Spin-1/2 invisible particles in heavy meson decays

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Abstract

The FCNC decay processes of the $B$ and $B_c$ mesons with the final states involving spin-1/2 particles are investigated. By considering the background of the Standard Model where $\nu\bar{\nu}$ contributing the missing energy and the experimental upper bounds for the branching fractions, we get the constraints of the coupling constants of the quark-antiquark and the assumed invisible particles $\chi\bar{\chi}$. The constraints of the coupling constants are then used to study the similar processes of the $B_c$ meson. At some specific region of $m_\chi$, the upper limit of $\text{BR}(B_c \rightarrow D(s)\chi\bar{\chi})$ is of the order of $10^{-6}$, while for $\text{BR}(B_c \rightarrow D'_s(\chi\bar{\chi})$, it is $10^{-5}$. The possibility of distinguishing $\chi$ to be a Majorana or Dirac fermion by the differential branching fractions is also discussed.

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I. INTRODUCTION

As the freeze-out mechanism [1, 2] can naturally interpret the observed dark matter abundance in our universe, the weakly-interacting massive particle (WIMP) is considered to be one of the most promising dark matter candidates. It is considered as a thermal relic from the local thermodynamic equilibrium early universe [3]. The observed dark matter relic abundance $\Omega_c h^2 = 0.1131 \pm 0.0034$ [4, 5] sets a lower bound for WIMP’s annihilation cross section. In specific models, the cross section can be connected to the mass of WIMP and coupling coefficients between WIMP and the Standard Model (SM) fermions. For example, the Lee-Winberg limit [6] demands its mass larger than a few GeV. However, this result is model-dependent. With different models or proper parameters selection, this constraint can be relaxed, which makes lower mass WIMP be possible. For example, the MeV-scale light dark matter (LDM) is proposed [7, 8] to explain the unexpected emission of 511 keV photons from the galaxy center.

Previous experiments most focus on the dark matter particle with large mass, namely hundreds of GeV to several TeV. But recent experiment [9] sets much stricter constraints on the parameter space for the WIMP with mass larger than several GeV. It provides a motivation to study the sub-GeV LDM through high-energy colliders. For example, CODEX-b at the LHCb experiment aimed to probe for GeV-scale long-lived particles [10]. Missing energy signals [11] in flavor-changing neutral current (FCNC) processes of heavy mesons provide a possible way to probe light WIMP. Within the SM, neutrinos $\nu\bar{\nu}$ in the final state make contribution to the missing energy. However, theoretical calculations of the branching fractions of $B \to h_f \nu\bar{\nu}$ are less than the experimental bounds of $B \to h_f \bar{E}$, where $h_f$ is the final meson and $\bar{E}$ is the missing energy. So there is still some allowed parameter space for the decays involving other light invisible particles.

Theoretically, spin of the invisible particle has several possibilities [12]. It can be a (pseudo)scalar [13], a fermion [14], or a hidden vector [15]. In the previous paper [16], we have considered the scalar and pseudoscalar cases. In this paper, we will focus on the spin-1/2 light dark matter particles. There are many models involving the fermionic dark matter particles, such as sterile neutrino [14], neutralino [17, 18], Higgs-portal [19, 20], Z-portal [21] and singlet-doublet [22–26]. Specifically, it can be either a Majorana or a Dirac fermion, as it is electrically neutral. Phenomenologically, the new invisible fermion can weakly interact with the SM fermions via a mediator, which can be a scalar [27], pseudoscalar [28], vector or axial-vector [29] particle. The mass of the mediator is usually considered to be hundreds of
GeV. In the energy level of heavy meson decays, namely several GeV, the branching ratios are greatly suppressed. However, as the FCNC processes in the SM are also highly suppressed, the contribution of the new physics maybe important, which has been extensively studied in the decays of mesons [30–36].

In this work, we will first introduce the effective operators to describe the coupling between quarks and the invisible fermions. The experimental upper bounds for the FCNC decay channels of the $B$ meson then provide constraints of the coupling constants, which will be applied to calculate the upper bounds of the similar decay processes of the $B_c$ meson. As we calculate the hadronic transition matrix elements, two methods are used: for the $B \to h_f$, we use the QCD light-cone sum rules (LCSR), while for the $B_c \to h_f$, the instantaneous Bethe-Salpeter (BS) method is applied. For the light invisible fermions, both the Majorana and Dirac cases are considered. As they interact differently with quarks, e.g. the Majorana fermion has neither vector nor tensor interactions, while Dirac fermion has both of them, the differential distribution will show slight difference.

