The form of the elastic core and areas of plastic flow base under the jucked stamp

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Abstract. The article provides the information about studies devoted to determining the position, shape and size of the elastic soil core and areas of plastic deformations formed in the soil foundation under the jucked stamp. Based on the results analysis of experiments carried out by third-party researchers and the results of calculations performed by the authors of the article, it was concluded that an elastic soil core in the form of a triangle is formed in the case when the indentation of a stamp is carried out into a base folded with loose ground, provided that $\phi > 25 - 30^\circ$ (this depends on the value of the lateral soil pressure coefficient), and the specific cohesion $C$ is quite small. In cohesive soils, the vertical section of the elastic soil core has a form close to that of half an ellipse, half or a segment of a circle. The exceptions are the results of the experiments by Kaganovskaya S E, where a triangular elastic core was obtained for clay soil. All these facts are in satisfactory agreement with Prandtl’s design scheme, and therefore, the use of calculation methods based on it is justified only for sandy bases. To solve the problem of calculating the bearing capacity of soil foundations composed of various types of soils, methods should be created that are not based on hypotheses about the shape of the elastic core and areas of plastic deformation.

1. Introduction

When determining the bearing capacity of foundations centrally loaded with normal force, as the most frequently used classical design scheme, the scheme proposed by Prandtl (figure 1 (a)) is used, which he applied when solving the problem of indenting an absolutely rigid stamp into a rigid-plastic body [1].

Later Hill [2] showed that Prandtl's solution is not the only one and proposed another slip field, which formed the basis of the design scheme, which also found application in soil mechanics (figure 1 (b)).

Figure 1. Design schemes by Prandtl (a) and Hill (b).
In addition, it was shown in [3] that it is possible to compile compilations from the solutions of Prandtl and R. Hill, however, they have not found their application in solving problems of the stability of foundations.

Analyzing the design scheme of Prandtl. The basis of this design scheme, corresponding to the limiting state of the foundation soil, is an elastic core formed under the stamp and having the shape of an isosceles triangle, the inclination angle of the lateral sides of which to the lower edge of the stamp depends only on the numerical value of the angle of internal friction of the soil $\phi$.

On both sides of it there are the strain range (SR) AKV and SK‘V, which are closed at the top of the core B. Above the SR, to the left and right of the symmetry axis of the stamp, there are Rankine zones - displacement zone (MK and M‘K’ – traces of sliding surfaces, see figure 1 (a)).

This design model is suitable for the case when the base is folded with loose ground $\phi\neq0$; specific cohesion $C\rightarrow0$. However, such foundations are rare in practice, and the calculator has to deal with cohesive soils ($\phi\neq0$; $C\neq0$).

Therefore, the question arises: what is the shape of the soil core in reality, and what is it determined by analytical and numerical solutions of problems on the stability of the soil foundation when the stamp is pressed? What is the shape of the strain range? Is Prandtl's design scheme applicable to any type of foundation? This article is devoted to find the answer to these questions.

2. Investigational study on elastic core shape

There are several experimental methods for determining the trajectories of soil particles and the outline of the boundaries of an elastic soil core. This includes photographing particles moving in the soil tray through the viewing window; and photographing through a transparent screen initially vertically and horizontally oriented, and then curved, in the course of the experiment, colored strips of soil; obtaining traces of particle movement on waxed or carbon-coated screens; laying in the material of models of bases at different depths near a transparent screen of thin strips of lead, the deformations of which can be used to judge the movement of soil particles, etc.

There are some of the results of investigational studies that we managed to find.

