Is the flip-flop behaviour of accretion shock cones on to black holes an effect of coordinates?

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ABSTRACT

We study numerically the relativistic Bondi–Hoyle accretion of an ideal gas on to a Kerr fixed background space–time on the equatorial plane with s-lab symmetry. We use both Kerr–Schild (KS) and Boyer–Lindquist (BL) coordinates. We particularly focus on the study of the flip-flop motion of the shock cone formed when the gas is injected at supersonic speed. The development of the flip-flop instability of the shock cone in the relativistic regime was reported recently for the first time. We reproduce the flip-flop behaviour found in the past when BL coordinates are used, and perform similar numerical experiments using horizon penetrating KS coordinates. We find that using KS coordinates the shock cone oscillates; however, such oscillations are not of the flip-flop type and their amplitude decreases with resolution.

Key words: accretion, accretion discs – black hole physics – hydrodynamics – shock waves.

1 INTRODUCTION

Bondi–Hoyle accretion is the evolution of a homogeneously distributed gas moving uniformly towards a central compact object (Bondi & Hoyle 1944). Depending on the relation existing between the velocity of the gas and the sound speed, a shock cone can be formed. Specifically, when the fluid is supersonic, a shock cone appears, otherwise it does not.

When the compact object rotates, an interesting phenomenon, called flip-flop instability, can appear (Matsuda, Inoue & Sawada 1987). This effect consists of the oscillation of the shock cone orientation. Various studies of this effect have been carried out under different conditions and for different purposes. For example, in Newtonian gravity, considering the density and velocity field to be non-uniform, they found that the shock cone might achieve a disc-like topology (Fryxell & Taam 1988; Taam & Fryxell 1988); also, the dependence of the appearance of the flip-flop behaviour on the size of the object has been studied (Sawada et al. 1989; Livio et al. 1991; Matsuda et al. 1991; Benensohn, Lamb & Taam 1997).

The astrophysical relevance of the flip-flop of the shock cone is associated, for example, with the fluctuations observed in the X-ray emission of pulsating systems such as Vela-x1 and EXO 2030+375, which are not associated with the orbital period of the binary system (Taam & Fryxell 1988).

The flip-flop effect can also happen within a relativistic scenario. Particularly interesting is the case when the compact object is a black hole. This case was first reported in Dönmez, Zanotti & Rezzolla (2011) and we consider that this is a very important subject to be studied in detail because the oscillations (of different nature) of a gas around a black hole may potentially explain the oscillatory behaviour of high-energy sources, such as quasi-periodically oscillating sources (QPOs) or bursts when the shock cone is unstable. Moreover, we consider the treatment involving the curved background space–time to be a very important step, which uses a truly black hole space–time background instead of the commonly used pseudo-Newtonian Paczyński–Wiita type potentials.

We are in a position to analyse the case where the gas is truly allowed to enter the black hole’s event horizon; this is what we do in this paper. In the relativistic Bondi–Hoyle accretion, the flip-flop instability in the equatorial plane was briefly reported by Dönmez et al. (2011). However, the coordinates used to describe the rotating Kerr black hole are not appropriate because the Boyer–Lindquist (BL) coordinates are singular at the event horizon, and as a consequence it is necessary – as in the Newtonian cases – to apply boundary conditions at an inner time-like artificial boundary outside and near the horizon.

In this work, we study the Bondi–Hoyle accretion process using the Kerr–Schild (KS) penetrating coordinates in the equatorial plane; in these coordinates, the fluid naturally falls into the black hole. We also study the case of using the singular BL coordinates in order to reproduce the flip-flop effect and measure its amplitude and phase. We reproduce the morphological behaviour in Dönmez et al. (2011), and measure the amplitude of the flip-flop oscillations using BL coordinates. However, using KS coordinates, we find some oscillations with a frequency that is not comparable to that observed using BL coordinates associated with the flip-flop effect; we observe that these oscillations are due to numerical artefacts because their amplitude converges to zero when resolution is increased.

In fact, there is a precedent within the Newtonian regime where it has been found that shock cone instabilities can be associated
with the implementation of boundary conditions at the surface of
the accretor, and may also influence the attached or detached nature
of the shock to the compact object or numerical parameters in the
calculations (Foglizzo, Galletti & Ruffert 2005).

