Spin-induced black hole spontaneous scalarization

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(Dated: November 18, 2020)

We study scalar fields in a black hole background and show that, when the scalar is suitably coupled to curvature, rapid rotation can induce a tachyonic instability. This instability, which is the hallmark of spontaneous scalarization in the linearized regime, is expected to be quenched by nonlinearities and endow the black hole with scalar hair. Hence, our results demonstrate the existence of a broad class of theories that share the same stationary black hole solutions with general relativity at low spins, but which exhibit black hole hair at sufficiently high spins ($a/M \gtrsim 0.5$). This result has clear implications for tests of general relativity and the nature of black holes with gravitational and electromagnetic observations.

Introduction: Direct and indirect detections leave little doubt that black holes (BH) exist in nature [1–8]. In general relativity (GR) the mass and the spin of an astrophysical BH fully determine its properties. An electric charge is also technically allowed, but is expected to be paltry for astrophysical BHs, see e.g. [9]. Any other quantity, hair in jargon, is not necessary according to no-hair theorems [10–12]. Future gravitational wave detectors will finally allow us to confront theorems and observations with unprecedented precision [13–15], improving upon current observations, which are perfectly compatible with hairless BHs [16, 17].

It is tempting to interpret an absence of BH hair as a vindication of GR minimally coupled to the Standard Model. However, new fundamental fields can be more elusive. It is illustrative to consider scalar fields: no-hair theorems exist for stationary BHs in scalar-tensor theories [18, 19], and static, spherically symmetric and slowly rotating BHs in shift-symmetric generalized (Horndeski) scalar-tensor theories [20, 21]. In fact, it turns out that there is a single coupling term in the Horndeski class that gives rise to hair: a linear coupling between the scalar and the Gauss-Bonnet (GB) invariant [21, 26], given by

$$G = R^\mu\nu\rho\sigma R_{\mu\nu\rho\sigma} - 4 R^\mu\nu R_{\mu\nu} + R^2. \quad (1)$$

Considering that the Horndeski class contains all actions for a massless scalar nonminimally coupled to gravity that yield second order equations upon variation, absence of hair actually seems to be the norm rather than the exception for scalar fields. Indeed, known hairy BH solutions circumvent theorems by evading one or more of their assumptions, see e.g. [21, 27–32].

A further complication in attempting to detect new fields through BH hair is the possibility that, even within the context of the same theory, only certain BHs might actually exhibit it. This was realized only recently, as the first models of BH scalarization appeared in the literature [33, 34]. For concreteness, consider the action

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left( R - \frac{1}{2} \nabla\mu \nabla^\mu \phi + f(\phi) G \right), \quad (2)$$

where $f$ is some function of $\phi$, and where we have also set (as in the rest of this paper) $8\pi G = c = 1$. Varying the action with respect to $\phi$ yields

$$\Box \phi = -f'(\phi) G, \quad (3)$$

where $f'(\phi) \equiv df/d\phi$. Assume that $f'(\phi_0) = 0$, for some constant $\phi_0$. Then solutions with $\phi = \phi_0$ are admissible and they are also solutions of GR. A no-hair theorem [33] ensures that they are unique if they are stationary, provided that $f''(\phi_0) G < 0$.

The fact that GR BHs are stationary solutions to this theory is not sufficient to conclude that there are no observable deviations from GR, as the perturbations over these solutions do not generally obey the GR field equations [35]. These perturbations may even grow unstable, thus rendering the GR solutions irrelevant. Indeed, one can think of $-f'' G$ as the (square of the) mass of the scalar perturbation on a fixed background. Hence, the condition above ensures that this effective (squared) mass is positive. If the condition is violated and the effective (squared) mass becomes sufficiently negative, the GR solutions suffer a tachyonic instability and the scalar develops a nontrivial profile.

A similar scalarization effect was shown to occur for neutron stars in a different class of scalar-tensor theories more that 25 years ago [30], and is triggered when the star compactness reaches a critical threshold. Related “dynamical” scalarization effects [37–40] are present in the

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1 No-hair theorems also exist for stars in shift-symmetric scalar tensor theories [22–25].
Black holes can scalarize if they have matter in their vicinity [28, 29], but the densities necessary to obtain a measurable effect are probably astrophysically unrealistic.

