Asymptotic Freedom and Dirichlet String Theory

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ABSTRACT

Fixed angle scattering at high energy in a string theory with boundaries satisfying Dirichlet conditions (Dirichlet strings) in $D = 4$ is shown to have logarithmic dependence on energy, in addition to the power-like behavior known before. High temperature free energy also depends logarithmically on temperature. Such a result could provide a matching mechanism between strings at long distance and asymptotic freedom at short distance, which is necessary for the reformulation of large-$N$ QCD as a string theory.
In order to reformulate the large-$N$ QCD as a string theory, assuming it is possible at all, we must find a string theory that contains only massive excitations (for pure QCD) and shares the characteristic features of asymptotic freedom, such as partonic behavior and logarithmic violation of scaling at high energy. The critical string theories we are mostly familiar with fail to meet these two requirements: they contain massless gravitons and massless Yang-Mill particles, and fixed angle scattering falls off exponentially with total energy. Liouville theory has been argued to be irrelevant for the problem of QCD, meanwhile the effective string picture advocated in [4], which without doubt is correct for long strings, has problems at short distances. Rigid strings at first sight seem to give the right behavior at high energy and high temperature, but they have the wrong sign of free energy at high temperature as compared with QCD, and furthermore there are problems with unitarity.

Another simple yet non-trivial modification of short distance behavior of strings is to add world-sheet boundaries satisfying Dirichlet boundary conditions – each boundary is mapped to a single point in space-time and the position is then integrated over. Such a theory was originally proposed as an off-shell extension of closed string amplitudes. Meanwhile the point-like states of the Dirichlet boundaries were proven to produce power-like behavior of fixed angle scattering at high energy. The point-like structure was then linked to partons. Developments of the Dirichlet string theory in modern language can be found in [10]. Recently Green points out that at high temperature the Dirichlet boundaries produce a free energy similar to that of the large-$N$ QCD found by Polchinski. Namely, in QCD the free energy of a long string per unit length goes as

$$\mu_{QCD}^2(\beta)/L^2 \sim -\frac{2g^2(\beta)N}{\pi^2\beta^4},$$

(1)
while for the Dirichlet string,
\[ \mu_{DS}^2(\beta)/L^2 \sim -\frac{8\pi^2 w_D g_{\text{open}}^2(\beta)}{\beta^4}, \]  
(2)

where \( w_D \) is related to Chan-Paton factor and here is chosen to be \( N \). We should think \( w_D g_{\text{open}}^2 \) as the weight of inserting a Dirichlet boundary. It is quite natural to study this string theory further to determine whether it has anything to do with large-\( N \) QCD.

The QCD result does not simply imply that the free energy is some power of temperature \( 1/\beta \). The gauge coupling constant \( g^2(\beta) \) in \( D = 4 \) depends logarithmically on the scale one is probing. The asymptotic freedom \[12\] is the most distinctive feature of QCD. It is the purpose of this paper to explore the logarithmic renormalization in the Dirichlet string theory, which was anticipated in Green’s work \[10\]. This study is interesting because it could provide a mechanism to make string theory “unstringy” at short distance, thus makes contact with asymptotically free large-\( N \) gauge theories: it can be a matching condition from a string theory and QCD, much as matching two effective field theories at some common scale. The particular model we are going to work on may of course not be the correct one, but it illustrates the point. Since this is a first work on higher order corrections in the Dirichlet string theory, we will learn how short distance behavior gets modified by loop effect.

The model we pick up is a bosonic critical string theory compactified on a torus \( T^{22} \) of radius \( R \) in each direction, with insertions of world sheet boundaries satisfying Dirichlet conditions for 4 large dimensions and usual Neumann conditions for the rest 22 dimensions. The string coordinates are thus denoted by \( X^\mu(\sigma, \tau) \) (\( \mu = 1, ..., 4 \)) and \( X^I(\sigma, \tau) \) (\( I = 5, ..., 26 \)). At the boundary \( \partial M \) of the world sheet \( M \), roughly speaking, \( X^\mu|_{\partial M} = x_\nu^\mu, \partial_n X^I|_{\partial M} = 0 \), here \( \partial_n \) means normal derivative. These boundaries are literally true instantons: they live for infinitely short duration in time.

