CKM2010 Working Group II Summary: 
Determination of $|V_{cs}|$, $|V_{cd}|$, $|V_{cb}|$ and $|V_{ub}|$

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We review the progress on the determination of the CKM matrix elements $|V_{cs}|$, $|V_{cd}|$, $|V_{cb}|$, $|V_{ub}|$ and heavy quark masses presented at the 6th International Workshop on the CKM Unitarity Triangle.

PRESENTED AT

Proceedings of CKM2010, the 6th International Workshop on the CKM Unitarity Triangle, University of Warwick, UK, 6-10 September 2010
1 Introduction

The CKM matrix elements must be determined precisely in order to constrain physics beyond the Standard Model. This working group report focuses on the most up-to-date results from theory and experiment used to obtain $|V_{cs}|$, $|V_{cd}|$, $|V_{cb}|$, and $|V_{ub}|$. We mainly concentrate on results from semi-leptonic $b$ and $c$ decays, though we also discuss leptonic decays of $B$ and $D$ mesons and the determinations of $b$ and $c$ quark masses.

2 Semi-leptonic $D$ decays and determination of $|V_{cs}|$ and $|V_{cd}|$

Semi-leptonic $D$ meson decays provide an opportunity to test lattice QCD calculations of the form factors $f_{K,\pi}^+(q^2 = 0)$ if one assumes Standard Model CKM unitarity. One can also turn this test around, using the lattice calculations of the form factors to directly determine the CKM matrix elements $|V_{cs}|$ and $|V_{cd}|$, thus testing the Standard Model via second row and second column unitarity. There are currently three groups using different lattice formulations to calculate properties of semi-leptonic $D$ decays. The HPQCD Collaboration has recently published an unquenched lattice result for the $D \to K\ell\nu$ form factor $f_{K}^{+}(0)$ using Highly Improved Staggered Quarks (HISQ) and used it to extract a value of $|V_{cs}|$ from experiment [1]. The older unquenched lattice calculation from the Fermilab Lattice and MILC Collaborations [2] has larger errors, but includes results at non-zero $q^2$, as well as the $D \to \pi\ell\nu$ form factor $f_{\pi}^{+}(q^2)$.

A preliminary lattice calculation of $f_{\pi}^{K,\pi}(q^2)$ from the ETM Collaboration with a quenched strange quark is also available and is in good agreement with the other two results [3]. Competitive results for the $D \to K$ and $D \to \pi$ form factors are also obtained from light-cone sum rules [4].

At CLEO-c, $D\bar{D}$ meson pairs are produced at threshold through the decays $e^+e^- \to \psi(3770) \to D\bar{D}$ at a center-of-mass energy (c.m.) near 3.770 GeV. The integrated luminosity of the $\psi(3770)$ sample is 818 pb$^{-1}$ corresponding to about 5.4 million $D\bar{D}$ events. By reconstructing the hadronic decay of one $D$ (tag side), the 4-momentum of the second charmed meson (signal side) is known. This allows to reconstruct a semileptonic decay with no kinematic ambiguity. For $D_s$ decays, CLEO-c uses a data sample taken at $\sqrt{s} = 4.170$ GeV equivalent to 600 pb$^{-1}$. The $D_s$ mesons used are from the reactions $e^+e^- \to D_{s}^{+}D_{s}^{-}$ or $D_{s}^{+}D_{s}^{-}$ at the $B$ factories Belle and BaBar, the production of charmed mesons is accompanied by additional particles from fragmentation. Belle performed a tagged analysis with full reconstruction of events $e^+e^- \to D_{tag}^{(*)}\bar{D}_{sig}^{*-}X$. This method gives high $q^2$ resolution at the price of low reconstruction efficiency. BaBar has chosen a different approach and reconstructs the signal charm meson only, the neutrino energy being evaluated from the
Figure 1: $f_+(q^2)$ comparison between isospin conjugate modes and with LQCD calculations [2]. The solid lines represent LQCD fits to the modified pole model. The inner bands show LQCD statistical uncertainties, and the outer bands the sum in quadrature of LQCD statistical and systematic uncertainties.

