On the Achievable Rate Regions for Interference Channels With Degraded Message Sets

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Abstract—The interference channel with degraded message sets (IC-DMS) refers to a communication model, in which two senders attempt to communicate with their respective receivers simultaneously through a common medium, and one sender has complete and a priori (non-causal) knowledge about the message being transmitted by the other. A coding scheme that collectively has advantages of cooperative coding, collaborative coding, and dirty paper coding, is developed for such a channel. With resorting to this coding scheme, achievable rate regions of the IC-DMS in both discrete memoryless and Gaussian cases are derived. The derived achievable rate regions generally include several previously known rate regions as special cases. A numerical example for the Gaussian case further demonstrates that the derived achievable rate region offers considerable improvements over these existing results in the high-interference-gain regime.

Index Terms—Cognitive radio, dirty paper coding, Gel’fand-Pinsker coding, interference channel, superposition coding.

I. INTRODUCTION

The interference channel with degraded message sets (IC-DMS) refers to a communication model, in which two senders attempt to communicate with their respective receivers simultaneously through a common medium, and one sender has complete and a priori (non-causal) knowledge about the message being transmitted by the other. Such a model generically characterizes some realistic communication scenarios taking place in cognitive radio networks or in wireless sensor networks over a correlated field [1]–[5].

From an information-theoretic perspective, the IC-DMS have been investigated in [1]–[5]. Specifically, several achievable results have been obtained in [1]–[5], and the capacity regions for two special cases have been characterized in [2]–[5]. The main achievable rate region in [1, Theorem 1] was obtained by incorporating Gel’fand-Pinsker coding [6] into the well-known coding scheme applied to the interference channel (IC) [7], [8]. In this coding scheme, each sender splits its message into two sub-messages, and allows its non-pairing receiver to decode one of the sub-messages. Knowing the two sub-messages and the corresponding codewords which sender 1 wishes to transmit, sender 2 applies Gel’fand-Pinsker coding to encode its own sub-messages by treating the codewords of sender 1 as known interferences. It has been also shown in [1, Corollary 2] that, an improved achievable rate region can be attained by time-sharing between the main rate region [1, Theorem 1] and a so called fully-cooperative rate point achieved by letting sender 2 use all of its power to transmit messages of sender 1. A different coding scheme was proposed in [2] and [3], in which neither of the senders splits its message into sub-messages, and receiver 2 does not decode any transmitted information from sender 1. Since sender 2 knows what sender 1 wishes to transmit, sender 2 is allowed to: 1) apply Gel’fand-Pinsker coding to encode its own message; and 2) partially cooperate with sender 1 using superposition coding. It has been proven in [2], [3] that, this is a capacity-achieving scheme for the Gaussian IC-DMS (GIC-DMS) in the low-interference-gain regime, in which the normalized link gain between sender 2 and receiver 1 is less than or equal to 1.

However, in practice, due to the mobility of wireless users or random geographic distributions of wireless sensors, sender 2 may be located near to receiver 2, as illustrated in Fig. 1. It is likely, in such a situation, that the normalized link gain between sender 2 and receiver 1 is greater than 1, which we term the high-interference-gain regime. In fact, the findings in this correspondence reveal that the achievable rate region that has been proven to be the capacity region in the low-interference-gain regime in [2] and [3], is strictly non-optimal for the Gaussian IC-DMS in the high-interference-gain regime.

In this correspondence, we develop a new coding scheme for the IC-DMS to improve existing achievable rate regions. Our coding scheme differs from the one proposed in [2], [3] in the way that, sender 2 splits its own message into two sub-messages, and encodes both sub-messages using Gel’fand-Pinsker coding. Moreover, receiver 1 is required to jointly decode the message from sender 1 and one sub-message from sender 2. Rate splitting is applied to enable receiver 1 to crossly observe partial information from sender 2, thus reducing the effective interference at receiver 1, whereas Gel’fand-Pinsker coding is applied to exploit the known interference at sender 2. With this coding scheme, we derive an achievable rate region for the discrete memoryless case, which is the main achievable rate result in the correspondence. We further show that our region includes several existing regions as special cases. Lastly, we extend the obtained regions from the discrete memoryless case to the Gaussian case, and show by a numerical example that our achievable rate region strictly improves the existing ones [2], [3] in the high-interference-gain regime.

