Hierarchical Radio Resource Optimization for Heterogeneous Networks with Enhanced Inter-cell Interference Coordination (eICIC)

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Abstract—Interference is a major performance bottleneck in Heterogeneous Network (HetNet) due to its multi-tier topological structure. We propose almost blank resource block (ABRB) for interference control in HetNet. When an ABRB is scheduled in a macro BS, a resource block (RB) with blank payload is transmitted and this eliminates the interference from this macro BS to the pico BSs. We study a two timescale hierarchical radio resource management (RRM) scheme for HetNet with dynamic ABRB control. The long term controls, such as dynamic ABRB, are adaptive to the large scale fading at a RRM server for co-Tier and cross-Tier interference control. The short term control (user scheduling) is adaptive to the local channel state information within each BS to exploit the multi-user diversity. The two timescale optimization problem is challenging due to the exponentially large solution space. We exploit the sparsity in the interference graph of the HetNet topology and derive structural properties for the optimal ABRB control. Based on that, we propose a two timescale alternative optimization solution for the user scheduling and ABRB control. The solution has low complexity and is asymptotically optimal at high SNR. Simulations show that the proposed solution has significant gain over various baselines.

Index Terms—Heterogeneous Network, Dynamic ABRB Control, Two Timescale RRM

I. INTRODUCTION

As HetNet provides flexible and efficient topology to boost spectral efficiency, it has recently aroused immense interest in both academia and industry. As illustrated in Fig. 1, a HetNet consists of a diverse set of regular macro base stations (BS) overlaid with low power pico BSs. Since this overlaid structure may lead to severe interference problem, it is extremely critical to control interference via RRM in HetNet. There has been much research conducted on RRM optimization for traditional cellular networks. In [1], [2], the authors considered power and user scheduling in single-carrier cellular networks. In [3], [4], the game theoretical approaches are proposed for distributed resource allocation. In [5], the authors proposed a dynamic fractional frequency reuse scheme to combat the inter-sector interference under a game-based optimization by each sector. The coordinated multipoint transmission (CoMP) [6] is another important technique to handle the inter-cell interference. For example, in [7], the authors exploited the uplink-downlink duality to do joint optimization of power allocation and beamforming vectors. In [8], a WMMSE algorithm is proposed to find a stationary point of the weighted sum-rate maximization problem for multi-cell downlink systems. While the above algorithms achieve comparably good performance, they require global channel state information (CSI) for centralized implementation [7] or over-the-air iterations and global message passing for distributed implementation [9]. It is quite controversial whether CoMP is effective or not in LTE systems due to large signaling overhead, signaling latency, inaccurate CSIT, and the complexity of the algorithm.

On the other hand, solutions for traditional cellular networks cannot be applied directly to HetNet due to the unique difference in HetNet topology. First, the inter-cell interference in HetNet is more complicated, e.g., there is co-tier interference among the pico BSs and among the macro BSs as well as the cross-tier interference between the macro and pico BSs. Furthermore, due to load balancing, some of the mobiles in HetNet may be assigned to a pico BS which is not the strongest BS [9] and the mobiles in the pico cell may suffer from strong interference from the macro BSs. To solve these problems, some eICIC techniques, such as the ABS control [9], have been proposed in LTE and LTE-A [10]. In [11], the authors analyzed the performance for ABS in HetNet under different cell range extension (RE) biases. However, they focused on numerical analysis for the existing heuristic eICIC schemes, which are the baselines of this paper. In [12], the authors proposed an algorithm for victim pico user partition and optimal synchronous ABS rate selection. However, they used a universal ABS rate for the whole network, and as a result, their scheme could not adapt to dynamic network loading for different macro cells.

In this paper, we focus on the resource optimization in the downlink of a HetNet without CoMP\textsuperscript{1} We consider dynamic ABRB for interference control and dynamic user scheduling to exploit multi-user diversity. The ABRB is similar to the ABS but it is scheduled over both time and frequency domain. Unlike [12], we do not restrict the ABS rate to be the same for all macro BSs and thus a better performance can be achieved. However, this also causes several new technical challenges as elaborated below.

\textsuperscript{1}While there are a lot of works using multi-antenna techniques (CoMP) to mitigate interference in HetNet [11], [13], such approaches require accurate knowledge of at least the cross-link CSIT at each macro and pico BS, which is not realistic in practice. As a result, the LTE-A working groups are actively studying eICIC techniques such as ABS for interference control of HetNet.
• **Exponential Complexity for Dynamic ABRB**: Optimization of ABRB patterns is challenging due to the combinatorial nature and exponentially large solution space. For example, in a HetNet with \( N_0 \) macro BSs, there are \( 2^{N_0} \) different ABRB pattern combinations. Hence, brute force solutions are highly undesirable.

• **Complex Interactions between dynamic user scheduling and dynamic ABRB**: There is complex coupling between the dynamic user scheduling and ABRB control. For instance, the ABRB pattern will affect the user sets eligible for user scheduling. Furthermore, the optimization objective of ABRB control depends on user scheduling policy and there is no closed form characterization.

• **Challenges in RRM Architecture**: Most existing solutions for resource optimization of HetNets require global knowledge of CSI and centralized implementations. Yet, such designs are not scalable for large networks and they are not robust with respect to (w.r.t.) latency in backhaul.

To address the above challenges, we propose a two timescale control structure where the long term controls, such as dynamic ABRB, are adaptive to the large scale fading. On the other hand, the short term control, such as the user scheduling, is adaptive to the local CSI within a pico/macro BS. Such a multi-timescale structure allows Hierarchical RRM design, where the long term control decisions can be implemented on a RRM server for inter-cell interference coordination. The short-term control decisions can be done locally at each BS with only local CSI. Such design has the advantages of low signaling overhead, good scalability, and robustness w.r.t. latency of backhaul signaling. While there are previous works on two timescale RRM [11], [12], those approaches are heuristic (i.e. the RRM algorithms are not coming from a single optimization problem). Our contribution in this paper is a formal study of two timescale RRM algorithms for HetNet based on optimization theory. To overcome the exponential complexity for ABRB control, we exploit the sparsity in the interference graph of the HetNet topology and derive structural properties for the optimal ABRB control. Based on that, we propose a two timescale alternative optimization solution for user scheduling and ABRB control. The algorithm has low complexity and is asymptotically optimal at high SNR. Simulations show that the proposed solution has significant performance gain over various baselines.

**Notations**: Let \( 1(\cdot) \) denote the indication function such that \( 1(E) = 1 \) if the event \( E \) is true and \( 1(E) = 0 \) otherwise. For a set \( S \), \( |S| \) denotes the cardinality of \( S \).

**II. SYSTEM MODEL AND HIERARCHICAL RESOURCE CONTROL POLICIES**

**A. HetNet Topology and Physical Layer Model**

Consider the downlink of a two-tier HetNet as illustrated in Fig. 1. There are \( N_0 \) macro BSs, \( N - N_0 \) pico BSs, and \( K \) users, sharing \( M \) OFDM subbands. Denote the set of the macro BSs as \( B_{\text{MA}} = \{1, \ldots, N_0\} \), and denote the set of the pico BSs as \( B_{\text{PI}} = \{N_0 + 1, \ldots, N\} \).

The HetNet topology (i.e., the network connectivity and CSI of each link) is represented by a topology graph as defined below.

**Definition 1** (HetNet Topology Graph). Define the topology graph of the HetNet as a bipartite graph \( G_T = \{\mathcal{B}, \mathcal{U}, \mathcal{E}\} \), where \( \mathcal{B} = \{B_{\text{MA}}, B_{\text{PI}}\} \) denotes the set of all Macro and Pico BS nodes, \( \mathcal{U} \) denotes the set of all user nodes, and \( \mathcal{E} \) is the set of all edges between the BSs and users. An edge \((k, n)\) is associated with a CSI label \( \{h_{m,k,n}, \forall m\} \), where \( h_{m,k,n} \) represents the channel coefficient between BS \( n \) and user \( k \) on subband \( m \). For each BS node \( n \), let \( \mathcal{U}_n \) denote the set of associated users. For each user node \( k \), define \( B_{\text{MA}}^{\mathcal{U}_n} = \{n : n \in B_{\text{MA}}, k \notin \mathcal{U}_n, (k, n) \in \mathcal{E}\} \) as the set of neighbor macro BSs and \( B_{\text{PI}}^{\mathcal{U}_n} = \{n : n \in B_{\text{PI}}, k \in \mathcal{U}_n, (k, n) \in \mathcal{E}\} \) as the set of neighbor pico BSs.

**Remark 1**. In the topology graph, \((k, n) \notin \mathcal{E} \) means that the path gain between user \( k \) and BS \( n \) is sufficiently small compared to the direct link path gain, and thus the interference from BS \( n \) will have negligible effect on the data rate of user \( k \).

We have the following assumption on the channel fading process \( \mathbf{H}(t) = \{h_{m,k,n}(t)\} \).

**Assumption 1** (Two timescale fading model). The channel fading coefficient has a two timescale structure given by \( h_{m,k,n}(t) = \sigma_{k,n}(t)W_{m,k,n}(t) \), \( \forall m, k, n \). The small scale fading process \( W_{m,k,n}(t) \) is identically distributed w.r.t. the subframe and subband indices \((t,m,n)\), and it is i.i.d. w.r.t. user and BS indices \((k,n)\). Moreover, for given \( t, m, k, n \), \( W_{m,k,n}(t) \) is a continuous random variable. The large scale fading process \( \sigma_{k,n}(t) > 0 \) is assumed to be a slow ergodic process (i.e., \( \sigma_{k,n}(t) \) remains constant for several superframes\(^2\)) according to a general distribution.

The two timescale fading model has been adopted in many standard channel models. The large scale fading \( \sigma_{k,n}(t) \) is usually caused by path loss and shadow fading, which changes much slowly compared to the small scale fading.

We consider the following biased cell selection mechanism to balance the loading between macro and pico BSs [9]. Let \( b_k \) denote the serving BS of user \( k \). Let \( \beta > 1 \) denote the cell selection bias and let \( P_n \) denote the transmit power of

\(^2\)One super-frame consists of \( L_S \) subframes
BS \( n \) on a single sub-band. Let \( \tilde{n}_m = \arg\max P_n \sigma^2_{k,n} \) and \( \tilde{n}_p = \arg\max P_n \sigma^2_{k,n} \) respectively be the strongest macro BS and pico BS for user \( k \). If \( \beta P_{n,k,n}^2 \geq P_{n,m} \sigma^2_{k,n,m} \), user \( k \) will be associated to pico cell \( \tilde{n}_p \), i.e., \( b_k = \tilde{n}_p \); otherwise \( b_k = \tilde{n}_m \).

If a user only has a single edge with its serving BS, it will not receive inter-cell interference from other BSs and thus its performance is noise limited; otherwise, it will suffer from strong inter-cell interference if any of its neighbor BSs is transmitting data and thus its performance is interference limited. This insight is useful in the control algorithm design later and it is convenient to formally define the interference and noise limited users.

Definition 2 (Interference/Noise Limited User). If a user \( k \) has a single edge with its serving BS \( b_k \) only, i.e., \( \sum_{n=1}^{N_k} 1 ((k, n) \in \mathcal{E}) = 1 \), then it is called a noise limited user (N-user); otherwise, it is called an interference limited user (I-user).

