In this letter we present a new N-flation model constructed by making use of multiple scalar fields which are being described by their own DBI action. We show that the dependence of the e-folding number and of the curvature perturbation on the number of fields changes compared with the normal N-flation model. Our model is also quite different from the usual DBI N-flation which is still based on one DBI action but involves many moduli components. Some specific examples of our model have been analyzed.

N-flation from multiple DBI type actions

Yi-Fu Cai\textsuperscript{a*} and Wei Xue\textsuperscript{b,a†}

\textsuperscript{a} Institute of High Energy Physics, Chinese Academy of Sciences, P.O.Box 918-4, Beijing 100049, P.R.China
\textsuperscript{b} School of Physics, Peking University, Beijing 100871, P.R.China

Inflation\textsuperscript{1,2} naturally resolves the flatness, homogeneity and primordial monopole problems, and predicts a scale-invariant curvature spectrum consistent with current cosmological observations\textsuperscript{3} very well. So it has become the prevalent paradigm to understand the initial stage of our universe. However, an inflationary model with a single scalar generally suffers from fine tuning problems on the parameters of its potential, such as the mass and coupling of this field.

It was firstly noticed by Liddle \textit{et al.}\textsuperscript{4} that, when a number of scalar fields are involved, they can relax many limits on the single scalar inflationary model. Usually, these fields are able to work cooperatively to give long inflationary stage, even none of them can sustain inflation separately. Models of this type have been considered later in Refs.\textsuperscript{5}--\textsuperscript{8}. The main results show that both the e-folding number $N$ and the curvature perturbation $\zeta$ are approximately proportional to the number of the scalars $N$. Later, the model of N-flation was proposed by Dimopoulos \textit{et al.}\textsuperscript{9}, which showed that a number of axions predicted by string theory can give rise to a radiatively stable inflation. This model has explored the possibility for an attractive embedding of multi-field inflation in string theory.

Over the past several years, based on the recent developments in string theory, there have been many cosmological studies on its applications to the early universe, especially to inflation. However, people still often encounter fine tuning and inconsistency problems when they try to combine string theory with cosmology. For example, Kofman and Linde in Ref.\textsuperscript{10} pointed out the deficiency of tachyon inflation: and there exists an $\eta$-problem in slow-roll brane inflation as reviewed in Ref.\textsuperscript{11}; and so on. Facing to these embarrassments, it is usually suggested that N-flation is able to relax these troubles and so can let stringy cosmology survive. A good example is that Piao \textit{et al.} have successfully applied assisted inflation mechanism to amend the problems of tachyon inflation\textsuperscript{12}. There are also many other works on investigating multi-field inflation models in stringy cosmology, for example see Refs.\textsuperscript{13}--\textsuperscript{16}.

Recently, an interesting inflationary model, which has a non-canonical kinetic term inspired by string theory, was studied intensively in the literature. This model is described by a Dirac-Born-Infeld-like (DBI) action\textsuperscript{17,18}. The inflation model with a single DBI field was investigated in detail\textsuperscript{19,20}, which has explored a window of inflation models without flat potentials. In this model, a warping factor was applied to provide a speed limit which keeps the inflaton near the top of a potential even if the potential is steep.

In this letter, we study a multi-field inflationary model, where each field is described by a DBI action and the total action is constructed by the sum of them. Therefore, it is worth emphasizing that our model is different from the usual DBI N-flation in which only multiple moduli fields are involved in one DBI action\textsuperscript{21,22,23}, but ours is constructed by multiple DBI type actions ("DBIs"). This action can be achieved if we consider a number of D3-branes in a background metric field with negligible covariant derivatives of field strengths and we assume that these branes are decoupled from others. Besides, we also need to neglect the backreaction of those branes on the background geometry as is usually done in brane inflation models. In this scenario, the scalars are able to work cooperatively like those in usual N-flation models. However, since their kinetic terms are of non-canonical form, the cumulative effect from multiple fields does not grow in linear form. From our analysis, the e-folding number $N$ is no longer proportional to $N$ but to $\sqrt{N}$ instead, and the curvature perturbation $\zeta$ is approximately proportional to $N^{3/2}$. Thus N-flation of this type shows quite different features from those in the usual N-flation model.

