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Dynamical analysis of fractional-order tobacco smoking model containing snuffing class

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Abstract The current pandemic situation caused by COVID-19 has affected human life globally at the economic, social and mental health levels. Specifically, tension has led an increasing number of people to the consumption of various types of tobacco. In this work, an existing tobacco smoking model with a specific class of tobacco snuffing is converted into a fractional order as many applications of fractional derivatives to recall the past history of smokers in the present model. For this purpose, we use fractional derivative in Caputo sense to study the model in the form of fractional order. Then Positivity, boundness and dynamics of the proposed model are investigated. For numerical results, the generalized “Adams–Bashforth–Moulton Method (GABMM) and fourth-order Runge–Kutta (RK4) method” are used to solve the proposed model and Matlab numerical computing environment is the current software used.
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1. Introduction

Mathematical models in fractional order have proven to be useful in manifesting wide range of phenomena mathematically than integer-order models because of fractional order reveals the past history and hereditary properties in models especially in the infectious diseases models. Mathematicians usually use, for simplicity, ordinary differential equations in integer order. In applied sciences, mathematical modeling has found widespread applications and in particular the fractional calculus, see [1–17]. Smoking is a cause of many diseases including many type of cancers In current pandemic of COVID-19 virus, smokers are at more risk to be affected by COVID-19 - because of many reasons including of smokers’ fingers are in touch with their lips regularly during smoking and this habit leads to increase the probability of transmission of virus from hand to mouth. Worldwide, those people who are smoking suffered increasingly from different disease like cancer of lungs, lips, throat. In this way the immune system
of smokers people weakens due to which they are easily exposed to serious disease like corona virus disease. Cigarette smokers are 2 to 4 times more likely to get heart disease than nonsmokers and also doubles a person’s risk for stroke and also higher risk to caught lung cancer. To increase the life expectancy of humans, scientists, doctors and mathematicians have tried to control smoking through modelling that contains media or education campaign or in the form of anti-nicotine medicine [18–31]. Mathematicians have tried to make different smoking models to represent cigarette smoking phenomena. This work was initiated by Castillo–Garsow et al. [18] in their model where they discussed the potential smokers represented by \( P \), smokers represented by \( S \), and quit smokers represented by \( Q \). Then a modified model of smoking that contained chain smokers class was presented by Sharami et al. [19]. Recently, researchers have designed several smoking models under various linear, saturated, square-root-type and harmonic-mean-type incidence rates [22,25–27,21,29–31]. Nowadays, researchers attempt to bring about different fractional order epidemic models. Due to a lot of applications, fractional calculus is applied in different scientific fields [32–38]. This research work demonstrates the smoking model in fractional order with snuffing class and determine the existence of an analytical and numerical solution of our proposed model, presented in [30] as:

\[
\begin{align*}
\frac{dX}{dt} &= \lambda - \beta_1 XH_1 - \mu X + \alpha Y, \\
\frac{dH_1}{dt} &= \beta_1 XH_1 - \beta_2 H_1 H_2 - (\rho + \mu)H_1, \\
\frac{dH_2}{dt} &= \beta_2 H_1 H_2 - (d + \omega + \mu)H_2, \\
\frac{dY}{dt} &= \omega H_2 - (x + \gamma + \mu)Y, \\
\frac{dZ}{dt} &= \gamma Y - \mu Z,
\end{align*}
\]

under the initial conditions:

\[
X(0) = e_1, \quad H_1(0) = e_2, \quad H_2(0) = e_3, \quad Y(0) = e_4, \quad Z(0) = e_5,
\]

for the parameters description see Table 1, of [30].

