The clustering of the SDSS-IV extended Baryon Oscillation Spectroscopic Survey DR16 luminous red galaxy and emission line galaxy samples: cosmic distance and structure growth measurements using multiple tracers in configuration space

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ABSTRACT

We perform a multi-tracer analysis using the complete Sloan Digital Sky Survey IV (SDSS-IV) extended Baryon Oscillation Spectroscopic Survey (eBOSS) DR16 luminous red galaxy (LRG) and the DR16 emission line galaxy (ELG) samples in the configuration space, and successfully detect a cross correlation between the two samples, and find $f\sigma_8 = 0.342 \pm 0.085$ ($\sim 25$ per cent accuracy) from cross sample alone. We perform a joint measurement of the baryonic acoustic oscillations (BAO) and redshift space distortions (RSD) parameters at a single effective redshift of $z_{\text{eff}} = 0.77$, using the auto- and cross-correlation functions of the LRG and ELG samples, and find that $D_N(z_{\text{eff}})/r_d = 18.93 \pm 0.37$, $D_M(z_{\text{eff}})/r_d = 19.65 \pm 0.54$, and $f\sigma_8(z_{\text{eff}}) = 0.445 \pm 0.038$, which is consistent with a $\Lambda$CDM model at 68% CL. Compared to the single-tracer analysis on the LRG sample, the Figure of Merit (FoM) of $\alpha_\perp, \alpha_\parallel$ and $f\sigma_8$ gets improved by a factor of 1.18 in our multi-tracer analysis, and in particular, the statistical uncertainty of $f\sigma_8$ is reduced by 11.6%.

Key words: large scale structure of the Universe; multi-tracer analysis; cosmological parameters

1 INTRODUCTION

Observations of the large-scale structure of the Universe provide an essential probe of the physics of the accelerating cosmic expansion, which was discovered by the observation of type Ia supernovae (Riess et al. 1998; Perlmutter et al. 1999). The clustering analysis of large-scale structure allows us to measure the cosmic expansion history and structure growth via signals of baryon acoustic oscillations (BAO) and redshift space distortions (RSD), respectively (Cole et al. 1995; Peacock et al. 2001; Cole et al. 2005; Hawkins et al. 2003; Eisenstein et al. 2005; Okumura et al. 2008; Percival & White 2009). The BAO, produced by the competition between gravity and radiation due to the coupling between baryons and photons before the cosmic recombination, leaves an imprint on the distribution of galaxies at late times. After the decoupling of photons, the acoustic oscillations are frozen at a characteristic scale around $\sim 100 \, h^{-1}\text{Mpc}$, which is determined by the comoving sound horizon at the drag epoch $r_d$. This feature corresponds to an excess in the 2-point correlation function, or a series of wiggles in the power spectrum (Percival et al. 2001; Cole et al. 2005; Eisenstein et al. 2005), making BAO a robust observable as a cosmic standard ruler. Measuring the BAO scale in the radial and transverse directions provides strong constraints on Hubble expansion rate and angular diameter distance, respectively. The RSD is produced due to peculiar motions of galaxies: galaxies tend to infall towards the local overdensity regions, thus the clustering along the line-of-sight (LOS) is enhanced on large scales (Kaiser 1987; Peacock et al. 2001). Thus measuring RSD effect set a constraint on the growth rate of cosmic structure.

The most precise BAO and RSD measurements to date were reported by the Baryon Oscillation Spectroscopic Survey (BOSS) collaboration using the final Data Release 12 (DR12) (Alam et al. 2015), which contains more than one million galaxies with spectroscopic redshifts. BOSS achieved a $1.0 - 2.5$ per cent BAO measurement precision and a $9.2 \, \text{per cent}$ RSD precision in the redshift range of $0.2 < z < 0.75$ (Alam et al. 2016), and extracted tomographic information of galaxy clustering in the past lightcone (Wang et al. 2017b; Zhao et al. 2017b; Wang et al. 2018b; Zheng et al. 2019), which is key for probing dynamical dark energy (Zhao et al. 2017a; Wang et al. 2018a). The BOSS DR12 data can provide high-precision constraints on cosmological parameters (Ivanov et al. 2020; D’Amico et al. 2020; Philcox et al. 2020). The extended BOSS (eBOSS) project, the successor of BOSS, aims to map the Universe using multiple galaxies at higher redshifts, covering the redshift range of $0.6 < z < 2.2$ (Dawson et al. 2016). It allows for BAO and RSD measurements at high redshifts, which is crucial to break degeneracy between key cosmological parameters, e.g. $H_0$ and $\Omega_m$ (Wang et al. 2017a).

However, the precision of the measurements of galaxy clustering is restricted by the cosmic variance on large scales due to the limited volume that a galaxy survey can map, and by the shot noise on small scales due to the discreteness of galaxies. One potential way to tackle the cosmic variance is to contrast multiple tracers of the dark matter field with different biases, i.e. the ‘multi-tracer’ technique (McDonald & Seljak 2009; Seljak 2009). In the ideal case with no shot noise, the ratio of over-densities of two tracers would be independent of the density field of dark matter, then the measurements of parameters related to the bias parameter can be immune to the cosmic variance, and thus they can be accurately determined. For practical applications, the gain from multiple tracers can be downgraded by various factors including the overlapping redshift ranges and sky regions, the ratio of biases, the Poisson noise of the 2-point function of each tracer, etc. Multi-tracer studies of galaxy surveys have been performed; for instance, Blake et al. (2013) found a $10 - 20$ per cent improvement on the RSD measurement via the multi-tracer analysis of the Galaxy and Mass Assembly survey (GAMA). This technique was also applied to analysing the galaxy clustering in the overlapping region between the BOSS and WiggleZ surveys (Ross et al. 2014; Beutler et al. 2016; Marin et al. 2016).

The eBOSS survey, which is a part of the Sloan Digital Sky Survey-IV (SDSS-IV) project (Blanton et al. 2017), used the 2.5-metre Sloan telescope (Gunn et al. 2006) located at the Apache Point Observatory in New Mexico. The spectra of samples are collected by the two multi-object fiber spectrographs (Smee et al. 2013). eBOSS is the first survey that can simultaneously observe multiple galaxies with large overlapping areas in a broad redshift range, which is ideal for a multi-tracer analysis. In this paper we present a multi-tracer analysis using the final eBOSS DR16 Luminous Red Galaxy (LRG) sample combined with the high redshift tail from BOSS DR12 CMASS sample, dubbed ‘LRGpCMASS’

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In this section, we briefly describe the eBOSS DR16 galaxy sample and the eBOSS DR16 Emission Line Galaxy (ELG) sample.

