The power-law expansion universe and the late-time behavior

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Abstract

Using the SNe Ia data we determine the three parameters in the power-law expanding universe model with time-dependent power $m$. Inputting $H_0$ and $t_0$, then we find the $\dot{a} - t$ evolution curve with $m = 5.0$ and $q_0 = -0.90$ can fit very well to that from SNe observation data. The model predicts the transition redshift $z \simeq 0.38$. The dark energy deduced from this model have phantom property but the universe doesn’t encounter the Big Rip singularity. Assuming that this model with the three parameters is valid for the future universe, then we predict that the total energy density of the universe is decreasing and will soon reach its minimum.

PACS numbers: 98.80.Cq, 98.80.Hm
I. INTRODUCTION

The supernovae Type Ia (SNe Ia) observation and the cosmic microwave background power spectrum measurement show the existence of dark energy and the flatness of the universe [1, 2]. In order to understand the nature of dark energy, many dynamical models have been proposed, such as, quintessence, tachyon, k-essence, etc. [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. A cosmological constant is the simplest model of dark energy, but no natural explanation for the origin of it can be given. The SNe Ia observations provide the currently most direct way to probe the dark energy through the luminosity-distance relation [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27].

In order to probe the evolution of dark energy, an appropriate and simple parametrization of them can take the advantage of being practical [28]. Despite the broad interest in proposing dynamical model of dark energy, their physical properties are still poorly understood at a fundamental level. As well known, almost all models of dark energy can fit to the observation data, but few models fitting well to the evolution of the universe in a very long history have been found. From a phenomenological point of view all known models of dark energy should be equivalent. We still have a long way from being able to give a complete explanation for the nature of dark energy. Nevertheless, It is possible to try to seek an appropriate form of solution to Friedmann equation independent of the concrete dark energy model.

Recently, we propose a power-law expansion universe model [29], \( a = a_0 t^{n(t)} \) with \( n(t) = n_0 + bt^m \), which for a spatially flat, isotropic and homogeneous universe leads to the consistent results with those given in some current researches such as [22]. Here, the parameters \( m, n_0 \) and \( b \) will be determined by fitting the evolution of the universe from the SNe data. The three cases are discussed, among which the best one has the parameters \( m = 5, q_0 = -0.9 \) and \( b = 2.76671 \times 10^{-54} \). The model with the three best parameters predicts \( z_T \simeq 0.38 \), favors the phantom dark energy for the current universe, and show the total energy density of the universe is decreasing.
II. DETERMINATION OF PARAMETERS BY SNE DATA

In order to describe the universe transition from deceleration to acceleration expansion we propose such a universe model described by

\( a = a_0 t^n = a_0 t^{n_0 + bt^m} \),

where \( a_0 \) is the scale factor for \( t = 1 \text{yr} \), \( n_0 \), \( b \) and \( m \) are three nonnegative parameters \[29\]. From (1), there is

\[ \dot{a} = a(n \ln t + \frac{n}{t}) = \frac{a}{t}[n_0 + bt^m(1 + mlnt)], \]

where a dot stands for the derivative with respect to \( t \). Giving the observed quantities, the current Hubble parameter \( H_0 \), the current deceleration parameter \( q_0 \) and the age of the universe \( t_0 \), then we have

\[ n_0 = H_0 t_0 - b(mx_0 + 1)t_0^m, \quad b = \frac{1 - (q_0 + 1)H_0 t_0}{m(mx_0 + 2)H_0 t_0 1 - m}. \]

In order to determine \( n_0 \) and \( b \), one needs to know \( H_0 = H(t_0) \), and choose the parameters \( q_0 = q(t_0) \) and \( m \). Fixing \( H_0 \) and \( t_0 \), using the different values of \( q_0 \) and \( m \), then one can obtain the different \( n_0 \) and \( b \), which yield different transition redshift \( z_T \). By adopting \( z_T = 0.5 \), then we can determine the values of \( m \), \( n_0 \) and \( b \)[29].

