How to trap a non-standard penguin? Isospin Symmetry Violations in $B$-decays with Enhanced Chromomagnetic Dipole Operator.

Alexey A. Petrov

*High Energy Theory Group*

*Department of Physics and Astronomy*

*University of Massachusetts*

*Amherst MA 01003*

Abstract

A recently proposed Non-Standard Model solution to the problem of low semileptonic branching ratio $B_{SL}$ which suggests a large branching ratio for the decay $b \rightarrow sg$ is critically examined. It is shown that the effects of the Enhanced Chromomagnetic Dipole Operator might lead to significant violations of isospin symmetry in rare radiative decays of $B$-mesons.
It has been recently proposed that effects of New Physics associated with the gluonic dipole operator \[O_{11}\] provide an elegant and simple solution to the problem of low semileptonic branching ratio of \(B\)-mesons \(B_{SL}\) simultaneously solving the problem of charm multiplicity \(n_c\). In these models, New Physics particles (e.g. techniscalars or supersymmetric particles) running inside of the penguin diagram loop significantly modify (increase) the Wilson coefficient \(C_{11}\) in front of the gluonic dipole operator \(O_{11} = \frac{g_s}{16\pi^2} m_b \bar{s}\sigma^{\mu\nu} \frac{1}{2} (1 - \gamma_5) t^a b G^a_{\mu\nu}\) in the effective Hamiltonian

\[H_{eff} = \frac{4G_F}{\sqrt{2}} \left[ V_{cb}V_{cs}(C_1(\mu)O_1(\mu)C_2(\mu)O_2(\mu)) - V_{tb}V_{ts}^* \sum C_i(\mu)O_i(\mu) \right] \tag{1}\]

increasing therefore the rate of the process \(b \to sg\). It is easy to see that increasing \(\Gamma(b \to sg)\) by the effects of New Physics we are simultaneously reducing \(B_{SL}\) and \(n_c\) as required by the experimental results:

\[B_{SL} = \frac{\Gamma(B \to Xl\nu)}{\Gamma_{tot}}, \quad n_c = \frac{\Gamma(B \to X_c) + 2\Gamma(B \to X_{cc})}{\Gamma_{tot}}, \quad \Gamma_{tot} = \sum_l \Gamma(B \to Xl\nu) + \Gamma(B \to X_c) + \Gamma(B \to X_{cc}) + \Gamma(b \to sg, b \to s\gamma, \ldots). \tag{2}\]

This, however, leads to the modification of the already (and not yet) observed processes by Enhanced Chromo-Magnetic Dipole Operators (ECMDO).

The direct effects of the ECMDO on the branching ratios of the decays of the type \(B^- \to \bar{K}\pi^-\) or \(B^- \to X_s\phi\) (which are dominated by the gluonic penguin diagram) have been recently considered in [2]. Surprisingly enough, nowadays none of the direct measurements could show the difference in the predictions of the Standard Model and ECMDO models [2]. The situation will certainly improve with new experiments and construction of the B-factory.

It is clear, however, that independent experimental tests are needed.

One of the possible decay modes to serve as a test ground is the exclusive \(B \to K^{*}\gamma\) decay (or relevant inclusive transition \(B \to X_s\gamma\)). The impact of ECMDO on this decay mode is indirect, for instance, through the renormalization of the \(b \to s\gamma\) vertex. The effect of it on the given branching ratio is relatively small, but might be larger on the portions of the photon spectrum. This, however, requires some experimental efforts in pushing down the threshold in the observed \(E_\gamma\).
In this paper we consider another possibility. Since hadrons are the asymptotic states of QCD there must exist bound state corrections to the calculated decay rate for $B \to K^{*}\gamma$ and $B \to X_s\gamma$. These “gluonic spectator effects” in the exclusive $B \to K^{*}\gamma$ and inclusive $B \to X_s\gamma$ have been recently considered in [3] [4]. It was shown that the charge (isospin) asymmetry

$$a = \frac{\Gamma_{B^{-}\to K^{*}\gamma} - \Gamma_{B^{0}\to K^{*}\gamma}}{\Gamma_{B^{-}\to K^{*}\gamma} + \Gamma_{B^{0}\to K^{*}\gamma}}$$  (3)

is fairly small in the Standard Model, typically of the order of a few percent. The primary sources of the isospin symmetry violations are long-distance effects (i.e. final state interactions or weak annihilation) [5] and “gluonic spectator effects” (diagrams where photon is radiated off the spectator quark). In the later case, the introduction of new gluonic penguin operator enhances these effects making the ratio (3) to be a potentially sensitive test of the ECMDO models. In what follows we consider isospin symmetry violations in the decay $B \to K^{*}\gamma$ for different values of magnitude and phase of the $C_{11}^{ECMD}$ Wilson coefficient.

