Investigations into Fine Grinding*

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Abstract

This paper presents four different studies from the French national research project that deal with the fine grinding of hydrargillite. These include pilot plant tests of dry grinding in an air jet mill and wet grinding in a stirred bead mill. Two other more fundamental studies are concerned with determining fragmentation schemes in fine grinding. The first investigation concerns theoretical predictions of breakage of complex agglomerate crystals such as hydrargillite, plus image-analysis-based methods of identifying different types of particle morphology. Finally, single-particle impact fragmentation is studied in a specially designed apparatus which allows observation of particle impacts on a target at up to 300 m/s. This has led to the identification of different regimes of breakage that affect complex crystals such as hydrargillite.

1. Introduction

It has been estimated that a considerable amount of the total world energy consumption is used for industrial particle size reduction processes. Metallurgical industries are based on mineral ores which necessarily pass through crushing and grinding operations. The materials used in the building industry all require a size reduction process at some stage or other. Furthermore, comminution theory is not well developed and the design of industrial plant is made on the basis of laboratory tests which are difficult to extrapolate, resulting in expensive over-sizing of equipment, thus adding to their inherent energy inefficiency. Finally, there is a lack of fundamental knowledge of fragmentation mechanisms that makes it impossible to predict product quality (particle size distribution, particle shape, physical properties) — all factors which are very important when fine grinding is used to produce special high-value products such as ceramics, pharmaceuticals, fillers, and pigments, etc.

For all these reasons, a co-ordinated research action has been set up in France to study modelling and design methods required for the production of fine powders with controlled properties, well-defined particle size distributions, and also to improve process energy efficiency. Four research teams are working on different aspects of the problem with the overall objective of extending knowledge and developing a methodology for modelling fine grinding operations.

All of the teams are working with the same material, hydrargillite; Al(OH)3. The programme includes pilot plant tests with two types of mill: An Alpine 100 AFG opposed jet mill (in Nancy), and a Drais Perl-Mill V/H stirred bead mill (in Toulouse). In parallel, there is a more fundamental study of fragmentation mechanisms in the high-velocity impact of jets of particles being carried out in Compiegne. Finally, the team in Vandoeuvre is examining theoretical fragmentation schemes and developing methods for characterising the debris.

2. Theoretical Aspects

2.1 Batch grinding

Batch grinding is generally described on the basis of the breakage (B) and selection functions (S) which represent the distribution of particle sizes resulting from the breakage of a mother particle and the aptitude of that particle to break, respectively. We thus have the well-known equation for batch grinding in cumulative form:

\[
\frac{dR(x, t)}{dt} = -S(x, t)R(x, t) + \int_{x}^{\infty} S(y)B(x, y) \frac{dR(y, t)}{dy} dy
\]

where \( R(x, t) = \int_{x}^{\infty} m(y, t) dy \)

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The general analytical solution to the integro-differential equation of batch grinding comprises a convergent series of iterated kernels. This solution is complicated and has found no practical application, leading workers to develop analytical solutions based on simplified representations of the selection and breakage functions. Kapur [2] in particular has shown that the grinding equation has a similarity solution when the breakage rate function is a power law and the breakage distribution function is self-similar, i.e. in cumulative form it is:

\[ B(x, y) = B(x/y) \]  

(2)

The interesting aspect of this solution is that the distributions generated at different times collapse into a single curve when they are plotted as a function of dimensionless particle size \( x/\mu_1(t) \) where \( \mu_1(t) \) is the first moment of the particle size distribution. Kapur has shown that the first moment, median or any other quantile size are all proportional to one another and that any characteristic length could be used instead of \( \mu_1 \). When the size spectra of material being ground in a mill is self-similar, the mechanism of size reduction probably remains constant during the operation, and extrapolation in time or any other operating variable becomes easy.

