Prediction of CME as an inverse problem

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Abstract. Magnetic reconnection has been invoked as the energy release mechanism for solar flares and coronal mass ejections release from the sun’s lower atmosphere. Theoretical models of magnetic reconnection have been traditionally developed within the frame work of magneto-hydrodynamics. However, in low density astrophysical plasmas such as in solar atmosphere, kinetic effects are expected to play a significant role. Here we have developed a kinematic theory for the prediction of CME which is a novel approach. Based on our data analysis we have assumed a general quadratic expression for the distribution function in velocity space and have looked for the complete class of self consistent magnetic fields, potential fields as well as density distributions. In our problem the occurrence of the complex roots reveal the existence of instability which can be interpreted as the CME.

1. Introduction
Coronal mass ejections (CMEs) are massive expulsions of magnetized plasma from the solar atmosphere due to a destabilization of the coronal magnetic configuration. As a consequence of this ejection, CMEs can form confined magnetic structures with both extremes of magnetic field lines connected to the solar surface extending far away from the sun into the solar wind while the coronal magnetic field is restructured in the lower corona. Depending on the solar cyclic phase about 3.5 CMEs are ejected per day during a solar maximum (minimum) and the ejected mass can reach values as high as $10^{15} - 10^{16}$ gm [1]. Transient interplanetary coronal mass ejections (ICMEs) have their origin in instability of the solar coronal field. The magnetic field ejected from the sun carries the magnetic helicity of its original field, and since helicity is not dissipated in the corona nor in the heliosphere, helicity will be contained in the ICME [2].

CMEs can carry greater than $10^{32}$ ergs of kinetic energy, so the most obvious question in studying these phenomena is where the energy comes from. Energy storage models are generally considered to be the most likely candidates at the present time. Highly non-potential coronal magnetic fields in active regions have been observed frequently, indicating that there is more than enough magnetic energy to drive coronal eruptions. This energy may be stored by photospheric motions shearing and twisting the coronal field, or the magnetic fields may already be twisted when they emerge from below the photosphere [3]. Despite years of study we still don’t understand the key aspects of CMEs: specifically how are they initiated in the solar corona, and how they evolve to produce the signatures that are measured with
interplanetary spacecraft. There is a lot of literature on CMEs in general and CME modeling in particular [4, 5, & 6].

In this paper we have developed a kinematic theory for the prediction of CME which is a novel approach. Based on our data analysis we have assumed a general quadratic expression for the distribution function in velocity space and have looked for the complete class of self consistent magnetic fields, potential fields as well as density distributions. In our problem the occurrence of the complex roots reveal the existence of instability which can be interpreted as the CME.

The paper is organised as follows. In Sec. 2 Data analysis and the angular distribution of the observed speeds of coronal mass ejections (CMEs), are discussed in detail. In Sec.3 we look at it as an inverse problem as in stellar dynamics [7, 8, 9, 10]. The two imaginary roots obtained from the cubic quadratic equation reveal the existence of instability which can be interpreted as the cause of CME. We also found that the ellipsoidal distribution of CME speeds arises due to the presence of magnetic fields in CMEs. Sec. 4 will summarize the present work.

2. Data Analysis

Over the past 12 years, coronal mass ejection have been routinely detected by visual inspection of each image from the large angle spectrometric coronagraph (LASCO) on board SOHO. Event catalogues have been assembled continuously and are made publicly available (http://cdaw.gsfc.nasa.gov/CME_list). These catalogues are used as a reference and form a valuable resource for our study. Using these catalogues we have studied the angular distribution of the observed speeds of coronal mass ejections of solar cycle 23, and we realize that instead of being isotropic it is elliptical in nature. Using this ellipsoidal velocity distribution we ask the question “If this distribution takes place, what should be the interaction within the system to generate this distribution”. Hence we call this as an inverse problem. Yurchyshyn et al studied the log normal distribution of the speeds of CMEs and they found the presence of non linear interactions and multiplicative processes in a system of many magnetic flux loops [11].
3. The Proposed Model: On the basis of inverse problem

The equations of the problem basically will be

$$\frac{df}{dt} + (V \cdot \nabla) f + \left[ \frac{F}{m} + \nabla \Omega + \frac{e}{mc} (V \times H) \right] \cdot \nabla f = 0$$  \hspace{1cm} (3.1)

