Research on extended hypoplastic model and its verification for deposits soil

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Abstract. In order to illustrate the mechanical properties of the coarse-grained deposits soil, a series of large scale triaxial tests of deposits soil from the southwest region of China were carried out. Based on the test data and the basic idea of hypoplasticity, the Wu-Bauer hypoplastic model was extended to describe the mechanical behavior of the deposits soil, so that the constructed equations were induced for developed analysis. The model parameter identification method was proposed for the establishment of extended hypoplastic constitutive model by using improved differential evolution algorithm, which was targeted to find the optimization of the error function between test and theoretical results instructed by the inverse analysis principle. Then, an optimization calculation program was written to achieve the identification of the constitutive model parameters, which showed that the proposed identification method was better than the conventional method of parameter determination. Moreover, the test data and numerical calculation of one-dimensional compression test and triaxial test were compared, which indicated that the extended Wu-Bauer hypoplastic model could reflect the different mechanical properties of the deposits soil under different initial conditions.

1. Introduction

Coarse-grained loose deposits is widely developed and distributed in the mountainous areas of western China. The special geological bodies are composed of various loose slope deposits, avalanche slope deposits and alluvial deposits. Its structure is disordered and poor in sortability which contains soil-rock mixtures [1]. Lots of engineering geological problems concerning the deposits have caused a series of geological disasters which threaten human survival, life and engineering safety. The mechanical properties of loose deposits are affected by many factors, such as the particle size, porosity, and the adhesion between the particles. The stress-strain behave of the material changes with different confining pressures and pore ratios. There is a big difference in the curve, for example, the dilatation occurs in the dense state, and the strain softening phenomenon occurs in the low confining pressure; however, these phenomena does not occur in the loose state [2-3]. The variety of influencing factors and the complexity of mechanical phenomena have brought great difficulties to the constitutive description of the cohesive soil-rock geomaterials.

Existing numerical simulations of actual engineering use various models to describe the mechanical properties of coarse-grained deposits and other soil-rock mixtures, such as the Duncan-Chang’s nonlinear model [4], the Shen’s double yield surface model [5] and the Tsinghua K-G nonlinear model [6] etc. However, the constitutive models still have the problems that the main mechanical characteristics
and parameters of coarse-grained soil cannot be described reasonably and comprehensively. Then various new constitutive model theories have been proposed one after another, which not only improves the level and quality of the simulation of various geotechnical materials, but also promotes the rapid development of non-classical plasticity theory. The representative non-classical plastic mechanics constitutive theories include multiple plastic potential models [7], boundary surface models with nested yield surfaces [8] and hypoplastic constitutive models [9].

Hypoplasticity theory is a new type of geotechnical constitutive theory based on thermodynamics developed in the late 1980s. Its basic idea was promoted and developed by Kolymbas [10]. Hypoplasticity theory discards the concepts of total strain decomposition into elastic strain and plastic strain, yield criterion, hardening law, plastic potential and flow rule in traditional elastoplastic mechanics and some artificial assumptions. It had established the function relationship between stress rate and strain rate directly using tensor formula in continuum mechanics. Based on the understanding of the mechanical properties of coarse-grained soil, it focuses on the introduction of a suitable one for the description of sand and other non-cohesive soils by WU & Bauer hypoplastic model [11]. The four parameters included in this model are only related to the physical properties of the material, but not to the density of the material. Considering the limitation of the model used to describe the characteristics of the deposits material, this article proposes an extended model of the W-B model, and conducts a comprehensive identification method study of its model parameters. The extended W-B hypoplastic model can be used at a higher density and pressure within the scope, a good quantitative description of the mechanical response of the accumulation body is made, which is in good agreement with the results of triaxial tests.

2. Extended hypoplastic modeling

2.1. General form of hypoplastic model

Hypoplasticity is a generalization of hypelasticity. Its stress-strain relationship does not need to be connected by a potential function, but is established by an isotropic nonlinear tensor value function directly [12-13]. In general, the basic equations of the hypoplastic model are expressed in increments of tensors as:

\[ \bar{\mathbf{T}} = \mathbf{H}(\mathbf{T}, \mathbf{D}) \]  \hspace{1cm} (2.1)

where \( \bar{\mathbf{T}} \) is the Jaumann-Zaremba stress rate tensor, which is defined as \( \bar{\mathbf{T}} = \dot{\mathbf{T}} + \mathbf{TW} - \mathbf{WT} \); \( \mathbf{W} \) is the rotation tensor; \( \dot{\mathbf{T}} \) is Cauchy Stress rate tensor; \( \mathbf{D} \) is the strain rate tensor. According to the basic assumptions and requirements of continuum mechanics [14], the general expression of the tensor value function of the hypoplastic constitutive model can be decomposed into a linear part and a nonlinear part.

\[ \bar{\mathbf{T}} = \mathbf{L}(\mathbf{T}, \kappa) : \mathbf{D} + \mathbf{N}(\mathbf{T}, \kappa) ||\mathbf{D}|| \]  \hspace{1cm} (2.2)

where \( \mathbf{L} \) is the fourth-order tensor operator; \( \mathbf{N} \) is the second-order tensor operator; \( ||\cdot|| \) represents the Euclidean norm; \( \kappa \) is the material state variable.

