Bound Soliton – Defect Spin States in Anisotropic Ferromagnetic Chains

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Abstract. The interaction of soliton with an impurity spin in a discrete anisotropic ferromagnetic chain is studied. The defect spin in our model is characterized by modification of the exchange interaction with its neighbors. A perturbed nonlinear Schrödinger (NLS) equation for the spin amplitude is derived on the basis of a semiclassical and a continuum approximation. A specific feature of this type of defect is that it leads to perturbations to all terms of the NLS equation.Localized soliton-defect spin solutions are obtained for the case of dark and bright solitons and their stability is analyzed.

1. Introduction
The study of solitary wave-type excitations in spin chains with different exchange interactions has attracted significant interest for many years [1-4]. The Heisenberg model for ferromagnets in semiclassical limit and continuum approximation is suitable to obtain soliton-like solutions and the results show that their evolution is governed by the family of nonlinear Schrödinger equations (NLSE) [5-6]. The inhomogeneity effect on the magnetic soliton has been analyzed in a number of studies [7-10]. In magnetic materials, it plays crucial role in determining the essential physical properties, and this allows many interesting technological applications such as magnetic recording and memory.

Inhomogeneity arises in real magnetic lattices due to the presence of various defects, for example imperfect grain boundaries, impurities, dislocations, etc. Defects are local deviations from a given ordered structure and the influence of different point defects on the propagation of nonlinear spin excitations in classical one-dimensional spin systems has been considered in references [11-13].

In this paper we investigate the stability of bound soliton-defect states in anisotropic ferromagnetic chains. The defect is determined by the change of the exchange coupling of an impurity spin with its nearest neighbors.

2. Hamiltonian of the system
The Hamiltonian describing an anisotropic ferromagnetic chain with an impurity spin in the nearest-neighbor approximation can be written as

$$\hat{H} = -J \sum_{n=1}^{N} \left[ 1 + \frac{\eta}{2} (\delta_{n,n_0} + \delta_{n+1,n_0}) \right] \left( \hat{S}_n^x \hat{S}_{n+1}^x + \hat{S}_n^y \hat{S}_{n+1}^y \right) - \tilde{J} \sum_{n=1}^{N} \left[ \frac{\tilde{\eta}}{2} (\delta_{n,n_0} + \delta_{n+1,n_0}) \right] \hat{S}_n^z \hat{S}_{n+1}^z,$$

(1)

where $\hat{S}_n^x, \hat{S}_n^y, \hat{S}_n^z$ are spin operators, $J > 0$ and $\tilde{J} > 0$ are the exchange integrals in the $x,y$-plane and in the $z$-direction, respectively. For $J = \tilde{J}$ the model is isotropic. The impurity spin is at site $n_0$ and the...
parameters $\eta$ and $\tilde{\eta}$ describe the different magnitudes of the exchange interaction with its neighbors. $\eta$ and $\tilde{\eta}$ can be of arbitrary sign and strength.

With the help of $\hat{S}_n^\pm = \hat{S}_n^z \pm i \hat{S}_n^x$, Hamiltonian (1) is reduced to

$$\hat{H} = -\frac{J}{2} \sum_{n=1}^{N} \left[ 1 + \frac{\eta}{2} (\delta_{n,n_0} + \delta_{n,n_0-1}) \right] (\hat{S}_n^z \hat{S}_{n+1}^z + \hat{S}_n^z \hat{S}_{n+1}^z) - \tilde{f} \sum_{n=1}^{N} \left[ 1 + \frac{\tilde{\eta}}{2} (\delta_{n,n_0} + \delta_{n,n_0+1}) \right] \hat{S}_n^z \hat{S}_{n+1}^z. \quad (2)$$

The corresponding equations of motion are found from $i \hat{S}_n^z = [\hat{S}_n^z, \hat{\mathcal{H}}]$. However, we can consider the linear term as a leading one and neglect the other two perturbed Ablowitz-Ladik equation considered in reference [14].