Fig. 2 illustrates an example of the HetNet topology graph. In Fig. 2(a), an arrow from a BS to a user indicates a direct link and the dash circle indicates the coverage area of each BS. An I-user which lies in the coverage area of a macro BS is connected to this macro BS, while a N-user does not have connections with the neighbor macro BSs in the topology graph as illustrated in Fig. 2(b).

B. Two Timescale Hierarchical Radio Resource Control Variables

We consider a two timescale hierarchical RRM control structure where the control variables are partitioned into long-term and short-term control variables. The long-term control variables are adaptive to the large scale fading \( \Sigma \) and they are implemented at the Radio Resource Management Server (RRMS). The short-term control variables are adaptive to the instantaneous CSI \( \mathbf{H} \) and they are implemented locally at each macro/pico BS.

1) Dynamic ABRB Control for Interference Coordination (Long-term control): ABS is introduced in LTE systems \([9]\) for interference mitigation among control channels in HetNet. It can also be used to control the co-Tier and cross-tier interference among the data channels. In LTE systems, ABS is only scheduled over time domain. In this paper, we consider dynamic ABRB control for interference coordination. The ABRB is similar to ABS but it is scheduled over both time and frequency domain. It is a generalization of ABS and enables more fine-grained resource allocation. When an ABRB is scheduled in a macro BS, a RB with blank payload will be transmitted at a given frequency and time slice and this eliminates the interference from this macro BS to the pico BSs and the adjacent macro BSs. Hence, as illustrated in Fig. 2, scheduling ABRB over both time and frequency domain allows us to control both the macro-macro BS and macro-pico BS interference. We want to control the ABRB dynamically w.r.t. the large scale fading because the optimal ABRB pattern depends on the HetNet topology graph. For example, when there are a lot of pico cell I-users, we should allocate more ABRBs at the macro BS to support more pico cell I-users. On the other hand, when there are only a few pico cell I-users, we should allocate less ABRBs to improve the spatial spectrum efficiency.

For any given subframe, define \( a_{m,n} \in \{0, 1\} \) to indicate if ABRB is scheduled (\( a_{m,n} = 0 \)) for subband \( m \) at macro BS \( n \). Let \( \mathbf{a}_n = [a_{m,1}, \ldots, a_{m,N_0}]^T \in \mathcal{A} \) be the ABRB pattern vector for subband \( m \) and \( \mathcal{A} \) is the set of all possible ABRB patterns. In the proposed dynamic ABRB control, each macro BS is allowed to dynamically change the ratio of ABRB transmission on each subband and this ratio can enable more fine-grained resource allocation. When an ABRB is scheduled, the neighbor macro BS 2 of the pico BS 3 transmits ABRB and the macro cell I-user 4 perceives a low interference RB. The ABRB can also be used to control the interference between macro cell users. For example, in the (t, 2)-th RB, macro BS 1 transmits ABRB and the macro cell I-user 5 perceives a low interference RB.

Figure 2: An example of topology graph and interference coordination using ABRB. We focus on the \( t \)-th subframe. The ABRB can be used to control the interference from macro BSs to pico cell users. For example, in the (t, 1)-th RB (i.e., subframe \( t \) and subband \( 1 \)) and (t, 3)-th RB, the neighbor macro BS 2 of the pico BS 3 transmits ABRB and the pico cell I-user 4 perceives a low interference RB. The ABRB can also be used to control the interference between macro cell users. For example, in the (t, 2)-th RB, macro BS 1 transmits ABRB and the macro cell I-user 5 perceives a low interference RB.\n
Definition 3 (Randomized ABRB Control Policy). An ABRB control policy of the \( m \)-th subband \( Q_m \) is a mapping from each of the \( N_0 \) macro BSs can either schedule an ABRB or not for the \((t, m)\)-th RB (i.e., subframe \( t \) and subband \( m \)), there are \( 2^{N_0} \) possible ABRB patterns. Hence, the size of \( \mathcal{A} \) is \( 2^{N_0} \).
the ABRB pattern space \( \mathcal{A} \) to a probability in \([0,1]\). At any subframe, the instantaneous ABRB pattern vector for subband \( m \) is stochastically determined according to the probabilities \( Q_m(a_m), \) \( \forall a_m \in \mathcal{A} \), where \( Q_m(a_m) \) denote the probability that the \( m \)-th subband is in ABRB pattern \( a_m \).

2) Subband Partitioning Control for Structural ABRB Design (Long-term control): To facilitate structural ABRB design, we partition the users into two types.

Definition 4 (Partitioning of User Set). The mobile user set is partitioned into two subsets \( \mathcal{U} = \{ \mathcal{U}_A, \mathcal{U}_B \} \), where \( \mathcal{U}_A \) denotes the set of Type A users and is defined as

\[
\mathcal{U}_A = \{ k : b_k \in \mathcal{B}_p \} \cup \left\{ k : \sum_{n=1}^{N} 1 ((k,n) \in \mathcal{E}) = 1 \right\},
\]

and \( \mathcal{U}_B = \mathcal{U} / \mathcal{U}_A \) denotes the set of Type B users.

The Type A users include all pico cell users and macro cell N-users, while the Type B users include all macro cell I-users. For Type A users, it will not lose optimality by imposing a synchronous ABRB structure where the transmissions of the ABRB at all macro BSs are aligned as much as possible. The formal definition of the synchronous ABRB structure is given in Theorem 2. As will be shown in Theorem 2 if there is only Type A users, imposing the synchronous ABRB structure can dramatically reduce the number of ABRB control variables from exponential large \((2^{N_0})\) to only \(N_0\) and this complex reduction is achieved without loss of optimality. On contrast, the performance of the macro cell I-users is very poor under the synchronous ABRB structure because aligning the data transmissions of all macro BSs will cause strong inter-cell interference for macro cell I-users. Motivated by these observations, we partition the \( M \) subbands into two groups, namely \( \mathcal{M}_A \) and \( \mathcal{M}_B \), and use different ABRB control policies for type A and type B users on these two groups of subbands respectively. The variable \( q_s = |\mathcal{M}_A|/M \) controls the fraction of Type A subbands.

3) Dynamic User Scheduling for Multi-user Diversity (Short-term control): At each subframe, each BS \( n \) dynamically selects a user from \( \mathcal{U}_n \) for each subband \( m \) based on the knowledge of current ABRB pattern \( a_m \) and channel realization \( \mathbf{H}_m \) to exploit multi-user diversity. Let \( \rho_{m,k} \in \{0,1\} \) be the user scheduling variable (of user \( k \) at BS \( b_k \)) of subband \( m \) and \( \rho_{m}^n = \rho_{m,k} | k \in \mathcal{U}_n \) be the associated vectorized variable. The set of all feasible user scheduling vectors at BS \( n \) for the \( m \)-th subbands with ABRB pattern \( a_m \) is given by

\[
\Gamma^m_n(a_m) = \left\{ \rho^m_{m} : \sum_{k \in \mathcal{U}_A} \rho_{m,k} \leq 1, \rho_{m,k} = 0, \forall k \in \mathcal{U}_A^c(a_m) \right\}, \quad \forall m \in \mathcal{M}_A
\]

\[
\Gamma^m_n(a_m) = \left\{ \rho^m_{m} : \sum_{k \in \mathcal{U}_A} \rho_{m,k} \leq 1, \rho_{m,k} = 0, \forall k \in \mathcal{U}_B^c(a_m) \right\}, \quad \forall m \in \mathcal{M}_B
\]

where \( \mathcal{U}_A^c(a_m) = \mathcal{U}_B \cup \left\{ k : b_k \in \mathcal{B}_m \right\} \) and \( \mathcal{U}_B^c(a_m) = \mathcal{U}_A \cup \{ k : b_k \in \mathcal{B}_p \} \), is the set of users that cannot be scheduled on a Type A subband under ABRB pattern \( a_m \); and \( \mathbf{I}_m^B(a_m) = \mathcal{U}_A \cup \{ k : b_k \in \mathcal{B}_p \} \). The physical meaning of \( \Gamma^m_n(a_m) \) is elaborated below. First, if a macro BS is transmitting ABRB, none of its associated users can be scheduled for transmission. Moreover, due to large cross-tier interference from macro BSs, a pico cell I-user cannot be scheduled for transmission if any of its neighbor macro BSs \( n \in \mathcal{B}_m^{MA} \) is transmitting data subframe (i.e., \( \sum_{n \in \mathcal{B}_m^{MA}} m,n > 0 \)). As will be seen in Section IV-A, explicitly imposing this user scheduling constraint for the pico cell I-users is useful for the structural ABRB design.

The user scheduling policy \( \pi_m \) of the \( m \)-th sub-band is defined below.

Definition 5 (User Scheduling Policy). A user scheduling policy of the \( n \)-th BS and \( m \)-th sub-band \( \pi^n_m \) is a mapping : \( A \times H \rightarrow \Gamma^m_n(a_m) \), where \( H \) is the CSI space. Specifically, under the ABRB pattern \( a_m \) and CSI realization \( \mathbf{H}_m \), the user scheduling vector of BS \( n \) is given by \( \rho^n_m = \pi^n_m(a_m, \mathbf{H}_m) \).

Let \( \pi_m = \{ \pi^n_m, n = 1,...,N \} \) denote the overall user scheduling policy on sub-band \( m \).

III. TWO TIMESCALE HIERARCHICAL RRM DESIGN

A. RRM Optimization Formulation

Assuming perfect CSI at the receiver (CSIR) and treating interference as noise, the instantaneous data rate of user \( k \) is given by:

\[
r_k(\{a_m\}, \mathbf{H}, \{\rho^n_m\}) = \sum_{m \in \mathcal{M}(k)} I_{m,k}(a_m, \mathbf{H}_m, \rho^n_m),
\]

where \( \mathcal{M}(k) = \mathcal{M}_A, \forall k \in \mathcal{U}_A, \mathcal{M}(k) = \mathcal{M}_B, \forall k \in \mathcal{U}_B \); \( I_{m,k}(a_m, \mathbf{H}_m, \rho^n_m) = \rho_{m,k} \log \left(1 + \frac{\mu_{m,k}^n}{\Omega_{m,k}}\right) \) is the mutual information of user \( k \) contributed by the \( m \)-th subband; and \( \Omega_{m,k} = 1 + \sum_{n \in \mathcal{B}_m} m,n \mu_{m,k,n}^n P_n + 1 (m \in \mathcal{M}_A) \sum_{n \in \mathcal{B}_m} |h_{m,k,n}|^2 P_n \) is the interference-plus-noise power at user \( k \) on subband \( m \).

For a given policy \( \Lambda = \{q_s, \{Q_m\}, \{\pi_m\}\} \) and large scale fading state \( \Sigma \equiv \{\sigma_{k,n}\} \), the average data rate of user \( k \) is given by:

\[
\tau_k(\Lambda) = E_r [r_k(\{a_m\}, \mathbf{H}, \{\rho^n_m\})/\Sigma] = \sum_{m \in \mathcal{M}(k)} I_{m,k}(Q_m, \pi_m),
\]

where the average mutual information on subband \( m \) is

\[
I_{m,k}(Q_m, \pi_m) = \sum_{a_m} Q_m(a_m) I_{m,k}(\pi_m, a_m),
\]

and \( I_{m,k}(\pi_m, a_m) = E_r [I_{m,k}(a, \mathbf{H}_m, \pi^n_m(a, \mathbf{H}_m)) / \Sigma, a] \). For conciseness, the ABRB pattern \( a_m \) for a specific subband \( m \) is denoted as \( a = [a_1,...,a_{N_0}] \) when there is no ambiguity.

