Heat current across a capacitively coupled double quantum dot for high magnetic field

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We study the heat current through two capacitively coupled quantum dots coupled in series with two conducting leads in the spinless case (valid for a high applied magnetic field). Our results are also valid for the heat current through a single quantum dot with strongly ferromagnetic leads pointing in opposite directions (so that the electrons with given spin at the dot can jump only to one lead) or through a quantum dot with two degenerate levels with destructive quantum interference and high magnetic field. Although the charge current is always zero, the heat current is finite when the interdot Coulomb repulsion is taken into account due to many-body effects. We generalize previous results for high temperatures and particular parameters obtained by Yadalam and Harbola [Phys. Rev. B 99, 195449 (2019)]. In particular we consider temperatures for which an orbital Kondo regime takes place. In contrast to previous results, we find that the heat current is finite even for \( U \to \infty \). In the Kondo regime, for temperatures much less than the Kondo energy scale, we obtain that the dependence of the thermal current with the temperature difference \( \Delta T \) is \( \sim (\Delta T)^\frac{3}{4} \) when the cold lead is at \( T_C \ll \Delta T \), and linear in \( \Delta T \) if \( T_C \gg \Delta T \). For large \( T_C \) the current saturates. As a function of Coulomb strength \( U \), for high \( \Delta T \) and \( T_C = 0 \), the charge current has a maximum for \( U \sim 3\Delta T \) and decreases with increasing \( U \) reaching a finite value for \( U \to \infty \). We also consider the case of different energy levels of the dots for which the device has rectifying properties.

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I. INTRODUCTION

In the last few years, the interest in the thermoelectric properties of nanodevices has increased and new relevant works were published which further contribute to our knowledge of these systems motivated by possible applications as well as fundamental interest.

Among some recent developments, it has been shown that molecules exhibiting quantum destructive interference effects, yield a higher thermopower. The differential Seebeck coefficient at finite applied bias voltage and difference of temperatures \( \Delta T \), linked to the proposal of quantum dots as possible nanoscale temperature sensors, is being studied. Systems of double quantum dots are also being studied. In particular, Yadalam and Harbola studied the full statistics of heat fluctuations in a system of two quantum dots in series, each one coupled to a corresponding lead (left dot with the left lead, right dot with the right lead) for spinless electrons (corresponding to a high magnetic field) and a Coulomb repulsion \( U \) between both dots (see Fig. 1).

The electrons cannot hop between the dots, and therefore the charge current is zero. Interestingly, for non-zero \( U \) (for which the model is not trivial) and \( \Delta T \) there is however a finite thermal current. A physical explanation of this is given in Section III This implies that the Lorenz number (the ratio of thermal to charge conductance divided by temperature) is infinite pointing to a strong violation of the Wiedemann-Franz law. For future discussions we denote this model as the “2-dot model”.

This model is equivalent to that of transport between two levels with destructive interference (DESINT) under high magnetic fields (the “DESINT model”). For example if a benzene molecule is doped with one electron or one hole, the many-body ground state has spin and orbital degeneracy. If the molecule is connected to one lead at an atom and the other lead is connected perpendicular to the first one so that it is coupled equally to a first and second nearest neighbor of the first atom, there is a linear combination of the many-body states that couples only to one lead and the other combination to the opposite lead. The mapping is explained in detail in Ref. and applies also for other molecular quantum dots with destructive interference.

The model is also equivalent to a spinfull model for one dot in which electrons with spin up can only hop to the left lead and electrons with spin down can only hop to the right lead (naturally the leads can be interchanged). This is the case for totally polarized ferromagnetic leads with opposite orientation (angle \( \pi \) between them). We

![Diagram](link)
denote this model as the “spinfull model” for future reference.

Yadalam and Harbola studied the model at high temperatures with two methods. For small coupling to the leads $\Delta_\nu$ with $\nu = L$ (left lead) or $\nu = R$ (right lead) they used the Lindblad quantum master equation approach, and for large $\Delta_\nu$ and small $U$ they used the saddle point approximation for the Schwinger-Keldysh coherent-state path integral, using a Hubbard-Stratonovich decoupling for the Coulomb repulsion. Unfortunately, due to the high temperatures used, the Kondo effect, which takes place at temperatures below the characteristic Kondo temperature $T_K$, is lost in their work.