2.1. Elastic cores of the delta shaped

For the first time laboratory studies of the shape of an elastic core were carried out, apparently, by Pigulevsky [4] on models of a base made of sand (see figure 2). In these experiments, a stamp with a diameter of $d = 5.7$ cm was pressed to a depth of $h = 3.5$ cm into a truncated sand cone (figure 2(a)) and into the horizontal surface of the sand layer (figure 2(b)), which in both cases contained colored interlayers. As a result, it was found that the stamp, when it is jucked into the sand, "creates a constant figure resembling" an inverted "cone with a base in the plane" of the stamp. "The deforming figure is created from the upper layers of the complex (sand with colored layers) without any redistribution of materials in them, as indicated by the preserved colored layers." This "deforming figure" in modern research is called the elastic core.

![Figure 2. The results of the experiments by M Kh Pigulevsky [4].](image)

Figure 3 shows the results of experimental studies carried out by Zaharescu [5], Bierez, Burela and Vaka [6], Mintskovsky [7] and Kaganovskaya [8]. It can be seen that with both central and eccentric loading of the punch, the elastic (compacted) soil plug has a triangular shape.
There is another example. Figure 4 shows photographs of elastic cores obtained by Professor Berezantsev [9; 10] during experiments on the indentation of buried and surface punches into a sandy base. It can be seen here, in both cases, the elastic core has a triangular shape.

![Figures](https://example.com/figures)

**Figure 3.** Photographs of the soil plug obtained by Zaharescu [5] (a), Biarez, Burela and Vaka [6] (b), Minckovskiy [7] (c, d) and Kaganovskaya [8] (e, f).

All pictures above demonstrate the elastic soil cores formed under the foot of the stamp in the sandy base. The exceptions are the photographs shown in figure 3(d); figure 3(f) [8] - these are the results of studies on the stability of a clay base using waxed screens, rubbed with graphite, made by the method of Remiznikov [11].

When carrying out this experiment, it was found that the elastic part of the core is formed already at a load of 30-40% of the critical one. It adjoins directly to the sole of the stamp and is similar in shape to an isosceles triangle (see figure 3(d)) with slightly curved lateral sides. The formation of the compacted core as a whole is completed when the load reaches a value close to the critical one, with a sharp increase in the intensity of the stamp settlement. Moreover, the height of the compacted core is approximately equal to the width of the stamp.

![Figures](https://example.com/figures)

**Figure 4.** Elastic soil cores of a triangular shape under a buried (a) and under a surface (b) stamp, obtained by Berezantsev (quoted from [9; 10]).
There is laid special emphasis on that this is the only result of the experiment carried out for clayey soils, among those that were able to find, in which the elastic soil core has a clear triangular shape.

2.2. Soil plug with a different shape
In works [12; 13] there are numerous photographs of elastic soil cores formed under the soles of round punches with a diameter of 4 cm to 60 cm, pressed or driven into sandy or clayey soils. Some of these photographs are shown in figure 5, and it follows that in all cases the elastic soil core has a shape close to a hemisphere, and its vertical section looks like a segment of a circle.

![Figure 5: Forms of an elastic soil plug according to experimental studies in sand under a round stamp d = 6 cm (a); in loam at d = 6 cm (b); in the sand under a stamp d = 60cm driven by a hammer (c); in sand under a 6*20cm stamp under conditions of plane deformation (d); in the sand under a stamp d = 15cm driven to a depth of 1d (d); 1.5d (e) and 2d (g); in the sand under a stamp d = 4cm with a concave sole (quoted from [12; 13]).](image)

In [14], the results of a full-scale experiment on the indentation of a stamp with a diameter of 9.5 cm are given, which indicate that the deformation zone of the soil below the base of the stamp has a shape close to a hemisphere (its vertical section has a shape close to a semicircle, see figure 6(a)). Investigating the nature of soil displacements under the stamp bottom, one can say with a high degree of confidence that the vertical section of the elastic soil plug has a shape close to the shape of a circle segment.

![Figure 6: The soil deformation zone under the round stamp d = 9.5 cm (a) (cited from [14]); photograph of the area of displacement of clay soil particles at the moment of completion of the compaction process under a pressed rigid rectangular stamp [15].](image)
An almost similar picture is shown in figure 6(b), which shows a photograph of the area of displacement of clay soil particles at the moment of completion of the compaction process under a pressed rigid rectangular stamp [15].