The performance of the HetNet is characterized by a utility function \( U(\mathbf{r}) \), where \( \mathbf{r} = [r_1,...,r_K] \) is the average rate vector. We make the following assumptions on \( U(\mathbf{r}) \).

Assumption 2 (Assumptions on Utility). The utility function can be expressed as \( U(\mathbf{r}) \equiv \sum_{c=1}^{C} \sum_{r \geq r_c} w_c u(r) \), where \( w_c \geq 0 \) is the weight for user \( k, u(\cdot) \) is assumed to be a concave and increasing function. Moreover, for any \( c, r \geq 0 \) such that \( cr \) and \( r \) belong to the domain of \( u(\cdot) \), \( u(r) \) satisfies

\[
u(cr) = f(c) \nu(r) + g(c),
\]

where \( f(c) > 0 \) and \( g(c) \) are some scalar functions of \( c \).
The above assumption is imposed to facilitate the problem decomposition in Section [II-B]. This utility function captures a lot of interesting cases below.

- **Weighted Sum Throughput**: The utility function is $U(\mathbf{r}) = \sum_{k=1}^{K} w_k r_k$.

- **α-Fair [14]**: α-Fair can be used to compromise between the fairness to users and the utilization of resources. The utility function is

$$U(\mathbf{r}) = \begin{cases} \sum_{k=1}^{K} \log(r_k), & \alpha = 1, \\ \sum_{k=1}^{K} (1 - \alpha)^{-1} r_k^{-\alpha}, & \text{otherwise.} \end{cases} \quad (3)$$

- **Proportional Fair [15]**: This is a special case of α-Fair when $\alpha = 1$.

Due to the statistical symmetry of the $M$ subbands, there is no loss of optimality to consider symmetric policy $\Lambda^s = \{q_s, Q_A, Q_B, \pi_A, \pi_B\}$, where $Q_m = Q_A (Q_B)$ and $\pi_m = \pi_A (\pi_B)$ if $m \in M_A (M_B)$.

**Lemma 1** (Optimality of Symmetric Policy). There exists a symmetric policy $\Lambda^s = \{q_s, Q_A^s, Q_B^s, \pi_A^s, \pi_B^s\}$ such that it is the optimal solution of the following optimization problem:

$$\mathcal{P}(\mathcal{G}_T) : \max_{\Lambda^s} U(\Lambda) \quad \text{s.t.} \quad \sum_{a \in A} Q_m(a) = 1; \quad Q_m(a) \geq 0, \quad \forall m. \quad (4)$$

Please refer to Appendix A for the proof.

Moreover, we have $\mathcal{I}_{m,k}(Q_A, \pi_A) = \mathcal{I}_{m',k}(Q_A, \pi_A), \forall m, m' \in M_A$ and $\mathcal{I}_{m,k}(Q_B, \pi_B) = \mathcal{I}_{m',k}(Q_B, \pi_B), \forall m, m' \in M_B$. As a result, the utility function under a symmetric policy $\Lambda^s$ can be expressed as:

$$U(\Lambda^s) = f(M)|A| U_A(Q_A, \pi_A) + g(M)(1 - q_s) \sum_{k \in U_B} w_k$$

$$+ f(M)(1 - q_s)U_B(Q_B, \pi_B) + g(M)q_s \sum_{k \in U_A} w_k,$$

where $U_A(Q_A, \pi_A) = \sum_{a \in A} w_k u_q(Q_m(a) \mathcal{I}_{m,k}(Q_A, \pi_A))$, $U_B(Q_B, \pi_B) = \sum_{k \in B} w_k u_q(Q_m(a) \mathcal{I}_{m,k}(Q_B, \pi_B))$, and $m_A \in M_A (m_B \in M_B)$ can be any Type A (Type B) subband. Finally, for a given HetNet topology graph $G_T = \{B, U, E\}$, the two timescale RRM optimization is given by:

$$\mathcal{P}(\mathcal{G}_T) : \max_{\Lambda^s} U(\Lambda^s) \quad \text{s.t.} \quad \sum_{a \in A} Q_A(a) = 1; \quad Q_A(a) \geq 0, \quad \forall a \in A \quad (5)$$

$$\sum_{a \in A} Q_B(a) = 1; \quad Q_B(a) \geq 0, \quad \forall a \in A \quad (6)$$

where (5) and (6) ensure that $Q_A(\cdot)$ and $Q_B(\cdot)$ satisfy the definition of probability mass function (pmf).

**B. Problem Decomposition**

Using primal decomposition, problem $\mathcal{P}(\mathcal{G}_T)$ can be decomposed into the following subproblems.

**Subproblem A (Cross-Tier Interference Control)**: Optimization of $\Lambda$ and user scheduling $\pi_A$.

$$\mathcal{P}_A(\mathcal{G}_T) : U_A^* \triangleq \max_{Q_A, \pi_A} U_A(Q_A, \pi_A) \quad \text{s.t.} \quad (5) \text{ holds.}$$

**Subproblem B (Co-Tier Interference Control)**: Optimization of $Q_B$ and user scheduling $\pi_B$.

$$\mathcal{P}_B(\mathcal{G}_T) : U_B^* \triangleq \max_{Q_B, \pi_B} U_B(Q_B, \pi_B) \quad \text{s.t.} \quad (6) \text{ holds.}$$

**Subproblem C (Subband Partitioning)**: Optimization of subband partitioning $q_s$.

$$\mathcal{P}_C(\mathcal{G}_T) : \max_{q_s} f(M)|A| U_A^* + g(M)(1 - q_s) \sum_{k \in U_B} w_k$$

$$+ f(M)(1 - q_s)U_B^* + g(M)q_s \sum_{k \in U_A} w_k.$$

Note that the solution of $\mathcal{P}_A/\mathcal{P}_B$ is independent of the value of $q_s$ because both $U_A(Q_A, \pi_A)$ and $U_B(Q_B, \pi_B)$ are independent of $q_s$. After solving $\mathcal{P}_A$ and $\mathcal{P}_B$, the optimal $q_s$ can be easily solved by bisection search. On the other hand, the optimization of $\pi_A/\pi_B$ is a stochastic optimization problem because $\mathcal{I}(m, k)$ involves stochastic expectation over CSI realizations and they do not have closed form characterization. Furthermore, the number of $\text{ABRB}$ control variables in $\mathcal{P}_A/\mathcal{P}_B$ is exponential w.r.t. the number of macro BSs $N_0$. We shall tackle these challenges in Section IV and V.

**IV. TWO TIMESCALE HIERARCHICAL SOLUTION FOR CROSS-TIER INTERFERENCE CONTROL (SUBPROBLEM A)**

In this section, we first derive structural properties of $\mathcal{P}_A$ and reformulate $\mathcal{P}_A$ into a simpler form with reduced solution space. Then, we develop an efficient algorithm for $\mathcal{P}_A$.

We require the following assumption to derive the results in this section.

**Assumption 3** (Assumptions for pico cell I-users). For any $n \in \mathcal{B}_p$, let $U_n^4 = \{k : k \in U_n, \sum_{n=1}^{N_4} 1 \left\{ (k, n) \in \mathcal{E} \right\} > 1 \}$ denote the set of all pico cell I-users in pico cell $n$. Then we have $\mathcal{B}_n^A = \mathcal{B}_n^A, \forall k, k' \in U_n^4$ and define $\mathcal{B}_n = \mathcal{B}_n^A, \forall k \in U_n^4$ as the set of neighbor macro BSs of pico cell $n$.

The above assumption states that a macro BS will interfere with all the I-users in a pico cell as long as it interferes with any user in the pico cell. This is reasonable since the coverage area of a macro BS is much larger than that of a pico BS.

**A. Structural Properties and Problem Transformation of $\mathcal{P}_A$**

We exploit the interference structure in the HetNet topology $G_T$ to derive the structural properties of $\mathcal{P}_A$. Throughout this section, we will use the following example problem to illustrate the intuition behind the main results.

**Example 1.** Consider $\mathcal{P}_A(\mathcal{G}_T)$ for the HetNet in Fig. 2 with $N_0 = 2$ Macro BSs and $N - N_0 = 1$ pico BS. The set of Type A users is $U_n = \{1, 2, 3, 4\}$ and the objective function is specified as $U_A(Q_A, \pi_A) = \sum_{k=1}^{K} \mathcal{I}(m, k)(Q_A, \pi_A)$ (i.e., we consider sum-rate utility). For illustration, we focus on the case when the marginal probability that a macro BS is transmitting $\text{ABRB}$ is fixed as $q_A = [q_A^1, q_A^2]^T = [0.7, 0.5]^T$. The marginal probability that macro BS $n$ is transmitting $\text{ABRB}$ is $q_n^A = \sum_{a \in \{a' : a'_m = 0\}} Q_A(a)$.
Define two sets of ABRB patterns $A_n$ and $\overline{A}_n = A_n/A_n$ for each BS $n$. For macro BS, $A_n$ is the set of ABRB patterns under which macro BS $n$ is transmitting data. For pico BS, $A_n$ is the set of ABRB patterns under which all of its neighbor macro BSs is transmitting ABRB. Using the configuration in Example 1 we have $A_1 = \{[1, 0], [1, 1]\}$, $A_2 = \{[0, 1], [1, 1]\}$ and $A_3 = \{[0, 0], [1, 0]\}$ (the formal definition of $A_n$ and $\overline{A}_n$ for general cases is in Appendix B).

In "Observation 1", we find that the ABRB pattern $a$ only affects the data rate of a Type A user in cell $n$ by whether $a \in A_n$ or not. Based on that, we find that the policy space for both the ABRB control $Q_A$ and user scheduling $\pi_A$ can be significantly reduced in "Observation 2 and 3". While these observations are made for the specific configuration in Example 1 they are also correct for general configurations and are formally stated in Lemma 2, Theorem 2 and Theorem 3 in Appendix B. Finally, using the above results, we transform the complicated problem $P_A$ to a much simpler problem with $q_A, \pi_A$ as the optimization variables.

**Observation 1 (Effect of ABRB on Mutual Information).** For given CSI $H_{m,a}$ and a feasible user scheduling vector $\rho_{m,a}$, the mutual information $I_{m,a,k}(a,H_{m,a},\rho_{m,a})$ of a Type A user $k$ in cell $n$ only depends on whether the ABRB pattern $a \in A_n$ or not, i.e., $I_{m,a,k}(a,H_{m,a},\rho_{m,a}) = I_{m,a,k}(a,H_{m,a},\rho_{m,a}) \forall a \in A_n$ (or $\forall a, a' \in A_n$). Moreover, we have $I_{m,a,k}(a,H_{m,a},\rho_{m,a}) \geq I_{m,a,k}(a',H_{m,a},\rho_{m,a})$ for any $a \in A_n, a' \in A_n$.

Let us illustrate the above observation using the configuration in Example 1. There are 4 type A users: $U_A = \{1, 2, 3, 4\}$. For user 1 in macro cell 1 (N-user), it is scheduled for transmission whenever macro BS 1 is not transmitting ABRB (i.e., $a_1 = 1$). Hence the mutual information is $I_{m,a,1}(a,H_{m,a},\rho_{m,a}) = a_1 \log(1 + P_1|h_{m,a,1,1}|^2)$ which only depends on whether $a_1 = 1$ (i.e., $a \in A_1$) or not. Moreover, the mutual information is higher if $a \in A_1$ because $I_{m,a,1}(a,H_{m,a},\rho_{m,a}) = 0 \forall a \in A_1$. For user 4 in pico cell 3 (I-user), if $a \in A_3$, its neighbor macro BS 2 is transmitting ABRB and the mutual information is $I_{m,a,4}(a,H_{m,a},\rho_{m,a}) = \rho_{m,a} \log(1 + P_3|h_{m,a,4,3}|^2)$; otherwise $a \not\in A_3$, macro BS 2 is not transmitting and we have $\rho_{m,a} = 0$ and $I_{m,a,4}(a,H_{m,a},\rho_{m,a}) = 0$. Similar observations can be made for user 2 and 3.

