Nonlinear predictive control of horizontal vibration of high speed elevator

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Abstract—This aims at the problem of horizontal vibration of high-speed elevators caused by the guide rail excitation, firstly, considering the nonlinear characteristics of the spring rigidity and damping of the guide shoe system, a four-degree-of-freedom high-speed elevator nonlinear horizontal vibration model is established. Secondly, the state equation of horizontal vibration of high-speed elevator is derived, and the system state is processed by Taylor expansion. The quadratic performance index of predictive control is introduced, and the active control law with analytic form is solved by optimizing the performance index. Finally, the simulation using MATLAB verifies the correctness of the model. By further solving the control law and simulation, the results prove that predictive control can effectively reduce the horizontal vibration of high-speed elevators.

1. Introduction

Nowadays, with the continuous increase of elevator speed, the problem of horizontal vibration caused by it becomes more and more obvious. Compared with the low-speed elevator, the displacement and acceleration of the center of mass of the high-speed elevator above the running speed are relatively large [1]. The problem of horizontal vibration control of high-speed elevators has become a research hotspot at this stage. Guide shoe as an important guide component, its nonlinear characteristics have a direct impact on the vibration behavior of the elevator car system, and many scholars treat the guide shoe system as a linear model. Zee et al. [2] proved through research and experiments that the guide shoe system should be a non-linear model, and cannot be simply used as a linear system, in which the displacement and force of the guide shoe system exhibit nonlinear characteristics. Zhang et al. [3] proposed a nonlinear constitutive equation of the rolling guide shoe associated with the high-speed elevator system, and by combining the Hertz contact theory and the Bouc-Wen hysteresis model, a nonlinear vibration model of the elevator car system was established. For the study of high-speed elevator horizontal vibration control, Feng et al. [4] considered the characteristics of non-linearity and parameter uncertainty, and applied the Lyapunov method to design the position controller and attitude controller respectively. Santo et al. [5] considered the nonlinearity of spring stiffness due to factors such as tilt, torsion, and rail deformation of the car system, and proposes a control strategy based on the state-dependent Riccati equation.

This article innovatively considers the nonlinear characteristics of the spring rigidity and damping of the guide shoe system, and establishes a four-degree-of-freedom high-speed elevator nonlinear horizontal vibration model. Continuous predictive control is used for single-step prediction to solve the control law...
with an analytical form. The system does not require online optimization and the calculation amount is small. Finally, MATLAB is used for simulation verification.

2. Nonlinear horizontal vibration model of high-speed elevator

The high-speed elevator car system mainly includes a car body and a car frame, which are connected by a rubber block [6]. The guide shoes are installed on the upper and lower sides of the car frame, and the guide system of the car is formed with the guide rail. To simplify the dynamic analysis and the complexity of the controller design, the nonlinear horizontal vibration model of the high-speed elevator is established as shown in Fig. 1, and the following assumptions

- Because the car body and the car frame belong to an elastic connection, the rubber block is regarded as a spring damping system.
- Because the roller is close to the surface of the guide rail, the mass of the guide wheel is ignored, and the guide shoe is simplified as a nonlinear spring damping system.
- Consider the deviation of the center of mass between the car and the car frame.
- Only the unevenness of the guide rail is considered to stimulate the car system.

In order to avoid complex mathematical equations, the following variables are defined

\[ z = [z_1, z_2, z_3, z_4, z_5, z_6]^T \]

Where, \( z_i = x_i - x_{cf} + l_i \theta \), \( x_{cf} \) is the horizontal displacement of the car body and car frame centroid, \( l_i \) and \( l_2 \) are the longitudinal dimensions of rubber block 1, 2 and 3, 4 to the car centroid \( O_c \), \( l_3 \) and \( l_4 \) are the longitudinal dimensions of the guide shoes 5, 6 and 7, 8 to the center of mass of the car frame \( O_f \), \( 5 \) and 6 are the longitudinal dimensions of the rubber blocks 1, 2 and 3, 4 to the centroid \( O_f \), \( x_{cf} (i=1,2,3,4) \) is the excitation signal of rail unevenness.

