Dynamical impact of the Planet Nine scenario: N-body experiments

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ABSTRACT
The Planet Nine hypothesis has now enough constraints to deserve further attention in the form of detailed numerical experiments. The results of such studies can help us improve our understanding of the dynamical effects of such a hypothetical object on the extreme trans-Neptunian objects or ETNOs and perhaps provide additional constraints on the orbit of Planet Nine itself. Here, we present the results of direct N-body calculations including the latest data available on the Planet Nine conjecture. The present-day orbits of the six ETNOs originally linked to the hypothesis are evolved backwards in time and into the future under some plausible incarnations of the hypothesis to investigate if the values of several orbital elements, including the argument of perihelion, remain confined to relatively narrow ranges. We find that a nominal Planet Nine can keep the orbits of (90377) Sedna and 2012 VP113 relatively well confined in orbital parameter space for hundreds of Myr, but it may make the orbits of 2004 VN112, 2007 TG422 and 2013 RF95 very unstable on time-scales of dozens of Myr, turning them retrograde and eventually triggering their ejection from the Solar system. Far more stable orbital evolution is found with slightly modified orbits for Planet Nine.

Key words: methods: numerical – celestial mechanics – Kuiper belt: general – minor planets, asteroids: general – Oort Cloud – planets and satellites: general.

1 INTRODUCTION
The discovery of 2012 VP113 (Trujillo & Sheppard 2014) has led the astronomical community to recognize that most, if not all, asteroids with semimajor axis, a, greater than 150 au and perihelion distance, q = a(1 − e), greater than 30 au —the extreme trans-Neptunian objects or ETNOs— display unusual patterns in the values of some of their orbital elements. The values of their argument of perihelion, ω, cluster about 0°—actually, −26°—(de la Fuente Marcos & de la Fuente Marcos 2014; Trujillo & Sheppard 2014), their longitude of the ascending node, Ω, about 134°(Batygin & Brown 2016; Brown & Firth 2016), their eccentricities, e, about 0.8, and their inclinations, i, about 20°(de la Fuente Marcos & de la Fuente Marcos 2014, 2016; de la Fuente Marcos, de la Fuente Marcos & Aarseth 2015). The clustering in e can be explained as resulting from a selection effect (if a=150 au and q=30 au, then e=0.8), but this is not the case of the ones observed in i, Ω and ω (de la Fuente Marcos & de la Fuente Marcos 2014). One of the theories proposed to explain the observed patterns places their sources in the perturbations due to yet undetected trans-Plutonian planets.

Perhaps the most popular incarnation of the trans-Plutonian massive perturber(s) paradigm is the so-called Planet Nine hypothesis. Based on results from extensive computer simulations, Batygin & Brown (2016) have predicted the existence of a massive planet located well beyond Pluto in order to explain the observed clustering in physical space of the perihelia of six ETNOs. The putative object responsible for inducing the clumping has been provisionally designated Planet Nine. Batygin & Brown (2016) have provided tentative values for the orbital parameters of the proposed 10 M⊕ planet: a = 700 au, e = 0.6, i = 30° and ω = 150°. The resonant scenario described in Batygin & Brown (2016) assumes that the orbits of the ETNOs are apsidally anti-aligned and nodally aligned with that of the perturber. The extrapolation of the Cassini data carried out by Fienga et al. (2016) indicates that Planet Nine as characterised by Batygin & Brown (2016) cannot exist in the interval of true anomaly f ∈ (−132, 106.5)°. In addition, Fienga et al. (2016) pointed out that from the perspective of the Cassini residuals, the most probable position of Planet Nine —assuming that Ω = 113°— is at f = 117.8°±0.5°. In the following, we define a nominal orbit of Planet Nine as the set of orbital elements: a = 700 au, e = 0.6, i = 30°, Ω = 113° and ω = 150°, with f = 117.8° at t = 0 (present time, see below). Regarding its origin, Planet Nine may have been scattered out of the region of the giant planets early in the history of the Solar system (see e.g. Bromley & Kenyon 2014, 2016) or even captured from another planetary system (Li & Adams 2016; Mustill, Raymond & Davies 2016).

