Non-monotonic potential description of alpha-$^{40}$Ca refractive elastic scattering

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Abstract. Experimental differential cross sections of $\alpha$ elastic scattering by $^{40}$Ca, over a wide range of incident energies, have been analyzed in terms of non-monotonic (NM) potentials, generated from the energy density functional (EDF) theory using a realistic two-nucleon potential coupled with an appropriate consideration of the Pauli principle. The Airy structure of the nuclear rainbow scattering data in the energy range of 36.1–42.6 MeV is well accounted for the first time by the shallow NM potential.

1. Introduction
Features of angular distributions of cross sections in elastic scattering of $\alpha$ particles and heavier projectiles, including anomalous large angle scattering (ALAS), have been well accounted for by three types of potentials. The first one is phenomenological having squared Woods-Saxon (SWS) geometry with the real volume integral per nucleon pair $J_R/(4A) \approx 300$ MeV.fm$^3$ [A being the target mass number], advocated by Michel and his collaborators [1-3, and references therein].

The second one pertaining to a microscopic double folding (DF) [4,5] of the effective nucleon-nucleon (N-N) potential. However, the folded potential, which is monotonic and has $J_R/(4A) \approx 300$ MeV.fm$^3$, needs renormalization. In the DF approach, potentials are generated from the N-N M3Y interaction [6] which does not contain a tensor component that is critical to our understanding of deuteron magnetic dipole and electric quadrupole moments. Moreover, the DF method neglects a proper consideration of the Pauli principle among the nucleons. As a consequence, the DF potentials are able to explain the elastic scattering data of $^6$Li, $^7$Li [7], only using an empirical energy-dependent renormalization factor. Sakuragi and his group [8,9] demonstrated elegantly that the renormalization may be done away with through the generation of a repulsive dynamic polarization potential (DPP) using a coupled discretized continuum channels (CDCC) method [9] in conjunction with the DF potential.

The third type of nucleus-nucleus potential being non-monotonic shallow with $J_R/(A_PA) \approx 100$ MeV.fm$^3$ [10-14] with $A_P$ as the projectile mass number. The real part of the latter has its roots in the energy-density functional (EDF) formalism [15,16] and is of molecular type with a repulsive core. As noted in [17,18], the basic ingredients for deriving nucleus-nucleus potentials using this method are: (i) a realistic N-N potential, (ii) experimental density distribution (DD) functions for each of the colliding nuclei, and (iii) local density approximation to determine the nucleonic mean-field that incorporates the Pauli principle. In the sudden approximation version of this method, the functional form of the nucleus-nucleus potential is non-monotonic (NM) as a consequence of the Pauli principle. The latter principle is

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not explicitly considered in other forms of the EDF theory, existing in the literature, such as those based on the Skyrme type of the N-N interaction [19].

As observed by Baye [20], both the deep and shallow nature of nucleus-nucleus potentials can fit accurately the same experimental data. However, unphysical bound states of the composite system of the colliding nuclei, supported by a deep potential, are eliminated in the energy spectrum microscopically calculated in the phase equivalent shallow potential with a repulsive core [21]. The features of the later are present in the EDF-generated potential.

The recent work of Basak et al. [22] demonstrated explicitly that the NM aspect of the real central potential in conjunction with an optimum imaginary part and an effective empirical spin-orbit part can successfully account for both the cross section (CS) and vector analyzing power (VAP) data of $^6$Li elastic scattering by a number of targets in the simple optical model (OM) analysis. The OM calculations has also been found to account well for the opposite signs of the VAP data of elastically scattered $^6$Li by $^{58}$Ni at about 20 MeV and by $^{120}$Sn at 44 MeV. In effect, the repulsive DPP as observed in [8,9] is taken care of by the repulsive core in the potential, generated in EDF with the Pauli principle. Moreover, the different excitation processes of $^6$Li and $^7$Li generate the DPP polarization potentials of opposite signs to produce opposite VAP for the two projectiles, as noted in [22].

The stupendous success of the NM real part of the $^6$, $^7$Li-nucleus potential in describing the elastic scattering CS and VAP data without the need of renormalization or adjustment of the EDF-derived potentials, has been attributed to the following aspects, incorporated in the EDF calculations:

(i) The density distribution (DD) functions can reproduce the correct binding energies (BE) of the colliding nuclear pairs.