Particle size is normally measured with instruments giving distributions in size classes, and it is often more convenient to use equation (1) in terms of discrete size classes as follows:

\[
\frac{dR_i}{dt} = -S_i R_i(t) + \sum_{j=1}^{i-1} (S_{j+1} B_{i,j} - S_j B_{i,j} ) R_j(t) - S_j B_{i,j} R_j(t)
\]

(3)

where \( R_i(t) = \sum_{j=1}^{i-1} m_j(t) \) and \( B_{i,j} = \frac{\sum_{k=j+1}^{i} b_{k,j}}{k!} \)

\( m_j(t) \) = the mass fraction of particles in size class \( i \).

\( S_i \) = the probability of breakage of particles of class \( i \).

\( b_{k,j} \) = mass fraction of particles of class \( j \) which break to end in size class \( i \).

This formalism was established over 25 years ago [1] and many methods have been developed to determine the breakage and selection functions, in particular by means of experiments with monosized sieve cuts. Unfortunately, it is difficult to prepare narrow sieve fractions in the particle size range covered by fine and ultrafine grinding, i.e. \(<50 \mu m\), and there are also difficulties in reconciling instrumental particle size analysis methods (Coulter Counter, Laser diffraction, etc.) with sieve sizing. These problems can be avoided by using the method proposed by Kapur [2-4], who gave an approximate solution of equation 3 in terms of two parameters \( G_j \) and \( H_j \), which are themselves functions of the functions \( B_{i,j} \) and \( S_j \).

\[ R_j(t) = R_j(0) \exp \left( \frac{G_j}{H_j} \frac{t^2}{2} \right) \]  

(4)

Equation (4) proposed by Kapur derives from an infinite series stopped at order 2, but by noting that \( R_j(t) = R(x_i,t) \), this may be generalized to any order \( p \) in terms of the particle size distribution at time \( t \), with respect to the initial particle size distribution [4]:

\[ \frac{R(x,t)}{R(x,0)} = \exp \left( \sum_{k=1}^{p} \frac{K^{(k)}(x)}{k!} \right) \]  

(5)

where \( K^{(k)}(x) = \sum_{j=1}^{i} (S_i B_{i,j} - S_j B_{i,j} ) (G_j - G_{j+1})^{k+1} R_j(0) \)  

(6)

Taking this expression to any given order, it is possible to obtain a mathematical representation of the evolution of the particle size distribution during batch grinding to any degree of precision.

2.2 Continuous grinding

In the case of a continuous grinding process, the relation between the feed and product particle size distributions will depend both on the grinding kinetics and on the time the material spends in the mill. If the solids move through the mill in plug flow, a mean residence time can be used in equation (5), but in general, the existence of a distribution of residence times (\( E(t) \)) must be taken into account. To a first approximation, which will be relaxed later on, it may be assumed that the residence time distribution is independent of particle size. Thus the change in particle size distribution between the inlet \( (R(x)) \) and the outlet \( (R_s(x)) \) of a continuous grinding process is given by the following:

\[ \frac{R_s(x)}{R_i(x)} = \int_0^∞ \exp \left( \sum_{k=1}^{p} \frac{K^{(k)}(x)}{k!} \right) E(t) dt \]  

(6)

Thus in addition to the representation of the kinetics it is also necessary to have an expression for the residence time distribution in the mill.
3. Stirred bead mill

3.1 Batch experiments

Figure 1 shows a schematic diagram of a stirred bead mill (Dyno-mill KDL) where the grinding medium consists of small beads which are violently stirred by the four stirrer discs. The particles to be ground are pumped through the mill as a slurry and ground by the mechanical action of the moving beads. Experiments have been performed to determine the change in particle size distribution of hydrargillite with time during grinding in a stirred bead mill. An example of the results is shown in Figure 2. For this run, the initial size distribution can be well described by a Rosin-Rammler-type equation:

\[ R(x,0) = \exp\left(\frac{X}{20.9}\right)^{1.1} \]  

(7)

These results can also be represented as shown in Figure 3, i.e. in terms of the variation of the mass fraction less than a certain particle size as a function of grinding time. Equation (5), taken to order 1, is now used to obtain a mathematical representation of the grinding kinetics. The lines in Figure 3 show the result using values of the coefficient \( K(x) \) as shown in Figure 4. Batch grinding kinetics can therefore be represented as follows:

\[ \frac{R(x,t)}{R(x,0)} = \exp(K(x)t) \]  

(8)

where \( K(x) = -kx^a \) with \( k = 0.0037 \) and \( a = 1.29 \).