Recently, a similar coding scheme has been proposed for the IC-DMS in the independent work [9]–[11]. The main differences between the coding scheme [9]–[11] and our coding scheme can be described as follows: 1) rate splitting is employed at both senders in [9]–[11], whereas in our coding scheme rate splitting is only employed at sender 2; 2) Gel’fand-Pinsker coding is applied twice in a successive manner in [9]–[11], whereas Gel’fand-Pinsker coding is applied twice in a parallel manner in our coding scheme. However, it is not clear how the differences in coding schemes affect achievable rate results.

II. THE CHANNEL MODEL

Consider the IC-DMS (also termed as the genie-aided cognitive radio channel in [1]) depicted in Fig. 2, where sender 1 wishes to...
transmit a message (or a message index), $w_1 \in M_1 := \{1, ..., M_1\}$, to receiver 1, and sender 2 wishes to transmit its message, $w_2 \in M_2 := \{1, ..., M_2\}$, to receiver 2. Typically, the discrete memoryless IC-DMS is described by a tuple $(X_1, X_2, Y_1, Y_2, p(y_1, y_2|x_1, x_2))$, where $X_1$ and $X_2$ are the channel input alphabets, $Y_1$ and $Y_2$ are the channel output alphabets, and $p(y_1, y_2|x_1, x_2)$ denotes the conditional probability of $(y_1, y_2) \in Y_1 \times Y_2$ given $(x_1, x_2) \in X_1 \times X_2$. The channel is discrete memoryless in the sense that

$$p(y_1, y_2|x_1, x_2, x_1', x_2', x_1, x_1' - 1, x_2, x_2 - 1, ...) = p(y_1, y_2|x_1, x_2),$$

for every discrete time instant $t$ in a synchronous transmission.

In view of the channel input-output relationship, the IC-DMS is the same as the IC. However, in the IC-DMS, sender 2 is able to non-causally obtain the knowledge of the message $w_1$, which will be transmitted from sender 1. This is the key difference between the IC-DMS and IC in terms of the information flow. We next present the following standard definitions with regard to the existence of codes and achievable rates for the discrete memoryless IC-DMS channel.

**Definition 1:** An $(M_1, M_2, n, P_e^{(n)})$ code for the discrete memoryless IC-DMS consists of

i) two encoding functions $f_1 : M_1 \rightarrow X_1^n$, and $f_2 : M_1 \times M_2 \rightarrow X_2^n$;

ii) two decoding functions $g_1 : Y_1^n \rightarrow M_1$, and $g_2 : Y_2^n \rightarrow M_2$;

iii) the average probability of error $P_e^{(n)} := \max\{P_e^{(n)}(1), P_e^{(n)}(2)\}$, where $P_e^{(n)}(i)$ denotes the average probability of error at decoder $i$, and is computed as

$$P_e^{(n)}(i) = \frac{1}{M_1 M_2} \sum_{w_1, w_2} p(\hat{w}_i \neq w_i | (w_1, w_2) \text{ sent}), \quad i = 1, 2.$$

**Definition 2:** A non-negative rate pair $(R_1, R_2)$ is said to be achievable for the IC-DMS, if there exists a sequence of $(M_1, M_2, n, P_e^{(n)})$ codes with

$$R_1 \leq \log M_1 \frac{n}{n}, \quad \text{and} \quad R_2 \leq \log M_2 \frac{n}{n},$$

such that $P_e^{(n)}$ approaches zero as $n \to \infty$. The capacity region of the IC-DMS is the set of all the achievable rate pairs, and an achievable rate region is a subset of the capacity region.

**III. An Achievable Rate Region for the Discrete Memoryless IC-DMS**

In this section, we present a new achievable rate region for the discrete memoryless IC-DMS, which is the primary result in this correspondence.

