CP Violation and Dilaton Stabilization in Heterotic String Models

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Abstract

We study the possibility of spontaneous CP violation in string models with the dilaton field stabilized at a phenomenologically acceptable value. We consider three mechanisms to stabilize the dilaton: multiple gaugino condensates, a nonperturbative Kähler potential, and a superpotential based on S-duality, and analyze consequent CP phases in the soft SUSY breaking terms. Due to non-universality forced upon the theory by requiring a non-trivial CKM phase, the EDM problem becomes more severe. Even if there are no complex phases in the VEVs of the SUSY breaking fields, the electric dipole moments are overproduced by orders of magnitude. We also address the question of modular invariance of the physical CP phases.
1 Introduction

At present, string theory and its extension, M-theory, provide the most promising schemes for the unification of the fundamental forces of nature. But before we can assign it the status of a Theory of Everything, we need to establish that the phenomenology of the Standard Model (SM) can be produced by some reasonable scenario. One of the outstanding problems is the origin of CP violation. In string theory CP is a gauge symmetry and can only be broken spontaneously by a vacuum expectation value (VEV) of some SM-singlet field. There is a variety of good candidates to do this job. These include the dilaton, $S$, which is related to the gauge coupling via

$$\text{Re}S \simeq \frac{1}{g_{\text{string}}^2},$$  

and the moduli which parameterize the size and shape of the compact dimensions, $T_i$ and $U_i$ (i=1,2,3). All of these generically attain complex VEVs and induce spontaneous breakdown of the CP symmetry. One must, however, make sure that this produces the right amount of CP violation. For instance, $\text{Im}S$ of order one induces very large electric dipole moments (EDMs) through a contribution to the QCD $\bar{\theta}$ parameter. On the other hand, it does not induce the Cabibbo-Kobayashi-Maskawa (CKM) phase. Therefore the dilaton alone can hardly do a good job. The moduli fields can, at least in principle, induce the CKM phase without violating the EDM bounds. In this paper we will consider the moduli fields $T_i$ as the source of observable CP violation and study to what extent such a scenario is phenomenologically viable.

An important property of weakly coupled heterotic orbifold models is the existence of an $SL(2,\mathbb{Z})$ symmetry: with integer $a,b,c,d$ and $ad - bc = 1$. The dilaton field is inert under this transformation at the tree level, but at the one loop level the cancellation of the modular anomaly requires it to transform as (up to a T-independent imaginary shift)

$$S \rightarrow S + \frac{3}{4\pi^2} \delta_{GS} \ln(icT + d),$$  

where $\delta_{GS}$ is the Green-Schwarz coefficient. In the weakly coupled heterotic string theory this symmetry is preserved to all orders in perturbation theory and provides important guidance for constructing low energy effective field theories. One however should keep in mind that this symmetry is broken spontaneously below the compactification scale and

$^1$Throughout this paper we shall assume that all moduli $T_i$ have the same value, $T$. 


is realized nonlinearly at the electroweak scale. The modular symmetry has important implications for CP violation as it sometimes allows us to eliminate unphysical CP phases.

Complex VEVs of the moduli fields as a source of CP violation have been considered before, see Ref. [3] and references therein. These models however are not fully realistic as they do not address the issue of dilaton stabilization. The observed unification of the gauge coupling constants at $\alpha_{\text{GUT}} = g^2/4\pi = 1/25$ implies that $\text{Re}S$ should take a value of around two. So, a consistent model must satisfy this requirement, which also has important implications for supersymmetry breaking. In this paper we shall examine scenarios which satisfy the following three requirements:

1. The dilaton field is stabilized at $\text{Re}S \sim 2$.
2. CP is violated.
3. The supersymmetry breaking scale is phenomenologically acceptable.

We find that these requirements are quite stringent and leave very few viable possibilities. A somewhat similar question in the context of effective Type I models was addressed in Ref. [4]. We note that one can also impose an additional phenomenological constraint that the flavor changing neutral currents are absent. However, as we will see, this constraint is often satisfied automatically in the class of models under consideration.

The paper is organized as follows. In section 2 we review supersymmetry breaking via gaugino condensation and discuss properties of the scalar potential possessing a modular symmetry. In section 3 we present our analysis of dilaton stabilization via multiple gaugino condensates, non-perturbative Kähler potential, and S-duality. The CKM phase in heterotic models is discussed in section 4. Section 5 is devoted to the discussion of the soft terms and modular properties of the CP phases. In section 6 we analyze various types of EDM contributions encountered in our models. Finally, the conclusions are presented in section 7.

All of the models we examine start with a single hidden sector gaugino condensate and then modify it to produce dilaton stabilization, so now we will take a short detour and give an introduction to this simple case.
2 Gaugino Condensation

Hidden sector gaugino condensation is one of the most popular schemes for the breaking of supersymmetry (see [4] for a recent review). This is realized in $E_8 \otimes E_8$ heterotic string theory where the condensate lives in one $E_8$, the other forming the observable sector. The Veneziano-Yankielowicz superpotential which describes the condensate is given by [6]:

$$W = \frac{1}{4} U \left( f + \frac{2}{3} \beta \ln U \right), \quad (3)$$

where $\beta$ is the one-loop coefficient of the beta function, $U = \delta_{ab} W^a e^{\alpha \beta} W^b$ is a chiral superfield whose lowest component corresponds to the gaugino condensate $\langle \lambda \lambda \rangle$, and

$$f = S + \left( 4 \beta - \frac{3 \delta_{GS}}{2 \pi^2} \right) \ln \eta(T), \quad (4)$$

is the gauge kinetic function.

In this paper we shall use a truncated superpotential, found by replacing $U$ by its value at $\frac{\partial W}{\partial U} = 0$. This approximates its value at the minimum of the scalar potential [7] and gives:

$$W = d e^{-\frac{3S}{2\beta}} \frac{e^{\frac{-3S}{2\beta}}}{\eta(T)^6 e^{-\frac{3\delta_{GS}}{4\pi^2 \beta}}} \quad (5)$$

with $d = -\beta/6e$. The (standard) Kähler potential is given by [8]:

$$K = -\ln Y - 3 \ln (T + \bar{T}), \quad (6)$$

where $Y = S + \bar{S} + \frac{3}{4\pi^2} \delta_{GS} \ln (T + \bar{T})$. The scalar potential is expressed as

$$V = e^G \left( G_i \left( G_i^j \right)^{-1} G^j - 3 \right), \quad (7)$$

where $G = K + \ln(|W|^2)$ and the subscripts (superscripts) denote differentiation and the sum over repeated indices runs over the (conjugate) fields in the system. Supersymmetry is broken by VEVs of the auxiliary fields ($j = S, T$):

$$F_j = e^{G/2} \left( G^i \right)^{-1} G_i, \quad (8)$$

String models typically contain hidden matter which can couple to the condensate. For scalar matter fields, $A$, with a gauge group $SU(N)$ and “quarks” with $M (N + \bar{N})$ representations, the condensate superpotential is:

$$W(S, T, A) = -N(32\pi^2 e)^{\frac{8 \pi^2 e}{12N}} \left( \det \mathcal{M} \right)^{\frac{8 \pi^2 e}{12N}} \frac{e^{-8 \pi^2 e \frac{S}{N}}}{\eta(T)^{\frac{8 \pi^2 e S}{N} - \frac{12 \delta_{GS}}{N}}}, \quad (9)$$
where $\mathcal{M}$ is a matrix containing the coefficients of the quarks in their trilinear terms in the superpotential, $\sum_{r,a,b} h_{rab} A_r Q_a Q_b = \mathcal{M}_{ab} Q_a Q_b$. We assume a generic singlet field $A = A_r$, so $\det \mathcal{M} = A^M$.

Since for a realistic case $\langle A^2 \rangle \ll S_R, T_R$, the scalar potential is dominated by the term proportional to $|\partial W/\partial A|^2$. So, to a good approximation, the minimum occurs at $\frac{\partial W}{\partial A} = 0$, and we can neglect the terms containing $A$ in the Kähler potential. Then we have:

$$W = \tilde{d} \frac{e^{-\frac{3S}{2\bar{\beta}}} \eta(T)^{\frac{9}{4\pi^2}}}{\eta(T)^{6 - \frac{9\delta_{GS}}{4\pi^2\beta}}}$$

where $\tilde{\beta} = \frac{3N-M}{16\pi^2}$ is the beta function and $\tilde{d} = (M/3 - N)(32\pi^2 e)^{3(M-N)}(M/3)^{3N-M}$.

This model does not lead to dilaton stabilization at a reasonable value, in fact at the minimum $\text{Re}S \to \infty$. So we must consider modifications. We shall study three models, one where corrections are made to the Kähler potential, one with a S-dual potential and another with two gaugino condensates.

Before we proceed, let us mention a few useful facts. First, if the scalar potential possesses an $SL(2, Z)$ symmetry, the fixed points under the duality group are always stationary. In practice, these fixed points are often minima. As we will see, if the modulus field is stabilized at a fixed point, the CKM phase vanishes and often there is no supersymmetry breaking. So our task will be to pull the minima away from the fixed points.