Based on Observation 1 the ABRB control policy space can be significantly reduced.

**Observation 2 (Policy Space Reduction for $Q_A$).** Consider $P_A(G_T)$ for the configuration in Example 1. The optimal ABRB control of $P_A(G_T)$ conditioned on a given marginal probability vector $q_A = [0.7, 0.5]^T$, denoted by $Q^{q_A}_A$, has the synchronous ABRB structure where the transmissions of ABRB at the macro BSs are aligned as much as possible. As a result, there are only 3 active ABRB patterns.

There are two macro BSs. Assume $q_1^A = 0.7$, $q_2^A = 0.5$ and the permutation is $\zeta(1) = 2, \zeta(2) = 1$. The active ABRB patterns are $A^A = \{[0, 0], [0, 1], [1, 1]\}$. The probabilities of the active ABRB patterns are illustrated below:

![Figure 3: Illustration of the structure of the optimal $Q_A^A$ conditioned on a given $q_A$](image)

In general, for a HetNet with $N_0$ macro BSs, there are only $N_0 + 1$ active ABRB patterns under synchronous ABRB structure, which is significantly smaller than the number of all possible ABRB patterns $2^{N_0}$. As a result, the optimization $P_A(G_T)$ w.r.t $Q_A$ (with $2^{N_0}$ variables) can be reduced to an equivalent optimization w.r.t. $q_A$ (with $N_0$ variables) with much lower dimensions. Observation 2 can be understood as follows. By Observation 1 a higher average mutual information can be achieved for user $k \in U_n$ under the ABRB patterns $a \in A_n$. Hence, for given marginal probabilities $q_A = [0.7, 0.5]^T$, the average mutual information region will be maximized if we can simultaneously maximize $\sum_{a \in A_n} Q_A(a)$ for all BSs $1 \leq n \leq 3$. For macro BSs $n = 1, 2$, we have $\sum_{a \in A_n} Q_A(a) = q_n^A$, which is fixed for given $q_A$. For pico BSs $n = 3$, we have $\sum_{a \in A_n} Q_A(a) = \min_{j \in B_3 \{1, 2\}} (q_j^A) = 0.5$, and the equality holds if and only if $Q_A$ has the synchronous ABRB structure in Fig. 3.

Similarly, we can reduce the user scheduling policy space using Observation 1.

**Observation 3 (Policy Space Reduction for $\pi_A$).** Consider $P_A(G_T)$ for the configuration in Example 1. For given CSI $H_{m,a}$ and ABRB pattern $a$, the optimal user scheduling at BS $n$ is given by

$$\pi^*_A(a, H_{m,a}) = \arg\max_{\rho_{m,a} \in \mathbb{N}_+} I_{m,a,k}(a, H_{m,a}, \rho_{m,a})$$

By Observation 1 if $\rho_{m,a}^*$ solves the maximization problem for certain $a \in A_n$, it solves for all $a \in A_n$. Hence, it will not lose optimality by imposing an additional constraint on the user scheduling such that $\pi^*_A(a, H_{m,a}) = \pi^*_A(a, H_{m,a}) \forall a, a' \in A_n$ (or $\forall a, a' \in \overline{A}_n$).

For convenience, let $\Xi_A$ denote the set of all feasible user scheduling policies satisfying the above constraint in Observation 3. The formal definition of $\Xi_A$ is given in Theorem 3. Then for given $q_A$, $\pi_A \in \Xi_A$ and under the synchronous ABRB, the corresponding objective function of $P_A$ can be rewritten as $U_A(Q^{q_A}_A, \pi_A) = \hat{U}_A(q_A, \pi_A)$, where

$$\hat{U}_A(q_A, \pi_A) = \sum_{k \in U_A} w_k u \left( I_{m,a,k}(q_A, \pi_A) \right)$$

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5The formal definition of the synchronous ABRB structure for general cases is in Theorem 2.
and $\overline{T}_{m_A,k} \left( Q^*_A, q_A^*, \pi_A \right)$ is the corresponding average mutual information given in (24) of Appendix B. As a result, the subproblem $P_A (G_T)$ can be transformed into a simpler problem with $O \left( N_0 \right)$ solution space.

**Corollary 1** (Equivalent Problem Transformation of $P_A (G_T)$). Let $q_A^*, \pi_A^*$ denote the optimal solution of the following joint optimization problem.

$$\max_{q_A^*, \pi_A^*} \hat{U}_A (q_A, \pi_A), \text{ s.t. } q_A \in Q_A, \pi_A \in \Xi_A^*,$$

(9)

where $Q_A = \{ q_A : q_{ij}^* \in [0, 1], \forall j \}$. Then $Q^*_A, q^*_A, \pi_A^*$ is the optimal solution of problem $P_A (G_T)$.

Furthermore, problem (9) is a bi-convex problem, i.e., for fixed $\pi_A$, problem (9) is convex w.r.t. $q_A$, and for fixed $q_A$, problem (9) is also convex w.r.t. $\pi_A$.

Please refer to Appendix C for the proof.

**B. Two Timescale Alternating Optimization Algorithm for $P_A$**

By Corollary 1, we only need to solve the equivalent problem of $P_A$ in (9). Since problem (9) is bi-convex, we propose the following Two Timescale Alternating Optimization (AO) algorithm. For notation convenience, time index $t$ and $T$ are used to denote the subframe index and super-frame index respectively, where a super-frame consists of $L_S$ subframes.

Algorithm AO$_A$ (Two Timescale AO for $P_A (G_T)$):

**Initialization**: Choose proper initial $q_A(0), \pi_A(0)$. Set $T = 0$.

**Step 1** (Short timescale user scheduling optimization): For fixed $q_A^{(T)}$, let $\pi_A^{(T)} = \{ \pi_A^{(T)}(n), n = 1, ..., N \}$, where $\pi_A^{(T)}$ is given by

$$\pi_A^{(T)}(a, H_{m_A}) = \arg\max_{\rho^*_m \in \Gamma_{m_A}(a) \cap \mathcal{U}_{m_A}} \frac{w_k}{\rho^*_m} \partial \mu \left( \frac{r \partial \mu}{\partial \rho} \right) |_{\rho = \rho^*_{m_A}, t = T},$$

where $\rho^*_k(n) = q_k M T_{m_A,k} \left( Q^*_A n, \pi_A^{(T)} \right)$ is the average data rate of user $k$ under $q_A^{(T)}$ and user scheduling policy $\pi_A^{(T)}$. For each subframe $t \in \{ T, T + 1, ..., T_L \}$, the user scheduling vector of BS $n$ is given by $\rho^*_m(n) = \pi_{A, m_A}^{(T)}(n, a, H_{m_A})$, where $H_{m_A}$ and $\pi_{m_A}$ are the CSI and ABRB pattern at the $t$-th subframe.

**Step 2** (Long timescale ABRB optimization): Find the optimal solution $q_A^{(T+1)}$ of problem (9) under fixed $\pi_A^{(T)}$ using e.g., Ellipsoid method. Let $T = T + 1$.

**Return to Step 1 until $q_A^{(T)} = q_A^{(T-1)}$ or the maximum number of iterations is reached.**

While (9) is a bi-convex problem and AO algorithm is known to converge to local optimal solutions only, we exploit the hidden convexity of the problem and show below that Algorithm AO$_A$ can converge to the global optimal solution under certain conditions.

**Theorem 1** (Global Convergence of Algorithm AO$_A$). Let $\left[ q_A^{(T)}, \pi_A^{(T)} \right] = \mathcal{F} \left( \left[ q_A^{(T-1)}, \pi_A^{(T-1)} \right] \right)$ denote the iterate sequence of Algorithm AO$_A$ at the $t$-th subframe, and denote the set of fixed points of the mapping $\mathcal{F}$ as $\Delta = \{ q_A \in Q_A, \pi_A \in \Xi_A^* : \mathcal{F} (q_A, \pi_A) = \mathcal{F} (q_A, \pi_A) \}$. For any $q_A \in Q_A, \pi_A \in \Xi_A^*$ that is not a fixed point of $\mathcal{F}$, assume that $\hat{U}_A (q_A, \pi_A) \neq \hat{U}_A (q_A, \pi_A)$, where $\hat{q}_A, \pi_A \in \mathcal{F} (q_A, \pi_A)$. Then:

1. Algorithm AO$_A$ converges to a fixed point $\left[ q_A, \pi_A \right] \in \Delta$ of $\mathcal{F}$.
2. Any fixed point $\left[ q_A, \pi_A \right] \in \Delta$ is a globally optimal solution of problem (9).

Please refer to Appendix D for the proof.

Step 1 of Algorithm AO$_A$ requires the knowledge of the average data rate $\tau_k^{(T)}$ under $q_A^{(T)}$ and $\pi_A^{(T)}$. We adopt a reasonable approximation on $\tau_k^{(T)}$ using a moving average data rate $R_k(t)$, $\forall t \in \{ T, T + 1, ..., T_L \}$, given by

$$R_k(t) = \frac{t - T}{T + 1} R_k(t - 1),$$

(10)

where $r_k(t)$ is the data rate delivered to user $k$ at subframe $t$.

**Remark 2**. If we replace $\tau_k^{(T)}$ in step 1 of Algorithm AO$_A$ with the approximation $R_k(t)$ in (10), the global convergence result in Theorem 1 no longer holds. However, it has been shown in [16] that $R_k(t)$ converges to $\tau_k^{(T)}$ as $t \to \infty$. Hence, with the approximation $R_k(t)$, Algorithm AO$_A$ is still asymptotically optimal for large super frame length $L_S$.

In step 2 of Algorithm AO$_A$, the average mutual information $\overline{T}_{m_A,k} \left( Q^*_A, q_A^{(T)}, \pi_A^{(T)} \right)$ in the optimization objective contains two intermediate problem parameters $e_k^{(T)} \left( \pi_A^{(T)} \right)$ and $\tau_k^{(T)} \left( \pi_A^{(T)} \right)$ defined under (24) in Appendix B. The calculation of $e_k^{(T)} \left( \pi_A^{(T)} \right)$ and $\tau_k^{(T)} \left( \pi_A^{(T)} \right)$ requires the knowledge of the distribution of all the channel coefficients, which is usually difficult to obtain offline. However, these terms can be easily estimated online using the time average of the sampled data rates delivered to user $k$ under ABRB patterns $A_{m_A}$ and $A_{m_A}$ respectively. The ABRB control $q_A^{(T)}$ is then obtained by solving the long timescale problem in step 2 with say, the ellipsoid method based on these estimates.

**V. TWO TIMESCALE HIERARCHICAL SOLUTION FOR CO-TIER INTERFERENCE CONTROL (SUBPROBLEM B)**

The number of ABRB control variables $Q_B (a), \forall a \in A$ is exponentially large w.r.t. $N$. To simplify $P_B$, we introduce an auxiliary variable called the ABRB profile and decompose $P_B$.