To include the past history or hereditary properties in our model, we establish the fractional order derivatives instead of integer order derivatives in system (1). As the term \( Z(t) \) does not appear in the first four equations of system (1), therefore without loss of generality, we can take out \( Z(t) \) from system (1). So, the following set of differential equations in fractional order can be written as a new system:

\[
\begin{align*}
0^\alpha D^\alpha_t X(t) &= \lambda - \beta_1 XH_1 - \mu X + \alpha Y, \\
0^\alpha D^\alpha_t H_1(t) &= \beta_1 XH_1 - \beta_2 H_1 H_2 - (\rho + \mu)H_1, \\
0^\alpha D^\alpha_t H_2(t) &= \beta_2 H_1 H_2 - (d + \omega + \mu)H_2, \\
0^\alpha D^\alpha_t Y(t) &= \omega H_2 - (x + \gamma + \mu)Y,
\end{align*}
\]

Here, the notation \( D^\alpha_t \) stands for derivative in Caputo sense with order \( 0 < \alpha \leq 1 \). The fractional order system is converted to ordinary differential equations system when \( \alpha = 1 \). System (3) leads to generalization for system (1). As integer-order epidemic models have established fruitful understanding for biological systems, more realistic biological models memory and after-effect properties are presented by fractional order models, especially in smoking dynamics. Therefore, the fractional-order derivatives are applied on system (1). The stability of the system is discussed as the same as proved in [30]. For basics of fractional calculus and fractional order differential equations (FODEs) see [39–45].

The arrangement of the rest of the paper is given in Section 2, where the dynamics of the fractional order model is presented. The intention of GABMM is presented in Sections 3, with a brief introduction for solution of fractional order smoking model. Section 4 is devoted to numerical simulation results of the GABMM, where comparisons of results obtained with GABMM and Runge-Kutta method (RKM) taken place in Section 5. Last section is devoted to a brief conclusion.

2. Dynamics of the fractional order model

Here, this part we derive results about positivity and boundedness. We define space by \( \mathcal{A}_1^H = \{(X, H_1, H_2, Y) | X, H_1, H_2, Y \geq 0 \} \).

**Theorem 1.** Let \((X_0, H_{10}, H_{20}, Y_0) \in \mathbb{R}_+^4 \) be initial values and \((X(t), H_1(t), H_2(t), Y(t)) \) be any solution. Then, the set \( \mathcal{A}_1^H \) is a positively invariant. Also one has

\[
\begin{align*}
\limsup_{t \to \infty} X(t) \leq X_\infty &:= \frac{\lambda + \alpha Y}{\mu}, \\
\limsup_{t \to \infty} H_1(t) \leq H_{1\infty} &:= \frac{\omega}{(\rho + \mu)w}, \\
\limsup_{t \to \infty} H_2(t) \leq H_{2\infty} &:= \frac{\omega H_2}{(x + \gamma + \mu)w}, \\
\limsup_{t \to \infty} Y(t) \leq Y_\infty &:= \frac{\omega H_2}{(x + \gamma + \mu)w}.
\end{align*}
\]

**Proof.** For the model (3), we have

\[
\begin{align*}
0^\alpha D^\alpha_t X(t)|_{x=0} &= \lambda + \alpha Y > 0, \\
0^\alpha D^\alpha_t H_1(t)|_{t=0} &= 0, \\
0^\alpha D^\alpha_t H_2(t)|_{t=0} &= 0, \\
0^\alpha D^\alpha_t Y(t)|_{y=0} &= \omega H_2 \geq 0.
\end{align*}
\]

Upon using generalized mean value theorem [46,47] together with (5), one has \( X(t), H_1(t), H_2(t), Y(t) \geq 0 \), for all values of \( t \geq 0 \). Equation first of the system (3) yields

\[
0^\alpha D^\alpha_t X \leq \lambda - \mu X + \alpha Y \infty.
\]

2nd and 3rd equations of the system (3) implies that

\[
0^\alpha D^\alpha_t (X + H_1 + H_2) \leq \lambda - \mu X + \alpha Y \infty - (\rho + \mu)H_1 - (d + \omega + \mu)H_2,
\]

which yields

\[
\limsup_{t \to \infty} X(t) + H_1(t) + H_2(t) \leq H_{1\infty}
\]

and

\[
\limsup_{t \to \infty} X(t) + H_1(t) + H_2(t) \leq H_{2\infty}.
\]
Accordingly, it follows the second and third estimate of (4). Now by the last equation of system (3), one has
\[ \frac{d}{dt} (xY + y) = -\mu x - \gamma y + \mu y \]
for large enough \( t \). Which leads the fourth estimate of (4). \( \square \)

