Broken Conformal Window

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We show that the edges of the conformal window of supersymmetric \( SU(N_c) \) QCD, perturbed by Anomaly Mediated Supersymmetry Breaking (AMSB), break chiral symmetry. We do so by perturbatively expanding around Banks–Zaks fixed points and taking advantage of Seiberg duality. Interpolating between the edges of the conformal window, we predict that non-supersymmetric QCD breaks chiral symmetry up to \( N_f \leq 3N_c - 1 \), while we cannot say anything definitive for \( N_f \geq 3N_c \) at this moment.

INTRODUCTION

Strongly correlated systems are difficult to study. They appear in high-\( T_c \) superconductors, heavy fermion systems, and frustrated spin systems in condensed matter physics, and quark confinement, chiral symmetry breaking [1, 2], and nuclear binding in nuclear physics. In all of these cases, the fundamental Hamiltonian or Lagrangian is believed to be “known,” while it is theoretically difficult to work out their consequences at long distances or infrared (IR) limits. Such systems cannot be studied within perturbation theory because we do not have a good candidate for the “zeroth order approximation” that captures the qualitative feature of the system. Once we have such a “zeroth order approximation”, we can identify how to approach a realistic system using perturbation theory.

Nuclear physics is believed to be a consequence of Quantum Chromo-Dynamics (QCD), a non-abelian gauge theory based on the \( SU(3) \) gauge group, with quarks and gluons as fundamental fields. It is known that QCD is an asymptotically-free theory, \( \text{i.e.} \), the coupling becomes weak at short distances but strong at long distances [3, 4]. It is also known that in nature, quarks have masses. Therefore, chiral symmetry cannot be an exact symmetry of the QCD Lagrangian. Yet it is difficult to understand how exactly this dynamical chiral symmetry breaking arises from the fundamental Lagrangian for QCD with massless quarks [1, 2]. It is responsible for making the pion mass light and allowing nuclei to bind beyond the size of individual nucleons. Path integrals over fields can be performed on supercomputers (\( \text{e.g.} \), lattice QCD), but they are limited in that the quark masses cannot be taken to zero.

One of the authors (HM) recently proposed [5] that the supersymmetric version of QCD (SQCD), which had been studied in seminal works by Seiberg [6, 7], combined with anomaly-mediated supersymmetry breaking (AMSB) [8, 9], provides an effective tool to study dynamics of QCD beyond perturbation theory. This tool may well be applicable beyond QCD, and in fact it led to new predictions on the IR dynamics of chiral gauge theories [10, 11] different from past conjectures. Furthermore, the application to \( SO(N_c) \) gauge theories led to the demonstration that both magnetic monopoles and mesons condensed in the ground state, practically a proof of confinement in these theories [12, 13]. It is also expected that the tool can be applied to lower dimensional systems such as for strongly correlated systems in condensed matter physics.

However, questions remained about the so-called “conformal window,” where the SQCD dynamics leads to an infrared (IR) fixed point for \( SU(N_c) \) gauge theories with number of flavors within the range \( \frac{3}{2}N_c < N_f < 3N_c \). Such theories have been shown to be superconformal in the long-distance limit [7]. Once perturbed by AMSB, it was immediately clear that the AMSB effects disappear asymptotically in the IR limit. However, it was not clear whether the theories reach IR fixed points in the presence of such a perturbation.

In this Letter, we argue that the AMSB effects are relevant and deflect the renormalization-group-equation (RGE) flow to chiral symmetry breaking. We demonstrate that the question can be studied unambiguously at the upper edge and lower edge of the conformal window for large \( N_c \), where IR fixed points are achieved within the validity of perturbation theory. The AMSB effects are shown to be relevant in these regimes, and therefore, the dynamics does not lead to superconformal fixed points. Instead, we show a convincing picture that the theories lead to chiral symmetry breaking.

