Short-Range Temporal Interactions in Sleep; Hippocampal Spike Avalanches Support a Large Milieu of Sequential Activity Including Replay

J. Matthew Mahoney1*, Ali S. Titiz2, Amanda E. Hernan1, Rod C. Scott1,3

1 Department of Neurological Sciences, University of Vermont College of Medicine, Burlington, Vermont, 05405, United States of America, 2 Department of Neurosurgery, University of California Los Angeles, Los Angeles, California, 90095, United States of America, 3 Neurosciences Unit, University College London Institute of Child Health, London, United Kingdom

* John.M.Mahoney@uvm.edu

Abstract

Hippocampal neural systems consolidate multiple complex behaviors into memory. However, the temporal structure of neural firing supporting complex memory consolidation is unknown. Replay of hippocampal place cells during sleep supports the view that a simple repetitive behavior modifies sleep firing dynamics, but does not explain how multiple episodes could be integrated into associative networks for recollection during future cognition. Here we decode sequential firing structure within spike avalanches of all pyramidal cells recorded in sleeping rats after running in a circular track. We find that short sequences that combine into multiple long sequences capture the majority of the sequential structure during sleep, including replay of hippocampal place cells. The ensemble, however, is not optimized for maximally producing the behavior-enriched episode. Thus behavioral programming of sequential correlations occurs at the level of short-range interactions, not whole behavioral sequences and these short sequences are assembled into a large and complex milieu that could support complex memory consolidation.

Introduction

Previous work has identified a remarkable phenomenon called sleep replay, in which hippocampal place cells fire in sequences during sleep that recapitulate sequences of firing that were repeated during repetitive behavior [1,2]. In addition to sleep replay, there have been multiple reports of sequential firing structure (temporal coding) in the awake state during resting periods while navigating. These observations include ‘forward replay’ [3] where a sequence of firing from previous navigation is reactivated during rest in the same order, ‘reverse replay’ [4], where a sequence is reactivated in the reverse order from previous navigation, as well as reactivations that represent novel trajectories through the environment, which include a bias toward
goal-oriented trajectories [5]. These phenomena provide evidence that neuronal firing sequences during burst activity in sleep are biased by observed, largely repetitive, behaviors during wake. However, normal awake experience is extremely complex and highly variable but nevertheless this complex behavior is consolidated into memory. The structure of a neural system that is flexible enough to encode complex experiences, and the systems level mechanisms that underpin this flexibility, remains unknown.

An interesting observation about replay is that the sequences are not always the same (e.g. there is some jitter about the replayed sequence). This could simply be an effect of noise or could be necessary for complex memory consolidation. If every time a neuron fired it were participating in a perfect reactivation of a behavioral firing sequence, then there would be no opportunity for that neuron to participate in any other firing sequence. It is already known that neurons participate in multiple sequences as evidenced by remapping of place cells when environments change (remapping). Because multiple environmental representations are multiplexed in the hippocampus, we hypothesize that sleep replay must be a complex mixture of sequences. We suggest that a network that can flexibly construct hi-fidelity, but not perfect, replay of a measured behavior can also construct sequences reflecting unmeasured behavior. To explore this possibility, we statistically characterize the structure of sequential firing during sleep and then establish that expected, behaviorally related sequences can be identified within this structure. This lends support to the idea that other sequences may also be behaviorally relevant. Specifically, we use a novel decoding strategy based on variable length Markov chains (VLMCs) to characterize the sequential firing of cells during sleep after rodents repetitively ran in a circular track. By modeling the bursting of neurons during sleep as Markov chains we make no explicit assumptions about the relationship between these cells firing activity during behavior and sleep. The Markov chain model characterizes the sequential correlations between cells during sleep and we then inspect the fitted model for behaviorally relevant sequential firing.

We report three key findings. First, we find that short-range temporal correlations between neural bursts are sufficient to explain the correlations in long sequences, indicating that the ensemble is structured in short sequences that are built into longer and more complex sequences, some of which are related to the expected replay sequence. Second, the sequential structures represented by the fitted Markov models are not ‘noisy cascades’ that prefer one or a few distinguished sequences, but rather they support a tremendous number of distinct sequences. Third, we see significant sequential correlations between pyramidal cells that had strong place-specific firing during the behavioral task and hence have clear behavioral sequences and other pyramidal cells that did not.

We suggest that at the system-level our results indicate that the brain is replaying fragments of experience and the dynamics of the ensemble mix these fragments together in a huge variety of ways to consolidate the memory of a huge variety of behaviors.

Results

Single unit activity from the CA1 region of the hippocampus was recorded from adult rats running in a circular track for 20 minutes (RUN) following which they slept for up to 1 hour (POST). We recorded 5 sessions in 3 animals and retained all pyramidal cells for analysis (Table 1). Analyses were performed on data collected during the entire motionless period after RUN.

Spike avalanches in neural activity during sleep

Prior to an analysis of temporal structure during burst firing activity it is important to establish that there is burst firing of pyramidal cells during sleep. Across all of the sessions and animals
the distribution of log-interspike intervals (ISIs) is bimodal, with a rapid firing regime (ISI < 50ms) and a 'long wait' regime (ISI > 50ms). Each pyramidal cell had a bimodal (log) ISI distribution and the average distribution across all cells shows a clear separation of time scales (Fig 1A). We define single unit bursts as sets of action potentials that are separated by less than maxISI = 50ms (c.f. Lee et al. [2]).

Across the neuronal ensembles, these bursts are correlated with each other in time, corresponding to spike avalanches where many neurons become simultaneously active [6,7]. We binned the spike trains and found spike avalanches, defined as consecutive time bins with at least one cell firing flanked by time bins with no activity from any cell (Fig 1B; see Materials and Methods and [7]). The timescale for binning was selected by computing the average waiting time for a spike from any cell in the ensemble; a spike avalanche therefore is a set of action potentials preceded and succeeded by a longer than average wait for an action potential from some neuron. These 'rate-normalized' time bins are data set dependent and allow for comparing recordings of different numbers of cell and action potentials, as for a fixed time bin recordings with more (fewer) action potentials would have a systematic shift toward larger (smaller) avalanches (c.f. [7,8]). The 'size' of a spike avalanche is defined as the number of spikes within the avalanche. Consistent with earlier reports [7], we find that the size distribution of spike avalanches is universal across rats and sessions and has a heavy tail relative to a null distribution of spike trains generated by independently randomizing interspike intervals (Fig 1C). 'Universality' means that the distribution of avalanche sizes is the same across all rats and sessions. The 'heavy tail' of the distribution means that large avalanches are much more common in the real data than in surrogate data of uncorrelated spike trains; action potentials happen in 'clumps' whose sizes are not predictable from what is expected of uncorrelated neurons, even after accounting for the 'burstiness' of individual neurons (ISI randomization). To quantify this difference in tails, we defined a large avalanche as an avalanche whose size exceeds 99% of all sizes in the null data. All of the measured distributions had significantly more large avalanches than expected under the null model (all p < 10^{-13}, binomial test; see Table 2).

Previous studies of episodic reactivation during sleep have focused on short (~150ms) epochs called 'sharp-wave ripples' (SWRs) during which the local field potential (LFP) has high power in the 'ripple band' (>150Hz). Many neurons become active during SWRs resulting in long sequences, but it is known that every neuron is also active outside of SWRs [2]. Indeed, there is a smooth gradation between avalanches with few spikes to those with many spikes (Fig 1C), indicating that there is not a categorical difference between the small and large avalanches. Furthermore, the SWR signal in the LFP is in part due to the activity of many cells firing at once and may serve as only a marker for large avalanches, but not their cause. Thus, avalanches are a unit of analysis that is conceptually distinct from firing activity only during SWRs. We note that other authors have used single unit criteria for identifying SWR periods that are essentially equivalent to looking for large spike avalanches (c.f. [2]).

