U(3)-Family Nonet Higgs Boson
and its Phenomenology

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Abstract

In a model where quark and lepton masses and family-mixings are caused not by a variety of Yukawa couplings $y_{ij}$ ($i, j = 1, 2, 3$: family indices) with one vacuum expectation value (VEV) $v = \langle \phi^0 \rangle_0$, but by a variety of VEV’s of a U(3)-family nonet Higgs boson $\phi_L$, $v^j_i = \langle \phi^0_{Lj} \rangle_0$, with a single coupling constant, the following problems are investigated: what constraints on the Higgs potential are imposed in order to provide realistic quark and lepton mass spectra and mixings and what constraints on the Higgs boson masses are required in order to suppress unwelcome flavor-changing neutral current effects. Lower bounds of the physical Higgs boson masses of $\phi_L$ are deduced from the present experimental data and new physics from the present scenario is speculated.

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1. Introduction

One of our dissatisfaction with the standard model is that for the explanation of the mass spectra of quarks and leptons, we are obliged to choose the coefficients $y_{ij}^f$ in the Yukawa coupling $\sum_f \sum_{i,j} f_{iL}^f \phi_{0j} \langle \phi_0^f \rangle \phi_{0i}^f$ ($f = \nu, e, u, d$, and $i, j$ are family indices) “by hand”. If we could understand the mass spectra from the vacuum expectation values (VEV’s) $\langle \phi_{0j}^f \rangle$ of U(3) family [1] nonet Higgs fields which couple with fermions as $\sum_f \sum_{i,j} f_{iL}^f \phi_{0j}^f \phi_{0i}^f f_{jR}$, we would be happy. Unfortunately, however, we know that the mass spectra of up- and down-quarks and charged leptons are not identical and the Kobayashi-Maskawa [2] (KM) matrix is not a unit matrix. Moreover, we know that in such multi-Higgs models, in general, flavor changing neutral currents (FCNC) appear unfavorably.

In the present paper, on the basis of a model where quark and lepton masses and family-mixings are caused not by a variety of Yukawa couplings $y_{ij} (i,j = 1, 2, 3$: family indices) with one vacuum expectation value (VEV) $v = \langle \phi_0 \rangle$, but by a variety of VEV’s $v_i = \langle \phi_{0i} \rangle$ with a single coupling constant, we investigate the following problems: what constraints on the Higgs potential are imposed in order to provide realistic quark and lepton mass spectra and KM mixings and what constraints on the Higgs boson masses are required in order to suppress unwelcome FCNC effects. It will be concluded that a special form of the Higgs potential $V(\phi_L)$, which leads to realistic quark and lepton mass spectra, can safely suppress unwelcome FCNC and the present experimental data put lower bounds of a few TeV on the physical Higgs boson masses.

The model we discuss is a seesaw-type quark and lepton mass matrix model [3], where the $6 \times 6$ mass matrix for fermions $f$ and $F$ are given by

\[
(\mathcal{F} F)_L \begin{pmatrix} 0 & m_L \\ m_R & M_F \end{pmatrix} \begin{pmatrix} f \\ F \end{pmatrix}_R ,
\]

where $f_L = (2, 1)$, $f_R = (1, 2)$, $F_L = (1, 1)$ and $F_R = (1, 1)$ of SU(2)$_L \times$SU(2)$_R$. We assume that $3 \times 3$ matrices $m_L$ and $m_R$ are universal for $f = u, d, \nu$ and $e$ (up-quark-, down-quark-, neutrino- and charged lepton-sectors), so that differences between quark and lepton sectors and between up- and down-sectors come only from the differences of $M_F$. We assume that the structure of $M_F$ is simply given by $[[\text{unit matrix}]+ b_f \text{ (a rank-one matrix)}]$, where $b_f$ is a complex parameter depending on $f$ (up- or down- and quark or lepton sectors). The SU(2)$_L$ [SU(2)$_R$] symmetry
breaking matrix $m_L \ [m_R]$ is given by $y_L \langle \phi^0_L \rangle_0 \ [y_R \langle \phi^0_R \rangle_0]$, where $\phi_L \ [\phi_R]$ belongs to $(2,1,8+1) \ [(1,2,8+1)]$ of $SU(2)_L \times SU(2)_R \times U(3)_{family}$. Note that the $U(3)$-family symmetry is badly broken by the heavy fermion mass matrix $M_F$, as we state in Sect.2.

Generally, the diagonalization of the mass matrix (1.1) transforms the vertex

$$
(F \ F)_L \begin{pmatrix} 0 & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix} \begin{pmatrix} f \\ F \end{pmatrix}_R ,
$$

into

$$
(F' \ F')_L \begin{pmatrix} \Gamma'_{11} & \Gamma'_{12} \\ \Gamma'_{21} & \Gamma'_{22} \end{pmatrix} \begin{pmatrix} f' \\ F' \end{pmatrix}_R ,
$$

where $\Gamma_{12} = y_L \phi^0_L$ and so on, and $(f', F')$ are mass eigenstates, so that the vertex $\Gamma'_{11}$ is not $\Gamma'_{11} = 0$ any longer. (The details are discussed in Sect.3.) Since the physical Higgs bosons $\phi_L$ are sufficiently light compared with the other Higgs bosons $\phi_R$ and so on, the contributions to FCNC in quarks and leptons will be dominated only by $\phi_L$. Therefore, in the present paper, we will concentrate our study on the Higgs boson $\phi_L$.

In the present paper, as a model of the Higgs boson $\phi_L$, we adopt a $U(3)$-family nonet Higgs boson model [4], which was proposed by one of the authors (Y.K.) in order to explain a charged lepton mass relation [5]

$$
m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 ,
$$

which predicts $m_\tau = 1776.969 \pm 0.001$ MeV for the input values [6] of $m_e$ and $m_\mu$.

He assumed a $U(3)_{family}$ nonet Higgs boson $\phi$ whose potential is given by

$$
V(\phi) = \mu^2 \text{Tr}(\phi \phi^\dagger) + \frac{1}{2} \lambda [\text{Tr}(\phi \phi^\dagger)]^2 + \eta \phi_s^* \phi_s^\dagger \text{Tr}(\phi_{oct} \phi_{oct}^\dagger) .
$$

Here, for simplicity, the $SU(2)_L$ structure of $\phi$ has been neglected, and we have expressed the nonet Higgs bosons $\phi^j_i$ by the form of $3 \times 3$ matrix,

$$
\phi = \phi_{oct} + \frac{1}{\sqrt{3}} \phi_s 1 ,
$$
where $\phi_{\text{oct}}$ is the octet part of $\phi$, i.e., $\text{Tr}(\phi_{\text{oct}}) = 0$, and $\mathbf{1}$ is a $3 \times 3$ unit matrix. For $\mu^2 < 0$, conditions for minimizing the potential (1.5) lead to the relation

$$v_s^*v_s = \text{Tr} \left( v_{\text{oct}}^\dagger v_{\text{oct}} \right),$$

(1.7)

together with $v = v^\dagger$, where $v = \langle \phi \rangle_0$, $v_{\text{oct}} = \langle \phi_{\text{oct}} \rangle_0$ and $v_s = \langle \phi_s \rangle_0$, so that we obtain the relation

$$\text{Tr} \left( v^2 \right) = \frac{2}{3} [\text{Tr}(v)]^2.$$

(1.8)

If we assume a seesaw-like mechanism for charged lepton mass matrix $M_\ell$, $M_\ell \simeq mM_E^{-1}m$, with $m \propto v$ and heavy lepton mass matrix $M_E \propto \mathbf{1}$, we can obtain the mass relation (1.4).

However, the model (1.5) is only a toy model, and here the SU(2)$_L$ structure of $\phi$ was not discussed explicitly. Moreover, the Higgs potential (1.5) brings unwelcome massless physical Higgs bosons into the theory. In the present paper, we investigate what potential form of $\phi_L$ is favorable in order to provide realistic fermion mass spectrum without contradicting the present experimental data. The outline of the model is presented in the next section 2.

In the section 3, we discuss the Higgs potential $V(\phi_L)$ of the U(3)-family nonet Higgs bosons $\phi_L \ (\phi_R)$ under an ansatz and the conditions for minimizing $V(\phi_L)$. In Sect.4, we calculate masses of the Higgs boson $\phi_L$, and in Sect.5, we estimate a lower bound of the mass of the Higgs bosons $\phi_{Li}^j \ (i \neq j)$ from the experimental data of the rare kaon decay $K_L \rightarrow e^\pm \mu^\mp$. Besides, the present model, in general, induces FCNC. In Sect.6, we will estimate lower bounds of the physical Higgs boson masses from the present experimental data of $K^0\bar{K}^0$- and $D^0\bar{D}^0$- mixings, and so on. Finally, in Sect.7, we will speculate a possible new physics which is expected from the present model.

2. Outline of the model

In our scenario, we prepare the following fermions: $f = \ell, q \ (\ell = (\nu, e), q = (u, d))$ and $F = N, E, U, D$, which belong to $f_L = (2, 1, 3), f_R = (1, 2, 3), F_L = (1, 1, 3)$, and $F_R = (1, 1, 3)$ of SU(2)$_L \times$SU(2)$_R \times$U(3)$_{\text{family}}$, respectively. Here, SU(2)$_L \times$SU(2)$_R$ are gauged, but U(3)$_{\text{family}}$ is not gauged. The global symmetry U(3)$_{\text{family}}$ will be broken not spontaneously, but explicitly. Up- and down-heavy fermions, $F_{\text{up}}$ and $F_{\text{down}}$, are distinguished by hypercharge $Y$ (note that $Y \neq B - L$ for the heavy fermions): Hypercharges of the heavy fermions ($N, E$) and ($U, D$) take
the values \((0, -2)\) and \((4/3, -2/3)\), respectively. The quantum numbers of those fields are listed in Table I.