It can easily be demonstrated that this equation is self-similar as is illustrated in Figure 5. Here, the cumulative size distributions are given as a function of the characteristic size \( x_{63.2} \) using \( x_{63.2} \) as the particle size corresponding to 63.2% of particles smaller than \( x_{63.2} \).
These results can be compared with other experiments in a 1-litre batch ball mill, and Figure 6 shows the evolution of the particle size in a typical run. For this experiment, the operating conditions were chosen to be similar to those commonly used in actual practice (mill speed =75 % of critical, 15 mm stainless steel balls, 20 % volume filling, 40 % powder filling, slurry density 50 %). The initial particle size was between 100 and 125 μm, and preliminary experiments — reported elsewhere [7] — established that these conditions would lead to minimum fineness. Figure 6 clearly shows that the size distributions exhibit self-similarity even after relatively short grinding times. Applying the Rosin-Rammler equation to these results leads to the following expression for the batch grinding kinetics in the ball mill:

\[
R(x, t) = \exp \left[ -\left( \frac{x}{x_{63.2}(t)} \right)^2 \right]
\]

(9)

In batch grinding, the characteristic particle size decreases as a function of the time of grinding, and Kapur has shown that it varies with time as in the Walker general energy-size equation:

\[
\frac{dx_{63.2}(t)}{dt} = -Cx_{63.2}^e(t)
\]

(10)

Integrating this with the initial conditions \( R(x, t) = R(x, 0) \) leads to an equation similar to equation (8). It was found that the experimental results could be correlated by the following equation:

\[
R(x, t) = R(x, 0)\exp (-2.4 \cdot 10^{-5}x^2t)
\]

(11)

Even though the two equations — (8) and (11) — for the batch grinding kinetics differ by an order of magnitude, the self-preserving character of the particle size distributions is demonstrated in both cases.

### 3.2 Continuous grinding

#### 3.2.1 Mass transport in a stirred bead mill

In order to characterise the transport of particles through a stirred bead mill, the standard grinding chamber of a Drais Perl Mill was replaced by a chamber which allows samples to be withdrawn from inside the mill. The mill is thus divided into five compartments numbered C1 to C5 as shown in Figure 7. Slurry samples can be taken by means of a syringe in each compartment — either near the centre at the stirrer shaft or from the periphery at the mill wall. An example of the change in particle size distribution within the mill during continuous grinding of hydargilite is shown in Figure 8. This shows that the particle size decreases linearly between the inlet and the outlet of the mill, and shows no evidence of classification by size in the mill. However, comparison of the size distributions of particles taken at the mill axis and at the wall (Figure 9) does demonstrate radial classification by the action of centrifugal force.

#### 3.2.2 Residence time distribution

As indicated above, if a continuous grinding process is to be treated, it is necessary to know the residence time distribution. As a first approximation, the radial classification of the particles may be neglected and the mill simulated as a series of stirred cells of volume defined by the separation between the stirrer discs.
The only adjustable factor here being the inter-cell recycle factor. Thus, the assumption that solids move through the mill in the same way as the carrier fluid allows the use of tracer experiments on the liquid to determine the residence time distribution. Such experiments have been presented elsewhere together with the transfer function of the model [6]. Combining equation (6) with the expression for the batch grinding kinetics (8) leads to:

\[
\frac{R_{\text{out}}(x)}{R_{\text{in}}(x)} = \int_0^\infty \exp(-kx^4t_\text{s})E(t_\text{s})dt_\text{s}
\]

