Hybrid synchronization of hyperchaotic n-scroll Chua circuit using adaptive backstepping control

Suresh Rasappan

Cogent Engineering (2015), 2: 1009273
Hybrid synchronization of hyperchaotic n-scroll Chua circuit using adaptive backstepping control

Suresh Rasappan

Abstract: In this paper, hybrid synchronization is investigated for n-scroll hyperchaotic Chua circuit using adaptive backstepping control. The theorem on hybrid synchronization for n-scroll hyperchaotic Chua circuit is established using Lyapunov stability theory. The backstepping scheme is recursive procedure that links the choice of Lyapunov function with the design of a controller and guarantees global stability performance of strict-feedback nonlinear systems. The backstepping control method is effective and convenient to hybrid synchronize the hyperchaotic systems which are mainly in this technique that gives the flexibility to construct a control law. Numerical simulations are also given to illustrate and validate the hybrid synchronization results derived in this paper.

Subjects: Science; Technology; Systems & Control

Keywords: synchronization; Chaos; adaptive backstepping control; n-scroll hyperchaotic Chua circuit

ABOUT THE AUTHOR

Suresh Rasappan obtained his PhD degree in Mathematics from Vel Tech Rangarajan Dr Sakunthala R & D Institute of Science and Technology, Chennai, Tamil Nadu, India in 2013, and MPhil degree in Mathematics from Bharathiar University, Coimbatore, Tamil Nadu, India in 2008. He is currently working as an associate professor in the Department of Mathematics, Vel Tech Dr RR & Dr SR Technical University, Chennai, India. He has published over 28 papers in international journals and book chapters. His research interest is differential equations.

PUBLIC INTEREST STATEMENT

Chaos synchronization can be applied in the areas of physics, engineering and biological science. Synchronization has been widely explored in a variety of fields including physical chemical, and ecological systems, secure communications etc. Synchronization of chaotic systems is a phenomenon that may occur when two or more chaotic oscillators are coupled, or when a chaotic oscillator drives another chaotic oscillator. Because the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a challenging problem. In most synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the master or drive system and another chaotic system is called the slave or response system, then the idea of synchronization is to use the output of the master system to control the slave system so that the output of the response system tracks the output of the master system asymptotically.
1. Introduction

Synchronization in chaos refers to the tendency of two or more systems which are coupled together to undergo closely related motion, even when the motions are chaotic.

The synchronization for chaotic systems has been widespread to the scope (Alligood, Sauer, & Yorke, 1997; Fujisaka & Yamada, 1983; Pecora & Carroll, 1990), such as generalized synchronization (Harmov, Koronovskii, & Moskalenko, 2005a; 2005b; Wang & Zhu, 2006), anti-synchronization, phase synchronization (Ge & Chen, 2006; Tokuda, Kurths, Kiss, & Hudson, 2008; Zhao, Lai, Wang, & Gao, 2004), lag synchronization, projective synchronization (Qiang, 2007), and generalized projective synchronization (Jian-Ping & Chang-Pin, 2006; Li, Xu & Li, 2007).

The property of anti-synchronization establishes a predominating phenomenon in symmetrical oscillators, in which the state vectors have the same absolute values but opposite signs.

When synchronization and anti-synchronization coexist, simultaneously, in chaotic systems, then that synchronization is called hybrid synchronization.

A variety of schemes to ensure the control and synchronization of such systems have been demonstrated based on their potential applications in various fields including chaos generator design, secure communication (Chen, 1996; Kanter, Kopelowitz, Kestler, & Kinzel, 2008; Yang & Chua, 1999), physical systems (Chern & Otsuka, 2012; Lakshmanan & Murali, 1996; Moreno & Pacheco, 2004), chemical reaction (Coffman, McCormick, Noszticzius, & Simoyi, 1987; Han, Kerrer, & Kuramoto, 1995), ecological systems (Blasius & Huppert, 1999), information science (Bauer, Atay, & Jost, 2010; Ghosh, Banerjee, & Chowdhury, 2007; Kocarev & Parlitz, 1995), energy resource systems, ghostbuster neurons (Wang, Chen & Deng, 2009), biaxial magnet models (Moukam Kakmeni, Nguenang, & Kofane, 2006), neuronal models (Che, Wang, Tsang, & Chen, 2010; Hindmarsh & Rose, 1984; Qi, Huang, Chen, Wang, & Shen, 2008), IR epidemic models with impulsive vaccination (Zeng, Sun, Li, & Sun, 2005), and predicting the influence of solar wind to celestial bodies (Junxa, Dianchen, & Tian, 2006; Suresh & Sundarapandian, 2012a).

So far a variety of impressive approaches have been proposed for the synchronization of the chaotic systems such as OGY method (Ott, Grebogi, & Yorke, 1990), sampled feedback synchronization method (Murali & Lakshmanan, 2003), time delay feedback method (Park & Kwon, 2003), adaptive design method (Lu, Wu & Han, 2004; Park, 2008; Park, Lee, & Kwon, 2007), sliding mode control method (Ya, 2004), active control method (Sundarapandian & Suresh, 2010). and backstepping control design (Suresh & Sundarapandian, 2012b; Wu & Lu, 2003; Yu & Zhang, 2006).

Recently, backstepping method has been developed and designed to control the chaotic systems. A common concept in the method is to synchronize the chaotic system. The backstepping method is based on the mathematical model of the examined system, introducing new variables into a form depending on the state variables, controlling parameters, and stabilizing functions. The difficult work of synchronizing the chaotic system is to remove nonlinearities which were done in the system and influencing the stability of state operation. The use of backstepping method creates an additional nonlinearity and eliminates undesirable nonlinearities from the system (Suresh & Sundarapandian, 2012c; 2013; Wang, Zhang, & Guo, 2010; Wang 2011a, 2011b).

The uncertainties are commonly in chaos synchronization and other control system problems. The uncertainties are one of the main factors in leading the adaptive-based synchronization. Adaptive control design is a direct aggregation of control methodology with some form of recursive system which identifies the system to determine the control of linear or nonlinear systems.