Second, for $\delta_{GS} = 0$ and the standard Kähler potential, $F_T = 0$ at the T-duality fixed points while $F_S = 0$ at the S-duality fixed points. This follows from the fact that $F_T \propto G_T$ and

$$G_T \propto \frac{1}{T + \bar{T}} + 2 \frac{\eta'(T)}{\eta(T)}\bigg|_{\text{f.p.}} = 0,$$

keeping in mind that a derivative of a modular invariant function vanishes at the fixed points. This, of course, equally applies to the S-dual potentials.

Third, let us mention a useful property of models with factorizable effective superpotentials, i.e. $W_{\text{eff}} = \Omega(S)/\Lambda(T)$. In such models, for $\delta_{GS} = 0$ an extremum (which is often a minimum) of the scalar potential occurs at

$$2S_RW_S - W = 0,$$

assuming the standard Kähler potential. Consequently, $F_S = 0$ at this point since it

\footnote{Another dilaton stabilization mechanism, in the context of type I models, was suggested in \cite{10}.}

\footnote{This also applies to $T$ and $W_T$.}
is proportional to precisely this combination. In the general case of \( \delta_{GS} \neq 0 \), there can be departures from this result.

Finally, in what follows we will consider generalized superpotentials consistent with the modular symmetry. That is, we will use the freedom to multiply the superpotential arising from gaugino condensation by a modular invariant function \( H(T) \):

\[
H(T) = [j(T) - 1728]^m j(T)^n P[j(T)] ,
\]

where \( j(T) \) is the absolutely modular invariant function, \( P[j(T)] \) is some polynomial of \( j(T) \), and \( m, n \) are integers. This is the most general modification consistent with T-duality and absence of singularities inside the fundamental domain. However, one should keep in mind that explicit examples where \( H(T) \) appears are lacking, so it is possible that we allow ourselves more freedom than there is in practice.

We will typically set \( P[j(T)] \) to one, noting that increasing the amount of \( j(T) \) in the potential typically forces the minima of \( T \) to the fixed points. In practice, the modulus field often gets stabilized at \( T_{\text{min}} = 1 \). Then, for \( m > 0 \), there is no supersymmetry breaking because \( H(1)|_{m>0} = 0 \), and such models must be discarded. The same applies to the other fixed points for \( n > 0 \) since \( H(e^{\pm i\pi/6})|_{n>0} = 0 \).

3 Dilaton Stabilization

As we have seen above, the simplest model of gaugino condensation does not lead to a finite value of the dilaton field. Since the VEV of the dilaton describes the gauge coupling constants, such a model is phenomenologically unacceptable. The simple model above might well be oversimplified and in more involved models dilaton stabilization can be achieved while retaining the main features of the single gaugino condensate model.

3.1 Models With Two Gaugino Condensates (“Racetrack Models”)

Generally, the hidden sector may contain non-semi-simple gauge groups. Given the right matter content, it is plausible that gauginos condense in each of the simple group factors. With a nonzero \( \text{Im}S \), these condensates may enter the superpotential with opposite signs thereby leading to dilaton stabilization.
Figure 1: Racetrack scalar potential with $H$ and $m=1$, $n=0$. $T$ is set to its minimum value, $T_{\text{min}} = 0.9850e^{0.5471i}$. The minimum in $S$ is at $S_{\text{min}} = 2.13 - 0.92i$.

Figure 2: Racetrack scalar potential with $H$ and $m=1$, $n=0$. $S$ is set to its minimum value, 2.13 – 0.92i. The minimum in $T$ is at $T_{\text{min}} = 0.9850e^{0.5471i}$.

Let us consider a model with two gaugino condensates. Suppose we have a gauge group $SU(N_1) \otimes SU(N_2)$ with $M_1(N_1 + \overline{N}_1)$ and $M_2(N_2 + \overline{N}_2)$ “quark” representations. The superpotential is simply the sum of that, (10), for each of the individual condensates [9]:

$$W = \tilde{d}_1 e^{-\frac{3S}{2\beta_1}} e^{-\frac{\eta_{GS}}{4\pi^2\beta_1}} + \tilde{d}_2 e^{-\frac{3S}{2\beta_2}} e^{-\frac{\eta_{GS}}{4\pi^2\beta_2}}.$$  (14)

For $N_1 = 6$, $N_2 = 7$ and $M_1 = 2$, $M_2 = 8$ we have dilaton stabilization at $S = 2.1 - 0.92i$ and $T = 1.23$ [12]. In this case the CKM phase is zero and we should consider modifications. Keeping the same condensing gauge groups and the matter content, we can multiply the superpotential by $H(T)$. The resulting minima are shown in Table 2.

The table shows $S$ and $T$ at the minima along with their auxiliary fields for various powers $m$ and $n$. SUSY breaking is given in Planck units and 0 indicates SUSY breaking.
much below the phenomenologically allowed range. For \( \delta_{GS} = 0 \), we see that \( S \) is always stabilized at a reasonable value and \( T \) is complex for \( m \geq 1 \). This is illustrated in Figs. 1 and 2. We can see that for \( m = 1, n = 0 \) the fixed point \( e^{i\pi/6} \) is a local maximum.

In all cases \( F_S = 0 \) due to Eq. 12, whereas \( F_T \) may be nonzero. In most cases the modulus is stabilized on the unit circle and SUSY remains unbroken. The presence of extrema on the unit circle can be seen from the fact that that there is always a stationary point at \( G_S = G_T = 0 \) and there is a point on the unit circle where \( G_T \) vanishes (since it is a derivative of a modular invariant function), whereas \( G_S \) is always zero. The minima on the unit circle (if they exist) away from the fixed points are typically lower than those at the fixed points because in the former case SUSY breaking is zero and the potential is negative while in the latter case the entire potential vanishes.

For \( m = n = 0 \) and \( m = 1, 2, n = 0 \) we have viable supersymmetry breaking \( (10^2 \text{ GeV} \leq F_T \leq 10^4 \text{ GeV}) \), but only in the latter case is there CP violation. However, even in this case \( T \) is close to the fixed point \( e^{i\pi/6} \) which results in a suppressed \( \sim \mathcal{O}(0.1) \)) CKM phase if \( \langle T \rangle \) is the only source of CP violation. We note that the dilaton also has a complex VEV in order to produce a relative sign between the condensates and \( \text{Im} S \) is fixed up to a discrete shift \[9\].

In all interesting cases, \( F_T \) receives a complex phase of order one. This occurs due to a rapid variation of \( G_T \) (Eq. 11) around the fixed points. However, \( F_T \) is not modular invariant by itself so this does not necessarily mean that the physical SUSY CP phases are also \( \mathcal{O}(1) \). We will discuss this subject separately in one of the subsequent sections.

Introduction of the Green-Schwarz term does not significantly change the situation, as should probably be expected. The only relevant change is that now \( F_S \) differs from zero due to the dilaton-modulus mixing (Table 3). In all cases \( F_S \) is of order \( F_T \) and also has an order one complex phase.

### 3.2 Models With Non-Perturbative Corrections to the Kähler Potential

One generally expects the Kähler potential to receive corrections from non-perturbative effects. Such effects may be responsible for dilaton stabilization. For instance, in Ref.\[13\] the following Kähler potential was suggested:

\[
K_S = \ln \left( \frac{1}{2\text{Re}S} + d(\text{Re}S)^{\frac{p}{2}} e^{-b\sqrt{\text{Re}S}} \right),
\]

(15)
where $d, p, b$ are certain constants ($p, b > 0$).

Figure 3: Scalar potential with non-perturbative Kähler potential, $m = n = 0$. $T$ is set to its minimum value, $e^{\pm i\pi/6}$. The minimum in $S$ is at 1.8.

Figure 4: Scalar potential with non-perturbative Kähler potential, $m = n = 0$. $S$ is set to its minimum value, 1.8. The minima in $T$ are at $e^{\pm i\pi/6}$. Note the invariance of the potential under the axionic shift $T \rightarrow T + i$.

The superpotential remains given by Eq.5. The second term under the log in Eq.15 represents non-perturbative corrections. Its form based on the natural assumptions that non-perturbative effects vanish in the limit of vanishing coupling constant ($\text{Re}S \rightarrow \infty$) and are zero at all orders of perturbation theory. Note that the scalar potential is a function of $\text{Re}S$ only, so $\text{Im}S$ remains undetermined.

In Table 4 we present the results of the potential minimization for $m$ and $n$ between 0 and 5 with $d = 7.8$ and $b = p = 1$. The presence of $H(T)$ does not visibly affect the value of $S$ at the minimum, but typically forces $T_{\text{min}}$ into the fixed points. At these points no CP violation is produced and SUSY remains unbroken unless $m = n = 0$ (recall that
Clearly, these minima are phenomenologically unacceptable. Having varied \( p, d, b \), we were unable to find viable CP-violating minima with a reasonable SUSY breaking scale. The same remains true for a nonzero \( \delta_{GS} \) (Table 5).

This however does not mean that this stabilization mechanism is altogether unattractive. It has a nice feature that dilaton-dominated SUSY breaking can be obtained and most of the soft CP phases can be eliminated. CP violation in this case may originate from fields other than the dilaton and moduli, for instance, Froggatt-Nielsen type fields. To avoid the SUSY CP problem, one needs to ensure that such fields do not break supersymmetry.

