Feasibility of a metamagnetic transition in correlated systems

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Abstract

The long-standing issue of the competition between the magnetic field and the Kondo effect, favoring, respectively, triplet and singlet ground states, is addressed using a cluster slave-rotor mean-field theory for the Hubbard model and its spin-correlated, spin-frustrated extensions in two dimensions. The metamagnetic jump is established and compared with earlier results of dynamical mean-field theory. This approach also reproduces the emergent super-exchange energy scale in the insulating side. A scaling is found for the critical Zeeman field in terms of the intrinsic coherence scale just below the metal–insulator transition, where the critical spin fluctuations are soft. The conditions required for metamagnetism to appear at a reasonable field are also underlined. Gutzwiller analysis on the two-dimensional Hubbard model and a quantum Monte Carlo calculation on the Heisenberg spin system are performed to check the limiting cases of the cluster slave-rotor results for the Hubbard model. Low-field scaling features for magnetization are discussed.

Keywords: metamagnetism, scaling behavior, slave rotor

(Some figures may appear in colour only in the online journal)

1. Introduction

The question of metamagnetism in strongly correlated systems is a much-studied subject. In the absence of applied magnetic field, the Hubbard model shows a metal–insulator transition (MIT) driven by local correlation or doping [1, 2]. The physics close to the MIT is likened to the formation and subsequent quenching of the moments in the Kondo impurity model [3]. The connection between the two is borne out in dynamical mean-field theory (DMFT), where the correlated lattice model is mapped onto an impurity model, exact in infinite dimensions. The quenching of the local spin fluctuations leads to Fermi liquid (FL) behavior at low temperatures, as shown in [4]. This self-consistent emergence of a low energy coherence is typical of strongly correlated systems. Close to a Mott transition, such systems show a residual antiferromagnetic (AF) exchange between local moments. How an external magnetic field interacts with the moments, especially in the correlated metallic regime where spin fluctuations are still extant, is a question of considerable interest. The experimental observation of metamagnetic transition (MMT) in a single crystal of bilayer perovskite metal Sr₂Ru₂O₇ [7, 8], an insulating magnet BiMn₂O₅ [9, 10], the multiferroic hexagonal insulator HoMnO₃ [11] and heavy fermions like MnSi [12] and CeRu₂Si₂ [13] has rekindled interest recently.

Gutzwiller approximation (GA)-based approaches [5, 6] motivated by the question of MMT in He³ showed that there is indeed an MMT beyond a critical correlation (lower than the critical Hubbard–U for MIT). The presence of an MMT is, however, contradicted by Weigers et al [14] who found a smooth variation of magnetization with applied field, in tune...
with Stoner’s [15–18] approach. However, Stoner’s theory, essentially a high temperature approach, underestimates the local correlation and misses the competition between the local moments and the magnetic field. The GA, on the other hand [5], is incapable of describing the ground state of the correlated metallic phase properly. However, it is only a theory for the ground state and neglects spin correlations entirely. Theoretical techniques [2, 19, 20] have been brought to bear upon this problem recently in order to understand it using more powerful tools. The emergence of DMFT has seen a major paradigm shift in the study of strongly correlated systems. These calculations reveal the presence of MMT in a half-filled Hubbard model. As DMFT captures the dynamics close to the transition in great detail [2] and treats the local spin fluctuations better, the corresponding low energy scale naturally emerges. In a weakly correlated metal, a smooth transition is observed from an unpolarized metal to a polarized band insulator. In the strong coupling limit (close to $U_0$), the phenomenon is distinctly different, though—showing a metamagnetic jump that drives the system from a strongly correlated metal to a field-driven band insulator. DMFT, however, could be numerically more intensive, depending on the choice of the impurity solver; moreover, the cluster extension of DMFT (C-DMFT) and the retrieval of the spin-fluctuation energy scale in the insulating side are fairly demanding tasks.