It is known that the firing order of neurons within SWRs contains information about behavior prior to sleep [2]. The time-ordered collection of single neuron bursts within an avalanche is a discrete sequence that describes the ensemble activity during these events (Fig 1B). We parsed each spike avalanche into 'words' using a method adapted from Lee et al. [2].

| Rat (session) | 1 (1) | 1 (2) | 2 | 3 (1) | 3 (2) |
|---------------|-------|-------|---|-------|-------|
| # Classical place cells (# cells) | 4 (10) | 3 (12) | 7 (15) | 3 (6) | 1 (6) |

Table 1. Classical place cell and other pyramidal cell numbers. Previous reports have indicated that approximately 30–50% of hippocampal pyramidal cells are place cells in any given spatial context. In our cohort we measured between 20% and 50%, in accordance with previous reports.

Rat (session) 1 (1) 1 (2) 2 3 (1) 3 (2)
# Classical place cells (# cells) 4 (10) 3 (12) 7 (15) 3 (6) 1 (6)
Fig 1. Burst structure of single units and spike avalanches. (A) The distribution of log-interspike intervals during sleep for a representative example (rat 1, session 1) is bimodal demonstrating a fast spiking regime with long gaps (bursting). The distributions for individual pyramidal cells from the session are shown in gray and the population average is in blue. Interspike intervals < 50ms (red line) indicate that two spikes belong to the same burst event. (B) Single unit bursts are correlated in time to produce ‘spike avalanches’ with many
Specifically, for a single neuron we considered spikes separated by less than 50 ms (maxISI) to be a single interval of activity represented by its initial spike time. We encode these activity intervals as symbols, e.g., '1' for neuron 1 and so on. The sequential behavior of the full ensemble is then represented as a sequence of symbols, each with a corresponding initial timestamp. To preserve the spike avalanche structure of the ensemble, we only parsed neurons into bursts within an avalanche. Between avalanches, we add a new symbol '0' for each empty time bin to indicate these silent periods in the ensemble activity. The parsed data are a sequence of burst words separated by '0's. We call this full sequence the burst sequence (Fig 1B). The burst sequence representation of the spiking dynamics is a coarse-graining of the spiking behavior where we ignore the fast timescale rate modulations of the spiking but we capture the latency to first spike for each neuron in the succession of bursts across the ensemble. This allows us to distinguish rate coding from temporal coding. We note that the burst sequence parsing depends on both the bin size for defining avalanches and the maxISI for defining a burst; different choices of these parameters will result in different sequences. However, our choices are natural and data-driven. The universality of avalanche distributions (Fig 1C) results from the rate-normalized time bins (as described above) indicating that an avalanche is a meaningful unit of ensemble activity and by extension that the binning timescale is meaningful. The choice of maxISI is dictated by the burstiness of pyramidal cells, which have a clear bimodal distribution of log-ISIs with maxISI = 50 ms splitting the two modes (Fig 1A).

We define the length of a burst word as the number of symbols (with repeats) in the word. In addition to the avalanche size distribution, the length distribution of burst words is also heavy-tailed relative to the null distribution. Analogous to large avalanches, we define long words as words with length exceeding 99% of the null distribution. Long words are generated by the true ensemble significantly more often than in null data (all p < 10^{-6}, binomial test; see Table 2). Thus, large avalanches are composed of more bursts, not just more spikes per burst.

### Second order sequential structure in spike avalanches

The burst sequence does not, in principle, have to have any statistical structure, i.e. bursts could happen independently according to their burst rates without any preference for neurons units active in a short period of time. An avalanche is defined as a sequence of time bins with spikes from some cell in the ensemble flanked by empty time bins. An example of parsing the burst activity within an avalanche into a discrete sequence is shown below. The boxed sequence is a 'large avalanche' that contains a subsequence that is a length-4 match to the RUN sequence (in red). (C) Rank-frequency plots of spike avalanche sizes (blue, population average over sessions in bold) compared to ISI-randomized data (black, average over sessions in bold) show that large avalanches are more common in the observed data than expected at random (all p < 10^{-13}, Table 2).

Table 2. Statistics of avalanche sizes and word lengths. The average number of spikes in a spike avalanche (size) is larger than expected from ISI-randomized null data. The number of large avalanches (>0.99-quantile of the null size distribution; see Materials and Methods) is significantly higher than expected in null data (binomial test). The average number of neural bursts (word length) is also larger than expected in the null distribution. Likewise, the number of long words is significantly larger than expected in null data.

| Rat (session) | Mean avalanche size (null mean ± std. err.) | # large avalanches (null expectation, p-value) | Mean word length (null mean ± std. err.) | # long words (null expectation; p-value) |
|--------------|--------------------------------------------|---------------------------------------------|------------------------------------------|-----------------------------------------|
| 1 (1)        | 4.81 (4.51±0.02)                           | 345 (105; 3.6e-66)                         | 2.76 (2.60±0.01)                         | 318 (110; 9.8e-60)                      |
| 1 (2)        | 4.59 (4.35±0.02)                           | 172 (67; 4.9e-17)                         | 3.33 (3.16±0.02)                         | 158 (67; 8.7e-22)                       |
| 2            | 4.96 (4.42±0.02)                           | 713 (255; 1.1e-84)                        | 2.91 (2.60±0.01)                         | 947 (293; 5.2e-205)                     |
| 3 (1)        | 5.29 (4.59±0.05)                           | 69 (20; 5.7e-14)                          | 3.49 (3.02±0.03)                         | 68 (22; 1.2e-15)                        |
| 3 (2)        | 5.03 (4.67±0.04)                           | 138 (51; 1.2e-16)                         | 3.65 (3.39±0.03)                         | 140 (59; 8.0e-20)                       |






to fire in particular sequences. To determine if the burst sequence had nontrivial sequential structure, we fit variable length Markov chains (VLMCs) of varying maximum depth to the burst sequence (see Materials and Methods).

A VLMC is a type of Markov chain, a class of stochastic models of discrete sequences that predict the symbols of a sequence as a function of history of the sequence. A Markov chain is parametrized by a matrix of transition probabilities with rows indexed by an exhaustive set of possible sequence histories (contexts) and columns indexed by the possible upcoming symbols (alphabet). The probability of occurrence of each upcoming symbol therefore depends on the context. For example, the probability of seeing a ‘2’ in the sequence depends on which symbols are immediately preceding. How much of the history is relevant and how these histories modify the symbol probabilities are learned from the data [9]. The maximum context length is called the depth of the model.

Unlike fixed order Markov models, VLMCs allow the lengths of the contexts to vary. In a $d^{th}$-order Markov model for an alphabet of size $A$, one has to estimate the transition probabilities for each of the $A^d$ possible histories, resulting in a model with $A \cdot A^d = A^{d+1}$ free parameters. This makes high order models impractical for sequences with even modest alphabet size. The central insight of VLMCs is that not every possible history is equally relevant for describing the sequential structure of the data. Some (potentially long) contexts frequently occur and are highly predictive of the future of the sequence, while others might not contribute predictive power over shorter contexts that generalize them. For example, does knowing that a ‘1’ instead of a ‘2’ happened 6 bursts ago appreciably modify our predictions about the next element of the sequence over and above the prediction we would make knowing only the last 5 bursts? For a general sequence, the answer to this question is nontrivial. VLMC learning algorithms identify the relevant contexts and learn transition parameters for them, solving this problem in a purely data-driven manner. We stress that this context learning is conceptually important even if (as we will see below) the optimal models are low depth; a priori a structured sequence could be structured through short- or long-range interactions. Identifying these interactions is critical to understanding the data sequence. Thus, VLMC models can, in principle, capture long contexts, say length 5, without the burden of having to fit a full $5^{th}$-order Markov chain [9] and are therefore highly flexible models for sequential data.