### 3.3 Models With S-Dual Potentials

![Figure 5: S-dual scalar potential with \( H \) and \( m=1, n=0 \). \( T \) is set to its minimum value, 0.9850e\(^{0.5471i} \). The minimum in \( S \) is at \( S_{\text{min}} = 1 \).](image1)

![Figure 6: S-dual scalar potential with \( H \) and \( m=1, n=0 \). \( S \) is set to its minimum value, 1. The minimum in \( T \) is at \( T_{\text{min}} = 0.9850e^{0.5471i} \).](image2)

It is feasible that the underlying theory can possess \( SL(2,Z) \) S-invariance in addition
to the well known T–modular invariance. S-self-dual models naturally exhibit dilaton stabilization as large and small values of S are related by S-duality \[14\]. In general, the gauge coupling is given by the gauge kinetic function \( f \):

\[
\text{Re} f = \frac{1}{g^2},
\]

so one can have \( g \to g \) under \( S \to 1/S \) if \( f \to f \), as opposed to the standard strong–weak duality. Since in the weak coupling limit \( (\text{Re} S \to \infty) \) \( f(S) \to S \), the simplest S-invariant kinetic function is \[14\]

\[
f_s = \frac{1}{2\pi} \ln(j(S) - 744).
\]

Note that this duality relates theories with the same coupling, but differing values of \( S \).

The standard Kähler potential

\[
K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T} - XX),
\]

where \( X^3 = W^\alpha W_\alpha \), then implies that the superpotential must have weight -1 with respect to the S-duality transformation:

\[
W = \frac{X^3}{\eta^2(S)} \left[ \frac{1}{2\pi} \ln(j(S) - 744) + 3b \ln(X\eta^2(T)/\mu) + c \right],
\]

where \( b = 2\beta/3 \) and \( \mu, c \) are constants. Integrating out the condensate using the truncated approximation, we get, after absorbing the constant \( c \) into \( \mu \),

\[
W = -\frac{2\beta\mu^3}{3e\eta(S)\eta(T)^6(j(S) - 744)^{3/\beta}}.
\]

In our numerical analysis, we consider a model with a \( SU(6) \) gauge group. The corresponding minima for the case without matter are shown in Table 6. The situation is very similar to the case of the racetrack models apart from the value of \( S_{\text{min}} \). \( F_S \) always vanishes since \( S \) is stabilized at the fixed point. For \( m \geq 1, n > 0 \), the minima in \( T \) are located on the unit circle where \( F_T \) vanishes. The only reasonable minimum appears for \( m = 1, n = 0 \), but again it is close to the fixed point where the CKM phase vanishes. Figs.5 and 6 illustrate the behaviour of the potential, which is very similar to what we have seen in the racetrack models.

We do not consider \( \delta_{GS} \neq 0 \) case since it is not clear whether one can maintain both T- and S- modular invariance at the one loop level.
4 The CKM Phase

In this section we will briefly discuss how the CKM phase can be produced in heterotic orbifold models. One of the crucial requirements any model should satisfy is that it reproduces the standard CKM picture of CP violation and the consequent CKM phase is of order one.

Complex Yukawa couplings in heterotic string models can be generated if the matter multiplets belong to the twisted sectors and the moduli fields attain complex VEVs. In general, this does not necessarily mean that a nonzero CKM phase is produced. The complex phases in the Yukawa matrices can often be removed by a basis transformation consistent with the $SU(2) \times U(1)$ symmetry. The proper measure of CP violation in the Standard Model is given by the Jarlskog invariant [15]:

$$J = \text{Im} \left( \det \left[ Y_u Y_u^\dagger, Y_d Y_d^\dagger \right] \right),$$

where $Y_u,d$ are the Yukawa matrices. A non-zero $J$ indicates the presence of the CKM phase.

The (renormalizable) Yukawa couplings can be calculated exactly in a given heterotic orbifold model. Often the Yukawa couplings have a very restricted flavour structure such that the complex phases are spurious and the Jarlskog invariant vanishes. That is the case for the prime order orbifolds, whereas for the non-prime orbifolds the CKM phase can be non-trivial [16]. Here we will give an example of the $Z_6$-I orbifold assuming that we have the freedom to assign a field to a fixed point of our choice [17, 18]. Note however that we do not attempt to reproduce the observed fermion masses and mixings, so this picture is not fully realistic. Nevertheless, it gives a fair idea of CP violation in the system.

Due to string selection rules, only fields belonging to particular fixed points can couple via the Yukawa interaction. This restricts the flavour structure of the Yukawa matrices. In the $Z_6$-I case, one of the allowed couplings is $\theta \theta^2 \theta^3$, where $\theta^i$ denote the twisted sectors. The corresponding $f_1 f_2 f_3$ Yukawa couplings are expressed as [18]

$$Y_{\theta \theta^2 \theta^3} = N \sqrt{l_2 l_3} \sum_{\vec{u} \in Z^4} \exp \left[ -4\pi T \left( \vec{f}_{23} + \vec{u} \right)^T M \left( \vec{f}_{23} + \vec{u} \right) \right],$$

where $f_{1,2,3}$ are the fixed points, $\vec{f}_{23}$ represents a projection of $f_2 - f_3$ onto the first two complex planes (corresponding to $T_1$ and $T_2$), $\vec{u}$ is a four-dimensional vector with integer components, $N$ is a normalization factor, $l_i$ are the “multiplicity” constants associated with
Table 1: \( Z_6 \)-I fixed point assignment for the observable fields.

| field | fixed point | \( l \) |
|-------|------------|-------|
| \( H_{1,2} \) | \((0,0) \otimes (0,0)\) | 1 |
| \( Q_1 \) | \((0,0) \otimes (0,0)\) | 1 |
| \( Q_2 \) | \((0,\frac{1}{3}) \otimes (0,\frac{1}{3})\) | 2 |
| \( Q_3 \) | \((0,\frac{1}{3}) \otimes (0,0)\) | 2 |
| \( U_1 \) | \((0,0) \otimes (0,0)\) | 1 |
| \( U_2 \) | \((0,\frac{1}{3}) \otimes (0,\frac{1}{3})\) | 3 |
| \( U_3 \) | \((\frac{1}{2},\frac{1}{3}) \otimes (0,\frac{1}{2})\) | 3 |
| \( D_1 \) | \((0,\frac{1}{2}) \otimes (0,0)\) | 3 |
| \( D_2 \) | \((0,0) \otimes (0,\frac{1}{2})\) | 3 |
| \( D_3 \) | \((0,0) \otimes (0,0)\) | 1 |

The fixed points, and the matrix \( M \) is given by

\[
M = \begin{pmatrix}
1 & -\frac{3}{2} & 0 & 0 \\
-\frac{3}{2} & 3 & 0 & 0 \\
0 & 0 & 1 & -\frac{3}{2} \\
0 & 0 & -\frac{3}{2} & 3
\end{pmatrix}.
\]

Clearly, this Yukawa coupling is complex for complex \( T \). Next, we need to assign the observable fields to the fixed points. One possible assignment producing a non-zero Jarlskog invariant is given in Table 1. Here the Higgs fields are assumed to belong to the \( \theta \) sector, quark doublets – to the \( \theta^2 \) sector, and the quark singlets – to the \( \theta^3 \) sector. We note that above the electroweak symmetry breaking scale the quark fields may also appear as linear combinations of the ones in Table 1, this however does not affect the Jarlskog invariant. Finally, the Yukawa couplings must be rescaled \( Y_{abc} \rightarrow Y_{abc} \check{W}^*/|\check{W}|e^{\check{K}/2}(K_aK_bK_c)^{-1/2} \) in order to have the canonical normalization and to be weight zero quantities under the modular transformation.

The corresponding Jarlskog invariant as a function of the modulus field \( T \) is presented in Fig. 4. We restrict \( T \) to be on the unit circle which is often the case of interest. The overall normalization of \( J \) is irrelevant for our purposes since we are not producing the observed quark mass hierarchy and mixings, however the figure provides important qualitative features. In particular, \( J \) vanishes at the fixed points of the modular group 1, \( \exp(\pm i\pi/6) \) due to the axionic shift invariance [16]. Away from the fixed points it is non-zero. If the Standard Model sector exhibited the T-duality invariance, the CKM phase would have to vanish everywhere on the unit circle [20]. However, typically the SM sector interactions are
not modular invariant\footnote{The duality symmetry is broken spontaneously at high energies and can only be non-linearly realized at the electroweak scale.}. This can be seen directly from the action of the duality transform on the fields at the fixed points \cite{16}. The fields necessary to restore full modular invariance are associated with heavy matter fields and decouple at low energies. This situation is analogous to what we encounter in GUT models, say $E_6$. The low energy spectrum does not form a representation of $E_6$ and to restore the symmetry one has to add extra heavy fields.

To conclude this section, we have argued that generally it is possible to generate a CKM phase at the renormalizable level through a complex VEV of the modulus field (away from the fixed points). For instance, order one CKM phase can be produced with $\text{Arg} T \sim O(\pi/12)$ for $T$ on the unit circle. Naturally, we expect that the CKM phase can be induced in a larger class of models if nonrenormalizable operators are taken into account\footnote{See, for instance, \cite{21} for a related discussion.}.

## 5 Soft SUSY Breaking Terms

In this section we will consider soft SUSY breaking terms for the minima obtained in the previous sections. The purpose of our analysis is to establish how much CP violation in the soft terms should generally be expected if both dilaton and moduli are stabilized by the underlying dynamics.

