Abstract: We support the idea that the baryon, B with mass $M_B$, couples to its current with a coupling $\lambda_B^2 \sim 0.71 M_B^6$ from an analysis of magnetic moment sum rules. And we find a sum rule among the experimental magnetic moments which is independent of the parameters of QCDSR.

Keywords: QCD sumrules, magnetic moments of baryons.

(1) Azad Physics Centre, Dept. of Physics, Maulana Azad College, Calcutta 700 013, India
(2) ICTP, Trieste, Italy 341000
(3) Work supported in part by DST grant no. SP/S2/K18/96, Govt. of India,
permanent address : 1/10 Prince Golam Md. Road, Calcutta 700 026, India, email : deyjm@iascl01.vsnl.net.in.
(4) Dept. of Physics, Presidency College, Calcutta 700 073, India
It was suggested recently [1], that the coupling of the octet and the decuplet baryons
to its interpolating current in the framework of the SVZ QCDSR (see [1] for earlier
references) scales with the baryons masses with a constant \( C = 1.37 \), as follows :
\[
\lambda_B^2 \sim C \, M_B^6 \tag{1}
\]
In our notation \( \lambda_B \) is defined through the matrix element
\[
\langle 0 | J_B | B \rangle = \lambda_B \, v^{(r)}
\tag{2}
\]
where \( J_B \) is the interpolating baryon current and \( v^{(r)} \) is the usual Dirac spinor for polarization \( r \), normalized to the baryon mass \( M_B \) as :
\[
\bar{v}^{(r)} v^{(r)} = 2 M_B.
\tag{3}
\]
We call eq. (1) the scaling law, (SL in short). Very general arguments as well as a specific
model was put forward in support of this SL in [1]. The SL is able to connect QCDSR
for various quantities (for example mass, magnetic moments, transitions or differences
thereof) of different baryons. These connections, which are the common sumrules, to
differentiate from QCDSR, are to be checked with experimental numbers. In the present
paper this task is achieved for the differences in magnetic moments of three multiplets :
\((p - n), (\Sigma^+ - \Sigma^-)\) and \((\Xi^- - \Xi^0)\), an experimentally verifiable sumrule is established
among these.

This SL was checked in [2], for decuplet baryons with a constant \( C = 0.71 \), in eq. (1)
and the \( \Sigma^*\), \( \Xi^* \) and \( \Omega \) couplings fit very well. The isobar \( \Delta \) is off the curve and Lee gets an
overestimate for the \( \Delta(1232) \) mass as well, so that it is not clear which mass to use in the
abscissa of the curve given by eq. (1). He observed that “Given the great importance of a
scaling law between baryon current couplings and their masses and its phenomenological
consequences, more investigations are clearly needed to resolve the deviations.”

The results of [1] were based on QCDSR for masses which are now modified partially.
In [3], uncertainties in the predictions of QCDSR calculations have been analysed comprehensively in \( \rho \)-meson and nucleon two-point functions. Indeed, the results are pretty
disturbing for QCDSR practitioners, as Ref.[3] indicates that reducing these uncertainties
requires both a better determination of the vacuum condensates and a better determination
of the sum rules themselves. The nucleon coupling deduced in [3] using Monte-Carlo
-based analysis still deviates from the SL [2]. To investigate the scaling law further, we
look into the QCDSR for magnetic moment differences.

Thus our outlook is quite different from that of [1, 2, 3] in so far as we do not look
at mass QCDSR, but at QCDSR for magnetic moment differences among octet -baryon
ismultiplets to find support for the scaling argument of the coupling . We propose
a magnetic moment sum rule which is experimentally verifiable and does not involve
QCDSR parameters.
Our starting point is the observation that the magnetic moments, for the baryons $B$, can be written in the form [5]
\[ \mu_B = \text{constant} \left(1 + \delta_B\right) \frac{e\hbar}{2cM_B} \]
(4)

where the constant is $8/3$ for proton $p$, $\Sigma^+$ and $-4/3$ for neutron $n$, $\Sigma^-$, $\Xi^-$ and $\Xi^0$ [5].

The $\delta_B$ are small numbers and for the differences $(\delta_p - \delta_n)$, $(\delta_{\Sigma^+} - \delta_{\Sigma^-})$ and $(\delta_{\Xi^-} - \delta_{\Xi^0})$ the QCD sumrules simplify very much [6]. The susceptibility terms cancel out. For completeness we write, as an example, the QCDSR for the nucleon (i.e. proton - neutron) from [6].

\[ \beta_N^2[\delta_p - \delta_n + (A_p - A_n)M^2].exp(-M_N^2/M^2) = \\
(7/2 - 2)\frac{M^2b}{192L^{4/9}g} + (-1/2 - 1)\frac{M^2b}{288L^4} + (-1/2 - 4)a^2L^4/72 \]
(5)

with
\[ \lambda_B^2 = 8\beta_B^2 \]
\[ a = -2\pi^2 <\bar{q}q> \]
\[ b = <g^2G^2> \]
\[ L = \frac{\ln(M^2/\Lambda_{QCD}^2)}{\ln(\mu^2/\Lambda_{QCD}^2)} \]
\[ \xi = \ln\left(\frac{M^2}{\Lambda^2}\right) - 1 - \gamma \]

where $\Lambda$ the cut-off, $\mu$ the renormalization point, $\Lambda_{QCD}$ and $\gamma$, the Euler-Mascheroni constant are 0.5 GeV, 0.5 GeV, 0.1 GeV and 0.577 respectively. Similar equations are given in [6] for $\Sigma$ and $\Xi$.