Adaptive control design is studied and analyzed in theory of unknown, but fixed parameter systems. The controller feedback gain could be depending on the system parameter.
2. Problem statement

Consider the chaotic system described by the dynamics

\[ \dot{x}_1 = F_1(x_1, x_2, \ldots, x_n, \alpha_i) \]
\[ \dot{x}_2 = F_2(x_1, x_2, \ldots, x_n, \alpha_i) \]
\[ \vdots \]
\[ \dot{x}_n = F_n(x_1, x_2, \ldots, x_n, \alpha_i) \]  \hspace{1cm} (1)

where \( x \in \mathbb{R}^n \) is the state of the system, in which the system (1) is considered as the master system; \( \alpha_i \) is the unknown parameter, \( \hat{\alpha}_i \) is the estimates as the parameter \( \alpha_i \).

The slave system is a chaotic system with the controller \( u = [u_1, u_2, u_3 \ldots, u_n]^T \) described by the dynamics

\[ \dot{y}_1 = G_1(y_1, y_2, \ldots, y_n, \alpha_i) + u_1(t) \]
\[ \dot{y}_2 = G_2(y_1, y_2, \ldots, y_n, \alpha_i) + u_2(t) \]
\[ \dot{y}_3 = G_3(y_1, y_2, \ldots, y_n, \alpha_i) + u_3(t) \]
\[ \vdots \]
\[ \dot{y}_n = G_n(y_1, y_2, \ldots, y_n, \alpha_i) + u_n(t) \]  \hspace{1cm} (2)

where \( u_i \) is the input to the system with parameter estimator \( \hat{\alpha}_i, i = 1, 2, \ldots, n \) and \( y \in \mathbb{R}^n \) is the state of the slave system and \( F_i, G_i (i = 1, 2, 3 \ldots n) \) linear or nonlinear functions with input from systems (1) and (2).

If \( F_i = G_i \) for all \( i \), then the system (1) and (2) are called identical and otherwise they are nonidentical chaotic systems.

The hybrid synchronization error is defined as

\[ e_i = \begin{cases} y_i - x_i & \text{if } i \text{ is odd} \\ y_i + x_i & \text{if } i \text{ is even} \end{cases} \]  \hspace{1cm} (3)

Then the synchronization error dynamics is obtained as

\[ \dot{e}_1 = G_1(y_1, y_2, \ldots, y_n, \alpha_i) - F_1(x_1, x_2, \ldots, x_n, \alpha_i) + u_1 \]
\[ \dot{e}_2 = G_2(y_1, y_2, \ldots, y_n, \alpha_i) + F_2(x_1, x_2, \ldots, x_n, \alpha_i) + u_2 \]
\[ \vdots \]
\[ \dot{e}_n = G_n(y_1, y_2, \ldots, y_n, \alpha_i) + (-1)^n F_n(x_1, x_2, \ldots, x_n, \alpha_i) + u_n \]  \hspace{1cm} (4)

The parameter estimation error is defined as

\[ e_{\alpha_i} = \alpha_i - \hat{\alpha}_i \]

The hybrid synchronization problem basically requires the global asymptotically stability of the error dynamics (4), i.e.

\[ \lim_{t \to \infty} ||e(t)|| = 0 \]  \hspace{1cm} (5)

for all initial conditions \( e(0) \in \mathbb{R}^n \).
Backstepping design procedure is recursive and guarantee global stability performance of strictfeedback chaotic systems. By using the backstepping design, at the $i$th step, the $i$th order subsystem is stabilized with respect to a Lyapunov function $V_i$ by the virtual control $\alpha_i$ and a control input function $u_i$.

Consider the global asymptotic stability of the system

$$\dot{e}_1 = G_1(y_1, y_2, \ldots, y_n) - F_1(x_1, x_2, \ldots, x_n, \alpha_i) + u_1$$  \hspace{1cm} (6)$$

where $u_i$ is control input, which is the function of the error vector $e_i$ and the state variables $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^n$. As long as this feedback stabilizes, the system (6) will converge to zero as $t \to \infty$, where $e_2 = \alpha_1(e_1)$ is regarded as a virtual controller.

For the design of $\alpha_1(e_1)$ is to stabilize the subsystem (6), the Lyapunov function is defined by

$$V_1(e) = e^T P_1 e_1 + \sum_{i=1}^{k} e_i^T R_i e_i$$  \hspace{1cm} (7)$$

where $P_1$, and $R_i$ are positive definite matrices.

The derivative of $e_i$ is

$$\dot{e}_i = -\hat{\alpha}_i$$  \hspace{1cm} (8)$$

Suppose the derivative of $V_1$ is

$$\dot{V}_1 = -e_1^T Q_1 e_1 - \sum_{i=1}^{k} e_i^T S_i e_i$$  \hspace{1cm} (9)$$

where $Q_1$, and $S_i$ are positive definite matrices.

Then $\dot{V}_1$ is a negative definite function.

Thus by Lyapunov stability theory, the error dynamics (6) is globally asymptotically stable.

The function $\alpha_1(e_1)$ is an estimative function when $e_2$ is considered as a controller.

The error between $e_2$ and $\alpha_1(e_1)$ is

$$w_2 = e_2 - \alpha_1(e_1)$$  \hspace{1cm} (10)$$

Consider the $(e_1, w_2)$ subsystem given by

$$\dot{e}_1 = G_1(y_1, y_2, \ldots, y_n, \alpha_i) - F_1(x_1, x_2, \ldots, x_n, \alpha_i) + u_1$$

$$\dot{w}_2 = G_2(y_1, y_2, \ldots, y_n) + F_2(x_1, x_2, \ldots, x_n, \alpha_i) - \hat{\alpha}_1(e_1) + u_2$$  \hspace{1cm} (11)$$

Let $e_3$ as a virtual controller in system (11).

Assume that when

$$e_3 = \alpha_2(e_1, w_2)$$  \hspace{1cm} (12)$$
the system (11) is made globally asymptotically stable.

Consider the Lyapunov function defined by

\[ V_2(e_2, w_2) = V_1(e_1) + w_2^TP_2w_2 + \sum_{i=k+1}^{m} e_i^TR_2e_i \]  

(13)

where \( P_2 \) and \( R_2 \) are positive definite matrices.

