Higher moments of net-proton multiplicity distributions in a heavy-ion event pile-up scenario

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High-luminosity modern accelerators, like the Relativistic Heavy Ion Collider (RHIC) at BNL and Large Hadron Collider (LHC) at CERN, inherently have event pile-up scenarios which significantly contribute to physics events as a background. While state-of-the-art tracking algorithms and detector concepts take care of these event pile-up scenarios, several offline analytical techniques are used to remove such events from the physics analysis. It is still difficult to identify the remaining pile-up events in an event sample for physics analysis. Since the fraction of these events is significantly small, it may not be as serious of an issue for other analysis as it would be for an event-by-event analysis. Particularly, when the characteristics of the multiplicity distribution are observable, one needs to be very careful. In the present work, we demonstrate how a small fraction of residual pile-up events can change the moments and their ratios of an event-by-event net-proton multiplicity distribution, which are sensitive to the dynamical fluctuations due to the QCD critical point. For this study we assume that the individual event-by-event proton and antiproton multiplicity distributions follow Poisson, negative binomial or binomial distributions. We observe a significant effect in cumulants and their ratios of net-proton multiplicity distributions due to pile-up events, particularly at lower energies. It might be crucial to estimate the fraction of pile-up events in the data sample while interpreting the experimental observable for the critical point.

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I. INTRODUCTION

The recent Beam Energy Scan (BES) program performed with STAR and PHENIX detectors at Relativistic Heavy Ion Collider (RHIC) and the upcoming upgrades to the STAR experiment for BES-II are motivated to explore the phase diagram of strong interaction. Quantum chromodynamics (QCD) predicts a phase transition from a hadron gas (HG) to a quark gluon plasma (QGP) phase in the temperature ($T$) and baryon chemical potential ($\mu_B$) plane of phase diagram $\mu_B$. Lattice QCD indicates a smooth crossover at $\mu_B \approx 0$, while other models predict a first order phase transition at higher baryon densities $\mu_B$. This suggests an existence of the QCD critical end point (CEP) as a termination point of the first order phase transition line at finite $\mu_B$ and $T$.\cite{7,8}

The event-by-event fluctuations of conserved quantities such as, net-baryon, net-charge, and net-strangeness are proposed as a useful observable to find the existence of CEP.\cite{3,11}. The correlation length ($\xi$) of the system is related to the moments of the multiplicity distribution of the above conserved quantities.\cite{12}. Thus, these moments can be used to look for phase transition and the CEP by varying the colliding beam energy.\cite{3,4}. The variance $\sigma^2$ of these distributions is related to $\xi$ as $\sigma^2 \sim \xi^2$.\cite{8}. The higher order moments such as skewness $S$ and kurtosis $\kappa$ are even more sensitive to $\xi$ as $S \sim \xi^{4.5}$ and $\kappa \sim \xi^7$.\cite{11,14}. The higher order moments have stronger dependence on the correlation length, hence, these moments are even more sensitive to the dynamical fluctuation.\cite{11,12}. The moments (mean, $\sigma$, $S$, and $\kappa$) of the net multiplicity distribution are related to the cumulants ($C_n$, $n = 1, 2, 3, 4$) as: $\langle M \rangle = C_1$, $\sigma^2 = C_2 = \langle (\Delta N)^2 \rangle$, $S = C_3/C_2^{3/2} = \langle (\Delta N)^3 \rangle/\sigma^3$ and $\kappa = C_4/C_2^2 = \langle (\Delta N)^4 \rangle/\sigma^4 - 3$, where $N$ is the multiplicity of the net distribution and $\Delta N = N - M$. The ratio of various $n$th-order cumulants $C_n$ of the distribution are related to the ratios and product of the moments as: $\sigma^2/M = C_2/C_1$, $S_\sigma = C_3/C_2$, $\kappa \sigma^2 = C_4/C_2$, and $S_\sigma^3/M = C_3/C_1$. One advantage of measuring the cumulant ratios is that the volume dependence of individual cumulants cancels out to first order. Further, the cumulant ratios can be related to the ratios of the generalized susceptibilities calculated in lattice QCD\cite{3,7,14}, and other statistical model calculations\cite{15}.

