The diversity-multiplexing tradeoff of the MIMO Z interference channel

Sanjay Karmakar    Mahesh K. Varanasi

Abstract

The fundamental diversity-multiplexing tradeoff (DMT) of the quasi-static fading MIMO Z interference channel (ZIC) with channel state information at the transmitters (CSIT) and arbitrary number of antennas at each node is derived. A short-term average power constraint is assumed. It is shown that a simple Han-Kobayashi coding scheme, where the 2nd transmitter’s signal depends on the channel matrix to the first receiver and the 1st user’s transmit signal is independent of CSIT, can achieve the full CSIT DMT of the ZIC. We also characterize the achievable DMT of a transmission scheme, which does not utilize any CSIT and show that for some range of multiplexing gains, the full CSIT DMT of the ZIC can be achieved by it. The size of this range of multiplexing gains depends on the system parameters such as the number of antennas at the four nodes (referred to hereafter as “antenna configuration”), signal-to-noise ratios (SNR) and interference-to-noise ratio (INR) of the direct links and cross link, respectively. Interestingly, for certain special cases such as when the interfered receiver has a larger number of antennas or when the INR is stronger than the SNRs, the No-CSIT scheme can achieve the F-CSIT DMT for all multiplexing gains. Thus, under these circumstances, the optimal DMT of the MIMO ZIC with F-CSIT is same as the DMT of the corresponding ZIC with No-CSIT. For other channel configurations, the DMT achievable by the No-CSIT scheme serves as a lower bound to the fundamental No-CSIT DMT of the MIMO ZIC.

Index Terms

DMT of MIMO Z interference channel, No CSIT, only CSIR, One sided MIMO IC.

S. Karmakar and M. K. Varanasi are both with the Department of Electrical Computer and Energy Engineering, University of Colorado at Boulder, Boulder, CO, 30809 USA e-mail: (sanjay.karmakar@colorado.edu, varanasi@colorado.edu).
I. INTRODUCTION

Significant progress in the direction of a complete understanding of the multiuser interference network has been made in the last few years, through the information theoretic performance analysis of various basic building blocks of this general network. Different performance metrics, such as the capacity, capacity within a constant number of bits, Generalized degrees of freedom (GDoF) and degrees of freedom (DoF), has been used to evaluate the performance of these basic channel models. While in terms of accuracy capacity is the best performance metric, it is also the hardest one to analyze. For instance, the capacity region of the most basic building block, the 2-user interference channel is known only for some very special cases \cite{2}, \cite{3}, \cite{4}, \cite{5}, \cite{6}, \cite{7}, \cite{8}, depending on the values taken by the various channel parameters.

The model used for the channel coefficients of a communication system is another important design parameter. Most of the capacity characterizations mentioned above or approximate capacity characterizations \cite{9}, \cite{10} assume time-invariant channel coefficients, whereas the channel coefficients of a practical wireless network vary with time and should be modeled as a fading one. Most of the works which assume a fading channel model use a much coarser performance metric such as the DoF or the GDoF. These metrics can only characterize the rate scaling factor/s with average SNR, of the corresponding channel and do not reveal any information about the reliability of communication. Diversity-multiplexing tradeoff introduced in \cite{11} for the point-to-point channel captures this relation between the rate and the reliability of the communication. Encouraged by the importance of knowing the best achievable reliability on a channel while communicating at a particular rate, in this paper we choose DMT as our performance metric. Further, as a first step towards understanding the general interference network, we choose the 2-user Z interference channel as our channel model in this paper. In the 2-user Z interference channel (ZIC), 2 transmitters communicate to their corresponding receivers via the same signal space, while only one of the transmitters interfere with the other receiver.

Besides being one of the basic building blocks of the general interference network, ZICs also emerge as the natural information theoretic model for various practical wireless communication scenario such as femto-cells \cite{12}. Also, the ZIC is a special case of the 2-user IC. Thus optimal (with respect to some metric) coding and decoding schemes on a ZIC might reveal useful insights for the 2-user IC also. For instance, the optimal DMT of the ZIC is an upper bound for the 2-user MIMO IC (with and without CSIT). These facts make the analysis of the ZIC an important step towards a better understanding of the general multiuser wireless system. Motivated by the aforementioned facts in this paper we analyze the DMT of the MIMO ZIC. However, unlike the DMT framework in a point-to-point (PTP) channel, \cite{11} where there is a single communication link which can be characterized by a single SNR, in a multiuser setting such as the one at hand, it is only natural to allow the SNRs and INRs of different links to vary with different exponentials with respect to a nominal SNR, denoted as $\rho$. This technique was first used in \cite{9} to analyze the DoF region which the authors referred to as the Generalized DoF (GDoF) region, of the 2-user SISO IC. Later, this technique was extended to the DMT scenario of the SISO IC in \cite{13} and \cite{14}. Following similar approach, we allow the different INR and SNRs at the receivers to vary exponentially with respect to $\rho$.
with different scaling factors. We refer to the corresponding DMT as the generalized DTM (GDMT) to distinguish it from the case when SNR=INR in all the links.

In this paper, we first derive the DMT of the MIMO ZIC with CSIT and arbitrary number of antennas at each node. The achievability is based on a simple Han-Kobayashi coding scheme, where the signal to be transmitted by the 2nd user depends only on the channel matrix to the first receiver, whereas the transmitted signal of the first user does not use any CSIT. The converse is proved by deriving a set of upper bounds to the achievable DMT, from a set of upper bounds to the capacity region of the ZIC. The set of upper bounds to the capacity region in turn is obtained assuming a genie aided interfered receiver. The computation of the DMT of the MIMO ZIC involves the asymptotic joint eigenvalue distribution of two specially correlated random Wishart matrices which was recently derived by the authors in [15] in a different context. Using this distribution result, the fundamental DMT of the MIMO ZIC channel with CSIT is established as the solution of a convex optimization problem. While it is argued that in general the optimization problem can be solved using numerical methods, closed-form solutions are computed for several special cases. Secondly, we characterize the achievable DMT of a transmission scheme which does not utilize CSIT. Comparing the achievable DMT of this scheme with the F-CSIT DMT of the ZIC, we identify two classes of MIMO ZICs on which the No-CSIT scheme can achieve the F-CSIT DMT. The first class of ZICs have equal number of antennas at all the nodes and a stronger INR than a certain threshold (e.g., Theorem 5) and the second class of ZICs have a larger number of antennas at the interfered node than a certain threshold (e.g., Theorem 4). The above result thus effectively characterizes the DMT of these channels without CSIT because the F-CSIT DMT of the channel is an upper bound to the No-CSIT DMT. For other channel configurations, this achievable DMT represents a lower bound to the fundamental No-CSIT DMT of the channel.

An early work in this direction is [16], where the authors derive an achievable DMT on a SISO ZIC with No-CSIT. The DMT of the SISO ZIC with F-CSIT can be obtained from [17], where the DMT (F-CSIT) of the 2-user SISO IC was derived. In this work, we focus on the MIMO case. In [18], an upper bound to the DMT of a 2-user MIMO IC with CSIT was derived for the case in which all nodes have same number of antennas and the direct and cross links have the same SNRs and INRs, respectively. This result, if specialized for the MIMO ZIC, provides only an upper bound. Our result will prove that for the special case considered in [18], this upper bound is actually tight on a ZIC with F-CSIT. However, the result of this paper on MIMO ZIC is much more general, in the sense that we consider arbitrary number of antennas at each node and arbitrary scaling parameters for the different SNRs and the INR of the system. Moreover, we also characterize the DMT of the channel with no CSI at the transmitters for some specific channel configurations.

**Notations:** We denote the conjugate transpose of the matrix $A$ as $A^\dagger$ and its determinant as $|A|$. $\mathbb{C}$ and $\mathbb{R}$ represent the field of complex and real numbers, respectively. The set of real numbers \{ $x$ $\in$ $\mathbb{R}$ : $a$ $\leq$ $x$ $\leq$ $b$ \} will be denoted by [$a, b$]. Furthermore, $(x \wedge y)$, $(x \vee y)$ and $(x)^+$ represent the minimum of $x$ and $y$, the maximum of $x$ and $y$, and the maximum of $x$ and 0, respectively. All the logarithms in this paper are with base 2. We denote the distribution of a complex circularly symmetric Gaussian random vector with zero mean and covariance matrix...
Any two functions $f(\rho)$ and $g(\rho)$ of $\rho$, where $\rho$ is the signal to noise ratio (SNR) defined later, are said to be exponentially equal and denoted as $f(\rho) \equiv g(\rho)$ if, $\lim_{\rho \to \infty} \frac{\log f(\rho)}{\log(\rho)} = \lim_{\rho \to \infty} \frac{\log g(\rho)}{\log(\rho)}$. The same is true for $\geq$ and $\leq$.

II. CHANNEL MODEL AND PRELIMINARIES

We consider a MIMO ZIC as shown in Figure 1 where user 1 ($Tx_1$) and user 2 ($Tx_2$) have $M_1$ and $M_2$ antennas and receiver 1 ($Rx_1$) and 2 ($Rx_2$) have $N_1$ and $N_2$ antennas, respectively. This channel will be referred hereafter as a $(M_1, N_1, M_2, N_2)$ ZIC. A slow fading Rayleigh distributed channel model is considered where $H_{ij} \in \mathbb{C}^{N_j \times M_i}$ represents the channel matrix between $Tx_i$ and $Rx_j$. It is assumed that $H_{11}$, $H_{21}$ and $H_{22}$ are mutually independent and contain mutually independent and identically distributed (i.i.d.) $\mathcal{C}\mathcal{N}(0, 1)$ entries. These channel matrices remain fixed for a particular fade duration of the channel and changes in an i.i.d. fashion, in the next. Perfect channel state information is assumed at both the receivers (CSIR) and both the transmitters (CSIT). Following [14], we also incorporate a real-valued attenuation factor, denoted as $\eta_{ij}$, for the signal transmitted from $Tx_i$ to the receiver $Rx_j$. At time $t$, $Tx_i$ chooses a vector $X_{it} \in \mathbb{C}^{M_i \times 1}$ and sends $\sqrt{P_i}X_{it}$ over the channel, where for the input signals we assume the following short term average power constraint:

\[ \frac{1}{N} \sum_{t=1+N(k-1)}^{N+N(k-1)} \text{tr}(Q_{it}) \leq 1, \forall k \geq 1, \ i = 1, 2, \text{where } Q_{it} = E\left(X_{it}X_{it}^\dagger\right), \]  

(1)

where $N$ represents the number of channel uses for which the channel matrices remain fixed, or in other words the fade duration.

**Remark 1:** Note that since the transmitters are not allowed to allocate power across different fades of the channel, the channel is still in the outage setting, i.e., the delay limited capacity of each of the links of the ZIC is zero [19]. Thus, the DMT characterization of this outage limited channel make sense.

The received signals at time $t$ can be written as

\[ Y_{1t} = \eta_{11}\sqrt{P_1}H_{11}X_{1t} + \eta_{21}\sqrt{P_2}H_{21}X_{2t} + Z_{1t}, \]

\[ Y_{2t} = \eta_{22}\sqrt{P_2}H_{22}X_{2t} + Z_{2t}, \]
where \( Z_{it} \in \mathbb{C}^{N_i \times 1} \) are i.i.d as \( \mathcal{CN}(0, I_{N_i}) \) across \( i \) and \( t \). The above equations can be equivalently written in the following form.

\[
Y_{1t} = \sqrt{\text{SNR}_{11}} H_{11} \hat{X}_{1t} + \sqrt{\text{INR}_{21}} H_{21} \hat{X}_{2t} + Z_{1t}; \\
Y_{2t} = \sqrt{\text{SNR}_{22}} H_{22} \hat{X}_{2t} + Z_{2t},
\]

where the normalized inputs \( \hat{X}_{is} \)s satisfy equation (1) with equality and \( \text{SNR}_{ii} \) and \( \text{INR}_{ji} \) are the signal-to-noise ratio and interference-to-noise ratio, respectively at receiver \( i \), which from now onwards will be denoted by \( \rho_{ii} \) and \( \rho_{ji} \), respectively. The performance of a link depends on the strength of the interference relative to the desired signal level and when interference strength is much less than the signal strength a better performance. This variation of performance due to relative difference in strengths of SNRs and INRs can not be captured through the DMT metric, if they differ by only a constant factor. To characterize the DMT under such a scenario we thus let the different SNRs and INRs to vary exponentially with respect to a nominal SNR, \( \rho \) with different scaling factors as follows:

\[
\alpha_{11} = \frac{\log(\text{SNR}_{11})}{\log(\rho)}, \quad \alpha_{22} = \frac{\log(\text{SNR}_{22})}{\log(\rho)}, \\
\alpha_{21} = \frac{\log(\text{INR}_{21})}{\log(\rho)}.
\]

For brevity, in the sequel we shall use the following notations: \( \text{SNR}_{ii} = \rho_{ii}, \text{INR}_{21} = \rho_{21}, \mathcal{H} = \{H_{11}, H_{21}, H_{22}\}, \rho = [\rho_{11}, \rho_{21}, \rho_{22}] \) and \( \bar{\alpha} = [\alpha_{11}, \alpha_{21}, \alpha_{22}] \).

