Prompt Fission Gamma-ray Spectra and Multiplicities for Various Fissioning Systems

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Abstract

The prompt fission gamma spectra (PFGS) and multiplicities (PFGM) are investigated from a Monte Carlo simulation of the fission fragment deexcitation. The fission fragment characteristics are sampled from mass, charge, kinetic energy, spin and parity distributions from experimental data or theoretical models. Initial excitation energy is shared between the two complementary fragments using a mass dependent temperature ratio law and a level density parameter law based on Ignatyuk’s prescription. Details can be found elsewhere in the literature. The deexcitation process can be performed with different calculation schemes. The first one is based on a Weisskopf model for neutron evaporation and nuclear transition sampling (from level density and strength function models) for gamma evaporation. In this case, the competition between neutrons and gammas is taken into account by using a spin dependent excitation energy limit under which gamma emission takes place. The second one is based on an Hauser-Feshbach model for neutron/gamma evaporation based on neutron transmission coefficients (from optical model calculations) and the same model as above for gammas. The n/\gamma competition is then automatically taken into account at the very beginning of the primary fission fragments evaporation process. Fission observables, especially related to prompt fission gammas are presented and discussed for spontaneous fission (\textsuperscript{252}Cf, \textsuperscript{240}Pu), thermal fission (\textsuperscript{235}U+\textsuperscript{n}_{\text{th}}) and fast fission (\textsuperscript{238}U+\textsuperscript{n}_{1.8\text{MeV}}). Comparisons with experimental data are shown when available.

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1. Introduction

This paper aims at the Monte Carlo simulation of fission fragment deexcitation with FIFRELIN code developed at CEA-Cadarache (Litaize and Serot, 2010), focusing on prompt gamma rays (spectrum \(N_{\gamma}(E)\), average multiplicity \(\overline{M}_{\gamma}\) and total average energy \(<E_{\gamma}^{\text{tot}}\>). The competition between neutrons and gammas is taken into account by the so-called Hauser-Feshbach approach (Hauser and Feshbach, 1952). These kind of simulations have been published recently by (Becker et al., 2013) or (Regnier et al., 2013a). Here we present recent calculations, compared with experiments, related to gamma-rays for spontaneous fission (\textsuperscript{252}Cf, \textsuperscript{240}Pu), thermal fission (\textsuperscript{235}U+\textsuperscript{n}_{\text{th}}) and fast fission (\textsuperscript{238}U+\textsuperscript{n}_{1.8\text{MeV}}). The case of \textsuperscript{239}Pu+\textsuperscript{n}_{\text{th}} is reported in (Serot et al., 2013).
2. Quick review of the model

Because we are dealing with a Monte Carlo simulation of the fission fragment deexcitation, we need to sample the initial characteristics of the fission fragments (before neutron emission): mass $A$, kinetic energy $KE$, nuclear charge $Z$, spin $J$ and parity $\pi$ (Litaize and Serot, 2010). Complete experimental input data sets can be used for mass and kinetic energy distributions but models are used for other characteristics. The excitation energy sharing between the two complementary fragments is performed through a $aT^2$ relation using the Ignatyuk prescription for the level density parameter (accounting for shell corrections and paring energy) and a mass dependent temperature ratio law ($R_T(A) = T_L/T_H$) where $T_L$ stands for the temperature of the light fragments group and $T_H$ for heavy fragments group. The part of energy coming from collective excitations is subtracted from the total excitation energy before partitioning the intrinsic energy. A simple rotating liquid drop model is used in this work to estimate the rotational part of this collective component. The deexcitation can be simulated with a Weisskopf theory based model (Weisskopf, 1937) for neutrons down to a spin dependent energy limit $E_{lim}(J)$. Under this limit, a Dicebox like model (Becvar, 1998) is used for gamma emission using level density and gamma strength function models coming from RIPL-3 nuclear data parameter library (Capote and al., 2009). A second model is based on an Hauser-Feshbach formalism (Hauser and Feshbach, 1952) allowing to account for the spin dependence of the various nuclear levels involved in a whole neutron and gamma coupled cascade. Neutron transmission coefficients are calculated through Talsys1.4 code (Koning et al., 2006) using different optical model parameters and the gamma transmission coefficients are calculated as in the first scheme. This last model has been recently implemented in FIFRELIN code and preliminary results have already been published (Regnier et al., 2013a), (Regnier et al., 2013b). The major gain of this algorithm is to take into account the competition between neutron and gamma emission with a better accuracy than with a Weisskopf model. Finally, five parameters can be considered as free inside the code. Two of them are related to the mass dependent temperature ratio : $R_T^{min}$ and $R_T^{max}$. One is related to the fraction of rigid spheroid moment of inertia $k_{rig}$ involved in the calculation of the rotational energy. The last two parameters are the average spin cut-off parameters for light and heavy fragments $\bar{\sigma}_L$ and $\bar{\sigma}_H$ used in the initial fission fragment spin distribution.

