Exact Symmetries realized on the Renormalization Group Flow

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Abstract

We show that symmetries are preserved exactly along the (Wilsonian) renormalization group flow, though the IR cutoff deforms concrete forms of the transformations. For a gauge theory the cutoff dependent Ward-Takahashi identity is written as the master equation in the antifield formalism: one may read off the renormalized BRS transformation from the master equation. The Maxwell theory is studied explicitly to see how it works. The renormalized BRS transformation becomes non-local but keeps off-shell nilpotency. Our formalism is applicable for a generic global symmetry. The master equation considered for the chiral symmetry provides us with the continuum analog of the Ginsparg-Wilson relation and the Lüscher’s symmetry.

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1 Introduction

The Wilsonian Renormalization Group (RG) is one of the most important achievements in modern physics\[1\]. In particular, the exact RG equations[2-4] have proved to be powerful both in perturbative and non-perturbative studies of field theories\[1\]. In a field theory, quantum fluctuations at shorter distances are integrated out to give an effective action for longer distances. For the well-defined integration, one needs to introduce some regularization procedure, which may be in conflict with symmetries in many important applications: for example, the presence of gauge symmetry or chiral symmetry is far from trivial. The incompatibility of symmetries and regularizations is a longstanding problem in the RG approach.

There have been several attempts[10-13] recently to investigate this problem based on a common recognition: a symmetry is broken at intermediate steps of the RG iteration, and is recovered only after the IR cutoff $k$ is removed. The breaking of the symmetry is controlled by the modified Ward-Takahashi (WT) identity, $\Sigma_k = 0[14]$. In practical calculations, one has to finely tune parameters in an effective action so that it satisfies the usual WT identity in the limit of $k \to 0$. This viewpoint, recovery of the symmetry by “fine tuning”, is due to Becchi[10] and extensively studied in refs. [11], [12] and [14].

Recent development in understanding chiral symmetry on the lattice has brought another important clue to our problem: Lüscher found an exact chiral symmetry on the lattice[15], relying on the Ginsparg-Wilson (GW) relation[16]. This provides us with the first non-trivial example of having an exact symmetry even after the regularization. The Lüscher’s chiral symmetry takes quite different form from that in the continuum limit.

Based on these observations, we shall give in this paper a general method to define global symmetry along a RG flow. It may be non-local and cutoff dependent, yet exact symmetry even for $k \neq 0$. We call this “renormalized symmetry”. Remarkably, our discussion applies to gauge symmetry as well by considering its global counterpart, the BRS symmetry.

We begin with a microscopic or UV action which is local and invariant under a symmetry transformation. In order to construct the effective action at low momentum, we consider the continuum analog of the blockspin transformation. This formalism developed in[4] introduces macroscopic fields (average fields), in terms of which the renormalized symmetry is realized. The important role of the macroscopic fields is also suggested by the GW relation and the Lüscher’s chiral symmetry. Since the blockspin transformation is a gaussian integral, we obtain an exact RG flow equation[4] for the effective action of the macroscopic fields. When expressed by the macroscopic fields and some source fields, the WT identity $\Sigma_k = 0$ takes the form of a master equation, from which we shall find the exact symmetry transformation for $k \neq 0$. We would like to emphasize here that our WT identity is for the exact renormalized symmetry, not for the broken or modified symmetry. This is the central issue of our formulation of renormalized symmetry. The flow equation for $\Sigma_k$ holds as a result of the algebraic relation between the operator specifying the RG

\[1\] See eg [5][6] for non-perturbative studies, [7-9] for reviews of the recent development.
flow and that appeared in the WT identity.

For gauge theories, the master equation is nothing but the one in the antifield formalism of Batalin and Vilkovisky. In order to see how the renormalized symmetry looks like, we give an effective action and a renormalized BRS symmetry for the Maxwell theory. As another test of our method, we consider chiral symmetry, and show that our master equation and associated renormalized symmetry are the continuum analog of the GW relation and the Lüscher’s symmetry. In our derivation the GW relation is regarded as an exact WT identity for the chiral symmetry.

