A novel method to investigate how the spatial correlation of the pump affects the purity of polarization entangled states

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We present an innovative method to address the relation between the purity of type-I polarization entangled states and the spatial properties of the pump laser beam. Our all-optical apparatus is based on a spatial light modulator and it offers unprecedented control on the spatial phase function of the entangled states. In this way, we demonstrate experimentally the relation between the purity of the generated state and spatial correlation function of the pump beam. © 2014 Optical Society of America

Spontaneous parametric down-conversion (SPDC) is a crucial process in the development of quantum technology, and represents one of the most effective sources of entangled photon pairs and of single photons [1,2]. For these reasons the spatial and the spectral properties of the downconverted beams have been extensively analyzed as a function of the coherence properties of the pump beam [3,4]. Less attention has been paid to the effect of the spatial properties of the pump on the purity of polarization entangled states, especially those generated by with type-I parametric down-conversion, since this may be revealed only by an accurate control of the phase profile of the output beams. In this letter, we exploit an all-optical innovative method based on a spatial light modulator (SLM) to gain an unprecedented control on the spatial phase function of the generated entangled states and demonstrate experimentally the relation between the purity of the generated state and the correlation function of the pump beam.

The downconverted state at the output of the two crystals, assuming that the spectra of the pump and of the parametric down-conversion are quasi-monochromatic, may be written as:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \int d\theta_s d\theta_i \sin(c(\frac{1}{2}k_\parallel L)) F(\Delta k_\perp)$$

$$\left[|H, \theta_s\rangle |H, \theta_i\rangle + e^{i\Phi(\theta_s, \theta_i)} |V, \theta_s\rangle |V, \theta_i\rangle\right],$$

(1)

where \(L\) is the crystals length and \(|P, \theta\rangle\) denote a single-photon state with polarization \(P = H, V\) emitted at angle \(\theta\). \(\Delta k_\parallel\) and \(\Delta k_\perp\) are the shifts respect to the phase matching condition of the longitudinal and transverse momentum of the two photons. The sinc function comes from the integration along the longitudinal coordinate inside the crystals, and the function \(F\) from the integration over the transverse coordinate: denoting by \(A_p(x)\) the complex amplitude of the pump, we have \(F(\Delta k_\perp) = \int dx A_p(x) e^{i\Delta k_\perp x}\). The phase term \(\Phi(\theta_s, \theta_i)\) arises from the optical path of the two photons generated in the first crystal inside the second crystal, and from the spatial walk-off between the \(H\) and the \(V\) beams of the down-conversion outside the crystals [1,2]. In general, we may write \(\Phi(\theta_s, \theta_i) = \phi(\theta_s) + \phi(\theta_i) + \phi_a\) where, up to first order, we have \(\phi(\theta) \propto n' k L / \cos((\theta_0 + \theta)/n') + k L \tan((\theta_0 + \theta)/n') \sin((\theta_0 + \theta) / 2) / n' \alpha_0 \theta_0\), where \(n'\) is the extraordinary index of refraction in the second crystal, \(k = 2\pi / \lambda\), \(\theta_0\) is the central angle, and \(L\) is the crystal length. The term \(\phi_a\) represents the additional phase possibly added by any external optical component, e.g. the SLM. The shifts \(\Delta k_\parallel\) and \(\Delta k_\perp\) are given by

$$\Delta k_\parallel = k_p - k_s \cos((\theta_0 + \theta_s)/n) - k_i \cos((\theta_0 + \theta_i)/n) \propto k\theta_0(\theta_s + \theta_i) = k\theta_0 \theta_+$$

$$\Delta k_\perp = k_s \sin((\theta_0 + \theta_s)/n) - k_i \sin((\theta_0 + \theta_i)/n) \propto k(\theta_s - \theta_i) = k\theta_-,$$

where \(\theta_+ = \theta_s + \theta_i\) and \(\theta_- = \theta_s - \theta_i\), and \(n'\) is the ordinary index of refraction. Using the new variables the overall phase function rewrites as

$$\Phi(\theta_-, \theta_+) = \phi_0 + \alpha_0 \theta_+ + \phi_a.$$

The purity of the state, which in this case equals the visibility, may be written as

$$p = \int \int d\theta_+ d\theta_- |\sin(\gamma \theta_-)|^2 |F(k\theta_-)|^2 \cos(\Phi(\theta_+, \theta_-)),$$

where \(\gamma = \frac{1}{2}k\theta_0 L\). The normalization condition is given by \(\int \int d\theta_+ d\theta_- |\sin(\gamma \theta_-)|^2 |F(k\theta_-)|^2 = 1\). Two cases are of special interest: if \(\Phi_a = -\phi_0\) the purity does not depend on \(F\) and we obtain the case that is usually described in the literature [1,2]. On the other hand, upon imposing \(\Phi_a = -\phi_0 - \alpha_0 \theta_+ + \beta \theta_-\) one obtains \(\Phi = \beta \theta_-\), i.e. the purity is now a function of \(F\). In this second case, using the Wiener-Khinchin theorem, we have:

$$p = \int d\theta_- |F(k\theta_-)|^2 \cos(\beta \theta_-) \propto \left\langle A^* \left( x + \frac{\beta}{k} \right) A(x) \right\rangle_x.$$
i.e. the purity of the state is proportional to the spatial correlation function of the pump beam.