Diversity order of a point-to-point channel [11] is defined as the negative SNR exponent of the average probability of error at the receiver. Since in the present channel model there are 2 receivers and more than one SNR and INR parameters, in the next subsection, we provide the definitions of diversity order and the multiplexing gains appropriate to the ZIC.

A. Definition of DMT of a MIMO ZIC

Let us start by defining the DMT of the channel formally. Let \( Tx_i \) is transmitting information using a codebook \( C_i(\rho) \) having \( 2^{L R_i(\rho)} \) codewords, each of length \( L \), at a rate \( R_i(\rho) \), then the corresponding multiplexing gain is denoted by \( r_i \) where

\[
r_i = \lim_{\rho \to \infty} \frac{R_i(\rho)}{\log(\rho)}, \text{ for } i = 1, 2.
\]

Remark 2: Note that the maximum asymptotic rate supportable by the first direct link is \( R_1^{\max} = \min\{M_1, N_1\} \log(\rho_{11}) \), when the second user is silent. Putting this into equation (6) we have

\[
r_i = \min\{M_1, N_1\} \alpha_{11} > \min\{M_1, N_1\}, \text{ if } \alpha_{11} > 1.
\]

Apparently, it seems that the direct link can support a multiplexing-gain strictly larger than \( \min\{M_1, N_1\} \). However, this is only a consequence of the fact that the multiplexing gain \( r_i \) in (6) is defined with respect to (w.r.t.) the nominal SNR \( \rho \) and therefore, a by product of the more general mathematical model that we assume in here in...
this paper. With respect to the direct link’s SNR $\rho_{11}$ the multiplexing-gain is still $\min\{M_1, N_1\}$, irrespective of the value of $\alpha_{11}$, i.e.,

$$\hat{r}_i = \lim_{\rho_{11} \rightarrow \infty} \frac{R_1(\rho)}{\log(\rho_{11})} = \min\{M_1, N_1\}. \quad (8)$$

Alternatively, this apparent difference can also be removed by equating the nominal SNR $\rho$ to the direct links SNR $\rho_{11}$ which amounts to setting $\alpha_{11} = 1$ in equation (7).

Now, to define the diversity order, let $P_{e,C_i}(\bar{\rho}, r_1, r_2)$ represents the maximum of the average probability of errors at the receivers (averaged over the random channel, Gaussian additive noise at the receivers and different codewords of a codebook) at a multiplexing gain pair $(r_1, r_2)$ and SNR of $\rho$, and $P^*_e(\bar{\rho}, r_1, r_2)$ represents the minimum $P_{e,C_i}(\bar{\rho}, r_1, r_2)$ among all possible coding schemes, i.e.,

$$P^*_e(\bar{\rho}, r_1, r_2) = \min_{\{\text{All possible coding scheme }, C_i(\rho)\}} P_{e,C_i}(\bar{\rho}, r_1, r_2), \quad (9)$$

then the corresponding diversity order $\alpha_i$ is defined as

$$d^*_{ZIC}(r_1, r_2) = \lim_{\rho \rightarrow \infty} \frac{-\log(P^*_e(\bar{\rho}, r_1, r_2))}{\log(\rho)}. \quad (10)$$

Note that the diversity order $d^*_{ZIC}(r_1, r_2)$ is a function of the relative scaling parameters of the different links $\bar{\alpha}$. However, for brevity of notation, we shall not mention them explicitly.

The typical approach to characterize the DMT of a communication channel, whose exact instantaneous end-to-end mutual information (IMI) is not known, is to find an upper and a lower bound to it. Then, from this upper and lower bound to the IMI a lower and upper bound to the appropriately defined outage event is derived, respectively. If the later set of bounds have identical negative SNR exponents then that represents the DMT of the corresponding channel. In this paper we adopt the same approach and therefore, need a subset and a superset to the IMI region of the channel, which we specify in the next subsection.

B. A Subset and a Superset to the instantaneous mutual information region

In this subsection, we shall first derive a set of upper bounds to the various end-to-end mutual information defining a superset to the IMI region of the MIMO ZIC. Next, we shall propose a simple superposition coding scheme, which can achieve a IMI region, with its various bounds within constant (independent of SNR and channel coefficients) number of bits to those of the superset. These bounds will then be used to derive the fundamental DMT of the channel.

Lemma 1: The IMI region of the 2-user MIMO ZIC with F-CSIT, for a given realization of channel matrices $\mathcal{H}$, denoted by $\mathcal{C}(\mathcal{H}, \bar{\rho})$, is contained in the set of real-tuples $\mathcal{R}^u(\mathcal{H}, \bar{\rho})$, where $\mathcal{R}^u(\mathcal{H}, \bar{\rho})$ represents the set of rate pairs $(R_1, R_2)$ such that $R_1, R_2 \geq 0$ and satisfy the following constraints:

$$R_i \leq \log \left| I_{N_i} + P_{ii} H_{ii} H_{ii}^\dagger \right| \triangleq I_{bi}, \quad i \in \{1, 2\};$$

$$R_1 + R_2 \leq \log \left| I_{N_1} + P_{21} H_{21} H_{21}^\dagger + P_{11} H_{11} H_{11}^\dagger \right| +$$

$$\log \left| I_{N_2} + P_{22} H_{22} \left( I_{M_2} + P_{21} H_{21}^\dagger H_{21} \right)^{-1} H_{22}^\dagger \right| \triangleq I_{bs}.$$
In what follows, we find a subset to the IMI region of the channel. Consider a coding scheme where the first transmitter uses a random Gaussian code book and the second user uses a superposition code as follows:

\[ X_2 = U_2 + W_2, \]  

(11)

where \( U_2 \) (hereafter mentioned as the private part of the message) and \( W_2 \) (public part of the message) are mutually independent complex Gaussian random vectors with covariance matrices as follows:

\[ \mathbb{E}(X_1 X_1^\dagger) = I_{M_1}, \quad \mathbb{E}(W_2 W_2^\dagger) = \frac{I_{M_2}}{2} \quad \text{and} \]
\[ \mathbb{E}(U_2 U_2^\dagger) = \frac{1}{2} \left( I_{M_2} + \rho^{\alpha_{21}} H_{21}^\dagger H_{21} \right)^{-1}. \]  

(12)

\textbf{Remark 3:} Note that this covariance split satisfies the power constraint in equation (1).

\textbf{Remark 4:} The above described coding scheme is clearly a special case of the Han-Kobayashi coding scheme where the first transmitter’s message does not have any private part. Also the DMT characterized in this paper represents the best DMT achievable on a ZIC when both transmitters have full CSIT. However, it will be shown shortly that the above coding scheme which use the knowledge of only \( H_{21} \) at \( T_x \) can achieve the F-CSIT DMT.

\textbf{Lemma 2:} For a given channel realization \( \mathcal{H} \), the above described coding scheme can achieve an IMI region \( \mathcal{R}_l(\mathcal{H}, \bar{\rho}) \), where \( \mathcal{R}_l(\mathcal{H}, \bar{\rho}) \) represents the set of rate pairs \((R_1, R_2)\) such that \( R_1, R_2 \geq 0 \) and satisfies the following constraints:

\[ R_i \leq \log \left( \left| I_{N_i} + \rho_{ii} H_{ii} H_{ii}^\dagger \right| \right) - 2N_i \triangleq I_{li}, \quad i \in \{1, 2\}; \]
\[ R_1 + R_2 \leq \log \left( \left| I_{N_1} + \rho_{21} H_{21} H_{21}^\dagger + \rho_{11} H_{11} H_{11}^\dagger \right| \right) + \]
\[ \log \left( \left| I_{N_2} + \rho_{22} H_{22} \left( I_{M_2} + \rho_{21} H_{21} H_{21}^\dagger \right)^{-1} H_{22}^\dagger \right| \right) \]
\[ - 2(N_1 + N_2) \triangleq I_{ls}. \]

It has been explained earlier that the superset, \( \mathcal{R}^u(\mathcal{H}, \bar{\rho}) \) given by Lemma [1] and the subset, \( \mathcal{R}_l(\mathcal{H}, \bar{\rho}) \), given by Lemma [2] to \( C(\mathcal{H}, \bar{\rho}) \) can be used to derive a set of upper and lower bounds to DMT of the channel, respectively. In what follows, we prove this fact formally. Let \( T_{x_i} \) is operating at a rate \( R_i(\rho) \) bits per channel use where the corresponding multiplexing-gain is \( r_i = \lim_{\rho \to \infty} \frac{R_i(\rho)}{\log(\rho)} \) for \( i = 1, 2 \) and the outage event \( \mathcal{O} \) is defined as follows:

\[ \mathcal{O} = \{ \mathcal{H} : (R_1, R_2) \notin C(\mathcal{H}, \bar{\rho}) \}. \]  

(13)

Following similar method as in [11] it can be easily proved that

\[ P^*_c(\bar{\rho}, r_1, r_2) = \Pr(\mathcal{O}), \]  

(14)
where $\mathcal{P}_c^* (\bar{\rho}, r_1, r_2)$ represents the minimum average probability of error achievable on the ZIC, as defined in subsection II-A. Now, from Lemma 1 and 2 for any realization of the channel matrices $\mathcal{H}$ we have,

$$\mathcal{R}(\mathcal{H}, \bar{\rho}) \subseteq \mathcal{R}^u(\mathcal{H}, \bar{\rho});$$

Or, $\{\mathcal{H} : (R_1, R_2) \notin \mathcal{R}^u(\mathcal{H}, \bar{\rho})\} \subseteq \{\mathcal{H} : (R_1, R_2) \notin \mathcal{R}(\mathcal{H}, \bar{\rho})\};$

Or, $\Pr \{\mathcal{H} : (R_1, R_2) \notin \mathcal{R}^u(\mathcal{H}, \bar{\rho})\} \leq \mathcal{P}_c^* (\bar{\rho}, r_1, r_2) \leq \Pr \{\mathcal{H} : (R_1, R_2) \notin \mathcal{R}(\mathcal{H}, \bar{\rho})\};$

Or, $\Pr \{\mathcal{H} : (R_1, R_2) \notin \mathcal{R}^u(\mathcal{H}, \bar{\rho})\} \leq \rho^{-d_{ZIC}^*(r_1, r_2)} \leq \Pr \{\mathcal{H} : (R_1, R_2) \notin \mathcal{R}(\mathcal{H}, \bar{\rho})\};$

Or, $\Pr \{\cup_i \{I_{bi} \leq R_i\}\} \leq \rho^{-d_{ZIC}^*(r_1, r_2)} \leq \Pr \{\cup_i \{I_{bi} \leq R_i\}\};$

where $R_s = (R_1 + R_2)$ and $I_{bi}$’s and $I_{li}$’s are as defined in Lemma 1 and Lemma 2, respectively. Note that, $I_{bi} = I_{li} + c_i$, where $c_i$ is a constant independent of $\rho$, for all $i \in \{1, 2, s\}$, which becomes insignificant at asymptotic SNR. Therefore, at asymptotic values of $\rho$ equation (16) is equivalent to

$$\rho^{-d_{ZIC}^*(r_1, r_2)} = \max_{i \in \{1, 2, s\}} \Pr \{I_{bi} \leq R_i\},$$

which can be written as

$$d_{ZIC}^*(r_1, r_2) = \min_{i \in \mathcal{I}} d_{O_i}(r_i), \text{ where}$$

$$d_{O_i}(r_i) = \lim_{\rho \to \infty} -\frac{\Pr (I_{bi} \leq r_i \log(\rho))}{\log(\rho)},$$

for all $i \in \mathcal{I} = \{1, 2, s\}$ and $r_s = (r_1 + r_2)$.

