Torsion-Gravity for Dirac fields and its effective phenomenology as the appearance of new physics

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We will consider the torsional completion of gravity for a background filled with Dirac matter fields, studying the weak-gravitational non-relativistic approximation, in view of an assessment about their effective phenomenology: we discuss how the torsionally-induced non-linear interactions among fermion fields in this limit are compatible with all experiments, and remarks on the role of torsion for the appearance of new physics are given.

Introduction

The torsional completion of gravity is essentially the result of not neglecting the torsion tensor within the most general connection of the spacetime; once torsion is allowed, it gives the possibility to couple the spin of matter fields in the same fashion in which curvature couples to the energy of matter fields: the existence of spin beside energy is no longer in debate ever since the introduction of spinors represented by the fermionic Dirac matter field.

What is in debate, however, is the way in which such coupling is realized, and that is, equivalently, the derivative order of the Lagrangian of the model: higher-order derivative Lagrangians have the general property for which torsion is differentially related to the spin of matter fields, a property usually encoded in the statement that torsion propagates out of the matter; the least-order derivative Lagrangian, that is the Lagrangian that is built with squared torsion and linear curvature, beside the first-order derivative Lagrangian, that is the Lagrangian that is built with squared torsion and linear curvature, beside the first-order derivative Dirac Lagrangian, usually called the Scialom-Kibble-Einstein-Dirac theory, or SKED theory for short, has the specific feature for which torsion is algebraically related to the spin, so that torsion does not propagate out of matter. The former type of models has therefore, from the point of view of torsion, a risky character, since the fact that torsion can propagate even in absence of matter ultimately means that it should be possible to detect torsion in vacuum: as experimental limits for torsion in vacuum are very strict [1, 2], then constraining a torsion that may propagate in vacuum means constraining torsion as a whole. The latter type of model, the SKED theory, does not have such a problem because the algebraic character of the torsion-spin coupling makes torsion algebraically zero in vacuum, so that torsion will always turn out to be compatible with such experimental limits no matter how strict they are outside matter.

Inside matter, however, is a different story: in fact, even in the SKED theory, where torsion is present only in matter, torsion would be constrained if it were possible to put limits by employing in-matter experiments, and very recently in reference [3], constraints for in-matter experiments are placed. Our purpose is to show that the SKED theory is still compatible with these constraints.

Because as a consequence of this fact, the SKED theory results to be more and more adequately defined as the sole theory still compatible with all known experiments, and since this is true even for the SKED theory in its most general form, we will proceed by discussing some of the possible consequences of this fact, especially for what concerns the role of torsion and the torsional constant.

I. THE SKED THEORY AND ITS EFFECTIVE INTERACTION

As we have specified in the introduction, any higher-order theory of gravity has in general a torsion-spin coupling differential field equation, so that whether or not spin is present, torsion is non-zero in general; in [2] the authors deal precisely with this type of situation by considering a very general Lagrangian for torsion in interaction with spinorial matter, so that their results are relatively model independent, and capable of including propagating torsion as well: since their results place stringent limits on torsion in vacuum, then they can be interpreted by stating that torsion can be assumed not to exist out of spinorial matter. But in theories in which torsion can propagate, it may be present even in absence of its spin source: then constraining torsion in vacuum signifies constraining torsion entirely, and these results can be interpreted by stating that torsion cannot be a propagating field. Or equivalently, since propagating torsion comes from higher-order field equations, they may be interpreted by stating that torsion cannot be described in terms of higher-order Lagrangians in general.

Therefore in the present paper we will focus on the least-order Lagrangian, generating torsion-spin coupling algebraic field equations, for which torsion may have whatever value can be assigned in spinorial matter without having to be different from zero also in vacuum, so that torsion may still be present inside matter even if it is always zero outside matter, and where inside of matter torsion results into a specific non-linear short-range type of interaction; in [1] the author considers such a theory, so that his results are specific to this model, which describes torsion as a short-range interaction: effects on the energy levels of atoms can be tested by means of the Hughes-Drever experiment, constraining this specific
type of torsion for short-range potentials.

