TRIPLY-CHARMED DIBARYON STATES OR TWO-BARYON SCATTERING STATES FROM THE QCD SUM RULES?

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Abstract

In this article, we construct the color-singlet-color-singlet type currents to study the scalar and axialvector \( \Xi_{cc} \Sigma_c \) dibaryon states with QCD sum rules in details by taking into account both the dibaryon states and two-baryon scattering states at the hadron side, and examine the existence of the \( \Xi_{cc} \Sigma_c \) dibaryon states. Our calculations indicate that the two-baryon scattering states cannot saturate the QCD sum rules, it is necessary to introduce the dibaryon states, the color-singlet-color-singlet type currents couple potentially to the molecular states, not to the two-particle scattering states, the molecular states begin to receive contributions at the order \( \mathcal{O}(\alpha_s^2) \), not at the order \( \mathcal{O}(\alpha_s^0) \).

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1 Introduction

A dibaryon state denotes an object with baryon number 2, the oldest known dibaryon state is the deuteron, which is a very loosely bound state of two baryons, a proton and a neutron, and is made of six light valence quarks. In 2014, the WASA-at-COSY collaboration established the narrow resonance structure \( d^*(2380) \) with \( I(J^P) = 0(3^+) \) as a genuine s-channel resonance using partial-wave analysis, and given the first clear-cut experimental evidence for the existence of a true dibaryon resonance \( \Xi_{cc} \Sigma_c \), the \( d^*(2380) \) was firstly observed in the double-pionic fusion to the deuteron \( \Xi_{cc} \Sigma_c \). The \( d^*(2380) \) may be a \( \Delta\Delta \) dibaryon state or a six-quark state, for more theoretical and experimental works on the light flavor dibaryon states, one can consult the comprehensive review \[3\].

On the other hand, many charmonium-like and bottomonium-like states were observed after the discovery of the \( X(3872) \) by the Belle collaboration in 2003 \[4\]. It is difficult to accommodate those exotic \( X, Y \) and \( Z \) states in the conventional meson spectrum, especially, the charged charmonium-like states are good candidates for the multiquark states, tetraquark states or molecular states \[5\]. The observation of the \( d^*(2380) \) triggers much theoretical interest on possible existence of the molecular states made of two heavy baryons. As the large masses of the heavy baryons reduce the kinetic energy of the two-baryon systems, which makes it easier to form bound states. In 2015, the LHCb collaboration observed two pentaquark candidates \( P_c(4380) \) and \( P_c(4450) \) in the \( J/\psi p \) mass spectrum \[6\]. In 2019, the LHCb collaboration observed a narrow pentaquark candidate \( P_c(4312) \) and confirmed the \( P_c(4450) \) pentaquark structure, which consists of two narrow overlapping peaks \( P_c(4440) \) and \( P_c(4457) \) \[7\]. The \( P_c(4380), P_c(4440) \) and \( P_c(4457) \) lie near the \( D\Sigma_c, D\Sigma_c^* \), \( D^*\Sigma_c \) and \( D^*\Sigma_c^* \) thresholds respectively, which leads to the conjecture that they are meson-baryon type molecular states. We can learn something about the \( D\Sigma_c, D\Sigma_c^*, D^*\Sigma_c \) and \( D^*\Sigma_c^* \) pentaquark molecular states from the \( \Xi_{cc}\Sigma_c, \Xi_{cc}\Sigma_c^*, \Xi_{cc}^*\Sigma_c \) and \( \Xi_{cc}^*\Sigma_c^* \) dibaryon states (or vice versa), which are connected with each other by heavy antiquark-diquark symmetry, if we assume the light-quark structures are almost identical \[8\]. In Ref.\[9\], the molecular states consist of a doubly charmed baryon and an S-wave charmed baryon are investigated in the one-boson-exchange model.

Exploring the hadron-hadron interactions plays an important role in understanding the meson-meson type, meson-baryon type, baryon-antibaryon type, baryon-baryon type molecular states. It is essential to make a detailed theoretical investigation of those molecular states to encourage or stimulate new experiments to search for some evidences. Theoretically, we can study the molecular states at the hadron level \[8,9,10\] or at the quark level \[11,12\].
In Ref. [11], Junnarkar and Mathur study the \( \Sigma_c \Xi_{cc}, \Omega_{cc}, \Omega_{bb}, \Omega_{ccbb} \) dibaryon states with \( J^P = 1^+ \) via the lattice QCD, and unambiguously find that the ground state masses of the dibaryon states \( \Omega_{cc}, \Omega_{bb}, \Omega_{ccbb} \) are below their respective two-baryon thresholds, but cannot obtain definitive conclusion about the existence of the \( \Sigma_c \Xi_{cc} \) and \( \Sigma_b \Xi_{bb} \) dibaryon states due to large systematic errors. The doubly or triply heavy baryon states have not been observed experimentally yet except for the \( \Xi_c \) [13]. So it is interesting to study the \( \Sigma_c \Xi_{cc} \) dibaryon states with the QCD sum rules, as we have experimental input to assess the bound energies. In Ref. [14], the \( H \)-dibaryon or \( \Lambda \Lambda \) dibaryon states are studied with the QCD sum rules. In Ref. [15], the \( d^*(2380) \) is assigned as a \( \Delta \Delta \) dibaryon state and studied with the QCD sum rules.