(ii) A proper consideration of the Pauli principle in the mean field [23] used in the EDF calculations leads to the NM nature of the interaction potential reflecting the repulsive core and eliminating the Pauli-forbidden unphysical states of the composite nucleus in a deep potential [21].

(iii) The mean field [23] in EDF calculations is based on the realistic N-N potential that accounts for the observed deuteron properties and two-nucleon scattering data up to the pion-production energy.

Goldberg and Smith [24] first pointed out that the nucleus-nucleus elastic scattering at angles beyond the grazing angle might exhibit a ‘nuclear rainbow’ phenomenon. Being a refractive process, such a nuclear scattering leading to the Airy structure in the angular distribution of elastic scattering is sensitive to the details of the nuclear potential in the nuclear interior. This is a consequence of the fact that the rainbow scattering arises from the interference of the partial L-values significantly lower than the grazing L-values responsible for the Fraunhofer diffraction scattering [25,26]. The broader angular widths and faster angular excursion with the incident energy of the rainbow oscillations are the distinctive features which make the refractive structure easily distinguishable from the diffractive scattering. Hence the nuclear rainbow scattering can probe the nuclear interior and eliminate the potential ambiguities, which triggered a considerable interest in finding its accountability since mid-seventies [6].

In the literature, application of non-monotonic potential to the analysis of the refractive structure is not available. It is claimed [6,27] that a ”deep attractive real part” in the optical potential, in addition to ”weak absorption”, is essential to generate and describe a refractive structure in the angular distribution of elastic scattering. Since the NM potential is shallow with a volume integral of $\sim 100 \text{ MeV fm}^3$, it would be interesting to examine the NM potential in the study of the refractive scattering data.

$^{40}$Ca is an ideal target, where there are substantial amount of $\alpha$- elastic scattering data [6,27,28] over a wide energy range. $\alpha$-particle is a simple composite projectile with zero spin and isospin having a compact structure in terms of binding energy (BE), which is around 28.3 MeV. The ($\alpha$, $^{40}$Ca) interaction system is expected to be ideal for the weak absorption and hence for studying refractive structure in the elastic scattering.

The purpose of the present study is to investigate how well the $\alpha+^{40}$Ca NM potential, with its parameters of the real part starting from those derived in the EDF formalism, can account for the
refractive structure along with the usual diffractive one in the angular distribution of the $\alpha + ^{40}\text{Ca}$ elastic scattering data over the energy range of $36.2 - 42.6$ MeV.

2. Optical potential parameters

The real part of the central nucleon potential for alpha particle generated using the density distribution (DD) functions in the EDF calculations, the details of which are given in [29]. The density dependence of the energy per nucleon, $E/A$, in the nuclear and nucleonic matter using realistic N-N potential with full consideration of the Pauli principle has been calculated by Brueckner et al. [23]. The density dependence of the calculated $\nu(\rho, \xi)$ has been parametrized analytically as

$$\nu(\rho, \xi) = b_1(1 + a_1\xi^2)\rho + b_2(1 + a_2\xi^2)\rho^{4/3} + b_3(1 + a_3\xi^2)\rho^{5/3}$$

(1)

The coefficients, $a_1 = -0.2$, $a_2 = 0.31$, $a_3 = 1.646$, $b_1 = -741.28$, $b_2 = 1179.89$ and $b_3 = -467.54$, are derived [30] from fitting the calculated curves for the density distribution of the energy per nucleon, $E/A$. Hence the nucleon potential in the mean field incorporates the exchange effect. The inhomogeneity correction with the parameter value of $\eta = 8.0$ has been found to reproduce the correct nuclear masses [22].

