Gravitational waves versus black holes
Trevor W. Marshall, School of Mathematics, University of Manchester,
Manchester M13 9PL, UK
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Abstract  It is argued that, in order for the gravitational field to be
propagated as a wave, it is necessary for it to satisfy a further set of field
equations, in addition to those of Einstein and Hilbert, and these equations
mean there is a preferred coordinate frame, called the Global Inertial Frame,
giving rise to a unique metric. The implication is that a true gravitational
field is not compatible with Einstein’s Principle of Equivalence, which is in con-
tradiction with his other fundamental concept of locality. The additional field
equations ensure that gravitational collapse does not go below the Schwarzschild
radius, thereby excluding the possibility of singular solutions (black holes) of the
Einstein-Hilbert equations. Such solutions would also violate Einstein’s locality
principle.

1 Introduction
In 1907, in two articles[1][2] separated by a few months, Albert Einstein stated
two principles to which he would subsequently give the names Principle of Local
Action (PLA), or in the original Prinzip der Nahewirkung, and Principle of
Equivalence (PE). The first of these states that nothing goes faster than light,
and it was considered by him to be the principle underlying what we now call
Special Relativity, Einstein’s theory of 1905. He restated it many times during
his later life[3][4][5], especially in his criticism of quantum theory, and in the last
of these citations he stated the view that it would be impossible to do science
without it. As for PE, although he classified it as ”the happiest thought of my
life” (6) p178, leading him, as it did, to the 1915 theory which we now know
as General Relativity (GR), the statement of 1907 was very modest compared
with what it later became; it stated that no observer can distinguish between a
uniformly accelerated frame and a frame at rest in a uniform gravitational field.

Even before the creation of GR, the two principles presented some compat-
ibility problems; both the gravitational red shift and the variation of the
free-space refractive index, predicted by Einstein in 1911 and resulting in the
bending of light beams, made the limiting value of $c$ somewhat ambiguous. But
the search from then to 1915, by Einstein in collaboration with Grossman[6] and
independently by Hilbert[7], for a group of transformations more general than
that of Lorentz under which the laws of field transmission should be invariant,
led eventually to a very strong formulation of PE. From that point on Einstein
was to state repeatedly that all the laws of physics should be independent of the
coordinate system. This strong form of PE is, I shall argue, totally destructive
of PLA.

During the formative years of GR he and Grossman tried to create a theory
of gravitation based on a more restricted group of transformations[6] than is
demanded by PE. An examination of the articles they wrote during this period shows that it was a desire to incorporate PLA, in the form of a tensor representing the local energy density of the gravitational field, that motivated the various twists and turns taken by them. Einstein had decided by 1915 that this was not possible, and in articles by Hilbert[8] and Schrödinger[9] the impossibility of constructing such a tensor was confirmed. Nevertheless, he returned in 1918 to the idea that gravitation, like any other field, must propagate at the velocity $c$. By imposing on the gravitational potential, which in conformity with PE is the same as the metric of the curved space, a certain restriction, he deduced a formula[10] for the *total* radiation emitted even though the *local* flux thereof remained ambiguous. It was not long before Eddington[11] drew attention to the fact that Einstein’s waves could be transformed out of existence by a coordinate change permitted under PE; he observed caustically that the waves "travel at the speed of thought". However, both Hilbert and Eddington ([11] pp40-41) showed themselves ready to modify PE in order to accommodate some notion of locality. The former[7] actually identified the related notion of "causality" as a further requirement for a physical theory of gravitation, and he stated a criterion to meet this requirement, that the signature of the metric $g_{ij}$ must be preserved as (+ − −−) throughout any evolution of a system. I shall show that Hilbert’s causality is a close relation of Einstein’s PLA, and that playing fast and loose with the signature of $g_{ij}$ is precisely what has led a substantial part of our community into the blind alley of black-hole theory. It should be noted that more recently[12] it was shown that coordinate transformations exist which transform away the Einstein energy loss globally as well as locally. A strong PE is not compatible with the existence of gravitational waves.

