Event-triggered control under time-varying rate and channel blackouts

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Abstract—This paper addresses the problem of event-triggered control of linear time-invariant systems over time-varying rate limited communication channels, including the possibility of channel blackouts, which are intervals of time when the communication channel is unavailable for feedback. In order to design an effective event-triggered controller that operates successfully even in the presence of blackouts, we analyze the channel data capacity, which is the total maximum number of bits that could be communicated over a given time interval. We provide an efficient real-time algorithm to estimate the channel capacity for a time-slotted model of channel evolution. Equipped with this algorithm we then propose an event-triggering scheme, which using prior knowledge of the channel information, guarantees exponential stabilization at a desired convergence rate despite intermittent channel blackouts. The contributions are the notion of channel blackouts, the effective control despite their occurrence, and the analysis and quantification of the data capacity for a class of time-varying continuous-time channels.

I. INTRODUCTION

Control under communication constraints is of great theoretical and practical importance and has motivated several streams of research, two of which are information-theoretic control and event-triggered control. Previously in [1], [2], we have sought to combine the strengths of the two areas to address the problem of channel blackouts - intervals of time during which the channel is unavailable to the control system. An example of the use of such a channel model is in scheduling shared communication resources.

Literature review: Comprehensive overviews of the literature on information-theoretic control may be found in [3], [4]. Early data rate results appeared in [5]–[7], which provided tight necessary and sufficient conditions on the data rate of the encoded feedback for asymptotic stabilization in the discrete-time setting. Since then, the problem has been studied under varying assumptions on the channels, some examples being [8]–[10]. In the continuous-time setting, the problem has been mainly studied under either periodic sampling or aperiodic sampling with known upper and lower bounds on the sampling period for single input systems [11], [12], nonlinear feedforward systems (single input systems [13], and switched linear systems [14], which also analyzes the incident convergence rate. It is not known if and how a best sampling period may be designed or if state-based aperiodic sampling can provide any advantage in efficiency and performance. In this context, [15] explores the stabilization problem under a state based aperiodic transmission policy, with the inter-transmission intervals being integral multiples of a fixed step size.

On the other hand, the event-triggered approach, see e.g. [16]–[18] and references therein, exploits the tolerance to measurement errors to design goal-driven state-based aperiodic sampling. The literature in this area mainly focuses on minimizing the number of transmissions while largely ignoring the quantization aspect. Some of the few exceptions include [19], [20], which utilize static logarithmic quantization and [21]–[23] (see also references therein) which use dynamic quantization. All these works guarantee a positive lower bound on the inter-transmission times, while [21]–[23] also provide a uniform bound on the communication bit rate (i.e., the number of bits per transmission). However, these references do not address the inverse problem of triggering and quantization given a limit on the communication bit rate.

Statement of contributions: In [1], [2], we combine the strengths of the two approaches to address the problem of event-triggered stabilization of continuous-time linear time-invariant systems under bounded bit rates. The event-triggered formulation allows us to guarantee a specified rate of convergence in the presence of non-instantaneous communication and possibly time-varying communication rate. The incorporation of information-theoretic aspects in our design also allows us to analyze sufficient average data rate, something not usually seen in the event-triggered literature. In this paper, we extend this line of research and consider time-varying channels. We also consider the possibility of channel blackouts, intervals of time during which the channel is unavailable for feedback. We assume that the encoder has knowledge of the time-varying channel properties sufficiently ahead in time, which it may use to plan the transmissions to guarantee exponential stabilization with a desired convergence rate despite the presence of blackouts. Though event-triggered control is often purported to make use of communication resources efficiently, a drawback is that typically the channel is assumed to be available for feedback on demand. Thus, our notion of scheduled channel blackouts and stabilization despite their occurrence is a key contribution in the context of event-triggered control. In addition, to effectively control despite the occurrence of blackouts, we need to capture and utilize the notion of channel data capacity - the maximum number of bits that may be communicated over possibly multiple transmissions during an arbitrary time interval. Quantifying this quantity for general time-varying channels is challenging. Hence, for

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a class of simple yet meaningful time-varying channels, we provide a real-time algorithm to effectively estimate the data capacity over an arbitrary time interval.

Notation: We let \( \mathbb{R}, \mathbb{R}_{\geq 0}, \mathbb{N}, \) and \( \mathbb{N}_0 \) denote the set of real, nonnegative real, positive integer, and nonnegative integer numbers, respectively. We denote by \( \| \cdot \|_2 \) and \( \| \cdot \|_\infty \) the Euclidean and infinity norm of a vector, respectively, or the corresponding induced norm of a matrix. For a symmetric matrix \( A \in \mathbb{R}^{n \times n} \), we let \( \lambda_m(A) \) and \( \lambda_M(A) \) denote its smallest and largest eigenvalues, respectively. For any matrix norm \( \| \cdot \|_1 \), note that \( \| e^{At} \|_1 \leq e^{\| A \|_1 t} \). For a function \( f : \mathbb{R} \to \mathbb{R}^n \), any \( t \in \mathbb{R} \), we let \( f(t^{-}) \) denote the limit from the left, \( \lim_{s \uparrow t} f(s) \). For a number \( a \in \mathbb{R} \), we let \( [a]_+ \triangleq \max\{0, a\} \).

II. PROBLEM STATEMENT

In this section, we first give the system description and then describe the channel and communication model, which is a key component of the control system. Finally, we describe the control objective.

System description: Consider a linear time-invariant control system,
\[
\dot{x}(t) = Ax(t) + Bu(t),
\]
where \( x \in \mathbb{R}^n \) denotes the state of the plant and \( u \in \mathbb{R}^m \) the control input, while \( A \in \mathbb{R}^{n \times n} \) and \( B \in \mathbb{R}^{n \times m} \) are the system matrices. We assume there exists a control gain matrix \( K \in \mathbb{R}^{m \times n} \) such that the matrix \( A = B + BK \) is Hurwitz. Under this assumption, \( u(t) = Kx(t) \) (with continuous-time feedback) renders the origin of (1) globally exponentially stable. We assume that the sensor and the actuator that the plant is equipped with are not co-located. The sensor can measure the state exactly, and that the actuator can exert the input to the plant with infinite precision. However, the sensor may transmit state information to the controller at the actuator only at discrete time instants of its choice and using only a finite number of bits. In this sense, we also refer to the sensor as the encoder and the actuator as the decoder.

Communication and channel model: Let \( \{t_k\}_{k \in \mathbb{N}} \) be the sequence of transmission times at which the sensor transmits an encoded packet of data, \( \{p_k\}_{k \in \mathbb{N}} \) be the sequence of reception times at which the decoder receives a complete packet of data, and \( \{\tilde{t}_k\}_{k \in \mathbb{N}} \) be the sequence of update times at which the decoder updates the controller state. At a transmission time \( t_k \), the sender sends \( np_k \) bits, which encode the plant state (\( p_k \) bits per dimension). Due to causality \( \tilde{t}_k \geq t_k \geq t_k \) and we call
\[
\Delta_k \triangleq t_k - t_k, \quad \bar{\Delta}_k \triangleq \tilde{t}_k - t_k,
\]
kth communication time and kth time-to-update, respectively. To keep things simple, we assume the encoder and the decoder have synchronized clocks and synchronously update their states at update times \( \{\tilde{t}_k\}_{k \in \mathbb{N}} \).

Remark 2.1: (On the distinction between \( \{r_k\} \) and \( \{\tilde{r}_k\} \)). The distinction between the reception times \( \{r_k\} \) and the update times \( \{\tilde{r}_k\} \) is a generalization of the model we adopted previously in [2], wherein the two sequences would essentially be identical. Decisions about when to update affect later decisions about when to transmit and so on. Thus, the introduction of a distinct update time sequence provides greater flexibility in the presence of time-varying channels, particularly in cases where there might be channel blackouts - intervals of time when the channel is unavailable.

The kth communication time \( \Delta_k \triangleq r_k - t_k \) is in general a function of \( t_k \) and the packet size (of \( np_k \) bits) represented by \( p_k \), and we assume \( \Delta_k = \Delta(t_k, p_k) \), where the function \( \Delta \) is a property of the time-varying channel. In this paper, we assume that
\[
\Delta(t, p) = \frac{p}{R(t)}. \tag{2}
\]

Remark 2.2: (Interpretation of \( R(t) \)). \( nR(t) \) has the connotation of instantaneous communication-rate and when we re-write (2) as
\[
nR(t) = \frac{np}{\Delta(t, p)} \tag{2}
\]
we see that \( nR(t) \) is the number of bits communicated per unit time of all the bits transmitted at time \( t \). Thus, for example, if \( R(t) = \infty \) then the packet sent at \( t \) is received instantaneously. We exclude \( n \) out of \( R(t) \) purely for notational convenience in the sequel.