Let’s consider a few more results of experiments on the indentation of rectangular stamps in the base model.

![Figure 7](image1.png)

**Figure 7.** The base of the load is applied through a rigid stamp before (a) and after (b) experimental studies [16].

The work [16] presents the results of experimental studies of the stability of the structures foundations made of non-rocky soils. The research flume has dimensions of 1.2m × 0.65m × 0.2m, the base material is dense sand of medium size with an angle of internal friction $\phi = 34^\circ$ and a bulk density $\gamma = 18.74 \text{ kN} / \text{m}^3$, there is no information on the value of specific cohesion (figure 7).

Examining figure 7(b), which shows a research flume after the completion of the experiment, the author of [16] concludes that the shape of the elastic plug does not depend on the size of the punch and the depth of its preliminary deepening. This, in our view, is a rather bold statement.

Analyzing the deformation of the test grid, it can be confidently asserted that the shape of the elastic core obtained in this experiment is close to the shape of a flattened semicircle or curvilinear trapezoid, but not a triangular.

Considering the results of two more field experiments. The first experiment to study the joint operation of rectangular foundations with a base was carried out on open sites in the South-West region of Odessa in the Tairov massif [17].

![Figure 8](image2.png)

**Figure 8.** The soil deformation zone under a rectangular footing [17] (a), the soil deformation zone at the base of the stamp with an area of 1 m², the scheme for laying down depth filling marks and the loop of the deformed zone according to the data of [18; 19] (b, c).

The purpose of the research was to study the process of deformation development and determine the actual boundaries of soil deformation zones, for which depth marks and determination of soil density were used in samples that were taken below the base of the foundation. The photograph (figure 8(a)) shows the boundaries of the soil deformation zone obtained as a result of this experiment.
Figure 8(b) shows a picture of the soil deformation zone at the base of the stamp with an area of \( A = 1 \text{m}^2 \), obtained by the authors of [18; 19] during a full-scale experiment in the field.

Comparison of soil deformation zones shown in figure 8(a) and figure 8(b), as well as the nature of the movements of the control strips located below the foot of the foundation, that the elastic cores formed under the foot of the stamp are not triangular.

3. Experiments to determine the shape of areas of plastic flow

Let us consider the results of experimental studies, in which, in addition to the image of the elastic plug, images of the strain range were obtained.

In the first two considered experiments, the authors chose paraffin as the base model material. The use of this material is explained by the fact that the physic-mechanical properties of paraffin have a certain analogy with the properties of cohesive soils. In addition, the paraffin changes color - whitens in places where plastic flow appears. This color change is due to the occurrence of microcracks, which are parallel to the maximum shear stress.

In the first series of experiments [20], the results of which are shown in figure 9, a rectangular steel bar was pressed into the paraffin sample.

![Figure 9](image)

**Figure 9.** Consecutive steps of microcracks zones development (strain range) when a metal stamp is pressed into a paraffin sample at a speed of pressing the stamp of 0.5 mm / min (cited from the work [20]).

In the second series of experiments [21], a spherical stamp was pressed into the base model made of paraffin. Physic-mechanical characteristics when testing paraffin samples on fast shear devices were obtained equal: \( \phi =10^5 \), \( c = 0.98 \text{ MPa} \), deformation modulus \( E_o=3.93 \text{ MPa} \) (as we can see, these characteristics are included in the range of variation of the corresponding values for cohesive soils). The final result of the experiment is shown in figure 10(a).

And finally, in the third experimental study [22], the deformation of the subgrade model is studied by the method of moiré grids. The base is made of an equivalent material, 90% of which is fine sand by weight. The load is applied using a 70×70mm non-recessed punch. The experimental results are shown in figure 10(b).

Analysis of the images shown in figure 9 and 10, means that the areas of plastic flow in all cases have a crescent shape, the only difference is in the first case, this "crescent" has a significant development in the vertical direction.