The potential $V(R)$ [32] between the $\alpha$ projectile and the $^{40}\text{Ca}$ target at a separation distance of $R$ is given by

$$V(R) = E[\rho(\vec{r}, R)] - E[\rho_\alpha(\vec{r}, R = \infty)] - E_T[\rho_T(\vec{r}, R = \infty)],$$

(2)

where $\rho$ is the DD function of the composite system. $\rho_\alpha$ and $\rho_T$ are respectively the DD functions for the projectile $\alpha$ and the target at $R = \infty$. In the sudden approximation, the density distribution function of the composite system is given by

$$\rho(r) = \rho_\alpha(r) + \rho_T(r).$$

(3)

For the alpha particle the DD function is given in [31] by

$$\rho_\alpha(r) = 4(\gamma/\pi)^{3/2}\exp(-\gamma r^2)$$

(4)

with $\gamma = 0.45 - 0.5$ [13]. The use of the width parameter $\gamma = 0.58$ leads to $\alpha$ central density of $\rho_0 = 0.317$ fm$^{-3}$, the RMS radius of $R_{rms} = 1.608$ fm and the binding energy of $BE = 12.93$ MeV. The latter is significantly lower than the experimental one. On the other hand, the value of $\gamma = 0.45$ yields $\rho_0 = 0.217$ fm$^{-3}$, $R_{rms} = 1.825$ fm and $BE = 20.96$ MeV. Although the $BE$ is reasonable, the central density is too low compared to the value of 0.32 fm$^{-3}$ deduced from experiments. $\gamma = 0.5$ leads to $\rho_0 = 0.254$ fm$^{-3}$, $R_{rms} = 1.732$ fm and $BE = 19.01$ MeV with $R_{rms}$ close to the range $R_{rms} = 1.67 - 1.70$ fm, as quoted in the compilation of de Vries et al. [33]. As noted earlier, the reproduction of $BE$ is crucial to the success of EDF-generated potentials. In view of this, $\gamma = 0.45$ has been used in the present work. The analytic DD for alpha shown in dotted line for $\gamma = 0.58$, in broken line for $\gamma = 0.45$, in solid line for $\gamma = 0.5$ and the experimental DD in solid circles in figure 1.
The dotted, dashed and solid lines denote the density distribution of $\alpha$ particle using, respectively, $\gamma=0.58, 0.45$ and 0.50 and the solid circles refer the experimental value.

Figure 1. The three-parameter Fermi (3pF) DD function for $^{40}\text{Ca}$ as given in [33]

$$\rho(r) = \rho_0 \left(1 + \frac{wr^2}{c^2}\right) \left[1 + \exp\left(\frac{r-c}{z}\right)\right]^{-1}$$

(5)

with the parameter values $w = -0.161$ fm, $c = 3.766$ fm, $z = 0.586$ fm, is taken from [33]. These parameters produce $\rho_0 = 0.1696$ fm$^3$ and $BE = 340.44$ MeV, which is very close to the experimental value of 342.05 MeV for $^{40}\text{Ca}$.

The real part of the nuclear potential for the $\alpha-^{40}\text{Ca}$ interaction, calculated from EDF, is given in solid circles in figure 2. The EDF-generated nuclear potential is parametrized in terms of the analytical expression, given by

$$V_N(R) = -V_0 \left[1 + \exp\left(\frac{R-R_0}{a_0}\right)\right]^{-1} + V_1 \exp\left[-\left(\frac{R-D_1}{R_1}\right)^2\right].$$

(6)

The first term is attractive with the WS geometry. The total $V_N(R)$ becomes non-monotonic with the inclusion of the second term which is repulsive with the shifted Gaussian form-factor. The broken and solid curves in figure 2 are the fits to the EDF potential with $D_1 = 0.0$, resulting in the parameter set EDF-1 and $D_1 \neq 0.0$ leading to set EDF-2, respectively, in table 1. The two sets differ mainly in the interior region of the target within about 3 fm. The total real part $V(R)$ of the $\alpha-^{40}\text{Ca}$ potential is obtained by adding the Coulomb potential $V_C(R)$ as

$$V(R) = V_N(R) + V_C(R).$$

(7)

$V_C(R)$ is assumed to be due to a homogeneously charged sphere of radius $R_C$ and is given by

$$V_C(R) = \begin{cases} \frac{2Z_T e^2}{2R_C}\left[3 - \frac{R^2}{R_C^2}\right] & \text{for } R \leq R_C, \\ 2Z_T e^2/R & \text{for } R > R_C \end{cases}$$

(8)

The imaginary part of the $\alpha$ potential is taken phenomenologically to be composed of volume and surface terms as

$$W_m(R) = -W_0 \exp\left[-\left(\frac{R}{R_W}\right)^2\right] - W_S \exp\left[-\left(\frac{R-D_S}{R_S}\right)^2\right]$$

(9)
Figure 2. Parametrization of the $\alpha-^{40}$Ca potentials from the EDF calculations (solid circles). The solid and dashed curves denote the real potentials using, respectively, the shifted ($D_1 \neq 0$) and unshifted ($D_1 = 0$) Gaussian repulsive core. The parameters are noted in table 1.

