Decaying coupled Fermions to curvature and the $H_0$ tension

David Benisty$^{1,2,*}$

$^1$Frankfurt Institute for Advanced Studies (FIAS), Ruth-Moufang-Strasse 1, 60438 Frankfurt am Main, Germany
$^2$Physics Department, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel

A formulation of cosmology driven by fermions $\psi$ is studied. Assumption of condensation for the spinor field simplifies the homogeneous solution of the Dirac equations and connects the spinor field with the scale parameter of the universe. With coupling between the Einstein term and spinor field, the possibility for a late time interaction emerges. In that way, the early universe agrees with $\Lambda$CDM model, but for the late universe the new integrating term dominates. From late time expansion data we obtain the $H_0$ from the SH0ES experiment. The data include the Pantheon Type Ia supernova, Quasars, Gamma Ray Bursts (for the Hubble diagram), cosmic chronometers and Byron Acoustic Oscillations. The decaying coupling reveals the capabilities of the scenario and makes it a good candidate for the description of nature. The tension can be reduced even further by including the local measurement of the Hubble constant.

I. INTRODUCTION

The standard model of cosmology, the $\Lambda$CDM model, requires a dark energy (DE) component responsible for the observed late-time acceleration of the expansion rate. The tension between the values of the Hubble constant $H_0$ obtained from the late universe measurements [1] and those from the Cosmic Microwave Background (CMB) by Planck Collaboration [2] is larger than $4\sigma$. This tension is one of the biggest challenges in modern cosmology that indicates a possible new physics [3–15]. Fermions in general relativity were studied in detail in [16] also with cosmological applications [17–19].

The search for constituents responsible for accelerated periods in the evolution of the universe is a big topic in cosmology. Several candidates has been tested for describing both the inflationary period and the present accelerated era: scalar fields, equations of state and cosmological constants. Fermionic fields has also been tested as gravitational sources of an expanding universe. In some of these models the fermionic field plays the role of the inflaton in the early period of the universe and of dark energy for the late universe. As we will see, under the simplest assumption of an homogeneous expansion the Noether symmetry yields the relation $\dot{\psi}\psi \sim 1/a^3$ where $\psi$ is the fermionic field, and $a$ is the scale factor of the universe. This relation impose a different scenario then the scalar fields in cosmology and gives another candidates for dark matter and dark energy. The corresponding theory that involve fermions instead of scalars are the "Fermions Tensor Theories" (FTT). Here we show that a simple coupling to curvature gives an exaltation to $H_0$ tension. This new coupling is dominant for the late universe and gives closer value to local measurement of the Hubble constant from [1]. For the early universe we get the $\Lambda$CDM model.

The plan of the work is as follows: Section II formulates the tetrad formalism for a homogeneous expansion. Section III suggests the FTT with the solved equations of motions. Section IV gives the extended theory we test. Section V test the extended model with the latest observations. Section VI summarizes the results.

II. FERMIONS IN CURVED SPACETIME

The tetrad formalism was used to combine the gauge group of general relativity with a spinor matter field. The tetrad $e^a_\mu$ and the metric $g_{\mu\nu}$ tensors are related through

$$g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}, \quad a, b = 0, 1, 2, 3,$$

(1)

with Latin indices refer to the local inertial frame with the Minkowski metric $\eta_{ab}$, while Greek indices denote the local coordinate basis of the manifold.

$\gamma^a$ are the Dirac matrices in the standard representation (flat spacetime). The Dirac matrices in curved space $\Gamma^a = e^a_\mu \gamma^\alpha$ are obtained using the tetrads $e^a_\mu$, labeled with a Latin index. The generalized Dirac matrices obey the Clifford algebra $\{\Gamma^a, \Gamma^b\} = 2g^{ab}$. The definition for the covariant derivative for spinors reads:

$$\partial_\mu \psi = \partial_\mu \psi - \Omega_\mu \psi, \quad \bar{\psi}_\mu = \bar{\partial}_\mu \bar{\psi} + \bar{\psi} \Omega_\mu,$$

(2)

$$\Omega_\mu = \frac{1}{4} g^{\beta\nu} \left\{ \frac{\nu}{\alpha\mu} - e^\nu_\alpha \partial_\mu e^\gamma_\alpha \right\} \gamma^\beta \gamma^\alpha,$$

(3)

where $\Omega_\mu$ is the spin connection, and $\{\nu_{\alpha\mu}\}$ is the Christoffel symbols:

$$\Gamma^P_{\mu\nu} = \frac{1}{2} g^{\rho\lambda} (g_{\lambda\mu,\nu} + g_{\lambda\nu,\mu} - g_{\mu\nu,\lambda}),$$

(4)

We apply the above fermionic model at a cosmological framework, focusing on early-time universe, and in particular on the inflationary realization. As usual we neglect standard matter. We consider the Friedman Lemaitre Robertson Walker (FLRW) homogeneous and isotropic metric

$$ds^2 = -dt^2 + a(t)^2 (dx^2 + dy^2 + dz^2),$$

(5)
with the scale factor $a(t)$, and thus through (1) the tetrad components are found to be
\[ e_0^\mu = \delta_0^\mu, \quad e_i^\mu = \frac{1}{a(t)} \delta_i^\mu. \] (6)
Moreover, the covariant version of the Dirac matrices are
\[ \Gamma^0 = \gamma^0, \quad \Gamma^i = \frac{1}{a(t)} \gamma^i, \] (7)
while the spin connection becomes
\[ \Omega_0 = \gamma^0, \quad \Omega_i = \frac{1}{2} \dot{a}(t) \gamma^i \gamma^0. \] (8)
This formulation is the basic mathmatics for the Fermion tensor theories (FTT).