In addition to bounded and time varying rate \( R(t) \), we also assume that for each time \( t \) there is an upper-bound on the packet size that can be successfully transmitted starting at \( t \), which we denote by \( np(t) \), i.e. \( p_k \leq \bar{p}(t_k) \) is imposed by the channel. Moreover, in order to model a channel with bounded capacity and in order to maintain synchronization between the encoder and the decoder, we also require that the encoder does not transmit a packet before a previous packet is received by the decoder and the controller updated, i.e. \( t_{k+1} \geq \tilde{t}_k \) for all \( k \in \mathbb{N}_0 \). We say the channel is busy at time \( t \) if \( t \in [t_k, r_k) \), for some \( k \in \mathbb{N} \). Alternatively, we say that a sequence of transmission times \( \{t_k\} \) and a sequence of packet sizes \( \{p_k\} \) are feasible if for every \( k \in \mathbb{N}_0 \),
\[
p_k \leq \bar{p}(t_k), \quad p_k \in \mathbb{N}_0 \tag{3a}
\]
\[
t_{k+1} \geq \tilde{t}_k \geq r_k, \quad r_k = t_k + \Delta(t_k, p_k). \tag{3b}
\]

Without loss of generality, we assume that \( \bar{p} \) is a piece-wise constant function with \( \bar{p}(t) \in \mathbb{N}_0 \) for all \( t \geq 0 \). And specifically, we refer to any interval during which \( \bar{p} = 0 \) as a channel blackout or simply a blackout. In this paper, we assume that the encoder knows both the functions \( t \to R(t) \) and \( t \to \bar{p}(t) \) a priori or sufficiently in advance.

Control objective: Consider the candidate Lyapunov function \( x \mapsto V(x) = x^TPx \), where \( P \) is the symmetric positive definite matrix that satisfies the Lyapunov equation
\[
P\bar{A} + \bar{A}^TP = -Q. \tag{4}
\]
where \( Q \in \mathbb{R}^{n \times n} \) is an arbitrary symmetric positive definite matrix and \( \bar{A} = (A + BK) \) is Hurwitz. We assume that
\[
V_d(t) = V_d(t_0)e^{-\beta(t-t_0)} \tag{5}
\]
with \( \beta > 0 \) (rate of convergence) is the desired “control performance”. We define the performance ratio function,
measuring the ratio of the quadratic Lyapunov function $V$ and the desired performance $V_d$,
\[ b(t) \triangleq \frac{V(x(t))}{V_d(t)}. \] (6)

Then we want to design an event-triggered control system that recursively determines the sequences of transmission times \( \{t_k\}_{k \in \mathbb{Z}_+} \), and update times \( \{r_k\}_{k \in \mathbb{Z}_+} \). We also want to design a coding scheme to encode messages and a rule to determine the number of bits \( np_k \) to be transmitted at \( t_k \) that respects the channel constraints. The control objective of the design is to ensure \( V(x(t)) \leq V_d(t) \ (b(t) \leq 1) \) holds for all \( t \geq t_0 \), which ensures exponential stability of the origin, in the presence of time-varying channel functions \( R \) and \( p \) and including the possibility of intermittent channel blackouts.

The rest of the paper is organized as follows - in the next section, we recollect some useful notation and results from our previous work. For control in the presence of channel blackouts, we need a notion of channel capacity over an arbitrary time interval, which is the maximum number of bits that may be communicated, possibly over multiple transmissions, during the interval. In Section IV, we treat this issue formally, where we first describe certain assumptions/approximations about the channel that enable us to compute the channel capacity and a method to obtain an estimate of the same in real time. We present the event-triggered control design, for achieving the control objective, in Section V.

### III. SOME USEFUL PRIOR RESULTS

In this section we recall some useful notation and results from our previous work [2], wherein the proofs of the results may also be found.

**Coding scheme:** We use dynamic quantization for finite-bit transmissions from the encoder to the decoder. In dynamic quantization, there are two distinct phases: the zoom-out stage, during which no control is applied while the quantization domain is expanded until it captures the system state at time \( r_0 = t_0 \in \mathbb{R}_0^\geq \); and the zoom-in stage, during which the encoded feedback is used to asymptotically stabilize the system. A detailed description of the zoom-out stage can be found in the literature, e.g. [24]. Here, we focus exclusively on the zoom-in stage, i.e., for \( t \geq t_0 \) for which we use a hybrid dynamic controller. We assume that both the encoder and the decoder have perfect knowledge of the plant system matrices. The state of the encoder/decoder is composed of the controller state \( \hat{x} \in \mathbb{R}^n \) and an upper bound \( d_{e} \in \mathbb{R}_0^\geq \) on the encoding error \( x_e \triangleq x - \hat{x} \). Thus, the actual input to the plant is given by \( u(t) = K \hat{x}(t) \). During inter-update times, the state of the dynamic controller evolves as
\[ \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) = \bar{A}\hat{x}(t), \quad t \in [t_k, t_{k+1}) \] (7a)

Let the encoding and decoding functions at \( k \)th iteration be represented by \( q_{E,k} : \mathbb{R}^n \times \mathbb{R}^n \mapsto G_k \) and \( q_{D,k} : G_k \times \mathbb{R}^n \mapsto \mathbb{R}^n \), respectively, where \( G_k \) is a finite set of \( 2^{np_k} \) symbols. At \( t_k \), the encoder encodes the plant state as \( z_{E,k} \triangleq q_{E,k}(x(t_k), \hat{x}(t_k^-)) \), where \( \hat{x}(t_k^-) \) is the controller state just prior to the encoding time \( t_k \), and sends it to the controller. This signal is decoded as \( z_{D,k} \triangleq q_{D,k}(z_{E,k}, \hat{x}(t_k^-)) \) by the decoder at the update time \( r_k \), at which time the sensor and the controller also update \( \hat{x} \) using the jump map,
\[ \hat{x}(r_k) = e^{\bar{A}\Delta t_k} \hat{x}(t_k^-) + e^{A\Delta t_k}(z_{D,k} - \hat{x}(t_k^-)) \triangleq \hat{x}(t_k^-), \] (7b)

where \( q_k : \mathbb{R}^n \times \mathbb{R}^n \mapsto \mathbb{R}^n \) is shorthand that represents the quantization that occurs as a result of the finite-bit coding. We allow the quantization domain, the number of bits and the resulting quantizer, \( q_k \), at each transmission instant \( t_k \in \mathbb{R}_0^\geq \) to be variable. Note that the evaluation of the map \( q_k \) is inherently from the encoder’s perspective because it depends on the plant state \( x(t_k) \), which is unknown to the decoder. Also, while the encoder could store \( \hat{x}(t_k^-) \), the decoder has to infer its value if \( \Delta_k > 0 \). We detail the specifics of both the encoder’s and the decoder’s procedures to implement (7b) in Algorithms 1 and 2, respectively, which describe how the encoder and the decoder update \( \hat{x} \) and \( d_{e} \) synchronously at the time instants \( \tilde{r}_k \). Then, in Lemma 3.1, we establish the consistency of the two algorithms.

**Algorithm 1: Update of encoder variables**

At \( t = t_0 = r_0 \), the encoder initializes
1: \( \delta_0 \leftarrow d_e(t_0) \{ \text{store initial bound on encoding error} \} \)

At \( t \in \{t_k\}_{k \in \mathbb{Z}_+} \), the encoder sets
2: \( z_k \leftarrow \hat{x}(t_k^-) \{ \text{store encoder variable} \} \)
3: \( z_{E,k} \leftarrow q_{E,k}(x(t_k), z_k) \{ \text{encode plant state with} \ np_k \text{bits} \} \)
4: \( \delta_k \leftarrow d_e(t_k)/2^{np_k} \{ \text{compute bound on encoding error} \} \)

At \( t \in \{\tilde{r}_k\}_{k \in \mathbb{Z}_+} \), the encoder sets
5: \( z_{D,k} \leftarrow q_{D,k}(z_{E,k}, z_k) \{ \text{decode plant state at} t_k \} \)
6: \( \tilde{x}(r_k) \leftarrow e^{A\Delta t_k}z_k + e^{A\Delta t_k}(z_{D,k} - z_k) \{ \text{update controller state} \} \)
7: \( d_{e}(r_k) \leftarrow ||e^{A\Delta t_k}||_\infty \delta_k \{ \text{update bound on encoding error} \} \)

**Algorithm 2: Update of decoder variables**

At \( t = t_0 = r_0 \), the decoder initializes
1: \( \delta_0 \leftarrow d_e(t_0) \{ \text{store initial bound on encoding error} \} \)

At \( t \in \{\tilde{r}_k\}_{k \in \mathbb{Z}_+} \), the decoder sets
2: \( z_k \leftarrow e^{-A\Delta t_k} \tilde{x}(r_k) \{ \text{compute encoder state at} t_k \} \)
3: \( z_{E,k} \leftarrow \{ \text{received from the encoder} \} \)
4: \( \delta_k \leftarrow \frac{1}{2^{np_k}}(||e^{A\Delta t_k}||_\infty - \delta_k - 1) \{ \text{compute bound on encoding error at} t_k \} \)
5: \( z_{D,k} \leftarrow q_{D,k}(z_{E,k}, z_k) \{ \text{decode plant state at} t_k \} \)
6: \( \tilde{x}(r_k) \leftarrow e^{A\Delta t_k}z_k + e^{A\Delta t_k}(z_{D,k} - z_k) \{ \text{update controller state} \} \)
7: \( d_{e}(r_k) \leftarrow ||e^{A\Delta t_k}||_\infty \delta_k \{ \text{update bound on encoding error} \} \)

**Lemma 3.1: (Consistency of Algorithms 1 and 2).** If initially the encoder and the decoder share identical values for \( \tilde{x}(t_0) \) and \( d_{e}(t_0) \), with \( ||\tilde{x}(t_0)||_\infty \leq d_{e}(t_0) \), then Algorithms 1 and 2 result in consistent \( \tilde{x}(t) \) and \( d_{e}(t) \) signals for all \( t \geq t_0 \). Further, \( t \mapsto \tilde{x}(t) \) evolves according to (7) and
and of its evolution as we use these properties in the rest of the paper. In the sequel, we make the following assumptions.

\[ W \triangleq \frac{\lambda_m(Q)}{\lambda_M(P)} - a \beta > 0, \]  

where \( a > 1 \) is an arbitrary constants. Assumption (10) is sufficient to guarantee with continuous-time and unquantized feedback a convergence rate faster than \( \beta \), in the absence of external disturbance. The following result provides an upper bound on the value of \( b \) that is convenient for our purposes.

