On Some Aspects of Study on Dimensions and Proportions of Church Architecture

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Abstract. Architecture forms and arranges the environment required for a comfortable life and human activity. The modern principles of architectural space arrangement and form making are represented by a reliable system of buildings which are used in design. Architects apply these principles and knowledge of space arrangement in regard to the study of special and regulatory literature when performing a particular creative task. This system of accumulated knowledge is perceived in the form of an existing stereotype with no regard for understanding of the form making and experience inherent to the architects and thinkers of previous ages. We make an attempt to restore this connection as the form-making specific regularities known by ancient architects should be taken into account. The paper gives an insight into some aspects of traditional dimensions and proportions of church architecture.

1. Introduction
The problem of the study is that each field of scientific knowledge about space gives own interpretation of its arrangement. The question about the role of architectural space and principles of its creation was raised in the beginning of XX c. by an architectural theorist N.A. Ladovsky. The doctrine of proportions was a doctrine of the beginning of a beginning. The proportion was called so, because the creation of the Universe was meant by the beginning and a concept of proportions was historically associated with the philosophy of Pythagoreans. Their thesis “a number is the essence of things” stated about a fundamental property of the nature that numbers reflect the essence of things; laws expressed by numbers show development and motion of processes and energies, taking place in the nature [1].

When observing and examining the nature, Pythagoreans believed that its different qualitative interconnections and phenomena exhibited the same mathematical properties. Based on these observations, they concluded that mathematical properties reflected the essence of phenomena and this essence was hidden under numbers and numerical ratios. Therefore, a number was treated as the beginning of everything, “unit of existence”. “Bodies” were composed from these fundamental qualityless units, forming in different combinations all possible geometrical figures, and as a result in the course of development “the units of existence” started to represent material objects in mathematics. And this qualityless number itself obtained a status of a “matter” (substance which has no qualities, as qualityless as the surrounding geometric space) [2].

A.F. Chernyaev in “The Basics of Russian Geometry” noted that in the nature there are no points, lines, planes or other geometrical shapes, studied by mathematics and geometry, no even properties,
studied by physics, chemistry or other sciences. In has bodies and corporal formations in the form of bounded blocks with properties, and these blocks are examined by different sciences. [2].

From ancient times church architecture reflected the ideas of people about the system of world organization and order. In its forms architects put a conceptual model of the Universe creation. The interconnection of the structure and forms of church architecture with the world structure has been specified by life processes and subjective changes in the social consciousness [3].

N.L. Pavlov developed a theory of reproducing a mechanism of the universal law in forms of ancient church architecture, considering it as the embodied in stone Universe [4]. He thought that abstract architectural forms had limited notional potential and they can be explained with the help of analogous forms of other related tradition, where they are developed at the later or more descriptive stage [4].

General ideas of the worldview in the era of Greek classics were based on numerical harmony and geometric universals. By the time of Pythagoras and the Pythagoreans the concept of symmetry expressed something uniform, commensurate, proportionally harmonious in the object. It was regarded as a way of coordination of many parts, with the help of which they are combined into the whole. The Greek term “symmetry” (symmetria) or “commensuration” was used along with the term “order”. There were two close Greek terms denoting order: taxis and kosmos. When translating from Greek Vitruvius introduced a Latin word “ordo” denoting a range, line, order, arrangement [3, p. 90-100]. The analysis on the systems of proportions – analogies conducted by Vitruvius in the third chapter of the treatise “On the Commensuration (Symmetry) of Churches” explained: “The proportion is correspondence between parts of the whole work and its whole towards a part taken as the initial, on which any regularity is based. The thing is that a church without commensuration and proportion can’t have the right composition if it doesn’t have the same exact articulation as a well-built man” [5].

Historians of architecture repeatedly testified that in the proportions of sides of the buildings there were geometric relations of the side and the square diagonal – a golden section. They also noted the presence of other proportional dimensions. The hypotheses appeared that forms of architecture originated from geometry and it was responsible for its proportional system [6]. Scientists of XX c. transformed the beliefs of the ancients about proportions into symmetry based on such antique concepts as uniformity, equality and identity. However, symmetry, embracing the basic part of natural phenomena, could not explain the essential natural phenomena, i.e. the change in dimensions, namely growth.

Symmetry is the concept of equal, uniform structure, the equality of keeping states. Proportion expresses the equality of equal, uniform changes. In the contemporary science the concept of proportion is closely related to symmetry. The proportion serves as the symmetry of similarities as one of the possible (the least studied and almost has no impact) cases [7].

The modern philosophical interpretation of the worldview in terms of symmetry is in sync with the well-known from the earliest times vivid ideas of a man and the Universe [7]. It is undeniable that the correlation of architectural sizes with the sizes of a man is the basics of comfort.

