Dissipation coefficients for supersymmetric inflatonary models

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Dissipative effects can lead to a friction term in the equation of motion for an inflaton field during the inflationary era. The friction term may be linear and localised, in which case it is described by a dissipation coefficient. The dissipation coefficient is calculated here in a supersymmetric model with a two stage decay process in which the inflaton decays into a thermal gas of light particles through a heavy intermediate. At low temperatures, the dissipation coefficient \( \propto T^3 \) in a thermal approximation. Results are also given for a non-equilibrium anzatz. The dissipation coefficient is consistent with a warm inflationary regime for moderate (\( \sim 0.1 \)) values of the coupling constants.

I. INTRODUCTION

Inflationary models have made a huge contribution to our understanding of the early universe and the origin of the basic cosmological features which are observed today in ever increasing detail [1, 2, 3]. This paper is concerned with the mechanism for converting the vacuum energy which dominates the universe during inflation into the radiation which dominates after inflation. Radiation can be produced during inflation, at the end of inflation or after inflation has finished. We will consider the first possibility in a particular class of supersymmetric models, to see whether a significant amount of radiation can be production during inflation. When this happens, the situation is described as warm inflation [4, 5] (see also [6]).

The production of radiation is balanced by dissipative terms in the equation of motion of the homogeneous inflaton field \( \phi \). The simplest possibility is a linear dissipation term,

\[
\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V_\phi = 0
\]

where \( V_\phi \) is the \( \phi \) derivative of the inflaton potential, \( H \) is the expansion rate of the universe and \( \Gamma \), which in general will depend on both \( \phi \) and temperature \( T \), is called the dissipation coefficient. The dissipation coefficient was originally introduced as an arbitrary parameter into discussions of reheating [7]. Explicit formulae for the dissipation coefficient were obtained from analysing particle production [8, 9] and linear response theory [10]. Later work has tended to use the Schwinger-Keldysh approach which we use here, eg [11, 12, 13, 14, 15, 16, 17, 18].

Dissipation can lead to important consequences when \( T > H \) (called the weak regime of warm inflation) and when \( \Gamma > H \) (the strong regime). In the weak regime, the primordial density perturbation spectrum is determined by thermal fluctuations rather than vacuum fluctuations. In the strong regime, restrictions on the gradient of the inflaton potential may be relaxed [17]. This is especially important in the context of \( F \)-term supersymmetric inflationary models (see [19] for a review). The potentials in these models contain supergravity corrections which rule out the ordinary type of inflation, but not warm inflation [14, 20].

On the other hand, the calculations which have been done so far have assumed that radiation remains close to thermal equilibrium, and this leads to additional consistency requirements:

- Thermal corrections to the inflaton potential have to be consistent with the restrictions on the gradient of the potential.
- The thermalisation time \( \tau \) of the radiation should be smaller than the expansion timescale of the universe.

These conditions lead to strong restrictions on the existence of warm inflation. Nevertheless, it is useful to investigate the close to equilibrium regime before moving on to the more difficult far from equilibrium regime.

The transfer of energy from the inflaton to another field \( \chi \) is dependent the coupling strength of the interaction, the relative sizes of the mass of the inflaton \( m_\phi \), the mass of decay product \( m_\chi \) and the temperature \( T \). The theory of reheating is well-developed for the post-inflationary era when the inflaton is oscillating and the parameter range \( m_\phi > m_\chi \). According to linear theory, the rate of decay of these oscillations is determined by the decay rate for

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processes such as $\phi \to 2\chi$ [3, 21, 22, 23]. The theory of preheating is the modification which takes into account non-linear effects close to the particle production thresholds [24, 25, 26].

The theory of the dissipation coefficient in the inflationary era is also quite well-developed, especially at high temperature, when $T > m_\chi$. This is the case in which the consistency requirements for an inflationary model are most severe, with large thermal corrections when the radiation is strongly coupled and large relaxation times when the radiation is weakly coupled [27].

Recently, there has been interest in the possibility that the inflaton can dissipate energy at low temperature via a two-stage decay $\phi \to \chi \to yy$, with the heavy field $\chi$ acting as an intermediary in producing a light field $y$ [28, 29, 30]. The dissipation in these models can become a significant feature if the coupling strengths of the inflaton to the heavy field is moderately large.

Inflation is usually associated with a very weakly coupled inflaton to protect the gradient of the potential from picking up large quantum corrections. However, in supersymmetric models the restrictions on the couplings are less severe. The gradient of the inflaton potential can be small enough to allow inflation whilst the thermalisation time for the radiation remains short, allowing warm inflation for moderate values of the coupling constants in these models [20, 31, 32].

In this paper, we shall calculate the dissipation coefficients in supersymmetric models which have an inflaton together with multiplets of heavy and light fields. Different contributions to the final result can be applied also to non-supersymmetric models.

In the high temperature regime $T > m_\chi$, our results are similar in many ways to those obtained previously for self-interacting [10] and coupled systems [33]. There are some differences for $T \sim m_\chi$ due to the details of the $\chi \to yy$ decay. These differences affect the temperature dependence of the dissipation coefficient. This could potentially have observable consequences, since studies of density perturbations in models with different forms of dissipation terms have shown that the spectrum of density perturbations is quite sensitive to the temperature dependence of the dissipation coefficient [34].

We have new results for the regime $H < T < m_\chi$ and $m_\phi < m_\chi$. Previous calculations in this regime have been, in effect, highly non-thermal [8, 30]. Our results differ substantially from these non-thermal calculations. We find that the leading order dissipation terms in the inflaton equation vanish in the low temperature limit, but they are sufficiently large to ensure that the temperature does not fall to zero during inflation. We shall see in section VI that warm inflation appears to be allowed for moderate values of the coupling constants.

## II. THERMAL FIELD THEORY

We begin with some of the basic notions and results from thermal field theory which will be used later on in this paper. More details and proofs can be found in standard texts, eg [35].

The ensemble average of a Heisenberg operator $\hat{A}$ can be written

$$
\langle \hat{A}(x) \rangle = \frac{\rho_{ii'} \langle \psi_i | \hat{A} | \psi_{i'} \rangle}{\rho_{ii'} \langle \psi_i | \psi_{i'} \rangle} = \frac{\text{tr}(\rho \hat{A})}{\text{tr}(\rho)}
$$

(2)

where $\rho$ is the density matrix and repeated indices indicate sums. We shall be using a shifted version of the Schwinger-Keldysh, or closed-time path formalism to evaluate ensemble averages [36, 37]. For a scalar field $\phi$, the generating functional

$$
Z[\phi_1, \phi_2, J_1, J_2] = \rho_{ii'} \langle \psi_i | T^* \exp \left( -i \int J_2 (\hat{\phi} - \phi_2) \right) T \exp \left( i \int J_1 (\hat{\phi} - \phi_1) \right) | \psi_{i'} \rangle,
$$

(3)

where $T$ denotes time ordering of the operators with the smallest time on the right and $T^*$ denotes time ordering of the operators with the smallest time on the left. Shifting the field operators by $\hat{\phi}_1$ and $\hat{\phi}_2$ is done for later convenience.

Following Calzetta and Hu [38], we combine the sources into a pair $J_a = (J_1, J_2)$ and introduce a metric $c_{ab} = \text{diag}(1, -1)$ to raise and lower the index $a$. This removes the inconvenience with the relative signs of the two source terms. The generating functional gives rise to four two-point functions,

$$
G_{ab} = -\frac{i}{Z} \frac{\delta^2 Z}{\delta J^a \delta J^b} |_{(J_1, J_2) = 0}
$$

(4)

If we let $\hat{n} = \hat{\phi} - \phi_a$, these two-point functions can be expressed in terms of the functions

$$
G^>(x, x') = G^<(x', x) = i \langle \hat{n}(x) \hat{n}(x') \rangle.
$$

(5)
The individual components are

\[
G_{11}(x, x') = \theta(t - t') G^>(x, x') + \theta(t' - t) G^<(x, x') \tag{6}
\]

\[
G_{22}(x, x') = \theta(t - t') G^<(x, x') + \theta(t' - t) G^>(x, x') \tag{7}
\]

\[
G_{12}(x, x') = G^<(x, x') \tag{8}
\]

\[
G_{21}(x, x') = G^>(x, x') \tag{9}
\]

The two-point function \(G_{11}\) is the thermal analogue of the Feynman green function \(G_F\). (Placing a factor of \(i\) in front of the expectation values in the definition of the two-point functions follows [39], but differs from most references.)