These equations represent the equations giving the distribution function of an electron gas moving in a background potential $\Omega$ provided by the positive ions. $F$ is an external force and $H$ is the magnetic field generated by the current. Based on our data analysis the angular distribution of the observed speeds of coronal mass ejections can be considered as ellipsoidal in nature. One could, therefore, assume that the ellipsoidal distribution plays an important role in the dynamics of coronal mass ejections (CMEs). This ellipsoidal surface on which the tips of the velocity vectors lie can give some information regarding the forces that exist in the system. Therefore we assume that the distribution function is an arbitrary function of a general quadratic surface in the velocity space or

$$f = f(\psi) \text{ Where}$$

$$\psi = \sum a_{ij} \psi_{ij}$$  \hspace{1cm} (3.2)

The exact structure of this function is not known. ‘Hence equation (3.1)’ now reads as

$$\frac{df}{dt} = \frac{df}{d\psi} \frac{d\psi}{dt} = 0$$  \hspace{1cm} (3.3)

We ask what are the possible velocity and magnetic fields which would admit $\psi$ as an integral of motion. If we consider a non zero $df/dt$, the above equation would demand that

$$\frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + (V \cdot \nabla) f + \left[ \frac{F}{m} + \nabla \Omega + \frac{e}{mc} (V \times H) \right] \cdot \nabla f = 0$$  \hspace{1cm} (3.4)

In this paper we are working in $\mu$ space (molecule space) and we shall formulate the problem in a six-dimensional orthogonal curvilinear coordinate system $\lambda$, $\mu$, $\nu$, $U$, $V$ and $W$, the first three being space coordinates and the remaining velocity coordinates.

We can then write ‘equation (3.5)’ as

$$\frac{\partial \psi}{\partial t} + \frac{U}{P} \frac{\partial \psi}{\partial \lambda} + \frac{V}{Q} \frac{\partial \psi}{\partial \mu} + \frac{W}{R} \frac{\partial \psi}{\partial \nu} + \frac{ \partial \psi}{\partial U} + \frac{\partial \psi}{\partial V} + \frac{\partial \psi}{\partial W} = 0$$  \hspace{1cm} (3.6)

We shall now define $\psi$ in a local co-ordinate system which is moving with a mean velocity $U_0, V_0, W_0$ and is of the generalized Schwarzschild type, i.e.

$$\psi(x, y, z, U, V, W, t) = \psi(Q + \sigma)$$  \hspace{1cm} (3.7)

Then $\psi$ can be written as

$$\psi = a(U - U_0)^2 + b(V - V_0)^2 + c(W - W_0)^2 + 2f(V - V_0)(W - W_0) + 2g(W - W_0)(U - U_0) + 2h(U - U_0)(V - V_0) + \sigma$$  \hspace{1cm} (3.8)
In the present case the angular distribution of the speeds of coronal mass ejections (CMEs) can be considered as ellipsoidal in nature and $U_0, V_0, W_0$ represents the centroid velocity of CME. For an ellipsoidal distribution $f = g = h = 0$. Hence

$$\psi = a(U - U_0)^2 + b(V - V_0)^2 + c(W - W_0)^2 + \sigma \tag{3.9}$$

'equation (3.9)' can then be written as

$$\psi = aU^2 + bV^2 + cW^2 - 2\Delta_1U - 2\Delta_2V - 2\Delta_3W - \chi \tag{3.10}$$

Where

$$\Delta_1 = aU_0 \quad \Delta_2 = bV_0 \quad \Delta_3 = cW_0$$

$$\chi = aU_0^2 + bV_0^2 + cW_0^2 + \sigma \tag{3.11}$$

In the above expressions $a, b, c, \Delta_1, \Delta_2, \Delta_3$ and $\chi$ are all functions of space and time which are to be determined. It may be noted that if we know $\Delta_1, \Delta_2, \Delta_3$ then we can get the average velocity of the local centroid as

$$\left( U_0, V_0, W_0 \right) = M^{-1} \left( \Delta_1, \Delta_2, \Delta_3 \right) \tag{3.12}$$

Where $M$ is the Symmetric matrix defined by

$$M = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} \tag{3.13}$$

We can express the acceleration term in 'equation (3.6)' from the Lagrangian equation of motion in classical electrodynamics.

$$\frac{d}{dt} \left[ \frac{\partial L}{\partial q_i} \right] - \frac{\partial L}{\partial q_i} = 0 \tag{3.14}$$

