Efficient multipartite entanglement concentration of electron-spin state with charge detection

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We present two entanglement concentration protocols (ECPs) for arbitrary three-electron W state based on their charges and spins. Different from other ECPs, in both two ECPs, with the help of the electronic polarization beam splitter (PBS) and charge detection, the less-entangled W state can be concentrated into a maximally entangled state only with some single charge qubits. The second ECP is more optimal than the first one, for by constructing the complete parity check gate, the second ECP can be used repeatedly to further concentrate the less-entangled state and obtain a higher success probability. Therefore, both the ECPs, especially the second one may be useful in current quantum information processing.

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I. INTRODUCTION

Quantum entanglement plays an important role in current quantum information processing and transmission [1]. The maximally entangled states may be the most important resources in practical quantum communication and computation tasks [1–12]. In most of quantum communication protocols, such as the quantum teleportation [2], quantum key distribution (QKD) [3, 4], quantum densecoding [5–7], and quantum state sharing (QSS) [10–12], they need the maximally entangled state to setup the quantum channel. Unfortunately, the practical transmission channel always contain noise, which will degrade the quality of the entanglement, and make the maximally entangled state become a mixed state or a less-entangled state [13].

The method for distilling a mixed state into a maximally entangled state is called the entanglement purification [13–14]. On the other hand, the way for distilling a pure less-entangled state into a maximally entangled state, which will be detailed here, is called the entanglement concentration, [15–22]. The first entanglement concentration protocol (ECP) was proposed by Bennett et al. in 1996, which is called as the Schmidt projection method [15]. In the protocol, they need some collective and nondestructive measurements, which are not easy to realize under current experimental condition. In 1999, Bose et al. showed that the entangle swapping could also be used to achieve the task of ECP [16]. This protocol was developed by Shi et al. in 2000 [17]. In 2001, two similar ECPs based on the linear optical elements were proposed by Yamamoto et al. and Zhao et al., respectively [18, 19]. The basic idea of these two ECPs is that they adopt the optical polarization beam splitter to complete the parity check measurement for photons. However, they require the sophisticated single-photon detectors to make the photon number detection, which is not likely to be available. These protocols were developed by Zhang et al. in 2008 [20]. They used the cross-Kerr nonlinearity to construct the quantum nondemolition detector to act the roles of both parity check measurement and single photon detector. In 2010, the ECP for single-photon entanglement was also proposed [21].

Up to now, most of the ECPs described above are focused on the bipartite entangled photon systems and there are only several multipartite ECPs. In 2003, Cao and Yang proposed an ECP for W state with joint unitary transformation [24]. In 2007, Zhang et al. proposed an ECP based on the collective Bell-state measurement [25]. In 2010, Wang et al. proposed an ECP for a special W state $|\alpha HV\rangle + \beta (|VHV\rangle + |VVH\rangle)$ [26], where $|H\rangle$ and $|V\rangle$ represent the horizontal and the vertical polarizations of photons, respectively. Later, Yildiz proposed an ECP for three-qubit asymmetric W states [27]. In 2012, Sheng et al. proposed an ECP for W state with the help of cross-Kerr nonlinearity [28].

For quantum communication, photons are the best candidates for carrying the information due to its fast transmission and easy manipulation. Actually, the conduction electrons are also good candidates in quantum information processing, especially in quantum communication. In 2004, by using charge detector, Beenakker et al. broke through the obstacle of the no-go theorem and constructed a controlled-not (CNOT) gate with conduction electrons [29, 30]. In their protocol, the charge detector can distinguish the occupation of the charge number 1 from 0 and 2, but it can not distinguish the charge number 0 and 2. Subsequently, protocols of

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preparing cluster states \[31\], entanglement purification and concentration \[32,37\], and other quantum information protocols were proposed \[38,40\]. As shown in Refs. \[11,43\], the interaction between the polarizations of photons and the electron spins of quantum dots in optical microcavities can also be used to perform the quantum information processing.

In this paper, we present two ECPs for three-electron less-entangled W state with the help of charge detection. In the first ECP, we adopt one pair of less-entangled W state and two auxiliary electrons to perform the concentration. With the help of the polarization beam splitter (PBS) and charge detection, the less-entangled W state can be distilled to the maximally entangled W state with some probability. In the second protocol, we construct the complete parity check gate and make the protocol can be used repeatedly to obtain a higher success probability.

This paper is organized as follows: In Sec. II, we explain the first ECP with the help of PBS and charge detection. In Sec. III, we explain the second ECP with the complete parity check gate. In Sec. IV, we make a discussion and summary.

