Klein–Nishina steps in the energy spectrum of galactic cosmic-ray electrons

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Abstract. The full Klein–Nishina cross-section of the inverse Compton scattering interactions of electrons implies a significant reduction of the electron energy loss rate compared with the Thomson limit when the electron energy exceeds the critical Klein–Nishina energy $E_K = \gamma_K m_e c^2 = 0.27 m_e^2 c^4 / (k_B T)$, where $T$ denotes the temperature of the photon graybody distribution. As a consequence, the total radiative energy loss rate of single electrons exhibits sudden drops in the overall $\dot{\gamma} \propto \gamma^2$-dependence when the electron energy reaches the critical Klein–Nishina energy. The strength of the drop is proportional to the energy density of the photon radiation field. The diffuse galactic optical photon fields from stars of spectral type B and G-K lead to critical Klein–Nishina energies of 40 and 161 GeV, respectively. Associated with the drop in the loss rate are sudden increases (Klein–Nishina steps) in the equilibrium spectrum of cosmic-ray electrons. Because the radiative loss rate of electrons is the main ingredient in any transport model of high-energy cosmic-ray electrons, Klein–Nishina steps will modify any calculated electron equilibrium spectrum irrespective of the electron sources and the spatial transport mode. To delineate most clearly the consequences of the Klein–Nishina decreases in the radiative loss rate, we chose as an illustrative example the simplest realistic model for cosmic-ray electron dynamics in the galaxy, consisting of the competition of radiative losses and secondary production by inelastic hadron–hadron collisions. We demonstrate that the spectral structure in the FERMI and HESS data is well described and even the excess measured by ATIC might be explained by Klein–Nishina steps.

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1. Introduction

Recent measurements of the energy spectrum of local galactic cosmic-ray electrons at energies above a few hundred GeV by the ATIC instrument (Chang et al. 2008) have reported a significant excess in the all-electron intensity that agrees, at lower energies, with the measurements of the PAMELA satellite experiment (Adriani et al. 2009), which has also observed a dramatic rise in the positron fraction starting at 10 GeV and extending up to 300 GeV. The significant ATIC excess has not been confirmed by the electron spectrum determinations with the FERMI satellite (Abdo et al. 2009) and the HESS air Cherenkov experiment (Aharonian et al. 2008, 2009), although these measurements indicate some spectral structure deviating from a pure power law behaviour in the ATIC energy range. These observations have motivated a large number of interpretations, from possible signatures of dark matter annihilation (e.g. Simet and Hooper 2009) to nearby astrophysical electron sources.

Here we explain the ATIC excess by a classical effect that has so far not been discussed in this context: during their galactic propagation, positrons and electrons with energies above 10 GeV are subject to synchrotron radiation losses in the galactic magnetic field of about 3 µG and inverse Compton radiation losses in galactic target photon fields listed in table 1, including the universal microwave background radiation field, infrared photons and optical stellar photons. The diffuse galactic optical photons can be characterized by the superposition of two graybody distributions (see the discussion in section 2.3 of Schlickeiser 2002):

1 photons from stars of spectral type G-K with energy density $W_G = 0.3\,\text{eV cm}^{-3}$ and temperature $T_G = 5000\,\text{K}$, corresponding to a mean photon energy $\langle \epsilon \rangle_G = 2.7k_B T_G = 2.327 \times 10^{-4}\,T_G = 1.16\,\text{eV}$;

2 photons from stars of spectral type B with energy density $W_B = 0.09\,\text{eV cm}^{-3}$ and temperature $T_B = 20\,000\,\text{K}$, corresponding to a mean photon energy $\langle \epsilon \rangle_B = 2.7k_B T_B = 2.327 \times 10^{-4}\,T_B = 4.65\,\text{eV}$.

