Interpreting cosmological tensions from the effective field theory of torsional gravity

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Cosmological tensions can arise within ΛCDM scenario amongst different observational windows, such as measurements of the $H_0$ and $\sigma_8$ parameters. These tensions, if finally confirmed by measurements, may indicate new physics beyond the standard paradigm. In this Letter, we report how to alleviate both the $H_0$ and $\sigma_8$ tensions simultaneously within torsional gravity from the perspective of effective field theory (EFT). We apply the EFT approach, which allows to investigate the evolution equations at the background and perturbation levels in a systematic way, and examine the conditions followed by the coefficients of various possibly involved operators such that cosmological tensions can be relaxed. Following these observations we construct concrete models of Lagrangians of torsional gravity. Specifically, we consider the parametrization $f(T) = -T - 2\Lambda/M^2 + \alpha T^{3/2}$, where two out of the three parameters are independent (in which an additional term of the form $cT^{1/2}$ can be added). This model can efficiently fit observations solving all statistical tensions. To our knowledge, this is the first time where a modified gravity theory can alleviate both $H_0$ and $\sigma_8$ tensions simultaneously, offering an additional argument in favor of gravitational modification.

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Introduction. — As the standard paradigm, the Lambda-Cold Dark Matter cosmology (ΛCDM) has been tested by various observations, from which the acceleration of today’s universe is interpreted to be sourced from a cosmological constant. However, the nature of cosmic acceleration remains mysterious. The possibility of a dynamical dark energy (DE), as well as the need of a stochastic process such as Inflation to generate initial conditions that seed the Large-Scale Structures (LLS), led to many proposals based either on the introduction of new fields [1, 2], or on gravity theories beyond General Relativity (GR) [3, 4].

With the accumulation of cosmological data, experimental tensions may arise within ΛCDM cosmology. If they were to remain under the increasing precision of experimental observations, they would constitute, in a statistical sense, clear indications of new physics beyond ΛCDM. One recently-well-debated tension relates the value of the Hubble parameter at present time, $H_0$, measured from the Cosmic Microwave Background (CMB) temperature and polarization data by the Planck collaboration to be $H_0 = 67.37 \pm 0.54$ km s$^{-1}$ Mpc$^{-1}$ [5], to the one from local measurements of the Hubble Space Telescope yielding $H_0 = 74.03 \pm 1.42$ km s$^{-1}$ Mpc$^{-1}$ [6]. Recent analyses with the combination of gravitational lensing and time-delay effects data reported a significant deviation at 5.3σ [7]. Another potential tension concerns the measurements of the parameter $\sigma_8$, which quantifies the gravitational clustering of matter from the amplitude of the linearly-evolved power spectrum at the scale of $8h^{-1}$Mpc. Specifically, a possible deviation was noticed between measurements of CMB and LSS surveys, namely between Planck [5] and SDSS/BOSS [8–10]. Nevertheless, the statistical confidence of this cosmological “tension” remains low and is not as manifest as the $H_0$ tension [11–13]. Although these two tensions could in principle arise from unknown systematics, the possibility of physical origin puts the standard lore of cosmology into additional investigations, by pointing to various extensions beyond ΛCDM.

There exist several proposals to alleviate the $H_0$ tension, such as early DE [14], interacting DE [15, 16], dark radiation [17], hot axions [18], modification of the details of Big Bang Nucleosynthesis (BBN) [19], effects of local inhomogeneities [20], or modified gravity [21, 22]. The $\sigma_8$ tension may be addressed by a hot dark matter (DM) component induced by sterile neutrinos [23], running vacuum models [24], a DM sector that clusters differently at small and large scales [25], or by modified gravity [13]. Additionally, there were attempts from non-conventional matter sector to address both tensions simultaneously, such as DM-neutrinos interactions [26], decaying DM [27], or DM-photon coupling [28]. Remarkably, both tensions can in principle be alleviated simultaneously via gravitational modifications. Indeed, the $H_0$ tension reveals a universe that is expanding faster at late times than that from a cosmological constant preferred by CMB data, while a lower value of $\sigma_8$ than the one of CMB most-likely ΛCDM would imply that matter clusters either later on or less efficiently. Hence, these two
observations seem to indicate that there might be less gravitational power" at intermediate scales.