In the static case $v = 0$ (3) is similar to the perturbed Ablowitz-Ladik equation considered in reference [14].

3. **Bound soliton-impurity solutions**

In the following part solitons in the form of amplitude-modulated waves

$$\alpha_n(t) = \varphi_n(t) e^{i(kn - \omega t)} \quad (4)$$

with a length scale much longer than the lattice constant (equals unit) are looked for. $k$ and $\omega$ are the wave number and the frequency of the carrier wave and the envelope $\varphi_n(t)$ is a real slowly varying function of position and time. So in the continuum limit and for $\varphi^2 \ll 1$, equation (3) transforms into the perturbed NLSE:

$$i \left( \frac{\partial \varphi}{\partial t} + 2JS[1 + \eta \delta(x-x_0)] \sin k \frac{\partial \varphi}{\partial x} \right) = [\omega_0 - \omega - 2S(J\eta \cos k - \tilde{f}\eta)\delta(x-x_0)]\varphi - JS[1 + \eta \delta(x-x_0)] \cos k \frac{\partial^2 \varphi}{\partial x^2} + [g + S(J\eta \cos k - \tilde{f}\eta)\delta(x-x_0)]|\varphi|^2 \varphi, \quad (5)$$

where $\omega_0 = -2g$ and $g = (J \cos k - \tilde{f}) S$.

In the static case $v = 0$ ($k = 0$), equation (5) takes the form

$$0 = \left[ \frac{1}{S} (\omega_0 - \omega) - 2 \left( J\eta - \tilde{f}\eta \right) \delta(x-x_0) \right] \varphi - J \left[ 1 + \eta \delta(x-x_0) \right] \frac{\partial^2 \varphi}{\partial x^2} + \left[ J - \tilde{f} + (J\eta - \tilde{f}\eta) \delta(x-x_0) \right] |\varphi|^2 \varphi. \quad (6)$$

Note that the exchange impurity introduces three $\delta$-function perturbing terms: a linear and a nonlinear terms $\sim (J\eta - \tilde{f}\eta)$ and a second derivative term proportional only to $\eta$. In this form equation (6) cannot be solved exactly. However, we can consider the linear term as a leading one and neglect the other two which are of order of $1/L^2$. In this case equation (6) has exact solutions as bound soliton-defect states which for $J < \tilde{f}$ are of the bright-soliton type

$$\varphi(x) = \varphi_0 \text{sech} \left( \frac{|x-x_0|}{L} + \Delta \right), \quad (7)$$

where

$$\varphi_0^2 = \frac{2J}{(J - \tilde{f})L^2}, \quad \omega = \omega_0 - \frac{JS}{L^2}, \quad \tanh \Delta = \frac{(J\eta - \tilde{f}\eta)L}{J}. \quad (8)$$
and for $J > J$ are of the dark-soliton type

$$\varphi(x) = \varphi_0 \tanh\left(\frac{|x-x_0|}{L} + \Delta\right),$$

(8)

with

$$\varphi_0^2 = \frac{2J}{(J-J)^L^2}, \quad \omega = \omega_0 + \frac{2JS}{L^2}, \quad \sinh(2\Delta) = -\frac{2J}{(J-J-\bar{\eta}/2)L}.$$  

The total strength of the linear impurity reads

$$\varepsilon = -2(J\eta - \bar{\eta})$$

(9)

and can be positive or negative. For the bright-soliton bound state $\varepsilon > 0$ acts as a repulsive defect, $\Delta < 0$ and the function (7) has two maxima at $x = x_0 \pm \Delta L$ ($L$ - width of the unperturbed soliton) while $\varepsilon < 0$ acts as attractive defect, $\Delta > 0$ and the function (7) has a single maximum at $x = x_0$. In the case of dark-soliton bound states we get the opposite. $\varepsilon > 0$ is attractive, $\Delta > 0$ and the function (8) has one minimum at $x = x_0$ while $\varepsilon < 0$ is repulsive, $\Delta < 0$ and the function (8) has two minima at $x = x_0 \pm \Delta L$.