2.1. The reproduction number and equilibrium points

Solving the following algebraic equations for finding the equilibria of the model (3),
\[ \begin{align*}
\lambda - \beta_1 X H_1 - \mu X + \alpha Y & = 0, \\
\beta_1 X H_1 - \beta_1 X H_2 - (\rho + \mu) H_1 & = 0, \\
\beta_2 H_1 H_2 - (d + \omega + \mu) H_2 & = 0, \\
\omega H_2 - (\gamma + \mu) & = 0.
\end{align*} \]
(6)

Two solutions to the system (6) are obtained via using some algebraic manipulations as \( E_0 = \left( \frac{1}{\mu}, 0, 0, 0 \right) \), and
\[ E^* = \left( X^*, H_1^*, H_2^*, Y^* \right), \]
where
\[ \begin{align*}
X^* & = \frac{\beta_1 X^* H_1^*}{\beta_1 X}, \\
H_1^* & = \frac{\beta_2 H_1^* (d + \omega + \mu)}{\omega X^*}, \\
H_2^* & = \frac{\omega H_2^*}{(\gamma + \mu)}, \\
Y^* & = \frac{(x + \gamma + \mu)}{(\gamma + \mu + \mu) H_1^* + (x + \gamma + \mu) H_2^*}.
\end{align*} \]
The Jacobian of system (2) is
\[ J = \begin{pmatrix}
-\beta_1 H_1 & -\beta_1 X H_1 & 0 & 0 \\
\beta_1 X - \beta_1 H_1 - (\rho + \mu) & -\beta_1 H_1 & 0 & 0 \\
\beta_2 H_1 & 0 & -\beta_1 H_1 - (d + \omega + \mu) & 0 \\
0 & \omega X & 0 & -(x + \gamma + \mu)
\end{pmatrix}. \]

Also at free equilibrium point \( E_0 \), the Jacobian is provided as
\[ J(E_0) = \begin{pmatrix}
-\mu & -\frac{\beta_1 X}{\mu} & 0 & x \\
0 & \frac{\beta_1 X}{\mu} - (\rho + \mu) & 0 & 0 \\
0 & 0 & -(d + \omega + \mu) & 0 \\
0 & 0 & \omega x & -(x + \gamma + \mu)
\end{pmatrix}. \]

Considering the given matrices to compute reproductive number
\[ F = \begin{pmatrix}
\frac{\beta_1 X}{\mu} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}, \]
\[ V = \begin{pmatrix}
(\rho + \mu) & 0 & 0 \\
0 & (d + \omega + \mu) & 0 \\
0 & 0 & -(x + \gamma + \mu)
\end{pmatrix}. \]

The maximum eigenvalue of \( FV^{-1} \) is \( \frac{\beta_1 X}{\mu(\rho + \mu)} \), so
\[ R_0 = \frac{\beta_1 X}{\mu(\rho + \mu)} \]
is the required reproductive number.

**Theorem 2.** Under the condition \( R_0 < 1 \), then the system (3) is locally stable and if \( R_0 > 1 \), then system (3) is unstable.

**Proof.** At \( E_0 \), the condition for local stability at the Jacobian of system (3) is given by
\[ J(E_0) = \begin{pmatrix}
-\mu & -\frac{\beta_1 X}{\mu} & 0 & x \\
0 & \frac{\beta_1 X}{\mu} - (\rho + \mu) & 0 & 0 \\
0 & 0 & -(d + \omega + \mu) & 0 \\
0 & 0 & \omega x & -(x + \gamma + \mu)
\end{pmatrix}. \]
which follows the eigenvalues \( \lambda_1, \lambda_2, \lambda_3 \) and \( \lambda_4 \) as
\[ \begin{align*}
\lambda_1 & = -\mu < 0, \\
\lambda_2 & = -(d + \omega + \mu) < 0, \\
\lambda_3 & = -(x + \gamma + \mu) < 0, \\
\lambda_4 & = (\rho + \mu)(R_0 - 1),
\end{align*} \]
implies that \( \lambda_2 < 0 \) if \( R_0 < 1, \lambda_2 = 0 \) if \( R_0 = 1 \) and \( \lambda_2 > 0 \) if \( R_0 > 1 \). \( \square \)