The paper is organized as follows: In Sec. II, we present the model-independent effective Lagrangian to describe the coupling between the light invisible fermions and quarks, and extract the constraints of the coupling coefficients. In Set. III, we calculate the upper limits of the branching fractions of $B_c$ decays, and give the differential decay rate as a function of the missing energy. Finally, We draw the conclusion in Sec. IV.

II. EFFECTIVE OPERATORS

The FCNC decay processes of heavy meson to spin-1/2 invisible particles $\bar{\chi}\chi$ are described in Fig. 1, where $q, q_f,$ and $\bar{q}'$ represent the quark and antiquark, respectively. The four-fermion vertex may be generated at the tree or loop level by introducing new physical

![FIG. 1. Feynman diagrams of decay channels involving invisible particles.](image-url)
mediators in specific models [27–29]. In this work, we follow Ref. [32] to introduce a model-independent effective Lagrangian,

$$\mathcal{L}_{\text{eff}} = \sum_{i=1}^{9} g_{fi} Q_i,$$

where $g_{fi}$ are the phenomenological coupling constants which are suppressed by the square of new physical energy scale $\Lambda^2$, and the subscript $f$ represent fermion which can be Majorana or Dirac type. There are 9 independent dimension-six effective operators $Q_i$s, which have the forms

$$Q_1 = (\bar{q}_f q)(\bar{\chi}\chi), \quad Q_2 = (\bar{q}_f \gamma^5 q)(\bar{\chi}\chi), \quad Q_3 = (\bar{q}_f q)(\bar{\chi}\gamma^5 \chi),$$

$$Q_4 = (\bar{q}_f \gamma^5 q)(\bar{\chi}\gamma^5 \chi), \quad Q_5 = (\bar{q}_f \gamma_\mu q)(\bar{\chi}\gamma^\mu \gamma^5 \chi), \quad Q_6 = (\bar{q}_f \gamma_\mu \gamma^5 q)(\bar{\chi}\gamma^\mu \gamma^5 \chi),$$

$$Q_7 = (\bar{q}_f \gamma_\mu q)(\bar{\chi}\gamma^\mu \chi), \quad Q_8 = (\bar{q}_f \gamma_\mu \gamma^5 q)(\bar{\chi}\gamma^\mu \chi), \quad Q_9 = (\bar{q}_f \sigma_{\mu\nu} q)(\bar{\chi}\sigma^{\mu\nu} \chi).$$

The upper limits of the coupling constants in the effective Lagrangian can be achieved by comparing the difference between theoretical predictions and the experimental data. As the corresponding detection of the $B_c$ meson is still missing, we cannot use the experimental data of $B_c$ meson to set constraints directly. Instead, the allowed region of the coupling constants can be obtained by considering the $B$ meson decay processes. These channels are $B^- \to K^-(K^*-\bar{E})$ and $B^- \to \pi^-(\rho^-) + \bar{E}$, which have the same vertex as that of the $B_c$ meson decays. The upper bounds of the $B$ meson decays involving missing energy are listed in the first column of Table I. The second column is the theoretical predictions, and the third one is the extracted upper limits for the decays involving the assumed particles.

One notices that they are of the same order as that of the SM background.

| Experimental bound [37–39] | SM prediction [40–43] | Invisible particles bound |
|---------------------------|-----------------------|---------------------------|
| $\text{BR}(B^\pm \to K^\pm \bar{E}) < 14$ | $\text{BR}(B^\pm \to K^\pm \nu\bar{\nu}) = 5.1 \pm 0.8$ | $\text{BR}(B^\pm \to K^\pm \chi\chi) < 9.7$ |
| $\text{BR}(B^\pm \to \pi^\pm \bar{E}) < 14$ | $\text{BR}(B^\pm \to \pi^\pm \nu\bar{\nu}) = 9.7 \pm 2.1$ | $\text{BR}(B^\pm \to \pi^\pm \chi\chi) < 6.4$ |
| $\text{BR}(B^\pm \to K^{*\pm} \bar{E}) < 61$ | $\text{BR}(B^\pm \to K^{*\pm} \nu\bar{\nu}) = 8.4 \pm 1.4$ | $\text{BR}(B^\pm \to K^{*\pm} \chi\chi) < 54$ |
| $\text{BR}(B^\pm \to \rho^\pm \bar{E}) < 30$ | $\text{BR}(B^\pm \to \rho^\pm \nu\bar{\nu}) = 0.49^{+0.01}_{-0.38}$ | $\text{BR}(B^\pm \to \rho^\pm \chi\chi) < 30$ |
A. χ is a Majorana fermion

