Drag Force, Jet Quenching, and AdS/QCD

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Abstract

In this note, two important transport observables in the RHIC experiment, damping rate and jet quenching parameter, are calculated from an AdS/QCD model. A quark moving in the viscous medium such as the Quark-Gluon-Plasma is modelled by an open string whose end point travels on the boundary of a deformed AdS\textsubscript{5} black hole. The correction introduced via the deformed AdS\textsubscript{5} is believed to help us better understand the data which is expected to be measured in the RHIC.

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1 Introduction

Observation of leading-particles with transverse momentum $p_T$ in the relativistic heavy ion collider (RHIC) provides significant insights into the matter formed at center of the collision, which is considered to evolve into quark-gluon plasma (QGP) for sufficiently high collision energy\[1\]. In Au+Au collisions at the center-of-mass energy $\sqrt{s_{NN}} = 200$ GeV at Brookhaven it has been observed a strong suppression of high $p_T$ in detectable hadron spectra relative to that in binary proton+proton collision, which is quantified by ‘the nuclear modification factor’ $R_{AA}(p_T)$ \[2\]. This suppression factor, characterizing in-medium effects, approaches an asymptotic value at the higher momentum region $p_T \geq 5$ GeV and become independent of detected particle species. It indicates that detected particles with high $p_T$ (hadron jet) have a partonic origin, i.e., only a quark from a $q\bar{q}$ pair produced near the surface of expanding QGP can escape to outside and form hadron jets, and another quark recoiled back to inside of matter suffers from medium-induced energy (momentum) loss, which accounts for the strong suppression of back-to-back jet production.

A measurable quantity sensitive to this in-medium energy loss is so-called jet quenching parameter $\hat{q}$. This quantity parameters a suppression factor $P_f(\hat{q}, L, \Delta E)$ which is the probability of the process that a hard quark radiates energy of $\Delta E$ to medium during propagation in path $L$. Since $P_f$ gives main contribution to $R_{AA}$, $\hat{q}$ nicely reflects the medium energy loss effect at the high-$p_T$ region\[2\]. Also, $\hat{q}$ is determined by the mean square transverse momentum of quark propagating in unit length (or effectively in mean-free path $\lambda_f$), $\hat{q} = \langle \vec{p}_\perp^2 \rangle / \lambda_f$. It should be noted that $\lambda_f$ here characterizes medium opacity, and is expected to be very small so is shear viscosity in QGP because $\eta_s \propto \lambda_f$. The ratio of $\eta_s$ to entropy density $s$ extracted from RHIC date is close to a conjectured minimum bound $\eta_s/s \geq 1/(4\pi)$ \[3\], implying that QGP is strongly coupled. Thus, if one tries to derive $\hat{q}$ from first principle in quantum field theory, non-perturbative calculation seems to be required to obtain a reasonable value.

Another observable quantity sensitive to the in-medium energy loss is a damping rate $\mu$ (or friction coefficient) defined by Langevin equation, $\dot{p} = -\mu p + f$, subject to a driving force $f$. $f$ is equivalent to a drag force up to sign provided $\dot{p} = 0$. This quantity also characterizes opacity, or equivalently energy loss in dissipative processes in medium. One might think some relation between $\hat{q}$ and $\mu$, both of which describe the energy loss. By definition, jet quenching parameter is related to momentum fluctuation $\hat{q} \propto \langle \vec{p}_\perp^2 \rangle$, and drag force is to an averaged momentum loss escaping to medium $\langle \dot{p} \rangle$. Therefore, if the medium provides a stochastic force as $f$ to quark moving therein, these two quantities are related to each other via the fluctuation-dissipation theorem.

