Apparent and Actual Shifts in Mass and Width of 
\( \phi \) Mesons Produced in Heavy-Ion Collisions

R. S. Bhalerao

*Nuclear Theory Center, Indiana University*

2401 Milo B. Sampson Lane, Bloomington, IN 47405, USA

and

*Theoretical Physics Group, Tata Institute of Fundamental Research*

Homi Bhabha Road, Colaba, Mumbai 400 005, India

S. K. Gupta

*Nuclear Physics Division, Bhabha Atomic Research Center*

Trombay, Mumbai 400 085, India

Abstract

We present a method of analyzing invariant-mass spectra of kaon pairs resulting from decay of \( \phi \) mesons produced in high-energy heavy-ion collisions. It can be used to extract the shifts in the mass and the width (\( \Delta M \) and \( \Delta \Gamma \)) of the \( \phi \) mesons when they are inside the dense matter formed in these collisions. We illustrate our method with the help of available preliminary data. Extracted values of \( \Delta M \) and \( \Delta \Gamma \) are significantly larger than those obtained with an earlier method. Our results are consistent with the experimentally observed \( p_T \) dependence of the mass shift. Finally, we present a phenomenological relation between \( \Delta M \) and \( \Delta \Gamma \). It provides a useful constraint on theories which predict the values of these two quantities.
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1 permanent address

E-mail: bhalerao@theory.tifr.res.in, Fax: 091 22 215 2110

E-mail: skgupta@magnum.barct1.ernet.in, Fax: 091 22 556 0750
1 Introduction

Properties such as mass, decay width, size, etc. of hadrons in general and light vector mesons ($\rho, \omega, \phi$) in particular, are thought to undergo a change when the surrounding medium is raised to a higher temperature and/or density; for recent reviews of finite-temperature and finite-density effects, see Refs. [1] and [2], respectively. Interest in this scenario arises largely due to its close connection with phase transitions in quantum chromodynamics (QCD). Recent years have witnessed intense experimental and theoretical activity in this area [3]. In this paper, we focus on the mass as well as the width of the $\phi$ meson.

Recently E-802 experimental collaboration at AGS (BNL) has reported preliminary results on the shift in the mass of the $\phi$ mesons produced in central $Si + Au$ collisions at 14.6 A GeV/c [4,5]. The mass was determined from the invariant-mass distribution of kaon pairs arising from the dominant decay mode $\phi \rightarrow K^+K^-$. It was studied as a function of the multiplicity of the collision events and the transverse momentum $p_T$ of the accepted $\phi$’s. It was found that in the events with the highest multiplicity (top 2% target-multiplicity-array TMA cut) the mass drops by $2.3 \pm 0.9 \pm 0.1$ MeV compared to the free-space value $1019.413 \pm 0.008$ MeV [6]. For the $\phi$ mesons with $p_T < 1.25$ GeV/c, the shift was even more ($3.3 \pm 1.0 \pm 0.1$ MeV), whereas for $p_T > 1.25$ GeV/c, there was no apparent shift. No numbers were reported for the shift in the decay width of the $\phi$ mesons. However, from the confidence contours for 1, 2 and 3 standard deviations in the observed mass versus width plot given in [5], it appears that the central value of the width is higher than the free-space value $4.43 \pm 0.06$ MeV [6] by about 0.78 MeV. If confirmed, this will be the first evidence of the modification of hadronic properties inside dense matter formed in heavy-ion collisions.

In a more recent publication [7], the E-802 collaboration has reported no shift in the mass and the width of the $\phi$ meson. This, however, does not contradict the earlier observations [5] because in Ref. [4] the top 7% of the highest-multiplicity events were clubbed together and no multiplicity dependence of the mass was reported. In the earlier work [5], the shift in the mass was seen when the TMA cut was applied at 2%.
2 Method

In this paper, we present a different method of analyzing the invariant-mass spectra of kaon pairs. We then use the data in Refs. [4-5] only to illustrate our method. We find that our method yields shifts in the $\phi$-meson mass and width which are several times larger than the values quoted above. We caution the reader that since these data are preliminary, our numerical results are obviously subject to change. *However, our main point is the method presented here which is independent of whether the published data [4-5] are eventually confirmed or not.* We think the present method has relevance to the analysis of future data on $\phi$ production. This acquires added importance in view of the possibility of a similar experiment being performed at CERN APS [8].