The elastic cores formed under the stamps also have very similar shapes: in the first case, it is almost half an ellipse, in the second, the elastic core has a crescent shape due to the indentation from the stamp, in the third - a segment of a circle, but not a triangle shape.
Analyzing the above data, one can conclude that:

1. An elastic solid soil plug in the form of a triangle is formed only when the indentation of the stamp is carried out into the base, folded with loose soil, moreover, $\varphi > 30^\circ$, and the specific cohesion $C$ is quite small.

2. When the base is composed of cohesive soils ($\varphi \neq 0; c \neq 0$) or when the material serving as the base in the experiment has physical and mechanical properties close to those of cohesive soil, in this case, the vertical section of the elastic soil core looks like half a circle, half ellipse or segment of a circle. The only exception to this rule was the results of experiments carried out by Kaganovskaya [8], in which it was possible to obtain an elastic core of a triangular shape for clay soil.

3. Strain range, enclosing the elastic soil core, after their closure have a crescent shape in all of their "images" found [20-22].

4. Analytical and numerical methods for constructing an elastic soil plug and strain range at the base of the stamp

Let us now consider the shapes of the elastic core and strain range, which are obtained as a result of their construction on the basis of relations and methods known and accepted in soil mechanics.

For an approximate construction of the strain range (SR) and the elastic core at the base during the indentation of the stamp, the Coulomb strength condition [23], written in the form

$$
(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2 = \left(\sigma_x + \sigma_y + \frac{2C}{\tan \varphi}\right)^2 \sin^2 \varphi,
$$

where: $\sigma_x; \sigma_y; \tau_{xy}$ is stress component.

This approach makes it possible to obtain precisely the approximate outlines of these objects, since it does not allow taking into account the transformation of the stress field in the soil mass after the appearance of the SR in it, which, in turn, will change the size and shape of the latter.

Figure 11 shows the conditional SR for ideally free-flowing (figure 11(a)) and ideally cohesive soil (figure 11(b)), constructed from “elastic” stresses for one of the examples given in [24].

Figure 10. Images of the elastic core (EC) and the strain range (SR) in the paraffin base when a spherical stamp is pressed into it [21] (a) and in the base from an equivalent material obtained by the method of moiré grids [22] (b).
The formula that determines the shape of plastic regions for ideally loose soil is written in the form (2), and for perfectly plastic - in the form (3).

\[
\frac{[\sigma_x - \sigma_y + (\xi_c - 1)\frac{\tau_{xy}}{\rho}]^2 + 4\tau_{xy}^2}{[\sigma_x + \sigma_y + (\xi_c + 1)\frac{\tau_{xy}}{\rho}]^2} = \sin^2 \varphi, \tag{2}
\]

\[
\tau_{\text{max}} = 0.5 \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} = C, \tag{3}
\]

where \(\gamma\) is density of soil.

It can be seen from these pictures that in the first case the vertical section of the elastic core has a shape very close to half of an ellipse, and in the second one it resembles a curvilinear trapezoid in shape strain range in both cases have a crescent shape.

Figure 12 (a), (b), (c) depict the conditional areas of strain range constructed on the basis of equation (1) using [26] for the example given in [27].

As one can see, the SR has a crescent shape of one shape or another, and the elastic soil core has the shape of an isosceles triangle with convex lateral sides and a curved trapezoid (convex lower base). The shape and size of the SR and the elastic core depend on the value of the lateral pressure coefficient of the soil \(\xi_c\) and, which is obvious, on the shear parameters \(c\) and \(\varphi\), which directly enter into the equation for the plasticity condition (1).