On the other hand, a way to approach this strongly coupled region of gauge theories is the AdS/CFT correspondence \[4\]. AdS/CFT allows us to investigate the strongly coupled regime of gauge theories where ‘t Hooft coupling $\lambda = g^2_{YM} N_c \gg 1$ by using classical gravity in effectively five dimensional AdS background, whose boundary corresponds to four dimensional gauge theory and one extra radial dimension should be interpreted as the energy scale of the gauge theory. In particular, one can consider a finite temperature gauge theory by introducing a black hole inside the AdS space\[5, 6\]. Using AdS/CFT, there are earlier attempts to addressing jet quenching parameter $\hat{q}$ \[7, 8, 13\] and damping rate $\mu$ \[9, 10, 11, 12, 13\]. Various
solutions of modified AdS background are discussed, namely, corresponding gauge theory are modified $\mathcal{N} = 4$ SYM’s. On the other hand, there are several works [14, 15, 16] to generalize the AdS/CFT correspondence to more realistic QCD. While there are several works to search elaborate string/supergravity solutions in full 10 dimensional space-time, whose holographic dual has properties similar to QCD, there are also many trials to directly construct five dimensional holographic dual of QCD by demanding to reproduce desired properties of QCD, which are referred to the AdS/QCD in our paper. In this paper, we reexamine the above two transport observables by using one of such AdS/QCD models. Especially, we adopt the model by Karch et al. [15] which possesses a confinement mechanism with the help of a non-trivial dilaton background. It is worth mentioning an alternative model proposed by Andreev and Zakharov [16]. They adopt a deformed $\text{AdS}_5$ metric but a trivial dilaton field. These two models are shown to be equivalent to each other as long as quadratic terms like $F^2$ in the effective action are concerned [16].

This paper is organized as follows. After we introduce the model being concerned in the next section, we warm up with a simple calculation of free energy of a static quark in thermal QGP. Then we carry out the calculation for drag force and jet quenching parameter in section 4. In comparison to the calculations based on the usual AdS/CFT, it is shown that the free energy has weaker dependence on temperature, and we observe bigger damping rate and smaller jet quenching parameter.

2 Model

The action of this model [15] is given by the following form,

$$S = \int d^5x \sqrt{-G} e^{-\Phi} \mathcal{L},$$

where $\mathcal{L}$ is a five dimensional Lagrangian density (whose detail is irrelevant in our calculation) and $\Phi$ is the dilaton field and is assumed to be proportional to the square of fifth coordinate $z$. The five dimensional metric $G_{MN}$ is given by the usual $\text{AdS}_5$ metric. It has been argued that this model reproduces Regge behavior $m_{n,S}^2 \sim \sigma_{\text{QCD}}(n + S)$. Where the QCD string tension $\sigma_{\text{QCD}} \simeq 0.93 \text{GeV}^2$ is properly adjusted by choosing a suitable value for the coefficient of $z^2$ in the dilaton field profile. In the later discussion, our calculation is based on the following correspondence between Wilson loop $W(C)$ in the gauge theory and on-shell worldsheet action $S$,

$$\langle W(C) \rangle \sim e^{-S}.$$

where the Nambu-Goto action $\mathcal{S}$ is given by

$$\mathcal{S} = \frac{1}{2\pi\alpha'} \int d\tau d\sigma e^{\phi/2} \sqrt{-\det \partial_a X^M \partial_b X^N G_{MN}}.$$  


1 The statement here seems to be conflicting. This is because we are comparing our result with the calculation based on different values of t’Hooft coupling $\lambda$, where $\lambda = 10$ was used for damping rate calculation and $\lambda = 6\pi$ for jet quenching parameter in their analysis.
Here we have assumed that the metric of this model is given in Einstein frame, thus we have
the extra dilaton contribution in front of the usual Nambu-Goto action.

In order to use the above formula, we must fix some parameters which are irrelevant in the
discussion of the paper \[15\]. The AdS metric and the dilaton field are given by \[2\]
\[
ds^2 = \frac{R^2}{z^2}(-dt^2 + dx^2 + dz^2),
\]
\[
\Phi(z) = cz^2,
\]
where the coefficient \(c\) is estimated as \(c = \sigma_{\text{QCD}}/4 \simeq 0.23\text{GeV}^2\) in the present model. \(R\) is the
radius of AdS space-time. One can fix the ratio \(R^2/\alpha'\) by matching the linear potential from
Wilson loop with the QCD string tension \(\sigma_{\text{QCD}}\). Such an estimation was performed in \[16\] for
another proposal of AdS/QCD. Here for the model in concern, we have
\[
\frac{R^2}{\alpha'} = \frac{4\pi}{ea^2c} \simeq 3.63,
\]
where \(a\) is a parameter in the Cornell potential for quark-antiquark relative separation \(r, V(r) = \frac{-\kappa}{r} + \frac{1}{a^2r} + C_0\) \[17\]. In the context of the original AdS/CFT, this ratio corresponds to
square root of t’Hooft coupling \(\lambda\). As a result, this value \(\lambda \sim \left(\frac{R^2}{\alpha'}\right)^2 \simeq 13.2\) is close to those in
the earlier calculations of transport observables where \(\lambda \sim 10^{-18}\). Notice that we have fixed
this value from the IR behavior in the confinement phase, instead of tuning it by hand to the
value in the energy scale at issue.