The measurement of net-proton \cite{16,17} and net-charge \cite{18,19} multiplicity distributions from BES at RHIC have drawn much attention from both the theoretical and experimental communities. There have been speculations that the non-monotonic behavior of $\kappa \sigma^2$ as a function of center-of-mass energy ($\sqrt{s_{NN}}$) in the net-proton multiplicity ($N_{\text{net}} = N_p - N_\bar{p}$) distribution measured by the STAR\cite{16} experiment may be an indication of the QCD critical point. Several studies have been carried out to estimate the excess of dynamical fluctuations such as the effect of kinematical acceptance\cite{20}, inclusion of resonance decays\cite{21,22}, exact (local) charge conservation\cite{23,24}, excluded volume corrections\cite{25,26}, and so forth to provide a proper thermal baseline for experi-
mental measurements [28, 32].

Recently, preliminary results from the STAR experiment on the net-proton multiplicity distribution show a large enhancement in $\kappa \sigma^2$ values at lower collision energies [33]. Several theoretical studies suggest that, the higher moments start to oscillate with temperature and $\mu_B$ near the QCD critical point [34–36]. The oscillating behavior observed in the experimental data motivated us to study the effect of residual pile-up events. Most of the pile-up events are removed using different experimental techniques, however one can not rule-out the possibility of a small fraction of “residual” pile-up events. The residual pile-up effect has never been considered while studying the cumulants in the experimental data. In the present work, we discuss the possibility of residual pile-up events as an artifact which can be present in these measurements and it’s influence on the results on higher moments of net-proton multiplicity distributions. This effect can be more pronounced at future heavy-ion experiments like CBM at FAIR, which will exceed collision rates up to 10 MHz [37]. The effect of residual pile-up events is important and should be considered before making any conclusion on critical point from the experimental data.

In high luminosity heavy-ion collisions, the contributions to background events may include the following [57–11]:

1. In-time pile-up events: If more than one collision occurs in the same bunch crossing in a collision of interest. This can be estimated by knowing the beam luminosity and collision cross-section at a particular $\sqrt{s_{NN}}$. For example, the average store luminosity of $1.3 \times 10^{26}$ cm$^{-2}$s$^{-1}$ and the collision cross-section of 9.6 barn has been measured by the STAR experiment for 19.6 GeV Au+Au running [42, 43]. Therefore, the collision rate was around $1.25 \times 10^3$ s$^{-1}$ ($\approx 1.25$ kHz). Further, the time difference between two bunches at RHIC is 109 ns; hence the contribution to the in-time pile-up events will be $109$ ns $\times 1.25$ kHz $= 1.36 \times 10^{-3}$ at $\sqrt{s_{NN}} = 19.6$ GeV. Similarly, the collision rates for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV are around 60 kHz, which leads to $\sim 6.5 \times 10^{-3}$ events as in-time pileup events.

2. Out-of-time pile-up events: If an additional collision occurs in a bunch crossing before and after the first collision. It may happen that the detectors are sensitive to several bunch crossings or their electronics integrate over more than the collision time period, and these collisions can affect the signal in a physics event. The STAR Time Projection Chamber (TPC) has a drift time of 40 $\mu$s, which leads to additional pile-up events of about 0.05 and 2.4 events as out-of-time pile-up for $\sqrt{s_{NN}} = 19.6$ and 200 GeV, respectively. It is to be noted that the high-resolution silicon vertex detector of STAR will further reduce the out-of-time pileup to almost zero.

3. Cavern background: Composed of mainly low energy neutrons and photons that can also cause radiation damage to detector elements and front-end electronics. The induced hits due to this background may increase the detector occupancy. This background is reduced by proper shielding of the detectors.

4. Beam halo events: As heavy ions are accelerated through the collider, the dispersion in the beam is called the beam halo, a less dense region of ions that forms outside the beam and gives rise to the background events.