To obtain more reliable shapes and sizes of strain range and an elastic soil core, it is necessary to solve the problem of a pressing-in of indenter in an elastoplastic formulation. Figure 12d shows a design model for the example considered in [27] and pictures of the SR obtained by the author as a result of solving the elastoplastic problem for a certain stage of loading. On the right SR (the figures are made in the same scale), the SR constructed according to Coulomb for the same loading stage using [26] is superimposed. It can be seen from the pictures that both plastic zones have practically the same size and shape. The only difference is that the Coulomb region is shifted slightly upward to the ground surface, leaving part of the "gray" region 1 free and taking a part of the plastic Rankine region 1a (see figure 12(d)). It can be assumed that with an increase in the load, the "gray" plastic regions will close along the foundation and then the formed elastic core seems probably, should take the shape of an isosceles triangle with concave lateral sides.
Figure 12. Strain range constructed on the basis of the Coulomb plasticity condition for loads corresponding to the maximum permissible values at: $\xi_0 = 0.95$ (a); $\xi_0 = 0.75$ (b); $\xi_0 = 0.55$ (c) for the design scheme and operating conditions [27]; calculation scheme from [27] (1 is plastic zone; 1a is plastic Rankine zone; 2 is elastic zone and plastic zone, constructed according to Coulomb, for this stage of loading) (d).

Figure 13 shows another calculation scheme for solving the problem of pressing-in of indenter in an elastoplastic statement [28]. The picture clearly shows that the plastic areas and the elastic soil core obtained by prof. V.G. Berezantsev, have practically the same shapes with the corresponding objects shown in figure 12(a), b and constructed according to Coulomb for stresses. The strain ranges are crescent-shaped, and the elastic soil core is in the shape of an isosceles triangle with convex lateral sides.

Figure 14 shows the results of mathematical modeling of the formation process of locally made deformation zones in geomaterials when the load is applied through a rigid stamp, described in [29]. To describe the deformation process beyond the elastic limit, a model with a non-associated flow rule and parameters depending on the pressure value and accumulated inelastic deformation, constructed by the author [29] on the basis of [30-35], were used.

In a porous rock (figure 14(a), (b)), destruction proceeds as follows. Under the stamp, where the average pressure is higher, a compression zone appears, in which the matrix solid material is destroyed. The vertical section of the compression zone (elastic core from our perspective) has a shape close to a segment of a circle, which is also observed in experiments for clay soils (see figure 5 to 10).

Figure 14 (c), (d) shows the results of modeling the pressing a stamp process into an ideally elastoplastic medium. It was found that at zero values of the coefficients of internal friction and dilatancy (this is equivalent to the use of the Prandtl – Reis model), the picture of deformation localization coincides with the well-known solution of L. Prandtl (figure 14 (c)). The solution of the
problem, taking into account internal friction and dilatancy, showed that with an coefficients increase, the localization zone expands noticeably (figure 14 (d)).

Figure 14. Formation of localized deformation zones in porous rock (a; b) and in dense rock with limited lateral deformation (c; d) (Cited from [29]).

Let us consider another work [36], which presents images of plastic deformation areas and an elastic soil core obtained by solving the problem of a pressing-in of indenter, the conditions of which are given in [37]. The computational domain with dimensions of 3.6m × 7.2m simulates a homogeneous, isotropic and weightless base, taking into account the symmetry about its central axis (right side of figure 15(a)). Half of the width of the flexible stamp $b / 2 = 1.44$ m. The clay soil of the base obeys the law of associated plastic flow and the Coulomb-Mohr strength condition, characterized by the following values of the design parameters: modulus of deformation $E_0 = 21.0$ MPa; lateral pressure coefficient $\xi_0=0.43$; $c=0.07$MPa, $\varphi=20^\circ$

Figure 15. Pictures of areas of plastic deformation at the base of flexible (a) and rigid (b) stamps, constructed for operating conditions [37], based on an approximate analytical solution of the mixed problem of the theory of elasticity and the theory of soil plasticity [38] and by the numerical method [36]; levels of values of the vertical component of the tensor of relative deformation $\varepsilon_{yy}$ at the limiting pressure on a flexible (c) and rigid (d) stamp.
As a result, the authors have constructed the isolines of the values of the component of the relative deformation tensor ν_{p3} at the limiting pressure of the flexible stamp \( p_r = 1.32 \) MPa on a weightless base. In this case, the "extreme" isolines, according to the authors, are the boundaries of the strain ranges (right side of figure 15(a)).