In this paper, we shall investigate this model with finite temperature. In order to introduce
temperature in the model, a natural guess is to incorporate black hole into the metric in \[6\]
whose Hawking temperature corresponds to the temperature at issue:
\[
ds^2 = \frac{R^2}{z^2}(-f(z)dt^2 + dx^2 + f(z)^{-1}dz^2),
\]
\[
f(z) = 1 - \frac{z^4}{z_T^4}.
\]
The function \(f(z)\) indicates the existence of a black hole with its event horizon at \(z = z_T\). As
in the dictionary of AdS/CFT correspondence, the Hawking temperature is given by the size
of horizon for large AdS black hole,
\[
T = \frac{1}{\pi z_T}.
\]

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2 One may consider a different AdS/QCD model as \[16\] based on the following modified AdS metric,
\[
ds^2 = h(z)\frac{R^2}{z^2}(-dt^2 + dx^2 + dz^2),
\]
with some suitable function \(h(z)\). However, in our calculation, \(h(z)\) will appear only in the combination of
\(h(z)e^{\Phi(z)/2}\) as one can easily seen from equation \[3\]. Though we only consider the case of \(h(z) = 1\) in the text,
we can easily recover the results for non trivial \(h(z)\).
Figure 1: Plot of red shift v.s. energy scale ($1/\pi z$). At a temperature of 200 MeV introduced by black hole (BH), we plot $V(z)f(z)$ for both AdS/CFT and AdS/QCD models. For comparison, we also plot those at zero temperature. Notice their differences in the low energy scale.

Figure 2: Regulated free energy of charm quark is identified as quark mass $m_c$ in thermal background. Position of flavor brane (relative to the horizon) in each model is adjusted to match experimental data $m_c \simeq 1.4$ GeV at $T = 318$ MeV. $\lambda = 10$ is chosen for the CFT model.

The property of strings in this geometry can be intuitively grasped by focusing on the effective redshift factor,

$$V(z)f(z) \equiv |e^{\Phi(z)/2}G_{tt}| = e^{cz^2 R^2/z^2}f(z). \quad (11)$$

In Fig. 1 we plot this effective redshift factor for several geometries. Note that, in any case, the geometries approach the usual AdS$_5$ at the boundary and its holographic dual gauge theory behaves like a conformal theory in UV region. On the other hand, the large effective redshift in the IR region of AdS/QCD model without black hole indicates the confinement of quarks.

3 Static quark solutions

Here we consider a static configuration of heavy quarks. Simply by argument based on symmetry, we learn that while one end of string is attached to the quark, the other end straightly
reaches to the horizon along z-direction. This static configuration gives rise to the expectation value of a Wilson line in thermal $N = 4$ super Yang-Mill theory and the free energy is given by the total string mass $\mathcal{S}$. This quantity, however, is divergent due to the infinite red shift close to the AdS$_5$ boundary. One way to render it finite is to locate quarks on a separate flavor brane at finite distance $z = z_m$, serving as a UV regulator. In this way, one obtains the free energy of static quark $F_q$ as a function of temperature at a given location of corresponding flavor brane $z_m$:

$$ F_q(T) = \frac{1}{2\pi\alpha'} \int_{z_m}^{z_T} e^{\Phi/2} \sqrt{-G_{tt}G_{zz}} dz. \quad (12) $$

In Fig. 2, it is shown that temperature dependence of regulated free energy of charm quark in the present AdS/QCD model is weaker than that in the usual AdS/CFT.

### 4 Moving quark solutions

#### 4.1 Drag force calculation

Here we consider a heavy quark moving on the boundary QCD but with a string tail into the AdS bulk. The dissipation of this quark in the QGP is described by the drag force, which is conjectured to be associated with a string tail in the fifth dimension. Now we consider a quark moving along constant $x^2$-$x^3$ plane and subject to a driving force $f_1$ such that

$$ \frac{dp_1}{dt} = -\mu p_1 + f_1. \quad (13) $$

One can read the drag force directly from $f_1$ for constant speed trajectory, whose corresponding string configuration in the effective five dimensions is given by

$$ x^1 = vt + \xi(z). \quad (14) $$

Here, the coordinates of string worldsheet $(\tau, \sigma)$ are chosen to be the spacetime coordinates $(t, z)$. The string action is given by

$$ S = \frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{\frac{f\xi'^2 - f^{-1}v^2}{1 + f\xi'^2 - f^{-1}v^2}}. \quad (15) $$