We firstly consider the situation that the invisible particle is a Majorana fermion. In such a case, the vector and tenor currents give no contribution, namely, $\chi\gamma^\mu\chi = 0$ and $\chi\sigma^{\mu\nu}\chi = 0$ (these are not true for the Dirac fermion). For the $0^- \rightarrow 0^-$ transitions, only three operators give non-zero contribution. The effective Lagrangian reads

$$L_1 = g_{m1}(\bar{q}_f q)(\bar{\chi}\chi) + g_{m3}(\bar{q}_f q)(\bar{\chi}\gamma^5\chi) + g_{m5}(\bar{q}_f \gamma_\mu q)(\bar{\chi}\gamma^\mu\gamma^5\chi),$$

where the subscript $m$ in $g_{mi}$ indicates that we are dealing with Majorana fermions. The hadronic transition matrix elements can be expressed as,

$$\langle M^- | (\bar{q}_f q) | M^- \rangle = \frac{M^2 - M_f^2}{m_q - m_{q_f}} f_0(s),$$

$$\langle M^- | (\bar{q}_f \gamma_\mu q) | M^- \rangle = (P + P_f)_{\mu} f_+(s) + (P - P_f)_{\mu} \frac{M^2 - M_f^2}{s} [f_0(s) - f_+(s)],$$

$$\langle M^- | (\bar{q}_f \sigma_{\mu\nu} q) | M^- \rangle = i [P_\mu (P - P_f)_\nu - P_\nu (P - P_f)_\mu] \frac{2}{M + M_f} f_T(s),$$

where $P$ and $P_f$ are the momenta of the initial or final mesons, respectively; $m_q$ and $m_{q_f}$ are the masses of quarks; $s$ is defined as $(P - P_f)^2$; $f_+$, $f_0$, and $f_T$ are form factors. Here we adopt the results of the LCSR method [44] to write the form factors as,

$$f_0(s) = \frac{r_2}{1 - s/m_{fit}^2},$$

$$f_+^{K,T}(s) = \frac{r_1}{1 - s/m_R^2} + \frac{r_2}{(1 - s/m_R^2)^2},$$

$$f_+^{\pi,T}(s) = \frac{r_1}{1 - s/m_R^2} + \frac{r_2}{1 - s/m_{fit}^2},$$

where the corresponding parameters $r_1$, $r_2$, $m_R$, and $m_{fit}$ are presented in Table II.

By finishing the three-body phase space integral, we get the branching ratio

$$BR = \frac{1}{512\pi^3 M_B^3 \Gamma_B^-} \int \frac{ds}{s} \frac{\lambda^{1/2}(M^2, s, M_f^2) \lambda^{1/2}(s, m_\chi^2, m_\chi^2)}{\lambda^{1/2}(M^2, s, M_f^2) \lambda^{1/2}(s, m_\chi^2, m_\chi^2)} \int d\cos \theta \sum_\lambda |M|, \quad (6)$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is the Källén function; $m_\chi$ is the mass of the invisible particle; $\theta$ is the angle between the three-dimensional momenta $\vec{P}_\chi$ and $\vec{P}_f$ in the center-of-momentum frame of the invisible particles; $\Gamma_B^-$ is the total width of $B^-$ meson; $\Omega = 2$ originates from the final two invisible particles being identical (Majorana fermion), and $\Omega = 1$ when $\chi$ is the Dirac fermion. In the square of the amplitude, there are interference terms which come from the contribution of two different operators. These terms
TABLE II. Parameters in the form factors of the $B \to \pi(K)$ processes [44].

| $F_i$ | $r_1$  | $r_2$  | $m_{fit}^2$ (GeV$^2$) | $m_R$ (GeV) |
|-------|--------|--------|-----------------------|--------------|
| $f_0^K$ | 0      | 0.330  | 37.46                 | -            |
| $f_+^K$ | 0.162  | 0.173  | -                     | 5.41         |
| $f_T^K$ | 0.161  | 0.198  | -                     | 5.41         |
| $f_0^\pi$ | 0      | 0.258  | 33.81                 | -            |
| $f_+^\pi$ | 0.744  | -0.486 | 40.73                 | 5.32         |
| $f_T^\pi$ | 1.387  | -1.134 | 32.22                 | 5.32         |

are proved to be zero when the invisible particles are (pseudo)scalars [16]. However, they are not all zero when $\chi$ is a fermion, which makes the calculations much more complicated. In Ref. [35], the same processes of $B$ meson are considered with all the interference terms assumed to be zero. In our work we will also calculate these terms, and actually for some of them, the contribution cannot be ignored.

