Sub-Nyquist receiver using sampling frequency and instant offsets

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Abstract
With rapid advance in analogue-to-digital converters, more analogue-to-digital converters in the market can cover multiple Nyquist zones. As a result, the implementation of a sub-Nyquist receiver designed to cover a bandwidth wider than what is dictated by Nyquist theory has become more feasible than ever. This article presents a sub-Nyquist receiver using both different sampling rates and an offset at sampling instant. Compared with other sub-Nyquist receivers, the proposed design is straightforward to implement and can avoid issues conventional sub-Nyquist receivers encounter at the presence of multiple signals.

1 | INTRODUCTION
To cover a broad bandwidth, a wideband receiver for surveillance purposes usually employs a channelized receiver design and one such example is illustrated in Figure 1 [1]. As shown in Figure 1, the incoming signal is first split and down-converted by several different local oscillators (LO) and then passed through low-pass filters. So, signals within different frequency ranges are down-converted to the baseband of different channels and each channel only covers a portion of the receiver’s working bandwidth. Afterwards, each channel is digitized by an analogue-to-digital (A/D) converter and a signal analyser can be used to process all digitized data. The Nyquist theorem states that if an analogue signal with a bandwidth less than $B$ Hz is sampled at a sampling rate higher than $2B$ Hz, it can be reconstructed from its samples [2]. Therefore, the A/D converter’s sampling rate and receiver’s working bandwidth determine the number of channels.

One obvious issue about channelized receivers is the cost and required space accompanying multiple channels. Each channel requires a local oscillator with frequencies different from other channels’, an A/D converter etc. Therefore, designing a receiver covering a broad bandwidth with sampling rate lower than what is required by Nyquist theory has been a topic luring the engineering community. If a low-sampling rate A/D converter can cover a broad bandwidth, not only the resulting receiver can be smaller and cheaper but such technology might also eliminate the need for immediate frequency (IF). Compressive sensing which uses either non-uniform sampling rate [3] or random modulation pre-integrator (RMPI) [4] to cover multiple Nyquist zones has received significant attention in recent years. Despite its mathematical elegance, the compressive sensing approach has issues such as the requirement of specially designed hardware and complicated calculation. Another approach usually referred to as the sub-Nyquist method covers multiple Nyquist zones by using multiple ADC of either different sampling rates or with time offset at sampling instants [5, 6]. The biggest advantage of the sub-Nyquist method is its easy implementation. With the rapid advances in the ADC technologies, it is not difficult to purchase ADC whose input bandwidth covers multiple Nyquist zones [7], thus making sub-Nyquist method a very feasible approach.

Although the sub-Nyquist method has been investigated, most works focus on the detection of single signal [5, 6]. Some of the more recent works use multiple sub-Nyquist channels and subspace method such as MUSIC or Prony to detect multiple signals [8, 9], but these method are computationally heavy and some method requires an A/D converter with variable sampling rate which might not be widely available [9]. This article demonstrates conventional sub-Nyquist receiver’s issues when...
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FIGURE 1 A channelized receiver diagram

FIGURE 2 Two conventional sub-Nyquist receivers. (a) Sub-Nyquist receiver based on different sampling rates. (b) Sub-Nyquist receiver based on sampling instant offset

handling multiple signals and proposes a revised sub-Nyquist receiver capable of processing multiple input signals. The proposed method’s implementation is straightforward and requires no special hardware, thus allowing engineers to conveniently realize a compact and economic wideband receiver.

2 PREVIOUS METHODS

Figure 2(a) and 2(b) illustrates two popular sub-Nyquist receiver approaches. The one illustrated in Figure 1(a) applies two different sampling rates, $F_{s1}$ and $F_{s2}$, to sample input data [5]. The two sampling rates should be coprime. Assuming the signal is converted to complex-valued signal before digitization [10], the size of FFT is $N$, the peak of top FFT magnitude spectrum is at bin $k_1$, the peak of bottom FFT magnitude spectrum is at bin $k_2$, the signal frequency, $f$, can be represented as

$$f = K' \cdot F_{s1} + \frac{F_{s1}}{N} (k_1 - 1)$$

$$f = K'' \cdot F_{s2} + \frac{F_{s2}}{N} (k_2 - 1).$$

Based on the receiver’s working range (i.e. the range of $f$), the value of $K'$ and $K''$ can be determined, thus the signal frequency, $f$, can be solved. This method is referred to as the $\Delta f$ method in later discussions.