Consider auxiliary random variables $W, U, V$, and a time-sharing random variable $Q$ defined on arbitrary finite sets $W, U, V$, and $Q$, respectively. Let $\mathcal{P}$ denote the set of all joint probability distributions $p(\cdot)$ that factor in the form of

$$p(q, w, x_1, u, v, x_2, y_1, y_2) = p(q|p(w, x_1|q)) p(u|w, q) p(v|w, q) \times p(x_2|u, v, w) p(y_1, y_2|x_1, x_2),$$

(1)

where $w, u, v, q$ are realizations of random variables $W, U, V$, and $Q$.

Let $\mathcal{R}(p)$ denote the set of all non-negative rate pairs $(R_1, R_2)$ such that the following inequalities hold simultaneously

$$R_1 \leq \min\{I(W; U, Y_1|Q), I(W; U; Y_1|Q)\}, \quad (2)$$

$$R_2 \leq I(U; V; Y_2|Q) + I(V; W|Q) - I(V; W|Q), \quad (3)$$

$$R_1 + R_2 \leq I(W, U; Y_1|Q) + I(W; V|Q) - I(V; W|Q), \quad (4)$$

$$0 \leq I(U, V; Y_1|Q), \quad (5)$$

$$0 \leq I(V, U, Y_2|W), \quad (6)$$

for a given joint distribution $p(\cdot) \in \mathcal{P}$.

Let $\mathcal{C}$ denote the capacity region of the discrete memoryless IC-DMS, and let

$$\mathcal{R} := \bigcup_{p(\cdot) \in \mathcal{P}} \mathcal{R}(p).$$

**Theorem 1:** The region $\mathcal{R}$ is achievable for the discrete memoryless IC-DMS, i.e.,

$$\mathcal{R} \subseteq \mathcal{C}.$$
apply Gel’fand-Pinsker coding to deal with the known interference, but also can cooperate with sender 1 to transmit \( w_1 \) using superposition coding. Let \( R_{21} \) and \( R_{22} \) denote the rates of \( w_{21} \) and \( w_{22} \) respectively, i.e., \( w_{2j} \in \{1, \ldots, 2^nR_{2j}\} \) and \( w_{2j} \in \{1, \ldots, 2^nR_{2j}\} \). If receiver 1 can decode \( w_1 \) and receiver 2 can decode both \( w_{21} \) and \( w_{22} \) with vanishing probabilities of error, then \((R_{1}, R_{21} + R_{22}\) is an achievable rate pair for the IC-DMS.

In the following proof, we will frequently use the notion of jointly typical sequences and joint asymptotic equipartition property [13, Section 14.2].

**Proof:** To prove that the entire region \( \mathcal{R} \) is achievable for the channel, it is sufficient to prove that \( \mathcal{R}(p) \) is achievable for a fixed joint probability distribution \( p(\cdot) \) in \( \mathcal{P} \).

### A. Random Code Generation

Consider a fixed joint distribution \( p(\cdot) \in \mathcal{P} \), and a random time-sharing codeword \( q \) of length \( n \). The codeword \( q \) that is revealed to both the senders and receivers, is assumed to be generated according to \( \prod_{j=1}^n p(q_j) \).

Generate \( 2^{nR_1} \) independent codewords \( \mathbf{W}(j), j \in \{1, \ldots, 2^nR_1\} \), according to \( \prod_{j=1}^n p(w_{1j}(q_j)) \), and for each \( w(j) \) generate one codeword \( \mathbf{X}_1(j) \), according to \( \prod_{j=1}^n p(x_{l1}(q_j)) \). Generate \( 2^{n(R_{21}+H(W|U)\pm\epsilon)} \) independent codewords \( \mathbf{U}(1), l_1 \in \{1, \ldots, 2^nR_{21}\} \), according to \( \prod_{j=1}^n p(u_{l1}(q_j)) \), and generate \( 2^{n(R_{22}+H(W|V)\pm\epsilon)} \) independent codewords \( \mathbf{V}(2), l_2 \in \{1, \ldots, 2^nR_{22}\} \), according to \( \prod_{j=1}^n p(v_{l2}(q_j)) \), where \( \epsilon \) denotes an arbitrarily small positive number. Lastly, for each codeword triple \( (\mathbf{u}(1), \mathbf{v}(2), \mathbf{w}(j)) \), generate one codeword \( \mathbf{X}_2(l_1, l_2, j) \) according to \( \prod_{j=1}^n p(x_{2j}(u_{l1}(q_j), v_{l2}(q_j), w_{1j}(q_j))) \). Now uniformly distribute the \( 2^{n(R_{21}+H(W|U)\pm\epsilon)} \) codewords \( \mathbf{u}(1) \) into \( 2^{nR_1} \) bins indexed by \( k_1 \in \{1, \ldots, 2^nR_{21}\} \), such that each bin contains \( 2^n(H(W|U)\pm\epsilon) \) codewords, and uniformly distribute the \( 2^{n(R_{22}+H(W|V)\pm\epsilon)} \) codewords \( \mathbf{v}(2) \) into \( 2^{nR_{22}} \) bins indexed by \( k_2 \in \{1, \ldots, 2^nR_{22}\} \) such that each bin contains \( 2^n(H(W|V)\pm\epsilon) \) codewords. The entire codebook is revealed to both the senders and receivers.