Suppose the derivative of \( V_2(e_1, w_2) \) is

\[ \dot{V}_2 = -e_1^TQ_1e_1 - w_2^TQ_2w_2 - \sum_{i=k+1}^{m} e_i^TS_2e_i \]  

(14)

where \( Q_1, Q_2, \) and \( S_2 \) are positive definite matrices.

Then \( \dot{V}_2(e_1, w_2) \) is a negative definite function.

Thus by Lyapunov stability theory, the error dynamics (11) is globally asymptotically stable. The virtual controller \( e_3 = \alpha_1(e_1, w_2) \) and the state feedback input \( u_2 \) make the system (11) asymptotically stable.

For the \( n \)th state of the error dynamics, define the error variable \( w_n \) as

\[ w_n = e_n - \alpha_{n-1}(e_1, w_2, \ldots, w_{n-1}) \]  

(15)

Considering the \( (e_1, w_2, \ldots, w_n) \) subsystem given by

\[ \begin{align*}
\dot{e}_1 &= G_1(y_1, y_2, \ldots, y_n, \alpha_i) \\
&\quad - F_1(x_1, x_2, \ldots, x_n, \alpha_i) + u_1 \\
\dot{w}_2 &= G_2(y_1, y_2, \ldots, y_n, \alpha_i) \\
&\quad + F_2(x_1, x_2, \ldots, x_n) - \dot{\alpha}_1(e_1) + u_2 \\
&\vdots \\
\dot{w}_n &= G_n(y_1, y_2, \ldots, y_n, \alpha_i) \\
&\quad - F_n(x_1, x_2, \ldots, x_n, \alpha_i) \\
&\quad - \dot{\alpha}_{n-1}(e_1, w_2, \ldots, w_{n-1}) + u_n
\end{align*} \]  

(16)

Consider the Lyapunov function defined by

\[ V_n(e_2, w_2, \ldots, w_n) = V_{n-1}(e_1, w_2, \ldots, w_{n-1}) + w_n^TP_nw_n + \sum_{i=m+1}^{n} e_i^TR_ne_i \]  

(17)

where \( P_n \) and \( R_n \) are positive definite matrices.

Suppose the derivative of \( V_n(e_2, w_2, \ldots, w_n) \) is

\[ \dot{V}_n(e_2, w_2, \ldots, w_n) = -e_1^TQ_1e_1 - w_2^TQ_2w_2 - \cdots - w_n^TQ_nw_n - \sum_{i=k+1}^{m} e_i^TS_2e_i \]  

(18)

where \( Q_1, Q_2, \ldots, Q_n, S_n \) are positive definite matrices.

Then \( V_n(e_1, w_2, \ldots, w_n) \) is a negative definite function on \( \mathbb{R}^n \).
Thus by Lyapunov stability theory (Hahn, 1967), the error dynamics (16) is globally asymptotically stable.

The virtual controller is

\[ e_n = \alpha_{n-1}(e_1, w_2, \ldots, w_{n-1}) \]  

and the state feedback input \( u_n \) makes the system (16) globally asymptotically stable.

Hence, the state of master and slave systems are globally and asymptotically synchronized.

3. System description

Recently, theoretical design and hardware implementation of different kinds of chaotic oscillators have attracted increasing attention, aiming real-world applications of many chaos-based technologies and information systems.

The \( n \)-scroll hyperchaotic Chua circuit (Yu, Lu, & Chen, 2007) is given by the dynamics

\[ \begin{align*}
\dot{x}_1 &= \alpha [g(x_2 - x_1) - x_3] \\
\dot{x}_2 &= \beta [-g(x_2 - x_1) - x_4] \\
\dot{x}_3 &= \gamma g(x_1 + x_3) \\
\dot{x}_4 &= \gamma x_2
\end{align*} \]  

where \( g(x_2 - x_1) \) is given by

\[ g(x_2 - x_1) = m_{N-1}(x_2 - x_1) + \frac{1}{2} \sum_{i=1}^{N-2}(m_i - m) \times |x_2 - x_1 + z_i - |x_2 - x_1 + z_i|| \]  

The recursive positive switching points \( z_i (i = 2, 3, \ldots, N - 1) \) can be deduced as

\[ \begin{align*}
z_2 &= \frac{(1+k_1) \sum_{i=1}^{m_{N-1}} m_i - m_{N-1}}{m_{N-1}} - k_1 x_1 \\
z_3 &= \frac{(1+k_2) \sum_{i=1}^{m_{N-1}} m_i - m_{N-1}}{m_{N-1}} - k_2 x_2 \\
&\vdots \\
z_{N-1} &= \frac{(1+k_{N-2}) \sum_{i=1}^{m_{N-1}} m_i - m_{N-1}}{m_{N-1}} - k_{N-2} x_{N-2}
\end{align*} \]  

and the \( k_i \) values are obtained as

\[ k_i = \frac{x_{i+1} - x_i^E}{x_i^E - x_i} \quad (1 \leq i \leq N - 2) \]  

in which \( x_i^E \) are the positive equilibrium points of \( g(x_2 - x_1) \).

3.1. Case 1: 2-scroll hyperchaotic attractor

The parameters of the systems (20) are taken in the case of hyperchaotic case as \( \alpha = 2, \beta = 20 \).

When \( N = 2 \), in Equation (20), the function \( g(x_2 - x_1) \) is given by

\[ g(x_2 - x_1) = m_1(x_2 - x_1) + \frac{1}{2}(m_0 - m_1) \times |x_2 - x_1 + z_1 - |x_2 - x_1 + z_1|| \]
When \( m_0 = -0.2, \ m_1 = 3 \) and \( z_1 < 1 \), the 2-scroll hyperchaotic attractor is generated. Figures 1–3 depict the 2-scroll hyperchaotic attractor.