With respect to black holes Oppenheimer and Snyder[13] showed in 1939 that a fairly simple metric describes what may plausibly be considered a set of dust particles under zero pressure which, in a finite time interval, collapses into a black hole, so called because light cannot escape from it. Objects with nonzero pressure are now widely believed to undergo similar collapse, subject to certain conditions of size and internal temperature, but the dust cloud remains the basis of the black-hole paradigm. It seems likely that Einstein knew of this work before it was published, because almost simultaneously he published an article[14] criticizing the concept, on the grounds that the crossing of the event horizon by the dust particles would violate his locality principle. In this he was also affirming his support for Hilbert’s notion of "causality", referred to above; at the point of crossing the event horizon, the signature of the metric changes from (+ − −−) to (− + −−). In other words PE, a principle which Einstein apparently still supported, predicted a process which violated locality; it seems that locality won the day as far as he was concerned!
2 The relativistic theory of gravitation

GR has only one set of field equations

\[
\mathcal{R}^{ij} - \frac{1}{2}g^{ij}g_{kl}\mathcal{R}^{kl} = 8\pi\kappa T^{ij} , \quad \kappa = \frac{G}{c^2} ,
\]

where \(\mathcal{R}^{ij}\) is the contracted curvature tensor derived from the metric tensor \(g_{ij}\), and \(T^{ij}\) is the local material stress, or energy density tensor. In his article on gravitational waves[10] Einstein introduced the noncovariant condition

\[
\partial_i\Phi^{ij} = 0 , \quad \Phi^{ij} = g^{ij}\sqrt{-g} .
\]

This is often referred to nowadays as a gauge condition, and it was further developed by de Donder[15] and later by Fock[16], who showed that, if the coordinates are cartesian and satisfy (2), then they also satisfy the harmonic condition

\[
\Box x_i = 0 ,
\]

where \(\Box\) is the generalized d’Alembertian operator

\[
\Box = \frac{1}{\sqrt{-g}}\partial_jg^{jk}\sqrt{-g}\partial_k .
\]

Fock showed that many complex calculations in GR are greatly simplified in harmonic coordinates. In particular he used them to calculate the rate at which a binary system, of which the only example for which he then had the relevant data was Sun-Jupiter, radiated away energy through gravitational waves. Although the effect was too small to observe in that system, his calculation formed the basis for such a system involving a neutron star, which made possible the calculation[17] of the radiative energy loss in a binary pulsar.

Although Fock criticized PE, and indeed considered that it was incorrect to label the Einstein-Hilbert theory "General Relativity", he did not consider the harmonic system of coordinates as in any way privileged. That was left to a later school of gravity theorists in the Soviet Union[12] who proposed that, within a given family of metrics classified as equivalent under PE, it is the harmonic one, which correctly describes the physical system. There is still a family of coordinate frames with this metric, but the group of coordinate transformations connecting these frames is the Lorentz group of the Special Theory of Relativity. They called the resulting theoretical structure the Relativistic Theory of Gravitation (RTG). A gravitating system studied in isolation, for which the distant field is Newtonian, is classified as an island system, and I shall call the harmonic frame for such a system, in which its centre of mass is at rest, the global inertial frame (GIF).

RTG differs significantly from General Relativity. In particular gravitational collapse does not go below the Schwarzschild radius, and so there are no black-hole singularities. At the same time the theory includes unambiguous expressions for the material and gravitational stress tensors, and the latter is
associated with a real physical field which propagates at the speed of light, just like the electromagnetic field of Faraday and Maxwell. The quantities $\Phi_{ij}$ are considered to be gravitational fields instead of being the gravitational potentials of GR, and the field equations of RTG consist of (1) together with (2). It is a radical departure to treat this latter equation as an essential, globally valid field equation, which, for a given source, results in a unique field, defined with respect to the GIF.