Our model is given by the following action

$$S = \int d^4x \sqrt{-g} \left[ \sum I P_I (X_I, \phi_I) \right],$$  \hspace{1cm} (1)

which involves $N$ scalar fields, with

$$P_I (X_I, \phi_I) = \frac{1}{f (\phi_I)} [1 - \sqrt{1 - 2f (\phi_I)X_I} - V_I (\phi_I)],$$  \hspace{1cm} (2)
where we define \( X_I = -i g^\mu \nu \partial_\mu \phi_I \partial_\nu \phi_I \) and the sign of metric is adopted as \((-\cdots,+,+,+\cdots)\) in this letter. This model involves multiple DBI type actions which give the effective description of D-brane dynamics (for example see Refs. \([18, 24]\)). Considering a system constructed by a number of D3-branes in a background metric field with negligible covariant derivatives of the field strengths and assuming that these branes are decoupled from each other, this system could be described by the above action which has a stringy origin as shown in Ref. \([25]\).

Here the scalar \( \phi_I \) is interpreted as the position of the \( I\)-th brane, and the warping factor \( f(\phi_I) = \tfrac{1}{\phi_I^2} \) is suitable for all scalars when we take on AdS-like throat and neglect the backreaction of the branes upon the background geometry. This assumption can be satisfied when the contribution of the background flux is much larger than that from the branes.

We now define a series of useful parameters, (i.e. the sound speeds), for the scalars

\[
c_{sl} \equiv \sqrt{1 - 2f(\phi_I)x_I} ,
\]

which lead to interesting features of the model. We assume spatial homogenity and isotropy, i.e. take a flat Friedmann-Robertson-Walker metric ansatz \( ds^2 = -dt^2 + a(t)^2 dx^i dx^i \), where \( a(t) \) is the scale factor of the universe. Then Eq. \((3)\) yields

\[
|\phi_I| = \phi_I^2 (\frac{1-c_{sl}^2}{\lambda})^{\frac{1}{2}} .
\]

If \( c_{sl} \sim 1 \), this model returns to the slow-roll version. However, if \( c_{sl} \sim 0 \), then \( |\phi_I| \approx \phi_I^2 / \sqrt{\lambda} \), and in this case there is an interesting relation for all the scalars:

\[
\Delta \phi_I^{-1} = \frac{\Delta t}{\sqrt{\lambda}} ,
\]

which means for a fixed time interval \( \Delta t \), the variations of \( \phi_I^{-1} \) for all the scalar fields are the same.

By varying with respect to the scalar, we obtain the equations of motion:

\[
\ddot{\phi}_I + 3H \dot{\phi}_I - \frac{c_{sl} \phi_I}{c_{sl} \phi_I - c_{sl} P_{I,I}} = 0 ,
\]

where “\( \cdot \)” denotes the derivative with respect to the scalar \( \phi_I \), and \( H \) is the Hubble parameter defined as \( \dot{a}/a \).

As an example, we focus on the case of IR type potential

\[
V_I = V_{0I} - \frac{1}{2} r_I^2 \phi_I^2 .
\]

The first part of the potential \( V_{0I} \) originates from the anti-brane tension from other throat. In IR DBI inflation, D-branes roll from the tip of the brane, thus the potential contains the terms like tachyon. We will assume \( s_I \equiv \frac{1}{\sqrt{g^I}} \) to be small numbers for simplicity, and take the normalization \( 8\pi G = 1 \). Due to the warping factor \( f(\phi_I) \), those scalars are able to stay near the top of their potentials, and so we have \( H^2 \simeq \tfrac{1}{2} \sum I V_{0I} \).

