String cosmological model in bianchi type IX inflationary universe with flat potential

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Abstract. String cosmology has been investigated in a spatially symmetric Bianchi -IX line element under consideration of flat potential. To illustrate the nonlinear system of field equations an appropriate relation between the metric coefficient and the hybrid expansion law (HEL) for the scalar factor \( a(t) \) is considered i.e. \( a(t) = a_0 t^{\gamma_1} e^{\gamma_2 t} \) where \( \gamma_1 \) and \( \gamma_2 \) are the non-negative constants and \( a_0 \) denotes current value of \( a(t) \). It has been observed that the energy density and particle density diverge initially and become finite for large \( t \). The tension in string clouds tends to infinite when \( t = 0 \) and constant value at a large value of \( t \) for \( m > 1 \). The proper volume is also increases with time in exponentially way favorable to inflationary criteria. Expansion and shear infinite large initially and approach to finite value at late time. Some structural aspects of the model and their importance are pointed.

Keywords: String Cosmology, Inflationary Universe, Particle Density, Flat Potential

1. Introduction

A study of inflationary cosmology provides a better explanation of the formation of galactic structure from the early universe. It is essential to study the problems in cosmology like horizon, monopole, and flatness in to develop a model mathematically of the present universe that accurately resembles the results of astronomical observations. The FRW model of the universe explains that the universe is purely isotropic and homogeneous in nature. The inflationary phenomenon has astrophysical significance to understand the cosmos evolution. The initial idea of expansion is suggested by Guth [1] with a reason for the false vacuum energy. The presences of the higg’s field in this discussion have major importance. Many cosmologists [2–5] have studied several aspects of the inflation phenomenon with the existence of the scalar field in different contexts. Explaining the precise dynamic state of our early universe is mysterious. The cosmic string is a constant defect in the topology caused by the phase transition at the beginning of the universe after the big bang is investigated by Kibble et al. [6] besides, the string cosmology has an important role to explain the structure of the physical universe. It is necessary to study that cosmic strings clouds provide the stabilization of the densities that cause the formation of galactic structures. The gravitational influence of the cosmic strings is important because they provide stress- energy and coexist with the gravitational field. Most of the cosmologists [7–13] have researched the universe’s string models at various points. The string cosmological model in the Bianchi IX was developed by Bali et al. [14–15]. Reddy [16] studied the Bianchi-II space with a enormous string source in general relativity. Katore and Chopade [17] have studied Bianchi Type VI inflationary space-time using a massless scalar field with flat potential. Krori et al. [18] resulted exact solutions in string cosmological models. Pradhan et al.[19] have derived locally symmetric rotationally Bianchi type II under influence of massive strings. Present work deals with Bianchi-IX space under consideration of flat potential in massive string source. To obtain a relativistic solution for a system of nonlinear fields eqs. We considered two supplementary conditions among metric coefficients as \( \alpha = \beta^m \) and hybrid expansion \( a(t) = a_0 t^{\gamma_1} e^{\gamma_2 t} \) where \( \gamma_1 \) and \( \gamma_2 \) are the non-
negative constants and $a_0$ denotes current value of $a(t)$. The structural parameters of the resulted model are also pointed.

2. The metric and field equations

The line element describes Bianchi type IX is obtained

$$ds^2 = -dt^2 + a^2dx^2 + \beta^2dy^2 + (\beta^2 \sin^2 y + \alpha^2 \cos^2 y)dz^2 - 2\alpha^2 \cos^2 ydxdz$$

where $\alpha$ and $\beta$ used for the coefficients and functions of t alone.

In case of gravity connected nominally to scalar region with Potential $V(\psi)$ is given by Lagrangian

$$l = \int \left( R - \frac{1}{2}\psi_i \psi_j g^{ij} - V(\psi) \right) \sqrt{-g} \, dx^4$$

Einstein field eq. is given by

$$R_i^j - \frac{1}{2} R g_i^j = -(\psi_i^j (\psi) + \psi_i^j (m))$$

with relation

$$\frac{1}{\sqrt{-g}} \partial_i (\sqrt{-g} \psi_i) = -\frac{dv(\psi)}{d\psi}$$

where

$$\psi_i = \frac{\partial \psi}{\partial x^i}, \psi_j = \frac{\partial \psi}{\partial x^j} \text{ and } \psi^k = g^{kj} \frac{\partial \psi}{\partial x^j}$$

(in geometrical unit $\frac{8\pi G}{c^4} = 1$)

where energy- momentum tensors related to scalar field with potential $V(\psi)$ and the massive strings source is given as

$$T_i^j (\psi) = \psi_i \psi_j - \left( \frac{1}{2} \psi_\alpha \psi^\alpha + V(\psi) \right) g_{ij}$$

$$T_i^j (m) = \rho \psi_i \psi_j - \Lambda x_i x^j$$

where $\psi_i \psi_j = -x_i x^j = -1$ and $x_i x^j = 0$

and

$$\rho = \rho_0 + \Lambda$$

Here $\rho$ denoted the energy density of string. The four velocity components of strings cloud and directions component of the strings is given as $\psi^i$ and $x^j$ respectively. $\rho_0$ be the rest energy density of elementary particles associated with the string clouds and $\Lambda$ denotes the tension density of strings (Letelier[9])