Note that in curved spacetimes $\mu_{\text{inv}}^2$ can be somewhat negative without necessarily developing a tachyonic instability.

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4 There is no advantage in using spheroidal harmonics, for which analytic expressions are unavailable, as they do not lead to a separable equation anyway.
where \( \langle j_1, j_2, m_1, m_2 | j_3, m_3 \rangle \) are the Clebsch-Gordan coefficients [53]. Note that the evolution of modes of different \( m \) decouples because of the axisymmetry of the problem. Moreover, because of reflection symmetry with respect to the origin, even-\( l \) and odd-\( l \) modes also decouple: the evolution of a mode \((l, m)\) is coupled to that of all the modes \((l + 2k, m)\), with \( k = 1, 2, 3, \ldots \).

To numerically evolve the system (7), we discretize the spatial grid and use a method of lines. By integrating in time using a fourth order explicit Runge-Kutta time-step inside the computational grid (as done e.g. in [51]), it becomes apparent that the equations are stiff for large \( \eta \), and that the numerical integration becomes unstable. To overcome this problem, we have used an Implicit-Explicit (IMEX) Runge-Kutta solver with adaptive time step, namely the IMEX-SSP3(3,3,2) and IMEX-SSP(4,3,3) schemes of [54]. Note that implicit methods [55], while effective at dealing with stiff problems, are typically less accurate and more computationally expensive. However, implicit-explicit algorithms, by employing explicit steps for the non-stiff terms and implicit steps only for the stiff ones, can tackle stiff problems with limited computational overhead. We successfully compared our code to results from both frequency-domain techniques [56] and similar time-domain codes [52]. Our implementation was also tested by analysing the convergence of the results (and their overall robustness) vs time-step and spatial-grid resolution.

**Results:** To investigate the possible presence of an instability, we evolve the scalar field by integrating the system given by Eq. (7), with \( l \) ranging from 0 to \( l_{\max} = 30 \) and \( |m| \leq l \), and with Gaussian initial conditions for each mode \( \psi_{lm} \). The results are robust against the choice of the cutoff \( l_{\max} \), as long as that is sufficiently large — and initial conditions, which only affect the early transient evolution of the scalar and not the unstable growth phase, if present. We consider BH spins \( a/M \sim 0.5 – 0.999 \) and qSGB coupling \( |\eta|/M^2 \sim 0.1 – 10^5 \).

From the simulations showing an exponential scalar growth, we extract the instability timescale \( \tau \) of the reconstructed field \( |\phi| = \left( \sum_{lm} |\psi_{lm}|^2 \right)^{1/2} \propto \exp(t/\tau) \) by fitting the time evolution of the scalar’s amplitude after the initial transient. The contours in Fig. 1 show \( \tau^{-1} \) as a function of \( a/M \) and \( |\eta|/M^2 \). The instability becomes stronger as either the spin or the coupling increases. Moreover, there is a minimum spin \( a_{\min} \) below which the instability disappears. For \( |\eta| \to \infty \), it appears that \( a_{\min}/M \to 0.5 \) (up to percent level numerical errors). The solid green line denotes the combinations of parameters for which the instability disappears (i.e. \( \tau \to \infty \)). With the blue dotted line we show the same marginal instability curve for the reconstructed field, but excluding the \( m = 0 \) modes. As can be seen, when the latter are excluded the parameter space region yielding an instability shrinks, i.e. the main contribution to the instability comes from the \( m = 0 \) modes. As a further test of this conclusion, we also computed the marginal instability curve for the \( m = 0 \) modes alone, and that does indeed match the solid green line in Fig. 1.

Even and odd parity modes (i.e. modes with even and odd \( l \)) automatically decouple in Eq. (7). In the \( m = 0 \) sector, which dominates the instability shown in Fig. 1, the odd and even modes give roughly comparable contributions. We have verified this by considering the marginal instability curves for the odd and even \( m = 0 \) modes separately, which are both very close to the solid green line of Fig. 1. As an example, the red dashed line in Fig. 1 represents the marginal instability curve for the \( m = 0 \) odd modes.