These parameters are chosen to reproduce some selected total average fission observables ('target' fission observables). Most of the time, these targets are the total average prompt neutron multiplicity for light and heavy fragments $\bar{\nu}_L$ and $\bar{\nu}_H$. When a good agreement with experimental results is achieved for these total average target observables, the set of selected parameters is used to study all the other fission observables (distributions, correlations, neutron and gamma spectra, multiplicities as a function of mass, kinetic energy, charge and so on). The preliminary results for various fissioning systems are presented in the following sections.

3. Spontaneous fission of Cf252

For $^{252}$Cf(sf) we used the data related to fission fragments mass and kinetic energies from (Varapai et al., 2005) as input data. The selected target fission observables are the total average prompt neutron multiplicities for light and heavy fragments ($\bar{\nu}_L = 2.051$ and $\bar{\nu}_H = 1.698$) from (Vorobyev et al., 2004). Several set of model parameters have been found that reproduce the measured values of $\bar{\nu}_L$ and $\bar{\nu}_H$. Some of them are reported in Tab. 1 with the corresponding results related to gamma average quantities. The total average prompt gamma multiplicity recently reported in (Billnert et al., 2013) is about $\bar{M}_\gamma \approx 8.3 \pm 0.1$. This value is consistent with previous experiments except for Skarsvag (Skarsvag, 1980) and Smith et al. (Smith et al., 1956) who found a higher multiplicity. Some specific experimental factors must be taken into account before any comparison: the energy threshold and the time coincidence window. For instance, in the work of Billnert et al., this time window was about 3 ns (Oberstedt, 2013). The threshold (at least the lowest energy used to integrate the spectrum and determine the multiplicity) was set to 100 keV. The default option in the code is 1 ms without threshold. In the code, the time window corresponds to the maximum half-life of nuclear levels coming from experimental data bases. Both threshold and prompt time window have a significant impact on average fission observables such as multiplicity, energy of a quantum and total gamma energy. For instance, the multiplicity increases by 4% from 10 ns to 1 ms (because more and more nano-isomers can decay). In the contrary the energy per quantum decreases by 2% and then the total gamma energy increases by 2%. If the
energy threshold increases from 0 to 100 keV, the multiplicity decreases by 8%, the energy per quantum increases by 8% and then the total gamma energy remains roughly constant (Regnier, 2013).

The first line of results in Tab. 1 calculated with FIFRELIN-W corresponds to a simulation using the Weisskopf statistical theory for neutron emission followed by gamma emission using level density and gamma strength function models for the calculation of gamma transmission coefficients. With this kind of simulation, the neutron/gamma competition is not properly taken into account partly because the spin of the different nuclear levels is not accounted for in the neutron emission cascade. Consequently it is very difficult to achieve a good agreement with measurements for both neutron and gamma average quantities. The spin distribution of secondary fission fragments (after neutron evaporation) is generally not consistent with the level scheme used during the gamma cascade (that is completed from experimental data at low energies by several level density and strength function models). Generally speaking the gamma spectrum using a Weisskopf model for neutron emission is harder compared to a Hauser-Feshbach model (FIFRELIN-HF). This can be observed in Fig. 1. This is essentially due to the fact that we must consider lower average spins for primary fission fragment distributions (initial spin cut-off is around $6\hbar$) compared to Hauser-Feshbach model (initial spin cut-off is around $10\hbar$) in order to reproduce the prompt neutron multiplicity (see 4th and 5th parameters in Tab. 1. Consequently the Yrast line is populated at lower spins in the case of a FIFRELIN-W simulation and then the amount of quadrupole electric transitions is lower (most of these transitions arise in the lower part of the spectrum).