**Theorem 3.** If \( R_0 < 1 \), then the system (3) is globally stable.

**Proof.** For proof of this theorem see [30]. \( \square \)

3. The generalized Adams–Bashforth–Moulton method

Here GABMM is presented in this section [48,49]. In this algorithm, the GABMM is derived for getting the numerical solution of the nonlinear FODEs. Let
\[ D_{\alpha}(t)y(t) = f(t, y(t)), \quad 0 \leq t \leq T, \]
(8)
with
\[ y^{(k)}(0) = y_{0k}, \quad k = 0, 1, \ldots, [a] - 1 \]
(9)
be a general problem of FODEs. We obtain the solution \( y(t) \) in view of application of fractional integral on both sides of Eq. (8)
\[ y(t) = \sum_{k=0}^{[a]-1} \frac{y_{0k}}{k!} t^k + \int_0^t \frac{(t - \tau)^{a-1}}{\Gamma(a)} f(\tau, y(\tau)) d\tau. \]
(10)

By setting \( h = \frac{T}{m}, t_n = nh, n = 0, 1, \ldots, m, \) Eq. (10) can be described as follows for some positive integer \( m \)
\[ y_{h}(t_{n+1}) = \sum_{k=0}^{[a]-1} \frac{y_{0k}}{k!} \frac{h^k}{(a + 1)} \left( \frac{t_{n+1}^a}{\Gamma(a + 2)} f(t_{n+1}, y_{h}(t_{n+1})) \right) + \frac{h^a}{\Gamma(a + 2)} \sum_{j=0}^{n} \alpha_{j,a+1} f(\tau, y_{h}(\tau)). \]
(11)
\[ a_{j,a+1} = \begin{pmatrix}
(n + 1)^{a+1} - (n + 1)^{a}, \\
(n - j + 1)j^{a+1} - 2(n - j + 1)^{a} + (n - j)^{a+1}, \\
1, \quad \text{if } j = 0, \\
\text{if } 0 < j \leq n, \\
\text{if } j = n + 1.
\end{pmatrix} \]
In which the predicted value \( y_{h}^{a}(t_{n+1}) \) may be derived as
\[ y_{h}^{a}(t_{n+1}) = \sum_{k=0}^{[a]-1} \frac{y_{0k}}{k!} \frac{h^k}{(a + 1)} + \frac{1}{\Gamma(a + 2)} \sum_{j=0}^{n} \beta_{j,a+1} f(\tau, y_{h}(\tau)). \]
(12)

in which
\[ \beta_{j,a+1} = \frac{h^a}{\Gamma(a + 2)} \left( n - j + 1 \right)^{a} - (n - j)^{a}. \]
The estimated error is
\[ \max_{j=0,1,\ldots,n} |y(t_j) - y_{h}(t_j)| = 0(h^p), \]
in which \( p = \min \{1 + a, 2\}. \)
4. Implementation of numerical simulation

Current part is related to numerical solution of the nonlinear fractional model using the GABMM method. The numerical scheme of model (3) with the help of GABMM is given as follows:

\[ X_h(t_{n+1}) = X_h + \frac{h}{\Gamma(\alpha)} \left[ f_h(t_n, X_h(t_n), H_h(t_n), Y_h(t_n)) + \sum_{j=0}^{\alpha-1} \beta_{\alpha-j} f_h(t_j, X_h(t_j), H_h(t_j), Y_h(t_j)) \right], \]

\[ H_h(t_{n+1}) = H_h + \frac{h}{\Gamma(\alpha)} \left[ f_h(t_n, X_h(t_n), H_h(t_n), Y_h(t_n)) + \sum_{j=0}^{\alpha-1} \beta_{\alpha-j} f_h(t_j, X_h(t_j), H_h(t_j), Y_h(t_j)) \right], \]