5. Beam gas events: Collisions that occur between the bunch and the residual gas inside the beam-pipe which generally occur off center in the detector.

In the experimental situation, several techniques are applied to reduce the background events. For example, while selecting good events for the physics analysis, $z$ coordinates of the collision vertex within $\pm 50$ cm for lower energies and $\pm 30$ cm for higher collision energies are applied in STAR measurements [10]. This ensures the suppression of the cavern background and that events are not biased toward one side of the detector coverage. Similarly, out-of-time pile-up events can be removed by making sure that all the tracks come from the same bunch-crossing. This is taken care of by ensuring that the events contain data from fast signals of the detector (like the STAR time-of-flight detector). Further, the beam halo events are mostly forward focused and hence do not produce significant background. However, in order to remove the background events from the beam halo and the beam pipe, a cut on the transverse $x$–$y$ coordinate of the vertex position is applied. Further, a reference number for the particle multiplicity, specific to the center-of-mass energy is used to reject pile-up events. Also, various correlations between the global detector subsystems are used to remove the pile-up events. In spite of all the mentioned procedures, one may not assert the complete removal of pile-up events from the physics data sample. As an example, if two peripheral collision events happen within the same bunch crossing, it is difficult to identify them. This can be misinterpreted as a semi central collision if their vertices are not further apart than the vertex resolution of the detector system.

Since the fraction of these residual pile-up events would be significantly small, it may not be as serious of an issue for other analyses as it is the case of an event-by-event analysis. However, it may have serious consequences on the results of higher moments. For example, a small modification in the number of protons and/or antiprotons at the tail of the event-by-event multiplicity distribution can modify the results significantly, which is demonstrated in the present work.

The paper is organized as follows: In the following section, we discuss the method which is used to artificially
include the pile-up events. In Sec. III, we show the results of net-proton multiplicity fluctuations, assuming the proton and antiproton multiplicity distributions as Poisson, negative binomial, and binomial distributions. Finally, we summarize our work and discuss its implications in Sec. IV.

II. METHOD USED FOR EVENT PILE-UP STUDIES

The method discussed for this study assumes the proton and antiproton multiplicities to be Poisson, negative binomial or binomial distribution. As mentioned in the previous section, if two collision events happen within a same bunch crossing, it may be difficult to disentangle them and it can be misinterpreted as a single event. We have adopted a simple Monte Carlo approach by generating two independent multiplicity distributions of proton ($p$) and antiproton ($\bar{p}$) using the corresponding mean values for (0–5%) centrality in Au+Au collisions at different $\sqrt{s_{NN}}$ as given in Ref. [16]. The mean values of $p$ and $\bar{p}$ for different collision energies are also listed in Table I.

First, we assume that a large sample of central physics events has a small fraction of events where two central events are piled up. The extra protons and antiprotons coming from a certain fraction of pile-up events are added to the original multiplicity distribution. Hence, out of all the accumulated events, some events will have higher multiplicities as compared to the usual multiplicity of a central collision event. These high multiplicity events are distributed toward the tail of the distribution. The presence of a small fraction of pile-up events can have substantial effect on the shape of the distribution, which are described by their higher moments and cumulants. As a second possibility, it may also happen that an event from a central collision mixes with an event from another centrality class to form a pile-up event. In such a case, we add the $p$ and $\bar{p}$ multiplicities from a small fraction of minimum bias events to the multiplicity distribution from central collisions. This may be more of a probable scenario which can happen in heavy-ion collisions. The minimum bias distribution for protons (antiprotons) is constructed by combining the multiplicity of protons (antiprotons) at different collision centralities ranging from 0 to 80% for each $\sqrt{s_{NN}}$. Figure 1 shows the minimum bias multiplicity distribution for protons and antiprotons at $\sqrt{s_{NN}} = 7.7$ and 200 GeV. Further, the multiplicity distribution of $p$ or $\bar{p}$ for different centralities are constructed using Poisson, negative binomial (NBD) or binomial distribution with the mean values given in Ref. [16]. The $N_{\text{diff}}$ distribution is obtained on an event-by-event basis using the modified $p$ and $\bar{p}$ multiplicities.