**Lemma 3.2:** (Upper Bound on Performance Ratio). Given \( t_k \in \mathbb{R}_{>0} \) such that \( b(t_k) \leq 1 \), then

\[ b(\tau + t_k) \leq \tilde{b}(\tau, b(t_k), e(t_k)), \]

for \( \tau \geq 0 \), where

\[ e(t) \triangleq \frac{d_e(t)}{c\sqrt{V_d(t)}}, \quad \tilde{b}(\tau, b_0, e_0) \triangleq \frac{f_1(\tau, b_0, e_0)}{f_2(\tau)}, \]

\[ f_1(\tau, b_0, e_0) \triangleq b_0 + \frac{W e_0}{w + \theta} (e^{(w+\theta)\tau} - 1), \quad f_2(\tau) \triangleq e^{w \tau}, \]

\[ e \triangleq \frac{W \sqrt{\lambda_m(P)}}{2\sqrt{\|PBK\|_2}}, \quad w \triangleq \frac{\lambda_m(Q)}{\lambda_M(P)} - \beta > 0, \quad \theta \triangleq \|A\|_2 + \frac{\beta}{2}. \]

Motivated by Lemma 3.2, we formally define the function

\[ \Gamma_1(b_0, e_0) \triangleq \min\{\tau \geq 0 : \tilde{b}(\tau, b_0, e_0) = 1, \frac{db}{d\tau} \geq 1\}, \]

which is a lower bound on the time it takes \( b \) to evolve to 1 starting from \( b(t_k) = b_0 \) with \( e(t_k) = e_0 \). The following result captures some useful properties of this function.

**Lemma 3.3:** (Properties of the Function \( \Gamma_1 \)). The following holds true,

(i) \( \Gamma_1(1, 1) > 0 \).

(ii) If \( b_1 \geq b_0 \) and \( e_1 \geq e_0 \), then \( \Gamma_1(b_1, e_1) \geq \Gamma_1(b_0, e_0) \). In particular, if \( b_0 \in [0, 1] \), then \( \Gamma_1(b_0, e_0) \geq \Gamma_1(1, e_0) \).

(iii) For \( T > 0 \), if \( b_0 \in [0, 1] \) and

\[ e_0 \leq \varphi_T(b_0) \triangleq \frac{(w + \theta)(1 - b_0)}{W(e^{(w+\theta)T} - 1)} + 1, \]

then \( \Gamma_1(b_0, e_0) \geq \min\{\Gamma_1(1, 1), T\} \).

**Channel-trigger:** We define the channel-trigger function

\[ h_{\text{ch}}(t) \triangleq \frac{c(t)}{\rho_T(b(t))} = \frac{d_e(t)}{c\sqrt{V_d(t)}\rho_T(b(t))}, \]

where \( T > 0 \) is a fixed design parameter. Lemma 3.3(iii) implies that, if \( h_{\text{ch}}(t_k) \leq 1 \), then \( b(t) \leq 1 \) for at least \( t \in [t_k, t_k + \min\{T; \Gamma_1(1, 1)\}] \). The next result provides an upper bound on the function \( h_{\text{ch}} \) and is useful later when establishing a uniform lower bound on the inter-transmission times for our design.

**Lemma 3.4:** (Upper Bound on Channel-Trigger Function). Given \( t_k \in \mathbb{R}_{>0} \) such that \( b(t_k) \leq 1 \), then for \( \tau \geq 0 \)

\[ h_{\text{ch}}(\tau + t_k) \leq \tilde{h}_{\text{ch}}(\tau, b(t_k), e(t_k), e(t_k)), \]

where

\[ \tilde{h}_{\text{ch}}(\tau, b_0, e_0, \psi_0) \triangleq \frac{\|e\|_\infty e_{\frac{\tau}{\psi_0}}}{\rho_T(b_0, e_0)}. \]

Additional results: The basic underlying idea behind our event-triggered design is to anticipate ahead of time the crossings of 1 by the performance-ratio function \( b(t) \) and the channel-trigger function \( h_{\text{ch}} \) after transmitting at most \( np \) number of bits. We can also show that

\[ h_{\text{ch}}(\tilde{r}_k) \leq \tilde{\Gamma}_2(b_0, e_0, \psi_0) \triangleq \min\{\tau \geq 0 : \tilde{h}_{\text{ch}}(\tau, b_0, e_0, \psi_0) = 1\}. \]

If we can ensure that \( h_{\text{ch}}(\tilde{r}_k) \leq 1 \), then the definition (12) of \( \Gamma_1 \) and Lemma 3.3 guarantee \( b \leq 1 \) until \( \tilde{r}_k \). For \( \tilde{r}_k \leq \min\{|\Gamma_1(1, 1), T\} \), we can anticipate \( h_{\text{ch}}(\tilde{r}_k) \leq 1 \), and hence \( \tilde{r}_k \leq \min\{\tau \geq 0 : \tilde{h}_{\text{ch}}(\tau, b_0, e_0, \psi_0) = 1\}. \)

Our problem then reduces to checking the zero-crossing of the functions \( \Gamma_1 - T_M \) and \( \Gamma_2 - T_M \). In the following two lemmas, we suggest equivalent ways to check these two conditions efficiently in real-time.

**Lemma 3.5:** (Algebraic Condition to Check if \( b < 1 \) for the next \( T^* \) units of time). Let \( T^* > 0 \). For any \( b_0 \in [0, 1] \), \( \Gamma_1(b_0, e_0) > T^* \) if and only if \( b(T^*, b_0, e_0) < 1 \). Further, the corresponding statement with the inequalities reversed and the one in which the inequalities replaced by equality are true.

Next, we make a similar observation about \( \Gamma_2 \).

**Lemma 3.6:** (Algebraic Condition to Check the Sign of \( \Gamma_2 - T^* \)). Let \( T^* > 0 \). For any \( b_0 \in [0, 1] \) and \( e_0 \in [\rho_T(b_0)], \Gamma_2(b_0, e_0, \psi_0) > T^* \) if and only if \( \tilde{h}_{\text{ch}}(T^*, b_0, e_0, \psi_0) < 1 \). Further, the corresponding statement with the inequalities reversed and the one in which the inequalities replaced by equality are true.

IV. CHANNEL DESCRIPTION AND CHANNEL CAPACITY

In this section, we go into more depth about the communication channel and on quantifying the channel capacity. In the overall scheme of the paper, we want the encoder to be able to estimate the channel capacity up to the beginning of a channel blackout period.

To this end, given feasible sequences \( \{t_k\}, \{p_k\} \) and \( \{\tilde{r}_k\} \), we first denote the number of bits (data) communicated (the...
data transmitted by the encoder and completely received by the decoder during the time interval \([\tau_1, \tau_2]\) as
\[
D(\tau_1, \tau_2, \{t_k\}, \{\hat{r}_k\}, \{p_k\}) = n \sum_{k=k_{\tau_1}}^{k_{\tau_2}} p_k,
\]
where \(\{\hat{r}_k\}_{k \in \mathbb{N}}\) satisfies (3) and
\[
k_{\tau_1} = \min\{k : t_k \geq \tau_1\}, \quad k_{\tau_2} = \max\{k : t_k + \hat{r}_k \leq \tau_2\}.
\]
Then, we denote the channel capacity during the time interval \([\tau_1, \tau_2]\) as \(D(\tau_1, \tau_2)\) and define it as the maximum data that could be communicated during the time interval. Thus,
\[
D(\tau_1, \tau_2) = \max_{\{t_k\}, \{p_k\}} D(\tau_1, \tau_2, \{t_k\}, \{\hat{r}_k\}, \{p_k\}) \quad \text{s.t.} \ (3) \text{ holds}.
\]
Notice that in (3), \(\hat{r}_k \geq r_k\) for all \(k \in \mathbb{N}_0\). In order to maximize the data communicated, we need to choose \(\hat{r}_k = r_k\). Further, since the data capacity should be independent of \(r_k \leq \Delta(t_k, p_k)\), we see that
\[
D(\tau_1, \tau_2) = \max_{\{t_k\}, \{p_k\}} D(\tau_1, \tau_2, \{t_k\}, \Delta(t_k, p_k), \{p_k\}) \quad \text{s.t.} \ (3) \text{ holds}.
\]
This is a constant
\[
\text{Lemma 4.1: (Channel-capacity under constant communication rate). Suppose } \forall t \in [\tau_1, \tau_2] \text{ (i) } R(t) = R \geq 0 \text{ and (ii) } \bar{p}(t) \geq 1 \text{ (no blackouts). Then, } D(\tau_1, \tau_2) = n[R(\tau_2 - \tau_1)].
\]
Proof: Since the communication rate \(R(t)\) is a constant for \(t \in [\tau_1, \tau_2]\), an optimal solution can be constructed by choosing \(p_k = 1\) and \(t_{k+1} - \hat{r}_k = r_k\) for all \(k \in \mathbb{N}_0\). Then, the statement of the lemma follows directly.

