Lyα Radiative Transfer: Modeling Spectrum and Surface Brightness Profiles of Lyα-emitting Galaxies at Z = 3–6

Hyunmi Song, Kwang-II Seon, and Ho Seong Hwang

1 Department of Astronomy, Yonsei University, 50 Yonsei-ro, Seodaemun-gu, Seoul 03722, Republic of Korea
2 Korea Astronomy & Space Science Institute, Daedokdae-ro 776, Yuseong-gu, Daejeon 34055, Republic of Korea
3 Astronomy and Space Science Major, University of Science and Technology, Daejeon 34113, Republic of Korea

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Abstract

We perform Lyα radiative transfer calculations to reproduce the Lyα properties of star-forming galaxies at high redshifts. We model a galaxy as a halo in which the density distributions of Lyα sources and H I plus dust medium are described with exponential functions. We also consider an outflow of the medium that represents a momentum-driven wind in a gravitational potential well. We successfully reproduce both the spectra and the surface brightness profiles of eight star-forming galaxies at z = 3–6 observed with the Multi-Unit Spectroscopic Explorer using this outflowing-halo model with Lyα scattering. The best-fit model parameters (i.e., the outflowing velocity and optical depth) for these galaxies are consistent with those in other studies. We examine the impacts of individual model parameters and input spectrum on the emerging spectrum and surface brightness profile. Further investigations of the correlations among observables (i.e., the spatial extent of Lyα halos and Lyα spectral features) and model parameters, and of spatially resolved spectra are presented as well. We demonstrate that the combination of spectrum and surface brightness profile provides strong constraints on model parameters and thus on the spatial/kinematic distributions of the medium.

Unified Astronomy Thesaurus concepts: Extragalactic astronomy (506); Lyman-alpha galaxies (978); Radiative transfer (1335); Radiative transfer simulations (1967); High-redshift galaxies (734); Interstellar medium (847); Circumgalactic medium (1879)

1. Introduction

Lyα emission is one of the most prominent emission features in the universe. It is usually generated by the interplay between atomic hydrogen (H I, the most common element in the universe) and ionizing photons from young stars. It was expected that strong Lyα emission would be detected from star-forming galaxies and could be used to probe even the first generation of galaxies at high redshifts (Partridge & Peebles 1967; Sunyaev et al. 1978). However, the detection of Lyα emission from either nearby galaxies or high-redshift galaxies has failed. Meier & Terlevich (1981) reported the first attempt to search for Lyα emission in nearby H II-selected galaxies, and found Lyα emission in only one galaxy; the first detection of high-redshift Lyα emitters that are not physically associated with quasars was made in 1998 by Cowie & Hu (1998).

The difficulties of detecting Lyα emission arise because of particular technical requirements; space telescopes for nearby sources and high detection sensitivities for high-redshift sources are necessary for a good detection. However, the difficulties are also due to the resonance-scattering nature of Lyα; Lyα photons are endlessly scattered off by neutral hydrogen atoms until they happen to be scattered into wing frequencies, which occurs quite rarely. Lyα photons are forced to travel long distances in the H I medium and have a high possibility of being destructed by dust, which eventually suppresses the observed intensity. In addition, this whole process is sensitive to the spatial and kinematic distributions of the H I medium, so it is not easy to understand the observed correlations between physical parameters (e.g., between Lyα intensity and metallicity, Lyα intensity and UV continuum flux, etc.) or to make observational predictions (Meier & Terlevich 1981; Hartmann et al. 1988; Neufeld 1990; Charlot & Fall 1993; Valls-Gabaud 1993; Kunth et al. 1998; Tenorio-Tagle et al. 1999; Mas-Hesse et al. 2003).

Nevertheless, there have been continuous efforts to search for Lyα emission from the observation side. There are targeted observations for individual objects as well as large photometric and spectroscopic surveys. Such Lyα surveys include the Large Area Lyman Alpha survey (LALA, Rhoads et al. 2000), the Lyα Reference Sample survey (LARS, Ostlin et al. 2014), the Subaru Deep Field survey (e.g., SILVERRUSH, Ouchi et al. 2018), and the survey with the Multi-Unit Spectroscopic Explorer (MUSE, Bacon et al. 2015). The MUSE at the ESO Very Large Telescope (VLT) is the latest technological advance with 24 integral field units and unprecedented sensitivity. There are several recent observations with MUSE that reveal ubiquitous Lyα emission in the universe (Bacon et al. 2015; Wisotzki et al. 2016; Drake et al. 2017; Leclercq et al. 2017).

Observations have shown that the shape of the Lyα line is diverse. It includes broad damped absorption profiles, P-Cygni profiles, double-peak profiles, pure symmetric emission profiles, and combinations thereof (Kunth et al. 1998; Mas-Hesse et al. 2003; Shapley et al. 2003; Möller et al. 2004; Noll et al. 2004; Tapken et al. 2004; Venemans et al. 2005; Wilman et al. 2005). This variety can be understood through a detailed radiative transfer calculation, which is analytically solvable only for simple cases (e.g., a static, plane-parallel slab by Harrington 1973 and Neufeld 1990, and a static uniform sphere by Dijkstra et al. 2006). Later, numerical algorithms based on Monte Carlo techniques were developed to solve radiative transfer for more general cases. Now theoretical studies mostly rely on them (e.g., Spaans 1996; Loeb & Rybicki 1999; Ahn et al. 2000, 2002; Zheng & Miralda-Escude 2002; Richling 2003; Cantalupo et al. 2005; Dijkstra et al. 2006; Hansen & Oh 2006; Tasić 2006;
Verhamme et al. 2006, 2015; Laursen et al. 2013; Behrens et al. 2014; Duval et al. 2014; Gronke et al. 2015; Smith et al. 2019; Lao & Smith 2020; Michel-Dansac et al. 2020). Meanwhile, a galaxy model needs to be constructed to perform such a radiative transfer calculation. One can adopt a realistic galaxy model from hydrodynamical simulations. Galaxies from such simulations can be useful for performing a statistical study of Lyα properties, but they cannot be directly used to model individual galaxies in observations. Therefore it would be better to adopt a simple but manageable toy model for the purpose of reproducing observations. One example for such models is a shell model, in which a central Lyα source is surrounded by a constantly expanding, homogeneous, spherical shell of H I medium with dust. Although this shell model has surprisingly well reproduced many observed Lyα line profiles (e.g., Ahn 2004; Schaerer & Verhamme 2008; Verhamme et al. 2008; Schaerer et al. 2011; Gronke et al. 2015; Yang et al. 2016; Gronke 2017; Karman et al. 2017), because of its extreme simplicity and contrivance, there is still room for improvement (e.g., see Section 7.2 in Yang et al. 2016; Orlitová et al. 2018).

On the other hand, there are other observables than the spectrum that are not yet fully understood. Dijkstra & Kramer (2012) reported one of the few studies that focused on reproducing observed Lyα absorption features in the spectra of background galaxies and Lyα halos around star-forming galaxies by considering a galaxy model with an outflowing clumpy medium. Here, Lyα halos denote the spatial distribution of Lyα emission, which is much more extended than that of the stellar UV continuum or Hα (e.g., Fynbo et al. 1999; Steidel et al. 2011; Matsuda et al. 2012; Hayes et al. 2013; Yang et al. 2017). This indicates a rich gas content in the circumgalactic medium (CGM). Thus, Lyα halos might provide us the information on the spatial distribution and kinematics of the CGM, which is important for understanding galaxy formation and evolution. However, Lyα halos are commonly detected in a stacked image because of their low surface brightness (Steidel et al. 2011; Momose et al. 2014, 2016). Thanks to the MUSE observations, which revealed ubiquitous Lyα halos around star-forming galaxies at high redshifts (Wisotzki et al. 2016; Leclercq et al. 2017), we can now study Lyα halos for individual galaxies, and even the spatial variations of their spectral properties (Erb et al. 2018; Claeyssens et al. 2019; Leclercq et al. 2020).

Therefore we are in a good position to fit the Lyα spectrum and the Lyα surface brightness profile for individual high-redshift star-forming galaxies, and to test a galaxy model with Lyα radiative transfer. In this study, we construct a galaxy toy model that is improved from the shell model to better represent real galaxies: a halo in which the density distributions of Lyα sources and H I plus dust medium, as well as medium bulk motion, are modeled with free parameters. We perform Lyα Monte Carlo radiative transfer calculations with this galaxy model for a number of model parameter sets. We then find a parameter set that best reproduces the observed Lyα spectrum and surface brightness profile of high-redshift star-forming galaxies reported in Leclercq et al. (2017; hereafter L17). We discuss the advantage of simultaneously using these two observables in constraining models, and examine various correlations among the model parameters and observables. We also explore spatially resolved Lyα spectra and the impact of input Lyα spectrum on emerging Lyα spectrum and surface brightness profile.

This paper is constructed as follows. In Section 2 we describe our Lyα Monte Carlo radiative transfer simulation, the galaxy model, the observational data to fit, and the fitting process. We present the best-fit results for the galaxy sample in Section 3. Further discussions and summaries are presented in Sections 4 and 5, respectively.

2. Method

2.1. Lyα Monte Carlo Radiative Transfer

We describe the Lyα Monte Carlo radiative transfer calculations we perform in this section. We use a code called LaRT (Lyα radiative transfer, Seon & Kim 2020). It is written in modern Fortran with the message-passing interface, and is enabled to consider arbitrary three-dimensional distributions for density, temperature, and kinematics of sources and medium on a regular Cartesian grid. We set a 1283 grid, which gives a physical resolution of ~0.2 kpc. In the code, we generate and track photons (or photon packets) in real and frequency spaces for a given simulation parameter set. The calculation is performed by distributing photons over processors and is later summed. The master-slave algorithm is used to implement dynamic load balancing. The code was extensively tested to reproduce the well-known benchmark cases for static slab and spherical geometries as well as for Hubble-like expanding spherical media. The test was performed not only in hydrogen-only media, but also in dusty media. The procedures performed in the code for each photon are listed below.

1. We generate a photon with a position vector \( r \), a direction vector \( \hat{k} \), and a frequency \( \nu \) drawn from the spatial distribution function of the Lyα source under consideration, an isotropic distribution function, and a Lyα input frequency distribution function of interest, respectively.

2. We randomly choose a \( \tau \) value following a probability distribution of \( \exp(-\tau) \) to determine the traveling distance \( \ell \) for the photon through the relation \( \tau = \int_{0}^{\ell} (\sigma_{H I}(x) + \sigma_{d}(x)) dx \), where \( \sigma \) and \( n \) represent the cross section and the number density of atomic hydrogen (H I) or dust (d), respectively. The optical depth due to hydrogen atoms is calculated using the Voigt routine in the VPFIT package (Carswell & Webb 2014).

3. The position of the photon is updated to \( r + \ell \hat{k} \). At the new position, we decide which of the two (hydrogen and dust) the photon will interact with by considering the probabilities given by their optical depths to the total optical depth. If the photon interacts with hydrogen (i.e., the photon being scattered), a new direction vector is drawn from the Rayleigh phase function. Then, the velocity components of the scattering atom are drawn from a function given by Equation (4) of Zheng & Mirafla-Escudé (2002) and the Maxwell–Boltzmann distribution (see Seon & Kim 2020, for more details). This velocity of the scattering atom and the old and new direction vectors of the photon determine a new frequency following the energy-momentum conservation law. If it interacts with dust, the photon will be either

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https://seoncafe.github.io/LaRT.html
scattered with the probability of \( a \) (dust albedo) or absorbed with a probability of \( 1 - a \). When the photon is scattered by dust, the frequency is not changed, and a new direction is drawn from a proper angular redistribution function (the Henyey–Greenstein phase function by default, Henyey & Greenstein 1941; Witt 1977).