Applying the relation \((5)\), the e-folding number of this multiple field inflation model can be evaluated as follows,

\[
N \equiv \int_0^f H dt \simeq \sqrt{\frac{\lambda}{3}} \sum_I V_{0I} \left( \frac{1}{\phi_I^2} - \frac{1}{\phi_I^4} \right)
\]

\[
\simeq \sqrt{N} \sqrt{\frac{\lambda}{3}} \langle V_0 \rangle \left( \frac{1}{\phi_I^2} \right) ,
\]

(\text{8})

under the assumption \( c_{sl} \sim 0 \). Here we define \( \langle O \rangle = (\sum_I O_I) / N \) which is the average value of the variables \( O_I \), and the subscript “\( i \)” and “\( f \)” represent the initial and final state respectively. Since in IR type models the scalars start rolling on the top of their potentials\(^2\), we have \( \phi_i ^f < \phi_i ^I \), and we can neglect the contribution of \( \phi_i ^f \) in Eq. \((8)\). Furthermore, from Eq. \((8)\) we can deduce that the e-folding number in multiple DBIs model is proportional to the square root of the number of scalars, i.e.

\[
N \propto \sqrt{N} .
\]

(\text{9})

This result is completely different from that obtained in slow-roll N-flation which gives \( N \propto N \). This difference shows that, in the inflationary model constructed by multiple DBI terms, although the fields work cooperatively, the cumulative effect from multiple fields does not grow linearly. This results in a lot of interesting phenomena.

We now investigate the curvature perturbation of N-flation constructed by multiple DBIs. In the calculation, we use the Sasaki-Stewart formulism \([28]\), in which the curvature perturbation on comoving slices can be expressed as the fluctuation of the e-folding number and thus can be given in terms of fluctuations of scalar fields \( \delta \phi_I = \frac{\delta \phi_I}{2\pi} \) on flat slices after horizon crossing. It is given by

\[
P_{\zeta}^4 = \sqrt{\sum_{I,J} N_{I,I} N_{J,J} \langle |\delta \phi_I \delta \phi_J| \rangle}
\]

\[
\simeq N^2 \frac{\sqrt{\lambda}}{6\pi} \langle V_0 \rangle \langle \phi^{-4} \rangle ,
\]

(\text{10})

where we have applied the general relation \( N_{I,I} = \frac{\mu}{\phi_I} \).

This result is consistent with single DBI inflation model when \( N = 1 \), but if one introduces more fields, \( P_{\zeta}^4 \) grows proportional to \( N^2 \) which is more rapid than

\(^{1}\) see Refs. \([20]\) for detailed analysis on this type DBI models.

\(^{2}\) The initial condition of inflation is essential, which is analyzed in \([24, 27]\).
that obtained in normal N-flation (for example see Refs. \[29\] and references therein). From Eqs. \[8\] and \[10\], we can establish the relation between the curvature perturbation and the e-folding number as follows,

\[
P_\zeta = \frac{N^4 N}{4\pi^2 \lambda} \frac{\langle \phi^{-4} \rangle}{\langle \phi^{-1} \rangle^3}. \tag{11}
\]

Moreover, for a set of the above uncoupled fields, we can derive the spectral index as follows,

\[
n_s - 1 = \frac{d \ln P_\zeta}{d \ln k} \approx -2\epsilon - \frac{\sum J_i (s_i + \eta_i)/ (c_s J_i^2)}{\sum J_i 1/c_s J_i^2}, \tag{12}
\]