A simple example of an island frame arises from considering the Schwarzschild metric for a point mass $m$ at the origin

$$ds^2 = \frac{r - 2m}{r} dt^2 - \frac{r}{r - 2m} dr^2 - r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right), \quad (5)$$

for which the d’Alembertian is

$$\Box = \frac{r}{r - 2m} \partial_r^2 - \frac{1}{r^2} \left[ r (r - 2m) \partial_r^2 + 2 (r - m) \partial_r + \partial_\theta^2 + \cot \theta \partial_\theta + \csc^2 \theta \partial_\phi^2 \right]. \quad (6)$$

The coordinate $z = r \cos \theta$ is not harmonic, while $Z = (r - m) \cos \theta$ is, so the metric associated with the GIF is

$$ds^2 = \frac{R - m}{R + m} dt^2 - \frac{R + m}{R - m} dR^2 - (R + m)^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right). \quad (7)$$

Using this metric instead of Schwarzschild’s gives a small correction[12] to the perihelion advance of a planet’s elliptic orbit, to the bending of light, and to the gravitational red shift, but they are all far too small to be measured.

### 3 The gravitational collapse of a dust cloud

Oppenheimer and Snyder[13] studied the metric

$$ds^2 = d\tau^2 - S_R^2 dR^2 - S^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right), \quad (8)$$

where

$$S = \left[ R^{3/2} - \frac{3}{2} \tau f(R) \right]^{2/3} \quad (9)$$

and

$$f(R) = \begin{cases} 1 & (R > 1) \\ \frac{R^{3/2}}{(R < 1)} & \end{cases} \quad (10)$$

Note that

$$S_R = \begin{cases} \sqrt{R/S} & (R > 1) \\ S/R & (R < 1) \end{cases} \quad (11)$$

which means that this metric has a discontinuity at $R = 1$. They showed that a frame defined by it is comoving in the sense that its material stress tensor,

$$T^{ij} = \frac{1}{8\pi} \left( R^{ij} - \frac{1}{2} g^{ij} g_{kl} R^{kl} \right) \quad (12)$$
has only a timelike nonzero component, that is
\[ T^{\alpha \beta} = T^{00} = 0, \quad T^{00} = \frac{ff'}{2\pi S^2 S_R}, \] (13)
for which the mass density is
\[ \rho dR d\theta d\phi = T^{00} \sqrt{-g} dR d\theta d\phi = \frac{ff'}{2\pi} \sin \theta dR d\theta d\phi. \] (14)

They used this metric to model the gravitational field of a spherically symmetric dust cloud with no rotation; the function \( f(R) \) contains the \( R \)-dependence of \( \rho \), and in the exterior region, \( R > 1 \), it is constant, making \( \rho \) zero. The comoving property of this frame means that the timelike coordinate \( \tau \) is a local proper time. It is a simple matter to establish that each particle in the cloud reaches the "event horizon", a point at which the \((+--\) signature of space-time changes, within a finite interval of \( \tau \). This was taken in [13] to show that the cloud collapses into a black hole, but we shall show, using the harmonic coordinates, that this was an incorrect conclusion.

For the exterior region, \( R > 1 \), the latter authors looked for a coordinate transformation \((\tau, R) \rightarrow (t, r)\) such that the above metric transforms into the Schwarzschild metric with \( 2m = 1 \). This leads to the result
\[ t = \frac{2}{3} \left( R^{3/2} - S^{3/2} \right) + 2 - 2\sqrt{S} + 2 \log \frac{\sqrt{S} + 1}{2} - \log (S - 1), \quad S = r. \] (15)

They then obtained an interior metric by imposing the conditions \( g^{tr} = 0 \) and \( S = r \) there also. They required in addition that \( t \), but not \( t_R \), be continuous at \( R = 1 \), and they deduced that
\[ t = -\log (y - 1) + 2 \log \frac{\sqrt{y} + 1}{2} + \frac{8}{3} y^{3/2} - 2\sqrt{y}, \] (16)
where
\[ y = \frac{R^2 - 1}{2} + \frac{r}{R}. \] (17)

The coordinates \((t, r)\) they refer to as "external coordinates" for both the exterior and the interior regions; this is by contrast with the comoving coordinates \((\tau, R)\). In the external frame \( t \) becomes infinite for the exterior at \( r = r_\infty = 1 \) and for the interior at
\[ r_\infty = \frac{3}{2} R - \frac{1}{2} R^3 \quad (R < 1), \] (18)
which is the event horizon.