We simplify the problem further by writing down the equations for $\delta_B - \delta_B'$ after operating with $(1 - M^2 \frac{\partial}{\partial M^2})$, where $M$ is the unknown of the QCDSR, the Borel parameter. The last operation, first used by Ioffe and Smilga, [7], gets rid of the intermediate state contributions marked $A_B, A_B'$ in [6]. Abbreviating by
\[ DL = \frac{4}{9}ln\left(\frac{\mu^2}{\Lambda_{QCD}^2}\right) \]
(6)
and
\[ E = exp\left(\frac{M_B^2}{M^2}\right) \]
(7)
the equations are:

\[ f_1(M^2) = \frac{M^2}{288}(\xi \frac{M_B^2}{M^2} E L^{-\frac{4}{9}} - \frac{E}{L^{\frac{4}{9}}} + \xi L^{\frac{16}{9}}) \quad (8) \]

\[ f_2(M^2) = \left[ \frac{M_B^2}{M^2} E.L^{-\frac{4}{9}} + D L.E.L^{-\frac{16}{9}} \right] \frac{M^2}{8} \cdot s \quad (9) \]

\[ f_3(M^2) = \left[ \frac{M_B^2}{M^2} E.L^{-\frac{4}{9}} + D L.E.L^{-\frac{16}{9}} \right] M^2 \cdot \frac{p}{192} \quad (10) \]

\[ f_4(M^2) = L^{\frac{4}{9}}(E + \frac{M_B^2}{M^2} E - DL.E.L) \cdot \frac{r}{72} \quad (11) \]

\[ (\delta_B - \delta_{B'}) = [f_1(M^2).b + f_2(M^2)m_s.a + f_3(M^2).b + f_4(M^2).a^2] / \beta_B^2 \quad (12) \]

The coefficients \( p, q, r, s \) are given in Table 1.

Thus the difference of magnetic moments are simple functions which only depend on the condensate values \( a \) and \( b \) and the contribution of various higher order terms and the intermediate excitations are easily gotten rid of. Of course there still are too many parameters since the various couplings \( \beta_B \) appears in the RHS of each equation. We suggest that one should use the SL to explore if the \( \beta_B \) can be expressed in terms of one another. And thus with three experimental mass differences one can determine the coefficient of the cubic SL connecting the \( \beta_B \), and \( a \) and \( b \).

We reemphasize that in this paper we find that the difference of magnetic moments is an ideal ground for searching for the truth of SL rather than mass sum rules which are riddled with many problems.

The next point we wish to stress is that over the years the value of the condensates \( a \) and \( b \) have changed a lot and now carry large uncertainties. Thus \[3, 2\] quote the values \( a = (0.52 \pm 0.05) \text{ GeV}^3 \) and \( b = (1.2 \pm 0.6) \text{ GeV}^4 \). In the following, we will attempt to find some suitable values of these parameters from magnetic moment differences.

RHS of eq. (12) is a function of the Borel parameter, \( M \). This is shown in Fig. 1 and the RHS matches the experimental numbers only at \( M = M_B \). We allow the \( \beta_B \) in Table 2 to have a small scatter from eq. (1) with \( C = 0.71 \), and find that with \( a \sim 0.47 \text{ GeV}^3 \) and \( b \sim 1.7 \text{ GeV}^4 \) we can fit the three magnetic moment differences. Next we proceed to find unique solutions of \( b \) and \( a \).

Ideally one should be able to work with any value of the Borel parameter \( M \) in QCDSR. But from Fig. 1 we see that this is not possible, the QCDSR curves for the differences in \( \delta_B \) will cross the experimental straight line only at one point. We accept the situation as such and only demand that the value of \( \delta_B - \delta_{B'} \) fits experimental numbers (Table 2) at \( M = M_B \). We observe from eq. (9) and Table 1 that \( m_s.a \) occurs only in \( \Xi : \)

\[ 0.42215 \cdot b - 4.8837 \cdot a^2 = (\delta_p - \delta_n) \]
Figure 1: Experimental $(\delta_B - \delta_{B'})$ are shown as a solid line, dotted line and crosses, the topmost for $B = \Sigma$, the middle one for $B = N$ and the lowest one for $B = \Xi$. The QCDSR results, as a function of Borel parameter $M$, crosses at the respective $M = M_B$.

\[-0.13416 b + 1.294a^2 = (\delta_{\Sigma^+} - \delta_{\Sigma^-})\]

\[0.235827 b - 1.613178 a^2 - 8.228(m_s a) = (\delta_{\Xi^-} - \delta_{\Xi^0}) \quad (13)\]

On solving first two of these equations one gets a set of values of $b$ and $a^2$, which by our choice of the couplings gives consistent results:

\[b = 1.695 GeV^4 \quad (14)\]
\[a = 0.475 GeV^3. \quad (15)\]

These values of $a$ and $b$ and the equation (13) combined give us the strange quark mass $m_s = 169.8$ MeV which is reasonable.