The only remaining step to characterize the DMT completely is to evaluate the probabilities in equation (18), which in turn requires the statistics of the mutual information terms $I_{bi}$’s. It will be shown in the next section that this statistics and therefrom the DMT of the channel can be characterized, if only the joint distribution of the eigenvalues of 2 mutually correlated random Wishart matrices are known.

### III. Explicit DMT of the ZIC

In this section, we shall evaluate the different SNR exponents, $d_{O_i}(r_i)$’s, of the various outage events given in equation (17), which would yield the explicit DMT expression for the ZIC. Substituting the right hand sides of the first and second bound’s in Lemma 1 in equations (18) we have

$$d_{O_i}(r_i) = \lim_{\rho \to \infty} -\frac{\Pr \left( \sum_{k=1}^{\min\{M, N_i\}} (\alpha_{ii} - v_{i,k})^+ \leq r_i \right)}{\log(\rho)}, \quad i \in \{1, 2\},$$

where $v_{i,k}$’s are the negative SNR exponents of the ordered eigenvalues of the matrices $H_{ii} H_{ii}^H$. The joint distribution of $\{v_{i,k}\}_{k=1}^{\min\{M, N_i\}}$ was specified in (11). Using this distribution and a similar technique as in (11), it can be shown
that
\[
d_{O_i}(r_i) = \min_{\min \{ M_i, N_i \}} \sum_{k=1}^{\min \{ M_i, N_i \}} (M + N + 1 - 2k)\nu_{i,k}
\]
subject to:
\[
\sum_{k=1}^{\min \{ M_i, N_i \}} (\alpha_{ii} - \nu_{i,k})^+ \leq r; \quad (20b)
\]
\[
0 \leq \nu_{i,1} \leq \cdots \leq \nu_{i,\min \{ M_i, N_i \}} . \quad (20c)
\]

Similar optimization problem will recur in the remaining part of the paper. So we formally state the solution of the above problem in the following Lemma for convenience of reference later.

**Lemma 3:** If \( d(r) \) represents the optimal solution of the optimization problem,
\[
\min \sum_{i=1}^{m} (M + N + 1 - 2i)\mu_{i}
\]
subject to:
\[
\sum_{i=1}^{m}(\alpha - \mu_{i})^+ \leq r; \quad (21b)
\]
\[
0 \leq \mu_{1} \leq \cdots \leq \mu_{m}, \quad (21c)
\]
then,
\[
d(r) = \alpha d_{M,N} \left( \frac{r}{\alpha} \right), \text{ for } 0 \leq r \leq m\alpha, \quad (22)
\]
where \( m = \min \{ M, N \} \) and \( d_{M,N}(r) \) represents the DMT of a \( M \times N \) point-to-point channel and is a piecewise linear curve joining the points \((M - k)(N - k)\) for \( k = 0, 1, \cdots m \).

**Proof:** Putting \( \mu_{i}' = \frac{\mu_{i}}{\alpha} \) in the optimization problem (21) we get,
\[
\frac{d(r)}{\alpha} = \min \sum_{i=1}^{m} (M + N + 1 - 2i)\mu_{i}'
\]
subject to:
\[
\sum_{i=1}^{m}(1 - \mu_{i})^+ \leq \frac{r}{\alpha}; \quad (23b)
\]
\[
0 \leq \mu_{1}' \leq \cdots \leq \mu_{m}'. \quad (23c)
\]
The solution of this modified optimization problem was derived in [11] and is given by
\[
\frac{d(r)}{\alpha} = d_{M,N} \left( \frac{r}{\alpha} \right), \text{ for } 0 \leq \frac{r}{\alpha} \leq m,
\]
or,
\[
d(r) = \alpha d_{M,N} \left( \frac{r}{\alpha} \right), \text{ for } 0 \leq r \leq m\alpha.
\]

The solution of the optimization problem (20) is now evident from Lemma 3 and is given by
\[
d_{O_i}(r_i) = \alpha_{ii} d_{M,N_i} \left( \frac{r_i}{\alpha_{ii}} \right), \forall r_i \in [0, \min \{ M_i, N_i \} \alpha_{ii}] \text{ and } i \in \{1, 2\}, \quad (24)
\]
where \( d_{m,n}(r) \) is the optimal diversity order of a point-to-point MIMO channel with \( m \) transmit and \( n \) receive antennas, at integer values of \( r \) and is point wise linear between integer values of \( r \). To evaluate \( d_{O_i}(r_i) \), we write
the bound $I_{bs}$ of Lemma 1 in the following way

$$I_{bs} = \log \left| (I_{M_1} + \rho_{11}H_{11}^\dagger (I_{N_1} + \rho_{21}H_{21}H_{21}^\dagger)^{-1}H_{11}^\dagger) \right|$$

$$+ \log \left| (I_{N_2} + \rho_{22}H_{22} (I_{M_2} + \rho_{21}H_{21}H_{21}^\dagger)^{-1}H_{22}^\dagger) \right|$$

$$+ \log \left| (I_{N_1} + \rho_{21}H_{21}H_{21}^\dagger) \right|,$$

where in step (a), $p = \min\{M_2, N_1\}$, $q_1 = \min\{M_1, N_1\}$, $q_2 = \min\{M_2, N_2\}$ and we have denoted the ordered non-zero (with probability 1) eigenvalues of $W_1 = H_{11}^\dagger (I_{N_1} + \rho_{21}H_{21}H_{21}^\dagger)^{-1}H_{11}$, $W_2 = H_{22} (I_{M_2} + \rho_{21}H_{21}H_{21}^\dagger)^{-1}H_{22}^\dagger$ and $W_3 = H_{21}H_{21}^\dagger$ by $\mu_1 \geq \cdots \geq \mu_{q_1} > 0$, $\pi_1 \geq \cdots \geq \pi_{q_2} > 0$ and $\lambda_1 \geq \cdots \geq \lambda_p > 0$, respectively. Now, using the transformations $\lambda_i = \rho^{-\upsilon_i}$, for $1 \leq i \leq p$, $\mu_j = \rho^{-\beta_j}$, for $1 \leq j \leq q_1$ and $\pi_k = \rho^{-\gamma_k}$, $1 \leq k \leq q_2$ in the above equation and substituting that in turn in equation (18) we get

$$\rho^{-d_{O_s}(r_s)} \geq \Pr \left( \sum_{i=1}^p (\alpha_{21} - \upsilon_i)^+ + \sum_{j=1}^{q_1} (\alpha_{11} - \beta_j)^+ + \sum_{k=1}^{q_2} (\alpha_{22} - \gamma_k)^+ < r_s \right).$$

(25)

To evaluate this expression we need to derive the joint distribution of $\gamma, \beta$ and $\upsilon$ where $\gamma = \{\gamma_1, \cdots, \gamma_{q_2}\}$ and similarly $\upsilon = \{\upsilon_1, \cdots, \upsilon_p\}$ and $\beta = \{\beta_1, \cdots, \beta_{q_1}\}$. Note that $W_1, W_2$ and $W_3$ are not independent and hence neither are $\gamma, \beta$ and $\upsilon$. However, this distribution can be computed using Theorems 1 and 2 of [15]. Using this joint distribution, equation (25) and a similar argument as in [11] $d_{O_s}(r_s)$ can be derived as the solution of an convex optimization problem as stated in the following Lemma.

**Lemma 4:** The negative SNR exponent of the outage event corresponding to the sum bound in Lemma 2 i.e.,
\( d_{O_s}(r_s) \), is equal to the minimum of the following objective function:

\[
\mathcal{J}^{\text{FCSIT}}(M_1, N_1, M_2, N_2) = \sum_{i=1}^p (M_2 + N_1 + M_1 + N_2 + 1 - 2i) \nu_i + \sum_{j=1}^{q_1} (M_1 + N_1 + 1 - 2j) \beta_j \\
+ \sum_{k=1}^{q_2} (M_2 + N_2 + 1 - 2k) \gamma_k - (M_1 + N_2) p \alpha_{21} \\
+ \sum_{k=1}^{q_2} \min((M_2-k), N_2) \sum_{i=1}^p (\alpha_{21} - \nu_i - \gamma_k)^+ + \sum_{j=1}^{q_1} \min((N_1-j), M_1) \sum_{i=1}^p (\alpha_{21} - \nu_i - \beta_j)^+; 
\tag{26a}
\]

constrained by:

\[
\sum_{i=1}^p (\alpha_{21} - \nu_i)^+ + \sum_{j=1}^{q_1} (\alpha_{11} - \beta_j)^+ + \sum_{k=1}^{q_2} (\alpha_{22} - \gamma_k)^+ < r_s; 
\tag{26b}
\]

\[
0 \leq \nu_1 \leq \cdots \leq \nu_p; 
\tag{26c}
\]

\[
0 \leq \beta_1 \leq \cdots \leq \beta_{q_1}; 
\tag{26d}
\]

\[
0 \leq \gamma_1 \leq \cdots \leq \gamma_{q_2}; 
\tag{26e}
\]

\[
(\nu_i + \beta_j) \geq \alpha_{21}, \forall (i+j) \geq (N_1+1); 
\tag{26f}
\]

\[
(\nu_i + \gamma_k) \geq \alpha_{21}, \forall (i+k) \geq (M_2+1). 
\tag{26g}
\]

**Proof:** The proof is relegated to Appendix A.

Differentiating the objective function with respect to \( \{\nu_i\} \), \( \{\beta_j\} \) and \( \{\gamma_k\} \), it can be easily verified that (26a) is a convex function of these variables. The constraints on the other hand are all linear. Therefore, equation (26) represents an convex optimization problem (e.g., see subsection 4.2.1 in [20]) and hence can be solved efficiently using numerical methods. Since we have already found expressions for \( d_{O_i} \) for \( i = 1, 2 \) as in equation (24), Lemma 4 provides the last piece of the puzzle required to characterize the DMT of the MIMO ZIC, by evaluating \( d_{O_s} \). This is stated formally in the following Theorem.

**Theorem 1:** The DMT of the \((M_1, N_1, M_2, N_2)\) ZIC, with FCSIT and short term power allocation scheme, is given as

\[
d_{O_sFC}^{ZIC}(r_1, r_2) = \min_{i \in \{1,2,s\}} d_{O_i}(r_i),
\]

where \( d_{O_i}(r_i) \) for \( i = 1, 2 \) and \( i = s \) are given by equation (24) and Lemma 4 respectively.

Although the computation of \( d_{O_i}(r_s) \) and hence characterization of the DMT of a general ZIC with arbitrary number of antennas at each node require application of numerical methods, in what follows, we shall provide closed form expressions for it for various special cases. Since the DMT with FCSIT acts as an upper bound to the DMT of the channel with no-CSIT or only CSIR, these expressions facilitates an easy characterization of the gap between perfect and no CSIT performances of the channel. The no-CSIT DMT of the channel will be characterized in section [IV] for different range of values of \( \alpha_{21} \) and no of antennas at different nodes.

The central idea in all of the following proof is the fact that the steepest descent method provides a global optimal value of a convex optimization problem: it is well known that the steepest descent method provides a local optimal value of the objective function. However, a local optimal solution is equal to a global one as well, for a convex
function \[20\]. Combining the above two facts we conclude that the value obtained by the steepest descent method is actually the global minimum of the objective function. The first case considered is a class of channels where all the nodes have equal number of antennas.