Here we show that the Lindblad quantum master equation method leads to the same result for the current as the atomic limit $\Delta_\nu \rightarrow 0$. In this limit the Kondo effect is not captured. On the other hand, the different possible Hubbard-Stratonovich decouplings also have problems in reproducing correctly the Kondo physics (see Section 4.1 of Ref. [40]).

The Kondo effect is one of the most paradigmatic phenomena in strongly correlated condensed matter systems. In its simplest version, for example for the spinfull model, the phenomenon is characterized by the emergence of a many-body singlet ground state formed by an impurity spin $1/2$ and the spin $1/2$ of the conduction electrons near the Fermi level, below the characteristic Kondo temperature $T_K$. As a consequence the spectral density of the impurity displays a resonance at the Fermi energy. This explains the widely observed zero-bias anomaly in transport through quantum dots with an odd number of electrons. The Kondo effect with spin $S > 1/2$ has also been observed.

The role of the impurity spin can be replaced by other quantum degrees of freedom that distinguishes degenerate states, such as orbital momentum. Orbital degeneracy leads to the orbital Kondo effect or to more exotic Kondo effects, like the SU(4) one, when both orbital and spin degeneracy coexist. Some examples are present in nanoscopic systems. Evidence of the orbital Kondo effect has also been observed in magnetic systems in which the spin degeneracy is broken. In our case for the 2-dot model when the on-site energy at both dots is the same, the occupancy of one dot or the other places the role of the orbital degree of freedom. This role is taken by the occupancy of one or the other of the degenerate levels in the DESINT model.

In this work we calculate the heat current of the model using different diagrammatic techniques that describe correctly the Kondo effect. We extend previous results for high temperatures to all temperatures and in particular smaller than $T_K$. We also consider the case of different energy levels of both dots. In this case, the device has the effect of rectifying the heat current, which is asymmetric for positive or negative heat bias. We compare the results at high temperatures with those of the atomic limit. We analyze the dependence on $U$ and show that there is a finite thermal current for $U \rightarrow \infty$. Most of the results presented were obtained using non-equilibrium perturbation theory up to second order in $U$, which is valid for small or moderate values of $U$. For infinite $U$ we use renormalized perturbation theory and the non-crossing approximation.

The paper is organized as follows. In Sec. II we describe the model, the equations for the particle and heat currents and the above mentioned theoretical methods. In Sec. III we discuss a physical picture for the heat transport. Sec. IV contains the results. Sec. V contains a summary and a discussion.

II. MODEL AND METHODS

A. Model

The Hamiltonian can be written as follows

$$H = \sum_\nu E_\nu d_\nu^\dagger d_\nu + U d_L^\dagger d_L d_R^\dagger d_R + \sum_{k \nu} \varepsilon_{k \nu} c_{k \nu}^\dagger c_{k \nu}$$

$$+ \sum_{k \nu \sigma} \left( V_{k \nu} c_{k \nu}^\dagger d_{\nu \sigma} + \text{H.c.} \right),$$

where $\nu = L, R$ refers to the left and right dot or leads. The first term describes the energy of an electron in each dot, the second term is the Coulomb repulsion between electrons in different dots, the third term corresponds to a continuum of extended states for each lead, and the last term is the hybridization between electrons of each dot and the corresponding lead.

For the DESINT model, the labels $L, R$ correspond to different degenerate levels of the same dot and the continua that hybridizes with each of them, and for the spinfull model, $L, R$ describe the different spin projections up, down.

In general, both leads are at different chemical potentials $\mu_\nu$ and temperatures $T_\nu$. For most of the results presented here we take $\mu_\nu = 0$.

The couplings to the leads, assumed independent of frequency are expressed in terms of the half width at half maximum of the spectral density in the absence of the interaction

$$\Delta_\nu = \pi \sum_k |V_{k \nu}|^2 \delta(\omega - \varepsilon_{k \nu}).$$

B. Equations for the currents

The equations for the particle and heat current, can be obtained using the Keldysh formalism in an analogous way to previous studies of transport through a single quantum dot, with appropriate modifications that take into account how the conducting leads are connected to the interacting part (the double dot for the 2-dot model or the single dot for the DESINT and spinfull models).
For the 2-dot model, the particle current flowing between the left lead and the dot can be written as

\[ J^L_N = \frac{2i\Delta_L}{\hbar} \int d\omega \left[ 2if_L(\omega)\text{Im}G^R_L(\omega) + G^\omega_L(\omega) \right], \quad (3) \]