The gold plated modes $D \to K \ell \nu$ and $D \to \pi \ell \nu$ are most useful for testing lattice QCD and determining the CKM matrix elements $|V_{cs}|$ and $|V_{cd}|$. The measurements of the $D \to K$ form factor are summarized in Table 1. These numbers agree with the most recent lattice QCD prediction, $f_+^K(q^2 = 0) = 0.747 \pm 0.019$ [1]. Theoretical calculations based on LQCD also reproduce the form factor shape at finite values of $q^2$, as shown in Fig. 1. As mentioned above, the form factor normalization from lattice QCD can be used to determine the CKM matrix elements. Using the measurement from CLEO-c [7] and the form factors from Refs. [1, 2], we obtain $|V_{cd}| = 0.234 \pm 0.007 \pm 0.002 \pm 0.025$ and $|V_{cs}| = 0.963 \pm 0.009 \pm 0.006 \pm 0.024$, where the third uncertainties are from the LQCD calculation of $f_+(0)$. On $|V_{cs}|$, the lattice error is thus 3% compared to an experimental uncertainty of 1%. For $|V_{cd}|$, the lattice error is about 10% while the experimental error amounts to 3%.

Table 1: Measurements of the $D \to K$ form factor assuming CKM unitarity. The uncertainties are statistical and systematic, respectively. Third error quoted by BaBar corresponds to the uncertainty in external inputs.

| Experiment     | $f_+(q^2 = 0)$               |
|----------------|------------------------------|
| Belle [5]      | 0.695 $\pm$ 0.007 $\pm$ 0.022 |
| BaBar [6]      | 0.727 $\pm$ 0.007 $\pm$ 0.005 $\pm$ 0.007 |
| CLEO-c [7]     | 0.739 $\pm$ 0.007 $\pm$ 0.005 |

In addition, CLEO-c has studied the $D \to V \ell \nu$ modes $D \to \rho \ell \nu$ and $D^+ \to \eta/\eta'/\phi \ell^+ \nu$ [8]. CLEO-c has also measured exclusive semileptonic decays of $D_s$ [9]
and the inclusive semileptonic rates of $D^0$, $D^+$ and $D_{s}^+$ \cite{10}. Two BaBar analyses study the decays $D_{s}^+ \to K^+K^-e^+\nu$ and $D^+ \to K^-\pi^+e^+\nu$ \cite{11,12}.

3 Leptonic $D$ and $B$ decays

The leptonic decay constants of $D$ and $B$ mesons can be calculated using lattice QCD and serve as important inputs to flavor physics studies. Again, three groups have recent results for these quantities: the Fermilab Lattice and MILC Collaborations, HPQCD, and the ETM Collaboration. All results are summarized in Table \ref{tab:2}, along with the world averages as determined by \cite{13}. In the determination of these lattice averages correlations are taken into account, the results obtained with a quenched strange quark are not included, and the PDG prescription for inflating errors for discrepant results is applied.

Table 2: Lattice results (in MeV) for heavy-light decay constants.

| Analysis                        | $f_D$  | $f_{D_s}$ | $f_B$  | $f_{B_s}$ |
|---------------------------------|--------|-----------|--------|-----------|
| HPQCD \cite{14,15}             | 213 ± 4| 248.0 ± 2.4| 190 ± 13| 231 ± 15  |
| FNAL/MILC \cite{16}            | 220 ± 9| 261 ± 9   | 212 ± 8 | 256 ± 8   |
| ETMC(quenched strange) \cite{17,18} | 197 ± 9| 244 ± 8   | 191 ± 14| 243 ± 14  |
| Average \cite{13}              | 213.9 ± 4.2| 248.9 ± 3.9| 205 ± 12| 250 ± 12  |

In the charm sector, the CKM matrix elements $|V_{cs}|$ and $|V_{cd}|$ are strongly constrained by CKM unitarity and leptonic $D$ decays thus allow to probe predictions of the charm decay constants $f_D$ and $f_{D_s}$ from lattice QCD assuming the Standard Model. CLEO-c \cite{19,20} uses its $E_{cm} = 3.770$ GeV and $E_{cm} = 4.170$ GeV data samples to study $D$ and $D_s$ leptonic decays, respectively. Again, the analysis strategy is to reconstruct a hadronic final state of the second charmed meson in the event. Belle \cite{21} measures $D_s^+ \to \mu^+\nu$ using a 548 fb$^{-1}$ data sample. In this analysis, $D_s$ mesons are inclusively reconstructed in events of the type $e^+e^- \to D_s^0 D^{\pm,0}K^{\pm,0}X$ and the $D_s$ 4-momentum is determined from the recoil system. A recent BaBar analysis \cite{22} uses a similar technique to measure $D_s^+ \to \mu^+\nu$ and $D_s^+ \to \tau^+\nu$. The numerical results of these analyses are summarized in Table \ref{tab:3} Comparing these numbers to lattice QCD theory (HPQCD \cite{14}, FNAL/MILC \cite{16}), one finds good agreement for $f_D$. For $f_{D_s}$, there is a slight 2$\sigma$ tension between experiment and the new HPQCD prediction.

