On the temperature prediction in a fire escape passage

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Abstract. Fire safety engineering requires a detailed understanding of fire behaviour and of its effects on structures and people. Many factors may condition the fire scenario, as for example, heat transfer between the flame and the boundary structures. Currently advanced numerical codes for the prediction of the fire behaviour are available. However, these solutions often require heavy calculations and long times. In this context analytical solutions can be useful for a fast analysis of simplified schematizations. After that, it is more effective the final utilization of the advanced fire codes. In this contribution, the temperature in a fire escape passage, separated with a thermally resistant wall from a fire room, is analysed. The escape space is included in a building where the neighbouring rooms are at a constant undisturbed temperature. The presence of the neighbouring rooms is considered with an equivalent heat transfer coefficient, in a boundary condition of the third type. An analytical solution is used to predict the temperature distribution during the fire. It allows to obtain useful information on the temperature reached in the escape area in contact with a burning room; it is useful also for a fast choice of the thermal characteristics of a firewall.

1. Introduction
To enable the occupants to move to a place of safety before the environment in any evacuation route becomes untenable from the effects of a fire, is the main objective of “fire hazard management” in buildings [1]. In general terms, in an event of fire inside a building the conditions of tenability in an evacuation route must be maintained for the period of time that occupants need to evacuate the building. These conditions have been synthesized in [2] as:

(i) the temperature will not endanger human life; and
(ii) the level of visibility will enable the evacuation route to be determined; and
(iii) the level of toxicity will not endanger human life”.

In the present paper we concentrate our attention on the tenability criterion for temperature. Purser [3, 4] gives a comprehensive review of the threshold times to temperature exposure for the human body, beyond which incapacitation, serious injury or death occurs. This tolerance time depends on the temperature (higher temperature / shorter tolerance time) up to a lower limiting value that is estimated around 50 °C. Based on available experimental data, Purser [3] was able to derive expressions for these tolerance times, intended as maximum exposure time, to incapacitation, serious injury / severe incapacitation, fatal exposure conditions with extensive third-degree burns. This information is particularly useful in a performance based design approach.

Differently, in a prescription based design approach is the “fire resistance” of building components...
and constructions to be commonly used. The building elements are classified in different fire resistance classes, or combinations of those, by specifying different performance criteria. Normally the fire resistance classification is expressed as a time limit in minutes (15, 30, 45, 60, 90, 120, 180, 240, or 360) which shows the time the performance criteria is fulfilled during a standardized fire test. Specific requirements for the classes are: R Load bearing capacity, E Integrity, I Insulation.

By following the Eurocode standard EN 1992-1-2 [5], "where compartmentation is required, the elements forming the boundaries of the fire compartment, including joints, shall be designed and constructed in such a way that they maintain their separating function during the relevant fire exposure. This shall ensure, where relevant, that:

- integrity failure does not occur
- insulation failure does not occur
- thermal radiation from the unexposed side is limited."

By following EN 1992-1-2 [5], “criterion “I” may be assumed to be satisfied where the average temperature rise over the whole of the non-exposed surface is limited to 140 K, and the maximum temperature rise at any point of that surface does not exceed 180 K.”

In a previous paper [6] we concentrated our attention on the insulation, I, that is the ability of a separating element of a building, when exposed to fire on one side, to restrict the temperature rise of the unexposed face below specified levels. In the present paper we now concentrate our attention to the tenability criterion for temperature, expressed in terms of tolerance time.

The predictions of the time-space temperature distribution in the separating wall and in the evacuation route are obtained by means of the analytical solution already discussed in [6]. The limits of this approach were also discussed in [6] and are not further analysed here. Now, the mathematical model is completed to take into account the presence of rooms surrounding the escape passage. This is a configuration often encountered in special buildings, like hospitals and multi-residence housing. To the best of our knowledge, there are no bibliographic references dealing with this subject.