With the help of a computer program [26], where an approximate analytical solution of the mixed problem of the elasticity theory and the soil plasticity theory [38] is shaped, we have constructed areas of plastic deformations for the same object and the same conditions for the case of weighty soil. In this case, the dimensions of the design scheme were 41.44 m x 7 m, and the width of the stamp was 1.44 m. The calculated values of the maximum permissible load intensity for a flexible and rigid stamp are, respectively, 2.2 MPa and 2.3 MPa, which is 40% and 24% more than the values given in [36].

An analysis of the graphic images of the plastic areas under flexible and rigid stamps (the left side of figure 15(a) and figure 15(b)) shows that:

1. The shape of the boundary of the plastic areas in both cases (the left side of figure 15(a) and figure 15(b)) practically coincide with the shape of the "extreme" isolines of the strain tensor, which are the boundaries of the plastic regions [36], shown in figure 15(a) (right side).
2. The elastic soil core in both cases has the shape of an isosceles triangle with curved concave lateral sides. Its base and height in the case of a flexible die is approximately 1.5 times the corresponding dimensions if the die is rigid.
3. Deflection angle of rebound surface trace when it reaches the daily level \( \alpha = \pi / 4 - \varphi / 2 = 35^\circ \), which corresponds to the value given in article [36] and L. Prandtl’s solution [1].
4. The calculated values of the maximum permissible load intensity for a flexible and rigid stamp are, respectively, 2.2 MPa and 2.3 MPa, which is 40% and 24% more than the values given in [36].

The written above means a satisfactory agreement between the results of solving this problem, obtained by different authors independently of each other by the analytical and numerical methods. This fact, in our opinion, may indicate a sufficient reliability of the results obtained.

If we interpret the images shown in figure 15(c), (d), in the context of [36], where they are presented, we can observe in figure 15(c) an elastic soil core in the form of an inverted isosceles triangle with a rounded lower apex (or an inverted curvilinear trapezoid?). And closed crescent-shaped "plastic regions" (three strain ranges, similar to figure 12 (c)). In figure 15(d), the processes of the formation of an elastic core and the closure of strain ranges have not yet been completed. But here, it can be assumed also that after the closure of three (two under the edges of the stamp and the third in depth on its vertical axis of symmetry) plastic regions, they will take a crescent shape, and the solid soil plug - a shape similar to that shown in figure 15(c).

5. Equation of lines - boundaries of strain ranges

As it was noted above, in order to obtain the most reliable shapes and sizes of plastic ranges, it is necessary to solve the problem of pressing-in of indenter in elastic-plastic formulation.

Figure 16 shows the strain ranges constructed for the operating conditions [36] by the method [38; 39] for loads \( q=0.5q_{\text{max}} \) (a), \( q=0.6q_{\text{max}} \) (b), \( q=0.85q_{\text{max}} \) (c) when \( \xi_i=0.43 \), where \( q_{\text{max}} \) is the maximum permissible value of the of a uniformly distributed load intensity on the base. The angle of internal friction of the foundation soil \( \varphi=20^\circ \). It also shows the curves enclosing the strain ranges, and, which, as can be seen from figure 16, coincide with almost 100% accuracy with the lower boundaries of the strain ranges. The equations is the following

\[
r = r_0 e^{\theta \tan \varphi}
\]

where \( r_0 \) is position vector of the first point, \( \theta \) is the angle of point from zero deviation, \( \varphi \) is angle of soil internal friction.

Therefore, these curves are logarithmic spirals.