Where the Lagrangian of the system is given by

$$L = T - \Omega + \frac{e}{c} (A \cdot V) \tag{3.15}$$

$A$ being the vector potential, $T$ the kinetic energy, and $\Omega$ the potential energy. 'Substituting (3.15) in (3.14)' we get

$$\dot{U} = \frac{1}{2P} \left[ \frac{V^2}{Q^2} \frac{\partial Q^2}{\partial \lambda} + \frac{W^2}{R^2} \frac{\partial R^2}{\partial \lambda} \right] - \frac{UV}{PQ} \frac{\partial P^2}{\partial \mu} - \frac{UW}{PR} \frac{\partial P^2}{\partial v} - \frac{e}{P} \frac{\partial \Omega}{\partial \lambda} - \frac{e}{c} \frac{\partial A_i}{\partial t}$$

$$- \frac{e}{c} \frac{V}{PQ} \frac{\partial}{\partial \lambda} (A_iQ) - \frac{\partial}{\partial \mu} (A_iP) - \frac{e}{c} \frac{W}{PR} \frac{\partial}{\partial \lambda} (A_iR) - \frac{\partial}{\partial v} (A_iP)$$
\[ \dot{V} = \frac{1}{2Q} \left[ \frac{U^2}{P^2} \frac{\partial P^2}{\partial \mu} + \frac{W^2}{R^2} \frac{\partial R^2}{\partial \mu} - \frac{UV}{PQ} \frac{\partial Q^2}{\partial \lambda} - \frac{VW}{QR} \frac{\partial Q^2}{\partial \nu} \right] - \frac{e}{c} \frac{\partial}{\partial \mu} (A_\mu) - \frac{\partial}{\partial \lambda} (A_\lambda) \frac{\partial}{\partial \nu} (A_\nu) \] 

(3.16)

\[ \dot{W} = \frac{1}{2R} \left[ \frac{U^2}{P^2} \frac{\partial P^2}{\partial \nu} + \frac{V^2}{Q^2} \frac{\partial Q^2}{\partial \lambda} - \frac{UV}{PQ} \frac{\partial Q^2}{\partial \lambda} - \frac{VW}{QR} \frac{\partial R^2}{\partial \mu} \right] - \frac{e}{c} \frac{\partial}{\partial \mu} (A_\mu) - \frac{\partial}{\partial \lambda} (A_\lambda) \frac{\partial}{\partial \nu} (A_\nu) \] 

In equation [3.16] the acceleration terms are all functions of \( \lambda, \mu, \nu, U, V, \) and \( W. \) If we substitute \( \dot{U}, \dot{V}, \dot{W} \) in equation (36) and collect cubic, square, linear and independent terms in and set them equal to zero, we get a set of 19 equations. For convenience we have expressed these equations in cartesian coordinates. These are arranged below in four groups as in [10]

\[ \begin{align*}
\frac{\partial a}{\partial x} &= 0, \frac{\partial b}{\partial y} = 0, \frac{\partial c}{\partial z} = 0, \frac{\partial a}{\partial y} = 0, \frac{\partial a}{\partial z} = 0, \frac{\partial b}{\partial x} = 0, \frac{\partial b}{\partial z} = 0, \frac{\partial c}{\partial x} = 0, \frac{\partial c}{\partial y} = 0
\end{align*} \] 

(3.17a)

\[ \begin{align*}
\frac{\partial \Delta_1}{\partial x} - \frac{1}{2} \frac{\partial a}{\partial t} &= 0, \frac{\partial \Delta_2}{\partial y} - \frac{1}{2} \frac{\partial b}{\partial t} = 0, \frac{\partial \Delta_3}{\partial z} - \frac{1}{2} \frac{\partial c}{\partial t} = 0
\end{align*} \] 

(3.17b)

\[ \begin{align*}
\frac{\partial \Delta_1}{\partial y} + \frac{\partial \Delta_1}{\partial c} &= \frac{e}{c} (b-a) H_z \\
\frac{\partial \Delta_2}{\partial x} + \frac{\partial \Delta_2}{\partial c} &= \frac{e}{c} (a-c) H_x \\
\frac{\partial \Delta_3}{\partial y} + \frac{\partial \Delta_3}{\partial c} &= \frac{e}{c} (c-b) H_x
\end{align*} \] 

