Joint Channel and Phase Noise Estimation in MIMO-OFDM Systems

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Abstract—The combination of MIMO techniques with OFDM, MIMO-OFDM, is a promising way of achieving high spectral efficiency in wireless communication systems. However, the performance of MIMO-OFDM systems is highly degraded by RF impairments such as phase noise. Similar to the SISO case, phase noise in MIMO-OFDM systems results in a common phase error (CPE) and inter carrier interference (ICI). In this paper the problem of joint channel and phase noise estimation in a system with multiple transmit and receive antennas where each antenna is equipped with its own independent oscillator is tackled. The technique employed makes use of a novel placement of pilot carriers in the preamble and data portion of the MIMO-OFDM frame. Numerical results using a 16 and 64-QAM modulation scheme are provided to illustrate the effectiveness of the proposed scheme for MIMO-OFDM systems.

Index Terms—MIMO-OFDM, Phase Noise, Common Phase Error (CPE), Inter Carrier Interference (ICI), LS, MMSE.

I. INTRODUCTION

Multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) systems are currently being deployed in the latest fourth generation (4G) wireless systems. Similar to the OFDM case, phase noise in MIMO-OFDM systems leads to common phase error as well as inter-carrier interference due to loss of orthogonality among the sub-carriers. CPE estimation schemes for MIMO-OFDM were derived in [1] and [2] by assuming that the channel state information (CSI) is perfectly known at the receiver and only for a case where only the receiver antennas are affected by the phase noise. Together with CSI estimation the problem of phase noise estimation especially for multiple antennas is only rarely discussed [3].

To the best of the authors’ knowledge, no publication exists in the up to date literature about the joint channel and phase noise estimation and compensation in MIMO-OFDM systems where both the transmit and receive antennas are equipped with an independent oscillator.

II. PHASE NOISE MODEL

The phase noise processes are given by $\theta_{m}^{[t]}(n) = \theta_{m}^{[t]}(n-1) + \Delta_{m}^{[t]}(n)$ and $\theta_{m}^{[r]}(n) = \theta_{m}^{[r]}(n-1) + \Delta_{m}^{[r]}(n)$, where $\theta_{m}^{[t]}(n)$ and $\theta_{m}^{[r]}(n)$ are the phase noise processes that are experienced by the $m$th transmit and receive antennas, respectively. $\Delta_{m}^{[t]}(n)$ and $\Delta_{m}^{[r]}(n)$ are the transmit and receive phase noise innovations which are modelled as i.i.d real Gaussian random variables with 0 mean and variances $\sigma_{\Delta_{m}^{[t]}}^{2}$ and $\sigma_{\Delta_{m}^{[r]}}^{2}$, respectively.

III. MIMO-OFDM WITH PHASE NOISE

For the MIMO-OFDM case, they may be $N_t$ transmit antennas and $N_r$ receive antennas, where each antenna is equipped with its own independent oscillator. The received time domain signal at the $q$th receive antenna in the presence of phase noise can be expressed as

$$y_q(n) = e^{j\theta_q^{[r]}(n)} \sum_{m=1}^{N_t} h_{q,m}(n) \ast e^{j\theta_m^{[t]}(n)} x_m(n) + w_q(n)$$

(1)

where $x_m(n)$ is the signal transmitted by the $m$th transmit antenna, $h_{q,m}$ describes the time domain channel impulse response between the $m$th transmit antenna and the $q$th receive antenna, $\theta_q^{[r]}(n)$ and $\theta_m^{[t]}(n)$ are the phase noise processes experienced at the $q$th and $m$th receive and transmit antennas, respectively. $w_q(n)$ is the additive white Gaussian noise and is assumed to be circularly symmetric complex Gaussian ($w_q(n) \sim \mathcal{N}(0,\sigma^2)$). The operator $\ast$ represents the convolution operation. Assuming that the cyclic prefix (CP) has been perfectly removed and after performing the DFT demodulation at the receiver, the signal received at the $q$th receive antenna at the $k$th sub-carrier is given by $Y_q(k)$

$$Y_q(k) = \sum_{j=0}^{N_r-1} P_q^{[r]}(k-j) \sum_{m=1}^{N_t} H_{q,m}(j) \sum_{i=0}^{N_r-1} P_m^{[t]}(j-i) X_m(i) + W_q(k)$$

$$= \sum_{m=1}^{N_t} H_{q,m}(k) P_q^{[r]}(0) P_m^{[t]}(0) X_m(k)$$

$$+ \sum_{m=1}^{N_t} \left[ \sum_{j=0}^{N_r-1} P_q^{[r]}(k-j) H_{q,m}(j) \sum_{i=0}^{N_r-1} P_m^{[t]}(j-i) X_m(i) \right]$$

$$- ICI_{q,m} + W_q(k)$$

(2)

In equation (2) above $P_q^{[r]}(k)$ and $P_m^{[t]}(k)$ are the receiver and transmitter phase noise Fourier coefficients, respectively. These are given by $P_q^{[r]}(k) = \frac{1}{N_c} DFT\{e^{j\theta_q^{[r]}(n)}\}$,
\[ P_m[k] = \frac{1}{N} DFT(e^{j\phi_m(n)}) \]

\[ H_{q,m} \] is the frequency domain channel impulse response between the \( q \)th receive and \( m \)th transmit antenna.