### B. Encoding and Transmission

We assume that two senders wish to transmit a message vector \((w_{11}, w_{21}, w_{22}) = (j, k_1, k_2)\). Sender 1 simply encodes the message as a codeword \( x_1(j) \) and sends the codeword in \( n \) channel uses.

Let \( \mathcal{A}_1^{nR_1} \) denote a jointly typical set. Sender 2 first needs to look for a codeword \( u(1, j) \) in bin \( k_1 \) such that \((u(1, j), w_{1j}(q_j), q_j) \in \mathcal{A}_1^{nR_1} \), and a codeword \( v(2, j) \) in bin \( k_2 \) such that \((v(2, j), w_{1j}(q_j), q_j) \in \mathcal{A}_1^{nR_1} \). If sender 2 finds such \( u(1, j) \) and \( v(2, j) \) successfully, the codeword \( x_{2}(l_1, l_2, j) \) is sent through \( n \) channel uses. Otherwise, sender 2 declares an encoding error.

### C. Decoding

Receiver 1 first looks for all the index pairs \((j, l_1)\) such that \((w_{1j}(q_j), u(1, j), y_1, q_j) \in \mathcal{A}_1^{nR_1} \). If \( j \in \{i, j\} \) all the index pairs found are the same, receiver 1 declares \( w_1 = j \). Otherwise, receiver 1 declares a decoding error. Receiver 2 will first look for all index pairs \((l_1, l_2)\) such that \((u(1, j), v(2, j), y_2, q_j) \in \mathcal{A}_1^{nR_1} \). If \( l_1 \) in all the index pairs found are indices of codewords \( u(1, j) \) from the same bin with index \( k_1 \), and \( l_2 \) in all the index pairs found are indices of codewords \( v(2, j) \) from the same bin with index \( k_2 \), then receiver 2 declares that the messages \((w_{21}, w_{22}) = (k_1, k_2) \) were transmitted. Otherwise, a decoding error is declared.

### D. Evaluation of Probabilities of Error

We now derive upper bounds for the probabilities of the respective error events which may happen during the encoding and decoding processes. Due to the symmetry of the codebook generation and encoding processing, the probability of error is not codeword dependent. Without loss of generality, we assume that the messages \((w_{12}, w_{21}, w_{22}) = (1, 1, 1)\) are encoded and sent. We further assume that the codewords \( u(1, 1) \) and \( v(2, 1) \) found in the respective bin 1 during the encoding process are \( u(1) \) and \( v(1) \), respectively. Hence, \( x_{1}(1) \) and \( x_{2}(1, 1, 1) \) are transmitted. We next define the following four types of events:

\[
\begin{align*}
E_{a,b}^u &= (U(a), W(b), q) \in \mathcal{A}_1^{nR_1}, \\
E_{a,b}^v &= (V(a), W(b), q) \in \mathcal{A}_1^{nR_1}, \\
E_{a,b} &= (U(a), W(b), Y_1, q) \in \mathcal{A}_1^{nR_1}, \\
E_{a,b} &= (U(a), W(b), Y_2, q) \in \mathcal{A}_1^{nR_1}.
\end{align*}
\]

Let \( P_{e}(\text{enc}2), P_{e}(\text{dec}1) \), and \( P_{e}(\text{dec}2) \) denote the probabilities of error at the encoder of sender 2, the decoder of receiver 1, and the decoder of receiver 2, respectively.