This work is one of a series of papers presenting results based on the final eBOSS DR16 samples. The multi-tracer analysis of the same samples is also performed in Fourier space to complement this work (Zhao et al. 2020b). For the LRG sample, produced by Ross et al. (2020), the correlation function is used to measure BAO and RSD in Bautista et al. (2020), and the analyses of BAO and RSD from power spectrum are discussed in Marin et al. (2020). The LRG mock challenge for assessing the modelling systematics is described in Graziano et al. (2020). The ELG catalogues are presented in Raichoor et al. (2020), and analyzed in Fourier space (DeMattia et al. 2020) and in configuration space (Tamone et al. 2020), respectively. The clustering catalogue of quasars is generated by Ross et al. (2020). The quasar mock challenge for assessing modelling systematics is described in Smith et al. (2020). The quasar clustering analysis in Fourier space is discussed in Neveux et al. (2020), and in configuration space in Hou et al. (2020). Finally, the cosmological implications from the clustering analyses is presented in Muller et al. (2020).

We introduce the galaxy samples and mock catalogues used in this paper in Sections 2 and 3, respectively. In Section 4, we describe the template of the full shape correlation function, and in Section 5, we show measurements of the correlation function. The methodology of parameter estimation and the fitting result are presented in Sections 6 and 7, respectively. We perform cosmological implications using in Section 8. Section 9 is devoted to the conclusion. In this paper, we use a fiducial $\Lambda$CDM cosmology with parameters: $\Omega_m = 0.307, \Omega_b h^2 = 0.022, h = 0.6777, n_s = 0.96, \sigma_8 = 0.8288$. The comoving sound horizon in this cosmology is $r_s^{\text{com}} = 147.74$ Mpc.

2 GALAXY SAMPLES

In this section, we briefly describe the eBOSS DR16 galaxy sample used in the work.

2.1 The eBOSS LRG and BOSS CMASS samples

The target sample of luminous red galaxies was selected from the optical SDSS photometry DR13 (Albareti et al. 2017) and the infrared photometry from the WISE satellite (Lang et al. 2016). The final algorithms of target selection and catalogue generation are described in Prakash et al. (2016) and in a companion paper (Ross et al. 2020). We use the LRG data of the complete 5 years of eBOSS in the redshift range of $0.6 < z < 1.0$. Its volume density distribution as a function of redshift is shown in red curves in Figure 1. The sky coverage of eBOSS DR16 LRG is $2475.51$ deg$^2$ in the North Galactic Cap (NGC) and $1626.80$ deg$^2$ in the South Galactic Cap (SGC), which are shown in red regions of Figure 2.

In order to correct for observational effects, the eBOSS DR16 LRG catalogue is assigned a set of weights, including weights for the redshift failure, $w_{\text{zf}}$, close pair due to fiber collisions, $w_{\text{cp}}$ and for systematics due to the effect of completeness, the variation of the mean density as a function of stellar density and galactic extinction, $w_{\text{sys}}$. In addition, the FKP weight to minimize the variance in the clustering measurement combining regions (Feldman et al. 1994) is added

$$w_{\text{FKP}} = \frac{1}{1 + n(z)P_0},$$

(1)

where $n(z)$ is the number density of galaxies, and $P_0$ is set to 10,000 $h^{-3}$Mpc$^3$. The total weight applied to each eBOSS LRG is (Ross et al. 2020)

$$w_{\text{tot}}^\text{LRG} = w_{\text{FKP}} \times w_{\text{sys}} \times w_{\text{cp}} \times w_{\text{zf}}.$$  

(2)

As the redshift range and sky coverage of eBOSS DR16 LRG sample overlap with the high redshift tail, i.e. $0.6 < z < 1.0$, and the footprint of the BOSS DR12 CMASS catalogue (Reid et al. 2016), which are shown in grey curves of Figure 1 and grey regions of Figure 2, respectively, a more complete sample is constructed by merging the eBOSS DR16 LRG catalogue with the BOSS DR12 CMASS catalogue. We denote this combined as ‘LRGpCMASS’ in this work. Note that the BOSS DR12 CMASS used different procedures for generating close-pair and redshift failure weights and the total weight is counted via (Reid et al. 2016)

$$w_{\text{tot}}^\text{CMASS} = w_{\text{FKP}} \times w_{\text{sys}} \times (w_{\text{cp}} + w_{\text{zf}} - 1).$$

(3)

The combined DR16 BOSS+eBOSS LRG catalogue includes the correct total weight for each LRG in order to avoid confusion (see Section 5.7 of Ross et al. 2020 for more details).

2.2 The eBOSS ELG sample

The target sample of emission line galaxies are selected from the DECam Legacy Survey (DECaLS) $grz$—photometry (Raichoor et al. 2017), which provides an imaging dataset at higher redshifts. The final large-scale structure catalogue creation is described in the companion paper (Raichoor et al. 2020). We use the complete eBOSS DR16 ELG catalogues in the redshift range of $0.6 < z < 1.1$, which is composed of 83, 769 galaxies in the NGC and 89, 967 galaxies in the SGC with spectroscopic redshifts. The redshift distributions in NGC and SGC are shown in blue solid and dashed curves in Figure 1. The eBOSS DR16 ELG sample overlaps with LRGpCMASS within $0.6 < z < 1.0$. The effective sky area of ELG is $369.4$ deg$^2$ in NGC and $357.5$ deg$^2$ in SGC, which are shown in blue regions of Figure 2. The overlapping area covered ELG and LRGpCMASS samples is $\sim 730$ deg$^2$.
The total weight assigned to each eBOSS ELG is

\[ w_{\text{tot}}^{\text{ELG}} = w_{\text{FKP}} \times w_{\text{sys}} \times w_{\text{cp}} \times w_{\text{zf}} ; \]

where \( P_0 = 4000 \, h^{-3} \, \text{Mpc}^3 \) in \( w_{\text{FKP}} \). A description of the observational effects is presented in Raichoor et al. (2020).

The ELG sample suffers from unknown angular photometric systematics, and this kind of angular contamination can bias the measurement of clustering. Burden et al. (2017) proposed a modified model of correlation function to null the angular modes from the galaxy clustering, such that the effect of angular contamination can be largely reduced. A sophisticated model is developed by Paviot et al. (2020), which is used for this analysis.

### 2.3 The radial integral constraint

The true radial selection function in spectroscopic surveys is difficult to determine from the survey itself, and it is commonly approximated from the redshift distribution of the actual data sample. When generating the corresponding random catalogue, the redshifts of data are assigned to the random catalogues, dubbed the shuffled scheme. Such that the average of density fluctuations along the LOS is zero. But this would lead to an impact on the galaxy clustering on large scales. This effect is called as the radial integral constraint (RIC). The scheme to correct the RIC effect in theory was proposed by de Mattia & Ruhlmann-Kleider (2019). This modelling method is used to account for the correction of RIC effect in the analysis of eBOSS DR16 ELG clustering (see e.g. DeMattia et al. (2020); Tamone et al. (2020)). Alternatively, we can subtract the RIC effect from the data measurement. Firstly we quantify the RIC effect using additional two sets of EZmocks without systematics (Zhao et al. 2020a). One set of mocks contains the RIC effect, in which the redshifts of the random catalogues are assigned by the redshifts of each mock data via the shuffled scheme. The other set is without the RIC effect, where 1000 mock datasets use a single random catalogue sampling the redshift distribution of data (dubbed the sampled scheme). The difference between these two sets of mocks provides an estimation of the RIC effect, which then can be subtracted from the data measurement. We are aware that this is an approximation, as the dependence of the RIC on cosmological parameters is not accounted for in this scheme. We performed a comparison with the forwarding modelling method and find the difference is negligible given the statistical uncertainty of the ELG sample.