II. EFFICIENT ECP WITH ELECTRONIC PBS

The schematic drawing of our ECP is shown in Fig. 1. Our ECP includes two steps. In the first concentration step, we suppose Alice, Bob, and Charlie share the three-electron less-entangled W state from \(S_1\), which can be described as

\[
|\Phi\rangle_{a1b1c1} = \alpha |\downarrow\rangle_{a1}|\uparrow\rangle_{b1}|\uparrow\rangle_{c1} + \beta |\uparrow\rangle_{a1}|\downarrow\rangle_{b1}|\uparrow\rangle_{c1} + \gamma |\uparrow\rangle_{a1}|\uparrow\rangle_{b1}|\downarrow\rangle_{c1},
\]

where \(\alpha\), \(\beta\) and \(\gamma\) are the initial entanglement coefficients, and \(|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1\). Here \(|\uparrow\rangle\) and \(|\downarrow\rangle\) represent spin up and spin down, respectively. Then \(S_2\) emits another single flying charge qubit and sends it to Alice with the form

\[
|\Phi\rangle_{a2} = \frac{\alpha}{\sqrt{|\alpha|^2 + |\beta|^2}} |\uparrow\rangle_{a2} + \frac{\beta}{\sqrt{|\alpha|^2 + |\beta|^2}} |\downarrow\rangle_{a2}.
\]

The less-entangled W state combined with the flying qubit can be described as

\[
|\Psi\rangle = |\Phi\rangle_{a1b1c1} \otimes |\Phi\rangle_{a2} = (\alpha |\downarrow\rangle_{a1}|\uparrow\rangle_{a2}|\uparrow\rangle_{c1} + \beta |\uparrow\rangle_{a1}|\downarrow\rangle_{a2}|\uparrow\rangle_{c1} + \gamma |\uparrow\rangle_{a1}|\uparrow\rangle_{a2}|\downarrow\rangle_{c1})
\]

\[
\otimes \left( \frac{\alpha}{\sqrt{|\alpha|^2 + |\beta|^2}} |\uparrow\rangle_{a2} + \frac{\beta}{\sqrt{|\alpha|^2 + |\beta|^2}} |\downarrow\rangle_{a2} \right)
\]

\[
= \frac{\alpha^2}{\sqrt{|\alpha|^2 + |\beta|^2}} |\downarrow\rangle_{a1} |\uparrow\rangle_{a2} |\uparrow\rangle_{b1} |\uparrow\rangle_{c1} + \frac{\beta^2}{\sqrt{|\alpha|^2 + |\beta|^2}} |\uparrow\rangle_{a1} |\downarrow\rangle_{a2} |\uparrow\rangle_{b1} |\uparrow\rangle_{c1} + \frac{\alpha \gamma}{\sqrt{|\alpha|^2 + |\beta|^2}} |\uparrow\rangle_{a1} |\uparrow\rangle_{a2} |\uparrow\rangle_{b1} |\downarrow\rangle_{c1} + \frac{\beta \gamma}{\sqrt{|\alpha|^2 + |\beta|^2}} |\uparrow\rangle_{a1} |\downarrow\rangle_{a2} |\uparrow\rangle_{b1} |\downarrow\rangle_{c1} + \frac{\alpha \beta}{\sqrt{|\alpha|^2 + |\beta|^2}} |\downarrow\rangle_{a1} |\downarrow\rangle_{a2} |\uparrow\rangle_{b1} |\uparrow\rangle_{c1} + \frac{\alpha \beta}{\sqrt{|\alpha|^2 + |\beta|^2}} |\downarrow\rangle_{a1} |\downarrow\rangle_{a2} |\downarrow\rangle_{b1} |\uparrow\rangle_{c1}.
\]

Alice lets the electrons in the spatial modes \(a_1\) and \(a_2\) pass through the PBS\(_1\), which can transmit the spin up \(|\uparrow\rangle\) and reflect the spin down \(|\downarrow\rangle\), respectively. From Eq. \((5)\), after the electrons passing through the charge detector \(C_1\), one can find that the item \(|\downarrow\rangle_{a1} |\uparrow\rangle_{a2}|\uparrow\rangle_{b1}|\uparrow\rangle_{c1}\) will lead \(C_1\) get none electron. The items \(|\uparrow\rangle_{a1} |\downarrow\rangle_{a2} |\downarrow\rangle_{b1}|\uparrow\rangle_{c1}\) and \(|\uparrow\rangle_{a1} |\uparrow\rangle_{a2} |\uparrow\rangle_{b1} |\downarrow\rangle_{c1}\) will lead \(C_1\) get both electrons. The items \(|\uparrow\rangle_{a1} |\uparrow\rangle_{a2} |\uparrow\rangle_{b1} |\downarrow\rangle_{c1}\) and \(|\downarrow\rangle_{a1} |\downarrow\rangle_{a2} |\downarrow\rangle_{b1} |\uparrow\rangle_{c1}\) will lead \(C_1\) get only one electron. Then, if Alice chooses the cases that the charge detector \(C_1\) only obtains one electron, Eq. \((5)\) will become

