The two-loop supersymmetric corrections to lepton anomalous magnetic and electric dipole moments

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Abstract

Using the effective Lagrangian method, we analyze the electroweak corrections to the anomalous dipole moments of lepton from some special two-loop topological diagrams which are composed of neutralino (chargino) - slepton (sneutrino) in the minimal supersymmetric extension of the standard model (MSSM). Considering the translational invariance of the inner loop momenta and the electromagnetic gauge invariance, we get all dimension 6 operators and derive their coefficients. After applying equations of motion to the external leptons, the anomalous dipole moments of lepton are obtained. The numerical results imply that there is a parameter space where the two-loop supersymmetric corrections to the muon anomalous dipole moments may be significant.

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I. INTRODUCTION

At both aspects of experiment and theory, the magnetic dipole moment of lepton as well as the electric dipole moment draw great attention of physicists because of their obvious importance. The anomalous dipole moments of muon not only can be used for testing loop effect in the standard model (SM), but also provide a potential window to detect new physics beyond the SM. The current experimental world average of the muon magnetic dipole moment is \[ a^{exp}_\mu = 11 659 203 \pm 8 \times 10^{-10}. \] (1)

Contributions to the muon magnetic dipole moment are generally divided into three sectors: QED loops, hadronic contributions as well as electroweak corrections. With the hadronic contributions which are driven from the most recent $e^+e^-$ data, we can get the following SM predictions\[2, 3, 4\]

\[ a^{SM}_\mu = 11 659 180.9 \pm 8.0 \times 10^{-10} \]
\[ a^{SM}_\mu = 11 659 175.6 \pm 7.5 \times 10^{-10} \]
\[ a^{SM}_\mu = 11 659 179.4 \pm 9.3 \times 10^{-10}. \] (2)

The deviations between the above theoretical predictions and the experimental data are all approximately within error range of $\sim 2\sigma$. Although this $\sim 2\sigma$ deviation cannot be regarded as a strong evidence for new physics, along with the experimental measurement precision and theoretical prediction accuracy being constantly improved, this deviation may turn more significant in near future.

In fact, the current experimental precision ($8 \times 10^{-10}$) already puts very restrictive bounds on new physics scenarios. In the SM, the electroweak one- and two-loop contributions amount to $19.5 \times 10^{-10}$ and $-4.4 \times 10^{-10}$\[5\] respectively. Comparing with the standard electroweak corrections, the supersymmetric corrections are generally suppressed by $\Lambda^2_{ew}/\Lambda^2_{NP}$, where $\Lambda_{ew}$ denotes the electroweak energy scale and $\Lambda_{NP}$ denotes the supersymmetric energy scale. However, there is a parameter space where the one-loop supersymmetric corrections are comparable to that from the SM\[6\]. Since the one-loop contribution can be large, the two-loop supersymmetric corrections are possibly quite important\[7\].

Utilizing the heavy mass expansion approximation (HME) together with the corresponding projection operator method, the two-loop standard electroweak corrections to muon
anomalous magnetic dipole moment (MDM) have been evaluated \cite{8}. Within the framework of CP conservation, the authors of Ref. \cite{9} present the supersymmetric corrections from some special two-loop diagrams where a close chargino (neutralino) or scalar fermion loop is inserted into those two-Higgs-doublet one-loop diagrams. Ref. \cite{10} discusses the contributions to muon MDM from the effective vertices $H^\pm W^\mp \gamma, h_0(H_0)\gamma\gamma$ which induced by the scalar quarks of the third generation.

In this work, we calculate the corrections from some special two-loop diagrams which are composed of internal neutralino (chargino) and (scalar) lepton lines. Since the electric dipole moment (EDM) of muon is also of special interest in both theoretical and experimental aspects \cite{11}, we as well present the lepton EDM here by keeping all possible CP violation phases. All the diagrams which we are going to calculate, were not discussed in literature. Besides, we first express our results in the form which explicitly satisfies the Ward identity requested by the QED gauge theory. In order to rationally predict the muon EDM, we certainly need to take the current upper experimental bounds on electron and neutron EDMs as rigorous constraints into account. Nevertheless, if we invoke a cancellation mechanism among different supersymmetric contributions \cite{13}, or assume those sfermions of the first generation to be heavy enough \cite{14}, the loop inducing lepton and neutron EDMs restrict the argument of the $\mu$ parameter to be $\leq \pi/(5 \tan \beta)$, but no constraints on other explicit CP violation phases are enforced.

Here, we apply the effective Lagrangian method to get the anomalous dipole momentums of lepton in this work. In concrete calculation, we assume that all external leptons as well as photon are off-shell, then expand the amplitude of corresponding triangle diagrams according to the external momenta of leptons and photon. Using loop momentum translational invariance, we write the sum of the triangle diagrams which correspond to the corresponding self-energy in the form which explicitly satisfies the Ward identity required by the QED gauge symmetry. Then we can get all dimension six operators together with their coefficients. After applying the equations of motion for external leptons, higher dimensional operators, such as dimension eight operators, also contribute to the muon MDM and EDM in principle. However, the contributions of dimension eight operators contain an additional suppression factor $m_\mu^2/\Lambda_{NP}^2$ compared to that of dimension six operators, where $m_\mu$ is the mass of muon. Setting $\Lambda_{NP} \sim 100\text{GeV}$, this suppression factor is about $10^{-6}$. Under current experimental precision, it implies that the contributions of all higher dimension operators
\( D \geq 8 \) can be neglected safely.

We adopt the naive dimensional regularization with the anti-commuting \( \gamma_5 \) scheme, where there is no distinction between the first 4 dimensions and the remaining \( D - 4 \) dimensions. Since the bare effective Lagrangian contains the ultra-violet divergence which is induced by divergent sub-diagrams, we give the renormalized results in the \( \overline{MS} \) scheme and on-mass-shell scheme \([15]\) respectively. The two-loop theoretical prediction certainly relies on our concrete choice of regularization scheme and renormalization scheme, however, our numerical results show that there is only tiny difference between the theoretical predictions by different regularization and renormalization schemes. We will discuss this problem in our other work.

Through repeating the supersymmetric one-loop results, we introduce the effective Lagrangian method and our notations in next section. We will demonstrate how to obtain the supersymmetric two-loop corrections to the lepton MDMs and EDMs in Section III. In the Section IV we study the dependence of the lepton MDMs and EDMs on the supersymmetry parameters numerically. Conclusions are presented in the last Section.

II. OUR NOTATIONS AND THE SUPERSYMMETRIC ONE-LOOP RESULTS

The lepton MDMs and EDMs that we will calculate can actually be expressed as the operators

\[
\mathcal{L}_{MDM} = \frac{e}{4m_i} a_i \bar{l} \sigma^{\mu\nu} l \, F_{\mu\nu},
\]

\[
\mathcal{L}_{MDM} = -\frac{i}{2} \bar{l} \sigma^{\mu\nu} \gamma_5 l \, F_{\mu\nu}.
\]

Here, \( l \) denotes the lepton fermion, \( F_{\mu\nu} \) is the electromagnetic field strength, \( m_i \) is the lepton mass and \( e \) represents the electric charge respectively. Note that the lepton here is on-shell.

In fact, it is convenient to get the corrections of the loop-diagrams to lepton MDMs and EDMs in terms of the effective Lagrangian method, if the masses of all the internal lines are much heavier than the external lepton mass. Assuming external leptons as well as photon all are off-shell, we expand the amplitude of corresponding triangle diagrams according to the external momenta of leptons and photon. Then we can get all higher dimension operators together with their coefficients. As discussed in the introduction, it is enough to retain only
FIG. 1: The one-loop self energy diagrams which lead to the lepton MDMs and EDMs in MSSM, the corresponding triangle diagrams are obtained by attaching a photon in all possible ways to the internal particles.

those dimension 6 operators in later calculations:

\[
\begin{align*}
O_1^\pm &= \frac{1}{(4\pi)^2} \bar{l}\left(\not{\! q}\right)^3 \omega_\mp l, \\
O_2^\pm &= \frac{e}{(4\pi)^2} (i\overline{D}_\mu l) \gamma^\mu F \cdot \sigma \omega_\mp l, \\
O_3^\pm &= \frac{e}{(4\pi)^2} \bar{l} F \cdot \sigma \gamma^\mu \omega_\mp (i\overline{D}_\mu l), \\
O_4^\pm &= \frac{e}{(4\pi)^2} \bar{l} (\partial^\mu F_{\mu\nu}) \gamma^\nu \omega_\mp l, \\
O_5^\pm &= \frac{m}{(4\pi)^2} \bar{l} (\not{\! q})^2 \omega_\mp l, \\
O_6^\pm &= \frac{cm}{(4\pi)^2} \bar{l} F \cdot \sigma \omega_\mp l,
\end{align*}
\]

(4)

with \( D_\mu = \partial_\mu + ieA_\mu \) and \( \omega_\mp = (1 \mp \gamma_5)/2 \). At one-loop level, there are two triangle diagrams which contribute to lepton MDMs and EDMs (Fig. 1). After expanding the amplitude according to external momenta, the triangle diagrams determine following dimension 6 operators together with their coefficients as

\[
L_{\chi_0^0 E_i}^{1L} = -\frac{(4\pi)^2 e^2}{2s_w c_w} \int \frac{d^4q}{i(2\pi)^4} \frac{1}{(q^2 - m^2_{\chi_0})(q^2 - m^2_{E_i})} \times \left\{ \left| (\xi_{\chi_0}^I)_{\alpha\beta} \right|^2 \left[ -m_{E_i}^2 \frac{q^2}{(q^2 - m^2_{E_i})^3} O_1^- + \frac{m_{E_i}^2}{4} \frac{q^2}{(q^2 - m^2_{E_i})^3} \left( \phi_2^- + \phi_3^- \right) \right] + \frac{1}{6} \frac{(q^2)^2}{(q^2 - m^2_{E_i})^3} O_4^- \right\}
\]

5
\[ \mathcal{L}^{UL}_{\chi^\pm_\alpha \tilde{\nu}_i} = -\frac{(4\pi)^2e^2}{s_w} \int \frac{d^4q}{i(2\pi)^4(q^2 - m_{\chi_\alpha}^2)(q^2 - m_{\tilde{\nu}_i}^2)} \]
\[ \times \left\{ |(\xi^I_{\alpha})_{i\alpha}|^2 \left[ \frac{m^4_{\chi_\alpha}}{\chi_\alpha^2} \frac{1}{q^2 - m_{\chi_\alpha}^2} \mathcal{O}^- + \frac{m^2_{\chi_\alpha}}{4} \frac{q^2}{(q^2 - m_{\chi_\alpha}^2)^3} \mathcal{O}_5^- + \mathcal{O}^+_5 \right] - \frac{1}{6} \left[ \frac{(q^2)^2}{(q^2 - m_{\chi_\alpha}^2)^3} - \frac{3q^2}{(q^2 - m_{\chi_\alpha}^2)^2} \right] \mathcal{O}^- \right\} + m^2_{\chi_\alpha} \frac{1}{2m_wc_\beta} |(\eta^I_{\alpha})_{i\alpha}|^2 \left[ \frac{m^4_{\chi_\alpha}}{\chi_\alpha^2} \frac{1}{q^2 - m_{\chi_\alpha}^2} \mathcal{O}^+_1 - \frac{m^2_{\chi_\alpha}}{4} \frac{q^2}{(q^2 - m_{\chi_\alpha}^2)^3} \mathcal{O}_5^+ + \mathcal{O}^+_5 \right] - \frac{1}{6} \left[ \frac{(q^2)^2}{(q^2 - m_{\chi_\alpha}^2)^3} - \frac{3q^2}{(q^2 - m_{\chi_\alpha}^2)^2} \right] \mathcal{O}^+_1 \]
\[ - \frac{m^4_{\chi_\alpha}}{\sqrt{2}m_wc_\beta} (\xi^I_{\alpha})_{i\alpha} (\xi^I_{\alpha})_{i\alpha}^* \left[ - \frac{m^2_{\chi_\alpha}}{\chi_\alpha^2} \frac{1}{q^2 - m_{\chi_\alpha}^2} \mathcal{O}_5^- + \frac{1}{2} \frac{q^2}{(q^2 - m_{\chi_\alpha}^2)^2} \mathcal{O}_6^- \right] - \frac{m^4_{\chi_\alpha}}{\sqrt{2}m_wc_\beta} (\xi^I_{\alpha})_{i\alpha} (\eta^I_{\alpha})_{i\alpha}^* \left[ - \frac{m^2_{\chi_\alpha}}{\chi_\alpha^2} \frac{1}{q^2 - m_{\chi_\alpha}^2} \mathcal{O}_5^+ + \frac{1}{2} \frac{q^2}{(q^2 - m_{\chi_\alpha}^2)^2} \mathcal{O}_6^+ \right] \right\}, \quad (5) \]

with

\[ (\xi^I_{\alpha})_{i\alpha} = (R^I_\chi)_I \left( (\mathcal{N}^i)_{\alpha_1} s_w + (\mathcal{N}^i)_{\alpha_2} c_w \right) - \frac{m_{\chi_\alpha}}{m_wc_\beta} (R^I_\chi)_{(3+I)i} (\mathcal{N}^i)_{\alpha_1} ; \]
\[ (\eta^I_{\alpha})_{i\alpha} = 2s_w (R^I_\chi)_{(3+I)i} (\mathcal{N})_{i\alpha} + \frac{m_{\chi_\alpha}}{m_wc_\beta} (R^I_\chi)_{Ii} (\mathcal{N})_{i\alpha} ; \]
\[ (\xi^I_{\alpha})_{i\alpha} = (R^I_\nu)_I (\mathcal{V})_{i\alpha} ; \]
\[ (\eta^I_{\alpha})_{i\alpha} = (R^I_\nu)_I (\mathcal{U}^t)^{2\alpha} \, . \] \quad (6) \]

Here, \( \mathcal{N}, \mathcal{V}, \mathcal{U} \) denote the mixing matrices of neutralinos and charginos respectively, \( I, J = 1, 2, 3 \) are the indices of generations. We also adopt the shortcut notations: \( c_w = \cos \theta_w, \ s_w = \sin \theta_w, \ c_\beta = \cos \beta \), where \( \theta_w \) is the Weinberg angle, and \( \tan \beta = v_2/v_1 \) is the ratio between the vacuum expectation values of two Higgs doublets. As for the mixing
matrices of sleptons and sneutrinos, $R_E$, $R_\nu$ are:

$$R_E^\dagger \begin{pmatrix} (M^2_{LL}) & (M^2_{LR}) \\ (M^2_{LR})^\dagger & (M^2_{RR}) \end{pmatrix} R_E = m^2_{E_i}, \quad (i, j = 1, \cdots, 6)$$

$$R_\nu^\dagger (M^2_\nu) R_\nu = m^2_{\nu_i}, \quad (i, j = 1, 2, 3).$$

Those $3 \times 3$ matrices are defined as

$$(M^2_{LL})_{ij} = (M^2_{LL})_{jj} + m^2_{\nu_i} \delta_{ij} + m^2_{\nu_e} (s^2_w - \frac{1}{2}) \cos 2\beta \delta_{ij},$$

$$(M^2_{RR})_{ij} = (M^2_{RR})_{jj} + m^2_{\nu_i} \delta_{ij} - m^2_{\nu_e} s^2_w \cos 2\beta \delta_{ij},$$

$$(M^2_{LR})_{ij} = -(\mu_{HL} m_{\nu_i} \tan \beta) \delta_{ij} + m_{\nu_i} (A_e)_{ij},$$

$$(M^2_{\nu})_{ij} = (M^2_{\nu})_{jj} + \frac{1}{2} m^2_{\nu_e} \cos 2\beta \quad (I, J = 1, 2, 3),$$

where $M^2_L$, $M^2_R$, $A_e$ are the bilinear and trilinear soft breaking parameters in the lepton sector separately, and $\mu_{HL}$ denotes the $\mu$-parameter in the soft supersymmetry breaking terms.