Literary sources contain information about the fact that from the earliest times buildings were measured with sazhens, elbows, palms, and spans. There were many proven examples of the fact that in Russia two or three measures were simultaneously used in the same building. This is attested to by archeological finds: a measuring cane, found in excavations of the cultural layer of the 12th c. in Novgorod [8]. The preserved in the archives projects of churches of XVIII-XIX cc. had a measuring scale in sazhens. A system of paired measures and a double square – geometrically correlated measuring scales, interdependent in relations and absolute values, is shown in proportions of Russian religious and residential buildings. The principle of geometric similarity established in the Old Russian measures, i.e. sazhens and geometrically subordinate lengths, from century to century was the basics of architect’s measures. Different ways of applying paired measures were reflected in the Russian church architecture [2].

Studying the process of cell division, a French mathematician B. Pascal discovered that it occurs by halving the mother cell and each subsequent cell is also divided in half, forming a geometric
progression. In the symmetric arrangement of numbers in a column appears something resembling a triangle: 1; 2; 4; 8; 16; … etc. The process of obtaining a geometric progression from the figure of two was called “Pascal’s triangle”. It’s interesting and important that the Old Russian commensurable instruments of sazhens were divided into smaller segments according to this technique (Figure 1). The architect A. Piletsky used the system of duplication and halving of Russian sazhens to build some Fibonacci series in the unified set of numbers [9].

![Figure 1. Sazhens.](image)

Where does the notion of a segment division into extreme and mean ratio, which makes it possible to obtain the golden number $\varphi$ and forming the proportion called by Leonardo da Vinci as “The Golden Ratio” and by Kepler “The Divine Proportion” come from is unknown. But in Ancient Egypt and Ancient Greece on the basis of the golden number $\varphi = 1,618…$ a series of 11 numbers was obtained by subsequently multiplying the basic 1 by $\varphi$ (a rising branch of the series) and dividing the basic 1 by $\varphi$ (a descending branch of the series), being called as the golden series (other variants: Greek or Egyptian series [10]). Let’s reproduce it:

…; 0,934; 0,056; 0,090; 0,146; 0,236; 0,382; 0,618; 1,00; 1,618; 2,618; 4,236; …

The golden number $\varphi = 1,618…$ is obtained in several ways, one of which is division of the segment into extreme and mean ratio (Figure 2). *Why the segment is divided into extreme and mean ratio and what the golden number $\varphi$ testifies is still unknown.* It is only known that this division creates an aesthetically complete image of those human works, where they enjoy use. We should note that the problem statement mentions division of one segment into two unequal parts $a$ and $c$ (Figure 2), so that the whole segment $(a + c)$ refers to a larger part of $c$ as $c$ to a smaller part of $a$. To get the golden proportion the segment $AC$ is divided into two unequal parts $AB = a$ and $BC = c$, so that its length $AC = AB + BC = (a + c)$ refers to a larger part of $c$ as $c$ refers to a smaller part of $a$. Let’s write down this ratio and make some more complicated calculations:

![Figure 2. Segment division into extreme and mean ratio.](image)
\[(a+c) = c / a\]  

(1.1)

The proportion (1.1) is called the golden proportion. In this case a finite in rational numbers segment length \((a + c)\), which is multiple of some measuring instrument, for example a meter, is implied. The statement of the problem does not mention the impossibility of its integer or linear fractional division or the irrationality of two formed in the division segments. It is a very important remark. It confirms an unexpected, probable or even accidental nature of division. Let us prove this random nature by replacing in (1.1) the ratio \(c/a\) to \(b:\)

\[b = c / a\]  

(1.2)

and substituting (1.2) with (1.1), we obtain a quadratic equation

\[b^2 - b - 1 = 0,\]  

(1.3)

by solving it, we find two values of \(b:\)

\[b_1 = \left(1 + \sqrt{5}\right)/2 = \phi = 1.6180339, \quad b_2 = \left(1 - \sqrt{5}\right)/2 = -1/\phi = 0.6180339\]  

(1.4)

The golden number \(\phi\) is an irrational number (concealed from accuracy number). That is a number, which infinite sequence cannot be calculated to the end, no matter how long it is calculated [11].

The monuments of church architecture remain mysterious to a great extent. They significantly differ from each other. The unifying element is the completion of temples and repetition of main compositional techniques. Of course, despite our limited knowledge we want to study these monuments, determine the uniqueness of their compositional techniques, decorative forms and elements, and try to set into their motives [129].

In architecture it is known that objects build in accordance with golden proportions have high aesthetic qualities and a balance of their parts. For the ancient Greeks, who widely used the golden proportions in the design of their structures, the presence of a proportional connection between all elements of the structure was a condition of harmony and steady perfection. And the word “harmony” means “connection” in translation from ancient Greek. The harmony of parts as a whole is expressed in terms of numerical relations, namely proportions. We can say that harmony is a system of qualities proportional to nature. In other words, it is a variety of elements (objects, figures, numbers, etc.) proportional to natural parameters. What is responsible for the appearance of harmony? What qualities indicate its presence?

To the harmoniously consistent artificial or numerical systems reflecting natural processes one can refer a combination of the following properties:

- Proportioning of elements to the golden number.
- The unity of the whole in diversity.
- The universal interrelation between elements.
- Structural properties of one ratio, allowing to restore the system using the minimum given data.
- The absence of random operations and elements in the system.
- Degree and arithmetic combinatorics of elements.
- Number memory (each element remembers about the rest), etc.

