ANALYTIC CALCULATION OF
BOSE-EINSTEIN CORRELATIONS AT LEP1 AND LEP2 *

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Theoretical results in the $\tau$-model are discussed in the context of Bose-Einstein
correlations in $e^+e^-$ annihilations at LEP1 and LEP2 as well as that of relativistic
heavy ion collisions at SPS and RHIC.

A current motivation$^1$ to study of Bose-Einstein$^2$ (or HBT$^3$ or GGLP$^4$)
correlations in $e^+e^-$ annihilations at LEP1 or LEP2 is to determine their
possible effects on the mass reconstruction of $W$ bosons at LEP2, see e.g.
ref.$^5$ for a recent review. The goal is to progress in 2 steps$^5$:

1) understand Bose-Einstein correlations at LEP1, at the $Z^0$ peak,
2) generalize these results to the case of $W^+W^-$ production at LEP2.

1. Bose-Einstein correlations in $\tau$-model at LEP1

A model of strongly correlated phase-space was developed in ref$^6$ to explain
the experimentally found invariant relative momentum $Q = \sqrt{-\left(k_1 - k_2\right)^2}$
dependence of Bose-Einstein correlations (BEC-s) in $e^+e^-$ reactions$^7,8$. As
the correlation function was expressed in terms of a distribution of a proper-
time $\tau$, let us call this model the $\tau$-model$^6,9,10$. It is based on

Assumption $i$): The momentum $k$ and the average position $\overline{x}$ are
strongly correlated: 
$\overline{x} = d k$ where $d$ is a constant of proportionality,
specified below, and $x = x^\mu = (t, \mathbf{r})$, $k \equiv k^\mu = (E_k, \mathbf{k})$.

Assumption $ii$): The space-time momentum-space correlation is nar-
rower than the proper-time distribution (when both are measured in di-
menionless units).

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Thus the emission function of the $\tau$-model is
\[ S(x, k) = \int_0^\infty H(\tau) \delta_\Delta(x - d k) N_1(k) \]
where $H(\tau)$ is the proper-time distribution, the factor $\delta_\Delta(x - d k)$ is describing the strength of the correlations between coordinate space and momentum space variables. In the original version of the $\tau$-model, these correlations were taken to be ideally strong, $\delta_\Delta(x - d k) = \delta(4)(x - d k)$, a four-dimensional Dirac-delta. The results of the $\tau$-model are unchanged if the $(x, k)$ correlations are described by a sufficiently, but not infinitely narrow function, whose width is characterized by some dimensionless and narrow scale $\Delta \ll \Delta \tau/\tau$, where $\Delta \tau$ is the width of $H(\tau)$. The distributions are normalized, $\int_0^\infty H(\tau) = 1$, and $\int d^4x \delta_\Delta(x - \tau k/m) = 1$.

If the expansion is 1+1 dimensional, then $d = \tau/m_t$, if the expansion is 1+3 dimensional then $d = \tau/m$, where the (longitudinal) proper-time is defined as ($\tau_l = \sqrt{t^2 - r_z^2}$ and) $\tau = \sqrt{t^2 - r_x^2 - r_y^2 - r_z^2}$, while the (transverse) mass is given as ($m_t = \sqrt{E_k^2 - k_z^2}$ and) $m = \sqrt{E_k^2 - \mathbf{k}^2}$. The former is relevant in 2-jet decays of $Z^0$ at LEP1, the latter is relevant in the case of fully hadronic $W^+W^-$ decays, mostly 4-jet events.

The experimentally measurable single-particle spectra $N_1(k)$ is
\[ \int d^4x S(x, k) = N_1(k) \]
arbitrary, and can be taken directly from the measurements.

The two-particle BEC-s $C_2(k_1, k_2) = N_2(k_1, k_2)/(N_1(k_1)N_1(k_2))$ are calculated with the help of the Yano-Koonin formula. The key step is that for any choice of $d$, one gets $\langle \mathbf{x}_1 - \mathbf{x}_2 \rangle (k_1 - k_2) = -0.5(d_1 + d_2)Q^2$.

Using a saddle-point integration based on $i$) and $ii$), we find that
\[ C_2(k_1, k_2) \approx 1 + \lambda \text{Re} \{ \bar{H}^2(w) \}, \]
where the argument is $w = Q^2/(2m)$ for 1+3 dimensional, and $w = Q^2/(2m_t)$ for 1+1 dimensional expansions, corresponding to the appropriate choice for variable $d$, and $\bar{H}(w) = \int_0^\infty d\tau H(\tau) \exp(iw\tau)$ is a Fourier-transform of $H(\tau)$. Thus an invariant relative momentum $Q$ dependent BEC appears, if the $(x, k)$ correlations between spacetime and momentum-space are strong enough and the proper-time distribution is broad enough.