### 2.4 The effective redshift

The effective redshift of the sample is determined via the following weighted pair-count,

\[ z_{\text{eff}} = \frac{\sum w_i^m w_j^n (z_i^m + z_j^n)/2}{\sum w_i^m w_j^n} , \]

where \( w_i \) is the total weight of the \( i \)-th galaxy at redshift \( z_i \). We compute the effective redshift over all galaxy pairs separated by a distance between 25 and 150 \( h^{-1} \, \text{Mpc} \), having \( z_{\text{eff}} = 0.70 \) for the combined sample of NGC and SGC LRGpCMASS when \( m = n = L \), \( z_{\text{eff}} = 0.845 \) for the ELG combined sample in NGC and SGC when \( m = n = E \), and \( z_{\text{eff}} = 0.77 \) for the cross galaxy pairs between LRGpCMASS and ELG samples, i.e. \( m = L \) and \( n = E \).

### 3 MOCK CATALOGUES

In this section we present the synthetic datasets used to validate our pipeline.

#### 3.1 MDPL2 mocks

To test our modelling against non-linear gravitational collapse and certain aspect of galaxy physics, we generate mock catalogue using the Multi-tracer Halo Occupation Distribution (MTHOD; Alam et al. 2019). The MTHOD approach introduces a new way to model multiple tracers in the same volume. In this approach each of the tracers can have its own occupation recipe for the central and satellite galaxies. MTHOD ensures that the joint probabilities of occupation are well behaved by limiting the total probability of central galaxies in a halo to 1 and makes sure that non-physical behaviour is forbidden, such as multiple types of galaxy.

The limits of separations have little effect on the value of the effective redshift.
ies at the centre of the same dark matter halo. The key parameters in MTHOD models involve the independent parameters for the occupation probability of central and satellite galaxies for each tracer. The MTHOD mock galaxy catalogue is created using the MultiDark Planck simulation (MDPL2; Prada et al. 2012) publicly available\(^2\) through the CosmoSim database. MDPL2 is a dark matter only N-body simulation using the Gadget-2 algorithm (Klypin et al. 2016). MDPL2 assumes a flat ΛCDM cosmology with \(Ω_m = 0.307, Ω_b = 0.048, h = 0.67, n_s = 0.96\) and \(σ_8 = 0.82\), and is a periodic box of side length \(1 h^{-1}\)Gpc sampled by 3840\(^3\) particles. A halo catalogue is generated using the ROCKSTAR halo finder (Behroozi et al. 2013) at an effective redshift of \(z = 0.86\).

The DM haloes are then populated using the following equations for central and satellite galaxies as a function of halo mass, \(M_{\text{halo}}\):

\[
P_{\text{cen}}^s(M_{\text{halo}}; \vec{θ}) = \sum_{\text{tr} \in \text{TR}} P_{\text{cen}}^{s\text{tr}}(M_{\text{halo}}; \vec{θ}^{\text{tr}}) \tag{6}
\]

\[
\langle N_{\text{sat}}^{s\text{tr}} \rangle (M_{\text{halo}}; \vec{θ}) = \sum_{\text{tr} \in \text{TR}} \langle N_{\text{sat}}^{s\text{tr}} \rangle (M_{\text{halo}}; \vec{θ}^{\text{tr}}) , \tag{7}
\]

where the sum is over all tracers in the list, \(\text{TR} = \{\text{LRG}, \text{QSO}, \text{ELG}\}\). This equation requires a constraint of \(p_{\text{cen}}^s \leq 1\) for any halo mass. The explicit forms of \(p_{\text{cen}}^s\) and \(\langle N_{\text{sat}}^{s\text{tr}} \rangle\) are given in equations 8-14 in Alam et al. (2019). The full list of parameters \((\vec{θ} = \{θ^{\text{LRG}}, θ^{\text{ELG}}, θ^{\text{QSO}}\})\) and best fit values obtained for the eBOSS samples are given table 1 of Alam et al. (2019). All three tracers (i.e. LRG, ELG and QSOs) are modelled within the MTHOD framework. However, we only use the LRG and ELG galaxies, and do not use the QSOs from the default in this paper. The number of LRG galaxy is 156, 800 and the number of ELG galaxy is 3, 301, 753. The volume density of ELG is much higher than that of LRG. We have two different models to populate the central galaxy called standard HOD (SHOD) and High Mass Quenched (HMQ) model. We create six realisations for each mock catalogue by projecting RSD along different axes of the cubic box.

### 3.1.1 The semi-analytic covariance matrix

In this analysis, we have six non-trivial combinations of correlation function multipoles, each of which has 25. This leads to a total of 11325 independent covariance matrix elements, thus the covariance requires significant computational power to compute. As an alternative, we consider semi-analytic methods, in particular the RascalC method (Philcox & Eisenstein 2019; Philcox et al. 2020), which is a fast algorithm for computing two- and three-point correlation function covariances in arbitrary survey geometries. This works by noting that, in the Gaussian limit, the covariance can be written as an integral of products of the correlation function over four copies of the survey window function, which can be rapidly evaluated using importance sampling and random particle catalogues. Non-Gaussianity can be added via a small rescaling of the shot-noise terms, shown to be an excellent approximation on BAO scales in O’Connell et al. 2016; O’Connell & Eisenstein 2019). Using RascalC it is possible to estimate covariance matrices from an observational dataset and window function alone, drastically reducing the dependence on mocks and hence the computational resources required.

Here, we estimate the covariances for the periodic MDPL2 mocks, using all non-trivial combinations of LRG, ELG and cross correlation functions. As an input we require estimates of the correlation function computed over a large range of radii; these are estimated from the mocks using bins of width \(Δr = 2 h^{-1}\)Mpc from \(r = 0 h^{-1}\)Mpc to \(r = 200 h^{-1}\)Mpc and ten angular bins. For efficient configuration-space sampling we use random particle catalogues, which, given the periodic geometry, are here simply sets of \(\sim 10^6\) particles uniformly placed on the cube for both LRGpC-MASS and ELG samples. In total, we sample \(\sim 4 \times 10^4\) quadruplets of points in configuration-space to build a smooth model, which requires \(\sim 400\) CPU-hours in total, significantly less than that required for traditional mock-based analyses.

### 3.2 The EZmocks

To estimate the covariance matrices of the clustering measurements of the eBOSS data, we rely on 1000 realisations of multi-tracer EZmock catalogues, for both LRGs and ELGs. These mocks are based on dark matter density fields generated using the Zel’Dovich approximation (Zel’dovich 1970). Galaxies are then sampled in the density field with effective bias descriptions. The bias models for LRGs and ELGs are calibrated separately to the eBOSS data, with four free parameters. Nevertheless, the underlying dark matter density fields for different tracers are evolved from the same initial conditions, to account for their cross correlations. As the result, the cross correlation function between the EZmock LRGs and ELGs are well consistent with that of the data on small scales (for details, see Zhao et al. 2020a).