While single-site DMFT and C-DMFT are perhaps some of the most efficient techniques for correlated systems, they have their own frailties in describing the insulating state with magnetic order. Incorporating spin correlations into these approaches is non-trivial. In addition, strongly correlated systems often come up with situations where the spin and charge of an electron appear to behave distinctly and their responses to external probes are quite disparate. Such situations are missed in DMFT-based theories. A natural separation of these two distinct degrees is the key to the slave-rotor (SR) approach. The Hilbert space of the physical electron is decoupled into the so-called ‘chargon’ (conjugate to rotor) and ‘spinon’ spaces and the unphysical states are eliminated via local constraints. The strongly correlated problem then maps onto interacting slave particles self-consistently coupled to a gauge field. The gauge fluctuations, being weak, provide a framework for studying the Mott–Hubbard physics at intermediate to large coupling [22, 23] in a straightforward manner.

This paper uses SR mean-field (SRMF) theory to investigate the MMT within the Hubbard–Heisenberg model. In what follows we consider only the Zeeman field, as the orbital contribution is much weaker. We not only find the MMT, but also address the question of why the metamagnetism (MM) arising out of competition between the Zeeman field and Kondo fluctuations remains so elusive experimentally. We predict a scaling behavior for the critical Zeeman field in terms of the Kondo scale in the strongly coupled metallic regime. A possible experimental realization of the transition in Mott–Hubbard systems is also prescribed. We show that tuning the system to strong coupling (via tensile strain, for example) can act as a precursor for field-driven MMT. We identify the regimes for the observation of MM at the emergent ‘spin-exchange’ energy scale in the insulating side. Possible experimental realization of MM in real materials or optical lattices, out of correlation and external driving field, is the primary focus of this work. Using extensive qualitative and quantitative arguments and a standard semi-analytical technique, we analyze the possible emergence of MM in correlated electronic systems. Indeed, there are various other slave-particle mean-field techniques used in the context of the Hubbard model [25, 27, 29]. As we see below, the SRMF theory gives good results at a nominal numerical cost, particularly in the strong coupling limit, where MM is most likely to be observed.

2. Model and formalism

We consider the $t − t′ − U − J$ model on a square lattice in the presence of external magnetic field. This is a general correlated model without particle–hole symmetry at half-filling, and with AF spin exchange built in. The various models studied below are different limiting cases of this, discussed as we go along.

$$ H = - \sum_{i,j,\sigma} t_{ij} c_i^{\dagger} c_j + U \sum_i n_i^\uparrow n_i^\downarrow + J \sum_{i,j} S_i S_j - h \sum_{\sigma} n_{i,\sigma}, \quad (1) $$

where $t_{ij} = t$, $t'$ are the nearest and next nearest neighbor hopping amplitudes, respectively. $c_i^{\dagger}(c_i)$ is the electron creation (annihilation) operator at a given site. $n_i^\uparrow(n_i^\downarrow)$ is the density operator for the up (down) spin. $J (>0)$ introduces AF spin exchange between spins at neighboring sites, while $h$ is the external Zeeman field. In terms of rotor and spinon operators, this model can be written as (see Flores et al [22] for the SRMF formulation)

$$ H_{SR} = - \sum_{i,j,\sigma} t_{ij} f_{i\sigma}^\dagger f_{j\sigma} e^{-i\phi} e^{i\theta} + U/2 \sum_i n_i^\theta (n_i^\theta - 1) + J \sum_{i,j} S_i^\dagger S_j^\dagger + J \sum_{i,j} S_i^\theta S_j^\theta \quad (2) $$

where $f_{i\sigma}$ is the spinon creation operator and the rotor creation (annihilation) operator is $e^{i\theta}$ ($e^{-i\theta}$). $n_i^\theta$ is the chargon density operator and $S_i^\dagger$ is $f_{i\sigma}^\dagger \sigma_{0 \beta} f_{j\beta}$. In the SRMF approximation the local constraint is relegated to a global constraint satisfied on the average

$$ \langle n_i^\dagger \rangle + \langle n_i^\uparrow \rangle + \langle n_i^\downarrow \rangle = 1. \quad (3) $$

where $\langle n_i^\dagger \rangle$ and $\langle n_i^\sigma \rangle$ are the average chargon and spinon density, respectively. Following straightforward algebra, the Hamiltonian equation (2) decouples into two coupled Hamiltonians solved self-consistently under the saddle-point approximation