The Polyakov functional integral\[2\] for strings with Dirichlet boundary conditions was
studied in [13]. We will follow the treatment of [14] for boundary conditions. Let us first look at the basic physical picture of Dirichlet strings [9, 10]. Amplitudes in ordinary closed string theory is formulated as sum over closed riemann surfaces with punctured holes representing positions of various vertex operators. A closed string coupling constant $g^2_c$ is added to distinguish contributions from different genus. In the present case the world sheet consists of additional boundaries, each of which is mapped to a single and different space time point. Those points are then integrated over the whole space time. It is important that the weight ($w_D g^2_{open}$ in (2)) of adding one such boundary is independent of $g^2_c$, while in ordinary open string theory the two are related due to unitarity. Thus we think about the Dirichlet string theory as a theory of closed strings or “glueballs”. One can of course simultaneously have other boundaries with Neumann conditions, which would be thought as adding “mesons”.

Assuming the Riemann surface we are considering has $B$ boundaries, $E$ vertex operators, and genus $G$, according to the general procedure of evaluating the Polyakov path integral [13], [15, 16], we have for the string amplitude

$$A_E(k_1...k_E) = C_M g^2_c E (w_D g^2_{open}) B \prod_{b=1}^{B} d^D x_b \int d^n \tau [det(P^i)]^{1/2} \frac{detH(P^i)}{detH(P)}^{1/2} \int DX e^{-S(x,\dot{x})} \prod_{e=1}^{E} d^2 \sigma_e V_e(\sigma_e, k_e).$$

The only difference from the usual string theory is the integration over $x_b^\mu$, apart from a trivial change of boundary conditions. We do not specify various symbols because they are either self-evident or will be given in specific cases later.

Let us comment on the operator formulation, which gives clearer physical picture [10]. Due to duality or conformal invariance, we can think of the string amplitude as a diagram with closed string propagating (with some initial or final boundary states) or with open string in the intermediate channel. The former interpretation uses the familiar closed string
propagator \((L_0 + \bar{L}_0 - 2)^{-1}\), while the latter is more interesting. \(X(\sigma, \tau)\) of an open string satisfying the above boundary condition is given by

\[
X^\mu = x_1^\mu + (x_2^\mu - x_1^\mu) \frac{\sigma}{\pi} + \sqrt{2\alpha'} \sum_n \frac{1}{n} \alpha_n^\mu \sin n\sigma e^{-in\tau},
\]

\[
X^I = x^I + \frac{2\alpha'm^I}{R} \tau + \sqrt{2\alpha'} \sum_n \frac{1}{n} \alpha_n^I \sin n\sigma e^{-in\tau}.
\]

(4)

\(L_0\) is then

\[
L_0 = \frac{1}{4\alpha'\pi^2} [(x_2 - x_1)^2 + (2\alpha'm/R)^2] + \sum_{n=1}^\infty \alpha_{-n} \cdot \alpha_n.
\]

(5)

The propagator is \((L_0 - 1)^{-1}\). The first term in (5) is most notable: it replaces the usual \(p^2\). The reader should consult [10] for more details.

The lowest order contribution of the instantons is of course one boundary insertion. Various aspects have been discussed thoroughly by Green [10, 11]. The world sheet in question has the topology of a disk. One can show that two tachyon scattering amplitude at fixed angle and high energy is proportional to \((stu)^{1/2}\), where \(s, t\) and \(u\) are Mandelstam variables. The growth of amplitudes at high energy is related to the existence of tachyons in the dual theory, and is expected to change when the correct vacuum is found. The high temperature free energy is already given in (2).

Our aim is to consider two instanton contribution. This is similar but physically distinct from the semi-off-shell amplitudes considered in [17]. The world sheet in a cylinder or annulus. The two boundaries are mapped to \(x_1^\mu\) and \(x_2^\mu\), respectively. We are interested in the case when the two instantons are close in spacetime, and they can be seen as a single instanton with renormalized strength. Conceptually this looks very much like how renormalization works in field theory. Since the bosonic string theory has various divergences, one must be careful about their interpretations. We are not worried about divergences caused by the closed string states, since they are related to the infrared behavior of the theory and can be cured. Rather the interesting divergences occur in the open string.
channel. In (5) there is a singularity outside the light cone, which is the reminiscence of
open string tachyon in the dual theory. We treat this as we do with the closed string
tachyon, that is, by analytical continuation. More interesting divergence occurs on the
light cone. Formally if we take $\alpha'$ to infinity, only light cone singularity remains [10]. It
is interesting that this is also the limit we would like to consider in high energy scattering
[18].