The conservation energy eq. is given by

$$\psi_i^j = -\frac{dV}{d\psi}$$

$$\dot{\rho} + \rho \left( \frac{\alpha_4}{\alpha} + 2 \frac{\beta_4}{\beta} \right) - \Lambda \frac{\alpha_4}{\alpha} = 0$$
Here sub-indices 4 denoted the ordinary derivative w.r.t. time, with the help of comoving coordinates explicit non-linear field equations (2.3) for line element (2.1) is obtained as

\[
\frac{\beta_1^2}{\beta^2} + 2\frac{\beta_2}{\beta} + \frac{1}{\beta^2} - \frac{3a^2}{4\beta^4} = -\frac{\psi^2}{2} + V(\psi) + \Lambda \tag{2.10}
\]

\[
\frac{1}{4}\frac{a^2}{\beta^4} + \frac{a_4\beta_4}{a\beta} + \frac{\beta_4}{\beta} + \frac{\Lambda a}{a} = -\frac{\psi^2}{2} + V(\psi) \tag{2.11}
\]

\[
2\frac{a_4\beta_4}{a\beta} - \frac{1}{4}\frac{a^2}{\beta^4} + \frac{\beta_4^2}{\beta^2} + \frac{1}{\beta^2} = \frac{\psi^2}{2} + V(\psi) + \rho \tag{2.12}
\]

From eq. (2.4) we have

\[
\psi_{44} + \psi_4 \left(\frac{a_4}{a} + 2\frac{\beta_4}{\beta}\right) = -\frac{dV}{d\psi} \tag{2.13}
\]

The physical terms expansion coefficient ($\theta$), proper volume ($V$), shear scalar ($\sigma^2$), hubble parameter ($H_0$) and the deceleration Parameter ($q$) can be obtained by following expression

\[
\theta = \frac{a_4}{a} + 2\frac{\beta_4}{\beta} \tag{2.14}
\]

\[
V = a^3(t) = \alpha \beta^2 \tag{2.15}
\]

where $a(t)$ be average scale factor

\[
\sigma^2 = \frac{1}{3}\left[\frac{a_4}{a} - \frac{\beta_4}{\beta}\right] \tag{2.16}
\]

\[
H_0 = \frac{1}{3}\left(\frac{a_4}{a} + 2\frac{\beta_4}{\beta}\right) \tag{2.17}
\]

\[
q = -\frac{\frac{a_4}{\beta_4/a}}{\left(\frac{a_4}{a}\right)} \tag{2.18}
\]

3. Solution of field equations

For deterministic solutions flat region is considered with constant potential i.e. $V(\psi) = \xi$

Eq. (2.13) reduces to

\[
\psi_{44} + \psi_4 \left(\frac{a_4}{a} + 2\frac{\beta_4}{\beta}\right) = 0
\]

which provides $\psi_4(\alpha \beta^2) = \psi_0$ \hfill (3.1)

where $\psi_0$ is integrating constant

from eqs. (2.10-2.12) we have

\[
\frac{\beta_4^2}{\beta^2} + \frac{\beta_4}{\beta} + \frac{1}{\beta^2} - \frac{a_4\beta_4}{a\beta} - \frac{\Lambda a}{a} = \Lambda \tag{3.2}
\]

\[
\frac{a_4}{a} + \frac{\beta_4}{\beta} + 3\frac{a_4\beta_4}{a\beta} + \frac{\beta_4^2}{\beta^2} + \frac{1}{\beta^2} = 2\xi + \rho \tag{3.3}
\]

Eqs. (3.1-3.3) are independent equations, we have to find five unknown $a, \beta, \Lambda, \rho$ and $\psi$. for this purpose we have to required two more conditions to acquire explicit solution of the fields eqs.we adopt that expansion is proportional to scalar shear which provide a relation between metric coefficients
\[ \alpha = \beta^m \quad m \neq 1 \quad (3.4) \]

where \( m \) is non-negative constant which takes care anisotropy of space time.

Berman [20] has proposed special Hubble law variation and Akarshu et. al [21] investigated hybrid expansion law(HEL) which give constant deceleration factor is given by

\[ a(t) = a_0 t^\gamma_1 e^{\gamma_2 t} \quad (3.5) \]

where \( \gamma_1 \) and \( \gamma_2 \) are non-negative constant and \( a_0 \) denotes current value of \( a(t) \)

using hybrid expansion law, we obtained result for matrix coefficient is given by

\[ \alpha = (a_0 t^\gamma_1 e^{\gamma_2 t})^\frac{2m}{\gamma_1 + \gamma_2} \quad (3.6) \]

\[ \beta = (a_0 t^\gamma_1 e^{\gamma_2 t})^\frac{1}{\gamma_1 + \gamma_2} \]

using suitable constant, eq. (3.1) become

\[ \psi = \psi_0 \int (t^\gamma_1 e^{\gamma_2 t})^{-3} dt + C_1 \quad (3.7) \]

here \( \psi_0 \) and \( C_1 \) are integrating constants.

