Noise sensitivity of a bio-inspired echolocation model

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Abstract. The Spectrogram Correlation And Transformation (SCAT) receiver is often cited as a model of bat auditory system that is responsible for echolocation signal processing and super resolution in range. We explore the response of a simplified baseband version of SCAT (BSCT) for two targets in white Gaussian noise. Knowing the loss of signal-to-noise ratio (SNR) relative to the matched filter (MF) is important for any practical applications of the algorithm. We show that this loss is 6–dB and increases if a critical noise level is surpassed.

1. Introduction

Bats' ability to resolve close targets has been attributed to a unique perspective of the world in which at local scale a group of reflectors is represented by the spacing between them [1]. Multiple behavioural experiments have demonstrated better resolution of two close targets than the conventional matched filter [2-5], and models of bat auditory system have been developed to reproduce such perceptual capabilities [6, 7]. These studies are limited to high to medium signal to noise ratios (SNR). The noise properties of the corresponding models are largely unexplored.

The Spectrogram Correlation And Transformation (SCAT) receiver is a model of the auditory system that was proposed by Saillant et al. for the bat Eptesicus fuscus [6] and further developed in [8]. It is a physiologically plausible model of the peripheral and the central auditory system of frequency modulating bats. Different aspects of this model are clarified and analysed in [9, 10]. The interference pattern arising from the return of two close targets is produced in a bank of filters and further processed to evaluate the distance between the targets.

A baseband version of the SCAT model (BSCT) was developed as a tool to study the SCAT model [11]. The noise-free response of the BSCT receiver was found analytically for two closely spaced scatterers and a waveform with a flat spectrum (Linear Frequency Modulated chirp) in [1]. It has been shown that such a receiver works in ultrasound and radio frequency experiments [1, 12].

Similar techniques that rely on interference of two signals to produce modulation of the spectral density have been introduced in the early ages of radar studies for range estimation in noise radars [13, 14]. The so called two-stage spectral analysis (TSSA) consists in cascading two physical spectrum analysers and processing the sum of the transmitted (zero delay) signal and the received signal. The first stage introduces frequency modulation of the spectrum related to the target delay and the second stage transforms the modulation into a range profile.

A fall-off in BSCT-like models performance at reduced SNR is expected as the noisy signal is correlated with a copy of itself rather than with a “clean” reference. The loss of Signal to Interference ratio for TSSA processing of slowly moving targets was derived in [15] and shown to be approximately 6 dB as compared to the cross correlation processing for selected example. A key paper...
on the subject of noisy references is [16] where the advantages for detection in passive MIMO radar networks are developed. Bats were trained to discriminate the closer of two targets. The background noise level was increased in 10-dB steps. The discrimination performance degraded slightly until 10-dB SNR was reached. After this threshold discrimination dropped sharply.

How big is the loss of SNR relative to the matched Filter (MF) is of major concern for any practical application of a new algorithm. The purpose of this paper is to analyse the loss of SNR for a BSCT receiver. Analytical expressions are derived and then confirmed through simulations.

2. The BSCT receiver

The BSCT receiver is described in details in [1]. This section summarizes the key results and relationship defining the model. For practical applications the BSCT procedure should be applied after signal compression, e.g. matched filtering.

The frequency profile $E[f_i]$ of the BSCT receiver is defined for $M+1$ frequencies $f_i$, $i = -\frac{M}{2}, \ldots, \frac{M}{2}$:

$$E[f_i] = \int_{f_i - \frac{B}{2}}^{f_i + \frac{B}{2}} |X(f)|^2 df,$$  \hspace{1cm} (1)

where $B$ is a parameter of the model, $X(t) \leftrightarrow X(f)$ is the input signal in time and frequency domain.