Motivated by this simple case, in the sequel, we make the assumption that the channel functions \(R\) and \(\bar{p}\) are piecewise constant so that the problem of finding a reasonable estimate of the \(D(\tau_1, \tau_2)\) is tractable while also ensuring that the overall problem of the paper is sufficiently interesting and useful. Specifically, we assume that
\[
R(t) = R_j, \quad \forall t \in [\theta_j, \theta_{j+1}) \quad \text{ (18a)}
\]
\[
\bar{p}(t) = \bar{p}_j, \quad \forall t \in [\theta_j, \theta_{j+1}) \quad \text{ (18b)}
\]
where \(\{\theta_j\}_{j=0}^{\infty}\) is a strictly increasing sequence of time instants. We also denote \(T_j = \theta_{j+1} - \theta_j\) as the length of the \(j\)th time slot \(I_j\). In order to concisely express the constraints in the optimization problem (17) we assume, without loss of generality, that \(\tau_1 = \theta_{j_0}\) and \(\tau_2 = \theta_{j_1}\), for some \(j_0, j_1 \in \mathbb{N}_0\). Now, we can think of the computation of \(D(\theta_{j_0}, \theta_{j_1})\) as an allocation problem - that of allocating the number of bits \(n\), with \(\phi_j \in \mathbb{N}_0\), to be transmitted in the time slots \(\{I_j\}\) for \(j \in \mathbb{N}_0^{j_j} = \{j_0, \ldots, j_j - 1\}\). It is clear that for optimality, the allocated bits in \(I_j\) need to be transmitted starting at the earliest possible time in \(I_j\) and that the channel not be idle until all the allocated bits \(n\phi_j\) are received. Thus, we can do away with the specific transmission sequences \(\{t_k\}\) in the optimization problem of data capacity.

In the sequel, for convenience, we let \(\phi_{j_0}^{j_j} \triangleq (\phi_{j_0}, \ldots, \phi_{j_j - 1})\), the vector of optimization variables. Now, we derive the constraints in the optimization problem. First, note that the maximum number of bits that can be transmitted in \(I_j\) is upper bounded as
\[
n\phi_j \leq n[R_j T_j] + n\bar{p}_j, 
\forall j \in \mathbb{N}_0^{j_j} \text{ s.t. } \bar{p}_j > 0 \quad \text{(19a)}
n\phi_j = 0, 
\forall j \in \mathbb{N}_0^{j_j} \text{ s.t. } \bar{p}_j = 0. \quad \text{(19b)}
\]
However, as noted in Lemma 4.1, only up to \(n[R_j T_j]\) of the bits are received within the time interval \(I_j\), while the remaining bits take up some of the time in \(I_{j+1}\). Thus, effectively the time available in \(I_{j+1}\) and consequently the upper bound on \(\phi_{j+1}\) is reduced. Moreover, in general, the number of bits transmitted in \(I_j\) has an effect on the number that could be transmitted in all subsequent intervals. Thus, for each \(j, j \in \mathbb{N}_0^{j_j}\), we introduce the functions
\[
\bar{T}_{j_1, j}(\phi_{j_0}^{j_j}) \triangleq \left( T_j - \sum_{i=j_1}^{j-1} \left( \frac{\phi_{j_0}^{i}}{R_i} - T_i \right) \right)
= \theta_{j+1} - \theta_{j_1} - \sum_{i=j_1}^{j-1} \frac{\phi_{j_0}^{i}}{R_i}. \quad \text{(20)}
\]
Now, we make the following observation.

\text{Lemma 4.2: (Available time in slot } I_j). \text{ Let } \bar{T}_j(\phi_{j_0}^{j_j}) \text{ be the time available in the slot } I_j \text{ given the allocation } \phi_{j_0}^{j_j}. \text{ Then,}
\[
\bar{T}_j(\phi_{j_0}^{j_j}) = \min_{j \in \mathbb{N}_0^{j_j}} \left\{ \bar{T}_{j_1, j}(\phi_{j_0}^{j_j}), T_j \right\}.
\]
\text{Proof: Observe that for each } j, j \in \mathbb{N}_0^{j_j}, \left[ \bar{T}_{j_1, j}(\phi_{j_0}^{j_j}) \right]_+ \text{ is an upper bound on the time available for transmission in the slot } I_j. \text{ Now, let}
\[
j_2 = \max\{i \in \mathbb{N}_0 \cap \{j_0, j - 1\} : \bar{T}_i(\phi_{j_0}^{j_j}) = T_i\}
\]
Then clearly, \(\{\phi_i\}_{i=0}^{j_2}\) are sufficient to determine \(\bar{T}_j(\phi_{j_0}^{j_j})\). Next, for the allocation \(\phi_{j_0}^{j_j}\), the bits transmitted during the time slots \(I_i\) for \(i \in \{j_2, j - 1\}\) are received by \(\theta_{j_2} + \sum_{j=j_2}^{j_1} \phi_{j_0}^{i}/R_i\) and thus in deed \(\bar{T}_j(\phi_{j_0}^{j_j}) = \min_{j_2 \leq j \leq j_1} \left[ \bar{T}_{j, j}(\phi_{j_0}^{j_j}), T_j \right]\). Finally, for each \(j \in \mathbb{N}_0 \cap \{j_0, j - 1\}\), \(\bar{T}_{j, j}(\phi_{j_0}^{j_j}) \geq \bar{T}_{j, j}(\phi_{j_0}^{j_j})\), which proves the lemma. \(\square\)

Therefore, for each \(j \in \mathbb{N}_0^{j_j}\) and \(j_1 \in \mathbb{N}_0 \cap \{j_0, j - 1\}\), consider the constraints
\[
n\phi_j \leq \left\{ \begin{array}{ll}
 n[R_j T_j \bar{T}_{j_1, j}(\phi_{j_0}^{j_j})] + n\bar{p}_j, & \text{ if } \bar{T}_{j_1, j}(\phi_{j_0}^{j_j}) > 0 \\
 0, & \text{ otherwise}
\end{array} \right. \quad \text{(21a)}
\]
Note that if \(\bar{T}_{j_1, j}(\phi_{j_0}^{j_j}) \geq T_j\), then the constraint (21a) is weaker than (19) and hence inactive. For \(\bar{T}_{j_1, j}(\phi_{j_0}^{j_j}) \in (0, T_j)\), the constraint reflects the reduced available time in
the time slot $I_j$ and if $\tilde{T}_{j,j}(\phi_{j0}^{T}) \leq 0$, for some $j_1 \in \mathbb{N}_0 \cap \{j_0, j-1\}$, then it corresponds to the case when the channel is busy for the whole of the time slot $I_j$ ($\tilde{T}_{j}(\phi_{j0}^{T}) = 0$). Thus (21a) accurately reflects the effect of possibly reduced available time during the slot $I_j$ due to prior transmissions.

Finally, since in the computation of $\mathcal{D}(\theta_{j0}, \theta_{j1})$, we are interested in the maximum number of bits that can be communicated (transmitted and received) during the time interval, we also require that any bits transmitted during the slot $I_j$ are received before $\theta_{j1}$, i.e. for each $j \in N_{j0}^{j1}$ and $j_1 \in \mathbb{N}_0 \cap \{j_0, j\}$

$$\frac{\phi_j}{R_j} \leq \left\{ \begin{array}{ll}
\tilde{T}_{j,j}(\phi_{j0}^{T}) + \theta_{j1} - \theta_{j+1}, & \text{if } \tilde{T}_{j,j}(\phi_{j0}^{T}) > 0 \\
0, & \text{otherwise}
\end{array} \right., \quad (21b)$$

which we obtain by similar reasoning on $\tilde{T}_{j}$ as for (21a).

Then, the channel-capacity is given as

$$\mathcal{D}(\theta_{j0}, \theta_{j1}) = \max_{\substack{\phi_j \in \mathbb{N}_0 \forall j \in N_{j0}^{j1} \text{ s.t. } (19), (21) \text{ hold}}} n \sum_{j=j_0}^{j-1} \phi_j.$$

Ignoring the fact that this is an integer program, the constraints (21) still make the problem combinatorial. Thus, in order to obtain a sub-optimal and efficient solution to the problem, we first make the following observation.

**Lemma 4.3:** If there exists some finite $J \in \mathbb{N}_0$ such that

$$\bar{\pi}_j < \sum_{i=j+1}^{j+J} T_i, \quad \forall j \in \mathbb{N}_0,$$

then, for any $j \in \mathbb{N}_0$, any bits transmitted in time slot $I_j$ would be received strictly before the end of the slot $I_{j+1}+j$.

The case of $J = 0$: The case of $J = 0$ is of special interest and will be addressed next. This is interesting because if $J = 0$ then for each $j$, $T_j > 0$ and hence $\tilde{T}_{j,j}(\phi_{j0}^{T}) > 0$ also for each $j_1 \in N_{j0}^{j1}$. Thus, for each $j \in N_{j0}^{j1}$ and $j_1 \in \mathbb{N}_0 \cap \{j_0, j-1\}$, the constraints (21a) reduce to

$$n \phi_j \leq n[R_j \tilde{T}_{j,j}(\phi_{j0}^{T})] + n \bar{\pi}_j.$$

In order to obtain an efficient, though sub-optimal, solution we enforce the stricter constraints, for each $j \in N_{j0}^{j1}$ and $j_1 \in \mathbb{N}_0 \cap \{j_0, j-1\}$,

$$n \phi_j + nR_j \sum_{i=j_1}^{j-1} \frac{\phi_i}{R_i} \leq nR_j(\theta_{j1} - \theta_{j_1}) + n \bar{\pi}_j - n,$$  \quad (23a)

where we have under-approximated the floor function, used (20) and rearranged terms.