Figure 2 shows the typical multiplicity distributions for protons, antiprotons, and net-protons for two different $\sqrt{s_{NN}} = 7.7$ and 200 GeV by taking their corresponding mean values. These two energies are considered to demonstrate the effect of pile-up event for a wider range of collision energies at RHIC. The multiplicity distributions are also compared with and without inclusion of pile-up events. In Fig. 2, the $p$ and $\bar{p}$ multiplicities from the central events are combined with 0.05% (five pile-up events in $10^4$ events) of the randomly selected $p$ and $\bar{p}$ multiplicities from minimum bias events, as shown in Fig. 1. Some fraction of excess protons due to pile-up events can clearly be seen as compared to a purely NBD distribution at $\sqrt{s_{NN}} = 7.7$ GeV. These excess events also reflect in the $N_{\text{diff}}$ distributions. Events with higher $p$ or $\bar{p}$ multiplicities will have larger pile-up effects, which can be observed in the proton multiplicity distribution. In Fig 2, the effect of pile-up events is more visible in the proton distribution at $\sqrt{s_{NN}} = 7.7$ GeV as compared to 200 GeV. Since at lower energies the mean number of protons is larger as compared to higher energies, therefore, the effect of mixing a central event with another central (or minimum bias) event is more pronounced. At higher energies, due to small mean multiplicity of $p$ and $\bar{p}$, the effect does not contribute much. However, in experimentally measured $p$ and $\bar{p}$ multiplicity distributions, it is not trivial to figure out these events, as the excess is very small and it may look like a real event multiplicity distribution which can be seen for $\sqrt{s_{NN}} = 200$ GeV in Fig. 2.

III. RESULTS AND DISCUSSIONS

Experimentally measured $p$ and $\bar{p}$ distributions are usually described by Poisson, negative binomial or binomial distributions. Poisson expectations reflect a system of totally uncorrelated, and statistically random particle production. The Poisson statistics is a limiting case of NBD, in which both the mean and variance of the distribution are same. Whereas in the case of NBD, the variance is larger than the mean of the distribution. In case of the binomial distribution, the variance is less than the mean. In the present study, a range of residual pile-up event fraction (0.01–2%) is considered, which may be realistic in experimental situations as discussed in Sec. I.
It is to be noted that this range of pile-up event fractions is based on the RHIC collision rates and detector response. Further offline analysis techniques can further reduce this number to an even smaller fraction. In the following subsections, we demonstrate the pile-up effect on the higher moments of net-proton multiplicity distributions.

A. Poisson distributions with event pile-up

The individual proton and antiproton distributions are independently generated assuming each of the distribution as Poisson with the measured mean values as given in Table I. The $N_{diff}$ distribution is constructed by taking $N_p$ and $N_{b}$ distributions on an event-by-event basis. The individual cumulants ($C_1$, $C_2$, $C_3$, and $C_4$) are calculated from the $N_{diff}$ distribution for different $\sqrt{s_{NN}}$.

Figure 2 shows the collision energy dependence of cumulants for different fractions of pile-up events. In this case, we have added the multiplicities from some fraction of the central collisions as pile-up events with the original multiplicities from the central collisions. It is observed that, a small fraction of pile-up events can have a significant effect on the cumulants and their ratios of the net-proton multiplicity distributions. Smaller fractions of pile-up events have minimal effect on lower moments (cumulants) such as $M$ ($C_1$) and $\sigma^2$ ($C_2$) of the distribution, where as larger effects are observed for higher cumulants ($C_3$ and $C_4$). Figure 3 shows the ratios of cumulants as functions of $\sqrt{s_{NN}}$ for different fractions of pile-up events. The $C_{32}(=C_3/C_2)$, $C_{42}(=C_4/C_2)$, and $C_{31}(=C_3/C_1)$ ratios show a strong dependence with energy, and the fraction of added pile-up events. Without the presence of pile-up events, $C_{32}$ remains constant at 1 for all $\sqrt{s_{NN}}$. Even a small fraction of pile-up events has a large effect on $C_{42}$ values. A similar study is performed by mixing the proton and antiproton multiplicities from minimum bias events as pile-up events with the $p$ and $\bar{p}$ multiplicities from the central collision events. Fig-

