D-brane annihilation, renormalization-group flow and non-linear $\sigma$-model for the ADHM construction

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Abstract

In this note $D9$- and anti-$D9$-brane annihilation in type I string theory is probed by a $D1$-brane. We consider the covariant Green-Schwarz or twistor formulation of the probe theory. We expect the theory to be $\kappa$-invariant after the annihilation is completed. Conditions of the $\kappa$-invariance of the theory impose constraints on the background tachyon field. Solutions to the constraints define tachyon values which correspond to type I $D5$-branes as remnants of the annihilation. As a byproduct we get a theory which lies in the same universality class as the non-linear $\sigma$-model for the Atiyah-Drinfeld-Hitchin-Manin construction.

1 Introduction

Recently a powerful apparatus to work with BPS excitations in superstring theory has been developed (see [1, 2] for some reviews). Despite this progress, we still have a poor understanding of the dynamics when SUSY is violated. In fact, even in the simplest situation of $D$-anti-$D$-brane ($D - \bar{D}$) systems we know only the details of their topological content rather than the dynamics of the annihilation process [3, 4, 5].

First, it is argued that the annihilation process of a $D - \bar{D}$-system is related to the tachyon rolling down to the bottom of its energy functional [3]. The tachyon field here describes the lowest energy excitations of strings stretched between the $D$- and $\bar{D}$-branes.

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This excitation has an imaginary mass which signals an instability in the $D - \bar{D}$-system and leads to the annihilation. Second, knowing which charges are excited, one could trace what kinds of branes should be remnants of the annihilation [3, 4, 5]. It is not clear, though, how to determine their position in their moduli space after the annihilation has happened. Moreover, it is not known how the tachyon rolls down to the bottom of its energy functional during the annihilation process. The main obstacle is the lack of knowledge of the tachyon potential [6].

Rather than looking for the tachyon potential in this note we would like to implement another approach. We know that in many occasions $D$-branes supply a good microscopic description of various low energy physics phenomena. In particular, the closest example to our approach is presented in ref. [7]. In this paper the $D$-brane description of instantons in SUSY Yang-Mills (SYM) theories is given. Concretely, the gauge connection corresponding to the YM instanton is recovered from a microscopic theory which describes a $D1$-brane in the $D5$-brane background in type I string theory. What is most important for us is that the $D1$-brane theory in question is absolutely restricted by its $(4,0)$ SUSY invariance [8].

Inspired by these considerations, we would like to probe a $D9-\bar{D}9$-brane annihilation by a $D1$-brane in type I string theory. Rather than dealing with the light-cone action for the $D1$-brane [1] we consider its covariant Green-Schwarz (GS) formulation [9]. For the theory to be non-anomalous it is necessary to consider the number of $D9$-branes to be the number of $\bar{D}9$-branes plus 32 [10, 11, 12].

The annihilation process is viewed on the $D1$-brane as a renormalization-group (RG) flow [13]. In fact, after the annihilation some of the strings stretched between the $D1$-brane and the $D9 - \bar{D}9$-system should become massive and decouple from the IR limit of the probe theory. In the limit the theory describes the $D1$-brane in the background of only 32 $D9$-branes with some gauge bundle on the latter.

Here we study the theory on the probe brane which is already a low energy intermediate step in the RG evolution. It is an approximation to an as yet unknown microscopic theory which contains both $D$- and $\bar{D}$-branes. Thus, we do not expect to recover from our probe theory explicitly the way the tachyon rolls down to the bottom of its energy functional. However, we could hope to extract from it some information about tachyon classical solutions which respect SUSY, i.e. tachyon values after the annihilation. In particular, one would hope that there is some symmetry which restricts possible background values of the tachyon after the annihilation [14]. In fact, before the annihilation there is some non-SUSY theory which describes the $D1$-brane in the presence of both $D9$- and $\bar{D}9$-branes. Via RG flow the theory evolves to a superconformal limit with a proper background value of the tachyon. We believe that there should be some hidden (non-linearly realized) SUSY of the theory, which forces it to flow in such a rigid way [14].

It is at this point that $\kappa$-invariance comes into the game. In fact, as is well established [15, 16], this symmetry is related to a linearly realized SUSY on the world-sheet: one could formulate the $D1$-brane theory with an explicit SUSY both on the world-sheet and in the target space. Then $\kappa$-symmetry appears from the world-sheet SUSY after the integration over auxiliary fields [17]. Hence, the presence of the $\kappa$-invariance is a sign that there is a linearly realized SUSY on the $D1$-brane world-sheet. In our case, we expect the invariance to be present only after the annihilation is completed.