According to the definition of the yield surface of the hypoplastic model, \( \forall \mathbf{T} \in \mathbb{T}, \exists \mathbf{D} \neq 0 \), when \( \bar{\mathbf{T}} = 0 \), the stress was said to enter the hypoplastic yield state. The stress field that satisfies the condition: \( \mathbf{T} \in \mathbb{T} = \{ \mathbf{T} | \bar{\mathbf{T}} = 0 \} \), constitutes a continuous surface in the stress space called the hypoplastic yield surface. At least one strain rate \( \mathbf{D} \) was existed corresponding to zero stress rate on the yield surface. Meanwhile, the volumetric strain rate disappears in the critical state as Cam-Clay’s model. From this, the yield surface expression can be derived.

\[ f(\mathbf{T}) = \mathbf{N}^T : (\mathbf{L}^{-1})^T : \mathbf{L}^{-1} : \mathbf{N} - 1 = 0 \]  \hspace{1cm} (2.3)

2.2. Wu-Bauer’s hypoplastic model

Wu and Bauer made some improvements to the specific form of linear and nonlinear terms on the basis of retaining the basic form of the early model of Kolymbas (1985). It makes up for some of the
deficiencies of the original model [15]. A practical Wu-Bauer hypoplastic model [11][16] was proposed for the first time with its specific form as follows.

$$\mathbf{T} = C_1 (\text{tr}\mathbf{T}) \mathbf{D} + C_2 \frac{T^3}{\text{tr}\mathbf{T}} \mathbf{T} + \left( C_3 \frac{T^2}{\text{tr}\mathbf{T}} + C_4 \frac{T^4}{\text{tr}\mathbf{T}} \right) \| \mathbf{D} \|$$

(2.4)

where $C_i (i = 1, 2, 3, 4)$ are dimensionless constants; $\mathbf{T}^*$ is the partial stress tensor.

The four parameters can be calibrated using the initial tangent modulus $E_s$, the initial Poisson’s ratio $\mu_s$, the internal friction angle $\varphi$ and the dilatancy angle $\psi$ from conventional triaxial test. The linear tensor operator $\mathbf{L}$ and non-linear tensor operator $\mathbf{N}^*$ can be derived in the principal stress space.

$$\mathbf{L} = \begin{bmatrix}
C_1 \text{tr}\mathbf{T} + C_2 \frac{T^2}{\text{tr}\mathbf{T}} & C_2 \frac{T^2}{\text{tr}\mathbf{T}} & C_2 \frac{T^2}{\text{tr}\mathbf{T}} \\
C_2 \frac{T^2}{\text{tr}\mathbf{T}} & C_1 \text{tr}\mathbf{T} + C_2 \frac{T^2}{\text{tr}\mathbf{T}} & C_2 \frac{T^2}{\text{tr}\mathbf{T}} \\
C_2 \frac{T^2}{\text{tr}\mathbf{T}} & C_2 \frac{T^2}{\text{tr}\mathbf{T}} & C_1 \text{tr}\mathbf{T} + C_2 \frac{T^2}{\text{tr}\mathbf{T}}
\end{bmatrix} ; \mathbf{N}^* = \frac{1}{\text{tr}\mathbf{T}} \begin{bmatrix}
C_3 T^2 + C_4 \left( \frac{2T_1 - T_2 - T_3}{3} \right)^2 \\
C_3 T^2 + C_4 \left( \frac{2T_2 - T_1 - T_3}{3} \right)^2 \\
C_3 T^2 + C_4 \left( \frac{2T_3 - T_1 - T_2}{3} \right)^2
\end{bmatrix}
$$

The Wu-Bauer hypoplastic model can not only describe the stress-strain relationship and yield characteristics during loading and unloading, but also describe the shear expansion and contraction behaviour.