We used 10-fold cross-validation to avoid overfitting to the burst sequence and to assess the maximum depth at which the statistical structure is captured. We define the normalized information, NI, as the fraction of total sequential information available in the data sequence captured by the depth-$d$ models (see Materials and Methods for mathematical definitions). NI has a value of zero if a model has no sequential structure at all (i.e. independent random sampling) and has a value of one if it is the optimal model. We see that across all rats and sessions the VLMC models saturate at depth 2 meaning that the best prediction of the upcoming symbol in the sequence depends most strongly on the previous two symbols. On average, first-order Markov chains captured around 90% of the sequential information and the second order structure captured a further 10% (Fig 2A). This saturation at depth 2 is potentially a feature of the measuring a small subset of neurons in the full hippocampus and it may be that measuring larger ensembles would reveal longer-range sequential structure. Indeed, the probabilities in the VLMC models are an estimation of the sequential correlations between the measured neurons and these correlations are certainly a function of the firing of other, unmeasured neurons. Nevertheless, a statistical description of a superset of the measured ensemble will have to produce the observed sequential correlations in the measured ensemble, but interleaved with bursts from other unmeasured cells. Cross-validation allows us to tune the fitted model to the robust correlations in the measured ensemble without overfitting.
The above shows that the burst sequence is sequentially structured, but there are two types of symbol in the sequence, symbols representing bursts (e.g. '1', '2', '3', etc.) and the symbol representing silence ('0'). To see if the fitted model contains nontrivial structure between neural bursts and not just structure related to the population bursting of avalanches, we scrambled the burst sequence by randomly permuting the bursts within the sequence but fixing the '0's. This preserves the burst rates of each neuron and all of the temporal structure between silence and bursting, but reduces all sequential correlations between bursts to noise. We fit VLMCs to these scrambled sequences. The models fitted on the scrambled sequences achieve only a fraction of the normalized information of the true data sequence (Fig 2A, inset). We emphasize that the scrambled sequences are structured; they have significant correlations due to the avalanche structure of the burst sequence, but those correlations alone do not account for all of the sequential information learned by the fitted models on the true data sequence. For subsequent analyses we used VLMCs of maximum depth 2 and we refer to this model as the fitted model.

Note that because VLMCs learn the relevant contexts, which have variable length, they are buffered against model misspecification, e.g. while fitting a depth-2 model on first-order data the algorithm 'prunes' the contexts back to first-order (for a detailed discussion of VLMC model learning see Begleiter et al. [9]).

The VLMC models make predictions of the rates of occurrence of subsequences within the burst sequence [9]. To visually assess the model fit, we computed the expected number of occurrences of each unique observed burst word under the fitted model and compared the
predicted frequencies to the observed frequencies of the words (Fig 2B, blue points). The independent model, which assumes the same symbol frequencies but no sequential correlations, makes much larger errors in predicting the word frequencies (Fig 2B, green points) demonstrating that sequential correlations are significant features of observed burst sequence. Thus, the VLMC models are capturing the longer-range sequential structure within the burst sequence, even though they only explicitly account for interactions across three consecutive letters (second order model). In other words, the short-range temporal interactions of the ensemble are sufficient to describe the long-range sequential structure.

Replay of behavioral sequences

Prior to sleep, the rats ran in a circular track for a food reward. During behavior the firing rates of pyramidal cells vary as a function of the rats’ position. The position-dependent firing can be highly localized in a single firing field or more diffuse with multiple firing fields (Fig 3 and S1–S4 Figs; Table 1). We call these two pyramidal cell subtypes classical place cells and other pyramidal cells respectively. It is unknown what hippocampal inputs are relevant for determining the spatial firing rates of individual pyramidal cells, but it is known that these rates depend on position, experimental context [10–12], and the cell assemblies to which the cell belongs [13]. Classical place cells fire in sequences during spatial navigation and repetitive behavior leads to repetitive activation of particular sequences, which allows us to define a behavioral ‘RUN sequence’ associated to the experimental behavior (Fig 3 and S1–S4 Figs). We define the RUN sequence as the sequence of classical place cells ordered by their firing order as the rat traverses the track; in Fig 3 the RUN sequence is 1-2-3-4-5-6-7. We note that the RUN sequence is defined exclusively from behavior and is completely independent of our analysis of firing during sleep.

The firing sequence of classical place cells during behavior is known to be related to the burst activity during sleep as it has been shown that sequential firing during sleep is biased to produce partial matches to the RUN sequence. Furthermore, sleep replay has been observed with similar numbers of recorded cells as in the current study [2]. Four of five sessions representing all three rats had multiple place fields allowing for defining a RUN sequence and looking for RUN sequence enrichment during sleep.

Consistent with reports of hippocampal replay, we find that the RUN sequence observed in each rat and session is more probable under the fitted model (Tables 3 and 4) than under a comparator model, the avalanche independent model (AIM), that has no sequential correlations between neurons but does preserve the burst rates and avalanche structure (see Materials and Methods). We define the likelihood ratio of a sequence as the ratio of the probability of the sequence under the fitted model to the probability under the AIM. When the likelihood ratio exceeds 1 the sequence is enriched, otherwise it is depleted. The likelihood ratio quantifies the extent to which the sequential structure of the fitted model is biased to produce the particular sequence relative to the AIM. A likelihood ratio of 2 indicates that the correlations in the data make the appearance of that sequence twice as likely as would be expected in sequentially uncorrelated data. The likelihood ratio for the RUN sequence was greater than one for all sessions, indicating that the RUN sequence is enriched in the sequential structure of sleep firing.

However, the likelihood ratios, as well as the computed RUN sequence probabilities under the Markov models, are functions of the training burst sequences, which inevitably have sampling noise. We can assess statistical significance of a likelihood ratio by considering the ‘jitter’ in the model fitting during cross-validation. We assign z-scores and p-values to sequences based on the variance of the ratios when some training data is held out (see Materials and Methods). Note that p-values for different sequences are correlated; so multiple hypothesis
testing corrections are likely to be conservative. The z-scores for 3 out of 4 sessions representing each of the rats were statistically significant.

In addition the RUN sequence, we define RUN-order sequences as sequences of place cells that occur in the behavior order but with gaps. For example, 1-3-4 is a RUN-order sequence for the RUN sequence 1-2-3-4-5-6-7 in Fig 3. These sequences allow us to ask whether partial
reactivations of the RUN sequence are enriched as well as the full sequence. Analyzing RUN-order sequences also allows us to answer whether we would have been able to identify RUN enrichment if we had not measured all of the cells we did measure. Conceptually this is an important positive control because every experiment is limited in how much of the system it measures.

We computed z-scores for all RUN-order sequences. For all sessions approximately 50% of all RUN-order sequences were significantly enriched (p < 0.05 Bonferroni correction for multiple hypothesis testing; Table 3). As described above, these significant enrichments include the full RUN sequence in three out of four sessions. (Rat 1, session 2 had a 3 cell RUN sequence, two length-2 subsequences of which are significantly enriched.) The histogram of z-scores for the 120 RUN-order sequences for Rat 2 shows that many (55/120) of those sequences as significantly enriched in the fitted models (Fig 4A). Note that there are also significantly depleted sequences in Fig 4A (large negative z-scores), indicating that the enrichment of the RUN-order sequences is not due to a general up-regulation of arbitrary place cell firing sequences.

The enrichment of the RUN-order sequences shows that the sequential structure during sleep produces the RUN-order sequence at rates higher than would be expected if the neuronal bursts within an avalanche were completely uncorrelated. Rat 2 had a long sequence of classical place cells (7/15 cells) that could be used to further explore the enrichment of the RUN sequence. We computed the enrichment of all 5040 permutations of the length-7 RUN sequence. (Note that these 5040 sequences are a tiny subset of all 15^7 = 170,859,375 possible

