Path to the 0.01% Theoretical Luminosity Precision Requirement for the FCC-ee (and ILC)

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Abstract

We present pathways to the required theoretical precision for the luminosity targeted by the FCC-ee precision studies. We put the discussion in context by reviewing briefly the situation at the time of LEP. We then present the current status and routes to the desired 0.01% targeted by the FCC-ee (as well as by the ILC).

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Our starting point will be the situation that existed at the end of LEP. At that time, the error budget for the BHLUMI4.04 MC \[1\] used by all LEP collaborations to simulate the luminosity process was calculated in Ref. \[2\]. We reproduce this result here in Table 1 for reference. In

| Type of correction/error | LEP1  | LEP2  |
|--------------------------|-------|-------|
| (a) Missing photonic $O(\alpha^2)$ \[3, 4\] | 0.10% | 0.027% |
| (b) Missing photonic $O(\alpha^3L_e^2)$ \[5\] | 0.015% | 0.015% |
| (c) Vacuum polarization \[6, 7\] | 0.04% | 0.04% |
| (d) Light pairs \[8, 9\] | 0.03% | 0.03% |
| (e) $Z$ and $s$-channel $\gamma$ \[10, 11\] | 0.015% | 0.015% |
| **Total** | **0.11%** | **0.061%** |

Table 1: Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, the above estimate is valid for a generic angular range within $1^\circ$–$3^\circ$ (18–52 mrads), and for LEP2 energies up to 176 GeV and an angular range within $3^\circ$–$6^\circ$. Total uncertainty is taken in quadrature. Technical precision included in (a).

In this table, we show the published works upon which the various error estimates are based as they are discussed in Ref. \[2\].

One way to address the 0.01% precision tag needed for the luminosity theory error for the FCC-ee is to develop the corresponding improved version of the BHLUMI. This problem is addressed recently in Ref. \[12\], wherein the path to 0.01% theory precision for the FCC-ee luminosity is presented in some detail. The results of this latter reference are shown in Table 2, wherein we also present the current state of the art for completeness, as it is discussed in more detail in Ref. \[12\].

| Type of correction / Error | Update 2018 | FCC-ee forecast |
|---------------------------|-------------|-----------------|
| (a) Photonic $O(L_e\alpha^4)$ | 0.027% | $1.0 \times 10^{-4}$ |
| (b) Photonic $O(L_e^2\alpha^4)$ | 0.015% | $6.0 \times 10^{-5}$ |
| (c) Vacuum polariz. | 0.014% \[13\] | $6.0 \times 10^{-4}$ |
| (d) Light pairs | 0.010% \[14, 15\] | $5.0 \times 10^{-4}$ |
| (e) $Z$ and $s$-channel $\gamma$ exchange | 0.090% \[10\] | $1.0 \times 10^{-4}$ |
| (f) Up-down interference | 0.009% \[16\] | $1.0 \times 10^{-4}$ |
| (g) Technical Precision | (0.027)% | $1.0 \times 10^{-4}$ |
| **Total** | 0.097% | $1.0 \times 10^{-4}$ |

Table 2: Anticipated total (physical+technical) theoretical uncertainty for a FCC-ee luminosity calorimetric detector with the angular range being 64–86 mrad (narrow), near the $Z$ peak. Description of photonic corrections in square brackets is related to the 2nd column. The total error is summed in quadrature.

The key steps in arriving at Table 2 are as follows. The errors associated with the photonic corrections in lines (a) and (b) in the LEP results in Table 1 are due to effects which are known
from Refs. [3, 4, 5] but which were not implemented into BHLUMI. In Table 2 we show what these errors will become after these known results are included in BHLUMI as discussed in Ref. [12]. Similarly, in line (c) of Table 1 the error is due to the uncertainty at the time of LEP on the hadronic contribution to the vacuum polarization for the photon at the respective momentum transfers for the luminosity process; in Table 2 we show the improvement of this error that is expected for the FCC-ee as discussed in Refs. [13, 17, 18].

Continuing in this way, in line (d) in Table 2 we show the expected [12] improvement, with reference to the LEP time for Table 1 in the light pairs error for the FCC-ee. As we explain in Ref. [12], the complete matrix element for the additional real $e^+ e^-$ pair radiation should be used, because non-photonic graphs can contribute as much as 0.01% for the cut-off $z_{\text{cut}} \sim 0.7$. This can be done with the MC generators developed for the $e^+ e^- \rightarrow 4f$ processes for LEP2 physics. With known methods [12], the contributions of light quark pairs, muon pairs and non-leading, non-soft additional $e^+ e^+ n \gamma$ corrections can be controlled such that the error on the pairs contribution is as given in line (d) for the FCC-ee. As noted, we also show the current state of the art [12] for this error in line (d) of Table 2.