The partial width can be written as

$$
\Gamma = \int dP S_3 \left( \sum_j g_{mj} T_j \right)^\dagger \left( \sum_i g_{mi} T_i \right) = \sum_{ij} g_{mj} g_{mi} \tilde{\Gamma}_{ij},
$$

(7)

where we have taken $g_{ij}$ to be real for simplicity, and defined $\tilde{\Gamma}_{ij} = \int dP S_3 T_j^\dagger T_i$, which is independent of the effective coupling constants. Some interference terms are zero by themselves or cancel each other out. The non-zero terms are $\tilde{\Gamma}_{11}$, $\tilde{\Gamma}_{33}$, $\tilde{\Gamma}_{55}$, and $\tilde{\Gamma}_{35}$. In Fig. 2, we plot them as functions of $m_\chi$. The solid and dashed lines represent the non-interference and interference terms, respectively. One can see that the two different channels have similar results, because the final mesons $K$ and $\pi$ have the same quantum number and small masses compared with that of the $B$ meson. The non-interference terms decrease when $m_\chi$ gets larger, because the phase space gets smaller. Detailed calculation shows that $\tilde{\Gamma}_{11}$ and $\tilde{\Gamma}_{33}$ are proportional to $(p_1 \cdot p_2 - m_\chi^2)$ and $(p_1 \cdot p_2 + m_\chi^2)$, respectively, where $p_1$ and $p_2$ are the momenta of two final invisible particles. So $\tilde{\Gamma}_{11}$ is smaller than $\tilde{\Gamma}_{33}$ except when $m_\chi = 0$. $\tilde{\Gamma}_{55}$ is less than $\tilde{\Gamma}_{33}$ as they are related to different effective operators. The interference terms $\tilde{\Gamma}_{35}$ and its complex conjugate $\tilde{\Gamma}_{53}$ are numerically equal. One can see that they are zero when $m_\chi = 0$ as they are proportional to $m_\chi^2$, which is quite different with $\Gamma_{ii}$. 

6
The hadronic transition matrix elements are parameterized by form factors $\epsilon$ where the pole structure is

$$P_A \epsilon_{ij} = \epsilon_{ij}$$

FIG. 2. $\tilde{\Gamma}_{ij}$ for $B \to K(\pi)\bar{\chi}\chi$ with $\chi$ being a Majorana fermion.

The effective Lagrangian for the $0^- \to 1^-$ process has the form

$$\mathcal{L}_2 = g_{m2}(\bar{q}_f \gamma^5 q)(\bar{\chi}\chi) + g_{m4}(\bar{q}_f \gamma^5 q)(\bar{\chi}\gamma^5 \chi) + g_{m5}(\bar{q}_f \gamma_{\mu} q)(\bar{\chi}\gamma^\mu \gamma^5 \chi) + g_{m6}(\bar{q}_f \gamma_{\mu} \gamma^5 q)(\bar{\chi}\gamma^\mu \gamma^5 \chi).$$

The hadronic transition matrix elements are parameterized by form factors $A_0$, $A_1$, $A_2$, $A_3$ $V$, $T_1$, $T_2$, and $T_3$ [45–47],

$$\langle M_f^- | (\bar{q}_f \gamma^5 q) | M^- \rangle = -i [\epsilon \cdot (P - P_f)] \frac{2M_f}{m_q + m_{q_f}} A_0(s),$$

$$\langle M_f^- | (\bar{q}_f \gamma_{\mu} q) | M^- \rangle = \frac{\epsilon \cdot (P - P_f)}{M + M_f} A_2(s)$$

$$- (P - P_f) \frac{M_f}{2s^2} [A_3(s) - A_0(s)]$$

$$\langle M_f^- | (\bar{q}_f \gamma_{\mu} q) | M^- \rangle = \frac{\epsilon_{\mu\nu\rho\sigma}}{2} P^\sigma (P - P_f) A_2(s)$$

$$\langle M_f^- | (\bar{q}_f \gamma_{\mu} q) | M^- \rangle = \frac{\epsilon_{\mu\nu\rho\sigma}}{M + M_f} V(s),$$

$$\langle M_f^- | (\bar{q}_f \gamma_{\mu} q) | M^- \rangle = \frac{\epsilon_{\mu\nu\rho\sigma}}{M + M_f} V(s),$$

where $\epsilon$ is the polarization vector of the final meson, and the $\epsilon^{0123} = 1$ convention is used.