We predict that non-supersymmetric QCD leads to chiral symmetry breaking all the way up to \( N_f < 3N_c \). Our analysis cannot be applied to the range \( N_f \geq 3N_c \) because the AMSB effects make the squark masses negative without a ground state in SQCD for \( N_f \geq 3N_c \).

ANOMALY MEDIATION

Here, we very briefly review Anomaly mediation of supersymmetry breaking (AMSB) [8, 9], which can be formulated with the Weyl compensator \( \mathcal{E} = 1 + \theta^2 m \) [14].
that appears in the supersymmetric Lagrangian as

$$\mathcal{L} = \int d^4\theta \mathcal{E}^* \mathcal{E} K + \int d^2\theta \mathcal{E}^3 W + c.c.$$  (1)

Here, $K$ is the Kähler potential and $W$ is the superpotential of the theory, and $m$ is the parameter of supersymmetry breaking. When the theory is conformal, $\mathcal{E}$ can be removed from the theory by rescaling the fields $\phi_i \rightarrow \mathcal{E}^{-1} \phi_i$. On the other hand, violation of conformal invariance leads to supersymmetry breaking effects. Solving for auxiliary fields, the superpotential leads to the tree-level supersymmetry breaking terms

$$\mathcal{L}_{\text{tree}} = m \left( \frac{1}{2} \frac{\partial W}{\partial \phi_i} \phi_i - 3W \right) + c.c. \quad \text{(2)}$$

Dimensionless coupling constants do not lead to supersymmetry breaking effects because of the conformal invariance at tree-level. However, conformal invariance is anomalously broken due to the running of coupling constants, and there are loop-level supersymmetry breaking effects in tri-linear couplings, scalar masses, and gaugino masses,

$$A_{ijk}(\mu) = -\frac{1}{2} (\gamma_i + \gamma_j + \gamma_k) (\mu)m, \quad \text{(3)}$$

$$m_i^2(\mu) = -\frac{1}{2} \gamma_i(\mu)m^2, \quad \text{(4)}$$

$$m_\lambda(\mu) = -\frac{\beta_\lambda}{2g^2}(\mu)m. \quad \text{(5)}$$

For the superpotential $W = \frac{1}{3!} \lambda_{ijk} \phi_i \phi_j \phi_k$,

$$\gamma_i = \mu \frac{d}{d\mu} \ln Z_i(\mu) = \frac{1}{8\pi^2} (-2g^2C_i + \frac{1}{2} \sum_{j,k} \lambda_{ijk} \lambda_{ijk}) \quad \text{(6)}$$

and $\gamma = \mu \frac{d}{d\mu} \gamma_i$. Here, $C_i = T^aT^a$ is the quadratic Casimir for the representation $i$. Note that we follow the notation $\beta_\gamma = \mu \frac{d}{d\mu} g^2$ etc. In general, physical masses are the sum of contributions from the superpotential (tree-level or non-perturbative), tree-level AMSB \[2\] and loop-level AMSB \[4\].

The most remarkable property of the anomaly mediated supersymmetry breaking is its ultraviolet-insensitivity. The expressions for the supersymmetry breaking parameters above depend on wave function renormalization and running coupling constants, which jump when heavy fields are integrated out from the theory. It turns out that the threshold corrections from the loops of heavy fields precisely give the necessary jump. Therefore the above expressions remain true at all energy scales and depend only on the particle content and interactions present at that energy scale. This point can be verified explicitly in perturbative calculations, and is very transparent in the \text{DR} scheme \[15\].