**Theorem 2:** Consider the MIMO ZIC with \( M_1 = M_2 = N_1 = N_2 = n \) and SNRs and INRs of different links are as described in Section \[11\] with \( \alpha_{21} = \alpha \) and \( \alpha_{22} = 1 = \alpha_{11} \). The best achievable diversity order on this channel, with F-CSIT and short term average power constraint \[1\], at multiplexing gain pair \((r_1, r_2)\), is given by

\[
d^*_{\text{ZIC},(n,n,n,n)}(r_1, r_2) = \min \{ d_{n,n}(r_1), d_{n,n}(r_2), d^\text{FCSIT}_{s,(n,n,n,n)}(r_s) \}
\]

where \( d_{\text{O}_s}(r_s) \) for the special channel configuration being considered is denoted by \( d^\text{FCSIT}_{s,(n,n,n,n)}(r_s) \), and if \( \alpha \leq 1 \), then

\[
d^\text{FCSIT}_{s,(n,n,n,n)}(r_s) = \begin{cases} 
\alpha d_{n,3n}(\frac{r_s}{\alpha}) + 2n^2(1 - \alpha), & \text{for } 0 \leq r_s \leq n\alpha; \\
2(1 - \alpha)d_{n,n}(\frac{r_s - n\alpha}{2(1 - \alpha)}), & \text{for } n\alpha \leq r_s \leq n(2 - \alpha).
\end{cases}
\]

and if \( 1 \leq \alpha \), then

\[
d^\text{FCSIT}_{s,(n,n,n,n)}(r_s) = \begin{cases} 
d_{n,3n}(r_s) + n^2(\alpha - 1), & \text{for } 0 \leq r_s \leq n; \\
(\alpha - 1)d_{n,n}(\frac{r_s - n}{\alpha - 1}), & \text{for } n \leq r_s \leq n\alpha.
\end{cases}
\]

**Proof of Theorem 2**  
The proof is relegated to Appendix \[3\].

**Remark 5:** Note that for \( \alpha = 1 \), the optimal DMT becomes \( d^\text{FCSIT}_{s,(n,n,n,n)}(r_s) = \min \{ d_{n,n}(r_1), d_{n,n}(r_2), d_{n,3n}(r_1 + r_2) \} \) which is exactly the upper bound derived in \[18\].

### A. The DMT of a Femto-Cell

A practical communication channel following the ZIC signal model appears in the so called Femtocell environment. The Femtocell \[12\] concept is an outcome of the telecommunication industry’s efforts to provide high-throughput, high quality services into the user’s home. Consider the scenario depicted in figure \[2\] where the larger circle represents the macro cell serviced by the macro cell base station (MCBS). Within this macro cell is the smaller circle represents a small area where the signal from the MCBS either does not reach with enough strength or does not reach at all, hereafter referred to as the Femtocell. To provide coverage in this region a smaller user deployed base station connected to the backbone can be used, which is called the Femto cell BS (FCBS). This FCBS can provide mobile services to the users within the Femtocell just like a WiFi access point. The basic difference between the FCBS and WiFi access point is that the former operates in a licensed band.

Now, let us consider the downlink communication on such a channel with one mobile user in both the Femtocell and the macro-cell. Note that since the MCBS signal does not reach the mobile user within the Femtocell, the signal input-output follows the ZIC model. To model the larger SNR of the Femtocell direct link we can assume that \( \alpha_{22} \geq 1 \). In what follows, we shall derive the DMT of this channel.

**Theorem 3:** Consider a ZIC with \( M_1 = M_2 = N_1 = N_2 = n, \alpha_{22} = \alpha \geq 1 \) and \( \alpha_{11} = \alpha_{21} = 1 \), F-CSIT and short term average power constraint given by \[1\]. The optimal diversity order of this channel at a multiplexing gain
Fig. 2: Femtocell channel model: down link.

pair \((r_1, r_2)\) is given by

\[
d_{\text{Femto}}^{(n,n,n,n)}(r_1, r_2) = \min \left\{ d_{n,n}(r_1), d_{n,n}(r_2), d_{s,(n,n,n,n)}^{\text{Femto}}(r_s) \right\}
\]

where \(d_{s,(n,n,n,n)}^{\text{Femto}}(r_s)\) for the special channel configuration being considered, is denoted by \(d_{s,(n,n,n,n)}^{\text{Femto}}(r_s)\), and

\[
d_{s,(n,n,n,n)}^{\text{Femto}}(r_s) = \begin{cases} 
  d_{n,3n}(r_s) + n^2(\alpha - 1), & \text{for } 0 \leq r_s \leq n; \\
  (\alpha - 1)d_{n,n}(r_s), & \text{for } n \leq r_s \leq n\alpha.
\end{cases}
\]

Proof: The proof is relegated to Appendix C.

Remark 6: Note that the fundamental DMT of the ZIC with single antenna nodes and \(\alpha_{22} = \alpha, \alpha_{11} = \alpha_{21} = 1\) was derived in [21]. This clearly is a special case of Theorem 3 and can be obtained by putting \(n = 1\).

Typically, in a multiuser communication scenario one end – say the base station in a cellular network – can host more antennas than the other, in what follows, we consider a case where \(M_1 = M_2 = M \leq \min\{N_1, N_2\}\).

Theorem 4: Consider the ZIC with \(M_1 = M_2 = M \leq \min\{N_1, N_2\}, \alpha_{11} = \alpha_{22} = \alpha_{21} = 1\), F-CSIT and short term average power constraint given by (1). The optimal diversity order achievable on this channel at a multiplexing gain pair \((r_1, r_2)\) is given by

\[
d_{M,N_1,M,N_2}^{\text{FCSIT}}(r_1, r_2) = \min \left\{ d_{M,N_1}(r_1), d_{M,N_2}(r_2), d_{s,(M,N_1,M,N_2)}^{\text{FCSIT}}(r_s) \right\}
\]

where \(d_{s,(M,N_1,M,N_2)}^{\text{FCSIT}}(r_s)\) for the special channel configuration being considered, is denoted by \(d_{s,(M,N_1,M,N_2)}^{\text{FCSIT}}(r_s)\), and

\[
d_{s,(M,N_1,M,N_2)}^{\text{FCSIT}}(r_s) = \begin{cases} 
  d_{M,(M+N_1+N_2)}(r_s) + M(N_1 - M); & 0 \leq r_s \leq M; \\
  d_{2M,N_1}(r_s); & M \leq r_s \leq \min\{N_1, 2M\}.
\end{cases}
\]

Proof: The proof is relegated to Appendix D.

Next we shall demonstrate the performance loss in terms of DMT, due to lack of CSIT. In Figure 3 explicit F-CSIT DMT curves for a few antenna configurations are plotted and compared against the performance of orthogonal schemes such as frequency division (FD) or time division (TD) multiple-access which do not require CSIT. It can
be noticed from the figure that the gain due to CSIT, over the orthogonal access schemes can be significant, particularly in MIMO ZICs. While using better coding-decoding schemes this gap can be reduced, in general, with CSIT a better performance can be achieved. However, to evaluate this gap in performance due to lack of CSIT, exactly it is necessary to know the best DMT achievable on the channel without any CSIT. Popular approaches of characterization of the No-CSIT DMT involves either evaluating the optimal instantaneous mutual information region of the channel without CSIT exactly or finding subsets and supersets of it which are within a constant number of bits to each other, as was done in the case of F-CSIT considered so far. For the no CSIT case either of the above two objectives are very hard to achieve. However, in the following section, we bypass this approach and characterize the No-CSIT DMT of the ZIC for some specific values of the different channel parameters such as the number of antennas at the different nodes and the INR parameter $\alpha$.

IV. DMT WITH NO CSIT

To avoid the above mentioned difficulties in this section, we use as the F-CSIT DMT derived in the previous sections, an upper bound to the No-CSIT DMT of the channel. Then, we derive the achievable DMT of a No-CSIT transmit-receive scheme, which for two special classes of ZICs meets the upper bound and therefore, represents the fundamental No-CSIT DMT of the corresponding ZICs. In what follows, we shall describe the No-CSIT transmit-receive scheme first.

Let both the users encode their messages using independent Gaussian signals. On the other hand, since $R_{x_1}$ is not interested in the signal transmitted by $T_{x_2}$ we consider a decoder at $R_{x_1}$ which does joint maximum-likelihood (ML) decoding of both the messages. However, the event where only the second user’s message is decoded incorrectly is not considered as an error event. $R_{x_2}$ uses an ML decoder to decode its own message. Hereafter, we will refer to this scheme as the Individual ML (IML) decoder and the encoding-decoding scheme as the Independent coding IML decoding scheme or the IIML scheme.
An achievable rate region of the individual ML decoder and independent Gaussian coding at both the users is given by the following set of rate tuples

$$ \mathcal{R}_{\text{IML}} = \left\{ (R_1, R_2) : R_1 \leq \log \det \left( I_{N_i} + \frac{\rho}{M_1} H_{11} H_{11}^\dagger \right) \triangleq I_{c_1}; \right. $$

$$ R_2 \leq \log \det \left( I_{N_2} + \frac{\rho}{M_2} H_{22} H_{22}^\dagger \right) \triangleq I_{c_2}; $$

$$ \left. (R_1 + R_2) \leq \log \det \left( I_{N_i} + \frac{\rho}{M_1} H_{11} H_{11}^\dagger + \frac{\rho^N}{M_2} H_{21} H_{21}^\dagger \right) \triangleq I_{c_d}; \right\} $$

Using the above expression for the achievable rate region, the corresponding achievable DMT of this transmit-receive scheme, i.e., mutually independent Gaussian coding at each transmitters and IML decoder at the interfered receiver, can be easily computed using standard techniques. The result is specified in the following Lemma.

**Lemma 5:** If we denote the achievable diversity order of the IML scheme, at multiplexing gain pair \((r_1, r_2)\), by \(d_{\text{IML}}^{(M_1, N_1, M_2, N_2)}(r_1, r_2)\) then,

$$ d_{\text{IML}}^{(M_1, N_1, M_2, N_2)}(r_1, r_2) \geq \min_{i \in \{1, 2, s\}} \left\{ d_{\text{IML}}^{(i, (M_1, N_1, M_2, N_2)}(r_i) \right\}, $$

where \(r_s = (r_1 + r_2)\) and

$$ d_{\text{IML}}^{(i, (M_1, N_1, M_2, N_2)}(r_i) = \lim_{\rho \to \infty} -\frac{\log (\Pr (I_{cs} \leq r_i))}{\log (\rho)}, \quad \forall \ i \in \{1, 2\}. $$

**Proof:** The proof is relegated to Appendix E.

Note that \(I_{cs}\) and \(I_{c_d}\) represents the mutual information of a point-to-point channel with channel matrices \(H_{11}\) and \(H_{22}\), respectively. Therefore, by the results of [11], we have

$$ d_{\text{IML}}^{(i, (M_1, N_1, M_2, N_2)}(r_i) = d_{M_i, N_i}(r_i), \quad 0 \leq r_i \leq \min\{M_i, N_i\}, $$

and \(i \in \{1, 2\}\). To analyze the outage event due to the third bound of the achievable rate region, we first approximate \(I_{cs}\) by another term which does not differ from it by more than a constant. Note that

$$ \log \det \left( I_{N_i} + \rho H_{11} H_{11}^\dagger + \rho^N H_{21} H_{21}^\dagger \right) - N_1 \log (\max\{M_1, M_2\}), $$

\( \leq I_{cs} \leq \log \det \left( I_{N_i} + \rho H_{11} H_{11}^\dagger + \rho^N H_{21} H_{21}^\dagger \right) \triangleq I'_{cs}. $$

Since a constant independent of \(\rho\), does not matter in the high SNR analysis, to compute \(d_{\text{IML}}^{(M_1, N_1, M_2, N_2)}\) we can use \(I'_{cs}\) in place of \(I_{cs}\). Next, we write \(I'_{cs}\) in the following manner:

$$ I'_{cs} = \log \det \left( I_{N_i} + \rho H_{11} H_{11}^\dagger + \rho^N H_{21} H_{21}^\dagger \right), $$

$$ = \log \det \left( I_{M_i} + \rho \bar{H}_{11} \bar{H}_{11}^\dagger \right) + \log \det \left( I_{N_i} + \rho^N H_{21} H_{21}^\dagger \right), $$

$$ \geq \left\{ \sum_{j=1}^{q_1} (1 + \rho^{\alpha_{11} \mu_j}) + \sum_{i=1}^{p} (1 + \rho^{\alpha_{21} \lambda_i}) \right\}, $$

where \(\bar{H}_{11} = \left( I_{N_i} + \rho^N H_{21} H_{21}^\dagger \right)^{-\frac{1}{N}} H_{11}\) and in step (a), \(p = \min\{M_2, N_1\}, \ q_1 = \min\{M_1, N_1\}\). Also, we have denoted the ordered non-zero (with probability 1) eigenvalues of \(W_1 = H_{11}^\dagger \left( I_{N_i} + \rho_{21} H_{21} H_{21}^\dagger \right)^{-1} H_{11}\) and
$W_2 = H_{21} H_{21}^{†}$ by $\mu_1 \geq \cdots \geq \mu_{q_1} > 0$ and $\lambda_1 \geq \cdots \geq \lambda_p > 0$, respectively. Now, using the transformations $\lambda_i = \rho^{-\upsilon_i}$, for $1 \leq i \leq p$, $\mu_j = \rho^{-\beta_j}$, for $1 \leq j \leq q_1$ in the above equation and substituting that in turn in equation (32) we get

$$p^{-d_{s, (M_1, N_1, M_2, N_2)}^{\text{IML}}(r_s)} = \Pr \left\{ \sum_{i=1}^{p} (\alpha_{21} - \upsilon_i)^+ + \sum_{j=1}^{q_1} (\alpha_{11} - \beta_j)^+ \right\} < r_s \right\}. \quad (38)$$

To evaluate this expression we need to derive the joint distribution of $\bar{\beta}$ and $\bar{\upsilon}$. Note that, since $W_1$ and $W_2$ are mutually correlated and so are $\bar{\beta}$ and $\bar{\upsilon}$. As already stated earlier, in general characterizing the joint distribution of the eigenvalues of such mutually correlated random matrices is a hard problem. However, in what follows, we show that this distribution can be computed using Theorems 1 and 2 of [15], which in turn facilitates the characterization of $d_{s}^{\text{IML}}(r_s)$.