The QCD sum rules approach is a powerful nonperturbative tool in studying the ground state tetraquark states, tetraquark molecular states, pentaquark states, pentaquark molecular states, and has given many successful descriptions of the hadronic parameters, such as the masses and decay widths [16, 17, 18, 19, 20, 21, 22, 23]. However, different voice arises, it is argued that the QCD sum rules are misused to study the tetraquark or molecule masses, all the four-quark currents can be rearranged into the color-singlet-color-singlet type currents, and couple potentially to the two-meson scattering states, the contributions to the four-quark states begin at the order \( \mathcal{O}(\alpha_s^2) \) [24, 25]. The dibaryon states consist of two color-singlet objects, they are another type molecular states. In this article, we will examine the applicability of the QCD sum rules to the molecular states.

In this article, we study the scalar and axialvector \( \Xi_{cc}, \Sigma_c \) dibaryon states with QCD sum rules in details. We take into account both the dibaryon states and two-baryon scattering states at the hadron side, and examine whether or not the QCD sum rules support the existences of dibaryon states.

The article is arranged as follows: we derive the QCD sum rules for the masses and pole residues of the triply-charmed dibaryon states and examine the applicability of the QCD sum rules in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusion.

### 2 QCD sum rules for the dibaryon states

In the following, we write down the two-point correlation functions \( \Pi_S(p) \) and \( \Pi_{\mu \nu}(p) \) in the QCD sum rules,

\[
\Pi_S(p) = i \int \frac{d^4x e^{ip \cdot x}}{x} \langle 0 | T \left\{ J(x) J^\dagger(0) \right\} | 0 \rangle ,
\]

\[
\Pi_{\mu \nu}(p) = i \int \frac{d^4x e^{ip \cdot x}}{x} \langle 0 | T \left\{ J_\mu(x) J_\nu^\dagger(0) \right\} | 0 \rangle ,
\]

(1)

where

\[
J(x) = J^T_c(x) C \gamma_5 J_{cc}(x),
\]

\[
J_\mu(x) = J^T_C(x) C \gamma_\mu J_{cc}(x),
\]

\[
J_c(x) = \epsilon^{ijk} q^T_i(x) C \gamma_\alpha q_j(x) \gamma_\alpha \gamma_5 c_k(x),
\]

\[
J_{cc}(x) = \epsilon^{ijk} c^T_i(x) C \gamma_\alpha c_j(x) \gamma_\alpha \gamma_5 q_k(x),
\]

(2)

the \( i, j \) and \( k \) are color indexes, the \( C \) is the charge conjugation matrix. The currents \( J(x) \) and \( J_\mu(x) \) have two color-neutral clusters and have the property under parity transformation,

\[
\hat{P} J(x) \hat{P}^{-1} = + J(\bar{x}),
\]

\[
\hat{P} J_\mu(x) \hat{P}^{-1} = - J^\mu(\bar{x}),
\]

(3)

where the coordinates \( x^\mu = (t, \vec{x}) \) and \( \bar{x}^\mu = (t, -\vec{x}) \).
We construct the Ioffe’s currents \( J_c(x) \) and \( J_{cc}(x) \) according to the Fermi-Dirac statistics and the attractive interactions originate from the one-gluon exchange. The currents \( J_c(x) \) and \( J_{cc}(x) \) have the \( J^P = \frac{1}{2}^+ \) and have the constituent quarks or valence quarks \( qqc \) and \( ccq \), respectively, just like the baryon states \( \Sigma_c \) and \( \Xi_{cc} \). The quantum field theory does not forbid the current-baryon couplings,

\[
\begin{align*}
\langle 0 | J_c(0) | \Sigma_c(p) \rangle &= \lambda_\Sigma U_\Sigma(p), \\
\langle 0 | J_{cc}(0) | \Xi_{cc}(p) \rangle &= \lambda_\Xi U_\Xi(p),
\end{align*}
\]

the \( U_\Sigma(p) \) and \( U_\Xi(p) \) are Dirac spinors. The values of the coupling constants or pole residues \( \lambda_\Sigma \) and \( \lambda_\Xi \) are not experimentally measurable quantities, we have to calculate those values with the QCD sum rules or lattice QCD.

The \( \Sigma_c \Xi_{cc} \) systems maybe form the \( \Sigma_c \Xi_{cc} \) dibaryon states, or maybe not form the \( \Sigma_c \Xi_{cc} \) dibaryon states (in other words, they are just the \( \Sigma_c \Xi_{cc} \) two-baryon scattering states). If they form the \( \Sigma_c \Xi_{cc} \) dibaryon states, the quantum field theory does not forbid the current-dibaryon couplings, the currents \( J(x) \) and \( J_\mu(x) \) couple potentially to the scalar and axialvector dibaryon states, respectively, or to the two-baryon scattering states with the spin-parity \( J^P = 0^+ \) and \( 1^+ \), respectively,

\[
\begin{align*}
\langle 0 | J(0) | D_S(p) \rangle &= \lambda_S, \\
\langle 0 | J_\mu(0) | D_A(p) \rangle &= \lambda_A \varepsilon_\mu, \\
\langle 0 | J(0) | \Sigma_c(q) \Xi_{cc}(p - q) \rangle &= \lambda_\Sigma \lambda_\Xi U_\Sigma^T(p) C \gamma_5 U_\Xi(p - q), \\
\langle 0 | J_\mu(0) | \Sigma_c(q) \Xi_{cc}(p - q) \rangle &= \lambda_\Sigma \lambda_\Xi U_\Sigma^T(p) C \gamma_\mu U_\Xi(p - q),
\end{align*}
\]

we use the \( D \) to denote the dibaryon states.