All the particles of the cloud go to their final positions, given by \( r_\infty (R) \) and stay there; this is a stationary state and it is surprising that Oppenheimer and Sneider did not acknowledge it as such. What happens on the wrong side of the event horizon, that is for proper times satisfying
\[ \tau_\infty > \frac{2}{3} - \frac{2}{3} \left( \frac{3 - R^2}{2} \right)^{3/2}, \] (19)
has no physical relevance at all. What seems to have caused their confusion was the discovery that the metric coefficients become singular near \( r = 1 \). There is a simple explanation for this, since the Schwarzschild metric in the exterior region has the same behaviour. Furthermore, the density \( \rho (r_\infty) \) becomes infinite at \( r_\infty = 1 \), as may be shown from the constant cumulative density in the comoving frame, namely

\[
P (R) = \int_0^R 4\pi \rho (R) \, dR = R^3.
\]

The corresponding quantity in the external frame is

\[
P (r_\infty) = [R (r_\infty)]^3 = 8 \cos^3 \left( \frac{\pi}{3} + \arccos r_\infty \right),
\]

and near \( r_\infty = 1 \) this gives an infinite density, namely

\[
4\pi \rho_\infty (r) \sim \sqrt{\frac{3}{2 - 2r}}.
\]

The infinite value of \( g_{rr} \) found by Oppenheimer and Snyder in the limit \( r \to 1 \) is simply a consequence of this mass concentration at the surface of the cloud. By contrast \( g_{tt} \) is zero at all points on the event horizon, which indicates an infinite red shift in the limit \( t \to \infty \).

Their exterior solution (15) is not unique, owing to the arbitrary function in their eqn.19 having been equated to one. Of course the absence of uniqueness is inevitable in General Relativity, it being a manifestation of PE, which ordains that no coordinate system is to be favoured. In RTG, by contrast, the external solution is unique, because it has to satisfy the harmonic condition; in that case the external solution is identical with (15), except (see the previous section) that \( S \) is equal to \( r + 1/2 \) instead of \( r \). As for the internal solution, the condition \( g^{fr} = 0 \) of Oppenheimer and Snyder somewhat fortuitously ensures that \( t_R/r_R \) is continuous at \( R = 1 \) if \( r \) and \( t \) are, and this then makes \( g_{tt} \) and \( g_{rr} \) continuous there, even though \( g_{RR} \) is not. The condition \( g^{fr} = 0 \) is arbitrary though allowable under PE, but it is not satisfied by the RTG solution. That is constructed by imposing the harmonic condition, and by requiring the continuity of both \( r_R \) and \( t_R \) at \( R = 1 \), thereby setting a constraint on the function \( f(R) \) in the internal region; this ensures by a different route the continuity of the metric.

In accordance with the latter condition, I choose for the interior density function

\[
f (R) = R^{3/2} e^{3X/2}, \quad X = 1 - R \quad (R < 1)
\]

giving

\[
S_R = S\xi, \quad \xi = \frac{X}{R} + \sqrt{\frac{R^3}{S^3}} \quad (R < 1)
\]

and the mass density

\[
\rho dR d\theta d\phi = \frac{3XR^2 e^{3X}}{8\pi} \sin \theta dR d\theta d\phi.
\]
The d’Alembertian operator in terms of the coordinates \((R, S, \theta, \phi)\) is
\[
\Box = \Box_1 - \frac{1}{S^2} \left( \partial_\theta^2 + \cot \theta \partial_\theta + \csc^2 \theta \partial_\phi^2 \right),
\]
where
\[
\Box_1 = \left( \frac{f^2}{S} - 1 \right) \left( \partial_S^2 + \frac{1}{S} \partial_S \right) - \frac{2}{\xi S} \partial_R \partial_S - \frac{1}{\xi^2 S^2} \partial_R^2 + \left[ \frac{f f'}{\xi S^2} - \frac{1}{S} \right] \partial_S
\]
\[
- \frac{1}{\xi^2 S^2} \left[ \frac{5 \xi}{2} - \frac{f'}{f} - \frac{\xi R}{\xi} \right] \partial_R.
\]

