Chiral and $U(1)_A$ restorations high in the hadron spectrum and the semiclassical approximation.

L. Ya. Glozman

Institute for Theoretical Physics, University of Graz, Universitätsplatz 5, A-8010 Graz, Austria

In quantum systems with large $n$ (radial quantum number) or large angular momentum the semiclassical (WKB) approximation is valid. A physical content of the semiclassical approximation is that the quantum fluctuations effects are suppressed and vanish asymptotically. The chiral as well as $U(1)_A$ breakings in QCD is a result of quantum fluctuations. Hence these breakings must be absent (suppressed) high in the spectrum and the spectrum of high-lying hadrons must exhibit symmetries of the classical QCD Lagrangian.

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If one neglects tiny masses of $u$ and $d$ quarks, which are much smaller than $\Lambda_{QCD}$ or the typical hadronic scale of 1 GeV, then the QCD Lagrangian exhibits the

\[ U(2)_L \times U(2)_R = SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A \]

(1)
symmetry. This is because the quark-gluon interaction Lagrangian in the chiral limit does not mix the left- and right-handed components of quarks and hence the total QCD Lagrangian for the two-flavor QCD can be split into the left-handed and right-handed parts which do not communicate to each other. We know that the $U(1)_A$ symmetry of the classical QCD Lagrangian is absent at the quantum level because of the $U(1)_A$ anomaly [1]. We also know that the chiral $SU(2)_L \times SU(2)_R$ symmetry is spontaneously broken in the QCD vacuum. That this is so is directly evidenced by the nonzero value of the quark condensate, \( \langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \approx -(240 \pm 10 \, \text{MeV})^3 \), which represents an order parameter for spontaneous chiral symmetry breaking. This quark condensate directly shows that in the QCD vacuum the left-handed quarks are correlated with the right-handed antiquarks (and vice versa) and hence the QCD vacuum breaks the chiral symmetry.

That the chiral symmetry is spontaneously broken is also directly seen from the low-lying hadron spectrum. If the chiral symmetry were intact in the vacuum, i.e. it were realized in the Wigner-Weyl mode, then all hadrons would fall into parity-chiral multiplets [2], i.e. multiplets of the $SU(2)_L \times SU(2)_R \times C_i$ group, where $C_i$ consists of identity and space inversion. In the baryon spectrum these multiplets are either parity doublets in $N$ and $\Delta$ spectrum that are not related to each other, or quartets that contain degenerate parity doublets in the nucleon and delta spectra with the same spin. From the low-lying nucleon and delta spectra we definitely conclude that there are no degeneracies of states of the same spin but opposite parity. This tells that chiral symmetry must be broken in the QCD vacuum, i.e. it is realized in the Nambu-Goldstone mode. Even more, there is no one-to-one mapping of the states with the same spin and opposite parity low in the spectrum. This suggests that low in the spectrum the chiral symmetry is not only strongly broken, but in addition realized nonlinearity [3]. Similar, in the meson spectrum the unbroken chiral symmetries would imply that e.g. $\pi$-mesons (\( I, J^P = 1, 0^- \)) and pure $n\bar{n} = \frac{3}{\sqrt{2}} f_0$ states (\( I, J^P = 0, 0^+ \)) would be systematically degenerate, level by level. Clearly it is not the case for the low-lying states (see Fig. 2).

The high-lying hadrons, however, show obvious signs of parity doubling. For example, all the excited nucleons around 1.7 GeV are well established states and we see here three approximate parity doublets with spins 1/2, 3/2 and 5/2. Similar, the lowest excitations with \( J = 9/2 \) are also well established states and they represent another good example of parity doubling. Not established states (i.e. candidates, that are marked as $^{*\,*\,*}$ and $^{\ast\,\ast\,\ast}$ states according to PDG classification [4]) also support parity doubling, though the uncertainties are very high (one should not take too seriously $^{\ast\,\ast\,\ast}$ states, of course). There is a well established state with \( J = 11/2 \), where so far no parity partner has been seen. Similar situation occurs in the delta spectrum. It has been suggested recently [5,2] that this parity doubling reflects effective chiral symmetry restoration in high-lying hadrons. Unfortunately from the baryon spectrum alone we cannot distinguish whether the parity doublets evidence the restoration of chiral or $U(1)_A$ symmetries\(^\dagger\).

\(^\dagger\)The restoration of the $U(1)_A$ symmetry, but not of the chiral symmetry, has been suggested by Jaffe as explanation of parity doublets in the baryon spectrum, as cited in ref. [2]. It has been shown in [2], however, that $U(1)_A$ cannot be restored

* e-mail address: leonid.glozman@uni-graz.at

\(^\ast\)This phenomenon has been referred to [6] as chiral symmetry restoration of the second kind in order to distinguish it from the chiral symmetry restoration in the QCD vacuum at high temperature or density.
The systematic data on high-lying mesons are still absent in the PDG tables. The results of the ongoing partial wave analysis [7–9] of high-lying mesons obtained in \( \bar{p}p \) annihilation at LEAR suggest a clear evidence [10,11] for the chiral symmetry restoration. This is well seen from the Fig. 2, where the high-lying \( \pi \) mesons and \( n\bar{n} \) mesons are shown. There are indications of simultaneous chiral and \( U(1)_A \) restorations in highly excited mesons [10], since the highly excited \( \pi \), \( a_0 \), and \( n\bar{n} \) mesons form approximately degenerate multiplets of the \( U(2)_L \times U(2)_R \) group.