where \( G^R_L(\omega) \) and \( G^\omega_L(\omega) \) is the retarded [lesser] Green function of the left dot and \( f_L(\omega) = \{ 1 + \text{exp}\left[ (\omega - \mu_L)/T_L \right] \}^{-1} \) the Fermi function. The corresponding charge current is \( J^L_C = eJ^L_N \), where \( e \) is the electronic charge. Similarly, the particle current flowing between the dot and the right lead is

\[ J^R_N = -\frac{2i\Delta_R}{\hbar} \int d\omega \left[ 2if_R(\omega)\text{Im}G^R_R(\omega) + G^\omega_R(\omega) \right]. \quad (4) \]

The heat currents \( J^R_Q \) flowing from the left lead to the dot and \( J^R_Q \) flowing from the dot to the right lead are

\[ J^R_Q = J^R_E - \mu_v J^R_N, \quad (5) \]

where \( J^R_E \) are the energy currents given by

\[ J^R_E = \pm \frac{2i\Delta_R}{\hbar} \int \omega d\omega \left[ 2if_v(\omega)\text{Im}G^R_v(\omega) + G^\omega_v(\omega) \right], \quad (6) \]

where upper (lower) sign corresponds to \( \nu = L \) (R).

These results can be extended to the equivalent DESINT or spinfull models. In the former case, the labels \( L \) and \( R \) denote two different energy levels, and in the spinfull case \( L \) denotes spin up, and \( R \) spin down.

In the stationary state, the charge and energy currents are uniform and should be conserved: \( J^L_N = J^R_N \) and \( J^E_N = J^E_R \). The heat current is not conserved under an applied voltage \( (\mu_L \neq \mu_R) \) due to joule heating of the interacting part of the system.\(^{20}\) For the models studied in this work also \( J^L_N = J^R_N = 0 \) because electrons cannot hop between left and right parts of the system.

C. Perturbation theory in \( U \)

For the Anderson model at equilibrium, with \( \Delta_L = \Delta_R = \Delta \), perturbation theory in \( U/(\pi\Delta) \) has been a popular method used for several years.\(^{69,70}\) also applied to nanoscopic systems.\(^{24-27}\) Comparison with Quantum Monte Carlo results indicate that the method is quantitatively valid in the symmetric case \( E_L = E_R = -U/2 \), for \( U/(\pi\Delta) \) as large as 2.42.\(^{78}\)

The method can be extended naturally to the nonequilibrium case using the Keldysh formalism.\(^{60,61}\) One shortcoming of the approach is that the particle and energy currents are not conserved. This means that in general the approximation gives \( J^L_N \neq J^R_N \) and \( J^E_N \neq J^E_R \) [see Eqs. \( (3), (4) \) and \( (5) \)], contrary to what one expects. In our case however \( J^L_N = J^R_N = 0 \) within numerical precision. In our calculations, presented in Section IV we represent the heat current defined as \( J^Q = (J^Q_N + J^Q_E)/2.0 \), where \( J^Q_N = J^Q_E \) because in our system \( J^Q_N = 0 \) [see Eq. \( (5) \)]. The relative deviation \( d = |J^Q_N|/J^Q - 1 = |J^Q_E|/J^Q - 1 \) is usually of the order of 2% or less, but reaches a value near 14% at high temperatures and the largest values of \( U \) used with this method \( (U = 7\Delta) \).

D. Renormalized perturbation theory

For \( U \gg \Delta \) the approach mentioned above fails but for energy scales below \( T_K \) one can use renormalized perturbation theory (RPT). The basic idea of RPT is to reorganize the perturbation expansion in terms of fully dressed quasiparticles, taking as a basis the equilibrium fermi liquid picture.\(^{28}\) The parameters of the original model are renormalized and their values can be calculated exactly using Bethe ansatz, or with high accuracy using numerical renormalization group.\(^{63,64}\) One of the main advantages is that the renormalized expansion parameter \( U/(\pi\Delta) \) is small, being usually below 1.1 even for \( U \rightarrow \infty \).\(^{63}\)

Our RPT procedure consists in using renormalized parameters for \( E_L = E_R \) and \( \Delta \) obtained at \( \mu_L = \mu_R = T_L = T_R = 0 \) by a numerical-renormalization-group calculation and incorporating perturbations up to second order in the renormalized \( U \). It has been shown explicitly that this non-equilibrium approach satisfies important Ward identities.\(^{20,64}\) At equilibrium, the method provides results that coincide with state-of-the-art techniques for the dependence of the conductance with magnetic field \( B \) \((c_B)^{63}\) and temperature \((c_T)^{64}\) to second order in \( B \) or \( T \). An analytical expression for \( c_T \) in terms of the renormalized parameters was provided.\(^{64}\)

However, for energy scales of the order of \( T_K \) or larger, the method loses accuracy and a complementary approach is needed.

