Small-x physics and heavy quark photoproduction in the semihard approach at HERA.

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Abstract

Processes of heavy quark photoproduction at HERA energies and beyond are investigated using the semihard ($k_\perp$ factorization) approach. The virtuality and longitudinal polarization of gluons in the photon-gluon subprocess as well as the saturation effects in the gluon distribution function at small $x$ have been taken into account. The total cross sections, rapidity and $p_\perp$ distributions of the charm and beauty quark photoproduction have been calculated. We obtained the some differences between the predictions of the standard parton model and the semihard approach used here.

1 Introduction

The heavy quark production at HERA is very interesting and important subject of study [1, 2, 3, 4]. Because it is dominated by photon-gluon fusion subprocess (Fig.1) one can study the gluon distribution functions $G(x, Q^2)$ in small $x$ region (roughly at $x > 10^{-4}$). Secondly this last issue is important for physics at future colliders (such as LHC): many processes at these colliders will be determined by small-x gluon distributions.

The heavy quark production processes are so-called semihard processes at HERA energies and beyond. By definition in these processes a hard scattering scale $Q$ (or heavy quark mass $M$) is large as compared to the $\Lambda_{QCD}$ parameter, but $Q$ is much less than the total center of mass energies: $\Lambda_{QCD} \ll Q \ll \sqrt{s}$. Last condition implies that the processes occur in small $x$ region: $x \simeq M^2/s \ll 1$. In such case parturbative QCD expansion have large coefficients $O(\ln^n(s/M^2)) \sim O(\ln^n(1/x))$ besides the usual renormalization group ones.
the heavy quark photoproduction cross section as result of photon-gluon fusion process has the form \[ \sigma_{\gamma g} = \sigma_{0g}^{\gamma g} + \alpha_s \sigma_{1g}^{\gamma g} + \ldots \], where \( \sigma_{0g}^{\gamma g} \) was calculated by Ellis and Nason [5]. The photon - gluon fusion cross section in low order decreases vs \( s \to \infty \) \( \sigma_{\gamma g} \sim M^2/s \ln(s/Q^2) \). But in the same limit \( \sigma_{1g}^{\gamma g} \to \text{Const} \), because of that the heavy quark photoproduction cross section is dominated by the contribution of the gluon exchange in the \( t \) - channel. That results breakdown the standard perturbative QCD expansion and the problem of summing up all contributions of the order \( (\alpha_s \ln(Q^2/\Lambda^2))^n \), \( (\alpha_s \ln(1/x))^n \) and \( (\alpha_s \ln(Q^2/\Lambda^2) \ln(1/x))^n \) in perturbative QCD appears in calculation of \( \sigma_{\gamma g} \).

It is known that summing up the terms of the order \( (\alpha_s \ln(Q^2/M^2))^n \) in leading logarithm approximation (LLA) of perturbative QCD leads to the linear DGLAP evolution equation for deep inelastic structure function. Resummation of the large contributions of the order of \( (\alpha_s \ln(1/x))^n \) leads to the BFKL evolution equation and its solution gives the drastical increase of the gluon distribution: \( xG(x, Q^2) \sim x^{-\omega_0}, \omega_0 = (4N_c \ln 2/\pi)\alpha_s(Q_0^2) \). The speed of the growth of the parton density as \( x \to 0 \) makes the parton-parton interactions very important, which in turn makes the QCD so-called GLR evolution equation essentially nonlinear.

The growth of the parton density at \( x \to 0 \) and interaction between partons induces substantial screening (shadowing) corrections which restore the unitarity constrains for deep inelastic structure functions (in particular for a gluon distribution) in small \( x \) region. These facts break the assumption of the standard parton model (SPM) about \( x \) and transverse momentum factorization for a parton distribution functions. We should deal with the transverse momentum factorization \( (k_\perp \text{ factorization}) \) theory. Making allowance for screening corrections at small \( x \) we have so-called semihard approach, which take into account the virtuality of gluons, their transverse momenta as well as the longitudinal polarizations of initial gluons.

In semihard approach screening corrections stop the growth of the gluon distribution at \( x \to 0 \). This effect was interpreted as saturation of the parton (gluon) density: gluon distribution function \( xG(x, Q^2) \) becomes proportional to \( Q^2 R^2 \) at \( Q^2 < q_0^2(x) \) and \( xG(x, Q^2) \sim 1/x \). The parameters \( R \) and \( q_0^2(x) \) can be considered as new phenomenological parameters: the \( R \) is related to size of hadron or of black spots in a hadron and the parameter \( q_0^2 \) is a typical transverse momentum of partons in the parton cascade of a hadron, which leads to natural infrared cut-off in semihard processes. The parameter \( q_0^2 \) increases with growth of \( s \). So, because of the high parton density in small \( x \) region a standard methods of perturbative QCD can not be used. In this point the semihard approach differs from the works where after choice of structure function at starting value \( Q_0^2 \) the QCD evolution are calculated using certain equations.