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TABLE I: Mean values of proton and antiproton distributions for most central (0–5%) Au+Au collisions at various $\sqrt{s_{NN}}$ measured by STAR experiment [14, 15] at RHIC.

| $\sqrt{s_{NN}}$ (GeV) | 7.7   | 11.5  | 19.6  | 27    | 39    | 62.4  | 200   |
|-----------------------|-------|-------|-------|-------|-------|-------|-------|
| Proton                | 18.918±0.009 | 15.005±0.006 | 11.375±0.003 | 9.390±0.002 | 8.221±0.001 | 7.254±0.002 | 5.664±0.001 |
| Antiproton            | 0.165±0.001  | 0.490±0.001  | 1.150±0.001  | 1.652±0.001  | 2.379±0.001  | 3.135±0.001  | 4.116±0.001  |

FIG. 2: The proton, antiproton and net-proton multiplicity distributions are shown with (open symbol) and without (solid line) pile-up events for $\sqrt{s_{NN}} = 7.7$ and 200 GeV. The proton and antiproton multiplicities from minimum bias events as pile-up events are added to the individual $p$ and $\bar{p}$ distributions, assuming each of the distributions are negative binomial distributions (NBD).
where \( \langle n \rangle \) is the mean of the distribution and \( k \) is an additional parameter. In the limiting case of \( k \to \infty \), the NBD reduces to a Poisson distribution. The individual proton and antiproton multiplicity distributions are constructed assuming each distribution is a NBD with their corresponding mean values. The \( k \) values are taken as 500 and 550 for \( p \) and \( \bar{p} \), respectively. A fixed \( k \) value is

\[
NBD(n) = \frac{\Gamma(n + k)}{\Gamma(n + 1)\Gamma(k)} \left( \langle n \rangle/k \right)^n (1 + \langle n \rangle/k)^{-n-k}
\]  

FIG. 3: Collision energy dependence of individual cumulants of net-proton distributions for different fractions of pile-up events for central (0–5%) Au+Au collisions. The individual \( p \) and \( \bar{p} \) multiplicity distributions are assumed to be Poisson. The pile-up events from central collisions are mixed with the original distribution from the same centrality.

FIG. 4: Collision energy dependence of cumulant ratios \((C_2/C_1, C_3/C_2, C_4/C_2, \text{and } C_5/C_1)\) of net-proton distributions for different fractions of pile-up events for central (0–5%) Au+Au collisions. The individual \( p \) and \( \bar{p} \) multiplicity distributions are assumed to be Poisson. The pile-up events from central collisions are mixed with the original distribution from the same centrality.

FIG. 5: Similar as Fig. 4. The pile-up events from minimum bias collisions are mixed with the original distribution from the same central collisions.
considered for $p$ or $\bar{p}$ in all the $\sqrt{s_{NN}}$ to avoid inclusion of additional correlation between the particles, which can change the shape of the NBD distribution.

The individual cumulants are calculated from the $N_{\text{diff}}$ distribution, which is constructed by taking individual $p$ and $\bar{p}$ multiplicity distributions. Figure 6 shows the $\sqrt{s_{NN}}$ dependence of cumulants for different fractions of pile-up events. Both the added pile-up multiplicities and the original multiplicity distributions are from the central Au+Au collisions. Like the case of the Poisson distribution, the effect of the pile-up events is larger for higher cumulant values ($C_3$ and $C_4$). Figure 7 shows the cumulant ratios as a function of $\sqrt{s_{NN}}$ for different fraction of pile-up events. In this case also, the $C_{32}, C_{42},$ and $C_{31}$ ratios show strong dependence on energy and the fraction of added pile-up events. Figure 8 shows the $\sqrt{s_{NN}}$ dependence of cumulant ratios for different fractions of pile-up events by mixing the pile-up multiplicities from the minimum bias events with the $p$ and $\bar{p}$ multiplicity distributions from the central collisions.

