1. Introduction

Type Ia supernovae are bright stellar explosions that are distinguished from other supernovae by a lack of hydrogen in their spectra. A widely accepted progenitor for these events is a close binary system in which the primary component is a degenerate C-O rich white dwarf that accretes from a low-mass companion. The explosion is believed to ignite in the center of the white dwarf when its center becomes sufficiently dense and hot. For a review of different explosion models, see [1] and references therein.

Constraints on the nature of the explosion come from studies of the supernova light curve and the composition of the ejecta. Multi-dimensional hydrodynamical simulations cannot include more than a few isotopes and are typically Eulerian. Here we report on our implementation of tracer particles into a simulation of a deflagrating C-O white dwarf. These particles are advected along streamlines and provide a Lagrangian description of the explosion that can be
post-processed with a realistic reaction network. We discuss our verification tests, a 2-D deflagration, and the resulting nucleosynthetic yields.

2. Simulation with Tracer Particles

The simulations of the deflagrating white dwarf were performed with the Flash code [2]. Flash solves the Euler equations for compressible flow and the Poisson equation for self-gravity on an adaptive mesh. These simulations employ a flame capturing scheme (see [3]) that advances the flame by evolving a passive scalar variable with an advection-reaction-diffusion equation. The flame model assumes that the flame propagates at a given speed \( v = \max(v_{\text{lam}}, v_{\text{turb}}) \), where \( v_{\text{lam}} \) is the laminar flame speed [4], and \( v_{\text{turb}} \) is a turbulent flame speed based on the assumption that the turbulent burning on macroscopic scales is driven by Rayleigh-Taylor instability [3]. The scalar variable is used to model the burning from \(^{12}\text{C}\) to \(^{24}\text{Mg}\). Further burning to Si-group and relaxation to NSE (here taken to be \(^{56}\text{Ni}\)) are modeled as two consecutive stages of burning following the burn from C to Mg.

In addition to the flame model, the simulation of the deflagration had 10,000 tracer particles added to track the evolution of the fluids. The technique of using tracer particles in an Eulerian scheme has been used in simulations of core-collapse supernovae [5,6] and Type Ia supernovae [7]. The initial spatial distribution of the particles follows the mass distribution. The tracer particles were evolved by a second-order predictor-corrector method [8], with the velocity at the particle position computed by parabolic interpolation.

2.1. Verification

We have partially verified the tracer particle module on a smooth hydrodynamic problem, the isentropic vortex. This test problem has a known analytic solution and is suitable for solution verification studies (see [9], and references therein). We used a uniform grid of 128x128 equidistant zones and 10,000 randomly distributed tracer particles. In our analysis we focused on a subset of three test particles located at \( r = 0.26, 0.767, \) and \( 3.873 \) cm. We evolved this system for \( t = 100 \) s, which corresponds to 21.3 orbital periods for the inner particle and about 15.4 orbital periods for the middle particle. The outer particle moves very slowly and traveled a negligible fraction of its orbit over the simulation.

Fig.1 (left panel) shows the relative error in the particle radius at the final time. The deviation from the nominal orbit is generally small and is less than 1% for \( t < 20 \) s, or about 4 orbits for the inner particle. At later times the error steadily grows but appears to be increasing for only the inner particle. The orbit of the inner particle is tight (resolved by about only 20 zones), and the general increase in radius likely results from diffusion in the hydrodynamic solver in addition to the intrinsic error in tracing the flow. While separating these effects is a future endeavor, we are confident that our implementation is correct and that tracer particles closely follow the streamlines.

2.2. A 2-D Deflagration

The two-dimensional deflagration simulation was performed in cylindrical coordinates and azimuthal symmetry on a domain with \( r = [0, 8192] \) km and \( z = [-8192, 8192] \) km and an effective resolution of 8 km. The initial model was an isothermal white dwarf of temperature \( T_{\text{wd}} = 5 \times 10^7 \) K, mass \( M_{\text{wd}} = 1.3 \, M_\odot \), and radius \( R_{\text{wd}} = 2130 \) km. The outer boundary conditions were hydrostatic outflow-only and reflecting boundary conditions were imposed along the
symmetry axis. The deflagration was ignited by making the innermost 100 km the white dwarf into completely burned material in hydrostatic equilibrium with its surroundings. The flame was then evolved by integrating the hydrodynamic equations supplemented by the advection-diffusion-reaction model described above. The deflagration was evolved for 2.0 s, by which time \( \approx 0.4 M_\odot \) had burned.

3. Nucleosynthesis

The isotopic abundances were calculated by integrating a reaction network along the trajectories in \((\rho, T)\) space followed by the tracers. We use a network consisting of 214 isotopes ranging from neutrons, protons, and \(\alpha\)-particles to \(^{70}\text{Zn}\). The coupled ODEs are integrated using an adaptive semi-implicit scheme with a sparse matrix solver \([10]\). We use the reaction rate compilation \textsc{reaclib} \([11]\), and incorporate the tabulated weak rates from \([12]\). Screening is computed according to the prescription of \([13]\). Since each particle is independent, the calculation is embarrassingly parallel, and large numbers of particles can be quickly post-processed on a multi-CPU machine.