![Excitation spectrum of the nucleon](image)

FIG. 1. Excitation spectrum of the nucleon. The real part of the pole position is shown. Boxes represent experimental uncertainties. Those resonances which are not yet established are marked by two or one stars according to the PDG classification. The one-star resonances with \( J = 1/2 \) around 2 GeV are given according to the recent Bonn (SAPHIR) results.

By definition an effective symmetry restoration means the following. In QCD the hadrons with the quantum numbers \( \alpha \) are created when one applies the local interpolating field (current) \( J_\alpha(x) \) with such quantum numbers on the vacuum \( |0\rangle \). Then all the hadrons that are created by the given interpolator appear as intermediate states in the two-point correlator

\[
\Pi_{J_\alpha}(q) = i \int d^4x \ e^{-iqx} \langle 0|T \{ J_\alpha(x) J_\alpha(0) \}|0\rangle, \quad (2)
\]

where all possible Lorentz and Dirac indices (which are specific for a given interpolating field) have been omitted, for simplicity. Consider two local interpolating fields \( J_1(x) \) and \( J_2(x) \) which are connected by chiral transformation, \( J_1(x) = U J_2(x) U^\dagger \), where \( U \in SU(2)_L \times SU(2)_R \) (or by \( U(1)_A \) transformation). Then if the vacuum is invariant under the given symmetry group, \( U|0\rangle = |0\rangle \), it follows from (2) that the spectra created by the operators \( J_1(x) \) and \( J_2(x) \) must be identical.

![Pion and \( n\bar{n} f_0 \) spectra](image)

FIG. 2. Pion and \( n\bar{n} f_0 \) spectra. The three highest states in both pion and \( f_0 \) spectra are taken from [7–9]. Since these \( f_0 \) states are obtained in \( p\bar{p} \) and they decay predominantly into \( \pi\pi \) channel, they are considered in [7–9] as \( n\bar{n} \) states.

We know that in QCD \( U|0\rangle \neq |0\rangle \). As a consequence the corresponding spectral densities \( \rho_1(s) \neq \rho_2(s) \). However, it may happen that the noninvariance of the vacuum becomes unimportant (irrelevant) high in the spectrum. Then the spectral functions \( \rho_1(s) \) and \( \rho_2(s) \) become very different because of the symmetry breaking in the vacuum, to the high-lying spectrum, where the asymmetry of the vacuum becomes unimportant and \( \rho_1(s) \approx \rho_2(s) \) (chiral symmetry restoration of the second kind). We stress that this effective chiral symmetry restoration does not mean that chiral symmetry breaking in the vacuum disappears, but only that the role of the quark condensates that break chiral symmetry in the vacuum becomes progressively less important high in the spectrum [2]. The valence quarks in high-lying hadrons decouple from the QCD vacuum.

In ref. [2] a justification for effective chiral symmetry restoration has been suggested. Namely, at large space-like momenta \( Q^2 = -q^2 > 0 \) the correlator can be adequately represented by the operator product expansion, where all nonperturbative effects reside in different condensates [12]. The only effect that spontaneous breaking of chiral symmetry can have on the correlator is via the
quark condensate of the vacuum, \( \langle \bar{q} q \rangle \), and higher-dimensional condensates that are not invariant under chiral transformation \( U \). However, the contributions of all these condensates are suppressed by inverse powers of momenta \( Q^2 \). This shows that at large space-like momenta the correlation function becomes chirally symmetric. In other words

\[
\Pi_n(Q) \rightarrow \Pi_n(Q) \quad \text{at} \quad Q^2 \rightarrow \infty. \tag{3}
\]

The dispersion relation provides a connection between the space-like and time-like domains for the Lorentz scalar (or pseudoscalar) parts of the correlator. In particular, the large \( Q^2 \) correlator is completely dominated by the large \( s \) spectral density \( \rho(s) \), which is an observable. Hence the large \( s \) spectral density should be insensitive to the chiral symmetry breaking in the vacuum and must satisfy

\[
\rho_1(s) \rightarrow \rho_2(s) \quad \text{at} \quad s \rightarrow \infty. \tag{4}
\]

This is in contrast to the low \( s \) spectral densities \( \rho_1(s) \) and \( \rho_2(s) \), which are very different because of the chiral symmetry breaking in the vacuum.