### 2 General formalism

Let $\varphi^A$ be a microscopic field with the Grassmann parity $\epsilon(\varphi^A) = \epsilon_A$ and $S[\varphi^A]$ a generic action. The microscopic or UV action is assumed to be invariant $\delta^a S[\varphi] = 0$ under an infinitesimal global transformation with parameters $\epsilon^a$, $\varphi^A \rightarrow \varphi^A + \delta^a \varphi^A \epsilon^a$, where $\epsilon(\delta^a) = \epsilon(\epsilon^a)$. The discussion to be given also applies to gauge theory: the action $S[\varphi]$ is a gauge fixed action and the relevant global transformation is the BRS transformation.

To specify a blockspin transformation, we introduce a function $f_k(p)$ with an IR cutoff $k$ in the Euclidean momentum space, and an invertible matrix $[R_k(p)]_{AB}$ satisfying $\epsilon([R_k(p)]_{AB}) = 0$, $[R_k(p)]_{AB} = (-1)^{\epsilon_A \epsilon_B} [R_k(p)]_{BA}$. For a boundary condition, we impose $f_k(p) \rightarrow 1$, $[R_k(p)]_{AB} \rightarrow \infty$ as $k \rightarrow \infty$. Possible choices of $f_k(p)$ and $[R_k(p)]_{AB}$ were discussed in [5], but we do not need to specify them here. Let $K^A_a$ be sources for the variations $\delta^a \varphi^A$: they will play an important role in our symmetry consideration. We may define an effective action for the macroscopic fields $\Phi^A$ in the presence of the sources by

$$
e^{-\Gamma_k[\Phi, K]} = \int \mathcal{D}\varphi e^{-S_k[\varphi, \Phi, K]},$$

$$S_k[\varphi, \Phi, K] = S[\varphi] + \frac{1}{2} (\Phi - f_k \varphi)^T R_k (\Phi - f_k \varphi)_+ + K^T_+ \delta^a \varphi_+,$$  

(2.1)

where $\Phi_\pm \equiv \Phi(\pm p)$ and their multiplication implies the integration over momentum as well as the sum over the index $A$, eg,

$$\Phi^T R_k \Phi_+ = \text{Str}(R_k \Phi_+ \Phi^T_+) \equiv \int_p \Phi^A(-p)[R_k(p)]_{AB} \Phi^B(p), \quad \int_p \equiv \int d^D p.$$  

(2.2)

The supertrace, Str, denotes a sum over momenta and indices. Note that $f_k[R_k]_{AB} \Phi^B$, a linear term of the macroscopic fields, acts as a source term for $\varphi^A$ in the path integral.

Since only the gaussian term depends on the cutoff $k$, one obtains the exact RG flow equation $\Gamma_k[\Phi, K]$:

$$\partial_k e^{-\Gamma_k[\Phi, K]} = - \left[ X + \frac{1}{2} \text{Str}(R_k^{-1} \partial_k R_k \mathbf{1}) + \text{Str}(\partial_k (\ln f_k) \mathbf{1}) \right] e^{-\Gamma_k[\Phi, K]},$$

$$X \equiv - \frac{1}{2} \frac{\partial^2}{\partial \Phi_+^T} (\partial_k R_k^{-1}) \frac{\partial^r}{\partial \Phi_+} + \partial_k (\ln f_k) \left[ \frac{\partial^r}{\partial \Phi_+^T} R_k^{-1} \frac{\partial^r}{\partial \Phi_+} + \Phi_+ \frac{\partial^r}{\partial \Phi_+} \right].$$  

(2.3)

\footnote{The index $A$ denotes kinds of fields and other indices as a whole, except field momentum.}
We consider now the symmetry property of the macroscopic action. Invariance of the microscopic action under the global transformation can be expressed as

\[
\int \mathcal{D}\varphi e^{-S_k[\varphi + \delta^a \varphi^a, \Phi, K]} - \int \mathcal{D}\varphi e^{-S_k[\varphi, \Phi, K]} = 0. \tag{2.4}
\]