**[Evaluation of \( P_e(\text{enc}2) \).]** An error is made if 1) the encoder at sender 2 cannot find \((u(1), w_{1j}(q_j), q_j) \in \mathcal{A}_1^{nR_1} \), and/or 2) it cannot find \((v(2), w_{1j}(q_j), q_j) \in \mathcal{A}_1^{nR_1} \). The probability of error at the encoder of sender 2 is bounded as

\[
P_e(\text{enc}2) \leq \text{Pr} \left( \bigcap_{U(1) \in \text{bin} 1} \left( (U(1), W(1), q) \notin \mathcal{A}_1^{nR_1} \right) \right) + \text{Pr} \left( \bigcap_{V(2) \in \text{bin} 1} \left( (V(2), W(1), q) \notin \mathcal{A}_1^{nR_1} \right) \right) = \prod_{U(1) \in \text{bin} 1} \left( 1 - \text{Pr}(E_{a,b}^u) \right) \prod_{V(2) \in \text{bin} 1} \left( 1 - \text{Pr}(E_{a,b}^v) \right) \leq \left( 1 - \text{Pr}(E_{a,b}^u) \right)^{2^n(H(W|U)\pm\epsilon)} + \left( 1 - \text{Pr}(E_{a,b}^v) \right)^{2^n(H(W|V)\pm\epsilon)}.
\]

As the time-sharing sequence \( q \) is predetermined, we have

\[
P_e(\text{enc}2) \leq \sum_{u(1, q_j) \in \mathcal{A}_1^{nR_1}} \text{Pr}(U(1) = u(1) \mid q) \text{Pr}(W(1) = w(1) \mid q) \geq \sum_{u(1, q_j) \in \mathcal{A}_1^{nR_1}} \text{Pr}(U(1) = u(1)) \text{Pr}(W(1) = w(1)) \geq 2^{n(H(U|W)\pm\epsilon) - n(H(W|U)\pm\epsilon)} = 2^{-n(U:W+\epsilon)}.
\]

Similarly, we can obtain \( P_e(\text{dec}1) \geq 2^{n(V:W+\epsilon)} \), and \( P_e(\text{dec}2) \geq 2^{n(V:W+\epsilon)} \).

**[Evaluation of \( P_e(\text{dec}1) \).]** An error is made if 1) \( \hat{E}_{1i_1} \) happens, and/or 2) there exists some \( j \neq 1 \) such that \( \hat{E}_{1j} \) happens. Note that the error events \( \hat{E}_{1i_1} \) with \( i_1 \neq 1 \) are not considered as error events,
and are excluded from the computation of the probability of error, as it is unnecessary for receiver 1 to correctly decode $l_1$. The probability of error at the decoder of receiver 1 can be upper bounded as

$$ P_e = \Pr(E_{1,1}^c) = \Pr(E_{1,1}^c) + \sum_{j \neq 1} \Pr(E_{j,j}^c) $$

Substituting (9) and (10) into (8), we obtain

$$ \Pr(E_{2,1}) \leq \Pr(E_{2,1})^c + 2nR_{21} + I(W;U|Y) + 4\epsilon \Pr(E_{2,2}). $$

Similarly, we obtain

$$ \Pr(E_{2,2}) = \sum_{(w, u, y_1, q) \in A_{2}^{(n)}} P(W(2) = w|q)P(U(1) = u, Y_1 = y_1|q) $$

$$ \leq 2^n(H(W;Y_1|Q) + I(W;U|Y) - I(W;Y_1|Q)) $$

Substituting (9) and (10) into (8), we obtain

$$ P_e(\text{dec}1) \leq \epsilon + 2^{-n(I(W;U,Y_1|Q) - R_1 - \epsilon)} $$

$$ R_1 \leq I(W;U,Y_1|Q), $$

and (12)

$$ R_1 + R_2 \leq I(W;U,Y_1|Q), $$

are satisfied.