Consider a system which remains close to local equilibrium. The natural starting point for perturbation theory is a free theory with an initially thermal distribution of states. The two-point functions for the interacting theory are given by Feynman diagram expansions with lines representing free theory propagators \(-iG_{0ab}\) and vertices derived from an interaction Lagrangian. The mass of the propagators and the vertices depend on the background fields \(\phi_a\), which may be time or space dependent. Each of these vertices has a label \(a\) attached, and the final expression for the diagram has a sum over the \(a\)'s. Later, we will make use of the following result,

- The maximum time rule. The contribution from a diagram vanishes if the time at any vertex is larger than all of the times on the external lines of the diagram.

This follows directly from the operator form of the two-point functions [39].

The next step is to consider the background fields \(\phi_a\). These can be identified with the ensemble averages of the field operator by setting

\[
\frac{\delta Z}{\delta J^a} = 0. \tag{10}
\]

We assume that this equation can be solved for \(J_a[\phi_0]\). The effective field equation is then \(F = 0\), where

\[
F[\phi](x) = -\left. \frac{\delta \Gamma[\phi_0]}{\delta \phi_0(x)} \right|_{\phi_0 = \phi} = 0, \tag{11}
\]

and the effective action

\[
\Gamma[\phi_a] = -i\ln Z[\phi_a, J_0[\phi_c]]. \tag{12}
\]

The effective action can be expanded, as usual, by a sum over 1-particle irreducible vacuum diagrams, with internal lines representing the free-theory propagators \(-iG_{0ab}\).

Dirac and Majorana fields can be included in the Schwinger-Keldysh formalism without any difficulty. We define the fermion two-point functions

\[
S^>(x, x') = i\langle \psi(x)\bar{\psi}(x') \rangle, \quad S^<(x, x') = -i\langle \bar{\psi}(x')\psi(x) \rangle \tag{13}
\]

and

\[
S_{11}(x, x') = \theta(t - t') S^>(x, x') + \theta(t' - t) S^<(x, x') \tag{14}
\]

\[
S_{22}(x, x') = \theta(t - t') S^<(x, x') + \theta(t' - t) S^>(x, x') \tag{15}
\]

\[
S_{12}(x, x') = S^<(x, x') \tag{16}
\]

\[
S_{21}(x, x') = S^>(x, x') \tag{17}
\]

where \(S^>(x, x') = -S^<(x', x) = \bar{S}^<(x, x')\). (For a Dirac matrix \(M\), let \(\bar{M} = \gamma^0 M^\dagger \gamma^0\) and \(\tilde{M} = (C^{-1}MC)^T\), where \(C\) is the charge conjugation matrix; \(\gamma^\mu = \tilde{\gamma}^\mu = -\gamma^\mu\)).

Finally, we consider an interacting system in thermal equilibrium with temperature \(T\). The background fields must then be constant, \(\phi_1 = \phi_2\) and the scalar two-point functions \(G_{ab}(x, x') \equiv G_{ab}(x - x')\). We set

\[
G_{ab}(P) = \int d^4x G_{ab}(x, x') e^{-iP \cdot (x - x')} \tag{18}
\]

where the 4-momentum \(P = (p, \omega)\). The vacuum polarization or self-energy \(\Pi_{ab}\) is defined by

\[
G^{-1a\bar{b}}(P) = (P^2 + m^2)\epsilon^{a\bar{b}} + \Pi^{a\bar{b}}. \tag{19}
\]

A diagrammatic representation for the two-point functions is shown in figure [1].
We shall often represent the dissipative part of the self-energy by a function $\alpha$, defined by

$$\alpha = i(\Pi_{21} - \Pi_{12})$$  \hspace{1cm} (20)

The function $\alpha$ is related to the relaxation time $\tau_p$ of the particle state via the relation $\alpha = 2\omega_p\tau_p^{-1}$, where $\omega_p$ is the energy of a state with momentum $p$.

In the case of fermions, we denote the self-energy by $\Sigma_{ab}$, and

$$S^{-1ab}(P) = (\gamma \cdot P + m)c_{ab} + \Sigma_{ab}.$$  \hspace{1cm} (21)

Further properties of the equilibrium propagators are listed in the appendix.

### III. DISSIPATION TERMS

#### A. Adiabatic approximations

We shall examine the effective field equation for an inflaton field which is interacting with radiation and evolving very slowly compared to other dynamical timescales. We shall construct an adiabatic approximation in the form of a derivative expansion.

Some conditions will be imposed on the radiation. The thermalisation timescale which governs the return to equilibrium of a small non-thermal fluctuation in the radiation will be denoted by $\tau_r$, the evolution timescale of the inflaton field will be denoted by $\tau_\phi$ and the expansion timescale of the universe by $\tau_a$. We take

- $\tau_r \ll \tau_\phi$
- $\tau_r \ll \tau_a$
- $T \gg H$

The first two conditions are necessary if the system is to remain close to thermal equilibrium. The second and third conditions allow us to neglect the expansion of the universe when applying the thermal field theory. Techniques exist for taking into account both expansion and thermal effects (see [29, 40] for example), but typically we expect either the thermal or expansion effects to predominate, and in this paper we consider the former only.

Consider the effective field equation,

$$F(x) = -\frac{\delta\Gamma}{\delta \phi_1(x)} \bigg|_{\phi_\alpha = \phi}$$  \hspace{1cm} (22)

where $\phi$ is spatially homogeneous and varies slowly about its value $\phi_t$ at time $t$. We can set $\delta \phi_\alpha = \phi_\alpha - \phi_t$ and expand $F$ by

$$F(x) = \sum_{n=0}^{\infty} F_n(x)$$  \hspace{1cm} (23)

where

$$F_n(x) = -\frac{1}{n!} \int d^4x_1 \ldots d^4x_n \frac{\delta^{n+1}\Gamma}{\delta \phi_1(x) \delta \phi_{\alpha_1}(x_1) \ldots \delta \phi_{\alpha_n}(x_n)} \bigg|_{\phi_\alpha = \phi_t} \delta \phi_{\alpha_1}(x_1) \ldots \delta \phi_{\alpha_n}(x_n)$$  \hspace{1cm} (24)
The first term $F_0$ represents the part of the field equations which contains no derivative terms, and can be expressed as the derivative of an effective potential which we denote by $V_\phi$.

The next term $F_1$ contains the derivative terms from the classical action and the equilibrium self-energy $\Pi^a_{\eta b}(k, t-t_1)$ of the inflaton perturbations $\eta$. According to the maximum time rule, we only need to integrate over times $t_1 < t$. The spatial integral is equivalent to evaluating the self-energy at zero spatial momentum, $F_1(x) = \ddot{\phi} + 3H \dot{\phi} - \int_{-\infty}^{t} dt_1 \Pi^a_{\eta a}(0, t-t_1) \delta \phi^a(t_1)|_{\phi_a=\phi}$ (25)

We shall assume that the self-energy introduces a response timescale $\tau$. If $\phi$ is slowly varying on the response timescale $\tau$, then we can use a simple Taylor expansion and write $\phi(t_1) = \phi(t) + (t_1 - t) \dot{\phi}(t) + \ldots$ (26)

The inflaton equation of motion including the linear dissipative terms is then $\ddot{\phi} + (3H + \Gamma) \dot{\phi} + V_\phi = 0$ (27)

with dissipation coefficient $\Gamma = 2 \int_0^{\infty} dt' \text{Re}(\Pi^a_{\eta a}(0, t')) t'$, (28)

where we have used $\Pi_{12} = -\Pi^*_{21}$.

This result is typical of linear response theory [41]. The range of applicability depends on the assumption that the system remains close to equilibrium. We also require $\tau < \tau_\phi$ for the existence of a local dissipation term. If this inequality breaks down, then we can replace the dissipation coefficient by the non-local expression (25), although the non-local expression can break down when the leading term $F_1$ is not a uniform approximation over large timescales [42].