Note that Lemma 4.3, with $J = 0$, guarantees the constraints (21b) are satisfied for all $j \in \{j_0, \ldots, j_f - 2\}$, while for $j_f - 1$ (21b) reduce to

$$\frac{\phi_{j_f-1}}{R_{j_f-1}} \leq \tilde{T}_{j_f,j}(\phi_{j0}^{T}).$$

and by expanding and rearranging the terms, we get the final constraints, for each $j_1 \in \mathbb{N}_0 \cap \{j_0, j_f - 1\}$,

$$\sum_{i=j_1}^{j_f-1} \frac{\phi_i}{R_i} \leq \theta_{j_f} - \theta_{j_1}. \quad (23b)$$

Now all the constraints, (19) and (23) are linear. Let

$$\phi^* = \arg\max_{\substack{\phi_j \in \mathbb{R}_+, \forall j \in N_{j0}^{j1} \text{ s.t. } (19), (23) \text{ hold}}} \sum_{j=j_0}^{j_f-1} \phi_j.$$

Then, it is clear that $\mathcal{D}_s(\theta_{j0}, \theta_{j1}) \leq \mathcal{D}(\theta_{j0}, \theta_{j1})$ and that

$$\phi^N \triangleq \lfloor \phi^* \rfloor \in \{\lfloor \phi_{r,0}^* \rfloor, \ldots, \lfloor \phi_{r-1,0}^* \rfloor\} \quad (24)$$

is a sub-optimal solution.

The case of $J > 0$: If $J > 0$, we forget optimality in favor of an easily computable lower estimate of the data capacity. We let

$$\tilde{\phi}_j = \lfloor R_j(\theta_{j+1} - \theta_j) \rfloor, \quad j \in \mathbb{N}_0,$$

which is the number of bits that can be communicated (transmitted and received) during the time slot $I_j = [\theta_j, \theta_{j+1}]$. Hence, $\{\phi_j\}_{j \in \mathbb{N}_0}$ is a feasible solution and

$$\mathcal{D}_s(\theta_{j0}, \theta_{j1}) \triangleq n \sum_{j=j_0}^{j_f-1} \phi_j^N$$

is a sub-optimal estimate of the data capacity.

Computing the channel-capacity in real-time: As mentioned earlier, we want the encoder to estimate the channel capacity up to the end of the next blackout period. However, the computation of $\mathcal{D}_s(\tau_1, \tau_2)$ involves solving a linear program and hence may not be suitable for real-time estimation. Thus, given $\mathcal{D}(\theta_{j0}, \theta_{j1})$ (or $\mathcal{D}_s(\theta_{j0}, \theta_{j1})$), we propose a simpler procedure to estimate $\mathcal{D}(t, \theta_{j1})$ (or $\mathcal{D}_s(t, \theta_{j1})$) for any $t \in [\theta_{j0}, \theta_{j0+1}]$. We present the procedure in the form a lemma in the following.

**Lemma 4.4:** (Real-time computation of channel-capacity). Let $\phi^*$ (or $\phi^N$) be any optimizing solution to $\mathcal{D}(\theta_{j0}, \theta_{j1})$ (or $\mathcal{D}_s(\theta_{j0}, \theta_{j1})$). Also, for any $t \in [\theta_{j0}, \theta_{j0+1}]$ let

$$\tilde{\mathcal{D}}(t, \theta_{j1}) \triangleq \lfloor n[\phi^* + R_{j0}(\theta_{j0} - t)] \rfloor + n \sum_{j=j_0+1}^{j_f-1} \phi_j^*$$

$$\mathcal{D}_s(t, \theta_{j1}) \triangleq \lfloor n[\phi^N + R_{j0}(\theta_{j0} - t)] \rfloor + n \sum_{j=j_0+1}^{j_f-1} \phi_j^N. \quad (25)$$

Then, $0 \leq \tilde{\mathcal{D}}(t, \theta_{j1}) - \mathcal{D}(t, \theta_{j1}) \leq n$ and $0 \leq \mathcal{D}_s(t, \theta_{j1}) - \mathcal{D}_s(t, \theta_{j1}) \leq n$.

Proof: Here we prove only the statements about $\mathcal{D}(t, \theta_{j1})$ as the proof of the statements for $\mathcal{D}_s(t, \theta_{j1})$ are exactly analogous to those of $\mathcal{D}(t, \theta_{j1})$.

First of all notice that for any $\tau_1 < \tau_2 < \tau_3$

$$\mathcal{D}(\tau_1, \tau_3) \geq \mathcal{D}(\tau_1, \tau_2) + \mathcal{D}(\tau_2, \tau_3).$$

Now, let $T_0 = \theta_{j0} + \frac{\phi_{j0}^N}{R_{j0}}$. Clearly, from the optimality of $\mathcal{D}(\theta_{j0}, \theta_{j1})$, it follows that

$$\mathcal{D}(\theta_{j0}, T_0) = n\phi_{j0}^N, \quad \mathcal{D}(T_0, \theta_{j1}) = n \sum_{j=j_0+1}^{j_f-1} \phi_j^*.$$
Thus, for the special choice of $T_0$, we have the stronger relation $D(\theta, \theta_i) = D(\theta, T_0) + D(T_0, \theta_i)$. Now, using (27) twice we get

$$D(\theta_{j_0}, \theta_{j_1}) \geq D(\theta_{j_0}, t) + D(t, \theta_{j_1})$$

$$\geq D(\theta_{j_0}, t) + D(t, T_0) + D(T_0, \theta_{j_1}),$$

which implies

$$D(\theta_{j_0}, \theta_{j_1}) - D(\theta_{j_0}, t) \geq D(t, \theta_{j_1}) \geq D(t, T_0) + D(T_0, \theta_{j_1}).$$

Notice that $D(t, T_0) + D(T_0, \theta_{j_1}) = D(t, \theta_{j_1})$. Now, we compute the difference between the upper and lower bounds on $D(t, \theta_{j_1})$

$$D(\theta_{j_0}, \theta_{j_1}) - D(\theta_{j_0}, t) - D(t, \theta_{j_1})$$

$$= D(\theta_{j_0}, T_0) + D(T_0, \theta_{j_1}) - D(\theta_{j_0}, t) - D(t, \theta_{j_1})$$

$$= n [R_{j_0}(T_0 - \theta_{j_0}) - |R_{j_0}(t - \theta_{j_0})| - |R_{j_0}(T_0 - t)|]$$

$$= n [-R_{j_0}(t - \theta_{j_0}) - |R_{j_0}(\theta_{j_0} - t)|]

\leq n,$$

where, in arriving at the second last relation, we have used the fact that $R_{j_0}(T_0 - \theta_{j_0}) = \phi_{j_0}^* \text{ is an integer.}$ The statement of the lemma now follows.

V. EVENT-TRIGGERED CONTROL UNDER DYNAMIC CHANNEL CAPACITY

A. No channel blackouts

At first we address the case of no channel blackouts, i.e. $ar{p}(t) \geq 1$ for all $t$, as it paves the way for the solution in the general case. In this special case of no channel blackouts, we do not require the function $t \mapsto R(t)$ to be piece-wise continuous. Though the function $t \mapsto \bar{p}(t)$ is, by its discrete nature, piece-wise constant.

In our proposed event-triggered control scheme, we need a uniform lower bound for $\bar{t}$ under certain conditions, which we provide in the following lemma.

Lemma 5.1: (Lower bound on $\bar{t}$). If $(2p, \psi_0) = \epsilon_0 \leq \rho_T(b_0)$ then $\bar{t}(b_0, \epsilon_0, \psi_0) \geq T^*(p)$ with

$$T^*(p) \leq \min\{\tau \geq 0 : g(\tau, p) = 1\}$$

$$g(\tau, p) \triangleq \left\lfloor \frac{2p}{\rho_T(b_0)} \right\rfloor e^{\frac{\tau}{\tau}} - e^{(\rho_T(b_0) - 1)\tau}. $$

Proof: From (6) and (13), we have

$$\rho_T(\bar{p}(\tau, b_0, \epsilon_0))$$

$$= (w + \bar{p})(1 - e^{-wT} + \frac{W e^{(w + \bar{p})T - 1}}{W e^{(w + \bar{p})T - 1}}) + 1$$

$$= \rho_T(e^{-wT}b_0) - \frac{e^{(w + \bar{p})T - 1} - e^{-wT} \epsilon_0}{e^{(w + \bar{p})T - 1}}$$

$$\geq \rho_T(e^{-wT}b_0) \frac{e^{(w + \bar{p})T - 1} - e^{(w + \bar{p})T}}{e^{(w + \bar{p})T - 1}}$$

where the inequality follows from the assumption that $\epsilon_0 \leq b_0$. Now, substituting this lower bound and the value of $\psi_0$ in (15) and noting the fact that $\rho_T(e^{-wT}b_0) \geq \rho_T(b_0)$ gives

$$\bar{t}(\psi_0, b_0, \epsilon_0, \psi_0) \leq g(\tau, p).$$

The claim now follows from the definition (16).

