Method of calculating the active part of axial shaped charge of a disintegrator

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Abstract. To organize the process of effective drilling, the technology of rock loosening by shaped charges is often used. It is known that not all energy of a shaped charge takes part in loosening the surrounding rock, but only the part of it that is closer to the charge funnel. It is called an active part of a shaped charge. [1]. The article is devoted to the development of a calculating method for the active part of a shaped charge. Analytical dependences for determining the mass of the active part of the charge are obtained.

To determine the expression that evaluates the active part (AP), let us consider an axial shaped charge with a cavity of arbitrary shape (Figure 1). Under the assumption of an instantaneous detonation of charge BB, expansive waves from the corresponding surfaces move simultaneously inside the charge with the same speed $C_t$.

![Figure 1. Shaped charge with an arbitrary shape cavity](image)

In the general case, in the coordinate system $0XYZ$, associated with the charge, the motion equations of expansive waves with speed $C_t$ from surfaces 1 and 2 (Figure 1) in parametric form will be as follows:
\[ y_1 = f_1(x, z, t, x_{0i}, y_{0i}, z_{0i}), \]  
\[ y_2 = f_2(x, z, t, x_{0j}, y_{0j}, z_{0j}), \]  
\[ i=1, 2, ..., n; j=1, 2, ..., m \] respectively,

where \( t \) is the time from the moment of charge detonation;

\( x_{0i}, y_{0i}, z_{0i}, x_{0j}, y_{0j}, z_{0j} \) are the initial coordinates of the \( i \)-th and \( j \)-th points of surfaces 1 and 2 respectively;

\( n, m \) is the number of considered points of corresponding surfaces.

Solving equation (2) with respect to time

\[ t = f_3(x, y, x_{0j}, y_{0j}, z_{0j}) \]

and substituting it into equation (1), we get an equation of the meeting surface of two expansive waves, which is the desired meeting surface of two expansive waves, and which is the desired surface of AP of the considered shaped charge

\[ y = f_4(x, z, y, x_{0j}, y_{0j}, z_{0j}). \]  

From formula (3) it can be seen that under the assumption of instantaneous detonation, the AP surface shape depends on the geometrical dimensions of the shaped charge and the shape of its cavity.

To define it, we will use an assumption about instantaneous detonation. Denoting \( 2 \alpha \) as the cavity half angle, we will consider the motion of expansive waves in the meridional section of a cylindrical axial charge with a spherical cavity (Figure 2). Let us note that when solving this task the casing can be neglected [2, 3].

After the instantaneous detonation, the equations of expansive waves from the corresponding charge sides in the rectangular coordinate system \( X0Y \), associated with the center of curvature of the charge cavity, are written in the following way:

\[ y = H + R \cos \alpha - C_i t \] - from line \( IG \),  
\[ x^2 + y^2 = (R + C_i t)^2 \] - from line \( ED \),  
\[ x = x_0 - C_i t \] - from line \( FG \),  
\[ y = R \cos \alpha + C_i t \] - from line \( DF \).

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Figure 2 shows that curve \( AB \) represents a set of intersection points of the expansive waves described by equations (4) and (5). Solving these equations in the first quadrant of coordinate system \( XOY \) and excluding \( CT \), we will get

\[
x^2 + y^2 = (R + H + R \cos \alpha - y)^2,
\]
whence we have the final equation of curve \( AB \)

\[
y_1 = \left( \eta^2 - x^2 \right) / 2\eta,
\]
where \( \eta = H + R(1 + \cos \alpha) \) is the charge parameter.

Similarly, for curve \( BC \), solving equations (5) and (6), we obtain

\[
y_2 = \sqrt{\zeta^2 - 2\zeta x},
\]
where \( \zeta = x_0 + R \) is the charge parameter,

and for \( DC \), solving equations (5) and (7) respectively, we have

\[
y_3 = \left( x^2 - \gamma^2 \right) / 2\gamma,
\]
where \( \gamma = R(1 - \cos \alpha) \) is the charge parameter.