\[
|\Psi\rangle' = \frac{\alpha^2}{\sqrt{|\alpha|^2 + |\beta|^2}} |\uparrow\rangle_{d1} |\uparrow\rangle_{d2} |\uparrow\rangle_{b1} |\downarrow\rangle_{c1} + \frac{\beta^2}{\sqrt{|\alpha|^2 + |\beta|^2}} |\downarrow\rangle_{d1} |\downarrow\rangle_{d2} |\uparrow\rangle_{b1} |\uparrow\rangle_{c1} + \frac{\alpha \beta}{\sqrt{|\alpha|^2 + |\beta|^2}} |\uparrow\rangle_{d1} |\uparrow\rangle_{d2} |\downarrow\rangle_{b1} |\uparrow\rangle_{c1},
\]

with the probability

\[
P_1 = \frac{|\alpha|^2(|\gamma|^2 + 2|\beta|^2)}{|\alpha|^2 + |\beta|^2}.
\]

|\Psi\rangle' can be normalized as

\[
|\Psi\rangle'' = \frac{\gamma}{\sqrt{|\gamma|^2 + 2|\beta|^2}} |\uparrow\rangle_{d1} |\uparrow\rangle_{d2} |\uparrow\rangle_{b1} |\downarrow\rangle_{c1}.
\]
Finally, Alice measures the electron in the detector D₁, which can make
\[ | \uparrow \rangle \rightarrow \frac{1}{\sqrt{2}} (| \uparrow \rangle + | \downarrow \rangle), \]
\[ | \downarrow \rangle \rightarrow \frac{1}{\sqrt{2}} (| \uparrow \rangle - | \downarrow \rangle). \]
(7)

Then Alice takes the Hadamard operation (H) on the electron in mode d₂, which can make
\[ \frac{\beta}{\sqrt{|\gamma|^2 + 2|\beta|^2}} | \downarrow \rangle d₁ | \uparrow \rangle d₂ | \uparrow \rangle b₁ | \uparrow \rangle c₁. \]
\[ + \frac{\beta}{\sqrt{|\gamma|^2 + 2|\beta|^2}} | \uparrow \rangle d₁ | \uparrow \rangle d₂ | \downarrow \rangle b₁ | \uparrow \rangle c₁. \]
(6)

If they got \( | \Phi \rangle_{d₁b₁c₁} \), one of the three parties, say Alice, Bob or Charlie only should perform a local phase rotation operation on her or his electron, they can get \( | \Phi \rangle_{d₁b₁c₁} \).

After Alice gets the electron in the detector D₁ on the \( \{| \uparrow \rangle, | \downarrow \rangle \} \) basis. It can be found if the measurement result is \( | \uparrow \rangle \), they will get
\[ | \Phi \rangle_{d₁b₁c₁} = \frac{\gamma}{\sqrt{|\gamma|^2 + 2|\beta|^2}} | \uparrow \rangle d₁ | \uparrow \rangle b₁ | \downarrow \rangle c₁ \]
\[ + \frac{\beta}{\sqrt{|\gamma|^2 + 2|\beta|^2}} | \downarrow \rangle d₁ | \uparrow \rangle b₁ | \uparrow \rangle c₁, \]
(8)

while if the measurement result is \( | \downarrow \rangle \), they will get
\[ | \Phi \rangle_{d₁b₁c₁} = \frac{\gamma}{\sqrt{|\gamma|^2 + 2|\beta|^2}} | \uparrow \rangle d₁ | \uparrow \rangle b₁ | \downarrow \rangle c₁ \]
\[ - \frac{\beta}{\sqrt{|\gamma|^2 + 2|\beta|^2}} | \downarrow \rangle d₁ | \uparrow \rangle b₁ | \uparrow \rangle c₁, \]
(9)

If they got \( | \Phi \rangle_{d₁b₁c₁} \), and if the measurement result is \( | \downarrow \rangle \), they will get
\[ | \Phi \rangle_{c₂} = \frac{\beta}{\sqrt{|\gamma|^2 + |\beta|^2}} | \downarrow \rangle c₂ + \frac{\gamma}{\sqrt{|\gamma|^2 + |\beta|^2}} | \uparrow \rangle c₂. \]
(10)