**A. Problem Decomposition of $P_B$**

We first define the ABRB profile.

**Definition 6** (ABRB Profile). The ABRB profile $A_B \subset A$ is a subset of ABRB patterns for Type B subbands.

Using the notion of ABRB Profile and primal decomposition, $P_B (G_T)$ can be approximated by two subproblems:

**Optimization of $Q_B$ and $\pi_B$ for a given ABRB Profile $A_B$.**

$$\mathcal{P}_B^1 (G_T; A_B) : \max_{Q_B (B), \pi_B} U_B (A_B) \triangleq \max_{Q_B (B), \pi_B} U_B (Q_B, \pi_B),$$

s.t. $Q_B (a) = 0, \forall a \notin A_B$, $Q_B (a) = 1, \forall a \in A_B$,
Optimization of ABRB profile $A^B$.

$$\mathcal{P}_B^I(\mathcal{G}_T) : \max_{A^B} U_B^I(A^B), \text{ s.t. } A^B \subseteq A; \mid A^B \mid \leq \mid U_B \mid.$$  

In $\mathcal{P}_B^I(\mathcal{G}_T)$, we restrict the size of $A^B$ to be no more than $\mid U_B \mid$. In Appendix E, we prove that at the asymptotically optimal solution of $\mathcal{P}_B(\mathcal{G}_T)$ as SNR becomes high, the number of active ABRB patterns is indeed less than or equal to $\mid U_B \mid$.

B. Two Timescale Alternating Optimization Algorithm for $\mathcal{P}_B^I(\mathcal{G}_T)$

Let $A^B_j$ denote the $j$th ABRB pattern in $A^B$. Then the average mutual information $\mathcal{I}_{m,b,k}$ can be rewritten as

$$\mathcal{I}_{m,b,k}(q_B, \pi_B) = \sum_{j=1}^{\mid A^B \mid} q^B_j I_{m,b,k}(\pi_B, A^B_j), \quad (11)$$

where $q^B = [q^B_1, ..., q^B_{\mid A^B \mid}]$ with $q^B_j = Q_B(a^B_j)$ denoting the probability that the ABRB pattern $a^B_j$ is used. Hence $\mathcal{P}_B^I(\mathcal{G}_T; A^B)$ can be reformulated as

$$\max_{q_B, \pi_B} \hat{U}_B(q_B, \pi_B) \triangleq \sum_{k \in U_B} w_k u(\mathcal{I}_{m,b,k}(q_B, \pi_B)) \quad (12)$$

s.t. $\sum_{j=1}^{\mid A^B \mid} q^B_j = 1$; and $q^B_j \geq 0, \forall j$.

Similar to [9], we propose a two timescale AO to solve for problem (12) w.r.t. $\pi_B$ and $q_B$.

**Algorithm AO_B (Two timescale AO for $\mathcal{P}_B^I(\mathcal{G}_T; A^B)$):**

**Initialization:** Choose proper initial $\pi_B^{(0)}, \pi_B^{(-1)}$. Set $T = 0$.

**Step 1** (Short timescale user scheduling optimization): For fixed $q_B^{(T)}$, let $\pi_B^{(T)} = \{n_B^{(T)}, n = 1, ..., N\}$, where $\pi_B^{(T)}$ is given by

$$\pi_B^{(T)}(a, H_B) = \arg\max_{\pi_B^{(T)}(a, H_B) \in \Pi_B^{(T)}} \sum_{n \in U_B} w_k \frac{\partial u(r)}{\partial r} I_{m,b,k}(a, H_B, p_m, p_n),$$

where $\tau_k^{(T)} = (1 - q_k) M I_{m,b,k}(q_B^{(T)}, \pi_B^{(T)})$ is the average data rate of user $k$ under $q_B^{(T)}$ and user scheduling policy $\pi_B^{(T)}$. For each subframe $t \in \{T_L, (T + 1) L - 1\}$, the user scheduling vector of BS $n$ is given by $p_m^{(T)} = \pi_B^{(T)}(a_m, H_{m,n})$, where $H_{m,n}$ and $a_m$ are the CSI and ABRB pattern at the $t$-th subframe.

**Step 2** (Long timescale ABRB optimization): Find the optimal solution $q_B^{(T+1)}$ of problem (12) under fixed $I_{m,b,k}(\pi_B^{(T)}, a^B_j)$'s using e.g., interior point method. Let $T = T + 1$.

**Return to Step 1** until $q_B^{(T)} = q_B^{(T-1)}$ or the maximum number of iterations is reached.

Using similar proof as that in Appendix D, it can be shown that Algorithm AO_B converges to the global optimal solution for problem (12) under similar assumption as in Theorem 1. The average data rate $\tau_k^{(T)}$ and the mutual information $I_{m,b,k}(\pi_B^{(T)}, a^B_j)$ can be estimated online in a similar way as in Algorithm AO_A.

C. Finding a Good ABRB Profile for $\mathcal{P}_B^I(\mathcal{G}_T)$

Problem $\mathcal{P}_B^I(\mathcal{G}_T)$ is a difficult combinatorial problem and the complexity of finding the optimal ABRB Profile is extremely high. In this section, we illustrate the top level method for finding a good ABRB profile. The detailed algorithm to solve $\mathcal{P}_B^I(\mathcal{G}_T)$ is given in Appendix E.

A good ABRB profile can be found based on an interference graph extracted from the topology graph.

**Definition 7** (Interference Graph). For a HetNet Topology Graph $\mathcal{G}_T = \{B, U, E\}$, define an undirected interference graph $\mathcal{G}_I(\mathcal{G}_T) = \{L, E_I\}$, where $L = \{l_k | \forall k \in U_B\}$ is the vertex set and $E_I = \{e(l_k, l_{k'}) | (0, 1), \forall k \neq k' \in U_B\}$ is edge set with $e(l_k, l_{k'})$ denoting the edge between $l_k$ and $l_{k'}$. For any $k \neq k' \in U_B$, if $\{(k, b_{k'}) \cup (k', b_{k'})\} \cap E \neq \Phi$, $e(l_k, l_{k'}) = 1$, otherwise, $e(l_k, l_{k'}) = 0$, where $\Phi$ is the void set.

Fig. 4 illustrates how to extract the interference graph from the topology graph using an example HetNet. Given an interference graph $\mathcal{G}_I(\mathcal{G}_T)$ for the HetNet, any two links $l_k, l_{k'}$ having an edge (i.e., $e(l_k, l_{k'}) = 1$) should not be scheduled for transmission simultaneously. On the other hand, we should “turn on” as many “non-conflicting” links as possible to maximize the spatial reuse efficiency. This intuition suggests that the optimal ABRB profile is highly related to the maximal independent set of the interference graph $\mathcal{G}_I(\mathcal{G}_T)$.

**Definition 8** (Maximal Independent Set (MIS)). A subset $\mathcal{V}$ of $\mathcal{L}$ is an independent set of $\mathcal{G}_I(\mathcal{G}_T) = \{L, E_I\}$ if $e(l_k, l_{k'}) = 0, \forall l_k, l_{k'} \in \mathcal{V}$, and $l_k$ $\neq$ $l_{k'}$. A maximal independent set (MIS) is an independent set that is not a proper subset of any other independent set. For any MIS $\mathcal{V}$, define $\mathcal{N} \{\mathcal{V}\} = \{n : \exists k \in U_B \cap U_{\mathcal{V}}, \text{s.t.} l_k \in \mathcal{V}\}$ as the maximal independent macro BS set corresponding to $\mathcal{V}$. Let $\Theta_T(\mathcal{G}_T)$ denote the set of all MISs of $\mathcal{G}_I(\mathcal{G}_T)$.

For example, the set of all MISs of the interference graph in Fig. 4 is $\Theta_T = \{\mathcal{V}_1 \triangleq \{l_1, l_3\}, \mathcal{V}_2 \triangleq \{l_1, l_4\}, \mathcal{V}_3 \triangleq \{l_2\}\}$. Define a set

$$\Psi \triangleq \{\Theta = \{\mathcal{V}_1, ..., \mathcal{V}_{|\Theta|}\} : \cup_{j=1}^{|\Theta|} \mathcal{V}_j = \mathcal{L}\}.$$
Then the top level flow of finding a good ABRB profile is summarized in Table I. In Step 2, we need to find a set of MISs \( \Theta^* \in \Psi \). For the example in Fig. 4, \( \Psi \) has a unique element given by \( \Theta_T \) and thus \( \Theta^* = \Theta_T \). In Step 3, the mapping from \( \Theta^* \) to the ABRB profile \( A_{BS}^* \) is \( A_{BS}^* = \{ a_{BS}^*(j) \}, j = 1, ..., |\Theta^*| \}, \) where \( a_{BS}^*(j) = [a_{BS}^1(j), a_{BS}^2(j), ..., a_{BS}^{|\Theta^*|}(j)] \) with \( a_{BS}^m(j) = 1, \forall n \in N(B_j^m) \) and \( a_{BS}^m(j) = 0, \forall n / N(B_j^m) \). For example, for the HetNet in Fig. 4, we have \( \Theta^* = \Theta_T \) and thus \( A_{BS}^* = \{ a_{BS}^1(1), a_{BS}^2(1), a_{BS}^3(3) \} \), where \( a_{BS}^1(1) = [1, 0, 1], a_{BS}^2(2) = [1, 0, 1] \) and \( a_{BS}^3(3) = [0, 1, 0] \). Since \( a_{BS}^1(1) = a_{BS}^2(2) \) in this case, \( A_{BS}^* \) can be reduced to \( A_{BS}^* = \{ [1, 0, 1], [0, 1, 0] \} \).

For a general HetNet, \( \Psi \) may have multiple elements and we need to find a \( \Theta^* \in \Psi \) such that the corresponding \( A_{BS}^* \) is a good solution of \( P_{BS}^* (G_T) \). The detailed algorithm (Algorithm B2) for finding such \( \Theta^* \) is given in Appendix E where we also show that the \( A_{BS}^* \) found by the proposed algorithm is asymptotically optimal for \( P_{BS}^* (G_T) \).

VI. RESULTS AND DISCUSSIONS

A. Summary of the Overall Solution and Implementation

Fig. 5 summarizes the overall solution and the inter-relationship of the algorithm components for the Hierarchical RRM. The solutions are divided into long timescale process and short timescale process. The long timescale processing consists of the step 2 of Algorithm AO_A and AO_B as well as the sub-band partitioning \( P_C (G_T) \). The short timescale processing consists of step 1 of Algorithm AO_A and AO_B as well as the generation of ABRB patterns. All the long timescale processes are implemented globally at the RRMS and all the short timescale processes are implemented locally at each macro and pico BS as illustrated in Fig. 5. At each super-frame (\( L_S \) subframes), \( (q_s, q_A, q_B) \) are computed from the RRMS and pass to the macro and pico BSs. Locally at each BS and each subframe, the ABRB patterns on Type A subbands \( a_{ms}^m, m \in M_A \) and that on Type B subbands \( a_{ms}^m, m \in M_B \) are generated from the distribution \( q_A \) and \( q_B \) respectively. Furthermore, at each subframe, the user scheduling \( \pi^B_n \) and \( \pi^B_q \) are determined at each BS \( n \) based on the instantaneous channel quality indicator (CQI) of the direct links from the BS to the users. At the end of the \( L_S \) subframes, the macro and pico BSs deliver the estimates of the data rates \( \{ e_k^A (\pi_A), e_k^A (\pi_A); I_{mb,k} (\pi_B, a_{B}^j) \} \) to the RRMS.