In this Letter we take the part to consider systematically the $H_0$ and $\sigma_8$ tensions, and report how to alleviate both simultaneously within torsional gravity. We exploit the effective field theory (EFT) of torsional gravity, a formalism that allows for a systematic investigation of the background and perturbations separately. This approach was developed early on in [29] for curvature gravity, and recently in [30, 31] for torsional gravity. EFT approaches have been widely applied with success in cosmology, for instance to inflation [32–35], to DE [36–41], or to bounce realization [42–47]. In order to address cosmological tensions, we identify the effects of gravitational modifications within the EFT on the dynamics of the background and of linear perturbations level. This will allow us to construct specific models of $f(T)$ gravity providing adequate deviation from ΛCDM that can alleviate $H_0$ and $\sigma_8$ tensions.

**Effective field theory.**– For a general curvature-based gravity, the action following the EFT approach in the unitary gauge, invariant by space diffeomorphisms [37], which lives on a flat Friedmann-Robertson-Walker (FRW) metric $ds^2 = -dt^2 + a^2(t) \delta_{ij}dx^i dx^j$, is given by

$$S = \int d^4x \left\{ \sqrt{-g} \left[ \frac{M_P^2}{2} \Psi(t)R - \Lambda(t) - b(t)g_{00}^{00} + M_2^2 \delta g_{00}^{00}2 - \bar{m}_2^2 \delta g_0^{00} \delta K - \bar{M}_3^2 \delta K^2 - \bar{M}_3^2 \delta K^\mu \delta K^\nu + \bar{m}_2^2 \delta \Psi \partial_\mu \delta \Psi \partial_\nu \Psi + \lambda_2 \delta R^2 + \lambda_3 \delta R_{\mu \nu} \delta R_{\mu \nu} + \mu_2^2 \delta g_{00}^{00} \delta R + \gamma_1 C^{\mu \nu \rho \sigma}C_{\mu \nu \rho \sigma} + \gamma_2 C^{\mu \nu \rho \sigma}C_{\mu \nu \rho \sigma} + \gamma_3 C^{\mu \nu \rho \sigma}C_{\mu \nu \rho \sigma} + \frac{1}{2} \sqrt{-g} \left[ \frac{M_4^4}{3} (\delta g_{00}^{00} - \delta m_2^2 \delta g_{00}^{00})^2 \delta K + ... \right] \right\}, \quad (1)$$

where $M_P = (8\pi G_N)^{-1/2}$ is the reduced Planck mass with $G_N$ the Newtonian constant. $R$ is the Ricci scalar corresponding to the Levi-Civita connection, $C^{\mu \nu \rho \sigma}$ is the Weyl tensor, $\delta K^\mu$ is the perturbation of the extrinsic curvature, and the functions $\Psi(t), \Lambda(t), b(t)$ are determined by the background evolution.

When the underlying theory includes also torsion [30], one can generalize the EFT action as follows,

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} \Psi(t)R - \Lambda(t) - b(t)g_{00}^{00} + \frac{M_2^2}{2} d(t)T^0 \right] + S^{(2)} \quad (2).$$

To compare with (1), one reads that at background level there is additionally the zeroth part $T^0$ of the contracted torsion tensor $T^\mu_\mu$, with its time-dependent coefficient $d(t)$. Furthermore, the perturbation part $S^{(2)}$ contains all operators of the perturbation part of (1), plus pure torsion terms including $\delta T^2$, $T^{0 \mu \nu} T^0_{\mu \nu}$, and $\delta T^{\mu \nu \rho \sigma} T^\rho_{\mu \nu \sigma}$, and extra terms that mix curvature and torsion, namely $\delta T \delta R$, $\delta g_{00}^{00} \delta T$, $\delta g_0^{00} \delta T^0$ and $\delta K \delta T^0$, where $T = T^{0 \mu \nu} T_{\mu \nu} + \frac{1}{2} T^{\rho \mu \nu} T_{\rho \mu \nu} - T^{\mu \nu \rho \sigma} T_{\mu \nu \rho \sigma}$ is the torsion scalar. Adding the matter action $S_m$ and then performing variation, one obtains the Friedmann equations to be [30]:

$$H^2 = \frac{1}{3 M_P^2} \left( \rho_m + \rho_{DE}^{eff} \right), \quad (3)$$

$$H = -\frac{1}{2 M_P^2} \left( \rho_m + \rho_{DE}^{eff} + P_m + P_{DE}^{eff} \right),$$

with dots denoting derivatives with respect to cosmic time $t$, and where

$$\rho_{DE}^{eff} = b + \Lambda - 3 M_P^2 \left[ H \Psi + \frac{dH}{t} + H^2 (\Psi - 1) \right], \quad (4)$$

$$P_{DE}^{eff} = b - \Lambda + M_P^2 \left[ 2 H \Psi + \frac{d}{t} + (H^2 + 2 H) (\Psi - 1) \right],$$

are respectively the effective DE density and pressure in the general torsional gravity. In the following, we treat the matter sector as dust, that satisfies the conservation equation $\dot{\rho}_m + 3 H \rho_m = 0$, which in terms of redshift leads to $\rho_m = \frac{3 M_P^2 H^2 \Omega_{m0}^2 (1+z)^3}{3 H^2}$, with $\Omega_{m0}$ the value of $\Omega_m \equiv 8 \pi G_N \rho_{m0} / (3 H^2)$ at present.

The above EFT approach holds for every torsional gravity, by making a suitable identification of the involved time-dependent functions. For instance, the well-known $f(T)$ gravity is characterized by the action $S = \frac{M_4^4}{3} \int d^4x f(T)$ [48], with $e = \det(e^{\mu}_{\nu}) = \sqrt{-g}$ and $e^\mu_{\nu}$ the vierbein, and thus by the Friedmann equations

$$H^2 = \frac{\rho_m + \rho_{DE}^{eff} + 3 H \psi + \frac{dH}{t} + H^2 (\psi - 1)}{2 M_P^2}, \quad \dot{H} = -\frac{1}{2 M_P^2} \left( \rho_m + \rho_{DE}^{eff} + \frac{dH}{t} + H^2 (\psi - 1) \right),$$

where $f(T)$ gravity can arise from the general EFT approach to torsional gravity by choosing $\psi = -f_T$, $\Lambda = \frac{M_4^4}{2} (T f_T - f)$, $b = 0$, $d = 2 f_T$ [30], and can restore GR by choosing $f(T) = -2 \Lambda / M_P^4$. Similarly, one can use the EFT approach to describe $f(R, T)$ gravity, and hence $f(T, B)$ gravity too, where $B = -2 \sqrt{\gamma} T^{\mu \nu}$ is the boundary term in the relation $R = -T + B$.

**Model independent analysis.**– In general, to avoid the $H_0$ tension one needs a positive correction to the first Friedmann equation at late times, that could yield an increase in $H_0$ compared to ΛCDM scenario. As for the $\sigma_8$ tension, we recall that in any cosmological model, at sub-Hubble scales and through matter epoch, the equation that governs the evolution of matter perturbations in the linear regime is [49–58]

$$\ddot{\delta} + 2 H \dot{\delta} = 4 \pi G_{eff} \rho_m \delta,$$

where $\delta \equiv \delta \rho_m / \rho_m$ is the matter overdensity and $G_{eff}$ is the effective gravitational coupling given by a generalized Poisson equation (see e.g. [39] for an explicit expression of $G_{eff}$ for the operators present in the action (1)). In general, $G_{eff}$ differs from the Newtonian constant
Since the friction term in (5) increases, the growth of \( \delta \) will be significant only at low redshift. Additionally, \( H_s \) is solved (one should choose \( \sigma = \delta(1)/\delta(a = 1) \)). Hence, alleviation of the \( \sigma_s \) tension may be obtained if \( G_{\text{eff}} \) becomes smaller than \( G_N \) during the growth of matter perturbations and/or if the “friction” term in (5) increases.