Here, we will still fix \( H_0 \) and \( t_0 \), and treat \( m \) and \( q_0 = q(t_0) \) as two input parameters. The improvement to our previous method is that we will not only require the model can describe the universe transition but hope that it may track the history of the universe evolution. From equation (2), one can have

\[ \dot{a}/a_0 H_0 = \frac{a}{a_0 H_0 t^n}[n_0 + bt^m(1 + mlnt)]. \]

FIG. 1 shows \( \dot{a}/H_0 - t \) curves with the same minimum for the three special cases with different values of \( m \) and \( q_0 \). FIG. 2 gives the comparison between \( \dot{a}/H_0 - a \) curves given from the SNe Ia data \[14, 18\] and FIG. 1. We find that among the three cases the third one \( c \), i.e., the curve with \( m = 5.0 \) and \( q_0 = -0.9 \) can best fit to the evolution curve from the SNe Ia data. This implies that the model predicts the deceleration parameter of the universe \( q_0 = -0.90 \). This value is bigger than some known results, such as, \( q_0 = 0.35 \pm 0.15 \) given in \[30\], but may be consistent with the results in \[31, 32\].
FIG. 1: The $\dot{a}/a_0H_0 - t$ figure for three special cases with different values of $m$ and $q_0$.

![Graph](image)

FIG. 2: The $\dot{a}/a_0H_0 - a/a_0$ figure. Curve B comes from the supernovae data set consisting of the gold sample (Ref. 18). E, D and C in (a), (b) and (c) correspond to the three curves in FIG. 1 $(q_0 = -0.638, m = 3.0)$, $(q_0 = -0.77, m = 4.0)$ and $(q_0 = -0.90, m = 5.0)$, respectively.

Now, we determine the universe transition redshift. Putting $m = 5.0$ and $q_0 = -0.90$ in equation 3 yields $b = 2.76671 \times 10^{-54}$, $n_0 = 0.804696$, $A = 2n_0 + 2m - 1 = 10.6094$ and
\( B = m(2n_0 + m - 1) = 28.047 \). Using equation (8) in [29] and letting \( q = 0 \), then we obtain

\[
b^2(1 + mx_T)^2t_T^{2m-2} + b(A + Bx_T)t_T^{m-2} + (n_0^2 - n_0)t_T^{-2} = 0,
\]

where \( x_T = \ln t_T \) and \( t_T \) denotes the transition time. Equation (5) yields \( t_T \simeq 9.71 \text{Gyr} \), and from the relation \( z = \frac{a(0)}{a(t)} - 1 \) we have the transition redshift

\[
z_T = \frac{a(t_0)}{a(t)} - 1 = \left( \frac{t_0^{n_0 + b_0^2}}{t_T^{n_0 + b_0^2}} \right) - 1 \simeq 0.383,
\]

which may be consistent with some known results, such as those given in [14, 19, 33].

In this section, by fitting to the SNe data we determine the parameters in the power-law expanding universe model with \((m = 5, n_0 = 0.804696, b = 2.76671 \times 10^{-54})\). Next, using the scale factor (11) with known parameters \( m, n_0 \) and \( b \) we will catch a glimpse of the evolution of dark energy and the universe.

### III. EVOLUTION PROPERTIES OF THE UNIVERSE

For the spatially flat, isotropic and homogeneous universe described by the scale factor (11), the total energy density is determined by

\[
\rho = \frac{3}{M_P^2} H^2, \quad H = \frac{\dot{a}}{a},
\]

with \( M_P = 1/\sqrt{8\pi G} = \frac{3H_0^2}{\rho_0} \) the reduced Planck mass. Assuming that the matter component is the perfect fluid, i.e., \( \rho_m = \rho_0 a(t_0)^3/a(t)^3 \), then the dark energy density is given by

\[
\rho_X = \rho_0 [(\frac{H}{H_0})^2 - a(t_0)^3/a(t)^3].
\]

FIG. 3 shows the phantom property of dark energy at the current epoch and FIG. 4 shows the minimum of Hubble parameter \( H_{\text{min}} \simeq 0.699 \times 10^{-10} \text{yr}^{-1} \) at \( t \simeq 14.36 \text{ Gyr} \), which yields the minimum of universe energy density \( \rho_{\text{min}} = (\frac{H_{\text{min}}}{H_0})^2 \rho_0 \simeq 0.997 \rho_0 \).