(12)

This particular case can be identified as the Laplace transformation of the function \(E(t_\text{s})\) and may therefore be written as:

\[
\frac{R_{\text{out}}(x)}{R_{\text{in}}(x)} = L(E(kx^4))
\]

(13)

Knowledge of the transfer function of the flow model in the mill can then be used in equation (13) to predict the output particle size distribution as a function of the input size distribution. Figure 10 shows an example of experimental results compared with three theoretical predictions where the stirred bead mill is considered to be (1) in plug flow, (2) a perfectly mixed stage, and (3) where the transfer function established experimentally [6] is used.

### 4. Opposed jet mill

An Alpine 100 AFG opposed jet mill with its integrated 50 ATP turbo-classifier was used. In this type of mill, the particles contained in the hopper are introduced into the grinding chamber by a screw feeder.
and are broken by their impact on one another under the action of three jets of compressed air. The particles are then swept upwards to the turbo-classifier where they are either allowed to exit from the mill or are returned to the grinding section. The size separation is controlled by the rotational speed of the classifying wheel. This type of mill can therefore be represented by the flow sheet shown in Figure 11.

All the flows shown in Figure 11, other than the feed and the product, are internal in the mill and cannot be directly measured. However, as will be shown later, it is possible to make independent measurements on the grinding section and on the classification section. Such information can then be coupled by mass balance over the grinding circuit to link the feed and product particle size distributions:

\[
R_0(x) = \frac{1}{I_p(x)} R(x) + \left[ \frac{1 - I_p(x)}{I_p(x)} \right] \int_0^x \frac{\Gamma(x)'}{1 - \Gamma(x)'} dx
\]

(14)

In this equation, \( I_p(x) \) is the batch grinding kinetics coupled with the residence time distribution, \( \Gamma(x) \) is the grade efficiency curve of the classifier, and \( E_T \) is the total efficiency of the classifier under the conditions of use. It is possible to eliminate \( E_T \) from this equation, but the result is far too complicated to be useful.

4.1 Grinding kinetics in a 100 AFG opposed jet mill

In an Alpine 100 AFG mill, it is not possible to make independent measurements of the particle size distribution of the material leaving the grinding section and entering the classifying section. However, short and quasi batch grinding experiments can be performed by loading the mill with a quantity of powder equal to the hold-up in continuous operation, setting the classifier to maximum speed to retain particles in the mill, and operating it under the same conditions of air pressure and flow rate as in continuous operation. Sampling at different intervals gives the time variation of the cumulative oversize distribution. An example of such results and their correlation to the Kapur approximation is shown in Figures 12 to 14 where:

\[
K^0(x) = a x \frac{1}{1 + (x/\gamma)^\beta}
\]

(15)

where

\[
a = 3.8 \cdot 10^{-4} \mu m^{-1} \ s^{-1}, \ \gamma = 106.4 \ \mu m, \ \beta = 2.8
\]
Fig. 14 The function $K(x)$ used in figure 13 and its fit to an exponential

This analysis is only valid for times less than 80 sec, which will be shown later to be adequate for effective residence times in continuous operation.

### 4.2 Classification in a 100 AFG opposed jet mill

The opposed jet mill can be converted to a 50 ATP classifier by removing the air jet section and replacing it with an air inlet section. This then allows independent determination of the characteristics of the classifier, and in particular determination of the grade efficiency curve $\Gamma'(x)$ and total efficiency $E_T$ for substitution in equation (14). An example of the result is given in Figure 15.