Thus we conclude that to get results for magnetic moments, consistent with experiment, at least for $M = M_B$, one must choose the parameters $a$ and $b$ which are close to the extreme limits of the error range given by [3, 2].

We now proceed to investigate the SL for decuplet baryons with the values of $a$ and $b$ given above. We find in Table 3 that in particular the isobar $\Delta(1232)$ has a much significantly smaller $\lambda_\Delta$ given by:

\[\lambda_\Delta^2 = \frac{4}{3} aE_1(M^2 \frac{M^2}{M_\Delta} L^\frac{3}{2} M^4 - \frac{2}{3} aE_0(M^2 \frac{M^2}{M_\Delta}) m_0^2 L^\frac{1}{2} M^2 - \frac{b}{18} aL^\frac{1}{2}|e_M^2 M^2 | / M_\Delta \quad (16)\]

where $E_n(x) = 1 - e^{-\Sigma x^n / n!}$, and $m_0^2 = 0.72 GeV^2$. Using the experimental mass, $\lambda_\Delta$ fits into the SL very well in Fig. 2. Reversing the argument one can state that if one were to trust the SL we advocate, the isobar mass problem may be in better shape. Note
that Lee [2] uses a coupling square in his tables which must be multiplied by a factor of 2 to compare with ours.

Now comes our central result: a sumrule which is independent of the steps of QCDSR through which it was derived.

We have shown that to get results for magnetic moments, (or at least the difference between the moments of two baryons in the same multiplet, consistent with experiment at \( M = M_B \), one must choose the parameters \( a \) and \( b \) which are close to the extreme limits of the error range given by [3, 2]. But then, we have these parameters given in terms of the differences \((\delta_p - \delta_n)\), \((\delta_{\Sigma^+} - \delta_{\Sigma^-})\) and \((\delta_{\Xi^-} - \delta_{\Xi^0})\). We can solve equations (13) either for \( a \) or \( b \) to get a consistent sumrule:

\[
(\delta_p - \delta_n) = -0.8153 - 1.2273 (\delta_{\Xi^-} - \delta_{\Xi^0}) - 5.3045 (\delta_{\Sigma^+} - \delta_{\Sigma^-}).
\] (17)

Note that magnetic moment experiments are being refined all the time and that \( \Sigma \) and \( \Xi \) magnetic moments were revised recently and are different from the ones quoted in [3, 3]. It will be interesting to see if the above sumrule will stand the test of time.

In summary we have used the scaling of the baryon coupling to its current to predict a sumrule among difference of octet baryon magnetic moments. We also indicate how the SL can be used to ameliorate other difficulties of QCDSR technique.

We believe that the SLs can be extended to charmed and beauty baryons. It may be possible to establish connections between decays from baryons and check them with experiments.

The authors are grateful for several e-mails from Drs. Lee and Leinweber. One of the authors (MD) wishes to thank the UNESCO, IAEA and ICTP for hospitality at Trieste, Italy.
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Table 1: Coefficients p, q, r and s

| System  | p  | q  | r  | s  |
|---------|----|----|----|----|
| p - n   | 1.5| -1.5| -4.5| 0  |
| $\Sigma^+ - \Sigma^-$| -1.5| 1.5| 4.5| 0  |
| $\Xi^+ - \Xi^0$   | 3  | -3  | -9 -18 f | -6  |

Table 2: Experimental numbers $\delta_B - \delta_{B'}$ for octet baryons along with our squared couplings $\beta_B^2$

| $M_B (GeV)$ | $\beta_B^2$ | System      | $\delta_B - \delta_{B'}$ |
|-------------|--------------|-------------|--------------------------|
| 0.940       | 0.0766       | p - n       | -0.388                   |
| 1.189       | 0.3037       | $\Sigma^+ - \Sigma^-$| 0.065                   |
| 1.315       | 0.3776       | $\Xi^+ - \Xi^0$ | -0.629                   |

Table 3: Decuplet couplings

| $M_B (GeV)$ | continuum $w$ (GeV) | Our $\lambda_B$ | $\lambda_B$ from [2] |
|-------------|---------------------|-----------------|----------------------|
| 1.232       | 1.65                | 1.458           | 2.26 ± 0.89          |
| 1.384       | 1.80                | 2.492           | 2.83 ± 0.32          |
| 1.533       | 2.0                 | 4.122           | 4.32 ± 0.47          |
| 1.672       | 2.3                 | 7.081           | 7.19 ± 0.75          |