After Charlie receives the single mobile electron, the whole system can be described as:
\[ | \Phi \rangle_{d₁b₁c₁} \otimes | \Phi \rangle_{c₂} \]
\[ = \frac{\beta \gamma}{\sqrt{|\gamma|^2 + 2|\beta|^2} \sqrt{|\gamma|^2 + |\beta|^2}} | \uparrow \rangle d₁ | \uparrow \rangle b₁ | \downarrow \rangle c₁ \]
\[ + \frac{\gamma^2}{\sqrt{|\gamma|^2 + 2|\beta|^2} \sqrt{|\gamma|^2 + |\beta|^2}} | \uparrow \rangle d₁ | \uparrow \rangle b₁ | \uparrow \rangle c₁, \]
\[ + \frac{\beta^2}{\sqrt{|\gamma|^2 + 2|\beta|^2} \sqrt{|\gamma|^2 + |\beta|^2}} | \downarrow \rangle d₁ | \uparrow \rangle b₁ | \downarrow \rangle c₁ \]
\[ + \frac{\beta \gamma}{\sqrt{|\gamma|^2 + 2|\beta|^2} \sqrt{|\gamma|^2 + |\beta|^2}} | \downarrow \rangle d₁ | \downarrow \rangle b₁ | \uparrow \rangle c₁, \]
\[ + \frac{\beta^2}{\sqrt{|\gamma|^2 + 2|\beta|^2} \sqrt{|\gamma|^2 + |\beta|^2}} | \uparrow \rangle d₁ | \downarrow \rangle b₁ | \uparrow \rangle c₁ | \downarrow \rangle c₂ \]
\[ + \frac{\beta \gamma}{\sqrt{|\gamma|^2 + 2|\beta|^2} \sqrt{|\gamma|^2 + |\beta|^2}} | \uparrow \rangle d₁ | \downarrow \rangle b₁ | \uparrow \rangle c₁ | \uparrow \rangle c₂. \]
(11)

Then, Charlie makes the electrons in the spatial modes c₁ and c₂ pass through the PBS₂. It is interesting to find that if the measurement result is \( | \uparrow \rangle d₁ | \uparrow \rangle b₁ | \downarrow \rangle c₁ | \uparrow \rangle c₂ \), they will lead both the two electrons in Charlie’s location in the spatial mode e₁, which makes the charge detector C₂ cannot detect any electrons. The items \( | \downarrow \rangle d₁ | \uparrow \rangle b₁ | \downarrow \rangle c₁ | \downarrow \rangle c₂ \) and \( | \uparrow \rangle d₁ | \downarrow \rangle b₁ | \uparrow \rangle c₁ | \downarrow \rangle c₂ \) will lead both the two electrons in the e₂ mode, which makes the charge detector C₂ detect two electrons. All the three items \( | \downarrow \rangle d₁ | \uparrow \rangle b₁ | \downarrow \rangle c₁ | \downarrow \rangle c₂ \), \( | \uparrow \rangle d₁ | \uparrow \rangle b₁ | \downarrow \rangle c₁ | \downarrow \rangle c₂ \) and \( | \uparrow \rangle d₁ | \downarrow \rangle b₁ | \uparrow \rangle c₁ | \downarrow \rangle c₂ \) make the spatial mode e₂ contains only one electron, which makes the detector C₂ detect exactly one electron. Then Charlie selects the items which makes C₂ detect only one electron, then the whole state will essentially collapse to the four-electron maximally entangled W state:
\[ | \Psi'' \rangle = \frac{1}{\sqrt{3}} (| \uparrow \rangle d₁ | \uparrow \rangle b₁ | \downarrow \rangle c₁ | \downarrow \rangle c₂ + | \downarrow \rangle d₁ | \uparrow \rangle b₁ | \uparrow \rangle c₁ | \downarrow \rangle c₂ + | \downarrow \rangle d₁ | \downarrow \rangle b₁ | \uparrow \rangle c₁ | \uparrow \rangle c₂), \]
(12)

with the probability of
\[ P₂ = \frac{3|\beta|^2|\gamma|^2}{(|\gamma|^2 + |\beta|^2)(|\gamma|^2 + |\beta|^2)} \]
(13)

where the subscription 2 means in the second concentration step.

Then, similar to the first step, Charlie performs the Hadamard operation on the electron in the e₂ mode. After the Hadamard operation, he detects the electron in the e₂ mode on the \( \{| \uparrow \rangle, | \downarrow \rangle \} \) basis by the detector D₂.