In this work we use three different sets of EZmocks. Two of them are free of observational systematics, with only survey footprint, veto masks, and radial selections applied, which are used to estimate the RIC effect mentioned in Sec. 2.3. The random catalogues for these two sets of mocks are generated using the sampled and shuffled schemes respectively. For the sampled random catalogues, the redshift distributions are sampled from the spline-smoothed \(n(z)\) of the data. While for the shuffled randomness, the redshifts are taken directly from the corresponding galaxy catalogues. The third set of EZmocks contain various observational effects, such as photometric systematics, fibre collisions, and redshift failures. These contaminated mocks are used to measure the covariance matrices of our analysis.

## 4 THE TEMPLATE FOR THE FULL SHAPE ANALYSIS

We use the ‘Gaussian streaming model’ (GSM) developed in Reid & White (2011) to compute the theoretical correlation function:

\[
1 + \xi(s_{\perp}, s_{\parallel}) = \int \frac{dy}{\sqrt{2\pi} \left[ \frac{1}{\sigma_{12}^2(r, \mu)} + \frac{1}{\sigma_{\text{FOG}}} \right]} \left[ 1 + \xi(r) \right] \times \exp \left\{ - \frac{s_{\parallel} - y - \mu v_{12}(r)}{2 \left[ \frac{1}{\sigma_{12}^2(r, \mu)} + \frac{1}{\sigma_{\text{FOG}}} \right]} \right\} , \tag{8}
\]

where \(s_{\parallel} \equiv s\mu\) and \(s_{\perp} \equiv s\sqrt{(1 - \mu^2)}\) denotes the separation of pairs along and across the LOS, respectively; \(\xi(r)\) is the real-space correlation function as a function of the real-space separation \(r\); \(v_{12}(r)\) is the mean infall velocity of galaxies separated by \(r\); and \(\sigma_{12}(r, \mu)\) is the pairwise velocity dispersion of galaxies. The parameter \(\sigma_{\text{FOG}}\) is used to marginalize over the Fingers-of-God (FoG) effect on nonlinear scales due to random motions of galaxies. The quantities \(\xi(r)\), \(v_{12}(r)\) and \(\sigma_{12}(r, \mu)\) are computed using

\[\text{https://www.cosmosim.org/cms/simulations/mdp12/}^2\]
the Convolution Lagrangian Perturbation Theory (CLPT) \(^3\) (Carlson et al. 2013; Wang et al. 2014),

\[
1 + \xi(r) = \int d^3 q M_0(r, q)
\]

(9)

\[
v_{12,i}(r) = [1 + \xi(r)]^{-1} \int d^3 q M_{1,i}(r, q)
\]

(10)

\[
\sigma_{12,i,j}^2(r) = [1 + \xi(r)]^{-1} \int d^3 q M_{2,i,j}(r, q),
\]

(11)

where \(v_{12,i}(r)\) is the component of mean pairwise velocity along the direction of pairwise separation \(\hat{r}_i\), and \(\sigma_{12,i,j}^2(r)\) is the velocity dispersion component along the pairwise separation vector \(\hat{r}_i\). \(M_0(r, q), M_{1,i}(r, q)\) and \(M_{2,i,j}(r, q)\) are integrals of the perturbation theory that depend on the linear matter power spectrum (see Wang et al. 2014 for more details).

As mentioned in Sec. 2.2, for the auto-CF of ELG, we need to account for a correction to the angular systematics in the modelling. Such a template of the modified CF, as shown below, developed by Paviot et al. (2020) can well mitigate the angular contamination.

\[
\xi(s_\perp, s_\parallel) = \xi(s_\perp, s_\parallel)_{\text{true}} - \frac{2}{n(\chi)} \int d\chi \int n(\chi) d\chi \int n(\chi) d\chi \int \frac{\xi(s_\perp, s_\parallel)_{\text{true}}}{n(\chi)} ds_\parallel/2 ds_\parallel
\]

(12)

where \(n(\chi)\) is the radial selection function of the survey, \(\chi\) is the comoving distance out to a galaxy at redshift \(z\), and the parameter \(z_{\text{peak}} = 0.84\) is determined by minimizing the difference between the mean of the modified CF multipole from two sets of ELG EZmocks (with and without systematics), as performed in Tamanwe et al. (2020).

The CLPT-GSM model can be easily generalised to model the cross-correlations between two tracers with different biases via the following transformation (Carlson et al. 2013; Wang et al. 2014),

\[
\langle F'' \rangle \rightarrow \frac{1}{2} \left( \langle F''_m \rangle + \langle F''_n \rangle \right)
\]

(13)

\[
\langle F'' \rangle \rightarrow \frac{1}{2} \left( \langle F''_m \rangle + \langle F''_n \rangle \right)
\]

(14)

\[
\langle F'' \rangle^2 \rightarrow \langle F''_m \rangle \langle F''_n \rangle
\]

(15)

\[
\langle F'' \rangle^2 \rightarrow \langle F''_m \rangle \langle F''_n \rangle
\]

(16)

\[
\langle F' \rangle \langle F'' \rangle \rightarrow \frac{1}{2} \left( \langle F'_m \rangle \langle F''_n \rangle + \langle F''_m \rangle \langle F'_n \rangle \right)
\]

(17)

Here the first local Lagrangian bias \(\langle F' \rangle\) is related to the Eulerian linear bias factor \(b\) via,

\[
b = 1 + \langle F' \rangle
\]

(18)

and the second local Lagrangian bias \(\langle F'' \rangle\) is fixed under the peak-background split assumption using the Sheth-Tormen mass function (Sheth & Tormen 1999).

The separation \(\ell = (s_\perp, s_\parallel)\) in the true cosmology might be different from those \((s_\perp, s_\parallel)\) in the fiducial cosmology, which is used to convert the redshifts to distances. This is known as the AP effect (Alcock & Paczynski 1979), which can be accounted for via the following relation:

\[
s_\perp' = \alpha_\perp s_\perp, \quad s_\parallel' = \alpha_\parallel s_\parallel.
\]

(19)

Here, two scaling factors \((\alpha_\perp, \alpha_\parallel)\) are introduced to parameterise the differences of distances (across and along the LOS) between the true and fiducial cosmology:

\[
\alpha_\perp \equiv \frac{D_M(z) r_{\perp}^{\text{fid}}}{D_M(z) r_{\perp}^{\text{true}}}, \quad \alpha_\parallel \equiv \frac{D_H(z) r_{\parallel}^{\text{fid}}}{D_H(z) r_{\parallel}^{\text{true}}},
\]

(20)

where \(D_M(z) \equiv (1 + z) D_A(z), \text{ and } D_A(z)\) is the angular diameter distance, \(D_H(z) = c/H(z), \text{ H(z) is the Hubble expansion parameter. The superscript ’fid’ denotes the corresponding values in the fiducial cosmology.}

5 MEASUREMENTS OF CORRELATION FUNCTIONS

We measure the auto-correlation functions for the ELG and LRGpCMASS samples using the Landy & Szalay (1993) estimator,

\[
\xi(s, \mu) = \frac{DD(s, \mu) - 2DR(s, \mu) + RR(s, \mu)}{RR(s, \mu)},
\]

(21)

where \(DD, DR \text{ and } RR\) are the weighted data-data, data-random and random-random pair counts with the separation \(s\) and the cosine of the angle between the pair and the LOS, denoted as \(\mu\).