The spacing profile is an inverse Fourier transform of the frequency profile with subtracted mean value:

$$e(t) = F^{-1}[E(f) - \text{mean}[E(f)]].$$  \hspace{1cm} (2)

For two targets convolved with a waveform with complex envelope $x_0(t)$ and central frequency $f_0$:

$$x(t) = [\delta(t - t_1)e^{-j2\pi f_0 t_1} + \delta(t - t_2)e^{-j2\pi f_0 t_2}] x_0(t);$$  \hspace{1cm} (3)

$$X(f) = X_0(f)e^{-j2\pi(f+f_0)t_1} + X_0(f)e^{-j2\pi(f+f_0)t_2}$$

$$= X_0(f)e^{-j2\pi f't_1}e^{-j\pi f^2 t_2} \cos \left( \frac{\pi f \tau + \frac{\psi_t}{2}}{2} \right);$$  \hspace{1cm} (4)

$$|X(f)|^2 = 2|X_0(f)|^2 \left[ \cos(2\pi f \tau + \psi_t) + 1 \right],$$  \hspace{1cm} (5)

where $t_1$ and $t_2$ are the time delays of the targets, $\tau = t_2 - t_1$ is the delay between the targets, $f' = f + f_0$ and $\psi_t = 2\pi f_0$.

Let's consider a waveform with flat spectrum with spectral density $A$ and bandwidth $B_C$:

$$|X_0(f)| = A \text{rect} \left( \frac{f}{B_C} \right);$$  \hspace{1cm} (6)

$$\text{rect}(f) = \begin{cases} 1, & |f| \leq \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$  \hspace{1cm} (7)

(i.e. an approximation of a band limited impulse with amplitude $A$, linearly frequency modulated chirp, stepped frequency signal or white noise signal). Then

$$|X(f)|^2 = 2A^2 \left[ \cos(2\pi f \tau + \psi_t) + 1 \right] \text{rect} \left( \frac{f}{B_C} \right)$$  \hspace{1cm} (8)

and equation (1) simplifies to

$$E[f_i] = 2A^2 \int_{f_i - B/2}^{f_i + B/2} \left[ \cos(2\pi f \tau + \psi_t) + 1 \right] \text{rect} \left( \frac{f}{B_C} \right) df$$

$$= 2A^2B \left[ \text{sinc}(\pi B) \cos(2\pi f_0 + \psi_t) + 1 \right],$$

for $|f_i| \leq (B_C - B)/2$, where $\text{sinc}(x) = \sin(\pi x)/(\pi x)$ [1]. For $|f_i| > (B_C + B)/2$, $E_E[i] = 0$. 

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The spacing profile for two targets is produced by replacing (9) in (2). A conventional MF range profile and a BSCT spacing profile are compared in figure 1 for two targets at 0.122 m, radio frequency linear chirp waveform with \( B_C = 4 \) GHz, and white Gaussian noise leading to SNR of 17 dB. The abscissa is in meters and in resolution gates, where one gate is equal to \( c/2B_C \), \( c \) is the speed of light. The single prominent peak in the spacing profile indicates the separation between the two targets, as opposed to the two peaks in the range profile that indicate the ranges to the targets.

![Graph showing matched filter range and BSCT spacing profiles](image)

**Figure 1.** Matched filter range and BSCT spacing profiles for simulated targets separated by 0.122 m, 4 GHz bandwidth and 17 dB SNR.

3. **SNR for a BSCT transformed signal**

The SNR is defined as the ratio of the peak amplitude squared of the compressed signal and the average noise power (the noise variance).

Let’s denote a noisy signal sampled at the Nyquist rate with \( s(n) \) and let \( w(n) \) is a band limited white Gaussian noise with variance \( \sigma_w^2 \):

\[
s = x + w; \tag{10}
\]

\[
S = X + W. \tag{11}
\]

First we will analyse a simplified version of BSCT without averaging (integration) in the frequency domain, i.e. frequency spacing and integration bandwidth are such that the frequency profile \( E[f_i] \) (1) is equal to the frequency energy of the processed echo \( |S(f_i)|^2 \) :
\[ E[f_i] = |S(f_i)|^2 = (X + W)(X + W)^* = XX^* + XX^* + WW^* + W^2 \]
\[ = |X|^2 + XX^* + (XX^*)^* + |W|^2 = |X|^2 + 2 \text{Re}(X W^*) + |W|^2 \]  
(12)

and for the two point target case, taking \( X \) from (4), (6) and \(|X|^2\) from (8), the frequency profile (12) becomes
\[ E[f_i] = 2 A^2 [\cos(2\pi f_i \tau + \psi_x) + 1] + 2 \text{Re}\left[2 A e^{-j k(f, \tau)} \cos\left(\pi f_i \tau + \frac{\psi_x}{2}\right) W^*\right] + |W|^2, \]  
(13)

where \( k(f, \tau) \) is a phase shift that we will ignore as it can go in the random phase of the noise. The noise energy \(|W|^2\) can be decomposed into a constant and a zero-mean part \(|W|^2\). Therefore,
\[ E = 2 A^2 [\cos(2\pi f_i \tau + \psi_x) + 1] + 4 A \cos\left(\pi f_i \tau + \frac{\psi_x}{2}\right) \text{Re}(W^*) + |W|^2 + \text{mean}(|W|^2). \]  
(14)