It is for this reason that we are looking for backgrounds for the $D1$-brane, which respect $\kappa$-invariance. Conditions that invariance imposes on the theory constrain the
possible values of the tachyon field. We find some equations which establish that this field is covariantly constant on light-like surfaces in the target space. The latter should be supplemented by integrability conditions so that if there is a gauge field background turned on the $D9$- and $\bar{D}9$-branes, the tachyon could be a non-trivial field rather than just a constant.

Let us explain why we have equations for the tachyon field which are linear rather than quadratic in differentials. In fact, $\kappa$-invariance of a superstring theory in a background of SYM fields puts the latter on mass-shell, i.e. one gets second order differential (classical) equations (of motion) for the fields $\gamma_\bar{\gamma}^{\bar{\gamma}\gamma}$. Hence, the appearance of the first order differential equation for the tachyon field might seem suspicious. As we already mentioned, however, the $\kappa$-invariance is related to SUSY of the world-sheet theory. Also the mere presence of the tachyon field explicitly violates SUSY in the theory. Thus, we expect SUSY to be linearly realized only for some specific tachyon values. That is the reason we get BPS like linear differential equations for the tachyon field.

Our main interest is in a soliton which is of co-dimension four within the $D9$-branes. Only in such a situation is there a SUSY vacuum in the probe theory \cite{ref}. In this case, the tachyon and background gauge field are functions of only four coordinates rather than ten. Then the integrability condition in question is just the self-duality equation for the gauge field. At large distances its instanton solution is represented as a pure gauge. The gauge matrix in the latter is equal to a non-trivial map (of a degree equal to the instanton charge) from $S^3$ at infinity to the group of Chan-Paton (CP) indexes. Specifically, we take the target of the map in question to be the diagonal $USp(4k)$ subgroup of the $USp(4k) \times USp(4k)$ group.

This choice could be clarified as follows. First, it should be stressed that we are considering a minimal construction of $D5$-branes from a $D9-\bar{D}9$-system. This construction is related to that due to Atiyah, Drinfeld, Hitchin and Manin (ADHM) for instantons in YM theories \cite{ADHM}. In principle, one could study other situations which lead to non-minimal generalizations of the ADHM construction \cite{ref}. For ”minimality” we consider a background gauge field on the $D9-\bar{D}9$-brane system respecting only the $USp(4k) \times USp(4k) \times SO(32)$ subgroup of the largest possible group with the same number of CP indexes. Second, we consider the symplectic groups as factors because we are looking for a minimal construction which leads to $k$ type I $D5$-branes. As is established in ref. \cite{ref, ref} each of the branes should have two CP indexes, taking values in $USp(2) \cong SU(2)$. Hence, $k$ type I $D5$-branes correspond to $USp(2k)$ group \cite{ref, ref}. Third, we consider $4k$ rather than just $2k$ indexes, because we have to embed two of the CP indexes of both $USp(4k)$’s into the tangent bundle of the target space (see \cite{ref} for such a construction).

Now if the tachyon is covariantly constant in the background in question it contains the aforementioned map. Hence, the tachyon field is a rectangular matrix ([4k] $\times$ [32+4k]) whose quadratic part ([4k] $\times$ [4k]) is the map in question. This is exactly the tachyon value we expect to get for $k$ type I $D5$-branes to appear as remnants of the $D9$-brane annihilation \cite{ref, ref}.

In conclusion, we have a $D9-\bar{D}9$-system with some excited Ramond-Ramond (RR) field corresponding to $k$ $D5$-branes before the annihilation. The RR field is encoded in terms of some gauge bundle on the $D9-\bar{D}9$-system \cite{ref, ref}. After the annihilation, the tachyon acquires a value which is covariantly constant on light-like surfaces in the gauge field background. In our case, such a tachyon value is proportional to the ADHM matrix \cite{ref}, which corresponds to the ADHM construction of instantons for the $SO(32)$ group.
Similarly as in ref. [8, 4], this matrix defines a mass term for the fermionic fields on the probe $D1$-brane.