The harmonic coordinates
\[
x_i = (t, r \sin \theta \cos \phi, r \sin \theta \sin \phi, r \cos \theta)
\]
satisfy
\[
\Box_1 t = 0,
\]
and
\[
\left( \Box_1 + \frac{2}{S^2} \right) r = 0,
\]
for which the exterior solutions, obtained by putting \(f = 1\) and \(\xi = \sqrt{R/S^3}\), are the same as (15) for \(t\), with \(r\) replaced by
\[
r = S - \frac{1}{2}.
\]

This is, as expected, the harmonic form of the Schwarzschild solution, as obtained already in the previous section. The interior coordinates are obtained by solving these same equations with \(r, t, R\) and \(t_R\) continuous at \(R = 1\), and \(t\) and \(r\) having the Correspondence Principle (CP) behaviour \(t \sim \tau, r \sim S\) as \(S\) tends to \(+\infty\). The latter conditions give the leading term for large positive \(S\), namely
\[
t \sim -\frac{2}{3} S^{3/2} e^{3Z/2} \sim -\frac{2}{3} S^{3/2} e^{3Z/2} \quad e^{-Z} = e^{X R},
\]
or equivalently
\[
r \sim r_0 e^{-Z}, \quad r_0 = \left( -\frac{3t}{2} \right)^{2/3},
\]
leading to a density
\[
\rho (r) \sin \theta dr d\theta d\phi \sim \frac{3r^2}{4\pi r_0^3} \sin \theta dr d\theta d\phi \quad (r < r_0).
\]
This gives us the collapse of a sphere of radius \(r_0 (t)\) with initially uniform mass density, and we see that the uniform density is preserved during the first stage.
of collapse, in agreement with the Newtonian behaviour for such a density. The leading relativistic correction is obtained from the next term, namely

$$t \sim -\frac{2}{3} S^{3/2} e^{3Z/2} - S^{1/2} \left(\frac{3}{2} \frac{Z}{2} + \frac{1}{2} e^{-3Z/2}\right), \quad r \sim S - \frac{3}{4} e^{-Z} + \frac{1}{4} e^{-3Z}$$

(34)

from which $S$ may be eliminated to give

$$r \sim e^{-Z} \left( r_0 + \frac{1}{4} - \frac{1}{4} e^{-2Z} \right), \quad r_0 = \left( -\frac{3t}{2} \right)^{2/3} - \frac{5}{2},$$

(35)

with the density

$$\rho (r) \sin \theta d\theta d\phi \sim \frac{3r^2}{4\pi r_0^2} \left( 1 - \frac{3}{4} r_0 + \frac{5r^2}{4r_0^2} \right) \sin \theta d\theta d\phi \quad (r < r_0).$$

(36)

This latter result shows a departure from the uniform density associated with Newtonian gravity; as the collapse progresses the density near the surface ($r = r_0$) increases faster than at the centre of the cloud.

To follow the evolution into the region of strong gravity requires a numerical approach, for which we make the further change of variables from $R$ to $X$ and $S$ to $Y = \log S + Z$, leading to

$$-S^2 \xi^2 \square_1 = \partial_X^2 - 2e^{-3X/2-3Y/2} \partial_X \partial_Y + (e^{-3X-3Y} - \xi^2 R^2 e^{2X-Y}) \partial_Y^2$$

$$+ \left( \xi^2 + \frac{1}{R^2} - \frac{3\xi X R}{2} e^{2X-Y} \right) \partial_Y$$

$$- \left[ \frac{5\xi}{2} + \frac{3}{2} \frac{3}{R} + \frac{2 + 3X}{2\xi R^2} \right] \left( \partial_X + \frac{X}{R} \partial_Y \right).$$

(37)

From this it is possible to derive more terms in the asymptotic series for $r$ and $t$, namely

$$t \sim \frac{2}{3} e^{3Y/2} - \left( \frac{3}{2} + \frac{1}{2} e^{-2Z} \right) e^{Y/2} + \sum_{n=0}^{\infty} t_n e^{-nY/2},$$