Now, for any $p \in \mathbb{N}_0$, let

$$T_M(p) = \sigma \min\{\Gamma_1(1, 1), T, T^*(p)\},$$

where $\sigma \in (0, 1)$ is a design parameter. Consider

$$L_1(t) \triangleq \bar{t} (T_M(\bar{p}(t)), b(t), \epsilon(t))$$

$$L_2(t, s) \triangleq \bar{h}(T_M(\bar{p}(t)), b(t), \epsilon(t), (\epsilon(t) / \bar{p}(t))).$$

We are ready to present the first main result of the section.

Theorem 5.2: (Event-triggered control when no blackouts). Suppose $t \mapsto \bar{p}(t)$ is piece-wise constant with a uniform dwell-time greater than $\delta_i > 0$ and uniform upper bound $p^\text{Max}$. Assume that

$$R(t) \geq \frac{p}{T_M(p)}, \quad \forall p \in \{1, \ldots, p\}, \quad \forall t.$$ (31)

Consider the system (1) under the feedback law $u = K \dot{x}$, with $t \mapsto \dot{x}(t)$ evolving according to (7) and the sequence $\{t_k\}_{k \in \mathbb{N}_0}$ determined recursively by

$$t_{k+1} = \min\{t \geq \tilde{t}_k : L_1(t) \geq 1 \lor L_2(t, t + \delta_1) \geq 1\}. \quad (32)$$

Let $\{t_k\}_{k \in \mathbb{N}_0}$ and $\{\tilde{t}_k\}_{k \in \mathbb{N}_0}$ be given as $\tilde{t}_0 = t_0$ and $\tilde{t}_k = t_k + \Delta_k$ for $k \in \mathbb{N}$. Assume the encoding scheme is such that (8) is satisfied for all $t \geq t_0$, further assume that $L_1(t_0) \leq 1$, $L_2(t_0, t_0) \leq 1$ and that (10) holds. Let $p_k$ be

$$p_k \triangleq \min\{p \in \mathbb{N} : \bar{h}(\frac{p}{R(t_k)}), b(t_k), \epsilon(t_k), \frac{(\epsilon(t_k)}{p_k} \leq 1\}. \quad (33)$$

Then, the following hold:

(i) $p_1 \leq \bar{p}(t_1)$. Further for each $k \in \mathbb{N}$, if $p_k \in \mathbb{N}_0 \cap \{p \mid p_k \leq \bar{p}(t_k)\}$, then $p_{k+1} \leq \bar{p}(t_{k+1})$.

(ii) the inter-transmission times $\{t_{k+1} - t_k\}_{k \in \mathbb{N}}$ and inter-update times $\{\tilde{t}_{k+1} - \tilde{t}_k\}_{k \in \mathbb{N}}$ have a uniform positive lower bound.

(iii) the origin is exponentially stable for the closed-loop system, with $V(x(t)) \leq V_0(t) e^{-\beta(t)}$ for all $t \geq t_0$.

Proof: The facts $V(x(t)) \leq V_0(t) \leq L_2(t_0, t_0) \leq 1$ in addition with the trigger (32) ensure that $L_2(t_1, t_1) \leq 1$. Then, (33) ensures that $p_1 \leq \bar{p}(t_1)$. Now, for each $k \in \mathbb{N}$, if $p_k \in \mathbb{N}_0 \cap \{p \mid p_k \leq \bar{p}(t_k)\}$ then

$$\tilde{t}_k - t_k = \frac{p_k}{R(t_k)} \leq \bar{p}(t_k) \leq T_M(\bar{p}(t)), \quad (34)$$

where the last inequality follows from (31), which in turn implies $\bar{h}(\tilde{t}_k) \leq 1$. This means $L_2(\tilde{t}_k, \tilde{t}_k) \leq 1$, from which it follows that $p_{k+1} \leq \bar{p}(t_{k+1})$.

Now, we prove (ii) - first observe that for any $k \in \mathbb{N}_0$, (32) ensures $L_1(t_k) \leq 1$, which as a consequence of Lemma 3.5 and (34) implies that $b(t) \leq 1$ for all $t \in [t_k, \tilde{t}_k]$. We have already seen that $\bar{h}(\tilde{t}_k) \leq 1$. Now, since in (29) $\sigma < 1$, there is a uniform positive lower bound for $t_{k+1} - t_k$, which proves (ii).

We have already seen that for any $k \in \mathbb{N}_0$, $b(t) \leq 1$ for all $t \in [t_k, \tilde{t}_k]$. Further, (32) also ensures that $b(t) \leq 1$ for
all \( t \in [t_k, t_{k+1}] \). Therefore \( b(t) \leq 1 \) (\( V(x(t)) \leq V(u(t)) \)) for all \( t \geq t_0 \), which completes the proof.

**Remark 5.3:** (Requirements on the knowledge of channel information in the scenario of no channel blackouts) Note that in the scenario with no channel blackouts, the encoder needs to know the channel information (\( R(t) \) and \( \tilde{p}(t) \)) only over a horizon of time \( \delta_1 \). Further, if a uniform lower-bound on \( t \mapsto \tilde{p}(t) \) is known then it is sufficient for the encoder to know only the current channel information. Also note that, by definition, if there are no channel blackouts then 1 is a uniform lower bound on \( t \mapsto \tilde{p}(t) \).

**B. Control in the presence of channel blackouts**

Now, we build on the previous subsection to address the scenario with channel blackouts. In this scenario, we assume that both \( R(t) \) and \( \tilde{p}(t) \) are piece-wise constant functions defined as in (18). We let \( B_k \triangleq [\tau_{k_1}, \tau_{k_2}] \) denote the \( k^{th} \) blackout slot, with \( k \in \mathbb{N}_0 \). Also, for any \( t \geq t_0 \), we let

\[
\tau_i(t) \triangleq \min\{s \geq t : \tilde{p}(s) = 0\},
\]

\[
\tau_i(t) \triangleq \min\{s \geq \tau_i(t) : \tilde{p}(s) > 0\},
\]

so that \( \tau_i(t) \) and \( \tau_i(t) \) give the beginning and the end times of the next channel blackout slot from the current time \( t \). When there is no confusion, we simply use \( \tau_i \), \( \tau_u \), and \( \tau_0 \), dropping the argument \( t \). Thus, for \( t \in [t_0, \tau_1] \), we have \( \tau_i(t) = \tau_i \) and \( \tau_u(t) = \tau_u \). Similarly for any \( k \in \mathbb{N}_0 \) and \( t \in (\tau_k, \tau_{k+1}] \), \( \tau_i(t) = \tau_i(t_{k+1}) \) and \( \tau_u(t) = \tau_u(t_{k+1}) \). Depending on the context, one or the other of these two equivalent notations is more convenient. Hence, in the sequel we use both the notations and switch between the two as when necessary.

The length of the blackout slot, \( \tau_u - \tau_i \), determines the upper bound on the encoding error \( d_e(\tau_i) \) or equivalently \( \epsilon(\tau_i) \) for effective operation of the system. We quantify this upper bound in the following lemma.

**Lemma 5.4:** (Upper bound on required \( \epsilon \) before blackout). Let \( T_b(t) \triangleq \tau_u(t) - \tau_i(t) \) and suppose the data transmissions are such that

\[
\epsilon(\tau_i(t)) \leq \epsilon_r(t) \triangleq \min \left\{ \frac{(e^{wT_b(t)} - 1)(w + \theta) + \epsilon^r}{w(e^{wT_b(t)} - 1) + \epsilon^r}, \frac{1}{e^{\theta T_b(t)}} \right\},
\]

(35)

where \( \theta \triangleq \|A\|_{\infty} + \frac{\theta}{2} \). If \( b(\tau_i) \leq 1 \) then \( b(s) \leq 1 \) for all \( s \in [\tau_i, \tau_u] \) and \( h_{ch}(\tau_u) \leq 1 \) in particular \( \epsilon(\tau_u) \leq 1 \).

**Proof:** From Lemma 3.3, we know \( \Gamma(1, \epsilon_r(t)) \geq \Gamma(1, \epsilon_r(t)) \). So, we need to show that \( \Gamma(1, \epsilon_r(t)) \geq T_b(t) \) or as per Lemma 3.5 that \( b(T_b(t), 1, \epsilon_r(t)) \leq 1 \). Direct computation shows that it is in deed the case, which proves \( b(s) \leq 1 \) for all \( s \in [\tau_i, \tau_u] \). The second claim also follows from direct computation as follows

\[
h_{ch}(\tau_u) \leq h_{ch}(T_b(t), 1, \epsilon_r(t), \epsilon_r(t)) \leq e^{\tilde{h}_{ch}(T_b(t), 1, \epsilon_r(t), \epsilon_r(t))} \leq e^{\tilde{h}_{ch}(T_b(t), 1, \epsilon_r(t), \epsilon_r(t))} \leq 1.
\]

(36)

The ability to ensure that \( \epsilon(\tau_i) \) is sufficiently small is determined by the channel capacity \( D(t, \tau_i) \). In order to have a real-time implementation we make use of the sub-optimal estimate \( \hat{D}_s(t, \tau_i) \) instead. However, notice that maximizing the data throughput and satisfying the primary control goal of exponential convergence may not be compatible in general. Hence, we need to impose an artificial bound on the allowed packet size in place of \( \tilde{p}(t) \). To this end, we first define \( \tilde{p}_j \triangleq \phi^J \) where \( \phi^J \) is the optimizing solution (24) of \( D_s(\theta_j, \tau_i(\theta_j)) \). Then, we define

\[
\Phi^J(t) \triangleq \min\{\tilde{p}(t), \Phi^J(t)\}
\]

(37)

be the artificial bound on the packet size for transmissions. Notice that \( \Phi^J(t) \) may at times be zero, even when \( \tilde{p}(t) > 0 \), which means letting \( \psi^J(t) \) be the bound on packet size may itself introduce artificial blackouts. However, we can say how large such artificial blackouts may be, which is the subject of the following lemma.