Assumed here is the translational invariance of the path integral measure, i.e., the absence of anomalies. For each independent parameter \(\varepsilon^a\), the WT identity reads

\[
\Sigma_{ka}[\Phi, K] \equiv -e^{\Gamma_{[\Phi,K]}(\Phi, K)} \Delta_a e^{-\Gamma_k[\Phi, K]} = -\langle K_b^- \delta_a \delta_b^k \varphi_+ \rangle_k, \tag{2.5}
\]

where the expectation value is taken with respect to the action \(S_k\) and the operator \(\Delta_a\) is defined by

\[
\Delta_a \equiv \text{Str} \left( f_k \frac{\partial^T}{\partial \Phi_-} \left( \frac{\partial^T}{\partial K_a^+} \right) \right). \tag{2.6}
\]

One obtains

\[
\Sigma_{ka} = \text{Str} \left( f_k \frac{\partial^T}{\partial \Phi_-} \left( \frac{\partial^T}{\partial K_a^+} \right) \right) + \Delta_a \Gamma_k. \tag{2.7}
\]

This takes the form of a master equation in the space of \((\Phi_A, K_A^a)\). As will be seen presently, for the BRS symmetry the source \(K_A(p)/f_k(p)\) can be identified with the antifield of the macroscopic field \(\Phi_A\), and (2.5) becomes the quantum master equation.

In order to obtain the flow equation for \(\Sigma_{ka}\) in our formulation, we notice that there is an algebraic relation between operator \(X\) in (2.3) and the operator \(\Delta_a\):

\[
[\Delta_a, X] = (\partial_k \Delta_a) \tag{2.8}
\]

on any Grassmann even quantity. This leads to the flow equation

\[
\partial_k \Sigma_{ka} = (e^\Gamma_X e^{-\Gamma_k}) \Sigma_{ka} - e^\Gamma_X (e^{-\Gamma_k} \Sigma_{ka}). \tag{2.9}
\]

It is easily seen that the r.h.s consists of the functional derivatives of \(\Sigma_{ka}\).

The above equations (2.5) \(~\sim~\) (2.8) hold quite generally. They also provide us with the transformation for the renormalized symmetry. In the following two sections we consider the BRS and global symmetry separately.

### 3 Renormalized BRS symmetry

#### 3.1 The master equation

For the BRS symmetry, the source \(K_A(p)/f_k(p)\) can be identified with the antifield \(\Phi_A^*\) for the macroscopic field \(\Phi_A\). Then, the operator \(\Delta\) in (2.6) and a bracket defined by

\[
(F, G) \equiv \int_p \left[ \frac{\partial^r F}{\partial \Phi_A(-p)} \frac{\partial^l G}{\partial \Phi_A^*(-p)} - \frac{\partial^r F}{\partial \Phi_A^*(-p)} \frac{\partial^l G}{\partial \Phi_A(-p)} \right], \tag{3.1}
\]

3
are exactly those in the antifield formalism of Batalin-Vilkovisky\cite{17}. Since the r.h.s of (2.5) vanishes because of the nilpotency $\delta^2=0$, one obtains the condition,
\[
\Sigma_k[\Phi,\Phi^*] = \frac{1}{2}(\Gamma_k[\Phi,\Phi^*], \Gamma_k[\Phi,\Phi^*]) + \Delta\Gamma_k[\Phi,\Phi^*] = 0,
\]
which is nothing but the quantum master equation. It is an algebraic equation which holds for any $\Phi$ and $\Phi^*$. The WT flow equation (2.9) tells us then that once it is satisfied at some cutoff $k = k_0$ it persists along the RG flow. This clearly demonstrates the presence of a cutoff dependent BRS symmetry, a renormalized BRS symmetry, in the macroscopic action. If the second term in the master equation vanishes, we may define the renormalized BRS transformation on $\Phi$ and $\Phi^*$ by
\[
\delta_r\Phi^A = (\Phi^A, \Gamma_k[\Phi,\Phi^*]), \quad \delta_r\Phi^*_A = (\Phi^*_A, \Gamma_k[\Phi,\Phi^*]).
\]

The cutoff dependent BRS transformation appeared earlier in a different context\cite{10}. The author took the viewpoint to finely tune the effective action for $k \neq 0$ with gauge non-invariant terms so that it satisfies the usual WT identity in $k \to 0$ limit. A series of papers followed to confirm this point of view perturbatively for various models\cite{11}. The “modified Slavnov-Taylor identity” and its flow equation are elegantly summarized in \cite{14}. However the presence of the exact BRS symmetry had not been understood.