Leptonic $B$ decays have been studied at the $B$ factories Belle and BaBar. The decay $B^+ \to \tau^+\nu$ has the largest branching fraction and is now well established. Increasingly stringent limits are being set on leptonic decays involving a light lepton.
Table 3: Measurements of the leptonic $D$ decay constants. The uncertainties are statistical and systematic, respectively.

| Experiment | $f_D$ (MeV) | $f_{D_s}$ (MeV) |
|------------|-------------|-----------------|
| CLEO \[19, 20\] | 206.7 ± 8.5 ± 2.5 | 259.0 ± 6.2 ± 3.0 |
| Belle \[21\] | 275 ± 16 ± 12 |
| BaBar \[22\] | 258.6 ± 6.4 ± 7.5 |

(electron or muon), $B^+ \to \ell^+\nu(\gamma)$. The missing neutrino(s) in the final state require a tagging technique. The analyses either use a hadronic tag, in which case the hadronic decays of the other $B$ meson in the event are fully reconstructed, or a semileptonic tag, in which case a charmed meson $D^{(*)}$ and a high momentum lepton from the other $B$ are required in the analysis. In summer 2010, Belle has presented a new measurement of the $B^+ \to \tau^+\nu$ branching fraction using the semileptonic tag technique, $(1.54^{+0.38+0.29}_{-0.37-0.31}) \times 10^{-4}$ \[23\]. BaBar quotes a new measurement using hadronic tags, $(1.80^{+0.57}_{-0.54} \pm 0.26) \times 10^{-4}$ \[24\]. All measurements of $B^+ \to \tau^+\nu$ are compatible and the Heavy Flavour Averaging Group quotes a combined branching ratio of $(1.64 \pm 0.34) \times 10^{-4}$. This value is in agreement with the Standard Model prediction of $(1.20 \pm 0.25) \times 10^{-4}$, calculated using the HPQCD value for $f_B$ of $190 \pm 13$ MeV and the HFAG value for $|V_{ub}|$ of $(4.32 \pm 0.16 \pm 0.29) \times 10^{-3}$ \[25\]. However, if this measurement is included in an overall fit to the CKM unitarity triangle, a tension appears as these fits prefer lower values of the $B^+ \to \tau^+\nu$ branching ratio.

4 Semi-leptonic $B$ decays and determination of $|V_{cb}|, |V_{ub}|$

The most accurate determinations of the matrix elements $|V_{cb}|$ and $|V_{ub}|$ are carried out through analysis of semi-leptonic $b \to u$ and $b \to c$ quark transitions. As the theoretical and experimental methods and uncertainties differ depending on whether the decay mode is exclusive or inclusive, these offer complementary ways of determining these matrix elements. On the theoretical side, the main challenge in exclusive decays such as $B \to D\ell\nu$ and $B \to \pi\ell\nu$ is to determine the non-perturbative form factors entering expressions for the decay rate, either in lattice QCD or light-cone sum rules. For inclusive decays, the $B \to X_c\ell\nu$ decay width can be reliably calculated using a straight-forward operator product expansion (OPE), while the experimental cuts needed in measurements of $B \to X_u\ell\nu$ introduce sensitivity to non-perturbative shape-functions and the theoretical treatment is more involved.
4.1 \textit{b- and c}-quark masses

An important input to the determination of the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$ from semi-leptonic decays are the \textit{b- and c}-quark masses. These heavy-quark masses can either be treated as external input and taken from lattice or QCD sum rule methods, or determined along with the CKM matrix elements in global fits for the inclusive decays. As the pole masses suffer from renormalon ambiguities of the order of $\delta m_{c,b} \sim \Lambda_{QCD}$, it is necessary that these determinations be carried out in short-distance schemes which are free of such ambiguities. Such schemes can be divided into two categories: the $\overline{\text{MS}}$ scheme, typically used in QCD sum rule and lattice determinations, and threshold schemes such as the kinetic, 1S, or shape-function schemes, typically used in global fits of semi-leptonic \textit{B} decays into \textit{c} or \textit{u} quarks.