If the measurement result is \( | \uparrow \rangle \), they will get
\[ | \Phi \rangle_{d₁b₁c₁} = \frac{1}{\sqrt{3}} (| \uparrow \rangle d₁ | \uparrow \rangle b₁ | \downarrow \rangle c₁ \]
\[ + | \downarrow \rangle d₁ | \uparrow \rangle b₁ | \uparrow \rangle c₁ + | \downarrow \rangle d₁ | \downarrow \rangle b₁ | \uparrow \rangle c₁), \]
(14)

Otherwise, if the measurement result is \( | \downarrow \rangle \), they will get
\[ | \Phi \rangle_{d₁b₁c₁} = \frac{1}{\sqrt{3}} (-| \uparrow \rangle d₁ | \uparrow \rangle b₁ | \downarrow \rangle c₁ \]
\[ + | \downarrow \rangle d₁ | \uparrow \rangle b₁ | \uparrow \rangle c₁ + | \downarrow \rangle d₁ | \downarrow \rangle b₁ | \uparrow \rangle c₁), \]
(15)

In this case, one of the three parties, say Alice, Bob or Charlie should perform a local phase rotation operation on her or his electron, then they can get \( | \Phi \rangle_{d₁b₁c₁} \). So far, the whole ECP is completed. We can calculate its
total success probability, which equals the product of the probability in each concentration step as

\[ P = P_1 P_2 = \frac{(|\gamma|^2 + 2|\beta|^2)}{|\alpha|^2 + |\beta|^2} \frac{3|\beta|^2|\gamma|^2}{(|\gamma|^2 + |\beta|^2)(|\gamma|^2 + 2|\beta|^2)} \]

\[ = \frac{3|\alpha|^2|\beta|^2|\gamma|^2}{(|\alpha|^2 + |\beta|^2)(|\gamma|^2 + |\beta|^2)}. \quad (16) \]

III. THE SECOND ECP WITH COMPLETE PARITY CHECK GATE

By far, we have described our ECP with PBSs and charge detectors. In fact, in the ECP, PBS combined with charge detectors acts the role of parity checking. During the above description, Alice and Charlie essentially pick up the even parity states |↑⟩|↑⟩ and |↓⟩|↓⟩, but discard the odd parity states |↑⟩|↓⟩ and |↓⟩|↑⟩, for after the PBS, the |↑⟩|↓⟩ and |↓⟩|↑⟩ will lead the two electrons both in the same spatial mode. We take Eq. (3) as an example. One can find that the item |↓⟩a1|↑⟩a2|↑⟩b1|↑⟩c1 will lead C1 get none electron, and the items |↑⟩a1|↓⟩a2|↓⟩b1|↑⟩c1 and |↑⟩a1|↓⟩a2|↑⟩b1|↓⟩c1 will lead C1 get both electrons. Therefore, as only one PBS combined with charge detector can only pick up the even parity states but has to discard the odd states, it cannot make a complete parity check.

![FIG. 2: A schematic drawing of our complete parity check gate (P). It is also shown in Ref. 26. It can completely distinguish the even parity states |↑⟩|↑⟩ and |↓⟩|↓⟩ from the odd parity states |↑⟩|↓⟩ and |↓⟩|↑⟩.](image)

In this section, we will describe another ECP with the complete parity check gate. Before we start to explain this ECP, we briefly explain the complete parity check, as shown in Fig. 2. In fact, this parity check has been studied for several years to construct the controlled-not (CNOT) gate [29], and perform the entanglement concentration and purification [32]. From Fig. 2, we suppose that two electrons |ψ1⟩ = α1|↑⟩a1 + β1|↓⟩a1 and |ψ2⟩ = α2|↑⟩b1 + β2|↓⟩b1 enter the complete parity check gate from the spatial modes a1 and b1, respectively. The whole state can be described as

\[ |\varphi_1⟩ \otimes |\varphi_2⟩ = (|α_1⟩|↑⟩a_1 + |β_1⟩|↓⟩a_1) \otimes (|α_2⟩|↑⟩b_1 + |β_2⟩|↓⟩b_1) \]

\[ = |\alpha_1\alpha_2⟩|↑⟩a_1|↑⟩b_1 + |\beta_1\beta_2⟩|↓⟩a_1|↓⟩b_1 + |\alpha_1\beta_2⟩|↑⟩a_1|↓⟩b_1 + |\beta_1\alpha_2⟩|↓⟩a_1|↑⟩b_1. \quad (17) \]

It is obvious that items |α1α2⟩|↑⟩a1|↑⟩b1 + |β1β2⟩|↓⟩a1|↓⟩b1 will lead C1 = 1. After the complete parity check gate, they will finally convert to another less-entangled W state of odd parity state |α1α2⟩|↑⟩a1|↓⟩b1 + |β1β2⟩|↓⟩a1|↑⟩b1. The items |α1β2⟩|↑⟩a1|↓⟩b1 + |β1α2⟩|↓⟩a1|↑⟩b1 will lead C1 = 0. After the complete parity check gate, they will become |α1β2⟩|↑⟩a1|↓⟩b1 + |β1α2⟩|↓⟩a1|↑⟩b1 from the odd parity state |α1β2⟩|↑⟩a1|↓⟩b1 + |β1α2⟩|↓⟩a1|↑⟩b1.