In the present model, differently from the standard \(SU(2)_L \times SU(2)_R \times U(1)_{B-L}\) model, we do not consider Higgs scalar fields which belong to \((2, 2)\) of \(SU(2)_L \times SU(2)_R\), so that there are no Higgs fields which couple with \(\bar{f}f\) at tree level. We assume only the following Yukawa interactions:

\[
H_{\text{Yukawa}} = y_0 \sum_{i,j} \bar{F}_i [\delta^i_j \Phi_0 + 3b_f (\Phi_X)_i] F_j \\
+ y_L \sum_{i,j} \left[ \bar{F}_i (\phi_L)_i^j F^\text{down}_R + \bar{F}_i (\phi_R)_i^j F^\text{up}_R + \text{h.c.} \right] + (L \leftrightarrow R) ,
\]

where \(\phi = (\phi^+, \phi^0)\) and \(\bar{\phi} = (\bar{\phi}^0, -\bar{\phi}^-)\). The scalar fields \(\phi_L\) and \(\phi_R\) belong to \((2, 1, 8 + 1)\) and \((1, 2, 8 + 1)\) of \(SU(2)_L \times SU(2)_R \times U(3)\) family, respectively, and the VEV’s \(\langle \phi^0_L \rangle_0\) and \(\langle \phi^0_R \rangle_0\) provide left- and right-handed weak boson masses \(m(W_L)\) and \(m(W_R)\), respectively. The fields \(\Phi_0\) and \(\Phi_X\) which belong to \((1, 1, 1)\) and \((1, 1, 8 + 1)\), respectively, do not contribute to weak boson masses \(m(W_L)\) and \(m(W_R)\), but play only a role of providing extremely large masses for vector-like fermions \(F\). Then, the mass matrices for fermions \((f, F)\) are given by

\[
(\bar{f} F)^{\text{up}}_L \begin{pmatrix} 0 & m_L^\dagger \\ m_R^\dagger & M_F \end{pmatrix} \begin{pmatrix} f \\ F \end{pmatrix}^{\text{up}}_R + (\bar{f} F)^{\text{down}}_L \begin{pmatrix} 0 & m_L \\ m_R^\dagger & M_F \end{pmatrix} \begin{pmatrix} f \\ F \end{pmatrix}^{\text{down}}_R + \text{h.c.} .
\]

In the present model, since we will choose \(m_L^\dagger = m_L\) and \(m_R^\dagger = m_R\) later, what we should do is to diagonalize the \(6 \times 6\) mass matrix

\[
M = \begin{pmatrix} 0 & m_L \\ m_R & M_F \end{pmatrix} .
\]

Under the approximation of \(M_F \gg m_L, m_R\), we obtain a seesaw-type mass matrix form

\[
M_f \simeq m_L M_F^{-1} m_R .
\]

The structures of \(m_L\) and \(m_R\) are common to quarks and leptons. The variety of \(M_f\) comes from structures of \(M_F\) which depend on \(F = U, D, N\) and \(E\).
In the present paper, we assume that

\[ \langle \phi^0_R \rangle_0 \propto \langle \phi^0_L \rangle_0. \]  

(2.5)
i.e., each term in \( V(\phi_R) \) takes the coefficient which is exactly proportional to the corresponding term in \( V(\phi_L) \). This assumption means that there is a kind of “conspiracy” between \( V(\phi_R) \) and \( V(\phi_L) \). However, in this paper, we will not go into this problem moreover.

On the other hand, the heavy fermion mass matrices \( M_F \) are given by

\[ M_F = y_0 \left[ \langle \Phi_0 \rangle_0 \mathbf{1} + \left( y_X^F / y_0 \right) \langle \Phi_X \rangle_0 \right], \]

where \( \langle \Phi_0 \rangle_0 = V_0, \langle \Phi_X \rangle_0 = V_X^F X \) and

\[
\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad X \equiv \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.
\]

(2.6)

Note that the U(3)-family symmetry is badly broken by \( \langle \Phi_X \rangle_0 \). The democratic term \( X \) in \( M_F \) may be understood, for example, by a permutation group \( S_3 \) \[8\]. However, in the present stage, since our interest is focused on the light Higgs boson \( \phi_L \), we do not touch the origin of the democratic term \( X \). Anyhow, we assume that the \( M_F \) is given by

\[ M_F = m_0 K_f \left( \mathbf{1} + 3 b_f X \right), \]

(2.7)

where \( m_0 K_f = y_0 V_0 \) and \( 3 b_f = (y_X^F / y_0) (V_X / V_0) \). Since the parameter \( K_f \) is given by \( K_f = y_0 \langle \Phi_0 \rangle_0 \), it is independent of \( f = u, d, e, \nu \), i.e., \( K_e = K_u = K_d \) at \( \mu = m_0 K \).

However, for numerical evaluations, we will treat \( K_f \) effectively as \( K_e \neq K_u \simeq K_d \), because of the evolution from \( \mu \sim m_0 K \) to \( \mu \sim M_W \).

The variety of the quark and lepton mass matrices \( M_f \) essentially originates in the parameter \( b_f \). Since we take \( v_L \equiv \langle \phi^0_L \rangle_0 \) and \( v_R \equiv \langle \phi^0_R \rangle_0 \) such as they are Hermitian, a \( CP \) violation phase can be included only in the heavy fermion mass matrix \( M_F \) (i.e., in the parameter \( b_f \)). For the charged leptons, considering the phenomenological success of the relation (1.8) [i.e., (1.4)], we put \( b_e = 0 \). Therefore, the mass matrix of the heavy leptons \( M_E \) is Hermitian, and \( CP \) violation does not manifest in the charged lepton sector.

The phenomenological study of the quark mass spectrum and family-mixings based on the seesaw-type mass matrix (1.1) with \( M_F \) of the form (2.7) has been done
by one of the authors (Y.K.) and Fusaoka [9]. They have found that the seesaw-type mass matrix with $M_F$ of the form (2.7) can provide an explanation why $m_t \gg m_b$, while $m_u \sim m_d$, by taking $b_u = -1/3$ ($\beta_u = 0$) and $b_d \simeq -e^{i\beta_d}$ ($\beta_d \simeq -20^\circ$), but with keeping $K_u = K_d$: The inverse of the matrix $[(\text{unit matrix})+(\text{democratic-type matrix})]$, $1 + 3b_f X$, also take a form $[(\text{unit matrix})+(\text{democratic-type matrix})]$, $1 + 3a_f X$, with $a_f = -b_f/(1 + 3b_f)$; The enhancement $m_t/m_b \gg 1$ comes from $|a_f| \to \infty$ in the limit of $b_f \to -1/3$, while $m_u \sim m_d$ comes from the feature that the democratic-type mass matrix [10] can provide a large mass only to the third family, i.e., the effect of $|a_u| \to \infty$ contributes mainly to $m_t$. Of course, by adjusting the parameter $\beta_d$, they [9] have also obtained reasonable KM matrix parameters as well as up- and down-quark masses. Here, we do not repeat their numerical results.

Since the VEV of $\phi^0_L$ is small compared with those of other Higgs fields $\phi^0_R$, $\Phi_0$ and $\Phi_X$, i.e., $(\text{Tr}(\phi_L)^2)^{1/2} \sim 10^2 \text{ GeV}$, we expect some observable effects of the physical Higgs bosons $\phi_L$ in the low energy ($10^2 - 10^3 \text{ GeV}$) experiments. The purpose of the present paper is to investigate the physics of the U(3)$_{\text{family}}$ nonet Higgs bosons $\phi_L$ from the phenomenological point of view. In the next section, we investigate a possible form of $V(\phi_L)$ which derives the relation (1.8) (therefore the charged lepton mass relation (1.4)). However, we will not touch what mathematical requirements can provide such a potential form. The purpose of the present paper is to study masses of the physical Higgs bosons $\phi_L$ and their interactions with gauge bosons and fermions when the fields $\phi_L$ are described by such a potential which can lead to the relation (1.8).

### 3. Higgs potential $V(\phi_L)$ and “nonet” ansatz

What is of great interest to us is a potential form of $\phi_L$, $V(\phi_L)$. Hereafter, we will omit the index $L$ and simply write $\phi_L$ as $\phi$. We do not consider mixings among Higgs scalar fields with hierarchically different VEV’s, i.e., among $\phi_L$, $\phi_R$ and $\Phi_0, X$. Then, the potential $V(\phi)$ is given by

$$V(\phi) = V_{\text{nonet}} + V_{\text{Oct-Singl}} + V_{\text{SB}} \, ,$$

where $V_{\text{nonet}}$ is a part of $V(\phi)$ which satisfies a “nonet” ansatz stated below, $V_{\text{Oct-Singl}}$ is a part which violates the “nonet” ansatz, and $V_{\text{SB}}$ is a term which breaks U(3)$_{\text{family}}$ explicitly.
The “nonet” ansatz is as follows: the octet component $\phi_{\text{oct}}$ and singlet component $\phi_s$ of the Higgs scalar fields $\phi_L$ ($\phi_R$) always appear with the combination of (1.6) in the Lagrangian. Under the “nonet” ansatz, the SU(2)$_L$ invariant (and also U(3)$_{\text{family}}$ invariant) potential $V_{\text{nonet}}$ is given by

$$V_{\text{nonet}} = \mu^2 \text{Tr}(\bar{\phi}\phi) + \frac{1}{2} \lambda_1 \sum_{i,j} \sum_{k,l} (\bar{\phi}_i^j \phi_j^i)(\bar{\phi}_k^l \phi_l^k)$$

$$+ \frac{1}{2} \lambda_2 \sum_{i,j} \sum_{k,l} (\bar{\phi}_i^j \phi_k^i)(\bar{\phi}_l^j \phi_l^k) + \frac{1}{2} \lambda_3 \sum_{i,j} \sum_{k,l} (\bar{\phi}_i^j \phi_k^i)(\bar{\phi}_j^k \phi_l^l),$$

(3.2)

where $(\bar{\phi}\phi) = \phi^- \phi^+ + \phi^0$. Here, for simplicity, we have taken only U(3)$_{\text{family}}$ singlet terms in which each two of four fields can make U(3)$_{\text{family}}$ singlets. A more general case, which includes terms $\sum_{i,j,k,l} (\bar{\phi}_i^j \phi_k^i)(\bar{\phi}_l^k \phi_l^l)$ and so on, is given in Appendix A.

In addition to $V_{\text{nonet}}$ which satisfies the nonet ansatz, we consider terms the following interaction terms between octet- and singlet-components $V_{\text{Oct.Singl}}$ which break the nonet ansatz:

$$V_{\text{Oct.Singl}} = \eta_1 (\bar{\phi}_s \phi_s) \text{Tr}(\bar{\phi}_{\text{oct}} \phi_{\text{oct}}) + \eta_2 \sum_{i,j} \left( (\bar{\phi}_s \phi_{\text{oct}})_i^j \right) \left( (\bar{\phi}_{\text{oct}})_i^j \phi_s \right)$$

$$+ \eta_3 \sum_{i,j} \left( (\bar{\phi}_s \phi_{\text{oct}})_i^j \right) \left( (\bar{\phi}_{\text{oct}})_i^j \phi_s \right) + \eta_3^* \sum_{i,j} \left( (\bar{\phi}_{\text{oct}})_i^j \phi_s \right) \left( (\bar{\phi}_{\text{oct}})_i^j \phi_s \right),$$

(3.3)

Note that the both potential terms $V_{\text{nonet}}+V_{\text{Oct.Singl}}$ are invariant under SU(3)$_{\text{family}}$ symmetry and the exchange $\phi_{\text{oct}} \leftrightarrow 1(\phi_s/\sqrt{3})$.