Additionally, we measure the cross-correlation between these two samples using the following estimator,

\[
\xi(s, \mu) = \frac{DB D_U - DB D_L - DR E + RE R_L}{RE R_L},
\]

(22)

where superscripts ‘E’ and ‘L’ represent the ELG and LRGpCMASS samples, respectively.

The Legendre projections of the correlation function is calculated to obtain the correlation function multipoles,

\[
\xi_\ell(s, \mu) = \frac{2\ell + 1}{2} \int_{-1}^1 d\mu \xi(s, \mu) L_\ell(\mu),
\]

(23)

where \(L_\ell(\mu)\) is the Legendre polynomial.

In Figure 3, we present measurements of the correlation function monopole and quadrupole, including the auto-correlation functions of LRG in blue (left panels) and ELG in green (right panels),

\[\text{https://github.com/wl1745881210/CLPT_GSRSD}\]
and their cross-correlation in red (middle panels), using a set of MDPL2 mock with the z LOS, which is produced via the multi-tracer HMQ HOD model. The correlation function multipoles are measured with a bin width of $5\, h^{-1}\text{Mpc}$ within the scale range of $25 - 150\, h^{-1}\text{Mpc}$. The error bar is estimated from the RascalC covariance matrix.

We show the correlation function multipoles measured from the DR16 galaxy samples and EZmocks in Figures 4 and 5 for measurements in the NGC and SGC, respectively. All the correlation function multipoles are measured with a bin width of $5\, h^{-1}\text{Mpc}$ within the scale range of $30 - 150\, h^{-1}\text{Mpc}$. The measurements of ELG are shown in upper panels, where the dashed-line and shaded areas display the $1\sigma$ regions evaluated from 1000 ELG EZmocks without and with removing the RIC effect, respectively; The black-line areas are the mean of ELG EZmocks with the $1\sigma$ standard deviation after further removing the angular systematics using Eq. 12; The black circles with the $1\sigma$ error bars are the multipoles measured from ELG samples with removing both the RIC effect and angular systematics in ELG data.

In the middle panels of Figures 4 and 5, we show measurements of cross-correlations between ELG and LRGpCMASS. The $1\sigma$ areas covered within the green dashed lines (RIC is not subtracted) and shaded regions (with RIC subtracted off) are evaluated from EZmocks. The green squares with the $1\sigma$ error bars are the measured multipoles from cross sample with the RIC effect removed. Within the $1\sigma$ region, the cross-correlation multipoles from EZmocks and data are mostly consistent on large scales.

The panels in the bottom of Figures 4 and 5 are the measured multipoles from LRGpCMASS sample and mocks. There is not much difference between blue dashed-line region (with RIC effect) and the blue shaded area (with removing RIC effect), which means that the RIC effect in LRGpCMASS data is negligible.

The covariance matrix can be estimated using the measurements of 1000 EZmocks,

$$
C_{ij}^{\ell'} = \frac{1}{N-1} \sum_k \left[ \bar{\xi}_i^\ell(s_k) - \bar{\xi}_i(s_k) \right] \left[ \bar{\xi}_j^{\ell'}(s_k) - \bar{\xi}_j^{\ell'}(s_k) \right],
$$

(24)

where the average multipole is given by

$$
\bar{\xi}_i(s_k) = \frac{1}{N} \sum_k \xi_i^\ell(s_k),
$$

(25)

here $N = 1000$ is the number of mock realisations. The normalised covariance matrices, i.e. $C_{ij}^{\ell'}/\sqrt{C_{ii}^{\ell} \times C_{jj}^{\ell'}}$, in NGC and SGC are displayed in the left and right panels of Figure 6, respectively.
Figure 5. Same as Figure 4, but in the SGC.

Figure 6. The correlation matrices between the correlation function monopole, quadrupole, and hexadecapole measured from 1000 EZmocks in the NGC (left) and SGC (right). For each measurement, $\xi_E^0$, $\xi_C^2$, or $\xi_L^4$, we show correlation for 24 bins linearly even spaced in separation $s$ between 30 to $150 h^{-1}$ Mpc.
We fit $\xi(\ell = 0, 2, 4)$ in the range $30 < s < 150 \, h^{-1}\text{Mpc}$ (72 data points for each sample). The matrix is $72 \times 72$ for each tracer, and so totally there is a $216 \times 216$ covariance matrix for the combined data vector of two auto-correlation and one cross-correlation measurements.

We show the 2D correlation function reconstructed from the measured monopole, quadrupole and hexadecapole using the ELG, LRGpCMaSS, and their cross samples in SGC in Figure 7, where the BAO ring at $\sim 100 \, h^{-1}\text{Mpc}$ and the squashing effect due to RSD is clearly visualised.

We quantify the Signal-to-Noise Ratio (SNR) of measurement on the cross-correlation between two tracers of eBOSS via

\[
\text{(SNR)}^2_{ij} = \sum_{s_i} \left[ C_{ij}^\ell (s_i) \right]^T F_{ij} C_{ij}^\ell (s_j),
\]

where $F_{ij}$ is the inverse covariance matrix for the measured cross-correlation. We obtain a detection of cross-correlation function at a significance of 15 $\sigma$.

6 PARAMETER ESTIMATION

We perform a global fitting in the following parameter space, i.e.

\[
p = \{ \alpha_\perp, \alpha_\parallel, b^m_{NGC}\sigma_8, b^m_{SGC}\sigma_8, f_8, \sigma_{\text{FoG}} \},
\]

where $m = E, L, \text{ or C}$ when using the ELG, LRGpCMaSS, or CROSS sample alone. We use different bias parameters for NGC and SGC. Namely for the fit to each sample, we have $N_p = 6$ free parameters in each case.

For the combined fits of two samples, e.g. ELG + LRGpCMaSS, the free parameters for bias factors are

\[
\{ b^m_{NGC}\sigma_8, b^m_{SGC}\sigma_8 \} = \{ b^E_{NGC}\sigma_8, b^L_{NGC}\sigma_8, b^E_{SGC}\sigma_8, b^L_{SGC}\sigma_8, b^E_{\text{FoG}}, b^L_{\text{FoG}}, \sigma_{\text{FoG}} \}.
\]

(28)

We totally have $N_p = 9$ free parameters. The number of free parameters in the cases of ELG+CROSS and CROSS+LRGpCMaSS are also $N_p = 9$.