E. Non-crossing approximation

For infinite \( U \) we also calculate the different Green functions entering Eqs. \( (3), (4) \) and \( (5) \), using nonequilibrium non-crossing approximation (NCA).\(^{63,64}\) The NCA technique is one of the standard tools for calculating these Green functions in the Kondo regime, where the total occupancy of the interacting subsystem (the double dot for the 2-dot model or the dot for the DESINT and spinfull models) is near 1 and with small fluctuations (the charge is well localized in the dot or dots). The NCA has being successfully applied to the study of a variety of systems such as C\(_{60}\) molecules displaying a quantum phase transition.\(^{68,81}\) a nanoscale Si transistor\(^{28}\) and the interplay between vibronic effects and the Kondo effect.\(^{39,83}\)
In spite of this success, the NCA has some limitations at very low temperatures (below ~ 0.1T_K). For example, it does not satisfy accurately the Friedel sum rule at zero temperature.\cite{64} In this sense it is complementary to RPT, which should be accurate for $T_L, T_R \ll T_K$.

In contrast to the RPT, the NCA conserves the charge current, as shown explicitly for the DESINT model in Ref. \cite{68}. We find that the NCA also conserves the energy current.

III. PHYSICAL PICTURE FOR THE THERMAL CURRENT

Since electrons cannot hop between left and right parts of the system, it is clear that the particle current is zero in our system. It might seem surprising that the heat current is nonzero under a finite temperature difference $\Delta T = T_L - T_R$ in spite of the fact that exchange of particles is not possible.

The aim of this section is to provide a simple picture for the transport of heat in the presence of interactions. We assume small $\Delta$, so that states with definite number of particles at each dot are relatively stable. Without loss of generality we can also assume $\Delta T > 0$. Let us take $E_L = E_R < \mu_L = \mu_R = 0$ and $E_\nu + U > 0$. For $\Delta > 0$ one of the possible ground states of the system has occupancies $(n_L, n_R) = (0, 1)$. Let us take this state as the initial state for a cycle of transitions that transport heat. For non-zero $\Delta$, if the left lead is hot enough, one can perform the first step of the thermal cycle (see Fig. \ref{fig:2}): i) take an electron from the left lead and occupy the left dot changing the state of the double dot to $(1,1)$ $\rightarrow (0,1)$. This costs energy $U + E_L$ which is taken from the left lead. Next (ii) take an electron from the right dot and transfer it to the right lead $\rightarrow (1,1)$. This relaxes the energy $U + E_R$ which is then transferred to the right lead. Next (iii) the electron from the left dot jumps to the corresponding lead $\rightarrow (0,0)$. This requires an energy $|E_L|$ taken form the left lead. Finally (iv) take an electron from the right lead and occupy the right dot closing the cycle $\rightarrow (0,0)$. This effect is finite even for infinite $U$, and therefore a scheme like that explained above can be applied for the quasiparticles.

The resulting thermal current depends on the probability per unit time of each process and requires an explicit calculation. In addition, while this picture provides a qualitative understanding for the general case, it is not enough to describe the thermal transport in the Kondo regime in which cotunneling events are important and does not explain what happens in the $U \rightarrow \infty$ limit. However, the physical grounds of the RPT explained in Section II.D allows us to apply the above picture to the Kondo regime even for $U \rightarrow \infty$. Following these ideas, it is convenient to think the low-energy excitations near the Fermi energy in terms of dressed quasiparticles rather than free electrons. These quasiparticles feel a renormalized repulsion $\tilde{U} \ll U$ which is finite even for infinite $U$, and therefore a scheme like that explained above can be applied for the quasiparticles.