Here we have seen that the transformation may be defined with the master equation in the antifield formalism, and the WT identity $\Sigma_k = 0$ is not a broken but exact identity. In the next subsection we shall give a simple model of the renormalized BRS symmetry for the Maxwell theory, where the above stated properties can be confirmed explicitly.

### 3.2 Abelian gauge symmetry

Let us consider the gauge-fixed Maxwell action in D=4 Minkowski space,
\[
S_0[\varphi,\varphi^*] = \int \left[ -\frac{1}{4} F^2 + B(\partial \cdot A + \frac{\alpha}{2} B) + i \partial^\mu \bar{c} \partial_\mu c + \varphi^* T \delta \varphi \right],
\]
where
\[
\varphi \equiv \begin{pmatrix} A_\mu \\ c \\ \bar{c} \\ B \end{pmatrix}, \quad \delta \varphi = \begin{pmatrix} \partial_\mu c \\ 0 \\ iB \\ 0 \end{pmatrix}, \quad \varphi^* \equiv \begin{pmatrix} A^*_\mu \\ c^* \\ \bar{c}^* \\ B^* \end{pmatrix}.
\]

The microscopic action $S_0$ satisfies the (classical) master equation, $(S_0, S_0) = 0$, for the antibracket defined in terms of $\varphi$ and $\varphi^*$: the $\varphi^*$ is the set of the antifields at the microscopic level\cite{3}. The macroscopic fields,
\[
\Phi \equiv \begin{pmatrix} A_\mu \\ c \\ \bar{c} \\ B \end{pmatrix},
\]
\footnote{Note that the BRS transformation in (3.4) is defined by the right derivative: $\delta \varphi^A = (\varphi^A, S_0)$.}
have an effective action defined in the relation,

\[ e^{i\Gamma_k[\Phi, \varphi^*]} = \int D\varphi e^{iS_k[\varphi, \varphi^*]}, \] (3.7)

where

\[ S_k[\varphi, \Phi, \varphi^*] \equiv S_0[\varphi, \varphi^*] + \frac{1}{2}(\Phi - f_k \varphi)^T R_k (\Phi - f_k \varphi) + \] (3.8)

with

\[ R_k(p) \equiv M_k^2(p) \begin{pmatrix} g^{\mu\nu} & i \\ -i & 1/\mu_k(p) \end{pmatrix}. \] (3.9)

We have chosen the blockspin kernel, the gaussian term, like a mass term: both \( M_k(p) \) and \( \mu_k(p) \) have the dimension of mass.

All the terms are bilinear so that we may obtain the macroscopic action explicitly,

\[ \Gamma_k[\Phi, \varphi^*] = \frac{1}{2}(\Phi^T, K_T[\varphi^*]) \begin{pmatrix} R_k - f_k^2 R_k D^{-1} R_k & f_k^2 R_k D^{-1} \\ f_k^2 R_k D^{-1} & -D^{-1} \end{pmatrix} \begin{pmatrix} \Phi_+ \\ K_+[\varphi^*] \end{pmatrix}, \] (3.10)

where \( D(p) \) is the matrix defined in the relation, \( S_0[\varphi, \varphi^*] = \frac{1}{2}\varphi^T (D - f_k^2 R_k) \varphi_+ + \varphi^* T \varphi_+ \), and \( K_+[\varphi^*] \) are the compact notations for the following vectors,

\[ K_{\pm}[\varphi^*] \equiv \begin{pmatrix} 0 \\ -i p \cdot A^*(\pm p) \\ 0 \\ i \bar{c}^*(\pm p) \end{pmatrix}. \]

Since \( \Delta \Gamma_k = 0 \), one obtains the renormalized BRS transformation for \( \Phi \) as

\[ \delta_r A_\mu(p) = f_k \frac{\partial \Gamma_k}{\partial A^*(-p)} = i p_\mu a(p)(f_k M_k)^2 C(p), \]

\[ \delta_r \bar{C}(p) = f_k \frac{\partial \Gamma_k}{\partial \bar{c}^*(-p)} = (f_k M_k)^2 b(p) \left[ p \cdot A(p) - i \left( \frac{f_k M_k}{\mu_k} \right)^2 B(p) - f_k \bar{c}^*(p) \right], \]

\[ \delta_r C(p) = \delta_r B(p) = 0, \] (3.11)

where

\[ a(p) \equiv \frac{1}{p^2 - (f_k M_k)^2}, \quad b(p) \equiv \frac{1}{p^2 - (f_k M_k)^2[\alpha + (f_k M_k/\mu_k)^2]} \]

