappa M^3} + e^{-\beta \kappa^{1/2} Q} \left( \frac{\beta}{3} \kappa^{1/2} Q + 1 \right) \right].
\]
The effective potential has no minimum so no chameleon mechanism is possible. Indeed, it is easy to demonstrate that $V_{q}(Q)$ is a decreasing function (for $\beta > 0$ which is clearly the case of physical interest) as $\exp\left(-\sqrt{6}q\right)$ is. Hence, this model is ruled out gravitationally.

C. Non-Renormalisable Potential

A class of potential with phenomenological interest can be obtained if the quintessence field $Q$ has a non-renormalisable superpotential. Although this is not what is expected from string theory, we will consider as it leads to very appealing quintessential properties. Therefore, we choose

$$W = -\frac{M^3}{n}(\kappa^{1/2}Q)^n,$$

Using Eq. (3), straightforward calculations lead to the following for $m$

Since $m$ gets large, the second term of the potential dominates and leads to acceleration in the matter era provided $2/3(n-3)^2 < 4$ i.e. $n \leq 3 + \sqrt{6}$. In this case, the future of our Universe would be with $\Omega_Q = 1$ with an equation of state

$$w_Q = -1 + \frac{2(n-3)^2}{9},$$

which is close to $-1$ when $n$ is close to 3. Finally, the effective potential for this model reads

$$V_{\text{eff}}(q) = \frac{1}{2} M_s \rho_{\text{mat}} e^{-\sqrt{\frac{2}{3}}q} - e^{\sqrt{\frac{2}{3}}q} e^{\sqrt{\frac{2}{3}}} \rho_{\text{mat}}.$$
IV. CONCLUSION: A NO-GO THEOREM FOR QUINTESSENCE?

We have presented models of moduli quintessence. Despite the large gravitational coupling of the moduli to matter in these models, a chameleon mechanism is at play and could render the models compatible with gravitational experiments. Unfortunately, in realistic cases such as gaugino condensation or non-renormalisable superpotentials, the chameleon phenomenon is not strong enough to save the models.

One can deduce a no-go theorem (modulo, of course, the assumptions made in this article, in particular that of the separate sectors) showing the incompatibility between quintessence in supergravity and gravity tests. Let us come back to the general structure of the scalar potential. As shown in Ref. [19], see Eq. (2.18), it can always be written as

\[ V_{\text{DE}}(Q) = \kappa M^6v_1(\kappa^{1/2}Q) + M_8 M^3 v_2(\kappa^{1/2}Q) + \frac{M^2}{\kappa} e^{\kappa K} (\kappa K^Q Q^1 - 3) + \sum_i |F_{zi}|^2 , \]  

(55)

where we have chosen to emphasize the various combinations of scales appearing in this expression and where, consequently, \( v_1(\kappa^{1/2}Q) \) and \( v_2(\kappa^{1/2}Q) \) are dimensionless functions, \( a \text{ priori} \) of order one at present time. The last term contains the \( F \)-terms of the hidden sector.

Let us consider first models where the Kähler potential can be expanded around \( \kappa \ll 1 \) for moduli and \( \kappa = 1 \) for the dilaton. Fine-tuning of the cosmological constant requires

\[ |W|^2 \sim \frac{1}{\kappa^n} |Q|^2 , \]  

(56)

where \( \cdots \) represent Planck suppressed operators which, at present time, are not necessarily negligible since we have \( \langle Q \rangle \sim m_{\text{Pl}} \) now. It is immediate to see that at leading order, the quintessence field picks up a soft breaking mass

\[ V_{\text{DE}}(Q) = \kappa M^6 v_1(\kappa^{1/2}Q) + M_8 M^3 v_2(\kappa^{1/2}Q) + m_3^2 |Q|^2 \sim M_8 M^3 v_2(\kappa^{1/2}Q) + m_3^2 |Q|^2 , \]  

(57)

where we have used that \( M_8 \propto m_{3/2} \) and have imposed \( \sum_i |F_{zi}|^2 = 3m_{3/2}^2 \kappa^{-1} \) in order to cancel the intolerably large contribution to the cosmological constant coming from the hidden sector. The last equality originates from the condition \( M_8 \gg \kappa M^3 \). From Eq. (57), we see that the potential acquires a minimum since, in general, the functions \( v_1 \) and \( v_2 \) are of the runaway type, \( i.e. \) decreasing with \( Q \). The value of the minimum is controlled by the scales \( M \), \( M_8 \) and \( m_{3/2} \). Due to the large value of \( m_{3/2} \) compared to the quintessence field, the minimum is generically small in Planck units. The scale \( M \) is tuned to get a minimum value for the potential of order \( \Omega_{\text{DE}} \). At this minimum, the mass of the quintessence field is \( m_{3/2} \) large enough to evade all the gravitational tests. Now, cosmologically, the steepness of the quadratic potential in \( Q \) implies that the field must have settled at the minimum before Big Bang Nucleosynthesis (BBN). If not, the energy density of the quintessence field would exceed the MeV energy scale of BBN. In practice, the potential is constant since BBN, \( i.e. \) equivalent to a cosmological constant. Notice that the coupling of the quintessence field to matter induces a correction to the potential in \( \kappa \rho_{\text{mat}} |Q|^2 \) which is negligible compared to \( m_{3/2}^2 |Q|^2 \), hence no chameleon effect.

One can circumvent this argument by taking singular potentials where the potential term in \( |W|^2 \) is constant. One can choose

\[ K_{\text{quint}} = -\frac{n}{\kappa} \ln \left[ \kappa^{1/2} (Q + Q^1) \right] . \]  

(58)

In this case, \( n=3 \) for moduli and \( n=1 \) for the dilaton. Fine-tuning of the cosmological constant requires

\[ \sum_i |F_{zi}|^2 = (3 - n)m_{3/2}^2 \kappa^{-1} , \]  

(59)

leaving

\[ V_{\text{DE}}(Q) = \kappa M^6 v_1(\kappa^{1/2}Q) + M_8 M^3 v_2(\kappa^{1/2}Q) \sim M_8 M^3 v_2(\kappa^{1/2}Q) . \]  

(60)

No mass term appears for the quintessence field. The dynamics are similar to the no-scale case with a contribution from the matter density. The mass of the quintessence field at the minimum of the matter-dependent potential is of order \( H_0 \). Moreover the thin-shell effect is only present for small values of the normalized scalar field \( q \), a situation which requires a non-generic quintessence superpotential (otherwise \( q \sim 1 \) generically).

We conclude that under broad circumstances, one cannot obtain a compatibility between quintessence and gravity tests in supergravity. Either the dynamics are equivalent to a cosmological constant or gravity tests are not evaded.
One possibility is to relinquish the assumption on the Kähler potential (three decoupled Kähler potentials). Work on this possibility (sequestered models and others) is in progress.

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