For electron Lorentz factors $\gamma$ much smaller than the critical Klein–Nishina Lorentz factor $\gamma_K = 0.27m_e c^2/k_B T = 1.58 \times 10^8 / T(\text{K})$ (see equation (3) below), the inverse Compton scattering cross-section of a single electron can be well approximated by the Thomson cross-section, resulting in the standard energy loss rate of single electrons $\dot{\gamma} = -4c \sigma_T W \gamma^2 / (3m_e c^2)$, where $\sigma_T = 6.65 \times 10^{-25}\,\text{cm}^2$ denotes the Thomson cross-section and $c$ the speed of light. However, for Lorentz factors $\gamma \geq \gamma_K$, the full Klein–Nishina cross-section has to be used (Jones 1965, Blumenthal and Gould 1970, Schlickeiser 1979, Prince and Schlickeiser 1979),
Table 1. Electromagnetic graybody radiation fields in the local interstellar medium.

| i | Comment | \( T_i \) (K) | \( W_i \) (eV cm\(^{-3}\)) | \( \gamma_{K,i} \) | \( E_{K,i} \) (GeV) |
|---|---|---|---|---|---|
| 1 | Spectral type B | 20 000 | 0.09 | \( 7.9 \times 10^4 \) | 40 |
| 2 | Spectral type G-K | 5000 | 0.3 | \( 3.2 \times 10^5 \) | 161 |
| 3 | Infrared | 20 | 0.4 | \( 7.9 \times 10^7 \) | 4.0 \( \times 10^4 \) |
| 4 | Microwave | 2.7 | 0.25 | \( 5.9 \times 10^8 \) | 3.0 \( \times 10^5 \) |

resulting in a significant reduction of the inverse Compton loss rate. For the two graybody optical photon distributions, the respective critical Klein–Nishina Lorentz factors are \( \gamma_{KN,G} = 3.2 \times 10^5 \), corresponding to an electron energy of \( E_{KN,G} = 161 \text{ GeV} \), and \( \gamma_{KN,B} = 7.9 \times 10^4 \), corresponding to an electron energy of \( E_{KN,G} = 40 \text{ GeV} \). We will demonstrate that this Klein–Nishina reduction of the inverse Compton energy loss rate leads to Klein–Nishina steps in the cosmic-ray electron equilibrium spectrum that describe the observed FERMI and HESS data well. In section 2, we determine the galactic synchrotron and inverse Compton energy loss rates in the full Klein–Nishina case. For the illustrative example of a purely secondary origin of galactic electrons, we show in section 3 the resulting Klein–Nishina steps in comparison with the recent electron spectrum observations.

2. Synchrotron and inverse Compton energy loss rates

The synchrotron energy loss rate of a single electron in a large-scale random magnetic field of constant strength \( B \) is (Crusius and Schlickeiser 1988)

\[
|\dot{\gamma}|_S = \frac{4\sigma_T c}{3m_e c^2} U_B \gamma^2,
\]

where \( U_B = B^2 / 8\pi = 0.22b_3^3 \text{ eV cm}^{-3} \) if we scale the galactic magnetic field strength as \( B = 3b_3 \mu\text{G} \).

In the appendix, we approximately calculate the inverse Compton energy loss rate of a single electron in one graybody photon field as

\[
|\dot{\gamma}|_C \simeq \frac{4\sigma_T c W}{3m_e c^2} \frac{\gamma_K^2 \gamma^2}{\gamma_K^2 + \gamma^2},
\]

where the critical Klein–Nishina Lorentz factor is given by

\[
\gamma_K = \frac{3\sqrt{5} m_e c^2}{8\pi k_B T} = \frac{0.27 m_e c^2}{k_B T}.
\]

For small electron Lorentz factors \( \gamma \ll \gamma_K \), the general inverse Compton energy loss rate (2) reduces to the Thomson limit

\[
|\dot{\gamma}|_C (\gamma \ll \gamma_K) \simeq \frac{4\sigma_T c W}{3m_e c^2} \gamma^2,
\]

whereas for large electron Lorentz factors \( \gamma \gg \gamma_K \), we obtain the energy-independent extreme Klein–Nishina limit

\[
|\dot{\gamma}|_C (\gamma \gg \gamma_K) \simeq \frac{4\sigma_T c W}{3m_e c^2} \gamma_K^2.
\]
The total radiative (synchrotron and inverse Compton) energy loss rate of a single electron is given by the sum of rate (1) and rate (2) for the four diffuse galactic radiation fields listed in table 1, yielding

$$|\dot{\gamma}|_R = \frac{4\sigma_T c U_B \gamma^2}{3m_e c^2} \left[ 1 + \frac{\sum_{i=1}^{4} W_i}{U_B \gamma^2 + \gamma_{K,i}^2 i} \right].$$