**Lemma 6:** The negative SNR exponent of the outage event corresponding to the sum bound in (30), i.e., $d_{s, (M_1, N_1, M_2, N_2)}^{\text{IML}}(r_s)$, is equal to the minimum of the following objective function:

$$d_{s, (M_1, N_1, M_2, N_2)}^{\text{IML}}(r_s) = \min_{\bar{\upsilon}, \bar{\beta}} \sum_{i=1}^{p} (M_2 + N_1 + M_1 + 1 - 2i) \upsilon_i + \sum_{j=1}^{q_1} (M_1 + N_1 + 1 - 2j) \beta_j$$

$$- M_1 p \alpha_{21} + \sum_{j=1}^{q_1} \sum_{i=1}^{\min\{N_1-j, M_1\}} (\alpha_{21} - \upsilon_i - \beta_j)^+; \quad (39a)$$

constrained by:

$$\sum_{i=1}^{p} (\alpha_{21} - \upsilon_i)^+ + \sum_{j=1}^{q_1} (\alpha_{11} - \beta_j)^+ \leq r_s; \quad (39b)$$

$$0 \leq \upsilon_1 \leq \cdots \leq \upsilon_p; \quad (39c)$$

$$0 \leq \beta_1 \leq \cdots \leq \beta_{q_1}; \quad (39d)$$

$$(\upsilon_i + \beta_j) \geq \alpha_{21}, \forall (i+j) \geq (N_1+1). \quad (39e)$$

**Proof:** The proof is relegated to Appendix F.

**Theorem 5 (A lower bound to the No-CSIT DMT of the ZIC):**

1) The optimal diversity order achievable by the IIML scheme described above, on a $(M_1, N_1, M_2, N_2)$ ZIC without CSIT is given as

$$d_{\text{LB, ZIC}}^{\text{No-CSIT}}(r_1, r_2) = \min_{i \in \{1, 2, s\}} d_{s, (M_1, N_1, M_2, N_2)}^{\text{IML}}(r_i),$$

where $d_{s, (M_1, N_1, M_2, N_2)}^{\text{IML}}(r_i)$ for $i = 1, 2$ and $i = s$ are given by equation (33) and Lemma 6 respectively.

2) This DMT also represents a lower bound to the No-CSIT DMT of the $(M_1, N_1, M_2, N_2)$ ZIC.

**Proof:** The first part of the Theorem follows from Lemma 5 and the second part of the Theorem follows from the fact that the IIML scheme is only one of the numerous transmit-receive schemes that are possible on the ZIC.

Although the computation of $d_{s, (M_1, N_1, M_2, N_2)}^{\text{IML}}(r_s)$ and hence characterization of the lower bound to the No-CSIT DMT of a general ZIC with arbitrary number of antennas at each node require application of numerical methods, in what follows, we shall provide closed form expressions for it for various special cases. We shall see that for two
special classes of ZICs this lower bound meets the upper bound, i.e., the FCSIT DMT of the channel. We start with the case where all the nodes have equal number of antennas.

**Lemma 7:** On a MIMO ZIC with $n$ antennas at all the nodes, $\alpha_{11} = \alpha_{22} = 1$ and $\alpha_{21} = \alpha$, the IML decoder can achieve the following DMT

\[
d_{\text{IML}}(n,n,n,n)(r_1, r_2) = \min \left\{ d_{n,n}(r_1), d_{n,n}(r_2), d_{s,n,n,n}(r_3) \right\}
\]

where $d_{s,n,n,n}(r_3)$ is given as

\[
d_{s,n,n,n}(r_3) = \begin{cases} 
  d_{n,2n}(r_3) + n^2(\alpha - 1), & 0 \leq r_3 \leq n; \\
  (\alpha - 1)d_{n,n}(\frac{r_3-n}{\alpha-1}), & n \leq r_3 \leq n\alpha.
\end{cases}
\]

**Proof:** The desired result is obtained by following the same steps as in the 2nd part of Theorem 2.

Figure 4 illustrates that the IML decoder can achieve the DMT (with F-CSIT) of the MIMO ZIC on a region of low multiplexing gains.

Comparing equations (29) to (40) we see that the IML decoder can achieve the F-CSIT DMT for high multiplexing gain values also, when $\alpha \geq 1$. This fact raises the natural question: is it possible for the IML decoder to achieve the F-CSIT DMT for all multiplexing gains and if it is, under what circumstances? It turns out that if the interference is strong enough then the IML decoder can achieve the F-CSIT DMT for all symmetric multiplexing gains. For example, Figure 5 illustrates this effect on a ZIC with 2 antennas at all the nodes.

This characteristics of the DMT on MIMO ZICs for general $n$ is captured by the following Lemma.

**Theorem 6:** The DMT specified in Lemma 7 represents the fundamental DMT of the channel with No-CSIT and $r_1 = r_2 = r$, if

\[
\alpha \geq 1 + \frac{d_{n,n}(\frac{n}{n})}{n^2}.
\]

**Proof:** Detailed proof will be provided in Appendix C.
Lemma 8: Consider the MIMO ZIC as in Theorem 4 but no CSI at the transmitters. The achievable diversity order of the IML scheme on this channel, at a multiplexing gain pair \((r_1, r_2)\), is given by

\[
d_{\text{IML}}^{(M,N_1,M,N_2)}(r_1, r_2) = \min \left\{ d_{M,N_1}(r_1), d_{M,N_2}(r_2), d_{2M,N_1}(r_s) \right\}.
\]

Proof: From Lemma 5 and equation (33) it is clear that to prove the Lemma it is sufficient to derive an expression for \(d_{s,(M,N_1,M,N_2)}^{\text{IML}}(r_s)\). Towards that, for convenience we use \(I'_{e_s}\) instead of \(I_{e_s}\) to evaluate the corresponding outage event since the two are within a constant number which does not matter at asymptotic SNR as was shown in equation (34). Using this in equation (32) along with the facts that \(M_1 = M_2 = M\) and \(\alpha = 1\) we get

\[
\rho^{-d_{s,(M,N_1,M,N_2)}^{\text{IML}}(r_s)} = \Pr \left\{ \log \det \left( I_{N_1} + \rho H_{11} H_{11}^H + \rho H_{21} H_{21}^H \right) \leq r_s \log(\rho) \right\},
\]

\[
= \Pr \left\{ \log \det \left( I_{N_1} + \rho H_{e} H_{e}^H \right) \leq r_s \log(\rho) \right\},
\]

(42)

where \(H_e = [H_{11}, H_{21}] \in \mathbb{C}^{N_1 \times (2M)}\) is identically distributed as the other channel matrices, since \(H_{11}\) and \(H_{21}\) are mutually independent. However the right hand side of the last equation represents the outage probability of an \(N_1 \times 2M\) point-to-point MIMO channel whose negative SNR exponent was computed in [11] and is given by \(d_{N_1,2M}(r_s)\). Using this in equation (42) we get,

\[
\rho^{-d_{s,(M,N_1,M,N_2)}^{\text{IML}}(r_s)} = \rho^{-d_{N_1,2M}(r_s)},
\]

or, \(d_{s,(M,N_1,M,N_2)}^{\text{IML}}(r_s) = d_{N_1,2M}(r_s)\).

Substituting this and equation (33) into equation (31) we obtain the desired result.

Figure 6 depicts the comparison of the achievable DMT of the IML decoder with that of the fundamental F-CSIT DMT of the channel on two different MIMO ZICs. Comparing the performance improvement of the IML decoder on the \((3, 4, 3, 3)\) ZIC with respect to that on the \((3, 3, 3, 3)\) ZIC, we realize that a larger number of antennas at
the interfered receiver can completely compensate for the lack of CSIT. Again by the argument that the FCSIT DMT represents an upper bound to the No-CSIT DMT of the channel, the observation from Fig. 6 imply that the DMT of Lemma 8 represents the fundamental DMT of the \((3, 4, 3, 3)\) ZIC with only CSIR. It turns out that, this channel is only a member of a large class of ZICs for which the No-CSIT DMT can be characterized. This class of channels is specified in the next Theorem.

**Theorem 7:** The DMT specified in Lemma 8 represents the fundamental DMT of the channel with No-CSIT and 
\[ r_1 = r_2 = r, \]
if
\[ N_1 \geq M + \frac{d_{M, \min\{N_1, N_2\}} \left( \frac{M}{M} \right)}{M}. \]

**Proof:** Comparing Theorem 4 and Lemma 8, the desired result can be obtained following the similar steps as in the proof of Theorem 6.

\[ \blacksquare \]

V. **Conclusion**

The DMT of the MIMO ZIC with CSIT is characterized. It is shown that the knowledge of \(H_{21}\) at the 2nd transmitter only is sufficient to achieve the F-CSIT DMT of the channel. The No-CSIT DMT of two special class of ZICs have been characterized revealing useful insights about the system such as: a stronger interference or a larger number of antennas at the interfered receiver can completely compensate for the lack of CSIT on a ZIC. Characterizing the DMT of the general ZIC with no CSIT is an interesting problem for future research.