Applying the equations of motion for leptons in Eq. (5), we can get the lepton MDMs as

$$\Delta a_{\chi^0_{\nu_i}} = -\frac{e^2}{2(s_w c_w)^2} \int \frac{d^4q}{i(2\pi)^4} \frac{1}{(q^2 - m^2_{\chi^0_{\nu_i}})(q^2 - m^2_{\nu_i})}$$

$$\times \left\{ m^2_{\nu_i} m^2_{\nu_e} \left( |(\xi^I_{\chi^0})_{\nu_i}|^2 + |(\eta^I_{\chi^0})_{\nu_i}|^2 \right) \frac{q^2}{(q^2 - m^2_{\nu_i})^3} - 2 m_{\nu_i} m_{\nu_e} m^2_{\nu_i} \Re \left( (\xi^I_{\chi^0})_{\nu_i} (\eta^I_{\chi^0})_{\nu_i}^\dagger \right) \frac{1}{(q^2 - m^2_{\nu_i})^2} \right\}$$

$$= -\frac{e^2}{12(4\pi)^2(s_w c_w)^2} \left\{ \left( |(\xi^I_{\chi^0})_{\nu_i}|^2 + |(\eta^I_{\chi^0})_{\nu_i}|^2 \right) x_{\nu_i} \rho_1(x_{\nu_i}, x_{\nu_e}) + 6(x_{\nu_i} x_{\nu_e})^{1/2} \Re \left( (\xi^I_{\chi^0})_{\nu_i} (\eta^I_{\chi^0})_{\nu_i}^\dagger \right) \rho_2(x_{\nu_i}, x_{\nu_e}) \right\},$$

$$\Delta a_{x^\pm_{\nu_i}} = \frac{e^2}{s_w} \int \frac{d^4q}{i(2\pi)^4} \frac{1}{(q^2 - m^2_{\chi^0_{\nu_i}})(q^2 - m^2_{\nu_i})} \frac{q^2}{(q^2 - m^2_{\nu_e})^3}$$

$$\times \left\{ m^2_{\nu_i} m^2_{\nu_e} \left( |(\xi^I_{\chi^0})_{\nu_i}|^2 + \frac{m^2_{\nu_i}}{2m^2_{\nu_e} c^2_{\beta}} |(\eta^I_{\chi^0})_{\nu_i}|^2 \right) \frac{q^2}{(q^2 - m^2_{\nu_i})^3} + \sqrt{2} m^2_{\nu_i} m_{\nu_e} \Re \left( (\eta^I_{\chi^0})_{\nu_i} (\xi^I_{\chi^0})_{\nu_i}^\dagger \right) \frac{q^2}{(q^2 - m^2_{\nu_i})^2} \right\}$$

$$= -\frac{e^2}{6(4\pi)^2 s_w c_\beta} x_{\nu_i} \left\{ \left( |(\xi^I_{\chi^0})_{\nu_i}|^2 + \frac{m^2_{\nu_i}}{2m^2_{\nu_e} c^2_{\beta}} |(\eta^I_{\chi^0})_{\nu_i}|^2 \right) \rho_1(x_{\nu_i}, x_{\nu_e}) - 6\sqrt{2} m_{\nu_i} \Re \left( (\eta^I_{\chi^0})_{\nu_i} (\xi^I_{\chi^0})_{\nu_i}^\dagger \right) \tilde{\varphi}_3(x_{\nu_i}, x_{\nu_e}) \right\},$$

(9)
with \(x_i = m_i^2/\Lambda_{NP}^2\), and \(\Lambda_{NP}\) denotes the new physics scale. The definitions of the functions \(\rho_{1,2}(x,y)\), \(\varphi_{1,2,3}(x,y)\) can be found in appendix C. If we change our notations to that of Ref. [6], one can notice that those expressions are completely the same as the corresponding equations given in Ref. [6]. In order to obtain the above expressions, we have used the following identities which originate from the loop momentum translational invariance:

\[
\int \frac{d^D q}{(2\pi)^D} \frac{q^2}{(q^2 - m^2)^2} - \frac{D}{2} \int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2 - m^2} \equiv 0 ,
\]

\[
\int \frac{d^D q}{(2\pi)^D} \frac{(q^2)^2}{(q^2 - m^2)^2} - \frac{D + 2}{2} \int \frac{d^D q}{(2\pi)^D} \frac{q^2}{q^2 - m^2} \equiv 0 .
\]  

In the CP conservation framework, the supersymmetric one-loop contribution is approximately given by

\[
\Delta a_{1L} \simeq 13 \times 10^{-10} \left( \frac{100 \text{ GeV}}{\Lambda_{NP}} \right)^2 \tan \beta ,
\]  

when all supersymmetric masses are assumed to be equal to \(\Lambda_{NP}\), and \(\tan \beta \gg 1\).

Correspondingly, the one loop supersymmetric contributions to the lepton EDMs can also be written as

\[
\Delta d_{\alpha i} = \frac{e^3}{2(s_w c_w)^2} m_{\chi_0^\alpha} m_{\tilde{E}_i} \text{Im} \left( (\eta_N^I)_{\alpha}^{(\xi_N^I)^\dagger} x_{\chi_0^\alpha}^{1/2} \rho_2(x_{\tilde{E}_i}, x_{\chi_0^\alpha}) \right) ,
\]

\[
\Delta d_{\alpha i} \simeq \frac{e^3}{\sqrt{2}(s_w c_w)^2 \Lambda_{NP}} \text{Im} \left( (\eta_C^I)_{\alpha}^{(\xi_C^I)^\dagger} x_{\chi_0^\alpha}^{1/2} \varphi_3(x_{\chi_0^\alpha}, x_{\tilde{E}_i}) \right) .
\]  

Certainly, supersymmetry inducing operators \(O_{2,3,6}^\mp\) also contribute to the lepton MDMs. As we have seen above, only the operators \(O_6^\mp\) contribute to the EDMs of lepton at one loop level. However, we will find that operators \(O_{2,3}^\mp\) also contribute to both lepton MDMs and EDMs at the two loop order.

**III. THE TWO-LOOP SUPERSYMMETRIC CORRECTIONS**

In this sector, we analyze the two-loop supersymmetric corrections to lepton anomalous dipole moments. The two-loop supersymmetric corrections to the coefficients of those
operators in Eq. (4) originate from the two-loop self-energy diagrams of leptons, which are depicted in Fig. 2. The corresponding dipole moment diagrams are obtained by attaching a photon to these diagrams in all possible ways. In these diagrams there is no new suppression factor, except a factor arising from loop integration, and the divergence caused by the sub-diagrams can be subtracted in the \( \overline{MS} \) or on-shell schemes safely. It turns out that for some regions of the parameter space the two-loop results are comparable with the one-loop contributions [9, 10]. The reason for this is that the dependence of the two-loop results on the relevant parameters differs from that of the one-loop results. Among those two-loop contributions which have been analyzed in the literature, the corrections to muon MDM from the effective vertices

\[
\gamma\gamma H_0, \gamma Z H_0
\]

induced by the scalar quarks of the third generation can be very well approximated by the formulae

\[
\Delta a_{\mu}^{\tilde{t},2L} \simeq -0.013 \times 10^{-10} \frac{m_t \mu \tan \beta}{m_{\tilde{t}} M_H} \text{sign}(A_t),
\]

\[
\Delta a_{\mu}^{\tilde{b},2L} \simeq -0.0032 \times 10^{-10} \frac{m_b A_b \tan^2 \beta}{m_{\tilde{b}} M_H} \text{sign}(\mu_H),
\]

(13)

where \( m_{\tilde{t}} \) and \( m_{\tilde{b}} \) are the masses of the lighter \( \tilde{t} \) and \( \tilde{b} \), \( A_{t,b} \) denote the trilinear soft breaking parameters of the \( t \) and \( b \) quarks, respectively, and \( M_H \) is the mass of the heavy CP-even Higgs bosons. As for the two-loop diagrams where a close chargino (neutralino) loop is inserted into those two-Higgs-doublet one-loop diagrams, the correspondingly contribution can be approximated as

\[
\Delta a_{\mu}^{\chi,2L} \simeq 11 \times 10^{-10} \left( \frac{\tan \beta}{50} \right) \left( \frac{100 \text{ GeV}}{\Lambda_{NP}} \right)^2 \text{sign}(\mu_H),
\]

(14)

if all supersymmetric masses are set equal, i.e. \( \mu_H = m_2 = M_A = \Lambda_{NP} \), and the \( U(1) \) gaugino mass \( m_1 \) relates to the \( SU(2) \) gaugino mass \( m_2 \) by the GUT relation \( m_1 = 5m_2/(3s_w^2 c_w^2) \) with the CP conservation assumption. Here \( M_A \) is the mass of CP-odd neutral Higgs. Although other contributions in Ref. [9, 10] cannot be neglected also, they cannot be approximated as the succinct formulae above.

Among the two-loop supersymmetric diagrams under investigation, the corrections to the coefficients of operators in Eq. (4) originate from three types of graphs: the lepton self-energy diagrams, where there are two neutralinos and two sleptons; a chargino, a neutralino and two sleptons or two charginos and two sleptons as virtual particles in the loop (Fig 2). In our previous works [10], we analyzed the contributions to the rare decay \( b \rightarrow s \gamma \) and neutron
FIG. 2: The two-loop self energy diagrams which lead to the lepton MDMs and EDMs in MSSM, the corresponding "triangle" diagrams are obtained by attaching a photon in all possible ways to the internal particles.

EDM from the same topological two-loop diagrams which are composed of gluino, chargino (neutralino), and squarks. Certainly, Figs.2 does not include all diagrams with internal slepton/neutralino (chargino) which contribute to the anomalous dipole moments of muon. Beside those diagrams in Fig.2 there are two-loop diagrams where a neutralino (chargino) one-loop self-energy composed of lepton-slepton or a slepton one-loop self-energy composed of lepton-neutralino (chargino) is inserted into those one-loop diagrams in Fig.[1] However, those diagrams belong to different topological classes, and we will analyze the corrections from those two-loop diagrams in our future works. We will adopt below a terminology where, for example, the "neutralino-neutralino contribution" means the sum of those triangle diagrams (indeed two triangles bound together), which have two neutralinos and two sleptons with a photon being attached in all possible ways to the internal lines. Because the sum of the "triangle" diagrams corresponding to each "self-energy" obviously respects the Ward identity requested by QED gauge symmetry, we can calculate the contributions of all the
"self-energies" separately.

Since the two-loop analysis is more subtle than the analysis at one-loop level, we show here in some detail how to evaluate all the processes, which contribute at two-loop level to the theoretical prediction of the lepton MDMs and EDMs. Taking the same steps, which we did in our earlier works [16], we obtain the following expressions for the relevant effective Lagrangian from the "neutralino-neutralino" self energy diagram:

\[
\mathcal{L}_{\tilde{\chi}_\alpha \tilde{\chi}_\beta} = -\frac{e^4 (4\pi)^2}{4(s_c c_w)^4} \int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{1}{D_{\tilde{\chi}_\alpha \tilde{\chi}_\beta}^{0}} \times \left\{ (\xi^I_N)_{j \beta} (\eta^j_N)_{i \beta} (\eta^j_N)_{\alpha j} (\xi^I_N)_{\alpha i} \sum_{\rho=1}^4 \left( \mathcal{N}^a_{\tilde{\chi}_\alpha \tilde{\chi}_\beta}^0 \right)_\rho \mathcal{O}^- \right. \\
+ (\eta^I_N)_{j \beta} (\xi^I_N)_{j \beta} (\xi^I_N)_{j \beta} (\eta^j_N)_{\alpha j} \sum_{\rho=1}^4 \left( \mathcal{N}^a_{\tilde{\chi}_\alpha \tilde{\chi}_\beta}^0 \right)_\rho \mathcal{O}^+ \\
+ m_{\tilde{\chi}_\alpha} m_{\tilde{\chi}_\beta} (\xi^I_N)_{j \beta} (\xi^I_N)_{j \beta} (\xi^I_N)_{j \beta} (\eta^j_N)_{\alpha j} \sum_{\rho=1}^4 \left( \mathcal{N}^b_{\tilde{\chi}_\alpha \tilde{\chi}_\beta}^0 \right)_\rho \mathcal{O}^- \\
- \frac{m_{\tilde{\chi}_\alpha}}{m_{\tilde{\chi}_\beta}} (\eta^I_N)_{j \beta} (\xi^I_N)_{j \beta} (\xi^I_N)_{j \beta} (\eta^j_N)_{\alpha j} \sum_{\rho=5}^6 \left( \mathcal{N}^c_{\tilde{\chi}_\alpha \tilde{\chi}_\beta}^0 \right)_\rho \mathcal{O}^+ \\
- \frac{m_{\tilde{\chi}_\alpha}}{m_{\tilde{\chi}_\beta}} (\xi^I_N)_{j \beta} (\eta^I_N)_{j \beta} (\eta^j_N)_{\alpha j} \sum_{\rho=5}^6 \left( \mathcal{N}^c_{\tilde{\chi}_\alpha \tilde{\chi}_\beta}^0 \right)_\rho \mathcal{O}^- \\
- \frac{m_{\tilde{\chi}_\alpha}}{m_{\tilde{\chi}_\beta}} (\xi^I_N)_{j \beta} (\xi^I_N)_{j \beta} (\eta^j_N)_{\alpha j} \sum_{\rho=5}^6 \left( \mathcal{N}^d_{\tilde{\chi}_\alpha \tilde{\chi}_\beta}^0 \right)_\rho \mathcal{O}^+ \right\} 
\]

with \( D_{\tilde{\chi}_\alpha \tilde{\chi}_\beta}^{0} = ((q_2 - q_1)^2 - m_{\tilde{\chi}_i}^2)(q_1^2 - m_{\tilde{\chi}_j}^2)(q_2^2 - m_{\tilde{\chi}_j}^2)(q_1^2 - m_{\tilde{\chi}_i}^2).
\] Those tedious expressions of the form factors \( \left( \mathcal{N}^{a, b}_{\tilde{\chi}_\alpha \tilde{\chi}_\beta}^0 \right)_\rho (\rho = 1, \cdots, 4) \) and \( \left( \mathcal{N}^{c, d}_{\tilde{\chi}_\alpha \tilde{\chi}_\beta}^0 \right)_\rho (\rho = 5, 6) \) are listed in appendix E. In order to express the sum of those corresponding triangle diagram amplitudes which satisfy the Ward identity required by the QED gauge symmetry explicitly, here we use the identities given in appendix A. In a similar way, we can rigorously verify the following equations with those identities:

\[
\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \left( \mathcal{N}^{a, b}_{\tilde{\chi}_\alpha \tilde{\chi}_\beta}^0 \right)_\rho \left( q_1 \leftrightarrow q_2, \alpha \leftrightarrow \beta, i \leftrightarrow j \right) 
\]
FIG. 3: The diagrams which cancel the ultra-violet divergence of the two-loop diagrams, \( \otimes \) represents the counter terms which originate from the corresponding one-loop diagrams. Here, the corresponding triangle diagrams are obtained by attaching a photon to the internal charged slepton or chargino.

\[
\begin{align*}
\equiv & \int \frac{d^{D} q_{1}}{(2 \pi)^{D}} \frac{d^{D} q_{2}}{(2 \pi)^{D}} \frac{\left( N_{a}^{b} \right)}{D_{\chi^{0}_{\alpha} \chi^{0}_{\beta}}} 2, \\
\int \frac{d^{D} q_{1}}{(2 \pi)^{D}} \frac{d^{D} q_{2}}{(2 \pi)^{D}} \frac{\left( N_{d}^{d} \right)}{D_{\chi^{0}_{\alpha} \chi^{0}_{\beta}}} 6 (q_{1} \leftrightarrow q_{2}, \alpha \leftrightarrow \beta, \ i \leftrightarrow j) \\
\equiv & \int \frac{d^{D} q_{1}}{(2 \pi)^{D}} \frac{d^{D} q_{2}}{(2 \pi)^{D}} \frac{\left( N_{c}^{c} \right)}{D_{\chi^{0}_{\alpha} \chi^{0}_{\beta}}} .
\end{align*}
\]

In fact, this is the direct consequence of the CPT invariance in the fundamental Lagrangian.

The coefficients of high dimensional operators in Eq. (15) contain ultra-violet divergence that is caused by those divergent sub-diagrams. In order to obtain physical predictions on lepton MDMs and EDMs, it is necessary to adopt a concrete renormalization scheme removing the ultra-violet divergence. In literature, the ultra-violet divergence is removed in
either the on-mass-shell renormalization scheme \[8, 9\], or the simpler \(\overline{MS}\) renormalization scheme \[10, 14, 16\]. As an over-subtracted renormalization scheme, on-shell scheme looks more physical than \(\overline{MS}\) renormalization scheme. However, there are at least two external legs of the divergent sub-diagram being the internal lines of the whole two-loop diagrams. This signifies that an artistic on-shell scheme is not more superior to the simpler \(\overline{MS}\) scheme in our case. Certainly, theoretical predictions on lepton anomalous dipole moments depend on the concrete renormalization scheme. Here, we present firstly the renormalized results which are obtained in the \(\overline{MS}\) renormalization scheme. We put the relatively complicated results which are obtained by the on-mass-shell scheme, in the appendix.