This paper is organized as follows. In Section 2, we describe
the system of the relativistic hydrodynamic equations and the metric
functions for each coordinate system. In Section 3, we describe the
numerical methods used for evolving the equations, initial wind
configurations and boundary conditions. In Section 4, we have a
comparison of the flip-flop behaviour between BL and KS coordinate
systems, and in Section 5 we present a discussion of our results
and some conclusions.

2 RELATIVISTIC HYDRODYNAMIC
EQUATIONS

We model the wind using the relativistic Euler equations for a
perfect fluid described by the stress–energy tensor

$$T^{μν} = ρ_0 h u^μ u^ν + pg^{μν},$$

where $ρ_0$ is the rest-mass density of a fluid element, $u^μ$ is the four-
velocity of the fluid elements, $h = 1 + ϵ + p/ρ_0$ is the specific
internal enthalpy, with $ϵ$ the rest-frame specific internal energy and
$p$ the pressure. The space–time background is described by the
metric $g_{μν}$, and we use units where $G = c = 1$ from now on. Additionally,
we assume that the fluid obeys an ideal gas equation of state
given by

$$p = ρ_0 ϵ (Γ - 1),$$

where $Γ$ is the adiabatic index or the ratio between specific heats.
Relativistic Euler equations are derived from the local conservation
of the rest-mass density and the local conservation of the stress–
energy tensor:

$$\nabla_μ (ρ_0 u^μ) = 0,$$

$$\nabla_μ (T^{μν}) = 0,$$

where $\nabla_μ$ is the covariant derivative consistent with $g_{μν}$ (Misner
et al. 1973). It is convenient to cast these equations in a flux balance
law fashion, as described in Martí, Ibáñez & Miralles (1991),
Banyouls et al. (1997) and Font et al. (2000). The only ingredient
left is the space–time background written within the $3+1$ decom-
position of the space–time formalism. Such a metric is written in
general as

$$ds^2 = -(α^2 - β^i β^i) dt^2 + 2β^i dx^i dt + γ_{ij} dx^i dx^j,$$

where $β^i$ is the shift vector, $α$ is the lapse function and $γ_{ij}$ are the
components of the spatial three-metric.

The background space–time corresponds to a rotating black hole;
however, since the main aim of the paper is the study of the influence
of a time-like internal artificial boundary used in previous studies,
we consider the case of the black hole being described with BL
coordinates, which is the usual approach, and the case of using
KS coordinates that have the property of penetrating the black hole
event horizon. In both cases, we describe the space–time using
spherical coordinates for the space $x^μ = (t, r, θ, φ)$.

In the first case, for BL coordinates the metric functions are

$$α = \left(\frac{\rho^2 Δ}{Σ}\right)^{1/2},$$

$$β^i = \left\{0, 0, -\frac{2Mr}{ρ_0}, 0\right\},$$

$$γ_{ij} = \begin{pmatrix}
\frac{ρ^2}{Δ} & 0 & 0 \\
0 & ρ^2 & 0 \\
0 & 0 & \frac{Σ}{ρ^2 sin^2 θ}
\end{pmatrix},$$

where

$$Δ = r^2 - 2Mr + a^2,$$

$$ρ^2 = r^2 + a^2 cos^2 θ,$$

$$Σ = (r^2 + a^2)^2 - a^2 Δ sin^2 θ,$$

and $M$ and $a$ are the mass and the angular momentum of the black
hole, respectively.

On the other hand, in KS coordinates the metric functions are

$$α = \left(1 + \frac{2Mr}{ρ_0}\right)^{-1/2},$$

$$β^i = \left\{0, 0, \frac{2Mr}{ρ_0}, 0\right\},$$

$$γ_{ij} = \begin{pmatrix}
1 + \frac{2Mr}{ρ_0} & 0 & -a \left(1 + \frac{2Mr}{ρ_0}\right) sin^2 θ \\
0 & ρ^2 & 0 \\
-a \left(1 + \frac{2Mr}{ρ_0}\right) sin^2 θ & 0 & \frac{Σ}{ρ_0 sin^2 θ}
\end{pmatrix}.$$

At this point, we assume that the system is cylindrically symmetric,
i.e. we only study the evolution of the gas on the equatorial plane
and set $θ = π/2$, which does not mean – as usually assumed – that
this is a sort of an infinitesimally thin morphology approximation.