$$ H_f = - \sum_{i,j,\sigma} t_{ij} B_{ij} f_{i\sigma}^{\dagger} f_{j\sigma} + J \sum_{i,j} S_i^\dagger S_j^\dagger - (\mu_f + h \sigma) n_{i,\sigma} \quad (4) $$
\[ H_0 = -2 \sum_{i,j,\sigma} t_{ij} \chi_{ij}(e^{-i\phi}e^{i\theta} + U/2\sum_i (n_i^\sigma)^2 - \mu_\theta \sum_i n_i^\sigma \] (5)

where \( B_0 = (e^{-i\phi}e^{i\theta})_0 \) and \( \chi_{ij} = \langle f^\dagger_{ij} f_{ji} \rangle_f \). \( \mu_f, \mu_\theta \) are Lagrange multipliers for the number constraint: two multipliers are generally used for convenience to control \((n_i^\sigma)\) and \((n_i^\sigma)\) separately while still satisfying equation (3). In the presence of a spin density wave (SDW) with a commensurate ordering wave vector \( \mathbf{Q} = (\pi, \pi) \), the mean-field spinon Hamiltonian is

\[ H_f^{\text{MF}} = \sum_{k,\sigma} \varepsilon_{k,\sigma} f^\dagger_{k,\sigma} f_{k,\sigma} - 2Jm \sum_k (f^\dagger_{k,\mathbf{Q}} f_{k,\mathbf{Q}} - f^\dagger_{k,\mathbf{Q}} f_{k,\mathbf{Q}}) \] (6)

where

\[ \varepsilon_{k,\sigma} = -2(tB + 3J\chi/4)(\cos k_x + \cos k_y) - 4t' B' \cos k_x \cos k_y - \mu_f - \eta \sigma \] (7)

and \( \chi_{k,\sigma} = \chi_{F,(k,\sigma)} + \chi_{F,(k,\sigma)} + \chi_{F,(k,\sigma) - (k,\sigma)} \). At half-filling, the corresponding self-consistency equations for the spinon sector are easily obtained and the magnetization is

\[ M = \frac{1}{2} \sum_{k,\sigma} \sigma \langle n_{F}(-\lambda_{k,\sigma}) \rangle + \langle n_{F}(\lambda_{k,\sigma}) \rangle \] (8)

where \( \chi, \chi' \) are, respectively, the spinon kinetic energies for the nearest and next nearest neighbor hoppings, and \( m \) is the staggered magnetization.

3. Results and discussions

To begin with, we briefly discuss our results on the Hubbard model at half-filling at \( t = 0 \) (all energies are given in units of \( t \)) and identify the Mott transition in SRMF. MMT results in this limit exist in DMFT \([2, 20]\) and a comparison of the same using SRMF theory is therefore in order. To date, no MMT results for this model that use SRMF theory (or any other slave-particle method) are available. We begin by studying the single-site and cluster approaches for the Hubbard Hamiltonian with and without field, respectively. We also observe how the spin-spin exchange interferes with correlation and how the spin fluctuations are affected in the proximity of MIT.

3.1. Hubbard \((t - U)\) model on a square lattice: MIT and metamagnetism

It is well-known that in the \( t - U \) model the local correlation drives the paramagnetic metal to a non-magnetic insulator in a Brinkman–Rice-like transition (the order parameter \( \phi \) going smoothly to zero (figure 1, left)). The divergence of the effective mass is signalled by the vanishing of the QP weight \( (Z = \phi^2) \) at the critical point. Although the nature of the insulating or metallic phase remains non-magnetic (unless we resort to a two-sublattice formalism similar to that of DMFT), irrespective of site or cluster analysis in the SR calculations, we note a different critical value of \( U/t \) for MIT in the site and cluster SRMF approach. As spatial correlations and non-local phase fluctuations are accounted for in the cluster, the critical \( U/t \) for MIT in a cluster approach is expected to be lower. The parameter \( B \) quantifying non-local fluctuations remains finite in the putative insulating phase and approaches zero in the \( U/t \rightarrow \infty \) limit. For single-site analysis, we find \( U_{c,\text{site}} = 6.483 \) while a two-site cluster gives a critical \( U_{c,\text{bond}} = 6.214 \) for MIT. As far as the reduction in \( U_{c} \) is concerned, \( B \) plays much the same role as a non-local spin–spin correlation. In the insulating phase, \( B \) is non-zero (figure 1, right) and effective hopping remains finite, i.e. the effective mass does not diverge at MIT. In the cluster extension of the
theory, therefore, spinons have a Fermi surface with finite Luttinger volume [28] even in the insulating phase.