**Lemma 5.5:** (Upper bound on the length of artificial blackouts). Let \( B_j \triangleq \{t \in I_j = [\theta_j, \theta_{j+1} : \psi^J(t) = 0\} \). Then, for each \( j \in \mathbb{N}_0 \), \( B_j \) is an interval and if \( \bar{\tau}_j > 0 \), then the length of \( B_j \) is less than \( 2/R_j = 2/R(\theta_j) \).

**Proof:** The fact that \( B_j \) is an interval follows directly from the definition (36). If \( \bar{\tau}_j > 0 \), then at any time \( t \in I_j \), \( \tilde{p}(t) < \bar{\tau}_j > 0 \), then, if \( \psi^J(t) = 0 \) for some \( t \in I_j \), then

\[
\tilde{p}_j + R_j(\theta_j - t) = \tilde{p}_j + R_j(\theta_{j+1} - T_j - t) < 1
\]

\[
\Rightarrow (\theta_{j+1} - t < 1) < \frac{1}{R_j} + \left( T_j - \frac{\tilde{p}_j}{R_j} < \frac{2}{R_j}ight)
\]

(38b)

where the second inequality follows from the optimality of \( \Phi^J(\theta_j, \tau_i(\theta_j)) \) because otherwise if \( R_j T_j - \tilde{p}_j \geq 1 \) then the optimality of \( \tilde{p}_j \) would imply that \( \tilde{p}_j = \tilde{p}_j + 1 \), which is a contradiction. This proves the lemma.

Now, we are very nearly in the framework of the previous subsection. However, to effectively overcome the artificial blackouts, we need to assume in the sequel that the signal \( t \mapsto R(t) \) is uniformly upper bounded by a known number \( R_{\max} \in \mathbb{N} \). We then define functions analogous to \( L_1 \) and \( L_2 \). In addition, we define one more function to capture the effect of the data capacity. Thus, consider the functions

\[
\hat{L}_1(t) = \tilde{b} \left( T_M(\psi^J(t)), b(t), e(t) \right)
\]

(38a)

\[
\hat{L}_2(t, s) = \tilde{h}_{ch} \left( T_M(\psi^J(t)), b(t), e(t), (e(t))/2^{\psi^J(s)} \right)
\]

(38b)

\[
L_3(t, e) \triangleq \min \left\{ \tilde{p}(t), \Phi^J(t) \right\}
\]

(37)

where \( \sigma_1 \in (0, 1) \) is a design parameter.

Clearly, we cannot satisfactorily control the system for an arbitrary channel characteristics with arbitrary channel blackouts. The following lemma presents a sufficient condition on the length of the blackout slots and the available data capacity.
Lemma 5.6: (Control feasibility in the presence of blackouts). Suppose \( t \mapsto R(t) \) and \( t \mapsto \tilde{p}(t) \) are piece-wise constant functions given as in (18). Let \( \{ (\tau_k, \alpha_k) \}_{k \in \mathbb{N}_0} \) be a sequence of channel blackout slots. Assume that \( \tilde{p}(t_0) \geq 0 \), \( \mathcal{L}_3(t_0, \epsilon(t_0)) \leq 0 \) and for each \( k \in \mathbb{N}_0 \) assume that \( \mathcal{L}_3(\tau_{uk}, 1) \leq 0 \). Then, there exists a transmission policy that would ensure \( \epsilon(\tau_k) \leq \epsilon_r(\tau_k) \) for each \( k \in \mathbb{N}_0 \), where \( \epsilon_r(\cdot) \) is given in (35).

Proof: Notice from the definition of \( \epsilon(t) \) in (11) and (8) that for any \( k \in \mathbb{N}_0 \) and \( s \in [r_k, r_{k+1}) \)
\[
\epsilon(s) = \left\| e^{A(s-t_k)} \right\|_{\infty} e^{\beta/2(s-t_k)} \epsilon(t_k) \leq \frac{e^{\beta(s-t_k)} \epsilon(t_k)}{2^{p_k}},
\]
which when recursively used gives us
\[
\epsilon(\tau_k) \leq \frac{e^{\beta(\tau_k-t_k)} \epsilon(t_k)}{2^{(\mathcal{B}(t, \tau_k)) / \epsilon_r(t)}},
\]
where \( \mathcal{B}(t, \tau_k) \) is the total number of bits communicated (transmitted and received) during the time interval \([t, \tau_k)\).

In other words, for any \( t \geq t_0 \), if
\[
\mathcal{B}(t, \tau_k) \geq n \log_2 \left( \frac{e^{\beta(\tau_k-t_k)} \epsilon(t_k)}{\epsilon_r(t)} \right)
\]
ensures that \( \epsilon(\tau_k(t)) \leq \epsilon_r(\tau_k(t)) \). Initially, \( \mathcal{L}_3(t_0, \epsilon(t_0)) \leq 0 \) ensures that there is enough data capacity, i.e. \( \mathcal{B}(t_0, \tau_0) \leq \mathcal{D}_s(t_0, \tau_0) \). Lemma 5.4 guarantees that for any \( k \in \mathbb{N}_0 \), if \( \epsilon(\tau_k) \leq \epsilon_r(\tau_k) \) then \( \epsilon(\tau_k) \leq 1 \). The final claim of the lemma simply follows from induction and the use of the fact that \( \mathcal{L}_3(\tau_{uk}, 1) \leq 0 \) for each \( k \in \mathbb{N}_0 \).

Now we are ready to present our next main result.

Theorem 5.7: (Event-triggered control in the presence of blackouts). Suppose \( t \mapsto R(t) \) and \( t \mapsto \tilde{p}(t) \) satisfy the assumptions of Lemma 5.6. In addition, assume that the piece-wise constant functions \( \tilde{p} \) and \( R \) are uniformly upper-bounded by \( \tilde{p}^{\text{Max}} \in \mathbb{R} \) and \( R^{\text{Max}} \in \mathbb{R}_{>0} \), respectively, and that they have a uniform dwell-time \( \tilde{T} > 0 \). Also, assume
\[
R(t) \geq \frac{(p + 2)}{T_M(p)}, \quad \forall p \in \{ 1, \ldots, \tilde{p}(t) \}, \quad \forall t.
\]

Consider the system (1) under the feedback law \( u = K \tilde{x} \), with \( t \mapsto \tilde{x}(t) \) evolving according to (7) and the sequence \( \{ t_k \}_{k \in \mathbb{N}_0} \) determined recursively by
\[
t_{k+1} = \min \left\{ t \geq \tau_k : \psi'(t) \geq 1 \land \left( \mathcal{L}_1(t) \geq 1 \lor \mathcal{L}_2(t, t+\delta) \geq 1 \lor \mathcal{L}_3(t, \epsilon(t)) \geq 0 \right) \right\},
\]
where \( \delta_t \in (0, \min \{ T, \tilde{T}/R^{\text{Max}} \}) \) is a design parameter. Let \( \{ \tau_k \}_{k \in \mathbb{N}_0} \) be given as \( \tau_0 = t_0 = 0 \) and \( \tau_k = t_k + \tilde{T}^k \) for \( k \in \mathbb{N} \). Let the update times \( \{ \tilde{r}_k \}_{k \in \mathbb{N}_0} \) be given as \( \tilde{r}_0 = 0 \) and for \( k \in \mathbb{N} \)
\[
\tilde{r}_k = \min \{ t \geq \tau_k : \psi'(t) \geq 1 \lor \tilde{p}(t) = 0 \}.
\]

Assume the encoding scheme is such that (8) is satisfied for all \( t \geq t_0 \). Further assume that \( \mathcal{L}_1(t_0) \leq 1, \mathcal{L}_2(t_0, 0) \leq 1 \) and that (10) holds. Let \( p_k \) be given by
\[
p_k = \min \{ p \in \mathbb{N} : \bar{h}_ch \left( T_M(p), b(t_k), \epsilon(t_k), \frac{\tilde{r}_k}{2^p} \right) \leq 1 \}.
\]

Then, the following hold:

(i) If \( p_k \leq \psi'(\tau_1) \). Further for each \( k \in \mathbb{N} \), if \( p_k \in \mathbb{N} \cap [p_k, \psi'(\tau_1)] \), then \( \tilde{p}_{k+1} \leq \psi'(\tau_{k+1}) \).

(ii) The inter-transmission times \( \{ t_{k+1} - t_k \}_{k \in \mathbb{N}} \) and inter-update times \( \{ \tilde{r}_{k+1} - \tilde{r}_k \}_{k \in \mathbb{N}} \) have a uniform positive lower bound.

(iii) The origin is exponentially stable for the closed-loop system, with \( V(x(t)) \leq V(x(t_0))e^{-\beta(t-t_0)} \) for all \( t \geq t_0 \).