Finally, if we go back to the Hauser-Feshbach model (FIFRELIN-HF), using a 100 keV threshold and 10 ns prompt time window, we obtain the following average quantities: $\langle M_\gamma \rangle \approx 9.8$, $\langle \epsilon_\gamma \rangle \approx 0.8$ MeV and $\langle E_{\text{tot}} \rangle \approx 7.9$ MeV (the statistical uncertainties are lower than 0.03).

Table 1. Average quantities related to prompt fission gamma rays for spontaneous fission of $^{252}$Cf(sf)

|        | $R_{\text{min}}$ | $R_{\text{L}}$ | $k_{\text{rig}}$ | $\pi_L$ | $\pi_H$ | $\nu_L$ | $\nu_H$ | $\nu$ | $M_\gamma$ | $<\epsilon_\gamma>$ | $<E_{\text{tot}}>$ | $E_N$ | $\Delta_t$ |
|--------|-----------------|----------------|-----------------|---------|---------|--------|--------|-----|------------|-----------------|-----------------|-------|-----------|
| (Vorobyev et al., 2004) | 2.051 | 1.698 | 3.76 |
| (Smith et al., 1956) | 10.3 | 0.79 | 8.2 | 0.040 | 0.3 µs |
| (Skarsvag, 1980) | 9.76 ± 0.40 | 0.72 | 6.99 ± 0.30 | 0.114 | 12 ns |
| (Billnert et al., 2013) | 8.30 ± 0.08 | 0.80 ± 0.01 | 6.64 ± 0.08 | 0.100 | 3 ns |
| (Verbinski et al., 1973) | 7.80 ± 0.30 | 0.88 ± 0.04 | 6.84 ± 0.30 | 0.140 | 10 ns |
| (Pleasonton, 1972) | 8.32 ± 0.40 | 0.85 ± 0.06 | 7.06 ± 0.35 | 0.085 |
| (Chyzh et al., 2012) | 8.14 ± 0.40 | 0.94 ± 0.05 | 7.65 ± 0.55 | 0.150 | 10 ns |
| FIFRELIN-W | 0.4 | 1.55 | 0.33 | 5.5 | 6.0 | 2.053 | 1.698 | 3.751 | 7.68 | 1.1 | 8.43 | 0.100 | 1 ms |
| FIFRELIN-HF | 0.3 | 1.45 | 0.6 | 10. | 9.5 | 2.060 | 1.696 | 3.756 | 10.44 | 0.79 | 8.22 | 0.100 | 1 ms |
| FIFRELIN-HF | 0.3 | 1.50 | 0.85 | 9.5 | 9.0 | 2.060 | 1.700 | 3.761 | 10.04 | 0.795 | 7.98 | 0.100 | 1 ms |
| FIFRELIN-HF | 0.3 | 1.50 | 0.85 | 9.5 | 9.0 | 2.060 | 1.700 | 3.761 | 9.8 | 0.8 | 7.9 | 0.100 | 10 ns |

Fig. 1. Prompt fission gamma spectrum for spontaneous fission of Cf252. Left part: up to 7 MeV. Right part: zoom up to 1.4 MeV highlighting the reproduction of the structures in the spectrum at low energies.
4. Fast fission of $^{238}\text{U}$