From formulas (8), (9) and (10) we will make a system of equations that defines the AP boundary of the shaped charge with a spherical cavity in the first quadrant

\[
y_1 = \left( \eta^2 - x^2 \right) / 2\eta;
\]
\[
y_2 = \sqrt{\zeta^2 - 2\zeta x};
\]
\[
y_3 = \left( x^2 - \gamma^2 \right) / 2\gamma.
\]

Knowing the boundary equation (11) of the active part of the shaped charge, it is easy to determine its volume and mass [8].

As an example, we will define the volume of AP of the axial cylindrical charge of infinite height with radius \( R \) of curvature of the cavity without a ring-type layer \( BB \). In this case, curves \( Y_1 \) and \( Y_3 \) degenerate into curve \( Y_2 \) and AP of the charge will have the shape shown in Figure 3.
Based on elementary geometrical considerations, we will define the volume of AP of the charge as the difference between a rotational figure formed by curve $Y_2$ and the spherical segment with the curve generator $Y_4$ with the corresponding limits of integration. We will write [6]

$$V_a = \pi \int_{y_4}^{y_A} [x(y)]^2 dy_2 - \pi \int_{y_4}^{y_A} [x(y_4)]^2 dy_4.$$  (12)

Taking into account that $y_2 = y$, we will express dependence $x(y_2)$ from (11). We have

$$x(y_2) = (\zeta^2 - \gamma^2)/2\zeta.$$  (13)

To determine the limits of integration $y_a$ and $y_A$ we will solve the system of equations

$$\begin{cases}
    x(y_2) = (\zeta^2 - \gamma^2)/2\zeta, \\
    x(y) = d_0/2
\end{cases} ;
\begin{cases}
    x(y_2) = (\zeta^2 - \gamma^2)/2\zeta, \\
    x(y) = 0
\end{cases},$$

from where we will get

$$y_a = \sqrt{R^2 - d_0^2}/4,$$

$$y_A = R + d_0/2.$$  

Curve $Y_4$ in the first quadrant is as follows

$$y_4 = \sqrt{R^2 - x^2},$$

then, knowing that $y_4 = y$, we will write

$$x(y_4) = \sqrt{R^2 - y^2}.$$  

Substituting the obtained results into expression (12), after performing the integration and simple transformations, we will get the desired formula

$$V_a = \{8\zeta^5 - 15\zeta^4\gamma + 10\zeta^2\gamma^3 - 3\gamma^5 - 40R^3\zeta^2 + 60R^2\gamma\zeta^2\} \pi/60\zeta^2.$$  (14)

where

$$\zeta = R + d_0/2;$$

$$\gamma = \sqrt{R^2 - d_0^2}/4.$$  

Formula (14) has a rather unmanageable look, but it already allows to make calculations of the AP volume of a cylindrical axial charge with a spherical cavity.

In the case when the cavity volume is equal to the hemisphere, that is, in formula (14) $R + d_0/2$, the volume of the active part is determined by the dependence

$$V_a = \pi d_0^3/20.$$  

When the cavity volume tends to zero, then from (14) we obtain the expression for determining the volume of AP of a cylindrical charge from its end (Figure 3):
\[ v_a = \pi d_0^3 / 24. \]

Sometimes it is more convenient to use another dependence, which is obtained from (14) after expressing curvature radius \( R \) through deflection \( h \) of the cavity and charge diameter \( d_0 \) [7]

\[ v_a = [8\zeta^6 - 15\zeta^4\gamma + 10\zeta^2\gamma^3 - 3\gamma^5] \pi / 60\zeta^2 - \pi h^2 [\gamma + 2h / 3] \]

here

\[ \zeta = (d_0^2 + 4h^2) / 8h + d_0 / 2; \]
\[ \gamma = (d_0^2 + 4h^2) / 8h - h. \]

If we denote the volume of a total cylindrical shaped charge through \( v_0 \), then it can be shown that the ratio value \( v_a / v_0 \), expressed as a percentage, depending on the amount of deflection \( h \) of the cavity lies in the range of 17–20% [4, 5].

Knowing the density of charge BB, the mass of the active part of a cylindrical shaped charge can be determined by the following formula

\[ m_{AU} = \rho_{BB} v_a. \]

Thus, using the above formulas, it is possible to calculate the mass of the active part of the axially symmetrical shaped charge.

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