4. We repeat steps 2 and 3 until either the photon escapes the system considered (i.e., a galaxy) or is absorbed by dust. Then, we go to step 1 for a new photon.

Although the absorption and scattering by dust can be explicitly simulated using our code with various choices of albedo and scattering phase function, as described in step 3, we postprocess the effect of dust to reduce the computational time. Whenever each photon escapes the galaxy, we record its initial position \( (r_i, \nu_i) \), initial frequency \( (\nu_i) \), cumulative (the photon has gone through) hydrogen optical depth \( (\bar{\tau}_{H_1}) \), cumulative hydrogen column density \( (\bar{N}_{H_1}) \), final (projected) position in the observed image \( (R_f) \), final frequency \( (\nu_f) \), and distance to an observer from the final position \( (\delta_{obs}) \). The initial position/frequency and the cumulative hydrogen column density are particularly recorded for postprocessing the spatial/frequency distributions of Ly\( \alpha \) source and dust effect. This postprocessing is adopted to reduce the number of simulation runs that are required to study the effect of the three. Following Gronke et al. (2015), we postprocess the spatial/frequency distributions of Ly\( \alpha \) and dust effect by adjusting the weight \( w \) of each photon to

\[
w = \frac{\mathcal{R}_f(r) \mathcal{S}_f(\nu)}{\mathcal{R}_i(r) \mathcal{S}_i(\nu)} \times \exp\left(- (1 - a) \bar{\tau}_{H_1} \bar{N}_{H_1} \frac{\Delta \nu}{\nu} \right) \text{DGR/DGR}_{MW}.
\]

(1)

\( \mathcal{R} \) and \( \mathcal{S} \) are the input spatial and spectral distributions of Ly\( \alpha \) sources, respectively. Here the subscripts \( i \) and \( f \) indicate the initial (i.e., chosen as an input for the simulation run) and final choices, respectively. DGR/DGR\(_{MW}\) is the amount of dust relative to gas (dust-to-gas ratio, DGR) normalized by the value of the Milky Way, and \( \bar{\tau}_{H_1} \) is the dust cross section per neutral hydrogen of the Milky Way, which is the product of \( \sigma_{d,MW} \) and DGR\(_{MW}\). Our choices of \( \mathcal{R}_i \) and \( \mathcal{S}_i \) are uniform distribution functions of \( r \) and \( \nu \), respectively, within given ranges, which can be easily modified for other distributions through postprocessing. The exponential term can be simply rewritten as \( \exp\left(- (1 - a) \bar{\tau}_{H_1} \right) \), which depicts the fraction of each photon that would not be destructed by dust but survive after its travel in the galaxy. In the postprocessing, we assume that dust scatters photons into the direction of its original propagation (perfect forward-scattering), the validation of which is presented in Appendix A. At far-UV wavelengths, scattering is indeed strongly forward-directed (Seon & Draine 2016). The dust extinction cross section and albedo of the Milky Way are \( \sigma_{d,MW} = 1.61 \times 10^{-21} \text{cm}^2/\text{H} \) and \( \alpha_{d,MW} = 0.325 \), respectively (Weingartner & Draine 2001; Draine 2003).

To make observables, we use the information of all photons at the moment when each of them escapes the galaxy. This method works because we set the system to be spherically symmetric (Section 2.2), and thus the observation from all directions will be identical.

### 2.2. Galaxy Model

We construct a galaxy model by improving the widely used shell model, which is more realistic but still simple. We adopt a spherically symmetric halo model for the distribution of hydrogen atoms (no deuterium) plus dust medium, whose density follows an exponential function with a scale radius \( (r_{\text{HI}}) \) that is a free parameter. The overall density level is another free parameter, that is, the optical depth from the center to the edge of the halo for a photon at the Ly\( \alpha \) central frequency \( (\tau_{HI}) \). We fix the medium temperature at \( 10^4 \text{K} \).

We assume that all the Ly\( \alpha \) photons are generated through recombinations of electrons and hydrogen ions that are ionized by UV photons from young stars (i.e., H II regions around young stars). We thus model the spatial distribution of Ly\( \alpha \) sources using the observed UV continuum surface brightness profiles. The UV surface brightness profile of the target galaxies in this study is well described by an exponential function of the projected distance from the galaxy center (see the rightmost panels of Figures 2 and 3 in L17), which is reconstructed as a modified Bessel function of the second kind with \( n = 0 \left( K_0(\chi) \right) \) when the distance is measured in three-dimensional space. We therefore assume that the spatial distribution of Ly\( \alpha \) sources follows \( K_0(r/r_{\text{cont}}) \), where \( r_{\text{cont}} \) is the scale radius of the UV continuum surface brightness profile. Ly\( \alpha \) photons are assumed to follow a Voigt profile with temperature \( 10^4 \text{K} \) (a typical value for H II regions) in frequency space. As mentioned in Section 2.1, the spatial and frequency distributions of Ly\( \alpha \) photons are postprocessed with Equation (1) by inserting the Bessel function and the Voigt function for \( \mathcal{R}_f \) and \( \mathcal{S}_f \), respectively.

We consider the bulk motion of the medium as most preexisting models do. An observed Ly\( \alpha \) emission line is typically singly red-peaked or red-peak dominated, which is expected to emerge from outflowing medium (e.g., Verhamme et al. 2006, 2008; Vanzella et al. 2010). There is other observational evidence such as the offsets between redshifts of interstellar absorption lines, Ly\( \alpha \) lines, and nebular emission lines (Steidel et al. 2010, and references therein), and blueshifted absorption and redshifted emission for resonance lines (Prochaska et al. 2011, and references therein). In particular, we employ an outflow model in which the velocity increases and then decreases as radial distance from the center increases. This is motivated by Dijkstra & Kramer (2012, see their Section 6.1 and references therein) to describe a momentum-driven wind from the center that is decelerating in the galactic gravitational potential well. However, instead of adopting a functional form by solving a momentum equation in a gravitational potential well as in Dijkstra & Kramer (2012), we rather consider a linearly increasing and then decreasing function for simplicity. It is expressed as

\[
V(r) = \begin{cases} V_{\text{peak}} r/r_{\text{peak}} & \text{if } r \leq r_{\text{peak}} \\ V_{\text{peak}} + \Delta V (r - r_{\text{peak}})/(r_{\text{max}} - r_{\text{peak}}) & \text{if } r > r_{\text{peak}}. \end{cases}
\]

(2)

where \( V_{\text{peak}} \), \( r_{\text{peak}} \), and \( \Delta V \) are the peak velocity, the radius at which the velocity reaches the peak value, and the velocity difference between those at \( r_{\text{max}} \) and \( r_{\text{peak}} \) (\( \Delta V \) is allowed in the range between \( -V_{\text{peak}} \) and 0) to represent decelerated

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5 The subscript, 0, indicates the Ly\( \alpha \) central frequency.
The MUSE spectral resolution and parameter values

Notes. If it is not mentioned, each parameter is in the unit stated here for the rest of the paper.

- a The scale radius of the medium density distribution that is described by an exponential function. It is in the normalized unit by the maximum scale of the system (rmax) we set in the simulation. All the length scales in this study are in the normalized unit by rmax.
- b The radius when the expanding velocity of the medium reaches its peak, in the normalized unit by rmax.
- c The peak velocity of the expanding velocity profile of the medium in units of km s\(^{-1}\).
- d The difference between the velocities at rmax and Vpeak in units of km s\(^{-1}\).
- e The optical depth measured at the Ly\(\alpha\) central frequency, which is given by the product of the hydrogen column density of the system and the cross section of a Ly\(\alpha\) photon at the central frequency with a neutral hydrogen atom.
- f Dust-to-gas ratio, which is the amount of dust relative to gas (in terms of mass). It is in the normalized unit by the value of the Milky Way (DGR\(_{\text{MW}}\)).
- g Redshift. \(z_0\) is an approximate estimate for the redshift of the system derived from observations.

outflowing motion but prevent inflowing motion), which are free parameters in our galaxy model. It should be noted that in our model, the initial generation of Ly\(\alpha\) photons is not from this expanding medium (i.e., nonmoving sources). Therefore, the frequency of Ly\(\alpha\) photons observed by a hydrogen atom in the medium will be seen shifted by the amount that is proportional to the medium velocity.

These five free parameters (\(r_{\text{sh}}\), and \(\tau_0\) for the medium density structure, \(V_{\text{peak}}\), \(r_{\text{peak}}\), and \(\Delta V\) for the medium kinematic structure) are simulation parameters, each of which is assigned a value from a range of interest for a simulation run. In addition, the relative amount of dust with respect to gas (DGR) and redshift (\(z\)) of a model galaxy are also considered as free parameters in our model. Their effects are implemented through postprocessing, the so-called postprocessing parameters. In postprocessing, the weight of each photon in a simulation output is adjusted following Equation (1) for a chosen DGR value, and their final wavelengths are redshifted by a factor of \(1 + z\) in the observer’s frame due to the cosmic expansion. Simulated observables are constructed in the observer’s frame.

Table 1 shows the parameter grid of the simulation. The simulation is run on the grid of the five simulation parameters, and outputs are additionally postprocessed on the grid of the two postprocessing parameters. In total, we run 13,230 simulations with \(10^6\) photons per simulation, and each simulation is postprocessed over additional 110 postprocessing parameter grid points.

| Parameter | Values |
|-----------|--------|
| \(r_{\text{sh}}\) \(^a\) | [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9] |
| \(r_{\text{peak}}\) \(^b\) | [0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6] |
| \(V_{\text{peak}}\) \(^c\) | [100, 200, 300, 400, 500] |
| \(\Delta V\) \(^d\) | [−500, −450, −400, ..., −100, −50, 0] |
| log \(\tau_0\) \(^e\) | [5.7, 6.0, 6.3, 6.6, 6.9, 7.2] |
| DGR\(^f\) | [0.0, 0.2, 0.4, ..., 1.6, 1.8, 2.0] |
| \(z\) \(^g\) | \(z_0 + [0.001, 0.002, 0.003, ..., 0.008, 0.009, 0.010]\) |

Notes.

1. The difference between the velocities at rmax and Vpeak in units of km s\(^{-1}\).
2. The optical depth measured at the Ly\(\alpha\) central frequency, which is given by the product of the hydrogen column density of the system and the cross section of a Ly\(\alpha\) photon at the central frequency with a neutral hydrogen atom.
3. Dust-to-gas ratio, which is the amount of dust relative to gas (in terms of mass). It is in the normalized unit by the value of the Milky Way (DGR\(_{\text{MW}}\)).
4. Redshift. \(z_0\) is an approximate estimate for the redshift of the system derived from observations.

2.3. Observational Data and Fitting Procedure

The targets that we aim to model with our simulation are taken from L17. They investigated 250 Lyman alpha emitters (LAEs) in the Hubble Ultra Deep Field, which were observed with the MUSE to study the extended Ly\(\alpha\) halo around individual galaxies. Because the MUSE covers a wavelength range between 4750 and 9350 Å, the targeted LAEs are mainly at a redshift range 2.75 ≤ z ≤ 6.5. L17 summarized all the measurements from their analysis in Table B.1. The measurements are good enough to fully reconstruct observed surface brightness profiles, but not spectra (only the equivalent width (EW) and the full width at half maximum (FWHM) of Ly\(\alpha\) lines are available in their table). Therefore, we use only 14 LAEs for which Ly\(\alpha\) spectrum and surface brightness profile are fully presented in their Figures 2 and 3 as a pilot study. Because we set an outflow model to reproduce red-dominated spectra that are typical Ly\(\alpha\) features in observations, we exclude three LAEs with a double-peak spectrum (MUSE 1087, 106, and 6297). We also exclude three LAEs with only a few photons in their surface brightness profiles, two of which have no counterparts in the UV continuum (MUSE 6498, 6534, and 218). As a result, we have eight LAEs, which are MUSE 1185, 82, 6905, 1343, 53, 171, 547, and 364. We read data points of their spectrum and surface brightness profile using Engauge Digitizer, a free software that extracts data points from images with graphs.