Fig. 15 Reduced grade efficiency curve

### 4.3 Residence time distribution and overall operation of an opposed jet mill

The only remaining factor is the residence time distribution in the mill. The practical difficulties involved in making tracer measurements have been circumvented by numerical testing of flow models of the mill. The simplest version which gives an adequate representation of the results is shown in Figure 16. As can be seen, this assumes an active grinding zone of 50 cm$^3$ included in the convergence of the 3 jets, followed by a plug flow transfer zone leading to and coming from the classifying section. Combination of these three elements (batch grinding kinetics, classification and residence time distribution), leads to a prediction of the overall action of the air jet shown in Figure 17 as compared with the experimental results in continuous grinding. It should be noted that this corresponds to a mean residence time of about 13 seconds, well within the limits of validity of the batch grinding equation (15).

Fig. 16 Flow model for residence time distribution in an air jet mill

Fig. 17 Comparison between experimental and computed (eq. 14) particle size distribution in continuous grinding in an air jet mill

### 5. Study of fragmentation mechanisms

Another separate part of the programme concerns a fundamental study of the fragmentation of particles in high-velocity impacts with the aim of gaining insight into the elementary processes occurring in an air jet mill and eventually a better understanding of the role of particle morphology. Two different aspects of this have been examined: an experimental apparatus has been built to observe high-velocity impacts (up to 300 m/s), and a theoretical study has been made to provide means of characterising debris and fragmentation routes for Al(OH)$_3$. 

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5.1 Fragmentation scheme for Al(OH)$_3$

SEM examination of particles of hydrargillite (Figure 18) leads to the proposed fragmentation scheme shown in Figure 19. In this, the original particles (shown at top left) can be broken down to the ultimate debris (bottom right) by several routes involving different mechanisms: rupture of grain joints, chipping, cleavage and grinding. The practical question raised is: Can the different routes occurring in a given grinding mill be determined and provide the grinding "signature" of a mill?

A given type of particle is situated in the fragmentation scheme by two parameters: particle size and particle morphology. The first of these can be determined by standard particle size analysis, for example, by laser diffraction. The second can be determined by image analysis using a sufficiently discriminating definition of particle morphology. The first method tried was fractal analysis by the Richardson-Mandelbrot method. This gave information on the detailed structure of particle contours, but presented problems of interpretation as soon as the scale of observation was close to the scale of the details used for discrimination.

Another method for characterising the morphology of ground material under development is based on the Hausdorff space or distance. In this, the contour of an irregular particle (A) is compared with a reference shape (R) in terms of the minimum distances between the real contour and the reference contour. The maximum of the minima from the reference to the particle is called the Hausdorff distance (H). The minima of the minima from the particle to reference is called the morphological distance (D).

$$H(A, R) = \text{Max}\{d(A, R), d(R, A)\} \quad \text{Hausdorff distance}$$

$$D(A, R) = \text{Min}\{d(A, R), d(R, A)\} \quad \text{Morphological distance}$$

These two values allow any given particle to be represented in a $\{H, D\}$ diagram. An example of these diagrams applied to debris of hydrargillite is shown in Figure 20. In addition, these characteristics can be used to define other morphological parameters such as: anisotropy, sinuosity, or roughness, and changes in these can be given as trajectories on the $\{H, D\}$, as are also shown in Figure 20, giving the results of simulations of changes in ideal systems. Finally, Figure 21 shows the results of the Hausdorff morphological analysis applied to hydrargillite subjected to 0, 10, 20, and 30 seconds grinding in an air jet mill.

6. Experimental study of high velocity impacts

6.1 Experimental equipment

Experimental equipment has been built to study the fracture of particles accelerated in a jet of air and impacting on a target [9]. As shown in Figure 22, it includes an air solids mixing unit, an accelerating nozzle, and a chamber containing the orientable target plate plus a cyclone/filter system for recovering the particles. The energy supplied to the particles is determined from the velocity of the particles, which is measured by two different techniques: an optical correlation method (Vector probe), and by video recording using a high-shutter-speed camera. This equipment allows experiments at velocities from 10 to 300 m/sec at solid loadings from 0.001 to 3.