Turning to line (e) in Table 2, we show the improvement of the error on the Z and s-channel $\gamma$ exchange for the FCC-ee as well as its current state of the art. In Ref. [12], a detailed discussion of all of the six interference and three additional squared modulus terms that result from the s-channel $\gamma$, s-channel Z, and t-channel Z exchange contributions to the amplitude for the luminosity process. It is shown that, if the predictions of BHLUMI for the luminosity measurement at FCC-ee are combined with the ones from BHWIDE [19] for this Z and s-channel $\gamma$ exchange contribution, then the error in the second column of line (e) of Table 2 could be reduced to 0.01%. In order to reduce the uncertainty of this contribution practically to zero we would include these Z and $\gamma_s$ exchanges within the CEEX [20] type matrix element at $O(\alpha^4)$ in BHLUMI. Here, CEEX stands for coherent exclusive exponentiation which acts at the level of the amplitudes as compared the original Yennie–Frautshi–Suura [21](YFS) exclusive exponentiation (EEX) that is used in BHLUM4.04 and that acts at the level of the squared amplitudes. It is expected to be enough to add the EW corrections to the LABH process in the form of effective couplings in the Born amplitudes. This leads to the error estimate shown in Table 2 in line (e) for the FCC-ee.

In line (f) in Table 2 we show the estimate of the error on the up-down interference between radiation from the $e^-$ and $e^+$ lines. Unlike in LEP1, where it was negligible, for the FCC-ee this effect, calculated in Ref. [16] at $O(\alpha^1)$, is 10 times larger and has to be included in the upgraded BHLUMI. Once this is done, the error estimate shown in line (f) for the FCC-ee obtains [12].

This brings us to the issue of the technical precision. In an ideal situation, in order to get the upgraded BHLUMI's technical precision at the level $10^{-5}$ for the total cross section and $10^{-4}$ for single differential distributions, one would need to compare it with another MC program developed independently, which properly implements the soft-photon resummation, LO corrections up to $O(\alpha^3 L^2)$, and the second-order corrections with the complete $O(\alpha^2 L^3)$. In principle, an extension of a program like BabaYaga [22, 23, 24], which is currently exact at NLO with a matched QED shower, to the level of NNLO for the hard process, while keeping the correct soft-photon resummation, would provide the best comparison to the upgraded BHLUMI to establish
the technical precision of both programs at the $10^{-5}$ precision level. During the intervening time period, a very good test of the technical precision of the upgraded BHLUMI would follow from the comparison between its results with EEX and CEEX matrix elements; for, the basic multi-photon phase space integration module of BHLUMI was already well tested in Ref. [26] and such a test can be repeated at an even higher-precision level.

In summary, we conclude that, with the appropriate resources, the path to 0.01% precision for the FCC-ee luminosity (and the ILC luminosity) at the Z peak is open via an upgraded version of BHLUMI.

**References**

[1] S. Jadach, W. Placzek, E. Richter-Was, B. F. L. Ward, and Z. Was, “Upgrade of the Monte Carlo program BHLUMI for Bhabha scattering at low angles to version 4.04”, *Comput. Phys. Commun.* **102** (1997) 229–251.

[2] S. Jadach, M. Melles, B. F. L. Ward, and S. A. Yost, “New results on the theoretical precision of the LEP / SLC luminosity”, *Phys. Lett.* **B450** (1999) 262–266, [hep-ph/9811245](https://arxiv.org/abs/hep-ph/9811245).

[3] S. Jadach, M. Melles, B. F. L. Ward, and S. A. Yost, “Exact results on O (alpha) corrections to the single hard bremsstrahlung process in low angle Bhabha scattering in the SLC / LEP energy regime”, *Phys. Lett.* **B377** (1996) 168–176, [hep-ph/9603248](https://arxiv.org/abs/hep-ph/9603248).

[4] S. Jadach, M. Melles, B. F. L. Ward, and S. A. Yost, “New results on the precision of the LEP luminosity”, *Acta Phys. Polon.* **B30** (1999) 1745–1750.

[5] S. Jadach and B. F. L. Ward, “Missing third order leading log corrections in the small angle Bhabha calculation”, *Phys. Lett.* **B389** (1996) 129–136.

[6] H. Burkhardt and B. Pietrzyk, “Update of the hadronic contribution to the QED vacuum polarization”, *Phys. Lett.* **B356** (1995) 398–403.