4. Numerical results

Further we have solved numerically system (3) for $N = 1000$, $J = S = 1$ and the spin with modified exchange interactions is placed at $n_0 = 500$. As an initial function we have put the bound state of the form (7) or (8) and investigated how the impurity parameters influence the soliton stability.

First, we investigated the strong anisotropic case $\bar{J} = \bar{\eta} = 0$. For this system equation (6) has bound soliton-defect solutions only of dark type (figure 1). The evolution of the bound soliton-defect function

![Figure 1](image.png)

**Figure 1.** Evolution of the dark soliton-defect solution (8) with $\bar{J} = \bar{\eta} = 0$ for a modified exchange impurity $\eta = -0.09$ (a) or $\eta = 0.09$ (a') and for a linear impurity $\varepsilon = -0.18$ placed on one site (b) or on three sites of the chain (b'). $J = S = 1$ and $L = 10$. The time is in units $1000/J$. 
(8) is stable for the attractive impurity with the total strength $\varepsilon = 0.18$ (figure 1(a)). For the repulsive impurity (total strength $\varepsilon = -0.18$) the solution is more sensitive to the perturbations associated with the defect spin and we observed small oscillations in the shape (figure 1(a')). We have compared our results with the case when a linear point defect $\varepsilon$ is placed at $n_0$ instead of the impurity spin with modified exchange interactions $\eta, \tilde{\eta}$. This can be described by a term $\varepsilon S \delta_{n,n_0} \alpha_n$ on the right side of (3) and the values of $\varepsilon$ are determined by (9). Figure 1(b) illustrates the evolution of the initial function under the influence of a real linear point defect with $\varepsilon = -0.18$. As expected the function remains unchanged. We performed further calculations with the same linear defect $\varepsilon = -0.18$ expanded on three chains sites (figure 1(b')) and observed the same oscillations in the shape of the function as in figure 1(a') which demonstrates the spatial extension of $\eta$.

$|\alpha_n|^2$  
(a)  
(b)  
(c)  

$|\alpha_n|^2$  

Figure 2. Evolution of the dark soliton-defect solution (8) with $\tilde{J} = 0.8$ for a modified exchange impurity with $\eta = \tilde{\eta} = -0.4$ (a), $\eta = \tilde{\eta} = 0.4$ (b) and $\eta = \tilde{\eta} = 0.2$ (c). All other parameters are the same as in figure 1.

Figure 2 shows the results for a model with $\tilde{J} < J$ (easy-plane anisotropy) and $\tilde{\eta} = \eta$. The bound soliton-defect solution is again of dark soliton type. The impurity is attractive for $\tilde{\eta} = \eta = -0.4$ (total strength $\varepsilon = 0.16$) and the initial pulse remains stable during the evolution [figure 2(a)]. In contrary the choice $\tilde{\eta} = \eta = 0.4$ leads to a total strength $\varepsilon = -0.16$ (repulsive impurity) for which the function exhibits shape oscillations (figure 2(b)). They become smaller for smaller impurity strength $\tilde{\eta} = \eta = 0.2$ (figure 2(c)).

Next we studied the case of easy-axis anisotropy ($\tilde{J} > J$) where the bound solutions are of the bright soliton type (7). The results for $\tilde{J} = 1.2$ are presented in figure 3. We have compared again the influence of the modified exchange interactions $\eta, \tilde{\eta}$ (figures 3(a), (a')) with the action of a corresponding linear defect $\varepsilon$ placed on three chain sites (figures 3(b), (b')). The relevant pictures for attractive (figures 3(a'), (b')) as well for repulsive (figures 3(a), (b)) values are similar.