As stated later, the potential which consists only of $V_{\text{nonet}}$ and $V_{\text{Oct.Singl}}$ cannot fix each value $v_i$ of the VEV’s $\langle \phi^0 \rangle_0 = v = \text{diag}(v_1, v_2, v_3)$ completely, although we can derive that the VEV’s $v$ should satisfy the relation (1.8). In order to fix three values of $v_i$ completely, we will add to an explicitly U(3)$_{\text{family}}$ symmetry breaking term $V_{SB}$. We consider that gauge symmetries are exact symmetries in the original Hamiltonian, so that those are broken only spontaneously, while global symmetries are phenomenological and approximate symmetries, so that the symmetries may be broken explicitly.
For a time, we neglect the term $V_{SB}$ in (3.1). For $\mu^2 < 0$, conditions for minimizing the potential (3.1) are as follows:

$$\left[\mu^2 + (\lambda_1 + \lambda_2)\text{Tr}(v^\dagger v)\right]v_s^* + \lambda_3 \text{Tr}(v^\dagger v^\dagger) v_s$$
$$+ (\eta_1 + \eta_2)\text{Tr}(v_{oct}^\dagger v_{oct}) v_s^* + 2\eta_3^* \text{Tr}(v_{oct}^\dagger v_{oct}) v_s = 0,$$  \(3.4\)

$$\left[\mu^2 + (\lambda_1 + \lambda_2)\text{Tr}(v^\dagger v)\right] v_{oct} + \lambda_3 \text{Tr}(v^\dagger v^\dagger) v_{oct} + (\eta_1 + \eta_2)v_s v_s^* v_{oct}^\dagger + 2\eta_3 v_s^* v_{oct} = 0,$$  \(3.5\)

and the similar equations with $v \leftrightarrow v^\dagger$ ($v_{oct} \leftrightarrow v_{oct}^\dagger$ and $v_s \leftrightarrow v_s^\dagger$). For simplicity, we consider the case $\eta_3^* = \eta_3$, so that $v^\dagger = v$. Then, we can readily obtain the desirable relation

$$v_s^2 = \text{Tr}(v_{oct}^2) = \frac{-\mu_s^2}{2(\lambda_1 + \lambda_2 + \lambda_3) + \eta_1 + \eta_2 + 2\eta_3},$$  \(3.6\)

which leads to the relation (1.8).

Note that the exact nonet form (1.6) is not always essential to provide the relation (1.8). In the modified potential

$$V_{nonet} = \mu_s^2 \bar{\phi}_s \phi_s + \mu_{oct}^2 \text{Tr}(\bar{\phi}_{oct} \phi_{oct}) + \frac{1}{2} \lambda_1 \sum_{i,j} \sum_{k,l} (\bar{\phi}_i^j \phi_j^i)(\bar{\phi}_k^l \phi_l^k) + \cdots,$$  \(3.7\)

where

$$\phi = \phi_{oct} + \frac{k}{\sqrt{3}} \phi_s 1,$$  \(3.8\)

we can also obtain the desirable relation (1.8) when the coefficient $k$ satisfies

$$k^2 = \mu_s^2/\mu_{oct}^2,$$  \(3.9\)

because the conditions for minimizing (3.7) leads to

$$k^2 v_s^2 = \text{Tr}(v_{oct}^2) = \frac{-\mu_s^2}{2(\lambda_1 + \lambda_2 + \lambda_3) + \eta_1 + \eta_2 + 2\eta_3}.$$  \(3.10\)

The essential assumption is that the terms $\bar{\phi}_s \phi_s$ and $\bar{\phi}_{oct} \phi_{oct}$ appear with the same relative weight in $V_{nonet}$ and in the Yukawa interactions with fermions. However,
there is no substantial difference between the cases of $k = 1$ and $k \neq 1$ for evaluation of physical quantities. Therefore, hereafter, we will investigate only the case of $k = 1$.

So far, we do not have any conditions more than (3.6) (therefore (1.8)) for $v$, although it is sufficient for deriving charged lepton mass relation (1.4). In order to fix each component of $v$, we must add some additional terms to the potential $V(\phi)$.

In general, we can choose such a family-basis in which $v$ is given by a diagonal form

$$v = \text{diag}(v_1, v_2, v_3) . \tag{3.11}$$

Then, under the replacement $\phi^0 \rightarrow \phi^0 + v$, seven components of $\phi^0_{\text{oct}}$, i.e., six components $\phi^0_{ij} \ (i \neq j)$ and a diagonal component (we denote it as $\phi^0_y$) can invariant, although the singlet component $\phi^0_s$ and the other diagonal component $\phi^0_x$ which is orthogonal to $\phi^0_y$ cannot be invariant:

$$\begin{align*}
\phi^0_s &\rightarrow \phi^0_s + v_s , \\
\phi^0_x &\rightarrow \phi^0_x + v_x , \\
\phi^0_y &\rightarrow \phi^0_y , \\
(\phi^0)^j_i &\rightarrow (\phi^0)^j_i \ (i \neq j) .
\end{align*} \tag{3.12}$$

Therefore, we add the following $U(3)_{\text{family}}$ symmetry breaking terms to the potential of $\phi$:

$$V_{SB} = \xi \left[ (\overline{\phi}_y \phi_y) + \sum_{i,j} (\overline{\phi}^0_i \phi^0_j) \right] . \tag{3.13}$$

We can easily see that the relation (3.6) is unchanged even by adding such the explicitly symmetry-breaking terms $V_{SB}$, because of (3.12). We would like to stress that the explicit $U(3)_{\text{family}}$ breaking (3.13) is a soft breaking, so that it does not spoil the Yukawa sector. We consider that the parameter $\xi$ satisfies

$$\xi + \mu^2 > 0 , \tag{3.14}$$

in order to guarantee $\langle \phi^0_y \rangle_0 = 0$ and $\langle \phi^0_{ij} \rangle_0 = 0 \ (i \neq j)$.

We can rewrite the mass terms $\mu^2 \text{Tr}(\overline{\phi}\phi) + V_{SB}$ into

$$\mu^2 (\overline{\phi}_s \phi_s) + \mu^2_{\text{oct}} \text{Tr}(\overline{\phi}_{\text{oct}} \phi_{\text{oct}}) + V'_{SB} , \tag{3.15}$$
where
\[ V'_{SB} = -\xi (\bar{\phi}_x \phi_x) , \]  
\[ (3.16) \]
and \( \mu^2_{\text{oct}} = \xi + \mu^2 \). The term \( V'_{SB} \) plays a role to fix the axis of the SU(3)_{family} breaking.

Here, for convenience of our discussions, we define the parameters \( z_i \) as
\[ v_i = v_0 z_i , \]  
\[ (3.17) \]
with \( z_1^2 + z_2^2 + z_3^2 = 1 \). Also we define the diagonal components of \( \phi_{\text{oct}} \), \( \phi_x \) and \( \phi_y \), as
\[ \phi_x = x_1 \phi_1^1 + x_2 \phi_2^2 + x_3 \phi_3^3 , \]  
\[ \phi_y = y_1 \phi_1^1 + y_2 \phi_2^2 + y_3 \phi_3^3 , \]  
\[ (3.19) \]
with \( \sum_i x_i = 0 \), \( \sum_i y_i = 0 \), \( \sum_i x_i^2 = 1 \), \( \sum_i y_i^2 = 1 \) and \( \sum_i x_i y_i = 0 \), where \( v_i = v_s/\sqrt{3} + x_i v_x \). Then, the coefficients \( x_i \) and \( y_i \) are given by the following relations:
\[ x_i = \sqrt{2} z_i - \frac{1}{\sqrt{3}} , \]  
\[ (3.20) \]
\[ (y_1, y_2, y_3) = \left( \frac{x_2 - x_3}{\sqrt{3}}, \frac{x_3 - x_1}{\sqrt{3}}, \frac{x_1 - x_2}{\sqrt{3}} \right) . \]  
\[ (3.21) \]

Some useful formulas for the parameters \( z_i \) are given in Appendix B.

Although in the present stage of the model, we must add an SU(3)_{family} symmetry breaking term \( V'_{SB} \) by “hand”, this does not mean that we must provide three values \((x_1, x_2, x_3)\). The independent parameter of \( x_i \) is only one (for example, see (B7) in Appendix B).

4. Higgs boson masses and interactions

For convenience, we rewrite the fields \( \phi^\pm \) and \( \phi^0 \) with the fields \( \chi^\pm \), \( \chi^0 \), and \( H^0 \) defined by
\[ \left( \begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right) = \frac{1}{\sqrt{2}} \left( \begin{array}{c} i \sqrt{2} \chi^+ \\ H^0 - i \chi^0 \end{array} \right) , \]  
\[ (4.1) \]
Then, the mass terms after the spontaneous symmetry break down are given by

\[ V_{\text{mass}} = V_m(\chi^\pm) + V_m(\chi^0) + V_m(H^0), \tag{4.2} \]

\[ V_m(\chi^\pm) = \xi \left[ \sum_{i \neq j} (\chi^-)^j_i (\chi^+)^i_j + \chi_y^0 \chi_y^0 \right] + (\lambda_2 + \lambda_3) \left[ \text{Tr}(v \chi^+) \text{Tr}(v \chi^-) - 2v_s^2 \text{Tr}(\chi^- \chi^+) \right] + (\eta_2 + 2\eta_3) \left[ v_s \chi_-^0 \text{Tr}(v \chi^+) + v_s \chi_+^0 \text{Tr}(v \chi^-) - 2v_s^2 \chi_-^0 \chi_+^0 - v_s^2 \text{Tr}(\chi^- \chi^+) \right], \tag{4.3} \]

\[ V_m(\chi^0) = \frac{1}{2} \xi \left[ \sum_{i \neq j} (\chi^0)^j_i (\chi^0)^i_j + \chi_y^0 \chi_y^0 \right] + \lambda_3 \left\{ \left[ \text{Tr}(v \chi^0) \right]^2 - 2v_s^2 \text{Tr}(\chi^0 \chi^0) \right\} + 2\eta_3 \left[ -v_s^2 \text{Tr}(\chi^0 \chi^0) + 2v_s \chi_y^0 \text{Tr}(v \chi^0) - 2v_s^2 \chi_y^0 \chi_y^0 \right], \tag{4.4} \]

\[ V_m(H^0) = \frac{1}{2} \xi \left[ \sum_{i \neq j} (H^0)^j_i (H^0)^i_j + H_y^0 H_y^0 \right] + (\lambda_1 + \lambda_2 + \lambda_3) \left[ \text{Tr}(v H^0) \right]^2 + 2(\eta_1 + \eta_2 + 2\eta_3) v_s H_s^0 \left[ \text{Tr}(v H^0) - v_s H_s^0 \right], \tag{4.5} \]

where we have used the relations (3.4) and (3.5).