The experimental setup is shown in Fig. 1. A linearly polarized cw 405nm diode laser (Newport LQC405 – 40P) passes through two cylindrical lenses, which compensate beam astigmatism, then a spatial filter composed by two lens and a pin-hole in the Fourier plane partially removes the spatial modes, and finally a telescopic system prepares a beam with the proper beam radius and divergence. A couple of 1-mm beta-barium borate crystals, cut for type-I down conversion, with optical axis aligned in perpendicular planes, are used as a source of polarization and momentum entangled photon pairs with $\theta_0 = 3^\circ$. In order to match the above theoretical model we use a compensation crystal on the pump, which removes the delay time between the vertical and horizontal polarization, and put a 10mm interference filter on the signal path, in order to reduce the spectral width of the generated radiation. A spatial light modulator (SLM), which is a liquid crystal phase film of the pump (left column) and its Fourier transform (middle column). We consider three relevant examples: in the first row we report the results obtained with a single mode Gaussian profile with a spot of 220µm. The phase function imposed by the SLM is given by $\Phi_a = -\phi_0 - a\theta_+$. Error bars on the experimental values are within the points. The solid black line is the theoretical prediction.

In Fig. 2 we report the typical behaviour of the visibility for $\beta = 0$, measured as a function of the parameter $\alpha$, which itself governs the phase function $\Phi_a = -\phi_0 - a\theta_+$ imposed by the SLM. In this configuration the overall phase function is given by $\Phi = (\alpha_0 - \alpha)\theta_+$ and thus it is possible to tune $\Phi_a$ and find the optimal value $\alpha = \alpha_0$, which removes the initial phase function and maximizes the purity. We find that this value is in good agreement with the expected theoretical value. The beam has a spot of 220µm: looking at the Fourier transform we see that we are not exactly dealing with a single mode Gaussian profile. However, this is not a problem since to fit data we use the square modulus of the Fourier transform obtained with the method of the cylindrical lens.

In order to complete the theoretical model we have to take into account the fact that the spatial coupling is not flat, but rather has a Gaussian profile with a FWHM of about 5mm. We thus insert this function when tracing out the spatial degrees of freedom in order to obtain the polarization state and its purity. In addition, there are some elements that introduce decoherence not compensable with the SLM: these are the gaps between the pixels of the SLM (3µm), the imperfect compensation of the delay time, the spectral effects of the parametric down-conversion, and the imperfect superposition between the amplitudes generated by the two crystals. In order to include these effects in the model we assume that the output state is described by the mixed state $\rho_{tot} = \rho p + (1 - m)\rho_{mix}$ where $\rho$ is the ideal superposition $\rho = |\Psi\rangle\langle\Psi|$ and $\rho_{mix}$ the corresponding mixture. The overall purity of the state is thus given by $\rho_{tot} = mp$ where $m$ depends only slightly on the characteristic of the pump.
function with a smaller width. In the last example, we place a grid with a step of 100\(\mu m\) in front of the two generating crystals. In the spatial profile we obtain two peaks, and this corresponds to a revival in the visibility after a collapse. Since the pump degrees of freedom represent a noisy environment for the polarization ones, upon modifying the spatial pump profile we are in fact inducing a non Markovian dynamics in the polarization degrees of freedom [10][11]. The small shift in the bottom right picture (about 0.2 rad/mrad) is probably due to an imperfect compensation (of about 30\(\mu m\)) of the spatial walk-off between the H and V polarization of the pump beam.

![Fig. 3. Spatial correlations of the pump beam and purity of the entangled output in three relevant cases: (first row) collimated pump beam of 220\(\mu m\), (second row) divergent pump beam of 220\(\mu m\), (third row) pump beam with two peaks. We report the spatial profile of the pump (left column), its Fourier transform (right column), and the visibility as a function of \(\beta\) (right column). The phase function imposed by the SLM is given by \(\Phi_a = -\phi_0 - \alpha_0 \theta_+ + \beta \theta_-\). Error bars on the experimental values are within the points. The solid black lines are the theoretical predictions (which include the effects of the walk-off in the two-peaks case).](image)

In conclusion, we have demonstrated the quantitative relation between the purity of type-I polarization-entangled states and the spatial properties of the pump. In order to obtain this result we exploited the unprecedented control of the spatial phase function of the generate states that is achievable by the use of a spatial light modulator. Our method may be used for entanglement engineering [12] and purification [7], and it paves the way for investigating fundamental effects in non Markovian open systems [13].

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