The temperature trend in the escape route is obtained in a simplified manner with a suitable choice of the boundary condition placed on the face of the wall not exposed to the fire. This boundary condition is obtained with an energy balance between the escape passage and the neighbouring rooms. In this configuration, dimensions and thermal characteristics of the walls (floor, ceiling and other unexposed walls) become influent for the evolution of the temperature. Numerical simulations obtained with specialized software, like FDS [7] or Smartfire [8], are certainly a right approach for the prediction of the temperature in complex escape routes, characterized by complex geometries and stratified wall. Nevertheless, our simplified analytical approach, with simplified geometry and walls characterization, can be certainly useful to individuate the driving parameters of the problem and to give them the right weight in a further detailed analysis.

The present work investigates the thermal performance of a fire resistant wall in an escape passage when exposed to a fire, in order to predict the temperature trend in the passage itself. The goal is the estimation of the tolerance time to temperature, it means the time needed to reach a value of temperature after that pains or hyperthermia occurs for a human body.

2. Formulation and solution
The problem is schematically shown in Fig. 1. In [6] we examined a fire resistant wall in the typical configuration used for the assessment of its fire resistance (Fig. 1.a). In the present paper the wall is now part of a fire-isolated passageway (Figs. 1.b and 1.c). This is a corridor of fire-resisting construction, "which provides egress to or from a fire-isolated stairway or fire-isolated ramp or to a road or open space“ [2]. The air temperature in the fire-isolated passageway is free to change and follows the evolution of the fire. The remaining walls of the fire-isolated passageway face rooms unaffected by the fire.

A sketch of the geometry considered for the mathematical formulation of the problem, is shown in Fig. 2. A single layer wall (thickness L) is exposed to a fire on one face (x = 0). The temperature in the fire room changes stepwise, with a final value typical of a post-flashover condition. The boundary condition on the surface exposed to fire is of the third type. As already demonstrated in [6], with a proper choice of the heat transfer coefficient, the temperature on this wall is that due to the standard
temperature-time curve suggested by EN 1991-1-2 [9]. The thickness of the wall is small against its height and width, so that the wall can be considered of unlimited extension and its heat flow is one-dimensional in the direction of the thickness. The properties of the fire resistant wall are uniform, constant and evaluated at a mean value of its temperature, significant for the solution of the problem. The prediction of the temperature in the wall is obtained with the procedure discussed in [6].

In the present application the thermal effect due to the presence of the neighbouring rooms, where the temperature is assumed to be maintained at a constant value in time, is simulated through an equivalent heat transfer coefficient. This approach is based on two assumptions:

- constant temperature in the neighbouring rooms;
- negligible thermal inertia of the walls separating the escape passage from the neighbouring rooms.

The first assumption is justified by considering that the time commonly involved in the evacuation of a building is relatively short and at the beginning of the fire. In [6] we discussed the cases described
by Fig. 1.a for applications similar to that analyzed here. In that cases the heat flux exchanged through the fire resistant wall is that highest possible. Nevertheless, during the initial phase of the fire, the heat flux transmitted by the wall is small for a time comparable to that requested for the evacuation. The thermal inertia of the fire resistant wall is large enough to accumulate the heat before transmitting it in the protected room. It follows that the neighbouring rooms are affected by the fire only after a significant time and that in the intermediate time their temperature can be assumed constant in time.

The second assumption is justified by the slow evolution of the temperature in the escape passage during the time necessary for the evacuation of the building, that is a direct consequence of the former assumption. It follows that during this time the heat transfer process between the escape passage and the neighbouring rooms can be modelled as quasi-steady.

Furthermore, a 2D effect (something like a fin effect) could be considered where floor and ceiling intersect the fire resistant wall. However, the low thermal conductivity of their materials and the significant thickness of these walls ensure a negligible contribution.