For the two-loop ”neutralino-neutralino” diagrams, the bare effective Lagrangian contains the following ultra-violet divergence

\[
\mathcal{L}_{\chi_0^0\chi_0^0} \in - \frac{\epsilon^4}{96s_w^4 c_w^4} \frac{1}{(4\pi)^2} \frac{\Gamma^2(1 + \epsilon)}{\epsilon} (4\pi)^2 (1 + \epsilon(3 + 2 \ln x_R))
\]

\[
\times \left\{ \rho_1(x_{\chi_0^0}, x_{\tilde{E}_j}) + \rho_1(x_{\chi_0^0}, x_{\tilde{E}_i}) + \epsilon \left( \varphi_1(x_{\chi_0^0}, x_{\tilde{E}_j}) + \varphi_2(x_{\chi_0^0}, x_{\tilde{E}_i}) \right) \left[ (\xi_N^I)_{j\beta} (\eta_N^J)_{i\alpha} (\eta_N^J)_{i\alpha}^\dagger (\xi_N^I)_{j\beta}^\dagger \left( \mathcal{O}_-^2 + \mathcal{O}_6^+ \right) + (\eta_N^I)_{j\beta} (\xi_N^J)_{i\alpha} (\xi_N^J)_{i\alpha}^\dagger \left( \mathcal{O}_2^+ + \mathcal{O}_3^+ \right) \right] \right\} \]

\[
+ \frac{m_{\chi_0^0}}{m_{\mu}} \left[ \rho_2(x_{\chi_0^0}, x_{\tilde{E}_i}) + \epsilon \varphi_3(x_{\chi_0^0}, x_{\tilde{E}_i}) \right]
\]

\[
\times \left[ (\eta_N^I)_{j\beta} (\xi_N^J)_{i\alpha} (\xi_N^J)_{i\alpha}^\dagger (\xi_N^I)_{j\beta}^\dagger \mathcal{O}_6^- + (\xi_N^I)_{j\beta} (\eta_N^J)_{i\alpha} (\eta_N^J)_{i\alpha}^\dagger \mathcal{O}_6^+ \right] \right\} + \cdots \right),
\]

with \(\epsilon = 2 - D/2\), where \(D\) denotes the dimension of time-space. Generally, the renormalization scale \(\Lambda_{\text{RE}}\) and the new physics scale \(\Lambda_{\text{NP}}\) should be of the same order in quantity, but there does not exist a compelling reason to make them equal. Here we keep the ratio \(x_R = \Lambda_{\text{RE}}^2 / \Lambda_{\text{NP}}^2\) as a free parameter in the expressions. In Eq. (17), we only retain the
operators $\mathcal{O}_{2,3,6}^\pm$ that contribute to the lepton anomalous dipole moments, and the ellipsis represents the convergent parts of those coefficients. Certainly, the ultra-violet divergence contained in the amplitude of counter diagrams (the first two diagrams in Fig. 3) will exactly cancel these divergences:

$$
\mathcal{L}_{\chi_0^0,\chi_{3,6}^0}^C = \frac{1}{(4\pi)^2 \Lambda_{NP}^2} \frac{e^4}{96 s_w^4 c_w^4} \frac{\Gamma^2(1+\epsilon)}{\epsilon} (4\pi)^2 \frac{\epsilon}{2} \left[ \rho_1(x_{\chi_0^0}^0, x_{\chi_{3,6}^0}) + \rho_1(x_{\chi_0^0}^0, x_{\chi_{3,6}^0}) \right] + \frac{\epsilon}{2} \left[ \rho_2(x_{\chi_0^0}^0, x_{\chi_{3,6}^0}) + \rho_2(x_{\chi_0^0}^0, x_{\chi_{3,6}^0}) \right] \]

Adding Eq. (18) and Eq. (15), we get the two-loop ”neutralino-neutralino” corrections to the lepton MDMs:

$$
\Delta a_{\chi_i^0, \chi_0^0} = -\frac{e^4}{(4\pi)^2 (s_w^2 c_w^2)^4} \left\{ x_{\chi_i^0} \frac{\Omega_{N,2,1}(x_{\chi_i^0}^0, x_{\chi_{3,6}^0}^0, x_{\chi_{3,6}^0}^0)}{\rho_2(x_{\chi_0^0}^0, x_{\chi_{3,6}^0}^0)} \times \left[ \text{Re} \left( (\xi_N^I)_{\beta \alpha} (\eta_N^J)_{\alpha \beta} (\eta_N^J)_{\alpha \beta} (\xi_N^I)_{\beta \alpha} \right) + \text{Re} \left( (\eta_N^I)_{\beta \alpha} (\xi_N^I)_{\beta \alpha} (\eta_N^J)_{\alpha \beta} (\xi_N^I)_{\beta \alpha} \right) \right] + \cdots \right\}.
$$
\[
\times \left[ \text{Re} \left( \left( \xi^I_N \right)_{\beta \gamma} \left( \xi^J_N \right)_{i \alpha} \left( \xi^J_N \right)_{i \alpha}^\dagger \left( \xi^I_N \right)_{\beta \gamma}^\dagger \right) + \text{Re} \left( \left( \eta^I_N \right)_{\beta \gamma} \left( \eta^J_N \right)_{i \alpha} \left( \eta^J_N \right)_{i \alpha}^\dagger \left( \eta^I_N \right)_{\beta \gamma}^\dagger \right) \right] \\
- (x_{i j} x_{i j}^0)_{\lambda \alpha}^{1/2} \Omega_{N,3}(x_{i j}^0; x_{E_i}^0; x_{E_j}^0; x_{E}^0; x_{E_j}^0) \text{Re} \left( \left( \eta^I_N \right)_{\beta \gamma} \left( \xi^J_N \right)_{i \alpha} \left( \xi^J_N \right)_{i \alpha}^\dagger \left( \xi^I_N \right)_{\beta \gamma}^\dagger \right) \\
- (x_{i j} x_{i j}^0)_{\lambda \alpha}^{1/2} \Omega_{N,4}(x_{i j}^0; x_{E_i}^0; x_{E_j}^0; x_{E}^0; x_{E_j}^0) \text{Re} \left( \left( \eta^I_N \right)_{\beta \gamma} \left( \eta^J_N \right)_{i \alpha} \left( \eta^J_N \right)_{i \alpha}^\dagger \left( \xi^I_N \right)_{\beta \gamma}^\dagger \right) \right) .
\] (19)

together with the lepton EDMs

\[
\Delta q_\alpha^{2L, \chi_0 \chi_3^0} = - \frac{e^5}{2(4\pi)^2(s_\alpha c_\alpha)^4 \Lambda_{NP}} \left\{ (x_{i j})^{1/2} \Omega_{N,1}(x_{i j}; x_{E_i}^0; x_{E_j}^0; x_{E}^0) \\
\times \left[ \text{Im} \left( \left( \xi^I_N \right)_{\beta \gamma} \left( \xi^J_N \right)_{i \alpha} \left( \xi^J_N \right)_{i \alpha}^\dagger \left( \xi^I_N \right)_{\beta \gamma}^\dagger \right) - \text{Im} \left( \left( \eta^I_N \right)_{\beta \gamma} \left( \eta^J_N \right)_{i \alpha} \left( \eta^J_N \right)_{i \alpha}^\dagger \left( \eta^I_N \right)_{\beta \gamma}^\dagger \right) \right] \\
+ (x_{i j} x_{i j}^0 x_{ \chi_3})^{1/2} F_{N,2}(x_{i j}; x_{E_i}^0; x_{E_j}^0; x_{E}^0) \\
\times \left[ \text{Im} \left( \left( \xi^I_N \right)_{\beta \gamma} \left( \xi^J_N \right)_{i \alpha} \left( \xi^J_N \right)_{i \alpha}^\dagger \left( \xi^I_N \right)_{\beta \gamma}^\dagger \right) - \text{Im} \left( \left( \eta^I_N \right)_{\beta \gamma} \left( \eta^J_N \right)_{i \alpha} \left( \eta^J_N \right)_{i \alpha}^\dagger \left( \eta^I_N \right)_{\beta \gamma}^\dagger \right) \right] \\
- x_{i j}^{1/2} \Omega_{N,3}(x_{i j}; x_{E_i}^0; x_{E_j}^0; x_{E_j}^0) \text{Im} \left( \left( \eta^I_N \right)_{\beta \gamma} \left( \xi^J_N \right)_{i \alpha} \left( \xi^J_N \right)_{i \alpha}^\dagger \left( \xi^I_N \right)_{\beta \gamma}^\dagger \right) \\
- x_{i j}^{1/2} \Omega_{N,4}(x_{i j}; x_{E_i}^0; x_{E_j}^0; x_{E_j}^0) \text{Im} \left( \left( \eta^I_N \right)_{\beta \gamma} \left( \eta^J_N \right)_{i \alpha} \left( \eta^J_N \right)_{i \alpha}^\dagger \left( \eta^I_N \right)_{\beta \gamma}^\dagger \right) \right) .
\] (20)

The form factors are expressed as

\[
\Omega_{N,1}(x_{i j}; x_{E_i}^0; x_{E_j}^0; x_{E}^0) = \frac{1}{24} \left\{ \left( 2 + \ln x_R \right) \left[ \rho_1(x_{E_i}^0; x_{E_j}) + \rho_1(x_{E_j}^0; x_{E_i}) \right] + \varphi_1(x_{E_i}^0; x_{E_j}) \right. \\
+ \varphi_2(x_{E_j}^0; x_{E_j}) - \frac{1}{2} \left[ x_{E_i} \frac{\partial^3}{\partial^3 x_{E_i}} \varphi_{2,2}(x_{E_i}^0; x_{E_j}) + x_{E_j} \frac{\partial^3}{\partial^3 x_{E_j}} \varphi_{2,2}(x_{E_j}^0; x_{E_j}) \right] \right\} \\
+ F_{N,1}(x_{i j}; x_{E_i}^0; x_{E_j}^0; x_{E}^0) ,
\]

\[
\Omega_{N,3}(x_{i j}; x_{E_i}^0; x_{E_j}^0; x_{E}^0) = - \frac{1}{4} \left\{ \left( 2 + \ln x_R \right) \rho_2(x_{E_i}^0; x_{E_i}) + \varphi_3(x_{E_i}^0; x_{E_i}) + \frac{1}{2} x_{E_i} \frac{\partial^2}{\partial^2 x_{E_i}} \varphi_{1,2}(x_{E_i}^0; x_{E_i}) \right\} \\
+ F_{N,3}(x_{i j}; x_{E_i}^0; x_{E_j}^0; x_{E_j}^0) ,
\]

\[
\Omega_{N,4}(x_{i j}; x_{E_i}^0; x_{E_j}^0; x_{E}^0) = - \frac{1}{4} \left\{ \left( 2 + \ln x_R \right) \rho_2(x_{E_j}^0; x_{E_j}) + \varphi_3(x_{E_j}^0; x_{E_j}) + \frac{1}{2} x_{E_j} \frac{\partial^2}{\partial^2 x_{E_j}} \varphi_{1,2}(x_{E_j}^0; x_{E_j}) \right\} \\
+ F_{N,4}(x_{i j}; x_{E_i}^0; x_{E_j}^0; x_{E_j}^0) ,
\] (21)

where

\[
\varrho_{m,n}(x_1, x_2) = \frac{x_1^m \ln^m x_1 - x_2^m \ln^m x_2}{x_1 - x_2}
\] (22)
and other functions $\rho_{1,2,3}$, $F_{N,i}$ ($i = 1, \ldots, 4$) are defined in appendix C. In Eq. (24), all terms in the brackets are correct only for the naive dimensional regularization scheme and $\overline{\text{MS}}$ renormalization scheme.

As for the two-loop "neutralino-chargino" corrections, we have

$$
\Delta a_{\mu, \chi^0}^{2L} = - \frac{e^4}{2(4\pi)^2 s_w c_w} \left\{ \frac{2\sqrt{2} r_l m_t}{m_w c_\beta} \text{Re}\left( (\lambda_N^I)^{\dagger}_{3i} (\zeta_C^I)^{\dagger}_{a_j} (\eta_C^I)^{\dagger}_{\alpha_i} (\eta_N^I)_{j\beta} \right) \times \Omega_{M,1} \left( 0; x_{\bar{\nu}_j}, x_{\chi^0}, x_{\chi^0}, x_{\chi^0} \right) + 4 r_l (x_{\chi^0})^{1/2} \text{Re}\left( (\lambda_N^I)^{\dagger}_{3i} (\zeta_C^I)^{\dagger}_{a_j} (\eta_N^I)_{j\beta} \right) \times \left[ F_{M,3} + F_{M,4} \right] \left( 0; x_{\bar{\nu}_j}, x_{\chi^0}, x_{\chi^0}, x_{\chi^0} \right) - 2 r_l (x_{\chi^0})^{1/2} \text{Re}\left( (\lambda_N^I)^{\dagger}_{3i} (\zeta_C^I)^{\dagger}_{a_j} (\eta_N^I)_{j\beta} \right) \times \Omega_{M,1} \left( 0; x_{\bar{\nu}_j}, x_{\chi^0}, x_{\chi^0}, x_{\chi^0} \right) \right\},
$$

(23)

as well as

$$
\Delta a_{\mu, \chi^0}^{2L} = - \frac{e^5}{2(4\pi)^2 s_w c_w^2 \Lambda_{NP}} \left\{ \frac{\sqrt{2} r_l m_t}{m_w c_\beta} \text{Im}\left( (\lambda_N^I)^{\dagger}_{3i} (\zeta_C^I)^{\dagger}_{a_j} (\eta_C^I)^{\dagger}_{\alpha_i} (\eta_N^I)_{j\beta} \right) \times \left[ F_{M,1} - F_{M,2} \right] \left( 0; x_{\bar{\nu}_j}, x_{\chi^0}, x_{\chi^0}, x_{\chi^0} \right) + 2 r_l (x_{\chi^0})^{1/2} \text{Im}\left( (\lambda_N^I)^{\dagger}_{3i} (\zeta_C^I)^{\dagger}_{a_j} (\eta_N^I)_{j\beta} \right) \times \left[ F_{M,3} - F_{M,4} \right] \left( 0; x_{\bar{\nu}_j}, x_{\chi^0}, x_{\chi^0}, x_{\chi^0} \right) - (x_{\chi^0})^{1/2} \text{Im}\left( (\lambda_N^I)^{\dagger}_{3i} (\zeta_C^I)^{\dagger}_{a_j} (\eta_N^I)_{j\beta} \right) \times \Omega_{M,1} \left( 0; x_{\bar{\nu}_j}, x_{\chi^0}, x_{\chi^0}, x_{\chi^0} \right) + \frac{(x_{\chi^0})^{1/2} m_t}{2 m_w c_\beta} \text{Im}\left( (\lambda_N^I)^{\dagger}_{3i} (\zeta_C^I)^{\dagger}_{a_j} (\eta_N^I)_{j\beta} \right) \times \Omega_{M,1} \left( 0; x_{\bar{\nu}_j}, x_{\chi^0}, x_{\chi^0}, x_{\chi^0} \right) \right\},
$$

(24)

where the couplings are defined as

$$(\zeta_C^I)_{\alpha_i} = \left( (R_E)^{3i} (U)_{\alpha_1} - \frac{m_t}{\sqrt{2} m_w c_\beta} (R_E)^{3i} (U)_{\alpha_2} \right),$$

$$(\lambda_N^I)_{\alpha_i} = (R_E)^{3i} \left( (N)_{1\alpha} s_w - (N)_{2\alpha} c_w \right).$$

(25)
With the naive dimensional regularization scheme and $\overline{MS}$ renormalization scheme, the form factors are written as

$$
\Omega_{M,1}(0; x_{\ell_i}, x_{\chi_{\alpha}^\pm}; x_{E_j}, x_{\chi_{\beta}^0}) \\
= -\frac{1}{12} \left\{ (2 + \ln x_R) \left[ \rho_1(x_{\ell_i}, x_{\chi_{\alpha}^\pm}) - \rho_1(x_{\chi_{\beta}^0}, x_{E_j}) \right] + \varphi_1(x_{\ell_i}, x_{\chi_{\alpha}^\pm}) \\
- \varphi_1(x_{\chi_{\beta}^0}, x_{E_j}) \right. \\
- \frac{x}{x_{\chi_{\alpha}^\pm}^2} \frac{\partial^3}{\partial^3 x_{\chi_{\alpha}^\pm}} \left[ 2 \frac{\partial_{2,2}}{\partial^3 x_{\chi_{\alpha}^\pm}} x_{\chi_{\alpha}^\pm}, x_{\ell_i} \right] - x_{E_j} \frac{\partial^3}{\partial^3 x_{E_j}} \left[ 2 \frac{\partial_{2,2}}{\partial^3 x_{E_j}} x_{\chi_{\beta}^0}, x_{E_j} \right] \right\} \\
+ [F_{M,1} + F_{M,2}] \left( x_{\ell_i}; x_{E_j}; x_{\chi_{\alpha}^\pm}; x_{E_j}, x_{\chi_{\beta}^0} \right), \\
\Omega_{M,3}(0; x_{\ell_i}, x_{\chi_{\alpha}^\pm}; x_{E_j}, x_{\chi_{\beta}^0}) \\
= \frac{1}{4} \left\{ (2 + \ln x_R) \varphi_2(x_{\ell_i}, x_{\chi_{\alpha}^\pm}) + \varphi_3(x_{\ell_i}, x_{\chi_{\alpha}^\pm}) - \frac{1}{2} \frac{\partial^2}{\partial^2 x_{\chi_{\alpha}^\pm}} \varphi_2(x_{\ell_i}, x_{\chi_{\alpha}^\pm}) \right. \\
+ F_{M,3}(0; x_{\ell_i}, x_{\chi_{\alpha}^\pm}; x_{E_j}, x_{\chi_{\beta}^0}) \right. \\
+ \frac{1}{4} \left\{ (2 + \ln x_R) \rho_2(x_{E_j}, x_{\chi_{\alpha}^\pm}) + \varphi_3(x_{E_j}, x_{\chi_{\beta}^0}) + \frac{1}{2} \frac{\partial^2}{\partial^2 x_{E_j}} \varphi_3(x_{\chi_{\beta}^0}, x_{E_j}) \right. \\
+ F_{M,3}(0; x_{\ell_i}, x_{\chi_{\alpha}^\pm}; x_{E_j}, x_{\chi_{\beta}^0}) \right\}.
$$