With the space–time background metric known, relativistic Euler
equations on a curved fixed background space–time in our coor-
dinates and for the two cases considered read (Martí et al. 1991;
Banyouls et al. 1997; Font et al. 2000)

$$∂_μ U + ∂_μ F^μ + ∂_μ F^φ = S = \frac{∂_μ √γ}{√γ} F^μ,$$

where $γ = det(γ_{ij})$ is the determinant of the spatial metric, $U$ is the
vector whose entries are conservative variables that depend on the
original primitive variables $ρ_0, v^i, p$ and $ϵ$. The vector $F^μ$ contains
the fluxes along the spatial directions, and $S$ is the source vector.
All these ingredients are specifically

$$U = \begin{pmatrix}
D \\
S_r \\
S_θ \\
S_φ \\
t\end{pmatrix} = \begin{pmatrix}
ρ_0 W \\
ρ_0 h W v_r \\
ρ_0 h W v_θ \\
ρ_0 h W v_φ \\
ρ_0 h W^2 - p - ρ_0 W
\end{pmatrix},$$

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where \( v' \) are the components of the velocity of the gas measured by an Eulerian observer, which are related to the spatial components of the four-velocity of the fluid elements by \( v' = u'/W + \beta'/\alpha \), with \( W \) the Lorentz factor, \( W = 1/\sqrt{1 - \beta^2/\alpha^2} \). The Christoffel symbols obtained from the space-time metric.

Finally, the system of equations (7)–(11) is a set of four equations for the variables \( \rho_\text{ini}, v', v^\phi, p \) and \( \epsilon \). Thus, in order to close this system of equations, we consider that the fluid obeys the equation of state (2).

### 3 NUMERICAL METHODS

We evolve numerically the system of equations (7)–(11) as a 2D initial-value problem using a finite-volume approximation on the equatorial plane. Specifically, we use the well-known high-resolution shock-capturing methods, with the HLL approximate Riemann solver (Harten, Lax & van Leer 1983; Einfeldt 1988), and the minmod piecewise reconstructor. For the update in time, we use the method of lines with a second-order Runge–Kutta TVD integrator along the time direction.

#### 3.1 Initial data

As initial data, we consider a homogeneous wind, which uniformly fills the whole domain, moving on the equatorial plane along the \( x \) direction with constant density and pressure. The initial velocity field \( v' \) can be expressed in terms of the asymptotic velocity, \( v_\infty \), as follows:

\[
v' = F_1 v_\infty \cos \phi + F_2 v_\infty \sin \phi,
\]

\[
v^\phi = -F_3 v_\infty \sin \phi + F_4 v_\infty \cos \phi,
\]

where

\[
F_1 = \frac{1}{\sqrt{\gamma r^2}},
\]

\[
F_2 = -\frac{F_3 F_4 v_\infty \cos \phi + F_1 F_2 v_\infty \sin \phi}{F_1 Y_{rr} + F_2 Y_{r\phi}},
\]

\[
F_3 = \frac{F_1 Y_{rr} + F_2 Y_{r\phi}}{\sqrt{(Y_{rr} Y_{r\phi} - Y_{\phi\phi}^2)(F_3^2 Y_{rr} + F_4^2 Y_{r\phi} + 2F_1 F_2 Y_{r\phi})}},
\]

\[
F_4 = -\frac{2Y_{r\phi}}{\sqrt{Y_{rr} Y_{r\phi}}}.
\]

And the relation \( v^2 = v_r^2 + v^\phi   \) is satisfied. Specifically, in the case of BL coordinates the above expressions for the velocity field reduce to

\[
v' = \sqrt{\gamma r^2} v_\infty \cos \phi,
\]

\[
v^\phi = -\sqrt{\gamma^{\phi\phi}} v_\infty \sin \phi.
\]

In order to choose the initial pressure profile, we introduce the asymptotic speed of sound \( c_{\text{ini}} \). Once we fix the value of \( c_{\text{ini}} \) and assume the density to be a constant \( \rho_\text{ini} = \rho_\infty \), the pressure can be found from the following expression:

\[
P_\text{ini} = \frac{c_{\text{ini}}^2 \rho_\infty (\Gamma - 1)}{\Gamma (\Gamma - 1) - c_{\text{ini}}^2 \Gamma}.
\]

As we can see, in order to avoid negative and zero pressures, the condition \( c_{\text{ini}} < \sqrt{\gamma r^2} - 1 \) has to be satisfied. Finally, the internal specific energy is reconstructed from the equation of state (2).