We proceed with the estimation of the effect for the exclusive transition $B \to K^{*}\gamma$ using Brodsky-Lepage (BL) perturbative QCD formalism [3]. In this formalism the decay amplitude can be written as

$$Amp = \int dx \ dy \ \phi_{K^*}^\ast(y)T_{\mu}(x, y) \ \xi^\ast \xi \phi_B(x),$$

$$\phi_{K^*}^\ast = \sqrt{3}f_{K^*}(1-y), \ \phi_B(x) = \frac{f_B}{2\sqrt{3}}\delta(x - 1 + \epsilon_B),$$

$$T_{\mu}(x, y) = \frac{1}{2} Tr\left[\xi^\ast(\phi_{K^*} + m_{K^*})t_{\mu}(x, y)\gamma_5(\phi_B - m_B)\right].$$  (4)

Here $x(y)$ is the momentum fraction carried by $b(s)$-quark inside of the meson. The hard scattering amplitude $t_{\mu}(x, y)$ is calculated from the diagrams presented in the Fig.1 and the asymptotic expressions for the hadronic wave functions are used [6]. There are four classes

1 Although the use of asymptotic wave functions leads to the underestimation of the branching ratios, hadronic uncertainties are cancelled to some extend in (3). Eventually one might study the effect using more model-independent methods, e.g. QCD sum rules.
of the spectator correction graphs involved [4]: {1} Photonic penguin diagrams. These give the leading contribution to the decay rate but do not involve gluonic dipole operators thus contributing to the asymmetry (3) through the interference terms. {2} “Triangle” and $W$-bremsstrahlung graphs. Their contributions are small compared to {1} and they do not modify the asymmetry. We drop these contributions hereafter. {3} Bremsstrahlung diagrams involving photon emission from external legs. These include spectator bremsstrahlung diagrams enhanced by large gluonic dipoles and thus largely responsible for the isospin symmetry breaking in the ECMDO models. {4} Weak annihilation graphs that contribute to the decays of $B^-$ but not to $B^0$ mesons thus forming the Standard Model “background” to the asymmetry.

We calculate the branching ratios according to the power counting of [3] modified for the leading non-zero $K^*$ mass effects. In particular, since the main focus here is the estimation of the isospin asymmetry, terms up to the order of $m_{K^*}/(m_B \epsilon_B)$ (scale of the weak annihilation contribution) [3], $\epsilon_B = \bar{\Lambda}/m_B \sim 0.065 - 0.1$ must be included. The power counting is governed by the peak approximation to $B$-meson distribution function and by the expansion of the pQCD amplitude in powers of $1/m_B$: terms $1/\epsilon_B$ scale like $m_B^1$ and $m_{K^*}/(m_B \epsilon_B)$ scale like $m_B^0$, so we must keep all of the terms up to the order $m_B^0$. [4] In this calculation we systematically neglect terms $\sim m_{K^*}^2$ with respect to the terms $\sim m_{K^*}$ in the amplitude. Gauge invariance implies that the decay rate can be written as

$$
\Gamma(B(p_B) \rightarrow K^*(p_{K^*}) \gamma(q)) = \frac{1}{16\pi} \frac{m_B^2 - m_{K^*}^2}{m_B^3} (|a|^2[3 - \frac{y}{x^2}(2x - 1)] + 2|c|^2x^2) \tag{5}
$$

with $x = p_B \cdot q/m_B^2$, $y = 1 - m_B^2(1 - x)^2/m_{K^*}^2$, and $a(c) = \sum_i a_i(c_i)$, and

$$
t_{\mu \nu} = a_i[g_{\mu \nu} - \frac{1}{xm_B^2} p_\mu p_\nu] - \frac{ic_i}{m_B^2} \epsilon_{\mu \nu \alpha \beta} p_\alpha q_\beta \tag{6}
$$

\[2\]In principle, any term proportional to $1/\epsilon_B$ gives $m_B/\bar{\Lambda}$ proportional to the non-perturbative parameter $\bar{\Lambda}^{-1}$, thus bringing non-perturbative uncertainty to the pQCD calculation. One can fix this uncertainty by fixing the value of $\bar{\Lambda}$ from other $B$-decays.
The dominant contribution comes from the diagrams of the class \{1\}.