**Evaluation of $P_e(\text{dec}2)$** An error is made if 1) $E_{1,1}^c$ happens, and/or 2) there exists some $(\hat{l}_1, \hat{l}_2)$ in which either $\hat{l}_1$ or $\hat{l}_2$ is not an index of any codeword from the respective bin 1 such that $E_{\hat{l}_1,\hat{l}_2}^c$ happens. The probability of the second case is upper bounded by the probability of the event, $E_{l_1,l_2}$, for some $(l_1, l_2) \neq (1, 1)$. Thus, the probability of error at the decoder of receiver 2 is bounded as

$$ P_e \leq \Pr(E_{2,1})^c + 2nR_{21} + I(W;U|Q) + 2\epsilon \Pr(E_{2,2}). $$

Following the same way as we derived upper-bounds of $\Pr(E_{2,1})$ and $\Pr(E_{2,2})$ in (9) and (10), we upper bound $\Pr(E_{2,1})$, $\Pr(E_{2,2})$, and $\Pr(E_{2,2})$ as

$$ \Pr(E_{2,1}) \leq 2^{-n(I(U,Y_1|Q) - 3\epsilon)}, $$

$$ \Pr(E_{2,2}) \leq 2^{-n(I(U,Y_1|Q) - 3\epsilon)}, $$

$$ \Pr(E_{2,2}) \leq 2^{-n(I(U,Y_1|Q) + I(W;V|Q) - 4\epsilon)}. $$

Substituting (14)–(16) into (13), we conclude that $P_e(\text{dec}2) \rightarrow 0$ as $n \rightarrow \infty$ if

$$ R_{21} \leq I(U;V,Y_2|Q) - I(W;U|Q), $$

$$ R_{22} \leq I(U;V,Y_2|Q) - I(W;V|Q), $$

$$ R_{21} + R_{22} \leq I(U;V,Y_2|Q) + I(U;V|Q) - I(W;U|Q) - I(W;V|Q), $$

are satisfied.

If (11)–(12) and (17)–(19) are satisfied, the average probabilities of error at both decoders diminish as $n \rightarrow \infty$. We hence conclude that a $(2^nR_1, 2^nR_{21} + 2^nR_{22}, n, P_e^{(n)})$ code with $P_e^{(n)} \rightarrow 0$ exists for the channel. Furthermore, we obtain (2)–(6) by applying Fourier-Motzkin elimination [15]–[17] on (11)–(12), (17)–(19), $R_{21} \geq 0$ and $R_{22} \geq 0$. Therefore, the rate region $\mathcal{R}(p)$ is achievable for a fixed joint probability distribution $p(\cdot) \in P$, and Theorem 1 follows.

**Remark 1** The proposed coding scheme exploits three coding methods to achieve any rate pair in the rate region, $\mathcal{R}$. The first method is cooperation that is realized by applying the superposition relationship between $w$ and $x_2$ via $p(x_2|u, v, w, q)$. The second one is collaboration, by which we mean that sender 2 separates its own message into two parts, i.e., $w_2 = (w_{21}, w_{22})$, and encodes $w_{21}$ at a possibly low rate such that receiver 1 can decode it. By doing so, the effective interference caused by the signals carrying the information from sender 2 may be reduced. The third one is *Gelfand-Pinsker coding* [6], which we apply to encode both messages, $w_{21}$ and $w_{22}$, from sender 2 by treating the codeword $w$ as known interference. This perhaps allows receiver 2 to be able to decode the messages from sender 2 at the same rate as if the interference caused by sender 1 was not present.

**IV. RELATING WITH SOME EXISTING RATE REGIONS**

In this section, we show that the achievable rate region $\mathcal{R}$ in Theorem 1 includes those derivated in [2], [3]. We further demonstrate with a Gaussian numerical example that, the improvement of $\mathcal{R}$ is strict in the high-interference-gain regime.