**Figure 1.** 2-scroll hyperchaotic attractor.

**Figure 2.** 2-scroll hyperchaotic attractor.

**Figure 3.** 2-scroll hyperchaotic attractor.
3.2. Case 2: 3-scroll hyperchaotic attractor

When \( N = 3 \), in Equation (20), the function \( g(x_2 - x_1) \) is given by

\[
g(x_2 - x_1) = m_2(x_2 - x_1) + \frac{1}{2}(m_0 - m_1) \times (|x_2 - x_1 + z_1| - |x_2 - x_1 + z_1|) + \frac{1}{2}(m_1 - m_2) \times (|x_2 - x_1 + z_2| - |x_2 - x_1 + z_2|)
\]

(25)

When \( m_0 = 3, m_1 = -0.8, m_2 = 3, z_2 = 1.8333 \) and \( z_1 < 1 \), the 3-scroll hyperchaotic attractor is generated. Figures 4–6 depict the 3-scroll hyperchaotic attractor.

3.3. Case 3: 4-scroll hyperchaotic attractor

When \( N = 4 \), in Equation (20), the function \( g(x_2 - x_1) \) is given by

\[
g(x_2 - x_1) = m_4(x_2 - x_1) + \frac{1}{2}(m_0 - m_1) \times (|x_2 - x_1 + z_1| - |x_2 - x_1 + z_1|) + \frac{1}{2}(m_1 - m_2) \times (|x_2 - x_1 + z_2| - |x_2 - x_1 + z_2|) + \frac{1}{2}(m_2 - m_3) \times (|x_2 - x_1 + z_3| - |x_2 - x_1 + z_3|)
\]

(26)

Figure 4. 3-scroll hyperchaotic attractor.

Figure 5. 3-scroll hyperchaotic attractor.
When $m_0 = m_2 = -0.7$, $m_1 = m_3 = 2.9$, $m_2 = 3$, $z_2 = 1.5289$, $z_3 = 3.0239$ and $z_1 < 1$, the 4-scroll hyperchaotic attractor is generated.

Figures 7–9 depict the 4-scroll hyperchaotic attractor.
4. Hybrid Synchronization of $n$-scroll hyperchaotic Chua circuits via backstepping control with recursive feedback

In this section, the backstepping method with recursive feedback function is applied for the hybrid synchronization of identical hyperchaotic $n$-scroll Chua circuits (Yu et al., 2007).

The $n$-scroll hyperchaotic Chua circuit is taken as the master system, which is described by

\begin{align}
\dot{x}_1 &= \alpha [g(x_2 - x_1) - x_3] \\
\dot{x}_2 &= \beta [-g(x_2 - x_1) - x_4] \\
\dot{x}_3 &= \gamma_0 (x_1 + x_3) \\
\dot{x}_4 &= \gamma x_2
\end{align}

where $g(x_2 - x_1)$ is given by

\begin{align}
g(x_2 - x_1) &= m_{n-1} (x_2 - x_1) + \frac{1}{2} \sum_{i=1}^{n-1} (m_i - m_{i-1}) \times (|x_2 - x_1 + z_i| - |x_2 - x_1 + z_i|) \tag{28}
\end{align}

where $x(t)(i = 1, 2, 3, 4) \in \mathbb{R}^4$ are state variables.

The $n$-scroll hyperchaotic Chua circuit is also taken as the slave system, which is described by

\begin{align}
y_1 &= \alpha [g(y_2 - y_1) - y_3] + u_1 \\
y_2 &= \beta [-g(y_2 - y_1) - y_4] + u_2 \\
y_3 &= \gamma_0 (y_1 + y_3) + u_3 \\
y_4 &= \gamma y_2 + u_4 \tag{29}
\end{align}

where $g(y_2 - y_1)$ is given by

\begin{align}
g(y_2 - y_1) &= m_{n-1} (y_2 - y_1) + \frac{1}{2} \sum_{i=1}^{n-1} (m_i - m_{i-1}) \times (|y_2 - y_1 + z_i| - |y_2 - y_1 + z_i|) \tag{30}
\end{align}

where $y(t)(i = 1, 2, 3, 4) \in \mathbb{R}^4$ are state variables.

The hybrid synchronization error is defined by

$$
\dot{e}_1 = \alpha [g(x_2 - x_1) - y_2] - e_3 \\
\dot{e}_2 = \beta [-g(x_2 - x_1) - y_4] - e_4 \\
\dot{e}_3 = \gamma_0 (x_1 + x_3) - y_1 \\
\dot{e}_4 = \gamma (x_2 - y_2)
$$

Figure 9. 4-scroll hyperchaotic attractor.
\[ e_1 = y_1 - x_1, \ e_2 = y_1 + x_2, \ e_3 = y_3 - x_3, \ e_4 = y_4 + x_4 \]  
(31)

The error dynamics is obtained as
\[ \dot{e}_1 = \alpha [g(y_2 - y_1) - g(x_2 - x_1)] - \alpha e_3 + u_1 \]
\[ \dot{e}_2 = -\beta [g(x_2 - x_1) + g(y_2 - y_1)] - \beta e_4 + u_2 \]
\[ \dot{e}_3 = \gamma (e_1 + e_3) + u_3 \]
\[ \dot{e}_4 = \gamma e_2 + u_4 \]
(32)

The modified error dynamics is defined by
\[ \dot{e}_1 = \alpha [g(y_2 - y_1) - g(x_2 - x_1)] - \alpha e_3 + u_1 \]
\[ \dot{e}_2 = -\beta [g(x_2 - x_1) + g(y_2 - y_1)] - \beta e_4 + u_2 \]
\[ \dot{e}_3 = \gamma (e_1 + e_3) + e_2 - y_2 - x_2 + u_3 \]
\[ \dot{e}_4 = \gamma e_2 + e_3 - y_3 + x_3 + u_4 \]
(33)

Now the objective is to find control law \( u_i, \ i = 1, 2, 3, 4 \) and for the parameter update law \( \dot{\alpha}, \dot{\beta}, \dot{\gamma}, \dot{\gamma_0} \) for stabilizing the system (32) at the origin.

First consider the stability of the system
\[ \dot{e}_4 = \gamma e_2 + e_3 - y_3 + x_3 + u_4 \]  
(34)

where \( e_3 \) is regarded as virtual controller.