The Lagrangian introduced in [2] is the starting point of the results discussed in [3] but now in terms of in-matter experiments; the results of [1] binding these short-range interactions, in [3] are improved: because for the torsionally-induced spin-contact interaction the effects of torsion on the interaction of spinors influence in-matter experiments, the results exhibited in [3], implying torsion to be very much constrained even inside matter, place strong bounds also on the SKED theory, the last theory that was still compatible with all experiments.

Our main purpose here is to consider the SKED theory, studying its effective interaction to better investigate the effects on in-matter dynamics, to see whether they really are incompatible with present experiments or not.

So to begin, we will introduce very briefly the formalism we intend to employ, exposed in [3], and where here we recall the most important notation: all along this paper we will work in a (1+3)-dimensional space-time with Riemann-Cartan geometry, described in terms of a metric tensor $g_{\mu\nu}$ and a torsion tensor $\xi_{\mu\nu\rho}$ which will be taken to be completely antisymmetric for the reasons explained in the reference above; the metric and torsion will construct the connection in terms of which we define the covariant derivatives $\nabla_{\mu}$ and $\nabla_{\mu}$ in the most general case and in the torsionless case, respectively, and where we have that $D_{\mu}g_{\alpha\beta}=\nabla_{\mu}g_{\alpha\beta}=0$ hold; then the curvature tensors $G_{\mu\nu\rho}=\nabla_{\rho}g_{\mu\nu}$ and $R_{\xi\mu\nu\rho}$ are defined as usually done in the most general case and in the torsionless case, respectively, and because of their symmetry properties we may also define $G_{\mu\nu}=G_{\nu\mu}$ with contraction given in terms of $G_{\mu\nu}g^{\nu\rho}=G$ and $R_{\mu\rho}=R_{\rho\mu}$ with contraction given by $R_{\mu\nu}g^{\nu\rho}=R$ called Ricci tensor and scalar and torsionless Ricci tensor and scalar. In Lorentz formalism, the metric is $g_{\mu\nu}=\epsilon_{\mu}^{\rho}\epsilon_{\nu}^{\rho}\eta_{\rho\eta}$ in terms of the basis of tetrad fields $\epsilon_{\alpha}^{\mu}$, and the constant metric $\eta_{ij}$ with Minkowskian structure and where $\omega^{\mu}_{\alpha}$ is the spin-connection; this formalism is equivalent to the previous one, but allows the possibility to introduce spinor fields. Here, the spinorial transformation will be taken in the spin representation, obtained after introduction of the $\gamma_{a}$ matrices verifying the Clifford algebra $\{\gamma_{a},\gamma_{b}\}=2\epsilon_{\alpha}^{\mu}\eta_{\rho\alpha}$ from which one may define the matrices $\sigma_{a\beta}=\frac{1}{2}\{\gamma_{a},\gamma_{\beta}\}$ as the infinitesimal generators of the spinorial transformation, and introducing the spinorial connection $\Omega_{a\beta}=\frac{1}{2}\omega^{\mu}_{\alpha}\sigma_{a\beta}$ we define spinorial covariant derivatives $D_{\rho}$ and $\nabla_{\rho}$ in the general and torsionless case, respectively, thus terminating the list of conventions we wanted to specify. With this kinematic background, we may proceed by defining the most general least-order derivative Lagrangian according to

$$L=(\frac{k}{2}Q_{\alpha\rho\sigma}Q^{\rho\sigma})+G-\frac{1}{2}\bar{\psi}(\gamma^{\mu}D_{\mu}\psi-\nabla_{\mu}\bar{\psi}\gamma^{\mu}\psi)+m\bar{\psi}\psi\quad(1)$$

where $k$ is the torsional coupling constant while the gravitational constant has been normalized to unity, and where $m$ is the mass of the matter field; we notice that such a torsional coupling constant is in general independent from the gravitational constant exactly in the same sense in which the torsion-squared term is independent from the linear curvature term: in the most general case both terms must be included, and correspondingly we have that in general two constants must be accounted.