B. Simple example

We shall illustrate the adiabatic approximation with a simple example which includes a massive boson field $\chi$ and a massless fermion $\psi$ with an interaction Lagrangian

$L_I(\phi, \chi, \psi) = -\frac{1}{2} g^2 \phi^2 \chi^2 - \frac{1}{\sqrt{2}} h \chi \bar{\psi} \psi$ (29)

The inflaton field $\phi$ is taken to be a slowly varying function of time.

![Diagram of vertices for interactions between the fields φ and χ.](2)

The Feynman diagram vertices are generated in the shifted Schwinger-Keldysh formalism by $L'_I(\phi, \eta, \chi, \psi) = L_I(\phi_1 + \eta_1, \chi_1, \psi_1) - L_I(\phi_2 + \eta_2, \chi_2, \psi_2)$ (30)

The vertices for interactions between $\eta$ and $\chi$ are shown in figure[2]. We are interested in the situation where $\chi$ as a heavy field, with mass $m_\chi \sim g \phi$. We shall therefore regard the first vertex as $O(g)$. 


The dissipation coefficient can be obtained from eq. (28). The contribution to the self-energy of the $\eta$ field at order $g^2$ is given by the first diagram in figure 3 with two $\chi$ propagators,

$$\Gamma = 4g^4 \phi^2 \text{Im} \int \frac{d^3k}{(2\pi)^3} \int_0^\infty dt' \,(G_{21}^\chi(t'))^2 t'.$$

This can also be written in terms of the spectral function $\rho_\chi$, using (A8),

$$G_{21}^\chi(t') = \int_{-\infty}^\infty \frac{d\omega}{2\pi} i(1 + n) \rho_\chi e^{-i\omega t'}.$$  \hspace{1cm} (32)

After applying the identity

$$\text{Im} \int_0^\infty dt' \, t' e^{-i\omega t'} = -\pi \delta'(\omega),$$  \hspace{1cm} (33)

we get

$$\Gamma = -2g^4 \phi^2 \int \frac{d^3k}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} \rho_\chi^2 n'. $$  \hspace{1cm} (34)

There are two contrasting limits in which the result can be replaced by an approximation. First there is the weakly coupled limit with small $h$ and fixed $T$. The energy integral is dominated by the point $\omega = \omega_k$,

$$\omega_k = \left(k^2 + m_\chi^2\right)^{1/2},$$  \hspace{1cm} (35)

which lies close to two poles in the spectral function (A10). These two poles are at $\omega = \omega_k \pm i\tau_\chi^{-1}$ (possibly on a different Riemann sheet due to branch cuts), where $\tau_\chi$ is the relaxation time for the $\chi$ boson. The integrand can be expanded about $\omega = \omega_k$ to obtain the result first derived by Hosoya and Sakagami [10],

$$\Gamma \approx g^4 \phi^2 \beta \int \frac{d^3k}{(2\pi)^3} \tau_\chi \rho_\chi m_\chi \int_0^\infty \frac{d\omega}{2\pi} \left(n + 1\right).$$  \hspace{1cm} (36)

The contribution to $\tau_\chi$ due to a single fermion loop is of order $h^{-2}$, as we shall see in a later section, so that $\Gamma = O(g^2 h^{-2})$.

The low temperature limit $\beta m_\chi \rightarrow \infty$ for fixed coupling $h$ gives a quite different approximation. In this case, exponential terms control the energy and momentum integrals and the dominant contributions come from $\omega \ll m_\chi$ and $k \ll m_\chi$. According to equation (A10), the spectral function can be replaced by

$$\rho_\chi \approx \frac{4\Gamma_\chi}{m_\chi^3}.$$  \hspace{1cm} (37)

where $\Gamma_\chi$ is the low-energy decay width of the $\chi$ boson. The dissipation coefficient given by equation (31) becomes

$$\Gamma \sim \frac{g^4 T^3 \phi^2}{m_\chi^6} \int \frac{d\omega}{2\pi} \frac{d^3k}{(2\pi)^3} 32\beta^3 \Gamma_\chi^2 \rho_\chi^2 n(n + 1).$$  \hspace{1cm} (38)

The magnitude of this term depends on the behaviour of the vacuum polarisation at small $\omega$. We shall give results for $\Gamma_\chi$ in a later section. The fermion contribution to the vacuum polarisation gives a contribution $\Gamma_\chi \sim h^2 K^2 m_\chi^{-1}$, and $\Gamma \propto T^7$. The dominant contribution to $\Gamma$ at low temperature comes from light scalar interactions, which we shall consider in section III.
C. Zero-temperature dissipation terms

The simple example of the previous section suggests that the dissipation coefficient in the inflaton equation vanishes at order $g^2$ in the zero temperature limit for a non-expanding universe. This contradicts some previous results [8,28,29,30], but it is consistent with the conclusions of Boyanovsky et al. [25], who found that their zero-temperature dissipation term could not be localised. We shall therefore examine this issue further.

Consider the simple model again, with the $O(g^2)$ contribution to the dissipation coefficient

$$\Gamma = 4g^4\phi^2 \Im \int \frac{d^3k}{(2\pi)^3} \int_0^\infty dt G_{21}^\chi(t)^2 t$$

(39)

In the vacuum, $G_{21}^\chi$ is the Wightman function, and includes corrections due to interactions with light fields. The spectral representation of the Wightman function is non-vanishing only when the energy lies in the physical particle spectrum,

$$G_{21}(\omega) = i\theta(\omega)\rho_\chi(\omega).$$

(40)

The dissipation coefficient becomes

$$\Gamma = 4g^4\phi^2 \Im \int \frac{d^3k}{(2\pi)^3} \int_{k}^\infty \frac{d\omega_2}{2\pi} \int_0^\infty \frac{d\omega_2}{2\pi} \int_0^\infty dt \rho_\chi(\omega_1)\rho_\chi(\omega_2) t' e^{-i(\omega_1+\omega_2-i\epsilon)}$$

(41)

which always vanishes.

In striking contrast, non-zero values of the dissipation coefficient can be obtained by using an approximation for the $\chi$ propagator, which we shall refer to as the exponential decay approximation,

$$G_{MS}(t) \approx \frac{i}{2(\omega_k - i\Gamma_\chi)} e^{-i(\omega_k - i\Gamma_\chi)t} \quad t > 0,$$

(42)

where $\omega_k = (k^2 + m_\chi^2)^{1/2}$ and $\Gamma_\chi$ is the $\chi$ decay width. This approximation gives a non-zero value for the dissipation coefficient $\Gamma_{MS}$ from eq. [39],

$$\Gamma_{MS} = 4g^2\phi^2 \Im \int \frac{d^3k}{(2\pi)^3} \frac{1}{(\omega_k - i\Gamma_\chi)^4}$$

(43)

This result was first obtained by Morikawa and Sasaki [8].

The difference between to results is due to the energy dependence of the width $\Gamma_\chi$ and the consequent failure of the exponential decay approximation for the zero-temperature propagator. The simplest example is when the $\chi$ propagator has a self-energy contribution $\Pi$ from a massless boson loop, which gives

$$\Pi = \frac{\gamma^2}{2\pi} \ln(k^2 - \omega^2 - i\epsilon)$$

(44)

where $\gamma$ is a constant. The Feynman green function

$$G_F(\omega) = (-\omega^2 + \omega_k^2 + \Pi)^{-1}$$

(45)

has branch cuts above and below the real $\omega$ axis which push the particle pole off the first Riemann sheet. After some contour manipulation,

$$G_F(t) = \int_0^\infty \frac{d\omega}{2\pi} \left\{ (-\omega^2 + \omega_k^2 - 2i\Gamma_\chi\omega_k)^{-1} - (-\omega^2 + \omega_k^2 + 2i\Gamma_\chi\omega_k)^{-1} \right\} e^{-i\omega t} \quad t > 0$$

(46)

where $\Gamma_\chi = \gamma^2/(2\omega_k)$ and the real part of the vacuum polarisation has been dropped. The result can also be obtained directly from the spectral representation of the Wightman function.