First, we discuss masses of the charged Higgs bosons $\chi^\pm$. From (4.3), the mass terms $\sum_{i,j} (\chi^-)^j_i M_0^2 (\chi^+)^{i_j}$ for the diagonal components of the fields $\chi^\pm$ are given by

\[ M_{ij}^2 = \xi y_i y_j + (\lambda_2 + \lambda_3)(v_i v_j - v_0^2 \delta_{ij}) + \frac{1}{3}(\eta_2 + 2\eta_3)(v_1 + v_2 + v_3) \left[ (v_i + v_j) - (v_1 + v_2 + v_3) \left( \frac{2}{3} + \delta_{ij} \right) \right], \tag{4.6} \]

where, from (3.20) and (3.21), $y_i$ is given by $y_i = \sqrt{2}(v_j - v_k)/\sqrt{3}v_0$ (($i, j, k$) are cyclic indexes of (1, 2, 3)).
The mass matrix (4.6) is diagonalized by transforming $(\phi_1^1, \phi_2^2, \phi_3^3)$ into

\[
\begin{pmatrix}
\phi_1 \\
\phi_2 \\
\phi_3
\end{pmatrix} = \begin{pmatrix}
z_1 & z_2 & z_3 \\
z_1 - \sqrt{\frac{2}{3}} z_2 - \sqrt{\frac{2}{3}} & z_3 - \sqrt{\frac{2}{3}} z_1 & z_3 - \sqrt{\frac{2}{3}} z_2 \\
\sqrt{\frac{2}{3}} (z_2 - z_3) & \sqrt{\frac{2}{3}} (z_3 - z_1) & \sqrt{\frac{2}{3}} (z_1 - z_2)
\end{pmatrix}\begin{pmatrix}
\phi_1^1 \\
\phi_2^2 \\
\phi_3^3
\end{pmatrix},
\] (4.7)

($\phi = \chi^\pm, \chi^0$ and $H^0$). From (3.20) and (3.21), we find

\[
\begin{pmatrix}
\phi_1 \\
\phi_2 \\
\phi_3
\end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix}
\phi_x + \phi_s \\
\phi_x - \phi_s \\
\sqrt{2} \phi_y
\end{pmatrix}.
\] (4.8)

Then, we obtain the masses of $\chi^\pm$ as follows:

\[
\begin{align*}
m^2(\chi^\pm_1) &= 0, \\
m^2(\chi^\pm_2) &= - (\lambda_2 + \lambda_3 + \eta_2 + 2\eta_3) v_0^2, \\
m^2(\chi^\pm_3) &= - \left[ \lambda_2 + \lambda_3 + \frac{1}{2} (\eta_2 + 2\eta_3) - \xi \right] v_0^2,
\end{align*}
\] (4.9)

where

\[
\xi = \xi / v_0^2, \tag{4.10}
\]

and $\chi^i_j$ denotes a boson with $i \neq j$.

Similarly, by the transformation (4.7), we obtain masses of $\chi^0$ and $H^0$:

\[
\begin{align*}
m^2(\chi^0_1) &= 0, \\
m^2(\chi^0_2) &= - 2(\lambda_3 + 2\eta_3) v_0^2, \\
m^2(\chi^0_3) &= - \left[ 2(\lambda_3 + \eta_3) - \xi \right] v_0^2, \\
m^2(\chi^0_{ij}) &= - \left[ 2(\lambda_3 + \eta_3) - \xi \right] v_0^2, 
\end{align*}
\] (4.11)

and

\[
\begin{align*}
m^2(H^0_1) &= [2(\lambda_1 + \lambda_2 + \lambda_3) + \eta_1 + \eta_2 + 2\eta_3] v_0^2, \\
m^2(H^0_2) &= - (\eta_1 + \eta_2 + 2\eta_3) v_0^2, \\
m^2(H^0_3) &= \xi v_0^2, \\
m^2(H^0_{ij}) &= \xi v_0^2.
\end{align*}
\] (4.12)
Since $\mu^2 < 0$, the positivity of $m^2(H_1^0)$ is guaranteed by (3.6), i.e.,
\[ 2(\lambda_1 + \lambda_2 + \lambda_3) + \eta_1 + \eta_2 + 2\eta_3 = -2\mu^2/v_0^2 > 0 \ . \quad (4.13) \]

The positivities of $m^2(H_2^0)$ and $m^2(\chi_2^\pm)$ require
\[ 2(\lambda_1 + \lambda_2 + \lambda_3) > -(\eta_1 + \eta_2 + 2\eta_3) > 0 \ , \quad (4.14) \]
and
\[ 2\lambda_1 + \lambda_2 + \lambda_3 + \eta_1 > -(\lambda_2 + \lambda_3 + \eta_2 + 2\eta_3) > 0 \ , \quad (4.15) \]
respectively. These relations are consistent with the positivity conditions of the potential for large values of the fields $\phi$. From (4.13) and (3.14), we obtain a constraint
\[ 2m^2(H_3^0) > m^2(H_1^0) \ . \quad (4.16) \]

If we suppose $\xi \to \infty$, the Higgs bosons $\chi_i^{\pm j}$, $\chi_3^\pm$, $\chi_i^0$, $\chi_3^0$, $H_i^{0j}$ and $H_3^0$ decouple from the present model, the physical Higgs bosons become only $H_1^0$, $H_2^0$, $\chi_2^0$ and $\chi_2^\pm$, so that the model becomes essentially identical with the two-Higgs-doublet model [11].

On the other hand, weak boson masses are obtained by calculating the kinetic term $\text{Tr}(D_\mu \phi D^\mu \phi)$ ($D_\mu$ is a covariant derivative). From the straightforward calculation, we can check that the Higgs bosons which are eaten by weak bosons $W^\pm$ and $Z^0$ are $\chi_1^\pm$ and $\chi_1^0$. The masses of weak bosons are given by
\[ m^2(W^\pm) = \frac{1}{2} g^2 v_0^2 \ , \quad (4.17) \]
\[ m^2(Z^0) = \frac{1}{2} g_z^2 v_0^2 \ , \quad (4.18) \]
where $g_z = g/\cos \theta_W$. Therefore, the value of $v_0$ is given by
\[ v_0^2 = \frac{m^2(W^\pm)}{g^2/2} = \frac{1}{4G_F/\sqrt{2}} = (174 \text{ GeV})^2 \ . \quad (4.19) \]

Since we take interest only in new effects which are caused by the existence of $\phi_L$, we neglect mixing of $\phi_L$ with $\phi_R$. Since, for a time, we deal only with tree-level physics, we calculate the interactions of $\phi_L$ by taking the unitary gauge.
Interactions of $\phi_L$ with gauge bosons are calculated from the kinetic term
$\text{Tr}(D_\mu \phi L D^\mu \phi_L)$. The results are as follows:

$$H_{EW} = + \frac{1}{2} \left( 2gm_W W^\mu_\mu + g_z m Z Z^\mu \right) H_1^0$$

$$+ i \left( eA_\mu + \frac{1}{2} g_z \cos 2\theta W Z_\mu \right) \text{Tr}(\chi^- \partial^\mu \chi^+) + \frac{1}{2} g_z Z_\mu \text{Tr}(\chi^0 \partial^\mu H^0)$$

$$+ \left( e^2 A_\mu A^\mu + eg_z \cos 2\theta W A_\mu Z^\mu + \frac{1}{4} g_z^2 \cos^2 2\theta W Z_\mu Z^\mu \right) \text{Tr}(\chi^- \chi^+)$$

$$+ \frac{1}{5} g_z^2 Z_\mu Z^\mu \left[ \text{Tr}(\chi^0 \chi^0) + \text{Tr}(H^0 H^0) \right]$$

$$+ \frac{1}{4} g^2 W^\mu_\mu \left[ 2\text{Tr}(\chi^- \chi^+) + \text{Tr}(\chi^0 \chi^0) + \text{Tr}(H^0 H^0) \right]$$

$$- \frac{1}{2} g \left( eA_\mu - g_z \sin^2 \theta W Z_\mu \right) \left\{ W^\mu + \text{Tr}(\chi^- \chi^+) + i \text{Tr}(\chi^- H^0) \right\} + \text{h.c.} \right\} , \quad (4.20)$$

where $g_z = g / \cos \theta_W$ and $\chi^\pm_1 = \chi^0_1 = 0$. Note that the interactions of the neutral Higgs boson $H^0_1$ with gauge bosons are completely identical with those of the neutral Higgs boson $H^0$ in the standard model.

Three-body interactions of $\phi_L$ are calculated from the potential (3.1):

$$H_{\phi\phi\phi} = \sqrt{2} \lambda_1 v_0 \text{Tr}(\chi^- \chi^+) H_1^0 + \frac{1}{\sqrt{2}} (\lambda_1 + \lambda_2) v_0 \text{Tr}(\chi^0 \chi^0 + H^0 H^0) H_1^0$$

$$- \frac{1}{\sqrt{2}} \lambda_3 v_0 \text{Tr}(\chi^0 \chi^0 - H^0 H^0) H_1^0 + \sqrt{2} i \lambda_4 \text{Tr}[(\chi^0 v - v \chi^0) \chi^- \chi^+]$$

$$+ \frac{1}{2\sqrt{2}} \lambda_5 \left[ \text{Tr} \left( (H^0 v + v H^0)(\chi^- \chi^+ + \chi^+ \chi^-) \right) - i \text{Tr} \left( (\chi^0 v - v \chi^0)(\chi^- \chi^+ - \chi^+ \chi^-) \right) \right.$$

$$\left. - 2 \text{Tr}[v(\chi^+ H^0 \chi^- + \chi^- H^0 \chi^+)] \right]$$

$$+ \frac{1}{2\sqrt{2}} \eta_1 v_0 \left[ (H_1^0 - H_2^0) \text{Tr}(2\chi^- \chi^+ + \chi^0 \chi^0 + H^0 H^0) \right]$$
As we discuss in Sect. 5 and Sect. 6, we consider that only the Higgs boson $H_1^0$ is light compared with the other Higgs bosons whose masses are of the order of TeV. The interaction (4.21) states the absence of the decays of these Heavy Higgs bosons into two $H_1^0$ states. Of course, the lightest Higgs boson $H_1^0$ cannot decay into multi-Higgs-boson states. The dominant decay modes of our Higgs bosons are those into two fermion states.