3. Analysis and results
The experimental data have been analyzed using the optical model code SFRESCO [34], which incorporates the coupled-channels code FRESCO 2.5 [35] coupled with the $\chi^2$-minimization code MINUIT [36]. The $\alpha+^{40}$Ca elastic scattering data for $E_\alpha = 36.2, 39.6, \text{and } 42.6 \text{ MeV}$ are taken from [1].

A systematic error of 15% has been assumed for the experimental cross section data normalized to the Rutherford cross-sections for the angular points without the error bars.

In the first stage of the analysis, the values of the parameters of the real part of EDF-1 and EDF-2 potential sets, obtained from the EDF calculations (table 1), were held fixed. The angular distribution data at the incident energy of 36.2 MeV was chosen first, as this is the lowest incident energy at which the experimental data cover a broad angular range as well as the ALAS and the refractive structure effects are substantial. Both the shifted ($D_1 \neq 0$) and unshifted ($D_1 = 0$) Gaussian forms of the surface imaginary potential in addition to the volume imaginary term in the optical potential, as given in (9), were tried on the angular distribution data. Both the potential sets, given in Tables 2 and 3, fail to produce a satisfactory description of the data over the entire angular range for the three energy points in the range from 36.2 to 42.6 MeV, as shown in figure 3. In particular, the large angle experimental data and the nuclear rainbow structure are not reproduced well. This is not surprising as the analytic density distribution (DD) shown in solid line in for alpha, employed in the EDF calculations, are very different from the experimental density distribution shown in solid circles, in the interior of the nucleus. Thus to improve the fits, EDF-generated potential parameters need empirical adjustments in the parameters for the real part for obtaining better fits.

In the second stage, the parameters of the real parts of set EDF-1 were adjusted to obtain the best possible fit to the data of the 36.2 MeV. Keeping the parameters of the real part and the geometry parameters of the surface imaginary part, $R_S$ and $D_S$ as well as the parameter $R_W$ of the volume imaginary one in (9) fixed, the depth parameters $W_s$ and $W_0$ were varied to obtain the best possible satisfactory fits to the data of higher energies e.g. 39.6 and 42.6 MeV. Final fits have been done visually after taking

Table 1. The non-monotonic potential sets EDF-1 and EDF-2 calculated from EDF method. $V_0$ and $V_1$ are in MeV; $R_0, R_1, R_C, a_0$ and $D_1$, in fm; and $J_R/(4A)$, in MeV.fm$^3$.

| Set   | $V_0$      | $R_0$ | $a_0$ | $V_1$ | $R_1$ | $D_1$ | $R_C$ | $J_R/(4A)$ |
|-------|------------|-------|-------|-------|-------|-------|-------|-----------|
| EDF-1 | 31.00      | 5.38  | 0.64  | 36.00 | 3.02  | 0.00  | 4.48  | 109.50    |
| EDF-2 | 29.85      | 5.38  | 0.64  | 16.00 | 2.15  | 1.75  | 4.48  | 113.60    |
Table 2. Non-monotonic EDF-1 potential parameters for $\alpha^{+40}$Ca. The parameter $D_1 = 0.00$ fm is used for all incident energies. $E_\alpha$, $V_0$, $V_1$, $W_0$ and $W_S$ are in MeV; $R_0$, $a_0$, $R_1$, $R_S$, $R_W$ and $R_C$ in fm. The volume integrals $J_R/4A_T$ and $J_I/4A_T$ are in MeV.fm$^3$. $\chi^2$ represents chi-square per point.