\[ t_1 = \frac{ig_s^2 C}{2Q^2 k_1^2} \mathcal{T} r \, \psi^* (\not{p} + m_{K^*}) \gamma_\alpha \bar{k}_1 V_{10} \cdot \xi^* \gamma_5 (\not{p} - m_B) \gamma_\alpha, \]

\[ t_2 = \frac{ig_s^2 C}{2Q^2 (k_2^2 - m_B^2 (1 - \epsilon_B)^2)} \mathcal{T} r \, \psi^* (\not{p} + m_{K^*}) V_{10} \cdot \xi^* [k_2 + m_B (1 - \epsilon_B)] \gamma_\alpha \gamma_5 (\not{p} - m_B) \gamma_\alpha. \]  

(7)

where \( C = \frac{16 \alpha_s}{3 \sqrt{2}} G_F V_{td} V_{ts} C_{10}^{eff}, \) \( C_{10}^{eff} = -0.32, \) \( V_{10} = m_b i \sigma_{\alpha \beta} (1 - \gamma_5) q_\beta, \) and \( Q_\alpha \) is the momentum of a gluon. Also,

\[ k_1^2 = -m_B^2 x (1 - x), \]

\[ k_2^2 - m_b^2 = -m_B^2 (1 - y - 2 \epsilon_B), \]

\[ Q^2 = -m_B^2 (x - y) (1 - x). \]  

(8)

Note that \( m_b = m_B (1 - \epsilon_B). \) Calculating the traces we arrive at

\[ a_1 = -\frac{1}{4} \frac{\alpha_s}{2 \pi} C f_B f_{K^*} \frac{m_{K^*}}{\epsilon_B m_B}, \]

\[ c_1 = -\frac{1}{2} \frac{\alpha_s}{2 \pi} C f_B f_{K^*} \frac{m_{K^*}}{\epsilon_B m_B}, \]

\[ a_2 = -\frac{1}{2} \frac{\alpha_s}{2 \pi} C f_B f_{K^*} \frac{m_B}{4 \epsilon_B} \left\{ \epsilon_B + \frac{m_{K^*}}{m_B} - 4 \epsilon_B \log \frac{1 - 2 \epsilon_B}{2 \epsilon_B} + 2 \epsilon_B \log \frac{1 - \epsilon_B}{\epsilon_B} - 2 i \pi \right\}, \]

\[ c_2 = -\frac{1}{2} \frac{\alpha_s}{2 \pi} C f_B f_{K^*} \frac{m_B}{2 \epsilon_B} \left\{ -\epsilon_B + \frac{m_{K^*}}{m_B} + 4 \epsilon_B \log \frac{1 - 2 \epsilon_B}{2 \epsilon_B} - 2 \epsilon_B \log \frac{1 - \epsilon_B}{\epsilon_B} + 2 i \pi \right\}. \]  

(9)

The diagram 1 of the class \{1\} was dropped in [3] in the approximation \( m_{K^*} = 0. \) The major contribution to the asymmetry [3] comes from the interference of the diagrams of class \{3\} (especially those involving spectator bremsstrahlung) and class \{1\} where the former includes the vertex for \( bsg V_{11}^{11}. \)

\[ V_{11}^{11} = F_1 (Q^2 g_{\alpha \beta} - Q_\alpha Q_\beta) \gamma_\beta (1 + \gamma_5) + F_2 m_b i \sigma_{\alpha \beta} Q_\beta (1 - \gamma_5) \]  

(10)

with \( F_i \) being the QCD-corrected Inami-Lim functions, \( F_2 = C_{11}. \) In the Standard Model \( C_{11}^{SM} = -0.159, \) \( |C_{11}^{ECMD}| \approx 7 |C_{11}^{SM}|. \) The denominators are

\[ k_5^2 = (1 - y) m_B^2 \left\{ 1 - y (1 - y) \frac{m_{K^*}^2}{m_B^2} \right\} \rightarrow (1 - y) m_B^2. \]
\[ k_6^2 = (1 - x) m_B^2 \{ -x + \frac{m_{K^*}^2}{m_B^2} \} \rightarrow -x(1 - x) m_B^2, \]
\[ Q^2 = m_B^2 (x - y) \{ x - \frac{m_{K^*}^2}{m_B^2} \} \rightarrow m_B^2 x(x - y). \] (11)