There are several advantages of the proposed hierarchical RRM. For example, each BS only requires the direct link CQI. Hence, this solution has low signaling overhead and good scalability on the complexity. Furthermore, only long term statistical information is needed at the RRMS. Hence, the solution is robust w.r.t. backhaul latency.

B. Simulation Performance

In this section, we consider a HetNet with 19 eNBs and 57 directional macro cells (three 120 degree sectors), as illustrated in Fig. 6. In each macro cell, there are 4 uniformly distributed pico BSs. There are 30 users in one macro cell, 2/3 of whom are clustered around the pico BSs, while others are uniformly distributed within the macro cell. The macro cells are separated in 500 meters, and the maximum transmit power of macro BSs and pico BSs are 46 dBm and 30 dBm, respectively. The PFS utility is considered. Key simulation parameters are summarized in Table III. The simulation was run over 1000 subframes. We compare the performance of the proposed algorithm with the following 3 baselines.

- **Baseline 1 (FFR with static synchronized ABS):** Proportional fair scheduling and static synchronized ABS are used. The ABS is transmitted synchronously among the macro BS with 1/8 blanking rate. Fractional frequency reuse 17 with factor 1/3 is applied to the outer zone of each macro to protect the cell edge users.
- **Baseline 2 (FFR with dynamic synchronous ABS):** Baseline 2 is the same as baseline 1, except that the ABS blanking rate is dynamically chosen to maximize the proportional fairness utility.
- **Baseline 3 (Clustered CoMP):** 3 neighbor macro BSs and the associated pico BSs form a cluster for cooperative zero-forcing (ZF) 18 with per BS power constraint.
Table II: Key simulation parameters.

| Parameters                      | Values                                                                 |
|---------------------------------|------------------------------------------------------------------------|
| Network layout                  | 19 eNBs, 3-cell sites, 4 picos per sector (site)                       |
| Number of UE per cell site      | 30                                                                     |
| BS transmit power               | Macro: 46 dBm, Pico 30 dBm                                             |
| Channel model                   | IMT-Advanced Channel Model (10 Annex B)                                |
| Scheduling                      | Proportional Fair                                                      |
| Thermal noise                   | -174 dBm/Hz                                                            |
| UE speed                        | 6 km/h                                                                 |
| Bandwidth                       | 10 MHz                                                                 |
| Number of subbands              | 35                                                                     |
| Cell selection bias             | 9 dB                                                                   |

Table III: Performance evaluations on a HetNet with asymmetric network topology, where in each macro cell, the number of pico BS and the number of users are Poisson distributed with mean $\lambda_p = 4$ and $\lambda_u = 30$, respectively.

| Parameters                      | Baseline 2 | Proposed | Gain  |
|---------------------------------|------------|----------|-------|
| Average cell capacity (Mbps)    | 27.7       | 28.8     | 4%    |
| Macro cell I-users (Kbps)       | 708        | 2351     | 232%  |
| Pico cell I-users (Kbps)        | 5498       | 5548     | 1%    |
| worst 10% users (Kbps)          | 715        | 908      | 27%   |

Figure 7: Throughput comparisons over different RRM schemes and in different backhaul latency scenarios. The proposed algorithm outperforms both baseline 1 and 2. It also outperforms baseline 3 when there is a backhaul latency.

1) Throughput Evaluations: Fig. 7 compares the throughput of different RRM schemes. The proposed scheme outperforms baseline 1 and 2 over all performance metrics. It also outperforms baseline 3 when 10 ms backhaul latency for the signaling among BSs is considered. These results demonstrate the superior performance and the robustness of the proposed hierarchical RRM scheme w.r.t. signaling latency in backhaul.

Table III summarizes the throughput performance of baseline 2 and the proposed scheme under asymmetric network topologies, where in each macro cell, the number of pico BS and the number of users are Poisson distributed with mean $\lambda_p = 4$ and $\lambda_u = 30$, respectively. The proposed scheme still outperforms baseline 2. In particular, it enjoys 27% throughput gain for the worst 10% users. As a comparison, the corresponding throughput gain for the worst 10% users is 6% in the symmetric topology in Fig. 7. This demonstrates that the proposed scheme can better adapt to dynamic network loading.

Remark 3. In the simulations, we have fixed the cell selection bias $\beta$ to be 9dB. For larger $\beta$, there will be more pico cell I-users and more severe cross-tier interference from macro BS to pico cell I-users. In this case, it is more critical to use better and more fine-grained eICIC schemes to control the cross-tier interference. As a result, the performance gap between the optimization based ABRB control and those heuristic ABS controls becomes larger as increases.

2) Convergence of the Proposed Hierarchical RRM Algorithm: Fig. 8 shows the utility $\hat{U}_A$ in (9) of the Type A users versus the number of super-frames. The utility increases rapidly and then approaches to a steady state after only $2 - 3$ updates. The figure demonstrates a fast convergence behavior of Algorithm AO_A. Similar convergence behavior was also observed for Algorithm AO_B and the simulation result is not shown here due to limited space.

C. Complexity

We compare the complexity of the baselines and proposed RRM algorithms. The complexity can be evaluated in the following 3 aspects.

1) For the short term user scheduling, the proposed scheme and baseline 1-3 have the same complexity order of $O(MK)$, while the baseline 4 has a complexity of $O(C_A MK)$ [18], where $C_A$ is a proportionality constant that corresponds to some matrix and vector operations with dimension $B_c$, and $B_c = 15$ is the number of BSs in each cooperative cluster.

2) For the long term ABRB control variables $q_A$ and $q_B$, as they are updated by solving standard convex optimization problems in step 2 of Algorithm AO_A and AO_B respectively, the complexities are polynomial w.r.t. the number of the associated optimization variables. Specifically, for control variable $q_A$, the complexity is polynomial w.r.t. the number of macro BSs $N_0$. For control variable $q_B$, the complexity
is polynomial w.r.t. the size of the ABRB profile $A^{B^*}$: $|A^{B^*}| \leq |U_B|$. In addition, they are only updated once in each super-frame.

3) The ABRB profile $A^{B^*}$ is computed using Algorithm B2 in every several super-frames to adapt to the large scale fading. In step 1 of Algorithm B2, the complexity of solving the convex problem (27) is polynomial w.r.t. $|\Theta^{(i)}| \leq |U_B|+1$. In step 2 of Algorithm B2, if the MWIS algorithm in [19] is used to solve problem (28), the complexity is $O(|E|)$, where $|E|$ is the number of edges in the interference graph $G_I(G_T) = \{L, E_I\}$.

VII. CONCLUSION

We propose a two-timescale hierarchical RRM for HetNet with dynamic ABRB. To facilitate structural ABRB design for cross-tier and co-tier interference, the $M$ subbands are partitioned into Type A and Type B subbands. Consequently, the two timescale RRM problem is decomposed into subproblems $P_A$ and $P_B$ which respectively optimizes the ABRB control and user scheduling for the Type A and Type B subbands. Both subproblems involve non-trivial multi-stage optimization with exponential large solution space w.r.t. the number of macro BSs $N_0$. We exploit the sparsity in the HetNet interference graph and derive the structural properties to reduce the solution space. Based on that, we propose two timescale AO algorithm to solve $P_A$ and $P_B$. The overall solution is asymptotically optimal at high SNR and has low complexity, low signaling overhead as well as robust w.r.t. latency of backhaul signaling.

APPENDIX

A. Proof of Lemma 2

Define the average rate region as

$$
\mathcal{R} \triangleq \bigcup_{A} \left\{ x = [x_k] \in \mathbb{R}^K_+ : x_k \leq \tau_k (A), \forall k \right\}.
$$

For any utility function that is concave and increasing w.r.t. to the average data rates, the optimal policy of $P (G_T)$ must achieve a Pareto boundary point of $\mathcal{R}$. Hence, we only need to show that any Pareto boundary point of $\mathcal{R}$ can be achieved by a symmetric policy. Define the average rate region under fixed $q_s$ as

$$
\mathcal{R} (q_s) \triangleq \bigcup_{\{Q_m, \pi_m\}} \left\{ x = [x_k] \in \mathbb{R}^K_+ : x_k \leq \tau_k (\{Q_m, \pi_m\}), \forall k \right\}.
$$

Then we only need to show that any Pareto boundary point of $\mathcal{R} (q_s)$ can be achieved by a symmetric policy $\Lambda^* = \{q_s, Q_A, Q_B, \pi_A, \pi_B\}$. Define the average mutual information region for subband $m \in \mathcal{M}_A$ as:

$$
\mathcal{R}_m \triangleq \bigcup_{\{Q_m, \pi_m\}} \left\{ [x_k]_{k \in U_A} \in \mathbb{R}^{|U_A|}_+ : x_k \leq \mathcal{I}_{m,k} (Q_m, \pi_m), \forall k \in U_A \right\}.
$$

Define the average mutual information region for subband $m \in \mathcal{M}_B$ as:

$$
\mathcal{R}_m \triangleq \bigcup_{\{Q_m, \pi_m\}} \left\{ [x_k]_{k \in U_B} \in \mathbb{R}^{|U_B|}_+ : x_k \leq \mathcal{I}_{m,k} (Q_m, \pi_m), \forall k \in U_B \right\}.
$$

It can be verified that $\mathcal{R}_m, \forall m \in \mathcal{M}_A$ is a convex region in $\mathbb{R}^{|U_A|}_+$ and $\mathcal{R}_m, \forall m \in \mathcal{M}_B$ is a convex region in $\mathbb{R}^{|U_B|}_+$. Moreover, due to the statistical symmetry of the subbands, we have

$$
\mathcal{R}_m = \mathcal{R}_m\prime, \forall m, m' \in \mathcal{M}_A,
$$

$$
\mathcal{R}_m = \mathcal{R}_m\prime, \forall m, m' \in \mathcal{M}_B.
$$

Let $\mathcal{R}_A = \mathcal{R}_m, \forall m \in \mathcal{M}_A$ and $\mathcal{R}_B = \mathcal{R}_m, \forall m \in \mathcal{M}_B$. From the convexity of $\mathcal{R}_m, \forall m$ and (15), we have

$$
\mathcal{R} (q_s) = |\mathcal{M}_A| \mathcal{R}_A \times |\mathcal{M}_B| \mathcal{R}_B.
$$

Hence, for any Pareto boundary point $\mathbf{r}^* = [r^*_1, ..., r^*_K]^T$ of $\mathcal{R} (q_s)$, $\mathbf{r}^* \in \mathbb{R}^{|U_A|}_+$ is a Pareto boundary point of $\mathcal{R}_A$ and $\mathbf{r}^* \in \mathbb{R}^{|U_B|}_+$ is a Pareto boundary point of $\mathcal{R}_B$. Due to the statistical symmetry of the subbands, there exists an ABRB control policy and a user scheduling policy $\{Q_A^*, \pi_A^*\}$ such that $\mathbf{r}^* \in \mathbb{R}^{|U_A|}_+$ can be achieved for all subbands $m \in \mathcal{M}_A$ and $\mathbf{r}^* \in \mathbb{R}^{|U_B|}_+$ can be achieved for all subbands $m \in \mathcal{M}_B$. Hence, $\mathbf{r}^*$ can be achieved using the symmetric policy $\Lambda^{**} = \{q_s, Q_A^*, Q_B^*, \pi_A^*, \pi_B^*\}$. This completes the proof.