To grasp the physical picture, we start with a simple case: \( b(t) = 0 \) and \( \Lambda(t) = \Lambda = \text{const.} \) (\( b \) and \( \Lambda \) are highly degenerate as shown in (4)), while \( \Psi(t) = 1 \). Hence, from (2), with the above coefficient choices, the only deviation from \( \Lambda \text{CDM} \) at background level comes from the term \( d(t)/T^0 \), and we remind that in FRW geometry \( T^0 = H \) when evaluated on the background. In this case, the first Friedmann equation in (3), using for convenience \( \Omega_m = \rho_m/(3M_p^2H^2) \) the matter density parameter. Accordingly, if \( d < 0 \) and is suitably chosen, one can have \( H(z \to z_{\text{CMB}}) \approx H_{\Lambda \text{CDM}}(z \to z_{\text{CMB}}) \) but \( H(z \to 0) > H_{\Lambda \text{CDM}}(z \to 0) \), i.e. the \( H_0 \) tension is solved (one should choose \( |d(z)| < H(z) \) and thus since \( H(z) \) decreases for smaller \( z \) the deviation from \( \Lambda \text{CDM} \) will be significant only at low redshift). Additionally, since the friction term in (5) increases, the growth of structure gets damped and therefore the \( \sigma_s \) tension is also solved (note that since we have imposed \( \Psi = 1 \), then \( G_{\text{eff}} = G_N \) as one can verify from (2) and (3), namely the contributions from \( T^0 \) vanish at first order in perturbations).

Furthermore, for typical values that lie well within (or to the closest of) the 1\( \sigma \) intervals of the \( H(z) \) redshift surveys, it is expected that CMB measurements to be sensitive to such deviation from \( \Lambda \text{CDM} \) scenario for non-vanishing \( T^0 \) at early times. Actually, the \( T^0 \) operator acts in a similar way as a conventional cosmological constant. Thus, it adds yet another new functional form to parameterize the background and leads to more flexibility in fitting redshift and clustering measurements. Due to the fact that \( \Lambda \) and \( T^0 \) are highly degenerate, an interesting possibility is to ask whether a universe without a cosmological constant but with a boundary term containing \( T^0 \) can fit well the data. To assess such a possibility, a fully consistent numerical analysis including both CMB and redshift measurements is required. This gives interesting consequences for various probes in the intermediate-to-high redshift range accessible to ongoing and near-future target surveys such as quasars, Lyman-\( \alpha \) or 21-cm lines.

\( f(T) \) gravity. Next, we further propose concrete models of torsional modified gravity that can be applied to alleviate the two cosmological tensions for which we provided a dictionary within the EFT. In particular, we focus on the well-known class of torsional gravity, namely the \( f(T) \) gravity, for which we already gave the correspondence with the EFT operators at lowest orders in perturbations.

We consider the following ansatz: \( f(T) = -[T + 6H_0^2(1 - \Omega_{m0}) + F(T)] \), where \( F(T) \) describes the deviation from GR (note however that in FRW geometry, apart from the regular choice \( F = 0 \), the \( \Lambda \text{CDM} \) scenario can also be obtained for the special case \( F(T) = cT^{1/2} \) too, with \( c \) a constant). Under these assumptions, the Friedmann equation becomes

\[
T(z) + 2\frac{F'(z)}{T'(z)}T(z) - F(z) = 6H_{\Lambda \text{CDM}}^2(z),
\]

where primes denote derivatives with respect to \( z \). In order to solve the \( H_0 \) tension we need \( T(0) = 6H_0^2 \approx 6(6H_0^2)^{CC} \), with \( H_{0,CC}^{CC} = 74.03 \text{ km s}^{-1} \text{ Mpc}^{-1} \) following the local measurements [6], while in the early era of \( z \gtrsim 1100 \) we require the universe expansion to evolve as in \( \Lambda \text{CDM} \), namely \( H(z \gtrsim 1100) \approx H_{\Lambda \text{CDM}}(z \gtrsim 1100) \), which implies \( F(z)|_{z \gtrsim 1100} \approx cT^{1/2}(z) \) (the value \( c = 0 \) corresponds to standard GR, while for \( c \neq 0 \) we obtain \( \Lambda \text{CDM} \) too). Note that, in this case the effective gravitational coupling is given by [59]

\[
G_{\text{eff}} = \frac{G_N}{1 + F_T}.
\]