![FIG. 3: A schematic drawing of our ECP with P gates. We adopt the P gates shown in Fig. 2 to replace the PBSs to reconstruct this ECP and make it have higher success probability.](image)

Now we use the complete parity check gate to substitute the PBS to reperform the ECP. We denote it as P gate in Fig. 2. From Eq. (3), Alice makes the electrons in a1 and a2 modes pass through the P gate. After the P gate, if the charge detector shows C1 = 1, they will get the same state in Eq. (4), which can be finally converted to Eq. (8) or Eq. (9) following the same step described above. On the other hand, if the charge detector shows C1 = 0, they will get a four-electron less-entangled state as

\[ |\Psi⟩_{d_1d_2b_1c_1} = \frac{α^2}{\sqrt{|α|^2 + |β|^2}} |↓⟩a_1 |↑⟩d_1 |↑⟩d_2 |↑⟩b_1 |↑⟩c_1 + \frac{β^2}{\sqrt{|α|^2 + |β|^2}} |↑⟩a_1 |↓⟩d_1 |↓⟩d_2 |↓⟩b_1 |↑⟩c_1 + \frac{βγ}{\sqrt{|α|^2 + |β|^2}} |↑⟩a_1 |↑⟩d_1 |↓⟩d_2 |↓⟩b_1 |↓⟩c_1. \quad (18) \]

Then, Alice measures the electron in d2 mode on the {|↑⟩, |↓⟩} basis. After the Hadamard operation, Eq. (18) will convert to another less-entangled W state of the form

\[ |\Psi⟩_{a_1b_1c_1} = α′|↓⟩a_1|↑⟩a_1|↑⟩c_1 ± β′|↑⟩a_1|↓⟩b_1|↑⟩c_1 ± γ|↑⟩a_1|↑⟩b_1|↓⟩c_1. \quad (19) \]
with

\[
\alpha' = \frac{\alpha^4}{\sqrt{|\alpha|^4 + |\beta|^4 + |\beta|^2 |\gamma|^2}},
\]
\[
\beta' = \frac{\beta^4}{\sqrt{|\alpha|^4 + |\beta|^4 + |\beta|^2 |\gamma|^2}},
\]
\[
\gamma' = \frac{\beta^2 \gamma^2}{\sqrt{|\alpha|^4 + |\beta|^4 + |\beta|^2 |\gamma|^2}}.
\]

(20)

'+' or '-' depends on the measurement results. If the measurement result of D1 is \(| \uparrow \rangle\), it is '+'; otherwise, it is '-' . The state \( |\Psi_1^+\rangle_{a1b1c1} \) can be reconcentrated with the same principle. Alice only needs to prepare another single electron state from \( S_2 \) of the form

\[
|\Phi\rangle_{a2} = \frac{\alpha'}{\sqrt{|\alpha'|^2 + |\beta'|^2}} |\uparrow\rangle_{a1} + \frac{\beta'}{\sqrt{|\alpha'|^2 + |\beta'|^2}} |\downarrow\rangle_{a2}.
\]

(21)

After making the electrons in the a1 and a2 modes pass through the \( P \) gate, if the charge detector shows \( C = 1 \), the combination \( |\Phi\rangle_{a2} \otimes |\Psi\rangle_{a1b1c1} \) will become

\[
|\Psi\rangle' = \frac{\alpha'\gamma'}{\sqrt{|\alpha'|^2 + |\beta'|^2}} |\uparrow\rangle_{a1} |\uparrow\rangle_{a2} |\uparrow\rangle_{b1} |\downarrow\rangle_{c1}
+ \frac{\beta'\gamma'}{\sqrt{|\alpha'|^2 + |\beta'|^2}} |\downarrow\rangle_{a1} |\downarrow\rangle_{a2} |\uparrow\rangle_{b1} |\uparrow\rangle_{c1}.
\]

(22)