In the region, where the transverse mass of heavy quark \( M_\perp \ll \sqrt{s} \), one need take into account the dependence of the photon-gluon fusion cross section on the virtuality and polarizations of initial gluon. Thus in semihard approach the matrix elements of this subprocess differs from ones of SPM (see for example).

We would like to remark that \( \alpha_s \) corrections in the semihard approach look quite different in comparison with usual and \( k_\perp \text{ factorization} \) ones. These corrections have been taken into account using new gluon density \( \Phi(x, q_1^2) \) (see below) which was calculated by Gribov, Levin and Ryskin in LLA where all large logs (of the types \( \ln(1/x) \) and \( \ln(Q^2/\Lambda^2) \)) were summed up. The another corrections which give contributions to \( K \)-factor have been taken into account in the normalization of function \( xG(x, Q^2) \) (see below also).
As far as the SPM calculations of the next-to-leading (NLO) cross section to the photo-production of heavy quarks that the review of its may found in the papers [2, 16]. The results [7] have been confirmed by Smith and Van Neerven [17]. The further results for the electro- and photoproduction of heavy quarks was obtained in Refs. [18, 19]. Authors in [16] notes that for beauty quark production the NLO corrections are large and various estimates of corrections lead to theoretical uncertainties of the order of a factor of 2 to 3. As was estimated in [20] the total cross section for beauty quark production at HERA will be only a few tens of percent large than the one-loop results [6].

For charm quark a theoretical uncertainties are even the higher. It is known that there is also the strong mass dependence of the results of calculations for charm quark production. Since the mass of the charm quark is small for perturbative QCD calculations the resumme procedure [11] is not applicable for charm quark production [20].

In Refs. [21, 22] we used the semihard approach in order to calculate the total and differential cross sections of the heavy quarkonium, $J/\Psi$ and $\Upsilon$, photoproduction. We obtained the remarkable difference between the predictions of the semihard approach and the SPM especially for $p_{\perp}$- and $z$-distributions of the $J/\Psi$ mesons at HERA.

In present paper we investigate the open heavy quark production processes in the semihard approach, which was used early in Ref. [10] for calculation of heavy quark production rates at hadron colliders and for prediction of $J/\Psi$ and $\Upsilon$ photoproduction cross sections at high energies in Refs. [21, 22].

2 QCD Cross Section for Heavy Quark Electroproduction

We calculate the total and differential cross sections (the $p_{\perp}$ and rapidity distributions) of charm and beauty quark photoproduction via the photon-gluon fusion QCD subprocess (Fig.1) in the framework of the semihard ($k_{\perp}$ factorization) approach [4, 14]. First of all we take into account the transverse momentum of gluon $\vec{q}_{2\perp}$, its virtuality $q_{2\perp}^2 = -\vec{q}_{2\perp}^2$ and the alignment of the polarization vectors along the transverse momentum such as $\epsilon_{\mu} = q_{2\perp}/|q_{2\perp}|$ [10, 12].

Let us define Sudakov variables of the process $ep \rightarrow Q\bar{Q}X$ (Fig.2):

$$
\begin{align*}
p_1 &= \alpha_1 p_1 + \beta_1 p_2 + p_{1\perp}, \\
q_1 &= x_1 p_1 + q_{1\perp}, \\
p_2 &= \alpha_2 p_1 + \beta_2 p_2 + p_{2\perp}, \\
q_2 &= x_2 p_2 + q_{2\perp},
\end{align*}
$$

(1)
where
\[ p_1^2 = p_2^2 = M^2, \quad q_1^2 = q_1^2 \perp, \quad q_2^2 = q_2^2 \perp, \]
p_1 and p_2 are 4-momenta of the heavy quarks, q_1 is 4-momentum of the photon, q_2 is 4-momentum of the gluon, p_1 \perp, p_2 \perp, q_1 \perp, q_2 \perp, are transverse 4-momenta of these ones. In the center of mass frame of colliding particles we can write P_1 = (E, 0, 0, E), P_2 = (E, 0, 0, −E), where \( E = \sqrt{s}/2 \), \( P_1^2 = P_2^2 = 0 \) and \( (P_1P_2) = s/2 \). Sudakov variables are expressed as follows:
\[
\begin{align*}
\alpha_1 &= \frac{M_1 \perp}{\sqrt{s}} \exp(y_1^\perp), \quad \alpha_2 = \frac{M_2 \perp}{\sqrt{s}} \exp(y_2^\perp), \\
\beta_1 &= \frac{M_1 \perp}{\sqrt{s}} \exp(−y_1^\perp), \quad \beta_2 = \frac{M_2 \perp}{\sqrt{s}} \exp(−y_2^\perp),
\end{align*}
\]
where \( M_{1,2 \perp} = M^2 + p_{1,2 \perp}^2 \), \( y_{1,2} \) are rapidities of heavy quarks, \( M \) is heavy quark mass.

