Feshbach Resonance due to Coherent $\Lambda$-$\Sigma$ Coupling in $^7\Lambda$He

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Abstract

Coherent $\Lambda$-$\Sigma$ coupling effect in $^7\Lambda$He is analyzed within three-body framework of two coupled channels, $\Lambda$-$t$-$t$ and $\Sigma$-$\tau$-$t$, where $\tau$ represents trinucleon which is either $^3$H or $^3$He. The hyperon-trinucleon ($Y\tau$) and trinucleon-trinucleon ($\tau\tau$) interactions are derived by folding $G$-matrices of $YN$ and $NN$ interactions with trinucleon density distributions. It is found that the binding energy of $^7\Lambda$He is 4.04 MeV below the $\Lambda+t+t$ threshold without $\Lambda$-$\Sigma$ coupling and the binding energy is increased to 4.46 MeV when the coupling effect is included. This state is 7.85 MeV above the $^6$He+$\Lambda$ threshold and it may have a chance to be observed as a Feshbach resonance in $^7\Lambda$Li ($e,e'K^+$) $^7\Lambda$He experiment done at Jefferson Lab.

Keywords:
Feshbach resonance: coherent $\Lambda$-$\Sigma$ coupling: hyperon-trinucleon interaction

PACS:

1. Introduction

Significance of $\Lambda$-$\Sigma$ coupling effect in binding mechanism of light $\Lambda$-hypernuclei has long been recognized and discussed in the references [1, 2]. Admixture of $\Sigma$ states in $\Lambda$-hypernuclei is probably an important aspect of hypernuclear dynamics. There are two coupling schemes namely incoherent and coherent $\Lambda$-$\Sigma$ couplings [3]. Incoherent $\Lambda$-$\Sigma$ coupling means a nucleon changes to an excited level after the interaction, while the other process where a nucleon remains in its ground state after converting $\Lambda$ to $\Sigma$, is called coherent $\Lambda$-$\Sigma$ coupling. In the latter case, all the nucleons have an equal chance to interact with the converted $\Sigma$ and coupling effect contributed from each nucleon is added coherently. Harada [4] has successfully fitted the experimental spectra of $^4$He (stopped $K^-$, $\pi^-$) [5] and $^4$He (in-flight $K^-$, $\pi^-$) [6] production reactions by taking into account the coherent $\Lambda$-$\Sigma$ coupling effect. Furthermore, all the s-shell $\Lambda$ hypernuclear binding energies are well reproduced only after the coherent $\Lambda$-$\Sigma$ coupling effect has been included [3, 7].

It has been found that the coherent coupling contribution is significantly large on the order of 1 MeV in $^4\Lambda$H and $^4\Lambda$He ground states.

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2. Coupled-channel three-body cluster model of $^7\Lambda$He

Having considered the above mentioned findings, we analyze a structure of $^7\Lambda$He in continuum by using three-body model of $\Lambda$-$t$-$t$, $\Sigma^0$-$t$-$t$ and $\Sigma$-$h$-$t$ coupled channels to investigate the coherent $\Lambda$-$\Sigma$ coupling effect. The coupling between $\Lambda$-$\Sigma^0$ gives coherent $\Lambda$-$\Sigma$ coupling, while Lane term of $\Sigma^0$-$\Sigma$-$h$ coupling plays a significant role in forming $^{3}_2H$ \cite{8}. All these couplings are included in our analysis. To solve three-body calculation, we employ Kamimura’s coupled rearrangement-channel method \cite{9}.

Three-body Hamiltonian of the $\Lambda$-$t$-$t$ diagonal part, which we explicitly show for explanation here, is

$$H_{\Lambda \Lambda} = -\frac{\hbar^2}{2\mu_c}\Delta_{\vec{k}} - \frac{\hbar^2}{2\mu_c}\Delta_{\vec{r}} + \{V_m(\vec{r}_1) + V_\Lambda(\vec{r}_2) + V_{\Sigma}(\vec{r}_3)\} + V_{\text{Pauli}}(\vec{r}_1, \vec{r}_2),$$

(1)

where $V_{\text{Pauli}}$ expresses Pauli exclusion effect between two tritons. In orthogonality-condition model (OCM) \cite{10},

$$V_{\text{Pauli}}(\vec{r}, \vec{r}') = \lim_{\lambda \to \infty} \lambda \sum_{\vec{r}} |\Phi_{\vec{r}}(\vec{r}) \rangle \langle \Phi_{\vec{r}}(\vec{r}')|,$$

(2)

where $\Phi_{\vec{r}}(\vec{r})$ is the Pauli forbidden state. Total wave function of the $\Lambda$-$t$-$t$ channel is expanded in Gaussian bases which are spanned over three rearrangement-channels as follows,

$$\Psi_{\Lambda \Lambda}(\vec{r}, \vec{R}) = \sum_{c=1}^{3} \sum_{l, j, \ell} A_{l, j, \ell}^{(c)} e^{-i(\vec{r} - \vec{R})^2} e^{-i(\vec{r}_1 - \vec{R})^2}.$$

(3)

Wave functions of the other channels $\Sigma^0$-$t$-$t$ and $\Sigma$-$h$-$t$ are treated in a similar way.

