Separation of Gravitational-Wave and Cosmic-Shear Contributions to Cosmic Microwave Background Polarization

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Inflationary gravitational waves (GW) contribute to the curl component in the polarization of the CMB. Cosmic shear—gravitational lensing of the CMB—converts a fraction of the dominant gradient polarization to the curl component. Higher-order correlations can be used to map the cosmic shear and subtract this contribution to the curl. Arcminute resolution will be required to pursue GW amplitudes smaller than those accessible by Planck. The finite cutoff in CMB power at small scales leads to a minimum detectable GW amplitude corresponding to an inflation energy near $10^{15}$ GeV.

Observation of acoustic oscillations in the temperature anisotropies of the cosmic microwave background (CMB; ) strongly suggests an inflationary origin for primordial perturbations . It has been argued that a new smoking-gun signature for inflation would be detection of stochastic background of inflationary gravitational waves (IGWs) . These IGWs produce a distinct signature in the CMB in the form of a contribution to the curl, or magnetic-like, component of the polarization . Since there is no scalar, or density-perturbation, contribution to these curl modes, curl polarization was considered to be a direct probe of IGWs.

There is, however, another source of a curl component. Cosmic shear (CS)—weak gravitational lensing of the CMB due to large-scale structure along the line of sight—results in a fractional conversion of the gradient mode from density perturbations to the curl component . The amplitude of the IGW background varies quadratically with the energy scale $E_{\text{infl}}$ of inflation, and so the prospects for detection also depend on this energy scale. In the absence of CS, the smallest detectable IGW background scales simply with the sensitivity of the CMB experiment—as the instrumental sensitivity is improved, smaller values of $E_{\text{infl}}$ become accessible . More realistically, however, the CS-induced curl introduces a noise from which IGWs must be distinguished. If the IGW amplitude (or $E_{\text{infl}}$) is sufficiently large, the CS-induced noise will be no problem. However, as $E_{\text{infl}}$ is reduced, the IGW signal becomes smaller and will get lost in the CS-induced noise. This confusion leads to a minimum detectable IGW amplitude .

In addition to producing a curl component, CS also introduces distinct higher-order correlations in the CMB temperature pattern. Roughly speaking, lensing can stretch the image of the CMB on a small patch of sky and thus lead to something akin to anisotropic correlations on that patch of sky, even though the CMB pattern at the surface of last scatter had isotropic correlations. By mapping these effects, the CS can be mapped as a function of position on the sky . The observed CMB polarization can then be corrected for these lensing deflections to reconstruct the intrinsic CMB polarization at the surface of last scatter (in which the only curl component would be that due to IGWs). In this Letter we evaluate how well this subtraction can be accomplished and study the impact of CS on experimental strategies for detection of IGWs.

To begin, we review the determination of the smallest detectable IGW amplitude in the absence of CS. Following Ref. , we consider a CMB-polarization experiment of some given instrumental sensitivity quantified by the noise-equivalent temperature (NET) $s$, angular resolution $\theta_{\text{FWHM}}$, duration $t_{\text{yr}}$ in years, and fraction of sky covered $f_{\text{sky}}$. We then make the null hypothesis of no IGWs and determine the largest IGW amplitude $T$, defined as $T = 9.2 V/m^4$, where $V = E_{\text{infl}}^4$ is the inflaton-potential height, that would be consistent at the 1σ level with the null detection. We then obtain the smallest detectable IGW amplitude $\sigma_T$ from

$$\sigma_T^{-2} = \sum_{l>180/\theta} \left( \frac{\partial C_{l,BB,GW}^B}{\partial T} \right)^2 (\sigma_{l,BB})^{-2},$$

where $C_{l,BB,GW}^B$ is the IGW contribution to the curl power spectrum, and

$$\sigma_{l,BB} = \sqrt{\frac{2}{f_{\text{sky}}(2l + 1)}} \left( C_{l,BB} + f_{\text{sky}} w^{-1} e^2 \sigma_s^2 \right),$$

is the standard error with which each multipole moment $C_{l,BB}$ can be determined. Here, $w^{-1} = 4\pi(s/T_{\text{CMB}})^2/(t_{\text{pix}} N_{\text{pix}})$ is the variance per unit area on the sky for polarization observations when $t_{\text{pix}}$ is the time spent on each of $N_{\text{pix}}$ pixels with detectors of NET $s$, and $\theta \approx 203 f_{\text{sky}}^{1/2}$ is roughly the width (in degrees) of the survey. In restricting the sum to $l > 180/\theta$, we have assumed that no information from modes with wavelengths...
FIG. 1. Minimum inflation potential observable at $1\sigma$ as a function of survey width for a one-year experiment. The left panel shows an experiment with NET $s = 25$ $\mu$K $\sqrt{\text{sec}}$. The solid curve shows results assuming no CS while the dashed curve shows results including the effects of an unsubtracted CS; we take $\theta_{\text{FWHM}} = 5'$ in these two cases. The dotted curves assume the CS is subtracted with $\theta_{\text{FWHM}} = 10'$ (upper curve) and 5' (lower curve). Since the dotted curves are close to the dashed curve, it shows that these higher-order correlations will not be significantly useful in reconstructing the primordial curl for an experiment similar to Planck’s sensitivity and resolution. The right panel shows results for hypothetical improved experiments. The dotted curves shows results with CS subtracted and assuming $s = 1$ $\mu$K $\sqrt{\text{sec}}$, $\theta_{\text{FWHM}} = 1'$, 2', and 5' (from top to bottom). The solid curve assumes $\theta_{\text{FWHM}} = 1'$ and $s = 1$ $\mu$K $\sqrt{\text{sec}}$, and no CS, while the dashed curve treats CS as an additional noise. The long-dash curve assumes CS subtraction with no instrumental noise ($s = 0$).

