Maximally entangled mixed states: Creation and concentration

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Using correlated photons from parametric downconversion, we extend the boundaries of experimentally accessible two-qubit Hilbert space. Specifically, we have created and characterized maximally entangled mixed states (MEMS) that lie above the Werner boundary in the linear entropy-tangle plane. In addition, we demonstrate that such states can be efficiently concentrated, simultaneously increasing both the purity and the degree of entanglement. We investigate a previously unsuspected sensitivity imbalance in common state measures, i.e., the tangle, linear entropy, and fidelity.

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By exploiting quantum mechanics it is possible to implement provably secure cryptography 1, teleportation 2, and super-dense coding 3. These protocols and most others in quantum information processing require a known initial quantum state, and typically have optimal results for pure, maximally entangled initial states. However, decoherence and dissipation may cause the states to become mixed and/or less entangled. As the success of a protocol such as quantum teleportation often hinges on both the purity and the entanglement of the initial state 4, it is important to study the interplay of these properties. Using a source of 2-qubit polarization entangled states 5, we investigate the creation of maximally entangled mixed states, and their concentration 6, 7, 8, 9.

Entangled states have been demonstrated in a variety of systems 10, 11, 12, 13, 14, 15. In fact, there are several classes of entangled states: maximally entangled and nonmaximally entangled pure states 5, 11, 16, nonmaximally entangled mixed states 17, and the special case of Werner states 18 (incoherent combinations of a completely mixed state and a maximally entangled pure state) have all been experimentally realized using optical qubits. For some time it was believed that Werner states possess the most entanglement for a given level of mixedness. However, Munro et al. 19 discovered a class of states that are more entangled than Werner states of the same purity. These maximally entangled mixed states (MEMS) possess the maximal amount of entanglement (tangle or entanglement of formation) for a given degree of mixedness (linear entropy) 20, 21.

By generating states close to the MEMS boundary, we have experimentally explored the region above the Werner state line on the linear entropy-tangle plane 22. We have also implemented a partial-polarizer filtration/concentration technique which simultaneously increases both purity and entanglement, at the cost of decreasing the ensemble size of initial photon pairs. Though the implementation requires initial state knowledge, we show that MEMS exist for which this “Procrustean” filtering technique 2, 8, 23 is much more efficient than other recent entanglement concentration schemes 24.

ρMEMS I = \left( \begin{array}{ccc}
\frac{i}{2} & 0 & 0 \\
0 & 1-r & 0 \\
0 & 0 & 0 \\
\frac{i}{2} & 0 & 0
\end{array} \right), \quad \frac{2}{3} \leq r \leq 1,

\rhoMEMS II = \left( \begin{array}{ccc}
\frac{i}{3} & 0 & 0 \\
0 & \frac{1}{3} & 0 \\
0 & 0 & \frac{2}{3}
\end{array} \right), \quad 0 \leq r \leq \frac{2}{3},

The exact form of the MEMS density matrix depends on the measures used to quantify the entanglement and mixedness 21: here we use the tangle (T(\rho) = |\max\{0, \lambda_1-\lambda_2-\lambda_3-\lambda_4\}|^2) 20, i.e., the concurrence squared; and the linear entropy (S_L(\rho) = \frac{1}{2} [1 - \text{Tr}(\rho^2)]) 1. Here \lambda_i are the square roots of the eigenvalues of \rho(\sigma_2 \otimes \sigma_2)\rho^*(\sigma_2 \otimes \sigma_2), in non-increasing order by magnitude, with \sigma_2 = \left( \begin{array}{cc}
0 & -i \\
i & 0
\end{array} \right). For this parameterization, where r is the concurrence, the MEMS density matrices exist in two subclasses 19, \rho_{MEMS I} and \rho_{MEMS II}, which have two and three eigenvalues, respectively.

Our creation of MEMS involves three steps: creating an initial state of arbitrary entanglement, applying local unitary transformations, and inducing decoherence. First, frequency degenerate 702-nm photons are created by pumping thin nonlinear Ba-Borate (BBO) crystals with a 351-nm Ar-ion laser. Polarization entanglement is realized by pumping two such crystals oriented such that their optic axes are in perpendicular planes. With a pump polarized at \theta_1, a variable entanglement superposition state \cos \theta_1 | HH \rangle + \sin \theta_1 | V V \rangle is created, where | HH \rangle represents two horizontally polarized and | V V \rangle two vertically polarized photons 2, 14. The pump polarization is controlled by a half-wave plate (HWP, in Fig. 1) set to \theta_1/2.

To create the MEMS I, we start by setting the ini-
FIG. 1: Experimental arrangement to create, and concentrate MEMS. A half-waveplate (HWP) sets the initial entanglement of the pure state. The φ-plate sets the relative phase between |HH⟩ and |VV⟩ in the initial state. HWP₂ and HWP₃ rotate the state into the active bases of the decoherers to adjust the amount of entropy. The tomography system uses a quarter-waveplate (QWP), HWP, and a polarizer in each arm to analyze in arbitrary polarization bases; the transmitted photons are counted in coincidence via avalanche photodiodes. The dashed box contains HWP₄ (oriented to rotate |H⟩ ↔ |V⟩ in the first arm of the experiment) and concentrating elements (a variable number of glass pieces oriented at Brewster’s angle to completely transmit |H⟩, but only partially transmit |V⟩).

The decoherer in each arm couples the polarization with the relative arrival times of the photons [30]. While two horizontal (|HH⟩) or two vertical (|VV⟩) photons will be detected at the same time, the state |HV⟩ will in principle be detected first in arm one and then in arm two, and vice versa for the state |VH⟩ (assuming the decoherer slows vertically polarized photons relative to horizontally polarized ones). Tracing over timing information during state analysis then erases coherence between any distinguishable terms of the state (i.e., only the coherence term between |HH⟩ and |VV⟩ remains). A sample tomography of a MEMS I is shown in Fig. 2(a).