Before we list the formulae for the soft breaking terms, let us make explicit our notation.
The soft SUSY breaking Lagrangian is given by
\[ L_{\text{soft}} = \frac{1}{2} (M_a \dot{\lambda}^a \chi^a + \text{h.c.}) - m_\alpha^2 \dot{\phi}_\alpha \dot{\phi}_\alpha - \left( \frac{1}{6} A_{\alpha \beta \gamma} \dot{Y}_{\alpha \beta \gamma} \dot{\phi}_\alpha \dot{\phi}_\beta \dot{\phi}_\gamma + B \dot{\mu} \dot{H}_1 \dot{H}_2 + \text{h.c.} \right), \] (23)
where \( \dot{Y}_{\alpha \beta \gamma} \) and \( \dot{\mu} \) are the Yukawa couplings and the \( \mu \)-term for the canonically normalized fields \( \dot{\phi} \). With the Kähler potential and the superpotential of the form
\[ K = \hat{K} + \tilde{K}_\alpha \phi^\alpha \dot{\phi}_\alpha + (Z H_1 H_2 + \text{h.c.}) \]
\[ W = \hat{W} + \frac{1}{6} Y_{\alpha \beta \gamma} \dot{\phi}_\alpha \dot{\phi}_\beta \dot{\phi}_\gamma, \] (24)
\( \hat{Y}_{\alpha \beta \gamma} \) and \( \hat{\mu} \) are given by [19]
\[ \hat{Y}_{\alpha \beta \gamma} = Y_{\alpha \beta \gamma} \frac{\hat{W}^*}{|\hat{W}|} e^{\hat{K}/2} \left( \tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma \right)^{-1/2}, \]
\[ \hat{\mu} = \left( m_{3/2} Z - F^m \partial_m Z \right) \left( \tilde{K}_{H_1} \tilde{K}_{H_2} \right)^{-1/2}. \] (25)
Here \( m = (S, T) \) and for definiteness we have assumed the Giudice-Masiero mechanism for generating the \( \mu \)-term [22]. The canonically normalized fields are obtained by the rescaling \( \dot{\phi}_\alpha = \tilde{K}_\alpha^{1/2} \phi_\alpha \). The gaugino masses, scalar masses, A-terms, and the B-term are expressed, respectively, as [19]:
\[ M_a = \frac{1}{2} (\text{Re} f_a)^{-1} F^m \partial_m f_a, \] (26)
\[ m_\alpha^2 = m_{3/2}^2 + V_0 - F^m \partial_m \partial_n \ln \tilde{K}_\alpha, \]
\[ A_{\alpha \beta \gamma} = F^m \left[ \tilde{K}_m + \partial_m \ln Y_{\alpha \beta \gamma} - \partial_m \ln (\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma) \right], \]
\[ B = \mu^{-1} \left( \tilde{K}_{H_1} \tilde{K}_{H_2} \right)^{-1/2} \left[ (2m_{3/2}^2 + V_0) Z - m_{3/2} \bar{F}^m \partial_m Z + m_{3/2}^2 F^m \left( \partial_m Z - Z \partial_m \ln (\tilde{K}_{H_1} \tilde{K}_{H_2}) \right) - F^m \bar{F}^n \left( \partial_m \partial_n Z - \partial_m Z \partial_n \ln (\tilde{K}_{H_1} \tilde{K}_{H_2}) \right) \right]. \]

Note that the gaugino masses computed with the kinetic function (4) do not appear to be modular invariant. An additional contribution from the massless fields of the theory is necessary to rectify this problem. Effectively this amounts to an addition of the non-holomorphic term \( 2\beta \ln (T + \bar{T}) \) to the kinetic function [24].

Before we proceed let us clarify our framework. In what follows, we will not restrict ourselves to a particular orbifold model. Instead, we will try to present some general features of models possessing modular invariance. At the same time, we will clarify some of our statements with explicit examples.
The Kähler function $\tilde{K}_\alpha$ is expressed as

$$\tilde{K}_\alpha = (T + \bar{T})^{n_\alpha}, \quad (27)$$

where $n_\alpha$ is a modular weight. Here we have assumed a diagonal Kähler metric which is almost always the case in phenomenologically acceptable models. The reason is that the space group quantum numbers typically prohibit an off-diagonal metric \[23\]. Moreover, our main results are unaffected even if we allow for a mixing among the fields belonging to the same twisted sector.

For non-oscillator states, we have \[24\]

$$n^{\text{untw.}}_\alpha = -1,$$
$$n^{\text{tw.}}_\alpha = -2 \quad (\text{three planes rotated}),$$
$$n^{\text{tw.}}_\alpha = -1 \quad (\text{two planes rotated}). \quad (28)$$

where we distinguished between the possibilities with the twists in all planes being nonzero and the twists in two planes being nonzero. The oscillator states usually appear as singlets and are not associated with the MSSM fields \[23\], so we will restrict our discussion to the non-oscillator states only.

It is interesting to note that in order to get nontrivial Yukawa textures and the CKM phase, the MSSM fields associated with different generations should belong to the same twisted sector. For instance, in our $Z_6$-I example all quark doublets belong to the $\theta^2$ sector, whereas all quark singlets are in the $\theta^3$ sector. If, say $Q_1$, belonged to the $\theta$ or $\theta^3$ sector, its coupling with $H_2 U_i$ or $H_1 D_i$ would be prohibited since only the coupling of the form $\theta \theta^2 \theta^3$ is allowed (if the Higgses are fixed to be in the $\theta$ sector). This would result in the Yukawa textures containing many zeros and the Jarlskog invariant would be likely to vanish.

This observation has implications for the Flavour Changing Neutral Currents (FCNC) since it implies that the modular weights and thus $m_\alpha^2$ are generation – independent. Excessive FCNC at low energies result from non-degeneracy of the squark masses and generally pose a problem for supersymmetric model building \[24\]. Here it is naturally avoided if we are to produce the CKM phase\[6\].

Similarly, in the A-terms the only generation-dependent piece comes from the Yukawas $(\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma$ is fixed). Nevertheless, this dependence can be strong because the term $\partial_m \ln Y_{\alpha\beta\gamma}$

\[6\]A flavour dependence in $A_{\alpha\beta\gamma}$ can also contribute to FCNC, however it generates left-right squark mass insertions proportional to the quark masses which are only loosely constrained \[24\].
can be significant and even dominant. In particular, it is easy to see from Eq.21 that for \( \text{Re}T \sim 1 \), the Yukawa coupling is dominated by one term. Consequently,

\[
\partial_T \ln Y_{\alpha\beta\gamma} \simeq -4\pi \frac{h_{23}^\dagger}{M} h_{23}
\]

(29)

for some (typically fractional) \( h_{23} \) depending on \( \alpha, \beta, \gamma \). Independently of \( T \), \( \partial_T \ln Y_{\alpha\beta\gamma} \) is an almost real number typically between -1 and -10 (for \( h_{23} = 0 \) it is zero). This creates a significant flavour dependence in the A-terms and thus a flavour universality of the minimal SUGRA model cannot be achieved. Note that if the correct fermion mass hierarchy is reproduced, this effect may become even stronger. These conclusions equally apply to other orbifolds.

If the MSSM fields are in the untwisted sector, the soft terms (apart from the gaugino masses) are universal. This possibility however is unattractive since the Yukawa couplings are either zero or one. In this case there is no fermion mass hierarchy and the CKM phase has to vanish. Even if non-renormalizable operators are taken into account this option is hardly phenomenologically viable [17].

The CP-violating phases appearing in the \( B- \) and \( \mu- \) terms critically depend on the \( \mu- \) term generation mechanism. The “bare” \( \mu \) parameter appearing in the superpotential would have to be of order Planck scale which is phenomenologically unacceptable. Thus a different mechanism is required. One of the attractive ways to produce the \( \mu- \) term of order \( m_{3/2} \) is the Giudice-Masiero mechanism [22]. This mechanism employs the Kähler symmetry of the theory so that a Kähler transformation induces an effective \( \mu- \) term in the Lagrangian even though it was not present initially. This requires the presence of the \( ZH_1H_2 \) term in the Kähler potential, which can be implemented in string models [26]. Such a term arises in even order orbifolds possessing at least one complex structure modulus \( U \). Specifically, in these models \( Z \) has the form

\[
Z = \frac{1}{(T_3 + T_3^\dagger)(U_3 + U_3^\dagger)}
\]

(30)

where \( T_3 \) is associated with the \( Z_2 \) plane of the orbifold and the Higgses are assumed to be untwisted. One can check that the Kähler potential has proper transformation properties up to \( O\left((H_1H_2)^2\right) \) under T-duality if the U-modulus transforms as \( U \to U - H_1H_2ic/(icT + d) \) [26].
5.1 SUSY CP Phases and Modular Invariance

In this subsection we will address the question of modular invariance of the physical CP phases. These phases are invariant under the $U(1)_R$ and $U(1)_{PQ}$ and are given by \[29\]:

$$\text{Arg}\left((B\hat{\mu})^* \hat{\mu} M\right), \text{Arg}\left(A^* M\right),$$

where the A-terms and the gaugino masses are assumed to have universal phases. For clarity of our presentation we will assume $\delta_{GS} = 0$. Also, as we have seen above, in all relevant cases $F_S = 0$, so henceforth we will set $F_S$ to zero. In what follows, we will consider in detail only the duality transformation $T \to 1/T$ since the discussion of the axionic shift invariance is quite trivial.