### III. FERMION TENSOR THEORIES

The framework of FTT has a similar form to Scalar Tensor Theories. The FTT read:
\[ \mathcal{L} = f(\phi) \frac{R}{16\pi G} + i \frac{1}{2} \left[ \bar{\psi} \Gamma^\mu \psi_{,\mu} - \bar{\psi} \Gamma^\mu \nabla^\mu \psi \right] - V(\phi) + \mathcal{L}_m, \] (9)
where $R$ is the Ricci scalar, $\psi$ and $\bar{\psi} = \psi^\dagger \gamma^0$ are the spinor field and its adjoint, respectively. The scalar $\phi \equiv |\bar{\psi}\psi|$ multiplies the fermionic field and it’s conjugate field. $f$ is the function that couples the Einstein term and $V$ the self-interaction potential density of the fermionic field. The kinetic term of the spinors is the same as the kinetic term from Dirac equation. However, FTT suggest a generic coupling function $f$.

[20] studies some cosmological solutions of FTT. For $f = 1$ and $V = m_\psi \phi$ the FTT reduce to the Dirac equation in curved spacetime. For the metric (5) the action (9) reduces to the form:
\[ \mathcal{L} = 3a\dot{a} \left( \frac{\dot{a}}{a} f(\phi) + a \phi f' \right) + \frac{i}{2} \left( \bar{\psi} \gamma^0 \psi - \bar{\psi} \gamma^0 \psi \right) + a^3 V \] (10)
The solution is obtained via the complete set of variations: the scale factor $a(t)$ and the spinor field $\psi$.

The variation with respect to field $\psi$ (extended Dirac equation) yields:
\[ \ddot{\psi} + \frac{3}{2} H \psi + i \gamma^0 \psi V' - 6i(\dot{H} + 2H^2) \gamma^0 \psi f' = 0, \] (11)
and similarly for $\bar{\psi}$. By multiplying the first equation by $\bar{\psi}$ from the left hand side and the second equation by $\psi$ from the right hand, the equations get the same form. The sum of those two equations gives:
\[ \frac{d}{dt}(\bar{\psi} \psi) + 3H \bar{\psi} \psi = 0, \] (12) with the solution:
\[ \phi := \bar{\psi} \psi = n_\psi / a^3. \] (13)
The variation with respect to the scale parameter $a$ yields:
\[ -\frac{\ddot{a}}{a} = -\frac{\rho_f + 3\rho_f}{6F} \] (14)
where:
\[ \rho_f = V - 3Hf' \phi, \] (15)
\[ p_f = -V + \phi f' + 2H \phi f' + f'' \phi^2 \]
\[ + \phi \left( V' - 3\phi (H + 2H^2) \right). \] (16)
We impose that the energy function associated with the Lagrangian (9) is null and we get:
\[ \frac{\partial \mathcal{L}}{\partial a} \ddot{a} + \bar{\psi} \frac{\partial \mathcal{L}}{\partial \psi} + \frac{\partial \mathcal{L}}{\partial \psi} \psi - \mathcal{L} = 0, \] (17)
yields the Friedmann’s equation:
\[ 3H^2 = \frac{V(\phi)}{f(\phi) - 3\phi f'(\phi)}. \] (18)
[21] uses the Noether symmetry in order to find the coupling function $f$. The paper suggests $f \sim \phi$ or $f \sim \phi^{1/3}$. However, for a cosmological solution it is possible to choose $f = 1$ and redefine the potential $V$.

### IV. DECAYING CURVATURE COUPLING

The $\Lambda$CDM model emerges from the simplest expression for the potential. A simple example is a linear expansion for both functions:
\[ f(\phi) = 1, \quad V(\phi) = \Lambda + m_\psi \phi. \] (19)
The corresponding Hubble function reads:
\[ H^2/H_0^2 = \Omega_m / a^3 + \Omega_\Lambda. \] (20)
with the dark matter component $\Omega_m = 3m_\psi n_\psi / 8\pi G$ (without other fields that come from $\mathcal{L}_m$). The extended example that we suggest in this paper is a power-law modification:
\[ f(\phi) = 1 - \frac{\phi}{6} \phi^{-1}, \quad V(\phi) = \frac{\Lambda}{8\pi G} + m_\psi \phi. \] (21)
The corresponding Hubble function yields:
\[ \left( \frac{H(z)}{H_0} \right)^2 = \frac{\Omega_\Lambda + \Omega_m (1 + z)^3 + \Omega_r (1 + z)^4}{1 + \xi / (1 + z)^{3\xi}}, \] (22)
The tension is reduced from 3σ to 0.59σ.