The form factors are parameterized by [45],

$$F_i(s) = P_i(s) \sum_k \alpha_k^i [z(s) - z(0)]^k,$$

where the pole structure is $P_i(s) = (1 - s/m_{R,i}^2)^{-1}$; $F_1$, $F_2$, $F_3$, $F_4$, $F_5$, $F_6$, $F_7$, and $F_8$ represent $A_0$, $A_1$, $A_{12}$, $A_3$, $V$, $T_1$, $T_2$, and $T_{23}$, respectively. $A_2$ and $T_3$ can be deduced from
the relations
\[ A_3(s) = \frac{M + M_f}{2M_f} A_1(s) - \frac{M - M_f}{2M_f} A_2(s) \]
\[ A_{12}(s) = \frac{(M + M_f)^2(M^2 - M_f^2 - s) A_1(s) - [(M + M_f)^2 - s] [M - M_f]^2 - s} {16 M M_f^2 (M + M_f)} A_2(s) \]
\[ T_{23}(s) = \frac{(M^2 - M_f^2)(M^2 + 3M_f^2 - s) T_2(s) - [(M + M_f)^2 - s] [(M - M_f)^2 - s]} {8 M M_f^2 (M - M_f)} T_3(s). \]

(11)

\[ z(s) \text{ is defined as } z(s) = \frac{\sqrt{s+ - s} - \sqrt{s+ - s_0}} {\sqrt{s+ - s} + \sqrt{s+ - s_0}}, \]

(12)

where \( s_\pm \equiv (M \pm M_f)^2 \) and \( s_0 \equiv s_+ (1 - \sqrt{1 - s_- / s_+}) \). The related parameters are listed in Table III.

For \( B \to K^*(\rho)\bar{\chi}\chi \), we plot \( \tilde{\Gamma}_{ij} \) as functions of \( m_\chi \) in Fig. 3. One notices that \( \tilde{\Gamma}_{66} \) is larger than the other terms. There is only one interference term \( \tilde{\Gamma}_{46} \) which is nonzero. Its contribution is negative. \( \tilde{\Gamma}_{22} \) and \( \tilde{\Gamma}_{55} \) are quite close to each other. For \( B \to K^*\bar{\chi}\chi \), these two terms are almost coincident.

![Fig. 3](image-url)

**FIG. 3.** \( \tilde{\Gamma}_{ij} \) for \( B \to K^*(\rho)\bar{\chi}\chi \) with \( \chi \) being a Majorana fermion.

**B. \( \chi \) is a Dirac fermion**

For the Majorana fermion, there is neither vector nor tensor interaction, while for the Dirac fermion, these two kinds of interactions also give contribution. When the invisible particle is a Dirac fermion, the effective Lagrangian has more operators. For the \( 0^- \to 0^- \)
TABLE III. Parameters in the form factors of the $B \rightarrow \rho(K^*)$ processes with $k_{max} = 2$ [45].

