Synchrotron Emission and Self-Absorption in GRB Afterglows

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Abstract. GRBs are the most energetic phenomena in the universe. For this, two types of shocks were suggested for the prompt emission and the jet within the medium environment. In this work we try to treat the radiation of the GRB afterglows by synchrotron emission mechanisms. Moreover, by ignoring the diffusion of the Inverse Compton scattering, the absorption effect like synchrotron self-absorption is also studied and discussed.

1. Introduction
The most important objective of the BeppoSAx mission was to allow a better localization of the gamma-ray bursts with accuracy of the order of $\sim 3' \times 3'$. That is allowed for the first time in 1997 the detection of the X-ray afterglow appearing a few hours after the gamma-ray burst GRB 970228 [1], than associated with the optical afterglow which was observed a few hours later. The X-ray afterglow presents decay in power law.

Some months later, and for the first time they measured the redshift, $z = 0.835$, of the burst GRB 970508 [2] which is provides definitively the cosmological origin of gamma-ray bursts.

Since then, theorists have focused their efforts on the development of theoretical models making it possible to obtain light curves matching with the observations. The strongly favored model which can describe these emissions well with successfully explanations of the majority of features of the GRBs is releasing $10^{51} \sim 10^{54}$ erg in a few seconds, with a variant the light curve is the fireball model [3][4][5][6][7][8][9][10]. The scenario in this model is when the duality of the fireball/blast-wave shocks the surrounding medium and emits radiation in X-ray, optical and radio bands with synchrotron emission are the main radiation mechanism . However, in reality we could not ignore the absorption effect when we talk about a low energy as a radio band . The goal of this paper is to show the importance of this effect when we study the GRB afterglows in low energies.

2. Dynamics and radiation
2.1. Hydrodynamic evolution
Feng model [11] assumes an evolution of the Lorentz factor $\Gamma$ in the fireball model [12] as follows:

$$\frac{d\Gamma}{dm} = \frac{\Gamma^2 - 1}{M_0 + m + U/c^2 + (1 - \varepsilon)\Gamma m}$$ (1)
where \( m \) is the swept-up mass, \( M_0 \) is the ejected initial mass and \( \varepsilon \) is the efficiency of radiation, the internal energy \( U \) in the fireball is approximately by the following expression [13]:

\[
dU = (1 - \varepsilon)dU_{ex} = (1 - \varepsilon)[(\Gamma - 1)dmc^2 + mc^2d\Gamma]
\]

(2)

Here \( U_{ex} \) is the internal energy produced in this expansion and radiation efficiency \( \varepsilon \) is given by [14]:

\[
\varepsilon = \varepsilon_e t_{syn}^{-1} t_{ex}^{-1}
\]

(3)

with \( t_{syn} = 6\pi m_e c/(\sigma_T B_{e,min}^2) \) and \( t_{ex} = R/(\gamma c) \) are the synchrotron cooling time and the co-moving frame expansion time, respectively. \( m_e \) being the electron mass, \( \sigma_T \) is Thomson cross section, \( B_{e,min} \) the magnetic energy density [11] and \( R \) is the radius of the blast-wave determined by

\[
\frac{dR}{dt} = \beta c\gamma(\gamma + \sqrt{\gamma^2 - 1})
\]

(4)

2.2. Synchrotron radiation and self-absorption

The distribution of the accelerated electrons by the external shock behind the blast wave in the absence of the radiation loss is generally assumed to be a power-low function of the electron energy [15]:

\[
N_e(\gamma_e) = \frac{dN_e}{d\gamma_e} = C\gamma_e^{-P}, \gamma_{min} \leq \gamma_e \leq \gamma_{max}
\]

(5)

where \( p \) is the index between 2 and 3, \( \gamma_{max} = a10^7(B'/1G)^{1/2} \) is the maximum Lorentz factor, with \( a \) is a factor taking its values between 1 and 10. Moreover, one can also define the minimum Lorentz factor as [16]:

\[
\gamma_{min} = \varepsilon_e(\Gamma - 1)\frac{m_p(p - 2)}{m_e(p - 1)} + 1
\]

(6)

The synchrotron radiation power at frequency \( \nu' \) from all the accelerated electrons in the comoving frame is given by [17]:

\[
P_\nu = \frac{2\sqrt{3}\epsilon^2 \nu_L}{c} \int_{\gamma_{min}}^{\gamma_{max}} N'_e(\gamma_e)F\left(\frac{\nu'}{\nu_L}\right) d\gamma_e
\]

(7)

where \( F(x) \) is the synchrotron function defined as [18]:

\[
F(x) = \int_x^{\infty} K_{5/3}(x')dx'
\]

(8)

\( K_{5/3} \) the second kind modified Bessel function.

and:

\[
\nu_e = \frac{2}{3}\nu_L\gamma_e^2
\]

(9)

where \( \nu_L \) is the Lamor frequency:

\[
\nu_L = \frac{1}{2\pi}\frac{eB'}{2mc}
\]

(10)

The radiation in the lab frame can be calculated using relativistic transformations as transformation as[17][19]:

\[
\nu = \frac{\nu'}{\gamma(\gamma + \sqrt{\gamma^2 - 1})}
\]

(11)
\[ \nu = \frac{(1 + \beta) \Gamma}{1 + z} \nu' \]  

(11)

\[ d\Omega = \frac{1}{(1 + \beta)^2 \Gamma^2} d\Omega' \]  

(12)

\[ t_{\text{obs}} = (1 + z) t \]  

(13)

The instantaneous intensity giving the curves of light at a frequency \( \nu \) in Jansky \((\text{erg.s}^{-1}.\text{cm}^{-3}.\text{Hz}^{-1})\) is:

\[ (F_\nu)_{\text{OTS}} = \frac{1}{4\pi D_L(z)^2} 4\pi \left( \frac{dP_\nu}{d\Gamma} \right)_{\text{OTS}} \]  

(14)

where \( D_L(z) \) is the luminosity distance calculated in the \( \Lambda \)CDM model with \( \Omega_M = 0.3, \Omega_\Lambda = 0.7 \) and \( H_0 = 71 \text{km.s}^{-1}\text{Mpc}^{-1} \).