The dynamic equation of the model obtained by applying Lagrange energy method is

\[ m_i \ddot{x}_i = 2 \sum_{i=1}^{2} (k \cdot z_i + c \cdot z_i) + u_i + u_e \]
\[ J_e \dot{\theta}_e = -2l_1 (k \cdot z_1 + c \cdot \dot{z}_1) + 2l_2 (k \cdot z_2 + c \cdot \dot{z}_2) + l_2 \cdot u_1 - l_2 \cdot u_2 \]

\[ m_j \ddot{x}_j = -2 \sum_{i=1}^3 (k \cdot z_i + c \cdot \dot{z}_i) + \sum_{i=3}^6 (k \cdot z_i + c \cdot \dot{z}_i + \Phi_i) - u_i - u_2 + u_3 + u_4 \]

\[ J_f \ddot{\theta}_f = -2l_1 (k \cdot z_1 + c \cdot \dot{z}_1) + l_1 \sum_{i=3}^6 (k \cdot z_i + c \cdot \dot{z}_i + \Phi_i) + 2l_1 (k \cdot z_1 + c \cdot \dot{z}_1) - l_3 \sum_{i=3,5} (k \cdot z_i + c \cdot \dot{z}_i + \Phi_i) - l_6 \cdot u_1 + l_6 \cdot u_2 + l_4 \cdot u_3 - l_4 \cdot u_4 \]

Where \( \Phi_i \) is the nonlinear term of the spring and damping system in the guide shoe. \( (i = 3, 4, 5, 6) \)

\[ \Phi_i = f_{\theta_i} + f_{\theta_i} = k_i \cdot z_i + c_i \cdot \dot{z}_i + c_2 \sqrt{z_i} \cdot \text{sgn}(z_i) \]  

The state vector is selected as \( x = [x_{01}, x_{02}]^T, x_{01} = [x_c, x_f, \theta_c, \theta_f]^T, x_{02} = [\dot{x}_c, \dot{x}_f, \dot{\theta}_c, \dot{\theta}_f]^T \). The control vector is selected as \( u = [u_1, u_2, u_3, u_4]^T \). The output vector is selected as \( y = x_{01} \). Then the state space equation of the system can be expressed as

\[
\begin{align*}
\dot{x}_{01} &= x_{02} \\
\dot{x}_{02} &= f_{01} (x) + Gu \\
y &= x_{01}
\end{align*}
\]

Where,

\[
f_{01} = \begin{bmatrix}
\frac{2}{m_e} \sum_{i=1}^3 (k \cdot z_i + c \cdot \dot{z}_i) \\
- \frac{2}{m_f} \sum_{i=1}^3 (k \cdot z_i + c \cdot \dot{z}_i) + \frac{1}{m_f} \sum_{i=3}^6 (k \cdot z_i + c \cdot \dot{z}_i + \Phi_i) \\
- \frac{2l_1}{J_e} (k \cdot z_1 + c \cdot \dot{z}_1) + \frac{2l_1}{J_e} (k \cdot z_2 + c \cdot \dot{z}_2) \\
- \frac{l_3}{J_f} \sum_{i=3,5} (k \cdot z_i + c \cdot \dot{z}_i + \Phi_i) + \frac{2l_1}{J_f} (k \cdot z_1 + c \cdot \dot{z}_1) \\
+ \frac{l_3}{J_f} \sum_{i=3,5} (k \cdot z_i + c \cdot \dot{z}_i + \Phi_i) - \frac{2l_1}{J_f} (k \cdot z_2 + c \cdot \dot{z}_2)
\end{bmatrix}
\]

\[
G = \begin{bmatrix}
\frac{1}{m_e} & \frac{1}{m_e} & 0 & 0 \\
- \frac{1}{m_f} & \frac{1}{m_f} & \frac{1}{m_f} & \frac{1}{m_f} \\
\frac{l_2}{J_e} & \frac{l_2}{J_e} & 0 & 0 \\
- \frac{l_6}{J_f} & \frac{l_6}{J_f} & \frac{l_4}{J_f} & \frac{l_4}{J_f}
\end{bmatrix}
\]