Table 1 shows the values of various orbital parameters for the six objects discussed in Batygin & Brown (2016) as well as relevant descriptive statistics; in this table, unphysical values are displayed for completeness. Both 2007 TG422 and 2012 VP113 are statistical outliers in terms of e as their values are outside the range (Q1 − 1.5IQR, Q3 + 1.5IQR), where Q1 is the first quartile, Q3 is

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the third quartile and IQR is the interquartile range or difference between them; we will see that, within the context of the Planet Nine hypothesis, 2007 TG$_{422}$ is particularly interesting. The data show that the mean value of the aphelion distance, $Q = a(1 + e)$, is 700 au and the lowest value of $a$ is 259 au. An orbit with $a = 700$ au and $q = 259$ au has $e = 0.63$. On the other hand, the highest value of $i$ is 30$^\circ$ and the mean values of $\Omega$ and $\omega$ are 102$^\circ$ and 314$^\circ$, respectively. The actual values of these two angular orbital elements are critical within the context of a nodal alignment and apsidal anti-alignment scenario. If the relative value of $\Omega$ should be 0$^\circ$ and the relative value of the longitude of perihelion, $\varpi = \Omega + \omega$, is expected to be equal to 180$^\circ$ (see e.g. Klaflke, Ferraz-Mello & Michtchenko 1992), then the values of $\Omega$ and $\omega$ for Planet Nine must be close to 102$^\circ$ and 134$^\circ$, respectively. The combination of values of $\Omega$ and $\omega$ may have a major impact on the overall stability of the ETNOs under Planet Nine’s gravitational perturbation, and it is therefore legitimate to question the plausibility of the values proposed so far.

Assessing the impact of the gravitational perturbation from a Planet Nine-like perturber on the short-term evolution of the known ETNOs is essential in order to decide if a candidate orbit is plausible or not. Here, we present the results of direct N-body calculations including the latest data available on the Planet Nine conjecture. The results from these calculations can help us improve our understanding of the dynamical effects of such a hypothetical object, or any other trans-Plutoian perturber, on the ETNO population. This Letter is organized as follows. Our N-body methodology is briefly described in Section 2. The evolution of the six ETNOs included in Table 1 subjected to the perturbation of Planet Nine moving along the nominal orbit described in Fienga et al. (2016) is studied in Section 3. Section 4 repeats the analysis for other orbits. Results are discussed and conclusions summarized in Section 5.

2 ASSESSING THE DYNAMICAL EFFECTS: AN N-BODY APPROACH

It is a well-known fact that the motion of the planets in the Solar system is chaotic; accurate predictions of their trajectories beyond a few tens of Myr are simply not possible (see e.g. Laskar 1989, 1990, 1994, 2008). Extensive computer simulations show that the Solar system as we know it remains at all times in a state of marginal stability; it is therefore reasonable to assume that the inclusion of Planet Nine must not change this current state of dynamical affairs. On the other hand, the known planets are not distributed randomly but organize themselves into planetary groups or collections of loosely connected mutually dynamically dependent planets. Innanen, Mikkola & Wieberg (1998) and Ito & Tanikawa (1999, 2002) have shown numerically that the terrestrial planets maintain their stability by sharing and weakening the secular perturbation from Jupiter. Tanikawa & Ito (2007) further extended this analysis concluding that the outer planetary group, or Jovian planets, forms a subsystem that is not affected by the inner planets; the motion of the group of the Jovian planets may not be perturbed to a significant degree if there is no inner planet group. This numerical evidence can be used to set up the most efficient yet still reliable physical model to perform N-body experiments that include an implementation of the Planet Nine hypothesis for testing. In other words, any realistic N-body study of the dynamical effects of the Planet Nine conjecture must include the Jovian planets, all of them, but can safely neglect the terrestrial planets. Under such physical model, we should expect accurate results within a few tens of Myr of $t = 0$; if a virtual ETNO becomes dynamically unstable in a few tens of Myr, this can be considered as a robust feature of that particular simulation and the same can be said about an orbit that remains fairly stable for a similar period of time.