By taking suitable constant, we obtain the line element (2.1) in the form

\[ ds^2 = -dt^2 + (t^\gamma_1 e^{\gamma_2 t})^\frac{6m}{\gamma_1 + \gamma_2} dx^2 + (t^\gamma_1 e^{\gamma_2 t})^\frac{6}{\gamma_1 + \gamma_2} dy^2 + \left[(t^\gamma_1 e^{\gamma_2 t})^\frac{6}{\gamma_1 + \gamma_2} sin^2 y + \right] dz^2 - 2(t^\gamma_1 e^{\gamma_2 t})^\frac{6m}{\gamma_1 + \gamma_2} dx dz \quad (3.8) \]

\section*{4. Physical and geometrical features}

Physical and geometric aspects of the model can be described

Proper volume \( (V) = (t^\gamma_1 e^{\gamma_2 t})^3 \quad (4.1) \)

\[ \text{Fig. 1 The plot of volume versus time } (\gamma_1 = 0.7, \gamma_2 = 0.5) \]

From Fig. 1 it is clearly observed that the proper volume increases as time increases and become infinite at late time, it represents the inflationary scenario in present universe. From Fig. 2 observed that scalar of expansion decreases with time
Average Hubble Parameter \((H_0) = \frac{\gamma_1}{t} + \gamma_2\) \hspace{1cm} (4.2)

Coefficient of Expansion \((\theta) = 3 \left[ \frac{\gamma_1}{t} + \gamma_2 \right]\) \hspace{1cm} (4.3)

Scalar of shear \((\sigma^2) = \left[ \frac{1}{3} \left( \frac{m-1}{m+2} \right) \left( \frac{\gamma_1}{t} + \gamma_2 \right) \right]^2\) \hspace{1cm} (4.4)

Anisotropic Parameter \((A_m) = \frac{1}{3} \sum_{n=1}^{3} \left( \frac{H_n}{H} - 1 \right)^2 = 8 \left( \frac{m-1}{m+2} \right)^2\) \hspace{1cm} (4.5)

The ratio of shear and expansion \(\left( \frac{\sigma}{\theta} \right) = \frac{1}{3} \sqrt{3} \left[ \frac{m-1}{m+2} \right]\) \hspace{1cm} (4.6)

The deceleration Parameter \((q) = \frac{1}{\gamma_1} \left( \frac{\gamma_2}{t^2} \right) - 1\) \hspace{1cm} (4.7)

The tension \(\Lambda\) in the string is given by

\[
\Lambda = \left[ -3 \left( \frac{m-1}{m+2} \right) \left( \frac{\gamma_1}{t} + \gamma_2 \right)^2 - \frac{\gamma_1^2}{t^2} \right] + \left( t^2 e^{y_1 t} \right)^m \left( t^2 e^{y_2 t} \right)^2
\] \hspace{1cm} (4.8)

The energy density term \(\rho\) is given by

\[
\rho = \left[ 3 \left( \frac{m+1}{m+2} \right) \left( \frac{\gamma_1}{t} + \gamma_2 \right)^2 - \frac{\gamma_1^2}{t^2} \right] + \left( t^2 e^{y_1 t} \right)^m \left( t^2 e^{y_2 t} \right)^2 - 2 \xi
\] \hspace{1cm} (4.9)

The particle density \(\rho_0\) of the string cloud is given by

\[
\rho_0 = \left[ 6 \left( \frac{m}{m+2} \right) \left( \frac{\gamma_1}{t} + \gamma_2 \right)^2 - \frac{\gamma_1^2}{t^2} \right] - 2 \xi + \left( t^2 e^{y_1 t} \right)^{\frac{6(m-2)}{m+2}}
\] \hspace{1cm} (4.10)

5. Conclusion and discussion

This paper deals with string influence on the cosmological study of the physical universe. The exact solution of the field’s eq. is obtained by using HEL and the relation between the metric coefficients with flat potential in string cosmology. The investigated model obeys the big-bang at an initial time and expansion becomes slows as the time increases. Proper volumes are increasing function of time in an exponential manner at the slow rate which agreed with eternal inflation. Since finite provides the model does not approach isotropy. At late time negative deceleration parameter show an accelerated scenario of the universe. The model has no initial singularities. Shear,
expansion and hubble parameter becomes divergent at initial epoch and leads to constant as $t \to \infty$. Energy density and particle density diverge initially and become finite for large $t$.

Tension in string clouds become infinite at $t = 0$ and constant value at late $t$ for $m > 1$. At $m = 1$ model leads to isotropic condition and become shear free. Inflationary cosmological model have been investigated by assuming relation between metric coefficient and hybrid law expansion to discussed some astrophysical facts in string cosmology.

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