Fig. 1 (right panel) shows the mass fractions of the isotopes synthesized in the deflagration. One notes that most of the original C-O did not burn and that roughly equal masses of \(^{56}\text{Ni}\) and \(^{28}\text{Si}\) were made. Only about 0.15 \(M_\odot\) of Ni-peak elements are synthesized. This is much less than that found in one-dimensional calculations such as W7 \([14]\) and recent 3-D octant calculations \([7]\). For our chosen configuration, the burning is incomplete. Large plumes form and rise toward the surface of the white dwarf. The large scale circulation arising from these flows tends to homogenize the composition as a function of radius: unburned \(^{12}\text{C}\) and \(^{16}\text{O}\), intermediate mass elements and Ni-peak elements are present at roughly constant abundances from the center to about 0.9 of the WD mass (as far as the rising plumes had risen for this simulation).

Figure 1. Left panel Relative error in the position of test tracer particles in the isentropic vortex verification test problem. Right panel Mass fractions of the isotopes synthesized by the deflagration. Dotted lines connect isotopes of the same element. The number beside each point refers to the mass number of that isotope.
We did repeat the calculations with a starting abundance that included $^{22}$Ne. From charge and mass conservation, one can show that the mass of $^{56}$Ni should decrease linearly with the initial mass fraction of $^{22}$Ne \cite{15}. Calculations with an initial $^{22}$Ne mass fraction of 0.02 and 0.06 also find a linear dependence of the mass fraction of $^{56}$Ni, but with a 10% steeper slope. Because the underlying hydrodynamical model was the same, however, we cannot draw further conclusions about the relationship between ejected $^{56}$Ni mass and progenitor metallicity. We also checked how the computed abundances scaled with the number of tracer particles. For a subset of 5000 randomly selected particles, the difference in the $^{56}$Ni mass from that computed with 10,000 particles is 8%.

We finally note that although only about 10% of the star is burnt to Fe-peak elements of that fraction, almost all of it is in the form of $^{56}$Ni; the mass fraction of $^{58}$Ni, $^{54}$Fe and $^{56}$Fe are 0.019, 0.009, and 0.0005, respectively. This is in contrast to 1-D models, for which electron captures convert the innermost $\approx 0.2 \, M_\odot$ to $^{56}$Fe \cite{16}. The rising plumes of hot ash carry the burnt material to lower density, where electron capture rates are lower. The effect of rising bubbles of hot fuel to mix the white dwarf and leave masses of C-O fuel unburnt has been noted by others (for example, \cite{17}) and indeed, is a motivation for suggesting a subsequent transition to detonation.

In conclusion, we have implemented particle tracers in an Eulerian, adaptive mesh hydrodynamics code, verified that they closely followed streamlines, and used them for post-processing a 2-D simulation of a deflagrating C-O white dwarf. Future 3-D simulations will use this capability for comparison against other simulations \cite{13} and observations.

REFERENCES

1. W. Hillebrandt and J. C. Niemeyer, ARA&A 38 (2000) 191
2. B. Fryxell, et al., ApJS 131 (2000) 273
3. A. M. Khokhlov, ApJ 449 (1995) 695
4. F. X. Timmes and S. E. Woosley, ApJ 396 (1992) 649
5. S. Nagataki, M. Hashimoto, K. Sato, and S. Yamada, ApJ 486 (1997) 1026
6. C. Travaglio, K. Kifonidis, and E. Müller, New Astro. Rev. 48 (2004) 25
7. C. Travaglio, W. Hillebrandt, M. Reinecke, and F.-K. Thielemann, A&A 425 (2004) 1029
8. W. J. Rider and D. B. Kothe A Marker Particle Method for Interface Tracking LA-UR-95-1740 (1995)
9. A. C. Calder, et al., ApJS 143 (2002) 201
10. F. X. Timmes, ApJS 124 (1999) 241
11. T. Rauscher and F.-K. Thielemann, At. Data Nucl. Data Tables75 (2000) 1
12. K. Langanke and G. Martínez-Pinedo, At. Data Nucl. Data Tables 79 (2001) 1
13. H. C. Graboske, H. E. Dewitt, A. S. Grossman, and M. S. Cooper, ApJ 181 (1973) 457
14. K. Iwamoto, F. Brachwitz, K. Nomoto, N. Kishimoto, H. Umeda, W. R. Hix, and F.-K. Thielemann, ApJS 125 (1999) 439
15. F. X. Timmes, E. F. Brown and J. W. Truran, ApJ 590 (2003) L83
16. F. Brachwitz et al., ApJ 536 (2000) 934
17. V. Gamezo, et al., Science 299 (2003) 77