For two targets that are not very close, i.e. \( B \gg 2 \), we can simplify \( \int_{-B/2}^{B/2} \cos(2\pi f f) df = 0 \). Therefore, the mean removed frequency profile \( \tilde{E} \) is
\[ \tilde{E} = E - \text{mean}(E) \approx 2 A^2 \cos(2\pi f_i \tau + \psi_x) + 4 A \cos\left(\pi f_i \tau + \frac{\psi_x}{2}\right) \text{Re}(W) + |W|^2. \]  
(15)

Equation (15) can be written as a sum of signal \( \tilde{E}_x \) and noise \( \tilde{E}_w \) components:
\[ \tilde{E} = \tilde{E}_x + \tilde{E}_w, \]  
(16)

where the signal part is
\[ \tilde{E}_x = 2 A^2 \cos(2\pi f_i \tau + \psi_x) \]  
(17)

and the noise part is
\[ \tilde{E}_w = 4 A \cos(\pi f_i \tau + \frac{\psi_x}{2}) \text{Re}(W) + |W|^2. \]  
(18)

Inverse Fourier transform of the signal part (17) is two shifted scaled impulses (or sincs) signal \( e_x \)
\[ e_x = F^{-1}(\tilde{E}_x) = A^2 \left( \delta(t-\tau) + \delta(t+\tau) \right) \]  
(19)

with peak power
\[ \text{max}|e_x|^2 = (A^2)^2. \]  
(20)

The power of a complex white Gaussian noise \( W \) with variance \( \sigma_w^2 \) has exponential distribution [18] with main statistics
\[ \text{mean}(|W|^2) = \sigma_w^2; \]  
(21)
\[ \text{var}(|W|^2) = \sigma_w^4. \]  
(22)

It is easy to switch to time domain considering that the Fourier transform of sampled in-band (complex) white noise is (complex) white noise and for \( M \) point signal [18]:
\[ \sigma_w^2 = M \sigma_w^2; \]  
(23)
\[ \text{var}[|W|^2] = \text{var}[|W|^2] = \sigma_w^4 = (M \sigma_w^2)^2; \]  
(24)
\[ \text{var}(F^{-1}|W|^2) = \frac{1}{M} \text{var}(|W|^2) = M \sigma_w^4. \]  
(25)

The Fourier transform of cosine modulated white noise is white noise with scaled power:
\[ \mathcal{F}^{-1}\left(A \cos(2 \pi f) \ W(f)\right) = \frac{A}{2} (w(t - \tau) + w(t + \tau)) \]

\[ \text{var}\left(\mathcal{F}^{-1}\left(A \cos(2 \pi f) \ W(f)\right)\right) = \left(\frac{A}{2}\right)^2 2 \sigma_w^2. \]

Therefore the cosine modulated white noise component in (18) has power

\[ \text{var}\left(\mathcal{F}^{-1}\left(4A \cos(\pi f_1 \tau + \psi_1)/2 \ \text{Re} \ W(f)\right)\right) = 4A^2 \sigma_w^2. \]  

The noise power of the BSCT transformed signal follows from (18), (25), and (26), considering that the covariance of a normal random variable with its square is zero:

\[ e_w = \mathcal{F}^{-1}(\hat{E}_w); \quad \text{var}(e_w) = 4A^2 \sigma_w^2 + M (\sigma_w^2)^2. \]

The output of a filter matching \( s \) with \( x_0 \) is:

\[ S_{MF} = S X_0^* = (X + W)X_0^* \]

and it can be shown that the MF SNR ratio is \( A^2/\sigma_w^2 \).