For example, let us consider two-jet events at LEP1. These are 1+1 dimensionally expanding systems, hence $d = \tau/m_t$. In case of the asymmetric Lévy $H(\tau)$ distribution, $H(\tau) \propto \frac{1}{(\tau - \tau_0)^{\lambda/2}} \exp(-\frac{4\Delta \tau}{\tau - \tau_0})$, we get
\[ C(Q) = 1 + \cos(R_0^2Q^2) \exp(-QR) \]
with $R_0^2 = \tau_0/m_t$ and $R^2 = \Delta \tau/m_t$. This form is similar to the observed data at LEP1. Currently known experimental constraints on BEC-s in $e^+e^-$ annihilations at LEP1 are summarized below. It is easy to show that the $\tau$-model satisfies all these properties\textsuperscript{9,10}:

1. BEC-s exist in both $Z^0$ decays at LEP1 and in fully hadronic decays of $W^+W^-$ events at LEP2.
2. The pion source is not spherically symmetric, but elongated along the thrust axis, when analyzed in the LCMS frame.
3. The shape of the BEC function is far from Gaussian at LEP1.
4. The effective source size decreases as $1/\sqrt{m}$ for pions, kaons, protons and $\Lambda$ particles, and as $1/\sqrt{m_t}$ for pions with different $m_t$.
5. The normalized three-particle cumulant correlation coincides with 1 within experimental errors in a large region of $Q_3$.
6. No significant isospin dependence is seen.
7. In $e^+e^-$ annihilations, the two-particle BEC depends on the relative momentum components only through the invariant momentum difference $Q = \sqrt{-(k_1 - k_2)^2}$.
8. After the peak, the BEC develops a shallow $C(Q) < 1$ dip region.

2. Bose-Einstein correlations from the $\tau$-model at LEP2

The key step is to introduce $\tau$ for $e^+e^- \rightarrow W^+W^-$ fully hadronic decays. This variable is now measured from the production point of the $W^+W^-$ pair (and not from the decay point of any of the $W$-bosons).

In these reactions the particle emitting source has two components,

$$S(x, k) = S^+(x, k) + S^-(x, k),$$

corresponding to the decays of $W^+$ and $W^-$, respectively. Thus the single particle spectrum has also two components, $N_1(k) = N_1^+(k) + N_1^-(k)$. When calculating the BEC-s for such a binary source, the key observation is $H^+(\tau) = H^-(\tau) = H_2(\tau)$. This simple equation has dramatic consequences for the BEC of fully hadronic decays of $W^+W^-$ pairs at LEP2:

$$C_2(k_1, k_2) = 1 + \lambda \text{Re}[\tilde{H}_2^2(Q^2/2m)],$$

where $\tilde{H}_2(w) = \int_0^\infty d\tau H_2(\tau) \exp(iw\tau) \equiv \tilde{H}^\pm(w)$ is the Fourier-transformed proper-time distribution at LEP2. If $H_2(\tau) \approx H_1(\tau)$, where the latter stands for the proper-time distribution of particle production in $e^+e^- \rightarrow Z^0$ hadronic decays at LEP1, then the BEC-s at LEP1 and LEP2 are also
approximately the same. This result implies no genuine inter-W Bose-Einstein correlations within the \( \tau \)-model, but the existence of a fully chaotic source with full symmetrization, see ref\(^{10} \) for further discussions.

3. Implications for heavy ion collisions

The BEC-s or HBT correlations at SPS and at RHIC were mostly analysed in terms of the Bertsch-Pratt (BP) or side-out-long variables, see refs\(^{11,12} \) for recent reviews. If we assume that the \( \tau \)-model and strong \((x,k)\) correlations are relevant also in heavy ion collisions, we can decompose the resulting correlation functions in the BP variables to find a surprising result: The variable of the correlation function, 
\[
Q^2 \Delta \tau / m_t = Q^2 R_s^2 + Q^2 R_o^2 + Q^2 R_l^2
\]
yields
\[
R_s^2 = \Delta \tau / m_t, \quad R_l^2 = \Delta \tau / m_t \quad \text{and} \quad R_o^2 = m^2 \Delta \tau / m_t^3,
\]
where subscripts \( \{s,o,l\} \) stand for side, out and long, respectively.

Experimentally, this implies \( R_s \approx R_l \propto 1/\sqrt{m_t} \), and \( R_o \) decreases with \( m_t \) faster than these. Thus very strong \((x,k)\) correlations in the \( \tau \)-model generate \( R_o < R_s \) dynamically! More detailed studies are needed to determine the proper variables of BEC-s in high energy heavy ion collisions. If the \( \tau \)-model is relevant in high energy heavy ion collisions, it may imply that \( R_o \ll R_s \) means a very broad proper-time distribution and strong \((x,k)\) correlations. Experimentally, this question can be decided by determining if the Bose-Einstein correlation function at RHIC depends on the relative momentum components only through the invariant relative momentum \( Q \), as in the case of \( e^+e^- \) collisions, or not.

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