2.3. Extended W-B hypoplastic model

The Wu-Bauer constitutive model has both advantages and disadvantages. It solves the problem of non-decreasing tangent stiffness of the Kolyma’s model in the triaxial compression experiment. It also can better describe the stress-strain characteristics of non-cohesive sand. However, the critical state parameter $C_3 = -C_4$ is derived under different stress paths, which causes the nonlinear terms of the constitutive equation (2.4) to be merged and degenerate into a three-parameter form.

$$\mathbf{T} = C_1 (\text{tr}\mathbf{T}) \mathbf{D} + C_2 \frac{T^3}{\text{tr}\mathbf{T}} \mathbf{T} + C_3 (\mathbf{T} + \mathbf{T}^*) \| \mathbf{D} \|$$

(2.5)

where $C_3' = C_3 / 3$.

This defect severely limits the universality of the model. For example, the initial Poisson’s ratio does not change with material changes, and it is not possible to simulate geotechnical media with large differences in density [17]. To this end, add the term $\text{tr} (\mathbf{D}^*) \mathbf{T}$, $\text{tr} (\mathbf{T}) \text{tr} (\mathbf{D}^*)$ disappears in the critical state ($\text{tr} \mathbf{D} = 0$), the addition of this item can describe the critical state under different stress paths, and a new four-parameter is obtained hypoplastic constitutive model.

$$\mathbf{T} = C_1 (\text{tr}\mathbf{T}) \mathbf{D} + \left( C_2 \text{tr}\mathbf{D} + C_3 \frac{T^3}{\text{tr}\mathbf{T}} \right) \mathbf{T} + C_4 (\mathbf{T} + \mathbf{T}^*) \| \mathbf{D} \|$$

(2.6)

It should satisfy both the stress and the volume strain to remain unchanged when the soil reaches the critical state according to the theory of critical state soil mechanics.

$$\mathbf{T} = 0 \quad \text{and} \quad \text{tr} \mathbf{D} = 0$$

(2.7)

Examining the spherical stress tensor and partial stress tensor in Equation 2.6.

$$\text{tr} \mathbf{T} = C_1 (\text{tr}\mathbf{T}) (\text{tr}\mathbf{D}) + \left( C_2 \text{tr}\mathbf{D} + C_3 \frac{T^3}{\text{tr}\mathbf{T}} \right) \text{tr}\mathbf{T} + C_4 (\text{tr}\mathbf{T} + \text{tr} \mathbf{T}^*) \| \mathbf{D} \|$$

(2.8)

$$\mathbf{T}^* = C_1 (\text{tr}\mathbf{T}) \mathbf{T}^* + \left( C_2 \text{tr}\mathbf{D} + C_3 \frac{T^3}{\text{tr}\mathbf{T}} \right) \mathbf{T}^* + 2C_4 \mathbf{T}^* \| \mathbf{D} \|$$

(2.9)

In the critical state, the condition (2.7) is satisfied, then the expressions (2.8) and (2.9) are both equal to zero, and can be reduced to (meanwhile using $\text{tr} \mathbf{T}^* = 0$).

$$\mathbf{T}^* (\text{tr}\mathbf{D}) + C_4 \text{tr} \mathbf{T}^* \| \mathbf{D} \| = 0$$

(2.10)

$$\frac{\mathbf{T}^*}{\| \mathbf{D} \|} = -\frac{\mathbf{T}^*}{\text{tr}\mathbf{T}} \frac{\mathbf{D}}{\| \mathbf{D} \|} = -\frac{T_1}{C_1} \frac{\mathbf{D}}{\| \mathbf{D} \|}$$

(2.11)

The new simultaneous equations 2.9 and 2.10 can solve the following expressions.

$$\frac{\mathbf{D}}{\| \mathbf{D} \|} = \frac{T_1}{C_1} \frac{\mathbf{T}^*}{\| \mathbf{T}^* \|}$$

(2.12)

By defining the unit strain rate tensor $\mathbf{D}^* \triangleq \frac{\mathbf{D}}{\| \mathbf{D} \|}$ and the unit partial stress tensor $\mathbf{R} \triangleq \frac{T^*}{\| \mathbf{T}^* \|}$, the yield surface equation can be written as $(\mathbf{D}^*; \mathbf{D}^* = 1)$.  

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The equation (2.13) is written as the general yield criterion \( \frac{J_2}{I_2} = k' \), it can be seen that this is the expression form of the Drucker-Prager yield criterion (as shown in Figure 2.1).

\[ \mathbf{R} : \mathbf{R} = \text{constant} \]  \hspace{1cm} (2.13)

In order to describe the shrinkage and dilation caused by the bulk characteristics of the accumulation body, the density factor function multiplier \( f_d \) is introduced into the nonlinear term of the extended hypoplastic model (2.6):

\[ f_d(e) = 1 - (1 - \omega)D_c \]  \hspace{1cm} (2.14)

where \( D_c \) is the modified relative density \( D_c \triangleq \frac{e_{cr} - e}{e_{cr} - e_{min}} \); \( \omega \in (0, 1) \) is the material parameter.