The sub-Nyquist receiver shown in Figure 2(b) uses two samplers of the same sampling rate, $F_{s1}$, but with an offset between two samplers’ sampling instants ($\Delta t$ in Figure 2(b)) [6]. The FFT is conducted upon two samplers’ outputs, respectively. When a signal is present, it should generate a peak at the same FFT bin of two FFT outcomes but with a phase difference ($\Delta \phi$) between two FFT peaks due to the sampling instant difference. The signal frequency can then be determined by the phase difference between FFT peaks of two FFT spectra as:

$$f = \frac{\Delta \phi}{2\pi} \frac{1}{\Delta t}.$$

As shown in Equation (2), the smaller $\Delta t$ is, the broader bandwidth this sub-Nyquist receiver can cover. In a noisy environment, the signal frequency estimated by Equation (2) might bear significant error. To solve this issue, one can apply the first part of Equation (1) to figure possible signal frequencies based on FFT magnitude spectrum and then use Equation (2) to determine the actual signal frequency. This method is referred to as the $\Delta t$ method in later discussions. It is worth notice that frequency estimation accuracy can be improved with the interpolation of FFT magnitude spectrum [11].

Although the two sub-Nyquist approaches illustrated in Figure 2(a) and 2(b), respectively, can successfully detect one signal over multiple Nyquist zones, both methods encounter issues when used to detect multiple signals. For the $\Delta f$ method, if there are multiple peaks in FFT spectra, it is difficult to associate peaks caused by the same signal. To explain it, assume that there are two concurrent signals with similar power and two peaks can be observed in two FFT spectra. To determine the signal’s actual frequency with Equation (1), peaks from two FFT spectra need to be paired. As there are two peaks in each spectrum, there are multiple possible combinations and a wrong paring can cause wrong frequency estimation. For the $\Delta t$ method, when two signals frequencies are separated by multiple of sampling rate, say $f$ and $f+F_{s1}$, there will be only one peak in two FFT spectra. As a result, the receiver based on the $\Delta t$ method fails to identify the existence of two signals.

To demonstrate issues of $\Delta f$ and $\Delta t$ methods, a simulation was conducted in Matlab®. The input signals are two complex-valued signals converted from two sinusoidal signals [10]. A complex-valued Gaussian noise is added and the signal-to-noise (SNR) is set to be 10 dB for both signals. This SNR value is used in another paper on sub-Nyquist receivers [8]. One signal’s (referred to signal 1) frequency sweeps from 2 to 13.52 GHz with resolution of 1 MHz and the other signal’s (referred to signal 2) frequency is arbitrarily fixed at 7.434 GHz. In other words, the working bandwidth of the receiver is set to 2–13.52 GHz. This receiver’s working bandwidth is chosen to be close to a typical wideband receiver’s working bandwidth. For the $\Delta f$ method, the two different sampling rates are 2.56 GHz ($F_{s1}$) and 2.199 GHz ($F_{s2}$). The sampling rate of 2.56 GHz has been used

$$f = \frac{\Delta \phi}{2\pi} \frac{1}{\Delta t}.$$
Another sampling rate ($F_{s2}$) needs to be coprime with 2.56 GHz (MHz as the unit) and it is intentionally chosen less than 2.56 GHz to emphasize the advantage of sub-Nyquist method, which allows a lower sampling rate. For the $\Delta f$ method, the value of $\Delta f$ needs to be smaller than $7.3964 \times 10^{-11}$ s to cover the whole bandwidth and, in this simulation, $\Delta f$ is arbitrarily set to be $3.6982 \times 10^{-11}$ s, half of maximum allowed value. Different $\Delta f$ values were also tried in this study and no significant performance difference was noticed. The sampled data is multiplied by a 256-point Blackman window before the 256-FFT operation to reduce spectral leakage. Both Blackman window and 256-FFT can be found used in the study of wideband receivers [11]. As the main focus of this article is on sampling rate rather than quantization, an ideal sampler with infinite precision is used in simulation.