**CONFORMAL WINDOW**

Seiberg established the conformal window of SQCD for $\frac{3}{2}N_c < N_f < 3N_c$, where the theory flows to IR fixed points with non-trivial superconformal dynamics. The $SU(N_c)$ electric theory ($N_f$ quarks $Q$ in the fundamental representation and $N_f$ anti-quarks $\bar{Q}$ in the anti-fundamental representation) and the $SU(N_f - N_c)$ magnetic theory ($N_f$ dual quarks $q$ and anti-quarks $\bar{q}$ together with a meson field $M$) are supposed to describe the same physics in the infrared (IR). We assume that the equivalence persists sufficiently near the IR fixed point, and take $m \ll \Lambda, \bar{\Lambda}$ to justify this assumption, where $\Lambda$ ($\bar{\Lambda}$) is the strong scale of the electric (magnetic) theory. The electric theory has no superpotential, while the magnetic theory has

$$W = \frac{1}{\mu_m} M^{ij} \tilde{q}_i q_j \quad \text{(7)}$$

with $M^{ij} = \tilde{Q}^i Q^j$. Here, $\mu_m$ is the matching scale that satisfies $\mu_m^{N_f} = \Lambda^{2N_c - N_f} \bar{\Lambda}^{3N_c - N_f}$. In either description, the theory has unbroken $SU(N_f)_{Q,q} \times SU(N_f)_{\tilde{Q},\bar{q}} \times U(1)_B$ symmetry.

For a superconformal theory, the conformal dimensions of chiral fields are determined completely by their conformal supercharges, $D(\phi) = \frac{2}{3} R(\phi)$. On the other hand, the $U(1)_R$ symmetry in supersymmetric QCD is determined by the anomaly-free condition and the charge conjugation invariance,

$$R(Q) = R(\tilde{Q}) = \frac{N_f - N_c}{N_f}, \quad R(q) = R(\tilde{q}) = \frac{N_f - \bar{N}_c}{N_f}, \quad R(M) = 2\frac{\bar{N}_c}{N_f}. \quad \text{(8)}$$

The anomalous dimension is $D(\phi) - 1$, and therefore the Kähler potential receives the wave function renormalization

$$K = Z_\phi(\mu) \phi^* \phi = \left( \frac{\Lambda}{\mu} \right)^{D(\phi) - 1} \phi^* \phi. \quad \text{(9)}$$

Here, $\mu$ is the renormalization scale, and $\Lambda$ is the energy scale where theory becomes nearly superconformal.

It is clear that AMSB effects asymptotically vanish towards the IR limit asymptotically because couplings no longer run,

$$m_{Q,q,M}^2(\mu) \rightarrow 0, \quad m_\lambda(\mu) \rightarrow 0. \quad \text{(10)}$$

However, the effects are relevant and change the IR dynamics unless

$$\frac{m_{Q,q,M}^2(\mu)}{\mu^2} \rightarrow 0, \quad \frac{m_\lambda(\mu)}{\mu} \rightarrow 0. \quad \text{(11)}$$
Unfortunately we do not have computational tools to answer this question for the entire range of the conformal window. Instead, we look at Banks–Zaks (BZ) fixed points [10] where the conformal dynamics can be studied using perturbation theory. This is possible at the upper edge or the lower edge of the conformal window as the IR fixed point couplings turn out to be perturbative, and we find that AMSB effects are relevant within the validity of such analysis.

**ELECTRIC BANKS–ZAKS FIXED POINT**

In this section, we show that the AMSB effects are relevant and modify the IR dynamics at the higher edge of the conformal window, using the electric BZ fixed point for $N_f = 3N_c/(1 + \epsilon)$, $0 < \epsilon \ll 1$. Note that the squark mass-squared is positive. The gauginos and squarks simply decouple and the theory reduces to the non-SUSY QCD at low energies, leaving no good handle on dynamics. Therefore, we will eventually need to make use of the magnetic description.

Running effects in the electric theory are given by

$$
\gamma_Q = \frac{1}{8\pi^2}(-2g^2C_F), \\
\beta(g) = \mu \frac{d}{d\mu} g^2 = \frac{-g^4}{8\pi^2} \frac{3N_c - N_f + N_f\gamma_Q}{3N_c - N_f},
$$

with $C_F = (N_c^2 - 1)/2N_c$. For $N_f = 3N_c/(1 + \epsilon)$, $N_c \gg 1$, and

$$
y = \frac{N_c}{8\pi^2} g^2,
$$

we find

$$
\mu \frac{d}{d\mu} y = -3g^2(\epsilon - y), \\
m_Q^2 = \frac{3}{4}y^2(\epsilon - y)m^2,
$$

the leading order in $\epsilon$. The approximation is valid when $y \approx \epsilon \ll 1$. An example of the solutions is shown in Fig. [1] (a).