On the simulation side, we construct a spectrum and a surface brightness profile by counting photons (with their weight) in a given wavelength bin and a radial bin, respectively. To better match the observation, it is necessary to follow the actual observational setups, including the aperture for spectrum, imaging bandwidth, spectral resolution, and point-spread function (PSF). Following the processes described in L17, we apply their aperture size (\(r_{\text{lim}}\)) to their Figures 2 and 3) and imaging bandwidth (the purple shaded range in the third column) when we count photons to construct the spectrum and surface brightness profile, respectively. We also convolve the spectrum with a Gaussian kernel with the MUSE spectral resolution of R = 3000, and convolve the surface brightness profile with a Moffat kernel with a fixed power index (\(\beta\)) of 2.8 and the MUSE PSF FWHM ~ 0.87. The MUSE spectral resolution and PSF FWHM are dependent on wavelength, but the dependence is weak (Bacon et al. 2017). Therefore we ignore their wavelength dependence for simplicity. The aperture size (\(r_{\text{lim}}\)) applied to each MUSE galaxy is summarized in Table 2 together with the scale radius of the UV surface brightness profile (\(r_{\text{cont}}\)). Both are in units normalized by rmax, i.e., the maximum angular extent of 5°, as displayed in Figures 2 and 3 in L17, which corresponds to the maximum angular size of the galaxy in the simulation.

To quantify how well the spectrum and the surface brightness profile of simulation describe the observational data, we define a likelihood as

\[
\ln L \propto -\frac{1}{2} \sum_i \left( \frac{O_i - M_i}{\sigma(O_i)} \right)^2.
\]

Here, \(i\) denotes the \(i\)th bin (a wavelength bin for the spectrum and a radial bin for the surface brightness profile), \(O\) and \(M\) denote values from the observation and model, respectively, and \(\sigma(O)\) is the error in observational data. Assuming that the spectrum and surface brightness profile are independent of each
Table 2
Observed Properties of Our Target Galaxies

| MUSE # | r_scaled | r_max | [\lambda_1, \lambda_2] | SN_sp | SN_SB | SN_tot |
|--------|----------|-------|------------------------|-------|-------|--------|
| 1185   | 0.041    | 0.24  | [6681.179, 6692.522]    | 7.747 | 7.051 | 7.482  |
| 82     | 0.017    | 0.20  | [5598.684, 5611.14]     | 3.344 | 6.025 | 4.327  |
| 6905   | 0.029    | 0.20  | [4982.524, 4990.045]    | 3.853 | 4.096 | 3.943  |
| 1343   | 0.016    | 0.20  | [6038.929, 6050.097]    | 1.427 | 2.070 | 1.748  |
| 53     | 0.030    | 0.24  | [7021.233, 7029.988]    | 10.158| 7.192 | 9.205  |
| 171    | 0.025    | 0.24  | [5934.927, 5941.174]    | 2.471 | 4.171 | 3.168  |
| 547    | 0.011    | 0.24  | [8477.564, 8486.28]     | 3.891 | 4.398 | 4.103  |
| 364    | 0.014    | 0.24  | [6004.97, 6009.919]     | 1.213 | 3.008 | 1.801  |

Notes.
a Scale radius of the UV continuum radial profile in units of r_max. Adopted from rs_cont in Figure 4 (also see Table B.2 in L17).
b Aperture size for a spectrum obtained by roughly measuring the radius of the HST segmentation mask that is convolved with the MUSE PSF (white contour in each panel of the second column of Figures 2 and 3 in L17).
c Wavelength range for the surface brightness profile (i.e., image bandwidth) that corresponds to the purple area in each panel of the third column of Figures 2 and 3 in L17.
d Mean signal-to-noise ratio of the observed spectrum data.
e Mean signal-to-noise ratio of the observed surface brightness profile data.
f Mean signal-to-noise ratio of the observed spectrum and surface brightness profile data.

The total likelihood \( \mathcal{L}_{\text{tot}} \) of a model with a given parameter set can be defined by a product of the likelihoods of the spectrum and surface brightness profile (\( \mathcal{L}_{\text{sp}} \) and \( \mathcal{L}_{\text{SBP}} \), respectively, and the subscripts \( \text{sp} \) and \( \text{SBP} \) stand for spectrum and surface brightness profile, respectively). By calculating \( \mathcal{L}_{\text{tot}} \) for all permitted parameter sets, we construct a surface of \( \mathcal{L}_{\text{tot}} \) in the seven-dimensional (five simulation parameters plus two postprocessing parameters) parameter space. Then, we derive a 1D (marginal) posterior distribution of a parameter and a 2D (marginal) posterior map of two parameters by marginalizing over other parameters assuming uniform priors. A best-fit parameter is found as the value where the maximum of its 1D posterior distribution appears, and the degeneracy between parameters is inferred from 2D posterior maps. We note that the normalizations of the simulated spectrum and the simulated surface brightness profile are adjusted arbitrarily to minimize the difference between model and observation. Because we compare only their shapes, a parameter that is more sensitive to the absolute levels (e.g., DGR) could be poorly constrained by our modeling.

The error of each best-fit value is roughly measured as follows. We linearly connect adjacent \( \mathcal{L} \) values and adjust the normalization so that the area below the \( \mathcal{L} \) profile within the allowed parameter range becomes 1. We then make a \( 1\sigma \) range that includes a best-fit value and covers 68% of the total area.

3. Results: Fit to Observed Ly\( \alpha \) Spectra and Surface Brightness Profiles

In this section, we show 1D and 2D posterior distributions of the seven parameters, constraints on their best-fit value, and the corresponding best model spectrum and surface brightness profile for each of our eight target galaxies. We first fit spectrum and surface brightness profile separately (Section 3.1), and later fit both simultaneously (Section 3.2).

3.1. To Fit Spectra and Surface Brightness Profiles Separately

Figures 1 and 2 show the posterior distributions of the parameters for the spectrum (\( \mathcal{L}_{\text{sp}} \)) and surface brightness profile (\( \mathcal{L}_{\text{SBP}} \)), respectively, of MUSE 1185. Note that posterior values are normalized by their maximum (denoted by \( \mathcal{L} \)). Overall, the spectrum provides tighter constraints on the parameters than the surface brightness profile does. Of the seven parameters, the redshift is most tightly constrained by spectrum, but neither spectrum nor surface brightness profile constrain the DGR. It should be noted that the surface brightness profile also provides a constraint on redshift. The results are similar for other targets.

The best-fit parameter sets for spectrum and surface brightness profile are found based on their 1D posterior distribution as the maximum posterior location for each parameter. We draw two sets of the model spectrum and surface brightness profile (solid line) with observational data (filled circles with error bars) in the left and middle panels: one with the best-fit parameter set for the spectrum (Figure 3), and the other with that for the surface brightness profile (Figure 4). Each best-fit parameter set reproduces well one and not the other.

The right panels show model profiles of the medium density and velocity for each given best-fit parameter set. It is clearly seen that the models for the spectrum and surface brightness profile do not necessarily agree with each other; the discrepancy is removed by fitting the spectra and surface brightness profiles simultaneously.

3.2. To Fit Spectra and Surface Brightness Profiles Simultaneously

To find a best-fit parameter set that simultaneously explains the spectrum and surface brightness profile well, we now explore the total posterior distributions, \( \mathcal{L}_{\text{tot}} \), which are given by the product of \( \mathcal{L}_{\text{sp}} \) and \( \mathcal{L}_{\text{SBP}} \). Figure 5 shows those for MUSE 1185 (we refer to similar figures in Appendix B for other target galaxies). The total posterior distributions become much sharper than those of the spectrum or surface brightness profile. The model spectrum (left) and surface brightness profile (middle) with the best-fit parameter set obtained from \( \mathcal{L}_{\text{tot}} \) are shown in Figure 6. This parameter set reproduces the spectrum and surface brightness profile nicely. The model profiles of medium density and velocity obtained from this simultaneous fit are shown in the right panel. The density profile is the same as that obtained from the surface brightness profile.
profile fit, while the velocity profile is different from the two that we obtained from the separate fits (see the right panels of Figures 3 and 4). The medium density profile tends to follow that determined by surface brightness profile, as indicated in Table 3, which summarizes the fitting results.

We summarize the best-fit parameter sets determined by $\mathcal{L}_{\text{sp}}$, $\mathcal{L}_{\text{SBP}}$, and $\mathcal{L}_{\text{tot}}$, respectively, in Table 3 for all the target galaxies. In most cases, the best-fit parameter set determined by $\mathcal{L}_{\text{tot}}$ is different from that by either $\mathcal{L}_{\text{sp}}$ or $\mathcal{L}_{\text{SBP}}$. In the case of MUSE 53, the parameters are constrained entirely by the spectrum, which can be expected from their posterior distributions given by the spectrum (Figure B8), which are much sharper than those by the surface brightness profile (Figure B9). Although the surface brightness profile does not alter the best-fit parameter values determined by the spectrum, it still tightens the parameter constraints. This is clearly seen in the total posterior distributions (Figure B4) in comparison with the spectrum posterior distributions (Figure B8).

The constraints of the parameters are, of course, largely affected by the data quality. The peak values of the spectrum and surface brightness profile of MUSE 1185, 82, 6905, and 53 are in the orders of $10^{-18}$ erg cm$^{-2}$ s$^{-1}$ Å$^{-1}$ and $10^{-17}$ erg cm$^{-2}$ s$^{-1}$ arcsec$^{-2}$, respectively (see the filled circles with error bars in Figures 6, B10, B11, and B13) with a mean signal-to-noise ratio $(S/N) \gtrsim 4$ (see the last three columns of Table 2). On the other hand, those of MUSE 1343, 171, 547, and 364 are lower by an order of magnitude with $(S/N) \lesssim 4$ (see Figures B12, B14, B15, B16, and Table 2). The posterior distributions for the target galaxies with stronger signals are sharper in general, and therefore the parameter constraints are tighter (e.g., Figures 5, B1, B2, and B4 versus Figures B3, B5, B6, and B7).

Table 3 also shows the reduced chi-square value of the model spectrum ($\chi^2_{\text{sp}}$) for a given best-fit parameter set. Similarly, the 11th column is that for a model surface brightness profile ($\chi^2_{\text{SBP}}$). The best-fit parameter set based on $\mathcal{L}_{\text{tot}}$ (the last row of each object) gives fairly good reduced chi-square values ($\lesssim 1$) for the spectrum and surface brightness profile, which quantitatively shows that the model successfully fits the observation.