Fig. \ref{fig:2} is also useful to represent the fluctuations involved in the Kondo effect. For the spinfull (DESINT) model they correspond to spin (orbital) fluctuations, and for the 2-dot model to the occupancy of one of the dots keeping the total occupancy of the dots in 1. The sequence of the two steps (i) and (ii) and its time-reversed sequence correspond to fluctuations through the virtual state with double occupancy. Note that a temperature gradient favors the sequence (i)-(ii) with respect to the reciprocal one. Similarly, the process (iii)-(iv) and the reciprocal one correspond to fluctuations through the virtual empty double dot and the former is favored by the temperature gradient. As a consequence of the inequivalence between the above mentioned direct processes and the time reversed ones, the occupancies of the left an right dots become different even if $E_L = E_R$, except for the symmetric case $E_L = E_R = -U/2$. This effect is similar to that caused by a magnetic field in the spinfull model.

IV. RESULTS

A. The atomic limit

In order to compare with some previous results,\cite{27} and our own ones at high temperatures, we discuss the limit

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig2}
\caption{(Color online) Schematic picture for the transport of heat in the presence of interactions.
}
\end{figure}
$\Delta_L, \Delta_R \to 0$.

One possible approach for this task would be to perform equations of motion for the different Keldysh Green functions, truncate them with some approximation and take the limit $\Delta_L, \Delta_R \to 0$. However, the presence of both interactions and the hopping terms renders the process very cumbersome. Therefore we start from the equations of motion for strictly $\Delta_L = \Delta_R = 0$ and solve some undetermined coefficients using conservation laws and general arguments.

For our Hamiltonian, the particles cannot jump between the dots and this implies that in the stationary state $J_N^0 = 0$ and therefore $J_Q^0 = J_E^0$.

In the atomic limit $\Delta_\nu = 0$, from equations of motion one has

\[
G^-_\nu(\omega) = \frac{1}{\omega - E_\nu} + \frac{n_\nu}{\omega - E_\nu - U},
\]

\[
G^\nu_\nu(\omega) = 2\pi i [a_\nu \delta(\omega - E_\nu) + b_\nu \delta(\omega - E_\nu - U)],
\] (7)

where $n_\nu = \langle d_\nu^\dagger d_\nu \rangle$ is the expectation value of the occupation of the dot $\nu$ and $\bar{\nu} = R$ ($L$) if $\nu = L$ ($R$). The functions $a_\nu$ and $b_\nu$ are undetermined for $\Delta_\nu = 0$ and one would need to include finite $\Delta_\nu$ in the equations of motion to determine them. This however introduces more involved Green functions and approximations are necessary to solve the resulting equations of motion. As an alternative we use conservation laws and a simple assumption to determine them.

Note that

\[
n_\nu = -\frac{i}{2\pi} \int d\omega G^\nu_\nu(\omega) = a_\nu + b_\nu. \tag{8}
\]

Replacing Eqs. (7) and (8) in Eqs. (2) and (3) and imposing $J_N^0 = J_Q^0 = 0$ one arrives at the following set of two equations

\[
n_\nu = (1 - n_\bar{\nu}) f_\nu(E_\nu) + n_\bar{\nu} f_\nu(E_\nu + U) \tag{9}
\]

from which $n_\nu$ can be determined. The result is

\[
n_\nu = \frac{f_\nu(E_\nu) - f_\bar{\nu}(E_\nu)D_\nu}{1 - D_L D_R}, \tag{10}
\]

\[
D_\nu = f_\nu(E_\nu) - f_\nu(E_\nu + U).
\]

Using Eqs. (10) the energy currents can be written as

\[
J^\nu_E = \pm \frac{4\pi \Delta_L U}{\hbar} [n_\nu f_\nu(E_\nu + U) - b_\nu]. \tag{11}
\]

Conservation of the energy current in the stationary state $J_E^R = J_E^L$ leads to an equation for $\Delta_L b_L + \Delta_R b_R$ at this time we make the assumption $b_L = b_R$. This is justified from the form of $G_\nu^-(\omega)$ [Eqs. (7)] and Eq. (8). One realizes that $b_L$ is the contribution to $n_L$ at an energy

$E_L + U$, which implies that the right dot is occupied (because of the presence of the Coulomb repulsion term). Therefore, one expects that $b_L$ is the probability of double occupancy and the same for $b_R$: $b_\nu = \langle d_\nu^\dagger d_L d_R^\dagger d_R \rangle$. Using $J^R_E = J^E_R$ and $b_L = b_R$ one obtains

\[
(\Delta_L + \Delta_R) b_\nu = \Delta_L n_R f_L(E_L + U) + \Delta_R n_L f_R(E_R + U). \tag{12}
\]