The energy integral can be done analytically,

$$G_F(t) = \frac{i}{2(\omega_k - i\Gamma_\chi)} e^{-i(\omega_k - i\Gamma_\chi)t} + f(\omega_k - i\Gamma_\chi, t) - f(\omega_k + i\Gamma_\chi, t)$$

(47)
with an error function part
\[ f(\omega, t) = \frac{e^{i\omega t}}{4\pi\omega} E_1(i(k + \omega)t) - \frac{e^{-i\omega t}}{4\pi\omega} E_1(i(k - \omega)t) \] (48)

We recognise the first term in (47) as the earlier approximation (42). This term is a good approximation to the full expression for early times up to \( t \sim \tau'_{\chi} \), where
\[ \tau'_{\chi} = \Gamma_{\chi}^{-1} \log \frac{m_{\chi}^2}{\Gamma_{\chi}} \] (49)

when the error functions begin to dominate and the propagator decays as a power law. When (47) is substituted into the formula for the dissipation coefficient (52), we find
\[ \Gamma = 4g^4 \phi^2 \text{Im} \int \frac{d^3k}{(2\pi)^3} \int_0^\infty dt' G_F(t')^2 t' = 0, \] (50)
due to a cancellation between the exponential and the error function terms.

A similar argument shows that the equilibrium propagator also has a power law decay at large time. However, it may be possible that an exponential decay law is the best representation the non-equilibrium propagators in the dissipation term, as has been suggested by Lawrie [17, 18]. There is some support for this suggestion from numerical investigations. A more suggestive way of rewriting (52) is to set
\[ F_{1}^{\text{non-loc}} = -4g^4 \phi^2 \text{Im} \int \frac{d^3k}{(2\pi)^3} \int_0^\infty \frac{d\omega_1}{2\pi} \frac{d\omega_2}{2\pi} \frac{d\omega_3}{2\pi} \rho_\chi(\omega_1) \rho_\chi(\omega_2) i(\omega_1 + \omega_2 - \omega_3 - i\epsilon)^{-1} \phi(\omega_3) \] (52)

A more suggestive way of rewriting (52) is to set
\[ F_{1}^{\text{non-loc}} = 2g^2 \text{Im} (g^2 \phi^2 K(\partial_\xi)\phi). \] (53)

where the kernel
\[ K(\zeta) = -\text{Im} \int \frac{d^3k}{(2\pi)^3} \int_0^\infty \frac{d\omega_1}{2\pi} \frac{d\omega_2}{2\pi} \rho_\chi(\omega_1) \rho_\chi(\omega_2) i(\omega_1 + \omega_2 + i\zeta + i\epsilon)^{-1} \] (54)

This would be a local derivative expansion if \( K(\zeta) \) was an analytic function. Although \( K(\zeta) \) is finite at \( \zeta = 0 \), there is a branch cut and \( K(\zeta) \) is non-analytic in the neighbourhood of the origin.

We can isolate a leading order logarithmic term in \( K(\zeta) \) by taking six derivatives with respect to \( \zeta \) and then taking a small \( \zeta \) approximation. For small \( \zeta \),
\[ \partial_\zeta^6 K(\zeta) \sim -120\rho_\chi(0)^2 \int \frac{d^3k}{(2\pi)^3} \int_k^\infty \frac{d\omega_1}{2\pi} \int_k^\infty \frac{d\omega_2}{2\pi} (\omega_1 + \omega_2 + iz)^{-6} \sim \frac{-i}{32\pi^2} \rho_\chi(0)^2 z^{-1} \] (55)

where \( \rho_\chi(0) \approx 4\Gamma_{\chi}/m_{\chi}^3 \). (At zero temperature, \( \rho_\chi \) vanishes for \( \omega < k \), and it is important to take the limit \( \omega \rightarrow k \) and then take the limit \( k \rightarrow 0 \).) Hence,
\[ 2\text{Im} K(\zeta) \sim K_0 + K_2 z^2 + K_4 z^4 - \frac{1}{6\pi^2} \frac{\Gamma_{\chi}^2}{m_{\chi}^6} z^4 \ln z \] (56)
where $K_0 \ldots K_4$ are constants. The important term for dissipation is the logarithm, which appears the field equations as

$$\ddot{\phi} + 3H \dot{\phi} + V_{\phi} - \frac{g^2}{3\pi^2} \frac{\Gamma_m^2}{m_\phi^6} g^2 \phi^2 \partial_\phi \ln(\partial_\phi) \phi = 0$$

(57)

This dissipative term represents the leading order adiabatic behaviour of the full expression (52).

The non-local operator $\ln(\partial_\phi)$ is fixed by linearity and causality as discussed in [44]. It can be expressed in integral form,

$$\ln(\partial_\phi) f = -\int_{-\infty}^{t} \ln(e^{\gamma(t-t')} \partial_{t'} f(t') dt'$$

(58)

where $\gamma$ is Euler’s constant. The integral can be evaluated for a broad class of functions, for example

$$\ln(m^{-1}\partial_\phi) \theta(t) \sin(mt) = \text{Si}(mt) \cos(mt) - \text{Ci}(mt) \sin(mt)$$

(59)

where $\text{Si}$ and $\text{Ci}$ are sine and cosine integrals. (This can be used to apply eq. (57) to the decay of an oscillating field $\phi = A \sin(mt)$, where the amplitude is a slowly varying function of time [45].)

**IV. SELF-ENERGY**

We shall examine the dissipative terms in the equations of motion for the inflaton with a range of interactions which are designed to be embedded easily in supersymmetric models. In this section we shall calculate the self energy for bosonic and fermionic massive $\chi$ fields interacting with light bosonic and fermionic fields of mass $m_y$.

The specific interaction terms for the $O(h^2)$ contributions to the self-energy are

1. For $\chi \to 2y$

$$\mathcal{L}_Y = -\frac{1}{\sqrt{2}} gh\phi(y^2 \chi + y^\dagger \chi^\dagger)$$

(60)

2. For $\chi \to 2\tilde{y}$

$$\mathcal{L}_Y = -\frac{1}{\sqrt{2}} h(\chi \bar{\psi}_y P_L \psi_y + \chi^\dagger \bar{\psi}_y P_R \psi_y)$$

(61)

3. For $\tilde{\chi} \to y\tilde{y}$

$$\mathcal{L}_Y = -\sqrt{2} h(y \bar{\psi}_\chi P_L \psi_y + y^\dagger \bar{\psi}_\chi P_R \psi_y)$$

(62)

where $P_L = \frac{1}{2}(1 + \gamma_5)$. The fermions used in the supersymmetric models are all Majorana.

**A. Interaction 1: $\chi \to 2y$**

The leading order contribution to the self-energy from the light boson loop is given by figure 4.
We shall calculate the dissipative part of the self-energy, and denote by $\alpha_y$ the function defined in equation (A5),

$$\alpha_y = i(e^{\beta\omega} - 1)\Pi_{12}$$

(63)

The loop diagram gives,

$$\alpha_y = -g^2h^2\phi^2(e^{\beta\omega} - 1)\int \frac{d^4K}{(2\pi)^4}G_{12}(K)G_{12}(P-K)$$

(64)

A spectral representation (A9) can now be applied,

$$\alpha_y = \frac{1}{2}h^2(e^{\beta\omega} - 1)\int \frac{dk^0}{2\pi} \frac{dk^i}{2\pi} \frac{d^3k}{(2\pi)^3}n(k^0)n(k^0)\delta(\omega - k^0 - k^0')\rho(k^0)\rho(k-k^0')$$

(65)

We can use the free bosonic spectral function given in the appendix (A10). The integral reduces to a different form for different ranges of energy. Let

$$\omega_c = (4m^2_y + p^2)^{1/2}.$$  

(66)

In the $\omega > \omega_c$ case, the integral reduces down to

$$\alpha_y = 2\pi g^2h^2\phi^2(e^{\beta\omega} - 1)\int \frac{d^3k}{(2\pi)^3} \frac{1+2n(\omega_k)}{4\omega_k\omega_{p-k}}\delta(\omega - \omega_k - \omega_{p-k}).$$

(67)

This integral can be done using a substitution

$$y = \omega_k + \omega_{p-k},$$

$$x = \omega_k - \omega_{p-k}.$$  

(68, 69)

The range of integration becomes $y > \omega_c$ and $-x(P) < x < x(P)$, where

$$x(P) = p\left(1 + \frac{4m^2_y}{P^2}\right)^{1/2}.$$  

(70)