In spite of the non-locality and the operator mixing, the renormalized BRS transformation is nilpotent on \( \bar{C}(p) \), which may be easily confirmed once we take account of the transformation of the antifield \( \bar{c}^*(p) \),

\[ \delta_r \bar{c}^*(p) = -f_k \frac{\partial \Gamma_k}{\partial \bar{c}^*(-p)} = if_k M_k^2 a(p) p^2 C(p). \] (3.12)
With similar calculations, one can obtain renormalized BRS transformations of other antifields and explicitly observe their (off-shell) nilpotency.

The above mentioned complication of \( \delta_r \) is partly caused by the bilinear term in the antifields appeared in \( \Gamma_k \). A canonical transformation may be used to make the action linear in antifields, which will not be discussed further.

## 4 Renormalized global symmetry

We now discuss other global symmetry. In this case, the r.h.s. of (2.5) does not vanish in general. Therefore, to obtain the WT identity we need to set \( K = 0 \) after taking the functional derivatives in (2.5):

\[
\Sigma_{ka}[\Phi] = \left[ \text{Str} \left( f_k \frac{\partial \Gamma_k}{\partial \Phi_-} \left( \frac{\partial \Gamma_k}{\partial K_+} \right)^T \right) + \Delta_a \Gamma_k \right]_{K=0} = 0. \tag{4.1}
\]

This is an algebraic relation to hold for any \( \Phi \). Unlike the BRS symmetry, we have no natural bracket structure\(^4\). Yet, since the operator \( X \) contains no functional derivatives with respect to \( K \), the WT flow equation (2.9) is unchanged: it persists again along the RG flow. Thus, the quantum master equation (4.1) ensures the presence of renormalized global symmetry. In the absence of the \( \Delta_a \Gamma_k \) term, the transformation for the renormalized symmetry is given by

\[
\delta_r \Phi^A(p) = f_k(p) \left[ \frac{\partial \Gamma_k}{\partial K_A^a}(-p) \right]_{K=0}. \tag{4.2}
\]

We now apply our formalism to the chiral symmetry. Let \((\psi, \overline{\psi}), (\Psi, \overline{\Psi})\) be microscopic and macroscopic fermion fields, respectively. We introduce the sources \((K, K)\) for the variations \(\delta \psi = i \gamma_5 \psi, \delta \overline{\psi} = i \overline{\psi} \gamma_5\). The macroscopic action is given by

\[
e^{-\Gamma_k[\psi, \overline{\psi}, \Psi, \overline{\Psi}, K, K]} = \int D\psi D\overline{\psi} e^{-S_k[\psi, \overline{\psi}, \Psi, \overline{\Psi}, K, K]},
\]

where

\[
S_k[\psi, \overline{\psi}, \Psi, \overline{\Psi}, K, K] = S[\psi, \overline{\psi}] + (\overline{\Psi} - f_k \overline{\psi}) - \alpha_k (\Psi - f_k \psi)_+ - \overline{\psi} \gamma_5 K - K \gamma_5 \psi.
\]

\(\alpha_k\) is a function of \(k\) and \(p\). The gaussian contains linear terms in \(\psi\) and \(\overline{\psi}\):

\[
- \alpha_k f_k \left[ (\overline{\Psi} - (\alpha_k f_k)^{-1} K \gamma_5) \psi_+ + \overline{\psi}_- \left( \Psi + (\alpha_k f_k)^{-1} i \gamma_5 K \right)_+ \right]. \tag{4.4}
\]