A preliminary study of the hydrodynamics of the jets showed that the particles do not follow the lines of gas flow near the impact zone if the criterion $\lambda$ is greater than 1 [8], where $\lambda$ is given by:
This limits the use of very small particles, but in all the experiments reported here $\lambda$ was always greater than 10.

It was further established that particle-particle interactions in the nozzle are minimised, and the results are comparable with single-particle impacts with very dilute jets which have a solids to air flow rate ratio of less than 0.1. However, the jet system has the advantage that a great number of particles are used and the results are statistically significant with respect to particle-particle variability.

By adding a second particle jet system, the equipment can be further extended for the study of double jet impacts.

6.2 Methodology

The experimental parameters are the air and the solids flow rates, $W_j$ and $W_s$, respectively, and the type of solid. Several different target materials and orientations have been used, but the discussion here will be limited to experiments with a silicon carbide target set at $90^\circ$ to the incident jet. The particle size distribution is measured before and after impact by sieving or with a laser diffraction instrument and related to the impact velocity $V_j$ or the specific energy $E_s = \rho_s V_j^2/2$. Several methods have been used to characterise the fragmentation (particle diameter or
6.3 Experimental results

Here, we report a selection of the results for 5 different types of particle: 3 inorganic powders (glass beads \( d_i = 129 \mu m \), sand \( d_i = 90-125 \mu m \), hydrargillite \( d_i = 93 \mu m \)) and 2 polymers (polyamide PI \( d_i = 123 \mu m \), polymethylmethacrylate PMMA \( d_i = 173 \mu m \)).

6.3.1 Inorganic particles

Figure 23 shows the results obtained with the glass beads; they are similar to those obtained with sand. That is, after a threshold value there is a progressive increase in \( d_i/d_f \) with particle velocity or with energy. The variation is linear with kinetic energy, meaning that the size reduction ratio varies with the square of the velocity of the particles. The existence of a threshold shows that in the low energy domain, the particles undergo plastic deformation on impact without breaking. It seems that this depends exclusively on the nature of the material, the particle size, and on the nature and surface state of the target, regardless of the fragmentation criterion that is used.

Figure 24 shows the effects of impacts on two particle sizes of hydrargillite. The smaller size is obtained by sieving the debris after impact of the larger size. It can be seen that in addition to a threshold value, there is a change in slope for the larger particles at a critical value of impact energy. This discontinuity is evidence of a change in impact behaviour corresponding to two fragmentation mechanisms. Reference to Figure 19 indicates that the first mechanism would seem to correspond to rupture by chipping or splitting of the grain joints of the hydrargillite where the weakest bonds are broken to produce smaller aggregates. The second mechanism would seem to correspond to cleavage of the particles and their destruction by grinding. The reduction in slope indicates that this second type of rupture is harder to produce.

Figure 24 also compares the behaviour of the initial hydrargillite particles (particle size 93 \( \mu m \)) with those of 37 \( \mu m \) particles obtained after a single impact at 15 kJ/kg. It can be seen that the smaller particles show a linear variation in size reduction ratio with impact energy and a slope identical to that of the second regime of the initial hydrargillite. Reasoning in terms of the total overall specific energy supplied to the particles in the two impacts (i.e. adding 15 KJ/kg) shows that the curve for the second impact is in line with the other results. This observation only holds true for two impacts.

Further experiments with four different sieve cuts of hydrargillite allow us to plot the variation of the threshold energy \( E_{so} \) and critical impact energy \( E_{sc} \) as defined in Figure 24 as a function of the particle size. The diagram shows 4 fragmentation zones corresponding to different mechanisms (Figure 25). All impacts below the \( E_{so} \) frontier are "pseudo-elastic". There is no significant reduction in particle size. The \( E_{sc} \) frontier indicates a limiting diameter of \( \approx 40 \mu m \), corresponding to the maximum size of
6.3.2 Organic particles

The experiments with organic particles demonstrate the difference in behaviour on impact between a semi-crystalline polymer (polyamide P11) and an amorphous polymer (PMMA). On the one hand, the semi-crystalline polyamide particles are easily broken and there is a significant size reduction ratio which, just as for sand and glass, varies in a linear fashion with the square of the impact velocity. And on the other hand, it is impossible to break the amorphous polymer PMMA with one impact. SEM observation shows that the particles are flattened at the point of impact and it seems that this plastic deformation engenders fissures which develop sufficiently over 5 and more impacts to lead to rupture.