larger than the survey size can be obtained; in fact, some information can be obtained, and our results should thus be viewed as conservative [7].

The second term in Eq. (2) is due to instrumental noise, and the first is due to cosmic variance. In the absence of CS, and for the null hypothesis of no IGWs, we set $C_{l}^{BB} = 0$, and the results for the smallest detectable IGW amplitude are shown as the solid curves in Fig. 1 for an experiment with detectors of comparable sensitivity to Planck’s (left) and a hypothetical experiment (right) with better sensitivity. The smallest detectable IGW amplitude $T_{l}$ scales as $s^{2}t_{\gamma}^{-1}$. For large survey widths, it scales as $\theta_{l}$, but at survey widths smaller than $\sim 5^\circ$ it increases because information from the larger-angle modes in the IGW-induced curl power spectrum is lost (cf. the IGW power spectrum in Fig. 2).

It is now easy to see how inclusion of CS affects these results. As discussed above, lensing of the gradient polarization at the surface of last scatter due to density perturbations leads to a CMB curl component with a power spectrum,

$$
\tilde{C}_{l}^{BB} = \frac{1}{2} \int \frac{d^{2}l_{1}}{(2\pi)^{2}} \left| l_{2} \cdot l_{1} \right|^{2} (1 - \cos 4\phi_{l_{1}}) C_{l_{2}}^{\phi\phi} C_{l_{1}}^{EE},
$$

where $l_{2} = 1 - l_{1}$ here and throughout, $C_{l}^{EE}$ is the power spectrum of the gradient component of polarization and $C_{l}^{\phi\phi}$ is the power spectrum of the projected lensing potential [10]. The latter is defined in terms of the potential fluctuations, $\Phi$, along the line of sight such that

$$
\phi(\hat{n}) = -2 \int_{0}^{r_0} \frac{dr}{dA(r)} \frac{dA(r_0 - r)}{dA(r_0)} \Phi(r, \hat{n}r),
$$

where $r$ is the comoving radial distance, or conformal look-back time, with $r_0$ at the last scattering surface, and $dA(r)$ is the comoving angular diameter distance. The
CS-induced curl power spectrum is shown as the dashed curve in Fig. 3. By the time these measurements are made, the cosmological parameters that determine this lensed curl power spectrum should be sufficiently well determined that this power spectrum can be predicted with some confidence. In that case, the CS-induced curl component can be treated simply as a well-understood noise for the IGW background. The smallest detectable IGW amplitude can then be calculated as above, but now inserting the lensed power spectrum, Eq. (3), in Eq. (2) for $C_{BB}$. The results are shown as the short-dash curves in Fig. 1. When lensing is included, the results no longer scale simply with $f_{\text{sky}}$, $s$, or $\tau_{\text{yr}}$, as there is now a trade-off between the instrumental-noise and CS-noise terms in Eq. (3). The left panel shows that the IGW sensitivity for an experiment with NET similar to Planck’s should not be affected by CS. This is because the IGW amplitudes that could be detectable by such experiments are still relatively large compared with the expected CS signal, especially at the larger angles that will be best accessed by Planck. However, CS will affect the ability of experiments more sensitive than Planck to detect unambiguously IGWs, as shown in the right panel. CS also shifts the preferred survey region to larger areas, as the IGW power spectrum peaks at larger angles than the CS power spectrum (cf. Fig. 2). Finally, note that if the CS curl is treated as an unsubtracted noise, it leads, assuming a no-noise polarization map, to a smallest detectable IGW is treated as an unsubtracted noise, it leads, assuming a spectrum (cf. Fig. 2). Finally, note that if the CS curl component can be subtracted by mapping correlations of galaxy ellipticities [11]. However, there will be some error in the CS reconstruction, leading to no residual lensing-induced curl component. Realistically, however, there will be some error in the CS reconstruction, from measurement error and also from cosmic variance, with a noise power spectrum, $C_{\phi \phi, \text{noise}}$, given by $N_l/l^2$. Since $C_{\phi \phi, \text{noise}} \gg C_{\phi \phi, \text{lensed}}$ along the damping tail of the CS power spectrum, $N_l$ converges to a finite value such that beyond a certain small angular scale, additional information from CMB provides no further information on CS. This leads to a finite limit on lensing extraction and, subsequently, a limit for the amplitude of IGW contribution that can be separated from CS.