For the joint fit of ELG, LRGpCMaSS and cross samples, as the bias of the cross sample can be derived from the biases of ELG and LRGpCMaSS via Eq. 13, in principle we do not need to introduce additional degrees of freedom for the bias factors for the cross sample. We only assign a new damping parameter, i.e. $\sigma_{\text{FoG}}$ to the cross-correlation function. So we have $N_p = 10$ free parameters for the joint fit. However, considering that the LRGpCMaSS and ELG of eBOSS DR16 are not fully overlapping, we also implement a fit by additionally introducing a set of bias parameters, i.e. $\{ b^E_{SGC}\sigma_8, b^L_{SGC}\sigma_8 \}$ for the cross sample. In this case the number of free parameters is $N_p = 12$.

We use a modified version of CosmoMC 4 (Lewis & Brindle 2002) based on a Markov Chain Monte Carlo (MCMC) technique to sample the parameter space $p$, and search for the minimum $\chi^2$,

\[
\chi^2(p) = \chi^2_{NGC}(p|b_{NGC}\sigma_8) + \chi^2_{SGC}(p|b_{SGC}\sigma_8),
\]

where

\[
\chi^2_{NGC}(p|b_{NGC}\sigma_8) \equiv \sum_{i,j} \left[ \xi^\ell (s_i, p|b_{NGC}\sigma_8) - \xi^\ell (s_j) \right] \left[ F^\ell_{ij, NGC} \left[ \xi^\ell (s_j, p|b_{NGC}\sigma_8) - \xi^\ell (s_j) \right] \right],
\]

(30)

and

\[
\chi^2_{SGC}(p|b_{SGC}\sigma_8) \equiv \sum_{i,j} \left[ \xi^\ell (s_i, p|b_{SGC}\sigma_8) - \xi^\ell (s_j) \right] \left[ F^\ell_{ij, SGC} \left[ \xi^\ell (s_j, p|b_{SGC}\sigma_8) - \xi^\ell (s_j) \right] \right],
\]

(31)

here $F_{ij}^\ell$ is the inverse of the covariance matrix in Equ. 24. An unbiased estimation for the inverse covariance matrix is given by

\[
C^{-1}_{ij} = \frac{N - N_b - 2}{N - 1} C^{-1}_{ij}.
\]

(32)

where $N_b$ is the number of the scale bins. In order to include the error propagation from the error in the covariance matrix into the fitting parameters (Percival et al. 2014) we rescale the covariance matrix, $\tilde{C}_{ij}$, by

\[
M = \frac{1 + B(N_b - N_p)}{1 + A + B(N_p + 1)},
\]

(33)

here $N_p$ is the number of the fitting parameters, and

\[
A = \left( \frac{N - N_b - 1}{N - N_b - 4} \right),
\]

(34)

\[
B = \left( \frac{N - N_b - 2}{N - N_b - 4} \right).
\]

(35)

\[\text{http://cosmologist.info/cosmomc/}\]
Figure 8. The best fits from measurements of LRG and ELG mock galaxy catalogue, and their cross correlation using MDPL2 mocks in (Alam et al. 2019). The shaded bands are an error of 1% on the $\alpha_\perp$ and $\alpha_|$ parameters and 3% on $f\sigma_8$, and the dashed lines in the middle of the shaded area are the fiducial values of parameters. The multi-tracer MDPL2 mock has two types of HOD models, i.e. standard (upper panels) and HMQ (lower panels) with the LOS of x, y, z, so we have six realisations in total. We fit the LRG auto-correlation (black), ELG auto-correlation (red), and their cross-correlation (blue), respectively, and then perform a joint fitting using these three sets of measurements (magenta). Note that for the MDPL2 mock sample, we do not need to assign bias parameters for NGC and SGC separately. So the number of free parameter for the fit of each sample is $N_p = 5$, and for the joint fit we have $N_p = 8$ free parameters.

Figure 9. The mean values with 1 $\sigma$ error bar from EZmock tests (left) and data fits (right) of different combinations, as shown in the legend.

7 MOCK TESTS

We validate our pipeline in this section, using two series of mock catalogues, namely, the N-body MDPL2 mocks and 1000 realisations of the EZmocks, as introduced in Sec. 3.

7.1 MDPL2 mock fits

Figure 8 shows the $\alpha_\perp$, $\alpha_|$ and $f\sigma_8$ parameters fitted to the MDPL2 mock. The multi-tracer MDPL2 mock has two types of HOD models, i.e. standard (upper panels) and HMQ (lower panels), and we consider that the LOS is along x, y or z axis, so we
have six realisations in total. We perform the fit to LRG auto-correlation, ELG auto-correlation, and their cross-correlation, respectively. The constraining results on the $\alpha_\perp, \alpha_\parallel$ and $f\sigma_8$ parameters from these three sets of measurement are displayed in black, red and blue, respectively. The fitted results are basically within the error of 1% for $\alpha_\perp$ and $\alpha_\parallel$, and the error of 3% for $f\sigma_8$. Then we perform a joint fit to these three sets of measurements together. The fitted results (magenta in Figure 8) are consistent with the expected values of the $\alpha_\perp, \alpha_\parallel$ and $f\sigma_8$ parameters.

7.2 EZmock tests

We apply our pipeline on the average of the correlation function multipoles, measured from 1000 realisations of the EZmocks, and present the marginalised mean values with 68% CL uncertainty of BAO and RSD parameters in Table 1 and in the left panel of Figure 9. As detailed previously, the ELG, LRGpCMASS and their cross correlation can be best modelled at effective redshifts of 0.845, 0.7 and 0.77, respectively, but for the joint fit, we make an assumption that all three correlation functions can be modelled using a fixed template at $z_{\text{eff}} = 0.77$, which is explicitly tested here.

As shown in Table 1, the observables of each tracer can be well fitted by a template created at their corresponding effective redshifts, and the bias of the fitting is well within 68% CL. The fitting to the ELG sample shows a relatively larger bias, especially in $f\sigma_8$, which means that the contamination from the systematics is not completely mitigated by our pipeline. As the bias for all parameters is within the statistical uncertainty, we validate our pipeline.

We then move on to tests of all observables at $z_{\text{eff}} = 0.77$, and find almost no change on the posterior of parameters. This well demonstrates that it is reasonable to model all three sets of observables at $z_{\text{eff}} = 0.77$, which is the effective redshift of the cross correlation. The joint fitting at $z_{\text{eff}} = 0.77$ successfully returns the input values of parameters with a marginal bias, which further validates our pipeline.

7.3 Data fits

We present measurements on the BAO and RSD parameters from the DR16 samples in Table 2 and in right panel of Figure 9, and find consistent BAO and RSD measurements from ELG and LRGpCMASS samples given their statistical uncertainties. Compared to results of the single-tracer analysis, the measurements of BAO and RSD from cross alone is consistent with ELG sample within the 1 $\sigma$ error bar. The LRGpCMASS gives a much smaller statistical uncertainty than that of ELG. The difference between $f\sigma_8$ values from LRGpCMASS and cross sample is 1.11 $\sigma$.

Combining one with another among ELG, LRGpCMASS, and cross samples, e.g. ELG + CROSS, ELG + LRGpCMASS, or CROSS + LRGpCMASS, we obtain the improved constraints. These measurements are well consistent within 1 $\sigma$ error.