C. Binomial distributions with event pile-up

The binomial distributions, used to explain the multiplicity distributions, are constructed using the mean ($C_1$) and variance ($C_2$) values of the proton and antiproton multiplicities as given in Refs. [16] [44]. Looking at the individual cumulants of proton and antiproton from Refs. [16] [44], the individual proton and antiproton distributions resemble the binomial distribution. The multiplicity distributions are assumed to be a binomial distribution as
where \( k \) is the observed multiplicity, \( n \) is the particles being produced and \( p \) is the probability to measure it. The \( C_1 \) and \( C_2 \) are related to the above parameters as \( C_1 = np \) and \( C_2 = np(1 - p) \). The net-proton distribution is obtained by assuming that both the proton and antiproton are produced binomially. Figure 9 shows the \( \sqrt{s_{NN}} \) dependence of cumulants of net-proton distributions for different fraction of pile-up events. The added pile-up multiplicities and the original multiplicity distributions are from 0–5% Au+Au collisions. The effects of pile-up events are larger for higher cumulants as in the cases of Poisson and NBD. Figures 10 and 11 show the cumulant ratios as a function of \( \sqrt{s_{NN}} \) for different fraction of pile-up events from central and minimum bias collisions, respectively mixed with the \( p \) and \( \bar{p} \) multiplicity distributions from the central collisions. The \( C_{32}, C_{42}, \) and \( C_{31} \) ratios show strong dependence on energy and the fraction of added pile-up events.

In all the cases, i.e., Poisson, NBD, and binomial, the higher order cumulant ratios have strong dependence on the fraction of pile-up events. The effect of event pile-up on \( N_{\text{diff}} \) distribution will be more crucial at lower \( \sqrt{s_{NN}} \), due to large asymmetry between proton and antiproton multiplicities. On the other hand, the event pile-up effect is not observed at higher collision energies, because the mean multiplicities of both \( p \) and \( \bar{p} \) are small and comparable. While constructing the net-proton multiplicity distribution, the excess pile-up effect gets neutralize for the high energy collisions, while at lower \( \sqrt{s_{NN}} \) this is not the case. At lower energies the mean multiplicity of pro-
tons is much larger than at higher energies. Therefore, while mixing a central event with central (or minimum bias) event, the effect is more pronounced as compared to higher collision energies. Recent preliminary results for net-proton multiplicity from the STAR experiment \[33\] observed that there is an increase in $\kappa_2^2 (= C_{42})$ values at lower collision energies (particularly at $\sqrt{s_{NN}} = 7.7$ and 11.5 GeV). The large value observed for $C_{42}$ of net-proton multiplicity distributions in central collisions originates partially from the efficiency correction. The measured uncorrected $C_{42}$ value, which would include pileup effects, is close to 1. Thus, any effect from pile-up events would be magnified by the efficiency correction. In the present analysis, we also observe an increase in the higher cumulant ratios due to the presence of residual pile-up events. For $\sqrt{s_{NN}} = 7.7$ and 11.5 GeV, the mean number of protons is higher as compared to other higher energy collisions, which causes the increase in the cumulants due to pile-up events. It is to be noted that, the pile-up effect will be more important for net-proton fluctuations as compared to net-charge fluctuations. At lower energies, the asymmetry between proton and antiproton multiplicities is larger, which is not the case for net-charge. Hence, it is important to know how much residual pile-up effect is present in the experimental data. One can make a more realistic estimate of the residual pile-up effect on the cumulant ratios by knowing the details in a real experimental environment.