### Table 3. Replay of RUN-order sequences.

| Rat (session) | # Enriched (Significant) RUN-order sequences (/ Total) | Full RUN sequence enriched (significant)? |
|---------------|------------------------------------------------------|------------------------------------------|
| 1 (1)         | 9 (5 / 11)                                           | Yes (Yes)                                |
| 1 (2)         | 4 (2 / 4)                                            | Yes (No)                                 |
| 2             | 91 (55 / 120)                                        | Yes (Yes)                                |
| 3 (1)         | 4 (2 / 4)                                            | Yes (Yes)                                |

doi:10.1371/journal.pone.0147708.t003

### Table 4. Enrichment of the RUN sequence during spike avalanches.

| Rat (session) | Independent rate, r_{AIM} (occurrences per symbol) | Fitted rate, r_{fit} (occurrences per symbol) | Maximal rate, r_{max} (occurrences per symbol) | r_{fit} / r_{AIM} | r_{max} / r_{AIM} |
|---------------|---------------------------------------------------|---------------------------------------------|-----------------------------------------------|-------------------|-------------------|
| 1 (1)         | 1.68e-07                                          | 2.49e-07                                   | 1.04e-03                                      | 1.48e+00          | 6.18e+03          |
| 1 (2)         | 7.11e-07                                          | 9.34e-07                                   | 3.04e-03                                      | 1.31e+00          | 4.27e+03          |
| 2             | 4.03e-13                                          | 8.71e-13                                   | 6.87e-05                                      | 2.16e+00          | 1.70e+08          |
| 3 (1)         | 1.92e-04                                          | 2.98e-04                                   | 1.15e-02                                      | 1.55e+00          | 5.97e+01          |

doi:10.1371/journal.pone.0147708.t004
length-7 sequences from the 15 measured cells.) The RUN sequence is in the top 9% of likelihood ratios for the permuted sequences showing that it is among the most strongly enriched sequences that can be built out of the 7 place cells (Fig 4B). Note that this distribution of likelihood ratios for permutations of the RUN sequence does not constitute a null distribution nor does the 0.09 quantile of the RUN sequence within this distribution constitute a p-value. The null model is the AIM and the RUN sequence as well as many of the permuted sequences has a likelihood ratio exceeding one, indicating enrichment. Also note that, as with Fig 4A, there are many likelihood ratios below 1, indicating depletion of certain sequences. Thus, the up-regulation of the RUN sequence and many permutations of it cannot be ascribed to a general up-regulation of arbitrary sequences of these place cells.

**RUN optimal Markov chains**

The trained behavior of running in a circular track for food is clearly a salient experience for the rats and the RUN sequence is enriched during sleep, but it is only one of many such sequences. Thus, the RUN sequence is not occurring at the theoretically maximum rate. It remains unclear, however, what that maximum rate would be and how the true rate compares to this maximum rate.

We constructed first-order Markov chains that maximized the rate of occurrence of the RUN sequence subject to the constraint that the burst rates of each neuron are equal to the observed rates. In effect, we identified the Markov chains that produce the maximum amount of RUN sequence possible given the burst rates. This optimized model allows us to characterize the capacity of a hypothetical ensemble with the measured burst rates and avalanche structure.
to produce the RUN sequence. This model describes the scenario where a particular spatial memory is prioritized over all other memory encoded by these cells and sleep is constrained to rigidly generating that specific cascade whenever any place cell fires. We compared this maximum rate \(r_{\text{max}}\) to the rate under the fitted model \(r_{\text{fit}}\) and the AIM rate \(r_{\text{AIM}}\) and we note that, while in all cases the RUN sequence appears more often under the fitted model than under the AIM, the fitted model produces the RUN sequence at only a tiny fraction of the rate possible for the measured burst rates (Table 4).

Note that the rates predicted by the models are inversely related to the length of the sequence and these rates are expected to go to zero as the sequence size grows. This is because for a larger number of cells there are more options for transitions. Thus we expect the discrepancy between the fitted rate for any sequence and the model optimized for that sequence to grow with the number of cells. Nevertheless, the RUN-optimal models characterize the capacity for the ensembles to produce the RUN sequence and provide a check for how well the fitted models approximate the perfect RUN cascades. These rates demonstrate that the ensembles are not simply ‘noisy’ RUN sequence generators during sleep and the place cells participate in many sequences other than the RUN sequence.

**Sequential structure between classical place cells and other pyramidal cells**

Pyramidal cells that are not classical place cells are typically removed from sequence-based analyses of sleep replay because of their complex spatial structure, whereas classical place cells yield distinct sequences of firing that can be expected as replay events [2]. Moreover, their distinction during behavior suggests that other pyramidal cells may play a fundamentally different functional role from classical place cells. Nevertheless, all pyramidal cells are bursty during sleep (Fig 1A). The sequential structure during sleep indicates that digrams (2-symbol sequences) of firing history are predictive of the upcoming neuron to fire. These digrams contain both classical place cells and other pyramidal cells, but they do not all appear with equal frequency. Some digrams are relatively common, while others are rare. We computed log-likelihood ratios for digrams under the fitted model and the AIM. Normalizing by the AIM allows for determining which digrams were enriched purely by the temporal structure between neurons in the fitted model and not because of the background firing rates of the neurons, which vary over two orders of magnitude (data not shown). If place cells preferentially fired in sequences with place cells, then we would see a rightward skew in the distributions of the place-place density relative to the place-other density, and likewise for other pyramidal cells. However the three densities are statistically indistinguishable (Fig 5A, all \(p > 0.05\), Kolmogorov-Smirnov statistic for 1000 random permutations of cell labels). Indeed, the sequential structure is mixed between both types of cell. Because a digram can only be enriched if that transition occurs more often in the true data than under the AIM, this demonstrates that classical place cells and other pyramidal cells are predictive of each other’s bursting within an avalanche and therefore correlated.

**Networks and enriched sequences**

The VLMC models encode the temporal structure of the burst sequence as a set of transition probabilities from each context (digram) to each possible next burst. We compared the transition probabilities of the fitted models to the AIM by taking log ratio of each transition probability in the fitted model to the corresponding probability in the AIM. This yields a rectangular conditional log-likelihood matrix, \(R\), which encodes how each context (indexing a row) up- or down-regulates the burst rates of each neuron relative to the uncorrelated AIM (see Materials and Methods). Analogous to the likelihood ratios for sequences, a transition for which this log
ratio exceeds 0 is an enriched transition otherwise it is a depleted transition. These values vary from 16-fold below background (i.e. some contexts suppress the firing of some neurons) to 4-fold above background with typical values lying between ±2-fold over background (Fig 5B).
The set of enriched transitions can be viewed as a network where nodes are states of the Markov chain and edges represent enriched transitions. This ‘enriched network’ encodes all enriched sequences; by construction every path in this network represents a sequence of transitions that occurs more often under the fitted model than the AIM. Fig 5C shows a representative example (Rat 2), however all sessions are similar. For clarity of display, we have used the first-order Markov model so that nodes represent neurons. It is immediately clear that the network is highly interconnected, with a tremendous number of edges indicating that each burst from a neuron modulates the immediate firing propensities of many other neurons in the ensemble. In addition, edges exist between both classical place cells (blue nodes) and other pyramidal cells (green nodes) indicating that these cells are functionally interacting to produce the observed burst sequence. While classical place cells and other pyramidal cells have potentially distinct functional roles, there is very little distinction between them in the networks of enriched transitions. In particular, the distribution of weighted node degree for the two classes of cells is not significantly different (data not shown). This suggests that each class of neuron is interacting with a similar fraction of the network.

As expected from the enrichment of RUN-order sequences, many of the RUN-order transitions between classical place cells are enriched transitions in the fitted VLMC models. We have highlighted these transitions within the network (Fig 5C, red arcs). Thus, many links from the behavior sequence are enriched during sleep, but each transition takes place within a large milieu of enriched transitions demonstrating that the RUN sequence is among many other sequences enriched during sleep. The RUN sequence is therefore privileged in the sense that it is enriched over the background firing rates, but it is only one of many such sequences.

Clustered contexts and the next element of the sequence

The second order structure in the Markov chains indicates that the history of bursting, i.e. the previous digram, is the relevant context for predicting the next neuron to burst in the sequence. For example, suppose that the two most recent bursts are the sequence $\sigma = 1\rightarrow 2$. The VLMC model encodes the probability for the ensemble to transition to the next symbol given that the history, $P_{\text{fit}}(s|\sigma)$. This conditional probability encodes how the firing history of the ensemble modulates the firing rates of neurons. We can compare these rates to the AIM, which has no sequential structure between neurons, to quantify the up- or down-regulation of a transition in the fitted model relative to the baseline firing rates. Specifically, we compute conditional log-likelihoods for each transition, which are naturally structured as a rectangular matrix with contexts (digrams) indexing rows and symbols indexing columns (see Materials and Methods for mathematical formulae). We denote these matrices by $R$.