In figure 1, we show the resulting radiative energy loss rate for the local galactic magnetic field and photon energy densities for relativistic electrons with energies between 1 and $10^7$ GeV. One clearly notices the four sudden drops whenever the electron energy reaches each of the critical Klein–Nishina energies. The strength of the drop is proportional to the energy density of the photon field. For an electron Lorentz factor below the smallest critical Klein–Nishina Lorentz factor, all four graybody photon fields plus the magnetic field energy density contribute to the loss rate. Once the electron Lorentz factor has exceeded the critical Klein–Nishina Lorentz factor of a particular photon field, this photon field no longer contributes to the radiative loss rate due to the much reduced inverse Compton loss rate in the Klein–Nishina limit. At Lorentz factors above the maximum critical Lorentz factor from the microwave background photons $\gamma_{k,4} = 5.9 \times 10^8$, only synchrotron losses contribute to the radiative loss rate.

3. Klein–Nishina steps in the electron equilibrium spectrum

In this section, we calculate the equilibrium spectrum of galactic cosmic-ray electrons above 10 GeV, taking into account the modified radiative loss rate (6), as well as non-thermal bremsstrahlung, adiabatic deceleration losses in a possible galactic wind with the velocity $v_{gw}$.
Figure 2. The equilibrium spectrum of cosmic-ray electrons as a function of the electron energy in GeV. The solid line represents the electron spectrum with Klein–Nishina corrections, and the dotted line shows the spectrum in the Thomson limit. For clearness of shape, the parameter \( k \) was chosen to be 3.6. The comparison of the shapes clearly illustrates the impact of the Klein–Nishina corrections.

and Coulomb and ionization losses (Pohl 1993). Because the radiative loss rate of electrons is the main ingredient in any transport model of high-energy cosmic-ray electrons, Klein–Nishina steps will modify any calculated electron equilibrium spectrum irrespective of the electron sources and the spatial transport mode. To delineate most clearly the consequences of the Klein–Nishina drops in the radiative loss rate, we chose as an illustrative example the simplest realistic model for cosmic-ray electron dynamics in the galaxy. Extensions to more sophisticated models of cosmic ray electron dynamics (the influence of localized point sources, spatial diffusion, convection and distributed re-acceleration), where the consequences of the modified inverse Compton losses also occur, are the subject of future work.

At electron energies above 10 GeV, the electron’s radiative loss time \( \tau_R = \gamma / |\gamma_R| \propto \gamma^{-1} \) is so short that the galaxy behaves as a thick target or fractional calorimeter (Völk 1989, Pohl 1993) for the electrons. The equilibrium energy spectrum of cosmic ray electrons \( N(\gamma) \) then results from the balance of electron production, expressed as injection spectrum \( Q(\gamma) \), and radiative energy losses from the solution of the balance equation

\[
\frac{d}{d\gamma} \left[ |\gamma_R(\gamma)| N(\gamma) \right] + Q(\gamma) = 0, \tag{7}
\]

implying

\[
N(\gamma) = |\gamma_R(\gamma)|^{-1} \int_\gamma^{\infty} dy \ Q(y). \tag{8}
\]

Moreover, we assume here that all electrons are secondaries resulting from inelastic hadron–hadron collisions of primary cosmic ray hadrons with interstellar gas atoms and
Figure 3. The equilibrium spectrum of cosmic-ray electrons as a function of the electron energy in GeV compared with observational data by HESS, ATIC and FERMI. The solid line represents the electron spectrum with Klein–Nishina corrections, and the dotted line shows the spectrum in the Thomson limit. Here, the full energy losses are taken into account, including non-thermal bremsstrahlung, adiabatic deceleration, Coulomb and ionization losses. The values of the parameters are $B = 3 \mu$G, $n_H = 0.3$ cm$^{-3}$, degree of ionization 0.2, $k = 3.6$ and $\text{div}(v_{gw}) = 10^{-13.13}$ s$^{-1}$.