Consider the Lyapunov function defined by
\[ V_1(e_4) = \frac{1}{2} e_2^2 + \frac{1}{2} e_4^2 \]  
(35)

Let define the parameter estimation error as
\[ e_\gamma = \gamma - \hat{\gamma} \]  
(36)

Differentiating the Equation (36)
\[ \dot{e}_\gamma = -\dot{\hat{\gamma}} \]  
(37)

Differentiate \( V_1 \) along with the Equation (37)
\[ V_1 = e_\gamma (\gamma e_2 + e_3 - y_3 + x_3 + u_4) + e_\gamma (-\dot{\hat{\gamma}}) \]  
(38)

Assume the controller \( e_3 = \alpha_1(e_4) \).

If
\[ \alpha_1(e_4) = -k_1 e_4, \text{ and } u_4 = y_3 - x_3 - \hat{\gamma} e_2 \]  
(39)

and the parameter update law \( \dot{\hat{\gamma}} \) is taken as
\[ \dot{\hat{\gamma}} = e_2 e_4 + k_2 e_\gamma \]  
(40)
then
which is a negative definite function. Hence, the system (34) is globally asymptotically stable.

The function \( \alpha_1(e_u) \) is an estimative function when \( e_3 \) is considered as a controller.

The error between \( e_3 \) and \( \alpha_1(e_u) \) is

\[
w_2 = e_3 - \alpha_1(e_u) = e_3 + k_1e_u
\]

Consider the \((e_1, w_2)\) subsystem given by

\[
e_1 = e_2e_3 + e_3
\]

\[
w_2 = \gamma_0(e_1 + w_2 - k_1e_u) + (k_1e_3 + 1)e_2
\]

\[
+ k_1(w_2 - k_1e_u) - y_2 - x_2 + u_3
\]

Let \( e_2 \) be a virtual controller in system (43).

Assume that when \( e_2 = \alpha_2(e_u, w_2) \) and the system (43) is made globally asymptotically stable.

Consider the Lyapunov function defined by

\[
V_2(e_u, w_2) = V_1(e_u) + \frac{1}{2}w_2^2 + \frac{1}{2}e_2^2
\]

Let us define the parameter estimation error as

\[
e_{\hat{\gamma}_0} = \gamma_0 - \hat{\gamma}_0
\]

Differentiating the Equation (45), we get

\[
e_{\hat{\gamma}_0} = -\dot{\gamma}_0
\]

The derivative of \( V_2(e_u, w_2) \) is

\[
\dot{V}_2 = \dot{V}_1 + w_2\dot{w}_2 + e_{\hat{\gamma}_0}\dot{e}_{\hat{\gamma}_0}
\]

\[
= e_2(e_2e_3 + w_2 - k_1e_u) + e_2(\dot{\gamma})
\]

\[
+w_2(\gamma_0(e_1 + w_2 - k_1e_u) + (k_1e_3 + 1)e_2
\]

\[
+ k_1(w_2 - k_1e_u) - y_2 - x_2 + u_3) + e_{\hat{\gamma}_0}(\dot{\gamma}_0)
\]

Substituting for \( e_3 \) from (42) into (47) and simplifying, we get

\[
\dot{V}_2 = -k_1e_u^2 - k_2e_2^2
\]

\[
+w_2(e_u + \gamma_0(e_1 + w_2 - k_1e_u) + (k_1e_3 + 1)e_2
\]

\[
+ k_1(w_2 - k_1e_u) - y_2 - x_2 + u_3) + e_{\hat{\gamma}_0}(\dot{\gamma}_0)
\]

Assume the virtual controller \( e_2 = \alpha_2(e_u, w_2) \)

\[
\alpha_2(e_u, w_2) = 0
\]

\[
u_3 = y_2 + x_2 - e_4 - k_1w_2
\]

\[
- k_4e_u - k_2w_2 - \gamma_0(e_1 + w_2 - k_1e_u)
\]

The parameter update law \( \dot{\gamma}_0 \) is

\[
\dot{\gamma}_0 = w_2(e_1 + w_2 - k_1e_u) + k_1e_{\hat{\gamma}_0}
\]
Then it follows that

\[ \dot{V}_2 = -k_1 e_4^2 - k_2 e_1^2 - k_3 w_2^2 - k_4 e_{\hat{\nu}_0} \] (51)

Thus, \( \dot{V}_2 \) is a negative definite function and hence the system (43) is globally asymptotically stable.

Define the error variable \( e_2 \) and \( \alpha_2(e_4, w_2) \) as

\[ w_3 = e_2 - \alpha_2(e_4, w_2) \] (52)

Consider the \((e_4, w_2, w_3)\) subsystem given by

\begin{align*}
\dot{e}_1 &= e_1 e_2 + e_3 \\
\dot{w}_2 &= (k_1 e_4 + 1)e_2 - e_4 - k_3 w_2 \\
&\quad + e_{\hat{\nu}_0}(e_1 + w_2 - k_1 e_4) \\
\dot{w}_3 &= -\beta [g(x_2 - x_1) + g(y_2 - y_1)] \\
&\quad - \beta e_4 + e_1 - y_1 + x_1 + u_2
\end{align*}

(53)

Let \( e_1 \) be a virtual controller in system (53).

Assume when it is equal to \( e_1 = \alpha_3(e_4, w_2, w_3) \), the system (53) is made globally asymptotically stable.

Consider the Lyapunov function defined by

\[ V_3(e_1, w_2, w_3) = V_2(e_1, w_2) + \frac{1}{2} w_3^2 + \frac{1}{2} e_\beta^2 \] (54)

Let us define the parameter estimation error as

\[ e_\beta = \beta - \hat{\beta} \] (55)

The derivative of (55) is

\[ \dot{e}_\beta = -\dot{\hat{\beta}} \] (56)

The derivative of \( V_3(e_3, w_2, w_3) \) is

\[ \dot{V}_3 = \dot{V}_2(e_1, w_2) + \dot{w}_3 w_3 + e_\beta \dot{e}_\beta \] (57)

i.e.

\[ \dot{V}_3 = e_4(e_1 e_2 + w_2 - k_1 e_4) + e_1(-e_2 e_4 - k_2 e_1) + w_2[(k_1 e_4 + 1)w_3 - e_4 - k_3 w_2] + e_{\hat{\nu}_0}(e_1 + w_2 - k_1 e_4) \]
\[ + e_{\hat{\nu}_0}[-w_2(e_1 + w_2 - k_1 e_4) - k_4 e_{\hat{\nu}_0}] \]
\[ + w_3[-\beta g(x_2 - x_1) + g(y_2 - y_1)] \\
&\quad - \beta e_4 + e_1 - y_1 + x_1 + u_2 + e_\beta(-\dot{\hat{\beta}}) \] (58)

