Model of unreliable devices system

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Abstract. The purpose of the research is to develop a model of a production system consisting of \( n \) parallel devices and having a standby device of lower reliability. Under complex failure conditions, a spare device in a cold standby allows quickly replacing the failed primary unit. The system operation is investigated from the beginning of work up to a certain point in time \( t_W \). A good approximation to reality is the acceptance that failure time follows general time distribution. The authored approach consists in drawing up an open scheme for the functioning system. Due to the complex failure of one of the devices, the process passes to the next state, which is characterized by residual failure time or residual time of the process. The calculation of residual durations is a feature of the investigated model, which, along with detailed schematization, makes it possible to calculate the set of state indicators and, on their basis, obtain the more general indicators of the functioning of investigated system. A numerical example is given for a system consisting of five devices.

1. Introduction

The authors consider the production system consisting of several parallel one-type unreliable devices. Failure of the device can lead to a decrease in efficiency and income to a deterioration of the company's reputation in the competitive market. A common practice is the use of spare parts that replace the failed elements. In some cases, the failed device is replaced entirely. Containment in the stock of the new device in general can be uneconomic, but it is possible to use the restored devices, the resource of which makes up some fraction of the new device resource. This reduces the need for spare parts and the time for replacement, since the installation of the device as a whole requires less time than for disassembly, replacement of the failed element and assembly of devices.

In the literature, extensive studies have been carried out on various types of redundancy with one or two units because of their frequent use in modern systems, for example [1–3]. According to the accepted classification, the restored device will be in the cold standby mode.

In the vast majority of papers have considered the Markov model of reliability, which has been developed from the point of view of the best descriptions of real situations. Thus, in [4, 5] the model takes into account the incomplete control of the incoming devices operability and the delay time in switching on the ‘cold’ standby device; \( m \) of the same type is considered in [6]. In [7], a closed solution for the reliability measures of a two-configuration system, 2-out-of-4 and 2-out-of-5, was obtained. A system with heterogeneous devices was studied in the work [8].
However, it is known that technical devices are aging systems and the use of the Markov model leads to a significant underestimation of reliability. Therefore, researchers used bivariate exponential distribution [9], Erlang distribution [10, 11] as the distributions of the time to failure.

Semi-Markov processes are most often used to analyze the systems reliability with an arbitrary distribution of failure time [12–14], which lead to the necessity of compiling and solving systems of integro-differential equations.

In the case of other reliability models of technical systems with arbitrary distribution failure time, serious difficulties arise both in the description of the process and in the solution of differential equations in particular derivatives, integro-differential and integral, for example [15–18]. Therefore, nowadays, simulation modeling is used to estimate reliability function (probability of failure) [19, 20]. However, the methods of statistical modeling can not be formalized, which does not allow creating a unified methodology for constructing such algorithms. A significant drawback is the poor visibility of the model and the consequent difficulty in finding optimal solutions.

Note that in the overwhelming number of studies, only steady-state characteristics of cold standby systems were obtained and only a number of papers analyze systems with more than two units [7, 10, 16].

In [21] a method is presented for describing random processes that are discrete in composition and continuous in time. We apply it to the analysis of the production system, consisting of five parallel unreliable devices with a cold standby unit.

2. Method Description and Model Assumptions

The authors consider the production system composed of five identical devices operating in parallel during a certain time of interest \( t_W \). In the standby, there is one device, the reliability of which is less than the reliability of the primary operating devices. The standby device replaces the first device, which has reached a complex failure, requiring significant time for repairs. The authors assume that the time \( t_W \) is less than the repair time due to the high complexity of the process to eliminate failure.

The initial data for the calculation are the distributions to the complex failure of primary and standby devices with arbitrary distribution functions \( E(t) \) and \( E_S(t) \) respectively, as well as the duration of the period of interest \( t_W \). To simplify the consideration of the problem, we assume that the distributions of primary devices are subject to the same distribution.

The scheme represents the states and transitions between them. The states are represented by the composition of device working to failure and the duration of time \( t_W \). Each state is numbered. The state ends up with the failure of one of the devices or the end of time \( t_W \). In this case, the general process enters another state. The arrow in brackets is written with the notation of the duration, the completion of which in the previous adjacent state determined the state under consideration. For groups of identical times to failure, states and transitions are combined. Then the beginning of the diagram will take the form shown in Figure 1. In the scheme, the second factor in the product indicates the number of failure time, the first factor is the number of identical operating times; index (number of state), indicates that the duration is residual from the previous state.