Heatmaps of the $R$ matrices show that the contexts cluster into groups according to how they predict the future bursting of the ensemble, i.e. how they modulate the instantaneous firing rates of the neurons in the ensemble. Fig 6A shows a representative example, but all sessions are similar (S5–S8 Figs). Two contexts are considered similar if they similarly regulate the ensemble (i.e. if their corresponding rows in $R$ are similar). We quantify this similarity by computing the cosine of the angle between the row vectors or $R$ corresponding to each pair of contexts (cosine similarity, see Materials and Methods). Hierarchical clustering shows that these pairwise similarities between the rows of $R$ have distinct clusters of contexts that similarly modulate the ensemble (Fig 6B). This clustering indicates that these multiple histories share similar futures, in that within a cluster each context is making similar predictions about which neurons are more or less likely to be the next to burst. This clustering is akin to how hippocampal networks are theorized to partition themselves into functional groups called cell assemblies [13–15], or groups of neurons with structured coactivation that potentially encode memory. In this
case, different sequential contexts precede futures with similar patterns of enrichment and depletion of particular neural bursts. Independent of the mechanisms by which cell assemblies are formed and activated, the fitted models show that measured ensembles are structured in such a way that certain cells preferentially fire after certain contexts, while the context clustering shows that multiple contexts share their preferences, indicating an robust evolution toward particular patterns of coactivation. This suggests that the sequential structure in the burst sequence is, in part, related to the functional organization of the hippocampus.

Discussion
Behavior, memory, and the criticality hypothesis

Sleep replay is postulated to be a phenomenon at the center of memory consolidation [16–19] and appears to be a single unit correlate of the reactivation of episodic memory that is subjectively experienced by humans during sleep [20]. The complexity of daily behavior that is encoded into memory requires a neural system that can capture regularities across disparate environmental exposures encoded by the same set of cells and can integrate those exposures with each other into coherent memories and useful insights that link the memories. A model of this consolidation process is that temporally structured, ‘episodic’, sequences of neural firing get ‘imprinted’ into the system during behavior and those sequences are repeated during sleep to either strengthen the connections within the hippocampus that encode that memory or to transmit a high fidelity copy of the memory to downstream cortical stores of long-term
memory. However, work in humans and rodents has shown that memory consolidation does not function simply to strengthen memory, but rather to integrate memory into preexisting associative networks by extracting the ’gist’ of the experience and to generate insight across multiple experiences [20]. It is unknown how the hippocampus manages the trade-off between high fidelity reactivation of distinct episodes and generalization across similar, but not identical, episodes. We provide evidence that, at the system level, short-range interactions are biased by behavior and that these short-range interactions are summed into long sequences, some of which are behaviorally relevant.

In principle, sequential correlations between bursts within an avalanche could have been trivial. Instead, we see enrichment for certain transitions, including those that generate replay sequences related to the RUN sequence (Figs 4 and 5C). We also observe depletion of other transitions. Our analysis shows that, in the ensembles measured, the optimum depth for modeling burst sequences during sleep is two, meaning that each burst can only directly influence the bursting of a cell at most two bursts in the future. Mathematically however, a depth-2 model can have a variety of dynamical properties. They can be strict cascades, where the previous two bursts strictly determine the future, or they can evolve with high probability to many possible symbols from each context. The fitted VLMCs are the latter (Figs 5C and 6). Thus, we provide direct evidence that ensemble activity during sleep is a complex mixture of sequential activity that is largely constructed out of short sequences, fragments of experience, that are pieced together, analogous to a language in which letters are joined in coherent ways to make words. This augments the observation of replay by placing replay events within a complex context of multiple enriched sequences.

The structure of avalanches in neural networks has been studied extensively in connection to the criticality hypothesis of neural dynamics [6]. At criticality, a neural network displays avalanches that span many orders of magnitude in size and duration. We show that spike avalanches in the hippocampus during sleep are distinguished from null data both in their size (number of spikes) and in the lengths of the burst words contained within them (Table 2). The existence of large avalanches and long words raises the question of whether it is adaptive for the brain to produce these long words. In our experiments, the long words in the burst sequence each appear only once during the entire sleep session. Meanwhile, the short-range statistics of these words are enriched for behavior. This suggests that the diversity of sequences generated by the avalanche dynamics during sleep is functioning to integrate all of these short sequences together in novel ways. This may be related to insight generation, the process of combining disparate pieces of information into a unified framework, a process known to be sleep-reinforced in humans [20]. Thus, the short-time interactions within the neural sequences appears as a mechanism for mediating the tradeoff between needing to imprint specific sequences (memory strengthening) and generating novel associations between memories (insight); behavior enriches certain short sequences, while the avalanche dynamics mix these pieces together in long events. More often than expected at random these mixtures will decode to RUN replay event. Although the only behaviorally relevant sequence that we have formally studied is the RUN sequence, inspection of the enriched networks shows that reverse replay transitions are also enriched in our models (Fig 5C).

Classical place cells vs. other pyramidal cells

Under the classic criteria [21], a cell is a ‘place cell’ if it has 1) a visually identifiable place field, 2) the place field contains at least 9 contiguous spatial pixels and 3) the spatial coherence is greater than 0.3. These cells are typically selected for sequence-based replay analyses, while cells with low spatial coherence or multiple place fields are discarded, as single firing field place
cells yield clear ‘episode sequences’ during behavior that can be identified as replay during sleep. The Markov chains we constructed, in contrast, are composed of bursts from all pyramidal cells measured during behavior. Nearly all pyramidal cells showed rate modulation as a function of position (Fig 3 and S1–S4 Figs), but most (63%) did not localize strongly in a single firing peak (Table 1). Thus, their firing is position dependent, but not to the extent of the distinct single field classical place cells. While other pyramidal cells do not show spatially localized firing during behavior, they are bursty during sleep (bimodal log- ISI distributions; Fig 1A) and become active in concert with the other cells in the measured ensemble. Thus, even though these cells do not undergo simple sequential firing during the experimentally measured behavior, they do fire within spike avalanches during sleep and are surely involved in behavioral encoding.

The hippocampal spatial code is a population code, i.e. the neural representation of position is distributed across multiple place cells. Standard tools for analyzing place cell ensembles are Bayesian decoding techniques that predict the location of the rat based on the ensemble firing pattern of place cells by inverting the spatial encoding using Bayes’ rule [22,23]. The trained decoding model is then used to interpret the sleep firing of the same ensemble and various properties of the decoded patterns are observed [24,25]. Bayesian decoding methods do work for pyramidal cells with multiple place fields (our ‘other pyramidal cells’); indeed, any spatial modulation of the firing rate of a cell provides some information about where the rat is. However, Bayesian decoding methods are supervised techniques and cannot be used in situations where it is unknown what features, e.g. a location in space, the neurons represent. Indeed, it is known that pyramidal cells that have high spatial coherence in one arena are often not classical place cells in other arenas [10–12]. Pyramidal cell ensembles therefore likely retain multiple features of the rat’s awake experience, presumably even features that are not under experimental control. If sleep replay of awake experience is a generic phenomenon used for all hippocampus-dependent memory consolidation it must be a complex milieu of memories supported on a shared neural substrate. Sequences that are coherent reactivations of traversing one enclosure could look nonsensical when interpreted as possible reactivations of sequential firing in another enclosure.

One of the virtues of sequence-based decoding of sleep replay is that it works with relatively modest numbers of measured cells as in the experiments reported here (c.f. [2]). If a set of place cells fires in a particular order during a repetitive behavior, then that sequence serves as a template for the reactivation of those episodes in sleep. During replay these cells will still fire in the same relative order independent of the number of measured cells. We augment this approach using phenomenological models (Markov chains) that describe the temporal statistics of the spike avalanches without explicit recourse to how these cells fire during behavior (i.e. independent of the behavioral encoding). These models yield three important observations. First, pyramidal cells are significantly sequentially correlated during sleep and those correlations are enriched for behavior-related sequential firing. Second, these sequential correlations exist between classical place cells (where behavioral sequences are clearly defined) and other pyramidal cells. Third, the sequential structure of bursts during sleep is not simply a ‘noisy’ replay of a singular experience, even in a case where that experience is highly repetitive and salient. Several authors have made similar points before [2,26,27]. We add to this observation by showing that, in fact, the sequential structure of firing during sleep produces many burst words that vary in frequency over several orders of magnitude and including a greater than expected number of long words (Fig 1C, Table 2). The structure of firing during sleep in this way resembles a full language more than a simple mixture of a few noisily repeated patterns. It is an open question for future research behavior modulates this large set of enriched sequences.
Conclusions

The paradigm of behavioral neuroscience is to relate patterns of neural firing to outcomes in behavioral experiments. Systems neuroscience augments this approach by considering the correlated firing of neuronal ensembles per se to gain insight into how behavior shapes neural dynamics and the magnitude of that effect. Using a systems neuroscience approach, we find that repetitive running in a circular track enriches short sequential firing patterns that sum into large sequences of hippocampal pyramidal cells but that these behavior-related patterns are embedded in a much larger network of such short sequences. These sequences include pyramidal cells for which sequential episodic encoding is less straightforward. This shows that sleep replay is fragmented. It coexists and is potentially integrated with many other firing patterns that are likely also to be replay of experiences other than the measured running behavior.