The tedious expressions of functions $F_{M,i}$ $(i = 1, \cdots, 6)$ are put in appendix C. With the naive dimensional regularization scheme and $\overline{MS}$ renormalization scheme, the resulting theoretical predictions on lepton anomalous dipole moments from two-loop "chargino-chargino" diagrams are similarly expressed as

$$
\Delta a_{2L, \chi_{\alpha}^\pm \chi_{\beta}^\pm} = \frac{\epsilon^4}{(4\pi)^2 s_w^2 t_w^2} x_{\ell_i} \left\{ \frac{2 m_w^2}{m_{\chi_{\alpha}^\pm}^2} \text{Re}\left[ \left( \frac{m_{\chi_{\alpha}^\pm}}{m_{\chi_{\beta}^\pm}^2} \text{Re}\left( \left( \eta_{\chi_{\alpha}^\pm}^{J}_i \eta_{\chi_{\beta}^\pm}^{J}_i \right) \overline{\chi_{\alpha}^\pm} \chi_{\beta}^\pm \right) \right) \Omega_{C,1}(x_{\ell_i}; x_{\chi_{\alpha}^\pm}; x_{E_j}, x_{\chi_{\beta}^0}) \right. \\
+ \frac{2 m_w^2}{m_{\chi_{\alpha}^\pm}^2} \text{Re}\left( \left( \eta_{\chi_{\alpha}^\pm}^{J}_i \eta_{\chi_{\beta}^\pm}^{J}_i \overline{\chi_{\alpha}^\pm} \chi_{\beta}^\pm \right) \Omega_{C,2}(x_{\ell_i}; x_{\chi_{\alpha}^\pm}; x_{E_j}, x_{\chi_{\beta}^0}) \right) \\
+ \text{Re}\left( \left( \eta_{\chi_{\alpha}^\pm}^{J}_i \eta_{\chi_{\beta}^\pm}^{J}_i \overline{\chi_{\alpha}^\pm} \chi_{\beta}^\pm \right) \Omega_{C,3}(x_{\ell_i}; x_{\chi_{\alpha}^\pm}; x_{E_j}, x_{\chi_{\beta}^0}) \right) \\
- \frac{m_{\chi_{\alpha}^\pm}^2 m_{\chi_{\beta}^\pm}}{\sqrt{2} m_{\chi_{\alpha}^\pm}^2 c_{\beta}^2} \text{Re}\left( \left( \eta_{\chi_{\alpha}^\pm}^{J}_i \eta_{\chi_{\beta}^\pm}^{J}_i \overline{\chi_{\alpha}^\pm} \chi_{\beta}^\pm \right) \Omega_{C,4}(x_{\ell_i}; x_{\chi_{\alpha}^\pm}; x_{E_j}, x_{\chi_{\beta}^0}) \right) \\
- \frac{\sqrt{2} m_{\chi_{\alpha}^\pm}^2}{m_{\chi_{\beta}^\pm} c_{\alpha} c_{\beta}^2} \text{Re}\left( \left( \eta_{\chi_{\alpha}^\pm}^{J}_i \eta_{\chi_{\beta}^\pm}^{J}_i \overline{\chi_{\alpha}^\pm} \chi_{\beta}^\pm \right) \Omega_{C,5}(x_{\ell_i}; x_{\chi_{\alpha}^\pm}; x_{E_j}, x_{\chi_{\beta}^0}) \right) \right\}.
$$

(27)
and
\[
\Delta a_{LL, x^+ x^\pm} = - \frac{e^5}{(4\pi)^2 s_w^2 \Lambda_{\text{NP}}} (x_{\ell 1})^{1/2} \left\{ \frac{m_{1}^2}{m_w^2 c_{\beta}^2} \text{Im} \left[ (\eta_{C}^I)_{j \beta} (\xi_{C}^J)_{j \alpha} (\xi_{C}^I)_{\alpha j} (\eta_{C}^J)_{\alpha i} \right] \Omega_{C,1}(x_{\ell 1}; x_{\ell 1}, x_{x^\pm}, x_{x^\pm}) \right.
\]
\[
- \frac{m_{1}^2}{m_w^2 c_{\beta}^2} \text{Im} \left[ (\xi_{C}^I)_{j \beta} (\eta_{C}^I)_{j \alpha} (\xi_{C}^I)_{\alpha j} (\eta_{C}^I)_{\alpha i} \right] \Omega_{C,2}(x_{\ell 1}; x_{\ell 1}, x_{x^\pm}, x_{x^\pm}) \right)
\]
\[+ 2(x_{x^\pm} x_{x^\pm})^{1/2} \left\{ \frac{m_{1}^2}{4m_w^2 c_{\beta}^2} \text{Im} \left[ (\eta_{C}^I)_{j \beta} (\eta_{C}^J)_{j \alpha} (\eta_{C}^J)_{\alpha j} (\eta_{C}^I)_{\alpha i} \right] \right\} \Omega_{C,4}(x_{\ell 1}; x_{\ell 1}, x_{x^\pm}, x_{x^\pm})
\]
\[+ \frac{m_{1}^2}{2\sqrt{2}m_w^2 c_{\beta}^2} \text{Im} \left[ (\eta_{C}^I)_{j \beta} (\eta_{C}^J)_{j \alpha} (\eta_{C}^J)_{\alpha j} (\eta_{C}^I)_{\alpha i} \right] \Omega_{C,3}(x_{\ell 1}; x_{\ell 1}, x_{x^\pm}, x_{x^\pm}) \right\} \right) (28)
\]

Here,
\[
\Omega_{C,1}(x_{\ell 1}; x_{\ell 1}, x_{x^\pm}, x_{x^\pm}) = \frac{1}{24} \left\{ \left( 2 + \ln x_{R} \right) \left[ \rho_1(x_{\ell 1}, x_{x^\pm}) + \rho_1(x_{\ell 1}, x_{x^\pm}) \right] \right. \]
\[
+ \varphi_2(x_{\ell 1}, x_{x^\pm}) - \frac{1}{2} \left[ x_{x^\pm} \frac{\partial^2}{\partial^2 x_{x^\pm}} \varphi_{2,2}(x_{x^\pm}, x_{x^\pm}) \right] \right\} \Omega_{C,3}(x_{\ell 1}; x_{\ell 1}, x_{x^\pm}, x_{x^\pm})
\]
\[+ \frac{1}{2} \left\{ \left( 2 + \ln x_{R} \right) \right. \varphi_3(x_{x^\pm}, x_{x^\pm}) + \frac{1}{2} \varphi_3(x_{x^\pm}, x_{x^\pm}) \right. \]
\[
+ \frac{1}{8} \frac{\partial^2}{\partial^2 x_{x^\pm}} \varphi_{2,2}(x_{x^\pm}, x_{x^\pm}) \right\} \Omega_{C,4}(x_{\ell 1}; x_{\ell 1}, x_{x^\pm}, x_{x^\pm})
\]
\[+ \frac{1}{2} \left\{ \left( 2 + \ln x_{R} \right) \right. \varphi_3(x_{x^\pm}, x_{x^\pm}) + \frac{1}{2} \varphi_3(x_{x^\pm}, x_{x^\pm}) \right. \]
\[
+ \frac{1}{8} \frac{\partial^2}{\partial^2 x_{x^\pm}} \varphi_{2,2}(x_{x^\pm}, x_{x^\pm}) \right\} \Omega_{C,4}(x_{\ell 1}; x_{\ell 1}, x_{x^\pm}, x_{x^\pm})
\]
\[+ \left. \left. \right. \right\} \] (29)

where the definitions of the functions $F_{C,i}$, $(i = 1, \cdots, 4)$ can be found in appendix C.

Thus, we obtain the MDMs and EDMs of leptons in the $\overline{\text{MS}}$ renormalization scheme. However, the on-shell renormalization scheme is also adopted frequently to remove the ultraviolet divergence which appears in the radiative electroweak corrections $^{15}$. As an over-subtract scheme, the counter terms include some finite terms which originate from those
renormalization conditions in the on-shell scheme beside the ultra-violet divergence to cancel the corresponding ultra-violet divergence in amplitude. In the concrete calculation performed here, we need the following counter terms to cancel the ultra-violet divergence in the one-loop corrections to the vertex $\tilde{E}_i^* \chi_\alpha \, l^l$

$$\delta C_{E_i^* \chi_\alpha \, l^l} = \left\{ \frac{e}{\sqrt{2} s_w c_w} \left[ \left( \frac{\delta e}{e} \delta l_{ij} + \frac{1}{2} (\delta Z^L_{E_{ij}}) \right) \delta_{\alpha \beta} \delta_{ij} + \frac{1}{2} (\delta Z^l_{E_{ij}}) \delta_{ij} \delta_{\alpha \beta} \right] + \frac{1}{2} \left( \delta Z^\alpha \right)_{\beta \alpha} \left( R^*_{E_{ij}} \right) N_{13} \right\} \omega_-$$

Here, $\delta e$ represents the renormalization correction to electrical charge, $\delta m_w$ and $\delta m_{\ell_{ij}}$ stand the renormalization corrections to the W-boson and lepton masses respectively, $\delta c_w$, $\delta s_w$ as well as $\delta c_\beta$ are the renormalization corrections to parameters $c_w$, $s_w$ and $c_\beta$, and $(\delta Z^L_{E_{ij}})$, $(\delta Z^R_{E_{ij}})$, $(\delta Z^\alpha)$ separately denote the wave function renormalization constants of leptons, sleptons, and neutralinos. In the on-shell scheme, we can fix those renormalization parameters by the mass-shell renormalization conditions [19, 20]. They include UV divergence which cancel the corresponding UV divergence in amplitudes and the finite contributions which are determined by the on-shell condition, to the resultant expression of the finite amplitudes. In a similar way, we can write the counter terms for the one-loop corrections to the vertex $\tilde{\nu}_i^* \chi_\alpha \, l^l$

$$\delta C_{\tilde{\nu}_i^* \chi_\alpha \, l^l} = -\frac{e}{s_w} \left[ \left( \frac{\delta e}{e} - \frac{\delta s_w}{s_w} \right) \delta_{l_{ij}} \delta_{ij} \delta_{\alpha \beta} + \frac{1}{2} (\delta Z^L_{E_{ij}}) \delta_{ij} \delta_{\alpha \beta} + \frac{1}{2} (\delta Z^R_{E_{ij}}) \delta_{ij} \delta_{\alpha \beta} \right] + \frac{1}{2} \left( \delta Z^\alpha \right)_{\beta \alpha} \left( R^*_{E_{ij}} \right) N_{13} \right\} \omega_+ \ . (30)
\[ + \frac{\delta m_w}{m_w} - \frac{\delta s_w}{s_w} \frac{\delta c_\beta}{c_\beta} \delta_{ij} \delta_{ij} \delta_{\alpha\beta} + \frac{1}{2} (\delta Z_\nu R_{ij})_{ll} \delta_{ij} \delta_{\alpha\beta} + \frac{1}{2} (\delta Z_\nu \tilde{\nu})_{ij} \delta_{IJ} \delta_{\alpha\beta} \]
\[ + \frac{1}{2} (\delta Z_{\nu}^*)_{ij} \delta_{\alpha\beta} \left( R_\nu^j \right) \beta \alpha \delta_{ij} \left\{ \omega_+ \right\} (31) \]

with \((\delta Z_\nu)_{ij}\), \((\delta Z_{\nu}^*)_{ij}\) are the wave function renormalization constants of sneutrino and chargino respectively. In order to shorten the length of text, we put the expressions of the theoretical predictions on the MDMs and EDMs of leptons in terms of the on-shell renormalization scheme in the appendix.

So far, we have obtained all the corrections from two-loop supersymmetric diagrams shown in Fig.2. Beside the two loop diagrams discussed here, it is well known that the two-loop Bar-Zee type diagrams also lead to significant contributions to the fermion MDMs and EDMs in the supersymmetric theory [14]. The Bar-Zee diagram corrections to muon MDM are discussed in [10], the contributions of Bar-Zee diagrams to muon EDM are analyzed in [17]. Beside those two-loop diagrams that have been analyzed in literature and the diagrams presented in this work, there are still large amounts of two-loop diagrams that have concrete contributions to muon MDM and EDM. The present status of two-loop calculations cannot be considered as a complete analysis on MDM and EDM of muon in the framework of supersymmetry. In the following section, we will only consider the corrections from those two-loop diagrams in Fig. 2 to the one-loop supersymmetric theoretical predictions on lepton MDM and EDM through numerical method with some assumptions on the parameter space of MSSM.

IV. NUMERICAL RESULTS

With the theoretical formulation derived in previous sections, we numerically analyze the dependence of the muon MDM and EDM on the supersymmetric parameters in this section. Especially, we will present the dependence of the muon MDM and EDM on some supersymmetric \(CP\) phases in some detail here. Within three standard error deviations, the present experimental data can tolerate new physics corrections to the muon MDM as \(-10 \times 10^{-10} < \Delta a_\mu < 52 \times 10^{-10}\). Since the scalar leptons \(\tilde{\nu}_\mu, \tilde{\mu}_{1,2}\) appear as the internal intermediate particles in the two-loop diagrams which are investigated in this work, the corrections of these diagrams will be suppressed strongly when slepton masses are much higher than the electroweak scale. To investigate if those diagrams can result in concrete
corrections to the muon MDM and EDM, we choose a suitable supersymmetric parameter region where the masses of the second generation sleptons are lying in the range $M_{\tilde{\mu}} < 1$ TeV. In this work, we neglect all other possible sources of flavor violation except those due to the CKM matrix, and try to avoid ambiguities of the unification conditions of the soft-breaking parameters at the grand unification scale in the mSUGRA scheme. The MSSM Lagrangian contains several sources of CP violating phases: the phases of the $\mu$ parameter in the superpotential and the corresponding bilinear coupling of the soft breaking terms, three phases of the gaugino mass terms, and the phases of the trilinear sfermion Yukawa couplings in the soft Lagrangian. As we are not considering the spontaneous CP violation in this work, the CP phase of the soft bilinear coupling vanishes due to the tree level neutral Higgs tadpole condition. Moreover, for the model we employ here, the mass of the lightest Higgs boson sets a strong constraint on the parameter space of the new physics. As indicated in the literature [18], the CP violation would cause changes to the neutral-Higgs-quark coupling, neutral Higgs-gauge-boson coupling and self-coupling of Higgs boson. The present experimental lower bound on the mass of the lightest Higgs bosons is relaxed to 60 GeV. In our numerical analysis we will take this constraint for the parameter space into account.

As a cross check, we have compared our one-loop supersymmetric prediction on muon MDM in CP conservation framework with that obtained with corresponding Fortran subroutine on muon MDM in the code *FeynHiggs* [21], and find a perfect agreement. In the two-loop sector, we check our Fortran subroutine on two-loop vacuum integrals with the corresponding programs in the package *FeynHiggs*, and also find that they agree with each other very well. For guaranteeing validity of the results, we also independently develop certain programs for those two-loop integrals to check our two-loop integrals in Fortran code.

Without losing too much generality, we will fix the following values for the supersymmetric parameters: $M_{\tilde{\mu} L} = M_{\tilde{\mu} R} = |A_{\tilde{\mu}}| = 500$ GeV, $|m_1| = |m_2| = 300$ GeV. Taking $\mu = 300$ GeV, we plot the MDM and EDM of muon versus the CP phase $\theta_{\tilde{\mu}} = \arg(A_{\tilde{\mu}})$ for $\tan \beta = 5$ or $\tan \beta = 20$ in Fig.4. As $\tan \beta = 5$, one-loop supersymmetric correction to the MDM of muon (Dash-Dot line) reaches $4 \times 10^{-10}$. With our choice for the parameter space, the two-loop supersymmetric corrections can approximately be as large as 30% of the one-loop results. For $\tan \beta = 20$, one-loop supersymmetric prediction on the MDM of muon (Solid line) is about $17.4 \times 10^{-10}$, whereas the relative correction from two-loop
supersymmetric contribution to one-loop result is approximated as 3%. In other words, the
corrections of these two-loop diagrams turn more and more insignificantly along with the
increase of $\tan \beta$. Actually, the third and fourth terms of neutralino-neutralino contribution
(Eq. 19) and the third term of neutralino-chargino contribution (Eq. 23) dominate the
corrections to muon MDM from the two-loop diagrams in Fig. 2 and those terms depend
on the parameter $\tan \beta$ very mildly. Because the CP phase $\theta_\mu$ affects the anomalous dipole
moments of muon through the mixing matrix $R_\mu$ for sleptons of second generation, this
leads to that the variation of the muon MDM versus $\theta_\mu$ is very gentle. Taking $\theta_\mu = \pm \pi/2$,
the theoretical prediction on muon EDM approximates as $1.8 \times 10^{-24}$ $(e \cdot cm)$ for $\tan \beta = 5$,
and $2.5 \times 10^{-24}$ $(e \cdot cm)$ for $\tan \beta = 20$. The muon EDM of this order can be detected
hopefully in near future experiments with experimental precision $10^{-24}$ $(e \cdot cm)$ \[11\].