![Figure 1](image-url)

**Figure 1.** Scheme of the system operation process, consisting of five primaries and one standby device.
The initial state 0 is represented by operating times of five devices. The devices are of the same type and are indicated in the diagram by 5·1. The letter W. defines the time tW. If during this time there is no complex failure of the machine, the system goes to the state 0* and the standby device is not used. Both during the time tW, a complex failure of one of the devices occurs, and the system goes to state 1. In this case, instead of five transitions, one is indicated on the diagram, and their possible number is ‘five’ to the left of the arrow.

In state 1, four primary devices operate for the residual time tW1 (the diagram indicates as W1). The devices are characterized by the residual time to complex failure T1 (given as 1). The standby device is switched on; the letter S. Accordingly indicates its operating time, there are three options for the process development. The first option is the end of the residual time tW1. At the same time, four primary and standby devices remain operational. In the scheme, this event is indicated by a transition to the state 1*.

The second option is a complex failure of any of the four primary devices. The system goes to state 2, characterized by the residual time T2 of the work of three unrevealed primary devices. The diagram indicates it as 3·1.2, where index 2 indicates the residual duration in the state 2. In addition, the state 2 is characterized by the residual failure time of the standby device (S2) and the residual time duration tW2.

The third option is a complex failure of the standby device; while the system goes to state 3, which is characterized by residual work up to failure of the four primary devices and the residual time tW3.

It follows from the scheme that in states 2 and 3 four devices operate, in states 4, 5, 6—three devices, i.e. the overall performance of the work decreases over time. That is why the process of system functioning was not considered further.

Durations V0,r and transition probabilities s0,r required to calculate the failure time of a system consisting of five elements, four elements, etc., taking into account the inclusion of a standby element.

In the adopted model, in particular, the distribution of the system operation time of just five devices is of interest. According to the scheme, this will be the time from the initial to the states 0*, 1*, 2, 3. To calculate the distribution of random durations V0,1*, V0,2, V0,3, auxiliary schemes are compiled. In the diagram for calculating the duration, for example, V0,3 segments with crosses indicate the operating time of the failed devices, segments with a line indicate the duration that did not end; the numbers 1—5 indicate the device numbers, S is the standby device, W is the system operation time (figure 2).

Figure 2. Auxiliary scheme for calculation of the duration V0,3.

The distribution function of operating time V0,3 is calculated by the formula

\[ S_{0,3}(t) = P(V_{0,3} \leq t) = P\left(T + T_S < t | T + T_S < \min(4T, T_W)\right). \]

Details of the preparation of auxiliary schemes and the calculation of them are set out in [22].

The distribution of the failure time of the five devices is calculated as

\[ F_S(t) = s_{0,0}S_{0,0}(t) + s_{0,2}S_{0,2}(t) + s_{0,3}S_{0,3}(t) + s_{0,1}S_{0,1}(t). \]

Distribution of failure time of at least four devices

\[ F_4(t) = \sum_{i=4}^{6}s_{0,i}S_{0,i}(t) + s_{0,2}S_{0,2}(t) + s_{0,3}S_{0,3}(t). \]

The sequence of transitions s0,i is determined by the scheme, for example, s0,3 = s0,1·s1,3.

To calculate the transition probabilities from the state k, it is necessary to know the residual durations representing this state. To calculate the residual time to the failure of the element j in the state r, Tj,r, in which the process passed from the state k due to the failure of the element i, from the set of elementary...
processes $M^{(k)}$ we single out two elementary processes $i$ and $j$ and form a subset of particular processes $U_{j-i} = M^{(k)} - i - j$. The informative relation for time $T_{j-i}$ can be represented in the form

$$T_{j-i} = \{T_j - T_{i<\text{ca}} \} | T_j > T_{i<\text{ca}}, \text{ where } T_{i<\text{ca}} = T_i < U_{j-i}. $$

For example, consider the state 1 in which the process has passed because of the failure of the primary device, and determine the residual working time. Then $j = w$, $r = 1$, $k = 0$, $i$ defines the primary unit. The informative relation for $T_{W:1}$

$$T_{W:1} = [T_W - T < \min(4T)] [T_W > T < \min(4T)].$$

Calculation of the residual durations to failure is discussed in detail in [21].

3. **Numerical analysis and discussion**

Calculations for $t_W=150$, $E(t) = 1 - \exp(-1.8 \cdot 10^{-10} t^4)$ with the mean time to the complex failure $\bar{t} = 246.5$ were conducted.

With the chosen $t_W$ and $E(t)$ it is received that:

- The mean time $\bar{t}_1$ of five devices system to the first complex failure of one of them is 117 units of time; therefore, the mean time for which a standby device is required will be $150 - 117 = 33$.
- With the probability $s_{0,0} = 0.64$ all five devices will work through the period under review without complex failures and no standby device will be required. Accordingly, with the probability $s_{0,1} = 0.36$ one of the primary devices will fail and a standby machine will be put into operation. Other probability states will depend on the distribution parameters $E_5(t)$.