Variation of this Lagrangian with respect to all fields involved yields the corresponding field equations, starting from the completely antisymmetric torsion-spin coupling field equations that are given in the following form

$$Q_{\mu\nu\rho}=-\frac{k}{2}V\alpha_{\mu\nu\rho}\quad(2)$$

which come together with the non-symmetric curvature-energy coupling field equations given according to

$$\frac{1}{2}G_{\mu\nu}Q^{\rho\sigma}Q_{\rho\sigma}+\frac{1}{2}Q^{\rho\sigma\pi}Q_{\rho\sigma\pi\mu
\nu\rho}+(G_{\rho\sigma}-\frac{1}{2}Gg_{\rho\sigma})=\frac{1}{4}(\bar{\psi}\gamma^{\rho}D_{\rho}\psi-D^{\rho}\bar{\psi}\gamma^{\rho}\psi)\quad(3)$$

complemented by the fermionic field equations

$$i\gamma^{\mu}D_{\mu}\psi-m\psi=0\quad(4)$$

as the most general system of field equations in which the completely antisymmetric torsion is coupled to the completely antisymmetric spin of the spinorial Dirac field in an algebraic way, and therefore we may employ such torsion-spin coupling field equations to substitute torsion with the spin of spinor fields in every expression.

Thus the action can be written in the form in which all curvatures and derivatives are decomposed in terms of their torsionless counterparts plus torsional contributions and since the torsion-spin coupling field equations are algebraic they can be used to have torsion substituted in terms of the spin of the fermionic fields yielding

$$L=R-\frac{1}{2}\bar{\psi}(\gamma^{\mu}\nabla_{\mu}\psi-\nabla_{\mu}\bar{\psi}\gamma^{\mu}\psi)+\frac{3}{16}\bar{\psi}\gamma^{\mu}\psi\bar{\psi}\gamma^{\mu}\psi+m\bar{\psi}\psi\quad(5)$$

which will eventually give the system of the field equations already in the form in which torsion has been replaced with spin-spin contact fermionic interactions and in which the torsional constant has the role of coupling constant giving the strength of these interactions.

By varying this action with respect to all fields that remain it is possible to get the field equations, which can also be obtained from the previous form of the field equations after decomposing all curvatures and derivatives in the corresponding torsionless curvatures and derivatives plus torsional contributions written as the spin of fermionic fields, yielding the symmetric curvature-energy coupling field equations according to the Einstein form

$$R_{\mu\rho\sigma}g^{\rho\sigma}=-\frac{1}{2}Rg^{\alpha\rho}=-\frac{1}{4}(\bar{\psi}\gamma^{\rho}\nabla_{\rho}\psi-\nabla^{\rho}\bar{\psi}\gamma^{\rho}\psi)+\bar{\psi}\gamma^{\rho}\nabla_{\rho}\psi-\nabla^{\rho}\bar{\psi}\gamma^{\rho}\psi-\frac{3}{16}\bar{\psi}\gamma^{\rho}\psi\bar{\psi}\gamma^{\rho}\psi\quad(6)$$

together with the fermionic Dirac field equations

$$i\gamma^{\mu}\nabla_{\mu}\psi-m\psi=0\quad(7)$$
as the most general system of field equations with torsion replaced by spin-spin contact fermionic interactions.

At last, we write the symmetric curvature-energy coupling field equations for the Ricci tensor given by

\[ R^{\alpha\beta} = \frac{1}{16} (\nabla^2 \gamma^\alpha \nabla^\beta \psi - \nabla^\beta \nabla^\alpha \psi + \nabla^\alpha \nabla^\beta \psi) \gamma^\mu \psi + \frac{1}{16} m \overline{\psi} \psi \sigma^{\alpha\mu} \]  

and with fermionic Dirac field equations unchanged

\[ i \gamma^\mu \nabla^\mu \psi - \frac{3k}{16} \overline{\psi} \gamma^\rho \psi \gamma^\mu \phi - m \psi = 0 \]  

showing that the spin-spin contact interaction disappear from the all field equations except from the spinorial field equations in which they have the form of Nambu-Jona–Lasinio potentials with undetermined coupling constant.