APPENDIX A

**Proof of Lemma 4**

Recall that, the negative SNR exponents of \(W_1\), \(W_2\) and \(W_3\) are denoted by \(\{\beta_j\}_{j=1}^{q_1}\), \(\{\gamma_k\}_{k=1}^{q_2}\) and \(\{\upsilon_i\}_{i=1}^{p}\), respectively. Interestingly, these matrices have exactly the same structure specified in Theorem 1 and 2 of [15].
except from the $\rho^{21}$ which can be easily taken care of. Therefore, using these Theorems we obtain the following conditional distributions:

$$f_{W_1|W_3}(\vec{\beta}\,|\,\vec{v}) = \begin{cases} \rho^{-E_1(\vec{\beta},\vec{v})} & \text{if } (\vec{\beta},\vec{v}) \in B_1; \\ 0 & \text{if } (\vec{\beta},\vec{v}) \notin B_1 \end{cases}$$

where

$$E_1(\vec{\beta},\vec{v}) = \left\{ \sum_{j=1}^{q_1} \left( (M_1 + N_1 + 1 - 2j)\beta_j + \sum_{i=1}^{\min\{(N_1-j),M_1\}} (\alpha_{21} - v_i - \beta_j)^+ \right) - M_1 \sum_{i=1}^{p} (\alpha_{21} - v_i)^+ \right\}$$

and

$$B_1 = \{ (\vec{\beta},\vec{v}) : 0 \leq v_1 \leq \cdots \leq v_p; 0 \leq \beta_1 \leq \cdots \leq \beta_{q_1}; (v_i + \beta_j) \geq \alpha_{21}, \forall (i+j) \geq (N_1 + 1) \}$$

and

$$f_{W_2|W_3}(\vec{\gamma}|\vec{v}) = \begin{cases} \rho^{-E_2(\vec{\gamma},\vec{v})} & \text{if } (\vec{\gamma},\vec{v}) \in B_2; \\ 0 & \text{if } (\vec{\gamma},\vec{v}) \notin B_2 \end{cases}$$

where

$$E_2(\vec{\gamma},\vec{v}) = \left\{ \sum_{k=1}^{q_2} \left( (M_2 + N_2 + 1 - 2k)\gamma_k + \sum_{i=1}^{\min\{(M_2-k),N_2\}} (\alpha_{21} - v_i - \gamma_k)^+ \right) - N_2 \sum_{i=1}^{p} (\alpha_{21} - v_i)^+ \right\}$$

and

$$B_2 = \{ (\vec{\beta},\vec{v}) : 0 \leq v_1 \leq \cdots \leq v_p; 0 \leq \gamma_1 \leq \cdots \leq \gamma_{q_2}; (v_i + \gamma_k) \geq \alpha_{21}, \forall (i+k) \geq (M_2 + 1) \}$$

These pdfs are easily obtained from equation (10) of [15] by changing it properly due to the presence of the $\rho^{\alpha_{12}}$ instead of $\rho$ in [15] and using the fact that

$$f_{W_3}(\vec{v}) = \begin{cases} \rho^{-\sum_{i=1}^{p} (M_2 + N_1 + 1 - 2i)v_i} & \text{if } 0 \leq v_1 \leq \cdots \leq v_p; \\ 0 & \text{otherwise}. \end{cases}$$

It was proved in [22] that given the non-zero eigenvalues of $W_3$, $W_1$ and $W_2$ are conditionally independent random matrices. Since the non-zero eigenvalues of $H_{21}H_{21}^\dagger$ and $H_{21}^\dagger H_{21}$ are exactly the same for each realization of the matrix $H_{21}$, given $\vec{v}$, $\vec{\beta}$ and $\vec{\gamma}$ are conditionally independent of each other. Intuitively, it is a well known fact in the literature of random matrix theory that the eigenvalues of $W_1$ are dependent on the matrix $W_3$ only through its eigenvalues. The same is true for $W_2$. Therefore, given the eigenvalues of $W_3$, the eigenvalues of $W_1$ and $W_2$ are conditionally independent. Using this fact we have the following:

$$f_{W_1,W_2,W_3}(\vec{\beta},\vec{\gamma},\vec{v}) = f_{W_1,W_2|W_3}(\vec{\beta},\vec{\gamma}|\vec{v}) f_{W_3}(\vec{v}),$$

$$(\text{a}) f_{W_1|W_3}(\vec{\beta}|\vec{v}) f_{W_2|W_3}(\vec{\gamma}|\vec{v}) f_{W_3}(\vec{v}).$$

\(^1\text{For a detailed proof of this fact the reader is referred to Lemma 1 of [22].}\)
Now, substituting equations (43), (46) and (49) into the above equation we obtain the joint distribution $f_{W_1, W_2, W_3}(\bar{\beta}, \bar{\gamma}, \bar{\upsilon})$.

When this joint distribution and the Laplace’s method is used following the similar approach as in [11] to evaluate equation (25) we obtain the following:

$$d_{O_s}(r_s) = \min \sum_{i=1}^{p} (M_2 + N_1 + 1 - 2i)\upsilon_i + \sum_{j=1}^{q_1} (M_1 + N_1 + 1 - 2j)\beta_j + \sum_{k=1}^{q_2} (M_2 + N_2 + 1 - 2k)\gamma_k$$

$$- (M_1 + N_2) \sum_{i=1}^{p} (\alpha_{21} - \upsilon_i)^+ + \sum_{k=1}^{q_2} \min\{M_2 - k, N_2\} \sum_{i=1}^{p} (\alpha_{21} - \upsilon_i - \gamma_k)^+$$

$$+ \sum_{j=1}^{q_1} \min\{N_1 - j, M_1\} \sum_{i=1}^{p} (\alpha_{21} - \upsilon_i - \beta_j)^+;$$

(52a)

constrained by:

$$\sum_{i=1}^{p} (\alpha_{21} - \upsilon_i)^+ + \sum_{j=1}^{q_1} (\alpha_{11} - \beta_j)^+ + \sum_{k=1}^{q_2} (\alpha_{22} - \gamma_k)^+ < r_s;$$

(52b)

$$0 \leq \upsilon_1 \leq \cdots \leq \upsilon_p;$$

(52c)

$$0 \leq \beta_1 \leq \cdots \leq \beta_{q_1};$$

(52d)

$$0 \leq \gamma_1 \leq \cdots \leq \gamma_{q_2};$$

(52e)

$$(\upsilon_i + \beta_j) \geq \alpha_{21}, \forall (i + j) \geq (N_1 + 1);$$

(52f)

$$(\upsilon_i + \gamma_k) \geq \alpha_{21}, \forall (i + k) \geq (M_2 + 1).$$

(52g)

Finally, the desired result follows from the fact that by restricting $\alpha_i \leq \alpha_{21}$ for all $i \leq p$ does not change the optimal solution of the above problem. This can be proved as follows: suppose the optimal solution have $\alpha_i > \alpha_{21}$ for some $i$. Now, since the objective function is monotonically decreasing function of $\alpha_i$ for all $i$, substituting $\alpha_i = \alpha_{21}$ does not violate any of the constraints but reduces the objective function. But, that means the earlier solution was not really the optimal solution. Therefore, in the optimal solution we must have $\alpha_i \leq \alpha_{21}$ for all $i \leq p$. However, with this constraint we have

$$(\alpha_{21} - \upsilon_i)^+ = (\alpha_{21} - \upsilon_i), \forall i \leq p.$$

Substituting this in equation (52a) we get the desired result.

**APPENDIX B**

**PROOF OF THEOREM 2**

The main steps of the proof can be described as follows. First, we simplify the optimization problem in equation (26) by putting specific values of the different parameters, such as $M_i$, $N_i$ and $\alpha_{ij}$ as stated in the statement of the Theorem, in equation (26). Then we calculate a local minimum of this simplified optimization problem for each value of $r$, using the steepest descent method, which by the previous argument then represents a global minimum. The later part of the problem, i.e., the computation of the local minimum will be carried out in two steps: in step
one, we consider the case when $\alpha \leq 1$ and then in the second step we consider the remaining case. Let us start by deriving the simplified optimization problem.

Substituting $M_1 = M_2 = N_1 = N_2 = n$, $\alpha_{11} = \alpha_{22} = 1$ and $\alpha_{21} = \alpha$ in the optimization problem of equation (26) we obtain the following:

$$\min \mathcal{O}_s \triangleq \min_{(\nu, \gamma, \beta, \bar{\alpha})} \sum_{i=1}^{n} (4n + 1 - 2i)\nu_i + \sum_{j=1}^{n} (2n + 1 - 2j)\beta_j + \sum_{k=1}^{n} (2n + 1 - 2k)\gamma_k$$

$$-2n^2\alpha + \sum_{k=1}^{n} \sum_{i=1}^{n-k} (\alpha - \nu_i - \gamma_k)^+ + \sum_{j=1}^{n} \sum_{i=1}^{n-j} (\alpha - \nu_i - \beta_j)^+; \quad (53a)$$

constrained by:

$$\sum_{i=1}^{n} (\alpha - \nu_i)^+ + \sum_{j=1}^{n} (1 - \beta_j)^+ + \sum_{k=1}^{n} (1 - \gamma_k)^+ < r_s; \quad (53b)$$

$$0 \leq \nu_1 \leq \cdots \leq \nu_n; \quad (53c)$$

$$0 \leq \beta_1 \leq \cdots \leq \beta_n; \quad (53d)$$

$$0 \leq \gamma_1 \leq \cdots \leq \gamma_n; \quad (53e)$$

$$(\nu_i + \beta_j) \geq \alpha, \quad \forall (i + j) \geq (n + 1); \quad (53f)$$

$$(\nu_i + \gamma_k) \geq \alpha, \quad \forall (i + k) \geq (n + 1). \quad (53g)$$

As stated earlier, in what follows, we solve this optimization problem in two steps; in the first step, we assume $\alpha \leq 1$.

A. Step 1: ($\alpha \leq 1$)

To apply the steepest descent method we first compute the rate of change of the objective function with respect to the various parameters. Differentiating the objective function in equation (53a) we obtain the following:

$$\left. \frac{\partial \mathcal{O}_s}{\partial \alpha_i} \right|_{\alpha_i = 0, \cdots, \alpha_{i-1} = 0, \alpha_{i+1} = 1, \cdots, \alpha_n = 1, \bar{\alpha}_1 = 1, \bar{\alpha}_2 = 1} = (4n + 1 - 2i), \quad 1 \leq i \leq n; \quad (54)$$

$$\left. \frac{\partial \mathcal{O}_s}{\partial \beta_1} \right|_{\alpha_1 = 1, \gamma_1 = 1} = (2n - 1) \leq (4n + 1 - 2i), \quad \forall 1 \leq i \leq n; \quad (55)$$

$$\left. \frac{\partial \mathcal{O}_s}{\partial \gamma_1} \right|_{\alpha_1 = 1, \beta_1 = 1} = (2n - 1) \leq (4n + 1 - 2i), \quad \forall 1 \leq i \leq n. \quad (56)$$

Note that it is sufficient, to consider the decay of the function with respect to (w.r.t.) $\beta_1$ and $\gamma_1$ only, because of the decreasing slope of the objective function with increasing index of $\beta$ and $\gamma$ and equation (53d) and (53e). Therefore, it is clear from the slopes of the function given above that for $(i - 1)\alpha \leq r_s \leq i\alpha$, the steepest descent of the function is along decreasing values of $\alpha_i$, while $\beta_1 = \gamma_1 = 1$. Note that for these values of $\beta_1$ and $\gamma_1$ the last two terms of equation (53a) vanishes and equations (53d), (53e) becomes redundant and as a result the optimization
problem of \((53)\) simplifies to the following:

\[
\begin{aligned}
\min & \sum_{i=1}^{n} (4n + 1 - 2i) v_i + 2n^2 (1 - \alpha); \\
\text{constrained by:} & \quad \sum_{i=1}^{n} (\alpha - v_i)^+ \leq r_s; \\
& \quad 0 \leq v_1 \leq \cdots \leq v_n;
\end{aligned}
\]  
(57a)

The solution of this optimization problem follows from Lemma 3 and is given by,

\[
d_s(r_s) = \alpha d_{n,3n} \left( \frac{r_s}{\alpha} \right) + 2n^2 (1 - \alpha), \quad 0 \leq r_s \leq n\alpha.
\]  
(58)

The above solution also imply that for \(\alpha \geq n\alpha\), the optimal solution have \(\alpha_i = 0, \quad \forall i\), which when substituted in equation \((53)\) we obtain the following problem.

\[
\begin{aligned}
\min & \sum_{j=1}^{n} (2n + 1 - 2j) \beta_j + \sum_{k=1}^{n} (2n + 1 - 2k) \gamma_k - 2n^2, \\
\text{constrained by:} & \quad \sum_{j=1}^{n} (1 - \beta_j)^+ + \sum_{k=1}^{n} (1 - \gamma_k)^+ < (r_s - n\alpha); \\
& \quad \alpha \leq \beta_1 \leq \cdots \leq \beta_n; \\
& \quad \alpha \leq \gamma_1 \leq \cdots \leq \gamma_n;
\end{aligned}
\]  
(59a)

Note that the last two summands in the objective function \((53)\) is zero because of equations \((59c)\) and \((59d)\). Now, from the symmetry of the optimization problem \((59)\) w.r.t. \(\beta_i\) and \(\gamma_i\), we can assume without loss of generality that the optimal solution will have \(\beta_i = \gamma_i, \quad \forall i\). Substituting this and \(\delta_i = \beta_i - \alpha\) in equation \((59)\) we get the following equivalent optimization problem:

\[
\begin{aligned}
\min & \quad 2 \sum_{j=1}^{n} (2n + 1 - 2j) \delta_j, \\
\text{constrained by:} & \quad \sum_{j=1}^{n} (1 - \alpha - \delta_j)^+ \leq \left( \frac{r_s - n\alpha}{2(1 - \alpha)} \right); \\
& \quad 0 \leq \delta_1 \leq \cdots \leq \delta_n;
\end{aligned}
\]  
(60a)