At the hadron side of the correlation functions \( \Pi_S(p) \) and \( \Pi_{\mu\nu}(p) \), we isolate the contributions of both the lowest dibaryon states and two-baryon scattering states,

\[
\Pi_{\mu\nu}(p) = \Pi_A(p) \left(-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}\right) + \cdots,
\]

\[
\begin{align*}
\Pi_S(p) &= \frac{\lambda_S^2}{M_S^2 - p^2} + \int_0^{s_0} \frac{ds}{s - p^2} \rho_{H,S}(s) + \cdots, \\
\Pi_A(p) &= \frac{\lambda_A^2}{M_A^2 - p^2} + \int_0^{s_0} \frac{ds}{s - p^2} \rho_{H,A}(s) + \cdots,
\end{align*}
\]

where

\[
\begin{align*}
\rho_{H,S} &= \frac{\lambda_S^2 \lambda_\Xi^2}{8\pi^2} \frac{\sqrt{\lambda(s, m_\Sigma, m_\Xi)}}{s} \left[s - (m_\Xi - m_\Sigma)^2\right], \\
\rho_{H,A} &= \frac{\lambda_S^2 \lambda_\Xi^2}{8\pi^2} \frac{\sqrt{\lambda(s, m_\Sigma, m_\Xi)}}{s} \left[s - (m_\Xi - m_\Sigma)^2 - \frac{\lambda(s, m_\Sigma, m_\Xi)}{3s}\right],
\end{align*}
\]

where \( \Delta^2 = (m_\Sigma + m_\Xi)^2, \lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ac, \) the \( s_0 \) are the continuum threshold parameters.

At the QCD side of the correlation functions \( \Pi_S(p) \) and \( \Pi_{\mu\nu}(p) \), we contract the \( q \) and \( c \) quark
fields with Wick theorem and obtain the results,

$$
\Pi_S(p) = -i \varepsilon^{ijk} \varepsilon^{lmn} e^{i'j'k'} e^{l'm'n'} \int d^4 x e^{ipx} \left\{ 4 \text{Tr} \left[ \gamma_0 S_{ij}^T(x) \gamma_\alpha C S_{\alpha T}^T(x) C \right] \quad \text{Tr} \left[ \gamma_\beta S_{\alpha T}^m(x) \gamma_\beta C S_{\beta T}^m(x) C \right] \\
-8 \text{Tr} \left[ \gamma_0 S_{ij}^T(x) \gamma_\alpha C S_{\alpha T}^T(x) C \right] \quad \text{Tr} \left[ \gamma_\beta S_{\alpha T}^m(x) \gamma_\beta C S_{\beta T}^m(x) C \right] \\
+16 \left[ \langle \gamma_\alpha \gamma_\beta \gamma_\gamma \gamma_\sigma \gamma_\epsilon \rangle \left( \langle \gamma_\alpha \gamma_\beta \gamma_\gamma \gamma_\sigma \gamma_\epsilon \rangle - \langle \gamma_\gamma \gamma_\gamma \gamma_\sigma \gamma_\epsilon \rangle \right) \right] \\
+8(\langle \gamma_\alpha \gamma_\beta \rangle + \langle \gamma_\gamma \gamma_\gamma \rangle) \left( \langle \gamma_\alpha \gamma_\beta \gamma_\gamma \gamma_\sigma \gamma_\epsilon \rangle - \langle \gamma_\gamma \gamma_\gamma \gamma_\sigma \gamma_\epsilon \rangle \right) \right\} 
$$

$$
\Pi_{\mu\nu}(p) = i \varepsilon^{ijk} \varepsilon^{lmn} e^{i'j'k'} e^{l'm'n'} \int d^4 x e^{ipx} \left\{ 4 \text{Tr} \left[ \gamma_0 S_{ij}^T(x) \gamma_\alpha C S_{\alpha T}^T(x) C \right] \quad \text{Tr} \left[ \gamma_\beta S_{\alpha T}^m(x) \gamma_\beta C S_{\beta T}^m(x) C \right] \\
-8 \text{Tr} \left[ \gamma_0 S_{ij}^T(x) \gamma_\alpha C S_{\alpha T}^T(x) C \right] \quad \text{Tr} \left[ \gamma_\beta S_{\alpha T}^m(x) \gamma_\beta C S_{\beta T}^m(x) C \right] \\
+16 \left[ \langle \gamma_\alpha \gamma_\beta \gamma_\gamma \gamma_\sigma \gamma_\epsilon \rangle \left( \langle \gamma_\alpha \gamma_\beta \gamma_\gamma \gamma_\sigma \gamma_\epsilon \rangle - \langle \gamma_\gamma \gamma_\gamma \gamma_\sigma \gamma_\epsilon \rangle \right) \right] \\
+8(\langle \gamma_\alpha \gamma_\beta \rangle + \langle \gamma_\gamma \gamma_\gamma \rangle) \left( \langle \gamma_\alpha \gamma_\beta \gamma_\gamma \gamma_\sigma \gamma_\epsilon \rangle - \langle \gamma_\gamma \gamma_\gamma \gamma_\sigma \gamma_\epsilon \rangle \right) \right\} 
$$

where

$$
S_{ij}(x) = \frac{i \delta_{ij} f}{2\pi^2 x^2} - \frac{\delta_{ij} x^2}{12} - \frac{g_s C_{ij}^a}{27648} - \frac{1}{8} \left( \delta_{ij} \sigma^{\mu\nu} q_i \right) \sigma_{\mu\nu} + \cdots
$$