In the working-group II session, new results for the heavy-quark masses using global fits of inclusive semi-leptonic \textit{b} and \textit{c} decays were presented, and will be discussed below in that context. In addition, there was a dedicated talk by A. Hoang on determining $m_c$ in the $\overline{\text{MS}}$ scheme from QCD sum rules and experimental data from charm production $e^+e^-$ collisions, based on work performed in \cite{26}. While the determination of $m_c$ from QCD sum rules is already in an advanced state, and recent calculations \cite{27,28} lead to a value of $m_c$ with very small errors, the purpose of the study was to re-examine several aspects of the current analyses. In particular, the full set of experimental data was included, and special attention was paid to the perturbative error analysis, accounting for all sources of scale variation and different ways of expanding the series. The final results of the analysis was an $\overline{\text{MS}}$ mass of \cite{26}

$$
\overline{m}_c(m_c) = 1.277 \pm 0.006_{\text{stat}} \pm (0.013)_{\text{syst}} \pm (0.019)_{\text{pert}} \pm (0.009)_{\alpha_s} \pm (0.002)_{\langle GG \rangle},
$$

where in the first line the first and second errors come from experimental uncorrelated and correlated uncertainties, respectively, the third error is the perturbative uncertainty, the fourth reflects the uncertainty in $\alpha_s(m_Z)$, and the last corresponds to non-perturbative effects from the gluon condensate. The central value does not differ significantly from that found in the analysis of \cite{28}, since the differences in the experiment and theoretical analyses largely cancel one another. However, the perturbative error estimate associated with the truncation of the series is an order of magnitude larger, leading to a total error which is larger by about a factor of two.

4.2 Semi-leptonic $b \rightarrow c$ decays and $|V_{cb}|$

\textit{Exclusive decays:} The determination of $|V_{cb}|$ from exclusive \textit{B} decay measurements requires the calculation of nonperturbative form factors, usually provided by lattice QCD. This conference saw an update of the Fermilab/MILC Collaborations’ lattice determination of the $B \rightarrow D^* \ell \nu$ form factor at zero recoil \cite{29}. The improvements
in the update are due mainly to increased statistics and the use of finer lattice spacings. Their new determination of the form factor is $F(1) = 0.908 \pm 0.17$ [29], and taking the latest Heavy Flavor Averaging Group (HFAG) update of $|V_{cb}| F(1) \times 10^3 = 36.04 \pm 0.52$ from experiment [30], the new value of $|V_{cb}|$ from exclusive $B \to D^* \ell \nu$ is $|V_{cb}| = 39.7(7)(7) \times 10^{-3}$ [31], where the errors are experimental and theoretical. The $F(1)$ form factor has also recently been calculated using zero recoil sum rules, yielding to $F(1) = 0.86 \pm 0.04$ and thus a larger value of $|V_{cb}|$ exclusive [32]. Improvements in the experimental error from semi-leptonic decays may eventually come from LHCb, as presented at this conference [33].

Inclusive decays: The theoretical tool for understanding inclusive $B$ decays is the Operator Product Expansion (OPE) which allows to express the transition amplitude as a double expansion in $\alpha_s$ and $\Lambda_{QCD}/m_b$. In the OPE, non-perturbative physics is expressed in terms of matrix elements of local operators, while the Wilson coefficients are perturbative. Expansions for inclusive observables in $B$ decays are available in two implementations, the kinetic [34, 35, 36] and the 1S scheme [37]. They both include terms up to $\mathcal{O}(\alpha_s^2 \beta_0)$ and $\mathcal{O}(1/m_b^3)$. A near term improvement is the implementation of the complete two-loop perturbative corrections in the kinetic scheme [38].