Let us first define strictly weight 2 modular functions (related to the Eisenstein function)

$$G_2(T) = \frac{1}{T + \bar{T}} + \frac{2\eta'(T)}{\eta(T)},$$

$$\tilde{G}_2(T) = \frac{1}{T + \bar{T}} + \frac{2\eta'(T)}{\eta(T)} - \frac{1}{3} \frac{H'(T)}{H(T)}.$$  \hspace{1cm} (32)

They indeed transform with a modular weight +2 since they are given by a logarithmic derivative of $(T + \bar{T})\eta^2(T)$ and $(T + \bar{T})\eta^2(T)H(T)^{-1/3}$. The auxiliary field $F^T$ is then given by

$$F^T = (T + \bar{T})^2 \tilde{G}_2(T)^* \times \text{modular invariant piece}.$$  \hspace{1cm} (33)

Since under duality $T + \bar{T} \to (T + \bar{T})/TT$ and $\tilde{G}_2 \to -T^2 \tilde{G}_2$, $F^T$ transforms as

$$F^T \to -\frac{1}{T^2} F^T.$$  \hspace{1cm} (34)

Let us now consider how $\hat{\mu}$ and $B\hat{\mu}$ transform under duality. For $U + \bar{U} = 1$ and $F^U = 0$, we have

$$\hat{\mu} = m_{3/2} + \frac{F^T}{T + \bar{T}},$$

$$B\hat{\mu} = 2m_{3/2}^2 + V_0 + m_{3/2} \frac{F^T + \bar{F}^T}{T + \bar{T}}.$$  \hspace{1cm} (35)

These expressions are apparently modular non-invariant. However, one must keep in mind that the U-modulus is not inert under duality. It provides the necessary terms to restore modular invariance. Indeed, the relevant terms in the Kähler potential are

$$\Delta K = -\ln(U + \bar{U}) + \frac{H_1 H_2 + \text{h.c.}}{(T + \bar{T})(U + \bar{U})}.$$  \hspace{1cm} (36)
Under duality $U \to U - H_1 H_2 / T$ \cite{20} and $\Delta K$ remains invariant up to $\mathcal{O}(H_1^2 H_2^2)$ terms which are consistently neglected in supergravity. In terms of the function $Z(T, U)$, this translates into the following “anomalous” transformation property

$$Z \to Z T \bar{T} - T,$$  \hspace{1cm} (37)

where we have set $U + \bar{U} = 1$. Using this fact and recalling that the Higgses are untwisted, one can show that under duality

$$\hat{\mu} \to -\frac{T}{T} \hat{\mu},$$

$$B \hat{\mu} \to -\frac{T}{T} B \hat{\mu}.$$  \hspace{1cm} (38)

One may be wondering whether such transformations with a T-dependent phase are consistent with modular invariance of the theory. To answer this question one should recall that the canonically normalized untwisted fields transform under duality as

$$\hat{\phi} \equiv \phi(T + \bar{T})^{-1/2} \to -i \left(\frac{T}{T}\right)^{1/2} \hat{\phi}.$$  \hspace{1cm} (39)

As a result, the interaction terms $\hat{\mu} \hat{H}_1 \hat{H}_2$ and $B \hat{\mu} \hat{H}_1 \hat{H}_2$ are exactly modular invariant. The same result can be established for the $\mu$-term generated non-perturbatively. We therefore see that $\text{Arg}((B \hat{\mu})^* \hat{\mu})$ is modular invariant.

The gaugino masses calculated with the kinetic function augmented by $2\beta \ln(T + \bar{T})$ \cite{24} are modular invariant. Indeed,

$$M_a \propto F^T G_2(T) \to M_a$$

since $F^T$ transforms with modular weight -2 (Eq.34). Thus the phase of $M_a$ is modular invariant. Note that the gaugino masses have a universal phase due to $F^S = 0$, whereas their magnitudes are proportional to the beta functions and thus are different.

The discussion of the A-terms is more involved. The complication comes from the term $\partial_T \ln Y_{\alpha\beta\gamma}$; as we know the Yukawas couplings have highly non-trivial transformation properties under duality. In fact, if we associate the MSSM fields with a subset of the fixed points of the orbifold, generally this subset does not transform into itself under duality \cite{16}. Consequently, the Standard Model interactions are generally non-invariant under duality, although the full set associated with all of the fixed points is invariant. So leaving aside
these flavour issues, the best we can do is to study the overall phase of the A-terms. As a matter of fact, this is a very good approximation for \( \text{Re}T \simeq 1 \). The reason is that the Yukawa couplings are dominated by one term, so \( \partial_T \ln Y_{\alpha\beta\gamma} \) is real to a very good degree (Eq. 29). The other relevant terms \( \hat{K}_m \) and \( \partial_m \ln(\bar{K}_\alpha \bar{K}_\beta \bar{K}_\gamma) \) are also real, so \( A_{\alpha\beta\gamma} \) has a universal phase up to small corrections, although its magnitude can be highly non-universal. This property remains valid under a duality transformation since again the sum is dominated by one term (if \( \text{Re}T \simeq 1 \)); this can also be seen from the expressions for the duality-transformed Yukawas [16]. We have checked numerically that the deviations from universality are within a few percent (apart from the suppressed elements \( A_{\alpha\beta\gamma} \simeq 0 \)) for the \( Z_6 \)-I model with \( T \) close to the unit circle. So for practical purposes we can treat \( \text{Arg} (A) \) as a universal phase.

Denoting by \( n_\alpha \) a modular weight of the relevant field, the A-terms can be cast in the following form

\[
A_{\alpha\beta\gamma} = F^T \partial_T \ln \left[ Y_{\alpha\beta\gamma}(T + \bar{T})^{-3-n_\alpha-n_\beta-n_\gamma} \right].
\] (41)

The modular weight of the Yukawa coupling is fixed by requiring the superpotential to transform with modular weight -3. So apart from the unitary transformation mixing the fields in each twisted sector, we have

\[
Y_{\alpha\beta\gamma} \rightarrow (iT)^{-3-n_\alpha-n_\beta-n_\gamma} Y_{\alpha\beta\gamma}.
\] (42)

It is easy to see that the A-terms stay invariant under duality,

\[
A_{\alpha\beta\gamma} \rightarrow A_{\alpha\beta\gamma}.
\] (43)

The corresponding interaction term \( A_{\alpha\beta\gamma} \hat{Y}_{\alpha\beta\gamma} \hat{\phi}_\alpha \hat{\phi}_\beta \hat{\phi}_\gamma \) also stays invariant if we recall

\[
\hat{\phi}_\alpha \rightarrow i^{n_\alpha} \left( \frac{T}{\bar{T}} \right)^{n_\alpha/2} \hat{\phi}_\alpha,
\]

\[
(\bar{K}_\alpha \bar{K}_\beta \bar{K}_\gamma)^{-1/2} \rightarrow (TT)^{(n_\alpha+n_\beta+n_\gamma)/2} (\bar{K}_\alpha \bar{K}_\beta \bar{K}_\gamma)^{-1/2},
\]

\[
e^{\bar{K}/2} \rightarrow (TT)^{3/2} e^{\bar{K}/2},
\]

\[
\frac{\hat{W}^*}{|\hat{W}|} \rightarrow (-i)^{-3} \left( \frac{T}{\bar{T}} \right)^{3/2} \frac{\hat{W}^*}{|\hat{W}|}.
\] (44)

Here we have used the fact that \( \hat{W} \) transforms with weight -3. In practice this is true only up to a \( T \)-independent phase which can always be absorbed into redefinition of the fields. One should keep in mind that here we have ignored the unitary transformation in the twisted
sector which accompanies the duality transformation, but this is justified as long as we are concerned with the overall phase of the A-terms.

The discussion simplifies if matter is untwisted. In this case the A-terms vanish and the requirement of modular invariance is trivially satisfied.

To summarize the results of this section, we find that even though the individual CP phases may not be modular invariant, the physical CP phases of Eq.31 are (at least under our assumptions).

Before we proceed to the numerical analysis, let us make an important comment. We would like to stress that there can be no CP violation induced by a VEV of the modulus field if it is stabilized at the fixed point, at least for \( \delta_{GS} = 0 \). The Jarlskog invariant vanishes at the fixed points regardless of the presence of the Green-Schwarz term, so there is no CKM phase. For \( \delta_{GS} = 0 \), \( F_T \) vanishes at the fixed points, so no soft phases are induced. Further, flavour-dependent complex phases in \( A_{\alpha\beta\gamma} Y_{\alpha\beta\gamma} \) arising from the phases in the Yukawa matrix can be removed by a phase redefinition of the quark superfields. In principle, CP violation can be induced by complex \( S \) and \( F_S \) but the dilaton does not distinguish flavours, so even in this case the CKM phase is zero. Thus, the conclusion is that no realistic CP violation can be produced for \( T \) at the fixed points. This of course is also true if \( T \) is sufficiently close to the fixed points. In our case \( T = 0.985 \, e^{0.5417i} \) which is close to \( e^{i\pi/6} \). The resulting non-removable phases in the Yukawa matrix are of order \( 10^{-1} \), so the CKM phase is suppressed.