B. Structural Properties of $P_A$ for General Cases

The formal definition of $A_n$ and $\overline{A}_n$ is

$$
A_n = \left\{ \{a \in A; and a_n = 1\}, \forall n \leq N_0, \left\{ \{a \in A; and a_n = 0, \forall n' \in B_n\}, \forall N > N_0, \overline{A}_n = A / A_n, \forall n \in \{1, ..., N\}. \right. \right. \right.
$$

The result in Observation 3 is formally stated in the following lemma.

**Lemma 2.** For given CSI $H_{m \times A}$ in $H$, BS index $n \in \{1, ..., N\}$ and user index $k \in U_n \cap U_A$, the following are true:

1) $\forall a, a' \in A_n$, we have $\Gamma^m_{a,n} (a) = \Gamma^m_{a',n} (a')$ and $\mathcal{I}_{m,k} (a, H_{m \times A}, \rho^m_{a,n}) = \mathcal{I}_{m,k} (a', H_{m \times A}, \rho^m_{a',n})$, $\forall k \in U_m$.

2) $\forall a, a' \in A_n$, $a' \in \overline{A}_n$ and user scheduling vector $\rho^m_{a,n} \in \mathcal{R}^m_{a,n}$, there exists $\rho^m_{a,n} \in \mathcal{R}^m_{a,n}$ such that $\mathcal{I}_{m,k} (a, H_{m \times A}, \rho^m_{a,n}) \geq \mathcal{I}_{m,k} (a', H_{m \times A}, \rho^m_{a',n})$.

**Proof:** By Definition 2 there is no inter-cell interference for macro cell N-users. By the definition of $\Gamma^m_{a,n} (a)$, a pico
cell I-user \( k \in \mathcal{U}_n \cap \mathcal{U}_A \) cannot be scheduled for transmission if any of the neighbor macro BSs in \( \mathcal{B}_n \) is transmitting data subframe (i.e., the current ABRB pattern \( a \in \mathcal{A}_n \)). On the other hand, if all of the neighbor macro BSs is transmitting ABRB (i.e., the current ABRB pattern \( a \in \mathcal{A}_n \)), the interference from the macro BSs is negligible. Finally, by Definition 2 there is no inter-cell interference for pico cell N-users. Then Lemma 1 follows straightforwardly from the above analysis and the definition of \( \mathcal{A}_n, \mathcal{A}_n \).

**Remark 4.** Note that in the proof of Lemma 2 we have used the fact that the inter-cell interference seen at a N-user is negligible. We have also used Assumption 3 which states that the sets of neighbor macro BSs of the pico cell I-users belonging to the same pico cell are identical.

For general cases, the result in Observation 2 is stated in the following theorem.

**Theorem 2** (Policy Space Reduction for \( Q_A \)). Given a marginal probability vector that each macro BS is transmitting ABRB \( q_A = \{q^j_A, j = 1, \ldots, N_0\} \), the optimal ABRB control policy of \( P_A (q_T) \) conditioned on \( q_A \), denoted by \( Q_{A|q_A}^* \), has the following synonymous ABRB structure:

(a) Let \( \varsigma \) be a permutation such that \( q^\varsigma_A(1) \leq q^\varsigma_A(2) \leq \cdots \leq q^\varsigma_A(N_0) \). The support of \( Q_{A|q_A}^* \) has only \( N_0 + 1 \) active ABRB patterns \( A^\varsigma(q_A) = \{a^\varsigma(1), \ldots, a^\varsigma(N_0 + 1)\} \), where \( a^\varsigma(j)(i) = [a^\varsigma(j)(i), a^\varsigma(j)(j) + 1], j = 1, \ldots, N_0 + 1 \) with \( a^\varsigma(1)(j) = 1, \leq i \leq N_0 + 1 - j \) and \( a^\varsigma(N_0 + 1)(j) = 0, N_0 + 1 - j < i \leq N_0 \).

(b) Define \( q^\varsigma_A(0) = 0, q^\varsigma_A(N_0 + 1) = 0, and \( \varsigma(N_0 + 1) = N_0 + 1 \). Then \( Q_{A|q_A}^* (a^\varsigma(j)) = q^\varsigma_A(j) - q^\varsigma_A(j + 1), j = 1, \ldots, N_0 + 1 \), and \( Q_{A|q_A}^* (a)(0) = 0, \forall a \in A(q_A) \).

**Proof:** By Lemma 2 for given marginal probabilities \( q_A \), the average mutual information region will be maximized if we minimize \( \sum_{a \in \mathcal{A}_n} Q_A(a) \) for all \( 1 \leq n \leq N \). For \( n = N \), we have \( \sum_{a \in \mathcal{A}_n} Q_A(a) = q^N_A \). For \( n > N \), we have \( \sum_{a \in \mathcal{A}_n} Q_A(a) \leq \min_q q^N_A \), where the equality holds if and only if \( Q_A \) has the structure in Theorem 2. This completes the proof.

An example of synchronous ABRB is illustrated in Fig. 3. The following theorem is a general version of Observation 2.

**Theorem 3** (Policy Space Reduction for \( \pi_A \)). There exists optimal user scheduling policy \( \pi_A^* \) for \( P_A(q_T) \) such that \( \pi_A^* \in \Xi_A^* \), where \( \Xi_A^* \) is a convex region. From (22-23) and the fact that \( \mathcal{I}_{m,k} (Q_A^A, \pi_A^*) \) is a Pareto boundary point of \( \mathcal{R}_{m,k}^n \), it follows that \( \mathcal{I}_{m,k} (Q_A^A, \pi_A^*) \) is a Pareto boundary point of \( \mathcal{R}_{m,k}^n(a), \forall a \in \mathcal{A}_n \) and \( \mathcal{I}_{m,k} (Q_{A|q_A}^* (a), \pi_A^*) \) is a Pareto boundary point of \( \mathcal{R}_{m,k}^n(a), \forall a \in \mathcal{A}_n \). Hence, there exists user scheduling policy \( \pi_A^* \) satisfying \( I_{m,k} (\pi_A^*, a) = \mathcal{I}_{m,k} (\pi_A^*, a), \forall a \in \mathcal{A}_n \) and \( I_{m,k} (\pi_A^*, a) = \mathcal{I}_{m,k} (\pi_A^*, a), \forall a \in \mathcal{A}_n \) for all \( k \in \mathcal{U}_A \). Then it follows that \( \mathcal{I}_{m,k} (Q_A^A, \pi_A^*) \) can be achieved by the control policy \( Q_{A|q_A}^* (a), \pi_A^* \in \Xi_A^* \).

**C. Proof of Corollary 2**

The first part of the corollary follows straightforward from Theorem 2 and 3. We only need to prove that problem \( [9] \) is bi-convex. The average mutual information in [9] can be expressed as

\[
\mathcal{I}_{m,k} (Q_A^A, \pi_A) = \sum_{a \in \mathcal{A}_n} Q_{A|q_A}^* (a) I_{m,k} (\pi_A, a) + \sum_{a \in \mathcal{A}_n} Q_{A|q_A}^* (a) I_{m,k} (\pi_A, a) = \sum_{a \in \mathcal{A}_n} Q_{A|q_A}^* (a) (I_{m,k} (\pi_A, a) + \sum_{a \in \mathcal{A}_n} Q_{A|q_A}^* (a) I_{m,k} (\pi_A, a))
\]

where \( Q_{A|q_A}^* (a) = I_{m,k} (\pi_A, a), \forall a \in \mathcal{A}_n \), is a concave function w.r.t. \( Q_A \) for fixed \( \pi_A \). Using the vector composition rule for concave function [20], the objective in [9] is also concave w.r.t. \( Q_A \) and thus problem [9] is convex w.r.t. \( Q_A \) for fixed \( \pi_A \). For fixed \( Q_A \), \( \mathcal{I}_{m,k} (Q_A^A, \pi_A) \) is a linear function of the user scheduling variables \( \{\pi_A (a, m, H_A)\} \). Hence problem [9] is also convex w.r.t. \( \pi_A \) for fixed \( Q_A \).
D. Proof of Theorem 1

It is clear that \( \hat{U}_A (q_A^{(T+1)}, \pi_A^{(T)}) \geq \hat{U}_A (q_A^{(T)}, \pi_A^{(T-1)}) \).

By the assumption in Theorem 1, we have \( \hat{U}_A (q_A^{(T+1)}, \pi_A^{(T)}) > \hat{U}_A (q_A^{(T)}, \pi_A^{(T-1)}) \) if \( q_A^{(T)}, \pi_A^{(T-1)} \) is not a fixed point of \( \mathcal{F} \). Combining the above and the fact that \( \hat{U}_A \) is upper bounded, AO_A must converge to a fixed point \( \{ q_A, \pi_A \} \in \Delta \). The rest is to prove that any \( \{ q_A, \pi_A \} \in \Delta \) is globally optimal for problem (9).

Note that problem (9) is equivalent to the problem

\[
\max \sum_{k \in \mathcal{U}_A} w_k u_k (I_k), \text{ s.t. } |I_k|_{\forall k \in \mathcal{U}_A} \in \mathcal{R}_{m,n}.
\]

Since the objective in (25) is a concave function w.r.t. \( I_k \), \( \forall k \in \mathcal{U}_A \), and \( \mathcal{R}_{m,n} \) is a convex region, the following Lemma holds.

**Lemma 3 (Optimality Condition for (9)).** A solution \( q_A^*, \pi_A^* \) is optimal for problem (9) if and only if its average mutual information \( \overline{T}_{m,n,k} \triangleq \overline{T}_{m,n,k} (Q_A^{(q_A^*, \pi_A^*)}) \), \( k \in \mathcal{U}_A \) satisfies

\[
\sum_{k \in \mathcal{U}_A} \frac{\partial u_k (r)}{\partial r} |_{r = \overline{T}_{m,n,k}} \left( \overline{T}_{m,n,k} - I_k \right) \geq 0, \forall |I_k|_{\forall k \in \mathcal{U}_A} \in \mathcal{R}_{m,n}.
\]

According to the step 1 of AO_A, for any ABRB pattern \( a_{m,n} \in \mathcal{A} \) and CSI realization \( H_{m,n} \in \mathcal{H} \), the user scheduling vector \( \rho_{m,n} \) of BS n under \( \pi_A \) is the optimal solution of

\[
\max \rho_{m,n} \in \mathcal{R}_{m,n} (a_{m,n}, H_{m,n}, \rho_{m,n}),
\]

where \( \hat{u}_{m,n} \) is the average mutual information under \( q_A, \pi_A \). Combining (26) and the fact that \( q_A \) is the optimal solution of problem (9) with fixed \( \pi_A \), we have \( \sum_{k \in \mathcal{U}_A} \hat{u}_{m,n} (I_k) = 0 \), \( \forall V \in \mathcal{A} \), \( \forall \pi_A \). This implies that \( q_A, \pi_A \) satisfy the optimality condition in Lemma 3 and thus is the globally optimal solution.

E. Optimization of ABRB Profile

For any MIS \( V \), define \( \tilde{I} (V) = |I_k (V)|_{\forall k \in \mathcal{U}_B} \), where

\[
\tilde{I}_k (V) = \begin{cases} \log (1 + P_n \sigma_n^2), & \text{if } l_k \in V; \\ 0, & \text{otherwise}, \end{cases}
\]

The ABRB profile optimization algorithm is given below.