Indeed, odd modes seem to have only marginally shorter instability times (by \( \sim 1 - 2\% \)) than even ones for high spins and large couplings. Conversely, in the region \( |\eta| < 1 \), \( a/M > 0.9 \) the even modes are slightly more unstable, as can be seen from the somewhat increased distance between the red dashed and solid green line curves.

Next we consider if some individual angular mode \( l, m \) gives the dominant contribution to the instability. To answer this question, we have to override the non-separability of the problem. To this end, we have force-
fully decoupled each $l$-mode in Eq. (7), suppressing “by hand” all the couplings between angular modes (i.e. $\langle lm|\mu_{am}^2 (r^2 + \chi^2)|jm\rangle$ with $l \neq j$) generated by the GB invariant; we have only kept active the contributions to the effective mass of the single $l$-mode. We have then let the system evolve, selecting Gaussian initial data for the chosen mode only. By this technique, we have isolated, for instance, the instability parameter space for the spherical mode $l = m = 0$, whose marginal instability curve is shown in Fig. 1 by a cyan dot-dashed line. However, we could not find any single $l, m$ mode for which the marginal instability curve obtained in this way matched, even roughly, the solid green line for the whole reconstructed field. We therefore conclude that the gravitational coupling between angular modes plays a fundamental role in the onset of the observed instability.

We now proceed to examine whether the instability is dominantly tachyonic or powered by superradiance. The growth times, as shown in Fig. 1, can be as small as $\sim 0.01M$. This seems to favor a tachyonic origin, as superradiance acts on longer timescales (see e.g. [50, 51]). Moreover, the fact that the instability is mostly due to the $m = 0$ modes, and that even the spherical mode $l = m = 0$ can be unstable (see cyan long-dashed critical line in Fig. 1) bodes ill for superradiance, as these modes can never satisfy the superradiance condition $\omega < m\Omega$ (with $\omega$ and $\Omega$ respectively the wave and horizon angular frequencies).

One may naively expect the spherical mode $l = m = 0$ not to suffer from a tachyonic instability either, since $\mu_{am}^2 = -\eta G$ is positive everywhere in a Schwarzschild spacetime when $\eta < 0$ (as considered here). However, the (squared) effective mass for the $l = m = 0$ mode is actually $-\eta \langle 0|G_{Ker}\rangle |0\rangle$, which only matches the naive estimate $-\eta G_{\text{Schwarzschild}}$ at leading order in spin, correcting it by terms $\mathcal{O}(a^2)$. This explains, in particular, why the spherical mode is stable at low spins.

To further confirm the tachyonic nature of the instabilities, we have conducted the following test. We re-run our simulations with the (squared) effective mass replaced by its absolute value, $\mu_{am}^2 \rightarrow |\mu_{am}^2|$. This is enough to suppress the instabilities, and further shows that the latter were due to the change of sign of the GB invariant close to the horizon. One can also look at the scalar fluxes through the event horizon after the initial transient. In Fig. 2, we compare the scalar field’s energy flux through the horizon for $\eta = -10M^2$ (blue) vs the same fluxes for minimally coupled scalar fields with imaginary (orange) and real (magenta) constant masses. Clearly, the flux for a scalar coupled to the GB invariant resembles more closely the tachyonic (i.e. imaginary mass) scalar field evolution, both in timescale and sign. Note that the constant, real mass case, whose evolution is unstable due to superradiance, shows a slower growth and negative energy fluxes. The latter are indeed the hallmark of a superradiant instability, which removes rotational energy and angular momentum from the BH.

The most plausible explanation for why Kerr BHs in $qSGB$ do not suffer from superradiant instabilities seems to be the rapid falloff of the GB invariant (thus of the effective mass) at large distances, $G(r \rightarrow \infty) \sim 1/r^6$. Scalar perturbations with a position-dependent mass were studied in [51], which showed that a steep decay of the mass with distance quenches the superradiant instability. This happens because the effective potential for scalar perturbations does not develop wells, and thus quasi-bound states, unless the mass remains relatively constant till at least $r \sim 2 - 3M$ [51].