With the help of the existence of a conserved energy-momentum current $\Pi$, one obtains the configuration of string tail as

$$ \xi' = \pm \frac{\Pi}{f} \sqrt{\frac{f - v^2}{f - \Pi^2}}. \quad (16) $$

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$^3$To be more precise, we consider a rectangular Wilson loop $C$ formed by propagation of a pair of static heavy quarks along time $T$. In the limit $T \to \infty$, the expectation value of Wilson line is given by $\langle W(C) \rangle \sim e^{-S} = e^{-TF_q}$, where $F_q$ is the free energy between this pair of quarks and could be understood as total mass of string connected in between quarks. While quarks are widely separated, with the appearance of black hole, a single heavy quark could be isolated due to a constant potential (deconfining phase) and $F_q$, upon regularization, could be seen as quark mass at finite temperature.
Given the boundary condition at $z = 0$ and $z \to z_T$, one is able to solve for $\Pi$ and thus integrate to gain $\xi$. Equivalently one can also impose the reality condition on $\xi'$ and find out that

$$\Pi = \mathcal{V}(z_v)v, \quad z_v^4 = (1 - v^2)z_T^4.$$  \hspace{1cm} (17)

The drag force is then given by [11]

$$-f_1 = -\frac{1}{2\pi \alpha'} \Pi = -\frac{\pi T^2 R^2}{2\alpha'} e^{\Phi(z_v)} \frac{v}{\sqrt{1 - v^2}},$$ \hspace{1cm} (18)

The first equality can be understood in the following way: the energy-momentum flow $\Pi$ along spatial worldsheet direction is exactly the momentum change of moving quark, hence the applying force. In the opaque environment such as the QGP, this momentum change is due to drag force. The work done by drag force could be better pictured as the energy-momentum flowing along the string tail, and finally dumped into the black hole with no return. The minus sign in front of $f_1$ refers to the convention for the drag force against the motion of quark. After applying the AdS/QCD model [15], the damping rate now becomes velocity-dependent, which has the form at each instantaneous moment

$$\mu(v, T) = \frac{\pi T^2 R^2}{2m\alpha'} e^{\frac{\sqrt{1 - v^2}}{2\pi T^2}},$$ \hspace{1cm} (19)

where $m$ is the mass of quark. This expression reduces to the one for AdS/CFT case with $c \to 0$ and $R^2/\alpha' \to \sqrt{\lambda}$ [9, 11]. In Fig. 3 we plot the time evolution of momentum for charm quark at temperature 200 MeV, where the initial quark momentum is estimated to be 10GeV/c. We see that QGP is more viscous in this model than that shown in [11].

### 4.2 Jet quenching parameter calculation

Here we consider a null-like rectangular Wilson loop $C$ formed by a pair of quark-antiquark with separation $L$ travelling along light-cone time duration $L^-$. The jet quenching parameter
\( \hat{q} \) is related to the Wilson loop expectation value by \[7, 23\]

\[
\langle W^A(C) \rangle \simeq \exp \left( -\frac{1}{4\sqrt{2}} \hat{q} L^2 \right), \tag{20}
\]

where the superscript \( A \) denotes the adjoint representation. Using AdS/CFT correspondence, one is able to calculate it in the fundamental representation as

\[
\langle W^F(C) \rangle \simeq e^{-S_I}, \tag{21}
\]

where \( S_I \) is the regulated finite on-shell string worldsheet action whose boundary corresponds to the null-like rectangular loop \( C \). The relation \( \langle W^F(C) \rangle^2 \simeq \langle W^A(C) \rangle \) holds for large \( N_c \) \[7\].