Consistently with the above discussion, our physical model includes the perturbations by the Jovian planets (Jupiter to Neptune). In order to compute accurate initial positions and velocities we used the heliocentric ecliptic Keplerian elements provided by the JPL On-Line Solar System Data Service$^1$ (Giorgini et al. 1996) and initial positions and velocities (for both planets and ETNOs) based on the DE405 planetary orbital ephemerides (Standish 1998) referred to the barycentre of the Solar system and to the epoch JD TDB 2457400.5 (2016-January-13.0), which is the $t = 0$ instant in our figures. In addition to the orbital calculations completed using the nominal elements of the ETNOs in Table 1, we have performed additional simulations using control orbits; the orbital elements of the control orbits have been obtained varying them randomly, within the ranges defined by their mean values and standard deviations (1σ). In our calculations, the Hermite integration scheme described by Makino (1991) and implemented by Aarseth (2003) is employed. The standard version of this direct N-body code is publicly available from the IoA website.$^2$ Relative errors in the total energy for the longest integrations are as low as $3 \times 10^{-12}$ or lower. The relative error in the total angular momentum is several orders of magnitude smaller. As pointed out in de la Fuente Marcos & de la Fuente Marcos (2012), the results from this code compare well with those from Laskar et al. (2011) among others. Fig. 1 shows the relative variations (difference between the values with and without Planet Nine divided by the value without the external perturber) of the values of the orbital elements $-a, e, i-$ of the four Jovian planets (Jupiter to Neptune, left to right) for $\pm 100$ Myr for the nominal orbit of Planet Nine. These deviations are consistent with the expectations as they remain very small for a few tens of Myr, growing afterwards. The average orbital evolution of the Jovian planets, which is time symmetric, is not significantly altered by the presence of Planet Nine as described by the nominal orbit (see above).

Batygin & Brown (2016) used the Mercury6 gravitational dynamics software package (Chambers 1999) to perform N-body simulations of 13 ETNOs subjected to the perturbation of the most massive known members of the Solar system. These calculations led them to select six out of 13 ETNOs as largely unaffected by the presence of Neptune; these objects are (90377) Sedna, 2004 VN$_{112}$, 2007 TG$_{422}$, 2010 GB$_{172}$, 2012 VP$_{113}$ and 2013 RF$_{96}$. The present-day orbital properties of these objects were subsequently used to constrain and validate the resonant perturbation mechanism behind the Planet Nine hypothesis. However, no attempt was made to confirm that, in the presence of their favoured incarnation of the Planet Nine hypothesis, the orbital evolution of the six ETNOs remained unaffected by Neptune. In the following, we perform such tests — evolving the present-day orbits of the six ETNOs backwards in time and into the future— within the framework described above and discuss the results obtained.

3 IMPACT OF THE ORBIT IN FIENGA ET AL. (2016)

Fienga et al. (2016) pointed out that from the perspective of the Cassini residuals, the most probable orbit for the object proposed

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1. http://ssd.jpl.nasa.gov/?planet_pos
2. http://www.ast.cam.ac.uk/~sverre/web/pages/nbody.htm
Impact of the Planet Nine hypothesis

Table 1. Values of various orbital parameters — $P$ is the orbital period, $\Omega^*$ and $\omega^*$ are $\Omega$ and $\omega$ in the interval $(-\pi, \pi)$ instead of the regular $(0, 2\pi)$— for the six objects discussed in Batygin & Brown (2016). The statistical parameters are $Q_1$, first quartile, $Q_3$, third quartile, IQR, interquartile range, OL, outer lower limit ($Q_1 - 1.5*IQR$), and OU, upper outlier limit ($Q_3 + 1.5*IQR$); see the text for additional details. (Epoch: 2457400.5, 2016-January-13.0 00:00:00.0 TDB. J2000.0 ecliptic and equinox. Source: JPL, Small-Body Database. Data retrieved on 2016 February 27.)