| $F_i$ | $B \rightarrow K^*$ | $m_{R,i}^{b\rightarrow s}$/GeV | $B \rightarrow \rho$ | $m_{R,i}^{b\rightarrow d}$/GeV |
|-------|---------------------|-----------------------------|---------------------|-----------------------------|
| $\alpha_{0}^{A_{0}}$ | 0.36 ± 0.05 | 0.36 ± 0.04 | 5.366 | 0.83 ± 0.20 | 5.279 |
| $\alpha_{1}^{A_{0}}$ | -1.04 ± 0.27 | 1.12 ± 1.35 | 0.27 ± 0.03 | 0.26 ± 0.03 |
| $\alpha_{2}^{A_{0}}$ | 0.30 ± 0.19 | 0.16 ± 0.41 | 5.829 | 0.39 ± 0.14 | 5.724 |
| $\alpha_{2}^{A_{1}}$ | -0.11 ± 0.48 | 0.46 ± 0.76 | 0.26 ± 0.03 | 0.30 ± 0.03 |
| $\alpha_{0}^{A_{12}}$ | 0.60 ± 0.20 | 0.60 ± 0.20 | 0.12 ± 0.84 | 0.46 ± 0.76 |
| $\alpha_{1}^{A_{12}}$ | -0.89 ± 0.19 | 2.37 ± 1.39 | 2.37 ± 1.39 | 1.80 ± 0.97 |
| $\alpha_{2}^{A_{12}}$ | 1.95 ± 1.10 | 1.45 ± 0.77 | 1.45 ± 0.77 | 1.45 ± 0.77 |
| $\alpha_{0}^{T_{1}}$ | 0.28 ± 0.03 | 0.28 ± 0.03 | 0.28 ± 0.03 | 0.27 ± 0.03 |
| $\alpha_{1}^{T_{1}}$ | 0.40 ± 0.18 | 5.829 | 0.47 ± 0.13 | 5.724 |
| $\alpha_{2}^{T_{1}}$ | 0.36 ± 0.51 | 0.58 ± 0.46 | 0.58 ± 0.46 | 0.58 ± 0.46 |
| $\alpha_{0}^{T_{2}}$ | 0.67 ± 0.08 | 0.67 ± 0.08 | 0.67 ± 0.08 | 0.75 ± 0.08 |
| $\alpha_{1}^{T_{2}}$ | 1.48 ± 0.49 | 1.48 ± 0.49 | 1.48 ± 0.49 | 1.90 ± 0.43 | 5.724 |
| $\alpha_{2}^{T_{2}}$ | 1.92 ± 1.96 | 2.93 ± 1.81 | 2.93 ± 1.81 | 2.93 ± 1.81 |

transition, it can be written as

$$L_3 = g_{d1}(\bar{q}_f q)(\bar{\chi}\chi) + g_{d3}(\bar{q}_f q)(\bar{\chi}\gamma^5 \chi) + g_{d5}(\bar{q}_f \gamma_\mu q)(\bar{\chi}\gamma^\mu \gamma^5 \chi) + g_{d7}(\bar{q}_f \gamma_\mu q)(\bar{\chi}\gamma^\mu \chi)$$

$$+ g_{d9}(\bar{q}_f \sigma_{\mu\nu} q)(\bar{\chi}\sigma^{\mu\nu} \chi),$$

where $g_{dij}$ are the phenomenological coupling constants between the invisible Dirac fermions and quarks. $\tilde{\Gamma}_{ij}$s are presented in Fig. 4. One notices that they are about a half of that in...
the Majorana case where $\chi$ and its antiparticle $\bar{\chi}$ are identical. In Fig. 4, one also notices that there are additional terms $\tilde{\Gamma}_{77}$, $\tilde{\Gamma}_{99}$ and $\tilde{\Gamma}_{79}$, which represent vector and tensor currents, since they are not zero when $\chi$ is a Dirac fermion. Like above, $\tilde{\Gamma}_{55}$ and $\tilde{\Gamma}_{77}$ have same value when $m_\chi = 0$. The $\tilde{\Gamma}_{99}$ term increases firstly, then decreases to zero when the phase space gets less. The interference term $\tilde{\Gamma}_{79}$ have the same trend as $\tilde{\Gamma}_{25}$, for they are proportional to $m^2_\chi$.

For $0^- \to 1^-$ processes, the effective Lagrangian can be written as,

$$L_4 = g_{d2}(\bar{q}_f \gamma^5 q)(\bar{\chi}\chi) + g_{d4}(\bar{q}_f \gamma^5 q)(\bar{\chi}\gamma^5 \chi) + g_{d5}(\bar{q}_f \gamma_\mu q)(\bar{\chi}\gamma^\mu \gamma^5 \chi) + g_{d6}(\bar{q}_f \gamma_\mu q)(\bar{\chi}\gamma^\mu \gamma^5 \chi) + g_{d7}(\bar{q}_f \gamma_\mu q)(\bar{\chi}\gamma^\mu \chi) + g_{d8}(\bar{q}_f \gamma_\mu q)(\bar{\chi}\sigma^{\mu\nu} \chi).$$

(14)

In Fig. 5, we plot the non-zero $\tilde{\Gamma}_{ij}$s as functions of $m_\chi$. One can see that the $\tilde{\Gamma}_{ii}$ terms are not equal to zero even when $m_\chi$ takes zero. As before, the interference terms begin from zero and end up with zero when $m_\chi$ increases. Comparing with Fig. 3, the additional term $\tilde{\Gamma}_{99}$ with different effective operator makes much larger contribution to the partial width.