| $E_\alpha$ | $W_s$ | $D_s$ | $R_s$ | $W_0$ | $R_0$ | $V_0$ | $V_1$ | $a_0$ | $-V_1$ | $R_1$ | $R_C$ | $J_R/4A_T$ | $J_I/4A_T$ | $\chi^2$ |
|------------|------|------|------|------|------|------|------|------|-------|------|------|-----------|-----------|------|
| 36.2       | 3.50 | 6.8  | 1.41 | 1.0  | 4.2  | 31.0 | 5.38 | 0.64 | 36.0  | 3.02 | 4.48 | 109.5     | 37.4      | 4.20  |
| 39.6       | 3.50 | 6.8  | 1.41 | 1.5  | 4.2  | 31.0 | 5.38 | 0.64 | 36.0  | 3.02 | 4.48 | 109.5     | 40.0      | 6.89  |
| 42.6       | 3.50 | 6.8  | 1.41 | 1.8  | 4.2  | 31.0 | 5.38 | 0.64 | 36.0  | 3.02 | 4.48 | 109.5     | 43.4      | 23.8  |

Table 3. Same as table 2 for set EDF-2 with $D_1 = 1.75$ fm.

| $E_\alpha$ | $W_s$ | $D_s$ | $R_s$ | $W_0$ | $R_0$ | $V_0$ | $V_1$ | $a_0$ | $-V_1$ | $R_1$ | $R_C$ | $J_R/4A_T$ | $J_I/4A_T$ | $\chi^2$ |
|------------|------|------|------|------|------|------|------|------|-------|------|------|-----------|-----------|------|
| 36.2       | 3.50 | 6.8  | 1.41 | 1.0  | 4.2  | 29.85| 5.38 | 0.64 | 16.0  | 2.15 | 4.48 | 113.6     | 37.4      | 4.20  |
| 39.6       | 3.50 | 6.8  | 1.41 | 1.5  | 4.2  | 29.85| 5.38 | 0.64 | 16.0  | 2.15 | 4.48 | 113.6     | 40.0      | 6.89  |
| 42.6       | 3.50 | 6.8  | 1.41 | 1.8  | 4.2  | 29.85| 5.38 | 0.64 | 16.0  | 2.15 | 4.48 | 113.6     | 43.4      | 23.7  |

Figure 3. Experimental differential cross-sections (solid circles) for the $\alpha^{+40}$Ca elastic scatterings at different energies are compared to the predictions using the EDF-1 (solid curves) and EDF-2 (broken curves) parameters of the non-monotonic potentials in Tables 2 and 3.

Table 4. Same as table 2 for Set-1 with $D_1 = 0.00$ fm.

| $E_\alpha$ | $W_s$ | $D_s$ | $R_s$ | $W_0$ | $R_0$ | $V_0$ | $V_1$ | $a_0$ | $-V_1$ | $R_1$ | $R_C$ | $J_R/4A_T$ | $J_I/4A_T$ | $\chi^2$ |
|------------|------|------|------|------|------|------|------|------|-------|------|------|-----------|-----------|------|
| 36.2       | 3.90 | 6.8  | 1.41 | 1.0  | 4.2  | 35.5 | 5.38 | 0.30 | 38.4  | 3.00 | 6.00 | 113.09    | 38.7      | 11.8  |
| 39.6       | 4.00 | 6.8  | 1.41 | 2.0  | 4.2  | 35.0 | 5.38 | 0.30 | 38.4  | 3.00 | 6.00 | 112.55    | 42.2      | 15.1  |
| 42.6       | 4.10 | 6.8  | 1.41 | 3.5  | 4.2  | 35.0 | 5.38 | 0.30 | 38.4  | 3.00 | 6.00 | 112.55    | 47.0      | 22.1  |

guidance from the $\chi^2$ fits, since it is more important to reproduce the features e.g., positions of the peaks etc., of the angular distributions than naively minimizing the $\chi^2$ only.

The procedure results in Set-1 noted in table 4 and the corresponding angular distributions compared to the experimental data in figure 4. This Set-1 potential parameter can explain the rainbow structure occurring at the scattering angles $\theta_{c.m.} \sim 100^\circ$ in the angular distribution of 36.2 MeV but fail to explain the rainbow structures at $\sim 80^\circ$ in those of 39.6 and 42.6 MeV. In this stage, the parameters of the real parts of set EDF-2 were adjusted to obtain the rainbow structure in the angular distribution of 39.6 and
Figure 4. Experimental differential cross-sections (solid circles) for the $\alpha + ^{40}$Ca elastic scatterings at different energies are compared to the predictions using Set-1 (solid curves) parameters of the non-monotonic potentials in table 4.