| Object          | $a$ (au) | $e$ | $i$ (°) | $\Omega^*$ (°) | $\omega^*$ (°) | $\sigma_\theta$ (°) | $q$ (au) | $P$ (yr) | $\Omega^*$ (°) | $\omega^*$ (°) |
|-----------------|---------|-----|---------|----------------|----------------|-------------------|--------|---------|----------------|----------------|
| (90377) Sedna   | 507.5603| 0.8501824| 11.92872 | 144.5463       | 311.4614       | 96.0077           | 76.0415| 939.0792| 1143.094       | -48.5386       |
| 2004 VN112     | 321.0199| 0.8525664| 25.56295 | 66.0107        | 327.1707       | 33.1814           | 47.3291| 594.7106| 575.810         | -66.0107       |
| 2007 TG322     | 492.7277| 0.9277916| 18.58697 | 112.9515       | 285.7968       | 38.7483           | 35.5791| 949.8764| 10937.517      | -74.2032       |
| 2010 GB99      | 371.1183| 0.8687090| 21.53812 | 126.1968       | 347.8124       | 118.4243          | 38.7483| 693.5121| 7149.512        | -12.1869       |
| 2012 VP113     | 259.3002| 0.6896024| 24.04680 | 90.8179        | 293.7168       | 24.5346           | 48.5386| 417.6061| 16706.361       | -66.2832       |
| 2013 RF98      | 309.0738| 0.8826022| 29.61402 | 67.5205        | 316.4991       | 24.0196           | 36.2846| 581.8631| 675.205         | -43.5009       |

Mean: 376.8000, 0.8452423, 21.87960, 102.0764, 313.7429, 55.8193, 54.0742, 699.5259, 7480.545, 102.0764, 2457.517
Standard deviation: 102.0532, 0.0813175, 6.13291, 32.7348, 22.5205, 40.8126, 19.5592, 206.5256, 3026.532, 32.7348, 22.5205
Median: 346.0691, 0.8606377, 22.79246, 101.8847, 313.9803, 35.9648, 48.0268, 644.1114, 6450.674, 32.7348, 22.5205
Q1: 312.0603, 0.8507784, 19.32476, 33.97264, 126.1968, 324.5028, 81.6929, 1316.6061, 16706.361, 126.1968, 324.5028
Q3: 462.3254, 0.8791289, 25.18391, 126.1968, 324.5028, 81.6929, 48.7245, 693.5121, 7149.512, 126.1968, 324.5028
IQR: 113.4623, 0.8507784, 19.32476, 33.97264, 126.1968, 324.5028, 81.6929, 1316.6061, 16706.361, 126.1968, 324.5028
OL: 867.7230, 0.9216547, 33.97264, 205.4747, 364.0277, 164.1877, 1316.6061, 16706.361, 205.4747, 4.0277
OU: 113.4623, 0.8507784, 19.32476, 33.97264, 126.1968, 324.5028, 81.6929, 1316.6061, 16706.361, 126.1968, 324.5028

Figure 1. Relative variations, with respect to the case without Planet Nine, of the values of the orbital parameters semimajor axis, eccentricity and inclination of the Jovian planets — Jupiter, left-hand panels, to Neptune, right-hand panels— subjected to the perturbation due to the nominal orbit of Planet Nine.