In the end, we get

$$\alpha_y = \frac{\hbar^2}{8\pi^2}g^2\phi^2\left(1 + \frac{4m^2_y}{P^2}\right)^{1/2} + \frac{\hbar^2}{4\pi^2}g^2\phi^2\frac{T}{p}\left(\frac{1-e^{-\beta(x(P)+\omega)/2}}{1-e^{-\beta(x(P)-\omega)/2}}\right),$$

$$\omega > \omega_c.$$  

(71)

Repeating the analysis for $0 < \omega < \omega_c$ gives $\alpha = 0$ for $p < \omega < \omega_c$ and

$$\alpha_y = \frac{\hbar^2}{4\pi^2}g^2\phi^2\frac{T}{p}\ln\left(\frac{1-e^{-\beta(x(P)+\omega)/2}}{1-e^{-\beta(x(P)-\omega)/2}}\right),$$

$$0 < \omega < p.$$  

(72)

These are quite a complicated expressions, but they can be interpreted simply in certain limits. At zero temperature only the first term from the $\omega > \omega_c$ result survives. Since $\alpha$ becomes the imaginary part of the vacuum polarisation, this term is related to the particle production probability and $\omega_c$ is the energy threshold for particle production.

At the other extreme, for high temperatures $\alpha = O(h^2m^2_y)$. This result can be checked by taking the Hard Thermal Loop approximation [55], which when applied here predicts that the imaginary part of the scalar self-energy should have no $O(T^2)$ contribution. This contrasts with the vacuum polarisation obtained from other types of vertex. Gleiser and Ramos [56] considered the scalar vacuum polarisation due to quadratic interactions, and their results suggest that $h^2\chi^2\gamma^2$ vertices would give $\alpha = O(h^4T^2)$ in the high temperature limit. We shall restrict ourselves to temperatures $T < h^{-1}m^2_y$ for which the $O(h^4T^2)$ terms can be neglected.

For light fields we can approximate the result by taking the massless limit $m_y = 0$,

$$\alpha_y = \frac{\hbar^2}{8\pi^2}g^2\phi^2\omega - p + \frac{\hbar^2}{4\pi^2}g^2\phi^2\frac{T}{p}\ln\left(\frac{1-e^{-\beta(\omega+p)/2}}{1-e^{-\beta(\omega-p)/2}}\right).$$

(73)

This approximation breaks down at the point $\omega = p$, but remains integrable.
B. Interaction 2: $\chi \rightarrow 2\tilde{y}$

The leading order contribution to the self-energy from a fermion loop is given by figure 5.

We shall calculate the dissipative part and denote this contribution by $\alpha_{\tilde{y}}$. The fermion loop diagram gives,

$$\alpha_{\tilde{y}} = \hbar^2 (1 - e^{\beta \omega}) \int \frac{d^4K}{(2\pi)^4} \text{tr}(S_{12}(K)S_{21}(K - P)).$$

(74)

A spectral representation can now be applied. In the $\omega > \omega_c$ case, the integral reduces down to

$$\alpha_{\tilde{y}} = -2\pi\hbar^2 \int \frac{d^3k}{(2\pi)^3} \frac{1 - 2n(\omega_k)}{4\omega_k \omega_{p-k}} (P^2 + 4m^2_y) \delta(\omega - \omega_k - \omega_{p-k}).$$

(75)

This integral can be done using the same substitution as before, resulting in

$$\alpha_{\tilde{y}} = -\frac{\hbar^2}{8\pi} P^2 \left(1 + \frac{4m^2_y}{P^2}\right)^{3/2} \frac{\hbar^2 T}{4\pi P} P^2 \left(1 + \frac{4m^2_y}{P^2}\right) \ln \left(\frac{e^{-\beta(\omega + x(P))/2} + 1}{e^{-\beta(\omega - x(P))/2} + 1}\right) \quad \omega > \omega_c.$$

(76)

Repeating the analysis for $0 < \omega < \omega_c$ gives

$$\alpha_{\tilde{y}} = -\frac{\hbar^2}{4\pi} \frac{T}{P} P^2 \left(1 + \frac{4m^2_y}{P^2}\right) \ln \left(\frac{e^{-\beta(x(P) + \omega)/2} + 1}{e^{-\beta(x(P) - \omega)/2} + 1}\right) \quad 0 < \omega < p.$$

(77)

The fermion loop, like the boson loop, gives $\alpha = O(\hbar^2 m^2_\chi)$ in the high temperature limit.

In the massless $m_y = 0$ limit,

$$\alpha_{\tilde{y}} = -\frac{\hbar^2 T^2}{8\pi} (\beta P)^2 \theta(\omega - p) - \frac{\hbar^2 T^2}{4\pi} \frac{1}{\beta P} (\beta P)^2 \ln \left(\frac{e^{-\beta(\omega + p)/2} + 1}{e^{-\beta(\omega - p)/2} + 1}\right).$$

(78)

The momenta have been scaled to display the dependence on the temperature in the low temperature limit. Note that, at low temperatures, the vacuum polarisation of the $\chi$ field is dominated by the bosonic loop contribution.

C. Interaction 3: $\tilde{\chi} \rightarrow y\tilde{y}$

The leading order contribution to the fermion self-energy is given by figure 6.
We shall calculate the function $\tilde{\alpha}$, given by equation (A6),

$$\tilde{\alpha} = -i(e^{\beta \omega} + 1)\Sigma_{12}. \tag{79}$$

From the diagram we have,

$$\tilde{\alpha} = 2\hbar^2(e^{\beta \omega} + 1) \int \frac{d^3k}{(2\pi)^3} G_{12}(K)S_{12}(P-K). \tag{80}$$

We proceed as before. For $\omega > \omega_c$,

$$\tilde{\alpha} = 4\pi\hbar^2 \int \frac{d^3k}{(2\pi)^3} (1 - n(\omega_k) + \hbar(\omega_{p-k})) (-\gamma^\mu(P_\mu - K_\mu) + m_y)\delta(\omega - \omega_k - \omega_{p-k}). \tag{81}$$

The substitution of the new variables $x$ and $y$ can be used again. The gamma-matrix terms become

$$-\gamma^\mu(P_\mu - K_\mu) = \frac{1}{2}\sigma_0 - \frac{x}{2p}\sigma_1, \tag{82}$$

where $\sigma_0$ and $\sigma_1$ are defined in (A13). The integral gives

$$\tilde{\alpha} = \tilde{\alpha}_0\sigma_0 + \tilde{\alpha}_1\sigma_1 + \tilde{\alpha}_m m_\chi, \tag{83}$$

with

$$\tilde{\alpha}_0 = \frac{m_\chi}{2m_y} \tilde{\alpha}_m = \frac{\hbar^2}{8\pi} \left( 1 + \frac{4m_y^2}{P^2} \right)^{1/2} + \frac{h^2}{8\pi} \frac{T}{p} \ln \left( \frac{1 - e^{-\beta(\omega+x(P))}}{1 - e^{-\beta(\omega-x(P))}} \right) \quad \omega > \omega_c, \tag{84}$$

and

$$\tilde{\alpha}_1 = \frac{\hbar^2}{8\pi} \left( 1 + \frac{4m_y^2}{P^2} \right)^{1/2} \frac{T}{p} \left( \frac{1 - e^{-\beta(\omega+x(P))}}{1 - e^{-\beta(\omega-x(P))}} \right) + \frac{\hbar^2}{8\pi} \left( \frac{T}{p} \right)^2 \left( \text{Li}_2(e^{-\beta(\omega-x(P))/2}) - \text{Li}_2(e^{-\beta(\omega-x(P))/2}) - \text{Li}_2(e^{-\beta(\omega+x(P))/2}) + \text{Li}_2(e^{-\beta(\omega+x(P))/2}) \right), \tag{85}$$

where $x(p)$ was given in eq. (80) and

$$\text{Li}_2(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^2} \tag{86}$$

is the dilogarithm function.