To obtain $|V_{cb}|$ with a precision of a few percent, the non-perturbative matrix elements are obtained from a global fit to experimental moments of inclusive $B$ observables. Currently the moments of the lepton energy and the hadronic mass in $B \to X_c \ell \nu$ and the moments of the photon energy spectrum in $B \to X_s \gamma$ are used. Recent measurements of these quantities were performed by Belle [39] and BaBar [40]. These analyses measure the inclusive spectra in hadronically tagged events and employ various techniques to correct for the distortions due to the measurement device.

The global fit to the experimental data is now performed by HFAG to combine data from different experiments and obtain optimal determinations of $|V_{cb}|$ and the $b$-quark mass $m_b$ from inclusive $b \to c$ decays. This fit uses a total of 66 measurements – 29 from BaBar, 25 from Belle and 12 from other experiments. The results in the kinetic and the 1S schemes are given in Tables 4 and 5. In both cases, the results with all moments and with $B \to X_c \ell \nu$ moments only are quoted. There is a $\sim 2\sigma$ tension between the inclusive and exclusive determinations of $|V_{cb}|$.

4.3 Semi-leptonic $b \to u$ decays and $|V_{ub}|$

Exclusive decays: The $B \to \pi \ell \nu$ decay rate is proportional to the combination $|V_{ub}|^2 f_+^2 (q^2)$, where $f_+ (q^2)$ is a nonperturbative form factor. To determine $|V_{ub}|$ from exclusive semi-leptonic decays thus requires measurements of the decay rates, and calculations of the form factor.

In the working-group session an update on the determination of the form factor using light-cone sum rules (LCSR) was given by P. Ball [41]. The LCSR calculations for the form factor are already in a mature state, and the focus was on a new cal-
Table 4: Results of the HFAG global fit in the kinetic scheme. The errors quoted are the results of the fit, where the covariance matrix includes experimental and estimated theoretical uncertainties. On $|V_{cb}|$, there are additional uncertainties from the $B$ lifetime and from an additional theoretical uncertainty of 1.4% in the expression of the semileptonic width, respectively.

| Input               | $|V_{cb}| \times 10^{-3}$ | $m_b^{\text{kin}}$ (GeV) | $\chi^2$/ndf. |
|---------------------|---------------------------|---------------------------|----------------|
| all moments         | 41.85 ± 0.42 ± 0.09 ± 0.59| 4.591 ± 0.031             | 29.7/59        |
| $X_c\ell\nu$ only  | 41.68 ± 0.44 ± 0.09 ± 0.58| 4.646 ± 0.047             | 24.2/48        |

Table 5: Results of the HFAG global fit in the 1S scheme. The errors quoted are the results of the fit, where the covariance matrix includes experimental and estimated theoretical uncertainties.

| Input               | $|V_{cb}| \times 10^{-3}$ | $m_b^{1S}$ (GeV) | $\chi^2$/ndf. |
|---------------------|---------------------------|-----------------|----------------|
| all moments         | 41.87 ± 0.25              | 4.685 ± 0.029   | 32.0/57        |
| $X_c\ell\nu$ only  | 42.31 ± 0.36              | 4.619 ± 0.047   | 24.2/46        |

culation of the $O(\alpha_s^2\beta_0)$ terms used in the perturbative part of the sum rule. The numerical effect of such corrections turns out to be small, which can be taken as an indication that the current results in LCSR \cite{42,43} are stable under higher-order radiative corrections. After the workshop, a new LCSR result appeared \cite{44}.

As sum rule calculations are subject to theoretical uncertainties which are difficult to quantify, much effort has been put into calculating these quantities using the model-independent methods of lattice QCD. While new results for the form factor $f_+(q^2)$ used in exclusive $b \to u$ transitions were not presented in this workshop, the existing results from Refs. \cite{45} and \cite{46} are used in experimental analyses.

New measurements from BaBar \cite{47,48} and Belle \cite{49} were presented at this workshop, along with updates of $|V_{ub}|$ determinations based on calculations of the form factors mentioned above. All analyses use an untagged technique, \textit{i.e.}, no requirements are made on the second $B$ meson in the event. Different methods are used to determine the values of $q^2$, however. The BaBar and Belle results for $|V_{ub}|$ are shown in Tables \ref{tab:vbub_babar} and \ref{tab:vbub_belle} respectively. Results from a model-independent determination of $|V_{ub}|$ obtained by simultaneously fitting the branching fraction data and the MILC lattice QCD form-factor after transforming to the so-called “z-parameterization” \cite{50} were also presented. The result of the fit to the BaBar data reads $|V_{ub}| = (2.95 \pm 0.31) \times 10^{-3}$, while the Belle data gives $|V_{ub}| = (3.43 \pm 0.33) \times 10^{-3}$. 