We first give a heuristic argument for the logarithmic renormalization in the Dirichlet
theory, based on analyticity in “position space” studied in [10]. We visualize a cylinder
diagram as sewing of several open string propagators (5) with some vertices that couple
the open string to closed string states, which are tachyons here. Now look at a particular
channel where the world sheet consists of a long tube along which two open strings prop-
agate, and closed string states at each side. The propagators of the two open strings are
the same as (5). So we have the following term

$$\int d^4\Delta x \frac{1}{(L_0 - 1)^2},$$

(6)

where $\Delta x^\mu = x_2^\mu - x_1^\mu$. Take the first excited state in $L_0$, we get logarithmic divergence.
This picture also prompts us to look at suitable corners of moduli space in more refined
treatment.

We begin a detailed discussion of fixed angle two tachyon scattering amplitude at high
energy on an annulus or cylinder. In (3), we need to know some determinants and the
propagator on the world sheet. The determinants are neatly worked out in both [14] in the
Polyakov approach and in [10] in operator formalism. The cylinder has one Teichmuller
parameter $l$ and we choose our background metric to be $\hat{g}_{ab}d\sigma^a d\sigma^b = (d\sigma^1)^2 + l^2(d\sigma^2)^2$.
Both $\sigma^1$ and $\sigma^2$ take range $[0, 1]$. The boundaries are at $\sigma_2 = 0, 1$. The vertex operators
are
\[ \prod_{i=1}^{E} \int d^2\sigma_i \sqrt{\hat{g}(\sigma_i)} \exp ik_{i\mu}X^\mu(\sigma_i), \] (7)
where \( k_i^2 = 8\pi T = 4/\alpha' \). The integration variables are \( x_1, x_2 \) and \( l \). The integration measure in (3) is, including contributions from matter sector,
\[ \frac{1}{(\alpha'l)^4} F_{1}^{22}(a, q)f(q^2)^{-24} \exp[-(x_2 - x_1)^2/(4\pi\alpha'l) + 4\pi l + i\sigma_i^2 p_i \cdot (x_2 - x_1) + ip_i \cdot x_1], \] (8)
where we have used the standard notations \( q = \exp(-2\pi l), a^2 = \alpha'/R^2 \), and [19]
\[ F_1(a, q) = \theta_3(0|\frac{\ln q}{2\pi i a^2}) = \sum_{n=-\infty}^{\infty} q^{n^2/2a^2}, \quad f(q^2) = \prod_{n=1}^{\infty} (1 - q^{2n}). \] (9)
\( F_{1}^{22}(a, q) \) arises from sum over the zero modes or momentum states of compactified dimensions. \( e^{4\pi l} f^{-24}(q^2) \) comes from oscillator summation of both matter and ghost. Notice the Dirichlet boundary condition prevents usual zero modes in uncompactified dimensions. This shows up as the factor \( 1/l^D \). And finally the exponential of \( x \) and \( p \) terms is the contribution of classical action.

We need to know the propagator on the cylinder. Since the tachyons involve \( X^\mu \) only, we just give the expression for Dirichlet condition \( X^\mu(\sigma_1, 0) = X^\mu(\sigma_2, 1) = 0 \). Note we have already extracted the zero mode contribution in (8). The propagator is easily found by the method of images to be [17]
\[ \langle X(\sigma_i)X(\sigma_j) \rangle = -\frac{l(\sigma_i^2 - \sigma_j^2)}{T} - \frac{1}{2\pi T} \ln \left| \frac{\theta_1(\nu_{ij}|2li)}{\theta_1(\bar{\nu}_{ij}|2li)} \right|, \] (10)
where \( \nu_{ij} = \sigma_i^1 - \sigma_j^1 + il(\sigma_i^2 - \sigma_j^2) \) and \( \bar{\nu}_{ij} = \sigma_i^1 - \sigma_j^1 + il(\sigma_i^2 + \sigma_j^2) \) are image points, and \( \theta_1(z|\tau) \) is the Jacobi \( \theta \)-function (see, e.g., [19]). When \( i = j \), we make one subtraction [19], and get
\[ \langle X(\sigma_i)X(\sigma_i) \rangle = -\frac{l(\sigma_i^2)^2}{T} - \frac{1}{2\pi T} \ln \left| \frac{\theta'_1(0|2li)}{\theta_1(2li\sigma_i^2|2li)} \right|. \] (11)

