Numerical Investigation of a New Single Vortex Generation Technique

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Abstract. In this work a new single vortex generation method based on the flow past a rotating cylinder is proposed. Two dimensional numerical simulation for a laminar flow at Re = 100 is conducted to investigate this method. An impulsive drop of the rotation speed appeared to trigger a vortex roll up process; one single vortex is growing behind the cylinder and detached from it. The process is studied for different rotation drop rates and several final rotation rates. Vortex characteristics are computed and compared for different cases. Increasing rotation drop rate generates a more rapidly growing vortex. Its trajectory is closer to the geometric centreline and its size and intensity are higher. This new vortex generation method allows the investigation of new experimental configurations for studying vortex flame interaction.

1. Introduction
Flame vortex interaction plays an important role in understanding basic mechanisms in turbulent combustion and combustion instability. The simple configuration of one vortex or a pair of vortices interacting with a premixed or diffused flame helps understanding characteristics like flame structure, flame instabilities, mixing, ignition and extinction of more complex combustion configurations observed in practical applications such IC engines and turbine combustors.

Flame vortex interaction has been extensively studied in the past years [1]. Experimental studies have provided a considerable amount of data in a wide variety of geometries using several dynamic methods. For all these experimental studies either vortex rings [2] [3] [4], vortex dipoles, Van Karman vortex street [5] or jet flows [6] [7] were used. These configurations gave a very important advancement in understanding flame/vortex dynamics; however some limitations exist rising the need to more improvements. In fact, all those experiments were limited by flame stability phenomena, reproducibility of the interaction, optical access, and quality of the vortical structure. For the last point, whereas the ideal and simple vortical structure seems to be a single coherent large scale two-dimensional span wise vortex with a well defined size and intensity, none of the configurations presented reached that ideal pattern. Other generation techniques could be investigated to reach this ideal structure with less considerations and parasitic phenomena.

One idea proposed in this paper is to modify the Van Karman Street shedded from a cylinder to generate a single two-dimensional vortex. The modification of the Van Karman Street by rotating the cylinder and suddenly decrease its rotation speed seems to be a potential configuration to reach that goal.
Flow past rotating cylinder attracted the attention of scientists few centuries ago and it was extensively studied since then. The rotation rate \( \alpha \) defined as the ratio of tangential cylinder velocity to the uniform flow velocity adds another control parameter to the flow, which, in the case of a Van Karman street, is only governed by Reynolds number. The first relevant works concerning flow past rotating cylinder were conducted principally by Prandtl [8], several authors pursued his path [9] [10] [11] [12] and agreed on some very attractive features of this flow: Flow topology is affected by the cylinder’s rotation. The wake behind the cylinder is modified and deflected towards the rotation direction; it shrinks gradually with rotation rate been increased. The strengths of the upstream and downstream vortices are not equal and The Von Karman Street is suppressed for a critical value that depends on the Reynolds number. For low Reynolds numbers, not only one steady state exists but actually two, with a narrow unsteady region between them. This unsteady region is featured by a one side single vortex shedding with a remarkably smaller frequency [13] and [14]. The exact point of onset being dependent on Reynolds number: with increasing Re the second shedding mode appeared at progressively smaller values of \( \alpha \) but will have a narrower region between the two steady states [15]. The two steady state regions present an interesting characteristic for our study. Because we are focusing on generating a single vortex, selected rotation rates must be inside these two regions. If not, the vortex shedding caused by the drop of the rotation speed (if any) will be accompanied by the vortex shedding of the unsteady state.

In this paper will numerically investigate the applicability of our idea to generate a single spanwise vortex in the laminar regime. The results will be the starting point of a set of experimental tests planned in the future.

2. Methodology
We will numerically study the effect of a sudden drop on the vortex generation process for a laminar flow. Unsteady two-dimensional laminar flow of a viscous incompressible fluid past a rotating cylinder is considered.

2.1. Governing equations
The flow is governed by the Navier-Stokes equations. The non-dimensional equations are given below:

\[
\frac{\partial U}{\partial \tau} + (U \cdot \nabla)U = -\nabla P + \frac{1}{\text{Re}} \nabla^2 U
\]

(1)

\[
\nabla \cdot U = 0
\]

(2)

Where \( U \) is the non-dimensional flow velocity vector in the Cartesian coordinate system. \( \tau \) is the non-dimensional time defined as \( \tau = tu_\infty/D \), \( P \) is the non-dimensional static pressure and \( \text{Re} \) is the Reynolds number of the flow given by \( \text{Re} = Du_\infty/\gamma \) where \( D \) is the diameter of the cylinder, \( u_\infty \) is the free stream inlet velocity and \( \gamma \) is the kinematic viscosity of the fluid. Non-dimensional rotational rate \( \alpha \) is defined as \( \alpha = \omega D/u_\infty \), where \( \omega \) is the angular velocity of the rotating cylinder.