The signal frequencies estimated with $\Delta f$ and $\Delta t$ methods and corresponding estimation errors vs actual signal 1 frequencies are illustrated in Figures 3 and 4, respectively. As shown in these figures, although both $\Delta f$ and $\Delta t$ methods can correctly identify signal frequencies in most cases, occasionally, significant estimation errors happen, thus limiting the applications of these two sub-Nyquist methods. The mean square error (MSE) defined in Equation (3) is calculated to quantize algorithm performance:

$$MSE = \sqrt{\frac{\sum_{i=0}^{N-1} (error_i)^2}{N}}$$

where $N$ is the number of tests in each simulation and $error_i$ is frequency error in MHz. Based on the simulation, the MSE of $\Delta f$ method is 1008.2 MHz for signal 1 and 68.5 MHz for signal 2. For the $\Delta t$ method, the MSE is 329.1 MHz for signal 1 and 330.2 MHz for signal 2.

### 3 PROPOSED METHOD

As described in the previous section, popular sub-Nyquist methods might encounter difficulty when multiple signals are present. To solve this matter, a modified sub-Nyquist receiver is
proposed and its diagram is shown in Figure 5. The proposed sub-Nyquist method combines both Δf and Δt methods and is referred to as the Δf+Δt method. The working principle of the Δf+Δt method is explained as follows. Signal frequencies are determined with Δf and Δt methods first, as described previously. In principle, signal frequencies determined with the Δt method are assumed to be correct except when the Δf method detects more signals than the Δt method. In this case, signal frequencies determined with the Δf method are considered accurate signal frequencies. The implementation of the proposed method is straightforward and its calculation complexity is moderate. Compared with other sub-Nyquist methods proposed to solve frequency ambiguity issues, the proposed method does not require time-consuming calculation such as the MUSIC method or special hardware like an A/D converter whose sampling rate can be adaptively changed.

To evaluate the effectiveness of this approach, the same simulation used to test Δf and Δt methods is repeated to test the Δf+Δt method and the results are illustrated in Figure 6. As shown in Figure 6, the proposed Δf+Δt sub-Nyquist method can successfully detect both signals with reasonable accuracy. There is no significant estimation error when the Δf+Δt method is used to detect signals. Compared with Δf and Δt methods, the new method reduces MSE from several hundred MHz to only 0.6 MHz. There are some moderate estimation error spikes (≤ 10 MHz) in certain signal frequency ranges which is worthy of explanation. As signal 2’s frequency is set at 7.434 GHz, when signal 1’s frequency is close to (7.434 ± k×Fs1) GHz or (7.434 ± k×Fs2) GHz, some FFT spectrum will see two peaks very close to each other, which result in increased frequency estimation error. If two signals’ frequency separation is less than the receiver’s frequency resolution, then the receiver might not separate them. This issue is about the receiver’s frequency resolution. For an FFT-based receiver, the straightforward solution is using a longer FFT window. However, using a longer window will sacrifice the receiver’s time resolution and increase system complexity. It also should be noted that as the frequency estimation errors of the three methods presented in this article are due to the fundamental design of receivers, such as frequency ambiguity for Δt and Δf method or limited frequency resolution for Δt ± Δf method, further increasing SNR will not improve system performance.

4 | CONCLUSION

This paper described a new sub-Nyquist method. Compared with conventional sub-Nyquist methods, the proposed method can identify multiple signals over a broad bandwidth without ambiguity. The hardware requirement and calculation complexity of the proposed method are moderate, thus making this method very practical.

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