Therefore, the perturbation equation (5) becomes

\[
\delta'' + \left[ \frac{T'(z)}{2T(z)} - \frac{1}{1 + z} \right] \delta' = \frac{9H_0^2\Omega_{m0}(1 + z)}{[1 + F'(z)/T'(z)]T(z)}\delta.
\]

Since around the time of the last scattering \( z \gtrsim 1100 \) the universe should be matter-dominated, we impose \( \delta'(z)|_{z \gtrsim 1100} \approx -\frac{1}{2}(1 + z)\delta(z) \), while at late times we look for \( \delta(z) \) that leads to a \( f \sigma_8 \) in agreement with redshift surveys observations.

By solving (7) and (9) with initial and boundary conditions at \( z \sim 0 \) and \( z \sim 1100 \), we can find the functional forms for the free functions of the \( f(T) \) gravity that we consider, namely \( T(z) \) and \( F(z) \), that can alleviate both \( H_0 \) and \( \sigma_s \) tensions. In the left panel of Fig. 1 we depict two such forms for \( F(T) \). Both models approach \( \Lambda \text{CDM} \) scenario at \( z \gtrsim 1100 \), with Model-1 approaching \( F = 0 \) and hence restoring GR, while Model-2 approaches \( F \propto T^{1/2} \) and thus it reproduces \( \Lambda \text{CDM} \). In particular, we find that we can fit well the numerical solutions of Model-1 by

\[
F(T) \approx 375.47 \left( \frac{T}{6H_0^2} \right)^{-1.65},
\]

and of Model-2 by

\[
F(T) \approx 375.47 \left( \frac{T}{6H_0^2} \right)^{-1.65} + 25T^{1/2}.
\]
Note that, the first term of Model-2, which coincides with Model-1, provides a small deviation to ΛCDM at late times, while decreases rapidly to become negligible in the early universe. Besides, we examine $G_{\text{eff}}$ given by (8), for the two models (10) and (11), which are displayed in the right panel of Fig. 1. As expected, at high redshifts in both models $G_{\text{eff}}$ becomes $G_N$, recovering ΛCDM paradigm. At very low redshifts $G_{\text{eff}}$ becomes slightly higher than $G_N$, increasing slightly the gravitational strength. This gravitational modification is in competition at late times with the accelerating expansion. It turns out that the effect of an increased cosmic acceleration with respect to ΛCDM in our $f(T)$ gravity models dominate over the stronger gravitational strength in the clustering of matter. We check that both models can easily pass the BBN constraints (which demand $|G_{\text{eff}}/G_N-1| \leq 0.2$ [60]), as well as the ones from solar system (which demand $|G_{\text{eff}}'(z=0)/G_N| \leq 10^{-3}h^{-1}$ and $|G_{\text{eff}}''(z=0)/G_N| \leq 10^5h^{-2}$ [61]).

We now show how Model-1 and Model-2 can alleviate the $H_0$ and $\sigma_8$ tension by solving the background and perturbation equations. In Fig. 2, we present the evolution of the Hubble parameter $H(z)$ and $f\sigma_8$ for the two $f(T)$ models, and we compare them with ΛCDM. We can see that the $H_0$ tension can be alleviated as $H(z)$ remains statistically consistent for all CMB, BAO and CC measurements at all redshifts. We remind the reader that the two $f(T)$ models, differing merely by a term $T^{1/2}$ which does not affect the background as explained before, are degenerate at the background level. At perturbation level, the two models behave differently as their gravitational coupling $G_{\text{eff}}$ differs. We can see that both models can alleviate the $\sigma_8$ tension, and fit efficiently BAO and LSS measurements. Note that at high redshifts ($z \geq 2$), Model-2 approaches ΛCDM slower than Model-1, however in a way that is statistically indistinguishable for present-to-day data. Nevertheless, future high-redshift surveys such as eBOSS for quasars and Euclid [67] for galaxies have the potential to discriminate among the predictions of $f(T)$ gravity and of ΛCDM scenario.