For the fast fission of $^{238}\text{U}$ at $E_n = 1.8$ MeV, we used the data related to fission fragments mass and kinetic energies from (Birgersson et al., 2009) as input data. These distributions can be reconstructed directly in FIFRELIN knowing the fission mode parameters (5 per mode) provided in (Birgersson et al., 2009) (35.21% for standard I and 64.79% for standard II). The selected target fission observables are the total average prompt neutron multiplicity and the total average prompt gamma energy from various nuclear data libraries: $\overline{\nu} = 2.58$ and $<E_{\gamma}^{tot}> = 7.0 \pm 0.4$ MeV (this latest energy is not very well estimated). We compared the prompt fission gamma spectrum of our simulation with the preliminary results from (Laborie et al., 2012) in which unfolding procedure is under progress (Fig. 2). As shown in Fig. 2, an overall good agreement is achieved up to 4 MeV. If we consider a 100 keV energy-threshold and a maximum nuclear level half-life of 10 ns then the average gamma quantities that are reported in Tab. 2 are: $\overline{\nu} = 9.0$, $<\epsilon_{\gamma} > = 0.78$ MeV and $<E_{\gamma}^{tot}> = 7.04$ MeV (the statistical uncertainties are negligible).

![Fig. 2. Prompt fission gamma spectrum for fast fission of $^{238}\text{U}+n_{1.8}$ MeV. The left part is a comparison between FIRELIN-HF and measurements from Laborie et al. The calculation is performed without energy-threshold, with a maximum nuclear level half-life of 1 ms. The encapsulated picture shows the different components, in the center of mass frame, of the transitions (type and multipolarity). The right part shows the calculated spectrum on a refined energy grid in the low energy range highlighting the structures that have been observed also for other fissioning systems.](image)

| $^{238}\text{U}+n_{1.8}$ MeV | $R_T^{\text{min}}$ | $R_T^{\text{max}}$ | $k_{sig}$ | $\overline{\nu}_L$ (h) | $\overline{\nu}_H$ (h) | $\overline{\nu}$ | $<\epsilon_{\gamma}>$ (MeV) | $<E_{\gamma}^{tot}>$ (MeV) | $E_{th}$ (MeV) | $\Delta t$ (ns) |
|---------------------------|-----------------|-----------------|---------|-----------------|-----------------|---------|-----------------|-----------------|----------------|------------|
| Target observables        | 0.4             | 1.6             | 1.0     | 7.5             | 11.5            | 2.58    | 9.0             | 0.78            | 7.04           | 0.100      | 10          |
| FIRELIN-HF                |                 |                 |         |                 |                 |         |                 |                 |                |            |             |

5. Thermal fission of $^{235}\text{U}$

The experimental mass and kinetic energy distributions from (Hambsch et al., 1989) were used to calculate the thermal fission of $^{235}\text{U}$. The target fission observables are the average prompt neutron multiplicities for light fragments: $\overline{\nu}_L = 1.42$ and for heavy fragments: $\overline{\nu}_H = 1.01$ from (Nishio et al., 1998). The spectrum is compared with recent measurements (Oberstedt et al., 2013) in Fig. 3. The parameters used to reproduce with a high accuracy the average prompt neutron multiplicities are reported in Tab. 3 and compared with (Oberstedt et al., 2013), (Verbinski et al., 1973), (Pleasonton et al., 1972) and (Peelle and Maireschein, 1971). The calculation seems to underestimate the PFGS below 300 keV (several lines have not the same amplitude in the experimental spectrum compared to the calculation) and above 4 MeV. This calculated spectrum shape immediately leads to an underestimation of the average gamma multiplicity (see Tab. 3) compared with results from Oberstedt et al.
Table 3. Prompt fission gamma rays average quantities for \( ^{235}\text{U} + n\text{th} \)

| \( ^{235}\text{U} + n\text{th} \) | \( R_{\text{min}} \) | \( R_{\text{max}} \) | \( k_{\text{L}} \) | \( \bar{\nu}_L \) | \( \bar{\nu}_H \) | \( \bar{\nu} \) | \( M_\gamma \) | \( < \epsilon_\gamma > \) | \( < E_{\text{tot}} > \) | \( E_{\text{lab}} \) | \( \Delta_t \) |
|-----------------|--------------|---------|--------|--------|--------|--------|-------------|---------|-------------|---------|-------|
| (Nishio et al., 1998) (target observables) | 1.42 | 1.01 | 2.43 | 8.19 ± 0.11 | 0.85 ± 0.02 | 6.92 ± 0.09 | 0.10 | 5 ns |
| (Oberstedt et al., 2013) | | | | 6.70 ± 0.30 | 0.97 ± 0.05 | 6.51 ± 0.30 | 0.14 | 10 ns |
| (Verbinski et al., 1973) | | | | 6.51 ± 0.30 | 0.99 | 6.43 ± 0.30 | 0.09 | 5 ns |
| (Pleasonton et al., 1972) | | | | 8.1 ± 0.8 | 0.90 | 7.0 ± 0.7 | 0.03 | 70 ns |
| (Pleasonton et al., 1972) | | | | 7.45 ± 0.32 | 0.96 | 7.18 ± 0.26 | 0.14 | 70 ns |
| (Peelle and Maienschein, 1971) | | | | 8.13 ± 0.35 | 0.87 | 7.25 ± 0.26 | 0.01 | 70 ns |
| FIFRELIN-HF | 0.7 | 1.35 | 0.8 | 6.0 | 10.0 | 1.424 | 1.012 | 2.436 | 7.37 | 0.88 | 6.65 | 0.10 | 10 ns |