$$
S_{ij}^T(x) = \frac{i}{(2\pi)^4} \int d^4k e^{-ikx} \left\{ \delta_{ij} \frac{g_s C_{ij}^a}{4} \left( \gamma_k + m_c \right) \left( \gamma_k + m_c \right) \sigma^{\alpha\beta} \right\}_k + \cdots
$$

$$
f^{\alpha\beta\mu\nu} = \left( \gamma_k + m_c \right) \gamma^{\alpha} \left( \gamma_k + m_c \right) \gamma^{\beta} \left( \gamma_k + m_c \right) \gamma^{\mu} \left( \gamma_k + m_c \right) \gamma^{\nu}
$$
and \( t^n = \frac{\lambda^n}{2} \), the \( \lambda^n \) is the Gell-Mann matrix \([17][20][24]\). In the full light quark propagator, see Eq. (11), we add the term \( \langle \bar{q}_j \sigma_{\mu \nu} q_i \rangle \) obtained from Fierz rearrangement of the \( \langle q_i \bar{q}_j \rangle \) to absorb the gluons emitted from other light quark lines and heavy quark lines to extract the mixed condensate \( \langle \bar{q}_j \sigma^{a} G q_i \rangle \) \([17]\).

In Fig. 1, we draw the lowest order Feynman diagrams, which correspond to the perturbative contributions in Eqs. (9)-(11). In other words, the free-quark contributions in the full quark propagators in Eqs. (11)-(12). The first diagram in Fig. 1 is factorizable or disconnected, the other diagrams are nonfactorizable or connected.

If we substitute the light-quark lines and heavy-quark lines in Fig. 1 with other terms in the full light-quark and heavy-quark propagators in Eqs. (11)-(12), we can obtain all the relevant Feynman diagrams. From the first diagram in Fig. 1 we can obtain both connected and disconnected Feynman diagrams, the connected contributions appear due to the operators \( \bar{q}_j G q_i, G \bar{q}_j G q_i \), which are of the order \( O(\alpha_s) \) and come from the Feynman diagrams shown in Fig. 2. From the operators \( \bar{q}_j G q_i, G \bar{q}_j G q_i \), we can obtain the vacuum condensate \( \langle \bar{q}_j \sigma G q_i \rangle^2 \), the \( g_\sigma^2 = 4\pi\alpha_s \) is absorbed into the vacuum condensate, so the diagrams in Fig. 2 can be counted as of the order \( O(\alpha_s^2) \). Those contributions should be taken into account as the QCD sum rules is a nonperturbative method, the connected Feynman diagrams appear at the order \( O(\alpha_s^0) \) or \( O(\alpha_s^1) \), not at the order \( O(\alpha_s^2) \) asserted in Refs. [24][25]. While from other diagrams in Fig. 1 we can obtain only connected Feynman diagrams.

In the similar systems, the four-quark systems, it is argued that the connected Feynman diagrams appear at the order \( O(\alpha_s^2) \), which contribute a Landau pole to indicate the existence of the tetraquark state or tetraquark molecular state \([24][25]\). In fact, the quarks and gluons are confined objects, they cannot be put on the mass-shell, it is questionable to say that the Landau equation is applicable in the nonperturbative QCD calculations involving bound states \([28]\). Furthermore, even the Landau poles appear, we cannot obtain the conclusion that there is a tetraquark state or tetraquark molecular state, as the Landau pole does not necessary equal a resonance, it is just a kinematical singularity, not a dynamical singularity \([5][29]\). If we insist on applying the Landau equation, we should choose the pole mass to warrant there exists a pole which corresponds to the mass-shell in pure perturbative calculations. In the case of the \( \eta_c \)-quark, the pole mass \( \hat{m}_c = 1.67 \pm 0.07 \text{ GeV} \) from the Particle Data Group \([32]\). \( 2\hat{m}_c = 3.34 \pm 0.14 \text{ GeV} > m_{\eta_c} \) and \( m_{J/\psi} \).

It is odd that the charmonium masses lie below the threshold \( 2\hat{m}_c \) in the QCD sum rules for the \( \eta_c \) and \( J/\psi \).

We can use the \( \hat{\Sigma}_c \) and \( \hat{\Xi}_{cc} \) to represent the color-singlet or color-neutral clusters \( cqq \) and \( ccq \) respectively in the currents. From those Feynman diagrams, we can observe that the initial color-neutral clusters \( \hat{\Sigma}_c \) and \( \hat{\Xi}_{cc} \) evolve to the final color-neutral clusters \( \hat{\Sigma}_c \) and \( \hat{\Xi}_{cc} \) with (without) interchanging colored objects, such as quarks and gluons, in the connected (disconnected) Feynman diagrams. The \( \Sigma_c \) (\( \Xi_{cc} \)) and \( \hat{\Sigma}_c \) (\( \hat{\Xi}_{cc} \)) have the same quantum numbers \( J^P = \frac{1}{2}^+ \), the quantum field theory allows nonvanishing couplings between the \( \Sigma_c \) (\( \Xi_{cc} \)) and \( \hat{\Sigma}_c \) (\( \hat{\Xi}_{cc} \)), irrespective of whether the \( \Sigma_c \) and \( \Xi_{cc} \) are the two-baryon scattering states or the \( \Sigma_c \Xi_{cc} \) components in the dibaryon states. We cannot obtain other information about the hadrons beyond that from the Feynman diagrams directly, because we carry out the operator product expansion in the quantum field theory, there are both perturbative and nonperturbative contributions, the vacuum condensates are highly nonperturbative quantities and embody the net collective effects. The nonperturbative contributions play an important role in the QCD sum rules, investigating the perturbative contributions of the order \( O(\alpha_s^2) \) alone cannot prove existence or nonexistence of the multiquark states, furthermore, no feasible QCD sum rules with predictions can be confronted to the experimental data are obtained in Refs. [24][25]. We should bear in mind that the Feynman diagrams at the quark-gluon level in the quantum field theory differ greatly from the Feynman diagrams in the intuitive potential quark models. If we insist on understanding the Feynman diagrams intuitively, then the disconnected Feynman diagrams give masses to the \( \Sigma_c \) and \( \Xi_{cc} \) baryons, the masses are not necessary the physical masses, while the connected Feynman diagrams contribute attractive
interactions to bind the massive $\Sigma_c$ and $\Xi_{cc}$ baryons to form molecular states.

