Free spectral range measurement of a fiberized Fabry-Perot etalon with sub-Hz accuracy

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Free spectral range measurement of a fiberized Fabry–Perot etalon with sub-Hz accuracy

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Abstract: In this work a narrow linewidth (1 kHz) laser source is used to measure the free spectral range of a fiberized Fabry-Perot etalon with sub-Hz accuracy ($10^{-8}$). A previously demonstrated technique based on the Pound-Drever-Hall error signal is improved in accuracy by the use of a narrow linewidth laser swept in frequency via an acousto-optic modulator, or single sideband generation. The sub-Hz ($10^{-8}$) accuracy attained enables the characterization of both the long-term drift and the polarization dependence of the free spectral range of the fiberized etalon.

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1. Introduction

Fabry-Perot etalons are multi-interference optical resonators having a periodic in frequency amplitude and phase response. In the past years the advancement of thin film and micro-toroid technologies have enabled the development of optical resonators having both high finesse ($Q$) and large frequency separation of successive resonances, or free spectral range (FSR) [1].
Etalons have been used to reduce the linewidth of lasers [2], for frequency multiplication of low repetition rate pulse trains [3], for astronomical instrument calibration [4], to suppress the timing jitter of harmonically mode-locked lasers [5], and recently to improve the performance of optoelectronic oscillators [6].

In most cases, it is advantageous to reference at least one optical tone to the etalon, or vice versa. This is performed by generating an error signal and feeding it back to either alter the frequency of the optical tone, or the center frequency of the resonator. Multiple setups have been demonstrated for the generation of the error signals. Referencing techniques include polarization-based effects [7] and spatial mode interference [8]. Yet, the most widely used cavity referencing mechanism uses phase modulation sidebands on the optical tone; it is called the Pound-Drever-Hall (PDH) technique [9,10].

For most applications based on optical resonators, precise knowledge of the FSR is required. Most FSR measurement techniques make use of the intensity response of etalons (Fig. 1 (a)), which have a flat response for small deviations from the center of the resonance [11,12]. On the other hand, a technique based on a modified PDH error signal has been demonstrated with accuracy of 1 part in $10^4$ [13]. The measurements performed were limited by the linewidth of the probing laser and the finesse of the etalon used.

In this work, the performance of the modified PDH technique described in [13] is analyzed revealing the requirement of a narrow linewidth frequency swept laser. This is achieved by incorporating a 1 kHz linewidth laser swept in frequency using an acousto-optic modulator (AOM). The developed scheme is used for the measurement of the FSR of an etalon with sub-Hz ($10^{-8}$) accuracy. The high accuracy achieved enables the experimental measurement of both the polarization dependence of the FSR and its long-term drift.

2. The Pound-Drever-Hall technique for free spectral range measurement

In the PDH technique a phase modulator (PM) is driven at a fixed frequency generating sidebands of opposite phase. The PDH error signal is formed when the frequency of the laser along with the phase modulation sidebands are swept through an etalon resonance. Upon reflection from the etalon the central optical tone experiences nonlinear amplitude and phase response, as seen in Fig. 1(a), while the optical sidebands are used as a reference. The reflected signal is photodetected and subsequently down-converted to the baseband using a frequency mixer and a low pass filter. The error signal generated is given by the formula:

$$V(v,\phi) \propto C \cdot \text{Re} \left\{ R(v) \cdot R\left(v + f_{PM}\right) - R(v) \cdot R\left(v - f_{PM}\right) \cdot e^{i\phi} \right\}, \text{ with } R(v) = \frac{r \cdot \left(e^{-2 \cdot \pi \cdot v/FSR} - 1\right)}{1 - r^2 \cdot e^{-2 \cdot \pi \cdot v/FSR}}, \quad (1)$$

where $v$ is the frequency of the laser, $f_{PM}$ is the phase modulation frequency, $\phi$ is the phase mismatch of the electrical arms at the mixer and $*$ denotes the complex conjugate. The proportionality constant $C$ in Eq. (1) is related to the responsivity of the photodetector and the electrical amplification of the signal among other parameters that do not affect its shape. $R$ is the etalon electric field reflection coefficient (Fig. 1(a)) and $r$ the mirror reflectivity, assuming identical etalon mirrors. Typically, the PDH error signal used in referencing is generated by using $\phi = 90^\circ$ and modulation frequency at 10-40% of the FSR. The generated error signal has a sharp discriminant at its center [10].