Fig. 3. Prompt fission gamma spectrum for thermal fission of \( ^{235}\text{U} \). Left part: whole energy range. Right part: Zoom on the low energy range.

6. Spontaneous fission of Pu240

The experimental mass and kinetic energy distributions from (Dematte et al., 1997) were used to calculate the spontaneous fission of \( ^{240}\text{Pu} \). The target fission observable was the total average prompt neutron multiplicity provided in several Exfor files: \( \bar{\nu} = 2.15 \pm 0.01 \) (EXFOR, 2013). Experimental results related to fission observables such as spectra or multiplicities are rather scarce. We have compared in Tab. 4 the average gamma quantities with the estimations from (Valentine, 2001) obtained from systematics. The calculated spectrum is shown on Fig. 4.

Table 4. Prompt fission gamma rays average quantities for \( ^{240}\text{Pu(sf)} \).

| \( ^{240}\text{Pu(sf)} \) | \( R_{\text{min}} \) | \( R_{\text{max}} \) | \( k_{\text{L}} \) | \( \bar{\nu}_L \) | \( \bar{\nu}_H \) | \( \bar{\nu} \) | \( M_\gamma \) | \( < \epsilon_\gamma > \) | \( < E_{\text{tot}} > \) | \( E_{\text{lab}} \) | \( \Delta_t \) |
|-----------------|--------------|---------|--------|--------|--------|--------|-------------|---------|-------------|---------|-------|
| (EXFOR, 2013) | 2.15 | | | | | | | | | | |
| (Valentine, 2001) | | | | | | | | | | | |
| FIFRELIN-W | 0.35 | 1.6 | 0.35 | 4.5 | 5.0 | 1.33 | 0.83 | 2.16 | 6.28 | 1.12 | 7.04 | 0.100 | 1 ms |
| FIFRELIN-HF | 0.4 | 1.35 | 0.85 | 7.0 | 7.0 | 1.23 | 0.92 | 2.15 | 7.56 | 0.89 | 6.75 | 0.100 | 1 ms |
| FIFRELIN-HF | 0.4 | 1.35 | 0.85 | 7.0 | 7.0 | 1.23 | 0.92 | 2.15 | 7.3 | 0.9 | 6.62 | 0.100 | 10 ns |

7. Conclusion

These preliminary results, especially related to prompt fission gammas, have been obtained with a Monte Carlo simulation of the fission fragment deexcitation based on statistical theory of Hauser-Feshbach. The calculated PFGS are in good agreement with experiments, reproducing the structures at low energy. The average quantities can be compared only if we know the energy threshold and the coincidence time window used in the experiments with a
Fig. 4. Prompt fission gamma spectrum for spontaneous fission of $^{240}$Pu. Left part: whole energy range. Right part: Zoom on the low energy range.

good accuracy. At the time being, it seems that we overestimate the average prompt fission gamma multiplicity by about $1/\text{fission}$ (leading to an overestimation of the total average energy dissipated by gamma-rays). This seems to be not the case for $^{235}\text{U}+\text{n}_{th}$: investigations are under progress. The initial spin distribution of the fission fragments directly impacts the low energy part of the spectrum but the parameters involved in this distribution can be adjusted to reproduce the neutron multiplicity for instance: this is another key point that will be investigated.

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