After computing those Feynman diagrams, we obtain the QCD spectral densities through dispersion relation. In this article, we carry out the operator product expansion to the vacuum condensates up to dimension-15, and take into account the vacuum condensates which are vacuum expectations of the quark-gluon operators of the order $O(\alpha_s)$. There are three light quark propagators and three heavy quark propagators in the correlation functions (9)-(10), if each heavy quark line emits a gluon and each light quark line contributes quark-antiquark pair, we obtain an operator $g_s G_{\mu
u} q_s G_{\alpha\beta} q_s G_{\lambda\tau} \bar{q} q q q q q$, which is of dimension 15, and can lead to the vacuum condensates $\langle \frac{\alpha_s}{\pi} G \rangle \langle q \bar{q} \rangle^2 \langle \bar{q} q \sigma G q \rangle$, $\langle g_s^3 G G G \rangle \langle q \bar{q} \rangle^3$ and $\langle \bar{q} q \sigma G q \rangle^3$, we take into account the vacuum condensate $\langle \bar{q} q, \sigma G q \rangle^3$ and neglect the vacuum condensates $\langle g_s^3 G G G \rangle \langle q \bar{q} \rangle^3$ and $\langle \frac{\alpha_s}{\pi} G \rangle \langle q \bar{q} \rangle^2 \langle \bar{q} q \sigma G q \rangle$. Compared to the $\langle \bar{q} q, \sigma G q \rangle^3$, the $\langle g_s^3 G G G \rangle \langle q \bar{q} \rangle^3$ and $\langle \frac{\alpha_s}{\pi} G \rangle \langle q \bar{q} \rangle^2 \langle \bar{q} q \sigma G q \rangle$ are neglectful due to the small values. The condensates $\langle g_s^3 G G G \rangle$, $\langle g_s^3 G G G \rangle \langle q \bar{q} \rangle$ and $\langle \bar{q} q, \sigma G q \rangle$ are not associated with the $\frac{1}{T}$, and play a tiny role in determining the Borel window, and they are neglected. Furthermore, we neglect the $\langle g_s^3 G G G \rangle \langle q \bar{q} \rangle^2$ due to the small value, it is also beyond the order $O(\alpha_s)$. In summary, we take into account the vacuum condensates $\langle q \bar{q} \rangle$, $\langle \frac{\alpha_s}{\pi} G \rangle$, $\langle \bar{q} q, \sigma G q \rangle$, $\langle \bar{q} q \rangle^2$, $\langle \bar{q} q \rangle^3 (\frac{\alpha_s}{\pi} G)$, $\langle \bar{q} q \rangle^3 (\bar{q} q \sigma G q)$, $\langle \bar{q} q \rangle^2 (\bar{q} q \sigma G q)^2$, $\langle q \bar{q} \rangle \langle \bar{q} q, \sigma G q \rangle^2$, $\langle q \bar{q} \rangle^2 (\bar{q} q \sigma G q)^2$, $\langle q \bar{q} \rangle^3 (\bar{q} q \sigma G q)^3$.

Now we take the quark-hadron duality below the continuum thresholds $s_0$ and perform the Borel transformation in regard to $P^2 = -p^2$ to obtain the QCD sum rules:

$$\chi_S^2 \exp \left( -\frac{M_S^2}{T^2} \right) + \int_{\Delta^2}^s ds \rho_{H,S} \exp \left( -\frac{s}{T^2} \right) = \int_{9m_c^2}^{s_0} ds \rho_S(s) \exp \left( -\frac{s}{T^2} \right), \quad (13)$$

$$\chi_A^2 \exp \left( -\frac{M_A^2}{T^2} \right) + \int_{\Delta^2}^s ds \rho_{H,A} \exp \left( -\frac{s}{T^2} \right) = \int_{9m_c^2}^{s_0} ds \rho_A(s) \exp \left( -\frac{s}{T^2} \right), \quad (14)$$

the very very lengthy expressions of the QCD spectral densities $\rho_S(s)$ and $\rho_A(s)$ are neglected for simplicity.
Figure 2: The connected Feynman diagrams originate from the first diagram in Fig[11] other
diagrams obtained by interchanging of the heavy quark lines (dashed lines) or light quark lines
(solid lines) are implied.