by Batygin & Brown (2016) could be: $a = 700$ au, $e = 0.6$, $i = 30^\circ$, $\Omega = 113^\circ$ and $\omega = 150^\circ$, with $f = 117.8$ at present time. We have computed the orbital evolution of the six ETNOs included in Table 1 subjected to the perturbation of Planet Nine for ±200 Myr. Our results are summarized in Fig. 2 that corresponds to the nominal orbits of the six ETNOs subjected to the action of Planet Nine as discussed in Fienga et al. (2016). They indicate that the orbital evolution of several of the objects displayed could be more chaotic into the future than it used to be in the past. This time asymmetry suggests that, within the Planet Nine hypothesis, the ETNOs might be a transient population that evolves from relatively stable orbits into rather unstable ones. In particular, the future orbital evolution of 2004 VN112, 2007 TG322 and 2013 RF98 is very unstable in this case. Using control orbits for the ETNOs slightly different from the nominal ones, but within $1\sigma$, produces similar results. The same can be said if it is assumed that Planet Nine is currently located at aphelion and/or that $\Omega \sim 90^\circ$ but leaving the values of the other orbital parameters unchanged. Fig. 2 clearly shows that the orbital evolution of several of the six ETNOs is eventually affected by Neptune (and other Jovian planets) as their eccentricities become close to 1 (2004 VN112 and 2007 TG322) or its semimajor axis becomes dangerously small (2013 RF98). Fig. 3 shows the pre-ejection evolution of one of the control orbits of 2007 TG322 that ends in ejection from the Solar system; the object turns retrograde following a dynamical pathway that resembles closely the one described for comet 96P/Machholz 1 in de la Fuente Marcos et al. (2015). Multiple orbit realizations of the ETNOs 2004 VN112, 2007 TG322 and 2013 RF98 evolve into retrograde objects and/or are ejected from the Solar system within the simulation time. This behaviour has been previously documented in simulations by Gomes, Soares & Brassier (2015) and Batygin & Brown (2016). Only the orbits of (90377) Sedna and 2012 VP113 remain relatively well confined in orbital parameter space in Fig. 2. However, longer integrations (not shown) indicate that, for this particular orbit, the six ETNOs could be ejected from the Solar system within 1.5 Gyr from now. It is clear that, in general, the assumption made by Batygin & Brown (2016) —that the singled out ETNOs were largely unaffected by the presence of Neptune— breaks up relatively rapidly once the perturbation from Planet Nine is taken into account. This is to be expected within the context of an apsidal anti-alignment scenario because the average value of $\Delta\varpi$ —using this hypothetical orbit and the data from Table 1— is nearly 153°, relatively far from the critical value of 180°. So the question is, how is the orbital evolution of the ETNOs affected if we enforce the $\Delta\varpi \sim 180^\circ$ condition?

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Figure 2. Orbital evolution of the six ETNOs in Table 1 subjected to the nominal Planet Nine perturbation (see the text for details): (90377) Sedna (black), 2004 VN112 (orange), 2007 TG422 (blue), 2010 GB174 (purple), 2012 VP113 (grey) and 2013 RF98 (red).

Figure 3. Detail of the pre-ejection evolution of 2007 TG422 for a simulation different from the one displayed in Fig. 2.

4 IMPACT OF OTHER SOLUTIONS

Here, we present results from two representative simulations that consider candidate Planet Nine orbits with $\Delta \Omega \sim 0^\circ$ and $\Delta \varpi \sim 180^\circ$.

4.1 Orbit A

Fig. 4 shows the orbital evolution of the same objects displayed in Fig. 2 but subjected to the perturbation of a 10 $M_\oplus$ planet moving along the orbit: $a = 701$ au, $e = 0.6$, $i = 33^\circ$, $\Omega = 89^\circ$ and $\omega = 142^\circ$, with $f = 180^\circ$ at $t = 0$. Now $\Delta \varpi = 185^\circ$ and the overall evolution is far more stable than that in Fig. 2; no objects turn retrograde and/or are ejected during the displayed time. The values of $e$ and $i$ remain within narrow limits, but $\omega$ circulates for two objects, 2010 GB174 and 2013 RF98. The values of the orbital elements of 2007 TG422, that was the most unstable in Section 3, remain well confined.

Figure 4. Similar to Fig. 2 but for one representative solution with $\Delta \varpi \sim 180^\circ$ (see the text for details).