5. Effective fermion-Higgs interactions and $K_L \rightarrow \mu^\pm e^\mp$ decay

Our Higgs particles $\phi_L$ do not have interactions with light fermions $f$ at tree level, and they can couple only between light fermions $f$ and heavy fermions $F$. However, since the fermion mass matrix (2.3) is diagonalized, the physical fermion states (mass eigenstates) are mixed states of $f$ and $F$. The physical fermion states are given by

$\left( \begin{array}{c} f^\text{phys}_L \\ F^\text{phys}_L \end{array} \right) = \left( \begin{array}{cc} U^f_L & 0 \\ 0 & U^F_L \end{array} \right) \left( \begin{array}{c} f^\text{phys}_L \\ F^\text{phys}_L \end{array} \right)$, \hspace{1cm} (5.1)

(and the similar relation with $L \rightarrow R$), where $6 \times 6$ unitary matrices $U_L^{(6 \times 6)}$ and $U_R^{(6 \times 6)}$ diagonalize the $6 \times 6$ mass matrix (2.3) as

$U_L^{(6 \times 6)} \left( \begin{array}{cc} 0 & m_L \\ m_R & M_F \end{array} \right) U_R^{(6 \times 6)\dagger} = \left( \begin{array}{cc} M_f & 0 \\ 0 & M'_f \end{array} \right)$, \hspace{1cm} (5.2)
and 3 × 3 unitary matrices $U^f_{L/R}$ and $U^F_{L/R}$ diagonalize the 3 × 3 mass matrices $M_f$ and $M'_F$ in (5.2) into the diagonal matrices $D_f$ and $D_F$ as

$$U^f_L M_f U^f_R \dagger = D_f , \quad U^F_L M'_F U^F_R \dagger = D_F , \quad (5.3)$$

respectively.

In the “seesaw” approximation $(m_L, m_R, \ll M_F)$, the 6 × 6 unitary matrices $U^{(6 \times 6)}_{L/R}$ are given by

$$U^{(6 \times 6)}_L \simeq \left( \begin{array}{cc} 1 & -m_L M^{-1}_R \\ M^{-1}_R m_L & 1 \end{array} \right) , \quad (5.4)$$

$$U^{(6 \times 6)}_R \simeq \left( \begin{array}{cc} 1 & -m_R M^{-1}_F \\ M^{-1}_F m_R & 1 \end{array} \right) , \quad (5.5)$$

where $m_L$ and $m_R$ are Hermitian in the present model. Therefore, an interaction vertex with the fermions $(f, F)$

$$\left( \begin{array}{c} \Gamma_{11} \\ \Gamma_{12} \\ \Gamma_{21} \\ \Gamma_{22} \end{array} \right) , \quad (5.6)$$

is also transformed into

$$\left( \begin{array}{cc} \Gamma^{\text{phys}}_{11} & \Gamma^{\text{phys}}_{12} \\ \Gamma^{\text{phys}}_{21} & \Gamma^{\text{phys}}_{22} \end{array} \right) , \quad (5.7)$$

where

$$\Gamma^{\text{phys}}_{11} \simeq U^f_L \left[ \Gamma_{11} - \Gamma_{12} M_F^{-1} m_R - m_L M_F^{-1} \Gamma_{21} + m_L M^{-1}_F \Gamma_{22} M^{-1}_F m_R \right] U^f_R \dagger$$

$$\simeq U^f_L \left[ \Gamma_{11} + \Gamma_{12} m_L^{-1} M_f + M_f m_R^{-1} \Gamma_{21} + M_f m_R^{-1} \Gamma_{22} m_L^{-1} M_f \right] U^f_R \dagger , \quad (5.8)$$

and so on. In (5.8), we have used the relation

$$M_f \simeq -m_L M_F^{-1} m_R . \quad (5.9)$$
For the effective Hamiltonian for the decay of the Higgs bosons ($H$), where we have used the relations

$$\Gamma_{\text{phys}}^{\phi_L} \approx U^{f}_{\phi_L} m^{-1}_L M f U^{\dagger}_R = U^{f}_{\phi_L} \langle \phi_L \rangle_0^{-1} U^{\dagger}_L D_f ,$$  

(5.10)

where we have used the relations $m_L = y_f \langle \phi_L \rangle_0$ and $U^{f}_L M f U^{\dagger}_R = D_f$.

For charged leptons $f = e$, since $U^{e}_L = U^{e}_R = 1$, $D_e = \text{diag}(m_e, m_\mu, m_\tau)$ and $\langle \phi_L \rangle_0^{-1} = v^{-1} = \text{diag}(v_1^{-1}, v_2^{-1}, v_3^{-1})$, the fields ($\phi_L$) couple to ($\tilde{e}_L^{\text{phys}}, \tilde{e}_R^{\text{phys}}$) with the effective coupling

$$\kappa_j \equiv \frac{m_j}{v_j} = \frac{1}{v_0} \sqrt{m_j^2(m_\tau + m_\mu + m_e)} = \frac{m_\tau + m_\mu + m_e}{v_0} \kappa_j .$$  

(5.11)

For example, for the interaction of $(H^0 - i\chi^0)/\sqrt{2}$, we obtain

$$\frac{1}{\sqrt{2}} \kappa_j (\tilde{e}^{L,R}_e e_R) (H^0 - i\chi^0)_i^j + \frac{1}{\sqrt{2}} \kappa_j (\tilde{e}^{L,R}_e e_L) (H^0 + i\chi^0)_i^j$$

$$= \frac{1}{2\sqrt{2}} \left\{ (\tilde{e}^{L,R}_e (a_{ij} - b_{ij} \gamma_5) e_j) (H^0)_i^j + i (\tilde{e}^{L,R}_e (b_{ij} - a_{ij} \gamma_5) e_j) (\chi^0)_i^j \right\} ,$$  

(5.12)

where

$$a_{ij} = \kappa_i + \kappa_j , \quad b_{ij} = \kappa_i - \kappa_j .$$  

(5.13)

Therefore, in the pure leptonic modes, the exchange of $\phi_L$ cannot cause family-number non-conservation. On the other hand, in quark sector, since $U^{q}_{L/R} \neq 1$, the family-number non-conservation is, in general, caused by the exchange of $\phi_L$.

Note that even in the limit of $U^{f}_L = 1$, the Higgs bosons ($H^0)_i^j$ ($i \neq j$) can sensitively affect rare decay modes $K_L \rightarrow e^+ \mu^-$, $K^+ \rightarrow \pi^+ e^- \mu^+$, $D^+ \rightarrow \pi^+ \mu^- e^+$, $B^+ \rightarrow \pi^+ e^- \tau^+$, $B^+ \rightarrow K^+ \mu^- \tau^+$, and so on. The most rigorous bound of the mass of the Higgs bosons ($H^0)_i^j$ ($i \neq j$) comes from the rare decay $K_L \rightarrow e^+ \mu^+$. The effective Hamiltonian for the decay $s \rightarrow d + e^+ + \mu^-$ is given by

$$H_{\text{eff}} = \frac{1}{m_H^2} (\overline{d} (a_{sd} + b_{sd} \gamma_5) s) (\overline{\mu} (a_{\mu e} - b_{\mu e} \gamma_5) e)$$

$$+ \frac{1}{m_\chi^2} (\overline{d} (b_{sd} + a_{sd} \gamma_5) s) (\overline{\mu} (b_{\mu e} - a_{\mu e} \gamma_5) e) ,$$  

(5.14)
where
\[ a_{sd} \simeq \frac{1}{2 \sqrt{2}} \left( \frac{m_s}{v_2} + \frac{m_d}{v_1} \right), \quad b_{sd} \simeq \frac{1}{2 \sqrt{2}} \left( \frac{m_s}{v_2} - \frac{m_d}{v_1} \right), \quad (5.15) \]

\[ a_{\mu e} = \frac{1}{2 \sqrt{2}} \left( \frac{m_\mu}{v_2} + \frac{m_e}{v_1} \right), \quad b_{\mu e} = \frac{1}{2 \sqrt{2}} \left( \frac{m_\mu}{v_2} - \frac{m_e}{v_1} \right), \quad (5.16) \]

and we have used \( U_d \simeq 1 \). By using the relation
\[ \langle 0 | (\bar{d} \gamma_5 s) | K^0(p) \rangle = -i \frac{f_K m_K^2}{m_s + m_d} \]

we obtain the decay amplitude
\[ A(K^0 \to e^- \mu^+) = \frac{f_K m_K^2}{m_s + m_d} \left[ \frac{b_{sd}}{m_H^2} (\bar{\mu}(a_{\mu e} - b_{\mu e} \gamma_5)e) + \frac{a_{sd}}{m_\chi^2} (\bar{\mu}(b_{\mu e} - a_{\mu e} \gamma_5)e) \right], \quad (5.18) \]

so that
\[ \Gamma(K_L \to e^\pm \mu^\mp) \simeq \frac{m_K}{256 \pi} \left( \frac{f_K m_K^2}{m_s + m_d} \right)^2 \left( \frac{1}{m_H^2} + \frac{1}{m_\chi^2} \right)^2 \left( \frac{m_s}{v_2} \right)^2 \left( \frac{m_\mu}{v_2} \right)^2, \quad (5.19) \]

where we have used the approximation for \((m_d/v_1)^2 \ll (m_s/v_2)^2\) and \((m_e/v_1)^2 \ll (m_\mu/v_2)^2\). The experimental lower bound \( B(K_L \to e^\pm \mu^\mp) < 3.3 \times 10^{-11} \) puts the constraint
\[ \left( \frac{1}{m_H^2} + \frac{1}{m_\chi^2} \right)^{-1} > (1.69 \text{ TeV})^2. \quad (5.20) \]

If \( m_H \sim m_\chi \), (5.20) leads to the lower bound
\[ m(H_3) \simeq m(\chi_3) > 2.4 \text{ TeV}. \quad (5.21) \]

Thus, Higgs scalar masses are expected to be a few TeV region.