From conservation laws we can easily obtain following conditions:
\[
q_1 \perp + q_2 \perp = p_1 \perp + p_2 \perp, \quad x_1 = \alpha_1 + \alpha_2, \quad x_2 = \beta_1 + \beta_2.
\]

The differential cross section of heavy quarks electroproduction has form:
\[
\frac{d\sigma}{d^2p_{1 \perp}}(ep \rightarrow Q\bar{Q}X) = \int dy_1^\perp dy_2^\perp \frac{d^2q_{1 \perp}}{\pi} \frac{d^2q_{2 \perp}}{\pi} \frac{|M|^2 Φ(x_1, q_{1 \perp}^2)Φ_p(x_2, q_{2 \perp}^2)}{16π^2(x_1x_2s)^2}
\]

For photoproduction process it reads:
\[
\frac{d\sigma}{d^2p_{1 \perp}}(γp \rightarrow Q\bar{Q}X) = \int dy_1^\perp \frac{d^2q_{2 \perp}}{\pi} \frac{Φ_p(x_2, q_{2 \perp}^2)|M|^2}{16π^2sx_2^2α_2}
\]

We use generalized gluon structure function of a proton \( Φ_p(x_2, q_{2 \perp}^2) \) which is obtained in semihard approach. When integrated over transverse momentum \( q_{2 \perp} (0, q_{2 \perp}, 0) \) of gluon up to some limit \( Q^2 \) it becomes the usual structure function giving the gluon momentum fraction distribution at scale \( Q^2 \):
\[
\int \Phi_p(x, q_{2 \perp}^2)dq_{2 \perp}^2 = xG_p(x, Q^2).
\]

We use in our calculation following phenomenological parameterization [10]:
\[
Φ_p(x, q_{1 \perp}^2) = C \frac{0.05}{x + 0.05}(1 - x)^3 f_p(x, q_{1 \perp}^2),
\]
where
\[
\begin{align*}
f_p &= 1, \quad q_{1 \perp}^2 ≤ q_0^2(x), \\
f_p &= \left( \frac{q_0^2(x)}{q_{1 \perp}^2} \right)^2, \quad q_{1 \perp}^2 > q_0^2(x),
\end{align*}
\]
and \( q_0^2(x) = Q_0^2 + Λ^2 \exp(3.56 \sqrt{\ln(x_0/|x|)}), Q_0^2 = 2\text{GeV}^2, Λ = 56 \text{ MeV}, x_0 = 1/3 \). The normalization factor \( C ≃ 0.97 \) mb of the structure function \( Φ_p(x, q_{2 \perp}^2) \) was obtained in [10] where bb-pair production at Tevatron energy was described.

The \( Φ(x_1, q_{1 \perp}^2) \) is well known virtual photon spectrum in Weizsacker-Williams approximation [23] before the integration over \( q_{2 \perp}^2 \):
\[
Φ(x_1, q_{1 \perp}^2) = \frac{α}{2π} \left[ \frac{1 + (1 - x_1)^2}{x_1 q_{1 \perp}^2} - \frac{2 m_e^2 x_1}{q_{1 \perp}^2} \right].
\]
The effective gluon distribution \( xG(x, Q^2) \) obtained from (7) increases at not very small \( x \) \((0.01 < x < 0.15)\) as \( x^{-\omega_0} \), where \( \omega_0 = 0.5 \) corresponds to the BFKL Pomeron singularity \( \tilde{\omega} \). This increases continuously up to \( x = x_0 \), where \( x_0 \) is a solution of the equation \( q_0^2(x_0) = Q^2 \). In the region \( x < x_0 \) there is the saturation of the gluon distribution: \( xG(x, Q^2) \simeq CQ^2 \).