At low frequencies synchrotron self-absorption (SSA) plays an important role. Where we can see a modification on the general shape of the light curve, the number of the cutoff and their values. So to estimate the self absorption frequency one requests the optical depth alongside the line of spectacle. A simple approximation is \( \alpha' R/\Gamma \) where \( \alpha' \) is the absorption coefficient \([20]\):

\[ \alpha' = \frac{(p + 2)}{8\pi m_e^2 \nu' N_e^0(\gamma_e) \gamma_e} \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} \gamma e \]  

(15)

with:

\[ P_\nu = \frac{2\sqrt{3} e^2 B}{m c^2} F\left(\frac{\nu'}{\nu_e}\right) \]  

(16)

So the instantaneous intensity for the SSA will be:

\[ (F_\nu)_{\text{SSA}} = \frac{1}{4\pi D_L(z)^2} 4\pi \left( \frac{dP_\nu}{d\Gamma} \right)_{\text{SSA}} \]  

(17)

\[ \left( \frac{dP_\nu}{d\Gamma} \right)_{\text{SSA}} = (1 + z)(1 + \beta) \left( \frac{dP_\nu}{d\Gamma'} \right)_{\text{SSA}} \]  

(18)

\[ \left( \frac{dP_\nu}{d\Gamma'} \right)_{\text{SSA}} = \left( \frac{dP_\nu}{d\Gamma'} \right)_{\text{OTS}} \alpha'_{\text{OTS}} (1 - e^{\alpha'_{\text{OTS}}}) \]  

(19)

3. Results and discussion

First, the evolution of the light curve is affected by the initial various parameters values: \( a = 4.0, p = 2.3, g = 0.0, \theta = 10^\circ, \varepsilon_e = 1.0, \varepsilon_B = 0.01, \Gamma_0 = 250 \) and \( M_0 = 2 \times 10^{-6} M_\odot \), with \( M_\odot \) is Solar mass. Here \( \theta \) is the jet half-opening angle. The GRB in our calculations is assumed to be at \( z = 1.0 \) corresponding to the luminosity distance about \( D_L = 6.82 \text{Gpc} \).

These previous parameters are fixed we change the others such as \( \lambda_{\text{obs}}^{-1} = 500.\text{cm}^{-1} \) related to UV emission and \( \nu_{\text{obs}} = 3 \times 10^8 \text{Hz} \) corresponding to the radio band. Here, we propose as an environment a homogeneous interstellar medium with \( n_0 = 1.0 \text{cm}^{-3} \), \( k = 0 \).

Figure 1 shows the light curve of the GRB afterglow in the two cases OTS and SSA emission for a radio frequency \( \nu_{\text{obs}} = 3 \times 10^8 \text{Hz} \). Here the synchrotron self-absorption appears regularly during the afterglow and it becomes really visible typically in the radio emission, so it arises in the radio afterglow. So far it was not able to be seen as in figure 2 when we have UV frequency \( \lambda_{\text{obs}}^{-1} = 500.\text{cm}^{-1} \). Figure 3 shows the ratio between the absorption coefficient for radio
Figure 1. Light curve of GRB Afterglow in the two cases OTS and SSA emission for radio frequency $\nu_{\text{obs}} = 3 \times 10^8 \text{Hz}$.

Figure 2. Light curve of GRB Afterglow in the two cases OTS and SSA emission for UV frequency $\lambda_{\text{obs}}^{-1} = 500 \text{cm}^{-1}$. 
Figure 3. Evolution of absorption coefficient for radio frequency $\nu_{obs} = 3 \times 10^8 Hz$ and UV frequency $\lambda_{obs}^{-1} = 500. cm^{-1}$.

Figure 4. Spectra of GRB Afterglow in the two cases OTS and SSA emission.
frequency \( \nu_{\text{obs}} = 3 \times 10^8 \text{Hz} \) and UV frequency \( \lambda_{\text{obs}}^{-1} = 500. \text{cm}^{-1} \). This result is confirmed in the figure 4 when the spectra of GRB Afterglow consist of a large absorption in low frequencies contrary to the same case in high frequencies.

4. Conclusion
Most of the observed gamma ray bursts (GRBs, i.e., very intense flashes of prompt, hard and very brilliant cosmologic electromagnetic radiations) are usually followed by afterglows which consist in remnant, softer, delayed radiations over a large frequency range extending from X-rays down to radio waves. The most popular model describing both two radiation types is the fireball model where two classes of violent collisions are assumed: (i) the internal shocks behind the GRB emission and (ii) the external shock producing the remnant afterglows. In this contribution, we report on a performed hydrodynamic modeling of the external shock that allowed us to calculate afterglow light curves by assuming the predominance of the synchrotron emission mechanism (OTS), which is justified mainly in the case of low electron densities (typically for \( n_e < 10^3 \text{cm}^{-3} \)), however at low frequencies the synchrotron self-absorption (SSA) has its effect on the cutoff of the low energy spectrum because of the important values of the absorption coefficient.

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