The problem is stated mathematically as follows:

Conservation of energy:
\[
\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \quad (1)
\]

Boundary conditions:
\[
x = 0 \quad h_1 [T_{t_1} - T(x = 0, t)] = -\lambda \frac{\partial T}{\partial x}(x = 0, t) \quad (2)
\]
\[
x = L \quad -\lambda \frac{\partial T}{\partial x}(x = L, t) = h_2 [T(x = L, t) - T_{t_2}] \quad (3)
\]

Initial condition:
\[
t = 0 \quad T(x, t = 0) = T_{a_2} \quad (4)
\]

Equations (1-4) are now expressed as dimensionless, according to the following dimensionless variables:
\[
\Theta = \frac{T(x, t) - T_{a_2}}{T_{t_1} - T_{a_2}} \quad \xi = \frac{x}{L} \quad \tau = \frac{t}{t_0}
\]

Equation (1) and related boundary and initial conditions, Eqs. (2-4), become:
\[
\frac{\partial \Theta}{\partial \tau} = \Theta \frac{\partial^2 \Theta}{\partial \xi^2} \quad (5)
\]

for \( \xi = 0 \)
\[
\left. \frac{\partial \Theta}{\partial \xi} \right|_{0, \tau} = B_{i_1} (\Theta(0, \tau) - 1) \quad (6)
\]

for \( \xi = 1 \)
\[
\left. \frac{\partial \Theta}{\partial \xi} \right|_{1, \tau} = -B_{i_2} \Theta(1, \tau) \quad (7)
\]

for \( \tau = 0 \)
\[
\Theta(\xi, 0) = 0 \quad (8)
\]

The dimensionless parameters \( B_{i_1}, B_{i_2} \) and \( T \), are given by:
\[
B_{i_1} = \frac{h_1 L}{\lambda} \quad B_{i_2} = \frac{h_2 L}{\alpha} \quad T = \frac{t_0}{L^2/\alpha}
\]

The solution of Eqs. (5-8) can be decomposed as follows:
\[
\Theta(\xi, \tau) = U(\xi) + V(\xi, \tau) \quad (9)
\]

When substituting Eq. (9) in Eqs. (5), it follows that the problem is now given by the solution of the following ordinary differential equations:
0 = \frac{d^2 U}{\xi^2} \quad \text{(10)}
\frac{\partial V}{\partial \tau} = T \frac{\partial^2 V}{\partial \xi^2} \quad \text{(11)}

where Eq. (10) deals with the Steady state component of the solution and Eq. (11) with its Time dependent component.

The Steady state component of the solution is coupled to the following boundary conditions:

\begin{align*}
\xi = 0 & \quad \frac{dU}{d\xi} \bigg|_{\xi=0} = Bi_i (U(0) - 1) \quad \text{(12)} \\
\xi = 1 & \quad \frac{dU}{d\xi} \bigg|_{\xi=1} = -Bi_i U(1) \quad \text{(13)}
\end{align*}

The solution of Eqs. (10) and (12-13) is given by:

\begin{equation}
U(\xi) = A + B\xi 
\end{equation}

where:

\begin{align*}
A &= \frac{Bi_i (1 + Bi_i)}{Bi_i + Bi_i + Bi_i Bi_i} \\
B &= -\frac{Bi_i Bi_i}{Bi_i + Bi_i + Bi_i Bi_i} \quad \text{(15)} \quad \text{(16)}
\end{align*}

The time dependent term of the dimensionless solution (Eq. (9)) comes from Eq. (11), to be solved with its dimensionless boundary and initial conditions:

for \( \xi = 0 \)
\begin{equation}
\frac{\partial V}{\partial \xi} \bigg|_{\xi=0, \tau} = Bi_i V(\xi = 0, \tau) \quad \text{(17)}
\end{equation}

for \( \xi = 1 \)
\begin{equation}
\frac{\partial V}{\partial \xi} \bigg|_{\xi=1, \tau} = -Bi_i V(\xi = 1, \tau) \quad \text{(18)}
\end{equation}

for \( \tau = 0 \)
\begin{equation}
V(\xi, \tau = 0) = -U(\xi) \quad \text{(19)}
\end{equation}

The solution of the differential problem given by Eq. (11) and Eqs. (17-19) is obtained through the well known method of separation of the variables [10-14]. The details are given in [6]; here only the final result is reported:

\begin{equation}
V(\xi, \tau) = 2\sum_{n=1}^{\infty} \left( Bi_i^2 + \beta_n^2 \right) \frac{AB_i - B}{\beta_n} + \left[ A + B + \frac{Bi_i B}{\beta_n^2} \right] \sin(\beta_n) + \frac{B - (A + B)Bi_i}{\beta_n} \cos(\beta_n) \\
\times \left[ Bi_i \sin(\beta_n \xi) + \beta_n \cos(\beta_n \xi) \right] e^{-T \beta_n^2 \tau} \quad \text{(20)}
\end{equation}