In this way, we obtain the probe theory which, on the level of massless modes, coincides with the non-linear $\sigma$-model for the ADHM construction [8, 14]. In other words, it flows in the IR limit to the same superconformal theory which describes the $D5$-brane background for the type I $D1$-brane theory as an instanton field of the $SO(32)$ group [22]. This time it is the latter field which encodes the information about corresponding RR charge. The former gauge field from the diagonal subgroup of $USp(4k) \times USp(4k)$, being a pure gauge at low energies (due to the non-zero tachyon vacuum expectation value (VEV)), decouples after the IR limit is taken.

Thus, without knowing the tachyon potential we could fix tachyon values after the annihilation. This is our main result.

## 2 Twistor formulation of the probe theory and $\kappa$-invariance

We consider a phase of type I string theory containing $32 + 4k$ $D9$-branes and $4k$ $\bar{D}9$-branes. The D-brane world-volumes fill the entire ten-dimensional space-time. We probe the annihilation of the $D9 - \bar{D}9$-system by a $D1$-brane.

In the GS or twistor formalism the probe theory contains the following fields at low energies. First, there are low-energy modes of strings attached by both their ends to the $D1$-brane. The modes are ten bosons $x_M$, $(M = 0, \ldots, 9)$ and ten-dimensional Majorana-Weyl fermions $\psi_A$, $(A = 1, \ldots, 16)$ [3, 23]. Second, there are low-energy modes of strings stretched between the $D1$- and $D9$-branes. These modes are two-dimensional Majorana-Weyl fermions $\lambda$ [25]. Third, there are also modes of strings stretched between the $D1$- and $\bar{D}9$-branes. Correspondingly these modes are two-dimensional Majorana-Weyl fermions $\chi$ of opposite to $\lambda$ chirality [3, 4]. If it were not for the presence of $\chi$, the $D1$-brane would have the same quantum numbers as the Heterotic $SO(32)$ string [23].

We are going to work with the twistor formulation of the theory [13, 16]:

$$
S = \int d^2 \sigma \left\{ P_M^{-} \left[ e_{++} \left( \partial^{a} x^{M} - \partial^{a} \psi^{A} \Gamma^{M}_{AB} \psi^{B} \right) - \varphi_{-} \Gamma^{M} \varphi_{-} \right] + \\
+ \text{Wess – Zumino term} + \\
+ \lambda^{\hat{A}} \left[ e_{--} \left( \delta^{pq} \partial_{a} - \partial_{a} x^{M} A_{M}^{pq}(x) \right) + \frac{1}{4} F_{ML}^{pq}(x) \Gamma^{ML} \psi^{A} \psi^{B} \right] \lambda^{\hat{A}} + \\
+ \lambda^{\hat{B}} \left[ e_{++} \left( \delta^{pq} \partial_{a} - \partial_{a} x^{M} D_{M}^{pq}(x) \right) + \frac{1}{4} H_{ML}^{pq}(x) \Gamma^{ML} \psi^{A} \psi^{B} \right] \lambda^{\hat{B}} + \frac{1}{2} \bar{\psi} \Gamma^{MN} \psi \right\}.
$$

(1)

Here $P_{M}^{a}$ and $\varphi_{-}$ are auxiliary fields. Their exact definition is not relevant for our further discussion and can be found in [15]. These auxiliary fields should obey a Cartan-Penrose condition: $P_{M}^{a} = e^{-1} \varphi_{-} \Gamma^{M} \varphi_{-}$. Now it is easy to see how after the integration over $P_{M}^{a}$ one recovers the standard GS formulation of the Heterotic string if $\chi$ is absent [15].

Also in this formula $e^{a}$, $a = 1, 2$ is a zweibein; $p = 1, \ldots, 32 + 4k$ and $\bar{p} = 1, \ldots, 4k$; $\Gamma_{M}$ ($\Gamma^{ML} = \Gamma^{M}, \Gamma^{L}$) are ten-dimensional $\gamma$-matrices in the Majorana-Weyl representation;
$T$ is a tachyon field which describes the lowest energy excitations of strings stretched between the $D9$- and $\bar{D}9$-branes [3]. It appears in (1) as an external field and transforms in the bi-fundamental representation under the $USp(4k) \times SO(32)$ and $USp(4k)$ groups. We choose the gauge fields $A_{M}^{pq}$ and $B_{M}^{\bar{p}q}$ on the $D9$- and $\bar{D}9$-branes (with the field strengths $F_{ML}^{pq}$ and $H_{ML}^{pq}$, correspondingly) respecting only this subgroup of the largest possible group with this number of CP indexes. These gauge fields couple to $\lambda$ and $\chi$ in the same way as a gauge field couples to Heterotic fermions [25]. All other fields on the $D9$- and $\bar{D}9$-branes are set to zero.