\[ \Omega_m = 3m_\psi n_\psi / 8\pi G, \quad \xi = \xi n_\psi^{-1}. \]  

The basis of the mechanism here is simple: for very small scale parameter, the field \( \phi \) goes to infinity. Therefore, if the function \( f(\phi) \) and the potential \( V(\phi) \) has the same form, so the cancellation between the nominator and the denominator in the Hubble function gives a constant Hubble function - that known as inflationary solution. For very large values of the scale parameter, the field \( \phi \) goes to zero. So if we introduce function that approaches 1 for very small values of \( \phi \), we get the standard Friedmann equations.

V. OBSERVATIONAL CONSTRAINTS

In order to constraint our model, we use couple of data sets: Cosmic Chronometers (CC) exploit the evolution of differential ages of passive galaxies at different redshifts to directly constrain the Hubble parameter [22]. We use uncorrelated 30 CC measurements of \( H(z) \) discussed in [23–26]. As Standard Candles (SC) we use uncorrelated measurements of the Pantheon Type Ia supernova [27] that were collected in [28], Quasars [29] and Gamma Ray Bursts [30]. The model parameters of the models are to be fitted with, comparing the observed \( \mu_{i,\text{obs}} \) value to the theoretical \( \mu_{i}^{th} \) value of the distance moduli which are the logarithms:

\[ \mu = m - M = 5 \log_{10}(D_L) + \mu_0, \]  

where \( m \) and \( M \) are the apparent and absolute magnitudes and \( \mu_0 = 5 \log (H_0^{-1} / \text{Mpc}) + 25 \) is the nuisance parameter that has been marginalized. The luminosity distance is defined by,

\[ D_L(z) = \frac{c}{H_0} (1 + z) \int_0^z \frac{dz'}{E(z')}, \]  

Here, \( \Omega_k = 0 \) (flat space-time). The last dataset we use is uncorrelated data points from different Baryon Acoustic Oscillations (BAO) collected in [31] from [32–43]. Studies of the BAO feature in the transverse direction provide a measurement of \( D_H(z)/r_d = c/H(z)r_d \), with the comoving angular diameter distance [44, 45]:

\[ D_M = \int_0^z \frac{c \, dz'}{H(z')}. \]
FIG. 2. The posterior distribution for different measurements with the \( \Lambda \)CDM model with 1\( \sigma \) and 2\( \sigma \) for \( \Omega_m \), \( \Omega_\Lambda \) and \( H_0 \).

In our database we also use the angular diameter distance \( D_A = D_M/(1 + z) \) and the \( D_V(z)/r_d \), which is a combination of the BAO peak coordinates above:

\[
D_V(z) = [zD_H(z)D_M^2(z)]^{1/3}.
\]  

(27)

\( r_d \) is the sound horizon at the drag epoch and it is discussed in the corresponding section. Finally for very precise "line-of-sight" (or "radial") observations, BAO can also measure directly the Hubble parameter [46].

We use a nested sampler as it is implemented within the open-source packaged Polychord [47] with the GetDist package [48] to present the results. The prior we choose is with a uniform distribution, where \( \Omega_m \in [0; 1] \), \( \Omega_\Lambda \in [0; 1 - \Omega_m] \), \( H_0 \in [50; 100] \text{Km/sec/Mpc} \), \( r_d \in [130; 160] \text{Mpc} \) and \( \xi \in [0; 0.1] \) for the additional parameter. Table I summarizes the best fit results for different models. For the cases the \( l \) is assumed to be a constant, 1 or 2 the Hubble parameter increases a bit from 68.81 ± 1.024 km/sec/Mpc to 70.64 ± 1.798 for \( l = 1 \) and 69.29 ± 1.155 for \( l = 2 \). For the case \( l \) is a free parameter in the range \([0; 3]\) we get the Hubble constant increases to 72.02 ± 3.081 km/sec/Mpc. The tension with the SH0ES experiment is reduced from 2.98\( \sigma \) to 0.59\( \sigma \). Still, if we approach to the early universe which refers to larger \( z \), the modified model is changes to \( \Lambda \)CDM model, since the new term \((1 + z)^{-3l}\) becomes negligible.

VI. DISCUSSION

In this work, we have studied in detail the phenomenology of a simple generalized fermionic field model for dark energy and dark matter. Assumption of condensation for the spinor field simplifies the homogeneous solution of dark energy and dark matter. Assumption of condensation for the spinor field simplifies the homogeneous solution of dark energy and dark matter. Assumption of condensation for the spinor field simplifies the homogeneous solution of dark energy and dark matter. Assumption of condensation for the spinor field simplifies the homogeneous solution of dark energy and dark matter.

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