6. Constraints on the Higgs boson masses from \( P^0 \bar{T}^0 \) mixings

As stated in the previous section, since \( U_L^T \neq 1 \) in quark sector, flavor changing neutral currents (FCNC), in general, appear by exchanging the Higgs bosons

\[ 19 \]
\( \phi_2, \phi_3 \) and \( \phi^i_j \ (i \neq j) \) (\( \phi = H^0 \) and \( \chi^0 \)), and they can sensitively contribute to the \( K^0\overline{K}^0 \), \( D^0\overline{D}^0 \) and \( B^0\overline{B}^0 \) mixings. In this section, we study the magnitudes of FCNC in details.

Note that as far as the physical Higgs boson \( H_1^0 \) is concerned, the interaction with quarks \( q_i \) is still diagonal type \( (\overline{q}_L q_i) H_{1}^0 \), because \( H_{1}^0 = \sum_i v_i (H_0^0)_{i}/v_0 \), so that

\[
\frac{1}{\sqrt{2}} v_0 (\overline{q}_L U_L^0 v (\phi_L)_{0}^{-1} U^*_L D q_R) H_{1}^0 + \text{h.c.} = \frac{1}{\sqrt{2}} \sum_i \frac{m_i^q}{v_0} (\overline{q}_i q_i) H_{1}^0. \tag{6.1}
\]

Therefore, the interactions of \( H_1^0 \) with quarks are identical with those of the physical neutral Higgs boson \( H_{SM}^0 \) in the standard model. Recall that the electroweak interactions of \( H_1^0 \) are also identical with those of \( H_{SM}^0 \). As far as \( H_1^0 \) is concerned, we cannot distinguish from the standard-model Higgs boson \( H_{SM}^0 \) experimentally.

The fermion-Higgs boson interactions are, from (5.10), given by

\[
H_{f\phi} = (\overline{f} L U v (\phi)_{0}^{-1} U^*_L D f_R) + \text{h.c.}
\]

\[
= \sum_{i,j} (\overline{f}_i L f_j R) \left( \sum_{k \neq l} \phi^l_k m_j U^k_i U^*_j + \sum_k \phi^k l m_j U^k_i U^*_j \right) + \text{h.c.}, \tag{6.2}
\]

where \( U \equiv U_L^f, \phi \equiv \phi^0 = (H^0 - i \chi^0)/\sqrt{2}, D \equiv D_f = \text{diag}(m_f^u, m_f^d, m_f^s) \) and \( f = u, d \).

The interactions with \( \phi^l_k \ (k \neq l) \) are rewritten as follows:

\[
\frac{1}{2\sqrt{2}} \sum_{i,j} \sum_{k \neq l} \overline{f}_i \left( (A_{ij}^{kl} - B_{ij}^{kl} \gamma_5) (H_0^0)_{k}^l + i(B_{ij}^{kl} - A_{ij}^{kl} \gamma_5) (\chi^0)_{k}^l \right) f_j, \tag{6.3}
\]

where

\[
A_{ij}^{kl} = \frac{1}{2} \left( \frac{m_i}{v_k} + \frac{m_j}{v_l} \right) U^k_i U^*_j, \quad B_{ij}^{kl} = \frac{1}{2} \left( \frac{m_i}{v_k} - \frac{m_j}{v_l} \right) U^k_i U^*_j. \tag{6.4}
\]

For the interactions with \( \phi^k_k \), by using the expression

\[
\phi^k_k = z_k \phi_1 + \left( \frac{1}{\sqrt{3}} z_k - \frac{2}{3} \right) \phi_2 + \sqrt{\frac{2}{3}} (z_l - z_m) \phi_3, \tag{6.5}
\]

\( z_l \) and \( z_m \) are introduced.
where \((k, l, m)\) are cyclic indices of \((1,2,3)\), we can obtain

\[
\sum_{i,j} m_j v_0 \left[ \delta_i^j (\phi_1 + \phi_2) - \sqrt{\frac{2}{3}} \phi_2 \sum_k \frac{1}{z_k} U_i^k U_j^{k*} + \sqrt{\frac{2}{3}} \phi_3 \sum_k \frac{z_l - z_m}{z_k} U_i^k U_j^{k*} \right] + \text{h.c.}
\]

\[
= \frac{1}{\sqrt{2}} \sum_i \frac{m_i}{v_0} \left[ \langle \bar{f}_i f_i (H_1^0 + H_2^0) - i(\bar{f}_i \gamma_5 f_i) \chi_2^0 \right]
\]

\[
- \frac{1}{\sqrt{3}} \sum_{i,j} \left[ \langle \bar{f}_i (a_{ij} - b_{ij} \gamma_5) f_j \rangle H_2^0 + i \langle \bar{f}_i (b_{ij} - a_{ij} \gamma_5) f_j \rangle \chi_2^0 \sum_k \frac{z_l - z_m}{z_k} U_i^k U_j^{k*} \right], \quad (6.6)
\]

where

\[
a_{ij} = \frac{1}{2} \left( \frac{m_i}{v_0} + \frac{m_j}{v_0} \right), \quad b_{ij} = \frac{1}{2} \left( \frac{m_i}{v_0} - \frac{m_j}{v_0} \right). \quad (6.7)
\]

The effective four-Fermi interactions of FCNC are given by

\[
H_{FCNC} = \frac{1}{3} \sum_{i \neq j} \left[ \frac{1}{m^2(H_0^0)} \left( \langle \bar{f}_i (a_{ij} - b_{ij} \gamma_5) f_j \rangle \right)^2 \right.
\]

\[
- \frac{1}{m^2(\chi_2^0)} \left( \langle \bar{f}_i (b_{ij} - a_{ij} \gamma_5) f_j \rangle \right)^2 \left( \sum_k \frac{1}{z_k} U_i^k U_j^{k*} \right)^2
\]

\[
+ \frac{1}{3} \sum_{i \neq j} \left[ \frac{1}{m_2(H_3^0)} \left( \langle \bar{f}_i (a_{ij} - b_{ij} \gamma_5) f_j \rangle \right)^2 \right.
\]

\[
- \frac{1}{m_2(\chi_3^0)} \left( \langle \bar{f}_i (b_{ij} - a_{ij} \gamma_5) f_j \rangle \right)^2 \left( \sum_k \frac{z_l - z_m}{z_k} U_i^k U_j^{k*} \right)^2
\]

\[
+ \frac{1}{2} \sum_{i \neq j, k \neq l} \left[ \frac{1}{m^2(H_k^0)} \langle \bar{f}_i (A_{ij}^kl - B_{ij}^kl \gamma_5) f_j \rangle \langle \bar{f}_i (A_{ij}^{lk} - B_{ij}^{lk} \gamma_5) f_j \rangle \right.
\]

\[
- \frac{1}{m^2(\chi_k^0)} \left( \langle \bar{f}_i (B_{ij}^kl - A_{ij}^kl \gamma_5) f_j \rangle \langle \bar{f}_i (B_{ij}^{lk} - A_{ij}^{lk} \gamma_5) f_j \rangle \right). \quad (6.8)
\]
If we suppose the case $\xi \to \infty$, we can neglect the contributions from $\phi_3$ and $\phi^l_k (k \neq l)$ to the $K^0\bar{K}^0$ mixing and so on, since these Higgs bosons decouple from the low energy effective theory. However, even then, the contributions form $\phi_2$ still remain. From the experimental values of $K^0-L^0$ and $D^0_1-D^0_2$ mass differences, the masses of $H_2^0$ and $\chi_2^0$ must be large than $10^5$ GeV. Considering from (4.14), (4.15) and $v_0 = 174$ GeV, it is unlikely that $H_2^0$ and $\chi_2^0$ have such large masses as far as we suppose that the coupling constants $\lambda_3, \eta_1, \eta_2$ and $\eta_3$ are of the order of one or less than it.

Note that the contributions form $\chi^0$ have the opposite signs to that from $H^0$. If we suppose $m^2(H^0_k) = m^2(\chi_k) \equiv m^2_{H^0_k} (k = 2, 3)$, which means

$$
\eta_1 + \eta_2 = 0 \ , \quad \lambda_3 + \eta_3 = 0 ,
$$

the contributions of $H_{\text{FCNC}}$ are considerably reduced:

$$
H_{\text{FCNC}} = \frac{1}{3} \sum_{i \neq j} \left[ \frac{1}{m^2(H^0_2)} \left( \sum_k \frac{1}{z_k} U^k_i U^k_j \right)^2 + \frac{1}{m^2(H^0_3)} \left( \sum_k \frac{z_l - z_m}{z_k} U^k_i U^k_j \right)^2 

- \frac{3}{2} \frac{1}{m^2(H^0_l)} \sum_k \left( \frac{1}{z_k} U^k_i U^k_j \right)^2 \right] \left[ (\bar{f}_i f_j)^2 - (\bar{f}_i \gamma_5 f_j)^2 \right]

= \frac{1}{3} \left( \frac{1}{m^2_{H^2}} - \frac{1}{m^2_{H^3}} \right) \sum_{i \neq j} \frac{m_i m_j}{v_0^2} \sum_k \left( \frac{1}{z_k} + \frac{z_l - z_m}{z_1 z_2 z_3} \right) (U^k_i U^k_j)^2

\times \left[ (\bar{f}_i f_j)^2 - (\bar{f}_i \gamma_5 f_j)^2 \right] ,
$$

where we have used the relations (B1)–(B4) given in Appendix B and

$$
U^i_j U^m_i U^{m*}_j = \frac{1}{2} \left[ (U^k_i U^{k*}_j)^2 - (U^i_j U^{*i}_j)^2 - (U^m_i U^{m*}_j)^2 \right] .
$$