7. Conclusion

A method of macroscopic modelling to predict the particle size distribution of product from continuous fine grinding has been developed and tested on two different types of mill: a stirred bead mill and an opposed air jet mill. This has been further extended to the case of a grinding circuit formed by an air jet mill with an integral classifier by combining batch grinding kinetics, a solids flow model, and the grade efficiency curve of the classifier.

A more fundamental investigation of fragmentation mechanisms in fine grinding of hydrargillite has led to the development of several methods of characterising debris morphology so as to identify fragmentation routes. Single-particle fragmentation experiments have identified at least two regimes of fracture of the hydrargillite which depend on impact energy.

Future work will attempt to combine these different approaches with the aim of finding a means of operating fine grinding processes to produce not only particles of a given size distribution, but also of a controlled particle morphology.

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Nomenclature

| Symbol | Description | Units |
|--------|-------------|-------|
| a      | constant    |       |
| B(x,y) | breakage function related to size x and y |       |
| B_i    | breakage function related to class i and j |       |
| D      | Morphological distance | m |
| d_i    | final mean diameter | m |
| d_j    | initial mean diameter | m |
| E(t)   | residence time distribution function | s^-1 |
| E_s    | specific energy | J/m^3 |
| E_so   | threshold specific energy | J/m^3 |
| E_sm   | mass specific energy | kJ/kg |
| E_t    | total efficiency of the classifier |       |
| G     | Kapur function, eq. (4) | s^-1 |
| H     | Hausdorff distance | m |
| H_i    | Kapur function, eq. (4) | s^-2 |
| I_p(x) | batch grinding kinetics coupled with residence time distribution |       |
| K(i)   | Kapur function related to class i | s^-k |
| m(y,t) | mass fraction for particle of size y, batch operation |       |
| R(x,t) | cumulative size distribution at time t, batch operation |       |
| R_i    | cumulative size distribution related to size i |       |
| S(y)   | selection function related to size y | s^-1 |
| S_i    | selection function related to class i | s^-1 |
| t      | time | s |
| V_p   | particle velocity | m/s |
| V_so  | threshold particle velocity | m/s |
| x     | particle size | m |
| y     | particle size | m |
| p(x)  | grade efficiency curve of the classifier |       |
| λ     | inertia number, eq.(16) |       |
| μ_i(t) | first moment of the particle size distribution |       |

indice

e  inlet
i  class
s  outlet
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Author's short biography

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John Dodds was awarded a degree in Chemical Engineering at Loughborough University of Technology (UK) in 1965 and a PhD from the same university in 1968 for work on the structure of particle packings and dewatering filter cakes. He then moved to work in France at the Laboratoire des Sciences du Génie Chimique, (Chemical Engineering Science Laboratory) in Nancy. His work there has covered: Ion exchange and adsorption processes, transport and capture of colloids in porous media involving colloid characterisation by Hydrodynamic Chromatography and Field Flow Fractionation as well as deep bed and membrane filtration processes. He now leads a team in Particle Technology working in packing models of porous media, instrumental methods for particle size analysis and fine grinding and classification processes. He is the author of over 200 papers and presentations at international conferences and the co-author of the book “Physics of Granular Media”.

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Christine Frances was born in 1965. She graduated from the Ecole Nationale Supérieure de Génie Chimique of Toulouse in 1988. She obtained the Doctor degree of the Institut National Polytechnique de Toulouse (France) in the field of crystallization. Since 1992 she is a researcher of CNRS (National Center of Research) in the field of comminution.

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