### 4. Discussion

#### 4.1. Interpretations of the Best-Fit Results

In this section, we compare the best-fit values of some of our model parameters, such as $V_{\text{peak}}$ and $\tau_0$, with those from other studies. Garel et al. (2012) constructed high-redshift LAEs in a dark matter-only cosmological simulation by implementing a semianalytic model with an expanding-shell model. Their model galaxies, which are tuned to match observed UV and Ly$\alpha$ luminosity functions well, have an expanding velocity of

![Figure 1. 1D (the top panel of each column) and 2D (the rest) marginal posterior distributions for the spectrum ($\mathcal{L}_{\text{sp}}$) of MUSE 1185. The tilde indicates that posterior values are normalized by their maximum. In the panel of $\Delta V-V_{\text{peak}}$, the dotted line indicates the allowed $\Delta V$ range (i.e., $-V_{\text{peak}} < \Delta V < 0$) for a given $V_{\text{peak}}$ value to represent decelerated outflowing motion but prevent inflowing motion.](image-url)
Figure 2. Similar to Figure 1, but showing (normalized) posterior distributions for the surface brightness profile ($\tilde{\mathcal{L}}_{\text{SBP}}$) of MUSE 1185.

Figure 3. Spectrum (left) and surface brightness profile (middle) are presented: filled circles with error bars are the observational data, and the solid line is the model with the best-fit parameter set for the spectrum of MUSE 1185 (denoted in the upper right corner of the middle panel). In the left panel, two dotted vertical lines denote the wavelength range we used for the surface brightness profile. The dash–dotted line denotes the Ly$\alpha$ central wavelength in the observer’s frame of the best-fit redshift (lower x-axis) and in the rest frame (upper x-axis, at 1215.67 Å). In the middle panel, the dotted line represents the surface brightness profile of the observed UV continuum that is used to reconstruct the spatial distribution of the Ly$\alpha$ source, which is an input in our model. In the right panel, the models for the density profile (solid line, y-axis on the left) and velocity profile (dashed line, y-axis on the right) of the medium for the given parameter set are presented.
We also compare our results with those of Verhamme et al. (2008) and Yang et al. (2016). Verhamme et al. (2008) reproduced the observed Ly$\alpha$ spectra of 11 high-redshift ($z \sim 3$) Lyman-break galaxies using an expanding-shell model, and they found the best models for these galaxies with an expanding velocity of $\sim 150$–200 km s$^{-1}$ and a H$\text{I}$ column density of $\sim 10^{20}$ cm$^{-2}$. Yang et al. (2016) also modeled 12 low-redshift ($z \sim 0.2$) green pea galaxies\footnote{Green pea galaxies are nearby compact starburst galaxies and are thought to be analogous to high-redshift star-forming galaxies.} as an expanding velocity of $\sim 150$–200 km s$^{-1}$ and a H$\text{I}$ column density of $\sim 2 \times 10^{19}$–$7 \times 10^{20}$ cm$^{-2}$. Yang et al. (2016) also modeled 12 low-redshift ($z \sim 0.2$) green pea galaxies\footnote{Green pea galaxies are nearby compact starburst galaxies and are thought to be analogous to high-redshift star-forming galaxies.} as an expanding velocity of $\sim 150$–200 km s$^{-1}$ and a H$\text{I}$ column density of $\sim 10^{20}$ cm$^{-2}$. We also compare our results with those of Verhamme et al. (2008) and Yang et al. (2016). Verhamme et al. (2008) reproduced the observed Ly$\alpha$ spectra of 11 high-redshift ($z \sim 3$) Lyman-break galaxies using an expanding-shell model, and they found the best models for these galaxies with an expanding velocity of $\sim 150$–200 km s$^{-1}$ and a H$\text{I}$ column density of $\sim 10^{20}$ cm$^{-2}$. Yang et al. (2016) also modeled 12 low-redshift ($z \sim 0.2$) green pea galaxies\footnote{Green pea galaxies are nearby compact starburst galaxies and are thought to be analogous to high-redshift star-forming galaxies.} as an expanding velocity of $\sim 150$–200 km s$^{-1}$ and a H$\text{I}$ column density of $\sim 10^{20}$ cm$^{-2}$.
Figure 6. Similar to Figure 3, but with the best-fit model obtained by considering the spectrum and surface brightness profile of MUSE 1185.

Table 3

| MUSE # | $\mathcal{L}$ | $r_{\text{fit}}$ | $r_{\text{peak}}$ | $V_{\text{peak}}$ | $\Delta V$ | log $t_0$ | DGR | $\chi^2_\nu$ a | $\chi^2_{\text{SBP}}$ a |
|--------|----------|---------------|----------------|---------------|-----------|----------|-----|----------------|----------------|
| 1185   | sp       | 0.7 ±0.2      | 0.4 ±0.2       | 300 ±43       | $-100^{+100}_{-104}$ | 7.2 ±0.6  | 1.0 ±0.8 | 4.49 ±0.001 | 12.1 ±16.18   |
|        | SBP      | 0.3 ±0.2      | 0.0 ±0.1       | 400 ±148      | 0 ±297     | 6.9 ±0.7  | 0.0 ±0.4 | 4.49 ±0.001 | 120.48 ±0.84  |
|        | tot      | 0.3 ±0.1      | 0.2 ±0.0       | 300 ±43       | $-150^{+150}_{-37}$ | 6.6 ±0.1  | 0.0 ±0.4 | 4.49 ±0.001 | 1.88 ±1.14    |
| 82     | sp       | 0.9 ±0.5      | 0.2 ±0.1       | 300 ±200      | $-197^{+197}_{-300}$ | 7.2 ±0.6  | 0.0 ±1.2 | 3.60 ±0.001 | 3.70 ±5.21    |
|        | SBP      | 0.7 ±0.2      | 0.0 ±0.1       | 500 ±275      | $-200^{+200}_{-300}$ | 5.7 ±0.5  | 0.0 ±1.4 | 3.60 ±0.003 | 44.19 ±10.41  |
|        | tot      | 0.5 ±0.4      | 0.1 ±0.0       | 300 ±43       | $-250^{+250}_{-300}$ | 6.6 ±0.1  | 2.0 ±1.3 | 3.60 ±0.000 | 0.94 ±1.16    |
| 6905   | sp       | 0.5 ±0.3      | 0.2 ±0.2       | 200 ±55       | $-150^{+150}_{-150}$ | 7.2 ±0.6  | 1.0 ±0.7 | 3.96 ±0.002 | 5.05 ±20.58   |
|        | SBP      | 0.3 ±0.2      | 0.0 ±0.2       | 500 ±253      | $-100^{+100}_{-119}$ | 6.0 ±0.6  | 2.0 ±1.4 | 3.96 ±0.002 | 48.80 ±0.38   |
|        | tot      | 0.1 ±0.0      | 0.0 ±0.1       | 300 ±53       | $-300^{+300}_{-43}$  | 6.3 ±0.3  | 0.0 ±1.3 | 3.96 ±0.002 | 0.82 ±0.49    |
| 1343   | sp       | 0.1 ±0.5      | 0.4 ±0.2       | 500 ±286      | $-200^{+200}_{-200}$ | 7.2 ±0.7  | 1.4 ±0.7 | 3.96 ±0.004 | 0.17 ±1.51    |
|        | SBP      | 0.9 ±0.4      | 0.3 ±0.2       | 400 ±164      | $-200^{+200}_{-200}$ | 7.2 ±1.0  | 1.0 ±0.7 | 3.96 ±0.003 | 0.17 ±0.34    |
|        | tot      | 0.8 ±0.1      | 0.4 ±0.2       | 200 ±168      | $-200^{+200}_{-125}$ | 6.9 ±0.9  | 2.0 ±1.4 | 3.96 ±0.004 | 0.96 ±1.36    |
| 53     | sp       | 0.4 ±0.0      | 0.1 ±0.0       | 300 ±43       | $-50^{+50}_{-50}$   | 6.3 ±0.1  | 2.0 ±1.0 | 4.77 ±0.000 | 1.65 ±1.50    |
|        | SBP      | 0.2 ±0.2      | 0.1 ±0.1       | 500 ±253      | $-237^{+237}_{-237}$ | 6.0 ±0.6  | 0.0 ±1.4 | 4.77 ±0.002 | 117.90 ±6.84  |
|        | tot      | 0.4 ±0.1      | 0.1 ±0.0       | 300 ±43       | $-50^{+50}_{-50}$   | 6.3 ±0.1  | 2.0 ±1.0 | 4.77 ±0.000 | 1.65 ±1.50    |
| 171    | sp       | 0.1 ±0.4      | 0.6 ±0.4       | 200 ±191      | $-100^{+100}_{-100}$ | 7.2 ±0.7  | 2.0 ±1.4 | 3.88 ±0.002 | 2.24 ±6.68    |
|        | SBP      | 0.5 ±0.3      | 0.0 ±0.1       | 500 ±259      | $-200^{+200}_{-200}$ | 6.6 ±0.6  | 2.0 ±1.4 | 3.88 ±0.004 | 10.22 ±0.79   |
|        | tot      | 0.8 ±0.4      | 0.0 ±0.1       | 200 ±51       | $-200^{+200}_{-175}$ | 6.3 ±0.2  | 2.0 ±1.4 | 3.88 ±0.003 | 1.20 ±0.81    |
| 547    | sp       | 0.9 ±0.5      | 0.2 ±0.2       | 200 ±219      | $-150^{+150}_{-150}$ | 6.6 ±0.7  | 0.0 ±1.4 | 5.97 ±0.002 | 13.46 ±12.09  |
|        | SBP      | 0.5 ±0.1      | 0.1 ±0.1       | 500 ±225      | $-500^{+500}_{-500}$ | 5.7 ±0.5  | 0.0 ±1.4 | 5.97 ±0.005 | 39.42 ±1.30   |
|        | tot      | 0.7 ±0.2      | 0.1 ±0.1       | 300 ±78       | $-200^{+200}_{-188}$ | 6.3 ±0.1  | 2.0 ±1.4 | 5.97 ±0.001 | 1.25 ±1.07    |
| 364    | sp       | 0.5 ±0.3      | 0.1 ±0.4       | 500 ±236      | $-231^{+231}_{-231}$ | 6.3 ±0.3  | 2.0 ±1.4 | 3.93 ±0.001 | 1.81 ±3.42    |
|        | SBP      | 0.5 ±0.2      | 0.0 ±0.1       | 400 ±200      | $-200^{+200}_{-174}$ | 6.6 ±0.6  | 0.0 ±1.4 | 3.93 ±0.008 | 3.55 ±0.64    |
|        | tot      | 0.9 ±0.4      | 0.3 ±0.4       | 200 ±77       | $-200^{+200}_{-300}$ | 6.0 ±0.3  | 2.0 ±1.4 | 3.93 ±0.001 | 0.68 ±0.36    |

Notes. Note that $r_{\text{fit}}$, $r_{\text{peak}}$, and $V_{\text{peak}}$ are in normalized units by $r_{\text{max}}$ and DGR is in units of the value of the Milky Way (DGR$_{\text{MW}}$).

a The marginal posterior distributions of parameters that is used to determine a given best-fit parameter set among those obtained using spectrum ($\mathcal{L}^\text{b}$), surface brightness profile ($\mathcal{L}_{\text{SBP}}$), and both ($\mathcal{L}^\text{c}$).

b The reduced chi-square value of the model spectrum with a given best-fit parameter set (of a given row in the table).

c The reduced chi-square value of the model surface brightness profile with a given best-fit parameter set.
expanding shell. Their best models for these galaxies give an expanding velocity of $\sim 30 \text{–} 350 \text{ km s}^{-1}$ and a H I column density of $\sim 10^{19} \text{–} 10^{20} \text{ cm}^{-2}$. All these results are similar to our results.