Substituting for \( e_2 \) from (52) into (58) and simplifying, we get
\[ V_1 = -k_1 e_1^2 - k_2 e_\gamma^2 - k_3 w_2^2 - k_4 e_\delta^2 + w_3 (k_1 e_1 + 1) \]
\[-\beta [g(x_2 - x_1) + g(y_2 - y_1)] - \beta e_4 + e_1 - y_1 + x_1 + u_2 + e_p (-\dot{\hat{\beta}}) \]  
(59)

Assume the virtual controller \( e_1 = \alpha_3(e, w_2, w_3) \).

\[ \alpha_3(e, w_2, w_3) = 0 \]
\[ u_2 = y_1 - x_1 + \beta e_4 + \beta [g(x_2 - x_1) + g(y_2 - y_1)] - k_5 w_3 - w_2 (k_1 e_\gamma + 1) \]  
(60)

The parameter update law \( \dot{\hat{\beta}} \) is
\[ \dot{\hat{\beta}} = -w_3 w_4 + k_6 e_{\delta} \]  
(61)

Then it follows that
\[ \dot{V}_1 = -k_1 e_1^2 - k_2 e_\gamma^2 - k_3 w_2^2 - k_4 e_\delta^2 - k_5 w_3^2 - k_6 e_\delta^2 \]  
(62)

Thus, \( \dot{V}_1 \) is a negative definite function and hence the system (53) is globally asymptotically stable.

Consider \((e, w_2, w_3, w_4)\) subsystem given by
\[ \dot{e}_1 = e_1 e_2 + w_2 - k_1 e_4 \]
\[ w_2 = (k_1 e_1 + 1) e_2 - e_4 - k_3 w_2 \]
\[ + e_r (e_4 + w_2 - k_1 e_4) \]
\[ w_3 = -e_\delta e_4 + e_1 - k_5 w_3 - w_2 (k_1 e_\gamma + 1) \]
\[ w_4 = a [g(y_2 - y_1) - g(x_2 - x_1)] - ae_3 + u_1 \]  
(64)

Consider the Lyapunov function defined by
\[ V_4(e_1, w_2, w_3, w_4) = V_3(e_1, w_2, w_3) + \frac{1}{2} w_4^2 + \frac{1}{2} e_\alpha^2 \]  
(65)

Let define the parameter error as
\[ e_a = \alpha - \hat{\alpha} \]  
(66)

The derivative of \( e_a \) is
\[ \dot{e}_a = -\dot{\hat{\alpha}} \]  
(67)

The derivative of \( V_4(e_3, w_2, w_3, w_4) \) is
\[ \dot{V}_4 = \dot{V}_3(e_1, w_2, w_3) + w_4 \dot{w}_4 + e_a \dot{e}_a \]  
(68)

i.e.
Choose the controller

\[
U_1 = -w_3 - a[g(y_2 - y_1) - g(x_2 - x_1)] + \hat{\alpha}e_3 - k_7w_4
\]

and the parameter update law \(\hat{\alpha}\) is

\[
\dot{\hat{\alpha}} = -e_3w_4 + k_8e_u
\]

Then

\[
\dot{V}_4 = -k_4e_3^2 - k_5w_4^2 - k_6e_\varphi^2 - k_7w_3^2 - k_8e_\theta^2
\]

Thus, \(\dot{V}_4\) is a negative definite function.

Thus, by Lyapunov stability theory (Hahn, 1967), the error dynamics (64) is globally asymptotically stable for all initial condition.

Thus, the states of master and slave systems are globally and asymptotically hybrid synchronized.

5. Theorem

The identical n-scroll hyperchaotic Chua’s circuit (27) and (29) are globally and asymptotically hybrid synchronized with the adaptive backstepping controls

\[
U_1 = -w_3 - a[g(y_2 - y_1) - g(x_2 - x_1)] + \hat{\alpha}e_3 - k_7w_4
\]

\[
U_2 = y_1 - x_1 + \hat{\beta}e_4 + \beta[g(x_2 - x_1) + g(y_2 - y_1)] - k_5w_3 - w_2(k_3e_\gamma + 1)
\]

\[
U_3 = -w_2 - \gamma_0e_1 - 2\gamma_0w_3 + e_4
\]

\[
U_4 = y_2 + x_2 - e_3 - k_4(w_2 - k_4e_\gamma) - k_3w_2 - \hat{\gamma}_0(e_4 + w_2 - k_4e_\gamma)
\]

and with the parameter update laws

\[
\dot{\hat{\alpha}} = -e_3w_4 + k_8e_u
\]

\[
\dot{\hat{\beta}} = -w_3w_4 + k_6e_\varphi
\]

\[
\dot{\hat{\gamma}} = w_2(e_3 + w_2 - k_4e_\gamma) + k_6w_4
\]

\[
\dot{\hat{\gamma}} = e_3e_4 + k_5e_\gamma
\]

6. Numerical simulation

For the numerical simulations, the fourth order Runge–Kutta method is used to solve the differential Equations (27) and (29) with the backstepping controls \(U_1\), \(U_2\), \(U_3\), and \(U_4\) given by (29).

6.1. Case 1: 2-scroll hyperchaotic attractor

The parameters of the systems (27) are taken in the case of hyperchaotic case as \(a = 2\), and \(\beta = 20\).

When \(N = 2\), \(m_0 = -0.2\), \(m_1 = 3\) and \(z_1 = 0.5\), the double scroll hyperchaotic attractor is generated.
The initial values of the master system (27) are chosen as $x_1(0) = 0.947$, $x_2(0) = 0.234$, $x_4(0) = 0.198$ and the initial values of the slave system (29) are chosen as $y_1(0) = 0.157$, $y_2(0) = 0.648$, $y_3(0) = 0.810$, $y(4) = 0.108$.

The initial values of the estimated parameters are $\hat{\alpha} = 2.3$, $\hat{\beta} = 4.9$, $\hat{\gamma} = 10$, $\hat{\gamma}_0 = 5.5$.