Otherwise, if \( C = 0 \), they will ultimately get another less-entangled W state as

\[
|\Psi\rangle''_{a1b1c1} = \alpha'' |\downarrow\rangle_{a1} |\uparrow\rangle_{b1} |\uparrow\rangle_{c1} \pm \beta'' |\uparrow\rangle_{a1} |\downarrow\rangle_{b1} |\uparrow\rangle_{c1}
\pm \gamma'' |\uparrow\rangle_{a1} |\downarrow\rangle_{b1} |\downarrow\rangle_{c1}.
\]

(23)

with

\[
\alpha'' = \frac{\alpha^4}{\sqrt{|\alpha|^4 + |\beta|^4 + |\beta|^2 |\gamma|^2}},
\]
\[
\beta'' = \frac{\beta^4}{\sqrt{|\alpha|^4 + |\beta|^4 + |\beta|^2 |\gamma|^2}},
\]
\[
\gamma'' = \frac{\beta^2 \gamma^2}{\sqrt{|\alpha|^4 + |\beta|^4 + |\beta|^2 |\gamma|^2}}.
\]

(24)

In this way, they can repeat the whole protocol to get a higher success probability. We can calculate the success probability in the second round as

\[
P^1_2 = \frac{|\alpha|^2 |\beta|^2 |\gamma|^2 + 2 |\gamma|^4}{(|\alpha|^4 + |\beta|^4) (|\alpha|^2 + |\beta|^2)}.
\]

(25)

We can also calculate the success probability in the third and other round as

\[
P^1_3 = \frac{|\alpha|^2 |\beta|^2 |\gamma|^2 + 2 |\gamma|^4}{(|\alpha|^8 + |\beta|^8) (|\alpha|^2 + |\beta|^2)},
\]

... 

\[
P^N_1 = \frac{|\alpha|^{2N} |\beta|^{2N-2} |\gamma|^2 + 2 |\beta|^{2N}}{(|\alpha|^{2N} + |\beta|^{2N}) (|\alpha|^{2N-1} + |\beta|^{2N-1}) (|\alpha|^2 + |\beta|^2)}.
\]

(26)

Here the superscription "1" means the first step performed by Alice. The subscription "1", "2", "3", ..., "N" is the iteration number. Then the total success probability of the first concentration step equals the sum of the probability in each round, which can be written as

\[
P^1 = \sum_{N=1}^{\infty} P^N_1.
\]

(27)

The same principle can also be used in Charlie's location. For example, after passing through the \( P \) gate, if \( C = 1 \), \( |\Phi\rangle_{a1b1c1} \otimes |\Phi\rangle_{c2} \) will become the maximally entangled W state. Otherwise, if \( C = 0 \), they will get

\[
|\Psi_2\rangle_{a1b1c1} = \gamma' |\uparrow\rangle_{a1} |\uparrow\rangle_{b1} |\downarrow\rangle_{c1} \pm \beta' |\uparrow\rangle_{a1} |\downarrow\rangle_{b1} |\uparrow\rangle_{c1}
\pm \alpha' |\downarrow\rangle_{a1} |\uparrow\rangle_{b1} |\uparrow\rangle_{c1}.
\]

(28)

\[
|\Psi_2\rangle_{a1b1c1} = \alpha'' |\downarrow\rangle_{a1} |\uparrow\rangle_{b1} |\uparrow\rangle_{c1} \pm |\beta'' |\downarrow\rangle_{a1} |\downarrow\rangle_{b1} |\uparrow\rangle_{c1}
\pm |\gamma'' |\uparrow\rangle_{a1} |\downarrow\rangle_{b1} |\downarrow\rangle_{c1}.
\]

(29)

After performing the Hadamard operation and measuring the electron in c2 mode, it will become

\[
|\Phi\rangle_{c2} = \frac{\beta^2}{\sqrt{|\gamma|^4 + |\beta|^4}} |\uparrow\rangle_{c2} + \frac{\gamma^2}{\sqrt{|\gamma|^4 + |\beta|^4}} |\downarrow\rangle_{c2}.
\]

(31)

Following the same principle described above, the combination state \( |\Psi_2\rangle_{a1b1c1} \otimes |\Phi\rangle_{c2} \) can also ultimately become the maximally entangled W state with some probability. In this way, we have proved that the second concentration step can also be repeated to get a higher success probability. We can calculate the success probability
in each concentration round as

\[ P_{1}^{2} = \frac{3|\beta|^{2}|\gamma|^{2}}{(|\gamma|^{2} + |\beta|^{2})(|\gamma|^{2} + 2|\beta|^{2})}, \]

\[ P_{2}^{2} = \frac{3|\beta|^{4}|\gamma|^{4}}{(|\gamma|^{2} + 2|\beta|^{2})(|\gamma|^{4} + |\beta|^{4})(|\gamma|^{2} + |\beta|^{2})}, \]

\[ P_{3}^{2} = \frac{3|\beta|^{8}|\gamma|^{8}}{(|\gamma|^{2} + 2|\beta|^{2})(|\gamma|^{8} + |\beta|^{8})(|\gamma|^{4} + |\beta|^{4})(|\gamma|^{2} + |\beta|^{2})}, \]

\[ \vdots \]

\[ P_{M}^{2} = \frac{3|\beta|^{2M}|\gamma|^{2M}}{(|\gamma|^{2M} + |\beta|^{2M})(|\gamma|^{2M-1} + |\beta|^{2M-1}) \cdots (|\gamma|^{2} + |\beta|^{2})}. \]