There are other estimates of the kinematics of the medium in galaxies using interstellar/circumgalactic absorption lines. Steidel et al. (2010) measured absorption lines in quasi-stellar object (QSO) spectra made by circumgalactic gas of star-forming galaxies at redshift 2 $\lesssim z \lesssim 3$ that are on the sightlines to the QSOs. They then examined the properties of absorption lines as a function of the galactocentric impact parameter up to $\gtrsim 100 \text{ kpc}$, and modeled them using outflowing halos with constant/ decelerating velocity profiles and a covering factor profile. A constant velocity profile gives an outflowing velocity of 650–820 km s$^{-1}$, and a decelerating velocity profile gives a mean outflowing velocity of $\sim 200 \text{–} 300 \text{ km s}^{-1}$. We could not directly compare because of different assumptions, but the estimate with a constant velocity profile is higher than our best-fit values and the estimate with a decelerating velocity profile is more or less similar to ours. Because the estimate of the outflowing velocity varies largely depending on which velocity profile is used, the disagreement between these estimates may not be an issue. It should be also noted that the best-fit velocity varies depending on the choice of the covering factor profile, and possibly other factors such as optical depth. Moreover, different emission/absorption lines trace different kinds of gas, thus different kinematics. Therefore, these suggest that our constraints on the outflowing velocity are consistent with previous studies in general.

We finally address the constraint on redshift. We set the redshift as a free parameter, and find that its best-fit values have shifts of 0.001–0.008 to lower redshifts than the estimates when the peak of the Lyα spectrum is assumed to be at the rest-frame Lyα central wavelength. This means that the Lyα line is more redshifted than that by the systemic redshift of galaxies, which is due to the resonance scattering of Lyα. Compared to the typical redshift error of the Sloan Digital Sky Survey (i.e., 30 km s$^{-1}$, Strauss et al. 2002), the shifts of 0.001–0.008 are non-negligible. More importantly, the accuracy of the redshift estimate is transferred to that of the estimates on $\tau_0$ and subsequently, to other parameters. We recall that Lyα-based redshift estimates are not always correct (e.g., Orlitová et al. 2018), therefore it is necessary to secure other reliable estimates of the redshift when available.

**4.2. Advantages of Using Both Spectrum and Surface Brightness Profile**

We examine the degeneracy between the parameters based on the 2D posterior distributions (e.g., see Figures 1 and 2). In the case of the spectrum, degeneracies of $\tau_0$–$r_{\text{peak}}$ and $z$–$r_{\text{peak}}$ appear prominently and commonly for all the target galaxies. A weak degeneracy between $z$ and $r_{\text{peak}}$ also appears in the majority of the target galaxies. The posterior distributions of the surface brightness profile also show degeneracies of $\Delta V$–$r_{\text{H}1}$, $\tau_0$–$V_{\text{peak}}$, $z$–$V_{\text{peak}}$, and $z$–$\tau_0$ for most target galaxies, which are overall less prominent than those of the spectrum. Interestingly, the degeneracy between $z$ and $\tau_0$ appears to be opposite for spectrum (negative correlation) and surface brightness profile (positive). Although the parameter degeneracy itself hinders precise parameter constraints, these opposite behaviors of the parameter degeneracy for the spectrum and surface brightness profile together can provide tighter constraints on $z$, $\tau_0$, and consequently, on other parameters as well.

The opposite behaviors of the parameter degeneracy are partly attributable to the fact that the posterior distributions of the spectrum and surface brightness profile prefer different parts of the parameter spaces. This results in quite different best-fit parameter sets for the spectrum and surface brightness profile, as summarized in Table 3. The difference is non-negligible, as seen in Figures 3 and 4; the best-fit parameter for one of the spectrum and surface brightness profile completely fails at reproducing the other. The comparison between $\chi^2_{\text{sp}}$ and $\chi^2_{\text{SBP}}$ with the best-fit parameter set for either the spectrum or surface brightness profile shows this failure in a quantitative way. The difference (i.e., ratio) between these two chi-squares can be as large as a factor of one hundred. This indicates that the spectrum and surface brightness profile are complementary and orthogonal in constraining our model parameters. Therefore, these two observables together can constrain the parameters far better, which is shown in the previous section with the total posterior distributions that become much sharper than those of the spectrum or surface brightness profile. Even the redshift constraint, which is expected to be done primarily by the spectrum, is affected when the surface brightness profile is taken into account. This constraining power of the surface brightness profile on redshift comes from the restriction on wavelength range for the surface brightness profile (i.e., image bandwidth). In the cases of MUSE 1343, 53, and 364, the surface brightness profile does not alter the redshift constraint by spectrum. However, the surface brightness profile still contributes to the parameter constraint by tightening it, as seen in the total posterior distribution for the redshift compared to its spectrum posterior distribution (e.g., Figure B4 versus Figure B8).

**4.3. Lyα Spectrum and Surface Brightness Profile Variation in Model Parameter Space**

We explore the variations of the Lyα spectrum and of the surface brightness profile in the model parameter space to better understand the fitting results and the parameter degeneracy in the previous sections. Here, we have convolved raw model spectrum and surface brightness profile with a Gaussian kernel and a Moffat kernel, respectively, as described in Section 2.3. However, we have applied neither the aperture size (to the spectrum) nor the image bandwidth (to the surface brightness profile) to first understand the pure impacts of the physical parameters. The effect of the aperture size is indirectly discussed in Section 4.6 with spatially resolved spectra. We examine the impacts of six parameters, except for the redshift. The results for $\tau_0$ and $r_{\text{H}1}$ are described in detail here, but those for other parameters are presented in Appendix C. It should be noted that we explore the impacts of parameters on the shapes of the spectrum and surface brightness profile.

Although the Lyα radiative transfer process is a bunch of random scattering events, we can roughly guess the shapes of the emerging Lyα spectrum and surface brightness profile by inferring the frequency change and the last scattering position of the photons, respectively. The rule of thumb is that the higher the effective optical depth, the stronger the frequency change in cross section in the photons that is caused by the relative motion between the photons and the medium in the rest frame of the medium.
change. A stronger frequency change results in a broader spectrum whose peak is farther away from the Lyα central wavelength, from which the correlation between peak shift and FWHM is naturally expected, as discussed in Section 4.5. The last scattering position can be guessed by inferring the location at which the cumulative effective optical depth reaches a certain value. We try to understand the impact of each parameter on the Lyα spectrum and surface brightness profile based on this rule of thumb. However, it should be noted that the details of the impact of each parameter can manifest themselves differently depending on the values of the other parameters.

We show changes in the Lyα spectrum and surface brightness profile with each parameter. We choose two galaxies for which the impact of a parameter on the spectrum and surface brightness profile is apparent most significantly. Our choice can differ depending on the parameter of interest. To illustrate this, we fix parameters other than the one under consideration at their best-fit values. However, the DGR is fixed at zero unless it is the parameter of interest (i.e., Figure C4) to make the problems simpler. The spectrum is normalized by its total intensity, and the surface brightness profile is normalized by its maximum value (i.e., the value at $r = 0$) for convenience.

We start from the changes in spectrum and surface brightness profile with $\tau_0$, which is easy to understand. Figure 7 shows the cases of MUSE 82 and 1343. The peak of the spectrum moves farther away from the central wavelength and the width becomes broader as $\tau_0$ increases. The surface brightness profile becomes flatter with increasing $\tau_0$. These trends arise because the number of scatterings in general increases accordingly.

Figure 8 shows the changes in Lyα spectrum and surface brightness profile with $r_{\text{HI}}$ for the same galaxies. The spectrum does not change significantly with $r_{\text{HI}}$, but the surface brightness profile becomes clearly flatter as $r_{\text{HI}}$ increases. In a less centrally concentrated medium (larger $r_{\text{HI}}$), Lyα photons can more easily diffuse out of the central region. In turn, the last scatterings tend to happen at larger radii, which results in a flatter surface brightness profile. Meanwhile, the behavior of the spectral shape with $r_{\text{HI}}$ is difficult to understand. Although the column density remains the same, redistributing matter can either effectively increase or decrease the optical depth depending on the velocity structure of the medium. Not only the effective optical depth, but also the last scattering positions change with $r_{\text{HI}}$, which means that the velocities at which Lyα photons are scattered off also changes. All of these factors together complicate predicting the change in spectral shape with $r_{\text{HI}}$.

Two competing factors play roles in forming the spectral shape in an expanding medium: (1) the Doppler frequency shift, and (2) the effective optical depth. In an expanding medium, the frequency of a photon that is scattered would be Doppler-shifted when transformed from the fixed frame to the medium frame; as a result, the final photon frequency after many scatterings will be shifted by an amount proportional to the expanding velocity. Therefore the frequency changes due to the Doppler effect (when photons undergo sufficiently many scatterings) will be more significant in a faster medium than in a slower medium. However, as the velocity of the medium increases, the effective optical depth will decrease, and thus the number of scatterings that the photons undergo before they escape the medium will decrease as well; consequently, we expect that a faster medium yields a smaller frequency change. This latter effect

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**Figure 7.** Spectra (left panel) and surface brightness profiles (right panel) with varying $\tau_0$ for two of our target galaxies. The MUSE id of the chosen galaxies is written in the top left corner of each spectrum panel. The parameters other than $\tau_0$ are fixed at their best-fit values, except for the DGR, which is fixed at zero. The dot-dashed line in the spectrum panels represents the Lyα central wavelength, 1215.67 Å, and the dotted line in the surface brightness profile panels represents the surface brightness profile of UV continuum (i.e., the Lyα source distribution).

**Figure 8.**
will be particularly significant in a fast-moving medium. We found, as shown in Appendix D, that the frequency shift due to the Doppler effect in an expanding medium is essential in a relatively slow-moving medium; however, the effective optical depth becomes more critical in a fast-moving medium.

For example, if the medium velocity is largely decelerated at large radii, which places more matter at larger radii (i.e., larger \( r_{\text{HI}} \)), it will result in a higher effective optical depth. As seen for the surface brightness profiles in Figure 8, more scatterings therefore tend to happen at larger radii, thus by medium at smaller velocities. However, this does not necessarily broaden the spectrum and causes its peak to lie farther away from the central wavelength. The two competing effects (the Doppler frequency shift and the effective optical depth) seem to cancel each other out in the case of MUSE 82, while the latter effect (i.e., enhanced effective optical depth by increasing \( r_{\text{HI}} \)) overweights the former effect (i.e., smaller frequency shift due to the lower medium velocity) in the case of MUSE 1343. The impact of \( r_{\text{HI}} \) on the spectrum could be more significant when there a non-negligible amount of Ly\( \alpha \) photons is emitted at large radii (i.e., larger \( r_{\text{cont}} \)). Indeed, all of our target galaxies are modeled to have a very compact Ly\( \alpha \) source distribution, as inferred from their UV distribution (see Table 2), which could limit the impact of \( r_{\text{HI}} \) on the spectrum.

As \( r_{\text{peak}} \) decreases, the spectrum becomes broader with higher intensities at the central wavelength and at the red wing, and the surface brightness profile tends to be steeper at the inner radii (Figure C1). Increasing \( V_{\text{peak}} \) has an effect similar to but stronger than that of decreasing \( r_{\text{peak}} \): the spectrum shows a much clearer peak shift toward the central wavelength, and the surface brightness profile becomes steeper over the entire range (Figure C2). Here, we need to note that the \( V_{\text{peak}} \) in Figure C2 is similar to the high velocities considered in Appendix D, for which no considerable scatterings happen, and thus the effect of the Doppler frequency shift is insignificant. As \( \Delta V \) decreases (i.e., the edge velocity decreases), the surface brightness profile becomes flatter, and the spectrum has a lower intensity at the central wavelength, with its peak shift toward the red side (Figure C3). When the edge velocity becomes zero, the emergent spectrum shows a small blue bump. The spectrum becomes slightly sharper and its peak approaches the central wavelength as the DGR increases; the impact of the DGR is more apparent when the optical depth is higher, as shown in the bottom row of Figure C4. The impact of the DGR on the surface brightness profile is minor in general. We refer to Appendix C for interpretations of these behaviors.