The rate change of the void ratio is directly related to the volume strain rate \( tr\mathbf{D} \) according to the law of conservation of mass expressed as follows.

\[ \dot{e} = -(1 + e)tr\mathbf{D} \]  \hspace{1cm} (2.15)

Then the relationship between the void ratio and the body strain can be obtained using the definite integral operation of (2.15).

\[ e = (1 + e_0)\exp(e_{v0} - e_v) - 1 \]  \hspace{1cm} (2.16)

where \( e_0 \) is the initial porosity ratio, \( e_v \) is the volumetric strain and \( e_{v0} \) is the initial volumetric strain.

The Wu-Bauer hypoplastic model was originally used to simulate non-cohesive soil without considering the effect of cohesion. The stress tensor \( \mathbf{T} \) in the extended constitutive equation is replaced with a new tensor \( \mathbf{\Omega} := \mathbf{T} - c\mathbf{I} \) containing cohesiveness parameters in this paper (\( c \) is the cohesiveness parameter).

A new expression of the constitutive equation (2.17) is obtained by introducing the multiplier \( f_d \) of the dense density factor function and replacing the original stress tensor.

\[ \overline{\mathbf{T}} = C_1 (tr\mathbf{\Omega})\mathbf{D} + \left( C_2 tr\mathbf{D} + C_3 \frac{\mathbf{D} \cdot \mathbf{D}}{tr\mathbf{D}} \right) \mathbf{\Omega} + C_4 (\mathbf{\Omega} + \mathbf{\Omega}^t)\|\mathbf{D}\|f_d \]  \hspace{1cm} (2.17)

The yield surface of this model is related to the porosity ratio, which means that the porosity ratio of the soil along different stress paths to failure is different, and the stress state will develop into a multi-yield surface area.
3. Parameter identification of Extended Hypoplastic Model

3.1. Numerical integration scheme of rate-type hypoplastic model

The equations of the metaplastic constitutive model are in the form of ordinary differential equations:

\[
\frac{d}{dt} T = H(t, T, \kappa)
\]  

(3.1)

According to the general form of the hypoplastic constitutive equation, the numerical integration format of the implicit format is used when it is used in the finite element calculation. Discrete the stress history into a finite number of equal steps that are sufficiently small. Knowing the stress value at time \( t \), the stress at time \( t + \Delta t \) can be obtained. The general integral form of iteratively updating stress within each time step \( \Delta t = [t_i, t_{i+1}] \) can be expressed by the following formula:

\[
T(t + \Delta t) = T(t) + \int_t^{t+\Delta t} H(T(\tau), D(\tau), e(\tau)) d\tau
\]  

(3.2)

For the above ordinary differential equations (3.1), the Runge-Kutta’s algorithm is used to solve iteratively within a limited time step. This method corresponds to the second-third-order Taylor expansion form.

\[
\begin{align*}
T(2)(t + \Delta t) &= T(t) + k_2 \\
T(3)(t + \Delta t) &= T(t) + \frac{1}{6} (k_1 + 4k_2 + k_3)
\end{align*}
\]  

(3.3)

where \( k_1, k_2, k_3 \) are expressed as below respectively:

\[
\begin{align*}
k_1 &= \Delta t \cdot H(t, T(t)) \\
k_2 &= \Delta t \cdot H \left( t + \frac{\Delta t}{2}, T(t) + \frac{k_1}{2} \right) \\
k_3 &= \Delta t \cdot H \left( t + \Delta t, T(t) - k_1 + 2k_2 \right)
\end{align*}
\]

The time step iteration convergence rules are as follows.

\[
\text{err} = \left\| T^{(3)}(t + \Delta t) - T^{(2)}(t + \Delta t) \right\| < TOL
\]  

(3.4)

In the formula (3.4), \( TOL \) is the preset tolerance value. Iteratively adopts adaptive time step technology. If the time step \( \Delta t \) reaches the error convergence standard, \( T^{(3)}(t + \Delta t) \) is the comprehensive understanding of the given time step \( \Delta t \), and the next new iteration time step can be determined as follows.

\[
\Delta t^{(\text{new})} = \min \left\{ 4\Delta t, 0.9\Delta t \left( \frac{TOL}{\text{err}} \right)^{1/3} \right\}
\]  

(3.5)

If the given time step is not as long as the convergence requirement, the next iteration time step is updated as follows:

\[
\Delta t^{(\text{new})} = \max \left\{ \frac{\Delta t}{4}, 0.9\Delta t \left( \frac{TOL}{\text{err}} \right)^{1/3} \right\}
\]  

(3.6)

In order to further control the calculation capacity and avoid too many iterations, the number of iteration steps is also controlled by ‘\( N_{\text{itermax}} \leq N_{\text{iterlimit}} \)’.