Now, we analyze the variation of supersymmetric corrections to the muon anomalous
dipole moments with the CP phase $\theta_1 = \arg(m_1)$. Taking $\mu_H = 300$ GeV, we plot the MDM
and EDM of muon versus the CP phase $\theta_1$ for $\tan \beta = 5$ or $\tan \beta = 20$ in Fig. 5. Here, we
find that muon MDM depends on the CP phase $\theta_1$ very gently. For $\tan \beta = 5$, the one-loop
correction to the muon MDM is about $4 \times 10^{-10}$. With our choice for the supersymmetric
parameters, muon MDM is approximated as $5.5 \times 10^{-10}$ when we include the corrections
from those two-loop diagrams in Fig. 2. As $\tan \beta = 5$, the one-loop contribution to the muon
EDM originates from the "neutralino-slepton" diagram, and two-loop contribution mainly
originates from the "neutralino-neutralino" diagrams. Because the concrete dependence of
one-loop result on the CP phase $\theta_1$ differs from that of two-loop result on $\theta_1$ drastically, it
is easy to understand why the correction to muon EDM from two-loop diagrams becomes
dominant. When $\tan \beta = 20$, the one-loop correction to muon MDM is enhanced, this leads
to that $\Delta a_\mu$ can reach $17.4 \times 10^{-10}$, the correction from those two-loop diagrams to muon
MDM turn insignificant now. However, the theoretical prediction on muon EDM exceeds
the precision of future experiment already.

Perhaps the most interesting subject to study is the variation of muon MDM and EDM
versus the CP phase $\theta_2 = \arg(m_2)$. Taking $\mu_H = 300$ GeV, we plot the MDM and EDM
of muon versus the CP phase $\theta_2$ for $\tan \beta = 5$ or $\tan \beta = 20$ in Fig. 6. Generally, the two-loop correction to the muon MDM is 30% approximately for $\tan \beta = 5$. In the largest CP violation ($\theta_2 = \pm \pi/2$) case, the EDM of muon is large enough and can be experimentally tested with the experimental precision in near future: $10^{-24}$ $(e \cdot cm)$. For
The supersymmetric corrections to the MDM and EDM of muon vary with the CP violating phase $\theta_\mu = \text{arg}(A_\mu)$ when $\mu_H = 300 \text{ GeV}$ and $\tan \beta = 5$ or $\tan \beta = 20$, where the dash-dot lines stand for the results of one loop with $\tan \beta = 5$, the dash-dot-dot lines stand for the results of two loop in $\overline{\text{MS}}$ scheme with $\tan \beta = 5$, the short dash lines stand for the results of two loop in mass shell scheme with $\tan \beta = 5$, the solid lines stand for the results of one loop with $\tan \beta = 20$, the dash lines stand for the results of two loop in $\overline{\text{MS}}$ scheme with $\tan \beta = 20$, and the dot lines stand for the results of two loop in mass shell scheme with $\tan \beta = 20$.

For $\tan \beta = 20$, the one-loop supersymmetric corrections to the MDM and EDM are enhanced drastically. Especially for the muon EDM, it reaches $2 \times 10^{-22} (e\cdot cm)$, which can be detected easily in the future experiment.

In the above analysis, we always suppose $\mu_H > 0$. It is well known that the sign of the one-loop contribution depends on the relative sign of $\mu_H$ and $m_2$. Assuming CP conservation with $\theta_1 = \theta_2 = \theta_\nu = 0$, we plot the muon MDM versus $\mu_H$ as $\tan \beta = 5$ or $\tan \beta = 20$ in Fig. 7. The plot shows that the correction from the two-loop diagrams can be neglected safely when $|\mu_H| \leq 100 \text{ GeV}$. Except the parameter $\mu_H$, another parameter $\tan \beta$ plays an important role in our analysis. Assuming CP conservation and setting $\mu_H = \pm 300 \text{ GeV}$, we
FIG. 5: The supersymmetric corrections to the MDM and EDM of muon vary with the CP violating phase \( \theta_1 = \text{arg}(m_1) \) when \( \mu_{\tilde{H}} = 300 \) GeV and \( \tan \beta = 5 \) or \( \tan \beta = 20 \), where the dash-dot lines stand for the results of one loop with \( \tan \beta = 5 \), the dash-dot-dot lines stand for the results of two loop in \( \overline{\text{MS}} \) scheme with \( \tan \beta = 5 \), the short dash lines stand for the results of two loop in mass shell scheme with \( \tan \beta = 5 \), the solid lines stand for the results of one loop with \( \tan \beta = 20 \), the dash lines stand for the results of two loop in \( \overline{\text{MS}} \) scheme with \( \tan \beta = 20 \), and the dot lines stand for the results of two loop in mass shell scheme with \( \tan \beta = 20 \).

plot the theoretical predictions on muon MDM versus \( \tan \beta \) in Fig. 8. Since the dominant contribution from the two-loop diagrams depends on \( \tan \beta \) weakly, the variation of two-loop correction is not very obvious with the increase of \( \tan \beta \). In other words, the correction from those two-loop diagrams turns insignificant in large \( \tan \beta \) case because the one-loop supersymmetric prediction on muon MDM is proportional to \( \tan \beta \).

All the numerical results are obtained under the assumption \( M_{\tilde{\mu}} < 1 \) (TeV). Since the supersymmetric corrections to the muon MDM and EDM contain a suppression factor \( \Lambda_{\text{EW}}^2/\Lambda_{\text{SUSY}}^2 \), the contributions from the considered diagrams Fig 2 become insignificant if the supersymmetric scale \( \Lambda_{\text{SUSY}} \gg \Lambda_{\text{EW}} \).
FIG. 6: The supersymmetric corrections to the MDM and EDM of muon vary with the CP violating phase $\theta_2 = \text{arg}(m_2)$ when $\mu_H = 300 \text{ GeV}$ and $\tan \beta = 5$ or $\tan \beta = 20$, where the dash-dot lines stand for the results of one loop with $\tan \beta = 5$, the dash-dot-dot lines stand for the results of two loop in $\overline{\text{MS}}$ scheme with $\tan \beta = 5$, the short dash lines stand for the results of two loop in mass shell scheme with $\tan \beta = 5$, the solid lines stand for the results of one loop with $\tan \beta = 20$, the dash lines stand for the results of two loop in $\overline{\text{MS}}$ scheme with $\tan \beta = 20$, and the dot lines stand for the results of two loop in mass shell scheme with $\tan \beta = 20$.

V. CONCLUSIONS

In this work, we analyze some two-loop supersymmetric corrections to the anomalous dipole moments of muon by the effective Lagrangian method. In our calculation, we keep all dimension 6 operators. We remove the ultra-violet divergence caused by the divergent sub-diagrams in the $\overline{\text{MS}}$ and on-shell renormalization schemes respectively. After applying the equation of motion for muon, we derive the muon MDM and EDM. Numerically, we analyze the dependence of muon anomalous dipole moments on CP violation phases. There is an experimentally allowed supersymmetric parameter space where those two-loop corrections
FIG. 7: The supersymmetric corrections to the MDM and EDM of muon vary with $\mu_H$ for $\tan \beta = 5$ or $\tan \beta = 20$ in CP conservation framework, where the dash-dot line stands for the results of one loop with $\tan \beta = 5$, the dash-dot-dot line stands for the results of two loop in $\overline{MS}$ scheme with $\tan \beta = 5$, the short dash line stands for the results of two loop in mass shell scheme with $\tan \beta = 5$, the solid line stands for the results of one loop with $\tan \beta = 20$, the dash line stands for the results of two loop in $\overline{MS}$ scheme with $\tan \beta = 20$, and the dot line stands for the results of two loop in mass shell scheme with $\tan \beta = 20$.

on the muon MDM are significant and cannot be neglected, meanwhile the EDM of muon can be large enough to be experimentally detected with the experimental precision of near future.

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FIG. 8: The supersymmetric corrections to the MDM and EDM of muon vary with $\tan \beta$ for $\mu_L = \pm 300$ GeV in CP conservation framework. Where the dash-dot line stands for the results of one loop with $\mu_L = -300$ GeV, the dash-dot-dot line stands for the results of two loop in $\overline{MS}$ scheme with $\mu_L = -300$ GeV, the short dash line stands for the results of two loop in mass shell scheme with $\mu_L = -300$ GeV, the solid line stands for the results of one loop with $\mu_L = 300$ GeV, the dash line stands for the results of two loop in $\overline{MS}$ scheme with $\mu_L = 300$ GeV, and the dot line stands for the results of two loop in mass shell scheme with $\mu_L = 300$ GeV.

APPENDIX A: THE IDENTITIES FOR TWO-LOOP INTEGRALS

$$
\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{1}{D_0} \left\{ \frac{q_1^2 (q_1 \cdot q_2)^2 - (q_1^2)^2 q_1 \cdot q_2}{(q_2 - q_1)^2 - m_0^2} + \frac{q_2^2 (q_1 \cdot q_2)^2}{q_2^2 - m_2^2} \right\} \equiv 0,
$$

$$
\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{1}{D_0} \left\{ \frac{q_1^2 q_2^4 (q_1 \cdot q_2)^2}{(q_2 - q_1)^2 - m_0^2} + \frac{(q_1^2)^2 q_2^4 q_1 \cdot q_2}{q_2^2 - m_2^2} - \frac{D(q_1^2)^2}{2} \right\} \equiv 0,
$$

$$
\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{1}{D_0} \left\{ \frac{q_1^2 q_2^2 (q_1 \cdot q_2)^2}{(q_2 - q_1)^2 - m_0^2} + \frac{D - 1}{2} q_1^2 q_1 \cdot q_2 \right\} \equiv 0,
$$

$$
\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{1}{D_0} \left\{ \frac{q_1^2 q_2^2 q_3^2 - q_2^4 (q_1 \cdot q_2)^2}{(q_2 - q_1)^2 - m_0^2} + \frac{q_1^2 q_1 \cdot q_2 q_3^2}{q_2^2 - m_2^2} - \frac{D + 1}{2} q_1^2 q_1 \cdot q_2 \right\} \equiv 0,
$$

$$
\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{1}{D_0} \left\{ \frac{(q_1 \cdot q_2)^3}{(q_2 - q_1)^2 - m_0^2} + \frac{(q_1 \cdot q_2)^3 q_1 \cdot q_2}{q_2^2 - m_2^2} \right\} \equiv 0,
$$

$$
\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{1}{D_0} \left\{ \frac{q_1^2 q_1 \cdot q_2 q_3^2 - (q_1^2)^2 q_2}{(q_2 - q_1)^2 - m_0^2} + \frac{q_1^2 q_1 \cdot q_2 q_3^2}{q_2^2 - m_2^2} - q_1^2 q_1 \cdot q_2 \right\} \equiv 0,
$$

$$
\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{1}{D_0} \left\{ \frac{(q_1 \cdot q_2)^3 - (q_1^2)^2 q_2}{(q_2 - q_1)^2 - m_0^2} + \frac{(q_1 \cdot q_2)^3 q_2}{q_2^2 - m_2^2} + \frac{D - 3}{2} q_1^2 q_1 \cdot q_2 \right\} \equiv 0,
$$

\text{27}
\[
\begin{align*}
\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{1}{D_o} & \left\{ (q_1 \cdot q_2)^2 - \frac{q_1^2 q_2^2}{2} \right\} = 0, \\
\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{1}{D_o} & \left\{ (q_1 \cdot q_2)^2 - \frac{q_1^2 q_2^2}{2} \right\} = 0, \\
\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{1}{D_o} & \left\{ (q_1 \cdot q_2)^2 - \frac{q_1^2 q_2^2}{2} \right\} = 0, \\
\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{1}{D_o} & \left\{ (q_1 \cdot q_2)^2 - \frac{q_1^2 q_2^2}{2} \right\} = 0, \\
\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{1}{D_o} & \left\{ (q_1 \cdot q_2)^2 - \frac{q_1^2 q_2^2}{2} \right\} = 0, \\
\int \frac{d^D q_1}{(2\pi)^D} \frac{d^D q_2}{(2\pi)^D} \frac{1}{D_o} & \left\{ (q_1 \cdot q_2)^2 - \frac{q_1^2 q_2^2}{2} \right\} = 0.
\end{align*}
\]

(A1)

with \( D_o = ((q_2 - q_1)^2 - m_0^2)(q_1^2 - m_1^2)(q_2^2 - m_2^2) \).

**APPENDIX B: THE FORM FACTORS**

\[
\begin{align*}
\left( N^\alpha_{\chi_0, \chi_0} \right) = \frac{-24}{D(D+2)} \frac{(q_1^2)^2 q_1 \cdot q_2 - (q_1^2)^2 q_2^2}{(q_1^2 - m_1^2)^3} + \frac{4}{D(D+2)} \frac{(q_1^2)^2 - q_1^2 q_2^2}{(q_1^2 - m_1^2)^2} \\
+ \frac{8}{D(D+2)} \frac{3q_1^2 q_1 \cdot q_2 q_2^2 - 2q_2^2 (q_1 \cdot q_2)^2 - (q_1^2)^2 q_2^2}{(q_1^2 - m_1^2)^2(q_2^2 - m_2^2)} \\
+ \frac{8}{D(D+2)} \frac{2(q_1 \cdot q_2)^2 q_2^2 - 3q_1^2 q_1 \cdot q_2 q_2^2 + q_1^2 (q_2^2)^2}{(q_1^2 - m_1^2)^2(q_2^2 - m_2^2)} \\
+ \frac{2}{D(D+2)} \frac{3q_1^2 q_1 \cdot q_2 - 2(q_1 \cdot q_2)^2 q_2^2}{(q_1^2 - m_1^2)(q_2^2 - m_2^2)} \\
+ \frac{2}{D(D+2)} \frac{q_1^2 q_1 \cdot q_2}{(q_1^2 - m_1^2)(q_2^2 - m_2^2)}.
\end{align*}
\]
\[
\left( N_{a, \alpha, x} \right)_2 = \frac{6}{D(D + 2)} \left( \frac{(q_1^2)^2 q_1 \cdot q_2 - (q_1^2)^2 q_2^2}{(q_1^2 - m_{E_j}^2)^3} + \frac{1}{D} \frac{q_1^2 q_2^2 - q_1^2 q_1 \cdot q_2}{(q_1^2 - m_{E_j}^2)^2} \right) \\
+ \frac{8}{D} \frac{q_1^2 q_2^2 - q_1 \cdot q_2 q_2^2}{(q_2^2 - m_{x, \alpha}^2)} - \frac{q_1^2 - q_1 \cdot q_2}{q_2^2 - m_{x, \alpha}^2},
\]

\[
\left( N_{a, \alpha, x} \right)_3 = \frac{6}{D(D + 2)} \left( \frac{(q_1^2)^2 q_1 \cdot q_2 - (q_1^2)^2 q_2^2}{(q_1^2 - m_{E_j}^2)^3} + \frac{1}{D} \frac{2(q_1^2)^2 - q_1^2 q_1 \cdot q_2 - q_1^2 q_2^2}{(q_1^2 - m_{E_j}^2)^2} \right) \\
+ \frac{2}{(q_1^2 - m_{E_j}^2)^2(q_2^2 - m_{x, \alpha}^2)} \left[ \frac{(q_1^2)^2 q_2^2 - q_1^2 q_1 \cdot q_2 q_2^2}{D(D + 2)} \right] \\
+ (D - 4) \cdot \frac{q_1^2(q_1 \cdot q_2^2) - (q_1^2)^2 q_2^2}{D(D - 1)(D + 2)} \\
- \frac{6}{D(D + 2)} \frac{q_1^2(q_2^2)^2 - q_1^2 q_1 \cdot q_2 q_2^2}{(q_1^2 - m_{E_j}^2)(q_2^2 - m_{x, \alpha}^2)^2} - \frac{1}{2D} \frac{3q_1^2 q_1 \cdot q_2 - 2q_1^2 q_2 q_2^2}{(q_1^2 - m_{E_j}^2)(q_2^2 - m_{x, \alpha}^2)} \\
- \frac{1}{(q_1^2 - m_{E_j}^2)(q_2^2 - m_{E_i}^2)} \left[ \frac{q_1^2 q_1 \cdot q_2}{2D} + \frac{(q_1 \cdot q_2)^2 - q_1^2 q_2^2}{D - 1} \right] \\
- \frac{6}{D(D + 2)} \frac{q_1 \cdot q_2(q_2^2)^2 - q_1^2(q_2^2)^2}{(q_2^2 - m_{x, \alpha}^2)^3} - \frac{1}{2D} \frac{2q_1^2 q_2 - q_1 \cdot q_2 q_2^2}{(q_2^2 - m_{x, \alpha}^2)^2} \\
- \frac{1}{2D} \frac{q_1 \cdot q_2 q_2^2}{(q_2^2 - m_{E_i}^2)} + \frac{q_1^2 + 2q_1 \cdot q_2}{4(q_2^2 - m_{E_i}^2)} + \frac{D - 2}{2D} \frac{q_1 \cdot q_2}{q_2^2 - m_{E_i}^2}. 
\]
\[
\left(\mathcal{N}^a_{\alpha,\chi,\gamma}\right)_4 = \frac{4}{D(D+2)} \frac{(q_1^2)^2 q_1 \cdot q_2 - (q_1^2)^2 q_2^2}{(q_1^2 - m_{\chi}^2)^3} - \frac{4}{D(D+2)} \frac{q_1 \cdot q_2 (q_2^2)^2 - q_1 (q_2^2)^2}{(q_2^2 - m_{\chi}^2)^3} \\
+ \frac{4}{(q_1^2 - m_{\chi}^2)^2 (q_2^2 - m_{\chi}^2)} \left[ \frac{(q_2^2)^2 q_2^2 - q_1^2 q_1 \cdot q_2^2}{D(D+2)} + \frac{q_1 (q_1 \cdot q_2)^2 - q_1 (q_2^2)^2}{(D-1)(D+2)} \right] \\
- \frac{4}{(q_1^2 - m_{\chi}^2) (q_2^2 - m_{\chi}^2)^2} \left[ \frac{q_2^2 q_2^2 - q_1^2 q_1 \cdot q_2 q_2^2}{D(D+2)} + \frac{q_1 (q_1 \cdot q_2)^2 - q_1 (q_2^2)^2}{(D-1)(D+2)} \right] \\
- \frac{1}{(q_2^2 - m_{\chi}^2)} \left[ \frac{q_2^2 q_1 \cdot q_2 - 2(q_1 \cdot q_2)^2}{D} - 2 \left( \frac{q_1 \cdot q_2)^2 - q_1 q_2^2}{D(D-1)} \right) \right] \\
+ \frac{1}{D} \frac{q_1^2 q_1 \cdot q_2}{(q_1^2 - m_{\chi}^2)^2} - \frac{1}{D(D+2)} \frac{q_1^2 - q_1 \cdot q_2}{q_1^2 - m_{\chi}^2} - \frac{1}{3} \frac{q_1^2 - q_1 \cdot q_2}{q_2^2 - m_{\chi}^2} \\
- \frac{2}{3D} \frac{6(q_1 \cdot q_2)^2 + 8q_1^2 q_2^2 - 11q_1 \cdot q_2 q_2^2}{(q_2^2 - m_{\chi}^2)^2} + \frac{1}{3D} \frac{2q_1^2 q_2^2 + q_1 \cdot q_2 q_2^2}{(q_2^2 - m_{\chi}^2)(q_2^2 - m_{\chi}^2)} \\
+ \frac{1}{q_2^2 - m_{\chi}^2} \left( \frac{q_2^2}{6} - \frac{2D + 3}{3D} q_1 \cdot q_2 \right) + \frac{D - 2}{3D} \frac{q_1 \cdot q_2}{(q_2^2 - q_1^2)^2 - m_{\chi}^2} ,
\]