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Table 6: Values of $|V_{ub}|$ ($10^{-3}$) derived from different $B \to \pi \ell \nu$ form factor calculations and two recent BaBar analyses [47, 48].

| $q^2$ (GeV$^2$) | Ref. [47] | Ref. [48] | Average |
|----------------|---------|---------|---------|
| HPQCD          | > 16    | 3.28 ± 0.13 ± 0.15$^{+0.37}_{-0.37}$ | 3.21 ± 0.17$^{+0.55}_{-0.36}$ | 3.23 ± 0.09 ± 0.13$^{+0.37}_{-0.37}$ |
| FNAL           | > 16    | 3.14 ± 0.12 ± 0.14$^{+0.35}_{-0.29}$ | 2.95 ± 0.31 | 3.09 ± 0.08 ± 0.12$^{+0.35}_{-0.29}$ |
| LCSR           | < 12    | 3.70 ± 0.07 ± 0.08$^{+0.54}_{-0.39}$ | 3.78 ± 0.13$^{+0.55}_{-0.40}$ | 3.72 ± 0.05 ± 0.09$^{+0.54}_{-0.39}$ |

Table 7: Values for $|V_{ub}|$ extracted from the Belle data [49] using different predictions of the partial width $\Delta \zeta$.

| $q^2$ (GeV$^2$) | $\Delta \zeta$ (ps$^{-1}$) | $|V_{ub}|$ ($10^{-3}$) |
|----------------|------------------|------------------|
| HPQCD          | > 16             | 2.07 ± 0.57      | 3.55 ± 0.09 ± 0.09$^{+0.62}_{-0.41}$ |
| FNAL/MILC      | > 16             | 1.83 ± 0.50      | 3.78 ± 0.10 ± 0.10$^{+0.65}_{-0.43}$ |
| LCSR           | < 16             | 5.44 ± 1.43      | 3.64 ± 0.06 ± 0.09$^{+0.60}_{-0.40}$ |

Inclusive decays: In principle, the same methods used for inclusive semi-leptonic decays into charm quarks described above can be used to determine $|V_{ub}|$ from $B \to X_u \ell \nu$ decays. However, in this case experimental cuts are required to suppress the background from decays into charm, and for very restrictive cuts the local OPE does not apply and theory predictions are sensitive to non-perturbative shape functions. Moreover, the dependence of the partial branching fractions in such a region of phase space depends much more strongly on $m_b$ than in the total inclusive rate, so parametric uncertainties become large.

Current determinations by HFAG [51] of $|V_{ub}|$ from inclusive decays are based on theory frameworks referred to as BLNP [52], DGE [53], GGOU [54], and ADFR [55]. Basically, these methods use a different set of theoretical assumptions, but BLNP, DGE, and GGOU are linked by the fact that they in one way or another produce the local OPE result for the total rate, which is however not the case for ADFR. More details can be found, for instance, in the report of the previous CKM workshop [56].

Compared to the previous CKM workshop, theoretical progress within the BLNP framework was made in [57], which included the next-to-next-to-leading order (NNLO) perturbative corrections to the leading term in the $1/m_b$ expansion within that framework. These corrections stabilize the dependence on the perturbative matching scales, and tend to raise the value of $|V_{ub}|$ compared to the current implementation of BLNP used by HFAG, which is based on next-to-leading-order calculations. Since these
corrections are part of the OPE prediction for the triple differential decay spectrum, namely the virtual plus real emission contributions in the soft and collinear limits, they could also be included in the GGOU and DGE frameworks. Whether this would be beneficial depends on whether the NNLO contributions from hard real gluon emission, which are power suppressed and not included in [57], give significant contributions in the regions of phase space where experimental measurements are made. Further efforts in understanding the structure of such power corrections within the BLNP framework were made through the calculation of the subleading jet functions at $O(\alpha_s)$ in [58]. This set of perturbative power corrections appears in a convolution with the leading-order shape function and is suppressed by $\alpha_s/m_b$ compared to the leading term. The implementation of the subleading jet functions in a numerical analysis within the BLNP framework should be relatively straightforward, but was not yet performed.