It is worth noting at this stage that the relaxation time of the $\chi$ field does not depend on the $\sigma_1$ terms. This is helpful because dissipation coefficient in the small coupling limit is largely determined by the relaxation time. If we use eq. (A15) for the thermal width $\Gamma_\chi$, with $m = m_\chi = g\phi$, we get

$$2\omega_p \Gamma_\chi = \frac{h^2}{4\pi} g^2 \phi^2 \left( 1 + \frac{2m_y}{m_\chi} \right)^{3/2} \left( 1 - \frac{2m_\phi}{m_\chi} \right)^{1/2} + \frac{h^2}{4\pi} g^2 \phi^2 \frac{T}{p} \left( 1 + \frac{2m_y}{m_\chi} \right) \ln \left( \frac{1 - e^{-\beta(\omega+x(P))}}{1 - e^{-\beta(\omega-x(P))}} \right). \tag{87}$$

We are interested mostly in very light fields $y$, for which we can use the limit $m_y \to 0$ when

$$2\omega_p \Gamma_\chi = \frac{h^2}{4\pi} g^2 \phi^2 \theta(\omega - p) + \frac{h^2}{4\pi} g^2 \phi^2 \frac{T}{p} \ln \left( \frac{1 - e^{-\beta(\omega+p)}}{1 - e^{-\beta(\omega-p)}} \right). \tag{88}$$

This is identical to the thermal width of the massive boson fields when both the light boson and light fermion loops are included.
V. MORE DISSIPATION TERMS

The calculation of the frictional terms in the inflaton equation of motion now needs to be extended to a more general class of models. The first generalisation we do is to take into account the coupling of the light fields to the inflaton which arises in eq. (60). The other generalisation is to consider dissipation terms which arise due to interactions between the inflaton and massive fermionic fields.

The interactions which we consider are all contained in a supersymmetric theory which has three superfields $\Phi$, $X$ and $Y$ with superpotential,

$$W = \frac{g}{\sqrt{2}}\Phi X^2 - \frac{h}{\sqrt{2}}XY^2.$$  \hfill (89)

The scalar components of the superfields are $\phi$, $\chi$ and $y$ respectively. This superpotential can easily be modified to produce a hybrid inflationary model by adding additional terms or by adding a $D$-term to the potential, neither of which affects the dissipation terms.

The interaction terms responsible for the inflaton decay are

1. For $\phi \to \chi$

$$\mathcal{L}_I = -g^2 \phi^2 \chi^\dagger \chi;$$  \hfill (90)

2. For $\phi \to \bar{\chi}$

$$\mathcal{L}_I = -\frac{1}{2}g\phi \bar{\psi}_\chi \bar{\psi}_\chi.$$  \hfill (91)

The scalar interaction vertices are given by figure 2.

A. Dissipation with an intermediate boson: $\phi \to 2y$ and $\phi \to 2\bar{y}$

Consider the dissipation coefficient given by eq. (28) with the equilibrium self energy diagram shown in figure 7. The dissipation coefficient in this case will be denoted by $\Gamma(\phi \to 2y/2\bar{y})$. It is given to order $g^2$ by

$$\Gamma(\phi \to 2y/2\bar{y}) = \text{Im} \int \frac{d^4k}{(2\pi)^4} \frac{d\omega_1}{2\pi} \frac{d\omega_2}{2\pi} \Gamma_2^{ab} \Gamma_1^{a'b'} G_a^{\chi}(\omega_1)G_b^{\chi}(\omega_2)G_d^{\chi}(\omega_1+\omega_2) \partial_{\omega_1} \delta(\omega_1 + \omega_2),$$  \hfill (92)

where the $\Gamma_{abc}$ are vertex factors. The vertex factors now include contributions from the light fields, as shown in figure 5. The vacuum loop inside the vertex factor is similar to the leading order correction to the $\chi$ self-energy, which allows to write the vertex factor in terms of $\alpha_y$, as

$$\Gamma_2^{ab}(\omega, \omega_1, \omega_2) = -2ig^2 \phi c_2^{ab} + \phi^{-1}(1 + n)\alpha_y(\omega_1)c_2^{ab} + \phi^{-1}(1 + n)\alpha_y(\omega_2)c_2^{ab} + O(h^4)$$  \hfill (93)

where $c_a^{bc} = \delta_a^{b} \delta_a^{c}(2\pi)^4\delta(K + K_1 + K_2)$, $\bar{I} = 2$ and $\bar{2} = 1.$
FIG. 8: Contributions to the $\eta$ vertex up to order $g^2h^2$.

The vertex factor can be substituted into eq. (92). We define

$$\sigma = 2 \text{Re} G_{11} = -2 \text{Re} G_{22}$$

and use the spectral representation $G_{21} = i(1+n)\rho_\chi$. The integral reduces down to

$$\Gamma(\phi \rightarrow 2y/2\tilde{y}) = \int \frac{d^3k}{(2\pi)^3} \int_0^{\infty} \frac{d\omega}{2\pi} \left( 4g^4\phi^2 \rho_\chi^2 + 4g^2 \rho_\chi \alpha_y \sigma + \frac{1}{2} \phi^{-2} \alpha_y^2 \sigma^2 \right) n' \cdot (95)$$

The first term recovers our result (34) from section IIIB, with an extra factor of two because $\chi$ is now a complex field.

FIG. 9: The dissipation coefficient $\Gamma(\phi \rightarrow 2y/2\tilde{y})$ plotted as a function of temperature for $h = 0.79$. The plot on the right shows the low temperature region where $\Gamma(\phi \rightarrow 2y/2\tilde{y}) \propto T^3$ and the small $h$ approximation breaks down.

Numerical values for the dissipation coefficient are plotted in figure 9. The small $h$ approximation works reasonably well even for $h \sim 1$, except in the low temperature regime.

For small coupling, $h$, the integrand is dominated by poles in the spectral function. The last two terms in eq. (95) are higher order in $h$ and can be dropped. The first term gives

$$\Gamma(\phi \rightarrow 2y/2\tilde{y}) \approx g^4\phi^2 \beta \int \frac{d^3k}{(2\pi)^3} \frac{\tau_\chi}{\omega_k^2} n(n+1).$$
where $\tau_\chi$ is the relaxation time of the $\chi$ field. For $h^{-1}m_\chi > T$, we can use eqs. (73) and (78) for the boson and fermion loop contributions to the self-energy,

$$\tau_\chi = \frac{2\omega_k}{\alpha_y(\omega_k, k) + \alpha_\tilde{y}(\omega_k, k)}$$  \hspace{1cm} (97)$$

When this is combined with a high temperature approximation, the dissipation coefficient becomes

$$\Gamma(\phi \rightarrow 2y/2\tilde{y}) \approx 0.691 \frac{g^2}{h^2} T \quad h^{-1}m_\chi > T \gg m_\chi.$$  \hspace{1cm} (98)$$

This is shown in figure 10.

In the low temperature limit the integral (95) is dominated by small $k$ and $\omega$. We can approximate $\rho_\chi \approx 2\alpha/m_\chi$ and $\sigma \approx 2/m_\chi^2$. The leading order behaviour is determined by the self-energy $\alpha_y$,

$$\Gamma(\phi \rightarrow 2y/2\tilde{y}) \approx \frac{g^2T^3\phi^2}{m_\chi^4} \int \frac{d\omega}{2\pi} \frac{d^3k}{(2\pi)^3} 34\beta^4 \alpha_y^2 \tilde{n}'.$$  \hspace{1cm} (99)$$

We can use the value of $\alpha_y$ given in eq. (23) to get

$$\Gamma(\phi \rightarrow 2y/2\tilde{y}) \approx 4.0 \times 10^{-2} g^2 h^4 \left(\frac{g\phi}{m_\chi}\right)^4 \frac{T^3}{m_\chi^2} \quad T \ll m_\chi.$$  \hspace{1cm} (100)$$

The value of the coefficient has been found by numerical integration.