The admissible values of the constant of separation \( \beta_n \) are the roots of the transcendental equation:

\begin{equation}
\frac{\sin(\beta_n)}{\cos(\beta_n)} = \frac{\beta_n (Bi_i + Bi_i)}{\beta_n^2 - Bi_i Bi_i} \quad \text{(21)}
\end{equation}

The search for these roots can be easily executed by means of a numerical procedure.
Finally, the temperature distribution is given by:

$$T(x,t) = T_{a2} + (T_{a1} - T_{a2}) \left(U(\xi) + V(\xi, \tau)\right)$$  \hspace{1cm} (22)

In the boundary condition given by Eq. (3), an equivalent overall heat transfer coefficient representing the overall heat transfer resistance between the wall surface facing the fire passageway and the neighbouring rooms not affected by the fire is used.

An energy balance written for the escape passage gives:

$$h_{2b}H(T(L,t) - T_{es}(t)) = K_f W(T_{es}(t) - T_{a2}) + K_c H(T_{es}(t) - T_{a2}) + K_{uw} H(T_{es}(t) - T_{a2})$$  \hspace{1cm} (23)

In terms of overall heat transfer, this can be written as:

$$h_{2b}H(T(L,t) - T_{es}(t)) = h_2 H(T(L,t) - T_{a2})$$  \hspace{1cm} (24)

The equivalent overall heat transfer coefficient, is then given by:

$$h_2 = \frac{1}{\frac{1}{h_{2b}H} + \frac{1}{K_fW + K_cH + K_{uw}H}}$$  \hspace{1cm} (25)

and in dimensionless form,

$$B_{i2} = \frac{1}{\frac{1}{\xi_H} + \frac{\xi_W}{K_f\xi_H + K_c\xi_W + K_{uw}\xi_H}}$$  \hspace{1cm} (26)

The dimensionless parameter $B_{i2}$ is now expressed as a function of a new set of dimensionless parameters:

$$B_{i2} = \frac{h_{2b}L}{\lambda} \xi_H = \frac{H}{L} \xi_W = \frac{W}{L} K_f = \frac{K_f L}{\lambda} \frac{K_c}{\lambda} = \frac{K_c L}{\lambda} \frac{K_{uw}}{\lambda}$$

Finally, the temperature in the fire isolated passageway is given by:

$$T_{es}(t) = T(x=L,t) - \frac{Q(t)}{h_{2b}H}$$  \hspace{1cm} (27)

where:

$$Q(t) = h_2 H(T(x=L,t) - T_{a2})$$  \hspace{1cm} (28)

In dimensionless form Eqs. (27-28) become:

$$\Theta_{es}(\tau) = \left(1 - \frac{B_{i2}}{B_{i2b}}\right) \Theta(1, \tau)$$  \hspace{1cm} (29)

$$\Gamma(\tau) = B_{i2b} / K_w \Theta(1, \tau)$$  \hspace{1cm} (30)

Finally, the dimensionless temperature in the fire isolated passageway depends on the following dimensionless parameters:

$$B_{i1} , T , B_{i2b} , \xi_H , \xi_W , K_f , K_c , K_{uw}$$

3. Results and Discussion

For the calculation of the tolerance time to be used as the tenability limit for design purposes, we refer to the correlation under midhumidity conditions given by Purser [3]:

$$t_{tol} = 2 \cdot 10^{31} T^{-16.963} + 4 \cdot 10^{6} T^{-3.7561}$$  \hspace{1cm} (31)

where $T$ is the ambient temperature (°C) and $t_{tol}$ is the tolerance time (min).
Figure 3. Wall temperature as a function of time for different values of $\xi_W$ and $Bi_2$.

Figure 4. Temperature in the escape room as a function of time for different values of $\xi_W$ and $Bi_2$.