The joint fits from ELG and LRGpCMASS auto-correlation functions and their cross correlation give the tightest constraints. For joint fits, we present the results in two cases, i.e. $N_p = 10$ denotes that we did not assign additional bias parameters for the cross samples, which are derived via Eq.13; $N_p = 12$ means that the cross sample has its own bias parameters. We find the BAO and RSD measurements in these two joint cases are in good agreement. Comparing with the fitted result from LRGpCMASS alone, we find the Figure of Merit (FoM) of the $\alpha_\perp, \alpha_\parallel$ and $f\sigma_8$ parameters, i.e. FoM = $1/\sqrt{\det \text{Cov}(\alpha_\perp, \alpha_\parallel, f\sigma_8)}$ from the joint ($N_p = 12$) fit is improved by a factor of 1.18. In particular, the improvement in the measurement precision of $f\sigma_8$ is 11.6% over using LRGpCMASS sample alone.

We also perform a fitting when the AP parameters are fixed to 1, as a consistency test of the fiducial cosmology. As expected, we get a tighter constraint on $f\sigma_8$ in this case, namely, the statistical uncertainty of $f\sigma_8$ with AP fixed is reduced by $\sim$ (25% - 40%) compared with results with AP parameters marginalized over. In cases with AP parameters fixed, we obtain a 9% improvement in the statistical precision of $f\sigma_8$ from the joint fit compared with the LRGpCMASS’s constraint. We compare our result on $f\sigma_8$ with AP parameters fixed with the forecast published in (Zhao et al. 2016), where the AP parameters are also fixed. Because the actual survey area is different from that used in the forecast, and the error on parameters is inversely proportional to the square root of the survey area, we perform a rescaling of the forecast using the areas, and find that the improvement on the precision of $f\sigma_8$ is 14%, which is slightly better than our actual analysis.

We derive the parameters $D_M/r_d = 18.93 \pm 0.37$ and $D_H/r_d = 19.65 \pm 0.54$ from the joint ($N_p = 12$) fitted results on $\alpha_\perp$ and $\alpha_\parallel$ in Table 2. The 1D posterior distributions of $D_M/r_d$, $D_H/r_d$, and $f\sigma_8$, and their 2D contour plots from the LRGpCMASS alone (blue) and the joint fit (black) are shown in Figure 10.

For the joint fits, the data vectors and covariance matrix for the $(D_M/r_d, D_H/r_d, f\sigma_8)$ parameters are given by

$$D \equiv 
\begin{pmatrix}
D_M/r_d \\
D_H/r_d \\
f\sigma_8
\end{pmatrix} = 
\begin{pmatrix}
18.93 \\
19.65 \\
0.445
\end{pmatrix}.$$
Table 1. The result of the fit to the mean of 1000 EZmocks. \( \Delta(p) \) shows the difference between the mean value from mock test and its expected value. The expected values of \( \alpha_\perp \) and \( \alpha_{\parallel} \) are 1. The expected values of \( f\sigma_8 \) at different \( z_{\text{eff}} \) are \( f\sigma_8(z_{\text{eff}} = 0.70) = 0.471 \), \( f\sigma_8(z_{\text{eff}} = 0.77) = 0.465 \), \( f\sigma_8(z_{\text{eff}} = 0.845) = 0.458 \).

| Catalogues     | \( z_{\text{eff}} \) | \( \Delta(\alpha_\perp) \) | \( \Delta(\alpha_{\parallel}) \) | \( \Delta(f\sigma_8) \) |
|----------------|----------------------|-----------------------------|-----------------------------|----------------------|
| ELG            | 0.845                | 0.030 ± 0.066               | 0.048 ± 0.080               | 0.068 ± 0.075       |
| ELG            | 0.770                | 0.034 ± 0.067               | 0.057 ± 0.080               | 0.061 ± 0.077       |
| LRGpCMASS      | 0.700                | 0.001 ± 0.022               | 0.010 ± 0.037               | −0.002 ± 0.045      |
| LRGpCMASS      | 0.770                | 0.001 ± 0.022               | 0.008 ± 0.038               | 0.002 ± 0.046       |
| CROSS          | 0.770                | 0.009 ± 0.053               | 0.045 ± 0.084               | 0.022 ± 0.083       |
| ELG+CROSS      | 0.770                | 0.025 ± 0.045               | 0.040 ± 0.060               | 0.032 ± 0.063       |
| ELG+LRGpCMASS  | 0.770                | 0.006 ± 0.022               | 0.011 ± 0.034               | 0.015 ± 0.038       |
| CROSS+LRGpCMASS| 0.770                | 0.003 ± 0.022               | 0.009 ± 0.036               | 0.010 ± 0.044       |

| Joint \( (N_p = 10) \) | 0.770              | 0.005 ± 0.022               | 0.013 ± 0.035               | 0.008 ± 0.039       |
| Joint \( (N_p = 12) \) | 0.770              | 0.007 ± 0.021               | 0.012 ± 0.034               | 0.013 ± 0.039       |

Figure 11. The evolution of \( D_M/r_d \), \( D_A/r_d \), and \( f\sigma_8 \) as a function of \( z \). For reference, the blue bands are the predictions from Planck 2018 in the \( \Lambda \)CDM cosmology (Aghanim et al. 2018).

8 COSMOLOGICAL IMPLICATIONS

In this section, we briefly discuss cosmological implications of our joint measurements from the multi-tracer analysis.

We use the distance measurement here to constrain the geometry of the Universe in the framework of an open \( \Lambda \)CDM cosmology, in which the Hubble expansion rate is

\[
H(z) = H_0 \sqrt{\Omega_{m0} (1+z)^3 + \Omega_{\Lambda0} + (1 - \Omega_{m0} - \Omega_{\Lambda0})}. \tag{38}
\]

To avoid the dependence on \( r_d \), we work in the parameter space of \((\Omega_{m0}, \Omega_{\Lambda0}, H_0r_d)\).

The BAO datatsets used here include the isotropic BAO measurements using MGS (Ross et al. 2015) and 6dFGRS (Beutler et al. 2011) galaxy samples; BOSS DR12 anisotropic BAO measurements in the low- and middle-redshift bins, i.e. \(0.2 < z < 0.5\) and \(0.4 < z < 0.6\) (Alam et al. 2016); the anisotropic BAO measurement from eBOSS DR16 quasars (Neveux et al. 2020; Hou et al. 2020), Lyman-\( \alpha \) forest (Bourboux et al. 2020), and our multi-tracer analysis of eBOSS DR16 ELG and LRGpCMASS.

In Figure 13, we present the 68 & 95% CL contour plots (black) for the cosmological parameters \((\Omega_{m0}, \Omega_{\Lambda0}, H_0r_d)\), and their one-dimensional probability distributions. The joint BAO data sets a strong constraint on dark energy density, i.e. \(\Omega_{\Lambda0} = 0.752 \pm 0.067\). The BOA alone favors the existence of dark energy at the significance of \(11\sigma\). Compared with the constraining result (i.e. blue contours in Figure 13) (Aata et al. 2018) using the isotropic BAO measurements using MGS (Ross et al. 2015) and 6dFGRS

\[ C = 10^{-3} \left( \begin{array}{c} 137.4645 -9.5231 7.3304 \\ 293.6931 -8.2139 1.4237 \end{array} \right) \tag{37} \]

which is used in the section of cosmological implications.