Multiple parameters show impacts on the spectrum or surface brightness profile. Therefore, a degeneracy between the parameters is naturally expected, and we mentioned some of them in the 2D posterior distributions of Section 4.2. We now can understand these degeneracies in a qualitative way from the spectral and surface brightness profile variations in the parameter space examined above. The most obvious degeneracy is found to be the one for \( \tau_0 \) and \( z \) in the spectrum. It is because their impacts on the spectrum are strong enough to manifest themselves consistently for any combinations of other parameters. A redshift \( z \) stretches the wavelength range by a factor of \((1+z)\). Thus, both \( \tau_0 \) and \( z \) cause a larger shift of the peak and a broader width when their value increases. This leads to the negative degeneracy between \( \tau_0 \) and \( z \), as noted in Section 4.2. Meanwhile, \( r_{\text{peak}} \) and \( V_{\text{peak}} \) also affect the spectral shape, but their impacts are not as significant as those of \( \tau_0 \) and \( z \). Moreover, the changes of the peak shift and the width with \( r_{\text{peak}} \) or \( V_{\text{peak}} \) do not appear consistently for all cases; sometimes a larger shift comes with a broader width and sometimes not. Therefore, a degeneracy involved with \( r_{\text{peak}} \) or \( V_{\text{peak}} \) could be either strong or weak. The parameter degeneracy in the surface brightness profile is weak, as in Figure 2. This is not because the parameters play unique roles in shaping the surface brightness profile, but because their roles are too simple.
Although most parameters have a clear impact on the surface brightness profile, the variation in surface brightness profile made by each parameter is mostly about the change in steepness. Such a simple variation is not enough to discriminate the effects of different parameters, which leads to poor parameter constraints from the surface brightness profile. Still, we can better constrain parameters using both spectrum and surface brightness profile, as seen in the posterior distributions of \( r_{\text{cont}} \) and \( \Delta V \), which are sharper than those of \( L_{\text{peak}} \) or \( L_{\text{SBP}} \) (e.g., Figure 5 versus Figures 1 or 2). This can be understood from the fact that the variations in spectrum and surface brightness profile with parameters are quite diverse. Some parameters change only one of the two, while others change both (e.g., \( r_{\text{HI}} \) versus \( r_{\text{peak}} \)); some parameters change the spectrum in a way similar to other parameters, but not the surface brightness profile (e.g., \( \tau_0 \) versus \( \zeta \)). These different behaviors tighten the total posterior distributions and break the parameter degeneracy.

### 4.4. Spatial Extent of Ly\( \alpha \) Halos and Its Dependence on the Model Parameters

One notable feature of Ly\( \alpha \) halos around high-\( \zeta \) star-forming galaxies is their size, which is typically much larger than the size observed in the UV continuum. The ratio of the scale radii of the brightness distributions in Ly\( \alpha \) and UV continuum \( (r_{\text{halo}}/r_{\text{cont}}) \) of our target galaxies covers a wide range from 3.5 to 21, with a mean value of 10.2. Indeed, the sizes of our target galaxies are much larger in Ly\( \alpha \) than in UV continuum.

We perform a simple analysis to examine the correlations between each model parameter and \( r_{\text{halo}}/r_{\text{cont}} \) for an indication which physical parameter is responsible for the property of the extended Ly\( \alpha \) halos.

Figure 9 shows the correlations between \( r_{\text{halo}}/r_{\text{cont}} \) and each of the six parameters, which are \( r_{\text{HI}}/r_{\text{cont}}, r_{\text{peak}}/r_{\text{cont}}, V_{\text{peak}}, V_{\text{edge}}/r_{\text{peak}}, \Delta V, \) and \( \tau_0 \) for our target galaxies (see the filled circles with error bars in each panel). We exclude the DGR and \( \zeta \) because they are certainly not responsible for the extent of the Ly\( \alpha \) emission. Errors are calculated from the likelihood distributions of model parameters, as summarized in Table 3. Although the errors are asymmetric, we assume symmetric errors by taking the mean of the lower and upper errors when we need to apply the error propagation for the final error estimation (except \( \Delta V \) and \( \tau_0 \)). In any case, the error estimation is rough, and thus it should not be taken at face value. Of the model parameters, \( r_{\text{HI}}/r_{\text{cont}} \) and \( r_{\text{peak}}/r_{\text{cont}} \) show non-negligible correlations with \( r_{\text{halo}}/r_{\text{cont}} \) (\( p \)-value lower than 0.1), and their Spearman rank-order correlation coefficients are 0.74 and 0.64, respectively. These correlations suggest that the large extent of the Ly\( \alpha \) emission results from the wide distribution of the neutral hydrogen medium and from the slow increase of outflowing velocity in the inner region. Another correlation that might be worth paying attention to is the one between \( r_{\text{halo}}/r_{\text{cont}} \) and \( V_{\text{edge}} \) (the Spearman rank-order correlation coefficient is –0.61 and the \( p \)-value is 0.11), which indicates that Ly\( \alpha \) emission is more extended when the velocity of the outer medium is lower (i.e., the velocity decrease in the outer.
region is larger). This is because more Ly$\alpha$ photons, especially those that freely escape from the inner region, can be scattered and escape at large radii, which results in more Ly$\alpha$ emission at large radii and thus a larger extent of Ly$\alpha$ halos.

To summarize, the large spatial extents of the Ly$\alpha$ halos seem relevant to the large spatial extents of the UV emission (a result of L17) and the neutral hydrogen medium, and the low bulk velocities of the medium at small and large radii. This needs to be further examined using a larger galaxy sample in future studies.

4.5. Correlation between the Peak Shift and the FWHM of the Ly$\alpha$ Spectrum

The correlation between peak shift and FWHM of the Ly$\alpha$ spectrum is naturally expected from the resonant-scattering process of Ly$\alpha$ photons. This correlation has been recognized as a way to derive a correct systemic redshift of a galaxy when only the Ly$\alpha$ emission line is available (e.g., Zheng & Wallace 2014). Recently, Verhamme et al. (2018) found an empirical relation between red-peak shift and FWHM using 45 galaxies for which lines other than Ly$\alpha$ line can be used for the systemic redshift estimate. They showed that this relation recovers the systemic redshift estimate with an accuracy of $\leq 100$ km s$^{-1}$.

We also confirm this correlation using all of our 13,230 simulations (blue density contours in Figure 10). However, the distribution of peak shifts and FWHMs of our simulated spectra has an offset from the empirical relation found by Verhamme et al. (2018, red line). Verhamme et al. (2018) compared in their Figure 4 the correlation from observations with that from models such as expanding shells, spheres, or biconical outflows (Schaerer et al. 2011; Zheng & Wallace 2014); an offset also exists between the two, but in the opposite direction of ours. This implies that the correlation between peak shift and FWHM can be a model discriminator.

We explore the correlations between model parameters and peak shift (top) or FWHM (bottom) in Figure 11. Both peak shift and FWHM show strong correlations with optical depth (rightmost panel in each row), which is the main driver of the correlation between peak shift and FWHM in Figure 10. While their correlations with other parameters are relatively weak, the correlation with velocity profile parameters appears interesting (middle panels); for example, a peak shift is negatively correlated with the medium velocity, and the FWHM, in contrast, is positively correlated. The negative correlation between peak shift and medium velocity arises because Ly$\alpha$ photons tend to experience fewer scatterings when the medium velocity is higher (the medium velocity in our model is relatively high, and thus the effect of the effective optical depth is stronger than that of the Doppler frequency shift). This may also cause a negative correlation between FWHM and medium velocity, but this does not seem to be the case. Because our velocity profile spans a range of velocities (unlike a constantly expanding velocity profile), it becomes broader as the peak velocity increases. Therefore, when the peak velocity is higher, Ly$\alpha$ photons are Doppler-shifted to a wider wavelength range in the rest frame of the medium, which ends up with a larger FWHM. This again indicates that the peak shift–FWHM correlation could play a role in discriminating models, especially regarding the velocity profile.
4.6. Spatially Resolved Lyα Spectra

One advantage of using the MUSE observational data is that we can obtain spatially resolved spectra. Unfortunately, only (spatially) integrated spectra are publicly available for now, but it is still worth examining simulated spectra as a function of (projected) distance from the galaxy center. Figure 12 shows the simulated Lyα spectra measured at annuli of different radii with the best-fit parameter set for four target galaxies. Each spectrum is normalized by the total number of Lyα photons at each annulus. Because the true intensity level of spectra is imprinted on the surface brightness profile, here we examine only their shape. The results show that the shape of these spectra varies significantly with the radius, indicating that such spatially resolved spectra can be additionally used to infer spatial and kinematic distributions of the medium in more detail.

In the case of MUSE 171, which is a relatively simple case with a constantly expanding velocity, the different spectra at different locations arise because of the difference in the number of scatterings that photons have undergone until their escape. The peak of the spectrum tends to appear at a longer wavelength when the spectrum is measured at a larger distance from the galaxy center. Therefore, a larger shift of the spectral peak at a larger distance indicates that these photons escape after more scatterings. This makes sense because these photons come across longer distances from the central part, being scattered more to reach that the larger radii, and then being able to escape there.9

The frequency shift (with respect to the initial frequency) of escaping photons could be larger not only when the optical depth is higher, but also when the medium moves faster, if the medium velocity is not too high (see Appendix D). For other target galaxies, the peak shift of the spectrum tends to become larger and then smaller as the radius increases. This behavior of the peak shift could be attributed to the velocity profile that increases and then decreases as the distance from the galaxy center increases. However, it should be noted that if the medium velocity is too high, photons can escape the system without many scatterings, so the behavior of the peak shift as a function of radius could not be easily understood. It should also be noted that the annuli where we obtain the spectra are in the projected space. Therefore, the spectrum at a small radius consists of photons that are emerging at a wide range of 3D distance, which may cause multiple peaks in the spectrum, as seen in most of our target galaxies. This feature will be useful for extracting kinematic information of the medium, especially its spatial variation, from observational data.

Although the parameters of the medium velocity field help us to understand the overall trend of spectra from different annuli, we need to consider other factors as well, especially $r_{\text{HI}}$ and $\tau_0$, to understand the phenomena in more detail. For example, the combination of small $r_{\text{HI}}$ and small $\tau_0$ for the case of MUSE 6905 leads to so few scatterings at large radii that the spectrum at the outermost annuli shows almost no shift even though the medium is completely static at the edge. The peak of spectra from the outer part of the galaxy shows a slight shift to the blue side because the expanding velocity profile of the medium always decreases (i.e., $r_{\text{peak}} = 0$).

In summary, Lyα spectra from different positions in and around a galaxy are quite distinguishable. To which degree this is so is determined by the interplay of multiple parameters regarding the spatial and kinematic distributions of medium. Therefore, spatially resolved Lyα spectra could give much tighter constraints on the parameters than an integrated spectrum. We will continue our work in this direction.