**B. Dissipation from an intermediate fermion: $\phi \rightarrow y\tilde{y}$**

The contribution to the dissipation coefficient from the decay $\phi \rightarrow \tilde{\chi} \rightarrow y\tilde{y}$ is given by

$$\Gamma(\phi \rightarrow y\tilde{y}) = g^2 \text{Im} \int \frac{d^3k}{(2\pi)^3} \int_0^\infty dt' \text{tr}(S_{21}(t')\tilde{S}_{21}(t'))t'$$ \hspace{1cm} (101)$$

where the fermion two-point functions and conjugates where defined following eq. (13). Using the spectral representation of the two-point functions in eq. (A11) gives,

$$\Gamma(\phi \rightarrow y\tilde{y}) = \frac{1}{2}g^2 \int \frac{d^3k}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} \text{tr}(\tilde{\rho}_\chi^2) \tilde{n}'.$$  \hspace{1cm} (102)$$

In the small coupling limit we can expand the integrand about the poles of the spectral function using (A15),

$$\Gamma(\phi \rightarrow y\tilde{y}) \approx g^2 m_\chi^2 \beta \int \frac{d^3k}{(2\pi)^3} \frac{\tau_\chi}{\omega_k^2} \tilde{n}(\omega_k)(1 - \tilde{n}(\omega_k)).$$ \hspace{1cm} (103)$$

The relaxation time $\tau_\chi = 1/\Gamma_\chi$, where $\Gamma_\chi$ is given by eq. (38). The main difference between this result and the contribution from the bosonic decay channel comes from the fermion distribution functions.

Figure 11 shows a comparison between the intermediate bosonic and fermionic decays of the inflaton. The total dissipation coefficient from all decays at high temperature is

$$\Gamma \approx 0.97 \frac{g^2}{h^2} T \quad h^{-1}m_\chi > T \gg m_\chi.$$ \hspace{1cm} (104)$$

When $T > h^{-1}m_\chi$, the two-loop contribution to the $\chi$ self-energy corresponding to decays such as $\phi \rightarrow m_\chi y\tilde{y}$ becomes important. The size of this contribution to the self-energy is known from previous work to be $O(g^2h^2T^2)$ [13], leading to the dissipation coefficient of Hosoya and Sakagami $\Gamma = O(h^{-2}m_\chi^2 T^{-1})$ [10].

In the low temperature limit the integrals are dominated by small $k$ and $\omega$. We have $\tilde{\rho}_\chi \approx 2\tilde{\alpha}/m_\chi$, and

$$\Gamma(\phi \rightarrow y\tilde{y}) \approx \frac{g^2T^3}{m_\chi^4} \int \frac{d^3k}{(2\pi)^3} \int_0^\infty \frac{d\omega}{2\pi} 8\beta^4 \text{tr}(\tilde{\alpha}^2)e^{-\beta\omega}$$ \hspace{1cm} (105)$$

Using the expression for $\alpha$ in (34) and (35),

$$\Gamma(\phi \rightarrow y\tilde{y}) \propto g^2 h^4 \frac{T^5}{m_\chi^4}$$ \hspace{1cm} (106)$$

The bosonic decay rate, which is $\propto T^3$, dominates at low temperatures.
C. Dissipation with the exponential decay propagator

The results have so far assumed that the heavy field has a thermal propagator. We consider now what happens if we use the exponential decay approximation for the propagator to calculated the friction coefficient $\Gamma_{MS}$ which was discussed in section IIIC. Although the heavy field is now not in equilibrium, the light field thermalises on a different timescale which we suppose is sufficiently short to keep the light field in equilibrium.

For the bosonic decay we obtain the contribution $\Gamma_{MS}(\phi \to 2y)$ to the dissipation coefficient from the decay $\phi \to \chi \to yy$ by using eq (43),

$$\Gamma_{MS}(\phi \to 2y) = 16g^4\phi^4 \int \frac{d^3k}{(2\pi)^3} \omega_k^2 \alpha_y,$$

The self-energy $\alpha_y$ is determined by the results of section IV. The dissipation coefficient can be obtained analytically,

$$\Gamma_{MS}(\phi \to 2y) = \frac{h^2g^2}{16\pi^2} \left( \frac{g^2\phi^4}{m_\chi^4} \right) \left( 1 + \left( \frac{4T}{m_\chi} \right)^5 f \left( \frac{2T}{m_\chi} \right) \right)$$

where

$$f(a) = \frac{2}{\pi} \int_0^\infty \frac{x^4}{(1 + a^2x^2)^4} \frac{1}{e^x - 1}.$$

Similarly, the contribution $\Gamma_{MS}(\phi \to 2\tilde{y})$ to the dissipation coefficient from $\phi \to \chi \to \tilde{y}\tilde{y}$ becomes

$$\Gamma_{MS}(\phi \to 2\tilde{y}) = \frac{h^2g^2}{16\pi^2} \left( \frac{g^2\phi^2}{m_\chi^2} \right) \left( 1 + \left( \frac{4T}{m_\chi} \right)^5 \tilde{f} \left( \frac{2T}{m_\chi} \right) \right)$$

where

$$\tilde{f}(a) = \frac{2}{\pi} \int_0^\infty \frac{x^4}{(1 + a^2x^2)^4} \frac{1}{e^x + 1}.$$

The main application of these results would be at low temperature and the temperature corrections are relatively small.
For the fermionic decay $\phi \to \tilde{\chi} \to y\tilde{y}$, we can make use of the symmetry between the decay width of the fermion $\tilde{\chi}$ given by eq. and the decay width of the boson $\chi$ when $m_\chi = m_{\tilde{\chi}}$. We obtain the contribution to the dissipation coefficient,

$$\Gamma_{MS}(\phi \to y\tilde{y}) = \frac{h^2 g^2}{8\pi^2} m_{\tilde{\chi}} \left( 1 + \left( \frac{2T}{m_{\tilde{\chi}}} \right)^5 f \left( \frac{T}{m_{\tilde{\chi}}} \right) \right)$$

(112)

In the zero-temperature limit, the total dissipation coefficient is then

$$\Gamma_{MS} = \frac{h^2 g^2}{16\pi^2} \left( 2 + \frac{g\phi}{m_\chi} + \frac{g^3\phi^3}{m_\chi^3} \right) g\phi.$$  

(113)

This is similar to the results obtained for non-supersymmetric models by Morikawa and Sasaki and by Berera and Ramos, although the coefficient calculated here is significantly larger due to the large number of decay channels available to inflaton in the supersymmetric theory. In addition, there are small numerical differences between different results due to slightly different prefactors in the exponential decay approximation.

VI. ENTROPY PRODUCTION DURING INFLATION

The main application for our results is to the inflationary universe. In this section we shall make some general remarks and check the consistency of some of the assumptions that we have made. During the slow evolution which is characteristic of inflation, we replace the inflaton equation by

$$3H(1 + r)\dot{\phi} + V' = 0$$  

(114)

We have parameterised the effectiveness of dissipation by taking the ratio of the dissipation coefficient to the expansion rate,

$$r = \frac{\Gamma}{3H}.$$  

(115)

The energy density during inflation is dominated by the inflaton potential energy $V$, and the expansion rate is given by

$$3H^2 = \frac{8\pi}{m_p^2} V$$  

(116)

By combining these equations, we can compare $\dot{\phi}$ to $H$,

$$\frac{\dot{\phi}^2}{H^2} = \frac{\epsilon m_p^2}{4\pi(1 + r)^2}$$  

(117)

where $\epsilon$ measures the slope of the potential,

$$\epsilon = \frac{m_p^2}{16\pi} \left( \frac{V'}{V} \right)^2.$$  

(118)

The second time derivative of the scale factor is positive for $\epsilon < 1 + r$.

The temperature of the radiation field can be determined from energy momentum conservation. The energy lost by the inflaton is converted into heat and increases the entropy density of the radiation $s$,

$$T \dot{s} + 3HT s = \Gamma \dot{\phi}^2$$  

(119)

The radiation is a relativistic gas, with

$$s = \frac{4}{3} a T^3$$  

(120)

where $a$ is the effective radiation constant, taking into account the number of degrees of freedom. We see immediately that for $\Gamma \propto T^3$, the temperature does not fall to zero, but reaches a lower limit $T_r$,

$$T_r s = r\dot{\phi}^2.$$  

(121)
If $\Gamma$ falls off faster than $T^4$, then the cosmological redshift would drive the temperature downwards and the radiation into the vacuum state.