Figure 17 shows the areas of strain ranges constructed for the operating conditions [40] by the method [38; 39] for loads \( q=0.9q_{\text{max}} \) (a), \( q=0.95q_{\text{max}} \) (b), \( q=q_{\text{max}} \) (c) with the value of the lateral pressure coefficient of the foundation soil \( \xi_c=0.4 \) and \( \varphi=44^\circ \).

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Comparing them with the strain ranges obtained [40] (figure 17(d)), one can see that they are congruent. It also shows curves enclosing the lower boundaries of the plastic areas, which, as in the previous case, are approximated by equation (4), meaning that, they are segments of a logarithmic spiral. It is noted that in this case, the angle of internal friction of the foundation soil is two times larger than the previous one.

Let's give one more example. Figure 18 shows the strain ranges (shear areas) obtained by the authors of [41] for sandy (a) and clayey (b) soil. Comparing the outlines of these areas with the outlines of the strain ranges given in figure 15(a), (b) and figure 17, one can see their similarity.

Let us note three main things that can be noted by analyzing figure 15, 16 (a), 17, 18:

a) the elastic soil core has a symmetrical triangular shape and is located directly under the stamp;

b) the volumes of the soil located above the upper boundaries of the plastic regions (shear), in which there are no shear deformations at all stages of loading, i.e. in a pre-limiting (undisturbed) state (in figure 17(d) and figure 18 they are designated by the number 1), are present in all figures;

c) the lower boundaries of the plastic regions shown in figure 17 (d) and figure 18 (a), can be approximated with a sufficient degree of accuracy by curves of the form (4), i.e. logarithmic spirals.
6. Conclusion

We have demonstrated not all examples, illustrating that the shapes and sizes of strain ranges and elastic soil cores obtained using physical and mathematical (based on analytical and numerical solutions of the problem of pressing a stamp into a homogeneous base) experiments, are very diverse.

Summarizing, it can be argued that:

1. An elastic soil core in the form of a triangle is formed only when the indentation of the stamp is carried out into the base folded with loose soil, provided that \( \theta > 25^\circ - 30^\circ \) (this depends on the value of the coefficient of lateral soil pressure), and the specific cohesion \( C \) is quite small. The same shape of the elastic soil core was obtained independently of each other by various authors in the analytical solution of the problem of foundation stability in a mixed (elastoplastic) formulation.

2. In cohesive soils or equivalent materials, whose physical and mechanical properties are close to those of cohesive soil, the vertical section of the elastic soil core has a form close to that of half an ellipse, half or a segment of a circle. The only exception to this rule was the results of experiments carried out by S.E. Kaganovskaya [8], where she managed to obtain an elastic core of a triangular shape for clay soil.

3. Numerous experimental data and the results of solving problems on the stability of foundations in the elastoplastic formulation indicate that in foundations composed of soils with low specific cohesion and high values of the angle of internal friction, plastic regions develop downward under the foundation. After the closure of the plastic areas, the latter develop to the sides from the axis of symmetry of the basement, forming zones of deep uplift (figure 15; 17; 18). Such a form of plastic regions cannot be obtained when using the Coulomb plasticity condition for their construction (1-3), no matter what physical and mechanical properties the soil is endowed with. The last statement is also true for bases composed of clayey soils with low values of the angle of internal friction.

4. The curves enclosing the lower boundaries of the plastic regions, which are discussed in Section 3, are practically exactly described by the equation of the logarithmic spiral.

Everything listed in paragraphs 1-4 to one degree or another is in accordance with the Prandtl design scheme, therefore, the use of design methods based on it can be justified for foundations composed of soils with high values of the angle of internal friction and low values of specific cohesion. The use of such methods for clayey soils with low values of the angle of internal friction should be treated with caution. Therefore, in our opinion, to solve the problem of calculating the bearing capacity of soil foundations composed of different types of soils, it is necessary to create methods that would not be based on hypotheses about one form or another of the elastic core and strain ranges. The creation of such methods is possible based on the solution of the corresponding elastoplastic (mixed) problems.

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