3.2. General method for determining parameters of hypoplastic model

Nine parameters are required for the extended hypoplastic constitutive equation, viz., \( C_1, C_2, C_3, C_6, \omega \). The five parameters \( \omega \) are dimensionless without definite physical implication, while other parameters \( c, e_0, e_{\text{min}}, e_{\text{crit}} \) have a clear physical meaning which can be measured by soil mechanics test. The extended W-B hypoplasticity equation (2.17) can be simplified to the following differential control equations.

\[
\begin{align*}
\dot{P}_1 &= C_1 (P_1 + 2T_2 - 3c)D_1 + \left[ C_2 (D_1 + 2D_2) + C_3 \frac{T_1D_1 + 2T_2D_2 - c(D_1 + 2D_2)}{T_1 + 2T_2 - 3c} \right] (T_1 - c) + \frac{C_4 (\frac{T_1 - 2T_2}{3} - c)}{\sqrt{D_1^2 + 2D_2^2} f_d(e(t))} \\
\dot{P}_2 &= C_1 (P_1 + 2T_2 - 3c)D_2 + \left[ C_2 (D_1 + 2D_2) + C_3 \frac{T_1D_1 + 2T_2D_2 - c(D_1 + 2D_2)}{T_1 + 2T_2 - 3c} \right] (T_2 - c) + \frac{C_4 (\frac{T_1 - T_2}{3} - c)}{\sqrt{D_1^2 + 2D_2^2} f_d(e(t))}
\end{align*}
\]  

(3.7)
As shown in Figure 3.1, the tangent modulus \( E \) and Poisson’s ratio \( \mu \) can be expressed by partial stresses, axial strain and radial strain at two points \( A \) and \( B \) from the experimental data.

\[
E_A = \frac{\tau_A^A - \tau_A^B}{\epsilon_1^A - \epsilon_1^B} = \frac{\tau_A^A}{\epsilon_1^A} ; \quad \mu_A = \frac{\sigma_2^A}{\sigma_1^A}
\]

\[
E_B = \frac{\tau_B^A - \tau_B^B}{\epsilon_1^B - \epsilon_1^A} = \frac{\tau_B^A}{\epsilon_1^B} ; \quad \mu_B = \frac{\sigma_2^B}{\sigma_1^B}
\]

(3.8)

![Figure 3.1 Typical triaxial test](image)

For a definite confining pressure and axial stress at two points, the quaternary linearity equation set (3.9) about the material constants \( C_i \) is obtained by substituting equation (3.8) into equation set (3.7). The parameters \( C_i \) can be determined by solving the equations. In this article, several groups of parameters were determined by the results of a traxial compression tests for the deposits soil with relative density 0.80 (shown in Table 3.1). The model parameters have large deviations depending on the position of the selected calculation point. Comparing the results of the triaxial consolidation drainage test under the confining pressure of 600kPa for the deposits soil, the numerical simulation results were calculated by substituting the above determined parameter groups into the extended hypoplastic model (shown in Figure 3.2).

| No. | Selected axial position | \( C_1 \) | \( C_2 \) | \( C_3 \) | \( C_4 \) |
|-----|------------------------|---------|---------|---------|---------|
| 1st | (0.08% 、 8% )         | -60.499 | -98.343 | -182.726| -124.470|
| 2nd | (0.5% 、 8.5% )        | -43.335 | -30.349 | -154.262| -89.671 |
| 3rd | (1.0% 、 9.0% )        | -24.160 | -57.379 | -70.926 | -49.016 |
| 4th | (1.5% 、 9.5% )        | -15.325 | -29.285 | -46.211 | -30.350 |

| Parameter | Value |
|-----------|-------|
| \( e_0 \) | 0.25 |
| \( e_{	ext{cfr}} \) | 0.21 |
| \( e_{\text{min}} \) | 0.02 |
| \( \omega \) | 0.72 |
| \( c \) | 62kPa |
The structural basis of the differential evolution algorithm is composed of three operations: selection, crossover and mutation, including: generating initial population (Initialization), mutation operation (Mutation), crossover operation (Crossection) and selection operation (Selection).

The nine parameters of the model were determined using the improved differential evolution algorithm and real experimental results from triaxial tests. Then, the optimal model parameter identification of extended Wu-Bauer hypoplastic model based on differential evolution algorithm

Differential Evolution Algorithm (Differential Evolution Algorithm) is enlightened from Darwin's biological evolution theory. R. Storn and K. Price [18] proposed a differential evolution algorithm for solving Chebyshev polynomial fitting problems. The differential evolution algorithm is widely concerned and used because of its few control parameters, high optimization efficiency, and good robustness.