In short summary, we conclude that the class of $f(T)$ gravity: $f(T) = - T - 2\Lambda/M_P^2 + \alpha T^{\beta}$, where only two out of the three parameters $\Lambda, \alpha$ and $\beta$ are independent (the third one is eliminated using $\Omega_m$), can alleviate both $H_0$ and $\sigma_8$ tensions with suitable parameter choices. Moreover, such kind of models in $f(T)$ gravity could also be examined through galaxy-galaxy lensing effects [68] and gravitational wave experiments [31].

Extensions in $f(T, B)$ gravity. - It is straightforward to generalize the EFT analysis into other torsional modifications that can also address the observational tensions. One such extension is $f(T, B)$ gravity, in which the Lagrangian is a function of both the torsion scalar $T$ and the boundary term $B = - 2\nabla^\mu T_{\nu}^{\\nu}$ [69] (note that in FRW geometry $B = 6\dot{H} + 18H^2$). Here we consider the subclass $f(T) = - T + F(B)$. In this case, the correspondence with the EFT parameters is: $\Psi(t) = 1, b(t) = 0, d(t) = 2\dot{\phi}^2 F(B)/(\dot{\phi} B\dot{t})$. By fixing $d(t) = \text{const.}$, we acquire that $F_B$ evolves linearly with cosmic time $t$. One can then solve the evolution equations for $d(t)$ and $\Lambda(t)$, imposing the observational measurements, in order to reconstruct the form of $f(T, B)$, as was done for $f(T)$ gravity. Extension to more general cases of gravity, where $\Psi(t), b(t)$, as well as higher-order operators, are left as free functions, can be considered along the lines developed here. We leave concrete model-building in $f(T, B)$ gravity and other modified gravity theories for future work.

Conclusions. - In this Letter we reported how theories of torsional gravity can alleviate both $H_0$ and $\sigma_8$ tensions simultaneously. Working within the EFT framework, torsional gravity theories can be identified to the EFT operators that allow us to extract the evolution equations of the background and of the perturbations
in a model-independent manner. This allows us to address in a systematic way how tensions amongst the observational measurements, such as the ones on $H_0$ and $\sigma_8$, can be relaxed. Following these considerations, we constructed concrete models from specific Lagrangians, describing cosmological scenarios where these tensions fade away. In particular, we investigated the well-known $f(T)$ gravity. Imposing initial conditions at the last scattering that reproduce $\Lambda$CDM scenario, and imposing the late times values preferred by local measurements, we reconstructed two particular forms of $f(T)$. These models are well described by the parameterization: $f(T) = -T - 2\Lambda/M_\text{Pl}^2 + \alpha T^\beta$. To our knowledge, this is the first time where both $H_0$ and $\sigma_8$ tensions are simultaneously alleviated by a modified gravity theory.

We mention that we used the simplest approach of EFT to torsional gravity, in the sense that we considered only operators present at the background level. Although we found parametrizations of the lowest order operators efficient in alleviating cosmological tensions, constructing more sophisticated scenarios by involving extra operators can lead to a fruitful phenomenology and thus inspire further investigations in many different directions. Namely, it would be interesting to perform an observational confrontation using various datasets to assess in a statistical way the performance of the modified gravity theory that we considered. Moreover, one can use the information from the EFT to construct gravitational modifications beyond the $f(T)$ class that can solve both tensions simultaneously, such as the $f(T,B)$ extensions, symmetric teleparallel gravity, $f(T,T_G)$ gravity, etc. These topics, while interesting and necessary, lie beyond the scope of this first investigation, and shall be addressed in follow-up works.

To end, we point out that our results can be put in the perspective of forthcoming LLS surveys that cover intermediate-to-high redshifts ranges, such as probes of quasars, Lyman-α or emission lines, where not only higher values of $H(z)$ could be detected, but also lower values of $f\sigma_8$ following the suppression of the structure growth from the early start of the accelerated expansion. These surveys will shed light and help on probing the observable effects predicted by torsional gravity.

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