\[
\left(\mathcal{N}^b_{\alpha,\chi,\gamma}\right)_1 = \frac{24}{D(D+2)} \frac{(q_1^2)^2 - q_1^2 q_1 \cdot q_2}{(q_1^2 - m_{\chi}^2)^3} + \frac{24}{D(D+2)} \frac{q_1 \cdot q_2 q_2^2 - (q_2^2)^2}{(q_2^2 - m_{\chi}^2)^3} \\
+ \frac{8}{D(D+2)} \frac{3q_1 q_1 \cdot q_2 - 4(q_1 \cdot q_2)^2 + q_1^2 q_2^2}{(q_1^2 - m_{\chi}^2)^2 (q_2^2 - m_{\chi}^2)} \\
- \frac{4}{D} \frac{q_1^2 - q_1 \cdot q_2}{(q_1^2 - m_{\chi}^2)^2} + \frac{8}{D(D+2)} \frac{5(q_1 \cdot q_2)^2 - 2q_1^2 q_2^2 - 3q_1 \cdot q_2 q_2^2}{(q_1^2 - m_{\chi}^2)(q_2^2 - m_{\chi}^2)^2} \\
+ \frac{2}{D} \frac{q_2^2 - q_1^2}{(q_2^2 - m_{\chi}^2)^2} - \frac{4}{D} \frac{q_1 \cdot q_2 - q_2^2}{(q_2^2 - m_{\chi}^2)^2} ,
\]

\[
\left(\mathcal{N}^b_{\alpha,\chi,\gamma}\right)_2 = -\frac{6}{D(D+2)} \frac{(q_1^2)^2 - q_1^2 q_1 \cdot q_2}{(q_1^2 - m_{\chi}^2)^3} + \frac{1}{D} \frac{q_1^2 - q_1 \cdot q_2}{(q_1^2 - m_{\chi}^2)^2} \\
- \frac{6}{(q_1^2 - m_{\chi}^2)^2 (q_2^2 - m_{\chi}^2)} \left[ \frac{q_1^2 q_1 \cdot q_2 - q_1^2 q_2^2}{D(D+2)} - \frac{(q_1 \cdot q_2)^2 - q_1^2 q_2^2}{(D-1)(D+2)} \right] \\
- \frac{2}{(q_1^2 - m_{\chi}^2) (q_2^2 - m_{\chi}^2)^2} \left[ 3 \left( \frac{q_1 \cdot q_2)^2 - q_1 q_2 q_2^2}{D(D+2)} \right) \right] \\
- \left( 2D + 1 \right) \frac{(q_1 \cdot q_2)^2 - q_1^2 q_2^2}{D(D-1)(D+2)} + \frac{1}{2D} \frac{q_1 \cdot q_2}{(q_1^2 - m_{\chi}^2)(q_2^2 - m_{\chi}^2)} \\
+ \frac{1}{2D} \frac{2q_1^2 - q_1 \cdot q_2}{(q_1^2 - m_{\chi}^2)(q_2^2 - m_{\chi}^2)} - \frac{6}{D(D+2)} \frac{q_1 \cdot q_2 q_2^2 - (q_2^2)^2}{(q_2^2 - m_{\chi}^2)^3} \\
+ \frac{1}{2D} \frac{q_1 \cdot q_2}{(q_2^2 - m_{\chi}^2)^2} + \frac{1}{2D} \frac{q_1 \cdot q_2}{(q_2^2 - m_{\chi}^2)(q_2^2 - m_{\chi}^2)} - \frac{1}{4(q_2^2 - m_{\chi}^2)} ,
\]
\[
\left( N_{x_0, \chi_0}^b \right)_3 = -\frac{6}{D(D+2)} \left( q_1^2 - q_2^2 \right)\frac{q_1 q_2 - q_1 q_2}{(q_1^2 - m_{\chi_0})^3} + \frac{1}{D} \left( q_1^2 - q_2 q_2 \right) + \frac{1}{D \left( q_1^2 - m_{\chi_0} \right)^2} \left( q_1^2 - q_2^2 \right) \frac{q_1 q_2 - q_2 q_2}{(q_1^2 - m_{\chi_0})^3} + \frac{1}{2D} \left( q_1^2 - m_{\chi_0} \right)^2 \left( q_2^2 - m_{\chi_0} \right)^2 (D+1)(D+2) + \frac{1}{2D} \left( q_1^2 - m_{\chi_0} \right)^2 \left( q_2^2 - m_{\chi_0} \right)^2 (D+1)(D+2)
\]

\[
\left( N_{x_0, \chi_0}^b \right)_4 = -\frac{4}{D(D+2)} \left( q_1^2 - q_2^2 \right)\frac{q_1 q_2 - q_1 q_2}{(q_1^2 - m_{\chi_0})^3} + \frac{1}{D} \left( q_1^2 - q_2 q_2 \right) + \frac{1}{D \left( q_1^2 - m_{\chi_0} \right)^2} \left( q_1^2 - q_2^2 \right) \frac{q_1 q_2 - q_2 q_2}{(q_1^2 - m_{\chi_0})^3} + \frac{1}{2D} \left( q_1^2 - m_{\chi_0} \right)^2 \left( q_2^2 - m_{\chi_0} \right)^2 (D+1)(D+2) + \frac{1}{2D} \left( q_1^2 - m_{\chi_0} \right)^2 \left( q_2^2 - m_{\chi_0} \right)^2 (D+1)(D+2)
\]

\[
\left( N_{x_0, \chi_0}^b \right)_5 = \frac{4}{D} \left( q_1^2 - q_2^2 \right)\frac{q_1 q_2 - q_1 q_2}{(q_1^2 - m_{\chi_0})^3} + \frac{4}{D \left( q_1^2 - m_{\chi_0} \right)^2} \left( q_1^2 - q_2 q_2 \right) + \frac{4}{D \left( q_1^2 - m_{\chi_0} \right)^2} \left( q_1^2 - q_2^2 \right) \left( q_1^2 - q_2^2 \right) \left( q_1^2 - m_{\chi_0} \right)^2 \left( q_2^2 - m_{\chi_0} \right)^2 (D+1)(D+2) + \frac{4}{D \left( q_1^2 - m_{\chi_0} \right)^2} \left( q_1^2 - q_2 q_2 \right) + \frac{4}{D \left( q_1^2 - m_{\chi_0} \right)^2} \left( q_1^2 - q_2^2 \right) \left( q_1^2 - q_2^2 \right) \left( q_1^2 - m_{\chi_0} \right)^2 \left( q_2^2 - m_{\chi_0} \right)^2 (D+1)(D+2)
\]

\[
\left( N_{x_0, \chi_0}^b \right)_6 = \frac{2}{D} \left( q_1^2 - q_2^2 \right)\frac{q_1 q_2 - q_1 q_2}{(q_1^2 - m_{\chi_0})^3} + \frac{2}{D \left( q_1^2 - m_{\chi_0} \right)^2} \left( q_1^2 - q_2 q_2 \right) + \frac{2}{D \left( q_1^2 - m_{\chi_0} \right)^2} \left( q_1^2 - q_2^2 \right) \left( q_1^2 - q_2^2 \right) \left( q_1^2 - m_{\chi_0} \right)^2 \left( q_2^2 - m_{\chi_0} \right)^2 (D+1)(D+2) + \frac{2}{D \left( q_1^2 - m_{\chi_0} \right)^2} \left( q_1^2 - q_2 q_2 \right) + \frac{2}{D \left( q_1^2 - m_{\chi_0} \right)^2} \left( q_1^2 - q_2^2 \right) \left( q_1^2 - q_2^2 \right) \left( q_1^2 - m_{\chi_0} \right)^2 \left( q_2^2 - m_{\chi_0} \right)^2 (D+1)(D+2)
\]

\((\mathcal{N}^d_{\alpha \nu \gamma})_5 = \frac{-4}{D} \frac{q_1^2 q_2 - q_1^2}{q_1^2} + \frac{4}{D} \frac{(q_1^2 q_2 - q_2^2)}{(q_1^2 - m^2 \epsilon_j)^2} \)
\((\mathcal{N}^d_{\alpha \nu \gamma})_6 = \frac{-4}{D} \frac{q_1^2 q_2 - q_1^2}{q_1^2} + \frac{2}{D} \frac{2 q_1^2 - q_1 q_2}{q_1^2 - m^2 \epsilon_j} + \frac{4}{D} \frac{q_1 q_2 q_3^2 - (q_2^2)^2}{(q_2^2 - m^2 \epsilon_j)^2} \)
\(2 + D \frac{q_1 q_2 - q_2^2}{q_2^2} \)
\(2 + D \frac{q_1 q_2 - q_2^2}{q_2^2} \)

APPENDIX C: THE FUNCTIONS

\[
\rho_1(x_1, x_2) = -6x_1 x_2 \frac{\ln x_1 - \ln x_2}{(x_1 - x_2)^4} + \frac{2x_1^2 + 5x_1 x_2 - x_2^2}{(x_1 - x_2)^3},
\]
\[
\rho_2(x_1, x_2) = 2x_1 x_2 \frac{\ln x_1 - \ln x_2}{(x_1 - x_2)^3} - \frac{x_1 + x_2}{(x_1 - x_2)^2},
\]
\[
\varphi_1(x_1, x_2) = -(2x_1^3 + 3x_1^2 x_2) \frac{\ln x_1 - \ln x_2}{(x_1 - x_2)^4} + \frac{28x_1^2 + x_1 x_2 + x_2^2}{6(x_1 - x_2)^3},
\]
\[
\varphi_2(x_1, x_2) = -(6x_1 x_2 - x_2^2) \frac{\ln x_1 - \ln x_2}{(x_1 - x_2)^4} - \frac{8x_1^2 - 37x_1 x_2 - x_2^2}{6(x_1 - x_2)^3},
\]
\[
\varphi_3(x_1, x_2) = x_2 \frac{\ln x_1 - \ln x_2}{(x_1 - x_2)^3} + \frac{x_1 - 3x_2}{2(x_1 - x_2)^2}.
\]

\[
\Psi_{3a}(x_0; x_A, x_B; x_0, x_0) = \frac{1}{24} \left\{ \left[ 2 \theta_{3,1} + \theta_{3,2} \right](x_A, x_B) + \frac{1}{(x_A - x_B)(x_0 - x_0)} \right\} \left[ x_A^3 x_0 \ln^2(x_A x_0) - x_A^3 x_0 \ln^2(x_A x_0) - x_B^3 x_0 \ln^2(x_B x_0) + x_B^3 x_0 \ln^2(x_B x_0) \right.
\]
\[
\left. + x_A^3 (x_0 - x_0) \Phi(x_0, x_A, x_0) - x_A^3 x_0 - x_0 + x_0) \Phi(x_0, x_A, x_0) \right. \left. + x_B^3 (x_0 - x_B + x_0) \Phi(x_0, x_B, x_0) + x_B^3 (x_0 - x_B + x_0) \Phi(x_0, x_B, x_0) \right\}.
\]

\[
\Psi_{3b}(x_0; x_A, x_B; x_0, x_0)
\]

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\[
\Psi_{3b}(x_0; x_A, x_B; x_\alpha, x_\beta) = \frac{1}{16} \left\{ \begin{array}{l}
\frac{1}{8} \left[ (x_\alpha + x_\beta) \left[ 2 \ell_{2,1} - \ell_{2,2} \right] (x_A, x_B) + \left( x_A^2 + x_A x_B + x_B^2 \right) \left[ 2 \ell_{1,1} - \ell_{1,2} \right] (x_\alpha, x_\beta) \\
+ \frac{1}{(x_A - x_B) (x_\alpha - x_\beta)} \left( x_A^3 x_\alpha + x_A^2 x_\alpha^2 - x_\alpha x_\beta^2 \right) \ln^2 (x_A x_\alpha) \\
- \left( x_A^3 x_\beta + x_A^2 x_\beta^2 - x_\alpha x_\beta^2 \right) \ln^2 (x_A x_\beta) - \left( x_B^3 x_\alpha + x_B^2 x_\alpha^2 - x_\alpha x_\beta^2 \right) \ln^2 (x_B x_\alpha) \\
+ \left( x_B^3 x_\beta + x_B^2 x_\beta^2 - x_\alpha x_\beta^2 \right) \ln^2 (x_B x_\beta) - x_A \lambda^2 (x_0, x_A, x_\alpha) \Phi(x_0, x_A, x_\alpha) \\
+ x_A \lambda^2 (x_0, x_A, x_\beta) \Phi(x_0, x_A, x_\beta) + x_B \lambda^2 (x_0, x_B, x_\alpha) \Phi(x_0, x_B, x_\alpha) \\
- x_B \lambda^2 (x_0, x_B, x_\beta) \Phi(x_0, x_B, x_\beta) \end{array} \right\}.
\]
\]

\[
\Psi_{2a}(x_0; x_A, x_B; x_\alpha, x_\beta) = \frac{1}{2} \left\{ \begin{array}{l}
\frac{1}{2 \ell_{2,1}} \left[ (x_A + x_B) \left[ 2 \ell_{1,1} - \ell_{1,2} \right] (x_\alpha, x_\beta) \\
+ \frac{1}{(x_A - x_B) (x_\alpha - x_\beta)} \left( x_A^2 \Phi(x_0, x_A, x_\alpha) - x_A^2 \Phi(x_0, x_A, x_\beta) \\
-x_B^2 \Phi(x_0, x_B, x_\alpha) + x_B^2 \Phi(x_0, x_B, x_\beta) \right) \end{array} \right\}.
\]

\[
\Psi_{2b}(x_0; x_A, x_B; x_\alpha, x_\beta) = \frac{1}{4} \left\{ \begin{array}{l}
\frac{1}{2 \ell_{2,1}} \left[ (x_A + x_B) \left[ 2 \ell_{1,1} - \ell_{1,2} \right] (x_\alpha, x_\beta) \\
+ \frac{1}{(x_A - x_B) (x_\alpha - x_\beta)} \left( x_A x_\alpha \ln^2 (x_A x_\alpha) - x_A x_\beta \ln^2 (x_A x_\beta) \\
-x_B x_\alpha \ln^2 (x_B x_\alpha) + x_B x_\beta \ln^2 (x_B x_\beta) + x_A (x_0 - x_A - x_\alpha) \Phi(x_0, x_A, x_\alpha) \\
-x_A (x_0 - x_A - x_\beta) \Phi(x_0, x_A, x_\beta) - x_B (x_0 - x_B - x_\alpha) \Phi(x_0, x_B, x_\alpha) \\
+ x_B (x_0 - x_B - x_\beta) \Phi(x_0, x_B, x_\beta) \right) \end{array} \right\}.
\]

\[
\Psi_{2c}(x_0; x_A, x_B; x_\alpha, x_\beta)
\]
\[
= \frac{1}{8} \left\{ \begin{array}{l}
(x_\alpha + x_\beta) \left[ 2 \theta_{1,1} - \theta_{1,2} \right] (x_A, x_B) + (x_A + x_B) \left[ 2 \theta_{1,1} - \theta_{1,2} \right] (x_\alpha, x_\beta) \\
+ \frac{1}{(x_A - x_B)(x_\alpha - x_\beta)} \left( (x_A + x_\alpha - x_0)x_A x_\alpha \ln^2(x_A x_\alpha) \\
- (x_A + x_\beta - x_0)x_A x_\beta \ln^2(x_A x_\beta) - (x_B + x_\alpha - x_0)x_B x_\alpha \ln^2(x_B x_\alpha) \\
+ (x_B + x_\beta - x_0)x_B x_\beta \ln^2(x_B x_\beta) - (x_\alpha - x_A - x_\beta)^2 \Phi(x_\alpha, x_A, x_\alpha) \\
+ (x_\alpha - x_A - x_\beta)^2 \Phi(x_\alpha, x_A, x_\beta) + (x_\alpha - x_B - x_\beta)^2 \Phi(x_\alpha, x_B, x_\alpha) \\
- (x_\alpha - x_B - x_\beta)^2 \Phi(x_\alpha, x_B, x_\beta) \right) \right\}. \\
\right.
\] (C7)