The $YN$ interaction used in our computations is a phase equivalent potential of the Nijmegen model-D $YN$ potential \cite{11}. Then, hyperon-trinucleon potentials are obtained by folding the effective interaction, i.e. $G$-matrix of the above $YN$ potential with trinucleon density distributions \cite{12}. They are expressed in five-range Gaussian form, the range and strength parameters of which are slightly modified so as to reproduce the empirical $\Lambda$ binding energy of $^{4}_1H(0^+)$ and $^{4}_\Lambda H(1^+)$, and the expansion coefficients for $I = 1/2$ are given in Table 1. Trinucleon-trinucleon ($\tau$-$\tau$) interaction is obtained by doubly folding $G$-matrix of Tamagaki’s OPEG $NN$ potential with trinucleon density distributions. This $\tau$-$\tau$ potential is spin-isospin dependent, and does not give any bound state of triton-triton two-body system in OCM treatment.

| State | $^4_1H(0^+)$ | $^4_\Lambda H(1^+)$ |
|-------|--------------|---------------------|
|       | $V_{\Lambda}(\Lambda \tau \tau)$ | $V_{\Sigma}(\Sigma \tau \tau)$ | $V_{\Lambda}(\Lambda \tau \tau)$ | $V_{\Sigma}(\Sigma \tau \tau)$ | $V_{\Lambda}(\Lambda \tau \tau)$ |
| $k$   | $V_{\Lambda}(\Lambda \tau \tau)$ | $V_{\Sigma}(\Sigma \tau \tau)$ | $V_{\Lambda}(\Lambda \tau \tau)$ | $V_{\Sigma}(\Sigma \tau \tau)$ | $V_{\Lambda}(\Lambda \tau \tau)$ |
| 1     | 1.7284       | 9.4720              | -3.5575                     | 0.36869                     | 1.9558                     | -0.16822                     |
| 2     | 50.838       | 69.234              | 4.2647                      | 43.237                     | 65.877                     | 7.1105                      |
| 3     | -63.595      | -105.09             | 32.682                      | -57.877                    | -44.391                    | 0.70109                     |
| 4     | 6.2861       | 10.130              | -4.0631                     | 5.1858                     | 2.1056                     | -1.3503                     |
| 5     | -1.1202      | -2.3001             | 0.8537                      | -0.86971                   | -0.61958                   | 0.12653                     |

Table 1: The hyperon-trinucleon interactions in MeV for $^4_1H(0^+)$ and $^4_\Lambda H(1^+)$. The range parameters are $\mu_1 = 1.00$ fm, $\mu_2 = 1.37$ fm, $\mu_3 = 1.87$ fm, $\mu_4 = 2.56$ fm, $\mu_5 = 3.50$ fm.
3. Results and discussions

From our calculation, a bound state is found to be at \(4.46\) MeV below the \(t + t + \Lambda\) threshold and about \(7.85\) MeV above the \(^6\text{He} + \Lambda\) threshold as shown in Fig. 1. It is a Feshbach resonance state \([13]\), because it lies in continuum region of the open channels such as \(^6\text{He} + \Lambda\), \(^6\text{He} + n\), \(^5\Lambda\text{He} + n + n\) and \(\alpha + n + n + \Lambda\) channels.

\[
\begin{array}{c|c}
\hline
\text{Channel} & \text{Energy (MeV)} \\
\hline
 t + t + \Lambda & 12.31 \\
 \Lambda^4\text{He} + t & 10.28 \\
 \hline
\end{array}
\]

Figure 1: The obtained Feshbach-resonance state of \(t + t + \Lambda\). It is shown together with the thresholds of various channels.

A possible way to populate this resonance state, \(^7\text{He}^*\), is through \((e, e'K^+)\) electro-production reaction on \(^7\text{Li}\) target. Formation of \(^7\text{He}^*\) through the \(tt\Lambda\) resonance is described with \(s\)-channel interaction model as shown in Fig. 2. Formation and decay spectra are analyzed, as explained in Ref. [14], by using Yamaguchi-type separable (i.e. \(s\)-channel) potential:

\[
\langle \vec{k} | V_{ij} | \vec{k}' \rangle = g_i(\vec{k}) U_{ij} g_j(\vec{k}'), \quad g_i(\vec{k}) = \frac{\Lambda^2_i}{\Lambda^2_i + \vec{k}^2},
\]

where \(i, j = t, \Lambda^4\text{He}, \Lambda^6\text{He}\).

Figure 2: Production and decay mechanisms of the \(tt\Lambda\) resonance state through \(^7\text{Li}\ (e, e'K^+)\) reaction.

Missing-mass spectrum and invariant-mass spectrum can be obtained by detecting emitted particles \(e'\) and \(K^+\) and decay particle \(\Lambda\), respectively. The effect of interaction range on the
missing-mass spectrum is investigated by varying the range parameter of $^4$H-$t$ interaction from 0.3 to 0.9 fm.

Figure 3 shows the missing-mass spectrum calculated with 3 MeV width of the $tt\Lambda$ resonance. We have compared this missing-mass spectrum with JLab experimental spectrum \cite{15}, where a peak structure is found at about 7 MeV above the $^6$He+$\Lambda$ threshold, which might correspond to our resonance state. A crude explanation of why a narrow peak appears in continuum region is such that; similarity in structures between $\alpha$-$t$ and $t-t$ may give a strong population of $tt\Lambda$ state, while different structures between $t-t$ and $^8$He ensure the formation of quasi-stable Feshbach resonance. However, a recent experimental spectrum of $^7$Li ($e, e'K^+$) $^7$He displays only a prominent peak below the $^6$He+$\Lambda$ threshold in bound region \cite{16}. In order to clarify the possible existence of Feshbach resonance in $^7$Li system, electro-production or equivalent experiments on $^7$Li target with high statistics are highly awaited.

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