We can work out the approximate solution near the fixed point as

$$
y(t) - \epsilon = (y(0) - \epsilon)e^{3\epsilon t}, \\
m_Q^2 = \frac{3}{4}e^{2(y(0) - \epsilon)}e^{3\epsilon t}m^2.
$$

Here $t = \ln \mu \to -\infty$ defines the IR limit. The AMSB effects are relevant when $3\epsilon^2 < 2$, or equivalently $N_f < 1.65N_c$. However, we cannot trust this upper bound since our approximations are no longer valid near it. Since $m_Q^2$ stays positive through all energy scales, there is no minimum along this direction.

Note that all $D$-flat directions are stabilized at the origin due to $m_Q > 0$.

![Figure 1. Running of the electric gauge coupling (above) and the AMSB quark mass squared (below) in units with $m = 1$ near the electric Banks–Zaks fixed point with $\epsilon = 1/20$ and $N_cg^2(0)/8\pi^2 = 0.03$. The dashed line represents the infrared fixed point.](image)

**MAGNETIC BANKS–ZAKS FIXED POINT**

We again show that AMSB effects are relevant and modify the IR dynamics, using the magnetic BZ fixed points for $N_f = 3N_c/(1 + \epsilon)$, $0 < \epsilon \ll 1$. Here we are at the lower edge of the conformal window because $N_f \approx 3N_c = 3(N_f - N_c)$ implies $N_f \approx \frac{3}{2}N_c$.

At the first sight, it appears that the dynamics is ambiguous because depending on the initial condition of coupling constants, $m_q^2$ and $m_Q^2$ are found to have either sign. However, in requiring that baryon direction [17] does not runaway to infinity in a way that is consistent with the electric theory, we have to assume that the initial conditions lie in the region of the parameter space where $m_q^2 \geq 0$. Otherwise the dual quark would roll down the potential to a minimum of $O(-m^4/(4\pi)^4)$ or possibly even $O(-m^2\Lambda^2/(4\pi)^4)$, and break the chiral symmetry $SU(N_f)_q \times SU(N_f)_\bar{q} \times U(1)_B$ to $SU(N_f - \tilde{N}_c)_q \times SU(N_f)_\bar{q}$.

It is not too surprising that the duality imposes a constraint on the magnetic theory, given that the electric theory has a single parameter $g$, while the magnetic theory two $g$ and $\lambda$. For them to describe the same physics, there must be a one-parameter constraint on the $(g, \lambda)$ parameter space. In the asymptotic IR limit, both theories reach the IR fixed point and there are no parameters.
Slightly away from the IR fixed point, the magnetic theory should follow a specific RG trajectory on the \((g, \lambda)\) plane. In the supersymmetric limit, we did not have tools to pick a trajectory. Together with the AMSB effects, now we can at least choose a half of the parameter space. It would be very interesting to see if further studies can pick a particular RGE trajectory.

Following [18], the magnetic RGEs are given by

\[
\begin{align*}
\gamma_q &= \frac{1}{8\pi^2} (-2g^2 C_F + \lambda^2 N_f), \\
\gamma_M &= \frac{1}{8\pi^2} \lambda^2 \tilde{N}_c, \\
\beta_g &= -g^4 \frac{3\tilde{N}_c - N_f + N_f \gamma_q}{8\pi^2 - \tilde{N}_c g^2}, \\
\beta_\lambda &= \lambda^2 (2\gamma_q + \gamma_M),
\end{align*}
\]

with \(C_F = (\tilde{N}_c^2 - 1)/2\tilde{N}_c\), and the squark and meson mass-squared are given by

\[
\begin{align*}
m_q^2 &= -\frac{1}{4} \gamma_q m^2 = \frac{1}{32\pi^2} (-2\beta g C_F + \beta_\lambda N_f) m^2, \\
m_M^2 &= -\frac{1}{4} \gamma_M m^2 = \frac{1}{32\pi^2} \beta_\lambda \tilde{N}_c.
\end{align*}
\]