We note that the value $U_{\text{c,cluster}}$ for MIT that we find is in good agreement with the C-DMFT prediction [21] of $U_{\text{c}} = 6.050t^6$. However, the AF nature of the insulating ground state of the half-filled Hubbard model remains beyond the scope of SRMF approximation. The paramagnetic insulator in the SRMF approximation is an artefact of the assumption that the correlation acts in the rotor sector only, i.e. on the charge degrees alone; spins are free, having only a renormalized effective mass. There is no a priori reason, therefore, why charge ordering would lead to spin ordering. As a consequence, the SRMF approach works better for systems with strong magnetic frustration, where a spin liquid insulator is likely. Both the single-site and cluster Mott transitions have a Brinkman–Rice nature. The QP weight goes to zero at critical Hubbard $U (U_{\text{c}})$ in a continuous fashion, and $Z$ scales perfectly with $1 - (U/U_{\text{c}})^2$ (figure 1-right panel).

3.1.1. Nature of MMT: single-site analysis. How the system reacts to an external magnetic field, for the whole range of $U$ (up to $U_{\text{c,site}}$), is shown in the phase diagram (figure 2(a)). For any finite $U$, the ferromagnetism shows a first order jump to its saturation value at some critical field, clearly an MMT, instead of a smooth enhancement to magnetic saturation (as predicted originally by Stoner and extended later by Weigers et al [14] using spin fluctuation theory). This abrupt jump in magnetization leads to an MIT, the order parameter going to zero ($\phi = 0$) in a highly discontinuous manner at the same instant. This is a field-driven first order transition, rather than being correlation driven. The field moves the up and down spin bands apart, leading to a weakly correlated, polarized band insulator [20]. The closer one approaches to the critical Coulomb repulsion, the more it becomes susceptible to such a transition at a lesser field (figure 2(b)). Physically, at such large values of $U$, the kinetic degrees of freedom are nearly quenched and effective mass is large—the strongly correlated metal is now susceptible to MIT. The presence of MM is confirmed for every finite $U < U_{\text{c}}$.

3.1.2. Nature of MMT: a two-site cluster analysis. In the single-site theory, the insulating ground state becomes ferromagnetic for an infinitesimal Zeeman field. The rotor Hamiltonian being local, there is no magnetic exchange scale. This is a well-known pathology of the single-site approximation in SR. In principle, one should look for the competing dynamics between an aligning field and spin fluctuation, recovered in the cluster version of the theory (figure 2(c)). Two diagrams, site and cluster, are plotted (figures 2(b) and (d)) to showcase the difference between the two schemes. The MMT in the metallic state is almost similar in the two cases—for any $U$ there is a transition. In the insulating state, however, any infinitesimal $h$ causes saturation in magnetization ($M$) in the single-site case, while a finite critical magnetic field (figure 3, main panel) representing the emergent AF spin-correlation scale $h^2/U$ (figure 1-right panel) is required for the cluster. The putative insulating state, where $\phi$ becomes zero, has interesting dynamics via inter-site correlation $B$ within the cluster approximation; the rotor is still coupled to the spins and the renormalized kinetic energies are finite in the insulating phase. In both cases, therefore, the vanishing of the spin-stiffness ($\chi$) signifies the metamagnetic jump: in the insulating region $h/U = 0$ in the single-site case, while for the cluster, $h_{\text{c}}/U$ has to be finite (figure 3, main panel) to overcome the spin-correlation scale.