**A. The Discrete Memoryless Case**

Let $\mathcal{P}_1^*$ denote the set of all joint probability distributions that factor in the form of

$$ p(q, w, x_1, v, x_2, y_1, y_2) = p(q)p(x_1, v|w)p(v|w, q)p(x_2|v, w, q) $$

$$ \times p(y_1, y_2|x_1, x_2). $$

Let $\mathcal{R}_{q_1}(p)$ denote the set of all non-negative rate pairs $(R_1, R_2)$ such that

$$ R_1 \leq I(W;Y_1|Q), $$

$$ R_2 \leq I(V;Y_2|Q) - I(V;W|Q), $$

for a fixed joint distribution $p(\cdot) \in \mathcal{P}_1^*$. Define

$$ \mathcal{R}_{q_1} := \bigcup_{p(\cdot) \in \mathcal{P}_1^*} \mathcal{R}_{q_1}(p). $$
Corollary 1: The region $R_{q1}$ is an achievable rate region for the discrete memoryless IC-DMS, i.e.,
\[
R_{q1} \subseteq R \subseteq C.
\]

Proof: Fixing the auxiliary random variable $U$ as a constant, we reduce (2)–(6) to (21) and (22), and the corollary follows immediately. ■

Remark 2: The achievable rate region $R_{q1}$ is identical to the region $R_{q2}$ reported in [3, Theorem 3.1], which is the discrete memoryless counterpart of the region given in [2, Theorem 4.1] and [3, Theorem 3.5]. It is shown in both [2] and [3] that $R_{q1}$ is the capacity region for the IC-DMS in the low-interference-gain regime.

Let $\mathcal{P}_2$ denote the set of all joint probability distributions $p(\cdot)$ that factor in the form of
\[
p(q, w, x_1, u, x_2, y_1, y_2) = p(q)p(x_1, w|q)p(u|q)p(x_2[u, u, q])p(y_1, y_2|x_1, x_2).
\]  
Let $R_{q2}(p)$ denote the set of all non-negative rate pairs $(R_1, R_2)$ such that
\[
R_1 \leq I(W; Y_1|U, Q),
\]
\[
R_2 \leq \min\left(I(U; Y_1|Q), I(U; Y_2|Q)\right),
\]
for a fixed joint distribution $p(\cdot) \in \mathcal{P}_2$. Define
\[
R_{q2} := \bigcup_{p(\cdot) \in \mathcal{P}_2} R_{q2}(p).
\]

Corollary 2: The region $R_{q2}$ is an achievable rate region for the discrete memoryless IC-DMS, i.e.,
\[
R_{q2} \subseteq R \subseteq C.
\]

Proof: It can be easily observed from (23) that $U$ and $W$ are conditionally independent given $Q$. Employing this fact and choosing $V$ as a constant in (2)–(6), we can readily obtain (24) and (25). ■

Remark 3: The Gaussian counterpart of $R_{q2}$ includes the set of achievable rate pairs in [2, Lemma 4.2] as a subset.

B. The Gaussian Case

With no loss of the information-theoretic optimality, any GIC-DMS can be converted to the GIC-DMS in the standard form through invertible transformations [2], [7], [16]. Hence, we only need to consider the GIC-DMS in the standard form described in the following
\[
Y_1 = X_1 + \sqrt{c_{21}} X_2 + Z_1,
\]
\[
Y_2 = X_2 + \sqrt{c_{12}} X_1 + Z_2,
\]
where $Z_i$, $i = 1, 2$, is the additive white Gaussian noise with zero mean and unit variance, and $\sqrt{c_{21}}$ and $\sqrt{c_{12}}$ are the normalized link gains in the GIC-DMS depicted in Fig. 4.

The transmitted codeword $x_i = (x_{i1}, \ldots, x_{in})$, $i = 1, 2$, is subject to an average power constraint given by $\frac{1}{n} \sum_{i=1}^{n} \|x_{ii}\|^2 \leq P_i$, $i = 1, 2$. Furthermore, we restrict our attention to the Gaussian codewords $X_i^* \leadsto x_{i1}, \ldots, x_{in}$, $i = 1, 2$, for the convenience of evaluation with continuous random variables.