For notational convenience, we define

\[
x = \frac{\tilde{N}_c}{8\pi^2} \lambda^2, \quad y = \frac{\tilde{N}_c}{8\pi^2} g^2.
\]

A numerical plot of this RG flow is shown in Figure 2, suprased with curves of \(m_q^2 = 0\) (red) and \(m_M^2 = 0\) (green). Notice that the flows appear to converge on the \(m_q^2 = 0\) line first, providing an indication of the finite meson expectation value in the ground state.

Imposing \(N_f = 3\tilde{N}_c/(1 + \epsilon)\), we can solve for the exact BZ fixed point to get:

\[
(x, y) = (2\epsilon, \frac{\tilde{N}_c^2 (7\epsilon + \epsilon^2)}{\tilde{N}_c^2 - 1}(1 + \epsilon))
\]

The eigenvalues of this matrix are \(14\epsilon\) and \(21\epsilon^2\). Therefore, the couplings approach the fixed point with the exponents \(e^{14\epsilon t}\) and \(e^{21\epsilon^2 t}\), where \(t = \ln \mu\). Notice that these exponential slow slower than \(\mu^2 = e^{\frac{21}{2}}\). Whenever \(14\epsilon, 21\epsilon^2 < 2\), which is indeed the case if \(7\epsilon \ll 1\) (even the faster exponent will be \(14\epsilon < 2\)). Therefore the AMSB effects are relevant for sufficiently small \(\epsilon\). An illustration of this running and its consequences for the squark and meson masses – in the regime where \(m_q^2 > 0\) is shown in Figure 2. It is reasonable to expect that the initial conditions are near the UV fixed point \(g = \lambda = 0\). For numerical plots, we took values already close to the IR fixed point to keep the amount of running manageable. Since \(m_q^2\) stays positive through all energy scales, there is no minimum along this direction.

Pushing \(\epsilon\) beyond the validity range, we see that AMSB effects are relevant if \(21\epsilon^2 < 2\), or equivalently \(N_f > 1.77\tilde{N}_c\). But we cannot trust this lower bound given that the approximations made by us are no longer valid there. While we cannot exclude the possibility that AMSB is irrelevant for \(N_f \sim 1.7\tilde{N}_c\), we will show a consistent picture below that it is likely relevant.

**CHIRAL SYMMETRY BREAKING**

We argued that the AMSB effects are relevant for the conformal window, which deflects the RGE flow away from the superconformal dynamics, and \(m_q^2 > 0\) so that there is no baryonic instability with the vacuum energy of deeper than \(-O(m_4^4/(4\pi^4))\). Even though strictly speaking we cannot show the latter point definitively away from the BZ fixed points, the consistent picture we derive below makes it highly plausible.
Given our observations, we now would like to understand what the IR limit of the theory is. We show that the ground state appears with a finite meson expectation value. After integrating out the dual quarks, the low-energy pure SYM develops a gaugino condensate with the exact non-perturbative superpotential

\[ W = \tilde{N}_c \left( \frac{3N_c - N_f}{\Lambda_m} \right)^{1/\tilde{N}_c} \left( \frac{M}{\mu_m} \right)^{1/\tilde{N}_c}. \]  

Writing \( M_{ij} = \lambda_{\mu\nu} \delta_{ij} \) for the canonically normalized \( \phi \), the AMSB potential is

\[ V_{\text{AMSB}} = -m \left( 3\tilde{N}_c - N_f \right) \frac{3N_f - N_f/\tilde{N}_c}{\Lambda_m} \left( \lambda \phi \right)^{N_f/\tilde{N}_c}. \]  