\[
\Psi_{2a}(x_0; x_A, x_B; x_\alpha, x_\beta)
= -\frac{1}{2} \left\{ \begin{array}{l}
\theta_{2,2}(x_A, x_B) + \theta_{2,2}(x_\alpha, x_\beta) + \frac{1}{(x_A - x_B)(x_\alpha - x_\beta)} \left( x_A x_\alpha \Phi(x_0, x_A, x_\alpha) \\
- x_A x_\beta \Phi(x_0, x_A, x_\beta) - x_B x_\alpha \Phi(x_0, x_B, x_\alpha) + x_B x_\beta \Phi(x_0, x_B, x_\beta) \right) \right\}. \\
\right.
\] (C8)

\[
\Psi_{1a}(x_0; x_A, x_B; x_\alpha, x_\beta)
= -\frac{1}{2} \left\{ \begin{array}{l}
\theta_{1,2}(x_\alpha, x_\beta) + \frac{1}{(x_A - x_B)(x_\alpha - x_\beta)} \left[ x_A \Phi(x_0, x_A, x_\alpha) \\
- x_A \Phi(x_0, x_A, x_\beta) - x_B \Phi(x_0, x_B, x_\alpha) + x_B \Phi(x_0, x_B, x_\beta) \right] \right\}. \\
\right.
\] (C9)

\[
\Psi_{1b}(x_0; x_A, x_B; x_\alpha, x_\beta)
= \frac{1}{4} \left\{ \begin{array}{l}
2 \theta_{1,1} - \theta_{1,2} \left[ x_A, x_B \right] + 2 \theta_{1,1} - \theta_{1,2} \left[ x_\alpha, x_\beta \right] \\
+ \frac{1}{(x_A - x_B)(x_\alpha - x_\beta)} \left( x_A x_\alpha \ln^2(x_A x_\alpha) - x_A x_\beta \ln^2(x_A x_\beta) \\
- x_B x_\alpha \ln^2(x_B x_\alpha) + x_B x_\beta \ln^2(x_B x_\beta) + (x_\alpha - x_A - x_\beta) \Phi(x_0, x_A, x_\alpha) \\
- (x_\alpha - x_A - x_\beta) \Phi(x_0, x_A, x_\beta) - (x_\alpha - x_B - x_\beta) \Phi(x_0, x_B, x_\alpha) \\
+ (x_\alpha - x_B - x_\beta) \Phi(x_0, x_B, x_\beta) \right) \right\}. \\
\right.
\] (C10)

\[
\Psi_6(x_0; x_A, x_B; x_\alpha, x_\beta)
= \frac{1}{(x_A - x_B)(x_\alpha - x_\beta)} \left[ \Phi(x_0, x_A, x_\alpha) - \Phi(x_0, x_A, x_\beta) \\
- \Phi(x_0, x_B, x_\alpha) + \Phi(x_0, x_B, x_\beta) \right]. \\
\] (C11)
\begin{align}
F_{N,1}(x_{i,j}; x_{E_i}^0, x_{x_0}^0; x_{E_j}^0, x_{x_0}^0) & = \left\{ \frac{1}{4} \frac{\partial^3}{\partial^3 x_{E_j}^0} \Psi_{3a} + \frac{1}{8} \frac{\partial^2}{\partial^2 x_{E_j}^0} \left[ \Psi_{2d} - \Psi_{2b} \right] + \frac{1}{4} \frac{\partial^3}{\partial^3 x_{x_0}^0} \Psi_{3b} + \frac{1}{3} \frac{\partial^3}{\partial^3 x_{E_j}^0} \partial x_{x_0}^0 \Psi_{3c}
- \frac{3}{8} \frac{\partial^2}{\partial x_{E_j}^0 \partial x_{x_0}^0} \Psi_{2b} + \frac{1}{8} \frac{\partial^2}{\partial x_{E_j}^0 \partial x_{x_0}^0} \Psi_{2d} + \frac{1}{4} \frac{\partial^2}{\partial x_{E_j}^0 \partial x_{E_i}^0} \left[ \Psi_{2b} - \Psi_{2d} \right] + \frac{1}{4} \frac{\partial}{\partial x_{E_j}^0} \Psi_{1a}
+ \frac{1}{8} \frac{\partial}{\partial x_{x_0}^0} \left[ 2\Psi_{1a} + \Psi_{1b} \right] \right\} (x_{i,j}; x_{E_j}^0, x_{x_0}^0; x_{E_i}^0, x_{x_0}^0) \\
- \left\{ \frac{1}{4} \frac{\partial^3}{\partial^3 x_{x_0}^0} \Psi_{3b} + \frac{1}{4} \frac{\partial^3}{\partial^3 x_{x_0}^0} \Psi_{3a} + \frac{1}{8} \frac{\partial^2}{\partial^2 x_{x_0}^0} \left[ 2\Psi_{2d} - \Psi_{2b} \right] \\
+ \frac{1}{8} \frac{\partial^2}{\partial x_{x_0}^0 \partial x_{E_i}^0} \Psi_{2b} \right\} (x_{i,j}; x_{E_i}^0, x_{x_0}^0; x_{E_j}^0, x_{x_0}^0). \quad (C12)\end{align}

\begin{align}
F_{N,2}(x_{i,j}; x_{E_i}^0, x_{x_0}^0; x_{E_j}^0, x_{x_0}^0) & = \left\{ \frac{1}{24} \frac{\partial^3}{\partial^3 x_{E_j}^0} \left[ \Psi_{2a} - \Psi_{2b} \right] + \frac{1}{8} \frac{\partial^2}{\partial x_{E_j}^0} \left[ \Psi_{2a} - \Psi_{2b} \right] - \frac{1}{24} \frac{\partial^2}{\partial x_{E_j}^0} \partial x_{x_0}^0 \Psi_{1a} \\
- 4\Psi_{2c} + \Psi_{2d} \right\} + \frac{1}{8} \frac{\partial^2}{\partial x_{E_i}^0 \partial x_{x_0}^0} \Psi_{1b} + \frac{1}{8} \frac{\partial^2}{\partial x_{E_j}^0 \partial x_{x_0}^0} \left[ 2\Psi_{1a} - \Psi_{1b} \right] \\
- \frac{1}{4} \frac{\partial}{\partial x_{E_i}^0} \Psi_{1a} \right\} (x_{i,j}; x_{E_i}^0, x_{x_0}^0; x_{E_j}^0, x_{x_0}^0) \\
+ \left\{ \frac{1}{8} \frac{\partial^2}{\partial^2 x_{E_j}^0 \partial x_{x_0}^0} \left[ \Psi_{2b} - \Psi_{2d} \right] + \frac{1}{24} \frac{\partial^2}{\partial^2 x_{x_0}^0} \left[ \Psi_{2a} - \Psi_{2b} \right] + \frac{1}{8} \frac{\partial^2}{\partial x_{x_0}^0} \Psi_{1b} \\
+ \frac{1}{8} \frac{\partial^2}{\partial x_{x_0}^0 \partial x_{E_i}^0} \Psi_{1b} \right\} (x_{i,j}; x_{E_i}^0, x_{x_0}^0; x_{E_j}^0, x_{x_0}^0). \quad (C13)\end{align}

\begin{align}
F_{N,3}(x_{i,j}; x_{E_i}^0, x_{x_0}^0; x_{E_j}^0, x_{x_0}^0) & = \left\{ - \frac{1}{4} \frac{\partial^2}{\partial^2 x_{E_j}^0} \left[ \Psi_{2a} - \Psi_{2b} \right] + \frac{1}{2} \frac{\partial}{\partial x_{E_j}^0} \left[ \Psi_{1a} - \Psi_{1b} \right] \\
- \frac{1}{6} \frac{\partial^2}{\partial x_{x_0}^0} \left[ 3\Psi_{2b} - 4\Psi_{2c} + \Psi_{2d} \right] + \frac{1}{2} \frac{\partial}{\partial x_{x_0}^0} \left[ \Psi_{1a} - \Psi_{1b} \right] \\
+ \frac{1}{4} \frac{\partial}{\partial x_{E_j}^0} \left[ \Psi_{1a} - \Psi_{1b} \right] \right\} (x_{i,j}; x_{E_j}^0, x_{x_0}^0; x_{E_i}^0, x_{x_0}^0) \\
+ \frac{1}{4} \frac{\partial^2}{\partial^2 x_{x_0}^0} \left[ \Psi_{2b} - \Psi_{2d} \right] (x_{i,j}; x_{E_i}^0, x_{x_0}^0; x_{E_j}^0, x_{x_0}^0). \quad (C14)\end{align}
\[
\begin{align*}
F_{N,4}(x_j; x_{E_i}, x_{\alpha_0}; x_{E_j}, x_{\beta_0}) &= \left\{ -\frac{1}{6} \frac{\partial^2}{\partial x_{E_j} \partial x_{\alpha_0}} \left[ 3\Psi_{2b} - 4\Psi_{2c} + \Psi_{2d} \right] - \frac{1}{4} \frac{\partial^2}{\partial x_{\alpha_0}^2} \left[ \Psi_{2a} - \Psi_{2b} \right] + \frac{1}{2} \frac{\partial}{\partial x_{E_j}} \\
&\quad+ \frac{3}{4} \frac{\partial}{\partial x_{\alpha_0}} - \frac{1}{4} \frac{\partial}{\partial x_{E_i}} + \frac{1}{4} \frac{\partial}{\partial x_{E_j}} \left[ \Psi_{1a} - \Psi_{1b} \right] \right\} (x_j; x_{E_i}, x_{\alpha_0}; x_{E_j}, x_{\beta_0}) \\
&\quad+ \frac{1}{4 \partial^2 x_{E_j}} \left[ \Psi_{2b} - \Psi_{2d} \right] (x_j; x_{E_j}, x_{\alpha_0}; x_{E_i}, x_{\alpha_0}) .
\end{align*}
\]

\[
\begin{align*}
F_{M,1}(0; x_{\nu_1}, x_{\chi_0}; x_{E_j}, x_{\beta_0}) &= \left\{ \frac{1}{4} \frac{\partial^3}{\partial x_{E_j}^2} \Psi_{3a} - \frac{1}{8} \frac{\partial^2}{\partial x_{E_j}^2} \left[ 2\Psi_{2a} - \Psi_{2b} - \Psi_{2d} \right] + \frac{\partial^3}{\partial x_{E_j} \partial x_{\chi_0}^2} \left[ \frac{1}{4} \Psi_{3b} + \frac{1}{3} \Psi_{3c} \right] \\
&\quad- \frac{1}{4} \frac{\partial^2}{\partial x_{E_j} \partial x_{\chi_0}^2} \left[ \Psi_{2b} - \Psi_{2d} \right] \right\} (0; x_{E_j}, x_{\alpha_0}; x_{\nu_1}, x_{\chi_0}) \\
&\quad+ \left\{ \frac{1}{8} \frac{\partial^2}{\partial x_{\chi_0}^2} \left[ \Psi_{2b} - \Psi_{2d} \right] - \frac{1}{4} \frac{\partial}{\partial x_{E_j}} \frac{\partial^2}{\partial x_{\chi_0}^2} \Psi_{3b} \\
&\quad- \frac{1}{4} \partial^3 \Psi_{3a} \right\} (0; x_{\nu_1}, x_{\chi_0}; x_{E_j}, x_{\beta_0}) .
\end{align*}
\]

\[
\begin{align*}
F_{M,2}(0; x_{\nu_1}, x_{\chi_0}; x_{E_j}, x_{\beta_0}) &= \left\{ \frac{1}{4} \frac{\partial^3}{\partial x_{E_j} \partial x_{\alpha_0}^2} \Psi_{3a} - \frac{1}{8} \frac{\partial^2}{\partial x_{E_j} \partial x_{\chi_0}^2} \left[ 2\Psi_{2a} - \Psi_{2b} - \Psi_{2d} \right] \\
&\quad+ \frac{\partial^3}{\partial x_{E_j} \partial x_{\alpha_0} \partial x_{\chi_0}^2} \left[ \frac{1}{4} \Psi_{2b} - \frac{1}{3} \Psi_{2c} \right] \\
&\quad+ \frac{1}{3} \Psi_{2d} \right\} (0; x_{E_j}, x_{\beta_0}; x_{\nu_1}, x_{\chi_0}) \\
&\quad+ \left\{ - \frac{\partial^3}{\partial x_{E_j} \partial x_{\alpha_0} \partial x_{\chi_0}^2} \left[ \frac{1}{4} \Psi_{3b} + \frac{1}{3} \Psi_{3c} \right] + \frac{1}{4} \frac{\partial}{\partial x_{E_j}} \frac{\partial^2}{\partial x_{\chi_0}^2} \Psi_{2b} \\
&\quad+ \frac{1}{8} \frac{\partial^2}{\partial x_{\chi_0}^2} \left[ \Psi_{2b} - \Psi_{2d} \right] - \frac{1}{4} \frac{\partial^3}{\partial x_{\chi_0}^2} \Psi_{3a} \right\} (0; x_{\nu_1}, x_{\chi_0}; x_{E_j}, x_{\beta_0}) .
\end{align*}
\]

\[
\begin{align*}
F_{M,3}(0; x_{\nu_1}, x_{\chi_0}; x_{E_j}, x_{\beta_0})
\end{align*}
\]
\[
\begin{align*}
\mathcal{F}_{M,4}(0; x_{\alpha}, x_{\beta}^-; x_{E_j}, x_{\chi_0}) & = \left\{ -\frac{1}{24} \sum_{E_j} \left[ \Psi_{2a} - \Psi_{2b} \right] - \frac{1}{24} \sum_{E_j} \left[ \Psi_{2b} - \Psi_{2d} \right] \right. \\
& \left. + \frac{1}{8} \sum_{E_j} \left[ \Psi_{2b} - \Psi_{2d} \right] \right\} (0; x_{E_j}, x_{\chi_0}^-; x_{E_j}, x_{\chi_0}^-) \\
& + \left\{ \frac{1}{24} \sum_{E_j} \left[ \Psi_{2b} - \Psi_{2d} \right] - \frac{1}{24} \sum_{E_j} \left[ \Psi_{2a} - \Psi_{2b} \right] \right\} (0; x_{E_j}, x_{\chi_0}^-; x_{E_j}, x_{\chi_0}^-). 
\end{align*}
\]

\[
\begin{align*}
\mathcal{F}_{M,5}(0; x_{\alpha}, x_{\beta}^-; x_{E_j}, x_{\chi_0}) & = \left\{ -\frac{1}{4} \sum_{E_j} \left[ \Psi_{2a} - \Psi_{2b} \right] - \frac{1}{6} \sum_{E_j} \left[ \Psi_{2b} - \Psi_{2c} \right] \right. \\
& \left. + \frac{1}{2} \sum_{E_j} \left[ \Psi_{2b} - \Psi_{2d} \right] \right\} (0; x_{E_j}, x_{\chi_0}^-; x_{E_j}, x_{\chi_0}^-) \\
& + \left\{ \frac{1}{4} \sum_{E_j} \left[ \Psi_{2b} - \Psi_{2d} \right] \right\} (0; x_{E_j}, x_{\chi_0}^-; x_{E_j}, x_{\chi_0}^-). 
\end{align*}
\]

\[
\begin{align*}
\mathcal{F}_{M,6}(0; x_{\alpha}, x_{\beta}^-; x_{E_j}, x_{\chi_0}) & = \frac{1}{4} \sum_{E_j} \left[ \Psi_{2b} - \Psi_{2d} \right] (0; x_{E_j}, x_{\chi_0}^-; x_{E_j}, x_{\chi_0}^-) \\
& + \left\{ -\frac{1}{4} \sum_{E_j} \left[ \Psi_{2a} - \Psi_{2b} \right] - \frac{1}{6} \sum_{E_j} \left[ \Psi_{2b} - \Psi_{2c} \right] \right. \\
& \left. + \frac{1}{2} \sum_{E_j} \left[ \Psi_{2b} - \Psi_{2d} \right] \right\} (0; x_{E_j}, x_{\chi_0}^-; x_{E_j}, x_{\chi_0}^-). 
\end{align*}
\]
\[
F_{C,1}(x_{ij}; x_{\varphi_i}, x_{\chi^\pm}; x_{\varphi_j}, x_{\chi^\pm}) = \left\{- \frac{1}{4} \frac{\partial^3}{\partial x_{\varphi_j}} \Psi_{2a} + \frac{1}{4} \frac{\partial^2}{\partial x_{\varphi_j}} \left[ \Psi_{2b} - \Psi_{2d} \right] - \frac{\partial^3}{\partial^2 x_{\varphi_j} \partial x_{\chi^\pm}} \left[ \frac{1}{4} \Psi_{2b} + \frac{1}{3} \Psi_{3a} \right] \right. \\
+ \frac{1}{4} \frac{\partial^2}{\partial x_{\varphi_j} \partial x_{\chi^\pm}} \Psi_{2b} - \frac{1}{4} \frac{\partial}{\partial x_{\varphi_j}} \left[ \Psi_{1a} - \Psi_{1b} \right] \right\} (x_{ij}; x_{\varphi_i}, x_{\chi^\pm}; x_{\varphi_j}, x_{\chi^\pm}) \\
+ \left\{ \frac{1}{4} \frac{\partial^3}{\partial x_{\varphi_j}} \Psi_{3b} - \frac{1}{8} \frac{\partial^2}{\partial x_{\varphi_j} \partial x_{\chi^\pm}} \left[ 2 \Psi_{2b} + \Psi_{2d} \right] - \frac{1}{8} \frac{\partial}{\partial x_{\varphi_j} \partial x_{\varphi_j}} \left[ 2 \Psi_{2b} - \Psi_{2c} - \Psi_{2d} \right] \\
- \frac{1}{8} \frac{\partial^2}{\partial x_{\chi^\pm} \partial x_{\varphi_j}} \Psi_{2a} + \frac{3}{8} \frac{\partial}{\partial x_{\chi^\pm}} \Psi_{1a} \right\} (x_{ij}; x_{\varphi_i}, x_{\chi^\pm}; x_{\varphi_j}, x_{\chi^\pm}), \tag{C22}
\]