We perform integration over \( x_i^\mu \) first, which imposes momentum conservation. We use the Mandelstam variables \( s = -(k_1 + k_2)^2, t = -(k_1 + k_3)^2, \) and \( u = -(k_1 + k_4)^2 \). Put
(7)-(12) in to (3) and use the on-shell condition we arrive at the expression

\[ A_4 = C g_c^4 (w_D g_{\text{open}}^2)^4 \int d^4 \Delta x \int_0^\infty dl \prod_{i=1}^4 (l \int d^2 \sigma_i) \frac{1}{(\alpha l)^2} F_{12}^{22}(a, q)f(q^2)^{-24} \]

\[ \exp[-(\Delta x)^2/(4\pi \alpha') + 4\pi l + i\sigma_i^2 p_i \cdot \Delta x] \prod_{j=1}^4 \left\{ \exp(2\pi l\sigma_j^2) \frac{\theta_1'(0|2li)}{\theta_1'(2l\sigma_j^2|2li)} \right\}^2 \]

\[ \left\{ \frac{\chi_{12}\chi_{34}}{\chi_{14}\chi_{23}} \right\}^{-s/4\pi T} \left\{ \frac{\chi_{13}\chi_{24}}{\chi_{14}\chi_{23}} \right\}^{-t/4\pi T} \left\{ \frac{\chi_{12}\chi_{13}\chi_{24}\chi_{34}}{\chi_{14}\chi_{23}} \right\}^{-4}, \]  

(12)

where

\[ \chi_{ij} = \exp\{+2\pi l\sigma_i^2\sigma_j^2\} \frac{\theta_1(\nu_{ij}|2li)}{\theta_1'(\nu_{ij}|2li)} \]  

(13)

(12) coincides with [17], apart from the \( \Delta x \) integration and compactification. We can of course integrate over \( \Delta x \) in (12), since it is a Gaussian. That will make closed string poles clear. Indeed the way the amplitude is written is suitable for discussing closed string intermediate states. Since we are interested in possible divergence in \( \Delta x \) integration, we will perform it at last step. It is convenient to make Jacobi transformation now, which is equivalent to a conformal transformation of the world sheet metric by a factor \( 1/l^2 \). (12) reads,

\[ A_4 = C g_c^4 (w_D g_{\text{open}}^2)^4 \int d^4 \Delta x \int_0^\infty \frac{dl'}{l'} \prod_{i=1}^4 (l' \int d^2 \sigma_i) \theta_3^{22}(0|2\pi a^2 l' i)f(e^{-2\pi l'})^{-24} \]

\[ \exp[-(\Delta x)^2 l'/(2\pi \alpha') + 2\pi l' + i\sigma_i^2 p_i \cdot \Delta x] \prod_{j=1}^4 \left\{ \frac{\theta_1'(0|l' i)}{\theta_1'(\sigma_j^2|l' i)} \right\}^2 \]

\[ \left\{ \frac{\chi_{12}\chi_{34}}{\chi_{14}\chi_{23}} \right\}^{-s/4\pi T} \left\{ \frac{\chi_{13}\chi_{24}}{\chi_{14}\chi_{23}} \right\}^{-t/4\pi T} \left\{ \frac{\chi_{12}\chi_{13}\chi_{24}\chi_{34}}{\chi_{14}\chi_{23}} \right\}^{-4}, \]  

(14)

where we have used \( l' = 1/2l \) and \( \chi_{ij} \) changes to

\[ \chi_{ij} = \left| \frac{\theta_1(-\nu_{ij} l' i)}{\theta_1(-\bar{\nu}_{ij} l' i)} \right|. \]  

(15)

It is clear that when \( s, t \) are both large, the last factor in (12) can be ignored, and the integral is dominated by saddle points [15]. In our case, they occur when all \( \sigma_i^2 \)'s approach the two boundaries of the cylinder, so that \( \chi_{ij} \) approaches 1. If we take one of the vertex
to the boundary, we find a \(1/\epsilon\) divergence, which is related to the singularity outside the light cone \([10]\). This divergence is removed by redefining our vertex operators \([20]\). In order to match the intuitive discussion before, we will also take the limit \(l \to 0(l' \to \infty)\). It is proven convenient to limit invariant distance between two vertex operators or between one vertex operator and one boundary to be larger than \(\epsilon\) \([20]\).

We take the limit \(\sigma_i^2 \to 0\). We simply Taylor expand various \(\theta\)-functions. Use the fact that \(\theta_1'/\theta_1 \sim i\pi\) and \(\theta_1''/\theta_1 \sim -\pi^2\) for the range of variables are are interested in, the \(s\) and \(t\) power terms simplify to \(\exp(\sum a_{ij}\sigma_i^2\sigma_j^2 s/4\pi T)\) and a similar expressions for the \(t\) term. If we take \(\sigma^2 \to 0\) faster than the other three, and change variables to

\[ z_1 = \sigma_3^2\sigma_4^2, \quad z_2 = \sigma_2^2\sigma_4^2, \quad z_3 = \sigma_2^2\sigma_3^2, \]  

(16)

(14) becomes

\[ A_4 \sim \ldots \int \frac{d\sigma_1^2}{(\sigma_1^2)^2} \int dz_1dz_2dz_3 \exp(z_1s/4\pi T + z_2t/4\pi T + z_3u/4\pi T) \sim (stu)^{1/2}. \]  

(17)

The above integral is divergent, which is cut off by \(\epsilon\) introduced before. One can add counterterms living on the boundaries to to cancel the divergence \([10, 17]\).