The solution of this optimization problem follows again from Lemma 5 and is given by

\[
d_s(r_s) = 2(1 - \alpha) d_{n,n} \left( \frac{r_s - n\alpha}{2(1 - \alpha)} \right), \quad 0 \leq \frac{r_s - n\alpha}{2(1 - \alpha)} \leq n(1 - \alpha) \\
= 2(1 - \alpha) d_{n,n} \left( \frac{r_s - n\alpha}{2(1 - \alpha)} \right), \quad n\alpha \leq r_s \leq n(2 - \alpha).
\]  
(61)

Combining equations \((58)\) and \((61)\) we obtain equation \((28)\) of Theorem 2 and we have completed the first step of this proof. In what follows we consider the remaining case, when \(\alpha \geq 1\).
B. Step 2: \((\alpha \geq 1)\)

Differentiating the objective function in equation (53a) we obtain the following:

\[
\frac{\partial O_s}{\partial \alpha_1} |_{\alpha_i = \alpha, \forall i \geq 2, \beta_i = 1, \gamma_i = 1} = \begin{cases} (4n - 1), & \text{for } \alpha_1 \geq (\alpha - 1); \\ (2n - 1), & \text{for } 0 \leq \alpha_1 \leq (\alpha - 1); \end{cases} \quad (62)
\]

\[
\frac{\partial O_s}{\partial \beta_1} |_{\alpha_i = \alpha, \forall i \geq 2, \beta_i = 1} = \frac{\partial O_s}{\partial \gamma_1} |_{\alpha_i = \alpha, \forall i \geq 2, \beta_i = 1} = \begin{cases} (2n - 1), & \text{for } \alpha_1 \geq (\alpha - 1); \\ (2n - 2), & \text{for } 0 \leq \alpha_1 \leq (\alpha - 1); \end{cases} \quad (63)
\]

\[
\frac{\partial O_s}{\partial \alpha_1} |_{\alpha_1 = (\alpha - 1), \alpha_i = \alpha, \forall i \geq 3, \beta_i = 1, \gamma_i = 1} = (4n - 3), \text{ for } \alpha_2 \geq (\alpha - 1). \quad (64)
\]

Comparing equations (62) and (63), we realize that for \(0 \leq r_s \leq 1\), the steepest descent is along the direction of decreasing \(\alpha_1\), while \(\beta_1 = \gamma_1 = 1\). On the other hand, comparing equations (62), (63) and (64) it is clear that beyond \(r_s = 1\), decreasing \(\alpha_2\) has the steepest descent than \(\beta_1, \gamma_1\) and even \(\alpha_1\). In the same way it can be proved that for \((i - 1) \leq r_s \leq i\), the steepest descent of the function is along decreasing values of \(\alpha_i\), while \(\beta_1 = \gamma_1 = 1\).

Note that for these values of \(\beta_1\) and \(\gamma_1\) the last two terms of equation (53a) vanishes and equations (53d)-(53g) becomes redundant and as a result the optimization problem of (53) simplifies to the following:

\[
\min \sum_{i=1}^{n} (4n + 1 - 2i)v_i - 2n^2(\alpha - 1); \quad (65a)
\]

constrained by:

\[
\sum_{i=1}^{n} (\alpha - v_i^+) \leq r_s; \quad (65b)
\]

\[
(\alpha - 1) \leq v_1 \leq \cdots \leq v_n; \quad (65c)
\]

Substituting \(v'_i = v_i - (\alpha - 1)\) for all \(i\) in the above set of equations we obtain the following equivalent optimization problem.

\[
\min \sum_{i=1}^{n} (4n + 1 - 2i)v'_i + n^2(\alpha - 1); \quad (66a)
\]

constrained by:

\[
\sum_{i=1}^{n} (1 - v'_i^+) \leq r_s; \quad (66b)
\]

\[
0 \leq v'_1 \leq \cdots \leq v'_n; \quad (66c)
\]

which in turn by Lemma 3 have the following optimal value

\[
d_s(r_s) = \alpha d_{n,3n}(r_s) + n^2(\alpha - 1), \quad 0 \leq r_s \leq n. \quad (67)
\]

It is clear from this solution that for \(r_s \geq n\), \(\alpha_i \leq (\alpha - 1)\) for all \(i\), and the optimization problem (53) reduces
to the following:

\[
\min \mathcal{O}_s = \min_{(\upsilon, \beta, \gamma, \bar{\alpha})} \sum_{i=1}^{n} (4n + 1 - 2i)\upsilon_i + \sum_{j=1}^{n} (2n + 1 - 2j)\beta_j + \sum_{k=1}^{n} (2n + 1 - 2k)\gamma_k
\]

\[
-2n^2\alpha + \sum_{k=1}^{n} \sum_{i=1}^{n-k} (\alpha - \upsilon_i - \gamma_k)^+ + \sum_{j=1}^{n} \sum_{i=1}^{n-j} (\alpha - \upsilon_i - \beta_j)^+ ;
\] (68a)

constrained by:

\[
\sum_{i=1}^{n} (\alpha - 1 - \upsilon_i) + \sum_{j=1}^{n} (1 - \beta_j)^+ + \sum_{k=1}^{n} (1 - \gamma_k)^+ \leq (r_s - n); \] (68b)

\[
0 \leq \upsilon_1 \leq \cdots \leq \upsilon_n \leq (\alpha - 1); \] (68c)

\[
0 \leq \beta_1 \leq \cdots \leq \beta_n; \] (68d)

\[
0 \leq \gamma_1 \leq \cdots \leq \gamma_n; \] (68e)

\[
(\upsilon_i + \beta_j) \geq \alpha, \ \forall (i + j) \geq (n + 1); \] (68f)

\[
(\upsilon_i + \gamma_k) \geq \alpha, \ \forall (i + k) \geq (n + 1). \] (68g)

Note the upper bound in equation (68c), which is different from equation (53c). Also, since we are seeking for the minimum value of the objective function which decreases with every \(\beta_i\) and \(\gamma_j\), in the optimal solution these parameters must take their minimum value, which from equations (68f) and (68g) is given by

\[
\beta_j |_{\text{min}} = \gamma_j |_{\text{min}} = \alpha - \upsilon_{n-j+1}, \] (69)

which along with the ordering among the \(\upsilon_i\)’s, \(\beta_j\)’s and \(\gamma_j\)’s imply that all the terms in the last two summands of the objective function are non-negative. Substituting these minimum values and thereby eliminating \(\beta_j\)’s and \(\gamma_j\)’s from the optimization problem we obtain the following equivalent optimization problem:

\[
\min \sum_{i=1}^{n} (2n + 1 - 2i)\upsilon_i, \] (70a)

constrained by:

\[
\sum_{i=1}^{n} (\alpha - 1 - \upsilon_i) \leq (r_s - n); \] (70b)

\[
0 \leq \upsilon_1 \leq \cdots \leq \upsilon_n \leq (\alpha - 1), \] (70c)
where the simplification of the objective function involves the following algebraic computations

\[
O_s = \sum_{i=1}^{n} (4n + 1 - 2i)v_i + 2 \sum_{j=1}^{n} (2n + 1 - 2j)(\alpha - v_{n-j+1}) - 2n^2\alpha \\
+ \sum_{k=1}^{n} \sum_{i=1}^{(n-k)} (\alpha - v_i - (\alpha - v_{n-k+1})) + \sum_{j=1}^{n} \sum_{i=1}^{(n-j)} (\alpha - v_i - (\alpha - v_{n-j+1}));
\]

\[
= \sum_{i=1}^{n} (4n + 1 - 2i)v_i + 2 \sum_{j=1}^{n} (2n + 1 - 2j)(\alpha - v_{n-j+1}) - 2n^2\alpha \\
+ n(n-1)\alpha - \sum_{i=1}^{n} 2(n-i)v_i - \sum_{k=1}^{n} (\alpha - v_{n-k+1}) - \sum_{j=1}^{n} (n-j)(\alpha - v_{n-j+1});
\]

\[
= \sum_{i=1}^{n} (2n + 1)v_i + 2 \sum_{j=1}^{n} (n + 1 - j)(\alpha - v_{n-j+1}) - n(n+1)\alpha \\
= \sum_{i=1}^{n} (2n + 1 - 2i)v_i + 2 \sum_{j=1}^{n} (n + 1 - j)\alpha - n(n+1)\alpha \\
= \sum_{i=1}^{n} (2n + 1 - 2i)v_i.
\]

The solution of this optimization problem follows again from Lemma 3 and is given by

\[
d_s(r_s) = (\alpha - 1)d_{n,n} \left( \frac{r_s - n}{\alpha - 1} \right), \quad 0 \leq (r_s - n) \leq n(\alpha - 1) \\
= (\alpha - 1)d_{n,n} \left( \frac{r_s - n}{\alpha - 1} \right), \quad n \leq r_s \leq n\alpha. \quad (71)
\]

Combining equations (67) and (71) we obtain equation (29) of Theorem 2

APPENDIX C

PROOF OF THEOREM 3

When we substitute the specific values of the different parameters in equation (26), it reduces to the following

\[
d^\text{Femto}_{n,n,n,n}(r_s) = \min \sum_{i=1}^{n} (4n + 1 - 2i)v_i + \sum_{j=1}^{n} (2n + 1 - 2j)\beta_j + \sum_{k=1}^{n} (2n + 1 - 2k)\gamma_k \\
- 2n^2 + \sum_{k=1}^{n} \sum_{i=1}^{(n-k)} (1 - v_i - \gamma_k)^+ + \sum_{j=1}^{n} \sum_{i=1}^{(n-j)} (1 - v_i - \beta_j)^+; \quad (72a)
\]

constrained by:

\[
\sum_{i=1}^{n} (1 - v_i)^+ + \sum_{j=1}^{n} (1 - \beta_j)^+ + \sum_{k=1}^{n} (\alpha - \gamma_k)^+ < r_s; \quad (72b)
\]

\[
0 \leq v_1 \leq \cdots \leq v_n; \quad (72c)
\]

\[
0 \leq \beta_1 \leq \cdots \leq \beta_n; \quad (72d)
\]

\[
0 \leq \gamma_1 \leq \cdots \leq \gamma_n; \quad (72e)
\]

\[
(v_i + \beta_j) \geq 1, \quad \forall (i+j) \geq (n+1); \quad (72f)
\]