Figure 5. Temperature in the escape passage (continuous line) and tolerance time (dashed line) as a function of time.
Due to the large number of dimensionless parameters characterizing the problem, the calculations are done for assigned values of a part of these. In particular, $Bi_1$, $T$, $Bi_2$, $xi$, are assumed constant. In particular these dimensionless values are: $T = 1$, $Bi_1 = 2.857$, $Bi_2 = 0.756$ and $xi = 30$.

The free dimensionless numbers are $xi$, $K_f$, $K_c$, $K_un$; with these parameters it is possible to discuss the effect on the temperature trend in the escape passage, of the thermal characteristics of the different walls of the corridor ($K_f$, $K_c$, $K_un$) and of their heat transfer surface ($xi$). In the reference configuration the conductance of ceiling, floor and unexposed wall are the same of the wall exposed to the fire. Due to the different total (convection and radiation) heat transfer coefficients (vertical, horizontal with upward heat flow, horizontal with downward heat flow) the transmittance of these walls are different ($K_f$, $K_c$, $K_un$). In the discussion of the results these parameters are incremented / decremented, at the same time, of the same amount.

In Figure 3 the dimensionless temperature of a wall heated by a fire is shown as a function of the dimensionless time. In particular, the dimensionless temperature refers to the unexposed face of the wall ($xi = 1$), and is given by two contribution, a steady one (Eq. (14)) and a time dependent one (Eq. (20)). The time dependent component is given by a summation of terms tending to zero for $\tau$ tending to infinity. Necessarily, the final temperature tends to the asymptotic steady value given by Eq. (14). This asymptotic steady value depends on $Bi_1$, $Bi_2$ and $T$. Since $Bi_2$ depends on a set of parameters through Eq. (26), for the same values of $Bi_2$, $xi$, $K_f$, $K_c$, $K_un$, the wall temperature depends only on $xi$. This behaviour is evident in Fig. 3 where, for constant values of $Bi_1$, $T$, $Bi_2$, $xi$, $K_f$, $K_c$, $K_un$, for increasing values of $xi$ a decrease of the asymptotic wall temperature can be observed. This is due to the better heat transfer conditions resumed in a higher $Bi_2$ and due to the larger heat transfer surface. Furthermore, for very large values of $xi$, the increase tends to reduce and the dimensionless wall temperature tends to a constant distribution, independent of $xi$. This is the case of Fig. 1.a, corresponding to the condition examined in [6] and used to determine the fire resistance of a wall in terms of Criterion "I", insulation of the wall.

The dimensionless temperature of the air in the escape room is shown in Fig. 4. For three values of $xi$, three curves are shown, calculated for 0.8$K_j$, $K_f$ and 1.2$K_j$ (the subscript $j$ means: f (floor), c (ceiling) and un (unexposed), respectively). For increasing values of $K_f$, the asymptotic dimensionless air temperature in the fire protected room decreases. A thermal insulation of the walls is the source of an increase of the temperature in the escape room. At the same time an increase of $xi$ produces a decrease of the asymptotic dimensionless air temperature in the fire protected room. This is clearly due to the combined effect of heat transfer surface and wall transmittance. Large corridors and slightly insulated walls are positive elements if we want to keep low the temperature in an escape, fire protected, corridor.

In Figure 5 the intersection of the temperature in a corridor with the tolerance time curve, Eq. (31), is detailed. The tolerance time, intended as the maximum exposure time before incapacitation, changes with the characteristics of the fire resistant wall, but also with that of the remaining walls of an escape passage. The results are shown in Tab. 1, where the dimensionless tolerance times are detailed for the same fixed data used to produce Fig. 6. For the examined wall exposed to fire, the tolerance time

| $xi$ | $K_f$ | $K_c$ | $K_un$ | $\tau_{tol}$ |
|------|------|------|-------|-------------|
| 18   | 0.220| 0.161| 0.201 | 0.153       |
| 18   | 0.274| 0.201| 0.252 | 0.158       |
| 18   | 0.329| 0.242| 0.302 | 0.162       |
| 36   | 0.220| 0.161| 0.201 | 0.163       |
| 36   | 0.274| 0.201| 0.252 | 0.171       |
| 36   | 0.329| 0.242| 0.302 | 0.178       |
| 72   | 0.220| 0.161| 0.201 | 0.184       |
| 72   | 0.274| 0.201| 0.252 | 0.196       |
| 72   | 0.329| 0.242| 0.302 | 0.208       |
varies from $\tau_{tol} = 0.15$ for the most insulated walls and the smallest width of the corridor, up to $\tau_{tol} = 0.21$ for the less insulated walls and the largest width of the corridor.