Using Eqs. (10) and some algebra one can verify that Eq. (12) leads to the correct result at equilibrium: for $\mu_L = \mu_R = 0$, $T_L = T_R = 1/\beta$ one has

\[
b_\nu = \langle d_L^\dagger d_R^\dagger d_R d_L \rangle = n_R f_L(E_L + U) = n_L f_R(E_R + U) \Rightarrow\frac{1}{1 + e^{-\beta(E_L + E_R + U)}} e^{-\beta(E_L + E_R + U)} \tag{13}
\]

Replacing Eq. (12) in Eq. (11) one obtains the final expression for the heat current

\[
J_Q = J_E^R = \frac{4\pi \Delta_L \Delta_R U}{\hbar(\Delta_L + \Delta_R)} [n_R f_L(E_L + U) - n_L f_R(E_R + U)], \tag{14}
\]

It can be checked that this expression is invariant under the replacement $E_\nu \to -E_\nu - U$, as expected from an electron-hole transformation of the Hamiltonian: $d_\nu \to d_{\nu}^\dagger$, $c_{\nu}^R \to -c_{\nu}^R$ with $\varepsilon_{\nu} \to -\varepsilon_{\nu}$.

For the symmetric case $E_\nu = -U/2$, one has $n_\nu = 1/2$ and Eq. (14) reduces to

\[
J_Q = \frac{2\pi \Delta_L \Delta_R U}{\hbar(\Delta_L + \Delta_R)} [f_L(E_L + U) - f_R(E_R + U)], \tag{15}
\]

which coincides with the expression obtained by Yadalam and Harbola (see the expression of $C_1$ in appendix A of Ref. 37, note that in their notation $C_\nu = 2\Delta_\nu$).

### B. Dependence of thermal current on $\Delta_\nu$

In the following, we take $E_L = E_R = E$, $\Delta_L = \Delta_R = \Delta$, $\mu_L = \mu_R = 0$ and $\Delta T = T_L - T_R > 0$.

In this short subsection we take parameters corresponding to Fig. 6 of Ref. 37 and calculate $J_Q$ as a function of $\Delta$ using perturbation theory in $U$ (see Section 1C) in the symmetric case $E = -U/2$ and for $\Delta > U/10$. For smaller values of $\Delta$ the method is not reliable. The result is represented in Fig. 3. For small $\Delta$ the heat current increases linearly with $\Delta$ [as expected from Eqs. (14), (15)]. For larger $\Delta$ the slope decreases and $J_Q$ reaches a maximum for $\Delta \sim 0.8 T_R$ and then decreases for larger $\Delta$. For all values of $\Delta$ the relative deviation
of the method in the conservation of the energy current
d = |JJQ/Q − 1| (see Section II C) is below 1%.

Although the method used is quite different from the
Schwinger-Keldysh coherent-state path integral in the
saddle point approximation used to represent the heat
current in Fig. 6 of Ref. 27, the result is very similar.
A possible reason for this similarity is that the param-
eters chosen correspond to very high temperatures, not
only in comparison with the Kondo temperature TK but
also in comparison with U. Then possible limitations of
the Hubbard-Stratonovich approximation to describe the
Kondo effect, 28 are not detected.

Below we discuss the thermal current at low tempera-
tures.

C. Dependence of thermal current on ΔT

In this subsection we take TR = 0 and analyze the depen-
dence of JQ on TL = ΔT, using perturbation theory
in U for the symmetric case E = EL = ER = −U/2 as
above. We consider several values of U within the valid-
ity of the perturbative approach. The results are shown
in Fig. 3. One signature of the limits of this perturbative
approach is the relative error in the conservation of the
energy current d = |JJQ/Q − 1| (see Section II C). While
it is negligible for very small temperatures and moderate
values of U, reaches a value of 12.6% for U = 7Δ and
TL ∼ 2Δ, decreasing slowly as TL increases.

For U = 7Δ, near the limit of validity of this approach,
the system has the characteristics of the Kondo regime
(−E, E + U ≫ Δ) at equilibrium. The spectral density
has a well defined peak at the Fermi energy (the Kondo
peak) separated from the charge-transfer peaks near E
and E + U. From the half-width at half maximum of the
Kondo peak one has an estimation of the Kondo tempera-
ture TK ∼ 0.27Δ. We find that for TL well below TK (we verify this for TL < 0.04Δ), the heat current behaves
as JQ ∼ (ΔT)4. This remains true as long as the smaller
temperature (TR in our case) is also much smaller than
TK. For large TR, JQ is linear in ΔT for small ΔT.