**Algorithm B2 (Algorithm for solving \( P_B^T (\Theta_T) \)).**

**Initialization:** Find initial \( \Theta_T^{(0)} = \{ \gamma_1^{(0)}, \ldots, \gamma_{|\Theta_T^{(0)}|}^{(0)} \} \) \( \in \Theta_T (G_T) \) such that \( \gamma_1^{(0)} \cup \gamma_2^{(0)} \cup \cdots \cup \gamma_{|\Theta_T^{(0)}|}^{(0)} \subseteq \mathcal{L} \). Set \( i = 0 \).

**Step 1** (Update the coefficients \( q_A^{(i)} \)): For fixed \( \Theta_T^{(i)} \), obtain the optimal solution \( q_A^{(i+1)} = \left\{ q_j^{(i+1)} \right\}_{j=1, \ldots, |\Theta_T^{(i)}|} \) of the following convex optimization problem

\[
\hat{U}_B (\Theta_T^{(i)}) = \max \sum_{k \in \mathcal{U}_B} w_k u_k \left( \sum_{j=1}^{|\Theta_T^{(i)}|} q_j \tilde{I}_k (V_j^{(i)}) \right),
\]

s.t. \( q_j \in [0, 1], \forall j \) and \( \sum_{j=1}^{|\Theta_T^{(i)}|} q_j = 1, \)

where \( q_j = [q_j^{(i)}]_{j=1, \ldots, |\Theta_T^{(i)}|} \), and \( V_j^{(i)} \) is the \( j \)th MIS in \( \Theta_T^{(i)} \).

**Step 2** (Update the set of MISs \( \Theta_T \)): Let \( \Theta_T^{(i+1)} = \left\{ V_j^{(i)} : q_j^{(i+1)} > 0 \right\} \cup \Theta_T^{(i+1)} \), where \( \Theta_T^{(i+1)} \) is given by

\[
\Theta_T^{(i+1)} = \arg\max_{V \in \Theta_T (G_T)} \sum_{k \in \mathcal{U}_B} q_k^{(i)} \tilde{I}_k (V),
\]

where \( q_k^{(i)} = w_k \frac{\partial u_k (r)}{\partial r} |_{r = \overline{T}_{m,n,k}^{(i)}} \).

If \( \hat{U}_B (\Theta_T (G_T)) - \hat{U}_B (\Theta_T^{(i+1)}) > \epsilon \), let \( i = i + 1 \) and return to Step 1. Otherwise, terminate the algorithm with \( \Theta_T^{*} = \left\{ V_1^{(i)}, \ldots, V_{|\Theta_T^{(i)}|}^{(i)} \right\} = \left\{ V_j^{(i)} : q_j^{(i+1)} > 0 \right\} \) and \( A^{B*} = \{ a_{b} (j), j = 1, \ldots, |\Theta_T^{(i)}| \} \) with \( a_{b}^* (j) = 1, \forall n \in \mathcal{N} (V_j) \) and \( a_{b}^* (j) = 0, \forall n \notin \mathcal{N} (V_j) \).

The convergence and asymptotic optimality of Algorithm B2 is proved in the following theorem.

**Theorem 4 (Asymptotically Optimal ABRB Profile).** Algorithm B2 always converges to an ABRB profile \( A^{B*} \) with \( |A^{B*}| \leq |\mathcal{U}_B| \). Furthermore, the converged result \( A^{B*} \) is asymptotically optimal for high SNR, i.e.

\[
\lim_{\|B\| \to \infty} \left( \frac{U_B (A^{B*}) - U_B (|\mathcal{U}_B|)}{|\mathcal{U}_B|} \right) = 0,
\]

where \( U_B = \alpha_n, \forall n = 1, \ldots, N_0 \) for some positive constants \( \alpha_n \)’s, and \( U_B^* \) is the optimal objective value of \( P_B (G_T) \).

**Proof:** Consider problem \( P_B (G_T) \) which is the same as \( P_B (G_T) \) except that there are two differences: 1) the fading channel \( H (t) \) is replaced by a deterministic channel with the channel gain between BS n and user k given by the corresponding large scale fading factor \( \sigma_n^2 \); 2) an additional constraint is added to the user scheduling policy such that any two links \( l_k, l_k' \) having an edge (i.e., \( e (l_k, l_k') = 1 \)) in the interference graph \( G_T \) cannot be scheduled for transmission simultaneously. It can be shown that the optimal solution of problem \( P_B (G_T) \) is asymptotically optimal for \( P_B (G_T) \) at high SNR. Moreover, using the fact that the achievable mutual information region in the deterministic channel is a convex polytope with \( \{ \tilde{I} (V), \forall V \in \Theta_T (G_T) \} \) as the set of Pareto boundary vertices, it can be shown that \( P_B (G_T) \) is equivalent to the following problem

\[
\max_{\Theta_T \subseteq \Theta_T (G_T)} \left\{ \max_{\Theta_T^{(i)}} \left( \left[ \sum_{k \in \mathcal{U}_B} w_k u_k \left( \sum_{j=1}^{|\Theta_T^{(i)}|} q_j \tilde{I}_k (V_j) \right) \right] \right) \right\},
\]

s.t. \( \Theta_T \subseteq \Theta_T (G_T) \), \( |\Theta_T| \leq |\mathcal{U}_B| + 1 \),

where \( V_j \) is the \( j \)th MIS in \( \Theta_T \). To complete the proof of Theorem 4 we only need to further prove that Algorithm B2 converges to the optimal solution of problem (29). Using the fact that any point in a \( |\mathcal{U}_B| - 1 \)-dimensional convex polytope can be expressed as a convex combination of no more than \( |\mathcal{U}_B| \) vertices, it can be shown that there are at most \( |\mathcal{U}_B| \) non-zero elements in \( q_A^{(i+1)} \) in step 1 of Algorithm B2. Hence \( \Theta_T^{(i)} \) is \( |\mathcal{U}_B| + 1 \), \forall i. Moreover, it can be verified that \( \hat{U}_B (\Theta_T^{(i+1)} > \hat{U}_B (\Theta_T^{(i)}) \), if \( \Theta_T^{(i)} \) is not optimal for \( \Theta_T^{(i)} \). Combining the above and the fact that \( \hat{U}_B (\Theta_T) \) is upper bounded by \( \hat{U}_B (\Theta_T (G_T)) \), Algorithm B2 must converge to the optimal solution of \( P_B (G_T) \). This completes the proof.

**Remark 5.** In step 2 of Algorithm B2, problem (28) is equivalent to finding a maximum weighted independent set.
(MWIS) in the interference graph $\mathcal{G}_I (\mathcal{G}_T)$ with the weights of the vertex nodes given by $\hat{\mu}_k^{(i)} \log \left(1 + P_n \sigma_{k,n}^2\right)$. The MWIS problem has been well studied in the literature [19]. Although it is in general NP hard, there exists low complexity algorithms for finding near-optimal solutions [19]. Although the Asymptotic global optimality of Algorithm B2 is not guaranteed when step 2 is replaced by a low complexity solution of (28), we can still prove its monotone convergence.

REFERENCES

[1] M. Ebrahimi, M. Maddah-Ali, and A. Khandani, “Throughput scaling laws for wireless networks with fading channels,” *IEEE Trans. Inf. Theory*, vol. 53, no. 11, pp. 4250 – 4254, Nov. 2007.

[2] D. Gesbert and M. Kountouris, “Rate scaling laws in multicell networks under distributed power control and user scheduling,” *IEEE Trans. Inf. Theory*, vol. 57, no. 1, pp. 234 – 244, Jan. 2011.

[3] D. Gesbert, S. Kiani, A. Gjendemsj et al., “Adaptation, coordination, and distributed resource allocation in interference-limited wireless networks,” *Proceedings of the IEEE*, vol. 95, no. 12, pp. 2393–2409, 2007.

[4] E. Altman, T. Boulgogne, R. El-Azouzi, T. Jimenez, and L. Wynter, “A survey on networking games in telecommunications,” *Computers & Operations Research*, vol. 33, no. 2, pp. 286–311, 2006.

[5] A. L. Stolyar and H. Viswanathan, “Self-organizing dynamic fractional frequency reuse in OFDMA networks,” in *INFOCOM 2008. The 27th Conference on Computer Communications*. IEEE, 2008, pp. 691–699.

[6] R. Irmer, H. Droste, P. Marsch, M. Grieger, G. Fettweis, S. Brueck, H. Mayer, L. Thiele, and V. Jungnickel, “Coordinated multipoint: Concepts, performance, and field trial results,” *IEEE Communications Magazine*, vol. 49, no. 2, pp. 102–111, 2011.

[7] H. Dahrouj and W. Yu, “Coordinated beamforming for the multicell multi-antenna wireless system,” *IEEE Transactions on Wireless Communications*, vol. 9, no. 5, pp. 1748–1759, 2010.

[8] Q. Shi, M. Razaviyayn, Z.-Q. Luo, and C. He, “An iteratively weighted MMSE approach to distributed sum-utility maximization for a MIMO interfering broadcast channel,” *IEEE Trans. Signal Processing*, vol. 59, no. 9, pp. 4331–4340, sept. 2011.

[9] LTE Advanced: Heterogeneous Networks, Qualcomm Incorporated, 2010.

[10] E-UTRA; Further Advancements for E-UTRA Physical Layer Aspects, 3GPP TR 36.814. [Online]. Available: http://www.3gpp.org

[11] Y. Wang and K. I. Pedersen, “Performance analysis of enhanced inter-cell interference coordination in LTE-advanced heterogeneous networks,” in *Vehicular Technology Conference (VTC Spring), 2012 IEEE 75th*. IEEE, 2012, pp. 1–5.

[12] J. Pang, J. Wang, D. Wang, G. Shen, Q. Jiang, and J. Liu, “Optimized time-domain resource partitioning for enhanced inter-cell interference coordination in heterogeneous networks,” in *Wireless Communications and Networking Conference (WCNC)*, 2012 IEEE. IEEE, 2012, pp. 1613–1617.

[13] M. Hong, R.-Y. Sun, H. Baligh, and Z.-Q. Luo, “Joint base station clustering and beamformer design for partial coordinated transmission in heterogeneous networks,” 2012. [Online]. Available: http://arxiv.org/abs/1203.6390

[14] J. Mo and J. Walrand, “Fair end-to-end window-based congestion control,” *IEEE/ACM Transactions on Networking*, vol. 8, no. 5, pp. 556–567, Oct 2000.

[15] F. Kelly, A. Maulloo, and D. Tan, “Rate control for communication networks: Shadow price proportional fairness and stability,” *J. Oper. Res. Soc.*, vol. 49, pp. 237–252, 1998.

[16] H. Kushner and P. Whiting, “Convergence of proportional-fair sharing algorithms under general conditions,” *IEEE Transactions on Wireless Communications*, vol. 3, no. 4, pp. 1250–1259, 2004.

[17] R. Ghaifar and R. Knopp, “Fractional frequency reuse and interference suppression for OFDMA networks,” in *Proceedings of the 8th International Symposium on Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks*, 2010, pp. 273–277.

[18] O. Somekh, O. Simeone, Y. Bar-Ness, A. Haimovich, and S. Shamai, “Cooperative multicell zero-forcing beamforming in cellular downlink channels,” *IEEE Trans. Inf. Theory*, vol. 55, no. 7, pp. 3206–3219, 2009.

[19] W. Brendel and S. Todorovic, “Segmentation as maximum weight independent set,” in *NIPS*, pp. 307–315, 2010.

[20] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.