Progress has also been made in estimating contributions from weak annihilation. Although at the level of the total rate weak annihilation can be treated within the OPE and is of the order $1/m_b^3$, its calculation at the level of differential decay spectrum is model dependent and its effect on partial decay rates used in the extraction of $|V_{ub}|$ is more uncertain. In Refs. [59, 60], recent CLEO data on semi-leptonic $D$ and $D_s$ decay [10] and heavy-quark symmetry [61] were used to estimate the potential effect of weak annihilation on extractions of $|V_{ub}|$. The main result of both of these studies is that the weak annihilation contribution on the fully inclusive measurement is only at most a 2% effect at the level of the total rate.

A further development has been the advent of the SIMBA collaboration, which aims at extracting $|V_{ub}|$ within the context of a global fit to $B \to X_s \gamma$ and $B \to X_u \ell \nu$ decays. A talk on the status of the SIMBA programme was given by F. Tackmann and is summarized in [62]. Roughly speaking, the theoretical framework underlying the approach is similar to that used in BLNP, although it differs in the treatment of non-perturbative shape functions [63]. The BLNP and GGOU approaches take a model for the shape function which is motivated by the shape of the $B \to X_s \gamma$ photon energy spectrum, and scan over many possibilities to determine uncertainties associated with this object. The intention of the SIMBA collaboration, on the other hand, is to use the available data from $B \to X_s \gamma$ and $B \to X_u \ell \nu$ decays to constrain the shape function, $|V_{ub}|$, and $m_b$ in a global fit. Preliminary results were given for a fit of the shape-function, $m_b$, and $|C_{11}^{incl}V_{tb}V_{ts}^*|$ (the normalization factor multiplying the photon energy spectrum) from the photon energy spectrum in $B \to X_s \gamma$ decays; a full study including also $B \to X_u \ell \nu$ decays is in progress. Since the goal is to extract not only the normalization of the decay rate, proportional to $|V_{ub}|$, but also its shape, determined at leading power by a single non-perturbative shape function, it would be very useful for this effort if the data on $B \to X_u \ell \nu$ decays were given in the form of differential spectra, rather than just partial decay rates for a few different choices of kinematical cuts.
Table 8: Results for $|V_{ub}| \times 10^3$ obtained with four theoretical calculations, taken from [64]. The uncertainties are experimental (i.e. sum of statistical and experimental systematic) and theoretical, respectively.

|       | BLNP         | DGE          | GGOU         | ADFR         |
|-------|--------------|--------------|--------------|--------------|
| Average | $4.30 \pm 0.16^{+0.24}_{-0.23}$ | $4.37 \pm 0.15^{+0.17}_{-0.16}$ | $4.30 \pm 0.16^{+0.13}_{-0.20}$ | $4.05 \pm 0.13^{+0.24}_{-0.21}$ |

On the experimental side, an update was given by C. Bozzi [64] on the status of measurements by Belle and Babar, and also the preliminary updates of inclusive $|V_{ub}|$ as extracted by HFAG. In addition to analyses which perform measurements in the endpoint region of the lepton energy spectrum, both collaborations also have measurements of partial rates with cuts on variables such as the hadronic invariant mass, the $q^2$ of the lepton pair, or the light-cone momentum $P_+ = E_X - |\vec{p}_X|$ of the hadronic system, which are carried out using recoil techniques [65, 66]. The preliminary results for the value of $|V_{ub}|$ extracted for a number of such measurements within the different theoretical frameworks can be found in [64]; the average over all measurements is shown in Table 8. As already mentioned, the theory predictions for the partial rates depend strongly on the heavy-quark parameters: the numbers in the table correspond to those determined by a global fit in the kinetic scheme, translated to the scheme needed by each method, where both $b \to c \ell \nu$ and $b \to s \gamma$ moments are used, giving $m_b(kin) = 4.591 \pm 0.031$ GeV, $\mu_\pi^2(kin) = 0.454 \pm 0.038$ GeV$^2$. There is an approximately 3σ discrepancy between the exclusive and inclusive determinations of $|V_{ub}|$. Further work is needed to understand this discrepancy.

**ACKNOWLEDGEMENTS**

We thank the organizers for a well-organized conference and the participants of Working Group II for helpful discussions about their work.

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