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molecules during their confinement in the galaxy. It is well established (Lionetto et al. 2005, Delahaye et al. 2009) that secondary production accounts for a substantial part of the observed galactic cosmic ray electrons at relativistic energies. The locally measured hadron spectrum (Antoni et al. 2004) at energies below $4.4 \times 10^{15}$ GeV is a power law $\propto \gamma^{-3}$, with spectral index $s = 2.74$. Using the hadron–hadron cross-section templates (Kelner et al. 2006), the resulting electron injection spectrum at energies above 10 GeV, $Q(\gamma) = Q_0 \gamma^{-s}$, then follows a power law with the hadron spectral index $s$. With this injection spectrum and the radiative energy loss rate (6), the equilibrium spectrum (8) becomes

$$N(\gamma) = \frac{Q_0 \gamma^{1-s}}{(s-1)|\gamma'|_R(\gamma)}$$

$$= \frac{3m_e c^2 Q_0 \gamma^{-s-1}}{4(s-1)\sigma_T c U_B} \left[1 + \sum_{i=1}^{4} \frac{W_i U_B \gamma_i^2 + \gamma_{K,i}^2}{\gamma_i^2 + \gamma_{K,i}^2} \right]^{-1},$$

which is shown in figure 3 in comparison with the observed energy spectrum of galactic cosmic ray electrons. The constant $Q_0$ has been chosen in order to reproduce the electron spectrum below 100 GeV. It can be seen that the spectral shape of the HESS and FERMI data is well fitted. Note that for the HESS data, only the published statistical errors are plotted. Additionally, there are systematic errors of order 15%, both in the derived electron intensities and in the electron
energies that arise from uncertainties in the modeling of hadronic interactions in the Earth’s atmosphere (see the discussion in Aharonian et al 2008). Bearing this in mind, our modeling fits the observations reasonably well.

The good model fit to the ATIC, HESS and FERMI electron data in figure 3 indeed supports the purely secondary origin hypothesis of high-energy cosmic-ray electrons used here for illustrating the effects of the Klein–Nishina steps. However, this simple illustrative model is not in accord with the measured positron excess in the electron data by the PAMELA experiment (Adriani et al 2009) because it would imply that the positron fraction is constant in energy and equal to the electron fraction. To account for the PAMELA measurements, a more sophisticated treatment of electron and positron propagation is required, including e.g. additional primary positron sources from young pulsars or dark matter decay (e.g. Pohl 2009) or different finite confinement times of positrons and electrons. These necessary modifications will be investigated in future. However, although not the final answer to galactic electron propagation, our chosen purely secondary model serves the important purpose of illustrating the importance of Klein–Nishina steps in such modified transport models. Such models including spatial diffusion of electrons should also include the high spatial inhomogeneity of the galactic magnetic field reported by Beck et al (2003). The then necessary spatial integration of equation (6) across regions with higher and lower magnetic field strength will then smear out the sharp Klein–Nishina steps and generate additional spectral variations. Future work is needed here to see how much of the reported ATIC excess can be accounted for.

4. Summary and conclusions

The full Klein–Nishina cross-section for the inverse Compton scattering interactions of electrons implies a significant reduction of the electron energy loss rate compared with the Thomson limit when the electron energy exceeds the critical Klein–Nishina energy