(38)

and

$$r \sim e^{-Z} \left( e^Y + \sum_{n=0}^{\infty} u_n e^{-nY/2} \right),$$

(39)

where the first few nonzero terms are

$$t_0 = \frac{2}{3} e^{-3X/2},$$

$$t_1 = \frac{2}{5} e^Z + \frac{15}{8} - \frac{1}{4} e^{-2Z} - \frac{1}{40} e^{-4Z},$$

$$t_2 = \frac{1}{2} e^{-3X/2} + \frac{3}{2} e^{X/2} (13 - 6X + X^2) - 20,$$

$$t_3 = \frac{43}{70} e^Z + \frac{29}{96} - \frac{1}{10} e^{-Z} - \frac{5}{32} e^{-2Z} + \frac{1}{160} e^{-4Z} + \frac{1}{3360} e^{-6Z},$$

(40)
\[ u_0 = -\frac{3}{4} + \frac{1}{4} e^{-2Z} \]
\[ u_3 = 20 - \frac{1}{2} e^{-3X/2} - \frac{3}{2} e^{X/2} (13 - 6X + X^2) = -t_z \]
\[ u_6 = \frac{5}{6} \left( \frac{15}{4} (X^2 - 4X + 9) e^{-X} - \frac{5}{12} e^{-3X} + \frac{100}{3} e^{-3X/2} \right). \] (41)

The partial differential equations (28) and (29) must now be integrated subject to the boundary conditions at \( X = 0 \), for which the first equation displays a logarithmic singularity at \( S = 1 \), that is \( Y = 0 \). This suggests there is a similar singularity along the characteristic \( Y = Y_0(X) \) through \( (0,0) \). This is in fact the event horizon in the \((X,Y)\) coordinates, and it is determined by the ordinary differential equation
\[ Y_0' = R\xi e^X - \frac{Y_0}{2} - e^{-3X/2} - 3Y_0/2 \]
\[ = X e^X - \frac{Y_0}{2} + (1 - X) e^{-X/2} - 2Y_0 - e^{-3X/2} - 3Y_0/2, \] (42)
which integrates numerically to give the function

| \( X \) | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \( Y_0 \) | .005 | .021 | .049 | .091 | .150 | .227 | .326 | .447 | .591 | .758 |

It may be noted that this is finite at \( X = 1 \), even though some coefficients of the partial differential equations are infinite there. This is a consequence of the choice made for the variable \( Y \). A further advantage of this choice is that, as \( Y \to +\infty \), the characteristics are \( Y = \text{constant} \), which aids the numerical integration. The latter is achieved by replacing each partial differential equation by a set of ordinary differential equations for the vectors \( r(X) = r(X,Y_i) \) and \( t(X) = t(X,Y_i) \), where \( Y_i \) are a set of 100 values of \( Y \) spaced equally between an upper value \( Y_0 \) at which the above asymptotic expansions apply (I took \( Y_0 = 5 \)) and a lower value \( Y_1 \) close to zero (I took \( Y_1 = .001 \)). The integration confirms that \( t \) becomes infinite as \( Y \) approaches \( Y_0(X) \), which means that we are seeing an extreme form of the gravitational red shift. The values of \( r(X) \) along this same curve, which we designate \( r_\infty(X) \), are

| \( X \) | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|-------|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \( r_\infty \) | .500 | .500 | .500 | .499 | .497 | .492 | .481 | .456 | .398 | .272 | 0 |

and this shows that gravitational collapse ceases at the radius \( r_\infty \), which takes the value 0.5 in our units. The latter function, or rather its inverse \( X(r_\infty) \), gives us the limiting density \( \rho_\infty(r) \), since the density in terms of the comoving coordinate \( X \) remains constant throughout, so that the cumulative density is
\[ P_\infty(r) = \int_0^r 4\pi \rho_\infty(r') dr' = \left[ 1 - X(r_\infty) \right]^3 e^{3X(r_\infty)}. \] (43)