Hence, the theory we are starting with is a non-SUSY two-dimensional $\sigma$-model. It evolves via the RG flow [3] to a superconformal theory in the IR if a proper background value of $T$ is standing in (1) [14]. In fact, some of the $D9$- and $\bar{D}9$-branes should annihilate leaving only the $D1$-brane in type I string theory, which contains only 32 $D9$-branes and, possibly, some non-trivial bundles on the latter. This $D1$-brane has the quantum numbers of the Heterotic string [25] and its theory is superconformal. Thus, one can be sure that if the theory in question eventually evolved to a superconformal limit for some value of $T$, this value really corresponds to a minimum of the tachyon energy functional.

As we explained in the introduction, the $\kappa$-invariance could help find tachyon classical solutions respecting SUSY. Let us, hence, impose conditions on the invariance of the action (1) under $\kappa$-transformations. In the conformal gauge the transformations look as follows [24, 16, 15]:

$$
\delta P_{M}^{a} = 0, \quad \delta \varphi_{-} = 0 \\
\delta \psi^{A} = 2i P_{M}^{a} \Gamma^{AB}_{M} \kappa_{B++} \\
\delta x^{M} = -i \delta \lambda^{A} \Gamma^{M}_{AB} \psi^{B} \\
\delta \left( A_{M} \partial_{-} x^{M} \right) = \partial_{-} \Lambda_{\kappa} + \left[ \Lambda_{\kappa}, A_{M} \partial_{-} x^{M} \right], \quad \delta \lambda^{p} = (\Lambda_{\kappa} \lambda)^{p}
$$

(2)

where $\Lambda_{\kappa} = \delta x^{M} A_{M}$ if only the background gauge field is non-zero. Also, by analogy with the transformations of $\lambda$ we could choose a natural transformation law [24] for $\chi$ to be:

$$
\delta \left( B_{M} \partial_{++} x^{M} \right) = \partial_{++} \Lambda_{\kappa}' + \left[ \Lambda_{\kappa}', B_{M} \partial_{++} x^{M} \right], \quad \delta \chi^{\bar{p}} = (\Lambda_{\kappa}' \chi)^{\bar{p}},
$$

(3)

and $\Lambda_{\kappa}' = \delta x^{M} B_{M}$. At the same time the tachyon field transforms under the $\kappa$-symmetry simply as follows:

$$
\delta T (x) = \partial_{M} T (x) \cdot \delta x^{M}
$$

(4)

It is necessary to supplement these transformations by Virasoro and SYM constraints [17, 15]. In our case, the latter are equivalent to the classical SYM equations of motion. Moreover, for (1) to be invariant under (2) the tachyon field $T$ should obey the following equation:

$$
\delta x^{AB} \cdot \hat{D}_{AB} T (x) = \\
= \delta x^{AB} \cdot \left\{ \hat{A}_{AB} T^{q\bar{p}} (x) + T^{q\bar{p}} (x) \cdot \hat{B}_{AB}^{q\bar{p}} (x) - \hat{A}_{AB}^{q\bar{p}} (x) \cdot T^{q\bar{p}} (x) \right\} = 0
$$

where $\delta x^{AB} = \hat{P}_{-}^{AC} \cdot \kappa_{C++} \cdot \psi^{B}$ and $\hat{P}_{A}^{AC} = P_{A}^{M} \Gamma_{M}^{AC}$

(5)
for any $\kappa$. Hence, this should be supplemented by integrability conditions:

$$
\begin{align*}
\hat{P} \cdot \kappa_{(1)} \cdot \psi \cdot \hat{D}, & \quad \hat{P} \cdot \kappa_{(2)} \cdot \psi \cdot \hat{D} \\
\delta x^{(1)} \cdot \delta x^{(2)} \cdot [\mathcal{D}^{(1)}, \mathcal{D}^{(2)}] &= 0.
\end{align*}
$$