$$E_K = \gamma_K m_e c^2 = 0.27 m_e^2 c^4 / (k_B T),$$

where $T$ denotes the temperature of the photon graybody distribution. As a consequence, the total radiative energy loss rate of single electrons exhibits sudden drops in the overall $\dot{\gamma} \propto \gamma^2$-dependence when the electron energy reaches the critical Klein–Nishina energy. The strength of the drop is proportional to the energy density of the photon radiation field. The diffuse galactic optical photon fields from stars of spectral type B and G-K lead to critical Klein–Nishina energies of 40 and 161 GeV, respectively. Associated with the drop in the loss rate are sudden increases (Klein–Nishina steps) in the equilibrium spectrum of cosmic ray electrons (see figure 2). Because the radiative loss rate of electrons is the main ingredient in any transport model of high-energy cosmic ray electrons, Klein–Nishina steps will modify any calculated electron equilibrium spectrum irrespective of the electron sources and the spatial transport mode. To delineate most clearly the consequences of the Klein–Nishina drops in the radiative loss rate, we chose as an illustrative example the simplest realistic model for cosmic-ray electron dynamics in the galaxy, consisting of the competition of radiative losses and secondary production by inelastic hadron–hadron collisions. We demonstrate that the spectral structure in the FERMI and HESS data is well described and even the excess measured by ATIC might be explained by Klein–Nishina steps.

After completing this work, we noticed the recent preprint by Stawarz et al (2009), who also explain the recently measured galactic electron spectrum by the Klein–Nishina suppression

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2 We are grateful to one of the referees for pointing this out.

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of the inverse Compton energy loss of relativistic electrons in an optical photon field. The differences to our work presented here are the following.

Firstly, we use the full set of radiative energy losses of relativistic electrons, namely non-thermal bremsstrahlung, adiabative deceleration losses, Coulomb and ionization losses, as well as inverse Compton and synchrotron losses, whereas Stawarz et al. (2009) take into account only the last two. Moreover, we use values of a few tenth of eV cm\(^{-3}\) for the energy densities of starlight (see table 1), whereas Stawarz et al. (2009) use a ten times larger energy density of \(u_{\text{star}} = 4 \text{ eV cm}^{-3}\) for their best fit of the observational data. Furthermore, our derivation of the Klein–Nishina steps is purely analytical. Beside further less significant differences of the two approaches, in the end neither Stawarz et al. (2009) nor we are able to clearly explain the sharp features claimed by the ATIC experiment.

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Appendix. Inverse Compton energy loss rate in graybody photon distributions

The inverse Compton power of a single electron in a general target photon field \(n(\epsilon)\) is (chapter 4.2 in Schlickeiser 2002)

\[
p_{C}(\epsilon_s, \gamma) = c \int_0^{\infty} d\epsilon n(\epsilon) \epsilon_s \sigma(\epsilon_s, \epsilon, \gamma),
\]

where \(\epsilon_s\) denotes the scattered photon energy. The differential Klein–Nishina cross-section (Blumenthal and Gould 1970) is given by

\[
\sigma(\epsilon_s, \epsilon, \gamma) = \frac{3\sigma_T}{4\epsilon \gamma^2} G(q, \Gamma)
\]

with

\[
G(q, \Gamma) = G_0(q) + \frac{\Gamma^2 q^2 (1 - q)}{2(1 + \Gamma q)},
\]

where

\[
G_0(q) = 2q \ln q + (1 + 2q)(1 - q)
\]

and

\[
\Gamma = \frac{4\epsilon \gamma}{mc^2}, \quad q = \frac{\epsilon_s}{\Gamma(\gamma mc^2 - \epsilon_s)}.
\]

By integrating over all kinematically allowed scattered photon energies, we find for the inverse Compton energy loss rate of a single electron

\[
|\dot{\gamma}| c = \frac{1}{mc^2} \int_0^{\infty} d\epsilon_s p_{C}(\epsilon_s, \gamma) = \frac{3c\sigma_T}{4mc^2 \gamma^2} \int_0^{\infty} d\epsilon \epsilon^{-1} n(\epsilon) \int_{\epsilon_s}^{\epsilon_{\text{max}}} d\epsilon_s \epsilon_s G(q, \Gamma).
\]
where $\epsilon_{\text{s,max}} = \Gamma \gamma mc^2/(\Gamma + 1)$ corresponds to $q = 1$. Using $q$ as an integration variable instead of $\epsilon_s$ results in

$$|\dot{\gamma}|_C = \frac{12c\sigma_T}{m_e c^2} \gamma^2 \int_0^{\infty} d\epsilon \, \epsilon \, n(\epsilon) J(\Gamma)$$

with

$$J(\Gamma) = \int_0^{1} dq \frac{q G(q, \Gamma)}{(1 + \Gamma q)^3}. \quad (A.5)$$