3.1.3. Scaling behavior for critical Zeeman field. We note that very close to the transition, for correlated models $k_{\text{F}}(U)/Z$ becomes independent of $U$. A strong Hubbard correlation of the order of $0.97U_{\text{c}}$–$0.995U_{\text{c}}$ shows that this ratio becomes a constant (figure 3, inset); and it is only within this range that $h_{\text{c}}$ has experimentally feasible values. It is interesting to note that $Zr^4$ measures the effective renormalized bandwidth of the dispersive QPs. The strong coupling limit of the problem renormalizes this number to a significantly smaller one, a measure of the diminishing coherence of the QPs. The $h_{\text{c}}$ in the strong coupling limit is the field required to destroy the coherence and favor a triplet spin state. Beyond this scale, the spin dynamics is determined only by $h_{\text{c}}$, thereby making $k_{\text{F}}(U)/Z$ universal, irrespective of the value of $U$. We find a

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**Figure 1.** Left: $\phi$ is plotted against $U/t$ for single-site (red dots are data points and the black continuous curve is a fit to the Brinkman–Rice picture) and cluster approximations (black dots are data points and the continuous green line is the Brinkman–Rice fit). The critical values of $U/t$ for MIT are 6.483 and 6.214 in single-site and cluster analyses, respectively. Right: the inter-site correlation from the cluster analysis in the insulating phase (red dots) and the fit to $t^2/U$ (black curve).

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There are two critical correlations in DMFT calculations [2], $U_{\text{c,1}}$ and $U_{\text{c,2}}$. The SRMF approach has only one critical $U$, which is physically the same as the $U_{\text{c,2}}$ of DMFT.
similar scaling feature for \( h_c \) in higher-dimensional bi-partite lattices. The scaling should hold true for bi-partite lattices in any dimension, as the hopping is renormalized by \( \sqrt{d} \) factor\( (t' = t/\sqrt{d}) \), where \( d \) is the dimension of the lattice. Hence the scaling is related to the competition between the coherence of the QPs and the external agent (field) trying to destroy the coherence and is independent of the dimensionality of the non-interacting bath.

3.1.4. Comparison with single-site DMFT, the GA and the quantum Monte Carlo (QMC).

In earlier studies of MMT [2, 5, 20], the value of the field \( (h) \) was limited below the hopping parameter \( t \). In such a situation, they looked for the value of \( U \) for which an MMT can be observed. The SRMF approach gives a critical field for MMT at any finite \( U \). For \( h < t \), we can compare our results with the previous work. For \( T = 0 \), we compare the value of \( U/U_c \) below which there is no MMT (as \( h \) rises to \( t \)): our single-site SRMFT gives a value close to 0.5, while in the GA it is about 0.44 [5] and in DMFT, it is about 0.61 [2, 20]. We also compare the GA with SR MFT. Vollhardt [5] studied the effect of an out-of-plane Zeeman field on the Hubbard model with a flat band using the GA. While we choose to do the same for a square lattice dispersion, an exact analytical solution is not possible in this case. The value of \( U_c \) for the Hubbard model on a square lattice is about 6.451 in the GA and with single-site SRMFT we find it to be 6.483. The agreement between these

Figure 2. Variation of magnetization showing the MMT in the site ((a), (b)) and cluster ((c), (d)) approximations. \( M - h/t \) plots close to the transition region of the site and cluster theories are shown separately in (b) and (d), respectively.

Figure 3. \( h_c/t \) in log scale against \( U/t \) to highlight the huge scale variations for the critical numbers in the site and cluster analyses (significantly different, not apparent from the phase diagram in figure 2). The inset shows that \( h_c \), scaled with the renormalized hopping, is constant with renormalized correlations.
two results motivates us to verify the critical value of the field \( h_c \) that induces a metamagnetic jump for a given \( U \) within the GA. This can be evaluated in two ways via the GA: plot magnetization versus field and search for the metamagnetic point, or find the point of flipping of the absolute minimum in the ground state energy \( (E_g) \). The \( E_g \) in the GA is calculated for the square lattice semi-analytically and it is a function of double occupancy \( (d) \), magnetization \( (M) \) and applied field \( (h) \). On the minimization of energy, an expression for \( m \) as a function of \( h \) and \( d \) is obtained. Putting it back in \( E_g \), the minimum is located numerically. The plot of \( E_g \) against \( d \) shows two minima, one at zero and the other at a finite value of \( d \) (figure 4). The minimum at non-zero \( d \) remains the absolute minimum up to some critical field. At a certain \( h_c \), the absolute minimum flips from a finite value to \( d = 0 \) (figure 4).