\(^4\)For a Grassmann odd symmetry such as supersymmetry, however, we may define a bracket in the space of \((\Phi^A, K^a_A)\).
Generically these are the only terms which act effectively as sources for \((\psi, \overline{\psi})\) in the path integral. If we assume the macroscopic action to be bilinear in the macroscopic fermions, it takes the form,

\[
\Gamma_k[\Psi, \overline{\Psi}, K, \overline{K}] = (\overline{\Psi} - (\alpha_k f_k)^{-1} \overline{K} i \gamma_5) (D - \alpha_k)(\Psi + (\alpha_k f_k)^{-1} i \gamma_5 K) + \overline{\Psi} - \alpha_k \Psi,
\]

where \(D\) denotes the Dirac operator for the macroscopic fields, defined as the coefficient of \(\Psi - \Psi^+\). Then the master equation (4.1) gives

\[
\Sigma_k[\Psi, \overline{\Psi}] = i \overline{\Psi} - \left[ D \gamma_5 (1 - \alpha_k^{-1} D) + (1 - \alpha_k^{-1} D) \gamma_5 D \right] \Psi^+ = 0 \quad (4.6)
\]

where we have used \(\text{tr}\{\gamma_5, D\} = 0\), which is legitimate in the absence of chiral anomalies. One obtains in this way the continuum analog of the GW relation:

\[
\{\gamma_5, D\} = 2 \alpha_k^{-1} D \gamma_5 D. \quad (4.7)
\]

Since the second term in (4.1) vanishes owing to \(\text{tr}\{\gamma_5, D\} = 0\), the chiral transformation on the macroscopic fields is readily given by

\[
\delta_r \Psi = f_k \partial \Gamma_k \partial K = i \gamma_5(1 - \alpha_k^{-1} D)\Psi, \\
\delta_r \overline{\Psi} = f_k \partial \Gamma_k \partial K = i \overline{\Psi}(1 - \alpha_k^{-1} D)\gamma_5, \quad (4.8)
\]

which is nothing but the Lüscher’s symmetry transformation. For the chiral symmetry, therefore, the master equation \(\Sigma_k = 0\) is identified with the GW relation. The flow equation (2.9) tells us that it persists along the RG flow. The Lüscher’s symmetry turns out to be the renormalized symmetry realized on the flow. It is probably worth pointing out that the variants in Lüscher’s symmetry are naturally understood in our formulation: an arbitrary vector perpendicular to \(\partial \Gamma_k / \partial \Phi^A\) may be added to the transformation since it does not change the condition \(\Sigma_k = 0\).

So far we have discussed for \(\delta_a S[\varphi] = 0\). Before closing this section, let us consider briefly the microscopic action with some non-invariant terms, \(\delta_a S[\varphi] \neq 0\). The presence of \(\delta_a S[\varphi]\) gives a new contribution in (2.5). Now \(\Sigma_{ka}[\Phi]\) does not vanish even after taking \(K = 0\). It should be remarked however that the non-vanishing term defined by

\[
\sigma_{ka}[\Phi] = -\langle \delta_a S[\varphi] \rangle_k \quad (4.9)
\]

still satisfies the flow equation

\[
\partial_k \sigma_{ka} = (e^{\Gamma_k X} e^{-\Gamma_k}) \sigma_{ka} - e^{\Gamma_k} X (e^{-\Gamma_k} \sigma_{ka}). \quad (4.10)
\]

This equation gives us some important information on the RG flow of the couplings for the non-invariant terms. It is straightforward to extend the eqs. (4.9) and (4.10) to the case of the BRS symmetry.
5  Summary

We have shown that a symmetry, not compatible with a given regularization, may survive exactly along the RG flow. The concrete realization of the symmetry reflects deformation due to the regularization. Naturally it reduces to the usual form in the $k \to \infty$ limit. In this letter we have presented a general formalism based on the “average action”, a continuum cousin of the blockspin transformation. The WT identity for the renormalized symmetry takes the form of the master equation, from which we may read off the associated transformation on the macroscopic fields.

The Maxwell theory was found to be a simple yet instructive example to understand the renormalized BRS transformation. As a result of the blockspin transformation it became non-local but still kept the off-shell nilpotency, as it should from our general argument. For the chiral symmetry in a continuum theory, we have identified the GW relation with the WT identity $\sum_k = 0$. Our formalism naturally leads us to identify the renormalized chiral symmetry with the Lüscher’s symmetry. This is regarded as another non-trivial example of the renormalized symmetries.

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