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$E_A = C_1(T_1^A + 2T_2^A - 3c) + \left[ C_2(1 + 2\mu_A) + C_3 \frac{T_1^A + 2T_2^A\mu_A - (1 + 2\mu_A)}{T_1^A + 2T_2^A - 3c} \right] (T_1^A - c)$

$0 = C_1(T_1^A + 2T_2^A - 3c) + \left[ C_2(1 + 2\mu_A) + C_3 \frac{T_1^A + 2T_2^A\mu_A - (1 + 2\mu_A)}{T_1^A + 2T_2^A - 3c} \right] (T_2^A - c)$

$E_B = C_1(T_1^B + 2T_2^B - 3c) + \left[ C_2(1 + 2\mu_B) + C_3 \frac{T_1^B + 2T_2^B\mu_B - (1 + 2\mu_B)}{T_1^B + 2T_2^B - 3c} \right] (T_1^B - c)$

$0 = C_1(T_1^B + 2T_2^B - 3c) + \left[ C_2(1 + 2\mu_B) + C_3 \frac{T_1^B + 2T_2^B\mu_B - (1 + 2\mu_B)}{T_1^B + 2T_2^B - 3c} \right] (T_2^B - c)$

Figure 3.2 Comparison of experiment and numerical simulation under serials of parameters
parameters and the minimum value of the objective function was found by means of differential evolution.

The optimization algorithm is used to minimize the formula (3.10) to obtain the optimal parameter value:

\[ f(\mathbf{x}) = \| \mathbf{v}'(\mathbf{x}) - \mathbf{v}(\mathbf{x}) \|^{2} \]  

(3.10)

In formula (3.10), \( \| \|^{2} \) is a 2-norm; \( \mathbf{x} \) is the parameter vector of the model to be determined; \( \mathbf{v}(\mathbf{x}) \) is the experimental observation value, and \( \mathbf{v}'(\mathbf{x}) \) is calculation value, such as the principal stress difference \((\sigma_{1}-\sigma_{3})\) and volume strain \(\varepsilon_{v}\) of triaxial test. The variable \( \mathbf{x} \) can only take values within a given range during optimization iteration. Equation (3.11) reflects the constraints.

\[
\min: f(\mathbf{x}) \\
\text{s.t.} \; \mathbf{x} \in \mathbf{X}^{*}
\]  

(3.11)

For the system of differential algebraic equations (DAEs) (Equation 3.7), let \( y = T_{1} \) in the differential equation and \( z = D_{2} \) in the nonlinear algebraic equation, then the system of equations is summarized as follows:

\[
\frac{dy}{dt} = f(y, z) \\
0 = g(y, z)
\]  

(3.12)

Equation \( 0 = g\left(y, z\right) \) is related to the total differential of time \( t \):

\[
0 = \frac{\partial g(y, z)}{\partial y} \frac{dy}{dt} + \frac{\partial g(y, z)}{\partial z} \frac{dz}{dt}
\]  

(3.13)

Substitute equation (3.13) into (3.12):

\[
\frac{dz}{dt} = -\left(\frac{\partial g(y, z)}{\partial y}\right)^{-1} \frac{\partial g(y, z)}{\partial z} f(y, z)
\]  

(3.14)

Substitute the nonlinear equation (3.13) into the above formula (3.14) as follows:

\[
\begin{align*}
\frac{\partial g(y, z)}{\partial z} &= C_{1}(T_{1} + 2T_{2} - 3c) + 2\left[C_{2} + \frac{C_{3}(T_{2} - c)}{T_{1} + 2T_{2} - 3c}\right](T_{2} - c) \\
&\quad - \frac{2C_{4}D_{2}[c+(T_{1}-4T_{2})/3]}{\sqrt{D_{1}^{2}+2D_{2}^{2}}} f_{d}(e(t)) \\
\frac{\partial g(y, z)}{\partial y} &= C_{1}D_{2} + C_{3}\left[\frac{D_{2}}{T_{1} + 2T_{2} - 3c} - \frac{T_{1}D_{1} + 2T_{2}D_{2} - c(D_{1} + 2D_{2})}{(T_{1} + 2T_{2} - 3c)^{2}}\right](T_{2} - c) \\
&\quad - \frac{C_{4}}{5\sqrt{D_{1}^{2} + 2D_{2}^{2}}} f_{d}(e(t))
\end{align*}
\]  

(3.15)

The parameters of extended hypoplastic model were calibrated by differential evolution global optimization algorithm using the conventional triaxial test data of deposits soil under the confining pressure of 400kPa. The program file is written by MATLAB software. The specific steps are as follows:

1. The five parameters to be optimized in the model are written into variable form evolution according to equation (2.7), ie \( \mathbf{x} = [C_{1}, C_{2}, C_{3}, C_{4}, \omega] \). Substitute it into the triaxial test differential algebraic equation system.