The commercial computational fluid dynamics (CFD) code ANSYS Fluent (version 15.0) is used to simulate the flow. A control-volume method is applied by the software to integrate the equations of motion. A second order upwind scheme is used to discretize the convective term in the momentum equations. SIMPLC (Semi-implicit method for pressure-linked equations Consistent algorithm is selected as the pressure-velocity coupling scheme. Finally, the time integration of the unsteady momentum equations is performed using a second-order approximation. The simulation is run with the initial rotation rate until a steady state is reached. The rotation rate is then impulsively reduced and the flow behavior is observed. To monitor the simulation evolution, Lift and drag coefficients along with the residuals are observed. A convergence criterion of \( 10^{-6} \) is used for continuity and \( x-y \) components of momentum equations. The Strouhal number is a non-dimensional parameter computed using the vortex shedding frequency \( f_s \) obtained by fast Fourier transform (FFT) of the lift fluctuating-time history. These parameters are used for validation and can give indications of flow characteristics including steadiness and vortex shedding. They are defined, respectively, as follows:
\[
C_D = \frac{2 F_D}{\rho u_\infty^2 D} \quad (3)
\]
\[
C_L = \frac{2 F_L}{\rho u_\infty^2 D} \quad (4)
\]
\[
S_t = \frac{f_s D}{u_\infty} \quad (5)
\]

where \( \rho \) is the fluid density; \( F_D \) and \( F_L \) are the total drag force and total lift force, respectively. \( \overline{C_D} \) and \( C_L' \) will be used for results validation, they represent the mean drag coefficient and the r.m.s. lift coefficient respectively.

2.2. Computational domain and boundary conditions
The geometry to be considered is a cylinder with diameter \( D \) submerged in a uniform fluid flow. The cylinder rotates about its axis with angular velocity \( \omega \). The computational domain used for the simulations is set at \( 24D \times 36D \) in \( x, y \) directions of a fixed Cartesian coordinate system. The center of the cylinder is located at a distance of \( 12D \) from the inlet boundary; \( 24D \) from the outlet boundary and an equidistance of \( 12D \) from top and bottom boundaries.

Uniform velocity is prescribed at the inlet; while at the outlet, Neumann type boundary condition is imposed for velocities. Pressure boundary condition is given as atmospheric at the outlet. The end wall effects are neglected in present work, no-shear boundary condition that corresponds to slip at top and bottom walls is prescribed. The cylinder rotates in the counter-clockwise direction and a no-slip boundary condition with rotating speed is imposed at the cylinder surface.

2.3. Grid independency and validation

Figure 1: Computational domain and mesh distribution

Figure 1 shows the detailed diagram of the grid system. An O-type grid structure with a non-uniform distribution was used; a finer grid was generated near the cylinder and gradually became coarser in the wake and the far field. The grid was clustered near the cylinder with an expansion rate of 1.2 in the radial direction from the cylinder surface. The distance between the first grid and the cylinder surface was 0.01D. The dimensionless time steps \( \tau = \frac{\tau_{u_\infty}}{D} = 0.0125 \) and \( 0.025 \) were used.

|                  | \( S_t \) | \( \overline{C_D} \) | \( C_L' \) |
|------------------|----------|----------------------|-----------|
| Williamson (1996) [16] | Exp      | 0.164                |           |
| Park, Kwon and Choi (1998) [17] | Num     | 0.165                | 1.33      | 0.23      |
| Sharman et al. (2005) [18] | Num     | 0.164                | 1.33      | 0.23      |
| Present study    | Num      | 0.168                | 1.36      | 0.238     |
Two dimensional flow around a fixed circular cylinder at Re=100 was computed for mesh independency and validation. With different grid numbers, grid independence tests were satisfied if the variation in the values of $S_t$, $C_D$ and $C_L$ were very small. Moreover, the two different time steps had negligible effect on the values above. As shown in table 1, the values of $S_t$, $C_D$ and $C_L$ of the present simulations case are all in good agreement with other published results of a circular cylinder. Based on these analyses, the mesh model with cell numbers of 22800 was adopted for capturing detailed vortex structures and concurrently saving the computational time. The zone close to the cylinder surface was meshed with 120 grids ($N_c=120$) uniformly distributed along the circumferential direction.

2.4. Vortex identification and tracking

To objectively and accurately determine the characteristics and dynamics of the vortical structures generated by the proposed method, a vortex core $\Sigma$ is identified Based on the Q-criterion [19]. To complete this description, specific parameters such as size, core position and intensity evolution are used. These relevant parameters are computed as follows [20]:

$$\Gamma = \iint_{\Sigma} \omega \, dx \, dy$$

(6)

Where $\omega$ refers to the vorticity field distribution in the core and $\Gamma$ is the vortex circulation.

The position $(x_c, y_c)$ of the vortex centroid is then given by:

$$x_c = \frac{1}{\Gamma} \iint_{\Sigma} x \omega \, dx \, dy$$

(7)

$$y_c = \frac{1}{\Gamma} \iint_{\Sigma} y \omega \, dx \, dy$$

(8)

Size and intensity of a vortex are difficult to define as a qualitative measurement. For a meaningful comparison, a vortex can be visualized as a rotating disc with a definite radius. Two quantities can be defined using this approximation: equivalent radius $R$ and tangential velocity $V_\theta$.