**Conclusions:** We have shown that a coupling, with a suitable sign, between a scalar and the GB invariant can lead to an instability triggered by rapid rotation. We have also demonstrated that this instability is not related to superradiance, but is instead tachyonic in nature. Nonlinear effects, which our approach does not capture, are expected to quench that instability and lead to a BH with scalar hair. The process is analogous to the more conventional spontaneous scalarization, but the threshold is controlled by the black hole rotation instead of its curvature.

The action that we use is sufficient for studying the onset of the instability that we have found for BHs. However, the endpoint of this instability, and hence the amount of hair a BH would carry, will strongly depend on nonlinear (self)interactions. There is no obvious reason to believe that this instability is restricted to BHs,

5 Stationary scalarized black hole solutions that constitute the end-point of the instability will be presented elsewhere [57, 58].
and it could well affect rapidly rotating stars as well. Hence, our results demonstrate that there is a broad class of theories where rotation might control deviations from GR. Our findings also have clear implications for searches of new physics in the strong-field regime. Black hole scalar hair induces vacuum dipole gravitational emission, which is potentially observable in the low frequency inspiral of binary system by gravitational wave interferometers [14, 15], deviations from GR in the spectrum of the gravitational wave ringdown [13] or in the electromagnetic spectrum of accretion disks [59], and it may also impact the black hole shadow observed by the Event Horizon Telescope [8].

We stress that we are not aware of any observational upper bounds on $\eta$, which we therefore allow here to reach very high values, for illustrative purposes and in order to excite higher modes. Note that slowly rotating black holes in qSGB would be identical to their GR counterpart. Compact stars can scalarize for $\eta < 0$ [33] and hence yield constraints. However, this effect could easily be quenched by adding a coupling between the scalar field and the Ricci scalar [41, 60]. The latter might be easily be quenched by adding a coupling between the scalar field and the Ricci scalar [41, 60]. The latter might be necessary to get a sensible cosmology [61], and would have no effect for black holes, thus leaving our analysis unaffected.

Acknowledgments. We thank Carlos Palenzuela and Miguel Bezares for insightful advice on technical aspects of the IMEX schemes and their validation, and Hector O. Silva for useful discussions on black hole scalarization. A.D., E.B. and N.F. acknowledge financial support provided under the European Union’s H2020 ERC Consolidator Grant “GRavity from Astrophysical to Microscopic Scales” grant agreement no. GRAMS-815673. T.P.S. acknowledges partial support from the STFC Consolidated Grant No. ST/P000703/1. We also acknowledge networking support by the COST Action GWverse Grant No. CA16104.