### 5.2 Numerical Results

In all interesting cases the modulus field is stabilized close to the fixed points. As we know, supersymmetry is unbroken at the fixed points, so \( F_T \) takes on a rather small value compared to \( m_{3/2} \) for \( T \simeq e^{i\pi/6} \). This leads to the problem of tachyons (see, e.g. \[\text{[5]}\]). Indeed, since

\[
V_0 \sim -3m_{3/2}^2 ,
\]

the soft sfermion masses in Eq.26 are dominated by the \( V_0 \) term and

\[
m_\alpha^2 \sim -2m_{3/2}^2 .
\]

This is of course a problem. One may impose the condition of the vanishing cosmological constant to begin with, but it would be extremely difficult to obtain dilaton stabilization, CP

\[\text{This can be seen explicitly from the phase factorization properties of the Yukawas (under } T \rightarrow T + i/2 \text{) of Ref. [16].}\]
violation, and correct SUSY breaking at the same time. For the lack of a better solution, we may simply assume the there is an additional contribution to the Kähler potential, \( K(X, \bar{X}) \), which allows us to set \( V_0 \) to zero [1]. This contribution will have an effect on all of the soft terms other than the gaugino masses and will help avoid tachyons. Another possibility is to include the effect of quantum corrections [27].

Another problem arises from the gaugino masses. In the absence of the dilaton SUSY breaking, the gaugino masses are suppressed by a loop factor \( \beta \). In addition to that, they are proportional to \( G_2(T) \) which is suppressed close to a fixed point. This results in a suppression factor of about \( 10^3 \). The problem is ameliorated in the presence of the Green-Schwarz term (for multiple gaugino condensates) which creates a non-zero \( F_S \) [28].

Concerning the magnitudes of the soft terms, for a representative point \( T = 0.985 \, e^{0.5417i} \) (racetrack \( \delta_{GS} = 0 \) and S-dual models, \( m = 1, n = 0 \)), we obtain

\[
M_a \sim 10^{-1} - 1 \text{ GeV} , \\
m_\alpha \sim 10^4 \text{ GeV} \text{ (tachyonic)} , \\
A_{\alpha\beta\gamma} \sim 10^3 \text{ GeV} , \\
\hat{\mu} \sim 10^4 \text{ GeV} , \\
\sqrt{B\hat{\mu}} \sim 10^4 \text{ GeV} .
\]

(47)

This SUSY spectrum as it stands is of course phenomenologically unacceptable. Significant modifications of the model are necessary. One possibility would be a mechanism producing a substantial dilaton SUSY breaking component, \( F_S \neq 0 \). This would certainly rectify the problem of light gauginos and help avoid tachyons. The presence of the Green-Schwarz term has a positive effect on the gaugino masses, however the scalar masses are still dominated by \( V_0 \) and the problem of tachyons persists.

Assuming that the above problems are solved one way or another, we can study, at least qualitatively, the CP phases in the model. For the racetrack model we have

\[
\text{Arg}(M_a) = 2.147 , \\
\text{Arg}(A_{\alpha\beta\gamma}) = -1.387 , \\
\text{Arg}(\hat{\mu}) = -0.041 , \\
\text{Arg}(B\hat{\mu}) = 0 .
\]

(48)
For the S-dual model the CP phases are very similar. The resulting physical phases are

\[
\text{Arg}\left( (B\mu)^*\hat{\mu}M \right) \approx 2.1,
\]

\[
\text{Arg}\left( A^*M \right) \approx 0.4.
\]

We see that generically the induced phases are \(O(1)\) and we encounter the SUSY CP problem which is the subject of our next section.

6 Electric Dipole Moments

In heterotic string models there are three types of contributions to the electric dipole moments. The first of them is the standard contribution from complex phases in \(F_{S,T}\) and \(\mu\). The second appears due to nonuniversality even if \(F_{S,T}\) and \(\mu\) are real. The last contribution is induced by \(\text{Im}S\) which generates the \(\bar{\theta}_{QCD}\) term. Let us consider each of these contributions in more detail.

i. Complex phases in \(F_{S,T}\) and \(\mu\).

These are the well known contributions originating from complex phases in the gaugino masses, A-terms, the \(\mu\) and \(B\mu\) terms. The electron, neutron, and mercury EDMs impose the following constraints on these complex phases (at the GUT scale) [29]:

\[
\phi_A \leq 10^{-2} - 10^{-1},
\]

\[
\phi_\mu \leq 10^{-3} - 10^{-2},
\]

\[
\phi_{\text{gaug.}} \leq 10^{-2}.
\]

(50)

Here \(m_{3/2}\) is assumed to be of order 200 GeV. To obtain each of these bounds all the phases except for the one under consideration have been set to zero. Clearly, the complex phases in Eqs.48 and 49 violate these bounds and induce large EDMs unless the soft masses are pushed up to 10 TeV.

ii. Nonuniversality.

In string models the SUSY CP problem appears to be more severe than in general supersymmetric models. The reason is that, if no spontaneously broken supergravity is assumed, the A-terms and the Yukawa matrices do not have to be related. One can treat these on different grounds and entirely separate the Standard Model from the rest of the MSSM. This is not the case in string models. Specifically, the A-terms have a contribution from the Yukawa couplings which is proportional to \(\partial_m \ln Y_{\alpha\beta\gamma}\). Since the Yukawa matrices have a
complicated flavour structure, the same is true for the A-terms. This is indeed what happens in our example: if we are to reproduce CP violation in the Standard Model, we are bound to place the quark fields at different orbifold fixed points. The corresponding Yukawa couplings are necessarily T-dependent and the A-terms non-universal. As we will show, this leads to unacceptably large electric dipole moments even if $F_{S,T}$ and the soft terms are completely real.

Let us first note that the relevant quantities appearing in the soft Lagrangian are

$$\hat{A}_{\alpha\beta\gamma} = A_{\alpha\beta\gamma} Y_{\alpha\beta\gamma}.$$  \hfill (51)

Clearly, these quantities are necessarily complex due to the complex phases in the Yukawa matrices. This would not be dangerous for the EDMs were the A-terms universal and real. What matters is the complex phases in the squark mass insertions in the super-CKM basis, i.e. in the basis where the Yukawa matrices are diagonal. To draw a correspondence between the supergravity notation we have used above and the “phenomenological” notation, let us fix the first index of the Yukawa to refer to the Higgs fields, the second index to be the generational index for the left-handed fields, and the last index to be that for the right-handed fields. For example, $Y_{H_i Q_j D_j} \equiv Y_{ij}^d$. Then, the super-CKM basis is defined by

$$\hat{U}_{L,R} \rightarrow V_{L,R}^u \hat{U}_{L,R}^*, \quad \hat{D}_{L,R} \rightarrow V_{L,R}^d \hat{D}_{L,R},$$

$$Y^u \rightarrow V_L^u Y^u V_R^u = \text{diag}(h_u, h_c, h_t),$$

$$Y^d \rightarrow V_L^d Y^d V_R^d = \text{diag}(h_d, h_s, h_b),$$ \hfill (52)

where $\hat{U}, \hat{D}$ are the quark superfields. The matrices $\hat{A}_{u,d}$ from Eq (51) are not diagonal in this basis and their diagonal elements generically contain order one complex phases. This is to be contrasted with the universal case where $\hat{A}_{u,d}$ and $Y^u,d$ are diagonal simultaneously with the former being real if $F_{S,T}$ are real.

Complex phases in the diagonal elements of $\hat{A}_{u,d}$ in the super CKM basis induce electric dipole moments of the quarks. In the universal case, the diagonal entries are proportional to the corresponding quark masses. For instance, $\hat{A}_{11}^u$ is much smaller than $\hat{A}_{22}^u$, etc. In this case, the complex phases are required to be less than $10^{-1} - 10^{-2}$ \cite{29}. The constraints become much stronger in the non-universal case. Indeed, the diagonal entries of $\hat{A}_{u,d}$ in the super-CKM basis are now proportional to some linear combination of the quark masses. For instance,

$$\hat{A}_{11}^u \propto m_u + \epsilon m_c + \epsilon' m_t.$$ \hfill (53)
This significantly increases the magnitude of \( \hat{A}_{11}^{u,d} \). Recall now that what matters for the EDMs is not just the phase of \( \hat{A}_{11}^{u,d} \) but its imaginary part. Clearly, even a small phase can be dangerous if the magnitude of \( \hat{A}_{11}^{u,d} \) is large.

In a non-universal case, the EDM constraints can be neatly expressed as constraints on the imaginary parts of the so called “squark mass insertions”. These are defined as 

\[
(\delta_{LR}^{u,d})_{ii} \sim \hat{A}_{ii}^{u,d} \langle H_{u,d} \rangle / \tilde{m}^2,
\]

where \( \tilde{m} \) is the average squark mass and the super-CKM basis is assumed. The neutron EDM constrains their imaginary parts to be no greater than \( \mathcal{O}(10^{-6}) \), whereas the mercury EDM constrains them to be less than \( \mathcal{O}(10^{-7}) \) \cite{29}. These bounds are violated in our case by orders of magnitude. Indeed, with \( \mathcal{O}(1) \) phases in the Yukawas and assuming \( \mathcal{O}(10^{-2}) \) mixing between the first and the third generations (i.e. \( (V_{L,R})_{13} \sim V_{13}^{CKM} \)), we typically get \( \text{Im}(\delta_{LR}^{u,d})_{11} \sim 10^{-4} \). Thus, the EDMs are overproduced by two-three orders of magnitude. Clearly, a similar effect occurs in the lepton sector if we allow non-diagonal lepton Yukawas.