$R$ is defined based on polar moments of vorticity:

$$R = \left[ \frac{1}{2\Gamma} \iint_{\Sigma} [(x-x_c)^2 + (y-y_c)^2] \omega \, dx \, dy \right]^{1/2}$$

(9)

$V_\theta$ is the tangential velocity at the periphery of the vortex core defined as:

$$V_\theta = \frac{\Gamma}{2\pi R}$$

(10)

All parameters previously defined will be normalized with respect to $D$ and $u_\infty$. Normalization will be indicated by superscript *.

3. Results and discussion:

3.1. Qualitative analysis

A qualitative description of the flow topology is conducted using instantaneous normalized vorticity field visualizations for Re=100 and an anticlockwise rotation rate drop from $\alpha_1 = 6$ to $\alpha_2 = 3$. Some selected instants are presented in figure 2.
The main observation is the generation of a single anticlockwise vortex from the cylinder. Once the rotation rate is reduced, positive and negative vorticity field growth is observed. The positive vorticity is gradually growing until the vortex is generated, shedded from the cylinder and advected away with the flow stream. No other vortices are observed; the flow is gradually stabilized and finally reaches the first steady state.

A vortex is rolling up and growing in size and strength as long as there is a circulation flux into it. This flux was possible because of the impulsive drop of the cylinder rotation speed. The initial high vorticity caused by the first rotation rate is now released in a lower speed region instead of being flushed by the highly rotated cylinder. The upper opposite circulation is also increasing and pushing away the generated vortex until it is totally shedded from the cylinder.

3.2. Quantitative analysis
In the previous section, the possibility of generating a single vortex by suddenly dropping the rotation speed was put in evidence. More quantitative analysis is now needed to understand the effect of reducing the rotation speed on the generating process and the vortex characteristics. To get the quantitative description of the generated vortex, equations (3) to (7) are used.

A non dimensional parameter called the rotation drop rate is defined as:

$$\Delta \alpha = \frac{\alpha_1 - \alpha_2}{\alpha_1}$$  \hspace{1cm} (11)

Starting from an initial rotation rate of $\alpha_1 = 6$, rotation speed is impulsively decreased to five different final values with a rotation drop rate of $\Delta \alpha_1 = 0.4$, $\Delta \alpha_2 = 0.5$, $\Delta \alpha_3 = 0.6$, $\Delta \alpha_4 = 0.7$ and $\Delta \alpha_5 = 0.8$ respectively. Results for different vortex tracking parameters are presented in figure 3.
In figure 3(a), three regions separated by two spatio-temporal discontinuities are identified. These regions are respectively characterizing the vortex’s formation, shedding and far-wake advection. The first region reflects the intensity of the circulation feeding process allowing the formation of the vortex. For the cases $\Delta \alpha_1$ and $\Delta \alpha_2$, the vortex shedding process is well defined; the circulation is observed to increase in this region. The size of the vortex is also increased causing the tangential velocity to drop. However, for other cases, a strange behavior is observed. Although the discontinuity reflects a vortex separation, the circulation is kept increasing meaning that the feeding process is still present. The circulation is appeared to be underestimated by the identification method presented in section 2. This behavior is due to the limitation of the velocity gradient based methods to identify the vortex region. These methods tend to eliminate patches from the vortex core when a high radial or axial stretching is present. The reader is referred to [21], and [22] for more details.

After the first discontinuity, the vortex is pinched off and started to move away from the cylinder. The second discontinuity is distinguished by a vortex-tail breakup. The vortex tail is elongated, pronouncing stretching and finally separating from the vortex core. This leads to a sudden drop of the vortex radius and an increase of the vortex core tangential velocity. The process is better observed for higher rotation drop rates and seems to totally disappear for the lowest drop rate case. The normalized radius is observed to converge to almost identical values but the vortex intensity has higher values for larger drop rates.

The vortex trajectory is also affected by the rotation drop rate. More the drop rate is significant, less the vortex core is deviated from the cylinder centerline.

4. Conclusion
In this work we numerically evaluate the possibility of generating single two dimensional vortex using flow past a rotating cylinder. The impulsive drop of the rotating speed is accompanied with a single vortex shedding. A vortex identification approach based on the Q criterion method was used to track this vortex. This method appeared to ignore some vortex regions for cases where a strong radial stretch
was present. In general, the vortex generation process is divided into three distinct regions separated by two discontinuities. The discontinuities reflect both the vortex shedding and the vortex-tail breakup processes. For the same initial rotation speed, the vortex is stronger for larger rotation drop rates. It is also shedded closer to the cylinder with a trajectory more adjacent to the cylinder centerline. These results give an interesting opportunity for new configurations to study the flame vortex interaction that could be experimentally tested in the future.

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