Let $M_0$ and $M_1$ be the rest masses and let $\Gamma_0$ and $\Gamma_1$ be the widths of the $\phi$ meson in the free space and in the dense medium, respectively. We define the shifts as

$$\Delta M = M_0 - M_1, \quad \Delta \Gamma = \Gamma_1 - \Gamma_0,$$

so that both are positive when the mass drops and the width increases with respect to their free-space values.

In Ref. [3], the invariant-mass spectrum of the $K^+K^-$ pairs was fitted by a function consisting of a background term and a relativistic Breit-Wigner (BW) resonant term convoluted with a Gaussian experimental mass resolution function. This procedure yielded the values of the shifts $\Delta M$ and $\Delta \Gamma$ given above. A background-subtracted mass spectrum is displayed in Fig. 1 where the histogram corresponds to 3% TMA and low-$p_T$ cuts and is taken from Fig. 2(b) of Ref. [3]. The dashed and solid curves represent single BW resonance terms convoluted with a Gaussian as stated above. Areas under the two curves are normalized to that under the histogram. The dashed curve corresponds to the scenario where there is no shift in the mass and the width. The solid curve corresponds to $\Delta M = 2.3$ MeV and $\Delta \Gamma = 0.78$ MeV. Clearly the data indicate a shift in the mass and the width of the $\phi$ meson. However, are the values of $\Delta M$ and $\Delta \Gamma$ obtained by fitting a single BW resonance term to the (background-subtracted) data correct? We think they are not.
Since the mean lifetime of $\phi$ in its rest frame is about 45 fm/c, a majority of $\phi$’s are expected to decay long after the dense medium in which they were produced has ceased to exist. That is, they will decay essentially in free space ($\Delta M = 0 = \Delta \Gamma$). The rest of the $\phi$’s, however, decay while still inside the dense medium. Hence a better procedure would be to fit the background-subtracted data with two instead of one BW terms, one unshifted and another shifted, added with appropriate weights. This we now proceed to do.

We work in the center-of-mass frame of the dense medium formed in the nucleus-nucleus collision. We employ the natural units $c = \hbar = 1$. Let $f$ be the fraction of $\phi$’s decaying inside the medium; then $(1 - f)$ is the fraction decaying in free space. We reanalyze the background-subtracted data of Ref. [5] by fitting them with two BW terms — an unshifted BW with mass $M_0$, width $\Gamma_0$ and weight $(1 - f)$ and a shifted BW with mass $M_1$, width $\Gamma_1$ and weight $f$:

$$\frac{dN_{K^+K^-}}{dM} = (1 - f) \ BW_c(M, M_0, \Gamma_0) + f \ BW_c(M, M_1, \Gamma_1).$$

(2)

Here $BW_c$ denotes the relativistic Breit-Wigner resonance term convoluted with a Gaussian experimental mass resolution function:

$$BW_c(M, M_0, \Gamma_0) = \int BW(M', M_0, \Gamma_0) \exp \left[ -\frac{1}{2} \left( \frac{M - M'}{\sigma} \right)^2 \right] \frac{dM'}{\sigma \sqrt{2\pi}}. \tag{3}$$

$$BW(M, M_0, \Gamma_0) = \frac{M_0 \Gamma_0(M)}{\pi} \frac{2M}{(M^2 - M_0^2)^2 + M_0^2 \Gamma_0^2(M)}. \tag{4}$$

The experimental mass resolution $\sigma$ is taken to be 2.2 MeV [9]. The energy-dependent width $\Gamma_0(M)$ is taken to be [10]

$$\Gamma_0(M) = \Gamma_0 \left( \frac{q}{q_0} \right)^3 \frac{2q_0^2}{q^2 + q_0^2}, \tag{5}$$

where the kaon momenta $q$ and $q_0$ are given by

$$q = (M^2/4 - M_K^2)^{1/2} \quad \text{and} \quad q_0 = (M_0^2/4 - M_K^2)^{1/2}. \tag{6}$$