By analogy we can compare the results with the impact of a nonlinear point defect $\gamma$

$$\gamma = -2(J\eta - \tilde{J}\tilde{\eta})$$  

(10)
Figure 3. Evolution of the bright soliton-defect solution (7) with $\tilde{J} = 1.2$ for a modified exchange impurity $\eta = \tilde{\eta} = 0.4$ (a) or $\eta = \tilde{\eta} = -0.4$ (a') and for a linear impurity placed on three sites of the chain $\epsilon = 0.16$ (b) or $\epsilon = -0.16$ (b'). All other parameters are the same as in figure 1.

Figure 4. Evolution of the bright soliton-defect solution (7) with $\Delta$ according to (11) for nonlinear impurity $\gamma = 0.16$ placed on one site (a) or on three sites (b) of the chain. All other parameters are the same as in figure 3.

that can be described by a term $\gamma S \delta_{n,n_0} \alpha_n \sqrt{1 - |\alpha_n|^2}$ on the right side of equation (3) and respectively would lead to a linear perturbing term $\sim \gamma$ and to a nonlinear perturbing term $\sim -\gamma/2$ in (6). In this case we have exact bound solutions of the bright-soliton type (7) with $\Delta$ of the following form:

$$\tanh \Delta = \frac{(J - \tilde{J})L}{\gamma} \left( \sqrt{1 - \frac{\gamma^2}{(J - \tilde{J})J} + \frac{\gamma^2}{(J - \tilde{J})^2L^2}} - 1 \right).$$

(11)

A comparison with the influence of a corresponding actual nonlinear repulsive defect $\gamma$ placed on one or three chain sites is also done (figure 4). The evolution of the initial function (7) when the space extension of the nonlinear impurity is taken into account is the same as for the exchange impurity (compare figure 4(b) and figure 3(a)).
Figure 5. Evolution of a bright soliton of the form (7) with $\Delta = 0$ placed on $n_0$ for a repulsive impurity with $\eta = \tilde{\eta} = 0.12$ (a), $\eta = \tilde{\eta} = 0.2$ (b); for an attractive impurity $\eta = \tilde{\eta} = -0.2$ (c) or for $\eta = \tilde{J} = 0.4$ (d). All other parameters are the same as in figure 3.

Figure 5 shows the influence of the parameters $\eta, \tilde{\eta}$ on the evolution of a bright soliton (7) with $\Delta = 0$ which is not exact solution of the perturbed system (6) and is localized on the impurity position $n_0$. For small impurity strengths the soliton only changes its form and remains trapped for both repulsive ($\varepsilon = 0.048$) and attractive ($\varepsilon = -0.08$) impurities [figures 5(a), (c)]. Larger values of the repulsive impurity ($\varepsilon = 0.08$) yield stronger oscillations and above a certain threshold the soliton is split into two smaller solitons propagating in opposite directions [figure 5(b)]. The soliton shape is not influenced by the impurity when $\eta = \tilde{J}$, i.e. $\varepsilon = 0$ [figure 5(d)].

Figure 6. Evolution of a bright soliton of the form (7) with $\Delta = 0$ placed on $(n_0 - 10)$ for an attractive impurity with $\eta = \tilde{\eta} = -0.4$ (a) or for a repulsive impurity with $\eta = \tilde{\eta} = 0.4$ (b). All other parameters are the same as in figure 3.
We observed the evolution of a static soliton (7) with $\Delta = 0$ fixed close to the impurity position $n_0$ (figure 6). The distance between soliton and impurity is comparable to the soliton width $L$. For attractive impurities the soliton is trapped and oscillate around $n_0$ [figure 6(a)]. For repulsive impurities the soliton is reflected [figure 6(b)].

5. Conclusion

Our numerical simulations showed that the single minima/maxima soliton - defect spin solutions are stable. The double minima/maxima solutions resulting from the balance between repulsive defect spin - spin nearest neighbor interaction and an attractive nonlinear interaction, are quite unstable. The spin with modified exchange interaction affects the two adjacent sites and hence this type of defect is effectively larger in size than a single on-site linear or nonlinear defect. This imposes more stringent restrictions on the width of the double minima/maxima soliton-defect spin solutions compared to point defects.

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