In order to get the non-relativistic limit we consider the Dirac equation at the second order derivative, obtained by applying to the Dirac equation another Dirac operator, giving a Klein-Gordon type of field equation

\[ \nabla^2 \psi + \frac{3k}{16} \overline{\psi} \gamma^\rho \psi \gamma^\mu \psi + \frac{1}{8} m \overline{\psi} \psi \sigma^{\mu\rho} \psi - \frac{9k^2}{16} \overline{\psi} \gamma^\rho \psi \gamma^\mu \psi + \frac{1}{16} m \overline{\psi} \psi + m^2 \psi = 0 \]  

in which incidentally we notice that even the absence of torsion encoded by assuming \( k \) null does not prevent gravitationally-induced non-linearities to occur, rendering the non-trivial the dynamics of spinors; on the other hand, keeping torsion while neglecting gravity means that we may take \( k \) to be much larger then unity, so that

\[ \nabla^2 \psi + \frac{9k^2}{16} \overline{\psi} \gamma^\rho \psi \gamma^\mu \psi + \frac{1}{8} m \overline{\psi} \psi \sigma^{\mu\rho} \psi - \frac{9k^2}{16} \overline{\psi} \gamma^\rho \psi \gamma^\mu \psi + \frac{1}{16} m \overline{\psi} \psi + m^2 \psi = 0 \]  

and showing that the non-linearity given by torsion are much more relevant: in this weak-gravity limit we may take stationary configurations of energy \( E \) subject to the low-speed regime \( E^2 - m^2 \approx 2m(E - m) \) and hence by writing everything in standard representation we have that the non-relativistic approximation is accomplished by the condition \( \overline{\psi} \approx (\phi^\dagger, 0) \) in terms of which we get

\[ \frac{1}{2m} \nabla \phi + \frac{9k^2}{96 \pi^2 m^2} |\phi|^2 \phi - \frac{1}{16} \phi^3 \phi \phi + (E - m) \phi = 0 \]  

as a pair of coupled Schrödinger field equations.

These field equations are those that have been obtained in reference [3], and next they will be compared to the results obtained in [3], in order to assess the way in which they could escaped from the constraints imposed in non-relativistic regimes by in-matter experiments.

And as it is clear, the Hamiltonian of such a system cannot have contributions of the Pauli type, so that the constraints placed in [3] do not apply.

To better see this point, it is best to reverse the perspective, considering [3] as reference; in this paper, the authors consider a very general action deriving the correspondingly general field equations, and taking the non-relativistic limit in which to write everything in terms of the effective Hamiltonian: then they compare such effective Hamiltonian with the results of measurements performed in an experiment employing polarized slow neutrinos in isotropic static liquid \(^4\)He as a medium.

By adapting the formalism of that paper to the present situation, the correction to the effective Hamiltonian is given by \( \delta H \approx -(\phi^3 \chi + \chi^3 \phi) \frac{9k^2}{16 \pi^2} \), where the torsion term is expressed via the temporal component of the axial vector of the spinor \( (\phi^1, \chi^1) \approx \overline{\psi} \) which is the fermion containing the information about \(^4\)He and, because it is static, we have that \( \chi = 0 \) and so that \( \delta H \approx 0 \) identically.

This is due to the fact that the probed term is proportional to the torsion of the medium, which is taken in the static case, where it vanishes in the present model.

As a consequence of this fact, we still have that the general SKED theory is the only gravitational theory that is compatible with all in-matter experiments.

II. THE TORSIONAL CONSTANT FOR THE NEW PHYSICS

So far we discussed that the theory presented in [3] is among all least-order derivative dynamical theories the most general one coupling the torsional completion of gravity to spinorial matter fields: this theory gives rise to Dirac matter field equations of Nambu-Jona–Lasinio type in which the torsional coupling constant is undetermined by any present empirical result; in particular its non-relativistic limit suppresses any spin-dependent contribution of the non-linear potentials, which are therefore unconstrained by any measurement conducted so far in terms of non-relativistic configurations even for in-matter experiments, and therefore in all experiments that have been conducted up to now. As the most precise measurements are performed by employing static media in which all anisotropic contributions due to the presence of torsion vanish, the SKED theory remains the only gravitational theory that is compatible with all such non-relativistic observations; to probe torsionally-induced non-linear interactions one is compelled to study in-matter experiments performed in the relativistic regimes, hence involving the high-energy scattering of particles, but relativistic scattering can probe models up to a few TeV solely, for the moment. Thus the spectrum ranging from the present to the Planck scale is still enormous.