In dimensional terms, for a wall characterized by $L = 0.10$ m, $\lambda = 1.19$ Wm$^{-1}$K$^{-1}$, $c = 1.3$ kJkg$^{-1}$K$^{-1}$, $\rho = 2200$ kgm$^{-3}$, $h_1 = 34$ Wm$^{-2}$K$^{-1}$, $h_{2a} = 9$ Wm$^{-2}$K$^{-1}$, and neighbouring walls with $U_f = 2.4$ Wm$^{-2}$K$^{-1}$, $U_c = 3.3$ Wm$^{-2}$K$^{-1}$, $U_{un} = 3.0$ Wm$^{-2}$K$^{-1}$, the characteristic time of the wall is $L^2/\alpha = 24034$ s. Taking this characteristic time as the reference $t_0$, for the values of $\tau_{tol}$ reported in Tab. 1, the dimensional values of tolerance time are calculated. For the largest corridor (W = 7.2 m) and the less insulated walls ($1.2$ $K_j$) the tolerance time is $t_{tol} = 83.3$ min; for the smallest corridor (W = 1.8 m) and the most insulated walls ($0.8$ $K_j$) the tolerance time is $t_{tol} = 61.3$ min. The difference amounts to 22 min. In terms of time useful for the evacuation of a building, the difference is significant.

4. Concluding Remarks

The analytical solution proposed for the study of the performance of a wall that separates a room with a fire from an escape space has provided useful results. With this solution we are able to easily calculate the air temperature in the escape route. This temperature depends on the boundary conditions, in particular on the width of the corridor and on the thermal transmittance of the walls separating the corridor from the neighbouring ambients.

The air temperature in the escape route varies significantly with geometry and boundary conditions. All this reflects into a tolerance time that significantly shorten if the walls are thermally insulated and the corridor is narrow.

Finally, the reference conditions used to determine the fire resistance class are those giving the longest tolerance time, while in the practical situations the dimensions the corridor and the thermal characteristics of the walls can reduce significantly this tolerance time.

5. Nomenclature

| Symbol | Definition |
|--------|------------|
| $A$    | constant coefficient |
| $B$    | constant coefficient |
| $Bi$   | Biot number |
| $h$    | heat transfer coefficient (Wm$^{-2}$K$^{-1}$) |
| $H$    | height of the wall (m) |
| $K$    | overall wall heat transfer coefficient (Wm$^{-2}$K$^{-1}$) |
| $L$    | thickness of the wall (m) |
| $t$    | time (s) |
| $Q$    | heat flow (W/m) |
| $T$    | temperature (°C) |
| $U$    | dimensionless steady state term of the solution |
| $V$    | dimensionless time dependent term of the solution |
| $W$    | width of the passage (m) |
| $x$    | horizontal coordinate |
| $y$    | vertical coordinate |

**Greek symbols**

- $\alpha$ thermal diffusivity (m$^2$/s)
- $\beta$ eigenvalue
- $\Gamma$ dimensionless heat flux, Eq. (30)
- $K$ dimensionless heat transfer coefficient
- $\lambda$ thermal conductivity (W/m°C)
- $\Theta$ dimensionless temperature
- $\tau$ dimensionless time
- $T$ dimensionless reference time
- $\xi$ dimensionless coordinate

**Subscripts**

- $a1$ air in the fire room
a2 air in the neighbouring rooms
b boundary
c ceiling
es escape space
f floor
H height of the escape space
tol tolerance
un unexposed wall of the corridor
w wall
W width of the escape space
1 upstream of the compartment wall
2 downstream of the compartment wall
0 reference

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