where we have defined the slow-roll parameters \(\epsilon \equiv -\frac{\dot{H}}{H^2}\), \(\epsilon_I \equiv \frac{\phi_I}{\sqrt{2c_s H}}\), and \(\eta_I \equiv \frac{2\phi_I}{c_s H}\). When there is only one scalar field, the above spectral index returns to the standard form of single DBI model \[31\]. Note that there is a relation \(\epsilon = \sum_I \epsilon_I^2 \approx \sum_I \frac{3s_I^2}{2c_s^2} / \sum_I V_0/\lambda\), and this quantity can be very small when \(\lambda\) is taken to be sufficiently large. And if \(\epsilon \ll 1\), each positive component \(\epsilon_I\) becomes negligible automatically. Explicitly, for the case of IR type potential we considered currently, the spectral index can be given by

\[
n_s - 1 \approx -\frac{4}{N} \frac{\langle \phi^{-1} \rangle}{\langle \phi^{-3} \rangle}. \tag{13}
\]

Although it is hard to judge in general whether the spectral index of our model is redder or bluer than that of its corresponding single scalar model, we may study their question in certain cases. For example, the spectral index coincides with that of the corresponding single field model when all the scalars at the horizon-crossing time have the same value \(\phi_I = \phi_0\).

Now let us consider some specific examples of this model. The simplest case is to choose all the scalars to have the same value: \(\phi_I = \phi_0\) for \(I = 1, \ldots, N\). Therefore we obtain

\[
P_\zeta = \frac{N^4 N}{4\pi^2 \lambda}, \quad n_s = 1 - \frac{4}{N}. \tag{14}
\]

As is known, we need the e-folding number for inflation \(N \simeq 60\) to explain the flatness of our universe. From the above equation, one can easily obtain a scale-invariant spectrum with an amplitude of order \(O(10^{-5})\) (required by cosmological observations, such as Ref. \[3\]) if \(\lambda/N \sim 10^{14}\).

Another interesting example is \(\phi_I = \phi_0 + I \cdot \Delta\) for \(I = 1, \ldots, N\). In order to make this case quite different from the first one, we assume \(\phi_0 \gg \Delta\) but \(N \cdot \Delta \gg \phi_0\). To solve this system, we need to apply the useful expression

\[
\langle \phi^{-1} \rangle = (-)^{l} \frac{\psi^{(l-1)}(1 + \phi_0/\Delta) - \psi^{(l-1)}(1 + \phi_0/\Delta + N)}{(l - 1)!\Delta N}, \tag{15}
\]

where \(\psi^{(l)}(z)\) is the \(l\)-th derivative of the digamma function \(\psi(z) \equiv \Gamma'(z)/\Gamma(z)\). We can use the Stirling formula to simplify the digamma function as \(\psi(z) \approx \ln z - \frac{1}{2z}\) when \(z\) is large enough. Accordingly, we obtain the results

\[
P_\zeta \approx \frac{N^4 N}{4\pi^2 \lambda} \frac{x^3}{3(\ln x)^2}, \quad n_s \approx 1 - \frac{6 \ln x}{N x}, \tag{16}
\]

with \(x \equiv (N \cdot \Delta)/\phi_0\) in this case. From Eq. \[16\], for a given the e-folding number \(N\), one can find that the tilt of the spectral index in multiple DBIs model is strongly suppressed by the variable \(x\). The dependence of \(n_s\) on the e-folding number \(N\) for different value of the variable \(x\) is plotted in Fig. \[1\]. From the figure, we can see that the spectrum of multiple DBIs model is generally closer to be scale-invariant when \(x\) is larger.

![FIG. 1: \(n_s\) as the function of the e-folding number \(N\) for different values of the variable \(x(\equiv N \cdot \Delta/\phi_0)\). The black solid line denotes the spectral index in the first case when all the scalars have the same value at horizon-crossing; the red dashed line denotes the spectral index in the second case with \(x = 10\); the orange dotted line \(x = 50\); the blue dash-dotted line \(x = 100\).](image)

The inflation model with multiple fields avoids some difficulties of single field inflation models, and so is regarded as an attractive implementation of inflation. In recent years, there have been a number of work studying this, such as Refs. \[31, 32, 33, 34, 35\], and there is a good review on this field Ref. \[36\]. In this letter, we have presented a new N-flation model in which a collection of DBI fields drives inflation simultaneously\(^3\). These scalars possess non-standard kinetic terms, and so