For all values of U, after the initial flat increase of the
thermal current with ΔT, for ΔT ∼ Δ, JQ increases ap-
proximately linearly with ΔT and when it reaches a few
times U if finally saturates. For U, ΔT ≫ Δ, the thermal
current is qualitatively described by Eq. 15, although
the saturation value is larger for this expression, particu-
larly for small values of U. In contrast, for small TL the
analytical expression has an exponential dependence and
falls below the value given by perturbation theory.

D. The limit U → ∞

Here we choose parameters corresponding to the
Kondo regime: EL = ER = −4Δ, and U → ∞, and
calculate the current using RPT and NCA (see Sections 11D, 11E) as a function of $\Delta T$, keeping $T_L = 0$ (RPT) or a small fraction of the Kondo temperature (NCA) so that the results are indistinguishable from those of $T_L = 0$.

In order to compare the results of both approximations, it is convenient to represent the results taking the magnitude. We have shown recently that extracting approximations differ, although they are of the same order of magnitude. We have shown recently that extracting $T_K$ from the temperature dependence of the conductance $G(T/T_K)$ of an equivalent model $H_{\text{eq}}$ at equilibrium is more reliable than fitting the spectral density or the non-equilibrium conductance. 87 This equivalent model consists in the usual spin-degenerate Anderson model for a dot connected to left and right leads with both spins. In particular for $\Delta_L = \Delta_R = \Delta$, $H_{\text{eq}}$ has coupling $\Delta/2$ for each spin and each lead. At equilibrium $H_{\text{eq}}$ has the same spectral density than our 2-dot model. However the transport properties are completely different because both models are connected in a different way to the leads and therefore the models differ out of equilibrium.

In any case we can use the mapping at equilibrium to define $T_K$. This task has been already done in Ref. 87 fitting a popular phenomenological expression for $G(T/T_K)$. The renormalized parameters for RPT were taken from previous calculations. 83,84 The result was $T_K = 0.00441\Delta$ for the RPT and $T_K = 0.00796\Delta$ for the NCA.

From the inset of figure 5, we observe that for much higher $\Delta T > 10\Delta$, the thermal current saturates to a finite value, as already found for other values of $U$ in the symmetric case (see Fig. 4). From the expression in the limit $\Delta \to 0$, Eq. (14) one might expect that $J_Q \to 0$ for $U \to \infty$ and very large but finite $\Delta T$. However, this expression is linear in $\Delta$ while the result shown in Fig. 5 for large $\Delta T$ is quadratic in $\Delta$. This suggests that Eq. (14) is valid to first order in $\Delta$ and a finite value of the thermal current can be obtained expanding the current to higher order near the atomic limit $\Delta_v \to 0$.

E. Dependence of thermal current on $U$

In Fig. 6 we represent the thermal current as a function of $U$ calculated by perturbation theory in the symmetric case $E_L = E_R = -U/2$, for different $\Delta T = T_L$, keeping $T_R = 0$. Since the thermal current strongly depends on $\Delta T$ for small $\Delta T$, the values have been multiplied by a factor indicated in the figure in order to represent them. In spite of the different magnitude, the different curves show a similar dependence, with a $U^2$ behavior for small $U$. At intermediate $T_L$, $(0.5\Delta$ and $\Delta$), the curves show a maximum within the interval of $U$ shown (determined by the validity of the perturbative approach).

According to the limit of small $\Delta$ [Eqs. (13), (15)], one expects that for large $\Delta T$ there is a maximum in the thermal current at an intermediate value of $U$. Since at high temperatures, the effects of correlations is expected to be less important, we have also calculated $J_Q$ for $\Delta T = 10\Delta$ as a function of $U$ for a larger interval, which in principle is beyond the validity of the approach and compare it with the result in the atomic limit $\Delta_v \to 0$ [Eq. (15)]. The result is shown in Fig. 7. Taking into account the limitations of both approximations, the results are surprisingly similar. In particular both approaches lead to a maximum in the thermal current for $U \sim 3\Delta T$. For
small $U$ the perturbative approach gives a quadratic dependence in $U$, while it is linear in Eq. (15).