In the case of the FSR measurement first described in [13], the error signal is observed as the modulation frequency is tuned for a phase mismatch $\phi = 0^\circ$. When the modulation sidebands are within the adjacent resonances of the etalon the PDH error signal resembles the shapes shown in the simulation in Fig. 1(b), which incorporates the effect of electrical noise with amplitude of 0.05 V. It should be noted that the shape of the error signal is preserved as the modulation frequency is varied through the FSR frequency, but it changes in amplitude. Specifically, Fig. 1(c) shows that the error signal vanishes when the modulation frequency matches the FSR and changes sign around the resonances. This effect is used for the measurement of the FSR.
Fig. 1. Simulation of the Pound-Drever-Hall error signal for an etalon having FSR = 100 MHz and $\tilde{\nu} = 100$. (a) Power and phase response of the transmitted beam as a function of the optical frequency. (b) Error signal voltage generated using phase modulation frequencies in the proximity of the etalon FSR. The simulation includes the effect of electrical noise with amplitude of 0.05 V. (c) Simulation of the peak-to-peak voltage of the error signal as the phase modulation frequency is varied for values close to the etalon FSR, with noise amplitude = 0.01 V. The red dot denotes the resolution limit of the simulation.

In practice, the peak-to-peak voltage ($V_{pp}$), is recorded as a function of the modulation frequency (Fig. 1(c)). The resolution limit of this technique is realized when the signal to noise ratio, approaches unity, as indicated by the red dot in Fig. 1(c). A significant advantage of the modified PDH technique is that the error signal is a product of the phase response of the resonator, which varies linearly at around the peak of the resonances in comparison to transmittance measurement techniques that have flat response in the same area of interest. Moreover, it should be noted that the slope of the phase response versus frequency detuning increases linearly with the resonator finesse (for a given FSR) as a result of a reduction in the resonance bandwidth. This effect implies that the relative accuracy of this technique depends on the resonator’s finesse. The high accuracy attained by the combination of the modified PDH technique with the frequency swept narrow linewidth laser enables the characterization of Fabry-Perot etalons and resonators with FSRs of up to tens of gigahertz, where phase modulators are available.

3. Experimental setup

Although the modified PDH technique has the potential for high accuracy measurements, it can be limited by the linewidth of the laser in use. In the previously published work a (semiconductor) distributed feedback (DFB) laser with MHz linewidth was used to probe etalons having $\tilde{\nu} = 100$, with FSR = 10 GHz, which corresponds to resonances having $\Delta f_{FWHM} = 100$ MHz [13]. In that work, the laser was frequency swept via current modulation. However, DFB lasers have inadequate linewidth for etalons having $\Delta f_{FWHM} < 1$ MHz, thus the FSR measurement of sharp resonances can only be enabled by the use of narrow linewidth lasers. In addition, the required narrow linewidth sources have to be frequency swept with sufficient linearity. In general, narrow laser linewidth is a product of cavity invariance though a frequency sweep implies a modulation of the laser cavity. Thus, for narrow linewidth lasers, the frequency sweep cannot be generated by an intrinsic property of the laser and has to be implemented externally.

In this work, a commercially available, narrow linewidth laser source at 1550 nm is used in conjunction with an external frequency sweep implemented using an AOM. The linewidth of the laser is measured via an optical heterodyne beat of two similar lasers and the deconvolved linewidth is measured to be 1 kHz for a measurement of 1 s (Fig. 2(a)), assuming a Lorentzian distribution.
The AOM used is fiberized and the maximum power in the 1st order deflected beam is obtained by a sinusoidal drive signal at 100 MHz. Scanning of the optical frequency is attained by sweeping the RF frequency driving the AOM. Nevertheless, a change in the driving frequency of the AOM results in a change of the direction of the diffracted 1st order, therefore a deviation from the optimized frequency results in a reduction of the beam power due to reduced fiber coupling. Therefore, a reasonable bandwidth of operation can be defined as the 3 dB, or full width at half maximum (FWHM) bandwidth, usually ~10% of the nominal frequency of the AOM, or 10 MHz for the AOM used. It should be noted that variations of the optical power input to the resonator do not affect the experiment significantly, since the RF signal generated by the photodetector is directed at the signal port of an electrical mixer. The mixer output is determined by the power at the local oscillator port (under saturation), which for the experiment presented is the phase modulation frequency designed to have sufficient power.