MEMS II are created by first producing the MEMS I at the MEMS I/II boundary, i.e., the state with r=2/3. In order to travel along the MEMS II curve, the optical path length difference in one arm must be varied from 140λ. This couples different relative timings to the |HH⟩ and |VV⟩ states, reducing the coherence between them. For instance, to make the MEMS II (B) in Fig. 2(a), 140λ decoherence was used in one arm, 90λ in the other. Fig. 2(a) indicates very good agreement between theory and experiment with fidelities of ~99% (the fidelity [31] between the target state ρₜ and the measured state ρₘ is given by F(ρₜ, ρₘ) = |Tr(√ρₜρₘ√ρₜ)|²).

The states (A) and (B) are shown in the Sₜ-T plane in Fig. 2(b), along with other MEMS we created. The states do not hit their Sₜ-T targets (shown as stars in the figure) within errors, even though the states have very high fidelities (~99%) with their respective targets. To explore the discrepancy, for each target we nu-
We concentrated a variety of MEMS. Fig. 3 shows the results for the MEMS I and II of Fig. 2 and an additional MEMS I (C). As the number of glass pieces is increased, the states initially become more like a pure maximally entangled state. For example, in the case of (A), the fidelity of the initial MEMS with the state $|\phi^+\rangle$ is 0.672. When the state is concentrated with eight glass slips per arm, the fidelity with $|\phi^+\rangle$ is 0.902: 45% of the initial photon pairs survive this filtering process. The theoretical maximum survival probability is 6.4%. Note a characteristic difference between the two MEMS subclasses: MEMS II cannot be filtered into a Bell state.

We now compare the theoretical efficiency of our local filtering scheme with the interference-based concentration proposal of Bennett et al. \cite{7}, assuming identical initial MEMS and the same number of photon pairs. We shall compare the average final entanglement of formation ($E_F$) \cite{26} (i.e., the $E_F$ of the concentrated state multiplied by the probability of success) per initial pair. The Bennett et al. \cite{7} scheme was recently approximated by Pan et al. \cite{24}, with CNOT operations replaced by polarizing beam splitters; however, due to incomplete Bell state analysis, the probability of successful concentration is only 50% of the original proposal (the recent scheme of Yamamoto et al. \cite{25} is unable to distill MEMS). The first step of both schemes is to perform a “twirling” operation \cite{33} to transform a general entangled state into a Werner state. However, this initial operation usually decreases the entanglement, and the scheme with twirling is efficient only when $r$ is close to 1.

In fact, MEMS I could also be distilled without the twirling operation, using the scheme of Pan et al. But then the probability of success depends on the parameter $r$. For most MEMS, the maximum distillation efficiency from filtration can exceed that achievable using the interference-based methods \cite{24}. For example, as shown in Table I, when the initial state is a MEMS with $r = 0.778$, the two-piece filtering technique has a theoretical $E_F$ per pair nearly three times higher than the interference scheme without twirling, even though a successful concentration produces nearly the same $E_F$.

In theory, using 2 to 5 slips achieves both higher entanglement of the successful state and better average entanglement yield. In practice, the filtration technique is much more efficient (see the final columns of Table I) \cite{34}.

We have demonstrated a tunable source of high fidelity...
MEMS. As a consequence of comparing the T-$S_L$ and fidelity values of generated MEMS with the theoretical targets, we identify and explain an unsuspected difference in sensitivity in these state measures. Furthermore, we have applied a Procrustean filtering technique to several MEMS, realizing a measured efficiency that is well above that achievable using other methods. However, in the limit of very strong filtering, small perturbations in the initial state will eventually dominate the process, yielding only product states (see Fig. 3). In practice, therefore, it may be optimal to combine both methods.

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TABLE I: Efficiency comparison of concentration technique of Bennett et al. using ideal CNOT [3], interference-based concentration [24] without twirling, and Procrustean filtering, for an initial MEMS with $r = 0.778$ and $E_F = 0.69$. The scheme of Bennett et al. requires a twirling operation that decreases the initial $E_F$ to 0.418 before the concentration [23]. In all schemes, except for the final column, we assume the ideal case, i.e., no loss and perfect detector efficiency. To calculate the no-loss result for our filtering scheme, we normalize the measured partial polarizer transmission coefficients (of a single glass piece) to $T_H = 0.740/0.990$ and $T_V = 1$. In the interference schemes, columns 2-4 assume the existence of the required two identical pairs, but in practice this requirement is difficult to achieve [23]. This limitation is reflected in column 5, which lists the average $E_F$ per initial pair achieved in our experiment, to be compared with the much lower value achievable with current interference method technology.

| Concentration method | Prob. of success | $E_F$ when successful | Ideal $E_F$ per pair | Exp. $E_F$ per pair |
|---------------------|-----------------|-----------------------|---------------------|---------------------|
| Twirling [7]         | 74.8%           | 0.51                  | 0.19                | NA                  |
| No Twirling [24]     | 35.2%           | 0.80                  | 0.14                | $\leq 10^{-5}$      |
| Procrustean          |                 |                       |                     |                     |
| 2 pieces             | 50.4%           | 0.81                  | 0.41                | 0.14                |
| 4 pieces             | 26.4%           | 0.88                  | 0.23                | 0.07                |
| 6 pieces             | 14.2%           | 0.93                  | 0.13                | 0.03                |

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fold coincidence data reported in [24]) in the interference schemes, which require 4 photons.