Figure 10 depicts the hybrid synchronization of 2-scroll hyperchaotic Chua's circuits (27) and (29).

Figure 11 depicts the hybrid synchronization error between 2-scroll hyperchaotic Chua's circuits (27) and (29).

Figure 12 depicts the parameter estimation of 2-scroll hyperchaotic Chua's circuits (27) and (29).
The estimated values of the parameters $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$, and $\hat{\gamma}_0$ converge to system parameters $\alpha = 2, \beta = 20, \gamma = 1.5, \text{ and } \gamma_0 = 1$. 

6.2. Case 2: 3-scroll hyperchaotic attractor

When $N = 3, m_0 = 3, m_1 = -0.8, m_2 = 3, z_2 = 1.8333, \text{ and } z_1 = 0.5$ the 3-scroll hyperchaotic attractor is generated.

The initial values of the master system (27) are chosen as $x_1(0) = 1.938, x_2(0) = 2.138, x_3(0) = 1.708, x(4) = 3.325$ and the initial values of the slave system (29) are chosen as $y_1(0) = 0.125, y_2(0) = 0.986, y_3(0) = 0.065, y(4) = 1.363$.

The initial values of the estimated parameters are $\hat{a} = 20.363, \hat{\beta} = 0.563, \hat{\gamma} = 10.613, \hat{\gamma}_0 = 0.9$.

Figure 13 depicts the hybrid synchronization of 3-scroll hyperchaotic Chua’s circuits (27) and (29).

Figure 14 depicts the hybrid synchronization error between 3-scroll hyperchaotic Chua’s circuits (27) and (29).

Figure 15 depicts the parameter estimation of 3-scroll hyperchaotic Chua’s circuits (27) and (29).

The estimated values of the parameters $\hat{a}, \hat{\beta}, \hat{\gamma}$, and $\hat{\gamma}_0$ converge to system parameters $\alpha = 2, \beta = 20, \gamma = 1.5, \text{ and } \gamma_0 = 1$. 

6.3. Case 3: 4-scroll hyperchaotic attractor

When $N = 4, m_0 = m_2 = -0.7, m_1 = m_3 = 2.9, m_2 = 3, z_2 = 1.5289, z_3 = 3.0239 \text{ and } z_1 = 0.5$ the 4-scroll hyperchaotic attractor is generated.

The initial values of the master system (27) are chosen as $x_1(0) = 1.938, x_2(0) = 2.138, x_3(0) = 1.708, x(4) = 3.325$ and the initial values of the slave system (29) are chosen as $y_1(0) = 0.125, y_2(0) = 0.986, y_3(0) = 0.065, y(4) = 1.363$.

The initial values of the estimated parameters are $\hat{a} = 20.363, \hat{\beta} = 0.563, \hat{\gamma} = 10.613, \hat{\gamma}_0 = 0.9$.

Figure 16 depicts the hybrid synchronization of 4-scroll hyperchaotic Chua’s circuits (27) and (29).
Figure 17 depicts the hybrid synchronization error between 4-scroll hyperchaotic Chua's circuits (27) and (29).

Figure 18 depicts the parameter estimation of 4-scroll hyperchaotic Chua's circuits (27) and (29).

The estimated values of the parameters $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$, and $\hat{\gamma}_0$ converge to system parameters $\alpha = 2$, $\beta = 20$, $\gamma = 1.5$, and $\gamma_0 = 1$.

7. Conclusion
In this paper, the adaptive backstepping control method has been applied to achieve global chaos hybrid synchronization for a family of n-scroll hyperchaotic Chua circuits. The backstepping control is a systematic procedure for hybrid synchronizing hyperchaotic systems and there is no derivative
in controller. The adaptive backstepping control design has been demonstrated to class of n-scroll hyperchaotic Chua circuits. Numerical simulations have been given to illustrate and validate the effectiveness of the proposed hybrid synchronization schemes of the chaotic circuit. The adaptive backstepping method is very effective and convenient to achieve global chaos hybrid synchronization.
Funding
The authors received no direct funding for this research.

Author details
Suresh Rasappan1
E-mail: mrpsuresh83@gmail.com
1 Department of Mathematics, Vel Tech University, No. 42 Avadi-Vel Tech Road, Avadi, Chennai 600062 Tamilnadu, India.

Citation information
Cite this article as: Hybrid synchronization of hyperchaotic n-scroll Chua’s circuits (27) and (29). Suresh Rasappan, Cogent Engineering (2015), 2: 1009273.

Cover image
Source: Author.

References
Alligood, K. T., Sauer, T., & Yorke, J. A. (1997). Chaos: An introduction to dynamical systems. Berlin: Springer-Verlag.
Bauer, F., Atay, F. M., & Jost, J. (2010). Synchronized chaos in network of simple unit. Europhysics Letters, 89, 20002. doi:10.1209/0295-5075/89/20002
Blasius, B., Huppert, A., & Stone, L. (1999). Complex dynamics and phase synchronization in spatially extended ecological system. Nature, 399, 354–359.
Che, Y. Q., Wang, J., Tsang, K. M., & Chen, W. L. (2010). Unidirectional synchronization for Hindmarsh–Rose neurons via robust adaptive sliding mode control. Nonlinear Analysis: Real world Application, 11, 1096–1104.
Coffman, K. G., McCormick, W. D., Noszticzius, Z., Simoyi, R. H., & Swinney, H. L. (1987). Universality, multiplicity and the effect of iron impurities in the Belousov–Zhabotinskii reaction. Journal of Chemical Physics, 86, 119–129.
Fujisaka, H., & Yamada, T. (1983). Stability theory of synchronized motion in coupled-oscillator systems. Progress of Theoretical Physics, 69, 32–37.
Ge, Z. M., & Chen, C. C. (2006). Projective synchronization of a new hyper chaotic Lorenz systems. Chaos, Solitons Fractals, 20, 639–647.
Ghosh, D., Banerjee, S., & Chowdhury, A. R. (2007). Synchronization between variable time-delayed systems and cryptography. Europhysics Letters, 80, 30006. doi:10.1209/0295-5075/80/30006
Hahn, W. (1967). The stability of motion. Berlin: Springer-Verlag.