\(^3\) The action of this model is similar to the ones considered in Refs. \[12, 57\], but with different motivations.
some non-linear information is involved when we investigate the background evolution and curvature perturbation. For example, the e-folding number of this model is no longer proportional to the number of scalars, but its square root instead as shown in Eq.\([\text{5}]\). In the detailed calculation, we considered a tachyonic potential and specifically chose two different cases. In the first case, we took all the scalars to have the same value at the horizon-crossing time, and the spectral index in this case coincides with that in the single DBI model; while in the second case, we assumed that the collection of the scalars at the horizon-crossing time is an arithmetical progression, and we found that the spectral index becomes closer to 1 if the height of this progression is much larger than the value of the first scalar.

Notice that, in this letter we merely studied the adiabatic perturbations during inflation. However, since in a model with a number of scalars involved, there should be entropy perturbations generated during inflation. This is an interesting issue deserving the future study. Here we just make some comments on this point. Since the kinetic terms are of non-linear form, the dispersion relations for entropy perturbations are usually modified. For example, the sound speed will affect the time of horizon-crossing. Therefore, if the entropy perturbations contribute to curvature perturbation at late times, they may lead to some new features of the primordial power spectrum. For example, a large local type non-Gaussianity is hard to generate from the adiabatic perturbations in DBI inflation, because the sound speeds decouple from local type non-Gaussianity and these perturbations in this respect become like those in standard slow-roll inflation model\([\text{39}]\). However, entropy perturbations with small sound speeds might be different. A good example is a model of DBI-curvaton proposed in Ref.\([\text{38}]\), where a sizable local non-Gaussianity was generated, and an even larger non-Gaussianity might be obtained in the model of multiple DBI-curvaton. A more detailed study is in preparation.

Acknowledgments We would like to thank Robert Brandenberger, Bin Chen, Yum-Song Piao, Yi Wang and Xinmin Zhang for useful discussions and valuable comments. This work is supported in part by National Natural Science Foundation of China under Grant Nos. 10533010 and 10675136 and by the Chinese Academy of Science under Grant No. KJCX3-SYW-N2.