We calculate the state parameters for the three distributions of time to failure of the standby device

$$E_{5,1}(t) = 1 - \exp(-1.3 \cdot 10^{-4} t^2), \quad E_{5,2}(t) = 1 - \exp(-4.3 \cdot 10^{-2} t), \quad E_{5,3}(t) = 1 - \exp(-4.1 \cdot 10^{-9} t^4).$$

The results of the calculations are summarized in Table 1.

| Indicators | $E_{5,1}(t)$, $\bar{t}_5 = 79$, | $E_{5,2}(t)$, $\bar{t}_5 = 119$, | $E_{5,3}(t)$, $\bar{t}_5 = 148$, |
|------------|---------------------------------|---------------------------------|---------------------------------|
| $s_{0,1}$  | 0.2397                          | 0.2662                          | 0.2717                          |
| $s_{0,2}$  | 0.0714                          | 0.0097                          | 0.0022                          |
| $s_{0,3}$  | 0.0455                          | 0.0097                          | 0.0022                          |
| $\bar{t}_5$| 147.6                           | 148.4                           | 148.5                           |
| $\bar{t}_4$| 149.6                           | 149.7                           | 149.8                           |

Consider the states in which five devices are operational, i.e. states $0^*$, $1^*$, $2$ and $3$. The meaning of the probability $s_{0,0}$ is described above. With a probability $s_{0,1}$ four primary devices and a standby device will operate without complex failures during the time $t_W$. With probability $s_{0,2}$ the second complex system failure will occur; with probability $s_{0,3}$ the standby device will fail; the average operating time will be $\bar{t}_3$.

In [1, 2] it is recommended that devices in the standby be selected in such a way that the mean failure time of the standby device is approximately equal to the mean time to the first failure of a group of working devices, $\bar{t}_S \approx \bar{t}_1$. Indeed, in this case the probability of failure of the standby device is extremely small ($s_{0,3} = 0.0097$). However, the values $s_{0,3}$ with lower values $\bar{t}_S$ is also small. Even with $\bar{t}_S \approx 0.5 \bar{t}_1$ the probability of failure of the standby device is 0.0455. The average operating time $\bar{t}_3$ of five devices is 147.6. If in the residual time (150 – 147.6) the complex failure of the first device is not eliminated, then four primary devices will operate no more than two units of time. Further, either the work efficiency
will decrease, or spare parts should be used. Fully using another standby device is inefficient. Thus, it is possible to use rather unreliable devices in cold standby.

Consider devices with mean time to failure of each \( t = 200 \). Then the mean time \( t_1 \) of five devices system to the first complex failure of one of them will be 100 units of time, the probability of this event— 0.78. In the absence of a standby device, four devices would have to work on average for 50 units of time. The results of the calculations are summarized in Table 2.

### Table 2. Indicators of system states with mean time to complex failure \( \bar{t} = 200 \)

| Indicators | \( E_{S,1}(t), \bar{t}_S = 79 \) | \( E_{S,2}(t), \bar{t}_S = 119 \) | \( E_{S,3}(t), \bar{t}_S = 148 \) |
|------------|---------------------------------|---------------------------------|---------------------------------|
| \( s_{0.2} \) | 0.3814                          | 0.4275                          | 0.4371                          |
| \( s_{0.3} \) | 0.1119                          | 0.0247                          | 0.0057                          |
| \( \bar{r}_5 \) | 134.9                           | 137.3                           | 137.8                           |
| \( \bar{r}_4 \) | 144.6                           | 146.0                           | 146.3                           |

While the mean time between failures \( \bar{t} \) of one device for the first and second cases differ not so significantly, 246 and 200, in the second case the probability \( s_{0.2} \) of the second complex failure of the primary device (for example, 0.38 versus 0.07 for distribution with \( \bar{t}_S = 79 \)), which may indicate the need for a second standby machine.

The transition probability to the next state due to the failure of the standby device is also not great. For the case \( \bar{t}_S = 0.5t \), the average remaining time when four machines without standby will work is 150–144.6=5.4, but the probability of such an event is \( s_{0.3}=0.11 \). For the other considered cases this probability is not great (less than 0.025).

### 4. Conclusion

Studies have shown the profitability of cold standby with reliability, lower reliability of the primary devices.

Using the method allows to study the process of the system operation in detail, in particular, to calculate the probabilities of each state, the time from the beginning of work to the end of any state, the duration of the work of the initial number of devices.

The development of research will be production system of model development with non-identical devices and with several standby devices.

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