We derive Eqs.(13)-(14) with respect to \( \tau = \frac{1}{T^2} \), then eliminate the pole residues \( \lambda_S \) and \( \lambda_A \) and obtain the QCD sum rules for the masses of the triply-charmed dibaryon states,

\[
M_{S}^{2} = \frac{-\frac{d}{d\tau} \left[ \int_{0}^{m_{c}^{2}} ds \rho_{S}(s) \exp \left( -s\tau \right) - \int_{\Delta_{s}^{2}}^{s_{0}} ds \rho_{H,S}(s) \exp \left( -s\tau \right) \right]}{\int_{0}^{m_{c}^{2}} ds \rho_{S}(s) \exp \left( -s\tau \right) - \int_{\Delta_{s}^{2}}^{s_{0}} ds \rho_{H,S}(s) \exp \left( -s\tau \right)}, \quad (15)
\]

\[
M_{A}^{2} = \frac{-\frac{d}{d\tau} \left[ \int_{0}^{m_{c}^{2}} ds \rho_{A}(s) \exp \left( -s\tau \right) - \int_{\Delta_{s}^{2}}^{s_{0}} ds \rho_{H,A}(s) \exp \left( -s\tau \right) \right]}{\int_{0}^{m_{c}^{2}} ds \rho_{A}(s) \exp \left( -s\tau \right) - \int_{\Delta_{s}^{2}}^{s_{0}} ds \rho_{H,A}(s) \exp \left( -s\tau \right)}, \quad (16)
\]

If the QCD sum rules can be saturated with the scalar and axialvector dibaryon states alone, we set \( \rho_{H,S} = \rho_{H,A} = 0 \) in Eqs.(13)-(16), we obtain the two sets of QCD sum rules for the dibaryon states, and refer those QCD sum rules as the QCDSR I, and refer the QCD sum rules in Eqs.(13)-(16) as the QCDSR II. On the other hand, if the QCD sum rules can be saturated with the scalar and axialvector two-baryon scattering states alone, we set \( \lambda_S = \lambda_A = 0 \), we obtain the two QCD sum rules,

\[
\int_{\Delta_{s}^{2}}^{s_{0}} ds \rho_{H,S} \exp \left( -\frac{s}{T_2} \right) = \kappa_S \int_{0}^{m_{c}^{2}} ds \rho_{S}(s) \exp \left( -\frac{s}{T_2} \right), \quad (17)
\]

\[
\int_{\Delta_{s}^{2}}^{s_{0}} ds \rho_{H,A} \exp \left( -\frac{s}{T_2} \right) = \kappa_A \int_{0}^{m_{c}^{2}} ds \rho_{A}(s) \exp \left( -\frac{s}{T_2} \right), \quad (18)
\]

where we introduce the parameters \( \kappa_S \) and \( \kappa_A \) to parameterize the deviations from the value 1. We refer the QCD sum rules in Eqs.(17)-(18) as the QCDSR III.

### 3 Numerical results and discussions

We choose the standard values of the vacuum condensates \( \langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3 \), \( \langle \bar{q}gsGq \rangle = m_0^2 \langle \bar{q}q \rangle \), \( m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2 \), \( \langle g_s^2 \sigma \rangle = (0.33 \text{ GeV})^4 \) at the energy scale \( \mu = 1 \text{ GeV} \) \[26, 30, 31\], and choose the \( \overline{\text{MS}} \) mass \( m_c(m_c) = (1.275 \pm 0.025) \text{ GeV} \) from the Particle Data Group \[32\], and
with \( m_u = m_d = 0 \). We take into account the energy-scale dependence of the input parameters,

\[
\langle \bar{q}q \rangle (\mu) = \langle \bar{q}q \rangle (1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{t}{\Lambda^2}},
\]

\[
\langle \bar{q}g_s \sigma Gq \rangle (\mu) = \langle \bar{q}g_s \sigma Gq \rangle (1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{t}{\Lambda^2}},
\]

\[
m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{t}{\Lambda^2}},
\]

\[
\alpha_s(\mu) = \frac{1}{b_0 t} \left[ 1 - \frac{b_1}{b_0^2} \log t + \frac{b_2}{b_0^4} \left( \log^2 t - \log t - 1 \right) + b_0 b_2 \right],
\]

(19)

where \( t = \log \frac{\mu^2}{\Lambda^2} \), \( b_0 = \frac{33-2n_f}{12\pi} \), \( b_1 = \frac{153-19n_f}{24\pi^2} \), \( b_2 = \frac{2857-693n_f+2n_f^2}{128\pi^4} \), \( \Lambda = 210\text{ MeV}, 292\text{ MeV} \) and \( 332\text{ MeV} \) for the flavors \( n_f = 5, 4 \) and 3, respectively [32, 33, 34], and evolve all the input parameters to the best energy scales to extract the dibaryon masses with the flavor \( n_f = 4 \).

In the article, we study the baryon-baryon type six-quark states (dibaryon states) or hexaquark states, which have three charmed quarks. Those six-quark systems are characterized by the effective charmed quark mass or constituent quark mass \( M_c \) and the virtuality \( V = \sqrt{M_D^2 - (3M_c)^2} \), where the \( D \) denotes the dibaryon states. We set the energy scales of the QCD spectral densities to be \( \mu = V \), it is a straight forward extension of the energy scale formula \( \mu = \sqrt{M_{X/Y/Z}^2 - (2M_c)^2} \) suggested for the hidden-charm tetraquark molecular states to the triply-charmed dibaryon states [19]. In this article, we choose the updated value \( M_c = 1.85\text{ GeV} \) [20], and take the energy scale formula,

\[
\mu = \sqrt{M_D^2 - (3M_c)^2},
\]

(20)

as a powerful constraint to satisfy.