Note that the vector $\hat{P} \cdot \kappa \cdot \psi$ has zero norm:

$$(\hat{P} \cdot \kappa \cdot \psi)^2 \sim P^2 \epsilon_{AB} \psi^A \psi^B.$$ 

Moreover, under variations of the parameter $\kappa$ (with $P$ and $\psi$ kept fixed) it sweeps an eight-dimensional hyperplane in the ten-dimensional space-time. In fact, as is well known, the matrix $\hat{P} \cdot \kappa$ has eight rather than sixteen non-zero eigen-values \[20\]. Hence, it defines eight real deformations of $\kappa$. Thus, there are eight varying components of the ten-vector in question, while the other two are fixed. This is the eight-dimensional hyperplane. At the same time under variations of $\hat{P}$ and $\psi$ all eight-dimensional hyperplanes are swept.

3 Solutions to the constraints and $D5$-branes as remnants of the annihilation

As follows from (6) when the background $A_M$ and $B_M$ are zero, the tachyon field should be a constant up to a gauge transformation. Unfortunately we can not derive from our formulae what kind of constant it should be. However, as a warm up exercise let us try to guess it \[14\]. Consider:

$$T_{[4k+32] \times [4k]} \sim \left(D_{[4k] \times [4k]} \oplus \mathbf{0}_{[32] \times [4k]}\right),$$

where $D$ is a diagonal matrix with all eigen-values of the order of the string scale. This tachyon value respects both $SO(32)$ and the diagonal $USp(4k)$ subgroups of the $SO(32) \times USp(4k) \times USp(4k)$ group. (See \[27\], \[28\] for discussion on this subject.) Gauge invariant expression for the tachyon VEV should be:

$$T^p \cdot T^q = \delta^{pq}.$$ 

Presumably the latter expression is related to the minimum of the tachyon potential: $\partial_T V(T)|_{T^2=1} = 0$, but its origin is not really important to us.

What is most important is that the formula (6) passes through the simplest check. In fact, consider the lowest energy excitations in the NS sector of the strings stretched between the $D1$-brane and the $D9-\bar{D}9$-system \[1\]. We denote these excitations as $Q^p$ and $\bar{Q}^p$, respectively. Their bare masses are equal to $\frac{1}{2}$ in string units \[ \square \]. Also their interactions with the tachyon field are $T^{pq} \cdot T^q \times Q_p \cdot Q_q$ and $T^{pq} \cdot T^q \times \bar{Q}_p \cdot \bar{Q}_q$. Hence, when the tachyon acquires the VEV as in (6),(8) we have the proper number of the NS modes with mass $\frac{1}{2}$ to describe the $D1$-brane in the background of $32$ $D9$-branes only. We are going to use eq. (8) later.

With the tachyon as in the eq. (7), the $\chi^\theta$ and $\lambda^\theta$ from (1) become massive, while $\lambda^n$, $n = 1, \ldots, 32$ are left massless: here $\lambda^p = (\lambda^\theta, \lambda^n)$. Because of IR effects in two dimensions, if such fields acquire masses there is no way for them to become massless. Hence, in the study of the RG evolution of the theory we can safely integrate these massive fields out, while leaving massless ones untouched. After the integration, we get the ordinary type
I D1-brane theory. This theory describes the background of only 32 D9-branes and is superconformal \(^2\). Thus, (3) is a proper VEV for the tachyon in this case. We just guessed the VEV in question but in the situation below we derive it.

Now let us study the case when there is a co-dimension four soliton left within the D9-branes after the annihilation. This corresponds to a solution of eq. (3) when \(A_M\), \(B_M\) and \(T\) are functions of four coordinates. Say the latter are \(x_6, \ldots, x_9\). In this case we expect a linear realization of SUSY in the IR limit\(^3\).

In the presence of the soliton in question, the ten-dimensional Lorentz invariance is broken: \(SO(9,1) \to SO(1,1) \times SO(4) \times SO(4)\). Here \(SO(1,1)\) corresponds to rotations along the D1-brane (directions \([0,1]\); one of the \(SO(4)\)'s is related to rotations along the soliton, but transverse to the D1-brane (directions \(2,\ldots,5\)), while another \(SO(4)\) corresponds to rotations in the directions transversal to the soliton \((6,\ldots,9)\).