[7] S. Eidelman and F. Jegerlehner, “Hadronic contributions to g-2 of the leptons and to the effective fine structure constant alpha (M(z)**2)”, *Z. Phys.* **C67** (1995) 585–602, [hep-ph/9502298](https://arxiv.org/abs/hep-ph/9502298).

[8] S. Jadach, M. Skrzypek, and B. F. L. Ward, “Analytical results for low angle Bhabha scattering with pair production”, *Phys. Rev.* **D47** (1993) 3733–3741.

[9] S. Jadach, M. Skrzypek, and B. F. L. Ward, “Soft pairs corrections to low angle Bhabha scattering: YFS Monte Carlo approach”, *Phys. Rev.* **D55** (1997) 1206–1215.

[10] S. Jadach, W. Placzek, and B. F. L. Ward, “Precision calculation of the gamma - Z interference effect in the SLC / LEP luminosity process”, *Phys. Lett.* **B353** (1995) 349–361.

† The upgrade of the BHLUMI distributions will be relatively straightforward because its multi-photon phase space is exact [25] for any number of photons.
[11] A. Arbuzov et al., “The Present theoretical error on the Bhabha scattering cross-section in the luminometry region at LEP”, Phys. Lett. B383 (1996) 238–242, hep-ph/9605239.

[12] S. Jadach, W. Placzek, M. Skrzypek, B. F. L. Ward, and S. A. Yost, Phys. Lett. B790 (2019) 314, 1812.01004.

[13] F. Jegerlehner, “$\alpha_{QED}(MZ)$ and future prospects with low energy e+e collider data”, FCC-ee Mini-Workshop, Physics Behind Precision https://indico.cern.ch/event/469561/.

[14] G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini, and F. Piccinini, “Light pair correction to Bhabha scattering at small angle”, Nucl. Phys. B547 (1999) 39–59, hep-ph/9811436.

[15] G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini, and F. Piccinini, “Light pair corrections to small angle Bhabha scattering in a realistic set up at LEP”, Phys. Lett. B459 (1999) 649–652, hep-ph/9905235.

[16] S. Jadach, E. Richter-Was, B. F. L. Ward, and Z. Was, “Analytical $O(\alpha)$ distributions for Bhabha scattering at low angles”, Phys. Lett. B253 (1991) 469–477.

[17] F. Jegerlehner, “Variations on Photon Vacuum Polarization”, 1711.06089.

[18] F. Jegerlehner, talk in 11th FCC-ee Workshop: Theory and Experiments, Jan. 8-11, 2019, CERN, Geneva, Switzerland.

[19] S. Jadach, W. Placzek, and B. F. L. Ward, “BHWIDE 1.00: $O(\alpha)$ YFS exponeniated Monte Carlo for Bhabha scattering at wide angles for LEP-1/SLC and LEP-2”, Phys. Lett.B390 (1997) 298–308, hep-ph/9608412.

[20] S. Jadach, B. Ward, and Z. Was, “Coherent exclusive exponentiation for precision Monte Carlo calculations”, Phys. Rev. D63 (2001) 113009, hep-ph/0006359.

[21] D. R. Yennie, S. C. Frautschi, and H. Suura,”The infrared divergence phenomena and high energy processes”, Annals Phys.13 (1961) 379–452.

[22] C. M. Carloni Calame, C. Lunardini, G. Montagna, O. Nicrosini, and F. Piccinini, “Large angle Bhabha scattering and luminosity at flavor factories”, Nucl. Phys. B584 (2000) 459–479, hep-ph/0003268.

[23] C. M. Carloni Calame, “An Improved parton shower algorithm in QED”, Phys. Lett. B520 (2001) 16–24, hep-ph/0103117.

[24] G. Balossini, C. M. Carloni Calame, G. Montagna, O. Nicrosini, and F. Piccinini, “Matching perturbative and parton shower corrections to Bhabha process at flavour factories”, Nucl. Phys. B758 (2006) 227–253, hep-ph/0607181.
[25] S. Jadach, B. F. L. Ward, and Z. Was, “The precision Monte Carlo event generator KK for two-fermion final states in e+ e- collisions”, *Comput. Phys. Commun.* **130** (2000) 260–325, Program source available from [http://jadach.web.cern.ch/](http://jadach.web.cern.ch/), hep-ph/9912214.

[26] S. Jadach and B. F. L. Ward, “Semianalytical third order calculations of the small angle Bhabha cross-sections”, *Acta Phys. Polon.* **B28** (1997) 1907–1979.