It is useful to introduce the asymptotic Mach number in terms of \( v_\infty \) and \( c_{\text{ini}} \) as \( \mathcal{M}_\infty = v_\infty/c_{\text{ini}} \). This quantity determines whether the flow is supersonic or subsonic. When \( \mathcal{M}_\infty \) is bigger than 1 the flow is supersonic, otherwise the flow is subsonic. In this paper, we are only interested in the supersonic case as we want to study the properties of the shock cone formed, in particular the possibility of flip-flop oscillations.

Other parameters we have to take into account are the black hole mass \( M \) and its angular momentum \( a \). In the case of the black hole mass, we chose units in which \( M = 1 \). On the other hand, we chose various representative cases for the rotation parameter \( a \). In Table 1, we summarize the set of initial parameters we consider in our analysis.

#### 3.2 Evolution

The numerical domain is \( \phi \in [0, 2\pi] \) and \( r \in [r_{\text{esc}}, r_{\text{max}}] \). The choice of both the interior boundary at \( r = r_{\text{esc}} \) and the exterior one at \( r = r_{\text{max}} \) deserves a careful analysis. First of all, the interior boundary may be space-like or time-like, depending on whether it lies inside the black hole event horizon or outside, respectively. In most of the previous analyses, such a boundary is time-like because the coordinates used are BL, which are singular at the event horizon, and as a way to avoid dealing with the coordinate singularity there, \( r_{\text{esc}} \) is chosen simply to be outside the event horizon; the price to pay with such a choice is that this internal boundary requires the implementation of boundary conditions for the fluid variables in a region where the gravitational field is strong and the gradients of the variables involved are big, which eventually implies the propagation of numerical errors.
that the choice of $r$ and, on the other, not all the materials have to be accreted by the black hole, which can happen by simply considering $r_{\text{acc}} < r_{\text{max}}$, where $r_{\text{acc}}$ is the radius of accretion within which all the materials are captured by the black hole and is defined in relativity by Petrich et al. (1989) as

$$r_{\text{acc}} = \frac{M}{c_{\text{acc}}^2 + v_{\infty}^2}.$$  \hspace{1cm} (21)

The numerical grid is uniformly spaced in the $r$ and $\phi$ directions. In particular, we use the base resolution of $\Delta r = 0.033$ in $r$ direction and $\Delta \phi = 0.014$ in $\phi$ direction for the evolution of all the models. We use a constant time step given by $\Delta t = \min (\Delta t_r, \Delta t_{\phi})$, where $C$ is a fixed and constant Courant factor estimated empirically as $C = 0.25$. Finally, in order to avoid the divergences in our variables along the evolution, due to the definition of the specific enthalpy, we introduce an artificial atmosphere, i.e. we impose the rest-mass density to be no smaller than the minimum value $1 \times 10^{-10}$.

4 RESULTS

4.1 Morphology

We have found similar results as done in the past; particularly, in the supersonic case, a shock cone characterized by a zone of high density shows up at the rare part of the black hole. This behaviour appears to be a sort of late-time attractor behaviour. In order to compare with previous research, we show various examples. As illustrative cases, we show in Fig. 1 two examples of the shock cone morphology using KS coordinates for two different values of the adiabatic index, $\Gamma = 4/3$ and 5/3. Several other cases were studied by changing the velocity of the wind and the rotation parameter of the black hole (see Table 1). The morphology of the rest-mass density is consistent with previous analyses (Petrich et al. 1989; Font & Ibáñez 1998b; Font, Ibáñez & Papadopoulos 1999).

Another important check is the measure of the mass accretion rate across a circle in the equatorial plane $\theta = \pi/2$, $\dot{M} = \int^{r_{\text{max}}}_{0} \alpha \sqrt{\rho D} (v^\prime - \beta^2/\alpha) d\phi$, as an indicator of the accretion process during the evolution. We measure the mass accretion rate in terms of the proper time $\tau$ at a given detector. In Fig. 2, we show the accretion rate for three different values of the initial velocity of the gas in order to illustrate the dependence of the accretion rate on the velocity of the wind. Our measurements are performed in a detector located at $r = 2.1$ for three different models. The figure shows that the higher the velocity of the wind, the smaller the accretion rate. Again these results are consistent with previous studies (Petrich et al. 1989; Font & Ibáñez 1998b; Font et al. 1999).