In most of the calculations presented before we have considered $E_L = E_R$, although the analytical results in the atomic limit $\Delta_c \to 0$ [Eq. (14)] are valid for arbitrary $E_c$. One effect of having different $E_c$ is the loss of the Kondo effect, in a similar way as the application of a magnetic field in the simplest impurity Anderson model. Another effect is that the current has a different magnitude for positive and negative $\Delta T$ of the same magnitude. This rectification effect might be important for applications.

In Fig. 8 we show an example of this rectification effect in the atomic limit. We have taken one level below but near to the Fermi energy and the other one below and separated from the Fermi energy. We obtain that the magnitude of the heat current is larger when the former level is next to the lead with the lower temperature. The ratio between both currents is larger than a factor two for small or moderate values of the temperature difference $\Delta T$.

In Fig. 9 we show similar results obtained with the NCA for infinite $U$. We define $\Delta = (\Delta_L + \Delta_R)/2$. To keep our convention $T_L > T_R$, we have inverted the device through the middle point instead of inverting the temperature. The result for the magnitude of the current is the same. While in the atomic limit used in the previous figure, the ratio of the currents is independent of the asymmetry between the $\Delta_c$ (only a multiplicative factor is affected), this is not the case of the NCA, although the dependence on the asymmetry is very weak for large temperature difference $\Delta T$. As in the previous case, the largest magnitude of the thermal current is obtained when the level nearest to the Fermi energy is next to the cold lead. Also the rectification ratio is larger than two for small or moderate $\Delta T$. A difference with the previous case is that for large $\Delta T$ a certain degree of rectification remains.

We have also done some calculations for $E_L \neq E_R$ using perturbation theory. However for small $U$ the rectification properties are too small, while for large $U$ the error in the conservation of the current increased rapidly and we considered that the results were not reliable enough.
V. SUMMARY AND DISCUSSION

We have studied the thermal current through a system of two capacitively coupled quantum dots connected in series with two conducting leads in the spinless case (corresponding to a high applied magnetic field). The system is also equivalent to one spinfull dot between two conducting leads fully spin polarized in opposite directions, and to a molecular quantum dot with two relevant levels connected to the leads in such a way that there is perfect destructive interference.

An interesting feature of the system is that charge transport is not possible, but heat transport is, due to the effect of the Coulomb repulsion between the electrons in the dots, leading to a strong violation of the Wiedemann-Franz law. A simple picture of the effect of the Coulomb repulsion in the heat transport is provided in Section III.

The system has been studied previously in the regime of high temperatures of both leads (including also the full counting statistics). We generalize the results in the limit of small coupling to the leads for arbitrary values of the other parameters, and considering all temperatures, and in particular the Kondo regime in which there is one particle strongly localized in the double dot, but fluctuating between both dots. For high temperatures of the leads, our results agree in general with the previous ones, displaying a non-monotonic behavior as a function of Coulomb repulsion and/or coupling to the leads, with a maximum at intermediate values.

For temperatures well below the Kondo energy scale $T_K$, we obtain that the heat current is proportional to the fourth power of the difference $\Delta T$ between the temperatures of both leads.

For infinite Coulomb repulsion, in contrast to the previous work, we find that the heat current is finite for all non-zero values of $\Delta T$. Within the Kondo regime, this result can be understood in the frame of renormalized perturbation theory: near the Fermi energy the main aspects of the physics can be understood in terms of dressed weakly interacting quasiparticles. Even if the original Coulomb repulsion $U \to \infty$, the renormalized one $\tilde{U}$ is small and comparable with the renormalized coupling to the leads.

When the energy of both dots $E_\nu$ or the the coupling to the the leads $\Delta_L$ are different, the system loses its inversion symmetry at the mid point of the dots, and therefore, one expects that the absolute value of the heat current $J_Q$ is different for positive or negative temperature difference $\Delta T$. This means that the device has some rectifying properties. In the case in which only the thermal gradient breaks inversion symmetry one has $J_Q(-\Delta T) = -J_Q(\Delta T)$. Our results suggest that the asymmetry in the couplings $\Delta_L \neq \Delta_R$ modifies the amplitude of the current but has little effect on the rectifying properties. Instead, when $E_L \neq E_R$ a factor larger than two between the current flowing in opposite senses can be obtained. It is possible that this effect might be increased adding more dots in series.

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