The constraint (6.9) rewrites the $\eta_1$- and $\eta_2$-terms in the potential (3.3) into

$$
\eta_i \sum_{i,j} \left[ \phi^0_s (\phi^0_{oct})_i^j - (\phi^s_{oct})_i^j \phi^0_s \right] \left[ \phi^- (\phi^-_{oct})_j^i - (\phi^-_{oct})_j^i \phi^- \right] ,
$$
where \((\phi^+\phi^0 - \phi^0\phi^+)\) and \((\phi^-\phi^0 - \phi^-\phi^0)\) belong to \(I = 0\) states. The constraint (6.10) leads the \(\lambda_3\) - the \(\eta_3\)-terms in the potential to

\[
\frac{1}{2} \lambda_3 \left[ \text{Tr}(\phi_{\text{oct}}\phi_{\text{oct}}) - \phi_s\phi_s \right] \cdot \left[ \text{Tr}(\phi_{\text{oct}}\phi_{\text{oct}}) - \phi_s\phi_s \right],
\]

(6.14)

where \(\phi_s\cdot\phi_s\) denotes the \(I = 0\) component in \((I = 1) \times (I = 1)\), i.e.,

\[
\phi_s\cdot\phi_s = \phi^-\phi^- - \phi^+\phi^+ - \phi^0\phi^0 + \phi^0\phi^0 - \phi^0\phi^0 + \phi^0\phi^0.
\]

(6.15)

Then, the physical Higgs boson masses are given by

\[
m^2_{H_1} \equiv m^2(H_0^0) = 2(\lambda_1 + \lambda_2)v_0^2,
\]

\[
m^2_{H_2} \equiv m^2(H_2^0) = m^2(\chi_2^0) = 2\lambda_3v_0^2,
\]

\[
m^2_{H_3} \equiv m^2(H_3^0) = m^2(H_3^0) = m^2(\chi_3^0) = m^2(\chi_3^0) = m^2(\chi_3^0) = \xi v_0^2,
\]

\[
m^2(\chi_2^0) = -(\lambda_2 + \eta_2 - \lambda_3)v_0^2,
\]

\[
m^2(\chi_3^0) = -(\lambda_2 + \frac{1}{2}\eta_2 - \xi)v_0^2.
\]

(6.16)

The mass difference between \(P_i^j\) and \(\overline{P}_i^j\), \(\Delta m_P = m(P_i^j) - m(\overline{P}_i^j)\), is given by [12]

\[
\Delta m_P = \eta_{QCD} B_Pj_{P}^2 m_P \left[ \left( \frac{m_P}{m_i + m_j} \right)^2 - \frac{1}{6} \right] \frac{1}{3} \left( \frac{1}{m_{H_2}^2} - \frac{1}{m_{H_3}^2} \right) K_{ij},
\]

(6.17)

where \(\eta_{QCD}\) is a QCD correction factor from hard gluon exchange, \(B_P\) is a parameter that characterizes the inaccuracy of the vacuum insertion approximation, and

\[
K_{ij} = \frac{m_i m_j}{v_0^2} \sum_k \left( \frac{1}{z_k^2} + \frac{z_k - z_l - z_m}{z_1 z_2 z_3} \right) (U_i^k U_j^k)^2.
\]

(6.18)

Although the \((\pi c)\) and \((\overline{D}s)\) currents involve the small factors \(m_u m_c/v_0^2 \simeq 2.8 \times 10^{-7}\) and \(m_d m_s/v_0^2 \simeq 6.5 \times 10^{-8}\), respectively, the contributions of them to the \(K_L\) - \(K_S\) and \(D^0\) - \(\overline{D}^0\) mass differences are not negligible because \(K_{ij}\) are given by \(K_{12} \simeq \ldots\)
\[ (U_L^d)^1_2 / z_1^2 \] for \( \Delta m_K \) and \( K_{12} \simeq (U_L^n)^1_2 / z_1^2 \) for \( \Delta m_D \), and the factor \( 1/z_1^2 \) takes a large value. The experimental values [6] \( m_{KL} - m_{K_S} = (3.510 \pm 0.018) \times 10^{-12} \) MeV, \( |m_{D_1} - m_{D_2}| < 2 \times 10^{10} \) h\(s^{-1} \), \( m_{B^+_u} - m_{B^+_d} = (0.51 \pm 0.06) \times 10^{12} \) h\(s^{-1} \), and \( m_{B^0_{sH}} - m_{B^0_{sL}} > 1.8 \times 10^{12} \) h\(s^{-1} \) lead to the constraints

\[
(1/m_{H_2}^2 - 1/m_{H_3}^2)^{-1/2} \simeq \left( \eta_{QCD}^K \right)^{1/2} |(U_L^d)^1_1(U_L^d)^1_2| \times 32 \text{ TeV} , \tag{6.19}
\]

\[
(1/m_{H_2}^2 - 1/m_{H_3}^2)^{-1/2} \simeq \left( \eta_{QCD}^D \right)^{1/2} |(U_L^u)^1_1(U_L^u)^1_2| \times 16 \text{ TeV} , \tag{6.20}
\]

\[
(1/m_{H_2}^2 - 1/m_{H_3}^2)^{-1/2} \simeq \left( \eta_{QCD}^B \right)^{1/2} |(U_L^d)^1_1(U_L^d)^1_3| \times 38 \text{ TeV} , \tag{6.21}
\]

and

\[
(1/m_{H_2}^2 - 1/m_{H_3}^2)^{-1/2} < \left( \eta_{QCD}^{B_S} \right)^{1/2} |(U_L^d)^1_2(U_L^d)^1_3| \times 88 \text{ TeV} , \tag{6.22}
\]

respectively, where we, for simplicity, have put the other contributions to \( P^0 - \overline{P}^0 \) mixing zero, and, we have used \( B_K = 0.65 \) [13] and \( f_K = 0.160 \) GeV [6] in (6.19) and \( f_D B_D^{1/2} = f_B B_B^{1/2} = f_{B_S} B_{B_S}^{1/2} = 0.2 \) GeV in (6.20) – (6.22).

The numerical estimates of the Higgs boson masses depend on the structures of \( U_L^n \) and \( U_L^d \). From the constraint \( V_{us} = (U_L^n)^{0t}_1 (U_L^d)^1_1 \simeq 0.22 \), we cannot consider a mass matrix model which provides \( (U_L^n)^2_1 \simeq 0 \) and \( (U_L^d)^2_1 \simeq 0 \) simultaneously. We suppose \( |(U_L^n)^2_1| \sim |(U_L^d)^2_1| \). If we take \( |(U_L^d)^1_1(U_L^d)^1_2| \simeq 0.22 \) by way of trial and \( \eta_{QCD} \simeq 1 \), the constraint (6.19) predicts \( (1/m_{H_2}^2 - 1/m_{H_3}^2)^{-1/2} \simeq 7.1 \) TeV. Only when \( m_{H_2} \simeq m_{H_3} \), the FCNC processes are highly suppressed. Since we have known \( m_{H_2} > 2.4 \) TeV from the data of \( K_L \to e^\pm \mu^\mp \), we cannot take too low a value of \( m_{H_2} \). For example, for \( m_{H_2} \simeq 2.5 \) TeV, the mass difference must be a very small value \( m_{H_3} - m_{H_2} \simeq 0.14 \) TeV.

### 7. Production and decays of new Higgs bosons

As we stated in Sect.4 and Sect.6, as far as the Higgs boson \( H_1^0 \) is concerned, its interactions with electroweak gauge bosons and with light fermions (quarks and leptons) are exactly the same ones as the physical Higgs boson \( H^0_{SM} \) in the standard model. From (4.16), the decays \( H_1^0 \to H_2^0 H_2^0, \chi_k^0 \chi_k^0 \) \( (k = 2, 3) \), \( (H^0)^i_j(H^0)^j_i \) and \( \chi_i^i \chi_j^j \) \( (i \neq j) \) are forbidden, so that the dominant decay mode is \( H_1^0 \to b\bar{b} \).
Therefore, in the present model, it is hard to distinguish the Higgs boson $H_1^0$ from $H_{SM}^0$ in the standard model.

The most distinguishable ones from the physical Higgs bosons in the standard model and/or in the conventional multi-Higgs model are $(H^0)_i^j$ and $(\chi^0)_i^j$ ($i \neq j$). If we suppose that they have masses of a several hundred GeV, we may expect a production

$$e^+ + e^- \rightarrow Z^* \rightarrow (H^0)_i^j + (\chi^0)_j^i,$$

$$\leftrightarrow f_i + \bar{f}_j \quad \leftrightarrow f_j + \bar{f}_i,$$

in a super $e^+e^-$ linear collider in the near future. Unfortunately, as discussed in Sect.5 and Sect.6, their masses must be larger than a few TeV, so that we cannot expect the observation in $e^+e^-$ collider.

Only a chance of the observation of our Higgs bosons $\phi_i^j$ is in a production

$$u + q(\bar{q}) \rightarrow t + (\phi)_i^3 + q(\bar{q}) \quad (q = u, d, s),$$

(7.2)

at a super hadron collider with several TeV beam energy, for example, at LHC, because the coupling $a_{tu} (b_{tu})$ is sufficiently large to produce (7.2):

$$a_{tu} \simeq \frac{m_t}{v_3} + \frac{m_u}{v_1} = 1.029 + 0.002.$$  

(7.3)

The dominant decay modes of $(H^0)_2^3$ and $(H^0)_1^3$ are hadronic ones, i.e., $(H^0)_2^3 \rightarrow c\bar{c}, s\bar{b}$ and $(H^0)_1^3 \rightarrow u\bar{t}, d\bar{b}$. Only for $(H^0)_1^3$, which is produced by the reaction $u + q \rightarrow c + (H^0)_1^3 + q$, the leptonic mode $(H^0)_1^3 \rightarrow e^- \mu^+$ has a visible branching ratio:

$$\Gamma(H_1^2 \rightarrow u\bar{c}) : \Gamma(H_1^2 \rightarrow d\bar{s}) : \Gamma(H_1^2 \rightarrow e^- \mu^+)$$

$$\simeq 3 \left[ \frac{(m_c)}{(v_2)} + \frac{(m_u)}{(v_1)} \right] : 3 \left[ \frac{(m_s)}{(v_2)} + \frac{(m_d)}{(v_1)} \right] : \left[ \frac{(m_u)}{(v_2)} + \frac{(m_c)}{(v_1)} \right]$$

$$= 73.5\% : 24.9\% : 1.6\%,$$

(7.4)

where we have used an approximate relation $U_L^f \simeq 1$ and have taken the quark mass values $m_q(\mu)$ at $\mu = 1$ GeV, $m_u = 5.6$ MeV, $m_d = 9.9$ MeV, $m_s = 199$ MeV and $m_c = 1.49$ GeV, as the quark masses inside ordinary hadrons.