The SNR after BSCT processing is

\[ \text{SNR} = \frac{\max(e_x)}{\text{var}(e_w)} = \frac{A^4}{4A^2 \sigma_w^2 + M (\sigma_w^2)^2} = \frac{A^2/\sigma_w^2}{4 + M \sigma_w^2/A^2} = \frac{\eta}{4 + M/\eta} \]

where \( \eta \) denotes the SNR of the compressed signal and \( M \) is the number of independent samples, i.e. characterises the time-bandwidth product for the signal, \( M = T B_c \) (e.g. see Sec. 2.4 in [19]).

From (29) we can see that the SNR after BSCT is lower than after MF. The relationship is shown on figure 2 with continuous lines for different values of \( M \). For high SNR the reduction is 6 dB or the ratio of the signal energy and the noise power is reduced 4 times. For low SNR the change in dB for BSCT is two times the change in dB of the SNR. The limit curves are

\[ \lim_{\eta \to \infty} \text{SNR}_{dB} = \eta_{dB} - 6; \]

\[ \lim_{\eta \to 0} \text{SNR}_{dB} = 2 \eta_{dB} - 10 \log_{10} M. \]

The limit curves intersect at \( \eta_{dB} = M_{dB} - 6 \) with \( \text{SNR}_{dB} = \eta_{dB} - 9 \), dB. The corresponding noise level could be considered critical for BSCT as the SNR degrades faster at higher values.

Higher values of the time-bandwidth parameter \( M \) lead to worse performance of BSCT. This means that BSCT is applicable only for short sequences which is fine as far as it is used for local targets resolution.

Integration of the spectral energy over a bandwidth \( B \) will reduce proportionally the noise power. At the same time the signal \( e_x \) amplitude is scaled by \( \text{sinc}(\tau B) \), i.e. the close targets are preserved and the distant targets are attenuated. The effect of BSCT model parameters on the SNR could be subject of future research.

4. Verification of results through simulations

The BSCT SNR is evaluated by simulation of a simple scenario – a band limited impulse waveform \( x_c = A \delta(t) \) and two point scatterer target \( x_\tau = \delta(t - \tau) + \delta(t + \tau) \).

The following steps were implemented and run in Python (3.5.2) with Numpy (1.13.3) and Matplotlib (1.5.3):

1) generate noise waveform sequence \( w[n] \) with variance \( \sigma^2 \), \( n = 0, 1, ..., M - 1 \);
2) generate signal \( s[n] \) containing two impulses with amplitude \( A \) at \( n = \pm p \);
3) calculate \( \text{SNR} = A^2/\sigma^2 \);
4) sum the noise and signal, $x[n] = w[n] + s[n]$;
5) apply BSCT transformation, $E = |\text{FFT}(x[n])|^2$, $y[n] = \text{IFFT}(E - \text{mean}(E))$, where (I)FFT is (Inverse) Fast Fourier Transform;
6) the BSCT peak is $A_{\text{BSCT}} = A^2$, (20), at index $2p$, and for high SNR $A_{\text{BSCT}} = y[2p]$;
7) calculate BSCT noise power $\sigma^2_{\text{BSCT}}$ as the variance of the signal $y[n]$ with discarded values at index $\pm 2p$;
8) the SNR for BSCT is $\text{SNR}_{\text{BSCT}} = A_{\text{BSCT}}^2 / \sigma^2_{\text{BSCT}}$.

The results of the simulations are shown in figure 2 as points. Each point represents an SNR average from 1000 simulations. There is a good fit between simulated and calculated results, though some bias is observed for small values of $M$.

![Figure 2. BSCT SNR as a function of signal SNR and the time bandwidth $M$. Continuous lines are calculated using (29). Points are averaged results from simulations with different values of $M$.](image)

5. Conclusion
The bat inspired BSCT receiver provides 6 dB lower SNR compared to the MF when the processing window is small enough. There is a critical noise level above which the SNR of BSCT falls two times faster in dB than the SNR for MF.

The frequency integration/averaging bandwidth of the BSCT receiver can be increased to reduce the SNR loss. A side effect is attenuation of distant spacings from the spacings profile.

Topics for future research are optimisation of the BSCT parameters, exploration of the noise behaviour for target spacing close to the resolution limits, physical experiments with real targets and study of the clutter interference.

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