IV. SUMMARY

To summarize the present work, we have emphasized the importance of residual pile-up events for net-proton higher moment analysis. It is demonstrated that even a small fraction of the pile-up events can change the higher cumulants significantly, especially at lower central-of-mass energies. This issue is even more important for the fixed target experiments like CBM where the collision rates will be even higher. Using a simple Monte Carlo simulation, we consider two scenarios, namely, when multiplicities from central collision are mixed with other central events and when the multiplicities from central collisions are mixed with less central events to mimic the pile-up scenario. In both cases, the resulting proton and antiproton multiplicities are modified according to the pile-up contribution and type, which are used to construct the event-by-event net-proton multiplicity distribution. To investigate the dependence on the nature of the probability distribution, the initial proton and antiproton distributions are assumed to be Poisson, NBD, or binomial. Qualitatively, all the choices show a significant increase in $C_{32}$, $C_{42}$, and $C_{31}$ ratios as the fraction of pile-up events is increased. The pile-up event has a tendency to increase the ratios of cumulants and is more significant at lower energies. This observation makes it critical to estimate the purity of the measured physics event sample for net-proton multiplicity analysis. Preliminary results from the STAR experiment \[33\] also show an increasing trend in these observables at lower $\sqrt{s_{NN}}$. The large increase in the net-proton cumulant ratios at lower energies from pile-up events makes it difficult to interpret the experimental observable for a critical point. The measurements from the STAR experiment may not have a significant contribution from pile-up events because of the high-resolution silicon vertex detector. Future high-luminosity experiments should be careful about the contribution of such events as it may influence the higher moment observables. It is important to estimate the effect of residual pile-up events before making any conclusion on the critical point while using higher moments of net-proton multiplicity distributions, as this may lead to a very different conclusion.