3. Design of nonlinear predictive controller

It is assumed that the state \( x \) of the system is smooth and continuous and its derivatives of any order exist. The control order of control vector \( u \) is \( \rho \). Let \( \Delta \tau > 0 \) be a real number, and the state of the nonlinear system in the prediction range is approximately Taylor series expansion.

\[
x_i (t + \tau) = x_i (t) + \tau \ddot{x}_i (t) + \frac{\tau^2}{2!} \dddot{x}_i (t) + \cdots + \frac{\tau^n}{n!} x_i^{(n)} (t) + \cdots (i = 1, 2, \ldots, m)
\]
Taking the first $\rho_i + 1$ terms, $x(t + \tau)$ can be approximated as

$$\begin{align*}
x(t + \tau) &= x(t) + T(x(t), \tau) + \Gamma(\tau)A(x(t))u(t) \\
\end{align*}$$

(9)

In the above formula, $\Gamma(\tau) = \text{diag} \left[ \frac{\tau^{\rho_1}}{\rho_1!}, \frac{\tau^{\rho_2}}{\rho_2!}, \ldots, \frac{\tau^{\rho_n}}{\rho_n!} \right]$, $T_i(x(t), \tau) = \tau D^{\rho_i}_{t}\left(f_i(t)\right) + \cdots + \frac{\tau^{\rho_n}}{\rho_n!} D^{\rho_n}_{t}\left(f_n(x(t))\right), T \in R^n$

$A(x(t)) = \left[ D_{\rho_1} \left( D^{\rho_1}_{t}\left(x_1(t)\right)\right), \ldots, D_{\rho_n} \left( D^{\rho_n}_{t}\left(x_n(t)\right)\right) \right], A \in R^{nxn}$.

Where, $D^{\rho_i}_{t}\left(f_i(t)\right) = \frac{\partial \left( D^{\rho_i}_{t}\left(f_i(t)\right) \right)}{\partial x_i} f_i$ is the n-th order Lee derivative of $f_i$ relative to $f$ [7], and

$$D^{\rho_i}_{t}\left(x_i(t)\right) = \frac{\partial \left( D^{\rho_i}_{t}\left(x_i(t)\right) \right)}{\partial x_i} g_i, (i = 1, 2, \cdots, k).$$

Formula (9) is essentially to take control role $u(\sigma) = u(t), \sigma \in [t, t + \tau]$, and the Taylor series expansion of $x_i(t + \tau)$ is appropriately truncated to obtain the predicted value of $x_i(t + \tau)$.

Predictive control requires the desired trajectory of the system state or output, so that the selected predictive index is optimized to obtain the control law. In this paper, the expected trajectory of the system state is represented by $x^d(t)$. Take performance index

$$J(x,u,t) = \frac{1}{2} \left[ \|u(t + \tau)\|_Q + \|u(t)\|_R \right]$$

(10)

Where, $e(t + \tau) = x(t + \tau) - x^d(t + \tau)$ is the prediction state error at time $t + \tau$, $Q$ is a positive definite symmetric matrix, $R$ is positive semidefinite symmetric matrix.

Solving predictive control problem $\min J(x,u,t)$, the control law is obtained from $\partial J / \partial u = 0$ is

$$u(t) = -\left[ \Gamma(\tau)A(x(t)) \right]^T Q \left[ \Gamma(\tau)A(x(t)) \right] + R \right]^{-1} \left[ \Gamma(\tau)A(x(t)) \right]^T \left( x(t) + T(x(t), \tau) - x^d(t + \tau) \right)$$

(11)

Considering the power of the actuator, its output control force is limited, that is

$$\left| u_i \right| \leq u_{\text{max}} (i = 1, 2, 3, 4)$$

(12)

4. Simulation analysis

In order to verify the applicability and effectiveness of the predictive control method, a nonlinear horizontal vibration simulation model for high-speed elevators was established based on MATLAB. Predictive controller parameter setting: predicting time domain $\tau = 0.1$, reference track $x^d(t)$ is $0$. Parameter matrix $Q = \text{diag} \left( 10^6, 10^6, 10^6, 10^4, 10^4, 10^4 \right)$, $R = \text{diag} \left( 10^{-4}, 10^{-4}, 10^{-4}, 10^{-4} \right)$.