4.2 Flip-flop behaviour

The flip-flop motion of the shock cone consists of the oscillation of the cone itself along the angular direction. In Fig. 3, we illustrate the snapshots of the shock cone morphology at various times for the model BL2. It can be clearly observed that the shock cone oscillates considerably in the angular direction. Similar results in the relativistic regime were reported recently for the first time in Dönmez et al. (2011).

Something that called our attention is the possibility that the flip-flop behaviour might be due to the numerical implementation of boundary conditions near the event horizon because in both
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Figure 1. The contour lines of the rest-mass density in order to compare the shock cone opening angle for models KS2 (top) and KS8 (bottom) in Table 1. It is also shown that the angle of the cone is bigger for bigger values of the adiabatic index (Table 1). The snapshot is shown for $t = 10000M$. The radial coordinate is in units of the accretion radius $r_{\text{acc}}$.

Figure 2. The mass accretion rates for models KS1, KS2 and KS3. We show how the accretion process depends on the velocity of the gas, the accretion rate being higher for slower winds, as expected.

Figure 3. Snapshots of the rest-mass density for the model BL2 at various values of the coordinate time (top-left panel $t = 7686$, top-right panel $t = 8062$, bottom-left panel $t = 8308$ and bottom-right panel $t = 8670$) are shown. The motion of the shock cone along the angular coordinate is called the flip-flop behaviour. We claim in this paper that the amplitudes of such oscillations are highly dependent on the coordinates used to describe the black hole space–time because they are singular at the event horizon. The units of the spatial coordinates are re-scaled with $r_{\text{acc}}$. Dönmez et al. (2011) and Fig. 3, BL singular coordinates at the event horizon are being used. Particularly important is that in order to simulate accretion processes it is necessary to use a radial domain such that $r \in [r_{\text{acc}}, r_{\text{max}}]$, with $r_{\text{acc}} > r_{\text{EH}}$, which implies that the inner boundary is time-like; moreover, if one wants to track the evolution of the gas up to regions near the black hole, the metric functions in non-penetrating coordinates tend to diverge and the gradients of the hydrodynamical variables are high. Then errors occurring due to the implementation of a time-like artificial boundary near the event horizon are expected to contaminate the simulations in the numerical domain.

In order to investigate this, we measure the oscillations along the angular direction of the shock cone when penetrating KS coordinates are used. In Fig. 4, we show the snapshots of what happens for model KS2, which is physically equivalent to that in Fig. 3. The oscillations are rather small compared to those observed when BL coordinates are used. In Fig. 5, we show the amplitude of the oscillations using the two coordinate systems for the two representative cases in Table 1. What we measure is the position of the maximum of the rest-mass density along the angular coordinate in radians on a circle of radius $r = 10M$; the maximum is located within the shock cone and is therefore a good quantity to be monitored; we plot it versus the proper time measured by a detector located at $r = 10M$, which appears in Figs 3 and 4 as a circle.

We identify flip-flop oscillations as those that clearly show the motion of the shock cone in the angular direction. In Fig. 5, we distinguish at least two important modes using BL coordinates, one with high frequency and low amplitude and the other with high amplitude and low frequency. The latter is the one associated with the flip-flop motion. The important observation here is that when KS coordinates are used, only the high-frequency mode is observed, i.e. the flip-flop mode does not occur. This clearly indicates that coordinates influence the motion of the shock cone, or what can be more interesting is that a true process of accretion, i.e. allowing the gas to really enter the black hole, implies that the flip-flop effect does not take place.

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In order to check the accuracy of the numerical code, in Fig. 6 we show the self-convergence of $\rho_0$, using three resolutions for the case of KS coordinates. The self-convergence was calculated for a fixed angle within the shock cone. This is a very strong test because along such a line the shock cone vibrates. We have a self-convergence better than first order when the shock cone has approached a nearly stationary stage, while first order is acceptable for systems that include the evolution of shocks.

In Fig. 5, we show that using KS coordinates the oscillations of the shock cone are not of flip-flop type, instead they seem to be vibrations. We go further and look at the amplitude of such oscillations. In Fig. 7, we show the trend of the amplitude of the vibrations of the shock cone with resolution. In order to have an idea of what would happen in the continuum limit, we calculate the average of the amplitude of the vibrations for the two resolutions and measure the amplitude of the vibrations with respect to such an average. The result for the exercise in Fig. 7 is that the amplitude around the average is $2.789 \times 10^{-2}$ for the resolution ($\Delta r = 0.022$, $\Delta \phi = 0.0093$) and $1.08 \times 10^{-2}$ for the resolution ($\Delta r = 0.0146$, $\Delta \phi = 0.0062$).