The relevant baryon masses and pole residues are \( m_{\Sigma^{++}} = 2453.97\text{ MeV}, m_{\Sigma^+} = 2452.9\text{ MeV}, m_{\Sigma^0} = 2453.75\text{ MeV}, m_{\Xi^+} = 3621.2\text{ MeV} \) from the Particle Data Group [32], \( m_{\Sigma} = 2.40\text{ GeV} \), \( m_{\Xi} = 3.63\text{ GeV} \), \( \lambda_{\Sigma} = 0.045\text{ GeV}^3 \) and \( \lambda_{\Xi} = 0.102\text{ GeV}^3 \) from the QCD sum rules [35, 36]. In this article, we choose the values \( m_{\Sigma} = 2.45\text{ GeV} \) and \( m_{\Xi} = 3.62\text{ GeV} \). We vary the baryon masses \( m_{\Sigma} \) and \( m_{\Xi} \) slightly, which leads to negligible changes. In this article, we take the continuum threshold parameter as \( \sqrt{\mathcal{S}_0} = m_{\Sigma} + m_{\Xi} + (0.5 \sim 0.7)\text{ GeV} = 6.1 \pm 0.6\text{ GeV} \) to take into account the contributions from the \( \Sigma_c \) and \( \Xi_{cc} \) baryon states sufficiently.

We define the pole contributions PC as

\[
\text{PC} = \frac{\int_{m^2_c}^{\infty} ds \rho_{S/A}(s) \exp\left(-\frac{s}{m^2_c}\right)}{\int_{m^2_c}^{\infty} ds \rho_{S/A}(s) \exp\left(-\frac{s}{T_{\text{PC}}^2}\right)},
\]

(21)

For the multiquark states, it is difficult to satisfy the pole dominance criterion. The energy scale formula, see Eq.(20), can enhance the pole contributions significantly, and also can improve the convergent behaviors of the operator product expansion substantially. If the operator product expansion is convergent, the contributions of the higher dimensional vacuum condensates \( D(n) \) with \( n \geq 10 \) play a minor important role,

\[
D(n) = \frac{\int_{m^2_c}^{\infty} ds \rho_{S/A,n}(s) \exp\left(-\frac{s}{m^2_c}\right)}{\int_{m^2_c}^{\infty} ds \rho_{S/A}(s) \exp\left(-\frac{s}{T_{\text{PC}}^2}\right)},
\]

(22)

where the subscript \( n \) in the QCD spectral densities \( \rho_{S/A,n}(s) \) denotes the vacuum condensates of dimension \( n \).
Table 1: The Borel parameters, continuum threshold parameters, energy scales, pole contributions, masses and pole residues for the triply-charmed dibaryon states from the QCD sum rules I and II.

| $J^P$         | $T^2$(GeV$^2$) | $\sqrt{s_0}$(GeV) | $\mu$(GeV) | pole            | $M$(GeV)     | $\lambda$(GeV$^3$) |
|--------------|--------------|-----------------|-----------|----------------|-------------|------------------|
| 0$^+(I)$     | 3.9 − 4.5    | 6.7 ± 0.1       | 2.5       | (38 − 63)$\%$ | 6.05 ± 0.13 | (1.45 ± 0.30) × 10$^{-2}$ |
| 0$^+(II)$    | 3.9 − 4.5    | 6.7 ± 0.1       | 2.5       | (38 − 63)$\%$ | 6.04 ± 0.13 | (1.39 ± 0.30) × 10$^{-2}$ |
| 1$^+(I)$     | 4.1 − 4.7    | 6.7 ± 0.1       | 2.4       | (38 − 62)$\%$ | 6.03 ± 0.13 | (1.44 ± 0.29) × 10$^{-2}$ |
| 1$^+(II)$    | 4.1 − 4.7    | 6.7 ± 0.1       | 2.4       | (38 − 62)$\%$ | 6.02 ± 0.13 | (1.39 ± 0.28) × 10$^{-2}$ |

We obtain the Borel windows and the best energy scales of the QCD spectral densities for the QCDSR I, which are shown in Table 1, via try and error. Now it is straightforward to get the pole contributions, the pole contributions are as large as (38 − 63)$\%$, it is large enough to extract the dibaryon masses. In the QCD sum rules for the multiquark states, the pole contributions are usually small due to the QCD spectral densities $\rho_{QCD}(s) \propto s^n$ with the largest value $n \geq 4$ in the zero quark mass limit.

In Fig.3 we plot the absolute values of the $D(n)$ for the central values of the input parameters. Although the perturbative contributions are not large enough to dominate the QCD sum rules, the vacuum condensate $\langle \bar qq \rangle^2$ with the dimension 6 serves as a milestone, the absolute values of the contributions of the vacuum condensates with $n \geq 6$ decrease monotonically and quickly with increase of the dimension except for the vacuum condensate $\langle \bar qq \rangle^3/\pi$, which plays a tiny role. The convergent behavior of the operator product expansion is very good.

Although the higher dimensional vacuum condensates play a minor important role in the Borel windows, they play an important role in determining the Borel windows, we should take them into account in a consistent way. In Fig.4 we plot the values of the $D(n)$ for $n \geq 8$ with variations of the Borel parameters for the central values of the other input parameters. From Fig.4 we observe that the contributions of the vacuum condensates of dimensions 8, 9, 10, 11 and 13 are large at the region $T^2 < 3.9$ GeV$^2$, we should choose $T^2 \geq 3.9$ GeV$^2$, while the contributions of the vacuum condensates of dimension 15 are very small in the whole regions. In fact, the vacuum condensate $\langle \bar q q \sigma G q \rangle^3$ is the vacuum expectation of the operator of the order $O(\alpha^3_s)$, which is beyond the truncation $O(\alpha^4_s)$ with $n \leq 1$.