### A. Channel Estimation

Fig. 1 shows the arrangements of pilots and data symbols in a MIMO-OFDM frame that can be used for effective channel and phase noise estimation. In the preamble section of the MIMO-OFDM frame, a transmitter transmits known pilots in two consecutive time slots while the rest of the transmitters do not transmit. The data section employs a comb structure such that each subcarrier carries data from a different antenna and thus effectively implementing a frequency division multiplexing scheme. To begin the initial channel estimation, let \( S_{m,1}^p \) denote the set of pilot indices in transmit antenna \( m \) during the first time instant whilst \( S_{m,2}^p \) denotes the set of indices during the second transmission instant. We note that \( |S_{m,1}^p \cup S_{m,2}^p| = N_c \), where \( N_c \) is the number of sub-carriers. Similarly \( S_{m,1}^n \) and \( S_{m,2}^n \) denotes the set of null sub-carriers of the \( m \)-th transmit antenna in the first and second transmissions, respectively. Let the received signal at the \( q \)th receive antenna when transmit antenna \( m \) transmits be given by \( Y_{q,m}^b(k) \) whilst the transmitted pilot sub-carrier is given by \( X_m(k) \). The estimated channel coefficient if we apply the LS method is then given by

\[ \hat{H}_{q,m}(k) = \frac{Y_{q,m}^b(k)}{X_m(k)} , \quad k \in S_{m,1}^p \cup S_{m,2}^p \]  

(3)

Since we do not know the variance of the sum of the ICI and AWGN, we assume it to be uncorrelated and Gaussian. We estimate the variance by calculating the average energy of the null sub-carriers as follows

\[ \sigma_q^2 = \frac{1}{N_z} \sum_{k \in S_{m,1}^n \cup S_{m,2}^n} |Y_{q,m}^b(k)|^2 \]  

(4)

where \( N_z \) denotes the cardinality of the set \( S_{m,1}^n \cup S_{m,2}^n \).

### B. CPE Estimation

A relative \( CPE_{q,m} \) for the data portion can be calculated from the channel estimates that would have been obtained from the preamble by applying the least squares to the comb structure of pilot arrangements in the data portion. This relative \( CPE_{q,m} \) is given by

\[ CPE_{q,m} = \frac{\sum_{k \in S_m^c} Y(c(k)\hat{H}_{q,m}^*(k))X_m^*(k)}{\sum_{k \in S_m^c} |H_{q,m}X_m(k)|^2} \]  

(5)

The channel estimate as seen by the data portion of the frame is relative to the preamble section is given by \( \hat{H}_{q,m} = \frac{H_{q,m}}{CPE_{q,m}} \).

Applying MIMO to sub-carrier index \( k \), the signal received at all the antennas can be written in the following form

\[ Y(k) = \hat{H}(k)X(k) + V(k) \]  

(6)

where \( Y(k) = [Y_1(k) \ Y_2(k) \cdots Y_{N_c}(k)]^T \) and \( X(k) = [X_1(k) \ X_2(k) \cdots X_{N_c}(k)]^T \). An estimate of the transmitted symbols \( X(k) \) can be obtained by using MMSE equalization as follows:

\[ \hat{X}(k) = C^H(K)Y(k) \]

where the MMSE equalizing matrix \( C(k) \) is given by

\[ C(k) = (\hat{H}(k)\hat{H}^H(k) + \Sigma(k))^{-1} \hat{H}(k) \]

The matrix \( \Sigma \) denotes the combined AWGN and ICI noise covariance matrix.

### IV. Simulation Results

Fig 2 show the symbol error rate results for 16-QAM and 64-QAM modulation schemes. The results show that the proposed scheme is very effective in estimating and compensating for phase noise. Results for 64-QAM shows that the higher the modulation scheme the more sensitive it is to the phase noise impairment. However, the results show that the proposed scheme is also effective for higher modulation orders.

### V. Conclusion

In this paper the problem of joint estimation of channel and phase noise in a MIMO-OFDM system where both the receive and transmit antennas are equipped with independent oscillators was tackled. The proposed scheme makes use of a novel placement of pilots and nulls in the preamble and data portion of the MIMO-OFDM frame to estimate the channel impulse response and the common phase error due to the phase noise. Simulation results show that the proposed scheme is quite effective in jointly estimating the channel and the phase noise impairment.

### REFERENCES

[1] T. C. W. Schenk, X.-J. Tao, P. F. M. Smulders, and E. Fledderus, “On the influence of phase noise induced ico in mimo ofdm systems,” IEEE Communications Letters, vol. 9, no. 8, pp. 682–684, Aug 2005.

[2] K. Nikitopoulos and A. Polydoros, “Decision-directed compensation of phase noise and residual frequency offset in a space-time ofdm receiver,” IEEE Communications Letters, vol. 8, no. 9, pp. 573–575, Sept 2004.

[3] H. Munn, N. Al-Dhahir, and Y. Li, “Optimal training signals for mimo ofdm channel estimation in the presence of frequency offset and phase noise,” IEEE Transactions on Communications, vol. 54, no. 10, pp. 1754–1759, Oct 2006.