Proof: Notice that (40) ensures that for any \( k \in \mathbb{N} \), \( \tilde{r}_k \geq \psi'(\tau_k) \). Also, \( 1/R^{\text{Max}} \) is a uniform lower bound on the dwell time of the piece-wise constant signal \( t \mapsto \psi'(t) \) and hence, so is \( \delta_t \). Now, notice from (41) that for any \( k \in \mathbb{N} \), \( \tilde{r}_k > r_k \) if and only if \( \psi'(r_k) = 0 \) and \( p_r(t) \geq 1 \). That is, \( \tilde{r}_k > r_k \) if and only if \( r_k \in [\tau_1, \tau_2) \), an artificial blackout interval. In all other cases, \( \tilde{r}_k = r_k \). Thus, it follows from Lemma 5.5 that \( \tilde{r}_k - r_k \leq \frac{n}{R(r_k)} \) for all \( k \in \mathbb{N} \). Hence, for all \( k \in \mathbb{N} \), we have
\[
\tilde{r}_k - r_k = (\tilde{r}_k - r_k) + (r_k - t_k) \leq 1 \frac{1}{R(r_k)} + \frac{p_k}{R(r_k)} \rightarrow \tilde{r}_k - r_k \leq \left\{ \begin{array}{ll}
np_k, & \text{if } \tilde{r}_k = r_k \\
\min \left( \frac{np_k}{R(r_k)}, \frac{np_k}{R(r_k)} \right), & \text{if } \tilde{r}_k > r_k.
\end{array} \right.
\]

In either case, it follows from (39) that \( \tilde{r}_k - t_k \leq T_M(p_k) \leq T_M(\psi'(\tau_k)) \) for all \( k \in \mathbb{N} \). Also observe that, by the construction of \( t \mapsto \psi'(t) \) in (37), we have
\[
\mathcal{D}_s(\tilde{r}_k, \tau_1) \geq \mathcal{D}_s(t_k, \tau_1) - np_k.
\]

Next, noting that
\[
\epsilon(\tilde{r}_k) = \left\| e^{A\tilde{T}_k} \right\|_{\infty} e^{\frac{\beta}{2}(\tilde{T}_k)} \epsilon(t_k) \leq \frac{e^{\beta(\tau_k-t_k)} \epsilon(t_k)}{2^{p_k}}
\]
we have
\[
n \log_2 \frac{e^{\beta(\tilde{T}_k-t_k)} \epsilon(t_k)}{\epsilon_r(t_k)} - np_k \leq \sigma_1 \mathcal{D}_s(\tilde{r}_k, \tau_1)
\]
where the second inequality follows from the fact that \( \mathcal{L}_3(t_k, \epsilon(t_k)) \leq 0 \), the third inequality is true because \( \sigma_1 \in (0, 1) \). Therefore, we see that \( \mathcal{L}_3(\tilde{r}_k, \epsilon(\tilde{r}_k)) \leq 0 \). Thus, using induction, we see that the proposed transmission policy ensures that by the beginning of the next blackout, \( t = \tau_1, \epsilon(\tau_1) \leq \epsilon_r \). Lemma 5.4 then implies that at the end of blackout, we have \( b_{ch}(\tau_k) \leq 1 \) and \( b(s) \leq 1 \) for all \( s \in [\tau_1, \tau_0] \). Therefore, we see that \( \mathcal{L}_3(\tilde{r}_k, \epsilon(\tilde{r}_k)) \leq 0 \).
\[ r_k - t_k = \frac{n_k}{R(t_k)} \geq \frac{1}{T_{\text{max}}} \] for each \( k \in \mathbb{N} \). Then, (ii) follows from the fact that \( t_{k+1} \geq t_k \geq r_k \).

Now we prove (iii). Notice (40) ensures that \( \hat{L}_1(t_k) \leq 1 \) for any \( k \in \mathbb{N} \), which as a consequence of Lemma 3.5 means that \( b(t) \leq 1 \) for all \( t \in [t_k, t_{k+1}] \) for any \( k \in \mathbb{N} \). Now, for \( t \in [t_k, t_{k+1}] \) for \( k \in \mathbb{N}_0 \), there are three cases. Case I: \( \psi(t) \geq 1 \). In this case, \( b(t) \leq 1 \) because \( \hat{L}(t) < 1 \). Case II: \( \psi(t) = 0 \) and \( \hat{p}(t) \geq 1 \), which corresponds to a time during an artificial blackout \([\tau_1, \tau_2]\). Recall from Lemma 5.5 that \( \tau_2 - \tau_1 = n/R(\tau_1) \), which using (39) then implies \( \tau_2 - \tau_1 \leq T_M(\psi(\tau_1)) \). Next, by design (41), \( \tilde{r}_k \notin [\tau_1, \tau_2] \) and hence \( r_k < \tau_1 \) and no transmission is in progress during \([\tau_1, \tau_2]\), which must mean \( \hat{L}_1(\tau_1^+) < 1 \). Lemma 3.5 then implies \( \Gamma_1(b(\tau_1), \epsilon(\tau_1)) \geq T_M(\psi(\tau_1)) \geq \tau_2 - \tau_1 \). Therefore, \( b(t) \leq 1 \) for all \( t \in [\tau_1, \tau_2] \). Case III: \( \psi(t) = \hat{p}(t) = 0 \), which corresponds to a time in a channel blackout slot. We have already seen in the proof of (i) that the proposed transmission policy ensures \( b(s) \leq 1 \) for all \( s \in [\tau_1, \tau_u] \) for any channel blackout \([\tau_1, \tau_u]\). Therefore, \( b(t) \leq 1 \) for \( V(x(t)) \leq V_d(t) \) for all \( t \geq 0 \) and claim (iii) follows.

Remark 5.8: (Requirements on the knowledge of channel information in the scenario with channel blackouts). In the scenario with channel blackouts, the encoder needs to know the time at which the next blackout is scheduled to occur \( \tau(t) \), the duration of the next blackout \( T_b \), from which \( \epsilon_r(t) \) may be computed. The encoder also needs to know the channel functions \( s \mapsto R(s) \) and \( s \mapsto \hat{p}(s) \) for all \( s \in [t, \tau(t)] \), from which the capacity may be estimated as \( D_s(t, \tau(t)) \).

VI. SIMULATION RESULTS

The simulation results we present here correspond to the design of Theorem 5.7 on the system given by (1) with

\[
A = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad K = \begin{bmatrix} 2 & -8 \end{bmatrix}.
\]

The plant matrix \( A \) has eigenvalues at 2 and 3, while the control gain matrix \( K \) places the eigenvalues of the matrix \( \tilde{A} = A + BK \) at -1 and -2. We select the matrix \( Q = I_2 \), for which the solution to the Lyapunov equation (4) is

\[
P = \begin{bmatrix} 2.2500 & -0.9617 \\ -0.9617 & 0.5833 \end{bmatrix}.
\]

The desired control performance is specified by

\[ V_d(t_0) = 1.2V(x(t_0)), \quad \beta = 0.8 \frac{\lambda_m(Q)}{\lambda_M(P)}. \]

We set \( a = 1.2 \) in (10), so that \( W > 0 \), and assume, without loss of generality, \( t_0 = 0 \). We chose the initial condition as \( x(t_0) = (6, -4) \), and the encoder and decoder use the information

\[
\dot{x}(t_0) = (0, 0), \quad \dot{e}_c(t_0) = 1.5||x(t_0) - \hat{x}(t_0)||_\infty.
\]

In (38), we chose \( \sigma_1 = 0.8 \). For these parameters, \( \Gamma_1(1, 1) = 0.5699 \) and chose \( T = 0.1 \times \Gamma(1, 1) \) and \( T_M(p) = 0.06 \times \min\{\Gamma(1, 1), T, T^*(p)\} \), where \( T^*(p) \) is as defined in Lemma 5.1. The time-varying channel functions \( n\bar{p} \) and \( \bar{R} \) are shown in Figures 1(a) and 1(b) respectively. Figure 1(a) also shows the number of bits transmitted on each transmission. In this simulation, the maximum possible number of bits transmitted on each transmission. Figure 2(a) shows the evolution of \( V \) and \( V_d \) and it is clear that the control goal is satisfied. Figure 2(b) shows the (interpolated) cumulative number of bits transmitted as a function of time. We see that there is a rush of transmissions just prior to 5 units of time, which we see from Figure 1(a) is the beginning of the first blackout. The number of transmissions in the 20 units of time in the simulation are 14, with the average inter-transmission interval as 1.44 and the minimum as 0.002. From Figure 2(b), we also see that on an average a little over 10 bits are transmitted per unit time.

VII. CONCLUSIONS

We have addressed the problem of event-triggered control of linear time-invariant systems under time-varying rate limited communication channels. The class of time-varying channels we consider is broad enough to include intermittent occurrence of scheduled channel blackouts (intervals of time when the communication channel is unavailable for feedback). We have designed event-triggered control schemes that, using prior knowledge of the channel information, guarantees the exponential stabilization of the system at a desired convergence rate in the presence of intermittent channel blackouts. Key enablers of our design are the definition and analysis of the channel capacity, which measures the maximum number of bits that can be communicated over a given time interval through one or more transmissions.
We have also provided an efficient real-time algorithm to estimate the channel capacity for a time-slotted model of channel evolution. Future work will explore the reduction of the conservatism of the proposed design, scenarios with bounded disturbances, and the extension of the proposed event-triggered stabilization schemes to stochastic settings with uncertain channel information.

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