To carry on the calculation based on our deformed AdS geometry, we rotate coordinate to light-cone one as \((t, x^1) \rightarrow (x^+, x^-)\), then the metric becomes

\[
ds^2 = \frac{1}{z^2} \left[-(1 + f)dx^+ dx^- + \frac{1}{2}(1 - f)[(dx^+)^2 + (dx^-)^2] + (dx^2)^2 + (dx^3)^2 + f^{-1} dz^2 \right]. \tag{22}
\]

We set the pair of quarks at \( x^2 = \pm \frac{L}{2} \) and choose the worldsheet coordinates \((\tau, \sigma)\) to be \((x^+, x^3)\). In this setup, we can ignore the effect of \( x^- \) dependence of the worldsheet and the string is simply configured by \( z(x^2) = z(\sigma) \) in the limit that \( L^- \) is much larger than \( L \). Then, the string action is given by

\[
S = \frac{R^2 L^-}{\sqrt{2 \pi \alpha'} z_T^2} \int_0^{L/2} d\sigma e^{\frac{\Phi(z)}{2}} \sqrt{1 + f^{-1} z'^2}. \tag{23}
\]

The equation of motion of \( z \) can be solved by

\[
E^{-2} = e^{-\Phi(z)} (1 + f^{-1} z'^2), \tag{24}
\]

where \( E \) is a normalized energy of motion. Plugging the above equation into the action, we have

\[
S = \frac{R^2 L^-}{E \sqrt{2 \pi \alpha'} z_T^2} \int_0^{L/2} e^{\frac{\Phi(z)}{2}} d\sigma = \frac{R^2 L^-}{\sqrt{2 \pi \alpha'}} \int_0^{z_T} \frac{e^{\Phi(z)} dz}{\sqrt{(z_T^4 - z^4)(e^\Phi - E^2)}}. \tag{25}
\]

Then the low-energy effective on-shell action becomes

\[
S \simeq \frac{R^2 L^-}{\sqrt{2 \pi \alpha'}} \left( \int_0^{z_T} \frac{e^{\frac{\Phi(z)}{2}} dz}{\sqrt{z_T^4 - z^4}} + \frac{E^2}{2} \int_0^{z_T} \frac{e^{-\frac{\Phi(z)}{2}} dz}{\sqrt{z_T^4 - z^4}} \right). \tag{26}
\]

This action needs to be subtracted by the inertial mass of quarks, given by two parallel flat string worldsheets along \( x^-z \) plane, that is,

\[
S_0 = \frac{2L^-}{2 \pi \alpha'} \int_0^{z_T} d\sigma e^{\frac{\Phi(z)}{2}} \sqrt{G_{zz} G_{zz}} = \frac{R^2 L^-}{\sqrt{2 \pi \alpha'}} \int_0^{z_T} \frac{e^{\frac{\Phi(z)}{2}} dz}{\sqrt{z_T^4 - z^4}}. \tag{27}
\]

\(^4\)The low energy approximation is justified by observing \( E(L) \) is small at the limit \( L \ll L^- \).
Notice that the integral is from the AdS boundary up to the horizon. Therefore, the net on-shell action is given by

$$S_I = S - S_0 \simeq \frac{R^2 L^2}{\sqrt{2\pi \alpha'}} \frac{E^2}{2} \frac{F(z_T)}{z_T},$$

(28)

where

$$F(z_T) \equiv z_T \int_0^{z_T} \frac{e^{-\Phi(z)}}{\sqrt{z_T^4 - z^4}} \, dz = z_T c^{1/2} \alpha^{-3/2} \left[ I^2_{-3/4} (c z_T^2) - I^2_{3/4} (c z_T^2) \right],$$

(29)

$E$ is also replaced by $L$ in the following way,

$$\frac{L}{2} = \int_0^{L/2} d\sigma = E z_T^2 \int_0^{z_T} \frac{dz}{\sqrt{(z_T^4 - z^4)(e^{\Phi(z)} - E^2)}} \equiv E z_T f(E; z_T).$$

(30)

Note that $f(E; z_T)$ is an increasing function of $E$ (regular at $E = 0$), and $f(0; z_T) = F(z_T)$ in Eq. (29).

To determine the jet quenching parameter, we only have to obtain the coefficient $a_1$ of linear term in expansion $E(L) = \sum a_n L^n$. From Eq. (30) one find

$$a_1 = \frac{\partial E}{\partial L} \bigg|_{L=0} = \frac{1}{2z_T} \left( f(E; z_T) + E \frac{\partial f(E; z_T)}{\partial E} \right)^{-1} \bigg|_{E=E(0)=0} = \frac{1}{2z_T f(0; z_T)}. \quad (31)$$