Here the ensemble average is taken independently over realizations of both the CMB and the intervening large-scale structure. In addition to these temperature estimators for the deflection angle, we also use analogous ones constructed from the polarization, as discussed in Ref. [13], although we do not reproduce those formulas here. The total noise in the estimator for the deflection angle can then be constructed by summing the inverses of the individual noise contributions. We thus determine the variance with which each Fourier mode of $\phi$ can be reconstructed.

With the deflection angle obtained this way as a function of position on the sky, the polarization at the CMB surface of last scatter can be reconstructed (details to be presented elsewhere [14]). In the ideal case, there would be no error in the CS reconstruction leading to no residual lensing-induced curl component. Realistically, however, there will be some error in the CS reconstruction, from measurement error and also from cosmic variance, with a noise power spectrum, $C_{\phi \phi, \text{noise}}$, given by $N_l/l^2$. Since $C_{\phi \phi, \text{noise}} \gg C_{\phi \phi, \text{lensed}}$ along the damping tail of the CMB power spectrum, $N_l$ converges to a finite value such that beyond a certain small angular scale, additional information from CMB provides no further information on CS. This leads to a finite limit on lensing extraction and, subsequently, a limit for the amplitude of IGW contribution that can be separated from CS.

The lensing reconstruction from CMB data only allows the extraction of $C_{\phi \phi}$ up to a multipole of $\lesssim 1000$ [13], but there is substantial contribution to the CS-induced curl component from lensing at smaller angular scales. We thus replace $N_l/l^2$ by $C_{\phi \phi}$ when the former exceeds the latter at large $l$. This provides an estimate to the noise expected in the reconstructed curl component that follows from implementing a filtering scheme where high-frequency noise in the CS reconstruction is removed to the level of the expected CS signal. The dot-dash curve in Fig. 3 shows the residual CS-induced curl component that remains after subtraction.

We can now anticipate the smallest IGW amplitude detectable by a CS-corrected polarization map by simply...
achieved by relaxing our assumption that power in the
of IGWs. Another improvement in this limit may be
angle CMB polarization \[15\] improving the detectability
functions and higher-order correlations may possibly im-
the inclusion of the
have used only the lowest-order temperature-polarization
this lower limit may be improved upon. First of all, we
small angular scales. There are several possible ways
here comes from the existence of finite CMB power on
reconstructable IGW amplitude using the techniques considered
an ultimate limit of roughly
higher sensitivity and resolution than Planck. An ul-
constructed with temperature and polarization maps of
likely to improve the IGW discovery reach of Planck. To
amplitudes. If there is no instrumental-noise limitation,
we would do just as well to simply treat the CS-induced
curl as a noise component of known amplitude. We can
expect to improve the discovery reach for IGWs by in-
correlations. We find that the CS reconstruction is un-
subtracted with a CS map obtained with higher-order
curl that is either modeled as an unsubtracted noise or
exists by mapping the curl component of the CMB po-
2
\times 10^{15} \text{GeV}, as discussed above. Correction for
small-scale power recently detected by CBI \[16\] comes
CMB drops exponentially at small scales. If the excess
from high redshifts, then there will be more small-scale
cohere patches with which to reconstruct the CS. In
this case, it is imaginable that a far more precise CS map
be reconstructed, but this might require even better
angular resolution and sensitivity.

During the preparation of the paper, we learned of
other very recently completed work by Knox and Song
\[17\] that performs a very similar calculation and reaches
similar conclusions. This work was supported in part by
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To conclude, we have studied the IGW amplitudes ac-
ning by mapping the curl component of the CMB po-
effects of CS with the CS map inferred from higher-
order correlations would allow us to access lower IGW
abilities. If there is no instrumental-noise limitation,
the sensitivity to an IGW signal is maximized by cov-
ering as much sky as possible, and the lowest accessible
inflaton potential, \(\sim 10^{15} \text{GeV}\), is obtained with a nearly
all-sky experiment.

To determine the ultimate limits of this class of
experiments, the long-dash curve in Fig. 1 shows the re-
results assuming perfect detectors (i.e., \(s = 0\)). If there
were no CS-induced curl, then we would have sensitiv-
ty to an arbitrarily small IGW amplitude, but the exis-
tence of a CS-induced curl provides an ultimate limit of
\(V^{1/4} \sim 4 \times 10^{15} \text{GeV}\), as discussed above. Correction for

\[V^{1/4} \sim 4 \times 10^{15} \text{GeV}\]

\[\text{for a hypothetical experiment with } s = 1 \mu \text{K} \sqrt{\text{sec}} \text{ and angular resolutions of } 5', 2' \text{ and } 1'. \text{ We now see there is a significant difference be-
tween the dashed curve and the dotted lines suggesting the}
increasing improvement with increasing resolution.

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