There is no need to recite all these properties, as they are included in proportioning according to the golden number [2].

Studying the numerical systems of architectural proportioning the architect A. Piletsky noted that numbers of Fibonacci series are characterized by a multivariate additivity of components with a resultant number in their own system [12]. For example:

\[3 + 5 = 8; \quad 5 + 8 = 13; \quad 3 + 5 + 13 = 21; \quad 21 + 13 = 34; \quad 3 + 5 + 13 + 34 = 55 \quad etc.\]
According to A.F. Chernyaev these arithmetic combinatorial properties of the series were the first sign of harmony of the system being created. He emphasized that commensurate instruments, i.e. the Old Russian sazhens in the amount of 15, reconstructed by A. Piletsky and named “Vsemer” was based on the golden number. The operations of commensuration carried out in ancient times with the help of ancient Russian instruments, i.e. sazhens, are still not properly understood not only by the general public, but also by specialists who renovate ancient buildings and temples.

It is not clear why there were many of them? Why was it necessary to use one sazhen for marking an object in height, another sazhen for width and the third one for length? Why are these sazhens disproportionate to each other? The logic of length required to use a uniform measuring instrument. The meter became such an instrument. However, meter is a standard for separating parts from the whole and it is not sufficient for commensuration. It is important only when measuring the already found proportions [2].

The peculiarity that distinguishes the Old Russian sazhens from all other instruments is that their fractions are obtained by splitting them (by dividing them in half). No fragmentation of elements into more than 2 parts was not allowed. In total, the sazhen included six elements (fractions) and the last, namely vershok was the 32th part of the sazhen. Each element had a duplicate name. For example, a great sazhen, Greek elbow and a church span, since none of them was equal in length to any other. Only elements of a small sazhen did not have a second name. Besides, each sazhen by the will of an architect could “grow” in terms of length by half, in two and two and a half times. Needless to say that parts of these sazhens could also increase proportionally.

The consecutive division of sazhens in two was performed in order that any part of a construction object had only integer number of sazhens and their elements. The layout of each object was carried out by an even number of sazhens. The volume of the object was formed starting from a height, which had an even number of one sazhen, then the width was split by an even number of other sazhens and finally, the length, by the same number of another sazhens. The inner parts of the object were marked in the same way, but with the sazhen elements. The smaller the room, the smaller part of the sazhen was used [2]. A.F. Chernyaev stated: “The space, according to the logic of inventors of the meter, is dead and that’s why it can be divided and combined in any proportions, without worrying about what will happen in architecture after joining. The result is the erection of formless spatial structures, inauthenticity of cities, accumulation of boxes, discomfort and psychological pressure of dwellers”.

For the ancient Russian architect, the external environment like the Earth, was alive and consisted of whole parts. Its mindless fragmentation became dismemberment of a single living body into parts. The architect in his work, to be precise in co-creation with the Supreme Forces, endowed the created structures with proportionality of sazhens and their fractions. The proportionality of sazhens and their fractions made the living space harmonious, gentle and long-standing, typical for the flow of living energy. The numbers of golden variety are dimensionless coefficients, which reflect the spatial change in quality. Apparently, they “work” only when there is a “standard” module, i.e. the first from the basic \(1\) number, which determines the process of ascending or descending the series. Module is like a coefficient of “change” of the dimensionality of space, its relationship to this space. The numbers of the golden section are “bars” of this movement, giving stability to the ongoing processes.

The study on the dimensions of building architecture will be incomplete if we exclude the impact of dimensionality on the mind. “If proportions of the architectural works that surround us belong to random families, as in most modern constructions, then a person finds himself in an environment, which proportional structure is not inherent in its symmetry”. Such an environment, which does not possess any of the groups specific to the human symmetry dimensions is in most cases not perceived by him. This is where the unfavorable influence of the environment created not according to the law of dimensions on the psychophysical state of a human is hidden [14]. The structural properties of a modern building determine the size of a structure and technological processes. The constructional capabilities of the material establish the maximum dimensions of the structures. For example, a cross section of the column specifies its possible height and span, that’s why the size of a building can be
determined by its design. At the same time, dimensions and proportions are the main objective of an architect in the search for artistic merits and beauty perceived by the human community [15].

2. Conclusion
Creating a religious building, architects of classical antiquity fulfilled the divine design, embodying in its architectural forms hidden knowledge. The secret content was embodied in real spatial structures and forms, in which the whole depth of unconscious intuitive comprehension of the world structure was revealed [4]. The model of the spatiotemporal world is ciphered in the architecture of ancient temples [3]. Relying on the principle of anthropometry, proportional relations of the outer and inner temple space are formed. A proportional measure for the building design was sazhen and its constituent parts. These values are commensurate with a well-built man, measures typical for proportions of natural objects [16]. In some cases, the heritage and deep knowledge of antiquity is available to the modern architect not at the level of awareness, but only in the form of formal laws and rules [16]. Any manipulations with forms are allowed for him as a sole creator. Making his place in this position, he often comes to complete arbitrariness in the space arrangement. The projected modern building ceases to be a reflection of the world structure, becoming a work of art.

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