Now we take into account all uncertainties of the input parameters, and obtain the values of the masses and pole residues of the triply-charmed dibaryon states from the QCDSR I and II respectively, which are shown explicitly in Table 1 and Figs.5-6 where the regions between the two vertical lines are the Borel windows. In Figs.5-6 we plot the masses and pole residues of the triply-charmed dibaryon states in much larger ranges than the Borel windows. Flat platforms appear at the Borel windows both for the masses and pole residues, it is reliable to extract the dibaryon masses. From Table 1 we can see that the central values of the dibaryon masses from the QCDSR I satisfy the energy scale formula $\mu = \sqrt{M^2_P - (3M_c)^2}$. If we take into account the two-baryon scattering states, we obtain a slightly smaller masses and pole residues for the dibaryon states from the QCDSR II, see Table 1 and Figs.5-6. The effects of the two baryon scattering states are rather small and can be neglected safely.

In Ref.11, the lattice QCD calculations indicate that the color-singlet-color-singlet type currents with $J^P = 1^+$ have non-vanishing couplings with the dibaryon states, which lie below the corresponding two-baryon thresholds. In Ref.9, the calculations based on the one-boson-exchange model indicate that there exist attractive interactions between the $\Xi_{cc}$ and $\Sigma_c$ in the channels $I(J^P) = \frac{1}{2}(0^+)$, $\frac{1}{2}(1^+)$ and $\frac{3}{2}(0^+)$. In Ref.8, the heavy antiquark-diquark symmetry is used to study the mass-splitting between the $J^P = 0^+$ and $1^+ \Xi_{cc}\Sigma_c$ dibaryon states in a model-independent way. In the present work, the predictions of the central values $M_S = 6.05$ GeV (6.04 GeV) and $M_A = 6.03$ GeV (6.02 GeV) from the QCDSR I (QCDSR II) are consistent with the previous works 8, 9, 11.
Figure 3: The absolute values of the contributions of the vacuum condensates of dimension $n$ for central values of the input parameters in the QCDSR I/II, where the $S$ and $A$ represent the scalar and axialvector dibaryon states respectively.

Figure 4: The contributions of the higher dimensional vacuum condensates with variations of the Borel parameters $T^2$ in the QCDSR I/II, where the $S$ and $A$ represent the scalar and axialvector dibaryon states respectively.
Figure 5: The masses of the dibaryon states with variations of the Borel parameters $T^2$ from the QCDSR I, where the $S$ and $A$ represent the scalar and axialvector dibaryon states respectively, the Pole+ΣΞ corresponds to the results from the QCDSR II.

Figure 6: The pole residues of the dibaryon states with variations of the Borel parameters $T^2$ from the QCDSR I, where the $S$ and $A$ represent the scalar and axialvector dibaryon states respectively, the Pole+ΣΞ corresponds to the results from the QCDSR II.
Figure 7: The parameters $\kappa$ with variations of the Borel parameters $T^2$ from the QCDSR III, where the $S$ and $A$ represent the scalar and axialvector dibaryon states respectively.

In Fig.7 we plot the parameters $\kappa_S$ and $\kappa_A$ with variations of the Borel parameters for the central values of the input parameters shown in Table 1. From the figure, we can see that the values $\kappa_S \ll 1$ and $\kappa_A \ll 1$, furthermore, the values of the $\kappa_S$ and $\kappa_A$ increase monotonically and quickly with increase of the Borel parameters, no flat platforms can be obtained. If we choose smaller energy scale, say $\mu = 1$ GeV (which is chosen in the QCD sum rules for the $\Sigma_c$ and $\Xi_{cc}$ [35, 36]), we can obtain very large values for the $\kappa_S$ and $\kappa_A$ with poor pole contributions, the values of the $\kappa_S$ and $\kappa_A$ decrease monotonically and quickly with increase of the Borel parameters, on the other hand, the convergent behaviors of the operator product expansion are very bad. We can draw the conclusion tentatively that the two-baryon scattering states cannot saturate the QCD sum rules and play a minor important role and can be neglected, which approves the observation obtained in Sect.2, the molecular states begin to receive contributions at the order $\mathcal{O}(\alpha_s^0)$, not at the order $\mathcal{O}(\alpha_s^2)$.

4 Conclusion

In this article, we construct the color-singlet-color-singlet type currents to interpolate the scalar and axialvector $\Xi_{cc}$, $\Sigma_c$ dibaryon states, and study them with QCD sum rules in details by carrying out the operator product expansion up to the vacuum condensates of dimension 15. At the hadron side, we take into account both the contributions of the dibaryon states and two-baryon scattering sates, and explore the existence or nonexistence of the $\Xi_{cc}$, $\Sigma_c$ dibaryon states in three cases. As there exist three valance $c$-quarks, the QCD sum rules are sensitive to the $c$-quark mass or the energy scale of the QCD spectral densities, we determine the best energy scales with the energy scale formula $\mu = \sqrt{M_D^2 - (3M_c)^2}$. The numerical results indicate that the two-baryon scattering states cannot saturate the QCD sum rules and play a minor important role, the dominant contributions come from the dibaryon states. Or it is necessary to introduce the dibaryon states to embody the net effects. The color-singlet-color-singlet type currents couple potentially to the molecular states, not to the two-particle scattering states. In the operator product expansion, the molecular states begin to receive contributions at the order $\mathcal{O}(\alpha_s^0)$, not at the order $\mathcal{O}(\alpha_s^2)$. We can search for the triply-charmed dibaryon states at the LHCb, Belle II, CEPC, FCC, ILC in the future.
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