Materials and Methods

Animals

3 Sprague Dawley male rats (300–350 g) were used in the study. All behavioral, pharmacological, and surgical procedures were done in accordance with National Institutes of Health guidelines and approved by the Dartmouth College Institutional Care and Use Committee.

Electrode preparation

32-channel electrode arrays manufactured in the late Robert U. Muller’s Laboratory (State University of New York, Downstate Medical Center, Brooklyn, NY) were used. Seven independently-drivable tetrodes consisting of bundles of 25 μm nichrome wire (A-M Systems, Sequim, WA) were individually inserted into polymicrotubing (Neuralynx, Bozeman, MT). Two additional 100 μm wires (California Fine Wire, Grover Beach, CA) were also included in the electrode array, one used to measure local LFPs and one inserted into the cerebellum as a reference. Two additional stainless steel wires (0.25mm) were soldered to 0.025 cm-thick wire and used for EMG (Plastics One, Roanoke, VA). Tetrodes were connected to a Mill-Max connector (Mill-Max, Oyster Bay, NY) and gold-plated to reduce impedences below 300kΩ.

Surgery

Rats were anesthetized with isoflurane (2% in O₂) until unresponsive to a tail pinch. An incision was made in the scalp, skull was exposed and five small holes were drilled. Three skull screws were affixed above the left parietal and left frontal cortex and the cerebellum. The ground wire was soldered to the cerebellar screw and the reference wire was inserted between the skull and the cerebellar parenchyma. The tetrode array was lowered so that the tip of the bundle was above the CA1 pyramidal cell layer (-3.8mm A/P, -2.0mm M/L, -1.8mm D/V). EMG wires were inserted bilaterally into the nuchal muscles. The entire complex was secured to the skull with dental cement. Animals were allowed to recover for a week prior to screening.

Screening and Data Acquisition

Tetrode assemblies were advanced 50μm three times per day with 4hr between recordings until waveforms of 100μV or larger were detected. The signal from the electrodes was preamplified directly from the rat's head by operational amplifiers and transmitted via a custom cable attached to a rotating commutator connected to a Neuralynx recording system (Neuralynx, Bozeman, MT). Tetrode signal was recorded at 32kHz. EEG was band-pass filtered 0.1-30Hz and EMG signal was filtered at 10-50Hz. Recordings were performed in an open arena placed
in a sound proof custom designed Faraday cage. The location of the rat was recorded and tracked using an LED attached to the preamplifier attached to the head of the rat.

Behavioral Task

Rats (N = 3) were trained to run in a familiar circular track in one direction. The diameter of the circular environment was 1 m, with a 10cm wide path and 60cm high walls. Rats were food restricted and rewarded with a food pellet when turning in the same direction to reinforce the behavior. Sessions where the rat did not turn in the same direction or did not complete the turn were not rewarded and considered incorrect. Animals were required to complete 20 or more correct turns in 15min. Once animals met this criterion, single units and LFPs were recorded during each 20 min session (RUN). After each session, animals were placed back in their home cages and single unit, LFP, and EMG were recorded for 1hr.

Sleep Scoring

Local EEG data were collected from the hippocampal electrodes and EMG data were collected from leads implanted in the nuchal muscles. EEG was filtered 1–35 Hz and EMG was filtered at 1050 Hz. Theta (4-12Hz) and delta (1-4Hz) power in the EEG signals was calculated. Behavioral states were determined using 10s time windows of EEG and EMG data; awake states were periods of movement on the EMG with a low delta/theta ratio, while sleep states were periods with loss of muscle tone and no activity on the EMG.

Offline Data Analysis

Action potentials (spikes) were clustered using Neuralynx Spike Sort 3D software (Bozeman, MT). We only retained cells that fired during both RUN and POST. We applied two firing rate filters to select cells for study. First, we only retained cells that fired more than 50 spikes during RUN. Second, to remove interneurons we selected only cells with firing rate less than 10 Hz during POST. Once cells had been selected, firing rate maps were computed in Matlab (MathWorks, Natick, MA) showing action potentials firing per pixel. Rate maps were established by dividing the firing map by the time the animal spent in each pixel. The rate maps showed the number of spikes per pixel per unit time [9,21,28]. Place cells were defined by the spatial localization of their firing fields by two criteria: coherence and concentration of spiking in angular coordinates of the circular track. First, place cell candidates were putative pyramidal cells that had a place field that was more than nine contiguous pixels and a firing rate three or more times greater than the session average [9,21]. Two-dimensional nearest-neighbor autocorrelation of each place cell’s firing rate within a 1x1 cm pixel with its eight nearest neighbors was defined as ‘spatial coherence’ [9,21]. Spatial coherence measures were used as an estimate of the spatial fidelity of cell firing within a place field [9,29]. Second, angular concentration was defined as having > 70% of spikes within a contiguous 40% of the angular distance around the track. Place cells were defined as putative pyramidal cell having coherence > 0.3 and being concentrated in angle as above.

To avoid recording the same cells twice, the tetrodes were advanced at least 25 μm at the end of each recording session. Thus, the neurons sampled in the animals with more than 1 recording were different.

Spike avalanches and burst word parsing

Spike avalanches are a phenomenon in neural systems where groups of action potentials are fired in bursts and the system becomes quiescent again [7,10–12]. To define the beginning and
end of spike avalanches, we first binned the spike trains of the ensemble during sleep. Spike avalanches were then defined as a set of consecutive time bins in which at least one cell had an action potential flanked by time bins during which no cell had an action potential. Because the ensembles varied in numbers of cells, we used a data-driven choice for bin size; the bin size was computed as the mean interspike interval (ISI) for the entire ensemble, i.e. all timestamps for all action potentials of all cells were pooled together [7,13].

The sequence of firing within an avalanche was further parsed into a burst word in two steps. First, for a single cell all spikes separated by less than \( \text{maxISI} = 50\,\text{ms} \) were grouped into a single unit burst represented by the timestamp of the initial spike (c.f. [2]). Next, these time-ordered bursts were converted into a discrete sequence of integers (symbols) representing the order of cell firing within the avalanche [2]. We concatenated the words associated with each avalanche by inserting one ‘0’ for each empty time bin between avalanches. This results in a single burst sequence of non-negative integers that encodes the sequential behavior of ensemble firing during POST.

**Spike randomization for avalanche null distributions**

We computed uncorrelated spike data for use as a null model by independently permuting the interspike intervals (ISIs) for each neuron separately. Specifically, for each neuron we built a random spike train by starting at zero and sampling from the ISI distribution for that neuron without replacement. We then placed a spike at that interval from the previous one and stopped when the ISI distribution was exhausted. Note that this choice of null model by construction preserves the firing rates and ISI distributions (burstiness) of each neuron but breaks all nontrivial correlations between neurons. We performed 100 ISI-randomizations to compute the mean and standard error for the null avalanche size and word length distributions.

We computed the statistical significance of the difference in tails between the null and observed avalanche size distribution using a binomial test. Specifically, we pooled the null size distributions from the 100 ISI-randomizations and defined the large avalanche threshold as the 0.99-quantile of the pooled null. We then counted the number of observed avalanches with size exceeding the threshold and computed the p-value using the binomial cumulative density function. Because the size distribution is discrete, the quantiles jump at each integer. We set the threshold as the last integer to have quantile \( q < 0.99 \) and used the corresponding value \( 1 - q \) as the probability for the binomial test. The test for the word length tails was identical.