While the argument above on the asymptotic symmetry properties of spectral functions is rather robust (it is based actually only on the asymptotic freedom of QCD at large space-like momenta and on the analyticity of the two-point correlator), it is not clear whether it can be applied to the bound state systems, which the hadrons are. Indeed, it can happen that the asymptotic symmetry restoration applies only to that part of the spectrum, which is above the resonance region (i.e. where the current creates jets but not isolated hadrons). So the question arises whether it is possible to prove (or at least justify) the symmetry restoration in highly excited isolated hadrons. We show below that both chiral and \( U(1)_A \) restorations in highly excited isolated hadrons must be anticipated as a direct consequence of the semiclassical approximation.

At large \( n \) (radial quantum number) or at large angular momentum \( L \) we know that in quantum systems the semiclassical approximation (WKB) work. Physically this approximation applies in these cases because the de Broglie wavelength of particles in the system is small in comparison with the scale that characterizes the given problem. In such a system as a hadron the scale is given by the hadron size while the wavelength of valence quarks is given by their momenta. Once we go high in the spectrum the size of hadrons increases as well as the typical momentum of valence quarks. This is why a highly excited hadron can be described semiclassically in terms of the underlying quark and gluon degrees of freedom.

A physical content of the semiclassical approximation is most transparently given by the path integral. The contribution of the given path to the path integral is regulated by the action \( S(q) \) along the path \( q(x, t) \)

\[
\sim e^{iS(q)/\hbar}. \tag{5}
\]

The semiclassical approximation applies when \( S(q) \gg \hbar \). In this case the whole amplitude (path integral) is dominated by the classical path \( q_{cl} \) (stationary point) and those paths that are infinitesimally close to the classical path. All other paths that differ from the classical one by an appreciable amount do not contribute. These latter paths would represent the quantum fluctuation effects. In other words, in the semiclassical case the quantum fluctuations effects are strongly suppressed and vanish asymptotically.

The \( U(1)_A \) symmetry of the QCD Lagrangian is broken only due to the quantum fluctuations effects. The \( SU(2)_R \times SU(2)_L \) spontaneous (dynamical) breaking is also pure quantum effect and is based upon quantum fluctuations. To see the latter we remind the reader that most generally the chiral symmetry breaking (i.e. the dynamical quark mass generation) is formulated via the Schwinger-Dyson equation. It is not yet clear at all which specific gluonic interactions are the most important ones as a kernel of the Schwinger-Dyson equation (e.g. instantons [13], or gluonic exchanges [14], or perhaps other gluonic interactions, or a combination of different interactions). But in any case the quantum fluctuations effects of the quark and gluon fields are very strong in the low-lying hadrons and induce both chiral and \( U(1)_A \) breakings. As a consequence we do not observe any chiral or \( U(1)_A \) multiplets low in the spectrum. However, if the quantum fluctuations effects are absent or suppressed due to some reasons, then the dynamical mass of quarks must vanish as well as effects of the \( U(1)_A \) anomaly.

We have just mentioned that in a quantum system with large enough \( n \) or \( L \) the quantum fluctuations must be suppressed and vanish asymptotically. Then it follows that in such systems both the chiral and \( U(1)_A \) symmetries must be restored. Hence at large hadron masses (i.e. with either large \( n \) or large \( L \)) we must observe symmetries of the classical QCD Lagrangian. This is precisely what we see phenomenologically. In the nucleon spectrum the doubling appears either at large \( n \) excitations of baryons with the given small spin or in resonances of large spin. Similar features persist in the delta spectrum. In the meson spectrum the doubling is obvious for large \( n \) excitations of small spin mesons (see Fig. 2) and there are signs of doubling of large spin mesons (the data are, however, sparse). It would be certainly interesting and important to observe systematically multiplets of parity-chiral and parity-\( U(1)_A \) groups (or, sometimes, when the chiral and \( U(1)_A \) transformations connect different hadrons [10], the multiplets of the \( U(2)_L \times U(2)_R \) group). The high-lying hadron spectra must be systematically explored. This experimental task is just for existing facilities like JLAB, BNL, SPRING 8, ELSA, as
well as for the forthcoming Japanese hadron facility and the proton-antiproton ring in Darmstadt.

The strength of the argument given above is that it is very general. Its weakness is that we cannot say anything concrete about microscopical mechanisms of how all this happens. For that one needs a detailed microscopical understanding of dynamics in QCD, which is both challenging and very difficult task. But even though we do not know how microscopically all this happens, we can claim that in highly excited hadrons we must observe symmetries of the classical QCD Lagrangian. The only basis for this statement is that in such hadrons a semiclassical description is correct.

As a consequence, in highly excited hadrons the valence quark motion has to be described semiclassically and at the same time their chirality (helicity) must be fixed. Also the gluonic field should be described semiclassically. All this gives an increasing support for a string picture of highly excited hadrons, where hadrons are viewed as strings with massless quarks of definite chirality at the end-points of the string [11].

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