Below we denote by \(\pm\) the left and right chirality under the \(SO(1,1)\) group. At the same time by \(\alpha\) and \(\dot{\alpha}\) we denote the indexes of the fundamental representation of the \(SU(2)_L\) and \(SU(2)_R\) subgroups of the aforementioned “transversal” \(SO(4)\) group. Besides that we embed two among the CP indexes \(p\) and \(\bar{p}\) into the tangent bundle of the target space. Thus, \(p = (\alpha, i, n)\) and \(\bar{p} = (\dot{\alpha}, i)\), where \(n = 1,\ldots,32\) and \(i = 1,\ldots,2k\): hence, both \(USp(4k)\)'s are broken to \(SU(2) \times USp(2k)\). In other words, the fermions on the D1-brane carry the following indexes \(\chi^\beta = \chi^{\dot{\alpha}i}\) and \(\lambda^p = (\lambda^{\alpha i}, \lambda^n)\).

If \(A_M, B_M\) and \(T\) are functions of the four coordinates only, then (3) and (8) take the form:

\[
d^{\alpha\dot{\alpha}}_{(1),(2)} D_{\alpha\dot{\alpha}} T(x) = 0 \quad \text{and} \quad d^{\alpha\dot{\alpha}}_{(1)(2)} q^{\dot{\beta}\beta} F_{\alpha\dot{\alpha}\beta\dot{\beta}}(x) = 0,
\]

where \(F_{\mu\nu} = [D_\mu, D_\nu], \quad F_{\alpha\dot{\alpha}\beta\dot{\beta}} = F_{\mu\nu} \cdot \tau^\mu_{\alpha\dot{\alpha}} \cdot \tau^\nu_{\beta\dot{\beta}}, \quad \mu = 6,\ldots,9\). Also here \(d^{(1),(2)}\) are constant (independent of \(x_\mu\)) vectors with zero norm in the complexified four-dimensional Euclidean space. They correspond to \(\hat{P} \cdot \kappa \cdot \psi\) with two different \(\kappa\)'s.

We consider complexification of the Euclidean space (use complex \(x_{\alpha\dot{\alpha}}\) coordinates rather than real \(x_\mu\)) and complexify the gauge group to show that there are non-trivial solutions to the eq. (9). What is most important, if \(\hat{P}\) and \(\psi\) are fixed, the vector \(d_{\alpha\dot{\alpha}}\) sweeps a complex two-plane (\(\beta\)-plane in the notation of ref. \(^2\)) in the complex four-dimensional space: In the case under study \(\hat{P} \cdot \kappa\) describes two complex deformations. At the same time, under variations of \(\hat{P}\) and \(\psi\) the vector \(d_{\alpha\dot{\alpha}}\) sweeps all light-like surfaces in the space.

Before going further we would like to remind that we are looking for a minimal construction of the D5-branes out of the D9–D9-system. As we mentioned in the introduction, to construct \(k\) type I D5-branes as a result of a D9-brane annihilation we need \(4k + 32\) D9-branes and \(4k\) \(\bar{D}9\)-branes. Besides that we take \(A^\mu_{\bar{m}}\) to be a pure gauge. Then, the second equation in (4) is equivalent \(^2\) to the self-duality condition for the YM connection \(\widetilde{A}^\mu_{\bar{m}} = A^\mu_{\bar{m}} - B^\mu_{\bar{m}}\) from the diagonal subgroup of \(USp(4k) \times USp(4k)\).

To find IR limit of the theory (4) we need to know the large distance behavior of the instanton solution to eq. (3). With the charge \(2k\) the solution behaves at infinity as follows:

\(^2\)We suppose, but can not prove, that in all other situations eq. (3) and (8) have trivial solutions.
\(^3\)From now on we fix the light-cone gauge.
\[ \tilde{A}^\alpha_{ij} \sim \left( \partial_\mu \hat{S} \right) \hat{S}^{-1} \bigg|_{\alpha\beta} \] where \( S^{ij}_{\alpha\alpha} = \delta^{ij} \cdot \frac{\hat{x}_{\alpha\alpha} - \hat{x}_{\alpha\alpha}^{(i)}}{|x - x^{(i)}|} \), \( |\hat{x}| \to \infty \). (10)

Here \( \hat{x}_{\alpha\alpha} = x_\mu \tau^\mu_{\alpha\alpha} \) and \( x^{(i)} \) are positions of the 2k instantons. This is not the most general behavior at infinity but we use it to clarify our idea.