In the experiment conducted, the AOM is driven by an RF signal frequency swept between 95 MHz and 105 MHz, as shown in Fig. 2(b). This RF swept signal is generated as the filtered intermodulation product of a mixed local oscillator at 155 MHz with an RF swept source (50 MHz to 60 MHz). This setup resulted in sufficient linearity of the RF frequency sweep, as seen in Fig. 2(b), without any spurious tones in the band of interest. The linear RF frequency sweep translates to a linear optical frequency sweep, using the AOM.

![Fig. 2. (a) Linewidth measurement of the laser source in use. The linewidth of the laser is measured using a heterodyne beat between a pair of similar laser sources to a deconvolved 1 kHz for 1 s. (b) Spectrogram of the frequency swept electrical drive of the acousto-optic modulator. Note the sufficient linearity of the driving signal for the generated 10 MHz sweep.](image)

![Fig. 3. (a) Experimental setup schematic. A narrow linewidth laser is frequency swept externally using an acousto-optic modulator driven by the appropriate frequency swept electrical signal. The swept laser source is used in a modified Pound-Drever-Hall setup to measure the free spectral range. VDC, voltage source; PC, polarization controller; AOM, acousto-optic modulator; BPF, band-pass filter; Amp, voltage amplifier; RF, radiofrequency synthesizer; PM, phase modulator; PS, phase shifter; Circ, circulator; PD, photodetector; LPF, low-pass filter. (b) Experimental results for the etalon free spectral range measurement. The peak-to-peak voltage of the error signal is recorded as a function of the phase modulation frequency. The FSR for the fiberized etalon is measured with sub-Hz accuracy 10^-8.](image)
4. Free spectral range characterization of a fiberized etalon

The complete system including the modified PDH setup, depicted in Fig. 3(a), is used for the measurement of the FSR of a commercially available fiberized etalon. The etalon is developed using ~1.02 m of dispersion shifted fiber having zero dispersion wavelength at ~1555 nm. The etalon is specified for FSR = 100 MHz with 1% accuracy and \( \Delta f \geq 100 \), which corresponds to resonances having \( \Delta f_{\text{FWHM}} \leq 1 \) MHz. The FSR of this etalon could not be measured using a DFB laser in the same technique, since the resonances are narrower than the linewidth of the DFB laser available.

The FSR of the fiberized etalon is first measured using sidebands that probe the resonance at \( \pm 5 \times \text{FSR} \), resulting in an accuracy of better than 1 kHz. Using this result, the etalon is subsequently probed with phase modulation sidebands at \( \pm 100 \times \text{FSR} \) to increase the accuracy. The results are presented in Fig. 3(b) divided by 100 to enable the calculation of the etalon FSR. As seen in the figure, the change in sign of \( V_{\text{PP}} \) can be detected for a minimum step of 1 Hz (100 Hz for the raw data). The FSR is experimentally measured to: \( \text{FSR} = 99.579 \pm 0.5 \) Hz, with a relative error of less than \( 10^{-8} \). This corresponds to an error in the measurement of the length of the meter-long fiber of ~5 nm, or 0.3% of the wavelength of the light used. It should be noted that the accuracy of the technique is extended beyond the limitation imposed by the linewidth of the laser, by using phase modulation sidebands at 100 \( \times \) FSR.

The increased accuracy attained by the use of the narrow linewidth laser is used to further study the properties of the fiberized etalon. In the above analysis the polarization dependence of the fiberized etalon FSR has been disregarded. Although the etalon is developed using non-polarization maintaining fiber, mechanical stress of the fiber can induce residual birefringence. Thus, the etalon exhibits two different polarization eigenstates which experience two slightly different optical path lengths in the fiberized etalon. This is demonstrated in Fig. 4(a) where the high resolution error signal of the two polarization etalon resonances is seen with varying amplitude depending on the laser polarization. For the laser at 1550 nm the two polarization resonances are separated by ~18.8 MHz, or ~19% of the FSR. It should be noted that the two FSRs of the polarization states cannot be measured unless a narrow linewidth laser is used.