Here the superscription "2" means the second step performed by Charlie. The subscription "1", "2", "3", \ldots "M" is the iteration number. The total success probability of the second concentration step equals the sum of the probability in each round, which can be written as

\[ P^{2} = \sum_{M=1}^{\infty} P_{M}^{2} \quad (33) \]

Finally, the total success probability of the whole ECP equals the product of the probability in each two concentration step as

\[ P_{T} = \sum_{N=1}^{\infty} \sum_{M=1}^{\infty} P_{N}^{1} P_{M}^{2}. \quad (34) \]

The total success probability of the two ECPs as a function of the \( \alpha^{2} \) is shown in Fig. 4. We choose \( \beta^{2} = \frac{1}{3}, \) and change \( \alpha^{2} \in (0, \frac{2}{3}) \). For the second ECP, we choose \( N = M = 3 \) for a good numerical simulation. It is shown that when \( \alpha^{2} \in (0, \frac{1}{3}) \) both success probability monotonic increase with \( \alpha^{2} \). They both have a maximally value when \( \alpha^{2} = \frac{1}{3} \). Moreover, it is obvious that by repeating the second ECP, the total success probability can be increased largely.

**IV. DISCUSSION AND SUMMARY**

By far, we have fully described our two ECPs for three-electron less-entangled W states. The two ECPs only need one pair of less-entangled W state, while other ECPs require two pairs of such states. It makes our two ECPs more economic than others. In the first ECP, we adopt the PBS combined with the charge detector to pick up the even parity states. However, the odd parity states have to be discarded. Therefore, it is not an optimal ECP. The second ECP is an improved protocol. In the second ECP, with the help of two PBSs and charge detectors, both the even parity states and the odd parity states can be remained. By repeating the ECP, the odd parity states can be recombined to the maximally entangled states with some probability. So the whole protocol can reach a higher success probability than the first protocol. Certainly, in both ECPs, we should know the exact coefficients \( \alpha, \beta \) and \( \gamma \) in advance to prepare the auxiliary single electron state. According to Ref. [23], in a practical concentration, the exact initial coefficients can be obtained by measuring enough amount of initial less-entangled samples[23].

Charge detection has played a prominent role in constructing the parity check gate. In fact, in Refs. [18, 19], the optical PBS can also act as the role of the parity check gate. However, in their protocols, after the photons passing through the PBS, one has to use the sophisticated single-photon detectors to ensure both spatial modes exactly contain only one photon. Moreover, the electrons will be destroyed if it is detected by the detector, which is so called the post-selection principle. Interestingly, in our first protocol, the charge detector essentially acts the similar role as the sophisticated single-photon detector. If its measurement result is \( C = 1 \), the two spatial modes will exactly contain one electron. Moreover, the charge detection can not destroy the electron, that is to say, after the charge detection, the electrons can be remained for other application. Under present experimental conditions, the charge detection can be realized by means of the point contacts in a two-dimensional electron gas [43]. Elzerman et al. once showed that the current achievable time resolution for the charge detection is \( \mu s \) [44]. Trauzettel et al. put forward a protocol for realizing such charge parity meter by using two double quantum dots along side a quantum point contact [38]. Mao et al. proposed a more universal method to realize such device [46]. Another important element of these ECPs is the PBS in spin. Ref. [47] described a such device with the help of
the beam splitters and the electric field. The total efficiency of the PBS can reach 100% in principle. In 2004, based on the PBS in spin, the electrical quantum computation protocol with mobile spin qubits was proposed.

In summary, we have proposed two ECPs for three-electron less-entangled W state with the help of the charge detection. Both the two protocols only require one pair of less-entangled W state, which makes them more economical. In the first ECP, by picking up the even parity states, but discarding the odd parity states, we can ultimately obtain the maximally entangled W state with some success probability. The second ECP is an improved one, for it can be used repeatedly to recombine the odd parity states and obtain a higher success probability. These features will make these ECPs have a practical application in current solid quantum computation and communication.

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