Nevertheless, this opens an interesting question about torsion, but before dealing with that, we would like to spend some words in order to clarify a misconception in the torsional completion of gravity: the torsional completion of gravity is achieved by not neglecting torsion beside curvature, as the two fundamental objects describing the character of the spacetime, and as a consequence we have the possibility to couple also the spin beside the energy of matter fields, thus obtaining the most exhaustive coupling of matter fields possible; however, because torsion is a tensor on its own, the action should not only have a curvature tensor implicitly containing torsion but torsion.
should also be explicitly present in terms of squared contributions, thus accounting for an independent constant that is different from the gravitational constant in general circumstances. Because Einstein gravity can be obtained variationally from the Lagrangian that is given by the Ricci torsionless curvature scalar $R$ people initially obtained the torsional completion of Einstein gravity from the variation of the Lagrangian that is given by the Ricci torsionfull curvature scalar $G$, but the action containing only $G$ has only one term, and therefore it cannot have more than one constant, which must then be nothing else but the Newton constant: overlooking the fact that more general Lagrangians were possible has laid the basis for the misconception that the torsional constant had to be the Newton constant, and such a misconception was eventually cemented along the decades. So we would like to take the opportunity here to stress out loud the fact that the Lagrangian given by $G$ is certainly the most straightforward but nonetheless not the most general Lagrangian, which is given by the Ricci torsionfull curvature scalar accompanied by quadratic torsion terms, therefore given in terms of two different constants, the gravitational one being the Newton constant but the torsional one being completely undetermined. As a consequence of the fact that the torsional constant is free not to be the Newton constant, we have that the torsionally-induced non-linearities within the Dirac matter field equations are free not to be relevant only at the Planck scale.

As a matter of fact, it may well happen by chance that the torsional constant is after all equal to the Newton constant, and thus the torsionally-induced non-linearities of the Dirac matter field equations are relevant only at the Planck scale, but this is not a necessity, and in general they will be relevant at larger distances; back to the problem of the boundaries on torsion, we have just recalled that, on the other hand of the allowed spectrum, the torsional constant cannot be larger than the Fermi constant, or else the torsionally-induced non-linearities of the Dirac matter field equations would have been relevant before the Higgs scale, but we have never detected them at those distances: this places the torsional constant between the Newton and Fermi constant, so not much of a constraint anyway. We know that the torsional constant must be smaller than the Fermi constant, but because it does not need to be as small as the Newton constant, it may happen that the torsional constant is just smaller than the Fermi constant, so now we may ask: if it were true that the torsional constant were just smaller than the Fermi constant, what would be the consequences of this situation? Would this be of any help in addressing some interesting open problem in physics?

For instance, in field theory, computing some quantities may lead to divergences unless a cut-off is introduced by hand, and even so it may well happen that a reasonable cut-off may still give exceedingly large results compared to observations; the whole idea of placing a cut-off beyond which computations cannot be done is interpretable by thinking that there is a limit beyond which new effects change the physics in such a way that the same computations done in terms of this new physics would give finite results: a theory with a torsional coupling constant that happens to be tuned just beyond these scales and corresponding torsionally-induced non-linear interactions does precisely this, if we interpret the torsional coupling constant as the effective limit encoded by the cut-off of the theory and the torsionally-induced non-linear terms as the new physics. All computations done in the standard context happen to work properly until the scale at which there is the cut-off because this is the scale in which the torsional coupling constant is still negligible, and beyond such scales calculations are no longer reliable because torsional effects change the effective phenomenology; the cut-off would be just beyond the present scales if the torsional coupling constant were to be a little smaller than the Fermi constant, and problems related to these divergences would not appear beyond this boundary. Or at least not necessarily.

Thus, if the torsional coupling constant happened to be tuned a little beyond the Fermi constant, it would mean that all of torsionally-induced non-linear interactions in high-energy scattering would become manifest soon after the scales we are probing in today’s accelerators, with the interesting consequence that such non-linearities might soon let new physics arise; this would be of some help in addressing problems that can be solved when new physics is necessary: these torsionally-induced non-linear potentials relevant soon beyond these scales might precisely be such new physics, solving these problems.

**Conclusion**

In this paper, we have shown that the general SKED theory is the only gravitational theory compatible with all experiments; we have stressed that this is true even if the torsional coupling constant happened to be just a little smaller than the Fermi constant, and therefore torsionally-induced non-linear potentials might be seen as new physics beyond such scale.

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