From the conserved equation for dark energy

\[
\dot{\rho}_X + 3H(\rho_X + p_X) = 0,
\]

where a dot denotes the derivative with respect to time, one can obtain the equation of state

\[
w_X = \frac{p_X}{\rho_X} = -\frac{\dot{\rho}_X}{3H\rho_X} - 1.
\]
FIG. 3: The $\rho_X - t$ figure is given for $m = 0.5$ and $q = -0.9$, which shows the evolution for dark energy in the past epoch for $\Omega_{m0} = 0.27, 0.30, 0.33$, respectively.

FIG. 4: The $H - t$ figure shows the Hubble parameter has the minimum $H_{\text{min}} \simeq 0.6991 \times 10^{-10} \text{yr}^{-1}$ at $t \simeq 14.36 \text{Gyr}$. 
FIG. 5: Curves (a),(b) and (c) denote $w_X$ for $\Omega_{m0} = 0.27, 0.30, 0.33$, respectively. The figure shows $w_X$ will reach its minimum value at $t = 18.5 \sim 19.5$ Gyr.

Assuming that the dark energy is slowly changing, then around the current epoch we can approximately have $w_X \simeq -\frac{\Delta \rho_X}{3H_0 \rho_{X0} \Delta t} - 1$, where $\Delta \rho_X = \rho_{X0} - \rho_X$ and $\Delta t = t_0 - t$. For $t = 1.399 \times 10^{10}$ yr, there are the energy density ratio $\rho_X/\rho_{X0} = 0.99956, 0.99946, 0.99936$, which lead to $w_{X0} = -1.21, -1.26, -1.30$ for $\Omega_{m0} = 0.27, 0.30, 0.33$, respectively.

From the Friedmann equations

$$H^2 = \frac{1}{3M_P^2}(\rho_X + \rho_m), \quad H^2 + \dot{H} = -\frac{1}{6M_P^2}(\rho_X + \rho_m + 3p_X), \quad (11)$$

one can have

$$w_X = \frac{p_X}{\rho_X} = -\frac{3H^2 + 2\dot{H}}{3H^2 - \rho_m/M_P^2}, \quad (12)$$

with $\dot{H} = b[m(m - 1)\ln t + 2m - 1]t^{m-2} - n_0 t^{-2}$ and $H = bt^{m-1}(1 + mlnt) + n_0 t^{-1}$. FIG. 5 illustrates the future evolution of dark energy, which will evolve to its minimum in the time interval $18.5 \sim 19.5$ Gyr, such as, $w_{Xmin} \simeq -1.738$ at $t \simeq 19.36$ Gyr for $\Omega_{m0} = 0.27$. For $t \gg 1$, $\rho_m$ may be neglected and for $m = 5$ equation (12) reduces to

$$w_X \simeq -1 - \frac{8}{15b} (t^5 lnt)^{-1}, \quad (13)$$
which implies that $w_X$ decreases rapidly to $-1$ for late time.

For the constant equation of state $w_X < -1$, the universe will encounter the sudden future singularity, i.e., Big Rip or Big Smash, but it may evade a Big Rip if the the equation of state falls off quickly. Clearly, the power-law expansion universe model considered currently describe such a phantom universe without the future singularity.

We study the power-law model with time-dependent power proposed in [29] by using the SNe data. For $m = 5$, $q_0 = -0.9$ and $b = 2.76671 \times 10^{-54}$, the model can well track the evolution of the supernova to a very high redshift, and the predicted transition redshift $z_T$ and deceleration parameter $q_0$ may be consistent with some known results. The model may be expected to be able to describe the future evolution of the universe in the near future since it can well track the past evolution history of the universe. Considering the matter as the perfect fluid, then the model predicts that the equation of state parameter for dark energy is decreasing and will go to its minimum value after about 5 Gyr for $\Omega_{m0} \sim 0.3$. Another interesting result given by the model is that the total energy density of universe is dropping and will soon approach its minimum value smaller slightly than the current one.

Obtaining a overall scale factor from a dark energy model should be what everyone yearns, who is working in this research realm. However, it is a pity that there are few models which can track a very long evolution history of the universe by fitting the observation data at one point. So, studying the scale factor in a direct way doesn’t interpret the nature of dark energy but should be very helpful for understanding the behavior of dark energy.

**Acknowledgement:** This work are supported by ITP Post-Doctor Project 22B580, Chinese Academy of Science of China, National Nature Science Foundation of China and Liaoning Province Educational Committee Research Project.

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