The gauge field (10) defines a map \( \hat{S} \) of the order 2k from \( S^3 \cong USp(2) \cong SU(2) \) at space infinity to the diagonal subgroup of \( USp(4k) \times USp(4k) \). In fact:

\[ \left( \partial_\mu \hat{S} \right) \hat{S}^{-1} \bigg|_{\alpha\beta} = 2\sigma^e_{\alpha\beta} \cdot \eta^{e\nu} \cdot \delta^{ij} \cdot \frac{x - x^{(i)}}{|x - x^{(i)}|^2}, \quad e = 1, 2, 3, \] (11)

where \( \eta^{e\nu} \) are t'Hooft symbols and \( \sigma^e \) are generators of the \( SU(2) \) group. This is just a singular instanton of the \( USp(4k) \) group with the charge 2k.

Plugging (11) into (12), we find that the tachyon field behaves at large distances as:

\[ T^j_{p\alpha} \sim \left\{ \frac{\hat{x}_{\alpha\alpha} - \hat{x}_{\alpha\alpha}^{(i)}}{|x - x^{(i)}|} \right\}, \quad |\hat{x}| \to \infty. \] (12)

This is the tachyon value found in [34]. It describes \( k \) singular \( D5 \)-branes within type I string theory after the annihilation.

Now let us consider the more general situation, i.e. deform the gauge field (10) to:

\[ \tilde{A}^\alpha_{ij} \sim \left( \partial_\mu \hat{S} \right) \hat{S}^{-1} \bigg|_{\alpha\beta} \] where \( S^{ij}_{\alpha\alpha} = \Delta^i (x) \cdot \hat{x}_{\alpha\alpha} \delta^{ij} - \hat{x}_{\alpha\alpha}^{(i)} \), \( |\hat{x}| \to \infty \)

and \( (\Delta^{-2})^{ij} = \left\{ x \hat{\delta}^{il} - X^{il} \right\}_\mu \left( x \hat{\delta}^{lj} - X^{lj} \right)_\nu - [X_\mu, X_\nu]^{ij} \right\} \tau^\mu \tau^\nu. \] (13)

Here \( \hat{S} \) defines a most general (up to gauge transformations) map of the order 2k from \( S^3 \) at spatial infinity to the diagonal subgroup of \( USp(4k) \times USp(4k) \). In this formula \( X^{ij}_{\alpha\alpha} \) is an arbitrary symplectic matrix from the diagonal subgroup of \( USp(2k) \times USp(2k) \), which obeys the reality condition \( \hat{X}^{\alpha\alpha} = e^{\alpha\beta} e^{\hat{\alpha}\hat{\beta}} \hat{X}^{\hat{\beta}} \).

With this value of the gauge field substituted into eq. (9), we find that the tachyon behaves at infinity as (14);

\[ T^j_{p\alpha} \Delta^i \left\{ \hat{x}_{\alpha\alpha} \cdot \delta^{ij} - X^{lj}_{\alpha\alpha} \right\} \oplus h^l_{\alpha} \right\}, \quad \text{where} \quad h^{\alpha\beta} = e^{\alpha\beta} e^{\hat{\alpha}\hat{\beta}} \left( h^{|n}_{\hat{\beta}} \right)^*, \quad |\hat{x}| \to \infty. \] (14)

It is a solution to eq. (8) up to the gauge transformation by the matrix \( \Delta^{ij} \) from the diagonal subgroup of \( USp(2k) \times USp(2k) \). Here \( e^{ij} \) is \( USp(2k) \) invariant tensor served to raise and lower \( i \) indexes.

So far \( h^{|n}_{\alpha} \) in eq. (14) is an arbitrary matrix, i.e. not fixed by the eq. (8). However, taking into account eq. (8), or \( T^j_{p\alpha} \cdot T^{p\beta} \sim \delta^{ij} \cdot \delta^{\hat{\beta}} \) in our case, the matrices \( X \) and \( h \) should obey the ADHM condition (18):
\[
(e^{\alpha\beta} \dot{X}_{\dot{\alpha}\dot{\beta}} \dot{X}_{\dot{\alpha}\dot{\beta}})^{ij} + h^m_{\dot{\alpha}} h^{nj}_{\dot{\beta}} = 0,
\]
(15)

with such a gauge choice as in eq. (14). Note that when \( h = 0 \) we recover the situation of the singular \( D5 \)-brane (12).