A corresponding uniform density would give
\[ P_U(r) = \frac{r^3}{\rho_\infty^3} = 8r^3, \] (44)
and a comparison may be made by plotting $P_\infty/P_U$ as a function of $r$. This shows that the process indicated by (36) above, namely a concentration of particles at the surface, continues and intensifies. For example, we find that $P_U$ would give less than 2% of the particle distribution in the range $0.497 < r < 0.5$, as compared with $P_\infty$ giving more than 28%, while, in $0.481 < r < 0.5$, $P_U$ gives less than 11%, compared with $P_\infty$ giving more than 61%. As $r_\infty$ tends to 0.5 the derivative of $P_\infty$, that is $\rho_\infty$ becomes infinite; this is natural, because the metric coefficient $g_{rr}$ must become infinite at $r = 0.5$ to match with its value in the exterior region $R > 1$. Also, associated with the infinite red shift at the event horizon, the determinant $g$ of the metric approaches zero there.

4 Conclusion

Our result shows that, contrary to the general result claimed by [13], there is, for a suitable choice of the density function $f(R)$, and of course a suitable coordinate frame, a stationary solution of the Einstein-Hilbert equation, and this state is approached in the limit $t \to +\infty$. It should be noted that, both in the present article and in [13], an additional condition had to be imposed in order to obtain an interior metric satisfying the Correspondence Principle, and that in both cases there is a singularity in the metric in the limit $t \to +\infty$.

More significant, however, is that the insistence on a global inertial frame as the vehicle for gravitational waves restores the gravitational theory of Einstein to the framework of his Special Theory. In particular the waves travel with the velocity of light and are consistent with his principle of locality. It is entirely natural then that RTG should also rule out gravitational collapse beyond the event horizon, leading, as it would, to an exotic topology in which the space and time coordinates would change places and to a complete demolition of causality.

The key to understanding what happens as a particle approaches the event horizon is that such an intense gravitational field produces a red shift which becomes infinite, that is all physical processes including gravitational collapse are infinitely slowed down. Effectively time is frozen. There is also an infinite mass density at the surface of the cloud. This is an extreme situation which none of us is likely to experience directly, but it ensures that, even in such circumstances, the world remains local and comprehensible.

References

[1] A. Einstein, *Ann. der Physik*, 23, 371-384 (1907)
[2] A. Einstein, *Jahrb. der Radioaktiv. und Elektronik*, 4, 411-462 (1907)
[3] A. Einstein, B. Podolsky and N. Rosen, *Phys. Rev.*, 47, 777 (1935)
[4] A. Einstein, *Dialectica*, 2, 320 (1948). English translation in *The Born Einstein Letters*, Macmillan, London (1971)
[5] A. Einstein, in *Albert Einstein: Philosopher-Scientist*, ed. P. A. Schilpp, Tudor New York (1949)

[6] A. Pais, *Subtle is the Lord... the Science and Life of Albert Einstein*, University Press, Oxford (1982)

[7] J. Mehra, *Einstein, Hilbert and the Theory of Gravitation*, D. Reidel, Boston (1974)

[8] D. Hilbert, *Goett. Nachrichten* 4, 21 (1917)

[9] E. Schrödinger, *Phys. Z.*, 19, 4 (1918)

[10] A. Einstein, *Sitzungsber. preuss. Akad. Wiss.*, 1, 154 (1918)

[11] A. S. Eddington, *The Mathematical Theory of Relativity*, University Press, Cambridge (1924)

[12] A. Logunov and M. Mestvirishvili, *The Relativistic Theory of Gravitation*, Mir, Moscow (1989)

[13] J. R. Oppenheimer and H. Snyder, *Phys. Rev.* 56, 455 (1939)

[14] A. Einstein *Ann. Math.* 40, 922 (1939)

[15] T. de Donder, *Théorie des Champs Gravitiques*, Gauthiers-Villars, Paris (1926)

[16] V. A. Fock, *The Theory of Space, Time and Gravitation*, Pergamon, New York (1959)

[17] M. Walker and C. M. Will, *Phys. Rev. Lett.*, 22, 1741 (1980)