\[
(v_i + \gamma_k) \geq 1, \quad \forall (i+k) \geq (n+1). \quad (72g)
\]
Differentiating the objective function with respect to $v_i$, $\forall i$, $\beta_1$ and $\gamma_1$ we find that
\[
\frac{\partial \mathcal{F}_{\text{Femto}}}{\partial v_i} \bigg|_{\beta_1=1, \gamma_1=\alpha} = (4n + 1 - 2i) \geq \frac{\partial \mathcal{F}_{\text{Femto}}}{\partial \beta_1} \bigg|_{\gamma_1=\alpha, v_k=0, \forall k<i, v_k=1, \forall k>i} = (2n - i),
\]
(73)
\[
\frac{\partial \mathcal{F}_{\text{Femto}}}{\partial \gamma_1} \bigg|_{\beta_1=1, v_k=0, \forall k<i, v_k=1, \forall k>i}, \forall i \leq n,
\]
(74)
where we have denoted the objective function by $\mathcal{F}_{\text{Femto}}$. It is clear from these values that, for $(i - 1) \leq r_s \leq i$, the steepest descent is along decreasing $v_i$ with $\beta_1 = 1$ and $\gamma_1 = \alpha$. Putting this in equation (72) we get
\[
d^\text{Femto}_{s,(n,n,n,n)}(r_s) = \min \sum_{i=1}^{n} (4n + 1 - 2i)v_i + n^2(\alpha - 1);
\]
(75a)
constrained by:
\[
\sum_{i=1}^{n} (1 - v_i)^+ \leq r_s; \quad 0 \leq v_1 \leq \cdots \leq v_n.
\]
(75b)
(75c)
Now, using Lemma 3 we obtain the minimum value of the the above optimization problem, which can be written as
\[
d^\text{Femto}_{s,(n,n,n,n)}(r_s) = d_{n,3n}(r_s) + n^2(\alpha - 1), \quad 0 \leq r_s \leq n.
\]
(76)
Next, we obtain the optimal value of the objective function for values of $r_s \geq n$. Note that for any $r_s \geq n$, $v_i = 0 \forall i$, which along with equation (72) imply that $\beta_j = 1$, $\gamma_j \geq 1 \forall j$. Putting this in equation (72) we get
\[
d^\text{Femto}_{s,(n,n,n,n)}(r_s) = \min \sum_{i=1}^{n} (2n + 1 - 2i)\gamma_i - n^2;
\]
(77a)
constrained by:
\[
\sum_{i=1}^{n} (\alpha - \gamma_i)^+ \leq (r_s - n);
\]
(77b)
\[
1 \leq \gamma_1 \leq \cdots \leq \gamma_n.
\]
(77c)
To bring the above problem into a form amenable to Lemma 3 we use the following variable transform in the above set of equations: $\gamma'_i = \gamma_i - 1$. This results in the following equivalent optimization problem.
\[
d^\text{Femto}_{s,(n,n,n,n)}(r_s) = \min \sum_{i=1}^{n} (2n + 1 - 2i)\gamma'_i;
\]
(78a)
constrained by:
\[
\sum_{i=1}^{n} (\alpha - 1 - \gamma'_i)^+ \leq (r_s - n);
\]
(78b)
\[
0 \leq \gamma'_1 \leq \cdots \leq \gamma'_n,
\]
(78c)
which in turn by Lemma 3 attains the following optimal value:
\[
d^\text{Femto}_{s,(n,n,n,n)}(r_s) = (\alpha - 1)d_{n,n} \left( \frac{r_s - n}{\alpha - 1} \right), \quad 0 \leq \frac{r_s - n}{\alpha - 1} \leq n
\]
(79)
\[
= (\alpha - 1)d_{n,n} \left( \frac{r_s - n}{\alpha - 1} \right), \quad n \leq r_s \leq n\alpha.
\]
(80)
Finally, combining equations (76) and (87) we obtain the desired result.
APPENDIX D
PROOF OF THEOREM 3

Substituting $M_1 = M_2 = M$ and $\alpha_{ij} = 1$ in equation (26), we obtain

$$d_{s,(M,N_1,M,N_2)}^{\text{FCST}}(r_s) = \sum_{i=1}^{M} (2M + N_1 + N_2 + 1 - 2i)v_i + \sum_{j=1}^{M} (M + N_1 + 1 - 2j)\beta_j + \sum_{k=1}^{M} (M + N_2 + 1 - 2k)\gamma_k$$

$$- (M + N_2)M + \sum_{k=1}^{M} (M-k) \left(1 - v_i - \gamma_i^+\right) + \sum_{j=1}^{M} \min\{(N_1-j,M)\} \left(1 - v_i - \beta_j^+\right); \quad (81a)$$

constrained by:

$$\sum_{i=1}^{P} (\alpha_{2i} - \nu_i)^+ + \sum_{j=1}^{q_1} (\alpha_{1i} - \beta_j)^+ + \sum_{k=1}^{q_2} (\alpha_{22} - \gamma_k)^+ < r_s; \quad (81b)$$

$$0 \leq v_1 \leq \cdots \leq v_p; \quad (81c)$$

$$0 \leq \beta_1 \leq \cdots \leq \beta_{q_1}; \quad (81d)$$

$$0 \leq \gamma_1 \leq \cdots \leq \gamma_{q_2}; \quad (81e)$$

$$(v_i + \beta_j) \geq 1, \forall (i+j) \geq (N_1+1); \quad (81f)$$

$$(v_i + \gamma_k) \geq 1, \forall (i+k) \geq (M+1). \quad (81g)$$

Differentiating the objective function with respect to $v_i$, $\forall i$, $\beta_1$ and $\gamma_1$ we find that

$$\left. \frac{\partial F_{a,FCSIT}}{\partial v_i} \right|_{\beta_1=1,\gamma_1=1} \geq \begin{cases} 
\left. \frac{\partial F_{a,FCSIT}}{\partial \beta_k} \right|_{\beta_1=1,\alpha_k=0, \forall k<i, \alpha_k=1, \forall k>i} & \forall i \leq n, \\
\left. \frac{\partial F_{a,FCSIT}}{\partial \gamma_i} \right|_{\beta_1=1,\alpha_k=0, \forall k<i, \alpha_k=1, \forall k>i} & \forall i \leq n,
\end{cases} \quad (82)$$

where we have denoted the objective function by $F_{a,FCSIT}$. It is clear from these values that, for $(i-1) \leq r_s \leq i$, the steepest descent is along decreasing $v_i$ with $\beta_1 = \gamma_1 = 1$ and $i \leq n$. Putting this in equation (81) we get

$$d_{s,(M,N_1,M,N_2)}^{\text{FCST}}(r_s) = \min_{i=1}^{n} \sum_{i=1}^{M} (2M + N_1 + N_2 + 1 - 2i)v_i + M(N_1 - M); \quad (83a)$$

constrained by:

$$\sum_{i=1}^{M} (1 - v_i)^+ \leq r_s; \quad (83b)$$

$$0 \leq v_1 \leq \cdots \leq v_n. \quad (83c)$$

Now, using Lemma 3 we obtain the minimum value of the above optimization problem, which can be written as

$$d_{s,(M,N_1,M,N_2)}^{\text{FCST}}(r_s) = d_{M,M+N_1+N_2}(r_s) + M(N_1 - M), \quad 0 \leq r_s \leq M. \quad (84)$$

Next, we evaluate the optimal value of the objective function for values of $r_s \geq M$. Note that for any $r_s \geq M$,
\( \nu_i = 0 \ \forall \ i, \) which along with equation (81g) imply that \( \gamma_j \geq 1 \ \forall j. \) Putting this in equation (81) we get

\[
d_{FCSIT}^{M,M_1,M_2}(r_s) = \min_{M} \sum_{j=1}^{M} (M + N_1 + 1 - 2j) \beta_j - M^2, \tag{85a}
\]

constrained by:

\[
\sum_{i=1}^{n} (\alpha - \gamma_i) \leq (r_s - n); \tag{85b}
\]

\[
1 \leq \gamma_1 \leq \cdots \leq \gamma_n. \tag{85c}
\]

To bring the above problem into a form amenable to Lemma 3 we use the following variable transform in the above set of equations: \( \gamma'_i = \gamma_i - 1. \) This results in the following equivalent optimization problem.

\[
d_{Femto}^{n,n,n,n}(r_s) = \min_{n} \sum_{i=1}^{n} (2n + 1 - 2i) \gamma'_i, \tag{86a}
\]

constrained by:

\[
\sum_{i=1}^{n} (\alpha - 1 - \gamma'_i) \leq (r_s - n); \tag{86b}
\]

\[
0 \leq \gamma'_1 \leq \cdots \leq \gamma'_n, \tag{86c}
\]

which in turn by Lemma 3 attains the following optimal value:

\[
d_{Femto}^{n,n,n,n}(r_s) = (\alpha - 1) \left( \frac{r_s - n}{\alpha - 1} \right), \quad 0 \leq \frac{r_s - n}{\alpha - 1} \leq n \tag{87}
\]

\[
= (\alpha - 1) d_{n,n} \left( \frac{r_s - n}{\alpha - 1} \right), \quad n \leq r_s \leq n\alpha. \tag{88}
\]

Finally, combining equations (76) and (87) we obtain the desired result.

**Appendix E**

**Proof of Lemma 5**

Let us denote the event that a target rate tuple \( (r_1 \log(\rho), r_2 \log(\rho)) \) does not belong to \( \mathcal{R}_{IML} \) by \( \mathcal{O}_{IML} \), i.e.,

\[
\mathcal{O}_{IML} = \left\{ \mathcal{H} : (r_1 \log(\rho), r_2 \log(\rho)) \notin \mathcal{R}_{IML} \right\}.
\]

Now, let us denote the maximum among the average probability of errors at both the receivers be denoted by \( \mathcal{P}_c \), then using Bayes’ rule we get

\[
\mathcal{P}_c = \mathcal{P}_{c|\mathcal{O}_{IML}} \mathcal{P}\{\mathcal{O}_{IML}\} + \mathcal{P}_{c|\mathcal{O}_{IML}^c} \mathcal{P}\{\mathcal{O}_{IML}^c\}, \tag{89}
\]

\[
\leq \mathcal{P}\{\mathcal{O}_{IML}\} + \mathcal{P}_{c|\mathcal{O}_{IML}^c}, \tag{90}
\]

where \( \mathcal{P}_{c|\mathcal{E}} \) denote the conditional average probability of error given the event \( \mathcal{E} \). Note that, the above equation holds for any SNR. When the target rate tuple belongs to \( \mathcal{R}_{IML} \), letting the block length be sufficiently large the probability of error given \( \mathcal{O}_{IML}^c \) can be made arbitrarily close to zero. Therefore, letting the block length of the
code goes to infinity at both side of the above equation we obtain

\[ P_e \leq \Pr \{ \{ I_{c_1} \leq r_1 \log(\rho) \} \cup \{ I_{c_2} \leq r_2 \log(\rho) \} \cup \{ I_{c_3} \leq r_3 \log(\rho) \} \}, \quad (91) \]

\[ \leq \sum_{i=1,2,3} \Pr \{ \{ I_{c_i} \leq r_i \log(\rho) \} \}, \quad (92) \]

\[ \overset{(a)}{=} \max_{i=1,2,3} \Pr \{ \{ I_{c_i} \leq r_i \log(\rho) \} \}, \quad (93) \]

\[ \overset{(a)}{=} \max_{i=1,2,3} \rho^{-d_{i,(M_1,N_1,M_2,N_2)}^{\text{FCSIT}}(r_i)} = \rho^{-\min_{i=1,2,3} d_{i,(M_1,N_1,M_2,N_2)}^{\text{FCSIT}}(r_i)} \quad (94) \]

where step (a) follows from the fact that in the asymptotic SNR the largest term dominates and the last step follows from equation (32). Finally, the desired result follows from the fact that \( P_e = \rho^{-d_{(M,N_1,M_2)}^{\text{IML}}(r_1,r_2)} \).

**APPENDIX F**

PROOF OF LEMMA 6

The joint distribution of \((\bar{\beta}, \bar{\upsilon})\) can be obtained from equation (50) substituting \( f_{W_2|W_1}(.) = 1 \). The rest of the proof follows the same steps as the proof of Lemma 4 and hence skipped to avoid repeating.

**APPENDIX G**

PROOF OF THEOREM 6

Since the FCSIT DMT is an upper bound to the No-CSIT DMT, it is sufficient to prove that when the condition of equation (41) is satisfied the expressions of Theorem 2 and Lemma 7 are identical.

It is clear from the comparison of equations (29) and (40) that for \( r_s = 2r \geq n \) they are identical for all values of \( \alpha \geq 1 \) and hence when \( \alpha \) satisfies equation (41). Now, it is only necessary to find a condition when the expressions in equations (29) and (40) are identical even when \( r_s < n \), which is what we derive next and turns out to be identical to the condition of equation (41).

It is clear from equation (40) and (29) that,

\[ d_{n,n}^{\text{FCSIT}}(r_s) < d_{n,n,n,n}^{\text{FCSIT}}(r_s), \quad \text{for } r_s \leq n. \]

Therefore, the DMTs given by Theorem 2 and Lemma 7 are identical only if for \( r_s \leq n \), the single user performance is dominating, i.e.,

\[ d_{n,n}(r) \leq \min \{ d_{n,n,n,n}(2r) \}; \]

\[ d_{n,n}(r) \leq d_{n,2n}(2r) + n^2(\alpha - 1), \quad (95) \]

where in the last step we have substituted the value of \( d_{n,n,n,n}^{\text{FCSIT}}(2r) \) from equation (40). Since \( d_{n,n,n,n}^{\text{FCSIT}}(2r) \) decays much faster than \( d_{n,n}(r) \) with increasing \( r \) and both are continuous functions of \( r \), equation (95) will be valid for all \( r \leq \frac{n}{2} \) if it is true for \( r = \frac{n}{2} \). Substituting this in equation (95) we get

\[ d_{n,n}(\frac{n}{2}) \leq n^2(\alpha - 1), \]

or \( \alpha \geq 1 + \frac{d_{n,n}(\frac{n}{2})}{n^2} \).

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