**Fitting variable length Markov chains**

Below we describe variable length Markov chains (VLMCs) in brief to provide relevant mathematical formulae. For a detailed account of the relevant background and inference algorithms, see Begleiter et al. [9].

Let \( S = \{0,1,2, \ldots , C\} \) be the set of symbols representing each of \( C \) cells and the silence character in the burst sequence and let \( s = s_1s_2 \ldots s_N \) be a sequence of length \( N \). For any Markov chain over the alphabet \( S \), the probability of the sequence \( s \) is given by

\[
P_M(s) = \prod_{i=1}^{N} P_M(s_i | s_1s_2 \ldots s_{i-1}),
\]

i.e. the probability of the upcoming symbol \( s_i \) is a function of the history of the sequence. Typically these conditional probabilities only depend explicitly on the most recent symbols and the amount of relevant memory can vary in length (see below and [9] for an extended discussion).
The log-loss of a model $M$ in predicting a sequence $s$ is given by

$$L_M(s) = -\frac{1}{N} \sum_{i=1}^{N} \log_2 \left( P_M(s_i | s_{i-1} \ldots s_1) \right)$$

The model $M$ is a good fit for the sequence $s$ if the log-loss is small; the log-loss is small if the probability of each symbol is high given the context. The log-loss function is a goodness-of-fit measure for Markov chain models. Log-loss is measured in bits.

We fit variable length Markov chain models (VLMCs) to the burst sequence using the Matlab software provided by Begleiter et al. as a companion to the paper [9]. Specifically, we used the Probabilistic Suffix Tree (PST) algorithm. In brief, the PST algorithm fits a VLMC model in two stages: first it filters possible contexts (up to a given maximum depth) by occurrence frequency in the data, then it tests how informative the contexts by iteratively considering the predictive power of longer contexts relative to shorter contexts contained within, i.e. it statistically compares the difference in predictive power of longer contexts compared to shorter ones.

The PST algorithm has 4 user parameters. Most importantly, the maximum depth parameter, $d$, defines the maximum amount of sequential memory used for predicting the future, i.e. the maximum context length allowed by the VLMC model. We fit VLMC models for depths $d = 0, 1, \ldots, 5$ and chose $d$ by 10-fold cross validation selecting the value of $d$ that produced the lowest average log-loss over the 10 testing folds (Fig 2A).

In addition to the maximum depth, the PST algorithm has hyperparameters defining the filtering stages of context identification. The PST algorithm only considers contexts that occur above a (small) minimum frequency, $P_{\text{min}}$, within the data. Furthermore, for each context, $\sigma$, there must exist a possible future symbol, $s$, for which the probability of transitioning to that symbol, $P(s|\sigma)$, exceeds a (small) user-defined threshold, $\gamma$. In addition, letting $\sigma'$ be the suffix of $\sigma$ obtained by shortening by one symbol, the PST algorithm requires that the ratio of probabilities $P(s|\sigma) / P(s|\sigma')$ (or its inverse) exceed a user-defined threshold $r > 1$. These last two parameters are used to prune the context tree and they ensure that the algorithm is only learning contexts that are relevant in the training data. We set $P_{\text{min}} = 0.006$, $\gamma = 0.001$, and $r = 1.05$. Manual tuning showed that the results were relatively robust to these choices.

### Sequential information and normalization

We fit depth-$d$ models to each burst sequence for $d = 0, \ldots, 5$. Let $L_d$ be the average log-loss of the depth-$d$ model over the 10 cross-validation rounds and let $L_{\text{opt}}$ the minimum average log-loss over all tested values of $d$. The difference, $I_d = L_0 - L_d$, between the unstructured 0th-order model and the depth-$d$ model is a measure of the amount of sequential structure captured by the depth $d$ model. We call $I_d$ the sequential information. We define $I_{\text{opt}}$ analogously. Like log-loss, $I_d$ is measured in bits. To make comparisons between animals and sessions, we define the normalized sequential information, $N I_d = \frac{I_d}{I_{\text{opt}}}$. $NI$ varies between 0, for a model with no learned sequential structure, and 1, for the optimal model.

### The avalanche independent model and sequence likelihood ratios

A sequence can either be enriched or depleted by the temporal structure of the fitted VLMC model meaning that it occurs at a rate higher or lower than in a ‘background model’. An obvious choice for background model is to use the fully independent model of independent, identically distributed random draws from the set of symbols in the data sequence. This model has no sequential structure at all and therefore breaks the avalanche structure of the burst sequence,
which produces longer words and longer periods of silence than expected under the fully independent model. An alternative background model we call the avalanche independent model, AIM, has no sequential correlations between neurons, but does preserve the avalanche structure.

We define the AIM to be the first-order Markov chain that preserves the rates of transition between silence and bursting but has no other structure. Specifically, we compute rates of transitions from silence to silence (0-0 in the sequence), silence to bursting (0-i, for some cell i), bursting to silence (i-0), and bursting to bursting (i-j). Denote these by $P_{00}, P_{0b}, P_{b0},$ and $P_{bb}$ respectively. Furthermore, let $P_1, P_2, \ldots, P_C$ be the burst rates of each cell normalized by the total number of bursts. The transition matrix of the burst independent model is then given by

$$
P_{AI}(0|0) = P_{00} \\
P_{AI}(i|0) = P_{ib}P_i \\
P_{AI}(0|i) = P_{0i} \\
P_{AI}(i|i) = P_i
$$

Note in particular that there is no sequential structure between bursts (last line). The avalanche independent model serves as a comparator for sequentially structured models that have the same transition rates to and from silence.

We define the likelihood ratio for a sequence $\tilde{s}$ as the ratio

$$
L(\tilde{s}) = \frac{P(\tilde{s})}{P_{AI}(\tilde{s})},
$$

where $P$ is the fitted model. Let $l(\tilde{s}) = \log_2(L(\tilde{s}))$ denote the log-likelihood ratio.

Because we cross-validated when we fit $P$, we compute statistical significance of likelihood ratios by standardizing the log-likelihood ratios. Let $\tilde{s}$ be a sequence (e.g. a RUN sequence) and let $l_i(\tilde{s})$ be the log-likelihood ratio for the $i$th cross-validation fold. Let $\langle \cdot \rangle$ denote averaging over $i = 1, \ldots, 10$. We define the z-score of a sequence as

$$
z(\tilde{s}) = \frac{\langle l(\tilde{s}) \rangle}{\sqrt{\langle l(\tilde{s})^2 \rangle - \langle l(\tilde{s}) \rangle^2}},
$$

i.e. the mean log-likelihood divided the standard deviation over the 10 cross-validation folds. We then compute a p-value by comparing the z-score to the standard normal distribution.

**Conditional log-likelihood ratios and enriched transitions**

A Markov chain is a conditional probability table. It defines transition probabilities given the current context. We wish to compare transition probabilities between two models, the fitted model and the AIM to discern which transitions occur more/less than expected in the fitted model. To do this, we define conditional log-likelihood ratios.

Let $\Sigma = \{\sigma_1, \sigma_2, \ldots, \sigma_K\}$ be the set of contexts for the fitted VLMC model. We define the conditional log-likelihood matrix $R$ as

$$
R(\sigma_i, j) = \log_2 \left( \frac{P(j|\sigma_i)}{P_{AI}(j|\sigma_i)} \right)
$$

The matrix $R$ is rectangular with rows indexed by the contexts of the fitted VLMC model and columns indexed by cells (and '0'). $R$ encodes the up- and down-regulation of the background bursting rates as a function of context. The logarithm treats up- and down-regulation on the same scale.
To visually inspect the structure of these matrices, we clustered R using cosine similarity and average linkage hierarchical clustering using the Matlab function 'linkage'. We drew cluster boundaries (Fig 6, black lines) using greedy modularity maximization[30]. We note that these cluster boundaries are only for visualization.

RUN-optimal Markov chains
To assess the capacity of the measured burst rates to produce the RUN sequence, we constructed first-order Markov chains that produced this word at the highest rate possible given the measured burst rates. We computed a RUN-optimal Markov chain by optimizing the transition probabilities of the chain to maximize the rate of producing the RUN sequence. The stationary distribution of the chain equal the long-run burst rates measured in the data and these are a function of all transition probabilities, not just those in the RUN sequence. We constrained the optimization so that the solution would have the measured burst rates. To preserve the avalanche structure of the data, we constrained the silence-to-silence transition rate to equal that measured in the data sequence (as we did with the AIM above). These are linear constraints on the transition matrix. We used the nonlinear optimization function ‘fmincon’ in Matlab to perform the optimization.