For the graybody photon distribution

$$n_G(\epsilon) = \frac{15 W}{\tau^4 (k_B T)^4 \epsilon^2 \exp[\epsilon / (k_B T)] - 1}, \quad (A.6)$$

the inverse Compton energy loss rate is then

$$|\dot{\gamma}|_C = \frac{20c\sigma_T W}{\pi^4 m_e c^2} \gamma^2 I(\gamma, T),$$

with

$$I(\gamma, T) = 9(k_B T)^{-4} \int_0^{\infty} d\epsilon \, \epsilon^3 \exp[\epsilon / (k_B T)] - 1. \quad (A.7)$$

Jones (1965) already noted that the double integral $I(\gamma, T)$ cannot be solved exactly, so that approximations (e.g. Petruk 2009) are required. It has been noted (Schlickeiser 2009) that the integral $J(\Gamma)$ is reasonably well approximated by

$$J(\Gamma) = \int_0^{1} dq \frac{q G(q, \Gamma)}{(1 + \Gamma q)^3} \simeq \frac{1}{9 + 2\Gamma^2}. \quad (A.8)$$

so that the double integral in equation (A.7) becomes

$$I(\gamma, T) = (k_B T)^{-4} \int_0^{\infty} d\epsilon \, \epsilon^3 \exp[\epsilon / (k_B T)] - 1 \frac{1}{1 + \frac{32\gamma^2 c^2}{9 m_e^2 c^4}}. \quad (A.9)$$

With the substitution $x = \epsilon / (k_B T)$, we find

$$I(A) = \int_0^{\infty} dx \, \frac{x^3}{1 + x^2} \frac{1}{\exp[x] - 1}, \quad (A.10)$$

where

$$A = \frac{3m_e c^2}{\sqrt{32}} (k_B T \gamma)^{-1}. \quad (A.11)$$

The series

$$\frac{1}{1 + (x^2/A^2)} = \sum_{k=1}^{\infty} (-1)^{k-1} x^{2(k-1)} A^{2(k-1)}$$

leads to

$$I(A) = \sum_{k=1}^{\infty} (-1)^{k-1} \frac{\Gamma [2k + 2] \zeta [2k + 2]}{A^{2(k-1)}}, \quad (A.12)$$
which, for $A \geq 1$ to lowest order in $A^{-2}$, yields
\[ I(A \geq 1) \simeq \Gamma [4] \zeta [4] = \frac{\pi^4}{15}. \] (A.13)

For $A < 1$, we approximate the integral (A.10) by
\[ I(A < 1) \simeq \int_0^A dx \frac{x^3}{e^x - 1} + A^2 \int_A^\infty dx \frac{x}{e^x - 1} \]
\[ \simeq A^2 \int_0^\infty dx \left( \frac{x}{e^x - 1} \right) - A^3 + \int_0^A dx x^2 \]
\[ = \frac{\pi^2 A^2}{6} - \frac{2A^3}{3} \simeq \frac{\pi^2 A^2}{6}. \] (A.14)

We combine the two expansions (A.13) and (A.14) to
\[ I(A) \simeq \frac{\pi^4}{15} \frac{1}{1 + \frac{2\pi^2}{5\pi^2}} \] (A.15)
valid at all values of $A$. The approximation (A.15) can be written as
\[ I(A) \simeq \frac{\pi^4}{15} \frac{1}{1 + \left( \frac{\gamma}{\gamma_K} \right)^2}, \] (A.16)
where we introduce the critical Klein–Nishina Lorentz factor
\[ \gamma_K \equiv \frac{3\sqrt{5}}{8\pi} \frac{m_e c^2}{k_B T} = \frac{0.27 m_e c^2}{k_B T}. \] (A.17)

Using this approximation in equation (A.7) readily yields the inverse Compton loss rate in equation (2).

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