\[ Y_h(t_{n+1}) = Y_h + \frac{h}{\Gamma(\alpha)} \left[ f_h(t_n, X_h(t_n), H_h(t_n), Y_h(t_n)) + \sum_{j=0}^{\alpha-1} \beta_{\alpha-j} f_h(t_j, X_h(t_j), H_h(t_j), Y_h(t_j)) \right], \]

in which

\[ X_h^0 = X_0 + \frac{1}{\Gamma(\alpha+1)} \sum_{j=0}^{\alpha} \beta_{\alpha-j} f_j(t_0, X(t_0), H(t_0), Y(t_0)) \]

\[ H_h^0 = H_0 + \frac{1}{\Gamma(\alpha+1)} \sum_{j=0}^{\alpha} \beta_{\alpha-j} f_j(t_0, X(t_0), H(t_0), Y(t_0)) \]

\[ Y_h^0 = Y_0 + \frac{1}{\Gamma(\alpha+1)} \sum_{j=0}^{\alpha} \beta_{\alpha-j} f_j(t_0, X(t_0), H(t_0), Y(t_0)) \]

are the required estimates at \( t_{n+1}, n = 0, 1, \ldots, m \).

5. Numerical and simulation results

In this section, the GABMM with initial and parameters’ values provided in Table 2, [30] is used for finding numerical results of fractional order system (3). This method is a very effective tool in obtaining numerical solutions of fractional order differential equations. In interval \([0, 60]\), some graphical results are presented for the numerical solutions of system (3). The other method for the solution of system (3) which uses \( \alpha = 1 \), is fourth-order RKM, the corresponding computed results are compared graphically with results obtained by GABMM. The selected step size is \( h = 0.0125 \). Approximate solutions for \( X(t), H_1(t), H_2(t), Y(t) \) and \( Z(t) \) are shown in Figs. 1–5 obtained by using GABMM and the fourth-order RKM, when \( \alpha = 1 \) and the solutions for \( X(t), H_1(t), H_2(t), Y(t) \) and \( Z(t) \) are shown in Figs. 6–10, by using GABMM for the different values of \( \alpha \). From the graphical results in Figs. 1–5, it can be seen that the results obtained using the proposed algorithm match the results of the RK4 method very well, which implies that the presented method...
can predict the behavior of these variables accurately in the region under consideration. Furthermore, other figures, show the approximate solutions for all considered classes obtained for different values of \( a \) using the proposed algorithm. From these graphical results, it is clear that the approximate solutions depend continuously on the time-fractional derivative.

6. Conclusions

In this manuscript, we have formulated and analyzed a new mathematical model for tobacco smoking with snuffing class. It ought to be emphasized that the model may be a generalization of a later published work proposed in [30]. Here, first, the fractional order tobacco smoking model with snuffing class is established. For a numerical solution of the proposed model, we accomplished the generalized Adams–Bashforth–Moulton method which resulted in excellent compatibility with solutions obtained with RK4 method. Also, the graphical results for the proposed model were presented. As a future work, the results could be expanded in this work to propose modern mathematical models for smoking with co-infections nature. Particularly, successful strategies to control smoking will be examined. It is noted here that analytical and numerical strategies for FDEs models arrangements are necessary.
Fig. 4 $Y(t)$ vs. time $t$: used solid line and dotted line for GABMM and RKM respectively.

Fig. 5 $Z(t)$ vs. time $t$: used solid line and dotted line for GABMM and RKM respectively.
Fig. 6  \(X(t)\) vs. time \(t\): used solid line, dashed line and dot-dashed line for \(x = 1.0, x = 0.85\) and \(x = 0.95\) respectively.

Fig. 7  \(H_1(t)\) vs. time \(t\): used solid line, dashed line and dot-dashed line for \(x = 1.0, x = 0.85\) and \(x = 0.95\) respectively.
Fig. 8  $H_2(t)$ vs. time $t$: used solid line, dashed line and dot-dashed line for $z = 1.0$, $z = 0.85$ and $z = 0.95$ respectively.

Fig. 9  $Y(t)$ vs. time $t$: used solid line, dashed line and dot-dashed line for $z = 1.0$, $z = 0.85$ and $z = 0.95$ respectively.
Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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