Table 5. Same as table 2 for Set-2 with $D_1 \neq 0.00$ fm.

| $E_\alpha$ | $W_s$ | $D_s$ | $R_s$ | $W_0$ | $R_0$ | $V_0$ | $R_0$ | $a_0$ | $-V_1$ | $R_1$ | $D_1$ | $R_C$ | $\chi^2$ |
|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 36.2      | 2.9   | 7.27  | 1.05  | 5.0   | 4.2   | 28.5  | 5.51  | 0.27  | 26.0  | 1.65  | 0.7   | 7.36  | 116.8 |
| 39.6      | 3.5   | 6.80  | 1.05  | 4.8   | 4.0   | 28.0  | 5.51  | 0.54  | 30.0  | 1.80  | 1.7   | 6.00  | 101.2 |
| 42.6      | 3.4   | 6.80  | 1.05  | 6.0   | 4.0   | 28.0  | 5.51  | 0.54  | 30.0  | 1.80  | 1.7   | 6.00  | 101.2 |

Figure 5. Experimental differential cross-sections (solid circles) for the $\alpha + ^{40}$Ca elastic scattering at different energies are compared to the predictions using the Set-2 (solid curves) parameters of the non-monotonic potentials in table 5.

42.6 MeV.

The whole procedure, described in the above paragraphs, was repeated for the set EDF-2 with $D_1 \neq 0.0$. The generated final set, Set-2, with parameters in table 5 produces the fits displayed in figure 5. In addition to reproducing the nuclear rainbow (refractive) structure in the angular interval $\theta_{c.m.} \sim 80 - 100^\circ$ in the energy range of 36.2 – 42.6 MeV, Set-2 describes the three angular distributions, comprising both the refractive and diffractive structures, at $E_\alpha = 36.2$, 39.6 and 42.6 MeV much better
than the Set-1 potential parameters.

4. Discussion and conclusions
The present work reports the results of investigation on the nuclear refractive structure in an elastic scattering in terms of NM potentials. The experimental angular distributions of $\alpha$-elastic scattering on the semi-transparent $^{40}$Ca nuclear medium are analyzed using the NM potential with Gaussian repulsive core in order to obtain satisfactory fits to the data at three incident energies in the range of 36.2 - 42.6 MeV. The initial parameters (table 1) of the real part of the NM $\alpha$-$^{40}$Ca potentials are generated by the EDF formalism using the sudden approximation in (3). Since the $\alpha$-particle DD, used in the EDF calculations, differs significantly from its experimental distribution (figure 1), the EDF-derived potential parameters are not expected to provide an adequate description of the experimental elastic scattering data and had to be adjusted empirically for obtaining satisfactory fits to the angular distributions. Two distinct sets of potential parameters marked by the volume integral $J_R/(4A)$, generated in such a way, are displayed in Tables 4 and 5. Of these two sets e.g. Set-1 and Set-2 (Tables 4 and 5), $\alpha$-$^{40}$Ca potential parameters, Set-1 (table 4) although is able to account for well the rainbow structures at angle $\theta_{c.m.} \sim 100^\circ$ for the incident energy 36.2 MeV arising from the nuclear refractive scattering, but fail to explain the absolute cross sections at forward angle and the rainbow structures at $\theta_{c.m.} \sim 80^\circ$ for the incident energies 39.6 and 42.6 MeV (see figure 4). Set-2 potential parameters in Table5 account for reasonably well the features of rainbow structures at all the energies (see figure 5).

The present study confirms all the claims made in the literature [6,27] about the occurrence of the rainbow structure in the angular distribution of elastic scattering excepting that “deep attractive real part in the optical potential, in addition to weak absorption, is essential” is to be rephrased for the case of NM potential to “absorption has to be adequately weak relative to the attractive part of the real potential, and the extent of its attractive zone and repulsive core is to be appropriately proportionate”.

In conclusion, the present study suggests that the NM nature of the $\alpha$-potential, although shallow in terms of the volume integral, is capable to describe reasonably the rainbow structure arising from the nuclear refractive diffraction process in addition to the Fraunhofer diffractive oscillations in the elastic scattering.

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