2. Define an m file, Function ‘fval = Hypoplasticity (x)’, enter the measured main stress difference \((\sigma_{1}-\sigma_{3})\), axial strain \(\varepsilon_{1}\) and volume strain \(\varepsilon_{v}\) in the defined Hypoplasticity.m file; and in the form of input vector \( \mathbf{x} \), Write the equation relationship in step (1).

3. Write an adaptive time step Runge-Kutta differential equation implicit numerical integration format solution program for Hypoplasticity.m to call.

4. Compare the test data \((\sigma_{1}-\sigma_{3}), \varepsilon_{v}\) with the results obtained by solving the hypoplastic model \((T_{1}-T_{3}), D_{2}\) in the ‘Hypoplasticity.m’ program file. Establish the multivariate error objective function after conversion, and return with fval value.

5. An improved differential evolution algorithm was written to identify parameters by interactively iterating ‘Hypoplasticity.m’ error multivariate objective function.

The specific program structure pseudocode is shown in Table 3.2.
Table 3.2 The parameters of extended hypoplastic equation by composed recognition

**P-code**: Parameter identification diagram of extended Wu-Bauer hypoplastic model based on typical differential evolution algorithm

|   |   |
|---|---|
| 1: | % Define parameter solution vector: \( x = \{ C_1, \ C_2, \ C_3, \ C_4, \ \omega \} \), vector dimension \( P = 5 \) |
| 2: | % Establishment of differential equations for triaxial test of hypoplastic model: \[
\frac{dT_1}{dt} = f_1(x, T_1, D_2) ; \quad \frac{dD_2}{dt} = f_2(x, T_1, D_2)
\]|
| 3: | % Establish multivariate objective function \( F(x) \) of calculated value and test data error. \[
\arg\min_F = \sum_j \left[ \tilde{v}_j(x) - \tilde{v}_j(x) \right]^2 ; \quad \tilde{v} = \{ T_1 - T_2, \int trD \} \]
| 4: | % Set the population size \( N \) of the differential evolution algorithm, the maximum number of iterations \( G_{\text{max}} \), crossection ratio \( CR \), et al. |
| 5: | \( x_i^1 = (C_1, C_2, C_3, C_4, \omega)^T = x_i,_{\text{min}} + \text{rand}(0,1) \cdot (x_i,_{\text{max}} - x_i,_{\text{min}}) \) % Initialization |
| 6: | While \( G \leq G_{\text{max}} \) % \( G = 1, 2, 3, \ldots G_{\text{max}} \) |
| 7: | \( x_m_i = x_i^G + \lambda (x_i^{z_1} - x_i^G) + (1 - \lambda) (x_i^{z_2} - x_i^{z_3}) \) % Mutation \( (i \neq z1 \neq z2 \neq z3) \) |
| 8: | \( x_c_i = \begin{cases} 
  x_m_i & \text{if } \text{rand}(0,1) \leq CR \\
  x_i^G & \text{otherwise} 
\end{cases} \) % Crossection |
| 9: | % Bring the individual parameter values of the population into the system of differential equations (RKs Numerical Integration).
   For \( i = 1 \) to \( N \)
   \[
x_i^{G+1} = \begin{cases} 
  x_c_i & \text{if } F(x_c_i) < F(x_i^G) \\
  x_i^G & \text{otherwise}
\end{cases} \]
   End |
| 10: | Get \( [x_{\text{best}}^G, F(x)_{\text{goal}}] = \min\{F(x_i^G)\} \) % Winner in this round |
|   | \( x_i^G = x_{\text{best}}^G \)
   \( G = G + 1 \) % Update the best candidate
   End |

Through the above comprehensive identification method of the hypoplastic model parameters, the calculated values of the parameters of the extended W-B constitutive model suitable for the deposits are obtained (see Table 3.3). Figure 3.3 shows the comparative test results of simulation calculations for obtaining parameter values, showing that the accuracy of the constitutive model parameters determined by the comprehensive identification method is higher than that of the general parameter determination methods.
Table 3.3 The parameters of extended hypoplasticity equation by composed recognition

| Parameter | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $\omega$ | $\epsilon_{\text{min}}$ | $\epsilon_{\text{crt}}$ | $\epsilon_0$ | $\sigma/kPa$ |
|-----------|-------|-------|-------|-------|---------|----------------|----------------|------------|-------------|
| Value     | -59.807 | -126.482 | -174.290 | -100.172 | 0.795 | 0.02 | 0.209 | 0.112 | 76.0 |
|           |        |        |        |        |        |                |                |            | 0.253 | 62.0 |