III. THE DECAY MODES OF THE $B_c$ MESON

In Sec. II, the LCSR method is adopted to calculate the hadronic transition amplitude in the FCNC processes of $B$ meson, where the final meson is light. While for the $B_c$ meson decay modes, both initial and final mesons are heavy. Under these circumstances, the BS method is a good choice to calculate the hadronic transition amplitude. In this method, we can safely make an instantaneous approximation when solving the Bethe-Salpeter equation.
fulfilled by the wave functions of the heavy mesons. Details of how to solve the instantaneous BS equation can be found in [48, 49]. The hadronic transition matrix element has the form

\[
\langle h^- | \bar{q}_1 \Gamma_{ij}^\xi b | B_c^- \rangle = \int \frac{d^3q}{(2\pi)^3} \text{Tr} \left[ \frac{\not{P}}{M} \varphi_{P_f}^{i+} (q_f) \Gamma_{ij}^\xi \varphi_{P_f}^{j+} (q) \right],
\]

(15)

where \( \varphi_{P_f}^{i+} \) and \( \varphi_{P_f}^{j+} \) are the wave functions of the initial and final mesons, respectively; \( \vec{q} \) and \( \vec{q}_f \) are the relative momentum of the quark and antiquark in the initial and final meson, respectively. In the Standard Model, \( \nu \) and \( \bar{\nu} \) lead to the missing energy in the decay processes \( B_c^- \to D'^{(*)}_{s} + E \) and \( B_c^- \to B'^{(*)}_{s} + E \). The branching ratio of former channels are of the order of \( 10^{-7} \sim 10^{-6} \), while for the later ones, it is of the order of \( 10^{-15} \sim 10^{-14} \). The exact results can be found in our previous paper [16].

A. \( \chi \) is a Majorana fermion

The decay processes of \( B_c \) meson to Majorana fermions are also described by the effective Lagrangians in Eq. (3) and Eq. (8). Using Eq. (15), we get \( \tilde{\Gamma}_{ij} \)'s as functions of \( m_\chi \), which are plotted in Fig. 6. Although a different method is used to parameterize the form factors, the results of \( B_c \to D(\star)_{s} \) and \( B_c \to D^*_{s} \) are quite similar to those in Fig. 2 and Fig. 3, respectively, because the main difference of these channels comes from the different spectator quarks. We also consider the processes \( B_c \to B^{(*)}_{s} \chi \). One notices that \( \tilde{\Gamma}_{ij} \)'s of the \( c \to u \) processes are two orders of magnitude less than these of the \( b \to d(\star) \) processes. This is because the phase space of the former channel is less than that of the later one. The \( \tilde{\Gamma}_{55} \) term is smaller than \( \tilde{\Gamma}_{33} \) in \( b \) decays while it is lager in \( c \) decays, which means this operator is less sensitive to the phase space.
The next step is to set constraints for the coupling constants and calculate the upper limits for the branching fractions of $B_c$ decays. In Sec.II, we have obtained $\tilde{\Gamma}_{ij}$ for the decay processes of $B$ meson. Considering the upper limits of the branching fractions of such channels, we can extract the allowed parameter space for the effective coupling constants. Here we use two different ways to make the calculation. Firstly, we assume just one effective
coupling constant is nonzero, and its upper bound can be easily achieved. Of course, different operators will give different results. Secondly, we will scan the whole parameter space spanned by all the coupling constants under all the constraints.

With the effective coupling constants achieved above, we calculate the upper limits for of the branching fractions of $B_c$ decays. The results are shown in Fig. 7, where the dashed lines represent those calculated in the first way and the solid line corresponds to that of the second way. One can see that the results of two different ways do not coincide in most $m_\chi$ regions. The difference comes from the contribution of the interference terms. For the $B_c^- \to P$ processes, the three cases $ij = 11$, $ij = 33$ and $Total$ have the same value when $m_\chi = 0$. At some points, $ij = 55$ coincides with $Total$. For the $B_c^- \to V\chi\chi$ processes, $ij = 66$ coincides with $Total$ when $m_\chi = 0$. The upper limits of the branching fractions of $B_c \to P\chi\chi$ are of one order of magnitude less than these of $B_c^- \to V\chi\chi$, which is mainly due to different experimental bounds. One notices that as $m_\chi$ increases, the branching ratios (for $Total$) firstly increases slowly, and then decreases rapidly. This is a result of the competition between the phase space and the effective coupling constants.