The square of matrix element of partonic subprocess \( \gamma^* g^* \rightarrow Q\bar{Q} \) can be written as follows:

\[
|M|^2 = 16\pi^2 e_Q^2 \alpha_s \alpha(x_1 x_2 s)^2 \left[ \frac{1}{(\hat{u} - M^2)(\hat{t} - M^2)} - \frac{1}{q_1^2 q_2^2} \left( 1 + \frac{\alpha_2 \beta_1 s}{\hat{t} - M^2} + \frac{\alpha_1 \beta_2 s}{\hat{u} - M^2} \right) \right]
\]

(10)

For real photon and off-shell gluon it reads:

\[
|M|^2 = 16\pi^2 e_Q^2 \alpha_s \alpha(x_1 x_2 s)^2 \left[ \frac{\alpha_1^2 + \alpha_2^2}{(\hat{t} - M^2)(\hat{u} - M^2)} + \frac{2M^2}{q_1^2 q_2^2} \left( \frac{\alpha_1}{\hat{u} - M^2} + \frac{\alpha_2}{\hat{t} - M^2} \right) \right] \]

(11)

where \( \alpha_2 = 1 - \alpha_1 \) and \( \hat{s}, \hat{t}, \hat{u} \) are usual Mandelstam variables of partonic subprocess

\[
\begin{align*}
\hat{s} &= (p_1 + p_2)^2 = (q_1 + q_2)^2, \\
\hat{t} &= (p_1 - q_1)^2 = (p_2 - q_2)^2, \\
\hat{u} &= (p_1 - q_2)^2 = (p_2 - q_1)^2, \\
\hat{s} + \hat{t} + \hat{u} &= 2M^2 + q_1^2 + q_2^2.
\end{align*}
\]

(12)

3 Discussion of the Results

The results of our calculations for the total cross sections of \( c \)- and \( b \)-quark photoproduction are shown in Fig.3 (solid curves). Dashed curves correspond to the SPM predictions with the GRV parametrization of the gluon distribution \( [24] \). The results of calculations for the SPM are shown without K-factor, which have the typical value \( K = 2 \) for hard scattering processes. (In the semihard approach K-factor is absent \( [10, 22] \)). The K-factor in the SPM may change the relation between the results of calculations in the semihard approach and SPM. But independently from it we see that the saturation effects more clearly are pronounced for charm quark photoproduction (at \( \sqrt{s_{\gamma p}} \approx 500 \text{ GeV} \)). In any case (with or without K-factor in SPM) the cross section for beauty quark photoproduction in the semihard approach is the higher than one in the SPM at \( \sqrt{s_{\gamma p}} > 200 \text{ GeV} \).

The \( p_\perp \) distributions for \( c \)- and \( b \)-quark photoproduction in the semihard approach (solid curves) and in the SPM (dashed curves) are shown in Fig.4. The curves are obtained in the semihard approach for charm quark photoproduction show the saturation effects in low \( p_\perp \) region \( (p_\perp < 2 \text{ GeV}/c) \). In middle \( p_\perp \) region \( (2 \text{ GeV}/c < p_\perp < 15 \text{ GeV}/c) \) the heavy quark photoproduction \( p_\perp \) distributions in the semihard approach are the higher ones of the SPM (with GRV parametrization of gluon distribution). At high \( p_\perp > 15 \text{ GeV}/c \) we have contrary relation between \( p_\perp \) distributions in the semihard approach and SPM.

Thus the SPM leads to over-estimated cross section in the low \( p_\perp \) region and under-estimated one in the middle \( p_\perp \) region as was noted as early as in Refs. \( [10, 13] \) (see also \( [24] \)). This behavior of \( p_\perp \) distributions in the \( k_\perp \) factorization approach is result from the off mass shell subprocess cross section \( [13] \) as well as the saturation effects of gluon structure function in semihard approach \( [10] \).
Fig. 5 show the comparison of the results for heavy quark rapidity distribution (in the photon-proton center of mass frame) in the different models: solid curve shows the y distribution in the semihard approach, dashed curve shows one in SPM. The discussed above effects are sufficiently large near the kinematic boundaries, i.e. at big value of $|y^*|$. We see that the difference between solid and dashed curvers can’t be degrade at all $y^*$ via change of normalization of SPM prediction.

4 Conclusions

We shown that the semihard approach leads to the saturation effects for the total cross section of charm quark photoproduction at available energies and predicts the remarkable difference for rapidity and transverse momentum distributions of charm and beauty quark photoproduction, which can be study already at HERA ep collider.

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Figure captions

1. QCD diagrams for open heavy quark photoproduction subprocesses
2. Diagram for heavy quark electroweak production
3. The total cross section for open charm and beauty quark photoproduction: solid curve - the semihard approach, dashed curve - the SPM
4. The $p_{\perp}$ distribution for charm and beauty quark photoproduction at $\sqrt{s_{\gamma p}} = 200$ GeV: curves as in Fig.3
5. The $y^*$ distribution for charm and beauty quark photoproduction: at $\sqrt{s_{\gamma p}} = 200$ GeV: curves as in Fig.3