4.2 Orbit B

The previous calculations suggest that the orbital evolution of the ETNOs subjected to Planet Nine perturbation could be more unstable into the future. For this reason, in this integration we neglect the past evolution of the ETNOs and focus on the future. The orbit of Planet Nine used here is: $a = 700$ au, $e = 0.6$, $i = 30^\circ$, $\Omega = 102^\circ$ and $\omega = 134^\circ$, with $f = 180^\circ$ at $t = 0$. In this case $\Delta \Omega = 0^\circ$ and $\Delta \varpi = 180^\circ$. Fig. 5 shows the orbital evolution of the same objects displayed in Figs 2 and 4. Now, 2007 TG422 becomes an eccentric but stable co-orbital trapped in the 1:1 mean motion resonance with Planet Nine after $\sim 160$ Myr (this is also observed sometimes for orbit A). Co-orbital motion is possible at high eccentricity (see e.g. Namouni, Christou & Murray 1999). The overall orbital evolution of the six ETNOs under this incarnation of Planet Nine is more stable than that in Fig. 2, but less stable than the one associated with orbit A (see Fig. 4). Several objects exhibit Kozai dynamics (Kozai 1962), including 2012 VP113 that also experiences this behaviour for the previous two orbits.

5 DISCUSSION AND CONCLUSIONS

The resonant coupling mechanism described in Batygin & Brown (2016), with some fine tuning, can shepherd some ETNOs and explain their unusual orbital patterns. It can also create dynamical pathways that may deliver objects to high inclination or even retrograde orbits, and place ETNOs within Kozai resonances. This last feature is shared with the scenarios discussed in Trujillo & Sheppard (2014), de la Fuente Marcos & de la Fuente Marcos (2014) or de la Fuente Marcos et al. (2015) although the origin is different. Within the Planet Nine paradigm, Kozai behaviour is a by-product of mean motion resonances.

Innanen et al. (1997) and Tanikawa & Ito (2007) performed numerical experiments to measure the strength of the gravitational coupling among the known planets against external perturbation.
They concluded that, for relatively short integrations, the two sub-systems pointed out above were able to absorb efficiently a perturbation several orders of magnitude stronger than that of a putative Planet Nine. This result can be used to guess the answer to the question, is Planet Nine alone or are there more? Planet Nine, if it exists, moves in an elongated orbit that may be vulnerable to long-term secular perturbations resulting from the Galactic tide or discrete events like close encounters with passing stars. In this context, a lone Planet Nine may not be able to survive in its present orbit for the age of the Solar system (see Li & Adams 2016), but a planet within a planetary group has better chances to be long-term stable. Therefore, if Planet Nine exists, it is probably not alone; planets similar to Uranus or Neptune (super-Earths) may also form at 125–750 au from the Sun (Kenyon & Bromley 2015, 2016).

In this Letter, we have explored the dynamical impact of the Planet Nine hypothesis proposed by Batygin & Brown (2016). This study has been performed using N-body techniques. Our main conclusions are as follows.

- The nominal orbit of Planet Nine as described in Fienga et al. (2016) can keep the orbits of (90377) Sedna and 2012 VP113 relatively well confined in orbital parameter space for hundreds of Myr but it may make the orbits of 2004 VN112, 2007 TG422, and 2013 RF98 very unstable on time-scales of dozens of Myr, turning them retrograde and triggering their ejection from the Solar system.

- Modifying the orbit in Fienga et al. (2016) by assuming that Planet Nine is currently located at aphelion and/or that $\Omega \sim 90^\circ$ does not make the orbits of 2004 VN112, 2007 TG422 and 2013 RF98 more stable and short-term ejections are still observed.

- Slightly modified solutions for Planet Nine’s orbit that enforce apsidal anti-alignment produce a far more stable dynamical evolution of 2004 VN112, 2007 TG422 and 2013 RF98.

- Orbital solutions with $\Delta \Omega = 0^\circ$ and $\Delta \sigma = 180^\circ$ induce Kozai dynamical evolution for several of the ETNOs studied here.

- The orbital evolution of 2007 TG422 is particularly sensitive to the details of the Planet Nine hypothesis. In some implementations the object turns retrograde to be eventually ejected from the Solar system, but in others it becomes a stable co-orbital of Planet Nine.

- For the nominal Planet Nine hypothesis, the ETNOs are a transient population with a typical lifetime of a few hundreds of Myr.

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