Supporting Information
S1 Fig. Spatial variation of pyramidal cells firing during RUN. Spatial rates maps for rat 1, session 1. (TIFF)
S2 Fig. Spatial variation of pyramidal cells firing during RUN. Spatial rates maps for rat 1, session 2. (TIFF)
S3 Fig. Spatial variation of pyramidal cells firing during RUN. Spatial rates maps for rat 3, session 1. (TIFF)
S4 Fig. Spatial variation of pyramidal cells firing during RUN. Spatial rates maps for rat 3, session 2. (TIFF)
S5 Fig. Clustering of VLMC contexts. Cluster analysis of VLMC contexts for rat 1, session 1. (TIFF)
S6 Fig. Clustering of VLMC contexts. Cluster analysis of VLMC contexts for rat 2. (TIFF)
S7 Fig. Clustering of VLMC contexts. Cluster analysis of VLMC contexts for rat 3, session 1. (TIFF)
S8 Fig. Clustering of VLMC contexts. Cluster analysis of VLMC contexts for rat 3, session 2. (TIFF)

Author Contributions
Conceived and designed the experiments: JMM AST RCS. Performed the experiments: AST. Analyzed the data: JMM AST AEH. Wrote the paper: JMM AST AEH RCS.
References
1. Skaggs WE, McNaughton BL. Replay of neuronal firing sequences in rat hippocampus during sleep following spatial experience. Science. 1996; 271: 1870–1873. PMID: 8596857
2. Lee AK, Wilson MA. Memory of sequential experience in the hippocampus during slow wave sleep. Neuron. 2002; 36: 1183–1194. PMID: 12495531
3. Diba K, Buzsáki G. Forward and reverse hippocampal place-cell sequences during ripples. Nat Neurosci. 2007; 10: 1241–1242. doi: 10.1038/nn1961 PMID: 17828259
4. Foster DJ, Wilson MA. Reverse replay of behavioural sequences in hippocampal place cells during the awake state. Nature. 2006; 440: 680–683. doi: 10.1038/nature04587 PMID: 16474382
5. Pfeiffer BE, Foster DJ. Hippocampal place-cell sequences depict future paths to remembered goals. Nature. 2013; 497: 74–79. doi: 10.1038/nature12112 PMID: 23594744
6. John M Beggs NT. Being Critical of Criticality in the Brain. Frontiers in Physiology. Frontiers Media SA; 2012; 3: 163. doi:10.3389/fphys.2012.00163 PMID: 22701101
7. Ribeiro TL, Copelli M, Caixeta F, Belchior H, Chialvo DR, Nicolelis MAL, et al. Spike Avalanches Exhibit Universal Dynamics across the Sleep-Wake Cycle. PLoS ONE. Public Library of Science; 2010; 5: e14129. doi:10.1371/journal.pone.0014129 PMID: 21152422
8. Beggs JM, Plenz D. Neuronal avalanches in neocortical circuits. The Journal of neuroscience. 2003.
9. Begleiter R, El-Yaniv R, Yona G. On Prediction Using Variable Order Markov Models. Journal of Artificial Intelligence Research. 2004;: 385–421.
10. Bostock E, Muller RU, Kubie JL. Experience-dependent modifications of hippocampal place cell firing. Hippocampus. 1991; 1: 193–205. doi:10.1002/hipo.450010207 PMID: 1669293
11. Anderson MI, Jeffery KJ. Heterogeneous modulation of place cell firing by changes in context. Journal of Neuroscience. 2003; 23: 8827–8835. PMID:14523083
12. Jeffery KJ, Gilbert A, Burton S, Strudwick A. Preserved performance in a hippocampal-dependent spatial task despite complete place cell remapping. Hippocampus. 2003; 13: 175–189. doi: 10.1002/hipo.10047 PMID: 12699326
13. Harris KD, Csicsvari J, Hirase H, Dragoi G, Buzsáki G. Organization of cell assemblies in the hippocampus. Nature. 2003; 424: 552–556. doi:10.1038/nature01834 PMID: 12891358
14. Buzsáki G. Neural Syntax: Cell Assemblies, Synapsembles, and Readers. Neuron. Elsevier Inc; 2010; 68: 362–385. doi: 10.1016/j.neuron.2010.09.023 PMID: 21040841
15. Harris KD. Neural signatures of cell assembly organization: Abstract. Nature Reviews Neuroscience. Nat Rev Neurosci. 2005.
16. Battaglia FP, Benchenane K, Sirotta A, Pennartz CMA, Wiener SI. The hippocampus: hub of brain network communication for memory. Trends Cogn Sci (Regul Ed). 2011; 15: 310–318. doi: 10.1016/j.tics.2011.05.008
17. Frankland PW, Bontempi B. The organization of recent and remote memories. Nat Rev Neurosci. 2005; 6: 119–130. doi:10.1038/nrn1607 PMID: 15685217
18. Mehta MR. Cortico-hippocampal interaction during up-down states and memory consolidation. Nat Neurosci. 2007; 10: 13–15. doi:10.1038/nn0107-13 PMID: 17189946
19. Dupret D, O’Neill J, Pleydell-Bouverie B, Csicsvari J. The reorganization and reactivation of hippocampal maps predict spatial memory performance. Nat Neurosci. 2010; 13: 995–1002. doi: 10.1038/nn.2599 PMID: 20639874
20. Wamsley EJ. Dreaming and offline memory consolidation. Curr Neurol Neurosci Rep. 2014; 14: 433. doi: 10.1007/s11910-013-0433-5 PMID: 24473388
21. Muller RU, Kubie JL, Ranck JB. Spatial firing patterns of hippocampal complex-spike cells in a fixed environment. J Neurosci. 1987; 7: 1935–1950. PMID: 3612225
22. Brown EN, Frank LM, Tang D, Quirk MC, Wilson MA. A statistical paradigm for neural spike train decoding applied to position prediction from ensemble firing patterns of rat hippocampal place cells. J Neurosci. 1998; 18: 7411–7425. PMID: 9736661
23. Zhang K, Ginzburg I, McNaughton BL, Sejnowski TJ. Interpreting neuronal population activity by reconstruction: unified framework with application to hippocampal place cells. J Neurophysiol. 1998; 79: 1017–1044. PMID: 9463459
24. Davidson TJ, Kloosterman F, Wilson MA. Hippocampal replay of extended experience. Neuron. 2009; 63: 497–507. doi: 10.1016/j.neuron.2009.07.027 PMID: 19709631
25. Bendor D, Wilson MA. Biasing the content of hippocampal replay during sleep. Nat Neurosci. 2012; 15: 1439–1444. doi: 10.1038/nn.3203 PMID: 22941111
26. Buhry L, Azizi AH, Cheng S. Reactivation, Replay, and Preplay: How It Might All Fit Together. Neural Plasticity. 2011; 2011: 1–11. doi:10.1152/jn.00897.2006

27. Gupta AS, van der Meer MAA, Touretzky DS, Redish AD. Hippocampal Replay Is Not a Simple Function of Experience. Neuron. Elsevier Ltd; 2010; 65: 695–705. doi:10.1016/j.neuron.2010.01.034 PMID: 20223204

28. Lenck-Santini P-P, Muller RU, Save E, Poucet B. Relationships between place cell firing fields and navigational decisions by rats. Journal of Neuroscience. Society for Neuroscience; 2002; 22: 9035–9047. PMID: 12388610

29. Zhou J-L, Lenck-Santini P-P, Zhao Q, Holmes GL. Effect of interictal spikes on single-cell firing patterns in the hippocampus. Epilepsia. 2007; 48: 720–731. doi:10.1111/j.1528-1167.2006.00972.x PMID: 17284294

30. Newman MEJ. Modularity and community structure in networks. Proc Natl Acad Sci USA. 2006; 103: 8577–8582. doi:10.1073/pnas.0601602103 PMID: 16723398