We recommend users to use the joint measurement \(^6\) reported in Eqs. 36 and 37 to perform constraints on dark energy or tests of gravity.

In Figure 11, we present our BAO and RSD measurements in this work with the \( \Lambda \)CDM prediction from Planck 2018 (Aghanim et al. 2018) together. Our measurement is well consistent with the prediction of Planck 2018 (Aghanim et al. 2018).

We also show our BAO and RSD measurements and the BAO distances favored by the reconstructed dynamical dark energy from a combined observational data (Wang et al. 2018a) together in Figure 12. There is no significant tension between the new measurement and the prediction of the reconstructed dynamical dark energy within 1 \( \sigma \) statistical error, although the measurement is more consistent with Planck 2018.

\(^6\) The multi-tracer BAO and RSD measurements and covariance matrix are available at https://github.com/ytcosmo/MultiTracerBAORSD/. This measurement can be used together with the BAO and RSD measurements in the first six \( z \) bins i.e. \(0.2 < z < 0.59\) from BOSS DR12 in (Wang et al. 2018b).
Table 2. The mean values with 68% CL error for the parameters, $\alpha_\perp$, $\alpha_\parallel$, $f\sigma_8$ from different datasets.

| Samples        | $\alpha_\perp$      | $\alpha_\parallel$ | $f\sigma_8$ | $\chi^2$/dof |
|----------------|---------------------|--------------------|-------------|--------------|
| ELG            | $1.097^{+0.074}_{-0.230}$ | $1.173^{+0.248}_{-0.219}$ | $0.439^{+0.135}_{-0.149}$ | 169/138      |
| ELG, SGC       | $1.033^{+0.071}_{-0.130}$ | $1.127^{+0.095}_{-0.163}$ | $0.431^{+0.104}_{-0.132}$ | 92/67        |
| ELG            | fixed               | fixed              | $0.433 \pm 0.045$       | 169/140      |
| LRGpCMASS      | $1.016 \pm 0.021$   | $1.007 \pm 0.028$  | $0.472 \pm 0.043$       | 161/138      |
| LRGpCMASS      | fixed               | fixed              | $0.448 \pm 0.032$       | 161/140      |
| CROSS          | $0.949 \pm 0.040$   | $1.118 \pm 0.118$  | $0.342 \pm 0.085$       | 147/138      |
| CROSS          | fixed               | fixed              | $0.443 \pm 0.050$       | 148/140      |
| ELG+LRGpCMASS  | $1.004 \pm 0.020$   | $1.020 \pm 0.028$  | $0.435 \pm 0.037$       | 309/279      |
| ELG, SGC +LRGpCMASS | $1.013 \pm 0.021$ | $1.014 \pm 0.028$  | $0.451 \pm 0.041$       | 232/208      |
| ELG+CROSS      | $0.970 \pm 0.037$   | $1.044 \pm 0.072$  | $0.402 \pm 0.062$       | 290/279      |
| CROSS+LRGpCMASS | $1.006 \pm 0.021$  | $1.016 \pm 0.029$  | $0.444 \pm 0.041$       | 298/279      |
| Joint ($N_p = 10$) | $1.003 \pm 0.020$  | $1.018 \pm 0.030$  | $0.439 \pm 0.039$       | 416/422      |
| Joint ($N_p = 10$, w/ AP fixed) | fixed | fixed | $0.444 \pm 0.029$ | 414/424 |
| Joint ($N_p = 12$) | $1.005 \pm 0.020$  | $1.016 \pm 0.028$  | $0.445 \pm 0.038$       | 416/420      |
| Joint ($N_p = 12$, w/ AP fixed) | fixed | fixed | $0.446 \pm 0.029$ | 414/422 |

Figure 12. The shaded bands are the uncertainties of angular diameter distance, $D_A(z)$ (left) and Hubble expansion rate, $H(z)$ (right) favored by the reconstructed dynamical dark energy in (Wang et al. 2018a). The data point with error bar is our measurement in this work. They are rescaled by the mean values in the $\Lambda$CDM predicted by Planck 2018 (Aghanim et al. 2018).

(Butler et al. 2011) galaxy samples; the anisotropic BAO measurement in three $z$ bins from BOSS DR12 (Alam et al. 2016); the isotropic BAO measurement from eBOSS DR14 quasars (Ata et al. 2018); and BOSS DR11 and DR12 Lyman-α sample (Font-Ribera et al. 2014; Bautista et al. 2017), the significance of non-zero dark energy density is improved by a factor of 1.67.

9 CONCLUSIONS

We perform a multi-tracer analysis in configuration space using the final eBOSS LRG sample combined with the BOSS CMASS sample, and the final eBOSS ELG sample.

We test the validity of the multi-tracer pipeline using the $N$-body MDPL2 mocks and EZmocks, before applying to the analysis of real data. We report a high-precision measurement on the cosmic expansion rate and growth of structure at the effective redshift $z = 0.77$, and find an improvement in the FoM of the $\alpha_\perp$, $\alpha_\parallel$, $f\sigma_8$ parameters of 18% over that using the LRGpCMASS sample alone. Note that the area covered by the LRGpCMASS sample is larger by a factor of 13 than that of the ELG sample, thus the LRGpCMASS dominates the information content in the joint analysis. But even in this case, a non-trivial improvement in the FoM is contributed by the ELG sample, demonstrating the efficacy of the multi-tracer method.

We combine our measurement with previous BAO distance measurements from MGS, 6dFGS, BOSS DR12, and new BAO distance measurements from eBOSS DR16 quasars and eBOSS DR16 Lyman-α sample, to test an open $\Lambda$CDM cosmology. It is found that a non-zero dark energy density is favored by BOA alone at a 11 $\sigma$ significance.

The stage-IV galaxy surveys, such as Dark Energy Spectroscopic Instrument (DESI)\(^7\) and EUCLID\(^8\), aim to observe multiple tracers with high density at higher redshifts. These surveys will explore the history of cosmic expansion and growth of structure with a higher precision, taking the advantage of the multi-tracer nature of

\(^7\) https://www.desi.lbl.gov/  \(^8\) https://www.euclid-ec.org/
Figure 13. The one-dimensional posterior distributions and the 68 and 95% CL contour plots for the cosmological parameters using MGS (Ross et al. 2015) + 6dFGS (Beutler et al. 2011) + BOSS DR12 (low-z and middle-z bins) (Alam et al. 2016) + eBOSS DR16 QSO (Neveux et al. 2020; Hou et al. 2020) + eBOSS DR16 Lyman-α forests (Bourboux et al. 2020) + our joint (N_p = 12) result (black), compared with the constraining result (blue) in eBOSS DR14 paper (Ata et al. 2018).

the survey. Admittedly, this requires efforts in the theoretical modeling and a deeper understanding of observational effects, to minimize the theoretical and observational systematics, respectively.

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