The parameters of 4m/s high-speed elevator are shown in Table 1. In the simulation, Gaussian white noise signal with mean value of 0 and standard deviation of 0.6 mm is used as the excitation signal of guide rail.

| TABLE I. SIMULATION PARAMETERS OF CAR SYSTEM |
|---------------------------------------------|
| Parameters       | Value    |
| Car body mass $m_c$ | 1.2e3 kg |
| Car frame mass $m_f$ | 8.0e2 kg |
| Car body moment of inertia $J_c$ | 1.3e3 kg·m² |
| Rotating inertia of car frame $J_f$ | 3.0e3 kg·m² |
| Rubber block stiffness $k$ | 4.0e5 N·m⁻¹ |
| Damping coefficient $c$ | 1.2e2 N·s/m |
| Stiffness coefficient $k_c$ | 2.0e4 N·m⁻¹ |
Damping coefficient $c_s$ 6.0e2 N·s/m
Damping modification coefficient $c_l$ 20 N·s/m
Non-linear coefficient of spring $k_n$ 2.0e6 N·m$^{-1}$
Shock absorber nonlinear modification coefficient $c_2$ 20 N·s/m
Distance $l_1$ 1.17 m
Distance $l_2$ 1.18 m
Distance $l_3$ 1.6 m
Distance $l_4$ 1.4 m
Distance $l_5$ 1.075 m
Distance $l_6$ 1.275 m
Maximum control force $u_{\text{max}}$ 20

Figure 2 Comparison graph of displacement of car body center of mass

Figure 3 Comparison graph of displacement of center of mass of car frame

The simulation results of the displacement of the center of mass of the car body and the car frame are shown in Fig. 2 and Fig. 3. It can be seen from the comparison that the maximum and root mean square displacement of the car body center of mass when the high-speed elevator has no control effect are 0.2095mm, 0.2706mm. The maximum and root mean square displacement of the center of mass of the car frame are 0.6355mm, 0.2615mm. The maximum value and root mean square of the displacement of the car body center of mass with predictive active control are 0.1971mm and 0.0869mm, respectively, which are 5.91% and 67.9% lower than those without control. The maximum and root-mean-square displacement of the frame's center of mass are 0.6273mm and 0.0844mm, respectively, which are 1.28% and 67.7% lower than those without control. It can be seen from this that the designed predictive controller can effectively reduce the horizontal vibration of the car body and the car frame and improve the comfort during riding.
Figure 4 Comparison graph of horizontal vibration acceleration of car body

Figure 5 Comparison graph of horizontal vibration acceleration of car frame

The simulation results of the car body and car frame centroid acceleration are shown in Fig. 4 and Fig. 5. It can be seen from the comparison that the maximum value and the root mean square of the acceleration of the car body centroid when the high-speed elevator has no control effect are 0.1427 m/s² and 0.055 m/s², respectively. The maximum and root mean square acceleration of the center of mass of the car frame are 0.2978 m/s² and 0.099 m/s². The maximum value and root mean square of the acceleration of the car body mass center predicted by active control are 0.058 m/s² and 0.0147 m/s² respectively, which are 59.4% and 73.1% lower than those without control. The maximum value and the root mean square of the acceleration of the center of mass of the car frame are 0.155 m/s² and 0.028 m/s², respectively, which are 48.0% and 71.5% lower than those without control. The use of predictive control method has a good effect on suppressing the horizontal vibration acceleration of the car body and the car frame, and has certain robustness.

5. Conclusions
In order to study the horizontal vibration of high-speed elevators, considering the nonlinear characteristics of the spring rigidity and damping of the guide shoe, the energy method is used to construct a four-degree-of-freedom car system dynamics model. On this basis, a continuous predictive controller is designed and simulated using MATLAB. The simulation results show that the center of mass displacement and acceleration of the cage body and cage frame are smaller than those without control, and the horizontal vibration of the cage system is obviously inhibited, which verifies the correctness of the model and the effectiveness of the predictive controller.

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