Han, S. K., Kerrer, C., & Kuramoto, Y. (1995). D-phasing and bursting in coupled neural oscillators. Physical Review Letters, 75, 3190–3193.

Harmov, A. E., Koronovskii, A. A., & Moskalenko, O. I. (2005a). Generalized synchronization onset. Europhysics Letters, 72, 901–907.

Harmov, A. E., Koronovskii, A. A., & Moskalenko, O. I. (2005b). Intermittent generalized synchronization in unidirectionally coupled chaotic oscillators. Europhysics Letters, 70, 169–174.

Hindmarsh, J. L., & Rose, R. M. (1984). A model of neuronal bursting using 3-coupled First order differential equations. Proceedings of the Royal Society London B: Biological Sciences, 221, 81–102.

Jian-Ping, Y., & Chang-Pin, L. (2006). Generalized projective synchronization for the chaotic Lorenz systems and the chaotic Chen system. Journal of Shanghai University, 10, 299–304.

Kanter, I., Kopelowitz, E., Kestler, J., & Kinzel, W. (2008). The stability of motion bursting using 3-coupled First order differential equations. Proceedings of the Royal Society London B: Biological Sciences, 221, 81–102.

Kocarev, L., & Parlitz, U. (1995). General approach for chaotic synchronization with application to communication. Physical Review Letters, 74, 5028–5030.

Lakshmanan, M., & Murali, K. (2003). Secure communication synchronization of a unified chaotic system. Chaos, Solitons Fractals, 810–816.

Lih, J. S., Chern, J. L., & Otsuka, K. (2012). Joint time-frequency analysis of synchronized chaos. Europhysics Letters, 57, 810–816.

Lu, J., Wu, X., & Han, X. (2004). Adaptive feedback synchronization of a unified chaotic system. Physics Letters A, 329, 327–333.

Moreno, Y., & Pacheco, A. F. (2004). Synchronization of Kuramoto oscillators in scale-free networks. Europhysics Letters, 68, 603–609.

Moukam Kakmeni, F. M., Nguenang, J. P., & Kofane, T. C. (2006). Adaptive generalized projective synchronization in different chaotic systems based on parameter identification. Physics Letters A, 367, 119–206.

Lih, J. S., Cher, J. L., & Otsuka, K. (2012). Joint time-frequency analysis of synchronized chaos. Europhysics Letters, 57, 810–816.

Suresh, R., & Sundarapandian, V. (2012). Global chaos synchronization of WINDMI and Coulef chaotic systems by backstepping control. Far East Journal of Mathematical Sciences, 67, 265–287.

Suresh, R., & Sundarapandian, V. (2012a). Hybrid synchronization of n-scroll Chua and Lu’s chaotic systems via backstepping control with novel feedback. Archives of Control Sciences, 22, 73–95.

Suresh, R., & Sundarapandian, V. (2012c). Global chaos synchronization of hyperchaotic ChenLee system using backstepping control via recursive feedback. In J. Mathew, P. Patra, D. K. Pradhan, & A. J. Kuttayamma (Eds), Lecture notes in computer science (Vol. 305, CCIS pp. 212–221). Berlin Heidelberg: Springer Verlag.

Suresh, R., & Sundarapandian, V. (2013). Global chaos synchronization of a family of n-scroll hyperchaotic Chua circuits using backstepping control with recursive feedback. Far East Journal of Mathematical Sciences, 73, 73–95.

Tokuda, I. T., Kurths, J., Kiss, I. Z., & Hudson, J. L. (2008). Predicting phase synchronization of non-phase-coherent chaos. Europhysics Letters, 83, 50003. doi:10.1209/0295-5075/83/50003

Wang, J., Chen, L., & Deng, B. (2009). Synchronization of ghostbursting neurons in external electrical stimulation via H∞ variable universe fuzzy adaptive control. Chaos, Solitons Fractals, 39, 2076–2085.

Wang, J., Dianchen, L., & Tian, L. (2006). Global synchronization for time delay WINDMI system. Chaos, Solitons and Fractals, 30, 629–635.

Wang, Z., Zhang, W., & Guo, Y. (2010). Robust consensus for uncertain multi-agent systems on directed communication topologies. In IEEE International Conference on Decision and Control (pp. 6317–6322). Atlanta, GA.

Wang, Z., Zheng, W., & Guo, Y. (2011a, June 29–July 1). Adaptive output consensus tracking of a class of uncertain multi-agent systems. American Control Conference. (pp. 3387–3392). San Francisco, CA.

Wang, Z., Zhang, W., & Guo, Y. (2011b, June 29–July 1). Adaptive backstepping-based synchronization of networked uncertain Lagrangian systems. In American Control Conference. (pp. 1057–1062). San Francisco, CA.

Wang, Y. M., & Zhu, H. (2006). Generalized synchronization of continuous chaotic systems. Chaos, Solitons Fractals, 27, 97–101.

Wu, X., & Lu, J. (2003). Parameter identification and backstepping control of uncertain chaotic system. Chaos Soliton Fractals, 18, 721–729.

Yu, H. T. (2004). Design of adaptive sliding mode controller for chaos synchronization with uncertainties. Chaos Soliton Fractals, 22, 341–347.

Yang, T., & Chua, L. O. (1999). Generalized synchronization of chaos via linear transformations. International Journal of Bifurcation Chaos, 9, 215–219.

Yu, S., Lu, J., & Chen, G. (2007). A family of n-scroll hyperchaotic attractors and their realization. Physics Letters A, 364, 244–251.

Yu, Y., & Zhang, S. C. (2006). Adaptive backstepping synchronization of uncertain chaotic systems. Chaos Soliton Fractals, 27, 1369–1375.

Zhou, L., Lai, Y.-C., Wang, R., & Guo, J.-Y. (2004). Limits to chaotic phase synchronization. Europhysics Letters, 66, 324. doi:10.1209/epl/i2003-10220-2

Zeng, G. Z., Chen, L. S., & Sun, L. H. (2005). Complexity of an SIR epidemic dynamics model with impulsive vaccination control. Chaos, Solitons and Fractals, 26, 495–505.