[1] B. L. Webster and P. Murdin, Nature 235, 37 (1972).
[2] M. J. Reid, A. C. S. Readhead, R. C. Vermeulen, and R. N. Treuhaft, Astrophys. J. 524, 816 (1999), arXiv:astro-ph/9905075 [astro-ph].
[3] R. Schoel et al., Nature 419, 694 (2002), arXiv:astro-ph/0210426 [astro-ph].
[4] M. J. Reid, K. M. Menten, R. Genzel, T. Ott, R. Schoedel, and A. Eckart, The Astrophysical Journal 587, 208 (2003).
[5] S. Gillessen, F. Eisenhauer, S. Trippe, T. Alexander, R. Genzel, F. Martins, and T. Ott, Astrophys. J. 692, 1075 (2009), arXiv:0810.4674 [astro-ph].
[6] B. P. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. Lett. 116, 061102 (2016), arXiv:1602.03837 [gr-qc].
[7] R. Abuter et al. (Gravity Collaboration), Astron. & Astrophys. 618, L10 (2018), arXiv:1810.12641 [astro-ph.GA].
[8] K. Akiyama et al. (Event Horizon Telescope), Astrophys. J. 875, L1 (2019), arXiv:1906.11238 [astro-ph.GA].
[9] E. Barausse, V. Cardoso, and P. Pani, Phys. Rev. D89, 104059 (2014), arXiv:1404.7149 [gr-qc].
[10] W. Israel, Phys. Rev. Lett. 16, 1776 (1966).
[11] B. Carter, Phys. Rev. Lett. 26, 331 (1971).
[12] D. C. Robinson, Phys. Rev. Lett. 34, 905 (1975).
[13] E. Berti, A. Sesana, E. Barausse, V. Cardoso, and K. Belczynski, Phys. Rev. Lett. 111, 101102 (2016), arXiv:1605.09286 [gr-qc].
[14] E. Barausse, N. Yunes, and K. Chamberlain, Phys. Rev. Lett. 116, 241104 (2016), arXiv:1603.04075 [gr-qc].
[15] A. Toubiana, S. Marsat, S. Babak, E. Barausse, and J. Baker, Phys. Rev. D 101, 104038 (2020), arXiv:2004.03626 [gr-qc].
[16] B. P. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. D100, 104036 (2019), arXiv:1903.04467 [gr-qc].
[17] M. Isi, M. Giesler, W. M. Farr, M. A. Scheel, and S. A. Teukolsky, Phys. Rev. Lett. 123, 111302 (2019), arXiv:1905.00859 [gr-qc].
[18] S. W. Hawking, Commun. Math. Phys. 25, 167 (1972).
[19] T. P. Sotiriou and V. Faraoni, Phys. Rev. Lett. 108, 081103 (2012), arXiv:1109.6324 [gr-qc].
[20] L. Hui and A. Nicolis, Phys. Rev. Lett. 110, 241104 (2013), arXiv:1202.1296 [hep-th].
[21] T. P. Sotiriou and S.-Y. Zhou, Phys. Rev. Lett. 112, 251102 (2014), arXiv:1312.3622 [gr-qc].
[22] E. Barausse and K. Yagi, Phys. Rev. Lett. 115, 211105 (2015), arXiv:1509.04539 [gr-qc].
[23] E. Barausse. Proceedings, 3rd International Symposium on Quest for the Origin of Particles and the Universe (KMI2017): Nagoya, Japan, January 5–7, 2017, PoS KMI2017, 029 (2017), arXiv:1703.05699 [gr-qc].
[24] A. Lekebel, E. Babichev, and C. Charmousis, JCAP 07, 037 (2017), arXiv:1706.04989 [gr-qc].
[25] K. Yagi, L. C. Stein, and N. Yunes, Phys. Rev. D 93, 024010 (2016), arXiv:1510.02152 [gr-qc].
[26] T. P. Sotiriou and S.-Y. Zhou, Phys. Rev. D90, 124063 (2014), arXiv:1408.1698 [gr-qc].
[27] E. Babichev and C. Charmousis, JHEP 08, 106 (2014), arXiv:1312.3204 [gr-qc].
[28] V. Cardoso, I. P. Carucci, P. Pani, and T. P. Sotiriou, Phys. Rev. Lett. 111, 111101 (2013), arXiv:1308.6587 [gr-qc].
[29] V. Cardoso, I. P. Carucci, P. Pani, and T. P. Sotiriou, Phys. Rev. D88, 044056 (2013), arXiv:1305.6936 [gr-qc].
[30] C. A. R. Herdeiro and E. Radu, Phys. Rev. Lett. 112, 221101 (2014), arXiv:1403.2757 [gr-qc].
[31] G. Antoniou, A. Bakopoulos, and P. Kanti, Phys. Rev. Lett. 120, 131102 (2018), arXiv:1801.03390 [hep-th].
[32] T. P. Sotiriou, Class. Quant. Grav. 32, 214002 (2015), arXiv:1505.00248 [gr-qc].
[33] H. O. Silva, J. Sakstein, L. Gualtieri, T. P. Sotiriou, and E. Berti, Phys. Rev. Lett. 120, 131101 (2018), arXiv:1711.02080 [gr-qc].
[34] D. D. Doneva and S. S. Yazadjiev, Phys. Rev. Lett. 120, 131103 (2018), arXiv:1711.01187 [gr-qc].
[35] E. Barausse and T. P. Sotiriou, Phys. Rev. Lett. 101, 099001 (2008), arXiv:0805.3433 [gr-qc].
[36] T. Damour and G. Esposito-Farese, Phys. Rev. Lett. 70, 2220 (1993).
[37] E. Barausse, C. Palenzuela, M. Ponce, and L. Lehner, Phys. Rev. D 87, 081506 (2013).
[38] C. Palenzuela, E. Barausse, M. Ponce, and L. Lehner,