Let us now check whether or not we have found a proper tachyon value which minimizes its energy functional. Specifically we are going to check whether or not theory (1) flows to a superconformal limit in the IR with such a tachyon value.

Substituting the value (14) for the tachyon field into the action (1) we get:

\[
L = L_{\text{kin}}(x, \psi, \chi, \lambda) + \chi^{\alpha j} \cdot \Delta^{ij} \cdot \left( \dot{X}_{\dot{\alpha}\dot{\alpha}} \cdot \delta^{\dot{\alpha} \dot{\alpha}} - \dot{X}_{\dot{\alpha}\dot{\alpha}} \right) \lambda_{\dot{\alpha}+} + h^{in}_{\dot{\alpha}} \chi^{\alpha n}.
\]
(16)

Here we showed spinor indexes to present a close similarity of our theory to that considered in ref. [7]. To understand how the theory (16) evolves under the RG flow, one must find massless fields among \( \lambda \) and \( \chi \). For this purpose it is necessary to look for a complete set of solutions to the equation [8, 7]:

\[
T^j_{\alpha \rho}(x) v^{\rho n}(x) = 0,
\]
(17)

for a general \( X^{ij} \) and \( h^{in}_{\dot{\alpha}} \) obeying (14). Once we have found all 32 solutions to these equations, it is possible to decompose \( \lambda_{\dot{\alpha}+} \) in their basis:

\[
\lambda_{\dot{\alpha}+}^n = \sum_{n=1}^{32} v^{\rho n} \chi_{\dot{\alpha}+}^n.
\]
(18)

Substituting this expression into (16) and integrating out massive modes, we get:

\[
L = L_{\text{kin}}(x, \psi) + \chi^{\alpha j} \cdot \Delta^{ij} \cdot \left( \dot{X}_{\dot{\alpha}\dot{\alpha}} \cdot \delta^{\dot{\alpha} \dot{\alpha}} - \dot{X}_{\dot{\alpha}\dot{\alpha}} \right) \lambda_{\dot{\alpha}+} + \partial_{\dot{\alpha}+} (x) \lambda_{\dot{\alpha}+}.
\]
(19)

where \( \dot{A}_{\dot{\alpha}\dot{\alpha}}^{nm}(x) = (v_{\dot{\alpha}}^n)^{-1} \partial_{\dot{\alpha}+} v_{\dot{\alpha}+} \). Note also that the gauge field (13), being a pure gauge at low energies (due to the non-zero tachyon VEV [8]), does not enter the IR Lagrangian (19). The theory (19) is superconformal [4, 8].

Taking into account (14), (15) and (17) we see that \( \dot{A}_{\dot{\alpha}\dot{\alpha}}^{nm} \) is the self-dual vector-potential corresponding to the ADHM “matrices” \( X \) and \( h \). This vector-potential describes \( k \) type I \( D5 \)-branes in some non-singular (but otherwise generic) point of their moduli space [7]. So, choosing the value of \( T \) as in (14) and (15), we arrive via RG at the superconformal theory (19). It describes the type I \( D1 \)-brane in the background of \( k D5 \)-branes.

Now let us discuss a difference between our theory and that considered in [7]. One immediately sees that the massive spectra of the two theories are different [14]. First, because of the factor \( \Delta^{ij} \), masses of the massive fermions in (16) are different from those of fermions in the theory from ref. [7]. Second, scalars which are present in [7] and correspond (along with \( \chi^{\alpha j} \) and \( \lambda_{\dot{\alpha}+}^n \)) to the strings stretched between \( D1 \) and \( D5 \)-branes, are absent in eq. (16). These scalars are massive at a generic point of the instanton moduli space [7].

It is worth mentioning at this point that one should not expect the theory (16) to properly reproduce all massive modes. In fact, this theory is a low-energy one, because it
depends on classical values of macroscopic fields such as $T$ and $A$ [14]. Hence, the theory does not contain microscopic degrees of freedom.

4 Conclusions and Acknowledgments

Thus, the theories from eq. (16) and from ref. [8, 7], while being different at high energies, flow to the same superconformal field theory in the IR. We believe in the existence of a microscopic theory underlying both of the theories in question [14]. After the integration of one type of its massive modes, the microscopic theory should lead to the theory considered in [7] as an intermediate step of the RG flow. However, the integration of another type of its massive modes leads the microscopic theory to the Lagrangian (16) as an intermediate step of the RG flow.

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