This problem is quite generic for heterotic string models. If we are to produce the CKM phase and fermion mass hierarchy, the Yukawas are bound to be nonuniversal and T-dependent\(^8\). This results in nonuniversal A-terms (unless \( F_T = 0 \) which is highly disfavoured by the dilaton stabilization mechanisms) inducing large EDMs even in the absence of complex phases in \( F_S \) and \( F_T \).

iii. QCD vacuum angle \( \bar{\theta} \).

In addition to the above sources of EDMs, there is a standard contribution to the \( \bar{\theta} \) parameter from \( \text{Im}S \). In the case of multiple gaugino condensates \( \text{Im}S \) is fixed (up to a discrete shift) by the potential minimization at an \( \mathcal{O}(1) \) value. So, generically this would overproduce the EDMs by many orders of magnitude. To rectify this problem one needs a stringy Peccei-Quinn mechanism which would set \( \bar{\theta} \) to zero regardless of its initial value. This requires an anomalous global U(1) symmetry which couples to the QCD anomaly but not to those of the other condensing groups. In string theory, such symmetries can arise from anomalous gauge U(1)’s and reasonable solutions can be obtained \cite{30}. Here we will not address this issue in detail and will simply assume that the strong CP problem is solved one way or another.

\(^8\)Corrections from non-renormalizable operators do not change this.
7 Conclusions

In this paper we have addressed the question whether it is possible to have spontaneous CP violation in heterotic string models at a phenomenologically acceptable level. In addition to having CP violation, we imposed the conditions of dilaton stabilization and a viable SUSY breaking scale. We find the following positive features in the models considered:

+1. CP can be broken by a VEV of the modulus field, while having phenomenologically acceptable values for the dilaton and the SUSY breaking scale.

+2. A non-trivial CKM phase can be produced.

Despite these encouraging general results, we encounter a number of difficulties to be solved in more realistic models:

-1. $O(1)$ complex phases in the Yukawa matrices and $F_T$ lead to the EDMs exceeding the experimental limits by orders of magnitude.

-2. $T$ is stabilized close to the fixed points of the modular group which results in a suppressed CKM phase.

-3. Generally there are tachyons and unacceptably light gauginos (the latter problem is mitigated in the presence of the Green-Schwarz term).

-4. $\text{Im} S$ gives a large contribution to $\bar{\theta}$ leading to the strong CP problem (which can be resolved in the presence of a global anomalous $U(1)$ symmetry).

In addition, it is very difficult to obtain a vanishing cosmological constant while retaining the positive features of the model. This however may not be a real problem since $V_0$ is not necessarily directly related to the cosmological constant.

Some of these problems can be solved if supersymmetry breaking is dilaton-dominated, i.e. $F_S \neq 0$ and $F_T \simeq 0$. This would suppress the EDM contributions originating from the non-universality and also help avoid tachyons and light gauginos. At the same time, in order to produce the CKM phase, the modulus field must be well away from the fixed points. Even if all this is the case, one has to ensure that there are no relative phases among $\mu$, $B\mu$, and the gaugino masses which is quite nontrivial even in the dilaton-dominated case. We find that this is hardly possible with the presently available dilaton stabilization mechanisms. In principle one can combine different mechanisms to obtain the desired features. We will report on this in a subsequent paper.

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| $m$ | $n$ | $S_{\text{min}}$ | $T_{\text{min}}$ | $F_S$ | $F_T$ |
|-----|-----|-----------------|------------------|-------|-------|
| 0   | 0   | $2.1299 - 0.9196i$ | 1.2346           | 0     | $2.16 \times 10^{-16}$ |
| 0   | 1   | $2.1299 - 0.9196i$ | 1.0000           | 0     | 0     |
| 0   | 2   | $2.1299 - 0.9196i$ | 1.0000           | 0     | 0     |
| 0   | 3   | $2.1299 - 0.9196i$ | 1.0000           | 0     | 0     |
| 0   | 4   | $2.1299 - 0.9196i$ | 1.0000           | 0     | 0     |
| 0   | 5   | $2.1299 - 0.9196i$ | 1.0000           | 0     | 0     |
| 1   | 0   | $2.1299 - 0.9196i$ | 0.9850 $e^{0.5417i}$ | 0     | $(-0.51 - 2.74i) \times 10^{-16}$ |
| 1   | 1   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.2401i}$ | 0     | 0     |
| 1   | 2   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.1913i}$ | 0     | 0     |
| 1   | 3   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.1642i}$ | 0     | 0     |
| 1   | 4   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.1462i}$ | 0     | 0     |
| 1   | 5   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.1331i}$ | 0     | 0     |
| 2   | 0   | $2.1299 - 0.9196i$ | 0.9922 $e^{0.5329i}$ | 0     | $(-1.08 - 5.94i) \times 10^{-15}$ |
| 2   | 1   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.2897i}$ | 0     | 0     |
| 2   | 2   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.2412i}$ | 0     | 0     |
| 2   | 3   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.2121i}$ | 0     | 0     |
| 2   | 4   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.1919i}$ | 0     | 0     |
| 2   | 5   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.1766i}$ | 0     | 0     |
| 3   | 0   | $2.1299 - 0.9196i$ | 0.9972 $e^{0.5159i}$ | 0     | $(-1.32 - 1.09i) \times 10^{-13}$ |
| 3   | 1   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.3168i}$ | 0     | 0     |
| 3   | 2   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.2705i}$ | 0     | 0     |
| 3   | 3   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.2416i}$ | 0     | 0     |
| 3   | 4   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.2209i}$ | 0     | 0     |
| 3   | 5   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.2049i}$ | 0     | 0     |
| 4   | 0   | $2.1299 - 0.9196i$ | 0.9960 $e^{0.5283i}$ | 0     | $(-0.94 - 5.24i) \times 10^{-12}$ |
| 4   | 1   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.3347i}$ | 0     | 0     |
| 4   | 2   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.2906i}$ | 0     | 0     |
| 4   | 3   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.2624i}$ | 0     | 0     |
| 4   | 4   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.2418i}$ | 0     | 0     |
| 4   | 5   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.2257i}$ | 0     | 0     |
| 5   | 0   | $2.1299 - 0.9196i$ | 0.9968 $e^{0.5274i}$ | 0     | $(-0.31 - 1.76i) \times 10^{-10}$ |
| 5   | 1   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.3478i}$ | 0     | 0     |
| 5   | 2   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.3056i}$ | 0     | 0     |
| 5   | 3   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.2782i}$ | 0     | 0     |
| 5   | 4   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.2579i}$ | 0     | 0     |
| 5   | 5   | $2.1299 - 0.9196i$ | 1.0000 $e^{0.2419i}$ | 0     | 0     |