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9 Although we assume a spatially extended Lyα source distribution, emission is mostly from the central region (i.e., the scale radius for the UV continuum emission of our target galaxies is smaller than 5% of the maximum extent of the system). Therefore, on-site emission at large radii accounts for only a small fraction, and the majority are those scattered from the central region, which have been scattered many times to reach large radii.
4.7. Impacts of the Ly$\alpha$ Input Spectrum on the Emerging Spectrum and the Surface Brightness Profile

To perform Ly$\alpha$ Monte Carlo radiative transfer calculations, we set the input Ly$\alpha$ spectrum as a Voigt profile with a temperature of $10^4$ K, the typical temperature of H II regions. All the results we show in this paper are based on this input spectrum. In other words, we performed the Ly$\alpha$ radiative transfer calculation only in the diffuse interstellar medium and the CGM, ignoring the radiative transfer in H II regions. However, in reality, for instance, some bulk outflowing motion induced by stellar winds in H II regions could give rise to asymmetric double-peak spectra (Kimm et al. 2019).

There are many possible options for the input spectrum, which include a Gaussian profile, a symmetric double-peak profile, and one including continuum with different line strengths (i.e., different equivalent widths). However, we consider only an asymmetric double-peak profile whose red peak is stronger than the blue one, as suggested by Kimm et al. (2019). We approximate the asymmetric double-peak profiles in Kimm et al. (2019) using a sum of two Voigt profiles; each of them is shifted by $\Delta \lambda$ toward the red and blue, respectively, and the red peak is twice stronger than the blue one. Although our approximation does not precisely mimic those in Kimm et al. (2019), we do expect to draw a qualitatively consistent conclusion with the case by adopting the exact form of their spectra.

We construct the Ly$\alpha$ spectrum and the surface brightness profile using the asymmetric double-peak profile as an input spectrum, which is easily done by adjusting the weight of each photon, following the procedure described in Section 2.1. We consider different cases with four peak separations (2$\Delta \lambda$, where $\Delta \lambda = 0, 5, 10, 15$, and $\Delta \lambda = 0$ corresponds to a pure Voigt profile), and implement each case by placing

\[ V(x + \Delta \lambda) + 0.5V(x - \Delta \lambda) \] in Equation (1), where $V(x)$ represents a Voigt profile. Figure 13 shows the Ly$\alpha$ spectra and surface brightness profiles of the four different peak separations for two galaxies with their best-fit parameter set. As the two peaks of the input spectrum are farther away from the central wavelength (i.e., as $\Delta \lambda$ increases), the emerging spectrum becomes sharper and the surface brightness profile becomes steeper. The photons at the red side of the input spectrum become even redder (i.e., farther away from the central wavelength) in the medium frame because of the expanding velocity; they are rarely scattered and emerge at their initial frequencies. On the other hand, the photons at the blue side become less blue (i.e., closer to the central wavelength); they escape after many scatterings, forming a broad component. As a result, the emerging spectrum tends to have a sharper peak with a larger shift as the peak separation increases. In the case of the surface brightness profile, as the two peaks of the input spectrum are farther away from the central wavelength, the majority (red photons) are less scattered, which results in a steeper profile. It should be noted that the details will be determined by the effective optical depth. When the effective optical depth is high, the change in the emerging spectrum due to the variation of the input spectrum will be small because a large number of scatterings tend to erase the information of the initial frequency distribution. (e.g., the case of MUSE 1343).

The best fit for each target galaxy will be quite different from that in Table 3 when we adopt such an asymmetric (red-dominant) double-peak spectrum as an input spectrum. We expect that a parameter set that gives a higher effective optical depth (i.e., higher optical depth and lower expanding velocities) will be preferred to remove a sharp peak with a large shift. It is important to have a better idea of the Ly$\alpha$ input spectrum for more accurate modeling on small spatial scales.
Moreover, we note that previous studies varied the width of the input Ly$_\alpha$ spectrum by incorporating turbulent motion. However, we were able to successfully model the data without considering this effect.

5. Summary

We perform Ly$_\alpha$ radiative transfer calculations with an outflowing-halo model that is improved from a simple, constantly expanding-shell model. We successfully reproduce the Ly$_\alpha$ properties (i.e., spectrum and surface brightness profile) of eight star-forming galaxies at $z = 3$–$6$ observed with MUSE, and the results are consistent with the results from other studies. We summarize our results as follows.

1. The constraints on the model parameters are largely improved by simultaneously using the spectrum and surface brightness profile. This is because the spectrum and surface brightness profile change diversely and independently of each model parameter, which helps breaking degeneracies between the model parameters.

2. We examine spatially resolved Ly$_\alpha$ spectra emerging at different distances from the galactic center. The spectral shape (e.g., the position and the number of spectral peaks and width) changes with distance. The changes can be understood with a given model parameter set, which provides a basis of using spatially resolved spectra to better infer density and velocity fields of the CGM from observations.

3. The individual model parameters change the Ly$_\alpha$ spectrum and surface brightness profile in various ways. The changes in Ly$_\alpha$ spectrum and surface brightness profile by each model parameter could be significant or not, depending on other parameter values. However, the impacts of optical depth and redshift on the Ly$_\alpha$ spectrum and surface brightness profile are always apparent.

4. There is degeneracy between the model parameters that can be understood from the changes in the Ly$_\alpha$ spectrum and surface brightness profile with each model parameter. The most prominent ones are optical depth—the radius of the peak expanding velocity and optical depth—redshift in determining the spectrum. The parameter degeneracy in determining the surface brightness profile appears weak in general, which is mainly because of the poor parameter constraints of the surface brightness profile.

5. Dust affects the shape of the Ly$_\alpha$ spectrum by differential intensity reductions depending on wavelengths. This is because the Ly$_\alpha$ spectrum consists of Ly$_\alpha$ photons that escape after different numbers of scatterings that systematically change with emerging wavelengths. However, these impacts of dust become significant when the effective optical depth is high.

6. We examine the correlations between the model parameters and the observed properties of high-redshift Ly$_\alpha$ galaxies. The spatial extent of Ly$_\alpha$ halos shows strong correlations with the spatial extents of UV emission and the neutral hydrogen medium, and the bulk velocities of the medium. The positive correlation between the peak shift and FWHM of the Ly$_\alpha$ spectrum is well reproduced by our simulations. The peak shift–FWHM correlation could provide an additional constraint on the velocity profile of the medium.

7. The Ly$_\alpha$ input spectrum is an important factor that determines the shapes of the emerging spectrum and surface brightness profile (in particular for H II regions with relatively low optical depth). We test the case of a
red-dominant double-peak input spectrum for various peak separations, which results in systematic changes in the emerging spectrum and the surface brightness profile. This suggests that model parameter constraints could vary largely depending on the choice of the Lyα input spectrum.

This study is the first attempt to model the observed Lyα spectrum and surface brightness profile simultaneously. We will extend our analyses to a larger sample to characterize the physical parameters of high-redshift Lyα-emitting galaxies with better statistics.

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Appendix A
Validation of the Postprocessing Dust Effect

We assume that dust scattering is perfectly forward-directed for the postprocessing of the dust effect. We examine whether this assumption is valid by comparing the escape fractions of Lyα photons when the effect of dust is fully taken into account in the simulation from the beginning ($f_{\text{esc}}^{\text{full}}$) and when it is implemented through postprocessing ($f_{\text{esc}}^{\text{PP}}$). The Lyα radiative transfer simulation is performed in an infinite static slab of medium at a temperature $10^4$ K for the ranges of $\tau_0$ and DGR of interest of this study. We consider a point, monochromatic source of Lyα. Figure A1 shows the ratio of $f_{\text{esc}}^{\text{PP}} / f_{\text{esc}}^{\text{full}}$ as functions of $\tau_0$ and DGR. The postprocessing approach tends to slightly underestimate the escape fraction, and the degree of the underestimation increases with $\tau_0$ and DGR. However, the amount of underestimation is less than $\sim 9\%$ even for the largest $\tau_0$ and DGR (i.e., $\log \tau_0 = 7.2$ and $\text{DGR} = 2\text{DGR}_{\text{MW}}$), and these two treatments of dust agree well in general. Therefore, the postprocessing approach can be claimed as a good approximation of the full simulation approach. This is expected because the angular redistribution function for dust scattering in the simulation is set to be the Henyey–Greenstein phase function, which is strongly forward-throwing. In summary, the assumption of perfect forward-scattering by dust is valid at least for the ranges of $\tau_0$ and DGR considered here.

Figure A1. Ratio of the two estimates on the escape fraction of Lyα photons that are emitted from a monochromatic point source and travel through an infinity slab of temperature $10^4$ K as functions of $\tau_0$ (x-axis) and DGR (denoted by colors): $f_{\text{esc}}^{\text{full}}$ is obtained when the effect of dust is fully simulated, and $f_{\text{esc}}^{\text{PP}}$ is obtained when the effect of dust is postprocessed.
and so the application of postprocessing the effect of dust is valid as well.

**Appendix B**

**Posterior Distributions of the Model Parameters**

(Continued)

We present only the total posterior distributions ($L_{\text{tot}}$) for the remaining seven target galaxies (Figures B1–B7) for the sake of brevity. However, we present the posterior distributions for the spectrum ($L_{\text{sp}}$) (Figure B8) and surface brightness profile ($L_{\text{SBP}}$) (Figure B9) for MUSE 53 because it is a special case because its best-fit parameter set is determined solely by the spectrum. Nevertheless, the parameter constraints become tighter by including surface brightness profile, which is revealed by comparing Figures B4 and B8. The best-fit model spectra and surface brightness profiles determined with $L_{\text{tot}}$ for the seven galaxies are also shown in Figures B10–B16.

![Figure B1. Similar to Figure 5, but for MUSE 82.](image-url)
Figure B2. Similar to Figure 5, but for MUSE 6905.
Figure B3. Similar to Figure 5, but for MUSE 1343.
Figure B4. Similar to Figure 5, but for MUSE 53.
Figure B5. Similar to Figure 5, but for MUSE 171.
Figure B6. Similar to Figure 5, but for MUSE 547.
Figure B7. Similar to Figure 5, but for MUSE 364.
Figure B8. Similar to Figure 1, but for MUSE 53.
Figure B9. Similar to Figure 2, but for MUSE 53.

Figure B10. Similar to Figure 6, but for MUSE 82.
Figure B11. Similar to Figure 6, but for MUSE 6905.

Figure B12. Similar to Figure 6, but for MUSE 1343.
Figure B13. Similar to Figure 6, but for MUSE 53.

Figure B14. Similar to Figure 6, but for MUSE 171.
Figure B15. Similar to Figure 6, but for MUSE 547.

Figure B16. Similar to Figure 6, but for MUSE 364.
Appendix C  
Lyman-α Spectrum and Surface Brightness Profile Variation in the Model Parameter Space (Continued)

In this section, we present the impact of the remaining model parameters on the shapes of the spectrum and surface brightness profile that are not fully presented in Section 4.3.

Figure C1 shows the changes in spectrum and surface brightness profile with $r_{\text{peak}}$. The spectrum tends to be broader with higher intensities at the central wavelength and at the red wing as $r_{\text{peak}}$ decreases (see the case of MUSE 6905). The higher intensity at the central wavelength arises because the inner medium becomes more transparent with a higher expanding velocity at a given radius when $r_{\text{peak}}$ is smaller, thus letting more photons at the central wavelength escape from the system without scatterings. The higher intensities at the red wing for smaller $r_{\text{peak}}$ arise because photons are more likely to be scattered by higher velocities, and thus are scattered into wavelengths farther away from the central wavelength. The surface brightness profile tends to be flatter at small radii when $r_{\text{peak}}$ is larger (because of more scatterings there), but its trend with $r_{\text{peak}}$ at large radii seems to depend on other parameters.