For \( U/U_z = 0.75 \), the critical value of field comes out to about 0.180. The value obtained from the SR analysis under the same set of parameters is 0.20, in reasonably good agreement with the GA. This similarity in the relevant scales emerging out of the GA and SRMFT is expected, as both of them work well in the strong coupling limit.

Finally, we perform a QMC analysis on an AF Heisenberg spin model on a square lattice, in the presence of perpendicular Zeeman field. On increasing the lattice sizes, we check that the \( M - h \) response for the system remains nearly the same for \( L = 24, 32 \) and 40 (where \( L^2 \) is the lattice size). We find the exponent \( (\beta) \) for magnetization against field for the \( L = 32 \) system and compare the number with the exponent \( (\delta) \) (figure 5) in the large-\( U \) limit of the Hubbard model. The exponents seem to agree well, which is expected since the Heisenberg model is the large-\( U \) limit of the Hubbard model at half-filling. Although the numerically found exponents from the two techniques are slightly different from one (1.02 and 0.98), within our numerical accuracy, it could well be that they are, in reality, just 1.0 in the low-field limit.

3.2. t – U – J model on a square lattice

As expected, our analysis did not produce an AF insulator in the Hubbard model without the explicit AF spin exchange necessary to generate it. Therefore we used the \( t – U – J \) model to search for a long-range spin-ordered state. It is interesting to note that an AF spin coupling \( J = t/4 \) causes a discontinuous transition from a uniform metallic state to an AF ordered insulating state in the case of a square lattice [22, 23] in the absence of field. The metallic state has a non-zero staggered magnetization \( (m) \), which decreases as the AF spin exchange decreases.

In this context, we may note that the picture is quite different for a triangular lattice: a magnetic order (the classical Neél state) and a sudden drop in QP weight appear at the same point (figure 6) [23]. The triangular lattice has no nesting, and no particle–hole symmetry at half-filling, and mitigates staggered magnetization in the metallic state. The square density of states, on the other hand, has a logarithmic divergence at

\[ \frac{1}{d} \]

and equals \( 0.20 \), in reasonably good agreement with the GA. This similarity in the relevant scales emerging out of the GA and SRMFT is expected, as both of them work well in the strong coupling limit.

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3.3. t’ – U – J model on a square lattice

The single-site analysis of \( t – t’ – U – J \) model on the square lattice shows a first order MIT at \( U_z = 5.131t \) in the absence of magnetic field. Switching over to a two-site cluster drives the critical correlation to a larger value 5.653t. In the single-site case the staggered magnetization saturates at the MIT and in the insulating side the spin and charge dynamics get quenched completely. In the cluster extension, however, the insulating state still has non-local phase fluctuations that lead to a finite spin stiffness and therefore the staggered magnetization never saturates. The finiteness of \( B \) for large correlation values implies that the AF order does not saturate. Hence, in the insulating side, on the application of external magnetic field, a minimum \( h = J \) is needed to destroy the AF order and revive ferromagnetism in the single-site theory. The metallic side of the problem in the single-site theory remains featureless, except that there is no AF metal now. A strong local correlation very close to MIT, however, competes against this frustration and produces a weakly AF metal.

The cluster analysis reveals interesting physics in both the metallic and insulating regimes. While the MIT becomes second order in this case, the Zeeman field competes with the exchange energy \( J \) as well as the emergent super-exchange
scale in the insulating side. However, the MMT for all finite values of local correlation still survives for both site and cluster analysis. The $M - U - \hbar$ phase diagram (figure 7) from the cluster analysis of the present model, zoomed in around the MIT region, reveals interesting details beyond the critical local correlation. There are two distinct jumps in the magnetization for $t = 0.85, U = 5.75$. These two manifestly separate locations of the jump correspond to the competition between two different scales with the external Zeeman field: the first one is where antiferromagnetism gets suppressed and ferromagnetism shows up, and the second one is where the spins are ferromagnetically saturated at an energy scale given by the residual kinetic energy ($\sim Bt / 2U$).