We shall consider this low temperature regime now in more detail, using the thermal expression for the dissipation coefficient,

$$\Gamma = \gamma g^2 h^2 \frac{T^3}{m^2}. \tag{122}$$

The temperature falls to the value $T_r$ given by eq. (121). When we eliminate $\dot{\phi}$ using (117),

$$\frac{T_r}{H} = \frac{\gamma g^2 h^4}{16 \pi a} \frac{\epsilon}{(1+r)^2 m^2} \tag{123}$$

Further information can be obtained from the amplitude of fluctuations in the cosmic microwave background. These have their origin as thermal fluctuations during inflation if $T > H$. Theory gives an amplitude $\Delta$ for a given mode, where

$$\Delta^2 = c H^2 T (HT)^{1/2} \frac{\dot{\phi}^2}{\phi^2}, \tag{124}$$

evaluated at the time that the mode crosses the horizon, and $c \approx 0.045$. The measured amplitude $\Delta \approx 5.4 \times 10^{-5}$ on a 500 MPc scale relevant to the large scale structure. We can use $\Delta$ to eliminate the expansion rate and obtain

$$\frac{T_r}{m} = \left( \frac{1}{256 \pi a c^2} \right)^{1/6} \frac{\epsilon^{1/2}}{(1+r) \Delta^{2/3}} \tag{125}$$

The range of temperatures for which the dissipation coefficient calculation can be applied and still be consistent with the low temperature approximation is $H < T < m$. This translates to limits on the values of $m$ which are consistent when

$$gh^2 > \left( \frac{4 \pi a}{c} \right)^{1/3} \gamma^{-1/2} \Delta^{2/3} \approx 5.1 \times 10^{-2}. \tag{126}$$

It is quite possible for supersymmetric models of inflation to have coupling constants in this range.

VII. CONCLUSIONS

We have taken a detailed look at the linear dissipation term in the equation of motion for an inflaton field embedded in a supersymmetric model. The dissipation was caused by the decay of the inflaton into light particles during inflation with a heavy field $\chi$ acting as intermediary.

- For temperature $T > m$, our results are consistent with a wide body of previous work. Some differences in the temperature dependence can be attributed to the special features of the interactions which play an important role for temperatures close to the heavy particle mass.

- For $H < T < m$, where $H$ is the expansion rate, we find significant differences between the dissipation coefficient in the thermal background and the dissipation coefficient obtained assuming exponential real-time decay of the $\chi$ propagator $\mathbb{R} \ 28 \ 29$. In particular, the dissipation coefficient in the thermal background does not approach a constant as $T \to 0$ and $H \to 0$, but instead behaves as a power law $\propto T^3$.

Thermal propagators decay exponentially for only a limited time before switching over to a power law decay. The switch over occurs at a logarithmic factor times the exponential decay time. A purely exponential decay might possibly represent an approximation to the non-equilibrium behaviour of the system if the dynamical timescale of the inflaton where shorter than the thermalisation timescale of the heavy field. We have therefore given results for both the thermal and exponential propagators.

We have argued that the low temperature dissipation coefficient obtained using the thermal propagator can still have an important effect on inflationary models. For a significant range of parameters, the dissipation heats the radiation to the $H < T < m$ range. Thermal fluctuations are then more important than vacuum fluctuations and become the source of primordial density perturbations $\mathbb{R} \ 3 \ 4 \ 11 \ 54 \ 46 \ 17$. 
Dissipation is far stronger for the exponential propagator and warm inflation is realised for a far larger range of parameters. This case has been studied in previous work which has divided up the parameter ranges into those leading to cold and warm inflation \cite{20, 31, 32}. There remains much to understand about the non-equilibrium physics of reheating during the inflationary era, but the results which we have obtained so far indicate that dissipation is important during inflation for supersymmetric models with moderate values of the coupling constants.

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APPENDIX A: PROPERTIES OF THERMAL PROPAGATORS

This appendix lists some important properties of the thermal equilibrium propagators. We begin with some of the most basic rules,

- Kubo-Martin-Schwinger periodicity relations,

\[ \Pi_{21} = e^\beta \omega \Pi_{12} \] \hspace{1cm} (A1)
\[ \Sigma_{21} = -e^\beta \omega \Sigma_{12} \] \hspace{1cm} (A2)

- The Kobes-Semenoff rules,

\[ \text{Im}(\Pi_{11}) = -\frac{i}{2} (\Pi_{12} + \Pi_{21}) \] \hspace{1cm} (A3)
\[ \text{Im}(\Sigma_{11}) = -\frac{i}{2} (\Sigma_{12} + \Sigma_{21}) \] \hspace{1cm} (A4)

The Kobes-Semenoff rules follow from the maximum time rule.

The periodicity relations imply that the off-diagonal components of the self-energies can be expressed in terms of functions \( \alpha(P) \) and \( \tilde{\alpha}(P) \),

\[ \Pi_{21} = -i(1 + n)\alpha \hspace{1cm} \Pi_{12} = -i n \alpha \] \hspace{1cm} (A5)
\[ \Sigma_{21} = -i(1 - \tilde{n})\tilde{\alpha} \hspace{1cm} \Sigma_{12} = i \tilde{n} \tilde{\alpha} \] \hspace{1cm} (A6)

where

\[ n(\omega) = \frac{1}{e^{\beta \omega} - 1}, \quad \tilde{n}(\omega) = \frac{1}{e^{\beta \omega} + 1} \] \hspace{1cm} (A7)

are occupation numbers for bosons and fermions. The off-diagonal components of the full two-point functions can also be expressed in terms of a real function, the spectral density \( \rho \),

\[ G_{21} = i(1 + n)\rho \quad G_{12} = i n \rho \] \hspace{1cm} (A8)
\[ S_{21} = i(1 - \tilde{n})\tilde{\rho} \quad S_{12} = -i \tilde{n} \tilde{\rho} \] \hspace{1cm} (A9)

If we discard the real part of the vacuum polarisation, then the Kobes-Semenoff rules imply that the spectral density \( \rho \) and the function \( \alpha \) are related by

\[ \rho = i(P^2 + m^2 + i\alpha)^{-1} - i(P^2 + m^2 - i\alpha)^{-1} \] \hspace{1cm} (A10)

The spectral density typically has a rich variety of poles and branch cuts. For a pole at \( \omega = \pm \omega_p \pm i\tau \), we can regard \( \omega_p \) as the energy of the particle state and \( \tau \) as the relaxation time. In perturbation theory, \( \omega_p = (p^2 + m^2)^{1/2} \) and \( \tau = 2\omega_p \alpha(p, \omega_p)^{-1} \).

For fermions, the spectral density

\[ \tilde{\rho} = i(\gamma^\mu P_\mu + m + i\tilde{\alpha})^{-1} - i(\gamma^\mu P_\mu + m - i\tilde{\alpha})^{-1} \] \hspace{1cm} (A11)
We can use rotational invariance to express $\tilde{\alpha}$ in terms of three scalar components,

$$\tilde{\alpha} = \tilde{\alpha}_0 \sigma_0 + \tilde{\alpha}_1 \sigma_1 + \tilde{\alpha}_m m$$  \hspace{1cm} (A12)

where the gamma-matrices have been adapted to a momentum frame,

$$\sigma_0 = \gamma^0 \omega - \gamma \cdot \hat{p} p$$  \hspace{1cm} (A13)

$$\sigma_1 = \gamma \cdot \hat{p} \omega - \gamma^0 p$$  \hspace{1cm} (A14)

These combinations satisfy $-\sigma_0^2 = \sigma_1^2 = P^2$ and $\{\sigma_0, \sigma_1\} = 0$. The poles of the spectral function are again given approximately by $\pm \omega_p \pm i \tau^{-1}$, but now

$$\omega_p \tau^{-1} = (\tilde{\alpha}_0 + \tilde{\alpha}_m) m^2$$  \hspace{1cm} (A15)

to leading order in $\tilde{\alpha}$. The relaxation time does not depend on $\tilde{\alpha}_1$.

Spectral densities for a free theory can easily be obtained by taking the limit $\alpha \to 0$,

$$\rho = 2\pi \text{sgn}(\omega) \delta(P^2 + m^2)$$  \hspace{1cm} (A16)

$$\tilde{\rho} = 2\pi \text{sgn}(\omega)(-\gamma^\mu P_\mu + m) \delta(P^2 + m^2).$$  \hspace{1cm} (A17)

The limit depends on the sign of $\omega$ because $\alpha(p, -\omega) = -\alpha(p, \omega)$.

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