On the other hand, the supersymmetric potential depends on the Kähler potential of \( \phi \) which receives the wave-function renormalization factor Eq. (32),

\[ Z_{\phi}(\mu) = c \left( \frac{\mu}{\Lambda_m} \right)^{1-3\tilde{N}_c/N_f}, \]  

with unknown overall normalization \( c \). The potential evaluated at \( \mu \approx \phi \) is therefore

\[ V_{\text{SUSY}}(\phi) = Z_{\phi}(\phi)^{-1} \left| \Lambda_m^{3-N_f/\tilde{N}_c} (\lambda \phi)^{N_f/\tilde{N}_c - 1} \right|^2. \]  

Combining Eq. (33) and (35), the potential has a stable minimum with

\[ V \propto -m^\alpha \Lambda_m^{4-\alpha}, \quad \alpha = 1 + \frac{N_f^2}{N_f^2 - 3N_f \tilde{N}_c + 3\tilde{N}_c^2}. \]  

The exponent \( \alpha \) is a consequence of the exact non-perturbative superpotential Eq. (32) and the exact anomalous dimension Eq. (34), and hence is also exact. It is important to recognize that the anomalous dimension factor (34) is crucial to avoid a potential run-away behavior for \( N_f > 2\tilde{N}_c, 2\tilde{N}_c \) by making sure that \( V_{\text{AMSB}} \) has a lower power than the supersymmetric potential \( V_{\text{SUSY}} \). Note that

\[ \frac{N_f^2}{N_f^2 - 3N_f \tilde{N}_c + 3\tilde{N}_c^2} = \frac{N_f^2}{N_f^2 - 3N_f \tilde{N}_c + 3\tilde{N}_c^2}. \]  

which demonstrates the equivalence of physics in either description (duality). The power in \( m \) goes from 4 (\( N_f = 2\tilde{N}_c \)) to 5 (\( N_f = 2\tilde{N}_c \)) back to 4 (\( N_f = 3\tilde{N}_c \)), see Fig. 5. This behavior is qualitatively consistent with the naive picture that SUSY particles decouple at \( \mu = m(\mu) \) perturbatively at either end with BZ fixed points, while SUSY breaking effects are power suppressed near \( N_f \sim 2\tilde{N}_c \) due to strong dynamics. Yet the quantitative understanding of the exponent based on this naive picture currently eludes the authors.

**CONCLUSION**

The main prediction of this Letter is that non-supersymmetric QCD leads to chiral symmetry breaking all the way up to \( N_f < 3\tilde{N}_c \). This prediction assumes that the phase of the dynamics is continuous from the lower edge to the higher edge of the conformal window,
where the AMSB effects are shown to be relevant, while we cannot conclusively determine whether the AMSB effects are relevant near the middle of the conformal window where the fixed point dynamics is non-perturbative.

Our analysis cannot be applied to the range $N_f \geq 3N_c$, because in SQCD for $N_f \geq 3N_c$ the AMSB effects make the squark masses negative without a ground state.

Our prediction appears to be consistent with recent lattice QCD simulations. Namely, the paper [19] suggests that $SU(2)$ gauge theories exhibit chiral symmetry breaking for $N_f < 6$, while $SU(3)$ gauge theory with $N_f = 8$ breaks chiral symmetry [21] and that with $N_f = 12$ appears to reach an IR fixed point [21]. These results are still subject to discussions, however [22].

The remaining issue is whether there is a phase transition as the size of the AMSB is increased from $m \ll \Lambda$ below the dynamical scale $\Lambda$ to $m \gg \Lambda$. The consistency of the analysis here and the lattice QCD results suggests that the two limits are continuously connected without a phase transition. It was suggested that an extension of the holomorphy argument may justify the absence of a phase transition [10, 11]. But further investigation is warranted to understand this question quantitatively.

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Figure 5. The power of the vacuum energy in $V_{\min} \propto -m^\alpha$, $\alpha = 1 + N_f^2/(N_f^2 - 3N_f N_c + 3 N_c^2)$.

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