Now we do the remaining integrals. When \(\sigma_1^1\) are far apart, our treatment of \(\theta\) functions is correct. After completing the integral we get divergence in powers of a space-time cutoff \(|\Delta x| > 1/\Lambda\). When, say, \(\sigma_1^1\) and \(\sigma_2^1\), and similarly \(\sigma_3^1\) and \(\sigma_4^1\) get close to the order of \(1/l',\) respectively, there is a change. In this case it is convenient to use \(\eta_{12}/l' = \sigma_1^1 - \sigma_2^1\) and \(\eta_{34}/l' = \sigma_3^1 - \sigma_4^1\). (12) is proportional to

\[ \int d^4\Delta x \int_0^{\infty} \frac{dl'}{l'} l'^2 \exp[-l'(\Delta x)^2/2\pi \alpha'], \]  

(18)

with again some \(\epsilon\) dependence. Strictly speaking the above equation should include sum of terms with \(\exp(2\pi nl')\) and \(\exp(2\pi a^2 nl')\), but they don’t give rise logarithmic divergence and as mentioned before we ignore them. The above integral becomes \(\ln(\Lambda^2/s)\). The \(s\)
factor is natural on dimensional grounds. It is not necessary for all four vertex operators to touch the same boundary. For example, if $\sigma_1^2, \sigma_2^1 \rightarrow 1$ while the other two go to zero, there will be a factor $\exp i (p_1 + p_2) \cdot \Delta x$ in (15) and (16), which is also logarithmic divergent.

Let’s write down the log–log divergent contribution to the amplitude (12)

$$A_{4D} \sim C_2 g^{4} (w_D g_{open}^2) \ln(\Lambda^2/s)(stu)^{1/2}.$$  \hspace{1cm} (19)

If we renormalize the Dirichlet coupling constant as

$$w_D g_{open}^2(\mu) = w_D g_{open}^2(\Lambda) + \frac{C_2}{C_1} (w_D g_{open}^2)^2 \ln(\Lambda^2/\mu^2),$$  \hspace{1cm} (20)

where $C_1$ as a factor from the disk diagram, we can write down the renormalization group equation

$$\frac{d(w_D g_{open}^2)}{d \ln \mu} = -\frac{2C_2}{C_1} (w_D g_{open}^2)^2,$$  \hspace{1cm} (21)

which is characteristic of asymptotically free theories (assuming the positivity of $C_1$ and $C_2$). This is the main result of this paper.

We may also compute the high temperature partition function by relating it to the mass squared of winding tachyon states. This is actually simpler than the one performed above, since the diagram involves only two vertex operators. We find the similar logarithmic divergence as (21), and we conclude the winding tachyon mass (2) contains logarithmic dependence on $\beta$ as well.

Some comments are in order. First of all, during our investigation we encountered power-like divergences at short distance in spacetime. In absence of an off-shell formulation, it is hard to deal with them. If the standard lore of renormalization applies, we would conclude they are not interesting because they can be cancelled by local counterterms. Logarithmic divergence is entirely different matter, of course. It is this that makes our result interesting. One interpretation of the power-like divergences would be that the Dirichlet string theory is too singular to be a consistent theory at short distance. At best
it is an effective theory, as the pion phenomenological Lagrangian [21]. The logarithmic dependence we found would serve as the matching condition between long distance string theory and short distance QCD. It is not ruled out, however, that the theory could be fully consistent once its true vacuum is found, or some other models, with for example more world sheet symmetries, are considered. We would like to see a more systematic treatment of other divergences in this string theory. To cancel these divergences we need to go beyond Weyl invariant theory in general [20, 17]. So an off-shell formulation is also needed here. Eventually this will help us to understand mass correction, gauge invariance and other issues.

In conclusion, the logarithmic divergence of Dirichlet string theory could be an important indication that a string formulation of large-$N$ QCD is possible. Certainly this deserves further study, and hopefully a new consistent string theory will emerge.

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