(3.17c)

\[ \begin{align*}
a \frac{\partial \Omega}{\partial x} + \frac{e}{c} (\Delta_1 H_x - \Delta_2 H_x) &= -\frac{\partial \Delta_1}{\partial t} + \frac{1}{2} \frac{\partial \chi}{\partial x} \\
b \frac{\partial \Omega}{\partial y} + \frac{e}{c} (\Delta_1 H_y - \Delta_2 H_y) &= -\frac{\partial \Delta_2}{\partial t} + \frac{1}{2} \frac{\partial \chi}{\partial y} \\
c \frac{\partial \Omega}{\partial z} + \frac{e}{c} (\Delta_1 H_z - \Delta_2 H_z) &= -\frac{\partial \Delta_3}{\partial t} + \frac{1}{2} \frac{\partial \chi}{\partial z}
\end{align*} \] 

(3.17d)

In the above set of equations \( H \) is written for \( \text{curl} \ A. \) Some of the features of the above equations are worth noting. The four classifications come from the coefficient of cubic, square, linear and independent terms in velocities. ‘Equations (3.17)’ are equations dealing with the field variables, such as the axes of
the ellipsoid, the mean centroid velocity, the potential and magnetic field. In the first set of equations, time derivatives do not appear at all. The second group of six equations involves the $\Delta$'s, time derivatives of the coefficients of velocity ellipsoid and components of $H$. These equations give the dependence of $\Delta$'s on the space coordinates. The last group of four equations is of a nature different from the rest and lead to six other integrability conditions.

'Equation (3.17b)' gives the six components of a symmetric tensor. In the matrix form 'equation (3.17b)' can be written as

\[
\begin{pmatrix}
\frac{\partial \Delta_1}{\partial x} + \frac{1}{2} \frac{\partial a}{\partial t} + \frac{\partial \Delta_2}{\partial y} + \frac{\partial \Delta_3}{\partial z} \\
\frac{\partial \Delta_2}{\partial x} + \frac{\partial \Delta_1}{\partial y} - \frac{1}{2} \frac{\partial b}{\partial t} + \frac{\partial \Delta_3}{\partial z} \\
\frac{\partial \Delta_3}{\partial x} + \frac{\partial \Delta_1}{\partial z} - \frac{1}{2} \frac{\partial c}{\partial t} + \frac{\partial \Delta_2}{\partial y}
\end{pmatrix}
\begin{pmatrix}
\frac{\partial \Delta_1}{\partial t} \\
\frac{\partial \Delta_2}{\partial t} \\
\frac{\partial \Delta_3}{\partial t}
\end{pmatrix}
= \begin{pmatrix}
0 & \frac{e}{c}(b-a)H_z & \frac{e}{c}(a-c)H_y \\
\frac{e}{c}(b-a)H_z & 0 & \frac{e}{c}(c-b)H_x \\
\frac{e}{c}(a-c)H_y & \frac{e}{c}(c-b)H_x & 0
\end{pmatrix}
\]

(3.18)

\[
\begin{pmatrix}
\frac{\partial \Delta_1}{\partial x} \\
\frac{\partial \Delta_2}{\partial y} \\
\frac{\partial \Delta_3}{\partial z}
\end{pmatrix}
= \begin{pmatrix}
\frac{\partial \Delta_1}{\partial x} + \frac{\partial \Delta_3}{\partial y} + \frac{\partial \Delta_2}{\partial z} \\
\frac{\partial \Delta_2}{\partial x} + \frac{\partial \Delta_1}{\partial y} + \frac{\partial \Delta_3}{\partial z} \\
\frac{\partial \Delta_3}{\partial x} + \frac{\partial \Delta_1}{\partial y} + \frac{\partial \Delta_2}{\partial z}
\end{pmatrix}
\]

(3.19)

'As per the equation (3.19)' it is clear that the magnetic field will generate the diagonal vectors and the off diagonal vectors. When we add the diagonal vectors we get the divergence condition of the vector $\Delta$.

\[
\frac{\partial}{\partial t}(a+b+c) = 2\nabla \cdot \Delta
\]

(3.20)

Where $\Delta$ is taken as a vector with components $\Delta_1, \Delta_2, \Delta_3$.

Hence the two vector field's $\Delta$ and $H$ can be expressed in terms of poloidal and toroidal vectors.
Thus the elliptical distribution of the speeds CME obtained from our analysis clearly indicates that magnetic field has a greater role in the dynamics of CME. If this distribution were spherical, clearly $a=b=c$ [7], then the magnetic field components vanishes.