In Fig. 8 we present the differential branching fractions as functions of $s$ which is defined as $s = (P - P_f)^2$. As examples, three cases with $m_\chi = 0$ GeV, $0.25(M - M_f)$, and $0.4(M - M_f)$, respectively, are considered. For comparison, the SM background with $\bar{\nu}\nu$ emission are also plotted as blue dashed lines, which are less than those of the $\bar{\chi}\chi$ emission channels in most regions of $s$. The left starting points of the curves is the lower bound of $s$, which are determined by the mass of $\chi$. The position of peaks of the distribution curves are almost independent of $m_\chi$, which is at the region $s = 16 \sim 18$ GeV$^2$. We can see that the peak value gets larger as $m_\chi$ increasing, because the branching ratio increases with $m_\chi$ (until reaches its maximum value around $m_\chi = 1.5$ GeV).

B. $\chi$ is a Dirac fermion

The similar analysis can also be applied to the Dirac fermions. The effective Lagrangians take the same forms as those in Eq. (13) and Eq. (14). In Fig. 9, we plot $\tilde{\Gamma}_{ij}$ as the function of $m_\chi$, which is about half of the corresponding one in the Majorana case. One can see there are several additional terms $\tilde{\Gamma}_{77}$, $\tilde{\Gamma}_{88}$, and $\tilde{\Gamma}_{99}$ which do not exist in the Majorana case. The effective coupling constants obtained by comparing with the experimental results are used to find the maximum values of the branching fractions which are plotted in Fig. 10. In
FIG. 7. The upper limits of branching ratios of $B_c$ decays to Majorana fermions.

Fig. 10(a) and Fig. 10(b), we give the results of the $B_c \to P\bar{\chi}\chi$ processes. If we only consider the contribution of $O_7$ or $O_9$, one can see the upper bound of the branching ratios, which are labeled by $ij = 77$ and $ij = 99$, respectively, are less than those resulted by other operators. This means that they do not affect the maximum branch fractions obtained by the second way, namely considering the operators altogether. This leads to the result that the upper limits of the branching ratios of such channels are the same as those in the Majorana case. Correspondingly, the differential branching fractions of two cases are also the same with each other.

In Fig. 10(c) and Fig. 10(d), the results of the $B_c \to V\bar{\chi}\chi$ processes are presented. The upper bound labeled by Total (solid blue line) is the same as that in Fig. 7(c) or Fig. 7(d) when $m_\chi$ is less than 1.2 GeV or 1.3 GeV. It becomes larger when $m_\chi$ continue to increase, as the $\tilde{\Gamma}_{77}$ term, which does not exist in the Majorana case, will give the main contribution. So in this range, there are some differences between the upper bounds obtained in two cases (Of course, if only $O_7$, $O_8$, and $O_9$ give contribution, the Majorana case is not allowed).
Correspondingly, the differential branching ratios, which are plotted in Fig. 11, should also show some differences with those in the Majorana case. When \( m_\chi = 0.4(M - M_f) \), the distribution curves have clearly different shapes from those in Fig. 8(c) and Fig. 8(d). This might provide a way to distinguish between them.

**IV. CONCLUSION**

We have studied the FCNC processes of the \( B \) meson decaying to the invisible spin-1/2 fermions. Both the Majorana and Dirac cases are considered. The effective Lagrangians are introduced to describe the coupling between invisible particles and quarks. By comparing the theoretical predictions of \( \text{BR}(B \to D^{(*)}_{(s)} \bar{\nu} \nu) \) and the experimental upper bounds for \( \text{BR}(B \to D^{(*)}_{(s)} E) \), we can get the constraints of the effective coupling constants. By scanning the allowed parameter space, we can get the upper limits of the branching fractions for the similar processes of \( B_c \) meson. When the final meson is a pseudoscalar, the upper limits of

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**FIG. 8.** The differential branching ratios of \( B_c \) decays to Majorana fermions.
the branching fractions is of the order of $10^{-6}$, and when the final meson is vector, it is of the order of $10^{-5}$. These results both are much larger than the SM background. The differential branching fractions of Majorana and Dirac invisible particles have different shapes when $m_\chi$ is larger than 1.2 or 1.3 GeV. This could provide a way to distinguish between the Majorana type particle from the Dirac one.

FIG. 9. $\tilde{\Gamma}_{ij}$ for $B_c \to h^{(*)}\bar{\chi}\chi$ with $\chi$ being a Dirac fermion.
FIG. 10. The upper limits of branching ratios of $B_c$ decays to Dirac fermions.

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FIG. 11. The differential branching ratios of $B_c$ decays to Dirac fermions.

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