Plugging the above expression into Eq. (28) and comparing it with Eq. (20-21), we obtain the jet quenching parameter,

$$\hat{q} = 2 \frac{R^2}{2\pi \alpha'} \frac{f^{-1}(0; z_T)}{z_T^3} = \sqrt{\lambda} \pi^2 F^{-1}(1/\pi T) T^3. \quad (32)$$

For trivial $\Phi (c \to 0)$, we have

$$F^{-1}(z_T) \to \frac{\Gamma(3/4)}{\sqrt{\pi \Gamma(5/4)}},$$

(33)

and this reproduces the result in [7],

$$\hat{q} \approx \frac{\pi^{3/2} \Gamma(3/4)}{\Gamma(5/4)} \sqrt{\lambda} T^3. \quad (34)$$

Since $F(z_T)$ in Eq. (29) is a decreasing function in $c$, the expression (32) shows that finite $c$ as a non-trivial dilaton effect enhances the jet quenching parameter for fixed $\lambda$ and temperature. Fig. 4 shows our result of jet quenching parameter. Note that the figure appears to show somehow that our $\hat{q}$ is reduced slightly in comparison with that from AdS/CFT. This is due to different values for $\lambda$ we have put in: $(R^2/\alpha')^2 = \lambda \simeq 13.2$ for ours from Eq. (7), while $\lambda = 6\pi \simeq 18.8$ for the AdS/CFT result [7].
Figure 4: Jet quenching parameter. Solid curve is our result for AdS/QCD model with $c = 0.23 \text{ GeV}^2$ and $R^2/\alpha' = 3.63$, and dashed one is for usual AdS/CFT model with $\lambda = 6\pi$. Straight dotted line is an evolution-time averaged value of $\hat{q}$ from RHIC data with a range 5-15 GeV$^2$/fm.

In RHIC experiment, $\hat{q}$ decreases temporally after the collision as temperature goes down during expansion of QGP, and its evolution-time averaged value is estimated to be $\bar{\hat{q}} \simeq 5 - 15 \text{ GeV}^2$/fm \cite{2 23}, which can be reproduced from Eq. (32) at $T \simeq 320 - 470 \text{ MeV}$. This range of temperature seems consistent with what one expects for initially equilibrated temperature $T_0 \simeq 360 \text{ MeV}$ at RHIC \cite{2 18}.

There are suggestions in \cite{7 23} which account for discrepancies between $\hat{q}$ calculated from the AdS/CFT correspondence and other non-conformal models such as ours: the first possibility is that the number of color adjoint degrees of freedom decreases by some factor as going from $N = 4$ SYM to a generic QCD model, then $\hat{q}$ is conjectured to decrease somewhat. Other possibilities in the context of AdS/CFT have also been explored\cite{8 23}: for example, while the $1/\lambda$ correction will reduce the $\hat{q}$, including chemical potential will enhance it. It may still worth exploring similar effects in the model being concerned.

To compare our result with experimental values explicitly, we employ the Bjorken expansion scaling \cite{19} for the temperature evolution as $T(\tau) = T_0(\tau_0/\tau)^{1/3}$ with $\tau_0 = 0.5 \text{ fm}$, and an evolution-time averaged value of $\hat{q}$ defined by $\bar{\hat{q}} = \frac{4}{(4\pi)^2} \int_{\tau_0}^{T_0 + L^-/\sqrt{2}} \hat{q}(\tau) d\tau$ \cite{23}. Our result produces $\bar{\hat{q}} = 3.5 \text{ GeV}^2$/fm provided $T_0 = 360 \text{ MeV}$ \cite{18} and $L^-/\sqrt{2} = 2 \text{ fm}$ \cite{2} for hard parton travelling, while AdS/CFT model \cite{23} gives $\bar{\hat{q}} = 3.9 \text{ GeV}^2$/fm with $\lambda = 6\pi$.

Regarding the experimental data from RHIC, it seems premature to make comparison with our result, and with AdS/CFT models as well, because of the large range of $\hat{q}$ from the experiment. There might be other energy loss sources beside gluon radiation, while only the later corresponds to $\hat{q}$ calculated in this paper. This is also suggested in \cite{20}.

Recently it has been argued that the string configuration used for jet quenching calculation may not be physical \cite{13 21 22 24}, thus physical meaning of the result obtained above remains an open question. It may be a little early to discuss the discrepancy between result based on AdS/CFT calculation and that of RHIC experiment at this moment, before a better understanding of physical configuration of a moving quark-antiquark pair inside the QGP is provided.
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