When the outer part of the galaxy is optically thick with higher intensities at the central wavelength and at the red wing as $r_{\text{peak}}$ decreases (see the case of MUSE 6905). The higher intensity at the central wavelength arises because the inner medium becomes more transparent with a higher expanding velocity at a given radius when $r_{\text{peak}}$ is smaller, thus letting more photons at the central wavelength escape from the system without scatterings. The higher intensities at the red wing for smaller $r_{\text{peak}}$ arise because photons are more likely to be scattered by higher velocities, and thus are scattered into wavelengths farther away from the central wavelength. The surface brightness profile tends to be flatter at small radii when $r_{\text{peak}}$ is larger (because of more scatterings there), but its trend with $r_{\text{peak}}$ at large radii seems to depend on other parameters. When the outer part of the galaxy is optically thick with large $r_{\text{StFl}}$, small $\Delta V$, and large $\tau_0$, the change in spectrum with $r_{\text{peak}}$ becomes negligible and the flattening of surface brightness profile by increasing $r_{\text{peak}}$ extends to a large radius (e.g., MUSE 1343).

We can speculate about the changes in the spectrum and surface brightness profile with $V_{\text{peak}}$ based on the analogy of those with $r_{\text{peak}}$. Increasing $V_{\text{peak}}$ has an effect similar to decreasing $r_{\text{peak}}$, it causes the medium to become more transparent at a given radius. Therefore, as $V_{\text{peak}}$ increases, a spectrum will have a sharper peak that is closer to the central wavelength and a broader red tail. A surface brightness profile becomes steeper with $V_{\text{peak}}$.

These trends are well shown in Figure C2, as expected. Compared to the case of $r_{\text{peak}}$, the impact of $V_{\text{peak}}$ on both the spectrum and surface brightness profile is more significant. This is because the maximum levels of the medium transparency and the frequency change that can be reached by varying $r_{\text{peak}}$ are limited by a given $V_{\text{peak}}$ value. Moreover, with a limit at $r < r_{\text{peak}}$, the medium becomes more transparent as $r_{\text{peak}}$ decreases. The medium at $r > r_{\text{peak}}$ becomes less transparent as $r_{\text{peak}}$ decreases. However, increasing $V_{\text{peak}}$ causes the whole medium to become more transparent.

The impact of $\Delta V$ on the spectrum and surface brightness profile is presented in Figure C3. The change in surface brightness profile with $\Delta V$ is obvious; as $\Delta V$ decreases (i.e., as the velocity at the edge decreases closer to zero), the surface brightness profile becomes flatter. This is because photons that were lucky enough not to experience many scatterings in the inner region are more likely to be scattered in the outer region as the medium becomes more static (i.e., less transparent) with decreasing $\Delta V$. The change in spectrum with $\Delta V$ appears less obvious, which can depend on other parameters. However, the spectrum tends to have a higher intensity at the central wavelength when $\Delta V$ is higher (i.e., higher velocity at the edge). One interesting feature in the spectrum is a small blue bump that appears when the velocity at the edge is zero. This blue bump may consist of lucky photons that have experienced almost no scattering until they reach the edge. However, these photons are scattered by the static medium at the edge to either redder or bluer wavelengths than the central wavelength, forming a (symmetric) double-peak spectrum. Because there are not many such lucky photons, such a double-peak feature is subdominant; the red peak of the double-peak spectrum is hidden in the dominant main body, and the blue peak appears as a small bump.

![Figure C1](image_url)  
Figure C1. Similar to Figure 8, but with varying $r_{\text{peak}}$.  

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Because dust absorbs Lyα photons and reemits photons at longer wavelengths (i.e., at mid- or far-infrared wavelengths), the main effect of dust is the reduction in the number of Lyα photons. Meanwhile, the probability of a photon being destructed by dust is proportional to the number of scatterings that the photon has experienced until its escape. Photons at wavelengths far away from the central wavelength are those that have been scattered frequently, and thus they are more likely to interact with dust. Therefore, the intensity at a longer wavelength (at the redder side of the Lyα central wavelength) might be reduced more than that at a shorter wavelength compared to the case without dust, implying that dust could...
have an impact on the shape of the spectrum as well as on the absolute intensity level. However, the spectra and surface brightness profiles with different DGRs show not much difference, as seen in the panels of upper two rows of Figure C4. This might be because the best-fit parameter set of each target galaxy is not optimal to see the effect of dust on the shapes of the spectrum and surface brightness profile. To maximize the dust effect, we use a parameter set that is thought to maximize the number of scatterings (large $\tau_0$, large $r_{\text{peak}}$, small $V_{\text{peak}}$, and small $\Delta V$). Now we can see some changes in the spectral shape with DGR in the bottom left panel of Figure C4; a greater intensity reduction at long wavelengths when the DGR is higher, resulting in a sharper spectrum with its peak closer to the central wavelength. It should be noted that each spectrum is normalized by the number of escaping photons to compare the spectral shape and not the intensity. The overall intensity, of course decreases with DGR. The effect of dust on the shape of the surface brightness profile appears to be less significant.

Figure C4. Panels in the upper two rows are similar to those in Figure 8, but with varying DGR. Panels in the bottom row show a parameter set that could maximize the number of scatterings (large $\tau_0$, large $r_{\text{peak}}$, small $V_{\text{peak}}$, and small $\Delta V$), which is adopted to show the impact of dust more clearly.

We did not vary one parameter that may have a strong impact on the Ly$\alpha$ transfer process: the medium temperature. We fixed the medium temperature at $10^5$ K in order to focus on the impact of other parameters such as the scale radius of the medium density and parameters for the medium velocity profile that have not been studied much so far. However, it is worth checking briefly how the medium temperature affects the shapes of the emerging Ly$\alpha$ spectrum and surface brightness profile. We perform four additional runs for a medium temperature of 101, 102, 103, and 105 K, fixing other parameters at their best-fit values to MUSE 1185. We note that hydrogen atoms are almost fully ionized at a temperature as high as 105 K. In our simulation, the medium temperature of 105 K should be regarded as an effective temperature that includes the effect of turbulent motion. Figure C5 shows that the spectrum becomes broader with a stronger peak shift, and the surface brightness profile becomes shallower as the medium temperature increases. This is because hydrogen atoms have a broader velocity distribution when the medium temperature is higher.
and they scatter Ly\(\alpha\) photons in a broader frequency range. As a result, a higher medium temperature leads to more scatterings (under the assumption that the ionization state of hydrogen atoms does not change with temperature). Similar to the case where we change the input spectrum, shown in Section 4.7, the best-fit parameter set for a galaxy will differ when we change the medium temperature. Nevertheless, our approach of fixing the medium temperature is useful for exploring the impact of other parameters more efficiently because we can reduce the dimension of the parameter space we are exploring and the resulting computation time. It will be our future work to consider the medium temperature together with all relevant quantities that include density, pressure, and ionization state to be self-consistent.

Appendix D

Peak Shift of the Ly\(\alpha\) Spectrum and the Peak Velocity of the Medium

In this section, we examine the impact of the velocity of the expanding medium on the emerging spectrum in more detail, assuming an expanding velocity that ranges from several dozen to several hundred km s\(^{-1}\). To focus on the impact of the medium velocity alone, we simplify the models by considering a central, monochromatic source in an expanding HI halo of uniform density (optical depth of 10\(^6\)) and uniform temperature (10\(^4\) K). We adopt three different types of velocity profile: (a) constant expansion, (b) Hubble-like expansion, and (c) accelerated and then decelerated expansion (as in this paper).

Figure D1 shows (left) the emerging spectra and (right) the distributions of the numbers of scatterings (\(N_{\text{scatt}}\)) for nine peak expanding velocities (denoted by different colors) for each velocity type (each row). From \(V_{\text{peak}} = 0\) to \(\sim 50\) km s\(^{-1}\) the red peak of the emerging spectrum appears to be progressively displaced redward. When \(V_{\text{peak}} \gtrsim 50\) km s\(^{-1}\), however, the peak appears to lie closer to the central wavelength (i.e., \(x = 0\)) as the velocity increases. This trend is consistent with the result shown in Figure 7 of Verhamme et al. (2006). The number of scatterings, on the other hand, consistently decreases as the velocity increases, which is attributable to the decreased effective optical depth. Therefore, as the medium velocity is too high, photons will escape without having chances to undergo a large number of scatterings; as a consequence, no significant frequency change is found in the emergent spectrum.

The behavior of spectral peak when \(V_{\text{peak}} \approx 50\) km s\(^{-1}\) can be understood with the help of the redistribution function, \(R(x_{\text{out}} | x_{\text{in}})\), which describes the probability of the “out” frequency (\(x_{\text{out}}\)) after a single scattering for a given “in” frequency (\(x_{\text{in}}\) the frequency before scattering).\(^\text{11}\) Figure 21 of Dijkstra (2017) shows that when the input frequency is farther away from the central frequency, so is the output frequency. As the velocity of the medium is higher, the input frequency is shifted farther away from the central frequency in the medium frame, which will end up with stronger frequency change after a number of scatterings despite the decreased effective optical depth. It should be noted, however, that this effect can occur for low medium velocities where enough number of scatterings can still occur; i.e., \(V_{\text{peak}} \lesssim 50\) km s\(^{-1}\) in our particular example. This critical expanding velocity will depend on the optical depth, on the temperature of the medium, and on the input spectrum.

\[^{11}\] See Section 7.3 of Dijkstra (2017) for more details.
Appendix E

Correlation between the Extent of the Lyα Source and the Lyα Halo

Figure E1 shows the correlation between $r_{\text{halo}}/r_{\text{cont}}$ and $r_{\text{cont}}$ for the eight target galaxies of this study (filled black circles) and for the full sample of L17 (filled black circles and open gray circles). $r_{\text{halo}}/r_{\text{cont}}$ is strongly anticorrelated with $r_{\text{cont}}$ (see the Spearman rank-order correlation coefficients and $p$-values given in the figure). It should be noted that this anticorrelation between $r_{\text{halo}}/r_{\text{cont}}$ and $r_{\text{cont}}$ is not derived from the simulation results, but from the observational results. This anticorrelation is interesting because $r_{\text{halo}}$ itself is positively correlated with $r_{\text{cont}}$ (see Figure 13 in L17). The positive correlation between $r_{\text{halo}}$ and $r_{\text{cont}}$ simply indicates that Lyα photons are generated where UV photons are (i.e., H II regions) through photoionization followed by recombination. However, the anticorrelation between $r_{\text{halo}}/r_{\text{cont}}$ and $r_{\text{cont}}$ implies that the $r_{\text{halo}}/r_{\text{cont}}$ correlation is not very strong but rather relatively weak. This weakness in the $r_{\text{halo}}/r_{\text{cont}}$ relationship suggests that other factors than $r_{\text{cont}}$ will be more critical in determining $r_{\text{halo}}$, the factor that mostly matters, the size of the Lyα halo, would be how significantly the Lyα photons diffuse out in space by resonance scattering, rather than the spatial size of the Lyα source.

Figure D1. Spectra (left panels) and numbers of scatterings (right panels) of photons that emerge from a central monochromatic source in an expanding H I halo of uniform density (optical depth of $10^6$) and uniform temperature ($10^4$ K) for nine peak expanding velocities (denoted by different colors) for three different velocity types: constantly expanding (top), Hubble-like expanding (middle), and accelerated and then decelerated expanding (bottom, $v_{\text{peak}} = 0.5$ and $\Delta V = -V_{\text{peak}}$).
Figure E1. Anticorrelation between $r_{\text{halo}}/r_{\text{cont}}$ and $r_{\text{cont}}$ of the eight target galaxies (filled black circles) and of the full sample of L17 (filled black circles and open gray circles). The Spearman rank-order correlation coefficients and $p$-values (in parentheses) for these two samples are given in the top right panel.

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