\begin{thebibliography}{99}
\bibitem{1} A. H. Guth, Phys. Rev. D \textbf{23}, 347 (1981); A. D. Linde, Phys. Lett. B \textbf{108}, 389 (1982); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. \textbf{48}, 1220 (1982).
\bibitem{2} For some early attempts we refer to: A. A. Starobinsky, Phys. Lett. B \textbf{91}, 99 (1980); K. Sato, Mon. Not. Roy. Astron. Soc. \textbf{195}, 467 (1981).
\bibitem{3} E. Komatsu et al. [WMAP Collaboration], \texttt{arXiv:0803.0517} [astro-ph].
\bibitem{4} A. R. Liddle, A. Mazumdar and F. E. Schunck, Phys. Rev. D \textbf{58}, 061301 (1998).
\bibitem{5} K. A. Malik and D. Wands, Phys. Rev. D \textbf{59}, 123501 (1999).
\bibitem{6} P. Kanti and K. A. Olive, Phys. Rev. D \textbf{60}, 043502 (1999).
\bibitem{7} E. J. Copeland, A. Mazumdar and N. J. Nunes, Phys. Rev. D \textbf{60}, 083506 (1999).
\bibitem{8} A. M. Green and J. E. Lidsey, Phys. Rev. D \textbf{61}, 067301 (2000).
\bibitem{9} S. Dimopoulos, S. Kachru, J. McGreevy and J. G. Wacker, JCAP \textbf{0808}, 003 (2008).
\bibitem{10} L. Kofman and A. Linde, JHEP \textbf{0207}, 004 (2002).
\bibitem{11} J. M. Cline, \texttt{arXiv:hep-th/0612129}.
\bibitem{12} Y. S. Piao, R. G. Cai, X. M. Zhang and Y. Z. Zhang, Phys. Rev. D \textbf{66}, 121301 (2002).
\bibitem{13} M. Majumdar and A. C. Davis, Phys. Rev. D \textbf{69}, 103504 (2004).
\bibitem{14} R. Brandenberger, P. M. Ho and H. C. Kao, JCAP \textbf{0411}, 011 (2004).
\bibitem{15} K. Becker, M. Becker and A. Krause, Nucl. Phys. B \textbf{715}, 349 (2005).
\bibitem{16} J. M. Cline and H. Stoica, Phys. Rev. D \textbf{72}, 126004 (2005).
\bibitem{17} O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, Phys. Rept. \textbf{323}, 183 (2000).
\bibitem{18} R. C. Myers, JHEP \textbf{9912}, 022 (1999).
\bibitem{19} E. Silverstein and D. Tong, Phys. Rev. D \textbf{70}, 103505 (2004); M. Alishahiha, E. Silverstein and D. Tong, Phys. Rev. D \textbf{70}, 123505 (2004).
\bibitem{20} X. Chen, Phys. Rev. D \textbf{71}, 063506 (2005); X. Chen, JHEP \textbf{0508}, 045 (2005).
\bibitem{21} M. X. Huang, G. Shiu and B. Underwood, Phys. Rev. D \textbf{77}, 023511 (2008).
\bibitem{22} D. Langlois, S. Renaux-Petel, D. A. Steer and T. Tanaka, Phys. Rev. Lett. \textbf{101}, 061301 (2008).
\bibitem{23} C. R. Contaldi, G. Nicholson and H. Stoica, \texttt{arXiv:0807.2331} [hep-th].
\bibitem{24} J. M. Maldacena, Adv. Theor. Math. Phys. \textbf{2}, 231 (1998).
\bibitem{25} W. Taylor and M. Van Raamsdonk, Nucl. Phys. B \textbf{573}, 703 (2000).
\bibitem{26} D. S. Goldwirth and T. Piran, Phys. Rept. \textbf{214}, 223 (1992).
\bibitem{27} R. Brandenberger, G. Geshnizjani and S. Watson, Phys. Rev. D \textbf{67}, 123510 (2003).
\bibitem{28} M. Sasaki and E. D. Stewart, Prog. Theor. Phys. \textbf{95}, 71 (1996).
\bibitem{29} S. Kim and A. R. Liddle, Phys. Rev. D \textbf{74}, 023513 (2006).
\bibitem{30} X. Chen, M. X. Huang, S. Kachru and G. Shiu, JCAP \textbf{0701}, 002 (2007).
\bibitem{31} Y. S. Piao, Phys. Rev. D \textbf{74}, 047302 (2006); I. Ahmad, Y. S. Piao and C. F. Qiao, JCAP \textbf{0806}, 023 (2008); \texttt{arXiv:0809.3333} [hep-th].
\bibitem{32} M. E. Olsson, JCAP \textbf{0704}, 019 (2007).
\bibitem{33} K. Y. Choi and A. R. Liddle, Phys. Rev. D \textbf{76}, 023502 (2007).
\bibitem{34} G. Panotopoulos, Phys. Rev. D \textbf{75}, 107302 (2007).
\bibitem{35} D. Battefeld, T. Battefeld and A. C. Davis, \texttt{arXiv:0806.1953} [hep-th]; T. Battefeld, \texttt{arXiv:0809.3242} [astro-ph].
\bibitem{36} D. Wands, Lect. Notes Phys. \textbf{738}, 275 (2008).
\bibitem{37} S. Thomas and J. Ward, Phys. Rev. D \textbf{76}, 023509 (2007).
\bibitem{38} S. Li, Y. F. Cai and Y. S. Piao, \texttt{arXiv:0806.2363} [hep-ph].
\end{thebibliography}