Expressions for $BW_c(M, M_1, \Gamma_1)$, $BW(M, M_1, \Gamma_1)$ and $\Gamma_1(M)$ are obtained by replacing $M_0$ by $M_1$ and $\Gamma_0$ by $\Gamma_1$ in Eqs. (3-6). We find that the numerical results with energy-dependent and energy-independent widths to be practically the same.
We now derive an expression for $f$. Let $\tau_0 = \Gamma_0^{-1}$ and $\tau_1 = \Gamma_1^{-1}$ be the mean lifetimes of $\phi$ at rest in the free space and in the dense medium, respectively. Then the corresponding lifetimes when the $\phi$ is in motion would be $\tau_0 \gamma$ and $\tau_1 \gamma$, due to the time dilation. Here $\gamma \equiv (1 - v^2)^{-1/2}$ is the usual Lorentz factor and $v$ is the average velocity of the $\phi$’s in the medium. Let $d$ be the “size” or “radius” or a typical linear dimension associated with the extent of the medium. The average time the $\phi$’s produced in the medium would take to traverse this distance is $d/v$. If $N_0$ is the number of $\phi$’s at time $t = 0$, then at time $t = d/v$, only $N_0 \exp(-\Gamma_1 d/v \gamma)$ of them would be left. Hence $f = 1 - \exp(-\Gamma_1 d/v \gamma)$. Now

$$v \gamma = p/M_1 = \sqrt{E^2 - M_T^2} / M_1 = \sqrt{M_T^2 \cosh^2 y_{cm} - M_T^2} / M_1,$$

where $p$, $E$, $M_T$ and $y_{cm}$ denote, respectively, the momentum, energy, transverse mass and center-of-mass rapidity of the $\phi$ traversing the medium. Hence

$$f(\Delta M, \Delta \Gamma) = 1 - \exp(-M_T \Gamma_1 d / \sqrt{M_T^2 \cosh^2 y_{cm} - M_T^2}).$$

(7)

Interactions of the outgoing $K^{\pm}$ with the medium were ignored in Refs. [4-5] and we too shall ignore them. This is a good first approximation because if these interactions were important the $\phi$ peak would have been washed out, whereas experimentally, a distinct, narrow peak is seen. Given the quality of the available data, we think, the present approach is adequate. In view of the possibility of better-quality data becoming available in the future, it is desirable to include these interactions by performing a detailed Monte-Carlo calculation. This would entail a considerable amount of extra work, and we leave it to the future. Such an elaborate calculation may change the values of $\Delta M$ and $\Delta \Gamma$ to some extent, but we do not expect it to change our conclusion that these will be more realistic than those resulting from the one-BW fit to the data. If the kaon-medium interaction is ignored, then $M_T$ and $y_{cm}$ of the $\phi$ are the same as those of the $K^{\pm}$ pair. In a Monte-Carlo simulation one can extract them from experimental data by modeling the interaction.
3 Results and discussion

Results of a least-squares fit to the same experimental data as in Fig. 1, with two instead of one BW terms, are shown in Fig. 2. The two dashed curves in Fig. 2 correspond to the two convoluted BW terms, one shifted and the other unshifted. The solid curve corresponds to their weighted sum as in Eqs. (2-6). The input parameters are \(d = 5\ \text{fm}, M_T = 1100\ \text{MeV}\) and \(y_{cm} = 0.3\) \cite{5}. The two fitted parameters are \(\Delta M = 6.0 \pm 1.9\ \text{MeV}\) and \(\Delta \Gamma = 5.4 \pm 3.9\ \text{MeV}\). The resultant value of \(f\) is 0.37. If \(M_T = 1200\ \text{MeV}\), the two fitted parameters are \(\Delta M = 6.2 \pm 2.4\ \text{MeV}\) and \(\Delta \Gamma = 7.7 \pm 5.0\ \text{MeV}\). The resultant value of \(f\) is 0.34. Note that the above values of the shifts are several times larger than those obtained with a one-BW fit. Secondly, even for an unrealistically large value of \(d\), say 10 fm, the fraction \(f\) of \(\phi\)'s decaying inside the dense matter is \(\sim 0.4\) to 0.5 showing that the assumption of a single BW resonance is questionable.

It is evident from Eq. (7) that as \(p_T\) increases the fraction \(f\) decreases. This means a larger fraction of \(\phi\)'s decay in free space and hence the mass shift decreases. This is exactly what has been observed experimentally \cite{5}.