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[1] M. A. Stephanov, K. Rajagopal and E. V. Shuryak, Phys. Rev. Lett. 81, 4816 (1998).
[2] Y. Aoki, G. Endrodi, Z. Fodor, S. D. Katz and K. K. Szabo, Nature (London) 443, 675 (2006).
[3] Z. Fodor and S. D. Katz, JHEP 0404, 050 (2004).
[4] M. G. Alford, K. Rajagopal and F. Wilczek, Phys. Lett. B 422, 247 (1998).
[5] M. A. Stephanov, Prog. Theor. Phys. Suppl. 153, 139 (2004); Int. J. Mod. Phys. A 20, 4387 (2005).
[6] A. Bazavov, H. T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, S. Mukherjee and P. Petreczky et al., Phys. Rev. Lett. 109, 192302 (2012).
[7] S. Ejiri, F. Karsch and K. Redlich, Phys. Lett. B 633, 275 (2006).
[8] M. A. Stephanov, K. Rajagopal and E. V. Shuryak, Phys. Rev. D 60, 114028 (1999).
[9] V. Koch, A. Majumder and J. Randrup, Phys. Rev. Lett. 95, 182301 (2005).
[10] M. Asakawa, U. W. Heinz and B. Muller, Phys. Rev. Lett. 85, 2072 (2000).
[11] M. Asakawa, S. Ejiri and M. Kitazawa, Phys. Rev. Lett. 103, 262301 (2009).
[12] M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009).
[13] R. V. Gavai and S. Gupta, Phys. Lett. B 696, 459 (2011).
[14] M. Cheng, P. Hegde, C. Jung, F. Karsch, O. Kaczmarek, E. Laermann, R. D. Mawhinney and C. Miao et al., Phys. Rev. D 79, 074505 (2009).
[15] F. Karsch and K. Redlich, Phys. Lett. B 695, 136 (2011).
[16] L. Adamczyk et al. [STAR Collaboration], Phys. Rev. Lett. 112, 032302 (2014).
[17] M. M. Aggarwal et al. [STAR Collaboration], Phys. Rev. Lett. 105, 022302 (2010).
[18] A. Adare et al. [PHENIX Collaboration], Phys. Rev. C 93, 011901 (2016).
[19] L. Adamczyk et al. [STAR Collaboration], Phys. Rev. Lett. 113, 092301 (2014).
[20] P. Garg, D. K. Mishra, P. K. Netrakanti, B. Mohanty, A. K. Mohanty, B. K. Singh and N. Xu, Phys. Lett. B 726, 691 (2013).
[21] D. K. Mishra, P. Garg, P. K. Netrakanti and A. K. Mohanty, Phys. Rev. C 94, 014905 (2016).
[22] V. V. Begun, M. I. Gorenstein, M. Hauer, V. P. Konchakovski and O. S. Zozulya, Phys. Rev. C 74, 044903 (2006).
[23] M. Nahrgang, M. Bluhm, P. Alba, R. Bellwied and C. Ratti, Eur. Phys. J. C 75, 573 (2015).
[24] A. Bzdak, V. Koch and V. Skokov, Phys. Rev. C 87, 014901 (2013).
[25] M. Nahrgang, T. Schuster, M. Mitrovski, R. Stock and M. Bleicher, Eur. Phys. J. C 72, 2143 (2012).
[26] J. Fu, Phys. Lett. B 722, 144 (2013).
[27] A. Bhattacharyya, R. Ray, S. Samanta and S. Sur, Phys. Rev. C 91, 041901 (2015).
[28] P. K. Netrakanti, X. F. Luo, D. K. Mishra, B. Mohanty, A. Mohanty and N. Xu, Nucl. Phys. A 947, 248 (2016).
[29] D. K. Mishra, P. Garg and P. K. Netrakanti, Phys. Rev. C 93, 024918 (2016).
[30] T. J. Tarnowsky and G. D. Westfall, Phys. Lett. B 724, 51 (2013).
[31] D. K. Mishra, P. Garg, P. K. Netrakanti, L. M. Pant and A. K. Mohanty, Adv. High Energy Phys. 2017, 1453045 (2017).
[32] P. Garg, D. K. Mishra, P. K. Netrakanti, A. K. Mohanty and B. Mohanty, J. Phys. G 40, 055103 (2013).
[33] X. Luo [STAR Collaboration], PoS CPOD 2014, 019 (2015) [arXiv:1503.02558 [nucl-ex]].
[34] M. A. Stephanov, Phys. Rev. Lett. 107, 052301 (2011).
[35] B. J. Schaefer and M. Wagner, Phys. Rev. D 85, 034027 (2012).
[36] X. Luo, Nucl. Phys. A 956, 75 (2016).
[37] C. H. öhne and P. Senger, The CBM Detector Concept, edited by B. Friman, C. Höhne, J. Knoll, S. Leupold, J. Randrup, R. Rapp, and P. Senger, Landolt-Boernstein New Series I, Vol. 23 (Springer, Heidelberg, 2010), Part V, Chap. 2, pp. 623629.
[38] Harnaire Ian, "A Study of Pile-up in 200 GeV Au+ Au Collisions at RHIC'', Doctoral dissertation, University of Illinois at Chicago, 2005.
[39] Z. Marshall [ATLAS Collaboration], J. Phys. Conf. Ser. 513, 022024 (2014).
[40] A. V. Fedotov, Nucl. Instrum. Meth. A 557, 216 (2006).
[41] A. Dreis, R. Fillier, H. Hsueh, W. MacKay and D. Trbojevic, Conf. Proc. C 0106181, 3141 (2001).
[42] Mid-Term Strategic Plan: 2006-2011 for the Relativistic Heavy Ion Collider at BNL, February 14, 2006.
[43] K. A. Dreis, L. Ahrens, M. Bai, J. Beebe-Wang, I. Blackler, M. Blaskiewicz, K. Brown, M. Brennan, D. Bruno, J. Butler, et al., Conf. Proc. C 110328, 2220 (2011).
[44] "Cumulants of proton and antiproton distribution measured by STAR experiment.
[45] G. J. Alner et al. [UA5 Collaboration], Phys. Lett. B 160, 913 (1985).
[46] T. Abbott et al. [E-802 Collaboration], Phys. Rev. C 52, 2663 (1995).
[47] F. Becattini, Z. Phys. C 69 (1996) 485.
[48] A. Adare et al. [PHENIX Collaboration], Phys. Rev. C 78, 044902 (2008).