In order to study the effects observed with BL coordinates, we show in Fig. 8 the position of the maximum of the density within the shock cone in time along the angular coordinate at a fixed radius. We ask whether the amplitude of the oscillations changes with resolution, and we find that the flip-flop behaviour is independent of the resolution, as one may expect that it could vanish in the continuum limit. The flip-flop behaviour is seen to be triggered with different phases for different resolution, which indicates that numerical discretization errors influence the appearance of the flip-flop that actually behaves as an instability.

5 DISCUSSION AND CONCLUSIONS

We study numerically the relativistic Bondi–Hoyle accretion on the equatorial plane on a space–time corresponding to a rotating black hole. Our analysis is focused on the shock cone motion of a supersonic gas that is being accreted on to the rotating black hole, and its dependence on the coordinates used to describe the space–time. We compare the cases of using BL non-penetrating horizon coordinates as done in previous research and KS penetrating coordinates.

We find that when BL coordinates are used, the shock cone oscillates in a flip-flop fashion with an amplitude of the order of a radian, whereas when penetrating coordinates are used such behaviour does not take place. In fact, considering the frequency of the flip-flop oscillations when BL coordinates are used, the oscillations observed with KS coordinates are found to be different. Instead, in the case of KS coordinates we interpret the oscillations within the shock cone to be of vibration or resonant type.

Furthermore, using various resolutions in our calculations, we show that the amplitude of the shock cone vibrations when KS coordinates are used decreases with resolution, thus suggesting that even the vibrations are not excited in the continuum limit. Our results are naturally restricted by the s-lab symmetry that we use in our calculations, which, however, has been also used to identify the shock cone oscillations with e.g. QPOs (Dönmez et al. 2011). In this

Figure 4. Snapshots of the rest-mass density of the shock cone, using KS coordinates for model KS2, are shown. The corresponding times associated with each panel are $t = 7686$ (top-left panel), $t = 8062$ (top-right panel), $t = 8308$ (bottom-left panel) and $t = 8670$ (bottom-right panel). We can see that the motion of the shock cone is very small compared with the case in which BL coordinates are used (see Fig. 3). By using the same coordinate time values as for the BL case to expose the snapshots, we do not intend to imply that the differences in the morphology are coordinate invariant. Instead, in order to compare the motion of the shock cones in the two coordinate systems, we use a detector and the proper time measured as explained below. The units of the spatial coordinates are re-scaled with $r_{\text{acc}}$.

Figure 5. We compare the position of the maximum of the rest-mass density along the angular coordinate $\phi$ during the evolution in terms of the proper time. The models we are considering to carry out this comparison are BL2 versus KS2 (top panel) and BL5 versus KS5 (bottom panel). In both cases, the maximum of $\rho_0$ is measured in a detector located at $r = 10M$. It is clear to see from these figures that when BL coordinates are considered, oscillations of high amplitude are presented, unlike the case of KS coordinates. $\phi$ coordinate is measured in radians.

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Figure 6. The order of self-convergence $Q$ of $\rho_0$ for a fixed angle centred at the shock cone. We calculate the $L_1$ norm of the differences between the value of the density for various resolutions. We show the self-convergence factor for resolutions $(\Delta r_1, \Delta \phi_1) = (0.033, 0.014), (\Delta r_2, \Delta \phi_2) = (\Delta r_1, \Delta \phi_1)/1.5$ and $(\Delta r_3, \Delta \phi_3) = (\Delta r_1, \Delta \phi_1)/(1.5)^2$.

Figure 7. The amplitude of vibrations of the shock cone around the average and how it decreases with resolution. We show only the oscillations of the maximum of the shock cone in a detector located at $r = 10M$ for the two highest resolutions used. The oscillations start at around $t \sim 500$. Before that stage we see a transient lapse that corresponds to the time it takes the shock cone to approach a nearly stationary regime. The top panel shows the result using medium resolution $(\Delta r = 0.022, \Delta \phi = 0.0093)$ and the bottom panel corresponds to high resolution $(\Delta r = 0.0146, \Delta \phi = 0.0062)$.

Figure 8. The position of the maximum of the shock cone at $r = 10M$ for various resolutions. We observe that the flip-flop effect persists; however, the oscillations change the phase for the BL2 model.

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