The difference between the FSRs of the two polarizations is measured to be: \( 29 \pm 0.5 \) Hz, or a \( \sim 3 \cdot 10^{-7} \) relative difference. This corresponds to a differential optical path length difference of ~300 nm, or approximately \( \frac{1}{5} \) of the wavelength. The specified polarization mode dispersion for the fiber of the etalon is 0.020 ps / km^{\frac{1}{2}} \), which corresponds to 190 nm path length difference

Fig. 4. (a) Polarization dependence of the free spectral range (FSR) of the etalon. Note that two resonance peaks appear on the black curve when the polarization input to the etalon is not adjusted, while a single peak of higher amplitude remains on the blue curve as the polarization input to the etalon is aligned to one of the polarization eigenstates of the etalon. (b) Variance of the FSR of the fiberized etalon as a function of time. Note that the FSR varies \( \sim 400 \) Hz in a 9 hour span.
for the round-trip length of the etalon, or $0.92 \times 10^{-7}$. This verifies the validity of the measurement, since the experimentally measured polarization mode dispersion demonstrated as the difference in the FSRs of the two polarization states is in the same order of magnitude with the fiber used in the etalon.

In addition, the high accuracy FSR measurement technique presented enables a study of the variation of the FSR due to the drift of the etalon fiber length. Shown in Fig. 4(b) is the FSR of the fiberized etalon as recorded during the working day with sub-Hz accuracy. It can be seen that the FSR of the etalon varies by ~400 Hz, or $4 \times 10^{-6}$, in 9 hours.

The thermal expansion coefficient of fused silica, the material used in the fiberized etalon, is specified to $\Delta L / L = 5.5 \times 10^{-7} \, ^\circ C^{-1}$, which corresponds to: $\Delta(FSR) = 55 \, Hz / ^\circ C$ for the ~99.58 MHz etalon. Thus, the 400 Hz variation of the etalon FSR corresponds to 7.3 $^\circ C$ change in the temperature of the etalon during the day, half of which happens during the “cold start” of the system. This temperature variation is reasonable, considering the laboratory conditions, also taking into consideration the power stored in the resonator, which is in the hundreds of mW.

The optical frequency sweep demonstrated is also used to probe the transmission peaks of the etalon by disengaging the PDH part of the setup. In this scheme the power transmitted through the etalon is measured as the frequency of the laser is linearly swept in time, as in [11]. The time can be converted to optical frequency, due to the linearity of the frequency sweep. The measurement is repeated for various sweep times and frequency spans and the resonance width is calculated as: $\Delta_f^{\text{FWHM}} = 0.624 \, \text{MHz} \pm 0.019 \, \text{MHz}$ which, given the FSR measured, corresponds to $f = 160 \pm 5$. Therefore, the difference in accuracy between the phase-sensitive modified PDH measurement ($10^{-8}$) and that of the direct power measurement ($5/160 = 3\%$) is demonstrated.

For etalons or resonators with narrow resonances, active locking of the laser to one of the transmission peaks will be beneficial, if not necessary, for both reducing the laser’s linewidth and minimizing the relative drift between the laser and the resonator. Moreover, the linear sweep of the laser can be performed using single sideband (SSB) generation, where a specialized phase modulator driven with the appropriate signal is used to shift an optical tone by the RF driving frequency [14]. A sweep in the RF frequency is translated to an optical frequency sweep, as with the AOM technique. In effect, this technique can surpass the frequency sweep limitation of the AOM (at the expense of increased complexity) and was experimentally implemented without any loss of accuracy in the measurements. From the results presented in this work, the SSB generation is used to provide the wide span (30 MHz) results of Fig. 4(a), which cannot be attained by the 10 MHz limited sweep of the AOM.

5. Conclusion

In this work a 1 kHz linewidth laser source is combined with an AOM (or a SSB generation setup) to induce a linear optical frequency sweep. The linearly swept narrow linewidth laser is used in a (previously demonstrated) modified PDH scheme to measure the free spectral range of a fiberized etalon with sub-Hz accuracy. The free spectral range of the etalon used is measured to be: 99 579 920.5 $\pm$ 0.5 Hz, or a relative error of less $10^{-6}$. The PMD of the fiber results in an experimentally measured 29 Hz difference between the free spectral ranges of the two polarization states. The thermal drift of the fiber is measured to be 400 Hz in a 9 hour interval, which corresponds to 7.3 $^\circ C$ change in the temperature of the etalon. The modified PDH technique offers increased accuracy due to the linear phase response of etalons around their resonances, while the use of a narrow-linewidth laser combined with an AOM-induced sweep improves upon the accuracy of the technique by a factor of $10^4$. In conclusion, this technique is expected to result in higher accuracy for resonators having higher finesse.

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