4. Experimental realizations

Metamagnetism has of late become a highly recurrent [7, 9–13, 26] phenomenon. The perpendicular or in-plane field required for such discontinuous, super-linear transitions in magnetization can be under 10 Tesla. All the strongly correlated systems mentioned earlier show MMT between 1–10 Tesla fields. However, the MM in each of these is materialspecific and often associated with some structural transitions. The role of correlation is not clear in most of them, while our concern is solely MM out of correlation, where the competition between the applied field and the local spin fluctuation is the key in driving the non-linear magnetization and the consequent first order jump.

On the long-standing issue of MM in liquid He$^3$, Georges and Laloux [19] hold the view that liquid He$^3$ should be viewed as a Mott–Stoner liquid and the Hubbard model with
about 8% vacancy offers a reasonable description of it. They predict \(\text{MMT} \atop \text{at about 26 bar in an 80 Tesla field} \), Weigers [14], however, did not find a metamagnetic jump in liquid \(\text{He}^3\) up to 200 Tesla. This may indicate [19] that \(\text{He}^3\) cannot be modelled by a half-filled Hubbard model. Vollhardt [5] puts liquid \(\text{He}^3\) in the intermediate coupling regime with \(U/E_F\) in the range of less than one (typically 0.8 or less). As we have shown above, the fields required for MMT are amenable only in the strong coupling regime and therefore Vollhardt’s estimates would imply that liquid \(\text{He}^3\) is not an ideal candidate for the observation of correlation-driven MMT within an accessible laboratory field.

The problem, therefore, is to find a material that can be modelled well by the Hubbard model or any of its extended incarnations with the desired range of parameters. In correlated systems the bare value of \(k_BT_F\) (\(T_F\) is Fermi temperature) is nearly 1–5 eV, which \((T_F)\) scales down to \(k_BT_F = Zt^*\) close to the Mott MIT. At exactly this range of parameters, as we showed, \(h_c\) for MMT becomes <100 T. The conclusion is driven by the fact that for \(t = \) 1 eV, in the site analysis, we obtain \(h_c/ht < 0.01\), which is equivalent to a field of 100 Tesla or less. We find that for a system with \(ht/ht\) of the order of 0.99, the typical value of the critical Zeeman field, by a similar analysis, would be about 20 Tesla \((h_c/ht < 0.002)\). A strongly correlated system which can be reasonably modelled by the single band Hubbard model, with its effective correlation \(U/U_c\) tuned (by pressure, for example) somewhere between 0.97–0.99, should, therefore, show an MMT within a reasonable magnetic field. It is also likely that cold atom systems in optical lattices can provide us with the option of observing this in the laboratory. The scaling analysis we discuss here, therefore, underlines the fact that systems with narrow correlated bands or orbital selectivity (in multi-orbital situations, for example) can facilitate MMT at an accessible field.

5. Conclusions

A ground state analysis of the Hubbard model and its spin-correlated, spin-frustrated versions has been performed in this paper in search of MM. The response to an externally applied Zeeman field in the SR mean-field formalism shows that there is indeed a regime of parameters where an MMT to a ferromagnetic state occurs for all finite local Hubbard correlations. However, it is clear that to observe MM in a laboratory field one would need to tune the system to a very narrow range close to the Mott transition and apply a fairly large magnetic field. These conditions make the observation of MM in such systems so elusive. On a fundamental level, on the other hand, one would like to know what happens to the spin-fluctuation and Kondo scales in a metal in the presence of a strong field favoring spin alignment. Clearly, it is the sharpest Kondo resonance peak, with the narrowest width (requiring close proximity to the Mott transition), that is most sensitive to external field and leads to the non-linear jump in magnetization and the consequent MMT.

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