Generally speaking, the achievable regions in Theorem 1, Corollary 1, and Corollary 2 can be extended to the discrete time Gaussian memoryless case with continuous alphabets by quantizing the channel inputs and outputs [18, Chapter 7]. In particular, the Gaussian extension of the rate region $R_{q1}$ in Corollary 1 has been given in [2, Theorem 4.1] and [3, Theorem 3.5]. We next outline how to extend $R$ to its Gaussian counterpart, while the Gaussian extension of $R_{q2}$ can be obtained in a similar manner. We first map the random variables involved in the joint distribution (1) to a set of Gaussian random variables with the following customary constraints:
\[
P1) \quad W \quad \text{distributed according to } \mathcal{N}(0, 1),
\]
\[
P2) \quad X_1 = \sqrt{c_{21}} W,
\]
\[
P3) \quad U \quad \text{distributed according to } \mathcal{N}(0, \alpha \beta P_2),
\]
\[
P4) \quad V \quad \text{distributed according to } \mathcal{N}(0, \alpha \beta P_2),
\]
\[
P5) \quad U = U + \lambda_1 W,
\]
\[
P6) \quad V = V + \lambda_2 W,
\]
\[
P7) \quad X_2 = U + U + \sqrt{\alpha P_2} W,
\]
where $\alpha, \beta \in [0, 1]$, $\alpha + \alpha = 1$, $\beta + \beta = 1$, and $\lambda_1, \lambda_2 \in [0, +\infty)$. The variables $W, U$, and $V$ are assumed to be mutually statistically independent. The mappings P3)–P6) are used to extend the Gel’fand-Pinsker coding to the Gaussian case. The coefficient $\lambda_1$ (or $\lambda_2$) determines the degree of correlation between the Gaussian random variables $W$ and $U$ (or $V$), which plays the same role as the variable $\alpha$ in [19]. The input-output relationship of the GIC-DMS can now be described by
\[
Y_1 = \left(\sqrt{c_{12}} \sqrt{\alpha} W + \sqrt{c_{21}} \sqrt{\beta} P_2 \right) W + \sqrt{c_{21}} \sqrt{\beta} V + Z_1,
\]
\[
Y_2 = U + U + \sqrt{\alpha P_2} W + Z_2.
\]

We fix the time sharing random variable $Q$ as a constant. The issue of how this time-sharing random variable affects the achievable rate region is well addressed in [20]. The rate region $R$ can be extended to its Gaussian counterpart by evaluating the respective mutual information terms in (2)–(6) with respect to the mappings defined by P1)–P7), (26), and (27). Due to limited space, the derivation details and final descriptions of the Gaussian extensions of $R$, $R_{q1}$, and $R_{q2}$ are omitted.

To illustrate improvements of our achievable rate regions over the existing achievable results [2] and [3], we provide the following numerical example (interested readers can refer to [21] for an additional numerical example that compares our rate result with those in [1, Theorem 1 and Corollary 2]).

Example: Fig. 3 compares the Gaussian extensions of rate regions $\mathcal{R}$, $R_{q1}$, and $R_{q2}$ in the high-interference-gain regime, i.e., $c_{21} > 1$. Note that the Gaussian counterpart of $R_{q2}$ includes the set of achievable rate pairs in [2, Lemma 4.2] as a subset. As can be observed in Fig. 3, our achievable rate region in Theorem 1 offers considerable improvements over the rate regions in [2] and [3] under two different parameter settings.

1 Recall that the Gaussian counterpart of $R_{q1}$ is the capacity region for the GIC-DMS in the low-interference-gain regime, i.e., $0 \leq c_{21} \leq 1$. 

Fig. 4. A Gaussian interference channel with degraded message sets.
Fig. 3. Achievable rate regions of the GIC-DMS extended from Theorem 1, Corollary 1, and Corollary 2 with two different settings: (I) $P_1 = 6$, $P_2 = 6$, $c_{21} = 2$, $c_{12} = 0.3$; (II) $P_1 = 6$, $P_2 = 1$, $c_{21} = 2$, $c_{12} = 0.3$.

V. CONCLUSIONS

In this correspondence, we have investigated the IC-DMS from an information theoretic perspective. We have developed a coding scheme that combines the advantages of cooperative coding, collaborative coding, and Gel’fand-Pinsker coding. With this coding scheme, we have derived a new achievable rate region for such a channel, which not only includes two existing results as special cases, but also exceeds them in the high-interference-gain regime.

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