Now we shall diagonalise the ‘matrix (3.18)’ and solve, we get a cubic quadratic equation.

$$\lambda^3 - \frac{e^2}{c^2} \left( (c-b)^2 H_z^2 + (a-c)^2 H_x^2 + (b-a)^2 H_y^2 \right) \lambda - 2 \frac{e^3}{c^3} (b-a)(c-b)(a-c) H_xH_yH_z = 0 \quad (3.21)$$

This equation is of the form

$$\lambda^3 + \alpha \lambda + \beta = 0 \quad (3.22)$$

‘Comparing equations (3.22) with (3.21)’

$$\alpha = -\frac{e^2}{c^2} \left( (b-a)^2 H_z^2 + (c-b)^2 H_x^2 + (a-c)^2 H_y^2 \right) \quad (3.23)$$

$$\beta = -2 \frac{e^3}{c^3} (b-a)(c-b)(a-c) H_xH_yH_z \quad (3.24)$$

If $\beta^2 + \frac{4\alpha^3}{27} > 0$, one root is real and two roots are imaginary

If $\beta^2 + \frac{4\alpha^3}{27} = 0$, three roots are real of which two are equal

If $\beta^2 + \frac{4\alpha^3}{27} < 0$, three roots are real and distinct

Since $a > b > c$ in the case of elliptical distribution, one root is real and other two roots are imaginary which reveals the existence of an instability inside magnetic flux tubes. Instability arising inside the flux tubes may thus interpreted as one of the causes of coronal mass ejections

‘Equations (3.17c) & (3.17d)’ can be written as

$$M \nabla \Omega + \frac{e}{mc} H \times \Delta = -\frac{\partial \Lambda}{\partial t} + \frac{1}{2} \nabla \chi \quad (3.25)$$

$$\Delta \cdot \nabla \Omega = -\frac{1}{2} \frac{\partial \chi}{\partial t} \quad (3.26)$$

Substituting the values of $H \times \Delta$ & $\frac{\partial \Lambda}{\partial t}$ in ‘equation (3.25)’ we get

$$\frac{dU_0}{dt} = \frac{b}{a} \omega V_0 \quad \& \quad \frac{dV_0}{dt} = -\frac{a}{b} \omega U_0 \quad (3.27)$$

Where $\frac{b}{a} = \varepsilon$, the eccentricity of the ellipse and is a positive definite quantity. It should be between 0 & 1

and $U_0, V_0, W_0$ represents the centroid velocity of CME

‘From equations (3.27)’

$$\frac{d^2 U_0}{dt^2} = -\frac{1}{\varepsilon} \frac{d \varepsilon}{dt} \left( \frac{d U_0}{dt} + \omega^2 U_0 \right) = 0 \quad (3.28)$$

Therefore it is assume that
\[ \varepsilon = \cos^2(\delta t) \]  

Substituting (3.29) in equation (3.28) we get

\[ \frac{d^2U_0}{dt^2} + 2\delta \tan(\delta t) \frac{dU_0}{dt} + \omega^2U_0 = 0 \]  

This is the equation of a damped harmonic oscillator and which indicates the presence of magnetic viscosity.

4. Results and Discussion

We have studied the angular distribution of observed speeds of coronal mass ejections of solar cycle 23. We found that the envelope of these distributions is ellipsoidal in nature and hence we look at it as an inverse problem. As in [8, 9] we got nineteen partial differential equations which break up into four distinct set of equations. The first group of nine equations involves only the coefficients of velocity ellipsoid \( (a, b, c) \). Further these equations do not involve any differentiations with respect to time. These nine equations give the dependence of the coefficients of the velocity ellipsoid on the space coordinates. The second group of six equations involves the \( \Delta \)'s, time derivatives of the coefficients of velocity ellipsoid and components of magnetic field \( H \). These equations give the dependence of \( \Delta \)'s on the space coordinates. We can represent these six equations in the form of a matrix, then diagonalise and solve we get a cubic quadratic equation. Since \( a > b > c \), one root is real and two roots are imaginary, these imaginary roots confirm the existence of instability in CMEs. [3.17b] gives the six components of a symmetric tensor and from this it is clear that the magnetic field will generate the diagonal vectors and off diagonal vectors. Thus the elliptical distribution of CME speeds obtained from our data analysis clearly indicates that magnetic field has a greater role in the dynamics of CME. The last group of four equations is of a nature different from the rest and lead to the equation of a damped harmonic oscillator, which indicates the presence of magnetic viscosity.

Future Work

We propose to perturb the flux tubes and are trying to find out the mechanism of ejection.

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