The formfactors for the spectator bremsstrahlung diagrams read
\[
\begin{align*}
  a_5 &= -q_{u,d} C' m_B^2 f_B f_{K^*} \frac{\alpha_s}{2\pi} \left\{ -\frac{3}{4} F_1 + \left[ -\frac{3}{4} + \log \frac{1 - \epsilon_B}{\epsilon_B} + i\pi \right] F_2 \right\}, \\
  c_5 &= -q_{u,d} C' m_B^2 f_B f_{K^*} \frac{\alpha_s}{2\pi} \left\{ -\frac{3}{2} F_1 + \left[ \frac{3}{2} - 2 \log \frac{1 - \epsilon_B}{\epsilon_B} - 2i\pi \right] F_2 \right\}, \\
  a_6 &= -q_{u,d} C' m_B^2 f_B f_{K^*} \frac{1}{1 - 2\epsilon_B} \frac{\alpha_s}{2\pi} \left\{ F_1 \left[ -\frac{1}{2} - \frac{m_{K^*}}{6 m_B} \right] + F_2 \left[ \frac{\epsilon_B}{2} + \frac{m_{K^*}}{6 m_B} \right] \right\}, \\
  c_6 &= -q_{u,d} C' m_B^2 f_B f_{K^*} \frac{1}{1 - 2\epsilon_B} \frac{\alpha_s}{2\pi} \left\{ F_1 \left[ -\frac{1}{2} - \frac{m_{K^*}}{6 m_B} \right] - F_2 \left[ \frac{\epsilon_B}{2} + \frac{m_{K^*}}{6 m_B} \right] \right\}. \quad (12)
\]

with \( C' = \frac{4}{\sqrt{2}} G_F V_{tb} V_{ts}^* \). In the Standard Model, the contribution of the operator
\[
O_{11} = \frac{g_s}{16\pi^2} m_b \bar{s} \sigma^{\mu\nu} \frac{1}{2} (1 - \gamma_5) \epsilon^a b G_{\mu\nu}^a
\] (13)
is suppressed, mainly because of the numerical smallness of the \( C_{11} \) coefficient in comparison with the photonic dipole coefficient \( C_{10} \): in the SM \( C_{10}/C_{11} \approx 2 \). In addition, bremsstrahlung diagrams are suppressed dynamically. In the ECMDO, the \( C_{11} \) is approximately one order of magnitude higher, therefore compensating to some extent the dynamical suppression.

Clearly, the relative phase between the ECMDO and SM \( C_{11} \)'s is not fixed, thus leaving some freedom, and, in principle, it can be chosen in a way that it does not affect \( B \to K^{*}\gamma \) SM predictions. This calculation, however, indicate that the isospin asymmetry is large on the large portion of the available parameter space therefore providing a good constraint on its value. The results of calculations are presented in the Fig.2.

Unfortunately, the isospin asymmetry has not yet been a target for the experimental investigation. Recent CLEO data [3], for instance, gives \( Br_{B \to K^{*}\gamma} = 3.8^{+2.9}_{-1.7} \pm 5.0 \) and \( Br_{B^0 \to K^{*}\gamma} = 4.4 \pm 1.0 \pm 0.6 \) providing only a rough estimate of the isospin symmetry violations in the range \( 0 - 50\% \) which is clearly unsatisfactory for singling out the ECMDO contribution. Hopefully, combined results from the isospin violation in \( B \to K^{*}\gamma \) and direct measurements would put strong constraints on the values of contributions from ECMDO models.
FIG. 1. Diagrams for the gluonic spectator corrections.

FIG. 2. Isospin violation asymmetry for different values of phase between SM and ECMDO coefficients.
REFERENCES

[1] A. L. Kagan, Phys. Rev. D51 (1995) 6194; M. Ciuchini, E. Gabrielli, G.F. Giudice, Phys. Lett. B388 (1996) 353; see also B. Grazadkowski, W.-S. Hou, Phys. Lett. B272 (1991) 383.

[2] A. L. Kagan, A. F. Perez, in preparation; A. F. Perez, Talk delivered at Phenomenology-96 Symposium, Wisconsin, unpublished; A. L. Kagan, Talk delivered at DPF-96 Meeting, Minnesota, to be published in the proceedings.

[3] C.E. Carlson, J. Milana, Phys. Rev. D51 (1995) 4950.

[4] J.F. Donoghue, A.A. Petrov, Phys. Rev D53 (1996) 3664.

[5] J.F. Donoghue, E. Golowich, A.A. Petrov, Preprint UMHEP-433, hep-ph/9609530, to be published in Phys. Rev. D; A. Ali, V.M. Braun, Phys. Lett. B359 (1995); H.-Y. Cheng, Phys. Rev. D51 (1995).

[6] R. Kass, “Recent results from CLEO”, Talk delivered at 1996 SLAC Summer Institute, to be published in the proceedings.