Finally, we present an empirical relation between \(\Delta M\) and \(\Delta \Gamma\). We define

\[
\overline{M} = (1 - f)M_0 + fM_1, \tag{8}
\]

\[
\Delta \overline{M} = M_0 - \overline{M}. \tag{9}
\]

On substituting Eqs. (1) and (9) in Eq. (8) and simplifying we get

\[
\Delta \overline{M} = f(\Delta M, \Delta \Gamma)\Delta M. \tag{10}
\]

Given the experimental histogram as in Fig. 1, it is straightforward to determine its centroid and it is reasonable to approximate \(\overline{M}\) by the centroid. Equation (9) can then be used to determine the “apparent” shift \(\Delta \overline{M}\). For a fixed \(\Delta \overline{M}\), Eq. (10) yields a curve \(\Delta \Gamma\) versus \(\Delta M\). We illustrate this in Fig. 3 for \(\Delta \overline{M} = 1, 2, 3\) and 4 MeV. The input parameters are as above, namely \(d = 5\ \text{fm}, M_T = 1200\ \text{MeV}\) and \(y_{cm} = 0.3\). Interestingly, as \(\Delta M\) increases, \(\Delta \Gamma\) decreases. This is easy to understand from Eq. (10) because for a fixed \(\Delta \overline{M}\), as \(\Delta M\)
increases, $f$ has to decrease, which from Eq. (7) requires $\Gamma_1$ and hence $\Delta \Gamma$ to decrease. The physics of this is also clear if one considers the three curves in Fig. 2. Equation (10) together with the experimentally determined $\Delta M$ provides a useful constraint on theories which predict the values of $\Delta M$ and $\Delta \Gamma$. For example, if the apparent shift $\Delta M$ extracted from experimental data is 2 MeV, then any model which claims to explain the data should have $\Delta M$ and $\Delta \Gamma$ not inconsistent with the curve labeled “2” in Fig. 3.

A variety of approaches, namely lattice QCD, QCD sum rules, Nambu-Jona-Lasinio model, quantum hadrodynamics, bag models, instanton-liquid model, etc. have been used in the literature to predict the masses and widths of the light vector mesons at finite temperatures ($T$) and/or densities ($\rho$). Here we focus on some of the recent calculations for the $\phi$ meson. Asakawa and Ko [11] have used the QCD sum rules to calculate the $\phi$-meson mass when both $T$ and $\rho$ are finite. For the values of $T$ and $\rho$ considered by them, the predicted drop in the mass is as high as a few hundred MeV. They have proposed a double-peak structure in the invariant-mass spectra of dilepton pairs arising from $\phi$ decay as a signal of the transition from quark-gluon plasma to hadronic matter. Finite-temperature effects on the $\phi$-meson mass and width have been studied recently by Shuryak and Thorsson, Haglin and Gale, Bhattacharyya et al. and Song [12].

4 Conclusions

In conclusion, (i) we have presented a method of analyzing invariant-mass spectrum of the kaon pairs; the resultant values of the shifts in the $\phi$-meson mass and width are significantly larger than those obtained with an earlier method. This difference arises due to the long lifetime of the $\phi$ which ensures that only a small fraction of them decay inside the dense matter. (ii) The model presented here is consistent with the experimentally observed $p_T$ dependence of the mass shift. (iii) We have presented a phenomenological relation between the shifts in the mass and the width.

It is worth reiterating that the data in [5] are preliminary. In view of the interesting conclusions that could be drawn from the data, it would be useful to have a more thorough
experimental investigation of the shift in the mass as well as the width of the $\phi$ meson. One would like to have data with better statistics, on a variety of targets and beams, and at various energies. One would like to see invariant-mass spectra of not only the kaon pairs but also the lepton pairs resulting from $\phi$ decay.

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FIGURES

FIG. 1. The background-subtracted invariant-mass spectrum for the $K^+K^-$ pairs. The histogram represents preliminary experimental data from [5]. The dashed and solid curves represent single BW resonance terms convoluted with a Gaussian as explained in the text. The dashed curve corresponds to the situation where there is no shift in the $\phi$-meson mass and width. The solid curve corresponds to $\Delta M = 2.3$ MeV and $\Delta \Gamma = 0.78$ MeV.

FIG. 2. The background-subtracted invariant-mass spectrum for the $K^+K^-$ pairs. Experimental data as in Fig. 1. The two dashed curves correspond to the two BW terms, one unshifted and the other shifted. The solid curve corresponds to their sum weighted by the factors $(1 - f)$ and $f$, respectively; see Eqs. (2-6).

FIG. 3. For a fixed $\Delta M$, Eq. (10) provides a relation between $\Delta \Gamma$ and $\Delta M$. This relation is represented by the solid lines labeled by the value of $\Delta M$ in MeV.
Figure 1
Figure 2

Counts

$M_{\text{inv}}$ (MeV)