![Figure 3.3](image_url)

**Figure 3.3** Comparison of triaxial experiment and numerical simulation under comprehensive identification of parameters

4. Simulation verification of extended W-B hypoplastic model test

The mechanical response of the model under specific boundary conditions can be obtained by numerical simulation for unit geotechnical test. Compared with the corresponding basic characteristics of the test, the main mechanical characteristics of the material can be reflected whether reasonably or not by the constitutive model. The unit test data of the one-dimensional confined compression and triaxial compression for the deposits compared with the numerical simulation results by the extended W-B hypoplastic equation was proposed in this paper (the model material parameters are shown in Table 3.3).

4.1. Validation of 1-dimensional lateral compression test model

The initial stress state is taken as the self-weight stress field, and the initial porosity ratio $\epsilon_0$ is 0.253 and 0.112, respectively. The calculation includes loading and unloading, first applying axial pressure to $P_2 = 300\text{kPa}$, and then unloading. Figure 4.1(a) shows the relationship between axial stress and radial stress. Figure 3.1(b) illustrates the relationship between the calculated axial strain and axial stress.
It can be seen from the figure above that the model reflects the inelastic and nonlinear characteristics well. The axial stress and the radial stress are close to a linear relationship in the loading stage otherwise nonlinearity in the unloading stage. The incremental ratio between the axial stress and the radial stress is defined as the static earth pressure coefficient in the unloading stage. The axial stress decreases faster than the horizontal stress during the loading phase. In the loading stage, the static earth pressure coefficient $K$ changes insignificantly which is close to the initial static earth pressure coefficient ($0.5$) calculated by the Jaky’s empirical formula.

4.2. **Validation of triaxial test model**

An axisymmetric unit of $30\text{cm} \times 60\text{cm}$ is used in the calculation. The initial isotropic pressure $T_{11} = T_{22} = T_{33}$ is applied, and then the confining pressure is kept constant, and the load is applied axially at a constant rate. The test model takes two different initial porosity states $e_0 = 0.253$ and $e_0 = 0.112$.
Figures 4.2 and 4.3 describe the relationship between the axial strain and the principal stress difference \((\sigma_1-\sigma_3)\) calculated at two different initial porosity ratios under confining pressures of 200kPa and 600kPa. It can be seen from the figure that the initial dense deposits soil \((e_0 = 0.112)\) exhibits strain softening and volumetric dilatancy characteristics under certain pressure, while the initial loose deposits soil body \((e_0 = 0.253)\) exhibits strain hardening and volumetric shrinkage characteristic. In addition, the failure strength of the deposits in the initial dense state is greater than that in the initial loose state. Therefore, the previous theoretical analysis of the extended hypoplastic model reflects some basic mechanical properties of deposits soil objectively.

Figure 4.4 illustrates the relationship between the corresponding axial strain and the porosity ratio. It can be seen from the figure that with the continuous development of the triaxial shear process, the void ratio of the deposits soil in the initial loose state continues to decrease, and the material becomes more and more dense. On the other hand, the void ratio of the deposits soil in the initial dense state behave a slight decrease first, and then increase so called phenomenon of dilatation. The critical porosity ratio was eventually approached under the corresponding stress state reflected in both curves.

In summary, the model uses the same set of model parameters at two different initial states \((e_0 = 0.253\) and \(e_0 = 0.112)\), which can reflect the different mechanical properties of the deposits soil material in different initial states. It is indicated that the initial state has a wide range of applications, which is difficult to describe with the four-parameter Wu-Bauer hypoplastic model without state variables.
5. Conclusions

(1) Explicit representation form are derived with advantages and disadvantages based on the in-depth study of the Wu-Bauer hypoplastic model. The model parameter determination method are also explained.

(2) The hypoplastic modelling makes an approach to describe deposits soil. The extended hypoplasticity equation was derived to explain the stress-strain relationship of deposits soil based on the Wu-Bauer hypoplastic model.

(3) The comprehensive parameters identification of extended constitutive model was proposed by means of differential evolution algorithm. An iterative solution program for the differential equations was compiled for the hypoplastic constitutive equation. This method can determine the constitutive parameters based on some of the obtained experimental data better than the conventional method.

(4) The numerical simulation of the conventional triaxial compression test based on the extended hypoplastic model was adopt to analyze some mechanical characteristics of the deposits in comparison with laboratory results.

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