Table 2: Minima for the racetrack models, $\delta_{GS} = 0$. 
\begin{table}[h]
\begin{center}
\begin{tabular}{|c|c|c|c|c|}
\hline
$m$ & $n$ & $S_{\text{min}}$ & $T_{\text{min}}$ & $F_S$ & $F_T$ \\
\hline
0 & 0 & 1.8843$-0.9196i$ & 1.2326 & $-1.43 \times 10^{-17}$ & $1.76 \times 10^{-17}$ \\
0 & 1 & 1.9295$-0.9196i$ & 1.0000 & 0 & 0 \\
0 & 2 & 1.9295$-0.9196i$ & 1.0000 & 0 & 0 \\
0 & 3 & 1.9295$-0.9196i$ & 1.0000 & 0 & 0 \\
0 & 4 & 1.9295$-0.9196i$ & 1.0000 & 0 & 0 \\
0 & 5 & 1.9295$-0.9196i$ & 1.0000 & 0 & 0 \\
1 & 0 & 1.9604$-1.0137i$ & 0.9918 $e^{0.5005i}$ & $(1.65 - 1.33i) \times 10^{-17}$ & $(-1.85 + 1.45i) \times 10^{-17}$ \\
1 & 1 & 1.9365$-0.9652i$ & 1.0000 $e^{0.2400i}$ & 0 & 0 \\
1 & 2 & 1.9340$-0.9559i$ & 1.0000 $e^{0.1912i}$ & 0 & 0 \\
1 & 3 & 1.9328$-0.9507i$ & 1.0000 $e^{0.1641i}$ & 0 & 0 \\
1 & 4 & 1.9321$-0.9473i$ & 1.0000 $e^{0.1462i}$ & 0 & 0 \\
1 & 5 & 1.9317$-0.9448i$ & 1.0000 $e^{0.1331i}$ & 0 & 0 \\
2 & 0 & 1.9604$-1.0163i$ & 0.9958 $e^{0.5117i}$ & $(3.55 - 2.92i) \times 10^{-16}$ & $(-3.95 + 3.20i) \times 10^{-16}$ \\
2 & 1 & 1.9396$-0.9746i$ & 1.0000 $e^{0.2897i}$ & 0 & 0 \\
2 & 2 & 1.9366$-0.9654i$ & 1.0000 $e^{0.2412i}$ & 0 & 0 \\
2 & 3 & 1.9350$-0.9599i$ & 1.0000 $e^{0.2121i}$ & 0 & 0 \\
2 & 4 & 1.9340$-0.9560i$ & 1.0000 $e^{0.1919i}$ & 0 & 0 \\
2 & 5 & 1.9333$-0.9531i$ & 1.0000 $e^{0.1766i}$ & 0 & 0 \\
3 & 0 & 1.9600$-1.0174i$ & 0.9998 $e^{0.5151i}$ & $(6.85 - 4.04i) \times 10^{-16}$ & $(-7.64 + 4.42i) \times 10^{-16}$ \\
3 & 1 & 1.9416$-0.9797i$ & 1.0000 $e^{0.3167i}$ & 0 & 0 \\
3 & 2 & 1.9384$-0.9709i$ & 1.0000 $e^{0.2704i}$ & 0 & 0 \\
3 & 3 & 1.9366$-0.9655i$ & 1.0000 $e^{0.2416i}$ & 0 & 0 \\
3 & 4 & 1.9355$-0.9615i$ & 1.0000 $e^{0.2208i}$ & 0 & 0 \\
3 & 5 & 1.9346$-0.9585i$ & 1.0000 $e^{0.2049i}$ & 0 & 0 \\
4 & 0 & 1.9608$-1.0176i$ & 0.9969 $e^{0.5180i}$ & $(4.03 - 3.93i) \times 10^{-13}$ & $(-4.47 + 4.32i) \times 10^{-13}$ \\
4 & 1 & 1.9429$-0.9831i$ & 1.0000 $e^{0.3347i}$ & 0 & 0 \\
4 & 2 & 1.9397$-0.9748i$ & 1.0000 $e^{0.2906i}$ & 0 & 0 \\
4 & 3 & 1.9379$-0.9694i$ & 1.0000 $e^{0.2624i}$ & 0 & 0 \\
4 & 4 & 1.9366$-0.9655i$ & 1.0000 $e^{0.2417i}$ & 0 & 0 \\
4 & 5 & 1.9357$-0.9624i$ & 1.0000 $e^{0.2256i}$ & 0 & 0 \\
5 & 0 & 1.9613$-1.0180i$ & 0.9959 $e^{0.5205i}$ & $(1.49 - 2.24i) \times 10^{-11}$ & $(-1.65 + 2.47i) \times 10^{-11}$ \\
5 & 1 & 1.9440$-0.9856i$ & 1.0000 $e^{0.3477i}$ & 0 & 0 \\
5 & 2 & 1.9408$-0.9776i$ & 1.0000 $e^{0.3056i}$ & 0 & 0 \\
5 & 3 & 1.9389$-0.9724i$ & 1.0000 $e^{0.2782i}$ & 0 & 0 \\
5 & 4 & 1.9376$-0.9686i$ & 1.0000 $e^{0.2579i}$ & 0 & 0 \\
5 & 5 & 1.9366$-0.9655i$ & 1.0000 $e^{0.2419i}$ & 0 & 0 \\
\hline
\end{tabular}
\end{center}
\caption{Minima for the racetrack models, $\delta_{GS} = 5$.}
\end{table}
Table 4: Minima for models with the non-perturbative Kähler potential, $\delta_{GS} = 0$. 

| $m$ | $n$ | $S_{min}$ | $T_{min}$ | $F_S$      | $F_T$ |
|-----|-----|-----------|-----------|------------|-------|
| 0   | 0   | 1.8       | $e^{i\pi/6}$ | $-5.8298 \times 10^{-7}$ | 0     |
| 0   | 1   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 0   | 2   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 0   | 3   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 0   | 4   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 0   | 5   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 1   | 0   | 1.8       | 1          | 0          | 0     |
| 1   | 1   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 1   | 2   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 1   | 3   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 1   | 4   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 1   | 5   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 2   | 0   | 1.8       | 1          | 0          | 0     |
| 2   | 1   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 2   | 2   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 2   | 3   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 2   | 4   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 2   | 5   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 3   | 0   | 1.8       | 1          | 0          | 0     |
| 3   | 1   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 3   | 2   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 3   | 3   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 3   | 4   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 3   | 5   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 4   | 0   | 1.8       | 1          | 0          | 0     |
| 4   | 1   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 4   | 2   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 4   | 3   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 4   | 4   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 4   | 5   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 5   | 0   | 1.8       | 1          | 0          | 0     |
| 5   | 1   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 5   | 2   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 5   | 3   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 5   | 4   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| 5   | 5   | 1.8       | $e^{i\pi/6}$ | 0          | 0     |
| $m$ | $n$ | $S_{min}$ | $T_{min}$ | $F_S$ | $F_T$ |
|-----|-----|-----------|-----------|-------|-------|
| 0   | 0   | --        | 0         | --    | --    |
| 0   | 1   | 1.9       | $e^{i\pi/6}$ | 0     | 0     |
| 0   | 2   | 1.9       | $e^{i\pi/6}$ | 0     | 0     |
| 0   | 3   | 1.9       | $e^{i\pi/6}$ | 0     | 0     |
| 0   | 4   | 1.9       | $e^{i\pi/6}$ | 0     | 0     |
| 0   | 5   | 1.9       | $e^{i\pi/6}$ | 0     | 0     |
| 1   | 0   | 1.8       | 1         | 0     | 0     |
| 1   | 1   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 1   | 2   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 1   | 3   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 1   | 4   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 1   | 5   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 2   | 0   | 1.8       | 1         | 0     | 0     |
| 2   | 1   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 2   | 2   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 2   | 3   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 2   | 4   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 2   | 5   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 3   | 0   | 1.8       | 1         | 0     | 0     |
| 3   | 1   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 3   | 2   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 3   | 3   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 3   | 4   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 3   | 5   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 4   | 0   | 1.8       | 1         | 0     | 0     |
| 4   | 1   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 4   | 2   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 4   | 3   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 4   | 4   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 4   | 5   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 5   | 0   | 1.8       | 1         | 0     | 0     |
| 5   | 1   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 5   | 2   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 5   | 3   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 5   | 4   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |
| 5   | 5   | 1.8, 1.9  | 1, $e^{i\pi/6}$ | 0     | 0     |

Table 5: Minima for models with the non-perturbative Kähler potential, $\delta_{GS} = 5$. For the parameter values considered, in the $m = n = 0$ case the extra dimensions become uncompactified.
| $m$ | $n$ | $S_{\text{min}}$ | $T_{\text{min}}$ | $\hat{F}_S$ | $\hat{F}_T$ |
|-----|-----|------------------|------------------|-------------|-------------|
| 0   | 0   | 1                | 1.2346           | 0           | $4.24 \times 10^{-16}$ |
| 0   | 1   | 1                | 1.0000           | 0           | 0           |
| 0   | 2   | 1                | 1.0000           | 0           | 0           |
| 0   | 3   | 1                | 1.0000           | 0           | 0           |
| 0   | 4   | 1                | 1.0000           | 0           | 0           |
| 0   | 5   | 1                | 1.0000           | 0           | 0           |
| 1   | 0   | 1                | $0.9850 e^{0.5417i}$ | 0           | $(-1.00 - 5.38i) \times 10^{-16}$ |
| 1   | 1   | 1                | 1.0000           | 0           | 0           |
| 1   | 2   | 1                | 1.0000           | 0           | 0           |
| 1   | 3   | 1                | 1.0000           | 0           | 0           |
| 1   | 4   | 1                | 1.0000           | 0           | 0           |
| 1   | 5   | 1                | 1.0000           | 0           | 0           |
| 2   | 0   | 1                | $0.9922 e^{0.5329i}$ | 0           | $(-0.21 - 1.17i) \times 10^{-14}$ |
| 2   | 1   | 1                | 1.0000           | 0           | 0           |
| 2   | 2   | 1                | 1.0000           | 0           | 0           |
| 2   | 3   | 1                | 1.0000           | 0           | 0           |
| 2   | 4   | 1                | 1.0000           | 0           | 0           |
| 2   | 5   | 1                | 1.0000           | 0           | 0           |
| 3   | 0   | 1                | $0.9972 e^{0.5159i}$ | 0           | $(-2.60 - 2.14i) \times 10^{-13}$ |
| 3   | 1   | 1                | 1.0000           | 0           | 0           |
| 3   | 2   | 1                | 1.0000           | 0           | 0           |
| 3   | 3   | 1                | 1.0000           | 0           | 0           |
| 3   | 4   | 1                | 1.0000           | 0           | 0           |
| 3   | 5   | 1                | 1.0000           | 0           | 0           |
| 4   | 0   | 1                | $0.9960 e^{0.5283i}$ | 0           | $(-0.18 - 1.03i) \times 10^{-11}$ |
| 4   | 1   | 1                | 1.0000           | 0           | 0           |
| 4   | 2   | 1                | 1.0000           | 0           | 0           |
| 4   | 3   | 1                | 1.0000           | 0           | 0           |
| 4   | 4   | 1                | 1.0000           | 0           | 0           |
| 4   | 5   | 1                | 1.0000           | 0           | 0           |
| 5   | 0   | 1                | $0.9968 e^{0.5274i}$ | 0           | $(-0.62 - 3.45i) \times 10^{-10}$ |
| 5   | 1   | 1                | 1.0000           | 0           | 0           |
| 5   | 2   | 1                | 1.0000           | 0           | 0           |
| 5   | 3   | 1                | 1.0000           | 0           | 0           |
| 5   | 4   | 1                | 1.0000           | 0           | 0           |
| 5   | 5   | 1                | 1.0000           | 0           | 0           |

Table 6: Minima for S-dual models, $\mu = 1.8 \times 10^{-3}$. 