\[
F_{C,2}(x_{ij}; x_{\varphi_i}, x_{\chi^\pm}; x_{\varphi_j}, x_{\chi^\pm}) = \left\{ \frac{1}{24} \frac{\partial^3}{\partial x_{\varphi_j}} \left[ \Psi_{2a} - \Psi_{2b} \right] + \frac{1}{24} \frac{\partial^2}{\partial x_{\varphi_j} \partial x_{\chi^\pm}} \left[ 3 \Psi_{2b} - 4 \Psi_{2c} + \Psi_{2d} \right] \right. \\
- \frac{1}{4} \frac{\partial^2}{\partial x_{\varphi_j}} \left[ \Psi_{1a} - \Psi_{1b} \right] - \frac{1}{4} \frac{\partial}{\partial x_{\chi^\pm}} \Psi_{9} \right\} (x_{ij}; x_{\varphi_i}, x_{\chi^\pm}; x_{\varphi_j}, x_{\chi^\pm}) \\
+ \left\{ \frac{1}{24} \frac{\partial^3}{\partial x_{\varphi_j} \partial x_{\chi^\pm}} \left[ \Psi_{2b} + 2 \Psi_{2c} - 3 \Psi_{2d} \right] + \frac{1}{8} \frac{\partial^2}{\partial x_{\chi^\pm} \partial x_{\varphi_j}} \right. \\
+ \frac{\partial^2}{\partial x_{\chi^\pm} \partial x_{\varphi_j}} \Psi_{1a} - \frac{1}{24} \frac{\partial^3}{\partial x_{\chi^\pm} \partial x_{\chi^\pm}} \left[ \Psi_{2a} - \Psi_{2b} \right] + \frac{1}{8} \frac{\partial^2}{\partial x_{\chi^\pm} \partial x_{\varphi_j}} \right. \\
+ \frac{1}{8} \frac{\partial^2}{\partial x_{\chi^\pm} \partial x_{\varphi_j}} \Psi_{1a} \right\} (x_{ij}; x_{\varphi_i}, x_{\chi^\pm}; x_{\varphi_j}, x_{\chi^\pm}). \tag{C23}
\]

\[
F_{C,3}(x_{ij}; x_{\varphi_i}, x_{\chi^\pm}; x_{\varphi_j}, x_{\chi^\pm}) = \left\{ \frac{1}{4} \frac{\partial^2}{\partial x_{\varphi_j}} \left[ \Psi_{2a} - \Psi_{2b} \right] + \frac{1}{6} \frac{\partial^2}{\partial x_{\varphi_j} \partial x_{\chi^\pm}} \left[ 3 \Psi_{2b} - 4 \Psi_{2c} + \Psi_{2d} \right] \right. \\
- \frac{\partial^2}{\partial x_{\varphi_j}} \left[ \Psi_{1a} - \Psi_{1b} \right] \right\} (x_{ij}; x_{\varphi_i}, x_{\chi^\pm}; x_{\varphi_j}, x_{\chi^\pm}) \\
- \frac{1}{4} \left\{ \frac{\partial^2}{\partial x_{\chi^\pm}} \left[ \Psi_{2b} - \Psi_{2d} \right] - \left[ \frac{\partial}{\partial x_{\chi^\pm}} + \frac{\partial}{\partial x_{\varphi_i}} \right] \Psi_{1a} \right\} (x_{ij}; x_{\varphi_i}, x_{\chi^\pm}; x_{\varphi_j}, x_{\chi^\pm}). \tag{C24}
\]
\[
\delta \left( \Delta a_{2L}^{a\chi^0} \right) = \Delta a_{2L}^{a\chi^0} \text{(on-shell)} - \Delta a_{2L}^{a\chi^0} (\overline{MS}) \\
= \frac{e^4}{(4\pi)^4 s_w^4 c_w^4} \left\{ \frac{x_f}{24} \left[ \ln x_{RE} + \partial_2(x_{iJ}, x_{\chi^a_\alpha}, x_{\xi^I_\beta}) \rho_1(x_{\chi^a_\alpha}, x_{\xi^I_\beta}) \right] \\
+ \left[ \ln x_{RE} + \partial_2(x_{iJ}, x_{\chi^a_\alpha}, x_{\xi^I_\beta}) \rho_1(x_{\chi^a_\alpha}, x_{\xi^I_\beta}) \right] \right\} \\
\times \left[ \text{Re} \left( \left( \xi^I_N \right)_{j\beta} \left( \eta^I_N \right)_{i\alpha} \left( \xi^I_N \right)_{\alpha j} \left( \eta^I_N \right)_{j\beta} \right) \right] \\
- \frac{x_f}{48} \left[ \theta_1(x_{iJ}, x_{\chi^a_\alpha}, x_{\xi^I_\beta}) \rho_1(x_{\chi^a_\alpha}, x_{\xi^I_\beta}) \right] \\
\times \left[ \text{Re} \left( \left( \xi^I_N \right)_{j\beta} \left( \eta^I_N \right)_{i\alpha} \left( \xi^I_N \right)_{\alpha j} \left( \eta^I_N \right)_{j\beta} \right) \right] \\
+ \frac{(x_f x_{\chi^a_\beta})^{1/2}}{4} \left[ \left( \xi^I_N \right)_{j\beta} \left( \eta^I_N \right)_{i\alpha} \left( \xi^I_N \right)_{\alpha j} \left( \eta^I_N \right)_{j\beta} \right] \\
+ \frac{(x_f x_{\chi^a_\beta})^{1/2}}{4} \left[ \left( \xi^I_N \right)_{j\beta} \left( \eta^I_N \right)_{i\alpha} \left( \xi^I_N \right)_{\alpha j} \left( \eta^I_N \right)_{j\beta} \right]
\]
\]
\[ \delta (\Delta d_{il}^{2L}, x^0 \chi^0) = \Delta d_{il}^{2L, x^0 \chi^0} (\text{on-shell}) - \Delta d_{il}^{2L, x^0 \chi^0} (\overline{MS}) \]

\[
\begin{align*}
&\left( \frac{x_{il} x_{i0} x_{i0}}{8} \right)^{1/2} 
- \frac{\rho_2(x_{E_i}, x_{E_j})}{8} \phi_1(x_{iJ}, x_{i0}, x_{E_j}) \\
&\times \text{Re} \left( \left( \eta_N^{J}_{i\beta} (\eta_N^{J})_{i\alpha} (\xi_N^{I})_{\alpha \beta} \right) \right) \\
&+ \left( \frac{x_{il} x_{i0} x_{i0}}{4} \right)^{1/2} \rho_2(x_{E_i}, x_{E_j}) \left( 1 + \ln x_{RE} + \phi_2(x_{iJ}, x_{i0}, x_{E_j}) \right) \\
&\left( \frac{x_{il} x_{i0} x_{i0}}{8} \right)^{1/2} \phi_1(x_{iJ}, x_{i0}, x_{E_j}) \rho_2(x_{E_j}, x_{i0}) \\
&\times \text{Re} \left( \left( \eta_N^{J}_{i\beta} (\eta_N^{J})_{i\alpha} (\xi_N^{I})_{\alpha \beta} \right) \right). \tag{D1}
\end{align*}
\]
\[
\delta \left( \Delta a_{t^L}^{2L} \chi^0 \chi^\pm \right) = \Delta a_{t^L}^{2L} \chi^0 \chi^\pm \text{(on-shell)} - \Delta a_{t^L}^{2L} \chi^0 \chi^\pm (\overline{MS}) \\
= \frac{\xi^5}{4(4\pi)^2 s_w^4 c_w^2} \left\{ \frac{x_{t^L} (x_{\chi^\pm} x_{\chi^\pm})^{1/2}}{3} \left[ \rho_1(x_{\chi^\pm}, x_{\chi^0}) \theta_{0,1}(x_{\chi^0}, x_{\bar{E}_j}) \\
- \theta_{0,1}(x_{\chi^0}, x_{\bar{E}_j}) \right] \Re \left( \langle \lambda^I_{N,\beta} (\lambda^I_{N,\beta} \zeta^I_{C} \zeta^I_{C} \rangle_{a_j} \eta^I_{C} \rangle \right) \\
+ \frac{\sqrt{2}}{3} \left[ (1 + \ln x_{RE} - \theta_{1,1}(x_{\chi^0}, x_{\bar{E}_j})) \rho_1(x_{\chi^0}, x_{\bar{E}_j}) \right] \right\} \times \frac{x_{t^L} m_{t^L}}{m_w c_\beta} \Re \left( \langle \eta^I_{N,\beta} (\lambda^I_{N,\beta} \zeta^I_{C} \zeta^I_{C} \rangle_{a_j} \eta^I_{C} \rangle \right) \\
+ (\varphi_3(x_{\chi^\pm}, x_{\bar{E}_j})(1 + \ln x_{RE} - \theta_{1,1}(x_{\chi^0}, x_{\bar{E}_j})) \\
- \frac{1}{4} x_{\chi^0} \theta_{0,1}(x_{\chi^0}, x_{\bar{E}_j}) \rho_2(x_{\bar{E}_j}, x_{\chi^0}) \right\} \times \Re \left( \langle \eta^I_{N,\beta} (\lambda^I_{N,\beta} \zeta^I_{C} \zeta^I_{C} \rangle_{a_j} \eta^I_{C} \rangle \right) \\
+ \frac{x_{t^L} m_{t^L}}{\sqrt{2} m_w c_\beta} \left\{ \rho_2(x_{\bar{E}_j}, x_{\chi^0}) \left[ (1 + \ln x_{RE} - \theta_{1,1}(x_{\chi^0}, x_{\bar{E}_j})) \right] \\
- \theta_{1,1}(x_{\chi^0}, x_{\bar{E}_j}) \right\} \times \Re \left( \langle \lambda^I_{N,\beta} (\lambda^I_{N,\beta} \zeta^I_{C} \zeta^I_{C} \rangle_{a_j} \eta^I_{C} \rangle \right) \right\}.
\]
\[-\frac{m_{l,t} x_{l,t}^{1/2}}{2\sqrt{2m_w c_\beta}} \left[ \rho_2(x_{\ell_j}, x_{\ell_i}) \left( 1 + \ln x_{\text{RE}} - \vartheta_2(x_{\ell_j}, x_{\ell_i}), x_{\ell_i} ) \right) \right. \\
- x_{\chi_a^\pm} \varphi_3(x_{\chi_a^\pm}, x_{\ell_i}) \theta_{a,1}(x_{\chi_a^0}, x_{\ell_j}) \right] \\
\times \text{Re} \left( \left( \xi_{\alpha_i}^I \right)_{j \beta} \left( \xi_{\alpha_j}^I \right)_{j \beta} \left( \eta_{\alpha_j}^J \right)_{\alpha_j} \left( \eta_{\alpha_j}^J \right)_{\alpha_j} \right) \right]. \\ 
\end{equation}

\[ \delta \left( \Delta a_{l,t}^{2L, \chi^\pm \chi^\pm} \right) = \Delta a_{l,t}^{2L, \chi^\pm \chi^\pm} (\text{on-shell}) - \Delta a_{l,t}^{2L, \chi^\pm \chi^\pm} (\text{MS}) \]
\[ = - \frac{e^4 x_{\ell_j}}{(4\pi)^2 s_w^4} \left\{ \left[ 1 + 12 \left( 1 + \ln x_{\text{RE}} - \vartheta_2(x_{\ell_j}, x_{\ell_i}), x_{\ell_i} ) \right) \rho_1(x_{\ell_j}, x_{\ell_i}), x_{\ell_i} ) \right] \\
+ \left( 1 + \ln x_{\text{RE}} - \vartheta_2(x_{\ell_j}, x_{\ell_i}), x_{\ell_i} ) \right) \rho_1(x_{\ell_i}, x_{\ell_i}) \right] \\
\times \left[ \frac{m_{l,t}^2}{m_{w}^2 c_\beta} \text{Re} \left( \left( \xi_{\alpha_i}^I \right)_{j \beta} \left( \eta_{\alpha_j}^J \right)_{\alpha_j} \right) \right] \\
+ \frac{m_{l,t}^2}{m_{w}^2 c_\beta} \text{Re} \left( \left( \eta_{\alpha_j}^J \right)_{j \beta} \left( \xi_{\alpha_j}^I \right)_{\alpha_j} \right) \right\} \\
\times \frac{1}{2 \sqrt{2}} \left[ \frac{m_{s_a^2} m_{l,t}^2}{m_{w}^3 c_\beta} \left( 1 + \ln x_{\text{RE}} - \vartheta_2(x_{\ell_j}, x_{\ell_i}), x_{\ell_i} ) \right) \varphi_3(x_{\ell_i}, x_{\ell_i}) \right] \\
- \frac{1}{2} \left[ \frac{m_{s_a^2} m_{l,t}^2}{m_{w}^3 c_\beta} \left( x_{\chi_a^\pm}, x_{\chi_a^\pm} \right) \right] \varphi_3(x_{\chi_a^0}, x_{\chi_a^0}) \right] \\
\times \text{Re} \left( \left( \eta_{\alpha_j}^J \right)_{j \beta} \left( \xi_{\alpha_j}^I \right)_{\alpha_j} \right) \\
- \left[ \frac{1}{\sqrt{2}} \frac{m_{s_a^2} m_{l,t}^2}{m_{w}^3 c_\beta} \left( 1 + \ln x_{\text{RE}} - \vartheta_2(x_{\ell_j}, x_{\ell_i}), x_{\ell_i} ) \right) \varphi_3(x_{\chi_a^0}, x_{\chi_a^0}) \right] \\
\times \text{Re} \left( \left( \eta_{\alpha_j}^J \right)_{j \beta} \left( \xi_{\alpha_j}^I \right)_{\alpha_j} \right) \right]. \quad (D5) \]
\[
\delta \left( \Delta d^{2L}_i, x^{\pm} \right) = \frac{e^5 x_i^{1/2}}{2(4\pi)^2 \Lambda_{NP}^4} \left\{ \frac{1}{12} \left[ \left( 1 + \ln x_{RE} - \vartheta_2(x_{iJ}, x_{x_{\frac{\alpha}{\beta}}}, x_{x_{\frac{i}{j}}}) \right) \rho_1(x_{\varphi_1}, x_{x_{\frac{\alpha}{\beta}}}) \right. \\
+ \left( 1 + \ln x_{RE} - \vartheta_2(x_{iJ}, x_{x_{\frac{\alpha}{\beta}}}, x_{x_{\frac{i}{j}}}) \right) \rho_1(x_{\varphi_1}, x_{x_{\frac{\alpha}{\beta}}}) \right. \\
\times \left[ \frac{m^2}{m^2 c^2_{\beta}} \text{Im} \left( (\xi^I_C)_{\beta \beta} (\eta^I_C)_{\alpha \beta} (\eta^J_C)_{\alpha j} (\xi^J_C)_{j \alpha}^\dagger \right) \\
- \frac{m^2}{m^2 c^2_{\beta}} \text{Im} \left( (\eta^I_C)_{\alpha \beta} (\xi^I_C)_{\beta \alpha} (\xi^J_C)_{j \alpha}^\dagger (\eta^J_C)_{j \alpha}^\dagger \right) \right. \\
+ \frac{1}{12} (x_{x_{\frac{\alpha}{\beta}}} x_{x_{\frac{i}{j}}})^{1/2} \vartheta_1(x_{iJ}, x_{x_{\frac{\alpha}{\beta}}}, x_{x_{\frac{i}{j}}}) \rho_1(x_{\varphi_1}, x_{x_{\frac{\alpha}{\beta}}}) \right. \\
\left. \left. \times \left[ \text{Im} \left( (\xi^I_C)_{\beta \beta} (\xi^J_C)_{\alpha j} (\xi^I_C)_{j \alpha}^\dagger \right) \\
- \frac{m^2}{m^2 c^2_{\beta}} \text{Im} \left( (\eta^I_C)_{\alpha \beta} (\xi^I_C)_{\beta \alpha} (\xi^J_C)_{j \alpha}^\dagger (\eta^J_C)_{j \alpha}^\dagger \right) \right]\right\}.
\]

(\text{D6})

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