8. Conclusion
In conclusion, inspired by the phenomenological success of the charged lepton mass relation (1.4), we have proposed a model with $U(3)_{\text{family}}$ nonet Higgs bosons $\phi_L$ and $\phi_R$ and vector-like heavy leptons $F$ ($F = U, D, N, E$) correspondingly to ordinary quarks and leptons $f$ ($f = u, d, \nu, e$), and have investigated its possible new physics.

The charged lepton mass relation (1.4) can derive only when the potential $V(\phi)$ takes a special form (3.1), which satisfies “$U(3)$-family nonet” ansatz. In order to avoid massless physical Higgs bosons, we must consider a term which explicitly breaks $U(3)$-family symmetry, (3.13) [or (3.16)].

In the low energy phenomenology, only the light Higgs boson $\phi_L$ plays a role. Of the 36 components of our Higgs boson $\phi_L$, the three, $\chi_1^\pm$ and $\chi_0^1$, are eaten by the gauge bosons $W^\pm$ and $Z^0$, respectively. The neutral Higgs boson $H_1^0$ has the same interactions with fermions and electroweak gauge bosons, so that it is hard to distinguish our Higgs boson $H_1^0$ from the neutral Higgs boson in the standard model experimentally.

If we take the $\xi \to \infty$ limit in the explicit $U(3)$-family symmetry breaking term $V_{SB}$, the Higgs bosons which have finite masses become only $H_1^0$, $H_2^0$, $\chi_2^0$ and $\chi_2^\pm$, so that the model becomes similar to the two-Higgs-doublet model. However, differently from the conventional two-Higgs-doublet model, our Higgs bosons $H_2^0$ and $\chi_2^0$ can contribute to the flavor-changing neutral current processes, so that their masses must be larger than $10^2$ TeV. We think that such a case is unnatural.

For the case of a finite $\xi$, we have 33 physical Higgs bosons given by (4.5), (4.11) and (4.12). In order to suppress the rare decay modes $K_L \to \mu^+e^\mp$, $K^+ \to \pi^+e^-\mu^+$, and so on, we must put the constraint (5.20), which leads to $m(H_3) \simeq m(\chi_3) > 2.4$ TeV for $m_H \simeq m_\chi$. In order to suppress the FCNC, the masses of $H_2^0$ and $\chi_2^0$ must be, in general, larger than $10^2$ TeV. Only the case which gives an acceptably lower values of the Higgs boson masses is the case $m(H) = m(\chi)$ and $m_{H2} \simeq m_{H3}$. Then, we can expect our Higgs bosons with masses of 2.5 TeV (except for $H_1^0$). In a top pair production at LHC, we may expect an observation of $t\bar{t}$ pair with a large $p_T$, due to the production $u + q \to t + \phi_1^3 + q$ and the subsequent decay $\phi_1^3 \to u + \bar{t}$.

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Appendix A

More general potential form of $V_{\text{nonet}}$ is given by

$$V_{\text{nonet}} = \text{r.h.s. of (3.2)} + \sum_{i,j,k,l} \left[ \frac{1}{2} \lambda_4 (\overline{\phi}_i^j \phi_j^k)(\overline{\phi}_k^l \phi_l^i) 
\right. \\
+ \frac{1}{2} \lambda_5 (\overline{\phi}_i^j \phi_j^i)(\overline{\phi}_k^k \phi_k^k) + \frac{1}{2} \lambda_6 (\overline{\phi}_i^j \phi_j^k)(\overline{\phi}_k^l \phi_l^i) + \frac{1}{2} \lambda_7 (\overline{\phi}_i^j \phi_j^i)(\overline{\phi}_k^k \phi_k^k) \left. \right] . \quad (A.1)$$

Then, for $\mu^2 < 0$, conditions for minimizing the potential (3.1) are as follows:

$$\left[ \mu^2 + (\lambda_1 + \lambda_2) \text{Tr}(v^\dagger v) \right] v_s^* + \lambda_3 \text{Tr}(v^\dagger v^\dagger) v_s + (\lambda_4 + \lambda_5 + \lambda_6 + \lambda_7) \frac{1}{\sqrt{3}} \text{Tr}(v^\dagger v^\dagger v)$$

$$+ (\eta_1 + \eta_2) \text{Tr}(v^\dagger_{oct} v_{oct}) v_s^* + 2\eta_3 \text{Tr}(v^\dagger_{oct} v_{oct}) v_s = 0 , \quad (A.2)$$

$$\left[ \mu^2 + (\lambda_1 + \lambda_2) \text{Tr}(v^\dagger v) \right] v_{oct} + \lambda_3 \text{Tr}(v^\dagger v^\dagger) v_{oct} + (\lambda_4 + \lambda_5)(v^\dagger v)_{oct}$$

$$+ \frac{1}{2} (\lambda_6 + \lambda_7)(v^\dagger v^\dagger v + vv^\dagger v^\dagger)_{oct} + (\eta_1 + \eta_2) v_s^* v^\dagger_{oct} v_{oct}^* + 2\eta_3 v_s^* v_{oct}^* v_{oct} = 0 , \quad (A.3)$$

and the similar equations with $v \leftrightarrow v^\dagger$ ($v_{oct} \leftrightarrow v_{oct}^\dagger$ and $v_s \leftrightarrow v_s^\dagger$), where $A_{oct}$ means an octet part of the $3 \times 3$ matrix $A$, i.e., $A_{oct} = A - (1/3)\text{Tr}(A) \cdot 1$. Only when

$$\lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 = 0 , \quad (A.4)$$

we can obtain the desirable relation (3.6).

What is of great interest to us is whether these terms ($\lambda_4 - \lambda_7$ terms) can generate additional masses of the neutral Higgs bosons $H_i^{0j}$ ($i \neq j$) or not, because these bosons become to massless in the limit of $\overline{\xi} \to 0$. Unfortunately, these terms cannot contribute to the masses except for those of $\chi_i^{\pm j}$ and $\chi_i^{0j}$ ($i \neq j$), because the $\lambda_4 - \lambda_7$ terms still respect the SU(3) family symmetry. Therefore, the existence of Goldstone bosons cannot be avoided by the introduction of these terms. The existence of the $\lambda_4 - \lambda_7$ terms does not improve the situation of our model and only makes our study intricate, so that we have omitted the study of the $\lambda_4 - \lambda_7$ terms from the present studies.
Appendix B

Since the parameters \( z_i = v_i/v_0 \) satisfy the relations

\[
z_1^2 + z_2^2 + z_3^2 = 1 , \tag{B.1}
\]

\[
z_1 + z_2 + z_3 = \sqrt{\frac{3}{2}} , \tag{B.2}
\]

we find the relations

\[
z_1 z_2 + z_2 z_3 + z_3 z_1 = \frac{1}{4} , \tag{B.3}
\]

\[
z_i z_j = \frac{1}{4} - \sqrt{\frac{3}{2}} z_k + z_k^2 , \tag{B.4}
\]

where \((i, j, k)\) are cyclic indices of \((1, 2, 3)\).

In the present seesaw-type mass matrix model, the values \( v_i^2 \) are proportional to the charged lepton masses \( m_i^e = (m_e, m_\mu, m_\tau) \), the values \( z_i \) are given by

\[
z_i = [m_i^e/(m_e + m_\mu + m_\tau)]^{1/2} , \tag{B.5}
\]

i.e.,

\[
\begin{pmatrix}
z_1 \\
z_2 \\
z_3
\end{pmatrix}
= \begin{pmatrix}
0.016473 \\
0.23687 \\
0.97140
\end{pmatrix} , \tag{B.6}
\]

so that

\[
\begin{pmatrix}
x_1 \\
x_2 \\
x_3
\end{pmatrix}
= \begin{pmatrix}
-0.55405 \\
-0.24237 \\
+0.79642
\end{pmatrix}
= \cos(\frac{\pi}{4} + \delta) \frac{1}{\sqrt{6}} \begin{pmatrix}
-2 \\
1 \\
1
\end{pmatrix}
+ \sin(\frac{\pi}{4} + \delta) \frac{1}{\sqrt{2}} \begin{pmatrix}
0 \\
-1 \\
1
\end{pmatrix} , \quad (\delta = 2.268^\circ) . \tag{B.7}
\]

The expression \( (B.7) \) suggests that the Higgs boson state \( \phi_x \) is almost given by a 45°-mixing between \( \lambda_3 \)- and \( \lambda_8 \)-components of \( SU(3)_\text{family} \). At the present stage,
the parameter $\delta$ is pure phenomenological one. If we can give only the value of $\delta$, we can fix the values of $(x_1, x_2, x_3; y_1, y_2, y_3; z_1, z_2, z_3)$. The open question why $\delta$ takes such a value will be answered in a future theory.

References and Footnotes

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Table I. Quantum numbers of fermions and Higgs bosons

|     | $Y$                     | SU(2)$_L$ | SU(2)$_R$ | U(3)$_{family}$ |
|-----|-------------------------|-----------|-----------|-----------------|
| $f_L$ | $(\nu, e)^{Y=-1}_L$, $(u, d)^{Y=1/3}_L$ | 2         | 1         | 3               |
| $f_R$ | $(\nu, e)^{Y=-1}_R$, $(u, d)^{Y=1/3}_R$ | 1         | 2         | 3               |
| $F_L$ | $N^Y_L=0$, $E^Y_L=-2$, $U^Y_L=4/3$, $D^Y_L=-2/3$ | 1         | 1         | 3               |
| $F_R$ | $N^Y_R=0$, $E^Y_R=-2$, $U^Y_R=4/3$, $D^Y_R=-2/3$ | 1         | 1         | 3               |
| $\phi_L$ | $(\phi^+, \phi^0)^{Y=1}_L$ | 2         | 1         | $8+1$           |
| $\phi_R$ | $(\phi^+, \phi^0)^{Y=1}_R$ | 1         | 2         | $8+1$           |
| $\Phi_F$ | $\Phi_0^{Y=0}$, $\Phi_X^{Y=0}$ | 1         | 1         | 1, $8+1$       |

\[ (2, 1, 8+1) \quad (1, 1, 1) \quad (1, 2, 8+1) \]

\[ \langle \phi_0^L \rangle_0 \equiv m_L/y_f^L \quad \langle \Phi_F \rangle_0 \equiv M_F/y_F \quad \langle \phi_0^R \rangle_0 \equiv m_R/y_f^R \]

Fig. 1. Mass generation of quarks and leptons $f$